Predictive Schemes for Bimaximal Neutrino Mixings

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Abstract

We present simple and predictive scenarios for neutrino masses which lead to a bimaximal pattern of mixings that apparently provides the best fit to the current solar and atmospheric neutrino data. It is shown that an approximate global \(L_e - L_\mu - L_\tau\) symmetry, broken only by Planck scale effects, can naturally explain the bimaximal mixing pattern. In one scenario, the solar neutrino oscillation parameters are induced entirely through radiative renormalization group effects. A one parameter model describing all neutrino data is presented, which predicts \(U_{e3} \approx \sqrt{m_e/m_\tau} \approx 0.017\). A second two parameter scheme predicts \(U_{e3}\) to be vanishingly small. We also show how these scenarios can be extended to accommodate the LSND observations within a 3+1 neutrino oscillation scheme by including a mirror sector of particles and forces.

I. INTRODUCTION

There appear to be persuasive arguments in favor of the possibility that both the solar and the atmospheric neutrino anomalies are explained by a specific pattern of neutrino mixings where both the \(\nu_e - \nu_\mu\) as well as the \(\nu_\mu - \nu_\tau\) mixing angles are nearly maximal, while the \(\nu_e - \nu_\tau\) mixing angle is small \(\frac{\Delta m^2_{\text{atm}}}{m_\tau} \approx 0.017\). A second two parameter scheme predicts \(U_{e3}\) to be vanishingly small. We also show how these scenarios can be extended to accommodate the LSND observations within a 3+1 neutrino oscillation scheme by including a mirror sector of particles and forces.
note, we suggest several simple scenarios which are quite predictive and therefore testable
in the near future in ongoing and proposed neutrino oscillation experiments.

Before proceeding to present our schemes, we wish to motivate some special forms of the
light neutrino mass matrix that lead to bimaximal mixings. These matrices are obtained
under some restricted but well motivated set of assumptions, mindful of possible approximate
symmetries that might be present in the lepton sector. Once identified, controlled breaking
of these approximate symmetries can lead to very predictive schemes for neutrino mixings,
as we shall demonstrate.

We work in a basis where the charged lepton mass matrix is diagonal. We define the
neutrino mixing matrix (the MNS matrix) $U$

$$
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix} = U 
\begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3
\end{pmatrix}
$$

(1)

where the states on the left and the right hand sides denote flavor and mass eigenstates.
The light neutrino mass matrix $M_\nu$ is given by the equation

$$
M_\nu = U M_d U^T
$$

(2)

where $M_d$ is diagonal and has the form $M_d = \text{Diag}(m_1, m_2, m_3)$. For the mass eigenvalues,
there are essentially three different possibilities of physical interest: (a) hierarchical form
where $m_1 \ll m_2 \ll m_3$; (b) inverted form where $|m_1| \simeq |m_2| \gg |m_3|$ and (c) degenerate
form where $m_1 = m$; $m_2 = m + \delta_2$; $m_3 = m + \delta_3$, with $\delta_{2,3} \ll m$. Since experiments
indicate that the $\nu_\mu - \nu_\tau$ mixing angle is nearly maximal, and that $U_{e3}$ is very small, we
choose the following approximate form for the mixing matrix $U$:

$$
U = \begin{pmatrix}
c & s & 0 \\
\frac{s}{\sqrt{2}} & -\frac{c}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\
\frac{s}{\sqrt{2}} & \frac{c}{\sqrt{2}} & -\frac{1}{\sqrt{2}}
\end{pmatrix}
$$

(3)

Here $s \equiv \sin \theta$ is the solar neutrino oscillation angle, which is of order one, while the
corresponding angle for atmospheric neutrino oscillation has been set to $1/\sqrt{2}$. In the exact
bimaximal limit, we will have $s = c = 1/\sqrt{2}$.

Consider case (a) in the exact bimaximal limit. Let us denote $m_3 = m$, $m_2 = \delta$ and set
$m_1 = 0$. The matrix $M_\nu$ is then give by

$$
M_{\nu}^{(a)} = \begin{pmatrix}
\delta/2 & -\delta/(2\sqrt{2}) & -\delta/(2\sqrt{2}) \\
-\delta/(2\sqrt{2}) & m/2 + \delta/4 & \delta/4 - m/2 \\
-\delta/(2\sqrt{2}) & \delta/4 - m/2 & \delta/4 + m/2
\end{pmatrix}
$$

(4)

For case (c), the form is the same as Eq. (4), except that there is an additional piece
proportional to the unit matrix, $\text{Diag}(1,1,1)m_0$, with $m_0$ setting the overall scale for the
degenerate mass. The relative hierarchy among the parameters in this case will be $m_0 \gg m \gg \delta$.

