Holographic Aspects of a Higher Curvature Massive Gravity

Shahrokh Parvizi\(^1\)
and Mehdi Sadeghi\(^{1,2}\)

\(^1\)Department of Physics, School of Sciences, Tarbiat Modares University, P.O.Box 14155-4838, Tehran, Iran
\(^2\)Department of Science, University of A. O. Borujerdi

June 13, 2017

Abstract

We study the holographic dual of a massive gravity with Gauss-Bonnet and cubic quasi-topological higher curvature terms. Firstly, we find the energy-momentum two point function and central charges of the 4-dimensional boundary theory, albeit the massive term acts as a relevant perturbation to the conformal field theory. Then we focus on a black hole solution in this background and derive the ratio of shear viscosity to entropy density for the dual theory. Results smoothly cover the massless limit.

Keywords: AdS/CFT duality, central charge, c-theorem, shear viscosity

1 Introduction

For two decades, the AdS/CFT correspondence \([1]-[3]\) has been at the center of attention in theoretical physics. It not only provides tools for performing calculations in strong coupling limit of field theories and condense matter phenomena, which otherwise undoubtedly was horrible if not impossible, but also opens new windows to understand different aspects of field theories and gravities as well. Much has been done for the Einstein gravity in the AdS bulk and investigated its CFT dual on the boundary. In the early of the AdS/CFT, the central charges of the boundary theory was found by the holography \([4]\). This was an important success which among others encouraged people to develop the duality for more complicated and realistic theories. In 4 dimensions, it is well known that CFT’s with Einstein gravity dual have two equal central charges \(a\) and \(c\) \([4,5]\). This means that the dual Einstein gravity gives information about a very special class of CFT’s. To distinguish between central charges and explore
more general conformal theories, one may add higher curvature (or higher derivative) gravities, which amongst them, the Gauss-Bonnet gravity serves as a simple model to study the duality. It has important features as its equation of motion includes only the second order derivatives, admits exact black hole solution and the corresponding dual theory is a CFT with two distinguished central charges $a \neq c$ [6]. It is possible to keep these advantages and add cubic curvature terms, the so called quasi-topological gravity which of course doesn’t generally admit a second order differential equation except for the AdS background which is in our interest [7, 8, 9]. The combination of Gauss-Bonnet and cubic quasi-topological gravity has not yet any stringy derivation, however as a toy model is rich enough to study different aspects of the dual conformal theory [9].

On the other hand, several studies have been performed for decades to generalize the graviton field to a massive one with different motivations from theoretical curiosity to phenomenological model buildings (for a recent review see [10]). Indeed, the problem of giving mass to the gravity is not an obvious one and was a challenge for several years. The first attempt was by Fierz and Pauli [11] proposing a linear massive model. Unfortunately, that doesn’t reduce to GR in the zero mass limit. Generalization to a nonlinear model in [12] was stopped by Boulware and Deser (BD) when they showed that this suffered from ghosts [13]. Finally in recent years, the BD ghost problem was resolved in [14, 15, 16] by a nonlinear massive gravity. This theory provides a fixed reference metric on which the massive gravity propagates. This breaks the general covariance with applications in holographic models with momentum dissipation [17, 18]. Then it was extended to include dynamics of this reference metric in the context of theories now known as bi-gravities [19, 20, 21] and higher dimensional massive graviton term is discussed in [22]. Many recent works include this massive gravity in the higher curvature gravities especially the Gauss-Bonnet (e.g., [23, 24, 25]) and various features mostly in the gravity side and some in the dual theories are derived.

Here our aim is to tackle the theory including Gauss-Bonnet cubic quasi-topological massive gravity. The important point about this combination is that while it has the rich structure of higher curvature theories, the presence of a mass scale will breaks the conformal symmetry on the boundary. One may consider the addition of massive gravity as a relevant perturbation of conformal symmetry. This may shed light on the boundary theory away from the fixed point.

In this regards, we study the boundary theory in section 2. Firstly we derive the two-point function of the boundary energy-momentum tensor and show that it includes a massive operator on the boundary. Then we find the trace of energy-momentum tensor. In addition to anomaly terms from which one can read central charges, we find a mass dependent term which is responsible for explicit conformal symmetry breaking. Of course it disappears in the zero mass limit.

We then look for an $\alpha$-theorem which is originally based on renormalization group flows [26, 27, 28, 29] and indicates the truncation of degrees of freedom when going toward an IR fixed point. In the context of AdS/CFT correspondence, $\alpha$-theorems are introduced by considering a generic background which asymptotes to AdS space [7, 30, 31, 32]. Then the $\alpha$-function approaches the $a$ charge (not the $c$) in the AdS limit. The function should be monotonically decreasing along the RG flow. This can be achieved by the null energy condition. In our case, we show that the same null
energy condition as the massless theory gives the correct monotonic $a$-function.

In section 3, we introduce an exact black hole solution in this background and derive its temperature and entropy then using the standard holographic methods to find the viscosity to entropy ratio. The latter is found to be $1/4\pi$ for the Einstein gravity with any matter content [33]. It was proposed by the KSS that this is a lower bound for relativistic quantum theories [34]. However, in the higher curvature gravities this bound is violated [35]. In [23] we showed that in the massive Einstein gravity there is no correction to $1/4\pi$ and addition of Gauss-Bonnet term is equivalent to rescaling the Gauss-Bonnet coupling $\lambda \rightarrow (1 + M_1)\lambda$ with $M_1$ a mass dependent parameter. Here we extend this calculation to include quasi-topological term and again we show that some mass dependent rescaling of the quasi-topological coupling gives the known result for the viscosity to entropy ratio. This may lower the ratio more than before. In section 4, we discuss on some bounds on parameters space. We consider the unitarity and causality bound on the boundary theory.

2 Quasi-Topological Massive Gravity and Holography

Let us start with a quasi-topological Gauss-Bonnet massive gravity in 5-dimensions with a negative cosmological constant. The action is given by [8, 23],

$$S = \frac{1}{2\ell_5^3} \int d^5x \sqrt{-g} \left( R + \frac{12}{\ell_5^2} + \frac{\lambda L_5^2}{2} \mathcal{L}_{GB} + \frac{7}{8} L_3^4 \mu L_3 + m_2^2 \sum_{i=1}^{4} c_i \mathcal{U}_i(g, f) \right)$$

where $R$ is the scalar curvature, $L$ the cosmological constant scale, $f$ a fixed rank-2 symmetric tensor known as reference metric and $m$ is the mass parameter. $\mathcal{L}_{GB}$ and $\mathcal{L}_3$ are respectively the Gauss-Bonnet and quasi-topology term with $\lambda$ and $\mu$ their dimensionless couplings. In (1), $c_i$’s are constants and $\mathcal{U}_k$ are symmetric polynomials of the eigenvalues of the $5 \times 5$ matrix $\mathcal{K}^\mu_\nu = \sqrt{g}^{\mu\rho} f_{\rho\sigma}^{\nu}$.

$$\mathcal{U}_1 = [\mathcal{K}], \quad \mathcal{U}_2 = [\mathcal{K}]^2 - [\mathcal{K}^2], \quad \mathcal{U}_3 = [\mathcal{K}]^3 - 3[\mathcal{K}] [\mathcal{K}^2] + 2[\mathcal{K}^3], \quad \mathcal{U}_4 = [\mathcal{K}]^4 - 6[\mathcal{K}^2][\mathcal{K}^2] + 8[\mathcal{K}^3][\mathcal{K}] - 3[\mathcal{K}^2]^2 - 6[\mathcal{K}^4]$$

The square root in $\mathcal{K}$ means $(\sqrt{A})^\mu_\nu (\sqrt{A})^\nu_\lambda = A^\mu_\lambda$ and the rectangular brackets denote traces[15].
This theory admits the AdS solution which can be derived as \( r \to \infty \) limit of the black hole solution in (38) with the radius of curvature

\[
\tilde{L}^2 = \frac{L^2}{f_\infty}
\]

where \( f_\infty \) is found from

\[
1 - f_\infty + \lambda f_\infty^2 + \mu f_\infty^3 = 0
\]

A simple derivation is to take \( r \to \infty \) limit of (46).

