Harvesting correlations from complex scalar and fermionic fields with linearly coupled particle detectors

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We explore entanglement harvesting with particle detectors that couple linearly to non-Hermitian fields. Specifically, we analyze the case of particle detectors coupled to a complex scalar quantum field and to a spin 1/2 fermionic field. We then compare these models with the well-known case of a real scalar field and previous results using quadratic models. Unlike quadratic models, the linear models are able to distinguish the particle from the anti-particle sector of the field. This leads to striking differences between quadratic couplings—which cannot be related to physically observable processes—and linear couplings—that can be related to high-energy physics. Finally, we use this analysis to comment on the role that the field statistics plays on the protocol.

I. INTRODUCTION

Quantum field theory (QFT) is one of the most successful frameworks of theoretical physics. Among its achievements are the foundations for the standard model and its many applications for the description of numerous other areas of physics ranging from condensed matter to quantum optics. However, our understanding of QFT is still far from complete, and there are many problems still to be fully understood. Examples of these problems are the formulation of a consistent measurement framework in QFT [11–6] and a precise way of quantifying local entanglement in quantum fields [7]. Fundamental studies of QFT often focus on the properties of the vacuum states. Indeed, due to the fact that the short distance behaviour of a QFT is determined by its vacuum state [8–11], it may be argued that exploring the properties of the vacuum is essential in order to acquire a deeper understanding of QFT.

One of the most remarkable features of the vacuum state of a quantum field is the presence of entanglement between spacelike separated regions [12–13]. However, quantifying entanglement in QFTs, or even defining a local notion of vacuum state is a non-trivial task. A possible way to approach this problem is to quantify the amount of entanglement that can be extracted by local probes that couple to the field at different spacetime regions. In fact, the setup in which two localized probes couple to the vacuum of a quantum field theory to spacelike separated regions has become known as entanglement harvesting [15–17].

The first work to study an entanglement harvesting protocol dates back from the 90’s, by Valentini [20]. This concept was revisited by Reznik et. al. [21, 22], where a pair of pointlike Unruh-DeWitt (UDW) detectors have been used to extract entanglement from the vacuum of a real scalar field. The simplified UDW detector models used by Reznik consisted of two-level systems that couple locally to the quantum field. Multiple different scenarios of entanglement harvesting have been considered during the last decades, including more general couplings [22, 23, 24], quantum fields [10] and in curved spacetimes [25–29].

In this work we study the entanglement harvesting protocol for non-Hermitian quantum fields. We compare the usual scalar field protocols with the case of a complex scalar field and a fermionic field when the detectors-field coupling is linear. Previous literature focused on quadratic couplings [23, 24] that are arguably more difficult to relate to physical processes. In contrast, particle detector models that couple linearly to non-Hermitian fields have recently been linked to physical processes [30, 31]. For instance, the fermionic particle detectors can be linked to the emission and absorption of neutrinos by nucleons [30]. Furthermore, besides computational simplicity and the ability to relate the models to physical processes, there is another advantage of the linear coupling compared to the quadratic one. Namely, the quadratic coupling does not distinguish the particle and the anti-particle sectors of the field, therefore the model cannot capture any effect that depends on the QFT’s particle versus anti-particle content [31].

We develop the entanglement harvesting formalism for the complex and fermionic fields in curved spacetimes, and then consider examples in flat spacetime, comparing the results with the well-known real scalar field case. We study the role that the particle and antiparticle sectors play in entanglement harvesting and how it relates to the physical process modelled by the detector. Moreover, we highlight what insights from the real scalar case carry through to the linear complex scalar and fermionic case, and which ones do not. Finally we explore the possible impact that statistics (bosonic versus fermionic) has on entanglement harvesting.

Our work is organized as follows. In Section [11] we
review the standard UDW model, i.e., a two-level system linearly coupled to a real scalar quantum field. We also review the protocol of entanglement harvesting and explicitly study the example where the detectors are initially in the ground and excited states. In Section II we review the linear UDW detectors that couple to a complex scalar field and in Section III we analyze the entanglement harvesting protocol with this model. In Section III we review the fermionic linear particle detector model and in Section IV we study entanglement harvesting using this model. Our conclusions can be found in Section VII.

II. THE UDW MODEL AND THE ENTANGLEMENT HARVESTING SETUP

In this section we will briefly review the well-known UDW particle detector model, and how it is used in entanglement harvesting scenarios. We then focus in the less commonly explored case where detectors in different states (ground-excited) couple to the quantum field.

A. The UDW Model

The Unruh-DeWitt detector model in its simplest form consists of a localized two-level quantum system linearly coupled to a free real scalar quantum field. This particle detector model has been used extensively in QFT in curved spacetimes and Relativistic Quantum Information in a plethora of scenarios ranging from quantum communication to the study of the Unruh and Hawking effects (see, among others, [34, 35]). This model has also been proven to capture the fundamental features of the light-matter interaction when exchange of angular momentum is not relevant [16, 44, 45]. In particular for our purposes, particle detectors have been used to study the entanglement structure of quantum fields through the protocol known as entanglement harvesting [15, 19].

In order to introduce the UDW model, consider a $D = n + 1$ dimensional globally hyperbolic curved spacetime $\mathcal{M}$ with a Lorentzian metric $g$. In this spacetime we introduce a real scalar quantum field $\hat{\phi}(x)$. We assume that the scalar field can be canonically quantized in terms of a normalized basis of solutions of the Klein-Gordon equation, $\{u_k(x), u^*_k(x)\}$, according to

$$\hat{\phi}(x) = \int d^n k \left( u_k(x) \hat{a}_k + u^*_k(x) \hat{a}^\dagger_k \right),$$

where the creation and annihilation operators, $\hat{a}_k$ and $\hat{a}^\dagger_k$, satisfy the bosonic canonical commutation relations

$$\left[ \hat{a}_k, \hat{a}^\dagger_{k'} \right] = \delta^{(3)}(k - k').$$

We then consider a particle detector moving in a time-like trajectory $z(\tau)$ in $\mathcal{M}$, where $\tau$ is the proper time parameter of the curve. The free Hamiltonian of the detector that generates time evolution with respect to its proper time is given by

$$\hat{H}_d = \Omega \hat{\sigma}^+ \hat{\sigma}^-,$$

where $\Omega$ is the energy gap between the ground and excited states ($|g\rangle$ and $|e\rangle$) of the detector and $\hat{\sigma}^\pm$ are the ladder operators. Namely, $\hat{\sigma}^+ = |e\rangle\langle g|$ and $\hat{\sigma}^- = |g\rangle\langle e|$. The coupling between the field and the detector is given by the following interaction Hamiltonian weight in the interaction picture [16, 47]:

$$\hat{h}_I(x) = \lambda \Lambda(\tau)\hat{\mu}(\tau)\hat{\phi}(x),$$

where $\lambda$ is the coupling strength, $\Lambda(\tau)$ is the spacetime smearing function, responsible for controlling both the spatio-temporal profile of the interaction and $\hat{\mu}(\tau) = e^{i\theta \tau} \hat{\sigma}^+ + e^{-i\theta \tau} \hat{\sigma}^-$ is the detector’s monopole moment in the interaction picture. The associated time evolution operator is then given by:

$$\hat{U}_I = \mathcal{T}_\tau \exp \left( -i \int dV \hat{h}_I(x) \right),$$

where $dV$ is the invariant spacetime volume element and $\mathcal{T}_\tau$ denotes the time ordering operation with respect to the proper time parameter $\tau$. It is important to notice that for smeared particle detectors, the time ordering operation, in principle, can depend on the time parameter chosen. However, in [47] it was shown that under the right conditions, any choice of time ordering is equivalent up to leading order in the coupling constant.

It is common to work perturbatively with the Dyson expansion for the time evolution operator $\hat{U}_I$, so that, to second order, it is given by

$$\hat{U}_I = \mathbb{1} + \hat{U}^{(1)}_I + \hat{U}^{(2)}_I + \mathcal{O}(\lambda^3),$$

with

$$\hat{U}^{(1)}_I = -i \int dV \hat{h}_I(x),$$
$$\hat{U}^{(2)}_I = -i \int dV dV' \hat{h}_I(x) \hat{h}_I(x') \theta(\tau - \tau'),$$

1 The Hamiltonian weight $\hat{h}_I(x)$ is related to the Hamiltonian density $h_I(x)$ by $h_I(x) = \hat{h}_I(x) \sqrt{-g}$.
2 It is usual to write the integral in Eq. (8) in terms of an integral in time of a Hamiltonian associated with a given foliation $\Sigma_t$ using

$$\int dV = \int dt \int_{\Sigma_t} d^n x \sqrt{-g},$$

where $t$ is a time parameter that parametrizes a foliation by spacelike surfaces $\Sigma_t$ for which we use an arbitrary spatial coordinate $x$ (for more details see [36, 37]). The Hamiltonian associated to this foliation is then given by

$$\hat{H}_I(t) = \int_{\Sigma_t} d^n x \sqrt{-g} \hat{h}_I(x).$$
where $\theta(\tau)$ denotes the Heaviside theta function that arises from the time ordering operation.

We assume the detector to start in a state $\hat{\rho}_{d,0}$ and to be completely uncorrelated with the field state, $\hat{\rho}_\phi$. That is, we consider the initial state of the detector-field system to be the density operator $\hat{\rho}_0 = \hat{\rho}_{d,0} \otimes \hat{\rho}_\phi$. After time evolution the final state of the full system will be given by

$$\hat{\rho} = \hat{U}_t \hat{\rho}_0 \hat{U}_t^\dagger.$$  \hfill (8)

We can obtain the evolved state of the detector by tracing out the field degrees of freedom. Up to second order in $\lambda$, we get

$$\hat{\rho}_d = \hat{\rho}_{d,0} + \hat{\rho}^{(1)}_d + \hat{\rho}^{(2)}_d + \mathcal{O}(\lambda^3),$$  \hfill (9)

where

$$\hat{\rho}^{(1)}_d = \text{tr}_\phi \left( \hat{U}_1 \hat{\rho}_0 \hat{U}_1^\dagger \right),$$

$$\hat{\rho}^{(2)}_d = \text{tr}_\phi \left( \hat{U}_2 \hat{\rho}_0 \hat{U}_1^\dagger + \hat{\rho}_0 \hat{U}_1^\dagger \hat{U}_1 \right).$$  \hfill (10)

where $\text{tr}_\phi$ denotes the trace over the field degrees of freedom.

At leading order, the excitation probability for a detector initially in the ground state and a field in an arbitrary state $\hat{\rho}_\phi$ is given by

$$p_{g\rightarrow e} = \text{tr}(\hat{\rho}_d | e\rangle \langle e |) = \lambda^2 \int dVdV' \Lambda(x) \Lambda(x') e^{i\Omega(\tau-\tau')} \langle \hat{\phi}(x') | \hat{\phi}(x) \rangle \hat{\rho}_\phi.$$  \hfill (11)

The excitation probability of the detector allows for the study of numerous features of quantum field theories, such as particle production by numerous effects, including Hawking radiation, detector acceleration and external effects in the field.

