On the quark scaling theorem and the polarisable dipole of the quark in a scalar field

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Summary
In this article the possible impact is discussed of two unrecognized theoretical elements on the present state of particle physics theory. These elements are the awareness that (a) the quark is a Dirac particle with a polarisable dipole moment in a scalar field and that (b) Dirac’s wave equation for fermions, if derived from Einstein’s geodesic equation, reveals a scaling theorem for quarks. It is shown that the recognition of these elements proves by theory quite some relationships that are up to now only empirically assessed, such as for instance, the mass relationships between the elementary quarks, the mass spectrum of hadrons and the mass values of the Z boson and the Higgs boson.

Keywords: polarisable Dirac dipole; quark scaling; hadron mass spectrum; Z boson; Higgs boson

1. Introduction
This article is aimed to discuss the possible impact that the awareness of a polarisable dipole moment of quarks in a scalar potential field of quarks, conceived as Dirac particles in a particular format, may have on the interpretation of particle physics theory. The discussion will be focussed on the bonds between quarks in mesons and baryons. As is well-known, canonical particle physics theory is captured in a rather abstract mathematical formalism. This formalism has been developed under adoption of some axiomatic attributes that have been unknown prior to the development of the Standard model. Among these are, for instance, weak isospin and hypercharge. They show up as quantum numbers attributed to the elementary fermions [1], such as listed in Table I.

Table I

| Fermions         | u  | d  | e^- | ν_e |
|------------------|----|----|-----|-----|
| s-spin quantum   | 1/2| 1/2| 1/2 | 1/2 |
| m_0-mass         | ?  | ?  | m_e | ≈0  |
| I_Z-weak isospin | 1/2| -1/2| -1/2| 1/2 |
| Y-hypercharge    | 1/3| 1/3| -1  | -1  |
| Q = I_Z + Y / 2  | 2/3| -1/3| -1  | 0   |

It is my aim to show how these attributes are related to those of a quark that has a polarisable dipole moment in a scalar potential field. Such a dipole moment is a unique
property of a particular non-electron format of Dirac’s particle, while it is absent in electron-type ones. As will be shown, this dipole moment could be the key for assigning reliable figures to the rest masses of elementary quarks and their hadron composites. It will be shown that a re-interpretation of these attributes allows a physical interpretation of isospin and removes the reason to accept the asymmetrical electric charge assignment to quarks. It will be shown as well that the number of elementary fermions would be reduced significantly.

Like all elementary fermions, quarks follow Fermi-Dirac statistics, obey the Pauli exclusion principle, have half integer spin and have distinct antiparticles. They can be modelled with the Dirac equation. The canonic formulation of Dirac’s particle equation reads as [2,3],

\[(ih\gamma^\mu\partial_\mu\psi - \beta m_0 c\psi) = 0,\]

in which \(\beta\) is a 4 x 4 unity matrix and in which the 4 x 4 gamma matrices have the properties,

\[\gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu = 0 \text{ if } \mu \neq \nu; \text{ and } \gamma_0^2 = 1; \gamma_i^2 = -1; \beta^2 = 1.\]

As usual, \(c\) is the vacuum light velocity, \(\hbar\) is the reduced Planck constant and \(m_0\) is the rest mass of the particle. While this equation captures a basic attribute as mass and attributes as spin state and particle/antiparticle state, it does not include quite some other properties of elementary fermions. It does not even include electric charge as an attribute, while Dirac’s theory is originally conceived for electrons. It includes mass \(m_0\) and spin \(S\), but the hypercharge and weak isospin are missing. These are rather artificial attributes, conceived in the mathematical standard model, in which empirical phenomena are captured by axiomatic abstraction.

While the spin quantum \(S\) can be physically understood in relationship with the quantization of the observable value of the angular momentum in integer values of the magnitude of the elementary angular momentum \(\hbar\), weak isospin has no known physical interpretation. Apart from its relationship with the electric charge as shown by the Gell-Mann-Nishijima formula [4,5] at the bottom line of Table I, it plays a role in the classification of hadrons, in interactions between nuclear particles and in the interaction with the omni-present energetic background field, known as the Higgs field. Weak isospin shows the same behaviour as spin \(S \in \{-s, -(s-1), ..., (s-1), s\}\) in the sense of being subject to the same algebra rules as spin, thereby in a “(iso)spin 1/2” doublet establishing an isospin triplet state \(|\pm 1\rangle\) and \(|0,0\rangle\) next to a singlet state \(|0,0\rangle\).

Particle physics theory has been developed over many decades of years. As is well-known, a major milestone in this development was set in 1961 by Gell-Mann and Ne’eman, dubbed as the Eightfold Way [6]. One may wonder how this scheme would have been set up if isospin would have been understood physically. Within the scope of this article, it is my aim to show that a physical interpretation allows a less heuristic alternative for the Eightfold Way.
To show that such a novel view might be a useful complement to present theory, some problems will be addressed that are difficult to solve with present-state theory. Examples of such problems are mass related, because present theory shows a weakness in that respect. In particular, it will be shown how the mass spectrum of hadrons can be calculated to a rather high precision, how the masses of the weak interaction bosons and the Higgs boson can be calculated and why, for instance, the mass of a charged pion is 4.6 MeV/c^2 larger than that of a neutral pion.

After the introduction in paragraph 2 of an unconventional non-electron format of a Dirac particle, the quark and the archetype meson (pion) will be profiled in the next two paragraphs in terms of this particle. In paragraph 5 a description will be given of the quark-scaling theorem as a basic quark property next to its polarisable dipole moment under a scalar potential. Paragraph 6 contains an assessment by theory of the Higgs boson mass, followed in paragraph 7 by an assessment of the Z boson by theory. In paragraph 8 the structural view as developed in this article will be compared and related with the mathematical view of the Standard Model. Paragraph 9 deals with the electroweak unification, followed (in paragraph 10) by a comparison between Weinberg’s model and the developed structural model. In paragraph 11, a short description of baryons is given on the basis of the developed theory. The subsequent paragraphs deal with the strong interaction in mesons (paragraph 12) and baryons (paragraph 13), including its relationship with Quantum Chromo Dynamics (QCD) and the role of gluons (paragraphs 14 and 15). Paragraph 16 contains a discussion and the conclusion. In these texts, quite some results are invoked from previously documented works in publications and preprints. The highlight on the quark-type Dirac particle and the quark-scaling theorem as two unrecognized theoretical principles will place those previous results in a better context.

2. Dirac particles with a polarisable dipole moment in a scalar potential field

The canonical set of gamma matrices in Dirac’s equation is given by,

\[
\gamma_0 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}; \gamma_1 = \begin{bmatrix} 0 & \sigma_1 \\ -\sigma_1 & 0 \end{bmatrix}; \gamma_2 = \begin{bmatrix} 0 & \sigma_2 \\ -\sigma_2 & 0 \end{bmatrix}; \gamma_3 = \begin{bmatrix} 0 & \sigma_3 \\ -\sigma_3 & 0 \end{bmatrix}; \beta = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.
\] (2)

The calculation of the excess energy of an electron in motion subject to a vector potential \( A(A_0, A_x, A_y, A_z) \), gives [2,7],

\[
\Delta E = \frac{e}{2m_0} \begin{bmatrix} \sigma & 0 \\ 0 & \sigma \end{bmatrix} \hbar \cdot \mathbf{B} + \frac{e}{2m_0} \begin{bmatrix} 0 & \sigma \\ \sigma & 0 \end{bmatrix} \hbar/c \cdot \mathbf{E},
\] (3)

in which \( \sigma \) is the Pauli vector, defined by
\[ \overline{\sigma} = \sigma_i \mathbf{i} + \sigma_j \mathbf{j} + \sigma_k \mathbf{k}, \]  

(4)

in which \((i, j, k)\) are the spatial unit vectors and in which \(\mathbf{B}\) and \(\mathbf{E}\) are generic field vectors (i.e. not necessarily of electromagnetic nature if \(\epsilon\) is a generic coupling factor) derived from the vector potential. The matrices are state variables with a real eigenvalue \(|\overline{\sigma}| = 1\), such that the angular momentum (associated with \(\mathbf{B}\)) can be conceived as a spin vector with eigenvalue \(|\mathbf{h}|\). Next to the angular momentum, a second momentum \(\mathbf{h}/c\) (associated with \(\mathbf{E}\)) can be identified with eigenvalue \(|\mathbf{h}/c|\). In terms of these two dipole moments, eq. (3) can be written as,

\[ \Delta E = \frac{e}{2m_0} |\mathbf{h}| \cdot \mathbf{B} + i \frac{e}{2m_0} |\mathbf{h}/c| \cdot \mathbf{E}. \]  

(5)

The electron has a real first dipole moment \((\epsilon \hbar/2m_0)\), known as the magnetic dipole moment, and an imaginary second dipole moment \((i\epsilon \hbar/2m_0c)\). The latter is one of the two anomalies of Dirac’s theory, pointed out by himself. He noticed a negative energy solution next to a positive energy solution. And he noticed a real magnetic moment next to an imaginary electrical dipole moment. About the first item he remarked that that the problem would disappear if the electron would change its polarity, but that “this is a phenomenon not yet observed”. About the second item he remarked that he doubted about the physical meaning of an imaginary electrical dipole moment. Curiously, like proven in [7] and its update [8], a different set of \(\gamma\) matrices may turn the imaginary electrical dipole moment into a real one.

To show this modality, let us start from the canonic format of Dirac’s equation as captured by,

\[ (i\hbar \gamma^\mu \partial_\mu - m_0 c \psi) = 0 \rightarrow (i\hbar \gamma^\mu \partial_\mu \psi - \frac{1}{i} m_0 c \psi) = 0, \]  

(6)

It can be rewritten after division by \(m_0 c\), in terms of wave function operators as,

\[ [\gamma_0 \hat{p}^\prime_0 + (\gamma \cdot \hat{p}^\prime) + I_4] \psi = 0, \]  

(7)

in which \(\hat{p}^\prime = \hat{p}^\prime(\hat{p}_1, \hat{p}_2, \hat{p}_3)\) with

\[ \hat{p}^\prime = \frac{1}{m_0 c} \frac{\hbar}{i} \frac{\partial}{\partial x_i} \quad \text{and} \quad \hat{p}^\prime_0 = \frac{1}{m_0 c} \frac{\hbar}{i} \frac{\partial}{\partial \tau}, \]  

(8)

and in which \(I_4\) is the \(4 \times 4\) identity matrix.
Note that the variables are signed by ‘ to emphasize their normalization on $m_0c$. Note also that the temporal parameter is written as proper time $\tau$ to emphasize the (special) relativistic nature of Dirac’s equation in free space. Rewriting (7) in the Weyl format gives,

$$
\begin{bmatrix}
1 & 0 & \hat{p}_0'\psi

0 & -I & \hat{p}_0'\chi
\end{bmatrix} + \begin{bmatrix}
0 & \bar{\sigma} & \hat{p}'\psi

-\bar{\sigma} & 0 & \hat{p}'\chi
\end{bmatrix} + \begin{bmatrix}
I & 0 & \psi

0 & I & \chi
\end{bmatrix} = 0, \tag{9}
$$

in which $\bar{\sigma} = \bar{\sigma}(\sigma_1, \sigma_2, \sigma_3)$ is the Pauli vector with the three Pauli matrices.

As known, Dirac’s equation is based upon a heuristic elaboration of the Einsteinean energy expression under use of particular properties of the $\gamma$ matrices. These properties can be summarized as,

$$
\gamma_{\mu}\gamma_{\nu} + \gamma_{\nu}\gamma_{\mu} = 0 \text{ if } \mu \neq \nu; \text{ and } \gamma_0^2 = 1; \gamma_i^2 = -1; \beta^2 = 1, \tag{10a}
$$

in which $\beta$ is the last matrix term in (9). Recognizing that the last term in the left hand part of (9) represent a matrix $\beta$ and that (6) is valid for a plus sign in front of $m_0$ as well, one should add in fact,

$$
\gamma_{\mu}\beta + \beta\gamma_{\mu} = 0; \beta = \pm 1, \tag{10b}
$$

which is trivial as long as $\beta$ is the identity matrix. The very same properties are met if (9) is modified into,

$$
\begin{bmatrix}
1 & 0 & \hat{p}_0'\psi

0 & -I & \hat{p}_0'\chi
\end{bmatrix} + \begin{bmatrix}
0 & -\bar{\sigma} & \hat{p}'\psi

\bar{\sigma} & 0 & \hat{p}'\chi
\end{bmatrix} + \begin{bmatrix}
I & 0 & \psi

0 & i & \chi
\end{bmatrix} = 0. \tag{11}
$$

Note that the $\beta$ is modified from the $4 \times 4$ identity matrix into the imaginary value of the “fifth” gamma matrix $\gamma_5$. The two representations (9) and (11) are equivalent. Both represent the common electron-type Dirac particle with a real magnetic dipole moment and an imaginary electric dipole moment. If $\beta$ would have been modified into the real value of $\gamma_5$, we would have obtained the tachyon format, which reads as,

$$
\begin{bmatrix}
1 & 0 & \hat{p}_0'\psi

0 & -I & \hat{p}_0'\chi
\end{bmatrix} + \begin{bmatrix}
0 & \bar{\sigma} & \hat{p}'\psi

-\bar{\sigma} & 0 & \hat{p}'\chi
\end{bmatrix} + \begin{bmatrix}
I & 0 & \psi

0 & I & \chi
\end{bmatrix} = 0. \tag{12}
$$

This tachyon format is studied in the context of the hypothetical existence of superluminal particles [9]. It does meet the constraint (10a), but it violates constraint (10b). Instead it meets,

$$
\gamma_{\mu}\beta + \beta\gamma_{\mu} = 0; \beta^2 = -1. \tag{13}
$$
Note the subtle difference between (10b) and (12). The dipole moments of the tachyon are similar to those of the electron-type: the equivalent magnetic one is real and the equivalent electric one is imaginary.

