Macroscopic nonlocal entanglement generation of two hybrid massive magnon systems

Da-Wei Luo,¹ Xiao-Feng Qian,¹,∗ and Ting Yu¹,†
¹Department of Physics and Center for Quantum Science and Engineering, Stevens Institute of Technology, Hoboken, New Jersey 07030, USA
(Dated: June 12, 2020)

We investigate dynamical generation of macroscopic nonlocal entanglements between two remote massive magnon-superconducting-circuit hybrid systems. Two fiber-coupled microwave cavities are employed to serve as an interaction channel connecting two sets of macroscopic hybrid units each containing a magnon sphere and a superconducting-circuit qubit. The structure of the hybrid system allows the existence of an optimal fiber coupling strength that requests the shortest amount of time to generate a systematic maximal entanglement, which is solely dependent on the couplings of the channel with both magnon and superconducting circuit. Our theoretical results are shown to be within the scope of specific parameters that can be easily achieved with current technology. The noise effects on the implementation of systems are also treated in a general environment suggesting the robustness of our results. Our discrete-variable qubit-like entanglement theory of magnons may lead to direct applications in various quantum information tasks.

Introduction.– Entanglement is an essential feature in various quantum communication, quantum cryptography, and quantum computation schemes [1]. In particular, nonlocal entanglement between two remote quantum objects is of crucial importance to quantum communication and quantum network [2]. Unfortunately, quantum entanglement is typically fragile due to the notorious effect of environment-induced disentanglement [3]. Therefore, the search for optimal physical systems that permit robust entanglement has never ceased. Macroscopic systems are believed to be one of the promising candidates. This has triggered various investigations of entanglement in massive quantum systems including superconducting circuits [4], quantum spin-state of Caesium gas samples [5], diamonds [6], quantum nano optical-mechanical cavities [7, 8], and even in classical optical systems [9].

Recently, collective spin excitations, termed as “magnons”, in macroscopic ferromagnetic materials such as Yttrium-Iron-Garnet (YIG) spheres shed a new light on developing a robust macroscopic quantum system that possesses certain favorable features for quantum information science and technology [10–29]. Two notable attributes of the magnon systems are their long lifetimes and great tunability [25]. In addition, these systems are demonstrated to be able to coherently exchange quantum information with other major types of qubit-candidate systems including microwave photons in the strong and even ultra-strong coupling regime through a cavity [11–13, 30, 31], superconducting circuits in the microwave regime [14, 28, 29], optical photons through magneto-optical interactions [15–19, 21, 23], etc. They can also induce Kerr nonlinear effects and display bi-stability [32] for quantum operation. Cavity-magnon-polariton has even been shown to be analogous to the dynamical Hall effect [33]. These interesting findings (mostly in a single-magnon system) may be extended to multi-magnon systems where the issue of entanglement emerges naturally. In the continuous-variable regime, generations of bipartite entanglement have been proposed for two magnons both in a single cavity [34] and in two separate cavities [35, 36], and tripartite entanglement has also been studied in detail in a hybrid system composed of a magnon, a cavity, and a mechanical oscillator [26]. The focus of continuous-variable entanglement is partially due to the magnon’s oscillator nature which makes it convenient to consider Gaussian and squeezed states. However, investigations of entanglement are still absent in the discrete-variable regime which is directly relevant to quantum information and computation tasks.

In this Letter, we provide a systematic analysis of discrete-variable entanglement of magnons at the low excitation regime in two cavity-based hybrid systems. In each hybrid unit, a magnon, together with a superconducting qubit (SQ) [14, 28, 29, 37–39], reside in a microwave cavity. The two cavity systems are coupled through a microwave fiber. Two types of remote macroscopic bipartite entanglements, i.e., magnon-magnon and magnon-SQ entanglements, are analyzed. Surprisingly, there exists an optimal fiber distance that can achieve maximal magnon-magnon and magnon-SQ entanglements with the shortest amount of time. We also show, by analyzing the physical parameters for the hybrid systems and the general environmental noise effects, that our hybrid system is realistically attainable with present-day technologies.

