Surprises in threshold antikaon-nucleon physics

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Low energy \( \bar{K}N \) interactions are studied within Unitary Chiral Perturbation Theory at next-to-leading order with ten coupled channels. We pay special attention to the recent precise determination of the strong shift and width of the kaonic hydrogen 1s state by the DEAR Collaboration that has challenged our theoretical understanding of this sector of strong interactions. We typically find two classes of solutions, both of them reproducing previous data, that either can or cannot accommodate the DEAR measurements. The former class has not been previously discussed.

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1. Low energy antikaon-nucleon interactions have been object of extensive study almost for the last 50 years. Based on early data on \( K^-p \) scattering, Dalitz and Tuan predicted \(^1\) in 1959 the existence of a sub-threshold \( \bar{K}N \) resonance, the \( \Lambda(1405) \), first seen experimentally three years later \(^2\). Despite this success, \( K^-p \) scattering is still challenging our understanding of strong interactions. First, this resonance, being too light, appears puzzling for now prevailing believes and claims of a shallow potential. This is of foremost importance as it is a way to obtain definite strongly attractive in contrast with the up to years later \(^2\). Despite this success, this resonance, being too light, appears puzzling for now prevailing believes and claims of a shallow potential. This is of foremost importance as it is a way to obtain definite strongly attractive in contrast with the up to

Second, there has been disagreement between the sign of the \( K^-p \) scattering lengths extracted either from scattering or from the 1s \( K^-p \) atomic level shift until 1998 when it was settled down by the KpX experiment at KEK \(^8\). Now, the around factor of two more precise DEAR measurement \(^4\) brings in a further disagreement with all previous theoretical results from SU(3) chiral dynamics, which are however compatible with the KEK measurement \(^8\). Third, the physical \( \Lambda(1405) \) has not yet been considered up to very recently \(^7\) as the admixture of two nearby poles, so that different reactions weighting more one pole or the other produce different resonant shapes peaking at different energies. For experimental evidences on this issue see \(^13\). Fourth, the recently discovered strange tri-baryons \( S^0(3115) \) and \( S^1(3140) \) have most likely shown that deeply bound states of \( K \), as predicted in \(^13\) and even deeper, do exist. The \( K \)-nucleus potential is therefore definitely strongly attractive in contrast with the up to now prevailing believes and claims of a shallow potential. This is of foremost importance as it is a way to obtain very dense nuclear matter \(^15\), \((3 \sim 4) \times \rho_0\) as well as to get kaon condensation in nuclear matter (e.g. neutron stars) \(^16\), or strangeness clusters in heavy-ion collisions. A sounder theoretical explanation of such strongly attractive \( K \)-nucleus potential is called for. Fifth, there is an exhaustive search of the strangeness content of the proton with positive results in several experiments worldwide \(^17\). Furthermore, the recent evaluation \(^18\) of the pion-nucleon sigma term \( \sigma_{\pi N} \), points toward a rather large strangeness content of the proton, with a contribution to the nucleon mass between 110 to 220 MeV. One can address this issue by calculating the proton scalar form factor, \( \langle p|\bar{s}s|p\rangle \), which by unitarity is related with the \( I = 0 \) S-wave \( KN \) amplitudes \(^19\), the subject of this letter. All these issues concern our basic knowledge of strong interactions and require as a necessary first step a precise understanding and calculation of the \( \bar{K}N \) strong amplitudes, specially at low and sub-threshold energies.

In the limit of massless \( u, d \) and \( s \) quarks, the QCD Lagrangian is symmetric under the chiral group \( SU(3)_L \times SU(3)_R \). Once this symmetry is spontaneously broken to the diagonal \( L + R \) subgroup there appear eight Goldstone bosons which acquire mass proportionally to the non-vanishing quark masses –pions, kaons and etas. Their low energies interactions are therefore fixed and can be cast in a Taylor expansion in powers of momenta and quark masses modulated by unknown coefficients. This is known as Chiral Perturbation Theory (CHPT) \(^20\). However, in a system like \( \bar{K}N \), where the \( \Lambda(1405) \) resonance is so close to threshold, CHPT needs to be supplied with a non-perturbative resummation scheme. We follow here the Unitary CHPT (UCHPT) \(^2\) pioneered in \(^21\). This set up was used in \(^11\) to study \( K\bar{N} \) scattering as well. There, the authors were not able to reproduce simultaneously previous \( \bar{K}N \) scattering data and the new precise DEAR measurement, and called for a possible inconsistency between the latter and former data. We will show below that this is not the case.

