The $B \to \pi \ell \nu$ semileptonic decay within the LCSR approach under heavy quark effective field theory

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The heavy quark effective field theory (HQEFT) provides an effective way to deal with the heavy meson decays. In the paper, we adopt two different correlators to derive the light-cone sum rules of the $B \to \pi$ transition form factors (TFFs) within the framework of HQEFT. We label those two LCSR results as LCSR-$U$ and LCSR-$R$, which are for conventional correlator and right-handed correlator, respectively. We observe that the correlation parameter $|\rho_{RL}|$ for the branching ratio $B(B \to \pi \ell \nu)$ is $\sim 0.85$, implying the consistency of the LCSRs under different correlators. Moreover, we obtain $|V_{ub}|_{LCSR-U} = (3.45^{+0.28}_{-0.20} \pm 0.13_{\text{exp}}) \times 10^{-3}$ and $|V_{ub}|_{LCSR-R} = (3.38^{+0.22}_{-0.19} \pm 0.12_{\text{exp}}) \times 10^{-3}$. We then obtain $\mathcal{R}_\pi|_{LCSR-R} = 0.68^{+0.10}_{-0.09}$ and $\mathcal{R}_\pi|_{LCSR-U} = 0.65^{+0.13}_{-0.11}$, both of them agree with the Lattice QCD predictions. Thus the HQEFT provides a useful framework for studying the $B$ meson decays. Moreover, by using right-handed correlator, the twist-2 term shall dominate the TFF deviation from the SM prediction. Those inconsistencies have aroused people’s great interests on searching the new physics (NP) beyond the SM from $B$ meson decays.

In the $B$ meson, the $b$ quark is much heavier than the light degrees of freedom in the initial heavy meson. While in the $D$ meson, the $c$ quark expansion is valid, such as: $\bar{\Lambda}p/\omega \ll 1$ and $v \cdot p/(2m_q) \ll 1$, where the $\Lambda$ is the effective mass of the light degrees of freedom in the initial heavy meson. In the $B$ meson, $b$ quark mass is much heavier than the other component. Therefore the energy of the light degrees of freedom is far less than the twice of the $b$ quark mass, whereas the second ratio varies roughly between

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I. INTRODUCTION

The $B$-meson semileptonic decay is an important channel for testing the heavy quark effective field theories. The mass of the $\tau$ lepton is much larger than the masses of the electron and muon, and the $\tau$ lepton is more sensitive to new physics associated with electroweak symmetry breaking. In recent measurements, the BABAR, Belle, and LHCb collaborations have observed large disagreement with the Standard Model (SM) predictions on the ratio $\mathcal{R}_{D^{(*)}} = B(B \to D^{(*)}\tau \nu)/B(B \to D^{(*)}\ell \bar{\nu})$, where $\ell$ stands for the light lepton $e$ or $\mu$. The weighted average of those measurements gives $\mathcal{R}_{D} = 0.407 \pm 0.039 \pm 0.024$ and $\mathcal{R}_{D^{*}} = 0.306 \pm 0.013 = 0.007$. The typical SM predictions are $\mathcal{R}_{D} = 0.299 \pm 0.011$2 and $\mathcal{R}_{D^{(*)}} = 0.252 \pm 0.003$. Meanwhile, the LHCb collaboration also measured the similar ratio $\mathcal{R}_{J/\psi} = 0.71 \pm 0.17 \pm 0.18$, which is $2 \sigma$ deviation from the SM prediction. Those inconsistencies have aroused people’s great interests on searching the new physics (NP) beyond the SM from $B$-meson semileptonic decays.

For example, it is helpful to know whether such kind of disagreement also occurs in $B \to \pi$ semi-leptonic decays. Using the Belle data set of $711 fb^{-1}$, the Belle Collaboration predicted the signal strength $\mu = 1.52 \pm 0.72$, where $\mu = 1$ corresponds to $B(B \to \pi \nu) = 1.0 \times 10^{-4}$. With the Belle measurement data, the Ref. 6 indicates the ratio is $\mathcal{R}^{\text{meas}}_{\pi} \simeq 1.05 \pm 0.51$. Theoretically, the Lattice QCD calculation of the RBC and UKQCD Collaborations gives $\mathcal{R}_{\tau} = 0.69 \pm 0.19$. Using the Lattice QCD prediction of the $B \to \pi$ transition form factors (TFFs) given by the Fermilab Lattice and MILC collaborations, Ref. 6 predicted $\mathcal{R}_{\tau} = 0.641 \pm 0.016$, which was alternated to $\mathcal{R}^{\text{2HDM}}_{\tau} = 1.01 \pm 0.04$ by using the type-II two Higgs doublet model (2HDM).

Before confirming the signal of NP, we need more effects to achieve an accurate SM prediction. The ratio $\mathcal{R}_{\tau}$ strongly depends on the non-perturbative $B \to \pi$ transition form factors (TFFs), which have been predicted by various approaches, such as the Lattice QCD approach, the perturbative QCD approach and the QCD light-cone sum rule (LCSR) approach, cf.Refs. 7,13,16,21. Each approach has its reliable application region, which can be extended to the physically allowable region via proper extrapolations. A consistent analysis can be achieved by properly connecting predictions under different approaches, which are suitable for various $q^2$-regions. Among them, the LCSR approach is applicable in both small and intermediate $q^2$ regions and provides an important role for studying the properties of $B \to \pi$ TFFs.

