$d_L(z)$ and BAO in the emergent gravity and the dark universe

Ding-fang Zeng

1Theoretical Physics Division, College of Applied Sciences, Beijing University of Technology

We illustrate that AMOND cosmology following from E. Verlinde’s emergent gravity idea which contains only constant dark energy and baryonic matters governed by linear inverse gravitation forces at and beyond galaxy scales fit with the luminosity distance v.s. redshift relationship, i.e. $d_L(z)$ of type Ia supernovae equally well as the standard $\Lambda$CDM cosmology does. But in a rather broad and reasonable parameter space, AMOND gives too strong baryon acoustic oscillation, i.e. BAO signals on the matter power spectrum contradicting with observations from various galaxy survey and counting experiments.

PACS numbers: 04.20.Cv, 98.65Dx, 95.35+d, 95.36+x

In a last month’s work [11], basing on insights from string theory, black hole physics and quantum information theory, Eric. Verlinde argues that the dark gravity effects observed in galaxies and clusters conventionally attributed to dark matters could be accounted for by the modified newtonian dynamics (MOND hereafter) following from the emergence feature of gravitation and space-time itself. Although Verlinde does not quotient concretly method for modifying the newton gravitation theory, so there is big arbitrarinesses in its prediction possibilities. His idea attracts much attention [2,3] as well as criticisms [10] due to its potential of kicking the longly non-measured dark matter contents out of our knowledge menus using first principle of quantum gravitation theories.

Historical research works [11–13] indicate that if at and beyond galaxy scales the square-inverse feature of newton gravitation is enhanced appropriately, e.g. the most simple way of enhancing to linear-inverse laws, then the galaxy rotation curves could be flattened properly just as observation requires. Verlinde argues that this square-inverse to linear-inverse transition should occur for general gravitation systems as long as their characteristic acceleration goes below the magic value,

$$a_{\text{acc}}^\text{magic} = \frac{1}{6}H_0 c$$

where $H_0$ is today’s hubble parameter. Since in a matter dominated universe $a \propto t^{2}, \, \bar{a} \propto -a^{-2}$, we expect that the universe as a gravitation system should also experience this square-inverse to linear inverse transition as its scale factor grows beyond

$$a_{\text{magic}}^{\text{eff}} \approx \left(\frac{a_0^2}{6}\right)^{\frac{1}{2}}$$

This reasoning immediately brings us two questions urgently. The first is, if the universe consists only of normal matters controlled by this modified gravitation theory and constant dark energy (AMOND hereafter), then could its late time expansion features still be similar to that observed in the type Ia supernovae’s distance-redshift relationship? The second is, could this AMOND model reproduce the $\Lambda$ plus Cold Dark Matter ($\Lambda$CDM here after) model’s beautiful prediction of large scale structure’s evolution and growth or not? At first glance, we may think that this two questions may not be answered properly before concrete MOND formulation thus well-established new gravitational field equation following from the emergent idea is spelled out. However, we will show in the following that this is not the case.

![FIG. 1: $o$ is arbitrary observer, while $m$ an arbitrary co-moving test particle in an isotropic and homogeneous expanding universe. The energy conserving condition obeyed by $m$ and the mass/energy contents inside the spherical region $M$ centered on $o$ simply has the form $\dot{m}a^2r^2 - \frac{GMm}{ar^2} = k$, with $k$ a constant determined by initial conditions.](image)

According to the standard textbook of S. Weinberg, in an isotropic and homogeneous universe

$$ds^2 = -dt^2 + a^2(t)\left[ \frac{dr^2}{1-k r^2} + r^2 d\theta^2 + r^2 sin^2 \theta d\phi^2 \right]$$

simple newton mechanic and energy conservation laws are sufficient in determining dynamics of the scale fac-
tor $a(t)$. Referring FIG.2 and captions there, according to which the conventional Friedman equation could be derived as follows

$$\frac{1}{2}m\dot{a}^2r^2 - \frac{4\pi G}{3} \frac{(\rho_m + \rho_\Lambda)a^3}{ar} = \ddot{k}$$  \hspace{1cm} (4)$$

$$\frac{\dot{a}^2}{a^2} - \frac{k}{a^2} = \frac{8\pi G}{3} (\rho_m + \rho_\Lambda)$$  \hspace{1cm} (5)$$

while the energy conservation requires

$$\rho_m a^3 = \text{const.1, } \rho_\Lambda = \text{const.2}$$  \hspace{1cm} (6)$$

Equations (4) and (5) constitute the full set of dynamic constraints for $a(t)$.

