S-wave charmed mesons in lattice NRQCD

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Heavy-light mesons can be studied using the $1/M$ expansion of NRQCD, provided the heavy quark mass is sufficiently large. Calculations of the S-wave charmed meson masses from a classically and tadpole-improved action are presented. A comparison of $O(1/M)$, $O(1/M^2)$ and $O(1/M^3)$ results allows convergence of the expansion to be discussed. It is shown that the form of discretized heavy quark propagation must be chosen carefully.

1. INTRODUCTION

Nonperturbative strong dynamics is typified by the mass scale $\Lambda_{\text{QCD}}$. Interactions involving a very heavy quark of mass $M$ can be studied systematically by expanding in $\Lambda_{\text{QCD}}/M$. Upon truncation of the expansion at some order, the resulting effective theory is not renormalizable and requires a momentum cutoff. If the required regularization is performed via a space-time lattice, then the cutoff is proportional to the inverse lattice spacing $1/a$. A useful effective theory must satisfy

$$\Lambda_{\text{QCD}} \ll 1/a < \sim M, \quad (1)$$

so the cutoff is large enough to include the bulk of the nonperturbative dynamics in the low-energy effective theory, but small enough that the truncation of the expansion remains sensible.

Is the charm quark heavy enough for a useful lattice effective theory? The present work addresses this question through a study of the masses of S-waves charmed mesons using quenched lattice NRQCD.\textsuperscript{[1]} Calculations are performed at two lattice spacings near 0.22 fm and 0.26 fm, and in each case results are given separately at $O(1/M)$, $O(1/M^2)$ and $O(1/M^3)$ in the effective theory. This work is an extension of results that have been reported previously\textsuperscript{[2]}, and further details can be found in that paper. Other authors have considered NRQCD up to $O(1/M^2)$\textsuperscript{[3]}.

2. ACTION

The lattice action has three terms: gauge action, light quark action and heavy quark action. The entire action is classically and tadpole-improved with the tadpole factor defined by

$$U_0 = \left\langle \frac{1}{3} \text{Re} \text{Tr} U_{pl} \right\rangle^{1/4}. \quad (2)$$

The gauge action includes a sum over $1 \times 2$ rectangular plaquettes as well as $1 \times 1$ elementary plaquettes. For light fermions, the Sheikholeslami-Wohlert action\textsuperscript{[4]} is used with the clover coefficient set to its tadpole-improved value. The heavy quark action is NRQCD.

A discretization of the NRQCD action leads to the following Green’s function propagation\textsuperscript{[1]}:

$$G_1 = \left( 1 - \frac{aH_0}{2n} \right)^n \frac{U_4}{U_0} \left( 1 - \frac{aH_0}{2n} \right)^n \delta_{x,0}, \quad (3)$$

$$G_{\tau+1} = \left( 1 - \frac{aH_0}{2n} \right)^n \frac{U_4^n}{U_0^n} \left( 1 - \frac{aH_0}{2n} \right)^n \times (1 - a\delta H)G_{\tau}, \quad \tau > 0, \quad (4)$$

where “$n$” should be chosen to stabilize the numerics, and the Hamiltonian is

$$H = H_0 + \delta H, \quad (5)$$

$$\delta H = \delta H^{(1)} + \delta H^{(2)} + \delta H^{(3)} + O(1/M^4), \quad (6)$$

$$H_0 = \frac{-\Delta}{2M}, \quad (7)$$

$$\delta H^{(1)} = -\frac{c_4}{U_0^2} \frac{g}{2M} \sigma \cdot \hat{B} + c_5 \frac{a^2 \Delta}{24M}, \quad (8)$$

where

- $U_0$ is the lattice unit interval
- $\Delta$ is the mass of the virtual quark
- $g$ is the coupling constant
- $\hat{B}$ is the magnetic field
- $\sigma$ is the Pauli matrix
- $c_4$ and $c_5$ are constants

The Hamiltonian is $H = H_0 + \delta H$, where $H_0$ is the classical Hamiltonian and $\delta H$ includes corrections up to $O(1/M^4)$. The form of discretized heavy quark propagation must be chosen carefully.
\[ \delta H^{(2)} = \frac{c_2}{g} \frac{iq}{8 M^2} (\Delta \cdot \mathbf{E} - \mathbf{E} \cdot \Delta) - \frac{c_6 a (\Delta^{(2)})^2}{16 n M^2} \]

\[ \delta H^{(3)} = -c_1 \frac{(\Delta^{(2)})^2}{8 M^3} + \frac{c_7}{g} \frac{g}{U_0^3} \frac{1}{8 M^3} \left\{ \Delta^{(2)}, \sigma \cdot \mathbf{B} \right\} \]

A tilde denotes removal of the leading discretization errors. Classically, the coefficients \( c_i \) are all unity, and their nonclassical corrections will not be discussed in this work.

