Accurate measurement of laser intensity using photoelectron interference in strong-field tunneling ionization

JIA TAN,1 YUEMING ZHOU,1,* MIN LI,1 MINGRUI HE,1 YALI LIU,1 AND PEIXIANG LU,1,2,3

1Wuhan National Laboratory for Optoelectronics and School of Physics, Huazhong University of Science and Technology, Wuhan 430074, China
2Laboratory of Optical Information Technology, Wuhhan Institute of Technology, Wuhhan 430205, China
3lupeixiang@hust.edu.cn

Abstract: Accurate determination of laser intensity is of fundamental importance to study various phenomena in intense laser-atom/molecule interactions. We theoretically demonstrate a scheme to measure laser intensity by examining the holographic structure originating from the interference between the direct and near-forward rescattering electrons in strong-field tunneling ionization. By adding a weak second-harmonic field with polarization orthogonal to the strong fundamental driving field, the interference pattern oscillates with the changing relative phases of the two-color fields. Interestingly, the amplitude of this oscillation in the photoelectron momentum spectrum depends on the parallel momentum. With the quantum-orbit analysis, we show that the amplitude of the oscillation minimizes when the time difference between the recollision and ionization of near-forward rescattering electron is half cycle of the fundamental driving field. This enables us to measure accurately the laser intensity by seeking the minimum of the oscillation amplitude. Moreover, we show that this minimum can be determined without scanning the relative phases, instead, by just monitoring the interference patterns for two relative phases. This facilitates the application of our scheme in experiment.

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OCIS codes: (190.7110) Ultrafast nonlinear optics; (260.3230) Ionization; (270.6620) Strong-field processes.

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1. Introduction

Tunneling ionization of atoms and molecules exposed to the strong laser fields is a fundamental process in intense laser-matter interactions [1–4]. The tunnelled electron wave packet further accelerated by the oscillating electric field of laser pulse may return back and rescatter or recombine with its parent ion, resulting in various nonlinear phenomena [5], such as high-order above-threshold ionization [6–8], high-order harmonic generation [9–12] and nonsequential double ionization [13–19]. Exploring the application of these nonlinear processes consists the main contents of the attosecond science. In the past decades, these rescattering processes have been extensively used for attosecond pulses generation [20–23], molecular orbit tomography [24–26] and probing electronic and nuclear dynamics [27–31]. For these applications, deep insights into the details of the processes are necessary and it has drew plenty attentions. For example, the ionization time and rescattering time of the tunneled electron wave packet have been determined with attosecond accuracy [32]. In these strong-field processes, the laser intensity is an important parameter. Uncertainty in the laser intensity could lead to misunderstanding on the underlying dynamics [33]. Therefore, accurate determination of the laser intensity is essential for fully understanding the strong-field processes and for further performing their applications. However, this is a longstanding and challenging task [34–36].

Traditionally, the laser intensity could be estimated by measuring the pulse energy, the spatial and temporal profiles. This method usually gives an uncertainty as large as 50% [37]. In strong-field ionization, various in situ methods have been proposed to measure laser intensity. For example, it has been shown that the laser intensity could be calibrated by comparing the experimentally measured ionization probability with these from the theories [38]. This method requires the accurate theory on calculating ionization probability. The laser intensity has also been usually estimated from the $2U_p$ or $10U_p$ ($U_p$ is the ponderomotive potential of the laser field) cutoff in the photoelectron energy spectrum [39]. However, in the actual experiments such cutoffs are usually not clear. Moreover, in the multiphoton ionization range, such cutoffs do not exist. Another method of calibration of laser intensity is based on the ac-stark shift of the ionization potential of the target [40–42]. By checking the shift of the above-threshold ionization (ATI) peaks in the energy spectrum, the laser intensity can be estimated on the accuracy of a few percents. While the existence of the resonant ionization pathway in which the ATI peaks are laser intensity-independent, impedes its application [43]. Moreover, in the tunneling region for the mid-infrared laser fields, the ATI peaks are not visible and thus a reliable calibration of laser intensity is prevented. For the elliptically and circularly polarized laser pulses, the laser intensity can be determined from the center position of the photoelectron momentum distribution (PEMD) or the width of transverse momentum distribution [33,44]. However, this method sensitively depends on either the adiabatic or nonadiabatic ionization picture used in calibration.

