Analysis of Arrhenius activation energy in magnetohydrodynamic Carreau fluid flow through improved theory of heat diffusion and binary chemical reaction

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Abstract
In this work, we addressed the characterization of stagnation-point Carreau fluid flow induced due to stretching of chemically reactive surface. The energy expression is incorporated with a new theory of heat diffusion named Cattaneo-Christov, which is an advanced form of Fourier’s heat flux formula. The additional term of thermal relaxation time is arisen in Cattaneo-Christov model of heat diffusion. We considered modified Arrhenius energy function with chemical reaction effect. The theory of boundary layer is employed to govern the mathematical phenomenon. The ordinary differential system is obtained by the implementation of suitable similarity variables. The governing system of mathematical expressions is solved by using Runge–Kutta based MATLAB bvp4c package. The derived solutions are sketched for various values of physical constraints on quantities of interest. A comparative study is presented for validating the results. It is found that the larger value of the thermal relaxation parameter act as non-conductor. It also noted that the destructive binary reaction is favorable to enhance the species concentration.

1. Introduction

The investigation on rheological features of non-Newtonian materials has made the interesting topic amongst all the researchers due to their broad involvement in engineering and food industries. Common examples of non-Newtonian materials are blood, shampoos, soaps, ketchup, polymer solutions, drilling muds, apple sauce, sugar solutions and many more. The non-Newtonian fluids potentially appeared in the manufacturing of optical fibers, foods, coated sheets, drilling muds, plastic polymers, etc. The scientists are facing difficulties to develop the single constitutive formula, which can be implemented to evaluate all the diverse properties of non-Newtonian materials. Various constitutive expressions of non-Newtonian fluids have been presented in the literature (see Bird et al [1]). The governing flow equations of non-Newtonian fluids are generally with higher order, complicated and are nonlinear in comparison of simple theory of Navier–Stokes. The power-law viscosity model has the limitation, i.e. it cannot adequately predict the viscosity for very small or very large shear rates. Due to such disadvantages of power law model, another viscosity model based on very high and very low shear rates was developed namely Carreau rheological model [2]. This model is very accurate to predict the shear thinning and shear thickening characteristics of fluids by using the various values of power law exponent. Also, this model overcomes the limitations of the power-law model identified above and appears to be gaining wider acceptance in chemical engineering and technological processes. Due to vast applications, the Carreau model has attained the interest of many investigators and engineers during the last few years. Chhabra and Uhlherr [3] and Bush and Phan-Thein [4] studied the flows of the Carreau fluid model over the spheres. Hayat et al [5]
examined the convective heating role in Carreau fluid induced due to movement of surface. Also, the following researchers considered the Carreau fluid model on the fluid flow problems with different geometries [6–9].

The description of fluid motion near stagnant region of solid surface occurs in both situations of moving and fixed body in a fluid is named ‘stagnation-point flow’. The investigation of such type flow is a major area of hotspot research in fluid mechanics and such situations generate when the flow impinges on solid surfaces. The investigators have paid keen interest due to its different applications in industry like flows over submarines and tips of aircrafts. Hiemenz [10] initially characterized the stagnant point flow of steady viscous fluid and elaborated the exact solutions. Homann [11] has further modified the work of [10] for axisymmetric case. Mahapatra and Gupta [12, 13] described the stagnant flows induced due to movement of surfaces. Soid et al [14] presented the magnetohydrodynamic stagnation point flow of viscous fluid generated by shrunk sheet. Chiam [15] discussed the steady viscous flow near a stagnation point and found that near the plate no boundary is formed when the velocities of stagnation flow and stretching plate are equal in the viscous free stream. Also, the following authors studied the magnetohydrodynamic flows over a stretching sheet and channel, like the magnetohydrodynamic boundary layer flow of a Carreau fluid due to shrinking surface with suction/injection is considered by Akbar et al [16]. In this study the authors analyzing the dual solutions on boundary layer flow and noted that the velocity profiles are depreciated by of power index parameter for both solutions. Elmaboud [17] analyzed the MHD free convection flow over a vertical porous channel. Later, he [18] considered the three dimensional nanofluid flow over an exponentially stretching sheet in the presence of uniform magnetic field. In this study the author considered three types of nanoparticles namely, copper (Cu), silver (Ag) and aluminum oxide (Al₂O₃).

