A NEW CHALLENGE TO SOLAR DYNAMO MODELS FROM HELIOSEISMIC OBSERVATIONS: THE LATITUINAL DEPENDENCE OF THE PROGRESSION OF THE SOLAR CYCLE

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ABSTRACT

The onset of the solar cycle at mid-latitudes, the slowdown in the drift of sunspots toward the equator, the tail-like attachment, and the overlap of successive cycles at the time of minimum activity are delicate issues in models of the $\alpha\Omega$ dynamo wave and the flux transport dynamo. Very different parameter values produce similar results, making it difficult to understand the origin of the properties of these solar cycles. We use helioseismic data from the Global Oscillation Network Group to investigate the progression of the solar cycle as observed in intermediate-degree global $p$-mode frequency shifts at different latitudes and subsurface layers, from the beginning of solar cycle $23$ up to the maximum of the current solar cycle. We also analyze those for high-degree modes in each hemisphere obtained through the ring-diagram technique of local helioseismology. The analysis highlights differences in the progression of the cycle below $15^\circ$ compared to higher latitudes. While the cycle starts at mid-latitudes and then migrates equatorward/poleward, the sunspot eruptions of the old cycle are still ongoing below $15^\circ$ latitude. This prolonged activity causes a delay in the onset of the cycle and an overlap of successive cycles, whose extent differs in the two hemispheres. Then the activity level rises faster, reaching a maximum characterized by a single-peak structure as opposed to the double peak at higher latitudes. Afterwards the descending phase shows up with a slower decay rate. The latitudinal properties of the progression of the solar cycle highlighted in this study provide useful constraints for discerning among the multitude of solar dynamo models.

Key words: dynamo – methods: data analysis – Sun: helioseismology

1. INTRODUCTION

The cyclic behavior of solar magnetic activity is ascribed to the dynamo process powered by the inductive action of the turbulent fluid in the Sun’s interior. A clear consensus has been reached on the $\Omega$ mechanism, which generates toroidal field by shearing a pre-existing poloidal field by differential rotation. Conversely it is still a matter of debate which $\alpha$-effect regenerates poloidal fields from toroidal ones. There are two main competitive mechanisms: (1) the $\alpha$-turbulent effect, which regenerates poloidal field from toroidal flux tubes by helical motion (Parker 1955); (2) the Babcock–Leighton mechanism, which is based on the observed decay of tilted, bipolar active regions, which act as poloidal field sources at the surface (Babcock 1961; Leighton 1964). While the $\alpha$-turbulent $\Omega$ dynamo offered a plausible explanation for the drift of sunspots toward equatorial latitudes via a dynamo wave, the Babcock–Leighton mechanism failed to reproduce the butterfly diagram. Therefore for several decades the $\alpha$-turbulent $\Omega$ dynamo has been favored over the Babcock–Leighton mechanism. As various observations have found a poleward surface meridional flow (Duvall 1979; Komm et al. 1993; Hathaway 1996), the inclusion of a poleward circulation along with an equatorward subsurface return flow initiated the development of the so-called flux transport dynamo (FTD) model. This new class of dynamo model revived the Babcock–Leighton mechanism, because the inclusion of this mechanism in FTD models has been successful in reproducing many global features of the solar cycle (Wang et al. 1991; Dikpati & Charbonneau 1999; Nandy & Choudhuri 2001). The resulting simulations showed that the butterfly diagram is produced by the equatorward subsurface return flow advecting the toroidal field toward the equator. $\alpha$-turbulent $\Omega$ FTD models have also been developed that operate in the tachocline (Dikpati & Gilman 2001), and an FTD that is simultaneously driven by an $\alpha$-turbulent effect located in the tachocline or in the whole convection zone, together with Babcock–Leighton type surface poloidal sources (Belucz & Dikpati 2013; Passos et al. 2014). The key question is whether the Babcock–Leighton mechanism is an active component of the dynamo cycle or a mere consequence of the decay of an active region. How well the FTD and/or dynamo wave models reproduce the features observed in the butterfly diagram might help to solve the puzzle.

