Field correlations and effective two level atom-cavity systems

S. Rebić∗, A. S. Parkins, and S. M. Tan

Department of Physics, University of Auckland, Private Bag 92019, Auckland, New Zealand

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We analyse the properties of the second order correlation functions of the electromagnetic field in atom-cavity systems that approximate two-level systems. It is shown that a recently-developed polariton formalism can be used to account for all the properties of the correlations, if the analysis is extended to include two manifolds - corresponding to the ground state and the states excited by a single photon - rather than just two levels.

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The fundamental challenge for nonlinear quantum optics is the realization of dissipation-free photon-photon interactions at the level of a few photons. In conventional nonlinear optical systems, Kerr nonlinearity gives rise to an effective photon-photon interaction that becomes important typically on the level of $10^{10}$ photons. Enhancement of the atom-field coupling using the techniques of cavity quantum electrodynamics (cavity QED) increases the Kerr nonlinearity. Two spectacular experiments have demonstrated that it is indeed possible to obtain large conditional phase shifts that arise from strong photon-photon interactions at the few photon level. The basic cavity QED scheme utilized in these experiments is based on the Jaynes-Cummings model (JC) of a two-level atom strongly coupled to a single cavity mode. Despite its success in demonstrating large single photon conditional phase shifts, this scheme appears to be fundamentally limited by the atomic and cavity dissipation. We note, though, the results of Hofmann et al. [3], suggesting that the use of one-sided cavity can significantly improve the phase shifts reported in [1]. Furthermore, Kojima et al. [4] analysed the nonlinear interaction of two photons and a two-level atom and explained bunching and antibunching effects in the output state of photons in terms of quantum interferences between different absorption and propagation processes.

One possible way to overcome the limitation due to dissipation is to study an effective two-level system, rather than a two-level atom. Schmidt and Imamoglu [5] have predicted that a four-level atomic scheme based on electromagnetically induced transparency (EIT) (called EIT-Kerr scheme) can give rise to several orders of magnitude enhancement in Kerr nonlinearity as compared to conventional two- and three-level schemes. In this scheme, atomic spontaneous emission is avoided through EIT. The prediction has been verified in a recent experiment by Kang and Zhu [6]. It has also been predicted that the presence of such large Kerr nonlinearities in a high-finesse cavity could result in photon blockade and effective two-level behavior of the cavity mode [7]. Recent progress in cavity QED demonstrates the experimental feasibility of the observation of photon blockade using state-of-the-art cavity QED techniques [8].

Another system predicted to exhibit photon blockade, proposed by Tian and Carmichael [11], is based on the JC model, but involves a single two-level atom strongly coupled to the cavity mode. If the atomic and cavity resonances coincide, and the external driving field is tuned to the lower (or upper) vacuum Rabi resonance, the system shows characteristic two-state behaviour.

In this Brief Report, we analyse the effective two-level behaviour as exhibited by EIT-Kerr and the extended JC schemes. By the extended JC model we mean a single two-level atom interacting with a single mode of a quantized cavity field, where the interaction of the atom with the field mode can be described by the JC Hamiltonian [2], extended by the driving and dissipation terms. The second order correlation function $g^{(2)}(\tau)$ has been established as a good measure of photon blockade [8], so our analysis concentrates on the properties of second order correlations. We show that the recently developed polariton formalism can be used to account for the properties of these correlations, provided that the model includes the entire first excitation manifold, rather than just two levels.

The Hamiltonian of an extended JC model in the absence of dissipation (Fig. (a)) is given by $H_{0}^{JC} = H_{int}^{JC} + H_{pump}$, where

$$H_{int}^{JC} = \hbar \theta \left( \sigma_z / 2 + a \dagger a \right) + i \hbar \left( ga^\dagger \sigma_+ - g^* a \sigma_- \right),$$

(1a)

$$H_{pump} = i \hbar \mathcal{E}_p (a - a \dagger).$$

(1b)

Here, $H_{int}^{JC}$ is the Hamiltonian of an atom interacting with a field mode, with $\sigma$‘s being the atomic pseudospin operators, $a$ and $a \dagger$ field annihilation and creation operators. Driving of the cavity by a classical field of amplitude $\mathcal{E}_p$ is described by the Hamiltonian $H_{pump}$. The atom and cavity mode (coupled with strength $g$) are assumed to be resonant, while we assume the driving field to be detuned by $\theta = \omega_k - \omega_{cav}$ from both atomic and cavity resonance. Including dis-
and excited atomic states, and numbers 0 and 1 denote the number of photons in the cavity mode. The splitting of the dressed states of the \( n \)'th excited manifold is found from Eq. (14), to be \( \epsilon_{\pm} = \frac{2g}{\hbar} n \pm \hbar \sigma \sqrt{n} \). If the laser field is tuned to the lower vacuum Rabi resonance (\( \theta = g \)), the system effectively behaves as a resonantly driven two-level system. Photon blockade occurs, since after the first photon excites the system, the second photon is detuned by \( \epsilon = (2 - \sqrt{2}) \hbar g \) from the resonance of the second excitation.

