Acoustogalvanic effect in Dirac and Weyl semimetals

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A nonlinear mechanism to generate a direct electric current by passing acoustic wave in inversion-symmetric Dirac and Weyl semimetals is proposed. It relies on pseudo-electromagnetic fields originating from dynamical sound-induced strains. Drawing on the similarity with the photogalvanic rectification, where a direct current is produced in a second order response to light, we called this phenomenon the acoustogalvanic effect. Unlike the standard acoustoelectric effect, which relies on the sound-induced deformation potential and the corresponding electric field, the acoustogalvanic one originates from the pseudo-electromagnetic fields, which are not subject to screening. Due to an interplay of the pseudoelectric and pseudomagnetic fields, the acoustogalvanic current shows a nontrivial dependence on the direction of the sound wave propagation. Being within the experimental reach, the effect can be utilized in investigations of the dynamical deformations and provides an effective probe of the pseudo-electromagnetic fields, which are yet to be experimentally observed in Weyl and Dirac semimetals.

Introduction.— The investigation of interplay between electric properties and sound waves has a long history and dates back to 1950s [1–5] (see also Refs. [6–8]). The generation of electric currents due to sound is known as the acoustoelectric effect. Its mechanism is related to a partial uncompensation of sound-induced dynamical deformation potential by electrons in solids. A sound wave drags charge carriers leading to a measurable current or voltage [3]. In low-dimensional systems, surface acoustic waves induced by piezoelectric substrate are routinely used to probe the acoustoelectric response [9–12]. Among them, a valley acoustoelectric effect driven by a surface acoustic wave was recently predicted in two-dimensional semiconductor [13]. All of the previous proposals can be summarized as follows: sound waves induce an electric field interacting with charge carriers and resulting in an electric current. However, the possibility to generate currents via “fictitious” strain-induced electromagnetic fields that average to zero over the whole sample was not investigated before.

In recent years, there have been a surge of interest in the fictitious or pseudo-electromagnetic fields in two-(2D) and three-dimensional (3D) strained Dirac materials. As an example, we mention pseudo-gauge field in graphene [14–17], bilayer graphene [18, 19], and transition metal dichalcogenides (TMDs) [20, 21]. A Hall current generated by a time-dependent pseudo-gauge field was also previously discussed in strained Dirac materials [22–24]. In 3D, axial gauge fields in strained Weyl semimetals can be noted [25–27]. However, to the best of our knowledge, the emergence of an electric current in Weyl and Dirac semimetals (WSMs and DSMs) due to sound-induced dynamical strain fields was not discussed before.

Dirac and Weyl semimetals represent a special class of solids with relativistic-like quasiparticles [28–30]. The valence and conduction bands touch at isolated Weyl nodes (Dirac points) allowing one to apply Weyl (Dirac) equations for the description of quasiparticle properties. If the time-reversal (T) symmetry is broken, each Dirac point splits into two Weyl nodes of opposite chiralities separated by the vector b (also known as the chiral shift [31]) in momentum space. As was shown in Refs. [25–27], mechanical strain in WSM can lead to the generation of the axial gauge field A5. This field couples to the quasiparticles of opposite chirality as if they have opposite electric charges. The time-dependent and nonuniform form A5(r, t) allows for the pseudo-electromagnetic fields (cE5, B5) = (−∂tA5, ∂r × A5). Certain Dirac semimetals, such as A3Bi (A = Na, K, Rb) and Cd3As2 [32, 33] contain two overlapping copies of T symmetry broken WSMs with nonzero chiral shifts pointing in opposite directions and also allow for pseudo-electromagnetic fields. (In fact, the former can be classified as Z2 Weyl semimetals [34].) These chirality-selective fields lead to many interesting phenomena [35]. Among them are the strain-
induced chiral magnetic effect and the “negative” pseudomagnetic resistivity [36–40], quantum oscillations in pseudomagnetic fields [41], the chiral torsional effect [42–44], unusual collective excitations [45–47], axial analogs of the chiral separation and anomalous Hall effects [40], the lensing of Weyl quasiparticles [48–50], etc.

In this Letter, similarly to the photogalvanic (or photovoltaic) effect [51–53], where a direct electric current (dc) is generated due to the rectification of a driving electromagnetic wave, we propose to use a sound (acoustic) wave instead of light to generate such a current. In analogy to the photogalvanic, we dub this phenomenon the acoustogalvanic effect. Acoustic waves leads to a dynamical local deformation in materials, which is modeled with a propagating displacement vector, \( \mathbf{u} = \text{Re}[\mathbf{u}_0 e^{i(\mathbf{q} \cdot \mathbf{r} - \omega t)}] \), where the sound frequency is \( \omega = v_0 q \), \( \mathbf{q} \) is the wave vector, and \( v_0 \) stands for the sound velocity. Then, the acoustogalvanic (AG) current is defined as the nonlinear response to the dynamical strain fields

\[
j^{\text{dc}}_a = \chi^{\text{AG}}_{abc} u_b u^*_c, \tag{1}
\]

where \( \chi^{\text{AG}}_{abc} \) is the acoustogalvanic susceptibility. As we already mentioned above, strains couple as effective oscillating pseudo-electromagnetic fields \( \mathbf{E}_5 \) and \( \mathbf{B}_5 \) in Weyl and Dirac semimetals. In terms of these fields, the AG current (1) can be rewritten as

\[
j^{\text{dc}}_a = \sigma_{abc} E_{5,b} E^*_{5,c} + \kappa_{abc} \text{Re}[E_{5,b} B^*_{5,c}] + \gamma_{abc} B_{5,b} B^*_{5,c}. \tag{2}
\]

Note that due to the combined effect of the Berry curvature and pseudoelectric fields, there will be also alternating currents (ac) in the first order response (see Sec. S II.A in the Supplemental Material [54] for the details). In particular, they are related to the pseudomagnetic analog of the chiral magnetic effect [37, 40]. However, due to their alternating nature and a different direction of these currents, they can be easily distinguished from the dc response and will not be considered here.

In what follows, we demonstrate that the pseudoelectromagnetic fields lead to a nontrivial AG response of Weyl and Dirac semimetals, where a dc current is generated in second order processes. By using the chiral kinetic theory [45, 55–58] as well as applying longitudinal sound waves, we calculate the intraband contribution to the AG current in a doped \( T \) symmetry-broken WSM and certain DSMs. The origin of strain-induced electric currents is related to the acoustoelectric drag effect, where the AG current vanishes when the wave vector of sound wave goes to zero \( q \to 0 \). In addition, an interplay of the strain-induced pseudoelectric and pseudomagnetic fields allows for a nontrivial dependence of the AG current on the direction of sound wave propagation. Finally, we provide estimations of the proposed acoustogalvanic effect. As for the practical implications, we believe that it can be useful for investigating dynamical deformations.

\textbf{Model.}—An effective Hamiltonian of strained WSMs in the vicinity of Weyl nodes is given by

\[
\mathcal{H}_\lambda = \lambda v_\mathbf{p} \sigma \cdot \left[ \mathbf{p} + \frac{e}{c} \lambda \mathbf{A}_5(r, t) \right] + D(\mathbf{r}, t), \tag{3}
\]

where \( \lambda = \pm \) is chirality, \( \sigma = (\sigma_x, \sigma_y, \sigma_z) \) is the vector of Pauli matrices, \( \mathbf{p} \equiv -i \hbar \partial_\mathbf{r} \) is the momentum, \( \mathbf{A}_5 \) is the strain-induced gauge field, and \( D \) is the deformation potential. It is easy to verify that the dispersion relation of undeformed system in the vicinity of Weyl nodes is linear, \( \epsilon^{(0)}_{\eta,p} = \eta v_p p \), where \( \eta = \pm \) correspond to the conduction and valence bands, respectively.

For definiteness, we consider a longitudinal sound wave, i.e., \( \mathbf{u}_0 = u_0 \mathbf{q} \) and, without the loss of generality, set \( \mathbf{b} = \mathbf{b}_z \). The corresponding model setup is presented in Fig. 1. The sound-induced deformation leads to the following axial gauge field [27, 36]

\[
A_{5,i} = -\frac{c h b}{e} \left[ \beta u_{i z} + \tilde{\beta}(b) \delta_{i z} \sum_j u_{j j} \right]. \tag{4}
\]

The deformation potential \( D \) can be represented as the series in displacement field \( D = \sum_n D^{(n)} \). For instance, the deformation potential in the first order \( n = 1 \) is \( D^{(1)} \propto \sum u_{j j} \). Note that \( u_{i j} = (\partial_i u_j + \partial_j u_i) / 2 \) is the linearized strain tensor as well as \( \beta(b) \) are related to the Gr"uneisen parameters. Similarly to graphene [17], we assumed that the deformation potential \( D \) is isotropic and momentum-independent. The electric current \( \mathbf{j} \) and charge \( \rho \) densities are defined as [55, 57, 59]

\[
\mathbf{j} = -e \sum_{\lambda,\eta=\pm} \sum_p \eta \left[ (\partial_t \mathbf{r}) L_{\eta,\lambda} f_{\eta,\lambda} + \partial_r \times (\epsilon_{\eta,p} \Omega_{\eta,\lambda} f_{\eta,\lambda}) \right], \tag{5}
\]

\[
\rho = -e \sum_{\lambda,\eta=\pm} \sum_p \eta L_{\eta,\lambda} f_{\eta,\lambda}
\]

with \( \sum_p \equiv \int d^3 p / (2\pi\hbar)^3 \). Note that \( L_{\eta,\lambda} = 1 - e\lambda (B_5 \cdot \Omega_{\eta,\lambda}) / c \) stands for the phase space volume, which is renormalized by the Berry curvature \( \Omega_{\eta,\lambda} = \sum_n \Omega^{(n)}_{\eta,\lambda} \). In undeformed systems, the latter has a monopole-like structure \( \Omega^{(0)}_{\eta,\lambda} = \lambda \eta p / 2p^2 \) (see Sec. S I in the Supplemental Material [54] for the fields-induced corrections to the Berry curvature). The last term in the current \( \mathbf{j} \) corresponds to the orbital magnetization \( (x \partial_r \times \mathbf{M}) \) (see, e.g., Ref. [59]). The distribution function \( f_{\eta,\lambda} \) for quasiparticles of each chirality is obtained by solving the Boltzmann equation in the presence of both pseudoelectric and pseudomagnetic fields

\[
\partial_t f_{\eta,\lambda} + (\partial_t \mathbf{p}) \cdot \partial_\mathbf{p} f_{\eta,\lambda} + (\partial_r \mathbf{r}) \cdot \partial_r f_{\eta,\lambda} = -\frac{f_{\eta,\lambda} - f^{(0)}_{\eta,\lambda}}{\tau}. \tag{6}
\]

