Color-Octet Charmonium Production in Top Quark Decays

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Abstract

We calculate the direct production rate of $J/\psi$ in top quark decays. The color-octet $J/\psi$ production via $t \rightarrow W^+bJ/\psi$ is shown to have a large branching ratio of order $1.5 \times 10^{-4}$, which is over an order of magnitude higher than that of the color-singlet $J/\psi$ production via $t \rightarrow W^+bJ/\psi g g$ or $t \rightarrow W^+b \chi_c J g$ followed by $\chi_c J \rightarrow J/\psi \gamma$. This result can be used as a powerful tool to test the importance of the color-octet mechanism in heavy quarkonium production.

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Since the discovery of charmonium in 1974, there have been a lot of attempts to interpret the production of these new states. Among many scenarios, the color-singlet model\[1\] gains more success than other alternatives\[2\] like the color-evaporation model\[1][3\]. Based upon the color-singlet model, it is possible to calculate the production rates from first principles by standard methods\[4\]. Indeed, the study of heavy quarkonium production may provide a suitable ground to precisely test quantum chromodynamics (QCD).

However, during the past few years, it is found that the color-singlet model also has some defects in describing the production of heavy quarkonium. Predictions for the S-wave charmonium production failed to explain the new data. In the 1992-1993 run, the \textbf{CDF} detector at Fermilab Tevatron\[5\] gave rates for prompt $\psi$ and $\psi'$ production at large transverse momentum which were orders of magnitude above the lowest order perturbative calculation within the color-singlet model\[6\]. Even after including the fragmentation contributions\[7][8][9\] which overwhelm the former when $P_T \geq 6 GeV\[10\]$, there is still left a big gap between theory and experiment.

Recently, a new effective field theory for bound states of heavy quark and antiquark was provided by Bodwin, Braaten, and Lepage\[11\] in the context of non-relativistic quantum chromodynamics(NRQCD). In this approach, the interaction operators are expanded in powers of $v$, the velocity of heavy quark and antiquark in the meson rest frame, and $\alpha_s$. Contributions of different orders in $v$ are separated according to the “velocity scaling” rules. In this new framework a heavy quarkonium state $H$ is not solely regarded as simply a quark-antiquark pair but rather a superposition of a series of Fock states:

$$|H(nJ^{PC})\rangle = O(1)|QQ(2S+1L_J, 1)\rangle + O(v)|QQ(2S+1(L \pm 1), 8)\rangle + O(v^2)|QQ(2S+1L_J, 8 \text{ or } 1)gg\rangle + \cdots,$$

(1)

where the angular momentum of the $Q\bar{Q}$ pair in each Fock state is labeled by $2S+1L_J$ with a color configuration of either $8$ or $1$. The pure $QQ$ state in color-singlet is only the leading term in the above expansion. Up to and including $O(v^2)$ in the Fock state expansion in describing $J/\psi(\psi')$ production, the color-octet matrix element $\langle O_{8}^{J/\psi}(3S_{1}) \rangle [\langle O_{8}^{\psi'}(3S_{1}) \rangle]$ should also be taken into consideration. Although these color-octet matrix elements are suppressed by order of $v^4$ relative to the corresponding color-singlet matrix elements $\langle O_{1}^{J/\psi}(3S_{1}) \rangle$,
\[ \langle \mathcal{O}_1^{\sigma}(S_1) \rangle \], they are enhanced by a factor of \(1/\alpha_s^2\) relative to the color-singlet process in the short-distance perturbative calculation. Therefore, the suppression in the color-octet matrix elements can be compensated. Treating the color-octet matrix elements as free parameters, the description of high-\(P_T\) \(J/\psi(\psi^\prime)\) production at the Tevatron can indeed be rescued \[12\] \[13\] \[14\], but clearly more work is needed before the new formalism is established as a successful theory of quarkonium production. To this end, a list of papers have been published \[15\] \[16\] \[17\], and still in this paper, we suggest using another important process to test the color-octet quarkonium production mechanism.

