Direct cavity detection of Majorana pairs

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No experiment could directly test the particle/antiparticle duality of Majorana fermions, so far. However, this property represents a necessary ingredient towards the realization of topological quantum computing schemes. Here, we show how to complete this task by using microwave techniques. The direct coupling between a pair of overlapping Majorana bound states and the electric field from a microwave cavity is extremely difficult to detect due to the self-adjoint character of Majorana fermions which forbids direct energy exchanges with the cavity. We show theoretically how this problem can be circumvented by using photo-assisted tunneling to fermionic reservoirs. The absence of direct microwave transition inside the Majorana pair in spite of the light-Majorana coupling would represent a smoking gun for the Majorana self-adjoint character.

Majorana quasiparticles are among the most intriguing excitations predicted in condensed matter physics[1]. The Majorana particle is equal to its own antiparticle, a property which opens up possibilities of non-abelian statistics[2] and topologically protected quantum computation[3] in condensed matter systems. Low frequency conductance measurements have prevailed so far in the experimental search for these exotic quasiparticles. In particular, hybrid structures combining semiconducting nanowires and superconductors have been intensively investigated[4–5]. The recent observation of zero energy conductance peaks and pairs of peaks with an oscillatory splitting are consistent with the existence of Majorana bound states (MBSs)[6–10, 12]. However, it is essential to find new tools to test more specifically the nature of these peaks[13–18].

Paradoxically, the self-adjoint character of MBSs, which draws so much interest, also makes them very difficult to detect. Photons trapped in a high finesse cavity are a priori very appealing for probing these elusive excitations[19]. Indeed, light, contrarily to conductance processes which involve only one partner of a Majorana doublet. During these transitions, photons with frequency \( \varepsilon \) are exchanged between the cavity and the Majorana pair, with a rate set by \( \beta \). However, transitions at frequency \( 2\varepsilon \) remain forbidden regardless of the circuit parameters. The purely longitudinal nature of \( \beta \), thereby revealed, would represent a direct signature, in the simplest setup, of the self-adjoint-character of MBSs.

Hybrid nanocircuits with superconducting parts have been coupled very recently to microwave cavities[36–39]. The light/matter coupling in such devices can be described generically with the Hamiltonian

\[
\hat{h}_{\text{tot}} = \hat{h}_N + \omega_0 \hat{a} \hat{a}^\dagger + \hat{h}_C (\hat{a} + \hat{a}^\dagger) \tag{1}
\]

with \( \omega_0 \) the cavity frequency. In a typical experiment, one determines the cavity frequency pull \( \Delta \omega_0 \) and damping pull \( \Delta \Lambda_0 \) from the cavity microwave response measured at frequency \( \omega_{RF} = \omega_0 \). In case of an electric coupling scheme, from a semiclassical linear response description, these signals correspond to \( \Delta \omega_0 + i \Delta \Lambda_0 = \chi(\omega_0) \), with \( \chi \), the nanocircuit charge susceptibility[37]. In a first approach, one can assume that the nanocircuit spectrum is discrete. Since \( \hat{h}_{N(C)} \) are quadratic, one can use \( \hat{h}_N = \sum_{\alpha} E_{\alpha} \hat{\gamma}^\dagger_{\alpha} \hat{\gamma}_\alpha \) and \( \hat{h}_C = \sum_{\alpha \beta} M_{\alpha \beta} \hat{\gamma}^\dagger_{\alpha} \hat{\gamma}_\beta + N_{\alpha \beta} \hat{\gamma}^\dagger_\alpha \hat{\gamma}_\beta + N^\dagger_{\alpha \beta} \hat{\gamma}^\dagger_\alpha \hat{\gamma}_\beta \), with \( E_{\alpha} > 0 \), \( \hat{\gamma}_\alpha \) Bogoliubov operators which combine electron and hole excitations, and \( \{ M_{\alpha \beta}, N_{\alpha \beta} \} \) matrix elements which depend on the overlap of the cavity photonic pseudopotential with the wavefunctions associated to \( \hat{\gamma}^\dagger_\alpha \) [34]. At zero temperature (\( T = 0 \)), this gives \( \chi^*(\omega_0) \simeq \sum_{\alpha \beta} |N_{\alpha \beta}|^2 (\omega_0 - E_\alpha - E_\beta + i0^+)^{-1}/2 \). Importantly, due to the Pauli exclusion principle, one has \( N_{\alpha \alpha} = 0 \). Hence, \( \chi(\omega_0) \) does not in-
exist a state degeneracy ωcircuit response at the inter-level separation [40]. However, having a nano-
circuit extended to a pair of MBSs described by self-adjoint creation operators in a finite portion of nanoconductor is driven to its topo-
cial to obtain MBSs. In elementary models [4, 5], when contacts [36, 41]. In contrast, lifting degeneracies is cru-
as observed in spin-degenerate superconducting atomic
conjugated operators involve transitions between electron and holes associated to
site in the nanocircuit placed in the microwave cavity. The DOS in S depends on the superconducting gap Δ and the broadening parameter Γt. The cavity microwave transmission b∥/b⊥ or reflection b⊥/b∥ is measured.

