Spectator and participant decay in heavy ion collisions

T. Gaitanos, H. H. Wolter

Sektion Physik, Universität München, Am Coulombwall 1, D-85748 Garching, Germany

C. Fuchs

Institut für Theoretische Physik, Universität Tübingen D-72076 Tübingen, Germany

We analyze the thermodynamical state of nuclear matter in transport calculations of heavy–ion reactions. In particular we determine temperatures and radial flow parameters from an analysis of fragment energy spectra and compare to local microscopic temperatures obtained from an analysis of local momentum space distributions. The analysis shows that the spectator reaches an equilibrated freeze-out configuration which undergoes simultaneous fragmentation. The fragments from the participant region, on the other hand, do not seem to come from a common fragmenting source in thermodynamical equilibrium.

PACS number(s): 25.75.-q,25.70.Mn

One of the major interests in the study of intermediate energy heavy–ion collisions is the understanding of the multifragmentation phenomenon and its connection with liquid–gas phase transitions [1]. For this it has to be assumed that in a heavy ion collision at some stage a part of the system is both in thermodynamical equilibrium and instable. Such a configuration is often termed a freeze-out configuration. The multifragmentation process would reflect the parameters of this source, i.e. its temperature, density, and perhaps collective (radial) flow pattern. Experimentally the fragment energy spectra are described in terms of such freeze-out models to extract these parameters. On the other hand, information on these quantities is also sought from other observables, in particular from isotope ratios of fragments, and from excited state populations. In the past the conclusions drawn from these different sources have often been in conflict with each other.

One way to study whether this scenario is applicable is to analyze the results of transport
calculations of heavy ion collisions \[2–6\]. Since such calculations reproduce reasonably well
the asymptotic observables it should be meaningful to look also into their intermediate time
behaviour to see whether, where, and when such freeze-out configurations exist. In ref. \[7\]
a method was developed to determine thermodynamical variables \textit{locally} by analyzing the
local momentum distribution even in the presence of possible anisotropies.

In the present work we apply this analysis to intermediate energy collisions of \(Au + Au\),
which were also studied extensively experimentally \[8–10\]. We want to establish whether the
concept of a freeze-out configuration used in the experimental analysis of fragment kinetic
energy spectra is supported in realistic transport calculations. Fragments are described
using a coalescence algorithm as detailed later. We study, in particular, the participant
and spectator regions which represent rather clean thermodynamical situations in a heavy
ion collision. In this way we also try to clarify some of the discrepancies between different
methods of temperature definition.

We base our investigation on relativistic transport calculations of the Boltzmann-
Nordheim-Vlasov type. In this work we use, in particular, the relativistic Landau-Vlasov
approach which was described in detail in ref. \[3\]. It uses Gaussian test particles in co-
ordinate and momentum space and thus allows to construct locally a smooth momentum
distribution. For the self energies in the transport calculation we have adopted the non-
linear parametrization NL2 \[11\]. In ref. \[12\] we compared this parametrization to more
realistic non-equilibrium self energies based on Dirac-Brueckner calculations. With respect
to the thermodynamical variables discussed here, we found no essential differences between
the two models, and thus we use the simpler NL2 here. A similar analysis with respect
to fragment kinetic energy spectra in central collisions has been performed previously in
ref. \[6\] in the framework of non-relativistic transport theory with special emphasis on the
dependence on assumptions of the equation of state and in-medium cross sections. Here also
a weak dependence of radial flow observables on particular choices of the microscopic input
was found.

