The Effective Magnetic Field Decay of Radio Pulsars: Insights from the Statistical Properties of Their Spin Frequency’s Second Derivatives

Yi Xie1,2  and Shuang-Nan Zhang2,3,4

1 School of Science, Jimei University, Xiamen 361021, Fujian Province, People’s Republic of China; xieyi@jmu.edu.cn
2 National Astronomical Observatories, Chinese Academy Of Sciences, Beijing 100012, People’s Republic of China; zhangsn@ihep.ac.cn
3 Key Laboratory for Particle Astrophysics, Institute of High Energy Physics, Beijing 100049, People’s Republic of China
4 Department of Modern Physics, College of Physics, University of Chinese Academy of Sciences, Beijing 100049, People’s Republic of China

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Abstract

We present a new method to investigate the effective magnetic field decay of isolated neutron stars, from the analysis of the long-term timing data of a large sample of radio pulsars. There are some differences between the distributions of frequency’s second derivatives of the pulsar spins with different effective field decay timescales. Kolmogorov–Smirnov tests are performed to reexamine the consistency of distributions of the simulated and reported data for a series of values of decay timescales. We show that the timescale of the effective field decay exceeds ∼5 Myr for pulsars with spindown age $\tau_c < 10^7$ yr or ∼100 Myr for pulsars with $10^7 < \tau_c < 10^8$ yr in the sample. The result does not depend on any specific theories of the field evolution, the inclination decay, or the variation in the moment of inertia. It is also found that the extent of the closed-line region of the magnetic field is close to the light cylinder $r_c$, i.e., the corotating radius $r_c \approx r_c$ is a good approximation for the observed pulsar population.

Key words: pulsars: general – stars: magnetic field – stars: neutron

1. Introduction

The magnetic field is probably one of the most important physical quantities affecting the evolution and the observational behaviors of radio pulsars. The field strength determines the loss rate of rotational energy, the luminosity of pulses, and thus the spin evolution and observability of a pulsar. The primary method used to determine the magnetic field is by measuring each pulsar’s spin parameters, which have actually provided a surprising amount of information on the nature of the pulsed radio sources. As such, the knowledge of the spin parameters is particularly valuable in elucidating whether magnetic field decay occurs in isolated neutron stars. Many impressive studies have been done on this issue during the past few decades (e.g., Ostriker & Gunn 1969; Gunn & Ostriker 1970; Lyne et al. 1982, 1985; Stollman 1987; Narayan & Ostriker 1990; Bhattacharya et al. 1992; Harrison et al. 1993; Han 1997; Lorimer et al. 1997; Cordes & Chernoff 1998; Tauris & Manchester 1998; Tauris & Konar 2001; Gonthier et al. 2002; Faucher-Giguère & Kaspi 2006; Igoshev & Popov 2015; Johnston & Karastergiou 2017). Unfortunately, the conclusions of pulsar population investigations have often been conflicting (see, e.g., Harding & Lai 2006; Ridley & Lorimer 2010; Lorimer 2011 for reviews). The lack of conclusive evidence on magnetic field decay is mainly due to the fact that the true age of a pulsar is unavailable, and the characteristic (spindown) age $\tau_c$ is normally significantly different from its true age (e.g., Zhang & Xie 2011). This makes the evolution of the magnetic field remaining as one of the most important unresolved issues of the physics of neutron stars.

Hobbs et al. (2010) studied the timing noise in the residuals of 366 pulsars that had been regularly observed for 10–36 yr, and showed that the magnitude of the frequency’s second derivatives of the pulsar spins, i.e., $|\ddot{\nu}|$, is much larger than that caused by magnetic braking of the neutron star, and the numbers of negative and positive $\nu$ are almost equal in the sample. It had also been noticed that the distributions between the positive and negative signs in the $\nu$–$\tau_c$ diagram show an approximate symmetry.

In this paper we present a new method to investigate the magnetic field decay of radio pulsars, from the analysis of the long-term timing data. We find that the decay timescale can be constrained by measuring the difference between the distributions of $\nu$ with different decay timescales in the $\nu$–$\tau_c$ diagram. This method can effectively avoid the “true age problem.” The method and its validity are described in Section 2, the revealed restrictions on the timescale of effective magnetic field decay are shown in Section 3, and the magnetospheric effects are also tested in the section. The physical implications of the decay timescales are discussed in Section 4, and the results are summarized and discussed in Section 5.

