Weak decay of swirling protons and other processes

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We investigate the weak-interaction emission of spin-1/2 fermions from decaying (and non-decaying) particles endowed with uniform circular motion. The decay of swirling protons and the neutrino-antineutrino emission from circularly moving electrons are analyzed in some detail. The relevance of our results to astrophysics is commented.

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I. INTRODUCTION

Recently the decay of uniformly accelerated protons as described from the point of view of inertial and coaccelerated observers was used as a paradigmatic example of the necessity of the Fulling-Davies-Unruh effect to the consistency of Quantum Field Theory. At the same time, it was shown that in addition to its conceptual importance, the decay of accelerated protons could be of “practical” relevance to astrophysics. It was estimated that about 1% of a bunch of protons with energy $10^{14}$ eV would decay (through weak interaction) if they were under the influence of a magnetic field of $10^{14}$ Gauss of a pulsar. The proton decay can be understood in this case as being induced by the centripetal force acting on the proton as it swirls around the magnetic field lines. The estimative above was obtained, however, by using the decay rate of uniformly accelerated protons rather than circularly moving ones. It was argued that this procedure should lead to good approximate results as far as the proton proper acceleration satisfies the constraint $a \gg \Delta M / R$, where $\Delta M \equiv M_n - M_p$ is the neutron-proton mass difference and $R$ is the local curvature radius of the proton trajectory.

Thus, as a step further, it would be desirable to refine our previous estimative by considering protons in circular motion indeed. For this purpose, we apply the formalism developed in Ref. (designed to study the weak-interaction emission of spin-1/2 fermions from classical and semiclassical currents) to the case of circularly moving particles with constant velocity (hereafter denominated uniformly swirling particles). We focus on the decay of uniformly swirling protons and on the neutrino-antineutrino emission from uniformly swirling electrons which is also relevant in some astrophysical situations as, e.g., in the cooling of neutron stars and in connection with high-energy neutrinos emitted from the magnetic

II. FORMALISM

Let us consider the following class of processes

$$p_1 \rightarrow p_2 \ f_1 \ f_2,$$

(2.1)

where a fermion-antifermion pair $f_1,f_2$ is emitted as the particle $p_1$ is supposed to evolve into the particle $p_2$. The $f_1,f_2$, $p_1$, and $p_2$ rest masses are $m_1$, $m_2$, $M_1$, and $M_2$, respectively. We will be interested here in cases where $m_1,m_2 \ll M_1,M_2$. The fermion emission will be assumed not to change significantly the four-velocity of $p_2$ with respect to $p_1$. This is called “no-recoil condition”, which is verified when the momentum of the emitted fermions (with respect to the inertial frame instantaneously at rest with particle $p_1$) satisfies $|k| \ll M_1,M_2$. Because $m_1,m_2 \ll M_1,M_2$, this implies that the energy of each emitted fermion satisfies $\omega \ll M_1,M_2$. As the typical energy $\omega$ of the emitted fermions is of the order of the proper acceleration $a$ of the particle $p_1$, the no-recoil condition can be recast as

$$a \ll M_1,M_2.$$

(2.2)
The particles $p_1$ and $p_2$ will be seen as distinct energy eigenstates $|p_1\rangle$ and $|p_2\rangle$, respectively, of a two-level system. The associated proper Hamiltonian $\hat{H}_0$ of the particle system satisfies, thus,

$$\hat{H}_0 |p_j\rangle = M_j |p_j\rangle, \quad j = 1, 2.$$  \hfill (2.3)

Hence, we describe our pointlike particle system by the semiclassical (vector) current

$$\hat{j}^\mu(x) = \hat{q}(\tau) \left[ u^\mu(\tau)/u^0(\tau) \right] \delta^3(x - x(\tau)), \quad (2.4)$$

where $x^\mu(\tau)$ is the classical world line associated with the particle system $p_1$-$p_2$, $u^\mu(\tau) \equiv dx^\mu/d\tau$ is the corresponding four-velocity, and $\hat{q}(\tau) \equiv e^{i\hat{H}_0\tau} \hat{q}_0 e^{-i\hat{H}_0\tau}$, where $\hat{q}_0$ is a self-adjoint operator evolved by the one-parameter group of unitary operators $\hat{U}(\tau) = e^{-i\hat{H}_0\tau}$.

