Coupling Faults Diagnosis of a Gear-Shaft-Bearing System with Crack and Tooth Wear Faults

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Abstract. Considering tooth wear, crack failure of a gear-rotor-bearing system, multiple faults diagnosis model are established. The gear rotor bearing system with coupled multiple faults is identified by using model-based signal-based identification method, and type of system fault are distinguished in order to raise more efficient and accurate fault identification algorithm. It verifies the theoretical correctness and reliability of the method by making the experiment of gear-rotor-bearing system on a gear-rotor test bench. This study has an important guiding significance and application value to accurately predict the faults of gear-rotor-bearing system, and improves the reliability of the gear rotor system.

1. Introduction
Gear rotor system is an important part of mechanical system, which often work under high speed and heavy load conditions. Fault diagnosis of gear system are very important for a whole machine [1]. Recently, the research on diagnosis of gear system has attracted great attention. Model of rotor dynamics, rotor crack, rule of crack propagation have been proposed and improved [2]. Tooth wear and tooth surface spalling theory have been also presented [3]. At present, there are two kinds of fault diagnosis methods for gear bearing rotor system. One is the model-based method, the other is the signal diagnosis method. Model-based method can determine the location of cracks based on the vibration signals. Signal diagnosis method judge the faults and identify the faults through the study of the time domain diagram, frequency spectrum, in which Fourier transform, Hilbert transform, frequency domain method are used.

Local wear faults of gear system occur only in individual teeth, so the vibration caused by the energy change in the entire gear vibration is very small, and often distribute in a wide frequency range. Wang et al. [4] used experimental method to collect vibration signals of normal gear and fault gear by sensors, and used wavelet analysis to process vibration signals. Parey et al. [5] set up a gear system fault model considering time-varying mesh stiffness and gear backlash. Using empirical mode method to diagnose tooth wear fault. Fakhfakh [6] diagnosed the faults of the gear system by spectrum and cepstrum analysis, and verified the correctness of the simulation analysis by the experiment. Dalpiaz [7] tested different degrees of tooth wear faults respectively, and used different signal analysis methods to analyze the vibration signals, the correctness and effectiveness of various methods were compared.

This paper will present a parameter identification method to identify multiple faults in gear system. Experimental study will be done to verify the reliability of theoretical model and algorithm.
2. Diagnosis model of faults in gear system

2.1. Diagnosis model of Crack Fault

Model-based method is used to diagnose crack fault in the gear system. By determining whether there are residual vibration equivalent forces on each node of rotor, the existence of cracks and location of the crack on the rotor can be effectively determined. Firstly, the vibration signal of the healthy rotor system are measured, then mass, stiffness and damping matrix are assembled and the dynamic equations of the system are established by the finite element method. Using the same method, the vibration state of rotor system with crack can be obtained. Finally, solving the residual vibration of the equivalent force by comparing the displacement, velocity and acceleration signals of the healthy and cracked rotor, the existence of cracks and location of the crack can be determined.

Assuming there is crack fault in a gear rotor system, dynamic characteristics of the whole system will be changed because of the existence of crack. Based on the finite element model of the system, the residual vibration model of the rotor system with crack can be written as:

$$M \{\ddot{X}\} + (C + G) \{\dot{X}\} + K \{X\} = P(t) + \Delta P(t)$$  \hspace{1cm} (1)

Where, $M$ is mass matrix of the system, $C$ is damping matrix, $G$ is gyroscopic matrix, $K$ is stiffness matrix, $P(t)$ is load matrix, $\Delta P(t)$ is the residual vibration force matrix.

By comparing the vibration value between the healthy rotor system and the fault rotor system, the residual vibration value of acceleration, velocity and displacement can be obtained.

$$\Delta \dot{X}(t) = \dot{X}(t) - \dot{X}_o(t)$$  \hspace{1cm} (2)

$$\Delta X(t) = X(t) - X_o(t)$$  \hspace{1cm} (3)

$$\Delta X_i(t) = X_i(t) - X_o(t)$$  \hspace{1cm} (4)

Where, $X_o(t)$ is displacement of healthy rotor, $X(t)$ is displacement of the fault rotor.

In order to obtain the vibration state of all degrees of freedom of the gear rotor system, modal expansion method is used. By measuring the residual vibration of the nodes, all the remaining vibration values are extended, and the relationship between them can be established.

$$\Delta X_M(t) = C_i \Delta X(t)$$  \hspace{1cm} (5)

Where, $\Delta X_M(t)$ is vibration value from the measured nodes, $\Delta X(t)$ is vibration value of all nodes, $C_i$ stands for a matrix formed from zero and one.

When the rotor is cracked, it is equivalent that an external torque applied to the rotor system. Therefore, the equivalent load is the direct basis for judging the existence of cracks. The model based method for the diagnosis of crack is to transform the crack into an equivalent load.

$$\Delta P(t) = M \Delta \ddot{X}_M + (C + G) \Delta \dot{X}_M + K \Delta X_M$$  \hspace{1cm} (6)

2.2. Diagnosis model of Tooth Wear Fault

In this paper, a new method to diagnosis the tooth wear fault is proposed. Firstly, using empirical mode method (EMD) to process the vibration signal with noise and choose appropriate intrinsic mode function (IMF), then using Hilbert transform (HHT) to obtain Hilbert spectrum and marginal spectrum. Finally, the frequency band information are analyzed from the marginal spectrum.