The $B$-meson is made up of a heavy $b$ (or $\bar{b}$) quark and one light quark (or antiquark), whose semileptonic decays can be described by using the heavy quark effective theory. For as the effective theory of heavy quark, there are some criteria23 to judge whether the heavy quark expansion is valid, such as: $\Lambda/(2m_Q) \ll 1$ and $v \cdot p/(2m_Q) \ll 1$, where the $\Lambda$ is the effective mass of the light degrees of freedom in the initial heavy meson. While in the $B$ meson, $b$ quark mass is much heavier than the other component. Therefore the energy of the light degrees of freedom is far less than the twice of the $b$ quark mass, whereas the second ratio varies roughly between
0 and 1/4 for \( m_\pi \leq v \cdot p \leq (m_B^2 + m_\pi^2)/(2m_B) \). These criteria are satisfied in this process. In addition to the heavy quark effective theory \([24,26]\), the heavy quark effective field theory (HQEFT) is also an effective theory which including the effects of the mixing terms between quark and antiquark fields. The HQEFT enables one to separate the long-distance dynamics from the short one via a systematic way \([27]\), which could lessen the number of non-perturbative wave functions or transition form factors to a certain degree and improve the precision of the calculations. The characteristic feature of HQEFT lies in that its Lagrangian has kept the ‘particle’, ‘antiparticle’, ‘small’ and ‘large’ field components. Basing the HQEFT framework, the LCSR approach can be adopted for a consistent description on the heavy to light meson semileptonic decays \([28,29,32]\). In the present paper, we shall give a detailed LCSR analysis of those TFFs within the framework of the HQEFT.

Usually, the current structure of the correlator is constructed by equating its quantum number \( J^P \) (total angular momentum \( J \) and parity \( P \)) as that of the \( B \) meson. Constructed via this way, the LCSR for the \( B \rightarrow \pi \) TFFs can be arranged following the twist structures of pion light-cone distribution amplitudes (LCDAs). However because the high twist LCDAs are still of high uncertainty, those terms involving high-twist LCDAs provide one of the main uncertainties to the LCSRs of the \( B \rightarrow \pi \) TFFs. It has been pointed out that by choosing a proper correlator, e.g. by using a chiral current, one can suppress the uncertainties from the high-twist LCDAs and achieve a more accurate LCSR prediction on the TFFs \([13,33,37]\). It is thus important to see whether the LCSRs under different choices of correlator are consistent with each other.

The remaining parts of this paper are organized as follows. In Sec. III we present the calculation technologies for deriving the key components, e.g. the TFFs \( f^\pm, f^0 \), of the \( B \rightarrow \pi \) semileptonic decays. In Sec. IV we present our numerical results and discussions. Section IV is reserved for a summary.

II. CALCULATION TECHNOLOGY

For the \( B \rightarrow \pi \) decays, its transition matrix element can be written as

\[
\langle \pi(p)|\bar{u}\gamma_\mu b|B(p+q)\rangle = f^+(q^2)\left[2p_\mu + \left(1 - \frac{m_B^2 - m_\pi^2}{q^2}\right) q_\mu\right] + f^0(q^2)\frac{m_B^2 - m_\pi^2}{q^2} q_\mu, \tag{1}
\]

where \( p \) is the momentum of pion and \( p+q \) is the momentum of \( B \) meson, \( f^{+,0}(q^2) \) are the two TFFs. Within the HQEFT framework, the \( B \rightarrow \pi \) transition matrix element can be expanded as \( 1/m_B \)-power series and at the leading-order accuracy, we have \([28,29]\):

\[
\langle \pi(p)|\bar{u}\gamma_\mu b|B(p+q)\rangle = \frac{\sqrt{m_B}}{\sqrt{4\Lambda_B}}(\pi(p)|\bar{u}\gamma_\mu b^+|B_\pi) = -\frac{\sqrt{m_B}}{\sqrt{4\Lambda_B}}Tr[\pi(v,\gamma_\mu b)B_\pi], \tag{2}
\]

where \( \Lambda_B = m_B - m_\pi, b^+ \) is the effective \( b \)-quark field, \( \pi(v, p) = \gamma^5[A(v, p) + i\not{p}B(v, p)] \), and the effective \( B \)-meson spin function \( B_\pi = -\sqrt{\Lambda}/(1+\not{v}\gamma^5/2) \) with \( v \) is the \( B \)-meson velocity, \( \not{p} = p^\mu/(v \cdot p) \), \( \Lambda \) is the binding energy, and \( \Lambda = \lim \Lambda_B \). \( A(v, p) \) and \( B(v, p) \) are scale-dependent coefficient functions characterized by the heavy-to-light transition matrix elements, where \( v \cdot p = (m_B^2 + m_\pi^2 - q^2)/(2m_B) \). Using those equations, we obtain the HQEFT \( B \rightarrow \pi\ell\bar{\nu}_\ell \) TFFs \( f^\pm(q^2), \) i.e.

\[
f^{\pm}(q^2) = \frac{\sqrt{\Lambda}}{\sqrt{m_BА_к}} \left[A(y) \pm \frac{m_B}{y} B(y)\right] + \cdots, \tag{4}
\]

where \( y = v \cdot p \), the symbol \( \cdots \) denotes the high-order \( O(1/m_\pi) \) contributions, and the TFF \( f^0(q^2) \) can be derived via the relation:

\[
f^0(q^2) = f^+(q^2) + \frac{q^2}{m_B^2 - m_\pi^2} f^-(q^2). \tag{5}
\]

