Abstract

Network calculus is an elegant theory which uses envelopes to determine the worst-case performance bounds in a network. Statistical network calculus is the probabilistic version of network calculus, which strives to retain the simplicity of envelope approach from network calculus and use the arguments of statistical multiplexing to determine probabilistic performance bounds in a network. One of the key results of deterministic network calculus is that the end-to-end performance measures determined using a network service envelope is bounded by $O(H)$, where $H$ is the number of nodes traversed by a flow. There have been many attempts to achieve a similar linear scaling of probabilistic performance bounds in statistical network calculus but with limited success. The main contribution of this paper is to establish an end-to-end stochastic network calculus with the notion of effective bandwidth and effective capacity from large deviations theory which provides end-to-end delay and backlog bounds that grows linearly in the number of nodes ($H$) traversed by the arrival traffic, under relatively general assumptions.

Keywords: Stochastic Network Calculus, Network Service Envelope, QoS, Effective Bandwidth, Effective Capacity
offered by the network. There exist many network theories which facilitate the modeling of traffic and service in data networks. Network calculus is one of the popular theories in recent times useful for the performance analysis of data networks. Network calculus uses deterministic arrival and service envelopes to bound traffic arrivals and the service offered at the nodes, respectively, to compute the worst case end-to-end performance bounds. However, most of the multimedia traffic observed in the Internet can tolerate some violation in its QoS requirements, and moreover, the statistical multiplexing in data networks smoothens the burstiness of the aggregate arrival traffic with high probability. Therefore the theory of network calculus was extended to the probabilistic domain, especially to benefit from the statistical multiplexing in data networks. The probabilistic version of network calculus is called statistical network calculus \(^1\) and it strives to retain many of the favorable characteristics of the network calculus, especially the simple envelope approach to derive probabilistic bounds. The raison d’être of network calculus is the possibility to model a network of nodes as a single abstract node using a network service envelope. In \(^1\), authors have shown that the end-to-end worst-case performance measures obtained by summing the per-hop results are bounded by \(O(H^2)\), while the end-to-end bounds obtained using network service envelope scales linearly in the number of nodes \(H\) connected in series. There have been many attempts to achieve similar linear scaling of end-to-end probabilistic performance bounds in statistical network calculus but with limited success. Most of these attempts use network service envelope with conservative envelope definitions \(^2\) or with some mathematical extensions like, rate correction factor \(^3\), delay threshold, busy period bounds \(^4\), time-domain extensions \(^5\). Most notably, in \(^3\) authors employed network service envelope with rate correction factor to compute the end-to-end performance measures that scales as \(O(H \log H)\). In \(^6\), authors have shown using stochastic network calculus with moment generating functions that, if the arrival traffic and the service offered at each network node are independent of one another, the end-to-end performance bounds can scale linearly, i.e., \(O(H)\). We direct the interested readers to the corresponding papers and to \(^4\) for an elaborate discussion on what makes statistical network calculus so difficult.

\(^1\)The terms statistical network calculus, stochastic network calculus and probabilistic network calculus are used interchangeably in the literature
Until now it has not been proven that the end-to-end probabilistic performance bounds computed using network calculus can actually scale as well as deterministic bounds. Based on the notion of effective bandwidth [7] and effective capacity [8] from large deviations theory, we develop an end-to-end stochastic network calculus which achieves the linear scaling of end-to-end performance measures without requiring any additional assumptions beyond the existence of effective bandwidth and effective capacity of arrival and service processes, respectively, and the stationarity of processes.

The rest of the paper is structured as follows: Section 2 introduces the arrival and service model used in the paper. In Section 3, we derive end-to-end backlog and delay bounds using stochastic network calculus with effective bandwidth and effective capacity functions. In Section 4, a numerical example using Markov Modulated On-Off traffic is presented for illustration. Brief conclusions are presented in Section 5.

