D-Instanton Generated Dirac Neutrino Masses

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We present a stringy mechanism to generate Dirac neutrino masses by D-instantons in an experimentally relevant mass scale without fine-tuning. Within Type IIA string theory with intersecting D6-branes, we spell out specific conditions for the emergence of such couplings and provide a class of supersymmetric local SU(5) Grand Unified models, based on the $Z_2 \times Z'_2$ orientifold compactification, where perturbatively absent Dirac neutrino masses can be generated by D2-brane instantons in the experimentally observed mass regime, while Majorana masses remain absent, thus providing an intriguing mechanism for the origin of small neutrino masses due to non-perturbative stringy effects.

I. INTRODUCTION

Until recently, no satisfactory mechanism was known for generating either Majorana neutrino masses (for a see-saw mechanism) or small Dirac masses within the Type IIA string theory context. Recent work [1, 2] has shown that Majorana masses can be generated non-perturbatively by D-brane instantons. In this letter we show that, as an equally plausible alternative, D-brane instantons may instead generate small Dirac neutrino masses at the observed scale without fine-tuning.

In the past year there have been intriguing insights into D-brane instantons [1, 2, 3, 4] which can generate perturbatively absent couplings of genuinely string theoretical origin with apparently no field theory analogs. (For reviews on the subject, see [5], [6].) In type II string compactifications with D-branes the specific superpotential couplings are typically forbidden due to perturbatively conserved “anomalous” $U(1)$ factors. However, under specific conditions, that ensure the correct number of D-instanton fermionic zero modes, these couplings can be generated with a strength that is exponentially suppressed by the D-instanton action. The mechanism was originally proposed for generating Majorana neutrino masses, the $\mu$-parameter and $R-parity$ violating terms [1, 2], as well as the study of supersymmetry breaking effects [4]. Further efforts in these directions focused on rational conformal field theory searches for global models with non-perturbatively realised Majorana masses [7], an explicit calculation of non-perturbative Majorana neutrino couplings within local orbifold constructions [8] and further studies of phenomenological implications for neutrino physics [9], as well as spelling out conditions for non-perturbatively induced Yukawa couplings $10 \times 10 \times 5_H$ within SU(5) Grand Unified Models (GUT’s) [10]. There have also been further analyses of non-perturbative supersymmetry breaking effects [11], [12].

While local realisations of models where this D-instanton mechanism were found [8], an important challenge was in the construction of globally consistent semi-realistic string vacua which realize such non-perturbative effects (for efforts within semi-realistic Gepner models, see [7]). The main difficulty seems to have been due to the specific Type IIA frameworks, where conformal field theory techniques are applicable. On the other hand, the T-dual Type I framework with magnetized D-branes, described by stable holomorphic bundles on compact Calabi Yau spaces, is amenable to algebraic geometry techniques. There, the first classes of semi-realistic globally consistent string vacua, where hierarchical couplings are realised by D1-brane instantons, were constructed [17].

The purpose of this paper is to present a new mechanism for small neutrino masses. Specifically, we present D-brane vacua with a Standard Model sector where perturbatively absent Dirac neutrino masses are generated non-perturbatively by D-instantons at a desired mass scale without fine-tuning, while at the same time ensuring the absence of non-perturbatively generated Majorana neutrino masses. This string mechanism should be contrasted with that for Majorana masses [1, 2]. Both types of masses can be generated by D-instantons that satisfy specific conditions and are thus restricted to specific string vacuum solutions. However, unlike D-instanton generated Dirac neutrino masses, the desired mass scale for Majorana neutrino masses can be achieved only after some fine-tuning of the volumes of the cycles wrapped by D-instantons.

For the sake of concreteness we focus on the Type IIA framework with intersecting D6-branes, though the T-dual formulation of conditions in the Type I framework with magnetized D9-branes on Calabi-Yau spaces can also be described employing techniques developed in [17]. In these cases the D-instantons can generate exponentially suppressed Dirac neutrino masses at experimentally relevant mass scales, while the Majorana masses are not generated. Thus the proposal provides a non-perturbative string realisation of the origin of small Dirac neutrino masses in the absence of a see-saw mechanism [20].

