A supernova constraint on bulk majorons

Steen Hannestad *, Petteri Keränen †, Francesco Sannino ‡
NORDITA, Blegdamsvej 17, DK-2100 Copenhagen, Denmark
(Dated: November 4, 2018)

PACS numbers: 11.10.Kk, 98.70.Vc, 12.10.-g

I. INTRODUCTION

Over the past few years there has been an enormous interest in models with extra dimensions beyond the 3+1 dimensions of the standard model. Such extra dimensions can be large if the gauge fields of the standard model are constrained to a 3+1 dimensional brane in the higher dimensional bulk space. This idea is particularly interesting because it may be connected with a low quantum gravity scale, perhaps only slightly above the scale of electroweak symmetry breaking. In Refs. [1, 2, 3] such a model was proposed in which the standard model brane is embedded in a bulk space of dimension δ which is toroidally compactified. Because gravity is allowed to propagate in the full bulk space it will look weaker to an observer confined to the brane. By Gauss’ law the effective four-dimensional Planck scale, \( M_P \), can be related to the true 4+δ dimensional energy scale of gravity, \( M \), by

\[
M_P^2 = (2\pi R)^\delta M^{\delta+2},
\]

where \( R \) is the common radius of all the extra dimensions. If \( R \) is sufficiently large \( M \) can be as small as the electroweak scale. Effectively gravity is weak because the field lines leak into the extra dimensions.

One of the most interesting features of these models is the presence of a tower of new modes for particles propagating in the bulk. The momentum of particles propagating in extra dimensions is discretised because the extra dimensions are compact. To an observer on the brane this effectively looks like a tower of new states (Kaluza-Klein) for each bulk particle, with a 4D mass related to the extra dimensional momentum. The energy spacing between these states is then in general of order \( R^{-1} \) which can be very small.

For the graviton this property has been used to put tight constraints on the possible radii of the extra dimensions. However, any particle which is a singlet in the standard model could in principle propagate in the bulk. Examples of this are sterile neutrinos and axions.

Some authors, for example, have investigated the possibility that right handed neutrinos might propagate in the extra dimensions and that their resultant suppressed couplings to the usual left handed neutrinos could account for the low scale of neutrino masses [4, 5]. Other authors [6, 7, 8, 9] have studied the constraints on such models due to experiment and observation. An alternate approach [10], corresponding to the conventional see-saw mechanism, has also been investigated by some authors. In this approach a Higgs singlet, which carries no standard model gauge quantum numbers, is allowed to propagate in the extra dimensions and give mass to right handed neutrinos which, for simplicity, are assumed to live on the brane.

Recently in Ref. [20] a model was studied in which the lepton number is spontaneously broken and the associated Goldstone boson (the “Majoron”) is present (see also [21]). This model was used to compute the decay rate for the intermediate vector boson \( Z \) to two neutrinos and the Majoron (denoted by \( J \)) or one of its “Kaluza-Klein” excitations [20]. Using the accurately known value of the \( Z \) width [22] bounds on the dimensionless coupling of the two neutrinos to the Majoron were obtained allowing for any number of extra dimensions and any intrinsic mass scale (see also [23]). The related neutrinoless double beta decay process \( n + n \rightarrow p + p + e^- + e^- + (J) \) has already been treated in a model of the present type [19]. A detailed discussion of supernova constraints in the 3+1 dimensional theory has very recently been given in [24].

In this paper we use this model [20] to compute processes of astrophysical interest. We first see that when compact extra dimensions are present new processes become relevant and can heavily affect supernovae dynamics. We then show that supernovae constraints on the dimensionless couplings are by many order of magnitude more stringent than the accelerators bounds. Our findings seem to discourage unnatural see-saw models of neutrino masses still allowed by accelerator bounds.

We summarize in section II the Majoron model extended to the extra-dimensional brane-world. In section III we briefly review the accelerator constraints. The Supernovae constraints are presented in section IV.
concluding section V we also discuss the possibility of the extra-dimensional Majorons to be source of dark matter.