Both dipole moments are real for a third modification of Dirac’s particle [7,8]. This modification reads as,

\[
\begin{pmatrix}
  I & 0 \\
  0 & -I 
\end{pmatrix}
\begin{pmatrix}
  \hat{p}'_{0}\psi \\
  \hat{p}'_{0}\chi
\end{pmatrix} + \begin{pmatrix}
  0 & \sigma \\
  -\sigma & 0
\end{pmatrix}
\begin{pmatrix}
  \hat{p}'\psi \\
  \hat{p}'\chi
\end{pmatrix} + \begin{pmatrix}
  0 & I \\
  I & 0
\end{pmatrix}
\begin{pmatrix}
  \psi \\
  \chi
\end{pmatrix} = 0. 
\]

(14)

As compared with the electron-type (11), the \( \gamma_0 \) matrix is made imaginary. It meets the constraints,

\[
\gamma_\mu \gamma_+ + \gamma_+ \gamma_\mu = 0 \quad \text{if} \quad \mu \neq \nu; \quad \gamma_\mu \beta + \beta \gamma_\mu = 0; \quad \gamma_0^2 = -1; \quad \gamma_i^2 = -1; \quad \beta^2 = -1. \tag{15}
\]

To understand the violations of the constraints (10) and the modifications into (13) and (15), it is instructive to solve the various formats (11), (12) and (14) of Dirac’s equation. In full expansion mode, (14) reads as

\[
\begin{pmatrix}
  1 & 0 & 0 & 0 & \hat{p}'_{0}\psi_0 \\
  0 & 1 & 0 & 0 & \hat{p}'_{0}\psi_1 \\
  0 & 0 & -1 & 0 & \hat{p}'_{0}\psi_2 \\
  0 & 0 & 0 & -1 & \hat{p}'_{0}\psi_3
\end{pmatrix} + \begin{pmatrix}
  0 & 0 & 0 & -i & \hat{p}'_{1}\psi_0 \\
  0 & 0 & 1 & i & \hat{p}'_{1}\psi_1 \\
  0 & -1 & 0 & 0 & \hat{p}'_{1}\psi_2 \\
  -i & 0 & 0 & 0 & \hat{p}'_{1}\psi_3
\end{pmatrix} + \begin{pmatrix}
  0 & 0 & 1 & 0 & \hat{p}'_{2}\psi_0 \\
  0 & 0 & 0 & -i & \hat{p}'_{2}\psi_1 \\
  0 & 1 & 0 & 0 & \hat{p}'_{2}\psi_2 \\
  0 & 1 & 0 & 0 & \hat{p}'_{2}\psi_3
\end{pmatrix} = 0
\]

and written differently,

\[
\begin{pmatrix}
  i\hat{p}'_{0} & 0 & \hat{p}'_{1} + 1 & (\hat{p}'_{1} - i\hat{p}'_{2}) & \psi_0 \\
  0 & i\hat{p}'_{0} & (\hat{p}'_{1} + i\hat{p}'_{2}) & -\hat{p}' + 1 & \psi_1 \\
  -\hat{p}'_{2} + 1 & (\hat{p}'_{1} - i\hat{p}'_{2}) & -i\hat{p}'_{0} & 0 & \psi_2 \\
  -(\hat{p}'_{1} + i\hat{p}'_{2}) & \hat{p}' + 1 & 0 & -i\hat{p}'_{0} & \psi_3
\end{pmatrix} = 0. \tag{16}
\]

Let \( \psi = u_\mu \exp\{i(\mathbf{k} \cdot \mathbf{r} - \omega t)\}; \quad \mathbf{k} = \mathbf{p}/\hbar; \quad \omega = W/\hbar. \tag{17} \]

Applying (17) on (16) gives after some elaboration,

\[
\begin{pmatrix}
  -iW & 0 & cp_3 + m_0c^2 & c(p_1 - ip_2) & \psi_0 \\
  0 & -iW & c(p_1 + ip_2) & -cp_3 + m_0c^2 & \psi_1 \\
  -cp_3 + m_0c^2 & -c(p_1 - ip_2) & iW & 0 & \psi_2 \\
  -c(p_1 + ip_2) & cp_3 + m_0c^2 & 0 & iW & \psi_3
\end{pmatrix} = 0. \tag{18}
\]

This homogeneous set of equations has the solution (17) indeed under the constraint of the determinant value

\[
W^2 = (m_0c^2)^2 - c^2|\mathbf{p}|^2. \tag{19}
\]

6
The canonical equations (9) or (11) show the same solution (17), but different from (19), under the constraint,

$$W^2 = E_{w}^2 = c^2|p|^2 + (m_0c^2)^2.$$

(20)

The tachyon equation (12) shows solution (17) for

$$W^2 = c^2|p|^2 - (m_0c^2)^2.$$  

(21)

For a meaningful wave function, \(\omega\) and \(k\), hence \(W\) and \(p\), must be real. Hence, let us consider the condition (19) more closely. It can be rewritten as,

$$\frac{W^2}{(m_0c^2)^2} = 1 + \frac{(v/c)^2}{(v/c)^2 - 1} = 0 \Rightarrow \frac{W^2}{(m_0c^2)^2} = \frac{1 - 2(v/c)^2}{1 - (v/c)^2}. $$

(22)

The condition for the momentum \(p\) evolves as,

$$\frac{c^2|p|^2}{(m_0c^2)^2} = 1 - \frac{W^2}{(m_0c^2)^2} = 1 - \frac{1 - 2(v/c)^2}{1 - (v/c)^2} \Rightarrow |p| = \pm \frac{m_0v}{\sqrt{1 - (v/c)^2}}. $$

(23)

Hence,

$$W = \pm m_0 c^2 \sqrt{\frac{1 - 2(v/c)^2}{1 - (v/c)^2}}; \quad |p| = \pm \frac{m_0v}{\sqrt{1 - (v/c)^2}}. $$

(24,25)

The similar elaboration for the tachyon format results into,

$$W = \pm \frac{m_0 c^2}{\sqrt{(v/c)^2 - 1}}; \quad \frac{|p|}{m_0 c} = \pm \frac{m_0v}{\sqrt{(v/c)^2 - 1}}. $$

(26,27)

The tachyon format shows real values for \(W\) and \(p\) under superluminal conditions. It is a reason for speculations on the potential existence of superluminal particles. It is not meaningful under subluminal conditions, because the real values turn into imaginary ones. The properties of the “third” format, though, as shown by (24,25) are real under subluminal conditions. The real value of its second dipole moment makes it of interest.

The interpretation of its \(W\) as energy, shown by (21) implies that this energy decreases if it goes from rest into motion. This is contra-intuitive, because in true empty space one would expect the opposite, such as expressed by the Einsteinean energy expression (20) that holds for the canonical case. There is, however, no compelling reason why, in spite of its dimensional appearance, \(W\) should be identical with the Einsteinean energy. The only thing that matters is its real value under subluminal condition, such as shown by (23). We are used
to the Einsteinean energy relationship that says that a particle gains energy when it moves from rest into motion. This happens, however, only if additional energy is fed into the particle. In a conservative system, like an orbiting electron, this additional energy is given as initial state. The quark described by (14), though, shows the opposite: it seems to lose energy if it moves from rest into motion. It inherits its motional energy from its rest mass. This is, possibly, in its fundament, not essentially different from adding some initial energy to the rest mass like in the case of an orbiting electron. Like proven in [7,8], such a particular Dirac particle has a polarisable dipole moment in a scalar field. Hence, under the hypothesis that the quark is a Dirac particle of the “third” type, it possesses a polarisable dipole moment $p_{bc}$ in a generic scalar nuclear field $\nabla A_0$, given by,

$$p_{bc} = \frac{g}{2m_0} \vec{\sigma} \, \hbar/c,$$

in which $g$ is a generic unknown nuclear coupling factor. An essential point to be made here is that the spin state of the polarisable dipole moment is an intrinsic property independent from its spatial orientation given by the dipole moment vector $\hbar/c$. The latter is subject to a potential field, while the spin state is an attribute without vectorial properties. The insensitivity of $\vec{\sigma}$ to either a magnetic field or the scalar part of the vector potential, implies that, similarly as in the case of particle/antiparticle state, quarks in the two states of $\vec{\sigma}$ can be regarded as two different quarks, which by convention can be indicated as an “up isospin” $u$ quark and a “down isospin” $d$ quark. More specifically, isospin can now be identified in terms of the eigen values $\pm 1$, of the matrix $\vec{\sigma}$. For a single particle as $\pm 1/2$, as being the projection of a spin vector on the vertical axis in the complex plane.

It will be clear now that isospin is closely connected with the spin of the elementary angular momentum. The latter will be denoted as nuclear spin. Whereas isospin is connected with the momentum vector $\hbar/c$, nuclear spin is related with the angular momentum vector $\hbar$ in the nuclear equivalent of the electron’s anomalous magnetic dipole moment,

$$p_h = \frac{g}{2m_0} \vec{\sigma} \, \hbar.$$  \hfill (28b)

In quite some textbooks $\vec{\sigma}$ is represented as a vector and $\hbar$ as a scalar quantity. Within the scope of this article, however, it is essential to emphasize that $\vec{\sigma}$ is a state variable and that $\hbar$ and $\hbar/c$ are vectors. It is essential to emphasize once more as well that $\hbar$ is an angular momentum and that $\hbar/c$ is a position vector. They have a different spatial dimensionality. This makes a difference in the spin interpretation of $\vec{\sigma} \, \hbar$ as compared to $\vec{\sigma} \, \hbar/c$. It makes the quantum numbers $\pm 1/2$ associated with isospin and associated with nuclear spin independent from each other because of the handiness of the angular motion.

For proper distinction, the up-state nuclear spin of an $u$ quark (or a $d$ quark) and the down-state of an $u$ quark (or a $d$ quark) will be indicated as, respectively, $u$ and $\bar{u}$ (or $d$ and $\bar{d}$ for $d$...
quarks). The particle state and the antiparticle state of an \( u \) quark will be indicated as, respectively \( u \) and \( \bar{u} \). Analogously \( d \) and \( \bar{d} \) for \( d \) quarks.

Summarizing: accepting the existence of “third-type” Dirac particles allows considering the quark as a particle with two real dipole moments in two mutually independent quantum states, in spite of their common origin from the same state variable.

3. Profiling a quark

Let us proceed by profiling a quark as a field. One of the issues to cope with in the context of this article are the different semantics of the field concept in classical physics, in quantum mechanical physics and in particle physics. In classical physics, the field is the static solution of an energetic wave equation. The field has an energetic interpretation. In quantum mechanics, the field is a solution of Schrödinger’s equation, which in fact is the non-relativistic limit of Dirac’s equation. This field has an probabilistic interpretation, because its integrated squared value is considered as the probability that a particle is at some moment in some spatial position. In particle physics theory, the two fields are unified in a single concept: the quantum field. This is done on the basis of (second) quantization. This allows the description of processes that are subject to an interchange between matter and energy, such as occur in decay processes and scattering processes, including, for instance, recoil. This Quantum Field Theory (QFT) is one of the pillars of the Standard Model of particle physics.

Because interchange between matter and energy is beyond the scope, the field view within the scope of this article is either classical, formalized as \( \Phi \), or quantum mechanical, formalized as a multi-component spinor \( \Psi(\mu, \nu) \) eventually reduced to a single component \( \psi \).

Modeling the quark’s field as a classical scalar \( \Phi \), it can be characterized by a Lagrangian density with the format

\[
L = -\frac{1}{2} \partial_{\mu} \Phi \partial^{\mu} \Phi + U(\Phi) + \rho \Phi, \tag{29}
\]

in which \( U(\Phi) \) is the potential energy of an energetic background field and in which \( \rho \Phi \) is the source term. If the background field would have the format,

\[
U(\Phi) = U_{DB} = \lambda_{DB}^{2} \frac{\Phi^{2}}{2}, \tag{30}
\]

the stationary format of the wave equation, obtained after application of the Euler-Lagrange equation from the Lagrangian, is the inhomogeneous Helmholtz equation [10], which for a pointlike source \( \rho \) shows the solution

\[
\Phi_{DB} = \Phi_{0} \frac{\exp(-\lambda_{DB} r)}{\lambda_{DB} r}. \tag{31}
\]
Such a field (31) is of a type as described by Debije [11] for a charged particle in an ionic plasma. The wave equation associated with this Lagrangian (29,30) has the same format as the Klein-Gordon equation, which originally has been conceived as a relativistic extension of the Schrödinger one. In present quantum field theory, this equation is considered as the field equation for a massive spin-zero particle. In a formal way, one might say that a background field of energy has given mass to a mass less boson. A more physical interpretation is the parallel with the scalar field expression for a charged particle in an ionic plasma. In Debije’s theory, it is the influence of the polarized ionic plasma composed by elementary dipoles that shields the mass less boson field from the electric charged pointlike source.

Similarly as an electron, the quark has an energetic monopole, represented by the source term $\rho \Phi$ in the Lagrangian (29). For an electron, the monopole is an electric point charge. For the quark it is the nuclear equivalent of the electric point charge. Next to the monopole, the electron and the quark have two dipole moments [2,7,8]. These dipole moments are the results from the elementary angular momentum $\hbar$ and the elementary mass dipole moment $\hbar/c$. In the case of an electron, these dipole moments give rise to, respectively, a real magnetic dipole and an imaginary electric dipole. In the case of a quark, these dipole moments give rise to, respectively, a real equivalent of the magnetic dipole and a real nuclear equivalent of the electric dipole. While, due to its imaginary value, the electric dipole moment of the electron cannot be polarized in a scalar potential field, the nuclear equivalent can, because of its real value.

Similarly as the monopole of the electron, the monopole of the quark, spreads a scalar potential field. This field is able to polarize the electric dipole equivalent of another quark. Such a dipole spreads an energetic potential with $x^{-2}$ dependency along the orientation axis of the dipole. As a consequence, an equilibrium of forces can arise between a repelling force from the $r^{-1}$ monopole field dependency and the attractive force with $x^{-2}$ dipole field dependency from suitably aligned dipoles of two quarks. Because nuclear forces have a short range, these potential fields must experience a shielding effect akin to the shielding of the field of an electric point charge in an ionized plasma. This shielding is known as the Debije effect. It occurs under influence of an omni-present fluidal field of energy. In particle physics such a background field is known as the Higgs field. Hence, in qualitative terms, the potential field of a quark along the axis of the polarisable dipole, can be expressed as,

$$\Phi(\lambda x) = \Phi_0 \exp(-\lambda x) \left\{ \frac{1}{(\lambda x)^2} - w \frac{1}{\lambda x} \right\}, \quad (32)$$

in which $\lambda$ (with dimension m⁻¹) is a measure for the range of the nuclear potential, in which $\Phi_0$ (in units of energy, i.e. joule) is a measure for the quark’s “charge”, and in which $w$ is a dimensionless weigh factor that relates the strength of the monopole field to the dipole field. The far field, decaying as $\exp(-\lambda x) / \lambda x$ is due to the monopole. As will be shown later, it can be seen as the major component of the weak interaction between the quarks. The near field, decaying as $\exp(-\lambda x) / (\lambda x)^2$ can be seen as the major component of the strong interaction between the quarks. This component is due the polarisable dipole.
While the spatial Debije format (31) of the field has been straightforwardly calculated from the functional expression of the background field (30), the spatial field expression of the quark’s field in this text has been derived indirectly. One may ask if it would be possible to arrive at the format (32) analytically from a functional field expression as well. Obviously, the simple unbiased symmetric background field expression (30), in which \( U(\Phi) = 0 \) for \( \Phi = 0 \), has to be modified for the purpose. The most simple approach is modifying (30) into

\[
U(\Phi) = -\frac{\mu_H^2}{2} \Phi^2 + \frac{\lambda_H^2}{4} \Phi^4. \tag{33}
\]

For positive values of \( \lambda_H^2 \) and \( \mu_H^2 \), it is a broken field that is zero for

\[
\Phi_0 = (\mu_H / \lambda_H)\sqrt{2},
\]

known as the vacuum expectation value.

(Note: this field format, originally conceived by Nambu [12] from quite a different perspective, has been dubbed later as the Higgs field because of its particular property to give mass to spin-1 particles, shown by Higgs [13] and Englert and Brout [14].)

Unfortunately the high non-linearity of this field prevents deriving an analytical solution \( \Phi(r) \) from (33) and (29). However, a numerical procedure allows deriving a two-parameter expression for \( \Phi(r) \) that closely approximates a true analytical solution. In this approach a generic Ansatz format is adopted for \( \Phi(r) \) from which an expression is retrieved of \( U(\Phi) \).

Subsequently, a fit of is searched on (33). In this approach, first of all, the Euler-Lagrange equation is applied on the static Lagrangian density (29). Hence, from

\[
\frac{\partial L}{\partial \Phi} - \partial_i \left( \frac{\partial L}{\partial (\partial_i \Phi)} \right) = 0, \tag{34}
\]

we have from (29),

\[
\partial_i \Phi \partial^i \Phi = \frac{d}{d\Phi} U(\Phi) + \rho. \tag{35}
\]

The Ansatz format of the field \( \Phi(r) \) is chosen as,

\[
\Phi(r) = \Phi_0' \exp(-\lambda r) \frac{\exp(-\lambda r)}{\lambda r} \left\{ \frac{\exp(-\lambda r)}{\lambda r} - 1 \right\}. \tag{36}
\]

The rationale behind the choice (36) is the assumption that the inter-quark potential will behave similarly as the inter-nucleon potential [15,16]. It is also nicknamed as the “liquid drop model”.
Substitution of (36) into (34) and subsequent calculation of $U(\Phi)$ gives a fit with (33) for $\mu_H^2$ and $\lambda_H^2$, such that

$$\frac{1}{2} \mu_H^2 = 1.06 \lambda^2 \quad \text{and} \quad \frac{1}{4} \lambda_H^2 = 32.3 \frac{\lambda^2}{\Phi_0^2}. \quad (37a)$$

A numerical calculation and a proof for this fit has been documented in [17].