volve transitions between electron and holes associated to
conjugated operators γα† and γα. This selection rule can be extended to T ≅ 0 or a level broadening smaller than the inter-level separation [40]. However, having a nanocircuit response at ω0 = 2Ea is possible provided there exists a state degeneracy Eα = Eα′ in the nanocircuit [40], as observed in spin-degenerate superconducting atomic contacts [36, 41]. In contrast, lifting degeneracies is cru-
ial to obtain MBSs. In elementary models [4, 5], when a finite portion of nanoconductor is driven to its topo-
logical phase, one non-degenerate Bogoliubov doublet (γ1†, γ1) approaches along the zero energy area to form a pair of MBSs described by self-adjoint creation operators \( \hat{m}_L = (\gamma_1^\dagger + \gamma_1)/\sqrt{2} \) and \( \hat{m}_R = i(\gamma_1^\dagger - \gamma_1)/\sqrt{2} \) such that
one has, at low energies, \( \hat{h}_N = \varepsilon \gamma_1^\dagger \gamma_1 = i\varepsilon \hat{m}_L \hat{m}_R + (\varepsilon/2) \) with \( \varepsilon = E_{11} \). An important signature of this scenario is the absence of direct microwave transitions in the Majorana subspace, i.e. \( \chi(\omega_0 = 2\varepsilon) \approx 0 \) due to \( N_{11} = 0 \).

Remarkably, this occurs even when the Majorana-cavity coupling is finite, i.e. \( \hat{h}_C \) contains a term in \( i\beta \hat{m}_L \hat{m}_R \) with \( \beta = M_{11} \). Using a spin analogy, the cavity and the MBSs can only have a longitudinal coupling, which is not able to change the state of the Majorana pair, i.e. \( \hat{h}_C \) and \( \hat{h}_N \) are represented by collinear vectors in the Bloch sphere associated to the Majorana subspace. This gives a smoking gun for the non-degenerate \( (\gamma_1^\dagger, \gamma_1) \) electron-hole conjugated pair, or equivalently, the pair of self-adjoint excitations \( (\hat{m}_L, \hat{m}_R) \). Importantly, the absence of direct microwave transitions in the Majorana doublet is meaningful only if one can confirm \( \beta \neq 0 \). Furthermore, the absence of direct transitions should be robust when the control parameters of the nanocircuit are varied, to discard any accidental cancellation of \( \chi(\omega_0) \). We use below a specific example to show how these tasks can be achieved.