Now, let us apply the above method to the MOND theory of E. Verlinde in which the matter-matter attractive force has linearly inverse law at scales beyond galaxies but the matter-dark energy force still satisfies the square inverse law,

$$V_{\text{num}} = -\frac{1}{\epsilon} GM_m m \propto -\nabla V = -\frac{GM_m m}{(ar)^{1+\epsilon}}$$  \hspace{1cm} (7)$$

$$V_{\text{m}} = -\frac{GM_m \Lambda m}{ar} \propto -\nabla V = -\frac{GM_m \Lambda m}{a^2 r^2}$$  \hspace{1cm} (8)$$

$$\epsilon = \frac{1}{2} + \frac{1}{2} \tanh \left( \frac{a_0}{a - a_{\text{magic}}} - \frac{a_0}{-a_{\text{magic}} - a} \right) \theta(a_{\text{magic}} - a)$$  \hspace{1cm} (9)$$

We introduce a simple $\epsilon(a)$ function here to implement the goal of changing the early time square-inverse law to the later time linear inverse law smoothly, where $\theta$ is the usual heaviside step function featured by $\theta(x < 0) = 0$, $\theta(0 < x) = 1$.

Substituting the above two potential formulas into the conservation equation (4), what we get will become

$$\frac{\dot{a}^2}{a^2} - \frac{k}{a^2} = \frac{8\pi G}{3} \left[ \rho_m \frac{a r}{\epsilon} + \rho_\Lambda \right]$$  \hspace{1cm} (10)$$

In the case of $k = 0$ and the matter dominated era, the function $a(t)$ could be explicitly solved out

$$a_{\text{mond}}(t) = \left( \frac{8\pi G r/\epsilon}{3} \right) \frac{2 + \epsilon}{2} \left[ \frac{r}{r^*} \right]_{r \to 0}$$  \hspace{1cm} (11)$$

Considering the fact that $\dot{a} \propto t^0 \to \text{const.}$, while in the standard Einstein/Newton cosmologies $\dot{a} \propto t^{-\frac{1}{2}} \to 0$, it is very surprising that the strengthened gravitation force does not lead to strengthened deceleration and recollapsing evolutions of the universe. Of course, when the constant dark energy is included, there existing recollapses or not will be relevant with the relative weight of $\rho_{m0}$, $\rho_{\Lambda 0}$ and $k$. Any way, this fact suggests us that this simple AMOND model may have radically different late time expansion features relative to the standard ΛCDM model.

Using equation (10) and the standard definition in conventional supernovae data analysis, we can derive out the luminosity distance v.s. redshift relation in the AMOND cosmology by the simple $\chi^2$-minimization method.

$$d_i(z) = \frac{a_0}{a} \frac{c}{|\Omega_k|^{\frac{1}{2}}} \sinh \left[ \frac{\Omega_k}{2} \int_0^z \frac{d\zeta}{H(\zeta)/H_0} \right]$$  \hspace{1cm} (12)$$

$$H^2(\zeta) = H_0^2 \left[ \frac{\Omega_m}{a_0^3} \left( \frac{a}{a_0} \right)^{1-\epsilon} + \Omega_\Lambda - \frac{\Omega_k a_0^2}{a^2} \right]$$  \hspace{1cm} (13)$$

where we defined $\frac{a}{a_0} \equiv \zeta + 1$, $\sinh[x] = \sin[x]$, $x$, $\sinh[x]$ as $\Omega_k > 0$, $= 0$ and $< 0$ respectively. We are very lucky that the annoying factors appearing in (10) could be simply absorbed into the definition of $\Omega_m$, so that we can now safely set $\epsilon = 0$ here. Now using observational data complied in the SCP Union2.1 [16] and minimizations of the following $\chi^2$-function

