Analysis of dynamic characteristics of nonlinear vehicle suspension system with five degrees of freedom

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Abstract: With the rapid development of the automobile industry, the performance requirements of the automobile has been becoming higher and higher which shows the great significance of the study on vehicle suspension characteristics. Based on the structural characteristics of the vehicle, a five degree of freedom vehicle model is established. The dynamic equation of the vehicle model is listed by using the vibration law of the multi degree of freedom damping system. The simulation model of five degree of freedom linear and nonlinear system of automobile is established by using MATLAB/SIMULINK. The simulation analysis of ride comfort is carried out based on simulated nonlinear stiffness: Compared with the results of linear and nonlinear parameters, the nonlinear stiffness coefficient alpha is monotonically decreasing in -0.4~0, which can improve the ride comfort of the vehicle. The nonlinear stiffness mainly affects the seat acceleration, the centroid acceleration and the pitching acceleration. The nonlinear system is optimized by the obtained law concluded from simulation. The results show that the performance of the suspension system can be greatly improved, the mean square root value of the centroid acceleration is reduced by 38.67 % and the mean square root value of the pitching acceleration is reduced by 44.54 %, the mean square root value of the seat acceleration is reduced by 11.31 %, and the improvement of vehicle ride comfort is very obvious.

1. Introduction
The structure of automobile suspension is usually composed of three parts: Elastic element, Shock absorber and Guiding Mechanism \cite{1}. The vehicle suspension is divided into active suspension, semi-active suspension and passive suspension. Passive suspension is a very traditional mechanical structure, the stiffness and damping coefficient are fixed, its structure is simple and its theory is relatively mature. With the advantages of reliable performance and relative lower cost, the passive suspension has been widely applied in related industry \cite{2}. The research on semi-active suspension began in 1973, first proposed by D.C. Karnopp and D.A. Crosby, realization control from the two aspects of changing the stiffness or damping of the suspension, improving the ride comfort and stability of a car \cite{3}. R Kalaivani and P Lakshmi proposed a simple robust adaptive neural fuzzy inference system for the vibration control of vehicle active suspension system, and it is more obvious that the system can restrain the acceleration of the body under the excitation of the random road \cite{4}. Kong Ling Bo and others in Changchun University use MATLAB optimization toolbox function to compile optimization program, which can get optimized process information and result in a short time \cite{5}. Jiang Peng of Zhejiang University is based on the seven degree of freedom model of the whole vehicle, the simulation research on the ride performance of the vehicle on the computer is realized \cite{6}.
This paper mainly studies the suspension of simulated nonlinear stiffness and nonlinear damping. The analysis results provided a way of thinking for designer.

2. Method
Simplify the car model, a vehicle model with five degrees of freedom is extracted. A five degree of freedom of 1/2 vehicle model is established, shown in figure 1. From the physical model and D’Alembert’s principle, the kinetic equation of the model can be listed as follows:

\[
M_1\ddot{Z}_1 = \left[ C_1\left(\ddot{Z}_1 - \dot{Z}_1 - L_1\dot{Z}_1\right) + K_1\left(\dot{Z}_1 - Z_1 - L_1\dot{Z}_1\right) + K_1\left(Z_d - Z_1\right) \right]
\]

\[
M_2\ddot{Z}_2 = \left[ C_2\left(\ddot{Z}_2 - \dot{Z}_2 + L_2\dot{Z}_2\right) + K_3\left(\dot{Z}_2 - Z_2 + L_2\dot{Z}_2\right) + K_3\left(Z_d - Z_2\right) \right]
\]

\[
M_3\ddot{Z}_3 = \left[ C_3\left(\ddot{Z}_3 - \dot{Z}_3 - L_3\dot{Z}_3\right) + K_4\left(\dot{Z}_3 - Z_3 + L_3\dot{Z}_3\right) + K_4\left(Z_d - Z_3\right) \right]
\]

\[
M_4\ddot{Z}_4 = K_5\left(Z_d - Z_4 + L_4\dot{Z}_4\right)
\]

\[
i_0\ddot{\theta}_0 = \left[ C_i L_i\left(\ddot{Z}_1 - \dot{Z}_1 - L_1\dot{Z}_1\right) + C_i L_i\left(\ddot{Z}_2 - \dot{Z}_2 - L_2\dot{Z}_2\right) + K_1 L_i\left(\dot{Z}_1 - Z_1 - L_1\dot{Z}_1\right) + K_2 L_i\left(\dot{Z}_2 - Z_2 - L_2\dot{Z}_2\right) + K_3 L_i\left(Z_d - Z_3 - L_3\dot{Z}_3\right) \right]
\]

![Figure 1. Five degree of freedom of 1/2 vehicle model](image)

The symbols in the figure 1 as follows:

- \(M_1, M_2\) — Un-sprung mass;
- \(M_3\) — Sprung mass;
- \(i_0\) — Pitching moment of inertia;
- \(K_1, K_2\) — Front and rear suspension stiffness;
- \(K_3, K_4\) — Tire stiffness;
- \(C_1, C_2\) — Damping coefficient of front and rear suspension;
- \(Z_1, Z_2\) — Vertical displacement of un-sprung mass;
- \(Z_3\) — The vertical displacement of the seat;
- \(Z_4\) — Body elevation angle;
- \(Z_0\) — The vertical displacement of the body center of the body;
- \(Z_{01}, Z_{02}\) — The inputting of the unevenness of the front and rear wheels;
- \(L_1, L_2\) — The distance between the body center of mass and the front wheel and the rear wheel;
- \(L=L_1+L_2\) — Vehicle wheelbase.

