Magnetic dilaton strings in anti-de Sitter spaces

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With an appropriate combination of three Liouville-type dilaton potentials, we construct a new class of spinning magnetic dilaton string solutions which produces a longitudinal magnetic field in the background of anti-de Sitter spacetime. These solutions have no curvature singularity and no horizon, but have a conic geometry. We find that the spinning string has a net electric charge which is proportional to the rotation parameter. We present the suitable counterterm which removes the divergences of the action in the presence of dilaton potential. We also calculate the conserved quantities of the solutions by using the counterterm method.

I. INTRODUCTION

The construction and analysis of black hole solutions in the background of anti-de Sitter (AdS) spaces is a subject of much recent interest. This interest is primarily motivated by the correspondence between the gravitating fields in an AdS spacetime and conformal field theory living on the boundary of the AdS spacetime [1]. This equivalence enables one to remove the divergences of the action and conserved quantities of gravity in the same way as one does in field theory. It was argued that the thermodynamics of black holes in AdS spaces can be identified with that of a certain dual conformal field theory (CFT) in the high temperature limit [2]. Having the AdS/CFT correspondence idea at hand, one can gain some insights into thermodynamic properties and phase structures of strong 't Hooft coupling conformal field theories by studying the thermodynamics of asymptotically AdS black holes.

On another front, scalar coupled black hole solutions with different asymptotic spacetime structure is a subject of interest for a long time. There has been a renewed interest in such studies ever since new black hole solutions have been found in the context of string theory.

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The low energy effective action of string theory contains two massless scalars namely dilaton and axion. The dilaton field couples in a nontrivial way to other fields such as gauge fields and results into interesting solutions for the background spacetime. It was argued that with the exception of a pure cosmological constant, no dilaton-de Sitter or anti-de Sitter black hole solution exists with the presence of only one Liouville-type dilaton potential [3]. Recently, the dilaton potential leading to (anti)-de Sitter-like solutions of dilaton gravity has been found [4]. It was shown that the cosmological constant is coupled to the dilaton in a very nontrivial way. With the combination of three Liouville-type dilaton potentials, a class of static dilaton black hole solutions in (A)dS spaces has been obtained by using a coordinates transformation which recast the solution in the schwarzschild coordinates system [4]. More recently, a class of charged rotating dilaton black string solutions in four-dimensional anti-de Sitter spacetime has been found in [5]. Other studies on the dilaton black hole solutions in (A)dS spaces have been carried out in [6, 7].

In this Letter, we turn to the investigation of asymptotically AdS spacetimes generated by static and spinning string sources in four-dimensional Einstein-Maxwell-dilaton theory which are horizonless and have nontrivial external solutions. The motivation for studying such kinds of solutions is that they may be interpreted as cosmic strings. Cosmic strings are topological structure that arise from the possible phase transitions to which the universe might have been subjected to and may play an important role in the formation of primordial structures. A short review of papers treating this subject follows. The four-dimensional horizonless solutions of Einstein gravity have been explored in [8, 9]. These horizonless solutions [8, 9] have a conical geometry; they are everywhere flat except at the location of the line source. The spacetime can be obtained from the flat spacetime by cutting out a wedge and identifying its edges. The wedge has an opening angle which turns to be proportional to the source mass. The extension to include the Maxwell field has also been done [10]. Static and spinning magnetic sources in three and four-dimensional Einstein-Maxwell gravity with negative cosmological constant have been explored in [11, 12]. The generalization of these asymptotically AdS magnetic rotating solutions to higher dimensions has also been done [13]. In the context of electromagnetic cosmic string, it has been shown that there are cosmic strings, known as superconducting cosmic strings, that behave as superconductors and have interesting interactions with astrophysical magnetic fields [14]. The properties of these superconducting cosmic strings have been investigated in [15]. It
is also of great interest to generalize the study to the dilaton gravity theory \[16\]. While exact magnetic rotating dilaton solution in three dimensions has been obtained in \[17\], two classes of magnetic rotating solutions in four \[18\] and higher dimensional dilaton gravity in the presence of one Liouville-type potential have been constructed \[19\]. Unfortunately, these solutions \[18, 19\] are neither asymptotically flat nor (A)dS. The purpose of the present Letter is to construct a new class of static and spinning magnetic dilaton string solutions which produces a longitudinal magnetic field in the background of anti-de Sitter spacetime. We will also present the suitable counterterm which removes the divergences of the action, and calculate the conserved quantities by using the counterterm method.

