Fly-by encounters between two planetary systems I: solar system analogues

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ABSTRACT
Stars formed in clusters can encounter other stars at close distances. These encounters strongly perturb planetary systems, directly causing ejection of planets or their capture by the intruding star, as well as exciting orbital eccentricities and leading to long-term destabilisation. Using extensive N-body simulations, we study fly-by encounters between two planetary systems, each a Solar System analogue with four giant planets from Jupiter to Neptune. We quantify the rates of loss and capture immediately after encounter, finding that one third of the planets lost from the host are indeed captured by the flying-by star. Interactions between planets during the fly-by encounters are found to be negligible. We then perform long-term (up to 1 Gyr) simulations investigating the ensuing post-encounter evolution, paying close attention to how captured planets interact with the original planetary systems. It is found that large numbers of planets are removed from systems due to planet–planet interactions. Captured planets are found to further enhance the frequency of system instability. While encounters between two planetary systems can initially leave a planetary system containing more planets immediately after a fly-by, the long-term evolution post encounter tends to lead to the ejection of planets as the system becomes unstable, thus leading in general to a reduction in the number of planets a system contains. We find that one half of the surviving captured planets end up on retrograde orbits. Thus, encounters between planetary systems are a channel to place planets on wide retrograde orbits that may be detectable through astrometry or direct imaging.

Key words: celestial mechanics – planet-star interactions – planetary systems – open clusters and associations: general

1 INTRODUCTION
Most stars, as well as our own Sun, form in a clustered environment together with many other stars (Lada & Lada 2003; Adams 2010). Though possibly negatively affecting planet formation by truncating the circumstellar disk from which planets accrete (for instance Adams et al. 2006; Pfalzner et al. 2005), embedding in a cluster does not necessarily stop planet formation around a star. Indeed, the occurrence rate for giant planets in star clusters has been found to be consistent with that of field stars (Brucalassi et al. 2017) and a few tens of planets are observed in clusters (see Cai et al. 2017, for a recent compilation).

However, a clustered environment is still detrimental to the planets. Self-consistent cluster simulations show that close encounters frequently happen between the stars (e.g., Malmberg et al. 2007; Adams et al. 2006). These stellar flybys may damage the planetary system. In a series of works, Laughlin & Adams (1998); Adams & Laughlin (2001); Adams et al. (2006) constructed the cross-sections for ejection and eccentricity excitation for the solar system giant planets. However, this is just the beginning: an otherwise stable multi-planet system (like our own solar system), if perturbed by a stellar encounter, may develop orbital instability in millions of years, long after the encounter (Spurzem et al. 2009; Malmberg et al. 2011; Hao et al. 2013).

We build this paper on the work of Malmberg et al. (2011) where the authors investigated fly-by encounters between a solar system (with four giant planets) and a single star. They found that the planets could be captured by the intruding star and the solar system is highly excited by the flyby. Given that exoplanets are commonly observed, with occurrence rates of a few tens of % (e.g., Cumming et al. 2008; Cassan et al. 2012; Zhu et al. 2018), we here ask what will happen if both encountering stars have planets. Hence, we will be modelling encounters between two planetary systems (both with planets), trying an answer, for instance, what is the planets’ role during the encounter and...
how the captured planets interact with the originals on long timescales.

The paper is organised as follows. In Section 2, we describe our encounter simulations. In Section 3, we simulate the long-term evolution of the post-encounter systems up to \(10^8\) or \(10^9\) yr. Section 4 is devoted to discussion of implications derived from this work. Finally we summarise our main results in Section 5.

## 2 ENCOUNTER PHASE

We start by introducing the abbreviations. Two types of encounters are investigated in this work. In type 1 (T1), we simulate encounters between a solar-mass star (with no planet around it) and a solar system (with four giant planets) while in type 2 (T2), two solar systems are involved. See Figure 1 for illustration. These are our “encounter phase” simulations.

### 2.1 Initial condition

For T1 encounters, we assume the initial distance between the flying-by star and the target solar system is 2000 au, much larger than 1000 au, the usual threshold for an encounter (e.g., Adams & Laughlin 2001). The relative velocity between the star and the barycentre of the target system is \(v_{\text{inf}} = 1\) km s\(^{-1}\), typical of young open star clusters (Adams 2010).

The maximum encounter pericentre distance is \(r_{\text{enc,max}} = 100\) au, because the emphasis of this paper is on the extreme encounters with immediate loss/capture and the long-term implications. For a given \(v_{\text{inf}}\) and stellar mass (one solar mass throughout this work), we consider gravitational focusing (see Armitage 2010, for instance) and calculate a maximum impact parameter \(b_{\text{max}}\) such that \(r_{\text{enc}} = r_{\text{enc,max}}\). Assuming that the impact parameter \(b\) follows a geometrical configuration and that the encounter is isotropic in the solid angle, we then randomly generate \(b \leq b_{\text{max}}\). Due to strong focusing, we largely obtain a flat probability density function (PDF) for \(r_{\text{enc}}\).

