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Nonlinear approximation in bounded orthonormal product bases. (English) Zbl 07739030

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Summary: We present a dimension-incremental algorithm for the nonlinear approximation of high-dimensional functions in an arbitrary bounded orthonormal product basis. Our goal is to detect a suitable truncation of the basis expansion of the function, where the corresponding basis support is assumed to be unknown. Our method is based on point evaluations of the considered function and adaptively builds an index set of a suitable basis support such that the approximately largest basis coefficients are still included. For this purpose, the algorithm only needs a suitable search space that contains the desired index set. Throughout the work, there are various minor modifications of the algorithm discussed as well, which may yield additional benefits in several situations. For the first time, we provide a proof of a detection guarantee for such an index set in the function approximation case under certain assumptions on the sub-methods used within our algorithm, which can be used as a foundation for similar statements in various other situations as well. Some numerical examples in different settings underline the effectiveness and accuracy of our method.

MSC:

41A50 Best approximation, Chebyshev systems
42B05 Fourier series and coefficients in several variables
65D15 Algorithms for approximation of functions
65D30 Numerical integration
65D32 Numerical quadrature and cubature formulas
65D40 Numerical methods for trigonometric approximation and interpolation
65Y20 Complexity and performance of numerical algorithms

Keywords:
sparse approximation; nonlinear approximation; high-dimensional approximation; dimension-incremental algorithm; bounded orthonormal product bases; projected coefficients

Full Text: DOI arXiv

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