Equilibration by quantum observation

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Abstract. We consider an unexplored regime of open quantum systems that relax via coupling to a bath while being monitored by an energy meter. We show that any such system inevitably reaches an equilibrium (quasi-steady) state controllable by the effective rate of monitoring. In the non-Markovian regime, this approach suggests the possible ‘freezing’ of states, by choosing monitoring rates that set a non-thermal equilibrium state to be the desired one. For measurement rates high enough to cause the quantum Zeno effect, the only steady state is the fully mixed state, due to the breakdown of the rotating wave approximation. Regardless of the monitoring rate, all the quasi-steady states of an observed open quantum system live only as long as the Born approximation holds, namely the bath entropy does not change. Otherwise, both the system and the bath converge to their fully mixed states.

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1. Introduction and background

The quantum Zeno effect (QZE), namely evolution slowdown of unstable quantum systems by frequent quantum measurements, has long been held to be a basic universal consequence of quantum mechanics for either isolated (‘closed’) [1]–[4] or ‘open’ quantum systems, i.e. systems coupled to their environment (a ‘bath’) [5]–[13]. Although for less frequent measurements there can be a reversal of the QZE, known as the anti-Zeno effect (AZE) [9]–[12], the universal onset of QZE under sufficiently frequent measurements is still unchallenged.

Here we show that a basic aspect of the dynamics of frequently measured (monitored) open quantum systems has been overlooked, namely their inevitable convergence to a steady state (equilibrium) that is determined by the measurement rate. We raise the questions: What is the essential difference between monitored closed systems, whose dynamics is well understood [14, 15], and their open-system counterparts? More specifically: How does the open-system convergence toward a steady state depend on the measurement rate? What role, if any, does the bath play in this convergence? How do the QZE and the AZE fit into this open-system dynamics?

In order to answer these fundamental questions, we conduct a thorough analysis of the monitoring dynamics (section 2) caused by repeated non-selective measurements of both open and closed systems. Indeed, in some respects measurements act on an open system as if they were caused by an additional bath. However, the crux of our analysis is the need to turn them on and off at specific time intervals in order to steer the equilibrium state in the desired manner. Such intermittent operations do not represent the action of a natural bath but rather man-made interventions to which we refer as non-selective measurements.

We find that it is essential to transgress three basic approximations commonly used for open systems (figure 1), namely the Markov approximation (section 1.2) [8], whereby the bath has no memory; the rotating wave approximation (RWA) (section 3.2) [16]–[18], whereby only
Figure 1. Steady states of the monitored open TLS as a function of $\tau$ for frequent (solid) and continuous (dotted) measurements under the complete Hamiltonian (without RWA) compared to their counterparts with RWA (dashed). A single period (vertical dashed green line) delineates the transition to the RWA regime. The bath memory (correlation) time, $t_c$ (vertical dotted black line), delineates the transition to the Markov regime. The no-measurement limit (thermal equilibrium) is the saturation value in the Markovian time domain. Inset: the corresponding rates of convergence $\gamma(\tau)$ for the three respective cases. The open system is a TLS with resonant frequency, $\omega_a$, coupled to an oscillator bath with the Lorentzian coupling spectrum $G_0(\omega) = \kappa/\omega_a(\omega / (1 + (\omega - \omega_0)^2 t_c^2))$, with the following parameters: scaled coupling strength $\kappa/\omega_a = 0.01$, scaled inverse temperature $\beta\omega_a = 2$, scaled correlation time $\omega_a t_c = 10$ and maximal coupling frequency $\omega_0 = 2\omega_a$.

secular (near-resonant) energy exchange between the system and the bath affects the dynamics; and the Born approximation [8], whereby the bath state remains constant (section 6). We show that deviations from these approximations have a dramatic influence on the convergence rate and steady state of a monitored open system. An important corollary of the analysis is the possible ‘freezing’ of states by choosing monitoring rates that set the equilibrium state to be the desired one (section 5). The findings are then summarized and connected to existing results [19]–[23] (section 6), particularly concerning pointer states and steady-state engineering.

1.1. Model

To answer the above questions, we first consider the well-studied model of a two-level system (TLS) coupled to a bath. With the free Hamiltonian $H_0 = \omega_a \sigma_z$, the TLS is weakly coupled to a bath via $H_{\text{open}} = -g \hat{B}(t) \sigma_x$, with $\hat{B}(t)$ being an arbitrary bath operator in the interaction picture.

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The reduced dynamics of the system is obtained to second order by a time-local non-Markovian master equation [17]. An initial state of the TLS that is diagonal in the $H_0$ basis remains diagonal for all later times $t$. The coupling in $H_{\text{open}}$ is off-diagonal ($\sigma_x$) to allow relaxation of the level populations. These evolve according to the equation [17]

$$\frac{\partial \rho_{ee}(t)}{\partial t} = -R_e(t) \rho_{ee}(t) + R_g(t) \rho_{gg}(t),$$

(1)

where $\rho_{ee}(t)$ and $\rho_{gg}(t)$ are the excited and ground state populations at time $t$ satisfying the normalization condition $\rho_{ee}(t) + \rho_{gg}(t) = 1$, with $R_e(t)$ and $R_g(t)$ being their respective transition rates.

Under the weak coupling assumption (the Born approximation), these rates are given as [17]

$$R_{\text{eq}}(t) = 2t \int_{-\infty}^{\infty} d\omega G_T(\omega) \text{sinc}[(\omega \mp \omega_a)t].$$

(2)

Here $G_T(\omega)$, the Fourier transform of the bath correlation (response) function $\langle \hat{B}(0)\hat{B}(t)\rangle_T$, is the temperature-dependent coupling spectrum of the bath. The sinc function signifies the time-dependent uncertainty of the TLS level splitting centered at $+\omega_a$ or $-\omega_a$.