For the giant planets, we acquire their ecliptic orbits from JPL Horizons [https://ssd.jpl.nasa.gov/|horizons] but assign random phase angles. So the target system is almost flat (Sun+4 planets) and the flying-by star is coming from a random direction.

In T2 encounters, the flying-by star is itself orbited by the four planets in their own ecliptic, assumed to be directed also randomly. Thus, the same encounter usually has two different inclinations as viewed from the ecliptics of two systems. We generate 10,000 sets of initial conditions for T1 encounters and 5,000 for T2 encounters. Hence, the total number of planets in the two encounter simulations is the same—40,000.

The integration is stopped at \(10^8\) yr by which time the distance between the two stars already becomes \(>1000\) au and the encounter finishes. Then we store the orbital elements and state vectors for all the objects. The inclination of a planet is measured against the ecliptic plane of a star and the orbit of a flying-by star is calculated in the same reference frame.

All N-body simulations are carried out with MERCURY (Chambers 1999) using the Bulirsch-Stoer algorithm with a tolerance of \(10^{-12}\).

### 2.2 Results

During a close encounter, the planets in the target system may be lost from the host system and some of those can be captured by the flying-by star. We confirm previous results (e.g., Malmberg et al. 2011; Pfalzner et al. 2005) that capturing a planet is only possible when \(r_{\text{enc}}\) is no larger than three times the planet’s semimajor axis, showing our choice of \(r_{\text{enc,max}} = 100\) au is sufficient.

Table 1 lists the rates for loss and capture for each of the four planets for T1 and T2 encounters. First we notice that the rates for T1 and T2 encounters are almost the same.

Table 1. Loss and capture rates for the four giant planets in type 1 and 2 encounters. Note any captured planet must be lost from its original host in the first place.

|         | Loss (%) | Capture (%) |
|---------|----------|-------------|
| T1      | T2       | T1          | T2          |
| Jupiter | 4.70\(^{+0.57}_{-0.53}\) | 4.45\(^{+0.47}_{-0.41}\) | 1.62\(^{+0.86}_{-0.70}\) | 1.74\(^{+0.42}_{-0.37}\) |
| Saturn  | 8.00\(^{+0.39}_{-0.38}\) | 7.87\(^{+0.37}_{-0.36}\) | 2.55\(^{+0.39}_{-0.37}\) | 2.64\(^{+0.39}_{-0.38}\) |
| Uranus  | 15.59\(^{+0.70}_{-0.64}\) | 15.55\(^{+0.69}_{-0.63}\) | 5.28\(^{+0.39}_{-0.37}\) | 5.36\(^{+0.39}_{-0.37}\) |
| Neptune | 23.98\(^{+0.90}_{-0.82}\) | 24.12\(^{+0.83}_{-0.80}\) | 8.88\(^{+0.51}_{-0.48}\) | 8.58\(^{+0.50}_{-0.49}\) |

(Chambers 1999)
of encounter. A key factor is apparently \( r_{\text{enc}} \). We plot these rates as a function of \( r_{\text{enc}} \) in Figure 2 for the four planets, all increasing at smaller encounter distances. Interestingly, the capture limit coincides with that for loss, meaning that during these distant encounters, the only way to relieve a planet of its host star is to capture it. This can be seen already, for instance, from figure 5 of Malmberg et al. (2011) where the authors showed that the maximum encounter distances for loss/capture were similar. When the encounter is deep reaching a planet \( r_{\text{enc}} \sim a_p \), the loss rate is \( \sim 0.5 \). This rate increases to \( \sim 0.8 \) when \( r_{\text{enc}} \ll a_p \) and the capture rate rises to 0.5. When normalised against the semimajor axes, the profiles of these curves seem similar.

In a subtler way, the inclination of the flying-by star \( i_{\text{enc}} \) also plays an important role in constraining the encounter geometry and thus the relative velocity between a planet and the flying-by star. In Figure 3 we show how loss/capture rates rely on \( r_{\text{enc}} \) and \( i_{\text{enc}} \) for Neptune. Seemingly two modes of capture, one characterised by large \( r_{\text{enc}} \) and low \( i_{\text{enc}} \) and the other by small \( r_{\text{enc}} \) with wider ranges of allowable \( i_{\text{enc}} \), emerge (and maybe some intermediate modes). This is a subset of encounters that lead to planet loss which also seem to have different modes. Prograde encounters are more effective in destabilising a planet in the target system (leading to loss or capture) at larger distances.

