The Research of Monte-Carlo Method for Aviation Materials Demand Forecast Based on the Life of the Reliability

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Abstract. With the rapid development of the aviation industry, the increasing number of missions and the impact of other new circumstances, the contradiction between supply and demand in aviation materials has become increasingly prominent. Consequently, more uncertainty and unpredictability in spare parts' demand, as well as the absence of accurate value of holding, overstocking and backordering costs, result in traditional inventory control policies ineffective. In order to make better use of the existing stock aviation material resources, it is necessary to study the optimization of aviation materials inventory management. In this paper, the reliability analysis of aviation materials maintenance data is conducted to grasp the occurrence rules of aviation materials failures, determine the failure distribution function, and establish a mathematical model of spare parts guarantee and demand based on stochastic process theory. The Monte-Carlo simulation method is used to calculate the aviation materials demand. The calculation results show that the method can achieve the purpose of controlling the inventory level through the demand under varying guarantee degrees, which makes the management of aviation materials more scientific.

1. Introduction

The cost of maintenance in the aviation industry is high, and there is always a continuous process of cutting costs, which must be done without interfering with safety and airworthiness. Therefore, aviation materials management issue has always been a meticulous focus on security topics in the aviation field. It is critical to make scientific and accurate predictions of aviation material consumption and demand, reasonably control inventory, and effectively improve the level of aviation materials management and the utilization rate of aviation materials reserve funds, to improve the economic efficiency of airlines [1-2].

Over the past few decades there has been a great deal of research in the area of maintenance modeling and optimization. Geisler, Brown and Hixon [3] summarized the earliest findings based on B-47 data collections. In this research they were unable to find a significant correlation between demands and activity levels. Shortly after their first publication Geisler and Brown conducted an analysis on demand data for B-47 airframe parts where they were able to recognize from the start that demand for aircraft spares exhibits an unexpectedly high variation. The research was conducted further to get a stronger grip on demand forecasting and inventory control. They realized that demand for aircraft spares were not adequately described the Poisson distribution. Youngs, Geisler and Mirkovich [4] research made the first step towards negative binomial probability distribution from Poisson distribution. The first clear and detailed reference in the early years on the use of negative binomial probability distributions to describe spare parts demand was in a research memorandum by...
Youngs, Geisler and Brown [5]. It was concluded that the method of conditional probabilities is much more precise than the straightforward calculation method for the low probabilities and frequencies of demand, which is important in the case of aircraft spares. Shortly after this, Geisler and Mirkovich [6] published analyses of spare parts demand on F-86D aircraft and discovered many similarities with those on B-47 aircraft.

The complexity of general aviation equipment and the uncertainty of failure make aviation material consumption to be certain. Therefore, it poses significant challenges for statistics, analysis and forecasting of aviation materials parts demand. The quantitative prediction method of aviation materials is a scientific statistical analysis method, which was developed based on large amount of historical data. This method is widely used because it can quantitatively describe the requirements of aviation materials [7]. This method mainly includes the aviation materials demand forecasting method based on time series model, the aviation materials demand forecasting method based on the aviation material life function, the aviation materials demand forecasting method based on maintenance theory, and the intelligent algorithm-based aviation materials demand forecasting method [8]. In the traditional inventory control theory, the circulation inventory of turn-around parts is mainly determined by Poisson distribution, to achieve inventory control of aviation materials [9].

Ensuring the supply of aviation materials is essential for aircraft maintenance services, thus the demand for aviation materials is closely related to the maintenance support method. Currently, with the establishment of the “reliability-centered” maintenance theory, parts have been developed from a single timing replacement to timing, condition-based, and monitoring. The demand probability (probability of failure) for working parts is not constant, the Poisson distribution assumption is no longer satisfied. Furthermore, when the parts of the turnaround time change, the circular inventory calculated by the average turnaround time will deviate greatly from the actual situation [10-12].

2. Reliability Analysis of Aviation Material Life

It can be known from the reliability theory that the failure time of a certain part of aircraft follows certain rules statistically, that is, the life of aviation materials obeys a certain distribution. The reliability of the aviation material maintenance data determines the failure distribution function (or life function) of the aviation materials and performs distribution tests, parameter estimates and life evaluations.

Aviation material spare parts life distribution forms can be generally divided into several types: exponential distribution (electronic components), normal distribution, logarithmic normal distribution (fatigue life of structural parts on aircraft), two-parameter and three-parameter Weibull distributions, (engine products) etc. several typical distributions [13].