### 2.1 Boundary Theory: The energy-momentum two-point function

Here we try to shed light on the dual boundary theory by the standard AdS/CFT prescription. Let’s start with the theory on the AdS\(_{d+1}\) background and try to find the correlation functions on the boundary. Since we are dealing with pure gravity in the bulk, it is natural to look for energy momentum two-point function on the boundary. In a conformal field theory, the symmetry dictates the form of two-point function to be [36]

\[
\langle T_{ij}(x)T_{kl}(x') \rangle_{\text{CFT}} = \frac{C_T}{(x-x')^8} I_{ij,kl}(x-x')
\]

where

\[
I_{ij,kl}(x) = \frac{1}{2} (I_{ik}(x)I_{jl}(x) + I_{il}(x)I_{jk}(x)) - \frac{1}{4} \eta_{ij} \eta_{kl} \quad \text{and} \quad I_{ij}(x) = \eta_{ij} - \frac{x_ix_j}{x^2}.
\]

According to the AdS/CFT prescription, one should perturb the metric in the bulk as \( g_{\mu\nu} \to g_{\mu\nu} + h_{\mu\nu} \) and solve the corresponding equation of motion for \( h_{\mu\nu} \) subject to the boundary condition \( h_{\mu\nu}^{(0)} \). Then the quadratic part of the action gives the boundary two-point function of the energy-momentum tensor. Of course this procedure involves divergences and some regularization is needed. However for our purposes, following [9], it is enough to consider \( r^2 h_{xy}(r,z)/L^2 \) perturbation in the AdS background and find the logarithmic behavior of the action in the momentum space as given in the following. Consider \( h_{xy} = \phi \), the action to second order is found to be,

\[
I_2 = \frac{1}{2f_p^3} \int d^5x \left( K_r (\partial_r \phi)^2 + K_z (\partial_z \phi)^2 + K_m \phi^2 + \partial_r \Gamma_r + \partial_z \Gamma_z \right)
\]

in which

\[
K_r = -\frac{r^3 \sqrt{f_\infty}}{2L^5} (1 - 2\lambda f_\infty - 3\mu f_\infty^2)
\]

\[
K_z = -\frac{r}{2\sqrt{f_\infty}L} (1 - 2\lambda f_\infty - 3\mu f_\infty^2)
\]

\[
K_m = -\frac{m^2 c_0 r}{4\sqrt{f_\infty}L^3} (3c_1 r + 2c_2 c_0)
\]
where \( \Gamma_z \) and \( \Gamma_r \) don’t contribute to the equation of motion. Take \( \phi = e^{ipz_\phi} \) \( \phi_p(r) \), one finds the equation of motion as,

\[
\phi_p''(r) + \frac{5}{r} \phi_p'(r) - \frac{A^2}{r^4} \phi_p(r) - \frac{B}{r^3} \phi_p(r) = 0
\]  

where

\[
B = \frac{3L^2c_0c_1m^2}{(1 - 2\lambda f_\infty - 3\mu f_\infty^2)}
\]

\[
A^2 = \frac{L^4(p^2 + \alpha_m^2)}{f_\infty}
\]

\[
\alpha_m^2 = \frac{2c_0^2c_m^2}{L^2(1 - 2\lambda f_\infty - 3\mu f_\infty^2)}
\]

The general solution can be found as

\[
\phi_p(r) = b_2 e^{-\frac{A}{r^2}} F_1 \left( \frac{B}{2A} + \frac{5}{2}, \frac{2A}{r} \right) + b_1 e^{-\frac{A}{r^2}} U \left( \frac{B}{2A} + \frac{5}{2}, \frac{2A}{r} \right)
\]

where \( U(a, b, x) \) and \( F_1(a; b; x) \) are the Tricomi and the first kind confluent hypergeometric functions, respectively. Applying the boundary condition of \( \phi_p(r_\infty) = 1 \), we take \( b_2 = 0 \) and

\[
b_1 = \frac{1}{6} (A^2 - B^2)(9A^2 - B^2) \Gamma(\frac{3}{2} + B/A)
\]

Substituting in the action (7), and using the equation of motion, we find,

\[
I_2 = \frac{1}{2\ell^3 p} \int d^5x \partial_r (K_r \phi \partial_r \phi)
\]

\[
= \frac{b_1^2}{2\ell^3 p} \int d^5x \partial_r \left[ K_r U \left( \frac{B}{2A} + \frac{5}{2}, \frac{2A}{r} \right) \partial_r U \left( \frac{B}{2A} + \frac{5}{2}, \frac{2A}{r} \right) \right]
\]

This results the two point function \( \langle T_{xy}(x)T_{xy}(x') \rangle \) of the boundary theory. Of course, the latter is not expected to be in conformal form in the presence of the mass term. So we should reach to some deformation of (5).

i) \( c_1 = 0 \):

For simplicity let us first take \( c_1 = 0 \) which corresponds to \( B = 0 \) in (5). The solution to the equation of motion reduces to the modified Bessel function of the second kind with a momentum modification \( p^2 \to p^2 + \alpha_m^2 \) in the solution of (9):

\[
\phi_p(r)|_{B=0} = \frac{A^2}{2p^2} K_2(\frac{A}{r})
\]

Now plug in the action and look for \( r \to \infty \) behavior, beside the divergent terms, we find the logarithmic part of the solution (11) as:

\[
\langle T_{xy}(x)T_{xy}(0) \rangle = \frac{1}{8f_\infty^3 \ell^3 p} \left( 1 - 2\lambda f_\infty - 3\mu f_\infty^2 \right) (p^2 + \alpha_m^2)^2 \left( \log (p^2 + \alpha_m^2) + N \right) + \mathcal{O}(\frac{1}{x^2})
\]
where $N$ is a constant number. On the other hand, the Fourier transform of the two-point function \([5]\), based on a CFT, read as \([36]\):

\[
\langle T_{xy} T_{xy} \rangle (CFT) = \frac{C_T}{640} p^4 \int d^4 x \frac{e^{i p \cdot x}}{x^4} = \frac{\pi^2 C_T}{640} p^4 \log p^2 + \cdots
\]

(15)

where \(\cdots\) stands for analytic terms in \(p\). Comparison with (14) shows the replacement of \(p^2 \rightarrow p^2 + \alpha^2_m\) which indicates the deviation from conformal symmetry by the dimensionful parameter \(m^2\). In terms of the inverse Fourier transform of (14), considering only the logarithmic term, we find

\[
\langle T_{xy} T_{xy} \rangle (p) = \frac{1}{8 f_{\infty}^{3/2}} \frac{L^3}{\ell_p^4} (1 - 2 \lambda f_{\infty} - 3 \mu f_{\infty}^2) p^4 \int d^4 x \frac{e^{-\alpha m x} e^{i p \cdot x}}{x^4}
\]

(16)

where the \(p^4\) factor stands for the tensorial part of the correlation function. One can read \(C_T\) in (15) from (16) which gives the central charge of the CFT.

\[
c = \frac{\pi^4}{40} C_T = \frac{\pi^2 L^3}{f_{\infty}^{3/2}} \frac{1}{\ell_p^4} (1 - 2 \lambda f_{\infty} - 3 \mu f_{\infty}^2)
\]

(17)

The exponential factor \(e^{-\alpha m x}\) shows the deviation from conformal invariance by a massive relevant operator with mass \(\alpha_m\).