The two coefficient functions \( A(y) \) and \( B(y) \) in the TFFs \( f^\pm(q^2) \) can be derived by using the LCSR approach. For the purpose, we need to design a correlator, which can be generally written as

\[
F_\mu(p, q) = i \int d^4xe^{i(q-m_\pi v)x}(\pi(p)|T\{j_\mu(x), j^\dagger_\mu(0)\}|0), \tag{6}
\]

where \( j_\mu(x) \) and \( j^\dagger_\mu(0) \) are usually chosen as \( j^\dagger_\mu(x) = \bar{u}(x)\gamma_\mu b^+(x) \) and \( j^\dagger_\mu(0) = i\not{m_\pi}b(0)^+\gamma_5q(0) \) \([13]\). It has been observed that by using \( j^\dagger_\mu(x) = \bar{u}(x)\gamma_\mu(1+\gamma_5)b^+(x) \) and \( j^\dagger_\mu(0) = \not{m_\pi}b^+(0)^+(1+\gamma_5)d(0) \), one can achieve more accurate LCSRs for the TFFs with less uncertain high-twist terms \([19]\). Even though by using the chiral currents, one may receive additional uncertainties from the scalar \( (1^+) \) \( B \) meson resonances, which however can be absorbed into the corresponding hadronic dispersion integral via a proper choice of continuum threshold \([14,13]\).

There are other choices of correlator. In the paper, we adopt the above two choices of correlators to show whether the LCSRs under different correlators are consistent with each other. For convenience, we label the LCSR for \( j^\dagger_\mu(x) \) and \( j^\dagger_\mu(0) \) as LCSR-\( U_\mu \), and the LCSR for \( j^\dagger_\mu(0) \) and \( j^\dagger_\mu(0) \) as LCSR-\( R_\mu \).

The calculation technology for deriving the LCSR-\( R_\mu \) and LCSR-\( U_\mu \) are the same. As an explicit example, we describe the procedures for deriving LCSR-\( R_\mu \) of the two coefficient functions \( A(y) \) and \( B(y) \) by using the correlator \( F^R_\mu(p, q) \) with two right-handed chiral currents \( j^\dagger_\mu(x) \) and \( j^\dagger_\mu(0) \).
First, we discuss the hadronic representation for the correlator. One can insert a complete series of the intermediate hadronic states in the correlator and isolate the pole term of the lowest pseudoscalar state to get the hadronic representation. That is, the correlator $F_R^\mu$ can be expressed as

$$F_R^\mu(p, q) = \frac{\langle \pi(p)|\bar{u}\gamma_\mu p^+|B_\mu\rangle\langle B_\mu|\bar{b}^+ i\gamma_5 d(0)\rangle}{m_B^2 - (p + q)^2} + \sum_{H_\pi} \frac{\langle \pi(p)|\bar{u} \gamma_\mu(1 + \gamma_5) b^+_H\gamma_5 d(0)\rangle}{m_{H_\pi}^2 - (p + q)^2}.$$  

(7)

Based on the original definition, the matrix element $\langle B_\mu|\bar{b}^+ i\gamma_5 d(0)\rangle = FTr[\gamma_\mu B_\mu]/2$, where $F$ represents the leading-order $B$-meson decay constant, which can be found in Refs. \[22, 24\]. With the help of the $B \rightarrow \pi$ matrix element \[23\], the correlator $F_R^\mu$ can be written as

$$F_R^\mu(p, q) = 2\pi \frac{A(y)\rho(y) + B(y)\rho^\mu}{2\Lambda_B - 2v \cdot k} + \int_{s_0}^{\infty} ds \frac{\rho(y, s)}{s - 2v \cdot k} + \text{Subtractions},$$  

(8)

where $k^\mu = P_B^\mu - m_B\nu^\mu$ is the heavy hadronic residual momentum. The spectral density $\rho(y, s)$ can be estimated by using the ansatz of the quark-hadron duality \[25\]. Contributions from the higher power suppressed subtraction terms can be suppressed or eliminated by applying the Borel transformation.

On the other hand, we need to deal with the corrector in large spacelike momentum regions $(p + q)^2 - m_B^2 \ll 0$ and $q^2 \ll m_B^2$ for the momentum transfer, which correspond to the small light-cone distance $x^2 \simeq 0$ and are required by the validity of operator product expansion (OPE). Based on the OPE, the correlator can be expanded into a power series of the pion LCDAs with various twists:

$$F_R^\mu(p, q) = i \int d^4xe^{ixv}e^{ix\cdot v} \int_{-\infty}^{\infty} dt \delta(x - vt) \times \langle \pi(p)|\bar{u}(x)\gamma_\mu \gamma_5 d(0)\rangle\rangle_0,$$  

(9)

where we have implicitly used the $B$-meson heavy-quark propagator within HQEFT, i.e. $S(x, v) = (1 + \gamma_\mu) x \times \int_{0}^{\infty} dt \delta(x - vt)/2$. For convenience, we present the needed pion LCDAs up to twist-4 accuracy in the Appendix, which are taken from Refs. \[31, 32\].