2. The Method

2.1. The Spindown Models

A basic model for a pulsar’s spindown is the magnetic dipole radiation model (Pacini 1968; Ostriker & Gunn 1969; Gunn & Ostriker 1970; Ruderman & Sutherland 1975). The standard dipole (SD) radiation model assumes that the pure magnetic dipole radiation in vacuum as the braking mechanism, i.e.,

$$\dot{\nu} = -K\nu^3, \tag{1}$$

where $\nu$ and $\dot{\nu}$ are the spin frequency and its first derivative, respectively. The parameter $K = 8\pi^2R^6B^2\sin^2\theta/3c^3I$ is a constant, $B = 3.2 \times 10^{19}\sqrt{-\nu/\nu^3}\sin\theta$ is the effective dipole magnetic field at equator, and $R (\approx 10^6\text{ cm})$ and $I (\approx 10^{45}\text{ g cm}^2)$ are the radius and moment of inertia, respectively. Xu & Qiao (2001) improved the model by incorporating the effect of a longitudinal current outflow (relativistic particle wind) powered by a unipolar generator into
the rotation energy-loss rate. This effect was confirmed by a few intermittent pulsars, whose rotation slows down faster when the pulsar is on than when it is off (e.g., Kramer et al. 2006). Further, Spitkovsky (2006) developed a numerical method for evolving time-dependent force-free MHD equations and applied it to solving a dynamic pulsar magnetosphere. Similarly, the dynamic magnetosphere (DM) model also included both the dipole radiation and the unipolar generator mechanisms to contribute the total braking torque of an oblique pulsar, and they found a formula that gives a very good fit to the oblique spindown for all inclinations. The formula has the same form with Equation (1) but with a different parameter $K$,

$$ K = \frac{4 \pi^2 B^2 R^6}{c^3 I} (1 + \sin^2 \theta). $$

The inferred effective magnetic field at the magnetic equator is then $B = 2.6 \times 10^{19} \sqrt{\nu} / (1 + \sin^2 \theta)^{1/2}$, which can be up to 1.7 times smaller than the estimate from the SD formula.

2.2. The Frequency’s Second Derivatives and the Revised Spindown Model

Both the SD and the DM models predict that the frequency second derivative of a pulsar spin $\nu_{SD} = \nu_{DM} = 3 \nu^2 / \nu > 0$. However, it is widely known that the observed $\nu$ for the majority of pulsars cannot be explained by these models with a constant field strength $B$. Particularly, the recent large-sample analysis showed (Hobbs et al. 2010; Zhang & Xie 2012a, 2012b) that $|\nu| > \nu_{SD}$, $\nu^+ (\tau_C) \approx -\nu^- (\tau_C)$, and $N (\nu^+) \approx N (\nu^-)$, where $N$ indicates the total number, the superscripts “+” and “−” indicate positive and negative signs of $\nu$, and $\tau_C = -\nu / 2 \nu$ is the characteristic age of a pulsar. All the pulsars in the sample are shown in the $|\nu| - \tau_C$ diagram in the left panel of Figure 1, in which 193 pulsars have $\nu > 0$ and the remaining 173 pulsars have $\nu < 0$.

Following Blandford & Romani (1988), we reformulate the braking law of a pulsar as $\dot{\nu} = -K(t)\nu^\lambda$, which assumes that the SD or DM model is responsible for the instantaneous spindown of a pulsar, but $K(t)$ is time-dependent. Generically and without depending upon any specific model for the time dependence, $K(t)$ can be decomposed into a long-term monotonic term plus a short-term perturbation to the monotonic term. Again assuming $R$ and $I$ are constants, the decomposition is equivalent to $B(t) = B_M(t) + B_O(t)$, where $B_M(t)$ is the long-term monotonic component and $B_O(t)$ is the short-term perturbation around $B_M$. The quasi-periodic oscillation structures, which are widespread in pulsar timing behaviors (Hobbs et al. 2004, 2010), can be phenomenologically described by $B_O(t)$ (Zhang & Xie 2013; Xie et al. 2015). One possible source of the perturbation may be their magnetospheric activities, which are known to influence their timing behaviors significantly (Lyne et al. 2010). Then after some simple algebra, we get