We show in Figure 4 the evolution of relative and escape velocities of Neptunes captured via the two modes with respect to the two stars. In both examples, Neptune is initially orbiting star 2–its relative velocity is smaller than escape velocity for that star: \( v_{\text{rel},2} < v_{\text{esc},2} \). After capture, it is bound to star 1 and \( v_{\text{rel},1} < v_{\text{esc},1} \).

In the top panel (large \( r_{\text{enc}} \) capture), before the closest approach at 0 yr, \( v_{\text{esc},1} \) steadily increases, meaning that star 1 is approaching Neptune. At around -70 yr, \( v_{\text{rel},1} < v_{\text{esc},1} \) but capture has not yet finished and the planet is still closer to star 2 (\( v_{\text{esc},1} < v_{\text{esc},2} \)). Then at -30 yr, \( v_{\text{esc},1} \) surpasses \( v_{\text{esc},2} \). From this time the gravitational pull of star 1 overtakes, dragging Neptune toward it. Finally at around 0 yr \( v_{\text{esc},2} < v_{\text{rel},2} \).

In the bottom panel (small \( r_{\text{enc}} \) capture), things happen more drastically. Before 0 yr, \( v_{\text{esc},2} > v_{\text{esc},1} \). i.e., Neptune stays closer to star 2 than to star 1. At 0 yr, the two stars are closest and star 1 begins to retreat as viewed from star 2. Now Neptune happens to be moving in the same direction as the motion of star 1. Hence, instantly \( v_{\text{rel},1} \ll v_{\text{rel},2} \) and \( v_{\text{esc},1} > v_{\text{esc},2} \); soon afterwards \( v_{\text{rel},1} < v_{\text{esc},1} \) capture is finished.

In addition to \( r_{\text{enc}} \) and \( i_{\text{enc}} \), other orbital parameters (e.g., the argument of pericentre) may also affect loss/capture (Pfalzner et al. 2018, 2005).

The orbits of planets captured through the two modes present different features. In Figure 5 we show the orbital elements of captured Neptunes as a function of \( r_{\text{enc}} \). At \( r_{\text{enc}} \lesssim 50 \) au, the resulting semimajor axis \( a \) is usually small <100 au and eccentricity \( e \) covers a large range from zero to unity. On the other hand, for small \( r_{\text{enc}} \) captures, \( a \) may reach >100 au, though there is a preference for small values.

**Figure 2.** Loss and capture rates as a function of \( r_{\text{enc}} \) for each of the planets. Solid lines are Loss rates and the dashed ones that of capture. The shaded region represent bootstrap errors at 95% confidence level.

**Figure 3.** Loss (left) and capture (right) rates as a function of encounter distance \( r_{\text{enc}} \) and \( i_{\text{enc}} \) (inclination of the encounter) for Neptune. Warmer colours mean higher chances. When integrated over \( i_{\text{enc}} \), this plot turns into Figure 2.

**Figure 4.** Two capture modes (large \( r_{\text{enc}} \), top panel and small \( r_{\text{enc}} \), bottom panel) exemplified by Neptune. In both panels, the Neptune is originally orbiting star 2 (relative velocity is smaller than escape velocity for that star \( v_{\text{esc},2} > v_{\text{esc},1} \)) while after, both are captured by star 1 (\( v_{\text{esc},1} > v_{\text{rel},1} \)). See text for details.
Meanwhile, the orbits are predominantly highly eccentric. With moderate $r_{\text{enc}}$, $a$ and $e$ show intermediate features.

Looking at the cumulative distribution function (CDF) of the elements in the right hand side panels of Figure 5, we find out that $\gtrsim 20\%$ are with $a > 100$ au. Distribution of $e$ is close to thermal but with an excess of large values, possibly due to a higher fraction of objects from small $r_{\text{enc}}$ captures.

We do not show the distribution for inclination and they are roughly symmetric with respect to $90^\circ$ (cf. Figure 12). This is because we measure the orbit of a captured planet with respect to the ecliptic of the new system, assumed to be oriented randomly.

Finally, in Figure 6, we present the orbital distribution for all the original and captured planets in the $(a,e)$ plane. The original planets are mostly moderately excited with $e$ up to 0.3 but large changes in $a$ and $e$ are also seen, overall in good agreement with existing studies (Laughlin & Adams 1998). As for the captured planets, the width of the orbits captured onto is positively related to the primordial semi-major axes–small initial often leads to small captured orbits.

### 3 POST ENCOUNTER LONG-TERM EVOLUTION

Having established that during the encounter phase planets play a minor role, we now proceed to investigate if their effect unfolds in the long-term post encounter evolution. We call the entire time span the post-encounter phase, as opposed to the encounter phase.

#### 3.1 Long-term evolution of a solar system perturbed by a flying-by star

##### 3.1.1 Initial condition

Irrespective of whether loss/capture of planets occurs, we randomly pick 1000 cases from T1 encounter simulations. The state vectors of the target star and the planets immediately after the encounter form the initial conditions for post encounter phase simulations. Each system is integrated in isolation for $10^8$ yr. These are referred to as T1 simulations.