Further, if the field starts in a Gaussian state with zero mean, the time evolution of the detector coupled to the field will depend on the field only through its Wightman function. For example, if the field starts in the vacuum state $|0\rangle$ defined by $\hat{a}_k |0\rangle = 0$ for all $k$, where $\hat{a}_k$ are the annihilation operators defined in the mode expansion, we get

$$\langle 0 | \hat{\phi}(x') \hat{\phi}(x) | 0 \rangle = \int d^3k \, u_k(x') u_k^*(x).$$  \hfill (12)

These considerations will become useful when we particularize our studies to vacuum entanglement harvesting.

### B. Ground-Ground Entanglement Harvesting Protocol

Now that we have introduced the interaction between one detector and the field, we will investigate the entanglement harvesting protocol \[15\]--\[19\]. In this protocol, instead of one, we must couple (at least) two particle detectors to the field. Our main interest is to analyze the harvesting of quantum correlations between space-like separated detectors.

Let us consider two detectors undergoing timelike trajectories $z_i(\tau_i)$. Each of the two detectors interact with the field as in Eq. (4). That is, in the interaction picture, the interaction Hamiltonian weight will then be given by

$$\hat{h}_{I,i}(x) = \lambda_i \Lambda_i(x) \mu_i(\tau) \hat{\phi}(x),$$  \hfill (13)

where $\lambda_i$ are the detectors respective coupling constants.

The full interaction Hamiltonian weight will then be given by

$$\hat{h}_I(x) = \hat{h}_{I,1}(x) + \hat{h}_{I,2}(x).$$  \hfill (14)

We assume that the field and the detectors have no correlations prior to the interaction. That is, the detectors-field system density operator will be given by $\rho_0 = \hat{\rho}_{d,0} \otimes \hat{\rho}_\phi$, where $\hat{\rho}_{d,0} = \hat{\rho}_{d_1,0} \otimes \hat{\rho}_{d_2,0}$. Here $d$ labels the two-detectors subsystem and $d_i$ labels the $i$-th detector. $\hat{\rho}_0$ will then evolve according to Eq. (6). As previously mentioned, the time evolution operator will depend on a notion of time ordering. Unlike the case where we consider one detector, in this general scenario with two detectors undergoing arbitrary motion, it is even more ambiguous what is the best notion of time ordering since their proper times may be radically different. However, as we discussed above, provided that the detectors start in either the ground or excited state, any notion of time ordering will yield the same result to leading order as shown in \[47\]. Given that there is no reason to pick time ordering with respect to one detector or the other and that to leading order the choice of time parameter for the ordering is irrelevant, we simply denote the time coordinate used to prescribe the time ordering by $t$, so that we can write

$$\hat{U}_t = \mathcal{T}_t \exp \left( -i \int dV \hat{h}_I(x) \right),$$  \hfill (15)

where $\mathcal{T}_t$ denotes the time ordering operator with respect to a parameter $t$, and $\hat{h}_I(x)$ is given by Eq. (14).

To leading order in $\lambda$, Eqs. (6) and (10) hold with $\hat{U}_t$ as in Eq. (15). We then obtain the final state of the two-detectors subsystem by tracing out the field state. If both detectors are initially in their ground state (that is $\hat{\rho}_{d,0} = |g_1 g_2\rangle \langle g_1 g_2 |$), their density operator in the $\{|g_1 g_2\rangle, |g_1 e_2\rangle, |e_1 g_2\rangle, |e_1 e_2\rangle\}$ basis will be

$$\hat{\rho}_d = \left( \begin{array}{cccc} 1 - \mathcal{L}_{11} & \mathcal{L}_{12} & \mathcal{L}_{13} & \mathcal{L}_{14} \\ -\mathcal{L}_{21} & 1 - \mathcal{L}_{22} & \mathcal{L}_{23} & \mathcal{L}_{24} \\ -\mathcal{L}_{31} & -\mathcal{L}_{32} & 1 - \mathcal{L}_{33} & \mathcal{L}_{34} \\ \mathcal{L}_{41} & \mathcal{L}_{42} & \mathcal{L}_{43} & 1 - \mathcal{L}_{44} \end{array} \right) + \mathcal{O}(\lambda^3),$$  \hfill (16)

$^3$ Here $\tau$ denotes the time-like Fermi normal coordinates associated with the trajectory $z(\tau)$. This is important for spatially smeared detectors and in fact $\tau = \tau(x)$ extends the notion of its proper time to a local region around the curve. More details can be found in \[16\]--\[17\].
where
\[
\mathcal{L}_{ij} = \lambda_i \lambda_j \int dV dV' \Lambda_i(x) \Lambda_j(x') e^{i \Omega_1 \tau_1 - i \Omega_2 \tau_2} \langle \hat{\phi}(x') \hat{\phi}(x) \rangle_{\hat{\rho}_0},
\]
\[
\mathcal{M} = -\lambda_1 \lambda_2 \int dV dV' \theta(t - t') \left( A_1(x) A_2(x') e^{i \Omega_1 \tau_1 + i \Omega_2 \tau_2} \langle \hat{\phi}(x') \hat{\phi}(x) \rangle_{\hat{\rho}_0} + A_2(x) A_1(x') e^{i \Omega_2 \tau_2 + i \Omega_1 \tau_1} \langle \hat{\phi}(x') \hat{\phi}(x) \rangle_{\hat{\rho}_0} \right),
\]
\[
\mathcal{E}_i = \lambda_i \int dV \Lambda_i(x) e^{-i \Omega_i \tau_i} \langle \hat{\phi}(x) \rangle_{\hat{\rho}_0}.
\]
Notice that the only term of the above that depends on the time ordering operation is \( \mathcal{M} \), which can be seen from the dependence on \( \theta(t - t') \). The details of these computations can be found in Appendix A.

There are several common entanglement measures that are typically used in entanglement harvesting. The two most popular are concurrence \( [18] \) and negativity \( [19] \). They both have their advantages and disadvantages. Here we will use the negativity that, unlike concurrence, can be readily used both for qubit detectors and higher dimensional ones (e.g., harmonic oscillator detectors \( [51, 53] \)).

The negativity of a bipartite density operator is defined as the sum of the absolute value of the negative eigenvalues of its partial transpose. In the case of the density operator in Eq. (16), the negativity is given by
\[
\mathcal{N} = \mathcal{N}^{(2)} + \mathcal{O}(\lambda^3),
\]
defined as the sum of the absolute value of the negative eigenvalues of its partial transpose. In the case of the density operator in Eq. (16), the negativity is given by
\[
\mathcal{N}^{(2)} = \max (0, |\mathcal{M} - |\mathcal{E}|^2| + |\mathcal{E}|^2 - \mathcal{L}).
\]

Notice that the only term of the above that depends on the time ordering operation is \( \mathcal{M} \), which can be seen from the dependence on \( \theta(t - t') \). The details of these computations can be found in Appendix A.

### C. Ground-Excited Entanglement Harvesting Protocol

We now consider the case (less frequently studied in the literature) where one detector starts in the excited state, while the other detector starts in the ground state. In this setup we can write the initial state of the detectors-field system, \( \hat{\rho}_0 \) as
\[
\hat{\rho}_0 = |e_1\rangle\langle e_1| \otimes |g_2\rangle\langle g_2| \otimes \hat{\rho}_0 = \hat{\rho}_4 \otimes \hat{\rho}_0.
\]

Performing the computations in the same fashion as we did in the ground-ground scenario, it is possible to obtain the time-evolved density operator associated with the two detectors. The full detail of this computation can be found in Appendix B. The time-evolved density matrix for the two detectors after coupling to the scalar field is
\[
\hat{\rho}_4 = \begin{pmatrix}
\hat{\mathcal{L}}_{11} & 0 & -i \mathcal{E}_1 & \hat{\mathcal{L}}_{12} \\
0 & 0 & -\mathcal{M}' & 0 \\
i \mathcal{E}_1^* & \mathcal{M}'^* & 1 - \hat{\mathcal{L}}_{11} - \hat{\mathcal{L}}_{22} & i \mathcal{E}_2 \\
\hat{\mathcal{L}}_{12} & 0 & -i \mathcal{E}_2^* & \hat{\mathcal{L}}_{22}
\end{pmatrix} + \mathcal{O}(\lambda^3),
\]
where \( \hat{\mathcal{L}}_{ij} \) and \( \mathcal{E}_i \) are given by Eq. (17) and the remaining components are explicitly given by
\[
\hat{\mathcal{L}}_{12} = \lambda^2 \int dV dV' \Lambda_1(x) \Lambda_2(x') e^{-i \Omega_1 \tau_1 - i \Omega_2 \tau_2} \langle \hat{\phi}(x') \hat{\phi}(x) \rangle_{\hat{\rho}_0},
\]
\[
\mathcal{M}' = \lambda^2 \int dV dV' \theta(t - t') \left( A_1(x) A_2(x') e^{i \Omega_1 \tau_1 - i \Omega_2 \tau_2} \langle \hat{\phi}(x) \hat{\phi}(x') \rangle_{\hat{\rho}_0} + A_2(x) A_1(x') e^{i \Omega_2 \tau_2 - i \Omega_1 \tau_1} \langle \hat{\phi}(x) \hat{\phi}(x') \rangle_{\hat{\rho}_0} \right),
\]
\[
\hat{\mathcal{L}}_{11} = \lambda^2 \int dV dV' \Lambda_1(x) \Lambda_1(x') e^{-i \Omega_1 (\tau_1 - \tau_2)} \langle \hat{\phi}(x') \hat{\phi}(x) \rangle_{\hat{\rho}_0}.
\]

When compared to the case of ground-ground initial state, the \( \mathcal{M}' \) terms play the role of the \( \mathcal{M} \) terms and the \( \hat{\mathcal{L}}_{12} \) term play the role of the \( \mathcal{L}_{12} \) terms and \( \hat{\mathcal{L}}_{11} \) represent the deexcitation probability of the first detector. Notice that \( \mathcal{M}' \) has different signs for the phases \( \Omega_1 \tau_1 \) and \( \Omega_2 \tau_2 \), while \( \mathcal{M} \) has the same sign for these phases. This is not surprising since the change \( \Omega_\gamma \rightarrow -\Omega_\gamma \) swaps the ground and the excited states (gap inversion). We can say that the integrand of \( \mathcal{M} \) is ‘faster rotating’, while the integrand of \( \mathcal{M}' \) is ‘slower rotating’. The opposite happens with the \( \hat{\mathcal{L}}_{12} \) terms, which are slower rotating, while the \( \hat{\mathcal{L}}_{12} \) terms from the ground-ground setup are faster rotating. For numerical purposes, faster rotating terms are harder to integrate when compared to slower rotating terms. In particular, this makes the calculation of the harvested entanglement easier when the initial state of the detectors is ground-excited.

The negativity for this choice of initial state is still given by a similar expression to \( (18) \). When the field’s state is such that \( \langle \hat{\phi}(x) \rangle_{\hat{\rho}_0} = 0 \) (hence, \( \mathcal{E}_1 = \mathcal{E}_2 = 0 \)) the
negativity is explicitly given by:
\[ N^{(2)} = \max \left( 0, \sqrt{\left| |\mathcal{M}'|\right|^2 + \frac{(\hat{L}_{11} - \hat{L}_{22})^2}{4} - \frac{\hat{L}_{11} + \hat{L}_{22}}{2}} \right), \]

but—related to the fact that \( \mathcal{M}' \) is slower rotating than \( \mathcal{M} \)—the negativity will be larger than what would be found in the ground-ground protocol.

