Extended SUSY SU(5) predicting type-III seesaw testable at LHC

Ram Lal Awasthi, Sandhya Choubey
Harish-Chandra Research Institute, Chhatnag Road, Jhunsi, Allahabad 211019, India

(Dated: May 11, 2014)

We propose an extension of the SUSY SU(5) which predicts LHC testable type-III seesaw. The supersymmetric SU(5) GUT model is extended by adding a 24-plet matter superfield along with a pair of 10_H and 10_H superfields. The 24-plet carries a triplet and a singlet fermion multiplet of SU(2)_L, which leads to type I+III seesaw. The additional 10_H (and 10_H) multiplets help in achieving gauge coupling unification while keeping the triplet fermion mass in the TeV range, making them accessible at LHC. We study the phenomenology of this model in detail. Large lepton flavor violation predicted in this model puts severe constraints on the Yukawa couplings of the triplet fermion. We show that this smoothes the possibility of observing the contribution of the heavy fermions in neutrino oscillation experiments. The presence of the additional 10_H and 10_H in this model not only gives gauge coupling unification, it also leads to very large lepton flavor violation.

I. INTRODUCTION

The standard model (SM) of elementary particles based on the gauge group SU(3)_C ⊗ SU(2)_L ⊗ U(1)_Y is now widely accepted as the low energy effective theory of a more complete model of particles. Amongst the strongest experimental reasons demanding the extension of the SM are the observation of masses and mixing of neutrinos, presence of excess of baryons over anti-baryons in the universe, and the existence of dark matter and dark energy of the universe, none of which can be partially or wholly explained within the realm of the minimal SM. Amongst the theoretical reasons which beg its extension are, the Higgs mass and Higgs vacuum stability problem, as well as the theoretical prejudice that all gauge couplings will eventually unify at some high scale. The extensions of the SM that predict the gauge coupling unification are the so-called Grand Unified Theories (GUTs). The first and simplest of such GUT models proposed by Glashow and Georgi is based on the SU(5) gauge group [1].

The minimal non-SUSY SU(5) [1] model contains the SM fermions in three generations of \(\mathbf{5} = (3, 1, 1/3) \oplus (1, 2, -1/2) \oplus (d^C, L)\) and \(\mathbf{10} = (3, 1, -2/3) \oplus (3, 2, 1/6) \oplus (1, 1, 1) \oplus (u^C, Q, e^C)\), where \(L\) and \(Q\) are the SM SU(2)_L lepton and quark doublets and \(d^C, u^C\) and \(e^C\) are the SM SU(2)_L singlets. The three dimensional numeric tuples in the above description correspond to SU(3)_C, SU(2)_L and U(1)_Y groups, respectively. The SM Higgs doublet is embedded in \(5_H \equiv (3, 1, -1/3) \oplus (1, 2, 1/2) \equiv (T, H)\), where \(H\) is the SM SU(2)_L Higgs doublet while \(T\) is a colored particle multiplet, absent in the SM. The SM singlet in the adjoint representation 24_H \(\equiv (8, 1, 0) \oplus (1, 3, 0) \oplus (3, 2, -5/6) \oplus (3, 2, 5/6) \oplus (1, 1, 0) \oplus (\Sigma_8, \Sigma_3, \Sigma_{(3, 2)}, \Sigma_{(\bar{3}, 2)}, \Sigma_0)\), triggers the breaking of SU(5) into SM, at the unification scale. The gauge bosons, as usual, are contained in the adjoint representation, 24_G. While 12 of these gauge bosons belong to the SM \((g, W, Z\) and \(\gamma)\), the rest, namely \(X(Q_{EM} = 4/3)\) and \(Y(Q_{EM} = 1/3)\), acquire masses of the order of the unification scale. This minimal SU(5) GUT is plagued with a variety of issues. Firstly, the gauge couplings of the SM fail to unify at a unique point in this set-up. Secondly, it cannot explain non-zero neutrino masses and should be extended with additional multiplets to achieve it. Thirdly, there is nothing to remedy the gauge hierarchy problem in this set-up. It also fails to provide a dark matter candidate, however we will not address this last issue in this paper.

The lacuna regarding the gauge coupling unification and the hierarchy problem can be easily remedied by invoking supersymmetry (SUSY) [2]. This allows to achieve the minimal supersymmetric standard model (MSSM) driven gauge coupling unification around \(10^{16.2}\) GeV, and also stabilizes the SM Higgs mass. The particle content of the the SUSY-SU(5) is extended to include the superpartners and an additional \(\mathbf{5}_H\) representation to avoid anomaly generation and also to generate masses for up-type quarks.

Gauge coupling unification can also be achieved in extended versions of non-SUSY SU(5) with additional multiplets. This is particularly relevant for models which extend the particle content of SU(5) by adding multiplets to explain non-zero neutrino masses. The additional multiplets contain the heavy particle which drives the seesaw mechanism for the generation of the tiny neutrino masses [3–7]. The corrections to the running of the gauge couplings coming from the presence of these additional multiplets can in some cases bring about their unification without having to implement SUSY. The extensions of SU(5) in this class of models that have recently gained popularity are the ones which predict LHC testable seesaw. One class of models extend the fermionic content of the minimal SU(5) by adding the adjoint representation 24 \(\equiv (\rho_8, \rho_3, \rho_{(3, 2)}, \rho_{(\bar{3}, 2)}, \rho_0)\) [8][10]. The heavy SM singlet and SU(2) triplet give rise to type I + type III seesaw which can explain the neutrino data. Demand for gauge coupling unification in these models predicts a triplet fermion within the reach
of the LHC, thus opening up the possibility of testing seesaw at the LHC in the context of a GUT model that connects it to proton decay. The second class of extensions encompass the models which extend the minimal SU(5) Higgs content with an additional symmetric representation $15_{H} = (1, 3, 1) \oplus (3, 2, 1/6) \oplus (6, 1, -2/3) \equiv (\Delta_{3}, \Delta_{(3,2)}, \Delta_{6})$ \cite{11}. The presence of the SU(2) triplet scalar in this multiplet allows for the type II seesaw mechanism to generate neutrino masses and mixing consistent with data. Here the triplet scalar is predicted to be in the LHC testable range in order to be consistent with a unified coupling at the GUT scale, hence in turn connecting it to bounds from proton decay.

