Electron loss from hydrogen-like highly charged ions in collisions with electrons, protons and light atoms

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Abstract

We consider electron loss from a hydrogen-like highly charged ion (HCl) in relativistic collisions with hydrogen and helium in the range of impact velocities \(v_{\text{min}} \leq v \leq v_{\text{max}}\) \((v_{\text{min}}\) and \(v_{\text{max}}\) correspond to the threshold energy \(\varepsilon_\text{th}\) for electron loss in collisions with a free electron and to \(\approx 5\ \varepsilon_\text{th}\), respectively) where any reliable data for loss cross sections are absent. In this range, where the loss process is characterized by large momentum transfers, we express it in terms of electron loss in collisions with equivelocity protons and electrons and explore by performing a detailed comparative study of these subprocesses. Our results, in particular, show that: (i) compared to equivelocity electrons protons are more effective in inducing electron loss, (ii) the relative effectiveness of electron projectiles grows with increase in the atomic number of a HCl, (iii) collisions with protons and electrons lead to a qualitatively different population of the final-state-electron momentum space and even when the total loss cross sections in these collisions become already equal the spectra of the outgoing electrons still remain quite different in almost the entire volume of the final-state-electron momentum space, (iv) in collisions with hydrogen and helium the contributions to the loss process from the interactions with the nucleus and the electron(s) of the atom could be rather well separated in a substantial part of the final-state-electron momentum space.

Keywords: electron loss, relativistic collisions, highly charged ions

(Some figures may appear in colour only in the online journal)

1. Introduction

Collisions of ions with atoms or molecules are accompanied by the number of basic atomic processes which not only are characterized by rich and interesting physics but also have important applications in other fields of physics (e.g. plasma physics, astrophysics, radiation physics), and other sciences (chemistry, biology, medicine). It is, therefore, not surprising that the various aspects of such collisions have been for long time and still remain a subject of extensive experimental and theoretical research.

In collisions between a bare projectile-nucleus and a target-atom the basic atomic processes are: (i) excitation or ionization of the atom, (ii) capture of electron(s) from the atom by the projectile and (iii) pair production (which becomes noticeable only at relativistic collision velocities). If an ion-projectile carries initially electrons then in collisions with an atom these electrons can also undergo transitions: the projectile can be excited or lose electron(s). These processes are termed projectile-electron excitation or loss and they have been intensively studied in the non-relativistic domain of impact energies and projectile charges (for a review see [1–3]).

With the advent of accelerators of relativistic heavy ions, in which the ions may reach velocities close to the speed of light \(c\), much higher impact energies and projectile charge states have become accessible for experiments on ion-atom collisions (for very recent experiments in this field see e.g. [4–6]). Highly charged projectiles produced at accelerators of heavy ions often carry one or more bound electrons which can be excited or lost in collisions with atoms. However, the
projectile-electron excitation and loss occurring in the relativistic domain of impact energies and projectile charges still remain much less explored than their non-relativistic counterparts. For instance, in sharp contrast to the situation in the non-relativistic domain for which theoretical descriptions started to appear as early as in the 1950th [7], even the first rigorous and self-consistent theoretical approaches for treating projectile-electron excitation and loss in the relativistic domain were developed not very long ago [8].

The first order perturbation theory in the interaction between the projectile and the target represents the simplest possible theoretical description of the projectile-electron excitation and loss. In order for this description to be valid, the colliding particles should represent comparatively weak perturbations for each other. The latter is the case when the following two conditions are fulfilled simultaneously: respectively, and (i) \( \alpha Z_f/v \ll 1 \) and (ii) \( \alpha Z_A/v \ll 1 \), where \( \alpha \approx 1/137 \) is the fine-structure constant, \( Z_f \) and \( Z_A \) are the atomic numbers of the projectile ion and target atom, respectively, and \( v \) is the collision velocity. Since the collision velocity cannot exceed the speed of light the conditions (i), (ii) set in general rather strong limitations on the maximum possible values of \( Z_f \) and \( Z_A \).

Recently some theoretical approaches have been proposed to address the process of projectile-electron loss in collisions where the above conditions (i), (ii) are violated. In particular, in collisions of very highly charged projectiles with heavy atoms occurring at not too high impact energies only the impact parameters, which are quite small on the scale of a neutral atom, noticeably contribute to the loss process. In such collisions the field of the atomic nucleus is too strong to be described within the first order perturbation theory but the role of the atomic electrons is negligible. Therefore, the problem of projectile-electron loss in collisions with a neutral atom can be reduced to a three-body problem (the electron, the nucleus of the projectile and the nucleus of the atom) and then treated by developing relativistic three-body Coulomb distorted-wave models [9, 10] (see also [11]).

