E2 component in subcoulomb breakup of $^8B$

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Abstract

We calculate the angular distribution and total cross section of the $^7$Be fragment emitted in the breakup reaction of $^8$B on $^{58}$Ni and $^{208}$Pb targets at the sub-Coulomb beam energy of 25.8 MeV, within the non-relativistic theory of Coulomb excitation with proper three-body kinematics. The relative contributions of the $E1$, $E2$ and $M1$ multipolarities to the cross sections are determined. The $E2$ component makes up about 65% and 40% of the $^7$Be total cross section for the $^{58}$Ni and $^{208}$Pb targets respectively. We find that the extraction of the astrophysical S-factor, $S_{17}(0)$, for the $^7$Be(p,γ)$^8$B reaction at solar energies from the measurements of the cross sections of the $^7$Be fragment in the Coulomb dissociation of $^8$B at sub-Coulomb energies is still not free from the uncertainties of the $E2$ component.

KEYWORDS: Coulomb dissociation of $^8$B, radiative capture of $p$ and $^7$Be, Astrophysical S-factors.

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The rate of the radiative capture reaction $^7\text{Be}(p, \gamma)^8\text{B}$ at solar energies, is of considerable interest in the quest for understanding the “Solar neutrino puzzle”. The $^{37}\text{Cl}$ and Kamiokande detectors are particularly sensitive to the flux of the high energy neutrinos emitted in the subsequent $\beta$ decay of $^8\text{B}$ [1]. Several attempts have been made in the past to measure the rate of this reaction at the lowest possible beam energies [2-7]. However, the measured cross sections disagree in absolute magnitude, and the S-factors ($S_{17}(0)$) extracted by extrapolating the data to solar energies ($\approx 20$ keV) differ from each other by about 30-40%.

An alternate indirect way to determine the radiative capture cross sections at low relative energies is provided by the Coulomb dissociation method [8], which is based on the fact that the dissociation of a projectile in the Coulomb field of a heavy target nucleus can be considered as its photodisintegration. By using the principle of detailed balance this cross section can be related to that of the radiative capture process (we refer to Ref. [9] for a comprehensive review).

Motobayashi et al. have performed the first measurements of the Coulomb dissociation of $^8\text{B}$ into the $^7\text{Be} - p$ low energy continuum in the field of $^{208}\text{Pb}$ with a radioactive $^8\text{B}$ beam of 46.5 MeV/A energy [10]. We have presented earlier a detailed analysis of this data [11], where the breakup cross sections of $^8\text{B}$ corresponding to $E_1$, $E_2$ and $M_1$ transitions were calculated using a theory of Coulomb excitation appropriate for intermediate beam energies. Considering only the $E_1$ component, the measured breakup cross sections were found to be consistent with a $S_{17}(0) = (15.5 \pm 2.80)$ eV barn, which is smaller than even the lowest value reported by the direct capture measurements. This is also appreciably smaller than the value used in the standard solar model calculations [1]. However, under the kinematical conditions of the experiment of Motobayashi et al., the $E_2$ component of breakup may be disproportionately enhanced. In fact, $E_2$ corrections calculated from one of the models [12] of the structure of $^8\text{B}$ may lead to a further reduction of approximately 20% in the value of $S_{17}(0)$ [11,13]. Nevertheless, the contributions of this component are strongly dependent on the model used to describe the structure of $^8\text{B}$, and it is difficult to draw any definite conclusion about the $E_2$ contributions to this data [11,14,15]. This has led to some of the authors of Ref. [10] to repeat this measurements with angular distributions extended to larger scattering angles where the cross sections are expected to be more sensitive to the $E_2$ component[16].

Recently von Schwarzenberg et al. [17] have measured the breakup of $^8\text{B}$ on the $^{58}\text{Ni}$ target at the beam energy of 25.8 MeV, well below the Coulomb barrier, where the $E_2$ component of the breakup is expected to be dominant. In contrast to the experiments reported in Refs. [10,16] where $^7\text{Be}$ and $p$ were measured in coincidence, these authors detect only the $^7\text{Be}$ fragment. In their analysis of the data, they have used the non-relativistic theory of Coulomb excitation [18] and the radiative capture cross sections of Kim, Park and Kim (KPK) [12] to estimate the $E_1$, $E_2$ and $M_1$ component of the breakup cross sections. However, the final state has been approximated as a two body system by these authors. This implies that the measured angles of $^7\text{Be}$ were equated to those of the $^7\text{Be}-p$ center of mass
(CM), which may not be correct. Furthermore, the Coulomb excitation functions needed in the calculations of the cross sections were obtained by interpolating the values given in the tables of Ref. [18], which may lead to inaccuracies.