II. BASIC EQUATIONS

Our starting point is the four-dimensional Einstein-Maxwell-dilaton action

\[
I_G = -\frac{1}{16\pi} \int_M d^4x \sqrt{-g} \left( R - 2\partial_\mu \Phi \partial^\mu \Phi - V(\Phi) - e^{-2\alpha \Phi} F_{\mu\nu} F^{\mu\nu} \right) - \frac{1}{8\pi} \int_{\partial M} d^3x \sqrt{-\gamma} \Theta(\gamma),
\]

where \( R \) is the scalar curvature, \( \Phi \) is the dilaton field, \( F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \) is the electromagnetic field tensor, and \( A_\mu \) is the electromagnetic potential. \( \alpha \) is an arbitrary constant governing the strength of the coupling between the dilaton and the Maxwell field. The last term in Eq. (1) is the Gibbons-Hawking surface term. It is required for the variational principle to be well-defined. The factor \( \Theta \) represents the trace of the extrinsic curvature for the boundary \( \partial M \) and \( \gamma \) is the induced metric on the boundary. While \( \alpha = 0 \) corresponds to the usual Einstein-Maxwell-scalar theory, \( \alpha = 1 \) indicates the dilaton-electromagnetic coupling that appears in the low energy string action in Einstein’s frame. For arbitrary value of \( \alpha \) in AdS space the form of the dilaton potential \( V(\Phi) \) is chosen as \[4\]

\[
V(\Phi) = \frac{2\Lambda}{3(\alpha^2 + 1)^2} \left[ \alpha^2 \left(3\alpha^2 - 1\right) e^{-2\alpha \Phi} + (3 - \alpha^2) e^{2\alpha \Phi} + 8\alpha^2 e^{\Phi(\alpha - 1/\alpha)} \right].
\]

Here \( \Lambda \) is the cosmological constant. It is clear that the cosmological constant is coupled to the dilaton field in a very nontrivial way. This type of the dilaton potential was introduced for the first time by Gao and Zhang \[4\]. They derived, by applying a coordinates transformation which recast the solution in the Schwarzschild coordinates system, the static dilaton black hole solutions in the background of (A)dS universe. For this purpose, they required the
existence of the (A)dS dilaton black hole solutions and extracted successfully the form of
the dilaton potential leading to (A)dS-like solutions. They also argued that this type of
derived potential can be obtained when a higher dimensional theory is compactified to four
dimensions, including various supergravity models [20]. In the absence of the dilaton field
the action (1) reduces to the action of Einstein-Maxwell gravity with cosmological constant.
Varying the action (1) with respect to the gravitational field $g_{\mu\nu}$, the dilaton field $\Phi$ and the
gauge field $A_\mu$, yields

$$R_{\mu\nu} = 2 \partial_\mu \Phi \partial_\nu \Phi + \frac{1}{2} g_{\mu\nu} V(\Phi) + 2e^{-2\alpha \Phi} \left( F_{\mu\eta} F^\eta_{\nu} - \frac{1}{4} g_{\mu\nu} F_{\lambda\eta} F^{\lambda\eta} \right),$$

(3)

$$\nabla^2 \Phi = \frac{1}{4} \frac{\partial V}{\partial \Phi} - \frac{\alpha}{2} e^{-2\alpha \Phi} F_{\lambda\eta} F^{\lambda\eta},$$

(4)

$$\partial_\mu \left( \sqrt{-g} e^{-2\alpha \Phi} F^{\mu\nu} \right) = 0.$$  

(5)

