Differential Effects of Virtual and Concrete Manipulatives in a Fraction Intervention on Fourth and Fifth Grade Students’ Fraction Skills

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ABSTRACT
Fraction knowledge has been found to predict later mathematical performance, but many students have difficulty with fractions. Virtual manipulatives (VM) and concrete manipulatives (CM) are effective approaches to teaching fractions, but previous research has not been able to reach a consensus on which manipulatives are the most effective. This quasi-experimental study employed a pre- and posttest design to investigate the differential effects of VM and CM in a fraction intervention on students’ fraction skills. In addition to fraction skills, students’ arithmetic fluency was measured. Fidelity of intervention, social validity, and time-efficiency of the manipulatives were also investigated. Fourth- and fifth-grade participants (N= 115) from Southern Finland were assigned to VM and CM intervention groups. The intervention was implemented during six 45-minute lessons over 2 weeks. Results revealed that the CM group outperformed the VM group in fraction skills, which suggests that CM should be favored in fraction interventions.

Fraction knowledge in terms of procedural understanding predicts students’ later mathematical performance even when controlling for IQ, socio-economic status, working memory, and broader mathematical understanding (Resnick et al., 2016; Siegler et al., 2012). Especially, knowledge in fraction magnitude (Booth et al., 2014; Liu, 2018) has been found to predict well mathematical performance in the final years of comprehensive school. Fraction knowledge is also central for daily activities, such as cooking and personal finance (Mazzocco et al., 2012). Still, many students have difficulty with fractions (Resnick et al., 2016). Students require effective and easy-to-use educational practices to fraction learning (Ennis & Losinski, 2019). It is also vital that these practices are experienced as meaningful and important by students to secure the benefit for later learning (Ledford et al., 2016; Sarama & Clements, 2016).

Previous research has shown that using manipulatives, whether virtual manipulatives (VM) or concrete manipulatives (CM), is effective to support students’ fraction learning (Corbonneau et al., 2013; Moyer-Packenham & Westenskow, 2013). However, prior research has not been able to determine whether manipulatives are differentially effective and which factors underlie possible differences (Mendiburo & Hasselbring, 2014; Moyer-Packenham et al., 2013). Since technology in the classroom is becoming increasingly employed (Bouck & Park, 2018; Shin et al., 2017), it is also necessary to provide knowledge for educators concerning the efficacy of VM and CM in order for them to choose the most suitable manipulatives for their students’ purposes (Sarama & Clements, 2016). This study aimed to investigate the possible differential effects of VM and CM in a fraction intervention on fourth- and fifth-grade students’ fraction skills to confirm and extend prior research.
Learning challenges with fractions

Previous studies have shown that students find learning fractions challenging due to their property of having multiple components (e.g., denominator and numerator) which need to be assessed simultaneously (Lortie-Forgues et al., 2015; Tian & Siegler, 2017). When fractions are introduced in primary mathematics education, it is the first time that students need to consider that properties associated with whole numbers do not apply to all numbers (Siegler et al., 2013; Tian & Siegler, 2017). According to Ni and Zhou (2005), many students experience whole number bias which refers to the fact that the properties of positive integers (e.g., single-unit counting scheme) are assumed to also apply to fractions. Also, limited growth in fraction skills between third and sixth grades has been shown to predict later failure in mathematical performance (Jordan et al., 2017).

Virtual and concrete manipulatives in teaching fractions

Sarama and Clements (2016) have suggested that the extent to which manipulatives can be manipulated and are meaningful representations of mathematical ideas, makes manipulatives effective. It is also preferable that those meaningful representations come in many forms rather than introducing students to several different manipulatives (i.e., examples of fractions in real life rather than providing students with a variety of different manipulatives; Sarama & Clements, 2016). Systematic investigations have discovered that manipulatives, whether CM or VM, are useful for learning various areas of mathematics (Corbonneau et al., 2013; Moyer-Packenham & Westenskow, 2013). Still, educators hold many generalizations about using CM or VM concerning for example whether specific manipulatives need to be used with certain age groups (Sarama & Clements, 2016). Manipulatives are not just visual representations, but rather objects that can be manipulated to promote the solving of mathematical problems (Moyer-Packenham & Bolyard, 2016). Concrete manipulatives (CM) are defined as physical objects that students can manipulate and grasp with their hands (Rosen & Hoffman, 2009). Another approach is the use of virtual manipulatives (VM), which can be defined as technology-enabled visual representations of mathematical objects with which students can interact (Moyer-Packenham & Bolyard, 2016).