In this letter, we present the results of an improved analysis of the data of Ref. [16] by using a proper three body kinematics (TBK), which avoids, automatically, equating the measured $^7\text{Be}$ angles with those of the CM of $^7\text{Be}-p$ system. Due to the difference in the masses of the two fragments, these two angles are expected to be different. Furthermore, we use a proper three body phase-space factor in the calculations of the cross sections.

In the first part of our presentation, we relate the triple differential cross section for the Coulomb breakup of a projectile (a) into its fragments (b and x) on a target A, ($A+a \rightarrow b+x+A$), to the cross section for the Coulomb excitation (to the continuum) of the projectile a, ($A+a \rightarrow a^*+A$), which is calculated within the Alder-Winther theory [18]. Using TBK (see e.g. Ref. [19]), the momenta $p_{bx}$ and $p_{a^*}$ describing the relative motion of the fragments b and x and the motion of their CM with respect to the target nucleus respectively, can be related to their individual momenta $p_b$ and $p_x$ as following

$$p_{bx} = \mu_{bx} \left( \frac{p_b}{m_b} - \frac{p_x}{m_x} \right),$$

$$p_{a^*} = p_b + p_x - \frac{m_b + m_x}{m_a + m_A} P,$$

where $m_i$ is the mass of the fragment $i$ and $P$ is the total momentum which is fixed by the conditions in the entrance channel. $\mu_{bx}$ is the reduced mass of the $b-x$ system. Now let the solid angles associated with the momenta $p_b$, $p_x$, $p_{bx}$ and $p_{a^*}$ be $\Omega_b$, $\Omega_x$, $\Omega_{bx}$ and $\Omega_{a^*}$ respectively, then we can write

$$\frac{d\sigma}{dE_b d\Omega_b d\Omega_x} = \frac{J}{4\pi} \frac{d\sigma}{dE_{bx} d\Omega_{a^*}} \frac{\partial E_x}{\partial E_{tot}},$$

where the total kinetic energy $E_{tot}$ is

$$E_{tot} = E_b + E_x + E_A$$

$$= E_{bx} + E_{a^*} + \frac{P^2}{2(m_a + m_A)},$$

is related to the projectile energy ($E_p$) and the reaction Q-value ($Q$) by $E_{tot} = E_p + Q$. In Eq. (4) $E_b$, $E_x$, and $E_A$ are the kinetic energies of the fragments b, x and recoiling target nucleus respectively, while $E_{bx}$ and $E_{a^*}$ are the kinetic energies of the relative motion of the fragments and that of their CM with respect to the target nucleus respectively. In Eq. (3), we have assumed that the angular distribution of fragments is isotropic in the projectile rest frame; the expressions without making this assumption are given in Ref. [20]. The last factor in Eq. (3) is given by

$$\frac{\partial E_x}{\partial E_{tot}} = m_A[m_x + m_A - m_x \frac{p_x \cdot (P - p_b)}{p_x^2}]^{-1},$$

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and the Jacobian $J$ is defined as

$$J = \frac{m_b p_b m_x p_x}{\mu_a p_{bx} \mu_{aA} p_{a}^*}.$$  \quad (7)

In Eq. (7) $\mu_{aA}$ is the reduced mass of the $a - A$ system. The cross section $d\sigma/dE_{bx}d\Omega_{a*}$ is related to the photo-dissociation cross section as,

$$\frac{d\sigma}{dE_{bx}d\Omega_{a*}} = \frac{1}{E_{bx}} \frac{dn_{\lambda}}{d\Omega_{a*}} \sigma(\gamma + a \rightarrow b + x),$$  \quad (8)

where $dn_{\lambda}/d\Omega_{a*}$ is the virtual photon number per unit solid angle $\Omega_{a*}$ for the relevant multipolarity ($\lambda$) in the breakup process, and this can be calculated within the Alder-Winther theory. The photodissociation cross section $\sigma(\gamma + a \rightarrow b + x)$ is related to the radiative capture cross section $\sigma(b + x \rightarrow a + \gamma)$ by means of the detailed balance theorem.

