Restriction on the energy and luminosity of $e^+e^-$ storage rings due to beamstrahlung

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The role of beamstrahlung in high-energy $e^+e^-$ storage-ring colliders (SRCs) is examined. Particle loss due to the emission of single energetic beamstrahlung photons is shown to impose a fundamental limit on SRC luminosities at energies $2E_0 \gtrsim 140$ GeV for head-on collisions and $2E_0 \gtrsim 40$ GeV for crab-waist collisions. With beamstrahlung taken into account, we explore the viability of SRCs in the $2E_0 = 240$–$500$ GeV range, which is of interest in the precision study of the Higgs boson. At $2E_0 = 240$ GeV, SRCs are found to be competitive with linear colliders; however, at $2E_0 = 400$–$500$ GeV, the attainable SRC luminosity would be a factor 15–25 smaller than desired.

The ATLAS and CMS experiments at the LHC recently reported $^1$ an excess of events at $M = 125$ GeV/c$^2$, which may be evidence for the long-sought Higgs boson. The precision study of the Higgs boson’s properties would require the construction of an energy-frontier Higgs boson. The precision study of the Higgs boson’s radiation energy losses, which are proportional to $e^{2E_0}$, would be considerably increased with energy.

LC projects are in advanced stages of development: the ATLAS and CMS experiments at the LHC recently reported $^1, 2$ an excess of events at $125$ GeV and luminosity-frontier properties would require the construction of an energy-frontier Higgs boson. The precision study of the Higgs boson’s radiation energy losses, which are proportional to $e^{2E_0}$, would be considerably increased with energy. Table I lists parameters of the recently proposed SRCs assuming a 1% energy acceptance: the critical photon energy for the maximum beam field $E_{c,max}$, the average number of beamstrahlung photons per electron per beam crossing $n_e$, and the beamstrahlung-driven beam lifetime. Please note that once beamstrahlung is taken into account, the beam lifetime drops to unacceptable values, from a fraction of a second to as low as a few revolution periods.

At the SRCs considered in Table I the beam lifetime due to the unavoidable radiative Bhabha scattering is 10 minutes or longer. One would therefore want the beam lifetime due to beamstrahlung to be at least 30 minutes. The simplest (but not optimum) way to suppress beamstrahlung is to decrease the number of particles per bunch with a simultaneous increase in the number of colliding bunches. As explained below, $E_{c,max}$ should be reduced to $\approx 0.001E_0$. Thus, beamstrahlung causes a great drop in luminosity, especially at crab-waist SRCs: compare the proposed $L$ and corrected (as suggested above) $L_{corr}$ rows in Table I.

To achieve a reasonable beam lifetime, one must make small the number of beamstrahlung photons with energies greater than the threshold energy $E_{th} = \eta E_0$ that causes the electron to leave the beam. These photons belong to the high-energy tail of the beamstrahlung spectrum and have energies much greater than the critical energy. It will be shown below that the beam lifetime is determined by such single high-energy beamstrahlung photons, not by the energy spread due to the emission of multiple low-energy photons.

The critical energy for synchrotron radiation $^3$

$$E_c = \hbar \omega_c = \hbar \frac{3\gamma^3 c}{2\rho},$$

where $\rho$ is the bending radius and $\gamma = E_0/mc^2$. The spectrum of photons per unit length with energy well above the critical energy $^3$

$$\frac{dn}{dx} = \sqrt{\frac{3\pi}{2}} \frac{\alpha \gamma}{2\pi \rho \sqrt{u}} du,$$
TABLE I. Parameters of LEP and several recently proposed storage-ring colliders \[6,7\]. “STR” refers to “SuperTRISTAN” \[8\]. Use of the crab-waist collision scheme \[11,12\] is denoted by “cr-w”. The luminosities and the numbers of bunches for all projects are normalized to the total synchrotron-radiation power of 100 MW. Beamstrahlung-related quantities derived in this paper are listed below the double horizontal line.