In this work, we introduce another novel in situ method based on the holographic pattern of the photoelectron momentum distribution (PEMD) in strong-field tunneling ionization to determine the laser intensity precisely. This holographic pattern originates from the interference of the direct electron (which reaches the detector directly after tunneling ionization) and the near-forward rescattering electron in strong-field tunneling ionization. This pattern has been experimentally observed several years ago [45] and it has been expected to be an effective tool in probing atomic and molecular structure and their ultrafast dynamics. In recent years, its application has been widely explored [46–52]. For example, it has been employed to extract the tunneling exit point [53] and the phase structure of the electron wave packet in molecular tunneling ionization [54,55]. Very recently, we have shown that with this photoelectron holographic interference the phase of the scattering amplitude can be extracted [56,57]. More fantastically, we have demonstrated
that this holographic interference is capable of measuring the attosecond charge migration in molecules where unprecedented attosecond and picometer spatio temporal resolutions could be achieved [58]. Here, we explore its application in determining the laser intensity.

We analyze the holographic interference pattern in the PEMD obtained by solving the time-dependent Schrödinger equation (TDSE). When a weak perturbative second harmonic (SH) with polarization orthogonal to the fundamental driving field is added, the holographic interference varies with relative phases of this orthogonal two-color (OTC) fields. We monitor the positions of interference minima. The results show that the positions of these minima oscillate in the transverse direction (perpendicular to the polarization of the fundamental field) periodically with the relative phases of the OTC fields. The amplitude of the oscillation depends on the parallel momentum (parallel to the laser polarization of the fundamental field) and minimizes at a certain parallel momentum. Employing the quantum-orbit analysis, we demonstrate how the amplitude depends on the parallel momentum and show that it minimizes when the traveling time (the time between recollision and tunneling ionization of the electron) is half cycle of the fundamental field. This establishes the relation between the parallel momentum where the amplitude minimizes and the traveling time and thus the tunneling ionization time. With this relationship, the intensity of the fundamental driving field is determined by seeking the minimum of the oscillation amplitude of interference minima. The results show that the positions of these minima oscillate in the transverse direction and outer (perpendicular to the polarization of the fundamental field) region smoothly by the splitting function $\frac{1}{2}(\max - \min)$.

\[ \begin{align*}
\Psi(t) &= \Psi(t)[1 - F_s(R_s)] + \Psi(t)F_s(R_s) = \Psi_1(t) + \Psi_2(t). 
\end{align*} \] (4)

2. Theoretical method

2.1. Numerically solving the time dependent Schrödinger equation

The PEMD was obtained by numerically solving the two-dimensional TDSE with single-active-electron (SAE) approximation (atomic units are used unless otherwise stated),

\[ \begin{align*}
i \frac{\partial \Psi(r, t)}{\partial t} &= H(r, t)\Psi(r, t), 
\end{align*} \] (1)

where $\Psi(r, t)$ represents wave function of system and $H(r, t)$ is Hamiltonian

\[ \begin{align*}
H(r, t) &= -\frac{1}{2}\nabla^2 + V_C(r) + V_L(r, t), 
\end{align*} \] (2)

where $V_C(r) = -(x^2 + y^2 + a)^{-1/2}$ is the effective soft-core potential with soft-core parameter $a = 0.92$ which yields the ionization potential of Xe ($I_p = 0.446$ a.u.). $V_L(r, t) = \mathbf{r} \cdot \mathbf{F}(t)$ represents the interaction of electron with the laser field. $\mathbf{r} = (x, y)$ denotes the position coordinate of electron. $\mathbf{F}(t)$ is the electric field of the OTC pulses, which is written as

\[ \mathbf{F}(t) = f(t)[F_s(t)\hat{e}_x + F_s(t)\hat{e}_y] = f(t)[F_1 \cos(\omega t)\hat{e}_x + F_2 \cos(2\omega t + \Phi)\hat{e}_y]. \] (3)