The microscopic determination of a local temperature from the phase space distribution
was discussed in detail in refs. [7]. Thus we only briefly review the procedure here. The local momentum distribution obtained from a transport calculation is subjected to a fit in terms of covariant hot Fermi–Dirac distributions of the form

\[
n(x, \vec{k}, T) = \frac{1}{1 + \exp \left[ -\left( \mu^* - k^*_\mu u^\mu \right) / T \right]} \tag{1}
\]

with the temperature \(T\), the effective chemical potential \(\mu^*(T)\) and \(k^*_0 = E^* = \sqrt{k^2 + m^2}\). For vanishing temperature eq.(1) includes the limit of a sharp Fermi ellipsoid with \(\mu^* = E_F = \sqrt{k^2_F + m^2}\). The local streaming four-velocity \(u^\mu\) is determined from the local 4-current \(j^\mu\) as \(u^\mu = j^\mu / \rho_0\), where \(\rho_0 = \sqrt{j_\mu j^\mu}\) is the local invariant density. Then the temperature \(T\) is the only fit parameter to be directly determined from the phase space distribution. In this procedure the effect of the potential energy is taken into account by way of effective masses and momenta and the Fermi motion of the correct density by the chemical potential \(\mu^*\). Thus this temperature is a local thermodynamic temperature, which in the following we denote as \(T_{loc}\).

Expression (1) is appropriate for a system in local equilibrium. In a heavy ion collision this is generally not the case and would lead to an interpretation of collective kinetic energies in terms of temperature. To account for anisotropy effects, e.g. in ref. [13] longitudinal and perpendicular temperatures have been introduced. In our approach we model anisotropic momentum distributions by counter-streaming or ‘colliding’ nuclear matter [7,14,15]; i.e., by a superposition of two Fermi distributions \(n^{(12)} = n^{(1)} + n^{(2)} - \delta n^{(12)}\), where \(\delta n^{(12)} = \sqrt{n^{(1)} \cdot n^{(2)}}\) guarantees the validity of the Pauli principle and provides a smooth transition to one equilibrated system. In [7,12] it has been demonstrated that this ansatz allows a reliable description of the participant and spectator matter at each stage of the reaction.

Experimentally much of the information about the thermodynamical behaviour in heavy ion collisions originates from the analysis of fragment observables. Thus also in the present analysis we will need to generate and analyze fragments. The correct and practical procedure how to properly describe fragment production is still very much debated [13,17]. Here we do not enter into this debate but use the simplest algorithm, namely a coalescence model,
as we have done and described it in ref. [12]. In brief, we apply phase space coalescence, i.e. nucleons form a fragment, if their positions and momenta \((\vec{x}_i, \vec{p}_i)\) satisfy 
\[ |\vec{x}_i - \vec{X}_f| \leq R_c \]
and 
\[ |\vec{p}_i - \vec{P}_f| \leq P_c. \]

\(R_c, P_c\) are parameters which are fitted to reproduce the observed mass distributions and thus guarantee a good overall description of the fragment multiplicities.

Fragment kinetic energy spectra have been analyzed experimentally in the Siemens-Rassmussen or blast model [8,9,18]. In this model the kinetic energies are interpreted in terms of a thermalized freeze-out configuration, characterized by a common temperature and a radial flow, i.e. by an isotropically expanding source. The kinetic energies are given by

\[
\frac{dN}{dE} \sim pE \int \beta^2 d\beta n(\beta) \exp(\gamma E/T) \times \left[ \frac{\sinh \alpha}{\alpha} \left( \frac{T}{E} - \frac{T}{E} \cosh \alpha \right) \right],
\]

where \(p\) and \(E\) are the center of mass momentum and the total energy of the particle with mass \(m\), respectively, and where \(\gamma^{-2} = 1 - \beta^2\) and \(\alpha = \gamma \beta p/T\). Various assumptions have been made for the flow profile \(n(\beta)\). A good parametrization is a Fermi-type function [10]. However, the results are not very different when using a single flow velocity, i.e. \(n(\beta) \sim \delta(\beta - \beta_f)\), which we also use here for simplicity. One then has two parameters in the fit, namely \(\beta_f\) and the temperature parameter in eq.(2), which we call \(T_{\text{slope}}\). It is, of course, not obvious that \(T_{\text{slope}}\) represents a thermodynamical temperature. One of the aims of this investigation is, in fact, to find its significance. The expression (2) has been applied to kinetic energy spectra of all fragment masses simultaneously, yielding a global \(T_{\text{slope}}(\text{global})\), or to each fragment mass separately, giving \(T_{\text{slope}}(A_f)\). If a global description was achieved, it was concluded that a freeze-out configuration exists. We will also test this procedure.