\( \hat{u}) \) \(c_1 \neq 0\):

In this case, similar calculation gives the logarithmic part of the energy-momentum two point function,

\[
\langle T_{xy}(x) T_{xy}(0) \rangle = \frac{1}{72 L^5} \sqrt{f_{\infty}} (1 - 2 f \lambda - 3 f^2 \mu)(9 A^2 - B^2)(A^2 - B^2) \log (p^2 + \alpha_m^2)
\]

(18)

In contrast to \(B = 0\), it is more difficult to find the inverse Fourier transform and extract the correct tensorial behavior. However, the shift in \(p^2\) indicates a factor of \(e^{-\hat{\alpha} m x}\) which in turn predicts presence of a massive operator on the boundary.

### 2.2 The trace anomaly and \(\alpha\)-theorem:

In this section we are looking for the trace of energy momentum tensor of the boundary theory. Let us start from a conformal field theory where in 4-dimensions, it includes two central charges, \(c\) and \(\alpha\) which can be derived by putting the conformal field theory on a curved background. Then the trace read as,

\[
\langle T^i_i \rangle = \frac{c}{16 \pi^2} I_4 - \frac{\alpha}{16 \pi^2} E_4
\]

\[
I_4 = R_{ijkl} R^{ijkl} - 2 R_{ij} R^{ij} + \frac{1}{3} R^2
\]

\[
E_4 = R_{ijkl} R^{ijkl} - 4 R_{ij} R^{ij} + R^2
\]

Here we assumed \(\alpha_m^2 > 0\). For \(\alpha_m^2 < 0\), \(\sin(\alpha_m x)\) instead of \(\exp(-\alpha_m x)\) appears.
where $E_4$ and $I_4$ are respectively the Euler density and the Weyl tensor squared. In the standard method for deriving central charges, one considers the Fefferman-Graham expansion of the metric near the boundary as \[4\]

$$\begin{align*}
\tilde{s}^2 &= \frac{\tilde{L}^2}{4 \rho^2} d\rho^2 + \frac{g_{ab}}{\rho} dx^a dx^b \quad \text{(21)}
\end{align*}$$

where the boundary is at $\rho = 0$ and

$$g_{ab} = g_{(0)ab} + \rho g_{(1)ab} + \rho^2 g_{(2)ab} + \cdots \quad \text{(22)}$$

in which next to the leading terms are determined by the EoM's. However, starting with a general background in a higher curvature theory is very difficult. Instead there is a nice trick by \[37\] as taking the boundary metric to be,

$$\begin{align*}
\tilde{s}^2 &= u(1 + \alpha \rho)(-r^2 dt^2 + \frac{dr^2}{r^2}) + v(1 + \beta \rho)(d\theta^2 + \sin^2 \theta d\phi^2) \quad \text{(23)}
\end{align*}$$

This is indeed the $AdS_2 \times S^2$ background with $\alpha$ and $\beta$ as perturbation which can be found from the EoM. It is known that the trace anomaly is found by the logarithmic part of the action as follows

$$I_{Ln} = -\frac{1}{2} \int \sqrt{g_0} < T^i_i > \quad \text{(24)}$$

where $Ln$ subscript means the logarithmic divergent part. Now simply extremize the action with respect to $\alpha$ and $\beta$ and plug them back to the action, one can easily read the central charges as coefficients of $I_4$ and $E_4$ where they appear as

$$I_4 = \frac{4}{3} \left( \frac{1}{u^2} + \frac{1}{v^2} - \frac{2}{uv} \right), \quad E_4 = -\frac{8}{uv} \quad \text{(25)}$$

So final form of the trace anomaly would be

$$< T^i_i > = \frac{c}{12\pi^2} \left( \frac{1}{u^2} + \frac{1}{v^2} - \frac{2}{uv} \right) + \frac{a}{2\pi^2} \frac{1}{uv} \quad \text{(26)}$$

This is an expression homogeneous in $u$ and $v$ with degree $-2$, i.e., involving $u^{-2}$, $v^{-2}$ and $(uv)^{-1}$ where two formers have equal coefficients.

Now consider the massive gravity. At the first glance introducing a mass scale seems to ruin the scale invariance of the theory and one may expect a non-zero energy-momentum trace because of explicit scale symmetry breaking. This is in addition to the conformal trace anomaly and some deviation from the functional form of (26) is expected.

To proceed, we need to introduce the reference metric $f_{\mu\nu}$ in the mass term of action \[1\]. The natural choice is the following

$$f_{ab} dx^a dx^b = c_0^2 \frac{u}{v} (1 + \alpha \rho) \frac{dr^2}{r^2} + c_0^2 (1 + \beta \rho)(d\theta^2 + \sin^2 \theta d\phi^2) \quad \text{(27)}$$
which is proportional to the spatial part of the metric (23) and gives a homogeneous trace anomaly as

\[ < T^i_i > = A_1 \left( \frac{1}{u^2} + \frac{1}{v^2} - \frac{2}{uv} \right) + A_2 \frac{1}{v^2} + A_3 \frac{1}{uv} \] (28)

with

\[ A_1 = \frac{1}{12 f^{3/2}_\infty \ell^3_p} \frac{L^3}{\ell^3_p} (1 - 2 \lambda f_{\infty} - 3 \mu f_{\infty}^2) \]
\[ A_2 = \frac{1}{12 f^{1/2}_\infty \ell^3_p} \frac{L^3}{\ell^3_p} c_2 c_0^2 m^2 (3 c_2 c_0^2 m^2 + 1 - 2 \lambda f_{\infty} - 3 \mu f_{\infty}^2) \]
\[ A_3 = \frac{1}{2 f^{3/2}_\infty \ell^3_p} \frac{L^3}{\ell^3_p} (c_2 c_0^2 m^2 + 1 - 6 \lambda f_{\infty} + 9 \mu f_{\infty}^2) \] (29)

In contrast to (26), the coefficient of the Euler density, \( A_3 \) is mass dependent and there is an extra term \( A_2 \) involving mass parameter. The latter term indicates the breaking of conformal structure of the energy-momentum trace. Taking the zero mass limit gives \( A_2 = 0 \) and

\[ c = \frac{\pi^2}{f^{3/2}_\infty \ell^3_p} \frac{L^3}{\ell^3_p} (1 - 2 \lambda f_{\infty} - 3 \mu f_{\infty}^2) \]
\[ a_1 = \frac{\pi^2}{f^{3/2}_\infty \ell^3_p} \frac{L^3}{\ell^3_p} (1 - 6 \lambda f_{\infty} + 9 \mu f_{\infty}^2) \] (30)

which are the well-known central charges of the massless theory [9].

