Passivity-Based Decentralized Criteria for Small-Signal Stability of Power Systems With Converter-Interfaced Generation

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Abstract—With the increasing penetration of converter-interfaced generation, it would be advantageous to specify local grid-connectivity criteria to ensure small-signal stability of the interconnected system, rather than carrying out centralized stability studies every time the connection of a new device is contemplated. Passivity of the device admittance, which is an example of a local criterion, has been used previously to avoid resonances between converter-interfaced systems and the lightly damped oscillatory modes of the network, but it is unsuitable in general due to the conservative nature of passivity. Therefore, a composite set of criteria are proposed in this paper, which include time-scale separation (slow-fast clustering) constraints on the transient behaviour, and distinct passivity constraints for the slow and fast clusters. The application of these criteria is demonstrated using a case study.

Index Terms—Distributed generation, droop control, inverter-based resources, passive systems, small-signal stability, virtual synchronous generator control.

LIST OF SYMBOLS

| Symbol | Description |
|--------|-------------|
| $\Omega$ | Complex frequency (in rad/s) |
| $u(t)$ | Input vector |
| $y(t)$ | Output vector |
| $x(t)$ | State variables |
| $S(x)$ | Storage function |
| $G(s)$ | Transfer function matrix |
| $G^R(j\Omega)$ | $G(j\Omega) + G^T(-j\Omega)$ |
| $\Omega_p$ | Poles on the imaginary axis |
| $i_D, i_Q$ | D-Q components of current absorbed by shunt devices |
| $i_{nD}, i_{nQ}$ | D-Q components of current injected into the network |
| $v_{nD}, v_{nQ}$ | Bus voltages |
| $Y_s(j\Omega)$ | Small-signal D-Q admittance of shunt devices |
| $Z_{nDQ}(s)$ | D-Q impedance of the T&D network |
| $Y_n(s)$ | D-Q admittance of the T&D network |
| $P, Q$ | Active and reactive power absorbed by shunt devices |
| $P_n, Q_n$ | Active and reactive power injected into the network |
| $\delta$ | Angular position of rotor |
| $\omega_r$ | Electrical speed of rotor |
| $M$ | Inertia |
| $E_g$ | Internal voltage magnitude |
| $x_g$ | Equivalent reactance |
| $\phi$ | Bus voltage phase angle |
| $V$ | Bus voltage magnitude |
| $V_n$ | Normalized bus voltage magnitude |
| $\tilde{\omega}$ | Calculated angular bus frequency |
| $\tilde{V}_n$ | Calculated rate-of-change of $V_n$ |
| $\tau$ | Derivative time constant |
| $k_{pf}, k_{pv}$ | Droop gain of active power with $\tilde{\omega}$ and $V_n$, respectively |
| $k_{qf}, k_{qv}$ | Droop gain of reactive power with $\tilde{\omega}$ and $V_n$, respectively |
| $J_n(s)$ | Wide-band Jacobian matrix of T&D network |
| $J_{sh}(s)$ | Wide-band Jacobian matrix of shunt devices |
| $J_{LF}$ | Steady-state Jacobian model of the network $= J_n(0)$ |
| $J_{sd}(s)$ | Approximate low-frequency model of the network $= J_{sh}(s) \times \frac{1+s\tau}{s}$ |
| $N_{sd}(s)$ | Inverse of $J_{sd}(s)$ |
| $N_{sd}(j\Omega)$ | $N_{sd}(j\Omega) + N_{sd}^T(-j\Omega)$ |
| $\Omega_l$ | Upper bound of the low frequency range |
| $\Omega_h$ | Lower bound of the high frequency range |
| $\lambda$ | Eigenvalues of the state-space model |
| $k_{cv}$ | $Q_n-V_n$ droop specified by network/system operator |
| $\Delta$ | Small deviations about the equilibrium point |
| $\omega$ | Quiescent value of the corresponding variables |

I. INTRODUCTION

A POWER system consists of a large number of devices connected to a Transmission and Distribution (T&D) network. Many of these devices are interfaced with the network using...
power electronic converters. Adverse dynamic interactions between the devices and the network lead to oscillatory instabilities such as low-frequency power swings, Sub-synchronous Resonance (SSR), Sub-synchronous Control Interactions (SSCI), Sub-synchronous Torsional Interactions (SSTI), Harmonic Resonances and Induction Generator effect [1]–[4]. These instabilities can lead to operational limitations or costly outages. Hence, stability assessment is an important exercise in the planning, design and operation of a power system. The aim is to anticipate, diagnose and mitigate such instabilities. Tools like time-domain simulation and eigenvalue computation are commonly used for this purpose. Large-scale stability studies can be quite involved, prompting the search for simpler alternatives.

A. Stability Assessment: A Decentralized Approach

For the assessment of stability one needs to consider a large set of operating conditions of the network and the connected devices. A fresh analysis has to be done whenever the connection of a new device to the network is contemplated. In general, the control strategies and parameters of all devices are needed to carry out the analysis. These onerous requirements can be alleviated to some extent by the use of reduced system equivalents around each device for specific device-network interactions. However, significant effort and engineering judgement are involved in determining the extent of the reduced system and the representation of the rest of the system by equivalents at its boundaries. Stability assessment has become even more challenging with the proliferation of distributed generation systems, many of which have non-standardized controllers. Therefore, the idea of having decentralized grid-connectivity criteria that would ensure the stability of the interconnected system has great appeal. The decentralized approach is depicted in Fig. 1. This involves checking compliance with the pre-defined criteria for each connected device individually. Ideally, the assessment of compliance would require only local information. Connection of a new device would involve the incremental effort of assessing only its compliance with the criteria. Decentralization also implies that the compliance assessment need not be carried out by a single entity, i.e., delegation of the task is possible.

Any scheme for ensuring stability that is based on local criteria is likely to be conservative (sufficient, but not necessary for ensuring stability). For the scheme to be of practical utility, the criteria should not be onerous or downright impossible to comply with. The criteria should also be such that compliance is easily verifiable through time-domain or frequency-domain analyses based on analytical/simulation models or measurements.

B. Decentralized Criteria Based on Passivity

Passivity [5] is a property of dynamical systems on which decentralized criteria can be based. It is well-suited for this role because of the following reasons.

a) Passive systems are stable.

b) The system formed by the negative feedback connection of passive sub-systems is also passive. Therefore the passivity of each sub-system can be individually ascertained with minimal information about the other sub-systems.

c) The model of a T&D network consisting of transmission lines, transformers, capacitors and inductors is inherently passive when it is formulated with currents and voltages as interface (input or output) variables. Topological changes in the network (for example, due to the addition/tripping of lines) or changes in the operating conditions do not affect the passivity of the network. The main objective then would be to have each of the sub-systems that are connected to the T&D network locally conform to the passivity constraint, in the manner of Fig. 1.

d) The definition of a sub-system can be flexible; it could be an individual device or may encompass a sub-network with several devices. What matters is whether the sub-system is passive as seen from its boundary buses.

e) For linear time-invariant (LTI) systems, passivity can be conveniently ascertained using frequency domain conditions. Note that LTI models are appropriate for the study of many adverse interactions, as these can often be traced to small-signal instabilities.