Figure 1 shows the main features of the butterfly diagram: (1) the onset of the cycle at mid-latitudes; (2) the sunspot drift toward the equator and its slowdown represented by a change in the slope of the butterfly wing (Maunder 1904; Li et al. 2001); (3) the tail-like attachment over the minimum phase that is more prominent when the activity is stronger, which might lead to the overlap of successive cycles; (4) the length of the overlap varies within 1–2 years. It characterizes only the minimum phase and it is confined at latitudes $15^\circ$ (Cliver 2014). This feature is also seen in torsional oscillations shown in the bottom panels of Figure 1 (Wilson et al. 1988; Howe et al. 2009). The rate of drift of sunspots toward the equator slows as the sunspot band approaches the equator, and halts at about $8^\circ$ latitude (Hathaway et al. 2003). The end of the migration does not correspond to the end of the activity because it produces the tail-like attachment. When the new cycle at mid-latitudes starts before the end of the old cycle at low latitudes, it causes successive cycles to overlap. FTD

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3 For example, the $\alpha$-turbulent effect is located at the base of the convection zone as in the models by Parker (1955) and MacGregor & Charbonneau (1997). These models are also known as thin-shell or interface dynamo.
models driven only by the Babcock–Leighton mechanism (Chatterjee et al. 2004), or along with the $\alpha$-turbulent effect operating in the bulk of the convection zone, currently have the best agreement with observations (Passos et al. 2014), because the length of the simulated overlap is short and it occurs only during the minimum at low latitudes. Conversely, thin-shell dynamo wave models (Moss & Brooke 2000; Schüssler & Schmitt 2004; Bushby 2006) or the thin-shell flux transport dynamo (Dikpati & Gilman 2001) tend to produce dynamo waves with too short a wavelength, leading to excessive overlap between adjacent cycles because this involves a wider range of latitudes. Furthermore they also fail to reproduce the tail-like attachment over the minimum phase. Moreover the direction of the migration of activity could also provide information on the nature of the $\alpha$ mechanism. Both formalisms make strong assumptions to initiate the sunspot cycle at mid-latitudes. The Babcock–Leighton FTD models assume that the deep equatorward meridional flow penetrates slightly below the convection zone to a greater depth than usually believed (Nandy & Choudhuri 2002), in order to prevent the onset and occurrence of a sunspot cycle above 45° as well as any other kind of cyclic activity. The same result is achieved with the $\alpha\Omega$ dynamo wave by inhibiting the $\alpha$-turbulent effect at higher latitudes (Schüssler & Schmitt 2004). Based on these assumptions, the magnetic activity in any type of FTD model starts at higher latitudes and then propagates only equatorward, while in the thin-shell $\alpha\Omega$ dynamo wave the magnetic activity can propagate poleward as well as equatorward (Bushby 2006). These two branches are also clearly seen in the torsional oscillation pattern

Figure 1. The top two panels show the butterfly diagram and the average daily sunspot area (taken from www.msfc.nasa.gov). The bottom left panel shows the torsional oscillations from Howe et al. (2009); the bottom right is the same but the butterfly diagram provided by Solar Influences Data Center (SIDC) using sunspot numbers is overplotted and rescaled to match the left panel. The plot extends outside the axes because the SIDC sunspot data have been overplotted from the descending phase of solar cycle 22, when GONG observations had not yet started (taken from https://landscheidt.wordpress.com/2009/02/25/latest-solar-differential-rotation-information/). The arrows identify the beginning of the equatorward branch at around 40°.
(e.g., Howe et al. 2009). This results from the solar-like differential profile, which is characterized by a sign change in \( \frac{dn}{d\ell} \) in the tachocline at high and low latitudes (Ruediger & Brandenburg 1995). This sign change, however, has not yet been confirmed by helioseismic observations.