Although the literature is full of examples of ground-ground entanglement harvesting, the ground-excited protocol is not so commonly analyzed. For this reason, besides considering the general case above we will also go into more explicit detail for two comoving inertial detectors in Minkowski spacetime. Specifically, let us consider the mode expansion of the quantum field to be given by
\[
u_p(x) = \frac{1}{(2\pi)^{\frac{d}{2}}} e^{i p \cdot x} \sqrt{2 \omega_p}, \tag{24} \]
where we assume inertial coordinates with \( x = (t, \mathbf{x}) \) and we write \( p = (\omega_p, \mathbf{p}) \), with \( \omega_p = \sqrt{p^2 + m^2} \), where \( m \) is the field’s mass.

The spacetime smearing functions for the detectors will be prescribed as
\[
\Lambda_1(x) = \chi(t) F(x), \tag{25} \\
\Lambda_2(x) = \chi(t) F(x - L), \tag{26} 
\]
where we split the spacetime smearing into a switching function \( \chi(t) \) and a smearing function \( F(x) \). The choices above assume the detectors to be comoving with the \( (t, \mathbf{x}) \) coordinate system with detector 1 located at the origin and detector 2 located at \( \mathbf{L} \). Without loss of generality, we assume \( \mathbf{L} = L e_z \), where \( e_z \) is the unit vector in the \( z \) direction. We also assume \( \Omega_1 = \Omega_2 = \Omega \), which makes the detectors identical. With these assumptions, we can rewrite the negativity using \( \hat{L}_{11} = \mathcal{L}(-\Omega) \) and \( \hat{L}_{22} = \mathcal{L}(\Omega) \), where
\[
\mathcal{L}(\Omega) = \frac{\lambda^2}{(2\pi)^n} \int \frac{d^d p}{2 \omega_p} \left| \hat{F}(p) \right|^2 \left| \tilde{\chi}(\omega_p + \Omega) \right|^2, \tag{27} \\
\mathcal{M}' = \frac{\lambda^2}{(2\pi)^n} \int \frac{d^d p}{2 \omega_p} \left| \hat{F}(p) \right|^2 e^{i p \cdot \mathbf{x}} L (Q(\Omega - \omega_p) + Q(-\omega_p - \Omega)), \tag{28} 
\]
where \( Q(\omega) \) is defined as
\[
Q(\omega) = \int dt dt' \theta(t - t') e^{i \omega(t - t')} \chi(t) \chi(t'), \tag{29} 
\]
and tilde denotes the Fourier transform, defined as
\[
\tilde{F}(p) = \int d^d x F(x) e^{i p \cdot x}, \quad \tilde{\chi}(\omega) = \int \chi(t) e^{i \omega t}. \tag{30} 
\]

In order to study an explicit example, we prescribe the switching and smearing functions according to
\[
\chi(t) = \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2\sigma^2}}, \tag{31} \\
F(x) = \frac{1}{(2\pi \sigma^2)^{\frac{d}{2}}} e^{-\frac{x^2}{2\sigma^2}}. \tag{32} 
\]
The parameters \( \sigma \) and \( T \) control the size of the interaction region and the duration timescale of the interaction, respectively. With these choices, we have
\[
Q(\omega) = T^2 e^{-\omega^2 T^2} (1 - \text{Erf}(\omega T)), \tag{33} \\
\tilde{\chi}(\omega) = T e^{-\frac{\omega^2}{2T^2}}, \quad \hat{F}(p) = e^{-\frac{p^2}{2T^2}}. \tag{34} 
\]

With the results above, it is possible to find analytical expressions for the integrals over the angular variables, so that \( \mathcal{L}(\Omega) \) and \( \mathcal{M}' \) can be written as a single integral over \( |p| \). We perform the computations in \( n \) dimensions in Appendix C. In the particular case of three space dimensions, we have:
\[
\mathcal{L}(\Omega) = \frac{\lambda^2}{2\pi^2} \int_0^\infty \frac{d|p|}{2 \omega_p} \left| \hat{F}(p) \right|^2 \left| \tilde{\chi}(\omega_p + \Omega) \right|^2, \\
\mathcal{M}' = \frac{\lambda^2}{2\pi^2} \int_0^\infty \frac{d|p|}{2 \omega_p} \left| \hat{F}(p) \right|^2 \left| \tilde{\chi}(\omega_p + \Omega) \right|^2 \left( Q(\omega_p - \Omega) + Q(\omega_p + \Omega) \right). 
\]

At last, the integral over \( |p| \) cannot be solved in terms of elementary functions, so we resort to numerical integration.

In Figs. 1 to 5 we plot the negativity of the two detector system as a function of the detectors separation for different values of the field mass. We fixed the detectors gap as \( \Omega = T \) and the detector size as \( \sigma = 0.1T \).

In Figs. 1 to 5 we plot the negativity of the two detectors state as a function of different parameters of the setup. In Fig. 1 we see the negativity as a function of the detectors’ separation for different values of the field mass. Notice that as \( L \) approaches \( \sigma = 0.1T \) the detectors’ spatial smearing overlap. We observe the following behaviour: The negativity decreases with the detector’s separation, and for sufficiently large masses the more massive the field is, the less entanglement can be extracted.

In Fig. 2 we study the dependence of the entanglement harvested with the field mass \( m \) and the detector gap \( \Omega \) when the detectors separation is given by \( L = 5T \), which guarantees that the spacetime regions of the interactions of the detectors are effectively spacelike separated (10\( \sigma \) to 20\( \sigma \) separation). It is possible to conclude the following:
Figure 2. (Top) Negativity of the two detector system as a function of the detector gap for different values of the field mass. We fixed the detector separation as $L = 5T$ and the detector size as $\sigma = 0.2T$ (this value was chosen to ease the numerical evaluation). (Bottom) Negativity of the two detector system as a function of the field mass for different values of $\Omega$. We fixed the detector separation as $L = 5T$ and the detector size as $\sigma = 0.1T$.

1) As a function of the detector gap, there is a peak in negativity that is controlled by the value of the field mass: the smaller the mass of the field, the higher the negativity peak. Also, for sufficiently small values of $\Omega$, we see a negativity peak for specific values of mass. 2) The less massive a field is, the more entanglement can be harvested. 3) Fields with higher mass allow entanglement to be harvested with smaller detector gaps. The overall behaviour of the negativity is similar to what is expected by the decay of correlations with mass. Notice that it is only possible to harvest entanglement from massless fields if the detector gap is large enough.

In Fig. 3 we analyze the behaviour of the negativity as a function of the detectors gap and size. For sufficiently large $\sigma$, the negativity is monotonically decreasing with $\Omega$. On the other hand, as the detector size decreases, the negativity starts oscillating as a function of $\Omega$. The oscillations can explicitly be seen in the top plot of Fig. 3. These oscillations are associated to the the curves of different $\Omega T$ crossing in the bottom plot of Fig. 3.

Figure 4 shows the negativity as a function of both $L$ and $\Omega$. The region where it is possible to harvest entanglement, as well as the overall behaviour of $N(2)$ presents differences when compared to what is seen in the literature for the ground-ground protocol for entanglement harvesting [15][19]. In the ground-ground case, negativity vanishes for sufficiently small values of $\Omega T$ to be zero for $\Omega$ below a certain threshold, while in the ground-excited case, we see that provided that the detectors are close enough (and definitely not spacelike separated [23]), they are able to get entangled. Finally, in Fig. 5 we consider the detectors interactions to be separated by a time interval $t_0$ and by a distance $L$. Same as in the case of the ground-ground protocol, the most entanglement appears...
when the detectors’ are within each other’s lightcones, although in light contact most of the entanglement is not due to harvesting and rather due to direct communication [30].

\[ - \]

\[ \text{III. THE COMPLEX SCALAR PARTICLE DETECTOR} \]

As shown in [31], it is possible to introduce a modification to the original UDW detector model to study some properties of the anti-particle sector of non-Hermitian fields without giving up on the linearity of the coupling. Consider an \((n+1)\)-dimensional spacetime \(\mathcal{M}\) with a complex scalar quantum field. The field \(\hat{\psi}(x)\) and its Hermitian conjugate, \(\hat{\psi}^\dagger(x)\) can be expanded in terms of a normalized basis of solutions of the Klein-Gordon equation, \(\{u_p(x), u_p^*(x)\}\), according to

\[
\hat{\psi}(x) = \int d^3p \left( u_p(x) \hat{a}_p + u_p^*(x) \hat{a}_p^\dagger \right), \tag{35}
\]

\[
\hat{\psi}^\dagger(x) = \int d^3p \left( u_p^*(x) \hat{a}_p^\dagger + u_p(x) \hat{a}_p \right), \tag{36}
\]

where \(\hat{a}_p^\dagger\) and \(\hat{a}_p\) are the creation and annihilation operators associated with particles and \(\hat{b}_p^\dagger\) and \(\hat{b}_p\) are the creation and annihilation operators associated with the anti-particle sector. Depending whether the field satisfies Bose-Einstein or Fermi-Dirac statistics, these operators will satisfy either canonical commutation relations or canonical anti-commutation relations. To encompass both options in our notation we write

\[
\left[ \hat{a}_p, \hat{a}_p^\dagger \right] = \delta^{(3)}(p-p'), \quad \left[ \hat{b}_p, \hat{b}_p^\dagger \right] = \delta^{(3)}(p-p'),
\]

with all other commutators/anti-commutators vanishing.

Among all the possible couplings of particle detectors to complex scalar fields, to the author’s knowledge, there are two proposals that have been studied in the literature: quadratically [23, 24, 56] and linearly coupled detectors [31]. In both cases, the detector’s quantum system is defined along a timelike trajectory \(z(\tau)\), where \(\tau\) is its proper time parameter.

The quadratic model considers a coupling with a Hermitian field observable that is quadratic on the field. The interaction Hamiltonian weight is prescribed as

\[
\hat{h}_I(x) = \lambda \Lambda(x) \tilde{\mu}(\tau) : \hat{\psi}^\dagger(x) \hat{\psi}(x) :, \tag{37}
\]

where \(\lambda\) is the coupling constant, \(\Lambda(x)\) is the real spacetime smearing function and \(\tilde{\mu}(\tau) = e^{i\Omega \tau} \hat{\sigma}^+ + e^{-i\Omega \tau} \hat{\sigma}^-\) is the detector’s monopole moment. The normal ordering operation in Eq. (37) has been shown to be necessary to regularize spurious divergences [50]. The response of this detector model to different field states and detector motions has been thoroughly studied in the literature [34]. In fact, in [23, 24], it has been shown that it is possible to use a pair of quadratically coupled particle detectors to harvest entanglement from the quantum vacuum.

However, if one is interested in the study of the properties of the field associated to its anti-particle content, this model does not suffice. In fact, a disadvantage of the interaction given by Eq. (37) is that the detector couples equally to the particle and anti-particle sector of the field. This is a direct consequence of the fact that the operator : \(\hat{\psi}^\dagger(x) \hat{\psi}(x) :\) is self-adjoint. This effectively makes the quadratically coupled detector unable to distinguish particles from anti-particles.