The application of passivity as the sole criterion for stability, however, encounters some difficulties, as discussed next.

C. Prior Work and Limitations of Existing Approaches

The “self-disciplined stabilization” concept presented in [6] is similar to the scheme envisaged in Fig. 1, but is applied to the Single-Input Single-Output devices in a DC microgrid. In three-phase AC systems, the devices are Multi-Input Multi-Output systems, and as a consequence the analysis is more involved. The idea of emulation of passive network characteristics using active power filters in order to avoid adverse network-device interactions in AC grids is proposed in [7], [8]. The passivity criterion has been applied in a limited and approximate manner for the stability analysis of voltage source converter (VSC) based devices [9]–[12] and HVDC converters [13]. In these papers, passivity is sought to be achieved locally around the network resonant frequencies only and is not explored over the entire frequency domain. This approach requires knowledge about the grid and other devices connected to it because the combined system determines the resonant frequencies.

The passivity of a system consisting of a T&D network with a STATCOM (in the low frequency range) is explored in [14]. It is shown that the coupled system can be passivated, but this is not a decentralized approach. Passivity based stabilization of
DC-AC grids with inter-linking converters is presented in [15]. The passivity criteria are presented in terms of the DC side variables (DC voltage and current) and AC side variables (active and reactive power and bus voltage magnitude and frequency). The analysis is restricted to lossless AC network models with decoupled active and reactive power flow equations. Furthermore, it is assumed that the DC side and AC side generator models (with these variables) are passive.

The passivity of converters and synchronous machines with their controllers is analysed in the authors’ previous work [16]. Passivity of the admittance in D-Q variables is shown to be a reasonable and achievable objective for the high-frequency models of the devices. It is also shown in the same paper that the admittances of typical active and reactive power injection devices inherently violate the passivity constraints at low frequencies, although this does not imply instability. Therefore, the passivity constraint on admittance is too restrictive for wide-band device models.

Overall, it can be said that there is a significant body of work on the passivation of power system components, but there are practical limitations in this approach due to the conservative nature of passivity. The holistic development of a practical passivity-based decentralized scheme is needed, which addresses the limitations of a single-criterion approach. This is the motivation and focus of this paper.

D. Contributions of This Paper

This paper proposes a composite set of criteria that overcomes the limitations of a single criterion based on passivity, while preserving the decentralized nature of the stability assessment scheme. The criteria are given below:

a) The poles of the device transfer function should be separable into two well-separated clusters based on their magnitude. In other words, the natural transients should exhibit a slow-fast separation in the time domain (low and high frequency separation in the frequency domain). This will permit a decoupled analysis of slow and fast transients.

b) The admittance transfer function obtained using the rectangular (D-Q) components of terminal currents and voltages should satisfy the frequency-domain passivity conditions in the high frequency range.

c) The transfer function between the derivatives of the polar components of voltage and the active and reactive power drawn should satisfy the frequency-domain passivity conditions in the low frequency range.

d) To avoid the problem of frequency drift and network non-passivity in the low frequency range, a few devices have to provide active power-frequency and reactive power-voltage dependence, as specified by the network/system operator.

Since most control strategies and transients in a power system exhibit time-scale separation characteristics, the first criterion is not unreasonable. Avoidance of grid resonances in the high frequency domain through passivity has already been demonstrated in earlier work [7]–[13]. It is shown in this paper that in the low frequency range and with the designated interface variables, devices having typical droop-control or virtual synchronous machine characteristics can be made passive. Hence, these criteria are expected to be reasonable for a large class of devices. Moreover, compliance can be verified numerically in the frequency-domain. Where analytical models or the internal details of the device and controller are not available, black-box simulation models or measurements may be used for obtaining the frequency responses.

The paper is organized as follows. The background and motivation for the passivity approach are presented in the next section. This is followed by Sections III and IV which describe the rectangular and polar formulations of the transfer functions of the network and devices. The limitations of using a single formulation to develop a passivity-based scheme are also brought out in these sections. The composite set of criteria to overcome the limitations is given in Section V, along with an illustrative case study in Section VI. A discussion on some open issues is presented in Section VII, followed by the conclusions.

II. PASSIVE SYSTEMS

A system having a set of inputs $u(t)$ and an equal number of outputs $y(t)$ is said to be passive [5] if,

$$u(t) \geq \frac{dS(x)}{dt}$$  \hspace{1cm} (1)$$

for all $u(t)$ and $x(t)$, and for all $t \geq 0$. $S(x)$ is a continuously differentiable positive semi-definite function of the states $x$, which is also called the storage function.

A. Passivity of LTI Systems (Frequency-Domain Conditions)

An LTI system represented by an $n \times n$ rational, proper transfer function matrix $G(s)$ is passive if

a) there are no poles in the right half $s$-plane (complex plane).

b) The matrix $G^R(j\Omega) = G(j\Omega) + G^H(j\Omega) = G(j\Omega) + G^T(-j\Omega)$ is positive semi-definite for all $\Omega \in (-\infty, \infty)$ which is not a pole of $G(s)$.

c) For all $j\Omega_p$ that are poles of $G(s)$, the poles must be simple [18] and $\lim_{\omega \to j\Omega_p} (s - j\Omega_p) G(s)$ should be positive semi-definite Hermitian.

Unlike (1) which requires us to find a suitable storage function $S(x)$ - which is not straightforward - the frequency domain conditions are convenient from a practical perspective since they can be directly evaluated. Note that the frequency response can be obtained either from an analytical model or by applying the frequency scanning technique [19] to a simulation model of the system, or from real-life measurements.

The superscripts $T$ and $H$ denote the transpose and conjugate-transpose operations respectively.
B. Useful Properties

The usefulness of the passivity criterion for stability assessment stems from the following properties.

a) Passivity is a sufficient condition for stability.

b) The inverse of a passive system is also passive, assuming that the state-space representation of the inverse system is well-defined.

c) A system formed by a negative feedback connection of passive sub-systems is also passive.

C. Application of Passivity to a Power System

Let us consider the application of the passivity based criterion to develop the decentralized scheme envisaged in Fig. 1. A power system is primarily a three-phase AC system. Devices like generators, loads, converters, etc. are connected in shunt to the T&D network. The interface variables (inputs and outputs) are the three-phase currents and voltages at the terminals of the devices. From Kirchoff’s Current Law it is evident that the overall system can be represented mathematically as a negative feedback connection of the devices to the T&D network as shown in Fig. 2. If the devices and the network are individually passive, then from the properties of passivity given in the previous sub-section the overall system is also passive.

Remark: A T&D network consisting of transmission lines, transformers, reactors and capacitors is inherently passive with terminal currents and voltages as the interface variables. This is because the storage function can be chosen to be the electromagnetic energy stored in the inductive and capacitive components; energy is dissipated in the resistive parts of these components, which results in (1) being satisfied. Topological changes in the network (due to the addition or removal of lines), or changes in the operating conditions do not affect the passivity of the T&D network.