In a similar way, 1000 systems are chosen from T2 encounter simulations and propagated for $10^8$ yr as well. Here we omit the captured planets in T2 encounters and call these T2 simulations. Hence, these systems are only briefly perturbed by the planets in the flying-by system during the encounter phase (thus they are flying-by planets). Last section shows that the flying-by planets cannot effect significant immediate disturbance on the target system. The purpose for this $\overline{T2}$ simulation is to examine whether these flying-by planets can have a delayed effect visible in the post encounter long-term evolution.

We then introduce a third set of simulation where we have the same systems as in $\overline{T2}$, the difference being that now captured planets are included. As shown in Table 1, the capture rates during these encounters are $\lesssim 10\%$. So in practice, we only need to rerun the simulations for about 100 systems, and for the remaining 900, no capture occurs and we take the results directly from $\overline{T2}$ simulations. These are our T2 simulations.

##### 3.1.2 Results

In Table 2 we show the survival rates at $10^8$ yr immediately after encounter and at $10^8$ yr of the T1 and T2 simulations. Being a down-sampling of those presented in Table 1, the statistics at $10^8$ yr agree in the two tables. Note here we present the survival rates whereas loss rates are shown in that table.

As already been pointed out (Malmberg et al. 2011; Hao et al. 2013), instability gradually develops in the post encounter evolution and the planets are destabilised severely.

Here, we observe that Uranus is the most vulnerable to instability, and is out-survived by Neptune, despite the fact that the latter is lost at higher rates during encounter phase. To be specific, while $1/3$ of Uranus’ loss occurs during encounter, the remaining $2/3$ gradually shows up post encounter. Notably, Saturn is characterised by a similar feature–greater extent of destabilisation during post encounter phase than encounter phase. Both two planets have
Table 2. Rates of stability of original planets immediately after encounter (IAE) and at 10⁸ yr into our long-term simulations for T2 (between two solar systems) and T1 encounters (between a solar system and a solar-mass star). In the T2 simulations, however, captured planets (if any) are omitted. Thus, the planetary systems in these simulation experience similar perturbation to those in T1 encounters. We refer to these as T2 but not T2.

|       | T1-IAE (%) | T1-10⁸ (%) | T2-IAE (%) | T2-10⁸ (%) |
|-------|------------|------------|------------|------------|
| Jup   | 95.3±1.2   | 95.1±1.2   | 95.6±1.2   | 95.6±1.2   |
| Sat   | 91.4±4.5   | 78.6±2.6   | 91.0±4.4   | 76.0±2.2   |
| Ura   | 84.0±2.3   | 51.6±3.3   | 84.6±2.8   | 51.6±3.3   |
| Nep   | 74.9±2.4   | 56.6±3.8   | 77.2±2.5   | 57.9±3.1   |

Figure 7. Distribution of \(a\) and \(e\) for the four planets at immediately after encounter (top row, IAE) and post encounter (bottom row, at 10⁸ yr). The left column show simulation results for T1 encounters and right T2 encounters (omitting captured planets, \(\tilde{T}2\)). See Figure 1 for colour coding.

more massive inner neighbours meaning that if they gain significant eccentricity, their orbits intersect these planets and may be ejected.

For Neptune, the fractional loss during the two phases is similar. Jupiter suffers from little instability during the post-encounter phase. This is to say that the only way to effectively destabilise Jupiter is to do that during the encounter phase. This is to say that the only way to effectively destabilise Jupiter is to do that during the post-encounter phase (see also Hao et al. 2013).

The majority of the destabilised planets end up ejected (2/3), often by Jupiter (Nesvorný & Morbidelli 2012). About 1/3 are engulfed by the Sun whereas planetary collisions are rare due to their mutual orbital inclinations (e.g., Rice et al. 2018).

Comparing the numbers for T1 and \(\tilde{T}2\) encounters, we find that the difference is negligible and we conclude that a planet that briefly flies by with its host star has little long-term influence on the target planetary system.

The distributions of \(a\) and \(e\) at 10⁸ yr immediately after encounter and at 10⁸ yr for T1 and \(\tilde{T}2\) simulations are shown in Figure 7. While those at 10⁸ yr are consistent with Figure 6, those at 10⁸ yr roughly agrees with figure 10 of Malmberg et al. (2011) where the authors studied the long-term evolution of the solar system’s giant planets after stellar encounters.

Looking at the Jupiters at at 10⁸ yr, we can identify a few subpopulations (see also Hao et al. 2013), one at its starting location \(a = 5.2\) au with small to intermediate \(e\), a group experiencing little post-encounter evolution and another at \(\sim 4.4\) au with moderate to large \(e\), caused by ejecting Saturn. Another small concentration shows up at 5 au with \(e\) slightly heated up, a result of the interaction with the icy planets.