In turn, in the linear model, the detector has \(U(1)\) charge and therefore it can couple to the field and its complex conjugate in a \(U(1)\) invariant way [31]. Moreover, the detector couples differently to \(\hat{\psi}(x)\) and \(\hat{\psi}^\dagger(x)\), so that the detector is able to distinguish particle and anti-particle content [31]. The Hamiltonian weight for this particle detector model is given by

\[
\hat{h}_I(x) = \lambda \left( \Lambda^c(x) e^{i\Omega \tau} \hat{\sigma}^+ \hat{\psi}^\dagger(x) + \Lambda^{c*}(x) e^{-i\Omega \tau} \hat{\sigma}^- \hat{\psi}(x) \right), \tag{38}
\]

where \(\Omega\) is the energy gap of the detector, \(\hat{\sigma}^\pm\) are \(SU(2)\) ladder operators and \(\Lambda^c(x)\) is now a complex spacetime smearing function. Once again, we denote the ground and excited states of the detector as \(|g\rangle\) and \(|e\rangle\), so that \(\hat{\sigma}^+ |g\rangle = |e\rangle |g\rangle\) and \(\hat{\sigma}^- |g\rangle = |e\rangle |e\rangle\).

Particle detector models linearly coupled to complex fields have been thought to be unphysical [34], especially due to the apparent break of the \(U(1)\) symmetry present in the free field theory. However, in recent studies [30, 31], it has been shown that these model can be used to approximate the coupling of nucleons with the...
neutrino field. The $U(1)$ symmetry issue is fixed through the introduction of a complex degree of freedom in the detector itself. In fact, the complex smearing function $\Lambda^{(c)}(x)$ must transform according to a $U(1)$ transformation, so that the full theory remains invariant \[31\].

As with all particle detector models, the leading order predictions for a zero-mean Gaussian state of the field will only depend on the field two-point functions. In particular, for the linear complex detector model interacting with the vacuum, the full state of the detector will depend on the following Wightman functions

$$
\langle 0|\hat{\psi}(x')\hat{\psi}^\dagger(x)|0\rangle = \langle 0|\hat{\psi}^\dagger(x')\hat{\psi}(x)|0\rangle = \int d^nu_k(x')u_k^*(x),
$$

(39)

because $\langle 0|\hat{\psi}(x')\hat{\psi}(x)|0\rangle = \langle 0|\hat{\psi}^\dagger(x')\hat{\psi}^\dagger(x)|0\rangle = 0$. Notice that the non-zero two-point functions above are the same as we had in the real scalar case in Eq. (12). In particular, this implies that the excitation probability of a single detector interacting linearly with a complex field in the vacuum state will be the same as if it were interacting linearly with a real scalar field. In fact, this was explicitly shown in \[31\], where these models were compared and the differences only arise when the field is in states that have non-zero particle or anti-particle content.

On the other hand, the entanglement harvesting capability of detectors that are linearly coupled to non-Hermitian fields has not yet been studied in the literature to the author’s knowledge. The model studied in this section is of particular importance, especially due to its similarities with the well-known UDW model. This allows for a comparison of the entanglement that can be harvested from the vacuum for non-Hermitian and real fields that is not polluted by the non-linear nature of the coupling.

IV. ENTANGLEMENT HARVESTING FROM COMPLEX FIELDS WITH LINEARLY COUPLED DETECTORS

In this section we explore an entanglement harvesting setup when two detectors couple to a complex scalar field linearly, according to the interaction Hamiltonian weight in Eq. (38). That is, we consider two anti-particle UDW detectors labelled by $j = 1, 2$ undergoing time-like trajectories $\mathbf{z}_j(\tau_j)$, where $\tau_j$ denotes their respective proper times. The interaction Hamiltonian weights between the detectors and the field can be written as

$$
\hat{H}_{1, j}(x) = \lambda \left( \Lambda_j^{(c)}(x) e^{i\Omega_j \tau_j} \hat{\sigma}^+ \hat{\psi}^\dagger(x) + \Lambda_j^{(c)}(x) e^{-i\Omega_j \tau_j} \hat{\sigma}^- \hat{\psi}(x) \right),
$$

(40)

where $\Omega_j$ is the $j$-th detector’s energy gap, $\hat{\sigma}^\pm$ are the respective two-level raising and lowering operators and $\Lambda_j^{(c)}(x)$ are the spacetime smearing functions associated with each detector. The full interaction Hamiltonian weight that governs the interaction will then be given by the sum of the individual interactions for $j = 1, 2$:

$$
\hat{h}_1(x) = \hat{h}_{1,1}(x) + \hat{h}_{1,2}(x).
$$

(41)

As we mentioned in Section \[11\] where the anti-particle detector model was reviewed, these detectors couple differently to the particle and anti-particle sector of the quantum field theory: the ground state of the detector couples to the anti-particle sector, while the excited state couples to the particle sector. This has meaningful consequences for the entanglement harvesting setup, and motivates the division of our analysis between the case where the detectors are both in the ground state and the case where the detectors start in different states. In the next two subsections we will discuss these two cases in detail.

A. Ground - Ground Protocol

We first assume that both detectors start in their respective ground states. Assuming the field to start in a state $\rho_0$, we can then write the full detectors-field initial state as

$$
\hat{\rho}_0 = |g_1\rangle\langle g_1| \otimes |g_2\rangle\langle g_2| \otimes \hat{\rho}_0 = \hat{\rho}_{4,0} \otimes \hat{\rho}_0.
$$

(42)

Using the time evolution operator of Eq. (5), with the interaction Hamiltonian weight in Eq. (11), it is possible to obtain the final state of the detectors quantum system. We perform the explicit computations in Appendix A and obtain the following density matrix

$$
\hat{\rho}_d = \left( \begin{array}{cccc} I - \mathcal{L}_{11} - \mathcal{L}_{22} & i\mathcal{E}_2 & i\mathcal{E}_1 & \mathcal{M}^* \\ -i\mathcal{E}_2^* & \mathcal{L}_{22} & \mathcal{L}_{21} & 0 \\ -i\mathcal{E}_1^* & \mathcal{L}_{12} & \mathcal{L}_{11} & 0 \\ \mathcal{M} & 0 & 0 & \mathcal{M} \end{array} \right) + \mathcal{O}(\lambda^3), \quad (43)
$$

where the matrix elements are given by

$$
\mathcal{L}_{ij} = \lambda^2 \int dV dV' e^{i\Omega_1 \tau_1 - i\Omega_2 \tau_2} \langle \hat{\psi}_i(x')\hat{\psi}_j^\dagger(x) \rangle_{\rho_0},
$$

$$
\mathcal{M} = -\lambda^2 \int dV dV' \theta(t - t') \left( e^{i\Omega_1 \tau_1 + i\Omega_2 \tau_2} \langle \hat{\psi}_i^\dagger(x)\hat{\psi}_j^\dagger(x') \rangle_{\rho_0} + e^{i\Omega_2 \tau_2 + i\Omega_1 \tau_1} \langle \hat{\psi}_i(x)\hat{\psi}_j(x') \rangle_{\rho_0} \right),
$$

$$
\mathcal{E}_i = \lambda \int dV e^{i\Omega_1 \tau_1} \langle \hat{\psi}_i(x) \rangle_{\rho_0},
$$

(44)

and we have defined

$$
\hat{\psi}_i(x) := \Lambda_j^{(c)}(x)\hat{\psi}(x), \quad \hat{\psi}^\dagger_i(x) := \Lambda_j^{(c)}(x)\hat{\psi}^\dagger(x). \quad (45)
$$

Notice the similarity with the case of the standard UDW model that couples to a real scalar field (Eq. (17)). Namely, with the replacement $\hat{\psi}(x) \rightarrow \hat{\phi}(x)$ and assuming $\Lambda_j^{(c)}(x)$ to be real, we recover the exact expressions from Subsection \[11\]. In particular, the negativity of the state
\(\hat{\rho}_0\) is given by the same expression that we had for the scalar case in Eq. \((18)\), and will only be non-zero if the \(\mathcal{M}\) component is positive and larger than the product \(\mathcal{L}_{11}\mathcal{L}_{22}\).

Despite this similarity, there are stark differences in the ability of the detector to harvest entanglement in the real and the complex scalar field. Indeed, for a complex scalar field we have

\[
(\hat{\psi}(x)\hat{\psi}(x'))_0 = (\hat{\psi}^\dagger(x)\hat{\psi}^\dagger(x'))_0 = 0. \tag{46}
\]

In particular, this implies that, to leading order, a pair of detectors that start in the ground state cannot harvest entanglement from the vacuum of a complex scalar field when linearly coupled according to the interaction in Eq. \((40)\). This can be intuitively understood by noticing that for charge conjugation invariant states, the field correlations appear only between the particle and anti-particle sectors. When both detectors start in the ground state, they couple only to the anti-particle sector, which does not contain self-correlations as per Eq. \((46)\). In fact, as we will see in the next Subsection, when one detector couples to the particle content and the other one couples to the anti-particle content of the field, it is possible to extract entanglement from the vacuum state. Notice that this ground-ground inability to harvest entanglement from the charge conjugation-invariant states at leading order is quite general and holds in curved space-times and independently of the Hilbert space representation of the field.

Finally, notice that although we have restricted our analysis to the case where both detectors start in the ground state, the same would apply to the case where both detectors start in the excited state. In fact, as discussed in [31], the linear complex scalar particle detector respects the symmetry \(\Omega \mapsto -\Omega\) when the field is in a charge conjugation invariant state, such as the vacuum. As a conclusion, if both detectors start in the ground state, the same would apply to the case where \(\mathcal{M}_2^\prime\) term is slow rotating, which simplifies the numerical evaluation of the negativity.

Specifically, the negativity for the case where \(\mathcal{E}_2 = \mathcal{E}_2^\prime = 0\) is given by Eq. \((23)\), where now \(\mathcal{M}_2^\prime = \mathcal{L}_{11}^\prime\) and \(\mathcal{L}_{12}^\prime\) are given by Eq. \((49)\). Namely,

\[
\mathcal{N}_2 = \max \left(0, \sqrt{|\mathcal{M}_2^\prime|^2 + \frac{(\mathcal{L}_{11} - \mathcal{L}_{22})^2}{4} - \frac{\mathcal{L}_{11} + \mathcal{L}_{22}}{2}} \right), \tag{50}
\]

where it should be noted that the \(\mathcal{M}_2^\prime\) term now involves different field correlators that couple the particle and anti-particle sector of the field. In particular, using the fact that the complex and scalar correlators in the vacuum state are the same (Eq. \((39)\)), it is easy to see that this detector model can harvest as much entanglement as the detector that couples to the real scalar field, since the \(\mathcal{E}_{ij}\) and \(\mathcal{M}_2^\prime\) terms are the same, and therefore, so is the negativity.

At last, notice that the explicit examples of Subsection III C yield the same results as they would for the linear complex particle detector models when choosing \(\Lambda(x) = \Lambda(x)\).

\[
\langle 0|\hat{\psi}(x')\hat{\phi}^\dagger(x)|0\rangle = \langle 0|\hat{\psi}^\dagger(x)\hat{\psi}(x)|0\rangle = \langle 0|\hat{\phi}(x')\hat{\phi}(x)|0\rangle = 0. \tag{51}
\]

That is, all conclusions drawn from the explicit example in Subsection III C are also valid for the linear complex particle detector models when choosing \(\Lambda(x) = \Lambda(x)\).

