Interaction of equivalence ratio fluctuations and flow fluctuations in acoustically forced swirl flames

Vincent Kather¹, Finn Lückoff¹, Christian O. Paschereit² and Kilian Oberleithner¹

Abstract
The generation and turbulent transport of temporal equivalence ratio fluctuations in a swirl combustor are experimentally investigated and compared to a one-dimensional transport model. These fluctuations are generated by acoustic perturbations at the fuel injector and play a crucial role in the feedback loop leading to thermoacoustic instabilities. The focus of this investigation lies on the interplay between fuel fluctuations and coherent vortical structures that are both affected by the acoustic forcing. To this end, optical diagnostics are applied inside the mixing duct and in the combustion chamber, housing a turbulent swirl flame. The flame was acoustically perturbed to obtain phase-averaged spatially resolved flow and equivalence ratio fluctuations, which allow the determination of flux-based local and global mixing transfer functions. Measurements show that the mode-conversion model that predicts the generation of equivalence ratio fluctuations at the injector holds for linear acoustic forcing amplitudes, but it fails for non-linear amplitudes. The global (radially integrated) transport of fuel fluctuations from the injector to the flame is reasonably well approximated by a one-dimensional transport model with an effective diffusivity that accounts for turbulent diffusion and dispersion. This approach however, fails to recover critical details of the mixing transfer function, which is caused by non-local interaction of flow and fuel fluctuations. This effect becomes even more pronounced for non-linear forcing amplitudes where strong coherent fluctuations induce a non-trivial frequency dependence of the mixing process. The mechanisms resolved in this study suggest that non-local interference of fuel fluctuations and coherent flow fluctuations is significant for the transport of global equivalence ratio fluctuations at linear acoustic amplitudes and crucial for non-linear amplitudes. To improve future predictions and facilitate a satisfactory modelling, a non-local, two-dimensional approach is necessary.

Keywords
Turbulent swirl flames, thermoacoustics, equivalence ratio fluctuations, turbulent transport

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Introduction
To fulfill emission regulations for nitrogen oxides (NOx) and other pollutants future gas turbines are envisioned to operate at lean premixed combustion. The mixture of high amounts of air with fuel upstream of the flame results in a lean uniform fuel-air mixture, yielding a reduction of NOx formation due to fewer local high temperature zones in comparison to fuel-rich diffusion flames.

However, lean premixed combustion comes with a technical challenge: its high susceptibility to thermoacoustic instabilities. These self-excited instabilities occur when a positive feedback cycle between unsteady heat release rate and pressure fluctuations is established. The resulting pressure oscillations can develop high limit-cycle amplitudes. State of the art gas

¹Laboratory for Flow Instabilities and Dynamics, Technische Universität Berlin, Berlin, Germany
²Chair of Fluid Dynamics, Technische Universität Berlin, Berlin, Germany
turbines produce static pressures of 10–50 bars within their combustion chambers. Oscillation amplitudes may reach 1–2% of the static pressure which cause severe structural damage and higher emissions and a far lower efficiency. Overall, the danger of thermoacoustic instabilities substantially limits the operational range of premixed combustion engines.

To comprehend the mechanisms underlying thermoacoustic instabilities, the positive feedback loop between pressure and heat release rate needs to be understood. If these two oscillations are in phase, energy is transferred from the flame to the acoustic field; if the amount of added energy exceeds the acoustic energy dissipation, the system becomes unstable. This self-excitation is largely determined by the flame response to acoustic perturbations which is driven by different coupling mechanisms. Kinematic effects like wrinkling and the movement of the flame surface are reasons for acoustically induced oscillations of the heat release rate especially in laminar flames. For turbulent propagation stabilized flames, the major excitation mechanisms originate from velocity perturbations and equivalence ratio fluctuations, the latter of which can only exist under the presence of mixture inhomogeneities. For auto-ignition stabilized flames, which are not considered in this work, other mechanisms come into place.

Perfectly premixed flames have a homogeneous mixture of air and fuel and thus do not trigger equivalence ratio fluctuations. At these conditions, thermoacoustic instabilities are driven by coherent velocity fluctuations that induce global heat release rate fluctuations in the flame. These velocity fluctuations are a result of acoustic velocity fluctuations that excite coherent structures in the shear layers and swirl fluctuations at the swirler. However, in industrial gas turbine combustors air and fuel are usually not perfectly premixed before reaching the flame, and the mixture fraction is therefore not constant. A homogeneous unburned mixture would require a relatively long mixing distance, which cannot be realized for safety reasons due to possible flashback or auto-ignition. Additionally, flames with a certain degree of mixture fraction inhomogeneity feature a broader operational range. This type of flame is usually referred to as partially premixed or, in the application-oriented literature, as technically premixed.

In case of partially premixed flames, equivalence ratio fluctuations play an important role in triggering thermoacoustic instabilities. Equivalence ratio fluctuations originate from pressure and velocity oscillations at the location of the fuel injection. These perturbations are transported to the flame, subjected to diffusion and dispersion, and may generate large oscillations in the global heat release rate. This is in particular the case for lean combustion, where flame properties such as the burning velocity are strongly susceptible to perturbations in the equivalence ratio. Moreover, strong temporal variations in the mixture fraction may lead to a dynamical displacement of the flame anchoring position, which then results in the generation of unsteady heat release rate.

To understand and model the thermoacoustic stability of turbulent partially premixed flames, the simultaneous measurements of velocity and equivalence ratio perturbations are required. In a number of studies, temporal equivalence ratio fluctuations during unstable combustion were measured using different laser absorption techniques, such as Direct Absorption Spectroscopy (DAS) and Tunable Diode Laser Absorption Spectroscopy (TDLAS). The results confirmed that temporal equivalence ratio fluctuations are directly linked to heat release rate fluctuations. In another experimental study, the response of the flame heat release rate to perturbations of equivalence ratios was investigated. The fuel flow was modulated using a siren. Simultaneous measurements of the fuel mass fraction at the injector using infrared absorption spectroscopy and of the flame response using CH* chemiluminescence were applied. The study confirmed that equivalence ratio fluctuations are convected to the flame at the bulk flow velocity. In a follow-up study, the same group investigated the response of a partially premixed flame to both velocity and equivalence ratio fluctuations based on chemiluminescence measurements. They proposed a linear reconstruction technique to separate both effects, but were not able to validate their approach since direct measurements of heat release rate fluctuations in a partially premixed flame were not available. Hermeth et al. conducted LES of a partially and a fully premixed flame showing that even for spatially well-premixed flames, temporal equivalence ratio fluctuations may not even out sufficiently and may dominate the flame response to acoustic forcing. In another study, it was observed that, depending on the phase lag between velocity and equivalence ratio fluctuations, non-linear effects such as a saturation of the flame response at high amplitudes may occur. This is a determining factor for the limit cycle pressure amplitude. Stoehr et al. experimentally studied the combined effects of velocity and equivalence ratio perturbations on thermoacoustic flame oscillations. They found that a periodic variation of the reactant mass flux rates and heat release in the flame indicated an additional convective delay, which caused local flame extinction. The results showed that the interaction of velocity and equivalence ratio fluctuations has a significant impact on the thermoacoustic response of turbulent swirl flames. This interaction depends mainly on the time delay between the two.
In a recent study, Blümner et al. conducted TDLAS and PIV measurements in the mixing section of a swirl stabilized combustor with the focus on the transport and mixing of temporal equivalence ratio fluctuations generated by linear acoustic perturbations. They measured a non-trivial frequency dependence of the mixing transfer functions, which they related to the coherent production of equivalence ratio fluctuations due to the excitation of coherent structures. This demonstrates the importance of the interaction of flow and equivalence ratio fluctuations even for linear acoustic forcing amplitudes, which is crucial for the prediction of flame stability.

In terms of thermoacoustic modeling, a key challenge is the prediction of the equivalence ratio fluctuations that are generated at the fuel injector and the modeling of their convection and diffusion in the mixing section between the injector and the flame. Regarding the generation of equivalence ratio fluctuations, Peracchio and Proscia introduced a time-domain model that assumes instant mixing of fuel and air and stiff fuel injection. It describes a mode-conversion process where the generated equivalence ratio fluctuation amplitudes are equal to the air fluctuations at the injector, with a phase difference of half a period. Regarding the transport of equivalence ratio fluctuations Sattelmeyer et al. proposed a model to account for dispersive effects by using convective delay times approximated from the mean velocity field in the mixing section. A similar approach was proposed by Xia et al. and Giusti et al. to model the dispersion of entropy waves based on a convection equation, neglecting the impact of diffusion. The authors argue that dispersion due to the non-uniform flow outweighs the impact of turbulent diffusion. A different approach is pursued by Wassmer et al. who derive a model from the 1D convection-diffusion equation. They show a good match with measured mixing transfer functions, however they noted that potential dispersive effects may have been implicitly accounted for by a higher diffusivity. With the attempt to model the simultaneous effect of mean shear dispersion and turbulent diffusion, Kaiser et al. developed a framework based on the two-dimensional linearized transport equation equipped with an eddy viscosity model. Comparison with DNS data of the mixing process in a turbulent channel flow reveal excellent agreement. The model could further qualitatively replicate the non-trivial frequency dependence of the linear mixing transfer function measured by Blümner et al.

