A novel integrated approach to the modelling and solving of the
Two-Echelon Capacitated Vehicle Routing Problem

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The paper presents a concept and implementation of a novel approach to the modeling and solving of the Two-Echelon Capacitated Vehicle Routing Problem (2E-CVRP). Multi-echelon distribution systems are quite common in supply-chain and logistic systems. Two environments, mathematical programming (MP) and constraint logic programming, in which constraints are treated in different ways and different methods are implemented, were combined to use the strengths of both. The proposed approach is particularly important for the decision models with an objective function and many discrete decision variables added up in multiple constraints. The 2E-CVRP is an extension of the classical Capacitated Vehicle Routing Problem (CVRP) where the delivery depot-customers pass through intermediate depots (called satellites). The 2E-CVRP was selected as a known and documented example for verification and evaluation of the effectiveness of the proposed approach. The presented approach will be compared with classical MP on the same data-sets in computational tests. The proposed approach will be used also for extensions of the modelled problem beyond the standard.

Keywords: vehicle routing; multi-echelon systems; constraint programming; mathematical programming; optimization

1. Introduction

There are two main distribution strategies: direct shipping and multi-echelon distribution. In the direct shipping, vehicles, starting from a depot, bring goods directly to demand points, while in the multi-echelon systems, the goods are delivered from the depot to the customers through intermediate points.

In two-echelon distribution systems, goods are delivered to an intermediate depot and, from this depot, to the customers.

The majority of multi-echelon systems presented in the literature usually explicitly consider the routing problem at the last level of the transportation system, while a simplified routing problem is considered at higher levels (Verrijdt & de Kok, 1995).

In recent years multi-echelon systems have been used in:

- Logistics enterprises and express delivery service companies.
- Hypermarkets products distribution.
- Multimodal freight transportation and supply chains.

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- E-commerce and home delivery services.
- City and public logistics.

The most of models of decision support and/or optimization in freight transportation and logistics industry have been formulated as the mixed integer programming (MIP) or mixed integer linear programming (MILP) problems and solved using the operations research (OR) methods (Kumar & Panneerselvam, 2012). Their structures are similar and proceed from the principles and requirements of mathematical programming (MP; Schrijver, 1998; Sitek & Wikarek, 2012b). It seems that better results will be obtained by the use of the constraint programming environments (CP/CLP) especially in modelling. The CP-based environments have the advantage over traditional methods of mathematical modelling in that they work with a much broader variety of interrelated constraints and allow producing ‘natural’ solutions for highly combinatorial problems. The CP/CLP environments have declarative nature (Apt & Wallace, 2006; Sitek & Wikarek, 2008).

The main contribution of this paper is novel approach (mixed CLP with MILP and problem transformation) and implementation (Integrated Solution Framework [ISF]) to modelling and optimization Two-Echelon Capacitated Vehicle Routing Problem (2E-CVRP) or a similar problem. In addition, some extensions and modifications to the standard 2E-CVRP, which are difficult and impossible to implement for the classical approach (MILP) are also presented.

Well known and documented in the literature, 2E-CVRP problem was chosen to test the proposed framework. The sets of test data (benchmarks) for 2E-CVRP are known. This study compares the integrated approach with the classical approach based on the same data and under the same calculation conditions.

The paper is organized as follows. In Section 2, the literature related to Multi-Echelon Vehicle Routing Problems has been reviewed. Next section is about motivation and contribution. In Section 4, the concept of novel approach to modelling and solving, and the solution environment have been presented. Then, the general description of Multi-Echelon Vehicle Routing Problems and mathematical model of 2E-CVRP has been discussed. Finally, test instances for 2E-CVRP and some computational results are discussed in Section 6.

2. Literature overview

The Vehicle Routing Problem (VRP) is used to design an optimal route for a fleet of vehicles to service a set of customers’ orders (known in advance), given a set of constraints. The VRP is used in supply chain management in the physical delivery of goods and services. The VRP is of the NP-hard type.

Nowadays, the VRP literature offers a wealth of heuristic and metaheuristic approaches, which are surveyed in the papers of (Bocewicz & Banaszak, 2013; Kumar & Panneerselvam, 2012; Perboli, Tadei, & Vigo, 2011) because exact VRP methods have a size limit of 50–100 orders depending on the VRP variant and the time-response requirements.