The conserved mass and angular momentum of the solutions of the above field equations
can be calculated through the use of the substraction method of Brown and York [21]. Such
a procedure causes the resulting physical quantities to depend on the choice of reference
background. A well-known method of dealing with this divergence for asymptotically AdS
solutions of Einstein gravity is through the use of counterterm method inspired by AdS/CFT
correspondence [22]. In this Letter, we deal with the spacetimes with zero curvature bound-
dary, $R_{abcd}(\gamma) = 0$, and therefore the counterterm for the stress energy tensor should be
proportional to $\gamma^{ab}$. We find the suitable counterterm which removes the divergences of the
action in the form (see also [23])

$$I_{ct} = -\frac{1}{8\pi} \int_{\partial M} d^3 x \sqrt{-\gamma} \left( -\frac{1}{l} + \frac{\sqrt{-6V(\Phi)}}{2} \right).$$

(6)

One may note that in the absence of a dilaton field where we have $V(\Phi) = 2\Lambda = -6/l^2$, the
above counterterm has the same form as in the case of asymptotically AdS solutions with
zero-curvature boundary. Having the total finite action $I = I_G + I_{ct}$ at hand, one can use the
quasilocal definition to construct a divergence free stress-energy tensor [21]. Thus the finite
stress-energy tensor in four-dimensional Einstein-dilaton gravity with three Liouville-type
dilaton potentials [2] can be written as

$$T^{ab} = \frac{1}{8\pi} \left[ \Theta^{ab} - \nabla \gamma^{ab} + \left( -\frac{1}{l} + \frac{\sqrt{-6V(\Phi)}}{2} \right) \gamma^{ab} \right].$$

(7)
The first two terms in Eq. (7) are the variation of the action (1) with respect to $\gamma_{ab}$, and the last two terms are the variation of the boundary counterterm (6) with respect to $\gamma_{ab}$. To compute the conserved charges of the spacetime, one should choose a spacelike surface $B$ in $\partial M$ with metric $\sigma_{ij}$, and write the boundary metric in ADM (Arnowitt-Deser-Misner) form:

$$\gamma_{ab}dx^a dx^b = -N^2 dt^2 + \sigma_{ij} \left( d\varphi^i + V^i dt \right) \left( d\varphi^j + V^j dt \right),$$

where the coordinates $\varphi^i$ are the angular variables parameterizing the hypersurface of constant $r$ around the origin, and $N$ and $V^i$ are the lapse and shift functions, respectively. When there is a Killing vector field $\xi$ on the boundary, then the quasilocal conserved quantities associated with the stress tensors of Eq. (7) can be written as

$$Q(\xi) = \int_B d^2 x \sqrt{\sigma} T_{ab} n^a \xi^b,$$

(8)

where $\sigma$ is the determinant of the metric $\sigma_{ij}$, $\xi$ and $n^a$ are, respectively, the Killing vector field and the unit normal vector on the boundary $B$. For boundaries with timelike ($\xi = \partial/\partial t$) and rotational ($\zeta = \partial/\partial \phi$) Killing vector fields, one obtains the quasilocal mass and angular momentum

$$M = \int_B d^2 x \sqrt{\sigma} T_{ab} n^a \xi^b,$$

(9)

$$J = \int_B d^2 x \sqrt{\sigma} T_{ab} n^a \zeta^b.$$  

(10)

These quantities are, respectively, the conserved mass and angular momenta of the system enclosed by the boundary $B$. Note that they will both depend on the location of the boundary $B$ in the spacetime, although each is independent of the particular choice of foliation $B$ within the surface $\partial M$.