In the following we apply the above methods to central \((b = 0 \text{ fm})\) and semi–central \((b = 4.5 \text{ fm})\) \(Au + Au\) collisions at \(E_{\text{beam}} = 0.25 - 0.8 \text{ AGeV}\). This reactions have been studied extensively by the ALADIN [11,12,20] and EOS [8] collaborations with respect to temperature and phase transitions. In ref. [7] we have previously studied this reaction at one energy, 600 MeV/A, only with respect to local temperatures and thermodynamical instabilities. In this work we perform the fragment analysis with the blast model to extract
and compare slope temperatures and we discuss a wider range of incident energies [21].

The spectator is that part of the system which has not collided with the other nucleus, but which is nevertheless excited due to the shearing–off of part of the nucleus and due to absorption of participant particles. In the calculation it is identified as those particles which have approximately beam rapidity. It was seen in ref. [7] that it represents a well equilibrated piece of nuclear matter at finite temperature.

In fig. 1 we show the evolution with time of the local temperature and the density for the spectator at various incident energies. After the time when the spectator is fully developed the properties are rather independent of incident energy which supports the freeze-out picture. Also after this time the density and temperature remain rather constant for several tens of fm/c, making it an ideal system in order to study the thermodynamical evolution of low-density, finite temperature nuclear matter. In ref. [7] we also determined pressure and studied the dependence of pressure on density. We found that after about 45 fm/c the effective compressibility $K \sim \partial P/\partial \rho |_T$ became negative indicating that the system enters a region of spinodal instability and should subsequently break up into fragments. At this time we find densities of about $\rho \sim 0.4 - 0.5 \rho_0$ and $T \sim 5 - 6$ MeV, which is in good agreement with findings of the ALADIN group based on isotope thermometers [19,20]. Recently the ALADIN group has also determined kinetic energy spectra of spectator fragments and has extracted slope temperatures using eq. (2). It was found that these are typically 10 to 12 MeV higher than those measured with the isotope thermometer.

Applying the coalescence model to the spectator we obtain kinetic energy spectra as shown in fig.2 at 600A MeV for nucleons ($A_f = 1$) and for fragments with $A_f \geq 2$ separately. We fitted these spectra with the model of eq. (2) in the rest frame of the spectator ($\beta_f = 0$). The $A_f \geq 2$ spectrum is well fitted with a temperature of $T_{\text{slope}} \sim (17 \pm 2)$ MeV. The nucleon spectrum, on the other hand, shows a two-component structure, as was also observed experimentally in ref. [20]. It is dominated by a low energy component with $T_{\text{slope,low}} = (7.3 \pm 3.5)$ MeV. The high energy component has rather poor statistics in our calculation and we interpret it as nucleons from the participant that have entered the spectator region.
The slope temperature of the low energy component is close to the local temperature $T_{loc} = (5 - 6) \text{ MeV}$ as discussed above with respect to fig. 1. Thus for nucleons both methods of temperature determination consistently are seen to yield about the same result. In fact, they should not necessarily be identical, since $T_{loc}$ is determined fitting the momentum distribution by a Fermi function while $T_{slope}$ in eq. (2) is based on a Maxwell-Boltzmann distribution, which are not the same at such low temperatures.

On the other hand the slope temperatures of the fragments are considerably higher than those of the nucleons. In fig. 3 we show the slope temperatures separately for the different fragment masses and also the local temperature for comparison. There is a rapid increase of $T_{slope}$ with fragment mass which saturates for $A_f \geq 3$ around $T_{slope} \sim 17 \text{ MeV}$, which was the temperature determined in fig. 2. The experimental values from ALADIN [20] also shown in fig. 3. were obtained by analogous blast model fits to the measured spectra. It can be seen that the slope temperatures from the theoretical calculations and from the data agree extremely well. Also the corresponding kinetic energies which range from 23.7 MeV ($A_f = 2$) to 28.1 MeV ($A_f = 8$) are in good agreement with the ALADIN data.