This work builds on the recent analytic and experimental findings and tends to reveal the complex interplay between the equivalence ratio fluctuations and hydrodynamic flow structures within a turbulent swirl combustor by means of quantitative measurements. Particular emphasis is placed on the role of vortical structures in the convection, dispersion and diffusion of equivalence ratio fluctuations at linear and non-linear acoustic forcing amplitudes. These processes are quantified through local and global mixing transfer functions that are determined by combining PIV, CO₂* and CH⁺ chemiluminescence and TDLAS. The goal of this work is two-fold, it provides data for future modeling of equivalence ratio fluctuation transport, revealing the two-dimensional structures involved in the mixing process as well as the mixing transfer functions, and this study extends the previously mentioned works in the sense that it is based on a highly turbulent swirl flame rather than simplified canonical flow cases.

In the following sections, the experimental setup, data acquisition for all of the measurements, and the methodology for post processing will be explained. Thereafter, a brief summary of current models for the generation and transport of equivalence ratio fluctuations is given that allow for a comparison of the empirical findings. The results will be analyzed with respect to the baseline configuration, linear transfer function response and non-linear mixing transfer functions.

**Experimental methods**

**Combustor test rig**

Measurements were conducted in an atmospheric combustor test rig, which is shown in Figure 1. The combustor was operated with natural gas. Four 18-inch woofers (B&C 18PS76, with a maximum electrical power of 600 W per speaker) with a variable upstream impedance of the combustor were used to realize high forcing amplitudes. To realize the latter, the length of the tube between the speakers and the burner inlet was controlled by a computer traversing system. This mechanism allows for the forcing of the woofers to be used in resonance over the desired frequency range. The facility can be operated both at perfectly premixed mode, where fuel is injected far upstream into the upstream plenum, and in partially (technically) premixed mode where fuel is injected into the swirler.

**Gas turbine model combustor**

The generic swirl combustor used in this measurement series generates swirl by injecting fluid tangentially into a mixing tube that terminates at the combustion chamber inlet. A technical drawing of the swirler is given in Figure 2. The burner was initially designed for hydrogen application. To avoid flashback, a central orifice was constructed, allowing for a jet that passes through the center, which can be adjusted by mounting different insets into the central opening of the swirler.
The combustor was operated at partially premixed mode, where the fuel was injected through 16 circumferential, equally spaced holes (diameter $d = 1.6\text{ mm}$) between the swirling flow and the axial jet. A 40 nm gas filter was installed in the fuel line to create a sufficiently high pressure loss to decouple the majority of the fuel supply from acoustic fluctuations. The filter was installed approximately 80 cm upstream of the fuel injector in the supply line, thereby leaving a compressible volume between the filter and the fuel injector, as the fuel injector itself was not choked and operated at a nominal Mach number of 0.12. As will be shown in this study, this arrangement is sufficient to achieve a stiff fuel injection. For all experiments considered in this work, the swirler was operated with a central orifice opening of
8.8 mm. The two fluids are united in the mixing tube, ranging from the swirler to the combustion chamber inlet with a diameter of $D = 34$ mm and a length of $3D$. The area jump to the combustion chamber with a diameter of $D = 200$ mm establishes a confined swirling jet in the combustion chamber. Since the swirl number in that jet is sufficiently large, a hydrodynamic phenomenon known as vortex breakdown occurs. As a result, a central recirculation zone (CRZ) is generated which induces an inner shear layer between the jet and the CRZ. Furthermore, an outer recirculation zone is induced due to the area jump, which leads to a stabilization of the flame in the resulting shear layers (see streamlines in Figure 2). The combustion chamber, with a length of $L = 300$ mm and a diameter of $D = 200$ mm, is made from silica glass in order to provide optical access for chemiluminescence and laser-based measurements. The water-cooled exhaust tube is 700 mm long and was equipped with an orifice at the downstream end. With this orifice the combustor is thermoacoustically stable over a wide operational range as the orifice increases the acoustic dissipation of the chamber mode, which has a velocity anti-node at this location.

**Multiple microphone method (MMM)**

To reconstruct the plane wave acoustic velocities upstream and downstream of the combustor, the multi-microphone-method was applied. The method is implemented using two microphone arrays (five water-cooled microphones each), located on either side of the combustor. The MMM allows for the measurement of the acoustic flame transfer function as well as a quantification of the acoustic perturbation amplitude. The MMM measurement error was calculated to be well below 5% for all results, based on a comparison of the reconstructed acoustic field with the measured data.

**Spatially resolved equivalence ratio measurements**

Over the last decades various studies have shown that there exists a fuel specific correlation between the equivalence ratio and different ratios of chemiluminescence emissions of species such as: $C_2^*/CH_*/OH*/CH_*/CO_2$. The chemiluminescence intensity of a single species is linearly dependent on the mass flow of the fuel-air mixture and depends exponentially on the equivalence ratio. Therefore, a ratio of two chemiluminescence emission signals is a function of the equivalence ratio only, which can serve as an accurate measure of the local equivalence ratio in premixed swirl flames. In many studies, the ratio of $CH^*$ and $OH^*$ is reported to be an accurate measure for the equivalence ratio. However, for quantitative measurements it requires laborious corrections for background $CO_2^*$ chemiluminescence measurements.

In this study, $CO_2^*$ and $CH^*$ chemiluminescence were measured to estimate the equivalence ratio fluctuations in the flame, exploiting the relationship $CH_*/CO_2 \propto \phi$ as shown by Bobush et al. The spatially resolved chemiluminescence data of both species was obtained with a single camera. This is possible because the focal length is almost identical for the respective filtered wavelengths, 407 nm ($CO_2^*$) and 431 nm ($CH^*$). These wavelengths are in the visible range, which does not require UV lenses to be used in the present experiments. Moreover, $CO_2^*$ and $CH^*$ chemiluminescence are independent of local strain rate, which makes these species applicable for the spatial equivalence ratio measurements in the present study.

Note that the technique of determining the local equivalence ratio indirectly from the chemiluminescence signal comes with an uncertainty of the spatial location of the measurement. This is because the chemiluminescence signal is emitted in the flame zone where the fuel is already burned while it correlates with the mixture fraction of the fresh unburned fuel upstream of the flame zone. Hence, this spatial uncertainty scales with the instantaneous flame thickness and the spatial gradient of the local equivalence ratio at the scale of the flame thickness. As we consider rather thin flames at high Reynolds numbers we expect this effect to be negligible, particularly within the context of large-scale (phase-averaged) equivalence ratio fluctuations which are the focus of this study.

The measurement of the spatial distribution of the two different chemiluminescent species was realized using an intensified Photron Fastcam SA 1.1 high-speed camera (1 Mpixel at 2 kHz double frame), which was installed perpendicular to the flow direction, facing the outlet of the mixing tube, which represents the inlet to the silica glass combustion chamber. The camera was equipped with a stereoscope (LaVision Image Doubler), which allows for the projection of an image pair onto a single camera chip. These two images capture the same field of view inside the combustion chamber, which can be seen in Figure 2. For each experimental configuration, a set of 2000 images was recorded at a rate of 2 kHz. Only for the measurement at an excitation frequency of 97 Hz, 2000 images were recorded at a rate of 1 kHz. Two optical bandpass filters were set to detect different chemiluminescent species: $CH^*$ at 431 nm (standard bandpass filter, center wavelength: 430 nm, FWHM: 10 nm) and $CO_2^*$ at 407 nm (standard bandpass filter, center wavelength: 405 nm, FWHM: 10 nm).
The chemiluminescence measurements were calibrated by conducting seven measurements with different $\phi_{set}$ values and recording calibration images. Figure 3 shows the measurements and the linear regression for the empirical relationship between the intensities of the CH$^+$/CO$_2$ chemiluminescence measurements and the set $\phi_{set}$ values. The intensities were derived from integrating the corresponding signals within the flame area. Measurements were only conducted with one air mass flow, as previous research by Bobusch et al.\textsuperscript{48} has shown the independence of the CH$^+$/CO$_2$ ratio to $\phi$ values versus the mass flow. The calibration procedure in combination with the deconvolution process allows the local mean and phase-averaged equivalence ratio to be determined within the flame area.

