Factorization of the charge correlation function in $B^0\bar{B}^0$ oscillations

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Abstract

Extraction of the mass difference $\Delta m$ from $B^0\bar{B}^0$ oscillations involves tagging of bottom flavour at production and at decay. We show that the asymmetry between the unmixed and mixed events factorizes into two parts, one depending on the production-tag and the other on the decay-tag.

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There is now a considerable body of experimental evidence for $B^0\bar{B}^0$ oscillations. The general strategy adopted in gathering it (for a review, see e.g., Ref. [1]) may be stated briefly as follows: (A) identify the $B^0$ decay events that have one or more out of a lepton $\ell^{\pm}$, a $D^{*\pm}$, and a $K^{\pm}$, (B) tag the bottom flavour at production by any convenient method: use of jet charge or high-$p_T$ lepton in the opposite-hemisphere, or $B\pi^{\pm}$ correlations in the same-side hemisphere, or asymmetry with polarized electron beam, (C) measure the displacement of decay vertex from the production vertex, (D) estimate the $B^0$ momentum, and (E) convert the displacement into propagation time of the neutral beam. From such measurements it has been demonstrated that the mixed-events occur with a sinusoidal time-dependence that is characteristic of oscillations. The frequency or mass-difference $\Delta m$ of $B^0\bar{B}^0$ mixing has been extracted [2]-[10] from the observed time-dependence.

Data on time-distribution are usually fitted to the charge correlation function between the numbers of mixed and unmixed events

$$C(t) = \frac{N_{\text{unmixed}} - N_{\text{mixed}}}{N_{\text{unmixed}} + N_{\text{mixed}}}.$$  

Here $t$ is the decay time of $B^0$ measured in its rest frame. The purpose of this note is to point out that $C(t)$ factorizes into a part that depends on the production tag and another that depends on the decay tag, provided we neglect terms that contribute to second order of $CP$ violation. The time dependence of $C(t)$ therefore will not be sensitive to the details and systematic errors of the production tag.

Let us suppose that the time-distribution of $B^0$ decay is determined by tagging a flavour-specific mode. Let the flavour at production be determined by the jet charge of the $b$-jet in the opposite-side hemisphere (jet charge is determined by a weighted sum of the charges of the individual tracks; for details, see, e.g., Ref. [4]). We denote the probability to find a $b$-jet with normal jet charge ($=-1/3$) by $P_n$. The jet from a $b$ can also have the abnormal charge ($=+1/3$) if the $b$ fragmented into $\bar{B}^0$ or $\bar{B}^0_s$ which oscillated to the conjugate meson having positive bottom flavour; let $P_a$ denote the probability to find a $b$-jet with abnormal jet-charge. Hence the probabilities associated with jet production are:

$$P_n = \text{Prob} (b \to J^-), \quad P_a = \text{Prob} (b \to J^+),$$  

(2)
\[ P_n = \text{Prob}(\bar{b} \rightarrow J^+) , \quad P_a = \text{Prob}(\bar{b} \rightarrow J^-); \] (3)

here the superscript \((\pm)\) on \(J\) denotes the sign of the jet charge \((\pm 1/3)\). Clearly, \(CP\) invariance is violated if the difference \((P_n - P_a)\) or \((P_a - P_n)\) is non-vanishing.

We take \(t = 0\) to be the instant at which the \(b\bar{b}\) pair is produced (we ignore the rare events with multiple \(b\bar{b}\) pairs). Let the \(\bar{b}\) fragmentation lead to a neutral beon \(B^0\) or \(B^0_s\) which decays at time \(t\) into a flavour-specific mode; this could, for instance, be the fully-reconstructible mode \(B^0 \rightarrow J/\psi K^{*0}\) for studying \(B^0 \bar{B}^0\) oscillations, or \(B^0_s \rightarrow D^- \ell^+ \nu\) for studying \(B^0_s \bar{B}^0_s\) oscillations [11]. In the following we focus on the inclusive decay mode \(B^0 / \bar{B}^0 \rightarrow [D^*(2010)^\pm + \text{anything}]\) and define the decay rates into normal and abnormal modes as

\[ D_n(t) = \Gamma(B^0(t) \rightarrow D^{*+} + \ldots), \quad D_a(t) = \Gamma(B^0(t) \rightarrow D^{*-} + \ldots); \] (4)

\[ \bar{D}_n(t) = \Gamma(\bar{B}^0(t) \rightarrow D^{*-} + \ldots), \quad \bar{D}_a(t) = \Gamma(\bar{B}^0(t) \rightarrow D^{*+} + \ldots). \] (5)

Here, the dots \((\ldots)\) indicate ‘anything’; \(\bar{B}^0(t)\) is the physical state which evolved from a \(\bar{B}^0\) state after a lapse of time \(t\); the decay rates corresponding to initial antiquark \((\bar{b})\) have a ‘bar’ on them. Decays classified as abnormal are due to \(B^0 \bar{B}^0\) oscillations. The conditions implied by \(CP\) invariance are \([D_n(t) - \bar{D}_n(t)] = 0\) and \([D_a(t) - \bar{D}_a(t)] = 0\).