The success of the Standard Model (SM) \[18\] \[19\] \[20\] suggests that the top quark must exit \[21\]. Recently, from the direct search at the Tevatron, the CDF and D0 groups confirmed the existence of a heavy top quark \[22\] \[23\], with a mass of \((176 \pm 8 \pm 10)\) GeV or \((199^{+19}_{-21} \pm 22)\) GeV. The next experimental studies will focus on the determination of its properties. Among others, the measurement of top quark decays into heavy quark mesons which are made of charm or bottom quark and antiquark, will be of special interest. In particular, the charmonium production in top quark decays will provide very useful information in testing the Standard Model.

In the Standard Model, charmonium (e.g. \(J/\psi\)) may be produced via the flavor changing neutral current transition \(t \to cg\) \[24\] followed by the charm quark fragmentation \(c \to J/\psi c\) or the gluon fragmentation \(g \to J/\psi\), but the rates of these processes are very small within the Standard Model and hence sensitive to the new physics beyond the Standard Model \[25\]. On the other hand, however, since in the Standard Model the dominant decay mode of top quark is \(t \to W^+b\) \[26\], the dominant direct charmonium production is expected to proceed via \(t \to W^+bg^*\) with the virtual gluon \(g^*\) fragmentation into charmonium. There are three subprocesses for \(J/\psi\) production via \(t \to W^+bg^*\): (i) color-octet gluon fragmentation \(g^* \to J/\psi\); (ii) color-singlet gluon fragmentation \(g^* \to J/\psi gg\); (iii) color-singlet gluon fragmentation \(g^* \to \chi_{cJ}g\) followed by \(\chi_{cJ} \to J/\psi \gamma\), where \(\chi_{cJ}(J = 0, 1, 2)\) are the \(P\)-wave states.

The direct charmonium production appears at orders \(\alpha_s^4\) or over in the color-singlet model \[27\], whereas the color-octet production process given by \(t \to W^+bJ/\psi\), as shown in Fig.1, is at order \(\alpha_s^2\). Its amplitude may be written as

\[
\mathcal{A}(t \to W^+bJ/\psi) = \frac{igg^2V_{tb}}{2\sqrt{2M}}T^a\epsilon^{a}_{\psi}\epsilon^{b}_{W}\bar{u}(P')\left[\gamma_{\mu}\frac{1}{P'-P-m_b}\gamma_\alpha(1-\gamma_5)\right] (2)
\]
respectively, and $M$ is the amplitude of all possible way of evolving to $J/\psi$ starting from a color-octet $Q\bar{Q}^{[3]}_{\text{S}}$ pair at short distances. It may be treated as phenomenological parameter which can be determined by fitting the data, e.g. from the $J/\psi$ production rate at the Tevatron $^{[4]}$. We define

$$f_1 = -(M^2 + 2m_b^2)(m_t^2 - m_b^2) + (m_t^2 + m_b^2 - 2m_w^2)m_w^2,$$

$$f_2 = -2(m_t^2 - m_b^2)^2(M^2 + m_b^2 + m_t^2) + 2m_w^2[(2M^2 + 3m_w^2)(m_b^2 + m_t^2)
- 2(M^2 + m_b^2)^2 - 4m_b^2m_t^2],$$

$$f_3 = -m_b^2 - m_t^2 - 2m_w^2,$$

$$f_4 = 2m_b^4 - 4m_b^2m_t^2 + 2m_t^4 - 4M^2m_b^2 + 2m_b^2m_w^2 + 2m_t^2m_w^2 - 4m_w^4,$$

$$f_5 = -(M^2 + 2m_b^2)(m_t^2 - m_b^2) + (m_t^2 + m_b^2 - 2m_w^2)m_w^2,$$

$$f_6 = 2(m_t^2 - m_b^2)^2 - 2(2M^2 - m_t^2 - m_b^2 + 2m_w^2)m_w^2,$$

$$f_7 = -2m_b^2 - 2m_t^2, \quad f_8 = -m_b^2 - m_t^2 - 2m_w^2,$$

where $M = 2m_c$ is the $J/\psi$ mass. Then, the differential decay rate for $t \rightarrow W^+bJ/\psi$ is given by

