Quark Molecular Model of the $S = 0$ Strange Pentaquark $(u\overline{s})$-$(uds)$ Baryon Spectrum

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We present a non-relativistic quark molecular model (QMM) of the strange crypto-exotic pentaquark baryon spectrum motivated by the recent data showing narrow, resonance enhancements in electromagnetic and hadronic production of kaons. Our model assumes color octet bonded quark molecular clusters forming exotic color singlet pentaquark baryons. We develop explicit molecular pentaquark wavefunctions that exhibit color interchange and cluster fermion antisymmetry. Our color electro-dynamics (CED) inspired Hamiltonian, which includes confinement, one gluon exchange and color magnetic interactions, is a natural generalization of the Isgur and Karl quark model Hamiltonian that reproduces the conventional meson ($q\overline{q}$) and baryon ($qqq$) spectrum [1]. We introduce the idea of Color Magnetic Confinement (CMC) which is employed to restrict the physical pentaquark spectrum and constrain the fundamental constants (color gyromagnetic ratios) of the QMM-CED Hamiltonian. Comparing our QMM spectrum predictions with the recent experimental data in associated strangeness photo- and electro-production reactions, we identify 4 candidate states that have masses and total widths consistent with exotic pentaquark structure.

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I. INTRODUCTION

Recent experimental data on electromagnetic and hadronic associated strangeness production suggest the possible existence of narrow multi-quark $S = 0$ [2,3] and $S = +1$ [4,5] states near 2.0 GeV. These pentaquark systems are in the realm of opening a complete new area of exciting physics on both experimental and theoretical fronts.

The existence and corresponding properties of exotic matter remains a puzzling open question in hadron spectroscopy. Resolving this mystery is important for understanding how QCD provides a consistent description for the binding of quarks and gluons into hadrons [6]. Exotic hadrons are color singlet multiquark combinations ($qq\overline{q}$, $qqqq\overline{q}$ . . . ) beyond the standard model quark-gluon-antiquark description of the normal hadrons (i.e., non-exotic quantum numbers). Examples include the $a_0/f_0(980)$ resonances, which are established scalar meson states that find their most natural interpretation as hadronic molecules, having wave functions with large components of lightly bound KK mesons [7,8]. The most stable configurations of $qqqq\overline{q}$ exotics are believed to be strange molecules, like $S^+_1(B^0\Lambda)$ and $S^-_1(B^0\Sigma)$, bound by long range color forces acting between color octet quark clusters (octet-bonded states). Exotic molecules such as ($qq$)-($qq\overline{q}$) can exist in the color 6-6 representations (sextet-bonded states), however these are expected to be much less stable because a color triplet $qq\overline{q}$ subsystem always contains a $q\overline{q}$ pair which can form a color singlet, allowing the sextet bonded $qqqq\overline{q}$ system to easily dissociate into a normal color singlet baryon ($qqq$) and meson ($q\overline{q}$) [9,10]. For a baryon $B \equiv qqq$ and a meson $M \equiv q\overline{q}$, if $B$ and $M$ are separately color singlets, the molecules $(B - M)$ will interact only via the relatively weak Van der Waals forces of QCD and thus will dissociate rapidly (and have large decay width) into a normal baryon and meson. However, if the molecular system $(B - M)$ is a color singlet with $B$ and $M$ being separately color octet states and possessing non-zero relative orbital angular momentum $L$, this system should be linked by very strong color forces and exhibit enhanced stability (i.e., long lifetime and narrow decay width compared to usual hadronic resonance states) since the anti-quark in $M$ is prevented by a centrifugal barrier from tunneling into $B$ where it can combine with a quark to form a color singlet meson. Additional stability is anticipated if at least one molecule cluster contains a strange (or charm) quark, since then decays to open non-strange ($\pi$, $\rho$, $\omega$) channels will be OZI [11] suppressed.

For $S = 0$ strange nucleon resonances, each color octet molecule cluster must have a strange quark, $B = qqs$ and $M = q\overline{s}$. This naively paradoxical feature of having $S = 0$ with strange exotic structure generates an interesting selection rule for detecting exotic baryons: the most stable pentaquark baryons will preferentially decay into strange meson and baryon channels by the OZI rule (such as $K^+\Lambda$, $K^+\Sigma$, etc.), since the strangeness in each molecular $B$ and $M$ cluster is conserved in the OZI enhanced final states. Consequently, the $S = 0$ pentaquark exotics will potentially show up as narrow nucleon resonances in kaon ($K$ or $K^*$) photo- and electro-production reactions, yet totally decouple from the non-strange $\pi$, $\rho$, $\omega$ production channels (or possibly make very small OZI evading/violating contributions).

This paper is the first work to suggest the possibility of exciting $S = 0$ pentaquark resonances in single kaon electromagnetic production from the nucleon (proton or neutron). The recently reported $S = +1$ pentaquark baryon resonance in $K^+K^-$...
photo-production from the neutron [4] is a crypto-exotic state with Λ quantum numbers (similar to the Λ(1405) which is widely accepted to be a $K^-p$ molecule). In this work we restrict our analysis to the $S = 0$ nucleon resonance sector, leaving the $S = \pm 1$ exotics for consideration in a follow up paper. Before passing into the formalism section, we note that previous authors claim that the $S = 0$ strange exotic baryons and their excited states are expected to fall on linear Regge trajectories [12,13].

The excited molecular pentaquarks presumably also have anomalously narrow decay widths in comparison to normal excited baryons since the same enhanced stability arguments apply. In contrast, our Quark Molecular Model (QMM) implementation permits only 4-low lying $S = 0$ pentaquark solutions, and dynamically excludes the possibility of additional excited states. This limited spectrum feature is a novel prediction of this QMM and stands in contrast to the usual outcome of quark model spectroscopy: to generate an overabundance of excited states, and providing theoretical bias for the experimental dilemma of “missing resonances”.

This paper focuses on two main areas: (1) a dynamical quark model of the $S = 0$ strange exotic baryon (u$\bar{s}$)-(uds) spectrum; and (2) new/recent experimental data that provides preliminary (albeit inconclusive) support for the possible existence of strange exotic resonances.

Section II describes the general formalism of our quark molecular model (QMM), and section III describes the current status of experimental data that suggest resonance enhancements due to the $S = 0$ pentaquark baryons. In forthcoming papers we will present a generalized hadrodynamical Lagrangian model that incorporates exotic resonances through strong final state interactions in kaon photo- and electro-production reactions [2,3], where we will demonstrate that the exotics have a strong forward angle production sensitivity.

II. FORMALISM

Our approach is to first construct totally antisymmetric spin-flavor-space-color pentaquark baryon wavefunctions, and then calculate the low lying crypto-exotic baryon spectrum using a non relativistic quark molecular model (QMM) basis for the spatial wavefunctions and energy eigenvalues. In this work, we assume the pentaquark crypto-exotic systems are heavy, color octet bonded ($B - M$) molecules with anomalous stability. Dynamical stability and explicit decay width calculations will be investigated in a future work. Based on the ideas of our introductory discussion, the lightest, stable $S = 0$ exotic baryons are expected to have two distinct features: (1) contain strange quark molecule clusters $B = qqs$ and $M = q\bar{s}$ to be OZI protected from non-strange decay channels [11]; (2) molecule clusters possess at least one unit of relative orbital angular momentum to generate a centrifugal barrier which suppresses quark rearrangement decay processes. In the derivation that follows, we show that both of these qualitative features naturally arise from an explicit minimal construction of pentaquark wavefunctions with color octet bonded molecular substructure when total antisymmetry (Fermi statistics) of quark interchange is enforced.

In order to construct a minimal color singlet pentaquark molecule with total fermion antisymmetry, first consider the normal 3-q baryon $SU(3)_F$ flavor octet and decuplet wavefunctions. Normal ground state 3-q baryons have totally antisymmetric quark combinations which are products of color [antisymmetric (A) singlet: $C_A^1$], flavor [mixed symmetry (MS) octet: $F_{MS}^8$; or symmetric (S) decuplet: $F_{10}^S$], spin [mixed symmetry (MS) spin $\frac{1}{2}$ doublet: $\chi_{MS}^\frac{1}{2}$; or symmetric (S) spin $\frac{3}{2}$ quartet: $\chi_S^\frac{3}{2}$], and spatial [symmetric (S) even parity: $\Phi_S^1$] wavefunctions, $\Psi_{symmetry}(color, flavor|spin|space)$:

$$\Psi_A(1, 8, 1^+_{\frac{1}{2}}) = C_A^1 \left( F_{MS}^8 \chi_{MS}^\frac{1}{2} \right)_S \Phi_S^1 \quad (\text{octet})$$

$$\Psi_A(1, 10, 3^+_{\frac{3}{2}}) = C_A^1 F_{10}^S \chi_S^\frac{3}{2} \Phi_S^1 \quad (\text{decuplet}),$$

where the large round brackets denotes the symmetric product of $MS$ flavor and spin functions. The corresponding anti-symmetric ($F_{\bar{\chi}}A$) product is excluded for ground states ($L = 0$, and even parity) because the color wavefunction is totally antisymmetric under quark interchange, thus the ground state 3-q baryons always have an overall symmetric product of flavor-spin-space wavefunctions. It is interesting to note that there is no flavor singlet (antisymmetric u.d.s wavefunction) 3-q state in the observed low lying baryon spectrum (the $\Lambda(1405)$ is not a candidate since it is interpreted as a $K^-p$ molecule, and the ground state $\Lambda$ is the isospin singlet member of the flavor octet).

In the quark model, it is easy to see why there is no flavor singlet baryon (at least, in the $\frac{1}{2}^+$ or $\frac{3}{2}^+$ ground states). Constructing the quark model as before, and requiring that all physical hadrons must exist in color singlet configurations, we obtain overall $MS$ and $S$ symmetry for the $\frac{1}{2}^+$ and $\frac{3}{2}^+$ color singlet $\otimes$ flavor singlets:

$$\Psi_{MS}(1, 1, 1^+_{\frac{1}{2}}) = C_A^1 F_A^1 \chi_{MS}^\frac{1}{2} \Phi_S^1$$

$$\Psi_S(1, 1, 3^+_{\frac{3}{2}}) = C_A^1 F_A^1 \chi_S^\frac{3}{2} \Phi_S^1.$$

Hence ground state flavor singlet baryons are inconsistent with the total anti-symmetry requirement. However, if the flavor singlet $F_A^1(u\bar{d}s) \frac{1}{2}^+$ and $\frac{3}{2}^+$ baryons have octet color symmetry, then they can bind with $q\bar{q}$ color octets to form physical color singlet baryon states. Our QMM is built on the assumption of color octet bonded $\alpha$-$\beta$ molecules between baryon number
We construct totally antisymmetric, color singlet pentaquark baryons which are bound color octet quark molecular clusters in the form: \( (\alpha \beta - \delta_\alpha), \) where \( \alpha \beta \) and \( \delta_\beta \) are generic labels referring to the color octet \( B = 0 \) \((u,d,s)\) and \( B = 1 \) \((u,d,s)\) clusters respectively. The central idea required to derive explicit wavefunctions in the \( \alpha - \beta \) QMM basis is to exploit the \( SU(3) \) symmetry of usual quark model color and flavor functions to generate pentaquark states that are color singlets and satisfy quark interchange antisymmetry.