Using manipulatives during fraction interventions for comprehensive school students has proven effective (Corbonneau et al., 2013; Moyer-Packenham & Westenskow, 2013). Many studies have also discovered that using manipulatives is an effective approach for struggling learners’ (Bouck & Park, 2018; Ennis & Losinski, 2019; Flores et al., 2018; Kim et al., 2015). Moreover, there have been two studies designed for comparing the effects of CM and VM in a fraction intervention to find out whether CM and VM are differentially effective for students’ learning (Mendiburo & Hasselbring, 2014; Moyer-Packenham et al., 2013). These studies, however, have led to contradicting results. Moyer-Packenham et al. (2013) studied the effects of CM and VM in a fraction intervention on third-and fourth-grade students fraction learning (N = 350). The intervention was carried out by the classroom teachers in the CM group and by university affiliated teachers in the VM group. Worksheets and the curriculum varied between the CM and VM group and various manipulatives (e.g., concrete and virtual fraction tiles and circles) were used within groups (i.e., manipulatives were different for each lesson). The frequency of intervention or time spent on individual practice varied between participants. They found no statistically significant differences between the students’ performance in CM and VM groups (Moyer-Packenham et al., 2013).

Mendiburo and Hasselbring (2014) examined the effects of VM and CM in a fraction intervention on fifth grade students fraction knowledge (N = 67) with an emphasis on fraction concepts. The intervention lasted for 10 days and was implemented by a researcher using lesson scripts with both groups following a similar curriculum. Time for exercises was set for both groups. Study limitations included different pre- and post-assessments, a lack of specificity as to the contents of the assessments, and no mention of assessment reliability. After completing the intervention, the VM group outperformed the CM group. The VM group also completed more tasks during the lessons, which
suggests that VM are more time-efficient and being able to conduct more practice tasks could facilitate better learning outcomes observed in the study (Mendiburo & Hasselbring, 2014). However, due to small sample size and other limitations in prior research, further investigation is needed to confirm these results.

**Social validity in using manipulatives in mathematics instruction**

Sarama and Clements (2016) have suggested that the meaningfulness and concrete nature (i.e., representation) of the manipulatives are important factors affecting student learning. Social validity has been used to refer to acceptance toward the intervention and manipulatives usefulness experienced by students (see Ledford et al., 2016).

In previous intervention studies, students have rated their intervention programs as highly accepted (see Ledford et al., 2016; Losinski et al., 2019; Sharp & Dennis, 2017). However, previous research on the effectiveness of VM and CM in fraction interventions has not assessed social validity (Mendiburo & Hasselbring, 2014; Moyer-Packenham et al., 2013). To deduce whether possible differential effects are related to the manipulatives meaningfulness, concrete nature or the acceptance toward the intervention, we adopted the construct of social validity in the present study by asking the students to rate social validity-related questions after the intervention was implemented.

**Present study**

Previous research on the differential effects of VM and CM on fraction learning has included multiple limitations such as including large differences between the VM and CM conditions in terms of intervention duration and using several different manipulatives (Moyer-Packenham et al., 2013), issues concerning the assessments reliability evidence and small sample sizes (Mendiburo & Hasselbring, 2014). Furthermore, due to these limitations, prior research has not been able to contribute to the understanding of the manipulatives features that make the manipulatives effective (i.e., manipulatives time-efficiency and social validity). (2014) 2001). Beyond the work of Mendiburo and Hasselbring (2014), little research has been conducted on whether manipulatives’ time-efficiency could provide additional knowledge in explaining differential effects of VM and CM in fraction learning (i.e., whether completing more practice tasks contribute to enhanced knowledge on fractions; Mendiburo & Hasselbring, 2014). To better understand whether manipulatives’ time-efficiency contributes to better learning outcomes further investigation is needed. Arithmetic fluency is a key component in mathematical learning and is widely used as an indicator of mathematical learning difficulties (Bartelet et al., 2014) and thus controlling for its impact allows tapping into the development of fraction skills. Previous research has not controlled for the impact of arithmetic fluency in students’ fraction learning although it plays a central role as a building block for later mathematical development (Kilpatrick et al., 2001).

