LIMITS ON THE NON-COMMUTATIVITY SCALE

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A non-vanishing vacuum expectation value for an antisymmetric tensor field leads to the violation of Lorentz invariance, controlled by the dimension (−2) parameter, $\theta_{\mu\nu}$. We assume that the zeroth order term in $\theta$-expansion represents the Standard Model and study the effects induced by linear terms in $\theta_{\mu\nu}$. If coupling to $\theta_{\mu\nu}$ is realized in strongly interacting sector of the theory, the clock comparison experiments place the limit on the possible size of this background at the level of $1/\sqrt{\theta} > \sim 5 \times 10^{14}$ GeV. If the interaction with $\theta_{\mu\nu}$ is initially present only in the QED sector, this limit can be relaxed to $10^{11} - 10^{12}$ GeV level. The strength of these limits obviates the inferiority of collider physics with regard to experimental checks of Lorentz invariance. Limits of similar strength are expected to hold in the case of mixed non-commutativity between four-dimensional and extra-dimensional coordinates. We also show that in certain models mixed non-commutativity can be interpreted as 4d CPT-violating background.

1 Introduction

Non-commutative field theories and their string theory realizations have been a subject of intensive theoretical research over the past few years (See, e.g. [1]). They can be defined either as theories on a space in which the coordinates are self-adjoint operators satisfying $[\hat{x}^\mu, \hat{x}^\nu] = i\theta_{\mu\nu}$ and $[\theta_{\mu\nu}, \hat{x}^\rho] = 0$ or as theories on an ordinary space but with a deformed multiplication law. It is easy to see that the operator $\theta_{\mu\nu}^{-1}[\hat{x}^\nu, \hat{x}^\mu]$ defines a derivation of the product of function on this space. This observation allows one to write generic field theory actions. Translation between the two approaches is done using the Weyl symbol

$$\hat{\Phi}(\tilde{x}) = \int d^Dx \int \frac{d^Dk}{(2\pi)^D} \phi(x) e^{i k_{\mu} \tilde{x}^\mu} e^{-ik_{\mu} x^\mu} . \quad (1)$$

If $V(\hat{\Phi}(\tilde{x}))$ is an arbitrary function of fields $\hat{\Phi}(\tilde{x})$ which we will interpret as potential, the equivalent action in terms of $\phi(x)$ is

$$S = \int d^Dx \left[ \frac{1}{2} (\partial_\mu \phi)^2 + V_*(\phi) \right] \quad (2)$$

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where $V_*(\phi)$ is $V(\hat{\Phi})$ in which we replace $\hat{\Phi}$ by $\phi$ and multiplication is given by the $*$-product

$$(\phi \ast \psi)(x) = \left. e^{\frac{1}{2}\theta_{\mu\nu} \partial_\mu(x) \partial_{\nu}(y) \phi(x) \psi(y)} \right|_{x=y}$$

(3)

One interesting phenomenological question is how large $\theta_{\mu\nu}$ could be without contradicting existing experimental data. To answer this, we start with the Standard Model and we include the external background $\theta_{\mu\nu}$ via the Moyal product. We take the ($*$-modified Standard Model and expand it once in the external $\theta$ parameter. The result is that SM is extended by the series of dimension 6 operators, composed from three or more fields:

$$SM(*) = SM + \sum_i \theta_{\mu\nu} O^{(i)}_{\mu\nu}.$$  

(4)

We consider this as an effective theory with a cutoff smaller than $1/\sqrt{\theta}$. The higher the energy/momentum transfer is, the larger the effect of $O_{\mu\nu}$ will be, so one can think of obtaining stringent limits on $\theta$ by using high-energy processes. We have considered in the Z-boson decay into a pair of photons, forbidden by Lorentz invariance and Bose statistics in the SM, but allowed when $\theta_{\mu\nu} \neq 0$. We obtained a very modest limit, $1/\sqrt{\theta} > 250$ GeV. This can be slightly improved by considering $\theta$-induced corrections to other high-energy processes, but the experiments do not have enough accuracy to produce sufficiently strong bounds on $\theta_{\mu\nu}$, so we turn our attention to low energy experiments. We show that low-energy precision experiments place the limit on the possible size of $\theta$ at the level of $1/\sqrt{\theta} > 5 \times 10^{14}$ GeV.

