An SZ Temperature Decrement - X-ray Luminosity Relation for Galaxy Clusters.

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ABSTRACT
We present the observed relation between $\Delta T_{SZ}$, the cosmic microwave background (CMB) temperature decrement due to the Sunyaev-Zeldovich (SZ) effect, and $L$, the X-ray luminosity of galaxy clusters. We discuss this relation in terms of the cluster properties, and show that the slope of the observed $\Delta T_{SZ} - L$ relation is in agreement with both the $L - T_e$ relation based on numerical simulations and X-ray emission observations, and the $M_{\text{gas}} - L$ relation based on observation. The slope of the $\Delta T_{SZ} - L$ relation is also consistent with the $M_{\text{tot}} - L$ relation, where $M_{\text{tot}}$ is the cluster total mass based on gravitational lensing observations. This agreement may be taken to imply a constant gas mass fraction within galaxy clusters, however, there are large uncertainties, dominated by observational errors, associated with these relations. Using the $\Delta T_{SZ} - L$ relation and the cluster X-ray luminosity function, we evaluate the local cluster contribution to arcminute scale cosmic microwave background anisotropies. The Compton distortion $\gamma$-parameter produced by galaxy clusters through SZ effect is roughly two orders of magnitude lower than the current upper limit based on FIRAS observations.

Key words: cosmology: observations — cosmology: theory — galaxies: clusters: general — cosmic microwave background

1 INTRODUCTION
The X-ray emission from galaxy clusters has enabled the study of what is assumed to be the largest virialized systems in the universe. The X-ray electron temperature, $T_e$, measures the depth of the galaxy cluster potential, while the X-ray luminosity, $L$, emitted as thermal bremsstrahlung by the intracluster plasma, measures primarily the baryonic number density within this potential. When combined with the fact that the electron temperature is a robust estimator of the galaxy cluster total mass, $M_{\text{tot}}$, the X-ray emission observations can be used in cosmological studies to derive constraints on the power spectrum of the initial density perturbations and on cosmological parameters (e.g., Eke et al. 1998; Oukbir & Blanchard 1997; Bahcall, Fan & Cen 1997). However, most of the important conclusions from such studies are dependent on the accuracy of used scaling relations between the cluster properties, such as the cluster X-ray luminosity, total mass, gas mass, and the electron temperature. Independent of the large scale distribution, baryonic component of the individual clusters has been used to constrain the mass density of the universe, $\Omega_m$ (e.g., Briel et al. 1992; White et al. 1993; White & Fabian 1995; David, Jones & Forman 1995; Evrard 1997), but such studies can be subject to variations in physical properties from one cluster to another, e.g., due to cluster cooling flows (Fabian et al. 1994; White, Jones & Forman 1997; Allen & Fabian 1998; Markevitch 1998).

Apart from X-ray emission observations, another well known probe of the intracluster gas distribution is the Sunyaev-Zeldovich (SZ) effect (Sunyaev & Zeldovich 1970, 1980). The SZ effect is the scattering of the cosmic microwave background (CMB) radiation via hot electrons of the X-ray emitting gas through an inverse-Compton process, producing a distortion in the CMB spectrum. In recent years, increasingly sensitive observations of galaxy clusters, first with single-dish telescopes and now with interferometers, have produced accurate maps of the CMB temperature change resulting from the SZ effect. Unlike the X-ray emission from galaxy clusters, the magnitude of the CMB temperature decrement due to the SZ effect is independent of the redshift, and allows a direct probe to the distant universe. It is likely that the potential use of the SZ effect as a cosmological probe to distant universe is yet to be fully real-
ized, and more work, both together and independent of the X-ray properties of galaxy clusters, is required. The combination of X-ray emission and SZ effect towards a given cluster can be used to determine the distance to that cluster, from which the Hubble constant can be derived. Also, the SZ effect, which measures that gas mass within galaxy clusters, can be used to constrain the total mass density of the universe (see, e.g., Myers et al. 1997). Other than cosmological uses, the SZ effect and the X-ray emission from clusters can be used to study cluster gas physics, since these two observables depends differently on the electron number density and temperature distributions.

