Low Density Code Design for Downlink NOMA System

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Abstract: A Low-Density Code Structure (LDC) spread transmission for Non-Orthogonal Multiple Access (NOMA) system based on modified combinatorial design is studied in this letter. To mitigate the multiuser access interference, the NOMA frequently needs complicated signal processing, which is difficult to be applied in practical scenarios. One potential solution is to design sparse multiple access structure for leveraging detection complexity. The sparse pattern of Balanced Incomplete Block Designs (BIBDs) provide an inherent sparse mapping for this system allowing simultaneous and more efficient usage of given. It is also able to form a bijection mapping with lower complexity and be reconstrued to achieve further sparsity. This new scheme allows a larger number of users transmitting in the downlink.

Keywords: NOMA, Downlink, Constellation design, Modified combinatorial design

Classification: Wireless Communication Technologies

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1 Introduction

In this letter, we studied a Low-Density Code design to build a code-domain NOMA system by taking advantages of highly structured BIBDs. In our design, different BIBDs provide NOMA with a flexible coding matrices design. Moreover, for further enlarging the massive Machine Type Communication (mMTC), a trade-off between the diversity and multiusers in one resource is built by expurgating excessive interference on coding matrices. For eliminating the surjection mapping introduced by the combing of multiuser data, a constellation forming paradigm is utilized. Instead of combining multiuser data in the same resource by numerically adding like conventional NOMA\cite{1}, the combination of the individual data is considered as a new constellation. As the NOMA pattern is known at the receiver end, the individual data can be decoupled. Furthermore, we illustrate that by applying the LDC design, it is possible to increase the number of users significantly compared to usual NOMA systems with comparable or better performance, sparsity and low design complexity.

2 System model

A single-cell multiuser NOMA system is considered, where a single antenna is equipped at the base station and users. In this system, \( b \) users and \( v \) resources will be considered, where \( v < b \).

In the downlink, the data bits of the \( j \)-th user \( d_j \) will be coded and modulated to a symbol \( m_j \) by using an \( M \)-quadrature amplitude modulation (QAM) constellation. The data of each user will be spread over \( k \) (\( k < v \)) resources, and each resource will be allocated the information of \( r \) (\( r < b \)) users. By combining \( r \) users’ data in the \( i \)-th resource, a new symbol \( x_i \) from a constellation with \( M^r \) symbols will be achieved.

After OFDM demodulation performed at the receiver, the received symbol \( y \in \mathbb{C}^v \times 1 \) is expressed as:

\[
y = \sum_{i=1}^{v} \sqrt{p_i} h_i x_i + \omega_i
\]  

where \( h_i \) and \( p_i \) denotes the diagonal channel coefficient and power distributed over the \( i \)-th frequency, respectively. \( \omega \sim \mathcal{N}(0, \sigma^2) \) denotes the additive white Gaussian noise (AWGN). Noticing that information of users is equal possible and linear combined over all frequencies, thus the \( p_i \)'s are equivalent, and without loss of generality, we assume \( |p_i| = 1 \). \( x_i \) denotes the information vector. After that, the signal is passed to the multiuser detector to decouple each user’s data.

3 Constellation design criteria and expurgation algorithm

In this section, we propose a constellation forming method based on the BIBD.

3.1 Constellation Design

Definition 1: A BIBD defined by \( D(v, b, r, k) \) is an ordered pair \( (V, B) \), where \( V \) is a set of \( v \)-element and \( B \) is a collection of \( b \) \( k \)-subsets of \( V \), called blocks. In BIBD, each element of \( V \) appears exactly in \( r \) out of \( b \) blocks.
between these parameters can be written as: \( r_v = k b \). Hence a binary \( v \times b \) matrix \( N_{v,b} \) is called the incidence matrix of a BIBD design as shown in Eq. (2).

\[
N_{v,b} = \begin{bmatrix}
n_{1,1} & n_{1,2} & \ldots & n_{1,b} \\
n_{2,1} & n_{2,2} & \ldots & n_{2,b} \\
\vdots & \vdots & \ddots & \vdots \\
n_{v,1} & n_{v,2} & \ldots & n_{v,b}
\end{bmatrix}
\]

where

\[
n_{i,j} = \begin{cases} 
1 & \text{if } i \text{ is in the } j \text{-th block of } B \\
0 & \text{if } i \text{ is not in the } j \text{-th block of } B
\end{cases}
\]

Then, the capability for a NOMA system transmitting more users compared to OMA [2] is related to the overload capacity (\( OC = \frac{\text{number of users}}{\text{number of resources}} \)). The BIBD determines this relationship between its structure and NOMA system. Thus, the overload capacity can be expressed numerically as \( OC = \frac{b}{v} = \frac{r}{k} \).

