MinT: A Fast Lightweight Envelope/Monte-Carlo Beam Optics Code for the Proton Beamlines of the Paul Scherrer Institute

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We report about the methods used in, and the performance of, the new fast and light-weight linear beam transport program MinT. MinT provides, beyond the usual linear ion optics, methods to compute the effects of beam degradation, multiple scattering and beam collimation. This is specifically important in facilities where the ion beam passes matter, for instance in proton therapy beamlines with an energy degrader as in the Proscan facility at PSI, but also for modeling the beam traversing the Muon- and Pion-production targets of the Paul Scherrer Institut’s high intensity proton accelerator (HIPA). MinT is intended to be useful as a support tool for the HIPA and Proscan control rooms. This requires to have useful results within a few seconds. Hence simplicity and speed of calculation is favored against higher accuracy.

MinT has been designed not only to replace the FORTRAN 77 codes TRANSPORT and TURTLE, but to combine and extent their capabilities. MinT is a byte-code-compiler which translates an input language, described by syntactic rules. This allows for control structures like “if-then-else” or “while”-loops, thus providing a high flexibility and readability.
I. INTRODUCTION

The Paul Scherrer Institute (PSI) in Villigen, Switzerland, is known for its high intensity proton accelerator (HIPA) \[1-3\] which held for many years the world record of proton beam power (up to 1.4 MW). PSI is also known for pioneering research of proton tumor therapy in the center for proton therapy (CPT) and its facility Proscan \[4\].

Both facilities are driven by cyclotrons and in both facilities the simulation of the proton beam optics faces the problem to describe beams of several ten to some hundred MeV passing through matter with subsequent beam collimation: In the HIPA facility, the 590 MeV proton beam has to pass through two graphite wheel targets before it is send to the SINQ spallation neutron source. In case of Proscan, the beam passes through an adjustable graphite wedge degrader to tune the beam energy and hence the range of the protons in the target tissue.

The HIPA facility, shown in Fig. 1, is driven by an accelerator chain consisting of a Cockcroft-Walton-type pre-accelerator and two isochronous separate sector cyclotrons, namely the 72 MeV Injector II and the Ring cyclotron, providing a 590 MeV proton beam of up to 2.4 mA. This high intensity beam is either send to the ultra-cold neutron source “UCN” \[5-11\] or via two graphite targets, the 5 mm thick Target M and the 40−60 mm thick Target E \[12-14\], to the Swiss spallation neutron source “SINQ” \[15-20\].

The Proscan therapy (PT) facility is driven by a 250 MeV superconducting compact cyclotron “COMET” with currents of (usually) up to 1 \(\mu\)A \[21-29\]. After extraction from COMET, the 250 MeV beam passes a double-wedge degrader which is used to reduce the beam energy to 74−230 MeV. The energy degradation increases the emittance.

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**FIG. 1.** Overview over HIPA facility at PSI. Here we are concerned specifically with the beamlines, the 870 keV injection beamline “BW870”, the 72 MeV-beamline “IW2”, connecting Injector 2 with the Ring cyclotron and the 590 MeV beamline, the so-called “p-Channel”, which guides the proton beam from the Ring cyclotron via Target M and Target E to beamdump and SINQ, respectively.
and energy spread by multiple Coulomb scattering. After energy degradation, the beam has to be collimated and the
energy spread reduced in order to match the acceptance of the beam transport system. Furthermore, the degraded
beam passes several thin monitors and vacuum windows before entering a treatment room.

In any accelerator facilities it is highly desired to have a fast beam-optics software to support control room work,
like the prediction of beam envelopes, the fit of envelopes to measured profiles or the development of new tunes. A
Beam dynamics code that is supposed to be useful for both, the HIPA and Proscan beamlines, must hence be able to
model the effects that degraders and collimators have on beam parameters like energy, emittance and beam divergence.
Up until recently, the only online beam dynamics code used in the HIPA control room was Urs Rohrer’s graphical
version of TRANSPORT [30–32]. The Proscan control rooms had no beam dynamics tool at all, since the use of
TRANSPORT and TURTLE [33, 34], though possible in principle, turned out to be too cumbersome to support daily
work in the control rooms. High-accuracy codes like OPAL [35–39], GEANT [40–42], or the Geant-based BDSIM [43],
do exist, but are better suited for high-accuracy offline studies, preferably executed on high performance computers.

MinT is a lightweight beam optics code designed to support control room activities, for instance for beam tuning
and development, that allows to provide reasonably accurate answers in short times (a few seconds up to a minute)
instead of highly accurate results within tens of minutes or hours. Though MinT has been specifically designed as an
online tool, it is useful offline as well, as a design-tool for the layout of new beam lines, when the speed of calculation
and the flexibility of the code are essential as well. Since modern control room computers are mostly using Linux
as operating system, MinT is a Linux-program as well. Other operating systems are not (yet) supported. However,
MinT is fast enough for the use in virtual machines.

In the following we report about the features and performance of MinT, the methods used in the program and
demonstrate it’s capabilities to model the beam optics of the beamlines which are part of PSI’s proton facilities.

II. THE BEAM OPTICS PROGRAM MINT

A precursor of MinT was developed for the Gantry 3 project at PSI [45–49]. The main idea was to model the
transition through matter (i.e. energy degrader) by fast and simple approximations and through collimators by
a “removal on hit” strategy with a Monte-Carlo-ensemble created from the matrix of second moments. This is
a pragmatic “engineering” kind of approach which often is sufficient in accuracy with respect to the transmitted
beam. MinT is a complete re-write of this first code, which is now controlled by a programming style input file with
capabilities for graphical output to screen and to all file formats supported by the GNU plotutils library (notably
Postscript, PNG and FIG).

A. Optics Calculations in MinT

The ion optics machinery of MinT is based on the linear beam optics methods known from the programs TRAN-
SPORT and TURTLE, i.e. the beam is described by it’s first and second moments in local co-moving coordinates. If

1 (M)int (i)s (n)ot (T)ransport [44].
$X = (x, x', y, y', z, \delta)^T$ are the usual six phase space coordinates in the co-moving frame, then the first moments are $X_i(s) = \langle X_i \rangle_s$ and second moments are given by a symmetric matrix $\Sigma_{ij}(s) = \langle (X_i - \langle X_i \rangle)(X_j - \langle X_j \rangle) \rangle_s$. The particle transport through beam optical elements, using a linear approximation and hard-edge magnets, is then given by a sequence of multiplications with symplectic matrices $M_k$. MiNT allows to control the stepsize in all beamline elements individually and can therefore be optimized by the user either for speed or for accuracy. It is possible, but not always desired, to compute the transfer matrix of some element in one step, since beam-loss typically occurs inside of quads and bends. Possible beam-loss in the center of magnets can only be estimated with reasonable accuracy, if the beam optics inside the element is calculated in small steps. MiNT evaluates beam losses only when the envelope is evaluated as well (i.e. after some step). MiNT possesses two modes, a pure envelope mode, corresponding to linear TRANSPORT calculations, and a Monte-Carlo assisted envelope mode (“sampled mode”), where the beam is represented by a Monte-Carlo generated ensemble of “rays” $X_i$. The user can switch between these modes by the insertion of dedicated elements.