The present study investigated the effects of CM and VM in a fraction intervention for fourth- and fifth-grade students to confirm and extend prior research. To address significant limitations in prior research, our intervention employed lesson scripts, similar worksheets across conditions, constant manipulatives, and a set timetable for all lessons. This ensured that all students received as similar treatment as possible to reliably compare the manipulatives efficacy. Furthermore, our study employed measures that were thoroughly developed, reliable and transparently reported as measures are a key factor in evaluating intervention effects (Crawford, 2014). As previous research has not controlled for other mathematical knowledge such as arithmetic fluency, our study aimed to address this limitation, as well. Fidelity of intervention was measured to evaluate that the intervention was implemented similarly in each class. Time-
efficiency of the use of manipulatives and social validity of the intervention material were also investigated to extend prior findings. Specifically, we focused on the following research questions:

1) Is there a statistically significant difference in student’s fraction performance (e.g., in magnitude comparison, converting mixed fractions into improper fractions) between CM and VM intervention groups from pre- to posttest when controlling for the impact of arithmetic fluency performance?

2) Is there a statistically significant difference in time-efficiency of the manipulatives used by CM and VM groups, as measured by the amount of tasks completed during intervention lessons?

3) To what extent do students in the CM and VM groups view the intervention and the manipulative used as socially valid and are there differences in the views between groups?

**Material and methods**

**Participants**

Participants were fourth- and fifth-grade students (N= 115) from six classes in four schools. Students from both grade levels were included as students had been previously introduced to our intervention topics and hence the intervention provided additional support and instruction. All schools were located in the same municipality area in Southern Finland. The participating general education classrooms were selected due to the classroom teachers’ contacts to the research group and willingness to participate in this study. A research permit was received from the municipality educational authorities. Letters containing a description of the study and a consent form were sent to students’ homes. Students who did not return the consent form or did not want to participate were excluded from the study (n= 7). Background data were also collected via the parental consent form and are presented in Table 1.

**Measures**

**Fraction skills**

There was no standardized test available to measure fraction skills in Finnish third to sixth graders and thus, we constructed a fraction measure for the purposes of this study. To secure validity for the developed measure, we describe the development process, test contents, internal consistency, item discrimination and relations to related and non-related variables (American Educational Research Association, American Psychological Assessment & National Council on Measurement in Education, 2014). We measured fractions skills firstly by taking four fraction concept items from a standardized 10SYS-DESI measure (Räsänen, 2004a) and then constructing items based on the fourth- and fifth-grade mathematics textbooks, textbook-based assessments, and the contents of the fraction intervention. As intervention contents concerned the basic skills required to learn fractions and that both

| Background variable               | VM    | CM    | Total |
|-----------------------------------|-------|-------|-------|
| Fourth grade                      | 15    | 21    | 36    | 31.3 |
| Fifth grade                       | 47    | 32    | 79    | 68.7 |
| Boys                              | 38    | 29    | 67    | 58.3 |
| Girls                             | 24    | 24    | 48    | 41.7 |
| Finnish as home language          | 59    | 51    | 110   | 95.7 |
| Finnish and some other as home language | 3   | 1    | 4    | 3.5 |
| Not Finnish as home language      | 0     | 0     | 1     | 0.9 |

