The Theory and Assessment of Spatial Straightness Error Matched New Generation GPS

X B Zhang, X L Sheng, X Q Jiang and Z Li

School of Mechanical Science and Engineering, Huazhong University of Science and Technology, Wuhan, China, 430074

E-mail: zhangxinbao1@mail.hust.edu.cn

Abstract. In order to assess spatial straightness error matched new generation Dimensional Geometrical Product Specification and Verification (GPS), the theory of spatial straightness error assessing is proposed and its advantages are analyzed based on metrology and statistics in this paper. Then, the assessing parameter system is proposed and it is testified in real application comparing to assessment result of the geometric tolerance theory. Statistical parameters of this assessing system post the different characteristics of spatial straightness error, and can reveal the impact of spatial straightness error on the accessory function more roundly to complement the single assessing parameter of geometrical tolerance for straightness error. The statistical spatial straightness tolerance and statistical spatial straightness error proposed in this paper is possible to be applied in evaluation of other error of form, orientation, location and run–out.

1. Introduction

Tolerance technology is an important techno-basis of mechanic and electronic industry. Tolerance can solve the conflict between the using demand of mechanic and electronic products and the producing demand and describe their accuracy quantitatively. In producing process of products, the tolerance embodies the relation of designing, producing, and measuring of products. And more, it develops rapidly in recent 10 years [1].

The old geometric tolerance standard, ISO R1101 standard, can’t fit the developing demand of tolerance technology [2]. It and its assessing system have disadvantages as following:

(1) Its theoretic basis is geometry. Geometry gives classical and certain mathematics method and, in advance, has theoretic shortages to describe the error of geometric dimension and error of form, orientation, location and run–out.

(2) Its measuring method is defined according to the old geometric tolerance standard and it is very heavy when measured error of form, orientation, location and run–out. Indeed, if measured strictly according to the definition of the old geometric tolerance standard, the measurement can hardly be done. Any measurement is a sampling process as a whole, so measurement result is certainly related with a believable probability.

(3) Function guaranteeing can not be defined according to the definition of the old geometric tolerance standard. The concept about error of form, orientation, location and run–out just reflects and limits the fluctuation range of extremum local points of the measured real feature, but these extremum local points are altered for probably wore or distorted in initial stages of accessory’s usage and they
can’t exactly reflect the extensive function of a accessory as a whole. Therefore, the old geometric tolerance standard isn’t able to be entirely compatible the development of CAD/CAM technology and automatic production.

International organization for standardization ISO/TC 213 has publicized new generation Dimensional Geometrical Product Specification and Verification (GPS), for example: ISO14660-1 and ISO14660-2 since 1999. Its theoretical basis is metrology. And more, the application of computer technology makes assessing error easily and is established new basis for ISO R1101 emending. New generation GPS has been imbibe the content of statistics to fit the new demand. Its main characteristic is considered the demand of designing, producing, and verifying and it is adapted for practice comparatively.

The straightness error measurement is one of the basic items in the GPS and is also an important one. The non-diffracting beam has better tolerance property to the drift of laser beam than that of the common collimated laser beam and is more suitable to be used as the datum-line for straightness error measurement. Using the non-diffracting beam produced by an axicon as the spatial datum-line for straightness error measurement, a new measurement method of spatial straightness error has been developed [3,4]. In order to assess spatial straightness error matched new generation GPS, the theory of spatial straightness error assessing is proposed in this paper.

2. The concept of the statistical tolerance and its theoretical basis
The statistical tolerance is the tolerance of the statistical assessing parameters.

2.1. the statistical tolerance characteristic
(1) The statistical tolerance isn’t used to limit the fluctuation range of a real feature, but is used to limit the fluctuation range of the statistical assessing parameters of a real feature.
(2) The value of the statistical tolerance is related with a specific believable probability. When the believable probability is changed, the value is varied.