$$ \dot{\nu} = 3\nu^2 / \nu + 2\nu \dot{B}_M / B_M + 2\nu \dot{B}_O / B_M = \nu_{SD} + \nu_M + \nu_O. $$

where $\nu_M = 2\nu \dot{B}_M / B_M$ and $\nu_M = 2\nu \dot{B}_O / B_M$. For relatively old pulsars without significant glitch activities, $\nu < 0$ and $B_M \leq 0$; therefore, $\nu_O \geq 0$. Given the case of a quasi-periodic oscillation, $\nu_O$ has a positive or a negative value with almost equal chances (Zhang & Xie 2012a, 2012b). Observationally, since generically and statistically $\nu^+ (\tau_C) \approx -\nu^- (\tau_C)$ and $N (\nu^+) \approx N (\nu^-)$ for a large number of pulsars, clearly $\nu_O$ in Equation (3) dominates the observed statistical properties of $\nu$. On the other hand, $\nu_{SD} > 0$ and $\nu_M > 0$ will cause some differences on the distributions of $\nu$ with different decay timescales, and some asymmetry between the observed $\nu^+$ and $\nu^-$. These effects might in turn provide clues of long-term magnetic field decay of pulsars, since $\nu_{SD}$ can be calculated from the observed $\nu$ and $\nu_O$.

3. Simulations and Tests

3.1. Monte Carlo Simulations

We assume that the magnetic fields of pulsars in the sample have a typical decay timescale. Defining the timescale as
\[ \tau_B \equiv -B_M/B_{\dot{M}}, \] we have \( \dot{v}_M = -2\dot{v}/\tau_B \) and
\[ \dot{v} = \dot{v}_O + \dot{v}_{SD} - 2\dot{v}/\tau_B. \] (4)
We can simulate the distributions of \( \dot{v} \) with different \( \tau_B \), as described below. Our strategy is then to search for the typical \( \tau_B \) that can maximize the \( p \)-value of the Kolmogorov–Smirnov test against the hypothesis that the reported distribution is the same as the simulated distribution in the \( |\dot{v}|-\tau_C \) diagram for pulsars in the sample.

We construct a phenomenological model for the dipole magnetic field evolution of pulsars with a long-term decay modulated by short-term oscillations,
\[ B(t) = B_0(t) \left( 1 + \sum k \sin \left( \phi + 2\pi \frac{t}{T} \right) \right), \] (5)
where \( t \) is the pulsar’s age, and \( k, \phi, T \) are the amplitude, phase, and period of the oscillation, respectively. \( B_0(t) = B_0 \exp(-t/\tau_B) \), in which \( B_0 \) is the field strength at the age \( t_0 \). Substituting Equation (5) into (1), we get the differential equation describing the spin frequency evolution of a pulsar as follows:
\[ \dot{\nu} = -AB(t)^2 \nu^3, \] (6)
in which \( A = \frac{8\pi^6 \sin^2 \theta}{3R^2} \) is a constant and \( R \approx 10^{13} \text{ cm} \), \( I \approx (10^{45} \text{ g cm}^2) \), and \( \theta \approx (\pi/2) \) are the radius, moment of inertia, and angle of magnetic inclination of the neutron star, respectively. The constant \( A \) and \( B(t) \) are actually inseparable in Equation (6), which means that the decay timescales of the effective magnetic fields can be attributed not only to the magnetic field evolution, by also the changes of the inclination angle, or the small changes in the moment of inertia.

