A deformable microswimmer in a swirl: capturing and scattering dynamics

Mitsusuke Tarama$^{1,2,3,4}$, Andreas M. Menzel$^2$, and Hartmut Löwen$^2$

$^1$Department of Physics, Kyoto University, Kyoto 606-8502, Japan
$^2$Institut für Theoretische Physik II: Weiche Materie, Heinrich-Heine-Universität Düsseldorf, D-40225 Düsseldorf, Germany
$^3$Institute for Solid State Physics, University of Tokyo, Kashiwa, Chiba 277-8581, Japan
$^4$Department of Physics, University of Tokyo, Tokyo 113-0033, Japan

(Dated: January 16, 2014)

Inspired by the classical Kepler and Rutherford problem, we investigate an analogous set-up in the context of active microswimmers: the behavior of a deformable microswimmer in a swirl flow. First we identify new steady bound states in the swirl flow and analyze their stability. Second we study the dynamics of a self-propelled swimmer heading towards the vortex center, and we observe the subsequent capturing and scattering dynamics. We distinguish between two major types of swimmers, those that tend to elongate perpendicularly to the propulsion direction and those that pursue a parallel elongation. While the first ones can get caught by the swirl, the second ones were always observed to be scattered, which proposes a promising escape strategy. This offers a route to design artificial microswimmers that show the desired behavior in complicated flow fields. It should be straightforward to verify our results in a corresponding quasi-two-dimensional experiment using self-propelled droplets on water surfaces.

PACS numbers: 47.63.Gd, 82.70.Dd, 47.32.Ef, 61.30.-v

I. INTRODUCTION

For many applications it is of key relevance to tune and control the motion of artificial and biological microswimmers [1–3] by external influences like confinement [4–9], solvent flow [10–13], or a magnetic field [14]. This can be exploited to construct motors and machines on the microscale [15–17] and artificial muscles [18], to mention just a few examples. In particular the motion of self-propelled particles in externally prescribed flow fields gives rise to significant changes in their swimming paths as shown by recent studies in planar Couette [13] and Poiseuille flow geometry [10–12].

Astonishingly the motion of a microswimmer in a swirl has never been considered so far, although swirl flows occur quite naturally in many situations, including turbulence. Here we address this problem and augment it by possible deformations of the particle that couple to the solvent flow. Using a theoretical description from our earlier work [19], we derive equations of motion for a swimmer in a swirl. The set-up of a swirl is similar to that of a scattering geometry and possesses therefore an analogy to the classical Kepler and Rutherford problem. In particular, one can discriminate between the two basic dynamic events of capturing and scattering: in the former, the swimmer is attracted by the swirl and cannot escape from it afterwards, while in the latter it escapes from the eddy by its own self-propulsion. For human swimmers, at high Reynolds numbers, it is important to propose a strategy to escape a swirl. We address this problem at low Reynolds numbers in an analogous way. In fact, the two events of capturing – which possibly leads to death – and scattering – corresponding to a successful escape and survival – depend on the impact parameter and the relative orientation of the swimmer with respect to the flow direction. In order to discriminate between these two results, we perform a theoretical stability analysis as well as a numerical solution of the corresponding equations of motion.

Our predictions can be tested for deformable swimmers in prescribed vortex flows. Experimental realizations are given by deformable droplets on fluid interfaces propelling due to chemical reactions [20, 21]. The vortex flow profile is of high practical relevance and experimentally easily accessible. In practice, a magnetic stir bar at the bottom of an artificial water tank is enough to steadily maintain it. Furthermore, it does not require periodic boundary conditions as they are in principle necessary for the typically studied example of planar Couette or linear shear flow. Nevertheless, we are not aware of any previous investigation of this set-up in the presence of active particles.

This paper is organized as follows: in the next section, the time-evolution equations of a deformable microswimmer in a swirl flow are described. We discuss the steady-state solutions and their stability in section III. Scattering and capturing dynamics, which actually constitute the central topic of this paper, are considered in section IV. Section V is devoted to a summary and conclusion. Finally, details of the analytical calculation carried out in section III are explained in the Appendix.

II. MODEL

In the following, we introduce the model equations for an active deformable microswimmer in a swirl flow. We consider a two-dimensional environment and denote the fluid flow field as $u(x,y)$. Our simple vortex flow (swirl)
is given by

\[ u(x, y) = \left( \mu \frac{-y}{x^2 + y^2}, \mu \frac{x}{x^2 + y^2} \right), \tag{1} \]

where \( \mu \) sets the strength of the vortex. It describes a rotational flow around a swirl center. Naturally, the swirl center defines the origin of our coordinate frame. A flow potential exists for this type of fluid flow such that \( \mathbf{u}(x, y) = -\nabla U(x, y) \), where \( U(x, y) = \mu \arctan(x/y) \).

Consequently, \( \nabla \times \mathbf{u} = 0 \), which implies that the fluid flow does not contain a local rotational contribution. This swirl flow can easily be realized in an experimental set-up. Surface swimmers on a fluid provide a reasonable realization of our two-dimensional considerations.