Saturn develops a small pile-up at 11 au, due to the fact that it cannot eject the ice planets effectively (e.g., Cloutier et al. 2015) and usually transport them inward (Fernández & Ip 1984; Malkhotra 1995); thus it gains angular momentum and jumps outward.

In terms of eccentricity, except Jupiter, all three seemingly become colder owing to removal of high-\(e\) components. Saturn, for example, is apparently eliminated severely beyond \(e > 0.3\). On the other hand, Jupiter is heated up and a large fraction achieve \(e > 0.3\) during the post encounter phase, a hint of the effect of stellar encounter, as otherwise self-excitation within the planets hardly boosts Jupiter to \(e > 0.3\) (c.f., Carrera et al. 2016).

The stability and orbital elements immediately after encounter both depend on the encounter distance during the encounter phase (Spurzem et al. 2009; Pfalzner et al. 2005). Is this information wiped out during post-encounter evolution? In Figures 8 and 9, we show the survival rate and median eccentricity of the planets as a function of \(r_{enc}\) for the two types of encounters.

Reading from the two figures, Jupiter can only be destabilised by encounters <10 au and only during encounter phase. Its eccentricity can be directly excited greatly during
encounter phase by such encounters or to a smaller extent in post-encounter phase by interplanetary interactions. In the former case, nonetheless, because the system is disrupted to a large extent, Jupiter has no planets to interact with and its $e$ is fossilised in post-encounter phase. Jupiter cannot feel more distant encounters ($r_{\text{enc}} > 30$ au) much.

For $r_{\text{enc}} < 10$ au, Saturn is further depleted during post-encounter phase but eccentricity distribution unchanged. A feature for Saturn is a strong depletion for moderate encounters (10 au $\lesssim r_{\text{enc}} \lesssim 30$ au) with a drop in $e$ by $10^3$ yr because of the removal of the excited ones. Though distant encounters with $r \gtrsim 30$ au cannot destabilise Saturn during encounter phase, they may induce strong interplanetary interactions that eliminate Saturn during post-encounter evolution.

The stability feature of the ice planets are similar to Saturn’s: encounters at larger $r_{\text{enc}}$ are able to cause damage post encounter though not during encounter. Uranus being affected the most severely. The evolution of $e$ is complicated and related to the elimination of high-$e$ components. In general, Neptune becomes cooler while Uranus may be heated up.

Due to post-encounter phase interaction, destabilisation of Saturn, Uranus and Neptune is less dependent on the encounter itself.

Again, we observe no difference between T1 and T2 simulations. Can captured planets change this picture? We now proceed to examine the results of our T2 simulations, where captured planets are included during post-encounter phase.

In Table 3, we present the survival rates for T2 simulations. Comparing those for original planets in T1 and T2 (Table 2), all original planets are more prone to instability, but only marginally and consistent with the error. Out of the 1000 systems in T2 simulations, only ~100 manage to capture planets from the flying-by star. Thus, the effects of the captured planets are greatly diluted. On the other hand, captured planets themselves are removed efficiently: except Jupiter, less than half survive till the end of the simulation. Hence, in the end we only have a few tens of captured planets left.

### 3.2 Long-term evolution of a solar system capturing one or more planets during an encounter

In order to obtain better statistics for the long-term evolution of the captured planets, we now perform new post-encounter phase long-term simulations to $10^3$ yr.

#### 3.2.1 Initial condition

From all T2 encounter phase simulations, we pick all planet-capture systems, a total of 1390. Two sets of simulations are carried out for these systems. In the first, we have both populations of captured planets (1783) and original ones (3559); this is our T2E simulations (“E” stands for extended). In the second, only the original ones are included; this is T2E.

Here the integration time is $10^3$ yr, because we want to resolve the instability at later times (e.g., Hao et al. 2013). In addition, these two sets of simulations are not to be directly compared with those in Section 3.1, since we are now using a biased subsample of the encounters (Figure 3).

#### 3.2.2 Results

As before, we first count the fraction of surviving planets in Table 4. Because of the bias toward close-in encounters, immediately after encounter, we already see a far greater extent of destruction.

Looking at the original planets in T2E simulations, as analysed in Section 3.1, a larger degree of disruption occurs during the post-encounter phase for Saturn and Uranus and this phenomenon looks more pronounced here. Similar to T2 simulations, the captured planets, except Jupiter, are also destabilised significantly and usually less than half survive, relative rates roughly consistent with Table 3. Uranus, for example, is removed by a relative fraction of 65% among the initial captured population. The fact that Jupiter is most resistant to interplanetary interactions agrees with previous studies (e.g., Hao et al. 2013).

