Representation of shifted vortex beams of arbitrary order as a combination of nonshifted vortices

A V Ustinov¹

¹Image Processing Systems Institute of RAS - Branch of the FSRC "Crystallography and Photonics" RAS, Molodogvardejskaya street 151, Samara, Russia, 443001

e-mail: andr@smr.ru

Abstract. In this paper, we investigate the focusing of beams with a displaced vortex of arbitrary order. For these purposes, we use the following mathematical model: the beam is represented as a combination of beams having a nonshifted vortex. It is shown theoretically that an optical vortex of an arbitrary integer order \( m \) displaced within an axisymmetric beam is equivalent to the finite sum of nonshifted vortices of orders from 0 to \( m \) inclusive. If the order of the displaced vortex beam is non-integer, then the sum is replaced by an infinite series. Numerical simulation was carried out under sharp focusing conditions in the Debye approximation. The obtained pictures of the focused displaced vortex beams, regardless of the order of the vortex and the magnitude of the displacement, have a qualitatively identical form - the shape of the Crescent. To obtain focal pictures of another type, the illuminating beam must contain an optical vortex of the opposite sign. The obtained results are relevant for multichannel communication systems based on the separation of laser beams carrying orbital angular momentum.

1. Introduction

Vortex beams are sensitive to various deviations in focusing optical systems – inclination, displacement from the optical axis, the presence of other aberrations. Note that the transformations introduced by non-ideal optical systems can have a positive meaning. In particular, the slope of the incident beam or lens [1-4], the presence of astigmatism or ellipticity [5,6], as well as the use of cylindrical lenses [7] introduce distortions into the vortex beam, allowing to visualize its topological charge. Converters based on astigmatic transformations allow to form vortex beams from non-vortex fields [8-10]. The presence of some other aberrations in the focusing system makes it possible to reduce the size of the focal spot in sharp focusing of vortex beams [11-13], and also used to improve the resolution of imaging systems [14-16] and to increase the depth of focus [17-19]. A similar effect is achieved by introducing asymmetry in the incident beam [20, 21] or the focusing element [22, 23], as well as by birefringence [24-27]. Misalignment in the optical system may lead to the "collapse" of high-order optical vortices into several first-order vortex components [28]. A similar effect is observed in the deviation of the wavelength of the illuminating beam from the base wavelength, as well as in the errors of the relief height of the diffraction optical element [29]. The first-order vortex beam in this case becomes asymmetric [28-30]. As a rule, the order of the optical vortex becomes fractional instead of the integer [29, 31-35].
The vortex beams displaced from the optical axis are also of interest in sharp focusing, since in this case a beam with a zigzag (rotating) trajectory of maximum intensity propagation is formed in the focal region [36]. Note that in [36] only the first order vortex beam was considered.

Off-axis vortex beams of different orders can be conveniently formed using binary fork-like gratings based on the interference of a vortex beam with an inclined plane beam (carrier spatial frequency) [37, 38]. In this case, it is possible to simultaneously form many different beams propagating at angles to the optical axis. Such multi-order (multichannel) optical elements are used for optical decomposition of laser fields on a certain basis, including angular harmonics [39-41].

In this paper, a theoretical and numerical study of the focusing of shifted vortex beams of arbitrary order is carried out. In numerical simulation, the sharp focusing of the displaced vortex field in the Debye approximation is considered [42]. In sharp focusing, not only the phase state, but also the polarization one of the beam becomes important [43-54]. In this case, there is an additional polarization transformation and the effect of interaction of the vortex phase (orbital angular momentum) with the state of circular polarization (spin angular momentum) [43-46, 51, 54]. In this regard, the picture of astigmatic transformation is further complicated. This effect is studied in detail in this paper. The obtained results are relevant for optical communication systems based on the separation of laser beams carrying orbital angular momentum [55-60].

