MHD Peristaltic Flow of a Couple - Stress with varying Temperature for Jeffrey Fluid through Porous Medium

Samah F. Jaber AL-khulaifawi and Dheia Gazal Salih Al–Khafajy

Department of Mathematics, College of Science, University of Al-Qadisiyah, Diwaneyah, Iraq.

E-mail: samah85.samah85@gmail.com, dheia.salih@qu.edu.iq

Abstract

This paper is intended for investigating the effects of magnetohydrodynamic on the couple stress unsteady flow of incompressible Jeffrey fluid with varying temperature through a cylindrical porous channel. The analytical expression of the axial velocity, stream function and gradient pressure, was created taking into account the effect of thermal diffusion on the flow of the fluid. The analytical formulas of the velocity, temperature have been illustrated graphically for significant various parameters such as magnetic parameter, couple stress parameter, permeability parameter.

Keywords: MHD, Jeffrey Fluid, peristaltic flow, couple stress, porous medium.

List of symbols and meanings:

| Symbol | The meaning |
|--------|-------------|
| $A$ | is the average radius of the undisturbed tube. |
| $B$ | is the amplitude of the peristaltic wave. |
| $\mathcal{L}$ | is the wavelength. |
| $s$ | is the wave propagation speed. |
| $\mathcal{t}$ | is the time. |
| $\nabla^2 = \frac{1}{r \partial r} \left( r \frac{\partial}{\partial r} \right)$ | is the Laplace operator. |
| $\mathbf{v}$ | is the velocity field. |
| $\rho$ | is the density. |
| $\mu$ | is the dynamic viscosity. |
| $k^*$ | is the permeability. |
| $\mathbf{B} = (0, B_0, 0)$ | is the inclined magnetic field. |
| $\mu_p$ | is the magnetic permeability. |
| $\tilde{\sigma}$ | is the Cauchy stress tensor. |
| $\zeta$ | is the constant associated with the couple stress. |
| $T$ | is the temperature of the fluid. |
1. Introduction

Peristaltic flows received a broad study by researchers because of interest in physiology and industry. The movement of blood in the bodies of living organisms is one of the applications of peristaltic movement that occupied the ideas of many researchers of its importance in blood transfusion. The arterial segment was contracted and extended periodically by spreading the progressive wave. And as a result of this, the researchers presented their scientific results related to peristaltic flow engineering, and among the first of these researchers in this specialization are: Latham [1]. In [2] he presented a detailed analysis of the peristaltic flow fluid in circular cylindrical tubes, in [3] he along on experimental results with a long wave approximation is adopted to analyze the problem of peristaltic pumping in a circular cylindrical tube. Moreover, peristalsis subjected to magnetic field effects is important in the treatment of hyperthermia, arterial flow, cancer treatment, etc.

We can consider detailed explanation of peristaltic fluids as well as experimental results with a long wave approximation dependent on a round cylindrical tube. It is very important to cast a magnetic field on peristalsis in the treatment of hyperthermia, arterial flow, cancer treatment, etc. Where the magnet is important in healing diseases of the uterus, ulcers, infections and intestine. On the other hand, the role of permeability is very important for the movement of the fluid, as is the case in extracting oil from wells and absorbing food in the intestine ... etc. Many researchers presented a study on the combined effect of the magnetic field and the presence of permeability the fluid flow channel, see [4-8]. At the present time, interest began to study the effect of temperature on the movement of liquids through a channel, as most researchers agreed that increasing the temperature increases the velocity of the fluid, see [9-14] for more details.

The present analysis is interested in discussing the effects of MHD on a couple's stress on Jeffrey fluid through a cylindrical porous medium duct. To date, studies have not found the presence of a magnetic field and the effect of varying temperatures from a couple's stress on the flow of a Jeffrey liquid through a porous channel in the cylindrical coordinates. This paper was divided into seven sections. The first section contains the flow channel form with the formulation of the governing equations and the formula for the equation for liquids fluid. As for the second section, it includes reviewing the boundary conditions with including non-dimensional transformations to facilitate the governing equations that assume there is a very small number of Reynolds or a very large wavelength to solve. As for sections 3 and 4, it is to solve problems and find a formula for temperature, velocity function, high pressure, and frictional force using Bissell functions and the regular ultra-high pressure measurement function. Whereas, the fifth section includes a discussion of the effect of the parameters on temperature, speed velocity, and pressure through detailed illustrations. The sixth section examines the phenomenon of trapping and the factors affecting it, whether increasing or decreasing, and in the last section it briefly presents the most important factors affecting the shape.

2. Mathematical Formulation

Consider a peristaltic flow of an incompressible Jeffrey fluid in a coaxial uniform circular tube. The Jeffrey fluid is a non-Newtonian non-compressible liquid model and it is a real fluid in which shear stress does not match the shear stress rate (or velocity gradient). The cylindrical coordinates are considered, where \( R \) is
along the radius of the tube and \( Z \) coincides with the axes of the tube as shown in figure 1, see [12].

**Figure 1** Geometry of the problem

The geometry of wall surface is described as:
\[
H(\tilde{Z}, \tilde{t}) = a + b \sin \left[ \frac{2\pi}{L} (\tilde{Z} - s\tilde{t}) \right]
\]

(1)

The basic equations governing of the problem (continuity, momentum and temperature equations) are given by:
\[
\nabla \mathbf{V} = 0
\]

(2)

\[
\rho (\mathbf{V} \cdot \nabla) \mathbf{V} = -\nabla p + \frac{\mu}{k} \mathbf{V} + \rho g \beta_1 (T - T_0) + \hat{\zeta} \nabla^4 \mathbf{V}, \ \text{see} \ [4], [12]
\]

(3)

\[
T_s \rho (\mathbf{V} \cdot \nabla) T = T_c \nabla^2 T - \nabla \cdot \mathbf{Q} - q (T - T_0)
\]

(4)

The constitutive equations for an incompressible Jeffrey fluid are given by:
\[
\tilde{\sigma} = -\tilde{p} \mathbf{I} + \tilde{\mathcal{S}}
\]

(5)

\[
\tilde{\mathcal{S}} = \frac{\mu}{1 + \lambda_1} (\tilde{\mathcal{X}} + \lambda_2 \tilde{\mathcal{X}})
\]

(6)

where \( \tilde{\mathcal{S}} \) is the extra stress tensor, \( \tilde{p} \) is the pressure, \( \mathbf{I} \) is the identity tensor, \( \lambda_1 \) is the ratio of relaxation to retardation times, \( \tilde{\mathcal{X}} \) is the shear rate, \( \tilde{\mathcal{X}} \) is material derivative, and \( \lambda_2 \) is the retardation time.

