Formation of algorithm of automatic parametric optimization of PI controller with variable parameters while using Internal Model Control

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Abstract. Known by its advantages, a proportional-integral controller (henceforth – a PI controller) with variable or switchable parameters, related to a class of controllers with a variable structure (henceforth – variable structure controllers, VSC) that do not use a sliding mode, is applied to control an object in the automatic system with a sufficient delay, reducing the performance quality of standard controllers (integral, proportionally integral, proportionally integral differential). The specifics of building this controller makes it more difficult to use analytical approaches to the parametric optimization of the system, which defines the use of algorithmic methods. The present work uses a gradient algorithm, in which the gradient constituents are calculated with the help of sensitivity functions with their definite advantages. To obtain initial values of VSC adjustable parameters, an adjustment method of a standard PI controller based on the internal model controller (IMC) is used. The gradient algorithm of an automatic parametric optimization (henceforth – an APO algorithm) has calculated optimal parameters of VSC starting from the closeness of the output coordinate of the object to the reference process with a minimum of the integral quadratic criterion. The correctness of the calculated vector of the controller adjustment by the formed APO algorithm is confirmed by necessary and sufficient extremum conditions of the chosen optimization criterion.

1. Introduction

Objects of transport systems with transport delay represent an important class [1-6]. With a ratio of τob/Tobm>1, τob is the magnitude delay, Tобm=max(Tоб1, Tоб2,…, Tобn) is the constant of controlled object time, serial continuous controllers (integral, proportionally integral, proportionally integral differential) do not provide the required quality of transient processes [1,4,6]. A way to control these objects is to apply controllers with a delay link in their structure (e.g., a predictive proportionally integral controller) [6-18]. An alternative way of compensating the object delay is to apply the switching of parameters without a sliding mode (henceforth – VSC) in the PI controller [2]. To bypass the switching of VSC structures in a steady-state mode due to interference, it is necessary to introduce a dead zone [4,5]. The presence of switching elements, a filter and the dead zone in these controllers makes it more difficult to apply analytical approaches to solve the problems of parametric optimization, consisting in the calculation of the VSC adjustment vector q according to the extremum value of the accepted optimality criterion on the basis of the reference model.
In the present work the parametric optimization algorithm, using the gradient procedure based on the sensitivity theory elements, is applied to VSC, and thus the parametric optimization problem for the considered automatic system is solved. To obtain the starting values of the VSC adjustment vector $q$, it is supposed to use a method on the basis of building a controller with an internal model (IMC) [6, 18-21].

2. Problem statement

The structural diagram of the considered automatic system is represented in Figure 1.

![Figure 1](image)

**Figure 1.** The structural diagram of the studied automatic system

Below we present a description of processes of the automatic system in Fig. 1 with the given PI controller with a dead zone [4, 5]:

\[
\begin{align*}
\varepsilon(t, q) &= \lambda(t) - x(t), \\
u(t, q) &= G_c(p, q^i)\varepsilon(t, q), (i = 1,2,3),
\end{align*}
\]

(1)

where $\varepsilon(t, q)$ is the switching system error; $\lambda(t)$ is the setting action; $u(t, q)$ is the controlling action; $x(t)$ is the output coordinate of the automatic control system (ACS); $G_c(p, q^1) = q_1 + \frac{q_2}{p}$, $G_c(p, q^2) = q_3 + \frac{q_4}{p}$, $G_c(p, q^3) = 0 + \frac{0}{p}$ is the operator of the classic PI controller with variable parameters; $G_p(p)$ is the controlled object operator; $p = \frac{d}{dt}$ is the differentiation operator; $\land, \lor$ are the logical operations.

Controlled object operation $G_p(p)$ is chosen in the form by which it is possible to describe processes of industrial facilities:
\[ G_p(p) = \frac{k_{ob}}{(T_{ob1}p + 1)(T_{ob2}p + 1)} e^{-\frac{\tau_{ob}}{p}}, \]  
(2)

where \( k_{ob} \) is the static amplification factor; \( T_{ob1}, T_{ob2} \) are the time constants; \( \tau_{ob} \) is the delay time.

