Investigation of the dynamic response of rock slopes at the entrance and exit section of a tunnel based on modal analysis

Lihu Dong¹, Danqing Song²⁎, Zhuo Chen³, Jianwei Zhang⁴

1. School of Electrical Engineering, Shenyang University of Technology, Shenyang, 110870, China; 2. State Key Laboratory of Hydrosience and Engineering, Tsinghua University, Beijing 100084, China; 3. College of Civil Engineering, Sichuan Agricultural University, Dujiangyan, Sichuan 611830, China; 4. School of Civil Engineering and Architecture, Henan University, Kaifeng, Henan 475004, China

* Corresponding author. E-mail address: songdq2019@mail.tsinghua.edu.cn (DQ. Song). ORCID: https://orcid.org/0000-0003-1015-9544 (DQ. Song)

Abstract. In the construction and operation of tunnel engineering in high intensity areas, the slope at a tunnel’s entrance and exit section has become a section prone to problems because of seismic load and tunnel excavation disturbance. To study the dynamic response characteristics of rock slopes at the tunnel entrance and exit section, considering the strong earthquake area in western China as the research area, a three-dimensional finite element (FE) model was established and the FE modal analysis of the slope at the tunnel entrance and exit section was conducted. The dynamic response characteristics of the slope body were investigated based on a frequency domain by analyzing the natural frequencies and corresponding vibration modes of different models. The results show that the tunnel excavation affects the slope’s dynamic response characteristics and natural frequency but has little effect on its vibration mode. The natural frequency ratio of slopes without and with a tunnel is 1.1:1.35 as a whole, under the same conditions. The slope gradient has a considerable effect on the natural frequency of the slope at the tunnel entrance and exit section. The natural frequency of the slope gradually increases with the gradient, and the changing trend is more obvious with an increase in the vibration mode order. The low-order vibration mode of the slope at the entrance of the tunnel is relatively simple, primarily demonstrating bending and torsional deformation. The vibration modes of the slopes with and without a tunnel is the same when the slope gradient is the same. The vibration modes of the two tunnel slope models with different gradients are also the same. This study investigated the dynamic response characteristics of the slope and tunnel structure based on the analysis of the inherent characteristics of the slopes at the tunnel entrance and exit section, which can provide a reference for the seismic design of the tunnel entrance section.

Keywords. Dynamic response; rock slope; tunnel entrance and exit section; natural frequency; modal analysis

1. Introduction
Tunnel construction has accelerated in China due to the introduction and implementation of several important national strategies [1]. The tunnel project construction will cut across steep and complex mountainous areas, and the tunnel’s entrance and exit section will encounter the severe challenges of...
landslide disaster under earthquakes, and the seismic stability of slopes at the tunnel’s entrance section containing complex geological conditions is particularly prominent [2]. Tunnel engineering is rapidly evolving, but it is also faced with various challenges posed by the changing natural environment and complex geological conditions [3-5]. Tunnel construction inevitably runs through the inner or surrounding areas of the landslide body, causing disturbance to the rock mass at the entrance of the tunnels and weakening the stability of slopes, leading to geological disasters such as landslides, collapse, and debris flow [6]. In recent years, with the rapid development of transportation infrastructure construction in China, numerous traffic tunnels and water diversion tunnels have encountered the deformation problem of the tunnel-landslide system [7]. According to the earthquake damage investigation of the highway tunnel in the Wenchuan Earthquake, the entrance section of the tunnel is second only to the severe seismic damage of the tunnel structure at the section across the fault fracture zone [8]. Therefore, investigating the dynamic response of the slopes at the tunnel entrance section for their seismic fortification is of significant importance.

Many scholars have researched the seismic dynamic response of rock slopes at the entrance section of tunnels. Jiang et al. (2018) investigated the dynamic response characteristics and seismic performance of slopes with a tunnel using a numerical method and shaking table test, and analyzed the influence of ground motion direction on the dynamic response of the slope and tunnel structure [9]. Wang et al. (2018) investigated the seismic dynamic characteristics of the slope at the entrance of a tunnel using shaking table model tests and analyzed the relationship between the superior frequency and the tunnel structure’s dynamic response characteristics [10]. Niu et al. (2018) analyzed the dynamic response characteristics of the slope with a tunnel using the finite element (FE) method and model test and further discussed the influence of ground motion parameters on the slope’s dynamic characteristics [11]. Wang et al. (2020) investigated the seismic dynamic response characteristics of the interface between hard and soft rock at the entrance of the tunnel using model tests and analyzed the influence mechanism of stratum lithology on the tunnel structure dynamic response [12]. Currently, modal analysis is widely used to investigate the structure’s dynamic characteristics and mechanical engineering. The dynamic deformation characteristics of engineering entities can be better reflected using the modal analysis method. However, there are few studies on the dynamic response of rock slope at the entrance section of the tunnel using the modal analysis method.