3. Method of solution

Let \( \mathbf{U} \) and \( \mathbf{W} \) be the respective velocity components in the radial and axial directions in the fixed frame, respectively. For the unsteady two - dimensional flow the velocity field, temperature function may be written as:
\[
\mathbf{V} = (\mathbf{U}(r, z), 0, \mathbf{W}(r, z))
\]

(7)

\[
T = T(r, z)
\]

(8)

By using the constitutive relations (5), (6) the equations of the problem (2)-(4) take the form:
\[
\frac{\partial \mathbf{U}}{\partial t} + \mathbf{U} \cdot \nabla \mathbf{U} = -\nabla p + \frac{\mu}{k} \mathbf{U} + \rho g \beta_1 (T - T_0) - \hat{\zeta} \nabla^4 \mathbf{U}
\]

(9)

\[
\frac{\partial \mathbf{W}}{\partial t} + \mathbf{U} \cdot \nabla \mathbf{W} = -\frac{\partial p}{\partial z} + \frac{1}{2} \frac{\partial (\tilde{R} \tilde{S}_{kk})}{\partial z} + \frac{\partial}{\partial z} (\tilde{S}_{kk}) - \frac{\dot{S}_{kk}}{\tilde{\nu}} - \frac{\mu}{k} \mathbf{U} - \sigma B_0^2 \mathbf{U} - \hat{\zeta} \nabla^4 \mathbf{W}
\]

(10)

\[
\frac{\sigma}{\partial t} + \mathbf{U} \cdot \nabla \sigma + \mathbf{W} \frac{\sigma}{\partial z} = \frac{T_c}{\tau_p} \left( \frac{\partial^2 \tau}{\partial R^2} + \frac{1}{2} \frac{\partial \tau}{\partial R} + \frac{\partial^2 \tau}{\partial z^2} + \frac{16 \sigma \tau \tau}{3k_0^2 \tau_p} \left( \frac{1}{\partial R} + \frac{\partial^2 \tau}{\partial R^2} - \frac{1}{\partial z} \nabla (T - T_0) \right) \right)
\]

(11)

\[
\frac{\partial \mathbf{W}}{\partial z} + \mathbf{U} \cdot \nabla \mathbf{W} + \mathbf{W} \frac{\partial \mathbf{W}}{\partial z} = \frac{T_c}{\tau_p} \left( \frac{\partial^2 \tau}{\partial R^2} + \frac{1}{2} \frac{\partial \tau}{\partial R} + \frac{\partial^2 \tau}{\partial z^2} + \frac{16 \sigma \tau \tau}{3k_0^2 \tau_p} \left( \frac{1}{\partial R} + \frac{\partial^2 \tau}{\partial R^2} - \frac{1}{\partial z} \nabla (T - T_0) \right) \right)
\]

(12)
The flow in the fixed coordinates \((\vec{R}, \vec{Z})\) between the two tubes is unsteady, it becomes steady at moving coordinates \((r, z)\) when the wave is the same speed in the \(Z\) -direction. The Transformations between the two frames is given by:

\[
\begin{align*}
\vec{r} &= \vec{R}, \quad \vec{z} = \vec{Z} - st, \\
\vec{u} &= \vec{U}, \quad \vec{w} = \vec{W} - s,
\end{align*}
\]  

(13)

Where \((\vec{u}, \vec{w})\) and \((\vec{U}, \vec{W})\) are the velocity components in the moving and fixed frames, respectively.

The appropriate boundary conditions are:

\[
\begin{align*}
\vec{w} &= -1, \quad \vec{u} = 0, \quad T = T_1 \quad \text{at} \quad \vec{r} = \vec{r}_1 = a_1, \\
\vec{w} &= -1, \quad \vec{u} = 0, \quad T = T_0 \quad \text{at} \quad \vec{r} = \vec{r}_2 (\vec{z}, t) = a_2 + b \sin(2\pi \vec{z}).
\end{align*}
\]  

(15)

In order to simplify the governing equations of the problem, we may introduce the following dimensionless transformations as follows:

\[
\begin{align*}
u &= \frac{\vec{u} \xi}{\alpha \Omega}, \quad w = \frac{\vec{w}}{\Omega}, \quad r = \frac{r}{\alpha a_2}, \quad z = \frac{z}{\xi}, \quad S = \frac{a_2^4}{\mu \xi}, \quad \delta = \frac{a_2}{\xi}, \quad Da = \frac{k}{\alpha^2}, \\
\mathcal{H} &= \frac{r - r_0}{r_1 - r_0}, \quad Re = \frac{\rho \alpha a^2}{\mu}, \quad M^2 = \frac{\alpha^2 \Omega^2}{\mu}, \quad Pr = \frac{\mu T_1}{\rho a_2}, \quad \Omega = \frac{a a_2^2}{\mu T_2},
\end{align*}
\]  

(16)

where \(\Omega\) the “amplitude ratio”, \(\alpha\) the “couple stress” fluid parameter indicating the ratio of the tube radius (constant) to material characteristic length \((\sqrt{\mu/\xi}\), has the dimension of length), \(Re\) the “Reynolds number is the ratio of inertia force to the viscous force”, \(Pr\) the “Prandtl number is ratio of kinematic viscosity to the thermal diffusivity”, \(Da\) the “Darcy number is the ratio of the permeability of the medium to the diameter of the particle”, \(Rn\) the “thermal radiation parameter”, \(Gr\) the “thermal Grashof number is a measure of buoyancy or free-convection effects in a flow”, \(M^2\) the “magnetic parameter is equal to the product of the square of the magnetic permeability, the square of the magnetic field strength, the electrical conductivity, and a characteristic length, divided by the product of the mass density and the fluid velocity”, \(\delta\) the “dimensionless wave number” \(\Omega\) “heat source/sink parameter”.

