Wake potential in collisional anisotropic quark-gluon plasma

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Within the framework of Boltzmann transport equation with a Bhatnagar-Gross-Krook (BGK) collisional kernel, we study the wake potential induced by fast partons traveling through the high-temperature QCD plasma which is anisotropic in momentum-space. We calculate the dielectric response function of a collisional anisotropic quark-gluon plasma (AQGP) for small $\xi$ (anisotropic parameter) limit. Using this, the wake potential for various combinations of the anisotropy parameter ($\xi$) and the collision rate ($\nu$) is evaluated both for parallel and perpendicular directions of motion of the fast parton. It is seen that the inclusion of the collision modifies the wake potential and the amount as well as the nature of the potential depends on the combinations of $\xi$ and $\nu$.

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I. INTRODUCTION

The aim of ultrarelativistic heavy-ion collision experiments at BNL RHIC and at CERN LHC is to understand the properties of very hot and dense partonic matter expected to be formed in these collisions. One of the objectives of such experiments is the identification and investigation of a phase transition from hadronic matter to what is known as the quark-gluon plasma (QGP). The hard jets created in hard parton scattering in the initial stages of heavy-ion collision will pass through the hot and dense matter and lose energy by collisional and radiative processes. This phenomenon is known as jet quenching where the high $p_T$ hadrons production has been found to be strongly suppressed \cite{1}. Other very important experimental evidence for the jet energy loss is the dihadron azimuthal correlation. The jet induced hadron pair distribution at RHIC shows a double peak structure in the away side \cite{2,3} for the intermediate $p_T$ particle. However, it is unclear how the parton-medium interaction affects the distribution. The possible explanation of this observation is that the coupling of jets to a strongly interacting medium may modify the angular distribution \cite{4–12}. In this scenario the partonic jets traveling through the QGP leads to formation of Mach cones \cite{6–8}, Cerenkov radiation \cite{9,10} and wakes \cite{11,13–16}, which may be observables as collective excitation of the medium.

Apart from the jet quenching many other signals of this nascent phase of matter have been proposed. These include electromagnetic probes (photon and dilepton) \cite{17}, $J/\psi$ suppression \cite{18}, collective flow \cite{19–21} and so on. However, the main difficulty in studying these probes lies in the determination of the initial conditions, such as, the isotropization/thermalization time vis-a-vis the initial temperature. In this sense, many properties of the QGP are poorly understood. The most pertinent question is whether the matter produced in relativistic heavy-ion collisions is in thermal equilibrium or not. The collective flow patterns observed at the RHIC provide strong evidence for rapid thermalization at less than 1 fm/c after the collision \cite{22}. On the other hand, using second-order transport coefficients consistent \cite{22} with conformal symmetry it has been found that the thermalization time has sizable uncertainties due to poor knowledge of the proper initial conditions. Plasma instabilities have been suggested to a play a major role in the isotropization process \cite{24}. Shortly after the collision, the rapid expansion of the matter along the beam direction causes faster cooling in the longitudinal direction than the transverse direction, leading to $\langle p_L^2 \rangle << \langle p_T^2 \rangle$ \cite{25} i.e., the phase space distributions of plasma particles become anisotropic in momentum space. At some later time, the system returns to an isotropic state due to the effect of the parton interactions which overcomes the plasma expansion rate. Therefore, it has been suggested to look for some observables which are sensitive to the early time after the collision. The effect of pre-equilibrium momentum anisotropy on various observables has been studied extensively in the last few years. The collective modes in an AQGP has been investigated in Refs. \cite{26,27}. The energy loss of parton in an AQGP has been studied in Refs. \cite{28,29}. The effect of anisotropy on the photon and dilepton yield have been investigated rigorously in Refs. \cite{31–35}.

When a high energy jet propagates through plasma, apart from the energy loss, it produces a wake in the plasma. The current and charge density wakes induced by a jet propagating in an isotropic and homogeneous QGP have been investigated by Ruppert and Muller \cite{11}. In the high temperature regime, the result shows the wake in both the
induced charge and current density due to the screening color charge cloud, but no Mach cones appear. On the other hand, in the quantum liquid scenario, if the jet travels supersonically, the wake exhibits an oscillatory behavior and Mach cones can appear. However, using the HTL approximation, the wake in the induced charge density shows an oscillatory nature in the backward direction at large parton velocity \[\beta p\] in case of isotropic plasma. The color response wake in viscous media has been investigated in Ref. [13]. In a previous paper [14], we studied the wake induced by the jet propagating through the anisotropic plasma. When the parton moves along the anisotropy direction with velocity less than the average plasmon speed \(v > v_p\). However, we do not find any oscillatory nature of the wake potential, when the parton moves in the transverse plane with respect to the anisotropy axis. In all the above works the effect of collision has been neglected. The inclusion of collision in the Boltzmann equation and its subsequent effect on the wake potential has been investigated in Ref. [14] assuming isotropic plasma. It is found that in case of collisional isotropic plasma, the wake behavior is less pronounced than in isotropic plasma [14]. Now we investigate the wake behavior of the plasma by taking into account collisions in the AQGP. The effect of the collision can easily be taken into account by approximating the collision term in the Boltzmann equation with the BGK description [36]. In this approach, the plasma by taking into account collisions in the AQGP. The effect of the collision can easily be taken into account by approximating the collision term in the Boltzmann equation with the BGK description [36]. In this approach, the dielectric function has been calculated in isotropic plasma [37] and the collective modes of anisotropic collisional plasma have also been investigated when the wave vector is parallel to the anisotropy direction. By using the BGK collisional kernel, we calculate the dielectric response function of a collisional AQGP for small anisotropy limit.