Coupled hybrid magnonics system.– A magnon [11, 30, 40] can generally be described by the collective spin in a ferromagnetic material such as YIG, whose self-Hamiltonian is given by $g\mu_B B_z \hat{S}_z$, where $g$ is the electron $g$-factor, $\mu_B$ is the Bohr magneton, and $B_z$ is the effective magnetic field. The collective spin operator $\hat{S}_{x,y,z}$ may be expressed in terms of bosonic creation and annihilation operators, by means of a Holstein-Primakoff transformation [41], $\hat{S}_+ = m^\dagger \sqrt{2S - m^\dagger m}$, $\hat{S}_- = \sqrt{2S - m^\dagger m} m$ and $\hat{S}_z = m^\dagger m - S$, where $m$
is a bosonic annihilation operator, \( \hat{S}_\pm = \hat{S}_x \pm i \hat{S}_y \) and \( S \) is the total collective spin number. When \( 2S \gg \langle m^1 m \rangle \), one has \( \hat{S}_z = \sqrt{2Sm} \). The magnon can couple to a microwave cavity mode via the Zeeman effect \([14, 30, 31]\), resulting in an interaction Hamiltonian \( H_{\text{int}}^{m-c} \propto \hat{S}_+ a + S_- a^\dagger \), where \( a \) is the annihilation operator of the cavity mode, under the usual rotating wave approximation (RWA). Therefore, the effective coupling can be seen as exchange between the magnon and microwave photon, \( H_{\text{int}}^{m-c} = g_{\text{eff}} (m^1 a + h.c.) \).

Likewise, as an artificial atom [42], a SQ may be fabricated to couple to the cavity mode via the exchange of microwave photons. While the magnon-cavity coupling is magnetic in nature, the SQ-cavity coupling is electric [14, 43, 44]. The canonical variables in a transmon SQ are the number operator \( n \) and the phase difference \( \varphi \), satisfying the commutation relationship \( [\varphi, n] = i \), akin to the canonical position and momentum operators. The transmon’s dipole coupling to the cavity is proportional to the charge \( (\text{number operator}) \) of the SQ, and the voltage of the cavity, \( H_{\text{int}}^{q-c} \propto n (a + a^\dagger) \). By rewriting \( \varphi, n \) in terms of bosonic creation and annihilation operators and taking the RWA, the coupling of the transmon to the cavity can be cast into a similar form as the magnon-cavity coupling.

As building blocks for a large scale network of coupled magnon cavities, let us consider two microwave cavities coupled by a fiber. Inside each cavity, there is a YIG magnon sphere and a SQ. The magnon and SQ are both coupled to the cavity mode, while the direct coupling between them is negligible [14, 28]. The model is schematically shown in Fig. 1, and the system Hamiltonian is given by

\[
H_s = \sum_{i=1,2} \left( \omega_a a_i^\dagger a_i + \omega_m m_i^\dagger m_i + \omega_q b_i^\dagger b_i \\
+ g_m (m_i^\dagger a_i + h.c.) + g_q (b_i^\dagger a_i + h.c.) \right) \\
+ J (a_1^\dagger a_2 + a_1 a_2^\dagger),
\]

(1)

assuming identical cavities, where \( a_i, m_i \) and \( b_i \) are the annihilation operators for the cavity mode, magnon and SQ respectively, \( \omega_{c(m, q)} \) is the frequency of the cavity (magnon, SQ)’s frequency, \( g_{m(q)} \) is the coupling strength between the magnon (SQ) and the cavity, and \( J \) is the coupling strength of the fiber connecting the two cavities. This coupling strength may be dictated by the leakage or decay rate of the cavity mode \( \Gamma_c \), estimated as \([45]\)

\[
J \approx \sqrt{8\pi c \Gamma_c / L}, \text{ where } c \text{ is the speed of light and } L \text{ is the fiber’s length. Note that under specific conditions } [14, 28], \text{ the cavity mode can be adiabatically eliminated to create an approximate effective SQ-magnon coupling through exchanging virtual cavity photons. Without specifying the range of this parameter, the Hamiltonian will be analyzed exactly. This setup may accommodate a remote generation of entanglement, between macroscopic objects (between two magnons) as well as a hybrid entanglement between a remote SQ and magnon (e.g. between } q_1 \text{ and } m_2 \). Since the system Hamiltonian (1) conserves the total number of excitations, we will focus on the single-excitation manifold and study the discrete entanglement dynamics. This allows us to treat the magnons, SQs and cavity modes effectively as two-level systems (qubits), which renders the systems an essential component for many quantum information processing devices.

**FIG. 1.** (Color online) Schematic representation of the system under consideration, consisting of two coupled microwave cavities. Inside each cavity is a YIG magnon sphere \( (m_{1,2}) \) and a SQ \( (q_{1,2}) \), both coupled to the cavity mode. Experimentally, the direct coupling between the magnon and the SQ is negligible.

