Implementing a Quantum CNOT and Quantum Memory Using a Quantum-Dot Cellular Automata Array

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Abstract—In this work, we develop a method to use Quantum-Dot Cellular Automata (QCA) for universal quantum computing. This method is based conceptually on refocusing in NMR systems. We show how an array of QCA cells can be used for isolated single qubit, as well as multi-qubit operations, such as the CNOT, by dividing the cells into active and passive sets such that the active cells undergo the transform while passive cells hold their quantum states. The same technique is used for developing a multi-qubit quantum memory. The effect of imperfect control parameters is discussed and the total time for a typical quantum operation is given. Using this approach, different quantum gates are implemented using pulse sequences applied to the QCA cells via control fields and potentials, while the layout remains the same.

Index Terms—QCA, quantum Computer, qubit, CNOT, quantum memory.

I. INTRODUCTION

QUANTUM computation was first introduced by Richard Feynman. Since then, many implementation for a universal quantum computer have been introduced [1]–[5]. One of the many potential implementations of quantum computing that has received less attention is quantum computing based on Quantum-Dot Cellular Automata(QCA). QCA was first introduced in [6] as a novel computation nanotechnology based on arrays of quantum dots and much of the research focus has been on implementations of digital logic circuits. In [7] the authors suggest the potential of QCA for universal quantum computing. In the current work, we extend the proposal in [7] by introducing a completely different method for multi-qubit quantum operations, such as CNOT. In the next section, we propose a QCA quantum CNOT. We’ll use a technique based on NMR refocusing [9]. This is possible because of the similarity between an array of QCA cells and NMR systems. Both can be modeled as coupled spin systems and described by a similar Hamiltonian. In Section 3 we discuss a method to realize a N qubit quantum memory. Section 4 is dedicated to isolated quantum operations on a QCA line. Section 5 explains briefly the effect of imperfect control parameters and finally we conclude this paper with a short discussion of the results we obtained.

II. CNOT OPERATION

The authors of [7] introduced methods for single qubit operations on QCA as well as a method for performing the CNOT operation using three QCA cells. This suggest the possibility of using a line of QCA cells as a quantum computer. However, when a single qubit operation is performed on one cell in a line of N QCA cells, the Hamiltonian of the whole system changes and this affects the state of the entire system because of the coupling terms in the Hamiltonian. In order to perform single qubit operations in isolation, one needs to decouple the controlled cell from the rest of the system which initially seems difficult since the coupling coefficients are fixed based on the physical layout of the circuit. As mentioned earlier, we propose a scheme based in refocusing, commonly applied in NMR. We attempt to emulate a NMR quantum computer using a linear array of QCA cells and will show that this enables QCA to be used as a universal quantum computer.

The assumption here is that the QCA implementation allows for system wide coherence and clock pulses that are much shorter than the decoherence time. It should be noted that estimates of decoherence in QCA suggest this may be difficult to realize, although recent progress in other related implantations a of quantum computing seem to ultimately allow us to bridge the gap.

Based on the model provided in [7] for a clocked array of two QCA cells, the Hamiltonian is:

$$\hat{H}_{\text{sys}} = -\gamma_1 \hat{\sigma}_x (1) - \gamma_2 \hat{\sigma}_z (2) - E_1 \hat{\sigma}_z (1) \hat{\sigma}_z (2) + E_0 P_{\text{bias},1} \hat{\sigma}_z (1) + E_0 P_{\text{bias},2} \hat{\sigma}_z (2).$$

(1)

The structure of a controllable two qubit QCA line in illustrated in Figure 1. Also it is shown in [8] that a CNOT operation can be decomposed into a set of unitary operations where

$$U_{\text{CNOT}} = e^{(-i \frac{\pi}{4} \hat{\sigma}_z (2))} e^{(i \frac{\pi}{4} \hat{\sigma}_z (1))} e^{(-i \frac{\pi}{4} \hat{\sigma}_z (2))} e^{(i \frac{\pi}{4} \hat{\sigma}_z (1))} e^{(-i \frac{\pi}{4} \hat{\sigma}_z (2))}.$$