2. Arrival and Service Models

In this section, we give a brief overview of the arrival and service models employed in this paper. Throughout this paper, we use a discrete time model $t \in \mathbb{N}_0 = \{0, 1, 2, \ldots\}$. We consider a network of $H$ nodes connected in series as shown in Fig. 1 with each node in the network having infinite-sized buffer serves the arrival traffic in a work-conserving fashion. Let the arrival and departure processes at a network node $h$ are modeled with bivariate real-valued left-continuous processes $A_h(s, t)$ and $D_h(s, t)$, respectively, which represents the cumulative amount of data seen in the interval $(s, t]$ for any $0 \leq s \leq t$. Let the service offered at hop $h$ for $h = 1, \ldots, H$ is characterized using a bivariate real-valued left-continuous process $S_h(s, t)$, which represents the cumulative amount of data served at the node in the interval $(s, t]$ for any $0 \leq s \leq t$. To simplify the notation, we denote $A_h(0, t) = A_h(t), D_h(0, t) = D_h(t), S_h(0, t) = S_h(t)$ for any $t \geq 0$. We assume that the network is causal, i.e., $A_h(t) \leq D_h(t)$ at any hop $h$ in the network for any $t \geq 0$, and there are no arrivals in the interval $(-\infty, 0]$. For an arrival process $A_h$ at a network node $h$, whose offered service is characterized by a stochastic service process $S_h$, the corresponding departure process $D_h$ satisfies for any fixed sample
path and all $t \geq 0$:

$$A_h \otimes S_h(t) \leq D_h(t) \quad (1)$$

where $\otimes$ denotes the $(\min, +)$ convolution of $A_h$ and $S_h$ which is defined as $A_h \otimes S_h(t) = \inf_{0 \leq s \leq t} \{A_h(0, s) + S_h(s, t)\}$. Any random process $S$ satisfying the above relationship (equation (1)) between arrival process and departure process for any fixed sample path is referred to as “dynamic F-server” [9].

Let $A = A_1$ be the arrival traffic at the node 1 or ingress of the network and $D = D_H = A_{H+1}$ represent the departure traffic from the node $H$ or egress of the network as shown in Fig. 1. The departure traffic $D_h$ from the node at hop $h$ becomes the arrival traffic $A_{h+1}$ to the downstream node at hop $h+1$, i.e., $A_{h+1} = D_h$ for all $h = 1, \ldots, H$. Since we assume the arrival and the service processes at each hop to be stationary, i.e., arrival and service processes depends only on the length of the interval $(s,t)$, but not on $s$ or $t$ itself, the arrival and the service processes for any $0 \leq s \leq t$ can be written as $A(s,t) = A(\Delta)$ and $S_h(s,t) = S_h(\Delta)$, respectively. In [9, 6, 10], authors show that the stochastic network service process $S_{net}$ describing the service offered in network of $H$ nodes connected in series, with stochastic service process $S_h$ for $h = 1, \ldots, H$ characterizing the corresponding service offered at hop $h$, for any fixed sample path is given by

$$S_{net} = S_1 \otimes S_2 \otimes \cdots \otimes S_H \quad (2)$$

To derive probabilistic performance measures using the stochastic arrival and service processes, one needs non-random functions characterizing the arrival and service processes. In [6], authors used moment generating function of the arrival traffic and conjugate moment generating function of the service process to derive the performance measures in a network. Here we adopt the popular notions of effective bandwidth ($\alpha_h$) [7] and effective capacity ($\beta_h$) [8] from large deviations theory to describe the stochastic arrival process and the service process characterizing the service offered at a network node $h$, respectively. The effective bandwidth of an arrival traffic $A_h$ from [7], for all $\theta, t > 0$, is given as

$$\alpha_h(\theta, t) = \frac{1}{\theta t} \log E \left[ e^{\theta A_h(t)} \right] \quad (3)$$

