MECHANISM FOR EXCITING PLANETARY INCLINATION AND ECCENTRICITY THROUGH A RESIDUAL GAS DISK

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ABSTRACT

According to the theory of Kozai resonance, the initial mutual inclination between a small body and a massive planet in an outer circular orbit is as high as \( \sim 39:2 \) for pumping the eccentricity of the inner small body. Here we show that with the presence of a residual gas disk outside two planetary orbits, the inclination can be reduced to as low as a few degrees. The presence of the disk changes the nodal precession rate and directions of the planet orbits. At the place where the two planets achieve the same nodal precession rate, vertical secular resonance (VSR) occurs so that the mutual inclination of the two planets will be excited, which might further trigger the Kozai resonance between the two planets. However, in order to pump an inner Jupiter-like planet, the conditions required for the disk and the outer planet are relatively strict. We develop a set of evolution equations, which can fit the \( N \)-body simulation quite well but can be integrated within a much shorter time. By scanning the parameter spaces using the evolution equations, we find that a massive planet (10 \( M_J \)) at 30 AU with an inclination of \( \sim 6^\circ \) to a massive disk (50 \( M_J \)) can finally enter the Kozai resonance with an inner Jupiter around the snowline. An inclination of \( 20^\circ \) of the outer planet to the disk is required for flipping the inner one to a retrograde orbit. In multiple planet systems, the mechanism can happen between two nonadjacent planets or can inspire a chain reaction among more than two planets. This mechanism could be the source of the observed giant planets in moderate eccentric and inclined orbits, or hot Jupiters in close-in, retrograde orbits after tidal damping.

Key words: celestial mechanics – planetary systems – planets and satellites: dynamical evolution and stability – planets and satellites: formation – planets and satellites: general – protoplanetary disks

Online-only material: color figures

1. INTRODUCTION

The Kozai mechanism is a kind of secular effect that occurs in hierarchical three-body systems (Lidov 1962; Kozai 1962; Naoz et al. 2011b). Within the limit of a circular restricted three-body model, the test particle in an inner orbit can be pumped to a highly eccentric or inclined orbit as long as its initial inclination relative to the outer massive perturber is \( \geq 39:2 \) (Kozai 1962; Innanen et al. 1997). Furthermore, Lithwick & Naoz (2011) and Katz et al. (2011) show that, when the massive perturber is in an eccentric orbit, the effect of octupole terms in the perturbing function will be effective so that the inner test particle may be expelled to retrograde orbits relative to the massive planet orbit.

One of the most prominent applications for the Kozai mechanism is the formation of orbital configurations for hot Jupiters (HJs). Recent observations of the Rossiter–McLaughlin (RM) effect (Rossiter 1924; McLaughlin 1924) show that most HJs might be in orbits misaligned with stellar spins. Actually, for the 53 HJs with RM effect measurements, at least eight HJs might be in retrograde motions (Winn et al. 2010; Triaud et al. 2010; Brown et al. 2012; Albrecht et al. 2012). As the classical core accretion scenario says that planets were formed in a protoplanetary disk surrounding the protostar, the existence of HJs in highly inclined orbits indicates that some dynamical mechanisms must exist after their formation in order to pump their inclinations. As the so-called disk migration scenario (Lin & Papaloizou 1986; Lin et al. 1996) failed to explain the existence of HJs in retrograde orbits, the Kozai mechanism was invoked to excite orbital inclinations (Wu & Murray 2003; Fabrycky & Tremaine 2007; Naoz et al. 2011a; Nagasawa et al. 2008).

Wu & Murray (2003) and Fabrycky & Tremaine (2007) proposed that a third massive body (either a binary or a brown dwarf companion) with a high orbital inclination (\( \geq 39:2 \)) can trigger the Kozai resonance so that the orbital eccentricity of inner planets can be pumped up to near 1, which can be damped at the periastron of the orbit with its excited inclination being preserved. However, population studies establish that only 10% of HJs can be explained by Kozai migration due to binary companions (Wu et al. 2007), while most of the HJ systems did not find any stellar or substellar companions. An alternative choice is whether the outer perturber can be replaced by a massive planet. Although this is possible, a very high mutual inclination between the two planets is required. For example, Naoz et al. (2011a) present a flipping example with a 3 \( M_J \) planet as the outer perturber, while the initial mutual inclination of the two planets is up to \( 71:5 \). Lithwick & Naoz (2011) show that if the outer perturbers are in more eccentric orbits, the relative inclination can be reduced but still must be as high as \( \sim 60^\circ \) for retrograde motion to occur. Thus, the origin of such a high mutual inclination itself merits explanations.

In this paper, we propose a mechanism to efficiently excite planetary eccentricities and inclinations with an outer residual gas disk. After gas giants have formed and swept away the inner part of the gas disk, the residual gas disk outside will perturb the architecture of inner planet systems. Due to the gravity of the residual disk, vertical secular resonances occur between the very massive outer planet and the inner ones at certain locations. Then the mutual inclination between the planetary orbits would be pumped. At this time, if the outer planet has a non-zero inclination relative to the disk midplane, which might have resulted from previous planetary scattering, the mutual inclination may increase to the Kozai critical value; then the Kozai effect between planets would be induced. As a result,
the eccentricities and inclinations of the inner planets would be excited to very high values.

The effect of the gas disk in excited planetary eccentricities was also studied by Nagasawa et al. (2003), Terquem & Ajmain (2010), and Teyssandier et al. (2013), etc. Our present work focuses on how, with the aid of a disk, a two-planet system will execute secular resonances between them in order to trigger the subsequent Kozai effect. We will also present the parameter studies with a set of evolution equations. The paper is organized as follows. In Section 2, we introduce the model and two examples and display the pumping region by scanning the $a_{1,0}$-$I_{2,0}$ plane. Then we show that the pumping mechanism is the secular resonance coupled with the following Kozai resonance and calculate the location of secular resonances in the case of small eccentricities and inclinations by timescale comparisons in Section 3. In Section 4, we deduce the changing rates of some crucial parameters in the pumping process relative to any inertial plane and compare them with the N-body simulation results. In Section 5, the influence of planetary parameters is investigated. According to that, we give the critical pumping conditions for a fixed gas disk. Section 6 displays situations in systems with more than two planets. Finally, discussions and conclusions are presented in Section 7.