Note. N= 115. Participants were on average 11.26 years old (SD = 0.56). The classes in Fourth grade included 15 and 21 students and the classes in fifth grade included 26, 17, 21, and 15 students.
fourth- and fifth-grade students are introduced to these concepts in a similar way, items were based on both grade level’s textbooks. The items were constructed and evaluated co-operatively by a team of educational professionals who gathered in five separate meetings to discuss the measure development. The constructed 44 items were tested with fourth-grade students before the current study to ensure that the items were not too easy (e.g., every student answered correctly on an item asking to color \( \frac{2}{3} \) from a circle divided in five parts) for fourth- and fifth-grade students. Three of the 44 items were deleted and six modified according to the results to make them more challenging. In total, after statistically analyzing and modifying the required items, we ended up with 45 items measuring procedural fractions skills: knowledge of fraction concepts (write the correct fraction represented in a fraction circle; 8 items), fractions magnitude comparison with symbols (mark >, < or = between \( \frac{5}{8} \) and \( \frac{7}{10} \); 8 items) and number line (mark \( \frac{1}{4} \) in a number line of \( \frac{1}{4} \) to \( \frac{3}{4} \); 3 items), converting mixed fractions into improper fractions and vice versa (convert \( \frac{3}{4} \) into a mixed fraction; 8 items), converting fractions into decimal numbers (convert \( \frac{1}{3} \) into a decimal number; 4 items), word problem solving (Simon has read \( \frac{2}{3} \) of his book. How much does he have left?; 6 items), and addition and subtraction (\( \frac{2}{3} + \frac{1}{4} \); 8 items). In the current study, internal consistency (Cronbach’s alpha) for fraction scale was high in both pretest and posttest (Table 3), and item discrimination from .12 to .93 in pretest. To test whether performance in fraction skills items was related to belonging into and intervention group, we used point-biserial correlation (American Educational Research Association, American Psychological Assessment & National Council on Measurement in Education, 2014). Point-biserial correlation coefficients between the intervention group and fraction skills items ranged from \(-.085 \) (\( p > .05 \)) to \(.357 \) (\( p < .001 \)). In addition, moderate to strong correlations between the fraction measure and arithmetic skills (Table 4) confirmed that fraction skills were correlated with a related variable, arithmetic skills (American Educational Research Association, American Psychological Assessment & National Council on Measurement in Education, 2014).

**Arithmetic performance**

We used an arithmetic fluency test, RMAT (Räsänen, 2004b), in pre- and posttest to assess participants’ arithmetic performance. RMAT is standardized for students in grades 3 to 6 and is widely used in Finland to examine students’ arithmetic performance. RMAT consists of 55 items, consisting of whole number addition, subtraction, multiplication and division, converting quantities, decimal number addition and multiplication, converting a fraction into a decimal number, fraction addition and subtraction, and first-degree equations. The test time was 10 minutes (Räsänen, 2004b), and despite the time limit, items had to have the correct answer to be counted as completed. RMAT’s reliability was also high at pretest (Table 3).

**Time-efficiency**

We measured time-efficiency of the manipulatives to provide additional information on the possible differential effects of the manipulatives. To measure time-efficiency, we collected the children’s worksheets used during individual practice time (10 minutes) and compared the number of tasks completed per lesson (Mendiburo & Hasselbring, 2014). To be counted, tasks did not need to be correctly solved, rather just completed in the manner defined in the worksheet directions (e.g., if the task requested a fraction and the student had replied with a whole number, the task was not counted as completed). The total number of tasks each student completed during the lesson was calculated and group means were compared. Every class had the same amount of time for individual practice, so that time-efficiency could be compared between the groups. Students completed the worksheets anonymously.
**Fidelity of intervention**

Fidelity of intervention was measured using a checklist adapted from Dane and Schneider (1998) to measure the extent to which the intervention was implemented within and across treatments. A research assistant observed fidelity of intervention. We adapted the items to align with the aims and contents of the intervention and study design. Also, we adapted the questions concerning the general quality of instruction criteria to suit the Finnish educational system. Checklist items consisted of adherence (i.e., the extent to which specific objectives of the intervention are met), exposure to instruction (i.e., the amount of time spent for instruction and other activity during the lessons), quality of delivery (i.e., the extent to which the instructor follows general quality of instruction criteria; e.g., positive and corrective feedback and active participation) and program differentiation (i.e., which manipulatives are used during the lesson).

**Social validity**

In order to determine whether possible differential effects between the intervention groups are related to the manipulatives’ meaningfulness, concrete nature or the acceptance toward the intervention, social validity was measured. During the last lesson of the intervention, participating students completed a questionnaire on the social validity of the intervention and the used manipulative. We designed the social validity measure for the purposes of this study. The questionnaire included a total of six questions concerning the acceptance of the intervention (Ledford et al., 2016). To examine the participants’ views, we asked the students about the usefulness and ease of using the manipulative, the amount of new skills learned from fractions and decimal numbers, and about the stories listened during intervention lessons and the intervention as a whole (see Table 7). The students provided their views on a 4-point scale formed from smiley faces or hand signals (e.g., thumb up). Social validity was analyzed item-by-item. Students completed the questionnaire anonymously. We tested the questionnaire in the piloting phase on fifth-grade students, and two open-ended questions were deleted because they did not produce appropriate information as most of the students responded to these questions by stating that they did not know.

