Policies for growth of influence networks in task-oriented groups: elitism and egalitarianism outperform welfarism

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Communication or influence networks are probably the most controllable of all factors that are known to impact on the problem-solving capability of task-forces. In the case connections are costly, it is necessary to implement a policy to allocate them to the individuals. Here we use an agent-based model to study how distinct allocation policies affect the performance of a group of agents whose task is to find the global maxima of NK fitness landscapes. Agents cooperate by broadcasting messages informing on their fitness and use this information to imitate the fittest agent in their influence neighborhoods. The larger the influence neighborhood of an agent, the more links, and hence information, the agent receives. We find that the elitist policy in which agents with above-average fitness have their influence neighborhoods amplified, whereas agents with below-average fitness have theirs deflated, is optimal for smooth landscapes, provided the group size is not too small. For rugged landscapes, however, the elitist policy can perform very poorly if the group size is large. In addition, we find that the egalitarian policy, in which the size of the influence neighborhood is the same for all agents, is optimal for both smooth and rugged landscapes in the case of small groups. The welfarist policy, in which the actions of the elitist policy are reversed, is always suboptimal, i.e., depending on the group size it is outperformed by either the elitist or the egalitarian policies.

I. INTRODUCTION

It has long been realized that the patterns of communication or influence that determine who can communicate with whom in a task-oriented group have a great impact on its problem-solving performance [1–6] (see [7–8] for more recent contributions). However, these studies have focused on imposed or fixed communication patterns (e.g., the hierarchical structure of the military and industrial organizations) thus excluding a priori the possibility of self-organization of the communication networks. A manner to introduce flexibility on the patterns of communication is to allow the agents to roam around an arena where they can interact with each other if the distance between them is less than a prespecified threshold [9]. This roaming scenario can avoid the so-called Groupthink phenomenon of social psychology that occurs when everyone in a group starts thinking alike, thus leading to catastrophic results [10]. Here we explore a simpler scenario of flexible communication patterns where immobile agents vary their radiiuses of interaction according to the (relative) quality of the solutions they offer to the problem posed to the group. This variation results in a time-dependent, adaptive directed network that links the agents to their influencers.

The information in social networks flows between individuals via social contacts and, in the cooperative problem-solving context, the relevant process is the imitative learning as expressed in this quote by Bloom “Imitative learning acts like a synapse, allowing information to leap the gap from one creature to another” [11]. Although social learning has inspired the design of a variety of optimization heuristics, such as the well-known particle swarm optimization algorithm [12] and the less known adaptive culture heuristic [13], a fully workable agent-based model to study group problem-solving via imitative learning was proposed only recently [14–15]. In this model, the agents perform individual trial-and-test searches to explore a fitness landscape and, more importantly, imitate a model agent – the best performing agent in their influence neighborhood at the trial. Hence the model exhibits the two critical ingredients of a collective brain, viz., imitative learning and a dynamic hierarchy among the agents [11]. In addition, the model bears upon cooperative problem-solving systems as the agents exchange messages informing each other on their partial success towards the completion of the task [17–18].

The task of the agents is to find the global maxima of smooth and rugged fitness landscapes generated with the NK model [19]. The performance of the group is measured by the properly scaled number of trials (or time) required to find those maxima. Our goal is to investigate how the policy of allocation of information to the agents affects the group performance. The amount of information allocated to an agent is determined by the number of agents in its influence neighborhood, which equals the number of incoming connections, and here we consider three distinct policies. The elitist policy in which the agents with above-average fitness amplify their influence neighborhoods whereas the agents with below-average fitness shrink theirs; the welfarist policy in which those actions are reversed; and the egalitarian policy in which the size of the influence neighborhood is the same for all agents, regardless of their fitness. Since the fitness of the agents change as they explore the state space of the fitness landscape, so do their influence neighborhoods, resulting in adaptive directed communication networks, which we characterize through the number and size of their strongly connected components.