In order to model the \( \nu \) distributions in the \( \tau_C-|\dot{\nu}| \) diagram, we first obtain \( \nu(t) \) by integrating the pulsar spindown law described as Equation (6), and the phase
\[ \Phi(t) = \int \nu(t') dt'. \] (7)
Then, these observable quantities, \( \nu, \dot{\nu}, \) and \( \ddot{\nu} \) can be obtained by fitting the phases to the third order of its Taylor expansion over a time span \( T_s \),
\[ \Phi(t) = \Phi_0 + \nu(t - t_0) + \frac{1}{2} \nu(t - t_0)^2 + \frac{1}{6} \nu(t - t_0)^3. \] (8)
We thus get \( \nu, \dot{\nu}, \) and \( \ddot{\nu} \) from fitting to Equation (8), with a certain time interval of phases \( \Delta T_{\text{int}} = 10^6 \) s.

We assume that \( k = 10^{-4.6} \sim 10^{-1.9} \) and \( T \) follows a uniform random distribution in the range from 0.1 to 10 yr, and the reasons will be shown in the next subsection. It is also assumed that the sample of the phase \( \phi \) of the field oscillations is uniformly distributed in the range from 0 to 2\( \pi \). Drawing randomly a data set \{\( \nu, \dot{\nu}, T_s \)\} from the reported sample space (i.e., from Table I of Hobbs et al. 2010), and calculating a corresponding start time \( t_0 \), we can obtain a rotation phase set \{\( \Phi(t_i) \)\} using Equation (7). Then the values of \( \nu, \dot{\nu}, \) and \( \ddot{\nu} \) can be obtained by fitting \{\( \Phi(t_i) \)\} to Equation (8). Hence, one has each \{\( \tau_C, |\dot{\nu}| \)\}. Repeat this procedure for \( N \) times, we will have \( N = (283) \) data points in the \( |\dot{\nu}|-\tau_C \) diagram. We exclude all the pulsars that have glitch records from the sample, since a small variation in the moment of inertial due to glitches may have an impact on the overall spindown evolution, for instance, a change on the amount of superfluid content of the star may impact the braking index (i.e., Ho & Andersson 2012), and actually the timing noise of younger pulsars can be mainly attributed to glitch recovery (Hobbs et al. 2010). Meanwhile, millisecond pulsars (\( \tau_C \gtrsim 10^5 \) yr or \( \dot{\nu} > 100 \) s\(^{-1} \)) are also excluded from the sample, since the characteristic age of millisecond pulsars is highly deceptive, and the millisecond period in these systems reflects the recycling spinup mechanism rather than secular spindown evolution.

As examples, we show the simulated \( \ddot{\nu} \) distribution with \( \tau_B = 10^6 \) yr in the left panel of Figure 1, the simulated distribution without field decay (\( \tau_B = 10^{15} \) yr is taken, which is longer than the age of the universe) in the right panel of Figure 1, and the reported \( \ddot{\nu} \) distribution in both panels. In the left panel, one can see that the simulated data are much more sparse in the lower part than the reported data, especially inside the triangular area surrounded by dashed lines. In the right panel, the two distributions are completely consistent.

Some of the simulated pulsars in the bottom right part of the sample may have turned off as a radio pulsar and have crossed the death line in the \( P-P^* \) diagram. In most models, the period \( P \) at turnover depends upon the structure and the magnitude of the neutron star’s surface magnetic field (Chen & Ruderman 1993; Zhang et al. 2000). Assuming a multipole magnetic field configuration, i.e., the polar cap area is similar to that of the pure dipole field, but with very curved field lines at the surface, and the radius of the curvature \( r_c \sim R = 10^{6} \) cm, and this field configuration will be discussed in Section 4. The theoretical death line of the pulsar is then (Chen & Ruderman 1993)
\[ 4 \log B_p - 6.5 \log P = 45.7. \] (9)
After taking into account this observational effect, about 30 simulated pulsars have been excluded from the simulated sample in the right panel of Figure 1.

The analysis on the scatter of \( \dot{\nu} \) versus \( \tau_C \) in Figure 1 is potentially misleading, since the two quantities may not be entirely independent. We carry out a similar analysis as Lyne et al. (1975) to assure the reader that inherent correlations could not be found by plotting random pairings of pulsars, i.e., the values of \( \dot{\nu}, \dot{\nu}, \) and \( \ddot{\nu} \) are randomly taken from different pulsars in the sample. In this case, there is no clear correlations between \( \ddot{\nu} \) and \( \tau_C \).