Here, $f(t)$ is pulse envelope. It has a trapezoidal shape, rising linearly during one cycle, then keeping constant for three cycles and decreasing linearly during the last one cycle of the fundamental pulse. The fundamental driving pulse $F_s(t)$ is polarized along the $x$ axis with amplitude $F_1$ and the SH field $F_s(t)$ is polarized along $y$ axis with amplitude $F_2$. In our calculations, the wavelength of the driving field is 1600 nm and the SH field is 800 nm. $\Phi$ is the relative phase of the two-color fields.

The split-operator spectral method on a Cartesian grid is used to solve the TDSE [59]. The initial wave function is obtained with imaginary-time propagation method [60]. The whole space is split into the inner $(0 - R_s)$ and outer $(R_s - R_{max})$ region smoothly by the splitting function $F_s(R_s)$ at the time $\tau$ [61]:

\[ \begin{align*}
\Psi(\tau) &= \Psi(\tau)[1 - F_s(R_s)] + \Psi(\tau)F_s(R_s) = \Psi_1(\tau) + \Psi_2(\tau). 
\end{align*} \] (4)
Here, \( F_s(R_s) = 1/[1 + e^{-(r-R_s)/\Delta}] \), \( \Delta \) is the width of crossover region and \( R_s \) represents the boundary of the inner space [62]. \( \Psi_1(\tau) \) is the wave function in the inner space which is numerically propagated under the full Hamiltonian, and \( \Psi_2(\tau) \) represents the wave function in the outer space which is analytically propagated under the Volkov Hamiltonian [61, 63]. Specially, at the time \( \tau \) during the propagation, the wave packet \( \Psi_2(\tau) \) is firstly transformed into momentum space \( C(p, \tau) \), and then it is propagated from time \( \tau \) to the end of laser pulse by

\[
\Psi_2(\infty, \tau) = \int C(p, \tau) e^{-ip \tau} \frac{d^2p}{2\pi},
\]

with \( C(p, \tau) = e^{-i \int_{t'}^\tau \frac{1}{2}[p + A(t')]^2 dt'} C(p, \tau) \). Here, \( p \) is the electron final momentum and \( A(t) = -\int_0^t F(t') dt' \) is the vector potential of the OTC fields. Therefore, we obtain the final momentum distribution related to the sum of the wave function in momentum space at \( \tau \),

\[
\frac{dP(p)}{dEd\theta} = \sqrt{2E} \left| \int C(p, \tau) \right|^2,
\]

where \( E = p^2/2 \) is electron energy and \( \theta \) is the angle between \( p \) and the direction of laser polarization. In our calculations, we choose time step \( \delta \tau = 0.05 \) a.u. At the end of the pulse, the wave function is propagated for one additional optical cycle to make sure the “slow” electrons reach the boundary \( R_s \) [64]. The boundary \( R_s \) is set to be 200 a.u. and \( \Delta = 8 \) a.u.

### 2.2. Quantum-orbit analysis

In this work, we focused on the holographic pattern in the PEMDs. This holographic pattern results from the interference of the direct and near-forward rescattering electrons, and it can be written as

\[
|M|^2 = |M_d|^2 + |M_r|^2 + 2|M_d||M_r|\cos(\Delta \varphi),
\]

where \( |M_d| \) and \( |M_r| \) are the amplitudes of the direct and rescattering electron wave packets, respectively. \( \Delta \varphi \) is the phase difference between the direct and rescattering electrons. Following [45], this phase difference can be calculated with the saddle-point approximation, which provides us the quantum orbit [65] to analysis the holographic interference. In the OTC fields \( \Delta \varphi \) can be written as

\[
\Delta \varphi = \varphi_r - \varphi_d = \frac{1}{2} \int_{t'_i}^{t'_f} [p_x + A_x(t)]^2 dt + \frac{1}{2} \int_{t'_i}^{t'_f} [p_y + A_y(t)]^2 dt - \frac{1}{2} \int_{t'_i}^{t'_f} [k_x + A_x(t)]^2 dt + \frac{1}{2} \int_{t'_i}^{t'_f} [k_y + A_y(t)]^2 dt + I_p(t'_r - t'_f).
\]