There are several variants and classes of VRP like the capacitated VRP (CVRP), VRP with Time Windows and Dynamic Vehicle Routing Problems, sometimes referred to as Online Vehicle Routing Problems, etc. (Kumar & Panneerselvam, 2012).

Different distribution strategies are used in freight transportation. The most developed strategy is based on the direct shipping: freight starts from a depot and arrives...
directly to customers. In many applications and real situations, this strategy is not the best one and the usage of a multi-echelon and particular two-echelon distribution system can optimize several features as the number of the vehicles, the transportation costs, loading factor and timing.

In the literature, the multi-echelon system and the two-echelon system, in particular, refer mainly to supply chain and inventory problems (Verrijdt & de Kok, 1995). These problems do not use an explicit routing approach for the different levels, focusing more on the production and supply chain management issues. The first real application of a two-tier distribution network optimizing the global transportation costs is due to (Jain & Grossmann, 2001) and is related to the city logistics area. They developed a two-tier freight distribution system for congested urban areas, using small intermediate platforms, called satellites (intermediate points for the freight distribution). This system is developed for a specific situation and a generalization of such a system has not already been formulated. The complete mathematic model of 2E-CVRP with the solution for sample test data in the classical approach has been proposed by Perboli et al. (2011), complemented with the method for boosting the computing efficiency (see Section 5).

3. CP-based environments

One of the most promising CP-based environments is constraint logic programming (CLP). CLP is a paradigm which represents a successful attempt to merge the best features of logic programming (LP) and constraint solving.

CLP is also a tool for solving constraint satisfaction problems (CSP) (Kumar & Panneerselvam, 2012; Verrijdt & de Kok 1995). For the important combinatorial case, CSP is characterized by following features:

- a finite set $S$ of integer variables $X_1, \ldots, X_n$, with values from finite domains $D_1, \ldots, D_n$;
- a set of constraints between variables. The $i$-th constraint $C_i(X_{i1}, \ldots, X_{ik})$ between $k$ variables from $S$ is given by a relation defined as subset of the Cartesian product $D_{i1} \times \ldots \times D_{ik}$ that determines variable values corresponding to each other in a sense defined by the problem considered;
- a CSP solution is given by any assignment of domain values to variables that satisfies all constraints.

The semantics of constraint logic programs can be defined in terms of a virtual interpreter that maintains a pair $<G, S>$ during execution. The first element of this pair is called current goal; the second element is called constraint store.

The current goal contains the literals the interpreter is trying to prove and may also contain some constraints it is trying to satisfy; the constraint store contains all constraints the interpreter has assumed satisfiable so far. At the beginning, the current goal is the goal and the constraint store is empty. The interpreter proceeds by removing the first element from the current goal and analysing it. In the end, this analysis should produce a successful termination or a failure. This analysis could involve recursive calls and addition of new literals to the current goal and new constraint to the constraint store. The interpreter backtracks if a failure is generated. A successful termination is generated when the current goal is empty and the constraint store is satisfiable. CLP can use Artificial Intelligence techniques to improve the search: constraint propagation, data-driven computation, ‘forward checking,’ and ‘lookahead’ (Verrijdt & de Kok, 1995).
CLP is a form of CP, in which LP is extended to include concepts from constraint satisfaction. A constraint logic program is a logic program that contains constraints in the body of clauses. Constraints can also be present in the goal. These environments are declarative.

4. Motivation

Based on our previous work Sitek and Wikarek (2008, 2012a, 2012b, 2013), Perboli et al. (2011), Relich (2011), Kumar and Panneerselvam (2012), Bocewicz and Banaszak (2013), and Dang and Nielsen (2013), we observed some advantages and disadvantages of both (CLP/MILP) environments.

An integrated approach of constraint programming (CP/CLP) and mixed integer programming (MIP/MILP) can help to solve optimization problems that are intractable with either of the two methods alone (Achterberg, Berthold, Koch, & Wolter, 2008; Jain & Grossmann, 2001; Milano & Wallace, 2010). Although OR and Constraint Programming (CP) have different roots, the links between the two environments have grown stronger in recent years.

Both MIP/MILP and finite domain CP/CLP involve variables and constraints. However, the types of the variables and constraints that are used, and the way the constraints are solved, are different in the two approaches (Achterberg et al., 2008).