**Particle image velocimetry**

The two components of the velocity field in the streamwise plane aligned with the combustor axis were measured using high-speed particle image velocimetry (PIV). The PIV system consists of a Photron Fastcam SA 1.1 high-speed camera (1 Mpixel at 2 kHz double frame) and a pulsed Nd:YLF diode pumped laser (Quantronix (Hamden, CT) Darwin Duo 527–100M, 527 nm wavelength and total pulse energy of 60 mJ). Analog to the CH$^+$/CO$_2$-chemiluminescence measurements mentioned above, a set of 2000 double-frame-images was recorded at a rate of 2 kHz and a time delay of 20 $\mu$s between the first and second PIV frame, for each configuration. Only for the measurement of an excitation frequency of 97 Hz, 2000 images were recorded at a rate of 1 kHz and a time delay of 40$\mu$s. The relatively short time delay was required to reduce the particle loss due to the strong out-of-plane component due to the high degree of swirl. A light sheet optic was used to generate an appropriate light sheet of approximately 1 mm thickness. This light sheet illuminated the heat resistant solid titanium dioxide (TiO$_2$) seeding particles of a nominal diameter of 2 $\mu$m in the measurement area. The particles were introduced to the flow far upstream of the burner by means of a brush-based seeding generator. The acquired particle snapshots were post-processed with a commercial PIV software (PIVTEC GmbH, PIVview) by applying a correlation scheme with multigrid refinement\textsuperscript{53}. The final window size was set to 16 $\times$ 16 pixels with an overlap of 50% in combination with spline-based image deformation\textsuperscript{54} and subpixel peak fitting. Finally, the calculated velocity fields were filtered for outliers and interpolated from adjacent interrogation windows.

**Quantitative light sheet method**

The PIV particle snapshots were furthermore used to estimate the mean and phase-averaged density fluctuations in the combustion chamber by means of quantitative light sheet method\textsuperscript{55–58}. The light intensity $I$, scattered by the particles, is estimated by subtracting a background image $I_B$ without seeding from the recorded raw scattered light intensity $I_{rec}$. This image is normalized by a time-averaged reference image $I_{ref}$ of uniform particle distribution taken from the uniformly seeded isothermal combustor flow. The corresponding normalization of $I_{rec}$ corrects for inhomogeneities of the light sheet, such that

$$I(x, y, t) = \frac{I_{rec}(x, y, t) - I_B(x, y)}{I_{ref}(x, y) - I_B(x, y)}$$

(1)

Additional influences on the detected light intensity such as multiple scattering or light extinction\textsuperscript{57} are neglected within the scope of this study.

Based on equation (1), the particle concentration $C(x, y, t)$ can be derived by employing the reference uniform particle concentration $C_{ref}$ according to

$$\frac{C(x, y, t)}{C_{ref}} = I(x, y, t)$$

(2)

By means of QLS an exact quantification of the number of particles is impossible. In general, the QLS technique only provides a relative information to a known quantity. Therefore, $C_{ref}$ is set to unity. Since the particle concentration depends on the fluid density $\rho$, it is used as a density measure within this study.

![Figure 3. Measured calibration curve for mean equivalence ratio over intensity ratio (symbols), linear fit (line).](image-url)
Under the assumption of a linear dependency, the relation
\[
\bar{p}(x, y) = \frac{C(x, y)}{C_{\text{ref}}} \rho_{\text{ref}}
\]
(3)
is derived, which includes the uniform density \( \rho_{\text{ref}} \) of the isothermal reference image\(^58\).

**OH\(^*\)-chemiluminescence**

For flame diagnostics, more specifically to visualize heat release rate fluctuations of the flame, the chemiluminescence of the OH\(^*\) radicals was simultaneously recorded (to the PIV) with a second Photon Fastcam SA 1.1 (1 Mpixel at 2 kHz single frame). The camera was equipped with an image intensifier and an optical bandpass filter that restricts the observed wavelengths to \( \lambda = 308 \) nm. The planar representation of the flame shape was reconstructed from the time-averaged OH\(^*\) signal using an Abel-deconvolution, assuming a perfectly symmetric flame shape with respect to the combustion chamber centerline.

**Tunable diode laser absorption spectroscopy**

The methane concentration and equivalence ratio inside the mixing tube was measured by Tunable Diode Laser Absorption Spectroscopy (TDLAS) making use of the wavelength modulation spectroscopy approach. Details to the present TDLAS arrangement can be found in Blümner et al.\(^33\). A fiber-coupled near-infrared tunable diode laser (Eblana Photonics EP1654-DM-DX1-FM) at a wavelength close to 1653 nm was modulated through a sinusoidal injection current at a frequency of 10 kHz. The laser light was guided through the mixing tube between two fiber-coupled collimators (Thorlabs TC12APC-1550). Freespace coupling was used, which is uncritical, since the selected wavelength is free from interference of other absorbing species. The laser beam was set to cross the mixing tube at a distance of 1.3D from the fuel injectors and a distance of 2 mm from the tube centerline (refer to Figure 2). The slight displacement of the beam from the tube centerline creates an incident angle of approximately 2 degrees to avoid total reflection-induced absorption errors within the glass mixing tube. The receiving collimator was fiber-coupled to a detector (Thorlabs PDA50B), which was connected to a lock-in amplifier (Signal Recovery 7270). It was used to recover the magnitude of the first harmonic 1f and second harmonic 2f of the transmitted laser signal simultaneously, at a lock-in time constant of 0.5 ms or a sampling rate of 2 kHz, respectively. The wavelength was current-tuned to the selected absorption feature by maximizing the 2f signal. It peaked at the center of the absorption line, which maximizes the signal-to-noise ratio. Finally, the laser signal was calibrated with respect to a set of perfectly premixed nominal equivalence ratios in a steady, unforced flow, measured at the mixing tube center line. The calibration curve showed a nearly linear trend up to a time-mean equivalence ratio of 1.5. Further details to the calibration procedure and measurement accuracy can be found in Blümner et al.\(^33\).

**Experiments and post-processing**

**Operating conditions and measurements**

Throughout this study, the combustor was operated at partially premixed conditions. Table 1 gives an overview of the considered operating conditions. For the considered experiments, the supplied air flow rate was 100 kg/h. This results in a Reynolds number of around \( \text{Re} = 60\,000 \) based on the kinematic viscosity of air with \( \nu = 14.45 \times 10^{-6} \text{m}^2/\text{s} \), the diameter of the combustor inlet with \( D = 34 \) mm, and the bulk flow velocity of \( u_0 = 25.5 \text{m/s} \) as derived from the mean air mass flow \( m_{\text{set}} \). The swirl number was set to \( S = 0.7 \). It is based on geometric quantities of the swirler and not on actual velocity measurements\(^59\). The equivalence ratio of the baseline case was set to \( \phi_{\text{set}} = 0.7 \). Note that the subscript "set" refers to the global set point of the fuel and air mass flow fed to the combustor which were metered by two coriolis mass flow meters at an uncertainty of ±1% each. The supplied air and fuel was not preheated. The combustor was operated at thermoacoustically stable conditions.

The measurement techniques mentioned above were applied starting with the MMM, followed by the PIV (and QLS) along with synchronized OH\(^*\)-chemiluminescence, followed by CH\(^+\)/CO\(_2^+\)-chemiluminescence along with synchronized TDLAS. All measurements were conducted for the acoustically unforced and forced conditions. Considering the linear system response, forcing was applied at acoustic velocity fluctuations of less than 10% of the bulk flow velocity within a frequency range of 50 to 500 Hz. For four selected frequencies (90, 202, 268, and 316 Hz) nonlinear forcing was applied with amplitudes exceeding

| Operating conditions for experiments. |
|--------------------------------------|
| Reynolds number \( \text{Re} \) [-] | \( \approx 60,000 \) |
| Air mass flow [kg/h] \( m_{\text{set}} \) [kg/h] | 100 |
| Swirl number \( \phi_{\text{set}} \) [-] | 0.7 |
| Equivalence ratio \( S \) [-] | 0.7 |
| Burner inlet temperature [K] \( T_{\text{in}} \) [K] | 293 |
| Exhaust temperature [K] \( T_{\text{ex}} \) [K] | \( \leq 1273 \) |
10% of the bulk flow velocity, which were achieved by exploiting the variable acoustic resonance frequencies of the combustor.

**Triple decomposition of experimental data**

Considering a turbulent flow subjected to acoustic harmonic forcing, the time-dependent signal (such as \( \phi \)) can be decomposed into three parts: the mean, the periodic and the turbulent stochastic part\(^{60} \)

\[ \phi = \overline{\phi} + \tilde{\phi} + \phi' \]  

(4)

The periodic part is obtained by subtracting the mean quantities from the phase-averaged quantities according to

\[ \tilde{\phi} = \langle \phi \rangle - \overline{\phi} \]  

(5)

where the phase-average of the acquired time signal is computed with respect to the acoustic forcing frequency \( f \), \textit{a posteriori} to the measurements.

The Fourier coefficients corresponding to the fundamental wave are calculated from the phase-averaged quantities as

\[ \hat{\phi} = \frac{1}{2\pi T} \int_0^T \langle \phi \rangle e^{-i2\pi f t} dt \]  

(6)

with \( T = 1/f \) representing the forcing period.

The line-of-sight-integrated chemiluminescence data was Abel-deconvoluted in post-processing in order to obtain a planar representation. This is only applicable to axisymmetric fields which applies to the mean and phase-averaged quantities, but not to the instantaneous snapshots.

