Thermal enhancement in the mixed convective flow of unsteady Carreau nanofluid with slip conditions: A numerical study

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Abstract
Nanofluids are formed by incorporating very small sized particles consist of metals, oxides, carbides, or carbon nano-tubes into base fluids like water, oil, ethylene glycol etc. Due to number of applications and amazing heat flow features of nanofluids, we are motivated to devote this article to explore the heat transform characteristics in transient flow of Carreau nanofluid over an inclined stretching cylinder. We have modeled the nonlinear mixed convection flow of Carreau nanoparticles in term of Lorentz force. Heat transfer mechanism in the flow is examined in view of nonlinear thermal radiation and non-uniform heat source/sink influences. Additionally, the effects of joule heating are also taken into account for examining the heat transport mechanism in the flow. Moreover, the effects of chemical reaction are also employed in concentration equation for investigation of mass transport phenomenon in the flow of nanofluid. To see the influence of involved physical parameters bvp4c numerical technique is employed. The numerical outcomes of physical parameters are assessed and depicted with logical discussion in the results and discussion section. The section of concluding remarks is designed to highlight the core findings of this study. It is revealed that the flow curves of nanofluid significantly grow up for escalating scales of nonlinear thermal convection constant. Moreover, an escalation is detected in the transport of thermal energy for growing scales of Eckert number and thermal radiation constant. Also, it is assessed that the flow velocity deteriorates by with an escalation in the magnitude of buoyancy force ratio parameter.

Keywords
Numerical solutions, transient flow, Carreau nanofluid, non linear thermal radiation, Joule heating, bvp4c technique

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Introduction
In various engineering systems, such as fuel cells and heat exchangers, fluids play an integral part in increasing the heat transfer rate. In addition, knowing that normal fluids have low thermal conductivity to improve the heat transfer rate, to get through this challenge, we need distinctive, high thermal conductivity fluids. They are called “nanofluids” for these special fluids. The word was first suggested by Choi. The main characteristics of nanofluids are that they have greater thermal...
conducivity. Many authors have based their studies on subjects Using nanofluid or normal fluid for thermal conductivity and viscosity. For this, Sheikholeslami et al. discussed the features of heat transfer of $Al_2O_3$-water nanofluid flow in a semi-annulus enclosure by employing Lattice Boltzmann method. Numerical investigations of mixed convection nanofluid flow over a lid driven cavity is driven by Selimendigil and Oztop. Qasim et al. discussed the phenomenon of slip flow of ferrofluid toward a stretching cylinder in the presence of Buongiorno’s model. Sheikholeslami et al. described the impact of MHD nanofluid flow between two rotating plates in the presence of nanoparticles. Hsiao discussed the radiative flow of nanofluid by utilizing mixed convection effects. Further, Dhanai et al. determined the dual branch solution of mixed convection nanofluid flow past an inclined cylinder in the presence of slippage effects. Hsiao numerically simulated the convective thermal transform in stagnation point flow of nanofluid with slip effects. Azam et al. numerically studied the behavior of stagnation point flow induced by an expanding cylinder in the presence of Brownian motion and thermophoresis effects. Hashim and Khan obtained the dual branch solutions of Carreau nanofluid induced by a shrinking cylinder. Hsiao also explored the characteristics of thermal extrusion in the flow on non-Newtonian nanofluid by considering the viscous dissipation effects. Tili et al. studied the mechanism of heat transfer in water-based nanofluids due to a horizontal circular cylinder by utilizing implicit finite difference scheme. By the utilization of chemical reactions the flow of nanofluid due to rotating frame was numerically inspected by Asma et al. Sohail and Naz explored the effects of Cattaneo-Christov theory on Sutterby nanofluid over a stretching cylinder. They examined that Brownian motion parameter boost the temperature field. Recently, Jamshed et al. computed the flow simulation of Casson nanofluid accelerated by stretching geometry. Mukhtar et al. incorporated the radiations effects to explore the heat flow features in the flow of Maxwell nanofluid. Some other recent studies on the investigation of nanofluid flows over different surfaces by utilizing several effects can be found in.