For case (b) with the inverted spectrum, we get (choosing for simplicity the case $c = s =
\frac{1}{\sqrt{2}}$ and $m_3 = 0$, $m_1 = m = -m_2 + \delta$),
\[ M^{(b)}_\nu = \begin{pmatrix} \frac{\delta}{2} & \frac{(2m - \delta)}{(2\sqrt{2})} & \frac{(2m - \delta)}{(2\sqrt{2})} \\ \frac{(2m - \delta)}{(2\sqrt{2})} & \frac{\delta}{4} & \frac{\delta}{4} \\ \frac{(2m - \delta)}{(2\sqrt{2})} & \frac{\delta}{4} & \frac{\delta}{4} \end{pmatrix}. \]  

(5)

Of these forms, Eq. (5) is particularly interesting since if we set \( \delta = 0 \) it reduces to

\[ M^{(0)}_\nu = \begin{pmatrix} 0 & m & m \\ m & 0 & 0 \\ m & 0 & 0 \end{pmatrix}. \]  

(6)

with a redefined \( m \). The mass matrix given in Eq. (6) has the interesting property that it has a global symmetry \( L_e - L_\mu - L_\tau \) (\( L_i \) is the \( i \)th lepton number) along with a permutation symmetry \( S_2 \) acting on the \( \nu_\mu, \nu_\tau \) flavors. This new symmetry unraveled in the leptonic world is perhaps analogous to the local \( B - L \) symmetry that is revealed by the seesaw mechanism for neutrino masses.

As it stands, \( M^{(0)}_\nu \) of Eq. (6) cannot lead to solar neutrino oscillations since \( \Delta m^2_{\odot} = 0 \). One needs to break the \( L_e - L_\mu - L_\tau \) symmetry in order to make the model realistic. Since the mass parameter \( m \) in Eq. (6) is fixed by the atmospheric data i.e., \( m \simeq \sqrt{\Delta m^2_{\text{atm}}} \), the strength of the \( L_e - L_\mu - L_\tau \) breaking should be of order \( \Delta m^2_{\odot}/\Delta m^2_{\text{atm}} \approx (3 \times 10^{-2} - 3 \times 10^{-3}) \). The smallness of this symmetry breaking parameter suggests that \( L_e - L_\mu - L_\tau \) as well as \( S_2 \) symmetries are indeed approximately conserved.

In this paper, we attempt to seek realistic gauge models that realize Eq. (6) as the leading approximation of the light neutrino mass matrix. The correction terms to the mass matrix will involve very few parameters, so that there is a certain predictive power for neutrino mass difference squares as well as mixings. The hope is that such models can then be tested in future experiments and may shed light into the nature of new physics operating in the neutrino sector and perhaps more generally within quark lepton unified schemes. We present several examples of such models. First we show that global \( L_e - L_\mu - L_\tau \) symmetry, broken only by Planck scale effects, can give a rather good fit to the neutrino data. Then we present a scheme where the neutrino sector has only one parameter which is fixed by the atmospheric neutrino data. Radiative renormalization effects from the charged lepton sector then determines the rest of the parameters in the neutrino mass matrix, making the model extremely predictive. For instance, in this model, we obtain the relation

\[ \Delta m^2_{\odot} \simeq \Delta m^2_{\text{atm}} 4\sqrt{2} r \sqrt{\frac{m_e}{m_\tau}}, \]  

(7)

where \( r \) is a calculable parameter which depends only on the scale of new physics (e.g. seesaw scale) governing the smallness of \( m_\nu \). In this model, the mixing parameter \( U_{e3} \) is predicted to be \( U_{e3} \simeq \sqrt{\frac{m_e}{m_\tau}} \approx 0.017 \). We also present a variation of this model with one more parameter, where \( U_{e3} \) can be larger (even close to the present experimental upper limit of 0.2). We present a second scenario which has two parameters. This model predicts bimaximal neutrino mixings with \( U_{e3} = 0 \). Thus any evidence for nonzero \( U_{e3} \) would rule out this scenario. Finally, we consider extensions of these schemes that may provide a simultaneous fit to solar, atmospheric as well as the LSND neutrino oscillation data. As is well known, this needs the introduction of a sterile neutrino. We achieve this in a manner
that introduces only two more parameters into the theory by invoking the mirror matter models, where all new parameters (except the weak scale and possibly the QCD scale) are fixed by an exact mirror symmetry \[8,9\].

