N = 1 Gribov superfield extension

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Abstract
We propose a mechanism displaying confinement, as defined by the behavior of the propagators, for four-dimensional, $N = 1$ supersymmetric Yang–Mills theory in superfield formalism. In this work we intend to verify the possibility of extending the known Gribov problem of quantization of Yang–Mills theories and the implementation of a local action with auxiliary superfields, like the Gribov–Zwanziger approach, to this problem.

Keywords: Gribov, superfield, confinement

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1. Introduction

The problem of gluon confinement is a challenging issue, which is relevant in the context of the general investigation of strongly coupled gauge theories.

Recently, this problem has received great attention from different approaches. One of these approaches arises from lattice simulation where the behavior of the gluon propagator in the infrared regime is studied [1–4]. These results display a positivity violation thus making a particle interpretation for gluon excitation at low energies impossible. This is understood as a strong signal of gluon confinement. From the analytical point of view, one possible approach to the confinement problem in the Yang–Mills theories (YM), comes from the analysis of the Gribov copies [5]. This is known as the Gribov problem, where the Gribov–Zwanziger (GZ) model [6–9], and this refined version, the so-called refined Gribov–Zwanziger (RGZ) model [10] and [11] for an alternative RGZ, take place. Also, a recently developed model based on the introduction of a replica of the Faddeev–Popov action enjoys a confined gluon propagator (replica model) [12]. Usually, the GZ model provides propagators behaving as:

$$G(p^2) = \frac{p^2}{p^4 + \gamma^4},$$  \hfill (1)

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where $G(p^2)$ is the gluon form factor in Euclidean spacetime and $\gamma$ is a mass parameter (in the GZ model this parameter is known as the Gribov parameter, which is directly associated with the restriction of the Feynman path integrals to the Gribov region). It is noteworthy that the lattice simulation mentioned earlier indicates that some condensates, which can be very important in obtaining gauge propagators like in the RGZ framework, are not covered in this paper.

Note that a propagator of type (1) can be seen as a propagation of two unphysical modes with imaginary squared masses $\pm i\gamma^2$ [13], which can be shown by rewriting it as

$$
\frac{p^2}{p^4 + \gamma^4} = \frac{1}{2} \left( \frac{1}{p^2 + i\gamma^2} + \frac{1}{p^2 - i\gamma^2} \right),
$$

(2)

being impossible to identify with a propagation of a physical particle. In other words, it is a suitable candidate to be a confined object. Another way to ensure this type of propagators are not associated with physical particles is the observation that it has negative norm contributions and thus it violates positivity [14]. Despite the fact that such propagators do not have an interpretation in the physical spectrum, it is still possible to construct composite operators, $\mathcal{O}[\mathcal{A}]$, whose correlation functions exhibit a Källén–Lehmann spectral representation:

$$
\langle \mathcal{O}(p)\mathcal{O}(-p) \rangle = \int_{\tau_0}^{\infty} d\tau \frac{\rho(\tau)}{\tau + p^2},
$$

(3)

where $\rho(\tau)$ is the positive spectral density and $\tau_0 \geq 0$ stands for the threshold. An important feature of the expression (3) is that we can move from the Euclidean to Minkowski space. Moreover, the positivity of the spectral density $\rho(\tau)$ enables us to give an interpretation of (3) in terms of physical states with positive norm [12, 13].

Although it is expected that the Gribov problem also occurs in supersymmetric theories [15–18], there is still a lack of studies on its implementation and consequences. One of the most interesting possible consequences of that propagator is that it could solve the problem of infrared singularity in the first component of the gauge superfield.

In relation to the confining behavior of the supersymmetric theories, $N = 1$, much has been done since Seiberg’s work with super QCD [19]. We refer to [20] and the references therein that cover recent developments with non-perturbative results and more. However, here we focus on pure super Yang–Mills theory (SYM) without matter.

Therefore, in this paper we will investigate the SYM ($N = 1$, $D = 4$) in the superfields formalism [21] addressing the Gribov problem as well as a GZ type action. The aim is to investigate how the superfield extension of these approaches can generate propagators of the Gribov type (1), and thus shed some light on how the quantization of gauge sector can affect the fermions, even in non-supersymmetric theories. We investigate this with the proviso that in the SYM theory that we study here, the fermions are on the adjoint representation of $SU(N)$, different from quarks in QCD for example.

