Waiting Time Distribution of Solar Energetic Particle Events Modeled with a Non-Stationary Poisson Process

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Abstract

We present a study of the waiting time distributions (WTDs) of solar energetic particle (SEP) events observed with the spacecraft WIND and GOES. The WTDs of both solar electron events (SEEs) and solar proton events (SPEs) display a power-law tail of $\sim \Delta t^{-\gamma}$. The SEEs display a broken power-law WTD. The power-law index is $\gamma_1 = 0.99$ for the short waiting times ($< 70$ hr) and $\gamma_2 = 1.92$ for large waiting times ($> 100$ hr). The break of the WTD of SEEs is probably due to the modulation of the corotating interaction regions. The power-law index, $\gamma \sim 1.82$, is derived for the WTD of the SPEs which is consistent with the WTD of type II radio bursts, indicating a close relationship between the shock wave and the production of energetic protons. The WTDs of SEP events can be modeled with a non-stationary Poisson process, which was proposed to understand the waiting time statistics of solar flares. We generalize the method and find that, if the SEP event rate $\lambda = 1/\Delta t$ varies as the time distribution of event rate $f(\lambda) = A \lambda^{\alpha} \exp(-\beta \lambda)$, the time-dependent Poisson distribution can produce a power-law tail WTD of $\sim \Delta t^{\alpha-3}$, where $0 \leq \alpha < 2$.

Key words: methods: statistical – Sun: particle emission

Online-only material: color figures

1. Introduction

Solar energetic particles (SEPs) are mainly accelerated during the processes of solar eruptions, namely flares and coronal mass ejections (CMEs), though, some of them may arise from other solar activities, e.g., jets or magnetic reconfiguration in high coronal sites (Klein et al. 2001; Pick et al. 2006). Even though a great deal of effort has been devoted to studies concerning the origins of SEPs, the relationship between SEPs and solar eruptions is still not quite understood (for some recent studies, refer to Aschwanden 2012b; Gopalswamy et al. 2012; Li et al. 2012; Li et al. 2013; Miroshnichenko et al. 2013; Reames 2013).

The time between events (waiting times, $\Delta t$) can provide critical information about how an individual event works, for instance, whether it is independent or connected/triggered by another event. The waiting time distributions (WTDs) are intensively applied in geophysics and astrophysics (Sotolongo-Costa et al. 2000; Lepreti et al. 2004; Wang & Dai 2013). In solar physics, WTDs have been studied over the last two decades for flares, CMEs, and radio bursts, etc., and a consensus has been reached that the WTDs display power-law-tail profiles (Pearce et al. 1993; Wheatland et al. 1998; Wheatland 2003; Eastwood et al. 2010).

The avalanche model of solar eruptions is described as a system of self-organized criticality (SOC), which predicts that WTD is a simple exponential profile consistent with a Poisson process (Bak et al. 1988; Lu & Hamilton 1991; Aschwanden 2012a). Different interpretations were suggested to understand the fact of the power-law WTD (Boffetta et al. 1999; Lepreti et al. 2001; Greco et al. 2009), and one of them is the time-dependent or non-stationary Poisson process (Wheatland 2000; Aschwanden & McTiernan 2010). If the rate of solar eruptions varies with time, the superposition of multiple exponential distributions can resemble a power-law WTD.

In principle, SEPs are representatives of solar eruptions. Therefore, from another standpoint, the study of WTDs of SEP events may provide important clues for understanding the relationship between SEPs and solar eruptions. Another aspect that motivates this study is the generalized model of Wheatland (2000) and Aschwanden & McTiernan (2010). We find that the generalized model fits well with the WTDs of SEP events, and it might also be applicable for the WTDs of other SOC systems.

2. Observations

SEPs mainly consist of electrons and protons, with a small amount of high-Z ions. The solar electron events (SEEs) are selected based on the observations of the WIND spacecraft, which was launched on 1994 November 1 and is still in operation, orbiting around Sun–Earth Lagrange 1 (L1) point. The three-dimensional Plasma and Energetic Particle instrument (3DPE; Lin et al. 1995) observes electrons with electron electrostatic analyzers in the energy range of $\sim 3$ eV–30 keV and with solid-state telescopes in the range of $\sim 25$–400 keV. Our survey of the data began in 1995 January and continued until 2012 December, this leads to the identification of 1594 SEEs with energies between $\sim 0.1$–310 keV. Note that the orbit of WIND passed through the Earth’s magnetosphere 73 times during the statistical period; it is necessary to remove these artificial waiting times. A statistical study of SEEs during solar cycle 23 has been done by Wang et al. (2012) and the candidates studied in the present work are extensions of the previous study.

