Lamb and ac Stark shifts in cavity quantum electrodynamics*

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Abstract
A quantum system consisting of a single-mode microcavity and a two-level quantum dot or atom placed inside this microcavity, the interaction of monoenergetic photons in the microcavity with the electron in the two-level quantum dot or atom being the allowed electrical dipole transition between two energy levels, is a basic element of prospective devices for quantum information processing. Its time evolution is governed by cavity quantum electrodynamics (CQED). The photon–electron coupling induces the shifts of energy levels of the electron in the two-level quantum dot or atom. If there is no photon in the microcavity, then the shift of electron energy levels is induced by the radiative corrections and it is called the Lamb shift. The presence of photons in the microcavity induces another type of electron energy level shift—the ac Stark shift. In this paper, we present the exact derivation of the formulae for the Lamb and ac Stark shifts of electron energy levels in the two-level quantum dot or atom placed inside a single-mode microcavity.

Keywords: microcavity, two-level system, cavity quantum electrodynamics, energy shift

Classification numbers: 2.01, 3.00, 3.01

1. Introduction

In quantum information (QI) technology, any two-state quantum system can be used as a quantum bit (qubit)—a basic element of any QI processing system [1]. The QI can be transmitted between two distant qubits through the intermediary of other quantum systems interacting with these qubits. Each qubit may be a two-level electron system with two non-degenerate energy levels, a two-level quantum dot or atom, for example, and with the allowed electrical dipole quantum transition between two levels. In this case, the exchange of a photon between two qubits may be used as a feasible physical mechanism of the QI transmission from one qubit to another [1, 2]. To enhance the coupling of an electron in a two-level system with photons, it was proposed to place two qubits inside a microcavity (MC). It is well known that photons in an MC must have a discrete energy spectrum. The photon–electron interaction processes inside a MC are governed by cavity quantum electrodynamics (CQED) [3].

QI processes in CQED have been studied intensively in many experimental and theoretical works [4–10].

The simplest quantum system in CQED consists of a single-mode MC and a two-level electron system placed inside this MC, the coupling between a single-mode photon in the MC and an electron in the two-level system being the electrical dipole transition between two energy levels. In the rotating wave approximation, the Hamiltonian of the above-mentioned complex quantum system is determined by the simple Jaynes–Cummings formula [11],

\[ H_0 = E e^g e + \gamma^* \gamma + f (\gamma^* g^* e + e^* \gamma), \]  

where \( e \) (\( e^* \)) and \( g \) (\( g^* \)) are the destruction (creation) of an electron in the excited and ground states, respectively, of a two level-system, \( \gamma (\gamma^*) \) is that of a photon, \( E \) is the difference in energies of the two levels, \( \Omega \) is the photon energy and \( f \) is the effective photon–electron coupling constant. We can choose the phases of electron wave functions such that \( f \) is a positive number.

The photon–electron coupling induces the shift in electron energy levels—the energy renormalization due to...
radiative corrections—the virtual emissions and absorptions of the photon by the electron. The electron energy level shift is called the Lamb shift if there is no photon in the MC. In the presence of photons in the MC, the electron energy level shift is called the ac Stark shift. A simple approximation of the ac Stark/Lamb shift was derived in [9,10] in the case of large detuning,

$$\frac{f^2}{|E - \Omega|} \ll 1.$$  \hspace{1cm} (2)

This work is devoted to the study of Lamb and ac Stark shifts without the large detuning condition (2). In section 2, we derive exact formulae for the Lamb and ac Stark shifts in the system with the Jaynes–Cummings expression (1) of a total Hamiltonian. The extension beyond the approximation determined by the use of the Jaynes–Cummings formula is presented in section 3. The expressions of the Lamb and ac Stark shifts are derived in the second-order approximation with respect to the ratio

$$\frac{f^2}{\Omega} \ll 1.$$  \hspace{1cm} (3)

In section 4, the conclusion and discussion are presented.

