How dark is the $\nu_R$-philic dark photon?

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ABSTRACT: We consider a generic dark photon that arises from a hidden $U(1)$ gauge symmetry imposed on right-handed neutrinos ($\nu_R$). Such a $\nu_R$-philic dark photon is naturally dark due to the absence of tree-level couplings to normal matter. However, loop-induced couplings to charged leptons and quarks are inevitable, provided that $\nu_R$ mix with left-handed neutrinos via Dirac mass terms. We investigate the loop-induced couplings and find that the $\nu_R$-philic dark photon is not inaccessibly dark, which could be of potential importance to future dark photon searches at SHiP, FASER, Belle-II, LHC 14 TeV, etc.
1 Introduction

Right-handed neutrinos ($\nu_R$), albeit not included in the Standard Model (SM), are a highly motivated dark sector extension to accommodate neutrino masses [1–5], dark matter [6–8], and baryon asymmetry of the universe [9]. Being intrinsically dark, $\nu_R$ might have abundant new interactions well hidden from experimental searches. In particular, it is tempting to consider the possibility that there might be a hidden gauge symmetry in the $\nu_R$ sector [10–14]. The new gauge boson arising from this symmetry does not directly couple to other fermions except for $\nu_R$ and naturally becomes a dark photon, which we referred to as the $\nu_R$-philic dark photon.

The $\nu_R$-philic dark photon is not completely dark. It may interact with normal matter via kinetic mixing [15], provided that the new gauge symmetry is Abelian; or, in the
presence of mass terms connecting $\nu_R$ and left-handed neutrinos $\nu_L$, via one-loop diagrams containing $W^{\pm}/Z$ and neutrinos. In the former case, the strength of dark photon interactions with quarks or charged leptons depends on the kinetic mixing parameter $\epsilon$ in $\mathcal{L} \supset \frac{\epsilon}{2} F^{\mu\nu} F'^{\mu\nu}$ where $F^{\mu\nu}$ and $F'^{\mu\nu}$ are the gauge field tensors of the SM hypercharge $U(1)_Y$ and the new $U(1)$, respectively. This case, being essentially independent of the neutrino sector, has been widely considered in a plethora of dark photon studies—for a review, see [16–19]. In the latter case, the loop-induced couplings depends on neutrino masses and mixing, and will be investigated in this work.

The aim of this work is to address the question of how dark the $\nu_R$-philic dark photon could be in the regime that dark-photon-matter interactions dominantly arise from $\nu_L$-$\nu_R$ mixing instead of kinetic mixing. We note here that the dominance might be merely due to accidentally small $\epsilon$, or due to fundamental reasons such as the SM $U(1)_Y$ being part of a unified gauge symmetry [e.g. $SU(5)$] in grand unified theories. We opt for a maximally model-independent framework in which generic Dirac and Majorana mass terms are assumed. The loop-induced couplings are UV finite as a consequence of the orthogonality between SM gauge-neutrino couplings and the new ones. Compared to our previous study on loop-induced $\nu_R$-philic scalar interactions [20], we find that the couplings in the vector case are not suppressed by light neutrino masses, and might be of potential importance to ongoing/upcoming collider and beam dump searches for dark photons.

The paper is organized as follows: In Sec. 2, we describe the relevant Lagrangian used in this work, reformulate neutrino interactions in the mass basis, and discuss generalized matrix identities for UV divergence cancellation for later use. In Sec. 3, we first derive model-independent expression for effective coupling of $Z'$ to charged leptons/quarks through one-loop diagram involving $Z$ and $W$ bosons, respectively. We then evaluate the coupling strength in three different examples. In Sec. 4, we present a qualitative discussion about possible connection between the $U(1)_R$ gauge coupling and the mass of $Z'$. In Sec. 5, we present constraints from a vast array of current and future experiments spanning from collider searches to astrophysical phenomena. We finally conclude in Sec. 6 with details of one-loop diagram calculations relegated to Appendix A.

2 Framework

We consider a hidden $U(1)$ gauge symmetry, denoted by $U(1)_R$, imposed on $n$ right-handed neutrinos. The gauge boson of $U(1)_R$ in this work is denoted by $Z'$. The relevant part of the Lagrangian for the $U(1)_R$ extension reads\(^1\):

\[
\mathcal{L} \supset \nu^\dagger_{Rj} \bar{\sigma}_\mu \gamma^\mu \nu_{Rj} + \left[ \frac{(M_R)_{ij}}{2} \nu_{Ri} \nu_{Rj} + (m_D)_{\alpha j} \nu_{L\alpha} \nu_{Rj} + \text{h.c.} \right] - \frac{1}{4} F'_{\mu\nu} F'^{\mu\nu} + \frac{1}{2} m_{Z'}^2 Z'_\mu Z'^{\mu},
\]

\(^1\)Throughout the main text we exclusively use Weyl spinors, while in the Appendix we use Dirac/Majorana spinors for loop calculations.
where \( \alpha = e, \mu, \tau; (i, j) = 1, 2, 3, \cdots, n; \) and
\[
D_j^\mu = \partial_\mu - ig_R Q_R,j Z^\mu_{(1)}. \tag{2.2}
\]

Here \( g_R \) is the gauge coupling constant of \( U(1)_R \) and \( Q_{R,j} \) is the charge of \( \nu_{R,j} \) under \( U(1)_R \).

Note that for most general forms of \( M_R \) and \( m_D \), both the Majorana and Dirac mass terms in Eq. (2.1) break the \( U(1)_R \) symmetry. In addition, for arbitrary charge assignments of \( \nu_{R,j} \) under \( U(1)_R \), the model would not be anomaly free. Nevertheless, one can construct complete models in which \( M_R \) and \( m_D \) arise from spontaneous symmetry breaking and the cancellation of anomalies can be obtained when several \( \nu_{R,j} \)’s have different charges with \( \sum_j Q^2_{R,j} = 0 \)—see the example in Sec. 3.2. In this section we neglect these model-dependent details and focus on the general framework proposed in Eq. (2.1).

The Dirac and Majorana neutrino mass terms in Eq. (2.1) can be framed as
\[
\mathcal{L}_{\nu\text{mass}} = \frac{1}{2}(\nu^T, \nu_R) \begin{pmatrix} 0_{3 \times 3} & m_D \\ m_D^T & M_R \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R \end{pmatrix}, \tag{2.3}
\]
where \( \nu_L = (\nu_{L,e}, \nu_{L,\mu}, \nu_{L,\tau})^T \) and \( \nu_R = (\nu_{R,1}, \nu_{R,2}, \cdots)^T \) are column vectors. The entire mass matrix of \( \nu_L \) and \( \nu_R \) can be diagonalized by a unitary matrix \( U \):
\[
\begin{pmatrix} \nu_L \\ \nu_R \end{pmatrix} = U \begin{pmatrix} \nu_{1,2,3} \\ \nu_{4,5,...} \end{pmatrix}, \quad U^T \begin{pmatrix} 0_{3 \times 3} & m_D \\ m_D^T & M_R \end{pmatrix} U = \begin{pmatrix} m_{1,2,3} \\ m_{4,5,...} \end{pmatrix}. \tag{2.4}
\]

Here \( \nu_i (i = 1, 2, \cdots, n + 3) \) denote neutrino mass eigenstates, with \( m_i \) being the corresponding masses. We refer to the basis after the \( U \) transformation as the \textit{chiral basis}, and the one before the transformation as the \textit{mass basis}.