Many thermal engineering, such as space craft and satellite technology, rockets, nuclear power plants, rocket propulsion, hybrid solar power systems, and industrial processes, thermal radiation outcomes in heat transfer have a great deal of scope. Interestingly, for producing thermal devices, linear radiation is not sufficient. Therefore, for many technical processes, non-linear radiation produced by the Rosseland approximation is used extensively. The convective cooling and thermal radiation effects on magneto-nanofluid along a stretched surface have been investigated by Rashidi et al. In the presence of slip and thermal radiation, the bidirectional flow of magnetized material is studied by Hayat et al. and Lin et al. are developing transient thin film flow of pseudo-plastic nanofluid over a stretched surface. With the existence of radiation impacts, Abbas et al. investigated the silent consequences of slip flow past a curved surface. Mustafa et al. explored the rotating flow of magneto-water nanofluid in the existence of non thermal radiation due to the stretching sheet. Hayat et al. presents the thermal aspects of magnetized nanofluid against a stretching surface. They discovered that the rate of heat transfer was lifted by the parameter of thermal radiation. Shit et al. approached the optimization of entropy in magneo-nanomaterial across an exponentially extended surface having thermal radiation effects. Madhu et al. examined the effect of thermal radiation over some stretching surface on unsteady non-Newtonian nanofluid. In the presence of thermal radiation, the influence of mixed convection on stratified Casson nanofluid flow over a stretching surface is addressed by Imtiaz et al. The characteristics of non-Newtonian fluid flow over an elongated surface with thermal radiation effects were discussed in comparison by Sandep and Sulochana. In the presence of suction/injection and thermal radiation effects, Eid et al. numerically examined the MHD flow of Carreau nanofluid past a nonlinear stretched surface. Hamid et al. used the Galerkin method to analyze nanofluid rotating flow activity through a stretched surface with thermal radiation and variable effects of thermal conductivity. Ullah et al. employed radiation effects to analyze the flow properties of Carreau nanofluid flow over radially shrinking sheet. Recently Khan et al. explored the radiation impacts on MHD and reactive flow of Jeffery nanofluid. They utilized bvp4c technique and presented numerical solutions. Waqas et al. explored the non-linear radiation effects on the biconvection flow of cross nanofluid with additional physical effects. They reported that the temperature distribution of cross nanofluid escalates for varying values of radiation parameter.

Different transmission structures in essence where certain mass and heat transfer with heat radiation arises are the result of buoyancy effects induced by diffusion of chemical species and heat. This mode of analysis is beneficial for improving a number of chemical technologies, such as food processing and polymer recycling. The occurrence of pure air and water is probable. Any environmental mass can be merged or naturally present with it. The power of chemical reactions with viscosity analysis to predict the reaction efficiency is to provide the system with a mathematical model. The study of chemical reaction, thermal, and mass transfer with radiation emitted is of considerable status in the chemical and thermochemical industries. It is feasible to promulgate processing of mineral as either
homogenous or heterogeneous structures. This depending on whether a single-phase volume reaction or an interface supposedly happened. Krishnamurthy et al.\textsuperscript{38} review the effect of melting and chemical reaction on Williamson nanofluid flow. The process of heat transport along the cylinder in the case of a chemical reaction became explored by Pal and Mondal.\textsuperscript{39} The affect of chemical reactions on nanofluid flow generated by a stretching cylinder was explored by Ramzan et al.\textsuperscript{40} They assessed that the chemical reaction parameter improves the thickness of the boundary layer. The nature of chemically reacting material in unsteady flow of Williamson fluid with nanoparticles was studied numerically by Hamid et al.\textsuperscript{41} In fact, Alshomrani et al.\textsuperscript{42} are evaluating the chemically reacting flow of Oldroyd-B fluid past a stretching cylinder. The combined effects of thermal slip and chemical reaction on non-Newtonian fluid on a stretched cylindrical surface in the presence of nanoparticles were determined by Tili et al.\textsuperscript{43} Zuhra et al.\textsuperscript{44} investigated the nanofluids film flow under the effects of cubic autocatalysis chemical reaction. Iqbal et al.\textsuperscript{45} analyzed the impacts of binary chemical reactions on the mass transport in flow of Carreau nanofluid and presented numerical solutions. Recently, Humane et al.\textsuperscript{46} explored the properties of chemical reaction on Casson-Williamson nanofluid flow. The process of heat transport is investigated. We further consider the ohmic radiation and mixed convection over the flow and heat transfer is examined. We also imposed a non uniform heat source sink with joule heat generation, and concentration of the fluid is kept constant at the surface of the cylinder. Finally, the stretching velocity of the cylinder is taken to be $u_w(x,t) = \frac{a}{c}t$, where $a$ and $c$ are constants.