In this work we aim at characterizing the different phases of the solar cycle at all latitudes and in the two hemispheres, because these properties can be used to constrain solar dynamo models. We use acoustic \( p \)-mode frequencies as a diagnostic tool to infer the progression of the 11 yr magnetic cycle. They are very well known to correlate strongly with solar magnetic activity (Elsworth et al. 1990; Libbrecht & Woodard 1990; Howe et al. 1999; Jain et al. 2000; Simoniello et al. 2010), and unlike many other proxies for solar activity they probe magnetic changes induced by weak as well as strong toroidal fields at all latitudes. In order to simultaneously track solar magnetic activity in both hemispheres separately, we further use localized high-degree frequencies from the ring-diagram technique. The paper is organized as follows: in Section 2 we describe the data analysis for both intermediate- and high-degree modes. The results are presented in Section 3, followed by the comparison with sunspot numbers in Section 4; the findings are discussed in Section 5.

2. DATA ANALYSIS

2.1. The GONG Data

In this work we look for temporal variations in \( p \)-mode frequencies caused by changes in magnetic activity levels as a function of latitude and subsurface layer. The mode frequencies analyzed here are obtained from the Global Oscillation Network Group (GONG)\(^4\) in two different degree ranges. The low- and intermediate-degree global mode frequencies are obtained for the individual \((n,\ell,m)\) multiplets, \(\nu_{n,\ell,m}\), where \(n\) is the radial order and \(m\) is the azimuthal order, running from \(-\ell\) to \(+\ell\). The mode frequencies for each multiplet were estimated from the \(m-\nu\) power spectra constructed from the time series of 108 days. The data analyzed here consist of overlapping data sets, with a spacing of 36 days between consecutive time series, covering the period from 1995 June to 2013 July in the \(\ell \) range \(20 \leq \ell \leq 147\) and frequency range \(1500 \mu\text{Hz} \leq \nu \leq 3900 \mu\text{Hz}\).

The high-degree mode frequencies are obtained from localized regions (\(15^\circ \times 15^\circ\)) on the solar surface using the GONG ring-diagram pipeline (Corbard et al. 2003). The analysis covers a period from 2001 July to 2014 June in the degree range \(180 \leq \ell \leq 1000\). In the ring-diagram method, localized regions on the solar surface are tracked with an average rotation rate at the solar surface for 1664 minutes. Each tracked area is apodized with a circular function, and then a three-dimensional fast Fourier transform is applied on both spatial and temporal directions to obtain a three-dimensional power spectrum. Finally, the corresponding power spectrum is fitted using a Lorentzian profile model to obtain acoustic mode parameters. The high-degree modes provide information about the outermost layer of the Sun’s interior.

2.2. Determination of the Frequency Shifts

We aim at characterizing the progression of the solar cycle at different latitudes. The sunspot cycle starts at mid-latitudes (\(\approx 30^\circ\)), it reaches its maximum at \(\approx 15^\circ\) latitude and stops at around \(8^\circ\) latitude. Modes of low to intermediate degree \(\ell\) are global, and sense the spherical geometry of the Sun. Therefore these are better described by spherical harmonics of the form \(Y_{n,m}(\theta, \phi) = P_{n,m}(\cos \theta)e^{i\phi}\), where \(P\) is a Legendre polynomial, \(\ell\) the spherical degree, and \(m\) the azimuthal order. The spherical harmonic degree \(\ell\) is the number of nodes along a circle at an angle \(\theta = \arccos(\frac{m}{\ell})\) at the equator. The azimuthal order \(m\) is the number of nodal lines crossing the equator. We therefore used the above ratio to select acoustic modes according to the upper latitude range to which they are most sensitive. It may be noted that acoustic modes with the same spherical degree \(\ell\) but different azimuthal order \(m\) increase their sensitivity to lower latitudes with increasing \(m\). In fact the sectoral modes \(((\ell, m) = \ell)\) are more sensitive to the regions near the equator while the zonal modes \((m = 0)\) have higher sensitivity at higher latitudes (Hill et al. 1991). We carry out this selection in five latitude ranges for \(0^\circ \leq \theta \leq 75^\circ\) spaced by \(15^\circ\). This allows us to split the cycle progression into latitude ranges. Since the acoustic waves travel throughout the interior and they are reflected by different layers depending on their frequencies, we further investigate the progression of the solar cycle based on their upper turning point \(u_p\).