As a final step, by applying the Borel transformation for the correlator $F_R^\mu(p, q)$, we obtain the LCSRs for the coefficient functions $A(y)$ and $B(y)$,

$$A(y) = -\frac{f_\pi}{2\pi} \int_0^{s_0} ds e^{2\lambda_B - s}/T \frac{1}{y^2} \left[ g_2(u) - \phi_{2;\pi}(u) \right] \left|_{u=1-\bar{\eta}} \right.$$

$$+ \left( \frac{1}{y} \frac{\partial}{\partial u} \right)^2 g_1(u) - \frac{1}{y^2} \frac{\partial}{\partial u} g_2(u) \right|_{u=1-\bar{\eta}}.$$  

(10)

$$B(y) = -\frac{f_\pi}{2\pi} \int_0^{s_0} ds e^{2\lambda_B - s}/T \left[ -\phi_{2;\pi}(u) \right.$$  

$$+ \left( \frac{1}{y} \frac{\partial}{\partial u} \right)^2 g_1(u) - \frac{1}{y^2} \frac{\partial}{\partial u} g_2(u) \right|_{u=1-\bar{\eta}}.$$  

(11)

Substituting them into Eqs. (11, 15), one can obtain the expressions for the needed two TFFs $f^+(q^2)$ and $f^0(q^2)$, i.e.

$$f^+(q^2) = -\frac{f_\pi\sqrt{T}}{2\pi m_B\Lambda_B} \int_0^{s_0} ds e^{2\lambda_B - s}/T \times \frac{1}{y^2} \left[ g_2(u) - \frac{m_B}{y} \left[ -\phi_{2;\pi}(u) \right.$$

$$+ \left( \frac{1}{y} \frac{\partial}{\partial u} \right)^2 g_1(u) - \frac{1}{y^2} \frac{\partial}{\partial u} g_2(u) \right] \right|_{u=1-\bar{\eta}},$$  

(12)

$$f^0(q^2) = -\frac{f_\pi\sqrt{T}}{2\pi m_B\Lambda_B} \int_0^{s_0} ds e^{2\lambda_B - s}/T \times \frac{1}{y^2} \left[ g_2(u) - \frac{m_B}{y} \left[ -\phi_{2;\pi}(u) \right.$$

$$+ \left( \frac{1}{y} \frac{\partial}{\partial u} \right)^2 g_1(u) - \frac{1}{y^2} \frac{\partial}{\partial u} g_2(u) \right] \right|_{u=1-\bar{\eta}},$$  

(13)

The Borel parameter $T$ and the continuum threshold $s_0$ are determined such that the resulting form factor does not depend too much on the precise values of these parameters; in addition the continuum contribution, which is the part of the dispersive integral from $s_0$ to $\infty$ that has
III. NUMERICAL ANALYSIS

A. Input parameters

As for the input parameters, the masses of pion, B-meson and b quark are taken as $m_{\pi^+} = 0.1395$ GeV, $m_B = 5.279$ GeV and $m_b = 4.75 \pm 0.05$ GeV, which are from the particle data group (PDG) \cite{40}. As for the decay constant, we take the pion decay constant $f_\pi = 0.1304$ GeV \cite{40} and the leading order $B$-meson effective decay constant $F = 0.34 \pm 0.04$ GeV$^{3/2}$ \cite{41}. The factorization scale $\mu$ is set as typical momentum of the $B$ meson decays, e.g. $\mu_0 = (m_B^2 - m_\pi^2)^{1/2} \approx 2.4$ GeV, and we set its error as $\Delta \mu = \pm 1$ GeV.

For the twist-2 LCDA $\phi_{2,\pi}(u, \mu^2)$, we adopt the Brodsky-Huang-Lepage (BHL) model \cite{42} to do this calculation. The BHL prescription of the hadronic wave function (WF) is obtained by connecting the equal-time WF in the rest frame and the WF in the infinite momentum frame. Meanwhile this LCDA can mimic the DA behavior from asymptotic-like to CZ-like naturally. Then with the experimental data on various processes, one may decide which is the right DA behavior possessed by the light pseudo-scalar mesons. And refs. \cite{43, 44} suggested that considering BHL model with the Wigner-Melosh rotation effects, they can get meaningful results which coincide with the experiment constrains. Furthermore, the process of the calculation of pion twist-2 LCDA can be found in the ref. \cite{42}. After integrating the transverse momentum part out, the twist-2 LCDA takes the form

\[
\phi_{2,\pi}(u, \mu^2) = \frac{\sqrt{3} A m_\pi \beta}{2 \pi^{3/2} f_\pi} \sqrt{u \bar{u}} \varphi(u) \\
\times \left\{ \text{Erf} \left( \sqrt{\frac{m_\pi^2 + \mu^2}{8 \beta^2 u \bar{u}}} \right) - \text{Erf} \left( \sqrt{\frac{m_\pi^2}{8 \beta^2 u \bar{u}}} \right) \right\},
\]

(14)

where $\bar{u} = 1 - u$, $\xi = 2u - 1$, $\varphi(u) = 1 + B \times C_2^{3/2}(\xi)$ and $\text{Erf}(x) = 2 \int_0^x e^{-t^2} dt / \sqrt{\pi}$. The light constitute quark mass $m_q \approx 0.30$ GeV, and the remaining three parameters $A$, $\beta$ and $B$ can be fixed by using three constraints \cite{42, 45}:

(i) The pion wave function normalization condition,

\[
\int_0^1 dx \int \frac{d^2 k_\perp}{16 \pi^3} \Psi_\pi(x, k_\perp) = \frac{f_\pi}{2\sqrt{6}}.
\]

(ii) The sum rule derived from $\pi^0 \to \gamma \gamma$ decay amplitude implies,

\[
\int_0^1 dx \Psi_\pi(x, k_\perp = 0) = \frac{\sqrt{3}}{f_\pi}.
\]

(iii) Generally, the twist-2 LCDA can be expanded as a Gegenbauer polynomial. We can get the second moment $a_2$ by using the orthogonality relation,

\[
a_n(\mu^2) = \frac{\int_0^1 du \phi_{2,\pi}(u, \mu^2) C_n^{3/2}(2u - 1)}{\int_0^1 du \bar{u} |C_n^{3/2}(2u - 1)|^2}.
\]