It is possible to obtain the effective Hamiltonian describing the photon blockade dynamics in terms of the two Rabi-split states \( |e_{\pm}\rangle \). We define the polariton operators \( q_{\pm} \) with \( |e_{\pm}\rangle = q_{\pm} |0, -\rangle \) and find

\[
q_{\pm} \exp (\pm i\hbar \epsilon_{\pm}) = \frac{1}{\sqrt{2}} (\sigma_{\pm} \pm \sigma_{-}) \ . \tag{5}
\]

Substituting the operators (3) in the Hamiltonian (2), transforming the Hamiltonian to a frame rotating at a laser frequency \( \omega_L = \omega_{\text{cav}} - \theta \), and performing a rotating-wave approximation, we arrive at the effective Hamiltonian

\[
H^{\text{eff}}_{\text{JC}} = 2\hbar \theta q_{+}^{\dagger} q_{+} + i \frac{\hbar \epsilon_{\pm}}{2} (q_{+}^{\dagger} q_{-} + q_{-}^{\dagger} q_{+}) - i\hbar \frac{\gamma}{2} (q_{+}^{\dagger} q_{-} + q_{-}^{\dagger} q_{+}) \ . \tag{6}
\]

In the following discussion, we assume \( \theta = g \). Hamiltonian (6) contains the effective two-level Hamiltonian of Tian and Carmichael [11], with two additional terms proportional to \( q_{+}^{\dagger} \) and \( q_{-} \). We note three key features: (i) This model is valid for large coupling \( g \) and weak driving \( \epsilon_{p} \), where the truncated (higher) manifolds do not influence the dynamics. The applicability of the effective model can be determined by the value of \( g^{(2)}(0) \), which should ideally be zero; (ii) Large amplitude oscillations in \( g^{(2)}(\tau) \), of frequency \( \sqrt{2} \epsilon_{p} \), are predicted by the effective Hamiltonian, to occur for \( \epsilon_{p} > (\kappa + \gamma)/2 \); (iii) Small amplitude modulations in \( g^{(2)}(\tau) \), of frequency \( 2\theta = 2g \) occur as a signature of the upper Rabi resonance. The last feature points at the shortcoming of the effective two-level model (also noted by Tian and Carmichael).

Hamiltonian (6) represents an effective two-manifold model, reducing the dynamics to transitions between the ground state and the entire first excited manifold.

Dressed states analysis for a single atom in the EIT-Kerr configuration has been carried out in Refs. [8, 9]. There are three states in the \( n = 1 \) manifold, one of which is resonant with the cavity mode. The second manifold contains four states. The outer two states are detuned far from the resonance and therefore their contribution to the system dynamics is negligible. The inner two states are also detuned, but lie closer to the resonance, with the size of the detuning determined by a coupling strength \( g_{2} \).
The three states in the first excited manifold are

\[
|\phi_0\rangle = \frac{|1, 1\rangle + (g_1/\Omega_c)|0, 2\rangle}{\sqrt{1 + (g_1/\Omega_c)^2}} \tag{7a}
\]

\[
|\phi_{\pm}\rangle = \frac{-(g_1/\Omega_c)|0, 1\rangle + i(\varepsilon_{\pm}/\Omega_c)|0, 3\rangle - |0, 2\rangle}{\sqrt{1 + (\varepsilon_{\pm}/\Omega_c)^2} + (g_1/\Omega_c)^2} \tag{7b}
\]

Again, the effective two-manifold model can be obtained by following the same method. Defining the polariton operators \(p_j\) with \(|\phi_j\rangle = p_j^\dagger|0, 1\rangle\), the following effective Hamiltonian emerges

\[
\mathcal{H}_{\text{eff}}^{EIT} = \hbar \varepsilon_{-} p_{-}^\dagger p_{-} + \hbar \varepsilon_{+} p_{+}^\dagger p_{+} + i\frac{\hbar \Omega_{+}}{2} (p_{-} - p_{-}^\dagger)
\]

\[
+ i\frac{\hbar \Omega_{-}}{2} (p_{+} - p_{+}^\dagger) + i\frac{\hbar \Omega_R}{2} (p_0 - p_0^\dagger)
\]

\[
+ i\hbar \Gamma_0 p_0^\dagger p_0 + i\hbar \Gamma_+ (p_{-}^\dagger p_{-} + i\hbar \Gamma_- (p_{+}^\dagger p_{+}\mathcal{a}))(8)
\]