### 3.2. Kolmogorov–Smirnov Tests
We perform a two-dimensional Kolmogorov–Smirnov (2DKS) test to reexamine the consistency of distributions of the simulated and reported \( \ddot{\nu} \) for a series values of \( \tau_B \). The 2DKS package\(^5\) (Peacock 1983) is adopted for the test. Our strategy is then to search for a typical \( \tau_B \) that can maximize the \( p \)-value of the 2DKS test against the hypothesis that the two distributions are consistent for the pulsars in the sample. We let \( \tau_B \) vary from \( 10^5 \) to \( 10^9 \) yr. The returned \( p \)-values are shown with solid lines in the top panel of Figure 2. The \( p \)-value 0.1 is considered as the threshold level with probability 95%. From the panel, one can see that the decay timescale can be well constrained with \( p \)-values larger than \( \sim 0.1 \). It is shown apparently that the decay timescale \( \tau_B \gtrsim 5 \times 10^5 \) yr. In the bottom panel, we show \( \tau_B^{-1} \) and \( \tau_B^{-1} \) as functions of \( \tau_B \), giving a constraint \( \tau_B \gtrsim 10^5 \) yr for 95% probability, which is
\[ \text{http://www.downloadplex.com/Scripts/Matlab/Development-Tools/two-sample-two-diensional-kolmogorov-smirnov-test_432625.html} \]
much looser than the constraint from 2DKS tests. It should be noticed that the 2DKS test has only an approximate and stochastic p-value in each simulation, thus very intensive tests (with $\log \Delta \tau_B = 0.01$) were performed, as shown in the top panel. We also checked the validity of 2DKS tests with the one-dimensional Kolmogorov–Smirnov (1DKS) test. A very similar result is obtained with 1DKS, but $\tau_B \gtrsim 10^6$ yr.

We also performed the 2DKS test only for young pulsars with $\tau_c < 10^7$ yr; the returned values also give $\tau_B \gtrsim 5 \times 10^6$ yr, as shown in the top panel of Figure 3. However, for the sample of middle-age pulsars with $10^7 < \tau_c < 10^9$ yr, the returned values give $\tau_B \gtrsim 10^{10}$ yr, as shown in the bottom panel of Figure 3. In addition, 23 millisecond recycled pulsars are excluded from our samples, and the number is too small to be tested independently with 2DKS.

It is very important to explore the parameter space of the simulations, especially regarding the dependence on the oscillation period $T$ and magnitude $k$. However, it is found that the method cannot place effective restrictions on $T$. For instance, there is no significant difference between the returned $p$-values for $T$ distributing uniformly from 5 to 25 yr or from 0.1 to 100 yr. There are extreme examples of magnetars with torque variations that could be interpreted as a change in the spindown magnetic field by a generous fraction within months (i.e., Archibald et al. 2015 in 1E 1048.1-5937). Thus, for a wider coverage the latter (0.1–100 yr) is chosen for all the simulations in this paper. For the magnitude $k$, the returned $p$-values for the lower limit and the upper limit ($k = 10^{-6}$) of the power index are shown in panel (a) of Figure 4. It can easily be seen that $3.9 \lesssim k_1 \lesssim 4.6$ and $1.9 \lesssim k_2 \lesssim 2.5$. Therefore, $k = 10^{-6}$ is taken for the wider coverage. As an example, the young pulsar PSR B1828-11 shows correlated shape and spindown changes (Stairs et al. 2019), and the observed 0.7% variation in $P$ implies a fractional change of similar magnitude in the oscillation magnitude $k \simeq 3.5 \times 10^{-3}$, which falls well within the limits.