Here \( f^{(0)}_{\eta,\lambda} = 1 / \left[ e^{\eta(\epsilon_{\eta,p} + D - \mu_\lambda)} + 1 \right] \) is the local equilibrium distribution function where the divergent vacuum
contribution was subtracted, \( T \) is temperature in the energy units, and the chemical potential \( \mu_\lambda \), which contains corrections from displacement fields, i.e., \( \mu_\lambda = \sum_n \mu_\lambda^{(n)} \) where \( \mu^{(0)} = \mu \). It can be decomposed into electric \( \mu^{(n)} \) and chiral \( \mu_\lambda^{(n)} \) parts: \( \mu_\lambda^{(n)} = \mu^{(n)} + \lambda \mu_\lambda^{(n)} \). Note also that the quasiparticle dispersion obtains additional field-induced corrections \( \epsilon_\eta, \mathbf{p} = \sum_n \epsilon_\eta, \mathbf{p}^{(n)} \) in the presence of pseudo-electromagnetic fields [60, 61] (for an explicit expression, see Sec. S I in the Supplemental Material [54]). For simplicity, we utilized a simple relaxation time approximation for the collision integral, where \( \tau \) is the intranode relaxation time and the inter-node processes were neglected. The equations of motion for the chiral quasiparticles are strongly modified by the Berry curvature and read as [59]

\[
\partial_t \mathbf{r} = \frac{\mathbf{v}_{\eta, \mathbf{p}} - e (E_\lambda \times \Omega_{\eta, \lambda}) - \frac{\lambda_5}{L_{\eta, \lambda}} (v_{\eta, \mathbf{p}} \cdot \Omega_{\eta, \lambda}) \mathbf{B}_5}{L_{\eta, \lambda}},
\]

\[
\partial_t \mathbf{p} = - \frac{e E_\lambda + \lambda_5 (\mathbf{v}_{\eta, \mathbf{p}} \cdot \mathbf{B}_5)}{L_{\eta, \lambda}},
\]

where \( \mathbf{v}_{\eta, \mathbf{p}} = \partial_p \epsilon_\eta, \mathbf{p} \) stands for the quasiparticle velocity. Note also that the effective pseudoelectric field is \( \mathbf{E}_\lambda = \lambda \mathbf{E}_5 + \partial_t (D - \mu_\lambda) / e \) and we took into account that for a longitudinal sound wave \( \mathbf{E}_5 \cdot \mathbf{B}_5 = 0 \).

Having defined the key aspects of the model, let us discuss how to calculate the current density. We assume that the deformations are sufficiently weak to allow for a perturbative solution to the Boltzmann equation (6). Since the propagation of sound distorts the ionic lattice, that the deformations are sufficiently weak to allow for a perturbative solution to the Boltzmann equation (6). The correspondents of the chiral potential \( \mu_\lambda \) compensate the deformation potential \( D^{(n)} \). For the model at hand, the compensation is exact in the first order \( n = 1 \). Then, the continuity relation for a chiral current allows for residual corrections to the chiral chemical potential that renormalize the effective electric field as \( \mathbf{E}_\lambda \to \lambda \mathbf{E}_5 \) where \( \mathbf{E}_5 = \mathbf{E}_5 - \partial_t \mu_5^{(1)} / e \) at \( n = 1 \) (see also the Supplemental Material [54]). The corresponding chiral chemical potential at \( \omega \tau \ll 1 \) reads as

\[
\mu_5^{(1)} \approx - \frac{e v_F^2}{3 \omega} (1 + i \omega \tau) (\mathbf{E}_5 \cdot \mathbf{q}) e^{i (\mathbf{q} \cdot \mathbf{r} - \omega t)} + c.c. \quad (9)
\]

Acosstogalvanic response. – In the case of an arbitrary direction of sound wave propagation, both the pseudoelectric \( \mathbf{E}_5 \) and pseudomagnetic \( \mathbf{B}_5 \) fields are generated. While the general expressions for the response tensors \( \sigma_{abc}, \kappa_{abc}, \) and \( \gamma_{abc} \) are given in Sec. S II.B in the Supplemental Material [54], here we focus on two limiting cases of sound wave propagation with respect to the chiral shift: (i) \( \mathbf{q} \parallel \mathbf{b} \) and (ii) \( \mathbf{q} \perp \mathbf{b} \). They correspond to \( \theta = 0 \) and \( \theta = \pi/2 \), respectively, in Fig. 1.

Case (i): Let us start with the case of the second-order response at \( \mathbf{q} \parallel \mathbf{b} \). As is easy to verify by using Eq. (4),

\[
\begin{align*}
\sigma_{zzz} & \approx \frac{e^3 v_F}{\hbar^2 v_s} \frac{\mu_\tau^2}{18 \pi^2 \hbar} (\omega \tau)^2 + \mathcal{O}[(\omega \tau)^3], \\
\kappa_{xzy} & \approx \frac{e^3 v_F}{\hbar^2 c} \frac{\mu_\tau^2}{12 \pi^2 \hbar} + G_1 (\mu, T, \tau_irr) + \mathcal{O}[(\omega \tau)^2], \\
\gamma_{xyy} & \approx (\omega \tau)^2 G_2 (\mu, T) + \mathcal{O}[(\omega \tau)^3].
\end{align*}
\]

![Fig. 2. The dependency of the acoustogalvanic susceptibility \( \chi_{AG}^{zzz} \) on the sound frequency \( \omega \) for \( \mathbf{q} \parallel \mathbf{b} \) (\( \theta = 0 \)) (solid red curve) and \( \mathbf{q} \perp \mathbf{b} \) (\( \theta = \pi/2 \)) (the other three curves). Here \( \chi_0 = 10^3 \text{ A/cm}^4 \).](image)

Intriguingly, we find that, due to the contribution of the chiral chemical potential (9), the nonlinear conductivity, here, scales as \( \tau^2 \) and it is independent of \( \omega \). Such a dependence on the frequency is clearly different from that for the conventional optical rectification, where \( \sigma^{(1)} \propto \tau \) or \( 1/\omega \) [62–66]. The corresponding component of the acoustogalvanic response function \( \chi_{AG}^{zzz} \) follows [54]

\[
\chi_{AG}^{zzz} = \frac{\omega^4 \hbar^2 b^2}{v_s^2 e^2 [\beta + \tilde{\beta}(b)]^2} \sigma_{zzz}. \quad (11)
\]

As one can see, \( \chi_{AG}^{zzz} \) grows with the sound frequency as \( \omega^4 \) owing to quadratic dependence of the pseudoelectric field, \( E_5 \propto \omega^2 \). Such a strong frequency dependence is one of the characteristic features of the acoustogalvanic response.

Case (ii): In the case \( \mathbf{q} \perp \mathbf{b} \) (without the loss of generality, \( \mathbf{q} \parallel \hat{\mathbf{x}} \)), both \( \mathbf{E}_5 \) and \( \mathbf{B}_5 \) are nonzero, which enriches the dynamics of the system. In the leading order in \( \omega \tau \), the following components of the response tensors are relevant [54]

\[
\begin{align*}
\sigma_{xzz} & \approx \frac{e^3 v_F}{\hbar^2 v_s} \frac{\mu_\tau^2}{30 \pi^2 \hbar} (\omega \tau)^2 + \mathcal{O}[(\omega \tau)^3], \\
\kappa_{xzy} & \approx \frac{e^3 v_F}{\hbar^2 c} \frac{\mu_\tau^2}{12 \pi^2 \hbar} + G_1 (\mu, T, \tau_irr) + \mathcal{O}[(\omega \tau)^2], \\
\gamma_{xyy} & \approx (\omega \tau)^2 G_2 (\mu, T) + \mathcal{O}[(\omega \tau)^3].
\end{align*}
\]
The explicit definitions of \( G_1(\mu, T, \Lambda_{\text{IR}}) \) and \( G_2(\mu, T) \) are given in Eqs. (S69) and (S70) in the Supplemental Material [54]. Note that \( \Lambda_{\text{IR}} \) is the infrared cutoff of the theory, which is discussed in the Supplemental Material [54]. As one can see, the leading order contribution in \( \omega \tau \ll 1 \) stems from the interplay between the pseudoelectric and pseudomagnetic fields quantified by \( \kappa_{\text{xxz}} \) in Eq. (13). Unlike the response to the pseudoelectric fields, where, as in the case (i), electric current is also directed along the chiral shift, \( \kappa_{\text{xxz}} \) is related to the Hall-like response \( \propto E_x \times B_y \).