$$\chi^2 = \sum_i \frac{|m_{\text{th}}(z_i, \Omega_m, H_0, \cdots) - m_{\text{ex}}(z_i)|^2}{\sigma_i^2}$$  \hspace{1cm} (14)$$

we find that the three parameter $\{\Omega_m, \Omega_\Lambda, H_0\}$ AMOND model fit with the observational data equally well with the standard ΛCDM model. But with radically different best fitting parameters

$$\text{AMOND : } \Omega_m = 0.81, \Omega_\Lambda = 0.58, H_0 = \frac{70.1 \text{ km}}{\text{s-mpc}}$$  \hspace{1cm} (15)$$

$$\text{ΛCDM : } \Omega_m = 0.29, \Omega_\Lambda = 0.76, H_0 = \frac{70.2 \text{ km}}{\text{s-mpc}}$$  \hspace{1cm} (16)$$
The former has $\chi^2 = 562.313$, while the latter has $\chi^2 = 562.40$. Just as we pointed out under equation (11) that the strengthened gravitational force in this model between matters does not make the acceleration of the universe difficult. Instead they make such accelerations more easier so that more less dark energy is need in accomplishing the observed acceleration!

The most big difficult a cosmological model without non-baryonic dark matter may encounter is that, it may lead to too strong baryonic acoustic oscillation (BAO) signal on the power spectrum of matter distributions such as those observed typically in 2dFGRS [22] and SDSS [17, 18] galaxy survey and counting experiments. Recalling that in the standard cosmological perturbation theory [19] galaxy survey and counting experiments. Recalling that in the standard cosmological perturbation theory [19]

$$d^2 = a^2(\eta)[1 + 2\Phi]\delta_{ij}dx^idx^j$$ (17)

$$k^2\Phi + \frac{3\dot{a}}{a}\left(\dot{\Phi} - \Psi\frac{\dot{a}}{a}\right) = 4\pi Ga^2\left(\rho_v, \delta_v + \rho_0, \Theta_0 + 4\rho_0, N_0\right)$$ (18)

$$k^2(\Phi + \Psi) = -32\pi Ga^2\left(\rho_0, \Theta_2 + \rho_0, N_2\right)$$ (19)

The last two equations follow from Fourier transformations of the linearised Einstein equation. Quantities on their right hand side are just the 1st and 2nd multipole expansions of the corresponding particle’s statistical distribution

photon : $f_0 = \left[\frac{1}{\sqrt{1 + 4\pi x^2/p^2}} - 1\right]^{-1}$ (20)

nutrino : $f_\nu = \left[\frac{1}{\sqrt{1 + 4\pi x^2/p^2}} + 1\right]^{-1}$ (21)

dark matter : $f_{BD} = \cdots$; (22)

baryon matter : $f_{BB} = \cdots$ (23)

$$\Theta_\eta(\bar{x}, t) \equiv -\frac{i}{2} \int_{-1}^{1} d\cos \theta \Theta(\bar{x}, \cos \theta, t) P_\eta(\cos \theta)$$ (24)

$N_0$, $N_2$, $\delta_0 \equiv \delta_{D0}$, $\delta_{B} \equiv \delta_{D0}$ similarly defined

Unlike $\Psi$ and $\Phi$, all these multipole’s evolution is controlled directly by the Boltzmann instead of Einstein equation

$$\frac{df}{dt} = C[f(\bar{p})]$$ (25)

The concrete form $C$ depends on the particle type and their mutual interactions. At first two levels, the component equation relevant with the baryon acoustic oscillation reads

$$\dot{\Theta}_0 + k\Theta_1 = -\dot{\Phi}, \tau(\eta) \equiv -n_e\sigma T a$$ (26)

$$\dot{\Theta}_1 - \frac{k}{3}\Theta_0 = \frac{k}{3}\Psi + \dot{\tau}(\Theta_1 - \frac{i\nu_a}{3})$$ (27)

$$v_b = -3i\Theta_1 + \frac{R}{T}(\dot{\nu}_a + \frac{\dot{a}}{a}\nu_a + ik\Psi)$$ (28)