II. A MAJORON MODEL IN EXTRA DIMENSIONS

In the original Majoron model of Chikashige, Mohapatra and Pececi \[25\] the lepton number associated with massive Majorana neutrinos is spontaneously broken. Here, the notations of Ref. \[29\] and Ref. \[21\] for the 3 + 1 and the extra dimensional case will be followed, respectively. In addition to the usual Higgs doublet

\[
\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \quad l = 0
\]

which has lepton number \(l\) equal to zero, the model contains an electrically neutral complex singlet field

\[
\Phi \quad l = -2.
\]

It is required that the Higgs potential constructed from \(\Phi\) and \(\phi\) conserves lepton number. The vacuum expectation values are:

\[
\langle \Phi \rangle = \langle \Phi^* \rangle, \quad \langle \phi^0 \rangle = \langle \phi^0 \rangle = \lambda \approx 2^{-\frac{1}{2}} G_F^{-\frac{1}{2}},
\]

where \(G_F\) is the Fermi constant and \(\langle \Phi \rangle\) (whose non-zero value violates lepton number) sets a new scale in the theory.

The three light physical two component neutrinos \(\nu_1, \nu_2, \nu_3\) acquire Majorana masses \(m_1, m_2, m_3\) which are of order \(\epsilon^2 \mathcal{M}_H\) with

\[
\epsilon = \mathcal{O} \left( \frac{D}{\mathcal{M}_H} \right).
\]

According to the standard “see-saw mechanism” \[27\] \(D/\lambda\) represents the “Dirac-type” coupling constants for the bare light neutrinos while the 3 \(\times\) 3 matrix \(\mathcal{M}_H/\langle \Phi \rangle\) represents the Majorana type coupling constants for the bare heavy (or “right handed”) neutrinos. Assuming the heavy scale \(\mathcal{M}_H\) to be substantially larger than the energy in play in the following we focus our attention only on the light neutrinos. The coupling of the Majoron \(J\), identified as \(J = \text{Im} \Phi\), to the physical neutrino fields \(\nu_1, \nu_2, \nu_3\) in 3 + 1 dimensions is:

\[
\mathcal{L}_J = \frac{J}{2} \sum_{a,b=1}^3 \nu_a^\dagger \sigma_2 g_{ab} \nu_b + \text{h.c.}.
\]

It turns out \[20\] that the coupling constants have the expansion:

\[
g_{ab} = -\frac{1}{\langle \Phi \rangle} m_a \delta_{ab} + \mathcal{O}(\epsilon^4 \mathcal{M}_H),
\]

where the leading term is diagonal in generation space. It is convenient to express this leading term using four component ordinary Dirac spinors

\[
\psi_a = \begin{pmatrix} \nu_a \\ 0 \end{pmatrix},
\]

in a \(\gamma_5\) diagonal representation of the Dirac matrices the relevant Lagrangian term reads:

\[
\mathcal{L}_J = i \frac{J}{2 \langle \Phi \rangle} \sum_{a=1}^3 m_a \left( \psi_a^T C^{-1} \frac{1 + \gamma_5}{2} \psi_a + \bar{\psi}_a \frac{1 - \gamma_5}{2} C \psi_a^T \right).
\]

Here \(C\) is the charge conjugation matrix of the Dirac theory.

The generalization of the present model to the case where the field \(\Phi\) propagates in \(\delta\) extra dimensions has been carried out in detail in Ref. \[20\]. These extra dimensions, denoted as \(y_i\) with \(i = 1, \ldots, \delta\), are assumed to be toroidally compactified with a radius \(R_i\). For simplicity we assume that all the radii \(R_i\) are equal to the same value \(R\). \(\Phi(x,y)\) continues to carry the “engineering dimension” one as it would in 3 + 1 dimensional space-time and via a Fourier expansion with respect to the compactified coordinates we have:

\[
\Phi(x,y) = \text{Norm} \sum_{n_1, \ldots, n_\delta} \Phi_{n_1, \ldots, n_\delta}(x) e^{\frac{i}{R} (n_1 y_1 + \cdots)},
\]

where \(\text{Norm} = [2\pi M R]^{-\frac{\delta}{2}}\) and \(M\) represents the intrinsic scale of the new theory.

