SUSY Dark Matter in the Universe- Theoretical Direct Detection Rates

J. D. VERGADOS

Theoretical Physics Section, University of Ioannina, GR-45110, Greece
E-mail: Vergados@cc.uoi.gr

Exotic dark matter together with the vacuum energy or cosmological constant seem to dominate in the Universe. An even higher density of such matter seems to be gravitationally trapped in our Galaxy. Thus its direct detection is central to particle physics and cosmology. Current fashionable supersymmetric models provide a natural dark matter candidate which is the lightest supersymmetric particle (LSP). Such models combined with fairly well understood physics like the quark substructure of the nucleon and the nuclear structure (form factor and/or spin response function), permit the evaluation of the event rate for LSP-nucleus elastic scattering. The thus obtained event rates are, however, very low or even undetectable. So it is imperative to exploit the modulation effect, i.e. the dependence of the event rate on the earth’s annual motion. Also it is useful to consider the directional rate, i.e. its dependence on the direction of the recoiling nucleus. In this paper we study such a modulation effect both in non directional and directional experiments. We calculate both the differential and the total rates using both isothermal, symmetric as well as only axially asymmetric, and non isothermal, due to caustic rings, velocity distributions. We find that in the symmetric case the modulation amplitude is small. The same is true for the case of caustic rings. The inclusion of asymmetry, with a realistic enhanced velocity dispersion in the galactocentric direction, yields an enhanced modulation effect, especially in directional experiments.

I. Introduction

In recent years the consideration of exotic dark matter has become necessary in order to close the Universe. Furthermore in order to understand the large scale structure of the universe it has become necessary to consider matter made up of particles which were non-relativistic at the time of freeze out. This is the cold dark matter component (CDM). The COBE data suggest that CDM is at least 60% of the non-exotic component. On the other hand during the last few years evidence has appeared from two different teams, the High-z Supernova Search Team and the Supernova Cosmology Project, which suggests that the Universe may be dominated by the cosmological constant Λ. As a matter of fact recent data the situation can be adequately described by a baryonic component Ω_B = 0.1 along with the exotic components Ω_{CDM} = 0.3 and Ω_Λ = 0.6 (see next section for the definitions). In another analysis Turner gives Ω_m = Ω_{CDM} + Ω_B = 0.4. Since the non exotic component cannot exceed 40% of the CDM, there is room for the exotic WIMP’s (Weakly Interacting Massive Particles). In fact the DAMA
experiment has claimed the observation of one signal in direct detection of a WIMP, which with better statistics has subsequently been interpreted as a modulation signal.

The above developments are in line with particle physics considerations. Thus, in the currently favored supersymmetric (SUSY) extensions of the standard model, the most natural WIMP candidate is the LSP, i.e. the lightest supersymmetric particle. In the most favored scenarios the LSP can be simply described as a Majorana fermion, a linear combination of the neutral components of the gauginos and Higgsinos.

II. Density Versus Cosmological Constant

The evolution of the Universe is governed by the General Theory of Relativity. The most commonly used model is that of Friedman, which utilizes the Robertson- Walker metric

\[(ds)^2 = (dt)^2 - R^2(t)\left[\frac{dr^2}{1 - \kappa r^2} + r^2(d\theta)^2 + \sin^2 \theta (d\phi)^2\right]\] (1)

The resulting Einstein equations are:

\[R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -8\pi G_N T_{\mu\nu} + \Lambda g_{\mu\nu}\] (2)

where \(G_N\) is Newton’s constant and \(\Lambda\) is the cosmological constant. The equation for the scale factor \(R\) becomes:

\[\frac{d^2 R}{dt^2} = -\frac{4\pi}{3} G_N (\rho + 3p) R = -\frac{4\pi G_N \rho}{3} R + \frac{\Lambda}{3}\] (3)

where \(\rho\) is the mass density. Then the energy is

\[E = \frac{1}{2} m \left(\frac{dr}{dt}\right)^2 - G_N \frac{m}{R} (4\pi \rho R^3) + \frac{\Lambda}{6} m R^2 = constant = -\frac{\kappa}{2} m\] (4)

This can be equivalently be written as

\[H^2 + \frac{\kappa}{R^2} = \frac{8\pi}{3} G_N \rho + \frac{\Lambda}{3}\] (5)

where the quantity \(H\) is Hubble’s constant defined by

\[H = \frac{1}{R} \frac{dR}{dt}\] (6)

Hubble’s constant is perhaps the most important parameter of cosmology. In fact it is not a constant but it changes with time. Its present day value is given by
\[ H_0 = (65 \pm 15) \text{ km/s } M_{pc}^{-1} \] (7)