The paper is organized as follows. In section 2, we present the Euclidean SYM theory and investigate some details relating to gauge-fixing. In section 3, we present a gauge-invariant local action with auxiliary (anti)chiral superfields of inspiration in GZ and find that it generates the desired behavior of the confining SYM propagators. Section 4 will be devoted to a brief summary and notation and useful formulas will be presented in the appendix.

2. Superfield approach to Gribov problem, $N = 1$, $D = 4$, SYM theory

2.1. $N = 1$, $D = 4$, Euclidean SYM theory

The pure $N = 1$ Euclidean SYM action on superspace is $S' = S_{SYM} + \overline{S}_{SYM}$, where $\overline{S}_{SYM}$ is the Osterwalder–Schrader (OS) conjugate of $S_{SYM}$ [22–24]. We highlight that the Euclidean
supersymmetry has its own peculiarities [25]. For example, note that Hermitian conjugation is replaced by OS conjugation. Without this we would have to work with complex fields or \( N > 1 \). For more details refer to the references above. Keeping this in mind we can work with \( S_{SYM} \) [21, 26] given by:

\[
S_{SYM} = \frac{1}{64g^2} \text{tr} \int d^4 x \, d^2 \theta W^a W_a. \tag{4}
\]

The field strength is given by:

\[
W_\alpha = D_\alpha^2 (e^{-gV} D_\alpha e^{gV}) \tag{5}
\]

and covariant derivatives:

\[
D_\alpha = \frac{\partial}{\partial \theta^\alpha} + i \sigma^\mu \bar{\theta} \gamma^\mu \partial_{\alpha}. \tag{6}
\]

\[
\bar{D}_\dot{\alpha} = -\frac{\partial}{\partial \bar{\theta} \dot{\alpha}} - i \theta \sigma^\mu \bar{\theta} \gamma^\mu \partial_{\dot{\alpha}}. \tag{7}
\]

The supermultiplet of gauge fields is given by the components of the superfield \( V = V^a T_a \) (with \( V \) real):

\[
V(x, \theta, \bar{\theta}) = N(x) + \theta \chi(x) + \bar{\theta} \bar{\chi}(x) + \frac{1}{2} \theta^2 M(x) + \frac{1}{2} \bar{\theta}^2 \bar{M}(x) + \theta \sigma^\mu \bar{\theta} \gamma^\mu \partial \lambda(x) + \frac{1}{4} \theta^2 \bar{\theta} \gamma^2 D(x). \tag{8}
\]

They belong to the adjoint representation of the gauge group \( SU(N) \). With \([T_a, T_b] = i f_{abc} T_c\).

The gauge transformations are implicitly defined by:

\[
e^{gV'} = e^{igL_g V} e^{-ig/\Lambda_1}. \tag{9}
\]

Or, for infinitesimal \( \Lambda \):

\[
\delta_{\text{gauge}} = -\frac{i}{2} L_{gV}(\Lambda + \bar{\Lambda}) - \frac{i}{2} (L_{gV} \coth(L_{gV}/2))(\Lambda - \bar{\Lambda}), \tag{10}
\]

with the Lie derivative \( L_{gV} X = [gV, X] \) and \( \Lambda = \Lambda^a T_a \) a chiral superfield (\( \bar{D}_\dot{\alpha} \Lambda = 0 \)).

See the appendix for notation and conventions.

2.2. Gauge-fixing

Being the action (4) gauge-invariant, its quantization is similar to YM theories. Therefore, the functional integration must be restricted to a gauge nonequivalent of the subset of fields, and the operator which appears in the bilinear term is not invertible over the space of all the field configurations so that the propagator necessary to make the perturbative theory cannot be defined unless the set of fields is restricted. In this case we have to fix the gauge and we can do so covariantly using the usual procedure of Faddeev–Popov (FP) [21, 27].

Here, we present some details of this derivation to show that the Gribov problem present in YM theories (see [29, 30] for a pedagogical reviews) also arises in the generalization of the Landau gauge and FP quantization to SYM theories.

