Dark Matter Halo Properties vs. Local Density and Cosmic Web Location

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ABSTRACT

We study the effects of the local environmental density and the cosmic web environment (filaments, walls, and voids) on key properties of dark matter halos using the Bolshoi-Planck ΛCDM cosmological simulation. The \( z = 0 \) simulation is analysed into filaments, walls, and voids using the SpineWeb method and also the VIDE package of tools, both of which use the watershed transform. The key halo properties that we study are the specific mass accretion rate, spin parameter, concentration, prolateness, scale factor of the last major merger, and scale factor when the halo had half of its \( z = 0 \) mass. For all these properties, we find that there is no discernible difference between the halo properties in filaments, walls, or voids when compared at the same environmental density. As a result, we conclude that environmental density is the core attribute that affects these properties. This conclusion is in line with recent findings that properties of galaxies in redshift surveys are independent of their cosmic web environment at the same environmental density at \( z \sim 0 \). We also find that the local web environment of the Milky Way and the Andromeda galaxies near the centre of a cosmic wall does not appear to have any effect on the properties of these galaxies’ dark matter halos except for their orientation, although we find that it is rather rare to have such massive halos near the centre of a relatively small cosmic wall.

Key words: Dark Matter Halos — Milky Way Galaxy — Cosmology — Simulations — Walls — Filaments — Voids — Large Scale Structure

1 INTRODUCTION

The basic structure of the cosmic web was described in the early 1970s as arising from the one-dimensional gravitational collapse of adiabatic fluctuations into pancakes/sheets and subsequently two- and three-dimensional collapse into filaments and nodes/knots (Zel’dovich 1970; Doroshkevich, Shandarin & Zeldovich 1983). These ideas could be realised in detail (e.g., Bond, Kofman & Pogosyan 1996) when the cold dark matter theory was developed (Blumenthal et al. 1984) and the ΛCDM density spectrum of adiabatic fluctuations was supported by the anisotropy structure of the cosmic background radiation and other observational evidence. The modern consensus is now that about 26% of the cosmic density is cold dark matter, ∼5% is ordinary (baryonic) matter, and ∼69% is dark energy perhaps in the form of a cosmological constant (e.g. Planck Collaboration et al. 2016). The history and evolution of these concepts has recently been summarized in a major conference (van de Weygaert et al. 2016). The various modern methods for determining the structure of the cosmic web have been compared in Libeskind et al. (2017), and reference therein. These methods generally agree on the range of cosmic densities corresponding to voids, with greater dispersion between the different methods in the densities assigned to sheets, filaments, and nodes. The cosmic densities assigned to these various cosmic
This paper looks at effects of the cosmic web environment on the properties of distinct dark matter halos (i.e., halos that are not sub-halos) in the Bolshoi-Planck cosmological simulation (Klypin et al. 2016; Rodríguez-Puebla et al. 2016) at redshift $z = 0$. The main tool that we use to define the web in the simulations is the Spine of the Web (SpineWeb) (Aragón-Calvo et al. 2010; Aragon-Calvo & Sjávarsson 2013), which starts by identifying voids. The boundaries of voids are walls/sheets, and the boundaries of the sheets are filaments. Other popular methods include the T-web and V-web, in which the cosmic web structures are identified by analysing the tidal and velocity shear fields; for example, voids in the V-web are characterized by diverging velocities, while nodes/knots are locations of converging velocities. The methods such as SpineWeb – which do not identify nodes but do identify filaments, walls, and voids – agree that more than 80% of the halo mass is in filaments, with less in walls, and least in voids (Libeskind et al. 2017). The volume fraction assigned to voids is about 40% in methods including SpineWeb and T-web, although the V-web assigns a volume fraction of about 70% to voids (Libeskind et al. 2017). To verify that our conclusions are robust, we also used the Void IDentification and Examination toolkit (VIDE) package (Sutter et al. 2015), based on the Zobov void finder (Neyrinck 2008), to determine the voids of the Bolshoi-Planck simulation at $z = 0$.

Since the cosmic web has its origin in the gravitational astrophysics of cosmic density and dark energy, a major subject of the many papers investigating the cosmic web has been to identify how the evolution and properties of dark matter halos are related to their locations within the cosmic web. For example, one area in which there is considerable agreement regards the orientation of the angular momentum of dark matter halos. In simulations the spin vector of halos in walls tends to lie in the walls, while the orientation of the spin of halos in filaments depends on the halo mass and the redshift, with lower-mass-halo spins tending to align with the filament while higher-mass-halo spins tend to be perpendicular (e.g. Aragón-Calvo et al. 2010; Hahn et al. 2007; Libeskind et al. 2013; Aragon-Calvo & Yang 2014). Galaxy observations have not yet yielded a consensus on such spin orientations, although it is true that the spin of the disk of the Milky Way does lie in the Local Wall (e.g. Navarro, Abadi & Steinmetz 2004; Neyrinck 2008; McCall 2014). The edge of this Local Wall is demarcated by a $\sim 4$ Mpc radius ring (filament) of large galaxies that McCall (2014) refers to as the "Council of Giants," with the Milky Way and the Andromeda galaxy (M31) located near the centre. We show in the present paper that the presence of even one such massive halo in such a small wall is rather unusual.

There is strong evidence that properties of dark matter halos including their masses, and of the galaxies that they host including their masses and luminosities, differ in different web locations (e.g., Eardley et al. 2015, and papers cited there). However, it is important to disentangle the effects of the environmental density and of the web environment. It is the main purpose of the present paper to do this for dark matter halos at $z = 0$, and we will show that the many halo properties that we investigate (except for their orientation) are entirely determined by the environmental density. That is – at least for the definitions of halo properties and environmental density that we adopt – these halo properties at a given environmental density are the same regardless of whether the halo is in a void, wall, or filament. In addition, we do not find evidence at $z = 0$ of special properties of halos as massive as those of the Milky Way and M31 in walls as small as the Local Wall.

A recent paper has shown that the mass function of dark matter halos at the same environmental density is independent of the halo’s location in the cosmic T-Web (Alonso, Eardley & Peacock 2015). We study different halo properties in the present paper: mass accretion rate, spin parameter, concentration, prolateness, scale factor of last major merger, and scale factor when the halo had half of its mass. The dependence of these and other halo properties on the halo’s environmental density has been discussed in detail in a recent paper Lee et al. (2017a), and the present paper extends this analysis to include the location of these halos within the cosmic web. Our results appear to be consistent with observational evidence that properties of nearby galaxies at a given environmental density do not depend on their cosmic web location (Yan, Fan & White 2013; Eardley et al. 2015; Brouwer et al. 2016). However, there are indications that location in the cosmic web may influence certain properties of galaxies even at the same environmental density, both nearby – e.g., Guo, Tempel & Libeskind (2015) find that SDSS galaxies in filaments have more satellite galaxies than those in other cosmic web locations – and at higher redshifts (e.g., Laigle et al. 2015, 2018).

This paper is organized as follows: In §2 we describe the Bolshoi-Planck cosmological simulation and the methods that we use to find and characterize the dark matter halos, their local densities, and their cosmic web locations. In §3 we compare many properties of dark matter halos in four mass bins as a function of both their environmental density and their locations within the cosmic web, and we find that both the median values and the distributions of these properties are all determined by the environmental density rather than the cosmic web location. In §4 we study how often halos as massive as those of the Milky Way and M31 occur in cosmic walls the size of the Local Wall. In §5 we summarize and discuss our conclusions. The Appendix contains figures that supplement those in the text. Appendix A shows that halo properties are similar in walls of various sizes, at the same cosmic density. Appendix B shows that changing the distance of halos from the centres of filaments has little effect on the halo properties that we study. Appendix C shows that the distances of halos in small walls to the centre of their walls have little effect on their angular momentum, which lies in the direction of the planes of their walls.

