Improved Estimator of Finite Population Variance Using Coefficient of Quartile Deviation

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Authors' contributions
This work was carried out in collaboration between all authors. Authors KJ, NJ, MH designed the study, and construct a new class of estimators. Authors MA and US performed the numerical analysis and wrote the first draft of the manuscript. Author AVGL managed the literature searches. All authors read and approved the final manuscript.

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ABSTRACT
This study introduces a new, better, class of ratio estimators for the estimation of population variance of the study variable by using the coefficient of quartile deviation of auxiliary variable. Bias and mean square error of the proposed class of estimators are also derived. The conditions of efficiency comparison are also obtained. Simulation and different secondary data sets are used to evaluate the efficiency of proposed class of variance estimators over existing class of estimators. The empirical study shows that the suggested class of estimators is more efficient the existing class of estimators for the population variance.

Keywords: Coefficient of quartile deviation; natural populations; simple random sampling; auxiliary variable.

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1. INTRODUCTION

Let us consider a finite population of size \( N \), and \( Y \) be the real variable under investigation. Estimations of the unknown population parameters are used in general when the sample information is only available. The finite population variance \( S^2 = \frac{1}{N-1} \sum_{i=1}^{N} (Y_i - \overline{Y})^2 \) is based on random sample selection from population. Many forms of population variances can be found in literature. In this study, our aim is to propose and investigate a better class of estimators of population variance in simple random sampling (SRS). We consider the helping information that the auxiliary variable offers; in sampling theory, we usually get the upgraded sampling design in order to have a more accurate analysis. We consider this supplementary information to increase the accuracy of population variance; see for details [1].

The first ratio estimator for population variance was introduced by [2] and many of the statisticians improved it in various ways for better performance. The notations used in this study are below:

\[ N : \text{Population size} \]
\[ n : \text{Sample size} \]
\[ Y : \text{Study variable} \]
\[ X : \text{Auxiliary variable} \]
\[ \overline{Y}, \overline{X} : \text{Sample mean of study and auxiliary variable} \]
\[ \overline{Y}, \overline{X} : \text{Population mean of study and auxiliary variable} \]
\[ s^2, s_x^2 : \text{Sample variance} \]
\[ S^2, S_x^2 : \text{Population variance} \]
\[ \rho : \text{Coefficient of correlation} \]
\[ C_y, C_x : \text{Coefficient of variations} \]
\[ Q_e : \text{Coefficient of quartile deviation} \]
\[ \beta_{2(y)} : \text{Coefficient of kurtosis of study variable} \]
\[ \beta_{2(x)} : \text{Coefficient of kurtosis of auxiliary variable} \]
\[ S^2_R : \text{Traditional ratio type variance estimator} \]
\[ \hat{S}^2_R : \text{Suggested estimator} \]
\[ B(\hat{S}^2_R) : \text{Existing estimator} \]
\[ Bias(.) : \text{Biases of estimators} \]
\[ MSE(.) : \text{Mean square errors of estimators} \]

2. EXISTING CLASS OF ESTIMATORS

Isaki [2] introduced a ratio (mean-per-unit) estimator of population variance when the \( S^2_x \) population variance of supplementary variable is known. The estimator introduced by [2] with its bias and mean squared error is given below:

\[ \hat{S}^2_R = S^2 \frac{s_x^2}{S_x^2} \]  \hspace{1cm} (1)

\[ B(\hat{S}^2_R) = \gamma S^2 \left[ (\beta_{2(x)} - 1) - (\lambda_{22} - 1) \right] \]  \hspace{1cm} (2)

\[ MSE(\hat{S}^2_R) = \lambda S^4 \left[ (\beta_{2(y)} - 1) + (\beta_{2(x)} - 1) - 2(\lambda_{22} - 1) \right] \]  \hspace{1cm} (3)

