Standard Model CP-violation and Baryon asymmetry

Part I: Zero Temperature

M.B. Gavela, a, M. Lozano b, J. Orloff c, O.Pène d

a CERN, TH Division, CH-1211, Geneva 23, Switzerland
b Dpto. de Física Atómica, Molecular y Nuclear, Sevilla, Spain
c Institut für Theoretische Physik, Univ. Heidelberg
d LPTHE, F 91405 Orsay, France

Abstract

We consider quantum effects in a world with two coexisting symmetry phases, unbroken and spontaneously broken, as a result of a first order phase transition. The discrete symmetries of the problem are discussed in general. We compute the exact two-point Green function for a free fermion, when a thin wall separates the two phases. The Dirac propagator displays both massive and massless poles, and new CP-even phases resulting from the fermion reflection on the wall. We discuss the possible quark-antiquark CP asymmetries produced in the Standard Model(SM) for the academic $T = 0$ case. General arguments indicate that an effect first appears at order $\alpha_W$ in the reflection amplitude, as the wall acts as a source of momentum and the on-shell one-loop self-energy cannot be renormalized away. The asymmetries stem from the interference of the SM CP-odd couplings and the CP-even phases in the propagator. We perform a toy computation that indicates the type of GIM cancellations of the problem. The behaviour can be expressed in terms of two unitarity triangles.

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Introduction

In ref. [1] we presented recently a summary of our ideas and results on Standard Model (SM) baryogenesis, in the presence of a first order phase transition. The aim of the present work is to describe in detail our zero temperature analysis. The finite temperature $T$ scenario is treated in the accompanying paper, ref. [14].

The baryon number to entropy ratio in the observed part of the universe is estimated to be $n_B/s \sim (4 - 6) \times 10^{-11}$ [3]. In 1967 A.D. Sakharov [4] established the three building blocks required from any candidate theory of baryogenesis:

(a) Baryon number violation,

(b) C and CP violation,

(c) Departure from thermal equilibrium.

The Standard Model (SM) contains (a) [5] and (b) [6], while (c) could also be large enough [3,8], if a first order $SU(2) \times U(1)$ phase transition took place in the evolution of the universe [4]. We will not enter the discussion on the latter: it will be assumed that a first order phase transition did take place, as strong as wished by the reader. An optimal sphaleron rate can be assumed as well. Our aim is to argue, on a quantitative estimation of the electroweak C and CP effects exclusively, that the current SM scenario is unable to explain the above mentioned baryon number to entropy ratio.

Intuitive CP arguments lead to an asymmetry many orders of magnitude below observation [10,11]. Assume a total flux of baryonic current, where all quark flavours are equally weighted. An hypothetical CP-violating contribution in the SM with three generations [6] should be proportional to

$$s_1^2 s_2 s_3 c_1 c_2 c_3 s_6 (m_t^2 - m_c^2)(m_t^2 - m_u^2)(m_c^2 - m_u^2)$$
$$\times (m_b^2 - m_s^2)(m_b^2 - m_d^2)(m_s^2 - m_d^2),$$

(1.1)

times some power of the electroweak coupling constant, to be determined. In eq. (1.1) the mixing angles and phase are the original Kobayashi-Maskawa ones [6]. In order to obtain a dimensionless result, the expression in (1.1) has to be divided by the natural mass scale of the problem $M_W$ or, at finite temperature, $T$, at the 12$^\text{th}$ power. It follows [11] that the resulting asymmetry is negligible, $n_B/s \sim 10^{-20}$ or smaller if other reduction factors are considered. When the physical boundary conditions are such that some specific flavour is singled out by nature (or experiment as, for instance, in the $K_0 - \overline{K_0}$ system, electric dipole moment of the neutron, etc.), some of the fermionic mass differences in (1.1) are no more compelling, and a stronger effect is possible. Nevertheless it seems difficult to compensate the 10 orders of magnitude difference in the above reasoning. Such could be the case if the desired observable came about as a ratio of (1.1) with respect to another process which contained by itself some fermionic suppression factors: an example is the CP-violating $\epsilon$ parameter in $K$ decays, where the CP-violating amplitude is compared to a CP-conserving denominator, $\sim \text{Re} (K_0 - \overline{K_0})$, which is a GIM suppressed object. This is not what happens with the baryon asymmetry of the universe where, as shown below, although nature may single out some specific flavours, there is nothing to divide with (other than the overall normalization...
to the total incoming flux, which bears no suppression factor). In this respect, the process has some analogy with the electric dipole moment of the neutron where the experiment preselects the up and down flavours, and of course the results are normalized to the total incoming flux of neutrons.

On the light of the above considerations, the reader may be astonished that we give the problem further thought. Our motivation is that the study of quantum effects in the presence of a first order phase transition is rather new and delicate, and traditional intuition may fail. A SM explanation would be a very economical solution to the baryon asymmetry puzzle. To discard this possibility just on CP basis suggests that new physics is responsible for it, without any need to settle whether a first order phase transition is possible at all. Furthermore, the detailed solution in the SM gives the “know how” for addressing the issue in any theory beyond the standard one where CP violation is first present at the one-loop level. On top of the above, the authors of ref. [12] have recently studied the issue in more detail and claim that, at finite temperature, the SM is close to produce enough CP violation as to explain the observed \( n_B/s \) ratio.

![Figure 1: Quarks scattering off a true vacuum bubble. Some notations in the text are summarised in the “zoomed-in” view.](image)

A first order phase transition can be described in terms of bubbles of “true” vacuum (with an inner vacuum expectation value of the Higgs field \( v \neq 0 \)) appearing and expanding in the preexisting “false” vacuum (with \( v = 0 \) throughout). We can “zoom” into the vicinity of one of the bubbles, see Fig. 1. There, the curvature of its wall can be neglected and the world is divided into two zones: on the left hand side, say, \( v = 0 \); on the right \( v \neq 0 \) and masses appear. The actual bubble expands from the broken phase \( (v \neq 0) \) towards the unbroken one \( (v = 0) \). We work in the wall rest frame in which the plasma flows in the opposite direction. Consider thus a baryonic flux hitting the wall from the unbroken phase. Far enough to the left no significant CP-violating effect is possible as all fermions are massless. In consequence, the heart of the problem lies in the reflection and transmission properties of quarks bumping on the bubble wall. CP violation distinguishes particles from antiparticles and it is \textit{a priori} possible to obtain a CP asymmetry on the reflected baryonic
The induced baryon asymmetry is at most \( n_B/s \sim 10^{-2} \Delta_{CP} \), in a very optimistic estimation of the non-CP ingredients [13].

The symmetries of the problem are analyzed in detail for a generic bubble. The analytical results correspond to the thin wall scenario. The latter provides an adequate physical description for typical momentum of the incoming particles \( |\vec{p}| \) smaller than the inverse wall thickness \( l \), i.e., \( |\vec{p}| \ll 1/l \). For higher momenta the cutoff effects would show up [17], but it is reasonable to believe that the accuracy of the thin wall approximation produces an upper bound for the CP asymmetry.

The precise questions to answer in the above framework are: 1) the nature of the physical process in terms of particles or quasi-particles responsible for CP violation, 2) the order in the electroweak coupling constant, \( \alpha_W \), at which an effect first appears, 3) the dependence on the quark masses and the nature of the GIM cancellations involved.

We shall consider the problem in two steps: \( T = 0 \) in the present paper, and finite temperature case in the subsequent one [14]. The cosmological first order phase transition is a temperature effect. In order to disentangle the physical implications of the presence of a wall, with the consequent breaking of translation invariance, from the pure thermal effects, we consider here an hypothetical world at zero temperature but with two phases of spontaneous symmetry breaking. We will see that this academic model is illuminating.

Intuition indicates that an existing CP violating effect already present at zero temperature will diminish when the system is heated, because the effective v.e.v. of the Higgs field decreases and in consequence the fermion masses do as well (only the Yukawa couplings already present at \( T = 0 \) remain unchanged). The expected decrease of CP asymmetries for increasing \( T \) follows, then, from the well known fact that the Kobayashi Maskawa CP violating effects of the standard model disappear when at least two fermion masses of the same charge vanish or become degenerate. This intuition can be misleading only if a new physical effect, absent at \( T = 0 \) and relevant for the problem, appears at finite temperature. Treating first the \( T = 0 \) case allows a clean analysis of the novel aspects of physics in a world with two phases of spontaneous symmetry breaking. The quantum mechanical problem is well defined and in particular the definition of particles, fields, in and out states, etc. is transparent.

At \( T = 0 \) the building blocks of the CP violating effect are threefold. First, the necessary CP-odd couplings of the Cabibbo-Kobayashi-Maskawa (CKM) matrix are at work. Second, there exist CP-even phases, equal for particles and antiparticles, which interfere with the pure CP-odd ones to make them observable. These are the reflection coefficients of a given particle hitting the wall from the unbroken phase. They are complex when the particle energy is smaller than its mass. Third, as argued below, the one loop self-energy of a particle in the presence of the wall cannot be completely renormalized away and results in physical transitions. The effect, thus, is present at order \( \alpha_W \) in amplitude. Such an effect is absent in the usual world with just one phase of spontaneous symmetry breaking, where the self-energy is renormalized away for an on-shell particle. The difference is easy to understand: the wall acts as an external source of momentum in the one-loop process.

The truly and essential non-perturbative effect is the wall itself. The propagation of any particle of the SM spectrum should be exactly solved in its presence. And this we do for a free fermion, leading to a new Feynman propagator which replaces and generalizes the usual one. With this exact, non-perturbative tool, perturbation theory is then appropriate in the
gauge and Yukawa couplings of fermions to bosons, and the one loop computations can be performed.

Strictly speaking, the gauge boson and Higgs particles propagators in the presence of the wall are required, and it is possible to compute them with a similar procedure \[16\]. Furthermore, at \( T = 0 \) the field theoretical problem is ill-defined in the unbroken phase where the fermions are massless, due to infrared divergences. Some cutoff is needed, although no dependence on it should remain when the cancellations between different diagrams contributing to a CP-violation effect are considered. By the time being, we work in a simplified case in which the one-loop effect itself is computed in one of the two phases. The final result describes a transition between any two flavours of the same charge, resulting in a CP violating baryonic flow for a given initial chirality, of order \( \alpha W^2 \) in the total rate. The GIM cancellations appear as powers of \( m_f^6/M_W^6 \) times logarithmic corrections, for the internal fermionic masses in loops. The dependence in the external masses is not trivial either, and its GIM implications are discussed in detail. It follows numerically that the CP factor is orders of magnitude smaller than the CP factor needed for baryon number generation. This \( T = 0 \) model is academic, and we develop it to sharpen our tools for the physical finite temperature case \[14\].

The scope of the present paper goes beyond the particular issue of baryon number generation in the SM. For instance, the above mentioned exact fermion propagator in the presence of a wall, which we derive, should be useful in other scenarios when a first order phase transition is present. The understanding at the quantum level of physical processes in the presence of phase transitions still is in its infancy, and we give it a modest try. CP violation in the SM is just an example of an effect which would disappear in a classical or semiclassical statistical physics approach.

2 Notations and symmetries

In this section we settle the formalism and express some general results. Consider several quark flavours in a gauge theory with a vacuum presenting a “wall” structure, parallel to the \( x-y \) plane. More precisely, assume that the Higgs field (or fields if there are several Higgses) has a vanishing v.e.v. for \( z \to -\infty \) and some constant v.e.v. for \( z \to \infty \). In between there is a “wall” with arbitrary width and profile. The interactions of quarks with the gauge and scalar bosons are given by some gauge invariant Lagrangian. As previously stated, the quark-boson interactions will be treated in perturbation, while the non-perturbative effect of the wall on the fermion propagation is solved exactly.

As a first step let us consider the non-interacting case and treat in first quantization formalism the free Dirac Hamiltonian, in the presence of the wall. The second quantized fields in the “interaction representation” are built subsequently.

The quarks/antiquarks hitting the wall from the unbroken phase are reflected or transmitted. The question is whether there exists a CP asymmetry that produces a different reflection probability for quarks and antiquarks.\(^3\)

\(^3\)Note that current conservation relates transmission to reflection, so that an asymmetry in the reflection probability automatically implies an asymmetry in the transmission probability.
In the unperturbed case, we will diagonalize the free Hamiltonian. The eigenstates provide then the reflection and transmission coefficients. The fields and the quark propagators are made out of these eigenstates. Furthermore, they will be the building blocks of the incoming (from the unbroken phase) and outgoing (back to the unbroken phase) wave packets when higher orders are considered later on.

The symmetries of the problem and the conserved or partially conserved quantum numbers are relevant to classify the eigenstates. It should be obvious that discrete symmetries are of particular relevance. In the second subsection we shall derive in the exact theory the consequences of the only unbroken discrete symmetry, CPT, and study in that context the effect of CP violation. Note that the consideration of a flux of particles approaching the wall with a non-zero average velocity (which will correspond later on to the physical process) breaks the CPT symmetry in a global sense, but the microscopic relations for two-particle transition amplitudes derived below will still hold.

