Monte Carlo Simulations of Star Clusters

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Abstract. A revision of Stodólkiewicz’s Monte Carlo code is used to simulate evolution of large star clusters. The survey on the evolution of multi–mass N–body systems influenced by the tidal field of a parent galaxy and by stellar evolution is discussed. For the first time, the simulation on the ”star–by–star” bases of evolution of 1,000,000 body star cluster is presented.

1. Introduction

Very detailed, recent observations of globular clusters suggest very close interplay between stellar evolution, binary evolution and dynamical interactions. This interplay is far from being understood. Monte Carlo codes, which use a statistical method of solving the Fokker–Planck equation provide all the necessary flexibility to disentangle the mutual interaction between physical processes important during globular cluster evolution. The codes were developed by Spitzer (1975, and references therein) and Hénon (1975, and references therein) in the early seventies, and substantially improved by Marchant & Shapiro (1980, and references therein) and Stodólkiewicz (1986, and references therein) and recently reintroduced by Giersz (1998, 2000), Heggie et al. 1999, Joshi et al. (1999a), Joshi et al. (1999b, hereafter JNR) and Rasio (2000, this volume). The Monte Carlo scheme takes full advantage of the undisputed physical knowledge of the secular evolution of (spherical) star clusters as inferred from continuum model simulations. Additionally, it describes in a proper way the graininess of the gravitational field and the stochasticity of the real N–body systems and provides as detailed as in direct N–body simulations information about movement of any objects in the system. This does not include any additional physical approximations or assumptions which are common in Fokker–Planck and gas models (for e.g. conductivity). Because of this, the Monte Carlo scheme can be regarded as a method which lies between direct N–body and Fokker–Planck models and combines most of their advantages. Moreover, Monte Carlo codes are simple, very fast, easily parallelized and easily scalable to the physical units. There is no need for special hardware or supercomputers to efficiently simulate evolution of realistic star clusters. However, as any numerical method, the Monte Carlo method suffers from some disadvantages. It can only deal with spherically symmetrical systems, and only small–angle two–body interactions. The galactic tidal field can only be approximated by the tidal cut-off, and unfortunately, cross–sections for some physical processes are needed (e.g. three–body binary collisions).
formation). Additionally, the physical processes, which evolve on time–scales comparable to the crossing time–scale, can not be properly investigated, and the method has difficulty with the proper definition of local parameters (e.g. density, velocity dispersion). Despite all of these disadvantages the Monte Carlo method can be easily and efficiently used to simulate evolution of realistic globular clusters. The comparison between numerical simulations and observations will help to infer the initial parameters of proto–clusters and help to disentangle the interplay between physical processes involved in cluster evolution. Moreover, the Monte Carlo method can be used to simulate dynamical formation of massive black holes in dense spherical stellar systems (e.g. galactic nuclei).

2. Results

The Monte Carlo code is described in detail in Giersz (1998), which deals with simulations of isolated single–mass systems. Here and in Giersz (2000) the Monte Carlo code is extended to include the following additional physical processes:

- multi–mass systems described by the power–law initial mass function:
  \[ N(m)dm = Cm^{-\alpha}dm, \quad m_{\text{min}} \leq m \leq m_{\text{max}}, \]
  where \( C \) and \( \alpha \) are constants.

- stellar evolution introduced according to prescription given by Chernoff & Weinberg (1990, hereafter CW) or Taut et al. (1997).

- three–body binaries described by the suitably modified Spitzer’s formula (Spitzer 1987, Giersz 2000).

- binary–binary interactions introduced according to Mikkola (1983, 1984) and Stodólkiewicz (1986).

- tidal field simulated by tidal cut-off with energy and/or apocenter criterion.

The results of Monte Carlo simulations of star cluster evolution will be presented in the next two subsections.

