Simplified and approximation autofocus back-projection algorithm for SAR

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Abstract: Due to accuracy limited inertial measurement unit systems, residual synthetic aperture radar (SAR) motion errors need to be compensated by autofocus method. Back-projection (BP) algorithm is a time-domain imaging process suitable for arbitrary trajectory and imaging on digital elevation model. However, traditional SAR autofocus algorithm requires a Fourier transform relationship between image domain and phase history domain. This study introduces an autofocus algorithm based on phase-error estimation. By selecting the image sharpness as target cost function, the exact phase error is directly estimated in closed form by solving the fourth-order polynomial with simplified approach. Another straightforward approximation autofocus method is also proposed to simplify calculation with high accuracy. The vast storage requirement for filtered BP data is reduced through a trade between time and space. Computer simulation and airborne experimental results validate the efficiency and efficacy of the proposed method.

1 Introduction
Back-projection (BP) algorithm is an accurate and real-time SAR imaging method for arbitrary trajectory. The image quality depends on the accuracy of antenna-phase centre (APC) to a certain extent. However, the common inertial measurement unit (IMU)/INS could not satisfy the requirement for high-resolution imaging. Therefore, autofocus BP algorithm is proposed for residual motion compensation. The main theme is to estimate and compensate the unknown phase error or slant range error of each pulse along the azimuth axis. Tan [1] considered polynomial curve fitting and introduced PGA into BP for SAR focusing, while the motion measurement absolute error is limited to 6 mm. In [2], a novel multiple aperture map drift with fast factorised BP (FFBP) is presented. The Fourier transform relationship between FFBP sub-aperture images and corresponding range-compressed phase history data is constructed through pseudo-polar BP coordinate. Both of the above methods rely on the assumption that a Fourier transform relationship exists between the image domain and range-Doppler domain. However, this assumption only holds for small aperture angle imagery. Another limitation of the method is the requirement of linear platform trajectory, where phase error could be estimated with linear FM Doppler chirp form in range-compressed data. Therefore in non-linear flight path geometry such as circular SAR [3] or missile-borne SAR, the uncompensated motion error is large and the method is not applicable.

Another more advanced BP autofocus method is proposed on image-quality optimisation. The object is to maximise or minimise the cost function which reflects the image focus. Typical cost function includes image sharpness, intensity, and entropy. The cost function also associates with the errors to be estimated, such as phase errors, slant range errors, or APC errors. In [4, 5], Ash proposed an autofocus routine for BP imagery from spotlight-mode SAR data. The per-pulse phase correction is optimised under the image sharpness criterion. The phase correction is deduced in closed form under ellipsoidal geometric in a coordinate descent framework. However, this method needs to store the per-pulse filtered back-projected data which requires large memory and the algorithm is more complicated. In [6, 7], minority pixels are selected for autofocus to estimate the phase error, which is used to obtain the APCs by solving a system of non-linear equations with optimisation method. Despite only three pixels are used to estimate phase error, the accuracy could easily be contaminated by noise and jammer. The optimised APCs also depend on the accuracy of the selected strong targets’ positions. Furthermore, its phase-error estimation strategy is same with Ash’s method.

Based on ABP algorithm in [4], this letter proposes a simplified and approximation autofocus BP algorithm. In this method, the exact per-pulse phase error is solved in an easier way under the coordinate descent optimisation framework. The approximate value is also given in closed form. The huge memory requirement for back-projected data is avoided by pre-BP imaging management. The remainder of this letter is organised as follows. Section 2 first illustrates the phase-errors model and coordinate descent framework. The proposed simplified and approximation algorithm is presented as follows. In Section 3, the point target simulation and real data experiment are implemented. The conclusion is given in Section 4.

2 Autofocus back-projection algorithm
2.1 Phase-errors model and coordinate descent framework
The convolution BP algorithm may be thought of as the ideal-matched filter for SAR imaging. No assumptions of flight trajectory are required for image formation. The key is the exactly known flight path. BP algorithm mainly includes three steps: range compression, interpolation, and azimuth accumulation. Defining the ground plane coordinates for pixel \(j\) \((j = 1, 2, \ldots, MN)\) is \(q_j\) and corresponding pixel value is \(z_j\). Then the focused image is

\[
z = \sum_{i=1}^{K} b_i
\]  

(1)

where \(b_i\) is the filtered BP of pulse \(k\) over all pixels \(j\) with real APCs. \(K\) is the total number of azimuth sampling points.

\[
b_{q_j} = s_{q_j}(2R_{q_j} c/k) \times \exp \left[ \frac{\pi R_{q_j}}{\lambda} \right]
\]  

(2)