| LEP | LEP3 | DLEP | STR1 | STR2 | STR3 cr-w | STR4 cr-w | STR5 cr-w | STR6 cr-w |
|-----|------|------|------|------|----------|----------|----------|----------|
| $2E_0$, GeV | 209 | 240 | 240 | 240 | 240 | 240 | 400 | 400 | 500 |
| Circumference, km | 27 | 27 | 53 | 40 | 60 | 40 | 40 | 60 | 80 |
| Beam current, mA | 4 | 7.2 | 14.4 | 14.5 | 23 | 14.7 | 1.5 | 2.7 | 1.55 |
| Bunches/beam | 4 | 3 | 60 | 20 | 49 | 15 | 1 | 1.4 | 2.2 |
| $N$, 10$^{11}$ | 5.8 | 13.5 | 2.6 | 6 | 6 | 8.3 | 12.5 | 25 | 11.7 |
| $e_{z}/\eta_{E}$, mm | 16 | 3 | 1.5 | 3 | 3 | 1.9 | 1.3 | 1.4 | 1.9 |
| $\eta_{E}$, mm | 58/0.25 | 20/0.15 | 5/0.05 | 23.3/0.09 | 24.6/0.09 | 3/0.011 | 2/0.011 | 3.2/0.017 | 3.4/0.013 |
| $\beta_{z}/\eta_{E}$, mm | 1500/50 | 150/1.2 | 200/2 | 80/2.5 | 80/2.5 | 26/0.25 | 20/0.2 | 30/0.32 | 34/0.26 |
| SR power, MW | 22 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| Energy loss/turn, GeV | 3.4 | 7 | 3.47 | 3.42 | 2.15 | 3.42 | 33.9 | 18.5 | 32.45 |
| $\mathcal{L}_{c}$, 10$^{34}$ cm$^{-2}$s$^{-1}$ | 0.013 | 1.3 | 1.6 | 1.7 | 2.7 | 17.6 | 6 | 1 | 2.2 |

where $u = \gamma_{E}/E_{c}, \alpha = e^{2}/hc$. To evaluate the integral of this spectrum from the threshold energy $\eta_{E0}$ to $E_{0}$ note that the minimum value of $u \gg 1$, the exponent decreases rapidly, and so one can integrate only the exponent and use the minimum value of $u$ outside the exponent. After integration and substitution of $\rho$ from Eq. 1 we obtain the number of photons emitted on the collision length $l$ with energy $E_{\gamma} \geq \eta_{E0}$:

$$n_{\gamma}(E_{\gamma} \geq \eta_{E0}) \approx \frac{\alpha^{2} \gamma}{\sqrt{6\pi a_{E} \gamma_{u}^{3/2}}} e^{-u}; \quad u = \frac{\eta_{E0}}{E_{c}}, \quad (3)$$

where $r_{c} = e^{2}/mc^{2}$ is the classical radius of the electron.

The regions of the beam where the field strength is the greatest contribute the most to the emission of the highest-energy photons. We need to find the critical energy for this field and the bunch size that yields an acceptable rate of beamstrahlung particle loss. The collision length $l \approx \sigma_{z}/2$ for head-on and $\approx \beta_{z}/2$ for crab-waist collisions. In the transverse direction, we can assume that the electron crosses the region with the strongest field with a 10% probability. The average number of beam collisions $n_{\text{col}}$ experienced by an electron before it leaves the beam can be estimated from $0.1n_{\text{col}}/\sigma_{z} = 1$, where $n_{\text{col}}$ is given by Eq. 3. Thus, $n_{\text{col}}$ and the beam lifetime due to beamstrahlung:

$$n_{\text{col}} \approx \frac{0.1}{\sigma_{z}} \frac{\sqrt{6\pi a_{E} \gamma_{u}^{3/2}} e^{-u}}{\alpha^{2} \gamma}; \quad \tau = \frac{\gamma^{2}}{n_{\text{col}} c}. \quad (4)$$

Assuming $E_{0} = 150$ GeV, $l = 0.1$ cm, $\eta = 0.01$, and a ring circumference of 50 km, from Eqs. 3 and 4 we get

$$u = \frac{\eta_{E0}}{E_{c}} \approx 8.5; \quad E_{c} \approx 0.12\eta_{E0} \sim 0.1\eta_{E0}. \quad (5)$$

The accuracy of this expression is quite good for any SRC because it depends on the values in front of the exponent in Eq. 4 only logarithmically.

