Crossed Andreev reflection in a $d$-wave superconductor with two quantum point contacts

S. Takahashi, T. Yamashita, and S. Maekawa

Institute for Materials Research, Tohoku University, Sendai 980-8577, Japan

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We theoretically study the crossed Andreev reflection in a hybrid nanostructure which comprises a $d$-wave superconductor and two normal-metal quantum wires. When the superconductor of the (110) oriented surface is in contact with the wires parallel and placed close to each other, the Andreev bound state is formed by the crossed Andreev reflection. When the contact barrier potential is sufficiently large, two sharp peaks appear in the conductance well below the gap structure, which originate from the bonding and antibonding Andreev bound states. We propose that these Andreev bound states form a two-level quantum system (qubit).

Quantum transport in nanostructures is of current interest in both theoretical and experimental studies. In a quantum wire, its width is so narrow that electron waves are strongly confined in the transverse direction and their transverse momentums are quantized. In a ballistic conduction of electrons through a quantum point contact with width comparable to the Fermi wavelength, a step-like structure with step of $2e^2/h$ appears in the conductance as a function of Fermi energy or width [1–3].

One of the fundamental consequences of superconductivity is the Andreev reflection at the interface of a normal-metal and a superconductor (SC) [4,5]. This phenomenon corresponds to an incoming electron from the normal side being reflected as a hole, thereby adding a Cooper pair in the superconducting condensate. In a tunnel junction of a normal-metal and a (110) oriented $d$-wave SC, the zero bias conductance peak appears due to the formation of the Andreev bound state at the interface [6,7]. However, when a single quantum wire of a single conducting channel is in contact with $d$-wave SC of the (110) oriented surface, the Andreev reflection is completely suppressed due to the quantum mechanical diffraction of electron waves at the narrow opening [8].

A basic question arises what happens if two quantum wires are in contact to the (110) oriented $d$-wave SC (see Fig. 1). When an electron is injected into SC from one of the wires, there is a possibility that the Andreev hole is reflected back into another wire due to the non-local effect called the crossed Andreev reflection (CAR) [9,10], thus providing an ideal system to study CAR.

In this Letter, we explore quantum-interference effects due to the crossed Andreev reflection in a $d$-wave SC with two quantum wires that are parallel and placed close to each other. It is shown that the resonance peak in the conductance is split into two sharp peaks at low energies when the barrier potential of the contact is sufficiently large. The lower and higher energy peaks correspond to the bonding and the antibonding Andreev bound states, respectively. This suggests that these Andreev levels form a two-level quantum system (qubit), whose population can be controlled by application of bias voltage and/or electromagnetic field, exhibiting a coherent oscillation of the Andreev qubit (Rabi oscillation).

We examine the quantum transport in a hybrid nanostructure of a $d$-wave SC and two normal-conducting quantum wires. Figure 1 shows a model structure in the $x$-$y$ plane with a two-dimensional (2D) $d$-wave SC occupying the left half space, and two quantum wires of width $w$, lead 1 and lead 2, which are parallel along $x$ and connected to SC at $y = \pm L/2$. The wave functions of electron and hole like quasiparticles with excitation energy $E$ in the electrodes are determined by the Bogoliubov-de Gennes equation

$$
\left( \begin{array}{cc} \mathcal{H}_0 & \Delta(k, r) \\ \Delta^*(k, r) & -\mathcal{H}_0 \end{array} \right) \left( \begin{array}{c} u_k(r) \\ v_k(r) \end{array} \right) = E \left( \begin{array}{c} u_k(r) \\ v_k(r) \end{array} \right),
$$

where $\mathcal{H}_0 = -\hbar^2 \nabla^2 / 2m - \epsilon_F$ is the single-particle Hamiltonian with the Fermi energy $\epsilon_F = \hbar^2 k_F^2 / 2m$. For simplicity, the Fermi wave number $k_F$ and the effective mass $m$ are common for all electrodes, and the amplitude of the gap function $\Delta(k, r)$ is uniform in SC and vanishes in the wires. In an anisotropic SC, the pair potential $\Delta(k, r)$ is a function of wave vector $k$, and its value is defined on the Fermi surface in the direction of $k$.

