Superconducting quantum refrigerator: Breaking and rejoining Cooper pairs with magnetic field cycles

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We propose a solid state refrigeration technique based on repeated adiabatic magnetization/demagnetization cycles of a superconductor which acts as the working substance. The gradual cooling down of a substrate (normal metal) in contact with the working substance is demonstrated for different initial temperatures of the substrate. Excess heat is given to a hot large-gap superconductor. The on-chip refrigerator works in a cyclic manner because of an effective thermal switching mechanism: Heat transport between N/N versus N/S junctions is asymmetric because of the appearance of the energy gap. This switch permits selective cooling of the metal. We find that this refrigeration technique can cool down a 0.3cm³ block of Cu by almost two orders of magnitude starting from 200mK, and down to about 1mK starting from the base temperature of a dilution fridge (10mK). The corresponding cooling power for a 1cm × 1cm interface are 25 nW and 0.06 nW respectively, which scales with the area of the interface.

The goal of building solid state refrigerators and heat engines working on quantum principles is an outstanding need for next generation quantum technologies [1]. It is known since the earlier days of superconductivity that the process of magnetizing a superconducting material quasistatically and adiabatically can reduce the temperature of the material substantially as it transitions to the normal state [2-4]. This is because a material in its superconducting state has more order and therefore less entropy, equivalent to that of a normal metal at a lower temperature. Hence when driven to the normal state adiabatically by an applied magnetic field, the achieved final state is much colder than the initial superconducting state. There were attempts in the past to try and implement adiabatic magnetization of a superconductor as an effective cooling technique, notably the early proposals by Mendelssohn and Moore [3], and by Keesom and Kok [2]. Recently Dolcini and Giazotto had studied the adiabatic magnetization of a superconductor by including dynamical dissipative effects such as eddy current losses, and suggested that this mechanism can still be used to achieve significant cooling for micro-refrigeration purposes [5].

Here we propose a cyclic superconducting refrigerator based on adiabatic magnetization of a superconductor, with a working mechanism similar to that of a domestic refrigerator. A conventional refrigerator operates by cyclically moving a working fluid between hot and cold reservoirs. Work is done by compressing a fluid, and letting it freely expand to a gas in a phase transition where it cools down and absorbs heat from the cold reservoir. The now hot gas is then re-compressed, liquifying it, and then dumps the excess heat to a hot reservoir, which is usually the environment that allows the fluid to thermalize and reset to its initial temperature. The cycle repeats many times such that a stable low final temperature is achieved in the cold reservoir. In comparison, the working fluid in our example is the electron gas in the working superconductor. The cold reservoir is a normal metal, and the hot reservoir is another superconductor having a larger gap. The superconducting state of electrons in the working substance is analogous...
to a compressed fluid. With an applied magnetic field, the electron fluid expands in a phase transition into the unpaired normal state at a lower temperature. Heat is then absorbed from the cold reservoir, and the electron fluid is re-compressed by reducing the applied magnetic field. The working substance, which is now hotter than the hot reservoir, has reduced electronic entropy in the paired state. The entropy of phonons has increased in the working substance in proportion, effectively holding the excess heat. Electron-phonon interactions in the working substance and a tunneling contact with the hot reservoir selectively removes hot electrons from the working substance, and facilitate reaching thermal equilibrium in the hot junction. This cycle repeats, establishing a low temperature steady state in the cold reservoir. See Fig. 1 and Fig. 2. There is significantly less back-flow of quasiparticles from the hot reservoir to the working substance, and from the working substance to the cold reservoir, when the former are hotter. This is due to the presence of an energy gap in the superconducting state, which exponentially suppresses the population of quasi-particle excitations in the superconductor.

We also assume that the Kapitza coupling across the tunnel junctions (see Supplementary Information) can be avoided by carefully choosing the disordered tunnel barriers such that it causes phonon mismatch, and prevents phonon mediated heat transport.

