IN MEMORY OF PROFESSOR ROUHUAI WANG (1924-2001):
A PIONEERING CHINESE RESEARCHER IN PARTIAL
DIFFERENTIAL EQUATIONS

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Research in partial differential equations had a late start in China. Among the older generation who already became full professors before 1949, there were only a handful of mathematicians such as Xinmou Wu, working in this vast field. In 1956 following the former Soviet model, the Chinese government mapped out a 12-year plan for the long-term development of science and technology. Partial differential equation was one of the key disciplines under development in this plan. Shortly thereafter, Soviet experts were invited to lecture in China. For example, Andrei Bitsadze was invited to give a series of lectures on PDE’s of mixed type. Chinese graduate Students were also sent to the former Soviet Union to study PDE’s. Upon return, they would train others in these fields. While in the former Soviet Union, Chaohao Gu, Yü-lin Chou, etc. learned about the theories of some branches of PDE’s that have widespread applications, such as nonlinear hyperbolic and nonlinear parabolic PDE’s. Subsequently, they promoted rapid development of research in these areas in China.

In the meantime, the theory of elliptic equations has undergone a much more substantial development in the last century and demonstrated a close tie with other branches of mathematics. Stimulated by Hilbert’s 19th and 20th problems, many profound yet fundamental results in this direction have since been produced. Rouhuai Wang was a pioneering Chinese researcher in this area. In 1963 he published a long article [4] proving the analyticity of solutions of higher-order nonlinear elliptic equations, extending Morrey’s results [3] to the case of the more general Lopatinski boundary condition.

It could be said that Professor Wang was entirely self-taught in his research area. His knowledge spanned broadly across many branches of mathematics, and he had a deep understanding of the many directions of research of PDE’s. He chose to work on problems that had both theoretical depth and future importance. In the early 1960s, Wang obtained Schauder estimates for solutions of higher-order parabolic equations [5], and then proved the $L^p$ estimates for solutions of these equations using Fourier method. Unfortunately, only an abstract of the paper of the $L^p$ estimates was published in “Scientia Sinica” before the outbreak of the Cultural Revolution [6], when the publication of all scientific journals ceased in China. Nonetheless, not only were his results published almost simultaneously with foreign counterparts, but his proofs were also more concise and elegant.

Despite the decade-long interruption during the Cultural Revolution, Professor Wang’s work soon caught up with the frontier of international research. He
started with pseudo-differential operators \[9\], and then moved toward Fourier integral operators and Maslov index theory. For Fourier integral operators he obtained the Leray formula of the corresponding Maslov index \[7, 8\]. A few years later, Caffarelli-Nirenberg-Spruck published a series work \[2\] on the Monge-Ampère equation. Because of the importance of such equations in geometry, fully nonlinear elliptic equations received much attention from the PDE community. Already over sixty years old, Rouhuai Wang together with his students courageously turned to this important direction. Using a method combining ingenuity and powerful computational skills, they obtained some pioneering results \[10, 11\]. His substantial research in two such disparate fields as a priori estimates and the index of complex Lagrange-Grassmann manifold demonstrates Rouhuai Wang's extraordinary ability.

Extensive knowledge, profound achievements, and elegant taste made him a respected leader and organizer of major Chinese academic activities in PDE's after the Cultural Revolution. In the 1980s, Professor Wang was an active supporter and participant in S.S. Chern's activities in the Chinese mathematical renaissance. Wang was the chief organizer of the Third International Symposium on Differential Geometry and Differential Equations (DD 3) held in Changchun (1982), the PDE year in Nankai (1985-1986), and DD 7 subsequently held in Tianjin. I was asked by Chern to help him during these activities, so I have deep understanding of the role Wang played in planning these activities.

DD 3 was an academic conference at the national level. To domestic scholars, this was basically a review of the existing state of the PDE community, which was recovering from the Cultural Revolution’s interruption to basic research. The foreign scholars invited to speak at the conference were leading experts in several important research directions. The conference organizers for DD 3 not only had the difficult task of meeting the needs and research interests of the mathematical community, but also played a guiding role. Without people like Rouhuai Wang who had a comprehensive understanding of the academic topics and the country’s research strength, it would have been very difficult to finalize a list of speakers that was agreeable to the entire PDE community.

In contrast, the goal of the Nankai PDE year was to guide domestic PDE research towards the international frontier. Professors and researchers from universities and research institutes, especially graduate students in PDE's, were invited to participate. Under Chern’s leadership, we identified several focal points of research at that time, and divided the participants into several groups accordingly. We invited several internationally-renowned scholars to give series of lectures. In order to help the domestic participants digest the material and subsequently carry out research, we arranged for the domestic experts to offer preparatory courses. We also organized seminars for the participating graduate students to systematically study the related literature. Each of the organizers of PDE-year was in charge of a group. Professor Wang was responsible for the micro-local analysis group in addition to his general organizing responsibilities. He undertook all of the complex organizational work personally. He worked tirelessly and thoughtfully, enabling people from around the country to find their appropriate place and to work in harmony for the success of the academic activities.