1.2. Thermal equilibration

The longest timescale of relevance is the bath memory (correlation) time, $t_c$, defined as the inverse width of $G_T(\omega)$, which marks the transition from the non-Markovian to the Markovian monitoring regime. In the Markovian regime, $t \gg t_c$, the sinc function is much narrower than the coupling spectrum $G_T(\omega)$ and can be approximated by a delta function in frequency, resulting in

$$\rho_{ee}(t \gg t_c) = \rho_{ee}(0) e^{-\gamma t} + (\rho_{ee})_{\text{Eq}} (1 - e^{-\gamma t}),$$

(3)

where the Markovian decay rate is given by

$$\gamma \equiv \frac{1}{t} [R_e + R_g](t \gg t_c) = 2\pi G_T(\omega_a).$$

(4)

$(\rho_{ee})_{\text{Eq}} = e^{-\beta \omega_a}/2 \cosh \beta \omega_a$ is the temperature-dependent, bath-induced, equilibrium excitation. Equation (3) shows the convergence is at the Golden-Rule rate $\gamma$, to the steady state, which is the thermal equilibrium state.

To examine whether the thermal equilibrium remains in the steady state under repeated measurements, we study below the dynamics of a monitored (measured) TLS in the presence of a bath.

2. Convergence to a steady state by monitoring

The problem at hand leads us to define the monitoring superoperator $\hat{M}_\tau$ composed of a unitary propagator over time $\tau$, $U(\tau)$, followed by a projector $\hat{P}$ corresponding to a non-selective, impulsive measurement in the basis of the system energy states $|j\rangle$ (of the free Hamiltonian $H_0$):

$$\hat{M}_\tau \rho = \hat{P} L_\tau \rho,$$

(5)
\[ \hat{L}_\tau \rho = U(\tau) \rho U^\dagger(\tau), \quad \hat{P} \rho = \sum_j |j\rangle \langle j| \rho |j\rangle \langle j|, \]  

(6)

where \( \rho \) is the combined system + bath density matrix and \( U(\tau) \) is the unitary propagator of the total Hamiltonian. The evolution due to \( n \) consecutive applications of (6) is given by

\[ \rho(t = n\tau) = \hat{M}_\rho^n \rho(0). \]  

(7)

We ask: What are the asymptotics of (7) as \( n \to \infty \), i.e. what are the steady states of the evolution, \( \chi(\tau) = \text{Tr}_B \left( \hat{M}_\rho^\infty \rho(0) \right) \), where \( \text{Tr}_B \) is the trace over the bath?

For the case of a TLS coupled to a bath and repeated projections of the total state onto the \( \sigma_z \) (energy) states of the TLS, the above-described convergence assumes the simple exponential form:

\[ \rho_{ee}(t = n\tau) = e^{-\gamma(\tau) t} \rho_{ee}(0) + \chi(\tau) \left( 1 - e^{-\gamma(\tau) t} \right). \]  

(8)

\[ \gamma_{\text{open}}(\tau) = \frac{1}{\tau} \int_0^\tau dt \left( R_g(t) + R_e(t) \right), \]  

(9)

\[ \chi_{\text{open}}(\tau) = \frac{\int_0^\tau dt e^{\gamma(\tau) t} R_g(t)}{\int_0^\tau dt e^{\gamma(\tau) t} \left( R_g(t) + R_e(t) \right)}, \]  

(10)

where \( \rho_{ee}(t) \) is the excited state population of the system’s density matrix. In multilevel systems (MLSs) for which the Born approximation holds, we also find exponential convergence toward a steady state (appendix A).

The derivation of (8) rests on the crucial fact that quantum non-selective, impulsive measurements obliterate any TLS coherence, namely ‘reset the clock’, so that consecutive steps of the evolution depend only on \( \tau \) and not on \( t \). The convergence rate \( \gamma(\tau) \) and the steady-state excitation \( \chi(\tau) \) are determined by the ‘free’ (unmeasured) evolution during the interval \( \tau \), which is unitary for a closed TLS and non-unitary for an open TLS due to the tracing-out of the bath. Remarkably, a similar convergence to a steady state holds for continuously monitored systems (appendix B).

For the Hamiltonian considered here, it is easy to show that an off-diagonal element \( \rho_{eg} \) will decay to zero at a rate \( \gamma(\tau)/2 \). Since the first measurement erases the off-diagonal terms, they are irrelevant for all subsequent evolution. Hence, there arises a natural choice of initial states that are diagonal in the free-Hamiltonian eigenbasis, which are the pointer states of the evolution [19].

We may now contrast the situation in (9) with the monitoring of a closed system where the dynamics of the TLS is governed by the Hamiltonian \( H_{\text{close}} = -\frac{\Omega}{2} \sigma_z \), with \( \Omega \) being the Rabi flipping rate between \( |e\rangle \) and \( |g\rangle \) and \( \sigma_z = |e\rangle \langle g| + |g\rangle \langle e| \). The convergence rate and the steady state are then (appendix A)

\[ \gamma_{\text{closed}}(\tau) = -\ln(|\cos(\Omega \tau)|)/\tau, \quad \chi_{\text{closed}} = \frac{1}{2} \quad \forall \tau. \]  

(11)

Namely, the timescale of the closed-TLS dynamics is the Rabi oscillation \( \Omega \), thus defining the QZE condition: for \( \Omega \tau \to 0 \) the evolution converges at the linear rate \( \gamma_{\text{closed}} \sim \Omega^2 \tau \to 0 \). Thus, for a fixed time and increasing number of measurements \( n \), the evolution indeed ‘freezes’.

However, there is necessarily a physical limit to the smallness of \( \tau \) [9]–[12]. The finiteness of \( \tau \) results in a convergence, albeit slow, to the only steady state of the evolution, which is the fully mixed state [14].

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3. Steady states

The full landscape of measurement-induced steady states shown in figure 1 relies on two features: (i) the interval between measurements is in the non-Markovian time domain; and (ii) the counter-rotating (CR) terms affect the system–bath interaction. The role played by each feature is detailed below.