Here, \( I_p \) is the ionization potential of Xe. \( t'_i \) is the ionization time of rescattering electron and \( t'_f \) is the ionization time of direct electron. \( t'_r \) is the recollision time of the rescattering electron. \( p_x \) and \( p_y \) are the parallel and transverse momenta, respectively. \( A_x(t) \) and \( A_y(t) \) are the vector potential of the fundamental field and SH field, respectively. \( k_x \) and \( k_y \) are the components of canonical momentum \( k \) of electrons before rescattering in the horizontal and vertical direction, respectively. The first and third terms account for the phase acquired for the motion of direct and rescattering electrons in the direction of SH field polarization. The second and forth terms represent the phase obtained from the motion of the direct and rescattering electrons parallel to the fundamental laser polarization. The fifth term is caused by the different ionization time.

In this study, the SH field is very weak (with the intensity of about 1% of the fundamental field), and thus the ionization time and the recollision time can be approximately determined from
the saddle-point equation in the fundamental driving field [32]. The evolution of the electron after tunneling ionization is affected by the SH field. Then, the saddle-point equation for the direct electron is

$$\frac{1}{2}[p + A_x(t_d)]^2 + I_p = 0,$$

and the saddle-point equations for the rescattering electron are

$$\frac{1}{2}[k_x + A_x(t_r)]^2 + I_p = 0,$$  \hspace{1cm} (9)

$$\frac{1}{2}[k_x + A_x(t_r)]^2 = \frac{1}{2}[p + A_x(t_r)]^2,$$  \hspace{1cm} (10)

$$\int_{t_d}^{t_r} [k_x + A_x(t)]\,dt = 0,$$  \hspace{1cm} (11)

$$\int_{t_d}^{t_r} [k_y + A_y(t)]\,dt = 0,$$  \hspace{1cm} (12)

where Eqs. (9) and (10) stand for the energy conservation of direct and rescattering electron at tunneling ionization, and Eq. (11) indicates the energy conservation during rescattering. Equations (12) and (13) present the return condition.

3. Result and discussion

Figure 1(a) shows the PEMD obtained by solving TDSE of Xe in 1600-nm single-color laser field. Figures 1(b)-1(d) display the PEMDs in the OTC laser fields combined a 1600-nm fundamental component and a much weaker 800-nm field, where the relative phases are $\Phi = 0.5\pi$, $\pi$ and $1.5\pi$, respectively. The intensity of the 1600-nm laser field is $1.5 \times 10^{14}$ W/cm$^2$. The intensity of the 800-nm field is $1 \times 10^{12}$ W/cm$^2$ and it can be considered as a weak perturbation. $p_x$ and $p_y$ are respectively the electron momentum along polarization directions of the 1600-nm and 800-nm
fields. The nearly horizontal fringes are clearly visible in the PEMDs. These fringes are the holographic pattern originating from the interference of the direct and near-forward rescattering electrons. For the single-color field, this interference structure is exactly symmetric about \( p_x = 0 \). However, for the OTC fields the fringes are distorted. The fringes shift along \( p_x \) direction and this shift depends on the parallel momentum \( p_x \). For example, at the relative phase \( \Phi = 0.5\pi \), the interference minima shift up at \( p_x = 1.0 \) a.u. and shift down at \( p_x = 1.9 \) a.u., as shown in Fig. 1(b). The shift of the fringes changes with relative phase. For example, the shift of the fringes is reversed for \( \Phi = 1.5\pi \), as compared to \( \Phi = 0.5\pi \).