III. MODELLING USING \( \Delta i_D, \Delta i_Q \) AND \( \Delta v_D, \Delta v_Q \) AS INTERFACE VARIABLES: “RECTANGULAR FORMULATION”

A. LTI Model of the System

Since the focus of this paper is on small-signal stability, the first step is to form the LTI models of the devices. Most power system devices can be made time-invariant or approximately so when they are expressed in the D-Q-o variables that are obtained using a synchronously rotating transformation [20] of the phase (a-b-c) variables. Two properties that facilitate passivity-based analysis in these transformed variables are derived in [16] and are stated below:

Property 1: Passivity of a sub-system is preserved under the D-Q-o transformation of the three-phase variables.

Property 1 ensures that the T&D network retains its passivity when the interface variables are chosen to be \((i_D, i_Q, i_o)\) and \((v_D, v_Q, v_o)\) instead of the corresponding a-b-c variables. Note that the zero sequence variables are generally stable, decoupled from the D-Q variables, and localized to a small part of the network. Therefore, they are not considered in the analysis that follows.
assess the passivity of the device admittances $Y_{sh}(s)$. Passivity compliance by the devices is sufficient to guarantee the passivity (stability) of the overall system.

B. Limitations of the Passivity-Based Scheme of Fig. 3

The scheme depicted in Fig. 3 is practical only if it is possible to make commonly-used devices passive in the rectangular formulation, if they are not so, to begin with. The results in [7]–[16] indicate that the frequency domain conditions of Section II-A can usually be satisfied by the D-Q admittance of many power system components in the high frequency range, typically above 10 Hz, through minor controller modifications. The conservatism associated with the decentralized assessment of passivity can be reduced in some cases by extending the boundary of the device to include a part of the resistance of the external T&D network, if available, as shown in Fig. 4 [16]. The aim is to increase passivity of the device as seen from the extended boundary while the external network passivity is still retained. However, it is also shown in [16] that despite these measures, the passivity conditions on the D-Q admittance are invariably violated in the low frequency range (typically 0 - 10 Hz) by many devices. The violation in the low frequency range is primarily due to the real and reactive power control strategies of devices, and therefore it cannot be alleviated by minor modifications in the controllers or by extending the device boundary. This passivity violation is exemplified by the low frequency behaviour of devices that emulate the Droop control strategy and the Virtual Synchronous Generator (VSG) control strategy.

Droop Strategy: The steady state active and reactive power drawn or injected by a wide class of power system components are often functions of frequency and voltage magnitude. The quasi-static model of a controlled device that follows a droop strategy is as follows.

$$\Delta P(s) = k_{pf} \Delta \omega(s) = k_{pf} s \frac{\Delta \phi(s)}{1 + sT}, \Delta Q(s) = k_{qm} \Delta V_n(s)$$

The polar components of the bus voltage ($\phi$, $V_n$) and the active ($P$) and reactive power ($Q$) absorbed by the device are related to the D-Q voltage and current components as follows.

$$\phi = \tan^{-1} \left( \frac{v_{nD}}{v_{nQ}} \right), V = \sqrt{v_{nD}^2 + v_{nQ}^2}, V_n = \frac{V}{V_o}$$

$$P = v_{nD}i_D + v_{nQ}i_Q, Q = v_{nD}i_Q - v_{nQ}i_D$$

The subscript 'o' denotes the quiescent value. $\tau$ is the time constant used to approximately compute the derivative of the phase angle and is generally quite small.

Property 3: The small-signal model of a converter which emulates (2) at low frequencies is non-passive in the rectangular formulation.

VSG Control Strategy: The VSG strategy is an alternative strategy for active and reactive power control of grid-connected converters. The controller is designed to emulate the characteristics of the classical model of a synchronous generator which is described by the following equations,

$$\frac{d\delta}{dt} = \omega_r - \omega_o,$$

$$M \frac{d\omega_r}{dt} = P_m - D_m \omega_r + P$$

$$v_{nD} + jv_{nQ} = E_g \alpha \delta + jx_g (i_Q + ji_D)$$

where $M, D_m, \omega_r, \delta, x_g$ denote the inertia, mechanical damping, electrical speed (in rad/s), rotor angle and transient reactance of the machine. Note that $E_g$ is constant. The schematic of the emulated system is shown in Fig. 5.

Property 4: The admittance corresponding to the small-signal model of a converter which emulates (4) at low frequencies is non-passive in the rectangular formulation.

The proofs of Properties 3 and 4 along with illustrative examples are given in [16]. Note that these properties hold regardless of the direction of the quiescent power flow. Since the low-frequency models given above represent the wide-band models at low frequencies, it follows that the wide-band models of the devices will also not satisfy the frequency-domain passivity conditions at low frequencies.

Remarks: Passivity is a sufficient criterion for stability and not a necessary one. The conservative nature of passivity can be an acceptable trade-off against the benefits of having a convenient-to-use local criterion, provided that passivity is an achievable objective. The foregoing analysis shows that frequency-domain passivity of the D-Q admittance matrix (rectangular formulation) is impossible to achieve with typical control strategies of converter-interfaced devices, although it is well known that an interconnected system with these devices can be operated stably. Since these control strategies are essential for voltage and frequency regulation, they cannot be abandoned or radically changed. Hence there is a need to study alternative formulations.
IV. MODELLING USING (ΔP, ΔQ) AND (Δω, ΔV^d) AS INTERFACE VARIABLES: “POLAR FORMULATION”

Since the low frequency behaviour of devices is often associated with strategies to regulate voltage magnitude and frequency, an alternative formulation with the active and reactive power drawn by the device (ΔP, ΔQ), and the derivatives of the polar components of the bus voltage (Δω, ΔV^d) as the interface variables is considered here. These variables are related to those given in (3) as follows.

\[ \Delta \omega(s) = \frac{s \Delta \phi(s)}{1 + s \tau}, \quad \Delta V^d_n(s) = \frac{s \Delta V_n(s)}{1 + s \tau} \]  

(5)

The computation of the derivatives in (5) is approximate, which preserves the proper-ness of the transfer functions. The derivative time constant \( \tau \) can be chosen to be approximately one period of the fundamental cycle i.e. 20 ms for a 50 Hz system. This representation is called the “Polar Formulation” in this paper.

A. Stability Analysis: Equivalence of the Rectangular and Polar Formulations

The following property clarifies the conditions under which the inferences regarding the stability of the overall system using the rectangular and polar formulations are equivalent.

**Property 5:** Stability of the closed-loop systems of the rectangular and polar formulations are equivalent, provided that the closed-loop system of the rectangular formulation does not have repeated poles at \( s = 0 \).

