Confidence intervals for the expected damage in random loadings: Application to measured time-history records from a Mountain-bike

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Abstract. The fatigue damage of a structure is usually estimated by calculating the damage values of a limited number of measured random time-histories. The limited damage values are generally not identical due to the sampling variability. In a recent work, confidence interval expressions have been proposed to bound the expected damage value. The method revealed a good agreement with simulations, thus suggesting its use also with real measurements. The present paper investigates the above-mentioned confidence intervals for expected damage by a real engineering application, in which few measured time-history records, or even only one, are available. This goal is achieved after measuring the random loadings acting on an instrumented Mountain-bike riding in a typical north Italian off-road track. As the whole ensemble of an infinite number of time-histories is not available and the expected damage is thus not known a priori, a sort of calibrator sample damage value, which is computed using a large number of measured time-history records, is used to estimate the expected damage. The obtained results confirm the accuracy of the proposed approach also with real measurements.

1. Introduction

Random loadings are the typical loads acting on mechanical structures and components subject to wind, waves, or surface irregularity. To avoid the failure of such structures and components, the fatigue damage is often calculated by a time-domain approach (rainflow counting method and Palmgren-Miner rule). The time-domain damage may be calculated from one measured random time-history record, which must be considered one sample value out of an infinite ensemble. If it is computed from another measured time-history of the same duration and statistical properties, the damage may change slightly. The damage values are usually not identical due to the sampling variability.

Indeed, the damage of a time-history with finite length is a random variable following a damage probability distribution. The variance around the expected damage is an important property of damage distribution. Mark and Crandall [1][2] and Bendat [3] have been developed theoretical approaches to estimate the variance of fatigue damage when an infinite ensemble of time-histories with finite length is available. Their methods were, however, limited to the linear oscillator system and to either odd or even integer values of inverse slope of the S-N curve. Madsen et al. [4] e Low [5] devised a more general method applied to any narrow-band Gaussian process, i.e. without restriction to the linear oscillator, see [6][7] for a survey of such approaches. To extend Low’s method from the Gaussian to the non-Gaussian case, Marques and Benasciutti [8] developed an approach based on a non-linear transformation. More recently, Marques et al. [7] proposed a simple and easy best-fitting equation to estimate the variance of
fatigue damage in Gaussian case. Though not derived by theoretical arguments and calibrated on the results of a few processes, this best-fitting expression is also applicable to wide-band process.

In their same work [7], confidence intervals have been derived to address the issue of estimating the variability of the damage when only few time-histories (Case 1) or even only one (Case 2) are available. In this situation, in fact, any of the methods mentioned above cannot be applied as they require an infinite ensemble of time-histories. For each case, two or more time-histories and only one time-history, a confidence interval expression to enclose the (unknown) expected damage was derived. The proposed approach revealed to be not only in a good agreement with numerical simulations, but also a useful tool for researchers dealing with real engineering applications.

The above-mentioned confidence intervals for the expected damage is here investigated by measuring the random loads acting on a Mountain-bike. To calculate the confidence intervals and the expected damage, several measured time-histories are obtained directly from the Mountain-bike in a typical north Italian off-road track. The stationarity hypothesis of all measured time-history records is also verified by different approaches, for example: comparison of loading (or cumulative) spectra, Short-Time Fourier Transform (STFT), and run test method. A sort of calibrator damage is used to estimate the unknown expected damage; it is the sample mean of several damage values, each of which is computed from a different measured time-history. The calibrator sample damage is needed to check whether the confidence intervals correctly encloses the expected damage. The results then confirm the accuracy of the proposed approach [7] when applied to real measurements.