The organization of the paper is as follows. In Sec. II, we derive the dielectric response function for collisional anisotropic plasma. In Sec. III we use the dielectric function to determine the wake potential in collisional AQGP and discuss the numerical results. Finally, we conclude in Sec. IV.

II. SELF-ENERGY IN COLLISIONAL ANISOTROPIC PLASMA

To obtain the gluon polarization tensor of a collisional QGP, we start from the Boltzmann transport equation:

\[ V_\mu \partial_X f_{g}^i(p, X) + g F_{\mu \nu}^{a \nu} F_{\nu \mu}^{a \nu} (X) \right|_{x} \times f_{g}^i(p) = \mathcal{C}_{a}^i(p, X), \]

where \( V^\mu = (1, p/|p|), \theta^i = \theta^a = 1 \) and \( \theta^i = -1 \). \( F^{\mu \nu} = \partial^\mu A^\nu - \partial^\nu A^\mu - ig[A^\mu, A^\nu] \) is the gluon field strength tensor with gauge field \( A^\mu = A^a_{\mu} T^a \) or \( A^\mu = A^a_{\mu} \tau^a \), where \( \tau^a, T^a \) with \( a = 1, \ldots, N_c^2 - 1 \) are the \( SU(N_c) \) group generators in the fundamental and adjoint representations with \( \text{Tr}[\tau^a, \tau^b] = \frac{1}{2} \delta^{ab} \). \( \text{Tr}[T^a, T^b] = N_c \delta^{ab} \) and \( g \) is the coupling constant. The scalar functions \( \delta f_{g}^i(p, X) \) are found by the projections: \[ \delta f_{g}^{q/\bar{q}}(p, X) = 2\text{Tr}[\tau_a n^{q/\bar{q}}(p, X)], \]

\[ \delta f_{g}^{\bar{q}}(p, X) = \frac{1}{N_c} \text{Tr}[T_a n^{\bar{q}}(p, X)], \]

with the colorless background fields \( n^{q/\bar{q}}(p) = f_{g}^{q/\bar{q}}(p) I \) and \( n^{q}(p) = f_{g}^{q}(p) I \). \( I \) and \( \bar{I} \) are unit matrices in the fundamental and adjoint representation, respectively. The scalar functions \( f_{g}^i(p) \) are also found by projections:

\[ f_{g}^{q}(p) = \frac{1}{N_c} \text{Tr}[n_{g}^{q}(p, X)], \]

\[ f_{g}^{\bar{q}}(p) = \frac{1}{N_c^2 - 1} \text{Tr}[n_{g}^{\bar{q}}(p, X)]. \]

In this paper, we will concentrate on the soft scale, i.e., \( k \sim gT << T \). The gauge field fluctuation is \( A \sim \sqrt{gT} \) and the derivatives are of the order \( \partial_X \sim gT \). We can neglect the higher order coupling constant and correspondingly \( D_X \rightarrow \partial_X \) and \( F^{\mu \nu} = \partial^\mu A^\nu - \partial^\nu A^\mu \). The BGK collisional term \( \mathcal{C}_{a}^i \) can be represented as

\[ \mathcal{C}_{a}^i(p, X) = -\nu \left[ f_{a}^i(p, X) - \frac{N_{eq}^i(X)}{N_{eq}^i} \right] f_{eq}^i(|p|) \]

with \( f_{a}^i(p, X) = f_{a}^i(p) + \delta f_{a}^i(p, X) \). This BGK collision term corresponds to an improvement of the relaxation time approximation for the collision term of the Boltzmann equation [37]. The collision rate \( \nu \) is independent of velocity and particle species. We define the particle number as,

\[ N_{eq}^i(X) = \int_{p} f_{a}^i(p, X), \quad N_{eq}^i = \int_{p} f_{eq}^i(|p|) = \int_{p} f_{a}^i(p), \]
with \( \int_p = \int \frac{d^3p}{(2\pi)^3} \). Thus the transport equation reads as:

\[
V\partial_X \delta f^i_a(p, X) + g^a V_p^I_F^{\mu\nu}(X)\partial^\rho(p) f^i(p) = -\nu \left[ f^i(p) + \delta f^i_a(p, X) - \left(1 + \frac{\int_p f^i_a(p, X)}{\int_p f^i(p)}\right) \right]
\]