According to the mathematical model of the vehicle suspension system, Selected the appropriate
block of program in Simulink. The linear model of the vehicle suspension system is established by MATLAB/SIMULINK using the dynamic equation. In a linear model, the expression of a spring is usually:

\[ F_S = KX \]

The \( F_S \) represents the spring force and the \( K \) represents the spring stiffness, \( X \) representing the spring displacement. The expression of damping force is usually:

\[ F_d = C \dot{x} \]

The \( F_d \) represents the damping force, and the \( C \) represents the linear damping, \( \dot{x} \) representing the relative velocity. In the dynamic equation of the nonlinear model, the \( KX, C \dot{x} \) term was replaced by \( KX + \alpha K \log(X + 1), C \dot{X} + \alpha C e^{(X-1)} \). The function module in SIMULINK can enter a variety of primary functions. Different values are given to the nonlinear coefficient \( \alpha \), Finally, the linear dynamic equation is transformed into a nonlinear dynamic equation with nonlinear coefficient. According to the new nonlinear dynamic equation, the corresponding nonlinear model is established.

3. Results and analysis:

It is assumed that the pavement serviceability is D grade. The speed is 40m/s; The simulation time is 20s; Running the program get the acceleration of suspension center of the car, the dynamic load diagram of the car seat and the tire, shown in figure 2-5:

GB4970-1996 is published in reference to international standard ISO2631 which is implemented in China. According to GB4970-1996, the simulated results were analysis. The value of the nonlinear term of stiffness is 0.4, 0.2, -0.2, -0.4. The pavement serviceability is D grade, The speed of the vehicle is 40m/s and the simulation time is 20s. The linear simulated model and the nonlinear stiffness simulated model of the vehicle suspension system are run. The above root mean square values were solved, shown in table 1. The conclusions can get through table 1 as follows:

(1) The nonlinear variation of the stiffness has little effect on the root mean square value of the tire, and has little effect.

(2) The nonlinear variation of the stiffness has an effect on the mean square root value, but the effect is less, and the smoothness becomes worse with the increase of the coefficient of the nonlinear stiffness factor alpha.

(3) The nonlinear variation of stiffness affects the root mean square value of seat acceleration, acceleration and pitching acceleration, and the ride comfort decreased with the increase of nonlinear coefficient \( \alpha \).

(4) The smoothness of the vehicle will become worse when the coefficient of the nonlinear
stiffness factor \( \alpha > 0 \), and the ride comfort of the car will become better when the \( \alpha < 0 \). And the absolute value of \( \alpha \) is bigger, the impact of the ride comfort of the car is bigger.

| Root mean square of output | \( \beta = 0.2 \) | \( \beta = -0.2 \) | Linear | \( \beta = -0.4 \) |
|---------------------------|-----------------|-------------------|--------|-----------------|
| Forward moving stroke (m) | 0.0763          | 0.0761            | 0.0764 | 0.0759          | 0.0759          |
| Rear moving stroke (m)    | 0.0762          | 0.0761            | 0.0762 | 0.0758          | 0.0758          |
| Seat moving stroke (m)    | 0.0876          | 0.0878            | 0.0879 | 0.0880          | 0.0881          |
| Centroid acceleration (m/s²) | 1.2944      | 1.2904            | 1.3273 | 1.2904          | 1.2904          |
| Seat acceleration (m/s²)  | 4.2051          | 4.2051            | 4.4346 | 4.2051          | 4.2051          |
| Pitching acceleration (rad/s²) | 0.8515       | 0.7384            | 0.7705 | 0.4724          | 0.5251          |

By using the same method, the root mean square of the linear model and the nonlinear damping model is also compared. When the nonlinear damping decreases with the value of -0.4~0.4 decreasing, the root mean square values of each volume increase or decrease, but the root mean square of tire and seat travel almost unchanged. The conclusions can get as follows:

1. The nonlinear change of damping has little effect on the root mean square root of the dynamic travel (response) of the seat tire.
2. The nonlinear variation of damping has a smaller effect on the acceleration and acceleration of the seat, but the nonlinear change of damping may cause the ride comfort of the vehicle to improve.
3. The smoothness becomes worse when the nonlinear damping factor \( \beta > 0 \), and the smoothness becomes better when the \( \beta < 0 \). The smaller the absolute value of \( \beta \) is, the greater the effect of the smoothness.

Reference:
[1] Shufeng Wang 2009 Automobile structure M. Beijing: National Defense Industry Press (In Chinese)
[2] Guiqing Xu 2010 Research status and development trend of automobile suspension J. High and new technology enterprises in China 15,35-36 (In Chinese)
[3] Zhigang Huang, Enrong Mao, Xincheng Liang, Hui Zhu and Qingpin Zhu 2006 Research on the development of automobile suspension J. Mechanical design and manufacture 11,67-69 (In Chinese)
[4] R Kalaivani and P Lakshmi 2013 Adaptive neuro-fuzzy controller for vehicle suspension system J. International Conference on Advanced Computing 3(1):236-240
[5] Lingbo Kong, Jing Liu, Hui Li and Shuang Lu 2004 Design of five degree of freedom vehicle suspension system based on MATLAB optimization toolbox J. Journal of Changchun University 14(16): 1-4 (In Chinese)
[6] Peng Jiang 2006 Simulation analysis and parameter optimization design of automobile suspension system D. Hangzhou: Zhejiang University (In Chinese)