By employing the theory of boundary layer approximation, we get the following equations.\textsuperscript{13,33,47}

\textbf{Continuity equation:}

$$\frac{\partial(ru)}{\partial x} + \frac{\partial(rv)}{\partial r} = 0. \quad (1)$$

\textbf{Momentum equation:}

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial r} = \frac{v}{r} \frac{\partial}{\partial r} \left[ 1 + \Gamma^2 \left( \frac{\partial u}{\partial r} \right)^2 \right] \frac{\partial^2 u}{\partial r^2}$$

$$+ \nu \Gamma^2 \frac{\partial^2 u}{\partial r^2} \left[ 1 + \Gamma^2 \left( \frac{\partial u}{\partial r} \right)^2 \right] \frac{\partial u}{\partial r}$$

$$- \frac{\sigma B^2(t)}{\rho} u + \nu(n-1) \Gamma^2 \frac{\partial u}{\partial r} \left[ 1 + \Gamma^2 \left( \frac{\partial u}{\partial r} \right)^2 \right] \frac{\partial^2 u}{\partial r^2}$$

$$+ g \left[ \Lambda_1(T - T_w) + \Lambda_2(T - T_w)^2 \right]$$

$$+ g \left[ \Lambda_3(C - C_w) + \Lambda_4(C - C_w)^2 \right] \cos \Phi. \quad (2)$$

\textbf{Development of mathematical model}

In this work an unsteady, incompressible flow of Carreau nanofluid over an inclined permeable stretching cylinder which makes an angle $\alpha$ with horizontal axis is examined. The impacts of non linear thermal radiation and mixed convection over the flow and heat transfer is investigated. We further consider the ohmic heating, velocity and thermal slip conditions on the surface of the cylinder. The flow phenomenon is described by depicting the geometry of the flow in Figure 1. To examine the flow analysis, we considered the cylindrical coordinates $(x, \theta, r)$ in such an arrangement that cylinder is stretched in $x$ direction while $r$ axis is considered as normal to the stretching. We also imposed a non uniform magnetic field with strength $B(t) = \frac{B_0}{\sqrt{1 - \epsilon t}}$ in such a way that cylinder is normal to it, where $B_0$ is a constant. Also the temperature $T_w$ and concentration $C_w$ of the fluid is kept constant at the surface of the cylinder.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{flow_geometry.png}
\caption{Flow geometry and coordinates system.}
\end{figure}

Energy equation:
\[
\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + \frac{\partial T}{\partial y} \left( \frac{\partial T}{\partial y} \right) = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) - k_s (C - C_w).
\] (3)

Concentration equation:
\[
\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = \frac{D_B}{r} \frac{\partial}{\partial r} \left( r \frac{\partial C}{\partial r} \right) - \frac{q}{\rho c_p} + \frac{1}{\rho c_p} \frac{\partial T}{\partial y}.
\] (4)

The physical realistic boundary conditions are
\[
u = \nu_0(x,t) + \nu_{slip}, \quad v = 0, \quad T = T_w + T_{slip},
\]
\[C = C_w \text{ at } r = R,
\]
\[u = 0, \quad T \rightarrow T_{\infty}, \quad C \rightarrow C_{\infty} \text{ at } r \rightarrow \infty.
\] (5, 6)

where
\[u_{slip} = \frac{M u_0}{\mu} \left[ 1 + \left( \frac{\partial u}{\partial r} \right)^2 \right]^{1/2}.
\] (7)

and the relation for non-uniform heat rise/fall is as follows
\[q'' = \frac{k u_0}{x_0} [A_s(T_w - T_0) + B_s(T - T_0)].
\] (8)

While the relation for radiative heat flux is given as
\[q_r = -\frac{16\sigma T^3}{3k} \frac{\partial T}{\partial y}.
\] (9)

where space and temperature dependent heat rise/fall is represented by \(A_s\) and \(B_s\), respectively. The case \(A_s > 0\) and \(B_s > 0\) express internal heat rise while \(A_s < 0\) and \(B_s < 0\) shows internal heat fall. Also \((k^*, \sigma^*)\) signifies the constant of mean absorption and Stefan-Boltzmann constant, respectively and other are mentioned in the nomenclature section.