Similarly, the effective capacity function of a stochastic service process $S_h$ at a node $h$ from [8], for all $\theta, t > 0$, is defined as

$$\beta_h(\theta, t) = -\frac{1}{\theta t} \log E \left[ e^{-\theta S_h(t)} \right] \quad (4)$$
3. Stochastic Network Calculus with Effective Bandwidth and Effective Capacity

In this section we apply the arrival and service models from Section 2 to derive end-to-end performance bounds using stochastic network calculus with effective bandwidth and effective capacity. The fundamental difference between the statistical network calculus and its deterministic counterpart is that the performance bounds are expressed as probabilistic tail bounds, i.e., the derived bounds are violated with some probability. The end-to-end backlog \( B \) and delay \( W \) processes at time \( t \) in network of \( H \) nodes connected in series as shown in Fig. 1 are given by

\[
B(t) = A(t) - D(t)
\]

and

\[
W(t) = \inf \{ d \geq 0 : A(t - d) \leq D(t) \},
\]

respectively. In the following we derive the probabilistic bound on the end-to-end backlog and delay using stochastic network calculus with effective bandwidth and effective capacity functions. It should be noted that no assumptions on the independence of arrival and service processes were made.

**Theorem 3.1.** Let \( A \) be the arrival traffic to a network of \( H \) nodes connected in series with effective bandwidth function \( \alpha \) and \( D \) be the departure traffic from the network. Assume \( S_h \) for \( h = 1, \ldots, H \) be the stochastic service process at each hop in a network of \( H \) nodes with their corresponding effective capacity \( \beta_h \). Then we have the following probabilistic bounds.

1. Backlog bound : The probabilistic bound on the backlog in a network is given, for all \( t \geq 0 \), by

\[
P\{B(t) > x\} \leq \inf_{\theta > 0} \prod_{i=1}^{H} \left( \sum_{u_i=0}^{t} e^{\frac{\theta u_i}{2}(\alpha_i(\theta,u_i) - \beta_i(\theta,u_i))} \right)^{\frac{1}{H}} e^{-\frac{\theta}{2} \frac{H}{x} x} \tag{5}
\]

2. Delay bound : The probabilistic bound on the delay in a network is given, for all \( t \geq 0 \), by

\[
P\{W(t) > d\} \leq \inf_{\theta > 0} \prod_{i=1}^{H-1} \left( \sum_{u_i=0}^{t} e^{\frac{\theta u_i}{2}(\alpha_i(\theta,u_i) - \beta_i(\theta,u_i))} \right)^{\frac{1}{H}} \left( \sum_{u_H=d}^{t} e^{\frac{\theta}{2}((u_H-d)\alpha(\theta,u_H-d)-u_H\beta_H(\theta,u_H))} \right)^{\frac{1}{H}} \tag{6}
\]
Proof: We now prove the probabilistic bound on backlog $B$. For any $t \geq 0$, we have

$$P \{ B(t) > x \} = P \{ A(t) - D(t) > x \} \leq P \{ A(t) - A \otimes S_{net}(t) > x \}$$

$$= P \{ A(t) - A \otimes S_1 \otimes \cdots \otimes S_H(t) > x \}$$

$$= P \left\{ \sup_{0 \leq k_1 \leq t} \{ A(t) - A(0, k_1) - S_1 \otimes \cdots \otimes S_H(k_1, t) \} > x \right\}$$

$$= P \left\{ \sup_{0 \leq k_1, k_2 \leq k_3 \leq \cdots \leq k_H \leq t} \{ A(k_1, t) - S_1(k_1, k_2) + S_2(k_2, k_3) + \cdots + S_H(k_H, t) \} > x \right\}$$

$$\leq P \left\{ \sup_{0 \leq k_1 \leq k_2 \leq t} \{ A(k_1, k_2) - S_1(k_1, k_2) \} \right\}$$

$$+ \sup_{0 \leq k_1 \leq k_2 \leq \cdots \leq k_H \leq t} \{ A(k_2, k_3) - S_2(k_2, k_3) \} + \cdots$$