3.3. The Magnetospheric Effects

Aside from the two prevailing models (SD and DM model), Contopoulos & Spitkovsky (2006) proposed a spindown formula that takes into account the magnetospheric particle acceleration gaps and the misalignment of magnetic and rotation axes, as well as the mechanism of the magnetic field reconnection around the equatorial extent $r_c$ of the closed-line region. This formula can be simply expressed as

$$\Omega = \frac{B^2 R^4 \Omega}{4 c \tau c^2} \left[ \sin^2 \theta + \left(1 - \frac{\Omega_{\text{death}}}{\Omega}\right) \cos^2 \theta \right],$$

where the corotating region follows the light cylinder as $r_c = r_{lc}(\Omega/\Omega_0)^{\alpha}$, $\Omega_0$ is the value of the angular velocity $\Omega$ at pulsar birth, and the parameter $\alpha$ ($0 < \alpha < 1$) depends on the efficiency of the reconnection around $r_c$. If the reconnection is very efficient, $r_c \approx r_{lc}$, i.e., $\alpha = 0$. However, if the reconnection is very inefficient, then the closed-line region cannot grow; thus, $r_c \approx \text{const.}$ (Contopoulos & Spitkovsky 2006). $\Omega_{\text{death}}(=2\pi/P_{\text{death}})$ describes a pulsar is “death,” i.e., the cessation of pulsar emission. $P_{\text{death}}$ can be written as

$$P_{\text{death}} = 8.1^{1/2-\alpha} \left( \frac{V_{\text{gap}}}{10^{12} \text{ V}} \right)^{-1/2-\alpha} \left( \frac{P_0}{1 \text{ s}} \right)^{-1/2-\alpha},$$

in which $V_{\text{gap}}$ is the gap potential.

We perform Monte Carlo simulations to confront the model with observations. The main procedures of the simulations are the same as in the previous subsections. We assume a lognormal distribution of polar magnetic fields with mean value $\langle \log(B/G) \rangle$ and standard derivation $\sigma_{\log B}$. Following Contopoulos & Spitkovsky (2006), $V_{\text{gap}} = 10^{12}$ V is taken, and the initial period $P_0$ is uniformly distributed between 10 ms and 0.2 s. We assume the distribution of inclination angle $\theta$ is also uniform from 0 to $\pi/2$. The returned $p$-values for

Figure 2. Top panel: the $p$-values of 2DKS for $|p|-\tau$ distributions from simulated data. The ranges of $p$-values larger than 0.1 are identified by the transverse line. The boundary of $p$-value $\geq 0.1$ indicates $\tau_B \gtrsim 5 \times 10^6$ yr. Bottom panel: the pulsar number ratios of the positive and negative simulated data. The ranges of $|\tau|$, and the observed 0.7% variation in $P$ implies a fractional change of similar magnitude in the oscillation magnitude $k \simeq 3.5 \times 10^{-3}$, which falls well within the limits.

Figure 3. Top panel: the $p$-values of 2DKS for $|p|-\tau$ distributions from young pulsars with $\tau_c < 10^7$ yr. Bottom panel: the $p$-values of 2DKS for $|p|-\tau$ distributions from young pulsars with $10^7 < \tau_c < 10^9$ yr.
of the upper limit \(k_1\) and the lower limit \(k_2\) of \(-\log k\). Panel (b): the \(p\)-values for various values of \((\log(B/G))\). Panel (c): the \(p\)-values for various values of \(\sigma_{\log B}\). Panel (d): the \(p\)-values for various values of \(\alpha\).

\[ \langle \log(B/G) \rangle, \sigma_{\log B}, \alpha \] are shown in panels (b)–(d) of Figure 4, respectively. The results show that \(12.05 \lesssim \langle \log(B/G) \rangle \lesssim 12.35, \sigma_{\log B} \lesssim 0.50, \) and \(\alpha \lesssim 0.11\).

The result of no floor for \(\sigma_{\log B}\) implies that the distribution width of \(\nu\) is determined by the oscillation magnitude \(k\), rather than by the distribution width of the magnetic field \(B\). The parameter \(\alpha \lesssim 0.11\) means that our method cannot prove or rule out the spindown law, but suggests that the reconnection of the north–south poloidal magnetic field around \(r_c\) is very efficient, and the extent of the closed-line region is close to the light cylinder. We thus propose that \(\alpha \sim 0\), or \(r_c \approx r_c\), is a good approximation for the observed pulsar population.

4. Physical Implications

The time-dependent behaviors of \(B(t)\), and thus the decay timescales of the effective magnetic fields, can be attributed not only to the magnetic field evolution, by also the changes of the inclination angle, or the small changes in the moment of inertia. However, since these tests only prescribe lower limits on the evolution timescales, the results are valid for all the three mechanisms.