For collisions with heavy atoms at noticeably higher impact energies, where the role of larger impact parameters increases and, as a result, the screening effect of the atomic electrons can no longer be neglected [11], a symmetric-eikonal model, in which the effect of the strong field of the neutral atom on the electron of the projectile is described by eikonal distortion factors, has been proposed [12]. In the limit \( \gamma \to \infty \), where \( \gamma = 1/\sqrt{1 - v^2} \) is the Lorentz factor of the collision, this model goes over into the so called light-cone eikonal distortion factors, has been proposed to address the process of projectile-electron loss in collisions where the above conditions (i), (ii) are fulfilled simultaneously: respectively, and (i) \( \alpha Z_f/v \ll 1 \) and (ii) \( \alpha Z_A/v \ll 1 \), where \( \alpha \approx 1/137 \) is the fine-structure constant, \( Z_f \) and \( Z_A \) are the atomic numbers of the projectile ion and target atom, respectively, and \( v \) is the collision velocity. Since the collision velocity cannot exceed the speed of light the conditions (i), (ii) set in general rather strong limitations on the maximum possible values of \( Z_f \) and \( Z_A \).

For example, let us suppose that \( U^{91+}(1s) \) loses an electron in the collision with an atom at an impact energy of 1 GeV/u (\( v \approx 120 \) a.u., \( \gamma \approx 2.07 \)). Applying equation (1) to this case we obtain that \( q_{\text{min}}^f \) and \( q_{\text{min}}^A \) exceed 40 a.u. and 19 a.u., respectively. It is obvious that if the atom is very light then \( q_{\text{min}}^A \) will be much larger than the typical orbiting momenta of all atomic electrons. It was shown in detail [15] that in such a case the motion of the atomic electron in the HCl-atom collision is governed by the field of the HCl and the cross section for excitation of or electron loss from the ion is approximated with very good accuracy by the following simple expression:

\[
\sigma = Z_A^\sigma_f + Z_A^\sigma_e,
\]

where \( Z_A \) is the atomic number of the atom, and \( \sigma_f \) and \( \sigma_e \) are the cross sections for the transition of an electron of the ion in collisions with equivelocity incident proton and electron, respectively. According to equation (2) the process of electron loss in collisions with an atom is very simply related to two basic loss processes occurring in collisions with protons and electrons.

3 \( \sigma_e \) should be averaged over the atomic Compton profile, if necessary.
In the present paper we consider electron loss from a hydrogen-like HCI in collisions with hydrogen and helium based on a detailed analysis of the electron loss in collisions with equivelocity protons and electrons. We shall focus on the range of impact energies in which the collision velocity \( v \) varies between \( v_{\text{min}} \) and \( v_{\text{max}} \), where \( v_{\text{min}} \) corresponds to the energy threshold of electron loss in collisions with a free electron and \( v_{\text{max}} \) refers to the kinetic energy of the free electron about five times larger than the threshold value.

This range has been chosen because of two main reasons. First, to our knowledge, for this range of impact velocities there have been no reliable calculations done for electron loss from HCIs in collisions with very light targets (like e.g. hydrogen and helium) and performing them would be of great interest from the theoretical point of view. Moreover, since (accurate) experimental data for collisions of HCIs with very light targets are also lacking in this range, such results could also serve as a guide for future experiments in this field.

Second, a comparative study of electron loss in collisions with equivelocity electrons and protons in the above described range of impact energies, where—unlike very high-energy collisions, in which the energy of the incident electron greatly exceeds the threshold value and in which equivelocity electron and proton projectiles yield already close loss cross sections—the differences between electron loss in collisions with these particles are expected to be quite large, is per se of great interest but, to the best of our knowledge, has never done before. Indeed, there exists a number of experimental and theoretical data on the total cross section for electron loss from hydrogen-like HCIs in collisions with electrons [16–22]. Besides, calculations are available for electron loss from such ions in collisions with various bare nuclei (see e.g. [9, 10, 23–26]) including protons. However the processes of electron loss by electron and proton projectiles have never been considered within the scope of the same paper. (We note that a comparative study of excitation of HCIs by equivelocity protons and electrons was performed in [27].)

The paper is organized as follows. In section 2 we describe two theoretical approaches. One of them is used for the consideration of electron loss from a HCI in collisions with electrons and the other one is applied to collisions with protons. Numerical results and discussion are presented in section 3. Section 4 contains conclusions.

The relativistic units \((\hbar = c = m_e = 1)\) are used throughout unless otherwise stated.

2. General consideration

2.1. Electron loss in collisions with an electron

Our description of electron loss from a hydrogen-like HCI by electron impact is based on the following main points. First, the field of the nucleus of the HCI is so strong that the interaction of both electrons with this field should be taken into account to all orders. Therefore, in our description the Furry picture is used in which the interactions with the nucleus are fully taken into account from the onset. Second, since the interaction between the electrons is relatively very weak it is sufficient to take this interaction into account within the first order of the corresponding perturbation theory\(^4\).