**Proof:** The notion of equivalence of the two formulations is depicted in Fig. 6. The expression of closed-loop transfer function \( G_R(s) \) is given as follows.

\[ G_R(s) = (Y_n(s) + Y_{sh}(s))^{-1}, \quad \text{where} \quad Y_n(s) = Z_{nDQ}(s)^{-1} \]  

(6)

For an \( m \) bus system, the following matrices are defined.

\[ v_{Do} = \text{diag}(v_{Do1}, ..., v_{Dmo}), v_{Qo} = \text{diag}(v_{Qo1}, ..., v_{Qmo}) \]

\[ i_{nDo} = \text{diag}(i_{nDo1}, ..., i_{nDmo}), i_{nQo} = \text{diag}(i_{nQo1}, ..., i_{nQmo}) \]

Note that \( \text{diag}(a, b) \) represents a diagonal matrix with \( a \) and \( b \) as the diagonal entries. The transfer functions of the individual components of the rectangular and polar formulations are related as follows.

\[ J_{nd}(s) = \frac{(1 + s \tau)}{s} \times J_n(s), \quad J_n(s) = (\mathcal{E} Y_n(s) + \mathcal{C}) \mathcal{F} \]

\[ J_{sd}(s) = \mathcal{N}_{sd}(s)^{-1} = \frac{(1 + s \tau)}{s} \times (\mathcal{E} Y_{sh}(s) - \mathcal{C}) \mathcal{F} / J_{sh}(s) \]  

(7)

where

\[ \mathcal{E} = \begin{bmatrix} v_{Do} & v_{Qo} \\ -v_{Qo} & v_{Do} \end{bmatrix}, \quad \mathcal{C} = \begin{bmatrix} i_{nDo} & i_{nQo} \\ i_{nQo} & -i_{nDo} \end{bmatrix}, \quad \mathcal{F} = \begin{bmatrix} v_{Qo} & v_{Do} \\ -v_{Do} & v_{Qo} \end{bmatrix} \]  

(8)

Note that \( Y_n(s) \) is defined in (6). The closed-loop transfer function \( G_P(s) \) is as follows.

\[ G_P(s) = (J_{sd}(s) + J_{nd}(s))^{-1} = \frac{s}{(1 + s \tau)} \times \mathcal{F}^{-1} G_R(s) \mathcal{E}^{-1} \]  

(9)

The expressions in (9) are derived using (7). Since \( \mathcal{F} \) and \( \mathcal{E} \) are both invertible (except for \( v_{Do} = v_{Qo} = 0 \)), the poles of both \( G_R(s) \) and \( G_P(s) \) are related as follows.

a) If there are no pole-zero cancellations on the RHS, then poles of \( G_P(s) \) are the same as of \( G_R(s) \), except for the additional stable pole at \( s = -\frac{1}{\tau} \). In this case, both models are equivalent from a stability perspective.

b) A pole-zero cancellation at \( s = -\frac{1}{\tau} \), if it were to occur, is not of concern since it is in the left half \( s \)-plane.

c) If \( G_R(s) \) has a pole at \( s = 0 \), which is cancelled by the zero at \( s = 0 \) in (9), then the stability is not affected provided that the cancelled pole is simple (non-repeated). Note that if there is a non-simple pole of \( G_R(s) \) at \( s = 0 \), then \( G_R(s) \) is unstable while \( G_P(s) \) can be stable.

**Remarks:** The closed-loop system of the rectangular formulation has non-simple (repeated) poles at \( s = 0 \) only if all devices emulating (4) have zero damping \( D_m \), and all devices emulating (2) have zero \( k_{pf} \). As positive \( D_m \) or \( k_{pf} \) is introduced in any device, one of the repeated poles at \( s = 0 \) moves towards the left, and the pole at \( s = 0 \) becomes simple. This can be understood in practical terms as follows: If \( D_m \) and \( k_{pf} \) are zero for all connected devices then the frequency will change monotonically if there is a load-generation imbalance. At least one device needs to have active power-frequency dependence in steady state for stability. The condition for stability equivalence derived here is, therefore, consistent with a well-known requirement for stable power system operation.

Despite the (conditional) equivalence of the closed-loop stability in both model formulations, the passivity characteristics of
the individual transfer functions $Z_{nDQ}(s), Y_{sh}(s), J_{nd}(s)$ and $N_{sd}(s)$ can be different.

B. Passivity of the T&D Network in the Polar Formulation

The following property which is proved in [21], implies that a wide-band T&D network model is not passive in the polar formulation.

Property 6: The wide-band model of any R-L-C network with $(ΔP, ΔQ) - (Δω, ΔV_n)$ as interface variables is not passive.

Low frequency model of the network: While the wide-band model of a T&D network in the polar formulation is not passive, it is instructive to study the passivity of the low frequency model. Note that if

$$\begin{bmatrix} ΔP_n(s) \\ ΔQ_n(s) \end{bmatrix} = J_n(s) \begin{bmatrix} Δφ(s) \\ ΔV_n(s) \end{bmatrix}$$  \hspace{1cm} (10)

then the transfer function of the network in the polar formulation is $J_{ndLF}(s) = \frac{(1+στ)}{s} J_n(0)$. For typical parameters, the natural modes of the T&D network model (in D-Q variables) usually lie in the high frequency range. Hence the low frequency model of the T&D network in polar variables, $J_{ndLF}(s)$, can be approximated as follows.

$$J_{ndLF}(s) \approx \frac{(1+στ)}{s} J_n(0) = \frac{(1+στ)}{s} J_{LF} \hspace{1cm} (11)$$

where $J_{LF}$ is the un-reduced static load-flow Jacobian matrix. $J_{ndLF}(s)$ has a simple pole at $s = 0$. For it to be passive, the residue evaluated at $s = 0$, i.e., $J_{LF}$, has to be positive semi-definite Hermitian (see Section II-A).

Normally, $J_{LF}$ is not Hermitian because of the resistances in the network. If we assume that losses are small in the network, then the resistances can be neglected in the low frequency model. A more significant issue is that $J_{LF}$ may not be positive semi-definite due to the shunt capacitances of the network [21]. To overcome this problem at least a few shunt connected devices should provide voltage regulation, as shown in Fig. 7. This may be achieved through simple droop control of reactive power [21]: $ΔQ = k_q  ω \Delta V_n$. The required droop factor $k_q$ that compensates the effect of the shunt capacitances should be specified and provisioned for by the network/system operator. With this compensation in place, the low frequency model of the network, $J_{LF}$ and therefore, $J_{ndLF}(s)$ can be considered as a passive sub-system.