Without loss of generality $\Phi'_0$ can be rescaled to the vacuum expectation value $\Phi_0$ (33) by modifying (37a) into

$$\frac{1}{2} \mu_H^2 = 1.06 \lambda^2 \quad \text{and} \quad \frac{1}{4} \lambda_H^2 = \frac{(1.06 \lambda)^2}{\Phi_0^2}. \quad (37b)$$

The two-parameter field is indistinguishable from the three-parameter field,

$$\Phi(r) = \Phi_0 \exp(-\lambda r) \{ \frac{1}{(\lambda r)^2} - w \frac{1}{\lambda r} \} \quad \text{for} \quad w = 1/0.555. \quad (38)$$

The quark’s field would show the characteristics as shown in the right-hand part of Figure 1. It is nicknamed as the “Mexican hat model”, owing to its shape if rotated around the vertical axis.

It would imply that a quark would be repelled by any other quark under influence of the far field, but attracted by the near field, thereby giving rise to mesons as stable two-quark junctions and baryons as three-quark junctions.

![Fig. 1](image-url) (Left) The quark’s scalar field $\Phi/\Phi_0$ as a function of the normalized radius $\lambda x$ ; (Right) The background field $U_H(\Phi) = -U(\Phi)$ retrieved from the spatial expression.

Unfortunately, this radial symmetric format is not viable, because it violates the renormalization constraint. However, comparing (38) with (32) reveals a striking
correspondence. Nevertheless, there is a major difference as well. While the derivation of (38) has been based upon a presupposed energetic monopole model for the quark, (32) is the result of a dipole moment next to a monopole. Hence, by restricting the validity of (38) to the dipole axis $x$, the renormalization problem is removed by rewriting (38) as a sum of a far field and a near field, such that,

$$
\Phi(x) = \Phi_F(x) + \Phi_N(x) \quad \text{with} \quad \Phi_F(x) = -w \Phi_0 \frac{\exp(-\lambda x)}{\lambda x} \quad \text{and} \quad \Phi_N(x) = \Phi_0 \frac{\exp(-\lambda x)}{(\lambda x)^2} \quad (39)
$$

The conclusion therefore is that the Higgs field has to be interpreted as the shielded radial symmetric field of an energetic monopole in conjunction with a one-dimensional dipole field. The quark, conceived as a subluminal tachyon-type Dirac particle, in this article further denoted as a pseudo-tachyon, is compatible with this model. The near field is due to the dipole and gives an interpretation for the near field that glues the quarks together in hadronic structures. The break in the field that spoils the symmetry of the Debye field, thereby modifying it into the Higgs field, can be ascribed to the quark’s polarisable dipole moment.

4. Profiling mesons and baryons

Let us suppose that we wish to build the archetype meson (quark plus antiquark) and the archetype baryons, i.e. the proton and the neutron (three quarks) by a single archetype quark. Would that be possible, and if not, what kind of theoretical instruments (read axioms) would be needed to do so? Figure 2 shows a schematic configuration between two elementary quarks.

![Figure 2](image_url)

Fig. 2. A quark has two real dipole moments, hence two dipoles. One of these (horizontally visualized) is polarisable in a scalar potential field. The other one (vertically visualized) is not. The dipole moments are subject to spin statistics. However, the polarity of the horizontal one is restrained by the bond: the horizontal dipoles are only oriented in the same direction: either inward to the centre or outward from the centre.

As noted before, the far field, decaying as $\exp(-\lambda x)/\lambda x$, due to the monopole, can be seen as the weak interaction between the quarks. The near field, decaying as $\exp(-\lambda x)/(\lambda x)^2$ is due the polarisable dipole. Unlike quark bonds, such lepton bonds don’t exist, because of
the lack of such dipole. In the figure it is supposed that the far field is repelling while the near field is attracting. In fact, the mechanism remains the same under an attracting far field and a repelling near field. Accepting the appearance of the archetype quark into two different modes $u$ and $d$, we may compose a classification scheme for the archetype mesons as shown in Table I. Under antiparallel nuclear spin condition, stable structures require compositions shown in the second column of Table I.

**Table I: archetype mesons**

| meson          | pseudoscalar coding | symb | isospin sum (Q) | vector mode coding | symb | isospin sum (Q) |
|----------------|----------------------|------|-----------------|------------------|------|-----------------|
| $\bar{u}d$     | $\pi^+$              | 1    | $\bar{u}d$      | $\rho^+$         | 1    |                 |
| $d\bar{u}$     | $\pi^-$              | -1   | $d\bar{u}$      | $\rho^-$         | -1   |                 |
| ($\bar{u}d + d\bar{u})/2$ | $\pi^0$          | 0    | ($\bar{u}d + d\bar{u})/2$ | $\rho^0$       | 0    |                 |

The possible parallel nuclear spin configurations are shown in the fifth column. Whereas the up-spin is supposed as being in isospin state $1/2$, the down spin state is in isospin state $-1/2$. The antiquarks, of course, have opposite signs. Note the additional neutral configuration $\omega$ of vector mesons and its difference with $\rho^0$. Note also that the picture shown in figure 2 and the associated coding scheme maintains its validity in the case that the monopoles are repelling and the isospin dipoles are attracting. Note that $u\bar{u}$ and (consequently) $u\bar{u}$ does not show up because of the Pauli constraint on isospin.

What about the archetype baryon? Figure 3 shows its basic configuration. It illustrates that the monopole fields of the quarks are balancing the fields of the polarisable dipole moments. Like explained before, the orientation of these dipoles is unrelated from their isospin status. This independence is illustrated in figure 4.

**Fig.3**: Left: the basic baryon structure as a harmonic oscillator. The polarisable dipole moments balance the fields of the monopoles. The vibra-rotations of the monopoles have an equivalent in the behavior of the center-of-mass. Right: illustration of the frame spin of the baryon.
In this picture, the isospin states are symbolized as either inward ($u$) or outward oriented ($d$) that either show up as nuclear spin 1/2 (two spins parallel, one anti parallel) or as nuclear spin 3/2 (three spins parallel). The possible configurations are shown in Table II. In ground state all three quarks have to be in a different condition, be it in nuclear spin or in isospin. As shown in the upper part, the ground state configuration with nuclear spin 1/2 shows only two different possible modes. The 3/2 spin state allows four possible modes, because the nuclear spin-spin interaction (to be discussed in this text later) brings any of the three quarks in a different state of energy with respect to any other. This relies on the same mechanism as known from the triplet states of electrons in molecular orbits. Comay [18] has given a detailed analysis for this phenomenon. From this point of view there is no need to invoke the QCD color charge as an additional axiom.

Fig.4: The basic baryon configurations. The arrows represent a symbolic representation of the isospins. The upper part holds for nuclear spin 1/2 (with dot) and for nuclear spin 3/2 (without dot). The nuclear spin 3/2 condition has two additional modes, shown in the lower part. The isospins oriented toward or outward the center-of-mass are regarded as, respectively, up spins ($u$) or down spins ($d$).

Inspection of the tables I and II shows that whereas in the case of mesons their electric charge is straightforwardly related with the isospin state, the isospin state of the baryons is extended with an additional term to obtain the same result. The reason why is suggested in the right hand part of figure 3. This shows that, unlike the meson frame, the baryon frame is subject to a possible additional spin, to be discussed later.

Unfortunately, the simple adoption of a single archetype quark in two different modes, is, by far, not adequate for the explanation on observations from mesons and baryons. A first extension to the considerations just given has led to the inevitable conclusion that next to the $u$ quark and the $d$ quark at least a third quark had to exist. There seemed being no escape than defining a new elementary particle that became known as the $s$ (trange) quark. In this article it will be shown, though, that its existence can be explained as a further consequence from the quark conceived as a polarisable Dirac particle. Before doing so, let us first extend the meson and baryon tables by including the $s$ quark. It can be done
systematically similar as in the case of the basic quarks by distinguishing between the nuclear spin antiparallel configuration (spin \(+/- 1/2\) for baryons) and the spin parallel configuration (spin \(+/- 3/2\) for baryons). The \(s\) quark appears being a particle with negative electric charge. The \(c\) (harm)quark, identified later in 1974, appeared being just positively charged. Because the electric charge of the hadron is an holistic attribute, one may still explain the charge of the hadrons as a consequence from asymmetrical charge splits or as symmetrical contributions on top of a bias.

Table II: archetype baryons

| baryon | scalar spin inward/outward | spin (nuclear) | isospin sum | frame spin | charge | symb |
|--------|---------------------------|----------------|-------------|------------|--------|------|
| \((ud)u\) | \(\pm 1/2\) | +1/2 | +1/2 | 1 | \(p\) |
| \((du)d\) | \(\pm 1/2\) | -1/2 | +1/2 | 0 | \(n\) |

(two isospins in different nuclear spin are allowed against a third particle)

| baryon | scalar spin inward/outward | spin (nuclear) | isospin sum | frame spin | charge | symb |
|--------|---------------------------|----------------|-------------|------------|--------|------|
| \((qqq)\) | \(\pm 3/2\) | +1/2 | +1/2 | 1 | \(\Delta^+\) |
| \((du)d\) | \(\pm 3/2\) | -1/2 | +1/2 | 0 | \(\Delta^0\) |
| \((uu)u\) | \(\pm 3/2\) | +3/2 | +1/2 | 2 | \(\Delta^{++}\) |
| \((dd)d\) | \(\pm 3/2\) | -3/2 | +1/2 | -1 | \(\Delta^-\) |

(two isospins in the same nuclear spin are allowed against a third particle)

From the rest masses of the hadrons, shown in the tables IV and V, it is obvious that the parallel nuclear spin configurations show significant higher values than the antiparallel configurations. The nuclear spin flip between parallel and antiparallel is known as strong interaction. The spin \(+/- 1/2\) table of the eight light \(u,d,s\) baryons is known as an octet, and the spin \(+/- 3/2\) table of the ten light \(u,d,s\) baryons is known as a decuplet. Note the mass difference between the \(\Lambda^0\) baryon and the \(\Sigma^0\) baryon. It makes a difference whether quarks with equal mass are in parallel or whether quarks with unequal mass are in parallel. In the \(+/- 3/2\) configuration the difference has disappeared and the two configurations coincide. Hence, it is obvious that de nuclear spin orientations have a major impact on the rest masses of the hadrons. More about this in quantitative terms will be discussed later in this article.

As noted before, whereas in the Standard Model the emergence of quarks heavier than the archetype \(u/d\) has been accepted by defining new elementary particles, it will be demonstrated now that such particles are a theoretical consequence of the Dirac quark as a polarisable particle. If so, the heavier quarks will no longer be elementary. From inspection of the basic meson and the basic baryon in figure 2, respectively figure 3, it will be clear that those stable structures show a two-quark (an)harmonic oscillator, respectively a three-quark (an)harmonic oscillator. Both structures can be analyzed by a one-body equivalent. Obviously, the meson is easier to handle than the baryon, albeit that the meson has to be
analyzed in its center-of-mass frame and subsequent relativistic correction. In that respect, the baryon is different. However, a three-body problem is notoriously difficult. One might oppose that these (an)harmonic oscillator structures are oversimplifications of the actual problem, because they suggest that the behavior of a quark can be captured in a non-relativistic Schrödinger-type wave function, while actually a relativistic Dirac-type wave function is required. But let us see where this road takes us.

### Table IV

| meson | pseudo scalar | symb | vector | isospin sum | Q | mass calculated (MeV/c²) |
|-------|---------------|------|--------|-------------|---|------------------------|
| $(u\bar{d})$ | $\pi^+$ | $(u\bar{d})$ | $\rho^+$ | 1 | 140/780 | 139/775 |
| $(d\bar{u})$ | $\pi^-$ | $(d\bar{u})$ | $\rho^-$ | -1 | 1 | |
| $(u\bar{d} + \bar{d}u)/2$ | $\pi^0$ | $(u\bar{d} + \bar{d}u)/2$ | $\rho^0$ | 0 | 0 | |
| $(u\bar{u} + \bar{d}d)/2$ | $\omega$ | $(u\bar{u} + \bar{d}d)/2$ | $\omega$ | 0 | 0 | |
| $(u\bar{s})$ | $K^+$ | $(u\bar{s})$ | $K^{*-}$ | 1 | 1 | |
| $(s\bar{u})$ | $K^-$ | $(s\bar{u})$ | $K^*$ | -1 | 1 | |
| $(d\bar{s})$ | $\bar{K}^0$ | $(d\bar{s})$ | $\bar{K}^0$ | 0 | 0 | |
| $(s\bar{d})$ | $\bar{K}$ | $(s\bar{d})$ | $\bar{K}$ | -1 | 0 | |
| $(s\bar{s})$ | $x$ | $(s\bar{s})$ | $\phi$ | 0 | 0 | 1032 | n.a./1020 |

### Table V

| baryon | spin $\pm 1/2$ | symb | spin $\pm 3/2$ | symb | isospin sum | frame spin | Q | mass calculated (MeV/c²) | mass actual (MeV/c²) |
|--------|----------------|------|----------------|------|-------------|------------|---|--------------------------|----------------------|
| $(u\bar{d})u$ | p | $(u\bar{d})u$ | $\Delta^+$ | 1/2 | 1/2 | 1 | 934/1237 | 938/1232 |
| $(d\bar{u})d$ | n | $(d\bar{u})d$ | $\Delta^0$ | -1/2 | 1/2 | 0 | |
| $(u\bar{u})u$ | $\Delta^+$ | 3/2 | 1/2 | 2 | |
| $(d\bar{d})d$ | $\Delta^-$ | -3/2 | 1/2 | 1 | |
| $(u\bar{d})s$ | $\Lambda^0$ | -1/2 | 1/2 | 0 | 1105 | 1115 |
| $(u\bar{s})u$ | $\Sigma^+$ | $(u\bar{s})u$ | $\Sigma^{**}$ | 1/2 | 1/2 | 1 | 1170/1377 | 1190/1385 |
| $(d\bar{s})d$ | $\Sigma^-$ | $(d\bar{s})d$ | $\Sigma^{*-}$ | -3/2 | 1/2 | -1 | |
| $(u\bar{s})d$ | $\Sigma^0$ | $(u\bar{s})d$ | $\Sigma^{*0}$ | -1/2 | 1/2 | 0 | |
| $(s\bar{s})u$ | $\Xi^0$ | $(s\bar{s})u$ | $\Xi^{*0}$ | -1/2 | 1/2 | 0 | 1314/1521 | 1314/1531 |
| $(s\bar{s})d$ | $\Xi^-$ | $(s\bar{s})d$ | $\Xi^{-}$ | -3/2 | 1/2 | -1 | |
| $(s\bar{s})s$ | $x$ | $(s\bar{s})s$ | $\Omega^{-}$ | -3/2 | 1/2 | -1 | n.a./1671 | n.a./1672 |
5. The meson

Conceiving the pion as a structure in which a quark couples to the field of the antiquark by the generic quantum mechanical coupling factor $g$ (it will turn out later that its value can be exchanged with $\Phi_0$ under invariance of the product $g\Phi_0$), the pion can be modeled as the one-body equivalent of a two-body oscillator, described by the equation for its wave function $\psi$,

$$-\frac{\hbar^2}{2m_m}\frac{d^2\psi}{dx^2} + \{U(d+x) + U(d-x)\}\psi = E\psi; \quad U(x) = g\Phi(x),$$

(40)

in which $\Phi(x)$ is the quark’s scalar field as derived before and eventually expressed by (32), $2d$ the quark spacing, $m_m$ the reduced mass that embodies the two massive contributions from the constituting quarks, $V(x) = U(d+x) + U(d-x)$ its potential energy, and $E$ the generic energy constant, which is subject to quantization.