It is convenient to give the basic consideration of electron loss using the rest frame of the HCI. We assume that the nucleus of the HCI is infinitely heavy and take its position as the origin.

Within the one-photon-exchange approximation the amplitude for electron loss reads

\[
U_{p_{\mu}m_{\alpha}p_{\mu}m_{\alpha}} = \alpha \cdot \int d^4r_1 d^4\bar{r}_2 \bar{\psi}_{p_{\mu}m_{\alpha}}(r_1) \psi_{p_{\mu}m_{\alpha}}(r_2) \gamma^\mu \gamma^\nu I_{p_{\mu}p_{\mu}}
\]

\[
\times (\varepsilon - \varepsilon(r_1), |r_1 - r_2|) \psi_{n_{\alpha}m_{\alpha}}(r_2) \gamma^\nu \gamma^\nu I_{n_{\alpha}n_{\alpha}}
\]

\[
\times (\varepsilon - \varepsilon(r_2), |r_1 - r_2|) \psi_{n_{\alpha}m_{\alpha}}(r_1) \psi_{p_{\mu}m_{\alpha}}(r_2).
\]

Here, \( r_1 \) and \( r_2 \) are the electron coordinates, \( \psi_{p_{\mu}} \) is the wave function for an electron in the continuum with an asymptotical momentum \( p \) (the corresponding energy is \( \varepsilon = (1 + p^2)^{1/2} \)) and polarization \( \mu \). The index 0 in the continuum state refers to the incident electron and the indices 1 and 2 to the scattered and emitted electrons, respectively. Further, \( \psi_{n_{\alpha}m_{\alpha}} \) represents the bound state wave function with the total energy \( \varepsilon_{n_{\alpha}} \), where \( n_{\alpha} \), \( f_{\alpha} \), \( l_{\alpha} \) and \( m_{\alpha} \) are the principal quantum number, the total angular momentum, the orbital quantum number of the upper component and the projection of \( f_{\alpha} \) respectively. In equation (3) \( \gamma^\nu \) are the gamma matrices \((i = 1, 2)\) and \( I_{n_{\alpha}n_{\alpha}} \) denotes the photon propagator. In the Feynman gauge the propagator is given by (see e.g. [28])

\[
I_{\gamma,\gamma'}(|\varepsilon - \varepsilon_{n_{\alpha}}, |r_1 - r_2|) = g_{\gamma\gamma'} \exp[i(|\varepsilon - \varepsilon_{n_{\alpha}}| |r_1 - r_2|)]
\]

\[
|r_1 - r_2|.
\]

(4)

where \( g_{\gamma\gamma'} \) is the metric tensor \((g_{11} = -g_{22} = -g_{33} = -g_{44} = 1 \text{ and } g_{12,13,14} = 0 \text{ if } \gamma_1 \neq \gamma_2)\).

Wave functions of the electrons from the continuum spectra \( \psi_{p_{\mu}} \) are described by the following expansion [29]

\[
\psi_{p_{\mu}}(r) = \int d^3r \sum_{\alpha, \beta} \psi_{p_{\mu}, e \beta \alpha}(r) a_{p_{\mu}, e \beta \alpha}(r),
\]

(5)

where \( \psi_{p_{\mu}, e \beta \alpha}(r) \) is a solution of the Dirac equation in the field of the HCI’s nucleus and the coefficients \( a_{p_{\mu}, e \beta \alpha}(r) \) have the following form

\[
a_{p_{\mu}, e \beta \alpha}(r) = \frac{(2\pi)^{3/2}}{\sqrt{p^4}} e^{i\phi_{p_{\mu}, e \beta \alpha}(r)} \delta(\varepsilon - \varepsilon(r)).
\]

(6)

In equation (6) the \( \phi_{p_{\mu}, e \beta \alpha}(r) \) is the Coulomb phase shift for the incident (emitted or scattered) electron, \( \Omega_{p_{\mu}}(p) \) is the spherical spinor (tensor spherical harmonic [30]) and \( \psi_{p_{\mu}}(p) \) is the spinor with projection \( \mu = \pm 1/2 \) on the electron momentum \( p \)

\[
\frac{p \sigma}{2p} \psi_{p_{\mu}}(p) = \mu \psi_{p_{\mu}}(p),
\]

(7)

where \( \sigma \) is the vector consisting of the Pauli matrices.

4 The limitations of the approach is discussed in section 2.3.
Assuming that the incident electron is unpolarized and polarizations of the scattered and emitted electrons are not detected, the fully differential cross section for electron loss is given by

\[
\frac{d\sigma}{d\Omega_1 d\Omega_2 d\Omega_{p1} d\Omega_{p2}} = \frac{\pi}{2} \sum_{\mu, \nu} |U_{\mu m_1 \nu m_2}|^2 \times \frac{\varepsilon_1 \varepsilon_2}{p_0} (2\pi)^6 \delta(\varepsilon_1 + \varepsilon_2 - \varepsilon_1 - \varepsilon_2), \tag{8}
\]

where \(\Omega_{p1}\) and \(\Omega_{p2}\) are momentum solid angles of the emitted and scattered electrons.

