Near threshold kaon-kaon interaction in the reactions $e^+e^- \rightarrow K^+K^-\gamma$ and $e^+e^- \rightarrow K^0\bar{K}^0\gamma$

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Abstract. Strong interactions between pairs of the $K^+K^-$ and $K^0\bar{K}^0$ mesons can be studied in radiative decays of $\phi(1020)$ mesons. We present a theoretical model of the reactions $e^+e^- \rightarrow \phi \rightarrow K^+K^-\gamma$ and $e^+e^- \rightarrow \phi \rightarrow K^0\bar{K}^0\gamma$. The $K^+K^-$ and $K^0\bar{K}^0$ effective mass dependence of the differential cross sections is derived. The total cross sections and the branching fractions for the two radiative $\phi$ decays are calculated.

1 Description of the theoretical model

The kaon-kaon strong interaction near threshold is largely unknown. Also the parameters of the scalar resonances $f_0(980)$ and $a_0(980)$ are still imprecise. The $\phi(1020)$ meson decays into $\pi^+\pi^-\gamma$, $\pi^0\pi^0\gamma$ and $\pi^0\eta\gamma$ have been measured, for the $\phi$ transition into $K^0\bar{K}^0\gamma$ only the upper limit of the branching fraction has been obtained in Ref. [1] but there are no data for the $\phi \rightarrow K^+K^-\gamma$ process.

In this paper we outline a general theoretical model of the $e^+e^-$ reactions leading to final states with two pseudoscalar mesons and a photon. At the beginning we derive the amplitude $A(m)$ for the $e^+e^- \rightarrow K^+K^-\gamma$ process. It is a sum of the four amplitudes corresponding to diagrams (a), (b), (c) and (d) in Fig. 1.

![Diagram](image)

Figure 1. Diagrams for the reaction $e^+e^- \rightarrow K^+K^-\gamma$ with final-state $K^+K^-$ interaction. The $K^+K^-$ elastic amplitude is labelled by $T$ and $R$ denotes the difference of the $K^+K^-$ amplitudes $T$ in Eq. [4].

\begin{equation}
A_a = 2i \int \frac{d^4k}{(2\pi)^4} \frac{J_e e^{i\pi T(k)}}{D(k)D(-k + p - q)},
\end{equation}

\begin{equation}
A_b = -4i \int \frac{d^4k}{(2\pi)^4} \frac{J_\mu e^{i\pi (k_\mu + q_\mu)T(k)}}{D(k + q)D(k)D(-k + p - q)},
\end{equation}

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\[ A_c = -4i \int \frac{d^4k}{(2\pi)^4} \frac{J_\mu \epsilon^\mu (k_\nu - \nu_\nu)k_\mu T(k)}{D(p-k)D(k)(-k + p - q)}, \]  
\[ A_d = -2i \int \frac{d^4k}{(2\pi)^4} \frac{J \cdot k \epsilon^\nu \bar{k}}{D(k)D(p-k)} \frac{[T(k - q) - T(k)]}{q \cdot \bar{k}}. \]

One can show that the amplitude \( A(m) = A_a + A_b + A_c + A_d \) is gauge invariant. In Eqs. \( 1-4 \), \( D(k) = k^2 - m^2_K + i\delta, \delta \to +0, \) is the inverse of the kaon propagator. \( m_K \) is the charged kaon mass, the four-vector \( \tilde{k} = (0, \hat{k}) \) with the unit three-vector \( \hat{k} = k/|k| \). In the above expressions \( q \) is the photon four-momentum, \( p = p_\nu + p_\nu \) is the \( \phi \) meson four-momentum, \( \epsilon^\nu \) is the photon polarization four-vector and \( J_\mu \) is defined as

\[ J_\mu = \frac{e^3}{s} F_{K}(s) \bar{\epsilon}(p_\nu) \gamma_\mu u(p_\nu), \]

where \( e \) is the electron charge, \( s = (p^2) \) is the Mandelstam variable, \( \nu \) and \( u \) are the \( e^+ \) and \( e^- \) bispinors, respectively, \( \gamma_\mu \) are the Dirac matrices and \( F_{K}(s) \) is the kaon electromagnetic form factor. The \( K^+K^- \) elastic scattering amplitude is given by

\[ T(k) = \langle K^- (k_1) K^+(k_2)|\bar{T}(m)|K^- (-k + p - q)K^+(k)\rangle, \]

where \( m^2 = (k_1 + k_2)^2 \) is the square of the \( K^+K^- \) effective mass and \( \bar{T}(m) \) is the \( K\bar{K} \) scattering operator. The on-shell \( K^+K^- \) amplitude can be expressed as \( T_{K^-K^+}(m) = \langle K^- (k_1) K^+(k_2)|\bar{T}(m)|K^- (-k_1) K^+(k_2)\rangle \). The four-momenta of kaons in the \( K^+K^- \) center-of-mass frame are: \( k_1 = (m/2, -\hat{k}_f) \) and \( k_2 = (m/2, \hat{k}_f) \), where \( k_f = \sqrt{m^2/4 - m^2_K} \) is the kaon momentum in the final-state. We can assume that \( T(k) \) is related to \( T_{K^-K^+}(m) \) as follows:

\[ T(k) = g(k)T_{K^-K^+}(m), \]

where \( g(k) \), as a real function of the modulus of the kaon three-momentum \( k \equiv |k| \), takes into account the off-shell character of \( T(k) \). From Eqs. \( 6, 7 \) one infers that \( g(k_f) = 1 \).