With increasing frequency, \(u_p\) approaches the surface (Basu et al. 2012). We divide frequency data sets into three groups: (i) the low-frequency range \(1500 \mu\text{Hz} \leq \nu \leq 2300 \mu\text{Hz}\) corresponds to \(0.9944 \leq R_\odot \leq 0.9987 \leq R_\odot\), (ii) the medium-frequency range \(2300 \mu\text{Hz} \leq \nu \leq 3100 \mu\text{Hz}\) to \(0.9987 \leq R_\odot \leq u_p \leq 0.9998 \leq R_\odot\), and (iii) the high-frequency range \(3100 \mu\text{Hz} \leq \nu \leq 3900 \mu\text{Hz}\) to \(0.9998 \leq R_\odot \leq u_p \leq 0.9999 \leq R_\odot\). Helioseismic observations have shown that the size of the variation in frequency with the solar cycle increases as \(u_p\) approaches the surface (Chaplin et al. 2001; Simoniello et al. 2013), so we then calculated the frequency shifts in the low, medium, and high frequency bands to investigate the properties of the solar cycle in different subsurface layers. Shifts in mode frequency \(\delta \nu_{n,\ell}(t)\) were defined as the differences between the frequencies observed at different times \(\nu_{n,\ell}(t)\) and the reference values of the corresponding modes \(\nu_{n,\ell}(\text{ref})\):

\[
\delta \nu_{n,\ell}(t) = \nu_{n,\ell}(t) - \nu_{n,\ell}(\text{ref}).
\]

\(\nu_{n,\ell}(\text{ref})\) was determined as the average frequency over the minimum between cycles 22 and 23. We took into account the period of observations from 1995 June up to 1996 May. While we have included the end of activity cycle 22, we have been very careful not to include in \(\nu_{n,\ell}(\text{ref})\) the beginning of solar activity at higher latitudes, because it would have led to significant differences in the size of \(\nu_{n,\ell}(\text{ref})\) at different latitudes, making difficult any comparison and interpretation of the properties of the solar cycle. We then determine the weighted frequency difference in the low, medium, and high frequency bands for each selected latitude. The weights \(\left(\frac{1}{u_p}\right)\) are the errors of the fitting procedure.

The frequency shifts of high-degree mode were obtained by analyzing the frequencies obtained from the ring-diagram technique. These frequencies in localized regions are affected by foreshortening as well as by the gaps in observation. Thus we have corrected these effects by modeling them as a two-dimensional function of the distance from the disk center and a linear dependence on the duty cycle (Howe et al. 2004;
Tripathy et al. (2013). The corrected frequencies are then used to compute frequency shifts; this is computationally similar to the calculation for global modes except for the choice of the reference frequency. For high-degree modes, the frequency difference of each mode is computed with respect to the average frequency of the same mode over the 189 dense-pack tiles (covering $\pm 60^\circ$ on the disk) that correspond to a magnetically quiet day (2008 May 11). In a similar manner, frequency shifts for each hemisphere and in different latitude ranges were computed with an appropriate reference frequency as described in Tripathy et al. (2015); e.g., for the northern hemisphere, the reference frequency was computed over only that hemisphere.

### 3. RESULTS

#### 3.1. Progression of the Solar Cycle at Different Latitudes

Since numerous examples have clearly shown the strong correlation between frequency shifts and the indices of magnetic activity over different timescales, we interpret frequency shifts as a measure of solar activity in the rest of the paper.