Consider a color octet baryon state (which, like colorful quarks, can not exist unbound in isolation) generated by a \( SU(3) \) Color \( \leftrightarrow \) Flavor symmetry transformation:

\[
\begin{align*}
C^A_\alpha(r, b, g) & \rightarrow C^8_{\alpha}(r, b, g) = F^8_{\alpha}(u \rightarrow r, d \rightarrow b, s \rightarrow g) \\
F^8_{\alpha}(u, d, s) & \rightarrow C^A_\beta(r, u \rightarrow d, b \rightarrow g),
\end{align*}
\]

where \( C^A_\alpha \) and \( F^8_{\alpha} \) are the standard antisymmetric color singlet and mixed symmetry flavor octet functions written in the literature describing details of the conventional quark model. The color-flavor \((C/F)\) transformation simply takes the color 

\( C \)

clusters respectively. The central idea required to derive explicit wavefunctions in the usual quark model color and flavor functions to generate pentaquark states that are color singlets and satisfy quark interchange antisymmetry.

\[
|\xi_{\alpha}(8,1)|_{1/2}^+ > = F^A_{\alpha}(u, d, s) \left( C^8_{\alpha}(r, b, g) \frac{1}{\Lambda_{\alpha}} \right) S \Phi^2_S.
\]

where the MS color and spin function product is symmetrized, and one color-decuplet, flavor-singlet baryon, \( \xi(\frac{3}{2}^+) \):

\[
|\xi_{10}(10,1)|_{3/2}^+ > = F^A_{\alpha}(u, d, s) C^{10}_{\beta}(r, b, g) \chi^3_S \Phi^2_S.
\]

Our goal is to produce overall antisymmetric pentaquark wavefunctions from color octet bonded molecular clusters \( \alpha(q \bar{s}) \) and \( \beta(u,d,s) \). We accomplish this by forming the anti-symmetrized product of \( \alpha \otimes \beta \) quark cluster wavefunctions with odd \( (L = 1) \) relative orbital angular momentum:

\[
\Psi^{X}_{\alpha \beta}(J^+) = |(\alpha \beta)_A||B = 1, I = \frac{1}{2}, Y = 1|J^+ > = |\alpha(q \bar{s})^0_{-1}||C^8_{\alpha}(\alpha) F^A_{\alpha}(\alpha) \chi^1_{\alpha, \beta}(\alpha) \Phi^2_S(\alpha) > \otimes |\beta(u,d,s)_{1/2}^+ ||C^8_{\beta}(\beta) F^A_{\beta}(\beta) \chi^1_{\beta, \alpha}(\beta) \Phi^2_S(\beta) > \otimes Y^{\alpha \beta}_{LM}(\theta, \phi)|_{L=odd} = C^A_{\alpha \beta}(X) F^2_{\alpha \beta}(X) \left[ \chi^{2S+1}_{S}(\bar{S}_X) = \bar{S}_\alpha + \bar{S}_\beta \right] \Psi_S(\alpha \beta |X) \otimes Y^{\alpha \beta}_{LM}(\alpha \beta |X)|_{L=odd} \right|_{S}^{J=L+S_X}.
\]

Here, we suppress Clebsch-Gordon factors and only express how overall fermion and color antisymmetry is generated in the QMM basis. The idea is to simultaneously couple the color octet \( \alpha \) and \( \beta \) functions to an overall antisymmetric \( (\alpha - \beta) \) color singlet function (to satisfy the quark confinement requirement). For notation when we consider experimental data later, we refer to the physical \( (\alpha - \beta) \) pentaquark molecules as the “\( X \)” states. In the QMM construction, one can see that these \( X \) states must have even parity and the \( B = 1, I = \frac{1}{2}, Y = 1 \) quantum numbers of nucleon resonances. The color singlet function \( C^1_{\alpha \beta}(X) \) (which couples \( 8_s \otimes S_\delta \rightarrow 1_X \)) is totally antisymmetric under color exchange. We construct explicit antisymmetric pentaquark color functions in the next subsection.

Note the flavor doublet \( F^2_{\alpha \beta}(X) \) and spin \( \chi^{2S+1}_{S}(\alpha \beta |X) \) functions individually have \( S \)-symmetry, and the total \( X \) angular momentum \( J = L + S_X \) couples total spin \( S_X = S_\alpha + S_\beta \) and orbital angular momentum \( L \) (with appropriate Clebsch-Gordon factors). A detailed analysis shows that the quarks in the \( \alpha(q \bar{s}) \) cluster can couple to either spin \( S = 0, 1 \), in which case the coupled spin-space functions for \( \alpha \) must be in \( A \) or \( S \) configurations respectively (for \( S_\alpha = 0, 1 \) spin state orthogonality). Fermion interchange antisymmetry arises from the product of \( A \)-symmetry color and \( S \)-symmetry flavor, spin, and space \((\alpha \beta |X) \) functions. The \( S \)-symmetry of the iso-doublet molecular flavor function \( F^2_{\alpha \beta}(X) \) is achieved from the product of two totally anti-symmetric flavor functions: the iso-doublet \( F^2_{\alpha \beta}(\alpha) \) and flavor singlet \( F^2_{\alpha \beta}(\beta) \). It is important to note here that interchange between quarks in \( |\alpha > \) and quarks in \( |\beta > \) only preserve the overall exact fermion antisymmetry if the \((\alpha - \beta) \) molecule has even
orbital angular momentum \( L \), because otherwise (for odd \( L \)) \( \alpha \leftrightarrow \beta \) quark exchange would generate a \( MS \) fermion exchange symmetry component due to the negative phase coming from the odd parity orbital \((\alpha - \beta)\) spherical harmonic \( Y_{L,M}(\alpha\beta|X) \) function. Our QMM restriction to odd-\( L \) states is essentially a dynamical constraint. Primarily, even-\( L = 0, 2, 4, \ldots \) states could not be formed in a cluster basis because certain quark exchanges (like \( s \) from \( |\beta\rangle \) exchanged with \( u \) or \( d \) from \( |\alpha\rangle \)) would generate wavefunctions which are not cluster factorized (products of \( |\alpha\rangle \otimes |\beta\rangle \) cluster functions) and hence destroys the chance for defining “basis states” with invariant multiplet quantum numbers. Furthermore, \( \alpha \leftrightarrow \beta \) quark exchange would allow \( s \) or \( \bar{s} \) tunneling and subsequent annihilation, which obviously destabilizes the pentaquark bound state. In QMM we prohibit \( \alpha \leftrightarrow \beta \) quark exchange by requiring the ground state to have one unit of relative orbital angular momentum, and excited states must take odd-\( L \) (to generate a centrifugal barrier to dynamically suppress quark rearrangement decay and to provide the even parity required for crypto-exotic nucleon quantum numbers). So the ground state \( L = 1 \), and \( L = odd \) excited state QMM wavefunctions have a restricted \( cluster \) fermion antisymmetry (that is, exact \( A \)-exchange symmetry for permutations within the \( \alpha(u,d,s) \) cluster and/or permutations within the \( \beta(q,\bar{q}) \) cluster, but \( MS \)-symmetry for interchanges between clusters). Exact overall fermion antisymmetry could be generated for \( \alpha \leftrightarrow \beta \) quark exchange in odd-\( L \) molecules if mixed color symmetry were allowed, but color confinement requires all physical states to be \( A \)-color singlets. From this more fundamental point of view, our QMM cluster fermion antisymmetry is the dynamical symmetry enforcing exact color confinement in a basis of factorized quark cluster functions with odd relative orbital angular momentum (and nucleon, \( N^* \) quantum numbers). Finally, we observe that for fermion antisymmetry within the \( \beta(uds) \) cluster, the \( \beta(\text{color-spin}) \) functions must be symmetrized, as indicated in equation (7), since the flavor singlet carries the overall antisymmetry in \( \beta(u,d,s) \).

**B. Pentaquark Wavefunctions**

1. Color Octet \( \beta(uds) \) States

In this work, we restrict our attention to \( B = 1 \) molecules, which must have the flavor singlet \( \xi(\frac{1}{2}^+) \) as its baryon constituent. The color decuplet \( \xi(\frac{3}{2}^+) \) cannot form \( B = 1 \) molecules since there is no possible construction of color decuplet \( q\bar{q}\) states, which would be necessary to form physical, color singlets. However, we note that the \( \xi(\frac{3}{2}^+) \) and its anti-particle \( \bar{\xi}(\frac{3}{2}^+) \) could form color singlet molecule states with baryon number \( B = 0 \) (\( \xi_{10} \) \( - \xi_{10} \) mesonic sextaquark) or \( B = 2 \) (\( \xi_{10} \) \( - \xi_{10} \) di-baryons). We write the explicit color-octet (spin-color symmetrized) flavor-singlet \( \xi(\frac{1}{2}^+) \) baryon wavefunctions labeled by color isospin \( (I^c,I_3^c) \) and color hypercharge \( (Y^c) \): \( |\xi(\text{spin-parity})|I^c,I_3^c,Y^c \rangle = \frac{1}{\sqrt{6}} [uds - u sd + sud - dsu + dus - dus] \otimes \frac{1}{3\sqrt{2}} [(2r_\uparrow r_\uparrow b_\uparrow - r_\uparrow r_\uparrow b_\uparrow - r_\uparrow r_\uparrow b_\uparrow) + \text{Perms}(1 \leftrightarrow 3, 2 \leftrightarrow 3)] \)
the C/F transformation previously introduced, we have the following quark quantum number assignments for color isospin, hypercharge, and CEC quantum numbers are natural analogs of the corresponding color electric charge is the sum over all quark contributions:

For all hadrons (including exotic multiquark systems) the total color confinement can be expressed as the condition that all physical hadrons must have total color electric which is the color generalization of the $U(1)$ electric charge (CEC) effective refer to the $\alpha$ with the corresponding algebra:

Next consider the $\alpha(qs)|8, 2A - 1, -1 >$ antisymmetric color-octet and flavor-doublet states $|\alpha_i Q^c (spin^\text{parity}) I^c, I^c, Y^c >$:

\[
\Sigma^c = |\Sigma^c(1/2)| 1, -1, 0 > = |\xi(8, 1)| 1/2 |bgb > \tag{20}
\]

\[
= \frac{1}{\sqrt{6}} [uds - usd + sud - sdu + dus - dus] \otimes \tag{21}
\]

\[
\Xi^0 = |\Xi^0(1/2)| 1/2, 1/2, -1 > = |\xi(8, 1)| 1/2 |ggr > \tag{22}
\]

\[
= \frac{1}{\sqrt{6}} [uds - usd + sud - sdu + dus - dus] \otimes \tag{23}
\]

\[
\Xi^- = |\Xi^-(1/2)| 1/2, -1/2, -1 > = |\xi(8, 1)| 1/2 |gbg > \tag{24}
\]

\[
= \frac{1}{\sqrt{6}} [uds - usd + sud - sdu + dus - dus] \otimes \tag{25}
\]

\[
\Lambda^0 = |\Lambda^0(1/2)| 0, 0, 0 > = |\xi(8, 1)| 1/2 |rgb(0) > \tag{26}
\]

\[
= \frac{1}{\sqrt{6}} [uds - usd + sud - sdu + dus - dus] \otimes \tag{27}
\]

\[
= \frac{1}{2\sqrt{3}} [(r_\uparrow b_\uparrow g_\uparrow + r_\downarrow b_\downarrow g_\downarrow) + \text{Perms}(1 \leftrightarrow 2, 3 \leftrightarrow 3)] \tag{28}
\]

The color octet states are named according to their flavor octet counterparts. In this naming convention, the $+, 0, -$ superscripts refer to the effective $U(1)$ color charge of the state, not the electric charge (which is zero for any flavor singlet). To be explicit, we write the octet color electric charge (CEC) operator $Q^c_i$ (for $i =$ quark index):

\[
\hat{Q}^c_i = (\hat{I}^c_i + \frac{1}{2} \hat{Y}^c_i) , \tag{29}
\]

which is the color generalization of the $U(1)$ electric charge quantization of $SU(3)$ flavor octet states. The color isospin, hypercharge, and CEC quantum numbers are natural analogs of the corresponding $SU(3)$ flavor quantum numbers. With the C/F transformation previously introduced, we have the following quark quantum number assignments for color isospin, hypercharge, and CEC as $|\text{color}(I^c, I^c, Y^c)|Q^c >$:

\[
| r \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{2}{3} & \frac{2}{3} & \frac{2}{3} \end{pmatrix} > \tag{30}
\]

\[
| b \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{3} & -\frac{1}{3} & \frac{1}{3} \end{pmatrix} > \tag{31}
\]

\[
| g \begin{pmatrix} 0, 0, \frac{2}{3} \\ 0, 0, \frac{1}{3} \end{pmatrix} > , \tag{32}
\]

with the corresponding algebra:

\[
\hat{Q}^c |r > = \frac{2}{3} |r > , \tag{33}
\]

\[
\hat{Q}^c |b > = -\frac{1}{3} |b > , \tag{34}
\]

\[
\hat{Q}^c |g > = -\frac{1}{3} |g > , \text{ etc.}. \tag{35}
\]

Color confinement can be expressed as the condition that all physical hadrons must have total color electric charge equal to zero (and overall color antisymmetry). For all hadrons (including exotic multiquark systems) the total color electric charge is the sum over all quark contributions: $Q^c = \sum_i Q^c_i$.