Heat transport mechanism is arisen due to variations in temperature between two various bodies. The heat transport has major role in industrial processes like production of energy, cooking of nuclear reactors and biomedical applications include drug targeting, heat conduction in tissues etc. Fourier [19] developed a formula to visualize the mechanism of heat transport which is known as ‘Fourier’s law of heat conduction’. Cattaneo [20] extended the Fourier theory of heat diffusion by considering the extra parameter of thermal relaxation. Christov [21] presented the modification in Cattaneo’s theory by using the time Oldroyd’s upper-convected time derivation formula. Meraj et al [22] visualized the effects of non-Darcy theory in hydrodynamic Jeffrey fluid flow by employing the theory of Cattaneo–Christov. Ramesh et al [23] described the properties of non-Fourier’s heat diffusion expression for laminar Casson fluid flow. Walter’s fluid B and second grade models of non-Newtonian group in presence of non-Fourier’s heat flux theory has been reported by Abbas et al [24]. The recent investigations on boundary layer flow using Cattaneo–Christov model is discussed by the authors in [25–27].

In 1889, the word activation energy is introduced by Svante Arrhenius. He defined that the energy that overcomes for a chemical reaction to occur is called activation energy. Activation energy can also be described as the minimum energy to start a chemical reaction. The idea of activation energy is generally useful in the areas pertaining to oil reservoir or geothermal engineering and in oil, water emulsions. The boundary layer fluid flow under binary chemical reaction was disclosed by Bestman [28]. He used perturbation approach to explore the role of activation energy in natural convected flow. The presence of binary chemical reaction under activation energy in rotating Maxwell fluid flow induced due to stretchable sheet has been described by Shafique et al [29]. They discussed that the rate of heat transport diminishes when fluid is subjected to higher rate of rotation. The impacts of nth order Arrhenius chemical reaction, suction/injection, thermal radiation and buoyancy force on time-dependent convective flow of incompressible fluid is reported by Makinde et al [30]. Awad et al [31] explored the importance of chemical reaction with finite Arrhenius activation energy in rotating fluid flow. Abbas et al [32] presented the numerical analysis of activation energy in Casson fluid flow induced by thermally radiative sheet.

In the present paper, we aim to explore the impact of binary chemical reaction with Arrhenius activation energy on hydromagnetic stagnation point flow of a generalized Newtonian Carreau fluid over a stretching surface. Energy equation is constructed by Cattaneo–Christov heat flux theory. The set of non-linear governing equations are solved by using MATLAB bvp4c package. Impact of various flow parameters on the velocity, temperature and concentration field are discussed in detail through graphs and tables.

2. Mathematical formulation

We assumed an incompressible, viscous and conducting laminar flow of generalized Newtonian Carreau fluid near stagnation-point over a stretching sheet coinciding with in the plane \( y = 0 \) through fixed stagnation point at \( x = 0 \). Cattaneo–Christov model of heat diffusion is employed to analyze the characteristics of heat transport. The components of velocity are assumed \((u, v)\) in direction of coordinate system \((x, y)\). We choose coordinate system such that \( x \)-axis is the along stretching sheet and \( y \)-axis is perpendicular to the sheet (see figure 1). A constant magnetic force of strength \( B_0 \) is implemented normal to direction of flow. The stretching velocity
\( u_a(x) = cx \ (c > 0) \), and free-stream velocity \( U_\infty (x) = ax \ (a > 0) \) assumed to be varying proportionally to \( x \) from stagnation-point in which \( a \) and \( c \) are constants. \( T_\infty \) and \( C_\infty \) are constant temperature and concentration, respectively. \( T_\infty \) and \( C_\infty \) are the ambient temperature and concentration, respectively. It is assumed that the induced magnetic field is neglected due to small magnetic Reynolds. Chemical reaction term is retained through modified Arrhenius function.