It will be clear from (40) that the potential energy $V(x)$ can be expanded as,

$$V(x) = U(d+x) + U(d-x) = g\Phi_0(k_0 + k_2x^2 + ....),$$

(41)

in which $k_0$ and $k_2$ are dimensionless coefficients that depend on the spacing $2d$ between the quarks.

Note that the effective mass $m_m$ of the two quarks is not necessarily the same as the constituent mass that results from an a-posteriori assignment from the non-observable rest mass of the pion calculated from the observable decay products. The constituent mass is mainly a result of the ground state energy of the oscillator, which is taken up from the field. Furthermore, it has to be kept in mind that this model holds in the center of mass frame, so that a lab frame interpretation will need a relativistic correction. To facilitate the analysis, (40) is normalized as,

$$-\alpha_0\frac{d^2\psi}{dx'^2} + V'(x')\psi = E'\psi,$$

(42)

in which $\alpha_0 = \frac{\lambda^2\hbar^2}{2m_m\Phi_0}$, $x' = x\lambda$, $d' = d\lambda$, $E' = \frac{E}{g\Phi_0}$, $U'(x') = \frac{U(\lambda x)}{g\Phi_0}$ and

$$V'(x') = U'(d'+x') + U'(d'-x') = k_0 + k_2x'^2 + .......$$

Invoking previous work [[19, eq. (24)], and to be confirmed in this text once more, we get for $\alpha_0$, 
\[ \alpha_0 = \frac{1}{4k_2}. \]  

(43)

Normalized quantities in this text will be indicated by a “prime” ('). The coefficients \( k_0(d') \) and \( k_2(d') \) can be straightforwardly calculated from (42) and (36) as,

\[
k_0 = 2 \frac{\exp(-2d') - \exp(-d')}{d'^2} \quad \text{and} \quad k_2 = \frac{\exp(-2d')}{d'^4} (6 + 4d'^2 + 8d') - \frac{\exp(-d')}{d'^2} (2 + d' + \frac{2}{d'}). \tag{44}
\]

The two quarks in the meson settle in a state of minimum energy, at a spacing \( 2\lambda d = 2d'_{\text{min}} \), such that \( [19,20] \),

\[ d'_{\text{min}} = \lambda d = 0.853; \quad k_0 = -1/2 \quad \text{and} \quad k_2 = 2.36. \tag{45} \]

Note: the field format (36) has been preferred above the indistinguishable field format (32) because (36) is a two-parameter format, while (32) is a three-parameter one.

**Table VI: meson excitations**

| Bottom level | \( E'_{\text{bind}} = -1/2 \) | mass ratio | mass in MeV/c^2 |
|--------------|-------------------------------|------------|-----------------|
| Ground state | \( E'_{1} - E'_{\text{bind}} = 0.84 \) | 1          | 137 (pion = 135-140) |
| First excitation | \( E'_{1} - E'_{\text{bind}} = 3.00 \) | 3.57       | 489 (kaon = 494-498) |
| Second excitation | \( E'_{2} - E'_{\text{bind}} = 6.06 \) | 7.21       | 988 (\( \eta' = 958 \)) |
| Third excitation | \( E'_{3} - E'_{\text{bind}} = 9.94 \) | 11.83      | ??? |

The archetype, the pion, is the two-quark oscillator in its ground state. The first excitation state transforms a pion into a kaon. The mass ratio between the two is the same as the mass ratio of the normalized energy constants \( E' - k_0 \). This is not trivial and it reflects the basic theorem of the theory. This theorem states that the energy wells of the two quarks are not massive. Instead, the mass attribute of two-quark junctions (mesons) and three-quark junctions (baryons) is made up by the vibration energy as expressed by the energy state of the quantum mechanical oscillator that they build. The distribution of this mass over constituent quarks is a consequence of this mechanism. Unfortunately, an analytical calculation of the \( E' - k_0 \) ratio of kaons over pions, is only possible for the quadratic approximation of the series expansion of the potential energy \( V'(z') \). A more accurate calculation requires a numerical approach. A procedure to do so has been documented in [19, Appendix C]. It shows that some simple lines of code in Wolfram’s *Mathematica* [21]
may do the job. The numerically calculated ratio of the energy constants appears to be 3.57 instead of 3 as it would have been in the harmonic case. The result explains the excitation of the 137 MeV/c² pion mass to the 490 MeV/c² mass of the pseudoscalar kaon. This result gives a substantial support for the viability of the theory as will be further developed in this article. This result also gives rise to the question if other mesons can be regarded as a result from enhanced excitation. Table VI gives a survey of the calculated ratios for higher excitation ratios. It gives the pseudoscalar \( \eta' \) meson as a candidate from second level excitation. The table gives no candidate for third level excitation. As shown in [22], the corresponding level of energy would imply a meson state with a positive value for the binding energy (as is reflected in the value of \( k_0 \)), which prevents a sustainable quasi-stable configuration.

6. The quark-scaling theorem and its impact

The wave equation of the simple pion model as shown in (42) is Schrödinger’s one, which, in fact, is the non-relativistic approximation of Dirac’s covariant wave equation. As is well known, Dirac adopted the Einsteinean energy formula as a starting point. He might have chosen Einstein’s geodesic equation instead. There is no reason why the momenta in the geodesic equation would not allow the same momentum-wave function transformation as in the energy equation. But why doing so? The consideration is that the geodesic equation may give additional results on top of those from the energy equation. The reason is that it contains an additional symmetry. Apart from energy conservation, it complies momentum conservation. As will be discussed in this paragraph, exploitation of this symmetry will reveal an interesting theorem.

Let us consider the quantum mechanical wave equation (42), once more in its denormalized format,

\[
-\frac{\hbar^2}{2m_m} \frac{d^2 \psi}{dx^2} + g\Phi_0 (k_0 + k_2 \lambda^2 x^2 + \ldots) \psi = E \psi.
\]

(46)

This represents an anharmonic quantum mechanical oscillator characterized by quantum steps \( \hbar \omega \) related with the effective mass \( m_m \), such that

\[
\frac{1}{2} m_m \omega^2 = g\Phi_0 k_2 \lambda^2 \rightarrow \frac{m'_m (\hbar \omega)^2}{(\hbar c)^2} = 2g\Phi_0 k_2 \lambda^2.
\]

(47)

Conventionally, \( m_m \) represents is the central mass of the oscillator. In the relativistic model described in chapter 4, it does not represent the individual masses of the two bodies, but, like stated before, it is an equivalent mass that captures the energy of the field. As usual, \( \omega \) is related with the vibration energy \( E_n = (n + 1/2) \hbar \omega \). Considering that the pion decays into a fermion via the weak interaction boson, it makes sense to equate the boson \( \hbar \omega \) with the weak interaction boson. Hence,
\( \hbar \omega_w = \hbar \omega. \)  

(48)

It also implies that the actual bond between the quark and the antiquark in a meson is sustained by the weak interaction boson. Hence, the spacing \( 2d_\lambda = 2d'_{\min} \) is expected about equal to a half wave length of the weak interaction boson \( \hbar \omega_w. \) Hence,
\[
\lambda = \frac{2(\hbar \omega_w)d'_{\min}}{\alpha \pi (hc)}, \quad (49)
\]
in which \( \alpha \) is a dimensionless correction factor of order 1. From (47)-(49), we have
\[
\Phi_0 = \frac{m'_m (\alpha \pi)^2}{8gk_2 d'^2_{\min}}. \quad (50)
\]

At this point, I would like to invoke a particular relationship from previously documented work [18]. In this work it has been shown that the 2D quantum mechanical wave equation as shown in (34) can equally well be derived from Dirac’s equation as commonly derived from the Einsteinean energy relationship, as well as derived from Einstein’s geodesic equation. The equivalence of the two approaches applied to the anharmonic quantum mechanical oscillator has revealed the relationship (see eq.(49) of [20]),
\[
\hbar \omega_w = 2g\Phi_0. \quad (51)
\]

Hence, from (49) and (51),
\[
\frac{g\Phi_0}{\lambda} = \frac{\alpha \pi (hc)}{2d'_{\min}}, \quad (52)
\]
in which \( k_0 = 1/2 \) and \( d'_{\min} = 0.853 \) as shown in (45). This ratio holds for all quarks. It means that the strength \( \Phi_0 \) as well as the range \( \lambda^{-1} \) of the potential field may be different for different quark flavors under invariance of the of \( \Phi_0 / \lambda \) ratio. In the basic meson configuration, i.e. the pion, \( \Phi_0 \) is fixed by the weak interaction boson cf. (52) and, like shown later, \( \lambda \) is fixed by the Higgs boson cf. (58). Scaled mesons, such as kaons have different values for \( \Phi_0 \) and \( \lambda \) under invariance of (52).

From (50) and (52), we have,
\[
m'_m = \frac{8k_2 d'^2_{\min}}{(\alpha \pi)^2} \left( \frac{g\Phi_0}{\lambda} \right) \lambda = \frac{8k_2 d'^2_{\min}}{(\alpha \pi)^2} \left( \frac{g\Phi_0}{\lambda} \right) \lambda = \frac{8k_2 d'^2_{\min}}{(\alpha \pi)^2} \left( \frac{g\Phi_0}{\lambda} \right) \lambda = \frac{2\hbar \omega_w}{\alpha \pi (hc)} d'_{\min}. \quad (53)
\]

Hence, after invoking (52),
From (53), (49) and (52) the dimensionless constant $\alpha_n$ in the normalized wave equation (42) can be readily calculated as (43) indeed. Note that $\alpha_n$ is different from $\alpha$ shown in (53). This unknown dimensionless constant $\alpha$ of order 1 has been assessed as $\alpha \approx 0.69$ in a study on the relationship between the gravitational constant and quantum physics [20]. In the next paragraph, its value will be discussed in a different context, which nevertheless will give the same result. It is tempting to suppose that $m_m'$ is the center-of-mass equivalent of the pion’s rest mass. Unfortunately, as will be shown later, this rest mass is highly influenced by the nuclear spin interaction, to be discussed in paragraph 8.

Note that $\Phi_0$ is a quantity that expresses the “strength” of the constituting quark next to the quantity $\lambda^{-1}$ that expresses the spatial range of its strength. Their product is constrained by the invariant (52). This identifies the scaling theorem of quarks. Different quark flavors may have different values for their strength and their spatial range, but always under the constraint of (52). Its relevance goes beyond pions. As to be discussed later in more detail, other mesons have a different spacing between the two quarks. Hence, (53) expresses the mass of these mesons as a function of this spacing $2d_0'$. Figure 4 shows the mass of the mesons as a function of the spacing parameter $d_0'$ between the quarks. The upper curve representing the mass is determined by $k_2(d_0')$. The lower curve is determined by $k_0(d_0')$ and represents the binding energy between the quarks.

\[
m'_m = \frac{8k_2d_0'^2}{(\alpha\pi)^2}\left(\frac{g\Phi_0}{\lambda}\right)2\hbar\omega_wd_0' = \frac{8k_2d_0'^2}{(\alpha\pi)^2}\hbar\omega_w. \tag{54}\]

In this calculation, the electromagnetic interactions have been ignored, because their influence is considered being of second order as compared to the nuclear interaction. More on the electromagnetic interaction will be subject of paragraph 10. Interestingly, the kaon energy, shown in Table VI does not only correspond with the energy of a pion in its state of
first excitation, but also with the ground state energy of a quark junction at smaller spacing, thereby composing the kaon as a \( u\bar{s} \) or a \( \bar{u}s \) bond, composed by the \( u \) quark next to the heavier \( s \) quark, such as illustrated in figure 5. The lower curve in the figure, representing the binding energy, should remain negative to allow a stable bond. Hence, the light sector \( (u, d, s) \) stops after second excitation from the ground state. However, the \( \varphi \) state \( (s, \bar{s}) \) is a new symmetrical state, from which new excitations may arise, and so on. The more elaborate calculation of the mass spectrum reported in [22] proves that the loss of binding energy inhibits the generation of quarks beyond the bottom quark. It makes the topquark \( (\approx 175 \text{ GeV}/c^2) \) a quark of a different nature. More on this in paragraph 15.

As noted before, the scaling theorem and the quark description as a pseudo tachyon are not part of canonic particle physics theory. In my previous studies it has been shown that the scaling theorem can be successfully applied for calculating the mass spectrum of mesons and baryons [20].

7. The Higgs boson

In the preceding chapters it has been demonstrated that the meson’s mass spectrum can be explained from the quark’s far field as defined in (39), supplemented by a near field from a dipole moment. The far field is a scalar field obtained from the steady state solution of a Proca-type wave equation with the format

\[ \frac{1}{c^2} \frac{\partial^2}{\partial t^2} r\Phi - \frac{\partial^2}{\partial r^2} r\Phi + \lambda^2 r\Phi = \rho_H(r,t), \]  

(55)

in which \( \rho_H(r,t) \) is a Dirac-type pointlike source that can be expressed as,

\[ \rho_H(r,t) = 4\pi \frac{\Phi_0}{\lambda} \delta^3(r) H(t), \]  

(56)

in which \( H(t) \) is Heaviside’s step function. Its solution is given by [23],

\[ r\Phi(r,t) \leftrightarrow \frac{\Phi_0}{\lambda} \frac{1}{s} \exp[-(\lambda r \sqrt{s^2/(\lambda c)^2 + 1})]. \]  

(57)

If, under violence of particle collisions, the equilibrium between the quarks is broken, the far field bosons will show up in decay channels of boson pairs, which will manifest themselves into a decay path of fermions. Momenta and energies of these fermions can be measured and can be traced back to numerical values for the energy of a nuclear boson pair. So, ultimately, the far field will show up as two quantum fields. The massive energy of the far field part, if interpreted as a single Higgs boson, would therefore be assigned as,

\[ m_H' \approx 2\lambda (hc). \]  

(58)

Subsequent application of (49) on this gives,
which, under consideration of $m_W' = 80.4$ GeV and $m_h' = 127$ GeV for the Higgs boson just gives $\alpha \approx 0.69$ quoted before. One might wonder about the factor 2. The Standard Model value $m_h'$ of the Higgs boson in natural units documented in [1] gives, under consideration of (37),

$$m_h' = \mu_h \sqrt{2} \approx 2\lambda \rightarrow m_h' = 2\lambda(hc).$$

This is just the same as (58). The result gives a strong support to the viability as developed in this article, including its compatibility with the Standard Model. However, unlike as in the Standard Model, the numerical value is now established by theory. Later in this article, the two bosons that make a single Higgs one, will be identified as gluons. See paragraph 14.

