Developing Lesson Design to Help Students’ Triangle Conceptual Understanding

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Abstract. The research was aimed to develop a lesson design so that students' triangle conceptual understanding could be achieved. The method that was used in this research was qualitative with applied didactical design research (DDR). The DDR consisted of three main steps, namely prospective analysis, metaphedidactic analysis, and retrospective analysis. From the three steps above, it was gained the empirical lesson design of triangle topic. The research results are: (1) there were some learning obstacles of students deal with the triangle topic, namely ontogenical, epistemological, and didactical obstacles; (2) implementation of the lesson was conducted under three main steps, namely action, formulation, and validation. For answering whether the design can be applied to other group of students, it was recommended that it could be investigated by doing advanced research.

1. Introduction
Triangle topic is one of the topics that should be taught on mathematics subject in school. This topic is essential in mathematics. It is said to be essential because this topic is used as a basis for learning of some other mathematical topics. Nevertheless, the facts on some schools show that some teachers have difficulties in developing mathematics learning model, including triangle topic, which is able to activate students in mathematical thinking process and teachers also have difficulties in contextualizing mathematical concept in their problem activities [1]. The difficulties faced by the teachers because of the lack of understanding of mathematical concepts, including the topic of triangle, should be taught. Thus understanding the concept of a triangle is very important for students of mathematics teacher candidates.

Some teachers state that people can be said to understand mathematics if they have memorized the concept and they are skilled at implementing the procedure in accomplishing mathematical tasks. Learning built on this view will produce students with discrete knowledge [2]. This is not in line with the purpose of mathematics learning which is expected to encourage the improvement of critical, critical, and logical thinking. There are two kinds of teaching beliefs regarding students’ learning: learning as knowing and learning as understanding [2] which can be briefly presented in Figure 1 below.
There are three main factors that influence a teacher's decision in determining learning components in the classroom: (1) knowledge, (2) beliefs, and (3) ways to access student cognitive development that involves observation of their behavior [3]. Mathematics teacher knowledge includes the content of mathematical material, pedagogy, and student cognition in mathematics. These three components interact with each other to produce specific knowledge according to the context or situation in the classroom [4]. The differences of teachers' belief about mathematics will result in differences of the teaching implementations. There is a high degree of consistency between the conception of the teacher and the teaching practices he or she performs. This is in accordance with the statement “.... Lynn, whose view of mathematics was best characterized as instrumentalist, taught in a prescriptive manner emphasizing teacher demonstrations of rules and procedures. Jeanne, on the other hand, viewed mathematics primarily as a coherent subject consisting of logically interrelated topics and, accordingly, emphasized the mathematical meaning of concepts and the logic of mathematical procedures. Finally, Kay, who held a problem-solving view of mathematics, emphasized activities aimed at engaging students in the generative processes of mathematics [5].

In relation to how students think, Suryadi [6, 7, 8] through DDR developed a metapedadidactical triangle, which includes pedagogical relationship (PR), didactical relationship (DR), and didactical-pedagogical anticipation (DAP) as shown in figure 2 following.

Pedagogical relationships are relationships related to how teachers develop pedagogical aspects based on teacher's knowledge of the characteristics of the students. While didactical relationship is a relationship associated with how students respond to the tasks it faces. Meanwhile, didactical-pedagogical anticipation is the anticipation prepared by teachers related to teaching materials and learning students’ obstacle. Metapedadidactic consists of three integrated components. The sequence of activities within the Metapedadidactic framework occurs before, during, and after learning. Teacher
thinking activities before learning emphasizes more on the activity of designing a didactic situation to be undertaken in the learning process. These activities include recontextualization, repersonalization, and prediction of student responses.

Based on the above description, this research question was, "How to create a lesson design of triangle and its implementation so that students' understanding on triangle concept can be achieved?"

2. Method

This study used a qualitative method by applying DDR consisting of three main stages, namely (1) didactical situation, a analysis before the learning (prospective analysis) in the form of hypothetical didactic design or hypothetical learning trajectory (HLT), (2) metapedadidactic analysis related to real situation of learning process, and (3) retrospective analysis. In the analysis, we explored ways of thinking through repersonalisation and recontextualization techniques that focus on triangle topics. From these three main stages, it was obtained an empirical didactic design. The subjects of this research were Mathematics Education students of one of the universities in Bandung who were taking Mathematics Selected Course, as many as 15 people. Students' learning were conducted through group discussions to develop meaningful learning experiences that would be useful in other mathematical topics. The data analysis was done through the analysis of the logbook made by the students and the discussion in the classroom.