We have similarity transformations (Iqbal et al.\(^{45}\))
\[\eta = \frac{v - R^2}{2R} \frac{u_0}{\mu} \sqrt{\frac{\nu}{T_{in}}} \sqrt[3]{Rf(\eta)},
\]
\[\theta(\eta) = \frac{T - T_{in}}{T_{in} - T_w}, \phi(\eta) = \frac{C - C_{in}}{C_w - C_{in}}.
\] (10)

The stream function \(\psi(r,\chi)\) is given by \(u = \frac{1}{r} \frac{\partial \psi}{\partial \chi}\) and \(v = -\frac{1}{r} \frac{\partial \psi}{\partial \chi}\), and by employing conversions in equation (10) in equations (1)-(4) we see that equation (1) is automatically satisfied while equations (2)-(4) take the following form
\[1 + 2\gamma \eta \left[ 1 + n We \left( f'' \right)^2 \right] \left[ 1 + We \left( f'' \right)^2 \right]^{1/2} \left( f'' \right)^{1/2} + \left( f'' \right)^2 - M^2 f'' - A \left( f'' + \frac{\eta}{2} \right)^2
\] (11)

\[1 + 2\gamma \eta \left( 1 + (1 + \beta, \theta) \theta + \Nt, (1 + \beta_2, \phi) \phi \right) \cos P = 0,
\]
\[\left[ (1 + R_d(1 + (1 - \theta - 1) \theta) \theta' (1 + 2\gamma \eta)) \right] + \Pr f f' + \Pr (Q'f') + \Pr (\theta)
\] (12)

\[+ \Pr(1 + 2\gamma \eta) (N \theta 
\]
\[\left\{ \begin{array}{l}
\theta(0) = 1 + \theta(0), \phi(0) = 1
\end{array} \right.
\] (13)

and by utilizing equation (10), boundary conditions in equations (5)-(6) are transformed as:
\[f(0) = 0, \quad f'(0) = 1 + \delta f'(0) \left( 1 + We \frac{f''(0)}{f''(0)} \right)^{1/2}, \quad \theta(0) = 1 + \delta \theta'(0), \phi(0) = 1
\] (14)

\[f'(\infty) \rightarrow 0, \theta(\infty) \rightarrow 0, \phi(\infty) \rightarrow 0.
\] (15)

Here, the local Weissenberg number \(We\), magnetic parameter \(M\), thermophoresis parameter \(N_t\), curvature parameter \(\gamma\), buoyancy ratio parameter \(N_b\), non linear thermal convection variable \(B_s\), mixed convection parameter \(R_d\), Brownian motion parameter \(N_b\), Eckert number \(Ec\), Prandtl number \(Pr\), Grasshopper number \(Gr_s\), Schmidt number \(Sc\), unsteadiness parameter \(A_s\), nonlinear convection variable for concentration \(B_c\), temperature ratio parameter \(\theta_0\), space dependent heat source/sink parameter \(Q'\), radiation parameter \(R_d\) and chemical reaction parameter \(\gamma_1\) are defined as follows:
\[We = \frac{\left( \frac{a^2 \lambda^2 \nu^2 \Gamma^2}{(1 - \nu^2)} \right)}{R^2}, \quad \gamma = \frac{\left( \frac{\nu(1 - ct)}{a R^2} \right)}{R^2}.
\]
\[N_t = \frac{\tau D_r (T_w - T_{in})}{\nu T_{in}}, \quad Pr = \frac{\mu c_p}{\nu},
\]
\[Gr_s = \frac{Gr_s}{Re_s}, \quad Gr_s = \frac{g \Lambda_1 (T - T_{in})}{\nu^3},
\]
\[R_d = \frac{Gr_s}{Re_s}, \quad N_t = \frac{Gr_s}{Re_s}, \quad A = \frac{c}{a},
\]
\[N_b = \frac{\tau D_b (C_w - C_{in})}{\nu}, \quad \beta_i = \frac{\beta_i}{\frac{A_s}{A_3}} \frac{T_w - T_{in}}{T_w - T_{in}},
\]
\[\beta_c = \frac{\Lambda_4 (C_w - C_{in})}{\Lambda_1}, \quad Sc = \frac{\nu}{\Lambda_3} \frac{\theta_0}{\frac{A}{A_3}} \frac{T_w - T_{in}}{T_w - T_{in}},
\]
\[Q' = \frac{A_s}{\nu (\rho c_p)}, \quad M = \left( \frac{\sigma B_0^2}{\rho a} \right)^{1/2}, \quad Q = \frac{B_s}{\alpha n (\rho c_p)},
\]
\[\gamma_1 = \frac{k_1}{U_0}, \quad R_d = \frac{16 a^3 T_{in}^3}{3k k^*}.
\] (16)
The skin friction and heat and mass transport coefficients are given by:

