Nuclear Tensor Force: origin and relativistic representation

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The relativistic origin of the Wigner tensor components of the nuclear force is presented in this work, associated with the Fock diagrams of Lorentz scalar and vector couplings. With this newly obtained relativistic formalism of Wigner tensor forces, more distinct tensor effects are found in the Fock diagrams of the Lorentz scalar and vector couplings, as compared to the Lorentz pseudo-vector and tensor channels. A unified and self-consistent treatment on both Wigner tensor and spin-orbit interactions, which dominate the spin-dependent features of the nuclear force, is then achieved by the relativistic models. Moreover, the analysis on the tensor strengths indicates the reliability of the relativistic representation of Wigner tensor forces in exploring nuclear structure, excitation and decay modes.

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Since the birth of nuclear physics, the nuclear force that binds protons and neutrons into an atomic nucleus is the most significant issue of the field. The earliest attempt in understanding the nature of the nuclear force was made by Yukawa with the meson exchange picture [1]. To a large extent, the nuclear force can be understood in terms of the exchanges of virtual mesons, which is the microscopic foundation of modern nuclear theories, such as the covariant density functional (CDF) theory [2]. At a very early stage, the nuclear force was recognized to contain not only central components but also the non-central ones, namely the Wigner tensor component that plays an essential role in binding the light nuclei [3–5]. Specifically, the electric quadrupole moment of the deuteron provides the most striking evidence of the nuclear tensor force [7].

As an important ingredient of the nuclear force, the tensor force is characterized by its spin dependent feature [8]. In the recent years, substantial impacts due to the nature of the tensor force were achieved in describing the nuclear multipole responses [16] and the citations [15], the non-charge exchange multipole responses of the Skyrme Hartree-Fock (SHF) plus random phase approximations [14–18] and decay modes [19]. For instance, within tensor force were also achieved in describing the nuclear excited Wigner form, $S_{12} = 3(\sigma_1 \cdot q)(\sigma_2 \cdot q) - \sigma_1 \cdot \sigma_2 q^2$, (1) where $S_{12}$ is a rank-2 Wigner tensor, with the momentum transfer $q = p_1 - p_2$. There still remain some unresolved problems, such as the origin of the nuclear tensor force and its coupling strength. For the later there exists an evident model dependence with respect to the widely used energy functionals such as the Skyrme forces [24], which might be partially due to the fact that the nuclear tensor force was included perturbatively. Within the CDF scheme, which provides a self-consistent treatment on the spin-orbit coupling, several attempts were also made to explore the tensor effects, e.g., in terms of $\omega$-tensor couplings [25]. However, these are Lorentz tensors and they give pure central type contributions in the limit of Hartree approach. Under the Yukawa scheme, the nuclear tensor force was recognized to originate from the exchanges of $\pi$ and $\rho$ (mainly tensor $\rho$) mesons [8, 26]. Only when the Fock terms of meson-nucleon couplings are included explicitly, the $\pi$ and $\rho$-tensor couplings can be efficiently taken into account, for instance, by the density dependent relativistic Hartree-Fock (DDRHF) theory [27–29], from which distinct tensor effects are revealed in nuclear structure properties [26, 28, 30]. Even though, the Fock terms of the Lorentz tensor couplings, e.g., the $\pi$ pseudo-vector and $\rho$ tensor couplings, are still mixtures of central and Wigner tensor forces [26].

Furthermore, a fully self-consistent treatment of the charge-exchange excitation modes, the GT and SD resonances has been achieved by the relativistic RPA based on DDRHF, from which is well demonstrated the crucial role played by the exchange (Fock) diagrams of the isoscalar $\sigma$ and $\omega$ couplings [31, 32]. Notice that these excitation modes were interpreted successfully by the Skyrme+Tensor models as well [12, 15], in which the tensor force was found to play a key role. As an indirect evidence, such consensus indicates that Wigner tensor components may exist in Fock diagrams of meson-nucleon couplings, not only the isovector ones ($\pi$ and $\rho$) but also the isoscalar ones ($\sigma$ and $\omega$).