Each general Kaluza-Klein field receives a mass squared increment

\[
\Delta m_{n_1, n_2, \ldots}^2 = \frac{1}{R^2} (n_1^2 + n_2^2 + \cdots n_\delta^2).
\]

The zero-mass Majoron \(J_{00, \ldots, 0}(x)\) corresponds to the previously studied 3 + 1 massless Majoron. The fields \(\Phi_{n_1, n_2, \ldots}(x)\) are expected to receive a substantial increment from the pure Higgs sector of the theory \[20\] and will be neglected in the following.

The intrinsic scale \(M\) and the compactification radius \(R\) can be related to each other when assuming in the “brane” model the graviton to propagate in the full \((3 + \delta) + 1\) dimensional space-time. Then the ordinary form of Newton’s gravitation law is only an approximation valid at distances much greater than \(R\). The Newtonian gravitational constant (inverse square of the Planck mass \(M_P\)) is obtained \[1, 2, 3\] as a phenomenological parameter from

\[
\left( \frac{M_P}{M} \right)^2 = (2\pi M R)^\delta = \frac{1}{\langle \text{Norm} \rangle^2}.
\]

Considering \(M_P\) as an experimental input (and approximating \(R_1 = R_2 = \cdots = R_\delta\), shows via \[12\] that \(M\) is the only free parameter introduced to describe the extra dimensional aspect of the present simple theory when \(\delta\) is fixed.
We expect $M_H/\langle \Phi \rangle$ to be very roughly of the order unity and $\langle \Phi \rangle$ of the order of $M$. Finally the Yukawa interactions of the Majoron and its Kaluza-Klein excitations with the light neutrinos are described by (c.f. (9)):

$$\mathcal{L}_J = \sum_{a=1}^3 \sum_{n_1, \ldots, n_3} i g_{aa;n_1, \ldots, n_3} J_{n_1, \ldots, n_3} \times$$

$$\left( \psi_a^T C^{-1} \frac{1 + \gamma_5}{2} \psi_a + \frac{1 - \gamma_5}{2} C \psi_a^T \right), (13)$$

to leading order in the neutrino masses, $m_a$ with

$$g_{aa;n_1, \ldots, n_3} = g_{aa} = \frac{m_a}{2 \langle \Phi \rangle M_P}. (14)$$

The vacuum expectation value $\langle \Phi \rangle$ (unless unnatural fine tuning is considered) is very roughly of the order of $M$, leading $g_{aa}$ to be naturally of the order of $m_a/M_P \approx 10^{-28}$ regardless of the number of extra dimensions [2]. Clearly the exact value of this universal coupling crucially depends on the unknown dynamics behind the spontaneous breaking of the lepton number [20].

### III. REVIEW OF CONSTRAINTS FROM ACCELERATORS

In [20] it has been shown that the accurately known value of the Z width can provide information about the coupling of two neutrinos to the Majoron. Both the 3 + 1 dimensional case and the case in which one adopts a “brane” world picture with the Majoron free to experience the extra dimensions have been studied. Bounds on the dimensionless coupling constants were obtained, allowing for any number of extra dimensions and any intrinsic mass scale. If the uncertainty of the Z’s invisible width $1.7 \times 10^{-3}$ GeV is roughly taken as an indication of the maximum allowed value for the total width into a Majoron and two neutrinos the following bounds on $|g_{aa}|$ were obtained in [20] for $M_S = 10^4$ GeV: $|g_{aa}| < 3.4 \times 10^{-12}$, $|g_{aa} | < 2.3 \times 10^{-10}$, $|g_{aa} | < 1.5 \times 10^{-8}$ for $\delta = 2, 3, 4$, respectively. These are much stronger bounds than the one obtained for the model in 3 + 1 space-time dimensions which is $|g_{aa}| < 0.11$.