2 METHODS

The following methods were used to study halo properties as a function of density in different web environments in the Bolshoi-Planck simulation with Planck parameters (Klypin et al. 2016; Rodríguez-Puebla et al. 2016): dark matter halos were found with RockSTAR (Behroozi, Wesccher & Wu 2013) and Consistent Trees (Behroozi et al. 2013); the cosmic dark matter density was Gaussian.
smoothed on different length scales (Lee et al. 2017a); and the Bolshoi-Planck simulation was grouped into filaments, walls, and voids using the SpineWeb method (Aragón-Calvo et al. 2010) and the VIDE method (Sutter et al. 2015).

2.1 Simulation & Halo Properties Studied

We use the Bolshoi-Planck simulation with 2048³ particles in a volume of (250 h⁻¹ Mpc)³ (Klypin et al. 2016; Rodríguez-Puebla et al. 2016). The Bolshoi N-body cosmological simulation was made with the Adaptive Refinement Tree (ART) code on the Pleiades supercomputer at NASA Ames Research Center. It uses the now-standard ΛCDM model of the universe and incorporates the results of the Planck Collaboration et al. (2014) with cosmological parameters: Ω_{m,0} = 0.693, Ω_{b,0} = 0.048, h = 0.678, n_s = 0.96 and σ_8 = 0.823. These cosmological parameters are compatible with the latest Planck results (Planck Collaboration et al. 2015).

We use the ROCKSTAR (Robust Overdensity Calculation using K-Space Topologically Adaptive Refinement) halo finder (Behroozi, Wechsler & Wu 2013) to identify dark matter halos in the Bolshoi-Planck simulation. ROCKSTAR is based on the adaptive hierarchical refinement of friends-of-friends groups of particles in six phase-space dimensions plus time. CONSISTENT TREES (Behroozi et al. 2013) generates merger trees and halo catalogues in a way that ensures consistency of halo mass, position, and velocity across time steps. This allows it to repair inconsistencies in halo catalogues, and add further information to properties found by ROCKSTAR.*

Out of the many halo properties found by ROCKSTAR and CONSISTENT TREES (Rodríguez-Puebla et al. 2016), the main dark matter halo properties that we study are:

(i) Specific mass accretion rate (dynamical time averaged) \( \dot{M}_{\text{dyn}} = \frac{dM_{\text{vir}}}{dt} \)

(ii) NFW concentration \( C_{\text{NFW}} \)

(iii) Spin parameter \( \lambda_{\text{B}} \)

(iv) Prolateness \( P \)

(v) Scale factor of the last major merger \( a_{\text{LMM}} \)

(vi) Scale factor when the halo had half of its mass \( a_{M/2} \)

The halo mass accretion rates averaged over a dynamical time are defined as

\[
\dot{M}_{\text{dyn}} = \frac{M_{\text{vir}}(t) - M_{\text{vir}}(t - \Delta t_{\text{dyn}})}{\Delta t_{\text{dyn}}},
\]

where the dynamical time of the halo is \( \Delta t_{\text{dyn}} = [G(\rho_{\text{vir}}(z)p_{\text{min}}(z))]^{1/2}, \rho_{\text{min}}(z) \) is the mean matter density at redshift \( z, \) and \( \Delta \) is the redshift-dependent virial overdensity (see, e.g., Rodríguez-Puebla et al. 2016, Fig. 2).

N-body simulations have shown that the density profile of most dark matter halos can be described by the Navarro, Frank & White (Navarro, Frenk & White 1996) profile:

\[
\rho_{\text{NFW}}(r) = \frac{4\rho_{\text{S}}}{(r/R_{\text{vir}})(1 + r/R_{\text{vir}})^{2}},
\]

where the scale radius \( R_{\text{vir}} \) is the radius where the logarithmic slope of the density profile is -2. The concentration parameter is defined as the ratio between the virial radius \( R_{\text{vir}} \) and the scale radius \( R_{\text{vir}} \):

\[
C_{\text{NFW}} = \frac{R_{\text{vir}}}{R_{\text{vir}}}. \]

Lee et al. (2017b) studied halos that has suffered significant mass loss due either to tidal stripping or to relaxation after mergers, and it shows that some such halos are not well described by Eq. (2).

The halo spin parameter (Bullock et al. 2001) is defined as

\[
\lambda_{\text{B}} = \frac{J}{\sqrt{2M_{\text{vir}}V_{\text{vir}}R_{\text{vir}}}},
\]

where \( J \) is the total angular momentum of a halo of mass \( M_{\text{vir}}, \) virial velocity \( V_{\text{vir}} \) and virial radius \( R_{\text{vir}}. \) Lee et al. (2017a) showed that the dependence of \( \lambda_{\text{B}} \) on density is similar to that of the Peebles spin parameter (Peebles 1969):

\[
\lambda_{\text{P}} = \frac{J}{GM_{\text{vir}}V_{\text{vir}}^{1/2}}.
\]

The prolateness of the spheroidal dark matter halo (Lee et al. 2017a) is defined as

\[
P = 1 - \frac{1}{\sqrt{2}} \left[ \left( \frac{b}{a} \right)^{2} + \left( \frac{c}{a} \right)^{2} \right]^{1/2},
\]

where \( a \geq b \geq c \) are the lengths of the largest, second largest, and smallest triaxial ellipsoid axes respectively, determined using the weighted inertia tensor method of Allgood et al. (2006). The prolateness of the simulated halos ranges from 0 (perfectly spherical) to 1 (maximally elongated, i.e. a needle), with most falling in the range 0.2 - 0.6 (Lee et al. 2017a).

2.2 Density and Cosmic Web Definition

Lee et al. (2017a) implemented a Gaussian smoothing procedure to compute the dark matter density of the full simulation volume smoothed on many different length scales. They convolved the 1/4 h⁻¹ Mpc cloud-in-cell (CIC) density cube with a 1-dimensional Gaussian kernel applied sequentially along each axis (x, y, z), and smoothed the box on scales of 1/2, 1, 2, 4, 8, and 16 h⁻¹ Mpc. Then they add to the information on each halo in the ROCKSTAR halo catalogue the CIC and smoothed density values corresponding to their locations in the simulation volume.

We used two different methods applied to the density field to delineate the cosmic web:

(i) SpineWeb (Aragón-Calvo et al. 2010) for filaments, walls, and small voids (MedianRadius\_Void  1.25 h⁻¹ Mpc) using the VIDE method (Sutter et al. 2015), which finds larger voids (MedianRadius\_Void  12.5 h⁻¹ Mpc) where MedianRadius\_Void is the median radius of all the voids in the simulation via the two different methods.
Aragón-Calvo et al. (2010) implemented SpineWeb using the Watershed Void Finder (Platen, van de Weygaert & Jones 2007) and the topology of the density field to delineate the cosmic web environment of a simulation into voids, walls and filaments. The identification of these web environments is done on three smoothing scales: 1, 2 and 4 $h^{-1}$ Mpc. For the analysis that is to follow, we only use the 2 $h^{-1}$ Mpc smoothing scale, for all analysis of the simulations.