Magnetic field are crucial for neutron stars’ activities. Understanding the long-term evolution of neutron stars’ magnetic fields might be key to unifying the observational diversity of isolated neutron stars (Viganò et al. 2013). The magnetic field in slow-rotating ultra-magnetized neutron stars, so-called magnetars (AXPs and SGRs), is believed to be decay on timescales of \(10^3–10^5\) yr (Thompson & Duncan 1996), since their rotational energy is not sufficient to power the observed emission. The isolated X-ray pulsars with spin periods longer than 12 s are still rarely observed. However, they are not subject to physical limits to the emission mechanism nor observational biases against longer periods. This puzzle could be well understood if their magnetic field is dissipated by one or even two orders of magnitude for 1 Myr, which is probably due to a highly resistive layer in the innermost part of the crust of neutron stars (Pons et al. 2013). For normal radio pulsars, some population synthesis studies suggest that \(\tau_B\) must be longer than 10 Myr (Hartman et al. 1997; Regimbau & de Freitas Pacheco 2001). However, there are also some other studies that claimed short decay timescales, i.e., \(0.1 \lesssim \tau_B \lesssim 10\) Myr (Lyne et al. 1985; Narayan & Ostriker 1990; Gonthier et al. 2004; Popov et al. 2010; Gullón et al. 2014; Igoshev & Popov 2014). The present method imposes a piecewise restriction on the decay timescale, i.e., \(\tau_B \gtrsim 5\) Myr for young pulsars (\(\tau_x < 10^7\) yr), and \(\tau_B \gtrsim 100\) Myr for middle-age pulsars (\(10^7 < \tau_x < 10^9\) yr), and may contribute to our understanding of actual mechanisms of the field decay and magnetic configurations in neutron stars.

Three avenues for the magnetic field decay in isolated neutron stars have been intensively studied, i.e., ohmic decay, ambipolar
diffusion, and Hall drift (e.g., Goldreich & Reisenegger 1992; Urpin & Shalabykov 1999; Geppert & Rheinhardt 2002; Hollerbach & Rüdiger 2002; Rheinhardt & Geppert 2002; Cumming et al. 2004; Pons & Geppert 2007, 2010; Kojima & Kisak 2012; Pons et al. 2012; Geppert et al. 2013; Gourgouliatos & Cumming 2014a). Depending on the strength of the magnetic fields, each of these processes may dominate the evolution. Ohmic decay occurs in both the fluid core and solid crust. It is inversely proportional to the electric conductivity and independent of the field strength. The Hall drift is nondissipative and thus cannot be a direct cause of magnetic field decay. However, it can enhance the rate of ohmic dissipation, since only electrons are mobile in the solid crust, and their Hall angle is large. This causes the evolution of magnetic fields to resemble that of vorticity, and then the fields undergo a turbulent cascade terminated by ohmic dissipation at small scales (Goldreich & Reisenegger 1992; Cumming et al. 2004). Compared with the Hall drift, the timescale of the ambipolar diffusion is much longer for normal pulsars; however, it may be very important for magnetars (Thompson & Duncan 1996). For a typical density and conductivity profile in the crustal region (e.g., Pons & Geppert 2007; Gourgouliatos & Cumming 2014b), the ohmic timescale is

$$\tau_{\text{Ohm}} \sim \frac{4 \pi \sigma L^2}{c^2} = 13.5 \left(\frac{\sigma}{3 \times 10^{24} \text{s}^{-1}}\right) \left(\frac{L}{\text{km}}\right)^2 \text{Myr},$$

(12)

where $\sigma$ is the electric conductivity, and $L$ is the characteristic length scale of magnetic field in the crust. For the Hall timescale, one reads

$$\tau_{\text{Hall}} \sim \frac{4 \pi e L n_e}{c B} = 16.8 \left(\frac{e}{B_{13}}\right) \left(\frac{n_e}{2.5 \times 10^{36} \text{cm}^{-3}}\right) \left(\frac{L}{\text{km}}\right)^2 \text{Myr},$$

(13)

in which $B_{13} \equiv B/(10^{13} \text{G})$. The combined effect, i.e., Hall cascade, could cause a fast field evolution on a timescale of the order of 10 Myr (Graber et al. 2015).