For further numerical experiments, the following basic values of parameters of the object (2) operator are taken, provided that the condition of the large delay of the object \( \tau_{ob}/T_{ob2} > 1 \) is satisfied:

\[ T_{ob1} = 10, T_{ob2} = 40, k_{ob} = 1, \tau_{ob} = 50. \]  
(3)

Requirements to the transient process (oscillability, response speed, maximal deviation, accuracy in the steady-state mode) can be implemented by the reference model presented by an oscillatory link [22]:

\[ G_{et}(p) = \frac{k_{et}}{T_{et}^2 p^2 + 2T_{et}\zeta_{et}p + 1} e^{-\frac{\tau_{et}}{p}}. \]  
(4)

To provide further examples of the calculation experiment, the following parameters of the reference model (4) are taken:

\[ \tau_{et} = \tau_{ob}, T_{et} = 40, k_{et} = 1, \zeta_{et} = 1.25. \]

In the present work, the value of the parameter filter \( T_f \) is chosen to be equal to the value of \( \frac{\tau_{ob}}{5 \div 6} \), which makes it possible to reduce phase retardation in the law of switching with a filter, compared to the law of switching without a filter, and not to “pass over” the first switching point \( t_0 \).

Let us represent an optimized integral criterion defining a degree of closeness of output coordinate \( x(t) \) to the reference process \( x_{et}(t) \):

\[ I = \int_0^\infty (x_{et}(t) - x(t, q))^2 dt, \]  
(5)

where \( x_{et}(t) = G_{et}(p)\lambda(t) \).

3. Optimization algorithm

The basis of the formed APO algorithm is comprised by the gradient procedure, and it is necessary to calculate the gradient constituents of the chosen optimization criterion, which in the present work are created with the help of sensitivity functions. Let us provide the sensitivity equations for the system (1) with criterion (5) [22,23,24]:

\[ \frac{\partial u(t, q)}{\partial q_j}(t) = G_p(p)\frac{\partial u(t, q)}{\partial q_j}(t), (j = 1,2,...,4). \]  
(6)

Let us accept the following notations in the present work to compactly represent expressions defining \( \frac{\partial u(t, q)}{\partial q_j} \):

\[ \Psi^+ = ((\Psi(t, \varepsilon(t, q)) > 0) \lor (0 < t < t_{ob})) \land (|\varepsilon(t, q)| > \alpha\lambda(t)); \]
\[ \Psi^- = ((\Psi(t, \varepsilon(t, q)) < 0) \land (|\varepsilon(t, q)| > \alpha\lambda(t)); \]
\[ \Psi^0 = |\varepsilon(t, q)| < \alpha\lambda(t). \]  
(7)
Due to the limited scope of the article, we present only an expression to calculate \( \frac{\partial u(t, q)}{\partial q_1} \):

\[
\begin{align*}
\frac{\partial u(t, q)}{\partial q_1} &= -\xi_i(t)(q_1 + \frac{q_2}{p}) + \varphi(t, q), \Psi^-; \\
\frac{\partial u_2(t, q)}{\partial q_1} &= -\xi_i(t)(q_3 + \frac{q_4}{p}), \Psi^-; \\
\frac{\partial u_3(t, q)}{\partial q_1} &= -\xi_i(t)(0 + \frac{0}{p}), \Psi^0.
\end{align*}
\]

(8)

According to the gradient algorithm in the process of optimization, the vector \( q \) of the adjustable parameters changes in accordance with the expression [22]:

\[ q[I] = q[I-1] - \Gamma \nabla_q I(x_{et}(t) - x(t, q[I-1])), \quad (l = 1, 2, \ldots), \]

(9)

where \( \Gamma = \{ \gamma_i \} \) is the vector of weighting factors; \( l \) is the algorithm step number; \( \nabla_q I(x_{et}(t) - x(t, q[I-1])) \) is the gradient vector (4). Let us represent an expression to define the gradient vector \( \nabla_q I(x_{et}(t) - x(t, q[I-1])) \), connected with the calculation of the vector of sensitivity function (7):

\[
\frac{\partial I(x_{et}(t) - x(t, q[I-1]))}{\partial q_i} = -2 \int_0^{\infty} (x_{et}(t) - x(t, q[I-1])) \xi_i (t) dt, (i = 1, \ldots, 4).
\]