2. Color Octet $\alpha(\text{us})$ States

Next consider the $\alpha(qs)|8, 2A - 1, -1 >$ antisymmetric color-octet and flavor-doublet states $|\alpha_i Q^c (spin^\text{parity}) I^c, I^c, Y^c >$: 

\[
\Sigma^c = |\Sigma^c(1/2)| 1, -1, 0 > = |\xi(8, 1)| 1/2 |bgb > \tag{20}
\]

\[
= \frac{1}{\sqrt{6}} [uds - usd + sud - sdu + dus - dus] \otimes \tag{21}
\]

\[
\Xi^0 = |\Xi^0(1/2)| 1/2, 1/2, -1 > = |\xi(8, 1)| 1/2 |ggr > \tag{22}
\]

\[
= \frac{1}{\sqrt{6}} [uds - usd + sud - sdu + dus - dus] \otimes \tag{23}
\]

\[
\Xi^- = |\Xi^-(1/2)| 1/2, -1/2, -1 > = |\xi(8, 1)| 1/2 |gbg > \tag{24}
\]

\[
= \frac{1}{\sqrt{6}} [uds - usd + sud - sdu + dus - dus] \otimes \tag{25}
\]

\[
\Lambda^0 = |\Lambda^0(1/2)| 0, 0, 0 > = |\xi(8, 1)| 1/2 |rgb(0) > \tag{26}
\]

\[
= \frac{1}{\sqrt{6}} [uds - usd + sud - sdu + dus - dus] \otimes \tag{27}
\]

\[
= \frac{1}{2\sqrt{3}} [(r_\uparrow b_\uparrow g_\uparrow + r_\downarrow b_\downarrow g_\downarrow + r_\downarrow b_\uparrow g_\downarrow) + \text{Perms}(1 \leftrightarrow 2, 3 \leftrightarrow 3)] \tag{28}
\]
where $q = u$ is the positive electric charge $I_3 = +\frac{1}{2}$, and $q = d$ is the negative electric charge $I_3 = -\frac{1}{2}$ state. As with the color octet $\beta(B = 1)$ states, the color octet members of the $\alpha(B = 0)$ cluster are named in correspondence with their flavor octet counterparts. The spin one $\alpha$-states are spin triplets having the same color and flavor structure $|\alpha^B_0 (spin^parity)|I^c, I_3^c, Y^c >$:

$$K^c_+ = |K^c_+(0^-)|1, \frac{1}{2}, \frac{1}{2}, 1 >$$

$$= \frac{1}{\sqrt{2}}(|r\bar{g} > - |\bar{g}r >) \otimes \frac{1}{\sqrt{2}}(\uparrow \downarrow - \downarrow \uparrow) \otimes \frac{1}{\sqrt{2}}(q\bar{s} - \bar{q}s) , \quad (36)$$

$$K^c_0 = |K^c_0(0^-)|1, \frac{1}{2}, -\frac{1}{2}, 1 >$$

$$= \frac{1}{\sqrt{2}}(|b\bar{g} > - |\bar{g}b >) \otimes \frac{1}{\sqrt{2}}(\uparrow \downarrow - \downarrow \uparrow) \otimes \frac{1}{\sqrt{2}}(q\bar{s} - \bar{q}s) , \quad (37)$$

$$\pi^c_+ = |\pi^c_+(0^-)|1, 1, 0 >$$

$$= \frac{1}{\sqrt{2}}(|r\bar{b} > - |\bar{b}r >) \otimes \frac{1}{\sqrt{2}}(\uparrow \downarrow - \downarrow \uparrow) \otimes \frac{1}{\sqrt{2}}(q\bar{s} - \bar{q}s) , \quad (38)$$

$$\pi^c_0 = |\pi^c_0(0^-)|1, 0, 0 >$$

$$= \frac{1}{\sqrt{2}}\left( \frac{1}{\sqrt{2}}(|r\bar{r} > - |r\bar{r} >) - \frac{1}{\sqrt{2}}(|b\bar{b} > - |b\bar{b} >) \right) \otimes \frac{1}{\sqrt{2}}(\uparrow \downarrow - \downarrow \uparrow) \otimes \frac{1}{\sqrt{2}}(q\bar{s} - \bar{q}s) , \quad (39)$$

$$\pi^c_- = |\pi^c_-(0^-)|1, -1, 0 >$$

$$= \frac{1}{\sqrt{2}}(|r\bar{b} > - |\bar{b}r >) \otimes \frac{1}{\sqrt{2}}(\uparrow \downarrow - \downarrow \uparrow) \otimes \frac{1}{\sqrt{2}}(q\bar{s} - \bar{q}s) , \quad (40)$$

$$\bar{K}^c_+ = |\bar{K}^c_+(0^-)|\frac{1}{2}, \frac{1}{2}, 1 >$$

$$= \frac{1}{\sqrt{2}}(|g\bar{b} > - |\bar{b}g >) \otimes \frac{1}{\sqrt{2}}(\uparrow \downarrow - \downarrow \uparrow) \otimes \frac{1}{\sqrt{2}}(q\bar{s} - \bar{q}s) , \quad (41)$$

$$\bar{K}^c_0 = |\bar{K}^c_0(0^-)|\frac{1}{2}, -\frac{1}{2}, -1 >$$

$$= \frac{1}{\sqrt{2}}(|g\bar{r} > - |\bar{r}g >) \otimes \frac{1}{\sqrt{2}}(\uparrow \downarrow - \downarrow \uparrow) \otimes \frac{1}{\sqrt{2}}(q\bar{s} - \bar{q}s) , \quad (42)$$

$$\bar{K}^c_- = |\bar{K}^c_-(0^-)|\frac{1}{2}, -\frac{1}{2}, -1 >$$

$$= \frac{1}{\sqrt{2}}(|g\bar{r} > - |\bar{r}g >) \otimes \frac{1}{\sqrt{2}}(\uparrow \downarrow - \downarrow \uparrow) \otimes \frac{1}{\sqrt{2}}(q\bar{s} - \bar{q}s) , \quad (43)$$

$$\eta^c_0 = |\eta^c_0(0^-)|0, 0, 0 >$$

$$= \frac{1}{2\sqrt{3}} \left( (|r\bar{r} > - |r\bar{r} >) + (|b\bar{b} > - |b\bar{b} >) - 2(|g\bar{g} > - |g\bar{g} >) \right) \otimes \frac{1}{\sqrt{2}}(\uparrow \downarrow - \downarrow \uparrow) \otimes \frac{1}{\sqrt{2}}(q\bar{s} - \bar{q}s) , \quad (44)$$

$$\eta^c_0 = |\eta^c_0(0^-)|0, 0, 0 >$$

$$= \frac{1}{2\sqrt{3}} \left( (|r\bar{r} > - |r\bar{r} >) + (|b\bar{b} > - |b\bar{b} >) - 2(|g\bar{g} > - |g\bar{g} >) \right) \otimes \frac{1}{\sqrt{2}}(\uparrow \downarrow - \downarrow \uparrow) \otimes \frac{1}{\sqrt{2}}(q\bar{s} - \bar{q}s) , \quad (45)$$

$$\eta^c_0 = |\eta^c_0(0^-)|0, 0, 0 >$$

$$= \frac{1}{2\sqrt{3}} \left( (|r\bar{r} > - |r\bar{r} >) + (|b\bar{b} > - |b\bar{b} >) - 2(|g\bar{g} > - |g\bar{g} >) \right) \otimes \frac{1}{\sqrt{2}}(\uparrow \downarrow - \downarrow \uparrow) \otimes \frac{1}{\sqrt{2}}(q\bar{s} - \bar{q}s) , \quad (46)$$

$$\eta^c_0 = |\eta^c_0(0^-)|0, 0, 0 >$$

etc. for the other octet members.
3. Color Octet Pentaquark States

With these \(\alpha(q\bar{q})\) and \(\beta(uds)\) basis states, we now form the totally antisymmetric \((\alpha - \beta)\) molecules. We proceed by coupling color, flavor, spin, and space wavefunctions to overall quantum numbers of the physical \((\alpha - \beta)\) states. As previously discussed, overall fermion antisymmetry in the \(\beta\)-cluster follows directly from the antisymmetry of the flavor singlet \(F^{3}_{A}(\beta)\). Construction of the symmetrized \(F^{3}_{A}(\alpha\beta|X)\) product function is straightforward yet tedious, so we omit the details. However, since antisymmetry of the pentaquark color function \(C^{A}_{A}(\alpha\beta|X)\) is crucial for color confinement, we show explicit details of the construction. To proceed, we note that each member of the anti-color triplet \((\bar{C}^{\alpha}, \bar{C}^{\beta}, \bar{C}^{\gamma})\) since antisymmetry of the pentaquark color function \(C^{A}_{A}(\alpha\beta|X)\) is crucial for color confinement, we show explicit details of the construction. To proceed, we note that each member of the anti-color triplet \((\bar{C}^{\alpha}, \bar{C}^{\beta}, \bar{C}^{\gamma})\) is color-equivalent to the anti-symmetrized product of two color states:

\[
|\bar{r}\rangle = \frac{1}{\sqrt{2}} (|b\rangle \langle g| - |g\rangle \langle b|) \quad (56)
\]

\[
|\bar{b}\rangle = \frac{1}{\sqrt{2}} (|r\rangle \langle g| - |g\rangle \langle r|) \quad (57)
\]

\[
|\bar{g}\rangle = \frac{1}{\sqrt{2}} (|b\rangle \langle r| - |r\rangle \langle b|) \quad (58)
\]

When the mixed symmetry color function from the \(\beta\)-cluster \(C^{8}_{MS}(\beta)\) is coupled to the mixed symmetry color function from the \(\alpha\)-cluster \(C^{8}_{MS}(\alpha)\), the \(SU(3)\) Clebsch-Gordon coefficients for many pairings will be zero because those products can not contribute to a zero color charge state. For example, consider the coupling of a \((rrb)\) color configuration in \(\beta\) with a \((r\bar{g})\) state in \(\alpha\). If we symbolically express the color Clebsch-Gordon coefficient as \(C_{G}(8_{\alpha} \otimes 8_{\beta} \rightarrow 1_{X}) = (\alpha||C^{1}_{1}, C^{2}_{2}||X^{1}||C^{3}_{1}, C^{4}_{2}, C^{5}_{3}||\beta)\) then

\[
C_{G}(\alpha^{8}(r\bar{g}) \otimes \beta^{8}(rrb)|X^{1}) = (\alpha||r, \bar{g}||X^{1}||r, r, b||\beta) = 0. \quad (59)
\]

This coupling must be zero (and thus \(|rrb \otimes \alpha|r\bar{g}\rangle\) is prohibited) since the net color electric charge of such a pentaquark state is not zero:

\[
\hat{Q}^{c} |\beta(rrb) \otimes \alpha(r\bar{g})\rangle = (Q_{r} + Q_{b} + Q_{g} + Q_{\bar{q}}) |\beta(rrb) \otimes \alpha(r\bar{g})\rangle = (2 + \frac{2}{3} - \frac{1}{3} + \frac{2}{3} + \frac{1}{3}) |\beta(rrb) \otimes \alpha(r\bar{g})\rangle = (2) |\beta(rrb) \otimes \alpha(r\bar{g})\rangle \neq 0. \quad (60)
\]

Next consider a typical coupling between \(\alpha - \beta\) color states that will contribute to a color neutral \(X\)-state:

\[
C_{G}(\alpha^{8}(r\bar{g}) \otimes \beta^{8}(ggb)|X^{1}) = (\alpha||r, \bar{g}||X^{1}||g, g, b||\beta) = \pm \mathcal{N}_{X}. \quad (63)
\]

where \(\mathcal{N}_{X}\) is the \(X\)-state pentaquark color normalization, and the sign will depend on the adopted phase convention. Because the pentaquark color singlet wavefunction must have overall \(A\)-symmetry, the normalization factor must have the same magnitude for all terms in the antisymmetric superposition. There are 9-primary (non-symmetrized) zero color combinations (only 8 independent) with non-zero \(8 \otimes 8 \rightarrow 1\) Clebsch-Gordon coefficients: \((g\bar{r}|rrb), (r\bar{g}|ggb), (b\bar{g}|ggr), (b\bar{r}|rgg), (g\bar{b}|bbr), (r\bar{b}|bgb), (r\bar{r}|rbb), (b\bar{b}|rbg), (g\bar{g}|rbg)\). Explicit color antisymmetry can be constructed in a sequence of color transformations starting with an arbitrary primary pentaquark color combination. For example, starting with the \((\alpha_{i-1}|\beta_{i-1} = (g\bar{r}|rrb)\) term, we first write the product of (non-normalized) MS-\(|\alpha\rangle\) and MS-\(|\beta\rangle\) color octet functions:

\[
C^{8}_{MS}(\alpha_{1}|g\bar{r}|X^{1}) C^{8}_{MS}(\beta_{1}|rrb|X^{1}) = (g\bar{r} - rg) (rrb - rbr + brr) \quad (64)
\]

and likewise for the 8-independent products. Next transform this first term by applying the color transformation \(r \leftrightarrow b\) (and subtract to anti-symmetrize):

\[
C^{8}_{MS}(g\bar{b}|X) C^{8}_{MS}(bbr|X) = (g\bar{r} - rg) (bbr - brb + rbb) \quad (65)
\]

