In-medium modification of $\rho$-mesons produced in heavy ion collisions.

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The mass shift $\Delta m_\rho$ and width broadening $\Delta \Gamma_\rho$ of $\rho$ mesons produced in heavy ion collisions is estimated using a general formula which relates the in-medium mass shift of a particle to the real part of the forward scattering amplitude $\text{Re} f(E)$ of this particle on constituents of the medium and $\Delta \Gamma$ to the corresponding cross section. It is found that in high energy ($E/A \gtrsim 100$ GeV) heavy ion collisions the $\rho$ width broadening is large and the $\rho$ (or $\omega$) peak could hardly be observed in $e^+e^-(\mu^+\mu^-)$ effective mass distributions. In low energy collisions ($E/A \sim$ a few GeV) a broad (a few hundred MeV) enhancement is expected at the position of the $\rho$ peak.

1. INTRODUCTION

The problem of how the properties of hadrons change in hadronic or nuclear matter in comparison to their free values has attracted a lot of attention. Among these properties of immediate interest are the in-medium particle’s mass shift and width broadening. Different models as well as ”model independent” approaches were used to calculate these effects both at finite temperature and finite density (for a review, see e.g. [1]). It is clear on physical grounds that the in-medium mass shift and width broadening of a particle are only due to its interaction with the constituents of the medium. Thus one can use phenomenological information on this interaction to calculate the mass shift and width broadening. In a recent paper[2] two of us argued that the mass shift of a particle in medium can be related to the forward scattering amplitude $f(E)$ of this particle on the constituents of the medium (in medium rest frame)

$$\Delta m(E) = -2\pi \frac{\rho}{m} \text{Re} f(E).$$

Here $m$ is the vacuum mass of the particle, $E$ is its energy in the rest frame of the constituent particle, and $\rho$ is the density of constituents. The normalization of the amplitude

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corresponds to the standard form of the optical theorem

\[ k\sigma = 4\pi \text{Im} f(E), \tag{2} \]

where \( k \) is the particle momentum. The width broadening is given by

\[ \Delta \Gamma(E) = \frac{\rho}{m} k\sigma(E). \tag{3} \]

The applicability criteria of Eqs. (1) and (3) were discussed in [2]. In short, they are:

1) The particle wave length \( \lambda \) must be much less than the mean distance between medium constituents \( d \), \( \lambda = k^{-1} \ll d \). This means that the particle momentum \( k \) must be larger than a few hundred MeV;

2) The particle formation length \( l_f \sim (E/m)/m_{\text{char}} \) \( (m_{\text{char}} \approx m_\rho) \) must be less than the nucleus radius \( R \);

3) \( \text{Re} f(E) \) which enters Eq.(1) must satisfy the inequality \( |\text{Re} f| < d \);

4) The scattering takes place mostly at small angles, \( \theta \ll 1 \). Only in this case the optical analogy on which Eqs. (1) and (3) are based is correct.

Eqs. (1) and (3) are correct also in cases when the medium constituents have some momentum distributions, e.g. the Fermi distributions for nucleons, or represent a gas at a finite temperature. In such cases, in the right-hand sides of Eqs. (1) and (3) an averaging over the momentum distribution of constituents must be performed. Eqs. (1) and (3) were derived in [2] basing on simple quantum mechanical arguments and the optical analogy. This approach allows one to formulate explicitly the applicability conditions presented above. When the medium is a gas in thermal equilibrium the equivalents of Eqs.(1) and (3) can be derived basing on thermal field theory. References [3,4] give few examples and the reference [5] gives a relativistic field-theoretic derivation.

In most of the papers on the in-medium hadron mass shifts the hadrons were considered at rest. As seen from Eq. (1) this restriction is not necessary theoretically. It is desirable to have theoretical predictions in a broad energy interval, since it extends possibilities of experimental investigations. As discussed in [6] for the cases of \( \rho \) or \( \pi \)-mesons embedded in nuclear matter the energy dependence of the mass shifts is rather significant at low energies, i.e. in the resonance region.