(2)

Our approach towards preforming this CNOT operation is through successive application of each part on our system (i.e., we first apply $e^{(-i \frac{\pi}{4} \hat{\sigma}_z (2))}$ and then $e^{(i \frac{\pi}{4} \hat{\sigma}_z (1))}$). Here we explain how each of these single and double rotations can be realized by the controllable Hamiltonian of Eq. 1. We start with the term $e^{(i \frac{\pi}{4} \hat{\sigma}_z (1))}$ which implements a two-cell rotation. To apply this unitary transform one simply needs to set all the biases and $\gamma$ coefficients to zero and therefore the unitary transform applied on the system will be
We can achieve our desired double rotation by varying $\Delta T$. In our case $\Delta T = \frac{15\pi}{16\hbar}$, where the phase $\frac{15\pi}{16}$ is effectively $\frac{\pi}{4}$ (considering the fact that a $2\pi$ rotation introduces a minus phase to the system).

In order to apply the term $e^{i\frac{\pi}{2}\sigma_z(i)}$, we simply need to eliminate the $\sigma_z(2)$ and the $\sigma_z(1)\sigma_z(2)$ terms in the Hamiltonian. As described in [8] [9],

$$
e^{i\frac{\pi}{2}\sigma_z(i)} = e^{i\frac{\pi}{2}\sigma_z(2)} e^{-i\frac{\pi}{2}\sigma_z(2)} = e^{i\frac{\pi}{2}\sigma_z(2)} e^{-i\frac{\pi}{2}\sigma_z(2)} e^{i\frac{\pi}{2}\sigma_z(2)} e^{-i\frac{\pi}{2}\sigma_z(2)} .$$

(3)

where $\Delta T = \frac{\pi\hbar}{8E_{\text{sys}}}$. Eq. 3 represents the unitary transform associated with the Hamiltonian in Eq 1 where all the $\sigma_z(2)$ terms have been eliminated. In order to apply $e^{i\frac{\pi}{2}\sigma_z(2)}$, which implements a $\pi$-rotation along the $x_2$-axis in Bloch sphere, we set all the control parameters except for $\gamma_i$ to zero and $\gamma_2 = \gamma_{\text{max}}$ (i.e., maximum possible value for $\gamma$) therefore we have $H_{\text{sys}} \simeq -\gamma_2\sigma_z(2)$ and the unitary transform applied on the state of the system for a time period $\Delta T_{\gamma_2}$ would be $e^{i\frac{\pi}{2}\sigma_z(2)} e^{i\Delta T_{\gamma_2}}$ where $\Delta T_{\gamma_2} = \frac{\pi\hbar}{2\gamma_2}$ for a $\pi$-rotation along the $x_2$-axis in the Bloch sphere. By substituting the result in Eq. 3:

$$e^{i\frac{\pi}{2}\sigma_z(1)} e^{i\frac{\pi}{2}\sigma_z(2)} e^{i\Delta T_{\gamma_2}} e^{-i\frac{\pi}{2}\sigma_z(2)} e^{-i\Delta T_{\gamma_2}} .$$

Note that $e^{i\frac{\pi}{2}\sigma_z(2)}$ can be achieved through exactly the same procedure but by replacing $\gamma_2$ with $\gamma_1$. Up to this point we have described all the necessary unitary transforms required for performing a CNOT except for the rotations along the $y$-axis, for example the first and last terms in Eq. 2. However, these rotations can also be achieved through rotations along the $x$-and $z$-axis and therefore an equivalent representation for CNOT would be

$$U_{\text{CNOT}} = \sqrt{e^{i\frac{\pi}{2}\sigma_z(i+1)} e^{i\frac{\pi}{2}\sigma_z(i+1)} e^{i\frac{\pi}{2}\sigma_z(i+1)} e^{i\frac{\pi}{2}\sigma_z(i+1)} e^{i\frac{\pi}{2}\sigma_z(i+1)} e^{i\frac{\pi}{2}\sigma_z(i+1)} .}$$