Repeated application of the various color transformations: \(r \leftrightarrow b, r \leftrightarrow g, b \leftrightarrow g\) generates the following (normalized) antisymmetric color cluster product expansion:

\[
C^{8}_{A}(\alpha|\beta|X) = \mathcal{N}_{X} \sum_{i=1}^{8} (-1)^{i+1} C^{8}_{MS}(\alpha_{i}|X) C^{8}_{MS}(\beta_{i}|X), \quad (66)
\]

where the \(|\alpha\rangle\)-color functions are:
\[ C_{MS}^{8}(\alpha_1|g\bar{r}|X) = (g\bar{r} - \bar{r}g) \]  
\[ C_{MS}^{8}(\alpha_2|g\bar{b}|X) = (g\bar{b} - \bar{b}g) \]  
\[ C_{MS}^{8}(\alpha_3|r\bar{b}|X) = (r\bar{b} - \bar{b}r) \]  
\[ C_{MS}^{8}(\alpha_4|b\bar{r}|X) = (b\bar{r} - \bar{r}b) \]  
\[ C_{MS}^{8}(\alpha_5|\bar{r}g|X) = (\bar{r}g - g\bar{r}) \]  
\[ C_{MS}^{8}(\alpha_6|\bar{b}g|X) = (\bar{b}g - g\bar{b}) \]  
\[ C_{A}^{8}(\alpha_7|r\bar{g}|X) = (r\bar{g} - \bar{g}r + b\bar{b} - \bar{b}b + g\bar{g} - g\bar{g}) \]  
\[ C_{A}^{8}(\alpha_8|r\bar{b}|X) = (r\bar{b} + \bar{b}r + b\bar{b} + g\bar{g} + g\bar{g}) \]  

and the \(|\beta>\) color functions are:

\[ C_{MS}^{8}(\beta_1|rrb|X) = (rrb - rbr + brr) \]  
\[ C_{MS}^{8}(\beta_2|bbr|X) = (bbr - bb\bar{r} + \bar{r}bb) \]  
\[ C_{MS}^{8}(\beta_3|bbg|X) = (bbg - bbg + \bar{b}gb) \]  
\[ C_{MS}^{8}(\beta_4|rrg|X) = (rrg - rgr + ggr) \]  
\[ C_{MS}^{8}(\beta_5|ggb|X) = (ggb - ggb + \bar{g}gb) \]  
\[ C_{MS}^{8}(\beta_6|grr|X) = (grr - grg + rgg) \]  
\[ C_{MS}^{8}(\beta_7|rbg|X) = (rbg + grb + grb + bgr + bgr) \]  
\[ C_{MS}^{8}(\beta_8|rbg|X) = (rbg - rbg + grb - bgr - bgr) \]  

Note within both the \(\alpha\) and \(\beta\) octets there are two terms that have zero net color charge. These color neutral octet member could be written with MS-symmetry, but to make the Clebsch-Gordon factors simple (and to demonstrate explicit 4\(\text{-}\)color symmetry) we show the two independent \(S\) and \(A\)-symmetry combinations. After the sum in Eq.(66) over the 8-octet states is performed, the color function will be totally antisymmetric in color interchange. This means that the interchange of any two colors will generate the opposite overall sign in the color cluster product expansion. For example, under the color exchange \(r \leftrightarrow b\):

\[ C_{MS}^{8}(\alpha_1|g\bar{r}|X) = (g\bar{r} - \bar{r}g) \]
\[ C_{MS}^{8}(\alpha_2|g\bar{b}|X) = (g\bar{b} - \bar{b}g) \]
\[ C_{MS}^{8}(\alpha_3|r\bar{b}|X) = (r\bar{b} - \bar{b}r) \]
\[ C_{MS}^{8}(\alpha_4|b\bar{r}|X) = (b\bar{r} - \bar{r}b) \]
\[ C_{MS}^{8}(\alpha_5|\bar{r}g|X) = (\bar{r}g - g\bar{r}) \]
\[ C_{MS}^{8}(\alpha_6|\bar{b}g|X) = (\bar{b}g - g\bar{b}) \]
\[ C_{A}^{8}(\alpha_7|r\bar{g}|X) = (r\bar{g} - \bar{g}r + b\bar{b} - \bar{b}b + g\bar{g} - g\bar{g}) \]
\[ C_{A}^{8}(\alpha_8|r\bar{b}|X) = (r\bar{b} + \bar{b}r + b\bar{b} + g\bar{g} + g\bar{g}) \]

The QMM pentaquark color antisymmetry is now explicit. There are a total of \((6 \times 2 \times 3) + (2 \times 6 \times 6) = 108\) terms in the antisymmetric sum, hence the color function normalization is: \(N_X = \frac{1}{\sqrt{108}} = \frac{1}{6\sqrt{3}}\).

**C. Color Magnetic Moments and Confinement**

We now turn our attention to a dynamical constraint that has two novel consequences: (1) to provide a selection rule that limits the QMM excitation spectrum (allowing only 4-physical states); (2) to provide dynamical relations between constants of the QMM Hamiltonian such that our calculated pentaquark barion mass spectrum has only one free parameter (the \(\xi\) mass). Using the color electric charge (CEC) operator described in the previous subsection, we now define the color magnetic moment (CMM) operator:

\[ \hat{\mu}^c = \sum_i \hat{\mu}_i^c \hat{\sigma}_{3i} = \mu_0^c \sum_i \hat{Q}_i^c \hat{\sigma}_{3i} \]

which is the color generalization of the \(U(1)\) magnetic moment operator of the conventional (non-exotic) quark model. The sum extends over all quarks and the color magnetic moment operator \(\hat{\mu}_i^c\) satisfies the \(SU(3)\) algebra:

\[ \hat{\mu}^c|r> = \mu_0^c \hat{Q}^c|r> = \frac{2}{3} \mu_0^c |r> = \mu_c^c |r> \]  
\[ \hat{\mu}^c|b> = \mu_0^c \hat{Q}^c|b> = \frac{1}{3} \mu_0^c |b> = \mu_b^c |b> \]  
\[ \hat{\mu}^c|g> = \mu_0^c \hat{Q}^c|g> = -\frac{1}{3} \mu_0^c |g> = \mu_g^c |g> \]
The constant $\mu_0$ is the fundamental color magnetic moment, which may be regarded as a free parameter of the QMM, however we take a different approach since its numerical value is not used in our spectrum calculations. The important point about the constant $\mu_0$ is that its value (proportional to the strong coupling constant $g_s = \sqrt{4\pi\alpha_s}$) must be the same for all quark colors to satisfy exact SU(3) color symmetry.

We can now state our hypothesis of color magnetic confinement (CMC):

**The net color magnetic moment is zero for all physical hadrons.**

If the CMC hypothesis is true, then the converse must also be true:

**Any hadron with non-zero color magnetic moment is unphysical.**

To see why the CMC conjecture is reasonable, we apply our hypothesis to a conventional quark model flavor octet state (the proton). First we take the direct product of standard $A$-color and $MS$-spin functions for the proton:

$$|p;\uparrow>_{2-3} = \frac{1}{\sqrt{6}} (rbg - rgb + bgr - brg + grb - gbr)$$

$$\times \frac{1}{\sqrt{2}} \uparrow (\uparrow\downarrow - \downarrow\uparrow),$$

where the $(2-3)$ subscript denotes $A$-symmetry for interchange between the second and third quarks in the spin function (and no other pairwise interchange symmetry). Obviously, the proton has CEC equal to zero. After applying the CMM operator, there are a total of 36 terms (12 terms with 3 quarks each) with exact cancellation between terms:

$$<\uparrow; p|\hat{\mu}^c|p;\uparrow> = <\uparrow; p|\mu_0 \sum_i \hat{Q}_i \hat{\sigma}_{3i}|p;\uparrow> = 0.$$ 

Note the cancellation is only exact if $\mu_0$ is the same for each color. The physical proton is built from a superposition of $MS$-symmetry flavor functions, but the exact cancellation in the CMM holds separately for each one, so the proton satisfies the CMC hypothesis. We can show that every member of the (non-exotic) baryon $(\frac{3}{2}^+)$ octet, $(\frac{1}{2}^-)$ decuplet and meson (0$^-$, 1$^-$) nonets also satisfy the CMC hypothesis (but omit the details here). It is possible that some (or many) of the excited states of the conventional (non-exotic) quark model may not satisfy the generalized CMC sum rule (including orbital CMM contributions) and therefore drop out of the excitation spectrum. The CMC principle may help to resolve the longstanding missing nucleon ($N^*$) resonance problem of baryon spectroscopy, or CMC may simply not be generally true, however we leave this question open for future consideration. We now proceed to apply the CMC conjecture to constrain the Hamiltonian of our pentaquark baryon molecule states.

### D. Color Magnetic Confinement (CMC) Constraint

The complication of applying CMC in the QMM pentaquark basis is that there must be at least one unit of relative orbital angular momentum between color octet $|\alpha(q\bar{s})>$ and $|\beta(uds)>$ clusters. Hence, there should be a CMM contribution from the colorful orbital motion of the molecular state. If CMC is generally true, then we must calculate the orbital contribution to the total molecular color magnetic moment. When this is accomplished, there will be consistency relations between the color spin-spin and spin-orbit coupling strengths in our QMM Hamiltonian so that overall CMC is satisfied.

Our molecular pentaquark spectrum is generated by the following Hamiltonian:

$$H = H_0 + C_F^\beta \left( \frac{\alpha_\beta}{r} \right) + \frac{A_{\alpha\beta}}{M_\alpha M_\beta} \vec{S}_\alpha \cdot \vec{S}_\beta \left( \frac{1}{r} \frac{dV}{dr} \right) + \frac{B_{\alpha\beta}}{M_\alpha M_\beta} \vec{L} \cdot \vec{S} \left( \frac{1}{r} \frac{dV}{dr} \right)$$

$$H_0 = -\frac{\nabla^2}{2M_{\alpha\beta}} + \frac{1}{2} \alpha^2 r^2$$

$$V(r) = C_F^\beta \left( \frac{\alpha_\beta}{r} \right) + \frac{1}{2} \alpha^2 r^2$$

$$\left( \frac{1}{r} \frac{dV}{dr} \right) = -C_F^\beta \left( \frac{\alpha_\beta}{r^2} \right) + k$$

where $\vec{S} = \vec{S}_\alpha + \vec{S}_\beta$ is the total spin, and $\vec{L}$ is the orbital angular momentum operator. This Hamiltonian has the same form as a non-relativistic reduction of the Dirac equation with a minimally coupled gauge field 4-vector potential (generating electric and magnetic interactions). Cluster confinement is enforced by the harmonic oscillator potential, which generates a convenient S.H.O. basis of radial wavefunctions for our spectrum calculations. As usual, $\alpha_s$ is the strong coupling constant of QCD [14] (fixed by conventional quark model spectroscopy $\alpha_s = 0.3$). We checked that using the running coupling $\alpha_s(r)$ gives an effect less than 1% in the radial expectation values calculated in this work. The factor $C_F^\beta$ multiplying the Coulomb term is the
The orbital color magnetic moment \( \vec{\mu}_L \) is expressed in terms of the total orbital angular momentum \( \vec{L} \) and effective orbital gyromagnetic ratio \( \gamma_L^{\alpha\beta} \):

\[
\vec{\mu}_L = \gamma_L^{\alpha\beta} \left( \frac{g_s}{2M_{\alpha\beta}} \right) \vec{L}.
\]

The origin of the orbital gyromagnetic ratio factor can be traced to the fact that the total orbital color magnetic moment is generated asymmetrically between the colorful clusters of different mass. Each cluster carries a different amount of orbital angular momentum relative to the center of mass of the molecular system (Fig. 1).

