Spontaneous spinning of a magnet levitating over a superconductor

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(Dated: March 22, 2022)

A permanent magnet levitating over a superconductor is found to spontaneously spin, overcoming resistance to air friction. We explain the physics behind this remarkable effect.

PACS numbers:

It is well known that a permanent magnet can levitate over a superconductor due to the Meissner effect. Furthermore, for type II superconductors, levitation over a flat superconductor is stable due to flux penetration and pinning of flux lines\cite{1}. However it appears not to be widely known that under appropriate conditions a levitating magnet has a strong tendency to start spinning: independent of initial conditions the rotation will reach a terminal velocity and persist, overcoming resistance due to air friction. After coming across this phenomenon independently we found out that this remarkable effect has been seen and discussed before\cite{2,3}. However its physical origin has not been completely elucidated before. The reader can see examples of this remarkable effect at the website \cite{4}.

A photograph of our experimental setup is shown in Figure 1. A $YBa_2Cu_3O_7$ disk of diameter $22mm$ and thickness $3mm$ rests on a metal base that is submerged in liquid nitrogen in a flat glass container. A permanent $Nd_2Fe_14B$ magnet disk of radius $R = 2.5mm$ and thickness $h = 1.2mm$ is placed vertically over the superconductor, as shown in the figures. The magnet is product number 64 – 1895 from Radio Shack Corporation. The height at which the magnet levitates over the superconductor is approximately $0.4mm$. It is determined by various parameters in the experimental setup such as the thickness of the superconductor, the mass of the magnet and the strength of the magnetization, as discussed in Ref.\cite{1}. The magnetization of the permanent magnet is perpendicular to the magnet disk surface, hence rotation of the magnet around a horizontal axis going through the disk center does not change the magnetic field.

If the magnet has a high degree of homogeneity it will start rotating almost immediately, while if it is inhomogeneous it will start rotational oscillations around an equilibrium orientation which increase in amplitude, until it 'goes over the top' and starts rotating either clockwise or counterclockwise at random depending on initial conditions. We have found this by experimentation with 8 different magnets, some with a high degree of homogeneity and some with less. The torque that makes the magnet oscillate and spin is to lowest order proportional to the magnet's angular velocity; if the magnet is initially at rest with zero angular velocity it is in unstable equilibrium, and any small perturbation will initiate the motion .

In agreement with previous work\cite{2,3} we find that the tendency to spinning is strongest when the magnet is situated in a region of large temperature gradient. In the setup of Fig. 1 with container height $1.2cm$ we find that a metal base of approximately $5mm$ is optimal for strongest spinning tendency. For such height the lower part of the magnet is still inside the container, while the upper part is outside; this presumably gives rise to largest temperature gradient. The terminal angular velocity for that case is approximately 4 revolutions/second. For a lower metal base the tendency to spinning and the terminal velocity is reduced, and if we use no metal base in this container, or if we use a deeper container, the tendency to spinning disappears. In such cases we would expect the entire magnet to be at a more or less uniform cold temperature due to proximity to the liquid $N_2$. Furthermore, we found that with the metal base and the flat container the spinning can be stopped by placing another container with liquid $N_2$ directly above the magnet, thus cooling the environment around the upper part of the magnet. Furthermore we have found that the tendency to spinning is reduced if the magnet is placed not vertically but at an angle; in that case, the temperature difference between top and bottom of the magnet is obviously reduced.

The observed effect is surprising because there is no obvious source for the kinetic energy of rotation that the magnet acquires, or for the torque that makes the magnet spin overcoming air friction. The effect does not depend on details of the experimental setup described above. We have observed it also with other superconducting samples of different shape, even with several small pieces of superconductor supporting the levitating magnet. The effect
is also seen when several of the magnets described above are put together making a cylinder of the same radius but larger height.

It was found in reference [2] that the effect does not originate in convective currents, i.e. 'wind', originating in the evaporating $N_2$, by placing the magnet inside a test tube and observing that the effect persists unchanged. We have repeated the same check. Additionally it was shown in ref. [3] that the effect persists when the air pressure is reduced. Furthermore it was conjectured in refs. [2,3] that the effect depends on the thermomagnetic properties of the magnet, namely on the reversible change of magnetization with temperature. However no direct experimental evidence was presented that the effect is connected to the magnetism of the magnet or the superconductivity of the superconductor. In principle one could imagine that the effect would occur also for a non-magnetic disk in a large temperature gradient if friction with the environment can be minimized.

However we have repeated the experiment with superconductors of different thicknesses, and observed a stronger tendency for spinning and a larger terminal angular velocity under the same conditions of temperature gradient for increasing thickness. This establishes that the superconductor plays a crucial role beyond simply levitating the magnet so that it can rotate without significant friction. Furthermore we have repeated the experiment with three identical magnets forming a cylinder of the same radius and three times the original height, and found that the tendency to spinning is only slightly reduced; when we then replaced the middle magnet with another magnet that had been previously heated to a temperature high enough to completely eliminate its magnetization, it was found that the tendency to spinning was now very significantly reduced. This establishes that the magnetism of the magnet plays a crucial role beyond simply levitating the magnet and that the spinning effect is larger for larger magnetization.

