The Geometrical Meaning of Time in the Presence of Matter

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Abstract. It is stated in many text books that the any metric appearing in general relativity should be locally Lorentzian i.e. of the type \( g_{\mu\nu} = \text{diag}(1, -1, -1, -1) \) this is usually presented as an independent axiom of the theory, which cannot be deduced from other assumptions. The meaning of this assertion is that a specific coordinate (the temporal coordinate) is given a unique significance with respect to the other spatial coordinates. It was shown that the above assertion is a consequence of requirement that the metric of empty space should be linearly stable and need not be assumed. In this work we remove the empty space assumption and investigate the consequences of matter on the stability of Lorentzian space-time. It will be shown that in the presence of matter an upper limit scale to the size of a locally Lorentzian universe exists which incidentally is about the size of the observable universe.

1. Introduction

It is well known that our daily space-time is approximately of Lorentz (Minkowski) type that is, it possesses the metric \( \eta_{\mu\nu} = \text{diag}(1, -1, -1, -1) \). The above statement is taken as one of the central assumptions of the theory of special relativity and has been supported by numerous experiments. But why should it be so?

Many textbooks \cite{3, 4} state that in the general theory of relativity any space-time is locally of the type \( \eta_{\mu\nu} = \text{diag}(1, -1, -1, -1) \), although it can not be presented so globally due to the effect of matter. This is a part of the demands dictated by the well known equivalence principle. The above principle is taken to be one of the assumptions of general relativity other assumption such as diffeomorphism invariance, and the requirement that theory reduce to Newtonian gravity in the proper regime lead to the Einstein equations\footnote{This specific form of Einstein equations is taken from Narlikar \cite{3}.}:\begin{equation}
G_{\mu\nu} = -\frac{8\pi G}{c^4} T_{\mu\nu}
\end{equation}
in which \( G_{\mu\nu} \) is the Einstein tensor, \( T_{\mu\nu} \) is the stress-energy tensor, \( G \) is the gravitational constant and \( c \) is the velocity of light.

Generic solution of Einstein equations for empty (and non-empty) spaces usually show different signatures as was shown by Eddington \cite[page 25]{5}. Among those possible solutions it was shown that for empty space-times \cite{1, 2} only the Minkowskian solutions are stable.

Thus one need not assume that space-time is locally Minkowskian based on an empirical (unexplained) fact, rather one can derive this property from the field equations themselves based
on the stability of the Minkowskian solutions. The number of assumptions needed to obtain
the celebrated results of general relativity is reduced, by making the theory more compact we
enhance its predictive strength. Moreover, it is shown that the existence of a temporal coordinate
is a necessary consequence of the geometrical structure of the four dimensional space and not a
separate ad-hoc assumption.

A different approach is due to Mukohyama & Uzan [6]. Those authors have derived a
Lorentzian metric assuming a that at the microscopic level the metric is Riemannian, i.e.,
locally Euclidean, and that the Lorentzian structure, that we usually consider as fundamental,
is in fact an effective property that emerges in some regions of a four dimensional space with a
positive definite metric. In such a model, there is no dynamics nor signature flip across some
hypersurface; instead, all the fields develop a Lorentzian dynamics in these regions because they
propagate in an effective metric. They have shown that one can construct a decent classical
field theory for scalars, vectors, and (Dirac) spinors in flat spacetime. And that gravity can be
included but that the theory for the effective Lorentzian metric is not general relativity but of
the covariant Galilean type. The constraints arising from stability, the equivalence principle,
and the constancy of fundamental constants are detailed and a phenomenological picture of the
emergence of the Lorentzian metric is also given. This construction, while restricted to classical
fields, offers a new view on the notion of time. Unfortunately this approach is not consistent
with Occam’s razor as one is forced to introduce several scalar fields (one of them a “clock field”) with fine tuned coupling in order to obtain the effective sign change.