The top panel of Figure 2 shows the variation of frequency shifts over solar cycle 23 and the ascending phase of solar cycle 24 at five latitude bands in the frequency range $1500 \mu Hz \leq \nu \leq 3900 \mu Hz$. All curves have been smoothed with a boxcar of 1 yr and the estimated uncertainties are of the order $10^{-3} \mu Hz$. It is clearly seen that the progression of the activity cycle is different at different latitudes. In particular, the variation in the magnetic activity below $15^\circ$ differs from that at higher latitudes: it rises faster, the maximum is characterized by a single-peak structure, and an excess of activity changes the evolutionary path of the descending phase around 2003 December. This difference is better highlighted in the bottom left panel of Figure 2, which compares the activity at $0^\circ \leq \theta \leq 15^\circ$ with $30^\circ \leq \theta \leq 45^\circ$. At $0^\circ \leq \theta \leq 15^\circ$, the declining phase lasted longer than at higher latitudes, delaying the time of the minimum and the onset of solar cycle 24 for about a year, which led to an overlap of cycle 23 with cycle 24. A similar long delay in the onset of solar cycle 23 caused an overlap between cycles 22 and 23. Furthermore, over both minimum phases, the activity level reached its deepest value at latitudes of $0^\circ \leq \theta \leq 15^\circ$. The bottom right panel highlights the similarity in the progression of solar activity at all latitudes above $15^\circ$. Here, the activity level at both minimum phases at all latitude bands is of comparable size. It rises with slightly different growth rates and reaches a maximum characterized by the typical double-peak structure, which has been interpreted as

![Figure 2](image-url)
Table 1
Onset of the Solar Cycle, Peak Amplitude, Rise and Decay Times, and Full Cycle Length in the Low, Medium, and High Frequency Ranges

| Latitude (deg) | Cycle 23 | Cycle 24 | Rise (1) | Rise (2) | Decay | Length |
|---------------|---------|---------|---------|---------|-------|--------|
|               | Min.    | Max. (1) | Max. (2) | Min.    | (Months) | (Months) | Max.-Min. | (Months) |
| Low Frequency Range |
| 0 ≤ θ ≤ 15 | 02/1998 | ... | 03/2002 | 03/2010 | ... | 49 | 96 | 145 |
| 15 ≤ θ ≤ 30 | 03/1996 | 06/2000 | 03/2002 | 11/2007 | 51 | 72 | 68 | 140 |
| 30 ≤ θ ≤ 45 | 01/1996 | 05/2000 | 03/2002 | 07/2007 | 52 | 74 | 64 | 138 |
| 45 ≤ θ ≤ 60 | 08/1996 | 04/2000 | 03/2002 | 07/2007 | 44 | 67 | 64 | 131 |
| 60 ≤ θ ≤ 75 | 09/1996 | 04/2000 | 03/2002 | 08/2007 | 43 | 66 | 65 | 132 |
| Medium Frequency Range |
| 0 ≤ θ ≤ 15 | 11/1997 | ... | 04/2002 | 01/2010 | ... | 53 | 93 | 146 |
| 15 ≤ θ ≤ 30 | 09/1996 | 07/2000 | 04/2002 | 04/2009 | 46 | 67 | 84 | 151 |
| 30 ≤ θ ≤ 45 | 06/1996 | 06/2000 | 03/2002 | 05/2008 | 48 | 69 | 74 | 143 |
| 45 ≤ θ ≤ 60 | 07/1996 | 06/2000 | 03/2002 | 11/2008 | 47 | 68 | 80 | 148 |
| 60 ≤ θ ≤ 75 | 08/1996 | 05/2000 | 03/2002 | 12/2008 | 45 | 67 | 81 | 148 |
| High Frequency Range |
| 0 ≤ θ ≤ 15 | 12/1997 | ... | 04/2002 | 01/2010 | ... | 52 | 93 | 145 |
| 15 ≤ θ ≤ 30 | 09/1996 | 06/2000 | 03/2002 | 03/2009 | 45 | 66 | 84 | 150 |
| 30 ≤ θ ≤ 45 | 09/1996 | 06/2000 | 03/2002 | 10/2008 | 45 | 66 | 79 | 145 |
| 45 ≤ θ ≤ 60 | 10/1996 | 06/2000 | 03/2002 | 11/2008 | 44 | 65 | 80 | 145 |
| 60 ≤ θ ≤ 75 | 11/1996 | 06/2000 | 03/2002 | 12/2008 | 43 | 64 | 81 | 145 |