2.1. Family \( \sim 1 \)

The initial conditions were chosen in a similar way as in a collaborative experiment (Heggie et al. 1999). The positions and velocities of all stars were drawn from a King model. All standard models have the same total mass \( M = 60000M_\odot \) and the same tidal radius \( R_c = 30 \) pc. Masses are drawn from the power–law mass function described above. The minimum mass was chosen as \( 0.1M_\odot \) and maximum mass as \( 15M_\odot \). Three different values of the power–law index were adopted: \( \alpha = 1.5, 2.35 \) and \( 3.5 \). The set of initial King models was characterized by \( W_0 = 3, 5 \) and \( 7 \). Additional models of CW’s Family 1 were computed to facilitate comparison with results of other simulations (minimum mass equal to \( 0.4M_\odot \), \( \alpha = 1.5, 2.5, 3.5 \) and total mass \( M = 90685M_\odot, 99100M_\odot, 103040M_\odot \), respectively).

In Table 1 the comparison between available results of \( N \)-body, Fokker–Planck and Monte Carlo simulations is presented. The standard models show a remarkably good agreement with \( N \)-body results (Heggie 2000). See columns
labeled by G and H-0.1 in Table 1. Only models with a flat mass function show some disagreement. These models are difficult for both methods. Violent stellar evolution and induced strong tidal striping lead to troubles with time-scaling for the N-body model and proper determination of the tidal radius for the Monte Carlo model. Generally, the same is true for Monte Carlo models of Family 1. Results of these models show good agreement with results of CW, Aarseth & Heggie (1998) and Takahashi & Portegies Zwart (1999). JNR's results, particularly for strongly concentrated systems, disagree with all other models. This can be connected with the fact, that JNR's Monte Carlo scheme is not particularly suitable for high central density and strong density contrast. Too large deflection angles adopted by JNR and consequently too large time-steps can lead to too fast evolution in these models.

### Table 1. Time of cluster collapse or disruption $^a$

| Model      | CW  | TPZ | JNR | H   | H-0.1 | G    | G-0.4 |
|------------|-----|-----|-----|-----|-------|------|-------|
| W3235/25   | 0.28| 2.2 | 5.2 | 2.1 | 11.3 | 6.3  | 0.7   |
| W335       | 21.5| 32.0| 31.0| >20.0| 16.0 | 17.6 | 26.0  |
| W515$^b$   | -   | -   | -   | 0.2 | 0.5  | 0.1  | 0.07  |
| W5235/25   | -   | -   | -   | 13.5| 7.0  | 6.8  | 13.2  |
| W535       | -   | -   | -   | >20.0| 6.0  | 7.0  | 26.1  |
| W715$^b$   | 1.0 | 3.1 | 3.1 | 1.2 | 3.4  | 2.1  | 2.8   |
| W7235/25   | 9.6 | 10.0| 3.0 | 11  | 1.7  | 1.9  | 9.8   |
| W735       | 10.5| 9.9 | 6.0 | 9.2 | 0.8  | 0.7  | 10.7  |

$^a$ Time is given in $10^9$ yr.,
The first number after W describes the King model and the following numbers the mass function power-law index.

CW — Chernoff & Weinberg (1990) — Family 1,
TPZ — Takahashi & Portegies Zwart (1999) — Family 1,
JNR — Joshi et al. (1999b) — Family 1,
H — Aarseth & Heggie (1998) — Family 1,
H-0.1 — Heggie (2000) — standard model — $m_{min} = 0.1 M_{\odot}$,
G — Giersz — standard model — $m_{min} = 0.1 M_{\odot}$,
G-0.4 — Giersz — Family 1,

$^b$ Cluster was disrupted, other models collapsed.