In envelope mode the quations are:

$$X(s + L) = M_k(L)X(s)$$
$$\Sigma(s + L) = M_k(L)\Sigma(s)M_k(L)^T$$

where $s$ is the position along the reference trajectory, $L$ is the length of the calculation step in element $k$ and $M_k$ is the transfer matrix of the $k$-th element. MiNT is equipped with an integrated symplectic Monte Carlo generator for multivariate Gaussian distributions. A distribution of $n$ trajectories is represented by a $6 \times n$-matrix $X_{ij}$, such that the center of the bunch is given by

$$X_i(s) = \frac{1}{n} \sum_{j=1}^{n} X_{ij}(s)$$

and the centered distribution $\tilde{X}_{ij}$ by

$$\tilde{X}_{ij} = X_{ij}(s) - X_i(s).$$

The matrix of second moments is then

$$\Sigma_{ij} = \frac{1}{n} \tilde{X} \tilde{X}^T$$

Without collimators, the evolution of the beam centroid, MC-sample and matrix of second moments is described by a symplectic transfer matrix $M(L)$ and is in Monte-Carlo mode given by:

$$X(s + L) = M_k(L)X(s)$$
$$\Sigma(s + L) = \tilde{X}(s + L)\tilde{X}(s + L)^T$$

In Monte-Carlo-mode, the $\Sigma$-matrix is obtained from the (possibly re-centered) Monte-Carlo-sample.

In the presence of collimators, the MC-sample is “filtered”: trajectories which hit a collimator, are removed from the ensemble $X$ which represents the beam. The current version of MiNT allows for circular, elliptic and rectangular

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2 Where the dash indicates the derivative along the beam path $x' = \frac{dx}{ds}$ and $\delta = \frac{d\delta}{ds}$ is the momentum spread.

3 MiNT, starting with version 0.50, supports second order calculation. But since second order effects are weak in the PSI proton beamlines, only first order has been compared and tested against measurements.
collimators and moveable horizontal or vertical, symmetric or asymmetric, slits. If \( X = (x, x', y, y', z, \delta = \frac{dp}{T})^T \) is the vector of local coordinates, then a trajectory passes an elliptic collimator with horizontal and vertical half-diameter \( a \) and \( b \), if

\[
\frac{(x + x_0)^2}{a} + \frac{(y + y_0)^2}{b} \leq 1
\]  

where \( x_0 \) and \( y_0 \) are the coordinates of the beam centroid.

No scattering calculations are done on collimators. But in many beam optical calculations, it is not the primary objective to obtain a detailed calculation of losses and activation, but the knowledge of the properties of the fraction of the beam which passes the collimator. The test Eq. 6 of 10^4 – 10^6 trajectories is computationally expensive, but the simplest general scalable method to solve the (strongly non-linear) collimation problem.

B. Beam Degradation

Besides collimation, the main task appearing for instance in proton therapy beamlines with beam degrader is the calculation of beam degradation effects, such as energy reduction and -straggling, and emittance increase by multiple scattering [51–55].

1. Energy Degradation

For the energy loss (and range) calculations the Bethe-Bloch-formula is used to describe the average energy loss per unit length of charged particles passing through matter [56]:

\[
\frac{dE}{ds} = -K \frac{z^2}{A} \frac{1}{\beta^2} \left[ \frac{1}{2} \log \left( \frac{2m_e c^2 \beta^2 \gamma^2 T_{\text{max}}}{I^2} \right) - \beta^2 - \frac{\delta}{2} \right].
\]

where \( s \) is the pathlength, \( m_e \) is the electron mass, \( z \) and \( M \) are the charge number and the mass of the projectile, \( N_A \) is Avogadro’s constant, \( Z \) and \( A \) are atomic number and mass of the medium (in g mol\(^{-1}\)), and \( K \) is the factor \( K = 4\pi N_A r_e^2 m_e c^2 \). Furthermore \( r_e \) is the classical electron radius \( r_e = \frac{e^2}{4\pi \varepsilon_0 c^2} \), \( \beta = \frac{v}{c} \) is the velocity in units of the speed of light \( c \), and \( \gamma \) is the known relativistic factor \( \gamma = \frac{1}{\sqrt{1 - \beta^2}} \). \( I \) is the mean excitation energy in units of eV, \( \delta \) is the density effect correction to ionization energy loss, and \( T_{\text{max}} \) is finally given by

\[
T_{\text{max}} = \frac{2 m_e c^2 \beta^2 \gamma^2}{1 + 2 \gamma m_e/M + (m_e/M)^2}.
\]

The current version of MINT neglects the density effect correction term (i.e. \( \delta \to 0 \)) as the contribution is negligible in the energy range below about a GeV [56].

In case of degraders made from composite materials (containing different nuclei), MINT computes the energy losses for each target component with the corresponding reduced density:

\[
\frac{dE}{ds} = -K \frac{z^2}{\beta^2} \sum_k \frac{Z_k}{A_k} \rho_k \left[ \log \left( \frac{2m_e c^2 \beta^2 \gamma^2}{I_k} \right) - \beta^2 \right],
\]

where the partial densities \( \rho_k \) have to sum up to the density of the compound \( \rho \):

\[
\rho = \sum_k \rho_k.
\]

For the convenience of use, MINT provides the possibility define arbitrary pure and composite materials.
2. Energy Straggling

The conventional 6th beam coordinate $\delta = \frac{\delta p}{p_0}$ is a relative quantity and with the degradation process, both the momentum spread $\delta p$ and the average momentum $p_0$, change. The former increases by energy straggling while the latter decreases by energy degradation.

The latter means that the final energy $E_f$ depends, after the passage through a slab of fixed thickness $\Delta s$, non-linearly on the initial energy $E_i$, so that

$$ \frac{dE_f}{dE_i} = \frac{dE_f/ds}{dE_i/ds} $$

and hence

$$ \sigma_{E_f}^2 = \left( \frac{dE_f/ds}{dE_i/ds} \right)^2 \sigma_{E_i}^2. \quad (12) $$

The combination with the stochastic straggling yields [57]:

$$ \sigma_{E_f}^2 = \left( \frac{M(E_f)}{M(E_i)} \right)^2 \left( \sigma_{E_i}^2 + M(E_i)^2 \int_{E_f}^{E_i} N(T) M(T)^2 dT \right), \quad (13) $$

where $M(E) = -\frac{dE}{ds}$ and $N(E)$ is the stochastic energy straggling for thin targets (in dependence on energy), which can be calculated by [58]:

$$ N(E) = \frac{d\sigma_E^2}{ds} = 4 \pi N_a r_e^2 (m_e c^2)^2 \rho \frac{Z}{A} \left( 1 - \frac{1}{2} \beta^2 \right) \gamma^2 $$

$$ = 4 \pi N_a r_e^2 (m_e c^2)^2 \rho \frac{Z}{A} \frac{1 + \gamma^2}{2} $$

where $\rho$ is the density, $Z$ the nuclear charge and $A$ the mass number of the scatterer. If we change the integration variable in Eqn. (13) we obtain in the limiting case $\Delta x \to 0$:

$$ \frac{\sigma_{E_f}^2}{M(E_f)^2} = \frac{\sigma_{E_i}^2}{M(E_i)^2} + \int_0^{\Delta s} \frac{N(E(x))}{M(E(x))^2} dx $$

$$ \frac{d}{ds} \left( \frac{\sigma_E^2}{M(E)^2} \right) = \frac{N(E)}{M(E)^2} $$

(15)

It is hence necessary to normalize the energy variance with the (absolute value of) the Bethe-Bloch function $M(E)$ in order to obtain, in Gaussian approximation, the desired differential equation. Eq. (15) can be integrated numerically to finally obtain $\sigma_E^2$ at the final energy $E_f$ and hence the final momentum spread $\delta$.

3. Deep Inelastic Scattering

MiNT enables to estimate the beam loss by deep inelastic scattering (DIS) by the formula provided in Ref. [59–61] for energies between a few MeV and a few hundred MeV. Any projectile that is subject to some inelastic process, is counted as lost. But MiNT does not remove tracks due to DIS, but simply calculates the surviving fraction of the beam. The beam current is then reduced along the beam path accordingly.
The DIS calculation requires that the user specifies the RMS radius $R_{rms}$ for the projectile and passed material. Then an inverse scattering length $\frac{1}{\lambda_{DIS}}$ can be written as

$$\frac{1}{\lambda_{DIS}} = \frac{\rho}{A} \sigma_{DIS} \quad (16)$$

The beam current change $dI$ for passing a slab of thickness $dx$ is then approximated by

$$\frac{dI}{ds} = -\frac{I}{\lambda_{DIS}} \quad (17)$$

Hence the surviving fraction of particles without deep inelastic process decreases exponentially with the thickness of the traversed matter.