with the effective Rabi frequencies of driving \(\Omega_{\pm}\),

\[
\Omega_{\pm} = -2\varepsilon_{\pm} \frac{g_1/\Omega_c}{\sqrt{1 + (\varepsilon_{\pm}/\Omega_c)^2} + (g_1/\Omega_c)^2}, \tag{9a}
\]

\[
\Omega_R = \frac{2\varepsilon_{\pm}}{\sqrt{1 + (g_1/\Omega_c)^2}}, \tag{9b}
\]

decay rates,

\[
\Gamma_0 = \frac{\kappa}{1 + (g_1/\Omega_c)^2}, \tag{9c}
\]

\[
\Gamma_+^{(\pm)} = \frac{\kappa g_{\pm}^2 + (\gamma_1 + \gamma_2)(\varepsilon_{\pm})^2}{g_{\pm}^2 + \Omega_{\pm}^2 + (\varepsilon_{\pm})^2}, \tag{9d}
\]

and energies \(\varepsilon_{\pm} = \delta/2 \pm \sqrt{\left(\delta/2\right)^2 + \Omega_{\pm}^2 + g_{\pm}^2}\). Key features of this effective two-manifold model can be identified in correspondence to those of the extended JC model. Large oscillations of frequency \(\Omega_R\) in \(g^{(2)}(\tau)\) are predicted to occur for \(\Omega_R > \Gamma_0/2\), or \(\varepsilon_{p} > (\kappa/4)/\sqrt{1 + g_{p}^2/\Omega_{c}^2}\). In addition, small amplitude modulations consisting of two frequencies \(\varepsilon_{\pm}\) will be observed. If \(|\varepsilon_+| \approx |\varepsilon_-|\), only one frequency will be visible, whereas if \(|\varepsilon_+| \neq |\varepsilon_-|\), oscillations with both frequencies should be apparent.

How well do these effective Hamiltonians describe the dynamics of the full system? We calculate the second order correlation function using a wave function simulations \(13\) of the original Hamiltonians \(4\) and \(3\). The photon space was restricted to 4 photons, resulting in a 10 (20) dimensional Hilbert spaces for the extended JC (EIT-Kerr) model. Results are depicted in Fig. 2. Then, using the same technique, we calculate the second order correlation function of the effective polariton Hamiltonians \(6\) and \(8\), requiring Hilbert spaces of only 3 and 4 dimensions respectively, and also plot the results in Fig. 2.

Identical values of couplings and decay rates are chosen in both schemes to enable better comparison. In the extended JC scheme, a significant antibunching (as measured by \(g^{(2)}(\tau = 0)\)) is found. The particular value of \(g^{(2)}(0)\) measures the validity of the truncation of dressed basis after the first manifold. To achieve even better agreement, a stronger coupling is needed \((g/\kappa \sim 20\) or larger) which is experimentally unavailing as of yet. We note the modulation of frequency \(2g\), as predicted by two-manifold model.

The other two curves show the simulation results for the EIT-Kerr system. They exhibit essentially the same values at the origin, which are now very close to zero. This means that the effective Hamiltonian \(8\) captures the significant dynamics well. The modulation of frequency \(\varepsilon_+ \approx |\varepsilon_-| \approx \sqrt{g_{\pm}^2 + \Omega_{c}^2}\) as predicted by the effective Hamiltonian is seen on the upper curve. The driving chosen to produce Fig. 2 is such that \(\Omega_R > \Gamma_0\), implying the presence of large oscillations in the correlation function. The opposite is true for the lower curve. Note also the existence of two modulation frequencies since the choice \(\delta = 4K\) implies \(\varepsilon_+ \neq |\varepsilon_-|\).

The two curves in the EIT-Kerr case show vastly
different coherence times – a difference attributable to the lifetime of the effective excited state \( \tau_{\text{a}} \). It follows from the decay rate \( \kappa_{\text{a}} \) that, given the fixed atom-field coupling \( g_1 \), the lifetime can be adjusted by the coupling laser (i.e., \( \Omega_{\text{c}} \)). As a consequence, the coherence time of the effective two-level system can be adjusted to virtually any prescribed value by varying the coupling \( \Omega_{\text{c}} \). Indeed, for larger values of \( g_1/\Omega_{\text{c}} \), such as the one depicted in the lower EIT-Kerr curve in Fig. 2, there is a comparatively slow recovery of the correlation function from the origin.

In conclusion, we have presented a polariton description of effective two-level atom-cavity systems in the strong coupling regime of cavity QED. It was shown how a reduction of a more sophisticated polariton structure to a lowest excited manifold can account for the properties of a second order correlation function of light leaking out of the cavity.

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