(6)

After Fourier transformation of \( \delta f^i(p, K) \) and \( F^{\mu\nu}(K) \), the above equation can be written as:

\[
\delta f^i(p, K) = \frac{-i g^a \theta V_p F^{\mu\nu}(K) \partial^\rho f^i(p) + i\nu (f^i_{eq}(p) - f^i(p)) + i\nu f^i_{eq}(p)(\int_p \delta f^i(p', K))/N_{eq}}{\omega - k.v + i\nu}
\]

(7)

\[
\delta f^i_0(p, K) = \left[-ig^a \theta V_p F^{\mu\nu}(K) \partial^\rho f^i(p) + i\nu (f^i_{eq}(p) - f^i(p))\right] D^{-1}(K, \nu)
\]

(8)

with \( D^{-1}(K, \nu) = \omega - k.v + i\nu \), \( \zeta(K) = \int_p \delta f^i_0(p, K) \) and \( \rho(K, \nu) = \frac{i\nu}{N_{eq}} \int_p f^i_{eq}(p) D^{-1}(K, \nu) \).

The induced current for each particle species \( i \) is

\[
J^\mu_{ind\ a}(K) = g \int_p V^\mu \delta f^i_a(p, K)
\]

\[
= g^2 \left[ \int_p V^\mu \partial^\beta f^i(p) \mathcal{M}_{\gamma \beta}(K, V) D^{-1}(K, \nu) A^\gamma_a(K) + g^a \theta \right]
\]

(9)

\[
= g^2 \left[ \int_p V^\mu f^i_{eq}(p)|D^{-1}(K, \nu)| \right] \int_p \partial^\beta f^i(p') \mathcal{M}_{\gamma \beta}(K, V') D^{-1}(K, \nu) A^\gamma_a(K)
\]

where \( \mathcal{M}_{\gamma \beta}(K, V) = g_{\gamma \beta}(\omega - k.v) - V, K, S^i(K, \nu) = \theta, \int_p V^\mu[f^i(p) - f^i_{eq}(p)]|D^{-1}(K, \nu)| \) and \( W_i(K, \nu) = 1 - \frac{\omega}{N_{eq}} \int_p f^i_{eq}(p)|D^{-1}(K, \nu) \). The total induced current is given by

\[
J^\mu_{ind\ a}(K) = 2 N_c g^a \delta^\mu_\gamma \mathcal{J}^\gamma_a(K) + N_f J^\mu_{ind\ a}(K) + J^\mu_{ind\ a}(K)
\]

(10)

Assuming the same distribution functions for the quarks and anti-quarks, the total induced current can be written as

\[
J^\mu_{ind\ a}(K) = g^2 \left[ \int_p V^\mu \partial^\beta f^i(p) \mathcal{M}_{\gamma \beta}(K, V) D^{-1}(K, \nu) A^\gamma_a(K) + 2 N_c g^a \theta S^\gamma(K, \nu)
\]

\[
+ \frac{g^2}{4\pi} \int_{k^\nu} V^\mu D^{-1}(K, \nu) \mathcal{M}_{\gamma \beta}(K, V') D^{-1}(K, \nu) W^{-1}(K, \nu) A^\gamma_a(K)
\]

\[
+ 2iN_c g^2 \nu^2 \int \frac{d\Omega}{4\pi} V^\mu D^{-1}(K, \nu) S^\gamma(K, \nu) W^{-1}(K, \nu).
\]

(11)

where \( W(K, \nu) = 1 - i\nu \int \frac{d\Omega}{4\pi} D^{-1}(K, \nu) \). From this expression of the total induced current the self energy can be obtained via

\[
\Pi_{ab}^{\mu\nu}(K) = \frac{\delta J^\mu_{ind\ a}(K)}{\delta A^\nu_b(K)}
\]

(12)

which gives

\[
\Pi_{ab}^{\mu\nu}(K) = \delta_{ab} g^2 \left[ \int p V^\mu \partial^\beta f^i(p) \mathcal{M}_{\gamma \beta}(K, V) D^{-1}(K, \nu) + \delta_{ab} \theta g^2 \nu \int \frac{d\Omega}{4\pi} V^\mu D^{-1}(K, \nu)
\]

\[
\times \int_{k^\nu} \partial^\beta f^i(p') \mathcal{M}_{\gamma \beta}(K, V') D^{-1}(K, \nu)
\]

\[
W^{-1}(K, \nu)
\]

(13)
which is a symmetric tensor, i.e., $\Pi^{\mu\nu}(K) = \Pi^{\nu\mu}(K)$ and transverse, i.e., $K_\mu \Pi^{\mu\nu}(K) = 0$. Since $\Pi^{\mu\nu}$ is gauge invariant one can evaluate it in any gauge. In the following we shall evaluate it in the temporal axial gauge where $A_0 = 0$.

