Membrane tension and manifest IIB S-duality

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ABSTRACT

A manifestly S-dual, and ‘12 dimensional’, IIB superstring action with an $Sl(2; \mathbb{R})$ doublet of ‘Born-Infeld’ fields is presented. The M-theory origin of the 12th dimension is the M-2-brane tension, which can be regarded as the flux of a 3-form worldvolume field strength. The latter is required by the fact that the M-2-brane can have a boundary on an M-5-brane.
1. Introduction

In the standard effective action of the 11-dimensional supermembrane [1], i.e. the M-2-brane, the tension is a fixed parameter. In [2] an alternative action was introduced in which the tension becomes a dynamical variable. This action is

\[ S = \int d^3 \xi \frac{1}{2v} [\det g + (\star G)^2] \]  

(1.1)

where \( \xi \) are the worldvolume coordinates, \( v \) is an independent worldvolume scalar density, \( g \) is the induced worldvolume metric, and \( \star G \) is the worldvolume dual of the 3-form

\[ G = dU - A. \]  

(1.2)

The 2-form \( U \) is an independent worldvolume gauge potential whereas \( A \) is the pullback of the 3-form gauge potential of D=11 supergravity*. Thus \( G \) is a type of ‘modified’ field strength 3-form. Note that the action (1.1) is scale invariant [2], a fact which motivated its construction. Also, for an appropriate choice of transformations of \( U \), the Lagrangian (and not just the action) is invariant under (super)isometries of the background. If \( g \) and \( A \) are interpreted as being induced from superspace tensors then the action is \( \kappa \)-symmetric provided that the background satisfies the superfield constraints of D=11 supergravity.

The \( U \) field equation of (1.1) implies that

\[ \star G = TV \]  

(1.3)

where \( T \) is a constant (with \( T \neq 0 \) spontaneously breaking scale invariance). The remaining field equations are then equivalent to those of the standard supermembrane action with tension \( T \). Thus, the membrane tension has been replaced by

* We use the same letter to denote superspace forms and their pullbacks since it should be clear which is intended from the context.
the flux of a 2-form gauge potential. This is analogous to the replacement of the cosmological constant of IIA supergravity by the flux of a 9-form gauge potential [3,4]. In that case, discontinuities in the 10-form field strength are associated with domain walls, i.e. 8-branes [5,4]. One could similarly associate discontinuities in $G$ with boundaries of the M-2-brane. The fact that the M-2-brane can have a boundary on a 5-brane [6,7] therefore provides a motivation for the new action (1.1). Note that the M-5-brane cannot have a boundary [8] so we should not expect to have to replace its tension with a 5-form gauge potential.

The purpose of this article is to point out some consequences, implied by duality, of the elevation of the M-2-brane tension to the status of a dynamical variable†. One consequence, following from double dimensional reduction [11], is that the IIA superstring tension should be replaced by 1-form gauge potential, as originally advocated in [12] (following an earlier suggestion [13]). It was shown in [2] that this 1-form gauge potential is a worldsheet Born-Infeld (BI) field. This may sound surprising because BI fields are usually associated with D-branes rather than ‘fundamental’ strings [14]. In fact, the quantized flux of the BI field on the D-string can be identified with the tension of a ‘fundamental’ IIB superstring [15]. In other words, the D-string effective action is actually the action for an arbitrary number of ‘fundamental’ IIB strings bound to a D-string [16].

Another consequence of the new M-2-brane action (1.1) is that the D-2-brane tension must be similarly replaced by a 2-form gauge potential. T-duality then implies that the tension of each D-p-brane should be replaced by a p-form worldvolume gauge potential. In particular, the D-string tension should be replaced by a 1-form gauge field. This cannot be the usual BI field because, as just noted, its flux is the tension of the ‘fundamental’ string. On the other hand, IIB S-duality implies that the new 1-form potential must be exchanged under S-duality with the BI field. In other words, the usual D-string action should be replaced by a manifestly S-dual one involving an $SL(2;\mathbb{R})$ doublet of 1-form gauge fields.

† Similar considerations were used in [9] to motivate a (2+2)-brane in 10+2 dimensions, also considered in [10] in a similar context.
As we shall see, the new IIB superstring action is 12-dimensional in an obvious sense. Supersymmetric theories in twelve dimensions have been the subject of speculation for a long time [17], and the recent suggestions [18] of a connection with IIB superstring theory have attracted considerable attention (see e.g. [19,20]). The author sees no direct connection of these ideas to the present work, but cannot exclude the possibility. It is perhaps worth pointing out that speculations concerning two time directions make sense only in the context of a postulated invariance under some 12-dimensional orthogonal group, since the number of time directions is related to the signature of this group. The new IIB superstring action presented here is only \( SO(9,1) \times Sl(2;\mathbb{R}) \) invariant, so the question of whether the twelfth dimension is spacelike or timelike does not arise.