**Intervention**

Intervention content and procedures were identical for the VM and CM group, apart from groups using different manipulatives. Each 45-minute lesson took place in the students’ own classrooms in a whole-group setting. Each lesson consisted of a single topic and included: (a) fraction concept and notation; (b) fraction magnitude comparison; (c) mixed fractions concept, notation and comparison; (d) word problems and multiple representations concerning previous fraction topics; and (e) decimal numbers concept and relationship to fractions and word problems concerning decimal numbers,

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**Table 2.** Intervention lessons structure, aims, and time spent on the activity.

| Lesson section                                      | Aim                                      | Number of minutes spent |
|-----------------------------------------------------|------------------------------------------|-------------------------|
| Introducing the lesson plan                         | Structure                                | 5                       |
| Revising previous lessons                           | Reviewing previous topics to build on that knowledge | 5                       |
| Listening to a prerecorded story                    | Introduce the new topic in a context, raise questions and provide a continuum for the lessons | 5                       |
| Teaching a new subject via manipulatives            | To explicitly teach the new subject through examples and engage students into learning by interactive instruction | 10                      |
| Modeling the use of manipulative in the tasks       | To explicitly show how the students’ manipulatives are best used in the tasks | 5                       |
| Individual practice                                 | To let the students individually practice via their own worksheets and use the manipulative, implemener providing feedback and individual assistance | 10                      |
| Word problem solved jointly                         | To review a word problem concerning the lessons topic and to present how to solve it | 5                       |
Table 3. Mean performance and ANOVA results for fraction skills and arithmetic fluency.

|                  | Total | VM | CM |
|------------------|-------|----|----|
|                  | N     | M  | SD | Range | a | n | M  | SD | Range | n | M  | SD | Range | F | df | p   |
| Fraction skills  |       |    |    |       |   |   |    |    |       |   |    |    |       |   |    |     |
| Pre              | 111   | 20.32 | 11.69 | 1–44 | .955 | 61 | 22.23 | 11.31 | 1–44 | 50 | 18.00 | 11.83 | 1–43 | 3.69 | 1,109 | .057 |
| Post             | 115   | 28.04 | 10.36 | 2–45 | .940 | 62 | 28.56 | 10.73 | 2–45 | 53 | 27.43 | 9.98  | 4–44 | 0.34 | 1,113 | .562 |
| Arithmetic fluency – Pre | 105   | 31.53 | 7.43 | 12–52 | .876 | 59 | 32.66 | 7.42 | 19–52 | 46 | 30.09 | 7.26  | 12–42 | 3.17 | 1,103 | .078 |
respectively. The lessons that contained word problems and multiple representations tasks focused on area and number line models for fractions and decimal numbers so that the students would understand relationships of the concepts, as well as for the students to extend their knowledge of different representations (Sarama & Clements, 2016). All tasks focused on procedural fraction knowledge. The theme of the intervention was space travel and it appeared in the stories provided during the lessons and in the word problems. Every intervention lesson also followed a consistent structure, presented in Table 2.

In the beginning of the lesson, the implementer introduced the students to the lesson plan and reviewed the previous lesson’s topic with a jointly solved task. To introduce the lesson’s topic, the implementer played a prerecorded story and then taught the topic explicitly through guided practice. To make clear how the students’ manipulatives were used in the tasks, the implementer used the manipulative when teaching the new topic and then explicitly showed how the manipulatives were best used in the tasks. After this, the students had time for individual practice via worksheets and their manipulatives. We constructed the worksheets for this study. The worksheets contained pictorial models, symbolic representations, and tasks for representing fractions with the CM or VM. This ensured that concrete, representational and abstract representations and tasks were available. The tasks became progressively more difficult toward the end of each worksheet and over the duration of the intervention. To avoid ceiling effects, the worksheets had up to 80 tasks to avoid students completing all items before the end of the practice period. All tasks were done on the worksheets and collected after the individual practice time. After individual practice, the implementer introduced a fraction word problem concerning the lesson topic that was solved jointly for review.