### 2.1 Free quarks in the presence of a wall.

Let us consider a static “wall” parallel to the $x - y$ plane. The Higgs vacuum expectation value depends only on the $z$ coordinate, resulting in hermitian mass matrices $m(z)$ and $m_5(z)$ for the quarks. The time independent Dirac Hamiltonian is then

$$H = \vec{\alpha} \cdot \vec{p} + \beta m(z) + i\beta \gamma_5 m_5(z),$$

(2.1)

where $\alpha_i = \gamma_0 \gamma_i$ and $\beta = \gamma_0$.

For the SM, in the basis where the mass matrices are diagonal, the study simplifies to the one flavour case and $m(z)$, $m_5(z)$ become real. Furthermore, it is always possible to rotate $m_5(z)$ away through a chiral rotation. In other models, a diagonalization of both matrices is not always possible and the matrix $m_5(z)$ may remain, for instance, in two-Higgs models when the relative phase between the corresponding v.e.v.'s changes with $z$ (see for example ref [15]).

Let $E, p_x, p_y, p_z$ denote the four-momentum of the quark. For a static wall the particle energy, $E$, is conserved, while $p_z$ is not when the fermion crosses the wall or bounces back, due to lack of translational invariance. The system is symmetric under reflections with respect to the $x - y$ plane and with respect to rotations around the $z$ axis, the latter implying conservation of total angular momentum in the $z$ direction, $J_z$. It is also invariant under Lorentz boosts parallel to the $x - y$ plane ($K_x$ and $K_y$ commute with $H$). From the two preceding symmetries follows a conserved quantum number for any wave function which is an eigenvector of $E, p_x$ and $p_y$. Boost the reference frame to obtain $p_x = p_y = 0$. In that frame, the helicity states of the incoming plane waves in the unbroken phase correspond to eigenstates of $J_z = S_z$. This quantum number is more convenient to use than helicity because it is conserved. Denote generically by $j_z$ ($j_z = \pm 1/2$) the corresponding eigenvalue of $J_z$: it will be used when labeling the eigenstates of the Hamiltonian.

Charge conjugation, $C$, is conserved when there exists a basis where both $m(z)$ and $m_5(z)$ are real matrices.

Define $P'$ as the product of parity times a rotation of angle $\pi$ around the $y$ axis, i.e., parity with respect to the $x - z$ plane, which will maintain the unbroken phase on the initial side of the $z$ axis. $P'$ is a symmetry when $m_5(z) = 0$ in some basis. $C P'$ is conserved when a
basis exists such that $m(z) + i\gamma_5 m_5(z)$ is a real matrix. The unperturbed SM Hamiltonian is invariant under $C$, $P'$ and $CP'$. In general, the following quantum numbers are conserved: $p_x, p_y, J_z, E$ and $CP'T$.

To be specific, we have assumed that the unbroken phase is on the negative side of the $z$ axis. We define the eigenstates $\psi_n$ by

$$E_n \psi_n = H \psi_n$$

where the “energy” $E_n$ may be positive or negative, and $n$ is a short hand notation for the quantum numbers that label the states. For the latter we choose $n = p_x, p_y, j_z, E_n, f, \alpha$. $p_x, p_y$ and $E_n$ are conserved real quantum numbers. They can take any positive or negative value provided $E_n^2 > p_x^2 + p_y^2$. $j_z$ is a discrete conserved quantum number defined above. $f$ is the flavour of the state in the broken region. $\alpha$ will be relevant when the dimension of the eigenspace is bigger than one, for fixed values of the other quantum numbers. The eigenspace has dimension one in the case of total reflection, i.e. when the energy is smaller than the height of the wall, $|E_n| < m(\pm \infty)$, where it is assumed that the basis is such that $m_5(\infty) = 0$. In the opposite case, when the energy is larger than the height of the wall, it has dimension two. $\alpha$ serves then to label the state in any convenient basis. Examples will be given in the case of the thin wall. With the above tagging, two states with different labels are linearly independent.

Given a complete set of orthonormal eigenstates of the Hamiltonian, fields are defined in the usual way,

$$\Psi(\vec{x}, t) = \sum_{n^+} b_{n^+} \psi_{n^+}(\vec{x}) e^{-iE_{n^+}t} + \sum_{n^-} d_{n^-}^\dagger \psi_{n^-}(\vec{x}) e^{-iE_{n^-}t},$$

where $n^+$ ($n^-$) label the positive (negative) energy states. $b_{n^+}$ and $d_{n^-}$ and their hermitian conjugates are annihilation and creation operators\footnote{The annihilation operators verify $b_n |0> = d_n |0> = 0$, the state $|0>$ being the ”vacuum” that contains the wall, i.e. with a Higgs expectation value that depends on $z$.} with the usual anticommutation relations. The fields $\Psi(\vec{x}, t)$ verify the Dirac equation in the presence of the wall. The completeness relation for the eigenstates $\psi_n(\vec{x})$ forces upon them the standard Fermi-Dirac anticommutation relations. They are the Heisenberg quark fields in the presence of the wall, when the coupling of quarks to the gauge and scalar bosons are switched off: they constitute the starting point of usual perturbation theory.

Strictly speaking, although we study transitions between two given fermions, the propagation of bosons in the presence of the wall is needed. The reason is that a CP-violating effect in the SM is at least a one-loop process. The bosonic propagators in the presence of the wall are thus pertinent and viable in analogy to the fermion treatment we are developing here\footnote{Strictly speaking, although we study transitions between two given fermions, the propagation of bosons in the presence of the wall is needed. The reason is that a CP-violating effect in the SM is at least a one-loop process. The bosonic propagators in the presence of the wall are thus pertinent and viable in analogy to the fermion treatment we are developing here.} for technical simplicity, we leave such a task for a forthcoming publication, as the subsequent modifications should not change neither the order in the electroweak coupling constant at which the effect first shows up, nor the type of GIM cancellations.

Table 1 shows the transformation laws for the discrete symmetries of the system.

### 2.2 Symmetries of the exact theories.

Consider now the exact theory with the quark-boson interactions incorporated. The states in eq. (2.2) are no more eigenstates of the total Hamiltonian. The fields defined in eq. (2.3) verify the Dirac equation in the presence of the wall. The completeness relation for the eigenstates $\psi_n(\vec{x})$ forces upon them the standard Fermi-Dirac anticommutation relations. They are the Heisenberg quark fields in the presence of the wall, when the coupling of quarks to the gauge and scalar bosons are switched off: they constitute the starting point of usual perturbation theory.
\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|}
\hline
symmetry & wave function & field & momentum & helicity \\
\hline
Identity & $\psi_n(\vec{x})$ & $\Psi(\vec{x}, t)$ & $p$ & $h$ \\
P & $i\gamma_0\psi_n(\vec{x})$ & $i\sigma\gamma_0\Psi(\vec{x}, t)$ & $\vec{p}$ & $-h$ \\
C & $i\gamma_2\psi_n^*(\vec{x})$ & $i\gamma_2\Psi^*(\vec{x}, t)$ & $p$ & $h$ \\
CP' & $\gamma_5\psi_n^*(\vec{x})$ & $\gamma_5\Psi^*(\vec{x}, t)$ & $\vec{p}$ & $-h$ \\
T & $-\sigma\gamma_5\psi_n(\vec{x})$ & $-\sigma\gamma_5\Psi(\vec{x}, -t)$ & $-p$ & $h$ \\
TCP' & $-\sigma\gamma_5\psi_n(\vec{x})$ & $-\sigma\gamma_5\Psi(\vec{x}, -t)$ & $-\vec{p}$ & $-h$ \\
\hline
\end{tabular}
\caption{Transformation laws for the wave functions and the fields under the discrete symmetries of the wall, with $\vec{p} = (p_x, -p_y, p_z)$. In the case of the symmetries C, CP' and CP'T the transformed wave function corresponds to a negative energy state. However the transformed momenta and helicities we quote are those of the anti-particles, i.e. opposite to those for negative energy states.}
\end{table}

2.3) are not Heisenberg fields of the full theory, but they can be used as the quark fields in the “interaction representation”. The discrete symmetries apply to the fields exactly as stated in Table 1. In general all these symmetries are broken except TCP'.

Consider, for definiteness, the unbroken phase. From Table 1 the TCP'-transformed of a state with flavour $f$, momentum $\vec{p}$ and helicity $h$ is given by

$$|f, \vec{p}, h >_{TCP'} = |\bar{f}, -\vec{p}, -h >, \quad (2.4)$$

where $\bar{f}$ corresponds to the antiquark with the same flavour. TCP' is an antiunitary operation, and invariance under it implies that

$$TCP' < b |e^{-iH(t'_1-t'_2)}|a >_{TCP'} = (< b |e^{-iH(t_1-t_2)}|a >)^*$$

$$= < a |e^{-iH(t'_1-t'_2)}|b >, \quad (2.5)$$

where $a, b$ are two given quark states and $t'_1 = -t_1$ and $t'_2 = -t_2$. The relation $e^{-iH(t'_1-t'_2)} = (e^{-iH(t_1-t_2)})^\dagger$ has been used. Equations (2.4) and (2.5) imply, for instance for a top to charm transition, that

$$< c, (p_x, p_y, -p_z), R |e^{-iH(t_1-t_2)}|t, \vec{p}, L > =$$

$$< t, (-p_x, p_y, -p_z), R |e^{-iH(t_1-t_2)}|\bar{c}, (-p_x, p_y, p_z), L >, \quad (2.6)$$

and the analogous formula for all flavours and helicities.

Letting $t_1 \to \infty$ and $t_2 \to -\infty$, two important consequences follow from this equation:

- i) All CP' asymmetries obtained by summing over all flavours, helicities and incoming momenta will necessarily vanish:

$$\sum_{f,f',h,p_{inc}} |A^2(f, h, p_{inc} \to f', -h, p_{out})|^2 - |A(\bar{f}, h, p_{inc} \to \bar{f}', -h, p_{out})|^2 = 0, \quad (2.7)$$

where $A(f, h, p_{inc} \to f', -h, p_{out})$ represents the amplitude for an incoming particle $f$ with helicity $h$ and momentum $p_{inc}$ to be reflected into an outgoing particle $f'$ with
helicity $-h$ and momentum $p^{\text{out}} = (p_x^{\text{inc}}, p_y^{\text{inc}}, -p_z^{\text{inc}}, E)$. This strong result does not kill, however, the scenario for baryogenesis during the electroweak phase transition, since the latter is based on the role of sphalerons, which is totally different for left-handed and right-handed quarks. In the following we will consider CP' asymmetries summed over flavours and momenta but only for a given initial helicity.

\[ \text{ii)} \quad \text{A straightforward consequence of equation } 2.7 \text{ is} \]

\[ \sum_{f,f',p^{\text{inc}}} |A(f,L,p^{\text{inc}} \rightarrow f', R, p^{\text{out}})|^2 - |A(\bar{f}, R, p^{\text{inc}} \rightarrow \bar{f}', L, p^{\text{out}})|^2 = \]

\[ \sum_{f,f',p^{\text{inc}}} |A(f,L,p^{\text{inc}} \rightarrow f', R, p^{\text{out}})|^2 - |A(f', R, p^{\text{inc}} \rightarrow f, L, p^{\text{out}})|^2, \quad (2.8) \]

which is a useful simplification, as it allows to express a CP' asymmetry without any use of anti-quarks.

As we are interested in the breaking of CP' symmetry, it is useful to express the consequences of an hypothetical theory invariant under this transformation:

\[ <c, (p_x, p_y, -p_z, R)| e^{-iH((t_1-t_2)} | t, \bar{p}, L > = \]

\[ <\bar{c}, (p_x, -p_y, -p_z, L) | e^{-iH((t_1-t_2)} | \bar{t}, (p_x,-p_y,p_z), R >, \quad (2.9) \]

and the analogous formulae for all flavours and helicities.

3 The thin wall.

From now on we consider the SM in the presence of a thin wall, which allows to perform easier analytic calculations. The unperturbed Hamiltonian is, for each flavour,

\[ H = \vec{\alpha} \cdot \vec{p} + \beta m\theta(z). \quad (3.1) \]

3.1 Eigenstates

Consider a positive energy fermion with given $p_x, p_y, E$.\(^5\)

On the left of the wall ($z < 0$), an incoming particle has a $z$-momentum $p_z = \sqrt{E^2 - p_x^2 - p_y^2} > 0$, and helicity $h$. Define

\[ p^{\text{inc}} = (p_x, p_y, p_z, E). \quad (3.2) \]

Upon hitting the wall, a reflected particle has $z$-momentum $-p_z$ and

\[ p^{\text{out}} = (p_x, p_y, -p_z, E), \quad (3.3) \]

while a transmitted one has $z$-momentum $p'_z$ given by

\[ p'_z = \sqrt{p_z^2 - m^2} \quad \text{if } p_z^2 > m^2 \]

\[ p'_z = i\sqrt{m^2 - p_z^2} \quad \text{if } p_z^2 < m^2. \quad (3.4) \]

\(^5\)E > \sqrt{p_x^2 + p_y^2}.\]
With these definitions, $e^{ip'_z z}$ is a falling exponential in the case of total reflection ($p'_2 < m^2$). We thus define the transmitted 4-momentum as

$$p_{tr} = (p_x, p_y, p'_z, E).$$  (3.5)

The above physical situation is described by the following eigenstate of the Hamiltonian, which will be denoted “incoming” eigenstate,

$$\psi^{inc}_{n^+}(\vec{x}) = \left( u_h(p'^{inc}) e^{i\vec{p}^{inc, \vec{x}}} + R u_h(p^{inc}) e^{i\vec{p}^{out, \vec{x}}} \right) \theta(-z) + (1 + R) u_h(p^{inc}) e^{i\vec{p}^{tr, \vec{x}}} \theta(z), \quad h = j_z. \tag{3.6}$$

It is directly related to an incoming wave packet. $u_h(p^{inc})$ is a solution of the Dirac equation in the unbroken phase,

$$\not{p}^{inc} u_h(p^{inc}) = 0,$$  (3.7)

and the reflection matrix $R$ is given by

$$R = \frac{m\gamma_z}{p_z + p'_z}. \tag{3.8}$$

The Dirac structure of $R$ reflects the opposite chirality of the incoming and reflected state. Note as well that $R$ is complex in the case of total reflection. Its imaginary phase does not change from particles to antiparticles, and constitutes an example of the CP-even phases which will later on interfere with the CP-odd ones to produce observable effects.