The cyclic superconducting refrigerator – Adiabatic magnetization of a superconductor preserves the total entropy of the material such that the entropies of the two phases are equal, $S^N(T_f, H = H_C) = S^S(T_i, H = 0)$, where $H$ is the applied magnetic field. This results in cooling of the material to a final temperature $T_f$ that is approximately proportional to $T_3$ [4–5]. For the cyclic superconducting refrigerator presented here, the crucial point is that the magnetic field inducing the phase transition, when applied quasi-statically, can be reversed quasi-statically to its initial value and therefore reversing the superconducting to normal phase transition of the working substance. This cycle can be performed repeatedly, where the working substance is driven between two different temperatures (hot and cold), envisaging a refrigeration cycle. The energy transfer is asymmetric. That is, energy flow has a preferred direction that is different for the different phases, as a consequence of the energy-structure of the N/S materials [10–12]. The proposed refrigerator is sketched in Fig. 1 and Fig. 2 (a).

We assume that the initial temperature of the working substance $T < T_c$ of the working substance, such that its specific heat in the superconducting state can be approximated by $C_S = 3\alpha T^3 + a\gamma T \exp(-bT_c/T)$, where $b = 1.44$, and $a = 9.14$. Here $\alpha$ and $\gamma$ are parameters specifying its specific heat at the normal state, $C_N = 3\alpha T^3 + \gamma T$. Here the common $T^3$ term is the phononic (Debye) contribution to the specific heat $C_S$. In the superconducting case, the exponential behavior of the electronic specific heat of the superconductor at low temperatures can be associated to the presence of a superconducting gap. The critical field as a function of temperature can be found from free energy differences $\Delta F_{N/S}$, which agrees reasonably well with the empirical formula, $H_C(T) = H_0(1 - T^2/T_0^2)$ for $T < T_c$, where $H_0$ is the zero-temperature critical field of the working substance [14].

We can calculate some ideal thermodynamic properties of the cyclic refrigerator. The temperature of the hot reservoir is $T_H = T_3$. The working substance is in thermal equilibrium with the hot reservoir initially and the following cycle occurs (See Fig. 1 and Fig. 2):

- **Step A:** A quasistatically applied magnetic field drives the working substance to the normal state. The transformation is iso-entropic and the working substance cools down to $T_1 = T_3^3/T_\ast^2 = T_3^3/T_\ast^2$. Here $T_\ast^2 = \frac{\gamma}{3\alpha}$, a characteristic temperature of the working substance. Magnetic work $W_{3,1}$ is done.

- **Step B:** The working substance is put in contact

\[ T_\ast^2 = \left(\frac{\gamma}{3\alpha}\right) \]
with the cold reservoir where it absorbs heat \(Q_C\). Since the electronic contribution to the entropy and specific heat dominates in the normal state, the transferred heat per unit volume can be approximated, \(Q_C = \int TdS = \frac{2}{\gamma_1}(T_C^3 - T_1^3)\).

The temperature \(T_C = T_2\) can be identified as the equilibrium final temperature between the cold reservoir and the working substance, approximated as \(T_C \approx \sqrt{t_2^3 + t_1^3} > T_1\), where \(T_1\) is the initial temperature of the substrate prior to the cycle. Maximum cooling power is obtained when \(T_1 = T_H\), and the cooling power tends to zero when \(T_1 \to T_1\). Here \(\gamma_1 T_1\) is the electronic entropy of the substrate.

- **Step C**: The electron fluid in the working substance is re-compressed by reducing the magnetic field quasi-statically and adiabatically, where it returns to the superconducting state at temperature \(T_3 = (T_C T_2^2)^{1/3}\). Magnetic work \(W_{2.4}\) is done.

- **Step D**: The working substance is put in contact with the hot reservoir. Since the reservoir has a high specific heat and bandgap, the final temperature achieved can be approximated to the temperature of the hot reservoir \(T_3 = T_H\). In this process, the amount of heat transferred to the hot reservoir per unit volume is given by, \(Q_B = \int TdS = \frac{2}{\gamma_1}(T_H^3 - T_1^3)\). We have approximated the entropy lines for the superconducting state to be only phononic, since the electronic contribution goes to zero exponentially at low temperatures.