8. The Z boson

The simple anharmonic oscillator model described by (40-42) enables the mass spectrum calculation of the pseudoscalar mesons as excitations from the pion state. The excitation mechanism stops beyond the bottom quark due to the loss of binding energy. The mass spectrum calculation of the vector mesons requires the inclusion of the impact of the nuclear spin shown in the upper part of figure 2. A spin flip marks the difference between the pseudoscalar pion and the vector type sisters rho. The massive energy difference $\Delta E$ between the two types is a consequence of a spin-spin interaction process. It is of a similar nature as the analysis of the interaction process between the spin of electron and the spin of the proton nucleus in a hydrogen atom. This requires a detailed quantum mechanical computation including recoiling. More on this can be found in [22]. Recognizing, though, that this is essentially a bosonic process, allows, in retrospect, a surprising simple approach. The step to be taken is conceiving the massive energy difference $\Delta E$ as a result of a bosonic interaction, mediated by $Z$ bosons in virtual state. Because of the asymmetry in the spin-spin interaction (−$3\hbar^2/4$ and $+\hbar^2/4$), we have,

$$m_\pi' = 2m_u' - 3m_Z'^\sigma \quad \text{and} \quad m_p' = 2m_u' + m_Z'^\sigma,$$

in which $m_\pi', m_p', m_u'$ and $m_Z'^\sigma$ are the energies of, respectively, the rest masses of pion and the rho meson, the constituent massive energy of the $u/d$ quark and the energy of the $Z$ boson in virtual state in the rest frame of mesons. The statement that the energy of the rest mass of the pion is equal to the non-relativistic equivalent of the energy of the $W$ boson enables to calculate the energy $m_Z'^\sigma$ of the $Z$ boson in virtual state as,
Under use of (61) and (62), the constituent rest mass energy $m'_u$ of the $u/d$ quark is calculated as

$$m'_u = \frac{1}{2} m'_{1} (1 + 3 \frac{m'_Z}{m'_{W}}).$$

From (63) the energy of the constituent of the $u/d$ quark is, under consideration of the energetic values of the weak interaction bosons $m'_{W} = 80.4$ GeV and $m'_Z = 91.2$ GeV, under adoption of the rest mass of the pion $m_{\pi} \approx 140$ MeV/c$^2$, calculated as 308 MeV. Under use of this value, the energy of the rho meson is calculated from (61) and (62) as $m'_\rho = 775$ MeV. This is a perfect fit with experimental evidence!

We may go a step further by invoking the mass relationships due to the nuclear spin interactions as derived in Griffith’s textbook in conjunction with the theoretically established ratio 3.57 of the kaon mass over the pion mass, listed in TableIV. The constituent mass value for the $s$ quark is calculated from,

$$m'_s = m'_u + m'_s - 3 \frac{m'_u}{m'_s} m'_s - m'_Z = 3.57 m'_s \rightarrow m'_s = 489 \text{ MeV}.$$  

The calculated values of the charmed quark ($c$) and the bottom quark ($b$) can be found in [22]. As noted before, the top quark ($t$) is out of scale. As to be discussed in paragraph 15, this is due to its different origin.

This means that the constituent masses of the quarks can be theoretically derived, next to the energy values of the weak interaction bosons from a single reference for which we have adopted the rest mass of the archetype meson. In principle even better than this, because the relationship between energies of the $W$ and $Z$ can be derived by theory as well, like shown in [22]. All this implies anyhow that there is no reason to consider the quark flavors as elementary. Note the difference with the calculation in Griffiths’ textbook, which is purely empirically based and denoted as “shaky”, but nevertheless rather accurate. It has to be taken into account, though, that the constituent massive energies of the quark flavors are somewhat artificial, because they represent the harmonic oscillator energy distributed over the quarks. For that reason the constituent quark energies in baryons are different from those in mesons. In the present state of theory the masses of hadrons are calculated by lattice QCD [37]. Different from using the rest mass of the pion as input reference, the bare masses of the quarks are used as reference. The value of these bare masses, however, are “tuned from the masses of mesons” [38]. It is fair to conclude that the quark model based upon the polarisable dipole moment of Dirac’s third particle and its application in the excitation model of hadrons shows undeniable intriguing relationships between the mass values of the quark flavors and the weak interaction bosons. It enables to calculate the mass spectrum of hadrons with great precision [22,35].
9. Relationship with the Standard Model

The given description of the bond between the $u$ quark and the $d$ quark has revealed the existence of two different binding forces that each can be modeled in terms of force interacting particles. The interacting force due to the polarisable dipole moment of a quark in a scalar potential field has been identified as the $W$ boson and the interaction force due to the nuclear spin interaction has been identified as the $Z$ boson. Let us now take a more abstract point of view, in which no knowledge is available about the actual physical mechanism of the nuclear force. This brings us to the basic question in particle physics, which in words is the simple one: how to describe the field of a quark in an ambient field of nuclear forces? The recipe is trying to find a covariant equivalent for the quark’s field in an ambient field from the quark’s field in free space. Because in the context of the Standard Model the quark has not been recognized as polarisable in a scalar field, physical knowledge of the nuclear forces could not been taken into account. While for electromagnetic interactions the gauge needed for covariance could be understood from a physical mechanism, the gauge for nuclear interactions had to be conceived from an abstract point of view. Its description might reveal the relationship between the structural view developed in this article with the gauge based view adopted in the Standard Model. This is the issue to be discussed in this paragraph. It touches the crux of the article, in which the view is taken that the SU(2) and SU(3) gauges have a physical structural basis of a similar kind as the physical structural basis of the U(1) gauge.

The interaction model between two quarks shown in the preceding paragraph is based upon spatial field descriptions. Such spatial descriptions are uncommon in the canonic field descriptions of the Standard model, in which fields are exclusively described in functional field parameters. It is instructive viewing the field of a nuclear particle as the mapping of its momenta $p_\mu$ on the amplitudes $\Psi_\mu$ of the four components of the solution of Dirac’s equation, i.e., as

$$\{p_0, p_1, p_2, p_3\} \rightarrow \{\Psi_0, \Psi_1, \Psi_2, \Psi_3\}. $$

This mapping is visualized in figure 6 for 1+2 dimensionality.

The left hand part is a geometric interpretation of the determinant condition (19),

$$W^2 = (m_0 c^2)^2 - c^{-2} |p|^2, \tag{64}$$

in which $p$ is the three-vector momentum ($ds/dt$, not be confused with the fourvector momentum $p$). It can be normalized as,

$$p_0^2 + p_1^2 + p_2^2 + p_3^2 = 1; \quad p'_i = \frac{p_i}{m_0 c}; \quad p'_0 = \frac{-W}{m_0 c^2}. \tag{65}$$
This allows representing the momentum space of a moving particle in free space as a sphere with unit radius.

Fig. 6. A visual interpretation of the mapping of the particle’s momenta into amplitudes of Dirac’s wave function solution. Note that this mapping is not 1 to 1.

The right-hand part is a geometric interpretation of the absolute values of the amplitudes $\Psi_\mu$ of the four components of the solution of Dirac’s equation. The amplitudes themselves are complex quantities. As a consequence of the semantics of the particle’s wave function, these amplitudes can be represented as orthogonal vectors in a unit sphere. Note that this mapping is not one-to-one. In momentum space, the angle $\mathcal{J}_\mu$ between the temporal momentum $p_0$ and the vector sum of the spatial momenta $p_i$ is a global invariant. As long as the particle’s energy is not changed by a field of force, the angle remains the same. In spinor space, there is a characteristic angle $\mathcal{J}_i$ between the component $\Psi_0$ associated with the temporal momentum and the vector sum of the components $\Psi_i$ associated with the spatial momenta. Although the mapping is not one-to-one, the angle $\mathcal{J}_i$ is globally invariant. Under influence of forces on the momenta, the angle $\mathcal{J}_\mu$ will change, while the radius of the momentum space will remain the same owing to the normalization. The angle in the spinor space will change as well and the radius of the spinor space will remain the same because of the wave function semantics. These angles play a role in the modification of Dirac’s free space equation into a covariant one. By definition, the covariant equation, valid for particles moving in a field of forces, has the same format as the free space equation after redefining the normal differential operators into covariant ones, i.e. $\partial_\mu \rightarrow D_\mu$. The prescription how to do it, is the modification of a global invariant quantity into a local invariant one. By modifying the global invariance of the Lorentz transform into a local invariant one, Einstein has been able to derive the transformation rule for the covariant derivatives that modified his equations of Special Relativity in free space into covariant equivalents for his equations of General Relativity. Paul Dirac’s prescription for making his equation (1) covariant in a conservative field of forces $A(A_0, A)$,
\[ p'_\mu \rightarrow p'_\mu + gA'_\mu, \quad \text{and} \quad p'_\mu \rightarrow \hat{p}_\mu \psi + gA'_\mu \psi; \quad \hat{p}_\mu = \frac{\hbar}{m_0 c} \frac{i}{\partial x_\mu}, \quad (66) \]

can be interpreted as the modification of the global invariance of \( \partial_x \) and \( \partial_t \) into local invariant ones, because (66) represents, as we shall see below, just infinitesimal rotations in, respectively, momentum space and spinor space. Effectively, these rotations takes place in 2D space, as long as a single particle is involved.

It is instructive to consider the particle’s antiparticle in this picture. The momentum amplitude of the antiparticle has the same value as the amplitude of the vector sum of the spatial momenta of the particle. And, in spite of the fact that the mapping from momenta to the values of the spinor components is non one-to-one, the absolute value of the amplitude of the spinor component associated with the temporal momentum is equal to the value of the vector sum in spinor space of the amplitudes of the spinor components associated with the spatial momenta of the antiparticle.

This picture allows to represent the particle-antiparticle bond as a 2 x 2 matrix \( \Psi_{pa} \),

\[
\Psi_{pa} = \begin{bmatrix} \psi_{1r} & \psi_{1s} \\ \psi_{2r} & \psi_{2s} \end{bmatrix}, \quad (67)
\]

in which \( \psi_{ir} \) represent the components associated with the temporal momenta and in which \( \psi_{is} \) represent those associated with the spatial momenta. Considering that the solution of Dirac’s equation in free space, like, for instance, shown on page 220 of Griffith’s textbook [1], implies that the absolute value of the amplitude of the temporal part and the absolute value of the sum of the amplitudes of the spatial part of the particle are antisymmetric with respect to those of the antiparticle, allows to conceive the particle-antiparticle bond of an \( u \) quark with the antiparticle of a \( d \) quark as an SU(2) Lie group. Its matrix has the following properties,

\[
\psi_{1r}^* \psi_{1r} + \psi_{1s}^* \psi_{1s} = 1; \quad \psi_{2r}^* \psi_{2r} + \psi_{2s}^* \psi_{2s} = 1; \quad \psi_{2r} = \psi_{1r}^*, \quad \psi_{2s} = \psi_{1s}^*. \quad (68)
\]

Because of this relationship, the matrix (67) is unitary, i.e.

\[
\Psi_{pa} \Psi_{pa}^T = 1, \quad (69)
\]

in which \( \Psi_{pa}^T \) is the transpose conjugate of \( \Psi_{pa} \).

Note that the elements of \( \Psi_{pa} \) are complex numbers.

A complex \( n \times n \) matrix has \( 2n^2 \) real parameters. The unitary condition on the rows removes \( n^2 \) of these and an additional one is removed by the constraint of unit determinant. That leaves 3 degrees of freedom for the SU(2) operator. The matrix (68) can then be generically represented as,
\[
\begin{pmatrix}
e^{i\theta} \cos \alpha & e^{i\gamma} \sin \alpha \\
e^{-i\gamma} \sin \alpha & e^{-i\theta} \cos \alpha
\end{pmatrix}.
\] (70)

Obviously, this matrix is unitary, thereby meeting the constraints as imposed by (68). Lie-group theory states that any matrix multiplication with the generic SU(2) format as defined in (70) leaves the object in the group. Hence, the transformation that maintains the desired property of Lagrangian equivalence is given by
\[
\Psi \rightarrow \Psi \exp(i\vec{\sigma} \vec{g}) \text{ with } \vec{g} = \vec{g}(\sigma_1, \sigma_2, \sigma_3) \text{ and } \vec{\sigma} = (\sigma_1, \sigma_2, \sigma_3),
\] (71)
as long as the matrices \(\vec{\sigma} = (\sigma_1, \sigma_2, \sigma_3)\) match with (70). The most simple ones are the three Pauli matrices,
\[
\sigma_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}; \quad \sigma_2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}; \quad \sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}.
\] (72)

From (75) should follow \(D_{\mu} \Psi \rightarrow D_{\mu} \Psi \exp(i\vec{\sigma} \vec{g})\). This is true if
\[
D_{\mu} \Psi = \partial_{\mu} \Psi - i \Psi \partial_{\mu} (\vec{\sigma} \vec{g}).
\] (73)

By identifying
\[
\vec{\sigma} \vec{g} = \sigma_k g^k = g_w \sigma_k W^k,
\] (74)
in which \(g_w\) is a generic dimensionless coupling factor,

we get,
\[
D_{\mu} \Psi = (\partial_{\mu} - ig_w \sigma_k W^k)\Psi; \quad k = 1,2,3.
\] (75)

Note: the subscript in \(g_w\) has been added for distinction from the coupling factor \(g\) introduced in (8) in a somewhat different context.

Because \(\sigma_k W^k\) are operations in the field domain with a complex number type, \(W^k\) cannot be identified as mappings of real valued momenta. Hence, it makes sense to redefine,
\[
W_1^+ = W_1 + iW_2; \quad W_1^- = W_1' - iW_2; \quad W_0 = W_3.
\] (76)

From (75) and (79),
\[
D_{\mu} \Psi = (\partial_{\mu} - ig_w (\tau_1 W_1^* + \tau_2 W_2^* + \tau_3 W_3^*))\Psi,
\] (77)
in which $\tau_k$ now are real valued matrices,

\[
\tau_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}; \quad \tau_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}; \quad \tau_0 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}.
\] (78)

Note that as yet the + sign and the − sign have no electrical meaning here. Note that this covariant derivative derived from SU(2) perspective contains the Pauli elements of Dirac’s equation as represented by (1). It includes the influence of the angular momentum. However, particular properties of the field of forces that necessitates a covariant derivative are not yet taken into account. The relevancy within the context of this article is that the recognition of the full-dimensionality of Dirac’s equation (which includes the spin phenomenon) in a bond of SU(2) particles reveals the existence of three interaction forces or, equivalently, the existence of interaction bosons in three modalities. It will be clear that apart from this conclusion, more is needed for proving the viability of a stable bond of SU(2) constituents. So, the next question to be addressed is: “what are the characteristics of the energetic background field that guarantees such a stable bond?”. The answer is a simple one: its Lagrangian density should remain locally invariant under substitution of a covariant derivative as specified by (75). One may expect that this condition will require certain properties of the interaction bosons $(W^+, W^-, W^0)$. This gives the recipe for defining a covariant derivative, formally dubbed as gauge, in Dirac-type wave equations of particle bonds. The gauge for particle bonds with a wave function (= field) that is subject to the unitary constraint, has been originally generically formulated by Yang and Mills [26].

Whereas the recipe for finding the answer to the problem is clear, finding the solution itself of is not a piece of cake. The GSW (Glashow, Salam, Weinberg) electroweak theory [27,28,29] has given the answer. Not surprisingly, the nuclear background energy field has the format of the Higgs field, such as introduced by (33). Under the influence of such a field, the bosons shown in (74), which are mass less in free space, gain mass. The proof for this can be found in many textbooks and tutorials. Whereas the mass of the bosons $W^+_\mu$ and $W^-_\mu$ is the same, the mass of the third boson is different, because in the GSW theory it evolves from a mix of contributions from the nuclear field and the electromagnetic field. It makes the neutral $Z$ boson. Omitting the influence of the electromagnetic field in the GSW theory would make the mass of the $Z$ boson the same as those of $W^+_\mu$ and $W^-_\mu$, which is in conflict with experimental evidence. Inspection of (74) however, makes clear that there is no reason why the coupling factor $g_W$ should be necessarily equal for all three bosons. Accepting this theoretical option would identify the $W^0_\mu$ as a $Z$ boson without being mixed up with electromagnetic energy, while still having a mass different from the $W$ bosons. More on this will be discussed in the next paragraph.

It is fair to conclude that this overview brings the origin of the bosons, as developed in the preceding paragraphs from a structural point of view, in agreement with the origin of the bosons conceived from a pure abstract point of view of gauging and the adoption of a heuristic format for the background field. Importantly and in addition to this, it has to be noted that, whereas the mass ratio of the $W$ bosons over the $Z$ boson in the structural view
could be assessed by theory, such as shown in the preceding paragraph, the very same ratio has in the GSW theory remained a value for which an empirical assessment is required. However, by omitting the electromagnetic field in the GSW theory, the unification of the nuclear forces with electromagnetism is now missing. How to include it, will be the subject of the next paragraph.