Thus, consider the functional integral for the real scalar gauge superfield \( V \):

\[
Z = \int \mathcal{D}V \, e^{-S_{SYM}(V)}, \tag{11}
\]

with \( S_{SYM} \) given by (4) and invariant over gauge transformations (10). Note that the operator that appears in the bilinear term is \( \bar{D}^2 \Pi_0 \), with the spin \( \frac{1}{2} \) projection operator given by (A.13), which is not invertible since it annihilates the chiral (anti-chiral) super-spin zero parts of \( V \) (\( \Pi_0 V = \frac{1}{16\pi} (\bar{D}^2 D^2 + D^2 \bar{D}^2) V \)). As a result, the operator has zero modes problems. And we can
verify that they are related to the gauge transformation. Considering a gauge transformation (10) for \( V = 0 \):

\[
\delta_{\text{gauge}} = \frac{i}{2}(\Lambda - \Lambda),
\]

we found that a set of chiral and anti-chiral fields appear similarly to the case of super-spin zero parts of \( V \). Therefore, we are integrating over gauge equivalent fields. As these give rise to zero modes, we ended up taking too many configurations into account. We have to choose gauge-fixing functions corresponding to the chiral (anti-chiral) gauge parameter \( \Lambda \) that can be taken away by an appropriate gauge transformation.

So we can then go ahead with the procedure of FP quantization, where the following identity (which also defines the FP determinant) is inserted into the functional (11):

\[
\Delta_F(V) \int_D D\Lambda \delta(F(V^\Lambda) - f)\delta(\bar{F}(V^\Lambda) - \bar{f}),
\]

for any chiral (anti-chiral) \( f, \bar{f} \) and gauge transformations \( F(V) \rightarrow F(V^\Lambda) \).

As the gauge-fixing will be a supersymmetric extension of the Lorentz (or Landau) gauge \( \partial^\mu a_\mu = 0 \) and noting that \( \partial^\mu a_\mu \) is a component of chiral (anti-chiral) superfield \( \bar{D}^2D^2V \), we must implement the conditions \( \bar{D}D^2V = 0, D\bar{D}V = 0 \). Therefore in the quantization procedure, \( F = D\bar{D}V \) and \( \bar{F} = D^2\bar{D}V \) are suitable gauge-fixing functions.

And following the usual procedure we ended with the action of gauge-fixing (Landau gauge)

\[
S_{gf} = -\frac{1}{16}\left\{ \text{tr} \int d^4x d^4\theta(c'\bar{D}^2sV + \bar{c}'\bar{D}^2sV) \right\},
\]

where the Faddeev–Popov ghost fields will be chiral (anti-chiral) superfield as the gauge parameter \( \Lambda \) (\( \bar{\Lambda} \)).

\[
c' = \epsilon^a T_a \quad \text{and} \quad c = \epsilon^a T_a \quad \text{are the antighost and the ghost respectively. And } s \text{ is the BRST nilpotent operator } (s^2 = 0).
\]

The total action \( S_{SYM} + S_{gf} \) is invariant under the BRST transformations [21, 28]:

\[
S_V = \delta_{\Lambda}V_{|_{\Lambda=ic}}
\]

\[
S_V = \left[ \frac{1}{2}L_{qV}(c + \bar{c}) + \frac{1}{2}(L_{qV}\coth(L_{qV}))(c - \bar{c}) \right]
\]

\[
= \left\{ -\bar{c} + c + \frac{1}{2}[gV, c + \bar{c}] + \ldots \right\}
\]

\[
sc = -\epsilon^a \left( sc^a = \frac{i}{2}f_{abc}c^b\epsilon^c \right)
\]

\[
s\bar{c} = -\bar{\epsilon}^2
\]

\[
s\bar{c}' = \bar{B}
\]

\[
sB = 0
\]

\[
s\bar{B} = 0.
\]

With \( s \) carrying ghost number 1.

The ghost part of gauge-fixing action becomes:

\[
S_{FP} = \frac{1}{16}\text{tr} \int d^4x d^4\theta(\bar{c}'D^2sV + \bar{c}'\bar{D}^2sV).
\]
An important detail is that the Jacobian $\Delta F(V)$ in (13) contains a determinant of type FP:

$$\begin{vmatrix}
\frac{\delta F(V^\Lambda)}{\delta \Lambda} + \frac{\delta \tilde{F}(V^\Lambda)}{\delta \Lambda} \\
\frac{\delta \tilde{F}(V^\Lambda)}{\delta \bar{\Lambda}} + \frac{\delta \bar{F}(V^\Lambda)}{\delta \bar{\Lambda}}
\end{vmatrix}.$$  

With the variational derivatives of $F$ and $\bar{F}$ evaluate at $\bar{\Lambda} = \Lambda = 0$. And so in our case we have the operators:

$$D^2D^2 \left[ -\frac{i}{2} L_{gV} \bullet \left( L_{gV} \text{coth}(L_{gV}/2) \right) \bullet \right] (17)$$

where the symbol $\bullet$ is to indicate that those operators act in chiral and anti-chiral superfields, which may have zero mode problems making the gauge-fixing still incomplete.

