We consider the problem of imitation from observation (IfO), in which the agent aims to mimic the expert’s behavior from the state-only demonstrations by experts. We additionally assume that the agent cannot interact with the environment but has access to the action-labeled transition data collected by some agent with unknown quality. This offline setting for IfO is appealing in many real-world scenarios where the ground-truth expert actions are inaccessible and the arbitrary environment interactions are costly or risky. In this paper, we present LobsDICE, an offline IfO algorithm that learns to imitate the expert policy via optimization in the space of stationary distributions. Our algorithm solves a single convex minimization problem, which minimizes the divergence between the two state-transition distributions induced by the expert and the agent policy. On an extensive set of offline IfO tasks, LobsDICE shows promising results, outperforming strong baseline algorithms.
We consider an environment modeled as a Markov Decision Process (MDP), defined by $M = (S, A, T, R, p_0, \gamma)$ [Sutton and Barto 1998], where $S$ is the set of states, $A$ is the set of actions, $T : S \times A \to \Delta(S)$ is the transition probability, $R : S \times A \to \mathbb{R}$ is the reward function, $p_0 \in \Delta(S)$ is the distribution of the initial state, and $\gamma \in (0, 1)$ is the discount factor. The policy $\pi : S \to \Delta(A)$ is a mapping from states to distribution over actions. For the given policy $\pi$, its state-action stationary distribution $d^\pi(s,a)$ and state-transition stationary distribution $\bar{d}^\pi(s,a)$ are defined as:

$$d^\pi(s,a) := (1-\gamma) \sum_{t=0}^{\infty} \gamma^t \Pr(s_t = s, a_t = a),$$

$$\bar{d}^\pi(s,s') := (1-\gamma) \sum_{t=0}^{\infty} \gamma^t \Pr(s_t = s, s_{t+1} = s'),$$

where $s_0 \sim p_0, a_t \sim \pi(\cdot|s_t), s_{t+1} \sim T(\cdot|s_t, a_t)$ for all timesteps $t \geq 0$. For brevity, the bar notation ($\bar{\cdot}$) will be used to denote the distributions for $(s,s')$, e.g. $\bar{d}^\pi(s,s')$.

Since we are dealing with offline IFO, we assume that direct online interaction with the environment is not allowed, and the policy should be optimized solely from a fixed offline dataset. We denote the dataset for state-only demonstrations collected by experts as $D^E = \{(s,s')\}_{i=1}^{N^E}$ and the dataset for state-action demonstrations by some imperfect agent as $D^I = \{(s,a,s')\}_{i=1}^{N^I}$. That is, we cannot access the actions taken by the expert, but instead, we have additional action-labeled transition data collected by some other agent with unknown level of optimality. We denote the empirical distributions of the datasets $D^E$ and $D^I$ by $\bar{d}^E$ and $\bar{d}^I$, respectively. For brevity, we will abuse $\bar{d}^I$ to represent $(s,a) \sim \bar{d}^I, (s,a,s') \sim \bar{d}^I, \text{and} (s,s') \sim \bar{d}^I$.

### 2 Preliminaries

#### 2.1 Markov decision process

We consider an environment modeled as a Markov Decision Process (MDP), defined by $M = (S, A, T, R, p_0, \gamma)$ [Sutton and Barto 1998], where $S$ is the set of states, $A$ is the set of actions, $T : S \times A \to \Delta(S)$ is the transition probability, $R : S \times A \to \mathbb{R}$ is the reward function, $p_0 \in \Delta(S)$ is the distribution of the initial state, and $\gamma \in (0, 1)$ is the discount factor. The policy $\pi : S \to \Delta(A)$ is a mapping from states to distribution over actions. For the given policy $\pi$, its state-action stationary distribution $d^\pi(s,a)$ and state-transition stationary distribution $\bar{d}^\pi(s,a)$ are defined as:

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Since we are dealing with offline IFO, we assume that direct online interaction with the environment is not allowed, and the policy should be optimized solely from a fixed offline dataset. We denote the dataset for state-only demonstrations collected by experts as $D^E = \{(s,s')\}_{i=1}^{N^E}$ and the dataset for state-action demonstrations by some imperfect agent as $D^I = \{(s,a,s')\}_{i=1}^{N^I}$. That is, we cannot access the actions taken by the expert, but instead, we have additional action-labeled transition data collected by some other agent with unknown level of optimality. We denote the empirical distributions of the datasets $D^E$ and $D^I$ by $\bar{d}^E$ and $\bar{d}^I$, respectively. For brevity, we will abuse $\bar{d}^I$ to represent $(s,a) \sim \bar{d}^I, (s,a,s') \sim \bar{d}^I, \text{and} (s,s') \sim \bar{d}^I$.

#### 2.2 Imitation learning and imitation learning from observation

**Imitation learning (IL)** aims to mimic the expert policy from its state-action demonstrations. IL can be naturally formulated as a distribution matching problem that minimizes the divergence between state-action stationary distributions induced by the expert and the target policy [Ho and Ermon 2016, Ke et al. 2020]. For example, one can consider minimizing KL-divergence [Kostrikov et al. 2020]:

$$\min_{\pi} D_{KL}(d^\pi(s,a)||d^E(s,a)) = \mathbb{E}_{d^\pi} \left[ \log \frac{d^\pi(s,a)}{d^E(s,a)} \right].$$

However, the standard IL requires action labels in the expert demonstrations, which may be a too strong requirement for various practical situations. **Imitation learning from observation (IFO)** relaxes the requirement on action labels, and aims to imitate the expert’s behavior only from the state observations. Since the expert’s action information is missing in the demonstrations, the distribution matching for state-action stationary distribution is no longer readily available. Therefore, IFO is reformulated as another distribution matching problem that minimizes the divergence between state-transition stationary distributions induced by the expert and the target policy [Yang et al. 2019, Torabi et al. 2019, Zhu et al. 2020]:

$$\min_{\pi} D_{KL}(\bar{d}^\pi(s,s')||\bar{d}^E(s,s')) = \mathbb{E}_{d^\pi} \left[ \log \frac{\bar{d}^\pi(s,s')}{\bar{d}^E(s,s')} \right].$$
We present offline imitation Learning from OBServation via stationary DIstribution Correction Estimation (DICE) methods for offline RL. LobsDICE essentially optimizes the stationary distributions of the target policy to match the expert’s state visitations. The derivation of our algorithm starts by rewriting the distribution matching problem (5) in terms of stationary distribution matching, whereas we consider the offline IfO (e.g. state-action distribution). The Bellman flow constraint (7) ensures $\bar{d}(\cdot, \cdot)$ to be the stationary distribution of an optimal policy, which best matches the state trajectories of the expert. Once we have computed the optimal solution $(d^\pi, \bar{d}^*)$, its corresponding optimal policy can also be obtained by $\pi^*(a|s) = \frac{d^\pi(s, a)}{\sum_{s'} d^\pi(s', a)}$.

Note that DemoDICE [Anonymous, 2022] considers a similar optimization problem to ours, but it deals with the offline IL (i.e. state-action stationary distribution matching), whereas we consider the offline IfO (e.g. state-transition stationary distribution matching). Accordingly, the optimization variable $\bar{d}(\cdot, \cdot)$ and the marginalization constraint (8) are added in our formulation.

Then, we consider the Lagrangian for the constrained optimization (6-8):

$$\max_{d, \bar{d}} \min_{\mu, \nu} - D_{KL}(\bar{d}||\bar{d}) - \alpha D_{KL}(d||\bar{d}) + \sum_{s} \nu(s)((1 - \gamma)p_0(s) + \gamma(T_\pi)(s) - (B_\pi)(s)) + \sum_{s, s'} \mu(s, s')(\bar{d}(s, s') - (T_\pi)(s, s')),$$

where $(B_\pi)(s) := \sum_{a} d(s, a)$ is a marginalization operator, and $(T_\pi)(s, s') := \sum_{\bar{s}, \bar{a}} T(\bar{s}|s, \bar{a})d(\bar{s}, \bar{a})$ is a transposed Bellman operator, and $(\bar{T}_\pi)(s, s') := \sum_{a} T(s'|s, a)d(s, a)$ is an operator to construct state-transition distribution from state-action distribution. The Bellman flow constraint (7) ensures $d(s, a)$ to be a valid state-action stationary distribution of some policy, where $d(s, a)$ can be interpreted as a normalized occupancy measure of $(s, a)$. The marginalization constraint (8) enforces $\bar{d}(s, s')$ to be the state-transition stationary distribution that is directly induced by $d(s, a)$. In essence, the constrained optimization problem (6-8) seeks the stationary distributions of an optimal policy, which best matches the state trajectories of the expert. Once we have computed the optimal solution $(d^\pi, \bar{d}^*)$, its corresponding optimal policy can also be obtained by $\pi^*(a|s) = \frac{d^\pi(s, a)}{\sum_{s'} d^\pi(s', a)}$.

Note that DemoDICE [Anonymous, 2022] considers a similar optimization problem to ours, but it deals with the offline IL (i.e. state-action stationary distribution matching), whereas we consider the offline IfO (e.g. state-transition stationary distribution matching). Accordingly, the optimization variable $\bar{d}(\cdot, \cdot)$ and the marginalization constraint (8) are added in our formulation.

