Simple cosmological de Sitter solutions on $dS_4 \times Y_6$ spaces

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Abstract

Explicit time-dependent solutions of the 10D vacuum Einstein equations are found for which spacetime is compactified on six-dimensional warped spaces. We explicitly work out an example where the internal manifold is a six-dimensional generalized space having positive, negative or zero scalar curvature, whose base can be a five-sphere $S^5$ or an Einstein space $T^{1,1} = (S^2 \times S^2) \rtimes S^1$. In this paper, inflationary de Sitter solutions are found just by solving the 10D vacuum Einstein equations. Our results further show that the limitation with warped models studied to date has arisen partly from an oversimplification of the 10D metric ansatz. We also give some explicit examples of a non-singular warped compactification on the de Sitter space $dS_4$.

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1. Introduction

The observed current acceleration of our universe [1] is among the most puzzling discoveries in modern cosmology. This somewhat unexpected result together with an inflationary epoch required to solve the horizon and flatness problems of the big bang cosmology needs to be understood in the framework of fundamental theories. While the basic principles of the early inflation are rather well established, with many of its predictions being supported by observational data from WMAP [2], it still remains a paradigm in search of a concrete theoretical model. Efforts are still underway to explain accelerating universes from string theory—a consistent quantum theory of gravity in $9 + 1$ spacetime dimensions.

In light of observational evidence for both an inflationary epoch in the distant past and a recent cosmic acceleration, it is of importance to construct explicit de Sitter solutions from 10D or 11D supergravity models which are the low-energy effective theories of superstrings. In recent years, various low energy versions of string theory have been widely used to understand and to probe the mathematical structures of quantum field theories and cosmologies at a
microscopic scale. String theory has been a source of new ideas and has greatly inspired novel scenarios of cosmology, but it has yet to confront data and make predictions. Examples of predictions which have received considerable interests include mechanism of supersymmetry breaking, techniques of generating a small positive cosmological constant [3] or quintessence [4] and general statements about the scale and properties of inflation [5]. Motivated by this success and observation-driven cosmology, it is natural to study string theory models in cosmological backgrounds [6–10].

The first requirement for any predictable model based on string theory (or theories of higher dimensional gravity) is to find a mechanism of dimensional reduction from 10 (or 11) dimensions to 4. The simplest scenario of this type is ordinary Kaluza–Klein compactification of a 10D supergravity theory with the geometry of the form \( M \equiv M_4 \times M_6 \), where \( M_4 \) is a 4D spacetime with Lorenzian signature and \( M_6 \) is a 6D compact manifold. Models in four dimensions obtained through ordinary KK reduction on a Ricci-flat space (or string theory compactified on Ricci flat Calabi–Yau spaces) are, however, far from being phenomenologically interesting as they are plagued at the classical level by a plethora of massless scalar fields \( \phi_i \), which are not observed in our real world. A viable 4D effective theory, like Einstein’s theory of general relativity, must feature a cosmological constant-like term or a nontrivial scalar field potential. The latter could on the one hand lift the moduli degeneracy, thereby making the theory predictive, and on the other hand select a vacuum state for our universe with some desirable properties, such as a small cosmological vacuum energy or gravitational dark energy.

An important problem in string or higher dimensional cosmology is to explain the asymmetry between the sizes of large and small spatial dimensions. A reasonable explanation for the observed asymmetry would be a cosmological evolution in which all dimensions are initially small (and symmetric) and are then subject to asymmetric expansion driven by the underlying cosmological dynamics. It would be interesting to see if inflationary cosmologies and/or the origin of three large spatial dimensions can be understood as purely an outcome of the vacuum Einstein equations in a cosmological (time-dependent) background.

To construct a phenomenologically viable model, one could either consider a compact internal space and then deal properly with the stabilization of the extra dimensional volume, or else consider a noncompact solution and then introduce a method of inducing finite-strength 4D gravity. Particularly, in the context of 10D warped supergravity models, some of the extra dimensions could extend along the physical 4D hypersurface. In this scenario, the effective 4D Newton’s constant can be finite because of a strong warping of extra dimensions in an Randall–Sundrum braneworld models [12].

In this paper, we report on new cosmological solutions that explain inflationary universes utilizing exact solutions of 10D Einstein equations. These solutions correspond to the dimensional reduction to four dimensions of the type II supergravity, where the spacetime is a warped product of a 6D generalized space \( Y_6 \) and \( M_4 ( \equiv \mathbb{R}^{1,3} ) \). We also outline possible observational consequences of these new exact solutions, which may have significant contribution in current efforts to make contact with cosmological observations.

2. Ricci-flat spaces and time-dependent solutions

Assuming that 10D supergravity is the relevant framework, we start with the following 10D metric ansatz (in the Einstein-conformal frame):

\[
\begin{align*}
    ds^2_{10} &= e^{-6\psi(x)} e^{2A(y)} g_{\mu\nu}(x) \, dx^\mu \, dx^\nu + e^{2\psi(x)} e^{-\sigma A(y)} g_{mn}(y) \, dy^m \, dy^n,
\end{align*}
\]

(2.1)
where $e^{\psi(y)}$ is the Weyl rescaling factor, $A(y)$ is the warp factor as a function of one of the internal coordinates, $y$, and $\alpha$ is a constant. For the present study, which will be based on 10D vacuum Einstein equations, the above metric ansatz is sufficiently general. The metric of the usual 3 + 1 spacetime (or a Friedmann–Robertson–Walker universe) is

$$
dx^2 = g_{\mu\nu} \, dx^\mu \, dx^\nu = -a^2 dt^2 + a^2 \, dx^2, \quad (2.2)$$

where $a(u)$ is a scale factor and $\delta$ is a constant. The cosmic time $t$ is defined by $dt = a^\delta du$. The metric of the internal 6D space $Y_6$ may be written as

$$
dx^2 = g_{mn}(y) \, dy^m \, dy^n = dy^2 + f(y) dx^2_{S_4}. \quad (2.3)$$

The 5D base space $X_5$ may be taken to be an Einstein space $T^{1,1} = (S^2 \times S^2) \times S^1$ with the metric

$$
dx^2_{X_5} = g_{ij} \, d\theta^i \, d\theta^j = \frac{1}{2} \left( e_\psi^2 + \frac{1}{2} \left( e_{\phi_1}^2 + e_{\phi_2}^2 + e_{\phi_3}^2 \right) \right), \quad (2.4)$$

where $e_\theta = d\theta_i$, $e_{\phi_i} = \sin \theta_i d\phi_i$, $e_\psi \equiv d\psi + \cos \theta_1 d\phi_1 + \cos \theta_2 d\phi_2 + \sin \theta_3 d\phi_3$ or some other compact manifolds, such as $S^5$. In the above, $(\theta_1, \phi_1)$ and $(\theta_2, \phi_2)$ are coordinates on each $S^2$ and $\psi$ is the coordinate of a $U(1)$ fiber over the two spheres. In this particular example, the internal space $Y_6$ becomes Ricci flat ($\tilde{R}(\tilde{g}) = 0$) when $f(Y) \equiv y^2$. The metric (2.3) then defines a standard 6D conifold widely studied in the literature, in the context of warped supergravity solutions; see, for example, [13].