MIP/MILP relies completely on linear equations and inequalities in integer variables, i.e. there are only two types of constraints: linear arithmetic (linear equations or inequalities) and integrity (stating that the variables have to take their values in the integer numbers). In finite domain CP/CLP, the constraint language is richer. In addition to linear equations and inequalities, there are various other constraints: disequalities, non-linear, symbolic (all different, disjunctive, cumulative etc.). In both MIP/MILP and CP/CLP, there is a group of constraints that can be solved with ease and a group of constraints that are difficult to solve.

The easily solved constraints in MIP/MILP are linear equations and inequalities over rational numbers. Integrity constraints are difficult to solve using MP methods and often the real problems of MIP/MILP make them NP-hard.

In CP/CLP, domain constraints with integers and equations between two variables are easy to solve. The system of such constraints can be solved over integer variables in polynomial time. The inequalities between two variables, general linear constraints (more than two variables), and symbolic constraints are difficult to solve, which make real problems in CP/CLP NP-hard. This type of constraints reduces the strength of constraint propagation. As a result, CP/CLP is incapable of finding even the first feasible solution.

Both approaches use various layers of the problem (methods, the structure of the problem, data) in different ways. The approach based on MP (MIP/MILP) focuses mainly on the methods of optimization and, to a lesser degree, on the structure of the problem. However, the data is completely outside the model. The same model without any changes can be solved for multiple instances of data. In the approach based on constraint programming (CP/CLP), due to its declarative nature, the methods are already built-in. The data and structure of the problem are used for its modelling in a significantly greater extent.

The motivation and contribution behind this work were to create a novel integrated method for constrained decision problems modelling and optimization instead of using MP or constraint programming separately.
It follows from the above that what is difficult to solve in one environment can be easy to solve in the other. Moreover, such an integrated approach allows the use of all layers of the problem to solve it.

The integrated method is not inferior to its component elements applied separately. This is due to the fact that the number of decision variables and the search area are reduced. The extent of the reduction directly affects the effectiveness of the method.

Unlike approaches presented in the literature (Hooker, 2011; Milano & Wallace, 2010), we integrate and transform the problem at the same time. In addition, the integration and transformation are performed at the implementation level using the proposed framework (Section 5).

In our approach to these problems we proposed the modelling, solving and optimization framework, where:

- knowledge related to the problem can be expressed as linear, logical and symbolic constraints;
- the decision models solved using the proposed framework can be formulated as a pure model of MIP/MILP or of CP/CLP, or it can also be an integrated (hybrid) model;
- the problem is modelled in the constraint programming environment, which is far more flexible than the MP environment;
- transforming the decision model to explore its structure has been introduced;
- constrained domains of decision variables, new constraints and values for some variables are transferred from CP/CLP into MILP/MIP/IP; and
- the efficiency of finding solutions to larger size problems is increased.

As a result, a more effective solution environment for a certain class of decision and optimization problems (2E-CVRP or similar) was obtained.

5. Integrated solution framework

Both environments have advantages and disadvantages. Environments based on the constraints such as CLPs are declarative and ensure a very simple modelling of decision problems, even those with poor structures if any. The problem is described by a set of logical predicates. The constraints can be of different types (linear, non-linear, logical, binary, etc.). The CLP does not require any search algorithms. This feature is characteristic of all declarative backgrounds, in which modelling of the problem is also a solution, just as it is in Prolog, SQL, etc. The CLP seems perfect for modelling any decision problem. Numerous MP models of decision-making have been developed and tested, particularly in the area of decision optimization. Constantly improved methods and MP algorithms, such as the simplex algorithm, branch and bound, branch-and-cost, etc. have become classics now.

The proposed method’s strength lies in high efficiency of optimization algorithms and a substantial number of tested models. Traditional methods when used alone to solve complex problems provide unsatisfactory results. This is related directly to different treatment of variables and constraints in those approaches (Section 3). This schema of the ISF and the concept of this framework with its phases (PH1–PH5, PHG1–PHG3) is presented in Figure 1. The names and descriptions of the phases and the implementation environment are shown in Table 1.
From a variety of tools for the implementation of the CP-based environment in ISF, ECLiPSe software (www.eclipse.org, 2013) was selected. ECLiPSe is an open-source software system for the cost-effective development and deployment of constraint programming applications. MP-based environment in ISF was LINGO by LINDO Systems (www.lindo.com, 2013). LINGO Optimization Modelling Software is a powerful tool for building and solving mathematical optimization models. ECLiPSe software is the environmental leader in ISF. ECLiPSe was used to implement the following phases of the framework: PH1, PH2, PH3, PHG1, PHG2, PHG3 (Figure 1 and Table 1).