**Entanglement dynamics.** Now, we turn to the entanglement dynamics of the system. Without loss of generality, the initial state of the system is prepared as a separable state, with one excitation in the first SQ, \( |\psi(0)\rangle = |001, 000\rangle \), where the basis of the system is \( |c_1 m_1 q_1, c_2 m_2 q_2\rangle \), where \( c_i \) denotes cavity \( i \) and the SQ (magnon) in cavity \( i \) is denoted as \( q_i(m_i), i = 1, 2 \). The system dynamics can be exactly solved, and the two-party reduced density operator of the combining SQs and the magnons takes an X-state [46], giving an analytical expression for the entanglement, measured by the concurrence [47]. In the case of a resonant configuration \( \omega_c = \omega_q = \omega_m \equiv \omega \), the self-Hamiltonian of the system only gives an overall global phase in the single excitation manifold, thus the choice of \( \omega \) does not affect the system dynamics. Then the magnon-magnon entanglement is obtained as

\[
C_{m_1, m_2} = 2 \frac{g_m^2 g_q^2 \left[ \sqrt{G_0} \cos \frac{\sqrt{G_0}}{2} \sin \frac{\delta t}{2} - J \cos \frac{\delta t}{2} \sin \frac{\sqrt{G_0}}{2} \right] J \sin \frac{\delta t}{2} \sin \frac{\sqrt{G_0}}{2} + \sqrt{G_0} \left( \cos \frac{\sqrt{G_0}}{2} \cos \frac{\delta t}{2} - 1 \right) }{G_0 \left( g_m^2 + g_q^2 \right)^2},
\]

(2)
where $G_0 = 4 \left( g_m^2 + g_q^2 \right) + J^2$. We plotted the entanglement dynamics for varying fiber coupling $J$ in Fig. 2. Surprisingly, stronger fiber coupling does not necessarily lead to faster entanglement generation. Indeed, there exists an optimal fiber coupling $J$ such that the time to generate peak entanglement is the shortest. From Eq. (2), it can be shown that the optimal time achieving peak entanglement is $t = 2n\pi/\sqrt{G_0}$, where $n$ is a positive integer, with a set of corresponding coupling strength $J$. The shortest time corresponds to setting $n = 1$, $t_{\text{opt}} = 2\pi/(3J_{\text{opt}})$, where the optimal fiber coupling $J_{\text{opt}} = \sqrt{(g_m^2 + g_q^2)/2}$.

Denoting the ratio between the SQ-cavity and magnon-cavity coupling rate $r = g_q/g_m$,

$$C_{m_1,m_2} = \frac{3\sqrt{3}r^2}{8 (r^2 + 1)^2},$$  \hspace{1cm} (3)

indicated by the red peaks in fig. 2. For varying magnon(SQ)-cavity coupling rate $g_m(q)$, the peak concurrence is maximized when $g_m = g_q = J$, at $C_{m_1,m_2}\max = 3\sqrt{3}/8$. This configuration corresponds to an equally-spaced energy spectrum of the system Hamiltonian, with a spacing equals to $J$, with a two-fold degeneracy at 0. A similar feature can also be observed for the SQ-SQ entanglement and the hybrid entanglement between the SQ and the remote magnon with shifted phases, representing a dynamical entanglement distribution [48, 49]. For the SQ-SQ entanglement, the peak entanglement is given by

$$\sqrt{(\eta - 1)(\eta + 3)}/8 (r^2 + 1)^2, \text{ where } \eta = \sqrt{8r^4 + 1}.$$  

The peak SQ-SQ entanglement thus approaches 1 when $r_q \rightarrow +\infty$ due to the fact that the initial excitation is on $q_1$, and the existence of the magnon in the cavity introduces a competition. It’s also worth pointing out that the system under consideration Eq. (1) does not actually create Gaussian continuous-variable entanglement due to the lack of squeezing terms, but the exchange of excitations is enough to generate a discrete-variable entanglement, as shown here.

For the hybrid entanglement between the remote SQ and magnons, although the system Hamiltonian is symmetric for SQ and magnon in each cavity, the choice of the initial state with one excitation at SQ 1 breaks the symmetry, consequently, the entanglement of $q_1-m_2$ deviates from that of $m_1-q_2$. It can be readily shown that $C_{q_1,m_2} = C_{q_1,q_2}/r_q$ and $C_{m_1,q_2} = r_q C_{m_1,m_2}$. The peak concurrence is plotted in Fig. 3 as a function of $r_q$. Interestingly, we observe that, while the SQ-SQ concurrence asymptotically increases with $r_q$, for the other types of entanglement there exists a maximum value for the peak concurrence at different coupling ratio $r_q$. All curves coincide at $r_q = 1$, which is expected since when $g_m = g_q$, the system dynamics does not distinguish between the SQ and magnon. In the limit of $r_q \rightarrow \infty$, the SQ-SQ peak entanglement approaches 1 while all other goes to 0. It should be notes that, while the time to generate peak concurrence is generically given by $t = 2n\pi/\sqrt{G_0}$, the corresponding fiber coupling $J$ is different and may require the solution of transcendental equations and do not process an analytical form. $C_{m_1,q_2}$ reaches its maximum $27/32$ at $r_q = \sqrt{3}$ while $C_{q_1,m_2}$ reaches $\approx 0.6922$ at $r_q \approx 0.6896$.

