PT symmetry via electromagnetically induced transparency

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Abstract: We propose a scheme to realize parity-time (PT) symmetry via electromagnetically induced transparency (EIT). The system we consider is an ensemble of cold four-level atoms with an EIT core. We show that the cross-phase modulation contributed by an assisted field, the optical lattice potential provided by a far-detuned laser field, and the optical gain resulted from an incoherent pumping can be used to construct a PT-symmetric complex optical potential for probe field propagation in a controllable way. Comparing with previous study, the present scheme uses only a single atomic species and hence is easy for the physical realization of PT-symmetric Hamiltonian via atomic coherence.

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OCIS codes: (270.0270) Quantum optics; (190.0190) Nonlinear optics.

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1. Introduction

In recent years, a lot of efforts have been made on a class of non-Hermitian Hamiltonian with parity-time (PT) symmetry, which in a definite range of system parameters may have an entirely real spectrum [12]. PT symmetry requires that the real (imaginary) part of the complex potential in the Hamiltonian is an even (odd) function of space, i.e., $V(r) = V^*(-r)$. Even though the Hermiticity of quantum observables has been widely accepted, there is still great interest in PT symmetry because of the motivation for constructing a framework to extend or replace the Hermiticity of the Hamiltonian in ordinary quantum mechanics. The concept of PT symmetry has also stimulated many other studies, such as quantum field theory [3], non-Hermitian Anderson models [4], and open quantum systems [5], and so on.

Although a large amount of theoretical works exist, the experimental realization of PT-symmetric Hamiltonian in the fields mentioned above was never achieved. Recently, much attention has been paid to various optical systems where PT-symmetric Hamiltonians can be realized experimentally by balancing optical gain and loss [6,9]. In optics, PT symmetry is equivalent to demand a complex refractive index with the property $n(r) = n^*(-r)$. Such refractive index has been realized experimentally using two-wave mixing in an Fe-doped LiNbO$_3$ substrate [10]. The optical realization of PT symmetry has motivated various designs of PT-synthetic optical materials exhibiting many intriguing features, including non-reciprocal or uni-directional reflectionless wave propagation [10-13], coherent perfect absorber [14,15], giant wave amplification [16], etc. Experimental realization of PT symmetry using plasmonics [17], synthetic lattices [18], and LRC circuits [19] were also reported.

In a recent work Hang et al. [20] proposed a double Raman resonance scheme to realize PT symmetry by using a two-species atomic gas with A-type level configuration. This scheme is quite different from those based on solid systems mentioned above [6,10-19], and possesses many attractive features. For instance, the PT-symmetric refractive index obtained in [20] is valid in the whole space; furthermore, the refractive index can be actively controlled and precisely manipulated by changing the system parameters in situ.

In the present article, we suggest a new scheme to realize the PT symmetry in a lifetime-
broadened atomic gas based on the mechanism of electromagnetically induced transparency (EIT), a typical and important quantum interference phenomenon widely occurring in coherent atomic systems [21]. Different from the two-species, double Raman resonance scheme proposed in [20], the scheme we suggest here is a single-species, EIT one. And due to the complexity of the susceptibility [20], it is difficult to design some PT potentials we wish, however, in our scheme, we can design many different periodic potentials and non-periodic potentials in light of our will, and the size of potential can also be adjusted conveniently. Especially, compared with the traditional idea that PT symmetric potential must be combined by the gain and loss parts, we utilize the atomic decay rate to design the imaginary part of PT potential, and use the giant cross-phase modulation (CPM) effect [21,22] of the resonant EIT system to realize the real part. We shall show that the cross-phase modulation contributed by the assisted field, the optical lattice potential provided by a far-detuned laser field, and the optical gain resulted from an incoherent pump can be used to construct a complex optical potential with PT symmetry for probe field propagation in a controllable way. The present scheme uses a single atomic species only and hence is simple for physical realization.

The rest of the article is arranged as follows. In the next section, a description of our scheme and basic equations for the motion of atoms and light field are presented. In Sec. III, the envelope equation of the probe field and its realization of PT symmetry are derived and discussed. The final section is the summary of our main results.

