1.1 Introduction to Dark Energy

Emptiness – the vacuum – is a surprisingly rich concept in cosmology. A universe devoid of all matter and radiation can still have evolution in space and time. In fact there can be very distinct empty universes, defined through their geometry. One of the great modern quests in science is to understand the hidden constituents of the universe, neither matter nor radiation, and their intimate relation with the nature of the quantum vacuum and the structure of spacetime itself.

Cosmologists are just beginning to probe the properties of the cosmic vacuum and its role in reversing the attractive pull of gravity to cause an acceleration in the expansion of the cosmos. The cause of this acceleration is given the generic name of dark energy, whether it is due to a true vacuum, a false, temporary vacuum, or a new relation between the vacuum and the force of gravity. Despite the common name, the distinction between these origins is of utmost interest and physicists are actively engaged in finding ways to use cosmological observations to distinguish which is the true, new physics. See Caldwell (2010) and Huterer (2010) in this volume for further details on the theoretical origins and observational probes, respectively, and Caldwell & Kamionkowski (2009); Durrer & Maartens (2010); Frieman, Turner, & Huterer (2008); Silvestri & Trodden (2009) for other recent reviews.
Here we will discuss how to relate the theoretical ideas to the experimental constraints, how to understand the influences of dark energy on the expansion and structure in the universe, and what frontiers of new physics are being illuminated by current and near-term data. In Sec. 1.2 we consider the vacuum, quantum fields, and their interaction with material components. The current level of our understanding about the properties of dark energy is reviewed in Sec. 1.3, and we relate this to a few, robust theories for the origin of dark energy. Looking to the frontiers of exploration, Sec. 1.4 anticipates what we may learn from experiments just now underway or being developed.

1.2 The Dynamics of Nothing

Emptiness, in general relativity, merely means that nothing has been put on the stage of space and time. The framework, however, the structure of space and time and their relation into spacetime, is part of the theory itself. A universe devoid of matter, radiation, all material contents still has geometry. We will consider here only the highly symmetrical case of a simply connected universe (no holes or handles) that is homogeneous (uniform among spatial volumes) and isotropic (uniform among spatial directions). A universe with only spatial curvature is called a Milne universe, or often just an empty universe. If even spatial curvature vanishes, then this is a Minkowski universe, a relativistic generalization of Euclidean space.

Suppose we now consider an energy completely uniform everywhere in space. One possibility for this is the energy of the spatial curvature itself, for example in the Milne universe. In evolving toward a lower energy state, the universe reduces the curvature energy, proportional to the inverse square radius of curvature, $a^{-2}$, by expanding. That is, the factor $a$ increases with time (and is often called the expansion factor or scale factor). Since the dynamical timescale of a self-gravitating system is proportional to the inverse square root of the energy density, then $a \propto t$. We see that there is no acceleration, i.e. $\ddot{a} = 0$, and the expansion continues at the same rate, $\dot{a} = \text{constant}$ forever.

Now imagine another uniform energy not associated with spatial curvature: a vacuum energy, a nonzero ground state level of energy. If this were negative, it would reduce the curvature energy and could counteract the expansion, possibly even causing collapse of the universe. That is, $a$ reduces with time until it reaches zero. Such a uniform
energy is called a negative cosmological constant. However suppose the vacuum energy were positive: then it would add to the energy and the expansion, increasing the rate such that we have $\ddot{a} > 0$ – acceleration.

Finally, remove the spatial curvature completely. With just the positive cosmological constant one still has the acceleration (recall the spatial curvature did not contribute to the acceleration positively or negatively). Nothing is in the universe but a positive, uniform energy. This is called a de Sitter universe. Most interesting, though, is what happens when we restore the matter and radiation into the picture. Matter and radiation have the usual gravitational attraction that pulls objects together, fighting against expansion. They act to decelerate the expansion. Depending on the relative contributions then between matter etc. and the vacuum, the final result can be either a decelerating or accelerating universe. One of the great paradigm shifts in cosmology was the realization and experimental discovery (Perlmutter et al., 1999; Riess et al., 1998) that we live in a universe that accelerates in its expansion, where gravity is not predominantly attractive.