We find that for both smooth and rugged landscapes
the welfarist policy is always suboptimal, i.e., it is outperformed either by the elitist or by the egalitarian policies. In addition, for small group sizes the egalitarian policy always yields the optimal performance. Except for small groups, the elitist policy is the optimal choice in the case of the smooth landscapes without local maxima, but it fails spectacularly for large groups in the case of rugged landscapes. In this case, it is even outperformed by the independent search, in which the agents explore the state space independently of each other, thus characterizing the Groupthink phenomenon since cooperation turns out harmful to group performance.

The rest of the paper is organized as follows. In Section II we present a brief description of the NK model of rugged fitness landscapes. In Section III we describe the rules for setting up the influence neighborhoods of the agents as well as the implementation of the imitative search to explore the state spaces of fitness landscapes. In Section IV we study the performance of the imitative search for both smooth and rugged landscapes focusing on the effects of the group size and of the policies of allocation of information to the agents. Finally, Section V is reserved for our concluding remarks.

II. NK-FITNESS LANDSCAPES

The NK model [19] is the choice computational implementation of fitness landscapes that has been extensively used to study optimization problems in population genetics, developmental biology and protein folding [20]. It was introduced originally to model the adaptive evolution process as walks on rugged fitness landscapes and its main advantage, which led to its widespread use in complexity science, is the possibility of tuning the ruggedness of the landscape by changing the two integer parameters that give the model its name, namely, $N$ and $K$. More pointedly, the NK landscape is defined in the space of binary strings of length $N$ and so the parameter $N$ determines the size of the state space, $2^N$. The other parameter $K = 0, \ldots, N - 1$ determines the range of the epistatic interactions among the bits of the binary string and influences strongly the number of local maxima on the landscape. In time, two bits are said to be epistatic whenever the combined effects of their contributions to the fitness of the binary string are not merely additive [19]. In particular, for $K = 0$ the corresponding (smooth) landscape has one single maximum whereas for $K = N - 1$, the (uncorrelated) landscape has on the average $2^N / (N + 1)$ maxima with respect to single bit flips [21].

In the NK model, each string $x = (x_1, x_2, \ldots, x_N)$ with $x_i = 0, 1$ has a fitness value $\Phi(x)$ that is given by the average of the contributions of each component $i$ in the string, i.e.,

$$\Phi(x) = \frac{1}{N} \sum_{i=1}^{N} \phi_i(x),$$

where $\phi_i$ is the contribution of component $i$ to the fitness of string $x$. It is assumed that $\phi_i$ depends on the state $x_i$ as well as on the states of the $K$ right neighbors of $i$, i.e., $\phi_i = \phi_i(x_i, x_{i+1}, \ldots, x_{i+K})$ with the arithmetic in the subscripts done modulo $N$. It is assumed, in addition, that the functions $\phi_i$ are $N$ distinct real-valued functions on $\{0, 1\}^{K+1}$ and, as usual, we assign to each $\phi_i$ a uniformly distributed random number in the unit interval [19]. Because of the randomness of $\phi_i$, we can guarantee that $\Phi \in (0, 1)$ has a unique global maximum and that different strings have different fitness values.

For $K = 0$ there are no local maxima and the sole maximum of $\Phi$ is easily located by picking for each component $i$ the state $x_i = 0$ if $\phi_i(0) > \phi_i(1)$ or the state $x_i = 1$, otherwise. However, for $K > 0$ finding the global maximum of the NK model is a NP-complete problem [22], which means that the time required to solve the problem using any currently known deterministic algorithm increases exponentially fast with the length $N$ of the strings. The increase of the parameter $K$ from 0 to $N - 1$ decreases the correlation between the fitness of neighboring strings (i.e., strings that differ at a single component) in the state space and for $K = N - 1$, those fitness values are uncorrelated.

