Muonium-Antimuonium Conversion in Models with Dilepton Gauge Bosons

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Abstract

We examine the magnetic field dependence of the muonium($\mu^+e^-$)-antimuonium($\mu^-e^+$) conversion in the models which accommodate the dilepton gauge bosons. The effective Hamiltonian for the conversion due to dileptons turns out to be in the $(V-A) \times (V+A)$ form and, in consequence, the conversion probability is rather insensitive to the strength of the magnetic field. The reduction is less than 20% for up to $B \approx 300$ G and 33% even in the large $B$ limit.

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Muonium, which is a bound state of $\mu^+$ and $e^-$, can be transformed to antimuonium, a bound state of $\mu^-$ and $e^+$, if there exists a lepton-number-non-conserving interaction [1]. Feinberg and Weinberg [2] studied the $M - \overline{M}$ conversion with a postulated effective Hamiltonian of $(V - A) \times (V - A)$ form. Later, this process has been studied within the left-right symmetric models and the models with doubly-charged Higgs bosons [3]-[7]. In these models, the effective Hamiltonian for the conversion is expressed either in the $(V - A) \times (V - A)$ form or in the $(V + A) \times (V + A)$ form. Thus far no $M - \overline{M}$ conversion has been observed [8].

Recently, an interesting class of models which have new $SU(2)_L$-doublet gauge bosons were proposed as extensions of the standard model [9]-[12]. In these models each family of leptons $(l^+, \nu_l, l^-)_L$ transforms as a triplet under the gauge group $SU(3)$ and the total lepton number defined as $L = L_e + L_\mu + L_\tau$ is conserved, while the separate lepton number for each family is not. The new gauge bosons $(X^\pm, X^{\mp\mp})$ carry lepton number $L = \pm 2$. Hence, hereafter, we refer to these gauge bosons as dileptons. The gauge group $SU(3)$ will be, for example, an $SU(3)_L$ in the $SU(15)$ grand unification theory model [10] or an $SU(3)_L \times SU(3)_C \times U(1)_X$ model [12].

The phenomenology on dilepton gauge bosons has been extensively studied. When the doubly-charged dilepton exists, the mixing of muonium and antimuonium is possible through the diagram illustrated in Fig. 1 and thus $M - \overline{M}$ conversion takes place [13]-[15]. In particular, the effective Hamiltonian for the mixing turns out to be in the $(V - A) \times (V + A)$ form. One of the present authors (K.S.) and Fujii and Nakamura calculated the probability for the $M - \overline{M}$ conversion in the models with dileptons and examined the lower mass bound on the doubly-charged dilepton $X^{\pm\mp}$ in Ref. [14]. But the analysis was done in the case of absence of magnetic fields. In this paper we consider the $M - \overline{M}$ conversion in static external magnetic fields and study the field dependence of the conversion probability.

The muonium or antimuonium system in the presence of static external magnetic field $\vec{B}$ is described by the following Hamiltonian,

$$\mathcal{H}_{int} = A_S e^S_{\vec{e}} \cdot \vec{S}_\mu + \mu_B g_e S^g_{\vec{e}} \cdot \vec{B} + \mu_B \frac{m_e}{m_\mu} g_\mu S^g_{\vec{\mu}} \cdot \vec{B}, \quad (1)$$

where $S^g_{\vec{e}}, m_e, g_e = -g_{e^+}$ and $S^g_{\vec{\mu}}, m_\mu, g_\mu = -g_{\mu^+}$ are spin, mass, the gyromagnetic ratio of electron (or positron) and $\mu^+$ (or $\mu^-$), respectively, and $\mu_B$ is Bohr magneton.
The first term of Eq. (1) is the source of $1S$ hyperfine splitting of the muonium (or antimuonium) system and $A = 1.846 \times 10^{-5}$ eV. Taking the magnetic field direction as the spin-quantization axis, we obtain the muonium energy eigenvalues as follows:

$$
E_M(1,+1) = \frac{A}{4} + P \\
E_M(1,-1) = \frac{A}{4} - P \\
E_M(1,0) = -\frac{A}{4}(1 - 2\sqrt{1+y^2}) \\
E_M(0,0) = -\frac{A}{4}(1 + 2\sqrt{1+y^2}), \quad (2)
$$

with

$$
P = \frac{1}{2} \mu_B B(g_e - g_\mu - m_e/m_\mu) \approx 5.76 \times 10^{-9} B (eV/G)$$

$$
y = \frac{1}{A} \mu_B B(g_e - g_\mu - m_e/m_\mu) \approx 6.30 \times 10^{-4} B (1/G). \quad (3)
$$

The corresponding eigenstates are expressed in a “natural” basis $|S_\mu^z S_e^z >$ as:

$$
|1,+1 >_M = |++>_M \\
|1,-1 >_M = |-->_M \\
|1,0 >_M = c|-->_M + s|+->_M \\
|0,0 >_M = -s|-->_M + c|+->_M. \quad (4)
$$

where $|+->_M$ means $|S_\mu^z = \frac{1}{2}, S_e^z = -\frac{1}{2} >_M$, etc., and

$$
c = \frac{1}{\sqrt{2}}[1 + \frac{y}{\sqrt{1+y^2}}]^{1/2} \\
s = \frac{1}{\sqrt{2}}[1 - \frac{y}{\sqrt{1+y^2}}]^{1/2}. \quad (5)
$$

It is noted that the $(J=1, J_z = 0)$ state among $1S$ triplet and $1S$ singlet state $(J = 0, J_z = 0)$, which are both energy eigenstates in the absence of external magnetic fields, mix with each other in the presence of $\overrightarrow{B}$ and they are not energy eigenstates any more. Thus it is understood that energy eigenstates $|1,0 >$ and $|0,0 >$ are the states which approach to $(J=1, J_z = 0)$ and $(J = 0, J_z = 0)$ states, respectively, when the magnetic field $\overrightarrow{B}$ vanishes. However, $(J=1, J_z = \pm)$ states among $1S$ triplet remain as energy eigenstates even in the presence of $\overrightarrow{B}$. 

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Energy eigenvalues and the corresponding eigenstates for the antimuonium system in the presence of external magnetic field $\vec{B}$ are obtained from Eqs. (2)(4) by interchanging $P \leftrightarrow -P$, $y \leftrightarrow -y$ and $c \leftrightarrow s$. Thus the energy eigenvalues for the antimuonium are

$$
E^M_{1+1} = \frac{A}{4} - P
$$

$$
E^M_{1-1} = \frac{A}{4} + P
$$

$$
E^M_{10} = -\frac{A}{4}(1 - 2\sqrt{1 + y^2})
$$

$$
E^M_{00} = -\frac{A}{4}(1 + 2\sqrt{1 + y^2}),
$$

and the corresponding eigenstates are

$$
|1, +1 \rangle_M = |++\rangle_M
$$

$$
|1, -1 \rangle_M = |-\rangle_M
$$

$$
|1, 0 \rangle_M = s |+\rangle_M + c |+\rangle_M
$$

$$
|0, 0 \rangle_M = -c |-\rangle_M + s |-\rangle_M.
$$

Now we consider the $M - \overline{M}$ conversion in the presence of static external magnetic fields. First we write down a useful formula for the $M - \overline{M}$ conversion which was derived by Feinberg and Weinberg a long time ago \cite{2}. If there exists an interaction $\mathcal{H}_{M\overline{M}}$ which would yield a matrix element for conversion of $M$ into $\overline{M}$ equal to

$$
< \overline{M}|\mathcal{H}_{M\overline{M}}|M> = \frac{\Delta^2}{2},
$$

the mass matrix for the $M - \overline{M}$ system is written as

$$
\mathcal{M}_{MM} = \left( \begin{array}{cc}
E_M & \frac{\Delta}{2} \\
\frac{\Delta}{2} & E_{\overline{M}}
\end{array} \right).
$$

Then the probability for a muonium atom of the state $|M>$ to decay as antimuonium of the state $|\overline{M}>$ at all is given by

$$
P(M) = \frac{\Delta^2}{2[\lambda^2 + (E_M - E_{\overline{M}})^2 + \Delta^2]};
$$

where $\lambda = G_F^2 m^5_{\mu}/192\pi^3$ is the muon decay rate and $G_F$ is Fermi constant.