It follows from the definition of $R$ that $(1 + R) u_h(p^{inc})$ is a solution of the Dirac equation in the broken phase,

$$(\not{p}^{tr} - m) (1 + R) u_h(p^{inc}) = 0,$$  (3.9)

as well as

$$\not{p}^{out} R u_h(p^{inc}) = 0.$$  (3.10)

In fact, eq. (3.9) was used to compute $R$. In the case of total reflection, $p'_2 - m^2 < 0$, the wave function (3.6) is the only positive energy eigenstate.

When transmission occurs, one more eigenstate is needed to span the eigenspace. A second linearly independent solution, to be called “outgoing” state (because directly related to the outgoing wave packets), is given by

$$\psi^{out}_{n^+}(\vec{x}) = \left( u_s(p^{out}) e^{i\vec{p}^{out, \vec{x}}} + R^\dagger u_h(p^{out}) e^{i\vec{p}^{inc, \vec{x}}} \right) \theta(-z) + (1 + R^\dagger) u_h(p^{out}) e^{i\vec{p}^{tr, \vec{x}}} \theta(z), \quad h = -j_z, \tag{3.11}$$

where

$$p^{br} = (p_x, p_y, -p'_z, E). \tag{3.12}$$

This wave function $\psi^{out}$ is however not orthogonal to $\psi^{inc}$. An orthogonal eigenstate is provided by the solution coming from the broken phase:

$$\psi^{br}_{n^+}(\vec{x}) = \sqrt{p_z/p'_z} \left[ \left( u_s(p^{br}) e^{i\vec{p}^{br, \vec{x}}} + J u_h(p^{inc}) e^{i\vec{p}^{tr, \vec{x}}} \right) \theta(z) + (1 + J) u_s(p^{br}) e^{i\vec{p}^{out, \vec{x}}} \theta(-z) \right], \tag{3.13}$$
where $J$ is the reflection matrix for a particle bouncing on the wall from the broken phase, given by
\[ 1 + J = \frac{p_z'}{p_z} (1 + R), \] (3.14)
and $s$ is a spin index dependent on $j_z$, such that $(1 + J) u_s(\vec{p}^{\text{br}}) = u_h(\vec{p}^{\text{out}})$ is a massless spinor with helicity $h = -j_z$, and $u_s(\vec{p}^{\text{br}})$ satisfies the Dirac equation in the broken phase
\[ (\vec{p}^{\text{br}} - m) u_s(\vec{p}^{\text{br}}) = 0. \] (3.15)

For later reference, we will name the solution in eq. (3.13) the “broken incoming” wave function.

The negative energy solutions, $\psi_{-n}$, describe the propagation of antiparticles. In terms of the quantum numbers for positive energy antifermions, the corresponding wave functions are
\[ \psi_{n-}^{\text{inc}}(\vec{x}) = \left( v_h(\vec{p}^{\text{inc}}) e^{i\vec{p}^{\text{inc}} \cdot \vec{x}} + \mathcal{R} v_h(\vec{p}^{\text{inc}}) e^{i\vec{p}^{\text{out}} \cdot \vec{x}} \right) \theta(-z) + (1 + \mathcal{R}) v_h(\vec{p}^{\text{inc}}) e^{i\vec{p}^{\text{tr}} \cdot \vec{x}} \theta(z), \quad h = j_z, \] (3.16)
\[ \psi_{n-}^{\text{out}}(\vec{x}) = \left( v_h(\vec{p}^{\text{out}}) e^{i\vec{p}^{\text{out}} \cdot \vec{x}} + \mathcal{R}^\dagger v_h(\vec{p}^{\text{out}}) e^{i\vec{p}^{\text{inc}} \cdot \vec{x}} \right) \theta(-z) + (1 + \mathcal{R}^\dagger) v_h(\vec{p}^{\text{out}}) e^{i\vec{p}^{\text{br}} \cdot \vec{x}} \theta(z), \quad h = -j_z, \] (3.17)
and
\[ \psi_{n-}^{\text{br}}(\vec{x}) = \sqrt{p_z/p_z'} \left[ (v_s(\vec{p}^{\text{br}}) e^{i\vec{p}^{\text{br}} \cdot \vec{x}} + \mathcal{J} v_s(\vec{p}^{\text{br}}) e^{i\vec{p}^{\text{tr}} \cdot \vec{x}}) \theta(z) + (1 + \mathcal{J}) v_s(\vec{p}^{\text{br}}) e^{i\vec{p}^{\text{out}} \cdot \vec{x}} \theta(z) \right], \] (3.18)

where $\mathcal{R} = -R$ and $(1 + \mathcal{J}) = p_z'/p_z (1 + \mathcal{R})$. $v_h(\vec{p}^{\text{inc}})$ is a negative energy Dirac spinor verifying the equation
\[ \vec{p}^{\text{inc}} v_h(\vec{p}^{\text{inc}}) = 0, \] (3.19)
with the helicity $h$ defined as $\vec{\sigma} \cdot \vec{p} v_h(\vec{p}) = -h v_h(\vec{p})$. The spin $s$ in eq. (3.18) is defined as for eq. (3.13), in such a way that $(1 + \mathcal{J}) v_s(\vec{p}^{\text{br}}) = v_h(\vec{p}^{\text{out}})$ with helicity $h = -j_z$. It follows from the definition of $\mathcal{R}$ that $(1 + \mathcal{R}) v_h(\vec{p}^{\text{inc}})$ is a solution of the Dirac equation in the broken phase:
\[ (\vec{p}^{\text{tr}} + m) (1 + \mathcal{R}) v_h(\vec{p}^{\text{inc}}) = 0, \] (3.20)
as well as
\[ \hat{p}^{\text{out}} \mathcal{R} v_h(\vec{p}^{\text{inc}}) = 0, \] (3.21)
and
\[ (\vec{p}^{\text{br}} + m) v_s(\vec{p}^{\text{br}}) = 0. \] (3.22)

It is convenient to consider the Fourier transformed wave functions
\[ \tilde{\psi}(q) = \int \frac{dz}{2\pi} e^{-iqz} \psi(z). \] (3.23)
From now on it will be assumed that the boost leading to \( p_x = p_y = 0 \) has been performed. We will only write here the \( z \) dependence, since the \( x - y \) part is trivial \((\delta(q_x)\delta(q_y))\). The results are

\[
\tilde{\psi}_{n^+}^{inc}(q_z) = \frac{1}{2i\pi} \left\{ \frac{1}{p_z - (q_z + i\epsilon)} - \frac{R}{p_z + q_z + i\epsilon} - \frac{1 + R}{p_z' - (q_z - i\epsilon)} \right\} u_h(\tilde{p}^{inc}), \tag{3.24}
\]

\[
\tilde{\psi}_{n^+}^{out}(q_z) = \frac{1}{2i\pi} \left\{ \frac{1}{-p_z - (q_z + i\epsilon)} - \frac{R^d}{-p_z + q_z + i\epsilon} - \frac{1 + R^d}{-p_z' - (q_z - i\epsilon)} \right\} u_h(\tilde{p}^{out}), \tag{3.25}
\]

\[
\tilde{\psi}_{n^+}^{br}(q_z) = \frac{1}{2i\pi} \sqrt{p_z/p_z'} \left\{ \frac{1}{-p_z' - (q_z - i\epsilon)} - \frac{J}{p_z' - (q_z - i\epsilon)} - \frac{1 + J}{-p_z' - (q_z - i\epsilon)} \right\} v_h(\tilde{p}^{br}), \tag{3.26}
\]

\[
\tilde{\psi}_{n^-}^{inc}(q_z) = \frac{1}{2i\pi} \left\{ \frac{1}{p_z - (q_z + i\epsilon)} - \frac{\overline{R}}{p_z + q_z + i\epsilon} - \frac{1 + \overline{R}}{p_z' - (q_z - i\epsilon)} \right\} \overline{u}_h(\tilde{p}^{inc}), \tag{3.27}
\]

\[
\tilde{\psi}_{n^-}^{out}(q_z) = \frac{1}{2i\pi} \left\{ \frac{1}{-p_z - (q_z + i\epsilon)} - \frac{\overline{R}^d}{-p_z + q_z + i\epsilon} - \frac{1 + \overline{R}^d}{-p_z' - (q_z - i\epsilon)} \right\} \overline{v}_h(\tilde{p}^{out}), \tag{3.28}
\]

\[
\tilde{\psi}_{n^-}^{br}(q_z) = \frac{1}{2i\pi} \sqrt{p_z/p_z'} \left\{ \frac{1}{-p_z' - (q_z - i\epsilon)} - \frac{\overline{J}}{p_z' - (q_z - i\epsilon)} - \frac{1 + \overline{J}}{-p_z' - (q_z - i\epsilon)} \right\} \overline{v}_h(\tilde{p}^{br}), \tag{3.29}
\]

with \( h = +j_z \) in \( \tilde{\psi}_{n^+}^{inc} \) and \( h = -j_z \) in \( \tilde{\psi}_{n^+}^{out} \).

### 3.2 Wave packets

The physical process under study is described by a wave packet coming from the unbroken phase, bouncing on the wall, and generating reflected and transmitted wave packets. The transmission probability follows from the reflection one. It is thus sufficient to concentrate on reflection properties.

Each wave packet has fixed flavour and helicity \( h \) (equivalent in the unbroken phase to \( j_z, -j_z \), respectively for the incoming, outgoing wave packets). Its energy and momenta are clustered around average values. Somewhere in the process the electroweak interactions will act as a perturbation, and induce flavour changes through loops, as will be discussed in Section 5.

The \( x \) and \( y \) components of the wave packet are not modified by the wall, and will be ignored in what follows. Consider the following incoming wave packet, approaching the wall from the unbroken phase,

\[
P(\tilde{p}^{inc}, z, t) = N \int \! dk_z e^{-\left( (k_z - p_z^{inc})^2 \frac{d^2}{x^2} \right)} e^{ik_z(z+Z-t-T)} u_h(\tilde{k}) \sim e^{ip_z^{inc}(z+Z-t-T)} e^{\left( \frac{(z+Z-t-T)}{c} \right)} u_h(\tilde{p}^{inc}), \tag{3.30}
\]

where \( N \) is a normalization constant, \( d \) denotes the spatial extension of the wave packet, which is located at time \( \sim -T \) (\( T > t \)) around position \( \sim -Z \) (\( Z > 0 \)), assuming \( Z/d >> 1 \), \( T/d >> 1 \). \( d \) is chosen so that \( p_z^{inc}d >> 1 \). Terms exponentially suppressed by \( \exp \left[ -(p_z^{inc}d)^2 \right] \) have been neglected in the second line of eq. \( (3.30) \).

At \( t \sim -T \), the wave packet is an almost monochromatic wave located far from the wall in the unbroken phase, with group velocity pointing towards the wall. To study its time
evolution when approaching the wall, an expansion on the eigenstates defined in subsection \ref{subsection2} is convenient. Up to exponentially suppressed terms, the result is

\[ P(\vec{p}^{\text{inc}}, z, t) = N \int dk_z e^{-(k_z - p_z^{\text{inc}})^2/2} \psi^{\text{inc}}_n(z) e^{-ik_z(t+T)}. \] (3.31)

In other words, the incoming wave packet totally expands on the eigenstates denoted by $\psi^{\text{inc}}_n$, thus justifying their name. Analogous conclusions hold for the outgoing wave packet in the unbroken phase. It follows that the physical amplitude can be expressed at order $\alpha_W$ as

\[ A(i \to f) = \langle \psi^{\text{out}}_n | H_{\text{int}} | \psi^{\text{inc}}_n \rangle, \] (3.32)

where $H_{\text{int}}$ is the interaction Hamiltonian induced by electroweak loops.

