Shortcuts in particle production in a toroidal compactified spacetime

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We investigate the particle production in a toroidal compactified spacetime due to the expansion of a Friedmann cosmological model in $\mathbb{R}^3 \times S^1$ outside a $U(1)$ local cosmic string. The case of a Friedmann spacetime is also investigated when torsion is incorporated in the connection. We present a generalization to toroidal compactification of $p$ extra dimensions, where the topology is given by $\mathbb{R}^4 \times T^p$. Some implications are presented and discussed. Besides the dynamics of spacetime, we also investigate in details the physical consequences of the topological transformations.

PACS numbers: 11.25.Mj, 11.27.+d, 42.50.Lc,

Introduction

Cosmic strings are a very interesting class of topological defects, which arise from spontaneous symmetry breaking, and could be generated by early universe phase transitions \[1,2,3\]. Depending on the broken symmetry, the type of topological defect may change. In particular, cosmic strings are related to the non-triviality of the first homotopy group of broken symmetry \[1\]. At the same time, up to now there is no consistent physical law that determines the topological shape of universe. So, a wide range of possibilities can be experimented, at least, theoretically.

Here we investigate some shortcuts concerned to the particle production due to an expansion of the universe given by the following metric ($G = c = h = 1$)

$$ds^2 = a(t)(-dt^2 + dr^2 + B^2r^2d\theta^2 + dz^2), \quad (1)$$

where the $z$ direction is compactified in a circle — $S^1$ topology — and the string, chosen to be along this direction, has a finite length $L$. In some sense, it is a complement of what was done in \[2\] and we shall refer to it latter.

The line element in Eq.\[1\] can be derived from Einstein’s equations with source given by the energy-momentum \textit{ansatz} \[\[\]

$$T_{\mu\nu} = \mu \text{diag} (1, 0, 0, 1), \quad (2)$$

where $\mu$ is the string linear density. For instance, this \textit{ansatz} stands for a straight string and the metric in Eq.\[1\] is obtained from Eq.\[2\] in linear gravity. However, it is possible to show that the form of the metric given in Eq.\[1\] is still valid for full Einstein’s equations, i.e., if gravity is input in the standard gauge Higgs theory, and Einstein’s equations are solved using the scalar field as source. In Eq.\[1\] the parameter $B$ is related to $\mu$ by $B = 1 - 4\mu$, and stands for the well known deficit angle of the cosmic string. In this paper we consider the GUT scale ($\mu \sim 10^{-6}$). It is clear that when $B \rightarrow 1$ — or correspondingly when $\mu \rightarrow 0$ — the metric in Eq.\[1\] is led to the Friedmann standard metric.

The last peculiarity of Eq.\[1\] we want to emphasize is the $S^1$ topology \[3\]. Our coordinate system obeys the equivalence class constraint $(t, r, \theta, z) = (t, r, \theta, z + mL)$, where $m$ is an integer. After all, Eq.\[1\] represents the line element outside a straight cosmic string in $r = 0$, parallel to the $z$-axis in a Friedmann cosmological model with $S^1 \times \mathbb{R}^3$ topology.

In what follows we also analyze the particle production due to an expansion in such spacetime, including the case where torsion is present, in the context of Eq.\[1\], in order to solve Klein-Gordon equation associated with a massless field in a Riemann-Cartan compactified spacetime. The investigation of this more general case is important, since torsion gravity theory can be considered as one of the most natural extensions of General Relativity \[5,6,7\] and, in the particular teleparallel connection we will use, the scalar field \textit{does} couple to torsion \[8,9\]. Also, recently there has been discovered a spin-1/2 matter field with mass dimension one, called ELKO spinor fields \[10,11,12\], which is closely related to standard scalar field theory presenting self-interaction term. Such class of spinor fields are prime candidates for dark matter and inflation \[12\]. Further, a global torsion field is one possible candidate for the CPT and Lorentz violation pa-
rameters [13]. Another great motivation to incorporate
torsion in this formalism is that it can be thought as
being a fundamental propagating field, which is charac-
terized by torsion mass and the values of the coupling
between it and fermions [3, 14]. It is well known that
effective quantum theory put severe restrictions on the
torsion parameters [14], and it could impose significant
characteristics concerning the geometric structures en-
dowing spacetime.

