Validation of the Longitudinal Motion Amplitude and Phase for Two Ships with Forward Speed at Close Proximity in Waves

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Abstract. The longitudinal motion amplitude and phase for two ships with forward speed at close proximity in waves are investigated by the frequency-domain potential flow theory, which is based on the mathematical kernel of three-dimension translating-pulsating source. On the foundation of numerical validation for a modified Wigley ship, the frequency-domain method is then extended to the kinematic characteristics for two ships. The two-ship model test is focused on the surge, heave and pitch mode motions. The numerical and experimental results shows that it is applicable to solve the hydrodynamic interaction problem for two advancing ships in waves by the introduction of translating-pulsating source Green function.

1. Introduction
From the view of fluid hydrodynamics, the alongside replenishment problem of two ships advancing at sea can be summarized as the hydrodynamic interaction effect between two floating bodies in waves. In the field of ship engineering, hydrodynamic interaction of two ships in waves is one of the classic problems in the field of ship dynamics. International Ship and Offshore Structures Congress (ISSC) had carried out extensive and in-depth discussions on two-ship hydrodynamic problem for many years [1-3]. The hydrodynamic problem of two ships at close proximity in waves is very different from the dynamics of single ship or multihull ship, which is focused on the motion response and wave force in only six degrees of freedom. However, coupled hydrodynamics of two ships advancing in waves involve the motion and load with twelve degrees of freedom.

In the traditional framework of slender body theory, free motion of three-dimension ship is approximated to that in two dimensions, in which the perturbation effect of ship longitudinal hydrodynamics on the three-dimension flow field is ignored. 2.5D+T theory shows a good stability in the numerical calculation of seakeeping performance for high-speed ships or multihull vessels [4, 5]. If the Brard number $\tau > 0.25$, the radiation and diffraction waves generated by advancing ships spread back. In the 2.5D+T theory, the forward ship-sections exert hydrodynamic interaction on the backward ones but there is no influence of backward ship-sections to the forward ones. Only the sway, heave, roll, pitch and yaw modes motion except the surge mode are taken into account in the slender body theory and 2.5D+T theory. The complete solution of longitudinal motion of ships must be applied by the three-dimension theory. Present work is focused on the validation of longitudinal motion amplitude and phase for two ships with forward speed at close proximity in waves, which is based on the frequency-domain potential flow theory. By the introduction of three-dimension translating-
pulsating source Green function, it is expected to delve into the hydrodynamic mechanics of two floating bodies in waves.

2. Mathematics of ship-motion amplitude and phase

In the defined coordinates as figure 1, it is assumed that the flow liquid is irrotational and incompressible ideal fluid, there is the velocity potential in the flow field that satisfies the Laplace equation. By the linear hydrodynamics theory, the unsteady velocity potential at the field point \( p (x, y, z) \) can be expressed as

\[
\Phi(p) = \Phi_h(p) + \Phi_d(p) + \Phi_0(p) = \sum_{j=1}^{7} \phi_j(p)\eta_j + \phi_d(p) + \phi_0(p) e^{-i\omega t}
\]  

(1)

In which, \( \Phi_h \) is the unsteady radiation potential at the point \( p \) due to the ship motions in oscillatory form; \( \Phi_d \) is the unsteady diffraction potential when the incident wave encounters with ship body; \( \Phi_0 \) is the unsteady incident potential. For the \( j \) th mode ship motion, \( \phi_j \) is the amplitude of radiation potential at unit oscillation and \( \eta_j \) is the first-order motion at six degrees of freedom, where the cases \( j = 1, 2, \ldots, 6 \) respectively correspond to the motion modes such as surge, sway, heave, roll, pitch and yaw. \( \phi_0 \) and \( \phi_d \) are the spatial components of incident and diffraction potentials respectively.

![Figure 1. Two ships with forward speed at close proximity in waves.](image)

In the framework of frequency-domain potential flow theory, the velocity potential \( \phi_j \) is solved by the reasonable Green-function distribution on the mean wetted ship surface, which are two types such as the point-source form and the hybrid source-and-dipole form. The former form is the source distribution method and the latter one is the direct potential approach. The velocity potential in present paper is expressed by the source distribution method [6]. Especially, the three-dimension translating-pulsating source Green function is characterized by the physical connotation of both translating and pulsating and its forward-speed effect is included in many boundary conditions and hydrodynamic coefficients. It is assumed that there are two dots of source point \((\xi, \eta, \zeta)\) and field point \((x, y, z)\) in the field. The uniform speed of translating-pulsating source is \( U \) and oscillating frequency is \( \omega \). Then this source in Bessho form can be derived by the following expression [7].