Before presenting some explicit solutions, we shall make some general comments about our choice of the 10D metric ansatz. In dimensions $D \geq 6$, and with $A(y) \neq \text{const.}$, one cannot absorb the factor $e^{-\alpha A(y)}$ into $g_{mn}(y)$ just by using some coordinate transformations unless that each and every components of the metric $g_{mn}(y)$ are equal or proportional to the same function, say $f(y)$. For brevity, let us take $D = 10$ and write the 6D metric as

$$
dx^2 = h(y) \, dy^2 + f(y) \, g_{mn} \, d\theta^m d\theta^n, \quad (2.5)$$

where $g_{mn}$ denote the metric components of the base space $X_5$, which are independent of the $y$ coordinate. In order to absorb the factor $e^{-\alpha A}$ inside $dy^2$ one could use the relation $e^{-\alpha A(y)} h(y) \, dy^2 = \tilde{d}y^2$ and also define a new function $X(\tilde{y})$ such that $e^{-\alpha A(y)} f(y) \equiv X(\tilde{y})$. The warp factor $e^{\alpha A(y)}$ multiplying the 4D part of the metric is now $\{X(\tilde{y})/f(\tilde{y})\}^{-2/\alpha}$. As is evident, the 10D metric still involves two unknown functions and the free parameter $\alpha$. Thus, in order to solve 10D Einstein equations in full generality, we should perhaps allow in the 10D metric ansatz one more free parameter, like $\alpha$ in the above example, than those considered in the literature, for example, [11, 14, 15].

It is easy to check that, with the metric (2.1), the 10D vacuum Einstein equations allow explicit solutions only when $\frac{\partial^2 \phi}{\partial \phi_1 \partial \phi_2} = 0$, which is the vanishing condition of the Ricci tensor $R_{\phi \phi}$. In this paper we only study explicit solutions of 10D vacuum Einstein equations, so we may consistently demand that either $A(y) = \text{const.}$ or $\varphi(t) = \text{const.}$ Of course, these conditions can be relaxed by introducing more complicated metric fluxes (or geometric twists) or by introducing external fluxes (or form fields) as present in various versions of supergravity or string theory. Alternatively, as in [9, 10], one could allow time-varying volume factors to each of the factor spaces, such as $X_5 = (S^2 \times S^2) \times S^1$.

First, let us assume that the Weyl factor $e^{\psi}$ is time varying, $d\psi/\, dt \neq 0$. In the vacuum case, the condition $R_{\phi \phi} = 0$ implies that $A(y) = \text{const.}$ In this case the exact solution to 10D Einstein equations following from equations (2.1)--(2.4) is given by

$$
a(u) \propto e^{cau}, \quad \varphi = \pm \frac{1}{2} \ln a, \quad \delta = 3, \quad (2.6)$$

where $c_0$ is arbitrary. The same result can be obtained by considering more general Ricci flat spaces, such as

$$
dx^2 = f_1(y) \, dy^2 + f_2(y) \, dx^2_{S_4}. \quad (2.7)$$
The Ricci-flatness condition $R_{6} = 0$ implies that $4f_{1}f_{2} = (df_{2}/dy)^{2}$, which is satisfied, for instance, with $f_{1}(y) \equiv f_{6}$ and $f_{2}(y) = f_{0}(y + y_{0})^{2}$. A more general 6D metric ansatz is

$$ds_{6}^{2} = \lambda^{2} f_{1}(y) dy^{2} + \frac{\lambda_{1}^{2}}{9} f_{2}(y) y^{2} e_{1}^{2} + \frac{\lambda_{2}^{2}}{6} y^{2} (e_{x}^{2} + e_{y}^{2}) + \frac{\lambda_{3}^{2}}{6} (y^{2} + 6b^{2}) (e_{x}^{2} + e_{y}^{2}),$$

(2.8)

where $\lambda$, $\lambda_{i}$ are arbitrary constants and $b$ is the resolution parameter. This metric becomes Ricci flat when

$$f_{1}^{-1}(y) = f_{2}(y) = \frac{y^{6} + 9b^{2}y^{4} + c}{y^{4}(y^{2} + 6b^{2})},$$

(2.9)

where $c$ is an arbitrary constant and $\lambda_{1} = \lambda_{2} = \lambda_{3} = \lambda$. In this case the metric (2.8) defines a standard 6D resolved conifold considered previously by Tseytlin et al [16]. The importance of taking a warped conifold metric has already been emphasized in the literature; see [10] for a discussion in a time-dependent (cosmological) background and [13, 16, 17] for some discussions in a static supergravity background.

In the case of $A(y) = \text{const}$ and a Ricci-flat 6D space (irrespective of its topology), the solution to 10D vacuum Einstein equations is given by

$$a \propto (t + t_{1})^{1/3}, \quad \psi = \psi_{0} \pm \frac{1}{2} \ln(t + t_{1}).$$

(2.10)

This gives only a non-accelerating universe. A more interesting solution can be found by replacing the FRW metric (2.2) by the metric of an open universe, in which case one has the following critical solution:

$$a(t) = t + t_{1}, \quad \psi(t) = \text{const}, \quad A(y) = \text{const}.$$  

(2.11)

A detailed analysis shows that one has to consistently set $\psi = \text{const}$ to get an inflationary de Sitter solution from 10D Einstein equations, at least, in pure supergravity models.

3. Generalized 6D spaces and exact de Sitter solutions

In this section we present a few explicit models of warped compactification for which inflationary de Sitter solutions are possible even in the vacuum case. These examples belong to standard warped compactifications for which the external fluxes are turned off or absent.