3.1. Beyond Markov

In the non-Markovian regime, namely for \( \tau \leq t_c \), the sinc function, representing energy uncertainty, and \( G_T(\omega) \) have comparable widths. This regime results in an oscillatory, non-monotonic dependence of the convergence rate on \( \tau \) for a given coupling spectrum. As \( \tau \) varies, \( \gamma(\tau) \) alternates between speedup, as expected from the AZE [9]–[12], and slowdown. Likewise, the steady state of the evolution has a non-monotonic dependence on \( \tau \) in this regime (figure 1). One can then reach a steady state that has higher or lower entropy than the thermal equilibrium, i.e. mix or purify the state upon choosing the appropriate measurement interval [17].

3.2. Beyond RWA

The shortest timescale of relevance is the natural oscillation period of the system, \( 1/\omega_a \). More frequent measurements, \( \tau \ll 1/\omega_a < t_c \), reveal another striking behavior:

\[
\gamma(\tau \ll 1/\omega_a) = 4\Gamma\tau, \quad \chi(\tau \ll 1/\omega_a) \approx \frac{1}{2},
\]

where \( \Gamma = \int_{-\infty}^{\infty} d\omega G_T(\omega) \) is the total system–bath coupling strength. This timescale reveals the closest analogue to the closed-system QZE, equation (11), in that the convergence rate is linear in \( \tau \) and the steady state of the evolution is the fully mixed state. This shows that the manifestation of the QZE in open quantum systems paradoxically results in a fully mixed steady state and not in ‘freezing’ the evolution.

On this timescale the energy spread \( 1/\tau \) exceeds \( \omega_a \); hence \(|e\rangle \) and \(|g\rangle \) become indiscriminate, so that the TLS may then absorb or emit quanta irrespective of the bath, at the expense of the system–bath interaction energy. Remarkably, the QZE and the ensuing fully mixed steady state require monitoring on such a short timescale, as follows from (12). This timescale entails the breakdown of the RWA [17]. This means that the non-Markovian dynamics by itself or, equivalently, the condition \( \tau \ll t_c \) does not suffice to ensure the QZE in open systems, contrary to the prevailing view [5]–[12].

Why is the RWA breakdown mandatory for the QZE? A combined system–bath Hamiltonian can always be decomposed into secular or rotating-wave (RW) and non-secular or CR parts (appendix C). The RWA neglects the CR terms since they are significant only at very short times, i.e. \( \omega_a t \ll 1 \). Yet, the QZE, whose signature is linear dependence of \( \gamma(\tau) \) on \( \tau \), requires, according to (12), such ultrashort measurement intervals and thus must account for the CR terms. Disregarding this fact and taking the RWA in the QZE regime, \( \omega_a \tau \ll 1 \), yields a convergence \( \propto \tau \), which has only half the magnitude of the convergence rate obtained from the complete Hamiltonian (appendix C). More importantly, the steady state under the RWA is wrong for such short \( \tau \): it strongly depends on the bath coupling spectrum, \( G_T(\omega) \), contrary to the steady state of the complete Hamiltonian that is the fully mixed state for the same \( \tau \) (appendix C) (figure 1).

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We can understand these novel results regarding the convergence to the steady state by considering the two competing aspects of open-system monitoring. The projective measurements of $\sigma_z$ have quantum non-demolition (QND) effects on the system and hence preserve its entropy, whereas the bath-induced evolution can either increase or decrease it, depending on the bath coupling spectrum, the evolution interval $\tau$ and the state of the system at that time. In this respect, the difference between the RW and CR dynamics is crucial. The RW terms preserve the combined system + bath state within the same energy subspaces, thus acting as if these were many closed systems. The coupling strength and measurement interval determine the relative weights of these subspaces. Hence, the RWA entails a strong dependence on the coupling spectrum and measurement interval, even in the QZE regime. By contrast, the CR terms mix the energy subspaces, thereby turning the entire system–bath state into that of a single closed system, which should thus conform to (11). The shorter the measurement time interval, the larger the contribution of the CR terms, resulting in an increasing entropy of the steady state.

4. Beyond Born

The major approximation in the above analysis is the Born approximation, under which the bath does not change from measurement to measurement and does not accumulate these changes in entropy. Thus, after each measurement, the relative contributions of the RW and CR terms are maintained, allowing a wide range of steady states, as discussed above. Yet, these are strictly speaking quasi-steady states if the Born approximation is not exact: since the bath changes are comparatively slower than the system dynamics, there could exist a timescale after which the corrections in the bath state and the quasi-steady states of the system are no longer negligible. This introduces another timescale, namely the Born time $t_B$, which depends on the bath size in a model-specific fashion: whereas for $t \ll t_B$ the Born approximation holds and the convergence to (9) applies, for $t \geq t_B$ the approximation breaks down and the accumulated bath changes start to influence the system dynamics (figures 2(a) and (b)).

The monitored (system + bath) dynamic equations (5)–(7) still hold without the Born approximation, but do they yield convergence to a steady state that depends on $\tau$? Repeated QND measurements of the system erase the system–bath coherences, thereby increasing the total entropy of the combined (system + bath) state. On the other hand, the free evolution between measurements is unitary in the combined system + bath Hilbert space and thus preserves the entropy. Where does this interplay lead to?

To answer this question, let us examine the structure of the combined density matrix, $\rho_{tot}$, of the system plus $N$-mode bath in two scenarios: (i) If the RWA applies (i.e. $\tau$ is not so short that (12) holds), then $\rho_{tot}$ is block-diagonal in $N \times N$ subspaces, with total energy $E_n$, $n = 0, \ldots, \infty$. The system + bath energy subspaces remain closed, and accumulate the entropy increase resulting from the measurements (figure 3(b)). The steady state of each such subspace conforms to closed-system dynamics (appendix A) i.e. is the fully mixed state, regardless of $\tau$ (figure 2, main panel). The overall steady state in this scenario depends on the relative contributions of the $E_n$-subspaces, which in turn depend solely on the bath coupling spectrum. Hence, only $G_1(\omega)$ (cf (2)) determines the steady state.