Before we study the dilepton contributions to the $M - \overline{M}$ conversion in the presence of static external magnetic fields, we review the case when the effective
Hamiltonian for $M - \overline{M}$ transition is written in the $(V - A) \times (V - A)$ form or $(V + A) \times (V + A)$ form \[13\]-\[17\],

$$
\mathcal{H}_{M\overline{M}} = \frac{G_{M\overline{M}}}{\sqrt{2}} [\not{p} \gamma_\lambda (1 \mp \gamma_5) c] [\not{p} \gamma_\lambda (1 \mp \gamma_5) c] + H.c., \tag{11}
$$

which arises in the left-right symmetric models and the models with doubly-charged Higgs bosons \[3\]-\[7\]. In this case matrix elements for conversion of $M$ into $\overline{M}$ are given in a “natural” basis $|S_z \mu S_z e\rangle$ as follows:

$$
\begin{align*}
\overline{M} < ++ | \mathcal{H}_{M\overline{M}} | ++ > M &= \overline{M} < -- | \mathcal{H}_{M\overline{M}} | -- > M \\
&= \overline{M} < ++ | \mathcal{H}_{M\overline{M}} | + - > M \\
&= \overline{M} < -- | \mathcal{H}_{M\overline{M}} | - + > M \\
&= \frac{\delta}{2}
\end{align*}
$$

other elements = 0, \tag{12}

with

$$
\delta = \frac{16 G_{M\overline{M}}}{\sqrt{2} \pi a^3}, \tag{13}
$$

where $a$ is the Bohr radius of the muonium $(m_r \alpha)^{-1}$ with $m_r^{-1} = m_\mu^{-1} + m_e^{-1}$. Thus we obtain,

$$
\overline{M} < 1, \pm 1 | \mathcal{H}_{M\overline{M}} | 1, \pm 1 > M = \frac{\delta}{2} \overline{M} < 0, 0 | \mathcal{H}_{M\overline{M}} | 0, 0 > M \\
&= \frac{\delta}{2} \overline{M} < 0, 0 | \mathcal{H}_{M\overline{M}} | 0, 0 > M
$$

for the matrix elements in the “energy eigenstate” representation. Now it is straightforward from Eqs.(2), (3), (11) and (14) to calculate the probability of a muonium in the $|1, \pm 1 >$, $|1, 0 >$ and $|0, 0 >$ states to decay as antimuonium. The results are \[16\]-\[17\],

$$
P^{(1, \pm 1)}(\overline{M}) = \frac{\delta^2}{2[\lambda^2 + 4P^2 + \delta^2]} \tag{15}
$$

for the $|1, +1 >$ and $|1, -1 >$ states and

$$
P^{(1, 0)}(\overline{M}) = P^{(0, 0)}(\overline{M}) \frac{\delta^2}{2[(1 + y^2)\lambda^2 + \delta^2]} \tag{16}
$$
for the \(|1,0>\) and \(|0,0>\) states.

It is noted that since the \((J = 1, J_z = 0)\) and \((J = 0, J_z = 0)\) states mix with each other in the presence of external magnetic fields, \(M - \overline{M}\) conversions from \(|1,0>_M\) to \(|0,0>_{\overline{M}}\) state and from \(|0,0>_M\) to \(|1,0>_{\overline{M}}\) state are also possible. Indeed, from the \(M - \overline{M}\) transition matrix elements

\[
\mathcal{M} < 0,0|\mathbf{H}_{M\overline{M}}|1,0>_M \quad = \quad \frac{y}{\sqrt{1+y^2}} \delta,
\]