Here we have:

\[ \lambda_{22} = \frac{\mu_y}{\mu_x} \]

\[ \beta_{2(x)} = \frac{\mu_x}{\mu_y} \]

Further, \( \lambda \) or \( \gamma \) has the same meaning, finite population correction factor. The estimation of variance plays, in general, a significant role in life sciences, as it is quite often used in sampling theory, while much effort have been made to enhance its estimated accuracy. Motivated by [3,4,5,6] suggested a class of estimators by using quartiles and some functions of quartiles of the supplementary variable, like the Inter-quartile range, Semi-quartile average and Semi-quartile range. In the following Table 1 we present some existing estimators along with their biases and means squared error (MSEs),
Where,

\[
A_{JI} = \frac{S^2}{S^2 + Q^2}, \quad A_{JG2} = \frac{S^2}{S^2 + Q^1}, \quad A_{JG3} = \frac{S^2}{S^2 + Q^3}, \quad A_{JG4} = \frac{S^2}{S^2 + Q^4}, \quad A_{JG5} = \frac{S^2}{S^2 + Q^5}
\]

and \( \gamma_x \) are the constant and for the estimator \( \hat{S}_{JI1}^2, \hat{S}_{JI2}^2, \hat{S}_{JI3}^2, \hat{S}_{JI4}^2 \), and \( \hat{S}_{JI5}^2 \) respectively.

### 3. PROPOSED CLASSES OF ESTIMATORS

In this study coefficient of quartile deviation is used for further improvement in existing estimators of population variance. The quartile deviation, which is a relative measure of dispersion, is known as the coefficient of quartile deviation. It is free of units of measurement and is a pure number [7]. Proposed estimators with their biases and MSEs are given Table 2.

### Table 1. Existing estimators with their bias and MSE

| Estimators | B() | MSE() |
|------------|-----|-------|
| \( \hat{S}_{JI1}^2 = s_j^2 \left[ \frac{S_j^2 + Q^2}{S_j^2 + Q^2} \right] \) | \( \gamma S_j^2 \left( A_{JI}(\beta_{21(j)}) - (\lambda_{22} - 1) \right) \) | \( \gamma S_j^2 \left[ (\beta_{21(j)} - 1) + A_{JI}(\beta_{21(j)} - 1) - 2A_{JI}(\lambda_{22} - 1) \right] \)
| \( \hat{S}_{JI2}^2 = s_j^2 \left[ \frac{S_j^2 + Q^1}{S_j^2 + Q^1} \right] \) | \( \gamma S_j^2 \left( A_{JG2}(\beta_{21(j)}) - (\lambda_{22} - 1) \right) \) | \( \gamma S_j^2 \left[ (\beta_{21(j)} - 1) + A_{JG2}(\beta_{21(j)} - 1) - 2A_{JG2}(\lambda_{22} - 1) \right] \)
| \( \hat{S}_{JI3}^2 = s_j^2 \left[ \frac{S_j^2 + Q^3}{S_j^2 + Q^3} \right] \) | \( \gamma S_j^2 \left( A_{JG3}(\beta_{21(j)}) - (\lambda_{22} - 1) \right) \) | \( \gamma S_j^2 \left[ (\beta_{21(j)} - 1) + A_{JG3}(\beta_{21(j)} - 1) - 2A_{JG3}(\lambda_{22} - 1) \right] \)
| \( \hat{S}_{JI4}^2 = s_j^2 \left[ \frac{S_j^2 + Q^4}{S_j^2 + Q^4} \right] \) | \( \gamma S_j^2 \left( A_{JG4}(\beta_{21(j)}) - (\lambda_{22} - 1) \right) \) | \( \gamma S_j^2 \left[ (\beta_{21(j)} - 1) + A_{JG4}(\beta_{21(j)} - 1) - 2A_{JG4}(\lambda_{22} - 1) \right] \)
| \( \hat{S}_{JI5}^2 = s_j^2 \left[ \frac{S_j^2 + Q^5}{S_j^2 + Q^5} \right] \) | \( \gamma S_j^2 \left( A_{JG5}(\beta_{21(j)}) - (\lambda_{22} - 1) \right) \) | \( \gamma S_j^2 \left[ (\beta_{21(j)} - 1) + A_{JG5}(\beta_{21(j)} - 1) - 2A_{JG5}(\lambda_{22} - 1) \right] \)