2. MODEL AND EXAMPLES

We consider a planet system with two giant planets (denoted as $m_1$ and $m_2$ for the inner and outer planets, respectively) orbiting around a central star, with a protoplanetary disk whose inner part had been swept out by giant planets (Zhang et al. 2008; Zhang & Zhou 2010a, 2010b). For simplicity, the gas disk is assumed to be a two-dimensional circular annulus with its mass distributed on the midplane. As both the mass and the angular momentum of the disk are much larger than those of the planets, we further suppose that the gravity of the planets has no influence on the disk, i.e., the disk is invariant. As the disk exerts gravity onto the planets, the equation for planet motion can be written as follows:

$$\frac{d^2 r_i}{dt^2} = -\frac{G(m_0 + m_i)}{r_i^2} \left(\frac{\mathbf{r}_i}{r_i}\right) + \sum_{j \neq i} N \frac{G m_j}{\left|\mathbf{r}_j - \mathbf{r}_i\right|^3} - \nabla \Phi,$$  \hspace{1cm} (1)

where $r_i$ is the position vector of the planets relative to the star and

$$\Phi = -G \int_{R_{in}}^{R_{out}} \Sigma(r) r dr \int_0^{2\pi} \frac{d\phi}{(r^2 + r_p^2 - 2rr_p\cos\phi \sin\theta_p)^{1/2}},$$ \hspace{1cm} (2)

displays the gravitational potential from the disk (Terquem & Ajmain 2010). $R_{in}$ and $R_{out}$ are the inner and outer borders of the disk. $(\alpha_p, \varphi_p, \theta_p)$ is the spherical coordinates of a planet in the coordinate system settled by the star and the disk midplane. $\Sigma(r)$ is the mass density of the disk, and we use the most commonly exponential density distribution of the disk radius $r$, $\Sigma(r) = \Sigma_0 (r/R_{eq})^{-\alpha}$. The total mass of the disk is given by $M_{\text{disk}}$ and the expression for $\Sigma_0$ is shown in Appendix B.

We apply the Runge–Kutta–Fehlberg 7(8) integrator to integrate Equation (1). Figure 1 gives a typical example, and its initial conditions are listed in Table 1. We set the star mass at $m_0 = 1 M_\odot$. The inner and outer boundaries of the outer gas disk are taken arbitrarily within the scope of disk observations. In order to satisfy the assumption that the angular momentum of the disk is overwhelming, we set the mass of the gas disk as $0.05 M_\odot$. Though it is much larger than the average mass (0.01 $M_\odot$) estimated by Williams & Cieza (2011), it is still within a reasonable range according to a recent transitional disk observation (such as LkCa 15; Kraus & Ireland 2012). The mass of the outer planet is moderately larger than the inner one in order to facilitate the excitation procedure, and the particular influence will be discussed in Section 5. We make the initial eccentricity and the inclination of the inner planet very small in order to show the pumping mechanism. The eccentricity of the outer planet is set very small in order to conveniently compare it with the results of the evolution equations (Section 4), and the non-zero eccentricity situation will be discussed in Section 5.

We can see from Figure 1 that the inclination of the inner planet relative to the disk midplane ($I_1$) goes up to nearly 50° within 0.3 Myr. After around 0.4 Myr, the eccentricity of the inner planet $e_1$ begins to rise, accompanied by the decline of the mutual inclination between planets ($I_{\text{tot}}$). Ascending nodes of the two planets precess at the same rates for most of the first 0.3 Myr, which implies that it is the secular resonance that raises $I_1$ and hence $I_{\text{tot}}$. This triggers the whole excitation process. The pericenter of the inner planet $\omega_1$ keeps librating when this occurs. We also note that for a while after 0.7 Myr, $I_1$ becomes larger than 90° while $e_1$ is close to 1. This provides a good opportunity for the planet to turn into a retrograde HJ after considering a tidal damping due to the central star. As a comparison, a case with the same initial conditions except that $e_1$ begins to rise, accompanied by the approaching nodes precession rates of the two planets. Then $e_1$ is pumped by the Kozai effect with the sign of $\omega_1$ librating.

The initial inclination $I_{2,0}$ and the semi-major axes are critical parameters for the pumping that occurs, so we scanned the phase space of $a_{1,0}$-$I_{2,0}$ and, for every case, extracted the maximum of $I_{\text{tot}}$, $I_1$, $e_1$, and $e_2$ (hereafter denoted by $I_{\text{tot,max}}$, $I_{1,max}$, $e_{1,max}$, and $e_{2,max}$) during the evolutions within 1 Myr (see the filled countlines in Figure 3). The cases here have the same parameters as Figure 1 except for the variables $a_{1,0}$ and $I_{2,0}$. Both $I_{1,max}$ and $I_{\text{tot,max}}$ have obvious minima at around $a_{1,0} = 3.5$ AU when $I_{2,0} = 0°$. The area above the contour line of $I_{\text{tot,max}} = 40°$ coincides with the area above the line of $e_{1,max} = 0.1$ (the discrepancy in their upper left corner is due

| Planet | Mass ($M_\oplus$) | Semimajor Axis (AU) | Eccentricity ($e$) | Inclination ($\alpha$) |
|--------|-----------------|---------------------|-------------------|-----------------------|
| $m_1$  | 1               | 3                   | 0.001             | 1                     |
| $m_2$  | 10              | 30                  | 0.001             | 30                    |

Disk

| Mass ($M_\odot$) | $R_{in}$ (AU) | $R_{out}$ (AU) | $\alpha$ |
|-----------------|---------------|---------------|---------|
| 50              | 50            | 1000          | 1       |
to a Kozai timescale, which is longer than 1 Myr, so there is not enough time for $e_1$ to rise), which implies that eccentricity pumping is attributed to the Kozai effect after inclinations have been excited. In Figure 3(d), the eccentricity of $m_2$ is also excited in the region where the planetary secular resonances occur (when $I_{2, 0} < 35^\circ$). Although $e_{2, max}$ becomes larger in some regions either due to the secular resonance or combined with the Kozai oscillations ($I_{2, 0} \geq 35^\circ$) from the gas disk (Terquem & Ajmia 2010; Teyssandier et al. 2013), it still maintains less than 0.1 in most cases, which is the basis of simplifications in the derivation of the evolution equations in Section 4.