2. Model and equations of motion

2.1. Model

The system under consideration is a cold, lifetime-broadened $^{87}$Rb atomic gas with N-type level configuration; see Figure 1. The levels of the system are taken from the D$_1$ line of $^{87}$Rb atoms, with $|1\rangle = |5S_{1/2}, F = 1\rangle$, $|2\rangle = |5S_{1/2}, F = 2\rangle$, $|3\rangle = |5P_{1/2}, F = 1\rangle$, and $|4\rangle = |5P_{1/2}, F = 2\rangle$. A weak probe field $E_p = e_z e^\delta_p(x, t) \exp [i(k_p z - \omega_p t)] + c.c.$ and a strong control field $E_c = e_z e^\delta_c \exp [i(-k_c y - \omega_c t)] + c.c.$ interact resonantly with levels $|1\rangle \rightarrow |3\rangle$ and $|2\rangle \rightarrow |3\rangle$, respectively. Here $e_j$ and $k_j$ ($\delta_j$) are respectively the polarization unit vector in the $j$th direction and the wave number (envelope) of the $j$th field. The levels $|l\rangle$ ($l = 1, 2, 3$) together with $E_p$ and $E_c$ constitute a well-known $\Lambda$-type EIT core.

Furthermore, we assume an assisted filed

$$E_a = e_z \delta_a(x) \exp [i(-k_a z - \omega_a t)] + c.c.$$  \hspace{1cm} (1)

is coupled to the levels $|2\rangle \rightarrow |4\rangle$, where $\delta_a(x)$ is field-distribution function in transverse direction.
tion. The assisted filed \( \mathbf{E}_a \), when assumed to be weak (satisfying \( \mathcal{E}_p \leq \mathcal{E}_d \ll \mathcal{E}_e \)), will contribute a CPM effect to the probe field \( \mathbf{E}_p \). Note that the levels \( |i\rangle \) (\( i = 1, 2, 3, 4 \)) together with \( \mathbf{E}_p \), \( \mathbf{E}_e \), and \( \mathbf{E}_a \) form a N-type system, which was considered firstly by Schmidt Imamoğlu \cite{22} for obtaining giant CPM via EIT.

In addition, we assume there is another far-detuned (Stark) optical lattice field

\[
\mathbf{E}_{\text{Stark}} = e \sqrt{2} E_s(x) \cos(\omega_L t) 
\]  

is applied to the system, where \( E_s(x) \) and \( \omega_L \) are respectively the field-distribution function and angular frequency. Due to the existence of \( \mathbf{E}_{\text{Stark}} \), a small and \( \chi \)-dependent Stark shift of level \( E_j \) to the state \( |j\rangle \) occurs, i.e., \( E_j \rightarrow E_j + \Delta E_j \) with \( \Delta E_j = -\frac{1}{2} \alpha_j \langle E_{\text{Stark}} \rangle_j = -\frac{1}{2} \alpha_j E_s(x) \left| \langle 3 | 2 \rangle \right|^2 \).

As will be shown below, the CPM effect contributed by the assisted field \( \mathbf{E}_a \) given by (11) and the Stark shift contributed by the far-detuned Stark field \( \mathbf{E}_{\text{Stark}} \) given by (2) will provide periodic complex refractive index to the evolution of probe-filed envelope. However, they are still not enough to obtain a refractive index with PT symmetry since a gain to the probe field is needed. Therefore, we introduce an incoherent optical pumping which can pump atoms from the ground-state level \( |1\rangle \) to the excited-state level \( |3\rangle \) with the pumping rate \( \Gamma_{31} \) [see equations (18a) and (18c) in Appendix]. Such optical pumping can be realized by many techniques, such as intense atomic resonance lines emitted from hollow-cathode lamps or from microwave discharge lamps \cite{23}.

In Fig. 1(a), \( \Gamma_{13}, \Gamma_{23}, \) and \( \Gamma_{24} \) are spontaneous emission rates denoting the population decays respectively from \( |3\rangle \) to \( |1\rangle \), \( |3\rangle \) to \( |2\rangle \), and \( |4\rangle \) to \( |2\rangle \); \( \Omega_p = (e_3 \cdot p_{13}) \mathcal{E}_p / \hbar \), \( \Omega_c = (e_3 \cdot p_{23}) \mathcal{E}_c / \hbar \), and \( \Omega_a = (e_3 \cdot p_{24}) \mathcal{E}_a / \hbar \) are respectively the half Rabi frequencies of the probe, control, and assisted fields, here \( p_{ij} \) signifies the electric dipole matrix element of the transition from state \( |i\rangle \) to \( |j\rangle \), \( \Delta_3 \), \( \Delta_2 \), and \( \Delta_4 \) are respectively one-, two-, and three-photon detunings in relevant transitions. Fig. 1(b) shows a possible experimental arrangement.