we obtain

\[
P^{(1,0)\rightarrow(0,0)}(\overline{M}) = P^{(0,0)\rightarrow(1,0)}(\overline{M}) \quad = \quad \frac{y^2 \delta^2}{2[(1+y^2)\lambda^2 + (1+y^2)^2 A^2 + y^2 \delta^2]},
\]

for the probability of a muonium of the \(|1,0>_M\) \((|0,0>_M\rangle\) state to decay as antimuonium through the state \(|0,0>_{\overline{M}}\) \((|1,0>_M\rangle\) state. However these probabilities are numerically extremely small and can be safely neglected in the following discussion.

The assumption that each state is produced with equal weight at the beginning gives

\[
P_{\text{Tot}}(\overline{M}) = \frac{\delta^2}{4(\lambda^2 + 4P^2 + \delta^2)} + \frac{\delta^2}{4[(1+y^2)\lambda^2 + \delta^2]},
\]

for the “total” probability of a muonium to decay as antimuonium. The magnetic field dependence of \(P_{\text{Tot}}(\overline{M})\) has been studied in Refs. [16],[17]. We plot the results for dependence of \(P_{\text{Tot}}(\overline{M})\), \(\frac{1}{2}P^{(1,1)}(\overline{M})\), and \(\frac{1}{2}P^{(1,0)}(\overline{M})\) on \(B\) in Fig.2. Note that the probabilities are normalized by \(P_{\text{Tot}}(\overline{M})|_{B=0}\) and \(G_{M\overline{M}}\) is taken to be 0.1\(G_F\).

In the presence of static external magnetic fields, the degeneracy between the \(|1,+1>_M\) and \(|1,+1>_{\overline{M}}\) states (the \(|1,-1>_M\) and \(|1,-1>_{\overline{M}}\) states) breaks down and the generated energy difference severely suppresses the conversion. In fact, the probability \(P^{(1,\pm 1)}(\overline{M})\) becomes negligibly small when \(B\) is in the order of \(10^{-1}\) G (see Fig.2-b). On the other hand, the \(|1,0>_M\) and \(|1,0>_{\overline{M}}\) states (the \(|0,0>_M\) and \(|0,0>_{\overline{M}}\) states) remain degenerate and thus the conversion persists up to the fields in the order of \(10^3\) G. In the limit of large \(B\), the \(|1,0>_M\) state becomes a pure \(|-+>_M\) while the \(|1,0>_{\overline{M}}\) state becomes a pure \(|+->_{\overline{M}}\), and thus the matrix element \(\mathcal{M} < 1,0|\mathbf{H}_{M\overline{M}}|1,0>_M\) vanishes. Hence the probability \(P^{(1,0)}(\overline{M})\) reduces to zero in this limit (see Fig.2-c below). By the same reason, \(P^{(0,0)}(\overline{M})\)
vanishes in the large $B$ limit. Finally we see from Fig.2-a that in the case of the effective Hamiltonian being in the $(V - A) \times (V - A)$ form or $(V + A) \times (V + A)$ form and $G_{M\overline{M}} = 0.1 G_F$, the $M - \overline{M}$ conversion probability is reduced to 50% at a field strength as low as 0.26 G, to 35.8% at $B = 1$ kG and to 1.2% at $B = 1$ T. The dependence of the normalized probabilities on the coupling strength $G_{M\overline{M}}$ is negligibly small for $G_{M\overline{M}} < 1 G_F$.