2. Theoretical consideration of the shifted vortex beams

In the Debye approximation, the electric field vector in the focal region is described by the following equation [42]:

\[
\mathbf{E}(u, v, z) = -\frac{i j}{\lambda} \int_{0}^{\theta_{\text{max}}} \int_{0}^{2\pi} B(\theta, \phi) T(\theta) P(\theta, \phi) \exp \left[ i k (x \sin \theta \cos \phi + y \sin \theta \sin \phi + z \cos \theta) \right] \sin \theta d\theta d\phi,
\]

where \((u, v, z)\) are Cartesian coordinates in the focal region, \((\theta, \phi)\) are spherical angular coordinates on the exit pupil of the focusing system, \(\theta_{\text{max}}\) is the maximal azimuthal angle, associated with the numerical aperture (NA) of the system, \(B(\theta, \phi)\) is a transmission function, \(T(\theta)\) is a pupil apodization function (for aplanatic systems \(T(\theta) = \sqrt{\cos \theta}\)), \(P(\theta, \phi)\) is the polarization vector in the exit pupil plane, \(k = 2\pi/\lambda\) is a wave number, \(\lambda\) is a wavelength of radiation, \(f\) is a focal distance.

For beams with radial symmetry of the amplitude and vortex phase of \(m\)-th order \(B(\theta, \phi) = R(\theta) \exp(\imath m\phi)\). Then equation (1) can be simplified [61]:

\[
\mathbf{E}_m(\rho, \varphi, z) = -\imath k f \int_{0}^{\theta_{\text{max}}} R(\theta) T(\theta) Q_m(\rho, \varphi, \theta) \sin \theta \exp(\imath k z \cos \theta) d\theta,
\]

where \((\rho, \varphi, z)\) are cylindrical coordinates in the focal region, and the components of the vector \(Q_m(\rho, \varphi, \theta)\) depend on the polarization vector of the incident beam and represent a superposition of Bessel functions of different orders [13].

In this report, we consider the input field as shifted optical vortices of arbitrary order inside a certain axisymmetric beam. Let's write it first in Cartesian coordinates:

\[
B(x, y) = R\left(\sqrt{x^2 + y^2}\right) \sum_{p} \left((x - x_p) + \imath (y - y_p)\right)^Q = R\left(\sqrt{x^2 + y^2}\right) \sum_{p} \left((x + \imath y) - (x_p + \imath y_p)^Q\right).
\]

For integer \(Q_p\), the expression (3) can be transformed using the binomial power formula:

\[
B(x, y) = R\left(\sqrt{x^2 + y^2}\right) \sum_{p} \sum_{l=0}^{Q_p} (-1)^l C_{Q_p}^{l} (x + \imath y)^Q_{p-l} (x_p + \imath y_p)^l.
\]

That is, a displaced vortex of order \(Q_p\) is equivalent to the finite sum of nonshifted vortices of orders from 0 to \(Q_p\) inclusively. From this it follows that from an algebraic point of view (if we ignore the bulkiness of the final expressions) we can use the already known analytical expressions obtained for the nonshifted vortex. If \(Q_p\) is a non-integer, then the finite sum is replaced by an infinite series, and will include nonshifted optical vortices of all orders of one sign.

Next, it is more convenient to proceed to the consideration in polar coordinates:
$$B(r,\phi) = R(r) \sum_p \left( r e^{i\phi} - r_p e^{i\phi} \right)^{Q_p} = R(r) \sum_p \sum_{l=0}^{Q_p} (-1)^l C_{Q_p}^l \left( r e^{i\phi} \right)^{Q_p-l} \left( r_p e^{i\phi} \right)^l.$$ \hspace{1cm} (5)

For one vortex and small values of $Q_p$, we write the representation explicitly:

$$B_1(r,\phi) = R(r) \left( r e^{i\phi} - r_0 e^{i\phi} \right); \quad B_2(r,\phi) = R(r) \left[ \left( r e^{i\phi} \right)^2 - 2r_0 e^{i\phi} r e^{i\phi} + \left( r_0 e^{i\phi} \right)^2 \right].$$

$$B_3(r,\phi) = R(r) \left[ \left( r e^{i\phi} \right)^3 - 3r_0 e^{i\phi} \left( r e^{i\phi} \right)^2 + 3 \left( r_0 e^{i\phi} \right)^2 \left( r e^{i\phi} - \left( r_0 e^{i\phi} \right) \right) \right].$$ \hspace{1cm} (6)

As can be clearly seen, the superposition consists of vortex beams with the same direction of rotation. In this case, the pictures, regardless of the order of the vortex and the magnitude of the displacement, look approximately the same asymmetrically - as a Crescent [36], resembling the pictures for fractional orders [62]. This is expected, since an optical vortex of fractional order can be represented as an infinite series of vortices with an integer order [16].

