Condensation Energy of a Spacetime Condensate

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(Dated: 24 November 2010)

Starting from an analogy between the Planck-Einstein scale and the dual length scales in Ginzburg-Landau theory of superconductivity, and assuming that space-time is a condensate of neutral fermionic particles with Planck mass, we derive the baryonic mass of the universe. In that theoretical framework baryonic matter appears to be associated with the condensation energy gained by spacetime in the transition from its normal (symmetric) to its (less symmetric) superconducting-like phase. It is shown however that the critical transition temperature cannot be the Planck temperature. Thus leaving open the enigma of the microscopic description of spacetime at quantum level.

INTRODUCTION — The theory of superconductivity proposed by Ginzburg and Landau in 1950 is based on Landau’s theory of second-order phase transition [1]. It was introduced as a phenomenological theory, but later Gor’kov showed that it can be derived from the full microscopic Bardeen Cooper and Schrieffer (BCS) theory in a suitable limit. Since its introduction the Ginzburg Landau theory was successfully used to describe superfluidity in Fermi systems (³He) and other physical systems where quantum phenomena are closely linked to phase transition thermodynamics.

Spacetime undergoes several phase transition during its cosmological evolution [2]. Our assumption is that at least one of them is a superconducting-like phase transition driven by processes of spontaneous symmetry breaking. This type of phase transition, allowed for any fermionic system exhibiting any type of weak attractive interaction, would leave spacetime in a condensed state with an energy gap and other critical parameters like temperature marking the transition from the normal to the superconducting state. Assuming also that gravitation is the underlying bounding interaction of the spacetime condensate, then further critical relevant parameters should be the condensate density where higher density means higher critical temperature and the gravitomagnetic field plays the role of the magnetic field in superconductivity. To remain in the condensed state the difference of free energy between the condensed and the normal spacetime state should always remain negative and could be expressed in terms of a thermodynamic critical gravitomagnetic field.

Having a spacetime condensate created in the process of a phase transition it should be possible to apply the standard Ginzburg Landau theory to spacetime. This means that two characteristic lengths scales should exist for spacetime, which would be the analog of the London penetration depth, and of the coherence length. It is natural to assign the coherence length to the Planck length and the penetration length to the London length characterizing the size of the universe. This allows also to associate a realistic spacetime energy density with the thermodynamic energy density of the condensate, i.e. as the geometric mean of two extreme energy densities, which we call the Planck-Einstein scale. Doing this we calculate from this theory, that the currently observed baryonic matter content of the universe, corresponds to the spacetime condensation energy gained in the transition from its normal to its new, less symmetric state.

CONDENSATION ENERGY IN SUPERCONDUCTORS The great advantage of the Ginzburg-Landau theory is its ability to solve many difficult problems in superconductivity, without any reference to the underlying microscopic BCS theory. In that sense Ginzburg Landau theory is more general and could be used to describe other solid state systems with spontaneously broken symmetry of ground state. It could explain basic superconducting properties of exotic superconductors, such as the high Tc cuprates, even though the original BCS theory does not seem to explain these systems.

We can analyze the phase diagram of superconductors in exactly the same manner as one would consider the well known thermodynamics of a liquid-gas phase transition problem such as given by the van der Waals equations of state. However, for the superconductor instead of the pair of thermodynamic variables P, V (pressure and volume) we have the magnetization $\vec{M}$ as the relevant thermodynamic parameter.

The Gibbs free energy $G(T,H)$ is generally the most convenient quantity to work with since the temperature $T$ and the external magnetic field $H$ are the variables which are most naturally controlled experimentally. Furthermore from $G(T,H)$ one can also reconstruct the Helmholtz free energy $F(T,M)$, $F = G + \mu_0 V \vec{H} \cdot \vec{M}$, with $\mu_0$ being the magnetic permeability, and volume $V$ the superconductor’s volume. The difference in free energies of superconducting and normal states at zero magnetic induction, $\vec{M} = -\vec{H}$, is

$$\Delta F_c = F_c(T,0) - F_n(T,0) = -\mu_0 V \frac{H^2}{2}$$ (1)