Let us express the critical energy $E_{c}$ via the beam parameters. In beam collisions, the electrical and magnetic forces are equal in magnitude and act on the particles in the oncoming beam in the same direction. Thus, we can use the effective doubled magnetic field. The maximum effective field for flat Gaussian beams $B \approx 2\eta_{E}/\sigma_{z} \beta_{z}$. The bending radius $\rho = pc/eB = \gamma mc^{2}/eB = \gamma \sigma_{z} \beta_{z}/2c$. Substituting to Eq. 4 we find

$$\frac{E_{c}}{E_{0}} = 3\gamma r_{c}^{2}N \sigma_{z}^{2}. \quad (6)$$

Combined with Eq. 5 this imposes a restriction on the beam parameters,

$$\frac{N}{\sigma_{z}^{2}} < 0.1\eta \frac{\alpha}{3\gamma r_{c}^{2}}. \quad (7)$$

This formula is the basis for the following discussion.

For Gaussian beams, the average number of beamstrahlung photons per electron for head-on collisions \[10\] $\langle n_{\gamma} \rangle \approx 2.12N\alpha_{E}/\sigma_{z}$, their average energy $\langle E_{\gamma} \rangle \approx 0.31\langle E_{c} \rangle$, and the average critical energy $\langle E_{c} \rangle \approx 0.42E_{c,\text{max}}$; hence, $\langle E_{c} \rangle \approx 0.13E_{c,\text{max}}$. Above we considered the maximum field, i.e., $E_{c}$ was equal to $E_{c,\text{max}}$. Then, for the condition in Eq. 7 we obtain

$$\langle n_{\gamma} \rangle = \frac{0.07\eta a^{2} \sigma_{z}}{r_{c} \gamma} \left( \frac{1}{E_{0}/100 \text{ GeV}} \right) \left( \frac{\eta}{0.01} \right), \quad (8)$$

$$\langle E_{\gamma} \rangle \approx 0.13 \times 0.1\eta_{E0} \approx 1.3 \times 10^{-2} \eta_{E0}. \quad (9)$$

For crab-waist collisions, $\langle E_{\gamma} \rangle$ is the same while the interaction length is shorter, $\beta_{z}$ instead of $\sigma_{z}$; therefore, the number of photons is proportionally smaller.
So, when $\tau$ is large enough $\tau$ is determined by the rare photons with energies $\gtrsim 8.5E_{\text{c, max}}$, a factor $8.5/0.13 = 65$ greater than $\langle E_{\nu}\rangle$.

The beam energy spread due to beamstrahlung can be estimated as follows. In the general case [14],

$$\frac{\sigma^2_{E}}{E_0^2} = \frac{\tau_{\nu}}{4E_0^2} \hat{\nu}_{\gamma}(E_0^2),$$

(10)

In our case, the damping time (due to radiation in bending magnets) $\tau_{\nu} \approx T_{\text{rev}} E_0/\Delta E_{\text{rev}}$, $\hat{\nu}_{\gamma} = \langle n_{\nu}\rangle/T_{\text{rev}}$, and $\langle E_{\nu}\rangle \approx 0.4(E_0^2)^2$ [15], which gives

$$\frac{\sigma^2_{E}}{E_0^2} \approx \frac{(\langle n_{\nu}\rangle/E_0^2)^2}{E_0 \Delta E_{\text{rev}}} = \frac{1.15 \times 10^{-9}(\sigma_z/mm)}{(E_0/100\text{GeV})/(\Delta E_{\text{rev}}/E_0)} \left(\frac{\eta}{0.01}\right)^3,$$

(11)

where $\Delta E_{\text{rev}}$ is the energy loss per revolution and $(\langle n_{\nu}\rangle)$ and $(E_{\nu})$ are given by Eqs. [8] and [9]. Taking the typical bunch length $\sigma_z = 5\text{ mm}$, $E_0 = 120\text{ GeV}$, and $\Delta E_{\text{rev}}/E_0 = 0.05$ we get an estimate for the energy spread due to beamstrahlung (under the condition in Eq. [7]) $\sigma_{E}/E_0 \approx 3 \times 10^{-4}(\eta/0.01)^{3/2}$.