The solar proton events (SPEs) represent more energetic solar eruptions and determine important properties of space weather. An SPE used to be defined as a flux of $\geq 10$ MeV protons greater than 1 pfu (particle flux unit, 1 pfu = 1 particle cm$^{-2}$ s$^{-1}$ sr$^{-1}$). For some practical reasons (see the discussion in Miroshnichenko 2003), the NOAA Space Environment Service Center (SESC) suggested that one should use...
an intensity of 10 pfu for $\geq 10$ MeV protons as a reliable signature of SPEs. In the context of representatives of more energetic solar eruptions, we use the NOAA SESC criterion for an SPE. According to the survey of NOAA SESC, a total number of 252 SPEs were recorded by the Geostationary Operational Environment Satellite (GOES) from 1976 January to 2013 December (ftp://ftp.ngdc.noaa.gov/STP/space-weather/solar-data/solar-features/GOES/SPE). The survey of data began in 1995 January through 2010 December and led to the identification of 1076 events.

3. METHODS AND RESULTS

3.1. Non-stationary Poisson Process

Theoretically, an SOC event occurs independently and randomly, follows a standard Poisson process, and predicts an exponential WTD:

$$P(\Delta t) = \lambda e^{-\lambda \Delta t},$$  \hspace{1cm} (1)

where $\Delta t$ describes the waiting time and $\lambda$ describes the mean event occurrence rate. If the event rate varies with time, $\lambda(t)$, following the derivation of Wheatland (2000) and Aschwanden & McTiernan (2010), the time-dependent or non-stationary Poisson process will give the WTD as

$$P(t, \Delta t) = \frac{\int_0^T \lambda(t) e^{-\lambda(t)\Delta t} dt}{\int_0^T \lambda(t) dt},$$  \hspace{1cm} (2)

where the observation time interval is $[0, T]$, if $t > T$, then $\lambda(t) = 0$.

Defining $f(\lambda)d\lambda = dt/T$, where $f(\lambda)$ is the fraction of time that the event rate is within the range $(\lambda, \lambda + d\lambda)$, in other words, $f(\lambda)$ is the time distribution of the event rate, a more tractable expression can be given:

$$P(\Delta t) = \frac{\int_0^\infty f(\lambda) e^{-\lambda \Delta t} d\lambda}{\int_0^\infty f(\lambda) d\lambda}.$$  \hspace{1cm} (3)

The mean event rate is $\bar{\lambda} = \int_0^\infty f(\lambda) d\lambda$ during the observation time interval $[0, T]$.

Comparing the solar cycle distribution of sunspots, the SEEs display two distinct features, as shown in Figure 1: (1) the maximum of the SEEs is delayed $\sim 1.5$ yr and (2) the high intermittency (the short-term fluctuations are stronger than that of the sunspot number) is clearly recognized. The high intermittency reflects SEEs that occur in a form of clustering, similar to the phenomenon of flare distribution (Aschwanden & McTiernan 2010). According to Wheatland (2000), the clustering suggests that the time distribution of the event rate, $f(\lambda)$, may follow an exponential function. Here we apply a form of $f(\lambda)$ as

$$f(\lambda) = A\lambda^{-\alpha} \exp(-\beta \lambda).$$  \hspace{1cm} (4)

This is a generalized form of $f(\lambda)$ that fits well with the real observations. This form includes the one of Wheatland (2000) who suggests an exponential form of $f(\lambda)$, where $\alpha = 0$. This form also includes Cases (4) and (5) of Aschwanden & McTiernan (2010), where $\alpha = 0$ and $\alpha = 1$, respectively.

Combining Equations (4) and (3), we derive the analytical expression of WTD:

$$P(\Delta t) = A \frac{\Gamma(3 - \alpha)}{\bar{\lambda}} (\beta + \Delta t)^{-(3 - \alpha)},$$  \hspace{1cm} (5)

where $\beta = (\Gamma(2 - \alpha)/\bar{\lambda})^{1/2-\alpha}$ and $0 \leq \alpha < 2$. The constant, $\Gamma$, corresponds to the so-called gamma function.

