Enhanced Joule Heating in Umbral Dots

Chandan Joshi, Lokesh Bharti and S.N.A. Jaaffrey
Astronomy and Astrophysics Laboratory, Department of Physics, University College of Science, Mohanlal Sukhadia University, Udaipur, 313001, India
(chandan-joshi@hotmail.com)

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Abstract.
We present a study of magnetic profiles of umbral dots (UDs) and its consequences on the Joule heating mechanisms. Hamedivafa (2003) studied Joule heating using vertical component of magnetic field. In this paper UDs magnetic profile has been investigated including the new azimuthal component of magnetic field which might explain the relatively larger enhancement of Joule heating causing more brightness near circumference of UD.

1. Introduction

Umbral dots (UDs) are bright features observed in the sunspot umbrae. Detailed study of such bright features play a key role in understanding the energy transport in sunspots and they exhibit most important physical parameters such as temperature, brightness, lifetime, magnetic field, size, mass outflows etc. For recent reviews on the subject, see Solanki (2003), Thomas and Weiss (2004).

There are two models for the UDs. First suggested by Parker (1979a, b) and then subsequently by Choudhuri (1986). Parker suggested in his sunspot model that the region below the visible surface termed as positive Wilson depression is made up of individual flaring flux tube embedded in the field free plasma. These tubes merge into an apex like single flux tube just above the umbral surface (negative Wilson depression level) of the sunspot. The expanding and rising up-flow of plasma creates a gap and develops UDs when it reaches to zero Wilson depression surface in sunspot. Further Choudhuri (1986) also showed that if the pressure of the plasma plume increases at the apex of the static configuration of a field free gas column, then it rises to certain height where the magnetic field pressure suddenly becomes negligible. Finally the trapped field free gas bursts in to a column at a speed of about 10 km s$^{-1}$ forming a UD. In another magnetohydrodynamic model, UDs are considered as tops of convective cells present in homogeneous magnetic field (Knobloch and Weiss, 1984). Later Degenhardt and Lites (1993) speculated that UDs are thin vertical magnetic tubes with a reduced magnetic field strength, a temperature enhancement and material upward flow from the bottom, embedded in a stationary sunspot umbra.

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Some experimental studies have revealed that the lifetime of UDs is in the range from 15 min to 2 hrs with average diameter of 150–300 km and the relative intensities with regard to umbra of sunspot vary from 1.1–2.6 (Lites et al., 1991; Ewell, 1992; Sobotka et al., 1997a; Tritschler and Schmidt, 2002; Sobotka and Hanslmeier, 2005). The number of investigators have also observed mass upward motion within the range of 0.3–1 km s\(^{-1}\) (Pahlke and Wiehr, 1990; Lites et al., 1991; Wiehr, 1994; Rimmele, 1997, 2004).

The role of magnetic profile has been unique for some specific properties of UDs and in one of the studies of magnetic profiles, Choudhuri (1986) established that there has been a reduced magnetic strength in the UD column relative to the surrounding umbra. Some spectroscopic observations of Adjabshirzadeh and Koutchmy (1983), Pahlke and Wiehr (1990), and Wiehr and Degenhardt (1993) have shown that positive magnetic field gradients in umbral dots are of about 20\%, whereas in the high resolution study of Lites et al. (1991) have revealed that magnetic strength is not significantly different from that the surrounding darker portions of the umbrae. Also recent study (Socas-Navarro et al., 2004) showed that there is more inclined fields in UDs and the vertical gradient of field may have opposite signs in the UD and dark background. Since all observations are obtained often with a low spatial resolution restricted for a small area so that the complete magnetic vector remains unknown. Thus these observational results point out that a good knowledge of magnetic vector field is crucially required. However, in recent simulation study of Schüssler and Vögler (2006), they have manifested that nearly field free upflow plumes and the UDs are a natural result of convection in a strong initially monolithic magnetic field.