2.3. Gribov problem

Similarly to YM let us now explicitly show that in the Landau gauge the gauge condition is not ideal as already noted above. Consider two equivalents fields $V \leftrightarrow V'$ connected by the gauge transformation (10), if both satisfy the same condition of Landau gauge $D^2D^2V = 0$, $D^3D^2V = 0 \leftrightarrow D^3D^2V' = 0$, we have

$$D^2D^2 \left( (\bar{\Lambda} - \Lambda) - \frac{i}{2}[gV, \bar{\Lambda} + \Lambda] + \ldots \right) = 0 (19)$$

And on-shell, the problem of zero modes becomes $(D^2D^2\Lambda = 16\partial^2\Lambda$ from (A.11)):

$$\left( -16\partial^2 \bullet -\frac{i}{2}[gV, 16\partial^2 \bullet] + \ldots \right) \Lambda = 0 (21)$$

Thus the existence of infinitesimal copies, even after FP quantization, is linked to the existence of zero modes of the operators above, which we can see as the supersymmetric generalization of FP operator.

For zero $V$ we can construct in the space of chiral and anti-chiral superfields the eigenvalue equations

$$-16\partial^2 \Lambda = \lambda_1 \Lambda$$

and

$$16\partial^2 \bar{\Lambda} = -\lambda_2 \bar{\Lambda}$$

that only has positive eigenvalues $\lambda_1 = \lambda_2 = 16p^2 > 0$. Thus for small values of $V$, we can expect that the eigenvalues $\lambda_1(V)$ and $\lambda_2(V)$ are greater than zero. However, for large $V$, this can no longer be guaranteed, and negative eigenvalues may appear for $V$ sufficiently large. And thus the above operators will also have null eigenvalues. This means that our gauge condition is not ideal. It is noteworthy that chiral and anti-chiral eigenvalues equations are present, e.g., in context of superinstantons [23, 31, 32].

To view the issue of zero modes let us take the equation (23) as an equation of eigenvalues to first order in $V$:

$$\left( -16\partial^2 \bullet -\frac{i}{2}[gV, 16\partial^2 \bullet] \right) \Lambda = \lambda \Lambda.$$

(25)
We remark that as in the non-supersymmetric case we can understand this equation as an Schrödinger type equation (being here supersymmetric), where the second member plays the role of a potential, and study its eigenvalues. For example see reference [33] with a delta potential and references therein.

Therefore, we have shown that Gribov problem also exists in SYM in the Landau gauge. We will see in the next section how to implement a possible solution to the Gribov problem, the local action of super GZ, and how it modifies the propagators for the Gribov type (1).

3. Gribov–Zwanziger local action on superspace

Now that we have shown that the FP quantization is incomplete in the sense of the previous section, it is necessary to improve the gauge-fixing. Gribov proposes for YM theories to be further restricted to a region of integration, the so-called Gribov region, which is defined as the region of gauge fields obeying the Landau gauge and for which the Faddeev–Popov operator is positive definite. Whereas in our case the operator is as it appears in (21) and (22). We then assume that this procedure can be generalized and thus the region of integration can be restricted in superspace.

To implement such a restriction we go directly to the GZ approach to construct an action which implements a restriction to the Gribov region order by order. We begin by observing that the operators in question are (within a factor ±i) the same as those that appear in the ghosts sector (16), i.e., the expectation value of the Faddeev–Popov operator is the inverse ghost propagator, which suggests how the auxiliary fields, Zwanziger-style, should appear in the action. This is achieved by introducing auxiliary superfields in the form of two quartets of BRST, one with chiral:

\[
s w' = u', \quad s u = w
\]

\[
s' u = 0, \quad s w = 0,
\]

(26)

and another with anti-chiral superfields:

\[
s \bar{w}' = \bar{u}', \quad s \bar{u} = \bar{w}
\]

\[
s' \bar{u} = 0, \quad s \bar{w} = 0.
\]

(27)

At this point it is important for our construction to show the ultraviolet dimension and ghost number of all fields and operators (see table 1).