2. Meson-baryon interactions are described to lowest order in the CHPT expansion, i.e. at \( \mathcal{O}(\mu) \), by the Lagrangian

\[
\mathcal{L}_1 = \langle iB\gamma^\mu[D_\mu,B] \rangle - m_0 \langle BB \rangle \\
+ \frac{D}{2} \langle B\gamma_\mu\gamma_5 u_B, B \rangle + \frac{F}{2} \langle B\gamma_\mu\gamma_5 u_B, B \rangle ,
\]

where \( m_0 \) stands for the octet baryon mass in the chiral limit. The trace \( \langle \cdot \cdot \cdot \rangle \) runs over flavor indices and axial-
We use $D = 0.80$ and $F = 0.46$ extracted from hyperon decays \(^{22}\). Furthermore, $u_a = i u^l (\partial_\mu U) u^l$, $U(\Phi) = u(\Phi)^2 = \exp(i2\Phi/f)$, with $f$ the pion decay constant in the chiral limit, and the covariant derivative $D_\mu = \partial_\mu + \Gamma_\mu$ with $\Gamma_\mu = [u^l, \partial_\mu u]/2$. The $3 \times 3$ flavor-matrices $\Phi$ and $B$ collect the lightest octets of pseudo-scalar mesons ($\pi, K, \eta$) and baryons ($N, \Sigma, \Lambda, \Xi$), respectively. At next-to-leading order (NLO) in CHPT, i.e. $O(p^2)$, the meson-baryon interactions are described by the Lagrangian

$$\mathcal{L}_2 = b_0 \langle BB \rangle (\chi^+) + b_D \langle B \chi^+, B \rangle + b_V \langle B \chi^+, B \rangle + b_L \langle B \{u, \mu, B\} \rangle + b_R \langle B \{u, \mu, B\} \rangle + b_s \langle B \{u, \mu, B\} \rangle + b_B \langle B \{u, \mu, B\} \rangle + \cdots \,. \quad (2)$$

Here ellipses denote terms that do not produce new contributions to S-wave scattering at $O(p^2)$. In addition, $\chi^+ = u^l \chi^+ u^l + u^l \chi^+ u$, $\chi = 2B_0 M_\pi$, $M_\pi$ is the diagonal quark mass matrix ($m_u, m_d, m_s$) and $B_0 f^2 = \langle 0|q\bar{q}|0\rangle$ the quark condensate in the SU(3) chiral limit. The $b_i$ are fitted to data.

3. We evaluate within CHPT at $O(p^2)$ all two-body scattering amplitudes with strangeness $S = -1$ corresponding to the ten coupled channels: $\pi^0\Lambda$, $\pi^0\Sigma^0$, $\pi^-\Sigma^+$, $\pi^+\Sigma^-$, $K^-p$, $K^0n$, $\eta K$, $\eta\Sigma^0$, $K^0\Xi^0$ and $K^+\Xi^-$, in increasing threshold energy order. Each channel is labeled by its position (1 to 10) in the previous list. We denote the CHPT amplitudes at $O(p)$ by $T_{ij}^{(1)}$ and at $O(p^2)$ by $T_{ij}^{(2)}$, with subindices $ij$ indicating the scattering process $i \to j$. We employ these perturbative amplitudes, given explicitly in \(^{23}\), as input to UCHPT at NLO. Two-body partial wave amplitudes can be written in matrix notation as \(^{7}\):

$$T(W) = [I + \mathcal{T}(W) \cdot g(s)]^{-1} \cdot \mathcal{T}(W) \,, \quad (3)$$

