Cellular Approach to Long-Range $p_t$ and Multiplicity Correlations in the String Fusion Model

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Abstract

The long-range $p_t$ and multiplicity($n$) correlations in high-energy nuclear collisions are studied in the framework of a simple cellular analog of the string fusion model.

Two cases with local and global string fusion is considered. The $p_t-n$ and $n-n$ correlation functions and correlation coefficients are calculated analytically in some asymptotic cases using suggested Gauss approximation.

It's shown that at large string density the $p_t-n$ and $n-n$ correlation coefficients are connected and the scaling takes place. The behavior of the correlations at small string density is also studied.

The asymptotic results are compared with results of the numerical calculations in the framework of proposed cellular approach.
1 Introduction.

The colour strings approach [1, 2] is widely applied for the description of the soft part of the hadronic and nuclear interactions at high energies.

In the framework of this approach the string fusion model was suggested in papers [3]. Later it was developed [4]-[6] and applied for the description of the long-range multiplicity and \( p_t \) correlations in relativistic nuclear collisions [7]-[9].

In the paper [11] we have formulated some simple cellular analog of the string fusion model, which enables explicit analytical calculations of the correlation functions in some asymptotic cases and can simplify calculations in the case of real nuclear collisions.

In that paper we have checked up the assumptions of the cellular approach and the validity of a suggested Gauss approximation in the simplest (no fusion) case when the explicit solution of the model can be found.

In the present paper in the framework of proposed cellular approach we calculate \( p_t-n \) and \( n-n \) correlation functions and correlation coefficients in two cases: with local and with global string fusion.

The calculations are done both numerically and in some asymptotic cases analytically using the Gauss approximation. The results of both calculations are in a good agreement, which proofs the validity of proposed Gauss approximation.

At large string density \( \eta \gg 1 \) the connection between the \( p_t-n \) and \( n-n \) correlation coefficients are found in both cases: with local or global string fusion. At that for the correlation coefficient the scaling takes place. It depends only on one combination \( \mu_0/\sqrt{\eta} \) of the variables \( \eta \) and \( \mu_0 \) (\( \mu_0 \) is the mean multiplicity emitting by a single string).

The paper organized as follows. In the next section we recall the formulation of the cellular approach in the case with a local string fusion.

In the section 3 we develop Gauss approximation at large string density \( \eta \) in the local fusion case.

In the section 4 the \( p_t-n \) and \( n-n \) correlation coefficients are calculated at large \( \eta \) in the case with local string fusion and connection between these two coefficients is found.

In the section 5 the \( p_t-n \) and \( n-n \) correlation functions and correlation coefficients are calculated at large \( \eta \) for "homogeneous" situation (a constant mean density of strings) in the case with local fusion. It's shown that in this situation the \( p_t-n \) and \( n-n \) correlation coefficients become equal and the \( \mu_0/\sqrt{\eta} \)-scaling takes place. The obtained results are compared with the results of numerical calculations using formulas of section 2.

In the section 6 we recall the formulation of the cellular approach in the case with a global string fusion. The exact closed formulas for the \( p_t-n \) and \( n-n \) correlation functions in this case are obtained.

In the section 7 we develop Gauss approximation at large string density \( \eta \) for the global fusion case. The \( p_t-n \) and \( n-n \) correlation functions and coefficients for this case are calculated and compared with the results of numerical calculations using formulas of section 6.

The behavior of the correlations at small string density is studied in Appendix A.
2 Cellular approach to the local string fusion.

Let us recall the formulas obtained for this case in [11]. We consider the collision of nuclei in two stage scenario when at first stage the colour strings are formed, and at the second stage these strings, or some other (higher colour) strings formed due to fusion of primary strings, are decaying, emitting observed particles.

In principle, one can consider two types of fusion. The case with a local fusion corresponds to the model, where colour fields are summing up only locally and the global fusion case corresponds to the model, where colour fields are summing up globally - all over the cluster area - into one average colour field, the last case corresponds to the summing of the sources colour charges. (In section 5 of [11] we have referred to these cases as A) and B) correspondingly.)

In the transverse plane depending on the impact parameter $b$ we have some interaction area $S(b)$. Let us split this area on the cells of order of the transverse string size. Then we’ll have $M = S(b)/\sigma_0$ cells, where $\sigma_0 = \pi r_0^2$ is the transverse area of the string and $r_0 \approx 0.2\, fm$ is the string radius.

In the case with a local fusion the assumption of the model is that if the number of strings belonging to the $i$-th cell is $\eta_i$, then they form higher colour string, which emits in average $\mu_0 \sqrt{\eta_i}$ particles with mean $p^2$ equal to $p^2 \sqrt{\eta_i}$, compared with $\mu_0$ particles with $\langle p^2 \rangle = p^2$ emitting by a single string.

If we denote by $n_i$ and $\bar{n}_i$ - the number and the average number of particles emitted by the higher string from $i$-th cell in a given rapidity interval, then

$$\bar{n}_i = \mu_0 \sqrt{\eta_i}$$

From event to event the number of strings $\eta_i$ in $i$-th cell will fluctuate around some average value $- \bar{n}_i$. Clear that in the case of real nuclear collisions these average values $\bar{n}_i$ will be different for different cells. They will depend on the position ($s$) of the $i$-th cell in the interaction area ($s$ is two dimensional vector in transverse plane). To get the physical answer we have to sum the contributions from different cells, which corresponds to integration over $s$ in transverse plane.

The average local density of primary strings $\bar{\eta}_i$ in the point $s$ of transverse plane is uniquely determined by the distributions of nuclear densities and the value of the impact parameter $- b$. They can be calculated, for example, in Glauber approximation. We’ll do this later in a separate paper. In present paper we consider that all $\bar{\eta}_i$ are already fixed from these considerations at given value of the impact parameter $- b$.

Let us introduce some quantities, which will play important role in our consideration:

$$N = \sum_{i=1}^{M} \eta_i, \quad \bar{N} = \sum_{i=1}^{M} \bar{\eta}_i$$

$$r = \sum_{i=1}^{M} \sqrt{\eta_i}, \quad \bar{r} = \sum_{i=1}^{M} \sqrt{\bar{\eta}_i}$$

then clear that $N$ is the number of strings in the given event and $\bar{N}$ is the mean number of strings for this type of events (at the fixed impact parameter $b$).