### 4. Emittance Increase by Lateral Straggling

MiNT provides two scattering models, the first follows the suggestion of Francis Farley [62, 63], which is a “local” approximation of the Moliere theory. Farley refers to Ref. [56] where the precision is claimed to be 11%. The theoretical description of the beam passage through the solid degrader material can best be understood, if one considers an idealized parallel beam (zero emittance) interacting with a single scatterer. The immediate effect of the scatterer is to change only angle (and energy) of the incident proton (or ion). The time derivative of the matrix $\sigma$ of second moments is then given (in one dimension) by [63]:

$$\frac{d\sigma}{ds} = \begin{pmatrix} 2\sigma_{12} & \sigma_{22} \\ \sigma_{22} & T(p) \end{pmatrix}, \quad (18)$$

where $T(p)$ is the scattering power. Farley used $T_{FF}(p) = \frac{K}{p^2\beta^2}$ as scattering power where $p = p(s)$ is the momentum, $\beta = \beta(s) = v(s)/c$ is the ratio of the projectile’s velocity to the velocity of light and $K$ is a material dependent constant that is given by Farley as:

$$K = 200 \text{ (MeV/c)}^2 \frac{Z^2}{X_0}, \quad (19)$$

with the charge of the projectile $Z$ and the radiation length of the target material $X_0$ [56] given in units of $[\text{g cm}^{-2}]$:

$$X_0 = \frac{716.4A}{Z(Z+1) \log(287/\sqrt{Z})} \quad (20)$$

Often an alternative radiation length $\tilde{X}_0 = X_0/\rho$ is used, which is divided by the target density $\rho$ and then has the unit $[\text{cm}]$.

According to Gottschalk, this corresponds to the scattering power of Fermi and Rossi [64]:

$$T_{FR} = \left( \frac{E_s Z}{p\beta} \right)^2 \frac{1}{X_0}, \quad (21)$$

where $E_s = 15\text{ MeV}$ approximately matches Farley’s formula. Gottschalk presented a detailed comparison of various theoretical models in Ref. [64] and derived a phenomenological model to match the experimental data. His scattering power is

$$T_{GS} = f_{dM} \left( \frac{E_s Z}{p\beta} \right)^2 \frac{1}{X_S}, \quad (22)$$
and the weight function $f$ user, allows to adjust the strength of angular straggling. The variable use either Farley-Fermi-Rossi or Gottschalk’s scattering power. A global variable named “MSFactor”, defined by the

$$x = \log \left( \frac{\rho}{\alpha r A} \right) - \frac{1}{2} \ln \left( 2 \pi r^2 \rho \right) + \frac{1}{2} \ln \left( \frac{33219 (A Z^{-1/3})}{2} - 1 \right)$$

and the weight function $f_{DM}$ is given by

$$l_1 = \log \left( \frac{p_{[MeV/c]}}{\beta} \right)$$

$$l_2 = \log \left( 1 + \varepsilon - \left( \frac{p \beta}{\rho t_1 \beta_1} \right)^2 \right)$$

$$f_{DM} = 0.5244 + 0.085773 l_2 + 0.1 l_1 - 0.004256 l_1 l_2$$

where $p_1$ and $\beta_1$ are momentum and velocity (in units of $c$) of the incident projectile. Gottschalk’s original formula is obtained for $\varepsilon = 0$. However, for very thin sheets of material, Gottschalk’s formula may result in large negative values for $l_2$. The parameter $\varepsilon$ enables the user to prevent unphysical results for these cases. The user may therefore use either Farley-Fermi-Rossi or Gottschalk’s scattering power. A global variable named “MSFactor”, defined by the

In any case, MINT integrates Eq. [18] to model the influence of multiple Coulomb scattering on the $\Sigma$-matrix of the beam. If MINT runs in Monte-Carlo-mode at the entrance of some degrader element, then the calculation is done twice in parallel: The existing ensemble is transported as in case of a drift and an initially zero $\Sigma$-matrix (a zero-emittance beam) is integrated according to Eq. [18] and then used to generate a second (scattered) ensemble. At
the end of the element, the matrices of both ensembles are added, which means that the direction of each trajectory is changed statistically.

III. EXAMPLES: MINT FOR PSI BEAMLINES

In this section we provide some example applications of MinT, namely to the PSI proton beamlines.

A. HIPA Beamlines

Fig. 1 gives an overview of the HIPA facilities [3]. The beam is generated in a compact ECR proton source [65] and extracted with a 60 kV extraction system which is located on the high voltage platform of the 810 kV Cockcroft-Walton accelerator. The resulting 870 keV DC beam with a current of 10 – 12 mA is send through a system of two bunchers, a 50 MHz first and a 150 MHz third-harmonic buncher, to the axial injection line of Injector II. Injector II is a 72 MeV high-current isochronous cyclotron with four separate sectors [66–71]. After extraction, the beam is send via the 72 MeV-beamline “IW2” to the so-called “Ring-cyclotron” [72–78] where it is accelerated to 590 MeV. A fraction of several ten µA can be split off from the 72 MeV-beam and send to the isotope production facility IP2. The 590 MeV beam is either transported to the ultra-cold neutrons at UCN (typically in “pulses” of a few seconds) or otherwise send to the muon/pion production graphite targets, first the 5 mm thick “Target M” and then to the 40 – 60 mm long “Target E”. After an appropriate collimation, the remaining useable beam is either send to the SINQ-target for neutron production or otherwise to a high-intensity beam dump ($I_{max} \approx 1.7$ mA).

Since TRANSPORT offers no convenient possibility to simulate the passage of high-energy particles through matter, the description of this beamline with TRANSPORT requires to split the optics calculation from Ring-cyclotron to SINQ (or beamdump) into at least 3 sections, from Ring to Target M, from Target M to Target E and from Target E to beamdump or SINQ, respectively. Each of this sections is then treated seperately and it is up to the user to verify the overall consistency. The MinT monte-carlo-mode (activated here behind Target M) allows not only to compute the beam optics in one go, but also to predict beam currents and “realistic” (non-Gaussian) beam profiles. In the following we provide some examples.

1. From Ring Cyclotron to the Beamdump

Fig. 2 shows the result of a MinT optical calculation from Ring cyclotron to the beamdump for two versions of Target E, 60 mm and 40 mm long (at $s \approx 61$ m). The symbols are measurements of the latter. The apertures of the respective beamline components are indicated by a colored background in PSI convention, i.e. blue for bends, red for quads, gray for drifts and yellow for steering magnets. The graphical output follows the TRANSPORT conventions, i.e. the vertical beam size ($2\sigma_{rms}$) is shown above axis (red) and the horizontal beam size (blue) below axis. The symbols indicate the beam size as measured by beam profile monitors.

The primary beam intensity was (60 mm target) 1840 µA, the beam intensity after collimation (measured by current monitor MHC5) was 1022 µA, the MinT prediction is 1090 µA using Gottschalk’s scattering power (1054 µA for the
modified version). In case of the 40 mm target, the measured (simulated) current after Target E was 1120 µA (1111 µA), using the modified version of Gottschalk’s scattering power.

These calculations used a Monte-Carlo sample of 50000 “macro”-particles (Monte-Carlo mode starting behind Target E) and took less than five seconds on an average laptop computer\(^4\). The prediction of the correct beam current depends on many details so that the agreement with the measured currents is quite satisfactory.