This is really quite striking a development, opening up whole frontiers of new physics. At its most personal, it reminds us of the “principle of cosmic modesty”. Julius Caesar (at least through George Bernard Shaw) defined a barbarian as one who “thinks that the customs of his tribe and island are the laws of nature.” After Copernicus we have moved beyond thinking the Earth is the center of the universe; with the development of astronomy we know that the Milky Way Galaxy is not the center of the universe; through physical cosmology we know that what we are made of – baryons and leptons – is not typical of the matter in the universe; and now we even realize that the gravitational attraction we take as commonplace is not the dominant behavior in the universe. We are decidedly on the doorstep of new physics.

How then do we elucidate the role of the vacuum? A first step is certainly to determine whether we are indeed dealing with a uniform, constant energy filling space. The vacuum is the lowest energy state of a quantum field. One can picture this as a field of harmonic oscillators, imaginary springs at every point in space, and ask whether these springs are identical and frozen, or whether they have some spatial variation and motion. An assemblage of values defined at points in space and time is basically a scalar field, and we seek to know whether dark energy is a true cosmological constant or a dynamical entity, perhaps one whose energy is not in the true ground state but is temporarily lifted above
zero and is changing with time.

The scalar field approach is a fruitful one since one can use it as an effective description of the background dynamics of the universe even if the origin of acceleration is from another cause. That is, one can define an effective energy density and effective pressure (determining how the energy density changes with time), and use that in the equations governing the expansion (although the growth of inhomogeneities can be influenced by other degrees of freedom). This description of the cosmic expansion holds even if there is no physical field at all, such as in the case of a modification of the gravitational theory. (We discuss some of the ways to distinguish between explanations in §1.4.)

Indeed, it is instructive to review some historical cases where dynamics indicated new physics beyond what was then known. In the 18th century, the motion of the planet Uranus did not accord with predictions of Newton’s laws of gravitation applied from the other material contents of the solar system. Two choices presented themselves: the laws were inadequate, or the knowledge of the material contents was incomplete. Keeping the laws intact and asking what new material content was needed to explain the anomaly led to the discovery of Neptune. In the 19th century, the motion of Mercury disagreed with the laws and material contents known. While some again sought a new planet, Einstein developed extensions to Newtonian gravity – the solution lay in new laws. For dark energy, we do not know whether we need to add new contents – a quantum scalar field, say – or an extension to Einstein gravity. However what is certain is that we are in the midst of a revolution in physics. While Einstein’s correction to Mercury’s orbit led to a minuscule $43''/\text{century}$ of extra precession, dark energy turns cosmology upside down by changing gravitational attraction into accelerated expansion, dominates the expansion rate, and determines the ultimate fate of the universe.

We can investigate dark energy’s dynamical influence in more mathematical detail through the scalar field language (without assuming a true, physical scalar field). The Lagrangian density for a scalar field is just

$$\mathcal{L} = \frac{1}{2} \phi_{\mu} \phi^{\mu} + V(\phi),$$

(1.1)

where $\phi$ is the value of the field, $V$ is its potential, and ; $\mu$ denotes derivatives with respect to the time and space coordinates. Using the Noether construction of the energy-momentum tensor, one can identify
the energy density $\rho$ and isotropic pressure $p$ (all other terms vanishing under homogeneity and isotropy) as

$$\rho = \frac{1}{2} \dot{\phi}^2 + \frac{1}{2}(\nabla \phi)^2 + V \quad (1.2)$$

$$p = \frac{1}{2} \dot{\phi}^2 + \zeta (\nabla \phi)^2 - V. \quad (1.3)$$

Here $\zeta = -1/6$ ($1/2$) depending on whether the field is treated as spatially incoherent or coherent. If the spatial gradient terms dominated, then the pressure to density ratio would be $-1/3$ (i.e. acting like spatial curvature) or $+1$ (i.e. acting like a stiff fluid or gradient tilt) in the two cases. However, in the vast majority of cases the spatial gradients are small compared to the other terms and are neglected.

It is convenient to discuss the scalar field properties in terms of the equation of state parameter

$$w \equiv \frac{p}{\rho} = \frac{(1/2)\dot{\phi}^2 - V}{(1/2)\dot{\phi}^2 + V}, \quad (1.4)$$

where the first equality is general and we neglect spatial gradients in the second equality, as in the rest of the article. When the kinetic energy term dominates, then $w$ approaches $+1$; when the potential energy dominates, then $w \to -1$, and when they balance (as in oscillating around the minimum of a quadratic potential) then $w = 0$, like nonrelativistic matter. Acceleration occurs when the total equation of state, the weighted sum (by energy density) of the equations of state of each component, is $w_{\text{tot}} < -1/3$. The Friedmann equation for the acceleration of the expansion factor is

$$\ddot{a}/a = -4\pi G (\rho_{\text{tot}} + 3p_{\text{tot}}) = -4\pi G \sum \rho_w (1 + 3w), \quad (1.5)$$

where $G$ is Newton’s constant and we set the speed of light equal to unity.