We now consider a semiconducting nanowire subject to spin-orbit coupling and a Zeeman field. The nanowire is tunnel coupled along its whole length to a superconducting contact S. It is also coupled at both ends to normal metal contacts \( N_L \) and \( N_R \) which enable the measurement of the nanowire density of states ν(ω). We describe this circuit with a phenomenological one-dimensional tight-binding chain \( \hat{h}_N = \sum_n (d_n^\dagger (E_x \sigma_y - \mu \sigma_0) d_n - \langle \gamma_1^\dagger \rangle \sigma_0 + \Delta \sigma_y) d_{n+1} + h.c.) \) with \( d_n = \{ d_n^\dagger, d_n \} \) and \( d_n^\dagger \) the creation operator for an electron with spin \( \sigma \) in site \( n \in [1, N_c] \) of the chain (Fig.1). We denote by \( E_x \) the Zeeman field on the sites, μ the sites chemical poten-
tial, which can be tuned with a gate voltage, \( t \) the hopping constant between the sites and \( \Lambda \) the spin-orbit constant. So far, studies on MBSSs coupled to cavities have reduced the effect of \( S \) to an effective pairing term added to \( \hat{h}_N \) \cite{20,23}. However, a realistic model must also take into account level broadening and dissipation. Therefore, we describe explicitly tunneling to \( S \) and \( N_{L,R} \) with an Hamiltonian \( \hat{h}_R \) (see \cite{10} for details). This leads us to introduce the tunnel rate \( \Gamma_N \) between site 1(\( N_r \)) and contact \( N_{L,L} \) and the tunnel rate \( \Gamma_S \) between site \( n \in [1, N_r] \) and contact \( S \). The DOS of \( S \) depends on the superconducting gap \( \Delta \) but also on a phenomenological parameter \( \Gamma_b \) which accounts for a broadening of the BCS peaks and a finite low energy DOS (Fig.1). Such effects can be caused by a finite magnetic field \cite{42,43}. We consider short chains and choose parameters such that few subgap levels are visible in the DOS of the nanowire, like in recent experiments\cite{10,12}. More precisely, for case A, one has \( N_r = 20, t = 2.5\Delta, \Lambda = 5\Delta, \Gamma_b = 0.1\Delta, \Gamma_N = 0.001\Delta, \mu = 6\Delta \) and for case B one has \( N_r = 8, t = 5\Delta, \Lambda = 4\Delta, \Gamma_b = 0.05\Delta, \Gamma_N = 0.2\Delta, \mu = 5.5\Delta \). We use \( E_z = \Delta, \Gamma_S = 5.5\Delta \) and \( k_B T = 0.01\Delta \) for both cases. The density of states \( \nu(\omega) \) at the ends of the nanowire reveals the occurrence of a pair of MBSSs above a critical Zeeman field (Fig.1(b)). These MBSSs show an oscillatory energy splitting with \( E_z \) (case B), or stick to zero energy for a longer chain (case A)\cite{11}.

We assume that the above nanocircuit is embedded in a microwave cavity. Hence, we use Hamiltonian \( \hat{h}_{tot} \) of Eq.(1) with \( \hat{h}_N = \hat{h}_W + \hat{h}_R \) and \( \hat{h}_C = g \sum_n \hat{a}_n^\dagger \hat{a}_n \). This last term means that cavity photons modulate the chemical potential of site \( n \) with a coupling constant \( g \). To treat on the same footing internal nanowire transitions and tunneling to the reservoirs, we use a Keldysh approach\cite{37,41,44,46}. We use Python and Numba, a LLVM-based Python compiler\cite{47}, to calculate numerically \( \Delta(\omega_0) \) and \( \chi(\omega_0) \) with \( \chi(\omega_0) = -i g^2 \int \frac{d\omega}{4\pi} \mathrm{Tr} \left[ \hat{S}(\omega) \hat{G}^\tau(\omega) \hat{S}(\omega) \right] \) (2) and \( \hat{S}(\omega) = \tau \left( \hat{G}^\tau(\omega + \omega_0) + \hat{G}^\tau(\omega - \omega_0) \right) \). Above, the retarded and advanced multisite Green’s functions \( \hat{G}^\tau/\hat{a} \) and the lesser self energy \( \Sigma^<(\omega) \) can be calculated from \( \hat{h}_N \), while \( \tau \) takes into account the structure of the photon/particle coupling in the Nambu⊗spin space\cite{40}.