Since the functions $\phi_i$ in eq. (1) are random, the ruggedness measures (e.g., the number of local maxima) of a particular realization of a NK landscape are not fixed by the choice of the parameters $N$ and $K$. In fact, those measures can vary considerably between landscapes characterized by the same values of $N$ and $K > 0$ [19], which implies that the performance of any search heuristic based on the local correlations of the fitness landscape depends on the particular realization of the landscape. Thus, in order to highlight the role of the parameters that are relevant to our goal of exploring the effect of flexible, fitness-dependent influence neighborhoods on group performance, here we compare the performance of the groups for the same realization of the NK fitness landscape. In particular, we consider two types of landscape: a smooth landscape with $N = 12$ and $K = 0$ and a rugged landscape with $N = 12$ and $K = 4$.

III. IMITATIVE LEARNING MODEL

We consider a system of $M$ agents placed in a square box of linear size $L$ with periodic boundary conditions. In the initial configuration, the coordinates $x$ and $y$ of each agent are chosen randomly and uniformly over the length $L$. The density of agents $\rho = M/L^2$, which we fix to $\rho = 1$ throughout the paper, yields the relevant spatial scale to measure the distance between agents on the square box. In fact, since the effective area of an agent is $1/\rho$, the quantity $d_0 = 1/\sqrt{\rho}$ can be viewed as the linear size or, for short, the size, of an agent and it will be our standard to measure all distances in our study. We note that the fixed value of the density $\rho$ is inconsequential, provided we use $d_0$ as the standard for measuring
distances in the square box. Each agent is represented by a binary string of length $N$, whose bits are initially drawn at random with equal probability for 0 and 1, so that each agent has an associated fitness value $\Phi_k$ with $k = 1, \ldots, M$. The fitness of the agents change with time as they explore the NK-fitness landscape aiming at finding its global maximum by flipping bits following the rules of the imitative learning search [15] as will be described next. Henceforth we will use the terms agent and string interchangeably.

The influence neighborhood of agent $k$ is comprised of all the agents located inside the circle of radius $d_k$ centered at the spatial coordinates of agent $k$. It is among those agents only that agent $k$ will select a model to imitate. Here we consider the prescription

$$d_k = d_0 \exp\left[\alpha \left(\Phi_k/\bar{\Phi} - 1\right)\right]$$

(2)

where $\bar{\Phi} = \sum_{k=1}^{M} \Phi_k / M$ is the mean fitness of the group at time $t$ and, for the sake of clarity, we have omitted the dependence on $t$ of the quantities $d_k$, $\Phi_k$ and $\bar{\Phi}$. The parameter $\alpha$ determines the radius of the influence neighborhood of each agent according to its relative fitness. For $\alpha > 0$, agents with fitness higher than average have a large influence neighborhood, i.e., they can see and eventually copy more agents in the group, whereas the agents with fitness lower than average have their influence neighborhoods downsized and are likely to become isolated for large $\alpha > 0$. We refer to this choice as the elitist policy, since the high-fitness agents have more opportunities to further improve their fitness through imitation. For $\alpha < 0$ the situation is reversed so that the lower-than-average fitness agents have their neighborhoods amplified and the above-than-average fitness agents have theirs curtailed. We refer to this choice as the egalitarian policy. Hence the case $\alpha = 0$, where the size of the influence neighborhoods is the same for all agents, is termed the egalitarian policy.

This scenario is illustrated in Fig. 1 that shows a snapshot of a system of $M = 100$ agents in the square box. Henceforth we refer to the network created by the union of the influence neighborhoods of all agents as the influence network. This directed network reduces to the classic undirected random geometric graph for $\alpha = 0$. This undirected graph was originally introduced to model wireless communication networks [23] and it was recently used as a face-to-face network in the modeling of the dynamics of human interactions [24] as well as in the study of the effects of mobility on cooperative processes [9].