The anisotropic phase-space distribution function can be obtained from any isotropic distribution function by rescaling only in one direction in momentum space by changing the argument $f(p) = f_{iso}(p^2 + \xi(p,\hat{n})^2)$. \hspace{1cm} (14)

where $\xi$ is an adjustable parameter which represents the strength of the anisotropy and the direction of anisotropy is determined by $\hat{n}$ (assumed to be in the beam direction). $f_{iso}$ is an arbitrary isotropic distribution function. Using this distribution function, the gluon polarization tensor of a collisional QGP within the BGK approach, is given by $\hat{n}$

$$
\Pi^{\mu\nu} (K) = m_D^2 \int \frac{d\Omega}{4\pi} \frac{v^\mu + \xi(v,\hat{n})n^\mu}{1 + \xi(v,\hat{n})^2} \left[ \delta^{ij} (\omega - k_i v^i) + v^i k^j \right] D^{-1}(K,v,\nu) + i\nu m_D^2 \int \frac{d\Omega'}{4\pi} (v')^i D^{-1}(K,v',\nu) \int \frac{d\Omega}{4\pi} \frac{v^\mu + \xi(v,\hat{n})n^\mu}{1 + \xi(v,\hat{n})^2} \left[ \delta^{ij} (\omega - k_i v^i) + v^i k^j \right] D^{-1}(K,v,\nu) W^{-1}(K,\nu) \hspace{1cm} (15)
$$

where $m_D$ is the isotropic Debye mass, represented by

$$
m_D^2 = - \frac{g^2}{2\pi^2} \int_0^\infty dp d^4p \frac{df_{iso}(p^2)}{dp}. \hspace{1cm} (16)
$$

In the limit $\nu \rightarrow 0$, the polarization tensor reduces to the self-energy in the anisotropic medium. Using the proper tensor basis $\Pi^{ij}$, one can decompose the self-energy into four structure functions as

$$
\Pi^{ij} = \alpha A^{ij} + \beta B^{ij} + \gamma C^{ij} + \delta D^{ij} \hspace{1cm} (17)
$$

where

$$
A^{ij} = \delta^{ij} - \frac{k_i k_j}{k^2}, \quad B^{ij} = \frac{k_i k_j}{k^2}, \quad C^{ij} = \hat{n}^i \hat{n}^j, \quad D^{ij} = k^i \hat{n}^j + k^j \hat{n}^i \hspace{1cm} (18)
$$

with $\hat{n}^i = A^{ij} n_j$ which obeys $\hat{n} \cdot k = 0$ and $n^2 = 1$. The structure functions $\alpha$, $\beta$, $\gamma$ and $\delta$ are determined by the following contractions:

$$
k_i \Pi^{ij} k_j = k^2 \beta, \quad \hat{n} \cdot \Pi^{ij} k_j = \hat{n} \cdot k^2 \delta, \quad \hat{n} \cdot \Pi^{ij} \hat{n}^j = \hat{n}^2 (\alpha + \gamma), \quad Tr \Pi^{ij} = 2\alpha + \beta + \gamma. \hspace{1cm} (19)
$$

The structure functions depend on $\omega$, $k$, anisotropy parameter($\xi$), collisional rate($\nu$) and the angle ($\eta$) between the anisotropy vector and the momentum. Now the dielectric tensor and the self-energy are related by the following relation (see Appendix A for derivation):

$$
\epsilon^{ij} = \delta^{ij} - \frac{\Pi^{ij}}{\omega^2} \hspace{1cm} (20)
$$

Using the relation $\epsilon(k,\omega) = \frac{k^i k^j \omega}{k^2} \omega \ln \left( \frac{k^2}{\omega^2} \right)$ \hspace{1cm} (21), we can calculate the dielectric function ($\epsilon(k,\omega)$) analytically for small $\xi$ limit. To linear order in $\xi$ we have from Eqs.(2) and (7)

$$
\epsilon(k,\omega) = 1 - \frac{m_D^2}{k^2} \frac{1}{k^2} \ln \left( \frac{z^2 + \omega^2}{z^2} \right) \left[ \Sigma + \xi \left( \frac{1}{6} (1 + 3 \cos 2\eta) + \Sigma \left( \cos 2\eta - \frac{z^2}{2} (1 + 3 \cos 2\eta) \right) \right) \right] \hspace{1cm} (21)
$$