III. QUANTUM MEMORY ON A QCA LINE

A $N$-qubit quantum memory device is a circuit for which the unitary evolution is represented by the identity operator over a certain time period and as a result, the state remains unchanged. Here we briefly describe how this can be done on a $N$-qubit QCA line using the technique described in the previous section. Consider the Hamiltonian of a line of $N$-QCA cells,

$$\hat{H}_{\text{sys}} = -\sum_i \gamma_i \hat{\sigma}_z(i) - \sum_i E_i \hat{\sigma}_z(i) \hat{\sigma}_z(i+1) + \sum_i E_0 P_{\text{bias}} \hat{\sigma}_z(i) .$$

(4)

Assuming the system $\hat{H}_{\text{sys}} = 0$ (i.e., a $N$-QCA Hamiltonian where all the $\gamma_i$'s are set to zero) is in state $|\phi\rangle$ right after a procedure and we want this state available at some time $T$, $T \geq \Delta T_{\text{memory}}$. Based on the commutation relations for Pauli matrices we know that

$$\left[ \prod_{j=1}^N e^{i\frac{\pi}{2}\sigma_z(i)} \right] e^{i\frac{\pi}{2}\sigma_z(i)} e^{i\frac{\pi}{2}\sigma_z(i)} e^{i\frac{\pi}{2}\sigma_z(i)} = I .$$

(5)

We have already discussed how $e^{i\frac{\pi}{2}\sigma_z(i)}$ can be produced by setting $\gamma_i = \delta_i$ and letting the time elapse for $\Delta T_{\gamma_i} = \frac{\pi\hbar}{2E_i}$ and therefore by successive application of the same method we can achieve $\prod_{j=1}^N e^{i\frac{\pi}{2}\sigma_z(i)}$. So general algorithm for the memory is as follows:

1) Apply $\prod_{j=1}^N e^{i\frac{\pi}{2}\sigma_z(i)}$ through the described procedure.
2) Set the Hamiltonian of the system back to $\hat{H}_{\text{sys}} = 0$ and let the time elapse for $\Delta T_{\text{memory}}$.
3) Apply $\prod_{j=1}^N e^{i\frac{\pi}{2}\sigma_z(i)}$.
4) Set the Hamiltonian to $\hat{H}_{\text{sys}} = 0$ for another $\Delta T_{\text{memory}}$.

Note that the total time the memory is held is $\Delta T_{\text{memory}} + 2 \sum_i \Delta T_{\gamma_i}$. One drawback of this memory is that it cannot be measured or used any time before the last unitary transform is completed and can only be measured or used in another memory cycle immediately after that.

One interesting characteristic of Eq. 5 is that $\sigma_z(i)$ and $\sigma_z(j)$ commute and therefore this equation can be rewritten as follows:

$$e^{i\sum_{j=1}^N \frac{\pi}{2}\sigma_z(i)} e^{i\Delta T_{\gamma_1}} e^{i\Delta T_{\gamma_2}} e^{i\Delta T_{\gamma_3}} .$$

(5-B)

This implies that all of the pulses can be applied simultaneously and if all of the $\gamma_i$'s are of the same amplitude and therefore $\Delta T_{\gamma_i}$ is constant, the total memory time will be $\Delta T_{\text{memory}} + 2 \Delta T_{\gamma_i}$. For the reminder of this work we will use Eq. 5-B instead of Eq. 5.

As an example, we will introduce a two qubit quantum memory. Note that the QCA circuit for such a quantum memory is not different than the QCA circuit for the two qubit CNOT gate (Figure 1). This is interesting because it shows that a QCA array is capable of performing different quantum operations and one only needs to vary the control parameters and not the layout.