II. $L_E - L_\mu - L_\tau$ SYMMETRY AND BIMAXIMAL MIXING

Suppose that $L_e - L_\mu - L_\tau$ symmetry is a global symmetry of the Lagrangian broken only by Planck scale induced terms. (Quantum gravity is expected to break all non gauge symmetries.) The Lagrangian in this case will have the general form:

$$\mathcal{L} = \frac{1}{M}(aL_e\phi L_\mu\phi + bL_e\phi L_\tau\phi) + \frac{f_{\alpha\beta}}{M_P}\ell(L_\alpha\phi L_\beta\phi) + h.c.$$

where $\alpha, \beta$ are flavor indices. Here $\phi$ is the standard model Higgs doublet. $M$ is the seesaw scale, which can be near the GUT scale $\sim 10^{15}$ GeV, so the first set of terms in Eq. (8) will dominate over the last set. Note that the $L_e - L_\mu - L_\tau$ symmetry is broken by the Planck scale induced terms. The leading contribution to $M_\nu$ is of the form in Eq. (6), with the two nonzero entries unequal in general (denote them $m_1$ and $m_2$). Upon diagonalizing, one obtains two nearly degenerate neutrino eigenstates with opposite CP parities with a common mass of $m = \sqrt{m_1^2 + m_2^2}$. If $M$ is near the GUT scale, $m \sim 0.03$ eV can be explained quite naturally. A bimaximal pattern of neutrino mixings will also be induced, with $U$ given as in Eq. (3).

At this stage, we cannot account for solar neutrino oscillation since $\Delta m_{\odot}^2 = 0$ from Eq. (6). Once we include the Planck scale corrections from the last term of Eq. (8), one gets a mass matrix

$$M_\nu = \begin{pmatrix} \delta & m_1 & m_2 \\ m_1 & \epsilon_1 & \epsilon_2 \\ m_2 & \epsilon_2 & \epsilon_3 \end{pmatrix}$$

with $\delta, \epsilon_i \approx 2 \times 10^{-5}$ eV. This leads to $\Delta m_{\odot}^2 \approx 10^{-6}$ eV$^2$, which is close to the required value for large mixing angle solar neutrino oscillation fit. It is encouraging that all the parameters are in the right range to fit solar and atmospheric neutrino oscillation given the assumption that the seesaw scale $M$ is near the GUT scale. It should be noted that in this scenario, the solar oscillation angle is given by $\sin^2 2\theta_\odot = 1 - \mathcal{O}(\Delta m_{\odot}^2/\Delta m_{\text{atm}}^2)^2$, which is very close to 1. Thus this scenario would prefer the LOW solution to the solar neutrino puzzle, although the LMA solution is not entirely excluded. As we shall see in the next section, small breaking of $L_e - L_\mu - L_\tau$ symmetry in the charged lepton sector would make this scenario nicely consistent with the LMA solution.

We now turn to making this type of models more predictive.

III. RADIATIVELY INDUCED SOLAR NEUTRINO OSCILLATIONS

In this section, we present a single parameter model for neutrino masses where the solar neutrino oscillations are generated by radiative renormalization group effects. We will provide a gauge theoretic derivation of the model in section IV. Consider the following
neutrino mass matrix defined at a (unification) scale much above the weak scale as in Eq. (6).

$$M_0^\nu = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} m .$$ (10)

Let us assume that the charged lepton mass matrix in the same basis is of the form:

$$M_{\ell^+} = \begin{pmatrix} 0 & 0 & x \\ 0 & y & 0 \\ x' & 0 & 1 \end{pmatrix} m_\tau .$$ (11)

For the most part we will take $x' = x$ so that $|x| \simeq \sqrt{m_e/m_\tau}$ and $y \simeq m_\mu/m_\tau$. We will also consider the possibility that $x \gg x'$ (the right handed singlet leptons multiply the matrix in Eq. (11) on the right). In the case $x = x'$, note that there is only parameter ($m$ of Eq. (10)) in the leptonic sector (both the charged leptons and neutrinos). We will show that this model can lead to a realistic description of the neutrino oscillations.