6. Two-Echelon Capacitated Vehicle Routing Problem
The 2E-CVRP was chosen to assess and evaluate the proposed approach and its implementation deliberately, as it is known in the literature and its models are published as
The 2E-CVRP is an extension of the classical Capacitated Vehicle Routing Problem (CVRP) where the delivery depot-customers pass through intermediate depots (called satellites). As in CVRP, the goal is to deliver goods to customers with known demands, minimizing the total delivery cost in the respect of vehicle capacity constraints. Multi-echelon systems presented in the literature usually explicitly consider the routing problem at the last level of the transportation system, while a simplified routing problem is considered at higher levels (Crainic, Ricciardi, & Storchi, 2004; Perboli et al., 2011).

Table 1. Description of phases.

| Phase | Name | Description |
|-------|------|-------------|
| PH1   | Implementation of decision model – [environment: CLP] | The implementation of the model in CLP, the term representation of the problem in the form of CLP predicates |
| PH2   | Transformation of implemented model for better constraint propagation (optional) – [environment: CLP] | The transformation of the original problem aimed at extending the scope of constraint propagation. The transformation uses the structure of the problem. The most common effect is a change in the representation of the problem by reducing the number of decision variables, and the introduction of additional constraints and variables, changing the nature of the variables, etc |
| PH3   | Constraint propagation – [environment: CLP] | Constraint propagation for the model. Constraint propagation is one of the basic methods of CLP. As a result, the variable domains are narrowed, and in some cases, the values of variables are set, or even the solution can be found |
| PHG1  | Generation of MILP/MIP/IP model – [environment: CLP] | Generation of the model for mathematical programming environment. Generation performed automatically using CLP predicates. The resulting model is in a format accepted by the MILP system |
| PHG2  | Generation of additional constraints (optional) – [environment: CLP] | Generation of additional constraints on the basis of the results obtained in step PH3 |
| PHG3  | Generation domains of decision variables and other values – [environment: CLP] | Generation of domains for different decision variables and other parameters based on the propagation of constraints. Transmission of this information in the form of fixed value of certain variables and/or additional constraints to the MILP |
| PH4   | Merging MILP/MIP/IP model – [environment: MILP] | Merging files generated during the phases PHG1, PHG2, PHG3 into one file. It is a model file format in MILP system |
| PH5   | Solving MILP/MIP/IP model – [environment: MILP] | The solution of the model from the previous stage by MILP solver. Generation of the report with the results and parameters of the solution |

MILP problems (Perboli et al., 2011). For benchmark tests of the 2E-CVRP, the data published in (www.orgroup.polito.it, 2013) were taken.

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In 2E-CVRP, the freight delivery from the depot to the customers is managed by shipping the freight through intermediate depots. Thus, the transportation network is decomposed into two levels (Figure 2): the first level connecting the depot (d) to intermediate depots (s) and the second one connecting the intermediate depots (s) to the customers (c). The objective is to minimize the total transportation cost of the vehicles involved in both levels. Constraints on the maximum capacity of the vehicles and the intermediate depots are considered, while the timing of the deliveries is ignored.

From a practical point of view, a 2E-CVRP system operates as follows (Figure 2):

- freight arrives at an external zone, the depot, where it is consolidated into the 1st-level vehicles, unless it is already carried into a fully-loaded 1st-level vehicle;
- each 1-level vehicle travels to a subset of satellites that will be determined by the model and then it will return to the depot; and
- at a satellite, freight is transferred from 1-level vehicles to second-level vehicles.