\[
YY_2 = \frac{-3R_d(1 + 2\gamma)(\theta_w - 1)^3(Y_4)^2(Y_3)^2 - 3R_d(1 + 2\gamma)(\theta_w - 1)(Y_3)^2 - 2\gamma Y_5 - 6R_d(1 + 2\gamma)(\theta_w - 1)(Y_4)^2(Y_3) - 2R_d\gamma Y_5 - 2\gamma R_d(\theta_w - 1)^3(Y_4)^2(Y_3) - 6\gamma R_d(\theta_w - 1)^2(Y_4)^3(Y_3) - 6\gamma R_d(\theta_w - 1)(Y_4)(Y_3)^2 - Pr M^2 Ec(y_2)^2 - Pr Y_1 Y_5}{(1 + 2\gamma)(1 + R_d + R_d(\theta_w - 1)^3(Y_4)^3 + 3R_d(\theta_w - 1)(Y_3) + 3R_d(\theta_w - 1)^2(Y_4)^2)}
\]

\[
YY_3 = \frac{-2\gamma Y_7 - Sc Y_7 Y_1 + Sc(\gamma Y_6 - (\frac{Nu}{Re})^2)(1 + 2\gamma)YY_2 + 2\gamma Y_1 Y_5}{(1 + 2\gamma)}
\]

subject to the boundary conditions

\[
Y_1(0) = 0, Y_2(0) = 1 + \delta_1 Y_3(0)
\]

\[
\{1 + \gamma_2 Y_2(0)\frac{\partial Y_2}{\partial \gamma}(0), Y_4(0) = 1 + \delta_2 Y_5(0), Y_6(0) = 1
\]

\[
Y_5(\infty) \rightarrow 0, Y_4(\infty) \rightarrow 0, Y_6(\infty) \rightarrow 0.
\]

**Validation of numerical code**

The numerical computations of \( f'(1) \) for value of unsteadiness parameter \( A \) and skin friction coefficient \( Re^{1/2}C_f \) for various scales of curvature parameter \( \gamma \) are compared with those of previously published studies and depicted in Tables 1 and 2 which show that the current study in reduced case is in good agreement with them and the bvp4c is valid numerical scheme.

**Results and discussion**

In this section we have employed bvp4c numerical technique to solve the ordinary differential equations (8)–(10) with corresponding boundary conditions mentioned in equations (11) and (12). The effects of all physical parameters on flow, thermal and concentration profiles are presented graphically through Figures 2 to 23. We fixed the default values for leading parameters such as \( A = 0.05, M = 0.2, R_l = 0.1, R_d = 0.3, We = 0.1, Bi = 1.0, \beta_c = 0.5, \gamma = 0.1, \gamma_1 = 0.1, Ec = 0.3, Nt = 0.2, Nc = 0.1, Nr = 0.3, Sc = 0.1, \theta_w = 3.0, \delta_1 = 0.8, \delta_2 = 0.8, Pr = 2.0, Q = 0.8, Q' = 1.0, \) and \( n = 0.5 \) during the entire computations.

**Table 1.** The numerical values of \( f'(1) \) for \( A = 0, Re = n = 1.0, We = R_l = \beta_c = \beta = N_t = M = 0. \)

| A  | Fang et al. | Khan et al. | Present study |
|----|-------------|-------------|---------------|
| 0  | -1.17775    | -1.17884    | -1.17769      |
Flow characteristics of Carreau nanofluid

