Various scalographic representation of electrocardiograms through wavelet transform with pseudo-differential operator like operators

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Abstract. This paper presents various experimental scalograms of electrocardiograms (ECG) from which the accuracy of discrimination between shockable and non-shockable arrhythmia is improved. To derive the scalograms, for the ECG signals the Gabor wavelet transform, having the various pseudo-differential operator like operators, is applied. Also, for the transformed signals, several nonlinear transforms by means of nonlinear functions are performed. These scalograms are analyzed by the normalized spectrum index (NSI) to find the statistical characteristics, and then the qualitative evaluation is performed to select the best pair of pseudo-differential operator and nonlinear function. Through the best pair selected, a good discrimination performance in the decision algorithm is guaranteed. The histogram is used in the decision stage to distinguish the shockable and non-shockable arrhythmia.

Keywords: Electrocardiograms, wavelet transform, pseudo differential operator, statistical method.

1. Introduction

Electrocardiogram (ECG) is an efficient noninvasive investigative tool which provides useful informations of the various states of heart, and these informations are used for the discrimination of diseases and treatment planning of patients [1]. In order to improve survival rate of the patients presenting heart attack symptom, it is important to develop quick and accurate discrimination procedures of typical features of the ECG signals for each of the symptoms. According to the report of the World health organization (WHO), cardiac arrhythmia is one
of the main reason of death worldwide. It is reported that the death caused by sudden cardiac arrest amounts 32% of all the deaths in the world [2]. In particular, every year more than 50,000 people die from sudden cardiac arrest in Japan [3], and 50% of the death are caused by cardiovascular disease in Europe [4].

In this circumstance, to increase the survival rate, the early access, the early cardiopulmonary resuscitation (CPR), the early defibrillation, and the early advance care are important. Especially, it is known that the early defibrillation plays the most important role to increase the survival rate [5]. The automated external defibrillator (AED) is an equipment to re-establish an effective rhythm by applying an electrical shock called defibrillation [6]. As the first-aid treatment, the ECG signal is analyzed to judge whether the defibrillation should be applied or not by the AED equipment. The important challenge of AED is to distinguish shockable and non-shockable arrhythmia in abnormal classes. Of the abnormal classes, ventricular fibrillation (VF) and ventricular tachycardia (VT) are the shockable arrhythmia which requires defibrillation to restart a heart for normal electrical function. In contrast, for the pulseless electrical activity (PEA), the defibrillation must not be applied since PEA is non-shockable arrhythmia. Precisely, if AED is applied to shock the patient with the PEA arrhythmia, then it would give harm to the patient [7]. Therefore, the discrimination of the shockable and non-shockable arrhythmia among the abnormal classes is crucially important.

Many researchers have analyzed the ECG signals through several conventional approaches of the signal analysis and tried to increase the discrimination accuracy. For e.g., Romero et al [8], analyze the ECG signals in the frequency domain and extract features for arrhythmia classification. In the frequency domain, one can find the distribution of the signal power on different frequencies but not get the time information for the corresponding frequency. Zhou et al [9], presents time domain algorithm architecture and define a classification rule for the VT and VF. In the time domain, one can see the signal amplitude over time, but can not take the informations of frequency. A number of researchers [10, 11, 12, 13, 14] analyze the ECG signals both in the time-frequency domain that is based on wavelet transform. The benefit of wavelet analysis of the ECG signal is to observe behaviors of the ECG signals in time and frequency domain simultaneously, through the scalogram, the time-frequency "spectrum".

In particular, [14], which has been announced by some of the authors of the present paper, gives a good distinction procedure between normal and abnormal rhythms, however it does not achieve an enough discrimination performance between the shockable and non-shockable arrhythmias in the class of abnormal rhythms (PEA, VF, and VT) (see Figures 3, 4 in subsection 2.3, and precisely see subsection 5.3).

In the present paper, to analyze the shockable and non-shockable ECG signals, from the previous work [14] we inherit to use the Gabor wavelet transform (see equation (4)). But here we add the new concept of the pseudo-differential operator like operators to the Gabor wavelet transform. Precisely, in section 2, by applying the different pseudo-differential operator like operators \( L \) with nonlinear transform \( H \) (see equation (5), (6)) to the Gabor wavelet transform we derived various scalograms of ECG signals. In section 3, these scalograms are analyzed by the normalized spectrum index (NSI) [14] (see equation (7), (8)) to find the statistical characteristics of the ECG signals. Then, we perform the qualitative evaluation, from which we select the best pair of pseudo-differential operator like operators
and non linear transformation. In section 4, we show that, through the best pair selected, a good discrimination performance in the decision algorithm is generated, where the histograms corresponding to the statistical features available form NSI (see Figures 15, 16) is used. Section 5, is the evaluation and validation section where we show that our proposed distinction procedure is better than the existing procedure provided by [14] (see Tables 2-6).