We estimate the \( \rho \) meson mass shift and width broadening in the case of \( \rho \)-mesons produced in heavy ion collisions. The most interesting case is the case of \( \rho^0 \), which can be observed through the decay \( \rho^0 \rightarrow e^+e^- (\mu^+\mu^-) \). We will assume that \( \rho \)-mesons are formed at the latest stage of the evolution of hadronic matter created in the course of a heavy ion collision when the matter can be considered as a noninteracting gas of pions and nucleons. (We will neglect the admixture of kaons and hyperons, which is known to be small [7], as well as heavy resonances.) This stage is formed when, during the evolution of the matter after collision, the total density of nucleons and pions becomes of order of the normal nucleon density in nuclei. The description of nuclear matter as a noninteracting gas of nucleons and pions of course cannot be considered as a very good one. So, it is clear from the beginning that our results may be only semiquantitative. The main ingredients of our calculation are the \( \rho\pi \) and \( \rho N \) forward scattering amplitudes and total cross sections as well as the values of nucleon and pion densities.
We consider here central heavy ion collisions and assume that the nucleon and pion momentum distributions in the gas are just the momentum distributions measured experimentally in such collisions. (Another model, in which nucleons and pions are assumed to be in the state of ideal gas at fixed temperature and chemical potential, will be considered in a subsequent publication [8].)

2. CALCULATION OF $\rho\pi$ AND $\rho N$ CROSS SECTIONS AND FORWARD SCATTERING AMPLITUDES.

Let us first focus on the amplitudes and cross sections. To determine these quantities we use the following procedure. In the low energy region we saturate the cross sections and forward scattering amplitudes by resonance contributions. At high energies we determine $\sigma_{\rho N}$ and $\sigma_{\rho\pi}$ from $\sigma_{\gamma N}$ and $\sigma_{\gamma\pi}$ using vector dominance model (VDM). $\sigma_{\gamma N}$ is well known experimentally [9], $\text{Re} f_{\gamma N}$ is determined from dispersion relation; $\sigma_{\gamma\pi}$ and $\text{Re} f_{\gamma\pi}$ can be obtained by the Regge approach. Since VDM allows one to find only the cross sections of transversally polarized $\rho$-mesons, we restrict ourselves to this case. As was shown in [2], at $E_\rho \gtrsim 2$ GeV $\Delta m$ and $\Delta \Gamma$ for longitudinal $\rho$-mesons in nuclear matter are much smaller than for transverse ones. At zero $\rho$-meson energy $\Delta m$ and $\Delta \Gamma$ for transverse and longitudinal $\rho$-mesons are evidently equal. In the case of scattering on low temperature gas they are comparable [10]. Therefore, for unpolarized $\rho$-meson our results should be multiplied by a factor ranging from 1 to 3/2.

To estimate $\text{Re} f_{\rho\pi}(s)$ at low energy we write in the center of mass (c.m.) frame

$$\text{Re} f_{\rho\pi}(s) = -\sum_R F^s_R F^i_R \frac{1}{2q_{cm}} \frac{B_R \Gamma_R(\sqrt{s} - m_R)}{(\sqrt{s} - m_R)^2 + \Gamma^2_R/4},$$

(4)

where $\sqrt{s}$ is the total c.m. energy, $m_R$ and $\Gamma_R$ are the mass and the total width of the resonance, $B_R$ is the branching ratio of its decay into $\pi\rho$ and $q_{cm}$ is the center of mass momentum,

$$q_{cm} = \frac{1}{2\sqrt{s}} \sqrt{[s - (m_\rho + m_\pi)^2][s - (m_\rho - m_\pi)^2]}. \tag{5}$$