C. Passivity of Shunt Devices in the Polar Formulation

1) Droop Strategy: The representation of converters emulating droop control strategy is given in (2). Substituting (5), these devices can be represented in terms of the polar model as follows.

$$\begin{bmatrix} Δφ(s) \\ ΔV_n(s) \end{bmatrix} = \begin{bmatrix} \frac{1}{k_{pf}} × \frac{(1+στ)}{s} \\ 0 \frac{1}{k_{qq}} \end{bmatrix} \begin{bmatrix} ΔP_n(s) \\ ΔQ_n(s) \end{bmatrix}$$  \hspace{1cm} (12)

Using (5), the device representation in terms of the polar input-output variables are as follows.

$$\begin{bmatrix} Δ\dot{ω}(s) \\ Δ\dot{V}_n^?(s) \end{bmatrix} = \begin{bmatrix} \frac{1}{k_{pf}} \frac{1}{k_{qg}} \frac{s}{(1+Ωτ)} & 0 \\ 0 & \frac{2}{k_{r}} \frac{2}{k_{re}} \frac{τΩ^2}{(1+Ωτ)^2} \end{bmatrix} \begin{bmatrix} ΔP(s) \\ ΔQ(s) \end{bmatrix}$$  \hspace{1cm} (13)

$$ΔN_{sd}^R(jΩ) = ΔN_{sd}(jΩ) + ΔN_{sd}(−jΩ)$$ is given by,

$$ΔN_{sd}^R(jΩ) = \begin{bmatrix} \frac{2}{k_{pf}} & 0 \\ 0 & \frac{2}{k_{qq}} \frac{2}{k_{re}} \frac{τΩ^2}{(1+Ωτ)^2} \end{bmatrix}$$  \hspace{1cm} (14)

Since $ΔN_{sd}^R(jΩ) > 0$ if $k_{pf}, k_{qg}, τ > 0$, a converter emulating the droop control strategy is passive.

2) VSG Strategy: Devices emulating VSG strategy can be modelled by the dynamical equations in (4). For such devices, $N_{sd}(s)$ is as given below.

$$N_{sd}(1,1) = \frac{1}{(Ms + D_m)} \left( \frac{s x_g (2V_o - E_g \cos ζ_o)}{E_g V_o (2V_o \cos ζ_o - E_g)} \right) \frac{1}{(1+στ)}$$

$$N_{sd}(2,1) = \frac{s x_g \sin ζ_o}{V_o (2V_o \cos ζ_o - E_g)} \frac{1}{(1+στ)}$$

$$N_{sd}(2,2) = \frac{s x_g \cos ζ_o}{V_o (2V_o \cos ζ_o - E_g)} \frac{1}{(1+στ)}$$

where $ζ_o = δ_o - tan^{-1}(\frac{2D_m}{M})$. $V_o$ denotes the quiescent terminal voltage magnitude, as given in (3). The detailed derivation of $N_{sd}(s)$ from the non-linear state-space equations is given in Appendix A. Since the poles of $N_{sd}(s)$ are $-\frac{D_m}{M}$ and $-\frac{1}{τ}$, it is stable.

If $στ$ is small, then $ΔN_{sd}^R(jΩ)$ is positive semi-definite because (a) its trace is $\frac{2D_m}{M} > 0$ for $M, D_m > 0$, and (b) its determinant is zero for all $Ω$. Hence $N_{sd}(s)$ of VSG based converters satisfies the frequency-domain passivity conditions at low frequencies.

Remarks: Although the low frequency model of the T&D network and converters (emulating droop control and VSG control strategies) satisfy the frequency domain passivity conditions in the polar formulation, the wide-band model of the T&D network is not passive in these variables (see Property 6 in Section IV-B). This restricts the utility of the polar formulation to the low frequency models only.

It is instructive to compare the polar formulation using the variables $[ΔP, ΔQ] - (Δω, ΔV_n)$ with a formulation that uses a related set of variables $[(ΔP, ΔQ) - (Δω, ΔV_n)]$ [15]. It is shown in [21] that with the $[(ΔP, ΔQ) - (Δω, ΔV_n)]$ formulation, a lossless T&D network is passive only if $[ΔP - ΔV_n]$ and $[ΔQ - Δω]$ dependencies are neglected, which may not always

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be realistic. Therefore, the polar formulation with \([\{\Delta P, \Delta Q\}, -\{\Delta \omega, \Delta V_{sh}^{dc}\}]\) is more suitable for the decentralized scheme.

### V. REvised CRiteria FOR THE DECENTRALIZED SCHEME

#### A. Time-Scale Separation to Facilitate Decoupled Analysis

Decentralized compliance with the frequency-domain passivity conditions cannot be achieved by a single wide-band model formulation, regardless of whether the overall system is stable or not. The models involving the rectangular and polar formulations, however, have mutually exclusive frequency ranges in which compliance with the passivity conditions is possible. Therefore, a decoupled analysis for the low and high frequency approximations of the system could make the scheme viable: the frequency-domain passivity criteria could be applied to the rectangular formulation at high frequencies, while they could be applied to the polar formulation at low frequencies.

Decoupled stability analysis of low and high frequency (slow and fast) approximations of the wide-band models is mathematically justifiable if they are non-interacting. A necessary condition for this is that the eigenvalues of the system should be separable into slow and fast clusters in the complex plane. The stability of the system with time-scale separated dynamics can then be predicted from the independent analysis of the eigenvalue clusters [22].

Slow-fast separation of dynamic behaviour is not an unreasonable requirement. Most power system components have this inherent feature. Generally, network resonant frequencies are well-removed from the modal frequencies associated with relative angular dynamics [1], [20], frequency, and voltage regulation. Controllers of power electronic converters generally have a hierarchical structure with faster “inner loops” associated with current control and firing angle generation, and relatively slower “outer loops” for voltage regulation and power modulation. It is commonly observed in power systems that the slow-fast eigenvalue separation of the individual devices and the network is preserved even when they are interconnected. Hence the separability of eigenvalues of individual devices can be imposed as an additional constraint to facilitate the decoupled passivity analysis in the polar and rectangular formulation. This is depicted in Fig. 8.

#### B. The Overall Criteria and Application Procedure

Based on the foregoing discussions, a composite set of criteria for the decentralized scheme are now presented along with the procedure to verify compliance.

1) **Tasks of the Network/System Operator:** (a) Specify the limiting value of frequencies for the slow and fast clusters in which the eigenvalues of the connected devices should lie, i.e., \(0 \leq |\lambda| < \Omega_l\) and \(|\lambda| \geq \Omega_h\). For a 50 Hz system and for T&D networks without fixed capacitor series compensation, the suggested values are 62.8 rad/s (10 Hz) for \(\Omega_l\) and 220 rad/s (35 Hz) for \(\Omega_h\). This is based on the fact that in networks without series capacitors, the network resonant frequencies in the D-Q frame are greater than or equal to the synchronous frequency (50 Hz). The thresholds suggested here are not hard and fast. When series capacitors are present, there are network modes present at sub-synchronous frequencies as well [3], typically around 30 Hz for commonly used levels of compensation. In such cases, it is suggested that \(\Omega_l\) and \(\Omega_h\) be reduced to 5 Hz and 20 Hz respectively in order to have sufficient margin between \(\Omega_l\) and the sub-synchronous network modes, at least for devices situated at locations where these modes are observable.

(b) For those devices that can provide ancillary services of voltage and frequency regulation, specify the values of \(k_{\text{QV}}\) and \(k_{\text{PF}}\). This will require the operator to examine the passivity of the Jacobian matrices for credible operating conditions. Note that \(k_{\text{QV}}\) is the value specified by the network/system operator to compensate for non-passivity of the T&D network due to shunt capacitances, as discussed in Section IV-B. \(k_{\text{PF}}\) is the power-frequency droop to avoid the presence of repeated poles at \(s = 0\) in the closed-loop system, as discussed in Section IV-A.