As a concrete application we construct a class of local models, based on the $Z_2 \times Z'_2$ toroidal orientifold, which explicitly realise this scenario. Within this local construction we do not address the moduli stabili-
lization issue, a difficult problem in string theory. The back-reaction of supergravity fluxes and/or strong gauge dynamics, responsible to for moduli stabilization, can also affect quantitative results for the proposed non-perturbative couplings (as well as global consistency constraints); however, this is beyond the scope of this paper.

The proposal is attractive since the classical instanton action enters the coupling at the exponentially suppressed level, proportional to the inverse string coupling and the volume of the cycles wrapped by the D-brane instanton. These couplings are thus generically extremely small, and thus generate tiny Dirac neutrino masses without any additional tuning of the volume of the cycles. This mechanism should be contrasted with the case of D-instanton generated Majorana neutrino masses [1, 2] and other such non-perturbatively generated couplings, such as the $10 \times 10_5$ Yukawa coupling of the SU(5) GUT [10], where some fine-tuning of the volumes of the cycles wrapped by D-instantons is needed in order to obtain the couplings in the desired regime, e.g. for Majorana masses in $10^{12 - 14}$ GeV range.

The specific focus are “semi-realistic” constructions within the Type IIA string theory framework with intersecting D6-branes wrapping three-cycles in the internal space (for reviews, see [13, 14]). Concrete realizations will be based on Grand Unified SU(5) models [20, 21, 22], with chiral families of quarks and leptons. However, again, we do not address the moduli stabilization. Specifically, the string vacuum constructions should have the property that the Yukawa couplings responsible for Dirac neutrino masses are absent perturbatively due to global $U(1)$ selection rules. Focusing on Type IIA theory, under suitable circumstances Euclidean D2-branes (E2-instantons) wrapping three cycles in the internal space can break these global $U(1)$ symmetries to certain discrete subgroups and generate $U(1)$ violating interactions, as spelled out in [1, 2, 4].

The Type IIA framework allows for a geometric formulation of the zero mode conditions in the presence of E2-instantons wrapping rigid three-cycles, and thus an explicit geometric interpretation of these non-perturbative effects. Namely, the intersection numbers of the of instanton and D-brane cycles, which specify the number of charged fermionic zero modes, are topological numbers. However, in order to illustrate the effects explicitly, we write expressions for these intersection numbers for a concrete local construction, based the $Z_2 \times Z_2^t$ toroidal orientifold, with Hodge numbers $(h_{11}, h_{12}) = (3, 51)$. We employ the notation of [23] (see also [24]), to which we refer for details of the geometry and the construction of rigid cycles. The orbifold group is generated by $\theta$ and $\theta'$ acting as reflections in the first and last two tori, respectively. The O6-plane parallel to the instanton is an O6$^+$ plane, whereas the three others are O6$^-$ planes.

The proposed framework requires three stacks $a$, $b$ and $c$ of D6-branes giving rise to a $U(5)_a \times U(1)_b \times U(1)_c$ gauge symmetry. The $U(5)_a$ splits into $SU(5)_a \times U(1)_a$, where the anomalous $U(1)_a$ gauge boson gets massive via the generalized Green-Schwarz mechanism and $U(1)_a$ appears as a global symmetry in the effective action. The matter transforming as $\mathbf{10}$ under $SU(5)_a$ arises at intersections of stack $a$ with its image $a'$; the matter fields transforming as $\mathbf{5}$ as well as Higgs fields $\mathbf{5}_H$ and $\mathbf{\bar{5}}_H$ are located at intersections of stack $a$ with $b$ and $b'$ or $c$ and $c'$. The Abelian gauge group factors $U(1)_b$ and $U(1)_c$ also acquire massive gauge bosons due to the generalized Green-Schwarz mechanism.