Then, we consider the Lagrangian for the constrained optimization (6-8):

$$\max_{d, \bar{d}} \min_{\mu, \nu} - D_{KL}(\bar{d}||\bar{d}) - \alpha D_{KL}(d||\bar{d}) + \sum_{s} \nu(s)((1 - \gamma)p_0(s) + \gamma(T_\pi)(s) - (B_\pi)(s)) + \sum_{s, s'} \mu(s, s')(\bar{d}(s, s') - (T_\pi)(s, s')),$$
where $\nu(s) \in \mathbb{R}$ are the Lagrange multipliers for the Bellman flow constraints (7), and $\mu(s, s') \in \mathbb{R}$ are the the Lagrange multipliers for the marginalization constraint (8). Note that the Lagrangian (9) cannot be naively optimized in an offline manner since it requires evaluation of $T(s'| s, a)$ for $(s, a) \sim d$, which is not accessible in the offline IO setting. Therefore, we rearrange the terms in (9) to eliminate the direct dependence on $d$ and $d$, introducing new optimization variables $w$ and $\bar{w}$ that denote stationary distribution correction ratios for $(s, a)$ and $(s', s'')$, respectively:

$$-E_d \left[ \log \frac{d(s, s')}{d(s, s'')} \right] - \alpha E_d \left[ \log \frac{d(s, a)}{d(s, a')} \right] + \sum_{s, s'} \nu(s) \left( (1 - \gamma) p_0(s) + \gamma (T_d(s) - (B_d)(s)) \right)$$

$$+ \sum_{s, s'} \mu(s, s') \left( \bar{d}(s, s') - \bar{T}(d)(s, s') \right)$$

$$= E_d [\mu(s, s') - E_{(s, a) \sim d} [\mu(s, s')]]$$

$$= (1 - \gamma) E_{p_0} [\nu(s)] + E_d \left[ \mu(s, s') - \log \frac{d(s, s')}{d'(s, s''')} \right] + E_d \left[ \nu(s) \left( (1 - \gamma) \right) p_0(s) + E_d [\gamma (T(d)(s) - (B_d)(s))] \right]$$

$$+ E_d \left[ \mu(s, s') \left( \bar{d}(s, s') - \bar{T}(d)(s, s') \right) \right]$$

$$= (1 - \gamma) E_{p_0} [\nu(s)] + E_d \left[ \nu(s) \left( (1 - \gamma) \right) p_0(s) + \mu(s, s') - \log \frac{d(s, s')}{d'(s, s'')}} \right] + E_d \left[ \mu(s, s') \left( \bar{d}(s, s') - \bar{T}(d)(s, s') \right) \right]$$

$$= \mathbb{L}_{LD}(w, \bar{w}, \mu, \nu).$$

For the first equality (10), we use the following properties of transpose operators:

$$\sum_{s} \nu(s) (B_d)(s) = \sum_{s, a} d(s, a) (B_d)(s, a),$$

$$\sum_{s} \nu(s) (T_d)(s) = \sum_{s, a} d(s, a) (T_d)(s, a),$$

$$\sum_{s, s'} \mu(s, s') (T_d)(s, s') = \sum_{s, a} d(s, a) (T_d)(s, a),$$

where $(T_d)(s, a) = \sum_{s'} T(s'| s, a) \nu(s')$, $(B_d)(s, a) = \nu(s)$, and $(\bar{T}(d))(s, a) = \sum_{s'} T(s'| s, a) \mu(s, s')$, with assumption $\bar{d}(s, s') > 0$ and $d(s, a) > 0$ when $d(s, s') > 0$ and $d(s, a) > 0$, respectively. For the second equality, we introduce the log ratio $r(s, s') = \log \frac{d(s, s')}{d'(s, s'')}$, in order to make an expectation for $d'$ (instead of $\bar{d}$), which is expected to have broader coverage than $\bar{d}$ in general. This log ratio can be easily estimated using a pretrained discriminator for two datasets $D_E$ and $D_I$, which will be explained in detail in the following section.

In summary, LobsDICE solves the following maximin optimization:

$$\max_{w, \bar{w} \geq 0} \min_{\mu, \nu} \mathbb{L}_{LD}(w, \bar{w}, \mu, \nu).$$

The optimal solution $(w^*, \bar{w}^*)$ of (13) represents stationary distribution corrections of an optimal policy $\pi^*$: $w^*(s, s') = \frac{\bar{d}(s, s')}{d'(s, s')}$ and $\bar{w}^*(s, s') = \frac{\bar{d}(s, s')}{d'(s, s')}$. 

### 3.2 Log ratio estimation via a pretrained discriminator

To optimize (12), an estimate of the log ratio $r(s, s') = \log \frac{d(E(s, s'))}{d'(s, s'')}$. is required. The log ratio estimation is straightforward for tabular MDPs since we can use empirical distributions from the datasets to estimate $\bar{d}(s, s')$ and $d'(s, s')$. For continuous MDPs, we train a discriminator $c : S \times S \rightarrow [0, 1]$ by solving the following minimization problem [Goodfellow et al. 2014; Zhu et al. 2020].

$$c^* = \arg \min_{c : S \times S \rightarrow [0, 1]} \mathbb{E}_{q_E} [\log c(s, s')] + \mathbb{E}_{q_I} [\log (1 - c(s, s'))].$$

(14)
It is proven that the optimal discriminator \( c^* \) satisfies \( c^*(s, s') = \frac{d^{\circ}(s, s')}{d^{\circ}(s, s') + d^{\circ}(s', s')}. \) Thus, \( r \) can be estimated by using the optimal discriminator \( c^* \) as follows:

\[
    r(s, s') = -\log \left( \frac{1}{c^*(s, s')} - 1 \right). 
\]

(15)

### 3.3 Maximin to min: a closed-form solution

Since the optimization (6) is an instance of convex optimization, we can reorder the maximin operator (13) to the minimax operator [Boyd et al., 2004]:

\[
    \min \max_{w, \bar{w} \geq 0} L_{LD}(w, \bar{w}, \mu, \nu). 
\]

(16)

Then, exploiting the strict convexity of \( x \log x \), we can derive a closed-form solution to the inner maximization for \((w, \bar{w}) \) in (16).

**Proposition 3.1.** For any \((\mu, \nu)\), the closed-form solution to the inner maximization of (16), i.e. \((w_{\mu, \nu}, \bar{w}_{\mu}) = \arg \max_{w, \bar{w} \geq 0} L_{LD}(w, \bar{w}, \mu, \nu), \) is given by:

\[
    w_{\mu, \nu}(s, a) = \exp \left( \frac{1}{\alpha} c_{\mu, \nu}(s, a) - 1 \right), 
\]

(17)

\[
    \bar{w}_{\mu}(s, s') = \exp (r(s, s') + \mu(s, s') - 1). 
\]

(18)

Finally, we reduce the nested min-max optimization of (16) to a single minimization by plugging the closed-form solution \((w_{\mu, \nu}, \bar{w}_{\mu})\) into \( L_{LD}(w, \bar{w}, \mu, \nu) \):

\[
    \min_{\mu, \nu} L_{LD}(w_{\mu, \nu}, \bar{w}_{\mu}, \mu, \nu) 
\]

\[
    = (1 - \gamma) E_{s \sim p_\nu}[\nu(s)] + E_{(s, s') \sim d^{\circ}} \left[ \exp \left( r(s, s') + \mu(s, s') - 1 \right) \right] + \alpha E_{(s, a) \sim d^{\circ}} \left[ \exp \left( \frac{1}{\alpha} c_{\mu, \nu}(s, a) - 1 \right) \right]. 
\]

We can even show that \( L_{LD}(w_{\mu, \nu}, \bar{w}_{\mu}, \mu, \nu) \) is a convex function of \( \mu \) and \( \nu \).

**Proposition 3.2.** \( L_{LD}(w_{\mu, \nu}, \bar{w}_{\mu}, \mu, \nu) \) is convex with respect to \( \mu \) and \( \nu \).

In short, by operating in the space of stationary distributions, offline I/O can, in principle, be resolved by solving a single convex minimization problem. This is in contrast to the existing I/O algorithms, which typically involve either an adversarial training that optimizes the policy and the discriminator [Torabi et al., 2019, Yang et al., 2019, Kidambi et al., 2021] or a nested min-max optimization for the policy and the critic [Zhu et al., 2020].

### 3.4 Policy extraction

In the previous sections, we have derived an algorithm that essentially solves the state-transition distribution matching problem via convex minimization. However, what we end up getting is \((\mu^*, \nu^*)\), which is an optimal solution of (19), not the optimal policy \( \pi^* \) in its form. Therefore, the remaining question is how we can extract the policy from \((\mu^*, \nu^*)\).

First, the stationary distribution correction of the optimal policy (i.e. \( w_{\mu^*, \nu^*} \)) is estimated by exploiting the closed-form solution (17), where \( w_{\mu^*, \nu^*}(s, a) = \frac{d^{\circ}_{\mu^*(s, a)}}{d^{\circ}_{\mu^*(s, a)}} \). Then, for discrete MDPs, the optimal policy can be easily obtained by:

\[
    \pi^*(a|s) = \frac{d^{\dagger}(s, a) w_{\mu^*, \nu^*}(s, a)}{\sum_a d^{\dagger}(s, a) w_{\mu^*, \nu^*}(s, a)} 
\]

(20)

This method is not directly applicable to continuous MDPs due to the intractability of marginalization in the denominator. For continuous MDPs, we adopt weighted behavior cloning (BC):

\[
    \max_{\pi} E_{(s, a) \sim d^{\star}} \log \pi(a|s) = E_{(s, a) \sim d^{\star}} \left[ w_{\mu^*, \nu^*}(s, a) \log \pi(a|s) \right] 
\]

where we weight the samples in the offline dataset \( D^f \) by \( w_{\mu^*, \nu^*}(s, a) \) while performing BC.
3.5 Practical algorithm

In practice, we estimate $L_{LD}(w, \bar{w}, \mu, \nu)$ in (12) using only those samples from the dataset distribution $d^I$. We denote each sample $(s, a, s')$ as $x$ for brevity.

\[
\min_{\mu, \nu} \max_{w, \bar{w} \geq 0} \hat{L}_{LD}(w, \bar{w}, \mu, \nu) := (1 - \gamma)E_{s \sim p_0}[\nu(s)] + \mathbb{E}_{x \sim d^I} \left[ \bar{w}(s, s')(r(s, s') + \mu(s, s') - \log \bar{w}(s, s')) + w(s, a)(\hat{\epsilon}_{\mu, \nu}(s, a, s') - \alpha \log w(s, a)) \right]
\]

where $\hat{\epsilon}_{\mu, \nu}(s, a, s') := -\mu(s, s') + \gamma \nu(s') - \nu(s)$ is a single-sample estimate of $\epsilon_{\mu, \nu}(s, a)$. Note that this sample-based objective function $\hat{L}_{LD}(w, \bar{w}, \mu, \nu)$ is an unbiased estimator of $L_{LD}(w, \bar{w}, \mu, \nu)$ as long as every sample $x = (s, a, s')$ in $D^I$ was collected by interacting with the underlying MDP.