Each emitted fermion will be associated with a spinorial field written as

$$\hat{\Psi}(x) = \sum_{\sigma = \pm, \sigma_1, \sigma_2} \int d^3k \left[ \hat{b}_{k\sigma} \hat{\psi}^{(+\omega)}_{k\sigma}(x) + \hat{d}_{k\sigma}^\dagger \hat{\psi}^{(-\omega)}_{-k\sigma}(x) \right], \quad (2.5)$$

where $\hat{b}_{k\sigma}$ and $\hat{d}_{k\sigma}^\dagger$ are annihilation and creation operators of fermions and antifermions, respectively, with three-momentum $k = (k^x, k^y, k^z)$, energy $\omega = \sqrt{k^2 + m^2}$ and polarization $\sigma$, and $\psi^{(+\omega)}_{k\sigma}$ and $\psi^{(-\omega)}_{-k\sigma}$ are positive and negative frequency solutions of the Dirac equation $i\gamma^\mu \partial_\mu \psi^{(\pm\omega)}_{k\sigma} - m_\sigma \psi^{(\pm\omega)}_{k\sigma} = 0$.

Next, we minimally couple the spinorial fields $\hat{\Psi}_1$ and $\hat{\Psi}_2$ (associated with the two emitted fermions $f_1$-$f_2$, respectively) to our semiclassical current $\hat{j}^\mu$ (that describes the particle system $p_1$-$p_2$) according to the weak-interaction action $10$-$11$

$$\hat{S}_I = \int d^4x \hat{j}_\mu \left\{ \hat{\Psi}_1 \gamma^\mu(c_V - c_A\gamma^5)\hat{\Psi}_2 + \hat{\Psi}_2 \gamma^\mu(c_V - c_A\gamma^5)\hat{\Psi}_1 \right\}, \quad (2.6)$$

where $c_V = c_A = 1$ in the cases here analyzed.

The transition amplitude for the process (2.1) at the tree level is given by

$$\mathcal{A}_{f_1 f_2} = \langle f_1 | \mathcal{O} \otimes \langle f_2 | \hat{S}_I | p_1 \rangle \otimes | p_1 \rangle, \quad (2.7)$$

and the differential transition probability is

$$\frac{d\mathcal{P}_{f_1 f_2}}{d^3k_1 d^3k_2} = \sum_{\sigma_1 = \pm, \sigma_2 = \pm} |\mathcal{A}_{f_1 f_2}|^2, \quad (2.8)$$

which leads to (see Ref. 3 for details)

$$d\mathcal{P}_{f_1 f_2} = \frac{2 G_{\text{eff}}^2}{(2\pi)^6 \omega_1 \omega_2} \int_{-\infty}^{+\infty} d\tau \int_{-\infty}^{+\infty} d\tau' e^{i\Delta M(\tau - \tau')} e^{i(k_1 + k_2)\cdot[x(\tau) - x(\tau')]} \times \left\{ \left[ 2k_1^\mu k_2^\nu + i e^{\mu\nu\alpha\beta} k_{1\alpha} k_{2\beta} \right] u_\mu(\tau)u_\nu(\tau') - k_1^\alpha k_2^\beta u^\mu(\tau)u_\mu(\tau') \right\}, \quad (2.9)$$

where $e^{\mu\nu\alpha\beta}$ is the totally skew-symmetric Levi-Civita pseudo-tensor (with $e_{123}^{123} = -1$), $k_1^\alpha k_2^\beta \equiv (k_1^x k_2^x + k_1^y k_2^y)/2$, $\Delta M \equiv M_2 - M_1$ and $G_{\text{eff}} \equiv |\langle p_2 | q_0 | p_1 \rangle|$ is the effective coupling constant.