It is interesting to note that the captured planets are not necessarily easier to destabilise than the originals. For
greater degree of destabilisation among the original ones, ac-
three become significantly stabler if we turn off the captured
the total number of planets evolve from
where no captives are considered. See text for details.
biased sample in terms of of encounter distance whereas in Table
10
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432
by solar system (because we are only interested in these
four original planets and no captives, being “40”. During the
number of originals and the second that of captives) fol-
each label comes with a two-digit number (first digit being
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whether it encounter a single star or another planetary sys-
spectively. So in this sense, for a given planetary system,
numbers, in
creases dramatically during the post encounter phase. For
instances, respectively. We note that as shown in Section 2,
Fly-by encounters between two planetary systems I
example, captured Uranus out-survive its original counter-
are not affected much from the captured planets. All together, the captured planets, though inducing greater degree of destabilisation among the original ones, act-
ually increase slightly the overall multiplicity both immedi-
ately after encounter and post encounter at 10^9 yr. Typically, the total number of planets evolve from 3.84±0.06 (2.56±0.07
originals+1.28±0.03 captives) immediately after encounter to
1.81±0.04 (1.35±0.04+0.46±0.04) at 10^9 yr. The corresponding numbers, in T2E simulations, are 2.56±0.06 and 1.68±0.05,
respectively. So in this sense, for a given planetary system, whether it encounter a single star or another planetary sys-
tem does not affect the number of planets in this system in the long term.
Figure 10 illustrates how the numbers of original and captured planets evolve during and post encounter. Here, each label comes with a two-digit number (first digit being the number of originals and the second that of captives) fol-
by, after the colon, another showing the number of cases in our simulations. Before encounter, all systems have four original planets and no captives, being “40”. During the encounter, all acquire at least one planet from the flying-
by solar system (because we are only interested in these systems here). Most frequently, for 432/1390 ~ 1/3 of the cases, one planet is captured without removing any of the four originals. The captive is predominantly Neptune and sometimes Uranus because capturing these two is possible at large encounter distances, without disrupting the origi-
nal planets. Also quite often, one original planet is ejected on capturing another (“31”) and this is observed 283 times.
Not surprisingly, “21” and “11” come next, with 198 and 122 instances, respectively. We note that as shown in Section 2,
to retain an original planet and to capture a planet from the flying-by system are two independent processes. Hence, in all “n1” cases, n being 1,2,3 or 4, the captured one is, in descending order of likelihood, Neptune, Uranus, Saturn or Jupiter. More than one planet can be captured and notably in 19 cases, all four planets hop from the one system to the other. And we point out that it is not rare to replace all original planets with captives, resulting in “0n” (n = 1,2,3 or 4) systems immediately after encounter.
As we have discussed before, the number of planets de-
creases dramatically during the post encounter phase. For example, only one system of “41” out of the 432 cases im-
mediately after encounter is able to keep all the five plan-
erts throughout the 10^9 yr post-encounter simulation. In this specific example, a Neptune is captured onto a wide orbit with a ~ 300 au during a r_{enc} ~ 50 au encounter. The original planets are almost not affected by the encounter while the captured Neptune, due to its small mass and large orbit, is unable to disrupt the originals (see, e.g., Inmanen et al. 1997). This is just one special case and a vast major-
ity of such systems lose the captured planet during the long term evolution, evolving to “n0” and actually only 11 systems managing to keep all originals (n = 4). This often occurs when the captive is Neptune or Uranus due to their small masses and thus vulnerability to instability. When Jupiter is captured, it likely survives to the end.
Finally, we have look at the planet numbers at the end of our post encounter simulation. Agreeing with Table 4, Fig-
ure 10 shows that losing the captured planets is the norm and more than half of the systems end up without a captive. Not surprisingly, the most common outcome is “10”, totalling 346. Among these, 302 are left with Jupiter only, usually on highly eccentric orbits (median eccentricity 0.35). A captured planet, if kept during the post-encounter phase, may coexist with an original, often Jupiter accompanied by a captured ice planet on a well separated orbit, forming a hierarchical system. Also, we observe over 200 systems that end up with one or more rarely two, captured planets, accounting for 20% of all systems.
Now we turn to orbital features of the planets. Because the T2E simulations consider all captured planets, we can compare their distribution immediately after encounter (bottom panel of Figure 6) and at 10^9 yr into post-encounter
main, by and large, unchanged. Those with extreme eccentricity close to unity may be preferentially eliminated and $i$ seems to shift toward prograde orbits slightly, but still nearly half survive on retrograde orbits. On contrary, we observe systematic variations in $a$. Both ice planets show more frequent presence on wider orbits (Malmberg et al. 2011). For example, 40% of surviving Neptunes have $a > 100$ au. Indeed, these planets are totally removed inside of $10-20$ au due to strong interaction with the gas giants. That for Saturn also develops more weight in its PDF on wider orbits at $10^8$ yr owing to elimination by Jupiter. That of Jupiter has not changed much except for the emergence of a small peak at $a \sim 4.4$ au, possibly owing to ejecting Saturn; a same but more pronounced feature is seen for original Jupiter (Figure 11).