We then apply the non-parametric closed-form solution for each sample $x = (s, a, s')$ in $D^I$:

\[
\hat{w}_{\mu, \nu}(x) = \exp \left( \frac{1}{\alpha} \hat{\epsilon}_{\mu, \nu}(s, a, s') - 1 \right),
\]

\[
\hat{w}(x) = \exp(r(s, s') + \mu(s, s') - 1).
\]

which is given similar to (17) (18). Plugging this into (21) yields a sample-based objective function for minimization:

\[
\min_{\mu, \nu} \hat{L}_{LD}(\mu, \nu) = (1 - \gamma)E_{s \sim p_0}[\nu(s)] + \mathbb{E}_{x \sim d^I} \left[ \exp \left( r(s, s') + \mu(s, s') - 1 \right) + \alpha \exp \left( \frac{1}{\alpha} (-\mu(s, s') + \gamma \nu(s') - \nu(s)) - 1 \right) \right].
\]

Still, the variable $\mu \in \mathbb{R}^{S \times S}$ is much higher dimensional than $\nu \in \mathbb{R}^S$. So, it works as the main bottleneck for the overall optimization. Fortunately, we can further simplify (24) by eliminating its dependence on $\mu$ via exploiting an additional closed-form solution.

**Proposition 3.3.** For any $\nu$, the closed-form solution to the minimization (24) with respect to $\mu$, i.e. $\nu = \arg\min_{\mu} \hat{L}_{LD}(\mu, \nu)$, is

\[
\mu_{\nu}(s, s') = \frac{1}{1 + \alpha} \left( -\alpha r(s, s') + \gamma \nu(s') - \nu(s) \right).
\]

By putting $\mu_{\nu}$ into (24), we obtain the following minimization problem:

\[
\min_{\nu} \hat{L}_{LD}(\nu) = (1 - \gamma)E_{s \sim p_0}[\nu(s)] + (1 + \alpha)\mathbb{E}_{x \sim d^I} \left[ \exp \left( \frac{1}{1 + \alpha} \hat{A}_{\nu}(s, a, s') - 1 \right) \right],
\]

where $\hat{A}_{\nu}(s, a, s') := r(s, s') + \gamma \nu(s') - \nu(s)$. Optimizing (26) in its form is not yet practical, because of its inclusion of an $\exp(\cdot)$, which causes numerical instability and the explosion of gradient. To resolve this issue, we propose to use a numerically-stable alternative of (26).

**Proposition 3.4.** Let $\hat{L}_{FD}(\nu)$ be the function:

\[
\hat{L}_{FD}(\nu) = (1 - \gamma)E_{s \sim p_0}[\nu(s)] + (1 + \alpha)\log \mathbb{E}_{x \sim d^I} \left[ \exp\left( \frac{1}{1 + \alpha} \hat{A}_{\nu}(s, a, s') \right) \right].
\]

Then, $\min_{\nu} \hat{L}_{LD}(\nu) = \min_{\nu} \hat{L}_{FD}(\nu)$ holds. Also, $\hat{L}_{FD}(\nu)$ is convex with respect to $\nu$.

Finally, our practical algorithm minimizes $\hat{L}_{FD}(\nu)$, which no longer suffers from numerical instability. To see this, consider the gradient $\nabla_x \log \mathbb{E}_{x \sim p_\nu}[\exp(h(x))] = \mathbb{E}_{x \sim p_\nu}[\frac{\exp(h(x))}{\mathbb{E}_{x \sim p_\nu}[\exp(h(x))]}] \nabla_x h(x)$, where $\exp(\cdot)$ is normalized by softmax and thus does not incur exploding gradients. Although this property of $\hat{L}_{FD}(\nu)$ is desirable, it is not yet clear whether we can extract an optimal policy from its optimal solution $\tilde{\nu}^* = \arg\min_{\nu} \hat{L}_{FD}(\nu)$. Fortunately, the following proposition reveals that $\tilde{\nu}^*$ and $\nu^*$ differ only by a constant shift.

**Proposition 3.5.** Let $V_{LD}$ and $V_{FD}$ be the set of optimal solutions of $\arg\min_{\nu} \hat{L}_{LD}(\nu)$ and $\arg\min_{\nu} \hat{L}_{FD}(\nu)$, respectively. Then, $V_{FD} = \{\nu^* + C| \nu^* \in V_{LD}, C \in \mathbb{R}\}$ holds.

Proposition 3.5 implies that stationary distribution corrections of an optimal policy can also be obtained from $\tilde{\nu}^*$.

\[
\hat{w}_{\mu_{\nu^*}, \nu^*}(x) = \exp \left( \frac{1}{\alpha} (-\mu_{\nu^*}(s, s') + \gamma \nu^*(s') - \nu^*(s)) - 1 \right)
\]

\[
= \exp \left( \frac{1}{1 + \alpha} \hat{A}_{\nu^*}(s, a, s') - 1 \right) \quad \text{(by (25))}
\]

\[
\propto \exp \left( \frac{1}{1 + \alpha} \hat{A}_{\nu^*}(s, a, s') \right) =: \tilde{w}_{\nu^*}(x)
\]
LobsDICE: Offline Imitation Learning from Observation via Stationary Distribution Correction Estimation

**Policy extraction** We must take caution when using $\bar{w}_\nu(x)$ since it is an unnormalized density ratio, i.e. $\mathbb{E}_{x \sim d_t} [\bar{w}_\nu(x)] \neq 1$. Therefore, to extract a policy, we perform weighted BC using self-normalized importance sampling [Owen 2013]:

$$\max_{\pi} \frac{\mathbb{E}_{x \sim d_t} [\bar{w}_\nu(x) \log \pi(a|s)]}{\mathbb{E}_{x \sim d_t} [\bar{w}_\nu(x)]}$$

(29)

which completes the derivation of our practical algorithm. To sum up, our practical implementation of LobsDICE solves $\tilde{\nu}^* = \arg \min_\nu \mathcal{L}_{FD}(\nu)$ of (27) via gradient descent, and extracts a policy via self-normalized weighted BC of (29).

**4 Related Works**

**Imitation learning from observation (IFO)** Recent approaches for IFO are mostly on-policy [Liu et al. 2018, Torabi et al. 2019, Yang et al. 2019, Sun et al. 2019, Liu et al. 2019] algorithms and are not directly applicable to the offline IFO setting considered in this work. **Mobile** [Kidambi et al. 2021] is a model-based IFO algorithm, but it encourages uncertainty in the online exploration, which is not suitable for the offline setting. **BCO** [Torabi et al. 2018] uses an inverse dynamics model (IDM) to infer the missing expert actions and performs BC on the generated expert’s state-action dataset. In addition to common issues by vanilla BC, BCO is not guaranteed to recover the expert’s behavior in general. **OPOLO** [Zhu et al. 2020] is a principled off-policy IFO algorithm, but it solves a nested optimization and requires evaluation on out-of-distribution action values during training, which suffers from numerical instability in the offline setting.

**Stationary Distribution Correction Estimation (DICE)** DICE-family algorithms perform stationary distribution estimation, and many of them have been proposed for off-policy evaluation [Nachum et al. 2019a, Zhang et al. 2020a, b], [Yang et al. 2020a, Dai et al. 2020]. Other lines of work consider reinforcement learning [Nachum et al. 2019b, Lee et al. 2021], offline policy selection [Yang et al. 2020b]. **ValueDICE** [Kostrikov et al. 2020] and **OPOLO** [Zhu et al. 2020] derive off-policy IL and IFO objectives using DICE. However, they suffer from numerical instability in the offline setting due to the nested min-max optimization and out-of-distribution action evaluation. **DemoDICE** [Anonymous, 2022] is an offline IL algorithm that directly optimizes stationary distribution as ours and reduces to solving a convex minimization. Yet, it requires expert action labels and is not directly applicable to IFO.

**5 Experiments**

We now present the empirical evaluation of LobsDICE on both tabular and continuous MDPs.

**Baselines** We compare LobsDICE with four baseline methods: BC on imperfect demonstrations, BCO [Torabi et al. 2018], and OPOLO [Zhu et al. 2020]. Additionally, we designed a strong baseline DemoDICEfO, which extends the state-of-the-art offline IL algorithm, DemoDICE [Anonymous 2022]. DemoDICEfO trains an inverse dynamics model, uses it to fill the missing actions in the expert demonstrations, and then runs DemoDICE using the approximate expert demonstrations and the imperfect demonstrations.

**5.1 Random MDPs**

We first evaluate LobsDICE and baseline algorithms on randomly generated finite MDPs using a varying number of expert/imperfect trajectories and different degrees of environment stochasticity. We follow the experimental protocol in previous offline RL works [Laroche et al. 2019, Lee et al. 2020, 2021] but with additional control on the stochasticity of transition probabilities. We conduct repeated experiments for 1K runs. For each run, (1) a random MDP is generated, (2) expert trajectories and imperfect trajectories are collected, and (3) each offline IFO algorithm is tested on the collected offline dataset. We evaluate the performance of each algorithm by measuring the total variation distance between state-transition stationary distributions by the expert policy and the learned policy, i.e. $D_{TV}(d^\pi(s, s')||d^{\pi_\theta}(s, s'))$. Our tabular LobsDICE optimizes (19) on the empirical MDP model constructed from the action-labeled dataset $D^I$ while extracting the policy through (20).