After the passage of Target M, the estimated beam energy is calculated to be 586.6 MeV, after the 40 mm (60 mm) long Target E it is 569.3 MeV (560.6 MeV). The emittance values assumed at the exit of the Ring cyclotron are \(\varepsilon_x = 0.154\) mmmrad and \(\varepsilon_y = 0.366\) mmmrad, after Target M they increase to \(\varepsilon_x = 0.468\) mmmrad and \(\varepsilon_y = 1.63\) mmmrad, after Target E (40 mm) to \(\varepsilon_x = 1.035\) mmmrad and \(\varepsilon_y = 5\) mmmrad. The MinT simulation results in a beam loss of 1.1% in Target M and of 8.24% in Target E, which agrees well with the numbers given in Ref. \[79\].

2. From Ring Cyclotron to SINQ

Since the installation of the SINQ, the beam dump is used only in times when the SINQ is not ready for data taking. In SINQ operation, the magnet AHL is active and the SINQ-beamline guides the beam downwards in order to inject it from below into the SINQ target (see Fig. 3\(^5\)). The SINQ-beamline is equipped with some large aperture quadrupoles. Rohrer’s version of TRANSPORT supports large aperture quads by a fringe field corrections derived from fringe field integrals\[30\] according to the approximation developed by Matsuda and Wollnik\[83\]. However, the suggested corrections are not symplectic. Furthermore the calculation of the fringe field integrals requires a precise knowledge of the field shape, which is not always available and/or reliable. Nonetheless MinT provides a symplectified version of this method, described in Sec. B. Furthermore MinT allows to choose another (fully symplectic) correction due to Baartman which does not require the knowledge of the fringe field integrals. In case of the SINQ beamline it provides an equivalently satisfying agreement with the measured beam profiles.

If \(f\) is the (uncorrected) focusing length of the quad \(R\) the pole radius and \(L\) the effective length, then the Baartman’s correction is based of the following change of the focusing length\[84\]:

\[
\Delta f = R^2/(2L)
\]

A more detailed description of the implementation in MinT is given in Sec. B of the appendix.

Both methods are able to describe the optics of the SINQ beamline with satisfactory accuracy. Fig. 4 shows the results using a MC sample of \(10^4\) particles with Baartman’s fringe field correction. The profile measurements between

\[^4\] The speed of calculation can vary significantly, depending on the used step size. The calculations shown here included a space-charge kicks at least every 100 mm.

\[^5\] More details on the collimators can be found in Refs. \[74, 82\].
MinT provides sufficient numerical accuracy to compute tunes which require little or no correction by manual fine-tuning by operators. However, approximate beam tunes can be elaborated and fine-tuned by a stepwise increase of the beam current to its production value. Even though the predicted losses can deviate from the measurements by a few percent, MinT allows for a reasonable online prediction of the beam optics. In a high current facility like HIPA, where even small losses, in the order of a permille, can overheat and melt components, the accuracy of the MinT model is certainly not sufficient to omit the fine tuning by operators. However, approximate beam tunes can be elaborated and fine-tuned by a stepwise increase of the beam current to its production value.

However, for (the commissioning or tuning of) low intensity beamlines like the ones used for proton therapy machine, MinT provides sufficient numerical accuracy to compute tunes which require little or no correction by manual fine-tuning.

3. Injector II and the 72 MeV-beamline

The 72 MeV beamline (“IW2”) connects the extraction of injector II with the injection of the Ring cyclotrons. As reported elsewhere, the Injector II cyclotron is operated in the space-charge dominated regime. MinT provides the possibility to define a sequence of elements as a “ring” so that the matched beam matrix $\Sigma_m$ can be derived from the matching condition, for given emittances and current. The specific problem in case of non-negligible space charge is the fact that the one-turn-transfer matrix depends on the strength of the space charge and hence on...
the beam size. If \( \mathbf{M} = \mathbf{M}(I, \varepsilon_i) \) is the one-turn-transfer-matrix, depending on beam current \( I \) and beam emittances \( \varepsilon_i \), then the beam is matched, iff

\[
\mathbf{S} = \mathbf{M} \mathbf{S} \mathbf{M}^{-1},
\]

where - with \( \mathbf{J} \) as the symplectic unit matrix - the matrix of second moments for the matched distribution is given as \( \Sigma_m = \mathbf{S} \mathbf{J} \). MnT makes use of the general decoupling/diagonalization methods described in Refs. [50, 93–95], which allow to determine \( \mathbf{S} \) and hence \( \Sigma \) for arbitrary symplectic transport matrix \( \mathbf{M} \) and given proper eigen-emittances. However, here \( \mathbf{M} \) depends itself on (elements of) the \( \Sigma \)-matrix, for instance on (square of the rms) beam size, then the problem can not be solved analytically and it is required to use an iterative scheme [89]. MnT is equipped with such a scheme and, for the beam conditions and currents of Injector 2, typically less than 10 iterations are required to find the matched beam. With low beam current or if space charge is ignored, ideal isochronous cyclotrons do not provide any longitudinal focusing and the matching is hence undefined in the longitudinal direction.

The matched beam is then used as a starting condition for a fit to the measured profiles of the 72 MeV beamline which connects Injector II and the Ring cyclotron. The MnT calculations confirmed that the matched beam assumption provides excellent starting conditions which allow to fit the beam envelope of the IW2-beamline in few steps. At high currents, the horizontal beam envelope requires almost no adjustments to match the measured beam sizes.

In linear approximation, the cyclotron specific space charge effect connect only horizontal and longitudinal motion, while the vertical motion is not affected. In other words, within Injector II, the vertical beam size is not directly coupled to the horizontal and longitudinal motion and hence can be fitted to the measured beam sizes without strong influence on the so-called “vortex motion” [89, 90, 92].

Fig. 5 shows the two stages of the fitting procedure. The darker colors show the beam envelopes of a two-parameter fit (varying only two beam emittance values), assuming a matched beam from Injector II. In the second step, the vertical initial beam parameters \( (\sigma_{33}, \sigma_{34}, \sigma_{44}) \) are varied to improve the fit to the vertical beam sizes. On the left the simulation starts with the last turn of Injector II (four bends) and ends on the right with the first turn of the Ring cyclotron (eight bends). For a comparison with OPAL see Ref. [85].

The results confirm that the horizontal and longitudinal beam parameters are due to the space-charge induced coupling, while the vertical beam parameters are linearly independent. This kind of horizontal-longitudinal “self-matching” works, as expected, only for sufficiently high beam currents. Their is no passage through matter and no significant beam collimation in the 72 MeV transfer line and MnT is therefore operated here in pure envelope mode.

4. The 870 keV-beamline

Fig. 6 gives a schematic overview of the 870 keV “BW870” injection line connecting the ECR ion source via Cockcroft-Walton DC preaccelerator with the center of Injector 2 [67, 70, 96–98]. Due to the strong space charge forces of the 10 mA DC beam, the beamline BW870 is most challenging in terms of ion-optical modelling. The specific difficulty is due to (partial) space charge compensation by electrons which are attracted and captured by the DC proton beam potential. Furthermore, the DC proton beam is bunched by a first and third harmonic buncher [71, 99]. The main task is here is to find a method to model the transition from a DC beam into a bunched beam. The bunching process involves all possible phases of the buncher and can therefore not be modelled in linear approximation.
MinT is adjusted accordingly, i.e. the energy change $\Delta \sigma$ is undefined. Walton type DC accelerator, the energy spread of the beam entering the buncher is very low and the bunch-length is which is an appropriate fraction of $z$. Hence a $\text{buncher re-samples the longitudinal distribution assuming a bunch length }$. In case of the 870 keV-beamline, one obtains $\epsilon_{x,y}$. The vertical (horizontal) envelopes are shown in dark red (blue). The “+” symbols indicate the respective beam sizes as measured by profile monitors and the light colored (red/blue) lines indicate the results of MinT envelope calculation before and after a fit of the axial beam parameters to fit the measured beam sizes. The corresponding initial (final) dispersion is shown in dark (light) green and the longitudinal beam dimension (scaled down by four) in (dark/light) orange. The computation, including all fits, takes a few seconds on an average personal computer. In case of the IW2 beamline, where space charge has to be included, the computation speed depends strongly on the settings. In the plot shown here, the distance between two subsequent space charge kicks was 50 mm.