In order to facilitate loop calculations, we need to transform neutrino interaction terms from the chiral basis to the mass basis. In the chiral basis, we have the following neutrino interaction terms:
\[
\mathcal{L} \supset \left[ \frac{\sqrt{2}}{2} W_{\mu}^\dagger \ell_{L,\alpha} \gamma^{\mu} \nu_{L,\alpha} + \text{h.c.} \right] + \frac{g}{2c_W} Z_{\mu}^\dagger \nu_{L,\alpha} \gamma^{\mu} \nu_{L,\alpha} + g_R Q_{R,j} Z_{\mu}^\dagger \nu_{R,j} \gamma^{\mu} \nu_{R,j}, \tag{2.5}
\]
where the first three terms are the SM charged and neutral current interactions, and \( \ell_L \) denotes left-handed charged leptons. Therefore, in the mass basis, after performing the basis transformation, we obtain:
\[
\mathcal{L} \supset \left[ (G_W)_{(\mu\nu)} W_{\mu}^\dagger \ell_{L,\alpha} \gamma^{\nu} \nu_j + \text{h.c.} \right] + (G_Z)_{(\mu\nu)} Z_{\mu}^\dagger \nu_{L,\alpha} \gamma^{\nu} \nu_j + (G_R)_{(\mu\nu)} Z_{\mu}^\dagger \nu_{R,j} \gamma^{\nu} \nu_j, \tag{2.6}
\]
where
\[
G_Z = \frac{g}{2c_W} U^\dagger \begin{pmatrix} I_{3 \times 3} \\ 0_{n \times n} \end{pmatrix} U, \quad G_R = g_R U^\dagger \begin{pmatrix} 0_{3 \times 3} \\ Q_R \end{pmatrix} U, \tag{2.7}
\]
\[
G_W = \frac{g}{\sqrt{2}} \begin{pmatrix} I_{3 \times 3} \\ 0_{3 \times n} \end{pmatrix} U. \tag{2.8}
\]

Here \( Q_R = \text{diag} (Q_{R,1}, Q_{R,2}, \cdots) \), \( I_{3 \times 3} \) is an identity matrix, and \( 0_{x \times y} \) is a zero matrix.

Notice that some products of the above matrices are zero:
\[
G_Z G_R = G_R G_Z = 0, \tag{2.9}
\]
\[
G_W G_R = G_R G_W^T = 0, \tag{2.10}
\]
which will be used in our loop calculations to cancel UV divergences.
Figure 1: Loop-induced $Z'$ couplings to charged fermions in the mass basis (upper panels) and in the chiral basis (lower panels).

3 Loop-induced couplings of $Z'$

At tree level, the $\nu_R$-philic $Z'$ does not directly couple to charged leptons or quarks. At the one-loop level, there are loop-induced couplings of $Z'$ generated by the diagrams shown in Fig. 1.

In the upper and lower panels, we present diagrams in the mass and chiral bases, respectively. The two descriptions are physically equivalent. The diagrams in the chiral basis imply that the loop-induced couplings are proportional to $m_D^2$, due to the two necessary mass insertions on the neutrino lines. Although in the mass basis this conclusion is not evident, technically our calculations are performed using the diagrams in the upper panel because of properly defined propagators.

Throughout this work, we work in the unitarity gauge so that diagrams involving Goldstone bosons can be disregarded. The detailed calculations are presented in Appendix A. The result for a single $W^\pm$ diagram with neutrino mass eigenstates $\nu_i$ and $\nu_j$ running in the loop reads:

$$i\mathcal{M}_W^{ij} = i \frac{G_W^2 (G_R^{\alpha i} G_W^{\beta j})^*}{16\pi^2} \mathcal{F}(m_i, m_j) \overline{u}(p_1) \gamma^\mu P_L u(p_2) \epsilon_\mu(q),$$

(3.1)

where $\overline{u}(p_1)$ and $u(p_2)$ denote the two external fermion states, $\epsilon_\mu(q)$ is the polarization vector of $Z'_\mu$, and

$$\mathcal{F}(m_i, m_j) \approx \frac{3}{2} + \frac{m_i^4 \log(m_j^2/m_W^2) - m_j^4 \log(m_i^2/m_W^2)}{(m_i^2 - m_j^2) m_W^2}.$$
because

Once again, we can see that the UV part cancels out during the summation of

holds. For a similar reason (\( \text{Eq. (3.3)} \) implies that we can safely ignore the second line in Eq. (3.2), as long as Eq. (2.10) takes the generalized measure

We have adopted dimensional regularization in the loop calculation so the loop integral

by the loop diagrams:

Hence only the first term in Eq. (3.5) needs to be taken into account.

Summing over \( i \) and \( j \) in Eq. (3.1), we obtain the following effective coupling generated by the loop diagrams:

Table 1: The values of \( Q_Z^{(f)} \) used in this work.

| \( f \) | \( \nu_L \) | \( e_L \) | \( u_L \) | \( d_L \) | \( e_R \) | \( u_R \) | \( d_R \) |
|-------|-------|-------|-------|-------|-------|-------|-------|
| \( Q_Z^{(f)} \) | \( \frac{1}{2} \) | \( -\frac{1}{2} + s_W^2 \) | \( \frac{1}{2} - \frac{2}{3}s_W^2 \) | \( -\frac{1}{2} + \frac{1}{3}s_W^2 \) | \( s_W^2 - \frac{2}{3}s_W^2 \) | \( \frac{1}{3}s_W^2 \) |

We have adopted dimensional regularization in the loop calculation so the loop integral takes the generalized measure

\begin{align}
\frac{d\beta_k}{(2\pi)^d} &\rightarrow \mu^{2\epsilon} \frac{d\beta_k}{(2\pi)^d} \quad \text{with} \quad d = 4 - 2\epsilon, \quad \text{which defines} \quad \mu \text{ and } \epsilon \text{ in Eq. (3.2)}.
\end{align}

Note that for each single diagram in the mass basis, the result is UV divergent. However, when we sum over \( i \) and \( j \), the UV divergence cancels out. This can be seen as follows:

\begin{align}
\sum_{ij} \frac{1}{\epsilon}(m_i^2 + m_j^2)G_W^j(G_R^iG_W^j)^* = \frac{1}{\epsilon}G_WM_R^2G_R^iG_W^j + \frac{1}{\epsilon}G_WG_I^iM_R^2G_W^j = 0, \quad (3.3)
\end{align}

where \( M_R^2 \equiv \text{diag}(m_1^2, m_2^2, m_3^2, \cdots) \) and in the second step we have used Eq. (2.10). Eq. (3.3) implies that we can safely ignore the second line in Eq. (3.2), as long as Eq. (2.10) holds. For a similar reason \( G_WM_R^iG_W^j = 0 \), the constant term \( \frac{3}{2} \) can also be ignored.

