Abstract

We explore a new class of simplified extensions to the Standard Model containing a complex singlet scalar as a dark matter candidate accompanied by a vector-like lepton as a mediator, both charged under a $Z_3$ symmetry. In its simplest form, the new physics couples only to right-handed electrons, and the model is able to accommodate the correct dark matter relic abundance around the electroweak scale up to several TeV evading the strongest constraints from perturbativity, collider and dark matter searches. Furthermore, the model is capable to enhance naturally positron fluxes by several orders of magnitude presenting box-shape spectra, and keeping sizable signals of gamma-rays. This framework opens up a lot of phenomenological possibilities depending on the quantum charge assignments of the new fields.

1 Introduction

Over the years, simple extensions to the SM have been proposed to account for the elusive dark matter (DM) particle, with many of them presenting distinctive observational signatures for different experimental setups. One of the simplest extensions to the SM to account for DM and distinctive indirect detection signals is the class of models with two fields transforming under a discrete $Z_2$ symmetry: real scalar DM plus a vector-like lepton, or a Majorana fermion plus a scalar singlet field \[ \psi \]. Focusing on the former, these class of models present distinctive gamma-ray signatures via Bremsstrahlung radiation sensitive to Fermi-LAT or even to the Cherenkov Telescope Array (CTA), but in terms of anti-particles, they lack of sizable fluxes.

Even when $Z_2$ symmetry is the simplest assumption to explore models with DM candidates, higher symmetries could shed light on interesting phenomenological aspects, without complicating things in a significant manner. For instance, a complex scalar with $Z_3$ symmetry disentangles the freeze-out relic values with direct detection observables \[ \psi \], recovering some parameter space of singlet DM at the electroweak (EW) scale in comparison to its $Z_2$ analogue real singlet \[ \psi \]. Ideas for fermionic DM \[ \psi \], multicomponent dark matter \[ \psi \], complex singlet plus a second doublet scalar \[ \psi \], neutrino mass generation \[ \psi \], inert doublets \[ \psi \] and the positron excess \[ \psi \] can be realized based on this new symmetry, to mention some of them.

In this work we explore for the first time a simple scenario in which we extend the SM model with two fields, a complex singlet scalar $S$ and a vector-like lepton (VLL) $\psi$, with both fields charged identically under the same $Z_3$ symmetry. Provided that $m_S < 2m_\psi$, the complex singlet becomes stable, becoming a DM candidate. The simplest realization of this scenario is when the new fields are $SU(2)_L$ singlets, leptophilic (also called lepton portal DM) $\psi$, and when the new fields couple only to right-handed electrons. We show that DM semi-annihilations play an important role in both relic density calculation and indirect detection signals. Even when direct detection appears at one-loop level, its effects are rather strong, but
since the couplings tend to be milder (contrary to the anal-
logue $Z_2$ version with a real scalar DM and a VLL mediator
\cite{13 50}, it is possible to evade these bounds keeping
DM at the GeV-TeV scale.

Interestingly, in some regions of the parameter space,
this minimal $Z_3$ leptophilic scenario may present sizable
fluxes of both gamma-ray and positron fluxes, with the lat-
ter featuring a box-shape spectra, then distinctive for ast-
rophysical probes. We constrain the model with bounds
from CTA projections and AMS-02 upper bounds data,
showing exclusions in some portions of the parameter space
below the TeV, and other regions requiring more sensible
experiments to be tested.

The paper is organized in the following way. In section 2 we introduce the model. In section 3 we present the most
relevant constraints on the model. In section 4 we present
the resulting parameter space of the model after imposing
the most relevant constraints, and finally in section 5 we
state the conclusions.

2 Model

Besides the SM particle content, we consider leptophilic
new physics consisting of a complex singlet scalar $S$ and
a vector-like lepton (VLL) $\psi$ with $Y = -1$, both charged
under a global $Z_3$: $S \rightarrow e^{i2\pi/3}S$ and $\psi \rightarrow e^{i2\pi/3}\psi$. We
assume that the new sector only couples to the first right-
handed lepton generation, in order to avoid lepton flavor
violating processes such as $\mu \rightarrow e\gamma$. In this way, the La-
grangian reads as

$$\mathcal{L} = \bar{\psi}(\partial + m_\psi)\psi + (y_\psi S \bar{\psi} e_R + \text{h.c.}) - V(H, S),$$

where the potential is given by

$$V(H, S) = \mu_H^2 |H|^2 + \lambda_H |H|^4 + \mu_S^2 |S|^2 + \lambda_S |S|^4 + \lambda_{SH} |S|^2 |H|^2 + \frac{\mu_3}{2} (S^3 + S'^3).$$

The parameters $m_\psi$, $y_\psi$ and $\mu_3$ can be made real by field
redefinitions. The singlet scalar does not acquire vacuum
expectation value (vev), and after EWSB (i.e. $\mu_H^2 < 0$)
the masses of the scalars become

$$m_H^2 = -2\mu_H^2,$$
$$m_S^2 = \mu_S^2 + \lambda_{SH} v_H^2/2,$$

where we recognize $m_h = 125$ GeV as the SM Higgs boson,
and $v_H = 246$ GeV. The relevant parameter space of the
model is then

$$\{m_\psi, m_S, g_\psi, \lambda_{SH}, \mu_3\}. \quad (5)$$

The $Z_3$ symmetry ensures the stability of the DM particle
provided $m_S < 2m_\psi$. Since we are dealing with a complex
scalar field, the DM candidate is not self-conjugate.