In the present paper we shall not discuss the above cross section (8) but instead in section 3 we consider the cross section differential in energy and solid angle of one of the outgoing electrons which is given by

\[
\frac{d\sigma}{d\Omega_1} = \int d\Omega_2 d\Omega_{p1} d\Omega_{p2} \frac{d\sigma}{d\Omega_1 d\Omega_2 d\Omega_{p1} d\Omega_{p2}} = \int d\Omega_1 d\Omega_2 d\Omega_{p1} d\Omega_{p2} \frac{d\sigma}{d\Omega_1 d\Omega_2 d\Omega_{p1} d\Omega_{p2}}. \tag{9}
\]

### 2.2. Electron loss in collisions with protons

Let us now consider electron loss from a hydrogen-like HCI in collisions with a proton. Like in the previous subsection, we shall give the basic consideration of this process using the perturbation theory in this interaction. Here, the process of electron loss can be treated within the Dirac particle. Since the interaction between the electron and the incident particle is again comparatively very weak, the description of the proton in our process can be drastically simplified if we remark that, because of its huge (compared to the electron) mass, it has an enormous momentum. As a result, the Coulomb–Dirac states for the incident \(|\Phi_{p,r}\rangle\) and scattered \(|\Phi_{p,f}\rangle\) proton can be taken in the eikonal approximation in which they read (see, for instance, [31])

\[
|\Phi_{p,r}\rangle = \frac{1}{\sqrt{2E_i}} \exp \left[ i \alpha E_i \ln (P_i R - \mathbf{P} \cdot \mathbf{R}) \right] \times \exp [i P_i \cdot \mathbf{R}] u_{P_{r0}}, \tag{11}
\]

\[
|\Phi_{p,f}\rangle = \frac{1}{\sqrt{2E_f}} \exp \left[ -i \alpha E_f \ln (P_f R + \mathbf{P} \cdot \mathbf{R}) \right] \times \exp [i P_f \cdot \mathbf{R}] u_{P_{f0}}, \tag{12}
\]

where \(u_{P_{r0}}\) and \(u_{P_{f0}}\) are constant bispinor amplitudes for the corresponding plane waves.

The next key point in our description of the proton is to remark that the changes in the proton momentum and energy in the collision are negligibly small compared to their corresponding initial values. Taking this into account enables us to perform analytically the integration of the transition probability over the momenta of the scattered proton and then sum and average the result over polarization of the final and initial states of the proton. By performing all these steps we obtain the following expression for the differential cross section of electron loss

\[
\frac{d\sigma}{d\Omega} = \frac{\varepsilon \varepsilon_1}{(2\pi)^3} \sum_{\mu, \nu} \int d^2 \mathbf{q}_1 \int d^2 \mathbf{q}_2 \left| \frac{\varepsilon_1 - \varepsilon_1}{\varepsilon_2} \right| \left| \psi_{p,\mu}( \mathbf{r} ) \exp \left[ i \mathbf{q} \cdot \mathbf{r} \right] \psi_{n_h,l_m}( \mathbf{r} ) \right|^2 \left| \begin{array}{c} \mathbf{q}_1 \mathbf{q}_2 \end{array} \right|^{\frac{1}{2}} , \tag{13}
\]

where \(\Omega\) is the momentum solid angle of the emitted electron and \(\mathbf{q}\) is the change in the momentum of the proton

\[
\mathbf{q} = (\mathbf{q}_1, q_{\min}), \quad q_{\min} = \frac{\varepsilon - \varepsilon_{n_h}}{v}, \tag{14}
\]

representing the momentum transfer to the HCI. The quantity \(\mathbf{q}'\), which also enters equation (13) and which is given by

\[
\mathbf{q}' = \left( \mathbf{q}_1, q_{\min} \right), \tag{15}
\]

has the meaning of the change in the momentum of the proton in the reference frame where the proton is initially at rest.

The integration in (13) runs over the two-dimensional vector of the transverse momentum transfer \(\mathbf{q}_1\). On the scale of the electron momentum the upper limit of the integration over the absolute value of \(\mathbf{q}_1\) in (13) can be safely set to infinity.

To conclude this subsection we note that the expression (13) can also be obtained by using the so called semi-classical
approximation in which the incident proton is regarded as a classical particle moving along a straight-line trajectory. It is remarkable that our more general treatment, in which the proton is described quantum-mechanically and in which its interaction with the nucleus of the HCI is fully taken into account, offering much more insight into the physics of the collision yields the same result for the cross section (13) giving, thus, a direct proof of the validity of the semi-classical approximation in our case.