These low energy features of such extensions disappear in the supersymmetrized versions of these models. Since the MSSM drives the gauge coupling unification once SUSY is invoked, it leaves almost no space for additional multiplets. Hence, addition of such multiplets significantly below the GUT scale spoils unification and as a result the particles in these multiplets are forced to have masses close to the GUT scale. The SUSY SU(5) model extended with the fermionic adjoint representation 24 giving type I + type III seesaw was studied in \cite{12, 13}, while the phenomenology of the SUSY SU(5) model with the symmetric $15_{H}$ Higgs extension giving type-II seesaw was discussed in \cite{14}. The seesaw scales in all these studies are usually at very high and hence their testability unforeseeable in any experiment. Also, addition of larger representations around TeV scale could lead to divergence of the gauge couplings before any unification. In this paper, we propose a model which not only predicts TeV scale type I and type III seesaw driven by particles that can be produced and probed at the LHC, but also contains a charged scalar singlet, with mass at the LHC testable scale. This is accomplished by adding a matter chiral superfield in the adjoint representation 24 and a pair of Higgs chiral superfields in the antisymmetric $10_{H}$ and $\overline{10}_{H}$ representations, to the minimal SUSY SU(5) field content. The fermions in 24 lead to type I+III seesaw mechanism, while the presence of $10_{H}$ and $\overline{10}_{H}$ allows us to modulate the running of the gauge coupling such the we get unification in the SUSY SU(5) GUT model with masses of the seesaw fermions in the TeV range. We study the phenomenology of this model in detail.

The paper is organized as follows. We begin by describing our model in Section II. In Section III we discuss gauge coupling unification and proton decay and the corresponding constraints on the particle mass spectrum. In Section IV we discuss the neutrino mass generation through the type I + type III seesaw mechanism. Constraints and predictions from lepton flavor violation and neutrinoless double beta decay are given in Sections V\textsuperscript{V} and VI\textsuperscript{V} respectively. Possibility of leptogenesis in this model is discussed in Section VII. We end in Section VIII\textsuperscript{V} with our conclusions.

II. MODEL

We propose an extension of the SUSY SU(5) where the field content with three generations of matter superfields 5 and 10, gauge superfield $24_{G}$, and scalar superfields $5_{H}$, $\overline{5}_{H}$ and $24_{H}$ of the minimal model, is augmented with one generation of matter superfields in the adjoint representation 24, and a pair of scalar superfields in $10_{H}$ and $\overline{10}_{H}$ antisymmetric representations. Note that our model is a further extension of the minimal adjoint SUSY SU(5) model proposed in the literature, with the addition of the antisymmetric $10_{H}$ and $\overline{10}_{H}$. The SM multiplet within $10_{H}$ are, $10_{H} = ((1, 1, 1) \oplus (3, 1, -2/3) \oplus (3, 2, 1/6)) \equiv (\chi_{S}, \chi_{T}, \chi_{Z})$. We need both $10_{H}$ and $\overline{10}_{H}$ for anomaly cancellation. Neutrino masses are generated via the type I + type III seesaw mechanism in the same way as in the adjoint seesaw scheme \cite{8–10}. Since we have only one 24-plet in this model, the neutrino mass matrix is of rank one at the renormalizable level. However, as has been noted before, this problem can be cured by including higher dimensional operators, to increase the rank of neutrino mass matrix from one to two, allowing for two massive neutrino states. Higher dimensional operators are required anyway in SU(5) to avoid $m_{d} = m_{\tilde{T}}$, which is experimentally untenable, both in the non-SUSY \cite{11} and SUSY \cite{2} versions of this GUT model. In what follows, we will see that in the model that we propose, the higher dimensional operators will also play a vital role in generating different mass scales for the additional multiplets, $(24, 10_{H}, \overline{10}_{H})$, and will be crucial for achieving the gauge coupling unification with the constraint that the masses of the seesaw mediating particles are within the LHC testable regime. And finally, the higher dimensional operators are also needed to create the mass separation between the SU\textsubscript{5} multiplets within 24, $10_{H}$ and $\overline{10}_{H}$, allowing for the triplet fermion $\rho_{3}$, the singlet fermion $\rho_{0}$ and the singly charged scalars $\chi_{S}$ and $\chi_{T}$ to be in the few 100 GeV to 1 TeV mass regime, allowing the possibility of producing them at the LHC.

The mass and Yukawa part of the Lagrangian involving the new field $10_{H}$ and $\overline{10}_{H}$ over and above those present in the earlier versions of the minimal adjoint SUSY SU(5) are

\begin{equation}
\mathcal{L} = Y_{\chi} \chi_{5} 5_{10_{H}}^{T} + Y_{\chi'} \chi'_{10} \overline{10}_{H} 24 + m_{\chi} \text{Tr}(\overline{10}_{H} 10_{H}) + \lambda'_{\chi} \text{Tr}(10_{H} \overline{10}_{H} 24_{H}) + \lambda_{\chi} \text{Tr}(10_{H} 24_{H}^{T} \overline{10}_{H}) + m_{\chi} \sigma_{10_{H}} \overline{10}_{H} 5_{H} + m_{\chi} \overline{10}_{H} 5_{H} \overline{10}_{H} 5_{H}
\end{equation}

where the first two terms on the right-hand side (RHS) are Yukawa couplings of the antisymmetric $10_{H}$ and $\overline{10}_{H}$ superfields with the matter superfields in $5_{H}, 10_{H}$ and 24 representations. Rest of the terms on the RHS of the above equation are mass and Yukawa interactions with other scalar superfields. Terms in the last line of the equation disappear due to their antisymmetric nature.
III. PARTICLE MASS SPECTRUM, GAUGE COUPLING UNIFICATION AND PROTON DECAY

Running of the SM gauge couplings from electroweak scale up to the unification scale in the MSSM scenarios, following a grand SUSY desert between the TeV and GUT scales, gives gauge coupling unification at $\sim 10^{16.2}$ GeV. This unification forbids the extensions of MSSM particle spectrum at low energy scales, partly because the additional particles contribute to the running of the gauge couplings and hence spoiling unification and partly because the gauge couplings could start diverging even before any unification is achieved. Thus, the conventional SUSY SU(5) models forbid any LHC predictable seesaw scale. In order to alleviate this problem and make seesaw testable at LHC, we keep a light SU(2)$_L$ triplet fermionic superfield, $\rho_3 \subset 24$. The present bound on triplet fermion from CMS experiment is $m_{\rho_3} > 180 - 210$ GeV \cite{15}. This triplet superfield will increase the slope of the SU(2)$_L$ gauge coupling, thereby misaligning the confluence at unification scale. To compensate for that, we require an equivalent change in SU(3)$_C$ and U(1)$_Y$ couplings. In what follows, we will see that the adjustment to the SU(3)$_C$ gauge coupling is made by making $m_{\rho_8} = 10^6$ GeV, while the running of the U(1)$_Y$ gauge coupling is tuned by demanding $m_{\chi^+ + \bar{\chi}} = 1$ TeV.