4 The quark propagator.

The free quark propagator is defined as usual:

\[ S(x, y) = \langle 0 | T(\Psi(x)\bar{\Psi}(y)) | 0 \rangle. \] (4.1)

It is not translational invariant due to the presence of the wall, and verifies the equation

\[ (\partial_{x_0} + iH(x))S(x, y) = \gamma_0 \delta_4(x - y), \] (4.2)

where $H(x)$ is the Hamiltonian in eq. (3.1). Furthermore, the time ordered product in equation (4.1) insures the Feynman boundary conditions. From eq. (2.3), we get

\[ S(x, y)\gamma_0 = \theta(x_0 - y_0) \sum_{n^+} \psi^{+}_{n^+}(\vec{x})\psi^{+\dagger}_{n^+}(\vec{y}) e^{-iE_{n^+}(x_0 - y_0)} - \\
\theta(y_0 - x_0) \sum_{n^-} \psi^{-}_{n^-}(\vec{x})\psi^{-\dagger}_{n^-}(\vec{y}) e^{-iE_{n^-}(x_0 - y_0)}, \] (4.3)

where $E_{n^\pm}$ is the positive/negative energy of the eigenstate, cf. eq. (2.2), and where the definition

\[ \sum_{n^\pm} \equiv \sum_{j, j', \alpha} \left( \frac{1}{2\pi} \right)^3 \int dt_x dt_y dt_0 E_{n^\pm} \] (4.4)

has been used.

The Fourier transformed propagator depends on two momenta, $q^i$ and $q^f$, unlike in the translationally invariant case,

\[ q^f = (E, q_x, q_y, q_z^f), \]
\[ q^i = (E, q_x, q_y, q_z^i). \] (4.5)

We can write

\[ S(q^f, q^i) = \int d\xi_x d\xi_y d(\xi_x' - \xi_x) d(\xi_y' - \xi_y) \int d\xi_0 d(\xi_0' - \xi_0) \ S(\xi', \xi) \times \\
e^{-i\xi^j_0 \xi^j_0 + iq^i_0 \xi_z - iq_x(\xi_x' - \xi_x) - iq_y(\xi_y' - \xi_y) + iE(\xi_0' - \xi_0)} \] (4.6)
where translation invariance in $x, y$ and time directions has been used. From now on we use the liberty of boosting in the $x, y$ plane to bring $q_x$ and $q_y$ to 0. At the end of the section we will return to the general case.

In terms of the momentum-space wave functions, the above translates into

$$S(q^f, q^i)\gamma_0 = \frac{-1}{i} \sum_{n=1}^{\infty} (2\pi)^6 \left[ \bar{\psi}^{inc}_n(q^f_z) \left( \bar{\psi}^{inc}_n(q^i_z) \right)^\dagger \frac{1}{E - E_{n+} + i\epsilon} \right.$$  

$$+ \bar{\psi}^{inc}_n(q^f_z) \left( \bar{\psi}^{inc}_n(q^i_z) \right)^\dagger \frac{1}{E - E_{n-} - i\epsilon} + \bar{\psi}^{br}_n(q^f_z) \left( \bar{\psi}^{br}_n(q^i_z) \right)^\dagger \frac{-1}{E - E_{n+} + i\epsilon} + \bar{\psi}^{br}_n(q^f_z) \left( \bar{\psi}^{br}_n(q^i_z) \right)^\dagger \frac{-1}{E - E_{n-} - i\epsilon} \right]. \tag{4.7}$$

Notice in the above expression that an orthonormal basis for the wave functions was used, i.e., the one spanned by $\bar{\psi}^{inc}$ and $\bar{\psi}^{br}$, given in eqs. (3.24), (3.27), (3.26) and (3.29). In particular, orthogonality requires to restrict the energy integration on the broken phase to $E > m$. We have explicitly verified the completeness relation for our system of eigenstates

$$\sum_{n=1}^{\infty} \left[ \bar{\psi}^{inc}_n(q^f_z) \left( \bar{\psi}^{inc}_n(q^i_z) \right)^\dagger + \bar{\psi}^{br}_n(q^f_z) \left( \bar{\psi}^{br}_n(q^i_z) \right)^\dagger \right] = \frac{1}{(2\pi)^3} \delta(q^f_z - q^i_z). \tag{4.8}$$

The obvious consequence of eqs. (2.2) and (4.8) is the Green’s function equation for the propagator:

$$-i(E - H) S(q^f, q^i)\gamma_0 = (2\pi)^3 \delta(q^f_z - q^i_z). \tag{4.9}$$

The Fourier transform of eq. (4.9) is:

$$\left( \partial_{\xi^0} + \vec{\alpha} \cdot \vec{\nabla}_{\xi} + i\beta m\theta(\xi_z) \right) S(\xi, \xi') = \gamma_0 \delta^4(\xi - \xi'). \tag{4.10}$$

Details on the computation of the propagator can be found in Appendix [x]. Let us summarise the results in 1+1 dimensions (time and z-directions). An interesting intermediate step is given by the expressions which follow. The contribution of states propagating exclusively on the left-hand side of the wall ($v = 0$, “unbroken” region) is

$$S(q^f, q^i)_{left} \gamma_0 = \left\{ \begin{array}{l}
\frac{1}{q^f_z - q^i_z + i\epsilon} \left( E + q^f_z \alpha_z \right) \frac{1}{(sE + i\epsilon)^2 - (q^f_z + i\epsilon)^2}
\frac{1}{s(1 + s\alpha_z)}
\frac{1}{sE + q^f_z + i\epsilon}
\end{array} \right\}. \tag{4.11}$$

\(^6\)The negative energy eigenfunctions describe the propagation of antiparticles: in the notation presently used an initial antiparticle will have momentum $q^f$, while a final antiparticle has momentum $q^i$. 

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\([x]\): This reference is not visible in the image. It is likely a citation or link to further information.
The contribution from states propagating exclusively on the right-hand side of the wall (\( v.e.v. \neq 0 \)), that is, in the “broken” region is

\[
S(q^f, q^i)_{\text{right} \gamma_0} = \left\{ \begin{array}{c}
\frac{-1}{q^f_z - q^i_z - i\epsilon (p'_z + i\epsilon)^2 - (-q^f_z + i\epsilon)^2} (E + q^f_z \alpha_z + m\gamma_0) \\
\frac{1}{(p'_z + i\epsilon) + (q^f_z - i\epsilon) p'_z + q^i_z + i\epsilon p'_z} \left(1 - \frac{p'_z}{E} \alpha_z + \frac{m}{E} \gamma_0 \right) \\
- \frac{1}{p'_z - q^f_z + i\epsilon p'_z + q^i_z + i\epsilon p'_z} \left(1 + \frac{p'_z}{E} \alpha_z + \frac{m}{E} \gamma_0 \right) \frac{sm\gamma_0}{sE + p'_0} \right\}.
\]

(4.12)

Finally, the contribution from states propagating across the wall is

\[
S(q^f, q^i)_{\text{across} \gamma_0} = s\left\{ \frac{1}{p'_z - q^f_z + i\epsilon sE - q^i_z + i\epsilon} \left(1 + \frac{sm\gamma_0}{sE + p'_0} \right) \frac{(1 + s\alpha_z)}{2} \\
+ \frac{1}{sE + q^f_z + i\epsilon p'_z + q^i_z + i\epsilon} \left(1 - s\alpha_z \right) \frac{(1 + s\gamma_0)}{2} \right\},
\]

(4.13)

where \( s \) is a parameter which takes values +1 or −1 for positive or negative energy particles, respectively. \( p'_0 \) is defined as follows

\[
p'_0 = +\sqrt{E^2 - m^2} + i\epsilon,
\]

(4.14)

and the factor \( sm\gamma_0/(sE + p'_0) \) originates from the reflection matrix \( R \), eq. (3.8).

Eqs. (4.11)-(4.13) show sometimes two \( i\epsilon \) terms with opposite signs present in the same denominator, for instance \( 1/((sE + i\epsilon) - (q^f_z + i\epsilon)) \) in the second line of eq. (4.11). The reason for this phenomenon is clear. The first \( i\epsilon \) in our example is there to implement the Feynman boundary conditions for the propagator in time direction. The second \( i\epsilon \) specifies in which spatial phase (\( z < 0 \) or \( z > 0 \)) this terms acts. This may seem ambiguous: is the pole located in the upper or lower half plane? Both prescription are not equivalent, unless the residue at the pole vanishes. In other words, since \( 1/(x \pm i\epsilon) = P(1/x) \mp \pi \delta(x) \), the ambiguity disappears only if the factor multiplying \( \delta(x) \) vanishes at \( x = 0 \). A close scrutiny of eqs. (4.11)-(4.13) shows that this is indeed the case. For instance, the residue at \( sE = q^f_z \) in the second line of eq. (4.11) is exactly compensated by the residue of the equally “ambiguous” term in the first line of eq. (4.11). It results that the sum of both terms is not ambiguous. It is possible to chose in a consistent way any sign for the \( i\epsilon \) in the first and second lines of eq. (4.11). Flipping from one convention to the other exchanges a term proportional to \( \delta(sE - q^f_z) \) between the first and second line of eq. (4.11), and the sum does not change. For later use denote “time \( i\epsilon \)” the usual Feynman convention and “space \( i\epsilon \)” convention the other one. In the following the “time \( i\epsilon \)” convention will be assumed unless specified otherwise.

The propagator is given by the sum of eqs. (4.11), (4.12) and (4.13) above,

\[
S(q^f, q^i)\gamma_0 = \left( S(q^f, q^i)_{\text{left} } + S(q^f, q^i)_{\text{right} } \right) + S(q^f, q^i)_{\text{across} } \gamma_0.
\]

(4.15)

In Appendix A it is explicitly shown that eq. (4.13) verifies the Dirac equation. Using the relations (5.38, 5.39), as well as the above mentioned freedom to shift certain poles, it is
possible to combine the various terms in a more compact and familiar way. One can either show that

\[
S(q^f, q^i) = -\left\{ \frac{1}{q^f - q^i + \imath \hat{q}^i} - \frac{1}{q^f - q^i - \imath \hat{q}^i} \right\} + \frac{1}{\hat{q}^i} \left[ \frac{1 + \imath \alpha_z}{2} \right] \left[ 1 - \frac{m \gamma_0}{E + p_z'} \right] s\gamma_0 \frac{m}{\hat{q}^i (\hat{q}^i - m)} - \frac{1}{\hat{q}^f - \hat{q}^i - m} \left[ 1 - \frac{m \gamma_0}{E + p_z'} \right] \left[ \frac{1 - \imath \alpha_z}{2} \right] s\gamma_0 \frac{m}{\hat{q}^i (\hat{q}^i - m)} \right\}, \tag{4.16}
\]

which simplifies the action of the \( q^i \)-Dirac operator, or that

\[
S(q^f, q^i) = -\left\{ \frac{1}{q^f - q^i + \imath \hat{q}^i} - \frac{1}{q^f - q^i - \imath \hat{q}^i} \right\} + \frac{m}{\hat{q}^i (\hat{q}^i - m)} \left[ 1 - \frac{m \gamma_0}{E + p_z'} \right] \left[ \frac{1 - \imath \alpha_z}{2} \right] s\gamma_0 \frac{1}{\hat{q}^i} - \frac{m}{\hat{q}^f - \hat{q}^i - m} \left[ 1 - \frac{m \gamma_0}{E + p_z'} \right] \left[ \frac{1 - \imath \alpha_z}{2} \right] s\gamma_0 \frac{1}{\hat{q}^i} \right\}, \tag{4.17}
\]

which prepares for the action of the \( q^f \)-Dirac operator. In fact, eq. (4.16) is the CP' conjugate of (4.17). An elegant expression, because more symmetric in the \( q^i \) and \( q^f \) dependences, can be obtained combining (4.16) and (4.17) and is given by

\[
S(q^f, q^i) = -1/2 \left\{ \frac{1}{q^f - q^i + \imath \hat{q}^i} - \frac{1}{q^f - q^i - \imath \hat{q}^i} \right\} \left[ \frac{1}{\hat{q}^f - \hat{q}^i - m} \right] + \frac{m}{\hat{q}^f - \hat{q}^i - m} \left[ 1 - \frac{m \gamma_0}{E + p_z'} \right] \left[ \frac{1 - \imath \alpha_z}{2} \right] s\gamma_0 \frac{m}{\hat{q}^i (\hat{q}^i - m)} \right\}. \tag{4.18}
\]

In (4.16), (4.17) and (4.18): all \( \hat{q} \)'s in the denominators are understood as regularized “à la Feynman”, i.e. as \( \hat{q} + \imath \epsilon \), the “time \( \epsilon \)" convention. This convention allows to check easily that our propagator obeys Feynman boundary conditions, i.e. that the positive frequencies \( (q_0 > 0) \) are “retarded”, and the negative ones \( (q_0 < 0) \) “advanced”.

If the “space \( \epsilon \)" convention was used instead, then the regularisation implies to consider all massless poles \( (1/\hat{q}^f, 1/\hat{q}^f) \) with \( q^f \to q^f \to q^i + \imath \epsilon \) and \( q^f \to q^i - \imath \epsilon \). Upon Fourier transform it corresponds to \( \theta(z_i) \) and \( \theta(\bar{z}_f) \) respectively, i.e. it locates the massless poles in the unbroken region. Analogously, all massive poles \( (1/(\hat{q}^f - m_f), 1/(\hat{q}^i - m_i)) \) are to be understood as \( q^f \to q^f \to q^i - \imath \epsilon \) and \( q^f \to q^i + \imath \epsilon \), locating them in the broken region. In this convention it is easy to check that the propagator in eqs. (4.16), (4.17) and (4.18) verifies the Dirac equation (4.10).
For some terms in eqs. (4.16), (4.17) and (4.18) the “time $i\epsilon$” and the “space $i\epsilon$” conventions are apparently in conflict. For instance,

$$\frac{\not{q}f}{q_0^2 - (q^2 + i\epsilon)^2} = \frac{\not{q}f}{q_0^2 - (q^2 + i\epsilon)^2} + \frac{2i\pi \not{q}f \theta(q_z^2)\delta(q^2_0 - (q_z^2)^2)}{\not{q}f - m}.$$  (4.19)

The two conventions differ by the last term on this equation. However, this term, as all matching terms of this type, do cancel among the different factors in any of the eqs. (4.16), (4.17) and (4.18). This cancellation is in fact the same phenomenon as the cancellation of the residues corresponding to “ambiguous $i\epsilon$’s” that we discussed after eqs. (4.11), (4.12) and (4.13). We are thus lead to the striking conclusion that the “time” and the “space” $i\epsilon$ conventions are strictly equivalent in eqs. (4.16), (4.17) and (4.18).