The factor $F_s$ is the spin factor, $F^i_R$ is the isospin factor. The latter is equal to 1/3 = (1/2) × (2/3) for $I_R = 1$. The first factor here reflects the fact that we are interested in the $\rho^0\pi$ scattering and only one of the two decay channels of a $I_R = 1$ resonance can contribute here: $R^\pm \rightarrow \rho^0\pi^\pm$, but not $\rho^\pm\pi^0$. The second factor corresponds to the assumption, that all three pion isospin states are equally populated in the gas. Similarly, for $I_R = 0$ the isospin factor is 1/9 = (1/3) × (1/3). We take into account the following resonances [9]: $a_1(1260)$, $\pi(1300)$, $a_2(1320)$ and $\omega(1420)$. The nearest resonance under the threshold, $\omega(782)$, does not give a considerable contribution due to its narrow width. For the spin factor we take $F^s_R = 1, 1, 2, 1$ correspondingly for the above mentioned resonances. (These factors take into account that we consider only transvers $\rho$). The amplitude in the pion rest frame is obtained from Eq. (4) multiplying it by the rescaling factor $k_\rho/q_{cm} = \sqrt{s}/m_\pi$, where $k_\rho = \sqrt{E_\rho^2 - m_\rho^2}$ is the $\rho$ momentum in the pion rest frame.
For $\sigma_{\rho\pi}$ we use the standard resonance formula

$$\sigma_{\rho\pi} = \sum_R F_s F_i (\pi) \frac{B_R \Gamma_R^2}{q_{cm}^2 (\sqrt{s} - m)^2 + \Gamma_R^2 / 4}.$$ (6)

As is known, the pion scattering amplitude on any hadronic target vanishes at zero pion energy in the target rest frame in the limit of massless pions (Adler theorem). In the framework of effective Lagrangians this can be achieved if the pion field enters through its derivative, $\partial \varphi / \partial x_\mu$. We assume that in the interaction describing the $\rho\pi$ scattering through the $a_1$ resonance, $\partial \varphi / \partial x_\mu$ is multiplied by the $\rho$-meson field strength tensor $F_{\mu\nu}$ and the field $a_{1\nu}$. This results in appearance of an additional factor in $a_1$ contributions to $\text{Re} f_{\rho\pi}$ and $\sigma_{\rho\pi}$ in Eqs. (4) and (6)

$$\left(\frac{s - m_\rho^2 - m_\pi^2}{(m_{a_1}^2 - m_\rho^2 - m_\pi^2)^2}\right),$$ (7)

which is normalized to 1 at $s = m_{a_1}^2$. At $s > m_{a_1}^2$ this factor was not taken into account. Similar factors were also introduced for contributions of other resonances.

In the high energy region we assume that the Regge approach is valid for $\gamma\pi$ scattering and apply VDM to relate $\rho\pi$ and $\gamma\pi$ amplitudes. As is well known, the Regge pole contributions to the forward scattering amplitude normalized according to Eq. (2) have the form

$$f(s) = -\frac{k}{4\pi s} \sum_i \frac{1 + e^{-i\pi\alpha_i}}{\sin \pi\alpha_i} s^{\alpha_i} r_i,$$ (8)

where $\alpha_i$ is the intercept of $i$-th Regge pole trajectory, $r_i$ is its residue, $k$ is the projectile momentum in the target rest frame. As follows from Eqs. (2) and (8),

$$\sigma(s) = \sum_i r_i s^{\alpha_i - 1},$$ (9)

$$\text{Re} f(s) = -\frac{k}{4\pi s} \sum_i \frac{1 + \cos \pi\alpha_i}{\sin \pi\alpha_i} r_i s^{\alpha_i}.$$ (10)

For $\sigma_{\gamma\pi}$ only the $P$ (pomeron) and $P'$ poles contribute [11,12]. The residues of $P$ and $P'$ poles in $\gamma\pi$ scattering were found by Boreskov, Kaidalov and Ponomarev (BKP) [12] using the Regge pole factorization theorem and the data on $\gamma p$, $\pi p$ and $p p$ scattering. Taking the BKP values for the $P$ and $P'$ residues we have

$$\sigma_{\gamma\pi}(s) = 7.48 \alpha \left[ \left(\frac{s}{s_0}\right)^{\alpha_P - 1} + 0.971 \left(\frac{s}{s_0}\right)^{\alpha_{P'} - 1}\right],$$ (11)