As McCall (2014) had measured the radius of the Local Wall as $\sim$ 4 Mpc ($2.7$ $h^{-1}$ Mpc) with $H_0 = 71.6 \pm 2.9$ km s$^{-1}$ Mpc$^{-1}$, we set a radius range of $2 - 3.4 h^{-1}$ Mpc to define walls like our Local Wall as we look for halos residing in such an environment. We need a smoothing scale that encompasses enough halos within this radius to do meaningful statistics for small walls like our Local Wall. In addition, we also wanted to do statistics with walls of all sizes, of up to 9.5 $h^{-1}$ Mpc and more, and the smoothing scale would have to encompass enough halos at these radii as well. As it turns out, the best smoothing scale that gives the widest range is the $2 h^{-1}$ Mpc smoothing scale. The $4 h^{-1}$ Mpc smoothing scale yielded too few halos at the lower wall radius bound below 4 $h^{-1}$ Mpc (most of the halos have been smoothed out). Similarly, the $1 h^{-1}$ Mpc smoothing scale yielded too few halos at the higher wall radius bound above 6 $h^{-1}$ Mpc (most of the halos had been grouped into smaller walls instead of larger walls).

Using SpineWeb, Figure 1 shows visualisations of a cosmic wall like our Local Wall containing two Milky Way mass halos (i.e., with mass $\sim 10^{12} M_\odot$), viewed from three different directions. Here the white spheres are dark matter halos and the two red spheres represent the Milky Way mass halos in this wall. The green dots show the wall, and the teal spheres represent the nearby filaments, including the filaments bounding the wall.

The SpineWeb filaments and walls have the thickness of single voxels, cubes of side $0.25 h^{-1}$ Mpc, in the Bolshoi-Planck simulation. We introduce a parameter $D$, the distance to filaments. We define this as a radius around a filament, forming a cylinder within which the halos are defined to be within a filament. We use $D = 0.25 h^{-1}$ Mpc for this paper. (We also tried $D = 0.75 h^{-1}$ Mpc and $D = 0.75 h^{-1}$ Mpc is shown in the Appendix B3 &B4.) We similarly assign halos to walls that are within a distance $0.75 h^{-1}$ Mpc, and not assigned to filaments.

To find and characterize the properties of larger voids in the Bolshoi-Planck simulation, we used the VIDE (Sutter et al. 2015) method, which similarly calculated a Voronoi tessellation for estimating the density field, and performed a watershed transform to construct voids.

2.3 Analysis

Lee et al. (2017a) determined how halo properties including the specific mass accretion rate, $\lambda_0$, and $C_{NFW}$ depend on density for halos in all cosmic web locations. We are extending that work by grouping the halos into different web environments of filaments, walls, and voids, and studying the effects of density on halo properties in those web environments. We did not look separately for halo properties in filaments as 62% of halos are already within a cylindrical radius of $D = 0.25 h^{-1}$ Mpc of the filaments. Instead, we analyzed the halos separated into these three cosmic environments: all web environments, walls, and voids.

Note that we used the exact same halos as Lee et al. (2017a), where high-mass halos were randomly removed in each mass bin in order to remove dependence of density on mass and get a flat mass distribution in each mass bin. This was done so that the halo properties would be dependent on density alone. Our results below show the effects of web location and density on halos of the same mass.

3 RESULTS

We split the presentation of the results into 2 subsections:

(i) General results: we look at the halo properties of dark matter halos across all filaments, walls, and voids. The results here can be generalised across all distinct halos and are not confined to just those of our own Local Wall.

(ii) Local Wall results: we then go on to look in more details at walls, in particular walls like our Local wall, as we are particularly interested to see if halos in walls like our own Local Wall have properties different from other web environments.

3.1 General Results

For the general results, we will be looking at the following:

(i) The plots of median distributions of halo properties in different cosmic web environments by the SpineWeb method.

(ii) The histograms of the full distribution of these halo properties in different cosmic web environments by the SpineWeb method.

(iii) The halo properties in larger walls found using the VIDE code.

3.1.1 Plots of SpineWeb Halo Properties

In Figures 2a and 2b, we present the plots of halo properties (specific mass accretion rate, spin parameter, NFW concentration, prolateness, scale factor at last major merger, and the half mass scale factor) against density for halos in all types of environment, in small ($2 - 3.4 h^{-1}$ Mpc) walls, and in voids of all sizes, across various dark matter halo mass bins. We define ‘small’ walls as having size $2 - 3.4 h^{-1}$ Mpc in order to determine properties of halos of walls like our Local Wall, as McCall (2014) has found the edge of our Local Wall to be at a $1 h^{-1}$ Mpc smoothing scale yielded too few halos at the lower wall radius bound above $6 h^{-1}$ Mpc of the filaments. Instead, we

It should be noted that only the first two smaller-mass bins yielded enough halos to allow meaningful statistical interpretation, as there are not enough halos of the two higher-
mass bins in the different web environments to make a robust comparison, especially in voids.

To balance each mass bin to have a flat mass-density relation, Lee et al. (2017a) did a 2 dimensional sub-binning by halo mass and a given local density parameter for each given mass bin, then randomly eliminated halos from appropriate sub-bins to force approximately equivalent mass distributions for each density sub-bin. We used the halos in these mass bins, and then plotted the median of each halo property as a function of density for each mass bin.

In addition, we show the lower and upper bound on the 95% confidence interval of the median, which can be seen in the plots as the thick yellow band. Moreover, we performed a smoothing for the plots using the moving median, where we took the median value of the halo properties at every few points as we move from left to right across the density axis, in order to remove noise.

We see some tiny deviations of halo properties in the different web location of all environments, walls, and voids, particularly as the log-density increases from 0.5 onwards. For example, in Figure 2a, we see that the accretion rate in voids appears to fall below that in all environments around log-density = 0.8. Conversely, we see that for NFW concentration, the halos in voids appear to have a larger concentration than those in all environments. However, these deviations could be misleading, particularly because there are fewer halos towards the high-density end of each plot, especially for voids. As a result, the deviations could have just arisen due to insufficient data. In order to see if there is any effect that the cosmic web environment has on these properties, we need to look at the full distributions, instead of the medians, where some of the information might have been smoothed away. For these full histograms, see Figures 3a & 3b. It should be stressed that the wall results in Figures 2 and 3 are for the geometric environments of small walls (2 – 3.4h⁻¹ Mpc), filaments, and voids, as we are interested in knowing whether our own Local Wall has any peculiarity affecting these halo properties. These figures are discussed further in §3.1.2.

In order to account for halo properties in different size walls, we made similar plots for walls of different sizes in Figure A1a and A1b in the Appendix. We see that for the different-sized walls

(i) Small walls (2 – 3.4h⁻¹ Mpc)
(ii) Medium walls (3.4 – 6.8h⁻¹ Mpc)
(iii) Large walls (6.8 – 9.5h⁻¹ Mpc)
(iv) Extra-large walls (> 9.5h⁻¹ Mpc)

where the number of halos per wall that we ended up analyzing is 27405, 8185, 1910 & 324 respectively, the halo properties mainly fall within the lower and upper bound on the 95% confidence interval of the median. For the full distribution of halo properties, we refer to the histograms of all walls in Figures A2a and A2b in the Appendix, which can be seen in the plots as the thick yellow band. We note that there do not appear to be any real deviations in halo properties between the different size walls when compared in the same mass bin. Thus, we find that halo properties in all web environments, small walls, and voids are essentially the same for walls of all sizes at the same environmental density.

3.1.2 Histograms of SpineWeb Halo Properties

In addition to the median distribution plotted in Figures 2a & 2b, we also studied the full distribution of the halo properties. In Figures 3a & 3b we present the histograms of these full distributions of the halo properties in small walls, voids, and all environments. We split the histograms up into regions of low to high density in order to study the overall effect of densities on the distribution. As there are not enough halos in the tails of the histogram for meaningful statistics, we concentrate on the peak of the full distribution by cutting off the tails of each histogram in the range of the x-axis shown in the diagrams. We then split each range into 40 bins. The density ranges from log₁₀ρᵢ/ρₐve = -0.5 to 2.