**Concrete and virtual manipulatives**

The CM group used plastic fraction circles as CM. The students got their own package of fraction circles each lesson when individual practice time started. The package contained 1/1, 2/2, 3/3, 4/4, 5/5, 6/6, 8/8, 10/10, and 12/12 circles. The pieces were not labeled. For mixed fractions lessons, we provided the students with paper wholes. The CM allowed students to show different parts of fractions and compare the magnitude by moving the circles on top of one another (i.e., when the fractions had different denominators).

The VM group used a virtual version of the concrete fraction circles. It was provided by the Math Learning Center (2017) and used mostly via iPads. One classroom could not provide every student with an iPad, so up to 30% of the students used a Chromebook. The VM allowed the students to display different numbers of fractional parts and color them in a desired way to represent the appropriate fraction. The VM also allowed the students to drag fraction circles on top of one another for a comparison (Math Learning Center, 2017). In terms of “manipulability,” VM allowed the students to present as many fraction circles and divide them into as many parts as they desired. Also, in contrast to CM, students were also able to drag the circles on top of another which was not that easy by using CM.

| Table 4. Correlation coefficients for fraction skills and arithmetic fluency. | 1      | 2 | 3 |
|--------------------------------------------------------------------------|-------|---|---|
| **VM**                                                                  |       |   |   |
| 1. Fraction skills – pre                                                | –     | .775** | .661** |
| 2. Fraction skills – post                                               | .775** | – | .615** |
| 3. Arithmetic skills                                                    | .661** | .615** | – |
| **CM**                                                                  |       |   |   |
| 1. Fraction skills – pre                                                | –     | .689** | .461** |
| 2. Fraction skills – post                                               | .689** | – | .464** |
| 3. Arithmetic skills                                                    | .461** | .464** | – |

**p < .01.
Table 5. Results from ANCOVA controlling for arithmetic fluency.

|                                | F (1, 101) | p     | partial $\eta^2$ |
|--------------------------------|------------|-------|------------------|
| Time * group                   | 4.13       | .045  | .039             |
| Time                           | 8.40       | .005  | .077             |
| Group                          | 0.06       | .804  | .001             |
| Time * Arithmetic fluency      | 64.74      | < .001| .391             |
| Arithmetic fluency             | 0.47       | .493  | .005             |

**Procedure**

We conducted a pilot study with the same design for testing the intervention and materials prior to this study. Only minor modifications were needed to the materials. During piloting of the intervention, it became clear that both groups would need paper worksheets due to the difficulties associated with using digital worksheets. Following the pilot, we added tasks to the worksheets so that every student had enough tasks for the time allocated to individual practice. Also, we added tasks guiding students to use more of their manipulatives.

This study employed a quasi-experimental pre- and posttest research design, and we divided the participating classrooms into two intervention groups: VM and CM intervention groups. The sampling occurred at the classroom level, due to practical (i.e., participating schools were not in close proximity to one another) and economical considerations (i.e., due to no external funding, there was only one intervention implementer available). The intervention was implemented for 45 min per session three times a week over two weeks. The first author implemented 71% of the intervention lessons. In addition to the first author, one classroom teacher implemented the intervention lessons for her own class and a research assistant implemented two lessons for another class. The lessons were scripted, and we printed and organized all the materials for the implementers before the lessons. The first author met with the two other implementers before the beginning of the intervention to go through the lesson plans, worksheets, and using the manipulatives so that lessons would be carried out in the same way. Both of them also had the opportunity to contact the research group and consult on any matter, as needed.

Data were collected before the intervention during the last week of January 2020 and one week after completing the intervention, during the last week of February 2020. A delayed posttest was scheduled for seven weeks after completing the intervention but was canceled due to the Covid-19 outbreak causing schools to move to remote instruction. The first author administered the fraction scale at both time points as a paper-and-pencil test in the students’ own classrooms. The classroom teacher was always present during the assessments. One 45-minute lesson was allocated for fraction scale and one for the RMAT. Both assessment sessions were administered within a three-day time period. Students were not allowed to use manipulatives during these assessments. Students completed the 10-minute paper-and-pencil social validity questionnaire during the last intervention lesson.

**Statistical analyses**

For answering the first research question, a 2 (time) × 2 (condition) mixed ANCOVA was conducted to determine statistically significant change in fraction skills over time and possible differences between groups. We included arithmetic fluency as a covariate to isolate the observed difference in fraction skills. 2 × 2 mixed ANCOVA is interpreted from the main effects and interaction.