$$+ \sup_{0 \leq k_H \leq t} \{ A(k_H, t) - S_H(k_H, t) \} > x \right\}$$

$$\leq E \left[ e^{\Theta \sup_{0 \leq k_1 \leq k_2 \leq t} \{ A(k_1, k_2) - S_1(k_1, k_2) \} + \sup_{0 \leq k_2 \leq k_3 \leq \cdots \leq k_H \leq t} \{ A(k_2, k_3) - S_2(k_2, k_3) \} + \cdots + \sup_{0 \leq k_H \leq t} \{ A(k_H, t) - S_H(k_H, t) \} \right] e^{-\Theta x}$$

$$\leq \prod_{i=1}^{H} E \left[ e^{H \Theta \sup_{u_i \leq t} \{ A(u_i) - S_i(u) \} \} \right]^{1/H} e^{-\Theta x}$$

$$\leq \prod_{i=1}^{H} \left( \sum_{u_i=0}^{t} E \left[ e^{H \Theta \{ A(u_i) - S_i(u_i) \} \} \right] \right)^{1/H} e^{-\Theta x}$$

$$\leq \prod_{i=1}^{H} \left( \sum_{u_i=0}^{t} E \left[ e^{2H \Theta A(u_i)} \right]^{1/2} E \left[ e^{-2H \Theta S_i(u_i)} \right]^{1/2} \right) \left( \sum_{u_i=0}^{t} e^{\theta u_i} \right)^{1/2} e^{-\theta x}$$

$$= \prod_{i=1}^{H} \left( \sum_{u_i=0}^{t} e^{\theta u_i} \right) e^{-\theta x}$$

The proof of the probabilistic bound on delay $W$ follows the similar steps. For any $t \geq 0$, we have

$$P \{ W(t) > d \} = P \{ A(t) - d - D(t) > 0 \} \leq P \{ A(t) - d - A \otimes S_{net}(t) > 0 \}$$

$$= P \{ A(t) - d - A \otimes S_1 \otimes \cdots \otimes S_H(t) > 0 \}$$
\begin{align*}
&= P \left\{ \sup_{d \leq k_1 \leq t} \{ A(t-d) - A(0, k_1) - S_1 \otimes \cdots \otimes S_H(k_1, t) \} > 0 \right\} \\
&= P \left\{ \sup_{d \leq k_1 \leq k_2 \leq k_3 \leq \cdots \leq k_H \leq t} \{ A(k_1, t-d) - \{ S_1(k_1, k_2) + S_2(k_2, k_3) + \cdots + S_H(k_H, t) \} \right\} > 0 \\
&\leq P \left\{ \sup_{0 \leq k_1 \leq k_2 \leq t} \{ A(k_1, k_2) - \{ S_1(k_1, k_2) \right\} + \sup_{0 \leq k_2 \leq k_3 \leq t} \{ A(k_2, k_3) - \{ S_2(k_2, k_3) \right\} + \cdots \\
&\quad + \sup_{d \leq k_H \leq t} \{ A(k_H, t-d) - S_H(k_H, t) \} \right\} > 0 \\
&\leq E \left[ e^{\Theta \sup_{0 \leq k_1 \leq k_2 \leq t} \{ A(k_1, k_2) - S_1(k_1, k_2) \} + \sup_{0 \leq k_2 \leq k_3 \leq t} \{ A(k_2, k_3) - S_2(k_2, k_3) \} + \cdots \\
&\quad + \sup_{d \leq k_H \leq t} \{ A(k_H, t-d) - S_H(k_H, t) \} \right] \\
&\leq \prod_{i=1}^{H-1} E \left[ e^{H \Theta \sup_{0 \leq u_i \leq t} \{ A(u_i) - S(u_i) \} \right] \left( \sum_{u_H = d}^{t} E \left[ e^{H \Theta (A(u_H - d) - S_H(u_H))} \right] \right)^{\frac{1}{H}} \\
&\leq \prod_{i=1}^{H-1} \left( \sum_{u_i = 0}^{t} E \left[ e^{2H \Theta A(u_i)} \right] \right)^{\frac{1}{2H}} \left( \sum_{u_H = d}^{t} E \left[ e^{-2H \Theta S(u_H)} \right] \right)^{\frac{1}{2H}} \\
&= \prod_{i=1}^{H-1} \left( \sum_{u_i = 0}^{t} e^{\frac{\theta u_i}{2}(\alpha(\theta, u_i) - \beta_i(\theta, u_i))} \right)^{\frac{1}{2H}} \left( \sum_{u_H = d}^{t} e^{\frac{\theta}{2}(u_H - d)(\alpha(\theta, u_H - d) - u_H \beta_H(\theta, u_H))} \right)^{\frac{1}{2H}}
\end{align*}