4 DISCUSSION

4.1 Cross-sections at different times

We begin our discussion with cross-sections. Here we have only simulated a single encounter and the long-term aftermath in a Monte-Carlo way. To put our simulation results into context, it is useful to calculate the so-called cross-sections. Following Adams et al. (2006), we define the interaction cross-section as

$$\sigma = \int_0^{r_{\text{max}}} p(r)2\pi rdr \left( 1 + \frac{v_{r,\text{esc}}}{v_{\text{inf}}} \right).$$

Here $p(r)$ is the probability that given an encounter at distance $r$ (not impact parameter $b$), a planet is destabilised/captured (and kept) during encounter/post-encounter phases. $r_{\text{max}}$ is the largest distance at which the two events may occur and is 100 au in our calculation. The factor inside the brackets quantify gravitational focusing (Malmberg et al. 2011), translating $r$ to impact parameter $b$; $v_{r,\text{esc}}$ is the escape velocity at $r$ and $v_{\text{inf}} = 1 \text{ km s}^{-1}$.

We calculate four cross-sectional areas for each planet. That is destabilisation immediately after encounter and at $10^8$ yr into post-encounter phase for original planets, and capture immediately after encounter and (stable) at $10^8$ yr for captured planets. For original planets, $T1$ and $T2$ are combined and for the captured planets, $T2E$ is used but truncated at $10^8$ yr in order to be consistent with the originals. The results are summarised in Table 5.

We note that the values obtained here immediately after encounter are larger than those presented in Adams et al. (2006); Li & Adams (2015); Laughlin & Adams (1998) by factors of a few to a few tens. We deem that the difference mainly stems from (1) the stellar masses of the stars: we only consider encounters between solar-type stars where a realistic spectrum of stellar mass was investigated in previous works and (2) the relative velocity of the two systems: here we fix it at $v_{\text{inf}} = 1 \text{ km s}^{-1}$, while these works adopted a distribution covering larger values (see also Hanse et al. 2017). While the absolute numbers in our work may not be directly applicable, of ratios for the values between different planets at immediately after encounter are consistent with literatures and the sizes are roughly linear with respect to $a$ (Li & Adams 2015).

Our long-term simulations allow us to evaluate the
We caution that our estimated cross-sectional areas at 10^8 yr may not be best constrained these long-term cross-sectional areas, which will lead to better knowledge on the birth cluster of the solar system (Adams 2010).

4.2 Miscellaneous

Recently, a so-called Planet-Nine hundreds of au away from the Sun was proposed to explain the orbital clustering of distant Trans-Neptunian objects (Batygin & Brown 2016). As previously suggested, it could be captured by the Sun from a flying-by star (Mustill et al. 2016). Such captured planets on wide orbits are also observed in our simulations. Additionally, we find that the original planets can be perturbed onto large-size orbits during an encounter (see also Malmberg et al. 2011). These captured/pumped-up planets may have their pericentres beyond the inner solar system and survive the long-term planet-planet interactions. Though, in our scenario (encounter distance r_enc < 100 au), there is a good chance that the outer solar system is greatly excited (Pfalzner et al. 2005).

Tens of planetary-mass objects on ~ 100 au orbits have been found by direct imaging and the occurrence is estimated at a few % (Ireland et al. 2011). For example, GSC 6214-210 b is orbiting a solar mass star at 240 au and an unlikely product of planetary scattering (Pearce et al. 2019). Here our result that shows such objects can be created during stellar encounters, either scattered or captured onto (see also Malmberg et al. 2011) and are stable post encounter.

The inner terrestrial planets are not accounted for in our simulations because inclusion of them would require much smaller time steps. Since immediately after encounter, the destabilisation cross-sections are proportional to the orbital size of a planet (see Table 5 and cf. Li & Adams 2015, for instance), the terrestrial planets are relatively invulnerable during encounter phase (Laughlin & Adams 1998). Extrapolating from Table 1, Earth only has a chance of <1% of being ejected during an encounter <100 au. However, these less massive inner planets are exposed to stronger destabilisation later during post-encounter phase (Laughlin & Adams 2000), when the giants develop instability (Mustill et al. 2017; Carrera et al. 2016). It was recently suggested that the appearance of close-in super earths might be positively correlated with that of outer cold Jupiters around solar type stars (e.g., Zhu & Wu 2018). If so, our results would imply that these (stable) systems might not have been born in dense cluster where encounters are frequent.