The analysis is generalized to the $\mathbb{R}^3 \times T^p$ topology,
where $T^p$ denotes the $p$-torus $S^1 \times S^1 \times \cdots \times S^1$. Although
the influence of the cosmic string background on the
particle production was investigated in, e.g., [13, 16, 17] we
are interested here in the way that the unusual topol-
ogy — achieved by $S^1$ and $T^p$ compactification — can
intervenes in this process.

The article is presented as follows: In Section I we
analyze the particle production using the solutions of the
Klein-Gordon equation to calculate the Bogoliubov coef-
cients. This paradigmatic Section establishes the stan-
dard tools for the subsequent analysis. In Section II
the case where torsion is incorporated in the connection
in investigated in details, and finally in Section (III) the
case without torsion is generalized for a $p$-dimensional
toroidal compactification.

I. PARTICLE PRODUCTION

When spacetime is static, even in a curved background
it is easy to define the vacuum state of the system. This
is because it is trivial to find a time-like Killing vector
that generates an one-parameter Lie group of isometries,
and so the vacuum together with all the Fock space can
be defined [15]. Let us analyze the particle production of
a scalar massless field in our model due to an expansion
of the universe, i.e., corresponding to a functional form
to $a(t)$ in Eq. (1). We should emphasize that this scalar
field have no relation to the Higgs-like scalar field that
generates the string.

The $in$ and $out$ states are defined by the static back-
ground (cosmic string) and the expansion shall be im-
plemented by a special function in the conformal factor.
As expected, the nontrivial topology implies a change in
physical events, and the procedure is very similar to the
one found in [15, 16].

The Klein-Gordon equation for the massless scalar field
$\phi = \phi(t, r, \theta, z)$ is

$$\left(\frac{\partial^2}{\partial t^2} + \frac{1}{a} \frac{\partial t}{\partial r}  - \frac{1}{r} \partial_r (r \partial_r ) - \frac{1}{B^2 r^2} \partial_\theta^2 - \partial_z^2\right) \phi = 0. \tag{3}$$

Separation of variables gives $\phi(x^\mu) = R(r) e^{i\lambda z} e^{i\alpha \theta} T(t)$, where
$T(t)$ and $R(r)$ respectively satisfy the following equations:

$$\frac{d^2 T(t)}{dt^2} + \frac{\dot{a}}{a} \frac{dT(t)}{dt} + w^2 T(t) = 0 \tag{4}$$

and

$$\frac{d^2 R(r)}{dr^2} + \frac{1}{r} \frac{dR(r)}{dr} + \frac{\alpha^2 R(r)}{B^2 r^2} + (\lambda^2 - w^2) R(r) = 0, \tag{5}$$

where $\dot{a} = da/dt$, and $w$ is a separation constant.

Note that $\alpha$ and $\lambda$ obys periodical constraints due to
the topology of the cosmic string and $S^1$, respectively.
Dirichlet boundary conditions are imposed at $r = \tilde{r}$
to keep the produced energy in a limited region [17]. Then
the solution of Eq. (5) is given by

$$R(r) = \frac{J_\nu(\sqrt{\lambda^2 - w^2} r)}{J_\nu(\sqrt{\lambda^2 - w^2} \tilde{r})} \frac{Y_\nu(\sqrt{\lambda^2 - w^2} r)}{Y_\nu(\sqrt{\lambda^2 - w^2} \tilde{r})}, \tag{6}$$