2. Derivation of the scalograms

In this section, we present our method by which we generate the scalograms. Here, we efficiently use the Gabor wavelet transform with pseudo differential operator like operators and non-linear transformation functions. We then select the best combination of pseudo differential operator and non-linear transformation function through the several trials (see Figures 3-12).

2.1. ECG dataset

For the present consideration, we use the ECG data given by the physio-bank database [15]. This database provides both shockable and non-shockable arrhythmia types. First, we separate original long ECG signals into pieces of five second signal segments and remove the linear trend from the segmented signals. Figures 1 and 2 show the five second different types of non-shockable and shockable signals. In Figure 1, the left is sinus rhythm (SR) and the right is pulseless electrical activity (PEA), both are known as non-shockable arrhythmia. On the other-hand, in Figure 2 the left is ventricular fibrillation (VF) and the right is ventricular tachycardia (VT) which are categorized as shockable arrhythmia.

![Figure 1: An example of non-shockable ECG (SR : Left, PEA : Right)](image1)

![Figure 2: An example of shockable ECG (VF : Left, VT : Right)](image2)
2.2. The Gabor wavelet transform with pseudo differential operator like operators

In this subsection, we briefly explain the framework of the wavelet transform with the pseudo differential operator like operators. The usual pseudo differential operators are defined in the framework of the Fourier analysis. We extend the notion of the pseudo differential operators to the wavelet analysis framework, and call them as the pseudo differential operator like operators, which are defined as follows. We are considering the real valued functions defined on $\mathbb{R} \equiv (-\infty, \infty)$, denoted by $f(t) \in \mathbb{R}$, as the observable signals. To investigate $f(t), \ t \in \mathbb{R}$, the Fourier transform of $f(t)$, denoted by $(\mathcal{F} f)(\xi)$ or $\hat{f}(\xi)$ defined below is a fundamental mathematical tool:

$$
\hat{f}(\xi) = (\mathcal{F} f)(\xi) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-it\xi} f(t) \, dt, \quad \text{for } f \in S'(\mathbb{R} \to \mathbb{R})
$$

where $i \equiv \sqrt{-1}$, and the space $S'(\mathbb{R} \to \mathbb{R})$ is the space of real Schwartz distributions. Then

$$
\mathcal{F} : S'(\mathbb{R} \to \mathbb{R}) \ni f \mapsto \hat{f} \in S'(\mathbb{R} \to \mathbb{R}),
$$

[16], and $\hat{f}(\xi)$ corresponds a decomposition of $f(t)$ in the space of the frequency. Correspondingly, let $\mathcal{F}^{-1}$ be the Fourier inverse transform such that

$$
(\mathcal{F}^{-1} g)(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{i\xi t} g(\xi) \, d\xi, \quad \text{for } g \in S'(\mathbb{R} \to \mathbb{R}).
$$

We denote $(\mathcal{F}^{-1} g)(t) = \check{g}(t)$. It then holds that

$$
(\mathcal{F}^{-1}(\mathcal{F} f))(t) = (\mathcal{F}^{-1} \hat{f})(t) = f(t), \quad \text{for } f \in S'(\mathbb{R} \to \mathbb{R}).
$$

One of an important formula in the framework of the Fourier transform is the following:

$$
(\mathcal{F} f') (\xi) = -i \xi \hat{f}(\xi), \quad \text{for } f \in S'(\mathbb{R} \to \mathbb{R}),
$$

(1)

where $f'(t) = \frac{d}{dt} f(t)$ (in the distribution sense).

Equation (1) can be generalized to the analysis of the pseudo-differential operators [17]. It is possible to consider, e.g. formally for each $\alpha \in \mathbb{R}$, the pseudo differential operator such that

$$
(- \frac{d^2}{dt^2} + 1)^\alpha f(t), \quad t \in \mathbb{R},
$$

(2)

of which Fourier transform is

$$
(\xi^2 + 1)^\alpha \hat{f}(\xi), \quad \xi \in \mathbb{R},
$$

(3)

(precisely, equation (2) is defined through (3)).