**Random MDP generation** Each random MDP $M$ is generated with $|S| = 20$ and $|A| = 4$. The transition connectivity is given by 4 for each $(s, a)$. The specific probabilities are determined by $[T(s'|s, a)]_{i=1}^4 = (1 - \beta) X + \beta Y$, where $X \sim \text{Categorical}(\frac{1}{5}, \frac{1}{5}, \frac{1}{5}, \frac{1}{5}, \frac{1}{5})$, and $Y \sim \text{Dir}(1, 1, 1, 1)$. $\beta \in [0, 1]$ is the hyperparameter to control transition stochasticity ($\beta = 0$ for deterministic MDPs and $\beta = 1$ for highly stochastic MDPs). More details can be found in Appendix B.1.
LobsDICE: Offline Imitation Learning from Observation via Stationary Distribution Correction Estimation

Figure 1: Performance of tabular LobsDICE and baselines in randomly generated MDPs. The first row indicates near-deterministic dynamics and the last row indicates highly stochastic dynamics. As the level of stochasticity increases, baselines fall into suboptimal, even the number of state-only expert demonstrations and imperfect demonstrations increases, while LobsDICE goes to optimal. For each algorithm, we measure the performance using total variation between state-transition stationary distributions of expert and learned policy. We plot the mean and standard error of total variations $TV(\bar{d}^\pi (s, s'), \bar{d}^\pi_E (s, s'))$ over 1000 random seeds.

Offline dataset generation For each random MDP $M$, we generate state-only expert demonstrations by executing a (softmax) expert policy and state-action imperfect demonstrations by a uniform random policy. We perform experiments for a varying number of expert demonstrations $N_E \in \{10, 100, 1000, 10000\}$ and imperfect demonstrations $N_I \in \{1, 3, 10, 30, 100, 300, 1000, 3000, 10000\}$.

Results Figure 1 presents the results for random MDP experiments. The first row corresponds to the case when $\beta = 0.01$ (nearly deterministic MDP). In this situation, the inverse dynamics disagreement will be close to zero, i.e. $D_{KL}(d^{\pi_1}(a|s, s')||d^{\pi_2}(a|s, s')) \approx 0$ for any two policies $\pi_1$ and $\pi_2$. Thus, the algorithms whose performance directly relies on the IDM’s accuracy (i.e. BCO and DemoDICEfO) even perform very well since it is very easy to learn a perfect inverse dynamics model in this scenario. OPOLO’s upper bound gap will also be close to zero, thus OPOLO directly minimizes the divergence of state-transition distributions. As a result, there is no performance gap among different algorithms, except for BCO whose performance is determined by the quality of imperfect demonstrations. Also, the performance of all algorithms (except for BCO) improves as more data is given, which is natural.

The second row and the third row in Figure 1 presents the result when $\beta = 0.1$ (weakly stochastic MDP) and $\beta = 1.0$ (highly stochastic MDP) respectively. In the stochastic MDPs, the IDM trained by the imperfect demonstrations fails to predict the expert’s actions accurately (more challenging as $\beta$ gets larger). As a consequence, BCO gets suboptimal and its suboptimality cannot be improved even if more data is given. DemoDICEfO performs better than BCO since it additionally considers the distributional shift by considering state distribution matching. However, it is still suboptimal due to its nature that directly depends on the quality of inferred action by the learned IDM. OPOLO does not rely on the learned IDM and outperforms both BCO and DemoDICEfO. Still, OPOLO can be inherently suboptimal due to its nature of optimizing the upper bound unless the underlying transition dynamics are deterministic and injective. This upper bound gap is not controllable by the algorithm and implies that OPOLO can be suboptimal even given an infinite amount of data with sufficient dataset coverage, which can be seen in the rightmost figures. Finally, our tabular LobsDICE using (19) essentially solves the exact state-transition distribution matching problem (as $\alpha \to 0$). LobsDICE is the only offline IFO algorithm that can asymptotically recovers the expert’s state demonstrations even though the underlying MDP is stochastic.
We present the empirical performance of LobsDICE and baselines on MuJoCo [Todorov et al., 2012] continuous control tasks. We build state-only expert demonstrations using 5 trajectories from expert-v2. For each task \((X, Y, Z)\) we construct imperfect demonstrations using \(X\) trajectories from expert-v2, \(Y\) trajectories from medium-v2, and \(Z\) trajectories from random-v2, respectively. Among all tasks, LobsDICE shows promising performance while BC, BCO, and OPOLO fail to achieve competitive performance in many tasks. We plot the mean and the standard errors (shaded area) of the normalized scores over five random seeds.

5.2 Continuous control tasks (Gym-MuJoCo)

We present the empirical performance of LobsDICE and baselines on MuJoCo [Todorov et al., 2012] continuous control tasks using the OpenAI Gym [Brockman et al., 2016] framework. We utilize the D4RL dataset Fu et al. [2020] for offline IFO tasks in four MuJoCo environments: Hopper, Walker2d, HalfCheetah, and Ant. Implementation details for LobsDICE and baseline algorithms are provided in Appendix B.2.

**Task setup** For each MuJoCo environment, we employ expert-v2, medium-v2, and random-v2 from D4RL datasets [Fu et al. 2020]. Across all environments, we consider three tasks, each of which uses different imperfect demonstrations while sharing the same expert observations. First, we construct the state-only expert demonstration \(D^E = \{(s, s')\}_{i=1}^{N_E}\) using the first 5 trajectories in expert-v2. Then, we use trajectories in expert-v2, medium-v2, and random-v2 to construct imperfect demonstrations with different ratios. We denote the composition of imperfect demonstrations as \((X, Y, Z)\) in the title of each subplot in Figure 2, which means that the imperfect dataset consists of \(X\) trajectories from expert-v2, \(Y\) trajectories from medium-v2, and \(Z\) trajectories from random-v2.

**Evaluation metric** For each environment, the normalized score is measured by \(100 \times \frac{\text{score}_{\text{random}} - \text{score}_{\text{expert}}}{\text{score}_{\text{random}} - \text{score}_{\text{expert}}}\), where the \text{score}_{\text{expert}} and \text{score}_{\text{random}} are average returns of trajectories in expert-v2 and random-v2, respectively. We evaluate average and standard error of normalized score over five random seeds.

**Results** Figure 2 summarizes that the empirical results of LobsDICE and baselines on continuous control tasks. We first remark that LobsDICE (blue) significantly outperforms OPOLO (green) in all tasks across all domains, although both LobsDICE and OPOLO are DICE-based algorithms. The failure of OPOLO comes from its numerical instability due to its dependence on nested optimization and using out-of-distribution action values during training. In contrast, LobsDICE solves a single minimization (27) while it does not involve any evaluation on out-of-distribution actions, thus it is optimized stably. Naive BC on imperfect demonstrations (black) is inherently suboptimal since it does not consider distribution-matching with the expert’s observation at all. While BCO (orange) exploits the expert’s demonstrations with the inferred actions by the IDM, its policy learning is done only on the very scarce expert dataset (i.e. 5 trajectories), which makes the algorithm perform not well. DemoDICEfO (red) exploits both expert demonstrations (where the missing actions are filled with the IDM) and the abundant imperfect demonstrations, but its performance is affected by...
the quality of the learned IDM. We empirically observe that the IDM error (on the true expert data) increases as the proportion of the non-expert data (i.e. medium-v2 and random-v2) increases, resulting in performance degradation of DemoDICEfO. Finally, LobsDICE is the only algorithm that was able to fully recover the expert’s performance regardless of the increase of non-expert data in the imperfect demonstrations, significantly outperforming baseline algorithms. This result highlights the effectiveness of our method that solves a state-transition stationary matching problem in a principled manner.

6 Conclusion

We presented LobsDICE, an algorithm for offline imitation learning from observations (IfO), which successfully achieves state-of-the-art performance on various tabular and continuous tasks. We formulated the offline IfO as a state-transition stationary distribution matching problem, where the stationary distribution is optimized via convex minimization. Experimental results demonstrate that LobsDICE achieves promising performance in both tabular and continuous offline IfO tasks.

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A Theoretical Analysis

A.1 Closed-form solutions

**Proposition 3.1.** For any \((\mu, \nu)\), the closed-form solution to the inner maximization of \((16)\), i.e. \( (w_{\mu, \nu}, \bar{w}_{\mu}) = \arg \max_{w, \bar{w} \geq 0} \mathcal{L}_{LD}(w, \bar{w}, \mu, \nu) \), is given by:

\[
\begin{align*}
    w_{\mu, \nu}(s, a) &= \exp \left( \frac{1}{\alpha} e_{\mu, \nu}(s, a) - 1 \right), \\
    \bar{w}_{\mu}(s, s') &= \exp (r(s, s') + \mu(s, s') - 1).
\end{align*}
\]

**Proof.** For \((s, a)\) with \(d^l(s, a) > 0\), we can compute the derivative \( \frac{\partial \mathcal{L}_{LD}}{\partial w(s, a)} \) of \( \mathcal{L}_{LD} \) w.r.t. \( w(s, a) \) as follows:

\[
\frac{\partial \mathcal{L}_{LD}}{\partial w(s, a)} = \sum_{s'} d^l(s, a, s')(e_{\mu, \nu}(s, a, s') - \alpha \log w(s, a) - \alpha) = 0
\]

\[
\Leftrightarrow \sum_{s'} T(s'|s, a)(e_{\mu, \nu}(s, a, s') - \alpha \log w(s, a) - \alpha) = 0
\]

\[
\Leftrightarrow \sum_{s'} T(s'|s, a)(e_{\mu, \nu}(s, a, s') - \alpha) = \alpha \log w(s, a)
\]

\[
\Leftrightarrow w(s, a) = \exp \left( \frac{1}{\alpha} \mathbb{E}_{s'\sim T|\cdot(s, a)}[e_{\mu, \nu}(s, a, s')] - 1 \right).
\]

Similar to the aforementioned derivation, when \(\bar{d}^l(s, s') > 0\), we can derive the derivative of \( \mathcal{L}_{LD} \) w.r.t. \( \bar{w}(s, s') \):

\[
\frac{\partial \mathcal{L}_{LD}}{\partial \bar{w}(s, s')} = \bar{d}^l(s, s')(\mu(s, s') - \log \bar{w}(s, s') + r(s, s') - 1) = 0
\]

\[
\Leftrightarrow (\mu(s, s') - \log \bar{w}(s, s') + r(s, s') - 1) = 0
\]

\[
\Leftrightarrow \log \bar{w}(s, s') = \mu(s, s') + r(s, s') - 1
\]

\[
\Leftrightarrow \bar{w}(s, s') = \exp (\mu(s, s') + r(s, s') - 1).
\]

**Proposition 3.2.** \( \mathcal{L}_{LD}(w_{\mu, \nu}, \bar{w}_{\mu}, \mu, \nu) \) is convex with respect to \( \mu \) and \( \nu \).