A rather simple method to model the effect of a buncher in combination with a DC beam has been implemented in MinT. This method presumes a sampled beam. Since the beam of the 870 keV beamline has passed a Cockcroft-Walton type DC accelerator, the energy spread of the beam entering the buncher is very low and the bunch-length is undefined.

The buncher element therefore has to introduce and (re-) define the bunch length using the buncher-frequency $f_b$ and the particle velocity $v$. Hence a MinT-buncher re-samples the longitudinal distribution assuming a bunch length $\sigma_z$ which is an appropriate fraction of $v/f_b$. In case of the 870 keV-beamline, one obtains

$$v/f_b = 255\text{ mm}$$

(27)

MinT uses typically 2 $\sigma$-values, and hence the bunch length is assumed to be approximately 4 $\sigma$ in total, so that

$$\sigma_z \approx \frac{1}{4} v/f_b.$$  

(28)

After generating a random longitudinal position with this $\sigma_z$, the energy (i.e. momentum deviation) of each particle is adjusted accordingly, i.e. the energy change $\Delta E_i$ of the i-th particle is modified by

$$\Delta E_i = -Q V_b \sin (2 \pi z_i/\sigma_z).$$

(29)
σ is shown in Fig. 8. The core of the formed bunches becomes longitudinally compact, and has a momentum spread of $\sigma_3 = 0.00682$.

Fig. 8 provides a qualitative and quantitative comparison of the beam profiles in the axial injection line. Since the dispersion at the location of the profile monitors is non-zero, the (strongly non-Gaussian) energy distribution induced by the two bunchers becomes visible in the beam profiles.

There have been simulations of the bunching process of a DC beamline with space-charge presented in the past [99]. The advantage of the method implemented in MinT is that it allows to compare the simulation results with beam profile measurements, and hence to validate the used model.

**B. The Proton Therapy Beamlines**

Fig. 10 provides an overview of the Proscan facility [100, 101], where proton beams in an energy range between 70 and 230 MeV are used to irradiate tumors for cancer therapy, taking advantage of the so-called bragg peak [51, 52, 53].

The compact isochronous cyclotron “COMET” provides a continuous wave (CW) beam of 250 MeV with currents of up to 800 nA [21, 22]. The beam energy is adjusted by means of a double-wedge-degrader made from high-density graphite, followed by a beam collimation system and the energy selection system (ESS). The collimation system consists of two multi-aperture collimators KMA3 and KMA5 and some fixed collimators (KMA4, KMA6 and
FIG. 8. Evolution of the longitudinal phase space of the 870 keV injection beamline between from buncher (upper left) up to the first turn of Injector 2 (lower right). The individual locations are (left to right), 50 MHz-buncher, before and after 3rd harmonic buncher, two positions in the vertical injection line (before QWB4 and after QWB6) and finally in the first turn of Injector 2. The 50 MHz-buncher is assumed to have a total voltage (both gaps summed up) of 17.1 kV, the 3rd harmonic buncher of −2.25 kV.

FIG. 9. Beam profiles of MWP26, MWP28 and MWP30 in the dispersive section of the axial injection line after the bunchers, for various voltages of buncher CWB with constant voltage (2.57 kV) of the third harmonic buncher CWB3 (left). Right: CWB at 17 kV with various voltages of CWB3. The measured (simulated) profiles are shown in the top (bottom) row. The profiles are centered and normalized to the same maximal value for better comparison.

KMA7) [102]. A fast kicker magnet and beam blocker BMA1 in front of the degrader are used to quickly switch the beam on or off.

The Proscan facility uses beams in the range from 70 MeV to about 230 MeV for patient treatment, controlled by the degrader wedge positions, but never the direct cyclotron beam. The transversal beam emittance therefore depends on the collimator geometry – even at the highest clinical energy of 230 MeV – but only weakly on the cyclotron beam emittance – as long as the beam is well-focused onto the center of the degrader wedges.

The (multivariate) Gaussian Monte-Carlo generator implemented in MinT approximates the beam distribution from the matrix of second moments by:

$$f(z) = \frac{f_{DIS}}{(2\pi)^2 \sqrt{|\Sigma_t|}} \exp\left(-z^T \Sigma_t^{-1} z/2\right),$$

Here only the transversal coordinates are of interest and therefore $z = (x, x', y, y')$ are the transversal coordinates and $\Sigma_t$ is the transversal matrix of 2nd moments. But since the collimators KMA3 and KMA5 select the beam in the vicinity of the forward direction, the transmitted intensity $I$ passing the collimators on axis (or better: close to the
axis) can be approximated by:

\[ I = f(0) d\Omega = \frac{f_{DIS}}{(2\pi)^2 \sqrt{\left| \Sigma_t \right|}} d\Omega = \frac{f_{DIS}}{(2\pi)^2 \varepsilon_x \varepsilon_y} d\Omega \]  

(31)

where \( d\Omega \) is the solid angle of the collimation system and \( \varepsilon_i \) are the eigenvalues of the \( \Sigma_t \)-matrix, i.e. the emittances of the transversal degrees of freedom. \( f_{DIS} \) is a beam loss factor that quantifies beam losses by deep inelastic (large angle-) scattering.

Hence the transmitted (forward) intensity \( I \) is approximately inversely proportional to the product of the emittances and hence depends directly on the strength of proton lateral straggling inside the degrader. The minimal emittances \( \varepsilon_i \) (and hence the maximal transmission) that can be achieved for a given energy at the degrader exit, depends also on the optical properties on the beam entering the degrader, but the emittances of the beam after the collimation system, is mostly determined by the solid angle defined by the apertures of the collimation system. Hence the beam focus as defined by the quadrupoles QMA1, QMA2 and QMA3 (located in front of the degrader), can be used as an additional knobs to control the beam current without having a strong effect on other beam parameters.

TRANSPORT is a powerful tool as long as beam losses and emittance increase along the considered beam transport system are negligible. Both of these conditions are not met in cyclotron driven proton therapy facilities, where energy degraders are used to adjust the beam energy, with the side effect to increase the beam emittance beyond the acceptance of the beamline. The use of a degrader implies the necessity to collimate the beam, both transversally (by beam collimation) and in energy spread by an energy selection system (ESS). However this scheme generates a
FIG. 11. Beam envelopes (2\(\sigma_{rms}\)) of the Proscan beamline from proton therapy cyclotron COMET to the entrance of Gantry 3 for an energy of about 100 MeV. The (negative) dispersion is shown in green. The beam width as measured by the proscan beam profile monitors [106, 107] is indicated by crosses. The calculated beam current (i.e. 10 log (I/\(\mu\)A)) is shown as a solid black line (measurements by gray filled circles). At the end of the beamline, the measured beam current is 90.4 pA agrees well with the MiNT result 89 pA.

strong energy dependence of the transmitted beam intensity. Uncompensated, the beam intensity varies between 70 and 250 MeV by roughly three orders of magnitude, for the same cyclotron current. In order to reduce this dynamic intensity range, the beam intensity change is (partially) “compensated” [104]: At low energies, the beam is well-focused onto the degrader in order to provide highest possible transmission. At energies above \(\approx\) 120 MeV, the beam is intentionally defocused on dedicated collimators to reduce the energy dependence of the transmitted intensity.