The other equation of motion is either the Friedmann expansion equation

$$H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3} \rho_{\text{tot}}, \quad (1.6)$$

where we can include any curvature energy density in $\rho_{\text{tot}}$, or the energy conservation or continuity equation

$$\dot{\rho} = -3H(\rho + p) \quad \text{or} \quad \frac{d \ln \rho}{d \ln a} = -3(1 + w). \quad (1.7)$$
The continuity equation holds separately for each individually conserved component. In particular, for a scalar field we can write the continuity equation as a Klein-Gordon equation
\[ \ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0. \] (1.8)

To examine the dynamics of the dark energy, one can solve for \( w(a) \); it is also often instructive to work in the phase space of \( w'-w \), where a prime denotes \( d/d\ln a \). For example, many models can be categorized as either thawers or freezers (Caldwell & Linder, 2005): their behavior either starts with the field frozen by the Hubble friction of the expanding universe (so kinetic energy is negligible and \( w = -1 \)), and then at late times the field begins to roll, moving \( w \) away from \(-1\), or the field starts off rolling and gradually comes to settle at a minimum of the potential, asymptotically reaching \( w = -1 \).

Since the dark energy does not always dominate the energy budget and expansion of the universe, it is also useful to examine the dynamics of the full system of components. One can define variables representing each contribution to the energy density, say, and obtain a coupled system of equations (Copeland, Liddle, & Wands, 1998). For example, for a scalar field
\[ x' = -3x + \lambda \sqrt{\frac{3}{2}y^2 + \frac{3}{2}x [2x^2 + (1 + w_b)(1 - x^2 - y^2)]} \] (1.9)
\[ y' = -\lambda \sqrt{\frac{3}{2}xy + \frac{3}{2}y [2x^2 + (1 + w_b)(1 - x^2 - y^2)]}, \] (1.10)
where \( x = \sqrt{\kappa \dot{\phi}^2/(2H^2)}, \ y = \sqrt{\kappa V/H^2}, \ \kappa = 8\pi G/3, \) and \( \lambda = -(1/V)dV/d(\phi\sqrt{3\kappa}) \), with \( w_b \) being the equation of state of the background, dominating component (e.g. matter, with \( w_b = 0 \), during the matter dominated era). To solve these equations one must specify initial conditions and the form of \( V(\phi) \), i.e. \( \lambda \).

The fractional dark energy density \( \Omega_w = x^2 + y^2 \), so its evolution is bounded within the first quadrant of the unit circle in the \( x-y \) plane (taking \( \phi > 0 \); it is simple enough to generalize the equations), and the dark energy equation of state is \( w = (x^2 - y^2)/(x^2 + y^2) \). So the dynamics can be represented in polar coordinates, with the density being the radial coordinate and the equation of state the angular coordinate (twice the angle with respect to the \( x \) axis is \( 2\theta = \cos^{-1} w \)). Figure 1.1 illustrates some dynamics in the \( y-x \) energy density component (or \( w-\Omega_\phi \) plane).
Fig. 1.1  The dynamics in the $y$-$x$, or potential-kinetic energy phase space for a thawing (PNGB) and a freezing (SUGRA) field. The dark energy density $\Omega_\phi$ acts as a radial coordinate, while the dark energy equation of state $w$ acts as an angular coordinate. Note the constancy of $w$ (i.e. the angle $\theta$) for SUGRA at early times, when it is on the attractor trajectory. The models have been chosen to have the same values today, $\Omega_{\phi,0} = 0.72$ and $w_0 = -0.839$ (where the curves cross). The curves end in the future at $a = 1.47$.

The term in square brackets in Eqs. (1.9) and (1.10) is simply $1+w_{\text{tot}}$. Another way of viewing the dynamics is through the variation of the dark energy equation of state

$$w' = -3(1-w^2) + \lambda(1-w)x \sqrt{2}. \quad (1.11)$$

One can readily see that $w = -1$ (and hence $x = 0$) is a fixed point, with $w' = 0$. It can either be a stable attractor (in the case of freezing fields) or unstable (in the case of thawing fields). Figure 1.2 illustrates some dynamics in the phase plane $w'$-$w$, an alternate view to Figure 1.1. Considerably more detail about classes of dynamics is given in [Caldwell & Linder (2005); Linder (2006)]. For example, through non-standard kinetic terms one can get dynamics with $w < -1$, sometimes called phantom fields ([Caldwell, Kamionkowski, & Weinberg, 2003]).