Source (Subramani and Kumarapandiyan 2012b)

### Table 2. Class of proposed estimators with their biases and MSE

| Estimators | B() | MSE() |
|------------|-----|-------|
| \( \hat{S}_{JG1}^2 = s_j^2 \left[ \frac{\sum_j Q^2}{\sum_j Q^2 + Q^2} \right] \) | \( \gamma S_j^2 \left( A_{JG1}(\beta_{21(j)}) - (\lambda_{22} - 1) \right) \) | \( \gamma S_j^2 \left[ (\beta_{21(j)} - 1) + A_{JG1}(\beta_{21(j)} - 1) - 2A_{JG1}(\lambda_{22} - 1) \right] \)
| \( \hat{S}_{JG2}^2 = s_j^2 \left[ \frac{\sum_j Q^2}{\sum_j Q^2 + Q^1} \right] \) | \( \gamma S_j^2 \left( A_{JG2}(\beta_{21(j)}) - (\lambda_{22} - 1) \right) \) | \( \gamma S_j^2 \left[ (\beta_{21(j)} - 1) + A_{JG2}(\beta_{21(j)} - 1) - 2A_{JG2}(\lambda_{22} - 1) \right] \)
| \( \hat{S}_{JG3}^2 = s_j^2 \left[ \frac{\sum_j Q^2}{\sum_j Q^2 + Q^3} \right] \) | \( \gamma S_j^2 \left( A_{JG3}(\beta_{21(j)}) - (\lambda_{22} - 1) \right) \) | \( \gamma S_j^2 \left[ (\beta_{21(j)} - 1) + A_{JG3}(\beta_{21(j)} - 1) - 2A_{JG3}(\lambda_{22} - 1) \right] \)
| \( \hat{S}_{JG4}^2 = s_j^2 \left[ \frac{\sum_j Q^2}{\sum_j Q^2 + Q^4} \right] \) | \( \gamma S_j^2 \left( A_{JG4}(\beta_{21(j)}) - (\lambda_{22} - 1) \right) \) | \( \gamma S_j^2 \left[ (\beta_{21(j)} - 1) + A_{JG4}(\beta_{21(j)} - 1) - 2A_{JG4}(\lambda_{22} - 1) \right] \)
| \( \hat{S}_{JG5}^2 = s_j^2 \left[ \frac{\sum_j Q^2}{\sum_j Q^2 + Q^5} \right] \) | \( \gamma S_j^2 \left( A_{JG5}(\beta_{21(j)}) - (\lambda_{22} - 1) \right) \) | \( \gamma S_j^2 \left[ (\beta_{21(j)} - 1) + A_{JG5}(\beta_{21(j)} - 1) - 2A_{JG5}(\lambda_{22} - 1) \right] \)

Whereas the constants are,

\[
K_j = \frac{S_j^2 Q^2}{S_j^2 Q^2 + Q_j}, \quad K_j = \frac{S_j^2 Q^2}{S_j^2 Q^2 + Q_j}, \quad K_j = \frac{S_j^2 Q^2}{S_j^2 Q^2 + Q_j}, \quad K_j = \frac{S_j^2 Q^2}{S_j^2 Q^2 + Q_j}, \quad K_j = \frac{S_j^2 Q^2}{S_j^2 Q^2 + Q_j}
\]

### 4. EFFICIENCY OF THE PROPOSED ESTIMATORS

From Table 1 the man square errors of existing class of estimators of population

Variance can be written as:

\[
MSE(\hat{S}_{JG1}^2) = \gamma S_j^2 \left[ (\beta_{21(j)} - 1) + A_{JG1}(\beta_{21(j)} - 1) - 2A_{JG1}(\lambda_{22} - 1) \right],
\]

(4)
where, \( i = 1, 2, 3, 4 \) and 5.