Using anonymous datasets, One-Way ANOVA was also used to answer the second and third research questions concerning manipulatives’ time-efficiency and differences in social validity between the intervention groups. Only one analysis of variance was ran for a specific dataset for answering the research questions and thus, no $p$-value correction was needed. All data were analyzed using a 95% confidence interval.
In preliminary analyses, missing data was analyzed using Little’s MCAR Test and normal distribution was examined. As sampling occurred at the classroom level, the difference between the classes in fraction knowledge gain scores (i.e., individual change between pre- to posttest scores) was analyzed using one-way ANOVA. Correlation coefficients were also investigated to determine whether performance in fraction skills and arithmetic fluency correlated.
From the 115 participants, 99 students completed the fraction skills and arithmetic fluency measures at both time points. Missing data analysis revealed that the missing data were missing completely at random (Little’s MCAR Test: $\chi^2 = 220.93, df = 254, p = .934$). All variables were normally distributed. There was no statistically significant difference in gain scores between the classes, F(5, 105) = 1.405, $p = .229$, which indicates that a specific class did not have an impact on the results when comparing the treatment groups.

Descriptive statistics and results from ANOVA for fraction skills and arithmetic fluency are presented in Table 3. According to means, the VM group outperformed the CM group in fraction skills at both time points, as well as in arithmetic fluency. However, these differences were not statistically significant. Change in fraction performance from pre- to posttest indicated that both groups made gains in fraction scale scores during the intervention.

Correlations for fraction skills and arithmetic fluency are presented in Table 4. All correlations were moderate, positive, and statistically significant. The strongest correlations were found between the pre- and posttests fraction skills. Also, the VM group presented stronger correlations between tests than the CM group. The correlation between fraction skills and arithmetic fluency in the CM group was smaller than in the VM group.

Fidelity of intervention was assessed for 20% of the intervention lessons by a research assistant using a fidelity of intervention checklist. Mean fidelity scores were: (a) adherence at 100%; (b) exposure to instruction at 92%; (c) program differentiation at 100%; and (d) quality of delivery at 92%. These results indicate that the groups received the same intervention apart from the type of manipulatives.

**Main analyses**

$2 \times 2$ mixed ANCOVA results are presented in Table 5. When the change from pre- to posttest in fraction skills was examined, a statistically significant interaction between the CM and VM group over time was found when controlling for arithmetic fluency (Figure 1). Even though the VM group outperformed the CM group in pretest, this result indicates that the CM group gained in fraction scale scores more from the intervention when controlling for arithmetic fluency.

Both groups made statistically significant gains during the intervention as a significant main effect of time was found (Table 5). Regardless of time, the scores did not differ between the VM and CM groups, as there was no statistically significant main effect (Table 5).

A statistically significant main effect for arithmetic fluency was found (Table 5) suggesting that arithmetic fluency performance had an impact on students’ scores in fraction skills and thus controlling for fluency was important. However, arithmetic fluency scores did not affect change in fraction skills over time, as no statistically significant interaction for time and arithmetic fluency was found (Table 5).

**Manipulatives time-efficiency**

The time-efficiency of manipulatives was calculated using worksheets completed in the lessons. Descriptive statistics and ANOVA statistics are presented in Table 6. According to means, the VM group completed more tasks during the lessons on four of the six lessons. Statistically significant differences between the VM and CM group were detected in lessons two, four, and five, with differences favoring the VM group.

**Social validity**

A total of 108 participants completed the questionnaire on social validity. It was coded so that the least desirable option (i.e., sad face) was coded as one and the most desirable (i.e., happy face) as 4. Descriptive statistics and ANOVA statistics are presented in Table 7. Means on the students’ social validity questionnaire indicate that students rated all but one question as above 3. The only question
where students’ mean ratings were below three was when students were asked about their manipulative’s usefulness. ANOVA statistics indicate that students responding on the social validity questionnaire in VM and CM groups differed on one question which concerned their views on the stories listened in the lessons.