At first sight it is surprising that $T_{slope}$ for nucleons and fragments differ from each other and also from $T_{loc}$. The difference has been interpreted in ref. [20] in terms of the Goldhaber model [22], as it has been applied to fragmentation by Bauer [23]. When a system of fermions of given density and temperature suddenly breaks up the fragment momenta are approximately given by the sum of the momenta of the nucleons before the decay. For heavier fragments the addition of momenta can be considered as a stochastic process which via the central limit theorem leads to Gaussian energy distributions which resemble Maxwell distributions and thus contribute to the slope temperature. As discussed by Bauer and also in ref. [20] this effect increases the slope temperatures by an amount which is of the order of the difference between the isotope and the slope temperatures.

We wanted to see whether a similar effect can explain the mass dependence seen in fig. 3. We therefore initialized statistically a system of the mass and temperature of the spectator, and subjected it to the same fragmentation procedure (coalescence) and to the
same fit by eq. (2) as we did for the heavy ion collision. These slope temperatures obtained from the statistical model are given in fig. 3 as a band, which corresponds to initializations between $\rho = 0.3\rho_0$ and $T = 6$ MeV and $\rho = 0.4\rho_0$ and $T = 5.5$ MeV, which cover the range of values in fig. 1. It is seen that the model qualitatively explains the increase in the slope temperature relative to the local temperature and the increase with fragment mass relative to that for nucleons. A similar conclusion was drawn in ref. [20] using the results from ref. [23]. This shows that $T_{\text{slope}}$ is not a thermodynamic temperature. The difference relative to the thermodynamic temperature can be understood from the fact, that to form a fragment the internal kinetic energies of the nucleons are limited by the coalescence condition. Since on the average all nucleons have the same momenta, this means that the collective momentum per nucleon of the fragment increases relative to the average. This simulates a higher temperature. This effect has been called "contribution of Fermi motion to the temperature". The purpose of using the Goldhaber model here was to demonstrate this effect. Whether a Goldhaber model applies to heavy ion collisions, can, of course, be debated, but our results are independent of this question.

Thus we seem to understand fairly well the kinetic energy spectra of the spectator fragments and we now turn to the participant region. The participant zone in a heavy ion collision constitutes another limiting, but still simple case for the investigation of the thermodynamical behaviour of nuclear matter. In contrast to the spectator zone one expects a compression-decompression cycle and thus richer phenomena with respect to fragmentation. The situation becomes particularly simple if we look at central collisions of symmetric systems which experimentally are selected using transverse energy distributions, charged particle multiplicities or polar angles near mid-rapidity [8,10,24].

We begin to characterize the calculated evolution of a collision for the case of $Au + Au$ at 600A MeV. A very well developed radial flow pattern appears after about 20 fm/c in agreement with findings of other groups [2,4,5]. The pressure in this reaction becomes isotropic at about 35 fm/c indicating equilibration. The number of collisions drops to small values at about 40 fm/c. This condition we shall call (nucleon) freeze-out. Thus equilibration
and freeze-out occur rather simultaneously. We find a density at this stage of about normal nuclear density and a (local) temperature of about $T_{\text{loc}} \sim 15$ MeV in the mid-plane of the reaction.

We now also apply the blast model of eq. (2) to fragment spectra generated in the coalescence model at the end of the collision at about 90 fm/c. The results for the slope temperature $T_{\text{slope}}$ and the mean velocity $\beta_f$ are shown in fig. 4 for a common fit to all fragments with $A_f \geq 2$ for different incident energies. These are compared to the corresponding values extracted by the EOS [8] and FOPI [9,10] collaborations by analogous blast model fits to charged particle spectra. Our results are in good agreement with the temperatures determined by the FOPI collaboration [10] and somewhat lower than those from EOS [8], in particular at higher incident energies. For the radial flow, the situation tends to be in reverse, in particular with respect to the FOPI results of ref. [9]. This is generally consistent with the findings of other groups. E.g. in ref. [8] in a similar approach results were obtained for the EOS data, which are close to the data for $T_{\text{slope}}$ and above the data for $\beta_f$. One should keep in mind, however, that $T_{\text{slope}}$ and $\beta_f$ are not independent fit parameters. Within the uncertainties of the description by blast model fits there is qualitative agreement between calculation and experiment.

