Entropy & viscosity bound of strange stars

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Abstract.
At finite temperature (T) there is a link with general relativity and hydrodynamics that leads to a lower bound for the ratio of shear viscosity and entropy density ($\eta/s$). We find that the bound is saturated in the simple model for quark matter that we use for strange stars at $T = 80\, \text{MeV}$, at the surface of a strange star. At this $T$ we have the possibility of cosmic separation of phases. We find that, although strongly correlated, the quark matter at the surface of strange stars constitute the most perfect interacting fluid permitted by nature. At the centre of the star, however, the density is higher and conditions are more like the results found for perturbative QCD.

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1. Introduction

We find that the ratio of the kinetic viscosity to entropy density of strange stars saturates the lowest possible bound. This is as perfect as an interacting fluid can be. Although it is not directly relevant to us - the background for the viscous bound conjecture of Kovtun, Son and Starinets [1] (KSS) will be briefly touched upon in this section because of its general interest. We will call this the KSS bound.

It is popularly known that black holes are endowed with thermodynamics. In higher dimensional gravity theories there exist solutions called black branes. They are black holes with translationally invariant horizons. For these solutions thermodynamics can be extended to hydrodynamics, the theory that describes long wavelength deviations from thermal equilibrium. In the holographic principle, where a black brane corresponds to a certain finite-temperature quantum field theory in fewer number of space time dimensions, and the hydrodynamic behaviour of black brane horizon is identical with the hydrodynamic behaviour in a dual theory.

The relevant arguments of KSS [1] for a generalization of the viscous bound $4 \pi \eta/s > 1$, is very interesting since it only invokes general principles like Heisenberg uncertainty relation for the typical mean free time of a quasi-particle and the entropy density $s$ which in turn is proportional to the density of the quasi particles. From here to our model is just one short step of identifying the quasi-particles to be the dressed quarks of a mean field description for a large colour effective theory. We describe the model in section 2 for the sake of completeness, emphasizing the possible observational checks of the model. In section 3 we describe the simplest possible calculation of the viscosity known to all and compare our results with other calculations. We present a brief discussion in section 4 and finally we present a summary and conclusion in section 5.

2. Strange stars at finite T

The chiral symmetry restoration (CSR) of our model is represented by an ansatz for density dependent quark mass:

$$M_i = m_i + M_Q \text{sech} \left( \frac{n_B}{N n_0} \right), \quad i = u, d, s. \quad (1)$$

where $n_B = (n_u + n_d + n_s)/3$ is the baryon number density, $n_0 = 0.17 \text{fm}^{-3}$ is the normal nuclear matter number density, and $N$ is a parameter.

The number density for the strange star in our model changes from the surface where it is between four and five times the normal nuclear matter number density of $n_0 = 0.17 \text{fm}^{-3}$ to about 15 times $n_0$ at the centre. At high $n_B$ the quark mass $M_i$ falls from a large value $M_Q$ to its current one $m_i$ which we take to be $2, 3$: $m_u = 4 \text{MeV}$, $m_d = 7 \text{MeV}$, $m_s = 150 \text{MeV}$ and $M_Q \sim 345 \text{MeV}$. Possible variations of the CSR can be incorporated in the model through $N$. The parameters of the modified Richardson potential with different scales for confinement ($350 \text{MeV}$) and asymptotic freedom ($100 \text{MeV}$) has been used to fit the octet and decuplet masses and magnetic moments [4, 5].
The calculation involves a density and temperature dependent gluon screening and thermal single particle Fermi functions with the modified two body quark - quark interaction. Along with the constraints of $\beta$ - equilibrium and charge neutrality in these calculations, it is found that energy minimum occurs at a density $\sim 4$ to 5 times the normal nuclear density $n_0$ till $T = 80 \text{ MeV}$. This is a relativistic, self consistent, mean field calculation. Strange quark matter is thus self bound by strong interaction itself. The energy density and pressure of this matter lead to strange quark star through the Tolman Oppenheimer Volkov (TOV) equation with mass and radius depending on the central density of the star. The results of the calculation as well as references to the many application of the model to astrophysical observations are given in Bagchi, et al. [6].

3. Calculations & Results

We have done the simplest possible calculation of the viscosity of a fluid using the expression based on a calculation by Clausius in 1860:

$$\eta = \frac{1}{3} m v n \lambda$$

(2)

where $m$ is the mass of the particles and $n$ is the number density. The mean free path $\lambda$ is given by

$$\lambda = \frac{3}{4 \pi d^2 n}$$

(3)

where $d$ is the interaction diameter. The interaction radius $r (= d/2)$ is calculated by assuming that the relevant particles occupy an effective volume $\frac{4}{3} \pi r^3$ :

$$r = \left[ \frac{3}{4 \pi n} \right]^{1/3}$$

(4)

In quark matter, $n = n_u + n_d + n_s$. We need to specify the momentum as follows :

$$m v = \sum_{i=u,d,s} m_i v_i$$

(5)

$$m_i v_i = \frac{\int_0^\infty k^3 f(k, U_i) dk}{\int_0^\infty k^2 f(k, U_i) dk}$$

(6)

where the Fermi distribution is

$$f(k, U_i) = \frac{1}{1 + e^{(U_i - \mu_i)/T}}$$

(7)

In Fig. (1) we find that at the surface of the strange star the KSS bound is saturated. In the model the strange star is self bound at a certain number density where a confinement to deconfinement transition takes place due to strong Debye screening. It is rather satisfying to see that a very simple evaluation of the viscosity and the entropy in the model leads to the KSS bound. One can envisage corrections to this scheme - for example a relativistic relative velocity correction at the upper limit can give a correction of 30% in a naive estimate. But the relative velocity range is from zero and for lower values the estimate gives smaller numbers.
The variation of $\eta/s$ with the coupling is counter intuitive as emphasized by Kovtun, Son and Starinets [1]. We wanted to check that the ratio in fact increases with decreasing coupling. To do this we needed the relevant $\alpha_s$ at each density.