In the subsequent Section 2 magnetic field profile for UDs has been discussed whereas in Subsection 2.2, we have tried to develop the modified magnetic field profile by introducing a proposed azimuthal component \(B_\phi(r)\) crucially required with vertical component \(B_z(r)\) and its effect on Joule heating. The resultant magnetic vector obtained for the UDs might be attributed to the increased current density which probably may enhance Joule heating power causing a relatively more brightness near the circumference as compared to the center of the UDs.

2. Magnetic Profile of Umbral Dots and Joule Heating

We still know very little about the structure and the nature of the magnetic field of UD, as there are few high-resolution evidences available (Lites et al., 1991; Socas-Navarro et al., 2004) for the vector magnetic field that can provide correct magnetic field structure in UDs. In spite of these observational challenges, the right choice of magnetic profile may produce better consistency with observations. In the next Subsection 2.1 we first give an overview
of vertical magnetic field and then we propose an additional azimuthal magnetic field with the vertical component in the subsequent Subsection 2.2.

2.1. Vertical component of magnetic field in umbral dots

As a matter of fact Joule heating power is partially responsible for brightness of the UDs and is directly governed by current density, which in principle is attributed to magnetic field profile (Garcia de la Rosa, 1987). Thus it was assumed for simplicity that magnetic field vector has only vertical parallel component in the UD column (Hamedivafa, 2003) and is a function of distance from axis of the UD column as

$$\mathbf{B} = B_z(r)\mathbf{a}_z$$  \hspace{1cm} (1)

where $\mathbf{a}_z$ is the unit vector along $z$-axis normal to the photosphere of the Sun and $r$ is the radial component in cylindrical coordinates $(r, \phi, z)$. Hamedivafa (2003) investigated Joule heating as brightening mechanism for umbral dots and he assumed one of the magnetic profiles given as

$$B_z(r) = B_0 - \gamma B_0 \exp \left( -\frac{r^2}{u^2} \right)$$  \hspace{1cm} (2)

where $B_0$ is saturated magnetic field in umbra of the sunspot, $\gamma$ is fraction of the field reduction and $u$ is the maximum radius of the UD column at any instant. Hamedivafa and Sobotka (2004) found direct observational evidence for Joule heating in some of the UDs.

2.2. Azimuthal component of magnetic field in UDs

A prudential overview of the existing models for the magnetic field of UDs is required since the magnetic nature of bright features in sunspot umbrae is not fully understood yet. The observational results of Lites et al. (1991) pointed out that UDs magnetic field strength is not very different from umbral field and also Socas-Navarro et al. (2004) showed that UDs have inclined fields. This promulgated the following two major assumptions which have been contemplated in theoretical investigations:

1. $B_z(r)$ is the force free axial vector field for the expanding and rising parallel axial flow of plasma plume in UD column. It develops cusp-like shape due to steeper pressure gradient of piled-up plasma in UDs leading to a drastic decrease of magnetic field strength in the upper layer of plumes than the proximate circumference of UD.

2. Steady flow of material in UD column does not give a temporally changing electric field i.e. there is no changing electric flux.
These assumptions deduce a basic magneto-hydrodynamic equation

$$(\nabla \times \mathbf{B})_z \propto \mu J_z$$

which is valid for azimuthal component. Where $\mu$ is the magnetic permeability across the active region and the current density $J_z$ is associated with a convectively unstable and oscillating vertical slab of plasma, sandwiched between the regions of $B_z(r)$ with vertical wavelength of 100 km and of period 100 s (Parker, 1979b). Moreover it may be expressed as a possible electric current parallel to the force free $B_z(r)$ within $r \leq u$ column as