where $\alpha_P = 1.0808$, $\alpha_{P'} = 0.5475$, $\alpha = 1/137$, $s_0 = 1$ GeV$^2$ and $\sigma$ in Eq. (11) is given in millibarns, if $s$ is in GeV$^2$. For the $P$ and $P'$ intercepts we take the Donnachie-Landshoff values [13]. Since in their fit to the data BKP assumed $\alpha_P = 1$ and $\alpha_{P'} = 1/2$, the values of residues in Eq. (11) are slightly changed in comparison with [12] in order to get
the same $\sigma_{\pi \gamma}$ at $s = 9$ GeV$^2$. From Eqs. (10) and (11) the real part of the forward $\gamma\pi$ scattering amplitude can be found:

$$\text{Re} f_{\gamma\pi}(s)_{\text{rest frame}} = -\frac{k}{4\pi} 7.48\alpha \left\{ -0.106 \left( \frac{s}{s_0} \right)^{\alpha_{\rho}-1} + 0.752 \left( \frac{s}{s_0} \right)^{\alpha_{\rho^*}-1} \right\},$$

where the momentum $k$ is in GeV and $\text{Re} f$ is in mb·GeV.

In VDM $\sigma_{\rho\pi}(s)$ is related to $\sigma_{\gamma\pi}(s)$ by (see [2])

$$\sigma_{\rho\pi}(s) = \frac{g_\rho^2}{4\pi \alpha} \left( 1 + \frac{g_\rho^2}{g_\omega^2} \right)^{-1} \sigma_{\gamma\pi}(s),$$

where $g_\rho^2/4\pi = 2.54$, $g_\rho^2/g_\omega^2 = 1/8$ and the $\varphi$-meson contribution is neglected. A similar relation holds also for $\text{Re} f_{\rho\pi}$. Unlike in [2], we prefer here to use direct Regge formulae for $\text{Re} f$ at high energies instead of finding it from $\sigma$ through the dispersion relation, since in the latter approach the results are sensitive to the low energy domain which is more uncertain.

The results of the calculations of $\sigma_{\rho\pi}$ and $\text{Re} f_{\rho\pi}$ as functions of $\rho$-meson energy in the pion rest frame are presented in Fig.1. As seen from Fig.1, the matching of low and high energy curves is satisfactory.

For the amplitude $\text{Re} f_{\rho N}$ at laboratory frame energies of $\rho$ above 2 GeV we use the results of Ref. [4] obtained using dispersion relation, VDM and experimental data on $\sigma_{\gamma N}$. At lower $E_\rho$ we again use the resonance approximation

$$\text{Re} f_{\rho N}(s) = -\frac{1}{4} \frac{1}{2q_{\text{cm}}} \sum_R (2J_R + 1) F_i \frac{\Gamma_R^\rho N (\sqrt{s} - m_R)}{(\sqrt{s} - m_R)^2 + \Gamma_R^2 / 4}$$

(14)

(the factor $(1/4)$ appears because we consider only transverse $\rho$). The isospin factors $F_i$ are 1/3 and 2/3 correspondingly for $N$ and $\Delta$ resonances. We take 10 well established $N$ and $\Delta$ resonances[9] with significant branchings into $\rho N$ and with masses above the $\rho N$ threshold and below 2200 MeV. The set of baryonic resonances taken into account is close to the set used in [14]. The main difference in comparison with [14] is that effective widths $\Gamma_{\text{eff}}^\rho N = \Gamma_R^\rho N (q_{\text{cm}}/q_{\text{cm},R})^{2l+1}$ were introduced only for the resonances close to the $\rho N$ threshold ($q_{\text{cm},R}$ is the value of $q_{\text{cm}}$ at the resonance). At $q_{\text{cm}} > q_{\text{cm},R}$ we put $\Gamma_{\text{eff}}^\rho N = \Gamma_R^\rho N$.