To see the real limitation of the median plots of Figures 2a & 2b, we shall point out that the entire y-axis range of

Figure 1. A cosmic wall and dark matter halos visualized in the Bolshoi-Planck simulation. The SpineWeb method (Aragón-Calvo et al. 2010) uses Voronoi tessellation and the discrete watershed transform method to delineate the cosmic web environment of the z = 0 Bolshoi-Planck simulation into cosmic voids, walls, and filaments. Here we see three views of a wall visualised in green with nearby dark matter halos shown as white spheres, where the radii of the spheres corresponds to the virial radii of the halos. The two Milky Way mass halos near the centre of the wall are shown in red. The nearby cosmic filaments, including those bounding the wall, are shown in teal.
Figure 2a. Halo properties in all web environments (purple medians and yellow dispersion), small walls (blue medians), and voids (black medians) as a function of density, where $\rho_\sigma$ is density used on that smoothing scale. LEFT TO RIGHT: The columns have been split into 4 mass bins of $\log_{10} M_{\text{vir}}/M_\odot = 11.20 \pm 0.375, 11.95 \pm 0.375, 12.70 \pm 0.375,$ and $13.45 \pm 0.375$, with density smoothing scales of $\rho_\sigma$ in these mass bins as 1, 2, 4, and 8 $h^{-1}$ Mpc respectively. The smoothing scale in each mass bin was chosen so that it is much larger than the halos in that mass bin, so that the surrounding environment rather than the halo itself mainly determines the density value. TOP TO BOTTOM: The median distribution of specific mass accretion rate, $\dot{M}/M$, and $C_{\text{NFW}}$ are plotted vs. density in the four mass bins. For these properties, we are following the plots in Figure 5 of Lee et al. (2017a), with the additional step of grouping the halos into all environments, small walls, and voids via the SpineWeb method of Aragón-Calvo et al. (2010). This allows us to study the median halo properties in the different cosmic web environments. We calculated the median using the moving median method, where we ranked the halo properties according to their density and calculated the median halo property of a number of halos as we move towards higher density across each subplot. The thick yellow band represents the 5th – 95th percentile dispersion of the median of each halo property for all environments. This confidence interval for each halo property is given by $\sqrt{n}/2 \pm 1.96 \sqrt{n}/4 \Rightarrow 1$, approximated from the binomial distribution, where $n$ is the number of halos in each bin. Within each subplot, we used plot lines and scatter plots to represent the median halo distribution. We note that for all these median plots, the halo properties seem unaffected by the cosmic environment (although there are some tiny fluctuations), and they seem to be controlled instead by the local density. Lastly, we note that for the specific mass accretion rate, in the two lowest-mass bins, only halos in filaments at high densities are losing mass, as indicated by the negative value range of 0 to -5. Halos at lower densities in filaments, walls, and voids appear to be mainly accreting mass, as indicated by their positive values. The median plots here are for halos in all environments, small walls, and voids only. In the Appendix, Figure B4 shows the same data as this figure, but with filament radius $D = 0.75 h^{-1}$ Mpc instead of $D = 0.25 h^{-1}$ Mpc used here.
Figure 2b. Halo properties in all web environments, small walls, and voids as a function of density. LEFT TO RIGHT: The columns have been split into 4 mass bins with corresponding smoothing scales, as in Fig. 2a. TOP TO BOTTOM: The median distribution of prolateness $P$, scale factor of last major merger $a_{LMM}$, and the scale factor when the halo had half of its $z = 0$ mass $a_{M/2}$ are compared with density in different mass bins. For these properties, we are following the plots in Figure 9 and 10 of Lee et al. (2017a), with the additional step of dividing the halos into different cosmic web environments. While these figures in Lee et al. plotted the properties using percentile binning, we instead used the same method as in Figure 5 of Lee et al. in order to see better the different environmental effects on these properties. The median plots here are for halos in all environments, small walls, and voids only. In the Appendix, Figures A1a & A1b show similar plots of median properties in walls of different sizes.

-5 to $5 \times 10^{-11}yr^{-1}$ for the specific mass accretion rate of Figure 2a fits into just the median 10 bins out of 40 bins of the full distribution of the accretion rate of Figure 3a. This full distribution of the specific mass accretion rate, as well as for the other halo properties of $\lambda_B$ and $C_{NFW}$, shows us that any tiny deviations that arise out of the differences in the distribution of halo properties in filaments, small walls, and voids are almost entirely negligible.

To quantify this, we made curve-fitting plots for the specific mass accretion rate $\dot{M}/M$, $\lambda_B$, and $C_{NFW}$. In Figure 5, we fitted the accretion rate to a Gaussian distribution, and...
Figure 3a. Histograms of halo properties in all web environments, small walls, and voids as a function of environmental density, for halo mass $\log_{10} M_{\text{vir}}/M_\odot = 11.20 \pm 0.375$ with $\rho_s$ density on the smoothing scale of $1 \ Mpc$. LEFT TO RIGHT: In addition to plotting all densities, we also split density into 3 different ranges to explore the effect of low, medium, and high densities on the halo properties, and we plot at the right the histograms for large densities in large voids found using VIDE. TOP TO BOTTOM: Specific mass accretion rate $\dot{M}/M$, $\lambda_B$, and $C_{\text{NFW}}$ are compared with density in small walls (radius $2 - 3.4 \ Mpc$) (in blue), voids of all sizes (grey) and environments of all sizes (orange). $\lambda_B$ and $C_{\text{NFW}}$ follow a log-normal distribution, while the accretion rate follows a Gaussian distribution as the variables that make up its properties are random. Their chi-squared goodness-of-fit is less than 1, and the curve-fitting is detailed in Figure 5 of this paper. To create the bins, we cut off the tail-ends of each histogram at the ranges shown above, as there are not enough halos in the tails for meaningful statistics. Then, we split the distribution into 40 bins per property. We note that for the accretion rate, halos in all environments, small walls and voids are losing mass (negative values) as well as gaining mass (positive values) with no distinction between environments.

In Figure 4, we plotted the standard deviation and mean of the specific mass accretion rate $\dot{M}/M$ in various cosmic environments. The dotted line in the plot shows the distribution under gradual increase in size of the walls, and while these parameters for walls appear to be sandwiched between those for voids and all cosmic web environments, the effect on the general Gaussian distribution of the histogram of $\dot{M}/M$ is tiny, showing that the cosmic web environment has little effect on this halo property.

$\lambda_B$ and $C_{\text{NFW}}$ to log-normal distributions (as in Rodriguez-Puebla et al. 2016).

In order to create a curve through our data points, we took the y-axis quantity of each halo property bin and treated that as a scattered point through which we drew a data curve. We then obtained the modelled curve via NUMPY CurveFit, where the best fit parameters (mean and standard deviation) were acquired by getting the best fitted curve (the model) given the histogram (our data).
Next, we found the chi-squared between the modelled curve against our data curve for all different geometric environments. In each environment, we found that the chi-squared between our data curve and the modelled curve is always less than 1. As a result of the similar values between the fitting parameters in different cosmic web environments (voids, walls, and all web environments), we can conclude that the entire distribution of properties in different web environments is similar, and that the cosmic environment does not appear to affect the distribution of halo properties that we studied.

The light blue vertical dotted lines in Figures 3a & 3b indicate the 10 halos in 5 walls that are most like the Local Wall; further details are in §3.2.2.