10. Electroweak unification

In this paragraph two different methods will be discussed to include electromagnetic interaction in the nuclear forces. The canonical way is its incorporation in the covariant derivative. This is done in the GSW theory, in which the asymmetry of electric charge between \( u \) quarks and \( d \) quarks is taken as granted. The second way is including the interaction in the force interaction model as developed in the preceding paragraphs. As will be demonstrated, the latter method allows a relative simple calculation. The purpose is to show that the structural view discussed in this article should not be considered as a conflicting theory but, instead, as a physical layer underneath the canonical theory, which reveals relationships that have so far been hidden.

In the analysis so far presented in this article, the electromagnetic interaction has been regarded as a second order effect. It will now be shown that the scaling theorem applied on quarks conceived as pseudo-tachyons is a powerful instrument for calculating the mass difference between the charged pion and the neutral pion. The task to be done is including the electromagnetic interaction into account as an additional force on the nuclear force between the quark and the antiquark. We may combine this additional force with the weak interaction force, implying that each quark feels a repulsive force \( F(r) \) from the other quark, such that

\[
F(r) = g \frac{d\Phi}{dr} + p \frac{(e/2)^2}{4\pi_0 r^2},
\]  

in which \( p \) is a dimensionless factor, which depends on the composition of the pion.

Conventionally, it has been taken for granted that \( u \) quarks and \( d \) quarks show a different electrical behavior. From empirical evidence it has been concluded that \( u \) quarks are charged as \( 2/3e \) and that \( d \) quarks are charged as \( -1/3e \). Within the view so far developed in this text, this asymmetry is an unexpected symmetry violation. As shown in Table III, the asymmetry disappears by hypothesizing a certain charge bias in the baryon configuration as a consequence of fermionic particle spin. In the theory of quarks, the origin of electric charge polarity is supposed being due to state of isospin. Unlike in the case of a meson, which can be conceived as a bosonic particle without fermionic particle spin, a baryon might have an additional contribution to the electric charge as a consequence from this spin. Accepting that electric charge is a holistic property of the hadron, rather than an individual property of a quark, the isospin contributions are symmetrical.

To include electromagnetic interaction under the constraint of symmetry, electric charge is distributed over the quarks as \( \pm e/2 \). Considering that equal charges are repelling, we have
in (79), \( p = 1 \) for the charged pion and \( p = -1 \) for the neutral pion. Let us rewrite (79) in terms of the electromagnetic fine structure relationship,

\[
e^2 = 4\pi e_0 \hbar c g_e^2,
\]

(80)

in which \( g_e^2 \) is the well-known fine structure constant \( g_e^2 = \alpha_{em} \approx 1/137 \). Because we have adopted \( g = \sqrt{g_e^2} \) as the nuclear coupling factor under invariance of \( g\Phi_0 \), the repelling interaction force \( F(r) \) between the quark and the antiquark, under consideration of equal contributions of charge can be written as,

\[
F(r) = g \frac{d\Phi}{dr} + \frac{p}{4} \frac{g^2 \hbar c}{r^2}.
\]

(81)

The far field potential \( \Phi_f(r) \) can now be written after including the influence of the electric interaction as,

\[
\Phi_f(r) = \frac{\Phi_0}{\lambda r} \exp(-\lambda r) + \frac{p}{4} \frac{g^2 \hbar c}{\lambda r}.
\]

(82)

Now, the potential \( \Phi(x') \) of the field built up by the quarks felt by the center of mass, expanded along the dipole axis, is built up by the near field \( \Phi_N(x') \) from the dipole moment, the far field \( \Phi_f(x') \) component of the weak interaction, and the electromagnetic potential \( \Phi_{em}(x') \), such that

\[
\Phi(x') = \Phi_N(x') - \Phi_f(x') - \Phi_{em}(x') \quad \text{with} \quad x' = \lambda x,
\]

\[
\Phi_N(x') - \Phi_f(x') = \Phi_0 (k_0 + k_2 x'^2 + \ldots),
\]

\[
\Phi_{em}(x') = p w \frac{g}{4} \frac{(hc)\lambda}{d'} (2 + \frac{2}{d'^2} x'^2 + \ldots), \quad d' = d\lambda.
\]

(83)

Note the appearance of the factor \( w \) in \( \Phi_{em}(x') \) as a consequence from the relative value of the far field strength as compared to the near field strength, such as explained by (32).

Note: \( \Phi_{em} \) holds under the assumption of equal charge distribution over the two quarks. Eq. (83) can now be written as,

\[
\Phi(x') = \Phi_0 (k_0' + k_2' x'^2 + \ldots), \quad \text{with}
\]

\[
k_0' = k_0(d') - \frac{1}{2} p w g \frac{(hc)\lambda}{d\Phi_0}; \quad k_2' = k_2(d') - \frac{1}{2} p w g \frac{(hc)\lambda}{d'^3\Phi_0},
\]

(84)
in which \( k_i (d') = k_i (d'_{\text{min}} + \delta) = k_i (d'_{\text{min}}) + \Delta k_i \).

As a consequence of the electric charge, the spacing between the two quarks in equilibrium is shifted apart by an amount of say \( 2\delta \), such that \( d'_0 = d'_{\text{min}} + \delta \). As a consequence of the shift, the mass formula (53) has to be evaluated under constraint of fixed values for \( \Phi_0 \) and \( \lambda \). The parameters affected by electromagnetic interaction are \( k_2 \) and \( d''_{\text{min}} \). Rather than using (53) for the purpose, the effect of the shift on the affected quantities, a return to the basic relationship as derived under general relativity \([17,19,20]\) is simpler and more effective. This relationship reads as,

\[
m'_{\text{eff}} = \frac{k_2 \lambda^2 (hc)^2}{2k_0^2 g\Phi_0}.
\]

Denoting \( m'_{\pi(\text{em})} \) as the massive pion energy (in the center of mass frame) under influence of charge and \( m'_\pi \) as the pion energy without electromagnetic interaction, we have

\[
\frac{m'_{\pi(\text{em})}}{m'_\pi} = \frac{k'_2 k_0^2}{k_2 k_0^2}.
\]

which can be expanded as,

\[
\frac{m'_{\pi(\text{em})}}{m'_\pi} = \frac{k'_2 k_0^2}{k_2 k_0^2} = \left( \frac{k_0^2}{k_2} \right) \left( k_2 + \Delta k_2 \right)^2 \approx \left( 1 + \frac{\Delta k_2}{k_2} - 2 \frac{\Delta k_0}{k_0} \right)
\]

From (84) it is clear that \( \Delta k_0 \) as well as \( \Delta k_2 \) are negative values, while we know that \( k_0 = -1/2 \) and \( k_2 \approx 2.36 \). Because \( \delta \ll d'_{\text{min}} \),

\[
\Delta k_0 \approx \frac{1}{2} p\omega g \frac{(hc)\lambda}{d'_{\text{min}}\Phi_0} ; \quad \Delta k_2 = \frac{1}{2} p\omega g \frac{(hc)\lambda}{d'_{\text{min}}\Phi_0} + \delta \left( \frac{\partial k_2}{\partial (d')} \right)_{d'_{\text{min}}}.
\]

Note that \( \Delta k_2 \) contains an additional term as compared with \( \Delta k_0 \). As shown in (19), this additional term, to be denoted as \( \delta k_2 \), considerably affects \( \Delta k_2 \).

Defining \( \Delta m'_n = m'_{\pi(\text{em})} - m'_\pi \), we have from (88) and (85),

\[
\Delta m'_n = \frac{\Delta k_2}{k_2} - 2 \frac{\Delta k_0}{k_0} m'_n = \frac{1}{2} p\omega g^2 \frac{hc}{d'_{\text{min}}} \left\{ - \left( \frac{1}{k_2 d''_{\text{min}}} + \delta k_2 \right) - 2 \frac{\lambda}{k_0} \frac{\partial}{\partial (d')} \right\} m'_\pi.
\]

Subsequent invoking (52), gives

\[
\Delta m'_n = p\omega \frac{1}{\alpha \pi} g^2 \left\{ - \left( \frac{1}{k_2 d''_{\text{min}}} + \delta k_2 \right) - 2 \frac{\lambda}{k_0} \right\} m'_n.
\]
The difference $\Delta m^\prime_{n0}$ between the charged pion ($p = 1$) and the neutral pion ($p = -1$) is twice as large as the absolute value $|\Delta m^\prime_\pi|$. Denoting the energy equivalent of the charged meson and the neutral meson, respectively, as $m^\prime_{\pi \pm}$ and $m^\prime_{00}$, we have:

$$\Delta m^\prime_{n0} = m^\prime_{\pi \pm} - m^\prime_{00} = 2|\Delta m^\prime_\pi| = 2 \frac{g^2}{\alpha \pi} \left(-\frac{1}{k_2 d^\prime_{\min}} + \delta k_2\right) - 2 \frac{2}{k_0} m^\prime_{\pi \pm}. \quad (91)$$

As shown in (19), the second contribution ($\delta k_2$) to $\Delta k_2$ is about twice as large as the first contribution.

Under neglect of $\delta k_2$, the result would be, under consideration of, $k_0 = -1/2$, $k_2 = 2.36$, $g^2 = 1/137$, $\alpha = 0.69$, $d^\prime_{\min} = 0.853$, $w = 1.55$, and $m^\prime_\pi = 140$ MeV, the result would be $\Delta^\prime_{n0} \approx 6.05$ MeV, while the accurate calculation, with the influence of $\delta k_2$ included, results into $\Delta^\prime_{n0} \approx 4.3$ MeV. The difference with the 2.3 MeV result reported is (19) is due to the presence of the factor $w$, which has not been recognized by me back in 2012 due to the unawareness of the quark’s second dipole moment at the time.

This is 4.3 MeV result is pretty close to the experimental evidence of 4.6 MeV. Note that this result has been obtained without a need to accept the 1/3-2/3 split of the elementary charge over $u$ and $d$ quarks. One may wonder now, without adopting an asymmetry between the basic quarks, how to explain the phenomenon that whereas charged pions have a larger mass value than neutral pions, the opposite is true for kaons. The explanation can be found in [19].

Rather than considering the electromagnetic interaction separately from the nuclear $W/Z$ interaction, one might prefer unifying the interactions in a single covariant derivative. This requires a modification of (77), which had to be modified anyway because of a possible different coupling factor of $W^0$ bosons to the Higgs field as compared with $W^\pm$ bosons. Moreover, the covariant derivative should allow for electromagnetic bosons that are not affected by the Higgs field. In the GSW theory this requirement is met by hypothesising the existence of an additional (mass less) interaction boson $B^\mu$, next to the $W^\mu$s, which mixes up with the $W^\mu_0$ boson, such that the interaction of the $W$ bosons and the $B$ boson with the Higgs field produces massive charged bosons $W^\pm_\mu$, a massive neutral boson $Z_\mu$ and a massless electromagnetic boson $A^\mu$. The components of this theory are, (a) a proper field format for the quark-antiquark junction conceived as an SU(2) group, (b) a proper format for the Higgs field operating on the quark junction and (c) a proper format for the covariant derivative.

The pion spinor as shown in (67) can be conceived as an SU(2) doublet $\Phi$ of two complex fields, symbolically represented as,
\[
\Phi = \begin{bmatrix} \Phi^+ \\ \Phi^0 \end{bmatrix}; \quad \Phi^+ = \frac{\varphi_1 + i \varphi_2}{\sqrt{2}}; \quad \Phi^0 = \frac{\varphi_3 + i \varphi_4}{\sqrt{2}},
\]

The (field) Lagrangian of the Higgs field can be represented as,

\[
\mathcal{L} = (\partial_\mu \Phi)^T (\partial^\nu \Phi) + V(\Phi), \quad \text{with} \quad V(\Phi) = \frac{\mu_H^2}{2} \Phi^T \Phi - \frac{\lambda^2_H}{4} (\Phi^T \Phi)^2,
\]

in which,

\[
\Phi^T \Phi = \begin{bmatrix} \Phi^* & \Phi^0 \\
\Phi^0 & \Phi^* \end{bmatrix} = \Phi^+ \Phi^* + \Phi^0 \Phi^0 = \frac{\varphi_1^2 + \varphi_2^2 + \varphi_3^2 + \varphi_4^2}{2}.
\]

The covariant derivative gets the format as defined before in (77).

\[
D_\mu \varphi = (\partial_\mu - ig_W (\tau_1 W^\mu_+ + \tau_2 W^\mu_- + \tau_0 W^\mu_0) - ig_Z B_\mu) \varphi,
\]

Note: whereas at this point a physical motivation for the conception of the SU(2) doublet of two complex fields is given from the pion spinor, such a motivation is lacking in most, if not all, texts on the GSW theory.

Replacing the normal derivatives in the field Lagrangian (93) by the covariant ones as defined in (94) and subsequent elaboration of the fields as variations around the field minimum of the Higgs field (known as the vacuum expectation value), produces new fields in which the bosons \(W^\pm\), have gained mass and in which a massive neutral boson \(Z\), shows up next to a mass less electromagnetic boson \(A\). Whereas in the GSW theory the \(Z\) boson is related with electromagnetism, in this article the \(Z\) boson has been related with the nuclear spin interaction between quarks. In spite of this difference, the GSW methodology can be maintained by defining the GSW covariant derivative (94) slightly different as,

\[
D_\mu \varphi = (\partial_\mu - ig_W (\tau_1 W^\mu_+ + \tau_2 W^\mu_-) + ig_Z \tau_0 W^\mu_0) \varphi.
\]

Note: in most texts on GSW the (unknown) coupling factors \(g_W\) and \(g_Z\) are written as \(g/2\) and \(g'/2\). As noted by Klauber [30], this is just by convention, because the factor 1/2 is sometimes omitted as well.

The potential \(V(\Phi)\) is minimum at \(\Phi_0\), found by differentiation of (93) as

\[
\Phi^T \Phi = \frac{\mu_H^2}{2} \frac{\lambda^2_H}{2} \Rightarrow \Phi_0 = \frac{\mu_H}{\lambda_H} \sqrt{2},
\]
corresponding to the vacuum expectation value indentified in (33). By substituting the covariant derivative (95) into the field Lagrangian (93), one may try to obtain field expressions in terms of \( \phi \), but, like shown in the GSW theory, it is more effective trying to obtain field expressions in terms of variations around the vacuum expectation value (96). How to do so, can be found in many texts [1,30]. Doing so in terms of the extended format (94) or in terms of the “naked” format (95) doesn’t make an essential difference. Elaboration shows that the mass less bosons gain mass, such that

\[
m'_w = g_w \Phi_0; \quad m'_z = g_z \Phi_0. \tag{97}
\]

Next to these boson masses, an additional boson is produced, known as the Higgs boson, which is considered as being the carrier of the background field, to the amount of,

\[
m'_h = \mu_h \sqrt{2}. \tag{98}
\]

Note that \( \lambda_h \) is dimensionless, while \( \mu_h \) has the dimension of energy. For \( m'_w \) and \( m'_h \) it are the same expressions as previously shown in (52), respectively in (58). While in the context of the GSW theory the assessment of the mass ratio \( m_z / m_w (= g_w / g_z) \) requires an numerical value for \( m_z \) from straight empirical evidence, this value for \( m_z \) can be found in the structural model from (61). Something similar holds for the Higgs boson. Rather than from straight empirical evidence, it can be found in the structural model from the relationship between \( m'_w \) and \( m'_h \) as shown by Eqs. (58-59). Note that these results do not reveal numerical values for the coupling constants \( g_w \) and \( g_z \), nor for the vacuum expectation value \( \Phi_0 \). The numerical values are related to \( m'_w \) as the reference. So far, we have only included the effect of the \( Z \) boson as the third weak interaction gauge boson in the covariant derivative while potentially still leaving electromagnetism as a second order add-on effect. In the full GSW theory, electromagnetism is made an integral part of it. It is done by unifying electromagnetism with the weak interaction on a theoretical fundament. This fundament is the adaption of the covariant derivative (95) into (94).