We have also observed that the effect depends on the thermal conductivity of the magnet. We did not have other magnets of different thermal conductivity but otherwise similar characteristics available, however, we observed that if the magnet is wrapped with thin Al foil, the effect becomes weaker or disappears altogether. The thermal conductivity of Al is much larger than that of $Nd_2Fe_{14}B$. This observation suggests that the effect will only occur if the thermal conductivity of the magnet is not too large.

Because the effect appears to persists indefinitely it does not seem possible that the energy to sustain the motion is supplied by the superconductor. Hence we conclude that it originates in heat from the environment, i.e. that the rotating magnet constitutes a simple heat engine as depicted in Figure 2. At any instant when the magnet is rotating, its upper half will be at a lower temperature than its environment, because half a period earlier it was in a region of much colder temperature. Similarly, its lower half will be at temperature higher than its environment because it was in a higher temperature region half a period ago. Hence the upper half will absorb heat from the environment and the lower half will release heat to the environment. If the heat absorbed by the upper half is more than the heat released by the lower half, the difference will be converted into work.

It remains to be explained how the conversion of heat to work depicted in Figure 2 occurs. We argue that it is a necessary consequence of the existence of a temperature gradient in the region where the magnet is and of the fact that the magnetization of the $Nd_2Fe_{14}B$ magnet is a decreasing function of temperature in the temperature range where the magnet is, as discussed in Refs.[2,3]. Qualitatively it works as follows: the superconductor exerts a force on each element of the magnet that is an increasing function of its magnetization. The magnetization in turn is an increasing function of temperature. An element of the magnet moving down has a higher temperature than its symmetric counterpart at the same distance from the superconductor that is moving up. The unequal force exerted by the superconductor on these two magnet elements leads to a net torque in the same direction as the existing angular velocity.

To make this argument concrete, consider the expression for the energy cost of putting vortices originating in a magnetic moment $m$ (parallel to the superconductor) through the superconductor, given in Ref. [1]

$$E = \frac{H_{c1}Lm}{\pi}d \equiv \frac{Cm}{d}$$

with $H_{c1}$ the lower critical field of the superconductor, $L$ the thickness of the superconductor and $d$ the distance from the superconductor to the magnetic moment. Let $(r, \varphi)$ be the radial and angular coordinates of an element of magnetic moment $d\vec{m}$ for which $d = d_0 + rcos\varphi$, with $d_0$ the distance from the superconductor to the center of the magnet disk. We measure $\varphi$ with respect to a vertical axis $z$. The torque on the magnet due to the

![FIG. 2: When the magnet rotates, the upper half is colder than its environment and the lower half is hotter than its environment. Heat $q_1$ flows into the upper half and heat $q_2$ flows out of the lower half, the difference is the work $w$ that makes the magnet spin.](image-url)
force exerted by the superconductor on this element of magnetic moment is

\[ d\tau = \frac{\partial E}{\partial \varphi} = \frac{C \times dm \times r \sin \varphi}{(d_0 + r \cos \varphi)^2} \]  \hspace{1cm} (2)

The element of magnetic moment is

\[ dm = M[T(r, \varphi)] hr dr d\varphi \]  \hspace{1cm} (3)

where \( M[T] \) is the local magnetization of the disk, which we assume to depend on the local temperature \( T(r, \varphi) \). If the magnet is rotating and has finite thermal conductivity we will have

\[ T(r, \varphi) \neq T(r, -\varphi) \]  \hspace{1cm} (4)

with \( T \) lower for the 'rising' side, hence there will be a net torque on the magnet. It is plausible to assume to lowest order

\[ T(r, \varphi) - T(r, -\varphi) = k r \sin \varphi \]  \hspace{1cm} (5)

with \( k \) a constant, hence the net torque is

\[ \tau = Ch_k \frac{\partial M}{\partial T} \int_0^R \int_0^\pi d\varphi \int_0^{\pi/2} \frac{r^3 \sin^2 \varphi}{(d_0 + r \cos \varphi)^2} \]  \hspace{1cm} (6)

The constant \( k \) will depend on the angular velocity \( \omega \) of the magnet and its thermal conductivity \( \kappa \), and in particular will go to zero both when \( \omega \to 0 \) and when \( \kappa \to \infty \). The torque \( \tau \) is a vector parallel or antiparallel to the angular velocity \( \vec{\omega} \) depending on whether \( \frac{\partial M}{\partial T} > 0 \) or \( \frac{\partial M}{\partial T} < 0 \) respectively. In the second case the torque opposes the motion and together with air friction will rapidly dampen any initial oscillation or rotation; in the first case instead, the torque will act in the presence of any small initial perturbation to increase the motion and the magnet will gain angular momentum and kinetic energy. In the presence of air friction the magnet will then reach a terminal angular velocity and continue its rotation indefinitely driven by this torque.