In Ref. [17], Eq. (8) has been used to get the total cross section of $^7$Be by integrating this equation over the relative energies $E_{bx}$ and the angular aperture ($\pm 6^\circ$) of the detectors (measuring $^7$Be) placed at 45$^\circ$ with respect to the beam direction. This procedure necessarily assumes that the angles of $^7$Be are the same as those $^8$B* ($^7$Be - p CM). Such an assumption is avoided if this cross section is obtained by integrating Eq. (3) over the energy of $^7$Be and the solid angles ($\theta$, $\phi$) of the (unobserved) proton and of $^7$Be. For given angles ($\theta_{7\text{Be}}$, $\phi_{7\text{Be}}$) and ($\theta_p$, $\phi_p$), and energies $E_{7\text{Be}}$ and $E_p$, one can use Eqs. (1) and (2) to determine the magnitude and directions of the momenta $p_{7\text{Be}}$ and $p_{8B^*}$. In this way the cross sections given by Eq. (3) can be determined from the Alder-Winther theory of Coulomb excitation.

The angular distributions of $^8$B* are obtained by integrating Eq. (8) over the relative energies $E_{bx}$. In our alternative approach, we used the procedure outlined in the previous paragraph to get the triple differential cross section $d\sigma/d\Omega_{7\text{Be}}dE_{7\text{Be}}d\Omega_p$ and by integrating them numerically over the solid angle $\Omega_p$ and the energy $E_{7\text{Be}}$, the angular distributions of the $^7$Be fragment have been obtained. In Figs 1a and 1b, we show the angular distributions (obtained by using the capture cross sections given by KPK model[12]) of $^8$B* and $^7$Be for the $^{58}$Ni target respectively. We note that the cross sections corresponding to the $E2$ component are larger than those of the other multipolarities in both cases (we have not shown here the $M1$ components as they are very small). In Fig 1a, the $^8$B* cross sections for the $E1$ multipolarity are smaller than those of the $E2$ at all the angles, which is in contrast to the results reported in Ref. (17). Moreover, the magnitudes of both $E1$ and $E2$ components are always larger than those given in this reference. This underlines the inadequacy of the interpolation method used in Ref. [17] to obtain the cross sections.

Although the angular distributions of the $^7$Be fragment look similar to those of $^8$B*, the magnitude of the $E2$ component in the former case is about 10% larger as compared to that in the latter case. The $E1$ component is 15% smaller.

The ratio of the experimental total breakup cross section of $^7$Be (obtained by integrating the breakup yields in the angular range, $(45\pm6)^\circ$, of the experimental
setup) to Rutherford elastic scattering of $^7$Be is reported to be $(8.1 \pm 0.8^{+2.0}_{-1.5}) \times 10^{-3}$, which is the only quantity measured in Ref. [17]. It is not possible to get the total breakup cross section of $^7$Be by directly integrating the angular distributions of various multipolarities shown in Fig. 1a, as the corresponding angles belong to $^8$B* and not to $^7$Be. Nevertheless, an approximate estimate of this cross section can be made (from Eq. 8) by noticing that the angles of $^7$Be can be related to those of $^8$B* for given values of proton angles and magnitudes of the $^7$Be and proton momenta (see Eqs. (1) and (2)). We find that for the $^7$Be angles in the range of $39^\circ$ - $51^\circ$, the $^8$B* angles vary between $41^\circ$ to $65^\circ$. We, therefore, determine the total cross section for the $^7$Be fragment from Fig. 1a by summing the contributions of $E_1$, $E_2$ and $M_1$ components which are obtained by integrating the corresponding angular distribution over this angular range. The ratio of this cross section to Rutherford elastic scattering of $^8$B is found to be $6.8 \times 10^{-2}$. This is about an order of magnitude larger than the experimental value. However, in the three-body case, where the total cross section of the fragment $^7$Be can be obtained in a straight-forward way by integrating the distributions shown in Fig. 1b over the angular range of the experimental setup, the value of this ratio is only $4.1 \times 10^{-2}$. It is interesting to note that the capture cross sections calculated by some other authors [21,22,23] could lead to somewhat smaller values for this ratio. For example, the model of Typel and Baur [21] gives a value of $2.6 \times 10^{-2}$. Therefore, the values of this ratio predicted by various theoretical models of the capture reaction are larger than the upper limit of its reported experimental value by factors of 3 to 4. Thus the uncertainty about the magnitude of the $E_2$ cross section calculated in various models is not eliminated by the measurement of Ref. [17], as this result is not reproduced by any existing model of $^8$B.