A manifestation of the quasi-biennial periodicity (Fletcher et al. 2010; Jain et al. 2011; Simoniello et al. 2012, 2013). Soon after the second maximum, the descending phase continued with comparable decay times, although around 2003 December the progression at latitudes 15° ≤ θ ≤ 30° changed slightly. To summarize the similarities and differences in the properties of the progression of the solar cycle at different latitudes and in the low, medium, and high frequency ranges, Table 1 lists (i) the epochs of minimum and maximum of the solar cycle in each latitudinal band, which have been defined as the timing corresponding to the lowest/highest value in the frequency shift at different latitudes; (ii) the rise and decay times; (iii) the full cycle length. The progression of the solar cycle shows common features in the three bands: it starts within the latitude range 30° ≤ θ ≤ 45° and follows at other latitudes. At 0° ≤ θ ≤ 15° latitude the onset of the new cycle is always delayed, but this time lag disappears at the time of the maximum, which occurs at the same epoch at all latitudes in each frequency range (Max. (2) in Table 1). This result further confirms that the rise in activity below 15° latitude is faster than at all other latitudes (Rise (2) in Table 1). The descending phase, instead, lasted longer below 15° latitude than at higher latitudes, leading to a slowdown in the progression of magnetic activity, which ended in the overlap of successive cycles (Decay in Table 1). This peculiar behavior ended up in a stronger asymmetry between the rise and decay times at latitudes below 15° than at higher ones. Even though the descending phase lasted longer at latitudes ≤15°, the faster rising phase made cycle lengths of comparable size at all latitudes in all frequency bands (Length in Table 1).

3.2. Sensitivity of the Progression of the Solar Cycle to the Subsurface Layers
To further study whether such behavior persists in the solar subsurface layers, each panel of Figure 3 shows the progression of the solar cycle in a selected frequency band and at all latitudes. As we can see, the properties of the two different modes of the solar cycle as described above do not change. To better compare the behavior and strength of activity in the three subsurface layers, each panel of Figure 4 compares the activity at the same latitude but in the three frequency ranges. As expected, the activity is stronger in the nearest subsurface layers than in deeper ones.

3.3. Progression of the Solar Cycle in the Northern and Southern Hemispheres
The top two panels of Figure 5 show the progression of the solar cycle in the northern and southern hemispheres as determined by the analysis of the high-degree modes in two different latitudinal bands. While in the northern hemisphere the activity is almost comparable at the two selected latitudes, the activity in the southern hemisphere is higher at the maximum at latitude 15° ≤ θ ≤ 30°, then reaches a comparable strength sometime after 2003 December. After 2006 September the activity at latitude 0° ≤ θ ≤ 15° is lower at latitude 15° ≤ θ ≤ 30° followed different patterns. In fact, while below 15° we observe a prolonged minimum until 2009 June with a consequent delay in the onset of the solar cycle in either hemisphere (shown in black), above 15° latitude the rising phase already started sometime after 2006 September (shown in red). When we compare the strength of activity at the same latitude but for different hemispheres (bottom two panels of Figure 5), we find that throughout the descending phase the southern hemisphere (shown in red) has been more active than the northern one (shown in black). In particular, we note an enhancement in the activity, which changes the evolutionary path of the descending phase at latitudes 0° ≤ θ ≤ 15° (bottom left panel). This deviation is stronger in the southern hemisphere. Interestingly where the excess of magnetic activity is more pronounced, the descending phase is consequently
slightly more prolonged, leading to a longer overlap of successive cycles. Table 2 lists the time of the minimum in both southern and northern hemispheres and at two latitudes. In the southern hemisphere the overlap of successive cycles lasted slightly longer than in the northern hemisphere. Comparing the timing of the minimum between the two hemispheres, we note that there is a delay between the two hemispheres of approximately one year.