As was done for the spectator we also apply the blast model separately for different fragment masses $A_f$. This is shown in fig. 5 at 600 AMeV in the left column. We observe that slope temperatures rise and flow velocities fall with fragment mass in contrast to the behaviour for the spectator fragments in fig. 3 where $T_{\text{slope}}$ was about constant. A similar behaviour has been seen experimentally at 1 A.GeV in ref. [8] and in calculations in ref. [5]. It can also be deduced from fragment spectra at 250 A.MeV shown in ref. [10], which yield values very close to the ones given here.

The fragment mass dependence of $T_{\text{slope}}$ is thus much stronger and qualitatively different than that for the spectator seen in fig. 3. Thus this behaviour cannot be interpreted as fragments originating from a common freeze-out configuration, i.e. from a fragmenting source. To arrive at an interpretation we have shown on the right column of fig. 5 the local
temperatures and flow velocities for different times before the nucleon freeze-out, i.e. for $t' = t_{\text{freeze-out}} - t$, with $t_{\text{freeze-out}} \sim 35\,fm/c$. (We recall that the values at the left of fig. 5 are obtained at the end of the reaction, i.e. about 90 fm/c.) It is seen that for $A_f = 1$ the values at freeze-out are close to the blast model ones, as required. However, for fragment masses $A_f > 1$ the slope temperatures and velocities behave qualitatively very similar to the local temperatures and flow velocities at earlier times. This would suggest to interpret the fragment temperatures and velocities as signifying that heavier fragments originate at times earlier than the nucleon freeze-out. This may not be unreasonable since in order to make a heavier fragment one needs higher densities which occur at earlier times and hence higher temperatures. However, this does not necessarily imply that the fragments are really formed at this time, since fragments could hardly survive such high temperatures, as also discussed in ref. [10]. But it could mean that these fragments carry information about this stage of the collision. In any case it means that in the participant region fragments are not formed in a common equilibrated freeze-out configuration, and that in such a situation slope temperatures have to be interpreted with great caution.

In summary fragmentation phenomena in heavy ion collisions are studied as a means to explore the phase diagram of hadronic matter. For this it is necessary to determine the thermodynamical properties of the fragmenting source. One way to do this experimentally is to investigate fragment kinetic energy spectra. In theoretical simulations the thermodynamical state can be obtained locally in space and time from the phase space distribution. In this work we have compared this with the information obtained from the generated fragment spectra. We apply this method to the spectator and participant regions of relativistic $Au + Au$-collisions. We find that the spectator represents a well developed, equilibrated and instable fragmenting source. The difference in temperature determined from the local momentum space (or experimentally from the isotope ratios) and from the kinetic energy spectra can be attributed to the Fermi motion in the fragmenting source as discussed in a Goldhaber model. In the participant region the local temperature at the nucleon freeze-out and the slope temperature from fragment spectra are different from those of the spectator.
The slope temperatures rise with fragment mass which might indicate that the fragments are not formed in a common, equilibrated source. These investigations should be continued using more dynamic methods of fragment formation.

We thank the ALADIN collaboration, in particular W. Trautmann and C. Schwarz, for helpful discussions. This work was supported in part by the German ministry of education and research BMBF under grant no. 06LM868I and grant no. 06TU887.