Thus, keeping in mind the non-supersymmetric approach, the super GZ action must be of the form:

\[
S_{SGZ} = \text{tr} \int d^4x d^4\theta s \left[ w' D^2 \left( \frac{1}{2} L_{\theta'} (u + \bar{u}) + \frac{1}{2} (L_{\theta'} \coth (L_{\theta'}))(u - \bar{u}) \right) \right]
\]

\[
+ \bar{w}' D^2 \left( \frac{1}{2} L_{\bar{\theta}'} (u + \bar{u}) + \frac{1}{2} (L_{\bar{\theta}'} \coth (L_{\bar{\theta}'}))(u - \bar{u}) \right)
\]

\[
+ \gamma^2 \text{tr} \int d^4x d^4\theta V(u - \bar{u}) + \gamma^2 \text{tr} \int d^4x d^2\bar{\theta} V u' + \gamma^2 \text{tr} \int d^4x d^2\theta \bar{V} \bar{u}'.
\]

(28)

Where \( \gamma^2 \) is a mass parameter, which should be determined by taking into account the full action, shown below, and must be nonzero.

Thus, the total action is:

\[
S = S_{SYM} + S_{gf} + S_{SGZ}.
\]

(29)
At this point it is important to emphasize that the term $\gamma^2 \text{tr} \int d^4x d^4\theta V(u) u$ breaks the BRST symmetry similarly to the non-supersymmetric GZ action. This term is a soft breaking term. There are many methods to ensure the ultraviolet renormalization of the action from the introduction of classical sources in order to treat this soft breaking term as an insertion \cite{8, 9}, to a mechanism that transforms this breaking into a classical linear breaking \cite{34–36}. It is not the purpose of this work to study the renormalizability of the action. In spite of that, the way to generalize the GZ action to a supersymmetric one is so close to the original GZ procedure, that we expect that the supersymmetric GZ is also ultraviolet renormalizable.

Now that the GZ action is generalized, it is important that the propagators calculation verifies that they have the expected behavior which occurs in non-supersymmetric GZ–YM and to analyze other possible features that are added to SYM.

### 3.1. The super gauge propagator

First, in order to calculate the propagator for the gauge superfield $V$, we need only the bilinear part of $S_{\text{SGZ}}$. We have:

$$S_{\text{SGZ}} = \text{tr} \int d^4x d^4\theta s \{ w' D^2 (u - \bar{u}) + \bar{w} \bar{D}^2 (u - \bar{u}) \} + \gamma^2 \text{tr} \int d^4x d^4\theta V (u - \bar{u})$$

$$\quad + \gamma^2 \text{tr} \int d^4x d^4\theta V u' + \gamma^2 \text{tr} \int d^4x d^4\theta \bar{V} \bar{u}$$

(30)

and for terms with $u, u'$

$$S_{\text{SGZ}} = \text{tr} \int d^4x d^4\theta \left( \frac{1}{4} u' \bar{D}^2 D^2 u + \frac{1}{4} \gamma^2 V \bar{D}^2 \bar{u} + \frac{1}{4} \gamma^2 V u' \right)$$

$$\quad + \text{tr} \int d^4x d^4\theta \left( \frac{1}{4} u' D^2 \bar{D}^2 \bar{u} - \frac{1}{4} \gamma^2 V D^2 u + \frac{1}{4} \gamma^2 V \bar{u} \right),$$

(31)

which gives one contribution to the bilinear term $(\bar{D}^2 D^2 u = 16 \delta^2 u)$

$$S_{\text{SGZ}} = -\text{tr} \int d^4x d^4\theta V \frac{\gamma^4}{2 g^2}$$

(32)

So, the free total action yields the field equation for $V$, inserting your source $J_V$:

$$\frac{1}{16} D^2 \bar{D}^2 D a V - \frac{\gamma^4}{2 g^2} V + \frac{1}{16} D^2 B + \frac{1}{16} \bar{D}^2 B = -J_V.$$  

(33)

Using transverse operator $\Pi_{\perp}$, and $\Pi_{\perp} V = V$, this action gives rise to a propagator of the form (in space coordinates):

$$\Delta_{\perp}^V (1, 2) = -\frac{2 a^2}{g^2 + \gamma^4} \Pi_{\perp} \delta^4 (x_1 - x_2) \delta^4 (\theta_1 - \theta_2).$$

(34)

Where $\delta^4 (\theta_1 - \theta_2) = (\theta_1 - \theta_2)^2 (\bar{\theta}_1 - \bar{\theta}_2)^2$.

