Transfer of water molecules in presence of low density water vapor

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Abstract. Problems of molecular transfer in gas environment are considered with assumption that part of water molecules may change their properties to provide transfer process. This change consists in increase of dimension of vector space, which used to describe transfer process, from 3 to 4. Other cases of extending of dimension of vector space are particular considered. At first, relations between affine group and Markov chain is considered. An action of Markov chain matrix as affine matrix is analyzed with result that point with coordinate number equal 1 is fixed point on straight line.

Movement of water molecule in conditions of rarefied gas environment or due to diffusion process are well-known limit cases of behavior of molecules in gas. However, it is interesting to consider an intermediate case when molecule is freely transferred at long distance much greater than free mean path. Transfer processes for mass, heat, charged particles in electrochemical science are well known but there is very little information about the molecular transfer.

Before to undertake experiments, it is necessary to possess some mathematical models of molecular transfer process. This task is resolved by an affine transformation group as more suitable tool to describe the transfer in physical space. The affine transformation group is multiplicative group with the group operation or composition rule which can be written as

\[ (B, b) \circ (A, a) = (AB, Ab + b) \]  

(1)

where A,B are scalars and a,b are vectors of \( \mathbb{R}^3 \) [1].

The affine transformation group is non-commutative, as one can see from (1), therefore its elements must be represented by matrices with matrix multiplication as the group operation. Simplest square matrix is 2x2 with four independent items. Number of elements may be reduced by using a subgroup of triangular matrices. Next simplification occurs if the element \( a_{22} \) will be equal 1. So, we obtain representation matrix in the next form with \( a_{11}, a_{12} \) still indefinite elements. These elements essentially define the affine transformation and can be taken from (1) \( a_{11} = A, B \) and \( a_{12} = a, b \).
The element $a_{11}$ always is scalar that gives stretching $a_{11}>1$ or pressing $a_{11}<1$ for every vector of $\mathbb{R}^3$. The element $a_{12}$ is any nonzero vector of $\mathbb{R}^3$ that defines the transfer from point $p$ to $p + v$ point where $p$ is the radius vector of selected point in $\mathbb{R}^3$.

The next problem of how to obtain the action of affine group on $\mathbb{R}^3$ is resolved below. If scalar $a_{11} = A$ in (2) is zero then both items of the first column become zero and it be possible to change the second matrix in (2) by vector.

$$
\begin{pmatrix}
B & b \\
0 & 1
\end{pmatrix}
\begin{pmatrix}
A & a \\
0 & 1
\end{pmatrix}
= 
\begin{pmatrix}
AB & aB + b \\
0 & 1
\end{pmatrix}
$$

(2)

These vectors essentially differ from $v \in \mathbb{R}^3$ because they have a scalar index 1 in additional one-dimensional space $C$, so water molecule obtain a representation suitable for transfer in space. One can consider these vectors as a collection of some elements of affine group which however, do not form subgroup [2].

The most known case of an increase of vector space dimension in $\mathbb{R}^3$ is given by the quaternion coordinate system which extends the physical three dimensional space (basis vectors $i, j, k$) to an abstract four dimensional vector space $\mathbb{R}^4$ with basis vectors $1, i, j, k$ [3].

There are two different approaches as to how to consider scalar variables in vector spaces. The first approach is based on assumption that for every $v \in \mathbb{R}^3$ and for every scalar $c$ both vector $v$ and $cv$ belong to the same vector space $\mathbb{R}^3$. This is customary point of view. The second one based on assumption that scalar variables form an additional one-dimension vector space $C$, so the overall dimension of space is increased and $cv \in \mathbb{R}^4$. This approach is used in constructions of complex and quaternion variables. There is another case, when the vector space dimension increase, it occurs due to using curve linear coordinate system in $\mathbb{R}^4$ [4].

The scalar index 1 is not unique it may be replaced by scalar index $C$ which controls length of vector $b$.

$$
\begin{pmatrix}
B & b \\
0 & 1
\end{pmatrix}
\begin{pmatrix}
1 \\
C
\end{pmatrix}
= 
\begin{pmatrix}
11 \\
C
\end{pmatrix}
$$

(3)

Now we can distinguish collections of water molecules using index $C$.

Let us consider a specific example in which molecules are transferred between two parallel solid surfaces. A coordinate system of our construction is a line perpendicular to both surfaces with coordinate origin lies outside of area between surfaces. The first surface has coordinate $s_1$, the second one $s_2$ with condition $s_2>s_1$. There are two possible ways to describe the transfer process using affine group transformation. The first relates with stretch of vector $s_1$ the second with addition some vector to $s_1$. Corresponding transformations may be written as

$$
\begin{pmatrix}
s_2/s_1 \\
0 & 1
\end{pmatrix}
\begin{pmatrix}
s_1 \\
1
\end{pmatrix}
= 
\begin{pmatrix}
s_2 \\
1
\end{pmatrix}
\begin{pmatrix}
1 & s_2-s_1 \\
0 & 1
\end{pmatrix}
\begin{pmatrix}
s_1 \\
1
\end{pmatrix}
= 
\begin{pmatrix}
s_2 \\
1
\end{pmatrix}
$$

(5)

Doubling of transfer process in case of linear moving may possess an experimental confirmation as doubling of intensity of transfer.
There is an unexpected relation between the particular example that was considered above and Markov chain because both formalisms are used $2 \times 2$ matrices of equal structure, so Markov matrix may be considered as affine matrix that was appropriated for some transfer. The matrix for Markov chain has the additional property that sum of items for every row is equal 1. This condition arises an interesting movement of considered object, the result is that no transfer occurs if molecule has coordinate value $s=1$.

$$\begin{pmatrix} k & 1-k \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$  \hspace{1cm} (6)

Matrix in (6) is given in form appropriated for the Markov chain matrix with scalar coefficient $k<1$. Moreover, one can expect that collection of molecules which coordinates lie near $C=1$ will show small displacement under action of the affine group.

In conclusion we note that a serious problem remains in relating the mathematical formalism developed above to physical understanding of their reality. However, we suppose that some properties given by mathematical constructions might arise within experimental studies. In particular, water molecules with 4D-description will not interact with ordinary water molecules and therefore have large the molecular mean free path.

**Conclusion**

In conclusion we note that a serious problem remains in relating the mathematical formalism developed above to physical understanding of their reality. However, we suppose that some properties given by mathematical constructions might arise within experimental studies. In particular, water molecules with 4D-description will not interact with ordinary water molecules and therefore have large the molecular mean free path.

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