(ii) In the other scenario where the RWA does not apply, the CR terms mix all the subspaces and effectively render the combined system + bath state that of a single closed system (figure 3(b)). The steady state of such an effectively closed system, subject to entropy-increasing measurements and entropy-preserving evolution, can only be the fully mixed state. Hence, under
Figure 2. Main panel: steady states of the monitored open TLS as a function of $\omega_a \tau$ obtained from dynamics with Born approximation (blue solid), exact dynamics (without Born approximation) but with RWA (red dashed), RWA with Born approximation (black dash-dot) and quasi-steady states of the exact dynamics of the full Hamiltonian (green dotted). The open system is a TLS with resonant frequency, $\omega_a$, uniformly coupled to a spin bath composed of $N$ spins with frequency $\omega_0$, whose coupling spectrum is $G_T(\omega) = N\kappa \delta(\omega - \omega_0)$, with the parameters: $\kappa/\omega_a = 0.2$, $\omega_0/\omega_a = 1.8$. Inset of (a): the excited-state population of the TLS according to the exact dynamics (without the RWA and Born approximation), under measurements performed at a fixed interval $\tau = 0.75/\omega_a$ upon varying the number of bath spins, $N = 2$ (red dashed), $N = 6$ (green dot-dashed) and $N = 20$ (blue solid). The dynamics exhibits exponential convergence to a quasi-steady state and a subsequent deviation from it towards the true steady state, i.e. the fully mixed state. As $N$ increases, the effective time that the quasi-steady state remains stable increases as well. Inset of (b): the same as (a) but for a fixed number of spins, $N = 20$, under measurements performed at different intervals: $\tau = 0.75/\omega_a$ (blue solid), $\tau = 0.7/\omega_a$ (green dot-dashed) and $\tau = 0.625/\omega_a$ (red dashed). The stability of the quasi-steady states (for a fixed bath size $N$) is seen to depend upon $\tau$. The lower the entropy of the quasi-steady state, the higher their stability.

exact dynamics, repeated measurements of a system coupled to a bath will inevitably force them to converge to a fully mixed state.

5. Freezing states by observations

A novel corollary of the non-Markovian analysis of open-TLS monitoring is the intriguing possibility to truly ‘freeze’ an initial state by choosing $\tau$ in (8) such that $\chi(\tau) = \rho_{ee}(0)$. The possibility of such freezing arises due to the presence of both the CR and co-rotating terms in

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Figure 3. (a) Schematic diagram of steady states in different scenarios. The range of steady states for an open system with the Born approximation but without RWA depends on both the coupling spectrum $G_T(\omega)$ and the measurement interval $\tau$, whereas for an open system without Born but with RWA it depends only on the coupling spectrum. (b) Schematic diagram of the combined system-bath density matrix for an open system without the Born approximation. Rotating-wave (RW) terms preserve the state within the same energy subspace, whereas counter-rotating (CR) terms mix them.

Remarkably, the same ‘freezing by observation’ mechanism holds also for frequently measured open MLSs, which show similar convergence to a steady state determined by the non-Markov $\tau$-dependent measurement-induced evolution (appendix A). The most interesting trend of such open MLSs is their non-thermal equilibration by observation. The MLS steady state has the following form in vector notation:

$$
\chi(\tau) = (I - F_D(\tau))^{-1} F_S(\tau)
$$

$$
F_D(\tau) = T_\tau e^{-\Gamma(\tau)\tau}
$$

$$
F_S(\tau) = \int_0^\tau dt' F_D(\tau) F_D(t')^{-1} r_g(t'),
$$

where $\chi(\tau) = \{\chi_n(\tau)\}$ is the $n$-level population vector, $F_D(\tau)$ is the single-observation decay factor, $T_\tau$ is the time-ordering operator, $\Gamma(\tau)$ is the matrix analogue of the TLS convergence rate.
Figure 4. Steady states of the monitored open four-level system are shown to have different, non-monotonic, dependence on $\tau$ for each level. Their non-thermal nature is evident: $\chi_1(\tau)$, dashed; $\chi_2(\tau)$, dotted; $\chi_3(\tau)$, solid. The same marking and bath spectrum as figure 1 with the parameters: $\omega_{n=1,2,3} = \omega_n\{0.7, 0.8, 1.0\}$, scaled coupling strength $\kappa/\omega_n = \{0.01, 0.01, 0.02\}$, scaled inverse temperature $\beta\omega_a = 2$, scaled correlation time $\omega_a t_c = \{20, 10, 20\}$ and maximal coupling frequency $\omega_0 = \omega_n\{1.5, 1.5, 2.0\}$. Inset: scheme of measurements addressing different levels.

$\gamma(\tau)$ and $r_g = \{R_g^{(n)}\}$ is the $n$-vector of downward $|n\rangle \rightarrow |g\rangle$ transition rates (see the derivation in appendix A).

As can be seen from equations (13)–(15) and figure 3, the multi-level open-system steady state populations strongly depend on $\tau$ and not only on the bath temperature (through the relaxation rates, equation (2)). Hence, multi-level equilibration by observation has a patently non-thermal nature and can be (non-monotonically) steered by the measurement interval. This is in contrast to the TLS steady state, which can always be cast in thermal (Gibbs) form, albeit with a measurement-dependent temperature.

Another situation that contrasts the steady-state behavior in an MLS to that of a TLS occurs when the spectrum of the MLS is degenerate. For example in a three-level system, with any two levels degenerate, measurements in the basis involving different orthogonal superpositions of these degenerate states may lead to different steady state values of the level populations. Thus choice of the basis in which the measurements are preformed plays a crucial role in determining the steady states of the degenerate MLS. Further consideration of this issue is beyond the scope of the present analysis.

With a fixed observation period, $\tau$, one cannot attain arbitrary ratios of excited-level populations. Yet, the repertoire of steady states can be extended by performing multiple non-selective measurements, each addressing another subset of levels at different time intervals (figure 4 (inset)). Alternatively, since the same convergence holds also for continuous measurements (appendix B), having different monitoring rates for each level can also drastically increase the repertoire of multilevel steady states.

Hence, choosing observation times such that

$$\chi(\tau) = \rho(0)$$

(16)
results in ‘freezing’ of the initial state for any number of measurements, even for non-thermal states. This ‘freezing’ is not related to the one associated with the QZE, since it does not require an extremely short measurement interval. On the contrary, this non-Markovian freezing applies only for a specific value of \( \tau \) and for any number of measurements, constituting a more robust and lasting state-shelving mechanism than the QZE.