We focus on the dissipative response \( \Delta_A = \text{Im}[\chi(\omega_0)] \) of the cavity, which should naturally reveal the effects of dissipative reservoirs. We first consider case A where \( \varepsilon \to 0 \) in the topological phase of the nanowire. The \( \omega_0 = E_z \) map of \( \text{Im}[\chi(\omega_0)] \), shown in Fig.2, reveals a wealth of features, sketched in Fig.2. Feature (1) is shown in more details in Fig.2 for constant values of \( \omega_0 \). It consists of a step at \( \omega_0 = \varepsilon \), which is the energy distance between one MBSS and the Fermi level of the reservoirs. For case A, the effects of the \( N_{L,R} \) reservoirs on \( \chi \) can be disregarded due to the vanishing \( \Gamma_N \). Therefore, feature (1) can be attributed to photo-induced tunneling between the MBSSs and the residual subgap DOS of \( S \), as represented in Fig.2. In practice, it is possible to have a well-grounded contact by realizing a direct connection between the cavity ground plane\cite{37,38}. In this case, feature (1) can exist only if the MBSSs are directly coupled to cavity photons, i.e. \( \beta \neq 0 \). In spite of this coupling, no transition occurs at \( \omega_0 = 2\varepsilon \) (red dotted line in Fig.2). The simultaneous presence/absence of a step/resonance at \( \varepsilon(2\varepsilon) \) occurs on a wide range of \( E_z \). We have therefore obtained a signature of the Majorana self-adjoint character\cite{50}. More precisely, these features indicate that we are in presence of a non-degenerate electron/hole conjugated pair, which is the natural precursor of a Majorana pair. It is then important to check from \( \nu(\omega) \) that \( \varepsilon \) vanishes with \( E_z \) (or shows several zero-energy crossings), as a signature of the spatial isolation of the two MBSSs formed out of the non-degenerate electron/hole pair.

We now demonstrate the robustness of our main results to variations of the nanowire spectrum. Figure 3(a) shows the \( \omega_0 - E_z \) map of \( \text{Im}[\chi(\omega_0)] \) in the case of a shorter nanowire (case B). Feature (1) persists in this limit, as indicated by the pink circle. Besides, the yellow circles indicate new steps caused again by photo-assisted tunneling from/to the MBSSs at \( \omega_0 = \varepsilon \). These steps can have a contrast significantly stronger than feature 1. Meanwhile, no resonant feature is visible along the \( \omega_0 = 2\varepsilon \) contour indicated by the red dotted line. This extends the possibilities for testing the longitudinal character of the coupling between the Majorana doublet and the cavity. Importantly, feature 1 of case A has an amplitude of \( 3 \times 10^{-4}g^2/\Delta \). With \( \Delta = 180 \mu eV \) and a site-cavity coupling \( g = 2 \mu eV \), this corresponds to 15 kHz. The pink circle features of case B have an amplitude of \( 6.3 \times 10^{-4}g^2/\Delta \) which corresponds to 340 kHz. These signals are small but within experimental reach\cite{51,52}, although the residual zero energy DOS in S is small, i.e. \( r \sim 5\%[2.5\%] \) in case A[B], which leads to the “hard gap” situation similar to Ref.\cite{53}. Note that in case B,
tunneling quasiparticles can be provided by both the S and $N_{L(R)}$ reservoirs, which contribute similarly to the broadening of the low-energy MBSs due to our choice of parameters. We will see later how to distinguish these contributions thanks to a finite bias voltage. Noticeably, an isolated zero energy crossing of two ordinary Andreev bound states could be caused by a trivial spin-degeneracy lifting. However, in this case, the frequency of feature 2, which corresponds to an internal nanowire transition inwards/outwards the pair, will not depend on $E_z$, in contrast with the Majorana case where it strongly depends on $E_z$ due to the topological phase transition.[40] Therefore, our setup is also able to rule out the case of a trivial superconducting wire with time reversal symmetry breaking impurities.

In practice, to measure experimentally signals similar to Figs. 2a and 3a, one must vary $\omega_0$. In principle, this is technically possible.[54–55]. However, it is useful to adapt our predictions for standard setups with a fixed $\omega_0$. In this case, other parameters must be changed to characterize the nanocircuit.[56]. In Fig. 4, we show $\Delta \Lambda_0$ versus $E_z$ and $\mu$, for case A. We use $\omega_0 = 0.15 \Delta$ which corresponds, with the gap $\Delta = 180 \mu$eV of Al, to the value $\omega_0 = 6.6$ GHz compatible with present microwave techniques. In these conditions, $\Delta \Lambda_0$ shows an ensemble of photo-assisted tunneling stripes which reveal the well known oscillations of $\epsilon$ with $E_z$ and $\mu$. The correspondence between Figs. 3a and b is given by the pink and yellow circles. The stripes are absent for low values of $E_z$, where the nanowire makes the transition to its non-topological phase and the MBSs thus disappear. The $\omega_0 = 2\epsilon$ contours are shown with red dotted lines in Fig. 4a. They do not correspond to any remarkable feature in the $\mu - E_z$ map of $\text{Im}[\chi(\omega_0)]$, contrarily to the $\omega_0 = \epsilon$ contours (pink dashed lines). Importantly, in an experiment, the red and pink contours can be determined independently from any theory, by performing a conductance measurement on $N_{L(R)}$ to get $\nu(\omega)$. We conclude that in the case where $\omega_0$ cannot be varied, a $\mu - E_z$ map of $\Delta \Lambda_0$ and $\nu(\omega)$ gives an efficient way to characterize the light-matter coupling in our circuit.