Under the so called tightly-coupling limit, Hu and Sugiyama show [20] that this equation array has simple oscillation solution

$$\Theta_0(\eta) + \Phi(\eta) = [\Theta_0(\eta) + \Phi(\eta)] \cos(kr_s)$$ (29)

$$+ \frac{k}{\sqrt{\beta}} \int_{0}^{\eta} dy \left[\Phi(y) - \Psi(y)\right] \sin\{k[r_s(\eta) - r_s(y)]\}$$

$$r_s \equiv \int_{0}^{\eta} dy_c(y), c_s \equiv (3 + 3R)^{-\frac{2}{3}}, R \equiv \frac{3\rho_0}{4\rho_\gamma}$$ (30)

This is just the baryon acoustic oscillation. It originates from the sound mode oscillation of the relativistic plasma in the early universe. At redshift $z \approx 1000$, the recombination occurs so that the big bang plasma becomes a neutral gas and the oscillation stops propagating any further. But periodic spatial inhomogeneity feature it brings continues to exist and evolves to the present time. In the standard CDM model, the baryonic and non-baryonic dark matters coexist even before the recombination, with the former to latter ratio equates about $\frac{1}{5}$. Since dark matters do not participate in the sound wave oscillation, the strength of baryonic acoustic oscillation signals on the power spectrum of total matters observed in the late time universe is very small. In cosmological models such as MOND where non-baryonic dark matters do not exist at all, to explain the smallness of of this signal is the main challenge.

![FIG. 3: In standard ΛCDM model (green line), the BAO signal manifests only as small wiggles on the matter power spectrum. But in the MOND model (red line), this signal manifests as strong oscillations of the power spectrum line. In the left panel, we replace the dark matter of ΛCDM with baryonic matters in MOND, while in the right panel, we replace it with dark energy. In both panels, $H_0$ is set as 70km/(s·Mpc).](image-url)
brings us suppressions of the BAO signal as required by observations. Our logic is, although we do not know what the full gravitational field equation grows like in the framework of emergent idea, in an exactly isotropic and homogeneous universe its key features are captured by (9)–(10), while its linear perturbation, as long as being second order partial differential equations, would then not deviate from the Einstein perturbations too much.

Under this logic, we integrate the whole system of Einstein-Boltzmann differential equations by the standard code of CAMB [21]. The results is displayed in FIG. 3 and 4 explicitly. From FIG. 3 we easily see that, replacing the conventional cold dark matter with either baryonic matter of MOND or dark energy of cosmological constant both bring us too strong BAO signals on the matter power spectrums measured observationally. Nevertheless, in the former case, ΛMOND and ΛCDM has approximately the same first peak positions on the power spectrum. This is expectable because it could be proved analytically basing on the tight-coupling approximation (29) whose validity has no relevance with assumptions of the ΛMOND model.

To obtain a suppressed BAO signal, we try in FIG. 4 using Ωm, ΩΛ parameters following from best fittings of the dℓ(z) relation of type Ia supernovae in the previous section, where spatial curvatures contribute remarkably heavier to the energy contents of the universe. However, even when we let the magic scale factor be a tuneable parameter, we do not obtain the required results. This means that, new mechanisms must be find to suppress the BAO signal in this ΛMOND cosmology to make it a competing model of ΛCDM.

Conclusion: we derive out dynamic equations controlling the evolution of scale factors in a simple ΛMOND cosmology which contains only constant dark energy and baryonic matters governed by linear inverse gravitation forces at and beyond galaxy scales. We find that the model fit with observational data type Ia supernovae’s luminosity distance v.s. redshift relationship equally well with the standard ΛCDM model does. However, since no dark matter is assumed, the model predicts too strong baryonic acoustic oscillation signals on the matter power spectrum than the standard ΛCDM does. Nevertheless, ΛMOND has the same position of first BAO peak as ΛCDM does if we replace dark matters in the latter with baryonic matters in the former. So a reasonable mechanism to suppress the strength of BAO signals may be the most urgent ingredient of ΛMOND in its road of growing into competing models of ΛCDM.

Acknowledgements

This work is supported by Beijing Municipal Natural Science Foundation, Grant No. Z2006015201001 and partly by the Open Project Program of State Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, China.