### 3.1.3 Void Halo Properties Using VIDE

The SpineWeb method splits the simulation into voids...
that are mostly rather small, with an average void radius of \( \sim 5 \text{ Mpc} \). For large voids, we used the VIDE (Void IDentification and Examination toolkit) method (Sutter et al. 2015), where the voids are similar in size to those found by VIDE on SDSS Release 9. With this VIDE method, the average void radius is \( \sim 13 \text{ Mpc} \). In the last column of Figures 3a & 3b, we find that even with these larger voids, the histograms with various sized walls remain similar to those with smaller voids using the SpineWeb method.

We note that the larger voids of VIDE can host more halos, and as a result their full distribution void histograms are less fragmented than those using SpineWeb on smaller sized walls. We find that using the VIDE method produces histograms of properties that are a closer fit with one another across different cosmic web environments than using the SpineWeb method. For example, in Figure 2b, we note that the halos in voids appear to produce a higher prolateness in the first and second mass bin. We note that for the full distribution of the histogram of prolateness in Figure 3b, the prolateness of halos in voids is less than the halos in all environment and walls. We also note that with our \( D = 0.25 h^{-1} \text{ Mpc} \) filament radius parameter, halos in voids only account for 2% of all halos. However, using the VIDE method we obtain larger voids and 30% of halos reside in them. We see the result of this higher percentage in the last column of Figure 3b, where there are no differences between the prolateness of halos in all environments, walls, and voids. Hence, we conclude that for full distribution using the VIDE method, the cosmic web environment does not significantly affect the halo properties.

3.2 Local Wall

3.2.1 Angular Momentum Orientation in Local Walls

Aragon-Calvo & Szalay (2013) have found that the SpineWeb method produced dark matter halos in walls whose angular momentum axes lie close to the plane of the wall. We wanted to check that this property still holds true when the SpineWeb is applied to the Bolshoi-Planck simulation. To verify this, we first look for walls like our own Local Wall by applying the following cuts to the initial 19281 walls of the Bolshoi-Planck simulation:

(i) Radius (wall): \( 2 - 3.4 h^{-1} \text{ Mpc} \)
(ii) Mass (halos): \( 0.7 - 1.3 \times 10^{12} M_\odot \)
(iii) Distance of halo to filaments: \( > 0.25 h^{-1} \text{ Mpc} \)

It should be noted that these 3 cuts do not include a requirement that wall halos be \( < 0.75 h^{-1} \text{ Mpc} \) away from walls, which is a criterion we used elsewhere in the paper to group halos to different environment. We did not use this criteria in the analysis of the Local Wall as we wanted to look for halos exactly in the plane of the wall here (McCall 2014).

These three criteria above bring the number of walls down to 594, and the number of dark matter halos in these walls down to 702. The first two cuts give us the halos the size of the Milky Way in a wall the size of our Local Wall. The remaining cut (iii) is used throughout this paper as part of the criteria to assign halos to their respective walls, as SpineWeb only assigns halos to filaments and walls that are \( 0.25 h^{-1} \text{ Mpc} \) thick around the halos (as the size of a voxel by SpineWeb is only \( 0.25 h^{-1} \text{ Mpc} \) on each side), although the length and planar dimensions are much longer and larger. By setting the distance of halo to filaments to be \( > 0.25 h^{-1} \text{ Mpc} \), we are using assignment criteria that more accurately reflects the cosmic environment found in surveys in terms of locating wall halos away from filaments, while keeping those halos located exactly in the voxels of their wall. Using the distance of halo to filaments to be \( > 0.25 h^{-1} \text{ Mpc} \) has the additional effect of pre-selecting halos to lie closer to the centre of their walls for small walls, as \( > 0.25 h^{-1} \text{ Mpc} \) is large compared with the radii \( 2 - 3.4 h^{-1} \text{ Mpc} \) of small walls. However, these halos in the wall might still be too close to filaments if we just set the distance of halo to filaments to be just \( > 0.25 h^{-1} \text{ Mpc} \), so we explored limiting the calculation of angular momentum to just the centre of these walls in Appendix C, and the results we found there are qualitatively similar to the results we will show later in this section.

These three cuts in this section gives us enough walls (594) to still do meaningful statistics. For the 594 walls that are similar enough to our Local Wall, to determine whether the angular momenta \( \omega \) of these dark matter halos in the walls lie within the plane of those wall, we applied the following procedure:

(i) We found a pair of nearby vectors that lie in a locally flat region around the dark matter halo, by choosing the few pixels closest to the halo that are in the plane of the wall and drawing a line between those pixels to form a vector.
(ii) The cross product of these vectors is orthogonal to the plane of the wall
(iii) Finally, we calculate the dot product between this orthogonal vector and the \( \omega \) vector.
Figure 5. Distributions of specific halo mass accretion rate $\dot{M}/M$, spin parameter $\lambda_B$, and NFW concentration $C_{NFW}$ of halos within $< 0.75 h^{-1}$ Mpc away from walls and $> 0.25 h^{-1}$ Mpc away from filaments. Corresponding to Figure 3a, we fitted a Gaussian distribution to the specific accretion rate, and lognormal distributions to $\lambda_B$ and $C_{NFW}$. As a result of the similar values between the fitting parameters in different cosmic web environments (voids, walls, and all web environments), we can conclude that the entire distribution of properties in different web environments is similar, and that the cosmic web environment does not appear to affect the distribution of halo properties. We note that the median mass accretion rate for the small walls is lower than for other size walls; we explore this in the Appendix B3.
If $\omega$ vector were lying exactly in the plane of the wall, then its dot product with the orthogonal vector should be 0. We expect the angular momentum to lie close to the plane of the wall, so the angle that the $\omega$ vector makes with the plane of the wall should be less than 45°, and the angle that it makes with the orthogonal vector should be more than 45° (their dot product tending towards 0 in Figure 6). We applied the above dot product procedure to all the dark matter halos that are near the centres of those 594 walls. Figure 6 shows the cumulative distribution of these angles. Then we took the mean of those dot products, and obtained the following results:

- Of all dark matter halos near the centres of 594 walls, the mean of the dot products between their $\omega$ vector and the orthogonal vector = 0.452
- As a result, the mean angle ($\omega$ vector) = \sim 60° to the vector that is orthogonal to the wall
- The mean angle of $\omega$ vector with respect to the plane wall is \sim 30°.

We can see these results visualised in the cumulative distribution of dot products (between the $\omega$ vector and the orthogonal vector) in Figure 6. The orange dots is an idealised flat distribution of dot products, with the true cumulative distribution represented in blue. We can see that 50% of the halos corresponds to the mean dot product of \sim 0.45, which is \sim 30° to the plane of the wall. Moreover, if the $\omega$ vector were lying at \sim 45° to plane of the wall, then its dot product with the orthogonal vector will be \sim 0.707, as indicated in the Figure by the vertical red dotted line. This corresponds to nearly 80% of all halos making an angle of less than \sim 45° with the plane of the wall.

Hence, our results here confirm that for walls similar to our own Local Wall, using the SpineWeb method, the mean angle between the halo angular momentum and the plane of the wall is about 30 degrees, i.e. the mean angular momentum axis of wall halos lies close to the plane of the walls.

### 3.2.2 Halo Properties in the Local Walls

McCall (2014) found that our Milky Way galaxy and the Andromeda galaxy lie near the center of a wall of \sim 4 Mpc in radius. By grouping halos in the $z = 0$ Bolshoi-Planck simulation into filaments, walls, and voids, we can look for walls that most closely resemble our own Local Wall and study the properties of the halos residing in such walls. If our own Local Wall has special halo properties, that is, if the halo properties in such Local Wall do not fall generally within the median of their known distribution, then that would suggest that the near-field cosmology of our own Local Group is peculiar. The properties that we can find observationally of the Milky Way and its dark matter halo could not then be generalised for all galaxies and their halo companions.

