The partonic origin of multiplicity scaling in heavy and light flavor jets

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Abstract: It has recently been shown that a KNO-like scaling is fulfilled inside the jets, which indicates that the KNO scaling is violated by complex vacuum-QCD processes outside the jet development, such as single and double parton scattering or softer multiple-parton interactions. In the current work we investigated the scaling properties of heavy-flavor jets using Monte-Carlo simulations. We found that while jets from leading-order flavor-creation processes exhibit a flavor-dependent pattern, heavy-flavor jets from production in the parton shower follow the inclusive-jet pattern. This suggests that the KNO-like scaling is driven by initial hard parton production and not by processes in the later stages of the reaction.

Keywords: High-energy hadron collisions; jets; KNO scaling; heavy-flavor; dead-cone effect

1. Introduction

Final-state multiplicities in small colliding systems are known to follow a negative binomial distribution (NBD) regardless of the collision species over several orders of magnitude of energy ranges [1–3]. It has also been observed in $e^+e^-$ collisions that the multiplicity distributions at different collision energies collapse into a single distribution when the so-called Koba-Nielsen-Olesen (KNO) scaling [4,5] is applied. The KNO-scaling was, however, found to be violated at higher energies and in more complex, hadronic collision systems [6,7]. The origin of the scaling, and the reason for its breakdown is still not understood completely, although many explanations have been proposed in the past decades [8–12]. In our earlier study we found that a KNO-like scaling is fulfilled within the jets. This indicates that KNO scaling is violated by complex quantum-chromodynamics (QCD) processes outside the jet development, such as single and double-parton scatterings or softer multiple-parton interactions (MPI) [13].

In our manuscript we investigate the scaling properties of heavy-flavor (HF) jets in comparison to an inclusive jet sample. Heavy flavor is mostly produced in hard (large momentum-exchange) processes, in the early stages of the collision. The most relevant perturbative QCD processes that contribute to the production cross section are leading-order (LO) flavor creation, and next-to-leading order (NLO) gluon splitting as well as flavor excitation [14]. The parton shower and fragmentation of heavy-flavor jets is different from light-flavor jets due to two main reasons: the color charge effect, that is, heavy flavor jets are initiated by quarks as opposed to light-flavor jets that are mostly gluon-initiated [15]; and the dead-cone effect, meaning that small-angle gluon radiations off a massive parton are forbidden in QCD, and as a consequence, heavy-flavor fragmentation is harder and results in different jet substructures [16–19].

In the current work we model both light and heavy-flavor jets using the PYTHIA 8 Monte-Carlo event generator [20], and we differentiate the samples according to the process the jets are created in. Whether the KNO-like scaling observed for inclusive jets is retained or violated in heavy-flavor jets can shed light on the origin of the scaling itself, and also on possible mechanisms that are responsible for the violation of the scaling in heavy-flavor jets. The methods we present can further be used to gain insight to the flavor-dependent evolution of the jets. Future measurements targeted on the scaling of light and heavy-flavor jets can also serve as a validation tool for heavy-flavor production and fragmentation models.
2. Analysis Method

We simulated proton-proton collisions at $\sqrt{s} = 7$ TeV center-of-mass energy utilizing the PYTHIA 8 (version 8.226) Monte-Carlo event generator with the Monash tune and HardQCD settings [20,21]. PYTHIA 8 is tuned to describe both the fundamental physical observables of the leading hard process and the underlying event, and it is known to reproduce final-state multiplicities well [22,23]. In PYTHIA 8 the hard parton scatterings and decays are simulated using LO matrix elements (ME). These are amended by initial and final state radiations, which create the parton shower (PS) in perturbative QCD calculations based on DGLAP splitting kernels [20], as well as soft and hard multiple-parton interactions integrated into a single framework [24]. The hadronic final state is then produced with the Lund string fragmentation model [25].