V. THE FERMIONIC PARTICLE DETECTOR

In this section we review the physical motivation behind the fermionic particle detector model introduced
in \[30\]. We then consider the more general fermionic detector that was studied in \[31\], and use it in an entanglement harvesting setup.

A. The Fermi Interaction

In \[30\], it was shown that the interaction of nucleons with the neutrino fields through a four-fermion interaction can be modelled by a particle detector model. The four-fermion theory effectively describes the interaction of neutrons, protons, electrons and neutrinos by treating these as spinor fields \(n, p, e\) and \(\nu_e\), respectively. The interaction is prescribed in terms of the Lagrangian density

\[
\mathcal{L}_{4F} = -2\sqrt{2} G_F \left( \bar{\nu}_e \gamma^\mu P_L e \right) (\bar{n} \gamma_\mu P_L p) + \text{H.c.}, \tag{52}
\]

where \(G_F \approx 1.16 \times 10^{-5} \text{ GeV}^{-2}\) is the Fermi constant and \(P_L\) denotes the projector in the left-handed component of the spinors, where the weak interaction takes place. This model is especially useful for the description of the \(\beta^-\)-decay, where a neutron decays into a proton, electron and anti-neutrino,

\[
n \rightarrow p + e^- + \bar{\nu}_e. \tag{53}
\]

The procedure to reduce the model from Eq. (52) to a localized particle detector comes in two steps. First we notice that the neutron and proton masses are much larger than both the electron and neutrino masses. Using the fact that the neutron and proton are always coupled to each other in the 4 fermion Lagrangian, we then replace their sector by a single two-level system, according to

\[
\bar{n} \gamma_\mu P_L p \rightarrow j_\mu(x) e^{i\Delta M \tau} |n\rangle |p\rangle, \tag{54}
\]

where we denote the neutron and proton states by \(|n\rangle\) and \(|p\rangle\), respectively. In Eq. (54), \(\Delta M\) is the mass difference of the neutron and proton and \(j^\mu(x)\) is a hadronic current, that gives the spatial profile of the neutron/proton system.

The next step is to notice that the electron mass is of the order of \(10^3\) times larger than the neutrino mass, so that the electrons involved in the process are considered to be localizable in terms of their interaction with the neutrino field (the interaction happens in a finite region of spacetime). We then make the following replacement in Eq. (52).

\[
P_L \hat{e} \rightarrow u(x) e^{-i\omega_e \tau} \hat{a}_e, \tag{55}
\]

where \(\omega_e\) is the energy scale of the electrons involved in the specific process considered, \(u(x)\) is an effective mode associated with the spatial profile of the electron and \(\hat{a}_e\) is the creation operator associated with the electron field that participates in the interaction.

With the two replacements of Eqs. (54) and (55) in Eq. (52), we end up with the following effective Lagrangian,

\[
\mathcal{L}_{4F} = -\frac{G_F}{\sqrt{2}} \left( e^{i(\Delta M - \omega_e) \tau} \bar{\nu}_e f(x) u(x) \hat{a}_e |n\rangle |p\rangle + e^{-i(\Delta M - \omega_e) \tau} \bar{u}(x) f(x) \nu_e \hat{a}_e^\dagger |p\rangle |n\rangle \right). \tag{56}
\]

Now, defining the ladder operators \(\hat{\sigma}^+ = \hat{a}_e |n\rangle |p\rangle\) and \(\hat{\sigma}^- = \hat{a}_e^\dagger |p\rangle |n\rangle\), we obtain a two-level system, whose excited state consists of a neutron and whose deexcited state consists of a proton and an electron. This two-level system is then localized by the spatial profile defined by the spinor \(f(x) u(x)\) and couples linearly to the neutrino field \(\nu(x)\). A more detailed discussion of this reduction can be found in the works \[30\], \[31\].

In terms of the raising and lowering operators \(\hat{\sigma}^-\) and \(\hat{\sigma}^+\), one obtains the following interaction Hamiltonian weight:

\[
\hat{h}_I(x) = 2\sqrt{2} G_F \left( e^{i\Omega \tau} \bar{\nu}_e \Lambda f(x) \sigma^+ + e^{-i\Omega \tau} \Lambda^\dagger f(x) \nu_e \sigma^- \right), \tag{57}
\]

where \(\Omega = \Delta M - \omega_e\) is the effective energy gap and \(\Lambda f(x) = f(x) u(x)\) is the spacetime smearing spinor, responsible for controlling both the spatial localization and time duration of the interaction. Effectively, we obtain a two-level fermionic particle detector model that interacts linearly with the neutrino field. For more details regarding this particle detector model and its symmetries, we refer the reader to the discussion in Subsection II B of \[31\].

It is important to notice that in this model the ground and excited states couple to different sectors of the neutrino field: the deexcitation of this detector happens through the emission of an anti-particle or absorption of a particle, while the excitation of the detector happens through the emission of a particle, or absorption of an anti-particle. In terms of the effective two-level model, the excited state couples to the particle sector, while the ground state couples to the anti-particle sector. This is a similar feature to that of the linear complex scalar detector model presented in Section III.

B. A particle detector probing a spin 1/2 field

We consider a fermionic quantum field \(\hat{\Psi}(x)\) in a \((3 + 1)\)-dimensional spacetime. We assume to have a basis of solutions of Dirac’s equations split under the typical convention into so-called positive and negative frequency solutions, \(\{u_{p,s}(x), \bar{v}_{p,s}(x)\}\), so that the fermionic field can be written as

\[
\hat{\Psi}(x) = \sum_{s=1}^2 \int d^3 p \left( u_{p,s}(x) \hat{a}_{p,s} + \bar{v}_{p,s}(x) \hat{b}_{p,s}^\dagger \right), \tag{58}
\]

\[
\hat{\Psi}(x) = \sum_{s=1}^2 \int d^3 p \left( \bar{u}_{p,s}(x) \hat{a}_{p,s}^\dagger + \bar{v}_{p,s}(x) \hat{b}_{p,s} \right). \tag{59}
\]
where \( \hat{\Psi}(x) \) is the conjugate spinor to \( \Psi(x) \) and \( \hat{a}_{p,s} \) and \( \hat{a}_{p,s}^\dagger \) are the annihilation operators associated with particles and anti-particles, respectively. They satisfy the fermionic canonical anti-commutation relations
\[
\{ \hat{a}_p, \hat{a}_{p'}^\dagger \} = \delta^{(3)}(p - p'), \quad \{ \hat{b}_p, \hat{b}_{p'}^\dagger \} = \delta^{(3)}(p - p'), \\
\{ \hat{a}_p, \hat{a}_{p'} \} = 0, \quad \{ \hat{b}_p, \hat{b}_{p'} \} = 0, \quad (60)
\]
\[
\{ \hat{a}_p^\dagger, \hat{a}_{p'} \} = 0, \quad \{ \hat{b}_p^\dagger, \hat{b}_{p'} \} = 0.
\]

In order to generalize and simplify the linear fermionic particle detector presented in the previous subsection, we consider a particle detector with spinor spacetime smearing function \( \Lambda^f_j(x) \) coupled to a general spinor field \( \Psi(x) \) via the following interaction Hamiltonian weight:
\[
\hat{h}_I(x) = \lambda \left( e^{i\Omega_j \tau^j} \hat{\Psi}(x) \Lambda^f_j(x) + e^{-i\Omega_j \tau^j} \bar{\Lambda}^f_j(x) \hat{\Psi}(x) \right).
\]

Notice that the Hamiltonian weight above is very similar to that of the linear complex particle detector. In fact, Eq. (61) follows from Eq. (38) with the substitution
\[
\hat{\psi}_j^\dagger(x) \Lambda^{(c)}(x) \mapsto \hat{\Psi}(x) \Lambda^f_j(x),
\]
where it should be understood that the spacetime smearing \( \Lambda^{(c)}(x) \), which is a complex function on the left-hand side, gets replaced by the spinor function \( \Lambda^f_j(x) \) on the right-hand side. We will use this fact to easily obtain analytical expressions for the linear fermionic detector.

VI. ENTANGLEMENT HARVESTING FROM A FERMIONIC FIELD WITH LINEARLY COUPLED DETECTORS

We now turn our attention to the setup of entanglement harvesting. For this purpose, we consider two fermionic detectors undergoing trajectories \( z_j(t_j) \), \( j = 1, 2 \), with energy gaps \( \Omega_j \), so that the interaction Hamiltonian weight associated with each of the two detectors is
\[
\hat{h}_{I,j}(x) = \lambda \left( e^{i\Omega_j \tau^j} \hat{\Psi}(x) \Lambda^{f}_j(x) + e^{-i\Omega_j \tau^j} \bar{\Lambda}^{f}_j(x) \hat{\Psi}(x) \right),
\]

where \( \Lambda^{f}_j(x) \) is the spacetime smearing spinor field associated with the \( j \)-th detector. The total interaction Hamiltonian weight is therefore
\[
\hat{h}_I(x) = \hat{h}_{I,1}(x) + \hat{h}_{I,2}(x).
\]

Given that all analytical expressions for the fermionic particle detector can be obtained from the results of Section IV upon the replacement \( \Lambda \rightarrow \Lambda^f \), the differences between the linearly coupled scalar complex field harvesting and the fermionic case will come from the corresponding two-point functions. Same as before, due to the fact that

in the vacuum state \( |0\rangle \),
\[
\langle 0 | \hat{\Psi}(x) \hat{\Psi}'(x') | 0 \rangle = 0,
\]
\[
\langle 0 | \hat{\Psi}(x) \hat{\Psi}'(x') | 0 \rangle = 0,
\]

we see again that if the two detectors start in the ground state (or both in the excited state), they will be unable to harvest entanglement from the vacuum of the fermionic field. The same is true for any charge-conjugation invariant state. The reason for this is the same that we had for the linear coupled complex scalar fields: the anti-particle sector is only correlated with the particle sector.

