Kinematics Simulation Analysis of Packaging Robot with Joint Clearance

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Abstract. Considering the influence of joint clearance on the motion error, repeated positioning accuracy and overall position of the machine, this paper presents simulation analysis of a packaging robot — 2 degrees of freedom (DOF) planar parallel robot based on the characteristics of high precision and fast speed of packaging equipment. The motion constraint equation of the mechanism is established, and the analysis and simulation of the motion error are carried out in the case of turning the revolute clearance. The simulation results show that the size of the joint clearance will affect the movement accuracy and packaging efficiency of the packaging robot. The analysis provides a reference point of view for the packaging equipment design and selection criteria and has a great significance on the packaging industry automation.

1. Introduction
Packaging industry automation is the inevitable result of the development of science and technology. The movement of packaging equipment, the core of the entire production line part, directly affects the operating efficiency. Now a great many of parallel robot play a role of a box robot in the production[1] as the result of high stiffness, high precision and dynamic performance characteristics of the parallel mechanism. However, because of the joint clearance and other factors, the advantages of the mechanism cannot fully play out[2-5]. Therefore, it is very important to choose the packaging robot in the case of kinematics simulation analysis with the joint clearance. Some progress[6-11] has been made in the study of the impact of motion pair clearance on the performance of the mechanism. The earliest study was the revolute model of the plane mechanism [12]. Biswas [13] introduced a planar mobile clearance model in the literature. Wang [14] first established the space rotation clearance model.

Based on the previous research results, this paper analyzes the motion error of a packaging robot - 2-DOF planar parallel robot with the joint clearance.

2. Working principle of packaging robots
2-DOF planar parallel robot is mainly composed of the frame, moving platform and two branches connecting the frame with moving platform, as shown in figure 1. The left branch consists of the active arm and the slave arm, and the slave arm is composed of two symmetrical single rods, which increases the stiffness and carrying capacity of the branch. The right branch is composed of the active arm, the active arm link, the hinge plate, the slave arm and the slave arm link, the link connecting the active arm with the active arm and the link connecting the slave arm with the slave arm form two parallelogram structures by hinge plates [15]. Two sets of parallelogram structure not only limit the rotation of the end, to achieve the end position and attitude of the decoupling. At the same time, the parallelogram structure can also improve the overall stiffness of the robot end in the vertical direction[16]. Both the active link and both ends of driven arm are hingedly connected with a smaller angle to reduce assembly errors causing by manufacturing errors. The whole parallel robot all use revolute pair to achieve a single plane translating along $x$, $y$ direction, to improve the speed and the acceleration of the moving platform and to satisfy a fast, stable packing operations requirements.

![Figure 1. 2-DOF planar parallel robot.](image)

3. Kinematic analysis of mechanisms

Taking the 2-DOF planar parallel robot as the object of study, the mechanism can be simplified into a symmetrical five link articulated mechanism in the case of the kinematic analysis of the parallel mechanism. The schematic diagram of the mechanism is shown in figure 2:

![Figure 2. Structure diagram of 2-DOF planar parallel robot.](image)
The symmetry center point of the two active arms is the origin, and the reference coordinate system \( O-xy \) is shown in figure 2. \( 2r \) is the distance between the rotating shaft of the active arm of the frame; \( l_1 \) is the robot's active arm length; \( \mu_1 \) \( \mu_2 \) are the unit vector of the left and right active arm; \( l_2 \) is the slave arm length; \( \sigma_1 \) \( \sigma_2 \) are the unit direction vectors of the left and right driven arms; \( O'(x, y) \) is the moving platform position vector; \( s \) is the position vector of \( O' \) with respect to \( O \). The joint angle of the robot's active arm is \( \alpha_1 \) and \( \alpha_2 \), and the rotation angle of the driven arm joint is \( \beta_1 \) and \( \beta_2 \).