For orbiting \((\alpha\beta)\) clusters in a particular spin configuration, each carries individual angular momentum:

\[
\vec{L}_{\alpha} = \vec{r}_{\alpha} \times \vec{p}_{\alpha} \quad \text{classical} \rightarrow \left( M_{\alpha} r_{\alpha}^2 \omega_{\alpha} \right) \vec{z}
\]

\[
\vec{L}_{\beta} = \vec{r}_{\beta} \times \vec{p}_{\beta} \quad \text{classical} \rightarrow \left( M_{\beta} r_{\beta}^2 \omega_{\beta} \right) \vec{z}
\]

where \( \vec{z} \) is the quantization axis of spin and orbital angular momentum, and the angular velocity (expectation value) \( \omega_{\alpha} \) must be the same for both clusters so the center of mass position is fixed in space (i.e., to satisfy conservation of linear momentum with no external forces). Also, the quantum two-body system satisfies Newton’s 3rd Law (with expectation values used for radial factors):

\[
M_{\alpha} r_{\alpha}^2 \omega_{\alpha}^2 = M_{\beta} r_{\beta}^2 \omega_{\beta}^2.
\]

Total orbital angular momentum is the vector sum of both cluster contributions, and naturally generates the standard reduced mass \( M_{\alpha\beta} \) factor of a CM separated two body system:

\[
\vec{L} = \vec{L}_{\alpha} + \vec{L}_{\beta}
\]

\[
= \left( M_{\alpha} r_{\alpha}^2 \omega_{\alpha} + M_{\beta} r_{\beta}^2 \omega_{\beta} \right) \vec{z}
\]

\[
= M_{\alpha\beta} \left( r_{\alpha} + r_{\beta} \right)^2 \omega_{\alpha} \vec{z}.
\]
Because of Newton’s 3rd Law, Eq. (106), each cluster angular momentum is proportional to the total angular momentum, depending on mass ratio factors:

\[ \vec{L}_\alpha = \left( \frac{M_\beta}{M_\alpha} \right) \vec{L}_\beta \]  
(110)

\[ \vec{L}_\alpha = \left( \frac{1}{1 + (M_\alpha/M_\beta)} \right) \vec{L} \]  
(111)

\[ \vec{L}_\beta = \left( \frac{M_\alpha}{M_\beta} \right) \left[ \frac{1}{1 + (M_\alpha/M_\beta)} \right] \vec{L} . \]  
(112)

Returning to the orbital color gyromagnetic ratio, we couple the separate \( \alpha \) and \( \beta \) orbital CMM contributions to the total orbital angular momentum:

\[ \gamma_{\alpha\beta}^{(z)} \left( \frac{g_s}{2M_{\alpha\beta}} \right) \vec{L} = \left( \frac{g_s}{2M_\alpha} \right) \vec{L}_\alpha + \left( \frac{g_s}{2M_\beta} \right) \vec{L}_\beta . \]  
(133)

Using Eqs. (111,112) we obtain:

\[ \gamma_{\alpha\beta}^{(z)} = \frac{M_\alpha M_\beta}{(M_\alpha + M_\beta)^2} \left[ \frac{M_\alpha}{M_\beta} + \frac{M_\beta}{M_\alpha} \right] . \]  
(114)

It is interesting to note the equal mass and infinite asymmetric mass limits:

\[ \lim_{M_\alpha=M_\beta} \gamma_{\alpha\beta}^{(z)} \rightarrow \frac{1}{2} \]  
(115)

\[ \lim_{M_\alpha>M_\beta} \gamma_{\alpha\beta}^{(z)} \rightarrow 1 \]  
(116)

\[ \lim_{M_\beta>M_\alpha} \gamma_{\alpha\beta}^{(z)} \rightarrow 1 . \]  
(117)

We now explore the effective color gyromagnetic ratio of the coupled molecular spin CMM, \( \gamma_{S\bar{S}}^{(z)} \). Using Eqs. (100,101,102), we have:

\[ (-)^{J-S} \gamma_{S\bar{S}}^{(z)} \left( \frac{g_s}{M_{\alpha\beta}} \right) \vec{S} = \gamma_\alpha \left( \frac{g_s}{2M_\alpha} \right) \vec{S}_\alpha + \gamma_\beta \left( \frac{g_s}{2M_\beta} \right) \vec{S}_\beta . \]  
(118)

The \( J \)-dependent phase factor is required for consistency because in the coupled quantum mechanical molecular system, \( (S_\alpha, S_\beta, S, S_c, L, J, \bar{J}) \) are “good” quantum numbers (operators commute with Hamiltonian), but \( (S_{\alpha z}, S_{\beta z}) \) are not. To obtain the required relations, first examine the classical \( +\bar{z} \) projection of Eq. (118) with \( (S = \frac{3}{2}|S_\alpha = 0, S_\beta = \frac{1}{2}) \):

\[ (-)^{J-\frac{3}{2}} \gamma_{S\bar{S}}^{(z)} \left( \frac{1}{2}|0, \frac{1}{2} \right) \left( \frac{g_s}{M_{\alpha\beta}} \right) \left( \frac{1}{2} \right) = \gamma_\alpha \left( \frac{g_s}{2M_\alpha} \right) \left( 0 \right) + \gamma_\beta \left( \frac{g_s}{2M_\beta} \right) \left( \frac{1}{2} \right) . \]  
(119)

Our first consistency relation for \( \gamma_{S\bar{S}}^{(z)}(S|S_\alpha, S_\beta) \) follows directly:

\[ (-)^{J-\frac{3}{2}} \gamma_{S\bar{S}}^{(z)} \left( \frac{1}{2}|0, \frac{1}{2} \right) = \left( \frac{M_{K^*_c\xi_c}}{M_{K^*_c}} \right) \gamma_{\xi_c} , \]  
(120)

where \( \alpha = K_c(q\bar{s}|0^-) \) (a spin 0\(^-\) color octet cluster with kaon spin-flavor composition), and \( \beta = \xi_c(u\bar{d}s|\frac{1}{2}^+) \), the previously discussed color octet, flavor singlet baryon cluster state. Because the \( K_c(0^-) \) cluster has spin zero, its color gyromagnetic ratio must equal zero: \( \gamma_{\xi_c} = 0 \).

Next consider a spin one (color octet) meson cluster \( (S_\alpha = 1) \), the \( \alpha = K^*_c(q\bar{s}|1^-) \) with vector kaon spin-flavor composition coupled with the \( \beta = \xi_c(u\bar{d}s|\frac{1}{2}^+) \):

\[ (-)^{J-\frac{3}{2}} \gamma_{S\bar{S}}^{(z)} \left( \frac{1}{2}|1, \frac{1}{2} \right) = M_{K^*_c\xi_c} \left( \frac{\gamma_{K^*_c\xi_c}}{M_{K^*_c}} - \frac{\gamma_{\xi_c}}{M_{\xi_c}} \right) . \]  
(121)

The last possible molecular spin combination is the vector \( \alpha = K^*_c(q\bar{s}|1^-) \) coupled to the \( \beta = \xi_c(u\bar{d}s|\frac{1}{2}^+) \) in a \( S = \frac{3}{2} \) configuration:

\[ (-)^{J-\frac{3}{2}} \gamma_{S\bar{S}}^{(z)} \left( \frac{3}{2}|1, \frac{1}{2} \right) = \left( \frac{1}{3} \right) M_{K^*_c\xi_c} \left( \frac{\gamma_{K^*_c\xi_c}}{M_{K^*_c}} + \frac{\gamma_{\xi_c}}{M_{\xi_c}} \right) . \]  
(122)
Finally, we apply the CMC constraint to develop consistency relations between the effective gyromagnetic ratios of the cluster states ($\gamma_{K_L},\gamma_{K_{z}},\gamma_{\xi}$) and the orbital factors. Eq. (99) can be written as:

$$\vec{\mu}_{\text{tot}} = \vec{\mu}_{\alpha\beta} + \vec{\mu}_{L} = (-)^{J-S} \gamma_{S}^\alpha \beta \left( g_{S} \over M_{\alpha\beta} \right) \vec{S} + \gamma_{L}^\alpha \beta \left( g_{L} \over 2M_{\alpha\beta} \right) \vec{L}$$  \hspace{1cm} (123)

$$= \gamma_{(J,L)}^\alpha \beta \left( g_{J,L} \over M_{\alpha\beta} \right) \vec{J} \underset{\text{CMC}}{\rightarrow} 0.$$ \hspace{1cm} (124)

So the CMC condition requires the color gyromagnetic ratio of any physical molecular state with quantum numbers $(J,J_L,L_s,S_z)$ to be identically zero: $\gamma_{(J,L)}^\alpha \beta = 0$. Examining the $\hat{z}$ projections for the different $(J|L,S)$ quantum couplings, we find the following consistency relations for $\alpha \beta$:

$$K_{c}\xi\left(\frac{1}{2},\frac{1}{2}\right): \quad \gamma_{S}^\alpha \beta \left(0,1\right) = \gamma_{L}^K c\xi$$ \hspace{1cm} (125)

$$K_{c}\xi\left(\frac{1}{2},\frac{1}{2}\right): \quad -\gamma_{S}^\alpha \beta \left(1,0\right) = -\gamma_{L}^K c\xi$$ \hspace{1cm} (126)

$$K_{c}\xi\left(\frac{1}{2},\frac{1}{2}\right): \quad \gamma_{S}^\alpha \beta \left(2,1\right) = \gamma_{L}^K c\xi$$ \hspace{1cm} (127)

$$K_{c}\xi\left(\frac{3}{2},\frac{3}{2}\right): \quad -\gamma_{S}^\alpha \beta \left(1,2\right) = -\gamma_{L}^K c\xi$$ \hspace{1cm} (128)

$$K_{c}\xi\left(\frac{1}{2},\frac{3}{2}\right): \quad -\gamma_{S}^\alpha \beta \left(3,\frac{3}{2}\right) = \frac{1}{3} \gamma_{L}^K c\xi$$ \hspace{1cm} (129)

$$K_{c}\xi\left(\frac{3}{2},\frac{3}{2}\right): \quad \gamma_{S}^\alpha \beta \left(2,1\right) = 0$$ \hspace{1cm} (130)

$$K_{c}\xi\left(\frac{5}{2},\frac{3}{2}\right): \quad -\gamma_{S}^\alpha \beta \left(\frac{3}{2},1\right) = -\frac{1}{3} \gamma_{L}^K c\xi$$ \hspace{1cm} (131)

Note the only consistent solution for Eqs. (129,130,131) is: $\gamma_{(J,L)}^\alpha \beta \left(\frac{1}{2},\frac{1}{2}\right) = 0$. But that would also require $\gamma_{L}^K c\xi \rightarrow 0$. Since $\gamma_{L}^K c\xi$ is uniquely determined by Eq. (114) (non-zero), the $K_{c}\xi(J|L,S) = \left(\frac{1}{2}|\frac{1}{2}\right), \left(\frac{3}{2}|\frac{1}{2}\right)$ states cannot consistently satisfy the CMC condition and are therefore unbound (unphysical states) in the QMM. Careful inspection will show the sign difference between the Eqs. (125,127) (or Eqs. (126,128)) we find the corresponding set of relations between the QMM gyromagnetic ratio factors:

$$\gamma_{S}^\alpha \beta \left(0,1\right) = \gamma_{L}^K c\xi = \left( M_{K_{c}\xi} \right) / M_{\xi} \gamma_{\xi}$$ \hspace{1cm} (132)

$$\gamma_{S}^\alpha \beta \left(1,0\right) = -\gamma_{L}^K c\xi = M_{K_{z}\xi} \left( \frac{\gamma_{K_{z}\xi}}{M_{K_{z}}} - \frac{\gamma_{\xi}}{M_{\xi}} \right),$$ \hspace{1cm} (133)

which we invert to express the unknown (phenomenological) cluster color gyromagnetic ratio factors $\gamma_{K_{z}}\xi$ and $\gamma_{\xi}$ in terms of the derived orbital factors $\gamma_{L}^\alpha \beta$, and colorful cluster masses $M_{\alpha} = (M_{K_{c}}, M_{K_{z}})$, $M_{\beta} = M_{\xi}$:

$$\gamma_{\xi} = \left( M_{K_{c}} + M_{K_{z}} \right) \gamma_{K_{c}\xi},$$ \hspace{1cm} (134)

$$\gamma_{K_{z}}\xi = \left( M_{K_{c}} + M_{\xi} \right) \gamma_{K_{c}\xi} + \frac{M_{K_{z}}}{M_{\xi}} \gamma_{K_{z}\xi} + \frac{M_{K_{c}}}{M_{\xi}} \gamma_{K_{c}\xi}.$$ \hspace{1cm} (135)

It is important to stress here that these parameter relations were derived analyzing the classical-$\hat{z}$ projection of the coupled spin $S_{z} = S_{az} + S_{bz}$, but we have already noted that $S_{az}$ and $S_{bz}$ are not good quantum numbers of the molecular system. We will refer to this parameter set as the “classical approximation” for the QMM-CMC constrained gyromagnetic ratios.

