Higher Twist Contributions to the Azimuthal Asymmetries in SIDIS

Yu-kun Song
School of Physics, Shandong University, Jinan, Shandong 250100, China
E-mail: songyk@mail.sdu.edu.cn

Abstract. This is intended to be a summary of a series of publications [1, 2, 3] carried out by our group. We showed that collinear expansion can be extended to the semi-inclusive deep inelastic scattering process \(e + p \rightarrow e + q + X\) as a systematic way of studying higher twist effects. We calculated the cross section up to twist-3 level for different polarized and unpolarized cases, and obtained the azimuthal asymmetry \(\langle \cos \phi \rangle\) in terms of gauge invariant twist-3 TMD parton correlation functions [1]. We further calculated the complete cross section up to twist-4 level for the unpolarized case, and obtained the azimuthal asymmetry \(\langle \cos 2\phi \rangle\) expressed in terms of gauge invariant twist-4 TMD parton correlation functions [3]. We also showed that the nuclear dependence of the TMD distributions can be obtained from the gauge link and studied the nuclear dependence of both \(\langle \cos \phi \rangle\) and \(\langle \cos 2\phi \rangle\) asymmetries with a Gaussian ansatz for transverse momentum dependence of parton correlation functions [2, 3].

1. Introduction

Inclusive and semi-inclusive deep inelastic scatterings (SIDIS) are important tools to investigate the structure of nucleon and nucleus governed by the Quantum Chromodynamics (QCD) for the strong interaction. The azimuthal asymmetries and their spin and/or nuclear dependence of the SIDIS cross sections are directly related to the parton distribution and polarization inside nucleon or nuclei and therefore are the subjects of intense studies both theoretically and experimentally. The azimuthal asymmetries \(\langle \cos \phi \rangle\) and \(\langle \cos 2\phi \rangle\) probe both the intrinsic transverse momentum distributions of partons (TMD) and perturbative QCD gluon bremsstrahlung, and have involved many theoretical and experimental efforts from almost 30 years ago. In small \(k_\perp\) region, many phenomenological analysis just employ simple parton model calculations [4] to account for the higher twist contributions. But as we know QCD multiple gluon interactions also contribute to these asymmetries, and they are indispensable in achieving gauge invariant results. Thus the applicability of naive parton model calculations is not appropriate, and exact gauge invariant higher twist contributions to azimuthal asymmetries in the framework of QCD are needed both in theory and phenomenology. The standard way of extracting higher twist contributions in DIS process is developed almost 30 years ago, named collinear expansion [5, 6, 7, 8, 9]. We extended this formalism to SIDIS process and extracted exact gauge invariant twist-3 and twist-4 contributions to azimuthal asymmetries, and further discussed nuclear dependence of these asymmetries [1, 2, 3]. In this report, I will briefly summarize the calculations and present the main results.
2. SIDIS up to twist-3 and $\langle \cos \phi \rangle$

For simplicity we considered the jet production process $e + p \rightarrow e + q + X$, which does not involve complications from hadronization process. To include higher twist contributions, we must take into account following diagrams with extra gluons exchanged between the struck quark and the spectator.

These diagrams are expressed as

$$
d\sigma \sim L_{\mu \nu}^2 d^2 \frac{W_{\mu \nu}}{k_{\perp}^2}$$

(1)

$$
d^2 W_{\mu \nu} = \int \frac{d^4 k}{(2\pi)^4} Tr \left[ \hat{H}_{\mu \nu}^{(0)}(k) \hat{\phi}(k) \right] + \sum_{c=L,R} \int \frac{d^4 k_1}{(2\pi)^4} \frac{d^4 k_2}{(2\pi)^4} Tr \left[ \hat{H}_{\mu \nu}^{(1)c}(k_1, k_2) \hat{\phi}_\rho(k_1, k_2) \right] + \sum_{c=L,M,R} \int \frac{d^4 k_1}{(2\pi)^4} \frac{d^4 k_2}{(2\pi)^4} \frac{d^4 k_3}{(2\pi)^4} \frac{d^4 k_4}{(2\pi)^4} Tr \left[ \hat{H}_{\mu \nu}^{(2)c}\rho\sigma(k_1, k_2, k_3, k_4) \hat{\phi}_\rho(k_1, k_2, k_3, k_4) \right] \cdots$$

(2)

The workflow of extracting higher twist contributions to SIDIS is as follows [1]. First we perform a Taylor expansion of hard parts $\hat{H}_{\mu \nu}^{(n)}$ at $k = x p$

$$
\hat{H}_{\mu \nu}^{(0)}(k) = \hat{H}_{\mu \nu}^{(0)}(x) + \frac{\partial \hat{H}_{\mu \nu}^{(0)}(x)}{\partial k_\rho} \omega^\rho k_\rho' + \frac{1}{2} \frac{\partial^2 \hat{H}_{\mu \nu}^{(0)}(x)}{\partial k_\rho \partial k_\sigma} \omega^\rho \omega^\sigma k_\rho' k_\sigma' + \cdots
$$