The key input in the construction of the local model is summarized in Tables I and II. Table I lists the wrapping numbers of the D6-branes wrapping bulk three-cycles $\Pi_a^3$, the building blocks of the local GUT models, and the wrapping numbers of the rigid three-cycle \$\Pi_\Sigma$ of the E2-instanton with the required zero mode structure.

| \(\text{nos} \) | \(n_a^1, m_b^1\) | \(n_a^2, m_b^2\) | \(n_a^3, m_b^3\) |
|---|---|---|---|
| $5_a$ | $(\nu_2, 2\nu_2/n_2)$ | $(1, 1)$ | $(0, -1)$ |
| $b_5$ | $(n_2, 2)$ | $(-1, 2)$ | $(-1, 1)$ |
| $c_5$ | $(-1, 0)$ | $(1, 1)$ | $(-1, -1)$ |
| $l_5^b$ | $(1, 0)$ | $(0, 1)$ | $(0, -1)$ |

Table I: Wrapping numbers of D6-branes wrapping bulk three-cycles and the wrapping numbers of the E2-instanton wrapping a rigid, orientifold action invariant, three-cycle on the $Z_2 \times Z_2^t$ toroidal orientifold. The wrapping numbers $n_2$ and $n_2$ are positive integers, co-prime with 2 and $2\nu_2/n_2$ integer, respectively. The simplest choice is $n_2 = \nu_2 = 1$. Another interesting (non-Abelian anomaly free) case is $n_2 = \nu_2 = 3$. The model is supersymmetric for the choice $\chi_1 = \chi_2 = 1/3\sqrt{3}$, $\chi_3 = n_2\sqrt{3}/2$ of the complex structure toroidal moduli $\chi_i \equiv (R_i/R_1)$, of the $i$th two-torus.

| \(\text{sector} \) | \(\text{number} \) | \(U(5)_a \times U(1)_b \times U(1)_c \) |
|---|---|---|
| (a, a') | $4(\nu_2 - 2\nu_2/n_2)$ | $10_{(2, 0, 0)} + 15_{(-2, 0, 0)}$ |
| (a, b') | $16\nu_2$ | $5_{(1, 1, 0)}$ |
| (a, c') | $16\nu_2$ | $5_{(-1, 0, -1)}$ |
| (b, c') | $16$ | $1_{(0, 1, 1)}$ |

Table II: Chiral matter spectrum for the local SU(5) GUT models with intersecting D6-branes on the $Z_2 \times Z_2^t$ orientifold. The wrapping numbers of the D6-branes are depicted in Table I. A special case $\nu_2 = n_2 = 3$ corresponds to the four family example and is free of non-Abelian anomalies. Another four family example corresponds to $\nu_2 = n_2 = 1$; however, in this case additional “filler” branes would have to be added to cancel non-Abelian anomalies.

We build a local model on the $Z_2 \times Z_2^t$ orientifold by wrapping D6 branes on the bulk cycles specified by wrapping numbers \(n_a^i, m_b^i\) (for further details see [23]).

The intersection numbers in the respective \((a, b)\) and \((a, b')\) sectors are:

\[
I_{ab} = 4 \prod_i (n_a^i m_b^i - n_a^i m_b^i), \quad I_{ab'} = -4 \prod_i (n_a^i m_b^i + n_a^i m_b^i)
\]

(1)
where we choose a convention that the chiral superfields in the $a, b$ representation correspond to $I_{ab} < 0$. The symmetric and anti-symmetric representations arise from the $(a, a')$ sector:

$$I_{a a'}^\text{antisymm} = \frac{1}{2}(I_{a a'} + I_{a O6}), \quad I_{a a'}^\text{symm} = \frac{1}{2}(I_{a a'} - I_{a O6}),$$

(2)

where

\begin{align*}
I_{a a'} &= -32n_a^a m_1^a n_2^a m_2^a n_3^a m_3^a \\
I_{a O6} &= -8n_1^a m_2^a m_3^a + 8n_2^a m_3^a m_1^a - 8n_3^a m_1^a m_2^a + 8n_3^a m_2^a m_1^a
\end{align*}

(3)

As shown in [8], the background of this model exhibits one class of so-called rigid $O(1)$ instantons. For completeness, in the conventions of [23], we give the full expression for the rigid three-cycle wrapped by the E2-instanton

$$\Pi_E = \frac{1}{4} \Pi_E^\theta - \frac{1}{4} \sum_{i,j,(13)\times(12)} \alpha_{ij,m}^\theta + \frac{1}{4} \sum_{j,k,(12)\times(12)} \alpha_{jk,n}^{\theta'} + \frac{1}{4} \sum_{i,k,(13)\times(12)} \alpha_{ik,m}^{\theta''},$$