To get a picture of another type, such as "chamomile" [40], it is necessary to form a superposition of optical vortices of different directions of rotation. For example, a shifted optical vortex of the opposite sign can be used as an illuminating beam:

$$B(r,\phi) = \left( r e^{i\phi} - r_0 e^{i\phi} \right)^{Q_s} \sum_p \left( r e^{i\phi} - r_p e^{i\phi} \right)^{Q_p}.$$ \hspace{1cm} (7)

In particular, for $Q_s=1$ and $Q_p=2$ we obtain:

$$B(r,\phi) = \left( r e^{i\phi} - r_0 e^{i\phi} \right) \left[ \left( r e^{i\phi} \right)^2 - 2r_0 e^{i\phi} r e^{i\phi} + \left( r_0 e^{i\phi} \right)^2 \right] = \left[ e^{i\phi} r^3 - 2r_0 e^{i\phi} r^3 + \left( r_0 e^{i\phi} \right)^2 \left( r e^{i\phi} - \left( r_0 e^{i\phi} \right) \right) \right].$$ \hspace{1cm} (8)

The second term up to the coefficient is similar to the illumination by a vortex-free beam. And the first term can be rewritten as:

$$e^{i\phi} r^3 - 2r_0 e^{i\phi} r^3 + \left( r_0 e^{i\phi} \right)^2 \left( r e^{i\phi} - \left( r_0 e^{i\phi} \right) \right) = 2 \left( r_0 e^{i\phi} \right)^2 \cos(\phi) r + e^{i\phi} \left( r^3 - r \left( r_0 e^{i\phi} \right)^2 \right) - 2r_0 e^{i\phi} r^2.$$ \hspace{1cm} (9)

The first term shows that the picture will contain a "dumbbell". Obviously, at larger values of $Q_s$ there will be "chamomiles" with different number of petals. It can be shown that the greatest possible number of petals is $2 \min(Q_s, \max Q_p)$.

### 3. Numerical simulation

This section presents the results of numerical simulation using expressions (1)-(2) for linear polarization of the incident field. Note that in the conditions of sharp focusing the field changes when polarization changes [63], but to observe the effects of displacement we may limit ourselves with one type of polarization.

![Figure 1. Sharp focusing of a linearly polarized centered vortex beam of the first order: a) the input phase, b) the longitudinal distribution of the total intensity, c) the transverse distributions of the total and component intensities at different distances.](image)

Figure 1 shows the results of sharp focusing of a first-order linearly polarized centered vortex beam. Obviously, linear polarization introduces a certain asymmetry [64] into the beam intensity even in the focal plane ($z=0$). Outside the focal plane, there are additional changes in the shape of the beam,
associated not only with the defocusing of the beam, but also different dynamics of changes in different components of the electromagnetic field. In figures 1c) and 2c) the first row shows the total intensity, and the second row shows the intensity of X-, Y- and Z-components (from left to right).

Figure 2 shows similar results for a first-order shifted vortex beam. At observing of the picture of the longitudinal distribution of the total intensity (Fig. 2b) it can be seen that the beam maximum propagates at an angle to the optical axis. A typical picture of a rotating Crescent is formed in the focal region at different distances [36], with the main contribution being determined by the X-component.

$$z = -\lambda, z = -0.5\lambda, Z = 0, z = 0.5\lambda, z = \lambda$$

**Figure 2.** Sharp focusing of a first-order linearly polarized shifted vortex beam: a) the input phase, b) the longitudinal distribution of the total intensity, c) the transverse distributions of the total and component intensities at different distances.

