Accurate Scallop Evaluation Method Considering Kinematics of Five-axis Milling Machine for Ball-end Mill

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Abstract. A new algorithm to evaluate the scallops left between consecutive tool tracks after five-axis machining of a complex-shaped part surface has been proposed. The algorithm has been developed for the ball-nose cutter. The novelty of the algorithm includes a variable plane to evaluate the effective tool profile and the part surface profile, the orientation of the tool as well as non-linear kinematics of the five-axis machine. The proposed algorithm has been specifically designed for and tested on the industrial Stereo lithography (STL) format representing complex shaped synthetic five-axis parts and a model of a crown of the molar tooth. The procedure has been tested against several modifications of the conventional curvature based method and the sphere intersection method. The ground truth is generated using the solid modeling engine of Vericut 8.2. The algorithm provides a tangible accuracy increase in terms of the average and the maximum error with regard to the reference methods.

1. Introduction
The scallops between the consecutive tracks of the five axis machining tool is a critical measure of the quality of the part surface. It is a paramount importance that a tool path generation algorithm has been integrated with an accurate scallop evaluation procedure. In many cases, it is imperative for the tool path structure and topology, consequently defines the quality of the output part. The conventional iso-parametric and the iso-planar method lead to conservative path intervals to control the scallops [1-2]. Only at some selected points, the scallops are compared with the prescribed tolerance. As the result, in some areas the part surface is unnecessarily accurate, and in some other areas the scallops are actually larger than that required by specification. While, having too small scallops is usually due to the redundancy of the tool path, the large scallops usually appear due to the errors in the scallop evaluation procedures integrated into toolpath generation. One of the first scallop evaluation formulas proposed by Suresh and Yang [3] is given by

\[ h = r - \left( r^2 - a^2 \right)^{1/2} \] (1)

for a flat surface and

\[ h = s \left\{ -R + \left[ (R + sr)^3 - a^2 \right]^{3/2} \right\} - \left( r^2 - a^2 \right)^{1/2} \] (2)

for a convex \((s = 1)\) and a concave \((s = -1)\) surface, where \(r\) is the tool radius, \(R\) is the surface curvature, and \(a\) is the half distance between two adjacent tool centers. However, the first order Taylor expansion formula ‘as in equations (1) and (2)’ is suitable only when the converted parametric interval is small. Therefore, Lin and Koren [4] propose a neat evaluation formula given by
\[ h = \frac{w^2(R + sr)}{8rR}, \]  

where \( h \) is the maximum allowable scallop height, \( w \) is CC path interval and \( s = 0, -1, 1 \) respectively for flat, concave and convex surface. Their technique is based on the second order Taylor series and an error compensation. The above formula has been later used by many authors (see for instance \([1, 5-9]\)). The above methods require evaluation of the curvature between the cutter contact (CC) points and a local approximation of the part surface by a sphere. Another version of this idea is a polynomial surface such as the Bezier patch, NURBS, etc.

A prominent example of the importance of the fast and accurate scallop evaluation is the family of the iso-scallop tool path generation methods (see a comprehensive survey of iso-scallop methods in \([10]\)). However, the majority of the methods do not consider the kinematics of the machine producing non-linear trajectory between each pair of CC points. It is important that the scallop between two consecutive tool positions also changes non-linearly and depends on the particular kinematics of the machine.

Further, the above algorithms are applied at the intersection of the effective tool profile and the part surface in a certain reference plane. Intuitively, the reference plane must contain the vector connecting neighboring cutter contact (CC) points and the vectors corresponding to the tool orientations at these CC points. However, constructing a plane passing through three linearly independent vectors is not possible. Therefore, the plane is defined using certain practical assumptions. For instance, when the surface is an explicit function of the horizontal coordinates, the CNC programmers often consider planes parallel to the \( z \)-axis. However, even in this simplest case, the ball-nose cutter produces the same scallop height for totally different orientations of the tool, whereas the actual scallop could be much smaller or larger.

Another version of this model constructs the reference plane passing through the CC point and perpendicular to the feed direction \([4]\). In this case, the plane is not perpendicular to the feed direction at the second CC point and may not even contain the second CC point. Hence, the algorithm considers a midpoint and an “average” feed direction at the midpoint.

A more sophisticated version of the scallop evaluation considers a set or reference planes rotated at the prescribed point such as the CC-midpoint, applying ‘equation (3)’ at each plane and considering maximum resulting scallop. However, it is not clear how to accurately evaluate the tool profiles at these plane. At some extreme positions of the planes, the actual 3D bodies of the tool on the consecutive tracks may not even meet. Finally, \([11]\) proposes to evaluate the scallop height for the ball-nose cutter using intersection of corresponding spheres follows a vector formed by center of intersection circle and the middle point between two neighboring CCs. However, the method can be inaccurate in case of sharp curvature variations. Moreover, it has been proposed and verified on a three-axis machine and only on parametric surfaces.