### 2.2. Maxwell-Bloch equations

Under electric-dipole and rotating-wave approximations, the Hamiltonian of the system in interaction picture reads

\[
\hat{H}_{\text{int}} = -\hbar \sum_{j=1}^{4} \Delta_j' |j\rangle \langle j| - \hbar \left( \Omega_p |3\rangle \langle 1| + \Omega_c |3\rangle \langle 2| + \Omega_a |4\rangle \langle 2| \right) + \text{h.c.},
\]

where h.c. denotes Hermitian conjugate, and

\[
\Delta_j' = \Delta_j + \frac{\alpha_j}{2\hbar} |E_s(x)|^2.
\]  

The motion of atoms interacting with the light fields is described by the Bloch equation

\[
\frac{\partial \sigma}{\partial t} = -\frac{i}{\hbar} [\hat{H}_{\text{int}}, \sigma] - \Gamma \sigma,
\]

where \( \sigma_{ij} \) is the density-matrix elements in the interaction picture, \( \Gamma \) is a \( 4 \times 4 \) relaxation matrix. Explicit expressions of Eq. (4) are presented in Appendix, in which an incoherent optical pumping (represented by \( \Gamma_{31} \)) from the level \( |1\rangle \) to the level \( |3\rangle \) is introduced [see equations (18a) and (18c)].

Under a slowly varying envelope approximation, Maxwell equation of the probe field is reduced to

\[
i \left( \frac{\partial}{\partial z} + \frac{1}{c} \frac{\partial}{\partial t} \right) \Omega_p + \frac{c}{2\omega_p} \frac{\partial^2 \Omega_p}{\partial x^2} + \kappa_{31} \sigma_{31} = 0,
\]  

(5)
where \( \kappa_{13} = N \omega_x |e_x \cdot p_{13}|^2 / (2 \varepsilon_0 \hbar c) \) with \( N \) being the atomic concentration. Note that, for simplicity, we have assumed \( \Omega \) is independent on \( y \), which is valid only for the probe beam having a large width in the \( y \)-direction so that the diffraction term \( \partial^2 \Omega_p / \partial y^2 \) can be neglected; in addition, we have also assumed that the dynamics of \( \Omega_a \) is negligible during probe-field evolution, which is a reasonable approximation because the assisted field couples to the levels \([2]\) and \([4]\) that have always vanishing population due to the EIT effect induced by the strong control field.

3. Realization of PT symmetric potential

3.1. Equation of the probe-field envelope

The Maxwell equation (5) governs the propagation of the probe field. To solve it one must know \( \sigma_{11} \), which is controlled by the Bloch equation (4) and hence coupled to \( \Omega_p \). For simplicity, we assume \( \Omega_p \) has a large time duration \( \tau_p \) so that \( \Gamma_{31} \tau_p \gg 1 \). In this case a continuous-wave approximation can be taken. As a result, the time derivatives in the Maxwell-Bloch (MB) equations (4) and (5) (i.e. the dispersion effect of the probe field) can be neglected, and only the diffraction effect of the probe field in \( x \) direction is considered. In addition, because the probe field is weak, a perturbation expansion can be used for solving coupled equations (4) and (5) analytically \([24,25]\).

We take the expansion \( \sigma_{ij} = \sigma_{ij}(0) + \varepsilon \sigma_{ij}(1) + \varepsilon^2 \sigma_{ij}(2) + \varepsilon^3 \sigma_{ij}(3) + \cdots, \Omega_p = \varepsilon \Omega_p(1) + \varepsilon^2 \Omega_p(2) + \cdots \). Here \( \varepsilon \) is a small parameter characterizing the typical amplitude of the probe field (i.e. \( \Omega_p \text{max} / \Omega_a \)). Substituting such expansion to equations (4) and (5), we obtain a series of linear but inhomogeneous equations for \( \sigma_{ij}^{(l)} \) and \( \Omega_p^{(l)} \) \((l = 1, 2, 3, \ldots)\) that can be solved order by order.