TABLE I: Parameters of the twist-2 LCDA $\phi_{2,\pi}(u, \mu_0^2)$ determined for $a_2(\mu_0^2) = 0.112 \pm 0.075$ \cite{12}.

| $a_2(\mu_0^2)$ | $A$ | $\beta$ | $B$ |
|----------------|-----|---------|-----|
| 0.039          | 23.50 | 0.61    | 0.075 |
| 0.112          | 24.63 | 0.59    | 0.010 |
| 0.185          | 23.50 | 0.63    | 0.141 |

FIG. 1: The behavior of LCDA $\phi_{2,\pi}(x, \mu_0^2)$ predicted by BHL model, in which the uncertainties are coming from all mentioned input parameters. As a comparison, we also present the JLab fit \cite{13}, QCD SR \cite{29}, CZ form \cite{46} and the Asymptotic form \cite{47}.

The Gegenbauer moment $a_2$ can be fixed via a global fit of various experimental data involving pion, and we adopt the result given in Ref. \cite{13}, e.g. $a_2(\mu^2 = 1\text{GeV}^2) = 0.112 \pm 0.075$. The determined parameters at the scale of 1GeV are presented in Table. I and one can get their values at any other scale by using the renormalization group evolution. At present, there is no definite conclusion on whether the pion LCDA $\phi_{2,\pi}(u, \mu_0^2)$ is CZ-form \cite{46}, asymptotic form \cite{47} or even flat-like \cite{48}. We present the twist-2 DAs in Fig. I where the asymptotic form, the CZ-form, the QCD SVZ sum rule prediction \cite{20} and the prediction by A. Khodjamirian et al. \cite{13} via using the Jefferson Lab data on $F_\pi$ are presented as a comparison. By varying $a_2$, the $\phi_{2,\pi}$ shape shall varies from a single peaked behavior to double humped behavior; The present adopted BHL-model provides a convenient way.
to mimic the pion twist-2 LCDA behavior and we shall adopt it together with error to discuss the theoretical uncertainty caused by the different choice of $\phi_{z;\pi}$.

### B. The $B \to \pi$ TFFs

As for the $B \to \pi$ TFFs $f^+/0(q^2)$, the allowable range of the Borel parameter $T$, and the effective continuous threshold $s_0$ are determined from the following three conditions:

- The continuum contribution is not higher than 35% of the total LCSR;
- All high-twist LCDAs contribution does not exceed 50% of the total LCSR;
- The derivatives of LSRs for TFFs with respect to $-1/T$ give LCSR for the $B$-meson mass $m_B$: we require the estimated $B$-meson mass to be fulfilled in comparing with the experiment one, e.g. $|m^b_{\exp} - m_B|/m^b_{\exp}$ less than 0.1%.

The flatness of the TFFs versus the Borel parameter $T$ provides an even weaker constraint for the range of $T$. With these criteria for the LSRs, the determined continuum threshold $s_0$ and the Borel parameter $T$ for the $B \to \pi$ vector TFF $f^+(q^2)$ are $s_0 = 2.0 \pm 0.1$ GeV$^2$ and $T = 1.4 \pm 0.1$ GeV$^2$ for the LCSR-$R$; and $s_0 = 2.2 \pm 0.1$ GeV$^2$ and $T = 1.5 \pm 0.1$ GeV$^2$ for the LCSR-$U$.

| Total | $\Delta_T$ | $\Delta\Delta$ | $\Delta T$ | $\Delta s_0$ | $\Delta(m_B; F)$ |
|-------|-------------|----------------|------------|--------------|----------------|
| $f^+_R(0)$ | 0.276 $^{+0.022}_{-0.026}$ | $^{+0.000}_{-0.002}$ | $^{+0.017}_{-0.016}$ | $^{+0.001}_{-0.001}$ | $^{+0.011}_{-0.011}$ |
| $f^+_U(0)$ | 0.282 $^{+0.020}_{-0.020}$ | $^{+0.000}_{-0.000}$ | $^{+0.009}_{-0.008}$ | $^{+0.002}_{-0.001}$ | $^{+0.007}_{-0.007}$ |

TABLE II: Uncertainties of the TFFs $f^+_R/U(q^2)$ at $q^2 = 0$. The uncertainties of total TFFs from the mentioned input parameters are summed up in quadrature.

In Table. III one show how the errors of the TFFs caused by the uncertainties of the parameters, the factorization scale $\mu$, the Borel parameter $T$, the continuum threshold $s_0$, the $b$-quark mass $m_b$, the $B$-meson effective decay constant $F$, and the twist-2 LCDA. Table III shows that the main errors come from the choices of $m_b$, $F$ and LCDA, whose effect could be up to $\sim 12\%$. Because $m_b$ and $F$ are correlated with each other, e.g. a larger $m_b$ could result in a smaller $F$ [30], we change their value simultaneously to discuss their effect to the TFFs.