2. Methodology

Our methodology is based on the direct estimation of the intersection of the spheres representing the ball-nose cutter which follows the non-linear trajectories produced by the inverse kinematic transformation of a particular five-axis machine.

2.1. Ball nose tool intersections

Consider a ball nose tool with the diameter \( d \). Consider an arbitrary tool path and two neighboring CL points \( p_1 \) and \( p_2 \). If \( D \equiv \| p_1 - p_2 \| > d \) the corresponding spheres do not intersect. Otherwise the intersection is a circle \( C'_{s} \) with the radius

\[ r' = \frac{1}{2} \left( d^2 - D^2 \right)^{1/2}. \]

Consider a parametric circle in the \( yz \)-plane given by \( C'_i(t) = \{0, r' \sin t, r' \cos t\} \). Define \( v_i = p_2 - p_1 \).

A plane \( P \) with the normal vector \( v_3 \) through an origin is given by \( (v_3, M) = 0 \), with \( M = (x, y, z) \).
Consider two arbitrary linearly independent vectors \( \mathbf{v}_1, \mathbf{v}_2 \in P \). Orthogonalizing \( \mathbf{v}_1, \mathbf{v}_2 \) and normalizing \( \mathbf{v}_3 \) produces a new orthonormal basis \( \{ \mathbf{v}_3, \mathbf{v}_1, \mathbf{v}_2 \} \) (in this order). Columns of the corresponding transformation matrix \( T \) are composed of the above basis vectors. Clearly, in the workpiece coordinates

\[
C_{12}(t) = TC_1(t) + \frac{D}{2} \mathbf{v}_3. 
\]  

The coordinate transformation is illustrated in ‘Figure 1(a)’, where \( P \) is a plane through \( p_1 \) and \( P' \) is a plane containing \( C_{12}(t) \).

2.2. Scallop estimation
The cutter contact (CC) points corresponding to the CL points \( p_1 \) and \( p_2 \) are given by

\[
p'_k = p_k - \frac{d}{2} t_k, \quad k = 1, 2, 
\]

where \( t_1 \) and \( t_2 \) are the tool orientation vectors.

Define the reference plane \( P_{12} \) by

\[
(n_c, M - p'_m) = 0, 
\]

where \( p'_m \) is the middle point between \( p'_1 \) and \( p'_2 \), \( n_c = n_m \times v_{CL} \), \( v_{CL} = \overrightarrow{p_1 p_2} \) and \( n_m \) is a vector approximating the normal to the surface at \( p'_m \). For instance, \( n_m = (n_1 + n_2) / 2 \), where \( n_1, n_2 \) are the normal vectors at \( p'_1 \) and \( p'_2 \) ‘Figure 1b’.

In order to find the intersection points \( A \) and \( B \) between \( C_{12} \) and \( P_{12} \), we use a built-in function ‘solve’ of MATLAB. The function finds two roots \( s_1 \) and \( s_2 \) so that \( A = C_{12}(s_1), \quad B = C_{12}(s_2) \).

Since \( \overrightarrow{AB} \in C_{12} \) and \( \overrightarrow{AB} \in P_{12} \), the scallop \( h \) is defined by the distance between the point \( B \) and the part surface along the vector \( \overrightarrow{AB} \) ‘see Figure I(b)’. Furthermore, we consider not only fixed CL points \( p_1 \) and \( p_2 \) but all spheres centered at the trajectories \( p_1(t), p_2(s) \) as long as \( \| p_1(t) - p_2(s) \| \leq d \). ‘Figure 1(c)’. In order to evaluate \( p_1(t), p_2(s) \), the inverse kinematic transformation of a five-axis machine is employed. The experimental machine is the vertical five-axis machining center Haas VF-2TR [7]. The machine configuration and the scheme of the inverse kinematics are illustrated in Figure 2.