The mathematical model (MILP) was taken from (Perboli et al., 2011). Table 2 shows the parameters and decision variables of 2E–CVRP. Figure 2 shows an example of the 2E-CVRP – transportation network.

$$\begin{align*}
\min & \sum_{i,j \in V_d \cup V_s} (c_{ij} \cdot X_{ij}) + \sum_{k \in V_s} \sum_{i,j \in V_s} (c_{ij} \cdot Y_{k,i,j}) + \sum_{k \in V_s} (s_k \cdot Ds_k) \\
& \sum_{i \in V_s} X_{0,i} \leq M_1 \\
& \sum_{j \in V_s \cup V_0, j \neq k} X_{j,k} = \sum_{i \in V_s \cup V_0, i \neq k} X_{k,i} \quad \text{for } k \in V_s \cup V_0 \\
& \sum_{k \in V_s, j \in V_c} Y_{k,j} \leq M_2 
\end{align*}$$

Figure 2. Example of 2E-CVRP transportation network.
Table 2. Summary indices, parameters and decision variables.

| Symbol | Description |
|--------|-------------|
| \( n_s \) | Number of satellites |
| \( n_c \) | Number of customers |
| \( V_0 = \{ v_o \} \) | Deport |
| \( V_s = \{ v_s1, v_s2, \ldots, v_{sn} \} \) | Set of satellites |
| \( V_c = \{ v_c1, v_c2, \ldots, v_{cn} \} \) | Set of customers |

**Indices**

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- \( n_c \): Number of customers
- \( V_0 = \{ v_o \} \): Deport
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- \( V_c = \{ v_c1, v_c2, \ldots, v_{cn} \} \): Set of customers

**Parameters**

- \( M_1 \): Number of the 1st-level satellites
- \( M_2 \): Number of the 2nd-level satellites
- \( K_1 \): Capacity of the vehicles for the 1st level
- \( K_2 \): Capacity of the vehicles for the 2nd level
- \( d_i \): Demand required by customer \( i \)
- \( c_{i,j} \): Cost of the arc \( (i,j) \)
- \( s_k \): Cost of loading/unloading operations of a unit of freight in satellite \( k \)

**Decision variables**

- \( X_{i,j} \): An integer variable of the 1st-level routing is equal to the number of 1st-level vehicles using arc \( (i,j) \)
- \( Y_{k,i,j} \): A binary variable of the 2nd-level routing is equal to 1 if a 2nd-level vehicle makes a route starting from satellite \( k \) and goes from node \( i \) to node \( j \) and 0 otherwise
- \( Q_{i,j}^1 \): The freight flow arc \( (i,j) \) for the 1st-level
- \( Q_{i,j}^2 \): The freight arc \( (i,j) \) where \( k \) represents the satellite where the freight is passing through.
- \( z_{k,j} \): A binary variable that is equal to 1 if the freight to be delivered to customer \( j \) is consolidated in satellite \( k \) and 0 otherwise

\[
\sum_{i \in V_c, j \in V_c} Y_{k,i,j} = \sum_{i \in V_c, j \in V_c} Y_{k,j,i} \quad \text{for } k \in V_s
\]

\[
\sum_{i \in V_0 \cup V_s, j \neq j} Q_{i,j}^1 - \sum_{i \in V_c, j \neq j} Q_{i,j}^1 = \begin{cases} 
D_{ij} - d_i & \text{if } j \text{ is not the deport} \\
\sum_{i \in V_c} d_i & \text{otherwise for } j \in V_s \cup V_0
\end{cases}
\]

\[
Q_{i,j}^1 \leq k_1 \cdot X_{i,j} \quad \text{for } i, j \in V_s \cup V_0, i \neq j
\]

\[
\sum_{i \in V_0 \cup V_s, j \neq j} Q_{i,j}^2 - \sum_{i \in V_c, j \neq j} Q_{i,j}^2 = \begin{cases} 
Z_{k,j}d_j & \text{if } j \text{ is not a satellite} \\
-D_j & \text{otherwise for } j \in V_0 \cup V_s, k \in V_s
\end{cases}
\]

\[
Q_{k,i,j}^2 \leq k_2 \cdot Y_{k,i,j} \quad \text{for } i, j \in V_s \cup V_c, i \neq j, k \in V_s
\]

\[
\sum_{i \in V_s} Q_{i,0}^1 = 0
\]
The objective function minimizes the sum of the routing and handling operations costs. The objective function is composed of three parts. The first is the cost of transport to the first level, next to the second level. The last part of the objective function is the cost of loading/unloading operations of a unit of freight in satellite. The loading at the depot, as well as unloading at the client is ignored. This results from the fact that they are fixed and do not affect the optimization process.