These pumping cases represent a possible scenario to excite efficiently the eccentricities and inclinations of planets when planets are far away from each other. The pumping critical angle is much lower than the Kozai critical angle because of the initial inclination excitation. From the nearly equal rates of change in the nodes of the two planets, we have deduced that it is secular resonance that excites the inclinations. We will further verify that in the next section by frequency and timescale comparisons.

3. CONDITIONS FOR SECULAR RESONANCES (ESR AND VSR)

Secular evolution dominates the dynamics of a planetary system when planets are far away from the star and they are not close to any low-order mean-motion resonances. In this context, once the precession frequencies of planets are integer multiples of each other, secular resonance occurs (Lithwick & Wu 2011; Nagasawa & Ida 2000). At the place where the timescales of the perihelion (nodal) processing rate of the two planets are equal due to the disk and mutual planetary perturbations, secular resonance occurs and is called eccentric (vertical, respectively) secular resonance, or ESR (VSR, receptively).

In order to obtain the timescales more explicitly, we first assume that the initial eccentricities and inclinations of both planets are small before they are effectively excited. We further assume that $m_2 \geq m_1$ and $a_2 \gg a_1$, so the evolution of $m_2$ is dominated by perturbations from the disk and the evolution of $m_1$ is mainly affected by perturbations from $m_2$ (see also Figure 5).

Under these assumptions, we use Lagrange equations (Murray & Dermott 1999) to derive the apsidal and nodal precession rate exerted by the disk gravity (see Appendix B for details):

\[ \dot{\Omega}_i, \text{disk} = \frac{3}{2n_i} K \cos I_i, \]

\[ \dot{\omega}_i, \text{disk} = -\frac{2}{n_i} K, \]

where $I_i$ and $\Omega_i$ are the inclinations and ascending nodes of the two planets ($i = 1, 2$) with respect to the disk midplane, $\omega_i$ is the argument of perihelion, $n_i$ is the angular velocity of
planetary mean motion,
\[ K = \frac{-\alpha + 2 - 1 + \eta^{-1-\epsilon} G M_{\text{disk}}}{1 - \eta^{-\epsilon/2} - 1 - \alpha} \frac{2 R_{\text{out}}}{\epsilon} \]  

is merely related to the disk parameters, \( \eta = R_{\text{in}}/R_{\text{out}} \), and \( \alpha \) is the exponential index of the disk profile.

In deriving Equations (3) and (4), terms with \( e^2 \) and \( \sin^2 I \) have been eliminated to simplify the expressions, which is suitable before the exciting of \( e \) and \( I \). Meanwhile, under these assumptions the semi-major axis \( a \), eccentricity \( e \), and inclination \( I \) of each planet have no secular trend from the disk gravity (see Appendix B). So the timescales of the outer planet can be separately estimated by \( 2\pi/\dot{\Omega}_1 \) and \( 2\pi/\dot{\Omega}_2 \). Then the timescales of the outer planet are

\[
\tau_{\Omega_2} = \frac{2\pi}{\Omega_{2,\text{disk}}}, \quad \tau_{\omega_2} = \frac{2\pi}{\dot{\omega}_{2,\text{disk}}}. \tag{6}
\]

Moreover, we apply the secular perturbation theory (Murray & Dermott 1999) to obtain the precession timescale of the inner planet due to planetary interactions. There are two eigenfrequencies \( g_1, g_2 \) (where \( g_1 > g_2 \)) for the \( e - \omega \) solution and one eigenfrequency \( f \) for the \( I - \Omega \) solution in two-planet systems. So

\[
\tau_{\Omega_1} = \frac{2\pi}{f} \quad \text{and} \quad \tau_{\omega_1} = \frac{2\pi}{g_1}. \tag{7}
\]

can be used to display the precession of \( \Omega_1 \) and \( \omega_1 \), respectively.

Figure 2. Same as Figure 1 except for \( a_{1,0} = 5.5 \text{ AU}, a_{2,0} = 35.5 \text{ AU}, \) and \( I_{2,0} = 10^\circ \).

(A color version of this figure is available in the online journal.)

Figure 4 shows these timescales versus the inner planet’s semi-major axis with the same initial conditions as Figure 3 except \( I_{2,0} = 0 \). \( \tau_{\Omega_1} \) and \( \tau_{\Omega_2} \), \( \tau_{\omega_1} \) and \( \tau_{\omega_2} \), respectively, have one cross point in Figure 4. The x-coordinates of the points display the value of \( a_{1,0} \) when VSR and ESR occur, which roughly match the location of pumping at \( I_{2,0} = 0 \) in Figure 3. The y-coordinates estimate the timescales of secular resonances, which are much less than the average age of the gas disk (Haisch et al. 2001).

When \( I_{2,0} > 0 \), \( \tau_{\Omega_2} \) becomes larger (Equation (3)), then the cross point of \( \tau_{\Omega_1} \) and \( \tau_{\Omega_2} \) moves inward along the \( \tau_{\Omega_1} \) line. Therefore, it only provides the estimation of the inner border of the excitation region in Figure 3. In order to estimate the excitation region more precisely, we will give the evolution equations of the elements in the next section.

4. EVOLUTION EQUATIONS AT ARBITRARY INCLINATIONS

To obtain the quantitative description of planetary orbits when secular resonance happens, we develop a set of simplified equations to describe the evolution of \( e_1, \omega_1, I_1, \Omega_1, I_2, \Omega_2 \), which are suitable for arbitrary inclinations (but are still required for a small \( e_2 \)). We set the disk midplane as the reference plane, which is assumed to coincide with the equatorial plane of the central star. So our derivations are different from Naoz et al. (2011b) in the context of three-body systems, as their reference plane is the invariant plane of the system.
Figure 3. Contours of the maximum mutual inclination between two planets $I_{\text{tot, max}}$ (a), the maximum inclination of the inner planet $I_{1, \text{max}}$ (b), the maximum eccentricity of the inner planet $e_{1, \text{max}}$ (c), and the maximum eccentricity of the outer planet $e_{2, \text{max}}$ (d) from full N-body simulations during the evolution of 1 Myr. Every point has a different initial inclination of the outer planet $I_{2, 0}$ (y-axis) and a different initial semi-major axis of the inner planet $a_{1, 0}$ (x-axis). The black lines in panels (a), (b), and (c) indicate the results of the evolution equations (8), which are integrated over 1 million years or truncated after $e_1 > 0.99$. The black stars in the two upper panels are used to label the positions whose coordinates are contoured in Figure 6. (A color version of this figure is available in the online journal.)