3 Constraints

In this section we review the most relevant constraints on
the model. We focus our attention to constraints that are
relevant for new physics in the mass range GeV-TeV, and
couplings bigger than $O(10^{-2})$. In most of the constraints
we assume $\lambda_{SH} = 0$, in order to emphasize the novelty of
the new physics related to the new Yukawa-like interaction
in eq. 1 and the cubic interaction in eq. 2.

3.1 Theoretical

We assume that the Yukawa-like coupling $y_\psi$ is positive
(negative values do not change the physical amplitudes)
and that perturbativity sets $y_\psi < \sqrt{4\pi}$. Additionally, the
stability of the EW vacuum state that

$$\lambda_H > 0, \quad \lambda_S > 0, \quad \lambda_{SH} > -\sqrt{\frac{2}{3}} \lambda_H \lambda_S. \quad (6)$$

Assuming perturbativity, we also assume that the maxi-
num values of each coupling in 6 is at most $4\pi$.

On the other hand, the value of $\mu_3$ can not be too large
because it enters in conflicts with the $Z_3$-breaking extrema,
threatening the vacuum stability of the scalar potential. In
\cite{37} was found that $\mu_3$ must fulfill the following relation

$$\max(\mu_3) \approx 2\sqrt{\frac{\lambda_S}{\delta}} m_S, \quad (7)$$

with $\delta$ a dimensionless parameter which regulates whether
the SM vacuum is or not a global minimum. Here we
simply take $\delta = 2$ (an absolute stable vacuum), and con-
sidering the maximum perturbative value for $\lambda_S$, in this
work we will have that $\max(\mu_3) \approx 2m_S$. 

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3.2 Collider

At hadron colliders, pair production of $\psi$ via Drell-Yan processes is the main production mechanism. Since we focus on the freeze-out of the complex scalars, we scan the Yukawa-like coupling in the range $g_\psi \sim O(10^{-2} - \pi)$, which makes all the subsequent decays of $\psi$ short-lived, then collider bounds do not depend on $g_\psi$, but only on the pair $(m_\psi, m_S)$. We use the limit projections obtained in [51] for the model in eq. [1] where proton-proton events were simulated at 14 TeV LHC using an integrated luminosity of 100 fb$^{-1}$. These bounds are in agreement to the recent results obtained in [52].

Additionally, we include bounds from compressed spectra, i.e. $m_\psi \approx m_S$, taken from [53]. These bounds are constructed by simulated proton-proton collisions at the LHC at $\sqrt{s} = 13$ TeV, with a real scalar plus a VLL with $Y = -1$, and both $SU(2)_L$ singlets. Even when these limits are obtained for the new physics coupled to muons of the SM, the bounds end up being similar to our case, in which the new physics couples only to $e^\pm$ [50].

Finally, if the Higgs portal is open and $m_h > 2m_S$, then the Higgs decay into $S(S^*S^*)$ contributes to the decay width $\Gamma_h$ of the Higgs given by [58]

$$\Gamma_{inv} = 2 \times \frac{\lambda_{SH}^2 v_H^2}{32 \pi m_h} \sqrt{1 - \frac{4m_S^2}{m_h^2}},$$

with the factor 2 in the right side account for the Higgs decay into both $SS$ and $S^*S^*$. Upper bounds on $Br(h \rightarrow inv) = \Gamma_{inv}/(\Gamma_{SM} + \Gamma_{inv})$, with $\Gamma_{SM} = 4.07$ MeV [54], set that $Br(h \rightarrow inv.) < 0.19$ at 95% C.L. [54], implying that $\lambda_{SH} \lesssim 10^{-4}$.

3.3 Relic Abundance

In the calculation of the relic abundance, we have implemented the model on LanHEP [55] and on MicrOMEGAS 5.2.7.a code [56]. We focus on the freeze-out of $S(S^*)$ particles, with the relic density determined by the processes presented in Fig. 1 plus Higgs portal contributions and higher order corrections. One of the novelties of this work is the presence and relevance of the diagram (a) of

![Figure 1: Leading processes producing the relic abundance of $S$ at freeze-out. Radiative corrections, Higgs portal contributions, and the corresponding CP transformed process are not shown.](image)

Fig. 1 with $\langle \sigma v \rangle_{SS \rightarrow e^+ e^-} \propto \mu_3^2 g_\psi^2$ (equivalent for its CP-conjugate process) occurring in the s-wave. To exemplify the influence of this process in the calculation of the relic abundance, in Fig. 2 (above) we show the values of the relic density calculation as a function of $m_S$ for different values of $\mu_3$, keeping the rest of the parameters shown at the top of the plot. Notice that deviations from $\mu_3 = 0$ curve (dashed black) in the range $m_S \lesssim m_\psi$, where the process (a) can be produced on-shell, although for $m_S \lesssim m_\psi/2$ annihilations of the type $SS(S^*S^*)$ are effective due to the thermal tail distribution. In this way, provided that $m_\psi \lesssim m_S \lesssim m_\psi$, the influence of the new physics dictated by diagram (a) may produce strong effects in the relic density calculation.