We are interested in harvesting entanglement, thus we will focus on the case where detector 1 starts in the ground state, while detector 2 starts in the excited state. In terms of the analogy with the physical model presented in Subsection VA, we would have one of the nucleons starting in the proton state, while the other starts in the neutron state. The density matrix associated with this process is then given by Eq. (48), and its components are given by Eq. (49), where the term \( \tilde{\Psi}_j(x) \) is given by
\[
\tilde{\Psi}_j(x) = \bar{\Lambda}^{f}_j(x) \hat{\Psi}(x).
\]

Notice that while the amount of entanglement that could be harvested in the complex scalar case was the same as the real scalar case, the fact that the Fermionic two-point functions are different imply noticeable changes in entanglement harvesting. In fact, the Wightman matrices for the fermionic field are given by
\[
W_0(x, x') = \langle 0 | \hat{\Psi}(x) \hat{\Psi}'(x') | 0 \rangle = \sum_{s=1}^{2} \int d^3p \, u_{p,s}(x) \bar{u}_{p,s}(x'),
\]
\[
\tilde{W}_0(x, x') = \langle 0 | \tilde{\Psi}(x) \tilde{\Psi}'(x') | 0 \rangle = \sum_{s=1}^{2} \int d^3p \, v_{p,s}(x') \bar{v}_{p,s}(x).
\]

In order to explicitly compare the difference between the entanglement harvested from complex scalar fields and fermionic fields, we consider an explicit example in flat spacetime. We choose the following basis of solutions to Dirac’s equations in Minkowski spacetime:
\[
u_{p,1}(x) = \frac{1}{(2\pi)^{3/2}} \sqrt{\frac{\omega^2 + m^2}{2\omega_p}} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ \frac{p_z m}{\omega_p + m} \frac{p_z m}{\omega_p + m} \end{pmatrix} e^{ip \cdot x},
\]
\[
u_{p,2}(x) = \frac{1}{(2\pi)^{3/2}} \sqrt{\frac{\omega^2 + m^2}{2\omega_p}} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ \frac{p_z m}{\omega_p} \frac{p_z m}{\omega_p} \end{pmatrix} e^{ip \cdot x},
\]
\[
u_{p,1}(x) = \frac{1}{(2\pi)^{3/2}} \sqrt{\frac{\omega^2 + m^2}{2\omega_p}} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} \frac{p_z m}{\omega_p + m} \frac{p_z m}{\omega_p + m} \end{pmatrix} e^{-ip \cdot x},
\]
\[ v_{p,2}(x) = \frac{1}{(2\pi)^2} \sqrt{\frac{\omega_p + m}{2\omega_p}} \left( \begin{array}{c} \frac{p_x - ip_y}{\omega_p + m} \\ \frac{\omega_p + m}{\omega_x} \\ 0 \\ 1 \end{array} \right) e^{-ipx}. \]  

(72)

This basis is normalized so that the canonical anti-commutation relations from Eqs. hold. These also satisfy the completeness relations

\[ \sum_{s=1}^{2} u_{p,s}(x)u_{p,s}(x') = \frac{1}{(2\pi)^3} \frac{e^{ip(x-x')}}{2\omega_p}(\rho + m), \]  

(73)

\[ \sum_{s=1}^{2} v_{p,s}(x)v_{p,s}(x) = \frac{1}{(2\pi)^3} \frac{e^{ip(x-x')}}{2\omega_p}(\rho - m). \]  

(74)

We pick the spacetime spinor smearing functions to be

\[ \mathcal{L}(\Omega) = \frac{\lambda^2}{(2\pi)^3} \int \frac{d^3p}{2\omega_p} \hat{\eta}_i(\rho + m)\eta_j|\tilde{F}(p)|^2|\tilde{\chi}(\omega_p + \Omega)|^2, \]  

\[ \mathcal{M}' = \frac{\lambda^2}{(2\pi)^3} \int \frac{d^3p}{2\omega_p} |\tilde{F}(p)|^2 e^{ip\cdot L} (\eta_1(\rho + m)\eta_2Q(-\omega_p - \Omega) + \bar{\eta}_1(\rho - m)\eta_2Q(\Omega - \omega_p)). \]  

(78)

The function \( Q(\omega) \) is given by Eq. 28, and the tilde denotes Fourier transform, as in Section IV.

We can now study the differences between harvesting from complex scalar fields and from fermionic fields with linear couplings. The major difference is related to the terms that involve \( \eta_i(\rho \pm m)\eta_j \), which show up in the integrals above. A consequence of this is the fact that the detectors can, in principle, have different transition probabilities depending on the choices of \( \eta_1 \) and \( \eta_2 \). The last noticeable difference is the units of the coupling constant in this model, which was dimensionless in Section III but has dimension of \( [E]^{-2} \) here.

In order to compute explicit examples, we must choose the spinors \( \eta_1 \) and \( \eta_2 \). We make the following choice:

\[ \eta_1 = \eta_2 = \begin{pmatrix} \Delta \\ 0 \\ 0 \\ 0 \end{pmatrix}. \]  

(79)

In the case of the interaction of nucleons with the neutrino fields, \( \Delta \) would be associated with the energy scale of the electrons involved in the interaction. In essence, the choices of \( \eta_1 \) and \( \eta_2 \) are associated with the electron states involved in the interaction with the neutrino field. The assumption that \( \eta_1 = \eta_2 \) above then indicates that the electron states involved in both processes are the same. This choice should generally increase the correlations between the detectors by making them associated to processes that involve electrons with the same energy and spin. In fact, Eq. 79 picks both electron states to have the spin in the positive \( z \) direction in the chosen coordinate system.

With the choices from Eq. 79, we obtain

\[ \eta_1\eta_2 = \Delta^2 \omega_p = \Delta^2 \sqrt{p^2 + m^2}, \]  

\[ \eta_1 m\eta_2 = \Delta^2 m. \]  

(80)

where \( \Delta \) is a constant with dimensions of \( [E]^{-2} \) that takes into account the units of \( \eta_1, \eta_2 \). The results above allow one to solve Eq. 78 analytically in the angular variables, so that once more we are left with one integral with respect to \( p \) that can be done numerically.

In Figs. 6, 8, 9 and 10 we plot the negativity of the detectors state as a function of the different parameters of the setup. We choose the same parameters that we did in the case of the scalar model in Subsection II C such that we can establish a direct comparison between the pairs of Figures 6 and 7, and 2 and 8 and 3 and 9 and 4 and 10 and 5.

In Fig. 6 we plot the negativity as a function of the detectors separation for different masses of the field. Overall, we see that the negativity decays rapidly with \( L \), same as in the scalar case. Analogously, the more mas-
The negative field is, the less entanglement can be harvested from the field.

In Fig. 7, we study the behaviour of the negativity of the detectors’ state as a function of their detectors gaps and the field mass when the detectors are separated by a distance $L = 5T$, which makes the effective interaction regions of the detectors spacelike separated with more than $10\sigma$ of separation. In the top subplot of Fig. 7, the negativity is plotted as a function of the detectors gap for varying values of the field mass. Unlike in the scalar case, if the field’s mass is small enough, we see oscillations, with no monotonic behaviour. In the bottom subplot, the negativity of the detectors state is plotted as a function of the field’s mass for different detectors energy gaps.

In Fig. 8, we plot the analogue of Fig. 3, where the behaviour of the negativity of the detectors state is studied as a function of the detectors gap $\Omega$ for different values of $m$. We fixed the detector separation as $L = 5T$ and the detector mass as $mT = 1$. (Bottom) Negativity of the two detector system as a function of the detector size $\sigma$ for different values of $\Omega$. We fixed the detector separation as $L = 5T$ and the detector mass as $mT = 1$. These are analogous to the plots in Fig. 3.

In Fig. 9, we fixed $m = 0$ and considered pointlike detectors with $\sigma = 0$. This is analogous to the plot in Fig. 4.

In Fig. 6, we fixed the detector gap as $\Omega T = 1$ and the detector size as $\sigma = 0.1T$. This is analogous to the plot in Fig. 1.

In Fig. 7, we fixed the detector separation as $L = 5T$ and the detector size as $\sigma = 0.2T$. (Bottom) Negativity of the two detector system as a function of the detector gap for different values of the field mass. We fixed the detector separation as $L = 5T$ and the detector size as $\sigma = 0.1T$. These are analogous to the plots in Fig. 2.

In Fig. 8, we plot the analogue of Fig. 3, where the behaviour of the negativity of the detectors state is studied as a function of the detectors gap $\Omega$ for different values of $\sigma$. We fixed the detector separation as $L = 5T$ and the detector mass as $mT = 1$. (Bottom) Negativity of the two detector system as a function of the detector size $\sigma$ for different values of $\Omega$. We fixed the detector separation as $L = 5T$ and the detector mass as $mT = 1$. These are analogous to the plots in Fig. 3.

In Fig. 9, we fixed $m = 0$ and considered pointlike detectors with $\sigma = 0$. This is analogous to the plot in Fig. 4.
for varying detector gaps and detector sizes. We conclude that the oscillations are stronger in the fermionic case as compared to the scalar case. In fact, in the scalar case, we could only see oscillations for detector sizes smaller than 0.15T (Fig. 3), while we see drastic oscillations in the fermionic case when the detector size is 0.2T. Also notice that the amplitude of the oscillations is suppressed by the detector’s size. Same as in the scalar case, mass decreases the amount of entanglement that can be harvested, but allows one to harvest using detectors with smaller energy gaps. Overall, in the fermionic case, we notice an important changes of behaviour as compared to the scalar case: The oscillations of the negativity as a function of Ω appear for all values of σ.

Figure 10 shows the negativity as a function of L and Ω. Comparing this plot to Fig. 4, we see the same oscillations as in Fig. 7. The region where entanglement can be extracted is also larger in the fermionic case than in the scalar case. Finally, in Fig. 10 we consider the situation where the centers of the detectors’ interactions are separated in time by τ₀ and in space by L. Same as in the scalar case, we observe that acquired entanglement peaks near the lightcone, although the peak most likely corresponds to entanglement acquired through communication rather than harvesting.

Overall, it is possible to identify similarities and differences between the complex scalar and the fermionic field. In both cases entanglement rapidly decreases with the detectors’ separation and with the field’s mass. However, the oscillatory behaviour is significantly more present in the fermionic case. This structure could potentially be exploited to optimize the extraction of correlations in possible fermionic field experiments and outperform its scalar analogue.

VII. CONCLUSIONS

We have studied entanglement harvesting with particle detector models that couple linearly to complex scalar and fermionic fields. These detector models are inspired by realistic high-energy physics processes. By using linear models we can avoid the persistent divergences that appear when studying entanglement harvesting with quadratic couplings. We have formally computed the density matrix of pairs of detectors moving on arbitrary trajectories in curved spacetimes when the field is in a general state. Then, we explicitly solved examples of comoving inertial detectors in Minkowski spacetime. In that scenario we quantified the amount of entanglement acquired by two linearly coupled detectors through the negativity of the detectors state after the interaction with the field. This allowed us to compare the fermionic and complex scalar models. When these particle detectors were compared, we found similarities as well as fundamental differences between the complex scalar and the fermionic case.

For complex scalar and fermionic detector models, we have discussed how the ground state only couples to the anti-particle sector of the field. Hence, two detectors that both start in the ground or excited state are unable to harvest entanglement from the vacuum state of a complex/fermionic quantum field. This result holds independently of the detectors trajectories, shape of the interaction or quantization scheme. We showed that this originates from the fact that all the correlations of non-Hermitian fields manifest through correlations of the particle and anti-particle sectors. On the other hand, if one detector starts in the ground state, while the other starts in the excited state, we showed that linearly coupled complex as well as fermionic particle detectors can harvest entanglement from the vacuum of a quantum field. Moreover, comparing with the known results for real scalar fields, we showed that two complex scalar detector models can harvest the same amount of entanglement as two UDW detectors coupled to a real scalar field.

The main difference between the fermionic case and the complex scalar at leading order in perturbation theory comes from the oscillations of the harvested negativity as a function of the detector’s gap and the separation between detectors. While the overall qualitative behaviour of complex scalars and fermions is similar, the presence of these oscillations could in principle be exploited to optimize the amount of harvested entanglement when working with fermionic setups.