4. CORRELATION WITH SUNSPOTS

4.1. STARA Data

In order to compare the variation of oscillation frequencies with known proxies of the solar activity, we use sunspot numbers calculated from the Sunspot Tracking And Recognition Algorithm (STARA: Watson et al. 2011). In the STARA sunspot catalog, the sunspot count does not include a factor for grouped sunspots and so the number is far lower than other sources. The sunspot numbers are calculated using the images from SOHO/MDI (the Michelson Doppler Imager on board the Solar and Heliospheric Observatory) for the period from 1996 June to 2010 October. Although there are some gaps in data due to SOHO’s vacation in 1998–99, the advantage of using this catalog over others is the availability of the location of sunspots on the solar disk, which is important in this analysis. The sunspot numbers beyond 2010 are also available but have been calculated using the images from HMI (the Helioseismic and Magnetic Imager), which have different spatial resolution, and no scaling has been performed yet. Figure 6 shows the sunspot numbers as measured by STARA; the gaps are due to unavailability of data. The sunspot numbers are the averages over the same period as the time series of the oscillation frequencies. We further grouped them into three latitude bands. In the analysis of global modes, the selection of a particular latitude range does not confine the modes’ sensitivity to the selected range. Instead it senses all latitudes in both hemispheres up to the highest selected latitude. Thus frequency shifts in the range $15^\circ \leq \theta \leq 30^\circ$, which covers modes at latitudes $\pm 30^\circ$ are compared with the sunspot data between $\pm 30^\circ$ latitude. There were no sunspots observed in latitude ranges $15^\circ \leq \theta \leq 30^\circ$ before 1996 August, while some were visible in the $0^\circ \leq \theta \leq 15^\circ$ bands since the beginning of the available data, i.e., 1996 June.

Over the minimum phase between cycles 22 and 23, we notice that a magnetic cycle started when the activity of the old cycle was still ongoing below $15^\circ$ latitude. We also note that the maximum is characterized by a double-peak structure for latitudes of $0^\circ \leq \theta \leq 30^\circ$, yet by a single-peak structure for $0^\circ \leq \theta \leq 15^\circ$, as already found in our helioseismic analysis of intermediate-degree modes. Furthermore, soon after 2003 December an excess of sunspots at latitudes of $0^\circ \leq \theta \leq 15^\circ$ changed the natural evolution of the activity during the descending phase.