A linear equation can be obtained by taking a logarithm of both sides of the cumulative distribution function equation for any of the above five statistical distributions. For a two-parameter Weibull distribution, the logarithm of both sides can be obtained:

\[ \ln \ln \frac{1}{1-F(x)} = -\beta \ln \sigma + \beta \ln x \]  

(1)

Set \( X = \ln x \), \( Y = \ln \ln \frac{1}{1-F(x)} \), \( A = -\beta \ln \sigma \) and \( B = \beta \), can have the following standard systems of linear equations:

\[ Y = A + BX \]  

(2)

For a set of known random variable data \( x_i (i = 1, 2, \cdots n) \) in ascending order, due to the data is limited, the trial probability value \( F_e(x_i) \) of the i-th data is calculated using the Median Rank:

\[ F_n(t_i) = \frac{i-0.3}{n+0.4} \]  

(3)

In the formula, \( n \) is the total number of samples, and \( i \) is the sequence number of life data from small to large. For any distribution, we can use experimental values instead of theoretical values and obtain a set of \( (X_i, Y_i) \) data. Point estimates of each distribution parameter were obtained by least squares fitting. Using the relationship between the linear correlation coefficient \( r \) and the \( t \) distribution,
a statistical test can be performed at a given significance level, and the optimal distribution form can be selected by comparing the correlation coefficient sizes.

3. Establishment of Aviation Material Demand Model

3.1. Determination of Aviation Material Inventory

The circular inventory reflects the overall inventory level of aviation materials and is an important factor that determines the quality of aviation materials inventory. Consumption law of aviation material inventory is random, and its demand is in the process of change, so the aviation materials department should reserve aviation materials with a suitable guarantee rate to prevent missing parts.

The aviation material support rate is an indicator that reflects the aviation material support level. Generally, the aviation material system's support rate is defined by the system service level, which plays a decisive role in the demand for spare parts. The total guarantee rate of aviation materials can be regarded as the probability that the spare parts can be provided in time, which can be understood as the probability that the spare parts will not exceed \( n \) failures in the cumulative working time \((0, T)\).

For non-repairable parts on the aircraft, if \( n \) spare parts are configured, the life of the spare parts is a random variable \( X \), and it is assumed that the life of the \( n \) spare parts is \( X_i \) \((i = 1, 2, \cdots, n)\), and are independently distributed. The distribution function and density function of spare parts life \( X \) are \( F(t) \) and \( f(t) \), then the cumulative working time:

\[
S_n = X_1 + X_2 + \cdots + X_n
\]  

Then, the required number of spare parts \( N(t) \) within a given cumulative working time \((0, t]\) is

\[
N(t) = \sup \{n | X_1 + X_2 + \cdots + X_n < t \}
\]  

\{\(N(t), t > 0\)\} is a random update process with an update distribution of \( F(t) \). In time \((0, t]\), the probability of \( k \) failures \( P(k) \) is

\[
P(k) = F^k(t) - F^{k+1}(t), k = 0, 1, 2, \cdots.
\]

In the formula, \( F^k(t) \) is a \( k \)-reconvolution function of the distribution function \( F(t) \).

Then the probability of the spare parts not exceeding \( n \) failures in the cumulative working time \((0, T]\), that is, the guarantee rate of the spare parts during this time period,

\[
P(N(T) < n) = \sum_{k=0}^{n} P(k) = 1 - F^{n+1}(T)
\]

For a given guarantee rate \( R_0 \), the amount of aviation material \( k \) in working time \((0, T]\) can be determined according to the above formula (7). In order to ensure that the guarantee rate of spare parts is not less than \( 1 - R_0 \) within a given cumulative working time \((0, T]\), the demand for spare parts should satisfy.

\[
P^{N_0+1}(t) = R_0
\]

3.2. Monte-Carlo Algorithm for Spare Parts Demand

The key to determining the demand for spare parts is to calculate the probability of the occurrence of the event \( A_k = \{X_1 + X_2 + \cdots + X_n \geq t \} \), and the calculation of the probability \( P(A_k) \) is very difficult. According to the law of large numbers, the Monte-Carlo method can be used to simulate the occurrence of the event \( A_k \) by sampling and estimate the probability of the even \( A_k \) to avoid complex convolution integration operations.