The one loop beta coefficients for renormalization group evolution of gauge couplings which are active be-

![Diagram](image-url)

FIG. 1. Two loop gauge coupling unification. The seesaw scale and $m_{\chi^+ + \bar{\chi}}$ are kept at TeV scale, while lepto-quarks have mass $m_{\rho_3}^0 \sim 10^{10}$ GeV yielding the unification at $M_G \simeq 10^{16}$ GeV.

| $\rho_3$ | $\rho_8$ | $\rho_{3,2} + \rho_{\tau,2}$ | $\chi_S + \bar{\chi}_S$ |
|---------|---------|----------------|------------------|
| (0 0 0) | (0 0 0) | 25/3 15 80/3 | 72/5 0 0 |
| (0 24 0) | (0 0 0) | 5 21 16 | 0 0 0 |
| (0 0 0) | (0 0 0) | 10/3 6 68/3 | 0 0 0 |

TABLE I. Two loop beta coefficients for beyond MSSM particles.

low GUT scale are, in SU(3)$_C \otimes SU(2)_L \otimes U(1)_Y$ format

$$b_{\text{SM}} = (-7, -19/6, 41/10),$$
$$b_{\text{MSSM}} = (-3, 1, 33/5),$$
$$b_{\rho_3} = (0, 2, 0), \ b_{\chi^+ + \bar{\chi}} = (0, 0, 6/5),$$
$$b_{\rho_8} = (3, 0, 0), \ b_{\rho_{3,2} + \rho_{\tau,2}} = (2, 3, 5). \ \ (2)$$

The corresponding two loop beta coefficients for Standard Model and Minimal Supersymmetric Standard Model can be read from \cite{16} and \cite{17}, while two loop beta coefficients for additional multiplets, present in the model, are listed in Table I. The two loop running of the gauge couplings from the electroweak scale to the GUT scale is shown in Fig. I. The particle spectrum of our model is shown in Table II. Since $\rho_3$ and $\rho_8$ masses are at low energy scales, mass of the lepto-quarks $\rho_{(3,2)}$ and $\rho_{(\tau,2)}$ is pushed at $m_{\rho_{(3,2)}, \rho_{(\tau,2)}} \leq M_G / \Lambda$ \cite{8}, where $M_G$ is the unification scale and $\Lambda$ is the energy scale where further new physics is expected to take over, like Planck or string scale. We observe that the changes in mass scale of the lepto-quarks $\rho_{(3,2)}$ and $\rho_{(\tau,2)}$ don’t change the value of the gauge coupling, $\alpha_G$ at unification. The change in mass scale of lepto-quarks also doesn’t alter the masses of other fields, very significantly. However, the unification scale $M_G$ depends on the value of $m_{\rho_{(3,2)}}$ and $m_{\rho_{(\tau,2)}}$. The masses of the lepto-quarks $m_{\rho_{(3,2)}, \rho_{(\tau,2)}} = 10^{10}$ GeV in our model while $m_{\rho_8} = 10^6$ GeV. Note that the constraint $m_{\rho_3} > 10^5$ GeV coming from cosmological bounds is consistent with our model \cite{18}.

| $\rho_3 \subset 24$ | $\sim 1$ TeV |
|------------------|------------------|
| $\chi_S + \bar{\chi}_S \subset 10_H + \bar{10}_H$ | $\sim 1$ TeV |
| $\rho_8 \subset 24$ | $\sim 10^6$ GeV |
| $\rho_{3,2} + \rho_{\tau,2} \subset 24$ | $\leq M_G / 100$ GeV |

TABLE II. BMSSM particle mass spectrum for type-III seesaw at TeV scale in our extended SUSY SU(5) model.

From the predicted unification scale $M_G$, unified gauge coupling $\alpha_G$ at $M_G$ and the above mentioned particle content, we estimate the proton decay life time from the
to the lepto-quark mass scale is depicted in Fig. 2. For nels. The variation of the proton lifetime with respect
that the mechanism suggested in [22] is operative to

\[ \alpha \]

This can be easily achieved by fine tuning the Eq. (1) and

\[ H \]

fields is

\[ \rho \]

multiplet

\[ \chi \]

expression [19]

\[ \frac{1}{m^{2}_D} \]

where \( F_q = 2(1 + |V_{ud}|^2)^2 \approx 7.6 \), \( A_R = 0.831 \) [20] and \( \alpha_H = 0.012 \) GeV \(^3\) [21], are renormalization factor and
hadronic matrix element, respectively. Here we assume
that the mechanism suggested in [22] is operative to
suppress Higgsino mediated dim-5 proton decay channels.
The variation of the proton lifetime with respect to the
lepto-quark mass scale is depicted in Fig. 2. For

\[ m_{\rho(3,2)} = 10^{30} \text{ GeV} \]

the proton lifetime is predicted to be \( \tau_p \approx 10^{35.6} \) years, in the range that can be probed in the future [23]. On the other hand we find that
the present experimental bound on proton lifetime

\[ \Gamma_{\rho 	o e^+ \pi^0}^{-1} \]

constrains lepto-quark mass to be \( m_{\rho_{3,2}} > 5 \times 10^8 \) GeV. Hence the lepto-quarks are far beyond any direct experimental reach.

The particle spectrum of the 24 fermionic multiplet has been discussed in detail in the literature before [5]. A short mention of the mass spectrum of the 10H multiplet is in order. From gauge coupling unification only the \( \chi_S \) and \( \overline{\chi_S} \) masses are required to be at low energy scale. This can be easily achieved by fine tuning the Eq. (1) and
non-renormalizable terms are not required. The mass spectrum for 10H fields is

\[ m_{\chi_S} = m_\chi - 6\Lambda = m_{\chi_T} - 10\Lambda = m_{\chi^c} - 5\Lambda \]

where \( \Lambda = (\lambda + \lambda^\prime)M_G/\sqrt{60} \).