Constraint (3) ensure, for \( k = v_0 \), that each 1-level route begins and ends at the depot, while when \( k \) is a satellite, impose the balance of vehicles entering and leaving that satellite. Constraint (5) force each 2nd-level route to begin and end to one satellite and the balance of vehicles entering and leaving each customer. The number of the routes in each level must not exceed the number of vehicles for that level, as imposed by constraints (2) and (4). The flows balance on each network node is equal to the demand of this node, except for the depot, where the exit flow is equal to the total demand of

\[
\sum_{j \in V_c} Q_{k,j,k}^2 = 0 \quad \text{for} \quad k \in V_s
\]

\[
Y_{k,i,j} \leq Z_{k,j} \quad \text{for} \quad i \in V_s \cup V_c, j \in V_c, k \in V_s
\]

\[
Y_{k,j,i} \leq Z_{k,j} \quad \text{for} \quad i \in V_s, j \in V_c, k \in V_s
\]

\[
\sum_{i \in V_s} Y_{k,i,j} = Z_{k,j} \quad \text{for} \quad k \in V_s, j \in V_c, i \neq k
\]

\[
\sum_{i \in V_s} Y_{k,j,i} = Z_{k,j} \quad \text{for} \quad k \in V_s, j \in V_c, i \neq k
\]

\[
\sum_{i \in V_s} Z_{i,j} = 1 \quad \text{for} \quad j \in V_c
\]

\[
Y_{k,i,j} \leq \sum_{i \in V_s \cup V_0} X_{i,j,k} \quad \text{for} \quad k \in V_s, i, j \in V_c
\]

\[
Y_{k,i,j} \in \{0, 1\}, Z_{k,j} \in \{0, 1\} \quad \text{for} \quad k \in V_s, i, j \in V_s \cup V_c, l \in V_c
\]

\[
X_{k,j} \in Z^+ \quad \text{for} \quad k, j \in V_s \cup V_0
\]

\[
Q_{i,j}^1 \geq 0 \quad \text{for} \quad i,j \in V_s \cup V_0; Q_{k,i,j}^2 \geq 0 \quad \text{for} \quad i, j \in V_s \cup V_c, k \in V_s
\]

\[
DS_k = \sum_{l \in V_s} (d_j \cdot Z_{k,j}) \quad \text{for} \quad k \in V_s
\]

\[
\sum_{i \in S_c} Y_{k,i,j} \leq |S_c| - 1 \quad \text{for} \quad S_c \subset V_c, 2 \leq |S_c| \leq |V_c| - 2
\]

\[
Q_{k,i,j}^2 \leq (k_2 - d_j) \cdot Y_{k,i,j} \quad \text{for} \quad i,j \in V_c, k \in V_s
\]

\[
Q_{k,i,j}^2 - \sum_{l \in V_s} Q_{k,j,l}^2 \leq (k_2 - d_j) \cdot Y_{k,i,j} \quad \text{for} \quad i,j \in V_c, k \in V_s
\]
the customers, and for the satellites at the second-level, where the flow is equal to the demand (unknown) assigned to the satellites which provide constraints (6) and (8). Moreover, constraints (6) and (8) forbid the presence of sub-tours not containing the depot or a satellite, respectively. In fact, each node receives an amount of flow equal to its demand, preventing the presence of sub-tours. Consider, for example, that a sub-tour is present between the nodes \(i, j\) and \(k\) at the 1st level. It is easy to check that, in such a case, does not exist any value for the variables \(Q_{1ij}\), \(Q_{1jk}\) and \(Q_{1ki}\) satisfying the constraints (6) and (8). The capacity constraints are formulated in constraints (7) and (9), for the first level and the second level, respectively. Constraints (10) and (11) do not allow residual flows in the routes, making the returning flow of each route to the depot (first level) and to each satellite (second level) equal to 0. Constraints (12) and (13) indicate that a customer \(j\) is served by a satellite \(k\) \((z_{kj} = 1)\) only if it receives freight from that satellite \((y_{kj} = 1)\). Constraint (16) assign each customer to one and only one satellite, while constraints (14) and (15) indicate that there is only one 2nd-level route passing through each customer and connect the two levels. Constraint (17) allows starting a second-level route from a satellite \(k\) only if a 1-level route has served it. Constraints from 17 to 20 result from the character of the MILP-formulated problem. Additional constraints were introduced by (Perboli et al., 2011) to increase the solution search efficiency. They strengthen the continuous relaxation of the flow model. In particular, authors in (Perboli et al., 2011) used two families of cuts, one applied to the assignment variables derived from the sub-tour elimination constraints (edge cuts) and the other based on the flows. The edge cuts explicitly introduce the well-known sub-tours elimination constraints derived from the Traveling Sales Problem. They can be expressed as constraint (21). The inequalities explicitly forbid the presence in the solution of sub-tours not containing the depot, already forbidden by the constraint (8). The number of potential valid inequalities is exponential, so that each customer reduces the flow of an amount equal to its demand \(d_i\) – constraints (22) and (23).