\(s_{q_j}(r, k)\) denotes the \(k\)th range compressed received signal at fast time \(r\). \(R_{q_j} = \|q_j - l_k\|\), is the range between the pixel \(j\) and \(k\)th real APC \(l_k\). Since the presence of trajectory measurement error, the defocused image should be written as \(\tilde{z}\).
\[ z = \sum_{k=1}^{K} \tilde{b}_k \]  

(3)

\[ \tilde{b}_k = b_k e^{i\phi_k} \]  

is the phase corrupted BP of pulse \( k \). Then, phase error \( \phi_k \)

\[ \phi_k \equiv \phi_{k,j} = 4\pi(R_{k,j} - R_{k})/\lambda \]  

(4)

\( R_{k,j} \) denotes the range between pixel \( j \) and \( k \)th measured APC. In (4), we assume that the phase error is spatial invariant and range error is limited within one range bin. This assumption is reasonable since in spotlight SAR the illuminated area is relatively small and IMU error could maintain the accuracy during synthetic aperture time.

To form a focused image from (3), the objective of autofocus BP algorithm is to estimate and compensate the pulse-by-pulse phase error \( \phi = (\phi_1, \ldots, \phi_k, \ldots, \phi_K) \). Therefore, the refocused image is

\[ \tilde{z} = \sum_{k=1}^{K} \tilde{b}_k e^{-i\phi_k} \]  

(5)

The cost function is a function that represents the image quality relative to phase error \( \phi \). Common cost functions include image sharpness, intensity, and entropy. Here, the maximal sharpness metric is considered which is suitable for derivation. Image sharpness \( s(\tilde{z}) \) is the summation of all pixels' intensity \( v_j \).

\[ s(\tilde{z}) = s(\phi) = \sum_{j=1}^{MN} v_j \sum_{j=1}^{MN} |z_{\tilde{c}j}^2| \]  

(6)

Optimisation algorithms usually begins at \( \phi_0 \) and generates a sequence of iterates \( \{\phi_j\}_{j=0}^{\infty} \) that terminate when it seems that a solution point has been approximated with sufficient accuracy [8]. There are two fundamental strategies for moving from the current point \( \phi_j \) to a new iterate \( \phi_{j+1} \): line search and trust region [9]. The two approaches differ in the order in which they choose the direction and distance of the move to the next iterate. Here, we choose the line search method which determines the search direction first.

According to different search directions, there are variant optimisation methods: Steepest descent method, Gradient descent method, Newton's method, and Coordinate descent method. Since the unconstrained optimisation in (6) is a function of multi-dimensional variable \( \phi \) from \( \mathbb{R}^K \) to \( \mathbb{R} \), the closed form of vector \( \phi \) is unavailable. Therefore, we choose the coordinate descent method.

The coordinate descent method is frequently used in practice to cycle through \( K \) coordinate directions \( c_1, c_2, \ldots, c_K \), using each in turn as a search direction. At the first iteration, we fix all except the first variable and find a new value of this variable that maximises the cost function. The process is repeated with the second variable on the next iteration. After \( K \) iterations, we return to the first variable and repeat the cycle. In this paper, we would derive the closed form of per-pulse phase error under coordinate descent framework. Define the 4th phase-error estimation of \( i \)th iteration is \( \phi_{4i}^i \), then the estimate at next iteration is

\[ \phi_{4i+1} = \arg \max_{\phi} \left( \phi_{4i} + \cdots, \phi_{4i+3} \right) \]  

(7)

2.2 Simplified autofocus BP algorithm

In [4], the phase estimation problem is reduced to ellipse interpretation and the closed form is presented. However, the approach is complicated and computational intensive, which includes basis ortho-normalisation, origin projection, and re-parametrisation, matrix eigen decomposition, and polynomial rooting. The store of per-pulse back-projected values for all pixels also results in heavy burden for memory. In this chapter, the simplified phase-error estimation would be given in closed form. Through BP imaging beforehand, only one pulse back-projected data needs to be stored.

Suppose the 4th phase optimising image at \( i \)th iteration is

\[ \tilde{z}(\phi) = \sum_{p=1}^{K} c^{-i\phi p} \tilde{b}_p + \sum_{p=K+1}^{K} c^{-i\phi p} \tilde{b}_p + c^{-i\phi k} \tilde{b}_k \]  

(8)

where \( y \) is the uncorrected back-projected data of pulse \( k \), \( x \) consists of the sum of phase-corrected back-projected data. The store of every \( \tilde{b}_p \) is sometimes unreasonable. Here, we define the latest updated corrected image with \( k \)-1 pulses at \( i \)th iteration is

\[ \tilde{z}_{k-1} = \sum_{p=1}^{K} c^{-i\phi p} \tilde{b}_p + \sum_{p=K+1}^{K} c^{-i\phi p} \tilde{b}_p + c^{-i\phi k} \tilde{b}_k \]  