IV. TYPE-III SEESAW PHENOMENOLOGY AND SIGNAL AT LHC

The complete neutrino mass matrix including singlet and neutral triplet fermion can emerge from the effective Lagrangian

\[ \mathcal{L}_\nu = L_i(y^i_{\rho_3} \rho_3 + y^i_{\rho_0} \rho_0)H + \frac{1}{2} m_{\rho_3} \rho_3^2 + \frac{1}{2} m_{\rho_0} \rho_0^2 \]

which gives

\[ M_\nu = \begin{pmatrix} 0 & m^T_D \rho_3^T \end{pmatrix} \]

where \( m_D = Y_D v_U, v_U \) being the vacuum expectation value of \( H_U \) while

\[ Y_D = \begin{pmatrix} y^1_{\rho_0} & y^2_{\rho_0} & y^3_{\rho_0} \\ y^1_{\rho_3} & y^2_{\rho_3} & y^3_{\rho_3} \end{pmatrix}, \quad M_\rho = \begin{pmatrix} m_{\rho_0} & 0 \\ 0 & m_{\rho_3} \end{pmatrix}. \]

This matrix is a complex symmetric matrix and is diagonalised by a unitary matrix which we denote by \( U_\nu \).
Though \( m_\nu \) is a 3 \( \times \) 3 matrix its rank=2 hence, it give nonzero masses to only two neutrinos. We can write [25]

\[ U^*_\nu m_\nu U^T_\nu \]

\[ m_D = i \sqrt{M_\rho R \sqrt{m_\nu U^T_\nu}}. \]

Here \( R \) is a 2 \( \times \) 3 complex matrix such that \( RR^T = I_2 \). Because one light neutrino is massless, we can express \( R \) in terms of only one complex parameter for normal hierarchy (NH) and inverted hierarchy (IH) as [26]

\[ R = \begin{pmatrix} \cos(z) & \pm \sin(z) \\ -\sin(z) & \pm \cos(z) \end{pmatrix} \]

\[ (\text{for NH}), \]

\[ R = \begin{pmatrix} \cos(z) & \pm \sin(z) & 0 \\ -\sin(z) & \pm \cos(z) & 0 \end{pmatrix} \]

\[ (\text{for IH}). \]

Hence, \( R^T R = (I_2, I_2) \) for NH and \( R^T R = (I_2, 0) \) for IH. Therefore we get Dirac-Yukawa couplings like

\[ y^1_{\rho_0} = i \sqrt{m_{\rho_0}} (\cos(z)\sqrt{m_{\rho_2}^2 U^*_{\nu 12}} \pm \sin(z) \sqrt{m_{\rho_2}^2 U^*_{\nu 13}}) / v_U, \]

\[ y^1_{\rho_3} = i \sqrt{m_{\rho_3}} (\sin(z) \sqrt{m_{\rho_2}^2 U^*_{\nu 12}} \pm \cos(z) \sqrt{m_{\rho_2}^2 U^*_{\nu 13}}) / v_U. \]

Similarly we can write Yukawa couplings for the inverted hierarchy (IH) case. Parameter \( z \) is in general a complex number hence \( Re(z) \) is periodic in \( [0, 2\pi] \) but \( Im(z) \) is free from any constraint and gives exponential variation in \( y \). The effect of \( Re(z) \) for large \( Im(z) \) becomes vanishingly small. The variation in \( y^1_{\rho_3} \) and \( y^1_{\rho_0} \) with respect to \( Im(z) \) is depicted in Fig. 8 where we see that in absence of \( Im(z) \), \( y_{\rho_3, \rho_0} \sim O(10^{-7} - 10^{-6}) \).
Together with the current status of neutrino masses and
mixings, the only unknown parameters are the two CP-phases and a complex $z$. The measurement of the Yukawa couplings can constrain the $z$ parameter severely.

The triplet fermions, both charged and neutral can be produced at the LHC through their gauge couplings. The production rate and subsequent decay of these exotic fermions has been widely studied in the literature and we refer the reader to [8, 28, 29] for in-depth study of the collider phenomenology of type III seesaw at LHC. Here we list the decay channels of $\rho_3$, which depend crucially on the Yukawa couplings [8, 29, 30]:

\begin{align}
\Gamma(\rho_3^- \to Z e_k^-) &= \frac{m_{\rho_3}}{32\pi} |y_{\rho_3}^1|^2 C_1 C'_1, \\
\Gamma(\rho_3^0 \to W^\pm e_k^\mp) &= \frac{m_{\rho_3}}{32\pi} |y_{\rho_3}^j|^2 C_2 C'_2, \\
\sum_k \Gamma(\rho_3^0 \to Z \nu_k) &= \frac{m_{\rho_3}}{32\pi} \left( \sum_k |y_{\rho_3}^k|^2 \right) C_1 C'_1, \\
\sum_k \Gamma(\rho_3^- \to W^- \nu_k) &= \frac{m_{\rho_3}}{16\pi} \left( \sum_k |y_{\rho_3}^k|^2 \right) C_2 C'_2;
\end{align}

where $C_1 = \left( 1 - \frac{m_{\nu}^2}{m_{\rho_3}^2} \right)^2$, $C'_1 = \left( 1 + 2 \frac{m_{\nu}^2}{m_{\rho_3}^2} \right)$, $C_2 = \left( 1 - \frac{m_{W}^2}{m_{\rho_3}^2} \right)^2$ and $C'_2 = \left( 1 + 2 \frac{m_{W}^2}{m_{\rho_3}^2} \right)$. This $\rho_3$ can also decay in to Higgs and light leptons through:

\begin{align}
\Gamma(\rho_3^- \to h e_k^-) &= \frac{m_{\rho_3}}{32\pi} |y_{\rho_3}^j|^2 \left( 1 - \frac{m_{h}^2}{m_{\rho_3}^2} \right)^2, \\
\sum_k \Gamma(\rho_3^0 \to h \nu_k) &= \frac{m_{\rho_3}}{32\pi} \left( \sum_k |y_{\rho_3}^k|^2 \right) \left( 1 - \frac{m_{h}^2}{m_{\rho_3}^2} \right)^2.
\end{align}

The predicted decay widths for all the above listed channels are given in Table III for two cases. The second column of the table gives the decay width where the $\text{Im}(z) = 0$ and we get the smallest possible value of the Yukawa couplings. Obviously this leads to the minimal decay width for the heavy fermion. In the third column of this table we give the decay widths calculated for the Yukawa couplings when $\text{Im}(z) = 10$. The decay widths are seen to significantly increase by many orders of magnitude with the increase of $\text{Im}(z)$ and hence the Yukawa couplings. While the Yukawa couplings and the decay widths will keep increasing with the value of $\text{Im}(z)$, we limit our discussion to $\text{Im}(z) = 10$ for reasons which will become clear in the following discussion on lepton flavor violation.