In this study, two types of three-dimensional (3-D) numerical models are used to perform modal analysis to investigate the relationship between the slopes natural frequency and their dynamic deformation characteristics, including the rock slopes without and with a tunnel, using a homogeneous rock mass slope at the entrance section of the tunnel as an example. This study provides a basis for seismic fortification of slope tunnel structures at the entrance of tunnels.

2. Methodology

Modal implies that the displacement of each particle in a system deviates from the original equilibrium position when the system freely vibrates according to its natural frequency, thus satisfying a certain proportional relationship. Mode refers to a system’s vibration mode when it vibrates at a certain natural frequency, in which the first mode is the dominant mode. Modal analysis is a crucial method of structural dynamic analysis and a form of dynamic frequency domain analysis, which mainly obtains natural mode shapes and frequencies. The FE method is a commonly used modal analysis method. Based on the elastic mechanics principle, the dynamic governing equation of modal analysis is as follows [13,14]:

$$[M]\ddot{\mathbf{U}}+[C]\dot{\mathbf{U}}+[K]\mathbf{U}+\{F\}=0$$  \hspace{1cm} (1)

where $[M]$, $[C]$, and $[K]$ are the mass matrix, damping matrix, and stiffness matrix, respectively. $\{F\}$ is the external force load function, which varies with time, and $\ddot{\mathbf{U}}$ and $\dot{\mathbf{U}}$ are the model’s acceleration and velocity vectors, respectively. $\mathbf{U}$ is the displacement model’s vector, which is used to describe the modal analysis of the mode shapes. In an ideal case, the influence of external force and the damping effect is not considered in the modal analysis. The modal analysis dynamic governing equation can be expressed as [13, 14]:

\begin{align*}
[M]\ddot{\mathbf{U}}+[C]\dot{\mathbf{U}}+[K]\mathbf{U}+\{F\} = 0,
\end{align*}
\[ [M]\{\ddot{U}\}+[K]\{U\}=0 \]  
(2)

The characteristic equation of Equation (2) is as follows [13, 14]:
\[ ([K] - \omega_i^2[M])\{U\} = 0 \]  
(3)

where \( \omega_i \) is the \( i \)-th natural circular frequency (\( i = 1, 2, 3..., n \)). The resulting natural frequency, \( f_i \), is as follows [13, 14]:
\[ f_i = \frac{\omega_i}{2\pi} \]  
(4)

The eigenvector corresponding to the eigenvalue is \( \{U\}_i \). \( \{U\}_i \) represents the mode shape of vibration at the natural frequency, \( f_i \). The first mode is the dominant mode, and the model’s dynamic characteristics are mainly controlled by the low mode. Only the first several modes are considered in the analysis. The frequency of an object is related to its hardness, mass, and external size, and the natural frequency, \( f \), is related to the stiffness, \( k \), and mass, \( m \), and the relationship is given as follows [13, 14]:
\[ f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \]  
(5)

3. Dynamic response characteristics of rock slopes at the tunnel entrance and exit section

3.1. Numerical model

Two types of FE numerical models are established based on the original size of the slope, including Model 1 (slope without tunnel) and Model 2 (slope with tunnel) (Figure 1). The overall slope in the research area is 20°–40°, and the homogeneous slope is considered the research object. Each type of slope includes five working conditions, including slope gradients (\( \alpha = 20^\circ, 25^\circ, 30^\circ, 35^\circ, \) and \( 40^\circ \)). The rock mass is modeled using a tetrahedral mesh. The lithology is basalt, and the physical and mechanical parameters of the rock mass are shown in Table 1. The slope deformation is a small strain problem in the FE modal analysis, and an elastic model is used in the modal analysis. The linear perturbation analysis step of the ABAQUS implicit solution function is used for modal analysis. Boundary conditions are set as the key factors influencing the slope dynamic analysis. The basis of the actual slope is infinite, but in the FE model, boundary size is limited; thus, how to utilize the FE model to simulate the actual infinite slope foundation and make the results more reasonable have been the important influencing factors in the FE dynamic analysis. In this study, the infinite element boundary method is used to simulate the slope’s infinite foundation. The boundary conditions of the model were reasonably set. The free and constrained boundary conditions were adopted on the model’s boundary, specifically as follows: Constraints were set in the X-direction for the left and right boundary conditions, and free boundary conditions were adopted in the Y- and Z-directions. Constrained boundary conditions are adopted in the X-, Y-, and Z-directions of the bottom surface of the slope, and free boundary conditions are adopted in the three directions of the slope’s front and back sides. The bottom and both sides of the model are set as the infinite element boundary. Two types of rock slopes were modeled, including the rock slopes without and with a tunnel.