If a technically natural see-saw model is adopted, the predicted coupling constants are far below these upper bounds. In addition, for this natural model, the effect of extra dimensions is to decrease the predicted partial Z width, the increase due to many Kaluza-Klein excitations being compensated by the decrease of their common coupling constant.

We shall see in the following that constraints from supernovae are much more stringent than the ones above.

### IV. CONSTRAINTS FROM SUPERNOVAE

**Supernova cooling** — The proto-neutron stars created by core-collapse supernovae are born with core temperatures of 30-50 MeV. The main cooling mechanism for these stars is thermal surface emission of neutrinos on a timescale of a few seconds. The total energy emitted is the order a few $10^{53}$ erg, with roughly equal amounts in all flavours of neutrinos and antineutrinos. This emission has been observed from SN1987A by Kamiokande [28], IMB [29] and Baksan [30], all observing $\nu_e$ events. The results from SN1987A are compatible with theoretical supernova models and therefore put a constraint on any non-standard cooling mechanism carrying away too much energy, and since majorons from the KK-tower will be produced in the supernova and carry away energy 1987A data can be used to constrain $g_{aa}$. This has been done several times in the literature for the 3 + 1 dimensional majoron models [31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42]. For these models, only a relatively small band in parameter space is excluded for the following reason: For large $g_{aa}$, the majorons are tightly coupled inside the neutron star, and only escape via surface emission, like neutrinos. Therefore they cannot carry away most of the energy and supernovae do not yield significant constraints. On the other hand, for small $g_{aa}$ the majorons do escape freely, but they are not produced in significant numbers. The end result is that a band of roughly $3 \times 10^{-7} < g_{aa} < 2 \times 10^{-5}$ is excluded [41].

For the $3 + 1 + \delta$ models the situation is different because each KK-mode is very weakly coupled. Therefore we are always in the limit where majorons escape freely once they are produced, and do not need to worry about possible surface emission. A fairly robust constraint on such “bulk emission” is the one proposed by Raffelt [43], that the emissivity of the neutron star medium should be

$$\epsilon \lesssim 10^{19} \text{ erg g}^{-1} \text{ s}^{-1}. (15)$$

In the 3 + 1 dimensional case where majorons are strictly massless the neutrino pair annihilation $\nu \bar{\nu} \rightarrow J$ is not allowed kinematically and the most important processes are $\nu \bar{\nu} \rightarrow J J$ and $\nu \rightarrow \bar{\nu} J$. However, in the present case, most of the emitted majorons have mass of order $T$, and the simple pair-annihilation is by far the most important process. The squared and spin-averaged matrix element for this process (per single Kaluza-Klein excitation) is

$$|A|^2 = \frac{1}{2} g_{aa} p_1 \cdot p_2. (16)$$

The emissivity per volume of the medium is then

$$Q(m_J) = \int d^3 p_1 d^3 p_2 d^3 p J(2\pi)^3 f_\nu(p_1)f_{\bar{\nu}}(p_2) \times |A|^2 \delta^4(p_1 + p_2 - p_J)(E_1 + E_2), (17)$$

where $d^3 p = d^3 p/(2\pi)^3 2E$ and $f_\nu$ are the thermal distributions of the incoming neutrinos, $f_{\bar{\nu}} \equiv e^{-p_\nu/T}$. Doing the integral gives

$$Q(m_J) = \frac{1}{128 \pi^3} g_{aa}^2 T^8 x^4 K_2(x) , \quad x \equiv m_J/T, (18)$$

where $K_2(x)$ is a Bessel function. However, we also need to sum over the KK-tower in order to obtain the total emissivity

$$Q = \frac{x^{\delta/2}}{\Gamma(\delta/2)/(2\pi)^{3/2}} \frac{M_p^5 T_5^{5+\delta}}{M_5^{2+\delta} g_{aa}} \int_0^\infty dx x^3 K_2(x) \quad (19)$$