The curious point about the energy-momentum trace (28) is that it only depends on mass through dimensionless combination \( c_2 c_0^2 m^2 \) where \( c_2 \) is dimensionless and \( c_0 \) is the length scale of the reference metric. So as a necessary (but not sufficient) condition for the conformal symmetry enhancement, one may look for cases where \( A_2 = 0 \). The first case is \( m = 0 \) which gives (30). Alternatively, we can take,

\[ 3 c_2 c_0^2 m^2 = -1 + 2 \lambda f_{\infty} + 3 \mu f_{\infty}^2 \] (31)

This may regard as deleting either of parameters in favor of the others. Condition (31) gives

\[ c = \frac{\pi^2}{f^{3/2}_\infty \ell^3_p} \frac{L^3}{\ell^3_p} (1 - 2 \lambda f_{\infty} - 3 \mu f_{\infty}^2) \]
\[ a_2 = \frac{2 \pi^2}{3 f^{3/2}_\infty \ell^3_p} \frac{L^3}{\ell^3_p} (1 - 8 \lambda f_{\infty} + 15 \mu f_{\infty}^2) \] (32)

where the \( c \) charge is the same as massless case, while \( a_2 \) is a new parameter. However we have not enough evidence to call \( a_2 \) a new central charge. It might be an accidental

\[ \text{indeed three mass dependent combinations } (c_1 c_0 m^2, c_2 c_0^2 m^2, c_3 c_0^3 m^2) \text{ appear in the massive gravity action. However only } c_2 c_0^2 m^2 \text{ is dimensionless and contributes to the trace anomaly.} \]
point in the parameter space for which the conformal symmetry breaking term in (28) vanishes but it is not sufficient to claim it as a new conformal symmetry fixed point. In addition recall from the previous subsection that there is a massive contribution to the energy-momentum two-point function with mass $\alpha_m$ where with condition (31) we have $\alpha_m^2 = -2/3L^2$. Appearance of $L$ scale indicates that the conformal symmetry is actually broken despite of the condition (31).

Finally it is worth to study the $c$-theorem of [30] in the context of massive gravity. Most of holographic $c$-theorems are based on the null energy condition in the bulk theory [38, 31, 32]. Let us consider it for the massive gravity. We start with the following metric,

$$ds^2 = e^{2A(r)}(-dt^2 + d\vec{x}^2) + dr^2$$  \hspace{1cm} (33)

In the large $r$ limit, we assume $A(r) = r/\tilde{L}$ and the metric becomes asymptotically $AdS$. Suppose a theory consisted of the action (11) as the gravity part with some matter source not included there, then the generalized Einstein equation has the following form,

$$G_{\mu\nu} - \frac{6}{L^2}g_{\mu\nu} + H_{\mu\nu} + Z_{\mu\nu} + M_{\mu\nu} = T_{\mu\nu}$$  \hspace{1cm} (34)

where $G$ is the Einstein tensor and $H$, $Z$ and $M$ are respectively variations of Gauss-Bonnet, quasi-topological and massive terms. On the other hand, we propose the following reference metric

$$f_{ab}dx^adx^b = \frac{1}{3}\tilde{L}^5c_0^2A''(r)A'(r)^5e^{2A(r)}(dx_1^2 + dx_2^2 + dx_3^2)$$  \hspace{1cm} (35)

Then we introduce,

$$a_1'(r) = -\frac{3\pi^2}{\ell_p^3A'(r)^4}A''(r)(1 - 2\lambda L^2A'(r)^2 - 3\mu L^4A'(r)^4)$$

$$= -\frac{\pi^2}{\ell_p^3A'(r)^4}(T_t^t - T_r^r) \geq 0.$$  \hspace{1cm} (36)

The second equality comes from the equation of motion and the inequality indicates the null energy condition for the matter field energy-momentum tensor. A simple integration of the above function gives

$$a_1(r) = \frac{\pi^2}{\ell_p^3A'(r)^3}(1 - 6\lambda L^2A'(r)^2 + 9\mu L^4A'(r)^4)$$  \hspace{1cm} (37)

In the AdS limit when $A(r) \sim r/\tilde{L}$, the $a_1$ central charge previously found in [30] will be recovered. Notice that this analysis is in the presence of the mass term in the action and gives the same functional form as in the literature for massless case [30]. This follows from the fact that the null energy condition used in [30] was based on $\xi^\mu\xi^\nu T_{\mu\nu} \geq 0$ with $\xi = (e^{-A(r)}, 0, 0, 0, 1)$ which has no component in non-vanishing directions of the reference metric.
3 Quasi-Topological Black Hole in Gauss-Bonnet Massive Gravity

In this section, we study a black brane solution to the action (1). We derive its metric, temperature and the entropy, then compute the viscosity of the hydrodynamic limit of the dual theory on the boundary.

3.1 The black brane solution:

We consider the following ansatz for the metric of five-dimensional planar AdS black brane,
\[ ds^2 = - \frac{r^2 N(r)^2}{L^2} f(r) dt^2 + \frac{L^2 dr^2}{r^2 f(r)} + r^2 h_{ij} dx^i dx^j, \] (38)

A generalized version of the reference metric \( f_{\mu
u} \) was proposed in [15] with the form \( f_{\mu
u} = \text{diag}(0, 0, c_0^2 h_{ij}) \), where \( h_{ij} = \frac{1}{r^2} \delta_{ij} \).

The values of \( U_i \) in (2) are calculated as below,
\[ U_1 = \frac{3 c_0}{r}, \quad U_2 = \frac{6 c_0^2}{r^2}, \quad U_3 = \frac{6 c_0^3}{r^3}, \quad U_4 = 0 \] (39)

Inserting this ansatz into the action (1) yields,
\[ I = \frac{1}{2 \ell^3} \int d^5 x \frac{3 N(r)}{L^5} \frac{d}{dr} \left[ r^4 \left( 1 - f(r) + \lambda f(r)^2 + \mu f(r)^3 + \frac{\Upsilon(r)}{r^4} \right) \right] \] (40)

in which
\[ \Upsilon(r) = r_0^4 \left( \frac{1}{3} m_1 \frac{r^3}{r_0^3} + m_2 \frac{r^2}{r_0^2} + 2 m_3 \frac{r}{r_0} \right) \] (41)
\[ m_1 = \frac{m^2 L^2 c_0 c_1}{r_0}, \quad m_2 = \frac{m^2 L^2 c_0 c_2}{r_0^2}, \quad m_3 = \frac{m^2 L^2 c_0^2 c_3}{r_0^3} \] (42)

where \( m_i \)’s are dimensionless mass parameters.

By variation of \( N(r) \) we have [8],
\[ \frac{d}{dr} \left[ r^4 \left( 1 - f(r) + \lambda f(r)^2 + \mu f(r)^3 + \frac{\Upsilon(r)}{r^4} \right) \right] = 0 \] (43)

\( f \) is given by solution of the following equation,
\[ r^4 \left( 1 - f(r) + \lambda f(r)^2 + \mu f(r)^3 + \frac{\Upsilon(r)}{r^4} \right) = b^4 \] (44)

in which \( b \) is a constant of motion and can be determined as a function of the radius of horizon at which \( f(r_0) = 0 \):
\[ b^4 = r_0^4 + \Upsilon_0 \] (45)
where $\Upsilon_0 = \Upsilon(r_0)$, then

$$1 - f(r) + \lambda f(r)^2 + \mu f(r)^3 = \frac{r_0^4}{r^4} + \frac{\Upsilon_0 - \Upsilon(r)}{r^4}$$  \tag{46}$$

That’s easy to show $N(r)$ is constant by variation of $f(r)$ from $N$. The speed of light in the boundary CFT is simply $c = 1$, thus we have $\lim_{r \to \infty} N^2 f(r) = 1$ so we take $N = 1/\sqrt{f_\infty}$.