2.2. the statistical tolerance rationality
(1) The statistical tolerance is based on probability theory and mathematics statistics (included Stochastic Process) and is suitable for the controlled real object that has a random characteristic. The statistical tolerance can completely use the theory and method of the statistical distribution, the statistical deduction, and the statistical sample.
(2) The measurement and assessment is done according to the statistical tolerance and is accorded with the theory and method of the statistical sample and the statistical assessment.
(3) The statistical tolerance is described the function in rationality. According to the regulation of the statistical tolerance, that describing the function is related with a specific probability is compatible to describe production usage comparatively.
(4) The statistical tolerance can reflect the demand to control the systematic error and random error in producing process and is compatible to describe real producing process comparatively to be propitious to do product statistics and analysis and to control product quality.
(5) if the specific believable probability is 1, the believable range of the statistical tolerance is corresponded to geometric tolerance. Therefore, geometric tolerance can be thought as the special case of the statistical tolerance.

3. The statistical assessing theory of spatial straightness error
The statistical assessing theory of spatial straightness error can be proposed considering reference [5] and the following ideas:
(1) Drawing straightness tolerance needs to be according to the demand of function. For example, for rotating shaft, the main demand is for rotating accuracy, so the rotating central axes is used as datum-line to limit the fluctuation of spatial straightness error. And yet, for rollers of plane supporting linear roller slide, the main demand is to control the identical distance of the two parallel tangential
planes of rollers. One tangential plane is used as datum-plane and its parallel tangential plane is used to contain roller to limit the fluctuation of spatial straightness error.

(2) Drawing straightness tolerance must consider producing technics and its characteristics. For example, for spatial straightness error of the central axes of rotating shaft, the theoretical central axes of a rotating shaft is suitable to be datum-line. In process, the displace of the point edge of cutting tool to the datum-line cause straightness error of shaft surface and then spatial straightness error of the central axes of a rotating shaft is formed. Because processing system has stochastic characteristic, the profile error curve of a rotating shaft is a stochastic function. Apparently, when drawing straightness tolerance, in order to consider how to control error characteristic of processing system, the theoretical central axes of a rotating shaft is suitable to be datum-line to limit the fluctuation of the central axes of a rotating shaft.

(3) Statistical straightness tolerance shall reflect extensive characteristics of the demand of product function and the demand of production.

The statistical assessing parameters of spatial straightness error is described as following

- spatial straightness error function
  
  It is a function that describes spatial straightness error distribution and their characteristic:
  \[ F(z) = (x, y) \quad (1) \]

  the probability of \((x, y)\) is \(P(x, y)\), the conditional probability of \(x\) is \(P(x)\) and the conditional probability of \(y\) is \(P(y)\).

- statistical straightness error
  
  It is the statistical assessing parameter that describes characteristic of spatial straightness error function:
  \[ C = S(x, y) \quad (2) \]

Such as, the variance of statistical straightness error:

\[ C = \sum (x_i - E(x))^2 + (y_i - E(y))^2 \cdot P(x, y) \quad (3) \]

where

\[
\begin{align*}
    E(x) &= \sum x_i P(x_i) \\
    E(y) &= \sum y_i P(y_i)
\end{align*}
\]

If

\[ E(x) = \int x P(x) dx, \quad E(y) = \int y P(y) dy \]

\[ C = \int_{x, y} [(x - E(x))^2 + (y - E(y))^2] \cdot P(x, y) \, dx \, dy \]

- the standard deviation of statistical straightness error:

\[ C = \sqrt{\sum (x_i - E(x))^2 + (y_i - E(y))^2 \cdot P(x, y)} \quad (5) \]

or

\[ C = \sqrt{\int_{x, y} [(x - E(x))^2 + (y - E(y))^2] \cdot P(x, y) \, dx \, dy} \]

- the maximum of statistical straightness error:

\[ C_{xk} = \left| x_{max}\right| \]

(6)

From the probability distribution of \((x,y)\), the probability distribution of \(r\) can be deduced and the math expectation of the deviation can be got:

\[ E(r) = \dn \cdot C_s \quad (7) \]

Where \(dn\) is a constant concerned with the probability distribution of \((x, y)\) and the sample number \(n\).