We present the contributions of different twist terms to the dominant $B \to \pi$ TFFs $f^+_R/U$ at the large recoil region $q^2 \approx 0$ GeV$^2$ in Table III. For the case of usual correlator $F^+_R$, contributions of the twist-3 terms are about 45% of the TFF $f^+_R(0)$. The twist-3 DAs are not known as well as the DAs of twist-2. And for the case of U correlator, Table III shows that the twist-2 and twist-3 terms have same size; thus, by using the R correlator, one can achieve a much better accuracy LCSR for the transition form factor. Table II shows the errors for the case of U correlator are seemingly small, this is because we do not include the uncertainty from twist-3 DAs in the present prediction. And twist-3 DAs uncertainties is not the point which this study focuses on. Meanwhile by using the chiral current correlator, we could get the precision twist-2 DAs information, in turn with the experiment data the twist-2 DAs would be determined exactly. And it is hard to distinguish the information of twist-2 or twist-3 in the usual current, because their contribution are the same size and blend with each other. While for the case of the right-handed chiral correlator $F^+_R$, contributions from the twist-3 terms are eliminated. Moreover, the twist-4 terms are negligible for both cases. Thus by taking the chiral correlator, uncertainties from the higher-twist LCDAs are greatly suppressed, leading to a more precise LCSR prediction. By summing the contributions from various twist terms, the total TFFs for both correlators agree well with each other with errors $^1$. The LCSR approximant should not depend on the choice of chiral correlator, and our present prediction for the TFFs provide one of such examples.

The LCSR prediction is valid when the final-state pion has large energy in the rest-system of $B$-meson, e.g. $E_\pi \gg \Lambda_{QCD}$. Using the relation, $q^2 = m_B^2 - 2m_BE_\pi$, we adopt a conservative range, $0 < q^2 < 12$ GeV$^2$, as the applicable range of the LCSR method. Thus before dealing with the $B \to \pi$ semi-leptonic decay, we need to extend the TFFs to the physically allowable range. We use the $z$-series parametrization to extrapolate the calculated TFFs to the physically allowable $q^2$-region [13], e.g.

$$f^+(q^2) = \frac{f^+(0)}{1 - q^2/m_B^2} \left\{ 1 + \sum_{k=1}^{N-1} b_k \left[ \frac{z(q^2)^k - z(0)^k}{N^k \left( z(q^2)^N - z(0)^N \right) - (-1)^N \frac{k}{N^k} \left( z(q^2)^N - z(0)^N \right) \right] \right\}$$

and

$$f^0(q^2) = f^0(0) \left\{ 1 + \sum_{k=1}^{N-1} b_k \left( z(q^2)^k - z(0)^k \right) \right\}. \quad (19)$$

$^1$ The differences between their central values, whose magnitude is comparable to the magnitude of the twist-4 terms, could be treated as the difference of the uncalculated high-twist terms.
The function $z(q^2)$ is defined as

$$z(q^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}},$$

(20)

where $t_\pm = (m_B \pm m_\pi)^2$, $t_0 = t_+ (1 - \sqrt{1 - t_+/t_-})$. The coefficients $b_0 = f^+(0)$, $b_1$, $b_2$ and $b_3$ are determined such that the quality of fit ($\Delta$) is less than 1%. The quality of fit is defined as:

$$\Delta = \frac{\sum |f^i - f^{\text{fit}}|}{\sum |f^i|} \times 100\%, t \in \{0, \frac{1}{2}, \ldots, \frac{23}{2}, 12\} \text{ GeV}^2.$$ 

| $b_1$ | $b_2$ | $b_3$ | $\Delta$ |
|-------|-------|-------|---------|
| $-1.600$ | $-1.453$ | - | $0.03\%$ |
| $-1.155$ | $2.075$ | $-3.377$ | $0.01\%$ |
| $-1.309$ | $-1.757$ | - | $0.50\%$ |
| $-1.197$ | $0.656$ | $-0.611$ | $0.06\%$ |

TABLE IV: Fitted parameters and qualities of fit of LCSR-$R$ and LCSR-$U$ TFFs $f^{+0}(q^2)$.

The determined parameters for the $z$-series and the quality of fit are listed in Table IV.

The extrapolated HQEFT LCSR for the $B \rightarrow \pi$ TFFs $f^{+0}(q^2)$ are presented in Fig.2, where left/right diagram stands for the LCSR-$R$/U, respectively. In Fig.2, the solid lines are TFFs with all parameters to be their central values, and the shade bands are TFF errors that are squared averages of the errors caused by all the error sources. Our predictions within errors for $f^{+0}(q^2)$ are consistent with the Lattice QCD predictions obtained by the HPQCD and Fermilab/MILC collaborations.

We adopt the correlation coefficient $\rho_{XY}$ to measure to what degree the TFFs under different choices of chiral correlators are correlated with each other, which is defined as

$$\rho_{XY} = \frac{\text{Cov}(X, Y)}{\sigma_X \sigma_Y},$$

(21)

where $X$ and $Y$ stand for the corresponding LCSR-$R$ and LCSR-$U$ TFFs, respectively. The covariance is $\text{Cov}(X, Y) = E[(X - E(X))(Y - E(Y))] = E(XY) - E(X)E(Y)$, where $E$ is the expectation value of $X$ or $Y$. $\sigma_X$ and $\sigma_Y$ are standard deviations of $X$ and $Y$. The range of correlation coefficient $|\rho_{XY}|$ is $0 \sim 1$. A larger $|\rho_{XY}|$ indicates a higher consistency between $X$ and $Y$. The correlation coefficient $|\rho_{R,U}|$ of LCSR-$R$ and LCSR-$U$ is 0.85 for the vector TFF $f^+(q^2)$, which changes to 0.52 for the scalar TFF $f^0(q^2)$. The correlation coefficient of both TFFs are larger than 0.5, implying those TFFs are consistent with each other.