The dynamics begins with the selection of a target agent at random, say agent $k$, at time $t = 0$ and proceeds as follows. A circle of radius $d_k$ is drawn around the target agent so that its influence neighborhood is determined (see Fig. 1). If the influence neighborhood is empty, i.e., there is no agent within a distance $d_k$ from agent $k$, or all agents in the influence neighborhood have fitness lower than or equal to the fitness of the agent $k$, then this agent simply flips a bit at random. We recall that due to the nature of the NK-fitness landscape – the fitness are real-valued random variables – two agents that have the same fitness must be identical (clones). If there are agents with fitness higher than the fitness of the target agent in its influence neighborhood, there are two possibilities of action. The first action, which happens with probability $1 - p$, consists of flipping a bit at random of the target string as before. Through the repeated application of this action, the agents can produce all the $2^M$ binary strings starting from any arbitrary string, which guarantees that the global maximum will eventually be reached for $p < 1$. The second action, which happens with probability $p$, is the imitation of a model string, which is the string of highest fitness in the influence neighborhood of the target agent. In this case, the model and the target strings are compared and the different bits are singled out. Then the target agent selects at random one of the distinct bits and flips it so that this bit is now the same in both strings. Hence, imitation results in the increase of the similarity between the target and the model agents, which may not necessarily lead to an increase of the fitness of the target agent if the landscape is not additive.

The parameter $p \in [0,1]$ is the imitation probability, which we assume is the same for all agents (see [25] for the relaxation of this assumption). The case $p = 0$ corresponds to the baseline situation where the $M$ agents explore the state space independently of each other. The case $p = 1$ corresponds to the situation where only the model strings explore the state space through random bit
flips, whereas the other strings simply follow the models in their influence neighborhoods. The imitation procedure described above was borrowed from the incremental assimilation mechanism used to study the influence of external media \cite{Buchanan2000, Axelrod1997} in Axelrod's model of social influence \cite{Axelrod1987}. This is the main feature that distinguishes our model from previous exploration (random bit flips) and exploitation (copy of fittest agent) models in which the copy mechanism is non-incremental so that the target agent is replaced by the model agent \cite{Buchanan2000}. As expected, this non-incremental mechanism may permanently trap the search in the local maxima of the fitness landscape.

After the target agent is updated, which means that exactly one bit of its string is flipped, we increment the time \( t \) by the quantity \( \Delta t = 1/M \). Then another agent is selected at random and the procedure described above is repeated. We note that during the increment from \( t \) to \( t + 1 \), exactly \( M \) string operations are performed, though not necessarily by \( M \) distinct agents. The collective search ends when one of the agents finds the global maximum and we denote by \( t^* \) the halting time. Here we measure the efficiency of the search by the total number of agent updates necessary to find the global maximum (i.e., \( M t^* \)), which is essentially the computational cost of the search. Since \( t^* \) typically scales with the size of the solution space \( 2^N \), it is convenient to present the results in terms of the rescaled computational cost, defined as

\[
C = M t^*/2^N.
\]

In the case of the independent search (\( p = 0 \)), it can be shown that the mean rescaled computational cost is given by\cite{Buchanan2000}

\[
\langle C \rangle = \frac{M}{2^N} \left[ 1 - (\lambda_N)^M \right],
\]

where \( \lambda_N \) is the second largest eigenvalue of a tridiagonal stochastic matrix \( T \) whose elements are \( T_{ij} = (1 - j/N) \delta_{i,j+1} + j/N \delta_{i,j-1} \) for \( j = 1, \ldots, N - 1 \), \( T_{i0} = \delta_{i,1} \), and \( T_{iN} = \delta_{i,N} \), where \( \delta_{i,j} \) is the Kronecker delta. The notation \( \langle \ldots \rangle \) stands for the average over independent searches on the same landscape. Notice that \( \langle t^*_M \rangle = 1/[1 - (\lambda_N)^M] \) is the expected number of trials for a group of \( M \) independent agents to find the global maximum. In particular, for \( N = 12 \) we have \( \lambda_{12} \approx 0.99978 \) and \( \langle t^*_M \rangle \approx 4545 \). Since \( (\lambda_{12})^M \approx e^{-M(1-\lambda_{12})} \) we have \( \langle C \rangle \approx \langle t^*_M \rangle /2^{12} \approx 1.11 \) for \( M \ll \langle t^*_M \rangle \) and \( \langle C \rangle \approx M/2^{12} \) for \( M \gg \langle t^*_M \rangle \).