Apart from these resonances, in the calculation of $\text{Re} f_{\rho N}$ two resonances with masses below the $\rho N$ threshold were also accounted for: $\Delta (1238)$ and $N (1500)$. It was assumed that VDM relates $\Gamma_{\rho N}$ and $\Gamma_{\gamma N}$ of these resonances in the following way. Since both resonances are close to the $\rho N$ threshold, we can write for each of them: $\Gamma_{\rho N} = q_{\text{cm}} \gamma_{\rho N}$, $\Gamma_{\gamma N} = k_{\text{cm}} \gamma_{\gamma N}$, where $q_{\text{cm}}$ and $k_{\text{cm}}$ are the $\rho N$ and $\gamma N$ c.m. momenta, respectively. Then we assume that $\gamma_{\rho N}$ and $\gamma_{\gamma N}$ are related by the VDM formula

$$\gamma_{\gamma N} = 4\pi \alpha \frac{1}{g_\rho^2} \left( 1 + \frac{g_\rho^2}{g_\omega^2} \right) \gamma_{\rho N}.$$  (15)

The values of $\gamma_{\gamma N}$ can be found from the values of $\sigma_{\gamma N}$ at the resonance peaks. The contributions of $\Delta (1238)$ and $N (1500)$ to $\text{Re} f_{\rho N}$ are essential at low energies of $\rho$ mesons; they comprise about $(-1) - (-0.5)$ fm at $E_\rho = 1 - 2$ GeV in the nucleon rest frame.
3. DETERMINATION OF THE MASS SHIFT AND WIDTH BROADENING OF THE $\rho$-MESON PRODUCED IN HEAVY ION COLLISION FROM THE DATA ON NUCLEON AND PION DISTRIBUTIONS

As was mentioned above, in heavy ion collisions we will consider only nucleons and pions as constituents of the medium. Therefore, in this case Eqs. (1) and (3) take the form

$$\Delta m_\rho(E) = - \frac{2\pi}{m} \{ \rho_N \text{Re} f_{\rho N}(E) + \rho_\pi \text{Re} f_{\rho \pi}(E) \} \ ,$$  \hspace{1cm} (16)

$$\Delta \Gamma_\rho(E) = \frac{k}{m} \{ \rho_N \sigma_{\rho N}(E) + \rho_\pi \sigma_{\rho \pi}(E) \} \ ,$$  \hspace{1cm} (17)

where $\rho_N$ and $\rho_\pi$ are the nucleon and pion densities in the medium formed at the latest stage of evolution of hadronic matter produced in heavy ion collisions.

We will restrict ourselves to consideration of head-on (central) collision with small values of impact parameter when the number of participants – the nucleons which experience considerable momentum transfer – is close to the total number of colliding nucleons.

Experimental data shows that the nucleons and pions produced in heavy ion collisions cannot be considered as thermal gas even at the final stage of evolution of hadronic matter created in the collisions. In order to demonstrate this let us discuss separately the cases of high energy ($E/A \sim 100$ GeV) and low energy ($E/A \sim$ a few GeV) heavy ion collisions. In case of high energy collisions the longitudinal (relative to the beam) and transverse
momenta of nucleons and pions are very different. In the experiment on S + S collisions at \( E/A = 200 \text{ GeV} \) [15] it was found (in the centre of mass frame) that \( \langle p_{TN}^n \rangle \approx 3.3 \text{ GeV}, \langle p_{TN}^\pi \rangle \approx 0.61 \text{ GeV} \) and \( \langle p_{LN}^n \rangle \approx 0.70 \text{ GeV}, \langle p_{LN}^\pi \rangle \approx 0.36 \text{ GeV} \). In other experiments on high energy heavy ion collisions (see e.g. [16,17]) the situation is qualitatively similar. This means that in this case one can by no means speak about thermal gas of final particles and their momentum distributions must be taken from experiment.