Below the unification scale, through the renormalization of the effective $d = 5$ neutrino mass operator the form of $M_0^\nu$ [10] will be modified. (Analogous corrections in $M_{\ell^+}$ is negligible.) The modified neutrino mass matrix at the weak scale is given by

$$M_\nu \simeq M_0^\nu + \frac{c}{16\pi^2} \ln(M_U/M_Z) \left( Y_\nu Y_\nu^T M_0^\nu + M_0^\nu (Y_\nu Y_\nu^T)^T \right) .$$ (12)

Here $c = -3/2$ for SM while $c = 1$ for SUSY, and $Y_\ell$ is the charged lepton Yukawa coupling matrix. We have absorbed the flavor–independent renormalization factor into the definition of $m$ in Eq. (10). Explicitly,

$$M_\nu \simeq \begin{pmatrix} 2rx & 1 + r(x^2 + y^2) & 1 + r(1 + 2x^2) \\ 1 + r(x^2 + y^2) & 0 & rx \\ 1 + r(1 + 2x^2) & rx & 2rx \end{pmatrix} m .$$ (13)

where

$$r \equiv \frac{c}{16\pi^2} Y_\tau^2 \ln(M_U/M_Z) .$$ (14)

Numerically, $r \simeq -3.7 \times 10^{-5}$ in the SM, while it is $r \simeq 0.022(\tan \beta/30)^2$ in SUSY. The parameters $x \simeq 0.017$ and $y \simeq 0.059$ are also much smaller than 1, facilitating an expansion in small $[r, x, y]$. Note that in the case of $x' \neq x$, the numerical value of $x$ can be larger than 0.017 by a factor even as large as 10. (The $e - \tau$ mixing will be still small if $x \simeq 0.17$, and will not upset the success of approximate $L_e - L_\mu - L_\tau$ symmetry.)

The eigenvalues of $M_\nu$ to leading order in $(r, x, y)$ are (in units of $m$)

$$m_1 \simeq \sqrt{2} + \frac{r}{2}(4x + \sqrt{2}) + \frac{r^2}{8} \sqrt{2}$$

$$m_2 \simeq -\sqrt{2} - \frac{r}{2}(-4x + \sqrt{2}) - \frac{r^2}{8} \sqrt{2}$$

$$m_3 \simeq -r^2 x ,$$ (15)
where terms of order $r^3, x^3, y^3$ etc have been dropped.

The leptonic mixing matrix $U$ is obtained from $U = V_\ell V_\nu^T$, where $V_\ell$ diagonalizes the charged lepton matrix and $V_\nu$ the neutrino mass matrix:

$$U = \begin{bmatrix}
\frac{1}{\sqrt{2}} - \frac{r}{2}(1 + \frac{x}{2}) & -\frac{1}{\sqrt{2}} - \frac{r}{2}(1 + \frac{x}{2}) & -\frac{rx}{\sqrt{2}} - \frac{x}{2}(1 - \frac{r}{2}) \\
\frac{1}{2} - \frac{r}{4}(1 + \sqrt{2}x) + \frac{r^2}{16} & \frac{1}{2} + \frac{r}{4}(-1 + \sqrt{2}x) & \frac{1}{2} - \frac{r}{2}x + \frac{3}{4}r^2 \\
\frac{1}{2} + \frac{r}{4}(1 + \sqrt{2}x) - \frac{3}{4}r^2 & \frac{1}{2} - \frac{r}{2}x + \frac{3}{4}(1 - \sqrt{2}x) - \frac{3}{4}r^2 & \frac{1}{2} - \frac{r}{2}x + \frac{3}{4}r^2 
\end{bmatrix}. \quad (16)$$