First we write the 10D metric in the following form (with $\alpha = 0$):

$$ds_{10}^{2} = e^{2A(y)} ds_{5}^{2} + \lambda^{2} \left(\frac{y^{2}}{2} ds_{X_{5}}^{2}\right).$$

(3.1)

One could in principle start with the metric of a generalized 6D space given in (2.8), but with $\alpha = 0$ the 10D vacuum Einstein equations admit an exact solution only when $\lambda_{1} = \lambda_{2} = \lambda_{3} = \lambda/\sqrt{2}$ and $b^{2} = c = 0$, for which the metric (2.8) reduces to the 6D part of the metric (3.1). The explicit solution to 10D Einstein equations is given by

$$A(y) = \frac{1}{2} \ln\left(\frac{3y^{2}}{8L^{2}}\right), \quad a(t) \propto e^{|t|L},$$

(3.2)

where $L$ is arbitrary. Interestingly enough, a warped spacetime as in (3.1) is also motivated from particle physics consideration in the manner of the Randall–Sundrum model [12], where one ignores the $X_{5}$ part of the 10D metric. There is however a crucial difference from that in RS models. In our case, the usual 4D spacetime and the internal 6D space both have positive curvature: $\tilde{R}_{(4)} = 6(\dot{a}/a + \dot{a}^{2}/a^{2}) = 12/(\lambda^{2}L^{2})$ and $\tilde{R}_{(6)} = 20/(\lambda^{2}y^{2})$. The 10D warped geometry is $ds_{10} \propto Y_{6}$. It is readily checked that, for the solution (3.2), the 10D Ricci scalar curvature vanishes, $R_{10} = e^{-2L} \tilde{R}_{4} + \tilde{R}_{6} - (4/\lambda^{2})(2A'' + 5A'^{2} + 10A'/y) = 0$. 

4
Next, we write the 10D metric in the following form (with $\alpha = 2$):
\[
\mathrm{d}s_{10}^2 = e^{2A(y)} \tilde{g}_{\mu\nu} \, \mathrm{d}x^\mu \, \mathrm{d}x^\nu + e^{-2A(y)} \, \mathrm{d}s_6^2.
\] (3.3)

The 10D Einstein equations admit an exact solution when the 6D metric takes the form
\[
\mathrm{d}s_6^2 = \lambda^2 \left( \mathrm{d}y^2 + 2y^2 \, \mathrm{d}s_4^2 \right),
\] (3.4)
which is actually obtained by taking $\lambda_1 = \lambda_2 = \lambda_3 = \sqrt{2} \lambda$ and $b^2 = c = 0$ in (2.8). In terms of the $u$-coordinate (i.e. with $\delta = 3$ in equation (2.2)), the explicit solution is given by
\[
A(y) = \frac{1}{4} \ln \left( \frac{3y^2}{2L^2} \right), \quad a(u) = \left( \frac{\lambda^2 L^2}{9u^2} \right)^{1/6}.
\] (3.5)

Note that the singularity of the scale factor at $u = 0$ is not a singularity of the usual 4D spacetime. This may be understood by looking at the same solution but expressed in terms of the 4D proper time $t$, which is nothing but $a(t) \propto \exp \left( t/\lambda L \right)$.

The above results clearly show that an accelerated expansion of the non-compact directions require nothing extraordinary other than a 6D Einstein space $Y_6$ (which may be positively curved as in (3.1) or negatively curved as in (3.4)) that supports not only a nontrivial warp factor but also a nonzero Hubble parameter. To our knowledge, this gives the first example where de Sitter solutions are found just by solving 10D vacuum Einstein equations, i.e., without introducing external fluxes or form fields.

Inflationary de Sitter solutions can also be obtained from warped compactifications on some Ricci flat 6D spaces, provided we write the 10D metric in a more general form. For illustration, let us make the following ansatz:
\[
\mathrm{d}s_{10}^2 = e^{2A(y)} \mathrm{d}s_6^2 + e^{-A(y)} \lambda^2 \left( \mathrm{d}y^2 + \alpha_1 y^2 \mathrm{d}s_4^2 \right),
\] (6.6)
where $\alpha$ and $\alpha_1$ are some constants. For this metric ansatz, we get
\[
\tilde{R}_{10} = e^{-2A} \left( \tilde{R}_4 - \mathcal{L}_A \right),
\] (3.7b)

where $\tilde{R}_4 = 6(\dot{a}/a + \dot{a}^2/a^2)$ and
\[
\mathcal{L}_A = \frac{e^{(\alpha+2)A}}{\lambda^2} \left[ \frac{3(\alpha + 2)}{32L^2} \right],
\] (3.8)

where $A' = \mathrm{d}A/\mathrm{d}y$. The 6D scalar curvature is $R_{(6)} = 20(1 - \alpha_1)/(\alpha_1 y^2)$. The effective 4D theory can have a nonzero cosmological constant-like term even when the 6D space is Ricci flat. This corresponds to the choice $\alpha_1 = 1$. To be precise, we can explicitly solve the 10D vacuum Einstein equations following from (3.6), whose solution is given by
\[
e^{(\alpha+2)A} = \frac{3(\alpha + 2) y^2}{32L^2}, \quad \alpha_1 = \frac{(\alpha + 2)^2}{8}, \quad a(t) \propto e^{(\alpha+1)L}.
\] (3.9)

Note that this solution will not change if we take $X_5 = S^5$ instead of $X_5 = T^{1,1}$. In the case of a Ricci-flat 6D space, we shall take $\alpha = -2 \pm 2\sqrt{2}$. Hence
\[
e^{A(y)} = \left( \frac{3y^2}{4L^2} \right)^{\pm1/2\sqrt{2}}, \quad \alpha_1 = 1.
\] (3.10)

One takes the $+ve$ exponent so as to avoid the divergence of the warp factor at $y = 0$.

In the context of string flux compactifications, Giddings et al [11] made the choice $\alpha = 2$ and $R_6 = 0$. This already belongs to a restrictive class of metrics, for which the solution of
10D vacuum Einstein equations is trivial: \( a(t) = \text{const}, A(y) = \text{const} \). In fact, as is clear from (3.9), when \( \alpha = 2 \), an inflationary de Sitter solution can be obtained by allowing \( Y_6 \) to have negative curvature \( R_6 = R_{\text{min}} g_{\text{min}} < 0 \), or by taking \( \alpha_1 = 2 \) in (3.6). Similarly, the model studied by Gibbons in [14] corresponds, in our case, to the choice \( e^{A(y)} = W(y) \) and \( \alpha = 0 \).

In this case, cosmological de Sitter solutions can be obtained by allowing \( Y_6 \) to have positive curvature \( R_6 > 0 \) or by taking \( \alpha_1 = 1/2 \) in (3.6).

Note that the solutions given above, equations (3.2) (3.5) and (3.9), are defined up to the rescaling \( y \rightarrow y + \epsilon \), where \( \epsilon \) is a shift parameter denoting the minimum size of \( X_6 \). The 10D Kretschmann scalar is given by

\[
K = R_{\text{abcd}} R_{\text{abcd}} \propto (y + \epsilon)^{-8/3(2 + \alpha)}.
\]

With \( \epsilon \rightarrow 0 \), the metric is smooth in the range \( 0 \leq y \leq \infty \). The only singularity is such that the warp factor \( e^{2A} \) goes to zero at \( y = -\epsilon \). However, since the \( g_{00} \) component of the metric decreases as \( y \rightarrow -\epsilon \), according to the strong version of the singularity theorem discussed in [15], a space-like singularity where \( e^{2A} \rightarrow 0 \) may be allowed as it can have a field theory interpretation. In any case, we can completely get rid of this pathology, i.e.: a causal treatment of bulk singularity, by expressing our solutions in some more suitable coordinate system.