The inequalities in the respective proofs for probabilistic backlog and delay bounds follow the similar reasons. The first inequality is from the property of stochastic network service process (equation (1)). The second inequality is due to the property of supremum operation, i.e., \( \sup_{0 \leq s \leq t} \{ X(s) + Y(s) \} \leq \sup_{0 \leq s \leq t} \{ X(s) \} + \sup_{0 \leq s \leq t} \{ Y(s) \} \). The third and fourth inequalities are
from Chernoff’s bound\(^2\) and Hölder’s inequality\(^3\), respectively. Application of Boole’s inequality leads to the fifth inequality. The sixth inequality is due to the application of Schwarz’s Inequality\(^4\). The final step is obtained by setting \(\theta = 2H\Theta\) and from the definition of effective bandwidth and effective capacity. Minimizing the expression over \(\theta\) proves our claim on probabilistic backlog and delay bound.\(\blacksquare\)

Since the use of Hölder’s inequality can be avoided for the independent random variables\(^5\), the performance bounds from Theorem 3.1 can be further improved if the arrival traffic process \(A\) and the stochastic service process \(S_h\) at each hop \(h\) for \(h = 1, 2, \ldots, H\) are statistically independent of one another and has been analyzed using statistical network calculus with moment generating functions in [6].

To analyze the linear scaling of end-to-end probabilistic performance bounds from Theorem 3.1 we consider a tandem network of \(H\) nodes connected in series as shown in Fig. 1, with each node offering similar service characterized by the stochastic service process \(S\) with the corresponding effective capacity \(\beta\). Then the probabilistic backlog and delay bounds from Theorem 3.1 for any \(t \geq 0\), will become

\[
P \{B(t) > x\} \leq \left( \sum_{u=0}^{t} e^{\frac{\theta u}{2} (\alpha(\theta,u) - \beta(\theta,u))} \right) e^{-\frac{H}{2} \theta x} \tag{7}
\]

\[
P \{W(t) > d\} \leq \left( \sum_{u=0}^{t} e^{\frac{\theta u}{2} (\alpha(\theta,u) - \beta(\theta,u))} \right) \frac{t}{H^{1/2}} \left( \sum_{u=d}^{t} e^{\frac{\theta (u-d)}{2} (\alpha(\theta,u-d) - u\beta(\theta,u))} \right)^{1/2} \tag{8}
\]

If the probabilistic backlog and delay bounds are violated at most with the probability \(\varepsilon\), then setting the bounds on the right-hand sides of equations

\(^2\)For random variable \(X\) and \(x, \theta \geq 0\), \(P\{X > x\} \leq E[e^{\theta X}] e^{-\theta x}\).

\(^3\)For random variables \(X, Y\) and \(a, b > 0\) with \(1/a + 1/b = 1\), \(E[XY] \leq E[X^a]^{1/2} E[Y^b]^{1/2}\).

\(^4\)For random variables \(X, Y\), \(E[XY] \leq E[X^2]^{1/2} E[Y^2]^{1/2}\).