2.3. Limitations of the approaches presented in sections 2.1 and 2.2

In the approaches, presented in the above subsections, the interaction between the incident particle (an electron, a proton) with the electron of the HCI is treated within the first order of perturbation theory. For collisions with HCIs this is in general an excellent approximation. However, it nevertheless breaks down when the emitted electron and the scattered particle have very close velocities.

In case of electron loss by proton impact such a situation would correspond to the so called electron capture into the projectile continuum and it can be described by using e.g. relativistic distorted-wave models (see [10, 32]). Since the momentum space available for the emitted electron is very large the capture to the projectile continuum affects the electron emission pattern only in a very small part of this space and has little influence on the total loss cross section.

If electron loss occurs in collisions with electrons and in the final state both electrons have very close velocities then the electron–electron repulsion can influence the shape of the electron spectra. In electron loss from HCIs by electron impact at collision energies sufficiently far from the threshold value this influence is of importance just for a small part of the final electron momentum space having, therefore, a very limited overall effect on the spectral shape and very weak impact on the total cross section. The simplest way to account qualitatively for the electron–electron repulsion in this case is to multiply the right-hand of equation (8) by the so called Gamov factor, \( G = \frac{2\pi\eta}{(\exp(2\pi\eta) - 1)} \) with \( \eta = \frac{e^2}{(\hbar\Delta V)} \), where \( e \), \( \hbar \) and \( \Delta V \) is the electron charge, the Planck constant and the absolute value of the difference in the electron velocities.

If electron loss by electron impact takes place at impact energies very close to the threshold then both the electron spectra and the total cross section may be strongly influenced by the electron–electron repulsion. It is obvious that the approach described in section 2.1 becomes inaccurate in this (so called Wannier [33]) regime of electron loss.

First, the approach for treating electron loss in collisions with atom, which we use, does not enable one to easily identify states of the atom after the collision. However, it is quite clear that if the electron initially bound in a HCI is removed by the interaction with an atomic electron the latter—because of a very large recoil—will leave the atom with probability essentially equal to one. Moreover, if the electron of the HCI is removed by the interaction with the nucleus of the atom, the atomic electron will still be likely to leave the atom because of its interaction with the strong field of the HCI’s nucleus.

Second, if expressions (9) and (13) are employed—according to formula (2)—to obtain results for collisions with atoms then the corresponding cross section will describe a situation where the summation/integration over all final states of the atom has been performed (see [15]) and be suited for a description of experiments in which the final state of the target atom is not detected.

3. Results and discussion

Let us now consider results of our calculations for the cross sections obtained by using equations (2), (9) and (13). However, before we proceed with considering results, two remarks may be appropriate.

3.1. Electron energy-angular distributions

We start with the cross sections \( \frac{d\sigma}{d\epsilon d\Omega} \), where \( \epsilon \) and \( \Omega \) are the kinetic energy and the momentum solid angle, respectively, of the electron in the final state. This cross section represents the energy-angular distribution for the outgoing electron, it is independent of the azimuthal emission angle. We note that in case of collisions with an electron the quantities \( \epsilon \) and \( \Omega \) refer to any of the two electrons in the final state (see equation (9)).

One should note that, while the total cross section for loss from HCIs by electron impact was calculated\(^7\) in the past [16, 18, 21], and the fully differential cross section had been explored for a close problem of ionization of the K-shell of heavy atoms by electron impact (see for review [34]), we are not aware of any previous calculations of the cross section \( \frac{d\sigma}{d\epsilon d\Omega} \) in case of electron projectiles.

In figure 1 we present the energy-angular distributions of outgoing electrons in collisions of \( U^{131}(1s) \) (\( \epsilon_{th} \approx 132 \text{ keV} \)) with 200 keV electrons and with equivelocity protons having an impact energy of 367.2 MeV. In this figure we show also results for collisions with atomic hydrogen (in case of collisions with randomly oriented hydrogen molecules the results for atomic hydrogen should simply be multiplied by 2) and helium. The latter two were obtained by using, according to equation (2), the corresponding results for collisions with equivelocity electrons and protons\(^8\). The energy-angular distributions were calculated for two reference frames: the rest frame of the HCI and the frame in which the incident electron (proton, atom) is initially at rest.

\(^6\) As, for instance, is illustrated by estimates given after formulas (1).

\(^7\) When calculating the total cross section the integration over the angles can be done analytically that makes the computation much easier and greatly saves the computation time.

\(^8\) We did not average the contribution of the atomic electron to the loss cross sections over the atomic Compton profile since for hydrogen and helium it is quite narrow and the averaging would make a difference only at impact velocities very close to the electron threshold where our approach, because of the reasons discussed in section 2.3, is no longer accurate.
The process of electron loss looks simpler when it is considered in the rest frame of the HCI. The general observation which follows from the results for the energy-angular distributions in this frame (see the left panel of figure 1) is that the shapes of the electron spectra in collisions with electrons and protons differ qualitatively. In collisions with protons the spectrum of electrons is extended to higher energies compared to collisions with electrons. This is expected since compared to an electron an equivelocity proton carries much more energy and, therefore, the kinematically available volume in the final-state-electron momentum space is orders of magnitude larger in collisions with protons. Although, due to the huge difference in masses and the absence of very high-frequency components in the field of the proton, just a tiny part of this volume can be noticeably populated, the resulting effective volume in collisions with protons still strongly exceeds that which is available in collisions with electrons.