For a simple swimmer, two basic kinds of activity can generally be distinguished: a spontaneous translational motion (self-propulsion), and a spontaneous rotation (spinning motion). The impact of active rotations has been investigated in detail \([12, 22–33]\). Here, for simplicity, we take into consideration only a spontaneous translational motion, i.e. self-propulsion. The time-evolution equations for a deformable active particle in a fluid flow field \( \mathbf{u} \) can then be derived from symmetry arguments as \([19]\)

\[
\frac{dx_i}{dt} = u_i + v_i, \tag{2}
\]

\[
\frac{dv_i}{dt} = \gamma v_i - (v_k v_k) v_i - a_1 S_{ik} v_k, \tag{3}
\]

\[
\frac{dS_{ij}}{dt} = -\kappa S_{ij} + b_1 \left[ v_i v_j - \frac{\delta_{ij}}{2} (v_k v_k) \right] + \nu_1 \left[ A_{ij} - \frac{\delta_{ij}}{2} A_{kk} \right], \tag{4}
\]

where \( \delta_{ij} \) is the Kronecker delta and summation over repeated indices is implied.

The variables in the above equations are as follows. \( x = (x_1, x_2) \) represents the position of the center of mass of the swimmer. It is parameterized as

\[ x = (r \cos \eta, r \sin \eta) \tag{5} \]

in polar coordinates.

When viewed from the laboratory frame, the microswimmer in total moves with the velocity \( \mathbf{u} + \mathbf{v} \). In this expression, \( \mathbf{u} \) is the imposed flow velocity of the fluid fixed by Eq. (1), while \( \mathbf{v} \) is the relative swimming velocity of the swimmer with respect to its fluid environment. We parameterize \( \mathbf{v} \) as

\[ \mathbf{v} = (v \cos \phi, v \sin \phi). \tag{6} \]

In a lowest-order approach, we take into account elliptic deformations of the swimmer and describe them by a symmetric traceless second-rank tensor \( S \). It can be directly related to the second Fourier mode of the shape changes of a deformable particle \([34]\). Furthermore it is of the same form as the order parameter used to characterize the state of nematic liquid crystals \([35]\). The components of \( S \) are parameterized as

\[
S_{11} = -S_{22} = s \cos 2\theta, \quad S_{12} = S_{21} = s \sin 2\theta. \tag{7}
\]

Here \( s \) characterizes the degree of elliptic deformation, while \( \theta \) measures the orientation of the symmetry axis of elliptic deformation when viewed from the laboratory frame. See Fig. 1 for the assignment of the angles \( \eta, \phi, \) and \( \theta \).

Finally, the tensor \( A \) in Eq. (4) characterizes the strain rate (dynamic elongational contribution) due to the fluid flow. Its components are given by

\[
A_{ij} = \frac{1}{2} \left( \partial_i u_j + \partial_j u_i \right). \tag{8}
\]

Depending on the coefficient \( \nu_1 \), the elongational part of the fluid flow can lead to stretching deformations of the deformable swimmer.

On the right-hand side of Eq. (3), the term with the coefficient \( \gamma \) is the crucial one for active swimmers. It describes self-propulsion with a net non-vanishing velocity of active swimming in the case of \( \gamma > 0 \). On the contrary, we do not include active deformations of the swimmer. It rather tends to relax back to the undeformed state. This is implied by the contribution with the coefficient \( \kappa > 0 \) in Eq. (4). In the limit of \( \kappa \to \infty \) we obtain \( S = 0 \) and our equations apply to the case of a rigid spherical swimmer in a swirl flow.

Finally, the terms with the coefficients \( a_1 \) and \( b_1 \) represent the leading coupling terms between the relative velocity \( \mathbf{v} \) and the deformation \( S \). Accordingly, deformations can influence the propulsion direction and the
swimming speed, whereas in turn propulsion can lead to shape changes. The impact of these terms has been studied in detail in the absence of external flow fields \[20, 44\]. One important result from these studies is that increased deformations lead to curved trajectories. Without the fluid flow, a bifurcation from straight to circular motion is found from Eqs. \[3\] and \[4\] at a critical value \(\gamma = \gamma_c\), where \[30\]

\[
\gamma_c = \frac{\kappa^2}{a_1 b_1} + \frac{\kappa}{2}.
\]

On the one hand, if \(a_1 < 0\) and \(b_1 < 0\), the symmetry axis of the elliptical deformation tends to be perpendicular to the propulsion velocity (“perpendicular case”) \[30\]. On the other hand, for \(a_1 > 0\) and \(b_1 > 0\), the two directions tend to be parallel to each other (“parallel case”) \[30\]. Not including any external flow field, Eqs. \[2\]–\[4\] were derived theoretically for an isolated deformable droplet self-propelling on a fluid interface due to a chemical reaction \[20\]. A recent theoretical investigation \[14\] suggests that such a system is a pusher in the perpendicular reaction \[20\].

**III. STEADY-STATE SOLUTIONS**

The swirl flow in Eq. \[1\] features a rotational symmetry. We therefore expect that circular steady-state solutions of closed circular loops exist, at least for passive particles. For rigid spherical passive particles, implying \(\gamma < 0\) and \(\kappa \to \infty\), Eqs. \[2\]–\[4\] reduce to \(\dot{x} = u\), \(\dot{v} = 0\), and \(S = 0\). Thus these objects are simply advected by the fluid flow on circles around the vortex center. The stability of the circular motion of any radius is marginal, with the radius only determined by the initial conditions. In the following, we investigate how deformability and self-propulsion change this result. For this purpose, we analyze the stability of the steady-state solutions of Eqs. \[2\]–\[4\].

**A. Passive circular motion**

We first consider a deformable passive particle, i.e. one that is not self-propelled. Consequently it is simply convected by the fluid flow, and the relative velocity with respect to the surrounding fluid vanishes, \(\dot{v} = 0\). Nevertheless, the particle can be deformed by the elongational component of the flow field.