A possibility to transform the model in the CLP environment (phase PH2) is an important aspect of that approach.

The our transformation of this model in this integrated approach focused on the re-sizing of \(Y_{k,i,j}\) decision variable by introducing additional imaginary volume of freight shipped from the satellite and re-delivered to it. Such transformation resulted in two facts. First of all, it forced the vehicle to return to the satellite from which it started its trip. Secondly, it reduced decision variable \(Y_{k,i,j}\) to variable \(Y_{i,j}\) which decreased the size of the combinatorial problem.

7. Computational tests: 2E-CVRP

For the final validation of the proposed approach and the ISF, the benchmark data for 2E-CVRP was selected.

The instances for computational examples were built from the existing instances for CVRP (Christofides & Eilon, 1969) denoted as E-n13-k4. All the instance sets can be downloaded from the website (http://www.oigroup.polito.it/, 2013). The instance set was composed of 5 small-sized instances with 1 depot, 12 customers and 2 satellites. The full instance consisted of 66 small-sized instances because the two satellites were placed over 12 customers in all 66 possible ways (number of combinations: 2 out of 12).

All the instances had the same position for depot and customers, whose coordinates were the same as those of instance E-n13-k4. Small-sized instances differed in the choice of two customers who were also satellites (En13-k4-1, En13-k4-5, En13-k4-9, En13-k4-12, etc.).
Numerical experiments were conducted for the same data in three runs. The first run was a classical implementation of model (1)–(20) and its solution in the MILP environment (LINGO). The second run used the same environment for model (1)–(23) with additional edge-cuts. In the final run, the model (1)–(20) and its solution were implemented in the proposed framework (ISF). The calculations were performed using a computer with the following specifications: Intel(R) Core(TM) 2 Quad CPU Q6600 @ 2 × 2.40 GHZ 2.4 GHZ RAM 198 GB. The analysis of the results for the benchmark instances demonstrates that the integrated approach may be a superior approach to the classical MP. For all examples, the solutions were found 4–16 times faster than they are in the classical approach (Table 3). In many cases, the calculations ended after 600 s as they failed to indicate that the solution was optimal.

As the presented example was formulated as a MILP problem, the ISF was tested for the solution efficiency. Owing to the integrated approach, the 2E-CVRP models can be extended over logical, non-linear and other constraints. At the next stage, logical constraints were introduced into the model. The logical relationship between mutually exclusive variables was taken into account, which in real-world distribution systems means that the same vehicle cannot transport two types of selected goods or two points cannot be handled at the same time. Those constraints result from technological, market-

| E-n13-k4-05 | FC | T   | C   | V(int V) |
|-------------|----|-----|-----|----------|
| E-n13-k4-10 | 268| 183 | 1262| 744(368) |
| E-n13-k4-12 | 290| 600a| 1262| 744(368) |
| E-n13-k4-14 | 228| 67  | 1262| 744(368) |
| E-n13-k4-16 | 238| 90  | 1262| 744(368) |
| E-n13-k4-20 | 276| 487 | 1262| 744(368) |
| E-n13-k4-30 | 290| 136 | 1262| 744(368) |
| E-n13-k4-40 | 254| 38  | 1262| 744(368) |
| E-n13-k4-50 | 280| 83  | 1262| 744(368) |
| E-n13-k4-60 | 326| 68  | 1262| 744(368) |
| E-n13-k4-66 | 400| 600a| 1262| 744(368) |

Note: FC, the optimal value of the objective functional; T, time of finding solution; and V(int V)/C, the number of variables (integer variables)/constraints.