In other words \( H_0^{-1} = (1.50\pm.35) \times 10^{10} \text{ y} \), which is roughly equal to the age of the Universe. Astrophysicists conventionally write it as

\[ H_0 = 100 \text{ km/s } M_{pc}^{-1} , \quad 0.5 < h < 0.8 \] (8)

Equations (3-5) coincide with those of the Newtonian theory with the following two types of forces: An attractive force decreasing in absolute value with the scale factor (Newton) and a repulsive force increasing with the scale factor (Einstein)

\[ F = -G_N \frac{mM}{R^2} \text{ (Newton)} \quad F = \frac{1}{3} \Lambda mR \text{ (Einstein)} \] (9)

Historically the cosmological constant was introduced by Einstein so that General Relativity yields a stationary Universe, i.e. one which satisfies the conditions:

\[ \frac{dR}{dt} = 0 \quad \frac{d^2 R}{dt^2} = 0 \] (10)

Indeed for \( \kappa > 0 \), the above equations lead to \( R = R_c = \text{constant} \) provided that

\[ \frac{1}{3} \Lambda R_c - \frac{4\pi}{3} G_N \rho R_c = 0 \quad \frac{1}{3} \Lambda R_c^2 - \frac{4\pi}{3} G_N \rho R_c^2 = \kappa \] (11)

These equations have a non trivial solution provided that the density \( \rho \) and the cosmological constant \( \Lambda \) are related, i.e.

\[ \Lambda = 4\pi G_N \rho \] (12)

The radius of the Universe then is given by

\[ R_c = \left( \frac{\kappa}{4\pi G_N \rho} \right)^{1/2} \] (13)

Define now

\[ \Omega_m = \frac{\rho}{\rho_c} , \quad \Omega_\Lambda = \frac{\rho_\Lambda}{\rho_c} , \quad \rho_v = \frac{\Lambda}{8\pi G_N} \text{ ("vacuum" density)} \] (14)

The critical density is

\[ \rho_c = 1.8 \times 10^{-23} h^2 \frac{g}{cm^3} = 10h^2 \frac{\text{nucleons}}{m^3} \] (15)
With these definitions Friedman’s equation $E = -\kappa \frac{m}{2}$ takes the form

$$\frac{\kappa}{R^2} = (\Omega_m + \Omega_\Lambda - 1)H^2$$

(16)

Thus we distinguish the following special cases:

$$\kappa > 0 \iff \Omega_m + \Omega_\Lambda > 1 \iff \text{Closed curved Universe}$$

(17)

$$\kappa = 0 \iff \Omega_m + \Omega_\Lambda = 1 \iff \text{Open Flat Universe}$$

(18)

$$\kappa < 0 \iff \Omega_m + \Omega_\Lambda < 1 \iff \text{Open Curved Universe}$$

(19)

In other words it is the combination of matter and "vacuum" energy, which determines the fate of the our Universe.

Before concluding this section we remark that the above equations do not suffice to yield a solution since the density is a function of the scale factor. An equation of state is in addition needed, but we are not going to elaborate further.

III. An Overview of Direct Detection - The Allowed SUSY Parameter Space.

Since this particle is expected to be very massive, $m_\chi \geq 30\text{GeV}$, and extremely non relativistic with average kinetic energy $T \leq 100\text{KeV}$, it can be directly detected mainly via the recoiling of a nucleus $(A,Z)$ in the elastic scattering process:

$$\chi + (A,Z) \rightarrow \chi + (A,Z)^*$$

(20)

($\chi$ denotes the LSP). In order to compute the event rate one needs the following ingredients:

1) An effective Lagrangian at the elementary particle (quark) level obtained in the framework of supersymmetry as described, e.g., in Refs. 1, 14.

2) A procedure in going from the quark to the nucleon level, i.e. a quark model for the nucleon. The results depend crucially on the content of the nucleon in quarks other than u and d. This is particularly true for the scalar couplings as well as the isoscalar axial coupling.

3) Compute the relevant nuclear matrix elements using as reliable as possible many body nuclear wave functions. By putting as accurate nuclear physics input as possible, one will be able to constrain the SUSY parameters as much as possible. The situation is a bit simpler in the case of the scalar coupling, in which case one only needs the nuclear form factor.
Since the obtained rates are very low, one would like to be able to exploit the modulation of the event rates due to the earth’s revolution around the sun \[24\], \[25\]− \[27\]. To this end one adopts a folding procedure assuming some distribution \[1\], \[25\], \[27\] of velocities for the LSP. One also would like to know the directional rates, by observing the nucleus in a certain direction, which correlate with the motion of the sun around the center of the galaxy and the motion of the Earth \[1\]− \[25\].