Using the possibility in PYTHIA 8 to restrict event generation to certain hard processes, we created four different sets of data. As the baseline for our study, we used an inclusive-jet sample, where any hard QCD scattering process was allowed above an appropriately selected value of the minimal momentum transfer in the hardest process ($p_T^\text{jet}$) depending on the jet transverse momentum ($p_T^\text{jet}$), as detailed in [26]. Next, we used samples with ME flavor creation, where hard 2 → 2 parton scatterings were allowed only with heavy-flavor outgoing partons: $gg \rightarrow b\bar{b}(cc)$ and $q\bar{q} \rightarrow b\bar{b}(cc)$. This provided wide-angle heavy-flavor jets created directly in the leading process of the event. Finally, we created a sample that is dominated by b-jets from the PS, by allowing only those 2 → 2 processes that do not directly create heavy flavor: $gg \rightarrow gg$, $gg \rightarrow q\bar{q}$, $ag \rightarrow q\bar{q}$, $q\bar{q} \rightarrow gg$, $q\bar{q} \rightarrow q'\bar{q}'$ (where incoming HF is allowed, but only light flavor exits), and finally three more processes: $q\bar{q} \rightarrow q\bar{q}$, $q\bar{q} \rightarrow q\bar{q}$ and $q\bar{q} \rightarrow q\bar{q}$ (where outgoing and incoming flavors are the same and $q$ and $q'$ may be of the same flavor) [27]. In this case the heavy quark pair is produced in a later step, e.g. in a $g \rightarrow b\bar{b}$ gluon splitting process, typically with smaller opening angles. The heavy quarks then often manifest in the final state as secondary jets besides the leading jet, or may even end up in the same jet.

In all cases, charged-particle jets were clustered from final-state charged pions, kaons and (anti)protons with $p_T > 0.15$ GeV using the anti-$k_T$ jet-clustering algorithm [28] with a resolution parameter of $R = 0.7$ in the mid-rapidity range $|\eta| < 1$ and full azimuth coverage. The reconstructed jets were categorized in 20 different $p_T^\text{jet}$ ranges, from 15 GeV up to 400 GeV. In the case of the charm and beauty jet samples, the corresponding heavy quark was required to fall within the cone of the selected jet, similarly to jet-tagging methods that are utilized in the experiment [29,30].

3. Results and Discussion

In Fig. 1 we plot the mean and RMS values of the event multiplicity ($N$) distributions at central pseudorapidity ($|\eta| < 1$), in function of $p_T^\text{jet}$, separately for inclusive jets, b-jets and c-jets from ME flavor creation as well as for b-jets from parton shower processes. As one expects, events having jets with a higher $p_T^\text{jet}$ contain more final-state hadrons on the average, and the distribution also gets broader toward higher $p_T^\text{jet}$. Heavy-flavor jets from ME flavor creation correspond to a lower average $p_T^\text{jet}$ than light-flavor jets from parton shower. Heavy-flavor jets from ME flavor creation correspond to a lower average multiplicity at a given $p_T^\text{jet}$, while heavy-flavor from the parton shower follows the trend of inclusive jets. The difference is especially prominent for higher $p_T^\text{jet}$.

As a next step we fitted the multiplicity distributions with a negative binomial distribution function in each of the jet transverse momentum ranges,

$$P_N = \frac{\Gamma(Nk + a)}{\Gamma(a)\Gamma(Nk + 1)} p^{Nk} (1 - p)^a,$$

where $a$, $k$ and $p$ are parameters related to the mean and dispersion of the distribution of the multiplicity $N$. In Fig. 2 we show the multiplicity distributions after all the $p_T^\text{jet}$ ranges have been scaled on top of each other using the NBD fits. The scaling approximately holds for all four jet samples we investigated. However, for jets containing charm or beauty from flavor creation, the data show minor departures from the NBD fits: the distribution is wider for larger $p_T^\text{jet}$, while narrower for smaller $p_T^\text{jet}$ values.

To quantify the deviations from the scaling behavior, and also to mitigate the effect of fluctuations, we calculated the higher moments of the multiplicity distributions in a similar manner to [13]. Here the $q^{th}$ moment in a given $p_T^\text{jet}$ window is defined as

$$\langle N^q \rangle = \sum_{N=1}^{\infty} P_N N^q,$$

(2)
where $P_N$ is the probability distribution corresponding to the event multiplicity $N$. If the scaling is fulfilled and the mean of the distribution scales with a factor $\lambda$, then it is expected that the $q^{\text{th}}$ moment scales with $\lambda^q$ as

$$\langle N^q(p_{T}) \rangle = \lambda^q \langle N^q(p_0) \rangle,$$

(3)

where $p_0$ is chosen so that the scaling factor is $\lambda(p_0) = 1$.