In the present paper, to investigate the ECG signals $f(t), \ t \in \mathbb{R}$, we use the Gabor wavelet transform with the modifications as follows: Let $L^2 \equiv L^2(\mathbb{R} \to \mathbb{C})$ be the space of the $\mathbb{C}$-valued, complex number valued, square integrable functions on the real line $\mathbb{R}$. For some given $\sigma > 0$ and $\omega_0 \in \mathbb{R}$, take the mother wavelet function $\psi(t)$ in $L^2$ as follows:

$$
\psi(t) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{t^2}{2\sigma^2}} e^{i\omega_0 t}, \quad t \in \mathbb{R}, \quad \text{with } i \equiv \sqrt{-1}.
$$
Then, for \( f \in L^2 \), define the Gabor wavelet transform \((Wf)(a, b)\) as follows:

\[
(Wf)(a, b) \equiv \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} f(t) \psi\left(\frac{t-b}{a}\right) dt, \quad a > 0, \quad b \in \mathbb{R},
\]

where, the variable \( \frac{1}{a} > 0 \) corresponds to the frequency of the function \( f \), and \( b \) corresponds to the time (shift). Next, we prepare two measurable functions \( L \) and \( H \) such that

\[
L : \mathbb{R}_+ \ni a \mapsto L(a) \in \mathbb{C}, \quad H : \mathbb{C} \ni y \mapsto H(y) \in \mathbb{C}.
\]

For \( f \in L^2 \), we then define our wavelet transform with pseudo differential operator like operator \( L \), and its (non-linear) transform by means of \( H \), which are \( \mathbb{C} \)-valued measurable functions with the variables \( a > 0 \) and \( b \in \mathbb{R} \), as follows:

\[
L(a) \cdot (Wf)(a, b), \quad H\left( L(a) \cdot (Wf)(a, b) \right).
\]

2.3. A suitable choice of the pair \( L(a) \) with \( H(\cdot) \) through the scalogram

We refine a conventional procedure of analysis the ECG signals by using the wavelet transform with the pseudo differential operator like operators \( L(a) \) and applying the non-linear functions \( H(\cdot) \) (see equation (5) and (6)) to the transformed signals, which are new development for the present work.

Roughly speaking, by applying the pseudo differential operator like operators to the Gabor wavelet transform defined by equation (4), similar to the case of the pseudo differential operators for the Fourier transform (see equation (2), (3)), we are able to get the information about the (fractional order of) differentiations of the input signals. A little precisely, for a given signal \( s(t) \), \( t \in \mathbb{R} \), a real valued measurable function which is bounded in any bounded region, let \( f(t) \equiv s(t) \chi_A(t) \), where \( A \) is a given bounded region and \( \chi_A(t) \) is the indicator function with \( A \), namely \( \chi_A(t) = 1 \) for \( t \in A \) and \( \chi_A(t) = 0 \) for \( t \in A^c \). Then in equation (4) for each \( a > 0 \), since the mother function \( \phi(\frac{t}{a}) \in L^p, 1 \leq p \leq \infty \), as a function with respect to \( t \in \mathbb{R} \), by Young’s inequality (cf. [18] section IX-4) the function \((Wf)(a, b)\) defined by equation (4), a convolution, we see that for each \( a > 0 \), \((Wf)(a, \cdot) \in L^r, 1 \leq r \leq \infty \), as a function with the variable \( b \in \mathbb{R} \). Hence, equation (5) and (6) are well defined for such \( f \), we then get much more fruitful information on a segment of the original signals \( f(t) \) than the one available through the simple application of the wavelet transforms.

In the sequel, we shall perform our analysis by using segmented signals \( f(t) \) given in the 5 seconds time interval, which we derived from the physio-bank database [15]. In the above indicator function \( \chi_A \), we shall set \( A = [0, 5] \) in the present paper (cf. also (7), (8)).

Moreover, by applying the non-linear functions \( H(\cdot) \) (see equation (5)) to the transformed signals, we can make bigger the part of the transformed signals which has a small energy and able to distinguish clearly the signals that have small differences, PEA and VT (see Figures. 7, 8). As a consequence, different energy over time leads to get better discrimination in the decision algorithm.