Figure 4. Precession timescales of the argument of pericenter $\omega$ and the longitude of the ascending node $\Omega$ of the two planets. The initial parameter is listed in Table 1 except for $I_{2,0} = 0$ and $a_1$ altering from 0 to 15 AU. Secular resonance for $e - \omega$ would take place around 3.3 AU, with $\tau_{e_1} = \tau_{e_2}$, and secular resonance for $I - \Omega$ around 2.7 AU, with $\tau_{I_1} = \tau_{I_2}$. (A color version of this figure is available in the online journal.)

At first, according to Mardling & Lin (2002), the secular evolution of the elements of $m_1$ affected by $m_2$ is expressed by the angular-momentum vector $\mathbf{h} = r \times \dot{r}$, the Runge–Lenz vector $\mathbf{e}$, and $\mathbf{\hat{q}} = \mathbf{h} \times \mathbf{\hat{e}}$ (see Equations (A3)–(A6)). (The hat indicates the unit vector.) And for $m_2$, the corresponding vectors are $\mathbf{H}$, $\mathbf{E}$, and $\mathbf{Q}$. Then time-averaging is executed, first over the inner orbit for eliminating eccentric anomaly $E_1$ and then over the outer orbit for removing $E_2$ (for the results see Equations (A9)–(A14)). Thereafter, we separately expand the two groups of unit vectors ($\mathbf{\hat{e}}$, $\mathbf{\hat{q}}$, $\mathbf{\hat{h}}$) and ($\mathbf{\hat{E}}$, $\mathbf{\hat{Q}}$, $\mathbf{\hat{H}}$) into terms with $I_1$, $\omega_1$, $\Omega_1$, and $I_2$, $\omega_2$, $\Omega_2$ (see Equation (A15)). This is the key step in developing the final formulae relative to an arbitrary plane rather than the invariable plane of two orbits. Finally, we obtain the evolution of the elements due to planetary perturbation up to the quadrupole terms without any reductions in eccentricities and inclinations (see Equations (A16)–(A21)). It is worth mentioning that the evolutions of $e_1$ and $\omega_1$ have no assumption of $\Delta \Omega = \pi$, so it has more terms than the quadrupole parts in formulae (C9) and (C5) of Naoz et al. (2011b).

The disturbance from the gas disk has been considered independently and details are given in Appendix B. The final
Figure 5. Evolution of \(\log[(dx/dt)_p/(dx/dt)_{\text{disk}}]\) (\(x\) represents \(I_1, \Omega_1, e_1, \omega_1, I_2, \) and \(\Omega_2\)) with time for the case in Figure 1. The red line is the boundary where \((dx/dt)_p = (dx/dt)_{\text{disk}}\).

(A color version of this figure is available in the online journal.)

Here \(x\) represents the six elements: \(I_1, I_2, \Omega_1, \Omega_2, e_1, \) and \(\omega_1\). We set \(e_2 = 0\) as \(e_2\) remains small in most cases (Figure 3(d)); then the six equations presented by the above one become closed (hereafter we call them “the evolution equations”).

We made comparisons for the two parts of the evolution equations by drawing \(\log[(dx/dt)_p/(dx/dt)_{\text{disk}}]\) from the true \(N\)-body simulation in Figure 5. As was expected, for the elements of \(m_1\), \((dx/dt)_p \gg (dx/dt)_{\text{disk}}\) in most cases, and for \(\Omega_2\), \((d\Omega_2/dt)_p \ll (d\Omega_2/dt)_{\text{disk}}\) at all times. As for \(I_2\), the influence from the disk is much smaller because of the small \(e_2\) (see Equation (B8)). These can be utilized in further deductions and simplifications.

Using the evolution equations, we can calculate the evolution of the inclination and eccentricity of the inner planet more quickly. The dashed line in Figure 1 displays the integration results from the evolution equations. Except for some delays, both the trend and the amplitude are fitted pretty well. Furthermore, we use the evolution equations to scan \(a_2, 0\), \(m_2\) and the scanned \(I_2, 0\) \(a_1, 0\). The filled color contour is composed of the values of the \(y\)-coordinates of the extremum \((I_2, 0, \text{min})\), which means the smallest inclination of \(m_2\) for the onset of the Kozai effect \((I_{\text{tot,max}} = 40^\circ}\) or for \(m_1\) retrograding \((I_{1, \text{max}} = 90^\circ}\). The solid line contour is built up by the \(x\)-coordinates of the extremum, which signify the locations of \(m_1\) when VSR between planets will occur.

Considering that a Jupiter-mass planet probably formed outside the snowline (2.7 AU for a 1 \(M_\odot\) star; see Ida & Lin 2004), we constrain the interesting scope beyond 2.7 AU for the solid contour in Figure 6. We can see that with a 0.05 \(M_\odot\)
gas disk ranging from 50 AU to 1000 AU, a Jupiter-mass planet at \( \sim 2.7 \) AU will be pumped by the Kozai effect with a 5\( m_J \) planet at 25 AU and inclined \( > 10^\circ \) relative to the disk midplane. Furthermore, it can be flipped into a retrograde orbit by a giant planet at \( \sim 25 \) AU with a mass of 5\( m_J \) and an inclination \( > 30^\circ \), or a mass of 10\( m_J \) with an inclination \( > 20^\circ \). As every \( a_2,0 - I_2,0 \) scan involved in Figure 6 is made up of the \( \leq 1 \) Myr integrations of the evolution equations, a general gas disk aged several million years is enough for the excitation process.

We also investigate the effect of the disk mass on the excitation process. Figure 7 gives scanned results similar to Figure 6(a) for \( M_{\text{disk}} = 20 M_J \) and \( M_{\text{disk}} = 100 M_J \). Other parameters remain the same as in Table 1 for simplicity. For the smaller disk mass (Figure 7(a)), the regions of \( I_{2,0,\text{min}} \) move toward the upper right relative to the same ones in Figure 6(a), which causes the lower \( I_{2,0,\text{min}} \) zone to become smaller. For the larger disk mass (Figure 7(b)), the \( I_{2,0,\text{min}} = 5^\circ \sim 7^\circ \) range extends to the less massive \( M_2 \) region as compared to Figure 6(a), and the \( I_{2,0,\text{min}} = 0^\circ \sim 5^\circ \) range appears in the upper region. So a more massive disk favors the pumping to some extent.