In fact, when the Fock diagrams are included, the nuclear force mediated by mesons exchanges is found to contain the characteristic spin-dependence of a tensor force. Associated with the nature of tensor force [8], the spin-orbit (SO) splitting will be essentially changed by the Wigner tensor couplings [see Eq. (1)], thus providing a direct test for the occurrence of nuclear tensor interaction. To simplify the notation, we take the SO splittings of neutron ($\nu$) $p$ and $d$ or-
bits of $^{48}\text{Ca}$ as the test examples. Figure [1] (a-d) shows the contributions to the SO splittings ($\Delta E_{\text{SO}} = V_{j_\nu, j_\nu} - V_{j_\rho, j_\rho}$) respectively from the neutron-neutron interactions of the total, Hartree and Fock terms, and the Fock terms of the isoscalar (namely $\sigma^E + \omega^E$). It is seen that the total $\Delta E_{\text{SO}}$ is essentially changed from $f_\sigma = f_\omega + 1/2$ to $f_\omega - 1/2$, which indicates that the neutron-neutron interactions are distinctly spin-dependent. In addition, such characteristic behaviors are dominated by the Fock diagrams, particularly the isoscalar contributions $\sigma^E + \omega^E$. This provides a concrete evidence for the existence of Wigner tensor components in the Fock diagram of meson-nucleon couplings, particularly in the isoscalar channels. On the other hand, it is confirmed that the Wigner tensor terms [1] are also found in the non-relativistic reduction of the Fock terms of isoscalar meson-nucleon couplings, similar as the isovector ones [33]. Therefore, the Fock diagrams can be considered as the mixture of central and tensor contributions, not only for the Lorentz tensor — $\pi$ pseudo-vector (PV) and $\rho$ tensor (T) coupling [26, 28, 33] but also for the Lorentz $\sigma$ scalar (S) and $\omega$ vector (V) ones, the new origin of nuclear tensor force.

![FIG. 1. (Color Online) Contributions to the spin-orbit splittings $\Delta E_{\text{SO}}$ = $V_{j_\nu, j_\nu} - V_{j_\rho, j_\rho}$ (MeV) of the nodeless neutron ($\nu$) orbits ($1p$ and $1d$) from the couplings with the neutron on the nodeless states ($j_\rho$) in $^{48}\text{Ca}$. In the plots (a-d) are shown the contributions of the total, Hartree terms, Fock terms, and the Fock terms of $\sigma$- and $\omega$-couplings (namely $\sigma^E + \omega^E$), respectively. The result are extracted from the calculations of DDRHF with PKA1.](image)

Notice that the spin operator $\hat{S} = \frac{1}{2}\sigma$ in Wigner tensor form [1] can be identified relativistically as $\hat{S} = \frac{1}{2}\Sigma = -\frac{1}{2}\gamma_0\gamma_\nu \gamma_\rho$, and $\gamma_0\gamma_\nu\gamma_\rho$ is the Dirac index of $\pi$-PV coupling. Inspired by the extraction of tensor contributions in one-pion exchange potential [26] and the non-relativistic reductions of the Fock terms, we present the following relativistic formalism to extract the Wigner tensor components hiding in $\pi$-PV, $\sigma$-scalar (S), $\omega$-vector (V) and $\rho$-tensor (T) couplings.

$$\mathcal{H}^{\sigma}_{\pi\psi} = -\frac{1}{2}\left[ f_{\sigma} \gamma_{\nu} \gamma_0 \gamma_\rho \right] \left[ f_{\sigma} m_\sigma \gamma_{\nu} \gamma_0 \gamma_\rho \right] \frac{D^{\pi\psi}_{T, \mu\nu}(1, 2)}{2}$$

(2)

$$\mathcal{H}^{\sigma}_{\sigma\psi} = -\frac{1}{4} \left[ g_{\sigma} \gamma_{\nu} \gamma_0 \gamma_\rho \right] \left[ g_{\sigma} m_{\sigma} \gamma_{\nu} \gamma_0 \gamma_\rho \right] \frac{D^{\sigma\psi}_{T, \mu\nu}(1, 2)}{2}$$

(3)

$$\mathcal{H}^{\sigma}_{\sigma\psi} = -\frac{1}{8} \left[ g_{\omega} \gamma_{\nu} \gamma_0 \gamma_\rho \right] \left[ g_{\omega} m_{\omega} \gamma_{\nu} \gamma_0 \gamma_\rho \right] \frac{D^{\sigma\psi}_{T, \mu\nu}(1, 2)}{2}$$