(3)

$$
\hat{H}_{\mu \nu}^{(1)L}(k_1, k_2) = \hat{H}_{\mu \nu}^{(1)L}(x_1, x_2) + \frac{\partial \hat{H}_{\mu \nu}^{(1)L}(x_1, x_2)}{\partial k_\rho} \omega^\rho k_\rho' + \cdots
$$

(4)

$$
\hat{H}_{\mu \nu}^{(2)L}(k_1, k_2, k_3) = \hat{H}_{\mu \nu}^{(2)L}(x_1, x_2, x_3) + \cdots
$$

(5)

where $\omega^\rho k_\rho' \equiv g^\rho_\rho' - \tilde{n} \rho n^\rho'$ is a projection operator so that $\omega^\rho k_\rho' = (k - x p)_\rho$. Then we decompose the gluon fields in the correlation matrix $\phi$ into collinear and non-collinear parts

$$
A^\rho = A^+ \frac{p^\rho}{p \cdot n} + \omega^\rho A^\rho
$$

(6)

Then we employ Ward identities to replace these derivatives and convert several terms

$$
\frac{\partial \hat{H}_{\mu \nu}^{(0)}(x)}{\partial k_\rho} = -\hat{H}_{\mu \nu}^{(1,L)\rho}(x_1, x_2) - \hat{H}_{\mu \nu}^{(1,R)\rho}(x_1, x_2),
$$

(7)

$$
\frac{\partial \hat{H}_{\mu \nu}^{(1,L)\rho}(x_1, x_2)}{\partial k_\sigma} = -\hat{H}_{\mu \nu}^{(2,L)\rho}(x_1, x_2, x_3),
$$

(8)

$$
\frac{\partial \hat{H}_{\mu \nu}^{(1,L)\rho}(x_1, x_2)}{\partial k_1\sigma} = -\hat{H}_{\mu \nu}^{(2,L)\rho}(x_1, x_2, x_3) - \hat{H}_{\mu \nu}^{(2,M)\rho}(x_1, x_2, x_1),
$$

(9)
The above formalism is further extended to twist-4 level, and we get the complete cross sections

\[ d\sigma = 2 \text{cross section of SIDIS with transversely polarized initial state hadron} \]  

Asymmetry

\[ \langle \cos \phi \rangle = -\frac{2(2-y)\sqrt{1-y} M f_\perp(x_B,k_\perp)}{1 + (1-y)^2} \]

Comparing with naive parton model results, \( \langle \cos \phi \rangle \) is proportional to an extra ratio of the twist-3 correlation function \( x_B f_\perp(x_B,k_\perp) \) to twist-2 distribution function \( f_1(x_B,k_\perp) \). Due to QCD interactions, this ratio might not be close to 1. This fact may have notable impact on phenomenological analysis of experimental data. On the other hand, the measurement of \( \langle \cos \phi \rangle \) provides a practical way of studying the twist-3 correlation function \( f_\perp(x_B,k_\perp) \).

**3. SIDIS up to twist-4 and \( \langle \cos 2\phi \rangle \)**

The above formalism is further extended to twist-4 level, and we get the complete cross sections
for unpolarized SIDIS process [3]. The procedure are quite the same, and here we just give the final results

\[
\frac{d\sigma}{dx_Bdyd^2k_\perp} = \frac{2\pi\alpha_m^2e_q^2}{Q^2y} \left\{ [1 + (1 - y)^2]f_1(x_B, k_\perp) - 4(2 - y)\sqrt{1 - y}\frac{|k_\perp|}{Q}x_Bf_1^{(1)}(x_B, k_\perp) \cos \phi \\
- 4(1 - y)\frac{|k_\perp|^2}{Q^2}x_B[\hat{\varphi}_1^{(1)}(x_B, k_\perp) - \hat{\varphi}_1^{(1)}(x_B, k_\perp)] \cos 2\phi \\
+ 8(1 - y)\left(\frac{|k_\perp|^2}{Q^2}x_B[\hat{\varphi}_1^{(1)}(x_B, k_\perp) - \hat{\varphi}_1^{(1)}(x_B, k_\perp)] + \frac{2x_B^2M^2}{Q^2}f_1(-)(x_B, k_\perp)\right) \\
- 2 \left[ 1 + (1 - y)^2 \right] \frac{|k_\perp|^2}{Q^2}x_B\hat{\varphi}_1^{(2)}(x_B, k_\perp) \right\}.
\]

Then we get the asymmetry \(\langle \cos 2\phi \rangle\)

\[
\langle \cos 2\phi \rangle = -\frac{2(1 - y)}{1 + (1 - y)^2}\frac{|k_\perp|^2}{Q^2}x_B[\hat{\varphi}_1^{(1)}(x_B, k_\perp) - \hat{\varphi}_1^{(1)}(x_B, k_\perp)]f_1^{(1)}(x_B, k_\perp). \tag{20}
\]

Again \(\langle \cos 2\phi \rangle\) is proportional to the ratio of the twist-4 correlation function \(x_B[\hat{\varphi}_1^{(1)}(x_B, k_\perp) - \hat{\varphi}_1^{(1)}(x_B, k_\perp)]\) to the twist-2 distribution function \(f_1(x_B, k_\perp)\). Both \(\langle \cos \phi \rangle\) and \(\langle \cos 2\phi \rangle\) will be recovered to parton model results for the special case \(g = 0\), which are consistent with previous parton model calculations [4].