$$I = \oint J_z \cdot da$$

Here $r$ is the radial distance from the UD axis; $u$ is the maximum radius of UD at which an undisturbed magnetic field strength of umbra exists. The high temperature of the hot unstable outflowing material would be sufficient to yield an ionized form, creating enough electrical current in these column. These concepts led us to believe that there should be azimuthal magnetic component $B_\phi(r)$ attached to the thin UD column, due to electric current of material at high temperature. Therefore, the magneto-hydrodynamic calculations under some boundary conditions for evolution of these flux tubes, diagnose UD$s$ of enhanced temperature and high intensity relative to surrounding umbra. However this current would be able to generate azimuthal magnetic field $B_\phi(r)$ with the help of Ampere circuitial law, Equation (3), and $B_\phi(r)$ may be expressed as

$$B_\phi(r) = \frac{\gamma \mu I r}{2\pi u^2} \quad \text{when} \quad r < u$$

$$B_\phi(r) = \frac{\gamma \mu I}{2\pi r} \quad \text{when} \quad r > u.$$  

Here $\gamma$ is the fraction of the field strength reduction on the central axis of the UD column with range $1 \geq \gamma \geq 0$. Hamedivafa (2003) has revealed that the magnetic field reduction is not beyond the radius of the bright UD$s$ but saturates after $r/u \approx 2$. The contribution of $B_\phi(r)$ is linear from the center to the circumference of the UD and then hyperbolically decreases beyond the UD column. The inclusion of $B_\phi(r)$ in the proposed model may be justified on the basis of following assumptions:

1. The center of the azimuthal component coincides with the symmetric axis of the UD$s$. It has been assumed to resolve the ambiguity between center of $B_\phi(r)$ and vertical component $B_z(r)$ which yields more or less fine radial magnetic structure inside UD$s$ producing a temperature stratification and a height independent values for magnetic field strength with respect to line of sight.
2. Magnetic field of the UDs consists of the two components, $B_\phi(r)$ and $B_z(r)$ to provide a consistent feature of magnetic profile in such a way that the magnetic field inside an UD is weaker than surrounding field and large magnetic field gradient is present at the boundary of UD. The component $B_\phi(r)$ might contribute more power to Joule heating.

3. $B_\phi(r)$ at the circumference of an UD becomes comparable to the observable magnetic field, $B_0$ of umbrae.

Thus resultant $\mathbf{B}(r)$ may be written as

$$\mathbf{B}(r) = B_\phi(r)\mathbf{a}_\phi + B_z(r)\mathbf{a}_z.$$  \hfill (7)

Normalised $B(r)$, $B_\phi(r)$ and $B_z(r)$ with $B_0$ are plotted as shown in Figure 1. The effective current density in UDs is calculated by the magnetic field as

$$\mathbf{J}_T = \frac{c}{4\pi} \nabla \times \mathbf{B}(r).$$  \hfill (8)
or the value of $\mathbf{J}_T$ is calculated in cylindrical co-ordinate system by the following determinant

$$
\mathbf{J}_T = \begin{vmatrix}
\frac{d}{dr} a_r & \frac{d}{dr} a_\phi & \frac{d}{dr} a_z \\
\frac{d}{d\phi} a_r & \frac{d}{d\phi} a_\phi & \frac{d}{d\phi} a_z \\
0 & \frac{\gamma \mu I r}{2\pi u^2} B_0 - \gamma B_0 \exp \frac{r^2}{u^2(t)} & \frac{\gamma \mu I}{8\pi^2 u^2} a_z
\end{vmatrix}.
$$

$$
\mathbf{J}_T = \left[ -\frac{c}{4\pi} \gamma B_0 \exp \frac{-r^2}{u^2} \right] a_\phi + \left[ \frac{c \gamma \mu I}{8\pi^2 u^2} \right] a_z
$$

Now the Joule heating power can be given as:

$$
Q' = \frac{4\pi}{c^2} \int_0^\infty \eta J^2_T (2\pi r) dr,
$$

where $\eta$ is electrical resistivity.