All these contradictory facts can be well understood by the natural assumption that the magnetic field is maintained by two current systems. The large-scale dipolar field that is responsible for the pulsar spindown is supported by long-living currents in the superconducting core. Currents in the crust support the small-scale multipolar fields that decay on timescales that are comparable to the pulsar spindown ages (Pons & Geppert 2007). The two current systems and the corresponding field configurations are particularly demonstrated in the burst activities of a low dipole magnetic field magnetar, SGR 0418+5729, which is expected to harbor a sufficiently intense internal toroidal component (Rea et al. 2010). The present result for middle-age pulsars, i.e., $\tau_B \gtrsim 100$ Myr, suggests that the dipole component that anchored in the crust are relatively low, and thus its decay has no observable influence on the spin frequency’s second derivatives of pulsars in the sample. In addition, the core-anchored field could be expelled and subsequently dissipated in the crust, and our result also implies that the timescale exceeds 5 Myr. This may be helpful to understand the poorly known physics at the crust–core boundary.

Our results are also suitable for changes of the inclination angle, which could be either due to rotation-magnetic axis alignment or three-dimensional magnetic field evolution (Philippov et al. 2014; Gourgouliatos & Hollerbach 2018). Using polarization data for a large number of isolated pulsars, Tauris & Manchester (1998) found that the magnetic beam axis aligned with the spin axis on a timescale of $\sim 10$ Myr. With new data, Young et al. (2010) found a shorter alignment timescale of $\sim 1$ Myr. Theoretically, the electromagnetic torque that brakes the rotation of a pulsar also tends to align the magnetic axis with the rotation axis (Davis & Goldstein 1970; Goldreich 1970). The electromagnetic alignment timescale is related to the spindown age as (Lander & Jones 2018)

$$\tau_A \equiv \frac{\sin \theta}{\frac{d}{dt} \sin \theta} = \frac{2 \sin^2 \theta}{\cos^2 \theta \tau_c},$$

(14)

For young or middle-age pulsars, our results imply the alignment timescale is most likely longer than $\sim 5$ or $\sim 100$ Myr, which is roughly consistent with the relation.

A small variation in the moment of inertia may have an impact on the overall spindown evolution, for instance, a decrease in the effective moment of inertia due to an increase on the amount of superfluid content as the star cools through neutrino emission may impact the braking index (Ho & Andersson 2012). However, most of the stars are typically young and glitching pulsars, which have been excluded from our sample. Our results imply that the populations without glitch record shows no long-term variation in the moment of inertia with a timescale shorter than 5 Myr.

5. Summary and Discussion

The perturbation from the long-term dipole magnetic field decay will produce some differences on the distributions for the second derivatives of pulsars’ spin frequency with different decay timescales. This in turn provides a new method to investigate the magnetic field decay of radio pulsars, which does not depend on any specific theories of field evolution or inclination decay. We made use of the published large-sample timing data of radio pulsars to find evidence of their magnetic field decay with 2DKS tests. The method imposes a piecewise restriction on the decay timescale, i.e., $\tau_B \gtrsim 5$ Myr for young pulsars with $\tau_c < 10^4$ yr, and $\tau_B \gtrsim 100$ Myr for middle-age pulsars with $10^4 < \tau_c < 10^5$ yr. It is also proposed that the corotating radius $r_c \approx r_k$ is a good approximation for the observed pulsar population. Though pulsars with major glitches have been excluded from the data, tiny glitch activities and other types of timing irregularities may still have some influence on the observed $\dot{\nu}$, which may cause a small deviation.

We expect to gain much deeper understanding of pulsars from future larger samples of radio pulsars with higher-precision data on $\dot{\nu}$, to be brought by China’s soon-to-be operating Five-hundred-meter Aperture Spherical radio Telescope (FAST) and the future Square Kilometre Array (SKA).

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ORCID iDs
Yi Xie @ https://orcid.org/0000-0003-2259-3655
Shuang-Nan Zhang @ https://orcid.org/0000-0001-5586-1017

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