The corresponding acoustogalvanic susceptibility contains three contributions \( \chi_{\text{xxz}}^\text{AG} = \chi_{\text{xxz}}^\sigma + \chi_{\text{xxz}}^\kappa + \chi_{\text{xxz}}^\gamma \). Explicit expressions for these terms are given in Eqs. (S66)–(S68) in the Supplemental Material [54]. Their numerical values are depicted in Fig. 2. The difference in magnitude between \( \chi_{\text{xxz}}^\sigma \) and \( \chi_{\text{xxz}}^\kappa \), as well as \( \chi_{\text{xxz}}^\gamma \), is related to the fact that the relative scaling of the latter is \((\omega \tau)^2\) for small \( \omega \tau \ll 1 \). Also, the strong frequency dependence of the acoustogalvanic susceptibility is clearly evident from the figure. Furthermore, we present the dependence of the current components \( j_x^\text{dc} \) and \( j_z^\text{dc} \) on the angle between the sound wave propagation and the chiral shift in Fig. 3. While the angular profile of the former component is \( \sim \sin \theta \left[ 1 + A_0 \cos (2 \theta) \right] \), \( j_z^\text{dc} \propto \cos \theta \left[ 1 - A_0 \cos (2 \theta) \right] \), where \( A_0 \) is a combination of functions \( G_1(\mu, T, \Lambda_{\text{IR}}) \), \( G_2(\mu, T) \), and terms \( \propto \mu \). The terms with \( \cos (2 \theta) \) cause a nontrivial modulation observed in Fig. 3. In particular, due to the interplay of the pseudoelectric and pseudomagnetic fields, \( j_z^\text{dc} \) attains its maximal values at \( \theta \approx \pi/4 \) and has a characteristic butterfly-like angular profile.

For our numerical estimates, we used the characteristic numerical parameters valid for the Dirac semimetal \( \text{Cd}_3\text{As}_2 \) [67–71]: \( v_F \approx 1.5 \times 10^5 \text{ cm/s}, \mu \approx 200 \text{ meV}, b \approx 1.6 \text{ nm}^{-1}, n_s \approx 2.3 \times 10^6 \text{ cm/s}, \tau \approx 1 \text{ ps}, \beta \approx 1 \). In addition, we assume \( T = 5 \text{ K} \) and \( \beta(b) \approx 1 \).

Discussion and Summary.– In this study, a nonlinear mechanism to generate a rectified electric current by passing sound waves in Weyl and certain Dirac semimetals is proposed. Unlike the acoustoelectric effect, this dc current is generated by the strain-induced pseudo-electromagnetic fields rather than real electric fields. Therefore, in analogy to the photogalvanic effect, we called this mechanism the acoustogalvanic rectification.

In contrast to the usual optical rectification, the sound-induced dc current quickly grows with a sound frequency. This profound difference is explained by the fact that the pseudo-electromagnetic fields are by themselves determined by the dynamics of deformation vector and, therefore, grow with frequency. By using the realistic model parameters, we estimated that the acoustogalvanic current should be experimentally observable for high frequencies (e.g., ultrasound) and amplitudes of the sound. Indeed, the order of magnitude of the current is \( j_{\text{dc}} \sim 100 \text{ nA} \) for \( \omega = 10 \text{ MHz} \), \( u_0 = 1 \text{ nm} \), and mm-sized crystals. For example, photocurrents of such magnitudes were recently observed in, e.g., Ref. [72]. As an additional advantage over the conventional acoustoelectric effect, which relies on usual electric field, pseudo-electromagnetic fields that drive acoustogalvanic currents are not subject to screening and could, in principle, attain significantly high values.

Let us briefly comment on the case of DSMs such as \( \text{Na}_3\text{Bi} \) and \( \text{Cd}_3\text{As}_2 \) as well as WSMs with multiple Weyl nodes such as transition metal monopnictides. In each copy of WSMs that constitute these DSMs, the direction of pseudo-electromagnetic fields is opposite and cancel the majority of the first-order effects such as, e.g., the anomalous Hall effect. However, the second order response will be doubled with respect to the case of a simple \( T \) symmetry broken WSMs. In the case of WSMs with multiple pairs of Weyl nodes, currents should be generated independently for each of the pairs, i.e., all components of the rectified current \( j_{\text{dc}} \) will be present regardless of the direction of the wave vector \( \mathbf{q} \).

While in this study we concentrated on the case of 3D WSMs, we believe that our qualitative results can be applied for 2D Dirac materials such as graphene and TMDs. Indeed, since dynamical strain also generates pseudoelectric fields in these materials, one can follow the same steps in the calculation of acoustogalvanic response as discussed in our study. Thus, a dc current could be also generated in 2D materials due to the acoustic drag effect.

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In this section, we present the key details of the chiral kinetic theory (CKT) valid up to the second order in (pseudo)electromagnetic fields. The Boltzmann equation of the latter reads as \[ \partial_t f_\lambda + \frac{1}{1 - \frac{e}{c} (\mathbf{B}_\lambda \cdot \mathbf{\Omega}_\lambda)} \left\{ \left[ -e \tilde{E}_\lambda - \frac{e}{c} (\mathbf{v}_p \times \mathbf{B}_\lambda) + \frac{e^2}{c} (\tilde{E}_\lambda \cdot \mathbf{B}_\lambda) \mathbf{\Omega}_\lambda \right] \cdot \partial_p f_\lambda \right. \\
+ \left[ \mathbf{v}_p - e (\tilde{E}_\lambda \cdot \mathbf{\Omega}_\lambda) - \frac{e}{c} (\mathbf{v}_p \cdot \mathbf{\Omega}_\lambda) \mathbf{B}_\lambda \right] \cdot \partial_\tau f_\lambda \right\} = I_{\text{coll}}(f_\lambda), \] (S1)

where \( \tilde{E}_\lambda = E_\lambda + \frac{1}{2} \partial_\tau \), \( \mathbf{B}_\lambda = B_\lambda + \lambda B_5 \), and \( \mathbf{\Omega}_\lambda \) is the Berry curvature monopole. The global equilibrium distribution (Fermi–Dirac) function is

\[ f_{\text{eq}}^\lambda = 1 - \frac{1}{e^{(\epsilon_p^{\text{(0)}} - \mu_{\lambda}^{\text{(0)}}) / T} + 1}. \] (S2)

Here \( \mu_{\lambda}^{\text{(0)}} = \mu^{\text{(0)}} + \lambda \mu_5^{\text{(0)}} \) is the effective chemical potential for the right-\( (\lambda = +) \) and left-handed \( (\lambda = -) \) quasiparticles, \( \mu^{\text{(0)}} \) is the electric chemical potential, \( \mu_5^{\text{(0)}} \) is the chiral chemical potential, \( T \) is temperature in the energy units, \( \epsilon_p^{\text{(0)}} = \eta v_F p \) is the dispersion relation, \( v_F \) is the Fermi velocity, and \( \eta = \pm \) corresponds to the conduction \( (\eta = +) \) and valence \( (\eta = -) \) bands, respectively. Superscript \( (0) \) stands for the undeformed system. Finally, the collision integral in the Boltzmann equation (S1) is denoted by \( I_{\text{coll}}(f_\lambda) \). In the relaxation time approximation it is given by \( I_{\text{coll}}^{\text{intra}} = -\left( f_\lambda - f_{\text{eq}}^\lambda \right) / \tau \), where \( \tau \) is the intra-node relaxation time and \( f_{\text{eq}}^{\text{(0)}} \) is the local equilibrium function, which will be defined in Eq. (S30). Since the inter-node relaxation usually require large momentum transfer, the corresponding relaxation time \( \tau_5 \) is much longer than that for intra-node processes. Therefore, for simplicity, we will neglect inter-node processes. It is convenient to separate the contribution of the filled states in the distribution function (S2) for the hole band \( (\eta = -) \), i.e.,

\[ f_{\text{eq}}^\lambda = \delta_{\eta, +} \frac{1}{e^{(\epsilon_p^{\text{(0)}} - \mu_{\lambda}^{\text{(0)}}) / T} + 1} + \delta_{\eta, -} \left[ 1 - \frac{1}{e^{(\epsilon_p^{\text{(0)}} - \mu_{\lambda}^{\text{(0)}}) / T} + 1} \right]. \] (S3)
As was shown in Refs. [S9, S10] the axial gauge fields are related to the deformation tensor $\mathbf{E}$ and the equations of the CKT presented above include both electromagnetic Hamiltonian of Weyl semimetals in the vicinity of Weyl nodes given in Eq. (3) in the main text. It is worth noting that these fields are expressed through the axial gauge fields $A$, $B$, and $\hat{A}$ in Refs. [S6, S7]. Its explicit formulation in the case of Weyl semimetals is given in Ref. [S8]. It is notable that the theory including the field-induced corrections to the Berry curvature and quasiparticle dispersion relation was derived in Refs. [S6, S7]. Its explicit formulation in the case of Weyl semimetals is given in Ref. [S8]. It is notable that the theory including the field-induced corrections to the Berry curvature and quasiparticle dispersion relation was derived in Refs. [S6, S7].

In what follows, the contribution of the filled states $\delta_{\alpha,-}$ will be subtracted leading to the multiplier $\eta$ in the expressions for the charge and current densities (S18) and (S19).

In the studies of the second order responses, one should use the appropriate CKT. In a general case, the kinetic theory including the field-induced corrections to the Berry curvature and quasiparticle dispersion relation was derived in Refs. [S6, S7]. Its explicit formulation in the case of Weyl semimetals is given in Ref. [S8]. It is notable that the theory including the field-induced corrections to the Berry curvature and quasiparticle dispersion relation was derived in Refs. [S6, S7].