### III. Static Magnetic Dilaton String

Here we want to obtain the four-dimensional solution of Eqs. (3)-(5) which produces a longitudinal magnetic fields along the $z$ direction. We assume the following form for the metric

$$ds^2 = -\frac{\rho^2}{l^2} R^2(\rho)dt^2 + \frac{d\rho^2}{f(\rho)} + l^2 f(\rho)d\phi^2 + \frac{\rho^2}{l^2} R^2(\rho)dz^2.$$  

(11)

The functions $f(\rho)$ and $R(\rho)$ should be determined and $l$ has the dimension of length which is related to the cosmological constant $\Lambda$ by the relation $l^2 = -3/\Lambda$. The coordinate $z$
has the dimension of length and ranges $-\infty < z < \infty$, while the angular coordinate $\phi$ is dimensionless as usual and ranges $0 \leq \phi < 2\pi$. The motivation for this curious choice of the metric gauge $[g_{tt} \propto -\rho^2$ and $(g_{\rho\rho})^{-1} \propto g_{\phi\phi}]$ instead of the usual Schwarzschild gauge $[(g_{\rho\rho})^{-1} \propto g_{tt}$ and $g_{\phi\phi} \propto \rho^2]$ comes from the fact that we are looking for a magnetic solution instead of an electric one. It is well-known that the electric field is associated with the time component, $A_t$, of the vector potential while the magnetic field is associated with the angular component $A_\phi$. From the above fact, one can expect that a magnetic solution can be written in a metric gauge in which the components $g_{tt}$ and $g_{\phi\phi}$ interchange their roles relatively to those present in the Schwarzschild gauge used to describe electric solutions [11]. The Maxwell equation (5) can be integrated immediately to give

$$F_{\phi\rho} = \frac{qle^{2\alpha\Phi}}{\rho^2 R^2}, \quad (12)$$

where $q$, an integration constant, is the charge parameter which is related to the electric charge of the rotating string, as will be shown below. Inserting the Maxwell fields (12) and the metric (11) in the field equations (3) and (4), we can simplify these equations as

\begin{align*}
2\rho^3 R^4 f' + 2\rho^4 R^3 f' R' + 2\rho^2 R^4 f + 8\rho^3 R^3 f R' + 2\rho^4 R^2 f R^2 \\
+ 2\rho^4 R^3 f R'' + \rho^4 R^4 V(\Phi) - 2q^2 e^{2\alpha\Phi} = 0, \quad (13) \\
2\rho^3 R^4 f' + \rho^4 R^4 f'' + 8\rho^3 R^3 f R' + 4\rho^4 R^3 f R'' + 2\rho^4 R^3 R' f' \\
+ 4\rho^4 R^4 f \Phi^2 + \rho^4 R^4 V(\Phi) + 2q^2 e^{2\alpha\Phi} = 0, \quad (14) \\
2\rho^4 R^3 R' f' + \rho^4 R^4 f'' + 2\rho^3 R^4 f' + \rho^4 R^4 V(\Phi) + 2q^2 e^{2\alpha\Phi} = 0, \\
\rho^4 R^4 \Phi' f' + \rho^4 R^4 \Phi' f + 2\rho^3 R^4 \Phi' f + 2\rho^4 R^3 R' \Phi' f - \rho^4 R^4 \frac{\partial V}{\partial \Phi} + \alpha^2 e^{2\alpha\Phi} = 0, \quad (15)
\end{align*}

where the “prime” denotes differentiation with respect to $\rho$. Subtracting Eq. (15) from Eq. (14) we get

$$2R' + \rho R'' + \rho R\Phi'^2 = 0. \quad (17)$$

Then we make the ansatz [5]

$$R(\rho) = e^{\alpha\Phi}. \quad (18)$$

Substituting this ansatz in Eq. (17), it reduces to

$$\rho \alpha \Phi'' + 2\alpha \Phi' + \rho (1 + \alpha^2) \Phi'^2 = 0, \quad (19)$$
which has a solution of the form

$$\Phi(\rho) = \frac{\alpha}{\alpha^2 + 1} \ln(1 - \frac{b}{\rho}),$$  \hspace{1cm} (20)

where $b$ is a constant of integration related to the mass of the string, as will be shown. Inserting (20), the ansatz (18), and the dilaton potential (2) into the field equations (13)-(16), one can show that these equations have the following solution

$$f(\rho) = \frac{c}{\rho} \left(1 - \frac{b}{\rho}\right)^{\frac{1-\alpha^2}{1+\alpha^2}} - \frac{\Lambda}{3} \rho^2 \left(1 - \frac{b}{\rho}\right)^{\frac{2\alpha^2}{\alpha^2+1}},$$  \hspace{1cm} (21)

where $c$ is an integration constant. The two constants $c$ and $b$ are related to the charge parameter via $q^2(1 + \alpha^2) = bc$. It is apparent that this spacetime is asymptotically AdS. In the absence of a nontrivial dilaton ($\alpha = 0$), the solution reduces to the asymptotically AdS horizonless magnetic string for $\Lambda = -3/l^2$ [12].