(4)

where $\Pi_E^\theta$ wraps the cycle $[(1,0) (0,1) (0,-1)]$. The twisted three-cycles $\alpha_{ij,n}^{\theta} [\alpha_{jk,m}^{\theta'}]$ can be understood as a product one-cycle $[a]^\theta ([b]^\theta')$ on the $\theta$-th two-torus and a two-cycle $S^2$, a blow-up of $(i,j) \in (1, 2, 3, 4) \times (1, 2, 3, 4)$ orbifold fixed points. (For further details see [23].) The intersection number in the $(a, E)$ specifies the number of chiral fermionic modes and is of the form:

$$I_{aE} = \prod_{i} (n_i^a m_i^E - n_i^E m_i^a).$$

(5)

Again the convention $I_{aE} < 0$ corresponds to chiral fermionic zero modes in the $(a, -1 E)$ representation. Note that since D6-branes wrap (non-rigid) bulk three-cycles, the intersection number in the $(a, E)$ sector does not depend on fractional twisted three-cycles of the instanton.

The supersymmetry conditions are ensured by requiring that the three-cycles are special Lagrangians with respect to the same holomorphic three-form. In the case of toroidal compactification these take the form:

\begin{align*}
m_1^a m_2^a m_3^a &- \sum_{i \neq j \neq k \neq i} n_i^a n_j^a n_k^a \chi_i \chi_j \chi_k = 0 \\
n_i^a n_j^a n_k^a &- \sum_{i \neq j \neq k \neq i} m_i^a m_j^a m_k^a \chi_i \chi_j \chi_k > 0
\end{align*}

(6)

where $\chi_i \equiv \left(\frac{R^*}{R}_i\right)$ is the complex structure modulus for the $i^{\text{th}}$ two-torus.

At the intersection of the $U(1)_b$ and $U(1)_c$, D6-branes the chiral matter corresponds to the right handed neutrinos $N_R$. We insist that there only exist chiral states with such $U(1)_b$ and $U(1)_c$ charges that they cannot couple perturbatively in Yukawa couplings $\lambda_{55} N_R$, i.e., $U(1)$ charges are perturbatively violated for such couplings, and thus Dirac masses corresponding to them are not present at this stage. In addition we want to ensure that the D-instanton zero modes induce the Dirac neutrino masses, while the Majorana neutrino masses are absent. These constraints can be ensured by the following specific signs for the intersection numbers:

\begin{align*}
I_{ab} &= 0, \quad I_{ab'} < 0, \\
I_{bc} &= 0, \quad I_{bc'} < 0, \\
I_{ac} &= 0, \quad I_{ac'} > 0.
\end{align*}

(7)

resulting in the following left-handed chiral superfield representations: $(5_a, 1_b), (1_b, 1_c)$, and $(\bar{5}_a, -1_c)$. Therefore, $N_R = (1_b, 1_c)$ (or any singlets in $(b, b')$ and $(c, c')$ sectors with respective charges $\pm 2_b$ and $\pm 2_c$) cannot couple perturbatively via Yukawa couplings to $5_a$ and $5_b$. We also assume that the states in the N=2 sector of $(a, b)$, and $(b, c)$ system are massive by wrapping the respective D-branes on parallel one-cycles that do not coincide. The wrapping numbers presented in the Table I comply with the above intersection number conditions and have the following specific values:

\begin{align*}
I_{ab} &= 0, \quad I_{ab'} = -16 \nu_2, \\
I_{bc} &= 0, \quad I_{bc'} = -16, \\
I_{ac} &= 0, \quad I_{ac'} = 16 \frac{\nu_2}{\nu_2},
\end{align*}

(8)

as are also listed in terms of the multiplicity of the states in the spectrum in Table II.

To generate the desired Yukawa couplings at the non-perturbative level the instanton has to have a spectrum of zero modes ensured by the intersection numbers:

$$I_{aE} = 0, \quad I_{bE} = 2, \quad I_{cE}^{\nu=2} = 1.$$ 

(9)

Note that for the non-chiral intersection with $I_{E} = 0$, however, the N=2 $(c, E)$ sector has one vector pair of massless modes. To ensure that the N=2 $(a, E)$ sector does not have massless modes, the parallel one-cycles on the third two-torus for the D6$_a$-brane and the E2-instanton do not coincide. We therefore ensure the correct structure of the zero modes, i.e., two zero modes $\sum_{c} \chi_{c} \chi_{c}$ in the representation $(-1_b, 1_E)$ and one vector pair $\lambda_{c} \chi_{c} \chi_{c}$, $(-1_c, 1_E)$ and $(1_c, -1_E)$.