where $\nu = \alpha/B$. Besides, the solution on the string is well
defined by imposing an additional vanishing boundary
condition at $r = r_0$, i.e., we impose a restriction on the
system. The solution is, then, valid in the range $r_0 < r < \tilde{r}$,
and the values of $w$ can be found by the transcendental
equation

$$J_\nu(Cr_0)Y_\nu(C\tilde{r}) - J_\nu(C\tilde{r})Y_\nu(Cr_0) = 0, \tag{7}$$

arising from the boundary conditions, where $C = \sqrt{\lambda^2 - w^2}$
and the roots are labeled by $k \in \mathbb{N}$. To find the
Bogoliubov coefficients we need to know the function $T(t)$ in the in and out regions. It is briefly recalled here,
and for more details see [18]. After a change in time
coordinate by $\tau = \int \frac{dt}{a(t)}$ we have

$$\frac{d^2 T(\tau)}{d\tau^2} + w^2 a^2(\tau) T(\tau) = 0, \tag{8}$$

leading to a well known case [15]. Now, suppose a smooth
expansion for this universe, e.g., $a(\tau) = \left(\frac{\Omega}{\rho} \sqrt{2} + 1 + (\Omega^2 - 1) \tanh(\rho \tau)\right)^{1/2}$, where $\Omega$ is a constant and $\rho$
gives the rate of expansion. Finally the solution is given in terms of
hyperbolic and hypergeometric functions

$$T_{in}(\tau) = \frac{1}{4\pi}(4\pi w_{in})^{-1/2} \times \exp (-iw_{in} \tau - i \frac{w_{in}}{\rho} \ln [2 \cosh(\rho \tau)]) \times 2F_1 \left(1 + i \frac{w_{in}}{\rho}, i \frac{w_{in}}{\rho}; 1 + i \frac{w_{in}}{\rho}; \frac{1}{2}(1 \pm \tanh(\rho \tau)) \right)$$

where $w_{in} = w$, $w_{out} = \rho \Omega$ and $w_{\pm} = \frac{w}{2} (\Omega \pm 1)$. The
linear transformations of hyperbolic functions give us the
desired relation between in and out states

$$\phi_{\lambda, \alpha, k}' = \gamma(\lambda, \alpha, k) \phi_{\lambda, \alpha, k} + \beta(\lambda, \alpha, k) (\phi_{\lambda, \alpha, k}^*)'$$

The terms $\gamma$ and $\beta$ above are the so-called Bogoliubov
coefficients given, in our case, by

$$\gamma(\lambda, \alpha, k) = \frac{\Gamma[1 - i w_{in}/\rho] \Gamma(-iw_{out}/\rho)}{\Gamma(-iw_{out}/\rho) \Gamma(1 - i w_{in}/\rho)} \tag{9}$$
and

\[ \beta(\lambda, \alpha, k) = \Omega \frac{\Gamma[1 - i\omega_m/\rho]\Gamma(i\omega_{out}/\rho)}{\Gamma(i\omega_+/\rho)\Gamma[1 + i\omega_+/\rho]}. \]  

(10)

According to the usual theory in curved spaces, the density of created particles per mode is \( |\beta(\lambda, \alpha, k)|^2 \), i.e.,

\[ |\beta(\lambda, \alpha, k)|^2 = \frac{\sin^2(\pi/\omega_+) \sinh(\pi/\omega_{in}) \sinh(\pi/\omega_{out})}{\sinh(\pi/\omega_{in}) \sinh(\pi/\omega_{out})}. \]  

(11)

We should remark that the effects of nontrivial topology are reflected on the excitation modes. In Eq.(11) the modes assigned by \( \alpha \) must obey the periodic constraint

\[ \alpha = 2\pi n. \]  

(12)

Besides, the \( S^1 \) compactification in the \( z \) coordinate implements another constraint, this time in \( \lambda \), since the string have a finite length \( L \) and these modes have a discrete spectrum

\[ \lambda = \frac{2\pi n}{L} \]  

(13)

where \( n, m \in \mathbb{Z} \). Note that this is an important result: the geometry of the source, as well as of the compactified dimension, modifies the mode of excitation from continue to discrete modes.