6. Discussion

Our analysis has revealed a cohesive picture of hitherto unnoticed, general anomalies of frequently probed quantum systems coupled to arbitrary (e.g. oscillator or spin) baths: (i) Whereas bath-induced inter-level transition rates in open systems exhibit slowdown under energy measurements that are frequent enough to conform to the QZE, the system does not retain its state as the measurements accumulate, but rather exhibits measurement-induced convergence toward an asymptotic steady state (equations (8)–(10)). (ii) In the QZE regime, the more frequent the measurements, the higher the asymptotic excitation and entropy. This reflects the remarkable fact that the QZE dynamics does not conform to the RWA and must account for CR terms in the system–bath interaction Hamiltonian (equations (8) and (10), (appendix C). (iii) Convergence to a maximal-entropy steady state may be avoided, within a limited range of initial excitations, by choosing the measurement rate capable of ‘freezing’ the initial state. (iv) Finally, at times beyond the Born time \( t_B \) (figure 2(a)), the steady state of a frequently measured open system no longer depends on the measurement interval. Rather, under the RWA, it depends solely on the bath coupling spectrum, whereas without the RWA, it can be the fully mixed state (figure 2(b)).

We stress that our findings can be tested in a variety of quantum systems coupled to baths whose non-Markovian timescales are experimentally accessible, e.g. ultracold atoms coupled to phonon baths [20], electron spins coupled to nuclear spins [21] or emitters with microwave transitions coupled to a cavity mode [22].

It is instructive to contrast the present findings with previous approaches to asymptotic steady states: (i) Since in our monitoring dynamics, any off-diagonal elements present in the initial state eventually decay to zero, one can identify the energy eigenstates of the system with the pointer basis [19]. However, our non-Markov treatment allows us to go a step further and show how one can choose specific pointer states that are maximally stable, for a given observation rate. (ii) Another approach [23] has been to engineer the quantum jump operators through which the system couples to the bath, so as to achieve the desired end state of the system evolution. Whereas this end state is reached via Markovian dynamics, our treatment draws on the richness of non-Markovian dynamics to achieve diverse steady states. Furthermore, manipulating the jump operators is equivalent to an external control over the system–bath interaction, assuming that the required control is at our disposal. In our approach, such control is not required: instead, manipulations are effected by the detector, which couples to the system alone and does not directly act on the system–bath coupling.

To conclude, the present findings shed new light on a fundamental issue at the interface of quantum measurement theory and the quantum dynamics of open systems: the interplay between measurement-induced and bath-induced rotating and counter-rotating non-Markovian dynamics. In particular, they essentially revise the notion of evolution freezing that has long been associated with the QZE.
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Appendix A. Driven MLS dynamics

A.1. Closed MLS

Consider a driven, closed MLS with a ground state and $N$ excited states, $|g\rangle$ and $|n\rangle$, respectively, with $n = 1, \ldots, N$ and energy separations $\hbar\omega_n$. Each excited level is driven by a resonant field, with a constant Rabi frequency $\Omega_1$. The corresponding Hamiltonian is (henceforth we take $\hbar = 1$):

$$H_{\text{close}} = \sum_{n=1}^{N} \omega_n |n\rangle\langle n| + \sum_{n=1}^{N} (\Omega_n |g\rangle\langle n| + \text{h.c.}). \tag{A.1}$$

The resulting rate equations are given by:

$$\dot{\rho}_{gg} = i \sum_{n=1}^{N} \Omega_n (\rho_{ng} - \rho_{gn}), \tag{A.2}$$

$$\dot{\rho}_{nn'} = i \Omega_n \rho_{gn'} - i \Omega_{n'} \rho_{ng}, \tag{A.3}$$

$$\dot{\rho}_{gn} = i \sum_{n=1}^{N} \Omega_{n'} \rho_{n'n} - i \Omega_n \rho_{gg}. \tag{A.4}$$

Since the Rabi frequencies are time independent, the dynamics can be solved by diagonalizing an $(N+1)^2 \times (N+1)^2$ matrix. The matrix eigenvalues are the oscillation frequencies of the system. The eigenvectors whose eigenvalues are equal to zero are the stationary states of the system.

We now show that for a three-level system, $N = 2$, the only diagonal linear combination of eigenstates with zero eigenvalue is the fully mixed state. The matrix eigenvalues are $\{0, \pm \sqrt{\Omega_1^2 + \Omega_2^2}, \pm \sqrt{\Omega_1^2 + \Omega_2^2}\}$, where $\Omega_1 = 2\Omega_1^2 + \Omega_1\Omega_2 + 2\Omega_2^2$ and $\Omega_2 = 4\Omega_1^2 + \Omega_1\Omega_2 + 4\Omega_2^2$. The three eigenstates with eigenvalue 0 are

$$\rho_a = \{0, 1, 1, 0, 0, 1, 0, 0\}, \tag{A.5}$$

$$\rho_b = \{\Omega_1/\Omega_2, 0, 0, 0, (-\Omega_2^2 + \Omega_1^2)/\Omega_1/\Omega_2, 1, 0, 1, 0\}, \tag{A.6}$$

$$\rho_c = \{1, 0, 0, 0, 1, 0, 0, 0, 1\}, \tag{A.7}$$

where $\rho = \{\rho_{gg}, \rho_{1g}, \rho_{2g}, \rho_{g1}, \rho_{g2}, \rho_{11}, \rho_{21}, \rho_{12}, \rho_{22}\}$. One can see that $\rho_c$ is the identity, i.e. the fully mixed state, and is the only steady state of the closed MLS, since no other linear combination of $\rho_a$ and $\rho_b$ can give a diagonal density matrix.