After using these transformations equations (13)-(14), substituting dimensionless equations (16) into problem equations (9)-(12) and boundary conditions (15), we get:

\[
\begin{align*}
\frac{\partial u}{\partial r} + u \frac{\partial u}{\partial r} + \frac{\partial w}{\partial z} &= 0, \quad \text{Re} \delta^3 \left( u \frac{\partial^2 u}{\partial r^2} + (w + 1) \frac{\partial u}{\partial z} \right) = - \frac{\partial u}{\partial r} + \delta \left[ \frac{1}{r} \frac{\partial}{\partial r} (r S_{rr}) + \frac{\partial}{\partial r} (S_{rz}) - \frac{5 \pi \rho a}{r} - \frac{\delta}{\alpha^2} \nabla^4 u - \frac{\delta}{\alpha^2} u - \delta M^2 u \right], \quad \text{Re} \delta \left( u \frac{\partial^2 u}{\partial r^2} + (w + 1) \frac{\partial w}{\partial z} \right) = - \frac{\partial u}{\partial r} + \frac{\partial w}{\partial r} (r S_{rr}) + \delta \frac{\partial}{\partial r} (S_{xz}) - \frac{1}{\alpha^2} \nabla^4 w - \left( M^2 + \frac{1}{\alpha} \right) \frac{\partial}{\partial r} \left( M^2 \frac{1}{\alpha} \right) w - \left( M^2 + \frac{1}{\alpha} \right), \quad \text{Gr} \mathcal{H}, \quad \text{Re} \delta \left( u \frac{\partial^2 u}{\partial r^2} + (w + 1) \frac{\partial w}{\partial z} \right) = \frac{1}{\text{Pr}^2} \frac{\partial^2 u}{\partial r^2} + \frac{1}{\text{Pr}^2} \frac{\partial^2 w}{\partial r^2} + \delta \frac{\partial^2 u}{\partial r^2} + \frac{4}{3 \alpha^2} \frac{1}{\text{Pr}^2} \left( \frac{\partial^2 u}{\partial z^2} \right) - \Omega \mathcal{H},
\end{align*}
\]  

(17)-(20)

where

\[
S_{rr} = \frac{2 \delta}{1 + \lambda_1} \left[ 1 + \frac{s \lambda_2 \delta}{a_2} \left( u \frac{\partial}{\partial r} + w \frac{\partial}{\partial z} \right) \left( \frac{\partial u}{\partial r} \right) \right]
\]  

(21)
The related boundary conditions regarding to the dimensionless variables in the wave frame are given by:

\[
\begin{align*}
\frac{\partial u}{\partial r} + u & + \frac{\partial w}{\partial z} = 0 \\
\frac{\partial p}{\partial r} = 0 \\
\frac{1}{\alpha^2} \nabla^4 w - \frac{1}{r} \frac{\partial}{\partial r} \left( r S_{rr} \right) + \left( M^2 + \frac{1}{\alpha a} \right) w & = - \frac{\partial p}{\partial z} - \left( M^2 + \frac{1}{\alpha a} \right) + GrH \\
\left( \frac{1}{pr} + \frac{4}{3\pi n} \right) \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial H}{\partial r} \right) - \Omega H & = 0
\end{align*}
\]

where \( S_{rr} = S_{\theta\theta} = S_{zz} = 0, \text{and} \ S_{rz} = \frac{1}{1 + \lambda_1} \left( \frac{\partial w}{\partial r} \right) \).

Equation (27) shows that \( p \) depends on z only. Replacing \( S_{rz} \) from equation (30) in equation (28), we have:

\[
\frac{1}{\alpha^2} \nabla^4 w - \frac{1}{r} \frac{\partial}{\partial r} \left( r S_{rr} \right) + \left( M^2 + \frac{1}{\alpha a} \right) w = - \frac{\partial p}{\partial z} - \left( M^2 + \frac{1}{\alpha a} \right) + GrH
\]

Assuming the components of the couple stress tensor at the wall to be zero, when Couple-stress, denoted by \( \bar{a} \) is defined as the ratio of the tube radius (constant) to material characteristic length (\( \sqrt{\frac{n}{\mu}} \) has the dimension of length), mathematically:

\[
\bar{a} = \alpha a_2 = \frac{\mu}{\sqrt{\eta} \bar{a}_2}
\]

Where, \( \mu \) is the dynamic viscosity, \( \eta \) is constant associated with couple stress, we can write The Couple-stress \( \eta V^4 \), \( \nabla \bar{V} \), see. [7], we have the following dimensionless boundary conditions:

\[
\begin{align*}
w & = 1, \frac{\partial \bar{w}}{\partial r} - \zeta \frac{\partial w}{\partial r} = 0 \text{ at } r = \epsilon \\
w & = 1, \frac{\partial \bar{w}}{\partial r} - \zeta \frac{\partial w}{\partial r} = 0 \text{ at } r = 1 + \Phi \sin(2\pi r)
\end{align*}
\]

Where \( \zeta = \frac{\eta}{\bar{a}} \) is a couple stress fluid parameter (\( \zeta \) and \( \zeta \) are constants associated with the couple stress, when \( \zeta \rightarrow 1 \) (i.e. \( \bar{a} \rightarrow \zeta \)) no couple stress effects, see [4], [5], and [6]).
4. Solutions of the Temperature Equations

The temperature equation (29), can be written as:

\[
\frac{\partial^2 \mathcal{H}}{\partial r^2} + \frac{1}{r} \frac{\partial \mathcal{H}}{\partial r} - \frac{\Omega}{r^2} \frac{1}{\text{Re}} \mathcal{H} = 0,
\]

Set \( \Lambda = -\frac{\Omega}{r^2} \frac{1}{\text{Re}} \), the equation (34) takes the form:

\[
r^2 \frac{\partial^2 \mathcal{H}}{\partial r^2} + r \frac{\partial \mathcal{H}}{\partial r} + \Lambda r^2 \mathcal{H} = 0
\]

The general solution of this equation “modified Bessel equation of zero-order”, with the boundary conditions equation (25) is:

\[
\mathcal{H} = B_1 I_0[r\sqrt{\Lambda}] + B_2 Y_0[r\sqrt{\Lambda}]
\]

where \( B_1 = \frac{J_0[\nu\sqrt{\Lambda}]}{Y_0[\nu\sqrt{\Lambda}]} \) and \( B_2 = \frac{J_0[\nu\sqrt{\Lambda}]}{Y_0[\nu\sqrt{\Lambda}]} \).

The general solution of motion equation (31) is:

\[
w = B_1 I_0(\beta r \sqrt{|s_1|}) + B_2 K_0(\beta r \sqrt{|s_1|}) + B_3 J_0(\beta r \sqrt{|s_2|}) + B_4 K_0(\beta r \sqrt{|s_2|}) - \frac{dp}{dz} \frac{Gr \mathcal{H} + (M^2 + \frac{1}{\text{Re}})}{M^2 + \frac{1}{\text{Re}}}
\]

Also \( I_0, K_0 \) are the modified Bessel functions of the first and second kind of zero order. By using the “MATHEMATICA 11” program and the boundary conditions equations (25) and (33) we have a constants \( B_1, B_2, B_3 \) and \( B_4 \).