(9)

From (10), we only need to update and store \( \tilde{z}_{k-1} \), calculate \( y \) to get \( x \). At first iteration with first pulse, the defocused image \( \tilde{z} \) is used instead of \( \tilde{z}_1 \) which is a time-space exchange strategy. Back to the optimisation function in (7), the \( j \)th pixel intensity of \( \tilde{z}(\phi) \) is \( v_j \)

\[ v_j = v_j^2 + v_j^2|v_j^2 + v_j^2|^2 \]  

(11)

Define \( X = e^{i\theta} \) and \( a = x \cdot y^* = [a_1, \ldots, a_j, \ldots, a_{MN}] \), where \( * \) represents the conjugate complex, \( \cdot \) is inner product of two vectors. Then, we get

\[ v = aX + a^*X^{-1} \]  

(12)

Since \( X = X^{-1} \), the cost function in (6) becomes a polynomial of \( X \)

\[ s(\phi) = s(X) = \| v + v \| = \| v + aX + a^*X^{-1} \| \]

\[ = \sum_{j=1}^{MN} (v_j + a_jX + a_jX^{-1})^2 \]

\[ = \sum_{j=1}^{MN} [v_j^2 + 2a_jv_j] + [X^2 a_j^2 + X^{-1} a_j^2] \]

(13)

To maximise \( s(X) \) with respect to \( X \), we consider the derivate of \( V s(X) \)

\[ V s(X) = 2AX - 2AX^{-1} + 2V a - 2V oX^{-2} \]  

(14)

where \( A = \sum_{j=1}^{MN} a_j^2, A^2 = \sum_{j=1}^{MN} a_j^2, V a = \sum_{j=1}^{MN} v_j a_j, v_{\phi} \in \mathbb{R}, a, X \in \mathbb{C}, \) \( s(X) \) reaches its maximum when \( V s(X) = 0 \), which is

\[ AX^2 + V aX - V oX - A^* = 0, X \neq 0 \]  

(15)
Equation (15) is a quartic polynomial equation about $X$. There are at most four complex roots. The roots of $\nabla s(X)$ may be found from the eigenvalues of the polynomial’s companion matrix or other numerical rooting algorithm.

Suppose the four complex roots are $X_1, X_2, X_3, X_4$. We found that if $A > V_a$, then $X_1 = X_2 = X_3 = X_4 = 1$ and they differ from each other. Else if $A \leq V_a$, there are $X_1 \neq 1, X_2 \neq 1, X_3 = X_4 = 1$. Since $v_{ij} > 0$, $A \leq V_a$ stands. Only two roots $X_2, X_3$ satisfy the requirement. Actually, $s(X_2) = -s(X_3)$. Then, $X$ is one of the two roots which maximises $s(X)$.

$$X = \arg \max_X \{s(X_2), s(X_3)\} \quad (16)$$

Correspondingly, the phase-error estimation $\phi$ is

$$\phi = \angle(X) \quad (17)$$

2.3 Approximate evaluation of phase error $\phi$

In last section, the accurate closed form of phase error $\phi$ is deduced by solving quartic polynomial equation. Here, we propose another approximation method which is more straightforward.

According to (11) and (12), we have $|A| > |V_a|$, $|X_1| = |X_2| = |X_3| = |X_4| = 1$ and they differ from each other. Else if $|A| \leq |V_a|$, there are $X_1 \neq 1, X_2 \neq 1, X_3 = X_4 = 1$. Since $v_{ij} > 0$, $|A| \leq |V_a|$. Therefore, only two roots $X_2, X_3$ satisfy the requirement. Actually, $s(X_2) = -s(X_3)$. Then, $X$ is one of the two roots which maximises $s(X)$.

$$X = \arg \max \{s(X_2), s(X_3)\} \quad (16)$$

Correspondingly, the phase-error estimation $\phi$ is

$$\phi = \angle(X) \quad (17)$$

$$s'(\phi) = -4|A| \sin(2\phi + \phi_a) + |V_a| \sin(\phi + \phi_v) = 0$$

$$\sin(\phi + \phi_v) = -\frac{|A|}{|V_a|} \sin(2\phi + \phi_a) = 0 \quad (19)$$

In (19), $\phi$ has two values. Substitute $\phi$ into (18), we choose $\phi = -\angle(V_a)$ which maximises $s(\phi)$.

Considering practical application, Fig. 1 illustrates the flowchart of the proposed algorithm. There are mainly three steps of the algorithm. Step 1: Select small regions that include relatively strong scatters. Small area could reduce the computational burden of BP and strong scatters enhance the accuracy of phase estimation with high SNR. Step 2: Obtain the defocused image with inaccurate trajectory. The trajectory could be measured by INS/POS or simply assumed as straight line. Step 3: Autofocus BP algorithm using phase-error optimisation which maximises the image sharpness under coordinate descent framework.