Given the ever increasing galaxy cluster SZ data, we have initiated a study to investigate the ways in which the SZ effect can be used both as a cosmological tool, as well as a tool to understand the distribution and physical properties of baryonic content within clusters. Here in this paper, we present the observed relation between the CMB temperature change due to the SZ effect, $\Delta T_{\text{SZ}}$, and the X-ray luminosity, $L$, of a sample of galaxy clusters. In Section 2, we present a brief introduction to the SZ effect and the X-ray emission, and formulate the expected relation between $\Delta T_{\text{SZ}}$ and $L$. In Section 3, we present the used cluster sample in which SZ effect has been observed and derive the $\Delta T_{\text{SZ}} - L$ relation, while in Section 4, we discuss the observed relation in terms of the galaxy cluster properties. In the same section, we use this relation as an application to study the cluster contribution to arcminute scale cosmic microwave background anisotropies.

## 2 THEORY

Briefly, the SZ effect is a distortion of the cosmic microwave background (CMB) radiation by inverse-Compton scattering of thermal electrons within the hot intrachuster medium (Sunyaev & Zeldovich 1970, 1980). The change in the CMB brightness temperature observed is:

$$\frac{\Delta T}{T_{\text{CMB}}} = f(x) \int \left( \frac{k_B T_e}{m_e c^2} \right) n_e \sigma_T dl,$$

where

$$f(x) = \left[ \frac{x(e^x + 1)}{e^x - 1} - 4 \right]$$

is the frequency dependence with $x = h \nu / k_B T_{\text{CMB}}$, $T_{\text{CMB}} = 2.728 \pm 0.002$ (Fixsen et al. 1996) and $n_e$, $T_e$ and $\sigma_T$ are the electron density, electron temperature and cross section for the Thomson scattering. The integral is performed along the line of sight through the cluster. We refer the reader to a recent review by Birkinshaw (1998) on the SZ effect and its observation. At the Rayleigh-Jeans (RJ) part of the frequency spectrum, $f(x) = -2$.

The other important observable of the hot intrachuster gas is the X-ray emission, whose surface brightness $S_X$ can be written as:

$$S_X = \frac{1}{4\pi(1+z)^3} \int n_e^2 \Lambda_e dl,$$

where $z$ is the redshift and $\Lambda_e(\Delta E, T_e)$ is the X-ray spectral emissivity of the cluster gas due to thermal Bremsstrahlung emission within a certain energy band $\Delta E$ ($\Lambda_e \propto T_e^{-1/2}$). By combining the SZ intensity change and the X-ray emission observations of a given cluster, the angular diameter distance to the cluster can be derived due to the different dependence of the X-ray emission and the SZ effect on the electron number density, $n_e$ (e.g., Cavaliere et al. 1977). Combining the distance measurement with redshift allows a determination of the Hubble constant, $H_0$, as a function of certain cosmological parameters (e.g., see, Hughes & Birkinshaw 1998). Other than the derivation of the Hubble constant, SZ effect and X-ray emission can in principle be used to constrain the cosmological parameters based on distance measurements of a large sample of galaxy clusters with a wide range of redshift.

The present paper discusses the relation between $\Delta T_{\text{SZ}}$ and $L$, the X-ray luminosity, for a sample of galaxy clusters. We can estimate the expected relation based on $n_e$ and $T_e$ dependences between the SZ effect and the X-ray emission. The SZ effect is due to the pressure integrated along the line of sight, i.e. $\Delta T_{\text{SZ}} \propto n_e T_e$, while the X-ray emission is due to the thermal Bremsstrahlung with $L \propto n_e^2 T_e^{1/2}$. Here, we ignore the contribution from X-ray line emission. By removing the $n_e$ dependence between $\Delta T_{\text{SZ}}$ and $L$, one gets $\Delta T_{\text{SZ}} \propto L^{1/2} T_e^{3/4}$. The relation between $L$ and $T_e$ has been well studied based on numerical simulations (e.g., Cavaliere et al. 1997) and observed data (e.g., Mushotzky & Scharf 1997; Allen & Fabian 1998; Arnaud & Evrard 1998). Assuming that $L \propto T_e^\alpha$, $\Delta T_{\text{SZ}} \propto L^{\gamma_1 + \gamma_2}$