5. Stream function

The corresponding stream functions \( u = -\frac{1}{r} \frac{\partial \psi}{\partial z} \) and \( w = \frac{1}{\rho} \frac{\partial \psi}{\partial r} \) is

\[
\psi = A_1 \frac{r^2}{4} - B_2 r K_0(r u_1) - B_4 r K_0(r u_2) + \frac{1}{2} B_1 r^2 \frac{\partial \mathcal{F}}{\partial z} \left[ 2, \frac{r^2 u_1^2}{4} \right] + \frac{1}{2} B_3 r^2 \frac{\partial \mathcal{F}}{\partial z} \left[ 2, \frac{r^2 u_2^2}{4} \right]
\]

\[
A_1 = -\left( \frac{dp}{dz} \frac{Gr \mathcal{H} + (M^2 + \frac{1}{\text{Re}})}{M^2 + \frac{1}{\text{Re}}}, u_1 = \beta r \sqrt{|s_1|}, u_2 = \beta r \sqrt{|s_2|}, K_0 \frac{\partial \mathcal{F}}{\partial z} \right)
\]

The instantaneous volume flow rate \( Q(z) = 2 \int_{r_1}^{r_2} \rho wdr \) is given by;

\[
\frac{dp}{dz} = \left( Gr \mathcal{H} - (M^2 + \frac{1}{\text{Re}}) \right) + \frac{(M^2 + \frac{1}{\text{Re}})}{r_2^2 - r_1^2} \left\{ Q(z) + \frac{2B_2}{u_1} [r_1 K_1(r_1 u_1) - r_2 K_1(r_2 u_1)] + \frac{2B_4}{u_2} [r_1 K_1(r_1 u_2) - r_2 K_1(r_2 u_2)] - B_1 \left[ \frac{r_2^2}{4} \mathcal{F}_1 \left[ 2, \frac{r_2^2 u_1^2}{4} \right] - \frac{1}{r_1^2} \mathcal{F}_1 \left[ 2, \frac{r_1^2 u_1^2}{4} \right] \right] - B_3 \left[ \frac{r_2^2}{4} \mathcal{F}_1 \left[ 2, \frac{r_2^2 u_2^2}{4} \right] - \frac{1}{r_1^2} \mathcal{F}_1 \left[ 2, \frac{r_1^2 u_2^2}{4} \right] \right] \right\}
\]

Following the analysis given by Shapiro et al.[14], the mean volume flow, \( q_2 \) over a period is obtained as

\[
q_2 = Q + \frac{1}{2} \left( 1 - \varepsilon^2 + \frac{\phi^2}{2} \right)
\]

This on using Eq. (38) yields
\[
\frac{dp}{dz} = \left( GrH - \left( M^2 + \frac{1}{D_D} \right) \right) + \left( M^2 + \frac{1}{r_D^2} \right) \{ q^2 - \frac{1}{2} ( 1 - \epsilon^2 + \phi^2 ) + \left( \frac{2B_i}{u_i} [ r_1 K_1 ( r_1 u_1 ) - r_2 K_1 ( r_2 u_1 ) ] + \right.
\]
\[
\left. \left( \frac{2B_i}{u_i} [ r_1 K_1 ( r_1 u_1 ) - r_2 K_1 ( r_2 u_2 ) ] - B_1 \left[ r_1^2 \phi_1 \left[ 2, \frac{r_1^2 u_1^2}{4} \right] - r_1^2 \phi_1 \left[ 2, \frac{r_1^2 u_1^2}{4} \right] \right] - B_3 \left[ r_2^2 \phi_1 \left[ 2, \frac{r_2^2 u_2^2}{4} \right] \right] \right) \right\}.
\]

where \( Y_1 \) and \( \phi_1 \) are the modified Bessel function of the second kind and hypergeometric function, respectively.

The pressure rise \( \Delta p \) and the friction force (at the wall) on the inner and outer tubes are \( F^I \) and \( F^O \), respectively, in a tube of length \( L \), in their non-dimensional forms, are given by:

\[
\Delta p = \int_0^1 \frac{dp}{dz} \, dz,
\]

\[
F^I = \int_0^1 r^2 \left( - \frac{dp}{dz} \right) \, dz,
\]

\[
F^O = \int_0^1 r^2 \left( \frac{dp}{dz} \right) \, dz.
\]

Substituting from equation (41) in equations (42) - (44) with \( r_1 = \epsilon, r_2 = 1 + \phi, \sin ( 2 \pi z ) \), and then evaluating the integrations by using the language of series for several values of the parameters included, by the “MATHEMATICA 11” program, and the obtained results are discussed in the next section.

6. Numerical Results and Discussion

In this section the numerical and computational results are discussed for the problem of an incompressible non-Newtonian Jeffrey fluid through porous medium with heat and mass transfer through the illustrations figures (2-39).

Based on equation (36), figures (2-3) shows that effects of the parameters \( \epsilon, \Omega, Rn \) and \( \phi \) on the temperature function \( H \), in figure 2, we notice that \( H \) increases with increasing \( \epsilon \) and \( \Omega \), while figure 3, illustrates the temperature function increases with increasing \( Rn \) and \( H \) decrease with increasing \( \phi \).

Based on equation (37), figures (4-9), illustrate the effect of the parameters \( \epsilon, \Omega, \alpha, \eta, \phi, \lambda, Gr, Da, M, q_2, Pr \) and \( Rn \) on the velocity distribution \( w \) vs. \( r \). We noticed that the velocity distribution starts to decrease and when it reaches point \( r=0.05 \) it starts to increase and for this, the general shape of the velocity distribution is a concave upward curve. Figure 4, illustrates the influence of the parameters \( \epsilon \) and \( \Omega \) on the velocity distribution function \( w \) vs. \( r \). It is found that the velocity \( w \) increases with the increasing \( \epsilon \) when \( r < 0.07 \), while \( w \) decreases with increasing of \( \epsilon \) when \( r > 0.07 \), and \( w \) decreases with increasing \( \Omega \). In the fifth plot, shows the behavior of \( w \) under the variation of \( \alpha \) and \( \eta \), one can describe here that \( w \) increases with increasing of \( \alpha \) and \( \eta \) at \( r > 0.2 \), while \( w \) decreases with increasing of \( \alpha \) and \( \eta \) at \( r < 0.2 \), Figure 6, we notice the rotation of the effects of the parameters \( \lambda \) and \( \phi \) on the velocity function, where the effect of parameter \( \lambda \) is direct in the region \( r < 0.2 \), while in the region \( r > 0.2 \) the effect of parameter \( \lambda \) is inverted, and vice versa for the parameter \( \phi \), we notice the decrease in the velocity when increasing \( \phi \) in the region \( r < 0.2 \) and the increase in the velocity with increasing \( \phi \) in the region \( r > 0.2 \). Figure 7 contains the velocity profile behavior under the parameters \( Gr \) and \( q_2 \), we see that the velocity profile goes down with the increases of \( Gr \) and \( q_2 \) when \( r < 0.2 \), and \( w \) increases with increasing of \( Gr \) and \( q_2 \) when \( r > 0.2 \). We notice the effect of the magnetic field and permeability on the velocity function in shape 8, we get the velocity decreases with an increase in \( M \) and \( Da \) at \( r > 0.2 \), while the velocity \( w \) increase with an increase in \( M \) and \( Da \) at \( r < 0.2 \). In the ninth plot, It is found that the velocity \( w \) increases with the increasing \( Pr \) and \( Rn \) in the region \( r > 0.2 \), while \( w \) decreases with increasing of \( Pr \) and \( Rn \) in the region \( r < 0.2 \).