$U$ has approximately the bimaximal form (compare Eq. (16) with Eq. (3)). Let us identify $m_1$ and $m_2$ to have masses of order 0.05 eV ($\sqrt{2}m \simeq 0.05eV$) so that the atmospheric neutrino oscillation is explained via $\nu_\mu - \nu_\tau$ oscillations. The solar mass splitting is then given by $\Delta m^2 = m_1^2 - m_2^2 \simeq 8\sqrt{2}r x m^2 \simeq 0.014rx eV^2$. In the SM, this is equal to $0.92 \times 10^{-8}$ eV$^2$ if $x = x'$. On the other hand, if $x = 10x'$, $\Delta m^2 \simeq 0.92 \times 10^{-7}$ eV$^2$, which is in the right range for LOW solution of the solar neutrino data. In SUSY, even with $x = x'$, we get $\Delta m^2 \simeq 4.7 \times 10^{-6}$ eV$^2$ for $\tan \beta = 30$, which is the right range for LMA solution. In the case of SUSY with $\tan \beta = 30$, if we choose $x = 0.2$, so that $U_{ee} \simeq 0.15$, which is near its experimental upper limit, we have $\Delta m^2 \simeq 5.5 \times 10^{-5}$ eV$^2$ and the solar mixing angle is given by $\sin^2 2\theta_\odot \simeq 0.88$. These values are nicely consistent with the large mixing angle MSW solution.

Neutrinoless double beta decay occurs in this model with an amplitude proportional to (see Eq. (16))

$$m_{\beta\beta 0v}^{\text{effective}} \simeq \left(\frac{1}{\sqrt{2}} - \frac{x}{2}\right)^2 m_1 + \left(-\frac{1}{\sqrt{2}} - \frac{x}{2}\right)^2 m_2 \simeq \sqrt{2}xm_1 \simeq 0.05 \times x \text{ eV}. \quad (17)$$

For $x = x' \approx 0.017$, this is $\simeq 10^{-3}$ eV, while for $x = 10x'$, this is 0.01 eV.

IV. A GAUGE MODEL

The model for neutrino masses described in the previous section can be derived by extending the standard model to include extra gauge singlets $S_0 \equiv (S_1, S_2)$ which transform as a doublet under an $S_3$ permutation group [12]. The left handed lepton doublets $\tilde{L} \equiv (L_2, L_3)$ of the second and third generation are assumed to transform also as doublets of $S_3$. The right handed lepton fields as well as all other standard model fields are assumed to be singlets under $S_3$. To obtain the desired charged lepton mass matrix we include two more pairs of standard model singlets, $S_{u,d}$ each transforming as a doublet under the $S_3$ group. We also assume a $Z_2$ symmetry under which $S_d, \tau_R$ are odd and all other fields are even.

The most general $L_e - L_\mu - L_\tau$ and $S_3$ invariant coupling of leptons is given by:

$$L_1 = \frac{1}{M^2} L_e \phi \tilde{L} \phi S_0 + \frac{1}{M} \tilde{L} \phi S_d \tau_R + \tilde{L} \phi S_d \mu_R + h.c. \quad (18)$$

Here $M$ is a fundamental scale close to the GUT scale. The singlets $S_0, S_u, S_d$ all acquire VEVs of order $M$. The $S_3$ invariant potential for these fields will admit the following structure of VEVs: $\langle S_0 \rangle = v_0 (1, 1), \langle S_u \rangle = v_u (1, 0)$ and $\langle S_d \rangle = v_d (0, 1)$ [12]. Such a VEV structure will lead to the charged lepton and neutrino mass matrices given in Eq. (10)-(11) except for the fact that $x = x' = 0$. To induce $x, x' \neq 0$, we need to introduce singlet Higgs fields that break the $L_e - L_\mu - L_\tau$ symmetry. This can also be done in simple ways.
V. A TWO PARAMETER MODEL FROM PERMUTATION SYMMETRY

In this section, we will present a two parameter model that can also emerge from the model of the previous section if we allow for terms that break $L_e - L_\mu - L_\tau$ symmetry. Consider the following Lagrangian:

$$\mathcal{L}_1 = \frac{1}{M^2} L_e \phi L \phi S_0 + \frac{1}{M} (L_\mu \phi L_\mu \phi + L_\tau \phi L_\tau \phi) + \bar{\phi} S \delta R + \bar{\phi} S u R + h.c.$$  \hspace{1cm} (19)

In this case, one obtains the Majorana neutrino mass matrix to be

$$M_\nu^0 = \begin{pmatrix} 0 & 1 & 1 \\ 1 & a & 0 \\ 1 & 0 & a \end{pmatrix} m.$$  \hspace{1cm} (20)

The charged lepton mass matrix is diagonal here. This mass matrix can be diagonalized to give the neutrino mixing matrix $U$, exactly as in Eq. (3). The solar neutrino oscillation is governed by $\Delta m^2_{\odot} \simeq 2m^2a$. This model predicts $U_{e3} = 0$ as can be seen from Eq. (3). Thus any evidence for nonzero $U_{e3}$ would rule out this model.

