Stress-Strain Measurement of Rubber with Optical Moiré Fringes

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Abstract. We use the moiré fringes to measure true stress and true strain of a rubber specimen in tension, and analyze the stress-strain relationship. With the printed straight-line pattern on a specimen, the moiré patterns appear when the specimen is stretched. The geometrical relationship of bright and dark fringes is used to calculate the strain values both in the axial and transverse directions. Together with optical image processing, which can also be used to obtain the instantaneous cross sectional area, we can determine true stress and true strain. The results from the moiré method yield the true stress and true strain in good agreement with those calculated from the standard engineering measurement. Additional benefits of the method include simultaneous measurement of stress and strain, their inhomogeneity, and shear strain.

1. Introduction

Optical metrology such as diffraction has been shown to successfully measure many physical properties due to its many advantages such as high precision and resolution, versatility, robustness and non-contact [1]. Moiré interferometry is a particular optical metrology based on the deformation of patterns, called moiré patterns, imprinted on an object. Hence, it is suitable for measuring physical properties of a deformable object due to its ability to detect the displacements of points. The moiré interferometry has been employed in many types of materials such as solids, films and composites in measurement of mechanical properties, such as stress and strain, as well as in the study of crack propagation and morphogenesis [2–6].

In Thailand, natural rubber has always been one of the national assets, and recently there is a national policy to develop more value-added rubber products. In doing so, it is important to know accurately the properties of rubbers. For instance, in designing high performance vehicle tires by finite-element analysis, true stress-true strain relation is required as an initial condition [7]. However, this relation is often approximated from the engineering stress-strain relation, which may not be accurate at large strain. The availability of direct measurement of true stress and true strain would undoubtedly be beneficial and allow the verification of the model outputs.

In the standard measurement method, e.g. ASTM D412, the rubber stress and strain are measured using the universal testing machine [8]. In such measurement, the specimen prepared in a dog-bone shape is stretched by axial load until failure. During the measurement, the specimen cross section is not simultaneously measured, so conventionally its initial value is used to determine the engineering
stress, whose maximum defines the tensile strength. In general, the true stress can be theoretically estimated from the engineering stress, and likewise strain, if the deformation is not too large. One can argue that this only gives average and macroscopic values of the desired tensile properties, where other mechanical properties of interest are derived.

In this research, we propose a method of measurement to determine true stress and true strain of a rubber specimen. The method is based on the aforementioned moiré interferometry. Implemented with two digital cameras, and signal image processing, the proposed method can determine the true stress and true strain at many positions on the specimen at the same time, leading to the determination of the specimen inhomogeneity. Here we demonstrate that such the method yields results in good agreement with true stress calculated from the standard engineering measurement. For its advantages, one can imagine that the proposed method is sufficiently robust and cost friendly to implement alone or jointly as a supplemented module to the standard testing machine, which together would give more accurate measurement of the specimen properties. More importantly, for materials such as natural rubbers having strain-induced crystallization, the engineering stress cannot give the accurate true stress at large elongation, but ours does.

2. Research Methodology
When a pattern is superimposed with itself, dark and bright fringes appear, as shown in figure 1 (left), which magnifies the underlying pattern [1,2]. Let \( p, p' \) denote the grating pitches (number of line-pairs per unit length) of the two patterns. If two patterns are superimposed with a relative angle \( \theta \), the fringe orientation \( \phi \) with respect to the predefined \( \theta = 0 \) axis, and the fringe spacing \( d \) obey geometrical relations, summarized in equations in figure 1 (right). If one of the patterns is deformed, the dark and bright fringes can conveniently be used to detect the displacement of two points.

\[
\sin(\phi) = \frac{p \sin(\theta)}{\sqrt{[p \sin(\theta)]^2 + [p \cos(\theta) - p']^2}}
\]

\[
d = \frac{p'\sin(\theta)}{\sqrt{[p \sin(\theta)]^2 + [p \cos(\theta) - p']^2}}
\]

Figure 1. (left) an example of two straight-line pattern superimposed, and (right) fringe orientation \( \phi \) and spacing \( d \) can be determined from geometrical relations.