Based on equation (41), figures (10-15), illustrate the effect of the parameters \( \epsilon, \Omega, \alpha, \eta, \phi, \lambda, Gr, q_2, Da, M, Pr \) and \( Rn \) on the distribution of \( dp/dz \) vs. \( z \). We noticed that \( dp/dz \) starts to increase and when it
reaches point $z=0.25$ it starts to decrease and for this, the general diagram of the distribution of $dp/dz$ is a concave downward curve. Figures 11, 13 and 14, illustrates the influence of the parameters $\alpha, \eta, Gr, q2, Da$ and $M$ on $dp/dz$. It is found that $dp/dz$ increases with the increasing $\alpha, \eta, Gr, q2, Da$ and $M$, respectively. Figures 10 and 15, illustrates the influence of the parameters $\Omega, \epsilon, Pr$ and $Rn$ on $dp/dz$. It is found that $dp/dz$ decreases with the increasing $\Omega, \epsilon, Pr$ and $Rn$, respectively. Figure 12, illustrates the influence of the parameters $\emptyset$ and $\lambda_1$, on $dp/dz$. It is found that $dp/dz$ increases with the increasing $\emptyset$ while $dp/dz$ decreases with the increasing $\lambda_1$.

Based on equation (42), figures (16-19) illustrates the effects of the parameters $\emptyset, \Omega, Da, \epsilon, \lambda_1, q2, \eta, Gr, Rn$ and $M$ on the pressure rise $\Delta p$. Figures (16-17) illustrates the effects of the parameters $\Omega, Da, \epsilon$ and $\lambda_1$ on the $\Delta p$ vs. $\emptyset$. We found that $\Delta p$ increases with increasing $Da$, and $\Delta p$ decreases with increasing $\Omega$ in figure 16. In figure 17 we notice that $\Delta p$ decreases with increasing $\epsilon$ in the region $(0,0.03)$ while $\Delta p$ increases with increasing $\emptyset$ when $\emptyset > 0.022$, while $\Delta p$ decreases with increasing $\lambda_1$ when $\emptyset < 0.022$. Figures (18-19) illustrates the effects of the parameters $\eta, Gr, M$ and $Rn$ on the pressure rise $\Delta p$ vs. $q2$, it is found that $\Delta p$ increases with the increasing for each $\eta, Gr, M$ and $Rn$.

Based on equation (43), figures (20-23) illustrates the effects of the parameters $\emptyset, \Omega, Da, \epsilon, \lambda_1, q2, \eta, Gr, Rn$ and $M$ on $F^i$. Figures (20-21) illustrates the effects of the parameters $\Omega, Da, \epsilon$ and $\lambda_1$ on $F^i$ vs. $\emptyset$. We found that $F^i$ decreases with increasing $Da$, and $F^i$ increases with increasing $\Omega$ in figure 20. In figure 21 we notice that $F^i$ increases with increasing $\epsilon$ in the region $(0,0.022)$ while $F^i$ decreases with increasing $\epsilon$ when $\emptyset > 0.022$, the $F^i$ decreases with increasing $\lambda_1$ when $0.021 < \emptyset < 0.1$ at $\epsilon = 0.15$, and $F^i$ increases with increasing $\lambda_1$ when $0 < \emptyset < 0.021$ at $\epsilon = 0.175$, while $F^i$ increases with increasing $\lambda_1$ otherwise. Figures (22-23) illustrates the effects of the parameters $\eta, Gr, M$ and $Rn$ on $F^i$ vs. $q2$, it is found that $F^i$ decreases with the increasing for each $\eta, Gr, M$ and $Rn$.

Based on equation (44), figures (24-27) illustrates the effects of the parameters $\emptyset, \Omega, Da, \epsilon, \lambda_1, q2, \eta, Gr, Rn$ and $M$ on $F^o$. Figures (24-25) illustrates the effects of the parameters $\Omega, Da, \epsilon$ and $\lambda_1$ on $F^o$ vs. $\emptyset$. We found that $F^o$ decreases with increasing $Da$, and $F^o$ increases with increasing $\Omega$ in figure 24. In figure 25 we notice that $F^o$ increases with increasing $\epsilon$ and $\lambda_1$ in the region $(0,0.023)$ while $F^o$ decreases with increasing $\epsilon$ and $\lambda_1$ when $\emptyset > 0.023$. Figures (26-27) illustrates the effects of the parameters $\eta, Gr, M$ and $Rn$ on $F^o$ vs. $q2$, it is found that $F^o$ decreases with the increasing for each $\eta, Gr, M$ and $Rn$.