The height \( d_0 \) at which the magnet levitates is proportional to \( L^{1/2} \), with \( L \) the thickness of the superconductor, hence from Eqs. (6) and (1) we expect the torque to increase as \( L^{1/2} \). This is in qualitative agreement with our experimental observations discussed earlier.

The expression Eq. (6) is valid for the case where the entire magnet has magnetization with the same \( \frac{\partial M}{\partial T} \). If as described in our experiment earlier, part of the magnet has no magnetization, this part does not contribute to the torque Eq. (6) but does contribute to the moment of inertia, thus reducing the tendency to spinning in agreement with our observations discussed earlier.

Next we discuss the principle on which this 'heat engine' is based. We consider the simplified example shown in Figure 3 of a magnetized 'pendulum' on top of a superconductor in the presence of a temperature gradient. The magnet has two possible positions, one characterized by height \( d_1 \) and environment temperature \( T_1 \) and the other by \( d_2 \) and \( T_2 \), with \( d_1 > d_2 \) and \( T_1 < T_2 \). Furthermore, the magnet's magnetic moment when it is at temperature \( T_i \) is \( m_i \), with \( m_2 > m_1 \). In the first step the magnet (at temperature \( T_1 \)) falls from \( d_1 \) to \( d_2 \) and the superconductor increases its energy by

\[ \epsilon_1 = C m_1 (\frac{1}{d_1} - \frac{1}{d_2}) \]  \hspace{1cm} (7)

In the second step at position \( d_2 \), the magnet comes to thermal equilibrium with its environment at lower temperature \( T_2 \) by releasing heat \( q_2 \), its magnetization increasing to \( m_2 \). The energy of the superconductor increases by

\[ \epsilon_2 = \frac{C}{d_2} (m_2 - m_1) \]  \hspace{1cm} (8)

In the third step, the magnet rises again to position \( d_2 \) and the superconductor lowers its energy by

\[ \epsilon_3 = C m_2 (\frac{1}{d_2} - \frac{1}{d_1}) \]  \hspace{1cm} (9)

and in the fourth step the magnet absorbs heat \( q_4 \) and the magnet heats to temperature \( T_1 \), and its magnetization decreases to \( m_1 \), while the superconductor decreases its energy by

\[ \epsilon_4 = \frac{C}{d_1} (m_2 - m_1) \]  \hspace{1cm} (10)

Note that \( \epsilon_1 + \epsilon_2 - \epsilon_3 - \epsilon_4 = 0 \) as required by conservation of energy for the superconductor. The difference in the energy increase of the superconductor when the magnet
falls and its decrease when the magnet rises is the gain in the kinetic energy of the magnet:

$$\Delta K = \epsilon_3 - \epsilon_1 = C(m_2 - m_1)(\frac{1}{d_2} - \frac{1}{d_1})$$  \hspace{1cm} (11)$$

and conservation of energy in a cycle requires that

$$q_1 - q_2 = \Delta K$$  \hspace{1cm} (12)$$
hence the magnet absorbs more heat than it releases, and the difference is converted into work, either increasing the kinetic or potential energy of the magnet or dissipated in air friction or both.

The process described above is an idealization but describes the essence of the mechanism giving rise to the spontaneous rotation. It is a necessary condition for the above analysis to be valid that the magnet disk does not conduct heat too fast, so that the side coming down has a higher temperature than the side going up. The thermal conductivity of $Nd_2Fe_{14}B$ is $\kappa = 9W/mK$, and its specific heat is $C = 0.5 \times 10^7$ ergs/grK. Using these values and the dimensions of our magnet we estimate the time it takes for heat to conduct across the magnet as approximately

$$\Delta t \sim \frac{C\mu}{\pi\kappa d} \sim 2.5 \text{ sec}$$  \hspace{1cm} (13)$$

where $\mu = 0.174\text{ gr}$ is the mass of the magnet. This is much larger than the observed rotation period, hence we conclude that indeed the temperature profile on the magnet is not equilibrated as required by the mechanism discussed here. Instead, the thermal conductivity of $Al$ is approximately 25 times larger than that of $Nd_2Fe_{14}B$, $\kappa = 237W/mK$. Hence when wrapped with $Al$ foil the magnet would have to turn faster than 10 revolutions/second to prevent thermal equilibrium throughout the magnet and allow this mechanism to operate. The resistance from air friction would however be much larger for such large angular velocity, which is presumably why the effect does not occur in that case.

In summary, we have explored the physics behind the unusual observation that a magnet levitating over a superconductor is found to spontaneously spin under the right conditions. We have explained the phenomenon as constituting a simple heat engine, where heat is absorbed from a high temperature source, a smaller amount is released to a lower temperature sink, and the difference is converted into work through the interaction between the magnet and the superconductor. Compared to other simple heat engines such as Carnot’s or Stirling’s the present one appears to be simplest, as it has only one moving part, the magnet itself, and all the steps in the cycle are combined in a single process, the rotation of the magnet. Possible applications should be explored.

Acknowledgments

The authors are grateful to B. Maple for kindly providing the superconducting sample and the liquid $N_2$ used in the experiments.

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