2. A manifestly S-dual IIB superstring

Following the steps in [12,2], it is not difficult to construct a new manifestly S-dual IIB superstring action. We first introduce two 1-form gauge potentials, \( V \) and \( \tilde{V} \), and their ‘modified’ 2-form field strength 2-forms

\[
F = dV - B \quad \tilde{F} = d\tilde{V} - \tilde{B}
\]  

(2.1)

where \( V \) and \( \tilde{V} \) are the two 1-form gauge potentials and \( B \) and \( \tilde{B} \) are the pullbacks to the worldvolume of the \( NS \otimes NS \) and \( R \otimes R \) two-form gauge potentials, respectively. The worldsheet Hodge duals \( \star F \) and \( \star \tilde{F} \) are worldsheet scalar densities. We can now write down the manifestly \( Sl(2;\mathbb{Z}) \) invariant (‘Einstein frame’) action

\[
S = \int d^2\xi \frac{1}{2v} \left\{ \det g + e^{-\phi} [\star F]^2 + e^{\phi} [\star (\tilde{F} - \ell F)]^2 \right\}.
\]  

(2.2)

The scalar \( \phi \) is the IIB dilaton field and \( \ell \) the axion. By rescaling to the ‘string-frame’ metric one can arrange for \( F \) to appear in the BI combination \( \det(g + F) \). Alternatively, one can scale to the dual D-string frame metric to arrange for \( \tilde{F} \) to
appear in this way, so either $V$ or $\tilde{V}$ may be interpreted as BI fields, but not both simultaneously. The complex field

$$\tau = \ell + i e^{-\phi}$$

transforms under $Sl(2; \mathbb{Z})$ via the fractional linear transformation

$$\tau \rightarrow \frac{a\tau + b}{c\tau + d}, \quad \left( \begin{array}{cc} a & b \\ c & d \end{array} \right) \in Sl(2; \mathbb{Z}).$$

The action (2.2) is then invariant if $V$ and $\tilde{V}$ transform as an $Sl(2; \mathbb{R})$ doublet

$$\left( \begin{array}{c} \tilde{V} \\ V \end{array} \right) \rightarrow \left( \begin{array}{cc} a & b \\ c & d \end{array} \right) \left( \begin{array}{c} \tilde{V} \\ V \end{array} \right)$$

Since $B$ and $\tilde{B}$ transform in the same way, the field strength 2-forms $F$ and $\tilde{F}$ also form an $Sl(2; \mathbb{R})$ doublet. The field equation of $\tilde{V}$ implies that

$$\star (\tilde{F} - \ell F) = e^{-\phi} v T$$

for constant T. If this is substituted into the remaining field equations * one recovers the usual (Einstein frame) super D-brane equations for a D-string of tension $T$.

No attempt will be made here to establish $\kappa$-symmetry. Instead, the action (2.2) will be interpreted as a purely bosonic one. Passing to the Hamiltonian form we then find the equivalent (bosonic) action

$$S = \int dt d\sigma \left\{ \dot{x} \cdot p + (\partial_t V_1) E + (\partial_t \tilde{V}_1) \tilde{E} + V_0 E' + \tilde{V}_0 \tilde{E}' + s x' \cdot p - \frac{1}{2} u \mathcal{H} \right\}$$

where $s$ is a Lagrange multiplier (shift function), $u = v/(x')^2$ is another Lagrange multiplier (lapse function), and $E$ and $\tilde{E}$ are the electric field variables conjugate

* To legitimate substitution into the action one would have to include a surface term, as discussed in [21] in a different context; when this is done one finds the same result as substitution into the field equations.
to $V_1$ and $\tilde{V}_1$, respectively. The Hamiltonian constraint function $H$ is
\[
H = (p + \tilde{E}\tilde{B} + EB)^2 + (x')^2[e^\phi(E + \ell\tilde{E})^2 + e^{-\phi}\tilde{E}^2]
\] (2.8)
where
\[
B_\mu = (x')^\nu B_{\mu\nu} \quad \tilde{B}_\mu = (x')^\nu \tilde{B}_{\mu\nu}.
\] (2.9)

A prime indicates differentiation with respect to the string's spatial coordinate $\sigma$. The constraints imposed by $V_0$ and $\tilde{V}_0$ imply that the electric fields $E$ and $\tilde{E}$ are independent of $\sigma$. Variation with respect to $V_1$ and $\tilde{V}_1$ shows that $E$ and $\tilde{E}$ will remain at their initial values. If $V$ and $\tilde{V}$ are taken to be $U(1)$ gauge fields then the values allowed to $E$ and $\tilde{E}$ are quantized. We shall suppose that the units are such that $E$ and $\tilde{E}$ are integers:
\[
E = m, \quad \tilde{E} = n.
\] (2.10)

If we now use this in (2.7), and discard surface terms†, we arrive at the action
\[
S = \int dt \int d\sigma [\dot{x} \cdot p + s x' \cdot p]
\]
\[
- \frac{1}{2} u \{ (p + m\tilde{B} + nB)^2 + (x')^2[e^\phi(m + n\ell)^2 + e^{-\phi}n^2] \}.
\] (2.11)

This is the Hamiltonian form of the action for an $(n, m)$ string. Setting $e^\phi = g_s$ (the string coupling) we find that the tension in the string frame is
\[
T = \sqrt{(n/g_s)^2 + (m + n\ell)^2}
\] (2.12)
as expected [22].