![FIG. 1. Schematic diagram of two-leg normal-conducting quantum wires in contact with a $d$-wave superconductor.](image-url)
function of an electron propagating along the x axis with wave number \( k \) in the nth subband is given by the product of the plane wave \( e^{ikx} \) and the nth transverse wave, \( \chi_n(y-L/2) \) in lead 1 and \( \chi_n(y+L/2) \) in lead 2, where \( \chi_n(y) = (2w)^{1/2} \sin \left[ (n \pi / w) \left( y + w/2 \right) \right] \), and has the energy \( E = \left( h^2 / 2m \right) |k^2 + (n \pi / w)^2| - \epsilon_F \). In the following, we restrict our calculation to the case where only the lowest subband (\( n = 1 \)) is occupied by electrons (or holes), which is realized for those wires satisfying \( \pi < k_F w < 2 \pi \). When an electron with energy \( E \) and wave number \( k_1 = [2mE + k_F^2 - (\pi/w)^2]^{1/2} \) is incident from lead 1 into SC, the wave functions in leads 1 and 2 are given by \( \Psi_1(x,y) = \varphi_1(x)\psi_1(y) \) and \( \Psi_2(x,y) = \varphi_2(x)\psi_2(y) \), where

\[
\varphi_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{ik_1x} + r_{11}^{ee} \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-ik_1x} + r_{11}^{eh} \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{ik_1x}, \\
\varphi_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-ik_1x} + r_{12}^{ee} \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{ik_1x}.
\]

(3)

Here, \( r_{11}^{ee} \) and \( r_{12}^{eh} \) are the amplitudes of the normal reflection (NR) and Andreev reflection (AR), respectively, while \( r_{12}^{ee} \) and \( r_{11}^{eh} \) are those of the crossed normal reflection (CNR) and the crossed Andreev reflection (CAR), respectively. A similar treatment is made for an incident electron from lead 2 into SC, the wave functions in leads 1 and 2 are given by \( \Psi_1(x,y) = \varphi_1(x)\psi_1(y) \) and \( \Psi_2(x,y) = \varphi_2(x)\psi_2(y) \), where

\[
\varphi_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{ik_1x} + r_{11}^{ee} \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-ik_1x} + r_{11}^{eh} \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{ik_1x}, \\
\varphi_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-ik_1x} + r_{12}^{ee} \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{ik_1x}.
\]

(2)

The barrier potential at the interface between the wires and SC is taken into account by the \( \delta \)-function-type potential with amplitude \( h^2 k_F / 2m ) Z \), \( Z \) being a dimensionless parameter [5]. The boundary conditions for the wave functions at the interfaces are \( \Psi_s(0,y) = \Psi_i(0,y) \) and \( [\partial_x \Psi_s(x,y) - \partial_x \Psi_i(x,y)]_{x=0} = k_F Z \Psi_1(0,y) \) for the \( \delta(x) \) potential. The matching technique to the boundary conditions [3] yields the reflection coefficients

\[
r_{11}^{ee} = -1 + \frac{k_1}{D_+} (k_1 + F - G_c - iZ),
\]

(5)

\[
r_{11}^{eh} = -\frac{k_1}{D_-} (1 - \frac{Z}{D_+}),
\]

(6)

\[
r_{12}^{eh} = -\frac{2k_1}{D_+ D_-} (G_c - iZ)^2 - (G_e^2 + G_s^2),
\]

(7)

\[
r_{12}^{ee} = -\frac{k_1}{D_-} (1 + \frac{Z}{D_+}),
\]

(8)

with \( k_1 = k_F / \sqrt{D_+} \) and \( D_+ = (\bar{F} + iZ) / (k_1 + F + iZ), \) \( F = \int_{-k_F}^{k_F} dp / 2 \pi E\sqrt{1 - (p / k_F)^2} \phi(p), \) \( G_c = \int_{-k_F}^{k_F} dp / 2 \pi E\sqrt{1 - (p / k_F)^2} \phi(p) \cos(p L), \) \( G_s = \int_{-k_F}^{k_F} dp / 2 \pi E\sqrt{1 - (p / k_F)^2} \phi(p) \sin(p L), \)