**Quantification of global equivalence ratio fluctuations**

The equivalence ratio of a mixture is defined as the ratio of fuel (subscript f) and oxidizer (subscript o) divided by their stoichiometric fraction, which depends on the fuel and oxidizer used. For a mass-based equivalence ratio this is commonly expressed as

\[ \phi = \frac{Y_f}{Y_oZ_{st}} = \frac{Y_f}{(1 - Y_f)Z_{st}} = \frac{1 - Y_o}{Y_oZ_{st}} \]  

(7)

where \( Y_f \) and \( Y_o \) are the mass fractions of fuel and oxidizer in the mixture and \( Z_{st} \) is their stoichiometric mass fraction. The expression \( \phi \) describes the local equivalence ratio at a given spatial location. To attain the global equivalence ratio \( \phi_{gl} \), which we define as the local equivalence ratio integrated over the cross-sectional area \( A \), we use a flux-based formulation of the equivalence ratio\(^{33} \), with

\[ \phi_{gl} = \frac{\bar{m}_{f,gl}}{\bar{m}_{o,gl}Z_{st}} \]  

(8)

consisting of the global mass flow for fuel

\[ \bar{m}_{f,gl} = \int_A \int Y_f \rho \hat{\phi} u dA = \int_A \int \frac{Z_{st}\phi}{1 + Z_{st}\phi} \rho u dA \]  

(9)

where for the second equality, expression (7) was used. Analogously, the global mass flow for the oxidizer reads

\[ \bar{m}_{o,gl} = \int_A \int Y_o \rho u dA = \int_A \int \frac{1}{1 + Z_{st}\phi} \rho u dA \]  

(10)

For better readability we neglect the term \( Z_{st}\phi \) as it is small for methane and air at lean conditions (\( Z_{st} \approx 0.04 \) for an equivalence ratio of 0.7) and the expression for the global equivalence ratio simplifies to

\[ \phi_{gl} = \frac{\int \rho \phi u rdr}{\int \rho u rdr} \]  

(11)

with \( u \) as the instantaneous local streamwise flow velocity and \( \rho \) the local density of air.

To further attain the phase-averaged \( \phi \) values using the triple decomposition from equations (4) and (5), equation (11) converts to

\[ \langle \phi_{gl} \rangle = \frac{\int \langle \rho \phi u \rangle rdr}{\int \langle \rho u \rangle rdr} \]  

(12)

which can be simplified to

\[ \langle \phi_{gl} \rangle = \frac{\int \left( \langle \rho \rangle \langle \phi \rangle u + \overline{\rho \phi u} \right) rdr}{\int \left( \langle \rho \rangle u + \overline{\rho u} \right) rdr} \]  

(13)

under the assumption that the terms \( \overline{\rho' \phi' u'} \) and \( \overline{\rho' \phi u'} \) can be neglected. Both terms have minor effect on the fluctuating part of the phase average. A detailed discussion and justification of this assumption is given in Blümner et al.\(^{33} \). Note that expression (13) contains
only phase-averaged quantities of the velocity, density and equivalence ratio, which are all accessible from the current experiments.

Definitions of spatial mixing transfer functions

The goal of this study is to quantify the generation of fuel fluctuations at the injector and their convection and diffusion between the injector and the flame. To adequately assess this, several spatial mixing transfer functions can be considered, which will be defined in the following.

Figure 4 displays a schematic depiction of the quantities that are available for this purpose. Utilizing the MMM measurements, the acoustic velocity of the planar acoustic waves was reconstructed at the combustor inlet. It is denoted as \( \bar{u}_{ac} \), which corresponds to the complex Fourier coefficient of the acoustic signal with respect to the fundamental forcing frequency. Assuming that the combustor is acoustically compact, which is justified for the considered frequencies, \( \bar{u}_{ac} \) serves as the reference velocity at the fuel injector, at the TDLAS measurement location, and in the flame area.

Equivalence ratio fluctuations were measured inside the mixing duct using the TDLAS technique at a distance of \( x/D = 1.3 \) downstream of the fuel injector. The TDLAS data was Fourier-transformed with respect to the applied forcing frequency, which yields the complex coefficient \( \phi_{gl}(f) \). As indicated by the subscript, the TDLAS data is expected to be equivalent to the global equivalence ratio fluctuations as it is based on a line-of-sight-integrated quantity. This assumption was validated in a related study where the subscript, the TDLAS data is expected to be equivalent to the considered frequencies, \( \bar{u}_{ac} \) serves as the reference velocity at the fuel injector, at the TDLAS measurement location, and in the flame area.

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The Fourier-transform of this quantity yields \( \hat{\phi}_{gl}(x,f) \), where the subscript indicates that this quantity is global in the sense that it does not depend on \( r \) (but still on \( x \)).

From these quantities, several transfer functions can be built that quantify the generation and streamwise transport of equivalence ratio fluctuations. Starting at the most upstream location, we define the fuel injector transfer function as

\[
F_{inj}^{inj}(f) = \frac{\dot{\phi}_{gl}/\bar{\phi}_{gl}}{\bar{u}_{ac}/u_{b}} \bigg|_{x=0}
\]

which relates the global \( \phi \)-fluctuations generated at the injector to the (acoustic) velocity fluctuations normalized by the bulk flow velocity. As \( \phi \)-fluctuation measurements are not possible at the injector location, this transfer function cannot be determined directly. However, the TDLAS data allows to quantify this transfer function in the mixing duct yielding

\[
F_{inj}^{duct}(f) = \frac{\dot{\phi}_{gl}/\bar{\phi}_{gl}}{\bar{u}_{ac}/u_{b}} \bigg|_{x=1.3D}
\]

which allows for the definition of the mixing transfer function

\[
F_{\phi_{gl}}^{duct}(f) = \frac{\dot{\phi}_{gl} \big|_{x=1.3D}}{\bar{\phi}_{gl} \big|_{x=0}} = F_{\phi_{gl}}^{duct} / F_{inj}^{inj}
\]

The corresponding gain and phase allows to quantify the convection and diffusion of the \( \phi \)-fluctuations that are generated by the acoustic forcing. The corresponding transfer function for the local \( \phi \)-fluctuations measured in the flame area \( A_{\text{flame}} \) is given as

\[
F_{\phi_{gl}}^{frame}(x, r, f) = \frac{\dot{\phi}_{gl} \big|_{(x, r) \in A_{\text{flame}}}}{\bar{u}_{ac}/u_{b}}
\]

Figure 4. Schematic measurement setup, showing the individual measuring areas (mixing duct and combustion chamber), along with the axial measurement positions of the variables \( \bar{u}_{ac}, \phi_{gl} \), and \( \phi \).
and the corresponding local mixing transfer function reads

$$F^\text{flame}_{\phi} (x, r, f) = \frac{\Phi_{\phi|x \in A_{\text{flame}}}}{\Phi_{\phi}|x=0} = F^\text{flame}_{u} / F^\text{inj}_{u}$$

(18)

which relates the equivalence ratio fluctuations in the flame to those generated at the injector. Finally, an analogous expression can be defined for the global $\phi$-fluctuations reading

$$F^\text{flame}_{\phi gl} (x, f) = \frac{\hat{\Phi}_{\phi gl}|x \in A_{\text{flame}}}{\Phi_{\phi gl}|x=0} = F^\text{flame}_{u} / F^\text{inj}_{u}$$

(19)

where $F^\text{flame}_{u}$ in this case is the global version of equation (17), reading

$$F^\text{flame}_{u} (x, f) = \frac{\hat{\Phi}_{\phi gl}|x \in A_{\text{flame}}}{u_{ac}/u_{b}}$$

(20)

Analog to $F^\text{duct}_{\phi}$, expression $F^\text{flame}_{\phi gl}$ represents a spatial mixing transfer function which relates the global (cross-sectional area-integrated) $\phi$-fluctuations at streamwise locations inside the flame area to the one generated at the injector (at $x = 0$).

### Modeling generation and transport of equivalence ratio fluctuations

#### Generation of equivalence ratio fluctuations

To model the generation of fuel fluctuations in gas turbine burners, a choked acoustically stiff fuel injector is commonly assumed. This implies that the pressure fluctuations in the combustion chamber do not couple with the fuel injector and the injected fuel mass flow remains constant. This behaviour is typically achieved by having a very high pressure loss across the fuel injector, which is the case in this experiment. With the assumption of instant mixing of fuel and oxidizer at the injector, the generation of $\phi$-fluctuations may be described by a mode conversion process between velocity fluctuations at the injector (i.e. acoustic velocity fluctuations) and phase-opposite equivalence ratio fluctuations$^{34}$, reading in time domain as

$$\frac{\hat{\Phi}_{\phi gl}}{\Phi_{\phi gl}} = \frac{1}{1 + \frac{u_{ac}}{u_{b}}} - 1$$

(21)

In the limit of small velocity fluctuations ($\bar{u} \ll u_{b}$), the corresponding transfer function reads$^{38}$

$$F^\text{inj}_{u} = \frac{\hat{\Phi}_{\phi gl}/\Phi_{\phi gl}}{u_{ac}/u_{b}} = e^{i\pi}$$

(22)

with a constant phase of $\pi$ and a gain of unity. For higher velocity fluctuation amplitudes, the gain increases and reaches infinity for $\bar{u}_{ac} = u_{b}$. While this model appears reasonable for small amplitudes the high gain at large acoustic amplitudes is questionable.