Apparently, above the defined statistical assessing parameters are all related to spatial straightness error function. When point estimating or interval estimating the statistical assessing parameters, they are all related to the sample number \(n\), so they are all related to the believable probability.
Referencing the statistical assessing parameters for surface roughness, more statistical assessing parameters can be set:

- the mean spacing of statistical straightness error:

For a rotating shaft, it reflects the frequency characteristic of rotating accuracy and is corresponded to mean spacing of the profile irregularities \( S_{m} \). The arithmetical means \( Z_{m} \) and \( \sigma_{m} \) are respectively mean distance spacing along z-direction and mean angular spacing along circumference-direction

\[
Z_{m} = \frac{\sum z_{i}}{m} \\
\sigma_{m} = \frac{\sum \theta_{i}}{\sum z_{i} / m}
\]

All these statistical assessing parameters can reveal the impact of spatial straightness error on the accessory function in different aspects more roundly, to complement the single assessing parameter of geometrical tolerance for straightness error.

4. The test data of spatial straightness error and its statistical assessment

The test data of spatial straightness error are obtained by the measurement method of spatial straightness error using non-diffracting beam. The sampling manner to measure the following test data is same as ordinary equi-space sampling manner. Because of the characteristic of ordinary equi-space sampling manner, \( z \) obeys Uniformity distribution. \( z_{i} \) is corresponding to \((x_{i}, y_{i})\), so:

\[
P(x_{i}) = P(y_{i}) = P(z_{i}) \quad \text{and} \quad P(x_{i}, y_{i}) = P(x_{i})P(y_{i})
\]

Let the unit of the offset \((x(z_{i}), y(z_{i}))\), \( x \) and \( y \) for short, to datum-line measured at position \( z_{i} \) be \( \mu \)m. Of \( \Delta x \) and \( \Delta y \) to the spatial datum-line of error evaluation be \( \mu \)m. Of \( r \) and \( \theta \) on the cylindrical coordinates be \( \mu \)m and radian respectively and of \( z \) be mm.

4.1. The test data of a straight rail of 1.5m and its assessment

The straight rail, GJZ optical bench, is produced by the workshop of HUST. Its work length is 1530mm and its straight error along the x-direction is less than 30\( \mu \)m/1m.

Let’s measure the straight rail under the same conditions as above. Set \( z_{0} = 400 \)mm (correspond to the position at the 100mm of the straight rail) and the increasing span per step is +70mm along z-direction. The measuring distance range is 0.4-1.8m behind the axicon. All of the offset \( r(z_{i}), (x(z_{i}), y(z_{i})) \), to datum-line measured at position \( z_{i} \) are shown in table 1. Figure 1 is the test curve of the spatial straightness error, the straightness error along x-direction and along y-direction, of GJZ optical bench.

Error evaluation based on minimum zone condition, the straightness error along x-direction:
\[
dx_{1max} = 28.5364 \mu \text{m}
\]

The test result of the straightness error along x-direction of GJZ optical bench is less than 30\( \mu \)m/1m.

\[\text{Figure 1. The test curve of GJZ optical bench.}\]

Based on least squares sum condition, the spatial datum-line of error evaluation is given [8,9] from table 1 as following:
Assessing straightness error on the statistical assessing theory.

- spatial straightness error function
  \((\Delta x, \Delta y) = F(z)\) is shown in table 2.

- statistical straightness error

The statistical assessing parameter: \(C_v = 6.5747\), where

\[
\begin{align*}
E(x) &= 0 \\
E(y) &= 0
\end{align*}
\]

| \(z\) (mm) | 400  | 470  | 540  | 610  | 680  | 750  | 820  |
|----------|------|------|------|------|------|------|------|
| \(x\) (µm) | 1.2  | 12.1 | 20.0 | 16.6 | 12.8 | 3.5  | -5.8 (min) |
| \(y\) (µm) | -0.3 | 7.6  | -2.6 | -6.8 | -20.5| -15.8 | -8.7 |

| \(z\) (mm) | 890  | 960  | 1030 | 1100 | 1170 | 1240 | 1310 |
|----------|------|------|------|------|------|------|------|
| \(x\) (µm) | 7.3  | 15.0 | 23.1 (max) | 3.7  | 0.8  | 12.5 | 21.2 |
| \(y\) (µm) | -6.2 | 1.5  | 2.9  | 9.3  | 7.7  | -5.9 | -9.3 |

| \(z\) (mm) | 1380 | 1450 | 1520 | 1590 | 1660 | 1730 | 1800 |
|----------|------|------|------|------|------|------|------|
| \(x\) (µm) | 17.1 | 11.5 | 3.6  | -4.1 (min) | 7.1  | 14.9 | 3.8 |
| \(y\) (µm) | -15.3 | -20.2 | -7.0 | -3.0 | 4.5  | 5.1  | -3.8 |