In those situations where the particle is accelerated by a background electromagnetic field, a full quantum-mechanical investigation would be, in principle, possible. In this case, any recoil effects associated with the fermion emission would be automatically taken into account. For instance, in Ref. 12 the quasiclassical approach was developed to consider $\gamma$-synchrotron radiation from an electron immersed in a classical background magnetic field with intensity $H \ll H_0 = 4.4 \times 10^{13}$ Gauss with the electron Lorentz factor satisfying $\gamma \gg 1$. A similar approach was applied to the neutrino-antineutrino emission in Sec. 6.1 of Ref. 13. Another very promising approach, which could be adapted to the present case, was recently developed by Higuchi who investigated the radiation reaction effect on accelerated charged in the context of Quantum Field Theory 13. In this vein, further developments of our formalism to naturally take into account back-reaction effects would be welcome. In spite of this, our semiclassical approach has the advantage of being applicable to a quite general class of processes irrespective to the acceleration source origin: electromagnetic, gravitational or some other one. Moreover, it agrees with the full quantum mechanical treatment used in the aforementioned cases when the no-recoil condition is satisfied (i.e., $\chi \ll 1$ in Refs. 12-13). Hence, our approach and the other ones in the literature 4-14 should be seen as complementing each other.

III. UNIFORMLY SWIRLING CURRENTS

The world line of a particle with uniform circular motion with radius $R$ and angular velocity $\Omega$, as defined by observers at rest in an inertial frame associated with inertial coordinates $(t, x)$, is

$$x^\mu(\tau) = (t, R\cos(\Omega t), R\sin(\Omega t), 0), \quad (3.1)$$
and the corresponding four-velocity is
\[ u^\mu(\tau) = \gamma(1, -R\Omega \sin(\Omega t), R\Omega \cos(\Omega t), 0) , \] (3.2)
where \( \gamma \equiv (1 - R^2\Omega^2)^{-1/2} = constant \) is the Lorentz factor (\( v \equiv R\Omega < 1 \)), \( t = \gamma \tau \), and \( a = \sqrt{-a^\mu a^\nu} = R\Omega^2\gamma^2 \) is the proper acceleration.

In order to decouple the integrals in Eq. (2.9), we define new coordinates,
\[ \sigma \equiv \gamma(\tau - \tau')/2 \quad \text{and} \quad s \equiv \gamma(\tau + \tau')/2 , \] (3.3)
and perform the change in the momentum variable
\[ k^\mu \rightarrow \tilde{k}^\mu = (\tilde{\omega}, \tilde{k}) , \] (3.4)
where
\[ \tilde{\omega} = \omega , \]
\[ \tilde{k}^x = k^x \cos(\Omega s) + k^y \sin(\Omega s) , \]
\[ \tilde{k}^y = -k^x \sin(\Omega s) + k^y \cos(\Omega s) , \]
\[ \tilde{k}^z = k^z , \]
which consists of a rotation by an angle \( \Omega s \) around the \( k^z \) axis. Hence, we obtain from Eq. (2.9) the following transition rate per momentum-space element of each emitted fermion:
\[
\frac{d\Gamma^p_{1} \rightarrow p_{2}}{d^3k_1 d^3k_2} = \frac{2 \gamma G_{\text{eff}}^2}{(2\pi)^6} \tilde{\omega}^2 \int_{-\infty}^{+\infty} d\sigma \exp \left( i \left( \Delta M \sigma / \gamma + (\tilde{k}_1 + \tilde{k}_2)^\mu X_\mu(\sigma) \right) \right) \]
\[ \times \left[ (\tilde{\omega}_1 \tilde{\omega}_2 + \tilde{k}_1 \cdot \tilde{k}_2) - R^2\Omega^2(\tilde{k}_1^2 \tilde{k}_2^2 - \tilde{k}_1^y \tilde{k}_2^y) + R^2 \Omega^2(\tilde{\omega}_1 \tilde{\omega}_2 - \tilde{k}_1^y \tilde{k}_2^y) \cos(\Omega \sigma) 
- 2 R\Omega (\tilde{\omega}_1 \tilde{k}_2^y + \tilde{\omega}_2 \tilde{k}_1^y) \cos(\Omega \sigma/2) 
- i R^2 \Omega^2 (\tilde{\omega}_1 \tilde{k}_2^z - \tilde{\omega}_2 \tilde{k}_1^z) \sin(\Omega \sigma) \right] , \] (3.5)