Using the rough equality $Q = \epsilon(\rho)$, $\langle \rho \rangle \approx 4 \times 10^{14}$ g cm$^{-3}$, this can be translated into a bound on $g_{aa}$ and $M$ which is

$$g_{aa} \lesssim X M_{\text{TeV}}^{1+\delta/2} T_{30}^{(5+\delta)/2}, \quad (21)$$

with the following values for $X$

$$X = \begin{cases} 1.0 \times 10^{-21} & \text{for } \delta = 2 \\ 2.1 \times 10^{-17} & \text{for } \delta = 4 \\ 4.6 \times 10^{-13} & \text{for } \delta = 6 \end{cases} \quad (22)$$

In the above, $T_{30} = T/(30 \text{ MeV})$. In all figures we use $T_{30} = 1$. The bounds are shown in Fig. 1 as a function of $M$. However, from considerations of graviton emission, strong bounds on $M$ already exist [44, 45, 46, 47, 48, 49]. These are shown as the thicker lines in the left part of the figure. Values of $M$ in this range are already excluded from graviton emission arguments [48].

Old neutron star excess heating — When considering graviton emission a much stronger bound on $M$ can be obtained by considering the decays of KK-gravitons produced in supernovae. Gravitons have a significant branching ratio into photons and the decays therefore produce gamma rays. All cosmological supernovae have therefore by the present produced a diffuse cosmic gamma background. Comparing this with observations of the diffuse gamma background by the EGRET instrument [50] yields a bound on $M$ significantly stronger than found from the supernova cooling argument [48]. More interestingly, the tightest constraint comes from observations of old, isolated neutron stars.

Most of the KK-modes emitted from the proto-neutron star have masses of order $3T$ and therefore also relatively low velocities. This again means that a large fraction (roughly one half) of the KK-modes have velocities lower than the escape velocity, and that neutron stars retain a halo of KK-modes with a typical radius of 2-3 $R_{\text{NS}}$. When these gravitons decay they heat the neutron star and can lead to excess surface emission from old neutron stars. This argument applies equally to KK-gravitons and majorons. For gravitons it was used in Ref. [50] to put an extremely strong limit on $M$, $M > 1600$ TeV ($\delta = 2$), 70 TeV ($\delta = 3$).

The KK-majorons only decay into neutrinos and not photons. However, the typical energy of the emitted neutrinos is 50-100 MeV, and the neutron star is not transparent to neutrinos of such high energy. Therefore the neutrinos hitting the neutron star surface will heat it just like photons do. The surface luminosity of the neutron star at late times should therefore reach a constant level corresponding to the energy deposited by decay neutrinos. This luminosity is given by

$$L_{\text{NS}} = f_J F_J E_{\text{TOT}} \langle \Gamma J F \rangle R_{\text{halo}}^2 R_{\text{NS}}^2, \quad (23)$$

where $f_J$ is the fraction of the total supernova energy emitted in majorons, $F_J$ is the fraction of the emitted majorons remaining bound to the neutron star, taken to be $1/2$, $E_{\text{TOT}}$ is the total SN energy, taken to be $3 \times 10^{53}$ erg, $\langle \Gamma \rangle$ is the average majoron decay rate, and $R_{\text{halo}}$ is the typical radius of the majoron halo, taken to be $2 R_{\text{NS}}$. The decay rate for non-relativistic majorons is given by