What is deficient in the above approach (and similar approaches) is that additional theoretical
structures & assumptions are needed in order to justify what appears to be a fundamental
property of space-time. In previous works [1, 2, 7] it was claimed otherwise. It was shown that
General relativistic equations and some ”old fashioned” linear stability analysis will lead to a
unique choice of the Lorentzian metric being the only one which is linearly stable in empty
space-time. Other metrics are allowed but are unstable and thus can exist in only a limited
region of empty space-time. The implications of this are discussed elsewhere [8, 9, 10]. It should
be mentioned that the choice of coordinates in the Fisher approach to physics can be also be
justified using the stability approach [7]. The nonlinear stability question of the Lorentzian
metric for empty space-times was settled by D. Christodoulou & S. Klainerman [12].

This paper assumes that space-time must have four dimensions, it does not explain why this is
so. For a possible explanation derived from string theory one can consult a paper by S. K. Rama
[13]. The analysis discussed in this work is classical, for quantum aspects of space time difference
one can consult a book by S. Hawking & R. Penrose [14]. Quantum field considerations made
by A. White, S. Weinfurtner and M. Visser [15] have shown that regardless of the underlying
classical theory, there are severe problems associated with any quantum field theory residing on
a signature-changing background which does not respect the space-time difference (Such as the
production of what is naively an infinite number of particles, with an infinite energy density).
Those authors raises the question as to whether signature change transitions could be fully
understood and dynamically generated within (modified) classical general relativity, or whether
they require the knowledge of a theory of quantum gravity.

The plan of this paper is as follows: in the first section we outline the stability analysis of
empty space-times, the next is dedicated to the modifications of those stability equations in the
presence of matter. Finally we deduce an upper scale for a stable Minkowskian space-time and
compare this scale with the size of the observable universe.

2. Stability analysis for empty space-times
In what follows I give an outline of the proof of the linear stability of Lorentzian space-time
and the linear instability of other possible flat space-times. Unfortunately the details cannot be
given here for the lack of space, but can be found elsewhere [1].
Consider a flat space-time, in this case one can find a set of coordinates such that the metric will be constant everywhere. A constant metric is nothing but a 4 by 4 symmetric matrix, therefore by a suitable choice of coordinates it can be made diagonal with eigenvalues all real. A final step would be to choose the scaling of the coordinates such that the metric will have the form:

\[ \eta = \text{diag} (\pm 1, \pm 1, \pm 1, \pm 1). \]

(2)

Notice that no zero eigen values are allowed by virtue of the four dimensionality of space-time.

Once the family of possible canonical flat metrics is established we make a small perturbation \( h_{\mu\nu} \) of the metric and consider a new metric

\[ g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \]

in which \( h_{\mu\nu} \ll 1 \). Inserting \( g_{\mu\nu} \) into equation (1) and keeping only first order terms in \( h_{\mu\nu} \) while taking \( T_{\mu\nu} = 0 \) we arrive at homogenous linear equations for \( h_{\mu\nu} \). Solving those equations with local but otherwise arbitrary boundary conditions we see that only in the case of a Lorentzian metric the perturbation does not exponentially grow while in any other case it does. This explains why space-time is Lorentzian and why time is unique among the coordinates of space-time.

3. Stability analysis for a Lorentzian non empty space-time

Let us now assume that space-time is locally Lorentzian and contains matter. We choose following Narlikar [3] the non-perturbed metric to be:

\[ \eta = \text{diag} (1, -1, -1, -1). \]

(3)

this metric is stable in empty space. We assume a fluid energy momentum tensor of the form [3]:

\[ T_{\mu\nu} = (p + \rho c^2)u_\mu u_\nu - pg_{\mu\nu} \]

In the above \( p \) is the pressure, \( \rho \) is the density and \( u_\mu = \frac{dx_\mu}{ds} \) in which the interval \( ds \) is defined as:

\[ ds^2 = g_{\mu\nu} dx^\mu dx^\nu \]

(5)

We notice that \( T_{\mu\nu} \) depend on metric perturbations both directly through the term \( -pg_{\mu\nu} \) and through the \( ds \) term. Taking only first order terms and assuming that only \( h_{00} \) is non zero we arrive at the equation:

\[ \eta^{00} \frac{\partial^2 h_{00}}{\partial x^0^2} + \eta^{ij} \frac{\partial^2 h_{00}}{\partial x^i \partial x^j} + 4\kappa \rho c^2 h_{00} = -2\kappa \rho c^2 \]