where
\[ G_{\mu\nu} \equiv -\frac{\partial I_l}{\partial X^\mu} \frac{\partial I_l}{\partial X^\nu} \] (3.8)
with
\[ I_l \equiv \int d^3k_l e^{i\tilde{k}_l X} / \tilde{\omega}_l , \quad l = 1, 2 , \] (3.9)
and \( \tilde{\omega}_l = \sqrt{\tilde{k}_l^2 + m_l^2} \), and
\[ \tilde{k}_l = k_l \cos \theta_l . \]
By doing so, we obtain
\[ I_l = \frac{4\pi}{|X|} \int_{m_l}^{+\infty} d\tilde{\omega} e^{i\tilde{\omega} X} \sin \left( \sqrt{\tilde{\omega}_l^2 - m_l^2} |X| \right) , \]
where \( |X| \equiv \sqrt{-X_1 X^1} \). Next, by redefining the fre-
quency variable as \( \tilde{\omega}_i \equiv m_i \cosh \xi \), we obtain

\[
I_l = \frac{-2\pi im_l}{|X|} \int_{-\infty}^{+\infty} d\xi e^{im_l(X^0 \cosh \xi + \sqrt{X^2 \sinh \xi})} \sinh \xi.
\]

Now, we perform the change of variable \( \xi \rightarrow \eta \equiv e^\xi \), leading to

\[
I_l = \frac{i\pi m_l}{|Y|} \int_0^{+\infty} d\eta (\eta^{-2} - 1) \exp \left[ \frac{im_l(Y^0 + |Y|)\eta}{2} + \frac{im_l(Y^0 - |Y|)}{2\eta} \right],
\]

where we have introduced a small positive regulator \( \epsilon > 0 \) in the integral as follows:

\[
X^\mu \rightarrow Y^\mu = (X^0 + i\epsilon, X^1, X^2, X^3).
\]

(Note that \( \text{Re}(Y^0) = |X^0| > |X| = |Y| \). Then, by using expressions (3.471.11) and (8.484.1) of Ref. [15], we obtain

\[
I_l = -\frac{2\pi^2 i m_l \text{sign}(\sigma) \mu(Y^0 \sqrt{Y^\mu Y_\nu})}{\sqrt{\mu(Y^0 \sqrt{Y^\mu Y_\nu})}},
\]

where \( \mu(Y^0 \sqrt{Y^\mu Y_\nu}) \) is the Hankel function of the first kind. As a result, by making the variable change \( \sigma \rightarrow \lambda \equiv -a\sigma/\gamma \) and by defining \( Z^\mu \equiv (a/\gamma)Y^\mu \), the transition rate (3.7) can be cast in the form (see also expression 8.472.4 in Ref. [15]).

\[
\Gamma_{p_1 \rightarrow p_2} = \frac{G^2e^{2\mu \delta a}}{8\pi^2} \int_{-\infty}^{+\infty} d\lambda e^{-i\Delta M/\lambda} Z^\mu \Delta^\nu A_{\mu\nu} \frac{H_2^{(1)}(z_1)}{z_1^2} \frac{H_2^{(1)}(z_2)}{z_2^2},
\]

where we have defined \( \Delta \equiv m_1/a, \Delta M \equiv \Delta M/a, \epsilon' \equiv a\epsilon/\gamma \ll 1, \epsilon \equiv -m_1 \text{sign}(\lambda) \sqrt{\Delta M \lambda}, \) and where \( Z^\mu = (-\lambda + i\epsilon', 0, -2 \gamma a/\gamma \sin(\Omega \lambda/2a), 0) \) with

\[
A_{\mu\nu} = \begin{bmatrix}
1 + R^2 \Omega^2 \cos(\Omega \lambda/\gamma) & 0 \\
0 & 1 - R^2 \Omega^2 \\
-2R \Omega \cos(\Omega \lambda/2a) & 0 \\
-iR^2 \Omega^2 \sin(\Omega \lambda/\gamma) & 0
\end{bmatrix}
\]

\[
H_2^{(1)}(z) \approx -\frac{4i}{\pi z} - \frac{i}{\pi} + O(z \ln |z|) \quad \text{for} \quad |z| \ll 1.
\]