2. Theoretical background

2.1. Expected damage

Under the Palmgren-Miner rule, the fatigue damage of a time-history \( x(t) \) of length \( T \) is the sum of the damage of every cycle counted:

\[
D(T) = \sum_{i=1}^{n(T)} d_i = \sum_{i=1}^{n(T)} \frac{s_i^k}{A}
\]

in which \( n(T) \) denotes the number of counted cycles, \( s_i \) is the stress amplitude of the \( i \)-th cycle, \( A \) and \( k \) are material constants of the S–N curve \( s^kN_f = A \). The rainflow counting method is commonly used to identify the counted cycles of a particular time-history \( x(t) \). The damage \( D(T) \) strictly depends on this time-history \( x(t) \) of length \( T \). In fact, \( D(T) \) may result in a different value if another \( x(t) \) of the same infinite ensemble is considered [6][7][8][9]. These damage values for a fixed length \( T \) may have a large scatter due to the randomness of the counted cycles \( n(T) \) and the stress amplitude \( s \).

The expectation of equation (1) provides the expected damage value:

\[
E[D(T)] = E \left[ \sum_{i=1}^{n(T)} d_i \right] = E[n(T)] \frac{E[s^k]}{A}
\]

where \( E[-] \) is the probabilistic expectation. The expectation of a random variable is a weighted average over all the possible values, which simply means the arithmetic mean of an infinite number of independent realizations of the same process. Indeed, equation (2) represents the damage value that would result from averaging all damage values. In other words, \( E[D(T)] \) may be computed when an infinite ensemble of time-histories is available. However, in a real application with a finite number of measured time-history records, \( E[D(T)] \) is never known exactly; it can only be approximated.

From now on, let us assume that the damage \( D(T) \) in equation (1) will be calculated by rainflow counting method and the Palmgren-Miner rule. Also, let us assume that its material constants of the S–N curve will be \( A = 1 \) and \( k = 3 \).
2.2. Confidence intervals for expected damage

The confidence interval for the expected damage by Marques et al. [7] is provided in two cases: few time-histories (Case 1) and only one time-history (Case 2). The damage \( D_i(T) \), \( i = 1, 2, \ldots, N \) is computed for each stationary random time-history \( x_i(t) \) of same duration \( T \).

Case 1 considers the set of \( N \geq 2 \) damage values to calculate the sample mean \( \overline{D}(T) = N^{-1} \sum_{i=1}^{N} D_i(T) \) and the sample variance \( \hat{\sigma}_D^2 = (N - 1)^{-1} \sum_{i=1}^{N} [D_i(T) - \overline{D}(T)]^2 \) of the damage. Following the definition of the confidence interval of a normally distributed random variable with unknown mean and unknown variance [10], the 100(1 - \( \beta \))% confidence interval for the expected damage is determined [7]:

\[
\overline{D}(T) - \frac{t_{dof,\beta/2} \cdot \hat{\sigma}_D}{\sqrt{N}} \leq E[D(T)] \leq \overline{D}(T) + \frac{t_{dof,\beta/2} \cdot \hat{\sigma}_D}{\sqrt{N}} \tag{3}
\]

where \( t_{dof,\beta/2} \) is the quantile of the Student's t-distribution with \( dof = N - 1 \) degrees of freedom.

Case 2 is based on the idea of dividing a single time-history into \( N_B \) disjoint (not overlapped) blocks of equal length, \( T_B = T/N_B \). The damage of the entire time-history \( x(t) \), see equation (1), sums up the damage of all blocks:

\[
D(T) = \sum_{i=1}^{N_B} D_{B,i}(T_B) = N_B \cdot \overline{D}_B(T_B) \tag{4}
\]

in which \( D_{B,i}(T_B), i = 1, 2, \ldots, N_B \) is the damage of each block and \( \overline{D}_B(T_B) = N_B^{-1} \sum_{i=1}^{N_B} D_{B,i}(T_B) \) is the sample mean of the damage of all blocks. A minimum number of blocks \( N_B \geq 2 \) is required and each block must have a minimum length \( T_B \) so to contain approximately \( 10^3 \) counted cycles.