The quantity $\mu_0 H_c^2/2$ is the condensation energy density. It is a measure of the gain in free energy per unit volume in the superconducting state compared with the normal state at the same temperature $T$ below the critical transition temperature $T_c$. The field $H_c^2$ is so-called thermodynamic critical field and can be expressed as function
of the coherence (healing) length $\xi$, and of the London penetration depth $\lambda$.

$$H_c = \frac{\Phi_0}{2\pi \mu_0 \sqrt{2} \xi \lambda}$$  \hspace{1cm} (2)

where $\Phi_0 = \hbar/2e$ is the magnetic flux quantum ($h$ is the Planck constant, and $e$ is the electron’s electric charge).

Although the thermodynamic critical field isn’t experimentally observable it indicates how stable the condensed state is; and it is also related with the observable critical fields, $H_1$ and $H_2$, in Type-II superconductors, being their geometrical mean.

**PLANCK-EINSTEIN SCALE** — The composition of the universe is largely unknown to modern physics. Nucleosynthesis gives a rather good estimate for the relative composition in baryonic matter. The relative energy density of baryonic matter, $\Omega_B$ with respect to the critical density,

$$\rho_c = 3H_0^2c^2/8\pi G \simeq 8.52 \times 10^{-10} \text{ [J/m}^3]$$  \hspace{1cm} (3)

where $H_0 \simeq 2.3 \times 10^{-18}$ [s$^{-1}$] is the Hubble constant, $G$ is universal gravitational constant, and $c$ is the speed of light in vacuum; reveals that baryonic matter only accounts for 4% of the total universe mass:

$$\Omega_B = \frac{\rho_B}{\rho_c} \sim 0.04$$  \hspace{1cm} (4)

Where $\rho_B$ is the density of observed baryonic matter in the universe. The other 96% of the universe mass is clearly unknown and is allocated to the so called dark matter and dark energy. Their respective relative densities with respect to the critical energy density are respectively:

$$\Omega_{DM} = \frac{\rho_{DM}}{\rho_c} \sim 0.23$$  \hspace{1cm} (5)

and

$$\Omega_{DE} = \frac{\rho_{DE}}{\rho_c} \sim 0.73$$  \hspace{1cm} (6)

We are living in a universe that exhibits accelerating expansion in (approximately) four space-time dimensions, with a de Sitter spacetime metric with cosmological constant $\Lambda = 1.29 \times 10^{-52}$[m$^{-2}$] \cite{3,8}. A small cosmological constant is equivalent to a small vacuum energy density, which could account properly for the dark energy density with equation of state $w = \rho/p = -1$ (showing a repulsive interaction) given by

$$\rho_{vac\Lambda} = \frac{c^4 \Lambda}{8\pi G} = 6.21 \times 10^{-10} \text{[J/m}^3].$$  \hspace{1cm} (7)

The fundamental scale, which could naturally host dark energy is the *Planck-Einstein scale*.

The Planck-Einstein scale corresponds to the geometric mean value between the Planck scale, $l_P = (hG/c^3)^{1/2}$, which determines the highest possible energy density in the universe, and the cosmological length scale, or Einstein scale, $l_E = \Lambda^{-1/2}$, which determines the lowest possible energy in the universe. The Planck scale is constructed out of the 4 fundamental constants, $c, h, G, k$, where $h = h/2\pi$ and $k$ is Boltzmann constant. The Einstein scale is built from the constants $c, h, \Lambda, k$. Thus the Planck-Einstein length $l_{PE} = \sqrt{l_PL_E}$ is the geometric mean of the two length scales in the universe, and involves the five fundamental constants $c, h, G, \Lambda, k$. All physical Planck-Einstein scales for energy, mass, time, and density, are calculated in a similar manner, Explicitly one has the following formulas \cite{9}:

$$E_{PE} = kT_{PE} = \left(\frac{c^7 h^3 \Lambda}{G}\right)^{1/4}$$  \hspace{1cm} (8)

$$m_{PE} = \frac{E_{PE}}{c^2} = \left(\frac{h^3 \Lambda}{cG}\right)^{1/4}$$  \hspace{1cm} (9)

$$l_{PE} = \frac{h}{M_{PE}c} = \left(\frac{hG}{c^3 \Lambda}\right)^{1/4}$$  \hspace{1cm} (10)

$$t_{PE} = \frac{l_{PE}}{c} = \left(\frac{hG}{c^3 \Lambda}\right)^{1/4}$$  \hspace{1cm} (11)

$$\rho_{PE} = \frac{E_{PE}}{l_{PE}^2} = \frac{c^4 \Lambda}{G}$$  \hspace{1cm} (12)