The assessment of the performance of the cooperative search is done by comparing its mean computational cost with the cost of the independent search, which is approximated very well by the constant \( \langle C \rangle \approx 1.11 \) for the typical group sizes \( M \) considered in the paper.

IV. RESULTS

As pointed out before, the performance of the imitative search is measured by the mean computational cost \( \langle C \rangle \), which is estimated by averaging the computational cost defined by eq. (3) over \( 10^5 \) searches on the same landscape realization. Since our main concern is the effect of the adaptive sizes of the influence neighborhoods on group performance, we will fix the imitation probability to \( p = 0.5 \) and vary the group size \( M \) and the parameter \( \alpha \) that appears in eq. (2) and determines the strength of the elitist (\( \alpha > 0 \)) and welfarist (\( \alpha < 0 \)) policies, i.e., determines how the radius of the influence neighborhood of an agent is affected by its relative fitness.

A. Smooth Landscape

The NK fitness landscape with \( K = 0 \) is an additive landscape (i.e., the fitness of a string is given by the sum of the fitness of its components) that exhibits a single maximum. Its fitness is \( q_{\text{max}} = 0.559 \) whereas the average fitness of the landscape is \( \Phi_{\text{AV}} = 0.415 \). The results of the performance of the imitative search for a landscape with \( N = 12 \) and \( K = 0 \) are summarized in Figs. 2 and 3 where the independent variable is \( M \) and \( \alpha \), respectively. It is convenient to begin the analysis of Fig. 2 with the results for \( \alpha = 30 \), where we observe an initial decrease of the computational cost with increasing \( M \) until the group size reaches the optimal value at \( M = 26 \). The subsequent increase of the cost for \( M \) greater than the optimum is probably due to the concentration of the strings in the vicinity of the model string and the consequent production of clones that reduce the efficiency of the search \cite{Buchanan2000}. The optimum group size decreases and the group performance degrades with decreasing \( \alpha \). For instance, for \( \alpha = -30 \), the mean cost reaches its minimum value for \( M = 3 \).

Figure 3 shows the computational cost against \( \alpha \) and reveals more clearly the interesting result that the group performance improves with increasing \( \alpha \), provided that \( M \) is not too small. This means that for the imitative search on a smooth landscape it is advantageous to allow the above-average fitness agents to enlarge their influence neighborhoods so they can inspect and eventually imitate more agents in the group. This elitist policy increases the chances of improvement of the agents which already have a high fitness and decreases those of the low-fitness agents, similarly to the so-called Matthew principle in which the rich get richer and the poor get poorer \cite{Matthew1909}. The opposite, welfarist policy in which the below-average agents enlarge their neighborhoods (i.e., \( \alpha < 0 \)) results in a much poorer performance, as illustrated in Figs. 2 and 3. Interestingly, however, for small groups, say \( M < 5 \) in Fig. 2 the best performance is achieved for the egalitarian policy (\( \alpha = 0 \)) where the sizes of the influence neighborhoods are not dependent on the fitness of the agents (i.e., \( d_k = d_{\text{opt}} \) for \( k = 1, \ldots, M \)). To conclude the
FIG. 2. Mean computational cost $\langle C \rangle$ as function of the group size $M$ for the imitative search on a smooth landscape. The imitation probability is $p = 0.5$ and the parameter that determines the radius of the influence neighborhood is $\alpha = -30, -5, 0, 5, 30$ as indicated. The parameters of the NK landscape are $N = 12$ and $K = 0$.