For the \( Z \) diagram, we have a similar amplitude for each single diagram. In the soft-scattering limit \( (q \to 0) \), we find

\begin{align}
iM_Z^{ij} = -\frac{igQ_Z^{(f)}G_W^j(G_R^iG_W^j)^*}{16\pi^2c_Wm_Z^2}F_2(m_i, m_j)\gamma^\muP_{L/R}u(p_1)\gamma^\muP_{L/R}u(p_2)e_\mu(q), \quad (3.4)
\end{align}

where \( f = \ell_{L/R}, u_{L/R}, \text{or } d_{L/R} \); and \( Q_Z^{(f)} \) is the \( Z \) charge of \( f \), defined in the way that the \( Z-f-f \) coupling can be written as \( gQ_Z^{(f)}/c_W \). The specific values of \( Q_Z^{(f)} \) used in this work are listed in Tab. 1. The \( F_2 \) function reads:

\begin{align}
F_2(m_i, m_j) \approx \frac{m_i^4\log(m_i^2) - m_i^4\log(m_i^2)}{(m_i^2 - m_j^2)} + \frac{1}{\epsilon} + \frac{1}{2} + \log \mu^2. \quad (3.5)
\end{align}

Once again, we can see that the UV part cancels out during the summation of \( i \) and \( j \) because

\begin{align}
\sum_{ij} \frac{1}{\epsilon}(m_i^2 + m_j^2)G_Z^j(G_R^i)^* = \frac{1}{\epsilon}\text{Tr}
\left[
M_R^2G_ZG_R^i + G_ZM_R^2G_R^i\right] = 0. \quad (3.6)
\end{align}

Hence only the first term in Eq. (3.5) needs to be taken into account.

Summing over \( i \) and \( j \) in Eq. (3.1), we obtain the following effective coupling generated by the loop diagrams:

\begin{align}
\mathcal{L}_{\text{eff}} = \left[g_{\ell_{L,R}}^{\ell_{L,R}}\sigma^\mu\epsilon_{L,\alpha} + g_{\ell_{R,L}}^{f_1f_2}\sigma^\mu f\right]Z_{\mu}. \quad (3.7)
\end{align}
where

\[ g_{\text{eff},W} = \sum_{ij} \frac{G_W^{\beta j}(G_W^{\alpha i}G_W^{\alpha i})^*}{16\pi^2} m_4^4 \log\left(\frac{m_i^2}{m_W^2}\right) - m_i^4 \log\left(\frac{m_j^2}{m_W^2}\right) \left(\frac{m_i^2 - m_j^2}{m_i^2 m_W^2}\right), \]  

\( (3.8) \)

\[ g_{\text{eff},Z} = \sum_{ij} \frac{gQ_Z^{(f)} G_Z^{ij}(G_R^{ij})^*}{16\pi^2 c_W} m_4^4 \log\left(\frac{m_i^2}{m_W^2}\right) - m_i^4 \log\left(\frac{m_j^2}{m_W^2}\right) \left(\frac{m_i^2 - m_j^2}{m_i^2 m_W^2}\right). \]  

\( (3.9) \)

3.1 Example A: \( 1 \nu_L + 1 \nu_R \)

First, let us consider the simplest case that there are only one \( \nu_L \) and one \( \nu_R \). The neutrino mass matrix \( M_\nu \) for the case can be diagonalized by a \( 2 \times 2 \) unitary matrix

\[ U^T \begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix} U = \begin{pmatrix} m_1 & 0 \\ 0 & m_4 \end{pmatrix}. \]  

\( (3.10) \)

This unitary matrix can be parametrized as follows

\[ U = \begin{pmatrix} -ic_\theta & s_\theta \\ is_\theta & c_\theta \end{pmatrix}, \quad \theta = \arctan\left(\sqrt{\frac{m_1}{m_4}}\right), \]  

\( (3.11) \)

where \( c_\theta = \cos \theta \) and \( s_\theta = \sin \theta \). Substituting the explicit form of \( U \) in Eqs. (2.7) and (2.8), we obtain

\[ G_Z = \frac{g}{2c_W} \begin{pmatrix} c_\theta^2 & ic_\theta s_\theta \\ -ic_\theta s_\theta & s_\theta^2 \end{pmatrix}, \quad G_R = g_R \begin{pmatrix} s_\theta^2 & -ic_\theta s_\theta \\ ic_\theta s_\theta & c_\theta^2 \end{pmatrix}, \]  

\( (3.12) \)

\[ G_W = \frac{g}{\sqrt{2}} \begin{pmatrix} -ic_\theta & s_\theta \end{pmatrix}. \]  

\( (3.13) \)

We can now perform the summation in Eqs. (3.8)-(3.9). Expanding the result as a Taylor series in \( s_\theta \) (assuming \( s_\theta \ll 1 \)) and only retaining the dominant contribution, we obtain

\[ g_{\text{eff},W} = -\frac{g^2 m_4^2 s_\theta^2}{32\pi^2 m_W^2} g_R, \]  

\( (3.14) \)

\[ g_{\text{eff},Z} = Q_Z^{(f)} \frac{g^2 m_4^2 s_\theta^2}{32\pi^2 m_W^2} g_R, \]  

\( (3.15) \)

where

\[ m_1 = ms_\theta^2, \quad m_4 = mc_\theta^2. \]  

\( (3.16) \)

Note that for \( m_1 \ll m_4 \),

\[ m_1 s_\theta^2 \simeq m_1 m_4 = m_D^2. \]  

\( (3.17) \)

Using \( G_F = \frac{\sqrt{2}g^2}{8m_W^2} \), we can rewrite Eqs. (3.14)-(3.15) as

\[ g_{\text{eff},W} = -\frac{\sqrt{2} G_F m_D^2}{8\pi^2} g_R, \]  

\( (3.18) \)

\[ g_{\text{eff},Z} = Q_Z^{(f)} \frac{\sqrt{2} G_F m_D^2}{8\pi^2} g_R. \]  

\( (3.19) \)
3.2 Example B: $1\ \nu_L + 2\ \nu_R$ with opposite charges

In this example, we construct a UV-complete model with one $\nu_L$ and two $\nu_R$ which have opposite $U(1)_R$ charges so that the model is anomaly free. The off-diagonal Majorana mass term does not violate the $U(1)_R$ symmetry and the Dirac mass term is generated by a new Higgs doublet $H'$ that is charged under $U(1)_R$. The $U(1)_R$ charges are assigned as follows:

$$Q_R(\nu_{R,1}) = +1, \ Q_R(\nu_{R,2}) = -1, \ Q_R(H') = -1, \ (3.20)$$

which leads to the following terms that fully respect the $U(1)_R$ symmetry:

$$L \supset y_\nu \tilde{H}^\dagger L \nu_R + \frac{M_R}{2} \nu_{R,1} \nu_{R,2} + h.c. \ (3.21)$$