Next we consider the $M - \overline{M}$ conversion in models with dileptons. The gauge interaction of dileptons with leptons is given by [18]

$$\mathcal{L}_{\text{int}} = -\frac{g_3}{2\sqrt{2}} X\mu l^T C \gamma^\mu \gamma_5 l - \frac{g_3}{2\sqrt{2}} X\mu l^T \gamma^\mu \gamma_5 C T^\mu + \frac{g_3}{2\sqrt{2}} X\mu l^T C \gamma^\mu (1 - \gamma_5) l + \frac{g_3}{2\sqrt{2}} X\mu l^T \gamma^\mu (1 - \gamma_5) C T^\mu,$$

(20)

where $l = e, \mu, \tau$, and $C$ is the charge-conjugation matrix. The gauge coupling constant $g_3$ is given approximately by $g_3 = 1.19e$ for the SU(15) GUT model [11] and by $g_3 = g_2 = 2.07e$ for the $SU(3)_L \times U(1)_X$ model [12], where $e$ and $g_2$ are the electric charge and the $SU(2)_L$ gauge coupling constant, respectively. It is noted that the vector currents which couple to doubly-charged dileptons $X^{\pm\pm}$ vanish due to Fermi statistics. Through the doubly-charged-dilepton-exchange diagram illustrated in Fig. 1, we obtain the following effective Hamiltonian for the $M - \overline{M}$ conversion, [18]

$$\mathcal{H}_{M\overline{M}}^{Di} = \frac{G_{M\overline{M}}^{Di}}{\sqrt{2}} [\overline{\mu} \gamma^\lambda (1 - \gamma_5) e] [\overline{\mu} \gamma^\lambda (1 + \gamma_5) e] + H.c.$$  

(21)

where $G_{M\overline{M}}^{Di}/\sqrt{2} = -g_3^2/(8 m_{X^{\pm\pm}}^2)$ and $m_{X^{\pm\pm}}$ is the doubly-charged dilepton mass. This form is obtained from Eq.(20) and with help of the Fierz transformation. It should be noted that the above effective Hamiltonian is in the $(V - A) \times (V + A)$ form. The most stringent lower mass bound for the doubly-charged dileptons at present is $(m_{X^{\pm\pm}}/g_3) > 340$ GeV (95%C.L.) [18]. This gives $G_{M\overline{M}}^{Di} < 0.13 G_F$.

With this effective Hamiltonian, we find that the matrix elements for conversion of $M$ into $\overline{M}$ are given in a “natural” basis $|S_\mu S_\nu^\ast>$ as follows:

$$\overline{M} < ++ |\mathcal{H}_{M\overline{M}}^{Di} | ++ M > = \overline{M} < -- |\mathcal{H}_{M\overline{M}}^{Di} | -- M > = \frac{\delta}{2},$$

$$\overline{M} < + - |\mathcal{H}_{M\overline{M}}^{Di} | + - M > = \overline{M} < - + |\mathcal{H}_{M\overline{M}}^{Di} | - + M > = -\frac{\delta}{2},$$

$$\overline{M} < + - |\mathcal{H}_{M\overline{M}}^{Di} | + + M > = \overline{M} < - + |\mathcal{H}_{M\overline{M}}^{Di} | - + M > = \frac{\delta}{2}.$$
other elements \( = 0 \), \( \text{(22)} \)

where

\[
\hat{\delta} = -\frac{8G^{D_i}}{\sqrt{2}\pi a^3}
\]  \( \text{(23)} \)

Since \( H_{MM}^{D_i} \) is in the \((V - A) \times (V + A)\) form, the matrix elements \( \overline{M} < + + |H_{MM}^{D_i}| + + >_M \) and \( \overline{M} < + - |H_{MM}^{D_i}| + - >_M \) take different values, and \( \overline{M} < + - |H_{MM}^{D_i}| - + >_M \) and \( \overline{M} < - + |H_{MM}^{D_i}| + - >_M \) do not vanish.

In terms of the “energy eigenstates”, the matrix elements for \( M - \overline{M} \) conversion are written as,

\[
\begin{align*}
\overline{M} < 1, \pm 1 |H_{MM}^{D_i}| 1, \pm 1 >_M & = \frac{\hat{\delta}}{2} \\
\overline{M} < 1, 0 |H_{MM}^{D_i}| 1, 0 >_M & = (1 - \frac{1}{2\sqrt{1 + y^2}}\hat{\delta}) \\
\overline{M} < 0, 0 |H_{MM}^{D_i}| 0, 0 >_M & = -(1 + \frac{1}{2\sqrt{1 + y^2}}\hat{\delta}) \end{align*}
\]  \( \text{(24)} \)

It is interesting to note that neither \( \overline{M} < 1, 0 |H_{MM}^{D_i}| 1, 0 >_M \) nor \( \overline{M} < 0, 0 |H_{MM}^{D_i}| 0, 0 >_M \) vanishes in the large \( B \) (i.e., large \( y \)) limit.