The calculation of this cross section has become pretty standard. One starts with representative input in the restricted SUSY parameter space as described in the literature \[12\], \[14\]. We will adopt a phenomenological procedure taking universal soft SUSY breaking terms at \(M_{\text{GUT}}\), i.e., a common mass for all scalar fields \(m_0\), a common gaugino mass \(M_{1/2}\) and a common trilinear scalar coupling \(A_0\), which we put equal to zero (we will discuss later the influence of non-zero \(A_0\)’s). Our effective theory below \(M_{\text{GUT}}\) then depends on the parameters:

\[m_0, \ M_{1/2}, \ \mu_0, \ \alpha_G, \ M_{\text{GUT}}, \ h_t, \ , \ h_b, \ , \ h_\tau, \ \tan\beta,\]

where \(\alpha_G = g^2_G/4\pi\) (\(g_G\) being the GUT gauge coupling constant) and \(h_t, h_b, h_\tau\) are respectively the top, bottom and tau Yukawa coupling constants at \(M_{\text{GUT}}\). The values of \(\alpha_G\) and \(M_{\text{GUT}}\) are obtained as described in Refs. \[12\]. For a specified value of \(\tan\beta\) at \(M_S\), we determine \(h_t\) at \(M_{\text{GUT}}\) by fixing the top quark mass at the center of its experimental range, \(m_t(m_t) = 166\text{GeV}\). The value of \(h_\tau\) at \(M_{\text{GUT}}\) is fixed by using the running tau lepton mass at \(m_Z\), \(m_\tau(m_Z) = 1.746\text{GeV}\). The value of \(h_b\) at \(M_{\text{GUT}}\) used is such that:

\[m_b(m_Z)_{\text{SM}} = 2.90 \pm 0.14 \text{ GeV}\]

after including the SUSY threshold correction. The SUSY parameter space is subject to the following constraints:

1.) The LSP relic abundance will satisfy the cosmological constrain:

\[0.09 \leq \Omega_{\text{LSP}}h^2 \leq 0.22 \]  \hspace{1cm}  (21)

2.) The Higgs bound obtained from recent CDF \[29\] and LEP2 \[30\], i.e. \(m_h > 113 \text{GeV}\). We should remember that the event rate does not depend only on the nucleon cross section, but on other parameters also, mainly on the LSP mass and the nucleus used in target. The condition on the nucleon cross section imposes

\[4 \times 10^{-7} \text{ pb} \leq \sigma_{\text{nucleon}}^{\text{scalar}} \leq 2 \times 10^{-5} \text{ pb} \]  \hspace{1cm}  (22)
severe constraints on the acceptable parameter space. In particular in our model it restricts $\tan \beta$ to values $\tan \beta \simeq 50$. We will not elaborate further on this point, since it has already appeared\textsuperscript{13}.

IV. Expressions for the Differential Cross Section.

The effective Lagrangian describing the LSP-nucleus cross section can be cast in the form\textsuperscript{15}

$$L_{\text{eff}} = -\frac{G_F}{\sqrt{2}} \left\{ (\bar{\chi}_1 \gamma^\lambda \gamma_5 \chi_1) J_\lambda + (\bar{\chi}_1 \chi_1) J \right\}$$  \hspace{1cm} (23)

where

$$J_\lambda = \bar{N} \gamma^\lambda \left( f_0^0 V + f_1^1 V \tau_3 + f_0^1 A \gamma_5 + f_0^1 A \gamma_5 \tau_3 \right) N , \quad J = \bar{N} (f_0^0 + f_1^1 \tau_3) N$$  \hspace{1cm} (24)

We have neglected the uninteresting pseudoscalar and tensor currents. Note that, due to the Majorana nature of the LSP, $\bar{\chi}_1 \gamma^\lambda \chi_1 = 0$ (identically).

With the above ingredients the differential cross section can be cast in the form\\textsuperscript{11,24,25}

$$d\sigma(u, v) = \frac{du}{2(\mu_r b v)^2} \left[ (\bar{\Sigma}_S + \bar{\Sigma}_V \frac{v^2}{c^2}) F^2(u) + \bar{\Sigma}_{\text{spin}} F_{11}(u) \right]$$  \hspace{1cm} (25)

$$\bar{\Sigma}_S = \sigma_0 \left( \frac{\mu_r (A)}{\mu_r (N)} \right)^2 \left\{ A^2 \left[ (f_0^0 - f_1^1 A - 2 Z A)^2 \right] \simeq \sigma_{p,\chi^0}^S A^2 \left( \frac{\mu_r (A)}{\mu_r (N)} \right)^2 \right\}$$  \hspace{1cm} (26)

$$\bar{\Sigma}_{\text{spin}} = \sigma_{p,\chi^0}^{\text{spin}} \xi_{\text{spin}} , \quad \xi_{\text{spin}} = \frac{\left( \mu_r (A)/\mu_r (N) \right)^2}{3(1 + f_1^1 f_0^0)^2} \left( \frac{v}{c} \right)^2$$  \hspace{1cm} (27)