| Type           | Density \( \rho \) (kN/m\(^3\)) | Poisson's ratio \( \mu \) | Elastic modulus \( E \) (MPa) | Internal friction angle \( \phi \) (°) | Cohesion \( c \) (kPa) |
|---------------|-------------------------------|--------------------------|-------------------------------|----------------------------------------|------------------------|
| Rock          | 23                            | 0.3                      | 75                            | 35.0                                   | 150                    |
| Lining structure | 24.0                        | 0.167                    | 34.5                          | 24.2                                   | 0.1                    |

3
3.2. Natural frequency analysis
The FE method was used to conduct modal analysis on the ten numerical models. Figure 2 shows the first ten natural frequencies of the rock slope without and with a tunnel under different slope gradient conditions. The figure shows that the natural frequencies of the two types of numerical models under different slope conditions increase with the order of mode shapes. It also shows that the natural frequencies of the first two steps of the mean slope and the tunnel slope under different slope gradient conditions are close to each other, whereas, with the increase of modal orders, the natural frequencies of the slopes with different slope gradients show significant differences after the third order. Further, when the slope gradient of the rock slope without and with a tunnel is 20°–40°, the natural frequency of the slope decreases with an increase in the slope gradient, indicating that the slope gradient impacts the slope’s natural frequency. In addition, to compare the influence of tunnel excavation on the natural frequency of slopes, the natural frequency ratio of the slopes without and with a tunnel is shown in Figure 3. The figure shows that under the same slope condition, the natural frequency ratio of the slopes without and with a tunnel is 1.1:1.35 on the whole, indicating that the slope’s natural frequency increases to a certain extent due to the tunnel’s existence. This is because the tunnel excavation alters the slope stiffness to some extent, changing its natural frequency.
3.3. Modal analysis

The vibration characteristics of engineering entities are often controlled using low-order modes. The influence of higher-order modes in mechanical and architectural structures is significant and cannot be ignored. Further, the geological structure of most rock mass slopes in geotechnical engineering is extremely complex, exhibiting strong discontinuity. According to the frequency domain analysis, the natural frequency of slopes not only includes low-order natural frequency but also incorporates several high-order natural frequencies, which is confirmed in literature [15-17]. The slope’s vibration mode is one-to-one corresponding to its natural frequency, and the vibration mode analysis can provide a reference for the slope’s dynamic failure mode. Figures 4–6 show that the main vibration modal feature of the two models is the bending deformation. Figure 4 shows that the first-order mode is mainly characterized by bending deformation in the slope crest area, indicating that the crest area of the rock slope without tunnel ($\alpha = 30^\circ$) is prone to deformation and failure under earthquake conditions. The second and third-order modes are the bending deformation at the top of the slope; the fourth and fifth-order modes are the bending deformation at the bottom and middle of the slope body. Figure 5 shows that the first-order mode of the slope containing tunnel ($\alpha = 30^\circ$) is characterized by bending deformation of the slope crest area, indicating that the top of the slope is prone to deformation and failure under earthquake conditions. In addition, the second to eighth-order modes of the slopes are characterized by bending deformation. Thus, when the slope gradient is the same, the vibration mode of the slopes with a tunnel is the same as that of the slopes without a tunnel. This implies that tunnel excavation has little impact on the slope’s vibration mode characteristics, and does not significantly alter the slope’s vibration mode. By comparing the vibration mode characteristics of the
two slope models incorporating tunnel ($\alpha = 30^\circ, 40^\circ$), the vibration modes of the two tunnel slope models are observed to be the same, suggesting that the slope gradient has little influence on the vibration mode of the rock slopes with a tunnel.

In addition, Figures 4–6 show that the first- and second-order vibration modes of the slopes are mainly viewed as the overall deformation characteristics, whereas those of the third-order and above are viewed as local deformation characteristics. This indicates that the low-order (first- and second-order) natural frequencies are primarily responsible for overall slope deformation characteristics, whereas high-order ($\geq$third-order) natural frequencies are responsible for the local slope deformation characteristics.