$$\frac{d^2\Gamma}{dx_1dx_2}(t \rightarrow W^+bJ/\psi) = \frac{g^2\alpha_s^2|V_{tb}|^2|\mathcal{M}_8(J/\psi)|^2}{24\pi M^2 m_b^2 m_w^2} \left\{ f_1 x_1^2 + f_2 x_1 x_2 + f_3 x_1^3 x_2 + f_4 x_1^2 x_2 + f_5 x_2^2 + f_6 x_1 x_2^2 + f_7 x_1^2 x_2 + f_8 x_1 x_2^3 \right\} / (x_1 x_2) \cdot (4)$$

Here, the variables $x_1 = m_b^2 - m_t^2 - m_w^2 + 2m_tE_{J/\psi}$, $x_2 = 2m_tE_{J/\psi} - M^2$. The physical limits of $x_1$ and $x_2$ are

$$x_1^\pm = \frac{1}{2(m_t^2 - x_2)} \left\{ (M^2 - x_2)(m_t^2 + m_b^2 - m_w^2 - x_2) \right\},$$

$$x_2^\pm = \frac{\lambda^2[(m_t^2 - x_2), m_b^2, M^2]\lambda^2[(m_t^2 - x_2), m_b^2, m_w^2]}{2} - M^2,$$

$$x_2^- = m_t^2 - (m_t - M)^2, \quad x_2^+ = m_b^2 + m_w^2.$$

Here $\lambda(x, y, z) \equiv (x - y - z)^2 - 4yz$. Setting $\alpha_s = 0.253$, $m_c = 1.5\text{GeV}$, $m_b = 4.9\text{GeV}$, $m_t = 176\text{GeV}$ $^{[2]}$, and $|\mathcal{M}_8(J/\psi)|^2 = 0.68 \times 10^{-3} \text{GeV}^2$ $^{[4]}$, we get the branching ratio of

$$B(t \rightarrow W^+bJ/\psi) \approx 1.46 \times 10^{-4}. \quad (6)$$

The dominant color-singlet prompt $J/\psi$ production process to be $t \rightarrow W^+bg^*$ with $g^* \rightarrow J/\psi g g$, and $g^* \rightarrow \chi_{cJ} g$ followed by $\chi_{cJ} \rightarrow J/\psi \gamma$, as shown in Fig.2. We can estimate
the partial width following the way in Ref. [28]. The differential decay rate of \( t \to W^+ bg^* \) is similar to Eq. (4), and can be easily obtained or found in Ref. [29]. With the definition

\[
\Gamma(g^* \to AX) = \pi \mu^3 P(g^* \to AX),
\]

the decay distribution \( P(g^* \to \chi_{cJ} g) \) and \( P(g^* \to J/\psi gg) \) for the gluon of virtuality \( \mu \) can be found in Ref. [3] and Ref. [30].

\[
\mu^2 P\left(g^* \to \chi_{c0} g\right) = \frac{r(1 - 3r)^2}{1 - r} C_p,
\]

\[
\mu^2 P\left(g^* \to \chi_{c1} g\right) = \frac{6r(1 + r)}{1 - r} C_p,
\]

\[
\mu^2 P\left(g^* \to \chi_{c2} g\right) = \frac{2r(1 + 3r + 6r^2)}{1 - r} C_p,
\]

\[
\mu^2 P\left(g^* \to J/\psi gg\right) = C_s r \int_{x^-}^{x^+} dx_{J/\psi} \int dx_1 f(x_{J/\psi}, x_1; r),
\]

where \( r \equiv M/\mu \), \( M \) is the mass of the relevant charmonium states, and

\[
C_p = \frac{8 \alpha_s^2 |R'_p(0)|^2}{9 \pi M^5}, \quad C_s = \frac{5 \alpha_s^3 |R_s(0)|^2}{27 \pi^2 M^3}.
\]