#### Transport and mixing of equivalence ratio fluctuations

The main focus of this work lies within the quantification of the streamwise transport of equivalence ratio fluctuations generated by velocity perturbations. The equivalence ratio fluctuations are thereby regarded as a passive scalar field. In this case “passive” refers to the fact that the flow field is independent of the scalar field.

We model the streamwise transport of global $\phi$-fluctuations by a one-dimensional convection-diffusion process$^{38}$, reading

$$\frac{\partial \Phi_{\phi gl}}{\partial t} + u_{c} \frac{\partial \Phi_{\phi gl}}{\partial x} - \Gamma_{\text{eff}} \frac{\partial^2 \Phi_{\phi gl}}{\partial x^2} = 0$$

(23)

where $u_{c}$ represents a (radially homogeneous and constant) streamwise convection velocity and $\Gamma_{\text{eff}}$ represents an effective diffusivity.

The capability of a 1D transport model to account for the convection and diffusion within a 3D swirled flow, as considered in this study, has been demonstrated within the context of entropy waves and equivalence ratio fluctuations$^{33,36,48}$. In fact, as shown by Blümner et al.$^{61}$, the 1D model performs better for turbulent swirling flows than the quasi-2D model suggested by Giusti et al.$^{37}$ which accounts for mean shear dispersion but neglects turbulent diffusion. In the current 1D model both these effects are accounted implicitly by the effective turbulent diffusivity$^{62}$. As a drawback, an estimate for this diffusivity cannot be easily obtained and it must be derived from a fit of measured mixing transfer functions. Moreover, for both models, the transport velocity $u_{c}$ is assumed to be constant in time, and the periodic part $\bar{u}_{c}$ is small, which is not necessarily the case for acoustically forced flows, as it is shown in this work.

To align the model with the experimental results, it is convenient to explore the spatial mixing transfer function $F_{\phi}(f) = \hat{\Phi}_{\phi gl}(x_{2}, f)/\hat{\Phi}_{\phi gl}(x_{1}, f)$, which reveals
the response rates between two streamwise positions in the frequency domain. Using equation (23) this transfer function reads

$$F_{\phi}(St) = e^{1/2Pe(1 - \sqrt{1 + 2St^4/Pe})} \quad (24)$$

where the Peclet number Pe represents the ratio between the diffusive time scale $\tau_d = \Delta x^2 / \Gamma_{eff}$ and the convective time scale $\tau_c = \Delta x^2 / u_c$. The Strouhal number St is defined as $St = (f\Delta x)/u_c$ and can be interpreted as the ratio between the convective distance $\Delta x = x_2 - x_1$ and the convective wavelength $\lambda_c = u_c/f$ of the equivalence ratio fluctuation. If not stated differently throughout this study, $x_1$ corresponds to the position of the fuel injector while $x_2$ is specified individually.

**Characterization of baseline configuration**

**Mean flow and flame shape**

Figure 5 shows the time-mean of the flow and flame inside the combustor at the baseline operating conditions (without acoustic forcing). The streamlines indicate the typical characteristic of swirl-stabilized combustor flows. Vortex breakdown creates an inner recirculation zone with the upstream stagnation point located somewhat downstream of the combustor inlet. The swirling jet entering the combustor is guided around this recirculation zone towards the combustion chamber walls (see Figure 5(a)). An external recirculation zone is created, due to the rapid area jump from the mixing tube cross section to combustion chamber cross section. Two shear layers exist, one between the jet and the inner recirculation zone and one between the jet and the outer recirculation zone.

The mean flame shape is indicated by the (Abel-deconvoluted) OH$^+$-distribution shown in Figure 5(b). It features a V-shaped flame that stabilizes in the inner shear layer between the recirculation zone and the incoming annular swirling jet. Note that the flame is relatively compact with substantial heat release rate taking place at the root of the flame near the combustion chamber centerline.

The local $\phi$-distribution is shown in Figure 5(c). It shows high $\phi$ values of up to $\phi = 1$ in the center and low values of $\phi \approx 0.5$ in the periphery, which differ substantially from the target of $\phi_{set} = 0.7$. The very high $\phi$ distribution on the center axis of the flame can be accredited for by the special combustor design that has produced similar results in previous measurement campaigns as well as numerical analyses$^{33,41,42}$. This confirms the expectation that the flame is only partially premixed.

In order to validate the accuracy of the $\phi$ measurements based on the chemiluminescence, these quantities are compared with the values set in the experiments. Figure 6 shows the mean global equivalence ratio $\bar{\phi}_{gl}$ as...
a function of streamwise distance. It is derived from the experimental data according to equation

$$\overline{\Phi}_{gl} = \frac{\int \rho \overline{\Phi} r dr}{\int \rho \overline{u} r dr}$$

The figure further shows the radially integrated CH$^+$ intensity as a function of streamwise distance to visualize the intensity distribution of the chemiluminescence signal emitted by the flame. Accordingly, the measured mean equivalence ratio remains approximately constant in regions of high chemiluminescence intensity. Only at the upstream and downstream boundaries of the flame area, significant scatter of the data is observed. The constant value thereby matches very well with $\phi_{set} = 0.7$, which was set as operating conditions of the experiments.

**Flow field dynamics at linear and non-linear forcing**

Figures 7 and 8 show the results from synchronized PIV and chemiluminescence measurements for two different forcing frequencies. The contours in gray-scale show the finite-time Lyapunov exponent (FTLE), a scalar quantity that can be extracted from phase-averaged velocity fields as determined from PIV. It is a suitable way to visualize Lagrangian coherent structures$^{63}$. The FTLE is a measure for the divergence of path lines in a fixed time interval, which can be calculate forward or backwards in time. To visualize the interfaces between the jet and the breakdown bubble, backward time FTLE is more suitable as it pronounces attracting structures. More details on how to compute FTLE from phase-averaged data is given in references$^{64,65}$. The FTLE is shown together with color contours of the phase-averaged local equivalence ratio $\langle \phi \rangle$ as determined from the chemiluminescence measurements.

Figure 7 shows two columns, on the left, the linear response of the flow field and equivalence ratio to the acoustic forcing at 202 Hz. Gray-scale shows FTLE determined from phase-averaged PIV images and colored contours show phase-averaged $\phi$. The phase-difference between rows is $2\pi/6$. Throughout
one period, the vortices are generated in the inner shear layer and transported downstream. At the flame base, $\phi$-fluctuations are strong, whilst only small values reach the flame tip. The gray-scale contour lines resulting from the FTLE imply borders through which no coherent mixing is taking place. Consequently, the $\phi$-fluctuation field aligns with these boundaries as can be seen in the figure.

The right column, depicting the non-linear responses, makes the described effects more apparent. Large vortex structures are formed, leading to strong radial mixing of $\phi$ values in the inner recirculation zone. As a result, low $\phi$ values are reached in the flame base (see e.g. phase No. 5). In turn, high $\phi$ values are transported to the flame brush and flame tip. At the fourth phase position, vortices entrain low $\phi$ value areas from the outer recirculation zone towards the high $\phi$ value areas in the center, inducing mixing of both areas. As a result, the $\phi$ value decreases clearly in the following phase-averaged representation (phase No. 5). Due to the forced vortical structures and corresponding vortex-induced mixing, $\phi$ values vary greatly throughout the flame area.

Figure 8 shows the linear and non-linear responses for a higher frequency of $f = 316$ Hz. The linear response depicted on the left side shows a wave-like movement of the annular flow field throughout one period, however no vortex formation is visible in contrast to the $f = 202$ Hz case. The lack of distinct vortices suggests a lower receptivity of the shear layers to the forcing frequency of $f = 316$ Hz than for lower frequencies. Fluctuations of $\phi$ values are far lower at the flame base, but higher in the flame tip, compared to the smaller forcing frequency. These fluctuations correlate well with the fluctuations of the flow field throughout the period.

In the non-linear case, shown in the right column of Figure 8, $\phi$ values exhibit an even stronger dependency of the flow field than for the linear case. Large vortices in the outer recirculation zone entrain fluid with low $\phi$ value, which induces strong $\phi$ fluctuations. The lack of vortices in the inner recirculation zone hinders the radial redistribution of the very high $\phi$ values arriving at the flame center. This causes highly fluctuating, local pockets with high $\phi$ values and strong concentration gradients in this region.

Comparing the two forcing frequencies, the most visual difference is the qualitative difference in the structure of the flow field fluctuations. These have, even for the linear response, a clear influence on the fluctuations of $\phi$ values.