Here $\hat{A}$ and $\hat{B}$ retain their form. However, the Berry curvature $\Omega_{\lambda}$, the quasiparticle energy $\epsilon_{\mathbf{p}}$, and the velocity $\mathbf{v}_{\mathbf{p}}$ will be modified. In the case of Weyl fermions, the corresponding expressions read as

$$\Omega_{\lambda} = \Omega_{\lambda}^{(0)} + \Omega_{\lambda}^{(1)} + \ldots,$$  
(S4)

$$\epsilon_{\mathbf{p}} = \epsilon_{\mathbf{p}}^{(0)} + \epsilon_{\mathbf{p}}^{(1)} + \epsilon_{\mathbf{p}}^{(2)} + \ldots,$$  
(S5)

$$\mathbf{v}_{\mathbf{p}} = \partial_{\mathbf{p}} \epsilon_{\mathbf{p}} = \mathbf{v}_{\mathbf{p}}^{(0)} + \mathbf{v}_{\mathbf{p}}^{(1)} + \mathbf{v}_{\mathbf{p}}^{(2)} + \ldots,$$  
(S6)

where the components are

$$\Omega_{\lambda}^{(0)} = \lambda \hbar \mathbf{p} \cdot \mathbf{p} / 2p^2,$$  
(S7)

$$\Omega_{\lambda}^{(1)} = -\frac{\hbar}{2} \left[ \frac{2}{c} \mathbf{p} (\mathbf{p} \cdot \mathbf{B}_{\lambda}) - \frac{1}{c} \mathbf{B}_{\lambda} + \frac{2\eta}{v_F} \left( \mathbf{E}_{\lambda} \times \mathbf{p} \right) \right]$$  
(S8)

and

$$\epsilon_{\mathbf{p}}^{(0)} = \eta v_F p,$$  
(S9)

$$\epsilon_{\mathbf{p}}^{(1)} = \lambda \frac{\hbar}{2c} \mathbf{B}_{\lambda} \cdot \mathbf{p},$$  
(S10)

$$\epsilon_{\mathbf{p}}^{(2)} = \frac{e^2 \hbar}{4c} \left\{ \frac{\eta v_F}{2} \left[ 2 \mathbf{B}_{\lambda}^2 - (\mathbf{B}_{\lambda} \cdot \mathbf{p})^2 \right] - \left( \mathbf{B}_{\lambda} \cdot \mathbf{E}_{\lambda} \times \mathbf{p} \right) \right\}.$$  
(S11)

By making use of Eq. (S5), the quasiparticle velocity $\mathbf{v}_{\mathbf{p}}^{(n)}$ equals

$$\mathbf{v}_{\mathbf{p}}^{(0)} = \eta v_F \mathbf{p},$$  
(S12)

$$\mathbf{v}_{\mathbf{p}}^{(1)} = \lambda \frac{\hbar}{2p^2} \mathbf{B}_{\lambda} - \lambda \frac{\hbar}{c} \mathbf{p} \times \mathbf{B}_{\lambda},$$  
(S13)

$$\mathbf{v}_{\mathbf{p}}^{(2)} = \frac{5\eta e^2 v_F \hbar^2}{16c^2 p^4} \mathbf{p} (\mathbf{B}_{\lambda} \cdot \mathbf{p})^2 - \frac{\eta e^2 v_F \hbar^2}{8c^2 p^4} \mathbf{B}_{\lambda} (\mathbf{B}_{\lambda} \cdot \mathbf{p}) - \frac{3\eta e^2 v_F \hbar^2}{8c^2 p^4} \mathbf{p} \mathbf{p} - \frac{e^2 h^2}{4cp^4} \mathbf{B}_{\lambda} + \frac{e^2 h^2}{4cp^4} \mathbf{B}_{\lambda} \cdot \mathbf{E}_{\lambda} \times \mathbf{p}.$$  
(S14)

Here $\mathbf{p} = \mathbf{p}/p$ and, for simplicity of notations, we omit explicit index $\eta$ at $\epsilon_{\mathbf{p}}$ and $\Omega_{\lambda}$. Also, we used an effective Hamiltonian of Weyl semimetals in the vicinity of Weyl nodes given in Eq. (3) in the main text. It is worth noting that the equations of the CKT presented above include both electromagnetic $\mathbf{E}$ and $\mathbf{B}$ as well as pseudo-electromagnetic $\mathbf{E}_5$ and $\mathbf{B}_5$ fields. As is discussed in the latter text, the latter can be induced by strain in Weyl and Dirac semimetals. These fields are expressed through the axial gauge fields $A_{0,5}$ and $A_5$ as $\mathbf{B}_5 = \partial_{\mathbf{r}} \times \mathbf{A}_5$ and $\mathbf{E}_5 = -\partial_{\mathbf{r}} A_{0,5} - \partial_{\mathbf{t}} \mathbf{A}_5/c$. As was shown in Refs. [S9, S10] the axial gauge fields are related to the deformation tensor $u_{ij}$ as

$$A_{0,5} = -\frac{1}{c} b_0 \beta \sum_j u_{jj},$$  
(S15)

$$A_{5,i} = -\frac{\hbar b}{c} \left[ \beta u_{iz} + \delta_{iz} \beta \sum_j u_{jj} \right].$$  
(S16)

Here $2b_0$ is the separation of Weyl nodes in energy, $b$ is the $z$-component of the momentum-space separation, i.e., the chiral shift (without the loss of generality, we assumed that $\mathbf{b} \parallel \hat{z}$),

$$u_{ij} = \frac{1}{2} \left( \partial_i u_j + \partial_j u_i + \sum_l \partial_l u_i \partial_l u_j \right) \approx \frac{1}{2} \left( \partial_j u_i + \partial_i u_j \right)$$  
(S17)

is the symmetrized strain tensor, and $\mathbf{u}$ is the displacement vector. The magnitude of the strain effects is parameterized by the Gr"uneisen parameter $\beta = -a \partial t/(t \partial a)$, where $t$ is the lattice hopping constant and $a$ is the lattice spacing. As...
we will show below, the last term in Eq. (S16) plays an important role when the direction of sound wave propagation and the chiral shift are not aligned. Microscopically, this term is related to the hopping probabilities between the same states (e.g., $s$ and $p$ states in a simple cubic lattice model [S9,S10]). To simplify calculations and present our qualitative results as clear as possible, let us neglect the second order in deformation vector terms in Eq. (S17). Such an approximation is also consistent with the linear form of strain-induced axial gauge fields in Eqs. (S15) and (S16).

The physical consistent current and charge densities can be represented as a sum of covariant charge and current as well as the Chern–Simons terms as $(\rho_\lambda, j_{\lambda}) = (\tilde{\rho}_\lambda + \rho_{\text{CS},\lambda}, \tilde{j}_\lambda + j_{\text{CS},\lambda})$. The covariant charge and current densities are defined as [S1,S2,S4]

\[
\tilde{\rho}_\lambda = -\sum_{\eta=\pm} e \eta \int \frac{d^3p}{(2\pi \hbar)^3} \left[ 1 - \frac{e}{c} (B_\lambda \cdot \Omega_\lambda) \right] f_\lambda,
\]

\[
\tilde{j}_\lambda = -\sum_{\eta=\pm} e \eta \int \frac{d^3p}{(2\pi \hbar)^3} \left\{ \mathbf{v}_p - \frac{e}{c} (\mathbf{v}_p \cdot \Omega_\lambda) B_\lambda - e \left( \tilde{E}_\lambda \times \Omega_\lambda \right) \right\} f_\lambda
- \sum_{\eta=\pm} e \eta \partial_r \times \int \frac{d^3p}{(2\pi \hbar)^3} f_\lambda \mathbf{e}_r \mathbf{p} \Omega_\lambda.
\]

(S19)

Here the last term in Eq. (S19) is the magnetization current and the overall prefactor $\eta$ originates from the fact that the contribution of the filled states (i.e., $\delta_{\eta,-}$ in Eq. (S3)) was ignored. The Chern–Simons terms in the electric charge and current densities are [S11,S12]

\[
\rho_{\text{CS}} = \frac{e^2}{2\pi^2 \hbar c} (\mathbf{b} \cdot \mathbf{B}) - \frac{e^3}{2\pi^2 \hbar^2 c^2} (\mathbf{A}_5 \cdot \mathbf{B}),
\]

\[
\tilde{j}_{\text{CS}} = \frac{e^2}{2\pi^2 \hbar} b_0 \mathbf{B} - \frac{e^3}{6\pi^2 \hbar^2 c^2} A_{0,5} \mathbf{B} - \frac{e^2}{2\pi^2 \hbar} \left( \mathbf{b} \times \tilde{E} \right) + \frac{e^3}{6\pi^2 \hbar^2 c} \left( \mathbf{A}_5 \times \tilde{E} \right).
\]

(S20)

(S21)

Their analog in the chiral current and charge densities reads as [S11]

\[
\rho_{\text{CS},5} = \frac{e^2}{6\pi^2 \hbar c} (\mathbf{b} \cdot \mathbf{B}_5) - \frac{e^3}{6\pi^2 \hbar^2 c^2} (\mathbf{A}_5 \cdot \mathbf{B}_5),
\]

\[
\tilde{j}_{\text{CS},5} = \frac{e^2}{6\pi^2 \hbar} b_0 \mathbf{B}_5 - \frac{e^3}{6\pi^2 \hbar^2 c} A_{0,5} \mathbf{B}_5 - \frac{e^2}{6\pi^2 \hbar} \left( \mathbf{b} \times \tilde{E}_5 \right) + \frac{e^3}{6\pi^2 \hbar^2 c} \left( \mathbf{A}_5 \times \tilde{E}_5 \right).
\]

(S22)

(S23)

As one of us advocated in Ref. [S12], the Chern–Simons terms in the current and charge densities are important to cure the anomalous local electric charge nonconservation in external electromagnetic and pseudo-electromagnetic fields. The consistent current and charge densities satisfy the following continuity relations [S11,S12]:

\[
\partial_t \rho_5 + \partial_k \cdot j_5 = -\frac{e^3}{2\pi^2 \hbar^2 c} \left[ (\mathbf{E} \cdot \mathbf{B}) + \frac{1}{3} (\mathbf{E}_5 \cdot \mathbf{B}_5) \right],
\]

\[
\partial_t \rho + \partial_k \cdot j = 0.
\]

(S24)

(S25)

Here the nonconservation of chiral charge determined by the right-hand side in Eq. (S24) is related to the chiral anomaly and its modification by pseudo-electromagnetic fields.