As the first step of the analysis, by setting several pairs \((L(a), H(\cdot))\), the pseudo differential operator like operator and non-linear transform function, we derive the scalograms corresponding to the ECG signals of SR, PEA, VF, and VT as follows:
i)  **(Figures 3, 4)** In these figures the scalograms with $L(a) = 1$ and $H(\cdot) = |\cdot|^2$, which is the conventional setting adopted by [14], are given. They show a good distinction between the normal (SR) and the abnormal (PEA, VF, and VT) signals. But the energies corresponding to PEA, VF, and VT are almost same over time on the scalograms.

ii)  **(Figures 5, 6)** In these figures the scalograms with $L(a) = a$ and $H(\cdot) = |\cdot|^{\frac{1}{4}}$ are given. By this setting, also the energies corresponding to PEA, VF, and VT are concentrated on the same level.

iii)  **(Figures 7, 8)** In these figures the scalograms with $L(a) = \frac{1}{a}$ and $H(\cdot) = |\cdot|^{\frac{1}{4}}$ are given. By this setting through the corresponding scalograms, PEA, VF, and VT are relatively clearly distinguished. In particular the differences of the maximum frequencies corresponding to PEA and VT is 7.2 (Hz) (randomly selected samples).

iv)  **(Figures 9, 10)** In these figures the scalograms with $L(a) = (\frac{1}{a})^2$ and $H(\cdot) = |\cdot|^{\frac{1}{4}}$ are given. By this setting, we do not have a better distinction than the one of the case iii).

v)  **(Figures 11, 12)** In these figures the scalograms with $L(a) = (\frac{1}{a})^2$ and $H(\cdot) = |\cdot|^{\frac{1}{4}}$ are given. By this setting, we see the small differences between PEA and VT.

From the above experimental results (also cf. Section 5, for the quantitative evaluations), for the subsequent considerations we henceforth adopt the pseudo differential operator like operators $L(a) = \frac{1}{a}$ with the non-linear transformation $H(\cdot) = |\cdot|^{\frac{1}{4}}$.

Figure 3: Generated scalograms by setting $L(a) = 1$ with $H(\cdot) = |\cdot|^2$ (SR : Left, PEA : Right)
Figure 4: Generated scalograms by setting $L(a) = 1$ with $H(\cdot) = |\cdot|^2$ (VF: Left, VT: Right)

Figure 5: Generated scalograms by setting $L(a) = a$ with $H(\cdot) = |\cdot|^1$ (SR: Left, PEA: Right)

Figure 6: Generated scalograms by setting $L(a) = a$ with $H(\cdot) = |\cdot|^1$ (VF: Left, VT: Right)
Figure 7: Generated scalograms by setting $L(a) = \frac{1}{\pi}$ with $H(\cdot) = |\cdot|^2$ (SR : Left, PEA: Right)

Figure 8: Generated scalograms by setting $L(a) = \frac{1}{\pi}$ with $H(\cdot) = |\cdot|^2$ (VF : Left, VT : Right)

Figure 9: Generated scalograms by setting $L(a) = (\frac{1}{\pi})^2$ with $H(\cdot) = |\cdot|^2$ (SR : Left, PEA: Right)
Figure 10: Generated scalograms by setting $L(a) = \left(\frac{1}{a}\right)^2$ with $H(\cdot) = |\cdot|^\frac{1}{2}$ (VF : Left, VT : Right)

Figure 11: Generated scalograms by setting $L(a) = \left(\frac{1}{a}\right)^\frac{1}{2}$ with $H(\cdot) = |\cdot|^\frac{1}{2}$ (SR : Left, PEA: Right)

Figure 12: Generated scalograms by setting $L(a) = \left(\frac{1}{a}\right)^\frac{1}{2}$ with $H(\cdot) = |\cdot|^\frac{1}{2}$ (VF : Left, VT : Right)
3. Analysis of scalograms

In this section, we present insights deduced from statistical features of the scalograms.