The dynamical view of dark energy identifies several key properties
Fig. 1.2 The dynamics in the $w'-w$ phase plane for a thawing (PNGB) and a freezing (SUGRA) field. The right or left curvature in Fig. 1.2 here translates into $w' > 0$ or $< 0$. The thawer starts in a frozen state ($w = -1$, $w' = 0$) and evolves away from the cosmological constant behavior, while the freezer starts at some constant $w$ given by an attractor solution and then evolves as its energy density becomes more substantial, eventually approaching the cosmological constant state. The x’s mark the present state, and the curves end in the future at $a = 1.47$.

that would lead to insight into the nature of the physics behind acceleration. Since $w = -1$ is a special state, we can ask whether the dark energy always stays there, i.e. is it a cosmological constant? Does dark energy act like a thawing (roughly $w' > 0$) or freezing (roughly $w' < 0$) field? Is it ever phantom ($w < -1$)? One could also look back at the continuity equation and ask whether each component is separately conserved or whether there is interaction. Keeping overall energy conservation, one could write

$$\frac{d \ln \rho_w}{d \ln a} = -3(1 + w) + \frac{\Gamma}{H} \quad \text{(1.12)}$$

$$\frac{d \ln \rho_m}{d \ln a} = -3(1 + w_m) - \frac{\Gamma}{H} \quad \text{(1.13)}$$

for the dark energy and (dark) matter components, where $\Gamma$ represents
the interaction. The impact of this is to shift each equation of state, such that $w_{\text{eff}} = w - \Gamma/(3H)$ and $w_{m,\text{eff}} = w_m + \Gamma/(3H)$.

Such interactions act as a fifth force violating the Equivalence Principle if dark energy responds to different components in different ways. Certainly interaction with baryons is highly constrained otherwise we would have found dark energy from particle physics experiments. The shift in equation of state could make dark energy that intrinsically has $w > -1$ look like a phantom field, and vice versa (see Wei (2010) for some current constraints). Dynamical analysis does allow us to make some general statements: for example, consider a phantom field arising from a negative kinetic term. The dynamical variable $y = \sqrt{\kappa V/H^2}$ has a fixed point when $y'_c = 0$, so the potential obeys $V'/V = 2H'/H \equiv 3(1 + w_{\text{tot}})$. However, such negative kinetic term fields roll up the potential so $V'$ is positive. Therefore $w_{\text{tot}}$ must be less than $-1$ and the field must remain asymptotically phantom, even in the presence of interactions.

1.3 Knowing Nothing

The existence of dark energy was first discovered through the geometric probe of the distance-redshift relation of Type Ia supernovae (Perlmutter et al., 1999; Riess et al., 1998). Such data have been greatly expanded and refined so that now the analysis of the Union2 compilation of supernova data (Amanullah et al., 2010), together with other probes, establishes that the energy density contribution of dark energy to the total energy density is $\Omega_{de} = 0.719 \pm 0.017$ and the dark energy equation of state, or pressure to density ratio, is $w = -1.03 \pm 0.09$ (assumed constant).

Other cosmological probes are now investigating cosmic acceleration, although none by themselves have approached the leverage of supernovae. Experiments underway use Type Ia and Type II supernovae, baryon acoustic oscillations, cosmic microwave background measurements, weak gravitational lensing, and galaxy clusters with the Sunyaev-Zel’dovich effect and X-rays. See Huterer (2010) for more detailed discussion.

Observables such as the distance-redshift relation and Hubble parameter-redshift relation, and those that depend on these in a more complex manner, can be used to test specific models of dark energy. For some examples of this, see Rubin et al. (2009); Sollerman et al.
However it is frequently useful to have a more model independent method of constraining dark energy properties. We have already seen in the previous section that one can classify many models into the general behaviors of thawers and freezers. There appears diversity within each of these classes, but de Putter & Linder (2008) found a calibration relation between the dark energy equation of state value and its time variation that defines homogeneous families of dark energy physics. Figure 1.3 illustrates both the diversity and the calibration.