We note that for \( |\lambda| \) large enough, \( |z| > 1 \), in which case the expansion (3.13) ceases to be a good approximation. For instance, for \( \gamma^2 \gg 1/\Delta \), we have that \( |z| > 1 \) for \( |\lambda| \geq 1/\sqrt{12}\Delta \) (\( l = 1, 2 \)), while for 1/\( \Delta \gg \gamma^2 \geq 1 \), we have that \( |z| > 1 \) for \( |\lambda| \geq 1/(\gamma \Delta) \). Notwithstanding, this will not be important because the error committed in this region will be small to affect the final result provided that \( \Delta \ll 1 \). Hence we write Eq. (3.13) in the form

\[
\Gamma_{p_1 \rightarrow p_2} \approx \frac{-G^2e^{2\mu \delta a}}{8\pi^2} \int_{-\infty}^{+\infty} d\lambda e^{-i\tilde{\Delta} \lambda \lambda} Z^\mu \Delta^\nu A_{\mu\nu} \frac{H_2^{(1)}(z_1)}{z_1^2} \frac{H_2^{(1)}(z_2)}{z_2^2} \left( \frac{16}{\gamma^4(Z^\lambda Z^\lambda)} + \frac{4(\tilde{m}_1^2 + \tilde{m}_2^2)}{\gamma^2 Z^\lambda Z^\lambda} \right),
\]

where

\[
Z^\lambda Z^\lambda = (\lambda - i\epsilon^2) - (2Ra/\gamma)^2 \sin^2(\Omega \lambda/2a).
\]

Eventually, Eq. (3.15) can be seen as the expansion of the reaction rate up to second order in \( \tilde{m}_1 \ll 1 \). In order to solve this integral, we expand \( Z^\lambda Z^\lambda \) for relativistic swirling particles [15-17], i.e., \( \gamma > 1 \) (recall that \( R = v^2 \gamma^2/a, \Omega = a/(v^\gamma), \) and \( v = \sqrt{1 - \gamma^{-2}})).

\[
Z^\lambda Z^\lambda \approx \frac{1}{2\gamma^2}(\lambda + i\sqrt{3}A_+)(\lambda + i\sqrt{3}A_-)(\lambda - i\sqrt{3}B_+)
\times (\lambda - i\sqrt{3}B_-),
\]

where

\[
A \mp \equiv 1 \pm \sqrt{1 + 2\epsilon/\sqrt{3}}
\]

and

\[
B \mp \equiv 1 \pm \sqrt{1 - 2\epsilon/\sqrt{3}}
\]

with \( \epsilon \ll 1 \). For \( |\lambda| \geq 2v^\gamma \), where the expansion ceases to be a good approximation, the integral contributes very
little again and, thus, will not have any major influence in the final result. Thus, the integral in Eq. (3.16) can be rewritten in the complex plane:

$$\Gamma_{p_1 \to p_2} \approx \frac{G_{\text{eff}}^2 a^5}{8\pi^4} \oint_C d\lambda \exp\left(-i \Delta M \lambda \frac{Z\gamma Z' A_{\mu \nu}}{(Z\lambda)^2}\right) \times \left(\frac{16}{\gamma^4 (Z\lambda)^2} - \frac{4(\tilde{m}_1^2 + \tilde{m}_2^2)}{\gamma^2 Z\lambda Z'\gamma}\right), \quad (3.19)$$

where we recall that this is valid for $\tilde{m}_1, \tilde{m}_2 \ll 1$ and $\gamma \gg 1$.