When comparing the phase position of the flow field fluctuations as well as the areas of constant $\phi$ values, the changes do not line up for the two forcing frequencies. The reason lies within the different origins, along with the different transport velocities of fuel and flow fluctuations. While the flow field fluctuations arise from vortex structures growing in the shear layers downstream of the combustor inlet, the $\phi$-fluctuations originate at the fuel injector. This yields contrasting time delays and in turn different interference patterns of the two mechanisms for different frequencies. Thus, even for the linear case, this effect would plausibly lead
to a frequency dependent interaction of flow field- and \( \phi \)-fluctuations.

**Linear mixing transfer functions**

**Generation and transport of \( \tilde{\phi} \) in the duct**

In this section, we investigate the generation and spatial mixing of equivalence ratio fluctuations based on the spatial mixing transfer function \( F_{\phi|u}^{\text{duct}} \) given in equation (16). As it was impossible to measure directly at the fuel injector, the fuel injector transfer function \( F_{\phi|u}^{\text{inj}} \) is approximated by the mode-convertion model equation (22). This will be justified \textit{a posteriori} by comparison with the data. Hence, the mixing transfer function \( F_{\phi|u}^{\text{duct}} \) is obtained by multiplying the measured transfer function \( F_{\phi|u}^{\text{duct}} \) as defined in equation (15) with \( e^{-\imath \pi} \). The mixing transfer function can then be directly compared to the 1D convection-diffusion model given in equation (24).

The gain and phase of the measured spatial mixing transfer function \( F_{\phi|u}^{\text{duct}} \) are shown in Figure 9 together with the model predictions. The measured gain values are approximately one for very low frequencies, somewhat above one for moderate frequencies, and drops off for higher frequencies. The overall shape of the gain is well reproduced by the 1D convection-diffusion model with \( Pe = 150 \). However, the gain shows some waviness indicative for an interference pattern which is not reproduced by the model.

Similar interference patterns in the mixing transfer function were observed in a related study based on TDLAS data inside a generic mixing duct. In this study, it was suggested that the constructive and destructive interference between two transport mechanisms is responsible for the interference pattern that may cause gain values higher than unity. One mechanism is related to the mean convection of \( \phi \)-fluctuations and the other is related to \( \phi \)-fluctuations induced by coherent flow structures that propagate at different velocity and potentially originate at different locations. This is also supported by the different phase positions of the flow and \( \phi \) fluctuations observed in Figure 7 in comparison to Figure 8. The second mechanism is not accounted for in the current model where only a single (and constant) transport velocity is assumed (see equation (23)). A recently proposed model of Kaiser et al.\textsuperscript{39}, based on a linearized two-dimensional transport equation may be more accurate in this regard.

Figure 9 further shows that the 1D model reproduces the measured phase very well for frequencies up to \( St = 1.7 \). It starts off at a phase value of zero for a frequency value of zero and decays nearly linearly with increasing frequency, which indicates a constant transport velocity (and convective time delay). The strong deviation of the model for high frequencies is likely to be caused by the aforementioned interference. Similar results were reported in the related study\textsuperscript{33}. However, note that for the model to fit the experimental data, the convection velocity \( u_c \) in the definition of the Strouhal number was set to \( u_c = 0.4 u_b \). This implies that the average convection velocity of the \( \phi \) fluctuations between the injector and the first measurement location is significantly lower than the mean bulk velocity. This is due to the geometry of the specific swirler, which generates a low momentum tangential inlet flow to which the fuel is injected and a high momentum central jet (see Figure 2).

The fact that the measured mixing transfer function approaches a gain value of one and a phase of zero towards zero frequencies (which is equivalent to zero \( \Delta x \), bearing the definition of \( St \) in mind) allows to conclude that the mode conversion model (equation (22)) applies for this combustor. Moreover, the good match of the 1D transport model in terms of gain and phase for a relatively wide frequency range allows to conclude that the mixing inside the duct is well reproduced by a 1D convection-diffusion process. In the current setup, the Peclet number of 150 is relatively high, which indicates that transport of equivalence ratio fluctuations is dominated by convection rather than diffusion\textsuperscript{38}.

**Transport of \( \tilde{\phi} \) fluctuations to the flame**

Figure 10 shows the spatial distribution of the local mixing transfer function \( F_{\phi|u}^{\text{flame}} \) measured in the flame. Analog to the previous section, it is derived from the product of the measured local transfer function \( F_{\phi|u}^{\text{flame}} \) and \( e^{-\imath \pi} \) assuming the mode conversion model in

![Figure 9](image-url)
equation (22) for the injector transfer function. This has been validated in the previous section. The local mixing transfer function is shown as contours of gain (left column) and phase (right column) values for five specific frequencies (rows). The streamlines indicate the mean flow.

Considering the gain contours, it is apparent for the low frequency cases that a gain value in the flame

Figure 10. Gain (left) and phase (right) of local linear mixing transfer function $F_{\text{flame}}$ measured in the flame for five different frequencies. Streamlines indicate mean flow.
center is approximately one, indicating that the $\phi$-fluctuations induced at the injector are not significantly attenuated before they reach the flame. The high values in the centerline are to be ignored, as this region is susceptible to large measurement errors following the data deconvolution process. Between 90 and 316 Hz, an overall increase in gain values inside the outer recirculation zone of the flame brush (outer part) can be noticed for increasing frequencies, speaking for a large influence of coherent structures, induced by acoustic fluctuations. At the highest frequency of $f = 470$ Hz the gain values are reduced throughout the flame, most likely resulting from an attenuation of $\phi$-fluctuations upstream of the flame.

The phase distribution displayed in the right column of Figure 10 suggests a convection-dispersion process taking place for the higher frequency cases. For low frequencies, the $\phi$-fluctuations seem to be dominated by the mean field convection, indicated by the radially homogeneous phase values. Starting at 290 Hz, the dispersion becomes apparent as the radial inhomogeneity of the phase distribution sets in. For the higher frequency cases these inhomogeneities are very pronounced, separating the fuel packets within the flame into individual regions. The same is visible in the gain values, depicted by local concentrations of high gain values. This observation can be qualitatively linked to the vortex dynamics observed in the FTLE fields shown in Figure 8. Accordingly, vortices in the outer shear layer entrain fluid with low equivalence ratio which generates $\phi$-fluctuations in the flame brush area that propagate along the outward directed swirling flow at the velocity of the vortical structures. In contrast, for the low frequency cases vortical structures entrain fluid with high equivalence ratio from the inner region (see Figure 7) which effectively homogenizes the radial distribution of the equivalence ratio. In summary, we observe a qualitative change of the local mixing transfer function between the low and high frequency cases which are linked to a qualitative change of the induced vortical structures.

After the discussion of the local mixing transfer function $F_{\text{flame}}$ in the flame, we now direct the attention to the global version $F_{\phi_{\text{flame}}}$, which relates the global $\phi$-fluctuations in the flame to the one at the injector. Analog to the local one, $F_{\phi_{\text{flame}}}$ is obtained by multiplying the measured transfer function $F_{\phi}$ with the reciprocal of the injector transfer function $e^{-it}$, as expressed in equation (19). This mixing transfer function is modeled using equation (24), assuming that the global $\phi$-fluctuations between the fuel injector and the flame is described by a 1D convection-diffusion process with a single transport velocity and a single effective diffusivity, analog to the duct.

**Figure 11.** Symbols: Mixing transfer function $F_{\phi_{\text{flame}}}$ measured in the flame at $x/D = 4.5$. Line: Convection-diffusion model equation (19) with $Pe = 300$ and $u_c = u_b$. The reference location was chosen based on the axial position with the highest chemiluminescence intensity, and hence, the best signal to noise ratio. However, since the frequency is normalized with the reference position, the plot applies to any streamwise distance to the injector location. The figure further shows the prediction of the convection-diffusion model using a Peclet number of 300. This is somewhat higher than for the mixing transfer function inside the duct, which suggests that the effective diffusion, which combines turbulent diffusion and dispersion, is less severe in the combustion chamber than in the duct. The experimental results further show the same wavy structure of the gain curve that was observed in the mixing transfer function measured in the duct. This is particularly pronounced for higher frequencies. Analog to the gain, the measured phase follows the monotonic decay predicted by the model for low and moderate frequencies and deviates strongly for higher frequencies. Note that for the best fit of experimental data, the convection velocity $u_c$ in the definition of the Strouhal number is now set to $u_c = u_b$, indicating that the mean convection velocity of $\phi$-fluctuations between the injector and the flame center is well approximated by the bulk flow velocity. This is in contrast to the region close to the injector where the transport velocity was found to be much smaller, as mentioned earlier. However, the deviations of the data from the model strongly suggest that the 1D convection-diffusion model can only be applied to low frequencies. For higher frequencies, the impact of flow fluctuations induced by turbulent coherent structures on the $\phi$-fluctuations alter the transfer function in a non-trivial way, which is not accounted for in the current
model. The frequency range, where these deviations are most pronounced, corresponds well with the radial inhomogeneities of the local $\phi$-fluctuations observed in Figure 10. This is indicative for the shortcoming of the 1D modelling approach. The good match for the very low frequency range in terms of phase and gain, however, confirms the validity of the mode-conversion model at the injector as already concluded from the measurements in the duct. This consistency brings also credibility to the two different measurement techniques.