FIG. 3. Mean computational cost $\langle C \rangle$ as function of the parameter that determines the radius of the influence neighborhoods $\alpha$ for group sizes $M = 10, 26, 100, 300$ as indicated. The imitation probability is $p = 0.5$ and the parameters of the NK landscape are $N = 12$ and $K = 0$.

Analysis of the group performance, we stress that the imitative search always outperforms the independent search for the smooth landscape.

Now we consider in detail some properties of the agent that found the global maximum and, consequently, halted the search. We refer to this agent as the winner. An instructive quantity is the distribution of probability of the number of agents in the winner’s influence neighborhood, $\Omega_w = 0, \ldots, M - 1$, at the instant just before it finds the global maximum, which we show in the upper panel of Fig. 4. For the purpose of comparison, we show in the lower panel of Fig. 4 the distribution of probability of the number of agents in the influence neighborhood of a randomly selected agent at the same instant. We note that $P(\Omega)$ gives effectively the distribution of the sizes of the influence neighborhood just before the search halts.

For $\alpha < 0$, these results indicate that the winner is very likely to be an isolated agent (i.e., $\Omega_w = 0$), as expected, though there are little more than 10% of isolated agents when the search halts. In fact, since the winner must differ of exactly one bit from the global maximum just before it flips the discordant bit, its fitness must be high and, consequently, its influence neighborhood must be small. The same reasoning applies for $\alpha > 0$, so the winner is highly likely to be connected to all the other agents (i.e., $\Omega_w = M - 1$), although, in this case, there are very few highly connected agents in the group as shown in the
lower panel of the figure.

In order to investigate whether the winners had an edge in the initial random setup of the group, we calculate the difference between the fitness of the winners at time $t = 0$, $\Phi_w(0)$, and the initial mean fitness of the group, $\Phi(0)$, for each search. Then we average this difference over the $10^6$ searches and show the result $\langle \Phi_w(0) - \Phi(0) \rangle$ in Fig. 5. Here the notation $\langle \ldots \rangle$ represents an average over different searches whereas $\cdots$ stands for an average over the agents in the group. The effect of the group size $M$ can be understood by noting that for small $M$ the chances that an agent—an outlier—is assigned a high fitness value at $t = 0$, $\Phi_w(0)$ does not differ much from the group average $\Phi(0)$. As $M$ increases, the chances that an outlier appears increase and the result that $\langle \Phi_w(0) - \Phi(0) \rangle > 0$ indicates that those outliers are more likely to be the winners, regardless of the value of $\alpha$. The dependence of $\langle \Phi_w(0) - \Phi(0) \rangle$ on $\alpha$ for fixed $M$ is more instructive, as it shows that the elitist policy ($\alpha > 0$) practically selects the future winners already in the initial generation by allowing the fittest agent to reap all the benefits of imitative learning.

To conclude our analysis of the influence of the adaptive communication patterns on the imitative search, it is of interest to characterize those patterns when the search halts. As pointed out, the network formed by the union of the influence neighborhoods of all agents is a directed graph where the agents are the nodes (see Fig. 1). Since the nub of the imitative search is to spread useful information (i.e., bits that increase fitness) among the members of the group, a relevant measure is the fraction of pairs of agents that are connected by directed paths so that they can directly or indirectly influence each other.

We recall that a directed graph is said to be strongly connected if every node is reachable from every other node, and a strongly connected component (SCC) of a directed graph is a maximal strongly connected subgraph $G_c$. Thus, following the usual line of analysis used to study percolation [33], we focus here on the number $N_c$ of SCCs as well as on the size $g_c$ of the largest SCC in the graph. By the size of a SCC we mean the number of nodes that belong to it. Figure 6 shows these two quantities in a properly normalized form, viz., $n_c = N_c/M$ and $g_c = G_c/M$. The fraction of SCCs reaches its minimum value for $\alpha = 0$ and is slightly asymmetric around that point, i.e., there is a bit more SCCs for positive than for negative $\alpha$. The size of the largest SCC exhibits this slight asymmetry too. The fact that the largest SCC occurs for $\alpha = 0$ for small group sizes and contains about 90% of the agents may be the reason that the egalitarian

FIG. 5. Average difference between the initial fitness of winner and the initial mean fitness of the group $\langle \Phi_w(0) - \Phi(0) \rangle$ as function of the parameter that determines the radius of the influence neighborhoods $\alpha$ for group sizes $M = 10, 26, 100, 300$ as indicated. The imitation probability is $p = 0.5$ and the parameters of the NK landscape are $N = 12$ and $K = 0$.