To see how the introduction of $S_{\text{SGZ}}$ illuminates the confinement of both bosons as fermions, we shall observe the propagators in field components.

Using (A.17) we can project the propagator for the gauge field $a_\mu$ and gaugino $\lambda^\alpha$:

$$\Delta_{a, a}^\perp (1, 2) = -\frac{2 a^2}{g^2 + \gamma^4} \left( \delta^4 (x_1 - x_2) \right) \delta^4 (\theta_1 - \theta_2)$$

(35)

Table 1. Quantum numbers of fields and operators.

| Fields and operators | $\theta^a$ | $D_u$ | $V$ | $c'$ | $c$ | $B$ | $w'$ | $w$ | $u'$ | $u$ |
|---------------------|-----------|------|-----|------|-----|-----|------|-----|------|-----|
| UV dimension        | $-\frac{1}{2}$ | $\frac{1}{2}$ | 0   | 1    | 0   | 1   | 0    | 1   | 0    | 1   |
| Ghost number        | 0         | 0    | 0   | $-1$ | 1   | 0   | $-1$ | 1   | $-1$ | 1   |
\[ \Delta^c_{\lambda,\bar{\lambda}}(1,2) = \frac{5 \, i a^2}{2 \, \delta^2 + \gamma^2} \delta^4(\theta_1 - \theta_2). \] (36)

We found that both show similar behavior as the gluons in non-supersymmetric theories (1). Namely, we obtain propagators with confining behavior in an integrated manner for the bosons and fermions in this model.

Another field component projection we analyze is the dimensionless and massless component of the gauge superfield \( V \) (the \( \theta = 0 \) component) whose propagator also becomes Gribov type

\[ \Delta^c_{HN}(1,2) = \frac{4}{\delta^2 + \gamma^2} \delta^4(x_1 - x_2). \] (37)

And so this naturally solves a problem characteristic of supersymmetrics theories in four dimensions, that is the appearance of a infrared singularity in this \( V \) component [37], here named \( N(x) \). This also indicates that the parameter \( \gamma \) must be different from zero, at least in this framework, to avoid this infrared singularity.

3.2. Ghost propagators and \( \gamma \) parameter

Since the action (29) only makes sense if the \( \gamma \) parameter is nonzero, we will now explicitly show that it is not independent in this theory. Its determination is closely linked to the restriction of the functional integration to the first Gribov region, which we will discuss in some detail here.

Firstly, it is noteworthy that in the literature dealing with the Gribov problem in YM theories, there has been a recent consensus on the scenario of dominance of configurations on the Gribov horizon on the Landau gauge [30], so that the restriction to the first Gribov region is, in practice, to take the configurations on the horizon, i.e. where occur the zeros modes of the FP operator, in our case given by equations (17, 18). Secondly, and as we have pointed out in the introduction of super GZ, to calculate the propagator of the ghosts is to take the inverse of these operators. Therefore, we focus on these calculus to one loop order to establish the one loop gap equation Gribov style.

In order to characterize the integration in the first Gribov region it is important to remember that the two point ghost function is essentially the inverse of the Faddeev–Popov operator and the zero eigenvalue of the Gribov equation corresponds exactly to the Gribov frontier. In this sense the two point ghost function continues to infinity at the Gribov frontier. This condition is the simplest way to obtain the gap equation for \( \gamma \). The procedure is explained in detail in [5] and is easily extended to the \( N = 1 \) supersymmetric case.

Firstly we need to calculate the two point ghost function, using perturbation theory at the first order of the form:

\[ \begin{align*}
\Delta^c_{\cdot,\bar{\cdot}}(1,2) &= \frac{1}{\delta^2} \overline{D^2} \delta^4(x_1 - x_2) \delta^4(\theta_1 - \theta_2) \\
\Delta^c_{c,c}(1,2) &= -\frac{1}{\delta^2} \overline{D^2} \delta^4(x_1 - x_2) \delta^4(\theta_1 - \theta_2).
\end{align*} \] (38) (39)
And we can define in momentum space, the one loop corrected ghost propagator as

\[ G_{c\bar{c}} = (G_{c\bar{c}}^{0} + G_{c\bar{c}}^{1}) \],

(40)

according to diagram above. With \( G_{c\bar{c}}^{0} \) given from (38):

\[ G_{c\bar{c}}^{0} = -\frac{\delta_{ab}}{p^2} \bar{D}^2 \delta^4(\theta_1 - \theta_2). \]  

(41)