[1] J. Pochodzalla, Prog. Part. Nucl. Phys. 39 (1997) 443.
[2] J. Aichelin, Phys. Reports 202 (1991) 233; R. Nebauer and J. Aichelin, Nucl. Phys. A 650 (1999) 65; P. B. Gossiaux, R. Puri, Ch. Hartnack, J. Aichelin, Nucl. Phys. A 619 (1997) 379.
[3] C. Fuchs, H.H. Wolter, Nucl. Phys. A 589 (1995) 732.
[4] J. Nemeth and G. Papp, Phys. Rev. C 59 (1999) 1802.
[5] A. Hombach, W. Cassing, S. Teis, U. Mosel, Eur. Phys. J. A 5 (1999) 157.
[6] F. Daffin, K. Haglin, and W. Bauer, Phys. Rev. C 54 (1996) 1375.
[7] C. Fuchs, P. Essler, T. Gaitanos and H.H. Wolter, Nucl. Phys. A626 (1997) 987.
[8] M. Lisa and the EOS collaboration, Phys. Rev. Lett. 75 (1995) 2662.
[9] W. Reisdorf and H. G. Ritter, Annu. Rev. Nucl. Part. Sci. 47 (1997) 663, and references there.
[10] W. Reisdorf and the FOPI Collaboration, Nucl. Phys. A 612 (1997) 493.
[11] B. Blättel, V. Koch, U. Mosel, Rep. Prog. Phys. 56 (1993) 1.
[12] T. Gaitanos, C. Fuchs, and H. H. Wolter, Nucl. Phys. A650 (1999) 97.
[13] J. Konopka, H. Stöcker and W. Greiner, Nucl. Phys. A 583 (1995) 357.
[14] L. Sehn, H.H. Wolter, Nucl. Phys. A 601 (1996) 473;
    C. Fuchs, L. Sehn, H.H. Wolter, Nucl. Phys. A 601 (1996) 505.

[15] R.K. Puri et al., Nucl. Phys. A 575 (1995) 733.

[16] S. Ajik, et al., Nucl. Phys. A 513 (1990) 187; J. Randrup et al., Nucl. Phys. A 514 (1990) 339.

[17] A. Guarnera, et al., Phys. Lett. B 373 (1996) 267; M. Colonna et al., Nucl. Phys. A 642 (1998) 449.

[18] P. J. Siemens and J. O. Rasmussen, Phys. Rev. Lett. 42 (1979) 880.

[19] V. Serfling and the ALADIN Collaboration, Phys. Rev. Lett. 80 (1998) 3928.

[20] W. Trautmann and the ALADIN Collaboration, see [21]; C. Schwarz, ibid.

[21] T. Gaitanos, C. Fuchs, H. H. Wolter, Proc. of the International Workshop XXVII on Gross Properties of Nuclei and Nuclear Excitations, Hirschegg, Austria, 1999.

[22] A. S. Goldhaber, Phys. Lett. B 53 (1974) 306; Phys. Rev. C 17 (1978) 2243.

[23] W. Bauer, Phys. Rev. C 51 (1995) 803.

[24] F. Rami and the FOPI Collaboration, Nucl. Phys. A 646 (1999) 367.
FIG. 1. Local temperature (top) and density (bottom) evolution in the spectator in semi-central \( Au + Au \) reactions (\( b=4.5 \) fm) at different beam energies as a function of time.
FIG. 2. Energy spectra for nucleons ($A_f = 1$) and fragments ($A_f \geq 2$) (filled diamonds and open circles, respectively) from the spectator for $Au + Au$ at $E_{beam} = 600$ AMeV. Blast model fits are shown (solid curves) which determine slope temperatures.
FIG. 3. Spectator slope temperatures for different fragment masses $A_f$ for the reaction as in fig. 2 (diamonds). Also shown is the nucleon local temperature (circle) and the temperature obtained from a statistical model (triangle and gray band, see text).
FIG. 4. Slope temperatures (top) and radial flow velocities (bottom) from blast model fits to fragment ($A_f > 1$) energy spectra for central collisions for different beam energies (filled dots). They are compared to data from [8–10].
FIG. 5. Slope temperatures (upper row) and flow velocities (lower row) for the same reaction as in fig. 4 at $E_{\text{beam}} = 0.6$ AGeV. In the left column for blast model fits for different fragment masses and also for $A_f > 1$ and for all fragments; in the right column the local values from the momentum distribution at times before the freeze–out (see text).