where $z = \frac{\omega + \xi}{k}$ and

$$
\Sigma = -1 + \frac{z^2}{2} \ln \left( \frac{z + 1}{z - 1} \right). \hspace{1cm} (22)
$$
The wake potential in configuration space due to the motion of a charge parton can be calculated with the help of the external color charge density associated with the test charge particle as \( \rho^{\text{ext}} \), where \( \rho^{\text{ext}} = \frac{m_D^2}{4k^2G} \left[ 4k^2 + \nu^2(\ln^2 R + \Theta^2) - 2k(\omega \ln R + 2\nu\Theta) \right] \) and 
\[
\begin{align*}
\text{Re} \epsilon(\omega, k) &= 1 + \frac{m_D^2}{4k^2G} \left[ 4k^2 + \nu^2(\ln^2 R + \Theta^2) - 2k(\omega \ln R + 2\nu\Theta) \right] \\
&+ \frac{m_D^2\xi}{24k^4G} \left[ -4k^4 + 2k^3\nu\Theta + 12k^2(\nu^2 - \omega^2) + 6k^2\cos 2\eta \left( 2k^2 - 6\omega^2 - k(2\omega \ln R + 3\nu\Theta) \right) \\
&+ \nu^2(6 + \ln^2 R + \Theta^2) \right] + 3(1 + 3\cos 2\eta) \left( \nu^2(\ln^2 R + \Theta^2)(\nu^2 - 3\omega^2) \\
&+ 2k(\omega^3 \ln R + 4\nu\omega^2\Theta - \nu^2\omega \ln R - 2\nu^3\Theta) \right) \right], \\
\text{Im} \epsilon(\omega, k) &= \frac{m_D^2\omega}{4k^2G} \left[ 2k\Theta - \nu(\ln^2 R + \Theta^2) \right] + \frac{m_D^2\xi}{24k^4G} \left[ 6\nu k^2 \cos 2\eta(2k\Theta - \nu(\ln^2 R + \Theta^2)) - (1 + 3\cos 2\eta) \right] \\
&\times \left( 2k^3\nu \ln R + 24k^2\omega\nu + 3\nu(3\nu^2 - \omega^2)(\ln^2 R + \Theta^2) + 6k\omega(-2\omega \ln R + \Theta(\omega^2 - 5\nu^2)) \right),
\end{align*}
\]

(23)

and

\[ R = \sqrt{\frac{(\omega + k^2 +\nu^2)}{(\omega \nu - k^2 +\nu^2)}, \Theta = \arccos \sqrt{\frac{\omega^2 - k^2 +\nu^2}{\omega^2 - k^2 +\nu^2 + 4k^2\nu^2}}, \text{ and } G = (k - \frac{\nu\Theta}{2})^2 + \frac{\nu^2}{4} \ln^2 R. \]

It is important to note that \( \text{Re} \epsilon \) gives the dispersion relation of the collective modes and \( \text{Im} \epsilon \) gives the damping or attenuation of these modes.

### III. WAKE POTENTIAL

In this section we derive the wake potential for collisional anisotropic \((\nu \neq 0, \xi \neq 0)\) plasma and compare it with collisionless anisotropic plasma \((\nu = 0, \xi \neq 0)\), collisional isotropic plasma \((\nu \neq 0, \xi = 0)\) as well as collisionless isotropic plasma \((\nu = 0, \xi = 0)\). The wake potential induced by the fast parton can be obtained from the Poisson equation as [40]:

\[ \Phi^a(k, \omega) = \frac{\rho^{\text{ext}}(k, \omega)}{k^2 \epsilon(k, \omega)} \]

(25)

where \( \rho^{\text{ext}}(k, \omega) \) is external color charge density.

Now we study the wake behavior of QGP interacting with a charge particle, \( Q^a \) moving with constant velocity \( \mathbf{v} \). The external color charge density associated with the test charge particle can be written as [40],

\[ \rho^{\text{ext}} = 2\pi Q^a \delta(\omega - \mathbf{k} \cdot \mathbf{v}). \]

(26)

The wake potential in configuration space due to the motion of a charge parton can be calculated with the help of Eqs. (25) and (26) and it reads as

\[ \Phi^a(r, t) = 2\pi Q^a \int \frac{d^3k}{(2\pi)^3} \int \frac{d\omega}{2\pi} \exp(i(k \cdot r - \omega t)) \frac{1}{k^2 \epsilon(k, \omega)} \delta(\omega - \mathbf{k} \cdot \mathbf{v}). \]

(27)

We evaluate the wake potential for the two special cases, (a) along the parallel direction of motion of the parton, i.e., \( r \parallel \mathbf{v} \) and (b) perpendicular to the direction of motion of the parton, i.e., \( r \perp \mathbf{v} \).

#### A. Wake potential along the parallel direction

The wake potential for the parallel case becomes

\[ \Phi^a_{||}(r, t) = Q^a m_D \int \frac{d^3k}{(2\pi)^3} \left[ \cos \gamma \frac{\text{Re} \epsilon(k, \omega)}{\Delta} + \sin \gamma \frac{\text{Im} \epsilon(k, \omega)}{\Delta} \right] \bigg|_{\omega = k \cdot \mathbf{v}} \]

(28)
where $\Gamma = m_D k (r - vt) \cos \theta$ and $\Delta = (\text{Re} \epsilon(k, \omega))^2 + (\text{Im} \epsilon(k, \omega))^2$. Note that $\cos \eta = \cos \lambda \cos \theta + \sin \lambda \sin \theta \cos \phi$, where $\lambda$ is the angle between $r$ and $z$ and $\theta$, $\phi$ are the polar and azimuthal angles corresponding to $k$.