**Proof.** Using the fact that \( \exp(\cdot) \) is a convex function, we can easily prove the convexity of \( \mathcal{L}_{LD}(w_{\mu, \nu}, \bar{w}_{\mu, \nu}, \mu, \nu) \). For brevity, let \( \mathcal{L}_{LD}(\mu, \nu) := \mathcal{L}_{LD}(w_{\mu, \nu}, \bar{w}_{\mu, \nu}, \mu, \nu) \). Then, for given \((\mu_1, \nu_1), (\mu_2, \nu_2)\) and \(t \in [0, 1]\),

\[
\begin{align*}
\mathcal{L}_{LD}(t\mu_1 + (1-t)\mu_2, t\nu_1 + (1-t)\nu_2) &\leq (1-t)\mathbb{E}_{p_0} [\nu_1(s) + (1-t)\nu_2(s)] + t\mathbb{E}_{d^l} [\exp (r(s, s') + t\mu_1(s, s') + (1-t)\mu_2(s, s') - 1)] \\
&\quad + \alpha t \mathbb{E}_{d^l} \left[ \exp \left( \frac{1}{\alpha} \mathbb{E}_{\nu'} \left[ -t\mu_1(s, s') - (1-t)\mu_2(s, s') + \gamma \nu_1(s') + (1-t)\nu_2(s') - tv_1(s) - (1-t)\nu_2(s) \right] - 1 \right] \right] \\
&\leq (1-t)\mathbb{E}_{p_0} [\nu_1(s)] + (1-t)(1-t)\mathbb{E}_{p_0} [\nu_2(s)] + t\mathbb{E}_{d^l} [\exp (r(s, s') + t\mu_1(s, s') + (1-t)\mu_2(s, s') - 1)] \\
&\quad + \alpha t \mathbb{E}_{d^l} \left[ \exp \left( \frac{1}{\alpha} \mathbb{E}_{\nu'} \left[ -t\mu_1(s, s') - (1-t)\mu_2(s, s') + \gamma \nu_1(s') + (1-t)\nu_2(s') - tv_1(s) - (1-t)\nu_2(s) \right] - 1 \right] \right].
\end{align*}
\]
Then, the dual problem of (30) is given by

\begin{align*}
\min_{\mu, \nu} \mathcal{L}_{LD}(\mu, \nu) := (1 - \gamma)\mathbb{E}_{p_0}[\nu_1(s)] + (1 - t)(1 - \gamma)\mathbb{E}_{p_0}[\nu_2(s)] \\
+ \mathbb{E}_{q'_t}[\exp(r(s, s') + \mu_1(s, s') - 1)] + (1 - t)\mathbb{E}_{q'_t}[\exp(r(s, s') + \mu_2(s, s') - 1)] \\
+ \mathbb{E}_{q'_t}[\exp(\frac{1}{\alpha}E_{\nu'}[\mu_1(s, s') + \gamma \nu_1(s') - \nu_1(s)] - 1)] \\
+ (1 - t)\alpha\mathbb{E}_{q'_t}[\exp(\frac{1}{\alpha}E_{\nu'}[- \mu_2(s, s') + \gamma \nu_2(s') - \nu_2(s)] - 1)] \\
= t\mathcal{L}_{LD}(\mu_1, \nu_1) + (1 - t)\mathcal{L}_{LD}(\mu_2, \nu_2).
\end{align*}

For the inequalities in the above formulation, we use the fact that \( \mathbb{E}_{q'_t}[\exp(\cdot)] \) is a instance of convex functions.

**Proposition 3.3.** For any \( \nu \), the closed-form solution to the minimization (24) with respect to \( \mu \), i.e. \( \mu_\nu = \arg \min_\mu \mathcal{L}_{LD}(\mu, \nu) \), is

\[ \mu_\nu(s, s') = \frac{1}{1 + \alpha}(- \alpha r(s, s') + \gamma \nu(s') - \nu(s)). \]  

**Proof.** For simplicity, let \( \hat{\mathcal{L}}_{LD}(\mu, \nu) := \hat{\mathcal{L}}_{LD}(w_{\mu, \nu}, \bar{w}_{\mu, \nu}, \mu, \nu) \) and \( x := (s, a, s') \). Then,

\[
\frac{\partial \hat{\mathcal{L}}_{LD}}{\partial \mu(x)} = d_1'(x) \left[ \exp(r(x) + \mu(x) - 1) - \exp \left( \frac{1}{\alpha}(- \mu(x) + \gamma \nu(s') - \nu(s)) - 1 \right) \right] = 0
\]

\[
\Leftrightarrow \exp(r(x) + \mu(x) - 1) = \exp \left( \frac{1}{\alpha}(- \mu(x) + \gamma \nu(s') - \nu(s)) - 1 \right)
\]

\[
\Leftrightarrow \alpha(r(x) + \mu(x)) = - \mu(x) + \gamma \nu(s') - \nu(s)
\]

\[
\Leftrightarrow (1 + \alpha)\mu(x) = - \alpha r(x) + \gamma \nu(s') - \nu(s)
\]

\[
\Leftrightarrow \mu(x) = \frac{1}{1 + \alpha}(- \alpha r(x) + \gamma \nu(s') - \nu(s))
\]

**Proof.** Using the fact that log-sum-exp is convex, we can prove the convexity in a similar way to prove Proposition 3.2.

**A.2 Fenchel dual formulation**

Let

\[ \delta_C(x) := \begin{cases} 0 & x \in C \\ \infty & \text{otherwise} \end{cases} \]

Then we can provide following proposition:

**Proposition A.1.** We can rewrite the optimization problem (26) as

\[
\max_{d \geq 0} - \delta_{(1 - \gamma)p_0}(- (\gamma T - B) d) - D_{KL}(\tilde{T} \| d) - \alpha D_{KL}(d_1 \| d_1').
\]

Then, the dual problem of (30) is given by

\[
\min_{\mu, \nu} \mathcal{L}_{FD}(\mu, \nu) := (1 - \gamma)\mathbb{E}_{p_0}[\nu_1(s)] + \log \mathbb{E}_{q'_t} \left[ \exp \left( r(s, s') + \mu(s, s') \right) \right] + \alpha \log \mathbb{E}_{q'_t} \left[ \exp \left( \frac{1}{\alpha}e_{\mu, \nu}(s, a) \right) \right].
\]

**Proof.** We first define following three functions

\[ f(\cdot) := \delta_{(1 - \gamma)p_0}(\cdot), \]

\[ g(\cdot ; r) := (\cdot ; - r) + D_{KL}(\| \tilde{T} \| d), \]

\[ h(\cdot ; \mu) := (\cdot ; \tilde{T} \mu) + \alpha D_{KL}(\| d_1 \| d_1'), \]

and conjugate functions,

\[ f_*(\cdot) := (1 - \gamma)\mathbb{E}_{p_0}[\cdot], \]

\[ g_*(\cdot ; r) := \log \mathbb{E}_{(s, s') \sim \tilde{T} \| d} \left[ \exp(\cdot + r(s, s')) \right], \]

\[ h_*(\cdot ; \mu) := \alpha \log \mathbb{E}_{(s, a) \sim d_1} \left[ \exp \left( \cdot - (\tilde{T} \mu)(s, a) \right) \right]. \]
Then, the dual formulation of the primal (30) can be derived as follows:

$$\max_{d \geq 0} - \delta(1-\gamma)p_0 \left( - (\gamma T - B) d \right) - \mathbb{E}_{(s,a) \sim d, s' \sim T^d(s,a)} \left[ \log \frac{(T_d d)(s, s')}{dE(s, s')} \right] - \alpha D_{KL}(d||d^t)$$

$$= \max_{d \geq 0} - f\left(- (\gamma T - B) d\right) + \mathbb{E}_{(s,a) \sim d, s' \sim T^d(s,a)} \left[ - \log \frac{(T_d d)(s, s')}{(T_d d)^t(s, s')} + \log \frac{dE(s, s')}{(T_d d)^t(s, s')} \right] - \alpha D_{KL}(d||d^t)$$

$$= \max_{d \geq 0} - f\left(- (\gamma T - B) d\right) - g(T_d d) - \alpha D_{KL}(d||d^t)$$

Proof.