In fact, for the metric (3.6), the singularity of the warp factor at \( y = 0 \) (with \( \alpha_1 \neq 1 \)) is a coordinate artifact. To quantify this, let us first consider the case \( \alpha = -2 \) and introduce a new coordinate \( z \) such that \( y \equiv e^{-\beta z} \), where \( \beta \) is a constant. The ranges of the coordinates are \(-\infty \leq z \leq \infty \) and \( 0 \leq y \leq \infty \). We then find that the metric

\[
ds_{10}^2 = e^{2A(z)} \left( -dt^2 + a(t)^2 \, dx^2 + 2 e^{-2A(z)} r_c^2 e^{-2\beta z} \left( \frac{\beta^2}{2} dz^2 + e^{\frac{1}{2}} e^{\frac{1}{2}} e^{\frac{1}{2}} + \frac{1}{6} \sum_{i=1}^{4} e_i^2 \right) \right)
\]

solves all of the 10D Einstein equations when

\[
A(z) = -\frac{\beta(z + z_0)}{2}, \quad a(t) \propto \exp \sqrt{\frac{2 t^2 e^{-2\beta z_0}}{3 r_c^2}},
\]

where \( r_c \equiv \lambda \) is the compactification scale. From the relation \( e^{-2A} = e^{2A} e^{2\beta(z+z_0)} \), we can easily see that it is the same warp factor \( e^{2A} \) that multiplies both the 4D and the 6D metrics in (3.11).

The Kretschmann scalars for the 6D and 10D parts of the metric are given by

\[
K = R_{\text{abcd}} R_{\text{abcd}} = 107 e^{2\beta(z+z_0)} / (3 r_c^2) > 0
\]

which are smooth everywhere, and the solution has no any physical singularities.

In the \( \alpha \neq -2 \) case, using the transformation \( y \equiv e^{-z/2} \) in (3.6), we find that the explicit solution to 10D Einstein equations is given by

\[
A = -\frac{(z + z_1)}{2 + \alpha}, \quad a \propto \exp \sqrt{\frac{32 t^2 e^{-z_1}}{3(2 + \alpha)^2 \lambda^2}}, \quad \alpha_1 = \frac{(2 + \alpha)^2}{8}.
\]

This solution is also regular everywhere.

Finally, we give one more explicit example of a completely nonsingular solution. We write the 10D metric in the form

\[
ds_{10}^2 = e^{2A(z)} ds_4^2 + e^{-A(z)} \lambda^2 ds_6^2,
\]

where the 6D metric has the form

\[
ds_6^2 = \sinh^2(z + z_0) \, dz^2 + \alpha_1 \cosh^2(z + z_0) \, d\bar{s}_y^2.
\]

Both the 6D Ricci scalar and the Kretschmann scalar \( K = R_{\text{abcd}} R_{\text{abcd}} \), i.e.

\[
R_6 = \frac{20(1 - \alpha_1)}{\alpha_1 \cosh^2(z + z_0)}, \quad K = \frac{8(5\alpha_1^2 - 10\alpha_1 + 17)}{\alpha_1^2 \cosh^4(z + z_0)}.
\]
are smooth everywhere. The 10D Einstein equations are solved for
\[
A(z) = \frac{1}{2 + \alpha} \ln \left( \frac{(2 + \alpha)^2 \cosh^2(z + z_0)}{32L^2} \right), \quad \alpha_1 = \frac{(\alpha + 2)^2}{8}, \quad a(t) \propto e^{t/\lambda L}. \tag{3.17}
\]
This provides an explicit example of non-singular warped compactification on dS4. The above results also reveal that inflationary cosmology is possible with every choice of \(\alpha\), provided that we do not restrict the spatial curvature of the internal space.

3.1. Einstein’s equations on a brane

In the following we specify a boundary condition such that the warp factor is regular at \(z = 0\) where we place a \(p\)-brane with brane tension \(\tau_p\). We also impose a \(Z_2\) symmetry around the brane’s position at \(z = 0\). The metrics like (3.11) and (3.14) will then have discontinuity in its first derivatives at \(z = 0\), implying that \(\partial|z|/\partial z = \text{sgn}(z)\). For illustration, we consider the metric (3.11) from which we derive
\[
G_{zz} = 0, \quad G_{MN} = -\frac{8}{\beta r_c^2} e^{-\beta z_0} \delta(z) g_{MN}, \tag{3.18}
\]
where \((M, N) = t, x_i, \theta_i, \phi_i, \psi\). The 10D Einstein equations can be written as
\[
G_{AB} = -\tau_p P(g_{AB}), \tag{3.19}
\]
where \(P(g_{MN})\) is the pull-back of the spacetime to the world volume of the \(p\)-brane (with \(p \geq 3\)) with tension \(\tau_p\).

In general, in a 10D warped background, one can consider a \(p\)-brane (with co-dimensions \(3 \leq p \leq 8\)) wrapped on the \(X_5\) space of the internal manifold. For instance, in the \(p = 8\) case, three of the extra dimensions could extend along the \(x^\mu\) directions. A situation like this could arise in the type IIA supergravity in ten dimensions where one finds NS5-branes as well as D6- and D8-branes. Similarly, in the type IIB supergravity in ten dimensions, one may consider both D3- and D5-branes.

Let us consider the special case that \(p = 8\), for which \(P(g_{AB}) = \delta(z) \delta^M_A \delta^N_B g_{MN}/\sqrt{g_{zz}}\). From equations (3.18)–(3.19) we then find that, with
\[
\tau_p = \frac{8}{r_c} e^{-\beta z_0/2}, \tag{3.20}
\]
the 10D Einstein equations are satisfied both at \(z = 0\) and away from \(z = 0\).

3.2. Some remarks on earlier no-go theorems

A couple of remarks are in order. The no-go theorems discussed earlier by Gibbons [14] and Maldacena and Nuñez [15] do not apply here, at least, in their original forms. The main reason for this is that the theorems discussed in [14, 15, 18] required all of the extra dimensions to be physically compact, the warp factor to be constant and the warped volume \(V_6^w \sim \int \sqrt{g_6} e^{(2-3\alpha)/A}\) to be finite (or constant) when integrated over \(Y_6\). In our examples the extra-dimensional volume is only suppressed as compared to the 4D volume, but it can be arbitrarily large. The previous authors also imposed an extra constraint, so-called a boundedness condition \(\int \nabla^2 e^{\alpha A} = 0\) (for any positive \(n\)), which is generally not satisfied by cosmological solutions, especially, in the presence of localized sources like branes. In the following we would like to be a bit more explicit on these remarks.