$$S(u) = \left[ \left( \frac{f_0^0}{f_A^1} \Omega_0(0) \right)^2 \frac{F_{00}(u)}{F_{11}(u)} + 2 \frac{f_0^0}{f_A^1} \Omega_0(0) \Omega_1(0) \frac{F_{01}(u)}{F_{11}(u)} + \Omega_1(0) \right]^2$$  \hspace{1cm} (28)

$$\bar{\Sigma}_V = \xi_{\text{spin}} V$$  \hspace{1cm} (29)

$$\xi_{\text{spin}} = \frac{\left( \mu_r (A)/\mu_r (N) \right)^2 A^2 \left(1 - \frac{f_1^1}{f_0^0} A - 2 Z A \right)^2 \left[ \frac{v_0}{c} \right]^2 \left[ 1 - \frac{1}{(2 \mu_r b)^2} \frac{2 \eta + 1}{(1 + \eta)^2} \frac{2 u}{v^2} \right]}{(1 + \frac{f_1^1}{f_0^0})^2}$$  \hspace{1cm} (30)

6
\( \sigma_{\text{p,} \chi_0} = \text{proton cross-section,} \ i = S, \text{spin,} \ V \) given by:

- \( \sigma^S_{\text{p,} \chi_0} = \sigma_0 \left( f^0_S \right)^2 \left( \frac{\mu_r(N)}{m_N} \right)^2 \) (scalar), (the isovector scalar is negligible, i.e. \( \sigma^S_{\text{p,} \chi_0} \approx \sigma^S_{\text{n,} \chi_0} \)),

- \( \sigma^{\text{spin}}_{\text{p,} \chi_0} = \sigma_0 \left( f^0_A + f^1_A \right)^2 \left( \frac{\mu_r(N)}{m_N} \right)^2 \) (spin), \( \sigma^{V}_{\text{p,} \chi_0} = \sigma_0 \left( f^0_V + f^1_V \right)^2 \left( \frac{\mu_r(N)}{m_N} \right)^2 \) (vector),

where \( m_N \) is the nucleon mass, \( \eta = m_x/m_N A \), and \( \mu_r(A) \) is the LSP-nucleus reduced mass, \( \mu_r(N) \) is the LSP-nucleon reduced mass and

\[
\sigma_0 = \frac{1}{2\pi} (G_F m_N)^2 \approx 0.77 \times 10^{-38} \text{cm}^2 \tag{31}
\]

\[ Q = Q_0 u \ , \quad Q_0 = \frac{1}{A m_N b^2} = 4.1 \times 10^4 A^{-4/3} \text{KeV} \tag{32} \]

where \( Q \) is the energy transfer to the nucleus, \( F(u) \) is the nuclear form factor and

\[
F_{\rho\rho'}(u) = \sum_{\lambda,\kappa} \frac{\Omega_\rho^{(\lambda,\kappa)}(u)}{\Omega_\rho(0)} \frac{\Omega_{\rho'}^{(\lambda,\kappa)}(u)}{\Omega_{\rho'}(0)}, \quad \rho, \rho' = 0, 1 \tag{33}
\]

are the spin form factors (\( \rho, \rho' \) are isospin indices) normalized to one at \( u = 0 \). \( \Omega_0(\Omega_1) \) are the static isoscalar (isovector) spin matrix elements. Note that the quantity \( S(u) \) is essentially independent of \( u \). So the energy transfer dependence is contained in the function \( F_{11}(u) \). Note also that \( S(u) \) depends on the ratio of the isoscalar to isovector axial current couplings. These individual couplings can vary a lot within the SUSY parameter space.

| \[ \Omega_0(0) \]^2 | 19F | 29Si | 23Na | 73Ge | 207Pb |
|----------------------|-----|-----|-----|-----|-----|
| \[ \Omega_1(0) \]^2  | 2.610 | 0.207 | 0.477 | 1.157 | 0.305 |
| \[ \Omega_0(0) \Omega_1(0) \] | 2.807 | 0.219 | 0.346 | 1.095 | 0.231 |
| \( \mu_{\text{th}} \) | 2.91 | -0.50 | 2.22 |
| \( \mu_{\text{exp}} \) | 2.62 | -0.56 | 2.22 |
| \( \mu_{\text{th}}(\text{spin})/\mu_{\text{exp}}(\text{spin}) \) | 0.91 | 0.99 | 0.57 |
FIG. 1.: The energy dependence of the coherent process, i.e. the square of the form factor, $(|F(u)|^2)$, for the isotopes $^{19}F$, $^{23}Na$ and $^{29}Si$. The allowed range of $u$ for the above isotopes is $0.011 \leq u \leq 0.17$, $0.015 \leq u \leq 0.30$, and $0.021 \leq u \leq 0.50$ respectively. This corresponds to energy transfers $8.9 \leq Q \leq 140$, $9.5 \leq Q \leq 190$, and $9.7 \leq u \leq 230$ KeV respectively.

