The existence of Josephson tunneling has been demonstrated between YBa$_2$Cu$_3$O$_{7-\delta}$ and Pb with the current flowing along the c-axis of YBa$_2$Cu$_3$O$_{7-\delta}$. This is presumed to come from an s-wave component of the superconductivity in YBa$_2$Cu$_3$O$_{7-\delta}$, perhaps induced by the orthorhombic distortion. This hypothesis by itself appears to be in contradiction with experiments on twinned samples whose tunneling current does not follow random statistics. We present a theory which depends on a competition between the intertwin d-wave coupling and a relatively enhanced s-wave component on the surface. This theory appears to explain, at least qualitatively, all observations made to date.

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Understanding the nature of the order parameter is one of the main challenges in the theory of high-T$_c$ superconductivity. In YBa$_2$Cu$_3$O$_{7-\delta}$ (YBCO), Josephson tunneling experiments with current flowing mainly in the a-b plane have made it clear that the dominant component is d-wave. However, a substantial body of work has also demonstrated the existence of Josephson tunneling along the c-axis from YBCO to a Pb electrode. This strongly suggests the presence of an s-wave component of the superconductivity of YBCO. This is to be expected: YBCO is orthorhombic and the crystal structure mixes the d-wave and s-wave components. This mixing has been adduced in several contexts: it can produce fractional vortices at grain boundaries, and appears to be necessary to explain features of the microwave response. Nevertheless, difficulties arise when the tunneling results are considered in detail. The main problem is that there is tunneling current even when the sample is twinned. Twins should occur in equal numbers of d + s and d − s, where the d-component is defined with respect to axes fixed in space. The net current would then be proportional to $\sqrt{N_T}$, where $N_T$ is the total number of twins. It is actually much larger, though twinning reduces the net current.

The theories of c-axis tunneling which have been proposed to date have serious drawbacks. The most detailed is that offered by Sigrist et al. Their picture involves no net tunneling from the twins themselves. The currents actually come from the twin boundaries where the order parameter is complex. Thus the state of the sample as a whole actually breaks time-reversal symmetry. These ideas do not seem to be consistent with the fact that the current in magnetic field is largely unaffected when the field is reversed. Furthermore, the current should increase when the number of twins increases. This is opposite to what is observed. In addition, microwave experiments on the junctions indicate that the tunneling to Pb is first-order Josephson tunneling.

Some other possibilities suggest themselves. For example, a proximity-effect d-wave gap may be induced in the Pb by the high-T$_c$. However, this is ruled out by the observation that the supercurrent follows a temperature dependence which mimics that of the Pb gap. Also, the coupling to the Pb electrode changes sign across a single twin.

The texturing of the twins, i.e., the fact that the boundaries run predominantly along one direction in space, might also be adduced as a factor that could produce a net current. The twin boundaries run along the diagonal of the a-b plane. Such a texture is equivalent to a rhombohedral distortion, which transforms according to the B$_2$ representation of the C$_{4v}$ group. Can such a distortion couple s-wave (A$_1$) and d-wave (B$_1$) order parameters? Any such coupling term belongs to the A$_1 \times B_1 \times B_2$ representation. But A$_1 \times B_1 \times B_2 = A_2$ which does not contain the identity representation. Therefore the introduction of the texture cannot produce a net Josephson current.

Our model takes its cue from the observation that the magnitude of $I_cR_n$ in general suggests a coupling to a high-T$_c$ gap of $\leq 1mV$, not 20$mV$. This suggests that the magnitude of the gap at an (001) surface is much reduced. This is consistent with the fact that, in spite of much effort, photoemission experiments (with resolutions of order 10 meV) have also never succeeded in seeing a gap at this surface. Furthermore, if this reduction is due to disorder, such as surface scattering, one would expect that the d-wave component is relatively much more suppressed than the s-wave. Similar results might occur due to an oxygen vacancy concentration gradient. The suggestion that the ratio of s to d increases as we approach the surface of the YBCO is due to Bahcall.