After spontaneous symmetry breaking, $H'$ acquires a vacuum expectation value: $\langle H' \rangle = (0, \ v'/\sqrt{2}$, leading to

$$L \supset m_D \nu_L \nu_R + \frac{M_R}{2} \nu_{R,1} \nu_{R,2} + h.c. \ (3.22)$$

Here $m_D = y_\nu v'/\sqrt{2}$. The neutrino mass matrix for this case can be diagonalized by a $3 \times 3$ unitary matrix:

$$U^T \begin{pmatrix} 0 & m_D & 0 \\ m_D & 0 & M_R \\ 0 & M_R & 0 \end{pmatrix} U = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_4 & 0 \\ 0 & 0 & m_5 \end{pmatrix} \ (3.23)$$

The texture of the mass matrix on the left-hand side of Eq. (3.23) leads to $m_1 = 0$ and $m_4 = m_5$, which is evident from its vanishing trace and determinant. This feature has been often considered in the literature on $\nu_R$ signals at the LHC—see e.g. [21] and references therein. The $3 \times 3$ unitary matrix can be parametrized as follows

$$U = \begin{pmatrix} -c_\theta & is_\theta & s_\theta \\ 0 & 1/s_\theta & 0 \\ is_\theta & -c_\theta & s_\theta \end{pmatrix}, \ \ \ \theta = \arctan \left( \frac{m_D}{M_R} \right) \ (3.24)$$

Using this form of $U$ in Eqs. (2.7) and (2.8), we obtain

$$G_Z = \frac{g}{2c_W} \begin{pmatrix} c_\theta^2 & 0 & -ic_\theta s_\theta \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -ic_\theta s_\theta & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \ \ \ G_R = g_R \begin{pmatrix} -s_\theta^2 & 0 & -ic_\theta s_\theta \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -ic_\theta s_\theta & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \ (3.25)$$

$$G_W = \frac{g}{\sqrt{2}} \begin{pmatrix} -c_\theta & is_\theta & s_\theta \end{pmatrix}. \ (3.26)$$

We can now perform the summation in Eqs. (3.8)-(3.9). Expanding the result as a Taylor series in $s_\theta$ (assuming $s_\theta \ll 1$) and only retaining the dominant contribution, we obtain

$$g_{eff,W} = \frac{g_R^2 c_\theta^2 m^2 s_\theta^2}{32\pi^2 m_W^2} g_R. \ (3.27)$$
\[ g_{\text{eff},Z} = -Q_Z^{(f)} \frac{g^2 c_{\theta}^2 m_\nu^2 s_{\theta}^2}{32 \pi^2 m_Z^2 c_W^2} g_R, \]  

(3.28)

where \( m \equiv \sqrt{m_D^2 + M_R^2} \) and

\[ m_D = m s_{\theta}, \quad M_R = m c_{\theta}. \]  

(3.29)

Expressing the results in terms of \( G_F \) and assuming \( s_{\theta} \ll 1 \), we obtain

\[ g_{\text{eff},W} = \sqrt{\frac{2}{2}} G_F m_D^{-1} g_R, \]  

(3.30)

\[ g_{\text{eff},Z} = -Q_Z^{(f)} \frac{\sqrt{2} G_F m_D^{-1}}{8 \pi^2} g_R. \]  

(3.31)

We comment here that the above UV-complete and anomaly-free model built on \( 1 \nu_L + 2 \nu_R \) can be straightforwardly generalized to \( 3 \nu_L + 2n \nu_R \) where half of the right-handed neutrinos have opposite \( U(1)_R \) charges to the other half. Such a generalization can accommodate the realistic three-neutrino mixing measured in neutrino oscillation experiments.

### 3.3 Example C: \( 3 \nu_L + 3 \nu_R \) with diagonal \( M_R \)

The most general case with three \( \nu_L \) and an arbitrary number of \( \nu_R \) is complicated and often impossible to be computed analytically. Here we consider an analytically calculable example with \( 3 \nu_L + 3 \nu_R \) and the following form of the neutrino mass matrix:

\[
\begin{pmatrix}
0_{3 \times 3} & m_D \\
m_D^T & M_R
\end{pmatrix} = 
\begin{pmatrix}
U_L^* & 0 \\
0_{3 \times 3} & m_D^{(d)}
\end{pmatrix}
\begin{pmatrix}
U_L & 0 \\
I_{3 \times 3}
\end{pmatrix},
\]  

(3.32)

\[ m_D^{(d)} = \text{diag}(m_{D1}, m_{D2}, m_{D3}), \quad M_R^{(d)} = \text{diag}(M_{R1}, M_{R2}, M_{R3}), \]

where \( U_L \) is a \( 3 \times 3 \) unitary matrix. Eq. (3.32) is not the most general form, but at least it can accommodate the realistic low-energy neutrino mixing responsible for neutrino oscillation.

The mass matrix in this case can be diagonalized by a \( 6 \times 6 \) unitary matrix:

\[ U'^T \begin{pmatrix}
0_{3 \times 3} & m_D^{(d)} \\
m_D^{(d)} & M_R^{(d)}
\end{pmatrix} U' = \text{diag}(m_1^2 s_{\theta_1}^2, m_2^2 s_{\theta_2}^2, m_3^2 s_{\theta_3}^2, m_1^2 c_{\theta_1}^2, m_2^2 c_{\theta_2}^2, m_3^2 c_{\theta_3}^2), \]  

(3.33)

where \((s_{\theta_1}, c_{\theta_1}) \equiv (\sin \theta_1, \cos \theta_1)\) and

\[ m_i = \sqrt{4m_{D_i}^2 + M_{R_i}^2}, \quad \theta_i = \frac{1}{2} \arctan \left( \frac{2m_{D_i}}{M_{R_i}} \right). \]  

(3.34)

The unitary matrix \( U' \) can be parametrized as follows

\[ U' = \begin{pmatrix}
-iC_{\theta} & S_{\theta} \\
-iS_{\theta} & C_{\theta}
\end{pmatrix}, \]  

(3.35)

where

\[ C_{\theta} = \text{diag}(c_{\theta_1}, c_{\theta_2}, c_{\theta_3}), \quad S_{\theta} = \text{diag}(s_{\theta_1}, s_{\theta_2}, s_{\theta_3}). \]  

(3.36)
Thus, the final unitary matrix $U$ that diagonalizes the original mass matrix is given by

$$ U = \begin{pmatrix} U_L & 0 \\ I_{3 \times 3} & \end{pmatrix} \begin{pmatrix} -iC_\theta & S_\theta \\ iS_\theta & C_\theta \end{pmatrix} = \begin{pmatrix} -iU_L C_\theta & U_L S_\theta \\ iS_\theta & C_\theta \end{pmatrix}. $$ (3.37)