† If the action is invariant under some symmetry, e.g. supersymmetry, then the process of discarding surface terms may lead to an action that is invariant up to a surface term. This is why the supermembrane action in flat space, for which the supersymmetry variation of the Wess-Zumino term is a surface term, can be replaced by the action (2.1) for which the Lagrangian, and not just the action, is invariant.
Note that the action (2.7) takes the form

$$S = \int dt \oint d\sigma [\dot{X} \cdot P - \lambda^I \mathcal{H}_I]$$  \hfill (2.13)

where \(X = (x^\mu, V_1, \tilde{V}_1)\) and \(P = (p_\mu, E, \tilde{E})\) and \(\mathcal{H}\) are a set of (first class) constraints. Thus, the action is 12 dimensional in an obvious sense. We conclude that the \textit{M-theory origin of the 12th dimension of IIB superstring theory is the M-2-brane tension}.

3. p-brane boundaries and worldvolume p-forms

It was noted above that, given a 2-form potential on the worldvolume of the D-2-brane, T-duality requires the existence of a p-form gauge potential on the worldvolume of each D-p-brane. The flux of its \((p+1)\)-form field strength through the worldvolume equals the D-p-brane tension. We have already used the fact that IIB S-duality requires a 1-form potential \((\tilde{V})\) on the IIB superstring worldsheet; similar reasoning shows that the IIB \(NS \otimes NS\), or `solitonic' 5-brane must have a 5-form gauge potential. In fact, once we introduce the 2-form gauge potential for the M-2-brane, the existence of a p-form gauge potential on almost all other p-branes follows by duality. An exception is the M-5-brane. Given a 4-form potential on the 4-brane worldvolume we \textit{cannot} deduce the existence of a 5-form potential on the M-5-brane worldvolume because the former has an alternative 11-dimensional explanation. This is just as well since we argued earlier that the M-5-brane action should not have such a field.

To see how the absence of a 5-form gauge field on the M-5-brane is compatible with the occurrence of a 4-form potential on the D-4-brane obtained by double-dimensional reduction, we note that \(x^{11}\) may first be replaced by its 4-form dual with 5-form field strength. The double-dimensional reduction ansatz now corresponds to a non-vanishing flux of this 5-form field strength through the D-4-brane worldvolume, so we may identify the 4-form potential on the D-4-brane as the dual
of the M-5-brane field $x^{11}$. Similarly, a 2-form potential on the D-membrane is not implied by a 1-form on the IIA superstring (although the reverse implication is valid) but it is implied by the existence of a 2-form on the D-2-brane, and the latter is implied by a combination of T-duality and IIB S-duality given the BI field on the D-string. Thus, by reversing the previous logic, we can use duality to deduce the existence of the 2-form gauge potential on the M-2-brane from known results on D-branes, but we cannot similarly deduce the existence of a 5-form gauge potential on the M-5-brane. For example, while the latter would be implied by a 5-form potential on the $NS \otimes NS$, or ‘solitonic’, 5-brane of the IIA theory there is no reason (in contrast to the IIB case) to suppose that there is such a field. Once one accepts the hypothesis that the M-2-brane has a 2-form gauge potential but the M-5-brane does not have a 5-form gauge potential it follows by duality† that a $p$-brane has a $p$-form gauge potential if and only if it can have a boundary on some other brane.

Finally, we wish to point out that the results reported here will likely have implications for one of the outstanding unsolved problems in the ongoing program to determine the full $\kappa$-symmetric actions of all superstring and M-theory $p$-branes, namely the IIB solitonic 5-brane. As we have seen, this action should have a 5-form gauge potential. It is tempting to suppose that there is a manifestly S-dual IIB 5-brane action analogous to the IIB string action given here but with an $Sl(2;\mathbb{R})$ doublet of 5-form gauge potentials. Note that there cannot be an $Sl(2;\mathbb{R})$ doublet of BI gauge fields in this case because a second BI field would disturb the balance of degrees of freedom. One suspects, therefore, that this manifestly S-dual IIB 5-brane action must involve the one BI field and its 3-form dual [23] in a symmetric way. If so this could make the action difficult to find.

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† At least for $p \leq 6$. Formally one could use T-duality to conclude that a D7-brane can have a boundary on a D9-brane but it is not clear to the author how this should be interpreted.
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