To get a divergence-free perturbation expansion, \( \sigma_{ij}^{(l)} \) and \( \Omega_p^{(l)} \) are considered as functions of the multiple scale variables \( z_l = \varepsilon^l z \) \((l = 0, 2) \) and \( x_1 = \varepsilon x \) \([24,25]\). In addition, we assume \( \Omega_a = \varepsilon \Omega_a^{(1)}(x_1) \), \( E_s = \varepsilon E_s^{(1)}(x_1) \). Thus we have \( d_{ij} = d_{ij}^{(0)} + \varepsilon d_{ij}^{(2)} \) with \( d_{ij}^{(0)} = \Delta_i - \Delta_j + i \Gamma_{ij} \) and \( d_{ij}^{(2)} = \left[(\alpha_i - \alpha_j) / (2h)\right]|E_s^{(1)}|^2 \).

At \( \varepsilon \text{O}(1) \)-order, we obtain non-zero density-matrix elements \( \sigma_{11}^{(0)} = 1 - (2 - X_1)X_2, \sigma_{22}^{(0)} = (1 - X_1)X_2, \sigma_{33}^{(0)} = X_2, \sigma_{12}^{(0)} = |\Omega_e^{(0)}|/(d_{32}^{(0)})^\star X_1X_2, \) with \( X_1 = \Gamma_{23}/|2 \text{Im}(|\Omega_e^{(0)}|/d_{32}^{(0)})^2| \) and \( X_2 = \Gamma_{31}/(\Gamma_{12} + \Gamma_{31}(2 - X_1)) \). It is the base state solution of the MB equations (i.e., the solution for \( \Omega_p = \Omega_a = 0 \)). We see that due to the existence of the incoherent optical pumping (i.e., \( \Gamma_{31} \neq 0 \)) there are populations in the states \([1], [2], \) and \([3]\). Because \( \Gamma_{31} \) takes the order of MHz in our model, the populations in \([2]\) and \([3]\) are small. In particular, \( \sigma_{22}^{(0)} = \sigma_{33}^{(0)} = 0, \sigma_{11}^{(0)} = 1 \) when \( \Gamma_{31} = 0 \).

At \( \varepsilon \text{O}(\varepsilon) \)-order, the solution is given by

\[
\begin{align*}
\Omega_p^{(1)} &= F e^{i \kappa_{13} x}, \\
\sigma_{21}^{(1)} &= \frac{\Omega_e^{(0)}\sigma_{33}^{(0)} - \sigma_{11}^{(0)} - d_{31}^{(0)} \sigma_{23}^{(0)}}{D_1} F e^{i \kappa_{13} x} = \alpha_{21}^{(1)} F e^{i \kappa_{13} x}, \\
\sigma_{31}^{(1)} &= \frac{K}{\kappa_{13}} F e^{i \kappa_{13} x} = \alpha_{31}^{(1)} F e^{i \kappa_{13} x}, \\
\sigma_{42}^{(1)} &= \frac{d_{42}^{(0)} \sigma_{22}^{(0)} + \Omega \sigma_{23}^{(0)}}{D_2} \Omega_a^{(1)} = \alpha_{42}^{(1)} \Omega_a^{(1)}, \\
\sigma_{43}^{(1)} &= \Omega_e^{(0)} \sigma_{23}^{(0)} + d_{43}^{(0)} \Omega_a^{(1)} = \alpha_{43}^{(1)} \Omega_a^{(1)},
\end{align*}
\]

with other \( \sigma_{ij}^{(2)} = 0 \). Here \( F \) is yet to be determined envelope function, \( D_1 = |\Omega_e| - d_{31}^{(0)} d_{21}^{(0)} \).
Fig. 2. The imaginary part \(\text{Im}K\) of \(K\) as a function of \(\Delta_3/\gamma_3\) for \(\Delta_2 = \Delta_3\). Solid (red), dashed (green), and dashed-dotted (blue) lines correspond to \((\Omega_c, \Gamma_{31}) = (0, 0), (5 \times 10^7 \text{ Hz}, 0), \) and \((5 \times 10^7 \text{ Hz}, 0.7\gamma)\), respectively. For illustration, the value of dashed-dotted (green) line has been amplified 7.8 times.

\[
D_2 = |\Omega_c|^2 - d_{42}^{(0)}d_{43}^{(0)}, \quad \text{and} \quad K = \kappa_{13}d_{21}^{(0)}(\sigma_{11}^{(0)} - \sigma_{33}^{(0)}) + \Omega_c\sigma_{21}^{(0)}.
\]

Obviously, in the linear case \(\Omega_p \propto e^{iKz}\), and \(K\) is complex. Thus \(K\), particularly its imaginary part, controls the behavior of the probe-field propagating along \(z\).