The temperature and the Hawking-Bekenstein entropy density can be found as $[39]$, $\begin{align*}
T &= \frac{1}{2\pi} \left[ \frac{1}{\sqrt{g_{rr}}} \frac{d}{dr} \sqrt{-g_{tt}} \right]_{r=r_0} = \frac{r_0^2}{4\pi L^2 \sqrt{f_\infty}} \frac{df}{dr} \bigg|_{r=r_0} = \frac{r_0}{\pi L^2 \sqrt{f_\infty}} (1 + M_1) \\
S &= \frac{1}{2\ell_p^2} \int d^3x \sqrt{-g} = 2\pi \left( \frac{r_0}{\ell_p L} \right)^3.
\end{align*}$

where in the first line we used $[40]$ to find $\frac{df}{dr} \bigg|_{r=r_0}$ with $\begin{equation*}
M_1 = \frac{1}{4} (m_1 + 2m_2 + 2m_3) \tag{48}
\end{equation*}$

### 3.2 The shear viscosity:

To find the shear viscosity we follow $[9]$ with a new coordinate $z = 1 - r_0^2/r^2$. The unperturbed metric read as $\begin{align*}
ds^2 &= \frac{r_0^2}{L^2 (1-z)} \left( \frac{f(z)}{f_\infty} dt^2 + dx_1^2 + dx_2^2 + dx_3^2 \right) + \frac{L^2}{4f(z) (1-z)^2} \frac{dz^2}{f(z)} \tag{49}
\end{align*}$

then we can perturb the metric by the following shifting: $\begin{align*}
dx_1 \to dx_1 + \varepsilon \phi(t) dx_2
\end{align*}$

The corresponding perturbed metric follows, $\begin{align*}
ds^2 &= \frac{r_0^2}{L^2 (1-z)} \left( -f(z) dt^2 + dx_1^2 + (1 + \varepsilon^2 \phi^2) dx_2^2 + 2\varepsilon \phi dx_1 dx_2 + dx_3^2 \right) + \frac{L^2}{4f(z) (1-z)^2} \frac{dz^2}{f(z)} \tag{50}
\end{align*}$

then, $\begin{align*}
\mathcal{K} &= \frac{c_0 \sqrt{1-z}}{r_0} \begin{pmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & (1 + \frac{3}{8} \varepsilon^2 \phi^2) & -\frac{1}{8} \varepsilon \phi & 0 \\
0 & -\frac{1}{8} \varepsilon \phi & (1 - \frac{1}{8} \varepsilon^2 \phi^2) & 0 & 0 \\
0 & 0 & 0 & 0 & 1
\end{pmatrix}
\end{align*}$

and the massive term in the Lagrangian up to second order is, $\begin{align*}
\sqrt{-g} U &= \frac{3H}{2} (m_1 + 2m_2 \sqrt{1-z} + 2m_3 (1-z)) + \frac{H}{8} (m_1 + 2m_2 \sqrt{1-z}) \varepsilon^2 \phi^2 + O(\varepsilon)^3 \tag{51}
\end{align*}$
where we have used

\[
H = \frac{r_0^4}{L_5\sqrt{f_\infty(1 - z)^{5/2}}}
\]

\[
\sqrt{-g} = \frac{r_0^4}{L_5\sqrt{f_\infty(1 - z)^3}} + O(\epsilon)^4
\]

\[
U_1 = 3\sqrt{\frac{1 - z}{r_0}}\left(1 + \frac{1}{12}\epsilon^2\phi^2 + O(\epsilon)^4\right)
\]

\[
U_2 = 6\sqrt{\frac{1 - z}{r_2}}\left(1 + \frac{1}{12}\epsilon^2\phi^2 + O(\epsilon)^4\right)
\]

\[
U_3 = 6\sqrt{\frac{1 - z}{r_3}} + O(\epsilon)^3
\]

\[
U_4 = 0 + O(\epsilon)^3.
\]

Taking \(\phi = \exp^{-i\omega t}\), the shear viscosity can be found from the residue of the Lagrangian as follows [40]

\[
\eta = -8\pi T \lim_{\omega, \epsilon \to 0} \frac{Res_{z=0}\mathcal{L}}{\omega^2 \epsilon^2}
\]

\[
= -8\pi T \lim_{\omega, \epsilon \to 0} \left[ \frac{Res_{z=0}\sqrt{-g}\left(R + \frac{12}{L_2^2} + \frac{\lambda L_2^2}{2} \mathcal{L}_{GB} + \frac{7}{8} L^4 \mu \mathcal{L}_3\right)}{\omega^2 \epsilon^2} + \frac{Res_{z=0}\sqrt{-g}U}{\omega^2 \epsilon^2} \right]
\]

\[(52)\]

The second term is given by \((51)\) and has no pole at \(z = 0\). The first term can be found as\[3\]

\[
\eta = \frac{r_0^3}{2L_3^3 L_5^3} \left[ 1 - 2\lambda f_0' + 9\mu (f_0'')^2 + 2f_0''^2 + 2f_0'(f_0''' - 3f_0'') \right]
\]

\[(53)\]

where \(f_0^{(n)} = d^n f(z)/dz^n |_{z=0}\) are derivatives of \(f\) at horizon and can be derived from \[(46)\] after rewritten in \(z\)-coordinate,

\[
f(z) - \lambda f^2(z) - \mu f^3(z) = z(2 - z) \left(1 + \frac{\Upsilon_0}{r_0^2}\right) - \frac{\Upsilon_0}{r_0^3} + \mathcal{Z}(z)
\]

\[(54)\]

where

\[
\mathcal{Z}(z) \equiv \frac{(1 - z)^2}{r_0^2} \Upsilon(z)
\]

\[
= \frac{1}{3} m_1 \sqrt{1 - z} + m_2 (1 - z) + 2m_3 (1 - z)^{3/2}
\]

\[(55)\]

\[3\]The following equation is the same as \[(9)\]. Although our temperature differs from the massless case, the mass parameters do not show up here and are implicit in \(f\)’s derivatives.
then

\[
\begin{align*}
 f' &= 2 \left( 1 + \frac{Y_0}{r_0^2} \right) + Z_0' = 2(1 + M_1) \\
 f'' &= 2\lambda f'^2 + 2 \left( 1 + \frac{Y_0}{r_0^2} \right) + Z_0'' = 2\lambda f'^2 - 2 + M_2 \\
 f''' &= 6\lambda f'f'' + 6\mu f'^3 + Z_0''' = 6\lambda f'f'' + 6\mu f'^3 + M_3
\end{align*}
\] (56)

in which

\[
\begin{align*}
 M_1 &= \frac{1}{4} (m_1 + 2m_2 + 2m_3), \quad M_2 = \frac{1}{4} (3m_1 + 8m_2 + 10m_3), \quad M_3 = \frac{1}{8} (-m_1 + 6m_3)
\end{align*}
\] (57)