As compared to the “naked” format shown in (95), the adapted format (94) contains an additional boson field to account for the electromagnetism that shows up in the SU(2) doublet field. Without an additional mechanism there is no reason why this additional boson field would not gain mass as a result of the same mechanism why the \( W \) bosons do. The only way to prevent this, is conceiving a preventing mechanism, be it an artificial one or a physically viable one. The hypothesis that such a mechanism exists became a corner stone in the GSW theory. From the straightforward covariant derivative (94), Weinberg constructed a modified one by hypothesizing a mechanism that mixes the (massless) boson fields \( W_\mu \) and \( B_\mu \) into two other (massless) bosons fields \( A_\mu \) and \( Z_\mu \) such that,

\[
\begin{bmatrix}
A_\mu \\
Z_\mu
\end{bmatrix} = 
\begin{bmatrix}
\cos \theta_w & \sin \theta_w \\
-\sin \theta_w & \cos \theta_w
\end{bmatrix} 
\begin{bmatrix}
B_\mu \\
W^0_\mu
\end{bmatrix},
\tag{99}
\]

which is equivalent with
\[ A_\mu = B_\mu \cos \theta_W + W^\mu_0 \sin \theta_W \]
\[ Z_\mu = -B_\mu \sin \theta_W + W^\mu_0 \cos \theta_W . \]  

(100)

Multiplying the upper equation with \( \sin \theta_W \), the lower one with \( \cos \theta_W \) and addition gives \( W^\mu_0 \). Multiplying the upper one \( \cos \theta_W \) etc., gives \( B_\mu \). Hence,

\[ W^\mu_0 = Z_\mu \cos \theta_W + A_\mu \sin \theta_W \quad \text{and} \quad B_\mu = -Z_\mu \sin \theta_W + A_\mu \cos \theta_W . \]  

(101)

The two are substituted in the covariant derivative (94), which is subsequently evaluated as before around the Higgs field minimum. It then appears that, if the Weinberg angle \( \theta_W \) has a suitable value, the mass less boson field \( Z_\mu \) gains mass, while the mass less boson field \( A_\mu \) remains mass less. In this condition the mass ratio \( m'_W / m'_Z \) happens to be,

\[ \frac{m'_W}{m'_Z} = \cos \theta_W . \]  

(102)

The discovery of neutral bosons \( m'_Z \approx 91.2 \) GeV with a different mass value from charged bosons \( m'_W \approx 80.4 \) GeV is considered as the viability proof of the GSW theory. This in spite of the inability to assess a numerical value for \( \theta_W \) by theory. It confirms Weinberg’s hypothesis that an SU(2) field, like the pion’s one, contains a particular physical mechanism that creates electromagnetism. Strictly spoken, the discovery shows that the third expected weak interaction boson is different in its charge transferring capabilities and has a somewhat different energy from the symmetrical weak interaction pair.

Let us proceed by considering how the covariant derivative (94) is influenced by the created \( A_\mu \) field. Let us evaluate (94) under consideration of (101), thereby omitting non-\( A_\mu \) containing terms for simplicity reason. Doing so, we have from (94) and (101),

\[ D_\mu \varphi = \{ \partial_\mu - i g_W \tau_0 W^\mu_0 - i g_e B_\mu \} + \ldots \varphi = \]
\[ \{ \partial_\mu - i g_W \tau_0 (Z_\mu \cos \theta_W + A_\mu \sin \theta_W) - i g_e (Z_\mu \sin \theta_W + A_\mu \cos \theta_W) \} + \ldots \varphi \rightarrow \]  

(103)

\[ D_\mu \varphi = \{ \partial_\mu - i (g_w \tau_0 + g_e g_W \cos \theta_W) A_\mu \sin \theta_W + \ldots \} \varphi. \]

This can be written as,

\[ D_\mu \varphi = (\partial_\mu - i q_{u,d} A_\mu + \ldots) \varphi , \]  

(104a)

in which

\[ q_{u,d} = (\pm I_z + \frac{Y}{2})(2 g_w \sin \theta_W) \quad \text{with} \quad I_z = \frac{\tau_0}{2} ; Y = \frac{g_e g_W}{g_w} \cos \theta_W . \]  

(104b)
Because of the eigen value $\tau = 1$, we have conveniently,

$$g_W = \frac{e}{2\sin \theta_W}; \quad Y = 1 = \frac{g_e}{g_W} \cos \theta_W \Rightarrow g_W = \frac{2\sin \theta_W}{\cos \theta_W},$$

(105)

This relationship is graphically illustrated in figure 7. The expression (104a) for the charge of the $SU(2)$ bond has been shown in Table I as an expression for the charge of the composing quarks with dedicated values for $I_z$ and $Y$. It has to be noted, though, that the GSW theory produces a holistic result for the quark-antiquark bond. That means that the assignment $Y = 1/3$ to $u$ and $d$ quarks is arbitrary, because its influence disappears in the junction of a quark and an antiquark. Considering that the bias $Y$ is a holistic property of the composite particle, there is no particular need to assign asymmetrical charges to the composing quarks. This view is reflected Table VII. By substitution of the covariant derivative (103) into (94), the masses of the Higgs boson, the masses of the $W^{+/-}$ bosons and the $Z$ boson are found, under consideration of the relationships (105) respectively as,

$$m_H' = \mu_H \sqrt{2}; \quad m'_W = \left(c_0 g'_W \right) \Phi_0; \quad m'_Z = m'_W / \cos \theta_W,$$

with $\Phi_0 = \frac{\mu_H}{\lambda_H} \sqrt{2} \text{ and } g'_W = \frac{e}{\sin \theta_W}$,

(106)

in which $c_0$ is a normalization constant.

![Fig.6. Graphical picture of the Weinberg angle.](image)

Whereas in structural view as well as in the “naked” format (95), the coupling constant $g_W$ is dimensionless, it is not the case in electroweak unification. Hence, effectively $c_0 g'_W = g_W$.

The derivation of these relationships can be found in many texts on the electroweak theory. Completeness-for-all, it might useful to note that in further elaboration of the electroweak theory the vacuum expectation value $\Phi_0$ has been given a numerical value by a relationship with Fermi’s constant $G_F$ such that $\Phi_0^2 = (G_F \sqrt{2})^{-1} = (246 \text{ GeV})^2$. In this respect, it is worthwhile to note here that the relationship $g_W \Phi_0 = m'_W$ in (40) and (101) allows an
arbitrary interchange between the quantities $g_w$ and $\Phi_0$. Either $g_w$ can be adopted as a reference, thereby fixing the value for $\Phi_0$, like I have done in my previous works by relating $g_w$ with the electromagnetic fine structure constant as $g_w = 1/\sqrt{137}$, or $\Phi_0$ can be chosen as the reference, thereby fixing $g_w$, like done in the Standard Model. In the latter case, we have [31,p.337],

$$g_w \frac{v}{2} = m'_w; \quad v = 2\Phi_0,$$

(107)

in which $v$ is related with Fermi’s constant $G_F$ as

$$\frac{1}{2v^2} = \left(\frac{G_F}{\sqrt{2}}\right)^2 = \frac{g_w^2}{8m_w^2} \rightarrow g_w \approx \frac{80.4}{246} \sqrt{2} = 0.46.$$  

(108)

It is instructive to compare Weinberg’s model, discussed in paragraphs 9-10, with the structural model shown in figure 2, discussed in paragraphs 5-8. This will be done in the next paragraph.

**Table VII**

| Fermions          | $u$  | $\bar{d}$ |
|-------------------|------|------------|
| S- spin           | 1/2  | 1/2        |
| $m_0$-mass        | ?    | ?          |
| $I_z$-weak isospin| 1/2  | 1/2        |
| $Y$-hypercharge    | $Y$  | $-Y$       |
| $Q$               | $(\pm I_z + Y)e$ | $(\pm I_z - Y)e$ |
| $Q_{\text{aul}}$  | $Q_u + Q_d = \pm 2I_z$ | $\pm e, 0$ |

11. Comparison of Weinberg’s model with the structural model

Weinberg’s model consists of a Lagrangian density composed by a doublet of two complex fields and a particularly shaped potential field (the Higgs field). From this density a wave function for the two fields can be derived as the solution of a wave equation described in terms of covariant derivatives. These covariant derivatives are composed as normal derivatives extended by gauge fields associated with a coupling factor. The substitution of these covariant derivatives in the Lagrangian density and subsequent evaluation reveals the properties of the gauge fields in terms of bosonic masses as dependencies of the Higgs field potential.

This model evokes a number of questions that are not discussed in textbooks. The first question is: What is the rationale behind the composition of Weinberg’s Lagrangian density? More specifically: Why adopting a doublet of two complex fields, i.e., why two, why complex
and why are the two fields intimately connected as an SU(2) group? The second question is: Why is this doublet subject to a potential field and why has this field the format of the Higgs field? The success of this model by experimental evidence, exposed by a pair of massive charged weak interaction bosons and one neutral one, is usually considered as proof enough, without a need for further explanation.

The structural model as introduced in this article, though, may give an answer to those otiose (?) questions. Obviously, the two complex fields in the Weinberg Lagrangian are (fermionic) fields of particles. Because these fields compose a doublet, the two particles are intimately connected. Such an intimately connected pair of particles is shown in the structural model of figure 2. The Weinberg model states that the two particles are subject to the same energy field. Where does this field comes from? The obvious answer is that the field experienced by one of the two particles is the energy field of the other particle. A remaining problem is how to explain the shape of the energy field. In the structural model, this energy field has three components that explains its particular format. It is a classical monopole field associated with a dipole field, both shielded by a polarisable ambient background field. This explanation requires to distinguish the fermionic fields of the two particles from their potential fields. The description of the particle’s fermionic field as a complex field is a model for its actual nature as a Dirac spinor. The two components of the complex field can be viewed as the two components of the Dirac spinor split up in a temporal related part and a spatial related part (discussed in paragraph 9). This model gives an adequate explanation for the origin of two massive weak interaction as the bond between the two particles. It is incomplete still, because a the nuclear spin interaction has still to be accounted for. After doing so the origin of a third weak interaction boson is explained.

As long as the origin of electric charge is ignored, there is no difference between the Weinberg model and the structural model, albeit that the models are conceived from a different point of view. Because of mathematical reasons, the Weinberg model needs a third boson, next to a paired one. Like shown in paragraph 9, it can, in principle, be introduced without taking electromagnetism into account, just by assigning a coupling factor to the third gauge field different in magnitude from the one assigned to the paired gauges. This means that the only essential difference between the Weinberg model and the structural model is the incorporation of electromagnetism in the covariant derivative. This is done by an artificial mechanism on the simple argument that somehow an explanation has to be found for the origin of electric charge in elementary nuclear particles. As noted before, experimental evidence of the existence of the Z boson cannot be considered as a proof for the correctness of this mechanism, because the third boson should show up anyway. In the structural model the charge of the W bosons is taken for granted. Within the scope of this article no attempt has been made to explain its origin. Nevertheless, the simple add-on allows a rather accurate calculation of the mass difference between a charged pion and a neutral pion. In a recent preprint, a more fundamental explanation for the origin of electric charge in the structural model is given [32].

The hypothetical axiomatic mechanism to explain the origin of the Z boson is equivalent with the spin-spin interaction mechanism as developed in this article from a structural point of view. However, the GSW theory is unable to relate the masses of numerical values of this
Z boson and the Higgs boson with the mass value of the W bosons, while the structural theory can. On the other hand, whereas in the structural theory the electric charge of the mesons is just accepted as a physical attribute related to isospin and unified with the weak force in a physical model that enables to calculate its influence on the mass of the quark-antiquark junction, the electroweak theory explains the origin of the electrical charge in a model that unifies the weak interaction with electromagnetism by a mathematical theory.

12. Strong interaction

One of the problems left is the unification of the GSW theory with strong interaction. This problem has given rise to the development of the QCD (Quantum Color Dynamics) theory, in which an additional boson is introduced, dubbed as gluon, as the carrier of an additional force next to the electromagnetic force and the weak interaction. This is done by extending the SU(2) model for the quark junction between the u quark and the \( \bar{d} \) quark toward the SU(3) model for three-quark junctions (baryons). Before discussing that model, let us consider the decay of the vector-type rho-meson into the pseudo-scalar type pion. As a rule of thumb, it is usually said that strong interaction decay comes first before weak interaction decay takes place. For that reason, the rho-pion decay is considered as a strong interaction mechanism. However, the structural view as developed in this article has revealed that this decay is nothing else but the nuclear spin flip in the spin-spin interaction process. Because we have identified the spin-spin interaction as the physical interpretation of Weinberg’s axiomatic mixing process, it is rather questionable if the rho-pion decay needs a description in terms of an additional bosonic carrier. It has been shown in this article so far, that a simplified structural physical model of two-quark junctions has given a comprehensible view on the rather abstract highly-sophisticated mathematical SU(2) electroweak theory. It might well be that such a simple physical model for three-quark junctions may do as well to give a comprehensible view on QCD.

13. Baryons

Whereas a meson can be conceived as the one-body equivalent of a two-body harmonic oscillator, a baryon can be conceived as the one-body equivalent of a three-body harmonic oscillator. The one-body equivalent of the three-body quantum mechanical oscillator can be analyzed in terms of pseudo-spherical Smith Whitten coordinates [33]. The Smith-Whitten system of coordinates is six-dimensional. Next to a (hyper)radius \( \rho \), the square of which is the sum of the squared spacings between the three bodies, there are five angles \( \varphi, \theta, \alpha, \beta, \gamma \), in which \( \varphi \) and \( \theta \) model the changes of shape of the triangular structure and in which \( \alpha, \beta \) and \( \gamma \) are the Euler angles. The latter ones define the orientation of the body plane in 3D-space. The planar forces between three identical interacting bodies not only are the cause of dynamic deformations of the equilateral structure, but also are the cause of a Coriolis effect that result in vibra-rotations around the principal axes of inertia of the three-body structure [34]. The application of this approach for baryons has been documented by the author in [35], showing that the wave equation of the quasi-equilateral baryon structure can be formulated as
This wave equation is the three-body equivalent of the pion’s two-body wave equation shown in (46). In the ground state we have \( m = 0 \). Hence,

\[
R(m,v,k) = 4m + |v - k| (4m + |v - k| + 4)
\]

The radial variable \( \rho \) is the already mentioned hyper radius. The potential field is just the threefold of the potential field in the wave equation of the pion. There are three quantum numbers involved. Two of those are left in the ground state, effectively bundled to a single one. The quantum number \( k \) allows a visual interpretation, while \( v \) is difficult to visualize. The impact of \( k \) is shown in the right hand part of figure 3. It illustrates the motion of the center of mass under influence of \( k \). Note that this rotation is quite different from a rotation of the triangular frame around the center of mass. It is the center of mass itself that rotates, while the frame does not. Actually, the small motions of the individual quarks are responsible for this motion. As shown in [35], this relatively simple wave function expression allows a pretty accurate calculation of the mass spectrum of baryons. The octet states in the baryon classification are the counterparts of the meson pseudoscalar states, the decuplet states the counterparts of the vector mesons. A single integer step in the quantum number \( l \), brings the \( p,n/\Delta \) level to the \( \Sigma / \Sigma^* \)-level, etc. The results of the mass calculations are shown in the most right-hand columns of the tables IV and V. As before, these tables are constructed by conceiving the \( u \) quark and the \( d \) quark as the archetype quark in a different state of isospin. The up-state nuclear spin of an \( u \) quark (or a \( d \) quark) and the down-state of an \( u \) quark (or or a \( d \) quark) are indicated as, respectively \( u \) and \( u \) (or \( d \) and \( d \) for \( d \) quarks). More on this mass spectrum, with inclusion of charmed and bottom quarks, can be found in [35].