This calibration provides a physical basis for a very simple but powerful relation between the equation of state value and time variation in the dark energy dynamics phase plane. The resulting parametrization

\[ w(a) = w_0 + w_a (1 - a) \]  

(1.14)
gives a highly accurate match to the observable relations of distance \( d(z) \) and Hubble parameter \( H(z) \). This form, emphatically not a Taylor expansion, achieves \( 10^{-3} \) accuracy on the observables and matches the \( w_0 - w_a \) parametrization devised to fit the exact solutions for scalar field dynamics (Linder, 2003).

Current data constrains \( w_0 \) to \( \sim 0.3 \) and \( w_a \) to \( \sim 1 \), which is insufficient to answer any of the questions raised in the previous section, e.g. whether dark energy is a cosmological constant or not, is thawing or freezing, etc. To give a clear picture of our current state of knowledge, Figure 1.4 displays the constraints from all current data in several different ways.

For example, for \( w \) held constant, Amanullah et al. (2010) find that the energy density contribution of dark energy to the total energy density is \( \Omega_{\text{de}} = 0.719 \pm 0.017 \) and the dark energy equation of state, or pressure to density ratio, is \( w = -1.03 \pm 0.09 \) (68% confidence level, including systematic uncertainties). While viewing the constraints on \( w \) under the assumption that it is constant (upper left panel) gives an impression of substantial precision, in fact none of the key physical questions have been answered. The upper right panel shows that when we leave open the values of \( w \) in different redshift ranges (redshift \( z = a^{-1} - 1 \)), then we have no reasonable constraints on whether \( w \) is in fact constant in time. Recall that for a simple scalar field, \( w \) is bounded from below by \(-1\), and must be less than \(-1/3\) to provide acceleration. So the panoply of current data does not give much evidence for or against constancy of \( w \).

The bottom left panel demonstrates that we have no constraints at
Fig. 1.3 [Top panel] Representative models exhibiting a diversity of dynamics are plotted for various parameter values in the $w$-$w'$ phase space, including braneworld/$H^\alpha$ models ($\alpha = 1$ DGP and $\alpha = 0.5$). [Bottom panel] Using the calibrated dark energy parameters $w_0$ and $w_a$, dark energy models and families lie in tightly homogeneous regions. Contrast this with the top panel, showing the same models before calibration (note $w_a$ has the opposite sign from $w'$). We here vary over all parameters in the potentials. Shading shows the effect of scanning over $\pm0.03$ in $\Omega_m$ (we omit the shading for $\phi^4$ and linear potential models to minimize confusion; the width would be about half that shown for PNGB). Distinctions between thawing and freezing models, and between freezing models, become highlighted with calibration. From de Putter & Linder (2008).
Fig. 1.4 Constraints from the Union2 supernova compilation, WMAP7 CMB, SDSS DR7 baryon acoustic oscillation, and Hubble constant data on the dark energy equation of state $w(z)$, in redshift bins. Top left plot appears to show that data have zeroed in on the cosmological constant value of $w = -1$, but this assumes $w$ is constant. When one allows for the values of $w$ to be different in different redshift bins, our current knowledge of dark energy is seen to be far from sufficient. Top right plot shows that we do not yet have good constraints on whether $w(z)$ is constant. Bottom left plot (note change of scale) shows we have little knowledge of dark energy behavior, or even existence, at $z > 1$. Bottom right plot shows we have little detailed knowledge of dark energy behavior at $z < 1$. Outer (inner) boxes show 68% confidence limits with (without) systematics. The results are consistent with $w = -1$, but also allow considerable variation in $w(z)$. Adapted from Amanullah et al. (2010).

all on dark energy above $z \approx 1.6$, neither knowing its properties nor even whether it exists. In the bottom right panel it is clear that the situation at low redshift (near the present time) is also quite uncertain: does $w$ differ from $-1$, and if so in which direction?

On the theoretical front, no consensus exists on any clear concept for the origin of dark energy. Any expansion history can be accommodated by a combination of potential and kinetic terms, but it is really not a case of an embarrassment of riches. There are two main problems: any
potential that one writes down should receive quantum corrections at high energies and so end up different from the original intent, and the energy scale corresponding to dark energy is much lower (by many tens of orders of magnitude) than scales associated with initial conditions in the early universe. How do we cue dark energy and cosmic acceleration to appear on the stage of the universe at the right moment? That is, one generically requires fine tunings to describe the universe today starting from high energy physics.