Based on the hypothesis that the damage values of each block, \( D_{B,i}(T_B) \), are independent and identically distributed [11][7], the variance \( \sigma_{DB}^2 \) of the damage of the whole time-history \( x(t) \) is defined as [11][7]:

\[
\sigma_{DB}^2 = \text{Var} \left[ \sum_{i=1}^{N_B} D_{B,i}(T_B) \right] = N_B \cdot \sigma_{DB}^2 \tag{5}
\]

where \( \sigma_{DB}^2 = \text{Var}[D_{B,i}(T_B)] \) is the variance of all blocks. Equation (5) involves the “true” (but unknown) variances; it can, however, be approximated as \( \sigma_{DB}^2 \equiv N_B \cdot \sigma_{DB}^2 \), in which the block variance is substituted by its sample estimate \( \sigma_{DB}^2 = (N_B - 1)^{-1} \sum_{i=1}^{N_B} [D_{B,i}(T_B) - \overline{D}_B(T_B)]^2 \). In analogy with Case 1 in equation (3), it is possible to define a confidence interval for the expected damage of blocks:

\[
\overline{D}_B(T_B) - \frac{t_{dof,\beta/2} \cdot \hat{\sigma}_{DB}}{\sqrt{N_B}} \leq E[D_B(T_B)] \leq \overline{D}_B(T_B) + \frac{t_{dof,\beta/2} \cdot \hat{\sigma}_{DB}}{\sqrt{N_B}} \tag{6}
\]

Multiplying equation (6) by the number of blocks \( N_B \) yields:

\[
N_B \left( \overline{D}_B(T_B) - \frac{t_{dof,\beta/2} \cdot \hat{\sigma}_{DB}}{\sqrt{N_B}} \right) \leq N_B E[D_B(T_B)] \leq N_B \left( \overline{D}_B(T_B) + \frac{t_{dof,\beta/2} \cdot \hat{\sigma}_{DB}}{\sqrt{N_B}} \right) \tag{7}
\]

Since \( N_B \) is deterministic [7], the expected value of equation (4) results in:

\[
E[D(T)] = E \left[ \sum_{i=1}^{N_B} D_{B,i}(T_B) \right] = N_B E[D_B(T_B)] \tag{8}
\]
This result can also be obtained by noting that \( E[D_B(T_B)] = (T_B/T) \cdot E[D(T)] \). Substituting into equation (7) and considering the approximation \( \delta_D \cong \sqrt{N_B} \cdot \delta_{DB} \) of equation (5), the confidence interval expression for \( E[D(T)] \) when considering only one time-history \( x(t) \) is [7]:

\[
D(T) - t_{dof,\beta/2} \cdot \delta_D \leq E[D(T)] \leq D(T) + t_{dof,\beta/2} \cdot \delta_D
\]

(9)

where \( D(T) \) denotes the damage of equation (4).

3. Methods and measurements

The primary purpose of this study was to obtain real measurements for estimating the expected damage and calculating the confidence intervals. This means that the present research was not intended in evaluating the safety of the Mountain-bike on an off-road track. By the apparatus described below, the Mountain-bike was, however, well-equipped to record time-histories in a typical north Italian off-road track and to use them to verify the methods described above.

The Mountain-bike is a 2010 Scott Sportster P6 (figure 1(a)), its frame is made of an aluminum 6061 alloy, and the rigid front fork is a Unicrown made of carbon steel. Two Rigida Cyber 10 size 700C wheels are coupled with 700 x 37c S207 semi-slick tires. The handlebar and the saddle are made by Scott Sports while the transmission, chain, and crankset are made by Shimano, Inc.

The bicycle front fork was instrumented with a Strain Gauge Bridge calibrated during static laboratory tests. To monitor the loads acting on Mountain-bike, a bending half-bridge was applied close to the middle of the front fork in the longitudinal plane, figure 2(b). Two strain gauges were placed symmetrically on the left tube. They were manufactured by HBM and the model was LY Linear Strain Gauges with 1 Measuring Grid (only one direction).