While the mesons in this article are considered as members of an SU(2) group, it makes sense considering the baryons as members of an SU(3) group. This is reflected in the wave function representation shown in (111). Because the archetype quarks are supposed to be identical, they hold each other in equilibrium by spatial momenta \( (p_x,p_y) \) with relative values of, respectively, \((\sqrt{3}/2,1/2),(\sqrt{3}/2,1/2)\) and \((0,1)\). These values are reflected in the spatial components of the wave function.

\[
\alpha_0 \left\{ \frac{d^2 \psi}{dp^2} + \frac{5}{\rho^2} \frac{d \psi}{dp} + \frac{R(m,v,k)}{\rho^2} \right\} + V'(\rho') = E'\psi,
\]

where

\[
\alpha_0 = \frac{\hbar^2 \lambda^3}{6mg\Phi_0}; \quad E' = \frac{E}{3g\Phi_0}; \quad V' = \frac{V}{3g\Phi_0}; \quad \rho' = \rho \lambda, \quad \text{and}
\]

\[
V(\rho') = 3g\Phi_0 (k_0 + k_2 \rho^2 + \ldots)
\]

(109)

\[
R(m,v,k) = 4m + |v - k| (4m + |v - k| + 4)
\]
\[
\Psi_{pa} = \begin{bmatrix}
\Psi_{1r} & \Psi_{1x} & \Psi_{1y} \\
\Psi_{2r} & \Psi_{2x} & \Psi_{2y} \\
\Psi_{3r} & \Psi_{3x} & \Psi_{3y}
\end{bmatrix} \rightarrow \begin{bmatrix}
-ib & a\sqrt{3}/2 & a/2 \\
-ib & -a\sqrt{3}/2 & -a/2 \\
-ib & 0 & -a
\end{bmatrix}.
\]

(111)

It is not difficult to prove that, under proper scaling of the amplitudes, this matrix is unitary (i.e. \( \Psi \Psi^\dagger = 1 \), in which \( \Psi^\dagger \) is the transpose conjugate of \( \Psi \)), and that its determinant is equal to 1 for any value of the ratio \( a/b \). In a conservative field of forces, like it is the case of interaction between the quarks as a consequence of their nuclear potential fields, the ratio \( a/b \) is subject to change. This implies that the nine-component spinor \( \Psi_{amp} \) may rotate over eight spatial angles \( \vec{\mathcal{G}}(\vartheta) \) in a nine-dimensional spinor space. This rotation is the equivalence of the weak interaction in the meson case. The bosons involved in SU(3) are known as gluons. Because the wave function shown in (109) is a single dimensional Schrödinger approximation of the generic nine-dimensional wave function (111), the fine nuances have disappeared. But, in fact, there is no conflict here with the Standard Model.

14. Relationship with Chromodynamics (QCD)

Because of the unawareness of the physical mechanism associated with the dipole properties of the archetype quark, the interaction between quarks in the baryons in the gauge theory of the Standard model has been conceived, similarly as in the case of mesons, from an abstract point of view. The bare fact that three basic quarks assemble a stable configuration is taken as a starting point. This view is reflected in the SU(3) representation of an overall spinor as shown in (111), the elements of which can be viewed as the amplitudes of three interacting subspinors that result from the solution of Dirac's equation. Similarly as the SU(2) modeling of mesons, described in paragraph 9, it embodies, in fact, both the properties of the nuclear spins (angular dipole moments), as well as the properties of the scalar dipole moments. Whereas the SU(2) spinor can be represented by a vector in a sphere, positioned by three angles, the SU(3) spinor can be represented by a vector in a hypersphere, positioned by eight angles. These angles represent the degrees of freedom of a complex \( n \times n \) matrix with \( 2n^2 \) real parameters. The unitary condition on the rows removes \( n^2 \) of these and an additional one is removed by the constraint of unit determinant. Whereas in the SU(2) case, three angles are taken up the covariant derivative as three independent gauge boson fields, in the SU(3) case eight angles are taken up as eight independent gauge boson fields. Under the same formalism as with SU(2), we have in SU(3),

\[
\Psi \rightarrow \Psi \exp(i\vec{\mathcal{A}} \vec{\mathcal{G}}) \text{ with } \vec{\mathcal{G}} = (\vartheta_1, \vartheta_2, \ldots, \vartheta_8) \text{ and } \vec{\mathcal{A}} = (\lambda_1, \lambda_2, \ldots, \lambda_8),
\]

(112)

in which \( \vec{\mathcal{A}} = (\lambda_1, \lambda_2, \ldots, \lambda_8) \) are the eight 3 x 3 Gell-Mann matrices [1]. From an abstract point view the eight angles can be captured in three quantum numbers. In QCD the three quantum numbers are visualized as colors. This allows visualizing the eight possible states of the SU(3) baryon as a color state. In the most simple condition, one of the quarks has a pure
blue charge, the second one has a pure green charge and the third one has a pure red charge. Any three quark combination with mixed colors may represent a physical state as well.

Over time, QCD has been developed as a successful theory, captured in a rather extensive set of rules. This, of course, is beyond the scope of this article. The reason for bringing it up is the question whether it is possible, similarly as in the case of mesons, conceived as an SU(2) meson group, to give it a structural interpretation. Answering the question would require, though, quite an extensive analysis. Within the scope of this article, in which baryons are only superfluously described, just a few arguments are given why a structural view on baryons is not in conflict with gauging on the basis of the axiomatic formalism in the Standard Model. The major argument is that, whereas in QCD three quantum numbers represent the state of baryons conceived as an SU(3) group, the structural view as summarized in paragraph 13 shows three quantum numbers as well. In previous work [35] it has been shown how different combinations of these quantum numbers not only results in the very same classification scheme as in Gell-Mann’s Eightfold Way, but also allows an accurate calculation of the mass spectrum in quantitative terms, like shown in table V and, more extensively, in[35].

It is useful to note that the right hand part of figure 3 shows a baryon in a basic state. From the mathematical representation (109) it will be clear that the three quantum numbers are representative for the quantum states of the three quarks. These quantum states express rather complicated vibra-rotations that are subject to the interacting bosons between the quarks. It may seem that the quark’s description as a monopole with two dipole moments is adequate for understanding the mass spectrum of mesons and baryons. This raises the question on the interpretation that has to be given to the gluons as they manifest themselves in nuclear experiments, in which they show up as the nuclear equivalents of photons. In an attempt trying to find the answer to this, let us consider a comparison between the quark as described in this article and an electron, as a stepping stone to the comparison between a gluon and a photon. This is the subject of the next paragraph.

15. The gluon

Similarly as the photon, the gluon will become physically manifest, either directly as a free mass less particle, or indirectly as a bound (“virtual”) one in the spin-spin interactions of the type discussed in paragraph 8. Whereas the (three) SU(2) bosons and the (eight) SU(3) bosons are indirect manifestations, which, like analyzed in this article, can be described by the behavior of the quark in terms of a monopole and two dipole moments, the direct manifestation shows up quite differently. Probably the most obvious one is the well-known “three-jet event”. This event shows up in a experiment in which a high energetic electron bundle scatters under 120° on a high energetic positron bundle. This produces bundles of quark pairs in separation, such as illustrated in figure 7. Because quarks are exclusively produced a pairs, a third energetic bosonic bundle is produced to conserve momentum and energy. Eventually the three bundles (“jets”) hadronize to observable particles. The existence of the third bundle is considered as the experimental confirmation of gluons [36]. How to explain this experiment in terms of the theory developed in this article?
The three-jet event reveals an interesting parallel between the gamma photon-electron relationship on the one hand and the gluon-quark relationship on the other hand. This parallel becomes clear if these relationships are described in the same terms. An adequate description is the time-dependent wave equation of the quark's far field as shown by (55). If $\lambda \to 0$, this is just the Maxwellian wave equation for the scalar part of the electromagnetic vector potential. It is interesting to consider the building of the quark's potential field from a sudden energy eruption from its source. This requires solving the Proca wave equation (55), described before in paragraph 7. Figure 8 shows how the building of this field evolves in time. The field evolves as the sum of a stationary component and a transient pulse. The opposite is true as well. A sudden disappearance of the gluon (for instance by annihilation) leaves the transient as a propagating boson. The comparison with the very same process as occurs in the case of a sudden appearance or disappearance of an electron makes clear that the transient pulse represents the equivalent of a gamma photon. However, unlike the gamma photon, the quark's transient pulse is subject to dispersion. The dispersion is due to the $\lambda^2$ term in the Proca wave equation (55). As shown by (57), the term is a consequence of the energetic ambient field, known as the Higgs field. The transient pulse, better known as gluon, propagates at light speed but dies quickly. The picture makes clear that experimental violence may produce a skewed quark pair and a gluon to conserve momentum. The gluon jet experiments do not allow identifying the skew of individual quark pairs nor the identification of single gluons. All what can be observed are bundles of hadronized fermions. Interpretation is done with a theory in mind.

The boson characteristics of the gluon are different from those of the weak interaction $W$/$Z$ bosons. Whereas the latter ones can be characterized as particles with a defined energy, expressed into a rest mass quantity or a defined quantum mechanical frequency (like mesons), the gluon energy is built up, similarly as the photon one, by the amplitudes of an ensemble of frequencies. The energy may have any value. Hence, the gluon is considered as a mass less particle. This, of course, is just a course qualitative view on the three-jet event.

![Fig. 7](image)

**Fig. 7.** The three jet event. The annihilation of a high energetic electron and a high energetic positron, resulting from bundles scattered under $120^\circ$, produces a photon, which in turn produces a separated quark pair and a gluon. Eventually the quark and the gluon bundles hadronize into observable particles. (From: unknown source on www).

A proof of equivalence with the QCD view, if possible, would need a challenging extensive elaboration beyond the scope of this article. On the other hand, there seems being no particular reason why the structural view on gluons as summarized in this paragraph as part of the structural view on particle physics developed in this article would be in conflict with
the axiomatically conceived QCD view. Both views support an SU(3) gauging, three quantum numbers for the baryon composition and the existence of gluons.

Fig.8. The building of the quark’s potential field as a result of a sudden energy eruption from its source. The field is the sum of the steady solution shown at the right and the transient pulse shown in the lower part of the figure. This pulse is the actual gluon. It propagates at light speed and it eventually disappears as a result of dispersion. If \( \lambda \) is zero, the transient is a never disappearing gamma photon and the stationary situation is shown by an unfinished rectangular shape of the upper most right graph. Note that the field is represented by \( r \Phi(r) \).

To illustrate that further elaboration might reveal additional relationships, let us consider the different behavior of the \( W/Z \) bosons and the gluon once more. Let us suppose that the meson-like behavior of the \( W/Z \) bosons is more than just an appearance and that they are mesons indeed in an hierarchical system, such as illustrated in figure 9. Like discussed in paragraph 8, the loss of binding energy in the excited anharmonic oscillator structure that binds the quarks together inhibits heavier quarks beyond \( u/d,s,c \) and \( b \). Nevertheless, there happens to be a topquark. Curiously however, the assigned rest mass of the topquark is out of scale. But if the topquark would be the constituent of the \( W/Z \) boson, its existence can be well understood. This is possible by invoking the relationship (61) as shown in paragraph 8 for the pion. Applying this on the \( W \) boson, gives

\[
m'_{W} = 2m'_{t} - 3m'_{Z},
\]

with \( m'_{W} \approx 80.4 \text{ GeV} \) for the \( W \) boson and \( m'_{Z} \approx 91.2 \text{ GeV} \) for the \( Z \) boson, the result is \( m'_{t} \approx 177 \text{ GeV} \). It explains the difference in the nature of a \( W \) boson as compared to a gluon and it explains the big mass gap between the \( u/d,s,c \) and \( b \) on the one hand and the topquark on the other hand.
The quantum of the bosonic field between two baryons is a meson in virtual state. The meson in real state is a quark-antiquark assembly bound by W/Z bosons in virtual state. The W/Z in real state behaves as mesonic topquark-antitop bond.

**16. Discussion and conclusion**

The theory described in this article is a structural view on particle physics with a physical interpretation on some of the axiomatic principles adopted in the mathematical formalism of the Standard Model. The model description of the mesons and the baryons is based upon a non-relativistic approximation of Dirac’s multi-dimensional fermionic wave equation to a single-dimensional Schrödinger one in the center of mass frame of hadrons, extended by separate additions of some second order effects not covered in the approximation and interpreted in the lab frame after relativistic correction. Although in this respect it lacks the rigidity of the conventional Standard Model description and although the scope of the presented theory is limited in the sense that scattering and decay processes are left beyond scope, the highlight on two additional principles that are not yet covered in the Standard Model, reveals useful complements to the present status of theory. The two basic principles highlighted in this article that can be added to the Standard Model are,

- a. The quark is an unrecognized Dirac particle that has, next to the well-known real dipole moment associated with the elementary angular momentum $\hbar$, a second real dipole moment associated with an elementary linear dipole $\hbar/c$, which, unlike as in the case of electrons, is polarisable in a scalar potential field.

- b. Deriving Dirac’s fermionic wave equation from Einstein’s geodesic equation rather than from Einstein’s energy expression reveals a complementary property to the quark conceived as a Dirac particle described in the first highlight. This property is the invariance of the frame-independent ratio $\Phi_0/\lambda$, in which $\Phi_0$, expressed in units of energy, is a measure of the quark’s potential and in which $\lambda$, expressed in m$^{-1}$, is a measure for the range of the quark’s potential. In the article, this property is dubbed as the quark-scaling theorem.

In the article it has been shown how these two unrecognized theoretical consequences from Dirac’s electron theory may influence the view on the Standard Model of particle physics without substantially affecting its basics of SU(2) and SU(3) gauging, electroweak unification and the mass generation mechanism from the Higgs field. Most of the presented results have been documented by me in literature in more detail before, some in journals, others in prepublications that met opposition because of a seeming conflict with common views that
are considered as proven in the wealth of studies and experiments in the high standard of present theory. The highlight on the two additional principles that are not yet covered in the Standard Model, may help showing that principles quite some problems can be tackled that so far have remained unsolved, like shown in the following non-exhaustive list,

1. The quark-antiquark model shown in figure 2 has allowed to express the Gravitational Constant $G$ into quantum mechanical quantities with a successful numerical proof [17,20].
2. Different from a theoretical axiomatic concept, isospin is a physical attribute associated with the quark’s polarisable dipole moment (this article).
3. The number of elementary quarks can be reduced to a single basic archetype [22]
4. The big gap between the rest masses of the $(u/d,s,c,b)$ quarks on the one hand and the topquark on the other hand is a consequence of the loss of binding energy between the quarks (this article).
5. The massive rest mass energies of the Higgs boson, the $W/Z$ bosons and the topquark can be assessed by theory. Experimental evidence is a confirmation instead of empirical axioms (this article).
6. The gluon-quark relationship is the equivalent of the photon-electron relationship (this article)
7. The mathematical axiomatic SU(2) and SU(3) gauges of particle physics theory can be replaced by physically based gauges similar to the electromagnetic U(1) gauge (this article).

Moreover, it has been demonstrated that the mass spectrum of hadrons can be calculated in a rather easy way, which can probably compete in accuracy with the results from cumbersome lattice QCD computations. It may seem as if the theory as described in this article is different from the Standard Model, or even be in conflict. If the latter would be true it has to be declined, because the Standard Model is well proven by an overwhelming amount of experimental evidence of its correctness. In fact, there is no basic conflict. Instead, it provides a physical basis for quite some underlying axioms that are accepted in present theory. There is no reason why such a basis would not allow application of the same powerful analytical techniques as used in the Standard Model. The structural view might be helpful for better understanding and a key for further progress, like demonstrated in the just given list.

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