To surmount these difficulties requires some symmetry to preserve the form of the potential, and some tracking mechanism to keep dark energy in the wings until the proper moment. Simple scalar fields fail on one or both of these counts (the cosmological constant fails on both). However there are a few possibilities that might offer guidance toward a more robust theory.

Some theories, such as the pseudo-Nambu Goldstone boson (PNGB) model (Frieman et al., 1995), impose a symmetry that protects the form. Such theories are known as natural theories. However to achieve acceleration at the right time still requires a restricted range of initial conditions. Attractor models where dark energy is kept off stage, but not too far off, for the radiation and matter dominated eras, are a useful class (Ratra & Peebles, 1988; Wetterich, 1988; Zlatev, Wang, & Steinhardt, 1999; Liddle & Scherrer, 1999). An intriguing class of models that incorporates both these advantages is the Dirac-Born-Infeld (DBI) action based on higher dimension theories (Alishahiha, Silverstein, & Tong, 2004; Martin & Yamaguchi, 2008; Ahn, Kim, & Linder, 2009, 2010). This employs a geometric constraint to preserve the potential and a relativistic generalization of the usual scalar field dynamics to provide the attractor property. The attraction to $w = -1$ actually occurs in the future, but prevents the dynamics from diverging too far from $w = -1$ at any time. Another class of interest, although not arising directly from high energy physics, is that of barotropic models. In the barotropic aether model the equation of state naturally transitions rapidly from acting like another matter component to being attracted to $w = -1$, thus “predicting” $w = -1$ for much of the observational redshift range and ameliorating the coincidence of recent acceleration (Linder & Scherrer, 2009); see Figure 1.5.

Theories along the lines of DBI or barotropic dark energy seem promising guideposts to a natural physical origin for acceleration, at least within the “new component” approach to dark energy. Inter-
estingly, both of them also make predictions for the microphysics of the dark energy distinct from simple scalar fields. Minimally coupled, canonical (standard kinetic term) scalar fields have a sound speed of field perturbations equal to the speed of light, and hence do not cluster except on near horizon scales. Both the DBI and barotropic theories have sound speeds that instead approach zero (and hence could cluster) for at least part of their dynamics. We explore this further in the next section, but Figure 1.6 shows limits due to current data on the sound speed $c_s$ for a barotropic-type model (DBI models have even weaker constraints on $c_s$).

1.4 The Frontiers of Nothing

From the previous section it may seem like our knowledge of nothingness is close to nothing. But the past dozen years of experimental work and theoretical investigation have ruled out large classes of models, albeit perhaps the simplest ones and of course the ones with the most obvious experimental signatures. The increasing difficulty has led some to pessimism, but the advancing network of diverse observational probes to be carried out over the next dozen years can be a source of hope.

Recall the story of Auguste Comte, who in 1835 declared that “we shall never be able to know the composition of stars”. It was only 14 years later that it was discovered that the spectrum of electromagnetic radiation encodes the composition of material. Perhaps within the next 14 years there will be an analogous breakthrough of theoretical and experimental techniques for dark energy. Such progress in scientific discovery has become gratifyingly habitual, as witness Richard Feynman’s quote:

Yesterday’s sensation is today’s calibration and tomorrow’s background.

Just as perturbations in the cosmic microwave background (CMB) radiation were undetected (and beginning to be despaired of) in the 1980’s, discovered in the 1990’s, and are today sometimes regarded as “background noise” relative to the signatures of galaxy cluster physics, so may the homogeneous background of dark energy and the value of $w(a)$ be treated in the future.

What lies beyond $w$? Even for the expansion history $a(t)$, i.e. the homogeneous dynamics of expansion, there is the question of whether
dark energy makes a contribution at high redshift, whether in an accelerating form or not. This is called early dark energy and current constraints are at the few percent level (Doran, Robbers, & Wetterich, 2007) – by contrast the cosmological constant would contribute a fractional density of $10^{-9}$ at the CMB last scattering surface at $z \approx 1090$. Within a few years, CMB data from the Planck satellite should tighten the constraints by a factor 10.

There is the issue of whether dark energy interacts with any other component other than through gravity. This could become apparent through a situation such as cosmological neutrino mass bounds being at variance with laboratory measurements (if dark energy interacts with neutrinos, e.g. Amendola, Baldi, & Wetterich (2008); Wetterich (2007)), or through features in the matter density perturbation power spectrum (if dark energy interacts with dark matter, e.g. Bean, Flanagan, & Trodden (2008)).