[1] E. Verlinde, “Emergent Gravity and the Dark Universe”, arXiv: 1611.02269
[2] R. S. Kunkolienkar, K. Banerjee, “Towards a dS/MERA correspondence”, arXiv: 1611.08581
[3] L. Liu, T. Prokopec, “Gravitational microlensing in Verlinde’s emergent gravity”, arXiv: 1612.00861
[4] M. Brouwer et al, “First test of Verlinde’s theory of Emergent Gravity using Weak Gravitational Lensing measurements”, to appear in MNRAS, arXiv: 1612.03034
[5] L. Iorio, “Are we close to put the anomalous perihelion precessions from Verlinde’s emergent gravity to the test?”, arXiv: 1612.03783
[6] P. Bueno et al, “Entanglement equilibrium for higher order gravity”, arXiv: 1612.04374
[7] A. Diez-Tejedor, A. Gonzalez-Morales, G. Niz, “Verlinde’s emergent gravity vs MOND and the case of dwarf spheroidal”, arXiv: 1612.06282
[8] S. Ettori et al, “Dark matter distribution in X-ray luminous galaxy clusters with Emergent Gravity”, arXiv: 1612.07288
[9] E. Barrientos, S. Mendoza, “MOND as the weak field limit of an extended metric theory of gravity with torsion”, arXiv: 1612.07970
[10] M. Milgrom, R. Sanders, “Perspective on MOND emergence from Verlinde’s ‘emergent gravity’ and its recent test by weak lensing”, arXiv: 1612.09582
[11] M. Milgrom, “A Modification of the Newtonian dynamics: Implications for galaxies”, Astrophys. J. 270 (1983)
[12] J. Bekenstein, M. Milgrom, “Does the missing mass problem signal the breakdown of Newtonian gravity?”, *ApJ.* 286 (1984) 7, [DOI: 10.1086/162570](https://doi.org/10.1086/162570)

[13] M. Milgrom, “MOND laws of galactic dynamics”, *Mon. Not. Roy. Astron. Soc.* 437 (2014), 2531, [arXiv: 1212.2568](https://arxiv.org/abs/1212.2568)

[14] M. Milgrom, “MOND theory”, *Can. J. Phys.* 93 (2015) no.2, 107, [arXiv: 1404.7661](https://arxiv.org/abs/1404.7661)

[15] R. Sanders, “A historical perspective on modified Newtonian dynamics”, *Can. J. Phys.* 93 (2015) no.2, 126, [arXiv: 1404.0531](https://arxiv.org/abs/1404.0531)

[16] N. Suzuki et al (SCP Collaboration), “The Hubble Space Telescope Cluster Supernova Survey: V. Improving the Dark Energy Constraints Above z=1 and Building an Early-Type-Hosted Supernova Sample”, *ApJ.* 746 (2012) 85, [arXiv:astro-ph/1105.3470](https://arxiv.org/abs/1105.3470)

[17] D. J. Eisenstein et al, “Detection of the Baryon Acoustic Peak in the Large-Scale Correlation Function of SDSS Luminous Red Galaxies”, *ApJ.* 633 (2005) 560, [arXiv: astro-ph/0501171](https://arxiv.org/abs/astro-ph/0501171)

[18] W. J. Percival et al, “The shape of the SDSS DR5 galaxy power spectrum”, *ApJ.* 657 (2007) 645, [arXiv: astro-ph/0608636](https://arxiv.org/abs/astro-ph/0608636)

[19] S. Dodelson “Modern Cosmology”, Academic Press, Version 2003, [SLAC BOOKS Database](https://slac.stanford.edu/books.html)

[20] W. Hu, N. Sugiyama, “Small Scale Cosmological Perturbations: An Analytic Approach”, *ApJ.* 471 (1996) 542, [arXiv:astro-ph/9510117](https://arxiv.org/abs/astro-ph/9510117)

[21] Lewis A., Challinor A., Lasenby A., *ApJ* 538 (2000) 473, [arXiv:astro-ph/9911177](https://arxiv.org/abs/astro-ph/9911177)

[22] S. Cole et al, 2dFGRS collaboration, “The 2dF Galaxy Redshift Survey: Power-spectrum analysis of the final dataset and cosmological implications”, *Mon. Not. Roy. Astron. Soc.* 362 (2005) 505, [arXiv: astro-ph/0501174](https://arxiv.org/abs/astro-ph/0501174)