As a simple example, we consider a TLS. The closed $\sigma_z$-driven TLS dynamics is described by Bloch equations, and in the absence of measurements it yields the Rabi oscillation

$$\langle \sigma_z(\tau) \rangle = \langle \sigma_z(0) \rangle \cos(\Omega \tau) + \langle \sigma_z(0) \rangle \sin(\Omega \tau). \tag{A.8}$$
The impulsive, non-selective measurements of $\sigma_\gamma$ eliminate the off-diagonal terms, resulting in $\langle \sigma_\gamma(0) \rangle = 0$ after every measurement. We then obtain

$$\rho_{ee}^{(m+1)}(\tau) = \cos(\Omega\tau)\rho_{ee}^{(m)} + \frac{1}{2} \left( 1 - \cos(\Omega\tau) \right).$$  \hspace{1cm} (A.9)

We can cast this recursion relation in the form of exponential convergence, equation (8), with convergence rate and steady state given by equation (11).

A.2. Open MLS

Consider an open $(N+1)$-level MLS that is weakly coupled to a thermal bath with mode energies $\hbar\omega_n$. The coupling to the finite-temperature bath may differ from one excited state to another. The total Hamiltonian is given by

$$H_{\text{open}} = H_S + H_B + H_{SB} \quad H_B = \sum_\lambda \omega_\lambda a_\lambda^\dagger a_\lambda,$$

$$H_S = \sum_{n=1}^N \omega_n |n\rangle \langle n|,$$

$$H_{SB} = \sum_{n=1}^N \sigma_{x,n} \hat{B}_n = \sum_{n=1}^N \sum_\lambda \sigma_{x,n} (\kappa_{\lambda,n} a_\lambda + \kappa_{\lambda,n}^* a_\lambda^\dagger),$$

where $\sigma_{x,n} = |n\rangle \langle g| + |g\rangle \langle n|$ is the nth X-Pauli matrix and $\kappa_{\lambda,n}$ is the coupling coefficient of level $n$ to the bath mode $\lambda$. Note that we do not invoke the RWA.

Henceforth we shall adopt the Born approximation [8], whereby the bath remains constant throughout the evolution.

Using Zwanzig’s projection-operator technique to trace out the bath in the Liouville equation of motion [17, 24] and then transferring to the interaction picture, where $\hat{\rho}(t) = e^{iH_S t} \hat{\rho}(0) e^{-iH_S t}$, we have derived, to second order in the system–bath coupling, the non-Markovian master equation:

$$\dot{\hat{\rho}} = \sum_{n,n'=1}^N \int_0^t d\tau \left\{ \Phi_{T,nn'}(t - \tau) \left[ \hat{S}_{n'}(\tau) \hat{\rho}, \hat{S}_n(\tau) \right] + \text{h.c.} \right\},$$

where $\Phi_{T,nn'}(t - \tau) = \langle \hat{B}_n(t) \hat{B}_{n'}(t') \rangle$ is the finite-temperature system–bath correlation function for levels $n, n'$, and $\hat{S}_n(t) = e^{i\omega_n t} |n\rangle \langle g| + \text{h.c.}$

One can write the density matrix in the following form:

$$\rho = \rho_{gg} |g\rangle \langle g| + \sum_{n,n'=1}^N \rho_{nn'} |n\rangle \langle n'| + \sum_{n=1}^N (\rho_{gn} |g\rangle \langle n| + \rho_{ng} |n\rangle \langle g|),$$

where we have deliberately separated the $\rho_{gg}$ terms from the $\rho_{nn'}$ terms. The equations for $\rho_{gg}$ are independent of the other terms in the density matrix. Hence, once they are set to zero by a measurement, they remain so (do not evolve) after the measurement and are henceforth disregarded.

For the vector defined as $\rho^D = \{\rho_{nn}\}^T$, $n = 1, \ldots, N$, the evolution between consecutive measurements of the energy separated by $\tau$ can then be solved, to second order in the system–bath coupling, in the matrix form

$$\rho^D(\tau) = F_D(\tau) \rho^D(0) + F_S(\tau),$$

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where $F_D(\tau)$ and $F_S(\tau)$ are given in equations (14) and (15), respectively. The elements of the matrix analogue of the TLS convergence rate are given by

$$\Gamma_{nn'}(t) = \frac{1}{\tau} \int_0^t dt' \left[ \delta_{nn'}(R_n(t') + R_g^{(n)}(t')) + R_g^{(n)}(t')(1 - \delta_{nn'}) \right].$$

(A.16)

where $\delta_{nn'}$ is Kronecker’s delta. The transition rates $R_n(t)$ from $|n\rangle$ to $|g\rangle$ and $R_g(t)$ from $|g\rangle$ to $|n\rangle$ are explicitly analyzed in [17].

Taking into account that a QND measurement of the MLS energy levels at $t = 0$ eliminates all off-diagonal terms in the density matrix, i.e. $\rho_{nn'}(0) = 0$ for $n \neq n'$, we can integrate the equations of motion. The resulting $\rho_{nn'}(t)$ are of fourth order in the system–bath coupling. Hence, we can neglect the off-diagonal elements in the dynamics of the diagonal elements.

Consecutive impulsive measurements of the energy levels $n$, separated by $\tau$, yield, using equations (13)–(A.16):

$$\rho_D(t = n \tau) = F_n^D(\tau) \rho_D(0) + (I - F_D(\tau))^{-1} \left(I - F_n^D(\tau)\right) F_S(\tau).$$

(A.17)

By virtue of (14) and $\Gamma(\tau)$ being a real and positive definite matrix, $F_n^D(\tau) \rightarrow 0$ as $n \rightarrow \infty$. The state $\rho_D$ therefore converges to a steady state

$$\chi(\tau) = (I - F_D(\tau))^{-1} F_S(\tau).$$

(A.18)

Hence, equations (A.13)–(A.17) show that an MLS weakly coupled to a bath always exponentially converges to a steady state, $\chi(\tau)$, as the number of energy measurements increases.