The Cauchy stress tensor for the generalized Newtonian Carreau rheological model is represented by the following equations:

\[
\tau = -pI + \eta \dot{\gamma},
\]

(1)

\[
\eta = \eta_\infty + (\eta_0 - \eta_\infty)[1 + (\Gamma\dot{\gamma})^{2 \frac{n-1}{2}}],
\]

(2)

here \( p \) the pressure, \( I \) the identity tensor, \( n \) the power law index, \( \eta_\infty \) the infinite shear rate viscosity, \( \eta_0 \) the zero shear rate viscosity, \( \Gamma \) the material parameter and \( A_1 = \text{grad} V + (\text{grad} V)^2 \) represents the first Rivlin—Erickson tensor in which \( V \) denotes the velocity vector.

The rate of shear \( \dot{\gamma} \) for flow is expressed as

\[
\dot{\gamma} = \sqrt{\frac{1}{2} \sum_j \sum j \ddot{\gamma}_{jj}} = \sqrt{\frac{1}{2} \text{tr}(A_1^2)}.
\]

(3)

For many practical cases we consider \( \eta_0 \gg \eta_\infty \). [33]

The constitutive relation for \( \eta_\infty = 0 \) is

\[
\eta = \eta_0[1 + (\Gamma\dot{\gamma})^{2 \frac{n-1}{2}}],
\]

(4)

The Carreau model with fluid index range \( 0 < n < 1 \) are commonly referred as shear thinning or pseudo plastic fluids and Carreau fluids with fluid index \( n > 1 \) are commonly referred as shear thickening or dilatants fluids. For \( n = 1 \), the Carreau model reduces to the Newtonian model.

The utilization of above assumptions leads to following governing expressions [34–36].

\[
\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = 0,
\]

(5)

\[
\begin{align*}
\frac{u}{\partial x} + \frac{v}{\partial y} &= U_\infty \frac{\partial U_\infty}{\partial x} + \frac{\eta_0}{\rho} \left[ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right] \left[ 1 + \Gamma^2 \left\{ 4 \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 \right\} \right]^{\frac{n-1}{2}} \\
+ \frac{\eta_0}{\rho} \left[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right] \left[ 1 + \Gamma^2 \left\{ 4 \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 \right\} \right]^{\frac{n-1}{2}} \\
+ \frac{\eta_0}{\rho} \left[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right] \left[ 1 + \Gamma^2 \left\{ 4 \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 \right\} \right]^{\frac{n-1}{2}} + \frac{\sigma B_0^2}{\rho} (U_\infty - u),
\end{align*}
\]

(6)

---

Figure 1. Physical model and co-ordinate system.
\[
\frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = \frac{\eta_0}{\rho} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) \left[ 1 + \Gamma^2 \left\{ \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial u}{\partial y} \right)^2 + \left( \frac{\partial v}{\partial x} \right)^2 \right\} \right]^{\frac{\alpha - 1}{2}}
\]

\[
+ 2 \frac{\eta_0}{\rho} \frac{\partial v}{\partial y} \frac{\partial^2 v}{\partial y^2} \left[ 1 + \Gamma^2 \left\{ \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial u}{\partial y} \right)^2 + \left( \frac{\partial v}{\partial x} \right)^2 \right\} \right]^{\frac{\alpha - 1}{2}}
\]

\[
+ \frac{\eta_0}{\rho} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \frac{\partial}{\partial x} \left[ 1 + \Gamma^2 \left\{ \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial u}{\partial y} \right)^2 + \left( \frac{\partial v}{\partial x} \right)^2 \right\} \right]^{\frac{\alpha - 1}{2}},
\]

(7)

\[
\rho C_p \left( \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = -\nabla \cdot \mathbf{q},
\]

(8)

\[
u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} - k^2 (C - C_\infty) \left( \frac{T}{T_\infty} \right)^m \frac{-E_a}{kT},
\]

(9)

where \( \rho, \sigma, C_p, u \) and \( v \) are the fluid density, electrical conductivity, specific heat and the velocity components in \( x \) and \( y \) directions, respectively.