Substituting it in Eqs. (2.7) and (2.8), we obtain

$$ G_Z = \frac{g}{2c_W} \begin{pmatrix} C_\theta^2 & iC_\theta S_\theta \\ -iC_\theta S_\theta & S_\theta^2 \end{pmatrix}, \quad G_R = g_R Q_R \begin{pmatrix} S_\theta^2 & -iC_\theta S_\theta \\ iC_\theta S_\theta & C_\theta^2 \end{pmatrix}, \quad Q_R = \text{diag}(Q_{R1}, Q_{R2}, Q_{R3}). $$ (3.38)

Next, we perform the summation in Eqs. (3.8)-(3.9), expand the result in $s_{\theta i}$, and retain the dominant contribution. The final result reads

$$ g_{\alpha \beta, W}^{\alpha \beta} = \sum_i -U_{Li}^2 (U_{L}^\dagger)^i Q_{Ri} \sqrt{2} G_F \frac{m_{D_i}^2}{8\pi^2} g_R, $$ (3.40)

$$ g_{\alpha \beta, Z}^{\alpha \beta} = \sum_i Q_{Zi}^{(f)} Q_{Ri} \sqrt{2} G_F \frac{m_{D_i}^2}{8\pi^2} g_R. $$ (3.41)

In the approximation that the $\nu_L$-$\nu_R$ mixing is small, the $3 \times 3$ unitary matrix $U_L$ is almost identical to the PMNS matrix. Due to the presence of off-diagonal entries in $U_L$, $g_{\alpha \beta, W}^{\alpha \beta}$ is generally not flavor diagonal and might lead to observable lepton flavor violation, which will be discussed in Sec. 5.

### 4 Dark photon masses and technical naturalness

In this section, we argue that despite being a free parameter, the mass of the $\nu_R$-philic dark photon $m_{Z'}$ is potentially related to the gauge coupling according to 't Hooft’s technical naturalness [22]. Generally speaking, from the consideration of model building and the stability of $m_{Z'}$ under loop corrections, we expect that $m_{Z'}$ is related to $g_R$ by

$$ m_{Z'} \gtrsim g_R \Lambda_{\text{breaking}}, $$ (4.1)

where $\Lambda_{\text{breaking}}$ stands for the symmetry breaking scale of $U(1)_R$. Although without UV completeness we cannot have a more specific interpretation of Eq. (4.1), we would like to discuss a few examples to show how $m_{Z'}$ is related to $g_R$.

First, let us consider that both $m_{Z'}$ and $M_R$ arise from a scalar singlet $\phi$ charged under $U(1)_R$ with $\langle \phi \rangle = v_R \neq 0$. This leads to $m_{Z'} \sim g_R v_R$ and $M_R \sim y_R v_R$ where $y_R$ is the Yukawa coupling of $\phi$ to $\nu_R$. In this case, we consider $v_R$ as the symmetry breaking scale $\Lambda_{\text{breaking}}$ so the tree-level relation $m_{Z'} \sim g_R v_R$ is compatible with Eq. (4.1). The Yukawa coupling has an upper bound from perturbativity, $y_R \lesssim 4\pi$, which implies that $m_{Z'}/M_R \sim g_R/y_R \gtrsim 4\pi g_R$, or

$$ m_{Z'}^2 \gtrsim \frac{g_R^2}{16\pi^2} M_R^2. $$ (4.2)
In the absence of a specific symmetry breaking mechanism, we can also obtain Eq. (4.2) purely from loop corrections to $m_{Z'}$. If $M_R$ breaks the $U(1)_R$ symmetry, the $Z'$-vacuum polarization amplitude generated by a $\nu_R$ loop is $\Pi^{\mu\nu}(q^2) \sim \frac{g^2_R}{16\pi^2} [\mathcal{O}(M_R^2)g^\mu - \mathcal{O}(1)q^\mu q^\nu]$, which implies that the loop correction to $m_{Z'}$ is of the order of $\frac{g^2_R}{16\pi^2} M_R^2$. Therefore, to make the theory technically natural, the physical mass should not be lower than the loop correction.

Note, however, that Eq. (4.2) is based on the assumption that $M_R$ breaks the $U(1)_R$ symmetry. If all the Majorana mass terms fully respect $U(1)_R$, such as Example B in Sec. 3, then the symmetry breaking scale can be lower, e.g., determined by $m_D$. Indeed, for the UV complete model in Example B, the symmetry breaking scale is determined by the VEV of the new Higgs doublet $H'$ so at tree level we have $m_{Z'} \sim g_R \langle H' \rangle$ and $m_D \sim y_D \langle H' \rangle$. Then using the perturbativity bound on $y_D$, we obtain

$$m_{Z'}^2 \gtrsim \frac{g^2_R}{16\pi^2} m_D^2.$$  \hspace{1cm} (4.3)

Finally, we comment on the possible mass correction from $Z-Z'$ mixing. According to the calculation in Appendix A, the vacuum polarization diagram leads to mass mixing between $Z$ and $Z'$:

$$L_{ZZ'}^{\text{mass}} = \frac{1}{2} Z, Z' \mu \begin{pmatrix} m_{Z_0}^2 & m_X^2 \\ m_X^2 & m_{Z_0}^2 \end{pmatrix},$$  \hspace{1cm} (4.4)

where $m_{Z_0}$ and $m_{Z_0}'$ denote tree-level masses and

$$m_X^2 = \frac{g_R Q_R g}{64\pi^2 \cos \theta_W} m_D^2.$$  \hspace{1cm} (4.5)

Here $m_X^2$ causes $Z-Z'$ mixing and the mixing angle is roughly $\frac{m_X^2}{|m_{Z_0}^2 - m_{Z_0}'^2|}$, which must be small. Otherwise, the SM neutral current would be significantly modified and become inconsistent with electroweak precision data. Taking the approximation $m_X^2 \ll |m_{Z_0}^2 - m_{Z_0}'^2|$, we obtain

$$m_Z^2 \simeq m_{Z_0}^2 + \frac{m_X^4}{(m_{Z_0}^2 - m_{Z_0}'^2)}, \quad m_{Z'}^2 \simeq m_{Z_0}'^2 - \frac{m_X^4}{(m_{Z_0}^2 - m_{Z_0}'^2)}.$$  \hspace{1cm} (4.6)

Hence we conclude that the mass correction from $Z-Z'$ mixing is

$$\delta m_{Z'}^2 \sim \frac{g^2_R}{(64\pi^2)^2} \frac{m_D^4}{|m_{Z_0}^2 - m_{Z_0}'^2|}.$$  \hspace{1cm} (4.7)

where we have neglected some $\mathcal{O}(1)$ quantities. This mass correction is generally smaller than the right-hand side of Eq. (4.3) because $m_D$ cannot be much above the electroweak scale.