![Fig. 2](image)

Fig. 2. (a) The interference term \( \cos(\Delta \varphi) \) as a function of \( p_y \) at \( p_x = 1.9 \) a.u. The blue dashed line represents the result extracted from the PEMD of single-color field. The yellow and green solid lines are the results extracted from the PEMDs in the OTC laser fields with relative phases \( \Phi = 0.5\pi \) and \( 1.5\pi \), respectively. The black dashed lines indicate the position of the first minimum of the interference term at \( p_y > 0 \). (b) The shift \( \Delta p_y \) of the first minimum of the interference term at \( p_y > 0 \) as a function of relative phase at \( p_x = 1.0 \) a.u. (the yellow solid line), 1.3 a.u. (the green solid line), 1.6 a.u. (the purple solid line) and 1.9 a.u. (the blue solid line). (c) The shift \( \Delta p_y \) obtained from the TDSE results as a function of relative phase for \( p_x \) ranging from 0.75 a.u. to 1.95 a.u. (d) The shift \( \Delta p_y \) obtained from Eq. (8) as a function of relative phase for \( p_x \) ranging from 0.75 a.u. to 1.95 a.u. The black solid lines in (c) and (d) indicate the maximum of the shift \( \Delta p_y \).

To show the shift of the interference fringes more clearly, we employ the procedure introduced in [56] to extract the phase difference \( \Delta \varphi \) between the direct and rescattering electrons from the PEMDs. In Fig. 2(a), we present the extracted interference term \( \cos(\Delta \varphi) \) as a function of transverse momentum at \( p_x = 1.9 \) a.u. It is shown that for \( \Phi = 0.5\pi \) the line shifts toward left and for \( \Phi = 1.5\pi \) it shifts toward right, as compared to the single-color field. To reveal this shift quantitatively, we seek the position of the first minimum of the interference term at \( p_y > 0 \) in the OTC laser fields and calculate its shift \( \Delta p_y \) relative to the position in the single-color field. In Fig. 2(b), we present \( \Delta p_x \) as a function of relative phase. It is shown that \( \Delta p_y \) oscillates with relative phase. In Fig. 2(b), we have traced \( \Delta p_x \) at different parallel momenta \( p_x \). It is interesting that the behaviors of \( \Delta p_y \) depend on the parallel momentum \( p_x \). For example, \( \Delta p_y \) maximizes at \( \Phi = 1.5\pi \) for \( p_x = 1.9 \) a.u. and it maximizes at \( \Phi = 2.0\pi \) for \( p_x = 1.0 \) a.u. Additionally, the amplitudes of oscillation of these lines also depend on the parallel momentum \( p_x \). These behaviors are more clearly seen in Fig. 2(c), where we display \( \Delta p_y \) as a function of relative phase \( \Phi \) for parallel momentum \( p_x \) ranging from 0.75 a.u. to 1.95 a.u. The solid black line marks the
maximum of $\Delta p_y$. It is obvious that the corresponding relative phase where $\Delta p_y$ maximizes changes gradually with parallel momentum.

We also calculate the oscillation $\Delta p_y$ of the position of the first minimum with Eq. (8) as shown in Fig. 2(d). Equation (8) is based on the strong-field approximation. It should be mentioned that Eq. (8) does not take into account the phase induced by the interaction between the parent ion and the rescattering electron (i.e., the phase of the scattering amplitude [56]), and thus its prediction of the holographic interference fringes deviates from the TDSE calculation and experimental data [66]. In order to correctly describe the holographic interferences, we should add a phase accounting for this interaction to Eq. (8) [56]. In our calculations, the orthogonal component is a weak perturbation, and thus this phase in the OTC laser fields is the same as that of the single-color field. Therefore, when considering the shift of the fringes induced by the OTC laser fields with respect to the single-color laser field, this phase does not matter. Thus, we can expect that the shift of the fringes can be accurately dealt with Eq. (8). This is confirmed by comparing Figs. 2(c) and 2(d), where the excellent agreement indicates that Eq. (8) is accurate in describing the shift of the fringes induced by the weak perturbation.