It may be remarked here that data for the angular distributions at larger angles may provide a better regime for determining the $E_2$ component in such an experiment. It can be seen from Fig. 1b that beyond $50^\circ$ the cross sections for this component are about 3 to 5 times larger than the corresponding $E_1$ cross sections.

It is suggested in Ref. [17], that the method employed in their experiment can be used to determine a precise value of $S_{17}$ if a heavy target is used in the experiment and the data is taken at the same incident energy and the angular range. This suggestion is, of course, based on the assumption that for heavy targets the $E_1$ component of breakup would be predominant under the similar kinematical conditions. We have examined the validity of this assumption in Fig. 2, where we show the results of calculations (performed using TBK) for $E_1$ and $E_2$ components of the reaction in which $^7$Be fragment is observed in the breakup of $^8$B on $^{208}$Pb target at the same beam energy. Indeed, the $E_1$ component is larger than the $E_2$ for certain angles. However, nowhere is the latter component negligible; it even takes over the $E_1$ component beyond $50^\circ$. In the angular region of $30^\circ$ - $40^\circ$, where $E_1$ component is large, $E_2$ cross sections still contribute up to $40\%$. Therefore, no regime of the $^7$Be angular distribution is completely free from the $E_2$ component of the breakup, thus a clean determination of the $S_{17}$
by performing an experiment similar to that done in Ref. [17] on a heavy target appears to be unlikely. In this regard, the experiments being carried out at GSI at beam energies of 200 MeV/A are more promising as has already been discussed in Ref. [11].

With the E2 contributions calculated with the TBK the dependence of the fraction \( f \left( \frac{\sigma_{\text{total}}^{\exp} - \sigma_{E1}^{\text{calculated}}}{\sigma_{E2}^{\text{calculated}}} \right) \) on \( S_{17} \) is different from that given in Ref [17]. The \( f \) vs. \( S_{17} \) curve looks more like that obtained in the similar analysis of the data of Ref. [10] (the dashed-dotted curve in Fig. 5 of Ref. [17]) by Shyam et al. [11].

In summary, we have analysed the recently measured data on the breakup of \(^8\text{B}\) on \(^{58}\text{Ni}\) target at the sub-Coulomb beam energy of 25.8 MeV. In this experiment only \(^7\text{Be}\) fragment has been detected. We found that with the proper three-body kinematics and phase-space factors used in the calculations, the theoretical total cross sections are still larger than the upper limit of the experimental data. Therefore, the present measurements do not completely eliminate the uncertainty in the \( E2 \) predictions of various models proposed to calculate the capture cross sections. Furthermore, the prospect of determining a precise value for the astrophysical S-factor \( S_{17} \) by performing the similar experiment with a heavy target does not seem to be very encouraging, as in no angular regime is the \( E2 \) component of the breakup negligible. A possible way to make the \( E2 \) component more definite would be to measure the angular distributions of \(^7\text{Be}\) in such an experiment, as those corresponding to \( E1 \) and \( E2 \) components are quite different in shape as well as in magnitude, and can then be easily separated from each other.

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Fig. 1a. Angular distribution for $^8\text{B}^*$ in the Coulomb excitation of $^8\text{B}$ on $^{58}\text{Ni}$ target at the beam energy of 25.8 MeV. The dashed and solid lines show the $E1$ and $E2$ cross sections respectively which are obtained by using Eq (8).
Fig. 1b. Angular distribution of the $^7$Be fragment emitted in the breakup reaction of $^8$B on $^{58}$Ni target at the beam energy of 25.8 MeV. The dashed and solid lines show the $E_1$ and $E_2$ cross sections respectively which are calculated by using Eq. (3) with the proper three body kinematics.
Fig. 2. Angular distributions of the $^7$Be fragment emitted in the breakup of $^8$B on $^{208}$Pb target at the beam energy of 25.8 MeV. The solid and dashed lines have the same meaning as in Fig. 1b.