### Table III. Decay widths of different tree level decay channels of triplet fermions. The lifetime $\tau = \frac{\hbar}{\Gamma}$, where $\hbar = 6.582119 \times 10^{-24}$ GeV·s.

| Decay channel | Decay width at $\text{Im}(z) = 10$ | Decay width at $\text{Im}(z) = 0$ | Decay width at $\text{Im}(z) = 10$ |
|---------------|----------------------------------|----------------------------------|----------------------------------|
| $\rho_3^0 \to Z \nu_k$ | $\text{Im}(z) = 10$ | $\text{Im}(z) = 0$ | $\text{Im}(z) = 10$ |
| $\rho_3^- \to W^- \nu_k$ | $\text{Im}(z) = 10$ | $\text{Im}(z) = 0$ | $\text{Im}(z) = 10$ |
| $\rho_3^0 \to W^+ e_k^+$ | $\text{Im}(z) = 10$ | $\text{Im}(z) = 0$ | $\text{Im}(z) = 10$ |
| $\rho_3^- \to h e_k^-$ | $\text{Im}(z) = 10$ | $\text{Im}(z) = 0$ | $\text{Im}(z) = 10$ |
| $\sum_k \rho_3^0 \to h \nu_k$ | $\text{Im}(z) = 10$ | $\text{Im}(z) = 0$ | $\text{Im}(z) = 10$ |

### V. LEPTON FLAVOR VIOLATION

![Lepton Flavor Violation](image)

Lepton flavor violation in this model is mediated even at the tree level by both the fermion triplet $\rho_3$ embedded in 24 and the charged scalar singlet $\chi_5$ in 10 of SU(5). The charged fermions belonging to the fermion triplet $\rho_3$ is mixed with the standard model charged leptons and could lead to lepton flavor changing decays of $\mu^-$ via the processes shown in Fig. 4. The non-standard effective vertex shown by the crosses in the figure is lepton number conserving, but lepton flavor violating dimension 6 coupling, and the branching ratio of this process over the
flavor conserving $\mu \to e \bar{\nu}_e \nu_\mu$ is given as\[30, 31\]

$$\text{Br}(\mu \to eee) = A v_D^4 \left( \frac{y_{\rho_3}^i}{m_{\rho_3}^2} \frac{1}{m_{\rho_3}^2} y_{\rho_3}^j \right)_{\mu e}, \quad (21)$$

where $A = 3 \sin^4 \theta_W - 2 \sin^2 \theta_W + 1/2 \approx 0.1980707$. Imposing the current limit for the $\mu \to eee$ of\[32\]

$$\text{Br}(\mu \to eee) < 1.0 \times 10^{-12}, \quad (22)$$
gives $|y_{\rho_3}| \leq 0.0061$ for $m_{\rho_3} = 1$ TeV. Marginally more stringent constraint comes from the experimental limits on $\mu \to e$ conversion in nucleus of $^{125}_{53}$ Ti, which gives

$$v_D^2 \left( \frac{y_{\rho_3}^i}{m_{\rho_3}^2} \frac{1}{m_{\rho_3}^2} y_{\rho_3}^j \right)_{\mu e} < 1.7 \times 10^{-7}. \quad (23)$$

This constrains $|y_{\rho_3}| \leq 0.0017$ for the same mass of $m_{\rho_3}$. These constraints on Yukawa couplings are stinger then constraints coming from $\text{Br}(l \to l' \gamma)$.

In addition to the contributions to lepton flavor violation discussed above, we could, in principle, have flavor violation coming from the standard running of the slepton mass matrices. The leading log contributions coming from presence of $\rho_3$ are given by\[12\]

$$(\delta_{LL})_{ij} = \frac{1}{8 \pi^2} \frac{3 m_3^2 + A_0^2}{m_L^2} \left( \frac{3}{2} y_{\rho_3}^i y_{\rho_3}^j \ln \left( \frac{M_G}{m_{\rho_3}} \right) + y_{\rho_0}^i y_{\rho_0}^j \ln \left( \frac{M_{\rho_0}}{m_{\rho_0}} \right) \right). \quad (24)$$

Assuming $m_L^2 \approx m_0^2$ and $A_0 \approx 0$ we get

$$(\delta_{LL})_{ij} \leq \mathcal{O}(10^{-5}) \text{ for } m_{\rho_0, \rho_3} \approx 1 \text{ TeV}. \quad (25)$$

The branching ratio of $l_i \to l_j \gamma$ is given by\[33\]

$$\frac{\text{Br}(l_i \to l_j \gamma)}{\text{Br}(l_i \to l_j \nu_j \bar{\nu}_j)} \approx \frac{a^3 \delta_{ij}^2}{G_F^2 m_i^4 \tan^2 \beta} \label{a} \quad (26)$$

$$\approx 2.9 \times 10^{-17}. \quad (27)$$

Hence we see that contribution of type III seesaw mediating $\rho_3$ to lepton flavor violating decays is almost negligible and can be ignored.

Lepton flavor violation at the tree level is also induced by the singly charged singlet scalar $\chi_S$ in our model through diagrams shown in Fig. 5. Note that the process $l_i^- \nu_j l_j^+ \nu_j \bar{\nu}_l$ mediated by the charged singlet scalar $\chi_S$ comes due to the $l_i^- \nu_j \chi_S$ vertex from the first term in Lagrangian given in Eq. 1 which is a Yukawa interaction between the charged singlet scalar and the neutral and charged leptons of the SM. Note that this Yukawa coupling matrix has to be antisymmetric. Therefore, the process shown in Fig. 5 are necessarily flavor violating, that is, $i \neq j$ and $k \neq l$. Unlike the Yukawa couplings $y_{\rho_3}$ which are constrained by the neutrino masses, the Yukawa couplings $Y_\chi$ is completely unconstrained from any other process and could potentially lead to large flavor violating decays of the charged leptons. Since the two neutrinos in this diagram are not observed, this process will give a contribution in the experiment that observe the standard flavor conserving decays of the charged leptons. This process is similar to the one in the Zee model\[34\] where the SM is extended with an additional scalar doublet and a charged scalar singlet. While unlike the Zee model we do not have any radiative correction to the neutrino mass matrix due to the different Higgs structure and SUSY nature of our model, we do have the $\chi_S$ mediated process shown in Fig. 5 as a result of the $Y_\chi$ Yukawa coupling in our model. Corrections to the four-Fermi interaction responsible for the weak decays such as $\mu^- \to e^- \bar{\nu}_e \nu_j$ has been calculated in the literature as $(G_F/\sqrt{2})(1 + \zeta)$\[35\], where $\zeta$ is given as