After applying the boundary-layer-investigation, the ruling equations of motion become:

\[
\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = 0
\]

(10)

\[
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = U_\infty \frac{\partial U_\infty}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2} \left[ 1 + \Gamma^2 \left\{ \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial u}{\partial y} \right)^2 \right\} \right]^{\frac{\alpha - 1}{2}}
\]

\[+ v(n - 1) \Gamma^2 \frac{\partial u}{\partial x} \left( \frac{\partial u}{\partial y} \right)^2 \left[ 1 + \Gamma^2 \left\{ \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial u}{\partial y} \right)^2 \right\} \right]^{\frac{\alpha - 1}{2}} + \frac{\sigma B^2}{\rho} (U_\infty - u),
\]

(11)

in which \( \nu = \frac{\eta_0}{\rho} \) represents the fluid kinematic viscosity.

In equation (9), the term \( \left( \frac{T}{T_\infty} \right)^m e^{-\frac{E_a}{kT}} \) denotes the modified form of Arrhenius function in which \( k = 8.61 \times 10^{-5} \text{ eV/K} \) represents the Boltzmann constant, \( m \) is the unit less exponent fitted rate constant lies in the range \(-1 < m < 1 \) and \( E_a \) is the activation energy.

The new heat flux model is known as Cattaneo-Christov model of heat diffusion and it has the following from [37]:

\[
q + \lambda \left( \frac{\partial q}{\partial t} + V \cdot \nabla q - q \nabla V + (\nabla \cdot V)q \right) = -k \nabla T.
\]

(12)

Here \( \lambda \) represents the heat flux relaxation time and \( k \) is the fluid thermal conductivity. It is noticed that for \( \lambda = 0 \), equation (12) reduces to classical Fourier’s law. For an incompressible fluids, \( \nabla \cdot V = 0 \) and equation (12) becomes

\[
q + \lambda \left( \frac{\partial q}{\partial t} + V \cdot \nabla q - q \nabla V \right) = -k \nabla T.
\]

(13)

Comparison of equations (8) and (12) and elimination of \( q \) lead to the following expression

\[
\frac{\partial T}{\partial t} + \frac{\partial T}{\partial y} + \lambda \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \left( \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) \frac{\partial T}{\partial x} + 2uv \frac{\partial^2 T}{\partial x \partial y}
\]

\[+ \left( \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) \frac{\partial T}{\partial y} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2}
\]

(14)

The associated boundary conditions are:

\[
\begin{align*}
& u = u_\infty(x) = \alpha x, \quad v = 0, \quad T = T_\nu, \quad C = C_\nu \quad \text{at} \quad y = 0 \\
& u \to U_\infty = ax, \quad T \to T_\infty, \quad C \to C_\infty \quad \text{as} \quad y \to \infty
\end{align*}
\]

(15)
3. Similarity transformations and solutions

Invoking the following transformations

\[ \psi(x, y) = x\sqrt{e} f(\eta), \quad \eta = y\sqrt{\frac{e}{V}}, \quad \theta(\eta) = \frac{T - T_\infty}{T_{w} - T_\infty}, \quad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}. \]  

Here \( \psi(x, y) \) is the stream function satisfying \( u = \frac{\partial \psi}{\partial y} \), \( v = -\frac{\partial \psi}{\partial x} \).