We have extracted the density dependent strong coupling constant $\alpha_s$ from the density dependent mass ansatz [7]. This is due to the simplified Schwinger-Dyson formalism of Bailin, Cleymans and Scadron [8] using the Dolan-Jackiw Real time propagator for the quark.

$$\alpha_s(r, n) = \frac{m_{dyn} - M_d(r, n)\pi}{\pi [u(r, n) + (u(r, n)^2 - M_d(r, n)^2)^{1/2}]^2}$$

We repeat the calculation here for the latest parameter sets [3] but essentially there is no fundamental change in $\alpha_s$, the variation being from 0.5 at low number density at the star surface to about 0.2 at the highest density in the centre. We find that $\eta/s$ is a decreasing function of $\alpha_s$ as discussed for example by Stephanov [9]. The RHIC is looking for this region of $\alpha_s$, i.e. large coupling-nonperturbative region. We see that in our case this happens at the surface of the strange star.

4. Discussions

Although Clausius’s equation of mean free path is essentially non-relativistic, we can show that the factor $4/3$ in the expression of relative velocity Clausius’s expression will be changed by a small amount. The factor becomes 1.328, 1.288, 1.218, 1.133 and 1.043 for $\beta (v/c)$
Table 1. Variation of the strong coupling constant $\alpha_s$ with increasing number density predicted by our quark mass ansatz.

| $n_B/n_0$ | 4    | 5    | 6    | 7    | 8    | 9    | 10   | 11   | 12   |
|-----------|------|------|------|------|------|------|------|------|------|
| $\alpha_s$ | 0.522| 0.532| 0.515| 0.486| 0.454| 0.422| 0.393| 0.365| 0.341|
| $n_B/n_0$ | 13   | 14   | 15   | 16   | 17   | 18   | 19   | 20   |      |
| $\alpha_s$ | 0.319| 0.299| 0.281| 0.265| 0.251| 0.238| 0.226| 0.215|      |

Figure 2. Variation of $\eta/s$ with strong coupling strength.

values of 0.1, 0.3, 0.5, 0.7 and 0.9 respectively. In our model, $\beta$ lies within 0.5-0.7. So we have neglected this relativistic correction.

We have used the classical expressions for viscosity and mean free path and treated them semi-classically to reach the non-perturbative regime of QCD. On the other hand, Heiselberg and Pethick [10] deduced an expression for viscosity by evaluating momentum relaxation rate and their applications to transport processes in degenerate quark matter within perturbative QCD limit. This is a variation on the previous calculation of Haensel and Jerzak [11]. The expression of Heiselberg and Pethick is quoted by several other authors [12, 13]. We used the viscosity expression of the reference [10] and find that at $T = 80\ MeV$, $n_B/n_0 = 5$ and 15, the $4\pi\eta/s = 3.85$ and 166.45 respectively. It increases with increasing density and decreasing temperature. For example, at $T = 50\ MeV$, $4\pi\eta/s = 13.99$ and 597.54 for $n_B/n_0 = 5$ and 15 respectively.

In a recent paper Lacey has given a very lucid and colourful representation of viscosity
bound for different fluids which we summarize here. As the \((T - T_c)/T\) varies from 0.5 to 0, \(\eta/s\) in (a) meson gas goes from 1.2 to 0.4, (b) water goes from 3.8 to 2.2 (c) liquid nitrogen from 3.4 to 0.8 and (d) liquid helium from 3.4 to 0.8. The matter in the strange star seems to be the first so called perfect interacting liquid where bound reaches the fraction \((4\pi)^{-1}\) and thus it may be the same fluid which Lacey marks as RHIC which stands for relativistic heavy ion collisions \([14]\).

5. Summary and Conclusions

We have found that \(\eta/s\) increases with increasing number density and with decreasing \(\alpha_s\). The transport here is radial hence the kinetic viscosity is the shear viscosity. At the surface of all strange stars we have an energy density which makes the pressure zero. This zero pressure is what makes the star self bound. And it is the strong interaction which is responsible zero pressure. The quark matter at the surface looks like a strongly correlated self bound system where \(4\pi \eta/s \sim 1\). It cannot be a mere co-incidence that such our simple model leads to such an interesting result, connecting the zero pressure energy density with that of RHIC. Further debate may be possible as to why it happens, but it is beyond the scope of this paper.

In summary we find that the strange star surface where the pressure is zero also turns out to be the region where the KSS viscosity bound is saturated, and perhaps is the same matter observed in RHIC.

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