with $W$ the total energy in the center of mass (CM) frame and $s = W^2$. This equation was derived in \(^{7}\) employing a coupled channel dispersion relation for the inverse of a partial wave $T_{ij}$. The unitarity or right hand cut is taken into account easily by the discontinuity of $T^{-1}(W)$ for $W$ above the $i$th threshold, given by the phase space factor $\delta_i g_i / 8\pi W$, with $g_i$ the CM tri-momentum. This is included in the diagonal matrix $g(s)$ where $g(s)_i$ is the $i$th channel unitarity bubble. The dispersion relation above is once subtracted so that we introduce a subtraction constant $\tilde{a}_i$ for each channel in the $g(s)_i$ function. In our problem, isospin symmetry reduces the number of subtraction constants from 10 to 6 \(^{12}\): $\tilde{a}_1, \tilde{a}_2, \tilde{a}_3 = \tilde{a}_3, \tilde{a}_5 = \tilde{a}_6, \tilde{a}_7, \tilde{a}_8$ and $\tilde{a}_9 = \tilde{a}_{10}$. Our $\tilde{a}_i$ satisfy $\tilde{a}_i \equiv a_i(\mu) - 2 \log \mu + 1$, with $a_i(\mu)$ the subtraction constants in \(^{7}\). The interacting kernel $\mathcal{T}(W)$ in \(^{8}\) is a $10 \times 10$ symmetric matrix incorporating local and pole terms as well as crossed channel dynamics contributions in the dispersion relation for $T^{-1}$. The matrix $\mathcal{T}$ ($\mathcal{T} = T_1 + T_2 + \cdots$, where subindices indicate the chiral order) is fixed by matching \(^{7}\) with the lower CHPT amplitudes $T_{ij}$, order by order \(^{7}\). At leading order, $O(p)$, $T_1 = T_1^{(1)}$ \(^{7}\) while at NLO, $O(p^2)$, $T_2 = T_2^{(2)}$. The matching can be done to any arbitrary order and for $O(p^3)$ or higher $T_n \neq T_n^{(n)}$.

4. The data we include in our fits are the $\sigma(K^-p \to K^-p)$ elastic cross section, the charge exchange one, $\sigma(K^-p \to K^0n)$, and several hyperon production reactions, $\sigma(K^-p \to \pi^+\Sigma^-)$, $\sigma(K^-p \to \pi^-\Sigma^+)$, $\sigma(K^-p \to \pi^0\Sigma^0)$ and $\sigma(K^-p \to \pi^0\Lambda)$. In addition, we also fit the precisely measured ratios at the $K^-p$ threshold:

$$\gamma = \frac{\sigma(K^-p \to \pi^+\Sigma^-)}{\sigma(K^-p \to \pi^-\Sigma^+)} = 2.36 \pm 0.04 \, ; \quad (4)$$

$$R_c = \frac{\sigma(K^-p \to \text{charged particles})}{\sigma(K^-p \to \text{all})} = 0.664 \pm 0.011 \, , \quad (5)$$

$$R_n = \frac{\sigma(K^-p \to \pi^0\Lambda)}{\sigma(K^-p \to \text{all neutral states})} = 0.189 \pm 0.015 \, ,$$

see \(^{7}\) for references. The first two ratios, being Coulomb corrected, are measured with 1.7% precision, i.e. of the same order as the expected isospin violation which neither we do fully consider here nor was in \(^{11}\). Indeed, all the other observables we fit have uncertainties larger than 5%. Since we just include S-wave amplitudes and P-waves start to contribute at higher momenta \(^{28}\), we only include in the fit the $K^-p$ cross sections low energy data points, namely, with laboratory frame $K^-p$ tri-momentum $p_t \leq 0.2$ GeV. This also enhances the sensitivity to the lowest energy region in which we are particularly interested and where UCHPT is more suitable. We also include in the fits the $\pi^+\Sigma^-$ event distributions from \(^{24}\) in average–this largely eliminates the $I = 1$ contribution. To calculate them we follow \(^{7}\). The number of data points included in each fit without the DEAR data is 94. Unless the opposite is stated, we also include in the fits the DEAR \(^{8}\) measurement of the shift and width of the 1s kaonic hydrogen level