Figures 2 to 9 disclose the flow features of Carreau nanofluid against varying magnitudes of \( \text{We} \), \( \text{A} \), \( \text{M} \), \( \text{Nr} \), \( \text{Ri} \) for both cases of flat plate \( (\gamma = 0) \) and cylinder \( (\gamma = 1) \). Figure 2 illustrates the influence of Weissenberg number \( \text{We} \) on flow curves of Carreau nanofluid. It is observed that the flow curves of nanofluid and the velocity thickness of the boundary layer significantly decline for developing amount of \( \text{We} \). Physically, Weissenberg number is defined as the ratio of relaxation time and specific process time and has direct relation with fluid relaxation time. Hence, larger Weissenberg number leads to enhance the relaxation time of fluid and consequently the fluid behaves like more viscous. Due to increase in viscous effects the drag force increases due to which fluid velocity diminishes. Figure 3 discloses the impact of unsteadiness parameter \( \text{A} \) on velocity profile of radiative Carreau nanofluid. The diminution in the flow curves of nanofluid is detected for varying scales of \( \text{A} \). These conclusions make sense physically, because the stretching coefficient \( (\alpha) \) has inverse relation with unsteadiness parameter \( \text{A} \). Thus for developing magnitude of \( \text{A} \), the amount of stretching parameter declines and therefore the velocity of the fluid depreciates. The impact of velocity slip parameter \( (\delta_1) \) on flow distribution of Carreau nanofluid is depicted in Figure 4. It has been noticed that the momentum boundary layer thickness and velocity of the nanofluid depreciates for larger degree of \( \delta_1 \). Actually, larger velocity slip parameter leads to rise in slip velocity and therefore, fluid velocity depreciates. It happens because, when slip condition arises, the velocity of the stretching cylinder will not remains same as the flow velocity near the

**Table 2.** Comparison of numerical computations of skin friction coefficient \( \text{Re}^{1/2} \text{Cf} \) with published studies\(^{49,50}\) in limiting case \( \text{M} = \text{A} = \text{We} = \text{Ri} = \text{Nr} = 0 \) and \( n = 1 \) for various scales of \( \gamma \).

| \( \gamma \) | Rangi and Ahmad\(^{49}\) | Hashim et al.\(^{50}\) | Present study |
|-------------|----------------|----------------|--------------|
| 0.0         | -1.0000        | -1.0000        | -1.000000    |
| 0.25        | -1.094378      | -1.094373      | -1.094365    |
| 0.5         | -1.188715      | -1.188727      | -1.188741    |
| 0.75        | -1.281833      | -1.281819      | -1.281812    |
| 1.0         | -1.459308      | -1.453373      | -1.453367    |

Figure 2. Illustration of \( f(\eta) \) for varying \( \text{We} \).

Figure 3. Illustration of \( f(\eta) \) for varying \( \text{A} \).

Figure 4. Illustration of \( f(\eta) \) for varying \( \delta_1 \).
cylinder. Figure 5 demonstrates the characteristics of velocity distribution under the influence of magnetic field \( M \). It is concluded from this plot that the velocity of the fluid deteriorates by increasing the magnetic field effect. This is because Lorentz force resists the fluid motion. To analyze the behavior of non linear thermal convection variable \( \beta_t \) against velocity profile Figure 6 is plotted. It is seen from this Figure. that velocity profile ascends for increasing scales of \( \beta_t \). In fact buoyancy force enhances with an increase in mixed convection variable which results in enhancement of velocity profile. To scrutinize the influence of buoyancy force ratio parameter \( N_r \) on flow curves of nanofluid, Figure 7 is plotted. From this Fig. it is assessed that the velocity of the fluid diminishes with an augmentation in the magnitude of \( N_r \). Figure 8 displays the characteristics of velocity distribution against the effect of buoyancy force parameter \( R_i \). It reveals that the velocity curves of the fluid grow for escalating scale of \( R_i \). As the buoyancy force has prevalent effects over viscous forces for greater scales of \( R_i \). Hence, the mixed convection variable is responsible to make the fluid flow more faster, due to this reason flow distribution of the fluid builds up. Moreover, the effect of non linear mixed convection variable for concentration \( \beta_c \) on velocity curves of nanofluid is depicted in Figure 9. It is noticed that the flow curves of Carreau nanofluid depict ascending trend for developing scales of \( \beta_c \). It is due to the buoyancy force effects.