Using the improved Feynman rules and D algebra \([21, 26, 38, 39]\) and after delta functions and D derivatives manipulations (and taking into account that \( f_{acd} f_{bcd} = N \delta_{ab} \) in \( SU(N = 2) \)), we have:

\[ G_{c\bar{c}}^{1} = -\frac{1}{2\pi^4} g^2 N \delta_{ab} \frac{1}{p^2} \bar{D}^2 (p) \delta^4(\theta_1 - \theta_2) \int \frac{d^4k}{(2\pi)^4} \frac{k^2}{k^4 + \gamma^4 (p - k)^2}. \]  

(42)

Next we define:

\[ \sigma(p^2, \gamma^2) = (2\pi)^4 g^2 N \int \frac{d^4k}{(2\pi)^4} \frac{k^2}{k^4 + \gamma^4 (p - k)^2}. \]  

(43)

Therefore, from (40):

\[ G_{c\bar{c}}^{ab} = -\frac{\delta_{ab}}{p^2} \bar{D}^2 \delta^4(\theta_1 - \theta_2) (1 + \sigma). \]  

(44)

Re-summing the one-particle irreducible diagrams gives:

\[ G_{c\bar{c}}^{ab} = -\frac{\delta_{ab}}{p^2} \bar{D}^2 \delta^4(\theta_1 - \theta_2) \frac{1}{1 - \sigma}. \]  

(45)

Now, as we are interested in the low momentum behavior we analyze the behavior of \((1 - \sigma)\) with \( k \approx 0 \), ie we get \( \sigma(0, \gamma^2) \):

\[ \sigma(0, \gamma^2) = (2\pi)^4 g^2 N \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^4 + \gamma^4}. \]  

(46)

Therefore we are able to define the one loop gap equation according to the above discussion of the scenario of dominance of configurations on the Gribov horizon, ie the ghost propagator (the inverse of FP operator) going to infinity, \((1 - \sigma) = 0\):

\[ (2\pi)^4 g^2 N \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^4 + \gamma^4} = 1. \]  

(47)

It is worth mentioning that for calculating the propagator to one loop for the anti-chiral fields \( G_{\bar{c}\bar{c}}^{ab} \) we have

\[ G_{\bar{c}\bar{c}}^{ab} = \delta_{ab} \frac{1}{p^2} D^2 \delta^4(\theta_1 - \theta_2) \frac{1}{1 - \sigma}, \]  

(48)

which has the same integral (47) which defines the \( \gamma \) parameter.

Note that the integral in equation (47) can be well defined with dimensional regularization and that similar calculations can be found in the literature \([29]\).

Thus the \( \gamma \) parameter is not independent as it is defined for the one loop gap equation (47). It is clear that in close analogy to the Gribov–Zwanziger procedure \([7–9]\) it is also possible to work directly with the gap equation \( \frac{d\gamma}{d\gamma} = 0 \). The results will be the same as in the more simple method explained in this section.
4. Conclusions

In this work we have studied the super Yang–Mills theory, $N = 1, D = 4$, considering the Gribov problem present in YM theories as well as a generalization of the Gribov–Zwanziger approach with auxiliary superfields in superspace. Using this approach we find that the confining behavior is induced by this supersymmetric theory. The results presented here are the first step toward a more extensive investigation and further suggest that the Gribov problem, and its solutions, can be treated consistently in supersymmetric theories. And by doing so one can shed light on the wider issues of these theories. In our case, this approach allows the solution of the well known problem of infrared supersymmetric theories in four-dimensional space and the theory becomes confining.

It is worth mentioning that we believe it is possible to make a renormalization of this theory since the inclusion of auxiliary superfields means the only term that breaks supersymmetry is a soft breaking term. This possibility is under investigation. A further possibility under investigation is that a supersymmetric breaking, together with the Gribov mechanism, can be important to the confinement of the fermions. The construction of a supersymmetric breaking that preserves the confining propagators for fermions and that leads them on fundamental representation of SU($N$) are the next steps to be completed in this direction and is also to be investigated. Other research possibilities in this framework are the supersymmetric version of RGZ and replica model mentioned in the introduction, as well as models with extended supersymmetry. It still would be interesting to check possible condensates.

Finally we would like to express our hope that the four-dimensional, $N = 1$ superfields formalism can provide some insight on non-supersymmetric theories such as QCD, where there is still a search for an integrated treatment for problems such as non-perturbative confinement of quarks and gluons and chiral symmetry breaking (see introduction of [40] for some discussion).