then

\[
\frac{4\pi \eta}{s} = 1 - 4(1 + M_1)\lambda - 36\mu \left[ 9 + 8M_1 + M_1^2 - 5M_2 - 3M_1M_2 + \frac{M_2^2}{2} + M_3 + M_1M_3 \\
-4(1 + M_1)^2(6M_1 - 5M_2 + 16)\lambda + 128(1 + M_1)^4\lambda^2 + 48(1 + M_1)^4\mu \\
= 1 - 4\lambda - 36\mu \left[ 9 + 8M_1 + M_1^2 - 5M_2 - 3M_1M_2 + \frac{M_2^2}{2} + M_3 + M_1M_3 \\
-4(1 + M_1)(6M_1 - 5M_2 + 16)\lambda + 128(1 + M_1)^2\lambda^2 + 48(1 + M_1)^4\mu \right]
\] (58)

with \(\tilde{\lambda} = (1 + M_1)\lambda\). Taking massless limit by \(M_i = 0\) we get,

\[
\frac{4\pi \eta}{s} = 1 - 4\lambda - 36\mu(9 - 64\lambda + 128\lambda^2 + 48\mu^2)
\] (59)

which is the same as result of [9]. If otherwise we put \(\mu = 0\) in (58),

\[
\frac{4\pi \eta}{s} = 1 - 4\tilde{\lambda} = 1 - 4(1 + M_1)\lambda \\
= 1 - \frac{4\pi L^2 \sqrt{f_\infty T}}{r_0}\lambda
\] (60)

This matches with our previous work [23].

Notice that the massive result (58) as a function of couplings, \(\lambda\) and \(\mu\), has the same structure as (59). So it is possible to introduce effective couplings as \(\tilde{\lambda}\) and \(\tilde{\mu} = \mu(\mu, \lambda, M_i)\) and replace them in (58) to find

\[
\frac{4\pi \eta}{s} = 1 - 4\tilde{\lambda} - 36\tilde{\mu}(9 - 64\tilde{\lambda} + 128\tilde{\lambda}^2 + 48\tilde{\mu}^2)
\] (61)

where \(\tilde{\mu}\) can be found as roots of a quadratic equation given by equating (61) to (58).

4 Physical constraints

Here we check some physical conditions on our parameters. The first one is unitarity which means that the norm of the energy momentum two-point function (16) is positive.
Therefore, the $c$-central charge should be positive, then

$$\quad 1 - 2\lambda f_\infty - 3\mu f_\infty^2 > 0 \quad (62)$$

We have from (46)

$$\quad \tilde{\Gamma}(f) \equiv 1 - f(r) + \lambda f(r)^2 + \mu f(r)^3 - \frac{r_0^4}{r^4} - \frac{\mathcal{Y}_0 - \mathcal{Y}(r)}{r^4} \quad (63)$$

$$\quad - \tilde{\Gamma}'(f_\infty) = 1 - 2\lambda f_\infty - 3\mu f_\infty^2 > 0 \quad (64)$$

This is exactly the ghost free condition for the graviton propagating on an AdS background [8].

The next constraint comes from the causality in the CFT. This is nontrivial since the 4-dimensional Lorentz symmetry is broken by the black hole background as well as by the reference metric. To investigate this, we consider the front velocity of signals in the dual CFT determined by $v_{\text{front}} = \lim_{|q| \to \infty} \frac{Re(\omega)}{q}$ which corresponds to the phase velocity of the short wavelength modes. So the necessary condition for causal behavior is $v_{\text{front}} \leq 1$. Define a new coordinate $\rho = \frac{r}{r_0}$ the metric (38) becomes as follows,

$$\quad ds^2 = \frac{r_0^2}{L^2 \rho^2} \left(- \frac{f(\rho)}{f_\infty} dt^2 + dx_1^2 + dx_2^2 + dx_3^2\right) + \frac{L^2 d\rho^2}{\rho^2 f(\rho)} \quad (65)$$

One can find linearized equations of motion by perturbing the metric with $h_{x_1 x_2} = \frac{r_0^2}{L^2 \rho} e^{-i\omega t + iq x_3} \phi(\rho)$, as follows,

$$\quad \partial_\rho \left(C^{(2)}(\rho, q^2) \partial_\rho \phi(\rho)\right) + C^{(0)}(\rho, q^2, \omega^2) \phi(\rho) + C^{(m)}(\rho, m^2) \phi(\rho) = 0 \quad (66)$$

The radial derivatives can be ignored since we are in the large momentum and frequency limit. Moreover the $C^{(m)}$ term comes from the mass term which doesn’t involve any derivatives so independent of $q^2$ and $\omega^2$. We therefore ignore the $C^{(m)}$ as well. In this way we find linearized equation as in the massless case [9]. It reduces to the following equation where only terms proportional to $q^2$, $\omega^2$ and their higher degrees are survived in the short wavelength limit,

$$\quad 0 = \omega^2 \left(1 - 2\lambda f(\rho) + \rho \lambda f'(\rho)\right) - \frac{f(\rho)}{f_\infty} q^2 \left(1 - 2\lambda f(\rho) + 2\rho \lambda f'(\rho) - \rho^2 \lambda f''(\rho)\right)$$

$$\quad - 3\mu \omega^2 \left[f(\rho) \left(f(\rho) - \rho f'(\rho) + \frac{3}{2} \rho^3 f^{(3)}(\rho) + \frac{3}{4} \rho^4 f^{(4)}(\rho)\right) + \frac{3}{8} \rho^2 f''(\rho) \left(\rho f''(\rho) + \rho^2 f^{(3)}(\rho)\right) + \frac{3}{8} \rho^4 f''(\rho) \right]$$

$$\quad + 3\mu \frac{f(\rho)}{f_\infty} q^2 \left[f(\rho) \left(f(\rho) - 2\rho f'(\rho) + \rho^2 f''(\rho) - \frac{3}{2} \rho^2 f^{(3)}(\rho) - \frac{3}{4} \rho^4 f^{(4)}(\rho)\right)\right]$$

$$\quad + \frac{\mu f(\rho)}{f_\infty} q^2 \left[f(\rho) \left(4 f'(\rho) - 3\rho f''(\rho) - 3\rho^2 f^{(3)}(\rho)\right)\right]$$

$$\quad - 6\mu\rho^2 \frac{f(\rho)}{f_\infty} q^2 f''(\rho) \left(\omega^2 - \frac{f(\rho)}{f_\infty} q^2\right) \quad (67)$$

\(^4\)Notice we differ from [9] by choosing $\rho = \frac{r}{r_0}$ instead of $\rho = \frac{r}{r_0^2}$. A simple change of variable transforms our linearized equations to those of [9].
Let us proceed step by step from the Einstein gravity where \( \lambda \) and \( \mu \) vanish. We thus have,

\[
v_f^2 \equiv \lim_{q^2 \to \infty} \frac{\omega^2}{q^2} = \frac{f(\rho)}{f_{\infty}},
\]

where \( f_{\infty} = 1 \) and

\[
f(\rho) = 1 + \frac{1}{3} m_1 \rho + m_2 \rho^2 + 2 m_3 \rho^3 - \rho^4
\]

(68)

For massless case it satisfies the causality condition, while in the massive gravity, the dominant term near the boundary is \( m_1 \) and should be negative to achieve the causality \( v_f^2 < 1 \). One may set \( m_1 = 0 \), then \( m_2 \) should be nonpositive and so on for \( m_3 \).