4.2. Frequency Shifts and STARA Sunspots

We aim at comparing STARA sunspot number with helioseismic observations to highlight similarities and differences in the progression of the cycle between the two activity proxies. They are sensitive to different magnetic field structures. Sunspots are the result of the strong toroidal fields located around equatorial latitudes, while acoustic waves sound the whole Sun at all latitudes. Therefore they are sensitive to strong and weak toroidal fields. To compare the behavior of the two activity proxies, we treated the STARA SSN data as we did the mode frequency. We determined the SSN reference values ($SSN_{ref}$) between 1996 December and 1997 March, the common period of quiet activity where SSNs at all latitudes are around two. Then we calculated the deviation $\delta SSN$ as the difference between the observed sunspot number at different epochs (SSN) and $SSN_{ref}$. Finally
δSSN and δν have been divided by the sum of the shifts/sunspots over the whole observational time, which gave us δSSN_{rel} and δν_{rel}. Figure 7 compares the behavior of solar magnetic activity from STARA data with frequency shifts determined in the high-frequency band at each selected latitude, because the high-frequency range sounds the closest layer to the solar surface. It is worth recalling that, as the reference values for δSSN and δν have been calculated over different periods of activity and the lengths of observations differ, we cannot directly compare the size of δSSN_{rel} and δν_{rel}; however the overall trend can be compared. Nevertheless we find that both activity proxies are characterized by a single-peak structure at latitudes below 15° yet by a double-peak structure above it. This further confirms that the single-peak structure is a signature of the solar magnetic activity at 0° ≤ θ ≤ 15°. Although the origin of the p-mode frequency shifts is still a matter of debate, magnetic fields in sunspots can widen or shrink the acoustic cavity (Schunker &
Cally 2006; Simoniello et al. 2010), shifting the mode frequency toward lower/higher values. We might argue that during the descending phase, soon after 2003 December, the emergence of an excess of sunspots produced the observed enhancement in the size of the shift predominantly in the southern hemisphere and at latitudes below 15°.

5. DISCUSSION

Changes in $p$-mode frequencies over the solar cycle are unique tools with which to investigate similarities and differences in the progression of the solar cycle at different latitudes and subsurface layers. In addition, high-degree modes calculated from local helioseismology techniques open a window on the solar hemispheric activity. In this work, therefore, we use intermediate- and high-degree acoustic modes to obtain a detailed description of the Sun’s global and hemispheric magnetism.

The latitudinal and frequency dependence of changes in $p$-mode frequency shifts over the solar cycle from intermediate-degree modes is a different representation of what is seen in the butterfly diagram and latitudinal inversions of the helioseismic
modes (Howe et al. 2002). However, this approach has highlighted important new details, which could be used to constrain the sources of the $\alpha$ mechanism.

The overall results have pointed out latitudinal differences in the progression of the solar cycle below and above 15° in both hemispheres. The onset of the cycle below 15° is delayed compared to that at higher latitudes, causing an overlap of successive cycles at the time of the minimum phase. In this regard the analysis of high-degree modes has identified and quantified, for the first time, differences in the length of the overlap of successive cycles in the two hemispheres.

Furthermore, in both hemispheres our findings confine the overlap to latitudes below 15°. Soon after the minimum, the activity level below 15° progresses with the fastest rise time, and it reaches a maximum characterized by a single peak. At higher latitudes the ascending phase is instead characterized by a slower rise time and it ends in a maximum characterized by a double-peak structure. Interestingly the single peak below 15° coincides with the second and higher peak at higher latitudes. The dynamo mechanism seems to synchronize the epoch of the maximum at all latitudes. During the descending phase we found latitudinal differences in the decay time, it being slower below 15° than at higher latitudes.

How can these observed properties help us in understanding the role of the Babcock–Leighton poloidal field sources with respect to the $\alpha$-turbulent one? For example, our findings have provided evidence that overlap occurs at latitudes below 15°. This confinement is better reproduced in FTD models including the Babcock–Leighton mechanism. Therefore we might envisage that to some extent the Babcock–Leighton mechanism might play a role in the solar dynamo (Cameron & Schüssler 2015), but to draw any conclusion all the features of the solar cycle need to be reproduced within this formalism. In fact the meridional flow speed sets the cycle period and the rise and decay times within Babcock–Leighton FTD. Within the $\alpha$-turbulent $\Omega$ dynamo the cycle period depends on (among other things) the $\alpha$-turbulent effect itself (Parker 1955). It would then be rather interesting to see how our findings will impose further constraints on the latitudinal dependence of the meridional flow speed and $\alpha$-turbulent effect. How well the resulting simulations will fit with observations, will be a...
valuable test to discern among the multitude of dynamo models and it will shed some light on the principal driving solar dynamo.

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