C. The $B \rightarrow \pi\ell\nu_\ell$ differential branching fraction, the matrix element $|V_{ub}|$ and the ratio $R_\pi$

The $B \rightarrow \pi\ell\nu_\ell$ differential decay widths can be written as

$$\frac{d\Gamma(B \rightarrow \pi\ell\nu_\ell)}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{192\pi^3 m_B^3} \lambda^{3/2}(q^2) |f_+(q^2)|^2 \times \left[ \frac{(q^2 - m_\pi^2)^2}{q^2} - \frac{(m_B^2 - m_\pi^2 - q^2)^2}{4q^2} \right] - m_\pi^2 \left\{ (m_\pi^2 + 2q^2) \right\},$$

(22)

where $m_l$ is the lepton mass. For the light lepton $l = e$ or $\mu$, the lepton mass can be safely neglected, and the differential decay width changes to

$$\frac{d\Gamma(B \rightarrow \pi\ell\nu_\ell)}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{192\pi^3 m_B^3} \lambda^{3/2}(q^2) |f_+(q^2)|^2,$$

(23)

where the $|V_{ub}|$ is the CKM matrix element, $G_F$ is the Fermi-coupling constant, and the phase-space factor $\lambda(q^2) = (m_B^2 + m_\pi^2 - q^2)^2 - 4m_B^2 m_\pi^2$. The branching fraction has the relation with the decay width, $d\Gamma/dq^2 = d\Gamma(B \rightarrow \pi\ell\nu_\ell)/dq^2 \times \tau_{B^0}$, where the total lifetime $\tau_{B^0} = 1.525 \pm 0.009$ ps [40]. Fig.4 indicates that both the LCSR-$R$ and the LCSR-$U$ predictions together with their errors (shown by shaded bands) are consistent with the BABAR and BELLE data.

Experimentally, people have measured the integrated branching fraction, $\Delta B(0, q^2_f) = \int_0^{q^2_f} (dB/dq^2) dq^2$ [53][54],

$$\Delta B(0, 12\text{GeV}^2) = (0.83 \pm 0.03 \pm 0.04) \times 10^{-4},$$

(BABAR 2012) (24)

$$\Delta B(0, 12\text{GeV}^2) = (0.808 \pm 0.06) \times 10^{-4},$$

(Belle 2013) (25)

Using their weighted averages, we can inversely determine the value of $|V_{ub}|$, e.g.

$$|V_{ub}|_R = (3.38^{+0.22}_{-0.16} \pm 0.12_{\text{exp}}) \times 10^{-3},$$

(27)

where the first error is squared average of those from the
FIG. 2: The extrapolated $B \to \pi$ TFFs $f^{+/0}(q^2)$ calculated under two different correlators, where the left/right one stands for the LCSR-$\mathcal{R}/\mathcal{U}$, respectively. The shaded bands are uncertainties from the mentioned error sources. As a comparison, we also present the predictions coming from Lattice QCD [7], LCSR [13, 17], pQCD [13] and HPQCD [51].

FIG. 3: The $B \to \pi\ell\bar{\nu}_\ell$ differential branching fractions for two different correlators, in which the left/right one stands for LCSR-$\mathcal{R}/\mathcal{U}$, respectively. As a comparison, we also present the BABAR data [53, 55], and the Belle data [54, 56].

input parameters and the second error is the experimental error of $\Delta B(0, 12\text{GeV}^2)$.

A comparison of $|V_{ub}|$ under various approaches is presented in Table. [V] Most of the predictions are consistent with each other within errors. Our HQEFT LCSR predictions show good agreement with the Belle measurements [54], as well as the usual LCSR predictions [13, 17].

The correlation coefficient for the branching fractions under the LCSR-$\mathcal{R}$ and LCSR-$\mathcal{U}$ TFFs is, $\rho_{\mathcal{RU}} = 0.85$, which confirms the previous observation that the physical observable should be independent to the choices of correlator. More explicitly, Table [V] shows a comparison of the branching fraction of $B \to \pi\ell\bar{\nu}_\ell$ with various experimental measurements, which shows that both LCSR-$\mathcal{U}$ and LCSR-$\mathcal{R}$ predictions are consistent with the data.

As a final remark, it is more useful to measure the following ratio, which avoids uncertainties from the input parameters such as $|V_{ub}|$ and could be used for a precision test of SM or to find new physics beyond SM,

$$R_\pi \equiv \frac{B(B \to \pi\tau\bar{\nu}_\tau)}{B(B \to \pi\ell\bar{\nu}_\ell)} = \frac{\int_{q^2_{\text{min}}}^{q^2_{\text{max}}} d\Gamma(B \to \pi\tau\bar{\nu}_\tau)/dq^2}{\int_{0}^{q^2_{\text{max}}} d\Gamma(B \to \pi\ell\bar{\nu}_\ell)/dq^2}.$$  \hspace{1cm} (28)

This ratio strongly depends on the behavior of the $B \to \pi$ TFFs. Roughly, due to the phase-space suppression, the decay width of $B \to \pi\tau\bar{\nu}_\tau$ shall be smaller than that of
which are proportional to of the magnitude of the chiral symmetry breaking terms for the TFFs, we obtain \( R_\pi \approx 0.95^{+0.08}_{-0.09} \) and \( R_{\pi}^{2\text{HDM}+\nu} = 1.02^{+0.01}_{-0.06} \). In this ref. \cite{62}, the author used the latest \( \sqrt{s} = 13 \) TeV CMS results to impose constraints on the charged Higgs \( H^\pm \) parameters within the (2HDM). There are still some parameter space for the 2HDM to live. Thus we get the same as that of Ref. \cite{62} that there may have new physics beyond the SM. However since the Belle measurements still have large errors, we still more accurate measurements to confirm whether this is really the case.