**Heat transfer aspects of nanofluid**

Figures 10 to 17 are being depicted to inspect characteristics of heat transfer for pertinent scales of \( R_i, \theta_w, Q, A, Ec, N_t, R_d, \) and \( Pr \) for both cases of flat plate \((\gamma = 0)\) and cylinder \((\gamma = 1)\). The influence of buoyancy force parameter \( R_i \) on thermal curves of nanofluid are depicted in Figure 10. From this graph it is examined that the heat transform curves and thermal thickness of boundary layer depreciate with augmentation in the magnitude of \( R_i \). Figure 11 is plotted to study the impact of temperature ratio parameter \( \theta_w \) against thermal distribution of nanofluid. Clearly, it is perceived that the heat transform in the flow rises and associated layer thickness grows for enhancing values of \( \theta_w \). These outcomes are according to our expectations, as for higher magnitude of \( \theta_w \) the thermal state of
nanoliquid rises which leads to strengthen the thermal curves of nanofluid. To disclose the characteristics of temperature distribution for heat generation parameter $Q$, Figure 12 is plotted. It is scrutinized that the higher values of $Q>0$ builds up the thermal distribution of nanofluid. The opposite situation is observed in heat absorption case ($Q<0$). It physically make sense that, obviously when more amount of heat is generated then temperature profile boost up and when heat is absorbed then temperature profile is depreciated. Figure 13 ensures the behavior of unsteadiness parameter $A$ regarding temperature field. It is observed that the temperature of the field and associated thickness of the layer reduces by an augmentation in the extent of $A$. Figure 14 exposes the enactment of thermophoresis parameter $N_t$ against temperature distribution. From this Figure it is assessed that the temperature of the field as well as the thermal thickness of the layer improve with an increment in the scale of thermophoresis parameter $N_t$. These findings relate with physical process of thermophoresis which elaborates the movement of fluid particles from warm space to cold corners hence, the fluid temperature boosts up. Figure 15, elucidate the characteristics of radiation parameter $R_d$
against temperature contours of Carreau fluid. An ascending trend of thermal curves is being assessed for increasing amount of $R_d$. Physically, more heat is produced when we intensify the radiations of the heat within the liquid which leads to augment the thermal curves of fluid. Figure 16, discloses the characteristics of temperature field against the Prandtl number ($Pr$). Depreciation in temperature is noticed for larger scales of Prandtl number. It occurs because of weaker thermal diffusivity. Since, thermal diffusion coefficient is inversely proportional to Prandtl number. Figure 17, discloses the impact of Eckert number ($Ec$) on thermal distribution of nanofluid. It is explored that temperature curves of nanofluid augment for improving degrees of $Ec$. Eckert number is the parameter which measures the loss of energy during flow configuration. It elaborates the relation among enthalpy difference and kinetic energy of the liquid particles. By the definition of Eckert number we seen that for its greater values temperature difference is reduces at the surface of cylinder. Physically because of frictional heating an extra amount on kinetic energy is stored in fluid particles, due to this reason boundary layer boosts for greater number of $Ec$. 

**Figure 12.** Illustration of $\theta(\eta)$ for varying $Q$.

**Figure 13.** Illustration of $\theta(\eta)$ for varying $A$.

**Figure 14.** Illustration of $\theta(\eta)$ for varying $Nt$.

**Figure 15.** Illustration of $\theta(\eta)$ for varying $Rd$. 

$Q = 1.0, 3.0, 5.0$

$Nt = 0.1, 0.3, 0.6$

$A = 0.1, 0.5, 0.9$

$Rd = 0.1, 0.3, 0.5$
Solutal aspects of nanofluid

Figures 18 to 20 are being sketched to examine the concentration distribution for pertinent magnitudes of Sc, Nt, and γ only for both cases of flat plate (γ = 0) and cylinder (γ = 1). Figure 18 depicted the effects of Schmidt number (Sc) for the concentration distribution of gasses hydrogen, helium, water vapors and oxygen. The estimation of Sc are taken 1.0, 3.0, 5.0, and it is concluded that concentration of the fluid depreciated by increase in Schmidt number. These findings are identical to the physical phenomenon, because Schmidt number is the ratio of mass diffusivity and momentum diffusivity (kinematic viscosity). It is the dimensionless number which describes liquid flows. Increase in Sc means decrease in molecular diffusivity and hence, the solutal transport of energy reduces. Therefore, the concentration of species is lesser for growing values of Sc. Hence presence of Schmidt number in the system explicitly modifies concentration profile throughout the region. Figure 19 is being designed to envision the solutal features of nanofluid under the influence of thermophoresis parameter (Nt). The ascending behavior of concentration distribution is noted for higher scales of...
These outcomes are identical with physical happening that away from the stretching cylinder a very high speed flow is created by the thermophoretic force which is generated due to change in thermal state of the system and as a consequence temperature declines and nano particles volume fraction increases for varying value of $N_t$. Figure 20 discloses the impact of chemical reaction parameter $g_1(\cdot)$ against concentration profile. It is analyzed that the solutal energy transport reduces for augmenting degrees of $g_1$. Some amount of energy is looses during chemical reaction that why this situation creates.