Then we study the general structure of the solution. It is easy to show that the Kretschmann scalar $R_{\mu\nu\lambda\kappa}R^{\mu\nu\lambda\kappa}$ diverges at $\rho = 0$ and therefore one might think that there is a curvature singularity located at $\rho = 0$. However, as we will see below, the spacetime will never achieve $\rho = 0$. Second, we look for the existence of horizons and, in particular, we look for the possible presence of magnetically charged black hole solutions. The surface $r = b$ is a curvature singularity for $\alpha \neq 0$. The horizons, if any exist, are given by the zeros of the function $f(\rho) = (g_{\rho\rho})^{-1}$. Let us denote the largest positive root of $f(\rho) = 0$ by $r_+$. The function $f(\rho)$ is negative for $\rho < r_+$, and therefore one may think that the hypersurface of constant time and $\rho = r_+$ is the horizon. However, the above analysis is wrong. Indeed, we first notice that $g_{\rho\rho}$ and $g_{\phi\phi}$ are related by $f(\rho) = g_{\rho\rho}^{-1} = l^2 g_{\phi\phi}$, and therefore when $g_{\rho\rho}$ becomes negative (which occurs for $\rho < r_+$) so does $g_{\phi\phi}$. This leads to an apparent change of signature of the metric from $+2$ to $-2$. This indicates that we are using an incorrect extension. To get rid of this incorrect extension, we introduce the new radial coordinate $r$ as

$$r^2 = \rho^2 - r_+^2 \Rightarrow dr^2 = \frac{r^2}{r^2 + r_+^2}dr^2. \hspace{1cm} (22)$$

With this coordinate change, the metric (11) is

$$ds^2 = -\frac{r^2 + r_+^2}{l^2}R^2(r)dt^2 + l^2 f(r)d\phi^2 + \frac{r^2}{(r^2 + r_+^2)f(r)}dr^2 + \frac{r^2 + r_+^2}{l^2}R^2(r)dz^2, \hspace{1cm} (23)$$
where the coordinates $r$ assumes the values $0 \leq r < \infty$, and $f(r)$, $R(r)$, and $\Phi(r)$ are now given as

$$f(r) = \frac{c}{\sqrt{r^2 + r_+^2}} \left(1 - \frac{b}{\sqrt{r^2 + r_+^2}} \right)^{\frac{1-\alpha^2}{2+\alpha^2}} - \frac{A}{3} \left(r^2 + r_+^2 \right) \left(1 - \frac{b}{\sqrt{r^2 + r_+^2}} \right)^{\frac{2\alpha^2}{\alpha^2 + 1}}, \quad (24)$$

$$R(r) = \left(1 - \frac{b}{\sqrt{r^2 + r_+^2}} \right)^{\frac{2\alpha^2}{1+\alpha^2}}, \quad \Phi(r) = \frac{\alpha}{1+\alpha^2} \ln \left(1 - \frac{b}{\sqrt{r^2 + r_+^2}} \right). \quad (25)$$