The Hamiltonian of this system is described in Eq. 1. After setting $P_{\text{bias1}}$, $P_{\text{bias2}}$, and $\gamma_1$ to 0 the Hamiltonian will be

$$\hat{H}_{\text{sys}} = -\gamma_2 \hat{\sigma}_z(2) - E_1 \hat{\sigma}_z(1) \hat{\sigma}_z(2) .$$

Now using Eq. 5 results in the following unitary transformation applied on our system:

$$U = e^{i\frac{\pi}{2}\sigma_z(2)} e^{i\frac{\pi}{2}\sigma_z(i)} e^{i\frac{\pi}{2}\sigma_z(i)} .$$
Using the Taylor series expansion and Pauli matrices commutation relations one can show that the above expression is the same as the identity matrix with the exception of a global phase factor. The steps required to apply this transformation are as follows:

1. Set $\gamma_2 = \gamma_{\text{max}}$ on for $\Delta T_{\gamma_2} = \frac{\pi}{2} T_2$ seconds to perform the $\pi$-rotation.
2. Set $\gamma_2 = 0$ and allow the system to evolve for $\Delta T$.
3. Repeat these two steps one additional time.

The total time for which the state of the system will remain constant is $2\Delta T_{\gamma_2} + 2\Delta T$, where $\Delta T$ is decided upon based on the desired rotation and $\Delta T_{\gamma_2}$ is inversely proportional to $\gamma_2$. Therefore, if we can implement a large square $\gamma$ pulse then $\Delta T_{\gamma}$ will become negligible.

IV. DECOUPLED ROTATIONS

One issue that we raised in the previous section was that performing single qubit quantum operation will affect the Hamiltonian of the whole system. In order to overcome this issue, one should divide the QCA array into two sets of **Active** and **Passive** cells. Figure 2 shows such division on an array of QCA cells. If this is possible, then single (or multi-) qubit operations can be applied on Active cells while Passive cells are held constant with the quantum memory operation. Therefore, any unitary operation that we apply must have the form:

$$U_{\text{sys}} = U_{\text{Active}} \otimes I_{\text{Passive}},$$

where $U_{\text{active}}$ is the unitary operation applied on active systems while $I_{\text{passive}}$ is just the identity operator. This can be done through a similar procedure as in Eq. 5 with modifications to alter the procedure in a way that ends up in $U_{\text{sys}}$ instead of $I$. One such modification is that the $\pi$-rotations will only be applied to the passive cells to decouple them from the rest of the system. In order to achieve the desired unitary transform we break $U_{\text{Active}}$ into two parts so we define a unitary operator, $V = e^{-i\frac{\pi}{2} \Delta T}$, such that:

$$e^{\sum_{\sigma_z(i) \in \text{passive}} -i \frac{\pi}{2} \sigma_z(i)} Ve^{\sum_{\sigma_z(i) \in \text{passive}} -i \frac{\pi}{2} \sigma_z(i)} = U_{\text{Active}} \otimes I_{\text{Passive}}.$$  

(6)

The reason we needed to break the unitary transform into two parts with the same $\Delta T$ is that we want to achieve the condition described for Eq. 5 for the passive cells. To be more precise, $\hat{H}$ must be of the following form. $\hat{H} = \hat{H}_1 - \sum_{\text{passive}} E_i \hat{\sigma}_z(i) \hat{\sigma}_z(i + 1)$. What happens in Eq. 7 with this choice of $\hat{H}$ is that the unwanted couplings will be eliminated using the refocusing technique. We’ll have the following in accordance with Eq. 8, assuming $P_{\text{bias}} = 0$ for passive cells:

$$e^{\sum_{i=1}^P -i \frac{\pi}{2} \sigma_z^{\text{passive}}(i)} e^{-\frac{\pi}{2} \sum_{i=1}^P E_i \sigma_z(i) \sigma_z(i + 1) \Delta T} e^{\sum_{i=1}^P -i \frac{\pi}{2} \sigma_z^{\text{passive}}(i)} = e^{\left(-i \frac{\pi}{2} \frac{\Delta T}{\gamma_2}\right)} e^{\left(-i \frac{\pi}{2} \frac{\Delta T}{\gamma_2}\right)} = U_{\text{Active}} \otimes I_{\text{Passive}}.$$  