Their ratio, however, is not changing very much. In fact actual calculations show that $3.0 \leq S(0) \leq 7.5$ for $^{19}F$, $0.03 \leq S(0) \leq 0.2$ for $^{29}Si$ and $0.4 \leq S(0) \leq 1.1$ for $^{23}Na$. The quantity $S(u)$ depends very sensitively on nuclear physics via the static spin ME. This is exhibited in Table (I). As we can see from Table (I) the spin matrix elements are very accurate. This is evident by comparing the obtained magnetic dipole moments to experiment and noting that the magnetic moments, with the exception of $^{23}Na$ are dominated by the spin. From the same table we see that $^{19}F$ is favored from the point of view of the spin matrix element. This advantage may be partially lost if the LSP is very heavy, due to the kinematic factor $\mu_r(A)$, which tends to favor a heavy target. The energy transfer dependence of the differential cross section for the coherent mode is given by the square of the form factor, i.e. $|F(u)|^2$. These form factors for the isotopes $^{19}F$, $^{23}Na$ and $^{29}Si$ were calculated by Divari et al and are shown in Fig (1). The energy transfer dependence of the differential cross section due to spin is essentially given by $F_{11}(u)$. These functions for the isotopes $^{19}F$, $^{23}Na$ and $^{29}Si$ were calculated by Divari et al and are shown in Fig (2). Note that the energy dependence of the coherent and the spin modes...
for light systems are not very different, especially if the PCAC corrections on
the spin response function are ignored.

FIG. 2.: The energy dependence of the spin contribution (spin response function $F_{11}(u)$) for the isotopes $^{19}$F, $^{23}$Na and $^{29}$Si. The allowed range of energy transfers is the same as in Table 1. In this figure we also plot $F_{11}(u)$ when the PCAC effect is considered.

V. Expressions for the Rates.

The non-directional event rate is given by:

$$
R = R_{\text{non-dir}} = \frac{dN}{dt} = \frac{\rho(0)}{m_x} \frac{m}{A m_N} \sigma(u, v) |v| \tag{34}
$$

Where $\rho(0) = 0.3 GeV/cm^3$ is the LSP density in our vicinity and $m$ is the
detector mass. The differential non-directional rate can be written as

$$
dR = dR_{\text{non-dir}} = \frac{\rho(0)}{m_x} \frac{m}{A m_N} d\sigma(u, v) |v| \tag{35}
$$

where $d\sigma(u, v)$ was given above.

The directional differential rate in the direction $\hat{e}$ is given by:
\[ dR_{\text{dir}} = \frac{\rho(0)}{m_\chi} \frac{m}{A m_N} v.eH(v,e) \frac{1}{2\pi} d\sigma(u,v) \]  \hspace{1cm} (36) 

where H the Heaviside step function. The factor of \(1/2\pi\) is introduced, since the differential cross section of the last equation is the same with that entering the non-directional rate, i.e. after an integration over the azimuthal angle around the nuclear momentum has been performed. In other words, crudely speaking, \(1/(2\pi)\) is the suppression factor we expect in the directional rate compared to the usual one. The precise suppression factor depends, of course, on the direction of observation. In spite of their very interesting experimental signatures, we will not be concerned here with directional rates. The mean value of the non-directional event rate of Eq. (35), is obtained by convoluting the above expressions with the LSP velocity distribution \(f(v,v_E)\) with respect to the Earth, i.e. is given by:

\[ \langle \frac{dR}{du} \rangle = \frac{\rho(0)}{m_\chi} \frac{m}{A m_N} \int f(v,v_E)|v| \frac{d\sigma(u,v)}{du} d^3v \]  \hspace{1cm} (37) 

The above expression can be more conveniently written as

\[ \langle \frac{dR}{du} \rangle = \frac{\rho(0)}{m_\chi} \frac{m}{A m_N} \sqrt{\langle v^2 \rangle} \langle \frac{d\Sigma}{du} \rangle \]  \hspace{1cm} (38) 

where

\[ \langle \frac{d\Sigma}{du} \rangle = \int \frac{|v|}{\sqrt{\langle v^2 \rangle}} f(v,v_E) \frac{d\sigma(u,v)}{du} d^3v \]  \hspace{1cm} (39) 