Under these assumptions, we investigate the steady-state solutions of Eqs. \[2\]–\[4\]. It turns out that the only remaining orientational degree of freedom is the relative angle \(\Theta\) between the deformation axis and the position vector of the particle, i.e. \(\Theta = \theta - \eta\). As explained in more detail in the Appendix, we find circular trajectories of fixed radius \(r = r_0\) characterized by

\[
\tan 2\Theta = \frac{\kappa r_0^2}{2\mu},
\]

and the detailed derivation given in the Appendix. The stability is marginal with respect to the radial direction and asymptotic with respect to the other degrees of freedom.

Eqs. \[12\]–\[14\] imply that for increasing stiffness of the particle the stability range of the circular steady-state solution increases. This can be seen as follows. Increasing stiffness corresponds to an increasing magnitude of \(\kappa > 0\). As \(\kappa\) approaches \(|a_1\nu_1|\) from below, the value of \(r_{0,\min}\) diverges. For \(\kappa \geq |a_1\nu_1|\), the condition for \(\gamma\) in Eq. \[12\] extends to the full range of \(\gamma < 0\). This corresponds to the natural requirement that the passive particle suffers from friction with its fluid environment, with \(\gamma < 0\) setting the friction parameter. Thus, in this case, circular steady-state trajectories of all radii are marginally stable.

Coming back to deformable passive particles, we tested our results by numerically solving the equations of motion Eqs. \[2\]–\[4\]. In the following, we always employed a forth order Runge-Kutta method of time increment \(\delta t = 10^{-3}\) and checked our results using still finer time steps. Here, we initialized the system by circular particles placed at different distances from the vortex center.

Fig. \[2\] shows corresponding numerical results for the system parameters chosen as \(\kappa = 0.5\), \(a_1 = b_1 = -1\), \(\nu_1 = 1\), and \(\mu = 1\), i.e. \(\kappa < |a_1\nu_1|\). This implies soft deformable passive particles. Indeed we find our analytical results confirmed. The solid (blue) line marks the stability limit derived in Eqs. \[12\] and \[14\]. Circular trajectories of radius \(r_0\) with a value of \(\gamma\) below this line are stable. On the contrary, if we initialize trajectories with \(\gamma < 0\) above the solid stability line (i.e. within the “dip” region in Fig. \[2\]), the system is expelled from that area as indicated by the horizontal arrows. Most of the
dense points in Fig. 2 on both sides of the dip follow from such expelled systems. As we can see, the expelled systems typically overshoot the stability line until they finally get stabilized. We checked numerically that the results are qualitatively the same when \(a_1 = b_1 = +1\), as it is predicted from our theoretical analysis.

**B. Active circular motion**

We now turn to steady-state active motions of \(v \neq 0\) for \(\gamma > 0\). The dynamic equations \([2, 4]\) were solved numerically for this purpose. In the absence of a fluid flow, an analytical solution of active straight motion had been obtained previously for \(0 < \gamma < \gamma_c\) \([36, \text{Eq. (9)}]\). We used it as an initial condition and place the particle at various distances from the vortex center for different values of \(\gamma > 0\). The system parameters were set to the same values as mentioned in the caption of Fig. 2 except that we considered both cases of \(a_1 = b_1 = \pm 1\).

As mentioned above, in the absence of the fluid flow, the deformation axis tends to be perpendicular to the propulsion direction for \(a_1 = b_1 = -1\), whereas it takes a parallel configuration for \(a_1 = b_1 = +1\) \([36]\).

On the one hand, for the perpendicular case \((a_1 = b_1 = -1)\) we find a steady-state active circular motion when \(0 < \gamma < \gamma_c\), where here \(\gamma_c = 0.5\). It is the analogue of the passive circular motion of \(\gamma < 0\). However, in contrast to the passive case, we now obtain only one stable diameter \(r_0\) for each value of \(0 < \gamma < \gamma_c\). This is indicated by the diamond symbols in Fig. 2. Depending on the initial conditions, the swimmer either asymptotically approaches this orbit, or it manages to escape from the swirl to infinite distance.

For \(\gamma > \gamma_c\), the situation becomes markedly different. Starting sufficiently close to the radius \(r_0\), we still observe the steady-state active circular motion as indicated in Fig. 2. However, another type of motion occurs depending on the initial conditions. We call it a lunar-type motion and depict a typical trajectory in Fig. 3(a) for \(\gamma = 1\). A short-time piece of the trajectory is emphasized by the thick solid red line. The black superimposed silhouettes show the particle orientations and degrees of deformation in an exaggerated way for illustration. We can understand this trajectory as the circular motion that already occurs in the absence of the swirl for \(\gamma > \gamma_c\) \([36, \text{Eq. (9)}]\) superimposed onto the circular convection due to the vortex flow. In this case, both rotational directions – the one of the smaller revolution, corresponding to the circular trajectory of the moon around the earth in our heliocentric picture, and the one of the larger revolution, corresponding to the trajectory of the earth around the sun – have the same sense of rotation as the fluid flow. The radius of the larger revolution depends on the initial condition.

On the other hand, in the parallel case \((a_1 = b_1 = +1)\) we did not observe a steady-state active circular motion. Instead, all particles escape far away from the flow center for \(0 < \gamma < \gamma_c = 0.5\). Therefore designing an active swimmer in the parallel configuration at low propulsion speed offers a promising strategy to allow escapes. In contrast, for \(\gamma > \gamma_c\), a particle again undergoes a lunar-type motion. We display a typical trajectory in Fig. 3(b). Here, however, the smaller revolution and the fluid flow have opposite sense of rotation, whereas the larger revolution and the fluid flow share the same sense of rotation. This is different from the perpendicular case considered above. Again, the radius of the larger revolution depends on the initial condition.