In the second step, consider the Gauss-Bonnet gravity, i.e. \( \lambda \neq 0 \) and \( \mu = 0 \). Then only the first line of (67) is nonvanishing. We may insert

\[
f(r) = f_{\infty} + f'(0) \rho + \frac{1}{2} f''(0) \rho^2 + \frac{1}{6} f^{(3)}(0) \rho^3 + \frac{1}{24} f^{(4)}(0) \rho^4 + \cdots
\]

(70)

where \( f \) derivatives can be found by (44) rewritten in \( \rho = r_0/r \) coordinate. Then

\[
v_f^2 = \frac{f(\rho)}{f_{\infty}} \left[ 1 + \frac{m_1 \rho}{3k^2} + \frac{m_1^2 \lambda^2}{9k^4} - \left( \frac{6 m_3 \lambda}{k^2} + \frac{2 m_1 m_2 \lambda^2}{k^4} + \frac{5 m_1^3 \lambda^3}{27 k^6} \right) \rho^2 \right] + O(\rho^4)
\]

(71)

where \( k = 1 - 2 \lambda f_{\infty} \). Near horizon the causality requires \( m_1 \lambda \leq 0 \). In the case of \( m_1 = 0 \), the \( m_3 \) term in the bracket survives and gives \( m_3 \lambda > 0 \).

The final stage is the full theory including the quasi-topological term. In the large momentum limit, the dominant term is the last line of (67). This is the same as Einstein gravity, this time including \( \lambda \) and \( \mu \),

\[
v_f^2 = \frac{f(\rho)}{f_{\infty}} = 1 + \frac{m_1 \rho}{3 f_{\infty}(1 - 2 \lambda f_{\infty} - 3 \mu f_{\infty}^2)}
\]

\[
+ \frac{m_1^2 (\lambda + 3 \mu f_{\infty}) + 9 m_2 (1 - 2 \lambda f_{\infty} - 3 \mu f_{\infty}^2)^2}{9 f_{\infty}(1 - 2 \lambda f_{\infty} - 3 \mu f_{\infty}^2)^3} \rho^2 + O(\rho^3)
\]

(72)

Since \( (1 - 2 \lambda f_{\infty} - 3 \mu f_{\infty}^2) \) is proportional to the central charge, it is positive by the unitarity. Thus for small \( \rho \), the \( m_1 \) should be non-positive. If we consider \( m_1 = 0 \), then

\[
v_f^2 = \frac{f(\rho)}{f_{\infty}} = 1 + \frac{m_2}{f_{\infty}(1 - 2 \lambda f_{\infty} - 3 \mu f_{\infty}^2)} \rho^2 + O(\rho^3)
\]

(73)

leads to \( m_2 \leq 0 \). We can go further and set \( m_1 = m_2 = 0 \), then we need to expand the front velocity to order \( \rho^3 \),

\[
v_f^2 = 1 + \frac{2 m_3}{f_{\infty}(1 - 2 \lambda f_{\infty} - 3 \mu f_{\infty}^2)} \rho^3 + O(\rho^4)
\]

(74)

This implies \( m_3 \leq 0 \). In summary, causality indicates mass parameters to be nonpositive. This can be seen as \( c_i \leq 0 \) in the action (11).
5 Conclusion

The higher curvature gravities are important in the holographic study of conformal field theories. In contrast to Einstein gravity which duals to four dimensional CFTs with equal central charges, higher curvature theories provide dual CFTs with two distinguished charges. A fundamental higher derivative gravity is expected to be derived from a string theory calculation, however the Gauss-Bonnet and cubic quasi-topological higher curvature gravities may be considered as toy models with rich structure to investigate the dual quantum field theory on the boundary [7, 8, 9]. Here we studied a higher curvature massive gravity. The later is important as generalization of the Einstein gravity and has phenomenological applications and theoretical consequences. It is known that when one considers massive gravity as a bulk theory in the context of AdS/CFT correspondence, the boundary theory violates the Lorentz invariance [17, 18]. This is related to introducing the reference metric. Here we investigated the conformal structure of the boundary theory. It was shown that adding a mass term to the bulk is equivalent to existence of a massive operator on the boundary. It appears like a short range Yukawa potential in the energy-momentum two-point function. The constant coefficient of this two-point function is proportional to the $c$ central charge and remains intact in the massive theory. To find the $a$ central charge of the CFT, we followed the method of [37], of course with subtleties regarding the mass term and the reference metric. We showed that the well-known $c$ and $a$ charges are recovered. There is also an explicit conformal symmetry breaking by a mass dependent term which vanishes in the zero mass limit.

In the second part of this article, we worked out an exact black brane solution. We derived the temperature to be mass dependent. Then we considered the hydrodynamic limit in the dual theory and calculated the shear viscosity to the entropy density, $\eta/s$. It was shown that by suitable rescaling of $\lambda$ and $\mu$ couplings (see (61)) one finds $\eta/s$ to be the same as massless case. Of course presence of mass alters the range of new couplings $\lambda$ and $\mu$. At the final stage, we investigated physical constraints as unitarity, ghost free and causality. The latter set condition on mass parameters $c_1$, $c_2$ and $c_3$. It was found that $c_1 \leq 0$, if $c_1 = 0$ then $c_2 \leq 0$ and so on.

Acknowledgment Authors would like to thank A. Imaanpur and M. M. Sheikh-Jabbari for useful discussions.

References

[1] J. M. Maldacena, “The Large N limit of superconformal field theories and supergravity,” Int. J. Theor. Phys. 38 (1999) 1113 [Adv. Theor. Math. Phys. 2 (1998) 231] [hep-th/9711200].

[2] E. Witten, “Anti-de Sitter space and holography,” Adv. Theor. Math. Phys. 2, 253 (1998) [hep-th/9802150].
[3] S. S. Gubser, I. R. Klebanov and A. M. Polyakov, “Gauge theory correlators from noncritical string theory,” Phys. Lett. B 428, 105 (1998) doi:10.1016/S0370-2693(98)00377-3 [hep-th/9802109].

[4] M. Henningson and K. Skenderis, “The Holographic Weyl anomaly,” JHEP 9807, 023 (1998) doi:10.1088/1126-6708/1998/07/023 [hep-th/9806087].

[5] M. Henningson and K. Skenderis, “Holography and the Weyl anomaly,” Fortsch. Phys. 48, 125 (2000) doi:10.1002/(SICI)1521-3978(20001)48:1/3<125::AID-PROP125>3.0.CO;2-B, 10.1002/(SICI)1521-3978(20001)48:1/3<125::AID-PROP125>3.3.CO;2-2 [hep-th/9812032].

[6] S. Nojiri and S. D. Odintsov, “On the conformal anomaly from higher derivative gravity in AdS / CFT correspondence,” Int. J. Mod. Phys. A 15, 413 (2000) doi:10.1142/S0217751X00000197 [hep-th/9903033].

[7] J. Oliva and S. Ray, “A new cubic theory of gravity in five dimensions: Black hole, Birkhoff’s theorem and C-function,” Class. Quant. Grav. 27, 225002 (2010) doi:10.1088/0264-9381/27/22/225002 [arXiv:1003.4773 [gr-qc]].

[8] R. C. Myers and B. Robinson, “Black Holes in Quasi-topological Gravity,” JHEP 1008, 067 (2010) [arXiv:1003.5357 [gr-qc]].