$$\Gamma_J = \frac{1}{64\pi} g_{aa}^2 m_J. \quad (24)$$

$f_J$ is a function of $M$ and $g_{aa}$, and can be found from the above cooling bound. The cooling bound, to a good approximation, corresponds to half the total energy being emitted in majorons ($f_J \simeq 1/2$). For gravitons the strongest bound comes from the neutron star PSR J0953+0755 [51, 52, 53], which is the oldest neutron star for which the thermal surface temperature has been measured. Its total surface luminosity is estimated to be $L \sim 10^{-5} L_{\odot}$ [54]. For majorons this neutron star also yields a strong constraint on $g_{aa}$ which is roughly

$$g_{aa} \lesssim X M_{\text{TeV}}^{1+\delta/2} T_{30}^{(5+\delta)/4}, \quad (25)$$

with the following values for $Y$

$$Y = \begin{cases} 3.3 \times 10^{-22} & \text{for } \delta = 2 \\ 4.8 \times 10^{-20} & \text{for } \delta = 4 \\ 7.0 \times 10^{-18} & \text{for } \delta = 6 \end{cases} \quad (26)$$

In all cases this bound is significantly stronger than the cooling bound, just as it is for gravitons. The bounds are summarized in Fig. 2. However, there is a limit to the applicability of the bound. The age of the neutron star PSR J0953+0755 is estimated to be $\tau = 1.7 \times 10^7$ yr [54]. If the majorons decay faster than this, they will have vanished already and cannot act as a heating source. From Eq. (24) one finds a lifetime of

$$\tau = 1.3 \times 10^{-21} g_{aa}^{-2} L_{\text{MeV}} / m_J \quad (27)$$

giving a rough upper limit on $g_{aa}$ of $1.5 \times 10^{-18}$, above which the limit of Eq. (23) does not apply.

Another possible problem is that majorons could be reabsorbed when they pass through the neutron star on a timescale much shorter than the age of the neutron star [54]. The most relevant process for reabsorption is inverse bremsstrahlung, $JN N \rightarrow N N$ which is induced via the electroweak interactions [22]. Since the majoron is a pseudo-scalar one can estimate the rate for this process in the same way as for axions [48]. The result is that reabsorption only happens on a timescale much longer than the lifetime of the neutron star PSR J0953+0755.
FIG. 1: Upper bounds on $g_{\alpha\alpha}$ from the supernova cooling bound, Eq. (21), for various $\delta$ and $M$. The bottom curve is for $\delta = 2$, the middle for $\delta = 4$, and the top for $\delta = 6$. The thick lines at low $M$ correspond to the excluded region of $M$ from graviton effects. The horizontal line corresponds to the upper limit of applicability of the neutron star heating limit.

FIG. 2: Upper bounds on $g_{\alpha\alpha}$ from the neutron star heating bound, Eq. (24), for various $\delta$ and $M$. The bottom curve is for $\delta = 2$, the middle for $\delta = 4$, and the top for $\delta = 6$. The thick lines at low $M$ correspond to the excluded region of $M$ from graviton effects. The horizontal line corresponds to the upper limit of applicability of the neutron star heating limit.

V. DISCUSSION

A. Majorons as dark matter?

By the same neutrino pair annihilation process as in supernovae, majorons are also produced in the early universe. This means that in principle one might obtain non-trivial bounds on $g_{\alpha\alpha}$ from considering cosmological production of majorons. For gravitons such considerations lead to extremely stringent bounds on $M$ and the maximum temperature, $T_{RH}$, of the radiation dominated epoch after inflation [53, 54, 55, 56]. If the fundamental scale $M$ is close to 1 TeV then the reheating temperature needs to be extremely low, typically in the MeV regime. However, there is a lower limit to how low $T_{RH}$ can be without disturbing big bang nucleosynthesis. Detailed calculations have shown that this limit is roughly $T_{RH} \gtrsim 0.7$ MeV [53, 54]. The number density of majorons in the universe can be found by solving the integrated Boltzmann equation

$$\dot{n}_J = -3Hn_J + \frac{g_{\alpha\alpha}^2}{128\pi^3}m_J^3TK_1(m_J/T),$$  \hspace{1cm} (28)