One readily notices that the numerical values of Planck-Einstein quantities correspond to typical time, length or energy scales in superconductor physics, as well as to typical energy scales for dark energy. In previous papers it has been pointed out \cite{10,12} that there could be a deeper reason for this coincidence: It is possible to construct theories of dark energy that bear striking similarities with the physics of superconductors. In these theories the Planck-Einstein scale replaces the Planck scale as a suitable cutoff for vacuum fluctuations.

**SPACETIME CONDENSATE AND BARYONIC MATTER**—

We will now draw an analogy between the Ginzburg-Landau theory of superconductivity, and the fundamental physical nature of space-time at the Planck scale. Let us consider that space-time at present time is an electrically neutral condensate created in a phase transition coupled with spontaneous symmetry break-down (possibly chiral symmetry breaking) in early period of universe evolution. The two basic length scales of Ginzburg-Landau theory and space-time condensate should be associated as following: the London penetration depth scale is equal to the Universe radius, and the coherence length corresponds to the Planck length.

$$\lambda \equiv 1/\sqrt{\Lambda}$$  \hspace{1cm} (13)
\[ \xi \equiv l_P \]  

Let us consider in addition that at the Planck scale, spacetime is a condensate formed by pairs of neutral fermionic particles with Planck mass \( m_P = (\hbar c/\Lambda)^{1/2} \), which we call \textit{Planck's condensate}. In that context the analog of the magnetic flux quantum \( \Phi_0 \) is the gravitomagnetic flux quantum for Planck mass \( \Phi_{0g} \).

\[ \Phi_{0g} = \frac{\hbar}{2m_P} \]  

The Planck condensate forms as soon as the universe cools down below a certain transition temperature \( \tau_c \), which we leave undefined. In other words space-time becomes a condensate for temperature \( T < \tau_c \). For temperatures higher than \( \tau_c \) physical spacetime acquires full symmetry and ceases to be a Planck’s condensate.

The analog of the magnetic variable corresponding to the magnetic field \( \vec{H} \) for superconductors becomes the gravitomagnetic field \( \vec{H}_g \) in the case of a spacetime condensate (and when gravitational field strength allows weak-field approximation). The magnetization \( \vec{M} \) is substituted by the gravitomagnetization \( \vec{M}_g \). The temperature \( T \) remains thermodynamic variable in both cases. The magnetic permeability \( \mu_0 \), must be accordingly substituted by the gravitomagnetic permeability \( \mu_{0g} = 4\pi G/c^2 \). \( V \) becomes the entire volume of the observable universe \( V_U \), which we assume to be spherical with radius equal to the Einstein length \( l_E = \Lambda^{-1/2} \).

The difference of free energy, \( \Delta F_{gc} \), between superconducting-like and normal states, at zero gravitomagnetic field and zero gravitomagnetization, \( \vec{M}_g = -\vec{H}_g \), is:

\[ \Delta F_{gc} = F_{gs}(T,0) - F_{gn}(T,0) = -\mu_{0g}V_U \frac{H_{gc}^2}{4} \]  

This means that making the universe less symmetric (spacetime condensate), will decrease its overall energy.

The quantity \( \rho^* \)

\[ \rho^* = \mu_{0g}H_{gc}^2/4 \]  

is the condensation energy density. It is a measure of the gain in free energy per unit volume in the superconducting-like state compared with the normal state at the same temperature \( T \) below the critical transition temperature \( \tau_c \).