[9] R. C. Myers, M. F. Paulos and A. Sinha, “Holographic studies of quasi-topological gravity,” JHEP 1008, 035 (2010) [arXiv:1004.2055 [hep-th]].

[10] C. de Rham, “Massive Gravity,” Living Rev. Rel. 17, 7 (2014) doi:10.12942/lrr-2014-7 [arXiv:1401.4173 [hep-th]].

[11] M. Fierz and W. Pauli, “On relativistic wave equations for particles of arbitrary spin in an electromagnetic field,” Proc. Roy. Soc. Lond. A 173, 211 (1939). doi:10.1098/rspa.1939.0140

[12] C. J. Isham, A. Salam and J. A. Strathdee, “F-dominance of gravity,” Phys. Rev. D 3, 867 (1971). doi:10.1103/PhysRevD.3.867

[13] D. G. Boulware and S. Deser, “Can gravitation have a finite range?,” Phys. Rev. D 6, 3368 (1972). doi:10.1103/PhysRevD.6.3368

[14] C. de Rham and G. Gabadadze, “Generalization of the Fierz-Pauli Action,” Phys. Rev. D 82, 044020 (2010) doi:10.1103/PhysRevD.82.044020 [arXiv:1007.0443 [hep-th]].

[15] C. de Rham, G. Gabadadze and A. J. Tolley, “Resummation of Massive Gravity,” Phys. Rev. Lett. 106, 231101 (2011) doi:10.1103/PhysRevLett.106.231101 [arXiv:1011.1232 [hep-th]].

[16] S. F. Hassan and R. A. Rosen, “Resolving the Ghost Problem in non-Linear Massive Gravity,” Phys. Rev. Lett. 108, 041101 (2012) doi:10.1103/PhysRevLett.108.041101 [arXiv:1106.3344 [hep-th]].

[17] D. Vegh, “Holography without translational symmetry,” [arXiv:1301.0537 [hep-th]].

[18] R. A. Davison, “Momentum relaxation in holographic massive gravity,” Phys. Rev. D 88, 086003 (2013) doi:10.1103/PhysRevD.88.086003 [arXiv:1306.5792 [hep-th]].
[19] S. F. Hassan and R. A. Rosen, “Bimetric Gravity from Ghost-free Massive Gravity,” JHEP 1202, 126 (2012) doi:10.1007/JHEP02(2012)126 [arXiv:1109.3515 [hep-th]].

[20] K. Hinterbichler and R. A. Rosen, “Interacting Spin-2 Fields,” JHEP 1207, 047 (2012) doi:10.1007/JHEP07(2012)047 [arXiv:1203.5783 [hep-th]].

[21] K. Nomura and J. Soda, “When is Multimetric Gravity Ghost-free?,” Phys. Rev. D 86, 084052 (2012) doi:10.1103/PhysRevD.86.084052 [arXiv:1207.3637 [hep-th]].

[22] T. Q. Do, “Higher dimensional massive bigravity,” Phys. Rev. D 94, no. 4, 044022 (2016) doi:10.1103/PhysRevD.94.044022 [arXiv:1604.07568 [gr-qc]].

[23] M. Sadeghi and S. Parvizi, “Hydrodynamics of a black brane in Gauss-Bonnet massive gravity,” Class. Quant. Grav. 33, no. 3, 035005 (2016) doi:10.1088/0264-9381/33/3/035005 [arXiv:1507.07183 [hep-th]].

[24] S. H. Hendi, S. Panahiyan and B. Eslam Panah, “Charged Black Hole Solutions in Gauss-Bonnet-Massive Gravity,” JHEP 1601, 129 (2016) doi:10.1007/JHEP01(2016)129 [arXiv:1507.06563 [hep-th]].

[25] S. H. Hendi, G. Q. Li, J. X. Mo, S. Panahiyan and B. Eslam Panah, “New perspective for black hole thermodynamics in Gauss-Bonnet-Born-Infeld massive gravity,” Eur. Phys. J. C 76, no. 10, 571 (2016) doi:10.1140/epjc/s10052-016-4410-4 [arXiv:1608.03148 [gr-qc]].

[26] A. B. Zamolodchikov, “Irreversibility of the Flux of the Renormalization Group in a 2D Field Theory,” JETP Lett. 43, 730 (1986) [Pisma Zh. Eksp. Teor. Fiz. 43, 565 (1986)].

[27] J. L. Cardy, “Is There a c Theorem in Four-Dimensions?,” Phys. Lett. B 215, 749 (1988). doi:10.1016/0370-2693(88)90054-8

[28] Z. Komargodski and A. Schwimmer, “On Renormalization Group Flows in Four Dimensions,” JHEP 1112, 099 (2011) doi:10.1007/JHEP12(2011)099 [arXiv:1107.3987 [hep-th]].

[29] Z. Komargodski, “The Constraints of Conformal Symmetry on RG Flows,” JHEP 1207, 069 (2012) doi:10.1007/JHEP07(2012)069 [arXiv:1112.4538 [hep-th]].

[30] R. C. Myers and A. Sinha, “Holographic c-theorems in arbitrary dimensions,” JHEP 1101, 125 (2011) doi:10.1007/JHEP01(2011)125 [arXiv:1011.5819 [hep-th]].

[31] J. T. Liu, W. Sabra and Z. Zhao, “Holographic c-theorems and higher derivative gravity,” Phys. Rev. D 85, 126004 (2012) doi:10.1103/PhysRevD.85.126004 [arXiv:1012.3382 [hep-th]].

[32] J. T. Liu and Z. Zhao, “A holographic c-theorem for higher derivative gravity,” arXiv:1108.5179 [hep-th].

[33] A. Buchel and J. T. Liu, “Universality of the shear viscosity in supergravity,” Phys. Rev. Lett. 93, 090602 (2004) doi:10.1103/PhysRevLett.93.090602 [hep-th/0311175].

[34] P. Kovtun, D. T. Son and A. O. Starinets, “Holography and hydrodynamics: Diffusion on stretched horizons,” JHEP 0310, 064 (2003) [hep-th/0309213].
[35] M. Brigante, H. Liu, R. C. Myers, S. Shenker and S. Yaida, “Viscosity Bound Violation in Higher Derivative Gravity,” Phys. Rev. D 77, 126006 (2008) doi:10.1103/PhysRevD.77.126006 [arXiv:0712.0805 [hep-th]].

[36] S. S. Gubser and I. R. Klebanov, “Absorption by branes and Schwinger terms in the world volume theory,” Phys. Lett. B 413, 41 (1997) doi:10.1016/S0370-2693(97)01099-X [hep-th/9708005].

[37] K. Sen, A. Sinha and N. V. Suryanarayana, “Counterterms, critical gravity and holography,” Phys. Rev. D 85, 124017 (2012) doi:10.1103/PhysRevD.85.124017 [arXiv:1201.1288 [hep-th]].

[38] R. C. Myers and A. Sinha, “Seeing a c-theorem with holography,” Phys. Rev. D 82, 046006 (2010) doi:10.1103/PhysRevD.82.046006 [arXiv:1006.1263 [hep-th]].

[39] Jacob D Bekenstein, “Black holes and entropy,” Phys. Rev. D7:2333-2346, 1973 doi:10.1103/PhysRevD.7.2333.

[40] M. F. Paulos, “Transport coefficients, membrane couplings and universality at extremality,” JHEP 1002, 067 (2010) doi:10.1007/JHEP02(2010)067 [arXiv:0910.4602 [hep-th]].