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Improvement of Vision Measurement Accuracy Using Zernike Moment Based Edge Location Error Compensation Model

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Abstract. This paper presents the Zernike moment based model developed to compensate edge location errors for further improvement of the vision measurement accuracy by compensating the slight changes resulting from sampling and establishing mathematic expressions for subpixel location of theoretical and actual edges which are either vertical to or at an angle with X-axis. Experimental results show that the proposed model can be used to achieve a vision measurement accuracy of up to 0.08 pixel while the measurement uncertainty is less than 0.36μm. It is therefore concluded that as a model which can be used to achieve a significant improvement of vision measurement accuracy, the proposed model is especially suitable for edge location of images with low contrast.

Keywords: vision measurement, zernike moment, edge location, error compensation model

1. Introduction
The accurate location of an edge is a very important function of the vision measurement and processing system. Traditional grads operators are high-pass operators sensitive to both the edge of an image and noise with their location accuracy is one pixel only, and they can not be used to satisfy higher precision required in such fields as integratal manufacturing, computer vision and photogrammetry Many other ways and means have therefore been proposed to this end, for example, equation parameter fitting method[1-2], frequency sampling method[3], nonlinearity interpolation method[4-5], dithering method[6], tangent information method[7], gray and space moment method[8-13], and of these methods, the arithmetical method based on moment is widely used for its less computation, high location precision, and less sensitivity to additive or multiplicative noise, it uses a theoretical two-grayscale instead of actual three-grayscale edge pattern, so that it is inevitable to incur theoretical errors. Although theoretical errors incurred have been addressed in some papers, the model suggested for rectification of location error take into consideration the edges vertical to X-axis, and no analysis is made for any actual edge at an angle with X-axis. Therefore a Zernike moment based model is presented in this paper for compensation of edge location error which is suitable for a two or three-grayscale edge pattern with edge vertical to or at an angle with X-axis.

2. Zernike moment based subpixel edge location method using two-grayscale edge pattern
The Zernike moment of image \( f(x,y) \) in \( n \) and \( m \) order can be expressed as:
\[
A_{nm} = \frac{n+1}{\pi} \int_{0}^{2\pi} \int_{-1}^{1} f(x, y) \cdot V_{nm}^*(\rho, \theta) \, d\rho \, d\theta 
\]

where, \( V_{nm}(\rho, \theta) \) is the Zernike polynomial in \( n \) and \( m \) order in the unit circle in polar coordinates, * is the complex conjugate.

When an edge is located using Zernike moment, only three Zernike polynomials in different orders of \( A_{00}, A_{11}, A_{20} \) are used, and their integral functions are \( V_{00} = 1 \), \( V_{11} = x + jy \), \( V_{20} = 2x^2 + 2y^2 - 1 \) respectively. According to the rotation invariance of complex moment amplitude of Zernike moment,

\[
A'_{nm} = A_{nm} e^{-jnm\phi} 
\]

where, \( A'_{nm} \) is the Zernike moment after rotation, \( A_{nm} \) is Zernike moment before rotation, and \( \phi \) is the rotation angle.

The relationships between the Zernike moments of an image before and after rotation are as follows:

\[
A'_{00} = A_{00}, \quad A'_{11} = A_{11} e^{j\phi}, \quad A'_{20} = A_{20} 
\]

After the edge is rotated through angle \( \phi \), the imaginary part of \( A_{11} \) is zero while the edge is parallel with Y-axis,

\[
\text{Im}[A'_{11}] = \sin(\phi) \text{Re}[A_{11}] - \cos(\phi) \text{Im}[A_{11}] = 0 
\]

where \( \text{Im}[A'_{11}] \) and \( \text{Re}[A'_{11}] \) are the imaginary and real parts of \( A_{11} \) after rotation respectively, so the rotation angle of the edge is:

\[
\phi = \tan^{-1} \left( \frac{\text{Im}[A_{11}]}{\text{Re}[A_{11}]} \right) 
\]

For the pattern shown in figure 1, the following expressions can be used for calculation of Zernike moment:

\[
A'_{00} = h\pi + \frac{k\pi}{2} - k \sin^{-1} l - kl\sqrt{1-l^2}, \quad A'_{11} = \frac{2k^3(1-l^2)}{3}, \quad A'_{20} = \frac{2k\sqrt{1-l^2}}{3} 
\]

According to equation (1), the vertical distance between the center of the circle and the edge can be expressed as:

\[
l = \frac{A_{20}}{A'_{11}} 
\]

The subpixel position of the image edge is:

\[
\begin{bmatrix}
  x'_s \\
  y'_s
\end{bmatrix} = \begin{bmatrix}
  x \\
  y
\end{bmatrix} + l \begin{bmatrix}
  \cos \phi \\
  \sin \phi
\end{bmatrix} 
\]
The subpixel edge location can be achieved using this location method with three Zernike moments of $A_{00}$, $A_{11}'$, $A_{20}$ only. When the grayscale is discrete, calculation of moment is a correlating calculation, to be exact, a convolution calculation of moment masks. So only three Zernike moment masks are used for the location of the edge and the computation is less than what is needed when the location method based on space moment is used.