Fig. 1(a) illustrates this ideal process. By the first law, we have \(W_{3.1} + Q_{1.2} + W_{2.4} + Q_{4.3} = 0\). Defining \(W = W_{3.1} + W_{2.4}\), we have \(W = -Q_{4.3} - Q_{1.2} = Q_H - Q_C\). The Coefficient of Performance (COP) is the ratio of heat taken from the cold reservoir \(Q_C\) to work \(W\) given by,

\[
\text{COP} = \frac{Q_C}{W} = \frac{t_H^0 - t_H^6}{\frac{2}{3}(t_C^{4/3} - t_H^{4/3}) - (t_C^2 - t_H^2)},
\]

where \(t_H = T_H / T_C, t_C = T_C / T_*\). Please see Fig. 2(c), where we plot the coefficient of performance as a function of \(t_H = T_H / T_C\).

In the above idealized analysis, we assume an on/off type energy exchange, so heat transfer to either a hot reservoir or a cold reservoir can be made on demand, like a piston operating a heat-transfer switch. While liquid-gas refrigerators can make a good approximation to this idealized description because of their ability to be freely moved around, solid state systems do not have such freedom. Instead, we must design appropriate physics to effectively turn on and off a switch of exchanging heat with either a hot reservoir or a cold reservoir in order to make an effective solid-state refrigerator. Below, we show that the asymmetry of heat transport between normal metals and superconductors has such a “switch” built in \([10]\), which permits selectively cooling down the cold reservoir, due to the presence of an energy gap in the superconductor.

**Continuous adiabatic cooling** – Here we provide a dynamical description for the gradual cooling of a substrate \(N_1\) in contact with the working substance \(S_2 / N_2\), which is subsequently in contact with a hot reservoir, \(S_H\). The quasiparticle tunneling across the interface and the dissipative effects determine the temperature evolution of the three regions, 1: substrate \((T_L)\), 2: working substance \((T_W)\), 3: hot reservoir \((T_R)\). The adiabatic description for cooling of the working substance with dissipative effects is governed by the relation \(\partial_t S_w = \frac{P_{qw}(t)}{T_w(t)}\), where \(S_w(T_w, t) = x_N(T_w, t)S_w^N(T_w) + (1 - x_N(T_w, t))S_w^P(T_w)\), and \(P_{qw}\) is the net dissipative power per unit volume in the working substance, due to thermal contacts and eddy currents, and \(T_w\) is the temperature of the working substance. Here \(x_N(T_w, t)\) is the fraction of normal metal in the working substance at time \(t\) given by,

\[
x_N(T_w, t) = 1 - n^{-1} \left(1 - \frac{H(t)}{H_C(T_w)}\right),
\]

where \(H(t)\) is the applied magnetic field and \(n\) is the demagnetization factor of the material \([15]\). Variation of \(x_N\) for our refrigeration protocol is shown in Fig. 3(b), which shows that the fraction increases from zero to one, and then falls back to zero in the proposed magnetization cycle. The dynamics of the refrigerator is described by the following set of simultaneous differential equations (assuming volume):

\[
C_{N_1}(T_L)\dot{T}_L = -x_N(T_w, t)P_{q}^{pp}(N_1, N_2) + P_{load} - (1 - x_N(T_w, t))P_{q}^{pp}(N_1, S_2)
\]

\[
C_w(H, T_w)\dot{T}_w = x_N(T_w, t)(P_{q}^{pp}(N_1, N_2) - P_{q}^{pp}(S_2, S_1)) + (1 - x_N(T_w, t))(P_{q}^{pp}(S_1, S_2) - P_{ede}) + P_{mag} + P_{eddy}.
\]

A similar dynamical equation exists for \(T_R\), but for a large volume of the hot reservoir, and coupling to a support at fixed initial temperature, we can safely assume that \(T_R = 0\). The specific heat \(C_w\) is the specific heat of the intermediate state, given by \([5]\), \(C_w(H, T_w) = x_NC_N(T_w) + (1 - x_N)CS(T_w) + C_{q}^{at}(H, T_w)\), where \(C_{q}^{at}(H, T_w) = \left(T_w H/\mu_0 H_C(T_w)S_w(T_w, 0) - S_w(T_w, 0)\right)^2\) corresponds to the latent heat of the phase transition. The competing cooling power is, \(P_{ede} = \frac{\omega_nT_wB^2}{\nu_C(T_w)H}\).