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Appendix. Notation, conventions and some useful formulas

We work with Euclidean metric: $\delta_{\mu\nu} = \text{diag}(++++)$, with Wick rotation from a theory in Minkowski space: $d^4x \rightarrow d^4x, x^4 \rightarrow ix^0$. Euclidean $\sigma$ - matrices ($\sigma_i$ - Pauli matrices) are defined as follows [41]:

$$\sigma^\mu = \sigma_\mu = (\sigma_i, i) \quad (A.1)$$

$$\bar{\sigma}^\mu = \bar{\sigma}_\mu = (-\sigma_i, i), \quad (A.2)$$

which are OS self-conjugate, and can include the following relations:

$$\bar{\sigma}^{\mu\alpha\alpha} = \epsilon^{\alpha\beta\gamma\delta} \epsilon^{\alpha\beta\gamma\delta} \sigma^\mu \quad (A.3)$$

$$\text{tr} \sigma_\mu \sigma_\nu = -2\delta_{\mu\nu} \quad (A.4)$$

$$\sigma^\mu_\alpha \bar{\sigma}^{\mu\beta} = -2\delta^\beta_\alpha \delta^\beta_\alpha. \quad (A.5)$$
Some supersymmetric conventions and useful formulas:
\[ \bar{\theta}^a \theta^b = -\frac{1}{2} \epsilon^{ab} \bar{\theta}^2 \] (A.6)
\[ \bar{\theta}^a \bar{\theta}^b = \frac{1}{2} \epsilon^{ab} \bar{\theta}^2. \] (A.7)

Covariant derivatives:
\[ D_\alpha = \frac{\partial}{\partial \theta^\alpha} + i \sigma^\mu_{a\bar{a}} \bar{\theta}^\alpha \partial_\mu \] (A.8)
\[ \bar{D}_\dot{\alpha} = -\frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}}} - i \theta^a \sigma^\mu_{a\bar{a}} \partial_\mu \] (A.9)
\[ [D_\alpha, \bar{D}_{\dot{\alpha}}] = -2i \sigma^\mu_{a\bar{a}} \partial_\mu \] (A.10)
\[ [D^2, \bar{D}^2] = -8i (D \sigma^\mu \bar{D}) \partial_\mu - 16 \bar{\theta}^2 \] (A.11)
\[ \int d^2 \theta = -\frac{1}{4} D^2, \quad \int d^2 \bar{\theta} = -\frac{1}{4} \bar{D}^2. \] (A.12)

Note that with these definitions our notation in superspace take the same form as in [42]. Finally we present the projection operators and their relations with delta functions:
\[ \Pi_1^+ = -\frac{D^2 \bar{D} 
^2 D_\alpha}{8 \bar{\theta}^2} = -\frac{\bar{D}^2 D^2 \bar{D}_\alpha}{8 \bar{\theta}^2} \] (A.13)
\[ \Pi_0^{++} = \frac{D^2 \bar{D}^2}{16 \bar{\theta}^2} \] (A.14)
\[ \Pi_0^{--} = \frac{D^2 \bar{D}^2}{16 \bar{\theta}^2} \] (A.15)
\[ \Pi_0 = \Pi^{++}_0 \pm \Pi^{--}_0, \quad \Pi_0 + \Pi_1^+ = 1 \] (A.16)
\[ \Pi_1^+ \delta^4(\theta_1 - \theta_2) = -\frac{1}{2 \bar{\theta}^2} e^{i(\theta_i \sigma^\mu \partial_\mu - \theta_i \bar{\sigma}^\mu \bar{\partial}_\mu)} (4 - \partial^2 \theta_1 - \theta_2)^2 (\bar{\theta}_1 - \bar{\theta}_2)^2 \] (A.17)
\[ \bar{D}_1^2 \delta^4(\theta_1 - \theta_2) = -4 (\theta_1 - \theta_2)^2 e^{i(\theta_i \sigma^\mu \partial_\mu - \theta_i \bar{\sigma}^\mu \bar{\partial}_\mu)} \] (A.18)
\[ D^2 \delta^4(\theta_1 - \theta_2) = -4 e^{-i(\theta_i \sigma^\mu \partial_\mu - \theta_i \bar{\sigma}^\mu \bar{\partial}_\mu)} (\bar{\theta}_1 - \bar{\theta}_2)^2. \] (A.19)

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