7. Trapping phenomena

The formation of an internally circulating bolus of fluid by closed streamlines is called trapping and this trapped bolus is pushed ahead along with the peristaltic wave. The effects of $\epsilon, \Omega, \emptyset, \lambda_1, Rn, Pr, Gr, M, Da, q2, \alpha$ and $\eta$ on trapping can be seen through 28-39. Figure 28 shows that the size of the trapped bolus decreases with the increase $\epsilon$ gradually in the middle of the channel while when we approach at the upper wall we notice the increase of the wave with the increase of $\epsilon$. The wave near the upper wall of the channel decreases with an increase of $\Omega$ in figure 29. In the Thirty plot shows that the size of the trapped bolus located in the center of the channel increases with the increase $\emptyset$ while when it is close to the upper wall we notice the decrease of the wave with the increase $\emptyset$. By figure 31 the size of the trapped bolus grow increase of $\lambda_1$ when it is close to the upper wall of the channel gradually. The effect of parameter $Rn$ on the trapped bolus in figure 32 is similar to the effect of parameter $\Omega$ on the trapped bolus in figure 29. By figure 33, we notice two trapped boluses, one in the center of the channel and the other at the upper wall both are decreases until it disappears with the increase $Pr$. In figure 34 the size of the trapped bolus decreases with the increase $Gr$ gradually at the upper wall. In figure 35 we notice the emergence and growth of the size of the trapped boluses, in addition to an increase in the wave at the upper wall of the channel when the value of $M$ increases. In figure 36, the size of the trapped bolus decreases with the increase $Da$ gradually at the upper wall of the
channel while its beginning to grow in the center with increase of $Da$. Figure 37 shows the effect of the parameter $q_2$ on the trapped bolus, as with the increase of $q_2$ the wave near the upper wall increases with the emergence of a new trapped bolus that caused the bolus to grow in the center of the channel. In figure 38 the size of the trapped bolus decreases with increase $\alpha$ in the wave near the upper wall. Finally in figure 39 we notice the effect of parameter $\eta$ on trapped bolus similar to that of trapped bolus $Da$ in figure 36.

**Figure 2:** The variation of temperature $H$ vs. $r$ at $Pr = 1$, $\varnothing = 0.2$, $Rn = 0.5$, $z = 0.1$

**Figure 3:** The variation of temperature $H$ vs. $r$ at $\varnothing = 0.2$, $\Omega = 1$, $Pr = 1$, $z = 0.1$

**Figure 4:** velocity distribution for various values of $\varepsilon$ and $\Omega$ with $\eta = 0.5$, $\alpha = 3.75$, $\lambda_1 = 0.1$, $\varnothing = 0.2$, $Gr = 2$, $q_2 = 0.5$, $M = 1.1$, $Da = 0.9$, $Sc = 0.5$, $Sr = 0.6$, $Pr = 2$, $Rn = 0.5$, $z = 0.1$.

**Figure 5:** velocity distribution for various values of $\eta$ and $\alpha$ with $\varepsilon = 0.2$, $\Omega = 0.9$, $\lambda_1 = 0.1$, $\varnothing = 0.2$, $Gr = 2$, $q_2 = 0.5$, $M = 1.1$, $Da = 0.9$, $Sc = 0.5$, $Sr = 0.6$, $Pr = 2$, $Rn = 0.5$, $z = 0.1$. 
Figure 6: velocity distribution for various values of $A1$ and $\phi$ with $\eta = 0.5$, $\alpha = 3.75$, $\varepsilon = 0.2$, $\Omega = 0.9$, $Gr = 2$, $q2 = 0.5$, $M = 1.1$, $Da = 0.9$, $Sc = 0.5$, $Sr = 0.6$, $Pr = 2$, $Rn = 0.5$, $z = 0.1$.

Figure 7: velocity distribution for various values of $Gr$ and $q2$ with $\eta = 0.5$, $\alpha = 3.75$, $A1 = 0.1$, $\phi = 0.2$, $\varepsilon = 2$, $\Omega = 1$, $M = 1.1$, $Da = 0.9$, $Sc = 0.5$, $Sr = 0.6$, $Pr = 2$, $Rn = 0.5$, $z = 0.1$.

Figure 8: velocity distribution for various values of $M$ and $Da$ with $\eta = 0.5$, $\alpha = 3.75$, $A1 = 0.1$, $\phi = 0.2$, $Gr = 2$, $\varepsilon = 0.2$, $\Omega = 0.9$, $q2 = 0.5$, $Sc = 0.5$, $Sr = 0.6$, $Pr = 2$, $Rn = 0.5$, $z = 0.1$.

Figure 9: Velocity distribution for various values of $Pr$ and $Rn$ with $\eta = 0.5$, $\alpha = 3.75$, $A1 = 0.1$, $\phi = 0.2$, $Gr = 2$, $\varepsilon = 0.2$, $\Omega = 0.9$, $q2 = 0.5$, $M = 1.1$, $Da = 0.9$, $Sc = 0.5$, $Sr = 0.6$, $z = 0.1$.

Figure 10: Distribution of $\frac{dp}{dz}$ vs. $z$ for various values of $\Omega$ and $\varepsilon$ with $\eta = 0.5$, $\alpha = 3.75$, $A1 = 0.1$, $\phi = 0.2$, $Gr = 2$, $Rn = 2$, $Pr = 1$, $q2 = 0.5$, $M = 1.1$, $Da = 0.9$, $Sc = 0.5$, $Sr = 0.6$, $z = 0.1$.

Figure 11: Distribution of $\frac{dp}{dz}$ vs. $z$ for various values of $\alpha$ and $\eta$ with $Sc = 0.5$, $Sr = 0.6$, $A1 = 0.1$, $\phi = 0.2$, $Gr = 2$, $\varepsilon = 0.2$, $\Omega = 0.9$, $q2 = 0.5$, $M = 1.1$, $Da = 0.9$, $Pr = 1$, $Rn = 2$, $z = 0.1$. 

Figure 12: Distribution of $\frac{dp}{dz}$ vs. $z$ for various values of $\varnothing$ and $\lambda_1$ with $\eta = 0.5$, $\alpha = 3.75$, $Sc = 0.5$, $Sr = 0.6$, $Gr = 2$, $\varepsilon = 0.2$, $\Omega = 0.9$, $q_2 = 0.5$, $M = 1.1$, $Da = 0.9$, $Pr = 1$, $Rn = 2$, $z = 0.1$.

Figure 13: Distribution of $\frac{dp}{dz}$ vs. $z$ for various values of $q_2$ and $Gr$ with $\eta = 0.5$, $\alpha = 3.75$, $Sc = 0.5$, $Sr = 0.6$, $\lambda_1 = 0.1$, $\varnothing = 0.2$, $\varepsilon = 0.2$, $\Omega = 0.9$, $M = 1.1$, $Da = 0.9$, $Pr = 1$, $Rn = 2$, $z = 0.1$.

Figure 14: Distribution of $\frac{dp}{dz}$ vs. $z$ for various values of $M$ and $Da$ with $\eta = 0.5$, $\alpha = 3.75$, $Sc = 0.5$, $Sr = 0.6$, $\lambda_1 = 0.1$, $\varnothing = 0.2$, $\varepsilon = 0.2$, $\Omega = 0.9$, $q_2 = 0.5$, $Gr = 2$, $Pr = 1$, $Rn = 2$, $z = 0.1$.