We now write the relative numbers of events \(N(ji)\), where the first sign \(j\) is the sign of the jet charge and the second sign \(i\) is the sign of the \(D^*\) charge:

\[ N(--) = P_n \bar{D}_n + \bar{P}_a D_a , \] (6)

\[ N(++) = \bar{P}_n D_n + P_a \bar{D}_a , \] (7)

\[ N(+-) = P_n D_a + P_a \bar{D}_n , \] (8)

\[ N(-+) = P_n \bar{D}_a + \bar{P}_a D_n . \] (9)

These expressions are easily interpreted: for instance, \(N(+-)\) refers to events with positive jet charge arising from the decay of either a \(\bar{b}\)-hadron which did not oscillate (called normal), or a \(b\)-hadron which did oscillate (called abnormal); the \(D^{*-}\) in the measurement-hemisphere can result from the decay following either the transition \(B^0(t) \rightarrow B^0\) or the transition \(\bar{B}^0(t) \rightarrow B^0\).
The relative number of unmixed events containing a bottom jet and the inclusive decay $B^0/\bar{B}^0 \to D^{*\pm} + \text{anything}$, is given by

$$N_{\text{unmixed}} = N(-) + N(++) \quad (10)$$

$$= \frac{1}{2}[(P_n + \bar{P}_n)(D_n + \bar{D}_n) + (P_a + \bar{P}_a)(D_a + \bar{D}_a)$$

$$- (P_n - \bar{P}_n)(D_n - \bar{D}_n) - (P_a - \bar{P}_a)(D_a - \bar{D}_a)] \quad (11)$$

$$\simeq \frac{1}{2}[(P_n + \bar{P}_n)(D_n + \bar{D}_n) + (P_a + \bar{P}_a)(D_a + \bar{D}_a)]. \quad (12)$$

The last step neglects terms that contribute to second order of $CP$ violation. The number of mixed events (namely, events having abnormal charge either at production or at decay, but not both) is similarly given by

$$N_{\text{mixed}} = N(+ -) + N(- +) \quad (13)$$

$$= \frac{1}{2}[(P_n + \bar{P}_n)(D_a + \bar{D}_a) + (P_a + \bar{P}_a)(D_n + \bar{D}_n)$$

$$- (P_n - \bar{P}_n)(D_a - \bar{D}_a) - (P_a - \bar{P}_a)(D_n - \bar{D}_n)] \quad (14)$$

$$\simeq \frac{1}{2}[(P_n + \bar{P}_n)(D_a + \bar{D}_a) + (P_a + \bar{P}_a)(D_n + \bar{D}_n)]. \quad (15)$$

wherein the neglected terms of second-order of $CP$-violation are different from those neglected in Eq. (12). Note that in Eqs. (10) and (13) the charge labels in $N(ji)$ depend on the tags used for production and decay.

The charge-correlation function $C(t)$ is $CP$-even. It contains the products $(P_i - \bar{P}_i)(D_j - \bar{D}_j)$, with $(i, j) = (n, a)$, which, being quadratic in $CP$-violation, are presumably small (the mass-matrix $CP$ violation is believed anyway to be too small to be relevant). Hence it is quite reasonable to substitute Eqs. (12) and (13) in Eq. (1). Thus $C$ takes the factorized form

$$C(t) \simeq \left[\frac{(P_n + \bar{P}_n) - (P_a + \bar{P}_a)}{(P_n + \bar{P}_n) + (P_a + \bar{P}_a)}\right] \times \left[\frac{(D_n + \bar{D}_n) - (D_a + \bar{D}_a)}{(D_n + \bar{D}_n) + (D_a + \bar{D}_a)}\right]. \quad (16)$$

The first (time-independent) factor involving $P$’s refers to the production tag and the second involving $D(t)$’s to the decay tag. Obviously Eq. (16) will be exact if the $CP$-violating differences $(P_i - \bar{P}_i)$ or $(D_i - \bar{D}_i)$ vanish for $i = n$ and $i = a$.