3.1. Characterization of scalograms through NSI

We performed a characteristic analysis on the scalograms through the normalized spectrum index (NSI). The NSI is a chronological representation of the movement of the "center of gravity of energies over frequencies” in a scalogram. For given \( H \) and \( L \), let \( H(L(a)(Wf)(a,b)) \) be the function defined in section 2.2. To derive the characteristics, we take \( NSI(f,L,W)(b) \) with respect to frequency. The definition of the NSI is given by (the continuous variable case)

\[
NSI(f,L,W)(b) = \frac{\int_{0}^{\infty} a \cdot H(L(a) \cdot (Wf)(a,b))da}{\int_{0}^{\infty} H(L(a) \cdot (Wf)(a,b))da}.
\]  

(7)

In the numerical treatments by means of MATLAB for the discretized wavelet transforms, we have to modify the formula (7) as follows. For simplicity, for given \( H \) and \( L \), let us denote, \( E(a,b) \equiv H(L(a)(Wf)(a,b)) \) and \( NSI(b) \equiv NSI(f,L,W)(b) \). Then, for the present discrete case the \( NSI(b) \) reads as

\[
NSI(b) \equiv \frac{\sum_{a} a \cdot E(a,b)}{\sum_{a} E(a,b)}.
\]  

(8)

Now we show the graphical representation of NSI for SR, PEA, VF, and VT signals (see Figures 13, 14). Here, the NSI obtained as a "time series” waveform from scalograms for each signal. In addition, the NSI waveform tends to change periodically and regularly for SR signal, while the changes are irregular for PEA, VF, and VT signals. As our objective, we mainly concentrate on discriminating non-shockable (PEA) versus shockable (VF and VT) arrhythmias. The maximum NSI value of the PEA signal is at the time near five second. On the other hand, the maximum NSI value appears at the time near one second for the VT signal. Inspecting the maximum over time, we get different NSI values for PEA and VT signals.

![Figure 13: NSI(b) of scalograms (SR : Left, PEA : Right)](image)
3.2. Statistical features extracted from scalogram through NSI

A list of statistical features derived from the scalograms through the NSI is given in Table 1. Here, eight features are extracted, and a matrix (8 × 4 × 1079) of scatter plots with histograms is created for each combination of variables, where 1079 samples are grouped into four classes by the grouping variable. It is not clear which features are effective for the discrimination of shockable and non-shockable arrhythmias. Therefore, we visualized the multivariate ECG classes in the different feature spaces shown in figure 15. The multi-variable plot matrix provides the graphical overview of the relations between all pairs of variables. Figure 15 shows the pairwise scatter plot in the lower and upper triangular and represent the histogram diagonally from top left to right. From the figure, we see that the histogram for mean of NSI gives separable class and the combination of the mean of NSI with all variables gives good separable class. Precisely, the combination of mean and variance of NSI gives a better separate class.

Table 1: List of features derived using $NSI(b)$

| No | Feature Name     | Symbol  |
|----|------------------|---------|
| 1  | Mean of NSI      | $\mu_{NSI}$ |
| 2  | Variance of NSI  | $\sigma^2_{NSI}$ |
| 3  | Slope of NSI     | $\lambda_{NSI}$ |
| 4  | Kurtosis of NSI  | $\kappa_{NSI}$ |
| 5  | Skewness of NSI  | $\gamma_{NSI}$ |
| 6  | Entropy-based index of NSI | $EBI_{NSI}$ |
| 7  | Power of NSI     | $P_{NSI}$ |
| 8  | Mode of NSI      | $M_{NSI}$ |
4. Discrimination by histogram

We use the histogram as a classifier of the groups to make the decision. The strategy of the histogram method in order to discriminate between the shockable and non-shockable arrhythmia is as follows.

(I) Let $K$ be the number of groups to be discriminated. Each of the groups corresponds to patients of SR, patients of VF, and so on.

(II) Suppose that we characterize the groups by using $r$ types of the features.

(III) Let \( x^{(m)} = (x^{(m)}_1, \ldots, x^{(m)}_r) \) be the data of \( m \)-th group for some \( n_m \in \mathbb{N} \), for \( m = 1, \ldots, K \).