Again using the formula (14), we obtain the following probabilities of a muonium to decay as antimuonium in the models with dileptons:

\[
P_{D_i}^{(1, \pm 1)}(\overline{M}) = \frac{\hat{\delta}^2}{2[\lambda^2 + 4P^2 + \hat{\delta}^2]} \]  \( \text{(25)} \)

for the \( |1, \pm 1 >_M \) states,

\[
P_{D_i}^{(1, 0)}(\overline{M}) = \frac{(2 - \frac{1}{\sqrt{1 + y^2}})^2\hat{\delta}^2}{2[\lambda^2 + (2 - \frac{1}{\sqrt{1 + y^2}})^2\hat{\delta}^2]} \]  \( \text{(26)} \)

for the \( |1, 0 >_M \) state and finally

\[
P_{D_i}^{(0, 0)}(\overline{M}) = \frac{(2 + \frac{1}{\sqrt{1 + y^2}})^2\hat{\delta}^2}{2[\lambda^2 + (2 + \frac{1}{\sqrt{1 + y^2}})^2\hat{\delta}^2]} \]  \( \text{(27)} \)

for the \( |0, 0 >_M \) state.
As before we assume that each state is produced with equal weight at the beginning, and we obtain,

\[
P_{Di}^{\text{Tot}}(\overline{M}) = \frac{3\hat{\delta}^2}{4[\lambda^2 + 4P^2 + \hat{\delta}^2]} + \frac{(2 - \frac{1}{\sqrt{1+y^2}})^2\hat{\delta}^2}{8[\lambda^2 + (2 - \frac{1}{\sqrt{1+y^2}})^2\delta^2] + \frac{(2 + \frac{1}{\sqrt{1+y^2}})^2\hat{\delta}^2}{8[\lambda^2 + (2 + \frac{1}{\sqrt{1+y^2}})^2\delta^2]}},
\]

(28)

for the “total” probability of a muonium to decay as antimuonium. In the limit of \( B = 0 \), we have

\[
P_{Di}^{\text{Tot}}(\overline{M})|_{B=0} = \frac{3\hat{\delta}^2}{8[\lambda^2 + \delta^2]} + \frac{9\delta^2}{8[\lambda^2 + 9\delta^2]}
\approx \frac{3\hat{\delta}^2}{2\lambda^2},
\]

(29)

which is the result first obtained in Ref. [14].