$$\Delta T_{\text{SZ}} \propto L^{1/2 + 3/4}.$$  

Based on numerical simulations, Cavaliere et al. (1997) predicts that $\alpha = 5$ at the scale of groups and flattens to $\alpha = 3$ for rich clusters, and saturates to $\alpha = 2$ for high temperature clusters. Currently, the SZ effect has been detected towards rich clusters with moderate to high electron temperatures, resulting in $\Delta T_{\text{SZ}} \propto L^{0.65}$ to $L^{0.88}$, when $\alpha$ varies from 5 to 2. The observed data currently suggest that $\alpha \sim 2.8$, less than previous studies which suggested values for $\alpha \sim 3$ to 3.3 (e.g., David et al. 1993).

Also, since $\Delta T_{\text{SZ}}$ is a measurement of the pressure along the line of sight to the cluster, the integral of $\Delta T_{\text{SZ}}$ through a cylindrical cut of the cluster is directly proportional to the gas mass within that cylinder. This assumes that the cluster gas is isobaric. If we expect the gas mass to scale with luminosity according to a relation of the form $M_{\text{gas}} \propto L^\gamma$, then

$$\Delta T_{\text{SZ}} \propto L^\gamma.$$  

The second estimate for the $\Delta T_{\text{SZ}} - L$ relation should be more accurate since the measurement of gas mass accounts for the integrated flux along the line of sight, while the former estimate relies slightly on the assumption of spherical geometry for galaxy clusters and that no cluster gas clumping is present.
Due to cluster cooling flows, so the values tabulated here have taken into account the variations in the X-ray luminosities. Fabian (1998) and Arnaud & Evrard (1998). These studies have used the published luminosity values from Allen & Fabian (1998). These studies have used the published luminosity values from Allen & Fabian (1998).

The accuracy of X-ray luminosities is the dependence on such corrections. Except for Cl0016, there is not heavily dependent on such corrections. Between 3% to 5%, but the relation between $\Delta T$ and $L$ is mainly important for clusters with high electron temperatures ($T_e > 10$ keV). Such corrections only change the SZ temperature decrement by about 3% to 5%, but the relation between $\Delta T_{SZ}$ and $L$ presented here is not heavily dependent on such corrections.

Another advantage of the tabulated values is the accuracy of X-ray luminosities. Except for Cl0016, the data at high electron temperatures ($T_e > 10$ keV). Such corrections only change the SZ temperature decrement by about 3% to 5%, but the relation between $\Delta T_{SZ}$ and $L$ presented here is not heavily dependent on such corrections.

In order to derive the observed relation between $\Delta T_{SZ}$ and $L$, we have compiled a sample of galaxy clusters for which SZ data are available from literature, and for which accurate X-ray results are also available. We list these clusters, X-ray luminosities in the 2 to 10 keV band, electron temperature, SZ temperature decrement and references to each of the X-ray luminosities in the 2 to 10 keV band, electron temperature, and SZ experiments in Table 1.

| Cluster Name | $z$ | $L_{\text{2-10 keV}}$ ($10^{44}$ ergs s$^{-1}$) | $T_e$ (keV) | $\Delta T_{SZ}$ (mK) | Reference |
|--------------|----|---------------------------------|----------|-------------------|-----------|
| Coma (A1656) | 0.023 | 11.3 | 8.2 ± 0.1 | -0.55 ± 0.10 | 1,1,2 & 3 |
| A2256 | 0.0601 | 11.1 | 7.5 ± 0.2 | -0.44 ± 0.09 | 1,1,2 |
| A2142 | 0.0899 | 30.4 | 9.3 ± 1.0 | -0.90 ± 0.14 | 4,4,2 |
| A478 | 0.09 | 24.2 | 8.1 ± 1.0 | -0.92 ± 0.15 | 4,4,2 |
| A1413 | 0.143 | 15.4 | 8.5 ± 1.0 | -0.96 ± 0.11 | 4,4,5 |
| A2204 | 0.152 | 34.6 | 9.2 ± 2.5 | -0.96 ± 0.28 | 4,4,15 |
| A2218 | 0.171 | 10.8 | 7.05 ± 0.35 | -0.75 ± 0.20 | 4,4,6 |
| A1689 | 0.180 | 32.2 | 10.0 ± 1.0 | -1.91 ± 0.30 | 4,4,8 |
| A665 | 0.182 | 17.8 | 9.03 ± 0.60 | -0.91 ± 0.09 | 4,4,9 |
| A773 | 0.197 | 14.8 | 9.29 ± 0.65 | -0.89 ± 0.10 | 4,4,10 |
| A2163 | 0.201 | 60.0 | 13.8 ± 0.8 | -1.93 ± 0.28 | 4,4,8 |
| A1835 | 0.252 | 44.9 | 9.8 ± 2.5 | -1.34 ± 0.15 | 4,4,15 |
| Z3146 | 0.291 | 37.3 | 11.3 ± 3.5 | -0.86 ± 0.14 | 4,4,15 |
| Cl0016 | 0.541 | 26.2 | 7.55 ± 0.7 | -1.21 ± 0.19 | 12 & 13,14,15 |