A mapping data matrix is formed and can be written as:

\[
M = \begin{bmatrix}
m_1 & \cdots & m_b \\
\vdots & \ddots & \vdots \\
m_1 & \cdots & m_b
\end{bmatrix}
\]

(3)

where the \( \bar{m} = \{m_1, m_2, \ldots, m_b\} \); \( m_i; i = 1, \ldots, b \) represents the users’ data. The resulting pre-coded data matrix, \( D \) is given by \( D = N_{v,b} \circ M \), where \( \circ \) denotes the Hadamard product operator. Noting that there are only \( r \) non-zero entries in each row of \( N_{v,b} \). Hence only \( r \) out of \( b \) users will be transmitted over a given resource.

Here, a constellation design is presented that can resolve the surjective problem caused by superimposing the user’s data and provides a bijective relation between constellation points and users’ data. For expressing the constellation forming algorithm, we define the index of the \( r \) non-zero entries of the \( i \)-th resource (row) as:

\[
J_i(1), \ldots, J_i(j), \ldots, J_i(b)
\]

(4)

where \( J_i(j) = j \) when \( n_{i,j} = 1 \). Replacing Eq. (3) into Eq. (4), then jointly merging \( x_j \)'s in row-wise, the modulated transmit vector can be written as:

\[
X = \begin{bmatrix}
\bigcup_{J_i(j) = J_i(1)} m_{J_i(j)} \\
\vdots \\
\bigcup_{J_i(j) = J_i(b)} m_{J_i(j)}
\end{bmatrix}
= \begin{bmatrix}
x_1 \\
\vdots \\
x_v
\end{bmatrix}
\]

(5)

where \( \bigcup_{J_i(j) = J_i(1)} m_{J_i(j)} = x_i \) are mapped to a QAM constellation with \( q^r \) points.

As \( m_{J_i(j)} \in \mathbb{M} \), the constellation alphabet corresponding to the \( x_i \) can be represented as \( \mathbb{M}^r \) symbols. Notice that Eq. (4) are uniquely determined by the matrix \( N_{v,b} \). Consequently, these indexes are identified at the receiver. For example, Eq. (6) below is the incidence of a 200% overload system with 13 users and 26 orthogonal frequencies and each user is modulated by binary phase shift keying (BPSK).
In Eq. (6) and (7) the shadowed ‘1’s shows the users on each resource and they will be explained later.

\[
N_{p=13, b=26} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]

(6)

The first matrix on the left of Eq. (7) is modulated to a $2^6(64)$-QAM constellations, needed to represent the $r = 6$ users in each resource in (6), if denoting $-1 \leftrightarrow 0$ and $+1 \leftrightarrow 1$. And then the middle matrix is the mapping process.

\[
\begin{bmatrix}
+1 & -1 & +1 & +1 & -1 & +1 \\
+1 & -1 & -1 & +1 & +1 & -1 \\
-1 & -1 & +1 & -1 & +1 & +1 \\
+1 & -1 & +1 & -1 & -1 & -1 \\
+1 & +1 & +1 & +1 & -1 & -1 \\
+1 & +1 & -1 & -1 & +1 & -1 \\
+1 & +1 & +1 & -1 & -1 & +1 \\
+1 & +1 & -1 & +1 & +1 & +1
\end{bmatrix}
\leftrightarrow
\begin{bmatrix}
101 & 101 & \\
100 & 110 & \\
001 & 011 & \\
110 & 101 & \\
110 & 011 & \\
101 & 110 & \\
100 & 001 & \\
110 & 100
\end{bmatrix}
\rightarrow
\begin{bmatrix}
110 & 011 \\
010 & 110 & \\
110 & 011 & \\
101 & 110 & \\
110 & 001 & \\
110 & 100 & \\
100 & 001 & \\
110 & 100
\end{bmatrix}

(7)

3.2 Expurgation algorithm

Designs with larger diversity are required to increase the detection accuracy, but this implies extended number of interferers per resource and therefore performance will be decreased. A solution for this is to expurgate the number of users per resource to obtain a structure with larger sparsity, then the number of interferers and the complexity of system is lowered [3]. Thus, LDC is proposed to achieve a larger overload capacity (OC) for transmitting more users in limited resources. Next, we will discuss the expurgating procedure. It is known that a basic BIBD is composed of $B$ blocks, \{$B_1, B_2, \ldots, B_B$\} and each block has $b$ elements. Owing of the balance property, number of users in each resource, which is $r$, and repeat number of each user over recourses, which is $k$, can be expurgated under the same ratio $\frac{b}{v} = \frac{r}{k}$.