Note that in [36] only the first order vortex beam was considered. To observe the focusing of several vortex beams of different orders, one can use multi-order DOE [39, 40]. Its transmission function is consistent with a set of optical vortices

$$\Psi(x, y) = \exp(i\phi) = \exp(i\tan^{-1}(y/x))$$

with different carrier frequencies ($\alpha, \beta$):

$$\tau(x, y) = \sum_{p=1}^{P} \Psi(x, y) \exp[i(\alpha_{p}x + \beta_{p}y)].$$

(10)

Figure 3 shows the results of sharp focusing of five linearly polarized vortex beams of orders 0, ±1, ±2 formed by a multi-channel optical element. From the pictures of the transverse distribution in the focal plane it can be seen that the vortex beams of the ±1st order provide the formation of the central focal spot in the longitudinal component (Fig. 3f), and ±2-order beams - in the Y-component of the initially X-polarized beam. This fact was noted in [65].

**Figure 3.** Sharp focusing of five linearly polarized vortex beams of orders 0, ±1, ±2 formed by the multi-channel optical element: the input phase (a), the longitudinal distribution of the total intensity (b), the transverse distribution in the focal plane to the total intensity (c) and the intensity of the X-component (d), Y-component (e) and Z-component (f).

Figure 4 shows similar results for the shifted optical element. In the picture of the longitudinal distribution of the total intensity (Fig. 4b) it can be seen that the maximum of the central beam, which has no phase singularity, propagates along the optical axis, while the maximums of the vortex beams propagate at an angle to the optical axis. Note that in the presence of displacement there is a dependence not only on the module of the order of the vortex, but also the sign of the order (direction
of rotation). This can be seen more clearly in Fig. 5, which shows the dynamics of the transverse distribution of vortex beams of different orders in the focal region at different distances.

![Figure 4](image)

**Figure 4.** Sharp focusing of five linearly polarized vortex beams of orders 0, ±1, ±2, formed by a shifted multi-channel optical element: the input phase (a), the longitudinal distribution of the total intensity (b), the transverse distribution in the focal plane to the total intensity (c) and the intensity of the X-component (d), Y-component (e) and Z-component (f).

![Figure 5](image)

**Figure 5.** Dynamics of changes in the transverse distribution at sharp focusing of several linearly polarized shifted vortex beams of different orders: transverse distributions of the total intensity at different distances.

As shown in Fig. 5, although there are certain differences in the formed patterns depending on the order of the displaced vortex, they are quite similar and represent a rotating Crescent. This is consistent with the theoretical predictions in section 2.

4. Conclusions
In the report, we theoretically showed that an optical vortex of an arbitrary integer order \( m \) shifted within an axisymmetric beam is equivalent to the finite sum of nonshifted vortices of orders from 0 to \( m \) inclusively. If the shifted vortex has a non-integer order, the sum is replaced by an infinite series including nonshifted optical vortices of all orders of one sign. Pictures of intensity regardless of the order of the vortex and the magnitude of the displacement will look about the same asymmetrically – have the shape of a Crescent. To get a picture of another type it is necessary to form a superposition of optical vortices of different directions of rotation. For this purpose, the illuminating beam must not be an axially symmetric, and contain the shifted optical vortex of the opposite sign.

To study the effects under consideration in the presence of several optical vortices of different orders, directions of rotation and polarization, it is possible to use a multi-order diffraction optical element. It is numerically shown (in the conditions of sharp focusing in the Debye approximation) that by changing the type of the illuminating beam and the order of the vortex phase singularity, the shape of rotating intensity distributions in the focal region can be significantly changed. The obtained results are relevant for communication systems based on the separation of laser beams carrying orbital angular momentum, taking into account the distortion and/or wandering of vortex beams in a turbulent or random medium.

5. References
[1] Bin Z and Zhu L 1998 Diffraction property of an axicon in oblique illumination *Applied Optics* 37 2563-2568

[2] Thaning A, Jaroszewicz Z and Friberg A T 2003 Diffractive axicons in oblique illumination: Analysis and experiments and comparison with elliptical axicons *Applied Optics* 42 9-17
[3] Khonina S N, Kotlyar V V, Soifer V A, Jefimovs K, Paakkonen P and Turunen J 2004 Astigmatic Bessel laser beams *Journal of Modern Optics* **51** 677-686

[4] Bendersky A, Perez Quintian F and Rebollo M A 2008 Modification of the structure of Bessel beams under oblique incidence *Journal of Modern Optics* **55** 2449-2456