To find walls like our own with the Milky Way and Andromeda galaxies in it, we performed the following cuts (with $h = 0.7$):

- (i) Radius (wall): 2 – 3.4 $h^{-1}$ Mpc
- (ii) Mass (Halos): 0.7 – 1.3 $\times 10^{12}$ $M_{\odot}$
- (iii) Distance (halo to filament): > 0.25 $h^{-1}$ Mpc
- (iv) Density (wall): 0.8 – 1.2 $x$ average cosmic density
- (v) Must occur in pairs of distinct halos
- (vi) Distance (between pairs): < 0.47 $h^{-1}$ Mpc
- (vii) Distance from centre of wall: < 1.75$h^{-1}$ Mpc

The first three items were used in §3.2.1 when we were determining the angular momentum. For this section, we extended these three cuts to include cosmic density (based on a survey of galaxies in the local region within about 10 Mpc Klypin et al. 2015), and that the pair of halos in the wall must be within 0.7 Mpc of each other, just like in our Local Wall. We present a flowchart of the different cuts we made to the simulation in Figure 7. From an initial 19281 walls, we are left with just 6 walls/12 halos that most resemble our Local Wall. On closer inspection, 2 of the halos were really just sub-halos of a group, unlike the Milky Way and Andromeda galaxies which are in separate distinct halos (being distinct means that though they are gravitationally bound together, their virial radii do not overlap). So we end up with just 5 walls/10 halos. We took these remaining 10 halos that are most like our own MilkyWay and Andromeda, and then further limit them to the centre of their walls, as...
mapped out by McCall (2014). This gives us just 3 walls/6 halos that are like ours in the Local Wall. Hence, we conclude that our Local Wall accounts for only 0.03% of all walls in the Bolshoi-Planck simulation.

For the 3 walls/6 halos whose environments are most like that of our own Local Wall, we study their halo properties. In Figures 8a and 8b, we looked at the distribution of halo properties in histograms of halos of the mass $log_{10} M_{200} = 11.95 \pm 0.375$ (~mass of our MilkyWay galaxy’s dark matter halo), in small walls the size of the Local Group wall and calculated at the smoothing scale of $r_s = 8h^{-1}$ Mpc where the average cosmic density is ~ 1 (Klypin et al. 2015).

We drew 6 vertical dotted green lines to indicate where the halo properties of these 3 walls/6 halos fall within the histogram. We also took a step backwards in the cut of halos in Local Walls and added in the vertical blue dashed lines of 4 halos to make up the 5 walls / 10 halos (that is, we do not restrict the halos to just being in the centre of their walls) to provide more information on the distribution of the halo properties in environments like our Local Wall. We note that these halo properties fall around the median of each histogram with known distribution (Gaussian for the specific mass accretion rate, and approximately log-normal distributions (Rodríguez-Puebla et al. 2016) for $L_B$ and $C_{NFW}$), while they fall randomly in histograms with no known distribution ($a_{LMM}$, Prolateness, and $a_{M1/2}$). Thus the presence of halos in walls like our Local Wall does not seem to affect the properties we studied. Hence we conclude that although our own Local Group environment is somewhat special, it has no effect on the halo properties we have examined except for the orientation of halo angular momenta (§3.2.1).

4 SUMMARY AND CONCLUSION

We found in this paper that at a given environmental density, the different cosmic web environment of filaments, walls and voids does not have significant effects on any of the halo properties that we studied at $z = 0$: the halo mass accretion rate (dynamical time averaged) $M_{\text{acc}}/M$, spin parameter $L_B$, NFW Concentration $C_{NFW}$, prolateness $P$, scale factor of the last major merger $a_{LMM}$, and scale factor when the halo had half of its mass $a_{M1/2}$. That is, the different locations of the cosmic web environment do not affect these core halo properties for halos of the same mass and at the same environmental density. We find that these halo properties are instead determined by the local environmental density of the halo.

In addition, we found that even though the presence of galaxies as massive as the Milky Way and Andromeda galaxies near the centers of walls as small as our Local Wall is quite rare (0.03% of all walls), it nevertheless appears to have essentially no effect on the halo properties that we studied. We also found that the angular momentum of halos in walls tends to lie within about 30$^\circ$ of the walls, in agreement with Aragon-Calvo & Szalay (2013).

Our results in this paper are consistent with observational evidence of Yan, Fan & White (2013), where the properties of galaxies at a given environmental density in the Sloan Digital Sky Survey do not depend on the cosmic web location, although as mentioned in §1 there are other authors who have found differences in galaxies in different cosmic web environments even at the same density, especially at higher redshifts.

It would be interesting to look at other halo properties in the Bolshoi-Planck or other simulations, to see if the web environment has any effects on them at constant density. In addition, it would also be interesting to examine galaxies in hydrodynamic simulations to see whether the galaxies have cosmic web dependences at fixed environmental density, unlike our results regarding dark matter halos.