Notes: column (6) indicates references for the X-ray luminosity, temperature and SZ measurement, respectively (for clusters with multiple SZ measurements, the second column under the same cluster refers only to the SZ data): 1. Arnaud & Evrard (1998); 2. Myers et al. (1997); 3. Herbig et al. (1995); 4. Allen & Fabian (1998); 5. Grainge et al. (1996); 6. Uyaniker et al. (1997); 7. Jones (1995); 8. Holzapfel et al. (1997); 9. Birkhain & Evrard (1998); 10. Grainge et al. (1993); 11. Carlstrom et al. (1996); 12. Neumann & Böhringer (1997); 13. Tsuru et al. (1996); 14. Hughes & Birkhain (1998); 15. Holzapfel (1996).

3 DATA

In order to derive the observed relation between $\Delta T_{SZ}$ and $L$, we have compiled a sample of galaxy clusters for which SZ data are available from literature, and for which accurate X-ray results are also available. We list these clusters, X-ray luminosities in the 2 to 10 keV band, electron temperature, SZ temperature decrement and references to each of the X-ray luminosities in the 2 to 10 keV band, electron temperature, and SZ experiments in Table 1.

Given that the tabulated SZ observations were made at different frequencies, we have scaled the SZ temperature decrement to the RJ part of the spectrum so that the comparison between the SZ effect and the X-ray luminosity of galaxy clusters is uniform. Also, some of the SZ experiments ignored the higher order corrections to the inverse-Compton scattering in evaluating the SZ temperature decrement. Using the relativistic corrections presented by Itoh et al. (1997), we have corrected for the SZ temperature decrement, which are mainly important for clusters with high electron temperatures ($T_e > 10$ keV). Such corrections only change the SZ temperature decrement by about 3% to 5%, but the relation between $\Delta T_{SZ}$ and $L$ presented here is not heavily dependent on such corrections.

Another advantage of the tabulated values is the accuracy of X-ray luminosities. Except for Cl0016, we have used the published luminosity values from Allen & Fabian (1998) and Arnaud & Evrard (1998). These studies have taken into account the variations in the X-ray luminosities due to cluster cooling flows, so the values tabulated here should be unbiased from cooling flow effects. However, we note that these luminosities may have both statistical and systematic uncertainties of the order 15%, resulting from poor calibration to uncertainties associated with removal of individual cluster cooling flow regions in calculating the luminosities. Our SZ cluster sample is primarily made of interferometric observations of the SZ effect (Carlstrom et al. 1996; Grainge et al. 1993), SuZIE observations at the Caltech Sub-mm Observatory (Holzapfel et al. 1997), and OVRO single-dish observations of the low redshift clusters (Myers et al. 1997).

The published SZ temperature decrement values in Myers et al. (1997) are not corrected for the beam dilution and switching. However, based on modeling of the cluster gas distribution for individual clusters, the authors have calculated various efficiencies in this process (see, Table 8 in Myers et al. 1997). We used these values to convert the observed SZ temperature decrements to beam-corrected values, which are presented in Table 1.