The intensity compensation at Proscan is done in two stages. The first stage is located upstream of the degrader: QMA3 is used to (de-) focus the beam and to smoothly reduce the transmission of high energies. In order to reduce the dynamic range by two orders of magnitude (from \(\approx\) 10\(^3\) to \(\approx\) 10), a single stage is not sufficient. A second stage uses the collimator KMA8 for Gantry 3 and, for historical reasons, another collimator KMB0 for Gantry 2. Yet again, the beam is focused to achieve high transmission at low energies and defocused the more the higher the beam energy. Without the second stage compensation, a single beamline tune (set of magnet settings), scaled by momentum, would suffice for all energies. The second compensation stage however requires a slightly different optics setting for each energy and hence a slightly different optical tune for all energies.

Note also that the proton therapy beamlines use some permanent monitors (profile and current) and thin vacuum windows that the beam has to pass [106, 107]. Hence the beam energy after the degrader is not exactly the same as the beam energy entering the nozzle. The effects of these monitors/windows are small but not completely negligible. The beam optics computation done with MiNT allows to take these effects into account.

The patient treatment planning system of Proscan always starts with the highest required energy (the deepest layer) and the reduces the energy stepwise for each layer. This is required to avoid hysteresis suppression cycles between
FIG. 12. Simulated (red lines) and measured (black lines) beam profiles corresponding to the measurement shown in Fig. [11]. The profiles have been equally scaled and centered for better comparison. (The last measured profile, MMDP18, is distorted due to a bad electrical contact.)

different layers in order to minimize layer switching and hence patient treatment time. The beam tunes should therefore preserve the ramping direction: a reduction of energy should, for all energies, correspond to a decrease of the field-settings of all quadrupoles. Without intensity compensation, the tunes would simply scale the field with the particle momentum and the requirement would be fulfilled automatically, but with active intensity compensation this requirement must be taken into account.

The energy selection system (ESS) consists of a double-bend achromat composed of two dipoles and four quadrupoles. The first dipole generates a non-zero dispersion that is compensated by the second dipole. A moveable collimator (FMA1) is located in the center between the dipoles where the dispersion is maximal, so that the energy spread of the beam can be reduced by adjusting the aperture of the horizontal moveable slit FMA1.

The beamlines to Gantry 1, OPTIS 2 and Gantry 2 are shown in gray, as we shall not discuss their optics here. Gantry 3 shares a beamline with the experimental area of the proton irradiation facility (PIF), which is used only during the night or on weekends, i.e. in times when no patient treatments take place.

MinT enables to model the beam tunes of this type of facility, both qualitatively and quantitatively with reasonable precision. The capability of the code to estimate beam-envelopes and -intensity simultaneously simplifies the design of beamline layout and tunes and allows to predict the transmitted current as well as the locations and the amount of beam loss. The Monte Carlo mode of MinT also enables to predict beam profiles. Significant deviations of the predicted beam profiles from measured profiles helped in the course of the Gantry 3 project to identify errors in the beamline model. Here we show how beam profile and intensity measurements taken during the commissioning shifts for Gantry 3 compare to recalculations lately done with MinT.

Fig. [11] shows a MinT calculation of the Gantry 3 beamline up to the coupling point of Gantry 3. This calculation requires less than 10 s with an Monte-Carlo ensemble of 1 Mio. particles on a standard laptop. Simulation results using OPAL of the same beamline have been shown in Ref. [48].
and effective way. the applicability of TRANSPORT type calculations to include the passage of matter and beam collimation in a fast MinT facility Proscan. In all cases measurements of the PSI proton facilities, firstly the high intensity facility HIPA, and secondly of the proton therapy MinT order calculations but the range of applicability has been expanded.

compare model and measurement. The accuracy of the ion optics calculation is comparable to TRANSPORT’s first space ensembles can be used as starting conditions in other codes like MinT also allows for the calculation of matched beams in ion (storage) rings, cyclotrons, and FFA’s for arbitrary couplings and given emittances including space-charge kicks. MinT also allows for the iterative calculation of linear matched beams, even for cyclotron specific couplings, i.e. the so-called “vortex effect”. The correspondings phase space ensembles can be used as starting conditions in other codes like OPAL MinT does not aim to compete with more “realistic” codes like OPAL or BDSIM. MinT is intended to replace (and extend the capabilities of) the fast and lightweight codes TRANSPORT and TURTLE, specifically as a control

FIG. 13. Same plot as is Fig. 11 but for an energy of ≈ 190 MeV. The intensity is reduced by defocusing the beam vertically with quadrupole QMA3 and after the ESS on collimator KMA8. The Monte Carlo mode starts after the degrader such that the collimator in front of the degrader (after QMA3) has no effect, whereas the collimators behind the degrader (at about s = 4 m) and collimator KMA8 (at about s = 18 m) reduce both, beam size and current, in agreement with the measurements.

IV. SUMMARY AND OUTLOOK

A description of the methods used in the new ion beam optics program MinT has been given, which allow to extend the applicability of TRANSPORT type calculations to include the passage of matter and beam collimation in a fast and effective way.

The results of various ion beam optics calculations with MinT have been compared with beam profile and intensity measurements of the PSI proton facilities, firstly the high intensity facility HIPA, and secondly of the proton therapy facility Proscan. In all cases MinT provides convenient and fast methods to simulate beam line optics and to compare model and measurement. The accuracy of the ion optics calculation is comparable to TRANSPORT’s first order calculations but the range of applicability has been expanded. MinT allows additionally for the simulation of slices of energy degraders or targets and – in the Monte-Carlo-mode – of collimators with simple geometry. It can predict beam losses and realistic beam profiles. The accuracy of these latter calculations is of course limited by the precision of the various multiple scattering approximations. The best agreement with data of the proton therapy facility was obtained using Gottschalk’s scattering model [64].
room tool for beamlines like those of PSI’s proton accelerator facilities. MinT has not been (directly) validated by comparison with other codes, but by comparison with profile and current measurements performed at PSI.

V. ACKNOWLEDGEMENTS

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MinT has been written in “C++” and compiled with the GNU©-C++ compiler on various Linux systems (Scientific Linux, Red Hat 6 & 7 & 8, OpenSuSE 15.X, Ubuntu). MinT uses the GNU© scientific library (GSL), and the GNU programs flex and bison. MinT version 0.50, described here, uses GNU libplot for graphical output.

Appendix A: Some Features of MinT

1. The Unit System of MinT

The main purpose of MinT is to provide an alternative to TRANSPORT for the PSI beamlines, both as an online-tool for the machine control rooms but also as a tool to develop tunes for Proscan or layouts for new beamlines.

MinT offers (and to a large degree requires) the definition and the use of units. The predefined unit system is the SI-system, but the user is free to define and use other units, for instance imperial units. MinT automatically scales variables accordingly and checks the consistency of calculations with respect to their physical units: MinT allows to add/subtract or assign only quantities of the same physical dimension. The user not only has the option but is obliged to define the units of all inputs. But he enjoys the freedom to define all parameters in his preferred units and there is no need for the user to convert units “manually”.

MinT comes with a considerable number of pre-defined (SI-) units. Also many “physical constants” are predefined in the form of physical units, for instance the speed of light ’c’, the elementary charge ’e’, the mass of the electron ’Me’ and proton ’Mp’ and many more. This allows to formulate physical equations in a convenient and readable manner. The MinT syntax expects units written in single quotes in order to distinguish, for instance, ’m/s’ from a quotient of variables.

Since certain variables, like for instance the pole tip field of a quadrupole magnet, make no sense unless they have the correct unit, MinT allows the user to define variables with a fixed physical meaning, like “MagneticField B” (instead of “Var B”). That is, the type of the physical quantity to be represented by a variable, can be generically taken into account. Hence, strictly speaking, MinT has at least as many basic types as there are different physical quantities and the unit system can be understood as a type checker.