To summarize, here we draw a less model-dependent conclusion that without fine-tuning, the $\nu_R$-philic dark photon mass is expected to be above the lower bound in Eq. (4.2) or Eq. (4.3), depending on whether $M_R$ breaks the $U(1)_R$ symmetry or not, respectively.
Figure 2: The $\nu_R$-philic dark photon confronted with known experimental constraints. Here $g_{\text{eff}}$ is the loop-induced coupling of $Z'$ to electrons. The quark couplings are of the same order of magnitude as $g_{\text{eff}}$ and we have ignored the difference between them when recasting constraints on quark couplings. The theoretically favored values of $g_{\text{eff}}$ are below the solid blue, orange, or green lines, assuming $U(1)_R$ breaks at the scale of $m_D = 246$ GeV, $M_R = 24.6$ TeV, or $M_R \sim 10^{14}$ GeV (Type I seesaw), respectively. The collider bound consists of BaBar, LHCb, LEP, and LHC 8 TeV limits—see the text or Fig. 3 for more details.

5 Phenomenology

In the previous two sections, we have derived the loop-induced couplings and also argued that from technical naturalness there is a lower bound on the dark photon mass. The results indicate the theoretically favored regime of the mass and the couplings. Therefore, to address the question of how dark the $\nu_R$-philic dark photon would be, we shall inspect whether and to what extent the theoretically favored regime could be probed by current and future experiments.

In our model, there are effective couplings to both leptons and quarks with comparable strengths. So the experimental constraints on this model are very similar to those on the $B - L$ model\textsuperscript{2}. Below we discuss a variety of known bounds that could be important for the $\nu_R$-philic dark photon. An overview of existing bounds is presented in Fig. 2, and the prospect of upcoming experiments in Fig. 3.

\textsuperscript{2}See e.g. Fig. 8 in [23], Fig. 3 in [24], and Fig. 13 in [18]
Figure 3: Sensitivity of future experiments (SHiP, FASER, Belle-II) on the $\nu_R$-philic dark photon. Here $g_{\text{eff}}$ is the loop-induced coupling of $Z'$ to electrons. The quark couplings are of the same order of magnitude as $g_{\text{eff}}$ and we have ignored the difference between them when recasting constraints on quark couplings. The theoretically favored values of $g_{\text{eff}}$ is below the solid blue or orange lines, assuming $U(1)_R$ breaks at the scale of $m_D = 246$ GeV or $M_R = 24.6$ TeV, respectively.

5.1 Experimental limits

5.1.1 Collider searches

With effective couplings to electrons and quarks, dark photons could be produced directly in $e^+e^-$ (BaBar, LEP) and hadron colliders (LHC), typically manifesting themselves as resonances in collider signals. For $m_{Z'} \gtrsim 175$ GeV ($t$ quark resonance), LHC data put the strongest bound via Drell-Yan production and detection of leptonic final states ($pp \to Z' \to \ell^+\ell^-$). At lower masses when $m_{Z'}$ is close to the $Z$ pole, electroweak precision tests (EWPT, including LEP measurement and other electroweak precision observables) become more important. A dedicated analysis on LHC and EWPT bounds and future prospects can be found in Ref. [25]. For $m_{Z'}$ below the $Z$ pole but above 10 GeV, according to the analyses in [18], the most stringent constraint comes from LHCb di-muon ($Z' \to \mu^+\mu^-$) measurements [26]. Below 10 GeV, the BaBar experiment [27] provides more stringent constraints via $e^+e^- \to \gamma Z'$ where $Z'$ may or may not decay to visible final states. In Figs. 2 and 3, we present all aforementioned constraints (for compactness in Fig. 2 they are labeled together as the collider bound).

5.1.2 Beam dump and neutrino scattering bounds

For $1 \text{ MeV} \lesssim m_{Z'} \lesssim 100$ MeV, beam dump (BD) and neutrino scattering experiments become important. BD experiments search for dark photons by scattering an electron/proton
beam on fixed targets and looking for dark particles that might be produced and subsequently decay after the shield to visible particles such as electrons. A compilation of existing BD bounds from SLAC E141, SLAC E137, Fermilab E774, Orsay, and KEK experiments can be found in [28]. Note that these BD bounds relies on $Z' \rightarrow e^+e^-$ decay, which implies that such bounds do not apply for $m_{Z'} \lesssim 2m_e$. Nonetheless, below 1 MeV there are much stronger bounds from cosmological and astrophysical observations hence for simplicity we do not show the invalidity of BD bounds below 1 MeV. The combined BD bound adopted in this work is taken from [24].

The dark photon in our model could contribute to elastic neutrino scattering by a new neutral-current-like process. Current data from elastic neutrino-electron (CHARM-II [29, 30], TEXONO [31], GEMMA [32], Borexino [33], etc.) and neutrino-nucleus (COHERENT [34]) scattering are all well consistent with the SM predictions. By comparing the results in Refs. [35–37], we find that the COHERENT bound is weaker than $\nu + e$ scattering bounds, among which the most stringent ones come from CHARM-II, TEXONO, and GEMMA. So the combined result from these experiments is taken from Ref. [37] and presented in Figs. 2 and 3.

5.1.3 Astrophysical bounds

Astrophysical bounds on dark photons are usually derived from energy loss in celestial bodies such as the sun, red giants, horizontal branch stars, and supernovae. Dark photons may contribute to stellar energy loss directly via dark photon free streaming or indirectly via neutrino production. The enhanced energy loss rate could alter stellar evolution on the horizontal branch in the Hertzsprung-Russell diagram. This sets the strongest limit for sub-MeV dark photons [38]. For smaller $m_{Z'}$, there are also similar bounds from the sun and red giants [38]. We adopt a combined bound from Ref. [23] with energy loss via neutrinos taken into account, and refer to it as the stellar cooling bound in Fig. 2.

The observation of SN1987A can be used to set strong limits on the effective coupling when $m_{Z'} \lesssim O(100)$ MeV [39]. The resulting bound further excludes the space below BD constraints by about three orders of magnitude.

5.1.4 Charged lepton flavor violation

The loop-induced couplings do not necessarily conserve lepton flavors, as indicated by Eq. (3.40). Note, however, that neither the $W$-diagram nor the $Z$-diagram causes flavor violation in the quark sector. In the presence of flavor-changing couplings of $Z'$ to charged leptons, there are strong constraints from charged lepton flavor violating (CLFV) decay such as $\ell_\alpha \rightarrow \ell_\beta \nu \bar{\nu}$, $\mu \rightarrow 3e$ [40], $\pi^0 \rightarrow e\mu$; from $\mu \rightarrow e$ conversion in muonic atoms [41], and from the non-observation of muonium-antimuonium transitions [42]. Constraints from $\ell_\alpha \rightarrow \ell_\beta \gamma$ are weaker since they arise only from two-loop contributions. We do not include CLFV bounds in Figs. 2 and 3 because such bounds depend on the flavor structure of $m_D$ which in the Casas-Ibarra parametrization [43]: $m_D = iU^*_L \sqrt{m_\nu^R} R^T \sqrt{M_R}$ where $R$ is a complex orthogonal matrix, depends not only on the PMNS matrix $U_L$ but also on the $R$ matrix. The effective flavor-changing couplings in the presence of non-trivial $R$ are more complicated and we leave them for future work.
5.1.5 Long-range force searches

Below 0.1 eV, laboratory tests of gravity and gravity-like forces provide highly restrictive constraints, including high precision tests of the inverse-square law (gravity $\propto r^{-2}$) [44, 45] and of the equivalence principle via torsion-balance experiments [46] and lunar laser-ranging (LLR) measurements [46, 47]. Besides, measurements of the Casimir effect [48] could set a limit that is slightly stronger than that from the inverse-square law when $0.05 \lesssim m_{Z'}/eV \lesssim 0.1$, which is not presented in Fig. 2. Also not presented here is the bound from black hole superradiance [49], which would only enter the lower left corner in Fig. 2. We refer to our previous work [20] for more detailed discussions on the long-range force searches and present only the dominant constraints from torsion-balance tests of the inverse-square law and the equivalence principle. We comment here that neutrino oscillation could also be used to probe long-range forces [50–53] but similar to the aforementioned CLFV bounds, the flavor structure cannot be simply taken into account by the PMNS matrix. Hence we leave this possibility to future studies.