## 2. Exact results for the system with the Jaynes–Cummings formula of the total Hamiltonian

We denote by $|g, n\rangle$ or $|e, n\rangle$ the quantum state of the system consisting of an electron in the ground or excited state, respectively, and $n$ photons in the MC. When the coupling constant $f$ vanishes, $f = 0$, they are the eigenstates of the Hamiltonian (1) with the following eigenvalues:

$$E_0^{(g,n)} = n\Omega, \quad E_0^{(e,n)} = E + n\Omega.$$  \hspace{1cm} (4)

Due to the photon–electron interaction, $f \neq 0$, mixing takes place between states in each pair $|g, n\rangle$ and $|e, n - 1\rangle$ and two eigenstates are formed,

$$|\psi_+^{(n)}\rangle = A_+^{(n)} |e, n - 1\rangle + B_+^{(n)} |g, n\rangle,$$

$$H |\psi_+^{(n)}\rangle = E_+^{(n)} |\psi_+^{(n)}\rangle,$$  \hspace{1cm} (5)

with eigenvalues

$$E_+^{(n)} = \frac{E_0^{(e,n-1)} + E_0^{(g,n)}}{2} \pm \frac{1}{2} \Delta^{(n)},$$  \hspace{1cm} (6)

$$\Delta^{(n)} = \sqrt{\left[\frac{E_0^{(e,n-1)} - E_0^{(g,n)}}{2}\right]^2 + 4nf^2}.$$  \hspace{1cm} (7)

One of the two eigenstates, $|\psi_+^{(n)}\rangle$, can be assigned to the eigenstate shifted from $|e, n - 1\rangle$, and the other one to that shifted from $|g, n\rangle$. We denote the eigenvalue of the former by $E^{(e,n-1)}$ and that of the latter by $E^{(g,n)}$. By definition, the Lamb shifts are determined by following formulae:

$$\delta_L E^{(e)} = E^{(e,0)} - E_0^{(e,n)},$$

$$\delta_L E^{(g)} = E^{(g,0)} - E_0^{(g,n)}.$$  \hspace{1cm} (8)

The assignment of each state from the pair $|\psi_+^{(n)}\rangle$ to the shifted state with the eigenvalue $E^{(e,n-1)}$ or $E^{(g,n)}$ depends on the sign of the difference $E - \Omega$. There are two different cases:

1. $E > \Omega$. In this case, $E^{(e,n-1)} > E^{(g,n)}$, $|\psi_+^{(n)}\rangle$ with higher eigenvalue is assigned to the shifted state of $|e, n - 1\rangle$, while $|\psi_-^{(n)}\rangle$ with lower eigenvalue to the shifted state of $|g, n\rangle$,

$$E^{(e,n-1)} > E^{(e,n)} > E^{(g,n)} > E\_.$$  \hspace{1cm} (9)

The shifts in energy levels due to the photon–electron interaction in this case are represented in figure 1. Note that $|g, 0\rangle$ is still an eigenstate of the Hamiltonian (1) with the eigenvalue equal to zero. The energy level of this state is not shifted. The exact formulae for the Lamb and ac Stark shifts are

$$\delta_L E^{(e)} = 0,$$  \hspace{1cm} (10a)

$$\delta_L E^{(g)} = \frac{E - \Omega}{2} \left(1 + \frac{4f^2}{(E - \Omega)^2} - 1\right).$$  \hspace{1cm} (10b)

$$\delta_S E^{(g,n)} = -\frac{E - \Omega}{2} \left(1 + \frac{4f^2}{(E - \Omega)^2} - 1\right).$$  \hspace{1cm} (10c)

It is obvious that

$$\delta_S E^{(g,n)} = -\delta_S E^{(e,n-1)}.$$  \hspace{1cm} (10d)

2. $E < \Omega$. In this case, $E^{(g,n)} > E^{(e,n-1)}$, $|\psi_+^{(n)}\rangle$ with higher eigenvalue is assigned to the shifted state of $|g, n\rangle$, while $|\psi_-^{(n)}\rangle$ with lower eigenvalue to the shifted state of $|e, n - 1\rangle$:

$$E^{(g,n)} > E\_ > E^{(e,n-1)} > E^{(e,n)}.$$  \hspace{1cm} (11)

$$\delta_L E^{(e)} = E^{(e,0)} - E_0^{(e,n)},$$

$$\delta_L E^{(g)} = E^{(g,0)} - E_0^{(g,n)}.$$  \hspace{1cm} (12)