In Fig. 3 we show the first nine moments of the multiplicity distributions divided by their order $q$ in function of the mean charged-hadron multiplicity $\langle N \rangle$ at $|\eta| < 1$, on a log-log scale. The four panels correspond to the four jet categories. The linear fits show a similar trend for all four cases, which means that the scaling is present also for heavy-flavor jets.

Fig. 4 summarizes the slopes of the fits for the first nine statistical moments, as well as the goodness-of-fit parameter $\chi^2/NDF$. All slopes are around unity within $\approx 5\%$. As expected, the goodness of the linear fits is worse for higher moments. The b-jets coming from parton shower processes follow the same trend as the inclusive jets. On the other hand, heavy-flavor jets coming from matrix element production in the simulations correspond to slope parameters that are slightly but significantly different from unity: in case of charm, slopes of the fits for moments $2 \leq q \leq 6$ tend to be lower than unity, while in the beauty case the slopes for moments $q \geq 7$ are larger than unity. Furthermore, the goodness of fits for HF ME production tends to be worse than for inclusive jets, $\chi^2/NDF \geq 10$ for any $q \geq 5$. This suggests that the KNO-like scaling originates from the hard parton production, and it is less influenced by the parton shower. The similar patterns of the inclusive jets and the b-jets from PS also indicate that event multiplicities are not driven by flavor-dependent jet fragmentation processes.

4. Conclusions

In this manuscript we summarize our results on the scaling properties of heavy-flavor jets from different production processes, and compare them to those on inclusive jets. We used PYTHIA 8 simulations to evaluate the charged-hadron event multiplicities at central pseudorapidity, in function of the charged-particle jet transverse momentum within the range $15 < p_{T}^{\text{jet}} < 400 \text{ GeV/c}$. We found that the multiplicity distributions satisfy a KNO-like scaling with $p_{T}^{\text{jet}}$ for charm and beauty jets similarly to what has been observed for inclusive jets. We note, however, that multiplicity distributions in events with jets initiated by charm and beauty directly from the leading hard process show some departure from the negative binomial shape, depending on the $p_{T}^{\text{jet}}$. Further analysis of the statistical moments of the multiplicity distributions shows that the scaling is
Figure 2. Charged-hadron multiplicity distributions with an NBD fit at $|\eta| < 1$, for all $p_T^{\text{jet}}$ ranges, scaled by the NBD fit means. The four panels from top left to bottom right correspond to inclusive jets, charm and beauty jets from ME flavor creation, and beauty jets from PS production.

A good description of hadron multiplicity distributions is a basic requirement for models and it is generally fulfilled by the most widely used event generators. However, multiplicities in function of the jet momentum for jets tagged with different flavors can provide means to further validate heavy-flavor production and fragmentation models. Also, while event multiplicity is a good proxy for jet multiplicity in case of jets coming from the leading hard process, this is not necessarily the case for jets that come from secondary hard processes or gluon radiation. An interesting extension of the current work in this direction could therefore be to evaluate the scaling in terms of the jet multiplicity instead of event multiplicity, and to see whether in that case the scaling of heavy flavor jets from the parton shower follows light or heavy jets.

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Figure 3. The first nine moments of the charged-hadron multiplicity distributions at $|\eta|<1$, in function of the average multiplicity corresponding to each $p_T$ range. The four panels are for inclusive jets, charm and beauty jets from ME flavor creation, and beauty jets from PS production. The distributions are normalized by their order $q$ on a log-log scale, and linear fits are applied.

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Figure 4. The slope parameters (top panel) and the goodness-of-fit parameters $\chi^2/NDF$ (bottom panel) of the linear fits for the first nine statistical moments of the multiplicity distributions, for charm and beauty jets from ME production and beauty jets from PS production, compared to that for inclusive jets, in function of the order of moments of the multiplicity distributions.

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