One can easily show that the Kretschmann scalar does not diverge in the range $0 \leq r < \infty$. However, the spacetime has a conic geometry and has a conical singularity at $r = 0$, since:

$$\lim_{r \to 0} \frac{1}{r} \sqrt{g_{\phi\phi} g_{rr}} \neq 1. \quad (26)$$

That is, as the radius $r$ tends to zero, the limit of the ratio “circumference/radius” is not $2\pi$ and therefore the spacetime has a conical singularity at $r = 0$. The canonical singularity can be removed if one identifies the coordinate $\phi$ with the period

$$\text{Period}_\phi = 2\pi \left(\lim_{r \to 0} \frac{1}{r} \sqrt{g_{\phi\phi} g_{rr}} \right)^{-1} = 2\pi (1 - 4\mu), \quad (27)$$

where

$$1 - 4\mu = \left\{ \frac{\Lambda l(r_+ - b)^{\frac{2\alpha^2}{\alpha^2 + 1}} [r_+ (\alpha^2 + 1) - b]}{3(\alpha^2 + 1)(b - r_+)r_+^{\frac{2\alpha^2}{1+\alpha^2}}} + \frac{lc(r_+ - b)^{\frac{1-\alpha^2}{\alpha^2 + 1}} [r_+ (\alpha^2 + 1) - 2b]}{2(\alpha^2 + 1)(b - r_+)r_+^{\frac{2\alpha^2}{\alpha^2 + 1}}} \right\}^{-1}. \quad (28)$$

The above analysis shows that near the origin $r = 0$, the metric (23) describes a spacetime which is locally flat but has a conical singularity at $r = 0$ with a deficit angle $\delta\phi = 8\pi\mu$. Since near the origin the metric (23) is identical to the spacetime generated by a cosmic string, by using the Vilenkin procedure, one can show that $\mu$ in Eq. (28) can be interpreted as the mass per unit length of the string [24].

### IV. SPINNING MAGNETIC DILATON STRING

Now, we would like to endow the spacetime solution (11) with a rotation. In order to add an angular momentum to the spacetime, we perform the following rotation boost in the $t - \phi$ plane

$$t \mapsto \Xi t - a\phi, \quad \phi \mapsto \Xi \phi - \frac{a}{l^2} t, \quad (29)$$
where $a$ is a rotation parameter and $\Xi = \sqrt{1 + a^2/l^2}$. Substituting Eq. (29) into Eq. (23) we obtain

$$
A^2 = -\frac{r^2 + r^2_+ R^2(r)}{l^2} \left( \Xi dt - a d\phi \right)^2 + \frac{r^2 dr^2}{(r^2 + r^2_+)^2} f(r) + l^2 f(r) \left( \frac{a}{l^2} dt - \Xi d\phi \right)^2 + \frac{r^2 + r^2_+}{l^2} R^2(r) dz^2, \quad (30)
$$

where $f(r)$ and $R(r)$ are given in Eqs. (24) and (25). The non-vanishing electromagnetic field components become

$$
F_{\phi r} = \frac{q \Xi l}{r^2 + r^2_+}, \quad F_{tr} = -\frac{a}{\Xi l^2} F_{\phi r}. \quad (31)
$$

The transformation (29) generates a new metric, because it is not a permitted global coordinate transformation. This transformation can be done locally but not globally. Therefore, the metrics (23) and (30) can be locally mapped into each other but not globally, and so they are distinct. Note that this spacetime has no horizon and curvature singularity. However, it has a conical singularity at $r = 0$. It is notable to mention that for $\alpha = 0$, this solution reduces to the asymptotically AdS magnetic rotating string solution presented in [12].

The mass and angular momentum per unit length of the string when the boundary $B$ goes to infinity can be calculated through the use of Eqs. (9) and (10). We obtain

$$
M = \frac{\alpha^2 (\alpha^2 - 1) b^3}{24 \pi l^3 (\alpha^2 + 1)^3} + \frac{(3 \Xi^2 - 2)c}{16 \pi l}, \quad (32)
$$

$$
J = \frac{3 \Xi c \sqrt{\Xi^2 - 1}}{16 \pi}. \quad (33)
$$

For $a = 0$ ($\Xi = 1$), the angular momentum per unit length vanishes, and therefore $a$ is the rotational parameter of the spacetime.