Here we present the numerical evaluation of the wake potential in parallel direction ($r \parallel v$) for two values of $\lambda$ i.e. $\lambda = 0$ and $\lambda = \pi/2$, where $\lambda = 0(\pi/2)$ corresponds to $r \parallel \hat{n}$ ($r \perp \hat{n}$). Numerical results of the wake potential along the parallel direction ($r \parallel v$) of motion of the parton are shown in Figs (1) and (2) with parton velocities $v = 0.55$ and $v = 0.99$, respectively. In these figures, the scaled parameter $\Phi_a^0$ is given by $2\pi \frac{\epsilon_m}{m_D}$ $\Phi_a^0$. The left (right) panels in Figs. (1) and (2) represent the case for $\lambda = 0 (\pi/2)$ i.e. $r \parallel \hat{n}$ ($r \perp \hat{n}$).

In Fig. (1a) we presents the wake potential when $\lambda = 0$, $v = 0.55$ and $(r - vt) < 0$ (backward direction). It is seen that for $\xi = 0$, $\nu = 0$ (collisionless isotropic plasma), the potential becomes Lennard-Jones type. For moderate values of $\xi$ with $\nu = 0$, the potential still remains as Lennard-Jones type, however the depth of the negative minimum decreases. If the value of $\xi$ is increased, the potential changes to modified Coulomb-like potential. This is the unique feature that we observe in this work. It is also seen that for collisional ($\nu = 0.2$) isotropic plasma, the depth of the negative minima increases compared to the isotropic case with $\nu = 0$. We also observe that when both the parameters are nonzero, the potential remains Lennard-Jones type and the depth of the negative minima decreases in these cases.
We thus concluded that because of the variation of the depth of the potential in the cases considered here there will be substantial effect on the observables such as heavy quarkonium suppression. Next we consider the behavior of the wake potential when \( \lambda = 0, v = 0.99 \) and \((r - vt) < 0\). This is shown in Fig. (2a). For isotropic collisionless plasma \((\xi = 0, \nu = 0)\) the potential is of Lennard-Jones type with oscillatory behavior. When \( \xi = 0.4 \) and \( \nu = 0 \), the form of the potential does not change. However, it becomes more oscillatory compared to the previous case for the reason explained in [16]. It is important to note that the depth of the negative minima remains unchanged in these cases. For collisional isotropic plasma \((\nu \neq 0, \xi = 0)\), the depth of the potential increases as compared to the case of collisionless isotropic plasma. The form of the potential remains unchanged with oscillatory behavior. If we increase the collision frequency, the depth of the potential becomes highest and most importantly the oscillatory behavior is smeared out. The combination \((\xi = 0.4, \nu = 0.2)\) leads to more oscillatory potential as compared to the case when \( \xi = 0 \) and \( \nu = 0.2 \). The overall observations in this case are as follows: The potential remains Lennard-Jones type in all the cases considered here. The depth of the negative minima varies depending upon the combinations of \( \nu \) and \( \xi \). For large value of \( \nu \), if we increase \( \xi \) the oscillatory nature of the potential is washed away.

We now turn our attention to the case when \( \lambda = \pi/2 \) i.e. \( r \perp \hat{n} \) and \( v = 0.55 \). The result is displayed in Fig. (1b), various combinations of \( \xi \) and \( \nu \) have been considered. In all the cases the potential is Lennard-Jones type and the depth of the negative minima is larger than the isotropic case if one of the parameters is nonzero or both are nonzero. Thus in this scenario the quarkonium state will be strongly bound and therefore, the dissociation temperature required to break the state will be higher as compared to the isotropic case. Moreover, it will certainly affect the conical flow and other observables. The wake potential corresponding to the previous case with larger velocity is shown in the Fig. (2b). As before, for collisionless isotropic plasma, the potential is of Lennard-Jones type with oscillatory behavior. With nonzero \( \xi \), the oscillatory behavior vanishes contrary to the case when \( r \parallel \hat{n} \) for a collisionless isotropic plasma. For collisional \((\nu = 0.2)\) isotropic plasma, the oscillatory behavior is observed again. If we increase \( \nu \), the oscillatory behavior again vanishes. The vanishing and reappearing of the oscillatory behavior for certain combinations of \( \xi \) and \( \nu \) is because of the fact that there is subtle competition between collisional isotropic plasma and collisionless anisotropic plasma. Finally we note that in the forward direction in all the above cases, the wake potential is a modified Coulomb-like potential.