Figure 2 shows the imaginary part \(\text{Im}K\) of \(K\) as a function of \(\Delta_3/\gamma_3\) for \(\Delta_2 = \Delta_3\). The system parameters used are \[26\] \(\gamma_1 = \Delta_1 = 0\text{ Hz}, 2\gamma_2 = 1 \times 10^3 \text{ Hz}, \Gamma_3 = 2\gamma_3 = 36\text{ MHz}, \kappa_{13} = 1.0 \times 10^{10} \text{ cm}^{-1} \text{ Hz}\). Solid (red), dashed (green), and dashed-dotted (blue) lines correspond to \((\Omega_c, \Gamma_{31}) = (0, 0), (5 \times 10^7 \text{ Hz}, 0), \) and \((5 \times 10^7 \text{ Hz}, 0.7\gamma)\), respectively.

From the solid line of Figure 2 we see that in the absence of the control field and incoherent pumping (i.e., \(\Omega_c = \Gamma_{31} = 0\)), the probe field has a very large absorption; however, when the incoherent pumping still absent but \(\Omega_c\) takes the value of \(5 \times 10^7 \text{ Hz}\), a transparency window is opened (as shown by the dashed line). This is well-known EIT quantum interference phenomenon induced by the control field \[21\]. However, there is still a small absorption (i.e., \(\text{Im}K > 0\), which can not be seen clearly due to the resolution of the figure). That is to say, although EIT can suppress largely the absorption, it can not make the absorption become zero.

The dashed-dotted line in Fig. 2 is the situation when the incoherent pumping \((\Gamma_{31} = 0.7\gamma)\) is introduced. One sees that a gain (i.e., negative \(\text{Im}K\)) in the region near \(\Delta_3 = 0\) occurs. Such gain is necessary to get a PT-symmetric optical potential for the probe-field propagation, as shown below.

At \(\mathcal{O}(\varepsilon^4)\)-order of the perturbation expansion, we obtain the closed equation for \(\Omega_p\):

\[
i\frac{\partial \Omega_p}{\partial z} + \frac{c}{2\Omega_p} \frac{\partial^2 \Omega_p}{\partial x^2} + \tilde{V}(x)\Omega_p = 0 \quad (8)
\]

after returning to original variables, with

\[
\tilde{V}(x) = \alpha_{12} \frac{|\mathbf{e}_y \cdot \mathbf{p}_{24}|^2}{\hbar^2} |\psi_0(x)|^2 + \alpha_{13} |\mathbf{E}_z(x)|^2 + K,
\]

\(K\) is a small correction term.
where $\Omega_p = \epsilon F \exp(iKz)$, the coefficients $\alpha_{12}$ and $\alpha_{13}$ are given in Appendix.

We now make some remarks about the potential $\tilde{V}(x)$ given by Eq. (9):

(1). The coefficients $\alpha_{12}$ and $\alpha_{13}$ are complex. We stress that the occurrence of a complex potential for the evolution of probe-field envelope is a general feature in the system with resonant interactions. The reason is that, due to the resonance, the finite lifetime of atomic energy states must be taken into account. As a result, the variation of the probe-field wavevector resulted by the external light laser fields (here the Stark and the assisted fields) are complex. It is just this point that provides us the possibility to realize a PT symmetric potential in our system by using the periodic external laser fields.

(2). If the incoherent pumping is absent, the probe field has only absorption but no gain and hence not possible to realize PT symmetry. With the incoherent pumping present, the parameter $K$ [given by the Eq. (7)] in the Eq. (9) is complex and has negative imaginary part in the region near $\Delta_3 = 0$, which can be used to suppress an absorption constant (i.e. the term not dependent on $x$) appearing in the previous two terms of $\tilde{V}(x)$.

(3). It is easy to show that if only a single external laser field (the Stark or the assisted field) is applied, it is impossible to realize a PT symmetry. That is why the two separated light fields (i.e. both the Stark and the assisted fields) have been adopted. We shall show below that the joint action between the Stark field, the assisted field, and the incoherent pumping can give PT-symmetric potentials in the system.