| Algorithm of expurgation over a given BIBD |
|--------------------------------------------|
| **Parameters:** The number of users can be expurgated: $k_e \times \frac{v}{b}$; $k - k_e \geq 0$; |
| **Input:** Given BIBD; |
| **Output:** expurgated BIBD with required sparsity. |
As a result, a sparser users’ incidence matrix than the original BIBD with the same \( OC \) can be obtained. As shown in example with Eq. (6), \( D(13,26,6,3) \) with \( r = 6 \) and \( k = 3 \). After expurgating; expurgated users shown in shadowed in Eq. (6) and (7), \( D_{e}(13,26,4,2) \) can be built, with the number of interferers per resource \( r = 6 \), decreased to \( r_{e} = 4 \) and the diversity per users \( k = 3 \) reduced to \( k_{e} = 2 \).

In Eq. (7), the \textbf{expurgated constellation} has a smaller size of constellation than \textbf{original one}.

Based on above discussion, the expurgated number of users over the resource can be expressed in (8) by \( r_{e} \),

\[
r_{e} = r - OC \times (k - k_{e})
\]

Furthermore, in other higher order BIBDs, the expurgating process has ability to reduce the constellation size to an acceptable range.

### 4 Multiuser detection design

The MUD design is presented in this section. We assume that channel state information (CSI) is known at receiver. Thus, the estimation of symbol for each resource, \( \hat{x}_{i} \) can be achieved form received vector \( y \) by applying maximum likelihood (ML).

Noting that Eq. (4) is uniquely determined by \( N \). Thus, for a specific \( N \), these indexes are known at the receiver. In each resource, \( \{ \hat{m}_{j}(r) \} \) will be allocated to the corresponding \( J_{i}(r) \) –th column. Since the weight of each row and column is constant, Maximum Ratio Combining (MRC) can combine the data of user \( j \) over all resources. Considering that \( k_{e} \) decoded replicas for each user. The estimated modulated symbol of user \( j \) can be written substituting Eq. (1) as:

\[
\hat{m}_{j} = \frac{\sum_{J(j)=1}^{J(j)=K} h_{j}(K) \hat{m}_{j}(K) h_{j}(K)}{h_{j}(K) h_{j}(K)}
\]

Noticing that this MUD design can be used in other NOMA patterns also. In our expurgation system, the diversity of \( k_{e} = 2 \) will be implemented.

### 5 Simulation Result

In this section, we present numerical simulation results to demonstrate the effect of new mapping pattern and expurgation algorithm on the average Bit Error Rate of all user in downlink NOMA.

Fig.1 shows the performance of our new NOMA constellation forming system by using the same size of SCMA and PDMA incidence matrices, \( (v = 4, b = 6, r = 3, k = 2) \); i.e., have 6 UE, 4 resources with overload 150%, under Rayleigh fading...
channel in the downlink. The data of each user is modulated by QPSK.

\[ \text{Figure 1. Uncoded Performance of } D\left(5,10,4,2\right), D\left(4,6,3,2\right), \text{De}(13,26,4,2) \]

The numerical result shows that our \( D\left(4,6,3,2\right) \) obtains a comparable performance to SCMA \([5]\) less than 0.5dB at \( P_e = 10^{-3} \). And our design slightly outperforms PDMA \([4]\). And the system \( D\left(13,26,6,3\right) \) after expurgating, \( \text{De}(13,26,4,2) \), can have the same \( r \) and \( k \) with \( D\left(5,10,4,2\right) \). Thus, the system complexity and performance will be acceptable.

Turbo coding with a code rate of 1/3 from LTE standard is utilized to this coded system.

\[ \text{Figure 2. Downlink Performance of different } D\left(b,v,r,k\right), \text{BPSK, Rayleigh channel, Turbo Coder} \]

Then, the Low-Density Code design after expurgating is straightforward. We display the system performances of before and after expurgating for transmitting large number of users in Fig.2 with solid and dash curves in the same color. We observe that after expurgating, large improvement of performance results for a variety size of users and \( OCs \) can be achieved.

6 Conclusion

The main contributions of this research are to solve the overlapped constellation points resulting from the surjective mapping when linearly combining multi-users data and then design a large size of system with more sparsity and flexibility. And an algorithm is proposed to construe sequences with more sparsity based on given combinatorial designs. These designs can also be applied in other NOMA systems to reduce the complexity of constellation design and multiusers detection.