II. THE FORMALISM IN A RIEHMANN-CARTAN SPACETIME

The Klein-Gordon equation for the massless scalar field in a Friedmann Riemann-Cartan spacetime, which geometry is endowed with a connection possessing a non null torsion tensor \( T^\nu_{\rho \lambda} \), is given by

\[ \frac{1}{\sqrt{|g|}} \partial_{\mu}(\sqrt{|g|}g^{\mu\nu}\partial_{\nu}\phi) + T^\nu_{\rho \lambda}\partial_\lambda \phi = 0. \]  

(14)

The metric in this case has the same form of Eq.(11), but it must be emphasized that to the connection a new term, corresponding to the torsion, must be added. It can be shown that this new solution of Einstein’s equation is diffeomorphic to the one found in Eq.(11), and since we are interested in the topological transformations, they are indeed equivalent.

After separation of variables, only the radial and temporal functions are modified by torsion effects, and we get

\[ \phi(t, r, \theta, z) = T(t)R(r)\Theta(\theta)Z(z), \]  

(15)

where

\[ R(r) = \frac{1}{\sqrt{1 - (C_1^2 + C_2^2) (z_n - z_{out})^2}} \]  

(16)

and in the particular case where the integration constant \( n \) is zero, \( R(r) \) reads

\[ R(r) = A_1 \exp(\alpha r) + A_2 \exp(-\alpha r). \]  

(17)

We have to choose in Eq.(10) the term which is regular at the origin, and the Dirichlet boundary conditions are implemented by the constants \( C_1 [A_1] \) and \( C_2 [A_2] \) for any value of \( n [n = 0] \).

The temporal equation is given by

\[ \frac{\partial^2 T(t)}{\partial t^2} + 2\dot{a}(t)\sqrt{a(t)} + \frac{\partial T(t)}{\partial t} + a(t)\omega^2 T(t) = 0, \]  

(18)

where \( \omega \) is a separation constant. Note that there is an extra term arising from torsion effects. There is not, in general, an analytical function which is a solution of the equation above. In fact, the set of the solutions of Eq.(18), that are not analytical explicit functions, is dense. However, in some particular cases of \( a(t) \) the solution is an analytical explicit function. We are concerned with a particular class of analytical solution \( a(t) \) which is a smooth expansion and satisfies \( 0 < b = \lim_{t \to \pm \infty} a(t) \), in order to the \( in \) and \( out \) states to make sense — again the system is restricted to a box. For instance, the solution when \( a(t) = (t^2 - 1)/(t^2 + 1) \) is given by

\[ T(t) = A \cos(f(t)^{1/4} \exp[2f(t)t] + \delta) \]  

(19)

where \( \delta \) is a phase and \( f(t) = \sqrt{\frac{t^2 + a}{t^2 + b}} \) (\( c = \text{cte.} \)). Also,

\[ \Theta(\theta) = D_1 \cos(\alpha \theta) + D_2 \sin(\alpha \theta) \]  

(20)

and

\[ Z(z) = \exp(i\lambda z), \]  

(21)

where the condition \( Z(z) = Z(z + mL) \) — in order to accomplish the toroidal compactification — is implicit in the form of Eq.(21). The Bogoliubov coefficient is given by \( (\phi_{in}, \phi_{out}) \) — the internal product between \( \phi_{in}, \phi_{out} \). Its explicit calculation is beyond the scope of the present article. Nevertheless, it is clear from the solutions that the geometry also shall transform the \( \lambda \) and \( \alpha \) modes in discrete ones.