$$
J^2_T = \mathbf{J}_T \cdot \mathbf{J}_T = \frac{c^2}{16\pi^2} \gamma^2 B_0^2 \frac{4r^2}{u^4} \exp \left( \frac{-2r^2}{u^2} \right) + \frac{c^2 \gamma^2 \mu^2 I^2}{64\pi^4 u^4}.
$$

From Equation (10) and (11), we get

$$
Q' = \frac{2\eta \gamma^2 B_0^2}{u^4} \left( \frac{u^4}{8} \right) + \frac{\eta \gamma^2 \mu^2 I^2 u^2}{4(4\pi^2 u^2)}.
$$

We assume that when $r \rightarrow u$ then $|B_\phi(r)| \rightarrow |B_0(r)|$, i.e. the maximum strength of azimuthal component at the boundary of UD is comparable with the undisturbed magnetic field $B_0$ of the umbra outside of the UD. Thus from Equation (5) we get

$$
B_0 = \frac{\gamma \mu I u}{2\pi u^2} = \frac{\gamma \mu I}{2\pi u}
$$

or

$$
\mu^2 I^2 = \frac{B_0^2 (2\pi u)^2}{\gamma^2}.
$$

Let

$$
U_{\gamma B} = \frac{\gamma^2 B_0^2}{8\pi}.
$$

With the help of Equations (12) – (15), the final power can be given as

$$
Q' = \frac{4\pi \eta U_{\gamma B}}{2} \left[ 1 + \frac{1}{\gamma^2} \right].
$$

The Joule Heating power is proportional to $2\pi \eta U_{\gamma B}$ whereas $Q = 2\pi \eta U_{\gamma B}$ is calculated by Hamedivafa (2003). The factor in the square bracket represents the enhancement in the power due to the current generated from
the magnetic field at $r$ close to $u$. For a smaller value of $\gamma$, larger would be the Joule heating power. This justifies the special features observed in the experimental studies of Lites et al. (1991) that the field strength within the umbrae vary at large scale (1400 – 2400 Gauss) and this large scale variation of field being inversely correlated with the umbral dot intensity.

3. Discussion and Conclusions

The result presented in this report depends upon the following crucial assumptions:

1. The $B_\phi(r)$ is developed just at the middle of UDs and increases linearly with $r$ inside the UD diameter up to the maximum value $B_0$. It decreases hyperbolically outside.

2. The current density $J_T$ is uniform within the UDs.

3. Magnetic profile of UDs consists of at least two magnetic components, $B_\phi(r)$ and $B_z(r)$ which stem not necessarily from the same geometrical height.

We conclude enhanced field on peripheral surface of UD as shown in Figure 1. The large peripheral flux is attributed to the additional azimuthal magnetic field component, which is produced by axial current $I$ due to intrusion of hot plasma. Thus modified UD model obtained by adding $B_\phi(r)$ into $B_z(r)$ helps to explain variation in brightness of the UDs. The relative Joule heating

$$\Delta Q = \frac{Q'}{Q} = 1 + \frac{1}{\gamma^2}$$

(17)

Table I. The relative Joule heating with different values of $\gamma$

| $\gamma$  | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
|-----------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| $\Delta Q$ | 101.0 | 26.0 | 12.11 | 7.25 | 5.00 | 3.78 | 3.04 | 2.56 | 2.23 | 2.00 |

Fractional Joule heating is much more at lower values of $\gamma$ but at higher values decreases slowly. The estimated Joule heating power is expected to justify the inverse correlation of brightness and magnetic field gradient (Lites et al., 1991) and the resultant magnetic vector supports more inclined magnetic field vector (Socas-Navarro et al., 2004) of the UD.

Moreover, these conclusions can be justified by high resolution spectropolarimetric data from ground based instrument such as Diffraction Limited Spectropolarimeter (DLSP) (Sankarsubramanian et al., 2004) at Dunn...
Solar telescope (NSO) and the spectropolarimeter onboard the satellite Hinode.

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