Here the \( i \)-th data \( x^{(m)}_i \) is of the form

\[
x^{(m)}_i = (x^{(m)}_{i,1}, \ldots, x^{(m)}_{i,r}).
\]

(IV) Now, let \( x^{(m)}_{\text{max},p} \) and \( x^{(m)}_{\text{min},p} \) be maximum and minimum value in the \( p \)-th feature (cf. (II)), respectively, for \( p = 1, \ldots, r \). That is,

\[
x^{(m)}_{\text{max},p} = \max_{1 \leq i \leq n_m} \{ x^{(m)}_{i,p} \}
\]

and

\[
x^{(m)}_{\text{min},p} = \min_{1 \leq i \leq n_m} \{ x^{(m)}_{i,p} \}.
\]

(V) Then, for each of the group, we take the bin width \( d^{(m)}_p \) of the histogram,

\[
d^{(m)}_p = \frac{1}{n_m} [x^{(m)}_{\text{max},p} - x^{(m)}_{\text{min},p}], \quad m = 1, \ldots, K.
\]

Figure 15: Multi-variable plot matrix with histogram
Now, label the frequency between the intervals and compute the histogram $H_{p}^{(m)}$ for the $p$-th feature of the $m$-th group.

(VI) Finally, suppose that we are given a test data, denoted by $x \equiv (x_1, \ldots, x_p)$, and composed with the $r$ number of features. Here, which group the patient belongs to is unknown. For the given test data $x$, we determine the successive intervals (symbol) for each of the group, take the weight values (symbol) for the corresponding interval and divide the weight value by the size $n_m$ (SR, PEA, VF, and VT) of corresponding group of data, the result of which denoted by $W_{p}^{(m)}(x)$.

(VII) As the decision by means of the histogram, the test data $x$ is judged to belong to the $m_o$-th group if $W_{p}^{(m)}(x), m = 1, \ldots, K$, takes the largest value at $m = m_o$.

5. **Evaluation and discussion**

In this section, we observe the effect of the different pseudo-differential operators with non-linear functions and discuss the evaluation results of our proposed method.

5.1. **General observation**

We demonstrate an intrinsic effect of $L(a)$ with $H(\cdot)$ using qualitative evaluation before the numerical decision. We thus select the qualitatively best two-variable scatter plot with histogram for the 1079 samples from all the combinations of $L(a)$ and $H(\cdot)$, as in Figure 16. In the figure, we observe that the distribution of abnormal signals (PEA, VF, and VT) are quite different from that of normal signal (SR), where the distribution of abnormal signals is at close distances for all settings of $L(a)$ with $H(\cdot)$. Among the different setting, $L(a) = \frac{1}{a}$ with $H(\cdot) = |\cdot|^2$ shows better distribution with respect to mean of NSI and the combination of mean and variance of NSI.
Figure 16: Effect of pseudo-differential operator $L(a)$ with nonlinear function $H(\cdot)$ on the setting 16-(a) $L(a) = a$ with $H(\cdot) = |\cdot|^\frac{1}{4}$, 16-(b) $L(a) = \frac{1}{a}$ with $H(\cdot) = |\cdot|^\frac{1}{4}$, 16-(c) $L(a) = \left(\frac{1}{a}\right)^2$ with $H(\cdot) = |\cdot|^\frac{1}{4}$, 16-(d) $L(a) = \left(\frac{1}{a}\right)^\frac{1}{2}$ with $H(\cdot) = |\cdot|^\frac{1}{4}$.

5.2. Evaluation process and metrics

For the evaluation of the proposed method, we consider three metrics: sensitivity (Sen%), specificity (Sp%) and accuracy (Acc%). Also, we performed k-fold cross validation [19] for stabilizing the performance of the proposed method. We have performed 4-fold cross validation that means four times iteration totally. The discrimination results of each iteration for the 1079 samples are in Tables (2-5). We have $Z_{total} = 1079$ samples where (SR (Non-shockable) $Z_{SR}^{total} = 491$), (PEA (Non-shockable) $Z_{PEA}^{total} = 134$), (VF (Shockable) $Z_{VF}^{total} = 299$) and (VT (Shockable) $Z_{VT}^{total} = 155$). Since, we performed 4-fold cross-validation, so the total of ($Z_{total} = 1079$) samples are randomly partitioned into 4 sub-samples of equal size. A single sub-sample, denoted by $T$, is used as the validation data for testing the model, and the remaining ($Z_{total} - T$) sub-samples are used as training data. Here, the $T$ samples are also selected randomly for each type of ECG signals. The cross-validation process is repeated 4 times and each validation process is denoted by class 1 – class 4.

5.3. Quantitative evaluation

We report our experimental discrimination performance for the wavelet transform with pseudo-differential operator like operators and nonlinear transformation function. Table 2 to 5 show the results of quantitative analysis by the proposed method. As shown in the Tables,
the proposed method presents good discrimination results. We see that the ratio of the successful discrimination for normal signal (SR) versus abnormal signals (PEA, VF and VT) is 100% for class 1 to 4.