Measured time-histories were collected through a Dewesoft data acquisition system and the model was MinitaurS Dewe-101 with 8 channels. Such a model contains an industrial power computer built directly in the unit. A filter was set with a cut-off frequency of 300 Hz, which was above the maximum frequency of interest [12]. A sampling frequency was fixed at 1000 Hz. The main “triangle” of the bicycle frame was exploited to fix the data acquisition on the inclined tube. The rechargeable supply battery was allocated behind the seat by a welded support.

In addition, a speedometer containing a sensor and magnet fixed on a spoke was used for monitoring the Mountain-bike speed. It was made by Marwi Group and model was a Union 8 Cycling Computer. The fully equipped Mountain-bike weighed about 12.2 kg.

![Figure 1. Mountain-bike: (a) overall view of the bicycle and components; (b) two strain gauges applied to the front fork; (c) portion of the off-road track (left side of the picture).](image)

The cycling conditions were set in order to obtain stationary random time-histories. The short off-road track of 0.5 kilometers length was plane (or almost plane) with a gravel surface. This typically north Italian track (left side of figure 1(c)) was located at the Ippodromo Comunale in Ferrara city. A rider of 59 kg mass guided the bicycle in a seated condition. The speed was kept practically constant at 15 km/h. These cycling conditions may not represent a critical situation in which a Mountain-bike on an off-road track is subject to. In fact, service loadings experienced by off-road bicycles are usually
rather irregular and also non-stationary (e.g. different tracks with various speeds). However, the same cycling conditions over time were needed to investigate the confidence intervals applied to only stationary random loadings.

A total of 41 measured time-histories were obtained under the same cycling conditions during 9 consecutive days. On Days 1-2, measurements were performed to collect 10 time-histories used for the confidence interval in Case 1. On the other hand, Case 2 required only one time-history, which was recorded on Day 3. The remaining 30 time-histories were measured on Days 4-9 and they are intended to be used for approximating the expected damage by the sample mean of 30 damage values. Note that the estimated expected damage was needed to verify the correctness of the proposed confidence intervals in the present work. However, the confidence intervals do not require the knowledge of the expected damage when they are applied in real cases. Consequently, the above-mentioned 30 measured time-histories are not needed in reality and are used here only for verification purposes. The minimum values of measured random time-history are $N = 2$ for Case 1 and $N = 1$ for Case 2.

All measured time-histories in this study were fixed at a time length $T = 300$ s and were normalized to have a zero mean $\mu_x = 0$ value and a variance equal to unity $\sigma_x^2 = 1$. An example is reported in figure 2 for the first measured time-history $x_1(t)$ from the bicycle front fork. In this particular time-history $x_1(t)$ it is possible to appreciate how there is no significant change in the mean and variance levels over time. Although the actual time-history values cannot be precisely predicted, which is a characteristic of the randomness of the loading, at least two well-separated frequencies are observed in the zoomed view of $x_1(t)$, see the right side of figure 2.

![Figure 2](image-url)  
**Figure 2.** Overall and zoomed view of the first measured time-history $x_1(t)$.

### 3.1. Stationary random loadings

The confidence interval formulae in equation (3) and (9) are only applicable to stationary random loadings, which have properties (e.g., frequency content, mean and standard deviation) that do not change over time. Different approaches (qualitative or quantitative) can be used to identify the stationarity [13][14]. Among them, the comparison of the loading (or cumulative) spectrum can be used to compare the statistical distribution of rainflow cycles when several measured time-histories are available. A comparison of loading spectra is demonstrated in figure 3(a) by using five measured time-histories.
A few low frequency components are usually detected in histories. A high number of blocks $N_B$, which results in a better time resolution. Consequently, each time-history was divided so to have $T_B = 10$ s and 75% of overlap fraction. The frequency resolution is artificially increased by adding zero values at the end of blocks (zero paddings). The application of the STFT on the first measured time-history $x_1(t)$ from the bicycle is presented in figure 3(b).