Importantly, D-instanton zero modes cannot generate Majorana masses for $N_R = (1_b, 1_c)$. We have also checked that in the concrete set-up there is no other $E$-instanton that could induce Majorana masses. Such an instanton would have to wrap a rigid three-cycle with the intersection numbers:

$$I_{EE} = 0, \quad I_{E\bar{E}} = 0, \quad I_{a\bar{E}} = 0, \quad I_{b\bar{E}} = 2, \quad I_{c\bar{E}} = 2.$$ 

(10)

The first three constraints require $(n_{\bar{E}}^E, m_{\bar{E}}^E) = (0, -1)$, however the last two constraints cannot be satisfied for any wrapping numbers $(n_{\bar{E}}^E, m_{\bar{E}}^E)$, corresponding to a rigid, supersymmetric three-cycle.
Note that specific conditions on the intersection numbers between D-branes (7) and the intersection numbers of the D-instanton with D-branes (9) apply to a general Type IIA set-up and ensure a mechanism that generates perturbatively absent Dirac neutrino masses due to E2-instantons, which cannot generate Majorana masses for $N_R$’s.

The contribution to the superpotential arises from the string amplitudes as shown in figure 1. These four fermionic zero modes $\lambda$ can be saturated via the two disk diagrams, thereby generating superpotential contributions to the Yukawa couplings $5\,5\,N_R$ of the type:

$$Y = \exp(-S_{\text{inst}}) = x \exp\left(-\frac{2\pi}{\alpha_{\text{GUT}} V ol_E^2} \frac{\text{Vol}_{E^2}}{\text{Vol}_{D6a}}\right) \quad (11)$$

where $\frac{\text{Vol}_{E^2}}{\text{Vol}_{D6a}} = (6\nu_a)^{-1}$ for the specific local construction.

The numerical factor, $x$, is expected to be of order 1. A more detailed conformal field theory calculation of the three-point and four-point disc amplitudes [8, 25] emerging from the geometric local set up in Figure 1 could in principle generate additional world-sheet instanton suppression terms, as were explicitly calculated for the D-instanton induced Majorana masses in [8]. In addition, further summation over $Z_2 \times Z_2^\prime$ images of the E-instanton can quantitatively affect $x$, as again was addressed in [8].

Taking $\nu_2 = 1$, $\alpha_{\text{GUT}} \sim \{2^{-1}, 30^{-1}\}$ and a VEV of the Higgs doublet $\sim 100$ GeV yields neutrino Dirac masses in the range

$$m_{\text{Dirac}} \sim \{2 \times 10^{-3}, 0.4\} \text{ eV} \quad (12)$$

which is a reasonable regime for the allowed range for the neutrino masses. Note, however, that the case $\nu_2 = n_2 = 3$ would require too small a value $\alpha_{\text{GUT}} \sim 10^{-2}$ to bring $m_{\text{Dirac}}$ to the $10^{-3}$ eV regime.

In conclusion, we have presented specific conditions for a concrete proposal, explicitly implemented within a local supersymmetric SU(5) construction with intersecting D6-branes, where string D-instantons (E2-instantons) can generate perturbatively absent Dirac neutrino masses while Majorana masses remain absent. The exponentially suppressed coupling can be engineered (without fine-tuning) to yield Dirac neutrino masses in the observed regime $\gtrsim 10^{-3}$ eV. While the concrete set-up was in the context of Type IIA string theory, realizations in the T-dual picture, namely the Type I framework, may be amenable to constructions of global models on compact Calabi-Yau spaces where string D-instanton couplings are realized within globally consistent models, à la [17].

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Small Dirac masses in supersymmetric field theory may also be generated by higher dimensional operators in the superpotential [13, 14] or Kähler potential [15], though this has not yet been implemented in a string construction. For a review, see [16]. If such terms were present in the proposed D-brane constructions they would merely be additive. Such higher dimensional operators have not been studied systematically in these classes of constructions.