FIG. 6. Ratio between the number of strongly connected components and the group size, $n_c$ (upper panel), and fraction of agents in the largest strongly connected component, $g_c$ (lower panel), for group sizes $M = 10, 26, 100, 300$ as indicated. The imitation probability is $p = 0.5$ and the parameters of the NK landscape are $N = 12$ and $K = 0$. 
FIG. 7. Mean computational cost ⟨C⟩ as function of the group size M for the imitative search on a rugged landscape. The imitation probability is p = 0.5 and the parameter that determines the radius of the influence neighborhood is α = −30, −5, 0, 5, 30 as indicated. The dashed curve is the analytical result for the independent search given by eq. (6). The parameters of the NK landscape are N = 12 and K = 4.

policy is optimal in this situation (see Fig. 2). For large M, however, there is no obvious link between g_c and the computational cost, since g_c is practically the same for α = 10 and α = −10, but the costs are very distinct (see Fig. 3).

It is interesting to note that, for | α | < 5, g_c goes to zero as M increases so that the directed graph is composed of a macroscopic number of microscopic SCCs since n_c is nonzero. For | α | > 5 we also have a macroscopic number of SCCs, but now at least one component is microscopic. In time, a quantity is macroscopic (microscopic) if it grows linearly (sublinearly) with M in the asymptotic limit M → ∞.

B. Rugged Landscape

The realization of the NK fitness landscape with N = 12 and K = 4 that we consider here has 53 maxima. The global maximum has fitness Φ^{max} = 0.7833 and the average fitness of the landscape is Φ^av = 0.508. Thus, finding the unique global maximum poses a difficult challenge to any hill-climbing type of search strategy. In fact, the presence of those local maxima makes the computational cost of the imitative search very susceptible to the choice of the group size M and of the policy to allot information to the agents, which is determined by the parameter α. Figures 7 and 8 illustrate the intricacies of this choice.

Whereas the best performance shown in Fig. 7 is achieved by the highly elitist policy (α = 30) for M ≈ 35, this policy gives the worst performance for both small and large group sizes. In particular, its performance for large M is even worse than that of the independent search, thus characterizing a Groupthink-like phenomenon in which cooperation harms the performance of the group. This happens because the agents are trapped in high fitness local maxima that are distant from the global maximum. The cost to escape those maxima can be very high for large groups due to the attractive effect of the clones of the model string. Interestingly, for small group sizes, the best performance is achieved by the egalitarian policy (α = 0) as in the case of the smooth landscape. Figure 8 shows that the welfarist policy (α > 0) is always suboptimal, regardless of the value of M. By suboptimal we mean that either α = 0 or α > 0 yield a better performance. This conclusion holds true for smooth and rugged landscapes as well.

Figure 8 shows the distribution of the number of agents in the influence neighborhood of the winner and of a randomly selected agent at the instant just before the winner finds the global maximum. The winner is almost certainly isolated for α < 0 or fully connected for α > 0, although the size of the influence neighborhood of a randomly selected agent is very little affected by the sign of α. An interesting characteristic of the search on the rugged landscape, which contrast with the smooth landscape, is that use of the elitist policy (α > 0) does not give any significant advantage to high-fitness outliers produced in the initial setup of the group as shown in Fig. 10 provided that M is fixed. This is probably because enlarging the influence neighborhood of an agent is not necessarily advantageous as the fitness of the model agents are not strongly correlated to the distances to the global maximum as happens for the smooth landscape (see Fig. 9).