$$\Delta E = 193 \pm 37 \,(\text{stat}) \pm 6 \,(\text{syst.}) \, \text{eV},$$

$$\Gamma = 249 \pm 111 \,(\text{stat.}) \pm 39 \,(\text{syst.}) \, \text{eV},$$

which is around a factor two more precise than the KEK \(^{8}\) measurement, $\Delta E = 323 \pm 63 \pm 11 \, \text{eV}$ and $\Gamma = 407 \pm 208 \pm 100 \, \text{eV}$. To calculate the shift and width of the 1s kaonic hydrogen state we use the results in \(^{25}\) incorporating isospin breaking corrections. We compare them with the ones from the Deser formula \(^{26}\). In addition, we keep the physical masses of mesons and baryons in the calculation of $g(s)$, which produces pronounced cusp effects. We further constrain our fits by computing several $\pi N$ observables calculated in baryon SU(3) CHPT at $O(p^2)$ with the values of the low energy constants determined in the fit. Unitarity corrections in the $\pi N$ sector are not as large as in the $S = -1$ sector and hence a calculation within pure SU(3) baryon CHPT is more reliable. Thus, we calculate $\omega_{3\pi}$, the isovector same $S$ wave scattering length, the pion nucleon

The \( \sigma \) term \( \sigma_{\pi N} \), and \( m_0 \) (from the value of the proton mass) at \( \mathcal{O}(p^2) \). We do not consider the isospin-odd \( \pi N \) scattering length \( a_{\pi N} \), since at this order is just given by \( g_A \), in good agreement with experiment \( ^{27} \). The \( \sigma_{\pi N} \) term receives sizable higher order corrections from the mesonic cloud which are expected to be positive and order to 10 MeV \( ^{28} \). Since we evaluate it just at \( \mathcal{O}(p^3) \), we enforce \( \sigma_{\pi N} = 20, 30 \) or 40 MeV in the fits (\( \sigma_{\pi N} = 45 \pm 8 \) MeV \( ^{29} \)). For the same reason, we enforce \( m_0 = 0.7 \) or 0.8 GeV. We also include the value \( a_{\pi N} = -(1 \pm 2) \cdot 10^{-2} m_0^{-1} \) in the fit procedure. This result after considering its experimental one \( a_{\pi N} = -(0.25 \pm 0.49) \cdot 10^{-2} m_0^{-1} \) and the theoretical expectation of positive \( \mathcal{O}(p^3) \) corrections around \( +1 \cdot 10^{-2} m_0^{-1} \) from unitarity \( ^{27} \). We stress that for all the fits we minimize strictly the \( \chi^2 \), that is, each data point is weighted according to its experimental error. We do not include the data from \( ^{31} \) in the \( \sigma(K^-p \to \pi^- \Sigma^+) \) cross section since they are incompatible with all the other data.

5. We typically find two classes of fits, namely, class A, which give rise to 1s kaonic hydrogen \( \Delta E \) and \( \Gamma \) around the DEAR measurement, and class B fits, which are at variance with the DEAR measurement but close to the results derived from Martin’s scattering lengths \( ^{10} \).