**Discussion**

This study investigated the differential effects of VM and CM in a fraction intervention on fourth- and fifth-grade students’ fraction skills. Our findings suggest that CM should be favored over this particular VM in fraction interventions as the CM group outperformed the VM group in fraction skills over the course of the intervention. Our findings contradict some previous results (Mendiburo & Hasselbring, 2014; Moyer-Packenham et al., 2013), but are in line with other previous fraction intervention studies, though these studies have been conducted in a special education context (Flores et al., 2018; Kim et al., 2015). Contradictory results to prior research may have resulted from controlling for the impact of arithmetic fluency as it is a central part of mathematical development (Kilpatrick et al., 2001) and prior research has not taken it into account (Mendiburo & Hasselbring, 2014; Moyer-Packenham et al., 2013).

Our findings on the manipulatives’ time-efficiency suggest that VM is more time-efficient than CM, which is consistent with previous research (Mendiburo & Hasselbring, 2014). Our results indicate that VM may be more time-efficient but the effects for learning were not as good as when using CM, which is in contradiction with previous research (Mendiburo & Hasselbring, 2014). Moyer-Packenham and Suh (2012) have argued that VM provides students with cognitive “off-load” and that CM can lead a student to get lost with one’s idea while browsing for example, the fraction circles and finding the right pieces. The property of providing cognitive “off-load” may well be the reason for VM being more time-efficient in our study. However, this property did not seem to contribute to better fraction knowledge.

Results from our social validity questionnaire indicated that students in both groups liked the intervention program, noted that they had learned new skills concerning both fractions and decimal numbers, and thought that their manipulative was easy-to-use. These results represent the same pattern as in previous fraction intervention studies (Losinski et al., 2019; Sharp & Dennis, 2017) and reflect the students’ acceptance toward the intervention (Ledford et al., 2016). However, students did not view their manipulatives as very useful for them, which causes concern. Clearly, we need more good quality studies about how and which kind of manipulatives meet the students’ needs. For example, in a recent study students preferred VM over CM though effects for learning were similar using either manipulative (Bone et al., 2021). As we compared the answers between the CM and VM group, we found that despite the fact that the CM group liked the stories more than the VM group, differences concerning other questions were not found. Previous research on the efficacy of VM and CM has not addressed social validity (Mendiburo & Hasselbring, 2014; Moyer-Packenham et al., 2013) and thus, our findings extend prior research on students’ views on a fraction intervention using VM and CM.

Our study had some limitations. First, we recognize that our sample was relatively small and selected and randomized studies concerning the differential effects of VM and CM in a fraction intervention are needed. Our intervention was also relatively short compared to previous studies on the effects of VM and CM (Mendiburo & Hasselbring, 2014; Moyer-Packenham et al., 2013) and extending it might affect the results. It is, however, notable that even this short intervention had a significant impact on the students’ fraction skills, regardless of the manipulatives used (Jordan et al., 2017). We recognize that including a business-as-usual control group would have provided us with stronger research design on drawing these implications (Thomas et al., 2004). Letting students use both manipulatives and choose themselves what to use could also result in different findings concerning the intervention’s social validity and could provide more insight on whether the students’ view both manipulatives as equally valid. Manipulatives used in this study were also somewhat restricted in
terms of representation and manipulability. For example, using linear representations such as number lines instead of circular models such as fraction circles would allow for a more explicit representation that all real numbers have magnitudes that can be located in the number line to facilitate understanding of magnitudes and part–whole relationships (Siegler & Pyke, 2013). Thus, investigating effects of different kinds of manipulatives that would allow less restricted use could provide more information. We acknowledge that the fraction and social validity measures were developed for this study and are not thereby supported by results in other studies, though we aimed to address this limitation by reporting validity evidence in an extensive manner by following the Standards for Educational and Psychological Testing (American Educational Research Association, American Psychological Assessment & National Council on Measurement in Education, 2014). In terms of analyzing, including social validity and manipulatives’ time-efficiency as predictor variables in a regression model and measuring time-efficiency by examining the time that was spent using the manipulative itself could have resulted in more sophisticated findings. However, as social validity and time-efficiency information was gathered anonymously, this was not possible with the current data.

Our findings are especially important for educators who are considering whether to use VM or CM in fraction instruction (Sarama & Clements, 2016). Our results implicate that CM can be favored during fraction interventions as students viewed them as valid and gained more from using them. Noticeably, our results indicate that both groups made statistically significant gains and as it may be that VM is more easily available, using VM rather than no manipulatives would also be appropriate if CM is not available or instruction is given remotely.

Disclosure Statement

No potential conflict of interest was reported by the authors.

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