We segment the sample into twins, which, if inclusions are neglected, may be thought of as forming approximately a one-dimensional array. (See Fig.1). It is known
of Josephson junctions at the surface, as shown in Fig. 1. Each is characterized by the phase of its s-component, denoted by $\phi_i$ for the $i$th twin. They are in an ’external field’ exerted by the coupling to the bulk below. This external field has an antiferromagnetic character due to the alternating nature of the twins from $d + s$ to $d - s$ relative to axes fixed in space. The alternation will not necessarily hold at the surface if the gap is more strongly s-like. The coupling across the twins at the surface will have the effect of aligning the s-component if its relative strength is large enough. In our model the intertwin coupling is different in the bulk and the surface, so we segment the sample into bulk and surface parts. The resulting picture is a one-dimensional array of Josephson junctions at the surface, as shown in Fig. 1. Each is characterized by the phase of its s-component, denoted by $\phi_i$ for the $i$th twin. They are in an 'external field' exerted by the coupling to the bulk below. This external field has an antiferromagnetic character due to the alternating nature of the twins from $d + s$ to $d - s$, and the fact that only the s-component will couple to the surface. Denote the phase of the s-wave component in the bulk by $\phi_i^b$. The 'junction' between the bulk and the surface is a complicated intervening region. The total energy of this region will depend on the twist of the surface. Denote the phase of the $s$ component throughout. The rotation of the $a$- and $b$-axes across the boundary implies that the twins alternate from $d + s$ to $d - s$ if the gap is large enough. In our model the relative strength is large enough. In our model the alternating nature of the twins from $d + s$ to $d - s$ relative to axes fixed in space. The alternation will not necessarily hold at the surface if the gap is more strongly s-like. The coupling across the twins at the surface will have the effect of aligning the s-component if its relative strength is large enough. In our model the intertwin coupling is different in the bulk and the surface, so we segment the sample into bulk and surface parts. The resulting picture is a one-dimensional array of Josephson junctions at the surface, as shown in Fig. 1. Each is characterized by the phase of its s-component, denoted by $\phi_i$ for the $i$th twin. They are in an 'external field' exerted by the coupling to the bulk below. This external field has an antiferromagnetic character due to the alternating nature of the twins from $d + s$ to $d - s$, and the fact that only the s-component will couple to the surface. Denote the phase of the s-wave component in the bulk by $\phi_i^b$. The 'junction' between the bulk and the surface is a complicated intervening region. The total energy of this region will depend on the twist of the surface.

$$J = J_0 \cos(\phi_i - \phi_i^b).$$

This, in the model, the coupling appears Josephson-like.

Fig. 1 suggests a number of interesting effects. For example, the couplings around a vertex which meets 4 separate regions, 2 surface and 2 bulk, may give rise to phase frustration. This permits the possibility of spontaneous flux running along the twin boundary. If this occurs, it would tend to wash out the phase alternation on the surface and to produce an overall Josephson coupling to the Pb. We shall assume that the appropriate Josephson penetration depth is too short to permit this, but it is actually difficult to estimate. Secondly, several papers have shown that, under certain circumstances, current may be expected along the boundary. This is unlikely to affect the c-axis tunneling results.

In the absence of an applied field, the energy for this model of the YBCO surface is:

$$F = -\sum_{i,j} K_{ij} \cos(\phi_i - \phi_j) - \sum_i J_i \cos(\phi_i - \phi_i^b),$$

where $\cos(\phi_i^b) = (-1)^i$. The total Josephson current to the Pb electrode is given by

$$I = \sum_i I_{ci} \sin(\phi_{pb} - \phi_i),$$

where the Pb phase is taken to be uniform - the twin width is short compared to the Josephson penetration depth. We restrict our considerations to low temperature, as experiments are only possible below the $T_c$ of Pb.

The ordered model is defined by the relations $K_{ij} = K$, $J_i = J$, and $I_{ci} = I$. The ratio of surface to bulk coupling is given by $K/J$. The mean field solution in the regime $J < 4K$ is $\cos(\phi) = (-1)^i J/4K$, with a maximum Josephson current

$$I_{max} = N_T I_c \sqrt{1 - \frac{J^2}{16K^2}}.$$

If $J > 4K$, then $\cos(\phi) = (-1)^i$, and $I = 0$. The supercurrent per unit area for a multitwin sample is therefore reduced from the value it would have in an untwinned sample, as is observed.