Spare parts \( X_i \) \((i = 1, 2, \cdots, n)\) are independent and identically distributed random variables. The Monte-Carlo method is used to simulate the generation of random data \( X_i \) \((i = 1, 2, \cdots, n)\) of the same type. To determine whether the event \( A_n \) occurred, if the event \( A_n \) occurs, it is counted as 1, otherwise it is counted as 0. Repeat the above simulation process \( N \) times, and record the cumulative number of occurrences of event \( A_n \) as \( N_1 \), then the frequency of event \( A_n \) is: \( \hat{P}(A_k) = N_1 / N \). If the number of simulations is large enough, there is \( \hat{P}(A_k) \to P(A_k) \).
For a given guarantee time $t$, as the value of $k$ increases, $P(A_k)$ increases monotonically and satisfies $\lim_{k \to \infty} P(A_k) = 1$. Therefore, there is a positive integer $k_0$ such that $P(A_{k_0-1}) < R_0 \leq P(A_{k_0})$. The interpolation method can be used to calculate the demand for spare parts under this type of statistical distribution.

$$n = k_0 - 1 + \frac{R_0 - p_{k_0-1}}{p_{k_0} - p_{k_0-1}}$$  \hspace{1cm} (9)

4. Analysis of Examples

The fault record data of the oil cooler (APU) on the three planes were collected, and 13 fault data were obtained, as shown in Table 1. The number of single-machine installations for this part is 2, the average annual flight hours of the aircraft is about 3000 hours, the turnaround time of the parts is about 30 days, and the planned guarantee rate is 95%.

| Sequence number | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   | 10  | 11  | 12  | 13  |
|-----------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Down time/h     | 45  | 88  | 121 | 187 | 250 | 290 | 316 | 324 | 339 | 348 | 352 | 584 | 737 |

Reliability analysis of oil cooler (APU) failure data is performed, supposing its life (time to failure) values obey normal distribution, log-normal distribution, exponential distribution, and two-parameter and three-parameter Weibull distributions. The least square method is used for inferences and hypothesis tests to compare the correlation coefficients of the five distributions and finally determine the distribution form from the best distribution. The estimated distribution parameters and correlation coefficients are shown in Table 2. The data distribution and fitting curve are shown in Figure 1. It can be seen from Table 2 and Figure 1 that the two-parameter and three-parameter Weibull distributions have the best correlation with actual fault points, and the correlation coefficient is the largest. Therefore, the two-parameter Weibull distribution is selected for Monte Carlo simulation.

| Distribution Type      | Parameter1 | Parameter2 | Parameter3 | Coefficient of Correlation |
|------------------------|------------|------------|------------|----------------------------|
| Normal Distribution    | $\mu = 306.23$ | $\sigma = 215.88$ |             | 0.9507                     |
| Exp Distribution       | $\lambda = 0.0033$ |             |            | 0.9214                     |
| Lognormal Distribution | $\mu = 5.504$ | $\sigma = 0.863$ |             | 0.9524                     |
| Two-Parameter          | $m = 1.462$ | $\eta = 353.56$ |             | 0.9801                     |
| Weibull Distribution   | $m = 1.462$ | $\eta = 353.56$ | $\gamma = 0$ | 0.9801                     |

Table 2. Distribution of various types of parameters and coefficient of correlation
The Monte-Carlo method was used for sampling. The sampling test found that the accuracy did not change after more than 3000 times. Within the calculation time \((0, t]\), the probability \(P(k)\) of the number of failures less than or equal to \(k\) times and the probability of \(k\) times are shown in Figure 2 and Figure 3. The analysis found that the probability of failure occurred 10 times and less than or equal to 20 times were 18.3% and 100% respectively during the flight time of 3000 hours. According to formula (9), the demand for aviation materials under different \(R_0\) can be calculated. As shown in Figure 4, the analysis found that as the planned guarantee rate \(R_0\) increases, the number of demand increases, and the demand increases in the \([0.1, 0.9]\) range smaller, the increase in the number of demand over 0.8 is obvious. From the perspective of economics, the plan guarantee rate can be set between 0.8-0.9, and the number of demands is set between 12-13.

**Figure 1.** Failure data and various distributions.

**Figure 2.** Probability distribution with number of failures less than or equal to \(k\).

**Figure 3.** Probability distribution with number of failures equal to \(k\).
5. Conclusion

Based on the reliability analysis of the historical maintenance data of aviation materials, the least square method is used to calculate the optimal distribution rule of the life of parts, after that the equation among the support rate, life reliability and inventory demand is calculated. Finally, the Monte Carlo method is used to calculate the aviation spare parts requirements under different guarantees. Through the analysis of examples, it is found that the model can be used to optimize the inventory amount to ensure economic efficiency while guaranteeing a certain guarantee rate.

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