How about the discrimination between shockable and non-shockable arrhythmia?. For VF versus PEA and VT versus PEA, the proposed method gives good discrimination performance for class 1 to class 4. Our proposed method is able to distinguish between VT and PEA with 87.84% average accuracy, 88.72% sensitivity, and 86.96% specificity. Similarly, for VF versus PEA, we have 95.32% average accuracy, 95.09% sensitivity, and 95.81% specificity.

### Table 2: Discrimination performance of the proposed method for class 1

| Group                              | Sen(%) | Sp(%) | Acc(%) |
|------------------------------------|--------|-------|--------|
| Normal (SR) vs Abnormal (PEA, VF and VT) | 100.00 | 100.00 | 100.00 |
| VF vs PEA                          | 96.55  | 96.43 | 96.51  |
| VT vs PEA                          | 88.95  | 81.82 | 85.90  |

### Table 3: Discrimination performance of the proposed method for class 2

| Group                              | Sen(%) | Sp(%) | Acc(%) |
|------------------------------------|--------|-------|--------|
| Normal (SR) vs Abnormal (PEA, VF and VT) | 100.00 | 100.00 | 100.00 |
| VF vs PEA                          | 95.71  | 96.67 | 96.00  |
| VT vs PEA                          | 87.50  | 90.62 | 89.06  |

### Table 4: Discrimination performance of the proposed method for class 3

| Group                              | Sen(%) | Sp(%) | Acc(%) |
|------------------------------------|--------|-------|--------|
| Normal (SR) vs Abnormal (PEA, VF and VT) | 100.00 | 100.00 | 100.00 |
| VF vs PEA                          | 95.00  | 96.00 | 95.29  |
| VT vs PEA                          | 87.50  | 88.89 | 88.24  |

### Table 5: Discrimination performance of the proposed method for class 4

| Group                              | Sen(%) | Sp(%) | Acc(%) |
|------------------------------------|--------|-------|--------|
| Normal (SR) vs Abnormal (PEA, VF and VT) | 100.00 | 100.00 | 100.00 |
| VF vs PEA                          | 93.10  | 94.12 | 93.48  |
| VT vs PEA                          | 90.91  | 86.49 | 88.14  |

**Comparison with conventional approach.** A goal of this experiment is to check the effectiveness of our proposed method (pseudo-differential operator like operator and nonlinear function) and to compare the existing results for shockable and non-shockable arrhythmia discrimination. Table 6 shows that the present method keeps the better performance of the discrimination than the one conducted by Okai et al [14].

We have to remark that [14] was carried out not only by using the scalogram corresponding to Figures 3 and 4, which gives the informations of the frequencies of ECG signals \( f \), but also some of the statistical features available from \( f \) directly. Moreover, in [14] the method of the Mahalanobis distance was used as the classifier of the groups, for which here we use the histogram method (see section 4).
From the table, we see that the discrimination performance is 100% for normal vs abnormal signals both by the methods of [14] and the present. On the other hand, the accuracy of shockable (VF and VT) versus non-shockable (PEA) arrhythmia for the existing method is 84.86% while present proposed method increases the accuracy to 91.58%, with 6.72% gain. As a consequence, the proposed method achieves the better result with respect to average sensitivity, specificity and accuracy.

| Method       | Group                                      | Sen(%) | Sp(%) | Acc(%) |
|--------------|-------------------------------------------|--------|-------|--------|
| Proposed     | Normal (SR) vs Abnormal (PEA, VF and VT)   | 100.00 | 100.00| 100.00 |
|              | Shockable (VF, VT) vs non-shockable (PEA)  | 91.90  | 91.38 | 91.58  |
| Okai et al. [14] | Normal (SR) vs Abnormal (PEA, VF and VT)   | 100.00 | 100.00| 100.00 |
|              | Shockable (VF, VT) vs non-shockable (PEA)  | 84.87  | 84.80 | 84.86  |

6. Conclusion

The major challenge for AED is to extract accurate information for the application of reliable shock therapy. Here, we accurately extract numeric information from abnormal ECG signals through our proposed method that helps in decision algorithm to get a better discrimination. From the scalographic representation and numerical experiments it is shown that the performance of the proposed method is better than the one of the existing methods.

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