The periodic oscillation in Figs. 2(b)-2(d) introduces us to fit the relative phase dependence of $\Delta p_y$ with

$$\Delta p_y = P_m \cos(\Phi - \Phi_m),$$

(14)

where the quantity $P_m$ describes the amplitude of this oscillation and the quantity $\Phi_m$ shows at which phase the shift $\Delta p_y$ maximizes. Technically, $P_m$ and $\Phi_m$ are determined by Fourier transforming $\Delta p_y$ with respect to $\Phi$ for each $p_x$ [67,68]. In Figs. 3(a) and 3(b), we display the obtained $P_m$ and $\Phi_m$ respectively as a function of $p_x$ for the data from TDSE results in Fig. 2(c). It is shown that the amplitude $P_m$ minimizes at $p_x = 1.3$ a.u. The obtained $\Phi_m$ depends on parallel momentum and varies from 2.0$\pi$ to 1.4$\pi$ when $p_x$ changes from 0.75 a.u. to 1.95 a.u. For comparison, the results extracted from Fig. 2(d) are also shown in Fig. 3. Both quantities agree with the TDSE results excellently. In the following, we will focus on the amplitude $P_m$, which enables us to determine the laser intensity accurately.

Given the agreement in Fig. 3, we reveal the origin of the minimum in $P_m$ using Eq. (8). For the near-forward rescattering electrons, we have $p_x \approx k_x$ through Eq. (11). Then it can be easily proved that the second, forth and fifth term in the right side of Eq. (8) can approximately cancel each other. In fact, this point has been confirmed by our numerical calculations (not shown here). Then the first and third terms in Eq. (8) are the dominant terms accounting for the phase difference [45]. With this in mind, the phase difference between the direct and rescattering
electrons is simplified as

$$\Delta \varphi \approx \frac{1}{2} \int_{t_i}^{t_f} \left\{ [p_y + A_y(t)]^2 - [k_y + A_y(t)]^2 \right\} dt. \quad (15)$$

Here, the superscript in $t_i$ has been omitted because $t_i' = t_i$. From Eq. (13), we have $k_y = -\int_{t_{r_i}}^{t_{r_f}} A_y(t) dt / (t_r - t_i)$. Obviously, $k_y = 0$ for the single-color field. From Eq. (15), we can obtain the transverse momentum $p_y$ of the first minimum in the single-color and OTC fields. Then the shift of the holographic interference fringes in the OTC fields with respect to that of the single-color is

$$\Delta p_y = p_y^{OTC} - p_y^S = -\frac{1}{2\omega} \int_{t_i}^{t_f} A_y(t) dt = -\frac{F_2}{2\omega} \int_{t_i}^{t_f} \sin(2\omega t + \Phi) dt.$$ \quad (16)

Here, $p_y^{OTC}$ and $p_y^S$ are the momenta of the first minimum of the holographic fringes in the OTC fields and the single-color fundamental field, respectively. Equation (16) shows that the shift of the holographic interference fringes in the OTC fields with respect to the fundamental field equals to $k_y$ ($k_y = -\frac{1}{2\omega} \int_{t_{r_i}}^{t_{r_f}} A_y(t) dt / (t_r - t_i)$). This indicates that $\Delta p_y$ depends on the relative phase, and also on the parallel momentum through $t_i$ and $t_r$.