To go to long-range rapidity correlations we have to consider two rapidity windows $F$ (forward) and $B$ (backward). Each event corresponds to a certain configuration
of strings and certain numbers of charged particles \( \{n_1, ..., n_M\} \) emitted by these strings in the forward rapidity window. Then the total number of particles produced in the forward rapidity window will be equal to \( n_F \):

\[
n_F = \sum_{i=1}^{M} n_i
\]

(3)

The probability to detect \( n_F \) particles in the forward rapidity window for a given configuration \( \{\eta_1, ..., \eta_M\} \) of strings is equal to

\[
P_{\{\eta_1, ..., \eta_M\}}(n_F) = \sum_{\{n_1, ..., n_M\}} \delta_{n_F, \sum_i n_i} \prod_{i=1}^{M} p_{\eta_i}(n_i)
\]

(4)

where \( p_{\eta_i}(n_i) \) is the probability of the emission of \( n_i \) particles by the string \( \eta_i \) in the forward rapidity window. By our assumption (1)

\[
\eta_i \equiv \sum_{n_i = 0}^{\infty} n_i p_{\eta_i}(n_i) = \mu_0 \sqrt{\eta_i}
\]

(5)

If we denote else by \( W(\eta_1, ..., \eta_M) \) the probability of realization of the string configuration \( \{\eta_1, ..., \eta_M\} \) in the given event, then the average value of some quantity \( O \) under condition of the production of \( n_F \) particles in the forward window will be equal to

\[
\langle O \rangle_{n_F} = \sum_{\{\eta_1, ..., \eta_M\}} \langle O \rangle_{\{\eta_1, ..., \eta_M\}, n_F} W(\eta_1, ..., \eta_M) P_{\{\eta_1, ..., \eta_M\}}(n_F)
\]

(6)

One has to omit in this \( M \)-fold sums one term, when all \( \eta_i = 0 \), which corresponds to the absence of inelastic interaction between the nucleons of the colliding nuclei (see details in Appendix A).

If the \( O \) in the number of particles produced in the backward rapidity window \( n_B \) in the given event, then we have to use for \( \langle n_B \rangle_{n_F} \) correlations:

\[
\langle n_B \rangle_{\{\eta_1, ..., \eta_M\}, n_F} = \mu_0 \sum_{i=1}^{M} \sqrt{\eta_i} = \mu_0 r
\]

(7)

If the \( O \) in the mean squared transverse momentum of particles produced in the backward rapidity window \( p_{t_B}^2 \) in the given event, then we have to use for \( \langle p_{t_B}^2 \rangle_{n_F} \) correlations:

\[
\langle p_{t_B}^2 \rangle_{\{\eta_1, ..., \eta_M\}, n_F} = \sum_{i=1}^{M} \frac{\sqrt{\eta_i}}{\sum_{i=1}^{M} \sqrt{\eta_i}} p_{t_B}^2 \sqrt{\eta_i} = p_{t_B}^2 \frac{\sum_{i=1}^{M} \eta_i}{\sum_{i=1}^{M} \sqrt{\eta_i}} = p_{t_B}^2 N
\]

(8)

Later we’ll assume that numbers of primary strings in each cell \( \eta_i \) fluctuate independently around some average quantities \( \bar{\eta}_i \) uniquely determined by the distributions of nuclear densities and the value of the impact parameter - \( b \) (see above), then

\[
W(\eta_1, ..., \eta_M) = \prod_{i=1}^{M} w(\eta_i), \quad \sum_{i=1}^{M} \eta_i w(\eta_i) = \bar{\eta}_i
\]

(9)

For clearness we’ll sometimes address to a simple ”homogeneous” case, when all \( \bar{\eta}_i \) (but not the \( \eta_i \), which fluctuate from event to event!) is equal each other in the

3
interaction area $\pi_i = \eta$ (a constant mean density of strings). The parameter $\eta$ coincides in this case with the parameter $\eta$ used in the papers [3, 4, 5] and has the meaning of the mean number of strings per area of one string ($\eta = (\text{mean string density}) \times \sigma_0$). In general case the parameters $\pi_i$ have the same meaning, but with mean string density depending on the point $s$ in the transverse interaction plane ($\pi_i = (\text{mean string density in the point } s) \times \sigma_0$).

As it was shown in [11] if we assume else the Poissonian form of $p_{\eta_i}(n_i)$ ($\rho(x)$ is the Poisson distribution with $x = a$):

$$p_{\eta_i}(n_i) = \rho_{\mu_0 \sqrt{\eta_i}}(n_i) = e^{-\mu_0 \sqrt{\eta_i}} \left(\frac{\mu_0 \sqrt{\eta_i}}{n_i!}\right)$$

then we find the Poissonian distribution for

$$P_{\{\eta_1, ..., \eta_M\}}(n_F) = \rho_{\mu_0 \sum_i \sqrt{\eta_i}}(n_F)$$

with $\langle n_F \rangle_{\{\eta_1, ..., \eta_M\}} = \mu_0 \sum_i \sqrt{\eta_i} = \mu_0 r = \langle n_F \rangle_r$ and $\sigma_{\eta F}^2 = \langle n_F \rangle_r = \mu_0 r$, which we can replace at large $\mu_0 \sum_i \sqrt{\eta_i}$ by Gauss distribution:

$$P_{\{\eta_1, ..., \eta_M\}}(n_F) = \frac{1}{\sqrt{2\pi \sigma_{\eta F}^2}} e^{-\frac{(n_F - \langle n_F \rangle_r)^2}{2\sigma_{\eta F}^2}}$$

It was also shown in the section 6 of [11] that if we assume the Binomial form of $p_{\eta_i}(n_i)$ then we find the Binomial distribution for

$$P_{\{\eta_1, ..., \eta_M\}}(n_F)$$

with $\langle n_F \rangle_{\{\eta_1, ..., \eta_M\}} = \mu_0 \sum_i \sqrt{\eta_i} = \mu_0 r = \langle n_F \rangle_r$ and $\sigma_{\eta F}^2 = \langle n_F \rangle_r (1 - \lambda) = \mu_0 r (1 - \lambda)$, where $\lambda \to 0$ corresponds to the Poisson limit and $\lambda \to 1$ corresponds to the case when each higher string emits fixed number of particles - $n_i = \pi_i = \mu_0 \sqrt{\eta_i}$ in each event.

Moreover appealing to the central limit theorem of the probability theory one can state that at large $M$ we’ll have formula (12) for any type of $p_{\eta_i}(n_i)$.

### 3 Gauss approximation at large string density for the local fusion.

Now we’ll evaluate $\sum_{\{\eta_1, ..., \eta_M\}}$ at the condition that all $\pi_i \gg 1$. At this condition we can use Gauss approximation for each $w(\eta_i)$

$$w(\eta_i) = \frac{1}{\sqrt{2\pi \sigma_{\eta_i}^2}} e^{-\frac{(\eta_i - \pi_i)^2}{2\sigma_{\eta_i}^2}}$$

with $\sigma_{\eta_i}^2 = \pi_i (1 - \lambda_\eta)$, where again $\lambda_\eta \to 0$ corresponds to the Poisson limit and $\lambda_\eta \to 1$ corresponds to the case with a fixed number of strings $N = N = \sum_{i=1}^M \pi_i$ (see the section 6 of [11]).