It should be noted that the separation of \( H_0 \) and \( \delta H \) in Eq. (4) is not unique. The heavy quark propagation of Eq. (4) uses a simple linear approximation to the true exponential dependence on \( \delta H \), while using a better-than-linear approximation for \( H_0 \). The present work will report on a generalization of this choice.

### 3. RESULTS

All data presented here correspond to a charmed meson with a light quark mass that is roughly twice the strange quark mass. More extensive results, including bottom mesons, can be found in Ref. [2]. The data sample includes 400(300) gauge field configurations at \( \beta = 6.8(7.0) \) corresponding to \( a \approx 0.26 \text{fm}(0.22 \text{fm}) \). All plots include bootstrap errors from 1000 ensembles.

Fig. 1 shows the simulation energy (as read from the plateau of an effective mass plot) of the \( ^1S_0 \) meson. Notice that at both lattice spacings the \( O(1/M^3) \) contribution is twice as large as the \( O(1/M^2) \) contribution, and that this large effect is dominated by the term containing \( c_{10} \) in the Hamiltonian, Eq. (10). This term is unique because it is the only term up to \( O(1/M^3) \) which contains a nonzero vacuum expectation value. The vacuum value can be calculated from our gauge field configurations, and as shown in Fig. 2, the simulation energy displays a very pleasing \( 1/M \) expansion after removal of the vacuum value.

The mass difference between \( ^3S_1 \) and \( ^1S_0 \) mesons is shown in Fig. 3. When the vacuum expectation value is not removed from the Hamiltonian, the \( O(1/M^3) \) contribution is twice as large as the \( O(1/M^2) \) contribution in magnitude, due to large effects from the terms containing \( c_7 \) and \( c_{10} \). Fig. 3 indicates that the \( c_{10} \) effect is entirely due to the vacuum expectation value.

This very substantial dependence of the spin splitting on the vacuum value should be disturbing, since the \( c_{10} \) term is spin-independent. Apparently the vacuum value is so large that it destabilizes heavy quark propagation and thereby introduces a spurious effect unless the vacuum value is removed from the Hamiltonian. To support this claim, we have redone the calculation after subtracting the \( c_{10} \) term from \( \delta H \) and adding it to \( H_0 \) in Eq. (4). When this is done, the triangles of Fig. 3 are reproduced regardless of whether the vacuum value is subtracted from the Hamiltonian. This is the physically-expected result.

Still, Fig. 3 contains a large \( O(1/M^3) \) contribution dominated by the term containing \( c_7 \). Although this term does not contain a nonzero vacuum expectation value, one wonders if the heavy quark propagation might be unstable for this term as well, unless a better-than-linear approximation is used for the \( c_7 \) term in Eq. (4).

Fig. 3 shows the effect of subtracting all of \( \delta H \) from its present location in Eq. (4) and putting the full Hamiltonian in place of \( H_0 \). That is,

\[ G_{\tau+1} = \left( 1 - \frac{a H}{2n} \right)^n U_0^\dagger \left( 1 - \frac{a H}{2n} \right)^n G_\tau, \]  

and we choose \( G_0 = \delta_{\tau,0} \). To maintain classical improvement, the following \( O(1/M^3) \) term must be added to the Hamiltonian:

\[ \delta H_{\text{new}} = -\frac{a}{4n} \left\{ \left( H_0 + \delta H^{(1)} \right), \delta H^{(2)} \right\}. \]  

Simulations with \( n = 5 \) and \( n = 7 \) are indistinguishable. At both lattice spacings, the large \( c_7 \) effect is found to be robust, and the contribution
from $c_9$ tends to increase. No discussion of the spin-independent terms containing $c_1$ and $c_{11}$ is presented here.

4. DISCUSSION

Instabilities can arise in the NRQCD expansion for charmed mesons due to the presence of a large vacuum expectation value. A better-than-linear approximation to heavy quark propagation is valuable for ensuring stability.

Substantial effects on spin splitting were found from $O(1/M^3)$ terms in the action. However further study, for example, of alternative definitions for the tadpole factor or of perturbative improvement for the NRQCD coefficients is needed before one can reach a definitive conclusion about the convergence of the NRQCD expansion for charmed mesons.

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