In a field $H$ is applied parallel to the twin boundaries, the maximum supercurrent is found by maximizing the expression

$$I = I_c \int \sin \left[ \phi_{pb} - \phi(x) + \frac{Ht}{\Phi_0} \right] dx$$

with respect to $\phi_{pb}$. Here $\Phi_0$ is the flux quantum and $t$ is the effective electrical junction width, including the penetration depths of the Pb and the YBCO. For a single twin this of course gives the usual Fraunhofer pattern. For two twins, there is a range of possible behaviors. For large $J/K$, there is a $180^\circ$ phase change and a symmetric field dependence: $I_{max}(H) = I_{max}(-H)$ and $I(0) = 0$. For intermediate $J/K$, we have the possibility of some asymmetry: $I_{max}(H) \neq I_{max}(-H)$ and some current even at zero field $I(0) \neq 0$. This last is due to breaking of time-reversal symmetry - the phase change across the boundary is less than $180^\circ$. These two possibilities are illustrated for twins of equal areas in Fig. 2. The experimental situation is likely to be close to the symmetric case. The areas of twins are large and one may expect the coupling to the underlying bulk to dominate. Experimentally, some asymmetry is indeed seen. This may also be attributed partly to unequal twin areas or self-field effects as well as time-reversal symmetry breaking found here.

A distinguishing feature of the present model is that, although time-reversal symmetry is broken by the competition between surface and bulk couplings, this does not necessarily show up in field-dependence experiments on multitwin samples. The overall alternation of the two types of domains will tend to wash out the asymmetric part of the signal. We illustrate this by a numerical solution of Eq. 3 for a sample with 500 twins. In Fig. 3(a) we show the field dependence of $I_c$ for an ordered sample with $J = 2K$. The pattern is the simple Fraunhofer one. In Fig. 3(b), the same size sample is used, but the twin area varies about the same mean. A gaussian distribution of fractional width 0.6 was chosen. The coupling to the bulk for each twin varies proportionately. The resulting field dependence shows no asymmetry at low fields. At high fields smaller length scales are probed. Because they are not subject to such complete averaging, asymmetry appears. This is an experimental prediction of the model.

Disorder in general will reduce the Josephson current. Some disorder is expected in every multitwin sample as
the areas of the twins vary to a certain extent, and the coupling to the bulk should increase as the area increases. This is illustrated in Fig. 4 for a sample with 100 twins and a distribution of couplings to the bulk. For a very disordered sample, no supercurrent can flow. Detailed comparison of theory and experiment in this regard is unfortunately difficult. The points in Fig. 4 correspond to different samples. In practice, samples with different twin densities have contact resistances which vary over an order of magnitude, showing that surface quality is highly dependent on preparation methods.

Our results may be summarized by a magnetic analogy. Think of the phases of the s-components at the surface as permanent magnetic moments confined to a plane. The surface coupling is ferromagnetic, while the coupling to the bulk gives rise to an antiferromagnetic external field. The maximum supercurrent at zero field represents the resultant moment. The maximum supercurrent at finite applied magnetic field represents the moment at a nonzero wavevector. When the surface coupling dominates the coupling of surface and bulk, we have ferromagnetism and a nonzero resultant moment. When the randomness in the external field becomes large, the individual moments become random in direction and there is no resultant.

The overall picture which results is that of nearly pure d-wave in the bulk and mixing of s-wave and d-wave near the surface, with s-wave perhaps predominating. This picture may well be specific to YBCO. In BSCCO-type high-Tc compounds, for example, it seems likely that the surface (as seen in photoemission) is representative of the bulk, and both are d-wave, at least in the optimally doped case. It is worth noting that the any disordered model which includes mixing will exhibit some sort of time-reversal symmetry breaking. No gauge transformation can restore all of the disordered phases to zero. Whether this breaking is visible in any particular experiment is a detailed question.

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FIG. 1. (a): schematic representation of the surface of a twinned sample, viewed along the c-axis. (b): the twin structure can be viewed as a one-dimensional alternation of different domains if the inclusions are ignored.

FIG. 2. The effects of an applied field on a junction with two twins of equal area for parameter values (a): J > 4K, and (b): J=2K. A large coupling to the bulk gives a symmetric field dependence, while a weaker bulk coupling results in asymmetry.

FIG. 3. (a): an ordered junction with J/K=2 and 500 twins of equal area. We obtain a symmetric field dependence, regardless of the time reversal breaking state which exists in the surface layer. (b): when a gaussian distribution of twin sizes and couplings J/K is used, asymmetry appears at higher fields.

FIG. 4. Ic(H = 0) for a junction of 100 twins with a gaussian distribution of bulk couplings J, as a function of distribution width. As the disorder increases, Ic(H = 0) is diminished.
(a) $I_c$ vs $H$

(b) $I_c$ vs $H$