A $Z_2$ version of this model has been studied in the past, consisting of a real scalar with a VLL mediator, both $SU(2)_L$ singlets (we call that model $Z2R$) [5, 13]. The resulting Lagrangian is similar to [1] without the $\mu_3$ term. In Fig. 2 (below) we compare the deviations in the Yukawa couplings in our model (blue curves) with respect to the ones obtained in $Z2R$ (grey lines), with the parameters subject to the correct relic abundance $\Omega_c h^2 = 0.12$ [57]. The new annihilation process from the $\mu_3$ term makes the Yukawa couplings in the $Z_3$ model milder, independent of the mass shift $\mu \equiv m_\psi/m_S$, provided $m_S \lesssim m_\psi$. This fact will have interesting effects on the phenomenology of this work.

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1. We have checked that the new physics contribution to the decay $Z \rightarrow e^+ e^-$ at one-loop results to be completely negligible to constraint the parameter space of the model.
charge-radius operator $\mathcal{L} \sim 2 h_\gamma \partial_\mu S^\dagger \partial_\nu S F^{\mu\nu}$ \cite{50,51,59}, with the total cross section for the DM-nucleon system given by

$$\sigma_{SN} = \frac{Z^2 e^2 C^2 (m_e, m_\psi) \mu_{SN}^2}{8 \pi A^2},$$

with $Z$ the atomic number, $e$ the electric charge, $\mu_{SN} = m_m N/(m_S + m_N)$, where $m_N = 0.94$ GeV is the nucleon mass, and $C(m_e, m_\psi)$ in the limit $m_e \ll m_\psi$ is given by

$$C(m_e, m_\psi) = -\frac{g_{\psi e}^2 e}{16 \pi^2 m_\psi} \left( 1 + \frac{2}{3} \log \left( \frac{m_e^2}{m_\psi^2} \right) \right).$$

In the case of Higgs portal interactions, the spin-independent cross section is present at tree-level. If the scalar DM gives the correct relic abundance, we have that the Higgs portal coupling must satisfy \cite{48}

$$\lambda_{SH} \leq \left( \frac{4 \pi m_b^4 m_\psi^2 \sigma_{Xe}}{f_N \mu_{SN}^2 m_N^2} \right)^{1/2},$$

with $\sigma_{Xe}$ the upper bounds given by XENON1T, and the Higgs-nucleon coupling is given by $f_N \approx 0.3$.

### 3.5 Indirect detection

Searches of gamma-rays and positrons are the most important constraints on this model from indirect detection. Anti-protons come from $SS^* \rightarrow e^+ e^- Z$ process via the decay and hadronization of the $Z$ boson, but bounds from anti-protons are expected to be even smaller than what is obtained in $Z2R$ \cite{13}, since once $m_S$ takes sizable values, the values of the Yukawa couplings tend to decrease to keep the correct relic abundance (see Sec. 3.3), in turn decreasing the annihilation of the aforementioned process. Therefore, we do not expect important constraints from anti-protons. In the following, we detail about gamma-ray and positron constraints exclusively in the mass regime $m_\psi/2 < m_S \leq m_\psi$.

#### Gamma-rays

Since we are interested in the mass regime previously described, this translate into $\mu \ll 10$, then one-loop annihilations $SS^* \rightarrow \gamma \gamma, \gamma Z$ are negligible in comparison to $SS^* \rightarrow e^+ e^- \gamma$ radiative process \cite{13} (see Fig. 3).
row). This latter process $2 \rightarrow 3$ is the leading gamma-ray source in the parameter space relevant for the present analysis here. Additionally, as it is shown in Fig. 2 below, in this mass regime we expect smaller Yukawa couplings with respect to the ones obtained in $Z2R$, then decreasing gamma-ray signals at some extent. To exemplify this fact, in Fig. 2(above) we show the corresponding values for the average cross section today into gamma-rays coming from $2 \rightarrow 3$ process for different $\mu$ values, fixing $g_\psi$ to the correct relic abundance value and $\mu_3 = 2m_S$. The blue lines, representing the predictions of the $Z_3$ model, tend to be reduced than the ones predicted by $Z2R$ (grey lines), then relaxing the bounds coming from Fermi-LAT and CTA. Notice that $\mu_3 = 2m_S$ is just an arbitrary election, and smaller $\mu_3$ will force to increase $g_\psi$ to maintain the correct relic abundance, automatically increasing the gamma-ray signal, and then approaching experimental bounds.

The predictions in Fig. 4(above) for the model of this work are understood by the fact that the relic density is obtained mainly by $\langle \sigma_{SS\rightarrow e^+e^-} \rangle$, then since $\Omega_S h^2 \sim Y_0 m_S$ with $Y_0 \sim \langle \sigma_{SS\rightarrow e^+e^-} \rangle^{-1} \propto m_S^2/g_\psi^2$, we have

$$\Omega_S h^2 \sim m_S^3/g_\psi^2. \quad (11)$$

This is, as $m_S$ increases, $g_\psi$ must increase too to keep the correct relic abundance, which is translated into the fact that higher DM masses produce stronger gamma-ray lines signals.