2) **Decentralized Tasks:** Device compliance checks may be undertaken by entities who have been assigned this task. The checks may be done by analyzing the frequency responses as given below.

a) Obtain the frequency response of the D-Q admittance matrix \(Y_{sh}(s)\) of the device. This may either be derived analytically or extracted numerically using the frequency scanning technique [19]. The scanning may be done over a sufficiently wide frequency range (a ballpark range is 0–200 Hz) so that it encompasses the controller bandwidth.

b) Obtain a state space model corresponding to \(Y_{sh}(s)\) of the device. A vector fitting algorithm (rectangular formulation) satisfying condition (b) of Section II-A in the high frequency range \(\Omega \geq \Omega_h\).

c) Ensure that the magnitude of the eigenvalues of the fitted state-space (wide-band) model of the device, |\(\lambda\)|, are well-separated into two clusters: \(0 \leq |\lambda| < \Omega_l\) and \(|\lambda| \geq \Omega_h\), with no eigenvalues satisfying \(\Omega_l \leq |\lambda| \leq \Omega_h\).

d) Ensure that the admittance model (rectangular formulation) satisfies condition (b) of Section II-A in the high frequency range \(\Omega \geq \Omega_h\).

e) Obtain \(J_{sh}(s)\) from \(Y_{sh}(s)\) using (7). For devices that can provide voltage regulation service ensure that the real part of the (2,2) entry of \(J_{sh}(j\Omega)\) is \(\geq k_{\text{QV}}\) in the low frequency range.
range (between 0 and $\Omega_l$), $k_{qv}^c$, being the value specified by the network/system operator. After subtracting $k_{qv}^c$, evaluate the transfer function $J_{sd}(s)$, and subsequently evaluate $N_{sd}(s)$.

f) For the device(s) which is(are) designated to provide frequency regulation, the real part of the (1,1) term of $J_{sd}(s)$ should be greater than or equal to the power-frequency droop $k_{pf}$ specified by the system/network operator in the low frequency range.

g) Ensure that $N_{sd}(s)$ satisfies condition (b) of Section II-A in the low frequency range.

The compliance procedure is depicted in Fig. 9. Note that compliance with the specified criteria may involve several iterations until all the conditions are achieved for the representative operating conditions of the device.

VI. CASE STUDY

Consider a 200 MVA VSC connected to a transmission network via a transformer having leakage reactance equal to 0.15 pu. The DC side of the device is connected to a voltage source which may be a battery or represent the output of a renewable energy system. Therefore, the VSC is capable of exchanging both active and reactive power with the grid. The VSC-based device is connected to the mid-point of a 500 kV transmission line, as shown in Fig. 10.

The transmission network (excluding the VSC) has resonant frequencies at 277 Hz, 377 Hz, 1032 Hz and 1132 Hz and is passive in the rectangular formulation. It is also passive in the polar formulation because of the voltage regulation provided by the sources at the two ends. The network does not have any resonant frequencies in the low frequency range, thereby satisfying the time-scale separation criterion as well.

The compliance of the device with the passivity criteria is investigated for two sets of the VSC controller parameters, one of which satisfies the proposed criteria, while the criteria are violated in the other case.

1) Device Controller: Let us first consider the VSC controller shown in Fig. 11. The controller parameters shown in the figure are expressed in per unit. Note that the current injected by the device is controlled by the well-known “vector control” scheme in the local (d-q) frame of reference [24]. The controller has the following supplementary blocks which are added to improve the passivity of the device:

a) Active series and shunt resistance emulation are included to improve the passivity of $Y_{sh}(s)$ in the high frequency range [25]. These are highlighted in blue and red respectively.

b) Voltage and frequency regulation via droop control is provided in the low frequency range.

2) Operating Conditions: Generally, a host of credible operating conditions should be considered in the assessment. For brevity, the results are shown only for two operating conditions. Case 1 represents an active power injection mode where the device injects active and reactive power of 0.7 pu and 0.25 pu respectively. Case 2 represents a reactive power compensation scheme (STATCOM application) where the device injects 0.7 pu reactive power.
3) Evaluation of the Frequency Response: The frequency response $Y_{sh}(s)$ of the device is obtained in the range (0.2–200) Hz using the frequency scanning technique [19]. The frequency response is fitted to a state-space model using the vector fitting algorithm [23]. The fitting is found to be accurate. To illustrate this, the frequency response of the (1,1) term of $Y_{sh}(s)$, denoted by $Y_{DD}(s)$, and the same evaluated from the fitted model are shown in Fig. 12.

4) Time-Scale Separation: The eigenvalues of the fitted state-space model are given in Table I. The poles are well-separated as per the proposed criteria. This facilitates the separate analysis of the faster and slower transients.

5) Passivity in the Rectangular Formulation: The passivity of the admittance is evaluated and is shown in Fig. 13. Note that $Y_{sh}(s)$ satisfies the passivity conditions in the high frequency range (≥32 Hz).

6) Passivity of the Polar Formulation: The real part of $J_{sh}(2,2)$ is greater than 2.0 in the low frequency range due to the voltage droop controller. Some of this can contribute towards voltage regulation as discussed in point (e) of Section V-B. With a $k_{vq}$ of 0.4 pu, the passivity of $N_{sd}(s)$ is assessed in the low frequency range and shown in Fig. 14. The eigenvalues of $N_{sd}(j\Omega)$ are non-negative in the low frequency range, indicating passivity compliance of $N_{sd}(s)$.

7) Transient Response: The performance of the device when it is connected to the transmission network is now simulated.

8) Eigenvalue Analysis: The eigenvalues of the interconnected system are evaluated and are given in Table II. The eigenvalues corroborate the inferences drawn from the simulation study.

9) Modified Controller: Violation of the Criteria: To illustrate the possible consequence of violation of the criteria, we
Fig. 17. The Modified VSC controller.

TABLE III
EIGENVALUES OF FITTED MODEL (MODIFIED CONTROLLER)

| \( P_s \) = 0.7 pu | \(-1.03, -3.09, -1.83 \pm j4.25, -37.11, -88.77, -0.48 \pm j269.63, -372 \pm j571.3, -510 \pm j949.6 \) |

Fig. 18. Eigenvalues of \( N_{sd}^T(s) \) and \( Y_{sh}^T(s) \).

Consider the modified controller shown in Fig. 17. The procedure described earlier is used again to check the compliance of the device with the decentralized criteria.

The eigenvalues of the fitted state space model of the device are shown in Table III. There are some eigenvalues whose magnitude fall in the intermediate range (between the proposed limits 10 Hz and 35 Hz) which violates the time-scale separation constraint. The passivity of the admittance matrix \( Y_{sh}(s) \) and \( N_{sd}(s) \) is evaluated and the corresponding eigenvalue plots are shown in Fig. 18. \( Y_{sh}(s) \) violates the passivity criterion in the high frequency range, although \( N_{sd}(s) \) satisfies it in the low frequency range.