The results presented in figure 1 also show that a considerable part of the momentum space in the final state, which is kinematically available in collisions with electrons, is considerably weaker populated in collisions with protons.

From the above two points it follows that in collisions with hydrogen and helium the contributions to the loss process from the interactions with the nucleus and the electron(s) of the atom could be rather well separated for a substantial part of the final-state-electron momentum space (that is clearly seen in figure 1).

For a given energy of the outgoing electron the shape of the angular distributions in collisions with electrons and protons is also quite different (see figures 1, 2). For instance, in collisions with electrons there is a strong increase at large angles which can be traced back to originate due to constructive interference between the direct and exchange contributions to the transition amplitude (3).
The doubly differential cross sections in collisions of $\text{U}^{91+}(1s)$ with 200 keV electrons and with protons are shown in the rest frame of the HCI as a function of the polar angle of the outgoing electron at fixed electron energies: 0.44 keV ( ), 2.3 keV ( ), 9.62 keV ( ) and 19.51 keV ( ). The angle is counted from the direction of the motion of the incident particle.

$\text{U}^{91+}(1s)$ with 600 keV electrons and equivelocity protons with the corresponding impact energy of 1.1 GeV. Figure 3 also contains results for collisions with 1.1 GeV atomic hydrogen and 1.1 GeV/u helium. Like in the previous case we observe in figure 3 that, compared to collisions with electrons, in collisions with equivelocity protons the spectra extend to higher energies. Besides, in the momentum space, kinematically available in collisions with electrons, there are parts which are substantially stronger populated in collisions with electrons than with equivelocity protons.

There are also noticeable differences between the shapes of the electron spectra presented in figures 1–4 which are especially visible in case of electron loss by electron impact. For instance, in collisions with 200 keV electrons the angular distribution of low-energy electrons is quite asymmetric with a pronounced maximum at 180° whereas at 600 keV impact energy the distribution of these electrons is more homogeneous. Also the separation between the maxima at low and high energies becomes better with increasing the impact energy reflecting a better separation between the electrons in the phase space that enables one to almost unambiguously identify the maximum at lower and higher energies with the emitted and scattered electron, respectively.

The interaction between the electrons can be considered as the sum of the (unretarded) Coulomb and the Breit interactions. The latter represents the (main) relativistic correction to the former and becomes of importance when the energy of the incident electron and/or the atomic number of the HCI are/is sufficiently high. In figures 5 and 6 we show the cross sections $\frac{d\sigma}{d\xi d\Omega}$ for electron loss from $\text{U}^{91+}(1s)$ by the impact of 200 and 600 keV electrons, respectively, where the cross section $\frac{d\sigma}{d\xi d\Omega}$ was calculated by ignoring the Breit interaction. These cross sections are shown as a function of the polar angle of the outgoing electron for few fixed energies. It follows from the figures that the Breit interaction has not just quantitative but also qualitative impact on the energy-angular distribution. Besides, the results shown in figures 5 and 6 quite clearly suggest that this interaction increases the total number of electron loss events.

We conclude this subsection by briefly considering the importance of the interaction between the outgoing particles (the so called post collision interaction—PCI) which is ignored in our approach that sets certain restrictions on it (see section 2.3). This importance is illustrated in figures 7 and 8 by plotting the relative ratio $\xi = \left(\frac{d\sigma^{PCI}}{d\xi d\Omega} - \frac{d\sigma}{d\xi d\Omega}\right) / \frac{d\sigma}{d\xi d\Omega}$, where $\frac{d\sigma^{PCI}}{d\xi d\Omega}$ and $\frac{d\sigma}{d\xi d\Omega}$ are the calculated differential cross sections with and without the inclusion of the PCI.

In figure 7 we show the relative ratio $\xi$ for electron loss from $\text{U}^{91+}(1s)$ in collisions with 367.2 MeV protons where the cross section $\frac{d\sigma^{PCI}}{d\xi d\Omega}$ was obtained by employing a relativistic continuum-distorted-wave-eikonal-initial-state (CDW-EIS) approach of [10]. This approach uses a fully relativistic description for the states of the electron in the field of the HCI.

It follows from the figure that the ratio is very small for the overwhelming part of the momentum space of the emitted electron and that the approach described in section 2.2 breaks down only when the electron velocity becomes very close to that of the projectile. In such a case the distorted-wave approach predicts a very strong increase in the spectrum of the emitted electron describing electron capture to projectile (proton) continuum.