We propose and easy way to construct $V$ for single qubit operations as well as for quantum CNOT operation. For single qubit operations we set $P_{\text{bias}} = 0$ apply the $\gamma$-pulses and then let the system evolve over half the rotation time resulting in the application of $V$, followed by a second sequence of $\gamma$-pulses and then letting the system evolve for another half rotation. Note that our Hamiltonians for these half rotations are of the form of $\hat{H}$ where $H_1$ is the actual rotation. It can be inferred from the Eq. 4 and Eq. 8 and the above explanation that this will result in an isolated single qubit rotation. For the isolated CNOT(ICNOT), we have one coupled and four single qubit unitary transforms according to Eq. 2. The isolated coupled unitary transform $e^{i \hat{\sigma}_z(1) \hat{\sigma}_z(2)}$ can be achieved by applying the proper $\gamma$-pulses, setting all the $P_{\text{bias}}$’s to zero and letting the system evolve enough to achieve the $e^{i \hat{\sigma}_z(1) \hat{\sigma}_z(2)}$ which is the half the two-cell rotation and needs $\frac{\Delta T}{2}$ to be applied. Following this, we apply another set of $\gamma$-pulses and let the system evolve for another $\frac{\Delta T}{2}$ to produce an isolated rotation between the desired cells for the total time $\Delta T$.

As an example, suppose we have a four qubit line and we want to achieve $I_3 \otimes U_{2,3} \otimes I_4$ or $I_3 \otimes U_{4,3}$ where $U_{2,3} = e^{(-i E_2 \sigma_2(2) \sigma_3(3) \Delta T)}$. Now in order to apply $U_{2,3}$ we first set the $P_{\text{bias}} = 0$ and apply two $\gamma$-pulses $e^{(i \frac{\pi}{2} \hat{\sigma}_z(1) + i \frac{\pi}{2} \hat{\sigma}_z(2))}$ and then we set $\gamma_{1.2} = 0$. The Hamiltonian of the system according to Eq. 4 is:

$$\hat{H}_{\text{sys}4} = -\sum_{i=1}^3 E_i \hat{\sigma}_z(i) \hat{\sigma}_z(i + 1).$$

We let the system evolve with this Hamiltonian for $\frac{\Delta T}{2}$ then we apply the $\gamma$-pulses again and let the system evolve with $\hat{H}_{\text{sys}4}$ for another $\frac{\Delta T}{2}$. Figure 3 shows the sequence of $\gamma$-pulses as described above. Let’s define $I(U) = U e^{\sum_{\sigma_z(i) \in \text{passive}} -i \frac{\pi}{2} \sigma_z(i)} U e^{\sum_{\sigma_z(i) \in \text{passive}} -i \frac{\pi}{2} \sigma_z(i)}$. Where, $U$ is the unitary evolution we want to apply to the system. Now based on the above explanation the ICNOT gate on adjacent qubits $i$ and $i + 1$ is as follows:

$$\text{ICNOT} = I \left( e^{i \frac{\pi}{2} \sigma_z(i + 1)} \right) I \left( e^{-i \frac{\pi}{2} \sigma_z(i + 1)} \right) I \left( e^{i \frac{\pi}{2} \sigma_z(i + 1)} \right) I \left( e^{-i \frac{\pi}{2} \sigma_z(i + 1)} \right) I \left( e^{-i \frac{\pi}{2} \sigma_z(i + 1)} \right) I \left( e^{-i \frac{\pi}{2} \sigma_z(i + 1)} \right) I \left( e^{i \frac{\pi}{2} \sigma_z(i + 1)} \right) I \left( e^{i \frac{\pi}{2} \sigma_z(i + 1)} \right) \left( e^{-i \frac{\pi}{2} \sigma_z(i + 1)} \right).$$

(8)

More complicated circuits can be built using the isolated single qubit operations and ICNOT gates.