Under a dominance of the pole at \( m = 2E_k \approx 2\sqrt{k^2 + m^2_K} \), the amplitude \( A(m) \) can be written in the following form:

\[ A(m) = \bar{\epsilon} \cdot \epsilon^* T_{K^-K^+}(m) [I(m) - I(m_\phi)], \]

where the integral \( I(m) \) reads

\[ I(m) = -2 \int \frac{d^3k}{(2\pi)^3} \frac{g(k)}{2E_k m - 2E_k} \left[ 1 - 2 \frac{|k|^2 - (k \cdot \hat{q})^2}{2p_0E_k - m^2_\phi + 2k \cdot \hat{q}} \right]. \]

In Eq. \( 9 \) \( p_0 = m + \omega \), where \( \omega \) is the photon energy in the \( K^+K^- \) center-of-mass frame, \( m_\phi \) is the \( \phi \) meson mass and \( \hat{q} = q/|q| \) is the unit vector indicating the photon direction.

In the model of Close, Isgur and Kumano \( 2 \) the momentum distribution of the interacting kaons has been expressed by the function

\[ \phi(k) = \frac{\mu^4}{(k^2 + \mu^2)^2}, \]

with the parameter \( \mu = 141 \) MeV. This function is normalized to unity at \( k = 0 \), however the function \( g(k) \) in Eq. \( 7 \) has to be normalized to 1 at \( k = k_f \), so the function \( g(k) \) corresponding to \( \phi(k) \) should be defined as

\[ g(k) = \frac{(k^2_f + \mu^2)^2}{(k^2 + \mu^2)^2}. \]
In Ref. [2] kaons are treated as extended objects forming a quasi-bound state. If the \( K^+ K^- \) system is point-like, like in Refs. [3] or [4], then the function \( g(k) \equiv 1 \). Let us note that both models can be treated as special cases of our approach.

Separable kaon-kaon potentials can be used to calculate the kaon-kaon amplitudes needed in practical application of the present model. Then the function \( g(k) \) takes the following form:

\[
g(k) = \frac{k^2 + \beta^2}{k^2 + \beta^2},
\]

(12)

where \( \beta \) is a range parameter. In Ref. [5] for the isospin zero \( K\bar{K} \) amplitude the value of \( \beta \) close to 1.5 GeV has been obtained. In order to get an integral convergence at large \( k \) in Eq. (9) we use an additional cut-off parameter \( k_{\text{max}} = 1 \) GeV.

2 Numerical results

In Fig. 2 we show the \( K^+ K^- \) effective mass distributions at the \( e^+ e^- \) energy equal to \( m_\phi \). On the left panel one observes some dependence on the form of the function \( g(k) \). A common feature is a presence of the maximum of the differential cross section situated only a few MeV above the \( K^+ K^- \) threshold. On the right panel we see a comparison of our model (solid line) with two other models, named the "no-structure" model [6] and the "kaon-loop" model, and described in Refs. [3] and [4]. The parameters of the latter models have been taken by us from experimental analysis of the data of the reaction \( \phi \to \pi^+ \pi^- \gamma \) [7] and then used in calculation of the results shown as dashed and dotted lines.

The \( K^0 \bar{K}^0 \) differential cross sections are presented in the left panel of Fig. 3. These cross sections are considerably lower than the \( K^+ K^- \) cross sections seen in Fig. 2. This is due to a smaller \( K^0 \bar{K}^0 \) phase space and to smaller absolute values of the transition amplitude \( T(K^+ K^- \to K^0 \bar{K}^0) \) which replaces in this case the elastic \( K^+ K^- \) amplitude in Eq. (8).
By integration of the $K^+K^-$ and $K^0\bar{K}^0$ effective mass distributions, shown as solid lines in left panels of Figs. 2 and 3, one can calculate the total cross sections which are equal to 1.85 pb and 0.17 pb, respectively. The corresponding branching fractions are $4.5 \cdot 10^{-7}$ and $4.0 \cdot 10^{-8}$. In the right panel of Fig. 3 we have plotted the contours of the branching fraction for the $\phi \rightarrow K^0\bar{K}^0\gamma$ decay as a function of the $a_0(980)$ resonance position. We see that it is possible to generate lower values of the branching fraction by a moderate change of not well known resonance mass and width.

![Figure 3](image.png)

**Figure 3.** Left panel: differential cross-section for the reaction $e^+e^- \rightarrow K^0\bar{K}^0\gamma$ as a function of the $K^0\bar{K}^0$ effective mass. The curves are labelled as in the left panel of Fig. 2. Right panel: contours of the branching fraction $Br$ for the decay $\phi \rightarrow K^0\bar{K}^0\gamma$ in the complex plane of the $a_0(980)$ pole position: $m_{a_0(980)}$ is the resonance mass and $\Gamma_{a_0(980)}$ is its width. The solid curve corresponds to the KLOE upper limit $Br = 1.9 \cdot 10^{-8}$, the dotted one to $Br = 1.0 \cdot 10^{-8}$, the dashed curve to $Br = 3.0 \cdot 10^{-8}$, the dashed-dotted one to $Br = 4.0 \cdot 10^{-8}$, and the dashed-double dotted one to $Br = 5.0 \cdot 10^{-8}$. The cross indicates the $a_0(980)$ resonance position on sheet $(+\pm)$ found in Ref. [8].

### 3 Conclusions

The above theoretical results for the reactions with charged and neutral kaon pairs indicate that the measurements of the $e^+e^- \rightarrow K^+K^-\gamma$ process can provide a valuable information about the pole positions of the $a_0(980)$ and $f_0(980)$ resonances. A coupled channel analysis of the radiative $\phi$ transitions into different pairs of mesons in the final state is possible after a relevant generalization of the present model.

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