However, we can derive a set of parameters consistent with the rules of quantum mechanics simply by taking expectation values of the spin operators associated with Eq. (118). Using the standard spin (and later orbital) expectation values:

$$< \vec{S} \cdot \vec{S} > = S(S + 1)$$ \hspace{1cm} (136)

$$< S_{z}^{2} > = S_{z}(S_{z} + 1)$$ \hspace{1cm} (137)

$$< S_{z}^{2} \cdot S_{\beta} > = S_{\beta}(S_{\beta} + 1)$$ \hspace{1cm} (138)

$$< S_{z}^{2} \cdot S_{z} > = \frac{1}{2}[S(S + 1) - S_{az}(S_{az} + 1) - S_{bz}(S_{bz} + 1)]$$ \hspace{1cm} (139)

$$< \vec{L} \cdot \vec{L} > = L(L + 1)$$ \hspace{1cm} (140)

$$< \vec{J} \cdot \vec{J} > = J(J + 1)$$ \hspace{1cm} (141)

$$< \vec{L} \cdot \vec{S} > = \frac{1}{2}[J(J + 1) - L(L + 1) - S(S + 1)],$$ \hspace{1cm} (142)
by taking the $\vec{S} \cdot \vec{S}$ projection of Eq. (118), we obtain the general quantum consistent parameter relations:

$$(-)^{J_{\alpha} - S} \gamma_{\vec{S}}(S|S_\alpha, S_\beta) =$$

$$\frac{M_{a\beta}}{S(S+1)} \left[ S_{\alpha}(S_{\alpha} + 1) \left( \frac{\gamma_{\alpha}}{2M_\alpha} \right) + S_{\beta}(S_{\beta} + 1) \left( \frac{\gamma_{\beta}}{2M_\beta} \right) + < \vec{S}_{\alpha} \cdot \vec{S}_{\beta} > \left( \frac{\gamma_{\alpha}}{2M_\alpha} + \frac{\gamma_{\beta}}{2M_\beta} \right) \right].$$

Applying Eq. (143) to the cluster $\alpha = K_\epsilon(0^-)$ ($S_\alpha = 0$, $S_\beta = \frac{1}{2}$, $S = \frac{1}{2}$) (and canceling the phase factor when coupling to different total angular momentum $J = \frac{1}{2}, \frac{3}{2}$ states), we obtain:

$$\gamma_{S}^{\alpha\beta} \left( \frac{1}{2} \right)^{1} = \gamma_{L}^{K_{\epsilon}} = \left( \frac{M_{K_{\epsilon}}}{M_{\epsilon}} \right)^{\gamma_{L}^{K_{\epsilon}}},$$

which is the same result as the classical approximation Eq. (132). A difference shows up when we apply Eq. (143) to the cluster $\alpha = K_\epsilon(1^-)$ ($S_\alpha = 1$, $S_\beta = \frac{1}{2}$, $S = \frac{1}{2}$):

$$\gamma_{S}^{\alpha\beta} \left( \frac{1}{2} \right)^{1} = \gamma_{L}^{K_{e}} = M_{K_{e}}^{\gamma_{L}^{K_{e}}} \left( \frac{2 \gamma_{K_{e}}}{3M_{K_{e}}} - \frac{1}{3M_{\epsilon}} \right),$$

Inverting as before, we obtain the quantum and CMC consistent gyromagnetic ratio parameter relations:

$$\gamma_{L}^{K_{e}} = \left( \frac{M_{L} + M_{K_{e}}}{M_{K_{e}}} \right) \gamma_{L}^{K_{e}}$$

$$\gamma_{K_{e}} \left( \frac{3}{2} \right) \left( \frac{M_{K_{e}} + M_{\epsilon}}{M_{\epsilon}} \right) \gamma_{L}^{K_{e}} + \frac{1}{2} \frac{M_{K_{e}}}{M_{\epsilon}} \left( \frac{M_{K_{e}} + M_{\epsilon}}{M_{\epsilon}} \right) \gamma_{L}^{K_{e}}$$

E. Color Electro-Dynamics (CED) QMM Hamiltonian

We now relate the color gyromagnetic ratios to the $A_{\alpha\beta}$ and $B_{\alpha\beta}$ coefficients in our QMM Hamiltonian Eq. (95). In the spirit of our CEC and CMC conditions, we treat the form of the color octet cluster spatial and spin dependent interactions assuming that the primary non-Abelian features of QCD are contained in the confinement term, leaving the spin-spin and spin-orbit terms in the same form as the usual electrodynamics electric and magnetic interactions. First consider the spin-orbit interaction of electrodynamics for a charged boson $\alpha$ and fermion $\beta$. Each cluster separately contributes to the total spin orbit interaction:

$$H_{so} = H_{so}^\alpha + H_{so}^\beta$$

$$H_{so}^\alpha = \frac{1}{2} \frac{1}{M_{a\beta}} \left( \frac{\gamma_{\alpha}}{2M_\alpha} \right) \left( \frac{1}{r} \frac{dV}{dr} \right) \vec{S}_{\alpha} \cdot \vec{L}$$

$$H_{so}^\beta = \frac{1}{2} \frac{1}{M_{a\beta}} \left( \frac{\gamma_{\beta}}{2M_\beta} \right) \left( \frac{1}{r} \frac{dV}{dr} \right) \vec{S}_{\beta} \cdot \vec{L}.$$

As already noted, the magnetic moment of a fermion is twice as large as a corresponding boson. The factor of $\frac{1}{2}$ in each term is due to the Thomas precession effect. The spin-orbit interaction is derived from the energy of a magnetic dipole $\vec{\mu}$ in a magnetic field $\vec{B}$:

$$H_{so} = -\vec{\mu} \cdot \vec{B}$$

$$\vec{\mu} = (\frac{g}{M_{a\beta}}) \vec{S}$$

$$\vec{B} = (\frac{g}{M_{a\beta}}) \left( \frac{1}{r} \frac{dV}{dr} \right) \vec{L}.$$

Here $g$ is the fundamental charge (electric or color), $M_{a\beta}$ is the reduced mass of the two body system, and for a Coulomb potential the derivative term generates a $r^{-3}$ radial dependence. We write the derivative term explicitly so that the QMM color generalization for the radial dependence can be modified by the confining term in the potential Eq. (97). Using S.H.O., confinement generates a constant term (the classical “spring constant”) in Eq. (98), but the extra term would involve a $r^{-1}$ term for exact linear confinement. We define the effective spin-orbit interaction in terms of a coefficient $B_{a\beta}$:

$$\left( \frac{B_{a\beta}}{M_{a}\bar{M}_{\beta}} \right) \vec{S} = \left( \frac{1}{2M_{a\beta}} \right) \left[ \left( \frac{\gamma_{\alpha}}{2M_\alpha} \right) \vec{S}_{\alpha} + \left( \frac{\gamma_{\beta}}{M_{\beta}} \right) \vec{S}_{\beta} \right].$$
then the total spin-orbit interaction takes the form expressed in the QMM Hamiltonian Eq. (95). Matching coefficients, we
define two constants \( (a, b) \) which depend on the gyromagnetic ratios \((\gamma_\alpha, \gamma_\beta)\) and mass ratios:

\[
B_{\alpha\beta} \vec{S} = a \vec{S}_\alpha + b \vec{S}_\beta
\]

\[
a = \left( \frac{M_\alpha + M_\beta}{2M_\alpha} \right) \left( \frac{\gamma_\alpha}{2} \right)
\]

\[
b = \left( \frac{M_\alpha + M_\beta}{2M_\beta} \right) \gamma_\beta .
\]

Projecting both sides of Eq. (156) with the total spin operator \( \vec{S} \), and evaluating the spin expectation values of Eqs. (136-139),
we find the spin-orbit Hamiltonian coefficient:

\[
B_{\alpha\beta} = \frac{1}{2} \left[ (a + b) + (a - b) \left( \frac{S_\alpha(S_\alpha + 1) - S_\beta(S_\beta + 1)}{S(S + 1)} \right) \right] .
\]

Note the \( B_{\alpha\beta} \) coefficient depends on the spin quantum numbers \((S, S_\alpha, S_\beta)\) of the molecular system.

Next consider the more complicated spin-spin (SS) interaction. The form of the SS interaction is different for the \( L = 0 \) and \( L \neq 0 \) cases. For completeness, we consider both terms. Our QMM is restricted to \( L = odd \), but we investigate the \( L = 0 \) Hamiltonian to see how it affects our dynamically restricted spectrum. The electrodynamics inspired form of the interaction is:

\[
H_{ss}^{L=0} = \alpha_s \left( \frac{8\pi}{3} \right) \left( \frac{\gamma_\alpha}{2M_\alpha} \right) \left( \frac{\gamma_\beta}{M_\beta} \right) \delta^3(\vec{r}) \vec{S}_\alpha \cdot \vec{S}_\beta \left( \frac{1}{r} \frac{dV}{dr} \right)
\]

\[
H_{ss}^{L\neq0} = \left( \frac{\gamma_\alpha}{2M_\alpha} \right) \left( \frac{\gamma_\beta}{M_\beta} \right) \left[ 3(\vec{S}_\alpha \cdot \hat{r})(\vec{S}_\beta \cdot \hat{r}) - \vec{S}_\alpha \cdot \vec{S}_\beta \right] \left( \frac{1}{r} \frac{dV}{dr} \right) .
\]

In the case of \( L = 0 \), the expectation value of \(< r^{-3} > \) in the derivative term diverges. However, the spin-spin term gets a \( \delta \)-function correction which regulates the divergence, hence the \(< r^{-3} >_{L=0} \) factor is replaced by \([14]:

\[
<r^{-3} >_{L=0} \rightarrow \frac{4\pi}{3} R_{00}(r = 0)^2 ,
\]

where \( R_{00}(r) \) is the normalized radial wavefunction (in the \( n = 0, L = 0 \) state) described in the next section. In the case of
\( L \neq 0 \), we must take the expectation value of the spin operator in Eq. (162). To simplify the evaluation (and recover the form
of the QMM Hamiltonian) we note that to preserve the CMC constraint with non-variable gyromagnetic ratio parameters, the spin
and orbital angular momentum must couple in maximally aligned or anti-aligned configurations (otherwise, the dipole
color magnetic fields of the quark cluster will not exactly cancel the orbital dipole field). Because the CMC condition locks the
physical color singlet molecule into maximally aligned (anti-aligned) configurations, all quantum substates with zero angular
momentum projections (i.e., \( M_L = 0, M_{S_\alpha} = 0, M_{S_\beta} = 0, M_S = 0 \)) are prohibited in our QMM. This physical restriction
(required for consistency with CMC) greatly simplifies the spin-spin operator, since the expectation value of any \( S_\alpha \) or \( S_\beta \) spin
projection will be identically zero:

\[
[3(\vec{S}_\alpha \cdot \hat{r})(\vec{S}_\beta \cdot \hat{r}) - \vec{S}_\alpha \cdot \vec{S}_\beta] \rightarrow 2 \vec{S}_\alpha \cdot \vec{S}_\beta .
\]

So we arrive at the QMM spin-spin Hamiltonian term expressed in Eq. (95). The \( A_{\alpha\beta} \) is therefore a simple product of the color
and \( \gamma_\alpha \gamma_\beta \) gyromagnetic ratios for the \( \alpha \) and \( \beta \) quark clusters:

\[
A_{\alpha\beta} = \gamma_\alpha \gamma_\beta .
\]

We now discuss the QMM spatial wavefunctions and resulting mass formula for the molecular pentaquark exotics.

F. QMM Spatial Wavefunctions and Expectation Values

To determine the molecular exotic baryon spectrum, we calculate the first order energy corrections to the exact energy
eigenvalues associated with the confining harmonic oscillator Hamiltonian \( \hat{H}_0 \):

\[
\hat{H}_0 = -\frac{\nabla^2}{2\mu} + \frac{1}{2}m\omega^2r^2 ,
\]

where in this subsection \( \mu \) is the reduced mass \( (\mu = M_{\alpha\beta} = M_\alpha M_\beta/(M_\alpha + M_\beta)) \), and \( \omega \) is the quantized oscillator frequency.
In this work, we use natural units where \( \hbar = c = 1 \). The angular eigenfunctions of \( \hat{H}_0 \) are the spherical harmonics, \( Y_{lm}(\theta, \phi) \).
The energy eigenvalues of the unperturbed oscillator are obtained from the radial Schrödinger equation which includes the
centrifugal barrier term:
\[ \chi'' + \left[ k^2 - \lambda^2 r^2 - \frac{l(l+1)}{r^2} \right] \chi = 0 , \] 
\hspace{1cm} (166) 

where 
\[ k = \sqrt{2\mu E} , \quad \text{and} \quad \lambda = \mu \omega , \] 
\hspace{1cm} (167) 

\( \chi(r) = r R(r) \) is the reduced radial wave function. Following the usual procedure, the small/large-\( r \) asymptotic behavior of Eq. (166) determines the form of the general solution:
\[ \chi(r) = r^{l+1} e^{-\frac{\lambda}{2} r^2} u(r) , \] 
\hspace{1cm} (168) 

and the function \( u(r) \) satisfies the differential equation:
\[ \xi u''(x) + \left( l + \frac{3}{2} - \xi \right) u'(x) + \frac{1}{2} \left( l + \frac{3}{2} - \frac{E}{\omega} \right) u(x) = 0 , \] 
\hspace{1cm} (169) 

expressed in terms of the dimensionless radial variable \( x = \lambda r^2 \). The regular solution of Eq. (169) is a confluent (or degenerate) hyper-geometric function \( \Phi[a, b; x] \) [15,16]:
\[ \Phi[a, b; x] = \sum_{j=0}^{\infty} \frac{\Gamma(a+j) \Gamma(b)}{\Gamma(a) \Gamma(b+j) \Gamma(1+j)} x^j \] 
\hspace{1cm} (170)
\[ = 1 + \frac{a}{b} x + \frac{a(a+1)}{b(b+1)} x^2 + \frac{a(a+1)(a+2)}{b(b+1)(b+2)} x^3 + ... \] 
\hspace{1cm} (171)
\[ u(x) = \Phi[\frac{1}{2}(l + \frac{3}{2} - \frac{E}{\omega}) , l + \frac{3}{2} , x] . \] 
\hspace{1cm} (172)