VI. MIRROR NEUTRINOS AND AN EXPLANATION OF LSND EXPERIMENT

So far we have ignored the LSND results [13] in our discussion of the positive neutrino oscillation signals. In order to accommodate the LSND results, one will have to include an extra neutrino species that is a standard model singlet (a sterile neutrino). In the presence of a sterile neutrino, there are two ways to understand the observations: one is the so-called 2+2 scheme [14] and the other is the 3+1 scheme [15]. The 2+2 scheme has the $\nu_{\mu,\tau}$ neutrinos with mass around an eV and $\nu_{e,s}$ with mass near $10^{-3}$ eV, with the latter explaining the solar neutrino data, the former explaining the atmospheric neutrino data and the gap between the two pairs explaining the LSND results. Recent SNO data disfavors the original version of the 2+2 model where all the missing solar $\nu_e$’s are converted via a small angle MSW mechanism only to the sterile neutrinos. Mixed 2+2 scenarios where there is substantial mixing between the two sectors separated by a gap seem to be consistent with all data [16].

In the 3+1 picture, on the other hand, it is assumed that the three active neutrinos are bunched together at a small mass value (say around $6 \times 10^{-2}$ eV or so), with the sterile neutrino at a mass near an eV. The atmospheric and solar neutrino data is explained by the oscillations among active neutrinos whereas the LSND data is explained by indirect oscillations involving the sterile neutrino [17]. The advantage of this picture in view of the SNO data is that the flat energy spectrum is explained by postulating bimaximal neutrino mixing pattern among the active neutrinos or the LOW solution, as discussed in previous sections of this paper. The question now is whether one can extend the model we analyzed to include one or more light sterile neutrinos and maintain at least some predictive power that we had in the three neutrino case.

One way to obtain light sterile neutrinos is to invoke a mirror sector for the particles and forces, as an exact replica of the standard model sector. One may then identify the neutrinos of the mirror sector as sterile neutrinos. The ultralightness of the sterile neutrinos is then
explained in a simple way by a mirror version of the seesaw mechanism. In this picture, the two sectors are connected only by gravity or possibly by very heavy particles that decouple from low energy physics. The mixing between neutrinos then owes its origin either to gravity or to operators generated by the exchange of the heavy particles that connect the two sectors.

In such theories there are two possibilities for the weak scale for the mirror sector: (i) it is the same as in the familiar sector or (ii) it is different. In order to explain the LSND results in the 3+1 scenario, we need to work with the second scenario, where mirror weak scale $v' \gg v$. We will consider a mirror embedding of the model discussed in section 2 which was based only on $L_e - L_\mu - L_\tau$ symmetry. We will assume that there is an identical symmetry i.e., $L'_e - L'_\mu - L'_\tau$ in the mirror sector and as in sec. II we will further assume that only Planck scale effects break these two symmetries. They are also responsible for mixing between the two sectors. The Lagrangian including the Planck scale breaking terms can then be written as:

$$
\mathcal{L} = \frac{1}{M} (a L_e \phi L_\mu \phi + b L_e \phi L_\tau \phi) + \frac{f_{\alpha \beta}}{M_{Pl}} (L_\alpha \phi L_\beta \phi' + \frac{1}{M'} (a L'_e \phi' L'_\mu \phi' + b L'_e \phi' L'_\mu \phi')
$$

As mentioned before, we work in a picture where the gauge symmetry breaking is not mirror symmetric. This then allows for the possibility that both the weak scale and the seesaw scale in the mirror sector are very different from those in the familiar sector. Suppose we assume that $\langle \phi' \rangle \equiv v' \simeq 5 \times 10^4$ GeV and the mirror seesaw scale $M' \simeq 10^9$ GeV or so. From Eq. (21) one sees that the two heavy neutrino states $\nu'_1, \nu'_2$ can have masses easily of the order of 1-10 GeV and the lightest mirror neutrino has a mass of $\sim$ eV and can therefore be used to understand the LSND results. The off diagonal $\nu - \nu'$ mixing terms are of order $vv'/M_{Pl} \approx 0.01$ eV, which is also of the right order for the purpose.