From Table 2, the MSEs of the proposed class of estimators for the population variance can be written as:

\[
MSE(i_i^2) = \lambda^2_{\gamma} \left[ (\beta_{2(i)} - 1)(\beta_{2(i) - 1}) - 2K_i^{-2}(\lambda_{22} - 1) \right],
\]

where \( i = 1, 2, 3, 4 \) and 5.

From equations (3) and (5), the efficiency condition has been derived, according to which the proposed class of estimators shows more efficient behavior than the traditional ratio estimator for the population variance. Similarly, from equations (4) and (5), the efficiency condition is also derived, showing again that the proposed class of estimators are more efficient than the existing class of ratio estimators for the population variance as given by [5]. These derived conditions are given below:

\[
\text{MSE}(i_i^2) < \text{MSE}(\hat{S}_R^2) \quad \text{if} \quad \left( \frac{\beta_{2(i) - 1}(K_i^2 + 1)^{1/2}}{2} \right)^2 < (\lambda_{22} - 1),
\]

\[
\text{MSE}(i_i^2) < \text{MSE}(\hat{S}_{2(z)}^2) \quad \text{if} \quad \left( \frac{\beta_{2(i) - 1}(K_i^2 + 2S_{2(z)}^2)}{2} \right)^2 < (\lambda_{22} - 1).
\]

5. EMPIRICAL STUDY

The simulated and secondary data are used to check the efficiency of the suggested class of estimators for the population variance over the existing class of estimators. The first data set is taken from [8], the second data set is taken from [9], and the third data set, concerning the production of rice crop for the period 1982-83 to 2014-15 in the Punjab, Pakistan, is taken from the Agricultural Statistics of Pakistan (Government of Pakistan, Ministry of Food, Agriculture and Livestock, 2014).

For population 1, 2 and 3 of size 80, 70 and 33 respectively, the sample size, descriptive statistics and constants required to find bias and MSE of new and existing estimators [5] of variance are calculated by using expression stated in Table 1 and Table 2. These values are given in Table 3. The biases and mean square errors for population 1, 2 and 3 are calculated by using these values and given in Table 4 and Table 5 to compare the efficiencies of the proposed and existing estimators.

| Parameters | Population 1 | Population 2 | Population 3 |
|------------|--------------|--------------|--------------|
| N          | 80           | 70           | 33           |
| \( n \)    | 20           | 25           | 10           |
| \( \bar{Y} \) | 51.8267      | 96.7000      | 2258.2       |
| \( \bar{X} \) | 11.2646      | 175.2671     | 1453.1       |
| \( S_{Y} \) | 18.3569      | 60.7140      | 839.0        |
| \( S_{Y} \) | 8.4563       | 140.8572     | 277.3504     |
| \( \rho \)  | 0.9413       | 0.7293       | 0.9690       |
| \( \beta_{2(z)} \) | 2.8664      | 7.0952       | 1.7109       |
| \( \beta_{2(y)} \) | 2.2667      | 4.7596       | 1.5718       |
| \( \lambda_{22} \) | 2.2209      | 4.6038       | 1.5117       |
| \( Q_{1} \) | 5.1500       | 80.1500      | 1221.7       |
| \( Q_{3} \) | 16.9750      | 225.025      | 1714.2       |
| \( A_{JG1} \) | 0.9328       | 0.9960       | 0.9843       |
| \( A_{JG2} \) | 0.8082       | 0.9888       | 0.9782       |
| \( A_{JG3} \) | 0.8581       | 0.9928       | 0.9936       |
Parameters | Population 1 | Population 2 | Population 3 |
--- | --- | --- | --- |
$A_{JG4}$ | 0.9236 | 0.9964 | 0.9968 |
$A_{JG5}$ | 0.8660 | 0.9924 | 0.9812 |
$K_1^*$ | 0.7986 | 0.9823 | 0.6392 |
$K_2^*$ | 0.6214 | 0.9520 | 0.5580 |
$K_3^*$ | 0.6333 | 0.9686 | 0.8146 |
$K_4^*$ | 0.7755 | 0.9840 | 0.8970 |
$K_5^*$ | 0.6486 | 0.9670 | 0.5958 |