### IV. SUMMARY

In the paper, we have made a detailed LCSR analysis on the \( B \to \pi \ell \bar{v}_\ell \) TFFs within the framework of HQEFT. As shown by Fig.\( \ref{fig2} \), the LCSR-\( \mathcal{U} \) and the LCSR-\( \mathcal{R} \) for two different correlators are highly correlated with each other, e.g. \( \rho_{\mathcal{R}\mathcal{U}} > 0.5 \), especially for the \( f^+(q^2) \), which provides dominant contributions to the \( B \to \pi \) semileptonic decays. The LCSR method is applicable in low and intermediate \( q^2 \) regions and the Lattice QCD method is applicable in large \( q^2 \) region. After extrapolation, our HQEFT LCSR predictions on the TFFs agree well with the Lattice QCD predictions derived by HPQCD and Fermilab/MILC Collaborations. As a byproduct, by using the chiral correlator, the twist-2 LCDA provides over 97% contribution to the TFFs, thus it provides a good platform for testing the properties of pion twist-2 LCDA. Moreover, by using the LCSR-\( \mathcal{U} \) and LCSR-\( \mathcal{R} \) TFFs, we obtain, \( |V_{ub}|/R = (3.45^{+0.28}_{-0.20} \pm 0.13_{\text{exp}}) \times 10^{-3} \) and \( |V_{ub}|/R = (3.38^{+0.12}_{-0.16} \pm 0.12_{\text{exp}}) \times 10^{-3} \), both of which are in accordance with experimental results within errors. The ratio \( R_\pi \) is a useful parameter to test the SM and confirm the new physics beyond the SM.

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Appendix: the nonlocal matrix elements and the twist-3 and 4 LCDAs

The relations between the pion LCDAs up to twist-4 accuracy and the nonlocal matrix elements are

\[
\langle \pi(p)|\bar{u}(x)\gamma_\mu\gamma_5d(0)|0\rangle = -ip_\mu f_{\pi} \int_0^1 du\epsilon^{up}\cdot x [\phi_{2,\pi}(u)]
\]

\[+ x^2 g_1(u) + f_\pi(x_\mu - \frac{x^2 p_\mu}{p\cdot x}) \int_0^1 du\epsilon^{up}\cdot x g_2(u), \tag{31}\]

\[
\langle \pi(p)|\bar{u}(x)i\gamma_5d(0)|0\rangle = \frac{f_\pi m_\pi^2}{m_u + m_d} \int_0^1 du\epsilon^{up}\cdot x \phi_{3,\pi}^0(u),
\]

\[\times \int_0^1 du\epsilon^{up}\cdot x \phi_{3,\pi}^0(u). \tag{32}\]

Here \(f_\pi\) is the pion decay constant, \(\phi_{2,\pi}(u)\), \(\phi_{3,\pi}^0(u)\), and \(\phi_{3,\pi}^0(u)\) are pion twist-2, 3 and 4 LCDAs, respectively. The twist-3 LCDAs take the form

\[
\phi_{3,\pi}^0(u) = 1 + \frac{1}{2} B_4(3\xi^2 - 1) + \frac{1}{8} B_4(35\xi^4 - 40\xi^2 + 3),
\]

\[
\phi_{3,\pi}^0(u) = 6\bar{u}u \left[ 1 + \frac{3}{2}C_2(5\xi^2 - 1) + \frac{15}{8}C_4(21\xi^4 - 14\xi^2 + 1) \right]. \tag{34}\]

where the parameters are [28, 29, 63, 64], \(B_2(\mu_5) = 0.29\), \(B_4(\mu_5) = 0.58\), \(C_2(\mu_5) = 0.059\) and \(C_4(\mu_5) = 0.034\). As for the twist-4 LCDAs, we adopt the ones derived by Ref. [31], e.g.

\[
g_1(u) = \frac{\bar{u}u}{6} [-5\bar{u}u(9h_{00} + 3h_{01} - 6h_{10} + 4h_{01}u + 10\bar{u}u) \times h_{10}u + a_{10}(6 + \bar{u}u(9 + 80\bar{u}u))] + a_{10}\bar{u}^3(10 - 15\bar{u} + 6u^2)\ln u + a_{10}u^5(10 - 15u + 6u^2)\ln u,
\]

\[
g_2(u) = \frac{5\bar{u}u(u - \bar{u})}{2} [4h_{00} + 8a_{10}\bar{u}u - h_{10}(1 + 5\bar{u}u) + 2h_{01}(1 - \bar{u}u)], \tag{35}\]

where

\[
h_{00} = -\frac{\delta^2}{3} \quad a_{10} = \delta^2\epsilon - \frac{9}{20}a_2^2m_\pi^2,
\]

\[
h_{01} = 2\frac{\delta^2\epsilon - \frac{3}{20}a_2^2m_\pi^2}{v_{10}} \quad v_{10} = \delta^2\epsilon,
\]

\[
h_{10} = 4\frac{\delta^2\epsilon + \frac{3}{20}a_2^2m_\pi^2}{v_{10}}
\]

where \(\epsilon(\mu_5) = 0.36\), \(\delta^2(\mu_5) = 0.17\text{GeV}^2\). 

[1] For the update of the average of \(\mathcal{R}(D)\) one can see the web page: \url{https://hflav-eos.web.cern.ch/hflav-eos/semi/summer18/RDLDAs.html}

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