(6)

in which \( \kappa = -\frac{8\pi G}{c^4} \). Taking a Fourier transform:

\[ h_{00} = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} A(x_0, \vec{k}) e^{i\vec{k} \cdot \vec{x}} d^3k, \quad \vec{k} = (k^1, k^2, k^3), \quad \vec{x} = (x^1, x^2, x^3) \]

(7)

We obtain the equation for \( A \):

\[ \frac{\partial^2 A}{\partial x^0^2} + (\vec{k}^2 + 4\kappa \rho c^2)A = -2\kappa \rho c^2 (2\pi)^2 \delta^3(\vec{k}), \quad \vec{k}^2 \equiv k^2 \]

(8)

In the above \( \delta^3(\vec{k}) \) is a three dimensional Dirac delta function. Since a solution of an inhomogeneous differential equation is a sum of a specific solution to the inhomogeneous differential equation plus a general solution to the homogeneous equation. For stability analysis it will suffice to look at the homogeneous equation:

\[ \frac{\partial^2 A}{\partial x^0^2} + (k^2 + 4\kappa \rho c^2)A = 0 \]

(9)
This shows that in the presence of matter stability can not be maintained unless:

\[ k > k_{\text{crit}} = \frac{4}{c} \sqrt{\frac{2\pi G \rho}{2}} \tag{10} \]

In terms of wavelengths we see that an upper scale for stable perturbations is:

\[ \lambda < \lambda_{\text{max}} = \frac{2\pi}{k_{\text{crit}}} = \frac{c}{2} \sqrt{\frac{\pi}{2G\rho}} \tag{11} \]

4. The scale of the observable universe

The density of the universe is estimated to be [16]: \( \rho \simeq 4.5 \cdot 10^{-28} \text{Kilogram/Meter}^3 \) which leads to a \( \lambda_{\text{max}} \simeq 1.08 \cdot 10^{27} \text{Meter} \). This is slightly larger than the radius of the observable universe: \( 14 \cdot 10^9 \text{Parsec} \simeq 4.32 \cdot 10^{26} \text{Meter} \). Hence in ”surprising” coincidence the diameter of the observable universe is about the size of the largest scale stable perturbation. At this time the size of the universe is not known [18] but it is suspected [9, 10] that above the stability scale the metric of space-time and hence physics may be quite different.

5. Conclusion

Mathematically speaking one of the main differences between time and space is encapsulated in the flat metric of our space-time which is \( \text{locally} \) of the Lorentzian type \( \eta_{\mu\nu} = \text{diag} (1,-1,-1,-1) \). But this is an empirical fact or a mathematical postulate thus unexplained. One can imagine also other flat metrics such as the Euclidian metric: \( \text{diag} (1,1,1,1) \). In Euclidian metrics there is no restriction on the speed of any moving body as the speed of light restricts the speed of propagation only in the presence of a Lorentzian metric. Why is our space-time Lorentzian and not Euclidean? The answer is that only the Lorentzian metric is stable [1] for an (almost) empty space. But space-time is not empty and thus the notion of time always progressing forward with the increase of entropy is probably just a consequence of the scales of reality that we are exposed to. In huge cosmological scale the Friedman-Lemaître-Robertson-Walker universe losses its Lorentzian character [9, 10]. The horizon problem related to the homogeneity of cosmic microwave background can be solved if one takes into account the superluminal motion of particles for \( r > r_c \) and the same particles moving into the \( r < r_c \) Lorentzian domain. What is shown here is that a \text{locally} Lorentzian type universe although stable for all scales in an empty universe has a size limitation for a non empty universe and hence the suggested solution to the horizon problem is not only plausible but in fact necessary.

It should be noted that the stability analysis given here is rather restricted as we only allowed \( h_{00} \) perturbations. A full stability analysis is expected to involve the roots of a sixth order polynomial and hence we speculate the existence of a lower (as well as an upper scale) [9] for a Lorentzian Space-Time. This may have some consequences for elementary particle physics.

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