Dynamic characteristics analysis of time-delay fractional order dynamic system

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Abstract. In this paper, a kind of time-delay fractional order dynamic system is studied. The classical integral sliding mode control method is used to design a single controller to control the time-delay fractional order Liu system from chaos to fixed point, and the single controller is further designed to control the time-delay fractional order Liu system from chaotic attractor to limit loop, so the chaos control of time-delay fractional order nonlinear dynamic system is achieved. On that basis, the dynamic characteristics of the time-varying delayed fractional-order Lorenz system are analyzed by the simulation, and the chaotic phenomena of the time-varying delayed fractional-order Lorenz system are verified under the provided time delay and order.

1. Introduction
Chaos control is one of the hot topics in the study of nonlinear dynamics, chaos has a certain application value in various engineering fields, such as secure communication, financial activity, aerospace, living system [1-5], and the most key link of chaos application is to control the chaos system, therefore, it is of a certain theoretical significance to study chaotic control theory.

Compared with ordinary differential equations, time-delay differential equations have more complex dynamic characteristics. For example, chaotic solutions of delay differential equations are easier to obtain [6-10]. Most of the traditional time-delay dynamic systems are modeled by integral order calculus, and if it is extended to the fractional order, it will inevitably produce many interesting phenomena. Some scholars tried to apply the time-delay fractional order dynamic system to the study of secure communication and neural network, and obtained some novel conclusions [11-13]. At present, the theoretical research of time-delay fractional order chaotic system is rarely involved in China, especially the chaos control and the chaotic synchronization of time-delay fractional order nonlinear dynamic system those are rarely reported [14-15]. Therefore, in this paper, the dynamic characteristics of the time-delay fractional order dynamic system are analyzed, the chaos of the time-delay fractional order system is controlled, and the Liu system is controlled form the chaotic motion to the state of fixed point and limit loop by integral sliding mode control. On that basis, the dynamic characteristics of time-varying delayed fractional order Lorenz system are analyzed to verify its chaotic phenomenon.
2. Chaos control of time-delay fractional order Liu system of single sliding mode control

In literature [14], a time-delay fractional order Liu system is proposed, and that system can be described as follows:

\[
\begin{align*}
D_\tau^{\alpha_1} x(t) &= a(y(t) - x(t - \tau)), \\
D_\tau^{\alpha_2} y(t) &= b(y(t) - \tau) - k_x(t)z(t) \\
D_\tau^{\alpha_3} z(t) &= -cz(t - \tau) + h\dot{x}^2(t),
\end{align*}
\]

(1)

It is pointed out in that literature that, in the case of \(\alpha_1 = 0.97, \alpha_2 = 0.97, \alpha_3 = 0.97,\) and \(\tau = 0.008,\) the system is in a chaotic state. In this paper, the time-delay fractional order Liu system is taken as the research object to verify whether control method has universality. The controller is slightly adjusted under the same conditions of the whole control form (single sliding mode control scheme), so that the time-delay fractional order Liu system added with the single sliding mode controller can be controlled from the chaotic motion to the state of fixed point and limit loop.

2.1. Liu chaotic system is controlled to the fixed point

Add a single sliding mode controller to the fractional order Liu chaotic system with external disturbance, and the system with controller is as follows:

\[
\begin{align*}
D_\tau^{\alpha_1} x(t) &= a(y(t) - x(t - \tau)), \\
D_\tau^{\alpha_2} y(t) &= b(y(t) - \tau) - k_x(t)z(t) + \Delta f(x, y, z) + d(t) + u, \\
D_\tau^{\alpha_3} z(t) &= -cz(t - \tau) + h\dot{x}^2(t),
\end{align*}
\]

(2)

The following control methods are provided:

The sliding mode surface is designed as follows:

\[
s = D_\tau^{\alpha_1 - 1} y(t) + \int_0^t k_1 y(\tau) d\tau
\]

The controller is designed as follows:

\[
u = -by(t - \tau) - k_1 y(t) + k_x(t)z(t) + k_2 \text{sign}(s)
\]

The effectiveness of the control scheme is verified by numerical simulation. The step length is \(h = 0.001, [a, b, c, h, k] \) are \([10, 40, 2.5, 4, 1],\) and the initial values are \([2.2, 2.4, 38].\) The simulation results are shown in Fig. 1. It can be seen in the figure that the system variable gradually converges to zero, which indicates that the system is controlled to the fixed point of zero.