2. The Class/Type System of MinT

The MinT input is based on a simple programming language but provides some features of object-oriented programming. There are predefined “objects”, namely beamline elements like drifts, bends and quads, but additional data fields can be defined and part of their functional behavior can be modified and some basic typing with feature
inheritance has been implemented. MiNT allows to create hierarchies of element types by the use of user-defined beamline element classes. If values are specified (assigned) within the type definition, then they are properties of the class and hence, if changed, are changed in all instances of that class. This is due to the fact that variables which are initialized within a type-definition, appear only once in memory and are therefore valid for all instances of the respective type.

```cpp
Type MyQuad(Quad) {
    SHAPE = CIRCULAR;
    Options = OPTLABEL;
    L = 368.0 'mm';
    DS = L/4.0;  // Step size
    R = 50.0 'mm'; // Pole tip radius
    RX = R-5.0 'mm'; // Hor. beam pipe radius
    RY = R-5.0 'mm'; // Vert. beam pipe radius
};
```

// Since "Current" and "MagneticField" are predefined physical quantities, they can be used as if they were basic data types:

```cpp
Type QMA(MyQuad) {
    Current I, Ilin=101.8 'A', Imax=150.0 'A';
    MagneticField b0 = 34.8 'G', b1 = 8886.5 'G',
        b2 = -671.4 'G', b3 = -80.5 'G';
    x:=abs(I)-Ilin)/(Imax-Ilin);
    B:= sign(I)*(b0 + b1 * abs(I)/Imax +
                (b2 * x^2 + b3 * x^3)*theta_h(abs(I)-Ilin));
};
```

The length of the quadrupole “QMA” is, as defined here, a property of the class “MyQuad”. If this length is redefined later, this changes the length of all quads of this class and hence the length of all quadrupole instances of this type. This allows to fit the parameters not only of individuals but of types; however it requires some care in the definition of the type hierarchy.

The actual definition of quadrupole “QMA1” of type “QMA” requires only to define the coil current to be completed. Since the field value “B” has been (re-) defined, in this example, as a function (indicated by “:=”), it is evaluated on every read access. When the ion optics machinery of MiNT reads the quadrupole field in order to compute the transfer matrix, the function “x” is evaluated and hence the field value is effectively a function of the (user-defined) current-variable “I”.

The excitation curve is approximated here by a third order polynomial and from this, the pole tip field is automatically computed, whenever it is used. Since MiNT computes the transfer matrix of each element just before the matrix is to be used, one might as well define:

```cpp
Type QMA(Quad) {
    MagneticField B0;
};
```

6 In the current version of MiNT, this is the only way for the user to define functions.
\[ B := B_0 \ast \left( \frac{PC}{(M_p \ast c \ast c)} \right); \]

where \( PC \) is the predefined variable of the beam’s actual momentum and \( “M_p” \) the projectile (here: proton) mass, so that now the quad field strength is automatically scaled to the momentum and equals \( “B_0” \) if the momentum (times the speed of light) is equal to \( m c^2 \) (that is, for \( \gamma = \sqrt{2} \) or \( \beta = 1/\sqrt{2} \)). Since MinT allows for stacked include statements, the user can build up type-libraries for specific kind of problems and easily switch between them by modifying merely the include-statements.

The input script is, like in high-level languages, devided into two parts. The first part provides the definition of element type and a description of the beamline(s). The main part is the “program”, i.e. contains the operations to be performed with the beamline. The main part of a mint-script is translated into a sequence of “byte-codes” which then allows to implement some standard loops (for-do, if-then-else, while-do, repeat-until). The overhead of generating a byte-code and runtime-linking does not significantly increase the execution time, since most of the execution time is usually spend with matrix multiplications.

In order to better compare results with TRANSPORT, MinT provides the possibility to export beamlines to a TRANSPORT type input file. However, due to the different concepts of the two programs, some minor adjustments by hand are usually required before the use with TRANSPORT.

3. Misalignments in MinT

MinT allows to define a number of misalignments as for instance offsets “Xofs” (horizontal position error) and “Yofs” (vertical position error) and misalignments of the axis Pitch, Yaw and Roll. If these variables are not initialized by the user, they are assumed to be zero. Trivially MinT can, beyond the envelope calculation, calculate, plot and fit the beam centroid as well. These features have been used to determine some (small) quadrupole misalignments within the Proscan facility. Details shall be reported elsewhere.

4. Fitting with MinT

The fitting and optimization functions of MinT are based upon the GNU scientific libraries (GSL) minimization routines. Each beamline-element in MinT is equipped with a malus-function which is supposed to quantify the disagreement between the actual state and the desired state of the beamline. Some elements, specifically monitors, have a predefined malus-calculation which computes the (squared) deviation of the calculated and measured beam size, which is activated whenever a non-zero measurement of the beamsize is defined by the user. The malus-functions of other beamline elements must be defined by the user according to their needs.

\[ \text{For instance, TRANSPORT does not accept layout calculations combined with space charge. The user then has to actively deactivate either of the two before running TRANSPORT.} \]
A typical malus-definition within an element-declaration might be:
\[
\text{Malus} := \text{maxError}(s_{11}, RX/2.0, 0.25 \text{ 'mm'}) + \text{maxError}(s_{33}, RY/2.0, 0.25 \text{ 'mm'})
\]
which increases the malus in case of a too large beam size, i.e. in cases where \(\sigma_x = s_{11}\) and/or \(\sigma_y = s_{33}\) are larger than half of the user-defined radius \(R_x\) and \(R_y\). The precision is also defined (here 0.25 mm). MintT profile monitors set the respective malus-variables to the square deviation, provided that the measured beam size and centroid positions are non-zero.

The malus of the beamline is the sum of the malus variables of all elements and the minimization of the malus is done by the beamline-method `Vary()`:
\[
\text{PKSINQ}::\text{Vary}(\text{@PKSINQ.Malus}, \text{@PKSINQ.QHG21.I1}, \text{@PKSINQ.QHG21.I2}, \text{@PKSINQ.QHG21.I3});
\]
where the first argument refers to the malus-variable to be minimized and all following parameters are to be varied (the number of parameters to be varied is arbitrary).

MintT is also equipped with an extension of this fitting routine which allows to vary multiply settings “at once”. This is the `MultiVary`-method, in which the malus is summed over a set of \(N\) calculations before the parameters are varied. This feature can for instance be used, if a parameter can not be fitted based on a single beamline setting. The syntax is:
\[
\text{G3}::\text{MultiVary}(\text{@iTune}, 17, \text{@G3.Malus}, \text{@G3.Protons.s11}, \text{@G3.Protons.c12}, \text{@G3.Protons.s22});
\]
where the first parameter, \(iTune\), is an integer variable that indicates the index of the “run”, the second parameter gives the number of “runs” and the remaining parameters are equivalent to `Vary()`, i.e. the malus-variable and the parameters to be varied. In the given example, the initial beamsize (of the initial beam named `protons`) is varied using 17 runs. The user then must take care that the respective beamline parameters are properly selected by the iterator “iTune”. Since MintT allows to define vectors, matrices and (simple) functions, the user can use a construction like the following:

```cpp
Int iTune=0;
Vector qma_fields[4]={ 3.7 'kG', 4.711 'kG', 5.1 'kG', 0.6123 'T' };

Type QMA(Quad) {
    [...]  
    B:=qma_fields[iTune];
};

Beamline BL {
    ...  
    QMA QMA1;
    ...  
};

begin
    BL::MultiVary(\text{@iTune}, 4, \text{@BL.Malus}, \text{@BL.SomeElement.SomeVariable},...);
end.
```

\(^8\) `maxError` and `minError` are predefined MintT-functions which compute the conditioned squared deviation: The result of `maxError` is zero if the respective value is below the limit.
FIG. 14. TRANSPORT style plot of the beam envelopes for a set of tunes for Gantry 2, starting from the cyclotron COMET (left to right) up to the isocenter. The second compensation collimator “KMB0” is located at $\approx 24.5\text{ m}$ downstream of profile monitor “MMBP1X”. Note that the Monte-Carlo mode is here activated behind the degrader so that the collimators before the degrader are ignored (behind QMA3). This is done on purpose to reduce computation time.