Factorization of $C(t)$ is valid under general conditions: There is no need to consider the production tag in a time-integrated version which is inherent to the jet-charge method; the
double-time distribution also will factorize. As for decay, any specific channel that can tag the bottom will suffice.

When a lepton tag is used either at the production end or at the decay end, factorization would automatically follow. This is because of the general result (see, e.g., Ref. [12]) that direct CP violation cannot show up in decay channels which include a lepton pair $\ell\nu$, provided we assume CPT-invariance, retain terms of lowest-order of the weak Hamiltonian and ignore the electroweak scattering phase-shifts. Recently the ALEPH group [3] has verified by simulations that the ‘lepton-signed jet charge’ $Q_{\ell H}(t)$ does factorize. This is to be expected because, if we now let the label $i$ in $N(ji)$ denote the lepton charge, the negative average of the product charges is given by

$$Q_{\ell H}(t) = - \frac{\sum_j \sum_i (j)(i) N(ji)}{\sum_j \sum_i N(ji)}$$

$$= C(t).$$

When the production-tag and the decay-tag are non-leptonic (for instance, $K^{\pm}$ and opposite-side jet, as in Ref. [7]), the terms that prevent factorization can be neglected as they involve second-order of CP violation. There is yet another case in which Eq. (16) is exact: this is when the production tag originates from a CP-invariant interaction. Examples of experimental interest are the asymmetry with polarized electrons [9] based on neutral-current electroweak interaction and the $B\pi^{\pm}$ correlations [10] based on strong interaction.

On the other hand, it may be noted that the function

$$\chi(t) = \frac{N_{\text{mixed}}}{N_{\text{unmixed}} + N_{\text{mixed}}} = \frac{1 - C(t)}{2}$$

(which is akin to the dilepton mixing ratio), does not factorize; the ratio ($N_{\text{mixed}}/N_{\text{unmixed}}$) also does not factorize.

In conclusion, when a pair of bottom particles is incoherently produced (as in $Z$ decays), the asymmetry $C(t)$ between the unmixed and mixed events is well-suited for studying $B^0\bar{B}^0$ or $B^0_s\bar{B}^0_s$ oscillations. It separates into two factors, one depending on the production-tag and the other on the decay-tag. For this reason, sensitivity to systematic errors associated with the production tag would be minimal if the frequency $\Delta m$ is extracted by using $C(t)$. 

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References

[1] S.L. Wu, Talk at the 17th International Symposium on Lepton-Photon Interactions at High Energies, Beijing, 1995, CERN preprint CERN-PPE/96-82 (1996).

[2] ALEPH Collaboration, D. Buskulic et al., Phys. Lett. B 313, 498 (1993); 322, 441 (1994).

[3] ALEPH Collaboration, D. Buskulic et al., CERN preprint CERN-PPE/96-102, July 1996, Submitted to Z. Phys. C.

[4] OPAL Collaboration, R. Akers et al., Phys. Lett. B 327, 411 (1994).

[5] OPAL Collaboration, R. Akers et al., Phys. Lett. B 336, 585 (1994); Z. Phys. C 66, 555 (1995); G. Alexander et al., Z. Phys. C 72, 377 (1996).

[6] DELPHI Collaboration, P. Abreu et al., Phys. Lett. B 338, 409 (1994).

[7] DELPHI Collaboration, P. Abreu et al., Z. Phys. C 72, 17 (1996).

[8] L3 Collaboration, M. Acciarri et al., Phys. Lett. B 383, 487 (1996).

[9] SLD Collaboration, K. Abe et al., Reports at the XXVIII International Conference on High Energy Physics, July 1996, Warsaw, Poland: SLAC-PUB-7228,7229,7230 (July 1996).

[10] CDF Collaboration, Reported at the XXVIII International Conference on High Energy Physics, July 1996, Warsaw, Poland: Fermilab preprint FERMILAB-Conf-96/175-E (Sept 1996).

[11] ALEPH Collaboration, D. Buskulic et al., Phys. Lett. B 377, 205 (1996).

[12] J.S. Bell, in High Energy Physics, Les Houches 1965, edited by C. DeWitt and M. Jacob, (Gordon and Breach, 1965), p. 403; P.K. Kabir, in Particle Interactions at High Energies, edited by T.W. Priest and L.L.J. Vick, (Oliver & Boyd, 1967), p. 248.