All of the above discussions have set \( e_2,0 = 0.001 \) in order to concentrate more easily on VSR. However, \( m_2 \) is more likely to be found on an eccentric orbit since planetary scattering has prompted a non-zero inclination. Figure 8 shows the same N-body simulations scanned as those in Figures 3(b) and (c) with a higher \( e_2,0 \). The remarkable difference in the higher \( e_2,0 \) situation is that the critical value of \( I_{2,0} \) for pumping gets smaller. The cases with \( I_{1,\text{max}} > 90^\circ \) and \( e_{1,\text{max}} > 0.99 \) appear even when \( I_{2,0} = 0^\circ \), which might be due to the strong coupling between ESR and VSR when \( e_2 \) is large. However, the effect is not obvious when \( e_{2,0} < 0.2 \).

In Figure 8, we also note that at small \( e_{2,0} \) (Figure 8(a)), planetary mean-motion resonances (4:1, 5:1, 6:1) cause an increase of \( I_{1,\text{max}} \) and \( e_{1,\text{max}} \). When \( e_{2,0} \) becomes larger, the regions outside 12 AU in Figure 8(b) and 8 AU in Figure 8(c) are full of cases with \( I_{1,\text{max}} > 130^\circ \) and \( e_{1,\text{max}} > 0.99 \). This occurs as the aphelion of the inner planet becomes comparable to the perihelion of the outer planet, and the planetary scattering dominates. We stop the simulation as long as any planet crosses the inner edge of the disk.
6. SYSTEMS WITH MORE THAN TWO PLANETS

With the help of VSR, a mutual inclination between planetary orbits much smaller than the Kozai critical value can eventually induce the pumping of the inner planet's eccentricity. However, the occurrence of VSR constrains the inner orbit to a narrow trigger range, and the chances are small that two adjacent planets happen to be in the VSR configuration. However, the above pumping mechanism can be extended to multiple planetary systems so that a wider trigger range can be achieved. Here we give two examples to show that the mechanism can also occur between two nonadjacent planets, as well as inspire a chain reaction among more than two planets. In the left panels of Figure 9, the innermost and outermost planets were in the right configuration to be excited in a two-planet situation, and after another planet is added between them, the excitation still occurs. The right panels in Figure 9 exhibit a chain reaction. The middle planet is located right in the VSR scope of the outermost planet so its inclination is pumped at first, which directly leads to an increase in the mutual inclination of the inner two planets. Lastly, the innermost planet is excited by the VSR with the middle planet. So the influence of the excitation of the outer planet can be spread to a more inward scope by a chain reaction. We do not explore the specific conditions or detailed influence of these more complicated operations of the mechanism and leave them to future works.

7. CONCLUSIONS AND DISCUSSIONS

In this paper, we proposed a mechanism to excite the eccentricities and inclinations of planets with a residual gas disk outside the planets. The excitation was the result of a coupling between the secular resonance and the Kozai effect. After several giant planets formed, the inner disk was assumed to have been swept out by gas giants during their accretion, and the outer part of the gas disk was thought to coexist with planets for as long as a million years. If the outermost planet has a moderately inclined orbit relative to the disk midplane, vertical secular resonance occurs between the planets. Then the mutual inclination between the two planets increases. Once it reaches $\sim 40^\circ$, the Kozai effect between the planets is induced, which can further pump the inner planet’s eccentricity and inclination to
Figure 9. Two cases of the evolution of the semi-major axis, inclinations, and eccentricities of three planets, which orbit the central star with a disk outside. The left plot has three planets with \(m_1 = 1m_J, m_2 = 1m_J, m_3 = 5m_J, a_1 = 10\) AU, \(a_2 = 20\) AU, \(a_3 = 40\) AU, \(I_1 = 1^\circ, I_2 = 1^\circ, I_3 = 30^\circ\). The disk parameters are the same as in Table 1. (A color version of this figure is available in the online journal.)

higher values (Figure 1). So this kind of mechanism is probably one of the origins of HJs on misaligned even retrograde orbits.

In order to describe the evolution of inclinations and the longitude of ascending nodes, we derived evolution equations, which are closed with the assumption of \(e_z = 0\) and are suitable for arbitrary inclinations. They are used to find out the locations and minimum initial inclinations for the occurrence of vertical secular resonance and Kozai resonance (Figures 3 and 6). The elements here are relative to the disk midplane, and the formulae are different from those of the elements relative to the invariable plane of the two orbits, so they can be utilized for situations with elements relative to any invariable plane.

From the evolution equations, we showed that, with a residual gas disk with a mass of 0.05 \(M_\odot\), located from 50 AU to 1000 AU, a Jupiter-mass planet will be pumped by an outer gas giant with 5\(m_J\) mass located out of 25 AU and 10° relative to the disk midplane at least. It could be flipped into a retrograde orbit by an outer gas giant with a 10\(m_J\) mass and an initial inclination of 20°. Such a mechanism can be also effective for a system with more than two planets, and the critical angles required might be more flexible with the presence of more planets.

We used a simple disk model in order to compare the results with the results of the evolution equations and fully discuss the effect of planetary parameters. One limitation is that the disk mass has to be much larger than the total mass of the planets in order to satisfy the angular-momentum-advantage assumption (Section 2), which restricts the discussion to the disk parameters. To verify that the pumping process is not irrelative to the disk model, we conducted the same simulations using a different disk model (Nagasawa et al. 2000; Zhao et al. 2012). We found that pumping still exists with similar structures and even locations of contour lines in Figure 3.

The mechanism described above has some resemblance to the mechanism in binary systems. For binary systems with two planets orbiting one of the stars, Takeda et al. (2008) created three distinct dynamical classes according to the differential nodal precessions of the two planets. The mechanism illustrated in our paper is similar to the so-called weakly coupled systems, with the same peculiarity that the planetary mutual inclination is excited by the secular resonance between the planets. This is actually a transitional case between “decoupled systems” and “dynamically rigid systems.” We illustrate this with three cases in Figure 10 where the semi-major axis of the inner planet is the only varying parameter. In the left panel, the planetary secular interaction is very weak and suppressed by perturbation from the disk, therefore, the secular nodal precession of the inner planet is much slower than that of the outer planet. In the right panel, the mutual effects between the planets become so strong that their nodal precesses coupled, and their maximum mutual inclination much smaller than the Kozai critical angle induced by secular resonance would not be limited in binary systems but can be extended to single-star systems.