[5] Cai Y and Lin Q 2004 Hollow elliptical Gaussian beam and its propagation through aligned and misaligned paraxial optical systems *J. Opt. Soc. Am. A* **21** 1058-1065

[6] Kotlyar V V, Khonina S N, Almazov A A, Soifer V A, Jefimovs K and Turunen J 2006 Elliptic Laguerre-Gaussian beams *J. Opt. Soc. Am. A* **23** 43-56

[7] Anguiano-Morales M 2009 Transformation of Bessel beams by means of a cylindrical lens *Applied Optics* **48** 4826-4831

[8] Abramochkin E and Volostnikov V 1991 Beams transformations and nontransformed beams *Optics Communications* **83** 123-135

[9] Beijersbergen M W, Allen L, van der Veen H E L O and Woerdman J P 1993 Astigmatic laser mode converters and transfer of orbital angular momentum *Optics Communications* **96** 123-132

[10] Courtial J and Padgett M J 1999 Performance of a cylindrical lens mode converter for producing Laguerre–Gaussian laser modes *Optics Communications* **159** 13-18

[11] Biss D P and Brown T G 2004 Primary aberrations in focused radially polarized vortex beams *Optics Express* **12** 384-393

[12] Singh R K, Senthilkumaran P and Singh K 2009 Structure of a tightly focused vortex beam in the presence of primary coma *Optics Communications* **282** 1501-1510

[13] Khonina S N, Ustinov A V and Pelevina E A 2001 Analysis of wave aberration influence on reducing focal spot size in a high-aperture focusing system *Journal of Optics* **13** 095702

[14] Reddy A N K, Sagar D K and Khonina S N 2017 Asymmetric apodization for the comma aberrated point spread function *Computer Optics* **41**(4) 484-488 DOI: 10.18287/2412-6179-2017-41-4-484-488

[15] Reddy A N K, Sagar D K and Khonina S N 2017 Complex pupil masks for aberrated imaging of closely spaced objects *Optics and Spectroscopy* **123** 940-949

[16] Reddy A N K, Martinez-Corral M, Khonina S N and Karpeev S V 2018 Focusing of light beams with the phase apodization of the optical system *Computer Optics* **42**(4) 620-626 DOI: 10.18287/2412-6179-2018-42-4-620-626

[17] Khonina S N 2012 Phase apodization of imaging system to increase the focal depth in coherent and incoherent cases *Computer Optics* **36** 357-364

[18] Khonina S N and Demidov A S 2014 Extended depth of focus through imaging system’s phase apodization in coherent and incoherent cases *Optical Memory and Neural Networks (Allerton Press)* **23** 130-139

[19] Khonina S N and Ustinov A V 2015 Generalized apodization of an incoherent imaging system aimed for extending the depth of focus *Pattern Recognition and Image Analysis* **25** 626-631

[20] Kovalev A A, Kotlyar V V and Fakhriev R F 2015 Asymmetrical Bessel modes of the first and second type and their superpositions *Computer Optics* **39**(1) 5-11 DOI: 10.18287/0134-2452-2015-39-1-5-11

[21] Kovalev A A, Kotlyar V V, Zaskanov S G and Kalinkina D S 2016 Laguerre-Gaussian beams with complex shift in Cartesian coordinates *Computer Optics* **40**(1) 5-11 DOI: 10.18287/2412-6179-2015-40-1-5-11

[22] Khonina S N, Nesterenko D V, Morozov A A, Skidanov R V and Soifer V A 2012 Narrowing of a light spot at diffraction of linearly-polarized beam on binary asymmetric axicons *Optical Memory and Neural Networks (Information Optics)* **21** 17-26

[23] Khonina S N, Karpeev S V, Alferov S V, Savel'ev D A, Laukkanen J and Turunen J 2013 Experimental demonstration of the generation of the longitudinal E-field component on the optical axis with high-numerical-aperture binary axicons illuminated by linearly and circularly polarized beams *J. Opt.**.** **15** 085704

[24] Hacyan S and Jáuregui R 2009 Evolution of optical phase and polarization vortices in birefringent media *J. Opt. A: Pure Appl. Opt.* **11** 085204
[25] Zusin D H, Maksimenka R, Filippov V V 2010 Bessel beam transformation by anisotropic crystals. *J. Opt. Soc. Am. A* **27** 1828-1833