All standard models, for which mass loss due to violent stellar evolution of the most massive stars does not induce quick cluster disruption, evolve in a very similar way. The rate of mass loss, evolution of the central potential, evolution of the average mass does not depend much on the initial central concentration of the system. They depend strongly on the index of the mass function. Models of Family 1, on the contrary show, as well, dependence on the initial concentration. Very high initial mass loss across the tidal boundary, connected with evolution of the most massive stars (for models of Family 1, there are more massive stars than for standard models), forces the system to substantial changes of its structure.
and in consequence to different evolution of the total mass, anisotropy, etc. Models which are quickly disrupted show only small signs of mass segregation. Models with larger central concentration survive the phase of rapid mass loss and then undergo core collapse and subsequent post-collapse expansion in a manner similar to isolated models. The expansion phase is eventually reversed when tidal limitation becomes important. As in isolated models, mass segregation substantially slows down by the end of the core collapse. After a core bounce there is a substantial increase in the mean mass in the middle and outer parts of the system, caused by the preferential escape of stars of low mass and tidal effects. Standard models, which are not quickly disrupted, show modest initial build up of anisotropy in the outer parts of the system. As the tidal stripping exposes inner parts of the system, anisotropy gradually decreases and eventually becomes slightly negative. The central part of the system stays nearly isotropic.

Models of Family 1, from the very beginning, develop in the outer parts of the system modest negative anisotropy. It stays negative until the time of cluster disruption, when it becomes slightly positive (during cluster disruption most stars are on radial orbits).

2.2. 1,000,000 body run

For the first time, the Monte Carlo simulation, on the "star–by–star" bases, of evolution of 1,000,000 body star cluster is presented. The initial conditions were as follows: total mass equal to $319,305 M_\odot$, tidal radius equal to $33.57 pc$, power–law index of mass function equal to $-2.35$, minimum and maximum mass equal to $0.1 M_\odot$ and $15.0 M_\odot$, respectively and King model parameter $W_0 = 5$.

The 1,000,000 body run shows basically the same features as, discussed above, models of Family $\sim 1$. As an example of the overall cluster evolution the time dependence of Lagrangian radii, core radius and tidal radius are presented in Figure 1. The three different phases of evolution can be clearly distinguished. First, short phase of violent mass loss due to stellar evolution leads to overall cluster expansion. Even the innermost Lagrangian radius expands, the contraction connected with mass segregation is not strong enough to dominate the expansion. Second phase is characterized by the slow core collapse. Tidal effects are small and cluster behaves in a similar way as an ordinary isolated system. Then in the third phase, post-collapse evolution is superposed with growing tidal stripping effects. The cluster nearly homogeneously contracts. The central parts of the system show clear signs of the gravothermal oscillations.

In Figure 2 the density profiles for the different epochs are presented. It is clear, that in the central parts of the system the density profile shows steeper slope than $-2.2$ (line labeled by 6), the standard value for single–mass systems. This is in agreement with results of CW. The power–law index is a function of the ratio of mass of the most massive stars to the average mass. The larger the ratio the smaller the power–law index. The core is mainly populated by the most massive stars (massive white dwarfs, neutron stars and black holes), whose masses are larger than the average mass in the vicinity of the core. So as a consequence, the power–law index is smaller than $-2.2$. The density profiles for an advanced collapse phase (lines labeled by 4, 5 and 6) show a bump in the middle part of the system (close to 0.08 in x–axis). The bump originates because of the growing influence of low mass stars on the determination of the
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Figure 1. Evolution of the Lagrangian radii, core radius and tidal radius, labeled by 0.5%, 10%, 50%, 99%, Rc and Rtide, respectively.

Figure 2. Density profiles for the different epochs (labels from 1 to 6 on the figure). The straight line indicates the power-law with exponent -2.2.
local density. The position and the size of the bump is in a good agreement with Fokker–Planck results (Takahashi & Lee 2000).

In order to perform simulations of real globular clusters several additional physical effects have to be included into the code. The tidal shock heating of the cluster due to passages through the Galactic disk, interaction with the bulge, shock–induced relaxation, primordial binaries, physical collisions between single stars and binaries are one of them. Inclusion of all these processes do not pose a fundamental theoretical or technical challenge. It will allow to perform detailed comparison between simulations and observed properties of globular clusters and will help to understand the globular cluster formation conditions and explain how peculiar objects observed in clusters can be formed. These kinds of simulations will also help to introduce, in a proper way, into future N–body simulations all necessary processes to simulate on the star–by–star basis evolution of real globular clusters from their birth to death.

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