In Fig. \( ^{11} \) we show the shift and width of the 1s kaonic hydrogen state in the first panel and the cross sections and event distribution data in the rest of panels. The solid and dashed lines correspond to the fits with \( \sigma_{\pi N} = 40 \) MeV and \( m_0 = 0.8 \) GeV, called \( A^+_1 \) and \( B^+_1 \), respectively – we discuss all the other fits in \( ^{28} \). Since the fit \( B_4 \) strongly disagrees with the DEAR measurement, we include in this fit the KKE measurement and not the DEAR one. In the first panel of Fig. \( ^{11} \) the solid circle on the left is for \( A^+_1 \) while the solid one on the right is for \( B^+_1 \). The empty circle is obtained using the Deser formula \( ^{28} \) with the \( K^-p \) scattering length from \( A^+_1 \). We observe a gentle correction to the Deser formula result when using the expression including the isospin breaking corrections from \( ^{28} \). The downward triangle is the result of using Martin’s scattering lengths \( ^{10} \) in \( ^{28} \). The squares correspond to the fits with \( \sigma_{\pi N} = 30 \) MeV and \( m_0 = 0.8 \) GeV, for details see \( ^{28} \). The isospin even \( \pi N \) scattering length results always around \( -1 \cdot 10^{-2} m_0^{-1} \). The values for the ratios in \( (4) \) from the fit \( A^+_1(B^+_1) \) are \( \gamma = 2.36(2.36), R_c = 0.628(0.655) \) and \( R_n = 0.172(0.195) \). Both fits reproduce data remarkably well, even for higher energies than included in the fit. The fitted parameters from \( A^+_1(B^+_1) \) are, in GeV units: \( f = 0.080(0.089), b_0 = -0.85(-0.32), b_D = 0.71(-0.10), b_F = -0.04(-0.31), b_1 = 0.60(-0.19), b_2 = 1.07(-0.27), b_3 = -0.19(-0.15), b_4 = -1.25(-0.28), a_1 = 0.37(-0.05), a_2 = 1.14(-0.54), a_3 = 0.22(-1.08), a_7 = 0.00(-0.05), a_8 = 0.31(-0.54) \) and \( a_9 = 1.38(0.64) \). Notice that the \( b_D, b_F \) values from the fit \( B^+_1 \) are close to the values obtained from \( \mathcal{O}(v^3) \) CHPT analysis of baryon 1/2+ mesons while for the fit \( A^+_1 \) this is not the case. However, we must stress that these couplings are not employed in the same formalism, UCHPT resums large contributions in this sector, and then there is no reason why the values should be the same. Indeed, a pure CHPT calculation of the lightest octet baryon masses is subject to huge higher order corrections and it is always questionable \( ^{28} \). The resulting \( K^-p \) scattering length is \( a_{K^-p} = (0.51 + 0.42) \) fm for the fit \( A^+_1 \) and \( (1.01 + i 0.80) \) fm for the fit \( B^+_1 \), i.e., a factor of two difference. Notice how the precise DEAR measurement places very severe constraints on the \( KN \) S-wave at threshold pointing to a less repulsive \( K^-p \) interaction. Indeed, this is also reflected by the (two) \( \Lambda(1405) \) pole positions which for the fit \( A^+_1 \) are at \( (1321 - i 43.5) \) and \( (1402 - i 39.6) \) MeV, around 30 to 40 MeV lower than the fit \( B^+_1 \) ones located at \( (1361 - i 29.9) \) and \( (1433 - i 31.7) \) MeV, respectively. This is crucial for \( K^-n \) potential calculations. We therefore also confirm the presence of two rather narrow poles conforming the \( \Lambda(1405) \) \( ^{12, 13} \) with this higher order calculation. We agree with the \( K^-p \) scattering length in \( ^{32} \) although not for \( a_0 \) and \( a_1 \) separately. In the isospin limit, we get \( a_0 = (-1.23 + i 0.45) \) fm and \( a_1 = (0.98 + i 0.35) \) fm for the fit \( A^+_1 \) and \( a_0 = (-1.63 + i 0.81) \) fm and \( a_1 = (-0.01 + i 0.54) \) fm for the fit \( B^+_1 \), where subindices refer to the \( KN \) isospin.

The S-wave and P-wave phase shifts difference at the \( \Xi^- \) mass has been recently determined from the measurement of the \( \Xi^- \to \Lambda n \) decay parameters. The results are \( \delta_p - \delta_S = (4.6 \pm 1.4 \pm 1.2) \) \( ^{33} \) and \( (3.2 \pm 5.3 \pm 0.7) \) \( ^{34} \). Neglecting the tiny P-wave phase shift \( ^{33} \), we obtain \( 2.0 \) \( ^{33} \) for the fit \( A^+_1 \) and \( 0.2 \) \( ^{33} \) for the fit \( B^+_1 \). Again the fit \( A_4 \) is in better agreement with the new \( S = -1 \) meson-baryon scattering data.

6. In summary, we have presented a NLO analysis of S-wave \( KN \) scattering within UCHPT, that combines the second order SU(3) CHPT meson-baryon amplitudes with a dispersion relation for the inverse of a partial wave amplitude \( ^{11} \). We have emphasized the strong constraints that these precise data imposes on the \( KN \) S-wave scattering amplitudes, implying a less repulsive \( K^-p \) interaction at threshold. This manifests in lower values for the two \( \Lambda(1405) \) resonance poles – whose presence we confirm at NLO. As a novelty we find a class of fits (class A) which show consistency between the DEAR and scattering data, both old and new \( ^{33, 34} \). Further exciting developments are foreseeable with the DEAR/SIDDHARTA experiment \( ^{34} \) which aims at an eV level measurement of the shift and width of kaonic hydrogen.

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FIG. 1: First panel: 1s kaonic hydrogen strong energy shift and width. In the rest, the solid lines correspond to the fit $A_4^+$ and the dashed ones to $B_4^+$. For further details see the text.

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