This equivalence is physically understandable. It means that all the poles for which the two conventions give conflicting “$i\epsilon$” signs have a vanishing residue: they correspond to physically non acceptable asymptotic states. For instance, consider a massless pole with a positive $q_z^2$ in eq. (A.7). Massless asymptotic states can only exist in the unbroken phase, i.e. on the left side of the wall. A final state in the unbroken phase must necessarily move away from the wall, i.e. with a negative $q_z^2$. It is then physically welcome that the residue vanishes when $q_z^2$ is positive. All other apparent conflicts are resolved in a similar way.

We shall denote the first line of expressions (4.16), (4.17) and (4.18) as the “non-homogeneous” part. The rest of the propagator in any of its versions will be denoted “homogeneous part”, as the latter is a solution of the Dirac equation without a source, contrary to the former. By the same token, it is possible to verify that the advanced and retarded Green functions in the presence of a wall differ from those for the usual Dirac propagator by terms which vanish upon the action of the Dirac operator.

The presence of both massless and massive poles reflects the fermion sailing between the unbroken and broken phases. It is trivial to verify in eqs. (4.16), (4.17) and (4.18) that the usual $i/\not{p}$ Feynman propagator for a massless fermion is recovered. Notice as well that, besides the usual $i\epsilon$ dependence which corresponds to absorptive contributions for on-shell fermions, new imaginary phases appear which are generated by the reflection coefficients of the fermion when $E^2 < m^2$. This CP-even phases, present for on-shell and off-shell fermions, are induced by the reflection matrix, eq. (3.8).

For completeness we give as well the fermion propagator in the presence of the wall for the 4-dimensional case, i.e., when $q_x$ and $q_y$ have not been boosted to zero values:

$$S(q^f, q^i) = -1/2\left\{\frac{1}{q^f_z - q_z^i + i\epsilon} \left( \frac{1}{\not{q}^f} + \frac{1}{\not{q}^i} \right) - \frac{1}{q^f_z - q_z^i - i\epsilon} \left( \frac{1}{\not{q}^f - m} + \frac{1}{\not{q}^i - m} \right) + \frac{m}{\not{q}^f - m \not{q}^i - m} \right\}$$

$$\times \left[ \frac{E\gamma_0 - q_z \gamma_x - q_y \gamma_y + \frac{msy}{p_x + p_z} (E\gamma_0 - q_z \gamma_x - q_y \gamma_y + s\gamma_z)}{p_z} \right] \frac{m}{\not{q}^f - m \not{q}^i - m}.$$  (4.20)

where $p_z = \sqrt{E^2 - q_x^2 - q_y^2 + i\epsilon}$ and $p_z' = \sqrt{E^2 - q_x^2 - q_y^2 - m^2 + i\epsilon}$. 

During the (long) preparation of this manuscript, we became aware of ref. [17] which derives the Dirac propagator in presence of a thick wall and uses it to compute the CP-asymmetry in the reflection process. The source of CP-violation they use is not a SM loop but rather the tree-level axial quark mass in a 2 scalar model. Although this difference makes the comparison between both works delicate, their results seem to support our statements that increasing the wall thickness would further reduce the asymmetry.

5 One loop computation.

The propagators in the presence of the wall will be depicted by a line screened by crosses (see Fig. 2), while the usual ones are drawn as solid lines. Consider a one-loop transition between two external flavours, $i$ and $f$, such as the ones in Fig. 2. The analogous diagrams with full lines, i.e., with the usual Feynman propagators, are completely renormalized away for on-shell external states. This will be changed by the presence of the wall, which acts on the diagram as a external source of momentum.

Figure 2: The kinky propagators for quarks, $W$’s and scalars can be assembled into loop diagrams. Vertices, being local, are unchanged in this formalism.

Let us review the properties expected in general, independently of the details of a given calculation. The process under consideration describes an asymptotically massless quark $i$, approaching the wall from the unbroken phase, which is reflected into the same phase as an asymptotic massless $f$ quark. A flip of chirality has thus necessarily occurred. The flavour change happens through one electroweak loop, and we will denote by $M$ the internal fermion mass.

First of all, the amplitude should not vanish at order $\alpha_W$. There is no symmetry reason known to us implying that the coefficient of the CP-violating operator should be zero at this order.

Second, the amplitude should vanish at least as $m_i m_f$ when the internal loop is computed in just one of the two phases, and Lorentz covariance is preserved. A priori this behaviour is not mandatory, as just one chirality flip is needed, and an amplitude vanishing with masses
as either \( m_i \) or would seem adequate. But there are more constraints: in the unbroken phase, the computation of the internal loop should behave as

\[
I_\mu(q_\mu) \propto \frac{1}{q_\mu}, \tag{5.1}
\]
due to Lorentz covariance. This loop is to be inserted on the propagator in the presence of the wall. Imagine that reflection has happened before the loop insertion (providing an \( m_i \) factor), but not afterwards. The outgoing state is then a plane wave verifying the Dirac equation for a massless particle. The action of eq. (5.1) on it gives zero. In the opposite case, when reflection happens only after the loop insertion, the analogous argument for the incoming massless state holds as well. Consequently, the action of the wall must be present before and after the internal loop insertion, and a non trivial dependence of the amplitude on both \( m_i \) and \( m_f \) results. The total effect should go to zero as some positive power of both of them, with an odd overall dependence on external masses, since chirality flips upon reflection. The argument can be generalized to the situation where the broken phase is considered inside the loop, the difference with eq. (5.1) being terms independent of \( \frac{1}{q_\mu} \), but proportional to the external masses, as it will be exemplified below.

Third, it is possible to argue the type of GIM cancellations for the quark masses inside the loop. The relevant terms in the reflection amplitude \( A(i \to f) \) correspond to the interference of diagrams with two different internal quark masses, \( M \) and \( M' \), to be summed upon. Each individual diagram can be written as

\[
A(i \to f) \propto F(M) = a + \frac{M^2}{M_W^2}(b + clog\frac{M}{M_W}) + \frac{M^4}{M_W^4}(d + e log\frac{M}{M_W}) + \ldots, \tag{5.2}
\]
with \( a, b, c, d, e \ldots \) depending only on external masses and momenta. Due to CKM unitarity (see eq. (5.23)), the CP observable is proportional to \( \sum_{M,M'} Im[F(M)F(M')^\ast] \). As each term in this sum is an antisymmetric function of \( M \) and \( M' \), the squared terms in the development (5.2) of the product cancel, while the cross-terms involving the constant piece \( a \) fall out once the sum is performed. The lowest order contribution is thus \( \sim \sum \frac{M^2M'^2}{M_W^4}log(M^2) \).

In the practical example below, we find \( \frac{M^2M'^4}{M_W^2} \log M'^2 \), i.e. a further internal mass suppression. It is important to notice that, whenever only the unbroken phase is considered inside the electroweak loop, \( c = d = e = 0 \), because the fermionic mass dependence stems from pure Yukawa couplings. No antisymmetric function in \( M, M' \) is viable, and the effect should vanish at order \( \alpha_W^2 \) in total rate. There is no reason, though, to neglect the broken phase, and we expect an \( \mathcal{O}(\alpha_W^2) \) contribution.

Finally, the observable should be proportional to the interference of the imaginary parts of the CKM couplings with the CP-even phases produced in the reflection on the wall.

The above considerations apply as well for the thick wall scenario. Some remarks specifically related to the wall thickness are appropriate, before entering into the details of our computation. The internal electroweak loop is in general a complicate function of \( q^2 \), and thus non-local. Its typical “inverse-size” is of the order of the dimensional parameters involved which means \( \geq M_W \), as we will see. To neglect its non-local character is only adequate

\(^7\)This dependence is in general a complicate function. For transparency, only the behaviour for \( M < M_W \) is described here. As the internal loop is IR convergent when \( M \to 0 \), no pure \( \log M \) terms can appear.
for wall thicknesses $l$ larger than the loop “size”, $l > M_W^{-1}$. For a thin wall, $l \to 0$, a local approximation for the internal loop is incorrect. Arbitrary large particle momenta are relevant, and it will be shown that indeed the non-local character of the internal loop is important. The role played by the latter effects in our calculation suggests that an even smaller result would follow in a more realistic thick wall computation, $l \gg M_W^{-1}$, where a local internal loop could be a good approximation.

We will see that the above four general characteristics do appear in the toy computation given below.

### 5.1 One particle irreducible loop

The complete computation of the diagrams in Fig. 2 requires not only the knowledge of the fermion propagator in the presence of the wall, but the corresponding one for gauge and Higgs bosons as well. They can be obtained in complete analogy with the computations of the previous section. However, they need rather lengthy calculations. Our goal here is rather to get a feeling of how the different building blocks work together than to solve exactly a problem which anyhow is academic to a large extent. We have chosen to simplify our task, leaving the full calculation for a forthcoming publication. What we compute in fact are the diagrams in Fig. 3 with the internal loop computed in the broken phase. Notice that a similar choice has been made in recent publications which try to solve the problem at finite temperature\cite{12}, where only the unbroken phase was considered inside the electroweak loops.
where $A$ and $B$ are matrices with 1 and/or $\gamma_5$ components, $A^{\text{subst.}}(q^2) = A(q^2) - A(0)$ and $B^{\text{subst.}}(q^2) = B(q^2) - B(0)$. Finally,

$$I^{\text{subst.}}(q_\mu) = \pi^2 c \int_0^1 dx \int_0^1 dy (qax + b) \frac{q^2 x (1 - x)}{q^2 xy (1 - x) - M_W^2 x - M_t^2 (1 - x)}. \quad (5.5)$$

Notice that with the above choice of substraction, the internal loop is completely renormalized away for an on-shell massless fermion which suffers no interactions with the wall. Concentrate for the moment in the case $i, f = d, s$ or $b$. The scalar-exchange contribution is then

$$I^{\text{scalar}}_s(q_\mu) = \frac{g^2}{2M_W^2 (2\pi)^4} K_{ij}^* K_{il} \int_0^1 dx \int_0^1 dy \frac{q^2 x (1 - x)}{q^2 xy (1 - x) - M_W^2 x - M_t^2 (1 - x)} \{xq_i m_f R + M_f^2 L - M_t^2 (m_f L + m_i R)\}, \quad (5.6)$$

where $L$ and $R$ denote the chiral projectors, $L = (1 - \gamma_5)/2$, $R = (1 + \gamma_5)/2$. $K$ describes the CKM couplings of the quarks and $M_i$ is the mass of the internal quark of $l$-th generation.

The $W$-exchange contribution to the substracted internal loop is

$$I^{\text{subtx}}_W(q_\mu) = \frac{g^2}{(2\pi)^4} K_{ij}^* K_{il} \int_0^1 dx \int_0^1 dy \frac{q^2 x (1 - x)}{q^2 xy (1 - x) - M_W^2 x - M_t^2 (1 - x)} xq_i L. \quad (5.7)$$

The above represent only one contribution among many that should be considered in the full calculation. When this internal loop is inserted below on our propagator, gauge invariance is preserved, as the fermion interaction with the wall is of Yukawa type and thus $SU(3) \times SU(2) \times U(1)$ gauge invariant. In the full theory, the substraction procedure will be more complicated, though.

Our aim in a toy computation is to get an idea of how the ingredients of CP violation, GIM mechanism, and wall reflection combine together, and to settle useful calculational tools for this type of problem.

### 5.2 Loop insertion in the propagator

The diagrams in Fig. 3 are computed inserting the internal loops given above on the fermion propagator in the presence of the wall, derived in the preceding section. The desired amplitude should describe the transition of an incoming massless flavour $i$ into an outgoing massless flavour $f$. The selection of these asymptotic states is obtained projecting onto the massless poles of the propagator, for the $4$-momenta appearing at both extremes of Fig. 3.