Although eccentricity pumping can occur with an initial mutual inclination much smaller than the Kozai critical angle in our mechanism, it is still within a rather narrow range of a disk, and planet configurations for a Jupiter-mass planet can be flipped. The efficiency for the occurrence of this mechanism in different systems will be investigated in future works. Compared to observations, the narrow range also implies that, first, most of the systems should have planets with moderate or low eccentricities and inclinations than the systems with retrograde HJs. Second, according to additional simulations, the more
Figure 10. Three cases representing three different kinds of evolution of the inner planet. The only different initial condition is the semi-major axis of the inner planet $a_1$, which is 0.56 AU, 4.33 AU, and 9.4 AU from left to right. Other parameters are the same: $m_1 = m_J$, $m_2 = 5 m_J$, $m_{\text{disk}} = 50 m_J$, $a_2 = 20$ AU, $R_{\text{in}} = 30$ AU, $R_{\text{out}} = 1000$ AU, $I_1 = 1^\circ$, $I_2 = 31^\circ$, $e_1 = e_2 = 0.001$, $\Omega_1 = \Omega_2$. All arguments are arbitrary. Black lines are for the elements of the inner planet and red lines are for those of the outer planet.

(A color version of this figure is available in the online journal.)

massive the inner planet is, the higher the initial outer inclination must be and the more massive the outer planet needs to be. We therefore speculate that the proportion of misalignment in Earth-like or Neptune-like planets is probably larger than that in Jupiter-like planets. All these need to be verified by further simulation statistics as well as observations.

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APPENDIX A

EVOLUTION OF THE ORBITAL ELEMENTS DUE TO PLANETARY PERTURBATION

We apply the Legendre polynomial expansion and the Runge–Lenz vector introduced in Mardling & Lin (2002) to deduce the elements’ evolution due to planetary interactions. The quadrupole contribution of the acceleration of the inner orbit produced by the third body is

$$ f_{1,p} = \frac{Gm_2}{R^3} (3x \hat{R} - r), \quad (A1) $$

and that of the outer planet from the inner one is

$$ f_{2,p} = -\frac{G\mu_{01}}{R^4} \frac{m_{012}}{m_{01}} \left[ \frac{3}{2} (5x^2 - r^2) \hat{R} - 3xr \right], \quad (A2) $$

where $r$ and $R$ are position vectors of the inner and outer planets in Jacobi coordinates, $x = r \cdot \hat{R}$, $m_{012} = m_0 + m_1 + m_2$, $m_{01} = m_0 + m_1$, $\mu_{01} = m_0 m_1 / m_{01}$.

The relations between the rates of change of the inner orbital elements and those of Runge–Lenz vectors are given by

$$ \frac{de_1}{dt} = \dot{e} \cdot \hat{e}, \quad (A3) $$

$$ \frac{d\omega_1}{dt} = -\frac{d\Omega_1}{dt} \cos I_1 + \frac{\dot{e}}{e_1} \cdot \hat{q}, \quad (A4) $$

$$ \frac{dI_1}{dt} = -\frac{(\sin \omega_1 \hat{e} + \cos \omega_1 \hat{q}) \cdot \hat{h}}{h_1}, \quad (A5) $$

$$ \frac{d\Omega_1}{dt} = \frac{(\cos \omega_1 \hat{e} - \sin \omega_1 \hat{q}) \cdot \hat{h}}{h_1 \sin I_1}, \quad (A6) $$

where

$$ \frac{de}{dt} = 2(f \cdot \dot{r} - \sin \omega \dot{q}) f - (f \cdot r) \dot{r} \frac{Gm_{01}}{r}, \quad (A7) $$
Here, $\mathbf{h} = \mathbf{r} \times \mathbf{r}$ is the orbital angular-momentum vector of the inner orbit, $\hat{e}$ is the Runge–Lenz vector, $\hat{q} = \hat{h} \times \hat{e}$, $r = a_1(\cos E_e - e_1)e + a_1\sqrt{1 - e_1^2}\sin E_e \hat{q}$, and $\mathbf{r} = -a_1n_1\sin E_e/(1 - e_1\cos E_e) + a_1n_1\sqrt{1 - e_1^2}\cos E_e/(1 - e_1\cos E_e)\hat{q}$. For the outer orbit, $\mathbf{r}$ would be replaced by $\mathbf{R}$, and the corresponding unit vector is $(\hat{E}, \hat{Q}, \hat{H})$.

We separately substitute expressions (A1) and (A2) for $\mathbf{f}$ into (A3)–(A6), and average first over the inner orbit then over the outer orbit, and simplify the results as follows:

$$\left(\frac{di_1}{dt}\right)_p = \frac{3Gm_2}{4h_1a_1^2}[(1 - e_1^2)^{3/2}((\cos \omega_1 \hat{e} - \sin \omega_1 \hat{q}) + e_1^2(4 \cos \omega_1 \hat{e} + \sin \omega_1 \hat{q})) \cdot [(\hat{h} \cdot \hat{E})\hat{E} + (\hat{h} \cdot \hat{Q})\hat{Q}], \quad (A9)$$

$$\left(\frac{d\Omega_1}{dt}\right)_p = \frac{3Gm_2}{4h_1\sin i_1a_1^2}[(1 - e_1^2)^{3/2}((\sin \omega_1 \hat{e} + \cos \omega_1 \hat{q}) + e_1^2(4 \sin \omega_1 \hat{e} - \cos \omega_1 \hat{q})) \cdot [(\hat{h} \cdot \hat{E})\hat{E} + (\hat{h} \cdot \hat{Q})\hat{Q}], \quad (A10)$$

$$\left(\frac{di_2}{dt}\right)_p = \frac{3Gm_2}{4h_2m_0a_1^2}[(1 - e_1^2)^{3/2}(\cos \omega_2 \hat{E} - \sin \omega_2 \hat{Q}) \cdot [(1 + 4e_1^2)(\hat{h} \cdot \hat{e})\hat{e} + (1 - e_1^2)(\hat{h} \cdot \hat{q})\hat{q}], \quad (A11)$$