As shown in figure 2, the position of the mechanism is solved with respect to the known active arm angle \( \alpha_i \) \( i = 1, 2 \), and the position vector \( O'(x, y) \) of the moving platform is solved. According to the mechanism geometry [17], the following equations are constructed in the reference coordinate system.

\[
\begin{align*}
- r + l_1 \cos \alpha_2 + l_2 \cos \beta_2 - l_1 \cos \beta_1 - l_2 \cos \alpha_1 + r &= 0 \\
l_1 \sin \alpha_2 + l_2 \sin \beta_2 - l_1 \sin \beta_1 - l_2 \sin \alpha_1 &= 0
\end{align*}
\]

(1)

The moving platform position vector reference point \( m \) can be expressed as:

\[
\begin{align*}
x &= - r + l_1 \cos \alpha_2 + l_2 \cos \beta_2 \\
y &= l_1 \sin \alpha_2 + l_2 \sin \beta_2
\end{align*}
\]

(2)

or

\[
\begin{align*}
x &= r + l_1 \cos \alpha_1 + l_2 \cos \beta_1 \\
y &= l_1 \sin \alpha_1 + l_2 \sin \beta_1
\end{align*}
\]

(3)

4. Kinematic error analysis considering the revolute clearance

For packaging robot, servo motor and the driving arm connection clearance is not obvious in the mechanical structure, and between the driving arm and the driven arm, a driven arm and moving platform with deep groove ball bearings, there is a clearance of a certain size. Therefore, the clearance between \( B_1, B_2 \) and \( O' \) is considered in the analysis.

4.1. Model of revolute clearance

![Clearance model](image1)

(a) Clearance model

![Equivalent length of slave arm](image2)

(b) Equivalent length of slave arm

**Figure 3.** Rotation clearance model.

Model of revolute clearance uses two-state motion model [6], that is, only consider contact and separation of two states for the revolute clearance, and the revolute clearance is equivalent to a non-quality virtual link, as shown in figure 3(a). Assuming that the radius and the axis radius of the
revolute pair are $R_B$ and $R_D$, respectively, the size of the motion pair clearance $c$ is

$$c = R_B - R_D$$

Due to the existence of the revolute clearance, the axis of the revolute pair can move freely in the hole, and the virtual massless link can be described by the virtual massless link length $N$ and the corner $M$. Their range of value is

$$r \in [0, c]; \beta \in [0, 2\pi]$$

Considering the existence of virtual massless link, the length of the follower arm changes $l_2'$ from $l_2$, and the angle changes $\beta_2'$ from $\beta_2$, the value of virtual massless link $l_2''$ has a range of

$$l_2 - (r_2 + r_2') \leq l_2'' \leq l_2 + (r_2 + r_2')$$

The link length error caused by the revolute clearance is

$$\Delta l_2 \in [-(r_2 + r_2'), (r_2 + r_2')] \square l_2$$

The length error of the link caused by the clearance is much smaller than the length of the link.

4.2. Model for revolute error transfer

The equation (2) and the equation (3) are integrated only in case that the clearance error of the revolute pair is taken into account.

$$\begin{align*}
x &= j r + l_1 \cos \alpha_i + l_2 \cos \beta_i & i = 1, j = 1 \\
y &= l_1 \sin \alpha_i + l_2 \sin \beta_i & i = 2, j = -1
\end{align*}$$

Partial derivation for $l_1$, $\beta_1$ and $O'(x, y)$ can derive

$$\begin{align*}
\Delta x &= \Delta l_2 \cos \alpha_i - l_2 \sin \beta_i \Delta \beta_i & i = 1, 2 \\
\Delta y &= \Delta l_2 \sin \alpha_i + l_2 \cos \beta_i \Delta \beta_i
\end{align*}$$