3.2. WTDs of SEEs and SPEs

Figure 2 shows the WTD of SEEs. We fit the WTD in two forms: a broken power law (left panel) and a non-stationary Poisson distribution with Equation (5) (right panel). The break of the power law occurs at $\sim 70 < \Delta t < \sim 100$ hr. The power-law index is $\gamma_1 = 0.99$ for waiting times $< 70$ hr and $\gamma_2 = 1.92$ for waiting times $> 100$ hr. An interesting phenomenon is that the break time is consistent with the periodical crossing of the stream interfaces (SIs) or the corotating interaction regions (CIRs), where a fast solar wind stream overtakes a leading slow one. A pair of shock waves (forward and reverse shocks) may form at the edges of CIRs (Gosling & Pizzo 1999), where charged particles can be energized (Reames 1999). This may explain the “bump” around the break time and the origin of the broken power law WTD of SEEs.

The best fit of WTD with Equation (5) gives $\alpha = 1.46$ (right panel of Figure 2). Note that the waiting times over 1000 hr are ignored in order to minimize the fitting errors. Figure 3 shows the fitting of WTD for waiting times $< 70$ hr (red dashed line), it leads to $\alpha_1 = 1.46$. Then, we fit the WTD for waiting times $> 100$ hr (blue dashed line) in a form of $P(\Delta t, \alpha_1 = 1.46) + P(\Delta t, \alpha_2)$, it leads to $\alpha_2 = 0.39$. It is clear that the same value of index 1.46 is obtained for all waiting times and waiting times $< 70$ hr. This indicates that the
Figure 2. WTD of SEEs, fitted with a broken power law (left panel) and a non-stationary Poisson distribution with Equation (5) (right panel). 
(A color version of this figure is available in the online journal.)

Figure 3. WTD of SEEs fitted with Equation (5) for waiting times <70 hr and >100 hr, respectively. 
(A color version of this figure is available in the online journal.)

deviation of WTD for waiting times >100 hr may arise from the same mechanism of the broken power law WTD caused by the modulation of CIRs.

A comparison of WTDs of SPEs and type II radio bursts is shown in Figure 4. Both WTDs are fitted in two forms: a power law for long waiting times and a non-stationary Poisson distribution with Equation (5). Very similar profiles of WTDs are recognized between SPEs and type II radio bursts. The power-law index, $\gamma$, is 1.82 for SPEs and 1.83 for type II radio bursts. The non-stationary Poisson distribution index, $\alpha$, is 0.87 for SPEs and 0.82 for type II radio bursts. This confirms the close relationship between the proton acceleration and the shock wave that is responsible for the formation of the type II radio bursts.

4. SUMMARY AND DISCUSSION

Waiting time statistics of SEP events are investigated in the present study. The WTDs of both SEEs and SPEs are consistent with a non-stationary Poisson process, which was proposed to explain the WTD of solar flares (Wheatland 2000; Aschwanden & McTiernan 2010). A generalized non-stationary Poisson distribution is derived to interpret the WTDs of SEP events. Our conclusions are as follows.

1. The solar cycle distribution of SEEs is featured as clustering. If the event rate $\lambda = 1/\Delta t$ varies with the time distribution of the event rate $f(\lambda) = A\lambda^{-\alpha}exp(-\beta\lambda)$, the non-stationary Poisson process gives the WTD in the form of Equation (5). This predicts a power-law tail $\sim \Delta t^{\alpha-3}$, where $0 \leq \alpha < 2$.

2. The WTD of SEEs show a broken power-law profile. The power-law index is $\gamma_1 = 0.99$ for waiting times <70 hr and $\gamma_2 = 1.92$ for waiting times >100 hr. This might be due to the modulation of CIRs.

3. The WTD of SPEs can be well fitted with Equation (5). Similar WTDs of SPEs and type II radio bursts are recognized, which indicates a close relationship between proton acceleration and the shock waves responsible for the formation of type II radio bursts.

Note that the power-law index was derived to be 2.16 for GOES soft X-ray flares (Wheatland 2000) and was $\sim 2.0$ for RHESSI hard X-ray flares (Aschwanden & McTiernan 2010). In the present study, the power-law index of the WTD of SEEs is 0.99 for short waiting times, which is much harder than the WTD of flares. This probably arises from the fact that SEEs are not only flare-related, but many of them appear to be associated with narrow CMEs, coronal jets, or energy releases in high coronal sites (Kahler et al. 2001; Klein et al. 2001; Pick et al. 2006; Li et al. 2011; Wang et al. 2012). The power-law index of WTD of SEEs is 1.92 for long waiting times,
taking into account the possible modulation of CIRs, which is comparable to the WTDs of flares and SPEs. This indicates the relationship between the solar eruption and the production of SEPs.

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Figure 4. WTDs of SPEs (left panel) and type II radio bursts (right panel). The blue line indicates power-law fitting and red line fitting with Equation (5). (A color version of this figure is available in the online journal.)