\[
G(x, y, z; \xi, \eta, \zeta) = \frac{1}{4\pi} \left( \frac{1}{r} - \frac{1}{r^\prime} \right) - \frac{i}{2\pi} K T(X, Y, Z)
\]  

(2)

In which,

\[
r = \sqrt{(x-\xi)^2 + (y-\eta)^2 + (z+\zeta)^2}, \quad r^\prime = \sqrt{(x-\xi)^2 + (y-\eta)^2 + (z+\zeta)^2},
\]

\[
k_i = \frac{1}{2\cos^2 \theta}(1 + 2\tau \cos \theta \pm \sqrt{1 + 4\tau \cos \theta}), \quad k_j = \frac{1}{2\cos^2 \theta}(1 + 2\tau \cos \theta \pm \sqrt{1 + 4\tau \cos \theta}),
\]

\[
T(X, Y, Z) = \int_{-\pi/2}^{\pi/2} \frac{1}{\sqrt{1 + 4\tau \cos \theta}} [k_x e^{i\pi} - \text{sgn} \cdot k_y e^{i\pi}] d\theta, \quad \bar{w} = Z + i(X \cos \theta + Y \sin \theta),
\]
\[ \alpha = \begin{cases} \cos^4 \frac{1}{4r} & (4r > 1) \\
 -i \cosh \frac{1}{4r} & (4r < 1) \end{cases} \]

\[ \varphi = \cos^4 \frac{X}{\sqrt{X^2 + Y}}, \quad \epsilon = \sinh^4 \frac{|Z|}{\sqrt{X^2 + Y}}, \quad \text{sgn} \epsilon = \text{sign}(\cos(\text{Re}(\theta))) \]

\[ X = K_1(x - \xi), \quad Y = K_1|y - \eta|, \quad Z = K_1(z + \zeta), \quad \tau = U \omega / g, \quad K_1 = g / U. \]

The generalized wave numbers here are represented by \( k_1 \) and \( k_2 \), and \( \text{sign} \) is the symbolic function.

If the ship is identified as a rigid body, the differential equation of ship motion response in waves can be derived by the rigid-body dynamics. The ship motion in six directions can be solved by the coupled equation in the seakeeping problem. Assuming that \( \eta_j^{\text{re}}, \eta_j^{\text{im}} \) are the real and imaginary part respectively for the \( j \)th mode, the ship-motion phase can be written as

\[ \alpha_j = \text{arg}(\eta_j^{\text{re}}, \eta_j^{\text{im}}) \quad j = 1 \sim 6 \]

The defined phase above is periodic and set to interval (-180°, 180°) in present paper.

### 3. Validation of motion amplitude and phase for single ship

As the object of modified Wigley ship, validation of single-ship motion amplitude and phase is applied at speed-Froude number \( Fr \) equalling to 0.2. The numerical results of motion amplitudes and phases for surge, heave and pitch modes are calculated with the mathematical kernel of three-dimension translating-pulsating source Green function, which are illustrated as Figures 2–7. The model-test results by Iwashita [8] are also given. If the oscillatory motion period \( T (T = 2\pi / \omega_j) \) corresponds to the encounter frequency, the peak of incident wave is positioned at the ship’s gravity centre on \( t / T = 0 \) and ship oscillatory motion of all modes can be defined as the cosine form. According to the thinking above, the sequential time form of ship motion in six degrees of freedom can be written as follows

\[ |\eta_j| \cos(\omega_j t + \alpha_j) \quad j = 1 \sim 6 \]

The following figures show that the calculations by three-dimension translating-pulsating source Green function are well in agreement with the experimental results for the surge, heave and pitch mode motions of modified Wigley ship, which verifies the feasibility in the numerical prediction of motion amplitude and phase for a single ship.

![Figure 2](image1.png)  
**Figure 2.** Amplitude of surge mode motion for the modified Wigley ship.

![Figure 3](image2.png)  
**Figure 3.** Phase of surge mode motion for the modified Wigley ship.
4. Validation of motion amplitude and phase for two ships

Validation test is carried out in a long tank and two models numbered as ship $a$ and $b$ are towed in head regular waves. The sway and yaw motions are constrained and surge, heave, roll and pitch motions are free for the two tested models. In the acquisition of physical quantities, optical system of Qualisys Tracker Manager (QTM) is to measure the dynamic motion response of ship $a$ and $b$ [9]. In order to identify the amplitude and phase of QTM signals with high precision in the two-model test, a new approach based on all-phase time-shift phase difference technology is applied to analysis the comprehensive features of collected signals [10]. According to the description of validation test, four modes of motion response are zero. Then, the coupled motion equations of two ships can be simplified as eight equations. The number of redundant equations is consistent with that of restrained motion modes for two ships. The specific parameters are described as follows: the speed-Froude number $Fr$ is 0.175, the transversal space $D_y$ is 0.372$L_a$ and the longitudinal space $D_x$ is 0. The results of calculation and experiment are illustrated as Figures 8–16 for the motion amplitude and phase of ship $a$ and $b$.