As in [11] we have then

$$\langle n_{B} \rangle_{n_F} = \mu_0 \frac{\int d\eta_1...d\eta_M \frac{1}{\sqrt{\pi}} e^{-\varphi(\eta_i, n_F)}}{\int d\eta_1...d\eta_M \frac{1}{\sqrt{\pi}} e^{-\varphi(\eta_i, n_F)}} (14)$$
and

\[ \langle p^2_B \rangle_{n_F} = p^2 \frac{\int d\eta_1...d\eta_M \frac{N}{\sqrt{r^*}} e^{-\varphi(\eta_i, n_F)}}{\int d\eta_1...d\eta_M \frac{1}{\sqrt{r^*}} e^{-\varphi(\eta_i, n_F)}} \]  \hspace{1cm} (15) 

Recall that \( N = \sum_{i=1}^{M} \eta_i \) and \( r = \sum_{i=1}^{M} \sqrt{\eta_i} \). Here

\[ \varphi(\eta_i, n_F) = \sum_{i=1}^{M} \frac{(\eta_i - \eta_i^*)^2}{2\eta_i(1 - \lambda_{\eta})} + \frac{(n_F - \mu_0 r)^2}{2\mu_0 r(1 - \lambda)} \] \hspace{1cm} (16)

Further we take out factors before exponent at the point, where \( \varphi \) is minimal, after that the rest integrals in the numerator and in the denominator are being reduced, and we find

\[ \langle n_B \rangle_{n_F} = \mu_0 r^* \] \hspace{1cm} (17)

and

\[ \langle p^2_B \rangle_{n_F} = \frac{p^2 N^*}{r^*} \] \hspace{1cm} (18)

The \( N^* \) and \( r^* \) are the values of \( N \) and \( r \) in the point \( \{\eta_1^*, ..., \eta_M^*\} \), where \( \varphi(\eta_i, n_F) \) is minimal:

\[ \frac{\partial \varphi(\eta_i, n_F)}{\partial \eta_i} = 0 \] \hspace{1cm} (19)

This leads to the system of equations:

\[ \frac{\eta_i^*}{\eta_i} - 1 = \frac{\mu_0 \kappa}{4\sqrt{\eta_i}} \left( \frac{n^2_F}{\mu^2_0 r^{*2}} - 1 \right) \] \hspace{1cm} (20)

Recall that \( r^* = \sum_{i=1}^{M} \sqrt{\eta_i} \) and

\[ \kappa = \frac{1 - \lambda_{\eta}}{1 - \lambda} \] \hspace{1cm} (21)

For the meaning of \( \kappa \) see the end of the previous section and the section 6 of [11], for both Poissonian distributions \( \kappa = 1 \) and \( \kappa \) is the relative width of the \( p(n_i) \) and \( w(\eta_i) \) distributions in other cases. The (20) defines \( \eta_i^* \) as function of \( n_F \).

Introducing short notations

\[ z_i = \sqrt{\frac{\eta_i^*}{\eta_i}}, \quad f = \frac{n_F}{\mu_0 r^*} = \frac{n_F}{\langle n_F \rangle}, \quad a_i = \frac{\mu_0 \kappa}{4\sqrt{\eta_i}} \] \hspace{1cm} (22)

we can rewrite (20) as

\[ z_i^3 - z_i = a_i \left( f^2 \frac{\tau^2}{r^{*2}} - 1 \right) \] \hspace{1cm} (23)

where \( \tau = \sum_{i=1}^{M} \sqrt{\eta_i} \); \( r^* = \sum_{i=1}^{M} z_i \sqrt{\eta_i} \) and \( N^* = \sum_{i=1}^{M} z_i^2 \eta_i \). The (23) defines \( z_i \) as function of \( f \): \( z_i = z_i(f) \). Then we can calculate \( \langle n_B \rangle_{n_F} \) and \( \langle p^2_B \rangle_{n_F} \) as a function of \( n_F \) using (17) and (18).
4  \( pt-n \) and \( n-n \) correlation coefficients at a large string density for the local fusion.

The correlation coefficients are defined in the same way as in the section 4 of \([11]\):

\[
b \equiv \frac{d\langle n_B \rangle_{n_F}}{dn_F} \bigg|_{n_F=\langle n_F \rangle} \tag{24}
\]

and

\[
\beta \equiv \frac{d\langle p^2_{tB} \rangle_{n_F}}{dn_F} \bigg|_{n_F=\langle n_F \rangle} \tag{25}
\]

or for "relative" quantities

\[
\overline{b} \equiv \frac{d\langle n_B \rangle_{n_F}/\langle n_B \rangle}{dn_F/\langle n_F \rangle} \bigg|_{n_F=\langle n_F \rangle} \tag{26}
\]

and

\[
\overline{\beta} \equiv \frac{d\langle p^2_{tB} \rangle_{n_F}/\langle p^2_{tB} \rangle}{dn_F/\langle n_F \rangle} \bigg|_{n_F=\langle n_F \rangle} \tag{27}
\]

(Note the same definition of \( pt-n \) correlation coefficient \( \overline{\beta} \) in \([12]\) (see formula (44) in \([12]\), see also remark in Appendix B).

In short notation using (17) and (18) we have:

\[
\overline{b} = \frac{1}{\overline{r}} \frac{dr^*}{df} \bigg|_{f=1} \tag{28}
\]

and

\[
\overline{\beta} = \frac{N}{\overline{r}} \frac{d(N^*/r^*)}{df} \bigg|_{f=1} \tag{29}
\]

We can’t solve the equations (23) for to find \( z_i = z_i(f) \) explicitly, but to calculate the correlation coefficients we need to know only \( z_i'(1) = \frac{dz_i(f)}{df} \bigg|_{f=1} \), which can be done explicitly.

We see that at \( f = 1 \) (23) has the obvious solution:

\[
f = 1, \quad z_i = 1, \quad \eta_i^* = \overline{\eta}_i, \quad r^* = \overline{r}, \quad N^* = \overline{N} \tag{30}
\]

We need to calculate \( z_i'(f) \) only at \( f = 1 \). Differentiating (23) on \( f \) and using then again (30) we find

\[
z_i'(1) = a_i \frac{4\overline{r}}{4\overline{r} + \mu_0\kappa M} \tag{31}
\]

with \( a_i = \mu_0\kappa/(4\sqrt{\overline{\eta}_i}) \). Then

\[
\overline{b} = \frac{1}{\overline{r}} \frac{dr^*}{df} \bigg|_{f=1} = \frac{1}{\overline{r}} \sum_{i=1}^{M} z_i'(1) \sqrt{\overline{\eta}_i} = \frac{\mu_0\kappa}{\mu_0\kappa + 4\overline{r}/M} \tag{32}
\]

and

\[
\overline{\beta} = \frac{1}{\overline{N}} \frac{dN^*}{df} - \frac{1}{\overline{r}} \frac{dr^*}{df} \bigg|_{f=1} = \frac{1}{\overline{N}} \frac{dN^*}{df} \bigg|_{f=1} - \overline{b} \tag{33}
\]
Using
\[ \frac{dN^*}{df} \bigg|_{f=1} = 2 \sum_{i=1}^{M} z'_i(1) \bar{\eta}_i = \frac{2\mu_0 \kappa \tau^2}{\mu_0 \kappa M + 4\tau} \] (34)
we have
\[ \bar{\beta} = \left( \frac{2\tau^2}{NM} - 1 \right) \frac{\mu_0 \kappa}{\mu_0 \kappa + 4\tau/M} = \left( \frac{2\tau^2}{NM} - 1 \right) \bar{b} \] (35)
We see the connection between \(pt-n\) and \(n-n\) correlation coefficients. Note that due to obvious inequality:
\[ \left( \sum_{i=1}^{M} \sqrt{\eta}_i \right)^2 \leq M \sum_{i=1}^{M} \eta_i \] (36)
we have \(\tau^2 \leq MN\) and hence always \(\bar{\beta} \leq \bar{b}\).