4. Nuclear dependence of azimuthal asymmetries

Azimuthal asymmetries serve as a new tool to probe nuclear properties. The formalism to include nuclear effects is first developed in [10], and then applied in [2] to study the nuclear dependence of \(\langle \cos \phi \rangle \) asymmetry. Nuclear dependence of \(\langle \cos 2\phi \rangle \) asymmetry is also studied in [3]. Because gauge link is generated by multiple scalar gluon exchange between the struck quark and the spectator, and these gluon lines can be connected to different nucleons in a large nucleus, the gauge link of nuclear parton distribution/correlation functions thus contain information of the distribution of a nucleon in the nucleus. This is the starting point of the formalism in [10]. Under the “maximal 2-gluon correlation approximation”, that is, they only keep those terms that have the maximum nuclear size enhancement, this formalism can give simple relations between parton distribution/correlation functions of nucleon and nucleus.

\[
f_1^A(x, k_\perp) \simeq \frac{A}{\pi\Delta_{2F}} \int d^2\ell_\perp e^{-\left(k_\perp - \ell_\perp\right)^2/\Delta_{2F}} f_1^N(x, \ell_\perp) \tag{21}
\]

\[
f_2^A(x, k_\perp) \simeq \frac{A}{\pi\Delta_{2F}} \int d^2\ell_\perp e^{-\left(k_\perp - \ell_\perp\right)^2/\Delta_{2F}} \frac{k_\perp \cdot \ell_\perp}{k_\perp^2} f_1^N(x, \ell_\perp) \tag{22}
\]

\[
\varphi_1^{(1)A}(x, k_\perp) \simeq \frac{A}{\pi\Delta_{2F}} \int d^2\ell_\perp e^{-\left(k_\perp - \ell_\perp\right)^2/\Delta_{2F}} 2\frac{(k_\perp \cdot \ell_\perp)^2 - \ell_\perp^2}{k_\perp^2} \varphi_2^{(1)N}(x, \ell_\perp) \tag{23}
\]

Adopting simple Gaussian ansatz for all transverse momentum dependence of parton distribution/correlation functions, and take identical parameters for these Gaussian distributions, we get in the following the simple result

\[
\frac{\langle \cos \phi \rangle_{eA}}{\langle \cos \phi \rangle_{eN}} \simeq \frac{\alpha}{\alpha + \Delta_{2F}} \quad \frac{\langle \cos 2\phi \rangle_{eA}}{\langle \cos 2\phi \rangle_{eN}} \simeq \left(\frac{\alpha}{\alpha + \Delta_{2F}}\right)^2 \tag{24}
\]

which show that azimuthal asymmetries are suppressed for nucleus involved SIDIS, and \(\langle \cos 2\phi \rangle\) is more suppressed than \(\langle \cos \phi \rangle\).
5. Conclusion

In this report I summarized a series of papers by our group on higher twist contributions to azimuthal asymmetries in SIDIS process, and discussed the nuclear dependence of these asymmetries. These asymmetries are proportional to the ratios of higher twist correlation functions to leading twist distribution functions, and may be quite different from naive parton model results. Measurements of these asymmetries provide a practical way of studying high twist correlation functions. Under a simple Gaussian ansatz for transverse momentum distribution, we showed that azimuthal asymmetries are suppressed for nucleus involved SIDIS process.

References
[1] Liang Z t and Wang X N 2007 Phys. Rev. D75 094002 (Preprint hep-ph/0609225)
[2] Gao J H, Liang Z t and Wang X N 2010 Phys. Rev. C81 065211 (Preprint 1001.3146)
[3] Song Y k, Gao J h, Liang Z t and Wang X N 2011 Phys. Rev. D83 054010 (Preprint 1012.4179)
[4] Cahn R N 1978 Phys. Lett. B78 269
[5] Ellis R K, Furmanski W and Petronzio R 1982 Nucl. Phys. B207 1
[6] Ellis R K, Furmanski W and Petronzio R 1983 Nucl. Phys. B212 29
[7] Qiu J W 1990 Phys. Rev. D42 30–44
[8] Qiu J w and Sterman G F 1991 Nucl. Phys. B353 105–136
[9] Qiu J w and Sterman G F 1991 Nucl. Phys. B353 137–164
[10] Liang Z t, Wang X N and Zhou J 2008 Phys. Rev. D77 125010 (Preprint 0801.0434)