The discussions of the no-go theorem in [11, 14, 15] appeared to be special at least from two reasons. First, the previous authors assumed, though only implicitly, that the extra
dimensional manifold is maximally symmetric, such as (with $D = 10, m = 6$)
\[
\begin{align*}
\text{d}x_6^2 &= \text{d}z^2 + \text{d}z_1^2 + \cdots + \text{d}z_5^2 \\
\text{d}x_7^2 &= \text{d}z^2 + \sin^2 z \, \text{d}\Omega_5^2 \\
\text{d}x_8^2 &= \text{d}z^2 + \sinh^2 z \, \text{d}\Omega_5^2
\end{align*}
\] 
(\epsilon = 0, T^6) 
\begin{align*}
\text{d}x_7^2 &= \text{d}z^2 + \sin^2 z \, \text{d}\Omega_5^2 \\
\text{d}x_8^2 &= \text{d}z^2 + \sinh^2 z \, \text{d}\Omega_5^2
\end{align*} 
(\epsilon = +1, S^6) 
(\epsilon = -1, H^6),
\] 
where $\text{d}\Omega_5^2$ is the metric of a five-sphere. This yields $R_{\mu\nu} = \epsilon (m - 1) g_{\mu\nu}$. In this case, the internal curvature has no free parameter that can be tuned or fixed according to the choice of $\alpha$ in the warp factor. Second, only some specific values of $\alpha$ were considered.

Although a choice as $\alpha = 2$ [3, 11] has often been motivated from earlier works by Klebanov and his collaborators [13, 16, 17], in the context of string/gauge theory dualities in a supersymmetric background, we find no particular reason to constraint the warp factor (and the 6D curvature), especially, in a general cosmological background.

Indeed, it is not difficult to see, even with a canonical choice of the metric as equation (3.21), that certain values of $\alpha$ (in the warp factor) allow a de Sitter solution to occur. In the above case, one has
\[
(10) R_{\mu\nu}(x, z) = (4) \tilde{R}_{\mu\nu}(x) - \tilde{g}_{\mu\nu} e^{2(\alpha+1)}(2(2 - \alpha)A^2 + \nabla_z^2 A).
\] 
(3.22) 
Thus, for some background solution with $^{(10)} R = 0$, one gets
\[
\nabla_z^2 e^{2(\alpha+1)} = \frac{(2 + \alpha)}{4} (4) R(\tilde{g}) - \frac{(2 - 3\alpha)}{2 + \alpha} e^{-2(\alpha+1)}(\partial_z e^{2(\alpha+1)})^2.
\] 
(3.23) 
While the condition like $\int \nabla_z^2 e^{2(\alpha+1)} = 0$ can be satisfied in some specific examples in AdS compactification, with suitable boundary conditions, it is quite restrictive. Localized sources like branes and orientifold planes will violate such a condition. We also refer to a recent paper [19] for some related discussions on warp factor dynamics.

Let us also consider the warped volume constraint
\[
\frac{1}{G_N^{\text{eff}}} \sim V_6 w \sim \int d^6 z \sqrt{\tilde{g}_6} e^{(2 - 3\alpha)A(z)}.
\] 
We may use this relation to fix an overall normalization of the warp factor such that $G_N^{\text{eff}}$ yields the 4D Newton’s constant at a boundary or 4D hypersurface where $e^A \sim 1$. Particularly, with a constant warp factor $e^{A(z)} \equiv e^{A_0}$, the warped volume $V_6 \sim e^{(2 - 3\alpha)A_0} \times \text{Vol}(Y_6)$ becomes finite once $\text{Vol}(Y_6)$ is fixed. But in a generic situation with a nonconstant warp factor, the warped volume can be arbitrarily large. We will return to this discussion below.

As is evident, some of the explicit solutions found in this paper might not respect at least one of the above-mentioned assumptions of the no-go theorem, especially, the boundary condition, $\int \nabla_z^2 e^{2(\alpha+1)} = 0$, which is typically an additional constraint on the warp factor. For illustration, consider the solution (3.14), for which
\[
(10) R_{\mu\nu}(x, z) = (4) \tilde{R}_{\mu\nu}(x) - \frac{\tilde{g}_{\mu\nu}}{\lambda^2} \sinh^2(z + z_0) e^{2(\alpha+1)}(2(2 - \alpha)A^2 + \nabla_z^2 A)
\] 
\[
= (4) \tilde{R}_{\mu\nu}(x) - \frac{\tilde{g}_{\mu\nu}}{\lambda^2} e^{2(\alpha+1)} \left[ \frac{2(2 - \alpha)A^2 + A''}{\sinh^2(z + z_0)} + \frac{10}{\sinh^2(z + z_0)} - \frac{\cosh(z + z_0)}{\sinh^2(z + z_0)} \right] \text{sgn}(z) A' 
\] 
\[= (4) \tilde{R}_{\mu\nu}(x) - \frac{\tilde{g}_{\mu\nu}}{4L^2} \left[ \frac{12}{L^2} + \frac{3(2 + \alpha)}{2L^2} \coth(z + z_0) \delta(z) \right].
\] 
(3.24) 
Tracing this we can see that a de Sitter solution with $\tilde{R}^{(4)} > 0$ is possible in our model.
3.3. How to get a finite Newton’s constant?

Although stabilization of the extra dimensional volume has been one of the key issues in the study of inflationary solutions from both classical supergravities and string theory, in the literature there are no simple constructions that give rise to a finite 4D Planck scale as well as a stabilized volume modulus. In our view, this is not a significant drawback in our model though one could always hope to find a robust example with all moduli fixed.

In [11], Giddings et al. found interest in string compactifications with a finite 4D Planck scale. However, their construction does not seem to reasonably produce a finite 4D Planck mass. To understand this, it is important to note that even when if one starts with a 6D compact manifold such as $S^6$ or compact Calabi Yau spaces, the 4D Planck mass is not finite on its own, especially, when the 10D metric background is warped or non-factorizable as in our examples. With a metric ansatz of the form (3.6), one has

$$\frac{1}{G_N^{\text{eff}}} \propto V_6^w \sim \int d^6y \sqrt{g_6} e^{(2-3\alpha)A(y)}. \quad (3.25)$$

For the choice made in [11], i.e. $\alpha = 2$, one has

$$V_6^w \sim \int d^6y \sqrt{g_6} e^{-4A(y)}. \quad (3.26)$$

As long as the warp factor is non-constant, which is obviously the case both in the models of string flux compactifications studied in [3, 11] and in the above examples, the 6D warped volume would have a nontrivial dependence on some function of $y$.