Write the above formula as a matrix

$$\begin{bmatrix}
1 & 0 & -l_2 \sin \beta_i & 0 \\
0 & 1 & l_2 \cos \beta_i & 0 \\
1 & 0 & 0 & l_2 \sin \beta_2 \\
0 & 1 & 0 & l_2 \sin \beta_2
\end{bmatrix}
\begin{bmatrix}
\Delta x \\
\Delta y \\
\Delta \beta_1 \\
\Delta \beta_2
\end{bmatrix} =
\begin{bmatrix}
\beta_1 & 0 \\
0 & \beta_1 \\
0 & \beta_2 \\
0 & \beta_2
\end{bmatrix}
\begin{bmatrix}
\Delta l_2 \\
\Delta l_2
\end{bmatrix}$$

Where $\Delta l_2$ is the virtual link length error caused by the revolute clearance. Through the above formula, the error from packaging robot and arm angle can be easily obtained.

4.3. Analysis on error simulation results

According to the contact model of the material parameters, the clearance size of the pair is determined to be 0.1mm, which is mainly for the error analysis of the moving platform of the box robot to the position of the grab.
It can be seen from figure 4 that the errors caused by the revolute clearance are symmetrical about the axis $y = x$, and the maximum error occurs at the positions where errors of the left and right slave arm is $\Delta l (0.2,0.2)$ and $(0.2,-0.2)$, where the angular variation of the left and right arms is $(-5.0142^\circ, 5.0142^\circ)$ rad and the direction error of the moving platform is $(-0.4111,0.4111)$ mm. The variation range of the moving direction is $(-0.2049,0.2049)$ mm; the total error range of the moving platform is $(0,0.4482)$ mm. The range on position error of robot movement platform is strictly control in 0.5 mm to meet the packing operation requirements.

### 5. Simulation analysis of packaging robots with the revolute clearance

After analyzing the kinematic error of the packaging robot, a model with a clearance of 0 mm, 0.1 mm, and 0.2 mm was established by Solidworks. In the Adams, the contact force function IMPACT is used to add the parameters such as stiffness, material damping, stiffness index, penetration depth, static friction speed, dynamic friction speed, static friction coefficient, dynamic friction coefficient and collision recovery coefficient of two contact parts. The influence of the motion error caused by the rotation clearance on the pose of the whole body is simulated.

The simulation results show that the curves of the driven arm angle and the moving platform position are shown in figure 5. From figure 5(a) and (b), it can be seen that the curve of the rotation angle from the slave arm is smooth and round when there is no clearance, and when the rotation clearance is 0.1mm and 0.2mm, there is a small floating up and down, but the overall basic coincidence for the
curve of the slave arm angle is based on the clearance-less curve; From figure 5(c), (d) and (e), it can be seen that when the clearance value is 0.1 mm and 0.2 mm, the moving plate along the x direction, the y direction and the position curve are basically coincident with the clearance-less curve, But there is a slight fluctuation. However, considering the above-mentioned variation curve, it can be concluded that the change of the pose of the whole body is increasing with the increase of the rotation clearance, and the moving platform is more obvious away from the ideal position. The size of the rotation clearance will affect the motion precision and packing efficiency. So the size of the clearance for hole and shaft conjugation should be strictly controlled in the machining process.

(a) Chang for left arm rotation angle

(b) Chang for right arm rotation angle

(c) The moving platform changes in the x direction
6. Conclusion
Through the theoretical research and simulation analysis of the mechanism on a packaging robot, the following conclusions are obtained:
1) In this mechanism, verify that the mechanism meets the packing requirements by considering the motion error analysis of the rotation clearance, Which provides a reference direction for improving the precision of the selected and designed packing equipment.
2) Through the simulation analysis of the whole robot, it is found that the motion accuracy becomes lower as the rotation clearance increases. Therefore, the size of the clearance for hole and shaft conjugation should be strictly controlled in the machining process.
The author only takes into account the error of the box robot caused by rotating the auxiliary gap, does not consider the error caused by the geometric size and assembly precision, and does not compensate for the error caused, which will become the further research direction.

7. Acknowledgments
This research was supported by the Natural Science Foundation of Hebei Province (grant no. E2015402130)

8. Reference
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