Does dark energy cluster? This could come about either through a low sound speed (although it also requires that $w$ deviate appreciably from $-1$) or a coupling to other components. Observationally this can be probed through detailed measurements of matter clustering on various length scales, using the next generation of galaxy surveys.

Perhaps the most intriguing possibility is new laws of physics: in the “Neptune vs. post-Newton” alternative to end up with extensions to the laws of gravitation beyond Einstein’s general relativity rather than a new quantum scalar field. It is not easy to find viable theories of gravity that accord with observations, and most of the ones that do exist are driven toward similarity with general relativity (GR). Again, we seek a model independent approach that might identify some key features that a fundamental extended theory would need.

The simplest generalization is to take a phenomenological approach of asking what feature of the observations could be shifted by a non-GR theory. As mentioned in §1.2, any modification of the expansion history is identical to an effective $w(a)$, so we must look further for an observational distinction. General relativity predicts a definite relation between the expansion history of the homogeneous universe and the growth history of energy density perturbations. Other theories of gravity may deviate from this relation so we can define a gravitational growth index $\gamma$ that accounts for effects on growth beyond the expansion influence, seeing if it is consistent with the GR prediction.

Parametrization of the growth of linear matter density perturbations
\( \delta \rho \) can be written as

\[
g(a) = e \int_0^a \left( \frac{[(\delta \rho/\rho)'/a']}{\Omega_m(a')^{\gamma-1}} \right) \, da'
\]

where \( g(a) = (\delta \rho/\rho)/a \). This separates out the expansion history (which enters \( \Omega_m(a) \)) effects on growth from any extra gravitational influences (entering \( \gamma \)). The gravitational growth index \( \gamma \) is substantially independent of other cosmological parameters and can be determined accurately. This form of representing deviations through \( \gamma \), a single constant, reproduces the growth behavior to within 0.1% accuracy for a wide variety of models (Linder, 2005; Linder & Cahn, 2007). Note that other changes to the gravitational driving of growth besides the theory of gravity, such as other clustering components or couplings, can also cause \( \gamma \) to deviate from its standard general relativity value of 0.55.

Moreover, gravitational modifications do more than affect growth: they alter the light deflection law in lensing and the relation between the matter density and velocity fields. This can introduce both time and scale dependent terms. In particular, the two potentials, appearing in the time-time and space-space terms of the metric, may no longer be equal as they are in general relativity, and the Poisson-type equations connecting them to the matter density and velocity fields could change. Among other approaches (e.g. Hu (1998); Hu & Sawicki (2007)), one can define new functions to account for these differences as (Daniel & Linder, 2010)

\[
-k^2(\phi + \psi) = 8\pi G_N a^2 \bar{\rho}_m \Delta_m \times \mathcal{G}
\]

\[
-k^2 \psi = 4\pi G_N a^2 \bar{\rho}_m \Delta_m \times \mathcal{V},
\]

where \( \phi \) and \( \psi \) are the metric potentials, \( \bar{\rho}_m \Delta_m \) the gauge invariant matter density perturbations, and \( G_N \) is Newton’s constant. In general relativity, the time and scale dependent functions \( \mathcal{G} \) and \( \mathcal{V} \) are identically unity.

Within a given theory of gravitation, the deviations \( \mathcal{G} \) and \( \mathcal{V} \) will be specified, but if we are searching for general deviations from Einstein gravity then we should take model independent forms for these functions. Allowing their values to float in bins in redshift and in scale (wavenumber) gives considerable freedom and does not prejudice the search for concordance or contradiction with general relativity. Figure 1.7 shows both the current constraints and those expected from the next generation galaxy redshift surveys.
Considerable current data exists to constrain gravity and cosmology, including the cosmic microwave background (CMB), supernova distances, weak gravitational lensing, galaxy clustering statistics, and crosscorrelation between the CMB photon and galaxy number density fields. Nevertheless, although this now constrains the sum of the potentials, and hence $G$ (see Eq. 1.16), fairly well, the growth of structure, in terms of $\mathcal{V}$, is still poorly known. This should change with the next generation of large volume, three dimensional galaxy mapping surveys. We see that the 8 “beyond GR” gravity parameters ($G$ and $\mathcal{V}$ each in two redshift and two wavenumber bins) could be determined to within $\sim 10\%$ or better.