3. **Zernike moment based subpixel edge location method using three-grayscale edge pattern**

3.1. **Mathematic expression for location of edge in parallel with Y-axis**

Although the foundation of equation (7) is based on the theoretical grayscale edge pattern shown in figure 1, the sensitization unit of CCD is limited in size, the sampling process is not an theoretical discrete sampling of a consecutive edge, and the optical measurement system is limited by optical diffraction, so there must be a transitional process up to the edge. The actual edge pattern is not a theoretical two-grayscale pattern, and it must be expressed by three-grayscale: background, object, and transition process as shown in figure 2. When the edge is parallel with Y-axis, for the new three-grayscale edge pattern, the mathematic expressions used for edge location are as follows:

**Figure 2.** Actual edge characterized by $h$, $\Delta k$, $k$, $l_1$ and $l_2$.

\[
A_{00} = 2 \int_{-1}^{1} \int_{0}^{\sqrt{1-x^2}} h \, dy \, dx + 2 \int_{h}^{l_1} \int_{0}^{\sqrt{1-x^2}} k \, dy \, dx + 2 \int_{l_1}^{1} \int_{0}^{\sqrt{1-x^2}} k \, dy \, dx
\]

\[= h\pi + k \left( \frac{\pi}{2} - l_2 \sqrt{1-l_2^2} - \arcsin l_2 \right) + \Delta k \left( l_2 \sqrt{1-l_2^2} - \arcsin l_2 - l_1 \sqrt{1-l_1^2} - \arcsin l_1 \right)\]

\[A_{11} = 2 \int_{-1}^{1} \int_{0}^{\sqrt{1-x^2}} h(x+iy) \, dy \, dx + 2 \int_{h}^{l_1} \int_{0}^{\sqrt{1-x^2}} k(x+iy) \, dy \, dx + 2 \int_{l_1}^{1} \int_{0}^{\sqrt{1-x^2}} k(x+iy) \, dy \, dx
\]

\[= -i \frac{4}{3} h + 2 \frac{\Delta k}{3} \left( (1-l_2^2)^3 - (1-l_1^2)^3 \right) + i \cdot \Delta k \left( l_2 - l_1 + \frac{l_2^3 - l_1^3}{3} \right)\]

\[+ 2k \frac{2}{3} \sqrt{1-l_2^2} - i \cdot k \left( \frac{2}{3} - l_2 + \frac{l_2^3}{3} \right)\]

\[A_{20} = 2 \int_{-1}^{1} \int_{0}^{\sqrt{1-x^2}} h(2x^2 + 2y^2 - 1) \, dy \, dx + 2 \int_{h}^{l_1} \int_{0}^{\sqrt{1-x^2}} k(2x^2 + 2y^2 - 1) \, dy \, dx
\]

\[+ 2 \int_{l_1}^{1} \int_{0}^{\sqrt{1-x^2}} k(2x^2 + 2y^2 - 1) \, dy \, dx
\]

\[= \frac{2\Delta k}{3} \left[ l_2 (l_2 - 1) \sqrt{1-l_2^2} - l_1 (l_1 - 1) \sqrt{1-l_1^2} - \frac{2k}{3} l_2 (l_2 - 1) \sqrt{1-l_2^2} \right] (9)\]

From equations (7), (8) and (9), subpixel location of actual three-grayscale edge pattern is:

\[
\begin{bmatrix}
x_s \\
y_s
\end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + l_m \begin{bmatrix} \cos \phi \\ \sin \phi \end{bmatrix}
\]

(10)
where, \( l_M = \frac{(\beta - 1)\sqrt{(l_2^2 - 1)^3 - \beta l_1^3(l_2^2 - 1)}}{\beta \sqrt{(1 - l_1^2)^3} + (1 - \beta)\sqrt{(1 - l_2^2)^3}} \), \( \beta = \frac{\Delta l}{k} \), \( 0 \leq \beta \leq 1 \).

