Quasiperiodicity, bistability and chaos in the Landau-Lifshitz equation

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The dynamics of an individual magnetic moment is studied through the Landau-Lifshitz equation with a periodic driving in the direction perpendicular to the applied field. For fields lower than the anisotropy field and small values of the perturbation amplitude we have observed the magnetic moment bistability. At intermediate values we have found quasiperiodic bands alternating with periodic motion. At even larger values a chaotic regime is found. When the applied field is larger than the anisotropy one, the behavior is periodic with quasiperiodic regions. Those appear periodically in the amplitude of the oscillating field. Also, even for low values of the driving force, the moment is not parallel to the applied field.

\[
\frac{1 + \eta^2}{g} \mathbf{M}_t = -(\mathbf{M} \times \mathbf{H}_{\text{eff}}) - \frac{\eta}{M_0} (\mathbf{M} \times (\mathbf{M} \times \mathbf{H}_{\text{eff}})).
\]

(2)

Where \( g \) is the local giromagnetic factor, \( \eta \) is the damping, \( M_0 \) the saturation magnetization, \( \mathbf{M} \) is the tridimensional local continuous magnetization, whose module is conserved \((\mathbf{M} \cdot \mathbf{M} = M_0)\), and \( \mathbf{H}_{\text{eff}} \) the effective field:

\[
\mathbf{H}_{\text{eff}} = \mathbf{H}_{\text{ext}} + \beta \mathbf{n} (\mathbf{n} \cdot \mathbf{M}) + \alpha \nabla^2 \mathbf{M} + \mathbf{H}_d.
\]

(3)

Here \( \mathbf{H}_{\text{ext}} \) is the external magnetic field, \( \beta \) is the anisotropy coefficient, \( \mathbf{n} \) is the unitary vector pointing in the anisotropy direction, \( \alpha \) the coefficient of exchange, and, \( \mathbf{H}_d \) the demagnetizing field that takes into account the dipolar interactions between local moments in the system.

Here we will deal with only one local magnetic moment, so that we will not consider exchange and dipolar interactions. This description could be relevant to dynamics of magnetic microwires with a high anisotropy.

The external field will be decomposed in two parts: \( \mathbf{H}_{\text{ext}} = \mathbf{H}_0 \cdot \sin(\omega t) \cdot \mathbf{n} \), perpendicular to \( \mathbf{n} \) and oscillating in time with frequency \( \omega \).

For practical purposes we rewrite the equation in adimensional form and expanded notation:

\[
\kappa \mathbf{m}_x = - (m_y (h_z + m_z) - m_z h_y) - \eta (m_x m_z (h_x + m_z) + m_x m_y h_y + (m_y^2 - 1) h_y),
\]

(4a)

\[
\kappa \mathbf{m}_y = (m_x (h_z + m_z) - m_z h_x) - \eta (m_y m_z (h_x + m_x) + m_x m_y h_x + (m_x^2 - 1) h_x),
\]

(4b)

\[
\kappa \mathbf{m}_z = - (m_x h_y - m_y h_x) - \eta (m_x m_z h_x + m_y m_z h_y + (m_z^2 - 1) (h_z + m_z)),
\]

(4c)

where \( \mathbf{m} = \mathbf{M}/M_0, \mathbf{h} = \mathbf{H}_{\text{ext}}/(\beta M_0), \kappa = 1 + \eta^2 \), and the dot represents the derivative with respect to an adimensional time \( \tau = g \beta M_0 t \). We take the \( z \) axis in the direction of \( \mathbf{n} \) and the \( x \) axis in the direction of \( \mathbf{h}_0 \), which makes \( h_y = 0 \).

\[13\]
These equations with appropriate initial conditions have been solved by using a fourth order Runge-Kutta scheme. We have observed that the condition that $|m| = 1$ is fulfilled with a precision of more than eight orders of magnitude ($10^{-8}$ in 1), for even more than $10^9$ integration steps.

When looking for chaotic behavior in driven systems, the usual quantities to vary are the frequency and the amplitude of the perturbation. Sweeping in frequency we have found that the interesting ones are those close to the resonance frequency, $\omega_r = 1$, in our units, and of the order of several GHz in real units. Thus possible perturbing signals are radio-wave sources. For simplicity in what follows will put the perturbing frequency fixed to the resonance, and sweep in the amplitude, $h = (h^2 + \eta^2)^{1/2}$.

A bifurcation diagram is shown in figure 2 for $h_z = 0.1$ and $\eta = 0.05$. Here only the components $m_\theta$ and $m_\phi$ are drawn, though in the numerical simulations we followed the equations 4a-c. The diagram shows the values of the components of $m$, at time intervals multiple of $T = 2\pi/\omega$ (Poincaré sections), with the value $h$ shown in the $x$ axis of the figure. There, various kinds of behaviors can be distinguished: periodic, quasiperiodic and chaotic motion. When there is only one point for a given value of $h$, it represents a periodic motion with period $T$; and when there is a continuum of points the behavior is quasiperiodic or chaotic. We will now try to describe the principal features of different critical points shown in figure 2. The changes in the diagram found when the dissipation ($\eta$) is changed, are mainly quantitative (changes the value of $h$ at which a given critical behavior is found). Also, the diagrams shown has been produced for the magnetic moment pointing in the direction of the external field at $t = 0$.