Next, we calculate the radiated power in form of each fermion,

$$W_{l \gamma}^{p_1 \to p_2} \equiv \int d^3 k_1 \int d^3 k_2 \, \omega_l \frac{d\Gamma_{p_1 \to p_2}}{d^3 k_1 d^3 k_2}, \quad (3.21)$$

where the index $l = 1, 2$ is used to distinguish which fermion we are referring to. We write Eq. (3.21) as

$$W_{l \gamma}^{p_1 \to p_2} = \frac{2 G_{\text{eff}}^2}{(2\pi)^6} \int_{-\infty}^{+\infty} d\sigma \, \epsilon^\lambda \Delta M / \gamma H_{\mu \nu} A_{\mu \nu}, \quad (3.22)$$

where we have chosen (with no loss of generality) $l = 1$, i.e., we are computing the radiated power associated with the fermion with mass $m_1$. Here

$$H_{\mu \nu} \equiv - \frac{\partial J_1}{\partial X^\mu} \frac{\partial J_2}{\partial X^\nu}, \quad (3.23)$$

where we recall that $z_1, Z\gamma$ and $A_{\mu \nu}$ are the same ones defined below Eq. (3.15). In order to perform this integral in the limit $\tilde{m}_1 \ll 1$, we use the approximation (3.15) and (3.19) and (see Ref. [17])

$$H_3^{(1)}(z_1) \approx - \frac{16i}{\pi z_1} - \frac{2i}{\pi z_1} - \frac{z_1 i}{4\pi} + O(z_1^2 \ln z_1) \quad (3.27)$$

for $|z_1| \ll 1$. Then, by letting Eqs. (3.15) and (3.27) in Eq. (3.20), we can perform the remaining integral in the complex plane along the path $C \equiv (-L, L) \cup \{ L \epsilon^{i\theta}, \theta \in [-\pi, 0]\}$ with $L \to \infty$, as for the reaction rate, and obtain the emitted power $W_{l \gamma}^{p_1 \to p_2}$. We present below the result for the leading term in $\gamma$ (see Ref. [18]):
\[ W_{1}^{p_{1} \rightarrow p_{2}} \approx \frac{G_{\text{eff}}^{2} e^{-2\sqrt{3} \Delta M}}{3456 \pi^{3}} \left[ 320 + 241 \sqrt{3} \Delta M + 246 \Delta M^{2} + 46 \sqrt{3} \Delta M^{3} + 12 \Delta M^{4} - 48(\tilde{m}_{1}^{2} + 5\tilde{m}_{2}^{2}) - 3\sqrt{3} \Delta M(17\tilde{m}_{1}^{2} + 65\tilde{m}_{2}^{2}) - 18 \Delta M^{2}(5\tilde{m}_{1}^{2} + 13\tilde{m}_{2}^{2}) - 24 \sqrt{3} \Delta M^{3}(\tilde{m}_{1}^{2} + 2\tilde{m}_{2}^{2}) \right] \]

where we recall that this is valid for \( \tilde{m}_{1}, \tilde{m}_{2} \ll 1 \) and \( \gamma \gg 1 \). Clearly, \( W_{2}^{p_{1} \rightarrow p_{2}} \) is obtained by permuting \( m_{1} \leftrightarrow m_{2} \) in Eq. (3.28).

### IV. PROTON DECAY

Seemingly, Ginzburg and Syrovatskii [19] were the first ones to comment about the decay of noninertial protons, but only recently Muller [20] presented the first estimate for the decay rate of the inverse \( \beta \)-decay

\[ p \rightarrow n e^{+} \nu \]  

by assuming that all the particles were scalars. Further, the authors used the semiclassical approach (where the leptons are described by fermionic fields indeed) to calculate the decay rate for uniformly accelerated protons. Here we analyze the case of swirling protons, which can model high-energy protons moving in the magnetosphere of a pulsar.