III. COMPACTIFIED EXTRA-DIMENSIONAL CASE

Once we have introduced the formalism for one compactified dimension, the generalization to extra dimensions is immediate. First of all, we introduce a topological structure with cylindrical symmetry in the hypersurface \((t, z_1, \ldots, z_p)\) constant. Our interest resides in the topological transformations of spacetime. We start with the following line element

\[ ds^2 = a(t) \left( -dt^2 + dr^2 + B^2 r^2 d\theta^2 + \frac{b(t)}{a(t)} dz_i dz_i \right), \]  

(22)
where \( i = 1, \ldots, p \). Now the topology is \( \mathbb{R}^3 \times T^p \) and we can investigate some shortcuts in particle production from a more generalized compactification formalism. Note that we are not considering a cloud of strings in the sense of [19]. Instead, we just deal with an unusual topology without any string in extra dimensions. Suppose that the universe, after the expansion analyzed before, passes to another expansion behavior, now characterized just by the dimensions of the \( p \)-torus. In other words, suppose that in a first moment \( b(t) = a(t) \), and all previous analysis (Sec. II) is still valid. After that, in \( t = t_0 \), consider an expansion in the extra dimensions and in the \( z \) direction. The Klein-Gordon equation for the field \( \phi = R(r)\Theta(\theta)T(t)Z(z_1, z_2, \ldots, z_p) \) in the geometry given by Eq. (22) reads

\[
\partial_t^2 \phi + \frac{1}{2} \frac{b}{b} \frac{\dot{b}}{b} \partial_t \phi + \frac{1}{r} \left[ \partial_r (r \partial_r) \right] \phi + \frac{1}{r^2} \frac{\dot{b}}{b} \partial_r \phi + \frac{\alpha(t)}{b(t)} \partial_{r} \phi = 0. \tag{23}
\]

Note that if \( b(t) = a(t) \) and \( p = 1 \) we recover the previous situation. If \( b(t) = a(t) \) and \( p \neq 1 \) the generalization is immediate: we obtain, after separation of variables in Eq. (23), the solution \( Z(z_1, z_2, \ldots, z_p) = \prod_{i=1}^{p} \exp(\lambda^i z_i) \). Of course, there will be another classification for each root of Eq. (4).

Now consider the case where \( a = \Omega \) and \( b(t) \) simulate an expansion until \( t = \tilde{t} \). Then, in the range \( t_0 < t < \tilde{t} \), after the expansion in all universe (\( a(t) \)) we have

\[
\frac{d^2 T}{dt^2} + \frac{1}{2} \frac{b}{b} \frac{\dot{b}}{b^2} T + \left( \frac{\Omega}{b} \lambda^2_1 + w^2 \right) T = 0 \tag{24}
\]

and

\[
\frac{d^2 R}{d \tau^2} + \frac{1}{r} \frac{dR}{d\tau} + \left( \frac{w^2}{r^2} - \frac{\alpha^2}{r^2 B^2} \right) R = 0. \tag{25}
\]

The separation of variables is slightly different from previous cases, and now \( t_0 \) and \( \tilde{t} \) give the asymptotic solutions. The solution for Eq. (25) is given by

\[
R(t) = C_1 J_{\langle \alpha/B \rangle}(wr) + C_2 Y_{\langle \alpha/B \rangle}(wr), \tag{26}
\]

where \( C_1 \) and \( C_2 \) are constants to be determined, while Eq. (24) gives, for a expansion implemented for instance by \( \exp(b_0 t) \), the solution

\[
T(t) = e^{-\frac{b_0 \bar{\nu}}{2}} \left[ \frac{J_{\bar{\nu}} \left( \frac{\xi e^{-\frac{b_0 \bar{\nu}}{2}}}{\bar{\nu}} \right)}{Y_{\bar{\nu}} \left( \frac{\xi e^{-\frac{b_0 \bar{\nu}}{2}}}{\bar{\nu}} \right)} - \frac{Y_{\bar{\nu}} \left( \frac{\xi e^{-\frac{b_0 \bar{\nu}}{2}}}{\bar{\nu}} \right)}{J_{\bar{\nu}} \left( \frac{\xi e^{-\frac{b_0 \bar{\nu}}{2}}}{\bar{\nu}} \right)} \right], \tag{27}
\]

where \( \bar{\nu} = -1/2 \sqrt{b_0^2 - 16w^2/b_0^2} \) and \( \xi = \frac{2\lambda^1}{b_0} \).