In Fig.3 we plot the magnetic field dependence of \( P_{Di}^{\text{Tot}}(\overline{M}) \), \( \frac{1}{2}P_{Di}^{(1,1)}(\overline{M}) \), \( \frac{1}{2}P_{Di}^{(1,0)}(\overline{M}) \), and \( \frac{1}{4}P_{Di}^{(0,0)}(\overline{M}) \). They are all normalized by \( P_{Di}^{\text{Tot}}(\overline{M})|_{B=0} \) and we take \( G_{MM}^{Di} = 0.1G_F \). As in the case of \( P^{(1,\pm 1)}(\overline{M}) \), the probability \( P_{Di}^{(1,\pm 1)}(\overline{M}) \) becomes negligibly small when \( B \) reaches the order of \( 10^{-1}G \) since the magnetic field breaks the degeneracy of the \( |1, +1 >_M \) and \( |1, +1 >_{\overline{M}} \) states (see Fig.3-b). However, the \( B \)-dependences of \( P_{Di}^{(1,0)}(\overline{M}) \) and \( P_{Di}^{(0,0)}(\overline{M}) \) are quite different from those of \( P_{Di}^{(1,0)}(\overline{M}) \) and \( P_{Di}^{(0,0)}(\overline{M}) \) (see Fig.3-c,d). Firstly, the \( M - \overline{M} \) conversion through the channel \( |0, 0 >_M \rightarrow |0, 0 >_{\overline{M}} \) is much preferred. Thus \( P_{Di}^{(0,0)}(\overline{M}) \) gives a dominant contribution to \( P_{Di}^{\text{Tot}}(\overline{M}) \). Secondly, \( P_{Di}^{(1,0)}(\overline{M}) \) and \( P_{Di}^{(0,0)}(\overline{M}) \) remain finite in the large \( B \) limit. This is due to the fact that the matrix elements \( \overline{M} < 1, 0 | H_{MM}^{Di} | 1, 0 >_M \) and \( \overline{M} < 0, 0 | H_{MM}^{Di} | 0, 0 >_M \) do not vanish in the large \( B \) limit when the effective Hamiltonian is in the \((V - A) \times (V + A)\) form. Interestingly enough, \( P_{Di}^{(1,0)}(\overline{M}) \) starts to increase around \( B = 1 \) kG and partially compensates the decrease of \( P_{Di}^{(0,0)}(\overline{M}) \) in the region \( B > 1 \) kG. Summing up each contributions, we find that \( P_{Di}^{\text{Tot}}(\overline{M}) \) is rather insensitive to the static external magnetic field. In fact Fig.3-a shows that \( P_{Di}^{\text{Tot}}(\overline{M}) \) is lowered to 83% in the region \( 0.2 \) G < \( B < 300 \) G and only to 67% in the large \( B \) limit. At \( B = 1 \) kG (1 T) the reduction is 22.4% (32.9%). Again the dependence of the normalized probabilities on the coupling strength \( G_{MM}^{Di} \) is negligibly small for \( G_{MM}^{Di} < 1G_F \).

In conclusion, we have studied the magnetic field dependence of the \( M - \overline{M} \) conversion in the models with dileptons. We have found that the conversion is rather
insensitive to the strength of the magnetic fields. If an experiment is performed in a magnetic field of 1 T and if a bound for the conversion probability $P(\overline{M}) < 10^{-10}$ is gained [17], then a bound for the coupling strength, $G_{M\overline{M}} < 1.8 \times 10^{-2} G_F$, is obtained for the usual $(V \mp A) \times (V \mp A)$ type-Hamiltonian. On the other hand, the models with dileptons give a more stringent bound $G_{M\overline{M}}^{Di} < 2.8 \times 10^{-3} G_F$.

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Figure caption

Figure 1
The doubly-charged dilepton exchange diagram for muon-antimuonium conversion. The arrows show the flow of lepton number.

Figure 2
The magnetic field dependence of the $M - \overline{M}$ conversion probability with an effective $(V \mp A) \times (V \mp A)$ type-Hamiltonian: (a) $P_{\text{Tot}}(\overline{M})$; (b) $\frac{1}{2} P^{(1,1)}(\overline{M})$; (c) $\frac{1}{2} P^{(1,0)}(\overline{M})$. They are all normalized by $P_{\text{Tot}}(\overline{M})|_{B=0}$ and $G_{MM} = 0.1G_F$ is assumed.

Figure 3
The magnetic field dependence of the $M - \overline{M}$ conversion probability in models with dileptons: (a) $P_{\text{Di}}^{\text{Tot}}(\overline{M})$; (b) $\frac{1}{2} P_{\text{Di}}^{(1,1)}(\overline{M})$; (c) $\frac{1}{4} P_{\text{Di}}^{(1,0)}(\overline{M})$; (d) $\frac{1}{4} P_{\text{Di}}^{(0,0)}(\overline{M})$. They are all normalized by $P_{\text{Di}}^{\text{Tot}}(\overline{M})|_{B=0}$ and $G_{MM}^{\text{Di}} = 0.1G_F$ is assumed.
This figure "fig2-1.png" is available in "png" format from:

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This figure "fig2-2.png" is available in "png" format from:

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