**Appendix B. Extensions to continuous measurements**

**B.1. Closed TLS**

The continuous measurement of $\sigma_z$ in a $\sigma_x$-driven closed TLS close system is described by the following ME, to second order in the coupling to the measuring device [8]

$$\dot{\rho}(t) = \frac{i}{2} \left[ \sigma_z, \rho(t) \right] - \frac{1}{2\tau} \left[ \sigma_z, \left[ \sigma_z, \rho(t) \right] \right],$$

(B.1)

where $\tau$ expresses the inverse of the measurement strength or, equivalently, the effective measurement interval. The solution after total monitoring time $T$ conforms to equation (8) with

$$e^{-\gamma_{\text{closed}}(\tau)T} = \frac{e^{-\mu_1T} - 1}{\mu_2 - \mu_1},$$

(B.2)

$$\mu_{1,2} = \frac{1}{\tau} \left( 1 \pm \sqrt{1 - (\Omega\tau)^2} \right),$$

(B.3)

$$\gamma_{\text{closed}} = \frac{1}{\tau}.$$

(B.4)

The real parts of the exponents in (B.2) are positive: $\mu_{1,2} > 0$.

We thus see that the monitoring dynamics is similar for impulsive frequent measurements and continuous measurements.

**B.2. Open TLS**

Continuous measurements of an open quantum system may be described as stationary random dephasing, at a rate $1/\tau$ [13, 25]. The Hamiltonian then has a randomly fluctuating energy $\sigma_z$.
term in addition to the term of $\sigma_z$ coupling to the bath:

$$H_{\text{open}} = \delta_r(t)\sigma_z - g\hat{B}(t)\sigma_x.$$  

(B.5)

The corresponding ME, to second order in the system–bath coupling, assuming an initial product state of the form $\rho_\Sigma(t) \otimes \rho_B$, can then be written as

$$\dot{\rho}(t) = \int_0^t dt' \left\{ \Phi(t - t') [\hat{S}(t), [\hat{S}(t'), \rho(t)]] + \text{h.c.} \right\}$$  

(B.6)

$$\hat{S}(t) = e^{-i\omega_a t - i \int_0^t dt' \delta_r(t')} \sigma_x + \text{h.c.},$$  

(B.7)

where the overbar denotes the ensemble average and $\Phi_1(t - t') = \langle \hat{B}(t)\hat{B}(t') \rangle$ is the bath correlation (response) function.

The Bloch equations derived from (B.6) have the form

$$\dot{\rho}_{ee}(t) = -\dot{\rho}_{eg}(t) = -R_e(t)\rho_{ee}(t) + R_g(t)\rho_{eg}(t).$$  

(B.8)

After time $t \gg \tau$, i.e. monitoring over time that is equivalent to several measurements, the transition rates can be shown to be time independent and acquire the following spectral form 

$$R_e(g)(\tau) = \frac{2}{\pi} \int_{-\infty}^{\infty} \frac{d\omega}{\pi} G_T(\omega) F_\tau(\omega) \exp(i\omega \tau),$$  

(B.9)

$$F_\tau(\omega) = \frac{\pi}{\pi + (\omega \tau)^2}.$$  

(B.10)

In the limit of a broad spectrum, i.e. white noise, the ensemble average causes the stochastic dephasing of the phase factor associated with $\delta_r(t)$: $e^{i\int_0^t dt' \delta_r(t')} = e^{-(t-t')/\tau}$, resulting in

$$F_\tau(\omega) = \frac{\tau}{\pi + (\omega \tau)^2}.$$  

(B.11)

Thus, continuous measurements, modeled by a stochastic dephasing process with correlation time $\tau$, are described by $F_\tau(\omega)$ being a Lorentzian of width $1/\tau$.

The exact solution to equation (B.8) is

$$\rho_{ee}(T) = e^{-\gamma(t)T} \rho_{ee}(0) + \int_0^T dt' e^{-\gamma(t)(T-t')} R_g(\tau),$$  

(B.12)

$$\gamma(t) = R_g(\tau) + R_e(\tau).$$  

(B.13)

The exponential decay of the first term in (B.12) implies convergence to the steady state, which conform to equations (8)–(10), with

$$\chi_{\text{open}}(\tau) = \frac{R_g(\tau)}{\gamma(\tau)}.$$  

(B.14)

B.3. Open MLSs

Similarly to the TLS scenario, continuous measurements of MLSs may be described as random dephasing [13, 25]. In the multilevel case, each level may experience different dephasing rate
1/\tau_n$, expressed as an additional factor in equation (A.11):

\[ H_S = \sum_{n=1}^{N} (\omega_n + \delta_{r,n}(t))|n\rangle\langle n| \]  

(B.15)

\[ \hat{S}_n(t) = e^{-i\omega_n t - i\int_0^t dt' \delta_{r,n}(t')}|n\rangle\langle g| + \text{h.c.} \]  

(B.16)

Furthermore, the continuous measurement-induced dephasing of each level is uncorrelated to the other levels’ dephasing, resulting in \( \hat{S}_n(t)\hat{S}_{n'}(t') = 0, \forall n \neq n' \). This results in the diagonal vector dynamics:

\[ \rho_D(t) = F_D(t)\rho_D(0) + F_S(t), \]  

(B.17)

with \( F_D(t) \) and \( F_S(t) \) given in equations (14), (15) and (A.16), with the transition rates \( R_n(t) \) and \( R_g(t) \) given by

\[ R_n(t) = 2\pi \int_{-\infty}^{\infty} d\omega G_{T\pm}(\omega) F_{T,n}(\omega - \omega_n), \]  

(B.18)

\[ R_g(t) = 2\pi \int_{-\infty}^{\infty} d\omega G_{T\pm}(\omega) F_{T,n}(\omega + \omega_n), \]  

(B.19)

\[ F_{T,n}(\omega) = \frac{\tau_n}{\pi} \frac{1}{1 + (\omega \tau_n)^2}. \]  

(B.20)

Appendix C. RWA and its breakdown

C.1. \( \sigma_x \)-coupling to a thermal bath in the RWA

For a TLS coupled to a thermal bath with mode energies \( \hbar \omega_{\lambda} \), the RWA Hamiltonian has the form

\[ H_S = \omega_e |e\rangle\langle e|, \quad H_B = \sum_\lambda \omega_{\lambda} \sigma_+^\dagger \sigma_\lambda, \quad H_I = \hat{B}_- \sigma_+ + \hat{B}_+ \sigma_-, \]  

(C.1)

\[ \hat{B}_- = \left( \sum_\lambda \kappa_\lambda \sigma_\lambda \right), \quad \hat{B}_+ = \left( \sum_\lambda \kappa_\lambda^* a_\lambda^\dagger \right), \]  

(C.2)

\( \sigma_+ = |e\rangle\langle g|, \sigma_- = |g\rangle\langle e| \) and \( a_\lambda(a_\lambda^\dagger) \) are the annihilation (creation) operators of mode \( \lambda \) of the bath.