Here, we can reorder the maximin to minimax and therefore, we can derive the

$$\min_{\nu, \mu} \max_{d \geq 0} \left\{ \max_{\nu} (- (\gamma T - B) d, \nu) - f_\nu(\nu) \right\} + g_\nu(\mu; r) - h(d; \mu)$$

$$= \min_{\nu, \mu} \left\{ \max_{d \geq 0} (\gamma T - B) d, \nu + f_\nu(\nu) + g_\nu(\mu; r) - h(d; \mu) \right\}$$

$$= \min_{\nu, \mu} L_{FD}(d, \nu) + f_\nu(\nu) + g_\nu(\mu; r) - h(d; \mu) := L_{FD}(d, \nu)$$

We can rewrite the last term as

$$\min_{\nu, \mu} \left( 1 - \gamma \right) \mathbb{E}_{s \sim p_0} [\nu(s)] + \log \mathbb{E}_{(s,a) \sim d^t, s' \sim T^d(s,a)} \left[ \exp(\mu(s, s') + r(s, s')) \right] + \alpha \log \mathbb{E}_{(s,a) \sim d^t} \left[ \exp \left( \frac{1}{\alpha} \varepsilon_{\mu, \nu}(s, a) \right) \right].$$

Lemma A.2. For given function $\mu : S \times S \rightarrow \mathbb{R}$ and $\nu : S \rightarrow \mathbb{R}$,

$$L_{FD}(\mu, \nu) = L_{FD}(\mu + C, \nu + C') \ \forall C, C' \in \mathbb{R}.$$ 

Proof.

$$L_{FD}(\mu + C, \nu + C') = (1 - \gamma) \mathbb{E}_{s \sim p_0} [\nu(s) + C'] + \log \mathbb{E}_{(s,a,s') \sim d^t} \left[ \exp(\mu(s, s') + C + r(s, s')) \right]$$

$$+ \alpha \log \mathbb{E}_{(s,a) \sim d^t} \left[ \exp \left( \frac{E_{s' \sim T^d(s,a)}[\gamma \nu(s') + C' - \mu(s, s') - C] - \nu(s)}{\alpha} \right) \right]$$

$$= (1 - \gamma) \mathbb{E}_{s \sim p_0} [\nu(s)] + (1 - \gamma) C' + \log \mathbb{E}_{(s,a,s') \sim d^t} \left[ \exp(\mu(s, s') + r(s, s')) \right] + C$$

$$+ \alpha \log \mathbb{E}_{(s,a) \sim d^t} \left[ \exp \left( \frac{E_{s' \sim T^d(s,a)}[\gamma \nu(s') - \mu(s, s')] - \nu(s)}{\alpha} \right) \right] + (\gamma C' - C - C')$$

$$= (1 - \gamma) \mathbb{E}_{s \sim p_0} [\nu(s)] + \log \mathbb{E}_{(s,a,s') \sim d^t} \left[ \exp(\mu(s, s') + r(s, s')) \right]$$

$$+ \alpha \log \mathbb{E}_{(s,a) \sim d^t} \left[ \exp \left( \frac{E_{s' \sim T^d(s,a)}[\gamma \nu(s') - \mu(s, s')] - \nu(s)}{\alpha} \right) \right]$$

$$= L_{FD}(\mu, \nu).$$
Finally, we can show that the relation between \( \text{arg min}_{\mu, \nu} \mathcal{L}_{LD} \) and \( \text{arg min}_{\mu, \nu} \mathcal{L}_{FD} \):

**Proposition A.3.** Let \( V_{LD} \) and \( V_{FD} \) be the set of optimal solutions \((\mu^*, \nu^*)\) of \( \text{arg min}_{\mu, \nu} \mathcal{L}_{LD}(w_{\nu, \mu}, \bar{w}_{\nu, \mu}, \mu, \nu) \) and \( \text{arg min}_{\mu, \nu} \mathcal{L}_{FD}(\mu, \nu) \), respectively. Then,

\[
V_{FD} = \{(\mu^* + C, \nu^* + C') | (\mu^*, \nu^*) \in V_{LD}, C \in \mathbb{R}, C' \in \mathbb{R}\}
\]

holds.

**Proof.** For brevity, we will denote \( \mathcal{L}_{LD}(w_{\nu, \mu}, \bar{w}_{\nu, \mu}, \mu, \nu) \) as \( \mathcal{L}_{LD}(\mu, \nu) \).

\((\subseteq)\) For given \((\hat{\mu}^*, \hat{\nu}^*) \in \text{arg min}_{\mu, \nu} \mathcal{L}_{FD}(\mu, \nu)\), we define two constants as follows:

\[
C = -\log \mathbb{E}_{(s,a,s') \sim d^{\mu}}[\exp(\hat{\mu}^*(s, s') + r(s, s') - 1)],
\]

\[
C' = \frac{\alpha}{1 - \gamma} \log \mathbb{E}_{(s,a) \sim d^{\mu}}\left[ \exp\left( \frac{\mathbb{E}_{s' \sim T(\cdot | s,a)}[\gamma \hat{\nu}^*(s') - \hat{\mu}^*(s, s') - \hat{\nu}^*(s')]}{\alpha} - 1 \right) \right] - \frac{C}{1 - \gamma}.
\]

Then, from the following two equations:

\[
\mathcal{L}_{LD}(\hat{\mu}^* + C, \hat{\nu}^* + C') = (1 - \gamma) \mathbb{E}_{s \sim \nu^*}[\hat{\nu}^*(s) + C'] + \mathbb{E}_{(s,a,s') \sim d^{\mu}}[\exp(\hat{\mu}^*(s, s') + C + r(s, s') - 1)]
\]

\[
+ \alpha \mathbb{E}_{(s,a) \sim d^{\mu}}\left[ \exp\left( \frac{\mathbb{E}_{s' \sim T(\cdot | s,a)}[\gamma \hat{\nu}^*(s') - \hat{\mu}^*(s, s') - \hat{\nu}^*(s')]}{\alpha} - 1 \right) \right] = (1 - \gamma) \mathbb{E}_{s \sim \nu^*}[\hat{\nu}^*(s)] + (1 - \gamma) C' + \mathbb{E}_{(s,a,s') \sim d^{\mu}}[\exp(\hat{\mu}^*(s, s') + r(s, s') - 1)] \exp(C')
\]

\[
+ \alpha \mathbb{E}_{(s,a) \sim d^{\mu}}\left[ \exp\left( \frac{\mathbb{E}_{s' \sim T(\cdot | s,a)}[\gamma \hat{\nu}^*(s') - \hat{\mu}^*(s, s') - \hat{\nu}^*(s')]}{\alpha} - 1 \right) \right] \exp\left( \frac{(1 - \gamma) C'}{\alpha} \right)
\]

and

\[
\mathcal{L}_{FD}(\hat{\mu}^* + C, \hat{\nu}^* + C') = (1 - \gamma) \mathbb{E}_{s \sim \nu^*}[\hat{\nu}^*(s) + C'] + \log \mathbb{E}_{(s,a,s') \sim d^{\mu}}[\exp(\hat{\mu}^*(s, s') + C + r(s, s'))]
\]

\[
+ \alpha \log \mathbb{E}_{(s,a) \sim d^{\mu}}\left[ \exp\left( \frac{\mathbb{E}_{s' \sim T(\cdot | s,a)}[\gamma \hat{\nu}^*(s') - \hat{\mu}^*(s, s') - \hat{\nu}^*(s')]}{\alpha} - 1 \right) \right] = (1 - \gamma) \mathbb{E}_{s \sim \nu^*}[\hat{\nu}^*(s)] + (1 - \gamma) C' + \mathbb{E}_{(s,a,s') \sim d^{\mu}}[\exp(\hat{\mu}^*(s, s') + r(s, s'))] + C
\]

\[
+ \alpha \log \mathbb{E}_{(s,a) \sim d^{\mu}}\left[ \exp\left( \frac{\mathbb{E}_{s' \sim T(\cdot | s,a)}[\gamma \hat{\nu}^*(s') - \hat{\mu}^*(s, s') - \hat{\nu}^*(s')]}{\alpha} - 1 \right) \right] \exp(C' - C)
\]

we can conclude that

\[
\mathcal{L}_{LD}(\hat{\mu}^* + C, \hat{\nu}^* + C') = \mathcal{L}_{FD}(\hat{\mu}^* + C, \hat{\nu}^* + C') = \mathcal{L}_{FD}(\hat{\mu}^*, \hat{\nu}^*) = \min_{\mu, \nu} \mathcal{L}_{FD}(\mu, \nu) = \min_{\hat{\mu}, \hat{\nu}} \mathcal{L}_{LD}(\hat{\mu}, \hat{\nu}).
\]

It means \((\hat{\mu}^* + C, \hat{\nu}^* + C') \in \text{arg min}_{\mu, \nu} \mathcal{L}_{LD}(\mu, \nu)\) and thus,

\[
(\hat{\mu}^*, \hat{\nu}^*) \in \{(\mu + C, \nu + C') | (\mu, \nu) \in \text{arg min}_{\mu, \nu} \mathcal{L}_{LD}(\mu, \nu), C \in \mathbb{R}, C' \in \mathbb{R}\},
\]

i.e.,

\[
\text{arg min}_{\mu, \nu} \mathcal{L}_{FD}(\mu, \nu) \subseteq \{(\mu + C, \nu + C') | (\mu, \nu) \in \text{arg min}_{\mu, \nu} \mathcal{L}_{LD}(\mu, \nu), C \in \mathbb{R}, C' \in \mathbb{R}\}
\]

\((\supseteq)\) Let \((\mu^*, \nu^*) \in \text{arg min}_{\mu, \nu} \mathcal{L}_{LD}(\mu, \nu)\). Then,

\[
\frac{d^*(s, a)}{d^{\hat{\mu}}(s, a)} = w_{\mu^*, \nu^*}(s, a) = \exp\left( \frac{\mathbb{E}_{s' \sim T(\cdot | s,a)}[\gamma \nu(s') - \mu(s, s')]}{\alpha} - 1 \right),
\]

\[
\frac{d^*(s, s')}{d^{\hat{\nu}}(s, s')} = \hat{w}_{\mu^*, \nu^*}(s, s') = \exp(r(s, s') + \mu^*(s, s') - 1).
\]
Let $C \in \mathbb{R}$ and $C' \in \mathbb{R}$. Then, we derive the following equation:

$$\mathcal{L}_{FD}(\mu^* + C, \nu^* + C') = \mathcal{L}_{FD}(\mu^*, \nu^*)$$

$$= (1 - \gamma)E_{s \sim p_0}[\nu^*(s)] + \log E_{{(s,a,s')} \sim d_t}[\exp(\mu^*(s, s') + r(s, s'))]$$

$$+ \alpha \log E_{(s,a) \sim d_t}[\exp \left( \frac{E_{s' \sim T(s, a)}[\gamma \nu^*(s') - \mu^*(s, s')] - \nu^*(s)}{\alpha} \right)]$$

$$= (1 - \gamma)E_{s \sim p_0}[\nu^*(s)] + \log E_{{(s,a,s')} \sim d_t}[\tilde{w}_{\mu^*, \nu^*}(s, s') \exp(1)] + \alpha E_{{(s,a) \sim d_t}}[w(s, a) \exp(1)]$$

$$= (1 - \gamma)E_{s \sim p_0}[\nu^*(s)] + \log E_{{(s,a,s')} \sim d_t} \left[ \exp(1) + \alpha \mathbb{E}_{(s,a) \sim d_t} \left[ \exp \left( \frac{\gamma \nu^* - \mu^*}{\alpha} \right) \right] \right]$$