(4)

$$\mathcal{H}^{\sigma}_{\rho\psi} = -\frac{1}{2} \left[ f_{\rho} \gamma_{\nu} \gamma_0 \gamma_\rho \right] \left[ f_{\rho} m_{\rho} \gamma_{\nu} \gamma_0 \gamma_\rho \right] \frac{D^{\sigma\psi}_{T, \mu\nu}(1, 2)}{2}$$

(5)

where $\Sigma = (\gamma^5, \Sigma)$, $M$ is the nucleon mass, and $T$ denotes the isospin operator of the nucleon ($\hat{T}$). The propagator terms $D^T$ read as,

$$D^{\pi\psi}_{T, \mu\nu}(1, 2) = \left[ \delta^T(1) \delta^T(2) - \frac{1}{3} g_{\mu\nu} m_\pi^2 \right] D_{\pi\psi}(1, 2)$$

$$- \frac{3}{8} g_{\mu\nu} \delta(1 - 2) m_\pi^2 D_{\pi\psi}(1, 2)$$

(6)

$$D^{\pi\psi}_{T, \mu\nu}(1, 2) = \left[ \delta^T(1) \delta^T(2) - \frac{1}{3} g_{\mu\nu} m_\pi^2 \right] D_{\pi\psi}(1, 2)$$

$$- \frac{3}{8} g_{\mu\nu} \delta(1 - 2) m_\pi^2 D_{\pi\psi}(1, 2)$$

(7)

where $\phi$ stands for the $\sigma$-S and $\pi$-PV couplings, and $\phi^T$ represents the $\omega$-V and $\rho$-T channels. For the $\rho$-$V$ coupling, $\mathcal{H}^{\sigma}_{\rho\psi}$, a corresponding formalism can be obtained simply by replacing $m_\omega$ ($g_{\rho}$) in eqs. [4] and [7] by $m_{\rho}$ ($g_\rho$) and inserting the isospin operator $\hat{T}$ in the interacting index. In keeping with the theory itself, the $\mu, \nu = 0$ components of the propagator terms will be omitted in practice, which amounts to neglecting the retardation effects. Compared to the Wigner tensor operator $S_{12}$ [see eq. [1]], it is clear that the differential part in $D^{\psi}_{T, \mu\nu}$ is consistent with the term $(\sigma_1 \cdot q)(\sigma_2 \cdot q)$ and the rest corresponds with $-\sigma_1 \cdot \sigma_2 q^2$.

To test the validity of the proposed formalism [eqs. [3-5]] as the relativistic representation of Wigner tensor components in Fock diagrams, Fig. [2] shows the relevant contributions to the SO splittings $\Delta E_{\text{SO}}$ of nodeless $1p$ and $1d$ orbits of $^{48}\text{Ca}$, namely the total Fock terms [plot (a,d)], tensor [plot (b, e)] and remaining central parts [plot (c, f)]. It is clearly shown that the spin dependence, the tensor feature in Fock diagrams [see Fig. [2]], can be extracted and quantified almost completely by the relativistic formalism [see Fig. [2], b, e]. By means of the contributions to the SO splittings, the interactions $V^{\pi\psi}_{j_\sigma, j_\rho}$ (or $V^{\pi\psi}_{1p, 1d}$) are found opposite to those $V^{\pi\psi}_{j_\sigma, j_\rho}$ (or $V^{\pi\psi}_{1p, 1d}$), consistent with the nature of tensor force [8]. Besides, the tensor effects contributed by the Fock diagrams of scalar $\sigma$ (pseudo-scalar $\pi$) and vector $\omega$ (vector and tensor $\rho$) meson are opposite and counteracted by each another, similarly to the cancellation between strong $\sigma$-attraction and $\omega$-repulsion. Compared to the isovector channels ($\pi$-$PV$, $\rho$-$V$ and $\rho$-$T$), more distinct tensor effects, with almost one order of magnitude larger, are brought
FIG. 2. (Color Online) Contributions to the spin-orbit splittings \(\Delta E_{SO}\) (MeV) from the Fock diagrams [plots (a, d)], and their tensor [plots (b, e)] and central [plots (c, f)] parts. The results are extracted from the calculations of DDRHF functional PKA1 \([23]\) by taking the nodeless neutron (\(v\)) orbits in \(^{48}\)Ca as examples. In plots (a-c) the filled (open) symbols denote the contributions from \(\sigma\)-\(S\) (\(\omega\)-\(V\)) couplings. In plots (d-f) are only shown the results of the nodeless neutron orbit \(v1d\) for \(\pi\)-\(PV\), \(\rho\)-\(V\) and \(\rho\)-\(T\) couplings.