3.2. Analysis of location errors

For a three-grayscale edge pattern relationship \( \frac{\Delta l}{k} = \frac{l_2 - l_1}{l_2 - l_1} \), then

\[ l = l_2 - \beta (l_2 - l_1) \]  

(11)

The location error resulting from the three-grayscale edge pattern is:

\[ E = l - l_M = \frac{\beta (1 - \beta) (l_2 - l_1) \left( \sqrt{(1 - l_1^2)^3} - \sqrt{(1 - l_2^2)^3} \right)}{\beta \sqrt{(1 - l_1^2)^3} + (1 - \beta)\sqrt{(1 - l_2^2)^3}} \]  

(12)

According to equation (12), the location error is zero when \( \beta (1 - \beta) \left( \sqrt{(1 - l_1^2)^3} - \sqrt{(1 - l_2^2)^3} \right) = 0 \), or \( \beta = 0, \beta = 1 \) and \( l_1 = -l_2 \).

3.3. Model used for location of edge at an angle with X-axis

Equation (10) is expression used for the subpixel edge location, and equation (12) is expression of the edge location error according to the three-grayscale edge pattern. Actually there is only one pixel point when the edge is parallel with Y-axis, and most edge points are at an angle with X-axis. According to equations (1), (2) and (9), the following is the mathematic expression used location error model for Zernike moment integral function of \( V_{00}, V_{11}, V_{20} \) when the edge is at angle \( \alpha \):

\[ E_{location} = l_2 - \beta (l_2 - l_1) - l' \]  

(13)

where, \( l' \), the distance between the sampling center and the edge based on Zernike moment when the edge is at an angle with X-axis, can be given by:

\[ l' = \frac{1}{3} \gamma_1 \gamma_2 \]  

(14)

\[ \gamma_1 = \cos^2 \alpha \left[ \beta l_1 (1 - l_1^2)^2 - \beta l_2 (1 - l_2^2)^2 + \frac{1}{3} l_2 (1 - l_2^2)^2 \right] \]

where,

\[ + \sin^2 \alpha \left[ \frac{1}{3} \beta l_1 (1 - l_1^2)^2 - \frac{1}{3} \beta l_2 (1 - l_2^2)^2 + \frac{1}{3} l_2 (1 - l_2^2)^2 \right] \]

\[ \gamma_2 = \cos \alpha \left[ \frac{2}{3} \beta (1 - l_1^2)^2 - \frac{1}{3} \beta (1 - l_2^2)^2 + \frac{2}{3} (1 - l_2^2)^2 \right] \]

4. Simulation and experimental analysis of edge location errors

4.1. Simulation analysis

According to 3.2 and 3.3 above, the error is 0 when \( \beta = 0, \beta = 1 \) or \( l_1 = -l_2 \), the simulation diagrams of the edge at an angle with X-axis are shown in figures 3, 4 and 5. It can be seen from these diagrams that the biggest location error caused by the angle is 0.52 pixel when \( \beta = 0, 1.25 \) pixel when \( \beta = 1 \), and 0.18 pixel when \( l_1 = -l_2 \). The location error is antisymmetric, so it has to be compensated while subpixel location of actual edge is calculated.
4.2. Experimental analysis

In order to evaluate the effectiveness of the proposed location error model, standard and actual images are used to locate the edges of these images, and a special object is measured by using the proposed model.

4.2.1. Location of standard straight line. The proposed model is used to locate the straight line edge of an image created with computer, which measures 256 pixel×256 pixel as shown in figure 6. The distance between the vertical line and the left edge of the image is 26 pixel, the distance between the horizontal line and the upper edge of the image is also 26 pixel. The gradient of the tilted line is 45°, the distance from the top-left corner to the tilted line is 122\sqrt{2} pixel. The locations of edges after been compensated using the proposed model are as shown in table 1.

4.2.2. Location of standard curves. The proposed model is used to locate the curves of an image created with computer, which measures 256 pixel×256 pixel as shown in figures 7. The location of
circle centre is (128 pixel, 128 pixel), and the radius of the circle is 80.0 pixels. The locations of edges after been compensated using the proposed model are as shown in table 2.

According to tables 1 and 2, the location error for a standard straight line is 0.05 pixel after been compensated using the proposed model, and it is 0.08 pixel for curves.

| Theoretical location | Actual location  | Location error along |
|----------------------|------------------|----------------------|
|                      |                  | X-axis     | Y-axis     |
| (35.00,26.00)        | (35.00,25.95)    | 0.00       | -0.05      |
| (36.00,26.00)        | (36.00, 25.95)   | 0.00       | -0.05      |
| (37.00,26.00)        | (37.00, 25.95)   | 0.00       | -0.05      |
| (38.00,26.00)        | (38.00, 25.95)   | 0.00       | -0.05      |
| (39.00,26.00)        | (39.00, 25.95)   | 0.00       | -0.05      |
| (26.00,35.00)        | (26.05,35.00)    | 0.05       | 0.00       |
| (26.00,36.00)        | (26.05,36.00)    | 0.05       | 0.00       |
| (26.00,37.00)        | (26.05,37.00)    | 0.05       | 0.00       |
| (26.00,38.00)        | (26.05,38.00)    | 0.05       | 0.00       |
| (26.00,39.00)        | (26.05,39.00)    | 0.05       | 0.00       |
| (120.00,124.00)      | (119.94,124.05)  | -0.06      | 0.05       |
| (121.00,123.00)      | (120.94,123.05)  | -0.06      | 0.05       |
| (122.00,122.00)      | (121.94,122.05)  | -0.06      | 0.05       |
| (123.00,121.00)      | (122.94,121.05)  | -0.06      | 0.05       |
| (124.00,120.00)      | (123.94,120.05)  | -0.06      | 0.05       |