With increasing \(\gamma \geq 0.7\), the situation gets still more complex in the parallel case. Depending on the initial condition, a multi-circular motion can emerge as illustrated in Fig. 3(c). The lighter gray, thick solid turquoise, and thick solid red lines show trajectory pieces of decreasing swimming-time intervals. To obtain the multi-circular motion, the swimmer was initially placed relatively close to the vortex center.

In summary, these observations suggest the following escape strategy for a deformable swimmer that cannot actively determine its swimming direction and was dragged into the swirl. If possible, a parallel configuration should be adopted. (Within the framework of Ref. 45 this corresponds to a puller-like propulsion mechanism.) Then the most effective way is not to try too hard to escape; in
other words, the effort of self-propulsion should be kept low \((\gamma < \gamma_c)\). In this combined situation we always observed that the swimmer manages to escape.

IV. CAPTURING AND SCATTERING DYNAMICS

We now study the “collision” of an active deformable swimmer with the swirl in analogy to a classical scattering experiment. Therefore the swimmer is initially not placed close to the vortex center, but at a comparatively large distance \(r_{\text{init}}\) away. If we moved it along the direction of its initial velocity orientation, it would miss the vortex center by a distance \(d_{\text{imp}}\), called the impact parameter. Furthermore, if the particle manages to escape from the vortex, we can measure the scattering angle of this event. For this purpose, we determine the angle \(\theta_{\text{cat}}\) between the initial velocity orientation and the final velocity orientation when the particle has reached a certain distance \(r_{\text{esc}}\) from the vortex center.

To keep the set-up simple and meaningful in the sense of a scattering experiment, we set the propulsion strength to values \(0 < \gamma < \gamma_c\). For these values, an active straight motion occurs in the absence of the flow field \([36]\). We provide this solution as an initial condition at a very large initial distance \(r_{\text{init}} = 1.5 \times 10^4\). After numerically integrating Eqs. (2)–(4) forward in time, we measure the scattering angle at the distance \(r_{\text{esc}} = 10^4\), if a scattering event occurs. We varied the values of \(\gamma\) and the impact parameter \(d_{\text{imp}}\), while the other parameters were chosen as before. Due to the swirl geometry, the events of passing the vortex center on one side is different from passing it on the other side. We therefore define \(d_{\text{imp}} > 0\) if the particle velocity is initially oriented towards the side of oppositely directed fluid flow. On the contrary, we set \(d_{\text{imp}} < 0\) if the swimmer initially propels towards the side of identically directed fluid flow.

The results of these measurements together with example trajectories are shown in Figs. 3 and 6 and discussed in the following. Again, we distinguish between a perpendicular configuration \((a_1 = b_1 = -1)\) and a parallel configuration \((a_1 = b_1 = +1)\).

A. Perpendicular case

For the perpendicular configuration \((a_1 = b_1 = -1)\), the particle tends to orient its deformation axis perpendicular to the propulsion velocity. As we saw in the previous section, this is not the best strategy to escape from the vortex. Indeed, as we will see shortly, in a finite interval of impact parameters, the particle gets captured by the swirl instead of being scattered. We discuss how the situation changes when the impact parameter increases.

Generally, the vortex makes the swimmer deviate from its straight trajectory of motion, see Fig. 3. For negative impact parameters \(d_{\text{imp}}\), the particle trajectory is only weakly deformed and the particle leaves the vortex geometry with basically the same velocity orientation as it entered the set-up. Thus the scattering angle \(\theta_{\text{cat}}\) vanishes. This is true even for weakly positive impact parameters, where the swirl flow guides the swimmer around the vortex center. The situation is illustrated in Fig. 3(a,b) for \(\gamma = 0.3\) and \(d_{\text{imp}} = 60\).

With further increasing impact parameter, the scattering process is more persistent. Now the velocity direction of the swimmer becomes permanently disturbed even after the swimmer has left the vortex geometry. We end up with a nonzero scattering angle \(\theta_{\text{cat}}\), see Fig. 3(a) and the trajectories for \(d_{\text{imp}} = 70\) and 85 in Fig. 3(b,c).

Then, as a function of increasing impact parameter, the swimmer more and more gets caught by the swirl. It can happen that the swimmer circles around the vortex center before it can finally escape from the swirl as depicted by the trajectory for \(d_{\text{imp}} = 89\) in Fig. 3(b,c).
the swirl any more. Example trajectories are depicted in Fig. 4(b,c) for \( d_{\text{imp}} = 90, 100, \) and 103. Interestingly, in all these cases the swimmer ends up on the same circular trajectory around the vortex center. This attractant type of motion corresponds to the active circular motion discussed in Section III B.

Finally, when the impact parameter is too large, the swimmer does not get close enough any more to the swirl center to be effectively captured. It now gets scattered again, passing the vortex center on the other side, however. To identify these events of passing on the other side of the swirl in Fig. 4(a), we shifted the corresponding scattering angles \( \eta_{\text{scat}} \) by \(-2\pi\). Such events are illustrated in Fig. 4(b,c) by the trajectories for \( d_{\text{imp}} = 104 \) and 110.