TABLE I. Interaction matrix elements \(V_{ij}^T\) of Wigner tensor components in the Fock diagrams of \(\sigma\)-\(S\) and \(\omega\)-\(V\) couplings with the limits that the spin partner states \(j_p\) and \(j_n\) share the same radial wave functions and the contributions of the small components of Dirac spinors are omitted. The results are extracted from the calculations of DDRHF with PKA1 for the neutron (\(v\)) orbits of \(^{48}\)Ca.

| \(V_{ij}^T\) | \(\sigma\)-\(S\) (10\(^{-1}\)MeV) | \(\omega\)-\(V\) (10\(^{-1}\)MeV) |
|-----------|-----------------|-----------------|
| \(v1p_{1/2}\) | 1.72 | 0.80 |
| \(v1d_{5/2}\) | 3.43 | 1.60 |
| \(v1f_{7/2}\) | 2.44 | 1.69 |
| \(v1p_{1/2}\) | -1.72 | -0.80 |
| \(v1d_{5/2}\) | -3.43 | -1.60 |
| \(v1f_{7/2}\) | -2.44 | -1.69 |

Similar tests are also performed for the relativistic formalism \([24, 25]\) of \(\pi\)-\(PV\) and \(\rho\)-\(T\) channels as well as the \(\rho\)-\(V\) one, and the tensor sum rules are obeyed in this limit.

On the other hand, it should be noticed that a nuclear tensor force emerges simultaneously with the presence of Fock diagrams of meson-nucleon coupling and the relevant tensor effects can be extracted completely by the proposed relativistic formalism \([25]\) without introducing any additional free parameters. From this point of view, the advantage of full relativistic Hartree-Fock (RHF) scheme based on meson exchange diagram of nuclear force, is then well demonstrated. Namely, the unified and self-consistent treatment of both tensor and SO interactions can be achieved by the RHF scheme, due to the Fock diagrams and Lorentz covariant structure of the theory itself. Moreover, direct constraints from the tensor-related observables are then feasible on understanding the nature of nuclear force with the relativistic representation of the Wigner tensor components involved by the Fock diagrams.

Not only on nuclear ground states \([9, 26]\), but also in nuclear excitations \([14, 15, 31, 32]\) and \(\beta\)-decay \([19, 36]\) there is a common understanding of the non-relativistic and relativistic models, for instance the SHF and RHF models. Both indeed share the success due to the presence of the tensor force, perturbatively enclosed in SHF or naturally involved in RHF. For the non-relativistic SHF models, the tensor contributions to the SO potential may originate from perturbative tensor terms and the exchange part of the central Skyrmie interaction and the tensor strength factors are determined as \(\alpha = \alpha_{\sigma} + \alpha_{\omega}\) and \(\beta = \beta_{T} + \beta_{C}\) \([9]\). Relativistically, these strength factors can be obtained approximately from the non-relativistic reduction of the relativistic formalism \([24, 25]\) as,

\[
\alpha = \frac{5}{12} \left[ \frac{1}{4} \alpha_{\sigma} \frac{1}{m_{\pi}^2} + q^2 - \frac{1}{8} \beta_{\omega} \frac{1}{m_{\rho}^2} + q^2 + \frac{1}{2} f_{T} \frac{1}{m_{\rho}^2} + q^2 \right],
\]

\[
\beta = \frac{5}{6} \left[ \frac{1}{2} \alpha_{\sigma} \frac{1}{m_{\pi}^2} + q^2 - \frac{1}{2} \beta_{\omega} \frac{1}{m_{\rho}^2} + q^2 \right],
\]