**Positrons**

In the annihilation of DM today, the following processes contribute to the positron fluxes:

1. $SS^* \rightarrow e^+e^-$,
2. $SS^* \rightarrow e^+e^-\gamma(Z)$,
3. $SS \rightarrow e^+e^-S^*$ (and its CP-conjugate process).

The process (1) annihilates in the p-wave (similar to Majorana DM), then velocity suppressed. Process (2) proceeds

![Diagrams](image)

**Figure 3:** Diagrams (a), (b) and (c) are Bremsstrahlung processes appearing from the $SS^*$ annihilation. The diagrams (d) and (e) correspond to the annihilation $SS^*$ (equivalent for $S^*S^*$), giving rise to the box-shape spectra for electrons and positrons.

in the s-wave but suppressed by $\alpha_{em}$ and by the phase-space, and additional pair of $e^\pm$ may be emitted from $\gamma/Z$. For $m_\psi/2 \lesssim m_S < m_\psi$, the process (3) results to be the dominant one, increasing $e^\pm$ fluxes by several orders of magnitude in comparison to the other two processes (provided $\mu_3 \neq 0$), since the diagram of Fig. 3(d) proceeds in the s-wave, without helicity nor electromagnetic suppressions.

As an example of the enhancement of the positron flux measured at earth, in Fig. 3(below) we show the energy flux of $e^+$ obtained with Micromegas 5.0.7a, for the parameter values specified in the plot. It is clear that as the blue and green curve fulfill $2m_S \gtrsim m_\psi$, the expected flux strongly increases in comparison to either $m_S < 2m_\psi$ (gray curve) or when $\mu_3 = 0$ (dashed green curve), with the latter two cases obtained mainly by process (2), and in a sub-leading proportion (near 1%) by process (1). Notice that the high fluxes presented here contain box-shaped positron spectra, since $\psi^+$ in diagram (d) decays in flight into $e^+S^*$ (for details of the kinematics see the Appx. A).

To our knowledge, constraints on the semi-annihilation $\langle \sigma_{DMDM\rightarrow e^+e^-DM^*} \rangle$ have not been constructed yet. However, in Ref. constraints have been set for different annihilation channels based on the data of AMS-02. Here, from the latter paper we take the bounds on $\langle \sigma_{DMDM\rightarrow e^+e^-} \rangle$.
Figure 4: (above) Average annihilation cross section $\langle \sigma_{e^+e^-\gamma} \rangle$ as function of the DM mass, with the blue curves representing the results obtained in the $Z_2R$ model, and the grey ones obtained in the $Z2R$ model. The Yukawa coupling $g_\psi$ has been fixed to the value leading to the observed relic abundance. The orange and red solid lines represent bounds from Fermi-LAT/HESS and expected sensitivity for CTA, respectively. (below) Positron flux as a function of the positron energy for the parameters shown in the figure.

as a reference to constraint $\langle \sigma_{\text{DMDM} \to e^+e^-} \rangle$. This is an estimation, but for simple kinematics, we expect that the real bounds on $\langle \sigma_{\text{DMDM} \to e^+e^-} \rangle$ be milder due to phase-space suppression.

4 Scan Results

In this section we show the viable parameter space of the model for some values of $\mu_3$ after imposing all the constraints previously described. We keep the Higgs portal equal to zero, and in the next section we discuss about the effects of this portal. In Fig. 4, we show the resulting parameter space for $\mu_3 = 0$, $m_S/2,2m_S$ (from top row to bottom row, respectively), keeping the LHC bounds as the black contour (LHC 14 TeV projection), the orange region (compressed LHC 13 TeV spectra) and the cyan for VLL searches (see Sec. 3.2), green regions in each plot are the parameter space where $m_S > m_\psi$, then $S$ being unstable, and the purple region showing the parameter space in which co-annihilations contribute with more than 20% of the relic generation. In the first column of plots we consider the parameter space after imposing perturbativity and the correct relic abundance, whereas in the second column we present the parameter space left after applying CTA bounds (in the first row) and AMS-02 bounds (in the second and third row). This seemingly arbitrary election between CTA and AMS-02 is simply due to the fact that in the first case are CTA bounds the leading ones, whereas in the other two cases is AMS-02. Finally, the third column shows the parameter space left after imposing XENON1T.