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References

Aligood B., Flores R. A., Primack J. R., Kravtsov A. V., Wechsler R. H., Faltenbacher A., Bullock J. S., 2006, MNRAS, 367, 1781
Alonso D., Eardley E., Peacock J. A., 2015, MNRAS, 447, 2683
Aragon-Calvo M. A., Platen E., van de Weygaert R., Szalay A. S., 2010, ApJ, 723, 364
Aragon-Calvo M. A., Szalay A. S., 2013, MNRAS, 428, 3409
Aragon-Calvo M. A., Yang L. F., 2014, MNRAS, 440, L46
Behroozi P. S., Wechsler R. H., Wu H.-Y., 2013, ApJ, 762, 109
Behroozi P. S., Wechsler R. H., Wu H.-Y., Klypin A., Primack J., 2013, ApJ, 763, 15
Blumenthal G. R., Faber S. M., Primack J. R., Rees M. J., 1984, Nature, 311, 517
Bond J. R., Kolman L., Pogosyan D., 1996, Nature, 380, 603
Brouwer M. et al., 2016, MNRAS, 462, 4451
Bullock J. S., Dekel A., Kolatt T. S., Kravtsov A. V., Klypin A. A., Porciani C., Primack J. R., 2001, ApJ, 555, 240
Doroshkevich A. G., Shandarin S. F., Zeldovich I. B., 1983, in IAU Symposium, Vol. 104, Early Evolution of the Universe and its Present Structure, Abell G. O., Chincarini G., eds., pp. 387–391
Eardley E. et al., 2015, MNRAS, 448, 3665
Guo Q., Tempel E., Libeskind N. I., 2015, ApJ, 800, 112
Hahn O., Porciani C., Carollo C. M., Dekel A., 2007, MNRAS, 375, 489
Klypin A., Kurachenkov E., Makarov D., Nasonova O., 2015, MNRAS, 454, 1798
Figure 7. Flowchart of the cuts made to find halo pairs like the Milky Way Galaxy and Andromeda. Our Local Wall is found to have a radius of $2.7 \ h^{-1} \text{ Mpc}$, while the masses of our Milky Way Galaxy and Andromeda are thought to be about $0.8 - 1.2 \times 10^{12} \text{ M}_\odot$. The cosmic density of the local ($\sim 10 \text{ Mpc}$ radius) environment is approximately equal to the average density of the universe (Klypin et al. 2015). By using the halos of Lee et al. (2017a), we found the density of the walls to be 1.02, 1.35 and 1.75 respectively for small halos, medium halos and large halos, all calculated with the $\rho_{\sigma}$ density on the smoothing scale of $8 \ h^{-1} \text{ Mpc}$. As SpineWeb assigns walls that are only a voxel thick (corresponding to only $0.25h^{-1} \text{ Mpc}$), we set a structural re-assignement criterion of assigning halos to walls if they they are $> 0.25h^{-1} \text{ Mpc}$ from filaments to more accurately reflect criteria found in surveys. Second to last, we wanted the wall to contain a pair of dark matter halos (MilkyWay + Andromeda), whose distance is less than about $0.47 \ h^{-1} \text{ Mpc}$ apart. We are left with 6 walls out of an initial 19282 Walls, which give us 0.06% of all walls in the Bolshoi-Planck, which are similar to our Local Wall. Lastly, if we further used the map of McCall (2014) to restrict halos to the centre of their walls like our Local Wall, we end up with just 3 walls / 6 halos left, which gives us 0.03% of all walls in the Bolshoi-Planck simulation.
Figure 8a. Histograms of halo properties in all web environments and small walls as a function of environmental density, for halo mass $\log_{10} M_{\text{vir}}/M_\odot = 11.95 \pm 0.375$ with $\rho_\sigma$ density on the smoothing scale of 8 $h^{-1}$ Mpc. LEFT TO RIGHT: In addition to plotting all densities, we also split density into 4 different ranges to explore the effect of low, medium, high and extra high densities on the halo properties. The 6 vertical green dotted lines (< 1.75$h^{-1}$ Mpc from the centre of mass of the walls) along with 4 vertical blue dashed lines (otherwise) make up the 10 halos most like our own MilkyWay Galaxy and Andromeda. TOP TO BOTTOM: Specific accretion rate $\dot{M}/M$, $\lambda_B$, and $C_{\text{NFW}}$ are compared with density in small walls (radius 2 – 3.4 $h^{-1}$ Mpc) (in blue) and environments of all sizes (orange). Unlike the histograms of Figures 3a and 3b, we did not plot the halo properties in voids here as there are too few halos in voids at the stated mass range calculated at the stated smoothing scale to do meaningful statistics: the number of halos in voids in all density ranges here is just 59, while the number of halos in all environment in the same density range is 74121, while that of walls is 2401. It should be noted these numbers in the all density range refer to halos in their respective environments after cuts made to their masses ($\log_{10} M_{\text{vir}}/M_\odot = 11.95 \pm 0.375$), after cuts made to the radius of the wall (2 – 3.4 $h^{-1}$ Mpc), after cuts made to being away from filament ($> 0.25h^{-1}$), and after cuts of reassigning halos further away in voids to walls instead ($> 0.75h^{-1}$ from walls). The first 3 cuts correspond to the first 3 criteria found in §3.1.2, and while we used < 0.75$h^{-1}$ Mpc for halos distance to walls to split the environment in the histograms, we did not use this criterion for halos in our Local Wall (the vertical lines), as we wanted to look for halos exactly in the plane of the wall there (McCall 2014). Also unlike the histograms of Figures 3a and 3b, the histograms here are not logged, due to the low overall density of halos measured at this smoothing scale. The histograms here corresponds to the mass range in the second column of Figures 2a, although it is calculated at the smoothing scale of 8 $h^{-1}$ Mpc, and not 2 $h^{-1}$ Mpc of Figure 2a. After additional cuts made to the number of halos listed as items v – vii in §3.1.2, we found 10 halos that are like those of the Milky Way galaxy and Andromeda (the methods that we used to select them are found in full in 3.2.2, and illustrated with a flowchart in Figure 7), which we have illustrated above as blue dotted lines. We note here that these 10 halos fall roughly in the median of these histograms with known Gaussian ($\dot{M}/M$) and lognormal distributions ($\lambda_B$ and $C_{\text{NFW}}$), indicating that these halo properties of the Milky Way galaxy and Andromeda in our Local Wall are not peculiar.
Figure 8b. Histograms of halo properties in all web environments and small walls as a function of density smoothed on a scale of $8h^{-1}$ Mpc, for halo mass $\log_{10} M_{\text{vir}}/M_\odot = 11.95 \pm 0.375$. LEFT TO RIGHT: As in Figure 8b. TOP TO BOTTOM: The distribution of prolateness $P$, scale factor of last major merger $a_{\text{LMM}}$, and scale factor when the halo had half of its $z=0$ mass $a_{M_1/2}$ are compared with density in small walls (radius $2 - 3.4h^{-1}$ Mpc) (in blue) and environments of all sizes (orange). The histograms here corresponds to the second column of Figure 2b, although it is calculated at the smoothing scale of $8h^{-1}$ Mpc, and not $2h^{-1}$ Mpc of Figure 2b. Like the histograms of Figure 8a, the density range here of $0.8 < \rho_\sigma/\rho_{\text{avg}} < 1.2$ corresponds to the average cosmic density (Klypin et al. 2015). We note that across all the different density bins, there is a general agreement in the distribution of halo properties between halos in all environment, and halos in walls. There appears to be a slight discrepancy for the low density range of $\rho_\sigma/\rho_{\text{avg}} < 0.8$, but this is due to the small number of halos in walls at this range for good meaningful statistics: there are only 146 halos in walls, while there are 13631 halos in all environment at the same range. For the other density ranges, there is a good agreement of properties across the different web environments. In the average cosmic density range of $0.8 < \rho_\sigma/\rho_{\text{avg}} < 1.2$, we found 10 halos that are most like those of the Milky Way galaxy and Andromeda, which we illustrated with green dotted lines (near centre of wall) and blue dashed lines (otherwise). The halo properties here do not fall along a median as those in 8a, as the properties here ($P$, $a_{\text{LMM}}$ and $a_{M_1/2}$) are not known to follow any particular distribution, while those in the preceding histograms do ($M/M$ follows a Gaussian distribution, while both $\lambda_B$ and $C_{\text{NFW}}$ follow a log-normal distribution). Here, we see that the properties of the 10 halos most like our Milky Way galaxy and Andromeda are randomly placed, again indicating that these halos properties of the Milky Way galaxy and Andromeda in our Local Wall are not peculiar.
APPENDIX A: WALLS OF ALL SIZES

This Appendix contains figures that supplement those in the text. Like Figures 2a & 2b, Figure A1a & A1b show the median halo properties in different cosmic environments, but now in walls of different sizes, ranging from small (2 – 3.4 h^{-1} Mpc) to extremely large (> 9.5 h^{-1} Mpc). We see that the halo properties are similar across walls of various sizes, indicating that the size of the walls does not affect those properties. They appear to be only affected by the local environmental density. Similarly, like Figures 3a & 3b, Figures A2a & A2b show the full distribution of halo properties in different cosmic environments, but again in walls of different sizes. Again, we do not see a difference in distribution of halo properties due to the sizes of the walls.

APPENDIX B: CHANGING PARAMETER D

For all of the distributions in the paper, we have used a parameter $D = 0.25 h^{-1} $ Mpc as the distance away from a filament, together with < 0.75 h^{-1} Mpc as the distance to a wall, to group halos into the environment we called a "wall"; those halos not in filaments or walls are considered to be in voids. In this appendix, we show the effect when we change $D$ to be $D = 0.75 h^{-1} $ Mpc instead. We can see that the effects are quite small, strengthening our argument that the cosmic environment has little effect on halo properties; it is density which governs the halo properties. It should be noted that we did not use < 0.75 h^{-1} Mpc as the distance to a wall when looking for halos like those in our Local Wall. Not using this criterion has the effect of limiting halos to just the plane of the walls, which is the case for the Milky Way and Andromeda galaxies (McCall 2014).