We treat the electron and phonon temperatures identical in Eq. 2, since electron-phonon relaxation occurs much faster compared to adiabatic magnetization, which is a slow process. Here \(P_{load}\) accounts for a small heating contribution from thermal coupling in the range 1pW-100nW as \(T_1\) is varied from 10mK to 200mK. The working substance can heat up due to eddy currents introduced by the magnetic field \(B = \mu_0 x_N(T_w, t)H_C(T_w)\) varies as \(P_{ede}(t) = \frac{A^2 B^2}{H_C}\), where \(A\) is the area which the normal
component of the field is passing through, and $R_w$ is the bulk resistance of the working substance. Eddy current effects can be reduced by a factor $\propto 1/N_w^2$, by subdividing the bulk into $N_w$ thin sheets \[16\]. Note that such a laminar formation occurs naturally in the effective description of the intermediate state where the metal and superconducting phases coexist with alternating thin strips of metal and superconducting phases, and the magnetic field lines pass only through the normal phase \[17\].

The quasiparticle power (energy exchange per unit time) transported between two normal metals is,

$$P_{N_1,N_2}^{qp} = \frac{2}{e^2 R} \int_0^\infty E \, dE (F_1(T_L) - F_2(T_w))$$

$$= \frac{1}{e^2 R} \frac{\pi^2 k_B^2}{6} (T_L^2 - T_w^2). \quad (3)$$

Here $F(T)$ is the Fermi-Dirac distribution at temperature $T$, $k_B$ is the Boltzmann constant, and $R = \frac{k_B}{T}$ is the normal state resistance of the junction. The specific resistance $R_s$ is assumed to be $2 \Omega \mu m^2$, and identical for both the junctions. As expected, good energy transfer is found (going as a power law of the temperature difference) because of the density of states-matching of the two normal metals. The maximum cooling power provided by the junction can be calculated from Eq. (3).

As noted previously, maximum cooling power is obtained when $T_L = T_i = T_H$, and $T_w = \frac{T_0}{2}$. Substituting, we obtain

$$P_{c,\max} = \frac{\gamma_2}{e^2 R} \frac{\pi^2 k_B^2}{6} t_H^2 (1 - t_H^4). \quad (4)$$

Using the parameters mentioned in the caption of Fig. 2 (c), we obtain $T_s = \sqrt{\frac{T_w}{\gamma_2}} = 11.6K$ for Tantalum. For a specific resistance $R_s = 2 \Omega \mu m^2$, a 10cm×10cm contact has resistance $10^{-4} \Omega$, yielding the cooling power at 10mK nearly equal to 6 nW. Further maximizing $P_{c,\max}$ over $t_H$ we obtain the optimal point of operation $t_H^\max = \frac{T_w}{T_s} > t_H^\max$, i.e., when

$$0.746 \frac{\Delta_2}{k_B \gamma_2} \frac{\alpha_2}{\gamma_2} > 1, \quad \text{using} \quad k_B T_c = \frac{\Delta_2}{1.764} \quad (5)$$

from the BCS theory \[18\]. The ideal refrigerator sketched in Fig 2 (c) has a COP = 1.65 at this optimal point.

Similarly, the quasiparticle power exchange between a normal metal and a superconductor is given by,

$$P_{N_1,S_2}^{qp} = \frac{2}{e^2 R} \int_0^\infty E \, dE \frac{E}{\sqrt{E^2 - \Delta_2^2}} (F_1(T_L) - F_2(T_w))$$

$$\approx \frac{2}{e^2 R} \left[ \left( \frac{\Delta_2^2 K_0}{k_B T_L} \left\{ \frac{\Delta_2}{k_B T_L} \right\} + \Delta_2 k_B T_L K_1 \left\{ \frac{\Delta_2}{k_B T_L} \right\} \right) \right.$$ \n
$$- \left( \frac{\Delta_2^2}{k_B T_w} \left\{ \frac{\Delta_2}{k_B T_w} \right\} + \Delta_2 k_B T_w K_1 \left\{ \frac{\Delta_2}{k_B T_w} \right\} \right) \right]. \quad (6)$$

Here $\Delta_2$ is the energy gap of $S_2$, and $K_{0,1}(x)$ are modified Bessel functions of order 0 and 1. A similar relation can be found for $P_{N_2,S_3}^{qp}$. In pursuing the integrals, we have assumed low temperatures such that the integrals are effectively approximated using Laplace transformations. For large $\Delta_2/k_B T_L$, the asymptotic expansion of Bessel function, $K_n(x) \sim e^{-x} \sqrt{\pi/2x}$, insures an exponential cut-off of the transport between the N/S junction, acting as the desired switch.