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Appendix A: Computations regarding the Ground - Ground Interaction of the Linear Complex Detector

Here we present the details of the computations regarding the general entanglement harvesting setup described in Section II B. For the ground-ground initial state, the initial state density operator is

\[
\hat{\rho}_0 = |g_1\rangle\langle g_1| \otimes |g_2\rangle\langle g_2| \otimes \hat{\rho}_\phi = \hat{\rho}_{d,0} \otimes \hat{\rho}_\phi. \tag{A1}
\]

Using the second order Dyson expansion for the time evolution operator in Eq. (15), we obtain

\[
\hat{U}_I = \mathcal{T} \exp \left( -i \int dV \hat{h}_I(x) \right) = \mathbf{1} + \hat{U}_I^{(1)} + \hat{U}_I^{(2)} + \mathcal{O}(\lambda^3), \tag{A2}
\]

where

\[
\hat{U}_I^{(1)} = -i \int dV \hat{h}_I(x), \tag{A3}
\]

\[
\hat{U}_I^{(2)} = - \int dV dV' \hat{h}_I(x) \hat{h}_I(x') \theta(t - t'), \tag{A4}
\]

and

\[
\hat{h}_I(x) = \lambda \left( \left( \Lambda_1(x) e^{i\Omega_1 \tau_1} \hat{\sigma}_1^+ \hat{\psi}^\dagger(x) + \Lambda_1^\dagger(x) e^{-i\Omega_1 \tau_1} \hat{\sigma}_1^- \hat{\psi}(x) \right) + \left( \Lambda_2(x) e^{i\Omega_2 \tau_2} \hat{\sigma}_2^+ \hat{\psi}^\dagger(x) + \Lambda_2^\dagger(x) e^{-i\Omega_2 \tau_2} \hat{\sigma}_2^- \hat{\psi}(x) \right) \right). \tag{A5}
\]

The time evolved density operator for the full system consisting of the two-detectors and the field then reads

\[
\hat{\rho} = \hat{U}_I \hat{\rho}_0 \hat{U}_I^\dagger. \tag{A6}
\]

We then find that, up to second order in \( \lambda \), the detector density operator is given by

\[
\hat{\rho}_d = \text{tr}_\phi(\hat{\rho}) = \hat{\rho}_{d,0} + \hat{\rho}_d^{(1)} + \hat{\rho}_d^{(1,1)} + \hat{\rho}_d^{(2,0)} + \hat{\rho}_d^{(0,2)}, \tag{A7}
\]

where

\[
\hat{\rho}_d^{(1)} = \text{tr}_\phi \left( U_I^{(1)} \hat{\rho}_0 + \hat{\rho}_0 U_I^{(1)} \right), \tag{A8}
\]

\[
\hat{\rho}_d^{(1,1)} = \text{tr}_\phi \left( U_I^{(1)} \hat{\rho}_0 U_I^{(1)} \right) = \int dV dV' \left\langle \hat{h}_I(x) \hat{\rho}_{d,0} \hat{h}_I(x') \right\rangle_{\hat{\rho}_\phi}, \tag{A9}
\]

\[
\hat{\rho}_d^{(2,0)} = \text{tr}_\phi \left( U_I^{(2)} \hat{\rho}_0 \right) = - \int dV dV' \theta(t - t') \left\langle \hat{h}_I(x) \hat{h}_I(x') \right\rangle_{\hat{\rho}_{d,0}}, \tag{A10}
\]

\[
\hat{\rho}_d^{(0,2)} = \text{tr}_\phi \left( \hat{\rho}_0 U_I^{(2)} \right) = - \int dV dV' \theta(t - t') \hat{\rho}_{d,0} \left\langle \hat{h}_I(x) \hat{h}_I(x') \right\rangle_{\hat{\rho}_\phi}. \tag{A11}
\]

Let us compute each of the above terms separately. The first order terms in the first parenthesis yields

\[
\hat{\rho}_d^{(1)} = -i\lambda \int dV \left( \Lambda_1(x) e^{i\Omega_1 \tau_1} \langle \hat{\psi}^\dagger(x) \rangle_{\hat{\rho}_\phi} |g_1 g_2\rangle\langle g_1 g_2| + \Lambda_2(x) e^{i\Omega_2 \tau_2} \langle \hat{\psi}^\dagger(x) \rangle_{\hat{\rho}_\phi} |g_1 g_2\rangle\langle g_1 g_2| - \Lambda_1^\dagger(x) e^{-i\Omega_1 \tau_1} \langle \hat{\psi}(x) \rangle_{\hat{\rho}_\phi} |g_1 g_2\rangle\langle g_1 g_2| - \Lambda_2^\dagger(x) e^{-i\Omega_2 \tau_2} \langle \hat{\psi}(x) \rangle_{\hat{\rho}_\phi} |g_1 g_2\rangle\langle g_1 g_2| \right). \tag{A12}
\]
Denoting \( \hat{\rho}^{(i,k)} = \hat{U}^{(i)}_t \hat{\rho}_0 \hat{U}^{(k)}_t \), we obtain the first term in the second parenthesis,

\[
\hat{\rho}^{(1,1)}_d = \lambda^2 \int dV dV' \left( \Lambda_1(x) \Lambda_1^*(x') e^{i\Omega_1 (\tau_1 - \tau_1')} \langle \hat{\psi}(x') \hat{\psi}^\dagger(x) \rangle_{\rho_0} |e_1 g_2 \rangle |e_1 g_2 \rangle 
+ \Lambda_1(x) \Lambda_2^*(x') e^{i\Omega_2 (\tau_2 - \tau_2')} \langle \hat{\psi}(x') \hat{\psi}^\dagger(x) \rangle_{\rho_0} |e_1 g_2 \rangle |g_1 e_2 \rangle 
+ \Lambda_2(x) \Lambda_2^*(x') e^{i\Omega_2 (\tau_2 - \tau_1)} \langle \hat{\psi}(x') \hat{\psi}^\dagger(x) \rangle_{\rho_0} |g_1 e_2 \rangle |g_1 e_2 \rangle \right). \tag{A13}
\]

In order to compute the first of the remaining terms, notice that

\[
\hat{h}_1(x) \hat{h}_1(x') \hat{\rho}_{4,0} = \lambda^2 (\Lambda_1(x) e^{i\Omega_1 \tau_1} \hat{\sigma}_1^+ \hat{\psi}^\dagger(x) + \Lambda_1^*(x) e^{-i\Omega_1 \tau_1} \hat{\sigma}_1^- \hat{\psi}(x) + \Lambda_2(x) e^{i\Omega_2 \tau_2} \hat{\sigma}_2^+ \hat{\psi}^\dagger(x) + \Lambda_2^*(x) e^{-i\Omega_2 \tau_2} \hat{\sigma}_2^- \hat{\psi}(x))
\]

\[
\times (\Lambda_1(x') e^{i\Omega_1 \tau_1'} \hat{\sigma}_1^+ \hat{\psi}^\dagger(x') + \Lambda_2(x') e^{i\Omega_2 \tau_2'} \hat{\sigma}_2^+ \hat{\psi}^\dagger(x') \hat{\rho}_{4,0}) \]

\[
= \lambda^2 \left( \Lambda_1(x) \Lambda_2(x') e^{i\Omega_1 \tau_1 + i\Omega_2 \tau_2} \langle \hat{\psi}(x') \hat{\psi}^\dagger(x) \rangle_{\rho_0} |e_1 g_2 \rangle |g_1 e_2 \rangle 
+ \Lambda_1(x) \Lambda_1^*(x') e^{i\Omega_1 \tau_1 - i\Omega_2 \tau_2} \langle \hat{\psi}(x') \hat{\psi}^\dagger(x) \rangle_{\rho_0} |g_1 e_2 \rangle |g_1 e_2 \rangle 
+ \Lambda_2(x) \Lambda_1^*(x') e^{i\Omega_1 \tau_1} \langle \hat{\psi}(x') \hat{\psi}^\dagger(x) \rangle_{\rho_0} |e_1 g_2 \rangle |g_1 e_2 \rangle 
+ \Lambda_2^*(x) \Lambda_2(x') e^{i\Omega_1 \tau_1 + i\Omega_2 \tau_2} \langle \hat{\psi}(x') \hat{\psi}^\dagger(x) \rangle_{\rho_0} |g_1 e_2 \rangle |g_1 e_2 \rangle \right). \tag{A14}
\]

and \( \hat{\rho}_d^{(0,2)} = \hat{\rho}_d^{(0,2)*} \).

We can then rearrange the final result, in order to write them in the \( \{|g_1 g_2\}, |g_1 e_2\}, |e_1 g_2\}, |e_1 e_2\} \) basis. We define \( \mathcal{M} \) according to Eq. \( \text{[17]} \) such that the \( \hat{\rho}_d^{(2,0)} \) and \( \hat{\rho}_d^{(0,2)} \) terms can be written as

\[
\hat{\rho}_d^{(2,0)} = \mathcal{M} |e_1 e_2\rangle |g_1 g_2\rangle 
- \lambda^2 \int dV dV' \theta(t - t') \left( \Lambda_2(x) \Lambda_2^*(x') e^{-i\Omega_2 (\tau_2' - \tau_2)} \langle \hat{\psi}(x') \hat{\psi}^\dagger(x) \rangle_{\rho_0} 
+ \Lambda_1(x) \Lambda_1^*(x') e^{-i\Omega_1 (\tau_1' - \tau_1')} \langle \hat{\psi}(x') \hat{\psi}^\dagger(x) \rangle_{\rho_0} \right) |g_1 g_2\rangle |g_1 g_2\rangle, \tag{A15}
\]

\[
\hat{\rho}_d^{(0,2)} = \mathcal{M}^* |g_1 g_2\rangle |e_1 e_2\rangle 
- \lambda^2 \int dV dV' \theta(t - t') \left( \Lambda_2^*(x') \Lambda_2(x) e^{i\Omega_2 (\tau_2' - \tau_2)} \langle \hat{\psi}(x') \hat{\psi}^\dagger(x) \rangle_{\rho_0} 
+ \Lambda_1^*(x') \Lambda_1(x) e^{i\Omega_1 (\tau_1' - \tau_1)} \langle \hat{\psi}(x') \hat{\psi}^\dagger(x) \rangle_{\rho_0} \right) |g_1 g_2\rangle |g_1 g_2\rangle \tag{A16}
\]

We further define \( \mathcal{L}_n \) and \( \mathcal{E}_i \) according to Eq. \( \text{[17']} \). Using these definitions, we can write the evolved state in matrix form as

\[
\hat{\rho}_d = \begin{pmatrix}
1 - \mathcal{L}_{11} - \mathcal{L}_{22} & i\mathcal{E}_2 & i\mathcal{E}_1 & \mathcal{M}^* \\
-i\mathcal{E}_2^* & \mathcal{L}_{22} & \mathcal{L}_{21} & 0 \\
-i\mathcal{E}_1^* & \mathcal{L}_{12} & \mathcal{L}_{11} & 0 \\
\mathcal{M} & 0 & 0 & 0
\end{pmatrix}. \tag{A17}
\]
Appendix B: Computations regarding the Ground - Excited Interaction of the Linear Complex Detector