Next, we consider a more generic case with non-resonant setups. In light of physical implementation of our system, we are primarily interested in the ranges of parameters that have been experimentally reported. We choose an experimentally accessible configuration [14] with the magnon-cavity coupling rate $g_m/2\pi = 21$ MHz, the SQ-cavity coupling rate $g_q/2\pi = 117$ MHz, and let the frequency of the SQ and magnon match. The de-
tuning δ = ω_c − ω_q is set to 183 × 2π MHz. With a typical cavity decay rate Γ_c/2π = 1.8 MHz, we estimate [45] the fiber coupling strength J ≈ 92.3 MHz, where the length of the fiber is 10 m. The entanglement between the magnons and the hybrid SQ-magnon entanglement is plotted in Fig. 4. It may be observed that a quasi-periodic oscillation exists. In Fig. 4 (a) we can see that due to the ∼ 5× difference between the magnon-cavity coupling and the SQ-cavity coupling, the generated magnon-magnon entanglement is less pronounced than the hybrid entanglement g_{1-m_2} in this configuration. In Fig. 4 (b) we show that by tuning the SQ coupling rate to be closer to the magnon coupling rate at g_q/2π = 30 MHz, the entanglement generation between the magnons may be greatly enhanced. Alternatively, we may also tune the system setup such that the magnon is strongly coupled to the cavity at around g_m/2π = 120 MHz [11], which aligns better with the SQ’s coupling, then, C_{m_1-m_2} can have a significant improvement up to 0.6.

While the total system Hamiltonian Eq. (1) exhibits several desirable features for coherence generation and transfer, the cavity system inevitably suffers dissipation. A mathematical treatment of the effects of environmental noise on entanglement generation may be studied by an open system approach [50]. For the distant entanglement generation case, two cavities may be affected by two separate local noises modeled by two multi-mode bosonic baths at zero temperature. More precisely, the Hamiltonians of the bath and interaction are given by $H_{\text{bath}} = \sum_j \sum_k \omega_k \hat{b}^\dagger_{j,k} \hat{b}_{j,k}$, and

$$H_{\text{int}} = \sum_j \sum_k g_k L_j \hat{b}^\dagger_{j,k} + g_q L_j^{\dagger} \hat{b}_{j,k},$$

where $H_{\text{int}}$ describes the interaction between the cavity system and bath, $\omega_k$ is the frequency of the $k$-th bath mode, $g_k$ is the corresponding coupling strength, $L_j$ ($j = 1, 2$) are the cavity-bath coupling operators and $\hat{b}_{j,k}$ denotes the $k$-th mode annihilation operator of bath $j$. For the leaky cavities with the coupling operators $L_j = \lambda \hat{a}_j$, we have numerically simulated the open system dynamics under generic colored noises by using a non-Markovian Schrödinger equation [51–54]. The reduced density operator for the open system is reliably recovered by averaging a large number of quantum trajectories generated by the stochastic Schrödinger equation encoded with a typical bath memory function $\alpha(t, s) = \sum_k g_k^2 e^{-\gamma(t-s)} = \exp(-\gamma|t-s|)/2$. The temporal entanglement is shown to be robust against environmental noises as displayed in Fig. 4(b) with realistic parameters $\gamma = 0.7$ and $\lambda/2\pi = 1.8$ MHz. When the cavity dissipation becomes stronger, one expects that an external quantum control must be utilized [55, 56].

Discussion and conclusion.— We study the feasibility of a coupled hybrid cavity system, where each microwave cavity contains one magnon and one SQ. It is shown that quantum entanglement can be reliably generated between remote (approximately 10 meters apart in the typical parameter regime) macroscopic magnons, and hybrid entanglement between remote SQ and magnon is also observed. Remarkably, there exists an optimal fiber coupling strength to achieve peak entanglement in the least amount of time, and stronger fiber couplings do not necessarily mean a faster entanglement generation. Notably, the peak entanglement is dependent only on the ratio between the SQ-cavity and magnon-cavity coupling strength, $r_q = g_q/g_m$. By encoding information with discrete variables rather than continuous variables, this hybrid system has provided a new framework to exploit the compatibility of SQs with a microwave cavity and the longevity, operability and potential scalability of magnon SQs. Since quantum entanglement is not a direct physical observable, quantum state tomography [57] may be applied on the qubit or magnon to measure and detect the entanglement generation in our system. Moreover, the effect of environmental noise is studied with a general colored noise approach. The proposed system is shown to be rather robust against the noise impact for a wide range of configurations.
This work is supported by NSF PHY-0925174.