3 Experimental results

In this section, we demonstrate results on both point target simulation and real data experiment.

3.1 Simulation results

The simulation parameters are listed in Table 1. The simple side-looking geometry generates the azimuth resolution of 0.29 m while range resolution is 0.30 m.

A random white Gaussian noise is added to the trajectory’s range direction where the maximum amplitude exceeds $\lambda$. The severely defocused image with trajectory error is illustrated in Fig. 2a. The azimuth mainlobe is totally split into several sidelobes. Fig. 2b shows the well-autofocused image with proposed algorithm in this letter. Fig. 3 demonstrates the comparison of azimuth slices with different algorithms. The green-dotted line of accurate slice shows the result of no motion error image. The proposed method plotted in red line with square marker offers

Table 1 Simulation parameters

| Parameter       | Value       | Parameter       | Value       |
|-----------------|-------------|-----------------|-------------|
| carrier frequency | 10 GHz      | slant range    | 3 km        |
| platform velocity | 100 m/s     | PRF            | 1000 Hz     |
| pulse width     | 2 μs        | synthetic time | 1.6 s       |
| bandwidth       | 500 MHz     | sampling rate  | 600 MHz     |
nearly the same PSLR and resolution except a shift in azimuth direction. This suggests a residual linear phase error uncompensated which is the same with PGA. The proposed approximation algorithm plotted in black dash-dot line also has the same performance with Ash’s method. Table 2 shows image sharpness versus iteration count for different processing algorithms. The second column represents the defocused image sharpness. We could see that the convergence rate is fast with coordinate descent optimisation. Moreover, the proposed simplified and approximation autofocus algorithm achieve the same performance as Ash’s method.

3.2 Real data results

A spotlight mode campaign was carried out with our airborne radar. Fig. 4a is the selected defocused BP image with low-accuracy IMU measurements. The scene size is $320 \times 320$ pixels with the interval of $0.1 \text{ m} \times 0.1 \text{ m}$. There are nine trihedral corner reflectors displaced with each other by $7.5 \text{ m}$ in range and azimuth direction. The autofocused BP image with proposed algorithm is shown in Fig. 4b where corner reflectors are well focused. The detailed comparison of azimuth slice of one reflector is plotted in Fig. 5a. The proposed method maintains the same performance with Ash’s in a more simple way. Fig. 5b illustrates the unwrapped estimated phase error. The total expansion of phase error exceeds $6 \pi$ which equals a range error of $1.5 \lambda$. To some extent, the range error would cause severe defocus in azimuth. However, the range error is negligible compared with range resolution, which means phase-error compensation is sufficient for BP autofocus. Figs. 6a and 6b show the BP imaging results before and after applying autofocus with phase error extracted in Fig. 5b. The scene size is $1000 \times 1000$ pixels with the interval of $0.2 \text{ m} \times 0.2 \text{ m}$. The result approves that the assumption of spatial invariant phase error is valid in this campaign. Obviously, the strategy of selecting a small region around strong targets for phase-error estimation first is successful. The required memory and processing time are simultaneously reduced for large area autofocusing.

4 Conclusion

In this paper, we demonstrate a simplified and approximation autofocus BP algorithm. The vast memory requirement of per-pulse filtered back-projected data is reduced in exchange with pre BP imaging. The algorithm processing flow is simplified compared with Ash’s method. Another approximation algorithm is proposed

Table 2  Image sharpness versus iteration count for different algorithms

| Iteration | Ash | proposed | approximation |
|-----------|-----|----------|--------------|
| 0         | 0.014 | 5.241   | 5.589        |
| 1         |        | 5.241   | 5.589        |
| 2         |        | 5.241   | 5.589        |
| 3         |        | 5.241   | 5.589        |

Fig. 2  Point target simulation of the proposed method
(a) Defocused image without autofocus BP algorithm, (b) Autofocus BP algorithm result

Fig. 3  Comparison of azimuth slices with different algorithms

Fig. 4  Real data processing of 9 corner reflectors
(a) Defocused real data BP image of 9 corner reflectors, (b) Autofocused BP image of 9 corner reflectors

Fig. 5  Azimuth slices and phase errors
(a) Comparison of azimuth slices with different autofocus algorithms, (b) Unwrapped estimated phase errors along azimuth axis

Fig. 6  Real data processing of 9 corner reflectors
(a) Defocused real data BP image of 9 corner reflectors, (b) Autofocused BP image of 9 corner reflectors
with high accuracy. The phase errors are both determined in closed form through optimization method. Simulation and real data processing validates the efficiency of the algorithm.

5 References

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