The beam energy spread due to synchrotron radiation (SR) in the bending magnets [14]:

$$\frac{\sigma^2_{E}}{E_0^2}_{\text{SR}} = \frac{55\sqrt{3}}{128\pi\sigma J_s} \frac{m^2 c^2 \Delta E_{\text{rev}}}{E_0} = \frac{0.016 \Delta E_{\text{rev}}}{J_s E_0 (\text{GeV}) E_0},$$

(12)

where $1 < J_s < 2$ is the partition number. For the projects in Table I $\sigma_{E}/E_0$ due to SR varies between 0.17% and 0.24%. For $E_0 = 120\text{ GeV}$, $\Delta E_{\text{rev}}/E_0 = 0.05$, $J_s = 1.5$ one gets $(\sigma_{E}/E_0)_{\text{SR}} \approx 2 \times 10^{-3}$. For the given example, the beamstrahlung energy spread becomes larger than that due to SR in rings at $\eta > 0.035$.

The energy spread due to beamstrahlung contributes to the beam lifetime (if the lifetime is large enough) when the energy acceptance $\eta \lesssim 6(\sigma_{E}/E_0)$; with [11] taken into account, this yields $\eta > 2.5(\Delta E_{\text{rev}}/10\text{GeV})/(\sigma_z/mm)$. For the typical $\Delta E_{\text{rev}} = 5\text{ GeV}$, $\sigma_z = 5\text{ mm}$, we get $\eta > 0.25$, which is much larger than the realistic storage-ring energy acceptance $\eta = 0.01–0.03$. Therefore, the beam energy spread due to beamstrahlung never causes the beam lifetime; the lifetime is always determined by the emission of single photons.

In the “crab waist” collision scheme [11, 12], the beams collide at an angle $\theta \gg \sigma_z/\sigma_{\gamma}$. The crab-waist scheme allows for higher luminosity when it is restricted only by the tune shift, characterized by the beam-beam strength parameter. One should work at a beam-beam strength parameter smaller than some threshold value, $\approx 0.15$ for high-energy SRCs [6].

In head-on collisions, the vertical beam-beam strength parameter (further “beam-beam parameter”) [14]

$$\xi_y = \frac{N r_{\nu} \beta_y}{2\pi \gamma \sigma_z \sigma_y} \approx \frac{N r_{\nu} \sigma_z}{2\pi \gamma \sigma_z \sigma_y} \text{ for } \beta_y \approx \sigma_z.$$  

(13)

In the crab-waist scheme [11],

$$\xi_y = \frac{N r_{\nu} \beta_y^2}{\pi \gamma \sigma_z \sigma_y} \text{ for } \beta_y \approx \sigma_z/\theta.$$  

(14)

The luminosity in head-on collisions

$$\mathcal{L} \approx \frac{N^2 f}{4\pi \sigma_z \sigma_y} \approx \frac{N f \gamma \xi_y}{2r_{\nu} \sigma_z},$$

(15)

in crab-waist collisions,

$$\mathcal{L} \approx \frac{N^2 f}{2\pi \sigma_y \sigma_z} \approx \frac{N^2 \beta_y f}{2\pi \sigma_y \sigma_z} \approx \frac{N f \gamma \xi_y}{2r_{\nu} \beta_y}.$$  

(16)

In the crab-waist scheme, one can make $\beta_y \ll \sigma_z$, which enhances the luminosity by a factor of $\sigma_z/\beta_y$ compared to head-on collisions. For example, at the proposed Italian SuperB factory [12] this enhancement would be a factor of 20–30.

Using Eqs. [13] and [13] and the restriction in Eq. [17] we find the minimum beam energy when beamstrahlung becomes important. For head-on collisions,

$$\gamma_{\text{min}} = \left(\frac{0.1 \eta \alpha \sigma_y}{6 \pi r_{\nu} \xi_y} \right)^{1/2} \approx \frac{\sigma_z^{3/4}}{\xi_y^{1/2} \gamma^{1/4}}.$$  

(17)

for crab-waist collisions,

$$\gamma_{\text{min}} = \left(\frac{0.1 \eta \alpha \beta_y^2}{3 \pi r_{\nu} \xi_y} \right)^{1/2} \approx \frac{\beta_z^{3/4}}{\xi_y^{1/2} \gamma^{1/4}}.$$  

(18)

In the crab-waist scheme, beamstrahlung becomes important at much lower energies because $\beta_y \ll \sigma_z$. This can be understood from Eq. [13] smaller $\beta_y$ corresponds to denser beams, leading to a higher beamstrahlung rate.