![Fig.1. System response of the Liu system after exerting control](image-url)
2.2. Liu chaotic system is controlled to the limit loop

Add a single sliding mode controller to the fractional order Liu chaotic system with external disturbance, and the system with controller is as follows:

\[
\begin{align*}
D^\alpha_{t} x(t) &= a(y(t) - x(t - \tau)), \\
D^\alpha_{t} y(t) &= by(t - \tau) - kx(t)z(t) + \Delta f(x, y, z) + d(t) + u, \\
D^\alpha_{t} z(t) &= -cz(t - \tau) + hx^2(t),
\end{align*}
\]

(3)

Control the system (3) to the limit loop, and the following control methods are provided:

The sliding mode surface is designed as follows:

\[
s = D^\alpha_{t} y(t) + \int_{0}^{t} k_1 y(\tau) + kx(\tau)z(\tau) - by(t - \tau)d\tau
\]

The controller is designed as follows:

\[
u = -k_1 y(t) + k_2 \text{sign}(s)
\]

Because the theoretical proof of Liu chaotic system controlling to limit cycle is too complex and difficult, the proof is not proved here and mainly verified by the way of numerical simulation. The numerical simulation results are shown in Fig. 2 and Fig. 3. It can be seen in Fig. 2 that the state variables of the system gradually turn into periodic motion, and a clear limit loop can be seen in Fig. 3. That simulation results show that the system (3) has been controled to the limit loop by the above control scheme and the expected control goal has been achieved.

3. Chaotic motion of time-varying delayed fractional order Lorenz system

Dynamic characteristics of time-delay differential equation not only depend on the current state of the system variables, but also are closely related to the historical states of some system variables. In the fractional order power system of which the state variables are in mutual coupling, the appearance of time delay will make the dynamic characteristics of system be more complex. However, when the time delay of the system is a function of time, its motion state would be more unpredictable. The time-varying delayed fractional order nonlinear dynamic system may have better prospect in the application of secure communication and so on [16-18]. Therefore, the time-varying delayed fractional order nonlinear dynamic system is studied theoretically, and its chaotic behaviors are explored.

3.1. Dynamic model of time-varying delayed fractional order Lorenz system

The dynamic model of integral order time-varying delayed Lorenz system is provided in literature [19], and the fractional order model of that system can be described as follows:
\[ \begin{align*}
D_t^\alpha x(t) &= a(y(t) - x(t)) \\
D_t^\alpha y(t) &= cx(t) - y(t) - x(t)z(t) \\
D_t^\alpha z(t) &= -bz(t - \tau(t)) + x(t)y(t)
\end{align*} \] (4)

Among them, \( x, y \) and \( z \) are state variables, \( a, b \) and \( c \) are parameters, which respectively are \([10, 8/3 \text{ and } 28]\), and \( \tau(t) \) is the time-varying delay.

3.2. Chaotic behavior of time-varying delayed fractional order Lorenz system

It is considered that the fixed time delay of time-varying delayed fractional order chaotic system is changed into time-varying delay, and the chaotic characteristics of time-varying delayed fractional order nonlinear dynamic system are studied. It has been pointed out in reference [19] that the order condition for the existence of chaos in the fractional order Lorenz system is \( q > 0.994 \), and, when the order \( q \) is less than 0.994, the system is a stable fixed point. Hereby, we find that the fractional order Lorenz system is in the order value of fixed point (\( q = 0.9 \)), and the introduction of time-varying delay can make the system return to chaos from the fixed point. For the time-delay fractional order Lorenz system (4), let \( \tau(t) = 30 |\sin(t)| \) and \( q = 0.9 \), and the phase diagrams of attractor of the system (4) are shown in Fig. 4. By calculation, the Lyapunov indexes [20] of the system (4) are \( \lambda_1 = -0.188541, \lambda_2 = -0.652877 \) and \( \lambda_3 = 0.0463628 \). It can be seen from the phase diagrams of attractor and the Lyapunov indexes that the time-varying delayed fractional order Lorenz system (4) is in a chaotic state under the conditions of the above time delay and order.

![Phase diagrams of attractor of the system (4)](image)

Fig. 4. Attractor of time-varying delayed fractional order Lorenz system, \( q = 0.9 \)

4. Conclusion

The time delay is ubiquitous and inevitable in the real world. The time-delay dynamic system has complex dynamic behaviors which are often used to describe some engineering phenomena with time delay. In this paper, two time-delay fractional order systems are taken as examples to analyze the dynamic behavior of some time-delay fractional order nonlinear dynamic systems, and the time-delay
fractional order Liu system is taken as the research object to find out a specific single sliding mode control scheme, so the time-delay fractional order Liu system is able to be controlled from chaotic motion to fixed point or limit loop respectively. Considering the application prospect of time-varying delayed fractional order nonlinear dynamic system in the future, the dynamic characteristics of time-varying delayed fractional order Lorenz system are further analyzed, and the relevant chaotic phenomenon is verified. Thus, it is of a certain theoretical value to study chaos control of time-delay fractional order system.

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