In Fig. 1, we show the observed SZ temperature decrement vs. the X-ray luminosity in the 2 to 10 keV band, to which we have fitted a relation of the form $\Delta T_{SZ} = aL^b$. For clusters with multiple SZ measurements, we have used the mean value of the reported temperature decrement but have appropriately scaled the uncertainty so that it covers the whole range suggested by the two separate measurements. Using a maximum-likelihood minimization, the data are best explained by:
which is very similar to the relation found by Shimasaku et al. (1992).

The observed $\Delta T_{SZ}$ vs. $L$, with the best fit relation shown as a solid line (Eq. 7). The open circles represent the low redshift clusters from Myers et al. (1997), while the filled circles represent the clusters between $0.1 < z < 0.21$. The solid triangle is C0016.

For the SZ observational data in Table 1, we derived $\Delta T_{SZ} \propto L^{0.61 \pm 0.18}$. The slope is consistent with the numerically simulated values for $\alpha$ ranging from 5 to 3, but slightly inconsistent with a $L - T$ relation of the form $L \propto T^2$, which is expected for clusters under the self-similar model. The current observational data suggest that $L \propto T^{2.6}$ (Markevitch 1998) to $T^{2.8}$ (Arnaud & Evrard 1998), which is consistent with the present estimate of the $\Delta T_{SZ} - L$ slope.

The $M_{gas} - L$ relation, as derived based on the observed mass data for a large sample of clusters in White et al. (1997) suggests a slope of $0.66 \pm 0.06$ to the $\Delta T_{SZ} - L$ relation, which is again consistent with the observed value. Since the SZ temperature decrement is a direct estimate on the cluster gas mass, and assuming that the measured gas mass is essentially similar to the gas mass the SZ experiments would observed based on the geometry, then $\Delta T_{SZ} \propto L^{0.66 \pm 0.06}$. However, effects due to cluster projections and other associated systematic errors (see, below) may produce a slightly different observed relation that one expected based on these simple arguments. We note that the observed $M_{gas} - L$ relation is based on the gas masses within inner $0.5$ Mpc from the cluster centers. Depending on the redshift and the instrumental properties (beam size, resolution etc), individual SZ experiments may be sensitive to a different radius from the cluster center. The $\Delta T_{SZ}$ values are derived by modeling the observed flux over a larger radius than $0.5$ Mpc. Thus, comparison of the two relations may be slightly problematic, however, since the total gas mass out to a larger radius would scale in a self-similar manner, we do not expect the slope to be affected. However, comparison of the normalizations are not currently possible. Also, we have used two different estimates of luminosities – luminosities in 2 to 10 keV for $\Delta T_{SZ} - L$ relation and bolometric luminosities for $M_{gas} - L$ relation — but here again, we don't expect the slope to have been affected systematically by these differences.

Other than statistical uncertainties associated with the measurement of SZ decrements and X-ray luminosities, it is likely that there are additional effects contributing to the observed scatter in the $\Delta T_{SZ} - L$ relation. Such contributions may come from projection effects of galaxy clusters, which are mostly modeled using spherical geometries, variations in gas mass fraction from one cluster to another, and effects due to gas clumping and nonisothermality.

For example, if a cluster is prolate and elongated along the line of sight, then the observed SZ temperature decrement would be larger than what is expected based on the X-ray properties of that cluster (see, Cooray 1998). In such a case, one may also find a substantially lower value for the Hubble constant. Since we are considering the galaxy cluster sample in a statistical manner, the derived relations such as $\Delta T_{SZ} - L$, should be unbiased of effects that may arise due to cluster projections. However, this requires that the clusters in the sample are distributed randomly, so that the whole sample is unbiased. This may not likely to be true with the current sample, as most of the current SZ targets are clusters which are likely to have been previously studied due to
certain properties. Such clusters, which may have been selected due to high X-ray luminosities or gravitational lensing effects, are likely to be prolate clusters elongated along the line of sight such that the observed decrement may be slightly higher than the value expected for the luminosity of that cluster. This is more likely to be the case with strong gravitational lensing clusters; the fact that both A1689 and A2163 have similar temperature decrements but different X-ray luminosities clearly suggests this possibility. Also, high luminosity clusters are likely to be strong cooling flows, but the luminosity values used here, except for Cl0016, have been corrected for such cooling flow contributions. For the present sample, if biased effects exist either at the low or high end of the luminosities, then the derived relation may have been affected.