The STFT exhibits no pronounced change in the frequency content (range from 0 to 50 Hz) over time. The amplitude of the STFT (intensity indicated in the color bar in figure 3(b)) also shows to be practically constant over time. These qualitative results were somehow expected due to the predisposed cycling conditions.

As the frequency content was shown to vary over time, it is possible to characterize the random loading in the frequency-domain by a one-sided Power Spectral Density (PSD). It also permits the differences in the distribution of power over frequencies to be appreciated best. To analyze the first measured time-history $x_1(t)$, the estimated PSD $\hat{S}_x(f)$ in figure 3(c) is computed by the Welch’s method [16]: Hanning block (or window), 75% overlapping and $T_B = 10$ s. It is interesting to note that the estimated PSD extends over a wide range of frequencies, which characterizes a so-called wide-band random loading. A few low frequency components are also observed in $\hat{S}_x(f)$ from 0 to 5 Hz and a high frequency component is perceived at about 28 Hz. In fact, low frequencies are usually detected in random time-histories by numerous small cycles while high frequency gives large cycles. Following the previously statement, at least two well-separated frequencies has the first measured time-history, small and large cycles in figure 2 are emphasized by low and high frequencies in figure 3.
A useful non-parametric method to quantify the stationarity of a random loading is the Wald–Wolfowitz run test or run test [15][17]. The method is based on the idea of dividing the time-history $x(t)$ into blocks $N_B$, similar to the approach proposed for the confidence intervals in Case 2 [7], the STFT, and the Welch’s method. The run test takes into account a sequence of blocks without overlapping. For every block $N_B$, a value is calculated for the statistical parameter that is being investigated. This sequence of observations (observed block values) defines a run $r$ that is followed and preceded by a different observation or no observation at all [15]. By classifying the observations as being above or below the sample median of all observations, the run distribution has a mean $\mu_r$ and variance $\sigma_r^2$ [15]:

$$\mu_r = 1 + n_a, \quad \sigma_r^2 = \frac{n_a(n_a - 1)}{2n_a - 1}$$

(10)

where $n_a$ is the number of observations above the median. Equation (10) is applied when the number of observations above the median $n_a$ equals the number of observations below the median $n_b$; so $n_a = n_b$. The run test assumes that the sequence of observations is independent observations of the same random variable [15]. The acceptance region for this hypothesis is [15]:

$$r_{n_a,1-\beta/2} < r \leq r_{n_a,\beta/2}$$

(11)

in which $r_{n_a,1-\beta/2}$ is the lower and $r_{n_a,\beta/2}$ the upper limit of the run test. These values are determined by $\mu_r$ and $\sigma_r^2$ in equation (10) or using a tabulation of run distribution [15]. If the observed number of runs falls inside the acceptance region, the random time-history $x(t)$ is presumed to be stationary. Otherwise, $x(t)$ is classified as non-stationary.

As an example, the run test method is demonstrated here for the first measured time-history $x_1(t)$. Rather than using the usual statistical parameters, e.g., the root-mean-square (RMS) value, as the output calculated in each block, it was decided to consider the damage $D_B(T_B)$ computed by the equation (1) for each block. By this approach, in contrast to the Rouillard’s one [18], the run test applied here can detect not only the changes in the RMS value, but also the changes of the mean and frequency content of the random signal. Another parameter that has a prominent effect in quantifying the stationarity is the block length $T_B$ of run test. A short block length increases the sensitivity of the analysis to local variations, whereas a wider length could not perceive the non-stationarity. For this reason, the same block length $T_B = 10$ s used for the STFT was exploited as a first attempt. Using the damage $D_B(T_B)$ (normalized to the median) as the observed block values, the run test is demonstrated in figure 4.