The function \( f \) in Eq. (11) is of the form [30]

\[
f(x_{J/\psi}, x_1; r) = \frac{(2 + x_2)x_2}{(2 - x_{J/\psi})^2(1 - x_1 - r)^2} + \frac{(2 + x_1)x_1}{(2 - x_{J/\psi})^2(1 - x_2 - r)^2} - \frac{1}{(2 - x_{J/\psi})^2}\left(\frac{6(1 + r - x_{J/\psi})^2}{(1 - x_2 - r)^2(1 - x_1 - r)^2} + \frac{2(1 - x_{J/\psi})(1 - r)}{(1 - x_2 - r)(1 - x_1 - r)r} + \frac{1}{r}\right),
\]

where \( x_i \equiv 2E_i/\mu \) with \( i = J/\psi, g_1, g_2 \) are the energy fractions carried by the \( J/\psi \) and two gluons in the \( g^* \) rest frame, and then \( x_2 = 2 - x_1 - x_{J/\psi} \). The limits of the \( x_1 \) integration in Eq. (11) are

\[
x_\pm = \frac{1}{2}(2 - x_{J/\psi} \pm \sqrt{x_{J/\psi}^2 - 4r}).
\]

We can evaluate the total decay rate of top quark to various color-singlet charmonium states, \( A \), via

\[
\Gamma(t \to W^+ bg^*; g^* \to AX) = \int_{M^2}^{m_{2/4}^2} d\mu^2 \Gamma(t \to W^+ bg^*(\mu^2)) \cdot P(g^* \to AX).
\]
In the numerical estimation, we take $\alpha_s = 0.253$, $m_c = 1.5 GeV$, $M = 2m_c$, $|R_s(0)|^2 = 0.999 GeV^3$, and $|R'_p(0)|^2 = 0.125 GeV^5$[31], and get

$$B(t \to W^+b\chi_{c0}g) \cdot B(\chi_{c0} \to J/\psi\gamma) = 2.49 \times 10^{-9},$$

(16)

$$B(t \to W^+b\chi_{c1}g) \cdot B(\chi_{c1} \to J/\psi\gamma) = 5.35 \times 10^{-6},$$

(17)

$$B(t \to W^+b\chi_{c2}g) \cdot B(\chi_{c2} \to J/\psi\gamma) = 1.88 \times 10^{-6},$$

(18)

$$B(t \to W^+bJ/\psi gg) = 1.39 \times 10^{-6}. $$

(19)

The $\chi_{cJ}$ production rates depend on the infrared cutoff. Here we take the cutoff $\mu^2_{\text{min}} = 2M^2$, which is the same as that in the fragmentation analysis[32]. Adding the branching ratios together, we obtain the total color-singlet prompt $J/\psi$ production rate to be $8.6 \times 10^{-6}$, which is about a factor of 20 smaller than that via the color-octet production mechanism.

In conclusion, we have considered the color-octet charmonium production in top quark decays, and found the branching ratio of this dominant process $t \to W^+bJ/\psi$ to be $1.46 \times 10^{-4}$, which is over an order of magnitude larger than that of color-singlet production processes. Such a large difference makes the process of charmonium production in top decay another important channel to identify color-octet qurkonium signals whenever there are enough top quark events at the Fermilab Tevatron, LHC (Large Hadron Collider), or NLC (Next Linear Collider) in the future.

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Figure Captions

Fig.1. Color-octet Charmonium production process in top quark decays.

Fig.2. Diagrams for color-singlet $J/\psi$ production. (a) via $g^* \rightarrow J/\psi gg$ (b) via $g^* \rightarrow \chi_{cJ} g \rightarrow J/\psi \gamma g$. For diagram (a) $x_i \equiv 2E_i/\mu$ with $i = J/\psi, g_1, g_2$ are the energy fractions carried by the decay products in $g^*$ rest frame normalized to $\mu \equiv m(g^*)$. 