Violation of the criteria implies that stability cannot be guaranteed when the device is connected to an external network. It is important to recognize that this does not necessarily imply that the interconnected system will be unstable, since the proposed criteria are sufficient but not necessary for stability. However, it turns out that for the particular external network parameters shown in Fig. 10, the system is indeed unstable when the modified controller is used. This is illustrated by a simulation study in which the controller is switched to the one shown in Fig. 17 at \( t = 1 \) s. The modified controller results in unstable interactions with the external network, which is manifested by the growing oscillations as shown in Fig. 19. It is instructive to note that the frequency of the oscillation is about 275 Hz, which is very close to the network resonant mode. It appears that in this particular case, the non-passive VSC device destabilizes this mode.

The eigenvalues of the interconnected system are shown in Table IV. The eigenvalues corroborate the instability seen in the simulated response when the modified controller is used. Interestingly, if the line length is reduced by 30%, then the system is stable with the modified controller despite the device not satisfying the criteria.

VII. DISCUSSION

A practical decentralized scheme for stability assessment based on passivity has been developed in the previous sections. This requires compliance with a set of criteria which are reasonable and easy to verify for converter-interfaced devices connected to a T&D network. While the criteria seem reasonable, there are certain assumptions/approximations involved, and also some practical limitations. These are discussed below.

1) Network Resistance: Passivity of the T&D network in the polar formulation (at low frequencies) requires the network to be lossless. The resistance of extra high voltage transmission lines and transformers is usually quite small compared to its reactance. Therefore, this is a reasonable assumption for such networks. In situations where the resistance may be significant, as in radial distribution lines, it is convenient to consider the resistance to be a part of the device as shown in Fig. 20. Recall that a similar extension of the device boundary was considered earlier in Section III-B to improve passivity of the device in the rectangular formulation. The extension of the boundary is also beneficial for the polar formulation at low frequencies as it facilitates the passivity of the (modified) distribution network; the device controller has to be adapted to ensure device passivity at the new boundary.

2) Universal Applicability/Legacy Issues: Although we have considered commonly-used control strategies associated with converter-interfaced generating systems, the proposed criteria
are not intended to be device-specific. However, it may not be possible to modify the controllers of some devices (to conform to the decentralized criteria) due to legacy issues. The high and low frequency models of some devices may also be inherently passivity-resistant in the rectangular (high frequency) and polar (low frequency) formulations respectively. For example, for the voltage and frequency dependent load with the following characteristic,

\[ \Delta P = k_{pf} \Delta \omega + k_{pu} \Delta V_n, \Delta Q = k_{qf} \Delta \omega + k_{qv} \Delta V_n \]

the transfer function \( N_{sd}(s) \) is given by,

\[ N_{sd}(s) = \frac{1}{(k_{pf} k_{qf} - k_{pu} k_{qf})} \begin{bmatrix} k_{qf} & -k_{pu} \\ \frac{1}{1+s \tau} & \frac{1}{1+s \tau} \end{bmatrix} \]  \( (15) \)

For such loads, \( N_{sd}(s) \) is not passive in general. For example, with \( k_{pu} = 0.07, k_{qv} = 0.5, k_{pf} = 0.006, \) and \( k_{qf} = 0.003 \), which are the steady-state parameters of a typical industrial motor [1], \( N_{sd}(j\Omega) \) is not positive semi-definite at \( \Omega = 0 \). In such scenarios, one may attempt to apply the criteria not to individual devices or loads, but to a sub-system consisting of several devices and a part of the network, whose transfer function is evaluated at its boundaries. This may be done with the expectation that the passive devices in the sub-system will have some “margins” to compensate for the non-passivity of the others, resulting in the satisfaction of the criteria at the boundaries. If this cannot be achieved, then a conventional stability study of the interconnected system will become necessary.

3) Stability and Passivity Margins: The notion of a quantitative passivity margin, which was alluded to in the previous point, could be useful as a measure of robustness to modelling approximations and variations in operating conditions that may not have been considered during the passivity assessment. The extent of non-negativity of the eigenvalues of \( G^R(j\Omega) \) (see Sec. II) could define a margin [26]; the sensitivity of this to various parameters could also be evaluated. However, whether this passivity margin can be readily transferred to another device which is passivity deficient, and how it can be effected, are open questions. The transfer of resistance between the systems as shown in Fig. 4 and Fig. 20 is indicative of the fact that this may be possible under specific situations.

Another question that is relevant from a power system stability perspective is: Other than the fact that passivity guarantees stability, is there a quantitative relationship between passivity (eigenvalues of \( G^R(j\Omega) \)) and stability (eigenvalues of the state-space system)? Unfortunately, at present, it appears that there is no direct/convenient way of doing this in a decentralized manner. Therefore, a pragmatic approach could be to have decentralized passivity compliance (as described in Section V-B) as the basic requirement for grid connectivity. Centralized analyses may be carried out periodically to assess the overall stability margins, but these analyses need not be done frequently as passivity compliance ensures that the basic requirement of stability is met.

4) Controllable Series (FACTS) Devices: The formulations presented in this paper considered a T&D network with devices connected in shunt. Devices capable of controllable series voltage injection such as Thyristor Controlled Series Capacitor (TCSC), Static Synchronous Series Compensator (SSSC) or both series voltage and shunt current injection capability such as Unified Power Flow Controller (UPFC) and Thyristor Controlled Phase Angle Regulator (TCPAR) may also be present in a network. For including these devices in the analysis, an augmented set of input-output variables needs to be defined. For the rectangular formulation, the D-Q components of injected series voltages \( (v_{brD}, v_{brQ}) \) and branch currents \( (i_{brD}, i_{brQ}) \) may be used at the locations where controllable series devices are present, as shown in Fig. 21. \( Y_n(s) \) is the admittance of the network (in D-Q variables) excluding the series devices and \( Y_{sh}(s) \) is the shunt device admittance, as defined before in (6). \( Z_{ser}(s) \) is the D-Q impedance of the series devices, while \( A_2 \) is the branch-to-bus incidence matrix of the series devices. For the polar formulation, the dual formulation described in [27] may be considered with the series active and reactive power injections and the polar components of the branch current as the interface variables. Investigations by the authors into the passivity properties of the augmented system are at a preliminary stage and hence are not reported in this paper.

VIII. CONCLUSION

This paper has developed decentralized grid-connectivity criteria for devices/sub-systems that ensure the small-signal stability of the interconnected power system. These can be evaluated locally and individually for each converter based device with minimal information about the rest of the system. The criteria...
are based on passivity, but distinct criteria are proposed for the high and low frequency models, with time-scale separation of transients imposed as an additional constraint. The compliance is easy to verify using frequency scanning techniques.

Although the criteria are reasonable, some limitations and exceptions still remain which may inhibit universal application of the scheme. Hence centralized stability studies cannot be completely done away with. Having most converter-interfaced devices that comply with the decentralized criteria may, however, reduce the need for frequently conducting these studies.

Since passivity is a conservative criterion for stability, future investigations could focus on finding measures to quantify the stability margins obtained through passivity compliance. The passivity properties of series-connected FACTS devices is another area that needs further investigation.