After performing the needed integrations over the velocity distribution, to first order in the Earth’s velocity, and over the energy transfer \(u\) the last expression takes the form

\[ R = \bar{R} t [1 + h(a,Q_{\text{min}}) \cos \alpha] \]  \hspace{1cm} (40) 

where \(\alpha\) is the phase of the Earth (\(\alpha = 0\) around June 2nd) and \(Q_{\text{min}}\) is the energy transfer cutoff imposed by the detector. In the above expressions \(\bar{R}\) is the rate obtained in the conventional approach by neglecting the folding with the LSP velocity and the momentum transfer dependence of the differential cross section, i.e. by

\[ \bar{R} = \frac{\rho(0)}{m_\chi} \frac{m}{A m_N} \sqrt{\langle v^2 \rangle} \bar{\Sigma}_S + \bar{\Sigma}_{\text{spin}} + \frac{\langle v^2 \rangle}{c^2} \bar{\Sigma}_V \]  \hspace{1cm} (41) 

where \(\bar{\Sigma}_i, i=S,V, \text{spin}\) have been defined above, see Eqs (26) - (29). It contains all the parameters of the SUSY models. The modulation is described
by the parameter $h$. Once the rate is known and the parameters $t$ and $h$, which depend only on the LSP mass, the nuclear form factor and the velocity distribution the nucleon cross section can be extracted and compared to experiment.

The total directional event rates can be obtained in a similar fashion by by integrating Eq. (36) with respect to the velocity as well as the energy transfer $u$. We find

$$R_{\text{dir}} = \frac{\bar{R}}{(t_0^*/4\pi)} |(1 + h_1(a, Q_{\text{min}}) \cos \alpha) e_z \cdot e - h_2(a, Q_{\text{min}}) \cos \alpha e_y \cdot e + h_3(a, Q_{\text{min}}) \sin \alpha e_x \cdot e|$$

We remind that the $z$-axis is in the direction of the sun’s motion, the $y$-axis is perpendicular to the plane of the galaxy and the $x$-axis is in the galactocentric direction. The effect of folding with LSP velocity on the total rate is taken into account via the quantity $t_0^*$, which depends on the LSP mass. All other SUSY parameters have been absorbed in $\bar{R}$. We see that the modulation of the directional total event rate can be described in terms of three parameters $h_l$, $l=1,2,3$. In the special case of $\lambda = 0$ we essentially have one parameter, namely $h_1$, since then we have $h_2 = 0.117$ and $h_3 = 0.135$. Given the functions $h_l(a, Q_{\text{min}})$ one can plot the the expression in Eq. (42) as a function of the phase of the earth $\alpha$.

VI. The Scalar Contribution- The Role of the Heavy Quarks

The coherent scattering can be mediated via the the neutral intermediate Higgs particles ($h$ and $H$), which survive as physical particles. It can also be mediated via s-quarks, via the mixing of the isodoublet and isosinlet s-quarks of the same charge. In our model we find that the Higgs contribution becomes dominant and, as a matter of fact the heavy Higgs $H$ is more important (the Higgs particle $A$ couples in a pseudoscalar way, which does not lead to coherence). It is well known that all quark flavors contribute, since the relevant couplings are proportional to the quark masses. One encounters in the nucleon not only the usual sea quarks ($u\bar{u}, d\bar{d}$ and $s\bar{s}$) but the heavier quarks $c, b, t$ which couple to the nucleon via two gluon exchange, see e.g. Drees et al. and references therein.

As a result one obtains an effective scalar Higgs-nucleon coupling by using effective quark masses as follows

$$m_u \rightarrow f_u m_N, \quad m_d \rightarrow f_d m_N, \quad m_s \rightarrow f_s m_N$$

$$m_Q \rightarrow f_Q m_N, \quad (\text{heavy quarks } c, b, t)$$

where $m_N$ is the nucleon mass. The isovector contribution is now negligible. The parameters $f_q$, $q = u, d, s$ can be obtained by chiral symmetry breaking.
terms in relation to phase shift and dispersion analysis. Following Cheng and Cheng we obtain:

\[
f_u = 0.021, \quad f_d = 0.037, \quad f_s = 0.140 \quad \text{(model B)}
\]

\[
f_u = 0.023, \quad f_d = 0.034, \quad f_s = 0.400 \quad \text{(model C)}
\]

We see that in both models the s-quark is dominant. Then to leading order via quark loops and gluon exchange with the nucleon one finds:

\[
f_Q = \frac{2}{27}(1 - \sum_q f_q)
\]

This yields:

\[
f_Q = 0.060 \quad \text{(model B)}, \quad f_Q = 0.040 \quad \text{(model C)}
\]

There is a correction to the above parameters coming from loops involving s-quarks and due to QCD effects. Thus for large \(\tan \beta\) we find:

\[
f_c = 0.060 \times 1.068 = 0.064, \quad f_t = 0.060 \times 2.048 = 0.123, \quad f_b = 0.060 \times 1.174 = 0.070 \quad \text{(model B)}
\]

\[
f_c = 0.040 \times 1.068 = 0.043, \quad f_t = 0.040 \times 2.048 = 0.082, \quad f_b = 0.040 \times 1.174 = 0.047 \quad \text{(model B)}
\]

For a more detailed discussion we refer the reader to Refs.