(10)

As is known while using the gradient procedure, it is necessary to indicate the initial values of the vector \( q[I] \), to solve this problem in the present work, it is supposed to use the building of the standard PI controller based on the controller with an internal model (a PI-IMC controller). According to [6, 25], for the object (4), initial conditions of the vector \( q[I] \) while using the PI-IMC controller will be the following:

\[ q_1 = q_3 = \frac{T_{ob1} + T_{ob2}}{k_{ob}(2\eta + \tau_{ob})}; q_2 = q_4 = \frac{1}{k_{ob}(2\eta + \tau_{ob})}, \]

(11)

where \( \eta \) is the parameter taking on values within the range from 0 to +\( \infty \).

Taking consideration of parameters of the object (3) and expressions (11) we define the boundaries of ranges of adjustable VSC parameters:

\[ q_1, q_3 \in (0; 1]; q_2, q_4 \in (0; 0.02]. \]

(12)

To make it possible to implement the problem to be solved using the computer, it is necessary to involve a condition of stopping the optimization process. In the present work, the stopping condition is formed according to the following principle [26]: in each \( n \) steps of algorithm (18), a minimum of the chosen optimization criterion (5) is checked, within the accuracy of two signs, compared to the previous value of the criterion. The algorithm (18) is performed while the condition is true:

\[ I_{\text{min}} (l \in [1; n]) > I_{\text{min}} (l \in [i \cdot n + 1; (i + 1)n]) > \ldots (i = 1, 2, \ldots), \]

(13)

where \( I_{\text{min}} \) is the minimal value of the criterion (5) for each \( n \) steps of the algorithm (10). In the course of testing the APO algorithm, the reference value of \( n=20 \) is chosen.
4. The optimization algorithm operability check
It is necessary to perform an operability test for the formed APO algorithm, which consists in the checking the credibility of the calculated values of the adjustable parameters of \( \mathbf{q}^* \) from the point of finding a local minimum of the criterion (5).

While starting the APO algorithm from different initial values of the vector of adjustable parameters \( \mathbf{q}_0 = (q_{1k}^0, q_{2k}^0, q_{3k}^0, q_{4k}^0) (k = 1, 2, \ldots) \), the corresponding final values at the optimum point \( \mathbf{q}_k^* = (q_{1k}^*, q_{2k}^*, q_{3k}^*, q_{4k}^*) \), computed by the APO algorithm, should provide the fulfillment of the necessary extremum condition at these points:

\[
\frac{\partial I(x_\alpha(t) - x(t, \mathbf{q}^*))}{\partial \mathbf{q}} = (0 \pm \Delta),
\]

where \( \Delta \) is the calculation error.

Also, at the optimum point, from the position of the minimum of the criterion (4), a sufficient optimality condition should be performed:

\[
\frac{\partial^2 I(x_\alpha(t) - x(t, \mathbf{q}^*))}{\partial q_i \partial q_j} > 0, (i, j = 1, \ldots, 4).
\]

The basis of the condition (14) is formed by a square Hessian matrix, consisting from second derivatives from the optimality criterion (5) by the adjustable parameters \( q_i \):

\[
\frac{\partial^2 I(x_\alpha(t) - x(t, \mathbf{q}^*))}{\partial q_i \partial q_j} = \begin{bmatrix}
\frac{\partial^2 I}{\partial q_i^2} & \frac{\partial^2 I}{\partial q_i \partial q_2} & \frac{\partial^2 I}{\partial q_i \partial q_3} & \frac{\partial^2 I}{\partial q_i \partial q_4} \\
\frac{\partial^2 I}{\partial q_2 \partial q_i} & \frac{\partial^2 I}{\partial q_2^2} & \frac{\partial^2 I}{\partial q_2 \partial q_3} & \frac{\partial^2 I}{\partial q_2 \partial q_4} \\
\frac{\partial^2 I}{\partial q_3 \partial q_i} & \frac{\partial^2 I}{\partial q_3 \partial q_2} & \frac{\partial^2 I}{\partial q_3^2} & \frac{\partial^2 I}{\partial q_3 \partial q_4} \\
\frac{\partial^2 I}{\partial q_4 \partial q_i} & \frac{\partial^2 I}{\partial q_4 \partial q_2} & \frac{\partial^2 I}{\partial q_4 \partial q_3} & \frac{\partial^2 I}{\partial q_4^2}
\end{bmatrix}
\]