$$\left(\frac{d\Omega_2}{dt}\right)_p = \frac{3Gm_2m_0a_1^4}{4h_m2m_0\sin i_1a_1^2}[(1 - e_1^2)^{3/2}(\sin \omega_2 \hat{E} + \cos \omega_2 \hat{Q}) \cdot [(1 + 4e_1^2)(\hat{h} \cdot \hat{e})\hat{e} + (1 - e_1^2)(\hat{h} \cdot \hat{q})\hat{q}], \quad (A12)$$

$$\left(\frac{de_1}{dt}\right)_p = -\frac{15m_2a_1^3}{4m_0a_1^2}n_1e_1\sqrt{1 - e_1^2}(1 - e_1^2)^{-3/2}[(\hat{e} \cdot \hat{E})(\hat{q} \cdot \hat{E}) + \hat{e} \cdot \hat{q} \cdot \hat{q}], \quad (A13)$$

$$\left(\frac{d\omega_1}{dt}\right)_p = -\left(\frac{d\Omega_2}{dt}\right)_p \cos I_1 + \frac{3m_2a_1^3}{4m_0a_1^2}n_1\sqrt{1 - e_1^2}(1 - e_1^2)^{-3/2}[4((\hat{e} \cdot \hat{E})^2 + (\hat{q} \cdot \hat{Q})^2) - (\hat{q} \cdot \hat{Q})^2 - (\hat{q} \cdot \hat{Q})^2] - 2]. \quad (A14)$$

The coordinates of the Runge–Lenz vectors relative to an arbitrary inertial plane are as follows:

$$\hat{e} = \begin{pmatrix} \cos \Omega_1 \cos \omega_1 - \sin \Omega_1 \sin \omega_1 \cos i_1 \\ -\sin \Omega_1 \cos \omega_1 + \cos \Omega_1 \sin \omega_1 \cos i_1 \\ \sin i_1 \sin \omega_1 \end{pmatrix},$$

$$\hat{q} = \begin{pmatrix} -\cos \Omega_1 \sin \omega_1 - \sin \Omega_1 \cos \omega_1 \cos i_1 \\ -\sin \Omega_1 \sin \omega_1 + \cos \Omega_1 \cos \omega_1 \cos i_1 \\ \sin i_1 \cos \omega_1 \end{pmatrix},$$

$$\hat{h} = \begin{pmatrix} \sin i_1 \sin \Omega_1 \\ -\sin i_1 \cos \Omega_1 \\ \cos i_1 \end{pmatrix},$$

and for $\hat{E}, \hat{Q}, \hat{H}$, the formulae are similar except for switching the subscripts from 1 to 2.

Then we derived the final simplified expressions for the rates of change of $I_1, I_2, \Omega_1, \Omega_2, e_1, \text{ and } \omega_1$

$$\left(\frac{dI_1}{dt}\right)_p = \frac{3m_2a_1^3n_1}{4m_0a_1^2}[(1 - e_1^2)^{-1/2}(1 - e_1^2)^{-3/2}[\cos I_1 \cos I_2 + \sin I_1 \sin I_2 \cos(\Omega_1 - \Omega_2)] \times \left\{ \sin I_1 \sin(\Omega_1 - \Omega_2) + \frac{1}{2}e_1^2(3 + 5 \cos 2\omega_1) \sin I_1 \sin(\Omega_1 - \Omega_2) \right. $$

$$+ 5 \sin 2\omega_1(\cos I_1 \sin I_2 \cos(\Omega_1 - \Omega_2) - \sin I_1 \cos I_2) \right\}], \quad (A16)$$

$$\left(\frac{dI_2}{dt}\right)_p = \frac{3m_2m_0a_1^3n_2}{4m_0a_1^2}[(1 - e_1^2)^{-2} \left\{ -\sin I_1 \sin(\Omega_1 - \Omega_2)[\cos I_1 \cos I_2 $$

$$+ \sin I_1 \sin I_2 \cos(\Omega_1 - \Omega_2)] + \frac{1}{2}e_1^2[-3 \sin I_1 \sin(\Omega_1 - \Omega_2)](\cos I_1 \cos I_2 $$

$$+ \sin I_1 \sin I_2 \cos(\Omega_1 - \Omega_2)] + 5 \cos 2\omega_1(\sin I_1 \cos I_2 - (1 + \cos^2 I_1) \sin I_2 \cos(\Omega_1 - \Omega_2)) + 5 \sin 2\omega_1(\sin I_1 \cos I_2 \cos(\Omega_1 - \Omega_2) - \cos I_1 \sin I_2 \cos(2\Omega_1 - \Omega_2)) \right\}], \quad (A17)$$
we set disk mass as an independent parameter and deduce the mass density from
as in observations, disk mass is a commonly estimated parameter rather than the radial distribution exponential or mass density, and
When \( \Omega_1 - \Omega_2 = \pi \), the latter two formulae turn to the quadrupole parts of (C9) and (C5) of Naoz et al. (2011b).

**APPENDIX B**

**EVOLUTION OF THE ORBITAL ELEMENTS DUE TO DISK GRAVITY**

As in observations, disk mass is a commonly estimated parameter rather than the radial distribution exponential or mass density, and we set disk mass as an independent parameter and deduce the mass density from

\[
\left( \frac{d\Omega_1}{dt} \right)_p = \frac{3m_2a_1^2n_1}{4m_1a_2^2\sin I_1} \left( 1 - e_1^2 \right)^{-1/2} (1 - e_2^2)^{-3/2} \left\{ \frac{1}{4} \sin I_1 \cos I_1 [2 \cos 2(\Omega_1 - \Omega_2) \sin^2 I_2 \\
- 3 \cos 2I_2 - 1] + \frac{1}{2} \cos 2I_1 \sin 2I_2 \cos(\Omega_1 - \Omega_2) + \frac{1}{2} e_1^2 \cos I_1 \cos I_2 \\
+ \sin I_1 \sin I_2 \cos(\Omega_1 - \Omega_2)](-3 + 5 \cos 2\omega_1)(\sin I_1 \cos I_2 \\
- \cos I_1 \sin I_2 \cos(\Omega_1 - \Omega_2)) + 5 \sin I_2 \sin 2\omega_1 \sin(\Omega_1 - \Omega_2) \right\}. \tag{A18}
\]