For example, consider the explicit solution (3.9), for which

$$V_6^w \sim V_5 \int dy (y + \epsilon)^5 e^{(2-3\alpha)A(y)} \propto V_5 \int dy (y + \epsilon)^{(14-\alpha)/(2+\alpha)}, \quad (3.27)$$

where $V_5$ is the physical volume of the base space $X_5$ (which is independent of $y$) and $\epsilon$ is the resolution parameter. In the particular case where $\alpha = 2$, we get

$$V_6^w \sim V_5 \int_{y_1}^{y_2} (y + \epsilon)^3 dy. \quad (3.28)$$

Apparently, if $y$ is allowed to range from 0 to $\infty$, then the 6D warped volume is not finite. This is indeed a familiar feature of almost every inflationary solutions arising from string theory or warped supergravity models [17], for which the radial modulus is left unfixed. This particular behavior of the solution is not changed (except some minor modifications) by introducing external form fields or supergravity fluxes [3, 5, 11]. The KKLT proposal [3] is not an exception. On a general basis, for any solution to be viable, the 6D volume should be small enough (preferably exponentially suppressed) compared to the 4D volume, at least, at late times ($t \to \infty$).

An apparent divergence of the 6D volume in the above examples may not be something that is phenomenologically ruled out or unpleasant. To be precise, let us first consider the solution (3.6)–(3.9), which can be written as

$$d\sigma_{10}^2 = e^{2A(y)} \left( -dr^2 + e^{2\alpha / L} \, dx_5^2 + \frac{32\alpha^2 L^2}{3(2 + \alpha)^2} \frac{dy^2}{(y + \epsilon)^2} + \frac{4L^2}{3} \, ds_{X_5}^2 \right). \quad (3.29)$$

All of the dimensions can be small and symmetric at $t = 0$. With the expansion of the usual $3 + 1$ spacetime, which is exponential in our example, the extra dimensional volume gets suppressed at late times, $t \to \infty$. Particularly, to a 4D observer, who uses the coordinates $x^\mu$.
In the above we have used the result
\[ V_5 = \frac{64\sqrt{2}}{27} \lambda^6 L^6 \left( 2 + \alpha \right) V_3 \int \frac{dy}{(y + \epsilon)} \]
\[ = \frac{2048\sqrt{2}}{729} \pi^3 \lambda^6 L^6 \frac{\ln(y + \epsilon)}{|(2 + \alpha)|}. \] (3.30)

In the above we have used the result \( V_5 \equiv \langle 1/108 \rangle \int d(cos \theta_1)d(cos \theta_2)d\phi_1d\phi_2d\psi = 16\pi^3/27 \).

An obvious drawback of the solution (3.29) is that the metric is singular at \( y = -\epsilon \). This is just a coordinate artifact, indicating that \( y \) is not a globally defined coordinate.

To get physical results, including finite 4D Planck mass, one may introduce some elements of the Randall–Sundrum-type braneworld models [12]. In this approach one first writes the metric solution in terms of the normalizable coordinate which may be related to the usual conifold coordinate \( y \) using the relations such as \( y \propto e^{-\beta z} \) (in the \( \alpha = -2 \) case) or \( y \propto \cosh z \) (in the \( \alpha \neq -2 \) case), or directly consider line elements of the form (3.11) or (3.14). Then by imposing a \( Z_2 \) symmetry around the brane’s position at \( |z| = 0 \), one can show that the extra dimensional volume is effectively finite.

As a physically more appealing example, we consider the non-singular solution (3.17), along with (3.14) and (3.15). The 10D metric takes the form
\[ \text{d}x_{10}^2 = e^{\Lambda(z)} \left( -\text{d}t^2 + e^{2\beta z} \text{d}x_3^2 + e^{-(2\alpha+\beta)z} \lambda^2 \text{d}x_6^2 \right) \]
\[ = e^{-\chi} \left( \frac{3(2 + \alpha)^2 \cosh^2 (z + z_0)}{32L^2} \right)^{2/(2\alpha)} \]
\[ \times \left( -\text{d}t^2 + e^{2\beta z} \text{d}x_3^2 + \frac{32L^2 \lambda^2}{3(2 + \alpha)^2} \tanh^2 (z + z_0) \text{d}z^2 + \frac{4L^2 \lambda^2}{3} \frac{\text{d}x_6^2}{\sin^2 z} \right), \] (3.31)

where \( e^\chi \) is arbitrary. Here, without loss of generality, we may set \( z_0 = 0 \), which corresponds to a rescaling of the radial coordinate \( z \). We thus obtain
\[ \text{d}x_{10}^2 = e^{-\chi} \left( \frac{3(2 + \alpha)^2 \cosh^2 z}{32L^2} \right)^{2/(2\alpha)} \]
\[ \times \left[ -\text{d}t^2 + e^{2\beta z} \text{d}x_3^2 + \frac{32L^2 \lambda^2}{3(2 + \alpha)^2} \tanh^2 z \left( \text{d}z^2 + \frac{(2 + \alpha)^2 \cosh^2 z}{8 \sinh^2 z} \text{d}x_6^2 \right) \right]. \] (3.32)

For this solution, as the result (3.16) also implied, both the 6D and the 10D curvature tensors are regular everywhere. If required, by choosing \( \alpha \) appropriately, such as \( \alpha < 2\sqrt{2} - 2 \), we can maintain a positively curved 6D manifold or a squashed six-sphere which is topologically compact. The radial modulus, which scales as \( |\tanh z| \), is finite in the limit \( |z| \to \infty \). To a 4D observer, who uses the coordinates \( x^\mu \) to measure rods and clocks, the 6D volume is
\[ V_6 = \frac{2048\sqrt{2}}{729} \pi^3 \lambda^6 L^6 \frac{\ln(z)}{|(2 + \alpha)|}. \] (3.33)

Presumably, there is no need to make any unnatural cutoff in the \( z \) coordinate. That is, as with the usual three non-compact directions \( (x_1, x_2, x_3) \), the range of the \( z \) coordinate may be chosen to be \( 0 \leq z \leq \infty \). It is interesting to note that, with an exponential expansion of the \( 3 + 1 \) spacetime, the 4D volume becomes infinitely large at late times \( (t \to \infty) \), while the extra dimensional volume grows only linearly with \( z \).
4. Effect of supergravity fluxes

Inflationary de Sitter solutions of the above type can be obtained in a more general class of warped compactifications or 10D supergravities by introducing a five-form flux of the form

$$F(y)$$

where \( F(y) \) is some function on \( Y_5 \) and \( \Omega_3 \) denotes a volume form for \( R^{3,1} \). In the case \( X_5 \equiv T^{1,1} \) or \( X_5 = S^2 \times S^3 \), one can also introduce a combined three-form flux \([11]\)

$$G_{(3)} = F_{(3)} - \tau H_{(3)}.$$  \hspace{1cm} (4.2)

The 10D axion \( \tau \) is generally allowed to vary over the compact manifold, \( \tau \equiv \tau(y) \).