We show below that applying a bias voltage to the hybrid nanocircuit-cavity system[57–59] enables one to discriminate processes involving the different fermionic reservoirs and to further check that the MBSs are well coupled to cavity photons. Fig. 4 shows the $\omega_0 - E_z$ contour map of $\text{Im}[\chi(\omega_0)]$ for case B, with a finite bias voltage $V$ applied simultaneously to $N_L$ and $N_R$. We observe clear differences with the case $V = 0$ of Fig. 3a. First, a new step marked by the black circle appears, due to tunneling between the MBSs and the $N_{L(R)}$ reservoirs, at $\omega_0 = \epsilon - eV$. Meanwhile, the step marked with the pink circle at $\omega_0 = \epsilon$ persists and is now only due to tunneling to $S$. The separation $\lambda V$ between the black and pink circles, which appears for a finite $V$, is also well visible in the $\mu - E_z$ map of $\text{Im}[\chi(\omega_0)]$ (see Fig. 4a, where $\lambda \approx \epsilon/\Delta(\partial \epsilon/\partial E_z)$). Second, photon emission revealed by $\text{Im}[\chi(\omega_0)] > 0$ appears for $V > \epsilon + \omega_0$, due to inelastic tunneling between $N_{L(R)}$ and the upper MBS (red areas in Fig. 4a). We conclude that the use of a bias voltage enables a differentiation of the processes involving the $N_{L(R)}$ and $S$ contacts. The persistence of the pink circle feature ensures that cavity photons modulate the potential difference between the MBSs and $S$. Interestingly, for $V > \epsilon$, the upper MBS becomes populated so that internal transitions to upper Andreev levels appear at remarkably low frequencies (see e.g. green circle in Figs. 4c and d). This represents another signature of the photon-MBS coupling, although it is not the coupling constant $\beta = M_{11}$ which is involved in this case but rather $N_{1\alpha}$ with $\alpha \neq 1$ and $E_\alpha > \epsilon$.

In any detection setup, false positive detection events can occur. In our case, a false positive detection of MBSs could happen in the (unlikely) case of a pair of extended non-degenerate Andreev bound states which would have accidentally an energy splitting with the same magnetic field dependence as the non-local pair of localized Majorana modes of Fig. 1a, and a non-conclusive feature 2. To rule out such a situation, one could perform supplementary tests readily accessible in our setup, such as non-local transport measurements using the $S$, $N_L$ and $N_R$ contacts, with, for instance, a varying pair splitting $2\epsilon$ (see for instance Refs.[63–67]).

To conclude, we have shown how to exploit photoinduced tunneling to check that a pair of MBSs is directly coupled to cavity photons. However, the direct microwave transitions inside the Majorana subspace remains forbidden in a wide range of parameters. This provides a means to check the self-adjoint character of MBSs. Importantly, this protocol is independent from any theory if the conductance of the nanowire is measured simultaneously with the cavity response to determine $\nu(\omega)$. Such crossed measurements are routinely achieved with mesoscopic QED devices.[37–40][58–61]. Our proposal relies on a nanocircuit geometry widely realized experimentally, and which has reproducibly revealed low energy conductance peaks. Furthermore, nanomaterials with superconducting contacts have been coupled to microwave cavities recently.[67–69]. Therefore, our proposal can be straightforwardly implemented with present ex-
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Supplemental Material

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In practice, the coupling between the fermionic reservoirs and the cavity photons can be characterized by tuning the nanocircuit in the adiabatic regime where tunneling is much faster than $\omega_\alpha$, and using a strong semiclassical microwave excitation, along the lines of Ref.19.

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