Examples: a) SuperB [12]: crab waist, $E_0 = 7\text{ GeV}$, $\sigma_y = 20\text{ mm}$, $\beta_y = 0.2\text{ mm}$, $\xi_y = 0.16$. Then, $\gamma_{\text{min}} = 29\text{ GeV}$, i.e., beamstrahlung is not important. b) The STR3 project (Table I): crab crossing, $E_0 = 120\text{ GeV}$, $\sigma_y = 50\text{ mm}$, $\beta_y = 0.25\text{ mm}$, $\xi_y \approx 0.2$. Then, $\gamma_{\text{min}} = 16.5\text{ GeV}$, a factor 7 lower than $E_0$; thus, beamstrahlung is very important. c) For projects STR1 and STR2: head-on, $E_0 = 120\text{ GeV}$, $\sigma_y = 500\text{ mm}$, $\sigma_z = 3\text{ mm}$, $\xi_y \approx 0.15$; $\gamma_{\text{min}} = 68\text{ GeV}$, beamstrahlung is important.

We have shown that beamstrahlung restricts the maximum value of $N/\sigma_z \sigma_y$ and becomes important at energies $E_0 \gtrsim 70\text{ GeV}$ for $e^+e^-$ storage rings with head-on collisions; when the crab-waist scheme is employed, this changes to the more strict $E_0 \gtrsim 20\text{ GeV}$. All newly proposed projects listed in Table I are affected as they have $E_0 \gtrsim 120\text{ GeV}$.

Now, let us find the luminosity $\mathcal{L}$ when it is restricted both by beam-beam strength parameter and beamstrahlung. For head-on collisions,

$$\mathcal{L} \approx \frac{(N f) N}{4\pi \sigma_z \sigma_y} \xi_y \approx \frac{N r_{\nu} \sigma_z}{2\pi \gamma \sigma_z \sigma_y} \frac{N}{\sigma_z \sigma_y} \equiv k \approx 0.1 \eta \frac{\alpha}{3\gamma r_{\nu}^2}.$$  

(19)
and \( \sigma_y \approx \sqrt{\delta_y \sigma_z} \). This can be rewritten as

\[
\mathcal{L} \approx \frac{(N_f)k \sigma_z}{4\pi \sigma_y}, \quad \xi_y \approx \frac{kr_c \sigma_z^2}{2\pi \gamma \sigma_y}, \quad \sigma_y \approx \sqrt{\delta_y \sigma_z}.
\] (20)

Thus, in the beamstrahlung-dominated regime the luminosity is proportional to the bunch length, and its maximum value is determined by the beam-beam strength parameter. Together, these equations give

\[
\mathcal{L} \approx \frac{(N_f)N \beta_y}{2\pi \sigma_y \sigma_z}, \quad \xi_y \approx \frac{N r_c \beta_y^2}{\pi \gamma \sigma_x \sigma_y}, \quad N \equiv k \approx 0.1 \eta \frac{\alpha}{3 \gamma r_c^2} \approx \frac{0.1 \eta \alpha}{3 \gamma r_c^2}.
\] (23)

and \( \sigma_y \approx \sqrt{\delta_y \sigma_z} \). Substituting, we obtain

\[
\mathcal{L} \approx \frac{(N_f)k \beta_y}{2\pi \sigma_y}, \quad \xi_y \approx \frac{k r_c \beta_y^2}{\pi \gamma \sigma_y}, \quad \sigma_y \approx \sqrt{\delta_y \sigma_z}.
\] (24)

The corresponding solutions are

\[
\mathcal{L} \approx \frac{(N_f)N}{4\pi} \left( \frac{0.2 \eta \alpha}{3} \right)^{2/3} \left( \frac{2\pi \xi_y}{\gamma \sqrt{\delta_y} \varepsilon_y} \right)^{1/3},
\] (25)

\[
\beta_y, \text{opt} = \varepsilon_y^{1/3} \left( \frac{3 \pi \gamma \sigma_x}{0.1 \eta \alpha} \right)^{2/3}.
\] (26)

We have obtained a very important result: in the beamstrahlung-dominated regime, the luminosities attainable in crab-waist and head-on collisions are practically the same. The gain from using the crab-waist scheme is only a factor of \( 2^{2/3} \sim 1 \), contrary to the low-energy case, where the gain may be greater than one order of magnitude. For this reason, from this point on we will consider only the case of head-on collisions.