The susceptibility of the probe field is given by $\chi(x) = 2e\tilde{V}(x)/\omega_p$. Because the potential $\tilde{V}$ is a complex function of $x$, which is equivalent to a space-dependent complex refractive index $n(x) = \sqrt{1 + \chi(x)} \approx 1 + e\tilde{V}(x)/\omega_p$ for the probe-field propagation. PT symmetry requires $\tilde{V}^*(-x) = \tilde{V}(x)$, which is equivalent to the condition $n^*(-x) = n(x)$.

3.2. The design of PT symmetric potential

Equation (8) is a linear Schrödinger equation with the “external” potential $\tilde{V}$). To realize a PT-symmetric model we assume the field-distribution functions in (1) and (2) taking the forms

$$E_a(x) = E_{a0}[\cos(x/R) + \sin(x/R)],$$

$$E_s(x) = E_{s0}\cos(x/R),$$

with $E_{a0}$ and $E_{s0}$ being typical amplitudes and $R^{-1}$ being typical “optical lattice” parameter. For convenience of later discussion, we write Eq. (8) into the following dimensionless form

$$i\frac{\partial u}{\partial s} + \frac{\partial^2 u}{\partial \xi^2} + V(\xi)u = 0,$$

with

$$V(\xi) = (g_{12} + g_{13}\sin 2\xi) + g_{13}\cos^2\xi + K_0,$$

where $u = \Omega_p/U_0$, $s = z/L_{\text{diff}}$, $\xi = x/R$, $g_{12} = \alpha_{12}E_{\Omega_p}L_{\text{diff}}/\hbar^2$, $g_{13} = \alpha_{13}E_{\Omega_p}^2L_{\text{diff}}/\hbar^2$, and $K_0 = KL_{\text{diff}}$. Here, $L_{\text{diff}} = 2\omega_pR^2/c$ is the typical diffraction length and $U_0$ denotes the typical Rabi frequency of the probe field.

PT symmetry of Eq. (12) requires $V^*(-\xi) = V(\xi)$. In general, such requirement is difficult to be satisfied because resonant atomic systems have very significant absorption. However, in the system suggested here the absorption can be largely suppressed by the EIT effect induced by the control field. The remainder small absorption that can not be eliminated by the EIT effect may be further suppressed by the introduction of the incoherent optical pumping. If the optical pumping is large enough, the system can acquire a gain. This point can be understood from Fig. 2 for the case of $(\Omega_p, \Gamma_{31}) = (5 \times 10^7 \text{ Hz, } 0.7\gamma)$ where near the EIT transparency window $\text{Im}K$ is negative, which means that the probe field acquires a gain contributed by the
optical pumping. Such gain can be used to suppress the imaginary parts of $g_{12}$ and $g_{13}$ through choosing suitable system parameters, and hence one can realize a PT symmetry of the system.

For a practical example, we select the D$_1$ line of $^{87}$Rb atoms, with the energy levels indicated in the beginning of Sec. 2.1. The system parameters are given by $2\gamma_2 = 1 \times 10^3$ Hz, $\Gamma_{3,4} = 2\gamma_{3,4} = 36$ MHz, $|p_{24}| = 2.54 \times 10^{27}$ C cm, $\omega_p = 2.37 \times 10^{15}$ s$^{-1}$. Other (adjustable) parameters are taken as $\kappa_{12} = 2.06 \times 10^{11}$ cm$^{-1}$s$^{-1}$, $R = 2.5 \times 10^{-2}$ cm, $\Omega_c = 4.0 \times 10^8$ s$^{-1}$, $\Delta_2 = -5.0 \times 10^5$ s$^{-1}$, and $\Delta_4 = 0$. Then we have $L_{\text{diff}} = 1.0$ cm, and

$$\delta_u(x) = 0.1 \left( \cos \xi + \sin \xi \right) \text{V/cm},$$

$$E_s(x) = 4.51 \times 10^5 \cos \xi \text{ V/cm},$$

$$\Gamma_{31} = 7.0 \times 10^5 \text{ Hz}. \quad (16)$$

Based on these data and the assisted laser field (14), the far-detuned laser field (15) and the optical pumping (16), we have $g_{12} = 0.01 + 0.4i$, $g_{13} = 1.00 + 0.03i$, and $K_0 = -11.7 - 0.4i$. Here, the imaginary parts of $g_{12}$ and $K_0$ can be alone controlled by $\delta_u(x)$ and $\kappa_{13}$, respectively. As a result, we obtain

$$V(\xi) = -11.7 + \cos^2 \xi + 0.4i \sin 2\xi + O(10^{-2}). \quad (17)$$

Equation (17) satisfies the PT-symmetry requirement $V^*(\xi) = V(\xi)$ when exact to the accuracy $O(10^{-2})$. The constant term $-11.7$ in $V(\xi)$ can be removed by using a phase transformation $u \rightarrow u \exp(-i11.7s)$. Equation (17) is a kind of PT-symmetric periodic potential. In fact, one can design many different periodic potentials or non-periodic potentials with PT symmetry in our system by using different assisted and far-detuned laser fields. Consequently, our system has obvious advantages for actively designing different PT-symmetric optical potentials and manipulating them in a controllable way.