(9)

(10)

(11)

and \( \phi(p) = \langle p | \chi_1 \rangle \) is the overlap integral of \( \chi_1(y) \) and \( e^{i p y} \):

\[
\phi(p) = \sqrt{8 w / \pi^2} \cos(p w / 2) / [1 - (p w / \pi)^2].
\]

(12)

Note that \( F \) in Eq. (9) represents the local coupling and is independent of distance \( L \), while \( G_c \) and \( G_s \) represent the non-local coupling and are dependent on \( L \). In the limit of \( L \rightarrow \infty \), where the two contacts are independent (\( G_c = G_s = 0 \)), one has \( r_{11}^{ee} = (k_1 - F - iZ) / (k_1 + F + iZ), \) \( r_{12}^{eh} = r_{12}^{ee} = 0, \) recovering the complete suppression of AR in a single quantum wire with a single transverse mode [8]. The reflection coefficients \( r_{11}^{ee}, r_{12}^{ee}, r_{11}^{eh}, \) and \( r_{12}^{eh} \) for an incident electron from lead 2 are obtained from those in Eqs. (5)-(8) by the replacement \( 1 \leftrightarrow 2 \) and \( L \rightarrow -L \). It follows from Eqs. (6)-(8) that, when there is no barrier potential (\( Z = 0 \)) at the interface, AR is absent (\( r_{11}^{ee} = 0 \)), whereas CNR and CAR are finite with the ratio \( r_{12}^{eh} / r_{12}^{ee} = G_c / G_s \).

When bias voltage \( V \) is applied to the two leads, the conductance \( G \) at zero temperature \( (T = 0) \) is given by

\[
G = \frac{4 e^2}{h} (1 - |r_{11}^{ee}|^2 - |r_{12}^{ee}|^2 - |r_{11}^{eh}|^2 - |r_{12}^{eh}|^2) E = e V, \]

(13)

where \( h \) is the Planck constant. Figure 2 shows the conductance \( G \) vs \( V \) for \( k_F w = 4, k_F L = 8, \) and different values of \( Z \). The conductance is normalized by the normal state value \( G_N = G(\Delta = 0) \), which takes \( G_N \sim 16(e^2 / h) k_F Z^2 (Z \gg 1) \) with \( F_N = F(\Delta = 0) \).
For a low barrier potential of small $Z$, the conductance decreases monotonically with decreasing $eV$ below $\Delta_0$. As the barrier potential becomes higher, a peak structure appears well below $\Delta_0$ and shifts towards lower $eV$, developing the double peak structure with increasing $Z$. If the conductance is plotted as a function of normalized voltage $ZV$ as shown in the inset, the conductance peaks fall into the same position, indicating that the resonance peak positions are scaled by $1/Z$.

Let us examine the origin of the double peak structure in the conductance for the tunneling case ($Z \gg 1$). Since $\Omega \approx |\Delta|/Z$ for $E \ll \Delta_0$, $F$, $G_c$, and $G_s$ in Eqs. (9)-(11) have the forms: $F \approx if/\Delta_0/E$, $G_c \approx ig_0\Delta_0/E$, and $G_s = ig_0\Delta_0/E$, where $f = (EF/\Delta_0)E_{\to 0}$, $g_c = (EG_c/\Delta_0)E_{\to 0}$, and $g_s = (EG_s/\Delta_0)$ are energy independent quantities, so that the Andreev reflection coefficients are calculated in the resonance forms

$$
\begin{align*}
G_{12}^{-} & \approx \frac{i\gamma}{E - E_+ + i\gamma} + \frac{i\gamma}{E - E_+ + i\gamma}, \\
G_{12}^{+} & \approx \frac{i\gamma}{E - E_- + i\gamma} - \frac{i\gamma}{E - E_- + i\gamma},
\end{align*}
$$