To clarify the possibility of the acoustogalvanic response in Dirac and Weyl semimetals, we consider the case in which the external electromagnetic fields are absent $\mathbf{E} = \mathbf{B} = \mathbf{0}$. Furthermore, we assume that the parity-inversion symmetry is not broken, i.e., $b_0 = 0$ and two Weyl nodes are separated by $2\mathbf{b}$ in momentum space. This model setup can be straightforwardly generalized to the case of certain Dirac semimetals, whose low energy spectrum contain Dirac points separated in momentum space. Among them are $A_3\text{Bi}$ ($A = \text{Na, K, Rb}$) and $\text{Cd}_3\text{As}_2$ [S13,S14], which can be also considered as $\mathbb{Z}_2$ Weyl semimetals [S15]. In particular, the second-order contributions from different copies of Weyl semimetals should simply add up in these systems.

We assume a longitudinal propagation of a sound wave with the displacement vector $\mathbf{u} = \frac{i}{2} \omega q e^{-i\omega t+iq\mathbf{r}} + c.c.$, where $\omega$ is the sound frequency, $q = \omega / v_s$ is the absolute value of the wave vector, and $v_s$ is the sound velocity. Then, by using the gauge potential in Eq. (S16), the pseudoelectric and pseudomagnetic fields are

\[
E_{5,j} = -\frac{ie\omega b}{2e} \left[ \beta u_{jz} + \delta_{jz} \tilde{\beta}(b) \sum I u_I \right] + c.c.
= \frac{h\omega q b_0}{2e} \left( \frac{\beta q j_z}{q} + \delta_{jz} \tilde{\beta}(b) q \right) e^{-i\omega t+iq\mathbf{r}} + c.c.,
\]

(S26)
and
\[ B_{5,j} = -\frac{e}{2\epsilon} \epsilon_{jmn} q_m b [\beta u_{nz} + \delta_{nz} \beta(b) \sum_l u_l] + \text{c.c.} = -\frac{e}{2\epsilon} \epsilon_{jmn} q_m b [\beta q_n q_z + \delta_{nz} \beta(b) q] e^{-i\omega t + i\mathbf{q}\mathbf{r}} + \text{c.c.} \]
\[ = \frac{eh_0}{2\epsilon} \epsilon_{jmn} q_m b \beta(b) q e^{-i\omega t + i\mathbf{q}\mathbf{r}} + \text{c.c.} \] (S27)

For the sake of brevity, we denote these fields as \( \mathbf{E}_5 = \frac{1}{2} \mathbf{E}_{5,0} e^{-i\omega t + i\mathbf{q}\mathbf{r}} + \text{c.c.} \) and \( \mathbf{B}_5 = \frac{1}{2} \mathbf{B}_{5,0} e^{-i\omega t + i\mathbf{q}\mathbf{r}} + \text{c.c.} \), where the subscript 0 denotes the amplitude of the oscillating field. It is worth noting that, as follows from Eqs. (S26) and (S27), pseudoelectric and pseudomagnetic fields are orthogonal, \( (\mathbf{E}_5 \cdot \mathbf{B}_5) = 0 \).

Like in conventional materials, deformations in Weyl and Dirac semimetals lead not only to the pseudo-electromagnetic fields \( \mathbf{E}_5 \) and \( \mathbf{B}_5 \), but modify the quasiparticle energy \( \epsilon_p \) due to the deformation potential term \( \sum_{i,j} D_{ij} = \sum_{n=1} \sum_{i,j} D_{ij}^{(n)}(\mathbf{p}) \) [S16-S18] i.e.,
\[ \epsilon_p \to \epsilon_p + \sum_{n=1} \sum_{i,j} D_{ij}^{(n)}(\mathbf{p}). \] (S28)

In the first order \( n = 1 \), \( D_{ij}^{(1)}(\mathbf{p}) \propto u_{ij} \). In order to simplify our calculations, we assume that the deformation potential does not depend on momentum, \( D_{ij}(\mathbf{p}) \approx D_{ij} \). Further, for our qualitative estimates, it is sufficient to ignore anisotropy, i.e., \( D_{ij} \approx D \delta_{ij} \). As will be shown below, the conventional acoustoelectric effect will be absent in this case.

The propagation of sound waves distorts the ionic lattice leading to deviations of local electric charge density from its equilibrium value. These deviations are (partially) screened by free charge carriers, which is captured by the following corrections to the chemical potential:
\[ \mu_\lambda \to \mu_\lambda = \sum_{n=0} \mu_\lambda^{(n)}, \] (S29)
where \( \mu_\lambda^{(0)} = \mu \). In general, deviations of the chiral chemical potential are also allowed in chiral systems such as Weyl semimetals. Up to the second order in weak perturbations, we have the following local equilibrium distribution function:
\[ f_\lambda^{(0)} = \frac{1}{1 + e^{(\epsilon_p - \mu_\lambda + V_\lambda^{(1)})/T}} \approx \frac{f_\lambda^{eq} + (\epsilon_\lambda^{(1)} + V_\lambda^{(1)}) \partial_{\epsilon_p} f_\lambda^{eq} + (\epsilon_\lambda^{(2)} + V_\lambda^{(2)}) \partial_{\epsilon_p} (\epsilon_\lambda^{(1)} + V_\lambda^{(1)})}{2} \frac{\partial^2 f_\lambda^{eq}}{\epsilon_\lambda^{(1)} + V_\lambda^{(1)}}, \] (S30)
where \( V_\lambda^{(n)} = D^{(n)} - \mu_\lambda^{(n)} \) and the global equilibrium distribution function \( f_\lambda^{eq} \) is given in Eq. (S2).

**S II. CURRENT DENSITY**

In this section, the current density up to the second order in strain-induced pseudo-electromagnetic fields is calculated. For our estimates, we use the numerical parameters valid for the Dirac semimetal Cd₃As₂. They are [S19-S23]
\[ v_F \approx 1.5 \times 10^8 \text{ cm/s}, \quad b \approx 1.6 \text{ nm}^{-1}, \quad \mu_0 \approx 200 \text{ meV}, \quad \tau_0 = 1 \text{ ps}, \quad v_s \approx 2.3 \times 10^5 \text{ cm/s}, \quad \beta \approx 1. \] (S31)

In addition, we assume that \( \mu_5 = 0, \ T = 5 \text{ K}, \) and \( \beta \approx 1 \). It is worth noting that since the single-band approximation is used, the electric chemical potential should be significantly higher than the frequency, i.e., \( \mu \gg \hbar \omega \), which is indeed the case for many Weyl and Dirac semimetals. By using the numerical parameters in Eq. (S31), we estimate that \( \mu/\hbar \approx 48.36 \text{ THz} \), which is well above typical ultrasound frequencies.

**S II.A. First order response**

In the first order in weak fields, we have the following Boltzmann equation:
\[ \partial_t f_\lambda^{(1)} = e [\mathbf{E}_\lambda + \frac{1}{c} (\mathbf{v}_p^{(0)} \times \mathbf{B}_\lambda)] \cdot \partial_{\epsilon_p} f_\lambda^{(1)} + (\mathbf{v}_p^{(0)} \cdot \partial_{\epsilon_p} f_\lambda^{(1)}) + (\partial_{\epsilon_p} f_\lambda^{eq}) \partial_t (\epsilon_\lambda^{(1)} + V_\lambda^{(1)}) = -\frac{f_\lambda^{(1)}}{\tau}. \] (S32)
It is straightforward to find the following solution for the above equation:

\[ f_{\lambda}^{(1)} = \frac{1}{2} f_{\lambda,0} e^{-i\omega t + iq r} + c.c., \]  

(S33)

where the amplitude of the distribution function reads as

\[ f_{\lambda,0}^{(1)} = \frac{1}{2} \left\{ \frac{\epsilon_{\lambda} \left[ V_{\lambda,0}^{(1)} \right]}{1 - i\omega t + (v_{\lambda}^{(0)} \cdot q)\tau} \right\} \]

(S34)

In the last expression, for simplicity, we assumed that both \( \omega t \ll 1 \) and \( v_{\lambda}^{(0)} \cdot q \ll 1 \) are small and expanded the denominator. According to the numerical parameters given in Eq. (S31), this approximation is indeed reasonable for realistic values of \( \omega \) and \( \tau \).

The charge and current densities in the first order in the fields read as

\[ \rho_{\lambda}^{(1)} = -e \sum_{\eta = \pm} \eta \int \frac{d^3p}{(2\pi\hbar)^3} f_{\lambda}^{(1)} - e \sum_{\eta = \pm} \eta \int \frac{d^3p}{(2\pi\hbar)^3} \left[ -\frac{e}{c} \left( B_{\lambda} \cdot \Omega_{\lambda}^{(0)} \right) f_{\lambda}^{eq} + \left( V_{\lambda}^{(1)} + \epsilon_{p,0}^{(1)} \right) (\partial_p f_{\lambda}^{eq}) \right] \]

and

\[ j_{\lambda}^{(1)} = -e \sum_{\eta = \pm} \eta \int \frac{d^3p}{(2\pi\hbar)^3} \left[ V_{\lambda,0}^{(1)} - e B_{\lambda} \left( \frac{q \times \Omega_{\lambda}^{(0)}}{c} \right) - e \left( \vec{E}_{\lambda} \times \Omega_{\lambda}^{(0)} \right) \right] \left[ f_{\lambda}^{eq} + \left( V_{\lambda}^{(1)} + \epsilon_{p,0}^{(1)} \right) (\partial_p f_{\lambda}^{eq}) \right], \]

respectively. By using expressions in Sec. for calculating the integrals over angles and momenta, it is straightforward to obtain the following amplitudes of the oscillating charge and current densities:

\[ \rho_{\lambda,0}^{(1)} = -\frac{e^2 \tau^2 v_{\lambda}^2}{3} \left( \frac{2}{3} + i\omega t \right) C_1 + e V_{\lambda}^{(1)} (1 + i\omega t - \omega^2 \tau^2) C_1 \]