Here, we apply Lemma A.4 to derive the first equality. Because

$$\min_{\mu, \nu} \mathcal{L}_{FD}(\mu, \nu) = \min_{\mu, \nu} \mathcal{L}_{LD}(\mu, \nu)$$

$$= \mathcal{L}_{LD}(\mu^*, \nu^*)$$

$$= (1 - \gamma)E_{s \sim p_0}[\nu^*(s)] + \mathbb{E}_{(s,a) \sim d_t} \left[ \exp \left( \frac{E_{s' \sim T(s, a)}[\gamma \nu^* - \mu^*] - \nu^*}{\alpha} \right) \right]$$

$$= (1 - \gamma)E_{s \sim p_0}[\nu^*(s)] + \mathbb{E}_{(s,a,s')} \left[ \tilde{w}_{\mu^*, \nu^*}(s, s') + \alpha \mathbb{E}_{(s,a) \sim d_t} \left[ \exp \left( \frac{\gamma \nu^* - \mu^*}{\alpha} \right) \right] \right]$$

$$= (1 - \gamma)E_{s \sim p_0}[\nu^*(s)] + \mathbb{E}_{(s,a,s')} \left[ 1 + \alpha \mathbb{E}_{(s,a) \sim d_t} \right]$$

we can conclude that $\mathcal{L}_{FD}(\mu^* + C, \nu^* + C') = \min_{\mu, \nu} \mathcal{L}_{FD}(\mu, \nu)$, i.e., $(\mu^* + C, \nu^* + C') \in \arg \min_{\mu, \nu} \mathcal{L}_{FD}(\mu, \nu)$. It means,

$$\arg \min_{\mu, \nu} \mathcal{L}_{FD}(\mu, \nu) \supseteq \{ \mu + C, \nu + C' \} \forall C, C' \in \mathbb{R}$$

Remark that the proposed objective (31) is stable and the aforementioned proposition allows us to use self-normalized weighted importance sampling to extract policy. However, as we discussed in Section 3.3 using $\mu$ is main bottleneck for the overall optimization and optimizing $\hat{\mathcal{L}}_{FD}(\nu)$ shows better performance in practice.

### A.3 Surrogate Objective

We first show a property of $\hat{\mathcal{L}}_{FD}$ that will be used to prove propositions:

**Lemma A.4.** For given function $\nu : S \rightarrow \mathbb{R}$,

$$\mathcal{L}_{FD}(\nu) = \hat{\mathcal{L}}_{FD}(\nu + C) \quad \forall C \in \mathbb{R}.$$

**Proof.**

$$\hat{\mathcal{L}}_{FD}(\nu + C)$$

$$= (1 - \gamma)E_{s \sim p_0}[\nu(s) + C] + (1 + \alpha) \log E_{{(s,a,s')} \sim d_t} \left[ \exp \left( \frac{1}{1 + \alpha} \hat{A}_{\nu + C}(s, a, s') \right) \right]$$

$$= (1 - \gamma)E_{s \sim p_0}[\nu(s) + C] + (1 + \alpha) \log E_{{(s,a,s')} \sim d_t} \left[ \exp \left( \frac{1}{1 + \alpha} \left( r(s, s') + \gamma (\nu(s') + C) - (\nu(s) + C) \right) \right) \right]$$

$$= (1 - \gamma)E_{s \sim p_0}[\nu(s) + C] + (1 + \alpha) \log E_{{(s,a,s')} \sim d_t} \left[ \exp \left( \frac{1}{1 + \alpha} \left( r(s, s') + \gamma \nu(s') - \nu(s) \right) \right) \exp \left( \frac{(\gamma - 1)C}{1 + \alpha} \right) \right]$$

$$= (1 - \gamma)E_{s \sim p_0}[\nu(s) + C] + (1 + \alpha) \log E_{{(s,a,s')} \sim d_t} \left[ \exp \left( \frac{1}{1 + \alpha} \left( r(s, s') + \gamma \nu(s') - \nu(s) \right) \right) \right] + (\gamma - 1)C$$

$$= (1 - \gamma)E_{s \sim p_0}[\nu(s)] + (1 + \alpha) \log E_{{(s,a,s')} \sim d_t} \left[ \exp \left( \frac{1}{1 + \alpha} \left( r(s, s') + \gamma \nu(s') - \nu(s) \right) \right) \right]$$

$$= \hat{\mathcal{L}}_{FD}(\nu).$$
Proposition 3.4. Let \( \hat{L}_{FD}(\nu) \) be the function:
\[
\hat{L}_{FD}(\nu) = (1 - \gamma)E_{s \sim p_{\nu}}[\nu^*(s) + C] + (1 + \alpha) \log E_{(s,a,s') \sim d^t}[\exp(\frac{1}{1 + \alpha} \hat{A}_\nu(s, a, s') - 1)]
\]
Then, \( \min_\nu \hat{L}_{LD}(\nu) = \min_\nu \hat{L}_{FD}(\nu) \) holds. Also, \( \hat{L}_{FD}(\nu) \) is convex with respect to \( \nu \).

Proof. First of all, from the fact that \( \log(x + 1) \leq x \) for all \( x > -1 \), we can easily conclude that \( \min_\nu \hat{L}_{FD}(\nu) \leq \min_\nu \hat{L}_{LD}(\nu) \). Now, we will show that \( \min_\nu \hat{L}_{LD}(\nu) \leq \min_\nu \hat{L}_{FD}(\nu) \). For given \( \nu^* \in \arg \min_\nu \hat{L}_{FD}(\nu) \), we define a constant \( C \) as follows:
\[
C = \frac{1 + \alpha}{1 - \gamma} \log E_{(s,a,s') \sim d^t}[\exp\left(\frac{1}{1 + \alpha} \hat{A}_\nu(s, a, s') - 1\right)]
\]
which implies
\[
\exp\left(\frac{(1 - \gamma)C}{1 + \alpha}\right) = E_{(s,a,s') \sim d^t}\left[\exp\left(\frac{1}{1 + \alpha} \hat{A}_\nu(s, a, s') - 1\right)\right].
\]
Then, from the following two equations:
\[
\hat{L}_{LD}(\nu^* + C) = (1 - \gamma)E_{s \sim p_{\nu}}[\nu^*(s) + C] + (1 + \alpha) \log E_{(s,a,s') \sim d^t}[\exp(\frac{1}{1 + \alpha} \hat{A}_\nu + C(s, a, s') - 1)]
\]
\[
= (1 - \gamma)E_{s \sim p_{\nu}}[\nu^*(s) + C] + (1 + \alpha) E_{(s,a,s') \sim d^t}[\exp(\frac{1}{1 + \alpha} \hat{A}_\nu(s, a, s') - 1) \exp\left(\frac{(\gamma - 1)C}{1 + \alpha}\right)]
\]
\[
= (1 - \gamma)E_{s \sim p_{\nu}}[\nu^*(s) + C] + (1 + \alpha) E_{(s,a,s') \sim d^t}[\exp(\frac{1}{1 + \alpha} \hat{A}_\nu(s, a, s') - 1) \exp\left(-\frac{(1 - \gamma)C}{1 + \alpha}\right)]
\]
and
\[
\hat{L}_{FD}(\nu^* + C) = (1 - \gamma)E_{s \sim p_{\nu}}[\nu^*(s) + C] + (1 + \alpha) \log E_{(s,a,s') \sim d^t}[\exp(\frac{1}{1 + \alpha} \hat{A}_\nu + C(s, a, s'))]
\]
\[
= (1 - \gamma)E_{s \sim p_{\nu}}[\nu^*(s) + C] + (1 + \alpha) \log E_{(s,a,s') \sim d^t}[\exp(\frac{1}{1 + \alpha} \hat{A}_\nu(s, a, s'))] + (\gamma - 1)C
\]
\[
= (1 - \gamma)E_{s \sim p_{\nu}}[\nu^*(s) + C] + (1 + \alpha) \log E_{(s,a,s') \sim d^t}[\exp(\frac{1}{1 + \alpha} \hat{A}_\nu(s, a, s'))]
\]
\[- (1 + \alpha) \log E_{(s,a,s') \sim d^t}[\exp(\frac{1}{1 + \alpha} \hat{A}_\nu(s, a, s') - 1)]
\]
\[
= (1 - \gamma)E_{s \sim p_{\nu}}[\nu^*(s) + C] + (1 + \alpha) \log E_{(s,a,s') \sim d^t}[\exp(\frac{1}{1 + \alpha} \hat{A}_\nu(s, a, s'))] - 1
\]
\[
= (1 - \gamma)E_{s \sim p_{\nu}}[\nu^*(s) + C] + 1 + \alpha,
\]
we can conclude that
\[
\hat{L}_{LD}(\nu^* + C) = \hat{L}_{FD}(\nu^* + C).
\]
From the Lemma A.2, we obtain
\[
\min_\nu \hat{L}_{LD}(\nu) \leq \hat{L}_{LD}(\nu^* + C) = \hat{L}_{FD}(\nu^* + C) = \min_\nu \hat{L}_{FD}(\nu).
\]
We show that \( \min_\nu \hat{L}_{FD}(\nu) \leq \min_\nu \hat{L}_{LD}(\nu) \) and \( \min_\nu \hat{L}_{LD}(\nu) \leq \min_\nu \hat{L}_{FD}(\nu) \), so \( \min_\nu \hat{L}_{LD}(\nu) = \min_\nu \hat{L}_{FD}(\nu) \).
Finally, similar to the proof steps for Proposition 3.2, we can easily show that \( \hat{L}_{FD}(\nu) \) is convex w.r.t. \( \nu \) (Remark that log-sum-exp is a convex function).
Proposition 3.5. Let \( V_{\text{LD}} \) and \( V_{\text{FD}} \) be the set of optimal solutions of \( \arg \min_\nu \hat{L}_{\text{LD}}(\nu) \) and \( \arg \min_\nu \hat{L}_{\text{FD}}(\nu) \), respectively. Then, \( V_{\text{FD}} = \{ \nu^* + C' \mid \nu^* \in V_{\text{LD}}, C \in \mathbb{R} \} \) holds.