The data on low energy heavy ion collisions also indicate that pions and nucleons cannot be described as gases in thermal equilibrium. The angular distributions of pions produced in Ni + Ni collisions at \( E/A = (1 - 2) \text{ GeV} \) show a considerable anisotropy [18]. If the pion angular distribution in the centre of mass frame is approximated by \( 1 + a \cos^2 \theta \), then it follows from the data that \( a \approx 1.3 \). Unfortunately, there is not enough information in the data about the nucleon angular and momentum distributions. We have checked the hypothesis of thermal equilibrium by assuming that the probability of production of a given number of particles is proportional to the statistical weight of the final state (Fermi-Pomeranchuk approach [19,20]). It is evident that this hypothesis is even more general than the hypothesis of thermal equilibrium. In this approach the pion/nucleon ratio \( R_\pi = n_\pi/N \) in central collisions can be predicted in terms of the main ingredient of the method – the volume per particle at the final stage of evolution (and, of course, the initial energy). The calculations show that the data [18] on the energy dependence of the ratio \( R_\pi \) are well described by the statistical model, but in order to get the absolute values of \( R_\pi \) in Ni + Ni as well as in Au + Au [21] collisions, it is necessary to put the volume per nucleon very small, about 5 times smaller, than the one in a normal nucleus, which is unacceptable.

Therefore, the only way to perform the averaging over momentum distributions of pions and nucleons seems to take the latter from experimental data on heavy ion collisions.

In calculation of the \( \rho \)-meson mass shift and width broadening the averaging must be performed over the \( \rho \)-meson direction of flight relative to nucleons and pions. Such calculation can be done only for real experimental conditions. For this reason we restrict ourselves to crude estimates.

Consider first the case of high energies and take as an example the experiment [15] on central collisions, where the ratio of pion to nucleon multiplicities was found to be \( R_\pi = 5.3 \). Suppose that in this experiment the \( \rho \)-meson is produced with longitudinal and transverse momenta in the lab. frame \( k_L = 3 \text{ GeV}, k_T = 0.5 \text{ GeV} \). We choose these values as typical for such experiment. At such values of \( \rho \) momenta the formation time of \( \rho \)-meson is close to the mean formation time of pions produced in heavy ion collisions. So, a necessary condition of our approach is fulfilled. Since the mean momenta of nucleons and pions in the experiment [15] are known (they were presented above), it is possible using the curves of Figs. 1 and 2 to calculate the mean values of \( \langle \text{Re}F_{\rho N} \rangle, \langle \text{Re}F_{\rho \pi} \rangle, \sigma_{\rho N} \) and \( \sigma_{\rho \pi} \) in \( \rho N \) and \( \rho \pi \) scattering. The results are (in the lab. frame):

\[
\langle \text{Re}F_{\rho N} \rangle \approx -1.1 \text{ fm}, \quad \langle \text{Re}F_{\rho \pi} \rangle \approx 0.03 \text{ fm}.
\]

\[
\langle \sigma_{\rho N} \rangle \approx 43 \text{ mb}, \quad \langle \sigma_{\rho \pi} \rangle \approx 20 \text{ mb}.
\]