Finally, we note that the quantities n_c and g_c are very similar to those shown in Fig. 5 for the smooth landscape. The only noticeable quantitative difference is that the
number of strongly connected components increases more steeply as $|\alpha|$ deviate from zero, so that for large $|\alpha|$ the final influence network exhibits much more SCC for the rugged than for the smooth landscape. However, this has no effect on the size of the largest SCC, which is quantitatively very similar for both landscapes.

V. DISCUSSION

Problem solving by task-groups (e.g., drug design, traffic engineering, software development) represents a substantial portion of the economy of developed countries nowadays. Hence the relevance of understanding the factors that influence the capability of a group of individuals to solve problems. Here we approach this issue by combining ideas from the theory of distributed cooperative problem-solving systems and organizational design.

In particular, we explore the effect of adaptive communication or influence networks, which determine who influences whom within the group, on the time the group takes to find the global maximum of a fitness landscape. The group performance is measured by a computational cost that essentially tallies the total number of bit flips performed on the $M$ binary strings (i.e., agents) that compose the group. The adaptive influence networks are directed graphs that result from the union of the fitness-dependent (and hence time-dependent) influence neighborhood of each agent. The size of the influence neighborhood of an agent is a measure of the amount of information it can use to decide which bit to flip, which brings forth the need to establish a policy for allocation of information to the agents based on their fitness. Here we consider three information allocation strategies that are determined by the sign of the parameter $\alpha$ in eq. (2). The first is the elitist policy in which agents with above-average fitness have their influence neighborhoods amplified, whereas agents with below-average fitness have theirs deflated. The second is the welfarist policy in which the actions of the elitist policy are reversed, and the third is the egalitarian policy in which the size of the influence neighborhood is the same for all agents.

Policies for allocation of information to the agents are of great importance when the links or connections between individuals are costly, as in the case of social networks of gregarious animals where there is a direct selection pressure to reduce the number of connections between entities because of their building and maintenance.
In the context of searching for the global optimum of a NK fitness landscape, a natural criterion to determine the amount of information allocated to an agent is its relative fitness, hence our proposal of the prescription to define the radiuses of the influence neighborhoods of the agents.

Somewhat surprising, we find that for small groups the egalitarian policy is optimal for both smooth and rugged landscapes (Figs. 2 and 7). In addition, we find that the elitist policy is optimal for smooth landscapes, provided the group size is not too small. However, this policy produces disastrous results for large group sizes in the case of rugged landscapes, which is akin to the Groupthink phenomenon of social psychology. The welfarist policy, on the other hand, is always suboptimal.

An interesting finding about the elitist policy regards its potential to select high-fitness outliers in the initial randomly generated group as the winners of the search, i.e., the agents that find the global maximum first. For smooth landscapes, the elitist policy picks winners with a much higher initial mean fitness than those of the other two policies (Fig. 5), in accordance with the Matthew principle that ‘the rich get richer and the poor get poorer’. However, for rugged landscapes the initial mean fitness of the winners is essentially the same for the three policies (Fig. 10). Thus the information allocation policies do not seem to affect the chances of an agent to win the search on a rugged landscape.

It is curious to note that even in a situation of strong welfare, say $\alpha = -30$ in Figs. 6 and Fig. 10, where the high-fitness agents are isolated and the low-fitness agents are allowed access to the entire group, the high-fitness outliers of the initial generation are still more likely to become winners, so our welfarist policy cannot reverse the random initial fitness inequality.

The characterization of the influence networks using the distribution of the sizes of the influence neighborhoods (lower panels of Figs. 4 and 9) and the statistics of the strongly connected components (Fig. 8) reveal the rich topology produced by the interplay between the network structure and the imitative search dynamics. We conclude that, except for very small groups, some degree of flexibility on the communication patterns among agents is beneficial to the group performance.

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