The continuity equation (5) is satisfied automatically, and equations (9), (11), (14) and (15) reduces to

\[ [1 + n\text{We}^2 (f''')^2][1 + \text{We}^2 (f''')^2]^{\frac{n-3}{2}} f''' + + ff'' - (f')^2 + \alpha^2 + M^2(\alpha - f') = 0, \]  

\[ \frac{1}{Pr} \theta'' + ff' - \gamma (f'f' + f^2 \theta'') = 0, \]  

\[ \frac{1}{Sc} \phi'' + 2f\phi' - \sigma\phi(1 + (\theta_{w} - 1)\theta)^{n}e^{\frac{E}{T_{w} + (\phi_{w} - 1)\phi}} = 0, \]  

\[ \frac{1}{W} \]
In above equations, $\alpha = a/c$ is the velocity ratio parameter, $M = \sqrt{\frac{\sigma B_0^2}{\rho c}}$ is the magnetic field parameter, $\text{We} = \sqrt{\frac{c T^2 \lambda^2}{\nu}}$ is the local Weissenberg number, $\text{Pr} = \frac{\mu C_p}{k}$ is the Prandtl number, $\gamma = \lambda c$ is the non-dimensional thermal relaxation time, $Sc = \nu/D$ is the Schmidt number, $\theta_w = T_w/T_\infty$ the temperature ratio parameter, $E = \frac{E_a}{k T_\infty}$ is the non-dimensional Arrhenius activation energy and $\sigma = k^2 / c$ is the rate of chemical reaction.

The important dimensionless physical quantities of interest are the local skin friction $C_f$, the local Nusselt number $Nu$ and the local Sherwood number $Sh$, which are given by

\begin{align}
 f(0) &= 0, \quad f'(0) = 1, \quad \theta(0) = 1, \quad \phi(0) = 1,
 f(\infty) &\to \alpha, \quad \theta(\infty) \to 0, \quad \phi \to 0.
\end{align}
where the wall shear stress \( \tau_w = \eta_b \frac{\partial u}{\partial y} \left[ 1 + \Gamma^2 \left( \frac{\partial u}{\partial y} \right)^2 \right]^{\frac{n-1}{2}} \bigg|_{y=0} \), the wall heat flux \( q_w = -k \left( \frac{\partial T}{\partial y} \right)_{y=0} \) and the wall mass flux \( j_w = -D \left( \frac{\partial C}{\partial y} \right)_{y=0} \).

In view of the above equations, an expression for the skin friction coefficient, local Nusselt number and local Sherwood number becomes

\[
C_f = \frac{\tau_w}{\rho U_m^2(x)}, \quad Nu_w = \frac{xq_w}{k(T_w - T_\infty)}, \quad Sh = \frac{xj_w}{D(C_w - C_\infty)},
\]

(21)

where \( \frac{x}{\nu} \) denotes the local Reynolds number.

\[
Re^{1/2} C_f = f'(0)\left[ 1 + We^2(f''(0))^2 \right]^{\frac{n-1}{2}}, \quad Re^{1/2} Nu = -\theta'(0) \quad \text{and} \quad Re^{1/2} Sh = -\phi'(0)
\]

(22)

\[\text{Figure 6. The influence of } \alpha \text{ on } \theta(\eta).\]

\[\text{Figure 7. The influence of } \alpha \text{ on } \phi(\eta).\]
4. Numerical results and discussion