The small value of \( \langle \text{Re}F_{\rho \pi} \rangle \) comes from the compensation of positive and negative \( \text{Re}F_{\rho \pi} \) at low and high energies (Fig. 1), i.e. from the scattering of \( \rho \) on pions moving in the same
direction as $\rho$ (comovers), or in the opposite one. Because of this compensation $\langle \text{Re} f_{\rho\pi} \rangle$ is badly determined, but since it is small this does not affect the final result. Using Eqs. (16) and (17) we can now find the mass shift and width broadening of $\rho$-meson. For the nucleon and pion densities we take
\begin{align}
\rho_N &= \frac{N}{V} = \frac{N}{Nv_N + nv_\pi} = \frac{1}{v_N(1 + R_\pi \frac{v_\pi}{v_N})}, \\
\rho_\pi &= \frac{n}{V} = \frac{n}{Nv_N + nv_\pi} = \frac{R_\pi \frac{1}{v_N(1 + R_\pi \frac{v_\pi}{v_N})}},
\end{align}
where $N$ and $n$ are the numbers of nucleons and pions at the final stage of evolution, $R_\pi = n/N$, $V$ is the volume of system at this stage. It is assumed that at this stage any participant (nucleon or pion) occupies a definite volume $v_N$ or $v_\pi$. We can write
\begin{align}
\rho_N &= \frac{\rho_0^N}{1 + R_\pi \beta}, \\
\rho_\pi &= \frac{\rho_0^N R_\pi}{1 + R_\pi \beta},
\end{align}
where $\rho_0^N = 1/v_N$ and $\beta = v_\pi/v_N$. For numerical estimates we take $\rho_0^N = 0.3$ fm$^{-3}$, almost two times the standard nucleon density. This number is probably one of the most uncertain ingredients of our calculations. Substitution of Eqs. (18), (19) and (22) into Eqs. (16) and (17) gives (for the experimental value of $R_\pi = 5.3$ and $\beta = 1$)
\begin{align}
\Delta m_\rho &= 18 - 2 = 16 \text{ MeV}, \\
\Delta \Gamma_\rho &\approx 150 + 400 = 550 \text{ MeV}.
\end{align}
The first numbers in Eqs. (23) and (24) refer to the contributions from $\rho N$ and second ones from $\rho \pi$ scattering. Because $\rho$-meson width broadening appears to be very large, the basic condition of our approach, $\Delta \Gamma_\rho \ll m_\rho$ is fullfilled badly. The other applicability condition of the method, $|\text{Re} f| < d$, is also not well satisfied, since in this case $d = 0.9$ fm. For these reasons the values of $\Delta m_\rho$ in Eq. (23) and of $\Delta \Gamma_\rho$ in Eq. (24) may be considered only as estimates.

The main conclusion from Eqs. (23) and (24) is that in the case of $\rho$-mesons produced in high energy heavy ion collisions with the longitudinal and transverse momenta chosen above, the mass shift is small, but the width broadening is large and hardly a $\rho$ (or $\omega$) peak would be observed in $e^+e^-$ or $\mu^+\mu^-$ effective mass distributions. Let us estimate how sensitive are the results to a variation of $k_L$ and $k_T$. It can be easily seen that the mass shift will be small in all cases (say, $\Delta m_\rho \lesssim 50$ MeV). If we put $k_T = 0$ instead of $k_T = 0.5$ GeV, this will only weakly affect the mean value of $\sigma_{\rho N}$ and decrease $\sigma_{\rho\pi}$ by 20%. The latter results in decrease of $\Delta \Gamma_\rho$ in Eq. (24) by 80 MeV, i.e. within the limits of accuracy. The variation of $k_{\rho L}$ between 1 GeV and 10 GeV also results in variation of the same order (10-20%) in $\Delta \Gamma_\rho$ in Eq. (24).

As was mentioned above, the main uncertainty in our approach comes from the value of the nucleon density at the final stage of evolution which we put to be $\rho_0^N = 0.3$ fm$^{-3}$. If this density at the time of the $\rho$-meson formation would be, say, two times smaller, then we would have $\Delta \Gamma_\rho \sim 250$ MeV and the $\rho$-meson could be observed as a broad peak in
$e^+ e^−$ or $\mu^+ \mu^−$ mass spectrum. It should be mentioned, however, that the chosen above value of $\beta = v_\pi/v_N = 1$ is not the only possible, or even the most plausible. If we assume that $\beta = (r_\pi/r_N)^3$, where $r_\pi$ and $r_N$ are pion and nucleon electromagnetic radii, $r_\pi = 0.66$ fm, $r_N = 0.81$ fm, then $\beta \approx 0.55$. This choice of $\beta$ increases $\Delta \Gamma_\rho$ by a factor of 1.6.

Our qualitative conclusion is the following. In the central heavy ion collisions at high energies, $E/A \sim 100$ GeV, where a large number of pions per participating nucleon is produced, the $\rho$- (or $\omega$) peak will be (if at all) observed in $e^+ e^−$ or $\mu^+ \mu^−$ effective mass distributions only as a very broad enhancement. Inspite of the fact that in our approach we used a few assumptions, which may raise some doubts (hypothesis of noninteracting nucleon and pion matter at the final stage of evolution, the numerical value of nucleon density, etc.), we believe that this qualitative conclusion is still intact.