From the above considerations, one can find the ratio of the luminosities of with and without taking beamstrahlung into account: it is equal to \( \sigma_z/\sigma_{z,\text{opt}} \) for head-on collisions and \( \beta_y/\beta_{y,\text{opt}} \) for crab-waist collisions and scales as \( 1/E_0^{1/3} \) for \( \gamma > \gamma_{\text{min}} \). In practical units,

\[
\sigma_{z,\text{opt}} \approx 2\xi_y^{2/3} \left( \frac{\xi_y}{\eta} \right)^{1/3} \left( \frac{E_0}{100 \text{ GeV}} \right)^{4/3} \frac{1}{\sigma_{z,\text{opt}}}; \quad \beta_{y,\text{opt}} \approx 0.63.
\] (27)

For example, for \( \xi_y = 0.15, \eta = 0.01, E_0 = 100 \text{ GeV} \) and the vertical emittances from Table \( \text{II} \) (\( \varepsilon_y = 0.01 \) to 0.15 nm), we get \( \sigma_{z,\text{opt}} = 2.5 \) to 6.4 mm.

According to Eq. \( \text{21} \) the maximum luminosity at high-energy SRCs with beamstrahlung taken into account

\[
\mathcal{L} \approx \frac{h N^2 f}{4\pi \sigma_x \sigma_y} \approx \frac{N f}{4\pi} \left( \frac{0.1 \eta \alpha}{3} \right)^{2/3} \left( \frac{2\pi \xi_y}{\gamma \sqrt{\delta_y} \varepsilon_y} \right)^{1/3},
\] (28)

where \( h \) is the hourglass loss factor, \( f = n_b c / 2\pi R \) is the collision rate, \( R \) the average ring radius, and \( n_b \) the number of bunches in the beam.

The energy loss by one electron in a circular orbit of radius \( R_b \) \( \delta E = 4\pi e^2 \gamma^4 / 3R_b \), then the power radiated by the two beams in the ring

\[
P = 2\delta E c N_{n_b} \approx \frac{4 e^2 \gamma^4 c n_b}{3R R_b}.
\] (29)

Substituting \( N_{n_b} \) from Eq. \( 28 \) to Eq. \( 28 \) we obtain

\[
\mathcal{L} \approx h \left( \frac{0.1 \eta \alpha}{32 \pi^2 \gamma} \right)^{2/3} \left( \frac{P}{3 e R_0 R_{100 \text{ MW}}} \right) \left( \frac{R}{R_{100 \text{ km}}} \right) \left( \frac{100 \text{ GeV}}{E_0} \right)^{1/3} \left( \frac{10^{34} \text{ cm}^{-2} \text{s}^{-1}}{\xi_y / \text{nm}} \right)^{1/3} \left( \frac{10^6 \text{ GeV}}{R_{32 \pi^2 \gamma} \xi_y / \varepsilon_y} \right)^{1/3}.
\] (30)

or, in practical units,

\[
\mathcal{L} \approx \frac{100 h \eta^2 \xi_y^{2/3} \varepsilon_y^{1/3}}{\left( E_0 / 100 \text{ GeV} \right)^{1/3} \left( \xi_y / \varepsilon_y \right)^{1/3}} \left( \frac{P}{100 \text{ MW}} \right) \left( \frac{2\pi R}{100 \text{ km}} \right) \left( \frac{R}{R_b} \right).
\] (31)

Once the vertical emittance is given as an input parameter, we find the luminosity and the optimum bunch length by applying Eq. \( 27 \). Beamstrahlung and the beam-beam strength parameter determine only the combination \( N/\sigma_z \); additional technical arguments are needed to find \( N \) and \( \sigma_z \) separately. When they are fixed, the optimal number of bunches \( n_b \) is found from the total SR power, Eq. \( 29 \).