In this part, we elaborated the role of physical constraints on fluid velocity, temperature and concentration. The obtained results are presented in the figures 2–15. Table 1 shows the correctness of the method used and verified with the prevailing results and found to be very good agreement with Mahapatra and Gupta [12], Khan et al [34], Nazar et al [38] and Ishak et al [39]. The friction factor coefficient, local Nusselt and local Sherwood numbers are derived and presented in table 2. In calculations the parametric values are selected as $We = 3$, $Pr = 0.71$, $\gamma = 0.1$, $\alpha = 0.1$, $n = 0.5$, $Sc = 0.3$, $\sigma = 3$, $\theta_w = 0.5$, $E = 0.5$, $M = 2$, $m = 0.5$. 
The impact of magnetic parameter \( (M) \) on fluid velocity \( (f'(\eta)) \), temperature \( (\theta(\eta)) \) and concentration \( (\phi(\eta)) \) is elaborated in the figures 2–4. From these plots, we visualized that \( f'(\eta) \) is a decreasing function of \( M \). In the physical view, the magnetic field parameter represents the ratio of magnetic force with the viscous force, so that the large value of \( M \) explicit the increase of Lorentz force. This is drag like force that produces more resistance to transport phenomena due to which fluid velocity reduces. Whereas the magnetic field parameter enhanced the fluid temperature and concentration profiles (see figures 3 and 4). This means the magnetic field works to increase the value of temperature and species concentration in the flow field. The influence of velocity ratio parameter \( (\alpha) \) on \( f'(\eta), \theta(\eta) \) and \( \phi(\eta) \) are plotted in figures 5–7. From figure 5, it is seen that for \( \alpha > 1 \), the fluid flow contains boundary layer behavior. Further, an enhancement in \( \alpha \) corresponds to thinner thickness.
of boundary layer. For fixed values of ε against the stretched surface, an increase in straining motion is appeared near region of stagnation that results the faster acceleration of external stream. An inverted structure of boundary layer is observed when α < 1. The free stream velocity of sheet becomes lesser to stretching velocity of external stream. An opposite structure is observed on q(η) and f(η) (see figures 6 and 7).

Figures 8–10 presents the velocity \( f'(\eta) \), temperature \( \theta(\eta) \) and concentration \( \phi(\eta) \) plots for various power law index parameter \( n \). It is noticed that the rising values of \( n \) accelerates the fluid velocity. Whereas,
depreciates the temperature and concentration profiles. Moreover, we observe that there is a corresponding increase in the momentum layer thickness while the opposite trend is observed on thermal and solutal boundary layers. We observed the accelerated motion for $n > 1$ while decelerated motion for $n < 1$.

Figure 11 elucidates the influence of thermal relaxation time parameter ($\gamma$) on $\theta(\eta)$. It is seen that both the fluid temperature and the thermal boundary layer thickness demonstrate a decelerating practice for the higher thermal relaxation parameter. Physically, an enhancement in thermal relaxation parameter causes less heat transfers from sheet to the fluid. With this indication that the thermal boundary layer will be thinner when the
behaves like a non-conductor. Fourier’s Law can be deduced from the present model by applying \( \gamma = 0 \). The influence of Weissenberg number (\( We \)) on velocity filed is plotted in figure 12. It is observed that \( f'(\eta) \) is increasing function of \( We \). Figure 13 reveals that the importance of activation energy (\( E \)) on the concentration distribution. It is clearly evident that the improving values of \( E \) accelerate the solutal boundary layer thickness which enhances the species concentration. Weaker rate of reaction is occurred due to higher energy activation and weaker temperature that resists the chemical reaction. Thus, the species concentration increases. The effect of temperature ratio parameter (\( \theta_{w} \)) on \( \phi(\eta) \) is depicted in figure 14. It is observed that an increase in \( \theta_{w} \) indicates poor concentration profiles and hence decreases the concentration boundary layer thickness. Figure 15 plots the concentration profiles for different values of chemical reaction rate constant (\( \sigma \)). As \( \sigma \) is gradually increased, the concentration profiles become thinner. The factor \( \sigma(1 + (\theta_{w} - 1)\theta)^{\frac{\gamma}{\gamma - 1}} \) is enhanced when we use the increment values either \( \sigma \) or \( \eta \). Here the destructive reaction is favorable due to which concentration rises.

Table 1 shows the comparison of the present results with the existing results of Mahapatra and Gupta [12], Khan et al [34], Nazar et al [38] and Ishak et al [39]. The variations of friction factor coefficient, the rates of heat and mass transfer coefficients for various flow parameters are presented in table 2. It is clear that the friction factor coefficient is an increasing function of \( n \), \( We \) and \( \alpha \) while decreases with an increase in \( M \). Also, increasing the values of \( M \) and \( \gamma \) improves the heat transfer rate but depreciates with the increasing values of \( n \), \( We \) and \( \alpha \). Furthermore, the mass transfer rate enhances with the rising values of \( n \), \( \sigma \), \( We \), \( \theta_{w} \) and \( \alpha \). Whereas, a reverse trend is observed for the increasing values of \( M \), \( \gamma \) and \( E \). The parameters \( E \), \( \theta_{w} \) and \( \sigma \) shows no effect on friction factor and heat transfer coefficients. However, \( \gamma \) shows no effect on \( f''(0) \).