In effect, we found that the swimmer gets scattered by the swirl and can escape for low enough impact parameters \( d_{\text{imp}} \). It gets captured by the swirl at intermediate impact parameters. For large enough impact parameters, it gets scattered again and can escape. Thus the dynamical behavior of getting scattered is reentrant as a function of the impact parameter \( d_{\text{imp}} \). Guided by this observation, we scanned the swimmer behavior in the parameter plane of the impact parameter \( d_{\text{imp}} \) and the active propulsion strength \( \gamma \). We distinguished between events of scattering or escape on the one hand and events of capturing on the other hand. The resulting dynamic phase diagram is shown in Fig. 5. Most interestingly, the phase behavior is not only reentrant as a function of the impact parameter \( d_{\text{imp}} \) for fixed propulsion strength \( \gamma \). Rather, at fixed intermediate impact parameter \( d_{\text{imp}} \), we also observe reentrance of the capturing event and a twofold reentrance of the scattering behavior with increasing propulsion strength \( \gamma \). We checked that our results only slightly vary with the initial distance \( r_{\text{init}} \) from the vortex center due to the inverse decay of the flow field. Qualitatively our results do not depend on the initial distance \( r_{\text{init}} \).

\[ \text{Fig. 4. Scattering dynamics for } a_1 = b_1 = -1 \text{ (perpendicular case): (a) scattering angle } \eta_{\text{scat}} \text{ as a function of the impact parameter } d_{\text{imp}} \text{ for various propulsion strengths } \gamma; \text{ (b,c) example trajectories for } \gamma = 0.3 \text{ and for different impact parameters } d_{\text{imp}}. \text{ Panel (c) shows details of the trajectories in panel (b) around the vortex center. Black superimposed silhouettes indicate the swimmer orientations and degrees of deformation. Arrows mark the direction of motion. Furthermore, the gray dotted lines illustrate the direction of the fluid flow with the vortex center given by the plus symbol. There is a finite interval of impact parameters for which the swimmer cannot escape from the vortex any more but gets captured by the swirl. (Other parameter values are } \kappa = 0.5, \nu_1 = 1, \text{ and } \mu = 1. \text{)} \]

These events correspond to reorientation processes that are more extreme than simple backscattering, and we indicate them by scattering angles \( \eta_{\text{scat}} > \pi \) in Fig. 4(a). The scattering angle seems to diverge when the impact parameter is further increased.

At still higher impact parameters, the swimmer eventually gets captured by the vortex. It cannot escape from

B. Parallel configuration

For the parallel configuration \((a_1 = b_1 = +1)\), the swimmer tends to orient its deformation axis along the direction of self-propulsion. This is a good strategy to avoid getting captured by the swirl. Indeed we never observed an event of permanent capturing for such swimmers that started far away from the vortex center with propulsion strengths \( \gamma < \gamma_c \). Again we discuss the changes in the dynamic behavior with increasing impact parameter.

While they are heading towards the vortex, the situation for active swimmers of parallel configuration is just the other way around as for those of perpendicular configuration; their trajectory gets curved into the opposite direction during this initial process. Therefore significant scattering now already takes place for negative impact parameters \( d_{\text{imp}} \) as demonstrated in Fig. 6.

First, for very negative impact parameters \( d_{\text{imp}} \), the
swimming trajectory is only slightly influenced by the swirl. The swimmer passes the vortex with only little net change in the propulsion direction, i.e. \( \eta_{\text{scat}} \approx 0 \), as displayed in Fig. 6(a, b) for \( \gamma = 0.3 \) and \( d_{\text{imp}} = -75 \). Increasing the impact parameter, the swimmer comes closer to the vortex center and the scattering angle \( \eta_{\text{scat}} \) increases, see the trajectory for \( d_{\text{imp}} = -71 \) in Fig. 6(b).

Interestingly, we observe a discontinuous jump of the scattering angle to values \( \eta_{\text{scat}} > 2\pi \) in Fig. 6(a) at higher impact parameters. The trajectory for \( d_{\text{imp}} = -70 \) in Fig. 6(b,c) shows the drastic event that occurs in this case and explains the jump in the scattering angle. The swimmer gets transiently caught by the swirl. Its trajectory describes a loop of more than a full rotation around the vortex center before the swimmer can escape with a net scattering angle \( \eta_{\text{scat}} > 2\pi \). As indicated by the trajectory for \( d_{\text{imp}} = -65 \) in Fig. 6(b,c), this behavior persists for further increasing impact parameters. However, the net scattering angle in Fig. 6(a) decreases and the loop around the vortex center does not describe a complete rotation of \( 2\pi \) any more.

Remarkably, a qualitative difference appears in the trajectories for higher impact parameters, although there is a continuous decrease in the net scattering angles in Fig. 6(a). As illustrated for \( d_{\text{imp}} = -45 \) and -36 in Fig. 6(b,c), the swimmer now first passes the vortex center on the opposite side. Still, however, the swimmer is transiently caught by the swirl and describes a loop around the vortex center as shown in Fig. 6(c).

FIG. 5. Dynamical phase diagram as a function of the impact parameter \( d_{\text{imp}} \) and the strength of active propulsion \( \gamma \) for the perpendicular case \( (a_1 = b_1 = -1) \). Two dynamic events are distinguished: getting scattered by the swirl with a subsequent escape (area outside the lines); and getting captured by the vortex (area between the lines). Our results do not qualitatively depend on the initial distance of the swimmer from the vortex center as can be seen when comparing panels (a) and (b) with each other as well as the solid with the dashed lines within each panel. Reentrance of the dynamic events can be observed in both directions of the parameter space.