**Figure 5.** Vibration mode of the modal analysis of the Model 2 ($\alpha = 30^\circ$): (a) first order; (b) second order; (c) third order; (d) fourth order; (e) fifth order; (f) sixth order; (g) seventh order; (h) eighth order

4. Conclusions

The modal analysis of the rock slopes at the tunnel entrance and exit section is performed using the FE model, and the following conclusions are drawn:

The slope gradient affects the natural frequency of the slopes at the tunnel entrance and exit section. When the gradient of the rock slopes without or with a tunnel is 20°–40°, the natural frequency of the slopes decreases with an increase in the gradient. In addition, tunnel excavation affects the slope’s natural frequency. Under the same conditions, the natural frequency ratio of the slope without and with a tunnel is 1.1:1.35, and tunnel excavation increases the slope’s natural frequency. The main mode of slopes with and without tunnels is bending deformation. Tunnel excavation and gradient have little influence on the slope’s vibration modal characteristics. The vibration modes of the slope with a tunnel are the same as those of the slope without a tunnel.
Acknowledgments
This work is financially supported by the National Natural Science Foundation of China (52109125), the China Postdoctoral Science Foundation (2020M680583), the National Postdoctoral Program for Innovative Talent of China (BX2020191), the Excellent Sino-foreign Youth Exchange Program of China Association for Science and Technology in 2020 (No. 58), and the Shuimu Tsinghua Scholar Program (2019SM058).

References
[1] Ding L Zhang L Wu X Skibniewski MJ Quanzhou Y 2014 Safety management in tunnel construction: case study of Wuhan metro construction in china Safety Science 62 8-15
[2] Ergün T 2018 Assessments on slope instabilities triggered by engineering excavations near a small settlement (turkey) Journal of Mountain Science 15(1) 114-129
[3] Song D Liu X Chen Z Chen J Cai J 2021 Influence of tunnel excavation on the stability of a bedded rock slope: a case study on the mountainous area in southern Anhui china KSCE Journal of Civil Engineering 25 114–123
[4] Song D Liu X Huang J Zhang J 2021 Energy-based analysis of seismic failure mechanism of a rock slope with discontinuities using Hilbert-Huang transform and marginal spectrum in the time-frequency domain Landslides 18 105–123
[5] Song D Liu X Li B Zhang J Bastos J 2020 Assessing the influence of a rapid water drawdown on the seismic response characteristics of a reservoir rock slope using time–frequency analysis Acta Geotechnica 1-22

Figure 6. Vibration mode of the modal analysis of the Model 2 ($\alpha = 40^\circ$): (a) first order; (b) second order; (c) third order; (d) fourth order; (e) fifth order; (f) sixth order; (g) seventh order; (h) eighth order
[6] Wang ZF, Shi FG, Li DD, Li H. 2020. Tunneling-induced deep-seated landslides: a case study in Gulin County, Sichuan, China. *Arabian Journal of Geosciences* 13(19):1039.

[7] Bandini A, Berry P, Boldini D. 2015. Tunnelling-induced landslides: the Val di Sambro tunnel case study. *Engineering Geology* 196:71-87.

[8] Wang G, Huang R, Louren OS, Kamai T. 2014. A large landslide triggered by the 2008 Wenchuan (M8.0) earthquake in Donghekou area: phenomena and mechanisms. *Engineering Geology* 182:148-157.

[9] Jiang X, Wang F, Yang H, Sun G, Niu J. 2018. Dynamic Response of Shallow-Buried Small Spacing Tunnel with Asymmetrical Pressure: Shaking Table Testing and Numerical Simulation. *Geotech Geol Eng* 36:2037-2055.

[10] Wang F, Jiang X, Niu J, Yang H. 2018. Experimental study on seismic dynamic characteristics of shallow-bias tunnel with a small space. *Shock and Vibration* 6412841.

[11] Niu J, Jiang X, Wang F. 2018. Stability analysis of rock slope with small spacing tunnel under earthquakes and influence of ground motion parameters. *Geotechnical Geological Engineering* 36:2437-2453.

[12] Wang DY, Yuan JX, Cui GY, Liu J, Wang HF. 2020. Experimental study on characteristics of seismic damage and damping technology of absorbing joint of tunnel crossing interface of soft and hard rock. *Shock and Vibration* 2020(1):1-13.

[13] Lee JH, Lee BS. 2012. Modal analysis of carbon nanotubes and nanocones using FEM. *Computational Materials Science* 51(1):1-42.

[14] Song DQ, Che AL, Zhu RJ, Ge XR. 2019. Natural frequency characteristics of rock masses containing a complex geological structure and their effects on the dynamic stability of slopes. *Rock Mechanics and Rock Engineering* 52(11):4457-4473.

[15] Zhou YF, Liu HX, Zhu X, Wen JH. 2020. Modal analysis of rock slope with a weak interlayer and its influence on seismic dynamic response of slope. *Earthquake Engineering and Engineering Dynamics* 40(01):223-232.

[16] Zhang XD, Yan ZX, Zhang S. 2010. ANSYS Numerical Analysis on Dynamic Response of Rock Slope Using ANSYS Software. *China Earthquake Engineering Journal* 32(02):117-121.

[17] Sun WY, Yan SH, Ou EF, Song XH. 2018. Analysis of influence factors on natural vibration characteristics of loess slope. *Journal of Railway Science and Engineering* 15(01):64-70.