[26] Khonina S N, Paranin V D, Ustinov A V and Krasnov A P 2016 Astigmatic transformation of Bessel beams in a uniaxial crystal *Optica Applicata* **46**(XLVI) 5-18

[27] Khonina S N and Porfirev A P 2018 Polarisation-dependent transformation of vortex beams when focused perpendicular to the crystal axis *Optics Communications* **428** 63-68

[28] Kotlyar V V, Almazov A A, Khonina S N, Soifer V A, Elfstrom H and Turunen J 2005 Generation of phase singularity through diffracting a plane or Gaussian beam by a spiral phase plate *J. Opt. Soc. Am. A* **22** 849-861

[29] Oemrawsingh S S R, van Houwelingen J A W, Eliel E R, Woerdman J P, Verstegen E J K, Kloosterboer J G and Hooft G W 2004 Production and characterization of spiral phase plates for optical wavelengths *Applied Optics* **43** 688-694

[30] Khonina S N, Porfirev A P and Ustinov A V 2015 Diffraction patterns with mth order symmetry generated by sectional spiral phase plates. *Journal of Optics* **17** 125607

[31] Berry M V 2004 Optical vortices evolving from helicoidal integer and fractional phase steps. *J. Opt. A: Pure Appl. Opt.* **6** 259-268

[32] Kotlyar V V, Kovalev A A and Soifer V A 2014 Diffraction-free asymmetric elegant Bessel beams with fractional orbital angular momentum *Computer Optics* **38**(1) 4-10

[33] Kovalev A A, Kotlyar V V, Porfirev A P and Kalinkina D S 2015 Analysis of the orbital angular momentum of superposition of diffraction-free Bessel beams with a complex shift *Computer Optics* **39**(2) 172-180 DOI: 10.18287/0134-2452-2015-39-2-172-180

[34] Kovalev A A, Kotlyar V V and Porfirev A P 2016 Transfer of orbital angular momentum from asymmetric Laguerre-Gaussian beams to dielectric microparticles *Computer Optics* **40**(3) 305-311 DOI: 10.18287/2412-6179-2016-40-3-305-311

[35] Kotlyar V V, Kovalev A A and Porfirev A P 2017 Asymmetric Gaussian optical vortex *Optics Letters* **42** 139-142

[36] Zhao X, Zhang J, Pang X and Wan G 2017 Properties of a strongly focused Gaussian beam with an off-axis vortex *Optics Communications* **389** 275-282

[37] Bazhenov V Yu, Soskin M S and Vasnetsov M V 1992. Screw Dislocations in Light Wavefronts *J. Mod. Opt.* **39** 985-990

[38] Khonina S N, Kotlyar V V, Soifer V A, Lautanen J, Honkanen M and Turunen J 1999 Generating a couple of rotating nondiffracting beams using a binary-phase DOE *Optik* **110** 137-144

[39] Kotlyar V V, Khonina S N and Soifer V A 1998 Light field decomposition in angular harmonics by means of diffractive optics *Journal of Modern Optics* **45** 1495-1506

[40] Khonina S N, Kotlyar V V, Soifer V A, Jefimovs K and Turunen J 2004 Generation and selection of laser beams represented by a superposition of two angular harmonics *Journal of Modern Optics* **51** 761-773

[41] Moreno I, Davis J A, Pascoguin B M L, Mitry M J and Cottrell D M 2009 Vortex sensing diffraction gratings *Optics Letters* **34** 2927-2929

[42] Khonina S N 2013 Simple phase optical elements for narrowing of a focal spot in high-numerical-aperture conditions *Optical Engineering* **52** 091711

[43] Helseth L E 2004 Optical vortices in focal regions *Optics Communications* **229** 85-91

[44] Zhao Y, Edgar J S, Jeffries G D M, McGloin D and Chiu D T 2007 Spin-to-orbital angular momentum conversion in a strongly focused optical beam *Phys. Rev. Lett.* **99** 073901

[45] Pu J and Zhang Z 2010 Tight focusing of spirally polarized vortex beams *Opt. Laser Technol.* **42** 186-191