![Figure 4](image-url)

**Figure 4.** Run test using the damage as the observed block values.

There are $r = 17$ runs (see integer values from 1 to 17 in figure 4) represented by the sequence of 30 observed block values. The upper and lower values in equation (10) were obtained by the tabulation in [15]. Accordingly, the acceptance region of run test results in $10 < 17 \leq 21$ for 95% level of
significance. The first measured time-history $x_1(t)$ is then quantified as stationary since $r = 17$ falls within $r_{15.0.075} = 10$ and $r_{15.0.025} = 21$. The run test was conducted for all the measured time-history records considered in this study. Although not shown here, the results quantified all time-histories as stationary random loadings.

4. Confidence intervals and expected damage using measurements

The confidence intervals for the expected damage in equation (3) and (9) are constructed from the damage values computed from the measurements in the Mountain-bike front fork, as those usually gathered in other similar engineering applications, see figure 5(a).

![Figure 5](https://via.placeholder.com/150)

**Figure 5.** Measured time-history records used for: (a) confidence intervals; (b) approximating the expected damage.

The intervals were constructed with a 95% confidence level by considering the values $D_i(T)$ and $D_{B,j}(T_B)$ used in Case 1 and Case 2, respectively. All time-histories $x(t)$ from which the damage is computed were measured with an equal time-length $T = 300 \text{ s}$, which guarantees at least $1 \cdot 10^4$ counted cycles. In Case 1, the confidence interval was computed for various amount of time-histories $N = 2, 3, \ldots, 10$. For all $N$ values, the same set was considered. To estimate the confidence interval in Case 2, another measured time-history was considered along with multiple blocks $N_B = 2, 3, \ldots, 10$.

In practice, the whole ensemble of measured time-histories is not available and, consequently, the expected damage of equation (2) is never known. However, a reasonable approximation to the expected damage was required to evaluate the proposed confidence intervals in equation (3) and (9) using real measurements. The expected damage was then estimated by considering a different set of stationary measured time-histories (see figure 5(b)) from the Mountain-bike. This means that the damage $D_i(T)$ were different values from those used for the confidence intervals. The sample mean of several damage values $\bar{D}(T) = N^{-1} \sum_{i=1}^{N} D_i(T)$ was calculated by the damage in equation (1). To approximate the expected damage $E[D(T)]$ with the sample mean damage $\bar{D}(T)$, a large finite set of $N = 30$ damage values was chosen. This sample mean damage value $\bar{D}(T)$ computed by $N = 30$ was assumed in this study to be the expected damage $E[D(T)]$. It is a sort of calibrator sample damage to which all other damage values from measurements are compared.
5. Results and discussions
The main values used for the confidence interval in Case 1 and Case 2, as well as the calibrator sample damage with its standard deviation, are presented in table 1. On the left, table 1 delivers the sample mean $\bar{D}(T)$ and sample standard deviation $\delta_D$ of Case 1 over the number of measured time-histories $N$. In the middle, it presents the damage $D(T)$ calculated by equation (4) and the standard deviation $\delta_D$ in equation (5) along with the different number of blocks $N_B$. On the right, table 1 provides the sample mean $\bar{D}(T)$ and sample standard deviation $\delta_D$ of the damage calibrator.

Table 1. The sample mean $\bar{D}(T)$ and sample standard deviation $\delta_D$ of Case 1 on the left side, $D(T)$ calculated by equation (4) and $\delta_D$ by equation (5) on the middle, and calibrator sample damage $\bar{D}(T)$ with its $\delta_D$ on the right.

| Case 1 | Case 2 | Calibrator |
|--------|--------|------------|
| $N$    | $\bar{D}(T)$ | $\delta_D$ | $N_B$ | $D(T)$ | $\delta_D$ | $N$ | $\bar{D}(T)$ | $\delta_D$ |
| 2      | 124608 | 9592       | 2     | 124737 | 7450       | 30  | 136671 | 9149       |
| 3      | 132845 | 15796      | 3     | 124573 | 10961      |     |         |            |
| 4      | 133955 | 13087      | 4     | 124554 | 12278      |     |         |            |
| 5      | 134387 | 11375      | 5     | 124559 | 12378      |     |         |            |
| 6      | 133485 | 10411      | 6     | 124222 | 12098      |     |         |            |
| 7      | 135022 | 10338      | 7     | 124216 | 9689       |     |         |            |
| 8      | 135511 | 9671       | 8     | 124387 | 15661      |     |         |            |
| 9      | 134326 | 9720       | 9     | 124290 | 12288      |     |         |            |
| 10     | 134180 | 9176       | 10    | 124365 | 14472      |     |         |            |