Considering the results of the first row of Fig. 4 (i.e. $\mu_3 = 0$) the correct relic abundance is obtained for $g_\psi\approx O(1-10)$, although for smaller $m_S$ the coupling $g_\psi$ tends to increase outside the perturbative limit. CTA discard a few points of the parameter space as it is shown in the plot in the middle, whereas XENON1T ruled out most of the parameter space up to 1 TeV. A small fraction of points in the purple region $m_S\approx m_\psi$ survive direct detection, similar to what is obtained in the analogue $Z2R$ model. In the latter model most of the exclusion comes from CTA (direct detection is suppressed at two-loops), whereas in the present model, CTA bounds are milder due to the smaller required $g_\psi$, but direct detection at one-loop makes the strongest exclusion.
Figure 5: Color maps for $g_\psi$ in the mass plane ($m_\psi, m_S$), for $\mu_3 = 0, m_S/2$ and $2m_S$ (rows from up to down, respectively). In all the plots we consider $\lambda_{SH} = 0$. The green regions in each plot is forbidden due to the fact that $S$ becomes unstable. The black solid line corresponds to the LHC projection bounds for 14 TeV, the cyan region corresponds to collider constraints for VLL, and the orange region is the exclusion from compressed spectra for LHC at 13 TeV. The dashed line is a reference in which above it the $SS(S^* S^*)$ annihilations start to become efficient, whereas the small purple region represent the place in which co-annihilations start to be effective.
For $\mu_3 \neq 0$ shown in the second and third rows of Fig. 5, notoriously smaller $g_\psi$ are obtained in the region $2m_S > m_\psi$. In these cases, CTA constraints are less strong due to the smallness of $g_\psi$, but constraints from AMS-02 become relevant in the low mass region of $2m_S > m_\psi$, excluding DM masses up to 200 GeV, approximately. Furthermore, for $\mu_3 = 2m_S$, XENON1T bounds do not rule out the parameter space in the region $2m_S > m_\psi$, whereas as $\mu_3 = m_S/2$ there is still some parameter space left.

In resume, a parameter space opens up below the TeV region passing the strongest constraints and presenting high sensitivity to experimental searches. As a mode of complementarity, in Fig. 6 we show the prospects of having DM with masses up to 10 TeV, where we have included XENONnT and CTA bounds, considering $g_\psi < \sqrt{4\pi}$. Notice that there is still a vast parameter space left after the imposition of the strongest constraints.

5 Discussion and Conclusions

In its simplest form, we have explored a novel model containing two new fields ($S, \psi$), both transforming under a $Z_3$ symmetry. As $m_S < m_\psi$, $S$ is stable becoming a DM candidate. The model allows a cubic interaction term for $S(S^*)$ leading to the appearance of a new scattering processes relevant for the calculation of the relic abundance and indirect detection, requiring milder Yukawa-like coupling unlike to previous similar constructions based on the $Z_2$ symmetry. This fact a priori makes that the model be perturbative at higher energies than its equivalent $Z_2R$. At the phenomenological level the characteristic of this model in its simplest form (Yukawa-like coupling only to electron/positrons) are remarkable, due to the fact that some regions of the parameter space not only survive near the EW scale up to several TeV evading strong direct detection at one-loop level, but also presenting sizable fluxes in both positrons and in gamma-rays, with the former presenting a distinctive box-shaped spectra. In this way, this new type of construction can be useful to explore multi-messenger DM, with possible signals correlations.

One of the assumptions made in this work was keeping $\lambda_{SH} = 0$. The consequences of having such portal with sizable values is equivalent to what occurs in these types of scenarios: new channels participating of the relic abundance calculation, new indirect detection signals, and direct detection at tree level. In order to keep the correct relic abundance passing all the previous constraints, the immediate effect of sizable $\lambda_{SH}$ is the reduction of $g_\psi$ or $\mu_3$ in order to keep the correct relic abundance, then affecting some results found in this work. However, couplings up to order $\lambda_{SH} \sim O(10^{-2})$ do not deviate the results already found in this work. Just to sketch, consider the parameter space point $(m_\psi,m_S) = (200,130)$ GeV, $g_\psi = 0.1$ and $\mu_3 = 2m_S$. We have checked that it is possible to set $\lambda_{SH} \lesssim 0.01$, evading all the constraints, without generating a significant deviation of the values presented in Fig. 5. Of course, higher couplings will affect the results obtained in this work, but a further analysis considering these effects is beyond the goals of this work.

Finally, all the treatment that has been carried out here is assuming a single coupling of the new physics to $e_R$ field. This we have done is for the sake of simplicity and illustration of the possible enhancement of the new annihilation processes of the complex DM, but certainly the coupling of the new sector to other lepton/quark families, simultaneously or not, and/or the dark sector fields in higher SM gauge representations could result in interesting phenomenology.
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A Electron/Positron kinematics

In this appendix we detail some of the kinematics for $e^\pm$ produced from the complex scalar DM annihilation. In order to exemplify the typical kinematics occurring in this model in the mass range $m_\psi / 2 < m_S < m_\psi$, we show in Fig. 7 the normalized spectra $dN/dE = \frac{1}{2} \frac{d\sigma}{dE}$ for the process $SS \rightarrow e^−e^+S^*$ obtained with CalcHEP code [61], assuming a center-of-mass momentum of 2 GeV, and the parameters indicated in the plot. The bumps and sharp peaks can be understood by kinematics. For instance, the process $SS \rightarrow e^−\psi$ (all the argumentation afterwards goes also for the CP-conjugate processes), neglecting the electron mass, present the following energies for the final states

$$E_{e^-} = m_S \left( 1 - \frac{m_\psi^2}{4m_S^2} \right), \quad E_\psi = m_S \left( 1 + \frac{m_\psi^2}{4m_S^2} \right).$$