From the analysis of figures below, surge, heave and pitch amplitudes of ship $b$ are less than those of ship $a$ at many wavelength conditions in the same height of incident wave. For instance, the heave-motion amplitude of ship $a$ is 1.35 times as that of ship $b$ in the resonant region. Through the compare with the solid lines and scattered points in following figures, the numerical calculations of two-ship motion amplitude and phase agree well with the tested data, which is validated the applicability of frequency-domain three-dimension translating-pulsating source Green function in the longitudinal motion problem for two ships advancing at close proximity in waves. Combined with the analysis of a single ship and two bodies, the surge-mode motion amplitude is relatively small in short waves and gradually increasing if the incident wave length is greater than a certain value. Obviously, there are some specific characteristics of two-ship problem, such as the two-ship heave amplitudes demonstrate a local peak in the resonant region nearby, which is related to the great amplitude of radiation force.
is a pity that the surge amplitude and phase are only captured in short waves during the two-model test, whose numerical calculations are fit with those in the validation of single-ship problem.

**Figure 8.** Compare for the amplitude of surge mode motion for ship a.

**Figure 9.** Compare for the amplitude of surge mode motion for ship b.

**Figure 10.** Compare for the phase difference of surge mode motion between ship a & b.

**Figure 11.** Compare for the amplitude of heave mode motion for ship a.

**Figure 12.** Compare for the amplitude of heave mode motion for ship b.

**Figure 13.** Compare for the phase difference of heave mode motion between ship a & b.
Figure 14. Compare for the amplitude of pitch mode motion response for ship $a$.

Figure 15. Compare for the amplitude of pitch mode motion for ship $b$.

Figure 16. Compare for the phase difference of pitch mode motion between ship $a$ & $b$.

5. Conclusions
Amplitude and phase are two important parameters for ship motion response in the seakeeping problem. The surge, heave and pitch amplitude and phase of the modified Wigley ship are validated in the broad wavelength range. The hydrodynamic interaction of two floating bodies with forward speed at close proximity in waves is related with many motion modes. The results above demonstrate that the proposed method with the integral kernel of three-dimension translating-pulsating source Green function can make good prediction of the longitudinal motion amplitude and phase for two-ship hydrodynamics in waves, which hints the availability of translating-pulsating source in the calculations of two-ship seakeeping problems. The next step will be focused on the relative motion between the two-ship supply points in practical replenishment engineering.

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References
[1] Committee I.2.Loads. Proceedings of the 17th International Ship and Offshore Structures Congress, Seoul, Korea, 2009
[2] Committee I.2.Loads. Proceedings of the 18th International Ship and Offshore Structures Congress, Rostock, Germany, 2012
[3] Committee I.2.Loads. Proceedings of the 19th International Ship and Offshore Structures Congress, Cascais, Portugal, 2015
[4] Zhao R, Faltinsen O. Slamming loads on high-speed vessels. Proceedings of 19th Symposium on Naval Hydrodynamics, Seoul, Korea, 1992, p 159-178
[5] Duan W Y, Hudson D A, Price W G. Theoretical prediction of the motions of fast displacement vessels in long-crested head seas. *Proceedings of 3rd International Conference for High Performance Marine Vehicles, Shanghai, China, 2000*, p 1-11

[6] Hess J L, Smith A M O. Calculation of nonlifting potential flow about arbitrary three dimensional bodies. *Journal of Ship Research*, 1964, 8(2), p 22-44

[7] Bessho M. On the fundamental singularity in the theory of ship motions in a seaway. *Memoirs of the Defense Academy Japan*, 1977, 17(8), p 95-105

[8] Iwashita H, Kashiwagi M, Elangovan M, et al. On an unsteady wave pattern analysis of ships advancing in waves. *Journal of the Japan Society of Naval Architects and Ocean Engineers*, 2011, 6(13), p 95-106

[9] Xiao W B, Dong W C, Wu H. Application of optical system QTM in the towing tank tests for two ship models. *Journal of Ship Mechanics*, 2014, 18(12), p 1424-1433

[10] Xiao W B, Dong W C. A frequency spectrum analysis method based on all-phase time-shift phase difference for the measured signals of ship model test. *Journal of Ship Mechanics*, 2013, 17(9), p 998-1008