In Table \( \text{II} \) we present the luminosities and beam parameters for the rings listed in Table \( \text{I} \) after beamstrahlung is taken into account. The following assumptions are made: SR power \( P = 100 \text{ MW}, R_b / R = 0.7, h = 0.8, \xi_y = 0.15, \eta = 0.01 \); the values of \( \varepsilon_y, \varepsilon_x \) and \( \beta_x \) are taken from Table \( \text{II} \).

Comparing Tables \( \text{I} \) and \( \text{II} \) one can see that at \( 2E_0 = 240 \text{ GeV} \) taking beamstrahlung into account lowers the luminosities at storage-ring colliders with crab-waist collisions by a factor of 15. Nevertheless, these luminosities are comparable to those at the ILC, \( \mathcal{L}_{\text{ILC}} \approx (0.55 - 0.7) \times 10^{34} \text{ cm}^{-2} \text{s}^{-1} \) at \( 2E_0 = 240 \text{ GeV} \). However, at \( 2E_0 = 500 \text{ GeV} \) the ILC can achieve \( \mathcal{L}_{\text{ILC}} \approx (1.5 - 2) \times 10^{34} \text{ cm}^{-2} \text{s}^{-1} \), which is a factor 15–25 greater than the luminosities achievable at storage rings.

In conclusion, we have shown that the beamstrahlung phenomenon must be properly taken into account in the design and optimization of high-energy e+e− storage rings colliders (SRC). We have demonstrated that beamstrahlung suppresses the luminosities as \( 1/E_0^{1/3} \) at energies \( E_0 \gtrsim 70 \text{ GeV} \) for head-on collisions and \( E_0 \gtrsim 20 \text{ GeV} \) for crab-waist collisions. Beamstrahlung makes the luminosities attainable in head-on and crab-waist collisions approximately equal above these threshold energies. At \( 2E_0 = 240-500 \text{ GeV} \), beamstrahlung lowers the luminosity of crab-waist rings by a factor of 15–40. Some increase in SRC luminosities can be achieved at rings with larger
TABLE II. Realistically achievable luminosities and other beam parameters for the projects listed in Table I at synchrotron-radiation power $P = 100$ MW. Only the parameters that differ from those in Table I are shown.

|      | LEP  | LEP3 | DLEP | STR1 | STR2 | STR3 cr-w | STR4 cr-w | STR5 cr-w | STR6 cr-w |
|------|------|------|------|------|------|-----------|-----------|-----------|-----------|
| $2E_0$, GeV | 209  | 240  | 240  | 240  | 240  | 240       | 400       | 400       | 500       |
| Circumference, km | 27   | 27   | 70   | 40   | 60   | 40        | 60        | 80        | 31        |
| Bunches/beam | $\sim$ 2 | $\sim$ 7 | 24   | 24   | 36   | 45        | 45        | 31        | 31        |
| $N$, $10^{11}$ | 33   | 5.9  | 2.35 | 3.9  | 4    | 0.4       | 0.34      | 0.6       | 0.65      |
| $\sigma_z$, mm | 8.1  | 5.9  | 5.7  | 6.9  | 6.9  | 3.4       | 6.7       | 7.8       | 9.6       |
| $\sigma_y$, $\mu$m | 1.4  | 1.1  | 0.53 | 0.78 | 0.78 | 0.19      | 0.27      | 0.36      | 0.35      |
| $\mathcal{L}$, $10^{34}$ cm$^{-2}$s$^{-1}$ | 0.47 | 0.31 | 0.89 | 0.55 | 0.83 | 1.1       | 0.12      | 0.16      | 0.087     |

radius, larger energy acceptance, and smaller beam vertical emittance.

We also conclude that the luminosities attainable at $e^+e^-$ storage rings (at one interaction point) and linear colliders are comparable at $2E_0 = 240$ GeV. However, at $2E_0 = 500$ GeV storage-ring luminosities are smaller by a factor of 15–25. Linear colliders remain the most promising instrument for energies $2E_0 \gtrsim 250$ GeV.

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