In figure 8 the relative ratio $\xi$ is displayed for electron loss from $\text{U}^{91+}(1s)$ in collisions with 200 keV electrons. The effect of the repulsion between the outgoing electrons was modelled by multiplying the fully differential cross section (9) by the Gamov factor (see section 2.3). We observe in the figure that the repulsion leads to a very small correction to the cross section which does not exceed 8% even when the outgoing electrons have the same energy.

Thus, both in collisions with electrons and protons the interaction between the outgoing particles leads to a very small overall effect on the cross section $\frac{d\sigma}{d\xi d\Omega}$. However, the details of this effect are quite different. In collisions with electrons the effect is always weak because a large range of angles (and absolute velocities) is accessible both for emitted and scattered particles and the effect is smeared out by the integration over the angles and energies of one of the particles. In contrast, in case of protons, which move in the collision practically with a constant velocity, such a smearing does not take place and the effect of the interaction between the proton and the electron becomes large when the outgoing electron moves under a small angle with an absolute velocity close to that of the proton.

9 And can be viewed as a generalization of the (semi-) relativistic CDW-EIS model of [32], in which these states are described by the Darwin and Sommerfeld-Maue (or Furry) approximations.
3.2. Total cross section for electron loss

In figure 9 we present results for the total cross section of electron loss from HCIs in collisions with equivelocity electrons, protons, atomic hydrogen (for collisions with randomly oriented hydrogen molecules these results should be multiplied by 2) and helium. Four different HCIs are considered: they range from Fe$^{25+}$ (1s) till U$^{91+}$ (1s) covering, thus, quite a broad range of the atomic numbers of HCIs. For each HCI the total cross sections are given as a function of the electron impact energy and the corresponding impact energy of an equivelocity proton. In each case the electron impact energy varies from the corresponding threshold value $\varepsilon_{th}$ to about 5 $\varepsilon_{th}$.

In this energy range a number of experimental and theoretical data is available for the total cross section for electron loss from hydrogen-like HCIs by electron impact. In case of electron loss from not very heavy HCIs our results are in good agreement with experimental data (see figure 9). Our results are also in good agreement with theoretical results of other authors [18, 20, 21].

The agreement is such that on the scale of figure 9 these results are practically indistinguishable.

Figure 3. Same as in figure 1 but for 600 keV electrons, 1.1 GeV protons, 1.1 GeV hydrogen, 1.1 GeV/u helium and 1.09 GeV/u U$^{91+}$ (1s).

Figure 4. Same as in figure 2 but for incident 600 keV electrons and equivelocity protons. Energies of the outgoing electron are: 3.05 keV (■), 15.78 keV (●), 37.50 keV (▲) and 134.4 keV (★).
The following main observations can be drawn from the results shown in figure 9. First, at impact velocities corresponding to the near-threshold values of the energy of the incident electron the protons are much more effective in producing electron loss than the equivelocity electrons. Second, with increasing the impact velocity the relative effectiveness of electron projectiles increases reaching almost ‘parity’ with that of equivelocity protons at the highest velocities shown in the figure. Third, an interesting feature of electron loss in collisions with equivelocity electrons and protons is that the relative effectiveness of electrons increases with the increase of the atomic number of the HCI. Besides, comparing results for electron loss from uranium in figures 9 and figures 3, 4 we may conclude that the equality of the total loss cross sections in collisions with equivelocity electrons and protons by no means implies the same (or even similar) shape of the differential cross sections.

It is of interest to compare the above observations with those which were made in [27] where the process of...
excitation of HCl ions into bound states by equivelocity electrons and protons was investigated. In particular, it was found there that:

(i) at electron impact energies near the excitation threshold the electrons are on average more effective in producing excitation than equivelocity protons, the cross section for excitation by electron impact reaches its maximum at the threshold;

(ii) with increase in the impact velocity the difference between equivelocity electrons and protons diminishes; and

(iii) the relative effectiveness of electrons in producing excitation increases when the atomic number of the HCl grows.

Thus, we see that the processes of loss and excitation in collisions with electrons have both similarities and differences.

The key difference is that the loss cross section is zero at the energy threshold for this reaction whereas the excitation cross section has a maximum at the excitation threshold. These qualitatively different behaviours of the cross sections are related to the fact that for the near-threshold collisions the smallness of the final-state phase space in case of excitation is fully compensated by the singularity in the Coulomb wave function of the slow outgoing electron whereas in case of electron loss, where there are two slow outgoing electrons whose energies are connected due to the energy conservation, such a compensation does not take place.