which depend on momentum transfer \(q\) due to the Yukawa propagators of meson exchanges and the baryon density \(\rho_b\) if the meson-nucleon couplings \((g_{\sigma}, g_{\omega}, g_{\rho}, f_{\pi} \text{ and } f_{T})\) are density-dependent. In the above expressions, the contributions of higher order terms are eliminated, e.g., the space parts of the Wigner tensor components [see Eq. \((4)\)] in \(\omega\)-\(V\) and \(\rho\)-\(V\) channels are of the order of \(1/M^2\), as well as the time component in the \(\rho\)-\(T\) coupling. To test the reliability of the relativistic formalism \([13]\) of Wigner tensor forces, Fig. 3 shows the tensor strength factors \(\alpha\) and \(\beta\) with respect to baryon density \(\rho_b\) and momentum transfer \(q\) determined by DDRHF functional PKO1, as compared to the Skyrme+Tensor forces SGII+Te3 \([34]\) and Skxta \([35]\), which are very successful respectively in describing nuclear excitations \([34, 37]\) and \(\beta\)-decay \([19]\). It is found that the factors \(\alpha\) and \(\beta\) determined by relativistic and non-relativistic models agree with one another.

As a test, the tensor sum rule \((2j_\perp + 1)\Delta_{j_\perp} = 0\) \([8]\) is verified with the relativistic formalism \([24, 25]\). Taking the neutron (\(v\)) orbits of \(^{48}\)Ca as examples, table I shows the interaction matrix elements \(V_{ij}^T\) of \(\sigma\)-\(S\) and \(\omega\)-\(V\) channels and the calculations are performed with the limit that the spin partner states \(j_p\) and \(j_n\) share the same radial wave function \([8]\) and the small components of Dirac spinors are omitted. It is found that the tensor sum rule is exactly fulfilled under this limit.
on the average. For Skyrme force SGII+Te3 that was applied in nuclear excitations, the agreements on the tensor strengths are found in lower density region with narrower range of momentum transfer $q$, as compared to Skxta that was utilized in exploring the $\beta$-decay. Referred to the consensus approached by the relativistic and non-relativistic models, it seems that nuclear tensor force exhibits its essential effects in describing nuclear excitations at lower density region with narrower range of momentum transfer than in describing $\beta$-decay. On the other hand, the tensor coupling strengths of the Skyrme forces, e.g., SGII+Te3 and Skxta, were adjusted according to varied applications which induce distinct model dependence. Based on the meson exchange picture, the relativistic representation of Wigner tensor force can get more correlations involved, for instance, the nuclear in-medium effects evaluated by the density dependence of the tensor couplings and the finite-range features carried by the Yukawa-type propagators. This may own the advantages in the extensive applications.

In summary, the relativistic representation of Wigner tensor components in nuclear force is proposed with the new origin associated with the Fock diagrams of Lorentz scalar ($\sigma$ and $\delta$) and vector ($\omega$ and $\rho$) couplings. The proposed relativistic formalism, which are utilized to quantify the Wigner tensor components in the Fock diagrams of meson-nucleon couplings, are confirmed to be identical with the nature of tensor force, in terms of the spin-orbit interactions as well as the tensor sum rule. Specifically more distinct tensor effects are found in the isoscalar than the isovector channels, which may interpret the success achieved by the DDRHF plus relativistic RPA scheme in describing nuclear excitation modes. Due to the self-consistence on the emergence of nuclear tensor force, unified and self-consistent treatment on both tensor and spin-orbit interactions can be achieved by the relativistic models with the presence of Fock diagrams, which is of special meaning in exploring the limits of existence of nuclear systems. Moreover, the relativistic model (DDRHF-PKO1) presents consistent tensor strengths ($\alpha$ and $\beta$) with the non-relativistic ones (e.g., SGII+Te3 and Skxta). Combined with the common successes achieved by both models, it well demonstrates the reliability of the relativistic representation of Wigner tensor force in describing nuclear structure, excitation and decay modes. In the future, as an efficient and feasible constraint, the tensor-related observable can be quantified by the proposed relativistic formalism and this paves a microscopic way to understand the interior nature of the nuclear force, which is instructive in developing modern nuclear theoretical models as well.

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