(S37)

and

\[ j_{\lambda,0}^{(1)} = \frac{e^2 \tau v_{\lambda} E_{\lambda}}{3} \left( 1 + i\omega t \right) C_1 + \frac{e \omega \tau v_{\lambda}^2 V_{\lambda}^{(1)}}{3} q \left( 1 + i\omega t \right) C_1 + i e^2 \frac{h \omega v_{\lambda}}{6c} B_{\lambda} \left( 1 + i\omega t \right) C_2 \]

\[- i \lambda e^2 \frac{\tau v_{\lambda}^2}{2} \left( q \times E_{\lambda} \right) \left( 1 + i\omega t \right) C_2 - \frac{e^2 h^2 v_{\lambda}^2}{2c} (q \times B_{\lambda}) C_3 - \lambda e^2 \frac{h^2 v_{\lambda}^2}{2c} B_{\lambda} C_2, \]

(S38)

respectively. The shorthand notations are

\[ C_1 = \sum_{\eta = \pm} \eta \int \frac{d^3p}{(2\pi\hbar)^3} \partial_p f_{\lambda}^{eq} = -\frac{2}{v_F} \sum_{\eta = \pm} \eta \int \frac{d^3p}{(2\pi\hbar)^3} \frac{1}{p} f_{\lambda}^{eq} = -\frac{1}{2\pi^2 h^3 v_F^2} \left( \mu_\lambda + \frac{e^2}{3} \right), \]

(S39)

\[ C_2 = \sum_{\eta = \pm} \eta \int \frac{d^3p}{(2\pi\hbar)^3} \frac{1}{p} \partial_p f_{\lambda}^{eq} = -\frac{1}{v_F} \sum_{\eta = \pm} \eta \int \frac{d^3p}{(2\pi\hbar)^3} \frac{1}{p^2} f_{\lambda}^{eq} = -\frac{\mu_\lambda}{2\pi^2 h^3 v_F^2}, \]

(S40)

\[ C_3 = \sum_{\eta = \pm} \eta \int \frac{d^3p}{(2\pi\hbar)^3} \frac{1}{p^2} \partial_p f_{\lambda}^{eq} = -\frac{1}{v_F} \sum_{\eta = \pm} \eta \int \frac{d^3p}{(2\pi\hbar)^3} \frac{1}{p^2} f_{\lambda}^{eq} = -v_F \sum_{\eta = \pm} \eta \int \frac{d^3p}{(2\pi\hbar)^3} \frac{1}{p^2} \partial_p f_{\lambda}^{eq} = -\frac{1}{2\pi^2 h^3 v_F}. \]

(S41)

To determine the correction to the chemical potential \( \mu_{\lambda}^{(1)} \) (recall that \( V_{\lambda}^{(i)} = D^{(i)} - \mu_{\lambda}^{(i)} \)), we enforce the continuity relation for both electric and chiral currents,

\[ \partial_t \rho_{\lambda}^{(1)} + \partial_r \cdot j_{\lambda}^{(1)} = 0, \]

(S42)

\[ \partial_t \rho_{\lambda}^{(1)} + \partial_r \cdot j_{\lambda}^{(1)} = 0. \]

(S43)

Note that since \( E_5 \cdot B_5 = 0 \), the chiral charge is also conserved. It is straightforward to check that the first equation leads to \( V^{(1)} = 0 \). This means that the deformation potential \( D^{(1)} \) is completely compensated by the deviations of the
electric chemical potential $\mu^{(1)}$ in the model at hand. On the other hand, Eq. (S43) allows for the following nontrivial solution:

$$ V_{5,0}^{(1)} = -\mu_{5,0}^{(1)} = \frac{e^{2\hbar v_F^2}}{3\omega} (\mathbf{E}_5 \cdot \mathbf{q}) \frac{1 - 2\omega^2 \tau^2 + 2i\omega \tau}{1 - \omega^2 \tau^2 + i\omega \tau - \frac{v_F^2}{2}\tau^2 (1 + 2i\omega \tau)} \approx \frac{e^{2\hbar v_F^2}}{3\omega} (\mathbf{E}_5 \cdot \mathbf{q}) (1 + i\omega \tau) + O(\omega^2 \tau^2). \quad (S44) $$

In the case of an exact expression for the distribution function given in the first line in Eq. (S34) and at $\mathbf{q} \parallel \mathbf{z}$, the amplitude $V_{5,0}^{(1)}$ reads as

$$ V_{5,0}^{(1)} = -\frac{v_F \tau e \dot{E}_{5,0} \left[\omega K_1(A_1, A_2) - v_F q_3 K_2(A_1, A_2)\right]}{2\omega + i\omega \tau \left[\omega K_0(A_1, A_2) - v_F q_3 K_1(A_1, A_2)\right]} \quad (S45) $$

Here

$$ K_0(A_1, A_2) = \int_{-1}^{1} d\cos \theta \frac{1}{A_1 + iA_2 \cos \theta} = \frac{2}{A_2} \arctan \left(\frac{A_2}{A_1}\right), \quad (S46) $$

$$ Q_1(A_1, A_2) = \int_{-1}^{1} d\cos \theta \frac{\cos \theta}{A_1 + iA_2 \cos \theta} = -\frac{2i}{A_2} \left[ A_2 - A_1 \arctan \left(\frac{A_2}{A_1}\right) \right], \quad (S47) $$

$$ K_2(A_1, A_2) = \int_{-1}^{1} d\cos \theta \frac{\cos^2 \theta}{A_1 + iA_2 \cos \theta} = \frac{2A_1}{A_2^2} \left[ A_2 - A_1 \arctan \left(\frac{A_2}{A_1}\right) \right] \quad (S48) $$

and we used $A_1 = 1 - i\omega \tau$ and $A_2 = v_F q_3 \tau$. Expression (S45) is valid even for $\omega \tau \approx 1$.

By using Eq. (S38), we derive the following amplitude of the oscillating electric current:

$$ j_0^{(1)} = \sum_{\lambda = \pm} j_0^{(1)}(\lambda) \approx -\frac{e^2 \hbar v_F^2}{3c} B_5 (1 - i\omega \tau) C_2 - i \frac{e^2 \hbar v_F^2}{3} (\mathbf{q} \times \mathbf{E}_5) (1 + i\omega \tau) C_2 + O[(\omega \tau)^2]. \quad (S49) $$

### S II.B. Second order response

Let us consider the second order response to the pseudo-electromagnetic fields $\mathbf{E}_5$ and $\mathbf{B}_5$. Note that since we used the strain tensor and the axial gauge in the linear approximation, the results for the second-order acoustogalvanic response should be considered as qualitative rather than quantitative. The second-order Boltzmann equation reads as

$$ -\frac{e}{c} \left( \mathbf{B}_\lambda \cdot \mathbf{\Omega}_\lambda^{(0)} \right) \frac{\partial}{\partial \tau} f^{(1)}_\lambda + \left( \mathbf{V}_\lambda^{(1)} + \epsilon^{(1)}_p \right) \frac{\partial}{\partial \mathbf{p}} f^{eq}_\lambda - e \left[ \mathbf{E}_\lambda^{(1)} + \frac{1}{c} \left( \mathbf{v}_p^{(0)} \times \mathbf{B}_\lambda \right) \right] \frac{\partial}{\partial \mathbf{p}} f^{(1)}_\lambda + \left( \mathbf{V}_\lambda^{(1)} + \epsilon^{(1)}_p \right) \frac{\partial}{\partial \mathbf{p}} f^{eq}_\lambda $$

$$ -\frac{e}{c} \left( \mathbf{v}_p^{(1)} \times \mathbf{B}_\lambda \right) \frac{\partial}{\partial \mathbf{p}} f^{eq}_\lambda + \left[ \mathbf{v}_p^{(1)} - e \left( \mathbf{E}_\lambda^{(1)} + \frac{1}{c} \left( \mathbf{v}_p^{(0)} \cdot \mathbf{\Omega}_\lambda^{(0)} \right) \right) \right] \nabla f^{(1)}_\lambda = -\frac{f^{(2)}_\lambda}{\tau} \quad (S50) $$

By using the definition of $\mathbf{v}_p^{(1)}$ in Eq. (S13), it is straightforward to show that the term $\left( \mathbf{v}_p^{(1)} \times \mathbf{B}_\lambda \right) \frac{\partial}{\partial \mathbf{p}} f^{eq}_\lambda$ in the second line in Eq. (S50) vanishes. Then, by solving Eq. (S50), we obtain the following amplitude of the distribution function that corresponds to direct current (dc) response:

$$ f^{(2)}_\lambda = -\frac{\tau}{\lambda} \left\{ - e \left[ \mathbf{E}_{\lambda,0}^{(1)} + \frac{1}{c} \left( \mathbf{v}_p^{(0)} \times \mathbf{B}_{\lambda,0} \right) \right] \right\} \frac{\partial}{\partial \mathbf{p}} f^{(1)}_\lambda + \frac{i e \omega}{c} \left[ B_{\lambda,0}^{(1)} \times \mathbf{\Omega}_\lambda^{(0)} \right] f^{(1)}_\lambda + \frac{i e \omega}{c} \left[ \mathbf{v}_p^{(1)} \right]^* \left( \mathbf{E}_{\lambda,0}^{(1)} + \frac{1}{c} \left( \mathbf{v}_p^{(0)} \times \mathbf{B}_{\lambda,0} \right) \right] f^{(1)}_\lambda $$

$$ + \frac{i e \omega}{c} \left[ B_{\lambda,0}^{(1)} \times \mathbf{\Omega}_\lambda^{(0)} \right] \left( \mathbf{v}_p^{(1)} + \epsilon^{(1)}_p \right) \frac{\partial}{\partial \mathbf{p}} f^{eq}_\lambda \right\} + c.c. \quad (S51) $$
Here the derivative with respect to momentum from $f^{(1)}_{\lambda,0}$ is