(16)

\[
\frac{\partial I(\varepsilon(t, \mathbf{q}^*)))}{\partial q_{ij}} = 2\int_0^\infty (\xi_i(t)\xi_j(t) - \xi_{ij}(t)(x_\alpha(t) - x(t, \mathbf{q})))dt, (i, j = 1, \ldots, 4).
\]

(17)

where \( \xi_{ij}(t), (i, j = 1, \ldots, 4) \) is the second-order sensitivity function.

Based on the things stated above in the present work, the indicator of performance of the formed APO algorithm is the fulfillment of criteria (14), (15) for \( \mathbf{q}^* \).

5. Study results
Let us present the operability check for the APO algorithm by practical examples with the setting action \( \lambda(t) = 1 \cdot 0.5(t) \). Figure 3 demonstrates the fact that the formed APO algorithm
of a system with the controller (1) provides the calculation of $q^*_k (k = 1,2,3)$ for different types of initial transient processes in the automatic system with different $q^0_k (k = 1,2,3)$. In this figure one can see that the obtained final processes sufficiently coincide with the reference model.

**Figure 3.** The graphs of transient processes at the initial (1) and final (2) points of the APO algorithm performance of the reference model (3)

Figure 3 shows that at the start of the APO algorithm from different initial values of the vector of adjustable parameters $q^0_k (k = 1,2,3)$, the corresponding final values at the optimum point $q^*_k (k = 1,2,3)$, calculated by the APO algorithm, provide the fulfillment of the necessary condition of the extremum (14) of the criterion (5) at these points.

**Figure 3.** The values of the constituents of the gradient $dl/dq_1(l)$, $dl/dq_2(l)$, $dl/dq_3(l)$, $dl/dq_4(l)$ of the optimization criterion (5) in the course of work of the APO algorithm with the parameters of the object (3)

Table 1 reflects the results of the APO algorithm work at different $q^0_k (k = 1,2,3)$ and presents a Hessian matrix.
Table 1. Results of the APO algorithm performance

| k  | $q_1^0$ | $q_2^0$ | $q_3^0$ | $q_4^0$ | $q_1^*$ | $q_2^*$ | $q_3^*$ | $q_4^*$ | $I(q^0)$ | $I(q^*)$ |
|----|--------|--------|--------|--------|--------|--------|--------|--------|---------|---------|
| 1  | 0,09   | 0,01   | 0,09   | 0,01   | 0,10588| 0,01855| 0,09553| 0,00020| 3,89     | 0,11    |
| 2  | 0,095  | 0,013  | 0,095  | 0,013  | 0,11892| 0,01840| 0,10228| 0,00032| 8,27     | 0,12    |
| 3  | 0,085  | 0,005  | 0,085  | 0,005  | 0,09585| 0,01854| 0,08900| 0,00013| 7,31     | 0,13    |

For the parameters of the object (3), according to Table 1, the Hessian matrix is positively defined, which is confirmed by the fact that the formed APO algorithm has found optimal parameters of the controller (1), based on the minimum of the criterion (5) while using the starting values of the vector $q^0$, obtained with the help of the IMC method.

6. Conclusion

The present work, with the help of the formed gradient algorithm, solves a problem of parametric optimization of a PI controller with variable parameters for an object with large delay at the given requirements for the quality of the transient process with filtration error in the law of switching and using initial values of the vector $q^0$, obtained with the help of the IMC method. The operability of the formed APO algorithm is proved by the fulfillment of the necessary and sufficient conditions of extremum of the criterion (5).

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