This will cause MiNT to sum up the malus-variable four times (starting from the initial value) each time, before @BL.SomeElement.SomeVariable is varied to minimize the malus. This is useful to fit for instance misalignment variables like quadrupole offsets to reproduce sets of position measurements for sets of quadrupole settings (“beam tunes”).

5. Multiple Plots in MinT

MiNT allows to plot a number of runs on a single graph.

Fig.14 shows for example 17 optimized tunes for the Gantry-2 beamline of Proscan covering the range from 70 MeV to 230 MeV in steps of about 10 MeV thus illustrating the use of the intensity compensation scheme by the use of two collimators located after QMA3 and QMA13, respectively. The energy dependence of the horizontal energy spread and hence the horizontal beam size in the dispersive region between AMA1 and AMA2 is nicely visualized in this plot.

The methods used to do this are:

```cpp
for (k=0;k<17;k+=1) {
    [...]  
    G2::Envelope("g2_optics_%d.env"<k);  
    G2::PushOptics();  
    [...]  
};
```
FIG. 15. Layout of the beamline IW2, including the last turn of Injector 2 and the first turn of the Ring cyclotron. (The corresponding beam optics is shown in Fig. 5). This plot was generated from a FIG-file (for the Linux program XFig) produced by MinT, but a direct Postscript output is possible as well.

6. Layout Calculations and Plots in MinT

MinT allows to compute the floor layout of beamlines and to produce figures illustrating the result. Fig. 15 shows the graphical output produced by MinT-layout routines.

The main purpose of the Layout routines, however, is not to generate beautiful graphics, but to provide numerical layout data to be compared survey data. An example of the output format:

| Element | Type | L     | S_ref | Z_vertex | Z_traj | X     | Y     | Z     |
|---------|------|-------|-------|----------|--------|-------|-------|-------|
| Sec     | ISEC| 5.494782 | 0.12875 | 0.        | 0.     | 0.    | 0.    | 0.    |
| Valley2b| ZS   | 1.834156 | 5.623532 | 6.540384 | 6.54061 | 568.19899 | 216.619893 | 1.5 |
| SM2     | SM   | 1.82647  | 7.457688 | 8.620229 | 8.370923 | 566.119154 | 216.619893 | 1.5 |
| Valley3a| ZS   | 1.834156 | 9.284158 | 10.700074 | 10.201236 | 566.119154 | 214.540048 | 1.5 |
| Valley3b| Drift| 1.351    | 11.118314 | 12.292652 | 11.793814 | 566.119154 | 212.94747 | 1.5 |
| RIZ     | ProfZ| 0.       | 12.469314 | 12.968152 | 12.469314 | 566.119154 | 212.27197 | 1.5 |
| Valley3b| Drift| 0.48778  | 12.469314 | 13.212042 | 12.713204 | 566.119154 | 212.02808 | 1.5 |
| SM3     | SM   | 1.82647  | 12.957094 | 14.618699 | 13.870329 | 566.119154 | 210.621423 | 1.5 |
| Valley4a| ZS   | 1.834156 | 14.783564 | 16.698643 | 15.700642 | 568.19899 | 210.621423 | 1.5 |
| Valley4b| ZS   | 1.834156 | 16.61772  | 18.532699 | 17.534798 | 570.033155 | 210.621423 | 1.5 |
| SM4     | SM   | 1.82647  | 18.451876 | 20.612544 | 19.365111 | 572.113   | 210.621423 | 1.5 |
| MXP1    | ProfX20| 0.       | 20.928346 | 22.425311 | 20.928346 | 572.113   | 212.43419 | 1.5 |
| AXA     | ABend| 0.6     | 20.978346 | 22.775534 | 21.278346 | 572.113   | 212.784412 | 1.5 |
| FXE     | RDrift| 0.       | 22.431346 | 23.928756 | 22.431346 | 572.221528 | 213.932516 | 1.5 |
| AXB     | ABend| 0.602   | 22.481346 | 24.292339 | 22.782346 | 572.255744 | 214.294486 | 1.5 |

The output data are aligned with the PSI convention, according to which there are two positions along the beam line. \( S_{ref} \) is the measured distance along the actual trajectory and follows the actual path in dipole magnets. \( Z_{traj} \)
is the length of the beam path if described as a polygon connecting the vertices of the bending magnets. It is little more than a convenient measure to compare results with geometric layout data and technical drawings.

Appendix B: Quadrupole Fringe Fields

1. TRANSPORT method (Matsuda/Wollnik)

Note that MINT provides only the linear approximation of the method proposed by Matsuda and Wollnik [83]. Provided the fringe field integrals \( I_1, I_2 \) and \( I_3 \) are known according to Rohrer’s TRANSPORT convention, the single quadrupole matrix \( M_q \) is embraced by two additional matrices at the quadrupole entrance \( M_i \) and exit \( M_f \):

\[
M_q \rightarrow M_f M_q M_i .
\]  

(B1)

The horizontal \( M_h \) and vertical \( M_v \) \( 2 \times 2 \)-matrices are given by:

\[
M_h = \frac{1}{a} \begin{pmatrix}
1 - x_0 & -x_1 \\
-x_2 & 1 + x_0
\end{pmatrix} \tag{B2}
\]

\[
M_v = \frac{1}{b} \begin{pmatrix}
1 + x_0 & x_1 \\
-x_2 & 1 - x_0
\end{pmatrix}
\]

where (upper sign for entrance, lower for exit):

\[
x_0 = \pm K R^2 I_1
\]

\[
x_1 = 2 K R^3 I_2
\]

\[
x_2 = K^2 R^3 I_3
\]  

(B3)

where \( K \) is the quadrupole strength \( K = \frac{B}{R(B_R)} \), which is positive for horizontally focusing quads. The variables \( a \) and \( b \) are both equal to 1 in the original calculation. However, \( 2 \times 2 \)-matrices are only symplectic, if they have unit determinant. Hence MINT uses:

\[
a = \sqrt{1 - x_0^2 - x_1 x_2}
\]

\[
b = \sqrt{1 - x_0^2 + x_1 x_2}
\]  

(B4)
2. Baartman’s approach to quadrupole fringe fields

Baartman’s correction \cite{84} is implemented by defining the following variables:

\[
\begin{align*}
\Delta \alpha_f &= f_0 k^2 R^2 / 2 \frac{S^2}{S + \alpha c} \\
\Delta \alpha_d &= f_0 k^2 R^2 / 2 \frac{S^2}{S + \alpha c}
\end{align*}
\]

\[
\begin{align*}
k_f &= k (1 - \frac{\Delta \alpha_f}{\alpha}) \\
k_d &= k (1 + \frac{\Delta \alpha_d}{\alpha})
\end{align*}
\]

(B5)

and from these new values the corrected solutions:

\[
\begin{align*}
\alpha &= k L \\
c' &= \cos (\alpha - \Delta \alpha_f) \\
s' &= \sin (\alpha - \Delta \alpha_f) \\
C' &= \cosh (\alpha + \Delta \alpha_d) \\
S' &= \sinh (\alpha + \Delta \alpha_d)
\end{align*}
\]

(B6)

and the corrected (focusing/defocusing) sub-matrices:

\[
\begin{align*}
M_f &= \begin{pmatrix}
c' & s' / k_f \\
-k_f s' & c'
\end{pmatrix}
\end{align*}
\]

\[
\begin{align*}
M_d &= \begin{pmatrix}
C' & S' / k_d \\
k_d S' & C'
\end{pmatrix}
\end{align*}
\]

(B7)

The results of this method for the SINQ-beamline are shown in Fig. 4. If quadrupole fringe fields are ignored, the optics can not be reproduced (See Fig. 16).

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