Figure 15: Distribution of $\frac{dp}{dz}$ vs. $z$ for various values of $Pr$ and $Rn$ with $\eta = 0.5$, $\alpha = 3.75$, $Sc = 0.5$, $Sr = 0.6$, $\lambda_1 = 0.1$, $\varnothing = 0.2$, $\varepsilon = 0.2$, $\Omega = 0.9$, $q_2 = 0.5$, $Gr = 2$, $M = 1.1$, $Da = 0.9$, $z = 0.1$.

Figure 16: Distribution of $\Delta p$ vs. $\varnothing$ for various values of $\Omega$ and $Da$ with $\varepsilon = 0.2$, $\lambda_1 = 0.1$, $Rn = 2$, $Pr = 2$, $Sc = 0.5$, $Sr = 0.1$, $q_2 = 0.5$, $Gr = 2$, $\alpha = 3.75$, $\eta = 0.5$, $M = 1.1$, $z = 0.1$.

Figure 17: Distribution of $\Delta p$ vs. $\varnothing$ for various values of $\lambda_1$ and $\varepsilon$ with $\Omega = 0.9$, $Rn = 2$, $Pr = 2$, $Sc = 0.5$, $Sr = 0.1$, $Gr = 2$, $\alpha = 3.75$, $\eta = 0.5$, $M = 1.1$, $Da = 0.9$, $q_2 = 0.5$, $z = 0.1$. 
Figure 18: Distribution of Δp vs. q2 for various values of Gr and η with \( \varepsilon = 0.2, \Omega = 0.9, \phi = 0.2, \lambda_1 = 0.1, \)
\( Rn = 2, Pr = 2, Sc = 0.5, Sr = 0.1, \alpha = 3.75, \)
\( M = 1.1, Da = 0.9, z = 0.1. \)

Figure 19: Distribution of Δp vs. q2 for various values of M and Rn with \( \varepsilon = 0.2, \Omega = 0.9, \phi = 0.2, \lambda_1 = 0.1, \)
\( Pr = 2, Gr = 2, \eta = 0.5, Sc = 0.5, Sr = 0.1, \alpha = 3.75, Da = 0.9, z = 0.1. \)

Figure 20: Distribution of \( F^i \) vs. \( \phi \) for various values of \( \Omega \) and Da with \( \varepsilon = 0.2, \lambda_1 = 0.1, Rn = 2, \)
\( Pr = 2, Sc = 0.5, Sr = 0.1, q_2 = 0.5, Gr = 2, \alpha = 3.75, \)
\( \eta = 0.5, M = 1.1, z = 0.1. \)

Figure 21: Distribution of \( F^i \) vs. \( \phi \) for various values of \( \lambda_1 \) and \( \varepsilon \) with \( \Omega = 0.9, Rn = 2, Pr = 2, Sc = 0.5, \)
\( Sr = 0.1, Gr = 2, \alpha = 3.75, \eta = 0.5, M = 1.1, Da = 0.9, q_2 = 0.5, z = 0.1. \)
Figure 22: Distribution of $F_i$ vs. $q^2$ for various values of $Gr$ and $\eta$ with $\varepsilon = 0.2, \Omega = 0.9, \phi = 0.2, \lambda_1 = 0.1, Rn = 2, Pr = 2, Sc = 0.5, Sr = 0.1, \alpha = 3.75, M = 1.1, Da = 0.9, q^2 = 0.5, z = 0.1$.

Figure 23: Distribution of $F_i$ vs. $q^2$ for various values of $M$ and $Rn$ with $\varepsilon = 0.2, \Omega = 0.9, \phi = 0.2, \lambda_1 = 0.1, Pr = 2, Gr = 2, \eta = 0.5, Sc = 0.5, Sr = 0.1, \alpha = 3.75, Da = 0.9, q^2 = 0.5, z = 0.1$.

Figure 24: Distribution of $F^o$ vs. $\Phi$ for various values of $\Omega$ and $Da$ with $\varepsilon = 0.2, \lambda_1 = 0.1, Rn = 2, Pr = 2, Sc = 0.5, Sr = 0.1, q^2 = 0.5, Gr = 2, \alpha = 3.75, \eta = 0.5, M = 1.1, z = 0.1$.

Figure 25: Distribution of $F^o$ vs. $\Phi$ for various values of $\lambda_1$ and $\varepsilon$ with $\Omega = 0.9, Rn = 2, Pr = 2, Sc = 0.5, Sr = 0.1, Gr = 2, \alpha = 3.75, \eta = 0.5, M = 1.1, Da = 0.9, q^2 = 0.5, z = 0.1$.

Figure 26: Distribution of $F^o$ vs. $q^2$ for various values of $Gr$ and $\eta$ with $\varepsilon = 0.2, \Omega = 0.9, \phi = 0.2, \lambda_1 = 0.1, Rn = 2, Pr = 2, Sc = 0.5, Sr = 0.1, \alpha = 3.75, M = 1.1, Da = 0.9, z = 0.1$.

Figure 27: Distribution of $F^o$ vs. $q^2$ for various values of $M$ and $Rn$ with $\varepsilon = 0.2, \Omega = 0.9, \phi = 0.2, \lambda_1 = 0.1, Pr = 2, Gr = 2, \eta = 0.5, Sc = 0.5, Sr = 0.1, \alpha = 3.75, Da = 0.9, z = 0.1$. 
Figure 28: Streamlines in the wave frame for various values of $\epsilon = \{0.2, 0.225, 0.25\}$ at $\Omega = 0.9$, $Da = 0.9$, $\phi = 0.2$, $\lambda_1 = 0.1$, $Rn = 2$, $Pr = 2$, $q_2 = 0.5$, $Gr = 2$, $Sc = 0.5$, $Sr = 0.1$, $M = 1.1$, $\alpha = 3.75$, $\eta = 0.5$.

Figure 29: Streamlines in the wave frame for various values of $\Omega = \{0.3, 0.4, 0.5\}$ at $\epsilon = 0.2$, $Da = 0.9$, $\phi = 0.2$, $\lambda_1 = 0.1$, $Rn = 2$, $Pr = 2$, $q_2 = 0.5$, $Gr = 2$, $Sc = 0.5$, $Sr = 0.1$, $M = 1.1$, $\alpha = 3.75$, $\eta = 0.5$.