For small values of $h$ ($h \sim h_z$) a discontinuity that corresponds to a folding bifurcation is found. This effect, consisting of two independent limit cycles, may be the experimental source for the observation of hysteresis when changing the amplitude of the perturbation ($h$). When increasing $h$ the jump in $\theta$ is the one shown in figure 2, but if $h$ is decreased, the jump to smaller $\theta$ would occur at a lower value of $h$. Experimentally, this phenomenon could manifest itself in a bistable behavior of a magnetic microwire near the resonance frequency.

Two bifurcations, identified as torus have been observed at $h = 0.60$ and $h = 0.75$, leading to two regions of complex (but mainly quasiperiodic) behavior: $h$ from 0.60 to 0.66, and from 0.75 to 0.87. The first one in the upper hemisphere, and the second one in the lower. In the torus bifurcations the stable limit circles become unstable and give rise to quasiperiodic motion on the surface of a torus. In figure 3 is shown the phase portrait, in coordinates $x$, $y$, and $z$, of the quasiperiodic attractor at $h = 0.64$. There, the Poincaré section changes from just one single point to a closed connected curve.

The region from 0.90 to 1.00 is a mixture of periodic and quasiperiodic behavior, and even chaotic motion. The chaos in this region is characterized by a chaotic attractor at $h \sim 0.9787$, which develops via a global bifurcation of the type of chaotic transients. This means that the system will evolve in the chaotic attractor for some time, and then, feeling the periodic or quasiperiodic stable orbits, will leave it. This is illustrated in figure 4 where the time evolution of the Lyapunov exponents is shown. Initially both exponents converge, one to a positive value, and the other to negative, as signature of chaos. But at a given time the positive exponent initiates a decrease towards 0, or even negative. The Poincaré sections for the initial and final time steps are also shown. Initially the trajectories follow a chaotic map, but after some time they eventually fall in a period-seven orbit. The time spent in the chaotic behavior becomes larger as the chaotic attractor is approached.

Next, there is a wide region of period doubling, with some higher period stripes. Finally, from 2.20 to 2.60, clear chaotic regions (see the Lyapunov exponents in figure 4) alternating with quasiperiodicity are observed. In figure 5 we show the Poincaré section of the chaotic attractor corresponding to $h=2.50$. In this case the route to chaos is also that of chaotic transients.

If the initial state for the magnetization is in the direction opposite to the external bias field, $h_z$, then, basically, the picture presented above holds. Nevertheless the folding disappears, and the stable period-one orbit evolves in the lower hemisphere. The same change of hemisphere happens for the period doubling region. The two (upper and lower) torus bifurcations are also preserved.

When the external applied bias, $h_z$, is larger than the anisotropy field (larger than 1 in our units) the behavior is slightly different. The bifurcation diagram is shown in figure 6. The same kind of structures repeats at $h = 3.25 \times n$, where $n$ is an integer. A magnification of those structures is shown in the inset, where it is seen that they consist of a torus bifurcation and several periodic windows. The magnetization does not remain at a value close to the saturation, but wanders over the whole sphere.

In conclusion, we have demonstrated that the dynamics governed by a driven Landau-Lifshitz equation in certain parameter region can be very complicated. We have observed manifestations of this behavior: the quasiperiodicity alternating with periodic motion, the transient chaotic behavior and finally a chaotic attractor. The experimental observation of these predictions is limited to the materials which could have a well defined single ferromagnetic resonance frequency, i.e. may be described by a single magnetic moment. Nevertheless, some features of this complicated behavior could be persistent in a coupled system. The estimation of the parameters show that even in the case of a well-defined single magnetic moment behavior, the experimental observation of the chaotic behavior would require a powerful source of radio-waves. Nevertheless, a folding bifurcation (bistable behavior) occurs for parameter values accessible by normal radio-wave antennas.

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14 This region may be called also of period bubbling because of the form the bifurcation diagram takes. It never proceeds further in doubling the period, but returns to period halving.

1 FIG. 1. Bifurcation diagram for the $\theta$ and $\phi$ components of the magnetization and Lyapunov exponents when $h_z = 0.1$ and $\eta = 0.05$.

2 FIG. 2. Quasiperiodic attractor at $h = 0.64$ in the diagram of figure 1.
FIG. 3. Time evolution for the Lyapunov exponents when $h = 0.9785$ in the diagram of figure 1. The left inset is the Poincaré section taken from $t = 0$ to $t = 1.8 \times 10^5$. The right one is taking $t = 4.5 \times 10^5$ to $t = 6.3 \times 10^5$.

FIG. 4. Chaotic attractor at $h = 2.5$ in the diagram of figure 1.

FIG. 5. The same bifurcation as in fig. 4 for $h_z = 2$. Insets show the magnification of the region of $h$ between 6 and 7.