$$\zeta = \frac{a^2 / 2 m_{\chi_S}^2}{G_F/\sqrt{2}}, \quad (28)$$

where $a$ is $e \mu$ element of the antisymmetric Yukawa matrix, $Y_\chi$, which is parametrized in terms of three complex parameters as

$$Y_\chi = \begin{pmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{pmatrix}, \quad (29)$$

and we have taken these parameters to be real for simplicity. The experimental observations return the constraint $\zeta < 10^{-3}$, which leads to the corresponding constraint on the $e \mu$ element of the Yukawa matrix $Y_\chi$ as $a^2 < 1.65 \times 10^{-8} m_{\chi_S}^2/(\text{GeV}^2)$. For a TeV scale $m_\chi$ we get $a < 0.128$ which is large enough to generate significant flavor violation. The branching ratio for $\mu \to e\gamma$ driven by $Y_\chi$ is\[35, 36\]

$$\text{Br}(\mu \to e\gamma) = \left( \frac{\alpha}{48 \pi} \right) \left( \frac{bc}{m_{\chi_S}^2 G_F} \right)^2 < 5.7 \times 10^{-13}, \quad (30)$$

which constrains the Yukawa couplings to $bc < 0.00126$. Similar constraints will come from $\text{Br}(\tau \to \mu\gamma) < 4.4 \times 10^{-8}$\[37\] and $\text{Br}(\tau \to e\gamma) < 3.3 \times 10^{-8}$\[37\], and, are $ab < 0.124$ and $ac < 0.093$ respectively. If
we saturate the $\tau \to \mu \gamma$ and $\tau \to e \gamma$ limits, we have $b < 1$ and $c < 0.72$, respectively, since $a > 0.128$. Therefore, from the bounds from radiative $\tau$ decays we have $bc < 0.72$, which is much weaker than the bound we have on this combination from $\mu \to e \gamma$.

In addition, we could have lepton flavor violation coming from the contributions of $\chi_S$ to the running of the slepton masses which could lead to radiative decays of the charged leptons. The leading log contribution in this case is given as [14]

$$
(\delta_{LL})_{ij} \approx \frac{1}{2\pi^2} \frac{3m^2 + A_0^2}{m^2_L} (Y_\chi)^i_j \ln \left( \frac{M_G}{m_{\chi_S}} \right).
$$

(31)

Without any further assumption and saturating the experimental bound we can find that

$$
\begin{align*}
    b &\approx \frac{B_r(\mu \to e \gamma)}{B_r(\mu \to \mu \gamma)} \frac{B_r(\tau \to e \nu_e\bar{\nu}_e)}{B_r(\tau \to \mu \nu_\mu \bar{\nu}_\mu)}, \\
    c &\approx \frac{B_r(\tau \to e \gamma)}{B_r(\tau \to \mu \gamma)} \frac{B_r(\tau \to \mu \nu_\mu \bar{\nu}_\mu)}{B_r(\tau \to \mu \nu_\mu \bar{\nu}_\mu)}.
\end{align*}
$$

(32)

The conclusion we can make from this analysis is that even though we do not get significant contribution to LFV from type III seesaw for possible parameter space, we still may explain it due to presence of extra charged singlet scalar. Also, the observation of $\mu \to e \gamma$ in near future constrains $\tau \to \mu \gamma$ and $\tau \to e \gamma$ to remain unobserved for very long.

VI. NEUTRINOLESS DOUBLE BETA DECAY

The symmetric mass matrix $M_\nu$, in Eq. [39] is diagonalized by the unitary $5 \times 5$ matrix [38]

$$
W = \begin{pmatrix}
N & U \\
T & V
\end{pmatrix}
$$

$$
\simeq \begin{pmatrix}
1 - \frac{1}{2}BB^\dagger & -B \\
-\frac{1}{2}B^\dagger B & 1 - \frac{1}{2}B^\dagger B
\end{pmatrix}
\begin{pmatrix}
U_\nu & 0 \\
0 & U_\rho
\end{pmatrix}
\tag{33}
$$

such that $W^T M_\nu W = \text{diag}(m_1, m_2, m_3, m_{\nu_\mu}, m_{\nu_\tau})$, and $B \simeq M_\rho^{-1} M_D$. Here $U_\nu$ diagonalizes the light neutrino mass matrix $m_\nu$ (cf. Eq. [39]), while $U_\rho$ diagonalizes heavy neutrino mass matrix.

The contribution to neutrinoless double beta decay [39] will come from the diagrams depicted in Fig. [6] wherein we have contributions from the exchange of both the light (left panel) and heavy (right panel) neutral fermion. The total amplitude for the process can therefore be written as [40]

$$
A \approx G_F^2 \left( \sum_i \frac{N_{ei}^2 m_i}{p^2} - \sum_j \frac{U_{ej}^2}{m_{\nu_j}} \right),
$$

(34)

where $i$ and $j$ run over first three and last two indices of the above eigenvalues, respectively. The half life of neutrinoless double beta decay can be written as [40]

$$
\Gamma_{0\nu\beta\beta} = G^2 \left| \frac{M_\nu}{m_e} \right|^2 \left( \sum_i \frac{N_{ei}^2 m_i}{p^2} + \sum_j \frac{U_{ej}^2}{m_{\nu_j}} \right)^2 \ln(2),
$$

(35)

where $G$ contains the phase space factors, $m_e$ and $M_\nu$ are electron mass and nuclear matrix element, respectively, and where $|p^2| \sim (190 \text{ MeV})^2$ [41] is the momentum exchanged in the process. The mixing matrices $N$ and $U$ are given in Eq. [33]. Elements inside the big modulus are the effective mass contributions to $0\nu\beta\beta$-decay due to light neutrinos and heavy neutral fermions, respectively. In the Fig. [7] we have plotted the contribution of the heavy fermions to the $0\nu\beta\beta$-decay effective mass, as a function of the seesaw parameter, $\text{Im}(z)$. The two horizontal lines in the figure represent the maximum achievable effective mass contribution due to light neutrino mediation, in normal and inverted hierarchy scenario, respectively. We reiterate that the lightest of the light neutrino in our model is massless. We note from the figure that the contribution of the heavy neutrinos to the effective mass in $0\nu\beta\beta$-decay dominates the one.