Behavior of skin friction, Nusselt number, and Sherwood number profiles

In this section we have computed the physical quantities like skin friction, rate of heat transport constant, and the rate of mass transport coefficient and depicted in Figures 21 to 23. The variation in skin friction coefficient ($\Re^{1/2}C_f$) in combination with magnetic parameter ($M$) for growing values of non linear thermal convection variable ($\beta_t$) have been demonstrated in Figure 21. A significant falloff in the skin friction coefficient is noticed for larger non linear thermal
convection variable. It physically implies that the higher magnitude of $\beta$, deteriorates the shear stress at the wall of the cylinder. The variation in dimensionless Nusselt number $(Re^{-1/2}Nu)$ incorporated with temperature ratio parameter $(\theta_w)$ is plotted in Figure 22 for different values of Eckert number $(Ec)$. It is seen that that magnitude of Nusselt number grows up with an escalation in Eckert number. The variation in dimensionless Sherwood number $(Re/C0)$ with temperature ratio parameter, unsteadiness parameter and temperature ratio parameter depreciate the nanoparticles volume fraction profile of nanofluid. It is noticed that the magnitude of Sherwood number is increased for pertinent values of $Nt$.

**Concluding remarks**

The main results of current study are mentioned below

- It is analyzed that the flow velocity of reactive nanofluid deteriorate for augmenting scales of slip parameter, Weissenberg number, and unsteadiness parameter while the larger mixed convection parameter and thermal convection parameter enhances the flow velocity.
- An escalation in radiation parameter and in Eckert number lead to augment the temperature distribution of nanofluid.
- A raise in Schmidt number and chemical reaction parameter depreciate the nanoparticles volume fraction profile of nanofluid.
- The friction coefficient was significantly decreased by larger non linear thermal convection variable.
- The Nusselt number raises for larger scales of Eckert number and temperature ratio parameter.
- A raise is in mass transfer rate is noted by increasing thermophoretic force parameter.
- The rate of heat transport in the flow is promoted by the magnifying scales of heat source parameter, unsteadiness parameter and temperature ratio parameter.

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Appendix

Notation

- $u$, $v$: velocity components (ms$^{-1}$)
- $x$, $r$: cylindrical coordinates (ms$^{-1}$)
- $T$: fluid temperature (K)
- $\mu$: generalized Newtonian viscosity (kg m$^{-1}$s$^{-1}$)
- $T_w$: ambient temperature (K)
- $C$: fluid concentration
- $c_p$: effective heat capacity (JK$^{-1}$ kg$^{-1}$)
- $C_w$: ambient concentration
- $D_B$: Brownian diffusion coefficient (m$^2$s$^{-1}$)
- $D_T$: thermophoresis diffusion coefficient (m$^2$s$^{-1}$)
- $u_w$: stretching sheet velocity (ms$^{-1}$)
- $k$: thermal conductivity (W/MK)
- $B_0$: magnetic field
- $f$: dimensionless stream function
- $N_b$: Brownian motion parameter
- $N_t$: thermophoresis parameter
- $k_s$: mean absorption coefficient
- $\phi$: dimensionless concentration function
- $We$: local Weissenberg number
- $Q^*$: space dependent heat source/sink
- $\sigma^*$: Stefan-Boltzmann constant
- $(Q)$: temperature dependent heat source/sink
- $\mu_\infty$: infinite shear viscosity
- $T_s$: surface temperature (K)
- $\Gamma$: relaxation time (s)
- $Re$: local Reynolds number
- $\rho$: surface heat flux
- $\rho$: fluid density (kg m$^{-3}$)
- $\alpha_m$: thermal diffusivity (m$^2$s$^{-1}$)
- $\kappa_r$: reaction rate $rac{1}{T}$
- $C_f$: skin friction coefficient
- $Pr$: Prandtl number
- $\gamma$: magnitude of deformation rate
- $\nu$: kinematic viscosity (m$^2$s$^{-1}$)
- $\psi$: stream function
- $T_w$: surface shear stress
- $\theta$: dimensionless temperature
- $\gamma_1$: chemical reaction parameter
- $Sc$: dimensionless similarity variable
- $(\rho c)_b$: heat capacity of the base fluid
- $\tau$: parameter defined as $\frac{(\rho c)_b}{(\rho c)_f}$
- $\alpha, c$: constants
- $Nu$: local Nusselt number
- $\gamma$: curvature parameter
- $\mu_0$: zero shear viscosity