The effective coupling constant for the inverse \( \beta \)-decay, \( G_{\text{eff}} = G_{p n} \), is obtained by imposing that the mean proper lifetime of inertial neutrons be 887 s [21], i.e.,

\[ \Gamma_{in}^{n \rightarrow p}(\Omega \rightarrow 0) = h/(887 \text{ s}) . \]  

Of course, we cannot use our expression (3.19) in this case since it is not valid when \( a < m_{c} \). Fortunately, however, \( \Gamma_{n \rightarrow p} \) can be integrated for inertial neutrons directly from Eq. (4.2), by making \( \Omega = 0 \) in Eq. (3.20).

This is achieved by a change of the momentum variables as shown in Eq. (3.3). After performing the corresponding integrations in the angular coordinates and in \( \tilde{\omega}_{e} \), we obtain

\[ \Gamma_{in}^{n \rightarrow p} = \frac{G_{p n}^{2}}{\pi^{3}} \int_{0}^{\Delta M - m_{c}} d\tilde{\omega}_{e} \tilde{\omega}_{e}^{2} (\Delta M - \tilde{\omega}_{e}) \times \sqrt{(\Delta M - \tilde{\omega}_{e})^{2} - m_{c}^{2}} , \]  

where we have assumed \( m_{\nu} = 0 \). By evaluating numerically Eq. (4.2) with \( m_{c} = 0.511 \text{ MeV} \) and \( \Delta M = (m_{n} - m_{p}) = 1.29 \text{ MeV} \), we end up with \( \Gamma_{in}^{n \rightarrow p} = 1.81 \times 10^{-3} G_{p n}^{2} \text{ MeV}^{5} \). As a result, in order to fit Eq. (4.2), we must set \( G_{p n} = 1.74 G_{F} \), where \( G_{F} = 1.166 \times 10^{-5} \text{ GeV}^{-2} \) is the Fermi coupling constant [21]. This phenomenological procedure has the advantage of by passing any uncertainties on the influence of the nucleon inner structure.

Now we are able to use Eq. (4.1) to plot the proton mean proper lifetime \( \tau(a) = 1/\Gamma_{p \rightarrow n} \) (see Fig. 1). We have plotted the proper lifetime \( \tau(a) \) rather than the laboratory lifetime \( t(a) \) in order to make it easier the comparison of this figure with Fig. 1 in Ref. [3]. We have only considered accelerations \( a < m_{p} = 938 \text{ MeV} \) in order to respect our no-recoil condition [2,3]. We notice that swirling protons decay somewhat faster than

FIG. 1: The proton mean proper lifetime \( \tau \) is plotted as a function of its proper acceleration \( a \), where we have assumed \( \gamma = 100 \). The result is not a very sensitive function of \( \gamma \) provided that \( \gamma \gg 1 \). \( \tau \propto 1/a^{3} \) for sufficiently large \( a \).

FIG. 2: \( W_{e} \) and \( W_{p} \) are plotted as functions of the proton proper acceleration with solid and dashed lines, respectively. Although we have assumed \( \gamma = 100 \) in the numerical calculation, the result is not a very sensitive function of \( \gamma \) provided that \( \gamma \gg 1 \).
uniformly accelerated protons with the same proper acceleration $a$. We also exhibit how much energy is carried out in form of electrons and neutrinos as calculated in Sec. II by plotting the emitted powers $W_l$ for $l = e^+, \nu$ in Fig. 2.

Astrophysics seems to provide suitable conditions for the observation of the decay of accelerated protons. Let us consider a cosmic ray proton with energy $E_p = \gamma m_p \approx 1.6 \times 10^{14}$ eV under the influence of a magnetic field $B \approx 10^{14}$ Gauss of a pulsar. Protons under these conditions have proper accelerations of $a_B = \gamma eB/m_p \approx 110$ MeV/$c^2 \gg m_e$. For these values of $E_p$ and $B$, the proton is confined in a cylinder with typical radius $R \approx \gamma^2/a_B \approx 5 \times 10^{-3}$ cm $\ll l_B$, where $l_B$ is the typical size of the magnetic field region. By using Eq. (3.19), we obtain that about 1.2% of a bunch of protons would decay in this way. Hence our original estimative achieved by assuming uniformly accelerated protons was roughly correct but still 2.7 times smaller than this more precise value. We note that we did not take into account the influence of the magnetic field on the emitted positron. Clearly a more precise estimation should take into account this effect as well as other ones as, e.g., the non-uniformity of the magnetic field and energy losses through electromagnetic synchrotron radiation. The latter, in particular, may not be a problem since extra energy may be furnished to the proton from dynamo processes. A comprehensive analysis of such astrophysical issues will be discussed elsewhere.