Figure 30: Streamlines in the wave frame for various values of $\phi = \{0.1, 0.125, 0.15\}$ at $\Omega = 0.9$, $\epsilon = 0.2$, $Da = 0.9$, $\lambda_1 = 0.1$, $Rn = 2$, $Pr = 2$, $q_2 = 0.5$, $Gr = 2$, $Sc = 0.5$, $Sr = 0.1$, $M = 1.1$, $\alpha = 3.75$, $\eta = 0.5$. 
Figure 31: Streamlines in the wave frame for various values of $\lambda_1 = \{0.07, 0.075, 0.08\}$ at $\phi = 0.2, \Omega = 0.9, \epsilon = 0.2, Da = 0.9, Rn = 2, Pr = 2, q_{2} = 0.5, Gr = 2, Sc = 0.5, Sr = 0.1, M = 1.1, \alpha = 3.75, \eta = 0.5$.

Figure 32: Streamlines in the wave frame for various values of $Rn = \{0.5, 0.6, 0.7\}$ at $\lambda_1 = 0.1, \phi = 0.2, \Omega = 0.9, \epsilon = 0.2, Da = 0.9, Pr = 2, q_{2} = 0.5, Gr = 2, Sc = 0.5, Sr = 0.1, M = 1.1, \alpha = 3.75, \eta = 0.5$.

Figure 33: Streamlines in the wave frame for various values of $Pr = \{1.5, 2, 2.5, 3\}$ at $\lambda_1 = 0.1, \phi = 0.2, \Omega = 0.9, \epsilon = 0.2, Da = 0.9, q_{2} = 0.5, Gr = 2, Sc = 0.5, Sr = 0.1, Rn = 2, M = 1.1, \alpha = 3.75, \eta = 0.5$. 
Figure 34: Streamlines in the wave frame for various values of $Gr = \{0.5, 0.75, 1\}$ at $\lambda_1 = 0.1, Rn = 2, \phi = 0.2, \Omega = 0.9, \varepsilon = 0.2, Da = 0.9, Pr = 2, q_2 = 0.5, Sc = 0.5, Sr = 0.1, M = 1.1, \alpha = 3.75, \eta = 0.5$.

Figure 35: Streamlines in the wave frame for various values of $M = \{1.3, 1.4, 1.5\}$ at $Gr = 2, Pr = 2, Rn = 2, \lambda_1 = 0.1, \varepsilon = 0.2, \phi = 0.2, Da = 0.9, \Omega = 0.9, q_2 = 0.5, Sc = 0.5, Sr = 0.1, \alpha = 3.75, \eta = 0.5$.

Figure 36: Streamlines in the wave frame for various values of $Da = \{1.1, 1.4\}$ at $M = 1.1, Gr = 2, Pr = 2, Rn = 2, \lambda_1 = 0.1, \varepsilon = 0.2, \phi = 0.2, \Omega = 0.9, q_2 = 0.5, Sc = 0.5, Sr = 0.1, \alpha = 3.75, \eta = 0.5$.
Figure 37: Streamlines in the wave frame for various values of $q2 = \{0.5, 0.52, 0.56\}$ at $Da = 0.9, M = 1.1, Gr = 2, Pr = 2, Rn = 2, \alpha_1 = 0.1, \varepsilon = 0.2, \phi = 0.2, \Omega = 0.9, Sc = 0.5, Sr = 0.1, \alpha = 3.75, \eta = 0.5$.

Figure 38: Streamlines in the wave frame for various values of $\alpha = \{3.1, 3.15, 3.2\}$ at $q2 = 0.5, Da = 0.9, M = 1.1, Gr = 2, Pr = 2, Rn = 2, \alpha_1 = 0.1, \varepsilon = 0.2, \phi = 0.2, \Omega = 0.9, Sc = 0.5, Sr = 0.1, \eta = 0.5, z = 0.1$. 
Figure 39: Streamlines in the wave frame for various values of $\eta = \{0.6, 0.65, 0.7\}$ at $\alpha = 3.75$, $q2 = 0.5$, $Da = 0.9$, $M = 1.1$, $Gr = 2$, $Pr = 2$ $Rn = 2$, $\lambda 1 = 0.1$, $\varepsilon = 0.2$, $\phi = 0.2$, $\Omega = 0.9$, $Sc = 0.5$, $Sr = 0.1$.

8. Concluding Remarks:

We briefly discuss the effect of different temperature on peristalsis MHD flow from a couple-stress Jeffrey fluid through the porous channel. Where we discussed the various parameters affecting the movement of the liquid and the pressure generated by the fluid movement, we list below the main points that we reached:

1. The velocity of the fluid increases with the increasing $\varepsilon$ and $\Omega$ when $r < 0.07$ and decreases otherwise.
2. The velocity of fluid decreases with the increasing $\eta$, $\lambda 1$, $\phi$, $Gr$, $q2$, $Rn$ and $Pr$ when $r < 0.2$ and increases otherwise.
3. The velocity of the fluid increases with the increasing $M$ and $Da$ when $r < 0.2$ and decreases otherwise.
4. The pressure variation $dp/dz$ increases with the increasing $\alpha$, $\eta$, $Gr$, $q2$, $M$ and $Da$, while $dp/dz$ decreases with the increasing $\varepsilon$, $\Omega$, $\lambda 1$, $\phi$, $Pr$ and $Rn$.
5. The pressure rise $\Delta p$ increases with the increasing $\eta$, $Gr$, $M$ and $Rn$, $\Delta p$ decreases with the increasing $\Omega$ and $Da$, while $\Delta p$ decreases with the increasing $\varepsilon$ and $\lambda 1$ when $\Theta < 0.03$, while $\Delta p$ increases with the increasing $\varepsilon$ and $\lambda 1$ when $\Theta > 0.03$.
6. The friction force at the wall $F^{I}$ and $F^{0}$ decreases with the increasing $\eta$, $Gr$, $M$ and $Rn$, $\Delta p$ increases with the increasing $\Omega$ and $Da$, while $\Delta p$ increases with the increasing $\varepsilon$ and $\lambda 1$ when $\Theta < 0.03$, and $\Delta p$ decreases with the increasing $\varepsilon$ and $\lambda 1$ when $\Theta > 0.03$.
7. The size of the trapped bolus decreases with the increasing $\varepsilon$, $\Omega$ and $Pr$ gradually in the middle of the channel while when we approach at the upper wall we notice the increase of the wave with the increasing $\varepsilon$, $\lambda 1$, $M$ and $q2$, respectively.
8. The size of the trapped bolus increases with the increasing $\Theta$, $Da$, $q2$ and $\eta$ in the middle of the channel while when we approach at the upper wall we notice the decrease of the wave with the increasing $\Theta$, $\Omega$, $Pr$, $Gr$, $Da$, $Rn$, $\alpha$ and $\eta$, respectively.

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