(12)

Considering $m_S = 100$ GeV and $m_\psi = 150$ GeV, we obtain in this $2 \rightarrow 2$ process that $E_{e^-} \approx 43$ GeV (green sharp peak in Fig. 7). Now, the energy distribution of $e^-/e^+$ is much more complex than the energies given in [12] as it can be seen in Fig. 7 since this continuous spectra for the outgoing particles comes from the fact that the complete process is $2 \rightarrow 3$ (see diagrams in Fig. 3), with the diagrams interfering. However, other interesting information can be obtained considering the following. If we consider only the s-channel diagram, and assuming that the emitted $e^+$ from the $SS$ annihilation process makes a $\theta$ angle with respect to the in-flight $\psi$, the energy of the positron is given by

$$E_{e^+} = \frac{(-m_S^2 + m_\psi^2)/2}{E_\psi - \sqrt{E_\psi^2 - m_\psi^2} \cos \theta},$$

(13)

with $E_\psi$ given in eq. [12]. Then, for $m_\psi = 150$ GeV, $E_{e^+} \approx [31, 56]$ GeV, which is exactly the place where the box-shaped spectra (continuous green line) is in the Fig. 7. The maximum energy of $e^-/e^+$ is independent of $m_\psi$, and is defined uniquely by $m_S$. Equivalently, the previous analysis can be made for the case of $m_\psi = 180$ GeV (blue lines), in which certainly the energy distribution of $e^-/e^+$ changes at some extent, moving apart the electron peak from the box-shaped spectra of the positron.

References

[1] N. F. Bell, J. B. Dent, T. D. Jacques, and T. J. Weiler, “Dark Matter Annihilation Signatures from Electroweak Bremsstrahlung,” *Phys. Rev. D* 84 (2011) 103517, arXiv:1101.3357 [hep-ph]

[2] T. Bringmann, X. Huang, A. Ibarra, S. Vogl, and C. Weniger, “Fermi LAT Search for Internal Bremsstrahlung Signatures from Dark Matter Annihilation,” *JCAP* 07 (2012) 054 arXiv:1203.1312 [hep-ph]

[3] L. Bergstrom, T. Bringmann, I. Cholis, D. Hooper, and C. Weniger, “New Limits on Dark Matter Annihilation from AMS Cosmic Ray Positron Data,” *Phys. Rev. Lett.* 111 (2013) 171101 arXiv:1306.3983 [astro-ph.HE]
[4] M. Garny, A. Ibarra, M. Pato, and S. Vogl, “Internal bremsstrahlung signatures in light of direct dark matter searches,” *JCAP* **12** (2013) 046, arXiv:1306.6342 [hep-ph]

[5] F. Giacchino, L. Lopez-Honorez, and M. H. G. Tytgat, “Scalar Dark Matter Models with Significant Internal Bremsstrahlung,” *JCAP* **10** (2013) 025, arXiv:1307.6480 [hep-ph]

[6] T. Toma, “Internal Bremsstrahlung Signature of Real Scalar Dark Matter and Consistency with Thermal Relic Density,” *Phys. Rev. Lett.* **111** (2013) 091301, arXiv:1307.6181 [hep-ph]

[7] T. Bringmann and F. Calore, “Significant Enhancement of Neutralino Dark Matter Annihilation from Electroweak Bremsstrahlung,” *Phys. Rev. Lett.* **112** (2014) 071301, arXiv:1308.1089 [hep-ph]

[8] F. Giacchino, L. Lopez-Honorez, and M. H. G. Tytgat, “Bremsstrahlung and Gamma Ray Lines in 3 Scenarios of Dark Matter Annihilation,” *JCAP* **08** (2014) 046, arXiv:1405.6921 [hep-ph]

[9] H. Okada and T. Toma, “Effect of Degenerate Particles on Internal Bremsstrahlung of Majorana Dark Matter,” *Phys. Lett. B* **750** (2015) 266–271, arXiv:1411.4858 [hep-ph]

[10] P. Agrawal, Z. Chacko, and C. B. Verhaaren, “Leptophilic Dark Matter and the Anomalous Magnetic Moment of the Muon,” *JHEP* **08** (2014) 147, arXiv:1402.7369 [hep-ph]

[11] J. Kopp, L. Michaels, and J. Smirnov, “Loopy Constraints on Leptophilic Dark Matter and Internal Bremsstrahlung,” *JCAP* **04** (2014) 022, arXiv:1401.6457 [hep-ph]

[12] C. Kelso, J. Kumar, P. Sandick, and P. Stengel, “Charged mediators in dark matter scattering with nuclei and the strangeness content of nucleons,” *Phys. Rev. D* **91** (2015) 055028, arXiv:1411.2634 [hep-ph]

[13] A. Ibarra, T. Toma, M. Totzauer, and S. Wild, “Sharp Gamma-ray Spectral Features from Scalar Dark Matter Annihilations,” *Phys. Rev. D* **90** no. 4, (2014) 043526, arXiv:1405.6917 [hep-ph]

[14] T. Toma, “Virtual internal bremsstrahlung of dark matter and connection with ams-02 result,” *Physics Procedia* **61** (2015) 188–192, https://www.sciencedirect.com/science/article/pii/S1875389214006439 13th International Conference on Topics in Astroparticle and Underground Physics, TAUP 2013.