*aCalculations stopped after 600 s, the feasible value of the objective function.
ing, sales or safety reasons. Only declarative application environments based on constraint satisfaction problem (CSP) make it possible to implement constraints such as. Table 4 presents the results of the numerical experiments conducted for 2E-CVRPs with logical constraints relating to the situation where two delivery points (customers) can be handled separately but not together in one route.

The next series of experiments involved the 2E-CVRP with time windows (2E-CVRP-TW). This problem is the extension of 2E-CVRP where time windows on the arrival or departure time at the satellites and/or at the customers are considered. The time windows can be hard or soft. In the first case, the time windows cannot be violated, while in the second one if they are violated a penalty cost is paid. Next experiments involved the implementation of the model (2E-CVRP-TW) with hard TW. The model and its optimization were implemented using the framework ISF.

The results for selected instances of the problem E-n13-k4 are shown in Table 5. There are no feasible solutions for certain values of TW. Table 6 shows the effects of the parameter TW on the value of the optimum solution. For certain values of TW, 

| E-n13-k4 | Fc  | T    | C    | V     | exCustomer⁴ |
|----------|-----|------|------|-------|-------------|
| E-n13-k4-05 | 232 | 7,55 | 21   | 788(785) | 1,3; 1,4; 1,6; 2,8 |
| E-n13-k4-09 | 252 | 8,42 | 21   | 788(785) | 1,3; 1,4; 1,6; 2,8 |
| E-n13-k4-12 | 290 | 12,43| 21   | 788(785) | 1,3; 1,4; 1,6; 2,8 |
| E-n13-k4-22 | 314 | 12,91| 21   | 788(785) | 1,3; 1,4; 1,6; 2,8 |

⁴Pairs of customers that cannot be served on one route.

Table 5. The results of numerical examples (Fc, T, C, V) for 2E-CVRP with time window (TW = 60).

| E-n13-k4 | Fc  | T    | C    | V     |
|----------|-----|------|------|-------|
| E-n13-k4-05 | 234 | 7,33 | 21   | 1082(1079) |
| E-n13-k4-09 | 244 | 7,30 | 21   | 1082(1079) |
| E-n13-k4-12 | NSF | 12,39| 21   | 1082(1079) |
| E-n13-k4-22 | NSF | 9,90 | 21   | 1082(1079) |

Note: Fc, the optimal value of the objective function; T, time of finding solution; V(int V)/C, the number of variables (integer variables)/constraints; NSF, no solution found (contradiction constraints); and TW, time window.

Table 6. The results of numerical examples (Fc) for 2E-CVRP with different time windows.

| E-n13-k4 | TW  |
|----------|-----|
| E-n13-k4-05 | 40  | 50  | 55  | 60  | 70  | 80  | 90  | 100 |
| NSF      | NSF  | NSF  | NSF  | NSF  | NSF  | NSF  | NSF  | NSF  |
| NSF      | NSF  | NSF  | NSF  | NSF  | NSF  | NSF  | NSF  | NSF  |
| NSF      | NSF  | NSF  | NSF  | NSF  | NSF  | NSF  | NSF  | NSF  |
| NSF      | NSF  | NSF  | NSF  | NSF  | NSF  | NSF  | NSF  | NSF  |

Note: Fc, the optimal value of the objective function; NSF, no solution found (contradiction constraints); and TW, time window.
solution is not found for other values are obtained optimal solution. To some extent, the value of TW it affect the resulting value of the objective function (Table 5).

8. Conclusion and discussion on possible extension

The efficiency of the proposed approach is based on the reduction of the combinatorial problem and the use of the best properties of both environments. The integrated approach (Tables 3 and 4) makes it possible to find solutions in the shorter time.

The benchmark tests for ISF (Table 3) showed that, compared with the MILP solver – LINGO, the solution was found several times faster.

In addition to solving larger problems faster, the proposed approach provides virtually unlimited modelling options with many types of constraints (logical, nonlinear, symbolic etc.). Therefore, the proposed solution is recommended for decision-making problems that have a structure similar to the presented models (Section 5). This structure is characterized by the constraints and objective function in which the decision variables are added together. Further work will focus on running the optimization models with non-linear and other logical constraints, multi-objective, uncertainty, etc. in the integrated optimization framework. The planned experiments will employ ISF for Two-Echelon CVRP with Satellites Synchronization (2E-CVRPSS) and 2E-CVRP with Pickup and Deliveries (2E-CVRP-PD).

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