Clear that at equal \(\eta_i \equiv \eta\) we have \(\tau = M\sqrt{\eta}, N = M\eta, \tau^2 = NM\) and
\[ \bar{\beta} = \bar{b} = \frac{\mu_0 \kappa}{\mu_0 \kappa + 4\sqrt{\eta}} \] (37)

From (32) we see that \(n-n\) correlation coefficient is always positive. Can \(pt-n\) correlation coefficient be negative? Let us consider nonhomogeneous situation when \(\eta_i = \eta_+\) at \(i = 1, ..., M_1\), \(\eta_i = \eta_-\) at \(i = M_1 + 1, ..., M, M_1 \sim M\) and \(\eta_+ \gg \eta_- \gg 1\). Then \(\tau = M_1\sqrt{\eta_+} + (M - M_1)\sqrt{\eta_-} \approx M_1\sqrt{\eta_+}, N = M_1\eta_+ + (M - M_1)\eta_- \approx M_1\eta_+\) and we have
\[ \bar{\beta} \approx \left( \frac{2M_1}{M} - 1 \right) \bar{b} \] (38)
We see that at \(M_1 < M/2\) we can have \(\bar{\beta} < 0\).

5 The \(\mu_0/\eta^{1/2}\)-scaling at large string density.

Let us consider for clearness homogeneous case, when all \(\eta_i\) is equal each other in the interaction area \(\eta_i = \eta\) (a constant mean density of strings). In this case we can explicitly calculate at large string density \(\eta\) not only the \(pt-n\) and \(n-n\) correlation coefficients, but also the corresponding correlation functions for the version with a local string fusion.

We have seen in the end of previous section that in this case the coefficients for \(n-n\) and \(pt-n\) correlations defined as (26) and (27) are coincide:
\[ \bar{\beta} = \bar{b} = \frac{\mu_0 \kappa}{\mu_0 \kappa + 4\sqrt{\eta}} \] (39)
where \(a = \mu_0 \kappa / (4\sqrt{\eta})\).

In this homogenous case \(\eta_i \equiv \eta\) we can also calculate the correlation functions \(\langle p_{tB}^2 \rangle_{n_F} \) and \(\langle n_B \rangle_{n_F} \) at any \(n_F\). Due to symmetry, the system of equations (23) have symmetrical solution \(z_i = z\) and can be reduced to one equation
\[ z^3 - z = a \left( \frac{f^2}{z^2} - 1 \right) \] (40)
because \( r = M \sqrt{\eta}, \ r^* = z M \sqrt{\eta} \) and \( N^* = z^2 M \eta \) with

\[
f = \frac{n_F}{\mu_0 M \sqrt{\eta}} = \frac{n_F}{\langle n_F \rangle}, \quad a = \frac{\mu_0 \kappa}{4 \sqrt{\eta}} \tag{41}
\]

The (40) defines the function \( z = z(f) \) and then using (37) and (38) we can calculate

\[
\langle n_B \rangle_{n_F} = \mu_0 r^* = \mu_0 M \sqrt{\eta} z(f) = \langle n_B \rangle z(f) \tag{42}
\]

and

\[
\langle p_{tB}^2 \rangle_{n_F} = p^2 N^*_{r^*} = p^2 \sqrt{\eta} z(f) = \langle p_{tB}^2 \rangle z(f) \tag{43}
\]

So we have at any \( n_F = \langle n_F \rangle_f \):

\[
\frac{\langle n_B \rangle_{n_F}}{\langle n_B \rangle} = \frac{\langle p_{tB}^2 \rangle_{n_F}}{\langle p_{tB}^2 \rangle} = z(f) \tag{44}
\]

From (39) and (40) we see that in this homogenous case at large string density \( \eta \) there is a remarkable scaling. The \( pt-n \) and \( n-n \) correlation coefficients and correlation functions depend only on one combination \( a = \mu_0 \kappa/(4 \sqrt{\eta}) \) of parameters.

We present the correlation function \( z(f) \) (44) in Figs. [12] and the correlation coefficient \( \beta = \frac{\eta}{\langle n \rangle} \) as function of \( \eta \) in Figs. [3, fig. 4] (the solid lines). We present also in Figs. [3, fig. 4] the results of our direct numerical MC calculations of the \( pt-n \) and \( n-n \) correlation coefficients in the local fusion case based on formulas (33) (empty and filled points correspondingly).

We see that in the case with local fusion at small string density we have large \( n-n \) correlations (the same as in the case without string fusion [11]) and no \( p_t-n \) correlations. (The analysis at very small values of \( \eta \leq 1/M \) see in Appendix A.)

At large string density in the homogeneous case the \( p_t-n \) and \( n-n \) correlation coefficients become equal and the \( \mu_0/\sqrt{\eta} \)-scaling takes place. We see also that in this limit our Gauss asymptotic is in a good agreement with results of the numerical calculations and \( M \)-independence takes place.

6 The global fusion at large string density. Exact solution.

In this case at first stage we also have \( M = S(b)/\sigma_0 \) cells (like in the case with local fusion) with \( \eta_i, \ i = 1, ..., M \) fluctuated around \( \eta \). Then (unlike the local fusion case) we have to find average \( \eta_c = \frac{1}{M} \sum_i \eta_i = \frac{N}{M} \) for given event, and then to generate particles from one cluster with average multiplicity equal to \( \mu_c \sqrt{\eta_c} = \mu_0 M \sqrt{\eta_c} = \mu_0 M \sqrt{N/M} = \mu_0 \sqrt{MN} \). The general formulae for this case was obtained in the section 5 of [11]:

\[
\langle O \rangle_{n_F} = \frac{\sum_{\{\eta_1, \ldots, \eta_M\}} \langle O \rangle_{\{\eta_1, \ldots, \eta_M\}, n_F} W(\eta_1, ..., \eta_M)p_{\mu_0 \sqrt{\sum_i \eta_i}}(n_F)}{\sum_{\{\eta_1, \ldots, \eta_M\}} W(\eta_1, ..., \eta_M)p_{\mu_0 \sqrt{\sum_i \eta_i}}(n_F)} \tag{45}
\]

where \( \mu_c = \mu_0 M \) with \( M = S(b)/\sigma_0 \). The assumption \( \eta_i >> 1 \) is essential, as only in this situation we can consider that the transverse area of the cluster \( \Delta S \) is equal to all interaction area \( S(b) \) \( (b \text{-impact parameter}) \).
The \( \langle O \rangle_{\{\eta_1, \ldots, \eta_M\}, n_F} \) is the rates of the backward production from configuration \( \{\eta_1, \ldots, \eta_M\} \). We have to use for \( \langle n_B \rangle_{n_F} \) correlations:

\[
\langle n_B \rangle_{\{\eta_1, \ldots, \eta_M\}, n_F} = \mu_c \sqrt{\eta_c} = \mu_0 M \sqrt{\frac{1}{M} \sum_i \eta_i}
\]

(46)

and for \( \langle p^2_{tB} \rangle_{n_F} \) correlations:

\[
\langle p^2_{tB} \rangle_{\{\eta_1, \ldots, \eta_M\}, n_F} = p^2 \sqrt{\eta_c} = p^2 \sqrt{\frac{1}{M} \sum_i \eta_i}
\]

(47)