[15] M. Duerr, P. Fileviez Perez, and J. Smirnov, “Gamma Lines from Majorana Dark Matter,” *Phys. Rev. D* **93** (2016) 023509, arXiv:1508.01425 [hep-ph]

[16] T. Bringmann, A. J. Galea, and P. Walia, “Leading QCD Corrections for Indirect Dark Matter Searches: a Fresh Look,” *Phys. Rev. D* **93** no. 4, (2016) 043529, arXiv:1510.02473 [hep-ph]

[17] F. Giacchino, A. Ibarra, L. Lopez Honorez, M. H. G. Tytgat, and S. Wild, “Signatures from Scalar Dark Matter with a Vector-like Quark Mediator,” *JCAP* **02** (2016) 002, arXiv:1511.04452 [hep-ph]

[18] J. Kumar, P. Sandick, F. Teng, and T. Yamamoto, “Gamma-ray Signals from Dark Matter Annihilation Via Charged Mediators,” *Phys. Rev. D* **94** no. 1, (2016) 015022, arXiv:1605.03224 [hep-ph]

[19] P. Sandick, K. Sinha, and F. Teng, “Simplified Dark Matter Models with Charged Mediators: Prospects for Direct Detection,” *JHEP* **10** (2016) 018, arXiv:1608.00642 [hep-ph]

[20] S. Baek, P. Ko, and P. Wu, “Top-philic Scalar Dark Matter with a Vector-like Fermionic Top Partner,” *JHEP* **10** (2016) 117, arXiv:1606.00072 [hep-ph]

[21] V. V. Khoze, A. D. Plascencia, and K. Sakurai, “Simplified models of dark matter with a long-lived co-annihilation partner,” *JHEP* **06** (2017) 041, arXiv:1702.00750 [hep-ph]

[22] N. F. Bell, Y. Cai, J. B. Dent, R. K. Leane, and T. J. Weiler, “Enhancing Dark Matter Annihilation Rates with Dark Bremsstrahlung,” *Phys. Rev. D* **96** no. 2, (2017) 023011, arXiv:1705.01105 [hep-ph]

[23] T. Bringmann, F. Calore, A. Galea, and M. Garny, “Electroweak and Higgs Boson Internal Bremsstrahlung: General considerations for Majorana
dark matter annihilation and application to MSSM neutralinos, \textit{JHEP} 09 (2017) 041 \arxiv{1705.03466 [hep-ph]}

[24] C. El Aisati, C. Garcia-Cely, T. Hambye, and L. Vanderheyden, “Prospects for discovering a neutrino line induced by dark matter annihilation,” \textit{JCAP} 10 (2017) 021 \arxiv{1706.06600 [hep-ph]}

[25] D. Barducci, A. Deandrea, S. Moretti, L. Panizzi, and H. Prager, “Characterizing dark matter interacting with extra charged leptons,” \textit{Phys. Rev. D} 97 no. 7, (2018) 075006 \arxiv{1801.02707 [hep-ph]}

[26] L. Calibbi, R. Ziegler, and J. Zupan, “Minimal models for dark matter and the muon $g-2$ anomaly,” \textit{JHEP} 07 (2018) 046 \arxiv{1804.00009 [hep-ph]}

[27] S. Colucci, F. Giacchino, M. H. G. Tytgat, and J. Vandecasteele, “Top-philic Vector-Like Portal to Scalar Dark Matter,” \textit{Phys. Rev. D} 98 (2018) 035002 \arxiv{1804.05068 [hep-ph]}

[28] S. Colucci, F. Giacchino, M. H. G. Tytgat, and J. Vandecasteele, “Radiative corrections to vectorlike portal dark matter,” \textit{Phys. Rev. D} 98 no. 11, (2018) 115029 \arxiv{1805.10173 [hep-ph]}

[29] S. Biondini and S. Vogl, “Scalar dark matter coannihilating with a coloured fermion,” \textit{JHEP} 11 (2019) 147 \arxiv{1907.08576 [hep-ph]}

[30] S. Junius, L. Lopez-Honorez, and A. Mariotti, “A feeble window on leptophilic dark matter,” \textit{JHEP} 07 (2019) 136 \arxiv{1904.07513 [hep-ph]}

[31] S. Baum, P. Sandick, and P. Stengel, “Hunting for scalar lepton partners at future electron colliders,” \textit{Phys. Rev. D} 102 no. 1, (2020) 015026 \arxiv{2004.02834 [hep-ph]}

[32] C. Arina, B. Fuks, L. Mantani, H. Mies, L. Panizzi, and J. Salko, “Closing in on $t$-channel simplified dark matter models,” \textit{Phys. Lett. B} 813 (2021) 136038 \arxiv{2010.07559 [hep-ph]}

[33] C. Arina, B. Fuks, and L. Mantani, “A universal framework for $t$-channel dark matter models,” \textit{Eur. Phys. J. C} 80 no. 5, (2020) 409 \arxiv{2001.05024 [hep-ph]}