From the present SZ sample, A2218, A773, and Cl0016 have been observed by multiple observational programs. The difference between these separate measurements are of the order 15% to 25%. Other than physical systematics effects discussed above, it is likely that the derived relation has an additional intrinsic dispersion as high as 25%. Such differences are likely to arise when clusters are modeled using different parameters and that the observational effects, such as the beam dilution produced by the instrument, are not properly taken into account. It is likely that the $\Delta T_{\text{SZ}} - L$ relation will be properly studied using carefully selected sample of galaxy clusters that are planned to be observed with ground-based interferometers, and space-based observatories such as PLANCK.

The current cluster sample used to derive the $\Delta T_{\text{SZ}} - L$ relation ranges in redshift from 0.02 to 0.54, with only one cluster beyond a redshift of 0.2. Therefore, we are unable to study any redshift evolution of this relation. However, we note that when using this relation to study clusters at high redshifts, it may be important to consider the possible evolutionary effects. However, if such effects exist in the $\Delta T_{\text{SZ}} - L$ relation, then it would be primarily due to the evolution of the X-ray luminosity function, but other effects, such as a systematic change in the cluster baryonic gas mass fraction with redshift, can also produce deviations in the $\Delta T_{\text{SZ}} - L$ relation.

In order to test the accuracy of the derived relation between SZ effect and X-ray luminosity of galaxy clusters, we now consider additional observable cluster properties. Since the X-ray luminosity and the gas mass of a cluster is a measurement of the baryonic content, the presented $\Delta T_{\text{SZ}} - L$ is primarily a probe of the baryonic mass distribution. When the $\Delta T_{\text{SZ}} - L$ relation is combined with scaling relations such as $L - T_e$ and $M_{\text{tot}} - T_e$ (Hjorth, Oukbir & Kampen 1998), one can constrain both the baryonic and dark matter distribution separately from each other. In Fig. 3, we show the observed temperature decrements as a function of the cluster electron temperatures in table 1. Even though the involved uncertainties are high, we find that the $\Delta T_{\text{SZ}} - T_e$ is:

$$\Delta T_{\text{SZ}} = -(0.56 \pm 0.51) \times 10^{-3} \left( \frac{T_e}{\text{keV}} \right)^{2.35 \pm 0.85} \text{ mK.}$$

(8)

The slope of this relationship is again consistent with simple scaling-law arguments.

The luminosity–gas mass relation observed from the X-ray measurements in Fig. 2, suggests $\Delta T_{\text{SZ}} \propto L^{0.64 \pm 0.06}$, which is in good agreement with the relation derived from the SZ observational data. As suggested earlier, this relation is more likely to be accurate since the integrated pressure along the line of sight is simply proportional to the gas mass within the line of sight, which is the gas mass within the cylinder defined by geometry. A direct probe of the total cluster mass along the line of sight is the mass derived based on gravitational lensing observations. Smail et al. (1997) studied lensing properties of a sample of galaxy clusters observed with the Hubble Space Telescope and measured both weak and strong lensing properties of the clusters. They derived a relationship between cluster luminosity, $L$, and mean shear, $\gamma$, of the form:

$$\gamma > = (0.074 \pm 0.017) \times \left( \frac{L}{10^{44} \text{ergs s}^{-1}} \right)^{0.58 \pm 0.23}.$$  

(9)