$$
\partial_\mathbf{p} f^{(1)}_{\lambda,0} = e \eta v_F [1 + i \omega \tau - iv_F \tau (\mathbf{q} \cdot \mathbf{p}) (1 + 2i \omega \tau)] \left\{ |\mathbf{E}_{\lambda,0} - \mathbf{p} (\mathbf{E}_{\lambda,0} \cdot \mathbf{p})| \left( \frac{\partial_\mathbf{p} f^{(1)}_{\lambda,0}}{p} \right) + \eta v_F \mathbf{p} (\mathbf{E}_{\lambda,0} \cdot \mathbf{p}) \left( \frac{\partial_\mathbf{p} f^{(1)}_{\lambda,0}}{p^2} \right) \right\} 
- i e \eta v_F^2 \left( \mathbf{E}_{\lambda,0} \cdot \mathbf{p} \right) |\mathbf{q} - \mathbf{p} (\mathbf{q} \cdot \mathbf{p})| \left( 1 + 2i \omega \tau \right) \left( \frac{\partial_\mathbf{p} f^{(1)}_{\lambda,0}}{p^2} \right) + i \lambda \eta \frac{e h v_F \omega \tau}{2c} \left[ 1 + i \omega \tau - iv_F \tau (\mathbf{q} \cdot \mathbf{p}) (1 + 2i \omega \tau) \right] 
\times \left[ |\mathbf{B}_{\lambda,0} - \mathbf{p} (\mathbf{B}_{\lambda,0})| \left( \frac{\partial_\mathbf{p} f^{(1)}_{\lambda,0}}{p^2} \right) + \lambda \eta \frac{\omega \tau v_F^2}{2c} \left( \mathbf{B}_{\lambda,0} \cdot \mathbf{p} \right) |\mathbf{q} - \mathbf{p} (\mathbf{q} \cdot \mathbf{p})| \left( 1 + 2i \omega \tau \right) \left( \frac{\partial_\mathbf{p} f^{(1)}_{\lambda,0}}{p^2} \right) \right] 
+ \frac{i \lambda}{2c} \frac{e h v_F^2 \tau}{2} \mathbf{p} (\mathbf{E}_{\lambda,0} \cdot \mathbf{p}) \left[ 1 + i \omega \tau - iv_F \tau (\mathbf{q} \cdot \mathbf{p}) (1 + 2i \omega \tau) \right] \left( \frac{\partial_\mathbf{p} f^{(1)}_{\lambda,0}}{p} \right) 
+ \frac{i \eta v_F \tau v_F (\mathbf{V}^{(1)}_{\lambda,0} \cdot \mathbf{p}) (\partial_\mathbf{p} f^{(1)}_{\lambda,0}) \left[ 1 + i \omega \tau - iv_F \tau (\mathbf{q} \cdot \mathbf{p}) (1 + 2i \omega \tau) + v_F \omega \tau^2 \mathbf{V}^{(1)}_{\lambda,0} \left( \frac{\partial_\mathbf{p} f^{(1)}_{\lambda,0}}{p} \right) (\mathbf{q} \cdot \mathbf{p}) \right] (1 + 2i \omega \tau) \right) .
$$

(S52)

Since we consider the dc response, continuity relations for electric and chiral current densities are automatically fulfilled. Therefore, the second order correction to the chemical potential $\mu^{(2)}$ can be determined from the condition of the electric charge neutrality $\rho^{(2)} = 0$. The corresponding correction does not provide any contributions to the dc current and, therefore, will not be considered.

The general expression for the second-order rectified current reads as

$$
\mathbf{j}^{(2)}_{\lambda} = -e \sum_{\nu = \pm} \frac{\gamma_{\nu}}{4} \int \frac{d^3 \mathbf{p}}{(2\pi \hbar)^3} \left\{ 2 \mathbf{v}^{(0)}_{\mathbf{p}} \mathbf{j}^{(2)}_{\lambda} + \left( \mathbf{v}^{(1)}_{\mathbf{p}} \cdot \mathbf{\Omega}^{(0)}_{\lambda} \right) \mathbf{B}^{\ast}_{\lambda,0} \mathbf{j}^{(1)}_{\lambda,0} - e \left( \mathbf{E}_{\lambda,0} \times \mathbf{\Omega}^{(0)}_{\lambda} \right) \mathbf{j}^{(1)}_{\lambda,0} 
+ \mathbf{v}^{(2)}_{\mathbf{p}} \mathbf{j}^{(2)}_{\lambda} - \frac{e}{e} \left( \mathbf{v}^{(0)}_{\mathbf{p}} \cdot \mathbf{\Omega}^{(0)}_{\lambda} \right) \mathbf{B}^{\ast}_{\lambda,0} \mathbf{j}^{(1)}_{\lambda,0} - e \left( \mathbf{E}^{(2)}_{\lambda,0} \times \mathbf{\Omega}^{(0)}_{\lambda} \right) \mathbf{j}^{(1)}_{\lambda,0} - e \left( \mathbf{E}_{\lambda,0} \times \mathbf{\Omega}^{(1)}_{\lambda,0} \right) \mathbf{j}^{(1)}_{\lambda,0} 
+ \left( \mathbf{v}^{(1)}_{\mathbf{p}} \right)^{\ast} \left( \mathbf{V}^{(1)}_{\lambda,0} + \epsilon^{(1)}_{\mathbf{p}} \mathbf{j}^{(1)}_{\lambda,0} \right) - e \left( \mathbf{E}_{\lambda,0} \times \mathbf{\Omega}^{(1)}_{\lambda,0} \right) \left( \mathbf{V}^{(1)}_{\lambda,0} + \epsilon^{(1)}_{\mathbf{p}} \mathbf{j}^{(1)}_{\lambda,0} \right) \left( \partial_\mathbf{p} f^{(1)}_{\lambda,0} \right) 
- e \left( \mathbf{E}_{\lambda,0} \times \mathbf{\Omega}^{(0)}_{\lambda} \right) \left( \mathbf{V}^{(1)}_{\lambda,0} + \epsilon^{(1)}_{\mathbf{p}} \mathbf{j}^{(1)}_{\lambda,0} \right) \left( \partial_\mathbf{p} f^{(1)}_{\lambda,0} \right) + \mathbf{c} \mathbf{c} . \right\}
$$

(S53)

After straightforward but tedious calculation, we derive the following current density $\mathbf{j}^{dc} = \sum_{\lambda = \pm} \mathbf{j}^{(2)}_{\lambda}$:

$$
\mathbf{j}^{dc}_{\lambda} = \sigma_{abc} E_{5, b} E_{5, c}^* + \kappa_{abc} \frac{1}{2} \left( E_{5, b} B_{5, c}^* + E_{5, b} B_{5, c}^* \right) + \gamma_{abc} B_{5, b} B_{5, c}^* .
$$

(S54)

Here the response tensors read as

$$
\sigma_{abc} = -e \sum_{\lambda = \pm} \frac{2}{4} \left\{ - q_0 \delta_{\lambda c} \frac{e v_F^2 \tau^2}{30} (1 + 2i \omega \tau) C_2 - \delta_{a b c} \frac{e v_F^2 \tau^2}{15} \left[ 4 e v_F \tau^3 \omega \frac{5}{3} - i \omega \tau \mathbf{V}^{(1)}_{\lambda} (1 + i \omega \tau) + \mathbf{V}^{(1)}_{\lambda} \right] C_2 
+ q_0 q_b q_c \frac{\tau v_F^2}{3} \left( \mathbf{V}^{(1)}_{\lambda} + \epsilon^{(1)}_{\mathbf{p}} \mathbf{j}^{(1)}_{\lambda,0} \right) \left( \partial_\mathbf{p} f^{(1)}_{\lambda,0} \right) + \mathbf{c} \mathbf{c} , \right\}
$$

(S55)

$$
\kappa_{abc} = -e \sum_{\lambda = \pm} \frac{2}{4} \left\{ \delta_{a b c} \frac{\lambda c e h v_F}{15} \left[ \frac{e v_F^2 \tau^2}{30c} \left( 18 + 31i \omega \tau - 2 \omega^2 \tau^2 \right) - \mathbf{V}^{(1)}_{\lambda} (5 + 1 + 2 \omega^2 \tau^2) - 2 \omega \tau \mathbf{V}^{(1)}_{\lambda} (1 + i \omega \tau) \right] \frac{i e v_F h^2}{12} \left( 1 + 2i \omega \tau \right) C_2 + \epsilon_{a b c} q_b q_c \left( \mathbf{V}^{(1)}_{\lambda} \right)^{\ast} \frac{i e v_F h^2}{12} \left[ 1 - 2 (\omega \tau)^2 \right] C_3 + \mathbf{c} \mathbf{c} . \right\}
$$

(S56)
and

\[ \gamma_{abc} = \frac{e}{4} \sum_{\lambda=\pm} q_a \delta_{bc} \frac{e^2 \hbar^2 v_F^4 \omega \tau^2}{60c^2} \left[ \frac{\omega \tau}{2} (1 + 2i \omega \tau) - (1 + 3i \omega \tau) \right] F + \text{c.c.} \]  

(S57)

Notice that the acoustic frequency is quite small and it is legitimate to expand it as small \( \omega \tau \). In this regime, we can approximate the above response functions as

\[ \sigma_{abc} \approx -q_a \delta_{bc} \frac{4e^3 v_F^3 \tau^4 \omega^2}{15} C_2 + q_b \delta_{ac} \frac{e^3 v_F^3 \tau^2}{9 \omega} C_2 - q_c \delta_{ab} \frac{e^3 v_F^3 \tau^2}{135 \omega} C_2 - \delta_{ab} q_c \frac{2e^3 v_F^3 \tau^2 q^2}{45 \omega} C_2 + O[(\omega \tau)^3], \]  