[34] J. T. Acuña, P. Stengel, and P. Ullio, “A Minimal Dark Matter Model for Muon $g-2$ with Scalar Lepton Partners up to the TeV Scale,” \arxiv{2112.08992 [hep-ph]}

[35] M. Becker, E. Copello, J. Harz, K. A. Mohan, and D. Sengupta, “Impact of Sommerfeld Effect and Bound State Formation in Simplified $t$-Channel Dark Matter Models,” \arxiv{2203.04326 [hep-ph]}

[36] F. D’Eramo and J. Thaler, “Semi-annihilation of Dark Matter,” \textit{JHEP} 06 (2010) 109 \arxiv{1003.5912 [hep-ph]}

[37] G. Belanger, K. Kannike, A. Pukhov, and M. Raidal, “$Z_4$ Scalar Singlet Dark Matter,” \textit{JCAP} 01 (2013) 022 \arxiv{1211.1014 [hep-ph]}

[38] J. M. Cline, K. Kainulainen, P. Scott, and C. Weniger, “Update on scalar singlet dark matter,” \textit{Phys. Rev. D} 88 (2013) 055025 \arxiv{1306.4710 [hep-ph]} [Erratum: Phys.Rev.D 92, 039906 (2015)].

[39] Y. Cai and A. P. Spray, “Fermionic Semi-Annihilating Dark Matter,” \textit{JHEP} 01 (2016) 087 \arxiv{1509.08481 [hep-ph]}

[40] J. Guo, Z. Kang, and P. Zhang, “WIMP Dark Matter Hidden behind its Companion,” \arxiv{2108.12964 [hep-ph]}

[41] C. E. Yaguna and O. Zapata, “Multi-component scalar dark matter from a $Z_N$ symmetry: a systematic analysis,” \textit{JHEP} 03 (2020) 109 \arxiv{1911.05515 [hep-ph]}

[42] C. E. Yaguna and O. Zapata, “Fermion and scalar two-component dark matter from a $Z_4$ symmetry,” \arxiv{2112.07020 [hep-ph]}

[43] G. Belanger, K. Kannike, A. Pukhov, and M. Raidal, “Impact of semi-annihilations on dark matter phenomenology - an example of $Z_N$ symmetric scalar dark matter,” \textit{JCAP} 04 (2012) 010 \arxiv{1202.2962 [hep-ph]}

[44] E. Ma, “$Z(3)$ Dark Matter and Two-Loop Neutrino Mass,” \textit{Phys. Lett. B} 662 (2008) 49–52 \arxiv{0708.3371 [hep-ph]}
[45] A. Aranda, D. Hernández-Otero, J. Hernández-Sanchez, V. Keus, S. Moretti, D. Rojas-Cifaló, and T. Shindou, “$Z_3$ symmetric inert (2+1)-Higgs-doublet model,” Phys. Rev. D 103 no. 1, (2021) 015023, arXiv:1907.12470 [hep-ph].

[46] Y. Cai and A. Spray, “Low-Temperature Enhancement of Semi-annihilation and the AMS-02 Positron Anomaly,” JHEP 10 (2018) 075, arXiv:1807.00832 [hep-ph].

[47] L. Bergstrom, T. Bringmann, and J. Edsjo, “New Positron Spectral Features from Supersymmetric Dark Matter - a Way to Explain the PAMELA Data?,” Phys. Rev. D 78 (2008) 103520, arXiv:0808.3725 [astro-ph].

[48] L. A. Cavasonza, H. Gast, M. Krämer, M. Pellen, and S. Schael, “Constraints on leptophilic dark matter from the AMS-02 experiment,” Astrophys. J. 839 no. 1, (2017) 36, arXiv:1612.06634 [hep-ph] [Erratum: Astrophys.J. 869, 89 (2018)].

[49] S. Profumo, L. Giani, and O. F. Piattella, “An Introduction to Particle Dark Matter,” Universe 5 no. 10, (2019) 213, arXiv:1910.05610 [hep-ph].

[50] J. Kawamura, S. Okawa, and Y. Omura, “Current status and muon g-2 explanation of lepton portal dark matter,” JHEP 08 (2020) 042, arXiv:2002.12534 [hep-ph].

[51] Y. Bai and J. Berger, “Lepton Portal Dark Matter,” JHEP 08 (2014) 153, arXiv:1402.6696 [hep-ph].

[52] G. Guedes and J. Santiago, “New leptons with exotic decays: collider limits and dark matter complementarity,” arXiv:2107.03429 [hep-ph].

[53] P. Athron, C. Balázs, D. H. J. Jacob, W. Kotlarski, D. Stöckinger, and H. Stöckinger-Kim, “New physics explanations of $g-2$ in light of the FNAL muon $g-2$ measurement,” JHEP 09 (2021) 080, arXiv:2104.03691 [hep-ph].

[54] CMS Collaboration, A. M. Sirunyan et al., “Search for invisible decays of a Higgs boson produced through vector boson fusion in proton-proton collisions at $\sqrt{s} = 13$ TeV,” Phys. Lett. B 793 (2019) 520-551, arXiv:1809.05937 [hep-ex].