This conclusion is in qualitative agreement with the measurement of $e^+ e^−$ pair production in heavy ion collisions [22], where no $\rho$ peak was found and only a smooth $e^+ e^−$ mass spectrum between 0 and 1 GeV was observed. If, however, such a peak will be observed in future experiments, this would indicate that the hadronic (nucleon and pion) density at the final stage of evolution, where $\rho$-meson is formed, is low, lower even than the normal nuclear density.

Recently preliminary data have been presented [22], where in studying the $e^+ e^−$ mass spectrum in Pb - Au collisions at $E/A = 160$ GeV it was found that the $\rho$-peak is absent for $k_T(e^+ e^−) < 400$ MeV, but reappears as a broad enhancement for $k_T(e^+ e^−) > 400$ MeV. We do not see a possibility for this in the framework of our approach in the case of central heavy ion collisions. Moreover, we believe that for central collisions the absence of $\rho$-peak at low $k_T$ and its reappearance at higher $k_T$ will be hard to explain in any reasonable model. The only explanation we see for this effect (if it will be confirmed), is that in this experiment the peripheral $\rho$-meson production plays an essential role. In this case the $\rho$-mesons with higher $k_T$ have a larger probability to escape the collision region and decay as free ones.

Let us turn now to the case of low energy heavy ion collisions, $E/A \sim$ a few GeV. Consider as an example the case of heavy ion collisions at $E_{kin}/A = 3$ GeV and production of $\rho$-meson with the energy $E_\rho^{tot} = 1.2$ GeV in the forward direction. (This energy of $\rho$-meson was chosen, because our approach works better at higher $E_\rho$, and $\rho$ of this energy can be kinematically produced at such heavy ion energy). The number of pions, produced in $E_{kin}/A = 3$ GeV collisions can be found by extrapolation of the data [18] on Ni + Ni collisions. The data shows that $R_\pi$ is with a good accuracy linear in $\sqrt{s}/2 − m$. We find $R_\pi = 0.48$. As follows from the analysis of the data [18] at $E_{kin}/A = 1.93$ GeV, the energies of produced pions are rather small, $E_\pi \sim 200 − 300$ MeV. At such low energies it is reasonable to suppose for pions $< p_{L\pi} >= < p_{\perp\pi} >= 0.2$ GeV. Assuming (this assumption does not influence essentially the final results) that the mean transverse momentum of nucleon participants produced in the collision is the same as at high energy, $< p_{TN} > = 0.61$ GeV [15] (see above), we can construct the momentum distributions of nucleons. Then we are in a position to calculate the mean values of $\langle R_{\rho N} \rangle$, $\langle R_{\rho\pi} \rangle$, $\sigma_{\rho N}$ and $\sigma_{\rho\pi}$ for this case. The results are:

$$\langle R_{\rho N} \rangle = -0.54 \text{ fm}, \quad \langle R_{\rho\pi} \rangle = 0.30 \text{ fm}, \quad \langle \sigma_{\rho N} \rangle = 46 \text{ mb}, \quad \langle \sigma_{\rho\pi} \rangle = 13 \text{ mb}. \quad (25, 26)$$
For the $\rho$ mass shift and width broadening we have (at the same value of $\rho^0_N$ as before and $\beta = 1$):

\begin{align}
\Delta m_\rho &= 37 - 10 = 27 \, \text{MeV} , \quad (27) \\
\Delta \Gamma_\rho &= 250 + 35 = 285 \, \text{MeV} . \quad (28)
\end{align}

(The first numbers in Eqs. (28) and (29) refer to $\rho N$ scattering, the second ones to $\rho \pi$). The conclusion is that at low energy heavy ion collisions $\rho$-peak may be observed in $e^+ e^-$ or $\mu^+ \mu^-$ effective mass distribution as a broad enhancement approximately at the position of $\rho$-mass.

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