We see that the difference with the case of local fusion consists in replacing \( \frac{1}{M} \sum_i \sqrt{\eta_i} \rightarrow \sqrt{\frac{1}{M} \sum_i \eta_i} \). As a consequence calculations in the case of global string fusion are much more simple, as we can reduce all sums \( \sum_{\{\eta_1, \ldots, \eta_M\}} \) to one sum \( \sum_N \), as in the no fusion case (see section 3 in [11]). So in the global fusion case we can write simple formulas:

\[
\langle n_B \rangle_{n_F} = \frac{\mu_0 \sqrt{M} \sum_N \sqrt{N W(N)} p_{\mu_0 \sqrt{M} \sqrt{N}}(n_F)}{\sum_N W(N) p_{\mu_0 \sqrt{M} \sqrt{N}}(n_F)}
\]

(48)

and

\[
\langle p^2_{tB} \rangle_{n_F} = \frac{p^2 \sum_N \sqrt{N W(N)} p_{\mu_0 \sqrt{M} \sqrt{N}}(n_F)}{\sum_N W(N) p_{\mu_0 \sqrt{M} \sqrt{N}}(n_F)}
\]

(49)

where \( W(N) \) is given by the formula

\[
W(N) = \sum_{\{\eta_1, \ldots, \eta_M\}} \delta_{N, \sum_i \eta_i} \prod_i w(\eta_i)
\]

(50)

We see that in the case of global fusion (one cluster at large \( \eta_i \) with area \( \Delta S \) being equal to all interaction area \( S(b) \)) \( n-n \) and \( pt-n \) correlations are connected

\[
\langle p^2_{tB} \rangle_{n_F} = \frac{p^2}{\mu_0 M} \langle n_B \rangle_{n_F} \quad \text{or} \quad \frac{\langle n_B \rangle_{n_F}}{\langle n_B \rangle} = \frac{\langle p^2_{tB} \rangle_{n_F}}{\langle p^2_{tB} \rangle}
\]

(51)

Note that unlike the local fusion case in this case we find this result without any assumptions on the properties of \( p(n_F) \) and \( w(\eta_i) \) distributions and for arbitrary (even nonequal) \( \eta_i \).

Clear that in this case the results can depend only on mean number of strings \( \overline{N} \) and on combination \( \mu_M \):

\[
\overline{N} = \sum_i \overline{\eta_i} \quad \mu_M = \mu_0 \sqrt{M}
\]

(52)

Below we’ll calculate numerically the correlation functions on formulas (48) and (49), but at first we would like to find explicit formulas for global fusion case in Gauss approximation. We’ll see that results really depend only on one combination of the variables (52), namely on

\[
\frac{\mu_M}{\sqrt{\overline{N}}} = \frac{\mu_0}{4 \sqrt{\overline{N}/M}}
\]

(53)

and the scaling takes place as in the case with local fusion.
7 Gauss approximation for global fusion at large string density.

Acting as in the no fusion case in section 4 of [11] (see also calculations in the Gauss approximation in the section 3 of the present paper) we find

$$\langle n_B \rangle_{n_F} = \mu_M \sqrt{N^*} = \mu_0 \sqrt{M \sqrt{N^*}}$$

(54)

or keeping in mind (31)

$$\frac{\langle n_B \rangle_{n_F}}{\langle n_B \rangle} = \frac{\langle p_{2B}^2 \rangle_{n_F}}{\langle p_{2B}^2 \rangle} = \sqrt{\frac{N^*}{N}}$$

(55)

The $N^*$ is the value of $N$ at which the function

$$\varphi(N, n_F) = \frac{(N - N^*)^2}{2N(1 - \lambda \eta)} + \frac{(n_F - \mu_M \sqrt{N})^2}{2\mu_M \sqrt{N}(1 - \lambda)}$$

(56)

gains its minimum. In short notations

$$z \equiv \sqrt{\frac{N^*}{N}}, \quad f = \frac{n_F}{\mu_M \sqrt{N}} = \frac{n_F}{\langle n_F \rangle}, \quad a = \frac{\mu_M \kappa}{4 \sqrt{N}} = \frac{\mu_0 \kappa}{4 \sqrt{N/M}}$$

(57)

we find the equation

$$z^3 - z = a \left( \frac{f^2}{z^2} - 1 \right)$$

(58)

which defines the function $z = z(f)$ and then using (55) we can calculate correlation functions

$$\frac{\langle n_B \rangle_{n_F}}{\langle n_B \rangle} = \frac{\langle p_{2B}^2 \rangle_{n_F}}{\langle p_{2B}^2 \rangle} = z \left( \frac{n_F}{\langle n_F \rangle} \right) = z(f)$$

(59)

and correlation coefficients for the global fusion case

$$\beta = b = z'(1) = \frac{a}{a + 1} = \frac{\mu_M \kappa}{\mu_M \kappa + 4 \sqrt{N}} = \frac{\mu_0 \kappa}{\mu_0 \kappa + 4 \sqrt{N/M}}$$

(60)

We see again that in Gauss approximation there is the same remarkable scaling. The $pt-n$ and $n-n$ correlations depend only on one combination of parameters: $a = \frac{\mu_0 \kappa}{4 \sqrt{N/M}}$.

Note that unlike the local fusion case in this case we find this result for arbitrary (even not equal) $\eta_i$.

In the homogeneous situation all $\eta_i = \eta$ and we have

$$\bar{N} = M \eta, \quad a = \frac{\mu_0 \kappa}{4 \sqrt{\eta}}$$

(61)

and

$$\beta = b = \frac{\mu_0 \kappa}{\mu_0 \kappa + 4 \sqrt{\eta}} = \frac{a}{a + 1}$$

(62)

We see that in the homogeneous situation in Gauss approximation the results with local and global fusion coincide (as we have expected in section 5 of [11]). We have the
same equations (40) and (58) with the same value of parameter $a$ (41) and (61). Note that in the no fusion case we have had very different equation for $z(f)$ (see section 4 of [11]).

Unlike the local fusion case in the global fusion case we can control the validity of Gauss approximation making calculations on exact formulas (48) and (49) at different values of $M$.

Along with the correlation function $z(f)$ (44), (59) in Figs. 1,2 and the correlation coefficient $\beta = b$ (39),(60) in Figs. 3-5, calculated on our scaling formulas, which in Gauss approximation are the same for local and global fusion (the solid lines), we present at the same pictures the results of exact calculations in the global fusion case on formulas (48) and (49) at different values of $M$ (the dotted and dashed lines).

We present also in Figs. 3-5 the results of our direct numerical MC calculations of the $pt$–$n$ and $n$–$n$ correlation coefficients in the global fusion case based on formulas (45-47) (half-filled squares).

We see that in this case the Gauss approximation works very well and the $\mu_0/\sqrt{\eta}$-scaling is not an artifact of this approximation. More over along with the $\mu_0/\sqrt{\eta}$-scaling at large $\eta$ we have also $M$-independence for correlation coefficients $\beta$ and $b$ (see Figs. 3-5) starting very early (from $M = 4$).

8 Conclusions.

In conclusion let us compare the results obtained in the present paper in the case with string fusion with results obtained in [11] in the case without string fusion.