Here, $\gamma$ is a measurement of the average tangential shear strength of galaxy clusters, and is directly proportional to the cluster total mass responsible for gravitationally lensing the background galaxies towards the foreground cluster. Thus, total mass can be written as $M_{\text{tot}} \propto L^{0.58 \pm 0.23}$. Since $M_{\text{gas}} \propto \Delta T_{\text{SZ}} \propto L^{0.66 \pm 0.06}$, we find that the ratio, $f_{\text{gas}} \equiv M_{\text{gas}} / M_{\text{tot}}$, is $\propto L^{0.08 \pm 0.24}$. This ratio measures the cluster gas mass fraction, which has been used in literature to constrain the total mass density of the universe based on baryonic-mass density as derived based on nucleosynthesis arguments (see, e.g., Evrard 1997). It is likely that this ratio is independent of the cluster luminosity, suggesting that the gas mass fraction within clusters is constant from one cluster to another. Recently, Arnaud & Evrard (1998) studied the changes in cluster gas mass fraction from one cluster to another and suggested that these changes are likely due to heating processes within clusters, such as due to winds. If such effects exist, then the $\Delta T_{\text{SZ}} - L$ relation would also be affected contributing to the observed dispersion.

We use the X-ray luminosity function (XLF) of clusters of galaxies from the ROSAT Brightest Cluster Sam-
ple (Ebeling et al. 1997), which is an X-ray selected, flux limited sample of 172 clusters compiled from the ROSAT All-Sky Survey data, to study the local cluster contribution to arc-minute scale cosmic microwave background (CMB) anisotropies. In order to compare with the current limits of the Compton $y$-parameter based on FIRAS observations, we convert our $\Delta T_{SZ} - L$ relation to a $y - L$ relation using:

$$y(L) = \frac{1}{2} \frac{\Delta T_{SZ}(L)}{T_{CMB}}.$$  \hfill (10)

The XLF is represented by:

$$\phi(L) dL = A L^{-\alpha} \exp \left(-\frac{L}{L_*}\right) dL,$$ \hfill (11)

where

$$A = (1.59 \pm 0.36) \times 10^{-7} h_{50}^2 \text{Mpc}^{-3} \times (10^{44} h_{50}^{-2} \text{ergs s}^{-1})^{\alpha - 1}.$$  \hfill (12)

$L_*$ is the luminosity in the 2 to 10 keV band. Using the luminosity function and the function $y(L)$, we can write the average $< y >$ parameter due to galaxy clusters as:

$$< y >= \int_{L_1}^{L_2} y(L) \phi(L) dV dL$$ \hfill (13)

where $L_1$ and $L_2$ are the lower and upper limits of XLF, respectively. In order to describe the cluster volume $dV$ we adopt a $\beta$-model for the cluster gas distribution with a core radius $R_c$. We assume a $\beta$ of $2/3$ to describe all clusters. We allow for the total cluster scale radius $R$ to vary with luminosity and the form, $R \propto L^\gamma$, where $\gamma$ is a free-parameter. According to Mohr & Evrard (1997), the X-ray size scales with temperature as $R \propto T^{0.93 \pm 0.11}$. With the $L - T$ relation, in the X-ray size is expected to scale with luminosity as $R \propto L^{0.2}$ to $L^{0.5}$. Using the tabulated data in White et al. (1997), we obtained a relation of the form:

$$R_c(L) = 0.46 \left( \frac{L}{10^{44} h_{50}^{-2} \text{ergs s}^{-1}} \right)^{0.33 \pm 0.15} \text{Mpc},$$ \hfill (14)

between the tabulated core radii for clusters, $R_c$, and their luminosities, $L$. We note that the core-radii from White et al. (1997) do not necessarily represent the true underlying scale-size of the X-ray emission. In their analysis, each cluster core-radius was treated as a parameter which was varied to obtain a flat temperature profile, under their assumption for the form of the gravitational potential. For the purpose of the present calculation, however, we are only interested in core-radii as a representation for the relative distribution of size-scales. Since we also vary most of the parameters, such as the slope in the $R - L$ relation, and the fact that our final results our presented as a ratio between the cluster radius and the core-radius, the use of a $R - L$ relation from data in White et al. (1997) should not affect the final conclusions. For the rest of the paper we assume that $\kappa$ ranges from 0 to 1.

In order to test the accuracy of derived $\Delta T_{SZ} - L$ relation, we also vary its slope $\gamma$, between 0 and 2.5.