Let us assume that the 10D metric spacetime takes the following convenient form:

$$ds^2_{10} = e^{2A(y)} g_{\mu\nu} \, dx^\mu \, dx^\nu + e^{-\alpha A(y)} g_{mn}(y) \, dy^m \, dy^n.$$  \hspace{1cm} (4.3)

The metric tensor \( g_{mn}(y) \) is arbitrary in the sense that the internal compact space \( Y_5 \) has positive, zero or negative Ricci curvature scalar. The noncompact components of 10D Einstein equations then take the form \([11]\)

$$R_{\mu\nu}^{(10)} = -\hat{g}_{\mu\nu} \left( \frac{G_{(3)}^2}{48 I\pi} + \frac{e^{-2(2\alpha + A)}}{4} (\partial F)^2 \right) + \frac{1}{M_{10}^2} \left( T_{\mu\nu}^{loc} - \frac{1}{16} \hat{g}_{\mu\nu} T^{loc} \right).$$  \hspace{1cm} (4.4)

With the metric ansatz (4.3), the \( \mu\nu \) components of the 10D Ricci tensor read

$$R_{\mu\nu}^{(10)} = (4) R_{\mu\nu}(\hat{g}) - \hat{g}_{\mu\nu} e^{(2\alpha + A)A} \left( \nabla^2 A + 2(2 - \alpha)(\partial_y A)^2 \right)$$

$$= (4) R_{\mu\nu}(\hat{g}) - \hat{g}_{\mu\nu} \left( \frac{2 + \alpha}{2 + \alpha} (\nabla^2 A) + \frac{2 - 3\alpha}{2 + \alpha} e^{-2(2\alpha + A)} (\partial_y e^{(2\alpha + A)A})^2 \right).$$  \hspace{1cm} (4.5)

Using this and tracing (4.4) we find

$$\nabla^2 A = \frac{1}{4} (4) R(\hat{g}) e^{-2(2\alpha + A)} - 2(2 - \alpha)(\partial_y A)^2$$

$$+ e^{-\alpha A} \frac{G_{(3)}^2}{48 I\pi} + \frac{e^{-(4\alpha + A)}}{4} (\partial_y F)^2 + \frac{e^{-\alpha A}}{8 M_{10}^2} \left( T^{loc} - T_{\mu\nu}^{loc} \right).$$  \hspace{1cm} (4.6)

or

$$\nabla^2 e^{(2\alpha + A)} = \frac{(2 + \alpha)}{4} (4) R(\hat{g}) - \frac{2 - 3\alpha}{2 + \alpha} \frac{e^{-2(2\alpha + A)} (\partial_y e^{(2\alpha + A)})^2}{4}$$

$$+ \frac{2 + \alpha}{4} e^{2A} \frac{G_{(3)}^2}{48 I\pi} + \frac{e^{-2(2\alpha + A)}}{4} (\partial_y F)^2 + \frac{(2 + \alpha)}{8 M_{10}^2} \left( T^{loc} - T_{\mu\nu}^{loc} \right).$$  \hspace{1cm} (4.7)

The choice made by the authors of \([11]\), i.e. \( \alpha = 2 \), is special for which the second term on the right-hand side of (4.6) vanishes. It is quite clear that, with a suitable choice of \( \alpha \), the 10D supergravity equations would allow a de Sitter solution \((4) R > 0\) with nonconstant warp factor even when the contribution from the second line in equation (4.6) or (4.7) vanishes.

The argument in \([11]\) that in the absence of localized brane sources the fluxes must be constant and the warp factor must also be constant does not hold, for instance, when \( \alpha < 2/3 \) even if \( \int \nabla^2 e^{(2\alpha + A)} = 0 \). A detailed discussion about the effects of fluxes, which is beyond the scope of this paper, will appear elsewhere.

Class. Quantum Grav. 27 (2010) 045011

I Neupane
5. Conclusion

We conclude the paper with a short summary of the results. We have presented new cosmological solutions which use generalized 6D Einstein spaces having positive, negative or zero scalar curvature. Our novel observation is that inflationary de Sitter solutions can arise with all three possibilities for the internal space curvature (i.e. zero scalar curvature. Our novel observation is that inflationary de Sitter solutions can arise

We conclude the paper with a short summary of the results. We have presented new cosmological solutions which use generalized 6D Einstein spaces having positive, negative or zero scalar curvature. Our novel observation is that inflationary de Sitter solutions can arise with all three possibilities for the internal space curvature (i.e. zero scalar curvature). Our novel observation is that inflationary de Sitter solutions can arise

A pertinent question to ask is: Can the 4D effective cosmological constant $\Lambda_4$ found in the above examples be tuned to be the present value of dark energy $\Lambda_0 \sim 10^{-120} M_{Pl}^2$? The answer to this question seems affirmative. In fact, all our solutions discussed in this paper are invariant under a constant rescaling $g_{mn} \rightarrow e^{-2z} g_{mn}$. As a result, there can appear an arbitrary coefficient multiplying the 10D metric spacetime. For example, consider the following 10D metric solution

$$ds_{10}^2 = e^{-2\beta |z| - \chi} \left( -dt^2 + \exp \left( 2 \frac{\chi}{3} e^{\alpha t} \right) dx_i^2 + e^{-2\Omega} \left( 2\beta^2 dz^2 + ds_i^2 \right) \right).$$ (5.1)

A constant warp factor like $e^{-x}$ in the above example does not change the cosmological behavior of our solutions, but it may affect the value of the effective 4D Planck mass. To be more specific, we may consider the following dimensionally reduced action:

$$\frac{d^2 s_{10}}{\Lambda_0^2} = \frac{M_{10}^8}{2(2\pi)^6} \int d^{10}x \sqrt{-g_{10}} (R_{(10)} + \cdots)$$

$$= \frac{M_{10}^8}{\pi^3} \times \frac{\sqrt{\gamma}}{432} e^{-4\beta - 6\Gamma} \int d^4x \sqrt{-g_4 (\dot{R}_{(4)} - 2\Lambda_4 + \cdots)},$$ (5.2)

where $\Lambda_4 \equiv 8 e^{2z}$. We have assumed the existence of some sort of ‘brane’ at the $z = 0$ boundary of spacetime, so $\int dz e^{-8\beta |z|} \sim \frac{1}{4\beta} O(1)$. The parameters like $e^{-x}$ and $e^{\alpha}$ may be fixed using some phenomenological constraints. As a simple possibility, we may demand that $\Lambda_4 = 8 e^{2z} \sim 10^{-120} M_{Pl}^2$ and $M_{10} > \text{TeV} \sim 10^{-15} M_{Pl}$. Then we would have to take $x \gtrsim 136$. If we do not require $\Lambda_4$ to be close to present value of dark energy density, then the parameters like $e^{-x}$ and $e^{\alpha}$ do not have to be fine-tuned precisely.