4. Conclusion

We have proposed a scheme to realize PT symmetry via EIT. The system we considered is an ensemble of cold four-level atoms with an EIT core. We have shown that the cross-phase modulation contributed by an assisted field, the optical lattice potential provided by a far-detuned laser field, and the optical gain coming from an incoherent pumping can be used to construct a PT-symmetric complex optical potential for probe field propagation in a controllable way. Comparing with previous study in [20], our scheme has the following advantages: (i) Our scheme uses only one atomic species, which is much simpler than that in [20]. (ii) The mechanism of realizing the PT-symmetric potential is based on EIT, which is different from that in [20] where a double Raman resonance was used. (iii) One can design many different PT-symmetric potentials at will in our scheme in a simple way.
Appendix

Explicit expression of Eq. (4)

Equations of motion for \( \sigma_{ij} \) are given by

\[
i \frac{\partial}{\partial t} \sigma_{11} + i \Gamma_{31} \sigma_{31} - i \Gamma_{13} \sigma_{33} + \Omega_p^* \sigma_{31} - \Omega_p \sigma_{31}^* = 0, \quad (18a)
\]

\[
i \frac{\partial}{\partial t} \sigma_{22} - i \Gamma_{23} \sigma_{33} - i \Gamma_{24} \sigma_{44} + \Omega_4^* \sigma_{32} - \Omega_4 \sigma_{32}^* + \Omega_p^* \sigma_{42} - \Omega_p \sigma_{42}^* = 0, \quad (18b)
\]

\[
i \frac{\partial}{\partial t} \left( \frac{\partial}{\partial t} + \Gamma_3 \right) \sigma_{33} - i \Gamma_{31} \sigma_{11} - \Omega_p^* \sigma_{31} - \Omega_p \sigma_{31}^* - \Omega_4^* \sigma_{32} - \Omega_4 \sigma_{32}^* = 0, \quad (18c)
\]

\[
i \frac{\partial}{\partial t} \sigma_{44} - \Omega_p^* \sigma_{42} + \Omega_p \sigma_{42}^* = 0, \quad (18d)
\]

\[
i \frac{\partial}{\partial t} \sigma_{21} + \Omega_4^* \sigma_{31} + \Omega_p^* \sigma_{41} - \Omega_p \sigma_{41}^* = 0, \quad (18e)
\]

\[
i \frac{\partial}{\partial t} \sigma_{31} + \Omega_p^* (\sigma_{11} - \sigma_{33}) + \Omega_p \sigma_{21} = 0, \quad (18f)
\]

\[
i \frac{\partial}{\partial t} \sigma_{41} + \Omega_p \sigma_{21} - \Omega_p \sigma_{43} = 0, \quad (18g)
\]

\[
i \frac{\partial}{\partial t} \sigma_{32} + \Omega_4 (\sigma_{22} - \sigma_{33}) + \Omega_p \sigma_{21}^* - \Omega_p \sigma_{43}^* = 0, \quad (18h)
\]

\[
i \frac{\partial}{\partial t} \sigma_{32} + \Omega_4 (\sigma_{22} - \sigma_{33}) + \Omega_4 \sigma_{32} - \Omega_p \sigma_{32}^* = 0, \quad (18i)
\]

\[
i \frac{\partial}{\partial t} \sigma_{33} + \Omega_4 \sigma_{32}^* - \Omega_p^* \sigma_{31} - \Omega_p \sigma_{31}^* = 0, \quad (18j)
\]

with \( d_{il} = \Delta'_i - \Delta'_l + i \gamma_{jl} \) (\( \Delta'_j \) is given by Eq. (3)), \( \gamma_{jl} = (\Gamma_j + \Gamma_l)/2 + \gamma_{jl}^{\text{dph}} \) \( (j \neq 3, l \neq 1) \), \( \gamma_{31} = (\Gamma_3 + \Gamma_{31})/2 + \gamma_{31}^{\text{dph}} \), and \( \Gamma_1 = \sum_{E_j \neq E_i} \Gamma_{jl} \). Here \( \gamma_{jl}^{\text{dph}} \) denotes the dipole dephasing rates caused by atomic collisions; \( \Gamma_{jl} \) is the rate at which population decays from \( |l\rangle \) to \( |j\rangle \). Especially, \( \Gamma_{31} \) is the incoherent pumping rate from \( |1\rangle \) to \( |3\rangle \).