which applies to a single majoron mode with mass $m_J$. $n_J$ is the number density of the single majoron mode $J$ and $H$ is the Hubble parameter. By summing over all KK-modes of the majoron in the same way as it was done for gravitons in Refs. [53, 54], one finds a present day density of

$$\rho_J = 8.3 \times 10^{-24} \frac{\pi^{5/2}}{T^{(\delta/2)}} \left(\frac{M_P}{T_{RH}}\right)^2 \left(\frac{T_{RH}}{M}\right)^{\delta+2} \text{GeV}^4 \times g_{\alpha\alpha}^2 \int_0^\infty dz z^{\delta-1} \int_z^\infty dq q^3 K_1(q),$$  \hspace{1cm} (29)

Requiring that this density is smaller than the critical density, $\rho_c = 8.1 \times 10^{-47}h^2$ GeV$^4$, then yields an upper bound on $g_{\alpha\alpha}$ as a function of $M$ and $T_{RH}$.

However, there is again a limit to the applicability of the bound because for large $g_{\alpha\alpha}$ the majorons will have decayed by the present. For the decay rate given in Eq. (24) and the requirement that $T \gtrsim 10^{10}$ yr one finds

$$g_{\alpha\alpha} \lesssim 3 \times 10^{-19}T_{RH,\text{MeV}}^{-1},$$  \hspace{1cm} (30)

assuming that the typical majoron mass is $\sim 3T_{RH}$, a fairly good approximation. In Fig. 3 we show the contours of $g_{\alpha\alpha}$ corresponding to critical density. Also shown is the lower bound on $M$ as a function of $T_{RH}$ from considering the decay of gravitons produced in the early universe. From this argument anything to the left of the thick grey lines in the figure is excluded. The limits used are the ones from Ref. [50] which are the most restrictive cosmological limits, combined with the limits from Ref. [14] which for low $T_{RH}$ can be more restrictive. Finally, we also show the upper bound on $g_{\alpha\alpha}$ from the above equation. In the case where the line from the graviton bound is to the right of the decay lifetime bound, majorons cannot possibly contribute to critical density. This is seen to be the case for both $\delta = 2$ and $\delta = 6$, and indeed for all values of $\delta$ from 2 to 6. Therefore the conclusion is that no non-trivial bound on $g_{\alpha\alpha}$ comes from cosmology, and further that majorons cannot contribute the dark matter of the universe [51]. Already, gravitons are excluded as a dark matter candidate because if they were to contribute critical density the photons produced by their decay would have been clearly visible.
FIG. 3: Contours of $\log(g_{\text{aa}})$ which corresponds to $\rho_J = \rho_c$. The upper panel is for $\delta = 2$ and the lower for $\delta = 6$. The thick line to the right is the lower bound on $M$ coming from considering graviton emission. The thick vertical line to the left corresponds to the maximum $g_{\text{aa}}$ for which majorons live longer than the age of the universe.

B. Conclusions

In the present paper we have shown that supernova constraints on brane-world scenarios for neutrino masses are many orders of magnitude stronger than the accelerator bound [20]. Even so the constraints found are somewhat weaker than what is naturally expected in see-saw models of neutrino mass.

We have also shown that bulk majorons cannot act as the dark matter in the universe, at least not within the present scenario with equal radii of all the extra dimensions.

Finally, our findings suggest that unnatural types of the “see-saw” mechanism for neutrino masses are unlikely to occur in nature, even in the presence of extra dimensions.

Acknowledgments

We wish to thank Jukka Maalampi, Georg Raffelt, Joseph Schechter, and Jose Valle for valuable comments. The work of F.S. is supported by the Marie-Curie fellowship under contract MCFI-2001-00181.
[61] It should be noted that 3+1 dimensional majoron models might still be of relevance for the dark matter problem. V. Berezinsky and J. W. F. Valle, Phys. Lett. B318, 360 (1993).