Table 4. Bias of Reviewed and New Estimators

| Estimators | Population 1 | Population 2 | Population 3 |
--- | --- | --- | --- |
$\hat{S}_{KG1}^2$ | 8.1745 | 362.2715 | 13032.7778 |
$\hat{S}_{KG2}^2$ | 3.9193 | 353.2657 | 12649.3720 |
$\hat{S}_{KG3}^2$ | 5.5035 | 358.2204 | 13616.5730 |
$\hat{S}_{KG4}^2$ | 7.8272 | 362.7572 | 13818.1863 |
$\hat{S}_{KG5}^2$ | 5.7702 | 357.7407 | 12839.9972 |
$\hat{t}_{k1}$ | 3.6289 | 345.3350 | -2577.0819 |
$\hat{t}_{k2}$ | -0.6399 | 308.5025 | -4516.4750 |
$\hat{t}_{k3}$ | -0.4141 | 328.5025 | 3867.0818 |
$\hat{t}_{k4}$ | 2.9600 | 347.3958 | 8000.6724 |
$\hat{t}_{k5}$ | -0.1135 | 326.5505 | -3694.5464 |

Table 5. MSE of Reviewed and New Estimators

| Estimators | Population 1 | Population 2 | Population 3 | Simulated Results |
--- | --- | --- | --- | --- |
$\hat{S}_{KG1}^2$ | 3480.3515 | 1427962.856 | 12548423583 | 3997.077 |
$\hat{S}_{KG2}^2$ | 2908.7734 | 1408850.951 | 12434849780 | 2629.099 |
$\hat{S}_{KG3}^2$ | 3098.2227 | 1419347.720 | 12724277603 | 3024.240 |
$\hat{S}_{KG4}^2$ | 3426.9869 | 1428997.768 | 12785803692 | 3852.322 |
$\hat{S}_{KG5}^2$ | 3133.1398 | 1418329.455 | 12491123595 | 3097.046 |
$\hat{t}_{k1}$ | 2878.6812 | 1392142.665 | 1031288122 | 2556.540 |
$\hat{t}_{k2}$ | 2668.7908 | 1316648.676 | 11003999666 | 2289.264 |
$\hat{t}_{k3}$ | 2662.0097 | 1357074.728 | 10399624186 | 2173.724 |
$\hat{t}_{k4}$ | 2813.5423 | 1396473.203 | 11199590496 | 2446.709 |
$\hat{t}_{k5}$ | 2657.7430 | 1353043.931 | 10623377994 | 2172.502 |
Table 4 presents the values of the biases of existing and proposed ratio type estimators. Each proposed ratio type estimator has a lower bias value compared to the bias value of the existing ratio type estimator. Similarly, each value of MSE of the new proposed estimators is also lower than the corresponding MSE value of the existing estimators, as given in Table 5.

6. CONCLUSION

In this study the class of ratio (mean-per-unit) type estimators is being modified by using some other parameter of auxiliary variable. The coefficient of quartile deviation is used. The product of coefficient of quartile deviation is used with the functions of quartiles. Each estimator of the class of the proposed estimators is compared to the corresponding existing estimator. Numerical explorations were used to show the behaviour of the efficiency condition, biases and MSEs of both the existing and the new estimators. It is then concluded that the proposed estimators are more efficient.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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