We have also examined the case for large \( \xi \). For \( v = 0.55 \) and \( \lambda = 0 \), the potential becomes coulomb-like. It is also seen that increasing the value of \( \xi \) the effect of collision becomes unimportant. However, for faster speed the potential remains Lennard-Jones type with no oscillatory behavior in case of collisional plasma. For \( \lambda = \pi/2 \), the potentials remain of Lennard-Jones type for both values of \( v \) considered here.
Next we consider the wake potential for the perpendicular case which reads as:

\[ \Phi_a^\perp (r, t) = Q^a m_D \int \frac{d^3k}{(2\pi)^3} \left[ \cos \Gamma' \frac{\text{Re} \epsilon(k, \omega)}{\Delta} + \sin \Gamma' \frac{\text{Im} \epsilon(k, \omega)}{\Delta} \right] \bigg|_{\omega = k \cdot v} \]

where \( \Gamma' = m_D k (r \cos \theta - v t \sin \theta \cos \phi) \).

We numerically evaluate Eq. (29) and the corresponding results (scaled wake potential) of the wake potential along the perpendicular direction of motion of the parton \( (r \perp \hat{n}) \) are shown in Figs. (3) and (4) for \( v = 0.55 \) and \( v = 0.99 \) respectively. We first discuss the case when \( \lambda = 0 \) \( (r \parallel \hat{n}) \), and \( v = 0.55 \). The corresponding results are shown in the left panel of Fig (3). It is seen that the wake potential is forward-backward symmetric. Let us first concentrate on the case of collisional isotropic plasma. For \( \nu = 0 \), the potential is of Lennard-Jones type as in the collisional isotropic plasma. However, the depth of the negative minima increases. If we increase \( \nu = 0.8 \), the potential turns into a modified Coulomb-like potential. However, for \( \xi \neq 0 \) (anisotropic plasma), the wake potential remains modified Coulomb-like potential irrespective of the value of the collisional frequency(\( \nu \)). On the other hand, for \( \lambda = \pi/2 \) \( (r \perp \hat{n}) \), the wake potential remains Lennard-Jones type up to \( \nu = 0.2 \) and \( \xi = 0.4 \) with the depth of the negative minima increasing with \( \xi \) and \( \nu \) (see right panel of Fig. (3)). For large values of \( \nu (0.8) \), the wake potential turns into Coulomb-like potential.

For the large velocity i.e. for \( v = 0.99 \), we find no oscillatory behavior for collisionless isotropic plasma for \( \lambda = 0 \) \( (r \parallel \hat{n}) \). (see left panel of Fig. (4)). Thus the wake potential created by the parton moving in transverse plane has different behavior than when it moves in the parallel direction. This should be reflected in the observables which have origin in the moving parton(such as di-hadron correlations). The results for \( \lambda = \pi/2 \) \( (r \perp \hat{n}) \) and \( v = 0.99 \) are shown in the right panel of Fig. (4). The potential remain of Lennard-Jones type for various combinations of \( \xi \) and \( \nu \) as long as collisional frequency is small. When \( \nu = 0.8 \), the potential turns into modified Coulomb-like potential. It is also seen that the depth of the negative minima decreases irrespective of the value of \( \xi \).

**IV. SUMMARY**

In this work we have investigated the behavior of the wake potential induced by a fast parton propagating through the collisional quark-gluon plasma which is anisotropic in momentum space. To simplify the analysis, we have calculated the dielectric response function in small \( \xi \) limit including the effect of BGK collisional kernel. We have
presented the wake potentials for both parallel ($\mathbf{r} \parallel \mathbf{v}$) and perpendicular ($\mathbf{r} \perp \mathbf{v}$) directions of motion of the fast parton with two different parton velocities. We see that when the parton moves in the parallel direction with $v = 0.55$, the potentials become of Lennard-Jones type in the backward region and Coulomb-like in the forward region for $\lambda = 0 (\mathbf{r} \parallel \hat{\mathbf{n}})$ and $\lambda = \pi/2 (\mathbf{r} \perp \hat{\mathbf{n}})$. However, the depth of the negative minimum varies with $\xi$ and $\nu$. For $v = 0.99$ the potential shows less oscillatory than collisionless isotropic and anisotropic QGP. We have also evaluated the wake potentials when $\mathbf{r} \perp \mathbf{v}$. In this case when $v = 0.55$, $\mathbf{r} \parallel \hat{\mathbf{n}} (\lambda = 0)$ and $\mathbf{r} \perp \hat{\mathbf{n}} (\lambda = \pi/2)$ it is observed that the potentials remain of Lennard-Jones type for $\nu$ up to 0.2 and for various values of $\xi$ (within the small $\xi$ limit). However, if we increase $\nu$ the potential becomes Coulomb-like irrespective of the values of $\xi$. For $v = 0.99$, $\mathbf{r} \parallel \hat{\mathbf{n}} (\lambda = 0)$ and $\mathbf{r} \perp \hat{\mathbf{n}} (\lambda = \pi/2)$ more or less similar behavior of the wake potential has been observed as in the case for $v = 0.55$. However, with any combinations of $\xi$ and $\nu$ we do not find any oscillatory behavior in the wake potential. Observations of the oscillatory and non-oscillatory behaviors of the wake potential for certain combinations of $\xi$ and $\nu$ imply that, as long as $\xi$ remains below a certain critical value, the effect of collision becomes dominant. But when $\xi$ is greater than this critical value, the anisotropy effect takes over, irrespective of the values of $\nu$, and the wake structure becomes similar to the case of collisionless AQGP. We also note that the anisotropic effect on the wake potential is speed dependent, and it has rich structure compared to the case of collisionless isotropic plasma. We observe that for $v = 0.99$ the oscillatory effect is more pronounced for $\xi \neq 0$ and $\nu = 0$. However, it decreases with the inclusion of collision. But for lower speeds the effect of anisotropy on the wake potential is minimal if the collision is included. We shall end by mentioning some of the phenomenological implications of the present work. In the present calculation, we see that the wake structure for collisional AQGP is less pronounced compared to the case of purely isotropic plasma, as well as collisionless AQGP. This will surely influence the conical flow and wave excitation in the plasma. Second, as the depth of the potential changes depending upon the values of $\xi$ and $\nu$, it can change the repulsive and attractive parts of the interaction potential of a fast parton in the plasma. In particular, in our work, we have shown that the depth of the negative minima becomes deeper for collisional anisotropic QGP as well as collisional isotropic QGP when $\mathbf{r} \parallel \hat{\mathbf{n}}$. This will lead to a stronger binding of diquarks propagating through the plasma requiring larger values of the dissociation temperature to separate the heavy quark bound state in such a scenario. Thus, the nuclear modification factors of charm and bottom mesons should be reevaluated, applying the concept of the present work. Moreover, it will also be interesting to explore whether such changes in the wake potential can influence the dihadron correlations.