FIG. 6. Scattering dynamics for \( a_1 = b_1 = +1 \) (parallel case): (a) scattering angle \( \eta_{\text{scat}} \) as a function of the impact parameter \( d_{\text{imp}} \) for various propulsion strengths \( \gamma \); (b,c) example trajectories for \( \gamma = 0.3 \) and for different impact parameters \( d_{\text{imp}} \). Panel (c) shows details of the trajectories in panel (b) around the vortex center, with further details given by the inset. Black superimposed silhouettes indicate the swimmer orientations and degrees of deformation. Arrows mark the direction of motion. In addition, the gray dotted lines illustrate the direction of the fluid flow. No capturing events are observed in this case, but the orbit can come very close to the vortex center. (Other parameter values are \( \kappa = 0.5 \), \( \nu_1 = 1 \), and \( \mu = 1 \).)
lighted by the inset of Fig. 6(c), the swimmer in all these cases always performs the loop around the center with the same rotational sense as the fluid flow. This is true independently of the side on which the swimmer first passes the vortex center. Consequently, in the latter two cases of $d_{\text{imp}} = -45$ and $-36$, the swimmer must switch the side that it exposes to the center of the swirl.

Finally, there is another discontinuous jump of the scattering angle in Fig. 6(a) at still higher impact parameters. Corresponding trajectories in Fig. 6(b) for $d_{\text{imp}} = -35$ and $-25$ reveal the reason for this jump. The swimmer does not perform a narrow loop around the vortex center any more. Instead its trajectory features a simple bend around the swirl. We observe events from close to backscattering up to practically no net scattering at all for large impact parameters. Again, for clarity, we shift the scattering angles corresponding to these events by $-2\pi$ in Fig. 6(a).

V. SUMMARY AND CONCLUSION

In summary, we studied the dynamic behavior of a deformable microswimmer in a swirl flow. To our knowledge this geometry has not been investigated before for active microswimmers, although it is a set-up of high practical relevance and straightforward to be realized in an experiment. Within our framework, we distinguished between two types of swimmers: those that tend to elongate perpendicularly to their propulsion direction and those that prefer a parallel configuration. Considering droplets propelling due to chemical reactions, a recent theoretical study suggests that the first ones can be classified as pushers while the second ones are pullers. They show different bound states in the swirl flow when they are initially placed close to the vortex center. Most significantly, however, when heading towards the swirl from far away, the perpendicular ones can get captured by the vortex flow, whereas the parallel ones were always observed to escape at low enough propulsion strengths.

We started by investigating the bound steady-state motions around the swirl center. Our approach naturally contains the situation of rigid spherical particles when the relaxation rate for deformations is large. A rigid spherical passive particle placed close to the vortex center is simply convected around the swirl on a circular trajectory. The radius of this circle is given by the initial position. In a first step, we found that this is no longer the case for the passive circular motion of deformable swimmers: depending on the friction of the swimmer with its fluid environment, some radii can become unstable.

For active deformable swimmers the situation becomes much more complex. The results qualitatively depend on whether the swimmer prefers the perpendicular or parallel configuration. On the one hand, for the swimmers of perpendicular configuration, there are two situations below a threshold value for the strength of self-propulsion and depending on the initial conditions. They either escape from the swirl, or they perform an active circular motion around the center of the swirl. However, in contrast to the passive case, only one radius of the active circular motion is stable for each value of the strength of self-propulsion. Above the threshold, these swimmers cannot escape from the swirl any more. Besides the active circular motion and depending on the initial conditions, a lunar-type motion can appear as the superposition of two circular trajectories of different radii. On the other hand, for the swimmers of parallel configuration, we did not obtain a bound steady-state motion below the threshold. Instead, they were always observed to escape from the swirl. Above the threshold, however, again a lunar-type motion appeared. For still higher strengths of self-propulsion, a multi-circular motion can also be found, depending on the initial conditions.

Second, we investigated how active swimmers interact with the swirl when they are heading towards the vortex from far away in a straight motion. We distinguished between two different dynamic events: getting captured by the swirl, or being scattered and managing to escape. In the latter case we measured the scattering angle as a function of the impact parameter in analogy to classical scattering experiments.

Remarkably, only for swimmers of perpendicular configuration we observed capturing events, occurring at intermediate impact parameters. In this case, the swimmer is caught by the swirl on a circular trajectory of prescribed radius around the vortex center. This corresponds to the active circular motion mentioned above. The capturing event is reentrant as a function of the strength of self-propulsion. For other impact parameters the swimmer is scattered. However, it may loop around the vortex center until it finally can escape. The scattering event is reentrant as a function of the impact parameter and twofold reentrant as a function of the strength of self-propulsion.

Active swimmers of parallel configuration and low enough propulsion strength were always observed to be scattered and escape from the swirl. At intermediate impact parameters, however, they can come very close to the vortex center in a loop trajectory. Interestingly, they always perform this loop with the same sense of rotation, independently of the side on which they first pass the swirl center. This may lead to a situation in which the swimmer changes the side that it exposes to the vortex center. The appearance and disappearance of the loop leads to a discontinuity in the scattering angle as a function of the impact parameter. In total we may conclude that the parallel configuration offers a promising escape strategy. Thus, to design an active deformable swimmer that is not captured by swirls, it should take the parallel configuration and propel slowly enough not to perform circular motions.

The swirl flow geometry is of high practical relevance and straightforward to be realized in an experiment. In principle, a magnetic stir bar on the bottom of a water tank is enough to create the vortex flow. Self-driven
drops on the water surface constitute appropriate deformable active swimmers [20–21] that propel in a quasi-two-dimensional environment. Therefore they should immediately allow to test and verify our predictions in an actual experimental set-up. This system should be easier to be realized than the typically investigated flow profiles of linear shear. Apart from that, also on the theoretical side several questions directly follow up on our study. First, the impact of thermal noise could be studied for small swimmer sizes [2, 46–48]. Next, and most notably, it will be worthwhile to analyze the swimmer behavior also in flow fields of non-vanishing local vorticity or even in turbulent flows. On the one hand, hydrodynamically enforced trapping and particle segregation can occur for passive particles in a vorticity flow [49, 50]. On the other hand, microswimmers were observed to create turbulence-like flows even by themselves in dense suspensions [51–53]. Finally, it has been reported about active swimmers that do not only self-propel but also actively rotate [12, 21–33, 54–57]. It will be very interesting to see how the active rotations interact with the rotational component of the flow fields and what their impact on the dynamics is. We thus hope that our results will stimulate further investigations both on the theoretical and the experimental side to elucidate the dynamics of active microswimmers in external flow fields.