\[
\left( \frac{d\Omega_2}{dt} \right)_p = \frac{3m_2n_1a_2^3}{4m_1a_2^3\sin I_2} \left( 1 - e_2^2 \right)^{-2} \left\{ \frac{1}{4} \sin I_2 \cos I_2 [2 \cos 2(\Omega_1 - \Omega_2) \sin^2 I_2 - 3 \cos 2I_1 - 1] \\
- 1 \sin I_1 \cos I_2 \cos(\Omega_1 - \Omega_2) + \frac{1}{2} e_1^2 \sin I_1 \cos I_2 \\
+ \sin I_1 \sin I_2 \cos(\Omega_1 - \Omega_2) \sin 2I_1 \cos 2I_2 \cos(\Omega_1 - \Omega_2) \\
+ 5 \sin 2\omega_1 \sin I_2 \cos(\Omega_1 - \Omega_2) \sin^2(\Omega_1 - \Omega_2) - \sin^2 I_2 \right\}, \tag{A19}
\]

\[
\left( \frac{de_1}{dt} \right)_p = \frac{15m_2a_1^3n_1}{8m_0a_1^3} e_1 \sqrt{1 - e_1^2} (1 - e_2^2)^{-3/2} \left\{ 2 \sin 2\omega_1 [\sin I_1 \cos I_2 - \cos I_1 \sin I_2 \cos(\Omega_1 - \Omega_2)]^2 \\
- \sin^2 I_2 \sin^2(\Omega_1 - \Omega_2)] - 2 \cos 2\omega_1 \sin I_2 \sin(\Omega_1 - \Omega_2) \sin I_1 \cos I_2 \\
- \cos I_1 \sin I_2 \cos(\Omega_1 - \Omega_2) \right\}, \tag{A20}
\]

\[
\left( \frac{d\omega_1}{dt} \right)_p = - \left( \frac{d\Omega_1}{dt} \right)_p \cos I_1 - \frac{3m_2a_1^3n_1}{4m_0a_1^3} \sqrt{1 - e_1^2} (1 - e_2^2)^{-3/2} \left\{ (1 - 5 \sin^2 \omega_1) \sin I_1 \sin I_2 \\
+ \cos I_1 \cos I_2 \cos(\Omega_1 - \Omega_2)]^2 + \sin^2(\Omega_1 - \Omega_2) \sin^2(\omega_1^2 + \sin^2 I_2 - 1) \\
- 5 \sin 2\omega_1 \sin(\Omega_1 - \Omega_2) \sin I_2 [\sin I_1 \cos I_2 - \cos I_1 \sin I_2 \cos(\Omega_1 - \Omega_2)] \\
+ 3 \sin^2 I_2 \sin^2(\Omega_1 - \Omega_2) - 1 \right\}. \tag{A21}
\]

When \( \Omega_1 - \Omega_2 = \pi \), the latter two formulae turn to the quadrupole parts of (C9) and (C5) of Naoz et al. (2011b).
We used Lagrange’s equations in Murray & Dermott (1999) to deduce the rates of change of elements due to disk gravity. First, we expanded the gravity potential in \( \frac{r_p}{r} \) to the quadrupole, like in Terquem & Ajmia (2010),

\[
\Phi = \frac{-\alpha + 2}{1 - \eta^{-\alpha+2}} \frac{GM_{\text{disk}}}{R_{\text{out}}} \left[ \frac{1 - \eta^{1-\alpha}}{1 - \alpha} + \frac{-1 + \eta^{-1-\alpha}}{1 + \alpha} \frac{r_p^2}{2R_{\text{out}}^3} \left( -1 + \frac{3}{2} \sin^2 \theta_p \right) \right].
\]  

(B3)

The first term in the square brackets has no contribution to derivation, so only the second one is retained. By defining

\[
K = \frac{-\alpha + 2}{1 - \eta^{-\alpha+2}} \frac{-1 + \eta^{-1-\alpha}}{1 - \alpha} \frac{GM_{\text{disk}}}{2R_{\text{out}}},
\]  

(B4)

and substituting the expressions with true anomaly \( f \) for \( r_p \) and \( \theta_p \), we obtained

\[
\Phi = K a^2 (1 - e^2)^2 \left[ \frac{1}{2} - \frac{3}{2} \sin^2 (\omega + f) \sin^2 I \right].
\]  

(B5)

We substituted the above one into Lagrange’s equations (6.148)–(6.150) in Murray & Dermott (1999), then averaged over true anomaly \( f \), and finally obtained the evolutions

\[
\left( \frac{da}{dt} \right)_{\text{disk}} = 0,
\]  

(B6)

\[
\left( \frac{de}{dt} \right)_{\text{disk}} = -\frac{15K e \beta}{4n} \sin 2\omega \sin^2 I,
\]  

(B7)

\[
\left( \frac{dI}{dt} \right)_{\text{disk}} = \frac{15K e^2}{8n \beta} \sin 2\omega \sin 2I,
\]  

(B8)

\[
\left( \frac{d\Omega}{dt} \right)_{\text{disk}} = \frac{3K \cos I}{4n \beta} (2 + 3e^2 - 5e^2 \cos 2\omega),
\]  

(B9)

\[
\left( \frac{d\omega}{dt} \right)_{\text{disk}} = \frac{K}{n \beta} \left\{ -2 - \frac{9}{8} e^2 + \frac{15}{4} e^2 \cos 2\omega + \sin^2 I \left[ \frac{9}{4} + \frac{9}{16} e^2 - \frac{15}{16} (2 + e^2) \cos 2\omega \right] \right\},
\]  

(B10)

where \( \beta = \sqrt{1 - e^2} \).

When \( i \approx 0 \) and \( e = 0 \), the expressions can be simplified into

\[
\left( \frac{da}{dt} \right)_{\text{disk}} = \left( \frac{de}{dt} \right)_{\text{disk}} = \left( \frac{dI}{dt} \right)_{\text{disk}} = 0,
\]  

(B11)

\[
\left( \frac{d\Omega}{dt} \right)_{\text{disk}} = \frac{3K \cos I}{2n},
\]  

(B12)

\[
\left( \frac{d\omega}{dt} \right)_{\text{disk}} = -\frac{2K}{n}.
\]  

(B13)
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