\section*{VII. Results and Discussion}

The three basic ingredients of our calculation were the input SUSY parameters (see sect. 1), a quark model for the nucleon (see sect. 3) and the velocity distribution combined with the structure of the nuclei involved (see sect. 2). We will focus our attention on the coherent scattering and present results for the popular target \(^{127}\text{I}\). We have utilized two nucleon models indicated by B and C which take into account the presence of heavy quarks in the nucleon. We also considered energy cut offs imposed by the detector, by considering two typical cases \(Q_{\text{min}} = 10, 20\, \text{KeV}\). The thus obtained results for the unmodulated total non directional event rates \(\bar{R}_t\) in the case of the symmetric isothermal model for a typical SUSY parameter choice are shown in Fig. 3.
Special attention was paid to the directional rate and its modulation due to the annual motion of the earth in the case of isothermal models. The case of non-isothermal models, e.g., caustic rings, is more complicated and it will not be further discussed here. As expected, the parameter $t_0$, which contains the effect of the nuclear form factor and the LSP velocity dependence, decreases as the reduced mass increases.

We will focus to the discussion of the directional rates described in terms of $t_0$ and $h_i, i = 1, 2, 3$ (see Eq. (42)) and limit ourselves to directions of observation close to the coordinate axes. As expected, the parameter $t_0$, decreases as the reduced mass increases. The quantity $t^0$ is shown in Fig. (4), for three values of the detector energy cutoff, $Q_{min} = 0, 10$ and 20 KeV. Similarly we show the quantity $h_1$ in Fig. (5). The quantities $h_2$ and $h_3$ are shown in Fig. (6) for $\lambda = 1$. For $\lambda = 0$ they are not shown, since they are essentially constant.
and equal to 0.117 and 0.135 respectively.

\[ \lambda = 0 \quad \lambda = 1 \]

**FIG. 4.** The dependence of the quantity \( t^0 \) on the LSP mass for the symmetric case (\( \lambda = 0 \)) as well as for the maximum axial asymmetry (\( \lambda = 1 \)) in the case of the target \( ^{127}I \). For orientation purposes three detection cutoff energies are exhibited, \( Q_{\text{min}} = 0 \) (thick solid line), \( Q_{\text{min}} = 5 \) keV (thin solid line) and \( Q_{\text{min}} = 10 \) keV (dashed line). As expected \( t^0 \) decreases as the cutoff energy increases.

\[ \lambda = 0 \quad \lambda = 1 \]

**FIG. 5.** The same as in the previous figure for the modulation amplitude \( h_1 \).
As expected, the parameter $t_0$, decreases as the reduced mass increases. It also decreases as the cutoff energy $Q_{\text{min}}$ increases. We notice that $t_0$ is affected little by the presence of asymmetry. On the other hand $h_1, h_2$ and $h_3$ substantially increase in the presence of asymmetry. Sometimes they increase as the cutoff energy increases (at the expense, of course, of the total number of counts. For the differential rate the reader is referred to our previous work \cite{15,16,17,18,19}. 

VIII. Conclusions

In the present paper we have discussed the parameters, which describe the event rates for direct detection of SUSY dark matter. Only in a small segment of the allowed parameter space the rates are above the present experimental goals. We thus looked for characteristic experimental signatures for background reduction, i.e. a) Correlation of the event rates with the motion of the Earth (modulation effect) and b) the directional rates (their correlation both with the velocity of the sun and that of the Earth.)

A typical graph for the total unmodulated rate is shown Fig. 3. We will concentrate here on the directional rates, described in terms of the parameters $t_0, h_1, h_2$ and $h_3$. For simplicity these parameters are given in Figs (4)-(6) for directions of observation close to the three axes $x, y, z$. We see that the unmodulated rate scales by the $\cos \theta_s$, with $\theta_s$ (the angle between the direction of observation and the velocity of the sun). The reduction factor, $f_{\text{red}} = t_0/(4\pi t_0) = \kappa/(2\pi)$, of the total directional rate, along the sun’s direction of motion, compared to the total non directional rate depends on the nuclear parameters, the reduced mass and the asymmetry parameter $\lambda$. We find $h_2$ (for $\lambda = 1$) and $h_3$ (for $\lambda = 1$)
that $\kappa$ is around 0.6 (no asymmetry) and around 0.7 (maximum asymmetry, $\lambda = 1.0$), i.e. not very different from the naively expected $f_{\text{red}} = 1/(2\pi)$, i.e. $\kappa = 1$. The modulation of the directional rate increases with the asymmetry parameter $\lambda$ and it also depends of the direction of observation. For $Q_{\text{min}} = 0$ it can reach values up to 23%. Values up to 35% are possible for large values of $Q_{\text{min}}$, but they occur at the expense of the total number of counts.