Again, by using some appropriate boundary condition, the Bogoliubov coefficient that supplies the vacuum excitation can be calculated. This last model is nothing but a toy model, however it may serve as a good laboratory to analyze some combined effects of extra dimensions and an unusual topology.

We emphasize that in this context \( b(t) \) is not understood as a radion field [20], since it has not a continuum behavior in time.

IV. CONCLUDING REMARKS AND OUTLOOKS

We have investigated how a topology, where one or more dimensions are compactified, can present signatures in a specific physical aspect, like particle production. It can be realized that the topology generated by the cosmic string is codified in the deficit angle and in the cylindrical symmetry, while the \( S^1 \) compactification is realized in the transformation from continuous modes to discrete ones. A general analysis of the Klein-Gordon massless field in the context of the Riemann-Cartan geometry is accomplished, formally ELKO Lagrangian is very similar to scalar field Lagrangian. ELKO spinor fields obey scalar field-like equations and it is shown in [10, 12] that they are prime candidates for inflation and dark matter. ELKO spinor fields are also more sensitive to the spacetime torsion [12], and from the formal viewpoint Section III gives the essential pre-requisites to investigate the relationship between particle production in a Riemann-Cartan Friedmann spacetime and the fields that are prime candidates to describe dark matter.

We also investigated, in Section III, the generalization of extra-dimensional toroidal-compactified models. One interesting particular case, when the topology is given by \( \mathbb{R}^3 \times T^2 \), can introduce a Kaluza-Klein theory over AdS spacetime, since \( \mathbb{R}^3 \times T^2 \simeq \mathbb{R}^3 \times S^1 \), and \( \mathbb{R}^3 \times S^1 \) corresponds to the AdS topology. In this topology it is also possible to use all the results arising from Eq. (22). In this case obviously the Kaluza-Klein modes are discrete, since we deal with compactified extra dimensions. The cases presented in Sections (II-III) give rise to the most direct generalization of our previous analysis and play an important role in this context, since they have interesting properties in the particle production processes. Vacuum effects in the field of multiple cosmic strings [21] in compactified dimensions shall be analyzed in a forthcoming paper.

Some interesting properties concerning scalar fields in the model shown in Sec. III were studied in [3], like \( \phi^2 \) fluctuations, where \( \phi \) is a massless scalar field. In this sense, the complete characterization of the quantum vacuum requires a deeper understanding about stress tensor vacuum fluctuations. Many problems appear in the formulation of such semiclassical system. Since the basic formalism was developed in the usual topology like in [22, 23], where the field propagator is well established, and the stress tensor computed, similar or analogous results were not achieved for the \( \mathbb{R}^3 \times S^1 \) topology, due to the inherent difficulties of such systems. For instance,
this topology gives origin to ultraviolet divergences, that must be renormalized by some method. Again, old questions on quantum field theory in curved spacetimes return: as 1) how to define a quantum state (in Fock space) in curved global background given by Eq. (1); 2) how to implement a reasonable cutoff from the physical point of view and 3) what kind of physically new and relevant information we can get from this unusual topology.

Finally, by accomplishing a toroidal compactification, there naturally arises a maximal number of covariantly constant spinor fields, since flat tori are the only manifolds with trivial holonomy. Each one of these spinor fields is related to a supersymmetry that remains unbroken by the compactification. Supersymmetric cosmic strings models involving toroidal compactification are beyond the scope of this paper.

V. ACKNOWLEDGMENT

Roldão da Rocha thanks to Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP) (PDJ 05/03071-0) and J. M. Hoff da Silva thanks to CAPES-Brazil for financial support.

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