The shifts in energy levels due to the photon–electron interaction in this case are represented in figure 1. Note that $|g, 0\rangle$ is still an eigenstate of the Hamiltonian (1) with the eigenvalue equal to zero. The energy level of this state is not shifted. The exact formulae for the Lamb and ac Stark shifts are

$$\delta_L E^{(e)} = E^{(e,0)} - E_0^{(e,n)},$$

$$\delta_L E^{(g)} = E^{(g,0)} - E_0^{(g,n)}.$$  \hspace{1cm} (13)

$$\delta_S E^{(g,n)} = -\frac{E - \Omega}{2} \left(1 + \frac{4f^2}{(E - \Omega)^2} - 1\right).$$  \hspace{1cm} (14)

$$\delta_S E^{(e,n-1)} = -\delta_S E^{(g,n)}.$$  \hspace{1cm} (15)
Lamb and ac Stark shifts: radiative corrections give the following contributions to the δ shown that, in this case, exact formulae (interaction in this case are represented in figure 2. The shifts in energy levels due to the photon–electron interaction are still valid. However, the algebraic values of δS E(i) and δS E(i) change the sign due to that of the difference E − Ω.

If the detuning is large, condition (2) holds, then formulae (13a)–(13d) are reduced to the expressions derived in previous works [8, 9] in the first-order approximation with respect to the small parameter in the lhs of the inequality (2). In the other extreme case of very small detuning, |E − Ω| ≪ f2,

formulæ (13a)–(13d) give the following results:

(i) E → Ω + 0
δδS E(e,n) ≈ f √n + 1,
δδS E(e,n) ≈ −f √n. (16)

(ii) E → Ω − 0
δδS E(e,n) ≈ −f √n + 1,
δδS E(e,n) ≈ f √n. (17)

The Lamb and ac Stark shifts are proportional to f, but not f2.

3. Radiative corrections beyond the Jaynes–Cummings approximation

The operator in the rhs of formula (1) is a very useful approximate expression of the Hamiltonian of the quantum system consisting of a single-mode MC and a two-level atom or quantum dot placed inside this MC, because it has exactly determined eigenstates and eigenvalues. A more exact formula for the Hamiltonian is

\[ H_0 = E e^+ e + \Omega \gamma^+ \gamma + f (\gamma^+ + \gamma)(e^+ g + g^+ e). \] (18)

This can be written as

\[ H = H_0 + H_1 \] (19)

with

\[ H_1 = f (\gamma^+ e^+ g + g^+ e \gamma). \] (20)

In the second-order approximation with respect to H1, the radiative corrections give the following contributions to the Lamb and ac Stark shifts:

\[ \delta \omega L_2 E(e,n) = - \frac{f^2}{\Delta(2)} \left\{ \Delta(2) - E - \Omega \Delta(2) - E + \Omega \Delta(2) - E + 3\Omega - \Delta(2) \right\} \] (21)

for any values of E and Ω,

\[ \delta \omega S_2 E(e,n) = \delta \omega L_2 E(e,n) \] (22)

for E > Ω and

\[ \delta \omega S_2 E(e,n) = \delta \omega L_2 E(e,n) \] (23)

for E < Ω, with

\[ \delta \omega S_2 E_{±(n)} = f^2 \left\{ (n + 1) \left( \frac{1}{\Delta(2)} - E - \Omega \right) - E - 5\Omega \pm \Delta(n) \right\} \] (24)

\[ \frac{f^2}{\Omega} \ll 1, \] (25)

4. Conclusion and discussion

The exact expressions of the Lamb and ac Stark shifts were derived for the energy levels of an electron in a quantum system consisting of a single-mode MC and a two-level atom or quantum dot placed inside this MC, with the total Hamiltonian determined by the Jaynes–Cummings formula. Between the ac Stark shift of the levels E(e,n−1) and E(e,n), there is the relation

\[ \delta \omega S_2 E(e,n−1) = -\delta \omega S_2 E(e,n) . \]

For large values of the difference E − Ω, our exact result is consistent with the approximate formula derived in previous works. At the resonance E = Ω, the Lamb and Stark shifts are proportional to the coupling constant f. The radiative corrections beyond the Jaynes–Cummings approximation were also calculated in the second order with respect to the small coupling constant f.

We have studied the above-mentioned quantum system without decoherence. The general theory of the decoherence of the complex systems like that mentioned above was presented in the review [12]. In general, decoherence leads not only to the broadening of the energy levels but also to their shifts proportional to the decoherence constants [12, 13].

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