4.2.3. **Edge location of actual image.** In order to evaluate the characteristics of the proposed location error model, experiments were made with two objects, a gauge block and a hole as shown in figures 8 and 9, using a high-precision image sampling and processing system. The computer is PIII 133MMX, the operating system is WinXP, the development environment is VC++6.0, and the illumination system is a ring-like reflective lamp. The objects are kept immovable, and their edges are sampled for several times before their edges are located using the proposed error model. The location results are shown in Tables 3 and 4. It can be seen from tables 3 and 4 that the edge location error is 0.08 pixel after been compensated using the proposed location error model.

| Theoretical location | Actual Location  | Location error along |
|----------------------|------------------|----------------------|
|                      |                  | X-axis     | Y-axis     |
| (50.00,145.77)       | (49.96,145.74)   | -0.04      | -0.03      |
| (51.00,149.70)       | (50.93,149.72)   | -0.07      | 0.02       |
| (52.00,152.98)       | (52.06,152.91)   | 0.06       | -0.07      |
| (53.00,155.84)       | (53.08,155.79)   | 0.08       | -0.05      |
| (54.00,158.40)       | (53.94,158.42)   | -0.06      | 0.02       |
| (145.77,50.00)       | (145.72,50.01)   | -0.05      | 0.01       |
| (149.70,51.00)       | (149.73,50.98)   | 0.03       | -0.02      |
| (152.98,52.00)       | (152.98,52.05)   | 0.00       | 0.05       |
| (155.84,53.00)       | (155.79,53.04)   | -0.05      | 0.04       |
| (158.40,54.00)       | (158.42,53.99)   | 0.02       | -0.01      |
4.2.4. Experimental measurement of object. Experimental measurements were made with a high-precision and high-efficiency measurement system developed by following the vision measurement principle for image capturing and processing, which consists of a CCD camera, an inspection microscope object lens, an optical fiber illuminator and a computer with an image capturing card. The object is imaged on the CCD sensor through the inspection microscope, and the computer captures the real-time images of the object through the capturing card and processes the image to acquire parameter of the object.

![Figure 8. Straight line used for experiment](image1)

![Figure 9. Curves used for experiment](image2)

![Figure 10. Image of object](image3)

The object is a slot with a nominal dimension of 3.030 mm, and the image captured by CCD is as shown in figure 10. The proposed method is used to locate and compensate the edge location, the method of least squares is used to fit the parallel edges to attain the parameter of the slot shown in table 5. It can be seen from table 5 that the measurement uncertainty is less than 0.36 \( \mu \text{m} \) after been compensated by the proposed location error model.

| Number of Measurements | 1   | 2   | 2   | 4   |
|------------------------|-----|-----|-----|-----|
| Subpixel location      | (164.26, 67.66) | (164.35, 67.60) | (164.31, 67.62) | (164.34, 67.62) |
| Number of Measurements | 9   | 10  |     |     |
| Subpixel location      | (164.32, 67.61) | (164.26, 67.63) | (164.31, 67.61) | (0.06,0.06) |

Table 5. measurement of slot in \( \mu \text{m} \)
| Measurement result | 3029.87 | 3029.96 | 3030.13 | 3029.89 |
|--------------------|----------|----------|----------|----------|
| Number of Measurements | 5        | 6        | 7        | 8        |
| Measurement result | 3030.46  | 3030.01  | 3030.23  | 3029.99  |
| Number of Measurements | 9        | 10       | Average  | Uncertainty(2σ) |
| Measurement result | 3029.94  | 3030.10  | 3030.058 | 0.36     |

### 4. Conclusion

It can be seen from the results and discussion above that the Zernike moment based model proposed for compensation of edge location error can be used to achieve a vision measurement precision of up to 0.08 pixel while the measurement uncertainty is less than 0.36 µm. It is therefore concluded that as a model which can be sued to achieve a significant improvement of vision measurement accuracy, the proposed model is especially suitable for edge location of images with low contrast.

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