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APPENDIX A

DERIVATION OF THE VSG MODEL

Consider the dynamical equations of the classical model of a synchronous generator, as given in (4). The active and reactive power drawn by the machine is given as follows.

\[ P = v_D i_D + v_Q i_Q, \quad Q = v_D i_Q - v_Q i_D \]  

The input and output variables for Model II formulation are \((\Delta P(s), \Delta Q(s))\) and \((\Delta \phi(s), \Delta V_n(s))\). The differential equations in (4) are linearized and can be written as follows.

\[
\begin{align*}
\frac{d}{dt} \begin{bmatrix} \Delta \delta \\ \Delta \omega_r \\ \Delta \omega_f \\ \Delta \omega_m \end{bmatrix} &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ -D_{\omega r} & 0 & 1 & 0 \\ -E_g & -E_g & 0 & 1 \\ D_{\omega m} \eta & -D_{\omega m} \eta & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta \omega_r \\ \Delta \omega_f \\ \Delta \omega_m \end{bmatrix} + \begin{bmatrix} V_o & 0 & 0 & 0 \\ 0 & V_o & 0 & 0 \\ 0 & 0 & V_o & 0 \\ 0 & 0 & 0 & V_o \end{bmatrix} \begin{bmatrix} \Delta P \\ \Delta Q \\ \Delta V_n \\ \Delta \omega_f \end{bmatrix} \\
\end{align*}
\]

Equation (16) is linearized and can be written as follows.

\[
\begin{align*}
\begin{bmatrix} \Delta P \\ \Delta Q \\ \Delta V_n \end{bmatrix} &= \begin{bmatrix} v_D \delta_{\phi o} \\ v_Q \delta_{\phi o} \\ E_g \phi_{\delta o} \end{bmatrix} \begin{bmatrix} 0 & -1 \eta & -E_g \sin \phi \delta_{\phi o} \\ 1 & 0 & -E_g \sin \phi \delta_{\phi o} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta I_D \\ \Delta I_Q \end{bmatrix} \\
\end{align*}
\]

where the subscript \(o\) denotes the equilibrium value of the corresponding variables. The algebraic equation in (4) is separated into real and imaginary parts, and is then linearized to obtain the following equation.

\[
\begin{align*}
\begin{bmatrix} \Delta I_D \\ \Delta I_Q \end{bmatrix} &= \begin{bmatrix} E_g \cos \phi \delta_{\phi o} & 0 \\ -E_g \sin \phi \delta_{\phi o} & 0 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta \omega_f \end{bmatrix} + \begin{bmatrix} 0 & \frac{1}{\delta_{\phi o}} \\ \frac{1}{\delta_{\phi o}} & 0 \end{bmatrix} \begin{bmatrix} \Delta \omega_r \\ \Delta \omega_m \end{bmatrix} \\
\end{align*}
\]

Substituting (19) in (18), we get

\[
\begin{align*}
\begin{bmatrix} \Delta P \\ \Delta Q \\ \Delta V_n \end{bmatrix} &= \begin{bmatrix} V_o E_g \cos \phi \delta_{\phi o} & -E_g \sin \phi \delta_{\phi o} \\ 0 \cos \phi \delta_{\phi o} & 0 \\ -E_g \sin \phi \delta_{\phi o} & 2V_o \cos \phi \delta_{\phi o} - E_g \cos \phi \delta_{\phi o} \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta \omega_f \end{bmatrix} + \begin{bmatrix} D_v \Delta v_D \\ \Delta v_Q \end{bmatrix} \\
\end{align*}
\]

and \(\phi_{\delta o} = \tan^{-1}\left(\frac{v_Q}{v_D}\right), \ \zeta_{\delta o} = \delta_{\phi o} - \phi_{\delta o}\). The rectangular voltage variables \((v_D, v_Q)\) are related to the polar voltage variables \((\phi, V_n)\) as follows.

\[
v_D = V_o V_n \sin \phi, \quad v_Q = V_o V_n \cos \phi
\]

Note that the quiescent value of \(V_n\) is 1. Linearizing we get,

\[
\begin{align*}
\begin{bmatrix} \Delta v_D \\ \Delta v_Q \end{bmatrix} &= \begin{bmatrix} V_o \cos \phi_{\delta o} & V_o \sin \phi_{\delta o} \\ -V_o \sin \phi_{\delta o} & V_o \cos \phi_{\delta o} \end{bmatrix} \begin{bmatrix} \Delta \phi \\ \Delta V_n \end{bmatrix} \\
\end{align*}
\]

Substituting (22) in (20), we get

\[
\begin{align*}
\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} &= \frac{V_o}{x_g} \begin{bmatrix} E_g \cos \phi_{\delta o} & -E_g \sin \phi_{\delta o} \\ -E_g \sin \phi_{\delta o} & 2V_o - E_g \cos \phi_{\delta o} \end{bmatrix} \begin{bmatrix} \Delta \phi \\ \Delta V_n \end{bmatrix} + D_{vsg} \begin{bmatrix} \Delta \phi \\ \Delta V_n \end{bmatrix} \\
\end{align*}
\]

where

\[
D_{vsg} = \frac{V_o}{x_g} \begin{bmatrix} E_g \cos \phi_{\delta o} & -E_g \sin \phi_{\delta o} \\ -E_g \sin \phi_{\delta o} & 2V_o - E_g \cos \phi_{\delta o} \end{bmatrix}
\]

Equation (23) can be re-written as follows.

\[
\begin{align*}
\begin{bmatrix} \Delta \phi \\ \Delta V_n \end{bmatrix} &= -\begin{bmatrix} \frac{1}{D_{vsg} C_m} & 0 \\ 0 & \frac{1}{D_{vsg} D} \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta \omega_r \end{bmatrix} + \begin{bmatrix} \frac{1}{D_{vsg} C_m} & 0 \\ 0 & \frac{1}{D_{vsg} D} \end{bmatrix} \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} \end{align*}
\]

The corresponding transfer function representation is

\[
\begin{align*}
\begin{bmatrix} \Delta \phi(s) \\ \Delta V_n(s) \end{bmatrix} &= N_{sh}(s) \begin{bmatrix} \Delta P(s) \\ \Delta Q(s) \end{bmatrix} \\
\end{align*}
\]

where \(N_{sh}(s) = C(sI - A)^{-1}B + D\), whose terms are:

\[
\begin{align*}
N_{sh}(1, 1) &= \frac{1}{(Ms^2 + D_m s)} + \frac{x_g (2V_o - E_g \cos \phi_{\delta o})}{V_o (2V_o \cos \phi_{\delta o} - E_g)} \\
N_{sh}(1, 2) &= N_{sh}(2, 1) = \frac{x_g \sin \phi_{\delta o}}{V_o (2V_o \cos \phi_{\delta o} - E_g)} \\
N_{sh}(2, 2) &= \frac{x_g \cos \phi_{\delta o}}{V_o (2V_o \cos \phi_{\delta o} - E_g)} \\
\end{align*}
\]

Note that \(N_{sd}(s) = \frac{s}{(s+\omega_{\delta})} \times N_{sh}(s)\).

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