Since physical solutions must have normalizable wave functions, the infinite series in Eq. (171) must be truncated after a finite number of terms \( a = -n \) (integer \( n \)), and hence the oscillator energy levels are quantized:
\[ \frac{1}{2} \left( l + \frac{3}{2} - \frac{E}{\omega} \right) = -n \quad (n = 0, 1, 2, 3, ...) . \] 
\hspace{1cm} (173)

Solving Eq. (173) we find the harmonic oscillator energy eigenvalues:
\[ E_{nl} = (2n + l + \frac{3}{2}) \omega , \] 
\hspace{1cm} (174)

associated with the eigenfunctions:
\[ \Psi_{nlm} = N_{nl} r^l e^{-\frac{\lambda}{2} r^2} \Phi[-n , l + \frac{3}{2} , \lambda r^2] Y_{lm}(\theta, \phi) . \] 
\hspace{1cm} (175)

With these basis functions, we compute the expectation values of the full Hamiltonian to determine the molecular exotic mass spectrum. In this work, we consider only the states with \( n = 0 \). Orbital excitations are prohibited by the CMC constraint, however radial excitation may be permissible. We do not explore the radial excitations and their consistency with CMC in this work. Since \( \Phi[0, b; x] = 1 \), the normalized wave functions are:
\[ \Psi_{nlm} = R_{nl}(r) Y_{lm}(\theta, \phi) = \left( \frac{2^{l+2}}{(2l+1)!!} \right)^{\frac{1}{2}} \lambda^{l+1} \frac{\lambda}{\pi} r^l e^{-\frac{\lambda}{2} r^2} Y_{lm}(\theta, \phi) . \] 
\hspace{1cm} (176)

Using these normalized wave functions, a general expression for the expectation value of the radial coordinate raised to an arbitrary power (\( p \)) can be evaluated:
\[ < r^p > = \int d\Omega \int_0^{\infty} r^{2+p} dr |\Psi_{nlm}|^2 \] 
\hspace{1cm} (177)
\[ = \left( \frac{2^{l+2}}{(2l+1)!!} \right)^{\frac{1}{2}} \lambda^{l+1} \frac{\lambda}{\pi} \Gamma(l + \frac{(3+p)}{2}) \frac{1}{2} \left( \frac{1}{\lambda} \right)^{(3+p)/2} \] 
\hspace{1cm} (178)

Specific expectation values for \( l = 0, 1 \) and \( p = 2, 1, -1, -2, -3 \) are evaluated in Table I.

The mass formula for the exotic molecular systems associated with the Hamiltonian of Eq. (95) is therefore:
with $s_\alpha = 0, 1$ and $s_\beta = \frac{1}{2}, \frac{3}{2}$. The oscillator frequency $\omega$ in Eq. (179) is given by the mass splitting between the proton and its first radial excitation using Eq. (175):

$$\omega = \frac{1}{2} [M_{N^*(1440)} - M_p(938)] = 0.251.$$  

The usual color factor between color triplet quarks forming a color singlet $C_F(3 \otimes 3 \rightarrow 1) = -4/3$ is not appropriate for one-gluon exchange between color octet clusters. Therefore, we will derive the correct factor $C_F^8 = C_F(8 \times 8 \rightarrow 1)$ for octet bonded clusters. Evaluating the Feynman diagram for gluon exchange between color octet (point-like) particles (Fig. 2), we have:

$$C_F^8 = \frac{1}{\sqrt{8}} \left( \frac{i}{2} \right)^2 f^{\alpha\beta\gamma} f^{\beta'\gamma'\alpha'} \delta_{\alpha\beta'} \delta_{\beta\alpha'} \delta_{\gamma\gamma'}$$

which involves a trace over the product of $SU(3)$ structure constants $f^{\alpha\beta\gamma}$, and projecting octet color functions $(a_s^\nu)$ onto singlet initial $|1>$ and final $|1'>$ state color functions:

$$|1> = (a_s^\alpha a_{\bar{\alpha}}^\beta)_1 = \frac{1}{\sqrt{8}} \delta_{\alpha\beta}$$

$$|1'> = (a_{\bar{\alpha}}^\alpha a_{\bar{\bar{\alpha}}}^\beta)_1 = \frac{1}{\sqrt{8}} \delta_{\bar{\alpha}\beta'}.$$  

Note the color octet-octet interaction generates a strongly attractive one gluon exchange potential, $V_{1g}(r) = C_F^8 a_s/r$.

The last step in constructing the QMM spatial wavefunctions involves forming coupling cluster spins $(S_\alpha, S_\beta)$, total spin $S$, and orbital angular momentum $L$ to overall angular momentum $J$. The angular momentum $J$-states are calculated using the superposition of $|L>$ basis states with CMC restricted Clebsch-Gordon coupling factors (all magnetic substates equal to zero are prohibited in the CMC restricted sum):

$$|\Psi_{\alpha,\beta}(r, \theta, \phi)||S, L, M_L, J, M_J > = \sum_{S\alpha, S\beta, M_S} \text{CMC}$$

$$C(J, M_J|L, M_L, S, M_S) C(S, M_S|S\alpha, M_{S\alpha}, S\beta, M_{S\beta}) R_{NL}(r) Y_{M_L}^{ML}(\theta, \phi).$$

Since the CMC principle restricts the angular momentum substates, the standard Clebsch-Gordon factors are renormalized in the QMM sum over states (details omitted). We now discuss our QMM parameter assignments and the resulting physical spectrum of pentaquark molecules.

**G. QMM Parameters and Spectrum Results**

The QMM cluster masses are not a priori known (since the color octet clusters can not be measured in unbound isolation). However, their masses should be comparable to the physical $K, K^*$ and $\Lambda$ masses due to the dominating contributions from constituent quark masses. Since the quarks inside each cluster combine to form a color octet, we anticipate a mass shift (increase) originating from the repulsive color factor of one gluon exchange between quarks in the octet color configuration. To estimate this mass shift, we calculate the color factor of one gluon exchange between $q\bar{q}$ coupled to a color octet, $C_F(3 \otimes 3 \rightarrow 8) = +\frac{3}{4}$, whereas the color factor for the color singlet is $C_F(3 \otimes 3 \rightarrow 1) = -\frac{4}{3}$. So the estimated mass shift due to one gluon exchange between quarks inside a color octet cluster is: $\delta M = \frac{3}{2} a_s < \frac{1}{2} >$.

For a typical hadronic size ($< r > \approx 0.5$ fm), we deduce the mass shift $\delta M \approx 0.1$ GeV. Since this is a small number compared to the bare cluster masses (i.e., physical $K, K^*$ and $\Lambda$ masses), and because the $\xi$-mass is an unknown adjustable constant which is fixed by the lightest pentaquark candidate, we simply use the physical $K, K^*$ masses for the $\alpha(q\bar{q})$ clusters and absorb the mass shift ambiguity into the unknown phenomenological $\xi$-mass, which is the only unconstrained parameter of QMM (Table II). The calculated QMM masses of the exotic (pentaquark) states are summarized in Table III.
III. EXPERIMENTAL STATUS

In this section, we review the current status on experimental data focusing on associated electromagnetic production of strangeness systems of protons. In particular, (re-)analysis of experiments were done in the spirit of finding potential narrow structures to support our QMM calculation.

A. Photo-production channel

Guided by the recent claim of three narrow resonances from the SPHINX collaboration [18–20], we re-examined the published data on K-meson photo-production using the \( p(\gamma, K)_0 \) reaction from the SAPHIR detector at ELSA [21].

The invariant mass distributions for both \( \Lambda \) and \( \Sigma \) hyperons are compared to the background generated by a well constrained hadrodynamical Lagrangian model prediction of [2,3] in Fig. 3. The various curves account for when the narrow baryon X-resonances are included (solid line), or not included (dot-dashed line). We assume relativistic Breit-Wigner shapes for the resonances, which are superimposed on the smooth background. The masses, total widths and signal strengths are adjusted to optimize \( \chi^2 \) agreement with the analyzed data (Table IV).

Overall, the qualitative agreement between the data and the Lagrangian model prediction is significantly improved for both hyperon production channels when the X-baryon resonances are included. A detailed description of the hadronic Lagrangian model employed is under preparation for submission [2,3].

B. Electro-production channel

The kaon photo-production results described in the previous section provided motivation to investigate the equivalent (virtual photon mediated) kaon electro-production reactions. The elementary process for \( p(e, e'K)_0 \) is well described at low \( Q^2 \) [22]. Analysis of recent Jefferson Lab data has shown that isobar based models provide a good description of electro-production observables [23,24]. We employ the same isobar model [2,3] to check for consistency with the photo-production results. We analyze dedicated kaon electro-production data that were taken in December 1999 during the second stage of experiment E91016 [25,26] at Jefferson Lab in the experimental Hall C using the 100% duty cycle of the CEBAF machine at a fixed beam energy of 3.245 GeV.

The scattered electrons were detected in the High Momentum spectrometer (HMS, \( \Delta P/P = \pm 10\% \), \( \delta P/P = 10^{-3} \), \( \Delta \Omega = 6.7 \) msr), set at 19.64\(^\circ\) and with a central momentum of 1.432 GeV/c. The K-mesons were detected in the Short Orbit spectrometer (SOS, \( \Delta P/P = \pm 20\% \), \( \delta P/P = 10^{-3} \), \( \Delta \Omega = 7.7 \) msr) along the virtual photon direction at 14.23\(^\circ\) with a central momentum of 1.290 GeV/c. The corresponding invariant mass and \( Q^2 \) settings were 1.925 GeV and 0.55 (GeV/c)^2, respectively. The detector packages in both spectrometers are similar and include: two planes of drift chambers (used for tracking information), four planes of plastic scintillators (used for trigger formation, and time-of-flight (TOF) information), a gas Čerenkov (for \( \pi/\pi \) separation), and a lead glass shower counter (for electron identification). The SOS had two additional Čerenkov counters to improve identification of the kaons: an aerogel Čerenkov (for \( \pi^+/K^+ \) separation), and a lucite Čerenkov (for \( p/K^+ \) separation). The detector performances achieved were similar to the ones obtained in earlier kaon experiments performed in this experimental hall [23,24].

In the hadron arm, the \( K^+ \) were identified using the aerogel, the lucite and TOF. In the electron arm, we used the gas Čerenkov and Pb-glass. A sample of 8262 coincident kaons were then reconstructed which includes subtraction of the background events from the target cell walls (0.6\%) and random coincident events (real-to-random ratio about 10/1).

The two hyperons produced during the reaction can be seen at their expected location (Fig. 4): \( M_{\Lambda(1116)} = (1116.40 \pm 0.66 \) MeV [\( \Gamma_\Lambda = (4.82 \pm 0.65 \) MeV] and \( M_{\Sigma^0(1192)} = (1192.7 \pm 0.12 \) MeV [\( \Gamma_\Sigma = (4.86 \pm 1.60 \) MeV].

The invariant mass distributions are shown in Fig. 5 in addition to our model differential cross section of [2,3] for the \( \Lambda \) (left) and \( \Sigma^0 \) (right) hyperons. The \( \chi^2 \) fits of the three enhancements using relativistic Breit-Wigner shapes are listed in Table V.

As a result of the different bin sizes used in the analysis of the data in [21] (30 MeV) and [25,26] (~12.55 MeV), the new Jefferson Lab electro-production invariant mass distributions are able to resolve two nearly degenerate resonances, \( X_1 \) and \( X_2 \), in the vicinity of the SAPHIR \( X_2 \) [21]. We believe that due to the large binning of the SAPHIR data, the first peak in \( \Sigma^0 \) production at 1.974 GeV averages over the JLab \( X_1 \) and \( X_2 \), which contribute in the same SAPHIR energy bin. The resulting averaged value, \( \overline{M}_{\Sigma^0(1192)} = (1794.1 \pm 6.7 \) MeV [\( \overline{\Gamma}_{\Sigma^0(1192)} = (97.7 \pm 55.1 \) MeV], is less than 1-\( \sigma \) of the extracted SAPHIR \( X_2 \) mass value, and the decay width is approximately equal to the sum of the two JLab \( X_1 \) and \( X_2 \) resonance widths. As in photo-production, we find optimum agreement between our model prediction and the electro-production data when the X-resonance degrees of freedom are included [2,3].
C. Summary

Although the resonances extracted have fairly poor statistics, there is a clear definite pattern between all of the experimental data. This gives us a strong confidence for the existence of the narrow states extracted. We summarize the results of our analysis for all experimental data sets in comparison with the QMM spectrum predictions in Table VI.