**Time delay analysis of $\phi$ and flow fluctuations**

As suggested here and in a previous study, the interference pattern observed in the gain of the mixing transfer function is possibly caused by an interference of flow and fuel fluctuations. To support this hypothesis, a time delay analysis is conducted. Therefore, the equivalence ratio perturbations are decomposed in two parts

$$\tilde{\phi} = \tilde{\phi}_b + \tilde{\phi}_h$$

(25)

where the first part corresponds to perturbations induced by the transport of equivalence ratio fluctuations with the bulk flow velocity (subscript $b$) and the second part corresponds to the perturbations induced by the hydrodynamic fluctuations (subscript $h$). This allows us to build a model for the streamwise mixing transfer function as

$$\frac{\tilde{\phi}_2}{\phi_1} = n_b e^{j\omega_1 \tau_1} + n_h e^{j\omega_1 \tau_2}$$

(26)

with a gain $n$ and time delay $\tau$ for each mechanism. For this expression it is assumed that magnitude and phase of the two parts are equal at the first reference point, which is most likely fulfilled at the fuel injector.

Rearranging the expression to

$$\frac{\tilde{\phi}_2}{\phi_1} = (n_b + n_h e^{j\Delta \omega \Delta \tau}) e^{j\omega_1 \tau_2}$$

(27)

with the difference in time delay $\Delta \tau = \tau_h - \tau_b$, it becomes obvious that the gain of the mixing transfer function depends on the phase difference expressed as $\Delta \phi = \omega \Delta \tau$. In the case of constructive interference, the phase angle must be zero or equal to an even multiple of $\pi$, while in the case of destructive interference the phase angle must be equal to an odd multiple of $\pi$. Based on these considerations, the frequencies of ideal constructive and destructive interference can be estimated from the known time delays.

To estimate the time delays of the two mechanisms, we base $\tau_b$ on a transport velocity of $u_b$, in accordance with the 1D convection-diffusion model (see Figure 11), and a convective distance of $4.5D$ which is the distance between the injector and the reference location (see Figure 4). The time delay $\tau_h$ is based on a transport velocity of $0.5u_b$, which is well within the range of the phase velocity of the hydrodynamic fluctuations measured inside the mixing duct in a previous study and a convective distance of $x/D = 4.5$.

As seen in Table 2, the frequencies determined from the time delays compare quite well with the locations of the local minima and maxima of the gain of the mixing transfer function shown in Figure 11. This supports the hypothesis that the different transport velocities of the hydrodynamic structures and the bulk flow cause the interference pattern in the mixing transfer function. Note that the frequencies based on a convective distance of $1.5D$ for the hydrodynamic fluctuations, which corresponds to the distance between the combustor inlet and the reference position, did not yield an acceptable agreement with the measurements, which implies that the flow perturbations are initiated at the swirler and not at the combustor inlet.

**Non-linear mixing transfer functions**

In this section, we investigate the generation and transport of $\phi$-fluctuations at non-linear acoustic forcing amplitudes with $|\tilde{u}_{ac} / \tilde{u}_b| > 0.1$. As mentioned earlier, these high acoustic velocities were achieved experimentally, by changing the length of the tube upstream of the burner (Figure 1), and thus, adjusting the resonance frequency of the test rig to the forcing frequency. As the setup only allows for a certain range in the length, only a certain interval of resonance frequencies can be achieved. Therefore, we only consider the four frequencies 97, 202, 268, and 316 Hz. These correspond to the Strohual numbers of 0.19, 0.35, 0.46, and 0.55 at the TDLAS measurement position ($x/D = 1.3$) and to 0.57, 1.2, 1.59, and 1.87 at the flame center ($x/D = 4.5$).

Regarding the generation of $\phi$-fluctuations at the injector, the mode-conversion model given in equation

| $\Delta \phi$ | $\pi$ | $2\pi$ | $3\pi$ | $4\pi$ | $5\pi$ | $6\pi$ |
|-------------|------|------|------|------|------|------|
| $\Delta \tau$ | 0.5 | 1 | 1.5 | 2 | 2.5 | 3 |
(21) predicts the $\phi$-fluctuations to increase in a non-proportional manner if the forcing amplitudes are increased, which would ultimately lead to infinite $\phi$-fluctuations for $|\dot{u}_{ac}| = n_\psi$. This model is compared to the TDLAS measurements conducted at $x/D = 1.3$, which is not too far away from the fuel injector.

Moreover, current research suggests that coherent structures created by non-linear acoustic forcing in the combustion chamber lead to additional mixing, which causes a dampening of the $\phi$-fluctuations when the forcing amplitude is increased. This hypothesis (enhanced turbulent mixing) proposed by Ćosić et al. is also validated in this section based on the local and global $\phi$-fluctuations measured in the flame.

**Local $\phi$-fluctuations in the flame**

Figure 12 displays the gain values of the local mixing transfer function $F_{\phi}^{\text{flame}}$, with each column representing a frequency and every row representing a forcing amplitude. To obtain this quantity from the measured transfer function $F_{\phi}^{\text{inj}}$, an injector transfer function of $F_{\phi}^{\text{inj}} = e^{i\pi}$ is assumed, which will be later justified.

For the lowest frequency case a growing of the flame area is clearly visible for increasing forcing amplitudes. This is due to stronger fluctuations of the flame in streamwise direction resulting from the higher forcing amplitudes, which leads to an increased mean flame length. Moreover, a reduction of the mixing transfer function gain with higher forcing amplitudes is clearly noticeable indicating a saturation of $\phi$-fluctuations. The forcing amplitudes achieved for the two lowest frequencies are very similar allowing for a direct comparison of the forcing impact on the mean flow field and mean flame. Compared to $f = 202$ Hz, the mean flame length is much more increased by the growing forcing amplitudes for $f = 97$ Hz. This is accompanied by a very distinct change of the mean flow field as shown for example by the growth of the outer

![Figure 12](image-url). Gain of local mixing transfer function $F_{\phi}^{\text{flame}}$ measured in the flame for four different frequencies at increasing acoustic forcing amplitudes. Streamlines indicate mean flow. Note that different forcing amplitudes were achieved for different frequencies.
recirculation zone. In case of \( f = 202 \, \text{Hz} \), the mean field changes are less pronounced. This observation indicates that the unforced mean flow field is more receptive to low-frequency forcing leading to more considerable flame fluctuations and higher gain values.

For the two highest forcing frequencies, the achievable maximum forcing amplitudes are lower due to acoustic dissipation of the test rig. Nonetheless, saturation of the mixing transfer function can also be observed, however, the modification of the mean flame area and the mean flow field are much less pronounced. Moreover, a comparison of cases with similar forcing amplitude shows that the saturation of mixing transfer function is more pronounced for the lower frequency case. Overall, it appears that the saturation of the local \( \phi \)-fluctuation transfer function with nonlinear forcing amplitudes is more pronounced for low frequencies, which is possibly related to the aforementioned receptivity of the shear layer.

Figure 13 shows the phase of the local mixing transfer function, providing information about the transport and dispersion of the equivalence ratio fluctuations as they travel through the flame region. For the lowest forcing frequency, the phase values indicate very long streamwise wavelengths, which extend beyond the flame length. Interestingly, the radial distribution of the phase does not seem to be disturbed considerably when increasing the forcing amplitude. It appears that the low frequency forcing primarily causes a streamwise bulk-movement of the \( \phi \)-field, without strong dispersive effects. This is somewhat different for the next higher forcing frequency of 202 Hz, where the phase becomes distorted in radial direction for higher forcing amplitudes. For the two highest frequencies, the phase remains nearly unaffected by the increase of forcing amplitude, which may however be attributed to the relatively low maximum achievable forcing amplitude at these high frequencies.

![Figure 13. Phase of local mixing transfer function \( F_{\phi}^{\text{flame}} \) measured in the flame for four different frequencies at increasing acoustic forcing amplitudes. Streamlines indicate mean flow. Note that different forcing amplitudes were achieved for different frequencies.](image-url)
Global mixing transfer functions

At this point, saturative effects have been observed in the local mixing transfer functions but their origin remains unknown. To further investigate the effects underlying the saturation and at what stage their impact is maximum, a study is conducted in which the separate transfer functions preceding and succeeding the injector are compared.