Upon integrating out the heavy neutrinos, one gets a 4×4 matrix with the following generic structure (neglecting terms smaller than $10^{-2}$ eV):

$$
\mathcal{M}^{(4)} = \begin{pmatrix}
0 & m_1 & m_2 & \epsilon_1 \\
m_1 & 0 & 0 & \epsilon_2 \\
m_2 & 0 & 0 & \epsilon_3 \\
\epsilon_1 & \epsilon_2 & \epsilon_3 & \delta
\end{pmatrix}
$$

For $\delta \gg \epsilon_i, m_i$, this matrix has one large eigenvalue $\approx \delta$, which mixes with $\nu_{e,\mu}$ with a mixing angle of order $\epsilon_i/\delta$; for $\epsilon_i \approx 0.02$ eV and $\delta \approx 0.2$ eV, the LSND results can be explained using indirect oscillation [18], with the probability given by $P(\nu_e \rightarrow \nu_\mu) \sim 4|\epsilon_1\epsilon_2/\delta^2|^2$.

VII. COSMOLOGICAL IMPLICATIONS

The mirror sector scenario has several interesting cosmological implication. First of all, the two heavy ($\approx$ GeV) mirror neutrinos annihilate into the lightest mirror neutrino early on and disappear. As a result they do not contribute to the expansion of the universe during big bang nucleosynthesis (BBN). The lightest mirror neutrino will however be in equilibrium during BBN. Thus, this model would predict one extra neutrino species at the
BBN. In principle, this could be tested once the He\textsuperscript{4} and D\textsuperscript{2} abundances are known more precisely.

A second interesting implication is that due to the heavy mirror scale, the mirror QCD scale is $\Lambda' \approx 7$ GeV (for the nonsupersymmetric case). This is calculated as follows. Mirror symmetry guarantees that the two QCD couplings are the same at very high energies. As they evolve to lower scales, due to the heavier masses of the mirror quarks $\alpha'_s$ will develop a higher slope around $10^5$ GeV corresponding to $t'$ decoupling and become larger than $\alpha_s$; this effect gets further reinforced at the $(b', c')$ masses. A simple calculation using $\alpha_s(M_Z) \simeq 0.118$ shows that $\alpha'_s \approx 1$ around 7 GeV. We define this as the mirror QCD scale. Thus from the neutrino oscillations we are able to predict the mirror QCD scale. Using the analogy with familiar QCD then one can predict that mirror proton mass is around 30-40 GeV.

The mirror electron mass is $m_{e'} \sim \frac{\nu'}{\nu} m_e \approx 500$ MeV. This would make the mirror hydrogen Bohr radius 1000 times smaller than the familiar hydrogen atom, leading to a completely new profile for the mirror sector of the universe.

The mirror baryons, being stable, can now become the dark matter of the universe. This picture is very similar to the one advocated in Ref. [19], with an important difference that here the mirror electron is about 10-30 times heavier. To see how this fits in with the dark matter picture [20], we note that in this scenario, to reconcile the presence of the mirror photon with the success of BBN, one needs to have the temperature of the mirror universe somewhat (say a factor of 2) lower than the temperature of the familiar sector [21]. One can then write

$$\frac{\Omega_{B'}}{\Omega_B} \simeq \left(\frac{T'}{T}\right)^3 \frac{m_{p'}}{m_p}. \quad (23)$$

For $m_{p'}/m_p \simeq 40$ and $T'/T \simeq 0.5$, we get $\Omega_{B'}/\Omega_B \simeq 5$. If we take $\Omega_B \simeq 0.05$, this leads to $\Omega_{DM} \equiv \Omega_{B'} \sim 25\%$, consistent with the current thinking in the field of dark matter physics.

**VIII. CONCLUSIONS**

We have presented several simple and predictive three neutrino oscillation scenarios that can explain the solar and the atmospheric neutrino data with a bimaximal mixing pattern that seems to be favored by current data, especially from the flat energy distribution for solar neutrinos. The measurement of the mixing parameter $U_{e3}$ and evidence for inverted pattern of masses would constitute tests of these models. We have also presented a mirror extension of the schemes that can accommodate the LSND results as well. We find that in the simplest mirror extension, the mirror QCD scale is predicted to be around 5-7 GeV. This leads to a mirror baryon mass around 40 GeV, which can then become a viable candidate for dark matter of the universe.

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