| \(\Delta r\) | 10.5380 | 12.2758 | 9.6706 | 6.6274 | 16.2623 | 13.1896 | 16.3672 | 3.1781 | 7.8095 | 15.2799 | 14.6019 |
| \(\Delta \theta\) | 3.8374 | 13.2997 | 14.0347 | 16.5538 | 5.9189 | 12.5833 | 8.3163 | 11.0504 | 4.2495 |
| \(\Delta r\) | 14.5203 | 0.2020 | -0.3561 | -1.4134 | -2.0960 | -2.8718 | -2.4953 | 0.8248 | 0.4821 | 1.9736 |
| \(\Delta \theta\) | 1.4535 | -0.3561 | -1.4134 | -2.0960 | -2.8718 | -2.4953 | 0.8248 | 0.4821 | 1.9736 |

Spatial straightness error function on the orthogonal coordinates is transformed into that on the cylindrical coordinates on the spatial datum-line of error evaluation, which shown in table 3. Then, the relation of the polar distance \(r\) with measuring distance \(z\) and that of the polar angle \(\theta\) with measuring distance \(z\) are got and shown in figure 2 and figure 3.

The mean spacing can be deduced from table 3, figure 2 and figure 3: \[
\begin{align*}
Z_m &= 295\text{mm} \\
\theta_m &= 2\pi / 560\text{mm}
\end{align*}
\]

Based on minimum zone condition, the straightness error along x-direction of the precise straight rail and GJZ optical bench are accorded with the verifying accuracy of these manufacturers.

Apparently, these straightness errors evaluation on the statistical assessing theory showed that these straightness errors on the statistical assessing theory are rounder than that based on minimum zone condition.
5. Conclusion
From the statistical assessing theory of spatial straightness error and its applications, the conclusion can be got as following:

![Figure 2](image1.png)  ![Figure 3](image2.png)

Figure 2. the relation curve of the polar distance r with measuring distance z of GJZ optical bench.

Figure 3. the relation curve of the polar angle \( \theta \) with measuring distance z of GJZ optical bench.

All these statistical assessing parameters proposed in this paper can reveal the impact of spatial straightness error on the accessory function in different aspects more roundly and can complement the single assessing parameter of geometrical tolerance for straightness error.

The relation of the statistical assessing parameters of spatial straightness error and the assessing parameter of geometrical tolerance for straightness error can be apprehended as:

- If the believable probability is 1, the maximum of statistical straightness error is corresponding to the assessing parameter of geometrical tolerance for straightness error on the spatial datum-line of error evaluation based on least squares sum condition. Therefore, the maximum of statistical straightness error can be used as the estimate value of the assessing parameter of geometrical tolerance for straightness error, and that, the assessing parameter of geometrical tolerance for straightness error can be thought as the special case of statistical straightness error.

The assessing theory on the statistical spatial straightness tolerance and statistical spatial straightness error proposed in this paper is possible to be applied in evaluation of other error of form, orientation, location and run–out.

References

[1] ZHANGHC, HUME 1992 Tolerancing techniques: the state of the art[J] International Journal of Production research 30 2111–35

[2] Gao Yunsheng, Feng Xiong and Li Zhu 2000 Statistical tolerance and the development process of statistical evaluation of surface roughness Journal of Engineering Design 68–71

[3] Zhang Qing, Zhao Bin and Li Zhu (in Chinese)1997 The application of non-diffracting beam in a straightness error measuring system Journal of Huazhong University of Science and Technology 25 1–3

[4] Zhang Xinbao, Zhao Bin and Li Zhu 2002 Study of the tolerance of the non-diffracting beam to laser beam deflection J. Opt. A: Pure Appl. Opt. 4 78–83

[5] Feng Xiong, Gao Yunsheng and Li Zhu 2001 Statistical roundness tolerance and the parameters for assessment Journal of Engineering Design 12–16

[6] Xiong Youlun 1989 Mathematics method of precision measurement (Beijing: China Metrology Press)

[7] Fei Ye-tai 2000 Error theory and data processing (Beijing: China Machine Press)