Testing gravity on cosmic scales is an area of intense interest at the moment; previous work using current data {Daniel et al., 2010; Bean & Tangmatitham, 2010; Zhao et al., 2010; Reyes et al., 2010} finds consistency with GR, although again deviations are certainly allowed. Better data from growth probes could play a key role in tightening constraints or uncovering new physics. A particularly exciting prospect is comparing the density, velocity, and potential field information through combining imaging and spectroscopic surveys {Jain & Zhang, 2008; Reyes et al., 2010; Jain & Khoury, 2010}. Extending our probes and understanding into the nonlinear density structure regime is another area of active exploration {Oyaizu, Lima, & Hu, 2008; Schmidt, 2009}.

1.5 Conclusions

Beyond the atoms and photons that make up our familiar world, and all the particles of the Standard Model of particle physics, the nature of the vacuum and spacetime is a mystery that has come to the forefront of physics. Gravity, the most familiar and omnipresent of all the forces, is not behaving as we expected. More than 70% of the energy density in the universe is made of nothing – nothing we have experienced before. Conditions are ripe for a true adventure in cosmology.

While current data are consistent with a cosmological constant as a source for dark energy, a cornucopia of other physical origins are in agreement as well. We do not yet know whether the dark energy is uniform, is dynamic, disappears at early times, is of a quantum origin or a gravitational origin. All are valid possibilities, and carry profound implications for the frontiers of physics and the fate of the universe.
A key question is whether we are dealing with a new physical ingredient or new physical laws – or both. For example, the dark energy may interact with neutrinos through a novel interaction; is dark energy really a completely separate sector of physics or are there new forces and symmetries as intricate as in known particle physics? We are very much at the beginning of our explorations of the frontier physics of dark energy and cosmic acceleration.

The exciting goal of future observations is to explore this wonderland of physics. We have few robust models but some general concepts, and some excellent model independent parametrizations. For the dynamical aspects of cosmic expansion, next generation measurements of the equation of state and its time variation, $w$ and $w'$, in the calibrated form of $w_0$ and $w_a$ describe the experimental reach to the subpercent level of observational accuracy. Comparison of tests of growth and expansion could give key clues to the underlying physics, as can contrasting the density, velocity, and gravitational potential fields of large scale structure. These should be enabled by a diverse network of future observations, delineating the physical properties of dark energy and testing general relativity. At the same time, these measurements deliver information of great value to many other astrophysical explorations as we map the structure, motion, and growth in our universe.

Settling the frontier will require challenging efforts by both observers and theorists. One must not only measure the expansion history $w(a)$, growth history $\gamma$, gravity $G$ and $V$, couplings, early dark energy etc. – but also understand them. Even if we fail to detect deviations from a cosmological constant, we cannot say the revolutionary physics of dark energy is known until we explain it. As two British Astronomers Royal said in the 19th century:

“I should not have believed it if I had not seen it!” – Sir G.B. Airy

and the reply

“How different we are! My eyes have too often deceived me. I believe it because I have proved it.” – Sir W.R. Hamilton

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Fig. 1.5  Barotropic models make a rapid transition from \( w = 0 \) at high redshift \((a \ll 1)\) to \( w \approx -1 \) more recently: the transition from \( w = -0.1 \) to \( w = -0.9 \) always takes less than 1.5 e-folds. This is inherent in the barotropic physics and, in distinction to quintessence, gives a prediction that observations of the recent universe should find \( w \approx -1 \). [Bottom panel] Effective potential corresponding to a barotropic model with \( c_s = 0 \). The x’s mark where the field is today and at \( a = 0.25 \), showing that it has reached the flat part of the potential, and so \( w \approx -1 \) for the last \( \sim 90\% \) of the age of the universe.
Fig. 1.6 68.3, 95.4 and 99.7% confidence level contours in the early dark energy model with constant sound speed $c_s$ and early dark energy density fraction $\Omega_e$. The constraints are based on current data including CMB, supernovae, LRG power spectrum and crosscorrelation of CMB with matter tracers. From de Putter, Huterer, & Linder (2010).
Fig. 1.7 Filled contours show 68% and 95% cl constraints on $\mathcal{V} - 1$ and $\mathcal{G} - 1$ for the two redshift and two wavenumber bins using mock future BigBOSS, Planck, and JDEM supernova data. The dotted contours recreate the 95% cl contours from Figs. 8 of Daniel & Linder (2010) using current data (note the offset from (0,0) may be from systematics within the CFHTLS weak lensing data) to show the expected improvement in constraints. The x’s denote the fiducial GR values. Adapted from Daniel & Linder (2010).
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