\(^5\)For any two independent random variables \(X, Y\), \(E[XY] = E[X]E[Y]\).
Figure 2: Network of $H$ concatenated nodes with cross traffic

Equations (7) and (8) to $\varepsilon$ and solving for $x$ and $d$ gives

$$x \geq \frac{2H}{\theta} \log \frac{1}{\varepsilon} \left( \sum_{u=0}^{t} e^{\theta u (\alpha(\theta,u) - \beta(\theta,u))} \right)$$  \hspace{1cm} (9)

$$- \log \left( \sum_{u=d}^{t} e^{\theta ((u-d)\alpha(\theta,u-d) - u\beta(\theta,u))} \right) \geq (H - 1) \log \left( \sum_{u=0}^{t} e^{\theta u (\alpha(\theta,u) - \beta(\theta,u))} \right)$$

$$- H \log \varepsilon \hspace{1cm} (10)$$

It is apparent from equations (9) and (10) that the end-to-end backlog and delay bounds using Theorem 3.1 grows linearly in the number of nodes $H$ a flow traverses in a network. The significance of the presented approach is that the stochastic information about the arrival and service processes are retained as long as possible using the concept of effective bandwidth and effective capacity, respectively, which allows the computation of end-to-end stochastic performance measures that grows linearly in the number of nodes $H$ traversed by the arrival traffic, whereas in other approaches as in [2, 3, 4, 5] the stochastic information about the arrival and service processes are lost as soon as statistical envelopes are fixed.

4. Numerical Example

In this section, we illustrate the benefits of end-to-end stochastic network calculus with effective bandwidth and effective capacity using the arrival traffic modeled as Markov modulated on-off (MMOO) process, especially we show that the end-to-end performance bounds computed using Theorem 3.1 are as good as the ones obtain using statistical envelopes [3] and also scales linearly in the number of nodes $H$. Markov modulated on-off process is commonly used to model voice [11] and video traffic [12] in the Internet. Markov modulated on-off process can be in "On" state or "Off" state.
for random time intervals which are negative exponentially distributed with averages $E[T_{on}]$ and $E[T_{off}]$, respectively. In "On" state, arrival traffic transmits data at a constant rate $P$ and no data is transmitted in "Off" state. The effective bandwidth of Markov modulated on-off process has an interesting property that $\alpha(\theta, t) \leq \alpha(\theta)$ and for any $\theta > 0$ is given by

$$\alpha(\theta) = \frac{1}{2\theta} \left( P\theta - r_{10} - r_{01} + \sqrt{(P\theta - r_{10} + r_{01})^2 + 4r_{10}r_{01}} \right)$$  \hspace{1cm} (11)$$

where $r_{10} = \frac{1}{E[T_{on}]}$ and $r_{01} = \frac{1}{E[T_{off}]}$.

For the analysis we consider a network of $H$ nodes connected in series with cross traffic as shown in Fig. 2. The queue at each hop $h$ is served in a work conserving fashion at a constant deterministic service rate $C$. The flow of interest is the one which traverses through the network of $H$ nodes connected in series and is termed through flow $A$. The flow which transits the network at each hop is termed cross flow $A_c$. Let $\alpha$ and $\alpha_c$ be the effective bandwidth functions of the through flow $A$ and the cross flow $A_c$, respectively. Let there be $N$ independent through flows at the ingress of the network and $M$ independent cross flows at each hop $h$ inside the network. For all $t \geq 0$ and any $\theta \geq 0$, the condition $C \geq N\alpha(\theta) + M\alpha_c(\theta)$ must be satisfied for stability. The service available to the $N$ through flows at hop $h$ can be characterized using leftover stochastic service process $S_h(t) = S(t) = Ct - MA_c(t)$ with effective capacity function $\beta_h(\theta) = \beta(\theta) = C - M\alpha_c(\theta)$ under general scheduling model \cite{6, 10} for $h = 1, \ldots, H$ and any $t, \theta \geq 0$. We evaluate the larger time interval $[0, \infty]$ instead of $[0, t]$ to compute end-to-end, closed-form performance measures using Theorem 3.1. The backlog $x$ and delay $d$ bounds which are violated at most with probability $\varepsilon$ can be computed from equations (9) and (10) using Theorem 3.1 for any $t \geq 0$ and $\theta > 0$, as