where $g = g_s/2f$, $E_{\pm} = (f \pm |g_c| + g_s^2/2f)\Delta_0/Z$, and $\gamma = (k_1/k_F)f\Delta_0/Z^2$. The other coefficients are $r_{11}^{\pm} \approx -1 - gr^{\pm}_{12}$ and $r_{12}^{\pm} \approx O(g^2)$. The resonance energies $E_{\pm}$ and intensity $g$ depend strongly on lead separation $L$, exhibiting damped oscillations with the period of the Fermi wave length $\lambda_F = 2\pi/k_F$. We note that the resonance positions $E_{\pm}$ and their separation $E_{+} - E_{-}$ are scaled by $1/Z$, while the line width $\gamma$ is scaled by $1/Z^2$. For $Z \gg 1$, the line width of the peaks is much smaller than the separation, and therefore a well-separated two-peak structure is formed as shown in the inset of Fig. 2. In this case, the conductance $G$ is written as the sum of two Lorentzians

$$
G_{GN} \approx \frac{I\gamma/\pi}{(eV - E_+)^2 + \gamma^2} + \frac{I\gamma/\pi}{(eV - E_-)^2 + \gamma^2},
$$

where $I = (\pi q^2 f/4F_N)$. The simple formula (16) reproduces the numerical result in the inset of Fig. 2, if the calculated values ($f = 0.405$, $g_s = -0.036$, $g_s = -0.0345$, and $F_N = 0.68$) are used in Eq. (16). Note that the peak height of $G_{GN}$ increases in proportion to $Z^2$.

To elucidate the formation of the Andreev bound states, we calculate the QP wave function $\Psi_s(x, y) = \psi_s^+(x, y)\Psi_e^+(x, y)\Psi_h^+(x, y)\Psi_h^+(x, y)$ in SC, where $\Psi_e^+$ and $\Psi_h^+$ are the electron and hole wave functions, respectively. Figure 3 shows the mapping of the absolute squares, $|\psi_s^+|^2$ and $|\psi_s^h|^2$, at the resonance energies on the $xy$ plane for $k_F w = 5$, $k_F L = 8$, and $Z = 50$, when an electron is incident from lead 1. It is clearly seen that the QP wave functions are strongly localized with very large peaks (red color) near the contacts due to the formation of the Andreev bound states, and that the decaying QP waves are traced to the (010) and (001) directions along which multiple reflections of electron and hole take place. It is also seen that the formation of the localized states, especially the relative position of the peaks with respect to the contacts, is different between the electron and hole QPs, and between the resonance energies $E_{\pm}$. For $Z \gg 1$ and around $E_{\pm}$, the explicit form of $\Psi_s$ inside SC is obtained as

$$
\psi_s^\pm = \frac{i\gamma}{E - E_\pm + i\gamma} \left( \psi_e^\pm \frac{1}{\sqrt{2}} \cos \frac{\pi q y}{2} \sin \frac{\sqrt{2}\pi q x}{2} \right),
$$

where, except very close to the interface ($x \approx 0$),

$$
\begin{align*}
\left( \psi_e^\pm \right) & = \frac{2g_i}{\pi} \int_0^{k_F} \frac{\Delta}{E} \cos \frac{pL}{2} \sin q x \left( \sin \frac{\sqrt{2}\pi q y}{2} \cos \frac{\sqrt{2}\pi q y}{2} \right) dp, \\
\left( \psi_h^\pm \right) & = \frac{2g_i}{\pi} \int_0^{k_F} \frac{\Delta}{E} \sin \frac{pL}{2} \sin q x \left( \cos \frac{\sqrt{2}\pi q y}{2} \sin \frac{\sqrt{2}\pi q y}{2} \right) dp,
\end{align*}
$$

with $q = \sqrt{k_F^2 - p^2}$ and $\Delta = 2\Delta_0 q g / k_F$. At lower resonance energy $E_-$, $\Psi_s$ is dominated by the first term whose electron (hole) wave function $\psi_e^-$ ($\psi_h^-$) is an odd (even) function of $y$. At higher resonance energy $E_+$, $\Psi_s$ is dominated by the second term whose electron (hole) wave function $\psi_e^+$ ($\psi_h^+$) is an even (odd) function of $y$. These results indicate that the electron (hole) wave functions at the lower and higher bound states have different parity with respect to $y$. It is noteworthy that the Andreev holes are reflected back into leads 1 and 2 in phase at $E_-$ and out of phase at $E_+$ (cf. Eqs. (14) and (15)).