**Figure 1.** Ball nose tool intersections and the scallop estimation.
Numerical experiments and testing against Vericut-“ground truth”

The proposed method called Spheres/Kinematics (SK) has been tested using a synthetic parametric surface, a synthetic STL surface and an STL model of the molar tooth crown. The reference methods are the curvature based method ‘equation (3)’ and a method based on intersection of the spheres [11] (see the introduction). The following procedure is employed: first a conventional tool path is generated. Next, the procedure evaluates the scallops between the tool tracks using the proposed method and the two reference methods. Finally, a cutting by the virtual five-axis machining center, Haas VF-2TR is performed by Vericut 8.2. The resulting solid model of the part surface $S_1$ is compared with the actual parametric surface $S(u,v)$ or with the STL model. Surface 1 (Saddle) is given explicitly by $z = x^2 / 20 - y^2 / 20 - 31.25$ ‘Figure 3(a)’. The size of the base of the part is 50x50 mm. The surface 2 (Waves) is an STL surface depicted in ‘Figure 4(a)’. The size of the base of the surface is 50x50 mm. Surface 3 is an STL model of a molar tooth crown in ‘Figure 5(a)’. The size of the base of the surface is 60x60 mm. A conventional iso-parametric zigzag was used to cut the surface 1 in ‘Figure 3 (b)’. Surface 2 and 3 were cut using iso-planar zigzag path shown in ‘Figure 4(b)’ and ‘Figure 5(b)’. All surfaces were cut by the ball-nose tool having the diameter 4 mm. The resulting solid models produced by Vericut are shown in ‘Figures 3, 4 and 5 (c)’.

(a) Machine configuration
(b) Kinematic chain

Figure 2. Five-axis milling machine with the rotary axes on the table.

3. Numerical experiments and testing against Vericut-“ground truth”

(a) Test parametric surface
(b) Zigzag tool path
(c) Solid model by Vericut

Figure 3. Surface 1 (Saddle).
Figure 4. Surface 2 (Waves).

Figure 5. Surface 3 (Molar tooth crown).

Table 1 shows scallop evaluated by the conventional techniques including the curvature method (C-method) and sphere-based method [11] (S-method). Table 2 shows the scallop evaluated by the SK-method and Vericut. The most important are the average and the maximum values.

Next, the scallop evaluation error is defined by

\[ \varepsilon_M = \left| \frac{h_M - h_G}{h_G} \right| \times 100\% \]

(8)

where, \( h_M \) denotes the scallop obtained by a method \( M \) (\( M = C,S,SK \) i.e. the curvature method, the spheres method and the proposed sphere-kinematics method respectively) and \( h_G \) is the ground truth scallop obtained by measuring the Vericut solid model.

Table 3 shows \( \varepsilon_M \) produced by SK and the two competing methods. Specifically, SK reduces \( \varepsilon_M \) in terms of the maximum scallop by 304% with the reference to the C-method and by 39% with regard to the S-method. In turn, SK reduces \( \varepsilon_M \) in terms of the average scallop by 12% with the reference to the C-method and by 11% with regard to the S-method. The advantage of the SK is explained by a new way of selection of the reference plane, taking into account the non-linear kinematics as well as the tool orientation.

| Method/Surface | Curvature method       | Spheres method       |
|----------------|------------------------|----------------------|
|                | Min | Max | Avr | Min | Max | Avr |
| Surface1       | 0.077 | 0.509 | 0.218 | 0.069 | 0.508 | 0.216 |
| Surface2       | 0.030 | 0.332 | 0.091 | 0.024 | 0.235 | 0.091 |
| Surface3       | 0.001 | 0.592 | 0.040 | 0.001 | 0.188 | 0.039 |
Table 2. Scallop evaluated by proposed method and Vericut (mm).

| Method/Surface | SK     |         |         | Vericut |         |         |
|----------------|--------|---------|---------|---------|---------|---------|
|                | Min    | Max     | Avr     | Min     | Max     | Avr     |
| Surface1       | 0.023  | 0.502   | 0.123   | 0.033   | 0.507   | 0.161   |
| Surface2       | 0.020  | 0.159   | 0.077   | 0.020   | 0.165   | 0.082   |
| Surface3       | 0.001  | 0.176   | 0.039   | 0.001   | 0.137   | 0.040   |

Table 3. Scallop evaluation error (%).

| Method/Surface | $\varepsilon_{SK}$ |         | $\varepsilon_{C}$ |         | $\varepsilon_{S}$ |         |
|----------------|---------------------|---------|-------------------|---------|-------------------|---------|
|                | Min     | Max     | Avr    | Min     | Max     | Avr    |
| Surface1       | 30.3    | 1.0     | 23.6   | 133.3   | 0.4     | 35.4   |
| Surface2       | 0.0     | 3.6     | 6.1    | 50.0    | 101.2   | 11.0   |
| Surface3       | 0.0     | 28.5    | 2.5    | 0.0     | 332.1   | 0.0    |

4. Conclusions
The proposed method shows much promise in terms of the accuracy, stability as well as reliability. The approach reduces the relative error up to 304% with regard to the maximum scallop and up to 12% with regard to the average scallop with the reference to conventional methods. We conjecture that the new approach can be used for efficient iso-scallop tool path generation.

5. References
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