$$x \geq \frac{2H}{\theta} \log \frac{1}{\varepsilon} \left( \sum_{u=0}^{\infty} e^{\frac{\theta u}{2}(N\alpha(\theta) - \beta(\theta))} \right)$$

$$\geq \inf_{\theta \geq 0} \frac{2H}{\theta} \log \frac{1}{\varepsilon} \left( 1 - e^{-\frac{\theta}{2}(C - N\alpha(\theta) - M\alpha_c(\theta))} \right)$$

$$- \log \left( \sum_{u=d}^{\infty} e^{\frac{\theta}{2}((u-d)N\alpha(\theta) - u\beta(\theta))} \right) \geq (H - 1) \log \left( \sum_{u=0}^{\infty} e^{\frac{\theta u}{2}(N\alpha(\theta) - \beta(\theta))} \right) - H \log \varepsilon$$  \hspace{1cm} (12)$$

$$-H \log \varepsilon \hspace{1cm} (13)$$
and after some reordering of equation (13) we obtain

\[ d \geq \inf_{\theta \geq 0} \frac{2H}{\theta(C - M \alpha_c(\theta))} \log \frac{1}{\varepsilon \left(1 - e^{-\frac{\theta(C - N \alpha(\theta) - M \alpha_c(\theta))}{(C - N \alpha(\theta) - M \alpha_c(\theta))}}\right)} \]  

(14)

It is apparent from equations (12) and (14) that the end-to-end backlog and delay bounds for MMOO traffic model using Theorem 3.1 grows linearly in the number of nodes \( H \) a flow traverses in a network. For the numerical experiment, we compute the end-to-end delay bound for \( N \) through flows in the network with a violation probability \( \varepsilon = 10^{-9} \). The capacity of the server \( C \) at each hop is set to 100 Mbps and for each Markov modulated on-off process, we choose the following values which are typically used to model voice flows [11]: \( P = 64Kbps, E[T_{on}] = 0.4s \) and \( E[T_{off}] = 0.6s \). The average arrival rate (\( m \)) of the Markov modulated on-off traffic using the given parameters is 25.6 Kbps.

In Figs. 3 and 4, the end-to-end delay bound computed using statistical envelope definitions from [3] and delay bound from equation (14) are plotted. Fig. 3 shows the probabilistic end-to-end delay bound with a violation probability (\( \varepsilon \)) of \( 10^{-9} \) as a function of increasing number of hops \( H \). At each hop, \( N = 781 \) through flows are multiplexed with \( M = 1953 \) independent cross flows. The plot validates the linear scaling of the end-to-end delay.
Figure 4: End-to-end statistical delay bounds for Markov modulated on-off traffic in a network of $H$ nodes with a violation probability $\varepsilon = 10^{-9}$ bounds determined using Theorem 3.1. In Fig. 4 we plot the probabilistic end-to-end delay bound for $N$ through flows in a network with $H = 1, 2, 5, 10$ hops for increasing $N + M$ number of flows at each hop while maintaining $N = M$. It can be observed that the delay bound from equation (14) yield a tighter delay bound than the ones computed using statistical envelopes even for single hop case. Though the difference between computed bound from equation (14) and bound using statistical envelopes become small for $H = 10$, the gap becomes large when the number of nodes $H$ traversed by the through flows is increased.

5. Conclusion

We presented an end-to-end stochastic network calculus with effective bandwidth and effective capacity functions. We then showed that such a formulation of network calculus results in end-to-end performance measures that grow linearly in the number of nodes traversed by the arrival traffic in a homogeneous network of nodes with each node offering similar service. Further we showed using numerical example with Markov modulated on-off arrivals traffic that the end-to-end delay bound computed using the presented stochastic network calculus is as good as the bounds obtained using statistical envelopes.
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