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1. For a review see e.g. G. Jungman et al., Phys. Rep. 267, 195 (1996).
2. G.F. Smoot et al., (COBE data), Astrophys. J. 396 (1992) L1.
3. E. Gawiser and J. Silk, Science 280, 1405 (1988); M.A.K. Gross, R.S. Somerville, J.R. Primack, J. Holtzman and A.A. Klypin, Mon. Not. R. Astron. Soc. 301, 81 (1998).
4. A.G. Riess et al, Astron. J. 116 (1998), 1009.
5. R.S. Somerville, J.R. Primack and S.M. Faber, astro-ph/9806228, Mon. Not. R. Astron. Soc. (in press).
6. Perlmutter, S. et al (1999) Astrophys. J. 517,565; (1997) 483,565 (astro-ph/9812133).
S. Perlmutter, M.S. Turner and M. White, Phys. Rev. Lett. 83, 670 (1999).
7. M.S. Turner, astro-ph/9904051, Phys. Rep. 333-334 (1990), 619.
8. D.P. Bennett et al., (MACHO collaboration), A binary lensing event toward the LMC: Observations and Dark Matter Implications, Proc. 5th Annual Maryland Conference, edited by S. Holt (1995);
C. Alcock et al., (MACHO collaboration), Phys. Rev. Lett. 74 , 2967 (1995).
9. R. Bernabei et al., INFN/AE-98/34, (1998); R. Bernabei et al., it Phys. Lett. B 389, 757 (1996).
10. R. Bernabei et al., Phys. Lett. B 424, 195 (1998); B 450, 448 (1999).
11. For more references see e.g. our previous report:
J.D. Vergados, Supersymmetric Dark Matter Detection- The Directional Rate and the Modulation Effect, hep-ph/0010151.
12. M.E. Gómez, J.D. Vergados, hep-ph/0012020.
M.E. Gómez, G. Lazarides and C. Pallis, Phys. Rev. D61 (2000) 123512 and Phys. Lett. B 487, 313 (2000).
13. M.E. Gómez and J.D. Vergados, hep-ph/0105115.
14. A. Bottino et al., Phys. Lett B 402, 113 (1997).
R. Arnowitt and P. Nath, Phys. Rev. Lett. 74, 4952 (1995); Phys. Rev. D 54, 2394 (1996); hep-ph/9902237.
V.A. Bednyakov, H.V. Klapdor-Kleingrotheaus and S.G. Kovalenko, Phys.
15. J.D. Vergados, *J. of Phys. G* 22, 253 (1996).
16. T.S. Kosmas and J.D. Vergados, *Phys. Rev. D* 55, 1752 (1997).
17. M. Drees and M.M. Nojiri, *Phys. Rev.* D47 (1993) 376.
18. M. Drees and M.M. Nojiri, *Phys. Rev. D* 48, 3843 (1993); *Phys. Rev. D* 47, 4226 (1993).
19. A. Djouadi and M. Drees, Phys. Lett. B 484 (2000) 183; S. Dawson, *Nucl. Phys. B359*, 283 (1991); M. Spira et al, *Nucl. Phys. B*453, 17 (1995).
20. T.P. Cheng, *Phys. Rev. D* 38, 2869 (1988); H.Y. Cheng, Phys. Lett. B 219, 347 (1989).
21. M.T. Ressell et al., *Phys. Rev. D* 48, 5519 (1993);
22. J.D. Vergados and T.S. Kosmas, *Physics of Atomic nuclei*, Vol. 61, No 7, 1066 (1998) (from *Yadernaya Fisika*, Vol. 61, No 7, 1166 (1998).
23. P.C. Divari, T.S. Kosmas, J.D. Vergados and L.D. Skouras, *Phys. Rev. C* 61 (2000), 044612-1.
24. J.D. Vergados, *Phys. Rev. D* 58, 103001-1 (1998).
25. J.D. Vergados, *Phys. Rev. Lett* 83, 3597 (1999).
26. J.D. Vergados, *Phys. Rev. D* 62, 023519 (2000).
27. J.D. Vergados, *Phys. Rev. D* 63, 06351 (2001).
28. K.N. Buckland, M.J. Lehner, G.E. Masek, in Proc. 3rd Int. Conf. on *Dark Matter in Astro- and part. Phys.* (Dark2000), Ed. H.V. Klapdor-Kleingrothaus, Springer Verlag (2000).
29. *CDF Collaboration*, FERMILAB-Conf-99/263-E CDF: http://fnalpubs.fnal.gov/archive/1999/conf/Conf-99-263-E.html.
30. P.J. Dorman, *ALEPH Collaboration* March 2000, http://alephwww.cern.ch/ALPHUB/seminar/lepcmar200/lepc2000.ps.