Note that the highest number of time-histories $N = 10$ gives the lowest standard deviation $\delta_D$ in Case 1. In Case 2, the damage $D(T)$ is almost constant (maximum difference of 0.5%) by varying $N_B$. Instead, the standard deviation $\delta_D$ in Case 2 results in a greater scatter (about 50%) than Case 1. The calibrator sample damage and its standard deviation (on the right of table 1) may be compared with the results from Case 1 (on the left of table 1) since the quantities $\bar{D}(T)$ and $\delta_D$ are obtained from the same procedure (see Section 2.2). The standard deviation $\delta_D$ presents a small reduction of about 0.3% between the calibrator and Case 1 for $N = 10$. It emphasizes the importance of the sample size $N$ with regards to the variability of damage. In other words, by increasing the number of measured time-histories $N$, the standard deviation $\delta_D$ decreases and the $\bar{D}(T)$ get closer to the “true” $E[D(T)]$. However, in practice, the standard deviation $\delta_D$ will never equal zero as well as the $\bar{D}(T)$ will never be coincident with the “true” $E[D(T)]$.

The confidence interval in Case 1 versus the number of measured time-histories $N$ is verified using the expected damage $E[D(T)]$ (approximated here with the calibrator sample damage), see figure 6(a). The confidence interval encloses $[D(T)]$ over all $N$, attesting the correctness of the proposed approach at least when applied to the stationary measured time-histories of this study. The sample damage $\bar{D}(T)$ in Case 1 approaches the $E[D(T)]$ as the number of measured time-histories increases. Furthermore, the greater is the number of time-histories, the narrower is the confidence interval of damage, which follows the same trend as that observed when the proposed approach was applied to simulated time-histories [7]. This suggests the need to use as many measured time-histories as possible to get a narrow confidence interval [7].
Figure 6. The confidence interval for Case 1 and Case 2, versus (a) the number of measured time-histories and (b) the number of blocks in one measured time-history.

Figure 6(b) compares the expected damage $E[D(T)]$ to the confidence interval in Case 2, as a function of the number of blocks $N_B$. The confidence interval containing the $E[D(T)]$ reveals to be very similar to its result using simulated time-histories [7]. This result confirms once more the correctness of the proposed confidence interval using real measurements.

Note that a structure would be designed unsafely if the expected damage $E[D(T)]$ were underestimated. The safe region is only that in which $D(T)$ in Case 1, or $D(T)$ in Case 2, is greater than $E[D(T)]$. It is then recommended to take the upper confidence limit (figure 6(a) and (b)) as the reference damage value to be used in the structure design [7].

6. Conclusions

Confidence intervals for the expected damage [7] has been derived and then verified by measured time-history records from a Mountain-bike. Several measured time-history records have been considered, from which the confidence intervals have been calculated to enclose the expected damage. The confidence intervals apply to the case in which a few time-histories or even only one are available. All measured time histories were qualified and quantified as being stationary random loadings when verified by different methods. The run test method applied here considers the fatigue damage as the observed value used to detect simultaneous changes in the frequency content, mean, and variance of the random loadings. The analysis results of one or more measured time-history records from the Mountain-bike confirm the correctness of confidence interval expressions. The confidence intervals were compared to the expected damage, which was approximated by the sample mean of several damage values (it represents a sort of calibrator sample value needed to check whether the confidence interval correctly encloses the expected damage). Results also showed that the damage computed from only one or few measured time-histories (thus ignoring its statistical variability) might lead to unsafe estimates of the expected damage. The use of the proposed approach [7] is then recommended in this case.
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