In Fig. 4 we show the derived $< y >$ values as a function of $R/R_c$. It is likely that cluster sizes, in general, are of the order 15 to 25 core radii. Each of the individual plots correspond to the different $R - L$ relation at steps of 0.25 between 0 and 1, while each of the curves represent the different $\gamma$ values at steps of 0.5 between 0 and 2.5. For our preferred values of $\kappa$ and $\gamma$, $\kappa \sim 0.25$ to 0.5 and $\gamma \sim 0.5$ to 1.0, we find that the cluster contribution to $y$-parameter through the SZ effect is at least two orders of magnitude less than the current upper limit based on FIRAS observations of $2.5 \times 10^{-5}$ (Mather et al. 1994). If all clusters have a constant size, independent of the X-ray luminosity, then the $< y >$ can be as high as $10^{-6}$, but this possibility is excluded by the observational data, which suggests that cluster size varies with the luminosity. In general, we find that the cluster contribution to $y$-parameter is $\sim 2 \times 10^{-7}$, which is consistent with previous estimates (see, e.g., Ceballos & Barcons 1994). The XLF used here has been calculated for clusters out to redshifts of 0.3, so that the derived results may be valid to clusters out to the same redshift. However, evidence for no evolution in the XLF out to redshifts of 0.8 has recently appeared on literature (Rosati et al. 1998), and thus, our results may be valid to a much higher redshift. As studied in Barbosa et al. (1996), in a low $\Omega_m$ universe, galaxy cluster contribution to Compton $y$ parameter may be as high as $10^{-5}$, with most of the contribution coming from clusters at $z > 1$.

Other than using the $\Delta T_{SZ} - L$ relation as a probe of cluster physics, the observed $\Delta T_{SZ} - L$ relation can also be used in the study of high redshift clusters, where SZ temperature decrements have been observed but no X-ray emission have been detected. For example, given that we now know the $\Delta T_{SZ} - L$ relation, it is reasonably possible to constrain the expected X-ray luminosity of such a detection, and then, perhaps, based on the observed X-ray flux threshold, put a lower limit on the redshift. Detection of such high redshift clusters has strong implications for cosmological world models, and based on the expected redshift and the mass of such clusters, one can constrain cosmological parameters with high accuracy (e.g, Bartlett, Blanchard & Barbosa 1998). The $\Delta T_{SZ} - L$ and $M_{gas} - L$ allows one to directly relate the observed temperature decrements to gas mass, and under the assumption of a constant baryonic fraction, to relate to the total mass of the cluster. It should
then be possible to apply the Press-Schechter (PS) formalism to number density of such cluster masses, and constrain the cosmological parameters, with little dependence on the X-ray observations. However, to perform such an analysis one requires data from a complete sample of galaxy clusters, and such SZ samples are currently not available; observations of a luminosity-selected unbiased sample of galaxy clusters would be useful in the future to constrain the cosmological parameters, using scaling relations such as the one presented here.

5 CONCLUSIONS

Based on observations of the Sunyaev-Zeldovich effect in galaxy clusters and the X-ray luminosity, we have derived a relation between the two observables. We studied this relation in terms of other cluster properties, and have found a relation between the two observables. We have studied this relation and the X-ray luminosity function for galaxy clusters, we have derived the local cluster contribution to the Compton y-parameter through the SZ effect. These values are at least two orders of magnitudes lower than the current upper limit based on FIRAS observations. Given the planned observations of SZ effect in large sample of galaxy clusters, such as with PLANCK and other ground based interferometers, it is likely that a relation such as $\Delta T_{SZ} - L$ would be useful both in making predictions and deriving important cosmological parameters. We leave such studied to be carried out in future, using complete and unbiased samples of galaxy clusters.

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$L_x \left(10^{44} \, h_{50}^{-2} \, \text{ergs s}^{-1}\right)$

$\Delta T_{\text{SZ}}$ (mK)
$M_{\text{gas}} \left(10^{12} h_{50}^{-2.5} M_{\odot}\right)$ vs $L_x \left(10^{44} h_{50}^{-2} \text{ergs s}^{-1}\right)$
FIRAS limit

\[ \frac{R}{R_{\text{core}}} = A L^0 \]
\[ R_{\text{core}} = A L^{0.25} \]
\[ R_{\text{core}} = A L^{0.5} \]
$R_{\text{core}} = A L^{0.75}$
\[ R_{\text{core}} = A L^1 \]