5.1.6 Prospect of upcoming experiments

Future hadron collider searches could significantly improve the experimental limits on heavy dark photons by almost one order of magnitude, as illustrated in Fig. 3 by the LHC 14 TeV and future 100 TeV collider sensitivity [25]. Moreover, several LHC-based experiments searching for displaced dark photon decays such as FASER [54], MATHUSLA [55, 56], and CodexB [57] will improve the BD bound in the low-mass regime. And the future SHiP experiment [58, 59] will substantially broaden the BD bound regarding both the dark photon mass and coupling. The current BaBar bound may be superseded by future bounds from Belle-II [60] and a muon run of NA64 [61, 62]. Hence a large part of the space that is often considered for dark photons ($20\text{MeV} \lesssim m_{Z'} \lesssim 10 \text{GeV}$ and $10^{-8} \lesssim g_{\text{eff}} \lesssim 10^{-3}$) will be probed by future experiments. Here we selectively present the sensitivity curves of SHiP, FASER, NA64$\mu$, and Belle-II from Ref. [18].

5.2 How dark is the $\nu_R$-philic dark photon?

Since the effective coupling $g_{\text{eff}}$ is proportional to $g_R$, by tuning down $g_R$ one can obtain arbitrarily small $g_{\text{eff}}$ to circumvent all constraints presented in Figs. 2 and 3. On the other hand, if $g_R$ is very small, then the lower bounds of $m_{Z'}$ discussed in Sec. 4 will also be alleviated, implying that the dark photon could be very light. Taking Eqs. (3.30), (3.31) and (4.3), we plot the blue lines in Figs. 2 and 3 with $m_D = v = 246 \text{GeV}$ and $g_R$ varying from 0 to $4\pi$. The space below the blues lines is the theoretically favored region if only the Dirac mass term breaks the $U(1)_R$ symmetry. This applies to the UV complete model in Sec. 3.2.

If the Majorana mass term also breaks the $U(1)_R$ symmetry, then the lower bound of $m_{Z'}$ is set by Eq. (4.2) instead of Eq. (4.3). In the standard type I seesaw, we have $M_R \sim m^2_D/m_{\nu}$ which implies that for $m_{\nu} = 0.1 \text{ eV}$ and $m_D = 246 \text{ GeV}$, the $U(1)_R$ symmetry breaks at a high energy scale around $10^{14} \text{ GeV}$. For this case, we plot the green curve in Fig. 2. As shown in Fig. 2, even though with $g_R \lesssim 10^{-11}$ the mass of $m_{Z'}$ could
be below the electroweak scale or lower, the effective coupling is many orders of magnitude below any of known experimental limits.

The inaccessibly large $m_{Z'}$ of the green curve is due to the underlying connection between $m_{\nu}$ and $M_R$ in the standard type I seesaw. In some alternative neutrino mass models, however, the scale of $M_R$ is decoupled from $m_{\nu}$—see e.g. [21] and references therein. The decoupling allows for a sizable $\nu_L$-$\nu_R$ mixing even when $M_R$ is reduced to the TeV scale, and has motivated many studies on collider searches for right-handed neutrinos [63–65]. Here for illustration we simply set $M_R = m_D / \sin \theta$ with $m_D = 246$ GeV and $\sin \theta = 10^{-2}$, which ensures that $\nu_R$ is sufficiently heavy to avoid all current collider bounds. The possibility of collider-accessible $\nu_R$ involves more complicated phenomenology which is beyond the scope of this work. The strength of $g_{\text{eff}}$ and the lower bound of $m_{Z'}$ in this case is presented by the orange lines in Figs. 2 and 3.

Now confronting the theoretically favored $g_{\text{eff}}$ and $m_{Z'}$ of the aforementioned three scenarios with the experimental limits, we can see that only when the $U(1)_R$ breaking scale is determined by $m_D$ or $M_R = m_D / \sin \theta$ with sizable $\sin \theta$, the $\nu_R$-philic dark photon could be of phenomenological interest. The former could potentially give rise to observable effects in long-range force searches, astrophysical observations, beam dump and collider experiments. The latter, albeit beyond the current collider bounds, might be of importance to future collider searches. In addition, the SHiP experiment will be able to considerably dig into the parameter space of the latter.

6 Conclusion

The $\nu_R$-philic dark photon $Z'$ which arises from a hidden $U(1)_R$ gauge symmetry and at the tree-level couples only to the right-handed neutrinos, interacts weakly with SM particles via loop-level processes—see Fig. 1. Assuming the most general Dirac and Majorana mass matrices, we have derived loop-induced couplings of $Z'$ to charged leptons and quarks. The results are given in Eqs. (3.8) and (3.9), which are applied to a few examples including a UV complete model. For a special case with three $\nu_L$ and three $\nu_R$, the loop-induced coupling are given by Eqs.(3.40) and (3.41). We have also discussed potential connections between the mass $m'_{Z}$ and the gauge coupling $g_R$ from the point of view of technical naturalness, which implies that $m'_{Z}$ should be generally above the lower bound in Eq. (4.2) if $M_R$ breaks $U(1)_R$, or the bound in Eq. (4.3) if only $M_D$ breaks the symmetry.

The theoretically favored values of the loop-induced couplings are confronted with experimental constraints and prospects in Figs. 2 and 3. We find that the magnitude of loop-induced couplings allows current experiments to put noteworthy constraints on it. Future beam dump experiments like SHiP and FASER together with upgraded collider searches will have substantially improved sensitivity on such a dark photon.