Here we show the details of the computations regarding the general ground-excited entanglement harvesting setup described in Section IV. Let us perform the same procedure as the one from Appendix A, but now considering the first detector to begin in the excited state $|e_1\rangle$. We then have that the initial density operator is

$$\hat{\rho}_0 = |e_1\rangle\langle e_1| \otimes |g_2\rangle\langle g_2| \otimes \hat{\rho}_\phi = \hat{\rho}_{A,0} \otimes \hat{\rho}_\phi. \quad (B1)$$

Expanding the unitary time evolution operator to second order in the Dyson expansion as in Appendix A, we obtain the following expression for the final state of the detectors

$$\hat{\rho}_d = \text{tr}_\phi(\hat{\rho}) = \hat{\rho}_{A,0} - i \int dV \left( \frac{\langle \hat{\rho}_{A,0} \hat{\rho} - \hat{\rho}_{A,0} \hat{\rho}_0 \rangle}{\rho_0} \right)$$

$$+ \int dV dV' \left( \frac{\langle \hat{\rho}_{A,0} \hat{\rho}_0 \hat{\rho}_0 \rangle}{\rho_0} - \theta(t-t') \left( \langle \hat{\rho}_{A,0} \hat{\rho}_0 \hat{\rho}_0 \rangle \right) \right), \quad (B2)$$

where

$$\hat{\rho}_d^{(1)} = -i \lambda \int dV \left( \Lambda_1^\dagger(x) e^{-i \Omega t_1} \langle \hat{\psi}(x) \rangle_{\rho_0} |g_1 g_2\rangle |e_1 g_2\rangle |e_1 e_2\rangle + \Lambda_2(x) e^{i \Omega t_2} \langle \hat{\psi}^\dagger(x) \rangle_{\rho_0} |e_1 e_2\rangle |e_1 g_2\rangle |g_1 g_2\rangle \right),$$

$$\hat{\rho}_d^{(1,1)} = \lambda^2 \int dV dV' \left( \Lambda_1^\dagger(x) \Lambda_1^\dagger(x') e^{-i \Omega t_1} \langle \hat{\psi}(x) \hat{\psi}(x') \rangle_{\rho_0} |g_1 g_2\rangle |e_1 g_2\rangle |e_1 e_2\rangle + \Lambda_2(x) \Lambda_2^\dagger(x') e^{i \Omega t_2} \langle \hat{\psi}(x) \hat{\psi}(x') \rangle_{\rho_0} |e_1 e_2\rangle |e_1 g_2\rangle |g_1 g_2\rangle \right), \quad (B3)$$

and

$$\hat{h}_d(x) \hat{h}_d(x') \hat{\rho}_{A,0} = \lambda^2 \left( \Lambda_1^\dagger(x) \Lambda_2^\dagger(x') e^{i \Omega t_1} \langle \hat{\psi}(x') \hat{\psi}(x') \rangle + \Lambda_2(x) \Lambda_1^\dagger(x') e^{i \Omega t_2} \langle \hat{\psi}(x') \hat{\psi}(x') \rangle \right) |g_1 g_2\rangle |e_1 g_2\rangle |e_1 e_2\rangle$$

$$+ \left( \Lambda_2(x) \Lambda_2^\dagger(x') e^{i \Omega t_2} \hat{\psi}(x') \hat{\psi}(x') + \Lambda_1(x) \Lambda_1^\dagger(x') e^{i \Omega t_1} \hat{\psi}(x') \hat{\psi}(x') \right) |e_1 g_2\rangle |e_1 e_2\rangle. \quad (B4)$$

The evolved density operator in matrix form is then given by

$$\hat{\rho}_d = \left( \begin{array}{ccc} \hat{\mathcal{L}}_{11} & 0 & -i \mathcal{E}_1 \\ 0 & \mathcal{M}' & 0 \\ -i \mathcal{E}_2^* & \mathcal{L}'_{12} & 0 \end{array} \right), \quad (B6)$$

where we used the definitions for $\hat{\mathcal{L}}_{11}$, $\hat{\mathcal{L}}_{12}$ and $\mathcal{M}'$ from Eq. (49).

Appendix C: Entanglement Harvested by the linear complex scalar model in $(n + 1)$-dimensional spacetime

In this appendix, we perform the computations of the $\mathcal{L}(\Omega)$ and $\mathcal{M}'$ terms of Section III that lead to Eqs. (27) and (34) in the case of 4 dimensional spacetime. In this appendix we will perform the computations in $(n + 1)$ dimensions, and later reduce them to $(3 + 1)$. Our final goal is to write both $\mathcal{L}(\Omega)$ and $\mathcal{M}'$ as integrals over one real variable, so we will assume that $n > 1$ in this appendix.
Both the $\mathcal{L}(\Omega)$ and $\mathcal{M}'$ terms involve $n$ dimensional integrals in the momentum variable $\mathbf{p}$. Given that most of the integrands depend only on $|\mathbf{p}|$, we pick spherical coordinates to solve the integrals. In spherical coordinates, it is then possible to write the integration measure $d^n\mathbf{p}$ as
\[
d^n\mathbf{p} = |\mathbf{p}|^{n-1}d|\mathbf{p}|d\Omega_{n-1} = |\mathbf{p}|^{n-1}(\sin \theta)^{n-2}d|\mathbf{p}|d\theta d\Omega_{n-2},
\] (C1)
where $d\Omega_m$ denotes the volume element of the $m$ dimensional unit sphere, $S^m$ and $\theta$ is the polar angle in $n$ dimensions. We have
\[
\int d\Omega_m = \frac{2\pi^{\frac{m+1}{2}}}{\Gamma\left(\frac{m+1}{2}\right)},
\] (C2)
where $\Gamma(m)$ is the Gamma function. This result allows us to solve for the $\mathcal{L}(\Omega)$ term, under the assumption that $\tilde{F}(\mathbf{p})$ only depends on $|\mathbf{p}|$ (real symmetric smearing function). We obtain:
\[
\mathcal{L}(\Omega) = \frac{\lambda^2}{(2\pi)^n} \int \frac{d^n\mathbf{p}}{2\omega_p} |\tilde{F}(\mathbf{p})|^2 |\tilde{\chi}(\omega_p - \Omega)|^2 = \frac{2\lambda^2}{(4\pi)^{\frac{n}{2}}\Gamma\left(\frac{n}{2}\right)} \int_0^\infty \frac{d|\mathbf{p}|}{2\omega_p} |\mathbf{p}|^{n-1} |\tilde{F}(\mathbf{p})|^2 |\tilde{\chi}(\omega_p - \Omega)|^2.
\] (C3)

However, even if $F(x)$ is spherically symmetric, the $\mathcal{M}'$ term depends on the inner product $\mathbf{p} \cdot \mathbf{L}$. By choosing coordinates for the momentum integral so that the $z$-axis is to be aligned with $\mathbf{L}$, we can write $\mathbf{p} \cdot \mathbf{L} = |\mathbf{p}| |\mathbf{L}| \cos \theta$, so that the $\mathcal{M}'$ term reads
\[
\mathcal{M}' = \frac{\lambda^2}{(2\pi)^n} \int \frac{d^n\mathbf{p}}{2\omega_p} |\tilde{F}(\mathbf{p})|^2 e^{i\mathbf{p} \cdot \mathbf{L}} (Q(\omega_p - \Omega) + Q(\omega_p + \Omega)),
\] (C4)
\[
= \frac{\lambda^2}{(2\pi)^n} \int_0^\infty \frac{d|\mathbf{p}|}{2\omega_p} |\mathbf{p}|^2 |\tilde{F}(\mathbf{p})|^2 (Q(\omega_p - \Omega) + Q(\omega_p + \Omega)) \int d\Omega_{n-1} e^{i|\mathbf{p}| |\mathbf{L}| \cos \theta}.
\]
Let $d\Omega_n$ be the area element of the hypersphere of dimension $n > 1$. Using angular coordinates $\{\theta_1, ..., \theta_n\}$ such that $\theta_1$ ranges from $0$ to $2\pi$ and $\theta_i$ ranges from $0$ to $\pi$ for $i = 2, ..., n$, the area element can be written as
\[
d\Omega_n = \prod_{i=1}^n (\sin \theta_i)^{i-1}d\theta_i.
\] (C5)

Then, notice that the area element of the hypersphere of dimension $n - 1$ is the area element of the hypersphere of dimension $n - 2$ times $d\theta (\sin \theta)^{n-2}$. In fact,
\[
d\Omega_{n-1} = d\theta_{n-1} (\sin \theta_{n-1})^{n-2} \prod_{i=1}^{n-2} d\theta_i (\sin \theta_i)^{i-1} = d\Omega_{n-2} d\theta_{n-1} (\sin \theta_{n-1})^{n-2}.
\] (C6)

Note that
\[
\int d\Omega_{n-1} = \frac{2\pi^{n/2}}{\Gamma\left(\frac{n}{2}\right)},
\] (C7)
\[
\int_0^\pi d\theta (\sin \theta)^{n-2} = \frac{\sqrt{\pi} \Gamma\left(\frac{n-1}{2}\right)}{\Gamma\left(\frac{n-1}{2}\right)},
\] (C8)

hence we can write
\[
\int d\Omega_{n-1} = \int_0^\pi d\Omega_{n-1} d(\sin \theta)^{n-2} = \frac{2\pi^{n-1}}{\Gamma\left(\frac{n-1}{2}\right)}.
\] (C9)

Considering all this, the integral over the angular variables can be solved analytically. Notice that for our expressions (e.g. (C1) and (C4)) we notated $\theta = \theta_{n-1}$. We hence obtain
\[
\int d\Omega_{n-1} e^{i|\mathbf{p}| |\mathbf{L}| \cos \theta} = \int d\Omega_{n-2} \int_0^\pi d\theta (\sin \theta)^{n-2} e^{i|\mathbf{p}| |\mathbf{L}| \cos \theta} = \frac{2\pi^{n-1}}{\Gamma\left(\frac{n-1}{2}\right)} \int_0^\pi d\theta (\sin \theta)^{n-2} e^{i|\mathbf{p}| |\mathbf{L}| \cos \theta}
\]
\[
= \frac{2\pi^{n-1}}{\Gamma\left(\frac{n-1}{2}\right)} \int_{-1}^1 du (1 - u^2)^{\frac{n-3}{2}} e^{i|\mathbf{p}| |\mathbf{L}| u} = 2\pi^{\frac{n}{2}} F_1\left(\frac{n}{2}, -\frac{|\mathbf{p}|^2 |\mathbf{L}|^2}{4}\right),
\] (C10)
where \( _0F_1(x) \) denotes the hypergeometric function. We then obtain an expression for \( M \) in terms of a single integral,

\[
M' = \frac{2\lambda^2}{(4\pi)^2} \int_0^\infty \frac{d|p|}{2\omega_p} |p|^2 |\tilde{F}(|p|)|^2 \, _0F_1 \left( \frac{n}{2}, -\frac{|p|^2 L^2}{4} \right) \left( Q(\omega_p - \Omega) + Q(\omega_p + \Omega) \right). \tag{C11}
\]

In the particular case of \( n = 3 \), the hypergeometric function reduces to

\[
_0F_1 \left( \frac{3}{2}, -\frac{|p|^2 L^2}{4} \right) = \frac{2}{\sqrt{\pi}} \frac{\sin(|p||L|)}{|p||L|} = \frac{2}{\sqrt{\pi}} \text{sinc}(|p||L|). \tag{C12}
\]