---

8What has been done above is close to the complete result in a limited extension of the standard model, with two Higgs's, one ($\Phi_d$) coupled to down quarks and the other ($\Phi_u$) to up quarks. We assume the following expectation values: $<\Phi_d(z)> = v_d \theta(z)$ and $<\Phi_u(z)> = v_u$, with $v_d \ll v_u$. In this model the equation (5.5) turns out to become the exact contribution from $W$ exchange, while eq. (5.6), or rather its Fourier transform, is modified by the following substitution: $m_i \rightarrow m_i \theta(z_i)$, $m_f \rightarrow m_f \theta(z_f)$, where $z_i$ ($z_f$) is the coordinate of the incoming (outgoing) vertex of the loop. An analysis of the changes in the computations runs parallel to the one performed above, but it does not seem pertinent to pursue this model here.
The relevant contributions of Fig. (3) are then those terms which contain both poles in \(1/q_i^i\) and \(1/q_{i^f}\). It can be shown that the non-homogeneous part of the fermion propagator in the presence of the wall, gives no contribution of this type. That is, among the four possible combinations of homogeneous and non homogeneous on both sides of the internal loop, only the homogeneous-internal loop-homogeneous contributes, yielding for a positive energy particle:

\[
A(i \rightarrow f) = \int \frac{d\tilde{q}_z}{2\pi} \frac{1}{\tilde{q}_i^+ \tilde{q}_{i^f}^+} \left[ \frac{1 + \alpha_z}{2} \right] \left[ 1 - \frac{m_f \gamma_0}{E + p_{i^f}^+} \right] \gamma_0 \frac{m_f}{\tilde{q}_i^+ (\tilde{q}_i^+ - m_f)} \left[ I_{\text{W}}^{\text{subt.}}(q_{\mu}) + I_{\text{scalarm}}^{\text{subt.}}(q_{\mu}) \right] \frac{m_i}{\tilde{q}_i^+ (\tilde{q}_i^+ - m_i)} \left[ 1 - \frac{m_i \gamma_0}{E + p_{i}^+} \right] \left[ 1 - \frac{\alpha_z}{2} \right] \gamma_0 \frac{1}{\tilde{q}_i^+},
\]

(5.8)

where

\[
p_{i}^{i^f} = \sqrt{E^2 - m_i^2} + i\epsilon \quad \text{(5.9)}
\]

and

\[
p_{i^f}^{i^f} = \sqrt{E^2 - m_{i^f}^2} + i\epsilon \quad \text{(5.10)}
\]

The \(q_z\) integration reflects the arbitrary off-shellness of the quarks upon reflection on the wall. The relevant integrals are the following ones:

\[
\int \frac{d\tilde{q}_z}{2\pi} \frac{1}{q_i^2 - m_i^2 q^2 - m_f^2 q^2 - \tilde{M}_i^2} = \frac{i}{2(m_i^2 - \tilde{M}_i^2)(m_f^2 - \tilde{M}_i^2)} \left[ \frac{1}{p_{i}^{i^f} p_{i^f}^{i^f}} (p_{i}^{i^f} + \tilde{M}_i^2 - m_i^2) - \frac{1}{\tilde{Q}_0} \right],
\]

(5.11)

and

\[
\int \frac{d\tilde{q}_z}{2\pi} \frac{1}{q_i^2 - m_i^2 q^2 - m_f^2 q^2 - \tilde{M}_i^2} = \frac{i}{2(m_i^2 - \tilde{M}_i^2)(m_f^2 - \tilde{M}_i^2)} \left[ \frac{1}{p_{i}^{i^f} p_{i^f}^{i^f}} (p_{i}^{i^f} \tilde{M}_i^2 + m_i^2 \tilde{M}_i^2 - m_i^2) - \frac{\tilde{M}_i^2}{\tilde{Q}_0} \right],
\]

(5.12)

where

\[
\tilde{M}_i^2 = x M_W + (1 - x) M_i^2,
\]

(5.13)

\[
\tilde{Q}_0 = \sqrt{E^2 - \tilde{M}_i^2} + i\epsilon.
\]

(5.14)

After some lengthy algebraic manipulations, the resulting amplitude for an incoming left asymptotic quark can be written as

\[
A(i_R \rightarrow f_L) = \frac{g^2}{2} \frac{\pi^2}{(2\pi)^4} K_{i R} K_{i^f} \int_0^1 dx \int_0^1 dy \frac{i}{y (m_i^2 - \tilde{M}_i^2)(m_f^2 - \tilde{M}_i^2)} \frac{1}{\tilde{q}_i^+ m_i m_f^2} \left[ \left( \frac{1}{p_{i}^{i^f} p_{i^f}^{i^f}} (p_{i}^{i^f} \tilde{M}_i^2 + m_f^2 \tilde{M}_i^2 - m_i^2) - \frac{\tilde{M}_i^2}{\tilde{Q}_0} \right) [x\tilde{a} + \tilde{b}] + \right.
\]

\[
\left. \left( \frac{1}{p_{i}^{i^f} p_{i^f}^{i^f}} (p_{i}^{i^f} \tilde{M}_i^2 - m_i^2) - \frac{1}{\tilde{Q}_0} \right) [x\tilde{c} + \tilde{d}] \right] \frac{1 + \alpha_z}{4} R \frac{1}{\tilde{q}_i^+}.
\]

(5.15)
where

\[
\tilde{a} = (2 + \frac{M^2_{W}}{M^2_{W}})(1 - \frac{p'^i_z}{E + p'^f_z}) + \frac{m^2_i}{M^2_{W}}(1 - \frac{p'^f_z}{E + p'^f_z}), \tag{5.16}
\]

\[
\tilde{b} = -\frac{M^2_{W}}{M^2_{W}}(1 + \frac{E - p'^i_z}{E + p'^f_z}), \tag{5.17}
\]

\[
\tilde{c} = -m^2_i \left[ \frac{E}{E + p'^f_z}(2 + \frac{M^2_{W}}{M^2_{W}}) + \frac{m^2_i}{M^2_{W}} E \right], \tag{5.18}
\]

\[
\tilde{d} = \frac{M^2_{W}}{M^2_{W}}(E^2 - p'^i_z p'^f_z)(1 + \frac{E - p'^i_z}{E + p'^f_z}). \tag{5.19}
\]

Notice that \(\tilde{Q}_0\), obtained from the convolution of the electroweak loop with the free fermion propagator in the presence of the wall, is in general complex, and will be one of the relevant CP-even phases which will interfere with the CKM complex couplings. This new phase stems from the non-local character of the internal electroweak loop, and would have disappeared in a linear approximation. Other contributions of the same type should be present in a full calculation with the wall accounted for inside the loop, and although cancellations are possible, they are not mandatory by any symmetry argument.

For an incident left-handed particle, the result is obtained from eqs. (5.15) to (5.19) replacing everywhere \(m_i\) by \(m_f\) and viceversa (and consequently \(p'^i_z\) by \(p'^f_z\)). The CKM couplings remain identical.

Consider transitions between charge \(-1/3\) quarks. Concentrate as well on quark energies below or equal to \(M_W\). The approximation \(m^2_i, m^2_f, E^2 < \tilde{M}^2_f\) is then justified, and eq. (5.15) simplifies to

\[
A(i_R \rightarrow f_L) = \frac{g^2}{2} \frac{\pi^2}{(2\pi)^4} K_{ii} K^*_{ij} \int_0^1 dx \int_0^1 dy \frac{i}{y M^4_{i}} \left[ \frac{1}{\tilde{q}^i} m_i m^2_f \left( \frac{1}{p'^i_z p'^f_z} - \frac{1}{i M_f} \right) \tilde{M}^2_f[xa + b] + \frac{1}{p'^i_z p'^f_z} \left( \frac{\tilde{M}^2_f}{p'^i_z + p'^f_z} - \frac{1}{i M_f} \right) [xc + d] \right] \frac{1 + \alpha_z}{4} R \frac{1}{\tilde{q}^i}. \tag{5.20}
\]

where \(a, b, c, d\), are given by eq. (5.19), neglecting terms in \(m^2_i/M^2_{W}, m^2_f/M^2_{W}\).

The amplitude can be rewritten as

\[
A(i_R \rightarrow f_L) = -i \frac{g^2}{2} \frac{\pi^2}{(2\pi)^4} K_{ii} K^*_{ij} \frac{1}{\tilde{q}^i} \frac{m_i m_f}{M^2_{W}} F(M) \frac{1 + \alpha_z}{4} R \frac{1}{\tilde{q}^i}. \tag{5.21}
\]

The function \(F(M)\) is defined as follows:

\[
F(M) = \frac{m_f}{p'^i_z + p'^f_z} (1 + \frac{E - p'^i_z}{E + p'^f_z})[I_1 - I_2] \tag{5.22}
\]

\[
+ i \frac{m_f}{M_W} \left[ (1 + \frac{E - p'^i_z}{E + p'^f_z})I_3 - (1 - \frac{p'^i_z}{E + p'^f_z})I_4 \right]. \tag{5.23}
\]
and the integrals \( I_{1,2,3,4} \) are

\[
I_n(M) = \delta_n \int_0^1 dx \left[ \frac{x(1-x)}{x + \frac{M^2}{M_{W}^2}(1-x)} \right]^{\gamma_n} x^{\beta_n}, \tag{5.24}
\]

with \( \delta_1 = 3, \delta_3 = M^2/M_{W}^2, \delta_2 = 3\delta_4/2 = 2 + M^2/M_{W}^2, \beta_n = (0,1,0,1) \) and \( \gamma_n = (1,1,3/2,3/2) \).

Consider \( i \neq f \), i.e., a flavour changing transition. The CP-violating observable is proportional to

\[
\sum_{l,l'} [ |\tilde{A}(i \to f_L)|^2 - |\tilde{A}(i_L \to f_R)|^2 ] = \left[ \frac{g^2}{4} \frac{\pi^2}{(2\pi)^4} \right]^2 \frac{(m_i m_f)^2}{M_{W}^2}
\]

\[
\times \left( -2 \right) \sum_{l,l'} Im \left[ K_{li} K_{lf}^* (K_{l'f}^* K_{l'f})^* \right] Im \left[ F(M_l) F^*(M_{l'}) \right], \tag{5.25}
\]

where \( \tilde{A}(i \to f) \) is obtained from \( A(i \to f) \) taking the residues of \( 1/\hat{q}_l^i \) and \( 1/\hat{q}_l^f \) at \( q_0 = q_0^i = -q_0^f \). \( \tilde{A}(i \to f) \) corresponds to the matrix element of the effective Hamiltonian between states \( |i \rangle \) and \( |f \rangle \), in a normalization that will be specified in the next subsection.

Eq. \((5.23)\) shows explicitly the interference between the CP-odd phases of the CKM couplings \( K \) and the CP-even phases of \( F \). The latter can have two different origins: either \( \hat{p}_l^i \) when \( i \) or \( f \) are totally reflected, or \( \hat{Q}_0 \), which leaves a trace of the non-locality of the internal loop in the \( i \) factor before \( I_3 \) and \( I_4 \).

It is well known that

\[
Im \left[ K_{li} K_{lf}^* (K_{l'f}^* K_{l'f})^* \right] = \eta J, \tag{5.26}
\]

where \( \eta = 1 \) if \( f - i = l' - l \ \ (\text{mod} \ 3) \), \( \eta = -1 \) when \( f - i \neq l' - l \ \ (\text{mod} \ 3) \) and where \( J \) is twice the area of the CM unitarity triangle, \( J = c_1 c_2 s_3 s_1 s_2 s_3 s_8 \). Let us now consider in the complex plane the triangle spanned by the three numbers \( F(M_l), l = u,c,t \). Let \( B \) be the area of this triangle with a sign +1 (-1) for a cyclic clockwise (anti-clockwise) ordering of \( u,c,t \) around the triangle. Then

\[
\sum_{l,l'} Im \left[ K_{li} K_{lf}^* (K_{l'f}^* K_{l'f})^* \right] Im \left[ F(M_l) F^*(M_{l'}) \right] = 8JB. \tag{5.27}
\]

This expression exhibits clearly the GIM mechanism for the internal flavours, since the area vanishes obviously whenever two points coincide. Additional vanishing happens when three points are aligned. The function \( F \) depends both on the internal and external masses as can be seen from eq. \((5.23)\). In order to exhibit the GIM mechanism for the external masses as well, \( Im \left[ F(M_l) F^*(M_{l'}) \right] \) can be expanded, leading to:

\[
\sum_{l,l'} [ |\tilde{A}(i_R \to f_L)|^2 - |\tilde{A}(i_L \to f_R)|^2 ] = \left[ \frac{g^2}{4} \frac{\pi^2}{(2\pi)^4} \right]^2 \frac{(m_i m_f)^2}{M_{W}^2}
\]

\[
\times \sum_{l,l'=l+1 \ (\text{mod} \ 3)} (-2) \sum_{jk} S_{jk}(M_l, M_{l'}) b_{jk}(E, m_i, m_f). \tag{5.28}
\]
In this expression, the function $S$ derives from the integrals $I_{1,2,3,4}$ in eq. (5.24), and contains the dependence on the masses for the quarks inside the loop,

$$S_{jk}(M, M') = I_j(M)I_k(M') - I_j(M')I_k(M),$$  \hspace{1cm} (5.29)

and the $b_{jk}$ are antisymmetric functions in $j, k$. They contain the dependence on the masses of the incoming and outgoing quarks. In the approximation used from eq. (5.20) on, they are equal to

$$b_{12}(E, m_i, m_f) = 0,$$  \hspace{1cm} (5.30)

$$b_{13}(E, m_i, m_f) = -\left(m_im_if\right)^2 \frac{m_j^2}{M_W^2} \frac{|2E + p_z^f - p_z^i|^2}{|E + p_z^f|^2} \text{Re}\left(\frac{1}{p_z^i + p_z^f}\right),$$  \hspace{1cm} (5.31)

$$b_{23}(E, m_i, m_f) = -b_{13}(E, m_i, m_f),$$  \hspace{1cm} (5.32)

$$b_{14}(E, m_i, m_f) = \left(m_im_if\right)^2 \frac{1}{M_W^2} \frac{1}{|E + p_z^f|^2} \times \text{Re}\left(\frac{(2E + p_z^f - p_z^i)(E + p_z^f - p_z^i)^*}{p_z^i + p_z^f}\right),$$  \hspace{1cm} (5.33)

$$b_{24}(E, m_i, m_f) = -b_{14}(E, m_i, m_f),$$  \hspace{1cm} (5.34)

$$b_{34}(E, m_i, m_f) = \left(m_im_if\right)^2 \frac{m_j^2}{M_W^2} \frac{E}{|E + p_z^f|^2} \text{Im}\left(p_z^f - p_z^i\right).$$  \hspace{1cm} (5.35)

For each $j, k$ pair, $\{I_j(M), I_k(M)\}$ can be seen as the coordinates of the point $M$ in the $(j, k)$-plane. Then the points $(m_u, m_c, m_t)$ span a triangle in this plane whose area $1/2 \sum_{l, l' = l+1 \pmod 3} S_{jk}(M_l, M_{l'})$ exhibits again the GIM mechanism for internal masses. Furthermore, because $\text{Im}\left[ (K_{i,l}K_{i,l'})^e (K_{i,l'}K_{i,l})^e \right]^*$ is antisymmetric in the exchange of $i$ and $f$, the sum over external flavours is proportional to

$$\propto \sum_{i, f = i + 1 \pmod 3} (b_{jk}(E, m_i, m_f) - b_{jk}(E, m_f, m_i)),$$  \hspace{1cm} (5.36)

which shows explicitly the GIM mechanism operating on external quarks. This pattern should be present in a complete computation, with pertinent modifications in the functions $S_{jk}, b_{jk}$.