Appendix A: Derivation of Eq. (20)

In this section we derive Eq. (20). For this, we note that the spatial component of induced current density is given by (see Eq. (11)),

$$J_{\text{ind}}^j(K) = g^2 \int_{\mathbf{p}} V^i \delta_{(p)} f(\mathbf{p}) M_{ijkl}(K,V) D^{-1}(K,v,\nu) A^i(K) + 2 N_c g \nu S^9(K,\nu)$$

$$+ g^2 (i\nu) \int \frac{d\Omega}{4\pi} V^i D^{-1}(K,v,\nu) \int_{\mathbf{p}'} \delta_{p'} f(\mathbf{p}') M_{ijkl}(K,V') D^{-1}(K,v',\nu) W^{-1}(K,\nu) A^i(K)$$

$$+ 2 N_c g^2 (i\nu) \int \frac{d\Omega}{4\pi} V^i D^{-1}(K,v,\nu) S^9(K,\nu) W(K,\nu). \tag{A1}$$

The spatial component of the polarization tensor can be written from Eq. (13) as:

$$\Pi^j(K) = g^2 \int_{\mathbf{p}} V^i \delta_{(p)} f(\mathbf{p}) M_{ijkl}(K,V) D^{-1}(K,v,\nu)$$

$$+ g^2 (i\nu) \int \frac{d\Omega}{4\pi} V^i D^{-1}(K,v,\nu) \int_{\mathbf{p}'} \delta_{p'} f(\mathbf{p}') M_{ijkl}(K,V') D^{-1}(K,v',\nu) W^{-1}(K,\nu) \tag{A2}$$

We also note that the thermal conductivity and the dielectric tensor is related by (37)

$$\epsilon^{ij}(K) = \delta^{ij} + \frac{i}{\omega} \sigma^{ij}(K) \tag{A3}$$
where \( \sigma^{ij}(K) = \frac{\delta f_{ij}(K)}{\delta E_i(K)} \) Using Eq. (A1) we derive

\[
\sigma^{ij}(K) = \frac{i g^2}{2} \int \frac{d^4p}{4\pi} V^i(p) \partial_jf(p)M_{jl}(K,V)D^{-1}(K,\nu) + \frac{i \nu}{4\pi} \int d^4\Omega V^i(K,\nu) \int d^4p' \partial_jf(p')M_{jl}(K,V')D^{-1}(K',\nu')W^{-1}(K,\nu)
\]  

(A4)

Here we have use the following relations in temporal axial gauge:

\[
E_i = F_0i = \partial_0A_i - \partial_iA_0 = -i\omega A_i.
\]  

(A5)

Using the above relations it is straightforward to show that

\[
\epsilon^{ij}(K) = \delta^{ij} - \frac{\Pi^{ij}(K)}{\omega^2}.
\]  

(A6)

The dispersion relation of the collective modes in temporal axial gauge can be written as [26]:

\[
\det\left[\left(k^2 - \omega^2\right)\delta^{ij} - k^i k^j + \Pi^{ij}(K)\right] = 0
\]  

(A7)

or equivalently [39]:

\[
\det\left[k^2\delta^{ij} - k^i k^j - \omega^2 \epsilon^{ij}(K)\right] = 0
\]  

(A8)

Thus, the modes can be obtained either by evaluating \( \Pi^{ij}(K) \) or \( \epsilon^{ij}(K) \).

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