Proof. We will prove this proposition by showing \( V_{\text{FD}} \subseteq \{ \nu^* + C' \mid \nu^* \in V_{\text{LD}}, C \in \mathbb{R} \} \) and \( V_{\text{FD}} \supseteq \{ \nu^* + C' \mid \nu^* \in V_{\text{LD}}, C \in \mathbb{R} \} \).

\((\subseteq)\) For given \( \nu^* \in \arg \min_\nu \hat{L}_{\text{FD}}(\nu) \), we define a constant \( C \) as follows:

\[
C = \frac{1 + \alpha}{1 - \gamma} \log \mathbb{E}_{(s,a,s') \sim d^t} \left[ \exp \left( \frac{1}{1 + \alpha} \bar{A}_{\nu^*}(s, a, s') - 1 \right) \right],
\]

which implies

\[
\exp \left( \frac{(1 - \gamma)C}{1 + \alpha} \right) = \mathbb{E}_{(s,a,s') \sim d^t} \left[ \exp \left( \frac{1}{1 + \alpha} \bar{A}_{\nu^*}(s, a, s') - 1 \right) \right].
\]

Then, by the proof steps of Proposition 3.4, we obtain

\[
\hat{L}_{\text{LD}}(\nu^* + C) = \hat{L}_{\text{FD}}(\nu^* + C) = \min_\nu \hat{L}_{\text{FD}}(\nu) = \min_\nu \hat{L}_{\text{LD}}(\nu).
\]

It means \( (\nu^* + C) \in \arg \min_\nu \hat{L}_{\text{LD}}(\nu) \) and thus,

\[
\nu^* \in \{ \nu^* + C \mid \nu^* \in \arg \min_\nu \hat{L}_{\text{LD}}(\nu), C \in \mathbb{R} \},
\]

i.e.,

\[
\arg \min_\nu \hat{L}_{\text{FD}}(\nu) \subseteq \{ \nu^* + C \mid \nu^* \in \arg \min_\nu \hat{L}_{\text{LD}}(\nu), C \in \mathbb{R} \}
\]

\((\supseteq)\) Let \( \hat{\nu}^* \in \arg \min_\nu \hat{L}_{\text{LD}}(\nu) \). Then,

\[
\frac{d^*(s,a)}{d^t(s,a)} = \frac{w^*_{\nu^*,\nu^*}(s,a)}{w^*_{\nu^*,\nu^*}(s,a)} = \exp \left( \frac{E_{\nu^* \sim T}(s,a)[\gamma \nu(s') - \mu(s,s')] - \nu(s) - 1}{\alpha} \right),
\]

\[
\frac{d^*(s',s')}{d^t(s,s')} = \frac{\hat{w}^*_{\nu^*,\nu^*}(s',s')}{\hat{w}^*_{\nu^*,\nu^*}(s',s')} = \exp(r(s,s') + \mu^*(s,s') - 1).
\]

Let \( C \in \mathbb{R} \). Then, we derive the following equation:

\[
\hat{L}_{\text{FD}}(\hat{\nu}^* + C) = \hat{L}_{\text{FD}}(\hat{\nu}^*)
\]

\[
= (1 - \gamma)E_{s \sim p_0}[\hat{\nu}^*(s)] + (1 + \alpha) \log \mathbb{E}_{(s,a,s') \sim d^t} \left[ \exp \left( \frac{1}{1 + \alpha} \bar{A}_{\nu^*}(s, a, s') \right) \right]
\]

\[
= (1 - \gamma)E_{s \sim p_0}[\hat{\nu}^*(s)] + (1 + \alpha) \log \mathbb{E}_{(s,a,s') \sim d^t} \left[ \exp \left( \frac{1}{1 + \alpha} \bar{A}_{\nu^*}(s, a, s') - 1 + 1 \right) \right]
\]

Here, we apply Lemma A.4 to derive the first equality. Because

\[
\min_\nu \hat{L}_{\text{FD}}(\nu) = \min_\nu \hat{L}_{\text{LD}}(\nu)
\]

\[
= \hat{L}_{\text{FD}}(\hat{\nu}^*)
\]

\[
= (1 - \gamma)E_{s \sim p_0}[\hat{\nu}^*(s)] + (1 + \alpha) \log \mathbb{E}_{(s,a,s') \sim d^t} \left[ \exp \left( \frac{1}{1 + \alpha} \bar{A}_{\nu^*}(s, a, s') - 1 \right) \right]
\]

\[
= (1 - \gamma)E_{s \sim p_0}[\hat{\nu}^*(s)] + 1 + \alpha,
\]

we can conclude that \( \hat{L}_{\text{FD}}(\hat{\nu}^* + C) = \min_\nu \hat{L}_{\text{FD}}(\nu) \), i.e., \( (\hat{\nu}^* + C) \in \arg \min_\nu \hat{L}_{\text{FD}}(\nu) \). Consequently,

\[
\arg \min_\nu \hat{L}_{\text{FD}}(\nu) \supseteq \{ \hat{\nu}^* + C \mid \hat{\nu}^* \in \arg \min_\nu \hat{L}_{\text{LD}}(\nu), C \in \mathbb{R} \}.
\]
B Experimental Details

B.1 Experimental details for random MDPs

We conducted experiments on randomly generated MDPs to investigate the

**Random MDP generation** For MDP $M = \langle S, A, T, R, \gamma, \rho_0 \rangle$, we first set $|S| = 20, |A| = 4, \gamma = 0.95, \rho_0(s) = 1$ for a fixed $s = s_0$. For each $(s, a)$, we sample four different states $\{s_1, s_2, s_3, s_4\}$. Then, we set the transition probability $(T(s'_1|s, a), T(s'_2|s, a), T(s'_3|s, a), T(s'_4|s, a)) = (1-\beta)X + \beta Y$, where $X \sim $ Categorical(0.25, 0.25, 0.25, 0.25) and $Y \sim \text{Dir}(1, 1, 1, 1)$. Then, when $\beta = 0$, the transition probability becomes one-hot vector, i.e., discrete MDP. In contrast, when $\beta = 1$, the transition of MDP becomes stochastic. Finally, the reward of 1 is only given to a state that minimizes the optimal value at initial state $s_0$; other states have zero rewards.

**Offline dataset generation** For each random MDP $M$, we generate state-only expert demonstrations by executing a (softmax) expert policy and state-action imperfect demonstrations by a uniform random policy. We perform experiments for a varying number of expert demonstrations $N_E \in \{10, 100, 1000, 10000\}$ and imperfect demonstrations $N_I \in \{1, 3, 10, 30, 100, 300, 1000, 3000, 10000\}$.

**Hyperparameters** We compare our tabular LobsDICE with BC, BCO, DemoDICEfO, and OPOLO. For KL-regularization hyperparameters of DemoDICEfO and LobsDICE, we use $\alpha = 0.01$

B.2 Experimental details for MuJoCo

For fair comparison, we use the same learning rate to train actors of BC, BCO, DemoDICEfO, and LobsDICE. We implement our network architectures for BC, BCO, DemoDICEfO, and LobsDICE based on the implementation of OptiDICE. For OPOLO, we use its official implementation without any modification to network architectures or hyperparameters. For stable discriminator learning, we use gradient penalty regularization on the $r(s, a)$ and $r(s, s')$ functions, which was proposed in Gulrajani et al. [2017] to enforce 1-Lipschitz constraint. To stabilize critic training, we add gradient L2-norm to the critic loss for the regularization. Detailed hyperparameter configurations used for our main experiments are summarized in Table 1.

| Hyperparameters                             | BC       | BCO      | DemoDICEfO | LobsDICE |
|---------------------------------------------|----------|----------|------------|----------|
| $\gamma$ (discount factor)                  | 0.99     | 0.99     | 0.99       | 0.99     |
| $\alpha$ (regularization coefficient)       | -        | -        | 0.1        | 0.1      |
| learning rate (actor)                       | $3 \times 10^{-4}$ | $3 \times 10^{-4}$ | $3 \times 10^{-4}$ | $3 \times 10^{-4}$ |
| network size (actor)                        | [256, 256] | [256, 256] | [256, 256] | [256, 256] |
| learning rate (critic)                      | -        | -        | $3 \times 10^{-4}$ | $3 \times 10^{-4}$ |
| network size (critic)                       | -        | -        | [256, 256] | [256, 256] |
| learning rate (discriminator)               | -        | -        | $3 \times 10^{-4}$ | $3 \times 10^{-4}$ |
| network size (discriminator)                | -        | -        | [256, 256] | [256, 256] |
| learning rate (inverse dynamics)            | -        | $3 \times 10^{-4}$ | $3 \times 10^{-4}$ | -        |
| network size (inverse dynamics)             | -        | [256, 256] | [256, 256] | -        |
| gradient L2-norm coefficient (critic)       | -        | -        | $1 \times 10^{-4}$ | $1 \times 10^{-4}$ |
| gradient penalty coefficient (discriminator) | -        | -        | 0.1        | 0.1      |
| batch size                                  | 256      | 256      | 256        | 256      |
| # of expert trajectories                    | 5        | 5        | 5          | 5        |
| # of training iterations                    | 1,000,000 | 1,000,000 | 1,000,000  | 1,000,000 |

Table 1: Configurations of hyperparameters used in our experimental results.

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[https://github.com/secury/optidice](https://github.com/secury/optidice)

[https://github.com/illidanlab/opolo-code](https://github.com/illidanlab/opolo-code)