[46] Khonina S N and Golub I 2012 Enlightening darkness to diffraction limit and beyond: comparison and optimization of different polarizations for dark spot generation *J. Opt. Soc. Am. A* **29** 1470-1474

[47] Khonina S N and Golub I 2012 How low can STED go? Comparison of different write-erase beam combinations for stimulated emission depletion microscopy *J. Opt. Soc. Am. A* **29** 2242-2246
[48] Khonina S N and Savelyev D A 2013 High-aperture binary axicons for the formation of the longitudinal electric field component on the optical axis for linear and circular polarizations of the illuminating beam Journal of Experimental and Theoretical Physics 117 623-630

[49] Zhou Z-H, Guo Y-K, and Zhu L-Q 2014 Tight focusing of axially symmetric polarized vortex beams Chin. Phys. B 23 044201

[50] Kotlyar V V, Stafeev S S and Porfirev A P 2015 Sharp focusing of linearly polarized asymmetric Bessel beam Computer Optics 39(1) 34-44 DOI: 10.18287/0134-2452-2015-39-1-34-44

[51] Savelyev D A and Khonina S N 2015 Characteristics of sharp focusing of vortex Laguerre-Gaussian beams Computer Optics 39(5) 654-662 DOI: 10.18287/0134-2452-2015-39-5-654-662

[52] Kharitonov S I and Khonina S N 2018 Conversion of a conical wave with circular polarization into a vortex cylindrically polarized beam in a metal waveguide Computer Optics 42(2) 197-211 DOI: 10.18287/2412-6179-2018-42-2-197-211

[53] Khonina S N, Ustinov A V and Volotovsky S G 2018 Comparison of focusing of short pulses in the Debye approximation Computer Optics 42(3) 432-446 DOI: 10.18287/2412-6179-2018-42-3-432-446

[54] Khonina S N 2019 Vortex beams with high-order cylindrical polarization: features of focal distributions Applied Physics B 125 100

[55] Gibson G, Courtial J, Padgett M J, Vlasnetso M, Pas’ko V, Barnett S M and Franke-Arnold S 2004 Free-space information transfer using light beams carrying orbital angular momentum Optics Express 12 5448-5456

[56] Čelechovský R and Bouchal Z 2007 Optical implementation of the vortex information channel New J. Phys. 9 328

[57] Bozinovic N, Yue Y, Ren Y, Tur M, Kristensen P, Huang H, Willner A E and Ramachandran S 2013 Terabit-scale orbital angular momentum mode division multiplexing in fibers Science 340 1545-1548

[58] Khonina S N, Kazanskiy N L and Soifer V A 2012 Optical vortices in a fiber: mode division multiplexing and multimode self-imaging. Chapter 15 Recent progress in optical fiber research (INTECH publisher, Croatia) p 450

[59] Gbur G and Tyson R K 2008 Vortex beam propagation through atmospheric turbulence and topological charge conservation J. Opt. Soc. Am. A 25 225-230

[60] Soifer V A, Korotkova O, Khonina S N and Shchepakina E A 2016 Vortex beams in turbulent media: review Computer Optics 40(5) 605-624 DOI: 10.18287/2412-6179-2016-40-5-605-624

[61] Khonina S N, Kazanskiy N L and Volotovsky S G 2011 Influence of vortex transmission phase function on intensity distribution in the focal area of high-aperture focusing system Optical Memory and Neural Networks (Information Optics), Allerton Press 20 23-42

[62] Zhang N, Davis J A, Moreno I, Lin J, Moh K-J, Cottrell D M and Yuan X 2010 Analysis of fractional vortex beams using a vortex grating spectrum analyzer Applied Optics 49 2456-2462

[63] Khonina S N and Ustinov A V 2018 Focusing of shifted vortex beams of arbitrary order with different polarization Optics Communications 426 359-365

[64] Khonina S N and Golub I 2011 Optimization of focusing of linearly polarized light Optics Letters 36 352-354

[65] Khonina S N, Kazanskiy N L and Volotovsky S G 2011 Vortex phase transmission function as a factor to reduce the focal spot of high-aperture focusing system Journal of Modern Optics 58 748-760

Acknowledgements
The work is executed at financial support of the Federal Agency of scientific organizations (agreement No. 007-GZ/CH3363/26).