ACKNOWLEDGEMENTS

This work was supported by the JSPS Core-to-Core Program “Non-equilibrium dynamics of soft matter and information”. M.T. acknowledges JSPS for a JSPS Research Fellowship. A.M.M. and H.L. thank the Deutsche Forschungsgemeinschaft for support of this work through the recently founded priority program on microswimmers SPP 1726.

Appendix: Stability analysis

In this Appendix, we describe the details of the stability analysis of the steady-state solutions of Eqs. (2)–(4). First, for the vortex flow specified by Eq. (1), the strain rate tensor of the flow field follows via Eq. (8). At the particle position $\mathbf{x}$, parameterized by Eq. (5), we obtain

$$\mathbf{A} = \begin{pmatrix} \mu r^{-2} \sin 2\eta & \mu r^{-2} \cos 2\eta \\ -\mu r^{-2} \cos 2\eta & -\mu r^{-2} \sin 2\eta \end{pmatrix}. \quad (A.1)$$

Inserting it together with the parameterizations Eqs. (5)–(7) into Eqs. (2)–(4), the equations of motion can be rewritten in the form

$$\frac{ds}{dt} = -ks + \frac{b_1}{2} v^2 \cos 2\Psi - \nu_1 \mu r^{-2} \sin 2\Theta, \quad (A.4)$$

$$\frac{d\Theta}{dt} = -\frac{b_1}{4s} v^2 \sin 2\Psi - \nu_1^{-1} \sin(\Theta - \Psi) - \mu r^{-2} \left(1 + \frac{\nu_1}{2s} \cos 2\Theta\right), \quad (A.5)$$

$$\frac{d\Psi}{dt} = \left(a_1 s - \frac{b_1}{4s} v^2\right) \sin 2\Psi - \nu_1 \frac{1}{2s} \mu r^{-2} \cos 2\Theta. \quad (A.6)$$

We here defined $\Theta = \theta - \eta$ and $\Psi = \theta - \phi$.

Following the general procedure, we investigate the stability of the steady-state solutions of these equations via the eigenvalues of the corresponding linear stability matrix. Its components are defined by

$$\mathcal{L}_{ij} = \frac{\partial}{\partial x_j} \left( \frac{dX_i}{dt} \right), \quad (A.7)$$

where $X = (r, v, s, \Theta, \Psi)$. We obtain the eigenvalues $\lambda$ of $\mathcal{L}$ as usual from the condition

$$\det(\mathcal{L} - \lambda \mathcal{I}) = 0, \quad (A.8)$$

with $\mathcal{I}$ the unity matrix. The corresponding motion is stable, if all $\lambda < 0$; marginally stable, if all $\lambda \leq 0$; and it becomes unstable, if at least one $\lambda > 0$.

From Eq. (A.3), there follow two types of steady-state solution. One of them describes a passive motion of $v = 0$, i.e. the particle is simply advected by the fluid flow. The other one corresponds to an active motion given by the relative speed $v = \sqrt{\Gamma}$ with respect to the surrounding fluid, where

$$\Gamma = \gamma - a_1 s \cos 2\Psi. \quad (A.9)$$

In the remaining part of this Appendix, we carry out the linear stability analysis of the passive motion.

For the passive motion $v = 0$, Eq. (A.2) implies that the distance $r$ from the vortex center remains constant. We thus obtain circular trajectories of fixed radius $r = r_0$ in the passive case. This is why we term this kind of motion the passive circular motion. The complete steady-state solution is found by setting the remaining time derivatives in the above dynamic equations equal to zero. Taking into account that $v = 0$, Eq. (11) in the main text then follows from Eq. (A.4). Likewise, Eq. (10) is obtained from Eq. (A.5) together with Eq. (A.4). The dynamic equation Eq. (A.6) can be ignored at this point because it determines the relative orientation of the relative velocity $\nu$, which vanishes in the case of passive circular motion $v = 0$. Naturally, it becomes important in the following when we study the bifurcation from the passive circular motion ($v = 0$) to other types of motion characterized by $v \neq 0$.

We determined the eigenvalues of the linear stability matrix for the passive circular motion via Eq. (A.8).
They are obtained as $\lambda_r = 0$, $\lambda_v = \Gamma$, $\lambda_\Psi = 2a_1s \cos 2\Psi$, and as the eigenvalues $\lambda_\pm$ of the submatrix

$$\mathcal{L}_{\text{sub}} = \begin{pmatrix} -\kappa & -2\nu_1\mu_r\cos 2\Theta \\ \frac{1}{2}\nu_1\mu_r s - 2s^2 \cos 2\Theta & \nu_1\mu_r s - s^2 \sin 2\Theta \end{pmatrix}. \quad (A.10)$$

On the one hand, since $s > 0$ (being the magnitude of deformation), we conclude from Eq. (11) that $\nu_1\mu \sin 2\Theta < 0$. Consequently, $\text{tr} \mathcal{L}_{\text{sub}} < 0$ and $\det \mathcal{L}_{\text{sub}} > 0$. This leads to $\lambda_+ < 0$, which is necessary for stability. On the other hand, the eigenvalue $\lambda_v = 0$ implies that the stability is at most marginal. However, we still need to consider the signs of the eigenvalues $\lambda_r$ and $\lambda_\Psi$. Both, $\lambda_r$ and $\lambda_\Psi$, depend on $\Psi$, so we now take into account Eq. (A.6).