The Andreev bound states appeared in the present system have the following implication. When the interface barrier potential is very high ($Z \gg 1$), the current flows through the system via the Andreev bound states with very sharp energy levels at $E_{\pm}$. When the bias voltage $V$ is set between $V_- = E_-/e$ and $V_+ = E_+/e$ of the conductance peaks, the lower bound state at $E_-$ is occupied by Andreev quasiparticles while the higher-energy bound state at $E_+$ is empty; the lower bound state is viewed as
prizes a d-wave superconductor and normal-conducting quantum wires. When a superconductor of the (110) oriented surface is in contact with the two-leg quantum wires via tunnel barriers, the resonance bound states are formed at low energies due to the crossed Andreev reflection. As a consequence, two well-separated sharp peaks appear in the conductance well below the superconducting gap structure. The lower and higher conductance peaks correspond to the bonding and the antibonding Andreev bound states whose wave functions have different parity. We propose that these Andreev bound states form a two-level quantum system (d-wave Andreev level qubit), which can be controlled by application of bias voltage and/or radiation of oscillating electric field.

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FIG. 3. Mapping of Andreev bound states on the xy plane, when an electron is incident from lead 1 to a d-wave SC. (a) and (b) are the absolute square of electron wave functions $|\Psi_0^s|^2$ at energies $E_-$ and $E_+$; (c) and (d) are that of hole wave functions $|\Psi_0^h|^2$ at $E_-$ and $E_+$.

The ground state and the upper bound state as the excited state, and therefore the system in this setup forms a two-level quantum system (qubit). For simplicity, we denote the ground state and excited state by $|0\rangle$ and $|1\rangle$, respectively. The Andreev bound states of electron (hole) QP at $E_-$ and $E_+$ have dipole moments $\langle 1\rangle |y|0\rangle$ due to the different parity of the states with respect to $y$. Therefore application of the electric field $eE_0 \cos(\omega t)$ oscillating along $y$ causes a significant Rabi oscillation $\sim eE_0 (1) |y|0\rangle /\hbar$ between the Andreev bound states, if $\omega$ is tuned to the resonance condition $\omega = E_+ - E_-$. and the damping rate $\gamma$ of the states is substantially smaller than $\hbar \omega$. The significant coherence oscillation requires a smaller damping $\gamma$, which is achieved by inserting tunnel barriers at the interfaces. The controlled evolution between the two states $|0\rangle$ and $|1\rangle$ is realized by applying resonant microwaves to the system. The two level Andreev bound states in the proposed system provides a new possibility of superconducting qubit (d-wave Andreev level qubit) [12].

The realization of quantum two-level systems (qubit) is fundamental issues in both physics and information technologies, because qubits are basic elements for quantum computation [13]. Recently, the coherent manipulation of the states in two-level systems has been demonstrated in nanostructured superconducting circuits [14,15].

In summary, we have theoretically studied the quantum-interference effects caused by the crossed Andreev reflection in a hybrid nanostructure which comprises a d-wave superconductor and normal-conducting quantum wires. When a superconductor of the (110) oriented surface is in contact with the two-leg quantum wires via tunnel barriers, the resonance bound states are formed at low energies due to the crossed Andreev reflection. As a consequence, two well-separated sharp peaks appear in the conductance well below the superconducting gap structure. The lower and higher conductance peaks correspond to the bonding and the antibonding Andreev bound states whose wave functions have different parity. We propose that these Andreev bound states form a two-level quantum system (d-wave Andreev level qubit), which can be controlled by application of bias voltage and/or radiation of oscillating electric field.

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