APPENDIX C: ANGULAR MOMENTUM ORIENTATION OF HALOS IN THE CENTRES OF WALLS

In §3.2.1, we used the following cuts to constraint the number of halos: walls of the Bolshoi-Planck:

(i) Radius (wall): 2 – 3.4 h^{-1} Mpc
(ii) Mass (halos): 0.8 – 1.2 \times 10^{12} M_{\odot}
(iii) Distance of halo to filaments: > 0.25 h^{-1} Mpc

Here, we show an alternative criterion for the halos, by limiting them to the centre of the their walls. We define here the halos to be near the centre of the wall with the following criteria:

(i) Radius (wall): 2 – 3.4 h^{-1} Mpc
(ii) Mass (halos): 0.8 – 1.2 \times 10^{12} M_{\odot}
(iii) Distance of halo to centre of mass of the wall: < 1.75 h^{-1} Mpc

where we calculated the centre of mass as the mean of the position voxels of each wall. These criteria limit the number of halos to 123 in 110 walls. This criteria of limiting halos to the centre of walls here produces fewer halos than those found using the methods in in §3.2.1, where we found 702 halos in 594 walls by reassigning halos in walls, but not limiting them to be near the centres of their walls. The results found here of the dot product between the orthogonal vector and the $\omega$ vector is 0.458, while those in §3.2.1 is 0.452. Hence, though we have limited the angular momentum to halos near the centres of walls, the qualitative result for the direction of the angular momentum vector lying close to the plane of the wall remains the same as that of for halos not stringently confined to the centres of their walls.
Figure A1a. Halo properties in all-sized walls as a function of density. LEFT TO RIGHT: The columns represent 4 mass bins with corresponding smoothing scales, as in Fig. 2a. TOP TO BOTTOM: The median distribution of specific halo mass accretion rate $\dot{M}/M$, spin parameter $\lambda_B$, and concentration $C_{NFW}$ are plotted vs. density in different mass bins across different size walls. The scatter plots in green to blue shading are the median halo properties corresponding to walls of different sizes. We split the walls up into different sizes because we are most interested in small $(2-3.4 \ h^{-1} \text{ Mpc})$ walls, as that corresponds to the size of our Local Wall (McCall 2014). These scatter plots fall within the thick yellow band, which represents the 5th - 95th percentile dispersion of the median of each halo property in all web environments. The right tail-end of each sub-plot has the fewest halos, and any differences at these tail-ends between the different sized walls are not statistically significant. RESULTS: We note that only the first two mass bins/columns have sufficient median halos in all walls for meaningful results. We see that although there are tiny deviations, the halo properties do not appear to be affected on the whole by different sized walls, but are affected by increasing density. This is analogous to the results of Figure 2a, where we looked at halo properties across different web environments including small walls. Similar to Figure2a, the horizontal blue dotted line in the specific accretion rate plots separates the halos gaining and losing mass.
Figure A1b. Halo properties in all-sized walls as a function of density. LEFT TO RIGHT: The columns represent 4 mass bins with corresponding smoothing scales, as in Fig. 2a. TOP TO BOTTOM: The median distribution of prolateness, scale factor of last major merger, and scale factor when the halo had half of its $z = 0$ mass are plotted vs. density in different mass bins across different size walls. RESULTS: We note that only the first two mass bins/columns have sufficient median halos in all walls for meaningful results. Similar to the results of Figure A1a, we see that the halo properties do not appear to be affected on the whole by the size of the walls. In addition, the scale factor of the last major merger does not seem to be affected by increasing density, although prolateness and the half-mass scale do. This is analogous to the results of Figure 2b, where we looked at halo properties across different web environments including small walls.
**Figure A2a.** Halo properties in walls of all sizes as a function of environmental density, for halo mass \( \log_{10} M_{\text{vir}}/M_\odot = 11.20 \pm 0.375 \). LEFT TO RIGHT: We split density into 3 different regions to explore the effect of low to high density on the halo properties. TOP TO BOTTOM: Specific accretion rate, \( \lambda_B \), and \( C_{NFW} \) are plotted vs. density in walls of all sizes. The histograms here correspond to those of Figure 3a of this paper, where we showed histograms of properties in different cosmic environments and small walls instead. RESULTS GENERAL: The halo properties do not appear to be affected by the different sizes of the walls. We note that that halo properties in the small and medium walls here appear to deviate from the distribution, taking on a more ‘jaggedy’ appearance, as we go towards larger densities due to lack of halos at these larger densities. This means that there is a lack of halos in small and medium walls at high densities, with the lack of halos contributing to the deviation. However, despite this deviation, we can see that the trend of the halo properties in small and medium walls generally follows that in walls of all sizes for each property.
**Figure A2b.** Histograms of halo properties in all-sized walls as a function of density, for halo mass \( \log_{10} M_{\text{vir}}/M_\odot = 11.20 \pm 0.375 \).

**LEFT TO RIGHT:** As in Figure A2a **TOP TO BOTTOM:** The full distribution of prolateness, scale factor of last major merger, and the scale factor when the halo had half of its \( z = 0 \) mass are plotted in walls of all sizes. The histograms here correspond to those of Figure 3b of this paper, where we showed histograms of properties in different cosmic environments and small walls instead. **RESULTS:** The halo properties do no appear to be affected by the different sizes of the walls. Similar to Figure A2a, the small and medium walls here appear to deviate from the distribution as we go towards higher densities due to lack of halos at these high densities. However, despite this, we can see the trend of the halos in small and medium walls generally following the halos in walls of all sizes for each property. Hence, along with the histograms of Figure A2a, we conclude that wall sizes do not significantly affect their halo properties.
Figure B3. Using parameter $D = 0.75 \, h^{-1} \, \text{Mpc}$ away from a filament, these are curve-fittings of the distributions of specific accretion rate $\dot{M}/M$, spin parameter $\lambda_B$, and concentration $C_{NFW}$ with halos $> 0.75 \, h^{-1} \, \text{Mpc}$ away from filaments, and $< 0.75 \, h^{-1} \, \text{Mpc}$ away from walls. RESULTS: With a larger distance away from filaments, the halos represented in this table are more concentrated in the centre of the walls than those in Figure 5. We note that the numbers tabulated here are very close to those found in Figure 5, indicating that the distribution of histograms in Figures 3a and 3b and the median plots in Figures 2a and 2b are very similar whether using the halo-to-filament distance parameter $D = 0.25 \, h^{-1} \, \text{Mpc}$ or $D = 0.75 \, h^{-1} \, \text{Mpc}$. We used a table of fits to show the similarity between the 2 different parameters, as the plots and histograms using these 2 parameters are too similar to see by eye the differences between them. We have, however, still included the median plots just for comparison in B4.
Figure B4. Using parameter $D = 0.75 \, h^{-1} \, \text{Mpc}$ away from a filament, these are halo properties in all web environments, small walls, and voids as a function of density, where $\rho_\sigma$ is density used on that smoothing scale. LEFT TO RIGHT: The columns have been split into 4 mass bins of $\log_{10} M_{\text{vir}} / M_\odot = 11.20 \pm 0.375, 11.95 \pm 0.375, 12.70 \pm 0.375,$ and $13.45 \pm 0.375$, with density smoothing scales of $\rho_\sigma$ in these mass bins as 1, 2, 4, and $8 \, h^{-1} \, \text{Mpc}$ respectively. TOP TO BOTTOM: The median distribution of specific accretion rate, $\lambda_B$, and $C_{\text{NFW}}$ are plotted vs. density in the four mass bins. RESULTS: The plots here uses the parameter $D = 0.75 \, h^{-1} \, \text{Mpc}$, while those in Figure 2a uses $D = 0.25 \, h^{-1} \, \text{Mpc}$. Looking between these 2 plots, it is difficult to see by eye the differences arising from using the different halo-distance-to-filaments parameter $D$. It is better to make the comparison using the quantities found in the table in Figure B3 instead.