Although a dedicated new high precision electroproduction experiment is planned to be performed for confirming and extracting the properties of these resonances [28], additional data in photo- and electro-production in that W range will be needed in the near future to shed light on the possible existence of the combined SPHINX, SAPHIR and Hall C data resonances discussed here. We would like to stress that the W distribution of the differential cross section would have to be kinematically constrained to center-of-mass angles between the virtual photon and outgoing kaon to be $\theta_{\gamma-K^+} \leq 20 - 30^\circ$ [2,3]) due to the (expected) strong forward peaking effect of these states on observable quantities.

IV. CONCLUSION

We have developed a quark molecular model (QMM) of $S=0$ pentaquark baryons assuming color-octet bond (qs)-(uds) molecular structure, and predict the low lying mass spectrum (4-states) using a color electrodynamics (CED) Hamiltonian. We have introduced the idea of color magnetic confinement (CMC) analogous to the well known color electric confinement (CEC) of traditional quark models. The CMC condition limits the allowed physical states in the QMM spectrum and generates consistency relations between the color gyromagnetic ratio factors of the QMM-CED Hamiltonian. This novel prediction stands in contrast to the usual outcome of quark model spectroscopy (which generate an overabundance of excited states) and provides a possible explanation of the dilemma of “missing resonances”.

To show the plausibility of our pentaquark spectrum predictions, we have reported experimental evidence of four narrow structures in the vicinity of $W = 2$ GeV in associated strangeness production reactions using the recent ELSA data in kaon photo-production [21] and Jefferson Lab data in kaon electro-production [25,26]. One resonance structure is not consistent with the QMM mass prediction and requires a typical (large $\Gamma \sim 200$ MeV) hadronic decay width, and hence is assumed to be a conventional (possibly missing) $N^*$ resonance, the $N^*(1898)$.

The suggested candidate pentaquark states of this analysis have average masses of 1.812, 1.962, 2.002 and 2.064 GeV, all of which have been identified with anomalously narrow decay widths (few tens of MeV). Two of those resonances (1.943 GeV and 2001.7 GeV) seen in the recent JLab data are believed to appear as an unresolved single resonance in the SAPHIR data at 1.974 GeV (due to relatively coarse energy binning). The fitted Breit-Wigner resonances align closely with the 4-predicted pentaquark states of the QMM, with masses: $X_0(1801)[1/2^+]$, $X_1(1967)[3/2^+]$, $X_2(1999)[1/2^+]$, and $X_3(2064)[3/2^+]$.

Additional high resolution experiments are required to establish the unambiguous existence and properties (i.e., masses, decay widths, branching fractions, hadronic coupling strengths, form factors, etc.) of the candidate $S=0$ pentaquark baryons suggested in this work. Our results provide a strong indication of possible crypto-exotic pentaquark baryon resonances in associated strangeness production channels, and support the need for investigating reaction channels other than non-strange ($\pi, \rho, \omega$, etc.) production to look for missing quark model resonances [29] and QCD allowed exotic states.

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\footnote{The SAPHIR [21] datum at 1974 GeV was not included in this average for consistency with the adopted analysis method of the data sets from the experiments discussed in this document.}
| $p$ | \( < r^p > \) | \( l = 0 \) | \( l = 1 \) |
|-----|----------------|---------------|---------------|
| 2   | \( \frac{\pi \sqrt{2 \lambda}}{2} \) | \( \frac{\pi \sqrt{2 \lambda}}{2} \) | \( \frac{\pi \sqrt{2 \lambda}}{2} \) |
| 1   | \( \frac{2 \sqrt{2 \lambda}}{2} \) | \( \frac{2 \sqrt{2 \lambda}}{2} \) | \( \frac{2 \sqrt{2 \lambda}}{2} \) |
| -1  | \( \frac{2 \sqrt{2 \lambda}}{2} \) | \( \frac{2 \sqrt{2 \lambda}}{2} \) | \( \frac{2 \sqrt{2 \lambda}}{2} \) |
| -2  | \( \frac{2 \sqrt{2 \lambda}}{2} \) | \( \frac{2 \sqrt{2 \lambda}}{2} \) | \( \frac{2 \sqrt{2 \lambda}}{2} \) |
| -3  | \( \frac{2 \sqrt{2 \lambda}}{2} \) | \( \frac{2 \sqrt{2 \lambda}}{2} \) | \( \frac{2 \sqrt{2 \lambda}}{2} \) |

TABLE I. Expectation values of the radial coordinate in the oscillator basis.

| Cluster \((\alpha, \beta)\) | Cluster Mass [GeV] | Gyromagnetic Ratio \(\gamma\) (Quantum Set) | Gyromagnetic Ratio \(\gamma\) (Classical Set) |
|---------------------------|--------------------|----------------------------------|----------------------------------|
| \(\alpha = K_\beta(0^-|q\bar{s})\) | \(M_{K_c} = M_K = 0.497\) | 0 | 0 |
| \(\alpha = K_\beta^*(1^-|q\bar{s})\) | \(M_{K_c^*} = M_{K^*} = 0.896\) | 2.189 | 2.462 |
| \(\beta = \xi_c(\frac{1}{2}^-|uds)\) | \(M_{\xi_c} = 0.982\) | 1.648 | 1.648 |

TABLE II. QMM parameters. The cluster color gyromagnetic ratios are fixed by the CMC constraint in terms of the cluster masses [GeV]. We fix the color octet cluster masses to equal the corresponding flavor equivalent \(K\) and \(K^*\) masses. The only free parameter is the flavor singlet color octet mass \(\xi_c\) (in bold type) adjusted to reproduce the lightest pentaquark candidate \((X_0(1800))\) observed in the SPHINX data [18–20].

| Name | \(X_{\alpha\beta}(J^\pi|L,S)\) | QMM Mass [GeV] Quantum Parameters | QMM Mass [GeV] Classical Parameters |
|------|-----------------------------|----------------------------------|----------------------------------|
| \(X_0(\frac{1}{2}^+)\) | \(X_{K_c\xi_c}(\frac{1}{2}^+,1,\frac{1}{2})\) | 1.801 | 1.801 |
| \(X_1(\frac{3}{2}^+)\) | \(X_{K_c\xi_c}(\frac{3}{2}^+,1,\frac{1}{2})\) | 1.967 | 1.967 |
| \(X_2(\frac{3}{2}^+)\) | \(X_{K_c\xi_c}(\frac{3}{2}^+,1,\frac{1}{2})\) | 1.999 | 1.972 |
| \(X_3(\frac{3}{2}^+)\) | \(X_{K_c\xi_c}(\frac{3}{2}^+,1,\frac{1}{2})\) | 2.064 | 2.049 |

TABLE III. The comparison between calculated QMM exotic masses [GeV] using quantum consistent and classical parameters for the gyromagnetic ratios. Note the different parameter sets only affect the \(K_\beta^*\) gyromagnetic ratio, so the two lightest exotics with \((K_c\xi_c)\) substructure are unaffected. The S.H.O. oscillator frequency was fixed by the \(N^*(1440) - N(938)\) mass splitting: \(\omega = 0.251\) GeV.
### TABLE IV. Masses and widths which characterize the narrow resonances fits to the $p(\gamma, K^-)\Lambda$ SAPHIR data [21]. $M_{X_i}$ are the masses, $\Gamma_{X_i}$ the widths, and $C_{X_i}$ the signal strengths. N/S: not seen in this channel. N/F: peak not fitted. C.L.: confidence level. See text for details.

| SAPHIR Data Fit | $p(\gamma, K^-)\Lambda$ | $p(\gamma, K^-)\Sigma^0$ |
|-----------------|-------------------------|-------------------------|
| $M_{X_0}$ [MeV] | N/S                     | N/S                     |
| $\Gamma_{X_0}$ [MeV] | N/S                     | N/S                     |
| $C_{X_0}$ | N/S                     | N/S                     |
| C.L. | -                       | -                       |
| $M_{X^-}$ [MeV] | 1898.4 ± 9.1            | N/S                     |
| $\Gamma_{X^-}$ [MeV] | 246.2 ± 57.0            | N/S                     |
| $C_{X^-}$ | 1.11 ± 0.04             | N/S                     |
| C.L. | -                       | -                       |
| $M_{X_1}$ [MeV] | N/S                     | 1974.2 ± 3.2            |
| $\Gamma_{X_1}$ [MeV] | N/S                     | 114.5 ± 11.3            |
| $C_{X_1}$ | N/S                     | 1.10 ± 0.04             |
| C.L. | -                       | > 99%                   |
| $M_{X_2}$ [MeV] | N/F                     | 2072.7 ± 4.7            |
| $\Gamma_{X_2}$ [MeV] | N/F                     | 113.5 ± 30.9            |
| $C_{X_2}$ | N/F                     | 1.27 ± 0.08             |
| C.L. | -                       | > 99%                   |

### TABLE V. Masses and widths characteristics of the narrow resonances fit to the $p(e, e'K^-)\Lambda$ Jefferson Lab (JLab) data [25,26]. N/F: peak not fitted. O/A: out of acceptance. C.L.: confidence level. Note the large width associated with the assumed $N^*$ peak (absent from the QMM spectrum).

| JLab Data Fit | $p(e, e'K^-)\Lambda$ | $p(e, e'K^-)\Sigma^0$ |
|---------------|-----------------------|-----------------------|
| $M_{X_0}$ [MeV] | N/F                   | O/A                   |
| $\Gamma_{X_0}$ [MeV] | N/F                   | O/A                   |
| $C_{X_0}$ | N/F                   | O/A                   |
| C.L. | -                     | -                     |
| $M_{N^*}$ [MeV] | 1896.5 ± 16.4         | N/F                   |
| $\Gamma_{N^*}$ [MeV] | 239.09 ± 121.4        | N/F                   |
| $C_{N^*}$ | 1.18 ± 0.01           | N/F                   |
| C.L. | > 99%                 | -                     |
| $M_{X_1}$ [MeV] | N/F                   | 1946.5 ± 4.2          |
| $\Gamma_{X_1}$ [MeV] | N/F                   | 43.6 ± 16.0           |
| $C_{X_1}$ | N/F                   | 1.53 ± 0.26           |
| C.L. | > 99%                 | -                     |
| $M_{X_2}$ [MeV] | N/F                   | 2001.7 ± 5.2          |
| $\Gamma_{X_2}$ [MeV] | N/F                   | 54.1 ± 23.2           |
| $C_{X_2}$ | N/F                   | 1.56 ± 0.29           |
| C.L. | > 99%                 | -                     |
| $M_{X_3}$ [MeV] | O/A                   | O/A                   |
| $\Gamma_{X_3}$ [MeV] | O/A                   | O/A                   |
| $C_{X_3}$ | O/A                   | O/A                   |
| C.L. | -                     | -                     |
TABLE VI. Experimental masses and widths of the narrow resonances fit to the $^{12}$C($p,K^+$)X SPHINX data [18–20], $p(\gamma,K^+)[\Lambda,\Sigma^0]$ SAPHIR data [21], and $p(e,e'K^+)[\Lambda,\Sigma^0]$ Jefferson Lab (JLab) data [25,26]. $M_X$ are the masses [MeV], $\Gamma_X$ the FWHMs [MeV], and $C_X$ the Breit-Wigner coupling strengths. N/S: not seen in this channel, O/A: out of acceptance. The QMM masses were calculated using the quantum consistent parameter set.
FIG. 1. The center-of-mass orbital angular momentum of the pentaquark system.

FIG. 2. Feynman diagram of the one gluon exchange between the color octet clusters.
FIG. 3. $\Lambda$ (left) and $\Sigma^0$ (right) invariant mass distributions in photo-production from [21] compared to the model prediction of [2,3] when the narrow baryon X-resonances are taken into account (solid line), and without X (dot-dashed line). The dotted line in the $\Lambda$ channel is the prediction from [27]. See text for details.
FIG. 4. Missing mass distribution for the kaon electro-production reaction [25,26]. Random coincidences and target wall contributions are subtracted.

FIG. 5. Invariant mass distributions of the Λ (left) and Σ⁰ (right) hyperons. The model prediction is compared to the experimental data [25,26] without (dot-dashed line) and with (dashed and solid line) coupling to the narrow X-resonances using the prediction of [2,3]. See text for details.
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