Professor Wang always put his hope in young people and cared very much about their academic growth. In the 1980s, the mathematical research of our country was in recovery. He encouraged researchers not only to overcome being afraid of difficulties, but also to master the latest and sharpest tools to attack the difficult
problems of importance. He highly praised the profound and delicate analytical
theories of pseudo-differential operators, Fourier integral operators, and micro-
local analysis. He demanded that the micro-local analysis group participants fully
master this set of analytical techniques, and not to be too anxious for quick results.
He joked that they should become a “fearless squad” and conquer this theoretical
height.

Later on in their research careers, many of the participants of the “fearless squad”
realized that this one year of academic activities had benefited them in a profound
and lasting way. Most notable among them is Fudan University’s Professor Shuxing
Chen. He was the captain of the fearless squad’s team. Under Wang’s guidance
and encouragement, he systematically mastered the ideas in micro-local analysis
through persistent learning. Using these ideas, he obtained remarkable results in
the area of higher-dimensional nonlinear equations of conservation laws. Conse-
quently he was invited to give a 45-minute talk at the 2010 International Congress
of Mathematicians. While Chen’s achievement should clearly be credited to his own
assiduous work, Wang’s strategic vision and guidance also played an indispensable
role.

Wang also practiced what he preached. During the Nankai PDE year, he switched
his focus at age 61 from linear PDE’s to the more difficult fully nonlinear ones. To
many mathematicians, it would have been simply unthinkable for someone of his age
to do so. Young professors and graduate students all enjoyed to stay close to him
for guidance and to listen to him discussing mathematics. Professor Wang was very
supportive of these talented young scholars. He would find all the opportunities he
could to promote their talents and to let them shine under the spotlight. Once,
there was an advisor who did not make good use of the talent of one of his young
advisees. Professor Wang criticized the advisor unsparingly, until the young scholar
eventually had a chance he deserved. Later, this young researcher obtained some
results of high impact.

In many ways, Professor Wang won everyone’s love and admiration!

I also have my own personal story to tell. The First National Conference on
Functional Analysis was held in Changchun in August, 1964. Professor Wang gave
a talk on Schauder estimates for higher-order parabolic equations, which was a
very important generalization of Agmon-Douglise-Nirenberg’s fundamental papers
on the $L^p$ theory for higher-order elliptic equations [1]. At that time I was just a
PDE beginner. My talk was on $L^2$ estimates for higher-order parabolic equations.
I used the Fourier method. After I finished, Professor Wang asked me what I was
going to do next. I said that I wanted to use the Michlin-Hörmander multiplier to
obtain the parabolic $L^p$ interior estimate, but did not know how to deal with the
boundary estimate. Immediately he asked me to stay one more week to work with
him.

I happily accepted the offer of course. I soon discovered that Wang had already
applied the Michlin-Hörmander multiplier method to get the interior estimate, and
had also made a lot of progress on the boundary estimates. After two days of
discussion, I went to his home on the morning of the third day. He told me that
during the previous night, he discovered a clever application of the Hilbert-Hardy
inequality to get the boundary estimate. Then he handed me the draft paper he
wrote. I carefully examined the proofs as he explained his work and found that
there were no errors.
As I prepared to leave Changchun, Wang proposed to publish the paper jointly with me. I thought the idea belonged wholly to Wang himself, and that I made no contribution to the research, so I insisted on declining the offer. However, he was very persistent. After I returned to Beijing he continued to put my name on the draft paper. Finally, I asked a senior mathematics professor at Peking University to write a letter to Professor Wang saying that I was still young, there were still opportunities to learn from Wang, and that it would not be too late to include my name once I made a real contribution. We finally persuaded him. The power of Professor Wang’s analytical skills and his noble character have been imprinted in my memory ever since. He continued to guide and encourage me as I developed my knowledge in PDE. The instances in which he helped me are uncountable. I've always thought of Professor Wang as my beloved mentor.

Professor Wang was a very gifted mathematician, but history did not give him the opportunity to fully display his talents. In his early years, his family was poor and he was orphaned at a young age. In his youth, he faced the national peril. In his prime came the Cultural Revolution, which wasted a whole decade. But he never blamed fate. He always kept his spirit high and bravely climbed new heights. In addition to contributing to mathematics through his extensive academic achievements and cultivating a cohort of outstanding scholars in his field, he left a precious spiritual wealth for the vast Chinese mathematical community. Whenever I think about Professor Rouhuai Wang, I deeply miss him and remember him with great fondness.

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