![Figure 4](image_url)

**Fig. 4.** (a) $\Delta p_y$ calculated by Eq. (16) as a function of relative phase for $p_x$ ranging from 0.75 a.u. to 1.95 a.u. The laser intensity is $1.5I_0$ ($I_0 = 1.0 \times 10^{14}$ W/cm$^2$). The black solid line indicates the maximum of $\Delta p_y$. (b) The amplitude $P_m$ of the oscillating $\Delta p_y$ as a function of $p_x$ at laser intensities $I = 0.75I_0$ (the blue lines), $1.0I_0$ (the green lines), $1.5I_0$ (the yellow lines) and $2.0I_0$ (the red lines), respectively. The solid lines represent the TDSE results and dashed lines stand for the results calculated by Eq. (16). (c) Left axis (the blue solid line): $P_m$ as a function of traveling time $\Delta t$. Here $T_1$ is the period of the fundamental field. Right axis (the green solid line): the parallel momentum $p_x$ as a function of the traveling time $\Delta t$. $A_1$ is the amplitude of the vector potential of the fundamental field. The laser intensity is $1.5I_0$. The black dashed lines stand for the position where the amplitude $P_m$ minimizes. (d) The same as (b) but with $p_x$ scaled by $A_1$.

In Fig. 4(a), we show $\Delta p_y$ calculated with Eq. (16) as a function of relative phase $\Phi$ for each $p_x$. This results are consistent with that shown in Fig. 2(c), confirming the validation of the
The ionization and rescattering time vary with laser intensity. Therefore, if one aims to determine the laser intensity more accurately, it is necessary to establish the map where the minimum of $P_m$ locates as a function of laser intensity. In Fig. 5(a), we show the traveling time at different $p_x$ as a function of the laser intensity of fundamental field. The solid line indicates the traveling time of the rescattering electrons is half cycle of the fundamental field. In Fig. 4(c), we plot $P_m$ as a function of traveling time. Clearly, the minimum locates at traveling time $\Delta t = 0.5T_1$. Since the traveling time is an injective function of the parallel momentum for the near-forward rescattering electron [the green solid line in Fig. 4(c)], one can obtain that the corresponding parallel momentum for $\Delta t = 0.5T_1$ is $0.6A_1$ ($A_1$ is the amplitude of the vector potential of the fundamental driving field). Thus, by seeking the position $p_x$ where the amplitude $P_m$ minimizes, $A_1$ of the driving field can be determined. Then the laser intensity is measured. It should be noted that the minimum of $P_m$ is not zero. This is due to the fact that the ionization time in Eqs. (9)-(11) is a complex number. In Fig. 4(d), we display the amplitude $P_m$ for different laser intensities. The data here are the same as those presented in Fig. 4(b) but with $p_x$ scaled by the vector potential $A_1$ of the fundamental driving field. It is shown that the position of minimum always locates around $0.6A_1$. It means that the laser intensity can be determined with the relation $p_x = 0.6A_1$. Thus, it provides a feasible way to measure the laser intensity.

Closer inspection of Fig. 4(d) shows that the position of minimum changes a little with laser intensity. This results from the fact that the ionization and rescattering time vary with laser intensity, as displayed by Eqs. (9)-(11). The corresponding traveling time changes and thus the position of the minimum in $P_m$ varies with laser intensity. Therefore, if one aims to determine the laser intensity more accurately, it is necessary to establish the map where the minimum of $P_m$ locates as a function of laser intensity. In Fig. 5(a), we show the traveling time at different $p_x$ as a function of the laser intensity of fundamental field. The solid line indicates the traveling time of $0.5T_1$, i.e, the position where the minimum of $P_m$ should be located. With this map, the intensity of the driving field can be accurately determined. As a comparison, we show the obtained minimum position of $P_m$ from the TDSE calculations at different laser intensities. The results are shown in Fig. 5(b). They are well located on the solid line predicted by Fig. 5(a). It confirms that the laser intensity can be preciously determined by seeking the minimum of $P_m$ and comparing it with the solid line in Fig. 5(a).