We have obtained in the paper [11] in the case without string fusion:

1. for $n$–$n$ correlations: $\overline{b} = \frac{a}{a+1}$ with $a = \mu_0\kappa$

2. for $pt$–$n$ correlations: $\overline{\beta} = 0$

In the present paper in the case with the global string fusion and in the local fusion case for homogenous situation ($\eta_i = \eta$) we find at large $\eta$:

1. for $n$–$n$ correlations: $\overline{b} = \frac{a}{a+1}$ with $a = \frac{\mu_0\kappa}{4\sqrt{N/M}} = \mu_0\kappa/(4\sqrt{\eta})$

2. for $pt$–$n$ correlations: $\overline{\beta} = \overline{b} = \frac{a}{a+1}$ with the same $a$

We see that with fusion the $n$–$n$ correlations became weaker, but now as compensation we have the $pt$–$n$ correlations of the same strength. We see also $\mu_0/\sqrt{\eta}$-scaling in this case.

For nonhomogeneous situation (different $\eta_i$) in the case with local string fusion we have find at large $\eta_i$:

1. for $n$–$n$ correlations: $\overline{b} = \frac{a}{a+1}$ with $a = \mu_0\kappa/(4\eta/M)$

2. for $pt$–$n$ correlations: $\overline{\beta} = \left(\frac{\eta}{\eta_i M} - 1\right)\overline{b}$ and hence $\overline{\beta} \leq \overline{b}$
\[ \mathcal{N} = \sum_{i=1}^{M} \eta_i \quad \text{and} \quad \mathcal{R} = \sum_{i=1}^{M} \sqrt{\eta_i} \]

As we have demonstrated above (see (33) and (35)), this leads to \( \beta \) smaller than \( b \): \( \beta \leq b \). It’s possible situation (38), in which \( \beta < 0 \).

At small string density as it’s shown in Appendix A the two types of limit at \( \eta \to 0 \) can be studied.

1. If one keeps \( M = \text{const} \), then we have \( N \to 1 \) (because the configurations with \( N = 0 \), are not considered as events) and we have nor \( p_t-n \) nor \( n-n \) correlation.

2. If one keeps \( \mathcal{N} = \text{const} \), then \( M \eta = \text{const} \) and \( M \to \infty \), hence the strings will be far separated in transverse plane and we’ll have the same results as in the case without string fusion [11].

Note that the results obtained in our cellular approach are in a good agreement with the results obtained in the framework of the real string fusion model taking into account detail geometry of strings overlapping [13].

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Appendixes:

A Correlations at small string density.

In this appendix we calculate the correlation functions and the correlation coefficients at small string density in the case with local string fusion. (In the case with global fusion this limit has no physical sense (see discussion in the section 5 of [11])).

For clearness we consider ”homogeneous” case, when all \( \eta_i \) is equal each other in the interaction area \( \eta_i = \eta \). Then in each cell \( i \) \( (i = 1, ..., M) \) the \( \eta_i \) fluctuate around this mean value according to Poisson law \( (\rho_a(x) \) is the Poisson distribution with \( \bar{\tau} = a)\):

\[ w(\eta_i) = \rho_\eta(\eta_i) \equiv e^{-\eta(\eta_i)^{\eta_i}} \eta_i! \]

(63)

We’ll assume also the Poissonian form of \( p_{\eta_i}(n_i) \):

\[ p_{\eta_i}(n_i) = \rho_{\mu_0\sqrt{\eta_i}}(n_i) \equiv e^{-\mu_0\sqrt{\eta_i}}(\mu_0\sqrt{\eta_i})^{n_i} n_i! \]

(64)

then we have the Poissonian distribution for

\[ P_{\{\eta_1,...,\eta_M\}}(n_F) = \rho_{\mu_0 \sum_i \sqrt{\eta_i}}(n_F) \]

(65)
with \( \langle n_F \rangle_{\{\eta_1,\ldots,\eta_M\}} = \mu_0 \sum_i \sqrt{\eta_i} = \mu_0 r = \langle n_F \rangle_r \).

Then we find from (3) for \( n-n \) correlations:

\[
\langle n_B \rangle_{n_F} = \frac{\mu_0 \sum_{\{\eta_1,\ldots,\eta_M\}} r \left( \prod_i w(\eta_i) \right) \rho_{\mu_0 r}(n_F)}{\sum_{\{\eta_1,\ldots,\eta_M\}} \left( \prod_i w(\eta_i) \right) \rho_{\mu_0 r}(n_F)}
\]

(66)

and for \( pt-n \) correlations:

\[
\langle p^2 B \rangle_{n_F} = \frac{p^2 \sum_{\{\eta_1,\ldots,\eta_M\}} N \left( \prod_i w(\eta_i) \right) \rho_{\mu_0 r}(n_F)}{\sum_{\{\eta_1,\ldots,\eta_M\}} \left( \prod_i w(\eta_i) \right) \rho_{\mu_0 r}(n_F)}
\]

(67)

Recall that

\[
N = \sum_{i=1}^M \eta_i \quad \text{and} \quad r = \sum_{i=1}^M \sqrt{\eta_i}
\]

The \( N \) is the number of strings in the given event.

One has to omit in all \( M \)-fold sums in (66) and (67) one term, when all \( \eta_i = 0 \), which corresponds to the absence of inelastic interactions between the nucleons of the colliding nuclei (we denote this fact by \( \sum' \)).

The probability \( P(n_F) \) to detect \( n_F \) particles in the forward rapidity window, which enters denominators of (66) and (67), is

\[
P(n_F) = C \sum_{\{\eta_1,\ldots,\eta_M\}}' \left( \prod_i w(\eta_i) \right) \rho_{\mu_0 r}(n_F)
\]

(68)

where from normalization condition we have

\[
C = \frac{1}{1 - w^M(0)} = \frac{1}{1 - e^{-M\eta}}
\]

(69)

Clear that this factor \( C \) is canceling in the numerator and in the denominator of (66) and (67), but if we calculate the mean number of strings \( \overline{N} \), we find

\[
\overline{N} = C \sum_{\{\eta_1,\ldots,\eta_M\}}' \left( \prod_i w(\eta_i) \right) N = C \sum_{\{\eta_1,\ldots,\eta_M\}}' \left( \prod_i w(\eta_i) \right) \left( \sum_i \eta_i \right) = \frac{M\eta}{1 - e^{-M\eta}}
\]

(70)

and for the \( \langle n_F \rangle \) at small \( \eta \ll 1 \) we have

\[
\langle n_F \rangle = \sum_{n_F} n_F P(n_F) = C \mu_{0F} \sum_{\{\eta_1,\ldots,\eta_M\}}' \left( \prod_i w(\eta_i) \right) r = \mu_{0F} \frac{M\eta}{1 - e^{-M\eta}} = \mu_{0F} \overline{N}
\]

(71)

Because for any \( \omega > 0 \) we have \( \sum \eta_i^\omega w(\eta_i) = \eta + O(\eta^2) \) at \( \eta \to 0 \), as the main contribution comes from the term \( \eta_i = 1 \).

There are two possibilities when \( \eta \to 0 \):

1. \( M = \text{const} \) and then \( M\eta \to 0 \) and \( \overline{N} \to 1 \) (see (70))

2. \( \overline{N} = \text{const} \) and then \( M\eta = \text{const} \) (see (70)) and \( M \to \infty \)
We’ll investigate both these possibilities.