In our construction, no energy conditions are violated in the 4D effective theory except the strong energy condition (SEC) $\dot{\hat{R}}_{(4)} > 0$, or equivalently $\rho + 3p > 0$, which applies only to ordinary matter. The SEC is violated anyway by a cosmological expansion satisfying $\dot{\hat{R}}_{(4)} \equiv -\frac{\dot{\rho}}{\rho} < 0$ and it may also be violated in spacetime dimensions $D < 10$. In the full 10D theory none of the energy conditions are violated (see also [20]).

Finally, we make a couple of remarks. In our view, for any cosmological model to be viable in large volume limits of some consistent string theory or supergravity compactifications, the solutions to 10D Einstein equations are required to be nontrivial (or cosmologically relevant) even when the external fluxes and or non-perturbative effects are absent. The present paper meets this requirement. Although in some specific models, for example, for the S-brane solutions derived in [6], the vacuum solution may not be obtained just by taking a zero-flux limit of some generic solutions, by shifting certain parameters in the solutions [7] or by writing a general ansatz for p-form gauge field strengths, consistent with the symmetries of the internal manifold, the zero flux case can be treated uniformly.

In string theory, certain background fluxes are known to be required to satisfy certain quantization conditions [11], especially, in a supersymmetric background, and also for some other good physical and mathematical reasons, such as, for fixing complex structure moduli associated with non-compact Calabi–Yau spaces [3]. These often place strong constraints on the local geometry of $Y_6$ and also on the warp factor. After all, supersymmetry is broken in
our universe and in the large volume limit, when the back reaction of the fluxes on Einstein’s
equations can be ignored (since their contribution to the stress tensor or the effective potential
is volume suppressed, the warp factor may be determined purely by 10D Einstein equations
alone, i.e. supergravity equations with zero flux. That is to say, any consistent solution of 10D
supergravity or string theory might survive in the limit the external fluxes are turned off, giving
rise to a smooth Einstein limit. One reason behind this expectation is that the effect of internal
space curvature scales as $e^{-2(2\alpha_a)A}$ while that the flux potential scales as $e^{-2(2\alpha_a)A \int X F^2}$ ($X$
is the internal manifold and $F$ is the field strength). Furthermore, in the large volume limit, all
possible non-perturbative effects, worldsheet $\alpha'$ corrections and string loop corrections can be
negligibly small.

In some ways our models look similar to Randall–Sundrum-type braneworld constructions
with the important difference that the effect of a 5D compact space or base manifold $X_5$ have
already been incorporated in the simple exact classical gravity solutions. It might be possible
to design string theory examples of inflationary cosmologies by generalizing the construction
in our paper. It is also quite plausible that Ricci non-flat warped spaces add richness to string
cosmology and may potentially lead to the realization of new cosmological scenarios.

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References

[1] Riess A G et al (Supernova Search Team Collaboration) 1998 Astron. J. 116 1009
Perlmutter S et al 1999 Astrophys. J. 517 565
[2] Spergel D N et al (WMAP Collaboration) 2003 Astrophys. J. Suppl. 148 175
Spergel D N et al (WMAP Collaboration) 2007 Astrophys. J. Suppl. 170 377
[3] Kachru S, Kallosh R, Linde A and Trivedi S P 2003 De Sitter vacua in string theory Phys. Rev. D 68 046005
Burgess C P, Cline J M, Stoica H and Quevedo F 2004 Inflation in realistic D-brane models J. High Energy
Phys. JHEP09(2004)033
[4] Neupane I P 2004 Accelerating cosmologies from exponential potentials Class. Quantum Grav. 21 4383
Ohta N 2005 Accelerating cosmologies and inflation from M/superstring theories Int. J. Mod. Phys. A 20 1
(arXiv:hep-th/0411230)
[5] Becker M, Leblond L and Shandera S E 2007 Inflation from wrapped branes Phys. Rev. D 76 123516
[6] Chen C M, Gal’tsov D V and Gutperle M 2002 Phys. Rev. D 66 024043 (arXiv:hep-th/0204071)
[7] Townsend P K and Wohlfarth M N R 2003 Accelerating cosmologies from compactification Phys. Rev.
Lett. 91 061302
Ohta N 2000 Accelerating cosmologies from S-branes Phys. Rev. Lett. 91 061303
Ohta N 2003 A study of accelerating cosmologies form superstring/M theories Prog. Theor. Phys. 110 259
[8] Chen C M, Ho P M, Neupane I P, Ohta N and Wang J E 2003 Hyperbolic space cosmologies J. High Energy
Phys. JHEP10(2003)058 (arXiv:hep-th/0306291)
[9] Neupane I P and Wiltshire D L 2005 Accelerating cosmologies from compactification with a twist Phys. Lett.
B 619 201
Neupane I P and Wiltshire D L 2005 Cosmic acceleration from M theory on twisted spaces Phys. Rev.
D 72 083509
[10] Neupane I P 2007 Accelerating universes from compactification on a warped conifold Phys. Rev. Lett. 98 061301
[11] Giddings S B, Kachru S and Polchinski J 2002 Hierarchies from fluxes in string compactifications Phys. Rev.
D 66 106006
[12] Randall L. and Sundrum R. 1999 A large mass hierarchy from a small extra dimension Phys. Rev. Lett. 83 3370
Randall L. and Sundrum R. 1999 An alternative to compactification Phys. Rev. Lett. 83 4690

[13] Klebanov I. R. and Tseytlin A. A. 2000 Gravity duals of supersymmetric $SU(N) \times SU(N + M)$ gauge theories
Nucl. Phys. B 578 123

[14] Gibbons G. W. 1985 Supersymmetry, Supergravity and Related Topics ed F. del Aguila, J. A. de Azcarraga and
L. E. Ibanez pp 123–46 (Singapore: World Scientific)

[15] Maldacena J. M. and Nuñez C. 2001 Supergravity description of field theories on curved manifolds and a no go
theorem Int. J. Mod. Phys. A 16 822

[16] Pando Zayas L. A. and Tseytlin A. A. 2000 3-branes on resolved conifold J. High Energy Phys. JHEP11(2000)028
(arXiv:hep-th/0010088)

[17] Klebanov I. R. and Strassler M. J. 2000 Supergravity and a confining gauge theory: duality cascades and $\chi$-SB-
resolution of naked singularities J. High Energy Phys. JHEP08(2000)052

[18] Wesley D. H. 2009 Oxidised cosmic acceleration J. Cosmol. Astropart. Phys. JCAP01(2009)041

[19] Douglas M. R. 2009 arXiv:0911.3378

[20] Neupane I. P. 2010 Extra dimensions, warped compactifications and cosmic acceleration Phys. Lett. B 683 88–95

[21] Neupane I. P. 2009 Accelerating universe from warped extra dimensions Class. Quantum Grav. 26 195008
(arXiv:0905.2774)