(S58)

\[ \kappa_{abc} \approx \epsilon_{abc} \frac{e^3}{12c} \left( 2v_F^3 \tau^2 C_2 - 6 \hbar^2 F + v_F \hbar^2 F \right) + \sum_{j=x,y,z} \epsilon_{ajc} q_j \frac{e^3}{18c} \left( v_F^5 \tau^4 C_2 - 3 \hbar^2 v_F^2 \tau^2 F \right) + O[(\omega \tau)^2], \]  

(S59)

\[ \gamma_{abc} \approx q_a \delta_{bc} \frac{e^3 v_F^3 \tau^2 \omega}{60c^2} F + O[(\omega \tau)^2]. \]  

(S60)

In addition to the shorthand notations in Eqs. (S39)–(S41), we introduced

\[ F = \sum_{\eta=\pm} \int \frac{d^3 p}{(2\pi \hbar)^3} \frac{1}{p^3} \partial_{\eta \mu} f^\text{eq}_{\lambda} = -\frac{1}{2\pi^2 \hbar^2 T} F_0 \left( \frac{\mu}{T} \right), \]  

(S61)

\[ \tilde{F} = \sum_{\eta=\pm} \int \frac{d^3 p}{(2\pi \hbar)^3} \frac{1}{p^3} f^\text{eq}_{\lambda} = \frac{1}{2\pi^2 \hbar^3} \int_{\Lambda_{\text{IR}}}^{\infty} dp \int_{\Lambda_{\text{IR}}}^{\infty} \frac{d\eta_F}{\eta} \int_{\Lambda_{\text{IR}}}^{\infty} \frac{d\eta}{\eta} \left[ \frac{1}{\Lambda_{\text{IR}}} f^\text{eq}_{\lambda} + \eta v_F \int_{\Lambda_{\text{IR}}}^{\infty} \frac{dp}{p} \partial_{\eta \mu} f^\text{eq}_{\lambda} \right] \]  

(S62)

We present the function \( F_0(x) \) in the left panel of Fig. S1. High- and low-temperature asymptotes of \( F_0(x) \) equal \( F_0(x) \approx 7\zeta(3)x/(2\pi^2) \approx 0.426x \) for \( x \to 0 \) and \( F_0(x) \approx x^{-1} \) for \( x \to \infty \), respectively. The function \( F_0(x) \) could be approximated by the Padé approximant of order \([5/6]\) as

\[ F_0(x) \approx \frac{7\zeta(3)}{2\pi^2} x + 0.03533x^3 + 0.0007432x^5 \]  

(S63)

The function \( \tilde{F}(x,y) \) is presented in the right panel of Fig. S1 for a few values of \( y \). It is clear that it has a \( 1/y \) dependence and quickly reaches a constant value at large values of \( x \). In our numerical calculations, we introduced the infrared cutoff \( \Lambda_{\text{IR}} \approx 10\sqrt{\hbar/eB_5}/c \). Such a cutoff separates the phase space of large momenta, where the semiclassical description provided by the chiral kinetic theory is valid, from the infrared region \( p < \Lambda_{\text{IR}} \), where such a description fails (for details, see also the discussion in Ref. [S3]). We believe that the appearance of such divergences is an artifact of the second order chiral kinetic theory, which is manifest for certain Hall-like responses \( \sim E_x \times B_\lambda \). On the other hand, since the presence of terms \( \sim \Lambda_{\text{IR}}^{-1} \) does not affect our qualitative conclusion regarding the possibility of the acoustogalvanic rectification, we leave the investigation of a proper treatment of such divergent terms for future studies.

**S II.C. Acoustogalvanic susceptibility**

Since the pseudo-electromagnetic fields \( E_5 \) and \( B_5 \) are secondary fields induced by the displacement vector \( u \), it is convenient to introduce the rewrite the electric current (S54) in terms of these fields as

\[ j_{\mu}^{\text{AC}} = \chi_{abc} u_b u_c^\ast, \]  

(S64)

where \( \chi_{abc}^{\text{AC}} \) is defined as the acoustogalvanic susceptibility. By using Eq. (S16), we derive the following component of the acoustogalvanic response function \( \chi_{zzz}^{\text{AC}} \)

\[ \chi_{zzz}^{\text{AC}} = \frac{\omega^4 \hbar^2 b^2}{e^2} \left[ \beta + \tilde{\beta}(b) \right]^2 \sigma_{zzz}. \]  

(S65)
The corresponding components of the acoustogalvanic tensor \( \chi^{AG}_{xxx} = \chi^\sigma_{xxx} + \chi^\kappa_{xxx} + \chi^\gamma_{xxx} \) for small \( \omega \tau \) are

\[
\begin{align*}
\chi^\sigma_{xxx} &= \frac{e \mu v_F b^2 \tilde{\beta}^2 r^4 \omega^6}{30 \pi^2 \hbar v_s^3}, \\
\chi^\kappa_{xxx} &= -\frac{e \mu v_F \tilde{\beta}^2 r^2 \omega^4}{12 \pi^2 \hbar v_s^3}, \\
\chi^\gamma_{xxx} &= \frac{\hbar^2 e^2 b^2 \tilde{\beta}^2 r^2 \omega^6}{e^2 v_s^3} G_2 (\mu, T).
\end{align*}
\]

Functions \( G_1 (\mu, T, \Lambda_{IR}) \) and \( G_2 (\mu, T) \) (see also Eqs. (13) and (14) in the main text) are related to the functions \( F_0 \) and \( \tilde{F}_0 \) defined in Eqs. (S61) and (S62) as

\[
\begin{align*}
G_1 (\mu, T, \Lambda_{IR}) &= \frac{e^3 v_F}{2 \pi^2 \hbar c T} \left( \tilde{F}_0 + \frac{1}{6} F_0 \right), \\
G_2 (\mu, T) &= -\frac{e^3 v_F^2}{120 \pi^2 \hbar c v_s T} F_0.
\end{align*}
\]

### S III. USEFUL FORMULAS

In this section, several useful formulas related to the integration over momenta and angles are presented. By making use of the short-hand notation for the global equilibrium Fermi–Dirac distribution function \( f_\lambda^{eq} = 1/[e^{(\epsilon_F - \mu_\lambda)/T} + 1] \) at \( \eta = + \), it is straightforward to derive the following formulas:

\[
\begin{align*}
\int \frac{d^3 p}{(2\pi \hbar)^3} p^{n-2} F(\theta) f_\lambda^{eq} &= -\frac{T^{n+1} \Gamma(n + 1)}{4\pi^2 \hbar^3 v_F^{n+1}} \text{Li}_{n+1} \left(-e^{\mu_\lambda/T}\right) \int_0^1 d \cos \theta F(\theta), \\
\int \frac{d^3 p}{(2\pi \hbar)^3} p^{n-2} F(\theta) \frac{\partial f_\lambda^{eq}}{\partial p} &= \frac{T^n \Gamma(n + 1)}{4\pi^2 \hbar^3 v_F^n} \text{Li}_n \left(-e^{\mu_\lambda/T}\right) \int_0^1 d \cos \theta F(\theta), \\
\int \frac{d^3 p}{(2\pi \hbar)^3} p^{n-2} F(\theta) \frac{\partial^2 f_\lambda^{eq}}{\partial p^2} &= -\frac{T^{n-1} \Gamma(n + 1)}{4\pi^2 \hbar^3 v_F^{n-1}} \text{Li}_{n-1} \left(-e^{\mu_\lambda/T}\right) \int_0^1 d \cos \theta F(\theta),
\end{align*}
\]

where \( T \partial f_\lambda^{eq}/\partial p = -v_F T \partial f_\lambda^{eq}/\partial \mu_\lambda = -v_F (e^{(\epsilon_F - \mu_\lambda)/T}/[e^{(\epsilon_F - \mu_\lambda)/T} + 1])^2, \ n \geq 0, \text{Li}_n(x) \) is the polylogarithm function, and \( F(\theta) \) is a function that depends only on the polar angle \( \theta \). The polylogarithm functions at \( n = 0, 1 \) can be rewritten...
in terms of the elementary functions

\[ \text{Li}_0 (-e^x) = - \frac{1}{1 + e^{-x}}, \]  
\[ \text{Li}_1 (-e^x) = - \ln (1 + e^x). \]  

(S74)

(S75)

The following identities are useful when summing over \( \eta = \pm \):

\[ \text{Li}_0 (-e^x) + \text{Li}_0 (-e^{-x}) = -1, \]  
\[ \text{Li}_1 (-e^x) - \text{Li}_1 (-e^{-x}) = -x, \]  
\[ \text{Li}_2 (-e^x) + \text{Li}_2 (-e^{-x}) = - \frac{1}{2} \left( x^2 + \frac{\pi^2}{3} \right). \]  

(S76)

(S77)

(S78)

Finally, by integrating over the angular coordinates, one can derive the following general relations:

\[ \int \frac{d^3p}{(2\pi\hbar)^3} pf(p^2) = 0, \]  
\[ \int \frac{d^3p}{(2\pi\hbar)^3} p(p\cdot a)f(p^2) = \frac{a}{3} \int \frac{d^3p}{(2\pi\hbar)^3} p^2 f(p^2), \]  
\[ \int \frac{d^3p}{(2\pi\hbar)^3} p(p\cdot a)(p\cdot b)f(p^2) = 0, \]  
\[ \int \frac{d^3p}{(2\pi\hbar)^3} p(p\cdot a)(p\cdot b)(p\cdot c)f(p^2) = \frac{a(b\cdot c) + b(a\cdot c) + c(a\cdot b)}{15} \int \frac{d^3p}{(2\pi\hbar)^3} p^2 f(p^2). \]  

(S79)

(S80)

(S81)

(S82)

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