The first one means that in the limit we have \( N = \overline{N} = 1 \) (because the configurations with \( N = 0 \), are not considered as events). Clear that in this situation we’ll have nor \( p_t\)-\( n \) nor \( n\)-\( n \) correlation, as we’ll have no fluctuations in the number of strings (see discussion in the end of the section 4 of \([11]\)). Detail calculations are presented below.

In the second case with \( \overline{N} = \text{const} \) we’ll have the fluctuations in the number of strings \( N \), but in the limit \( \eta \to 0 \) we’ll have \( M \to \infty \), the strings will be far separated in transverse plane and the strings fusion will plays no role. So in this case we’ll have the same results as in the no fusion case, considered in \([11]\): large \( n\)-\( n \) correlation with a correlation coefficient equal to \( b = \mu_0/(\mu_0 + 1) \) and no \( p_t\)-\( n \) correlation. (See calculations below.)

**Detail calculations.** Let us evaluate the \( M \)-fold sums in (66) and (67) at \( \eta \to 0 \) keeping all terms of order \((M\eta)^k\), \((M\eta)^k\eta\) with any \( k \) and omitting all terms of order \((M\eta)^k\eta^2\) and higher.

The terms of order \((M\eta)^k\) originates from the summands in (66) and (67) with \( \eta_{i_1} = \ldots = \eta_{i_k} = 1 \) and other \( \eta_i = 0 \). The terms of order \((M\eta)^k\eta\) originates from the summands in (66) and (67) with \( \eta_{i_1} = 2 \), \( \eta_{i_2} = \ldots = \eta_{i_k} = 1 \) and other \( \eta_i = 0 \). Keeping this into mind we find for \( P(n_F) \):

\[
P(n_F) = C(G_0 + \frac{\eta}{2} G_1) \tag{72}
\]

and for \( n\)-\( n \) correlations:

\[
\langle n_B \rangle_{n_F} = \mu_{0B} \frac{N_0 + \frac{\eta}{2} N_1}{G_0 + \frac{\eta}{2} G_1} \tag{73}
\]

For \( p_t\)-\( n \) correlations we have:

\[
\langle p_{tB}^2 \rangle_{n_F} = p^2 \frac{P_0 + \frac{\eta}{2} P_1}{G_0 + \frac{\eta}{2} G_1} = p^2 \left( 1 + \frac{\eta}{2} \frac{P_1 - G_1}{G_0} \right) \tag{74}
\]

Here

\[
G_0 = P_0 = \sum_{k=1}^{M} C_M^k \eta^k \rho_{\mu_0 k}(n_F) \tag{75}
\]

\[
N_0 = \sum_{k=1}^{M} k C_M^k \eta^k \rho_{\mu_0 k}(n_F) \tag{76}
\]

\[
G_1 = \sum_{k=1}^{M} k C_M^k \eta^k \rho_{\mu_0 (k+\gamma)}(n_F) \tag{77}
\]

\[
N_1 = \sum_{k=1}^{M} (k + \gamma) k C_M^k \eta^k \rho_{\mu_0 (k+\gamma)}(n_F) \tag{78}
\]

\[
P_1 = \sum_{k=1}^{M} \frac{k + 1}{k + \gamma} k C_M^k \eta^k \rho_{\mu_0 (k+\gamma)}(n_F) \tag{79}
\]

where \( \mu_0 \equiv \mu_{0F} \) and \( \gamma = \sqrt{2} - 1 \).

Note that at \( M \gg 1 \) and \( M\eta = \text{const} \) we have for \( P(n_F) \):

\[
P(n_F) = C e^{-M\eta \frac{\mu_0}{n_F}} (G_0 + \frac{\eta}{2} e^{-\mu_0 \gamma} G_1) \tag{76}
\]

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and for $n$–$n$ correlations:

$$
\langle n_B \rangle_{nF} = \mu_0 \frac{N_0 + \frac{n}{2} e^{-\mu_0} N_1}{G_0 + \frac{n}{2} e^{-\mu_0} G_1}
$$  \hspace{1cm} (77)

For $pt$–$n$ correlations we have:

$$
\langle p_t^2 \rangle_{nF} = p^2 \frac{P_0 + \frac{n}{2} e^{-\mu_0} P_1}{G_0 + \frac{n}{2} e^{-\mu_0} G_1} = p^2 \left(1 + \frac{n}{2} e^{-\mu_0} \frac{P_1 - G_1}{G_0}\right)
$$  \hspace{1cm} (78)

Here

$$
G_0 = \frac{G_0}{P_0} = \sum_{k=1}^{\infty} k^{n_F} \frac{d^k}{k!}
$$  \hspace{1cm} (79)

$$
N_0 = \sum_{k=1}^{\infty} k^{n_F+1} \frac{d^k}{k!}
$$

$$
G_1 = \sum_{k=1}^{\infty} (k + \gamma)^{n_F} \frac{d^k}{(k-1)!}
$$

$$
N_1 = \sum_{k=1}^{\infty} (k + \gamma)^{n_F+1} \frac{d^k}{(k-1)!}
$$

$$
P_1 = \sum_{k=1}^{\infty} (k + \gamma)^{n_F-1}(k + 1) \frac{d^k}{(k-1)!}
$$

where $d = \frac{2M\eta e^{-\mu_0}}{\eta}$.

For the control of calculations we have used also the following explicit formulas at $n_F = 0, 1, 2$:

$$
G_0(0) = \frac{G_0}{P_0}(0) = e^d - 1
$$  \hspace{1cm} (80)

$$
N_0(0) = G_1(0) = G_0(1) = P_0(1) = de^d
$$

$$
N_1(0) = G_1(1) = (d + \sqrt{2})de^d
$$

$$
P_1(0) = \sum_{k=1}^{\infty} \frac{k + 1}{k + \gamma} \frac{d^k}{(k-1)!}
$$

$$
N_0(1) = G_0(2) = \frac{P_0}{P_1}(0) = (d + 1)de^d
$$

$$
N_1(1) = G_1(2) = (d^2 + d(1 + 2\sqrt{2}) + 2)de^d
$$

$$
P_1(1) = (d + 2)de^d
$$

$$
N_0(2) = (d^2 + 3d + 1)de^d
$$

$$
N_1(2) = (d^3 + d^2 + 3(1 + \sqrt{2}) + d(7 + 3\sqrt{2}) + 2\sqrt{2})de^d
$$

$$
P_1(2) = (d^2 + d(3 + \sqrt{2}) + 2\sqrt{2})de^d
$$

The results of the calculations using the asymptotic formulas (73-75) in the first case ($M = \text{const}, \eta \to 0, M\eta \to 0$ and $N \to 1$) are shown in the Figs. 6-7 (lines) together with results of direct MC numerical calculations using formulas (66) and (67) (points).

As we have expected in this case both $n$–$n$ and $pt$–$n$ correlation coefficients go to zero when $N \sim 1$, i.e. at $\eta < 1/M$. Remember that we have $M$-independence for the correlation coefficients at large $\eta$. Now we see that in this limit it disappears at
\( \eta \leq 1/M \). This is also the reason for nonlinear dependence of the correlation coefficients on \( \eta \) in this region which one can see in Figs. 6-7 for \( \mu_0 = 4 \).