The heat exchange between the two superconducting elements has two contributions, the quasiparticle power exchange, $P_{S_2,S_3}^{qp}$, and a term depending on the relative phase of the superconductors $P_{S_2,S_3}$. \[19\] \[20\]. The quasiparticle tunneling power across the $S_2/S_3$ junction is approximated \[21\],

$$P_{S_2,S_3}^{qp} \approx \frac{1}{e^2 R} \sqrt{\frac{\pi}{\Delta_3}} \Delta_3^{5/2} \left( \frac{k_B T_w}{\Delta_3} \right)^{1/2} \left( \frac{\Delta_3}{(k_B T_w)} \right) \times \cosh \left( \frac{\hbar \gamma_0}{2k_B T_w} \right) - \sqrt{\frac{k_B T_w}{\Delta_3}} \left( \frac{\Delta_3}{(k_B T_w)} \right), \quad (7)$$

where we have assumed that the difference $\Delta_3 = \Delta_2$ is
much bigger than the thermal energies of quasiparticles which help reduce the back-flow of heat from the reservoir to the working substance. The magnitude of the φ dependent term is always smaller than $P_{S_2 S_3}^{ph}$ and is given by $P_{S_1 S_2}^{ph} = -\Delta_{S}/\Delta_{S} P_{S_2 S_3}^{ph} \cos \phi$ \cite{21}. In the examples considered, we have taken $T_c = 4.48K$ for Tantalum, $T_{Cr} = 9.29K$ for Niobium. The refrigerator operates below 0.1 $T_c w$ in the examples presented, where the superconducting gap remains constant at its zero temperature value. In Fig. 3 we have set $\phi = 0$, and $\dot{\phi} = 0$. In general, the relative phase between the superconductors provides another control knob in the problem, and is significant in determining the cooling power when the magnetizing cycles are applied faster than the thermal relaxation time of the Josephson junction \cite{22,23}.

**Discussions** – We proposed a cyclic superconducting refrigerator using the principle of adiabatic magnetization of a superconductor. The refrigerator action is similar to a conventional kitchen refrigerator. Here, the working fluid is the electron gas in a superconductor switching between normal (expanded) and superconducting (compressed) states in an applied magnetic field. Substantial cooling down of a substrate is predicted, as depicted in Fig. 3(b) for different equilibrium initial temperatures of the refrigerator. We conclude by noting that many variations on this proposal are possible. For example, if another set of metal/superconductor junctions is placed on the other side of the metal to be cooled, then out-of-phase double-action refrigeration is possible, where one side continues to cool the metal down while the other side is heating up and ejecting its excess heat. Our solid state refrigeration technique can be very effective for achieving significant cooling in superconducting circuits, and for applications such as superconducting single photon detectors \cite{24} and sensors \cite{25}.

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Here we propose two alternative implementation schemes for the refrigeration protocol discussed in this letter. Both use a suspended membrane to reduce the Kapitza coupling in order to inhibit phonon thermal transport between interfaces (See Fig. 4). The Kapitza coupling for an interface between materials $j$ and $k$, with phonon temperatures $T_{ph_j}$ and $T_{ph_k}$ is given by [7–9],

$$ P_{ph_j,ph_k} = KA(T_{ph_j}^4 - T_{ph_k}^4), $$

where $A$ is the area of the interface, and $K$ is the coupling $\sim 200$ W m$^{-2}$ K$^{-4}$, for typical metal interfaces. We stress that in general, different temperatures for electrons and phonons can be investigated in this scheme, as marked in the figures. Negligible Kapitza coupling, and fast electron-phonon interaction relative to the adiabatic magnetization process ensures that we can essentially treat the electron/phonon temperatures the same during the quasi-static operation of the refrigerator, as described in Eq. (2) of the main text.