V. NEUTRINO EMISSION FROM UNIFORMLY SWIRLING ELECTRONS

Let us, now, consider the emission of neutrino-antineutrino pairs from accelerated electrons,

$$e^- \rightarrow e^- \nu_e \bar{\nu}_e \ ,$$

and compare our results in the proper limit with the ones in the literature obtained in the particular case where the electrons are quantized in a background magnetic field [1, 2].

The emission rate and the total radiated power of neutrino-antineutrino pairs can be calculated from the Sec. IV results by assuming $\Delta M = m_\nu = 0$:

$$\Gamma_{e\bar{\nu}} = \frac{\sqrt{3}}{3458\pi^4} G_{\text{ev}}^2 a^6 \left(98 + 31/\gamma^2\right) + O(\gamma^{-4}) \ , \quad (5.2)$$

and

$$W_{e\bar{\nu}} = \frac{G_{\text{ev}}^2 a^6}{135\pi^3} \left(25 + 7/\gamma^2\right) + O(\gamma^{-4}) \ , \quad (5.3)$$

where $G_{e\nu}$ is the corresponding effective coupling constant.

In order to determine the value of $G_{e\nu}$, we compare Eq. (5.3) with the neutrino-antineutrino radiated power obtained in the particular case where the electron is uniformly swirling in a constant magnetic field $B$, provided that its proper acceleration $a = \gamma eB/m_e \ll m_e$ (no-recoil condition). This can be easily calculated from the differential emission rate given, e.g., in Ref. [9] or Ref. [13] (see Eq. (6.6) in Ref. [13] for the final result below),

$$W_{e\bar{\nu}}^{LP} = \frac{5}{108\pi^3} \left(2 C_\text{ev}^2 + 23 C_\text{A}^2 \right) G_F^2 m_\nu^2 \chi^6 \ , \quad (5.4)$$

where the vector and axial contributions to the electric current are $C_\text{ev}^2 = 0.93$ and $C_\text{A}^2 = 0.25$ [22], respectively, and $\chi = a/m_e \ll 1$. Thus, by comparing $W_{e\bar{\nu}}^{LP} = 1.14 \times 10^{-2} G_F^2 a^6$ with our Eq. (5.3), we obtain $G_{e\nu}^2 = 1.38 G_F$, which is 40% smaller than the one obtained with our original estimative with uniformly accelerated electrons. In Figs. 3 and 4 we plot Eqs. (5.2) and (5.3), respectively, for swirling electrons with $a \leq m_e$ and $\gamma = 100$. We note that for the same electron proper
VI. DISCUSSION

We have investigated the weak-interaction emission of spin-1/2 fermions from decaying (and non-decaying) uniformly swirling particles. As a particular application, we have focused on the inverse $\beta$-decay of uniformly swirling protons. We have shown that high-energy protons in background magnetic fields may have a considerably short lifetime. By restricting our semiclassical current to behave classically, i.e., by making $\Delta M \rightarrow 0$, we were able to use our formalism to investigate the neutrino-antineutrino pair emission from uniformly accelerated electrons and compare our results with the ones in the literature obtained by quantizing the electron field in a background magnetic field. By comparing the results obtained for uniformly accelerated and swirling particles, we conclude that depending on the accuracy level required, one can use directly the formulas derived for uniformly accelerated currents to make a reasonable estimation for reaction rates and emitted powers associated with processes involving accelerated particles as the ones treated here. This may be particularly useful in some astrophysical situations.

Finally, it is worth mentioning that the approach of treating here. This may be particularly useful in some astrophysical situations.

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