Finally, we compute the electric charge of the solutions. To determine the electric field one should consider the projections of the electromagnetic field tensor on special hypersurface. The normal vectors to such hypersurface for the spacetime with a longitudinal magnetic field are

$$
u^0 = \frac{1}{N}, \quad \nu^r = 0, \quad u^i = -\frac{V^i}{N},
$$

and the electric field is $E^\mu = g^{\mu \rho} e^{-2 \alpha \Phi} F_{\rho \nu} u^\nu$. Then the electric charge per unit length $Q$ can be found by calculating the flux of the electric field at infinity, yielding

$$
Q = \frac{q \sqrt{\Xi^2 - 1}}{4 \pi l}. \quad (34)
$$
It is worth noting that the electric charge is proportional to the rotation parameter, and is zero for the case of static solution. This result is expected since now, besides the magnetic field along the $\phi$ coordinate, there is also a radial electric field ($F_{tr} \neq 0$). To give a physical interpretation for the appearance of the net electric charge, we first consider the static spacetime. The magnetic field source can be interpreted as composed of equal positive and negative charge densities, where one of the charge density is at rest and the other one is spinning. Clearly, this system produce no electric field since the net electric charge density is zero, and the magnetic field is produced by the rotating electric charge density. Now, we consider the rotating solution. From the point of view of an observer at rest relative to the source ($S$), the two charge densities are equal, while from the point of view of an observer $S'$ that follows the intrinsic rotation of the spacetime, the positive and negative charge densities are not equal, and therefore the net electric charge of the spacetime is not zero.

V. CONCLUSION AND DISCUSSION

In conclusion, with an appropriate combination of three Liouville-type dilaton potentials, we constructed a class of four-dimensionl magnetic dilaton string solutions which produces a longitudinal magnetic field in the background of anti-de Sitter universe. These solutions have no curvature singularity and no horizon, but have conic singularity at $r = 0$. In fact, we showed that near the origin $r = 0$, the metric (23) describes a spacetime which is locally flat but has a conical singularity at $r = 0$ with a deficit angle $\delta \phi = 8 \pi \mu$, where $\mu$ can be interpreted as the mass per unit length of the string. In these static spacetimes, the electric field vanishes and therefore the string has no net electric charge. Then we added an angular momentum to the spacetime by performing a rotation boost in the $t - \phi$ plane. For the spinning string, when the rotation parameter is nonzero, the string has a net electric charge which is proportional to the magnitude of the rotation parameter. We found the suitable counterterm which removes the divergences of the action in the presence of three Liouville-type dilaton potentials. We also computed the conserved quantities of the solutions through the use of the counterterm method inspired by the AdS/CFT correspondence.

It is worth comparing the solutions obtained here to the electrically charged rotating dilaton black string solutions presented in [5]. In the present work I have studied the magnetic spinning dilaton string which produces a longitudinal magnetic field in AdS spaces
which is the correct one generalizing of the magnetic string solution of Dias and Lemos in dilaton theory [12], while in [5] I constructed charged rotating dilaton black string in AdS spaces which is the generalization of the charged rotating string solutions of [25] in dilaton gravity. Although solution (21) of the present paper is similar to Eq. (16) of Ref. [5] (except the sign of c) and both solutions represent dilaton string, however, there are some different between the magnetic string and the electrically charged dilaton black string solutions. First, the choice of the metric gauge $g_{tt} \propto -\rho^2$ and $(g_{\rho\rho})^{-1} \propto g_{\phi\phi}$ in the magnetic case which is quite different from the Schwarzschild gauge $[(g_{\rho\rho})^{-1} \propto g_{tt}$ and $g_{\phi\phi} \propto \rho^2]$ proposed in [5]. Second, the electrically charged dilaton black strings have an essential singularity located at $r = 0$ and also have horizons, while the magnetic strings version presented here have no curvature singularity and no horizon, but have a conic geometry. Third, when the rotation parameter is nonzero, the magnetic string has a net electric charge which is proportional to the rotation parameter, while charged dilaton black string has always an electric charge regardless of the rotation parameter.

The generalization of the present work to higher dimensions, that is the magnetic rotating dilaton branes in AdS spaces with complete set of rotation parameters and arbitrary dilaton coupling constant is now under investigation and will be addressed elsewhere.

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