We see also that using asymptotic formulas (73–75) we can calculate \( n-n \) correlation coefficients in wider region of small \( \eta \), than \( p_t-n \) correlation coefficients, because we have the contributions of order \( (M\eta)^k \) and \( (M\eta)^k\eta \) for \( n-n \) correlations and only first non-trivial contribution of order \( (M\eta)^k\eta \) for \( p_t-n \) correlations. Note the very good agreement between results of the calculations on the asymptotic formulas (73–75) (lines in the Figs. 6-7) and the results of direct MC numerical calculations on formulas (66) and (67) (points in the Figs. 6-7).

In the second case when \( \eta \to 0 \) we keep \( \overline{N} = \text{const} \) and then due to (70) \( M\eta = \text{const} \), so \( M \to \infty \). We can use formulas (77–79) with \( d = \text{const} \), then in the limit \( \eta \to 0 \) we find for \( n-n \) correlations:

\[
\langle n_B \rangle_{n_F} = \mu_0 B \frac{\overline{N}_0}{C_0} = \mu_0 B \frac{\sum_{k=1}^{\infty} k \rho_{M\eta}(k) \rho_{\mu_0 k}(n_F)}{\sum_{k=1}^{\infty} \rho_{M\eta}(k) \rho_{\mu_0 k}(n_F)}
\]

(81)

Here we multiply both the numerator and the denominator by \( e^{-M\eta} \mu_0^{n_F}/n_F \).

Recall that \( \rho_n(x) \) is the Poisson distribution with \( \overline{\pi} = a \), then we see that formula (81) coincides with the formula for \( n-n \) correlations obtained in the paper \( [11] \) in the case without string fusion (see formula (23) in \( [11] \)). So we’ll have the same result for the \( n-n \) correlation coefficient \( b = \mu_0/(\mu_0 + 1) \) as in the no fusion case (see Figs. 6-7).

For \( p_t-n \) correlation coefficients in this limit we find from (78) and (79) with \( d = \text{const} \):

\[
\langle p_{tB}^2 \rangle_{n_F} = p^2 (1 + O(\eta))
\]

(82)

So we have no \( p_t-n \) correlation as in the case without string fusion \( [11] \).

\section{On the difference between \( \langle n_B \rangle \) and \( \langle n_B \rangle_{n_F=\langle n_F \rangle} \).}

It’'s possible instead of (26) and (27) to use the following definitions for the correlation coefficients:

\[
\overline{B} = \frac{d\langle n_B \rangle_{n_F}/\langle n_B \rangle_{n_F}}{dn_F/\langle n_F \rangle} \bigg|_{n_F=\langle n_F \rangle}
\]

(83)

and

\[
\overline{\beta} = \frac{d\langle p_{tB}^2 \rangle_{n_F}/\langle p_{tB}^2 \rangle_{n_F}}{dn_F/\langle n_F \rangle} \bigg|_{n_F=\langle n_F \rangle}
\]

(84)

where

\[
\langle n_B \rangle_{n_F} = \langle n_B \rangle_{n_F=\langle n_F \rangle}
\]

(85)

and

\[
\langle p_{tB}^2 \rangle_{n_F} = \langle p_{tB}^2 \rangle_{n_F=\langle n_F \rangle}
\]

(86)

Clear that

\[
\langle n_B \rangle = \sum_{n_F} P(n_F) \langle n_B \rangle_{n_F} \approx \langle n_B \rangle_{n_F=\langle n_F \rangle} \sum_{n_F} P(n_F) = \langle n_B \rangle_{n_F=\langle n_F \rangle} \equiv \langle n_B \rangle_{\langle n_F \rangle}
\]

(87)

and the same for \( \langle p_{tB}^2 \rangle_{n_F} \equiv \langle p_{tB}^2 \rangle_{n_F=\langle n_F \rangle} \approx \langle p_{tB}^2 \rangle \).

In our Gauss approximation these two types of quantities coincide with each other.
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\[ z(f) = \frac{\langle p_{T} B \rangle}{nF} = \frac{\langle n B \rangle}{nF} \]

with fusion, \( \mu_0 / \sqrt{\eta} \)-scaling

- global, \( \mu_0 = 1, \eta = 1, M = 4 \)
- global, \( \mu_0 = 1, \eta = 1, M = 128 \)
- global, \( \mu_0 = 2, \eta = 4, M = 4 \)
- global, \( \mu_0 = 2, \eta = 4, M = 128 \)
- local and global, \( \mu_0 / \sqrt{\eta} = 1 \), Gauss

Figure 1: The \( p_{T} - n \) and \( n-n \) correlation functions. Solid line - the Gauss approximation for local and global fusion at the value of scaling variable \( \mu_0 / \sqrt{\eta} = 1 \). Dashed and dotted lines - the results of exact calculations in the global fusion case at different values of \( \mu_0, \eta \) and \( M \). The \( \mu_0 / \sqrt{\eta} \)-scaling and \( M \)-independence.

\[ z(f) = \frac{\langle p_{T} B \rangle}{nF} = \frac{\langle n B \rangle}{nF} \]

with fusion, validity of Gauss approx.

- global, \( \mu_0 = 4, \eta = 2, M = 4 \)
- global, \( \mu_0 = 4, \eta = 2, M = 128 \)
- local and global, \( \mu_0 / \sqrt{\eta} = 2.83 \), Gauss

Figure 2: The same as in Fig. 1, but at different values of \( \mu_0 \) and \( \eta \).
Figure 3: The $p_t$–$n$ and $n$–$n$ correlation coefficients at $\mu_0 = 1$. Solid line - the Gauss approximation for local and global fusion $\bar{b} = \bar{\beta} = \mu_0 / (\mu_0 + 4\sqrt{\eta})$. Dashed and dotted lines - the results of exact calculations on the formulas (48-49) in the global fusion case at different values of $M$ (half-filled squares - the same by means of direct numerical MC calculations on formulas (45-47)). The empty and filled points - the results of direct numerical MC calculations in the local fusion case based on formulas (6-8) for the $pt$–$n$ and $n$–$n$ correlations correspondingly. The filled circle - the $n$–$n$ correlation coefficient $\bar{b} = \mu_0 / (\mu_0 + 1)$ in the case without string fusion [11].

Figure 4: The same as in Fig. 3 but at $\mu_0 = 2$. 
Figure 5: The same as in Fig. 3 but at $\mu_0 = 4$.

Figure 6: The $n-n$ correlation coefficient at small values of $\eta$ for $\mu_0 = 1$ and $\mu_0 = 4$. The lines - results of the calculations using the asymptotic formulas (73) and (75) at $M = \text{const}$ $(N \to 1)$. The points - results of direct MC numerical calculations using formula (66). The arrows show the value of the $n-n$ correlation coefficient $\bar{b} = \mu_0 / (\mu_0 + 1)$ in the case without string fusion [11], which corresponds to the limit $\overline{N} = \text{const}$ $(M \to \infty)$. 
Figure 7: The $p_t-n$ correlation coefficients at small values of $\eta$ for $\mu_0 = 1$ and $\mu_0 = 4$. The lines - results of the calculations using the asymptotic formulas (74) and (75) at $M = \text{const}$ ($\overline{N} \to 1$). The points - results of direct MC numerical calculations using formula (67).