In the case of passive circular motion, the steady-state solution of Eq. (A.6) follows as

$$\sin 2\Psi = \frac{\kappa}{a_1\nu_1 \sin 2\Theta}. \quad (A.11)$$

This expression does not determine the sign of $\cos 2\Psi$.

Thus there is always a solution that guarantees the condition $\lambda_\Psi = 2a_1s \cos 2\Psi \leq 0$ necessary for marginal stability. Nevertheless, we must satisfy $\sin^2 2\Psi \leq 1$ for the steady-state solution to exist. This leads to the condition $r_0 \geq r_{0,\text{min}}$ with $r_{0,\text{min}}$ given by Eq. (13) in the main text. Taking into account Eqs. (A.9) and (A.11) together with the last eigenvalue $\lambda_v = \Gamma$, we need to require

$$\lambda_v = \Gamma = \gamma + a_1 \cos 2\Psi \frac{\nu_1\mu_r s}{\kappa} \sin 2\Theta < 0 \quad (A.12)$$

for the solution $\nu = 0$ to be stable. Together with Eqs. (A.11) and (10), we obtain $\gamma < \gamma_-$ with $\gamma_- \text{ given by Eq. (14) in the main text.}$ When $r_0$ approaches $r_{0,\text{min}}$ from above, we find that $\gamma_-$ tends to zero.

Finally, when $r_0 < r_{0,\text{min}}$, the steady-state solution for $\Psi$ does not exist. Eq. (A.6) then implies that $\Psi$ monotonically increases or decreases, depending on the parameters. Then, on average, $\cos 2\Psi$ vanishes, and Eq. (A.12) reduces to $\gamma < 0$ for $r_0 < r_{0,\text{min}}$. We tested and confirmed this observation numerically (see also Fig. 3). The results are summarized in Eq. (12) in the main text.
[35] P. G. de Gennes and J. Prost, *The Physics of Liquid Crystals* (Clarendon Press, Oxford, 1993).
[36] T. Ohta and T. Ohkuma, Phys. Rev. Lett. **102**, 154101 (2009).
[37] T. Hiraiwa, M. Y. Matsuo, T. Ohkuma, T. Ohta, and M. Sano, Europhys. Lett. **91**, 20001 (2010).
[38] T. Hiraiwa, K. Shitara, and T. Ohta, Soft Matter **7**, 3083 (2011).
[39] M. Tarama and T. Ohta, Eur. Phys. J. B **83**, 391 (2011).
[40] Y. Itino, T. Ohkuma, and T. Ohta, J. Phys. Soc. Jpn. **80**, 033001 (2011).
[41] Y. Itino and T. Ohta, J. Phys. Soc. Jpn. **81**, 104007 (2012).
[42] A. M. Menzel and T. Ohta, Europhys. Lett. **99**, 58001 (2012).
[43] M. Tarama, Y. Itino, A. M. Menzel, and T. Ohta, Eur. Phys. J. Special Topics (in press).
[44] K. Shitara, T. Hiraiwa, and T. Ohta, Phys. Rev. E **83**, 066208 (2011).
[45] N. Yoshinaga, [arXiv:1307.3120](https://arxiv.org/abs/1307.3120).
[46] J. R. Howse, R. A. L. Jones, A. J. Ryan, T. Gough, R. Vafabakhsh, and R. Golestanian, Phys. Rev. Lett. **99**, 048102 (2007).
[47] S. van Teeffelen and H. L"owen, Phys. Rev. E **78**, 020101 (2008).
[48] B. ten Hagen, S. van Teeffelen, and H. L"owen, J. Phys.: Condens. Matter **23**, 194119 (2011).
[49] E. Calzavarini, M. Cencini, D. Lohse, and F. Toschi, Phys. Rev. Lett. **101**, 084504 (2008).
[50] S. Martens, A. V. Straube, G. Schmid, L. Schimansky-Geier, and P. H"anggi, Phys. Rev. Lett. **110**, 010601 (2013).
[51] H. H. Wensink, J. Dunkel, S. Heidenreich, K. Drescher, R. E. Goldstein, H. L"owen, and J. M. Yeomans, Proc. Natl. Acad. Sci. USA **109**, 14308 (2012).
[52] H. H. Wensink and H. L"owen, J. Phys.: Condens. Matter **24**, 464130 (2012).
[53] J. Dunkel, S. Heidenreich, K. Drescher, H. H. Wensink, M. B"ar, and R. E. Goldstein, Phys. Rev. Lett. **110**, 228102 (2013).
[54] P. Tierno, R. Golestanian, I. Pagonabarraga, and F. Sagu"es, Phys. Rev. Lett. **101**, 218304 (2008).
[55] Y. Wang, S.-t. Fei, Y.-M. Byun, P. E. Lammert, V. H. Crespi, A. Sen, and T. E. Mallouk, J. Am. Chem. Soc. **131**, 9926 (2009).
[56] S. Ebbens, R. A. L. Jones, A. J. Ryan, R. Golestanian, and J. R. Howse, Phys. Rev. E **82**, 015304 (2010).
[57] L. F. Valadares, Y.-G. Tao, N. S. Zacharia, V. Kitaev, F. Galembeck, R. Kapral, and G. A. Ozin, Small **6**, 565 (2010).