2 Experimental limits on $\theta_{\mu\nu}$

The most notable feature of the effective Lagrangian (4) is the explicit violation of Lorentz invariance. Such a feature is experimentally excluded to a very high accuracy (see, e.g. [5]). This violation is controlled in our case by the size of $\theta_{\mu\nu}$ and its effect first shows up in dimension 6 operators. One can show that the non-commutative QED leads to the interaction of atomic angular momentum $J_i$ with the “magnetic” component $\epsilon_{ijk} \theta_{lj}$. At this point it becomes clear that similar effects will appear in the hadronic physics, when we consider the ($*$)-extended interaction of quarks and gluons. In the most interesting case, this interaction leads to the effective coupling of the nucleon spin with $\theta$,

$$\langle N|L_{QCD}(|N) = \theta - \text{independent terms} + \frac{d_0}{2} \theta_{\mu\nu} \sigma_{\mu\nu} N, \quad (5)$$
which in non-relativistic limit simply becomes $d_\theta(\theta_B \cdot \frac{\xi}{\lambda})$. The size of $d_\theta$ is now given by the third power of a characteristic hadronic scale, $d_\theta \sim \Lambda_{\text{had}}^3$. One can further discuss the issue whether $\theta_{\mu\nu}\bar{q}\sigma_{\mu\nu}q$ can be generated through loops at the quark level. In this case, one can hope to substitute two powers of $\Lambda_{\text{had}}$ by the scale of the effective cutoff, so that $d_\theta \sim (\text{loop factors})^n \Lambda_{\text{had}}^2 \Lambda_{\text{cutoff}}^n$, where $n$ is the number of loops involved. It is likely that $n = 2$. Since the value of $\Lambda_{\text{cutoff}}$ is quite uncertain and could be low, the loop level does not present serious advantage over $\Lambda_{\text{had}}^3$ and we choose to limit our discussion to the tree level.

To estimate $d_\theta$ we use a simplified version of the nucleon three-point function QCD sum rules. We compute the two point function $\Pi(p) = i \int d^4xe^{ip \cdot x} \langle 0|\eta(0)\bar{\eta}(x)|0 \rangle$ of generalized spin 1/2 nucleon interpolating currents: $\eta(x) = 2\epsilon_{abc}\left[ u^aT(x)C\gamma_5d^b(x) \right] + \beta \left[ u^aT(x)Cd^b(x) \right] \gamma_5 u^c(x)$ in the presence of the external $\theta_{\mu\nu}$-background. $\beta$ is a free parameter. Here we use $\beta = 0$ correlator that has the best overlap with the nucleon ground state. By quark-hadron duality, the result must also be represented as a sum of resonance contributions which can be created by $\eta$ current from vacuum, $\langle 0|\eta|N \rangle = \lambda u_N$. To extract the desired information about the ground state nucleon, we need to be in the intermediate regime where $-p^2$ is high enough that asymptotically free calculations make sense, while $-p^2$ should be low enough such that on the “phenomenological” side only the ground state survives and the contribution of the excited states is suppressed. We match the two sides at 1 GeV to obtain an estimate of $d_\theta$ for nucleons.

On the OPE side we can use an asymptotically free description, and thus include the $\theta_{\mu\nu}$-piece as the correction to a free massless quark propagator:

$$S(x,0) = \frac{i\gamma^\mu}{2\pi^2x^4} \frac{\gamma^\nu}{2\pi^2x^4} d^a G_{\nu\mu}^a \theta_{\mu\nu}. \quad (6)$$

It is then straightforward to compute the two-point function and express the result as the combination of a pure perturbative piece and non-perturbative condensates. Choosing $\sigma_{\mu\nu}\theta_{\mu\nu}$ Lorentz channel, performing Borel transformation and equating OPE side to the phenomenological part we obtain:

$$(\theta \cdot \sigma) \frac{1}{108\pi^2} m_0^2 \langle \bar{q}q \rangle + \mathcal{O}(1/M^2) = \lambda^2 \frac{2m_N^2 d_\theta}{M^4} e^{-\frac{m_N^2}{M^2}} (\theta \cdot \sigma) + \mathcal{O}(e^{-\frac{m_N^2}{M^2}}) \quad (7)$$

with $m_0^2 = \langle \bar{q}\sigma \cdot Gq \rangle / \langle \bar{q}q \rangle \simeq 0.8 \text{ GeV}^2$, and $(2\pi)^4 \lambda^2 \simeq 1 \text{ GeV}^6$, from which we get our estimate of $d_\theta$:

$$d_\theta \sim \frac{1}{54\pi^2} m_0^2 \langle \bar{q}q \rangle \frac{M^6}{m_N^4} e^{-\frac{m_N^2}{M^2}} \frac{1}{\lambda^2} \simeq 0.1 \text{ GeV}^3 \quad (8)$$
This estimate can be improved by calculating next order terms in OPE, including the anomalous dimensions, and fixing the size of unknown "susceptibility" condensates that would necessarily arise in the presence of $\theta_{\mu\nu}$ background.