2.1. True Stress and True Strain Determination from Moiré Fringes
By definition, the strain \( \varepsilon \) is calculated from \( \varepsilon = \frac{d \varepsilon}{\varepsilon} = \frac{d}{\ell} \); thus, \( \varepsilon \) is obtained from integration of \( \Delta \varepsilon = \Delta / \ell \) which can be determined from small deformation. With a small angle \( \theta \approx 0 \) such small displacements can measured, so that \( \varepsilon \approx p - p' \)/\( p = p/d \) [1]. From the moiré fringes, the strain can be determined in terms of dark and bright fringes as \( \varepsilon_y = p(\partial n_x/\partial y) \) and \( \varepsilon_x = p(\partial n_y/\partial x) \), where \( n_x \) and \( n_y \) are the orders of the dark and bright fringes, respectively, at any point. Also, the shear strain \( \gamma = \varepsilon_y (\partial \varepsilon_x/\partial y) + \varepsilon_x (\partial \varepsilon_y/\partial x) \) can be obtained. Similarly, the true stress \( \sigma_T = F/A \) can be determined from the actual load \( F \) and the instantaneous cross-sectional area \( A \) during the deformation.
2.2. Experimental Procedure
To prepare a pattern on a rubber specimen, laser-printed stripes on paper are imprinted onto the specimen using acetone. Five designated points are marked on the specimen for easy tracking by digital camera. The patterned specimen is then placed on a movable track connected to a variable load. Another set of pattern on a transparency is placed over the specimen, over which a digital camera is set to take pictures. A schematic top view of the setting is shown in figure 2(a). Also, the size of the specimen is small enough that the coverage area is small; hence, the top transparency does not bend.

Figure 2. (a) a schematic setting of the experiment from top view, and an inset (b) showing an example of a specimen with moiré fringes and marked dots for image processing.

When the specimen is deformed by the load, the pattern is changed as seen, for example, in figure 2(b). In the above setting, one digital camera will record both the deformed pattern on the \( xy \) plane (i.e. the specimen plane), and another camera detects the thickness which will be used to calculate the instantaneous cross sectional area. We use image processing software to measure displacements of the marked points; subsequently, the stress and strain are determined as described in Section 2.1.

We performed measurements up to 30% strain due to available load, which we believe is sufficient to demonstrate our method and provide convincing evidences. As the applied load increases, the specimen is more deformed; hence, this method should measure the deformed length and cross sectional area more accurately, so it should remain valid and applicable. In contrast, the theoretical true stress and strain calculated from the standard measurement are no longer accurate. Moreover, the shear strain and inhomogeneity can be simultaneously measured.

3. Results and Discussion
The measured true stress and strain values at five selected points are shown in figure 3, which demonstrate the feasibility of the method, and its ability to measure stress and strain inhomogeneity.

Figure 3. Strain (left) and stress (right) at five sampled points (P1-P5) on the specimen subject to the same applied loads. The different measured values indicate inhomogeneity in the specimen.
Figure 4. Stress-strain relationships at points P1–P5 on the specimen. As strain increases, inhomogeneity can result in different curves.

Figure 5. Stress-strain relationship as compared with the engineering stress-strain (circles) and their theoretically calculated true stress-strain (squares).

From figures 4-5, the stress-strain relations are nearly the same for low strain, but deviate visibly as the strain increases. Their variances can be used as an indicator of the specimen inhomogeneity.

The stress and strain from the proposed method show good agreement with those from the standard measurement for low strain. It is evident that our method can measure the true stress and strain satisfactorily, and it can remain effective for large strain where the theoretical conversion from engineering to true stress-strain is no longer valid. Moreover, advancement in image processing and load variation will only improve the precision and accuracy of the method.

4. Conclusion
We demonstrate the feasibility of using optical method with the moiré interferometry to determine the true stress and strain values of a rubber specimen under tension. Simple geometrical relations from the deformed patterns allow us to calculate instantaneous displacement at any point on the specimen, as well as the cross sectional area. As demonstrated, this method can determine true stress and strain with good agreement with the theoretical ones calculated from the engineering stress-strain by the standard measurement. The proposed method can potentially be used to measure shear strain values, as well as stress-strain inhomogeneity of a specimen. Together with convenient image processing, robustness and adaptability, we believe this method can provide advantages in measuring rubber properties.

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