V. IMPERFECT $\gamma$ PULSES AND TOTAL COMPUTATION TIME

As one might know Eq. 4 is the reduced form of a 16 state Hamiltonian [9,10,11]. Where $\gamma_i$ and $E_i$ can be calculated directly from that Hamiltonian and are polynomial functions of the on site energies which are fixed and gate voltages which are variable. In theory they can be precisely determined by
changing the gate voltages. Note that for this purpose eight gates are needed per cell, four to control on site energies and four to control the tunneling energies of a pair of adjacent sites on a cell. We claim that \( \gamma_i \) can become sufficiently small or completely turned off. Here is how: According to [14, 15] for a pair of potential wells tunneling probability is proportional to overlap of the single electron wavefunction of each potential well. We can change this overlap using a pair of gates/electrodes with positive or negative charges to bend the electron orbitals of a quantum dot towards or away from each other (Figure 4). However in practice \( E_i = 0 \) cannot be achieved and therefore perfect \( \gamma \)-pulses do not exist. A realistic Hamiltonian in which we assume a fixed minimum \( E_i \) for when one applies the \( \gamma_j \)-pulse (for a \( \pi \)-rotation along \( X_j \)) will be as follows:

\[
\hat{H}_{\gamma_j} = -\gamma_j \hat{\sigma}_z(j) - \sum_{i \neq j}^{N} \epsilon_i \hat{\sigma}_z(i) - \sum_{i}^{(N-1)} E_i \hat{\sigma}_z(i) \hat{\sigma}_z(i+1),
\]

where \( \epsilon_i = \epsilon E_i \), and replacing \( \gamma_j E_j \) with \( \gamma_j \) so the strength of \( \hat{\sigma}_z(j) \) is a product of \( E_j \). we can have a simpler form for the above equation:

\[
\hat{H}_{\gamma_j} = -\gamma_j E_j \hat{\sigma}_z(j) - \sum_{i \neq j}^{N} \epsilon_i \hat{\sigma}_z(i) - \sum_{i}^{(N-1)} E_i \hat{\sigma}_z(i) \hat{\sigma}_z(i+1).
\]

(9)

Also we know that for a \( \pi \)-rotation we must have \( \Delta T_{\gamma_j} = \frac{\hbar}{\pi \gamma_j E_j} \) which is inversely proportional to \( \gamma_j \) therefore, for a large \( \gamma \) this value is comparatively short. We should estimate the differences between the unitary evolution for the imperfect \( \gamma \)-pulse and a perfect \( \gamma \)-pulse. From [12] we know for an arbitrary norm \( || \cdot || \):

\[
|| e^{i\Delta T_{\gamma_j} \frac{\hbar}{\gamma_j E_j}} - e^{i\Delta T_{\gamma_j} \frac{-\gamma_j E_j \hat{\sigma}_z(j)}{\hbar}} || \leq \Delta T_{\gamma_j} \frac{\hbar}{\gamma_j E_j} || e^{i\Delta T_{\gamma_j} \frac{\hbar}{\gamma_j E_j}} ||
\]

In order to go any further with this we need to chose a particular norm. It is typical to consider the absolute value of the largest eigenvalue of a matrix as a norm. This choice results in:

\[
|| e^{i\Delta T_{\gamma_j} \frac{\hbar}{\gamma_j E_j}} - e^{i\Delta T_{\gamma_j} \frac{-\gamma_j E_j \hat{\sigma}_z(j)}{\hbar}} || \leq || \pi (N-1) \frac{\hbar}{\gamma j} \frac{\pi (N-1)}{2 \gamma j} ||
\]