| \( \alpha \) | Mahapatra and Gupta [12] | Khan et al [34] | Nazar et al [38] | Ishak et al [39] | Present study |
|---|---|---|---|---|---|
| 0.01 | – | –0.998 028 | – | –0.9980 | –0.998404 |
| 0.1 | –0.9694 | –0.969 387 | –0.9694 | –0.9694 | –0.969436 |
| 0.2 | –0.9181 | –0.918 107 | –0.9181 | –0.9181 | –0.9181113 |
| 0.5 | –0.6673 | –0.667 262 | –0.6673 | –0.6673 | –0.667264 |
| 2.0 | 2.0175 | 2.017 487 | 2.0176 | 2.0175 | 2.017503 |
| 3.0 | 4.7293 | 4.729 260 | 4.7296 | 4.7294 | 4.729282 |

| \( M \) | \( n \) | \( We \) | \( \gamma \) | \( \alpha \) | \( E \) | \( \sigma \) | \( \theta_{w} \) | \( f''(0) \) | \( -\theta'(0) \) | \( -\phi'(0) \) |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.695 727 | 0.073 000 | 0.951 751 |
| 2 | 0.788 996 | 0.074 422 | 0.948 750 |
| 3 | 0.868 208 | 0.075 620 | 0.946 416 |
| 0.2 | 1.499 424 | 0.077 176 | 0.943 163 |
| 0.8 | 0.825 894 | 0.073 986 | 0.949 654 |
| 2.0 | 0.572 802 | 0.071 349 | 0.955 632 |
| 1 | 0.703 020 | 0.072 902 | 0.952 009 |
| 3 | 0.598 052 | 0.071 589 | 0.955 051 |
| 5 | 0.542 905 | 0.070 774 | 0.957 180 |
| 0.1 | 0.641 343 | 0.048 910 | 0.954 559 |
| 0.3 | 0.641 343 | 0.099 839 | 0.952 566 |
| 0.5 | 0.641 343 | 0.162 705 | 0.950 240 |
| 0.1 | 0.917 647 | 0.087 700 | 0.931 152 |
| 0.3 | 0.799 286 | 0.077 703 | 0.941 461 |
| 0.5 | 0.641 342 | 0.072 185 | 0.953 633 |
| 0.5 | 0.641 342 | 0.072 185 | 0.891 637 |
| 1.5 | 0.641 342 | 0.072 185 | 0.741 552 |
| 2.0 | 0.641 342 | 0.072 185 | 0.692 314 |
| 1 | 0.641 342 | 0.072 185 | 0.775 656 |
| 3 | 0.641 342 | 0.072 185 | 1.106 607 |
| 5 | 0.641 342 | 0.072 185 | 1.364 936 |
| 1 | 0.641 342 | 0.072 185 | 0.876 088 |
| 2 | 0.641 342 | 0.072 185 | 1.013 964 |
| 3 | 0.641 342 | 0.072 185 | 1.107 073 |
5. Final remarks

The consequences of binary chemical reaction and activation energy on MHD stagnation point flow of a Carreau fluid are visualized in this research. In addition, the Cattaneo-Christov model of heat diffusion is employed to address the heat transport process. Based on the present study, the following conclusions are made.

1. The friction factor coefficient and the mass transfer coefficients increases for shear thickening ($n > 1$) and shear thinning ($0 < n < 1$) fluids. However, the opposite trend was observed on heat transfer coefficient.

2. The velocity ratio parameter enhances the fluid velocity significantly for $\alpha > 1$.

3. Weissenberg number improves the momentum layer thickness.

4. Fluid temperature is retarded for the enhancing values of $\gamma$.

5. Low temperature and high activation energy leads to smaller reaction rate.

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