According to the definition of EMD, a signal $x(t)$ can be decomposed by the following steps.

$$x(t) = \sum_{i=1}^{n} c_i(t) + r_n(t)$$  \hspace{1cm} (7)

Where, $x(t)$ is the original signal, $c_i(t)$ is intrinsic modal component, $r_n(t)$ is residual signal component.

Hilbert-Huang transform is a method of obtaining the best local signal by changing the frequency and amplitude of the sinusoidal curve.

$$H[c_i(t)] = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{c_i(t)}{t - \tau} d\tau$$  \hspace{1cm} (8)

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Amplitude function and phase function of the signal can be solved through Hilbert-Huang transformation.

\[ a_i(t) = \sqrt{c_i^2(t) + H^2[c_i(t)]}, \quad \phi_i(t) = \arctan \frac{H[c_i(t)]}{c_i(t)} \]  

(9)

Where, \( a_i(t) \) is amplitude function of \( x(t) \), \( c_i(t) \) is mode function of signal, can be obtained from EMD decomposition. \( \phi_i(t) \) is phase function of the signal.

Then, the original signal can be expressed as:

\[ x(t) = \text{Re} \sum_{i=1}^{n} a_i(t)e^{i\phi_i(t)} \]  

(10)

Hilbert energy spectrum is defined as:

\[ H(\omega, t) = \text{Re} \sum_{i=1}^{n} a_i(t)e^{i\omega(t)\omega} \]  

(11)

By Hilbert transformation, the marginal spectrum of the Hilbert transformation is defined. Marginal spectrum can be obtained by integrated along the time axis.

\[ h(\omega) = \int_0^T H(\omega, t) \, dt \]  

(12)

Where, \( T \) is time length of the original vibration signal analysis.

3. Coupling faults diagnosis method

Coupling faults of gear box include two types of the typical failures, crack rotor and gear wear. Model-based method is used to diagnose crack fault in the gear system. By solving the residual vibration of the equivalent force, the existence of cracks and location of the crack can be determined. Hilbert transform method is used to diagnose gear wearing fault. After analyzing the marginal spectrum of Hilbert transformation, it can be found that the change of amplitude of the meshing frequency and its harmonic components, which used to diagnosis the tooth wear fault.

4. Analysis and discussion

Figure 1 shows a gear rotor system test bench, the length of axis is 350mm, modulus of elasticity is \( 2 \times 10^{11} \) Pa, Poisson's ratio is 0.3, which supported by four 7306 ball bearings. Torque of gear transmission is 200N·m. The tests of the gear rotor system with coupling faults of crack and wear have been done in this bench. The high-speed shaft and low-speed shaft have been divided into ten nodes, respectively. Eddy current sensors are installed in high speed shaft, which can measure displacement of four nodes. The rotating speed is adjusted to 3600rpm by a frequency converter.

In the test bench, there is a crack between the third node and fourth node in the high-speed shaft to simulate the crack fault. Teeth wear fault were simulated by installing some hard iron in the gear teeth. First, the first to fifth order modes of the high speed and low speed shafts in horizontal and vertical directions are calculated respectively. Then, the test with the health of the rotor has been done, the residual vibration displacement of each node are obtained according to the modal expansion method. Using the model of diagnosis method based on residual vibration calculation of each node, crack fault is diagnosed.

![Figure 1. Gear rotor system test bench](image-url)
Figure 2 shows vibration modes from first order to fifth order of high-speed shaft, low-speed shaft in gear rotor system. According to the modal expansion method, the residual vibration displacement of each node are obtained. Residual vibration velocity and residual vibration acceleration of all nodes are solved by differential method.

![Vibration modes of the gear-rotor system](image)

**Figure 2.** Vibration modes of the gear-rotor system

The residual vibration forces are calculated and are shown as Figure 3, Figure 4. From Figure 3, it can be clearly seen that there is the maximum residual vibration force between the third node and fourth node in the high speed shaft. Figure 4 shows there is a small residual vibration force in the horizontal direction due to influence of high speed shaft, but there is no residual force in the vertical direction. Therefore, it can be inferred that there is a crack between the third node and fourth node in the high-speed shaft.

![Residual vibration force of high-speed shaft](image)

**Figure 3.** Residual vibration force of high-speed shaft

![Residual vibration force of low-speed shaft](image)

**Figure 4.** Residual vibration force of low-speed shaft
The second step is to diagnose the wear fault of the tooth surface. First, the decomposed modal functions and residues are obtained by EMD method. The marginal spectrum of each IMF component is given in Figure 5. It can be seen that the central frequency of the marginal spectrum of each IMF component is gradually reduced, which is in accordance with the characteristics of the EMD decomposition. Figure 6 gives the marginal spectrum after Hilbert transformation.

![Figure 5. Marginal spectrum of IMF component](image1)

![Figure 6. Marginal spectrum of gear wear fault signal](image2)

From Figure 5 and Figure 6, it can be seen clearly that the amplitude of rotating frequency of input shaft is much larger than other frequency components, and the amplitude of its meshing frequency is also very large. Because of the interference of noise signal, the rotating frequency the output shaft can hardly be observed. Therefore, it can be concluded that the gear wear fault occurs on the input shaft.

5. Conclusions
Aiming to the crack, gear wearing faults of gear rotor system, a multi fault identification algorithm for gear rotor system is proposed. The shaft crack fault and tooth wear gear bearing rotor system coupled by the sensors, vibration displacement, using model-based method to calculate the residual vibration based on the vibration displacement signals sampled by the sensors, the crack fault and the position of crack can be judged correctly. Using Hilbert-Huang transform to get the marginal spectrum, wear fault and the wear position are determined by the frequency spectrum. Experiments prove the effective of the methods.

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