Perturbation expansion of the MB equations

The coefficients of Eq. (9) are given by

\[
\alpha_{41} = -\frac{k_{13} \Omega_c}{D_1} a_{41}^{(2)} + \frac{k_{13} \Omega_c}{D_1} a_{23}^{(2)} + \frac{k_{13} d_{21}^{(0)}}{D_1} \left( \alpha_{11}^{(2)} - a_{23}^{(2)} \right), \quad (19a)
\]

\[
\alpha_{31} = \frac{k_{13} (\alpha_3 - \alpha_1)}{2 h D_1} d_{21} a_{31}^{(1)}, \quad (19b)
\]
where

\[ \alpha_{22F}^{(2)} = \frac{-2\Gamma \text{Im} \left[ \frac{\alpha_{22}^{(1)}}{d_{32}^{(0)}} \right] - 2(\Gamma_{23} - X_3)\text{Im}(\alpha_{31}^{(1)})}{\Gamma X_3 - \Gamma_{31}(\Gamma_{23} - X_3)}, \] (20a)

\[ \alpha_{33F}^{(2)} = \left[ 2\text{Im}(\alpha_{31}^{(1)}) - \Gamma_{31}\alpha_{22F}^{(2)} \right] / \Gamma, \] (20b)

\[ \alpha_{22G}^{(2)} = \frac{2\text{Im}(\alpha_{31}^{(1)}) \alpha_{22F}^{(2)} + 2\text{Im}(\alpha_{42}^{(1)}) + 2\Gamma_{31}(\Gamma_{23} - X_3)\text{Im}(\alpha_{42}^{(1)})/\Gamma_4}{\Gamma X_3 - \Gamma_{31}(\Gamma_{23} - X_3)}, \] (20c)

\[ \alpha_{44}^{(2)} = \frac{2}{\Gamma_4} \text{Im}(\alpha_{42}^{(1)}), \] (20d)

\[ \alpha_{41}^{(2)} = \frac{\alpha_{41}^{(1)} - \alpha_{21}^{(1)}}{d_{41}^{(0)}}, \] (20e)

\[ \alpha_{33G}^{(2)} = (-\Gamma_{31}\alpha_{44}^{(2)} - \Gamma_{31}\alpha_{22G}^{(2)}) / \Gamma, \] (20f)

\[ \alpha_{11F}^{(2)} = \frac{\Gamma_{13}\alpha_{33F}^{(2)} - 2\text{Im}(\alpha_{31}^{(1)})}{\Gamma_{31}}, \] (20g)

\[ \alpha_{11G}^{(2)} = \frac{\Gamma_{13}\alpha_{33G}^{(2)} / \Gamma_{31}}{\Gamma_{31}}, \] (20h)

\[ \alpha_{23F}^{(2)} = \frac{(-\alpha_{21}^{(1)} + \Omega_+ \alpha_{33F}^{(2)} - \Omega_- \alpha_{22F}^{(2)}) / (d_{32}^{(0)})^*}{\Gamma X_3 - \Gamma_{31}(\Gamma_{23} - X_3)}, \] (20i)

\[ \alpha_{23G}^{(2)} = \frac{(\alpha_{43}^{(1)} + \Omega_+ \alpha_{33G}^{(2)} - \Omega_- \alpha_{22G}^{(2)}) / (d_{32}^{(0)})^*}{\Gamma X_3 - \Gamma_{31}(\Gamma_{23} - X_3)}, \] (20j)

with \( \Gamma = \Gamma_{13} + \Gamma_{31}. \)

**Acknowledgments**

This work was supported by the NSF-China under Grant Nos. 11074221, 11174080, and 11204274, and by the discipline construction funds of ZJNU under Grant No. ZC323007110; and in part by the Open Fund from the State Key Laboratory of Precision Spectroscopy, ECNU.