All $b_{jk}$ but $b_{12}$ require the existence of the CP-even phase in $\tilde{Q}_0$ found above, and depend as well on the complex reflection behaviour of the initial and final quarks, contained in $p_z^i$ and $p_z^f$. $b_{12}$ depends only on the latter, and could be $\neq 0$ only when $m_i, m_f$ and $E$ are not neglected in front of $M_W$.

Note that all $b_{jk}$ vanish when either $m_i$ or $m_f$ go to zero, respecting the second point of the general behaviour announced at the beginning of this section. In each sector of quark charges ($-1/3$ or $+2/3$) the heavier masses will dominate the effect, as expected from intuition. Note that a two-threshold structure is present, corresponding to $m_i, m_f$.

In our numerical results we use the exact values of the functions $S_{jk}(M, M')$. It is instructive, though, to show a fit for the particular case $M, M' << M_W$, an appropriate expansion for all quarks but the top,
\[
S_{1,2}(M, M') \to \frac{M^2 M'^2}{M_W^2} \left[ \frac{M^2}{M_W^2} \log \frac{M^2}{M_W^2} - \frac{M'^2}{M_W^2} \log \frac{M'^2}{M_W^2} \right],
\]
and the ratio of the remaining \( S_{j,k}(M, M') \) to the result in eq. (5.37) is given by \( S_{1,3}/S_{1,2} \to -\frac{7}{15}, S_{1,4}/S_{1,2} \to +1.5, S_{2,3}/S_{1,2} \to -\frac{1}{30}, S_{2,4}/S_{1,2} \to -\frac{1}{4}, \) and \( S_{3,4}/S_{1,2} \to +\frac{1}{6} \), in the same limit. This behaviour is consistent with the third point of the general expectations developed at the beginning of this section.

### 5.3 Transition amplitude in terms of wave functions

The computation of the amplitude described in the previous subsection was obtained by projecting over the massless poles of the propagator in the presence of the wall, once a flavour-changing electroweak loop was inserted.

The result should be equivalent to the more direct one in terms of incoming and outgoing wave functions (wave packets), as given by eq. (3.32), with no need to consider the full propagator in the presence of the wall derived in Section 4, as only the usual propagators were used inside loops. To prove that such is the case, consider the incoming wave function \( \tilde{\psi}_{n+}^{inc} \) in eq. (5.27). The following identities\(^9\)

\[
\frac{1}{\hat{q}} \gamma_0 = \frac{1}{p_z - q_z + i\epsilon} \frac{1 + \alpha_z}{2} + \frac{1}{p_z + q_z + i\epsilon} \frac{1 - \alpha_z}{2},
\]

\[
\frac{1}{\hat{q} - m} \gamma_0 = \frac{1}{p_z} \frac{1}{2} \frac{[1 + p_z + p'_z \alpha_z + m \gamma_0]}{p_z - q_z + i\epsilon} + \frac{1}{2} \frac{1}{p'_z + q_z + i\epsilon} \gamma_0,
\]

lead to the equivalent expression

\[
\tilde{\psi}_{n+}^{inc}(q_z) = \left[ -\frac{1}{2\pi i} \frac{m}{\hat{q}(\hat{q} - m)} (1 - \frac{m \gamma_0}{p_z + p'_z}) \gamma_0 + \delta(p_z - q_z) \right] \frac{1 + \alpha_z}{2} u_h(\vec{p}^{inc}).
\]

In the neighbourhood of the massless poles of the propagator of a given quark,

\[
S(q^f, q^i) \quad \to \quad 2\pi i \sum_h \tilde{\psi}_{n+}^{inc}(q^f) u_h(\vec{p}^{inc}) \frac{1 + \alpha_z}{2} \delta(p_z - q_z),
\]

\[
S(q^f, q^i) \quad \to \quad 2\pi i \sum_h u_h(\vec{p}^{out}) \tilde{\psi}_{n+}^{out}(q^f) \frac{1 + \alpha_z}{2} \delta(p_z + q_z).
\]

In eq. (5.40) the first term corresponds to the residue at the massless pole of the “homogeneous” part of the propagator, while the term in \( \delta(p - q_z) \) corresponds to the residue at the same pole of the “non homogeneous” part of the propagator. For the particular choice of subtraction (at \( q^2 = 0 \)) in subsection 5.1, the latter obviously gives no contribution. The conclusion found in the previous subsection, namely that only the homogeneous parts of the propagator participated in the amplitude under consideration, gets thus a simple interpretation in terms of the wave function contributions to \( <\psi_n^{out}|H_{ind}|\psi_n^{inc}> \). And the equivalence of both methods to compute the amplitude is proved.

\(^9\)Recall that \( p_z = \sqrt{E^2} = q_0 \) in this case.
6 CP Asymmetry

6.1 Reflection probability

The task is to estimate the probability of reflecting a quark of flavour \(i\) into another flavour \(f\), for a given flux of incoming particles (antiparticles), i.e. the reflected flux per unit incoming flux. Let us consider a finite box of length \(L_z\) and unit section in the plane \(x - y\), which incorporates the wall. For simplicity, we will ignore in the following the \(x\) and \(y\) directions. We normalize the wave functions \(\psi_{\text{inc}}\) and \(\psi_{\text{out}}\) in that box. Denote the matrix element of the interaction Hamiltonian, generated by the loop between the later states, by

\[
<\psi_{nf}^{\text{out}}|H_{\text{int}}|\psi_{ni}^{\text{inc}}> = \tilde{A}(i \rightarrow f) / L_z
\]

where \(\tilde{A}(i \rightarrow f)\) is dimensionless, see eqs. (5.25), (5.28). For any time interval \(T\) such that \(\tilde{A}(i \rightarrow f) T / L_z \ll \infty\),

\[
<\psi_{nf}^{\text{out}}|e^{-iHT}|\psi_{ni}^{\text{inc}} > \sim e^{-iE_T} \frac{\tilde{A}(i \rightarrow f)}{L_z} \int_{-T/\varepsilon}^{T/\varepsilon} dt e^{i \frac{1}{2}(E_f - E_i) t}
\]

whose modulus squared gives the reflection probability per particle \((E = (E_f + E_i)/2\) in this equation). Taking now the usual limit:

\[
\left[ \int_{-T/\varepsilon}^{T/\varepsilon} dt e^{i \frac{1}{2}(E_f - E_i) t} \right]^2 \rightarrow \pi \delta(E_f - E_i). \tag{6.3}
\]

For an incoming flux \(\Phi\) per unit time of particles with velocity \(v\), the number of particles in the volume will be \(\Phi v L_z\). The reflection probability per unit time and unit flux is then:

\[
P(i \rightarrow f) \sim \sum_{p_f} \frac{\tilde{A}(i \rightarrow f)^2}{L_z} v 2\pi \delta(E_f - E_i). \tag{6.4}
\]

In the infinite volume limit, \(\sum_{p_f} / L_z \rightarrow \int dp_f / (2\pi)\), and with \(\int dp_z \delta(E_f - E_i) = 1/v\), it follows that

\[
P(i \rightarrow f) = |\tilde{A}(i \rightarrow f)|^2. \tag{6.5}
\]

The number of particles reflected per unit time (or reflected flux) is \(\Phi_r = |\tilde{A}(i \rightarrow f)|^2 \Phi_i\), and thus related to the incoming flux through the previously computed \(|\tilde{A}|^2\).

Assume that the probability of the incoming state \(i\) in the unbroken phase is given by a distribution \(\rho(i)\), which depends in general on the flavour, chirality and energy of \(i\). For instance, at finite temperature close to equilibrium, this distribution will be close to the Fermi-Dirac one, with proper modification of the dispersion relation between momentum and energy. We define useful CP asymmetries by:

\[
\Delta_{CP} = \frac{\sum_{i,f} \rho(i)(P(i \rightarrow f) - P(\bar{i} \rightarrow \bar{f}))}{\sum_{i'} 2\rho(i')}, \tag{6.6}
\]
where the sum over $i, f$ in the numerator implies the sum over the energies and flavours for the isosinglet quarks (or the isodoublet quarks), while the sum $i'$ in the denominator acts over flavour and helicities and the factor 2 stems from summing over quarks and antiquarks.

In the case of zero temperature, if we assume an equal incoming probability for all the flavours and helicities, and considering the asymmetry for one given energy (for instance the most favorable one) the formula (6.6) simplifies to:

$$\Delta_{CP}(\omega)^{T=0} = \sum_{i,f} \Delta_{CP}(\omega)^{T=0}_{i,f},$$

where

$$\Delta_{CP}(\omega)^{T=0}_{i,f} = \frac{P(i \rightarrow f) - P(i \rightarrow \bar{f})}{4N_{fl}}$$

is the contribution to the asymmetry of a given pair of flavours. $N_{fl}$ denotes the numbers of flavours, 6. The colour factor has been obviated as it will finally cancel between nominator and denominator.

### 6.2 Numerical results

Consider first the reflection asymmetries in the “down” quark sector. The numerical contribution of the functions $S_{jk}(M_l, M_f)$, which contain the dependence on the internal quark masses ($l, l' = u, c$ and $t$), is the following one:

$$\sum S_{1,2}(M_l, M_f) = 3.82 \times 10^{-5} \quad \sum S_{1,3}(M_l, M_f) = -1.84 \times 10^{-5}$$
$$\sum S_{1,4}(M_l, M_f) = 3.68 \times 10^{-5} \quad \sum S_{2,3}(M_l, M_f) = -9.96 \times 10^{-7}$$
$$\sum S_{2,4}(M_l, M_f) = 5.74 \times 10^{-7} \quad \sum S_{3,4}(M_l, M_f) = 6.81 \times 10^{-7}, \quad (6.9)$$

where the sums on $l, l'$ are constrained by the relation $l' = l + 1$ (mod 3).

The following values have been used for the particle masses: $M_W = 80.22$GeV, $m_u = 5$MeV, $m_d = 10$MeV, $m_s = 200$MeV, $m_c = 1.3$GeV, $m_b = 4.7$GeV, $m_t = 120$GeV. Our numerical results are presented in Fig. 4. As expected the heavier quark masses dominate. This is well understood from the analytical behaviour eq. (5.35). The dominant contribution for charge $-1/3$ quarks is thus given by the $s-b$ pair. The two spikes in the figures correspond to the thresholds for the external quark masses involved. Indeed, the analytical formulae in eq. (5.33) display such a structure, with for instance in the case $s_R \rightarrow b_L$:

$$b_{12} = 0, \quad b_{14} = -\left(\frac{m_s m_b}{M_W^2}\right)^2 \frac{m_s}{M_W}, \quad \text{at } E = m_s$$
$$b_{13} \sim -\left(\frac{m_s m_b}{M_W^2}\right)^2 \frac{m_s}{M_W}, \quad b_{14} \sim +\left(\frac{m_s m_b}{M_W^2}\right)^2 \frac{m_b}{M_W} \frac{1}{2} \frac{m_s^2}{m_b^2}, \quad \text{at } E = m_b \quad (6.10)$$

while for the complementary transition $b_R \rightarrow s_L$:

$$b_{12} = 0, \quad b_{14} = +\left(\frac{m_s m_b}{M_W^2}\right)^2 \frac{m_s}{M_W}, \quad \text{at } E = m_s$$
$$b_{13} \sim -\left(\frac{m_s m_b}{M_W^2}\right)^2 \frac{9}{4} \frac{m_s^2}{m_b M_W^2}, \quad b_{14} \sim +\left(\frac{m_s m_b}{M_W^2}\right)^2 \frac{3}{2} \frac{m_b^2}{m_s M_W}, \quad \text{at } E = m_b. \quad (6.11)$$

The reason not to add in the asymmetry the isosinglet and isodoublet quarks is the TCP’ symmetry, eq. (2.7), which implies that their incoming probability $\rho(i)$ are equal. The eventual effect of sphalerons to transform the CP asymmetry generated by the wall into a baryon number asymmetry, is anyhow different for doublets and singlets. We thus need to distinguish the asymmetry for doublets and singlets.
The total asymmetry, eq. (6.7), is clearly dominated by the $s_R \rightarrow b_L$ contribution, with a maximum value $\Delta_{CP}^{T=0}(\omega \sim m_b) = 5 \times 10^{-26}$ around the $m_b$ threshold.