Figure 14 shows the gains of three different transfer functions plotted over the acoustic amplitude: the transfer function $F_{\text{duct}}$ (top) relating the global fluctuations in the mixing duct at $x/D = 1.3$ to the acoustic velocity fluctuations at the fuel injector, the transfer function $F_{\text{flame}}$ (bottom) relating the global fluctuations in the flame center at $x/D = 4.5$ to the acoustic velocity fluctuations at the injector, and the ratio of the two (middle), relating the fluctuations in the flame to the one in the duct. For each of the three, four frequencies are compared: 97, 202, 268, and 316 Hz (identical to the frequency cases in the previous sections). To account for the different range of amplitudes for the different measured forcing frequencies, the horizontal axis is plotted logarithmically.

Considering the transfer function in the duct, first, (Figure 14 (top)) all of the different measurements (frequency cases) follow a similar trend: the gain values are constantly 1 until about $|\tilde{u}_{ac}| = 0.08$, where a saturation sets in that follows a continuous trend to zero. The three higher frequency cases show no sign of frequency dependence, as measurements of identical forcing amplitudes produce identical gain values, independent of the forcing frequency. The 97 Hz case, however, produces gain values approximately 30% larger than the rest. Generally, the decay of the gain for higher forcing amplitudes may either be attributed to non-linear effects in the mode conversion process at the injector or to enhanced mixing between the injector and the TDLAS measurement location in the duct. The fact that the attenuation of the gain is frequency independent for three out of four frequencies speaks for the first explanation, as turbulent diffusion is expected to be frequency dependent. Nonetheless, what can be stated with high confidence is that the mode conversion model (21) cannot be applied for non-linear forcing amplitudes as it predicts increasing gain values (see black line).

Considering the mixing transfer function between the duct and the flame next (Figure 14 (middle)), we observe a clear frequency dependence for the non-linear behaviour of the gain. For the lowest frequency case the gain saturates only at relatively high amplitudes somewhat in agreement with the gain values of the 202 Hz case. The 268 Hz case shows saturation at very low amplitudes starting at $|\tilde{u}_{ac}| = 0.02$, while the highest frequency case (316 Hz) yields an increase of the gain values exceeding 1 in the same range of acoustic amplitudes. When comparing the different measurement positions for identical frequency cases, only for the 268 Hz case, a similar linear descending trend can be seen. In the other cases, the results diverge, thus a connection between the frequency cases extending over the different measurement positions cannot be made.

Finally Figure 14 (bottom) shows the transfer function between acoustic fluctuations and $\phi$-fluctuations in the flame. Similar to the transfer function in the duct it shows a somewhat more monotonic saturation, setting in at $|\tilde{u}_{ac}| = 0.5$ for all frequency cases except for the 268 Hz case, which, in accordance with the preceding transfer function, produces gain values far smaller and a saturation setting in far earlier than for the remaining three frequency cases. Once again a strong frequency dependence among the gain values is apparent. By comparison with the mixing transfer function in the duct, this stems from the mixing process between the duct and the flame.

All the three presented transfer functions reveal a pronounced frequency dependence of saturating effects. The reasons underlying the different saturating mechanism can be enhanced turbulent diffusion and dispersion along with non-linear effects already triggered at the injector. The model prediction of an ever increasing gain for higher forcing amplitudes is not backed by the measurements. Moreover, the observed non-trivial frequency dependence of the mixing transfer
function at higher forcing amplitudes cannot be represented by a 1D convection-diffusion process as given in equation (23), which would generally predict a decreasing gain with higher frequencies. Comparing the measurement results shown in Figure 14, both the 202 Hz as well as the 316 Hz cases produce gain values exceeding 1 in both diagrams. As mentioned earlier, a constructive interference between the $\vec{u}$ and the $\phi$ fields is a probable cause for this. This emphasizes the role of coherent flow structures for the estimation of the non-linear saturation of $\phi$-fluctuations.

**Summary and conclusions**

The present work studies the generation and transport of temporal equivalence ratio fluctuations in acoustically forced, technically premixed swirl flames in regards to linear response and non-linear saturation. Different measurement techniques are employed to visualize interactions of local fuel ratio fluctuations and coherent vortical structures, both of which are affected by acoustic forcing and lead to global equivalence ratio fluctuations; among them PIV, chemiluminescence measurements, TDLAS, and MMM. The experimental results are compared to a 1D convection-diffusion model.

Through acoustic perturbations, phase-averaged, spatially resolved flow and equivalence ratio fluctuations are obtained to determine local and global mixing transfer functions inside the mixing duct between the injector and the flame and in the flame area.

The results show that for linear forcing amplitudes, the generation of equivalence ratio fluctuations at the injector is well approximated by a simple mode-conversion model, but it fails for non-linear forcing amplitudes. Measurements show a decay of equivalence ratio fluctuations with higher acoustic forcing amplitudes instead of an increase as predicted by the model. This strongly suggests a non-linear saturation process at the fuel injector which should be considered in future attempts to model the non-linear flame response.

The application of a 1D transport model shows that the average transport velocity of the equivalence ratio fluctuations between the injector and the flame is well approximated by the bulk flow velocity. Moreover, the model reveals a much higher effective diffusivity in the mixing duct than in the combustion chamber. This suggests that the length of the mixing duct strongly influences the phase and gain of the equivalence ratio fluctuations. Considering the mixing process for non-linear forcing between the mixing section and the flame, the measurements reveal a decrease of local equivalence ratio fluctuations with higher forcing amplitudes. This suggests enhanced turbulent mixing induced by stronger acoustic forcing, which could be modelled by the 1D transport equation with an amplitude-dependent turbulent diffusivity.

However, considering the global equivalence ratio fluctuations for linear and non-linear forcing amplitudes, the experiments show an increase for the gain of the mixing transfer function, which cannot be explained by a 1D transport process.

The deviations are ascribed to interference patterns emanating from a two-dimensional and non-local interaction of coherent flow fluctuations and fuel fluctuations, which occur at linear and non-linear forcing amplitudes. Only for very low frequencies this effect seems negligible.

A simple time-delay analysis further allows to predict the interference pattern of the mixing transfer function and supports the relevance of coherent structures interfering with the transport of equivalence ratio fluctuations.

Future modelling should therefore address the two-dimensional non-local nature of the mixing process, which does not seem to be sufficiently characterized by an effective turbulent diffusion concept alone.

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**ORCID iDs**

Finn Lückoff https://orcid.org/0000-0002-6610-0851
Kilian Oberleithner https://orcid.org/0000-0003-0964-872X

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Appendix

Notation

\[ A \] \text{ cross-sectional area} \\
\[ A_{\text{flame}} \] \text{ streamwise flame area} \\
\[ D \] \text{ Swirler diameter} \\
\[ f \] \text{ acoustic forcing frequency} \\
\[ FTF \] \text{ flame transfer function} \\
\[ l \] \text{ length} \\
\[ m \] \text{ air mass flow} \\
\[ m_f \] \text{ local fuel mass flow} \\
\[ m_o \] \text{ local oxidizer mass flow} \\
\[ M \] \text{ molar mass} \\
\[ MMM \] \text{ Multi Microphone Method} \\
\[ Pe \] \text{ Peclet number} \\
\[ PIV \] \text{ Particle Image Velocimetry} \\
\[ Re \] \text{ Reynolds number} \\
\[ S \] \text{ geometric swirl number} \\
\[ St \] \text{ Strouhal number} \\
\[ t \] \text{ time} \\
\[ T \] \text{ period of acoustic forcing signal} \\
\[ \tau_c \] \text{ convective time scale} \\
\[ \tau_d \] \text{ diffusive time scale} \\
\[ TDLAS \] \text{ tunable diode laser absorption spectroscopy} \\
\[ TF \] \text{ transfer function} \\
\[ u, v \] \text{ streamwise and transversal (radial) flow velocity} \\
\[ u_b \] \text{ bulk flow velocity} \\
\[ u_{\text{ac}} \] \text{ acoustic reference velocity} \\
\[ u_c \] \text{ convection velocity} \\
\[ x, r \] \text{ streamwise and radial coordinate} \\
\[ \nu \] \text{ kinematic viscosity} \\
\[ Z_{\text{st}} \] \text{ stoichiometric mass fraction} \\
\[ (\cdot) \] \text{ flow} \\
\[ (\cdot)’ \] \text{ fluctuation in time domain of value} \\
\[ (\cdot) \hat{\cdot} \] \text{ Fourier coefficient of value} \\
\[ (\cdot)_{\text{gl}} \] \text{ global values, integrated over cross-sectional area } A \\
\[ (\cdot)_{\text{mean}} \] \text{ mean value} \\
\[ (\cdot)_{\text{periodic}} \] \text{ periodic part of value} \\
\[ (\cdot)_{\text{phase-averaged}} \] \text{ phase-averaged part of value} \\
\[ (\cdot)_{\text{set}} \] \text{ values set by the experiments} \\
\[ \rho \] \text{ density} \\
\[ \varphi \] \text{ phase angle} \\
\[ \phi_{\text{set}} \] \text{ global equivalence ratio set in experiment} \\
\[ \phi \] \text{ local equivalence ratio} \\
\[ \psi \] \text{ scalar field}