Therefore for our system to be almost error free we must have: \( \gamma \gg N \). Note that in the above we assumed the largest eigenvalue of \( -\sum_{i \neq j}^{N} \epsilon_i \hat{\sigma}_z(i) - \sum_{i}^{(N-1)} E_i \hat{\sigma}_z(i) \hat{\sigma}_z(i+1) \) is \( \sum_{i}^{(N-1)} E_i = (N-1)E \). In order to test our criteria for \( \gamma \) we ran a simulation for ten cells; we set the value for \( \epsilon \) as 0.1 and we increased \( \gamma \) from 10 to 50 the result is depicted in Figure 5. This figure shows how the greatest eigenvalue of the error matrix varies with respect to an increasing value of \( \gamma \).

The total time required for implementing an algorithm can be computed considering the amount of time for each operation. Also having parallel operations on the QCA line helps in minimizing the time and one must design their algorithms based on this fact and a new programming paradigm seems to be necessary particularly for this device. The time for single qubit operations is \( T_{\text{operation}} = O(\frac{\hbar}{\gamma_i E_i}) \) and for two qubit operations \( T_{\text{operation}} = O(\frac{\hbar}{\gamma_{ij} E_{ij}}) \). Through out the article we described the time needed for various quantum operations. Here we compare a double rotation time of an implementation with the coherence time of such system. Imagine a QCA line of arbitrary length. As we described earlier the amount of time needed for a decoupled rotation is \( 2\Delta T_{\gamma} + 2\Delta T \) where we assumed all of the gamma pulses are applied at the same time. we also mentioned in section II that the time needed for a double rotation is \( \frac{15\pi \hbar}{4\gamma} \) where the double rotation is being applied on the cell \( i \) and \( i+1 \). For a specific implementation of molecular QCA [13], we know \( E_i \) and \( \gamma_i \) are of the order of a portion to a few electron volts. Therefore \( 2\Delta T_{\gamma} + 2\Delta T \propto 10^{-15} \text{ seconds} \). It is reasonable to think that the coherence time for a molecular QCA is of \( 10^{-9} \text{ seconds} \) order. Another implementation could be the dangling bonds on silicon surface as introduced in [16] where the decoherence is calculated to be \( \propto 10^{-8} \text{ seconds} \) while the tunneling rate is \( \propto 10^{14} \text{ seconds}^{-1} \). The method that we explained in the previous section can be used for controlling \( \gamma_i \).

VI. Conclusions

Two of the key components of a quantum computer, i.e. quantum CNOT and quantum memory, have been realized theoretically on a QCA circuit through refocusing. This suggests that a universal quantum computer can be built on a fixed layout of an array of QCA cells. The refocusing is performed by applying high amplitude \( \gamma \)-pulses. The dependence of the proposed method on high amplitude \( \gamma \)-pulses can be an issue for long arrays as the lower bound for \( \gamma_{\text{max}} \) is linearly proportional to the length of the array. This issue however would be solved by choosing QCA implementations that allow large \( \gamma_{\text{max}} \).
Fig. 3. The sequence of $\gamma$ pulses and the unitary operator, $V$, for the example system.

$$V = e^{\frac{i}{\hbar} \alpha \frac{\Delta T}{2}}$$

Fig. 4. Proposed method for changing $\gamma_i$ in dangling bond qubit implementations. The dots are dangling bonds and the electron presence is depicted inside and outside the silicon. Two charged electrodes are being used for reshaping the electronic profile.

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Fig. 5. $L - 2$ norm of error for $\gamma$ pulse in a single QCA cell with respect to the amplitude of $\gamma_{max}$ normalized by $E_0$.

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