Substituting eq. (13), eq. (14), eq. (15), and the gravitomagnetic permeability \( \mu_{0g} \) in eq. (2), we obtain the analog equation for the critical gravitomagnetic field \( H_{gc} \).

\[ H_{gc} = \frac{1}{8\pi\sqrt{2}} \left( \frac{c^2\Lambda}{G^3} \right)^{1/2} \frac{1}{m_P} \]  

Substituting the explicit expression of the Planck mass, in the critical gravitomagnetic field, eq. (18), we obtain the constant cosmological critical gravitomagnetic field expressed in function of the universal fundamental constants, \( G, \hbar, c, \Lambda \).

\[ H_{gc} = \frac{1}{8\pi\sqrt{2}} \left( \frac{c^6\Lambda}{G^2} \right)^{1/2} \]  

Substituting eq. (19) into eq. (17) we obtain the constant cosmological density of condensation energy associated with the emergence of the superconducting-like phase of spacetime:

\[ \rho^* = \frac{1}{128\pi} \frac{c^4\Lambda}{G} = 3.88 \times 10^{-11} \ [J/m^3] \]  

One sees that \( \rho^* \) is proportional to the Planck-Einstein density of energy Eq. (12). Comparing the density of condensation energy, eq. (20), with the critical density, eq. (3), we find it to be in nearly exact coincidence with the cosmological relative density of Baryonic matter, eq. (4).

\[ \Omega^* = \Omega_B = \frac{\rho^*}{\rho_c} = \frac{1}{48} \frac{c^2\Lambda}{P^2} = 0.0456 \]  

Therefore if the condensation energy of spacetime is the Baryonic matter content of our universe then the total excess of free energy produced during the superconducting condensation of space-time is the total mass of the Baryonic matter, \( M_{BU} \), present in our cosmos. Substituting eq. (19) in eq. (18) and dividing by \( c^2 \) we obtain:

\[ M_{BU} = \frac{\Delta F_{gc}}{c^2} = \frac{1}{96} \frac{c^2}{G} \sim 1.3 \times 10^{51} \ [Kg] \]  

**Discussion and Conclusions** — If we attempt to understand the space-time condensation energy from the microscopic composition of the Planck condensate we are directly lead into the well known cosmological constant problem. This can be easily demonstrated through an analogy with BCS theory: BCS theory shows that the condensation energy density \( \rho_F \) per atom in the superconductor is of order:

\[ \rho_F = kT_c g(\epsilon_F). \]  

Where \( k \) is Boltzmann constant, \( T_c \) is the critical temperature and \( g \) is the density of states at the Fermi level.

Since we considered in the previous section that spacetime condensates when the universe cools down below the critical temperature \( \tau_c \), if we take \( \tau_c \) equal to the Planck temperature \( T_P = k^{-1}(\hbar c^3/\Lambda)^{1/2} \), in eq. (23), the analog of the condensation energy per atom, \( \rho_F \) in eq. (24), will be played by the gained density of condensation energy, i.e., the baryonic density of the universe \( \rho^* \), eq. (20). Carrying out this analogy eq. (20) becomes:

\[ \rho^* = g E_P. \]  

Multiplying both sides of eq. (20) by the Planck volume we deduce that the baryonic energy density content of
Planck cells is 125 orders of magnitude smaller than the total Planck energy density:

\[
\frac{\rho^* l_P^3}{E_P} = 8.37 \times 10^{-125}
\]  

(25)

Since \(\rho_{DE} \sim 18 \rho^*\) it is easy to see that eq. (25) is a possible statement of the so called cosmological constant problem [14]. This might also indicate that the critical transition temperature \(\tau_c\) of the superconducting like spacetime seems not to be the Planck temperature.

Presently we do not have yet a complete theory of quantum gravity from which we could deduce the analog of density of states \(g\) for spacetime. Before BCS theory was formulated similar problems where raised in the context of the Ginzburg Landau theory, to explain the mysterious tiny condensation energy per atoms in superconductors. Only BCS theory could explain that this energy is so small because \(kT_c\) is three orders of magnitude smaller than the fermi energy, \(\epsilon_F\). Will the theory of quantum gravity be the equivalent of the BCS theory for superconductivity? Although the use of Ginzburg-Landau concepts in the framework of cosmology seems promising, this question is still open [15][16].

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