The main similarity is that in both these processes the effectiveness of electrons compared to that of equivelocity protons grows with the increase in the atomic number of the

Figure 9. The total cross sections for electron loss from $\text{Fe}^{25+}+(1s)$, $\text{Xe}^{53+}+(1s)$, $\text{Au}^{78+}+(1s)$ and $\text{U}^{91+}+(1s)$ in collisions with equivelocity electrons (solid curves), protons (dotted curves) as well as with atoms of hydrogen (dashed curves) and helium (dash-dot curves). In the right lower panel the open circle with error bars is the total cross section for electron loss from 405 MeV/u $\text{U}^{90+}(1s^2)$ colliding with $\text{H}_2$ measured in [35] (which we scaled to collisions of hydrogen-like uranium with atomic hydrogen by dividing their result by 4). All the other experimental data, shown in this figure, were measured for electron–ion collisions and are taken from [21] ($\text{Fe}^{25+}(1s)$, ★), [20] ($\text{Au}^{78+}(1s)$, ▲), [17] ($\text{U}^{91+}(1s)$, ■) and [19] ($\text{U}^{91+}(1s)$, ◦).
HCI. Since according to the non-relativistic consideration the relative effectiveness must be independent of the HCI’s atomic number (that can be very simply shown by scaling the Schrödinger equation to $Z_0$), this grows should be attributed to relativistic effects in the motion of the electrons and the interaction between them.

In figure 9 shown also are available experimental results on the total loss cross section from HCIs in collisions with electrons (to our knowledge, such data are absent for collisions with protons and atomic hydrogen). Besides, in this figure we also display the cross section for electron loss from $405 \text{ MeV/u } U^{90}_{\text{II}}$ in collisions with $H_2$ measured in [35] which we have divided by 4 in order to scale it to collisions of $405 \text{ MeV/u } U^{91}_{\text{III}}$ with atomic hydrogen.

For electron loss from very heavy hydrogen-like HCIs the experimental data are very scarce (see figure 9) and do not seem to be very accurate. Besides, in case of electron loss from $U^{91}_{\text{III}}(1s)$ by electron impact experimental data of different groups are not in agreement with each other.

4. Conclusion

We have considered the process of electron loss from hydrogen-like HCIs in collisions with hydrogen and helium in the range of impact velocities $v_{\text{min}} \leq v \leq v_{\text{max}}$, where $v_{\text{min}}$ and $v_{\text{max}}$ correspond to the threshold energy $\varepsilon_0$, for the loss in collisions with electrons and to $\approx 5 \varepsilon_0$, respectively. In this range, where reliable data for loss cross sections were absent, the process is characterized by momentum transfers which are large on the atomic scale and is very simply related to electron loss in collisions with electrons and protons. Therefore, a detailed information about electron loss in collisions with hydrogen and helium is obtained by performing a comparative analysis of the latter two basic processes (which is per se of great physical interest).

In the process of excitation by electron impact near the threshold for this reaction the smallness of the momentum space of the outgoing electron is fully compensated by the Coulomb singularity in its wave function [27]. In contrast, in the process of electron loss near the threshold the smallness of the momentum spaces of the two outgoing electrons is not compensated by the Coulomb singularity in their wave functions. Thus, unlike the process of excitation, the Coulomb attraction between the electrons and the nucleus of the HCI cannot ‘compete’ with the advantage of a very large phase (momentum) space available in collisions with protons and, therefore, the latter turn out to be more effective, compared to equivelocity electrons, in producing electron loss. However, similar to excitation, the relative effectiveness of electron projectiles increases when the atomic number of the HCI grows. This increase is a purely relativistic effect.

In collisions with protons the spectra of outgoing electrons extend to much larger energies. Besides, by analysing the electron energy-angular distribution we found out that in the final-state-electron momentum space, kinematically available in collisions with electrons, there are considerable parts which are weaker populated in collisions with protons than with electrons. Therefore, in collisions of HCIs with hydrogen and helium targeta the contributions to the electron loss process, which are caused by the interactions of the electron of the HCI with the nucleus and the electron(s) of the target, could be rather well separated from each other in the energy-angular distribution for a substantial part of the final-state-electron momentum space and could be explored in experiment independently.

It also follows from our results that even when, with increase in the impact energy, the total loss cross sections in collisions with electrons and protons become already equal, the spectra of the outgoing electrons still remain substantially different in almost the entire volume of the final-state-electron momentum space.

Accurate experimental results for the total cross section of electron loss from heavy hydrogen-like HCIs in collisions with very light targets are absent. To our knowledge, there is also no experimental data on the electron loss spectra in such collisions. Further, in case of electron loss from very heavy hydrogen-like HCI (like e.g. gold, uranium) by electron impact experimental data are quite scarce even for the total cross section (sometimes contradicting to each other). Besides, we are not aware of any experimental results on electron spectra in such collisions. All this suggests that new experiments in this field would be very desirable and the present theoretical results (as well as the availability of accurate methods for calculating electron loss from HCIs in collisions with electrons, protons and very light atoms) could become a guide for further experimental activities in this field, in particular, for experiments on ion–atom collisions planned at the GSI (Darmstadt, Germany) and in the Institute of Modern Physics (Lanzhou, China).

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