Using the same procedure as in [13, 17, 24, 26], one arrives at the following transition rates, \( R_{e(g)}^{\text{RWA}}(|e\rangle \rightarrow |g\rangle) \) and \( R_{g(e)}^{\text{RWA}}(|g\rangle \rightarrow |e\rangle) \):

\[ R_{e(g)}^{\text{RWA}}(t) = 2\pi t \int_0^\infty d\omega G_{T\pm}^{\text{RWA}}(\omega) \text{sinc} [(\omega \mp \omega_n)t], \]  

(C.3)

where

\[ G_{T+}^{\text{RWA}}(\omega) = (n_T(\omega) + 1) G_0(\omega), \]  

(C.4)

\[ G_{T-}^{\text{RWA}}(\omega) = n_T(-\omega) G_0(-\omega), \]  

(C.5)
\[ G_0(\omega) = |\kappa_\lambda|^2 \delta(\omega - \omega_\lambda), \]  
\[ n_T(\omega) = (e^{\beta\omega} - 1)^{-1}, \]  
where \( \beta = 1/k_B T \) is the inverse temperature of the bath.

Thus, the \( |g\rangle \rightarrow |e\rangle \) rate in the RWA is the overlap of \( G_T \) and the sinc over positive frequencies. At zero temperature, it survives due to the vacuum contribution (spontaneous decay rate). The \( |e\rangle \rightarrow |g\rangle \) rate in the RWA is the overlap of \( G_T \) and the sinc over the negative frequencies. It vanishes at zero temperature since it does not have the vacuum contribution.

Clearly, the two transition rates are influenced by different coupling spectra, \( G_{T+}^{\text{RWA}} \) and \( G_{T-}^{\text{RWA}} \) in the RWA.

C.2. \( \sigma_x \)-coupling to a thermal bath without RWA

Without RWA,
\[ H_I = \sigma_z \hat{B}, \quad \hat{B} = \sum_\lambda \left( \kappa_\lambda a_\lambda + \kappa_\lambda^* a_\lambda^\dagger \right), \]
where \( \sigma_z = |e\rangle\langle g| + |g\rangle\langle e| \). This \( H_I \) can be decomposed into secular (RW) and non-secular (CR) parts:
\[ H_I = (\hat{B}_+ \sigma_+ + \hat{B}_- \sigma_-) + (\hat{B}_x \sigma_x + \hat{B}_y \sigma_y). \]

Now, the \( |e\rangle \rightarrow |g\rangle \) and \( |g\rangle \rightarrow |e\rangle \) transition rates are given by
\[ R_{e(g)}(t) = 2\pi t \int_{-\infty}^{\infty} \text{d}\omega G_{T}(\omega) \text{sinc}\left((\omega \pm \omega_a)t\right), \]
\[ G_{T}(\omega) = (n_T(\omega) + 1) G_0(\omega) + n_T(\omega) G_0(-\omega). \]

Hence, both transition rates are given by the overlap over the entire range of frequencies (positive and negative) and therefore are influenced by the same coupling spectra.

C.3. Comparison of RWA and non-RWA decoherence rates

We compare the resulting transition rates with and without the RWA in two limits: (i) the Markovian long time limit, where sinc\([(\omega \pm \omega_a)t]\) \( \approx \delta(\omega \pm \omega_a)/t \), and (ii) the QZE ultrashort time limit, where sinc\([(\omega \pm \omega_a)t]\) \( \approx 1 \).

(i) The Markovian limit. In the Markovian long-time regime, one has the following results:
\[ R_{e}^{\text{RWA}} = R_{e} = 2\pi n_T(\omega_a) G_0(\omega_a), \]
\[ R_{g}^{\text{RWA}} = R_{g} = 2\pi n_T(\omega_a) G_0(\omega_a), \]
\[ \frac{R_{e}}{R_{g}} = e^{\beta\omega_a}. \]

One can draw two conclusions: (a) The rates obtained with and without the RWA are the same in the Markovian long-time regime. (b) The ratio of these rates conforms to the thermal equilibrium (Gibbs) state of the system.
(ii) The QZE limit. In the ultrashort time limit, one obtains the following results:

\[ R_{e}^{\text{RWA}} = 2\pi t \int_{-\infty}^{\infty} d\omega(n_{T}(\omega) + 1)G_{0}(\omega), \quad (C.15) \]

\[ R_{g}^{\text{RWA}} = 2\pi t \int_{-\infty}^{\infty} d\omega n_{T}(\omega)G_{0}(\omega) \quad (C.16) \]

\[ \frac{R_{e}^{\text{RWA}}}{R_{g}^{\text{RWA}}} = e^{\beta_{RWA}a_{a}}, \quad R_{e} = R_{g} = R_{e}^{\text{RWA}} + R_{g}^{\text{RWA}}, \quad \frac{R_{e}}{R_{g}} = 1. \quad (C.17) \]

One can draw the following conclusions: (a) The RWA changes the results drastically in this limit, indicating that the RWA does not hold there. (b) The RWA effective temperature is finite and depends on the characteristics of the bath, whereas the non-RWA effective temperature is infinite and does not depend on the bath. These features can be explained by the fact that at ultrashort times after a measurement, the energy uncertainty is so large that the TLS couples to all modes of the bath, at both positive and negative energies, and therefore there is no distinction between the ground and excited states as regards their coupling to the bath. By contrast, the RWA increasingly distinguishes between the two as \( t \) decreases.

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