As demonstrated above, to determine the position where $P_m$ minimizes, one needs to scan the...
relative phases of the OTC fields to obtain the PEMDs. Experimentally, this is a very tedious task. In the following, we demonstrate that the position where $P_m$ minimizes can be determined by just analyzing the PEMDs at two relative phases separated by $0.5\pi$. According to Eq. (14), the shift $\Delta p_y$ at $p_x$ for the relative phases $\Phi$ and $\Phi + 0.5\pi$ have the relation:

$$\Delta p_y(\Phi; p_x) + \Delta p_y(\Phi + 0.5\pi; p_x)^2 = P_m^2(p_x).$$

Thus, by extracting the shift of holographic structure at two relative phases, the amplitude $P_m$ of the oscillating $\Delta p_y$ can be determined. In Fig. 6(a) we show the the shift $\Delta p_y$ calculated by Eq. (16) for $\Phi = 0.1\pi$ and $\Phi = 0.6\pi$. The amplitude $P_m$ obtained through Eq. (17) is shown by the open circles. There is a clear minimum at $p_x = 1.3$ a.u. In Fig. 6(b), the data $P_m$ at another pair of relative phases $\Phi = 1.3\pi$ and $\Phi = 1.8\pi$ are shown. The obtained $P_m$ is the same as those in Fig. 6(a). The minimum also locates at $p_x = 1.3$ a.u. Thus, by analyzing the holographic pattern for an arbitrary pair of relative phases $\Phi$ and $\Phi + 0.5\pi$, the minimum position of $P_m$ can be determined. Then, the laser intensity can be measured. Figure 6(c) displays the determined laser intensities through this two-relative-phase measurement. The data are well located around the accurate value shown by the solid line, confirming the validity and accuracy of our method.

Our scheme of determining the laser intensity is based on seeking the minimum of $P_m$. This minimum is still observable when the laser focal volume effect is taken into account, as shown in Fig. 7 where a Gaussian beam profile is considered. Thus our method can be used in real experiment. Note that in our scheme, the measured laser intensity is not the peak intensity of the laser pulse, but the effective intensity which is most relevant for the strong-field phenomena. In Fig. 7, the peak intensity we used is $2.25 \times 10^{14}$ W/cm$^2$. From the minimum of $P_m$, we can obtained that the effective laser intensity for the ionization signal is $1.13 \times 10^{14}$ W/cm$^2$. In the results above, a pulse shape with flat top is used in our calculation. For the realistic Gaussian-shape pulses (not shown here), the minimum in $P_m$ is also clear, and thus our scheme of determining
The amplitude $P_m$ as a function of the parallel momentum $p_x$. Here laser focal volume effect has been taken into account by considering a Gaussian beam profile. The peak laser intensity is $2.25 \times 10^{14}$ W/cm$^2$. The vertical dashed line indicates the minimum of $P_m$.

### 4. Conclusion

In summary, we have investigated the holographic pattern in strong-field tunneling ionization in OTC fields. Our results show that the positions of the interference minima oscillate with the relative phase. The amplitude of this oscillation depends on the parallel momentum. With the quantum-orbit analysis based on saddle-point approximation, we showed that the amplitude minimizes when the traveling time of the rescattering electron is half cycle of the fundamental driving field. This predicates that the minimum for oscillation amplitude locates at the parallel momentum of $0.6A_1$, which is confirmed by our TDSE calculations. Therefore, scanning the relative phases of OTC fields and seeking the minimum of the oscillation amplitude of the interference pattern, the vector potential and thus the intensity of the fundamental driving field can be accurately determined. Moreover, we demonstrated that the amplitude of the oscillations as a function of parallel momentum can be determined by just analyzing the PEMDs at two relative phases separated by $0.5\pi$. Thus the laser intensity can be measured by just monitoring the PEMDs at two relative phases.

### Funding

National Natural Science Foundation of China (11622431, 61475055, 11604108, 11627809); Program for HUST Academic Frontier Youth Team.

### Acknowledgments

Numerical simulations presented in this paper were carried out using the High Performance Computing Center experimental testbed in SCTS/CGCL (see http://grid.hust.edu.cn/hpcc).