![FIG. 6. Feynman diagrams for neutrinoless double beta decay.](image-url)
coming from the light neutrinos when $\text{Im}(z) > 8.5$ and 10, for normal and inverted neutrino mass hierarchy, respectively. Fig. 3 reveals that for these values of $\text{Im}(z)$, $y_{\rho_3} > 0.001$ and 0.01 for normal and inverted neutrino mass hierarchy, respectively. However, we had seen in the last section that such large values of the Yukawa couplings are ruled out by the current constraints on lepton flavor violation. Therefore, the $0\nu\beta\beta$-decay signal in this model is expected to be driven by the standard light neutrino mass term.

VII. LEPTOGENESIS

A simple explanation to baryonic asymmetry of universe is provided by the Baryogenesis via Leptogenesis. The leptonic asymmetry is generated by out of equilibrium decay of seesaw generating particles. Usually in type-I or type-III seesaw with more then one generations, TeV scale leptogenesis may occur through resonance in self-energy contribution [42]. But, in a hybrid seesaw through type-I+III [43], as we discussed above, CP-asymmetry is generated only through vertex correction, because with the present experimental bound on $m_{\rho_3}$ self-energy contributions are absent to avoid the braking of $SU(2)_L$ symmetry at this scale. The vertex correction, gives asymmetry of the type [43] [44]

$$
(\epsilon_{\rho_3})_{i} = \frac{\text{Im} \left[ y^{*}_{\rho_3} y_{\rho_0} \right]}{8\pi (y^{*}_{\rho_3} y_{\rho_0})_{11}} f \left( \frac{m_{\rho_0}^2}{m_{\rho_3}^2} \right)
$$

(36)

where

$$
f(x) = \sqrt{x} \left[ (1 + x) \ln \left( \frac{1 + \frac{1}{x}}{1} \right) - 1 \right]
$$

(37)

Substituting $\rho_3 \leftrightarrow \rho_0$ will give asymmetry generated by singlet fermion. Because $f(x) < 1$, $\forall x$; this asymmetry does not lead to a resonance effect, like we get for the self-energy term. As we see in the Fig. 3 that for $y_{\rho_3}, y_{\rho_0} >> 10^{-6}$ the imaginary part is the major contribution to the Yukawa couplings, and that $y^{*}_{\rho_3} - y_{\rho_0} \simeq \Delta y^i$. For the Yukawa couplings saturating the flavor violating bounds, for $m_{\rho_0} \simeq m_{\rho_3}$ we get

$$
(\epsilon_{\rho_3})_{i} \sim \frac{\text{Re}(\Delta y^i)\text{Im}(y^i)}{8\pi} f \left( \frac{m_{\rho_0}^2}{m_{\rho_3}^2} \right) << 10^{-10}.
$$

(38)

The total asymmetry $\epsilon_{\rho_3} = \sum_i (\epsilon_{\rho_3})_{i}$ is constrained as [45] as

$$
\epsilon_{\rho_3} \lesssim \frac{3}{8\pi} \frac{m_{\rho_3} m_{\rho_0}}{v_u^2} \sim 10^{-13}.
$$

(39)

With this small CP asymmetry we do not get any significant amount of baryonic asymmetry, because

$$
\eta_B \simeq 10^{-2} \sum_i \epsilon_i \kappa_i (w \to \infty),
$$

while

$$
\eta_B^{\text{CMB}} = (6.2 \pm 0.15) \times 10^{-10}
$$

(40)

and as the efficiency factor $\kappa_i << 1$, the model prediction never reaches the experimental value $\eta_B^{\text{CMB}}$. Here, $10^{-2}$ is the dilution factor coming from converting the CP asymmetry to the baryonic asymmetry and $w = m_{\rho_3}/T$, where $T$ is the temperature. Hence, leptogenesis is expected to fail in this model and we need to look alternative ways to explain the baryonic asymmetry of the universe. A significant CP asymmetry requires both real and imaginary parts of the Yukawa couplings to be large, which is possible only when the masses of the triplet and singlet fermion are increased, well beyond the reach of the LHC. Therefore, any observation of triplet fermion and SUSY in near future at the LHC/ILC, could be taken as a hint for non possibility of leptogenesis within these classes of type-I+III models.

VIII. DISCUSSION AND CONCLUSION

We proposed an extension of the SUSY SU(5) GUT model which allows to test seesaw at the LHC. This is accomplished by adding a fermionic 24 matter superfield and a pair of $10_H$ and $\overline{10}_H$ Higgs superfield to the model. The triplet and singlet fermions from the 24 representation lead to type I+III seesaw in this model. The presence of the charged singlet scalar in $10_H$ and $\overline{10}_H$ allows the controlling of the gauge coupling running in such a way that unification is achieved in this model with the triplet fermions and the charged singlet scalar masses being around 1 TeV. This opens up the possibility of producing and testing these particles at the colliders.

We studied the seesaw phenomenology of this model and showed how the additional freedom from the seesaw framework allows for very large Yukawa couplings $y_{\rho_3}$ for the heavy fermions even for TeV scale masses. We parametrized this freedom in terms of a complex variable $z$ and showed that the $\text{Im}(z)$ component could give very large Yukawa couplings, leading to enhanced decay width and hence shorter lifetimes for the heavy fermions at the collider. We studied the lepton flavor violation predicted in this model. Constraints from $\mu \to eee$ and $e \to e e e$ conversion was shown to severely constrain the Yukawa couplings $y_{\rho_3}$ in this model. We also studied the contribution of the heavy fermion exchange in neutrinoless double beta decay fermions. We showed that the constraints from lepton flavor violation completely smother any chances of seeing these contribution in the neutrinoless double beta decay experiments.

In addition to the exotic fermion, we also have an exotic single charged scalar in this model. We showed that very large lepton flavor violation at the tree level is induced in this model by the charged singlet scalar. We studied these processes and put constraints on the coupling of the charged singlet scalar with the SM fermions.
In conclusion, our model predicts a testable seesaw within a SUSY GUT framework. In addition, it has a rich phenomenology in lepton flavor violation experiments. In case SUSY as well as fermionic triplets are observed at the LHC or the future colliders, it will be a test for type-III seesaw within a SUSY framework and this model will be a viable candidate as the beyond the standard model theory to explain this result.