The interaction of the form (5) creates a Zeeman-type splitting of atomic multiplets relative to the fixed vector $\theta_B$ and can be searched through the sidereal variations of the Zeeman frequencies. Thus, one can use the results of the most precise clock comparison experiments. The presence of $\theta_{\mu\nu}$-background affects primarily nuclear spin. Thus the magnetic field, measured by mercury atom, will be corrected due to the interaction of the nuclear spin with $\theta_B$ to a larger extent than the magnetic field measured by paramagnetic Cs. The absence of sidereal variations in the difference of magnetic field measured by Cs and Hg verified at the level of 100 nHz translates into the extremely tight bound: $1/\sqrt{\theta} > 5 \cdot 10^{14}$ GeV. If non-commutativity is introduced initially only in the QED sector, the interaction (6) is still generated, although suppressed by $(\alpha/\alpha_s)^2$ compared to the QCD case. This lowers the limit to the level of $\theta < (10^{11} - 10^{12}$ GeV)$^{-2}$ which is still beyond the reach of any conceivable high-energy experiment.

3 Breaking CPT by mixed non-commutativity

We now discuss the phenomenological consequences of mixed non-commutativity $\theta_{\mu M}$, where $\mu$ is a normal 4-d index and $M$ is along extra dimensions. An intriguing possible consequence of this interaction is the breaking of the four-dimensional CPT symmetry. To understand the origin of this breaking, one can consider the effective loop-generated interaction between a fermion $\psi$ and 5d non-commutative background. $\theta_{MN}\bar{\psi}\sigma_{MN}\psi$. Specifying $M$ and $N$ to $\mu$ and 5, we get $\theta_{\mu 5}\bar{\psi}\gamma_5\psi$ which is explicitly CPT-odd in four dimensions.

In order to get a quantitative estimate of the magnitude of the effective CPT violation we consider a 5-dimensional non-commutative theory with all fields propagating in all 5 dimensions, compactify the fifth dimension on a circle and integrate out the heavy Kaluza-Klein modes to obtain the low energy effective action for the zero-modes. Computing one-loop diagrams in the 5-d non-commutative U(1) theory we obtain CPT non-invariant $dim = 6$ operators in four dimensions:

$$\mathcal{L} = \frac{5e^2\epsilon}{48\pi^2} \left[ 1 + \frac{9}{10} \frac{e_i}{e} \right] \theta_{\mu 5}(\bar{\psi}_i\sigma^{\alpha\beta}\gamma_5\psi_i)\partial_\mu F_{\alpha\beta}, \tag{9}$$

which is independent of the mass of the heavy KK mode. Phenomenological consequences of this interaction are obtained by specializing (8) to the case of the light quarks. The relevant matrix elements over nucleon and nuclear states
turn out to be rather simple in this case. We refer a reader to our original paper for more details. In the end, the effective operator translates to the interaction of the form \( I_i \theta_{i5} \), where \( I \) is the nuclear spin. Using again the results of the two most sensitive experiments, we deduce the level of sensitivity to the presence of mixed non-commutativity \( |\theta_{i5}| \lesssim (5 \cdot 10^{11} \text{GeV})^{-2} \). It should be mentioned here that CPT-violating terms can be avoided in orbifold models or in models where fermions are localized to a 3-dimensional domain wall.

4 Conclusions

Extreme precision of clock comparison experiments impose severe restrictions on the possible scale of non-commutativity. Non-commutative QCD requires \( 1/\sqrt{\theta_{ij}} \gtrsim 5 \times 10^{14} \text{GeV} \), non-commutative QED implies \( 1/\sqrt{\theta_{ij}} \gtrsim 10^{11} - 10^{12} \text{GeV} \) while the limits on mixed non-commutativity are \( 1/\sqrt{\theta_{\mu5}} \gtrsim 5 \times 10^{11} \text{GeV} \).

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