Hence as the answer to the question proposed in the title, we conclude that the $\nu_R$-philic dark photon might not be inaccessibly dark and could be of importance to a variety of experiments!
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A Explicit calculation of loop diagrams

In this appendix, we compute loop diagrams presented in Fig. 1 in the mass basis. In the main text, we use two-component Weyl spinors for conceptual simplicity. However, technically it is more convenient to convert them to four-component Dirac/Majorana spinors so that the standard trace technology can be employed. Following the same convention as Ref. [20], we rewrite Eq. (2.6) as

\[ \mathcal{L} \supset (G_Z)^{ij} Z_{\mu} \overline{\psi}_i \gamma_\mu \psi_j + (G_R)^{ij} Z'_{\mu} \overline{\psi}_i \gamma_\mu \psi_j + \left[ (G_W)^{\alpha\mu} W_{\mu}^{-} \psi_\alpha \gamma_\mu \psi_i \right] + \text{h.c.}, \]  

(A.1)

where \( P_L = \frac{1}{2} (1 - \gamma_5) \), \( \gamma_L^\mu \equiv \gamma^\mu P_L \), and

\[ \psi_\alpha = \begin{pmatrix} \ell_{L,\alpha} \\ \ell_{R,\alpha}^\dagger \end{pmatrix}, \quad \psi_i = \begin{pmatrix} \nu_i \\ \nu_i^\dagger \end{pmatrix}. \]  

(A.2)

For simplicity, we symbolically denote the relevant product of neutrino-gauge couplings by \( G_X \) (it may stands for different quantities in different diagrams), which will be replaced by specific couplings when actually used.

A.1 The Z diagram

The diagram is presented in the upper right panel in Fig. 1. We first compute the vacuum polarization part of the diagram (i.e. without the external fermion lines):

\[ i \mathcal{M}_{\mu\nu} = G_X \int \frac{d^4 k}{(2\pi)^4} \text{Tr} \left[ \gamma_\mu P_L \Delta_i (q - k) \gamma_\nu P_L \Delta_i (k) \right], \]  

(A.3)

where \( q \) is the momentum of \( Z' \) and

\[ \Delta_i (p) = \frac{i}{p - m_i}. \]  

(A.4)

Taking into account the Lorentz structure of the amplitude, this can be further decomposed as:

\[ i \mathcal{M}_{\mu\nu} = -\frac{i G_X}{16\pi^2} \left[ \mathcal{F}_1 (m_i, m_j, q^2) q_\mu q_\nu + \mathcal{F}_2 (m_i, m_j, q^2) g_{\mu\nu} \right], \]  

(A.5)

where

\[ \mathcal{F}_1 (m_i, m_j, q^2) = \frac{5 m_i^4 - 22 m_i^2 m_j^2 + 5 m_j^4}{9 (m_i^2 - m_j^2)^2} + \frac{2 m_i^4 (3m_j^2 - m_i^2)}{3(m_i^2 - m_j^2)^3} \log \left( \frac{m_i^2}{m_j^2} \right) \]  

and
By applying the result of Eq. (A.3) to Eq. (A.8), we obtain
\[ F_2(m_i, m_j, q^2) = \frac{m_i^2 + m_j^2}{2} - m_j^4 \log \left( \frac{m_i^2}{m_j^2} \right) + (m_i^2 + m_j^2) \left[ \frac{1}{\epsilon} + \log \left( \frac{\mu^2}{m_i^2} \right) \right] + \mathcal{O}(q^2) , \quad (A.7) \]

The full amplitude of the $Z$ diagram can be written as
\[ iM_Z = -i \ G_X \int \frac{d^4 k}{(2\pi)^4} \text{Tr} \left[ \gamma_\mu P_L \Delta_j(q - k) \gamma_\nu P_L \Delta_i(k) \right] \Delta_Z^{\mu\nu}(q) \overline{u(p_1)\gamma_\nu P_L/Ru(p_2)} , \quad (A.8) \]

where the most general form of $\Delta_Z^{\mu\nu}(q)$ in $R_\xi$ gauges is
\[ \Delta_Z^{\mu\nu}(q) = \frac{-i}{q^2 - m_Z^2} \left[ g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2 - \xi m_Z^2} (1 - \xi) \right]. \quad (A.9) \]

We proceed with the unitarity gauge corresponding to $\xi \to \infty$, and the soft-scattering limit $q \ll m_Z$:
\[ \Delta_Z^{\mu\nu}(k) \xrightarrow{\xi \to \infty, \ q \ll m_Z} \frac{i g^{\mu\nu}}{m_Z^2} . \quad (A.10) \]

By applying the result of Eq. (A.3) to Eq. (A.8), we obtain
\[ iM_Z = -i \ \frac{G_X}{16\pi^2 m_Z^2} \left[ F_1(m_i, m_j, q^2) \ g_\rho q_\nu + F_2(m_i, m_j, q^2) \ g_\mu q_\nu \right] \overline{u(p_1)\gamma_\nu P_L/Ru(p_2)} , \quad (A.11) \]

where $F_1$ and $F_2$ were already given in Eqs. (A.6) and (A.7), respectively.

**A.2 The $W$ diagram**

The diagram is presented in the upper left panel in Fig. 1. The amplitude reads:
\[ iM_W = -i \ G_X \int \frac{d^4 k}{(2\pi)^4} \overline{u(p_1)\gamma_\rho P_L} \Delta_j(k - p_1) \gamma_\rho P_L \Delta_i(p_2 - k) \gamma_\mu P_L u(p_2) \Delta_W^{\rho\mu}(k) , \quad (A.12) \]

where
\[ \Delta_i(p) = \frac{i}{p^2 - m_i^2} , \quad (A.13) \]
\[ \Delta_W^{\rho\mu}(k) = \frac{-i}{k^2 - m_W^2} \left[ g_{\rho\mu} - \frac{k_\rho k_\mu}{k^2 - \xi m_W^2} (1 - \xi) \right] . \quad (A.14) \]

Similar to the $Z$ diagram, we take the unitarity gauge ($\xi \to \infty$) and the soft-scattering limit ($q \to 0$). The quantity in the loop integral is proportional to
\[ \int \frac{d^4 k}{(2\pi)^4} \gamma_\rho P_L \Delta_j(k - p_1) \gamma_\rho P_L \Delta_i(p_2 - k) \gamma_\mu P_L \Delta_W^{\rho\mu}(k) = C_a \gamma_\rho P_L + C_b \gamma P_L p_1^2 + C_c \gamma P_L p_2^2 . \quad (A.15) \]

Here ($C_a, C_b, C_c$) are functions of scalar invariants $p_1^2$ and $p_2^2$. The last two terms are suppressed when imposing the on-shell conditions. Focusing only on the $\gamma P_L$ term, we obtain
\[ iM_W = i \ \frac{G_X}{16\pi^2} F(m_i, m_j) \overline{u(p_1)\gamma_\rho P_L u(p_2)} , \quad (A.16) \]
where
\[
\mathcal{F}(m_i, m_j) = \frac{2m_i^2 + 2m_j^2 + 3m_W^2}{2m_W^2} + \frac{m_i^4 \log \left( \frac{m_i^2}{m_W^2} \right) - m_j^4 \log \left( \frac{m_j^2}{m_W^2} \right)}{(m_i^2 - m_j^2) m_W^2}
\]
\[+ \frac{m_i^2 + m_j^2}{m_W^2} \left[ \frac{1}{\epsilon} + \log \left( \frac{\mu^2}{m_W^2} \right) \right].
\] (A.17)

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