We capture a few hundreds of Jupiters in our simulations. Among these, one is captured onto a highly eccentric orbit with pericentre distance <0.05 au and a = 2.3 au which is lost later due to interactions with its original equivalent. If the original Jupiter did not exist, this Jupiter would have survived and it would be circularised by tides onto a 5.3 day orbit, turning into a hot Jupiter. Brucalassi et al. (2016, 2017) showed an excess of such planets in dense clusters. So our result points to a new formation channel, though probably less likely than some others (e.g., Shara et al. 2016). We note, as Jupiter is captured with random orbital inclinations, our model may be particularly relevant for retrograde hot Jupiters.

Many exoplanets reside on orbits significantly tilted against the equator of the host star (often close-in and observed via, e.g. Rossiter-McLaughlin effect; see Triaud 2018). A few tens are even rotating with projected inclinations > 90° (Breslau & Pfalzner 2019). In our simulations, both captured and original planets can become retrograde rotators.

In our long-term simulations for plant-capturing systems, 597 systems possess captured planets at the end. The occurrence rates for prograde and retrograde captives are similar (Figure 12), so roughly 300 systems (out of the 10000 in all encounters) have counter-rotating planets, meaning an occurrence rate of ~ 3%.

On the other hand, the original planets can also become retrograde, mainly under the direct effect of the flying-by star during encounter (see also Breslau & Pfalzner 2019) but not due to the interplanetary interactions (cf. Chatterjee et al. 2008; Hao et al. 2013). In our long-term post encounter simulations, 68 out of 2000 systems retain retrograde
originals—the occurrence rate for a system with retrograde originals is $\sim 3\%$.

Suppose a certain fraction of solar system analogues, say $\alpha$, experience an encounter $< 100$ au with another. This means that $>0.06\alpha$ of such have retrograde planets.

While likely inaccessible to transit-based Rossiter-McLaughlin effect, retrograde planets created in our simulations are detectable through techniques that probe the system's motion in the plane of the sky. The reflex astrometric motion of the host star provides the full orbital geometry of a planet, up to a degeneracy in separating the ascending from the descending node of the orbital plane (Perryman et al. 2014; Ranalli et al. 2018). A ten-year Gaia mission lifetime will enable the detection of a Jupiter-mass planet at 4 au around stars out to 70 pc (Ranalli et al. 2018), with its orbital motion reliably determined up to the nodal degeneracy. Captured planets rarely end up on such tight orbits (Figure 11), but the original Jupiter remains in half of the systems with surviving captured planets. The captured planets may be detected by complementary direct imaging, which also determines the full orbit with the same nodal degeneracy (Alzner & Argyle 2012). Current direct imaging instruments such as SPHERE can detect substellar objects with a contrast ratio of $10^{-4}$ at 0.3 arcsec (Beuzit et al. 2019), or 20 au at 70 pc, while ELT-CAM has a goal of a contrast of $5 \times 10^{-6}$ at 0.1 arcsec (7 au at 70 pc). Thus, even where direct imaging itself cannot discover the inner prograde planet(s), its combination with Gaia astrometry will indeed enable the identification of such retrograde systems.

5 CONCLUSION

We have performed $N$-body simulations looking into fly-by encounters between two solar system analogues, each carrying four giant planets. Our simulations consist of two phases: the instantaneous evolution during encounter and the long-term evolution post-encounter. Our main findings are:

(i) Planets can jump from one star to the other during a close stellar encounter. This is not rare and the capture (immediate loss) ratio is about 1/3.

(ii) Planet-planet interactions are negligible during encounter, affecting neither loss nor capture.

(iii) A flying-by star, if approaching the target solar system in the same direction as the rotation of the planets, can more effectively destabilise/capture planets at further encounter distances.

(iv) During an encounter, a planet can be scattered/captured onto orbits orders of magnitude wider than its initial orbit. Largely decoupled from the inner system, such a planet can be stable for Gyr.

(v) Post-encounter, interplanetary interactions induce great extent of planetary destruction. Less massive planets (those other than Jupiter) are especially vulnerable.

(vi) Except in the case of Jupiter, capturing a planet does not say much about retaining it. Using cross sectional area estimated, we show that to keep a captured planet is several times as difficult as to capture it.

(vii) A captured planet significantly increase the degree of instability. As a result, in a statistical sense, whether a flying-by star has planets (and hence can/cannot be captured) does not affect the multiplicity of the target planetary system.

(viii) Flybys can place wide-orbit planets onto retrograde orbits, either by capturing a planet directly onto a retrograde orbit, or by flipping the orbit of an existing wide-orbit planet. In many such systems at least one inner planet survives on a prograde orbit. Retrograde systems will soon become detectable through direct imaging supplemented by astrometry of the stellar reflex motion.

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\[1\) See https://www.eso.org/sci/facilities/eelt/docs/ESO-193104_2_Top_Level_Requirements_for_ELT-CAM.pdf

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