APPENDIX

Renormalization group equations

The one-loop beta coefficients for Yukawa couplings, soft scalar masses and trilinear terms are standard and straightforward to evaluate [17, 47] by hand or using a package [48]. A sample of slepton soft mass RGE is

$$\beta_{m_L^2} = 2\sqrt{3} h_{p_3} h_{p_0} + 2 h_{p_0} h_{p_0}^\dagger + 8 h_L h_L^\dagger + 2 h_l h_l^\dagger$$

$$+ 6 I \left( \frac{1}{5} |m_1|^2 g_1^2 - |m_2|^2 g_2^2 \right)$$

$$- \frac{3}{5} g_2^2 I \left( m_H^2 - m_a^2 - m_\chi^2 + m_\chi^2 + \text{Tr}(m_k^2) \right)$$

$$- 2 \text{Tr}(m_a^2) - \text{Tr}(m_k^2) + \text{Tr}(m_a^2)$$

$$+ \sqrt{3} (2 m_{Y_2} y_{p_3} y_{p_0} + y_{p_0} y_{p_0} m_L^2 + m_L^2 y_{p_0} y_{p_0})$$

$$+ 2 m_{Y_2} y_{p_0} y_{p_0} + 2 m_{Y_2} y_{p_0} y_{p_0} + y_{p_0} y_{p_0} m_L^2$$

$$+ m_L^2 y_{p_0} y_{p_0} + 8 m_\rho_0^2 Y_\chi Y_\chi^\dagger + 4 Y_\chi Y_\chi^\dagger$$

$$+ 8 Y_\chi m_L^2 T Y_\chi^\dagger + 4 m_\chi^2 Y_\chi Y_\chi^\dagger + 2 m_\chi^2 y_l y_l^\dagger$$

$$+ y_l y_l^\dagger + 2 y_m e T y_l^\dagger + m_\chi^2 y_l y_l^\dagger \right). \quad (41)$$

Experimental bound on LFV processes constraints the couplings of triplet and singlet fermionic superfields to be very small. Hence, we explicitly write the new contributions to soft mass beta coefficients due to $\chi S, \bar{\chi} S$ only. With the constraint $y_{p_0} m_0 << Y_\chi$, the leading new contributions beyond MSSM are

$$\beta_{m_Q^2} = 0,$$

$$\beta_{m_Z^2} = \frac{4}{5} g_2^2 m_\chi^2 - \frac{4}{5} g_1^2 m_\chi^2,$$

$$\beta_{m_L^2} = -\frac{2}{5} g_1^2 m_\chi^2 + \frac{2}{5} g_1^2 m_\chi^2,$$

$$\beta_{m_S^2} = 2 h_L^2 h_L^\dagger - \frac{6}{5} g_2^2 m_\chi^2 + \frac{6}{5} g_1^2 m_\chi^2 + 2 m_\chi^2 Y_\chi Y_\chi^\dagger$$

$$+ 2 m_\chi^2 Y_\chi Y_\chi^\dagger + 2 Y_\chi + Y_\chi^\dagger + 2 m_\chi^2 Y_\chi^\dagger$$

$$+ 8 Y_\chi m_L^2 T Y_\chi^\dagger + 4 m_\chi^2 Y_\chi^\dagger \right). \quad (42)$$

Acknowledgements

S.C. acknowledges partial support from the European Union FP7 ITN INVISIBLES (Marie Curie Actions, PITN-GA-2011-289442).

[1] H. Georgi and S. L. Glashow, Phys. Rev. Lett. 32 (1974) 438.
[2] S. Dimopoulos and H. Georgi, Nucl. Phys. B 193 (1981) 150. N. Sakai, Z Phys. C 11 (1981) 153.
[3] S. Weinberg, Phys. Rev. Lett. 43 (1979) 1566.
[4] P. Langacker and M. x. Luo, Phys. Rev. D 44 (1991) 817.
[5] P. Minkowski, Phys. Lett. B 67 (1977) 421. T. Yanagida, proceedings of the Workshop on Unified Theories and Baryon Number in the Universe, Tsukuba, 1979, eds. A. Sawada, A. Sugamoto; S. Glashow, in Cargese 1979, Proceedings, Quarks and Leptons (1979): M. Gell-Mann, P. Ramond, R. Slansky, proceedings of the Supergravity Stony Brook Workshop, New York, 1979, eds. P. Van Nieweuwkuizen, D. Freeman; R. Mohapatra, G. Senjanović, Phys. Rev. Lett. 44 (1980) 912.
[6] M. Magg and C. Wetterich, Phys. Lett. B 94 (1980) 61. G. Lazarides, Q. Shafi and C. Wetterich, Nucl. Phys. B 181 (1981) 287; R. N. Mohapatra and G. Senjanović, Phys. Rev. D 23 (1981) 165.
[7] R. Foot, H. Lew, X. G. He and G. C. Joshi, Z. Phys. C 44 (1989) 441.
[8] B. Bajc and G. Senjanovic, JHEP 0708 (2007) 014 [arXiv:hep-ph/0612029]; B. Bajc, M. Nemevsek and G. Senjanovic, Phys. Rev. D 76 (2007) 055011 [arXiv:hep-ph/0703080].
[9] I. Dorsner and P. F. Perez, JHEP 0706 (2007) 029 [arXiv:hep-ph/0612216].
[10] H. Georgi and C. Jarlskog, Phys. Lett. B 86 (1979) 297; J. R. Ellis and M. K. Gaillard, Phys. Lett. B 88 (1979) 315.
[11] I. Dorsner and P. F. Perez, Nucl. Phys. B 723 (2005) 53 [arXiv:hep-ph/0502476]; I. Dorsner, P. F. Perez and R. Gonzalez Felipe, Nucl. Phys. B 747 (2006) 312 [arXiv:hep-ph/0512068].
[12] E. Ma, Phys. Rev. Lett. 81 (1998) 1171 [arXiv:hep-ph/9805219]; P. F. Perez, Phys. Lett. B 654 (2007) 189 [arXiv:hep-ph/0702287]; P. F. Perez, Phys. Rev. D 76 (2007) 071701 [arXiv:hep-ph/0705.3589]; C. Biggio and L. Calibbi, JHEP 10 (2010) 037 [arXiv:1007.3750 [hep-ph]].
[13] I. K. Cooper, S. F. King and C. Luhn, Phys. Lett. B 690.
[48] R. M. Fonseca, Comput. Phys. Commun. 183 (2012) 2298 [arXiv:1106.5016 [hep-ph]]; F. Staub, Comput. Phys. Commun. 182:808,2011 [arXiv:1002.0840 [hep-ph]].