Consider now charge $-2/3$ quarks. Strictly speaking, our computation is not a good approximation for this sector as in eq.(5.20) we have neglected factors of $m_q^2/M_W^2$, $m_f^2/M_W^2$, etc., which is not legitimate for the top quark. Nevertheless the type of GIM cancellations, which we justified in terms of general arguments in Sect. 3, should still hold. In particular, the analytical behaviour for the internal mass dependence ($d,s,b$ in this case), given in eq. (5.37), are specially accurate because all internal masses are really small in front of $M_W$.

The exact numerical result for these quantities is

\[
\begin{align*}
\sum S_{1,2}(M_l, M_{l'}) &= 2.37 \times 10^{-10} & \sum S_{1,3}(M_l, M_{l'}) &= -1.12 \times 10^{-10} \\
\sum S_{1,4}(M_l, M_{l'}) &= 3.33 \times 10^{-10} & \sum S_{2,3}(M_l, M_{l'}) &= -7.37 \times 10^{-12} \\
\sum S_{2,4}(M_l, M_{l'}) &= 4.91 \times 10^{-11} & \sum S_{3,4}(M_l, M_{l'}) &= 3.45 \times 10^{-11},
\end{align*}
\]  

(6.12)

where the sums now run on $l,l'=d,s,b$ under the usual cyclicity constraint $l'=l+1 \pmod{3}$.

In this toy computation, the “up” type quarks contribution is expected to be about the same as the “down” type one: while the external mass dependence gives a bigger factor, the internal GIM cancellation produces the opposite effect. Applying for the sake of the argument our analytical formulae to this sector, gives an external mass dependence at the dominant threshold $\sim m_q^5/M_W^5$, while the internal GIM cancellation goes as $\sim m_q^6/M_W^6$. The contributions of both types of quarks should not differ much. Using our formulae at face value results in a dominant $c_R \rightarrow t_L$ contribution of order $\Delta_{CP}^{T=0}(\omega \sim m_t) \sim 2.5 \times 10^{-25}$. In the present case $b_{12}$ is not zero, which reinforces the effect.

As previously discussed, in a complete computation, an external suppression factor $\sim (m_q/m_W)^2$ could disappear, leading to a large enhancement for certain flavours (up to 13 orders of magnitude for the $d-s$ pair, still insufficient for the observed asymmetry), although this may be very optimistic as the pattern of unitarity triangles must persist.

7 Conclusions

Realistic models of baryogenesis imply the inclusion of finite temperature effects. We consider scenarios with a first order phase transition. It is interesting to disentangle whether temperature effects, other than the existence of the transition itself, are responsible for possible modifications of standard CP arguments. We thus revert in a first step to the analysis of particle propagation at $T=0$.

The essential non perturbative element is the wall separating the two phases of spontaneous symmetry breaking. The propagation of any particle of the SM spectrum should be exactly solved in its presence. And this we have done for a free fermion and a thin wall, leading to a new Feynman propagator which replaces and generalizes the usual one. It contains both massless and massive poles, and new CP-even phases, present both for on-shell and off-shell fermions. They stem from the tree-level reflection of fermions on the bubble wall. This propagator can be useful in physical process other than baryogenesis, where a first order phase transition is relevant.
When electroweak loops are considered, a CP-asymmetry is found in the one-loop reflection properties of quarks and antiquarks. The one-loop self-energies cannot be completely renormalized away for on-shell fermions in the presence of a wall, as the latter acts as a source of momenta. The asymmetry is then found at order $\alpha_W$ in amplitude and requires the interference of the mentioned CP-even phases with the CP-odd CKM ones. The desired effect is thus present without any consideration of temperature effects in the quark propagation.

A complete one-loop SM calculation is beyond the scope of the present paper. We have instead performed a toy computation in which only the broken phase is considered inside the loops, in a Lorentz covariant way. The result is many orders of magnitude below what observation requires, and it can be expressed in terms of two unitarity triangles. They show the internal and external GIM cancellations and the interplay between CP-even and CP-odd phases. This pattern of triangles should survive in a complete computation, which we leave for later publication. We argue that suppression factors in external masses could be absent then, due to the breaking of Lorentz invariance in the loops analysis, although the quantitative enhancement is insufficient.

With the expertise acquired in this paper, we face in ref. [14] the physical finite temperature case, where we discuss the common building blocks and new features of the scenario.

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A Feynman boundary conditions for the fermion propagator

We prove here that the propagator derived in eqs. (4.16), (4.17) and (4.18) is a unique solution of the inhomogeneous Dirac equation (4.10). In a first step, the equivalence between eqs. (4.16) and (4.17) will be established. We then prove that the two $'i\epsilon$” conventions are indeed equivalent, as stated in section (4). Next we show, in the “space $i\epsilon$” convention, that the propagator satisfies the inhomogeneous Dirac equation. Finally, in the (Feynman) “time $i\epsilon$” convention, we demonstrate that the Feynman boundary conditions are indeed satisfied.

A.1 Equivalence between eqs. (4.16) and (4.17).

Let us consider the difference between the right hand sides of eq. (4.17) and eq. (4.16). The difference between the first lines of these equations is

$$\frac{1}{q^i - q^i_0} + i\epsilon \left( \frac{1}{q^i - \not{q}^i} - \frac{1}{q^i - \not{q}^i_0} \right) - \frac{1}{q^i_0 - q^i_0} - i\epsilon \left( \frac{1}{q^i - m} - \frac{1}{q^i - m} \right), \quad (A.1)$$
where the Feynman convention is understood in all all $1/q^\gamma$, $1/(q - m)$'s.

The difference between the second and third lines of the same equations is

$$- \frac{1}{q^f s\alpha_s s\gamma_0} \frac{1}{q^i} + \frac{1}{q^f - m} s\alpha_s s\gamma_0 \frac{1}{q^i - m}. \quad (A.2)$$

$q^f$ and $q^i$ differ only in their $z$ component, implying that $q^f - q^i = -\gamma_z(q^i_z - q^f_z)$. Using this equality, together with $1/q^i - 1/q^f = 1/q^f(q^f - q^i)1/q^i$, it is straightforward to show that the sum of eqs. (A.1) and (A.2) vanishes, whence the equality of r.h.s. of (4.16) and of (4.17).

### A.2 Equivalence between the two “iε” conventions.

The usual “time iε” simply amounts to $q_0^2 \rightarrow q_0^2 + i\epsilon$, while the “space iε” convention has been defined in Sec. 4. The difference between the two “iε” conventions has been exhibited in eq. (A.7).

Consider the equality

$$- \frac{m}{q^f(q^f - m)} = - \frac{1}{q^f - m} + \frac{1}{q^f}. \quad (A.3)$$

Applying it to eq. (4.17), we can separate the terms containing $1/q^f$ and those containing $1/q^f - m$. Let us dismiss momentarily the latter, which will be discussed later.

It is possible to prove that the substitution of all $1/q^f$ factors in eq. (4.17) by $2i\pi q^f \theta(q^f_z)\delta(q_0^2 - (q^i_z)^2)$ gives a vanishing result. Indeed, from

$$q^f \theta(q^f_z)\delta(q_0^2 - (q^i_z)^2) = s|q_0|\gamma^0(1 - s\alpha_z)\theta(q^f_z)\delta(q_0^2 - (q^i_z)^2), \quad (A.3)$$

the third line in eq. (4.17) gives then a vanishing coefficient, since $(1 - s\alpha_z)(1 + s\alpha_z) = 0$. Applying now in the second line the equality

$$(1 - s\alpha_z) \left[ 1 - \frac{ms\gamma_0}{E + p_z'} \right] \frac{1 - s\alpha_z}{2} = 1 - s\alpha_z, \quad (A.4)$$

it follows that

$$(1 - s\alpha_z) s\gamma_0 \frac{1}{q^i} = s(1 - s\alpha_z) \frac{q^i_z + q^i_z}{(q^f_z)^2 - (q^i_z)^2 + i\epsilon} = s(1 - s\alpha_z) \frac{1}{q^f_z - q^i_z + i\epsilon}, \quad (A.5)$$

where we have used $sq_0 = q_z^i$ (since $sq_0 > 0$ by definition of $s$, and $q_z^i > 0$ due to the $\theta(q_z^f)$ factor). Finally, using once more eq. (A.3), the second line gives

$$q^f \theta(q^f_z)\delta(q_0^2 - (q^i_z)^2) \frac{1}{q^f_z - q^i_z + i\epsilon}, \quad (A.6)$$

which obviously cancels the contribution from the first line.

Let us now turn to the $1/(q^f - m)$ term. The equivalent of eq. (A.7) is
The proof proceeds in a way similar to the preceding one but slightly more involved. It can be shown that

\[
(\frac{q^f + m}{q^0_0 - (q_z^f)^2} - m) \theta(-q_z^f) \delta(q_0^2 - (q_z^f)^2 - m^2) = s|q_0^0|\gamma^0 \left(1 + \frac{p_z^f \alpha_z + m\gamma^0}{s|q_0^0|}\right),
\]

(A.8)

where \(p_z^f = \sqrt{q_0^2 - m^2}\). The last bracket in (A.8) is twice a projector that plays the same role as \(1 - s\alpha_z\) in the preceding derivation.

Using

\[
\left(1 + \frac{p_z^f \alpha_z + m\gamma^0}{s|q_0^0|}\right) \left[1 - \frac{m\gamma_0}{E + p_z^f}\right] \frac{1 - s\alpha_z}{2} = 0,
\]

(A.9)

it follows that the contribution from the second line of (4.17) vanishes. From

\[
\left(1 + \frac{p_z^f \alpha_z + m\gamma^0}{s|q_0^0|}\right) \left[1 + \frac{m\gamma_0}{E + p_z^f}\right] = \left(1 + \frac{p_z^f \alpha_z + m\gamma^0}{s|q_0^0|}\right)
\]

(A.10)

it is possible to check that the third line cancels the contribution from the first one, which ends the proof for the for \(1/(q^f - m)\) term.

The \(1/(q^i - m)\) and \(1/q^i\) may be treated in a similar way leading to the announced conclusion: the quark propagator in eq. (4.17) has the same value whichever “\(i\epsilon\)” convention is used. From the equality derived in the preceding subsection, this applies also to eqs. (4.16) and (4.18).

### A.3 The inhomogeneous Dirac equation

We prove here that the propagator verifies the inhomogeneous Dirac equation. Let us consider the equation (4.10). Multiplying both sides by \(i\gamma_0\), the Dirac operator becomes

\[
\theta(-\xi_z^f)i\bar{\psi} + \theta(\xi_z^f)(i\bar{\psi} - m).
\]

(A.11)

We use the “space \(i\epsilon\)” convention, so that \(1/(q^f - m)\) has a pole in the \(q_z^f\) complex plane below the real axis. Its Fourier transform has then a pole above the real axis,

\[
\int dq_z^f e^{iq_z^f\xi_z^f} \frac{1}{q^f} \propto \theta(-\xi_z^f)[i\bar{\psi}]^{-1}.
\]

(A.12)

Similarly, \(1/(q^f - m)\) has a pole above the real axis, leading to

\[
\int dq_z^f e^{iq_z^f\xi_z^f} \frac{1}{q^f - m} \propto \theta(\xi_z^f)[i\bar{\psi} - m]^{-1}.
\]

(A.13)

As a consequence, the operator (A.11) applied to \(m/q^f (q^f - m)\) gives zero. It follows that the second and third lines in eq. (4.17) vanish under the action of the Dirac operator. Its action on the first line gives
\[- \frac{1}{q_z^f - q_z^i + i \epsilon} + \frac{1}{q_z^f - q_z^i - i \epsilon} = 2i\pi \delta(q_z^f - q_z^i). \quad (A.14)\]

This ends the proof. The same argument holds when the Dirac equation is applied on the right hand side of the propagator. In this case, it is more convenient to perform the analysis on eq. (A.14), sticking again to the “space $i\epsilon$” convention.

### A.4 Feynman boundary conditions.

In the “time $i\epsilon$” convention, it is trivial to proof that the propagator verifies the Feynman boundary conditions. Indeed, the latter state that the positive frequencies are “retarded” while the negative ones are “advanced”. By definition, the “time $i\epsilon$” prescription achieves it, as it amounts to the replacement $q_0^2 \rightarrow q_0^2 + i\epsilon$, which leads to poles in the $q_0$ complex plane below (above) the real axis when its real part is positive (negative).

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Figure 4: Asymmetries for incoming down quarks, as a function of energy.