3. Result and Discussion

The focus of developing the lesson design was directed to the re-contextualization aspect of the concept and terms of a triangle, by exploring the facts about the relation of three positive real numbers that can be the size of the sides of a triangle. Lecturing was conducted by assigning tasks in order to build concepts and conjectures about the facts/principles of triangle. Students was asked to focus on the thinking activity in completing the tasks. Students' completion were intended to provide a thinking experience. This lecturing had three main stages: action, formulation, and validation.

3.1. Action

At the stage of action, the researcher showed an image of a 3 x 3 square like figure 3 and asked the students to estimate what kind of tasks will be assigned.

![Figure 3. 3x3 square](image)

Some of the student responses that emerged were; (1) searching for the number of squares, (2) searching for the number of rectangles, looking for the area, (4) looking for the Circumference square, (5) filling by letters, (6) filling numbers, (7) placing the number so that the sum of rows equals the sum of columns equals the sum of diagonals. Furthermore, the researcher said that the task is to fill the boxes with the numbers 1, 2, 3, 4, 5, 6, 7, 8, and 9 so that the sum of rows equals the sum of columns equal to the sum of diagonals within 5 minutes. For students who had successfully filled the boxes were asked to give hint to the students who had difficulty, not by way of telling directly, but none of the students who do it because she/he was afraid of giving information directly. Furthermore, the researchers conveyed several questions that were faced to help students who have difficulty. The questions were: (1) of the nine boxes, was there a middle position? (2) if the numbers were sorted, which number was in the middle and Was there a link between the location of the box and the sequence of numbers? With the clue, the students filled the middle box with the number 5 and by
experiments, they filled in the other boxes with the corresponding numbers successfully. One of the students' work is as shown in Figure 4 below.

|   |   |   |
|---|---|---|
| 4 | 9 | 2 |
| 3 | 5 | 7 |
| 8 | 1 | 6 |

*Figure 4. Filled 3x3 square*

The researcher conveyed that there were 8 combinations of the numbers in the square that had sum of 15, namely: \{4, 9, 2\}, \{3, 5, 7\}, \{8, 1, 6\}, \{4, 3, 8\}, \{9, 1, 5\}, \{2, 7, 6\}, \{4, 5, 6\}, and \{2, 5, 8\}. The next researcher states that if each number represents a line segment with the same unit, such as cm, which set is the size of the sides of a triangle? Do it in 10 minutes! Generally the students drew a triangle and come to the conclusion that not all of the 8 sets could form a triangle and only three sets could form triangles, namely \{3, 5, 7\}, \{2, 7, 6\}, and \{4, 5, 6\}.

3.2. Formulation

The researcher assigned the students to find the requirement that the set of three numbers be the size of the sides of the triangle. Students were required to complete it within 10 minutes. There are several variations of how the students wrote these terms: (1) Three positive real numbers would be the sides of a triangle if the two smallest numbers were summed up was larger than the third. (2) Three positive real numbers would be the size of the sides of a triangle if the largest number was less than the sum of the other two numbers. When they were asked to write the condition (s) with mathematical sentences, initially the students were quite difficult, but after they discussed in the group they finally got the mathematical sentence as follows: Let \(a, \ b, \ c\) positive real numbers. If \(a \leq b \leq c\) and \(c < a + b\) then \(\{a, \ b, \ c\}\) is the size of the sides of a triangle.

3.3. Validation

The conclusions obtained by the students need to be validated. Therefore, the students were asked to determine the set of these numbers which could be the sides of a triangle. A. \{24 cm, 25 cm, 7 cm\}, b. \{12 cm, 13 cm, 4 cm\}, c. \{12 cm, 13 cm, 25 cm\}, d. \{11 cm, 14 cm, 15 cm\}, And e. \{9 cm, 11 cm, 22 cm\}. This task could be completed quickly by the students. The next task was the students should determine all possible natural number \(k\) if \{11 cm, 15 cm, \(k\) cm\} was a set of sides of a triangle. This task could be completed quickly by the students.

The results of observation in the preliminary study it appeared that the learning process that occurs more provided information about the nature of the concept, not how to explore the concept. This fact is suspected as the cause of learning obstacle, either ontogenical obstacle, didactical obstacle, or epistimological obstacle. The emergence of ontogenical obstacle was seen in the absence of the enhancement of students' level of thinking. The emergence students’ didactical obstacle was seen from the results of observations that described how the teacher organizes learning in the classroom, especially on triangle lesson. Meanwhile, students’ epistimological obstacle was seen from the narrowness of context that students understood related to triangle matter. Such learning barriers were overcome by inviting students to make a repersonalization on triangle lesson. From the research results revealed that the implementation of this lecturing through three stages, namely action, formulation, and validation. At the stage of the action, students are challenged to fill the boxes will be able to provoke students' curiosity. This is an important part of mathematical proficiency [9]. It was revealed that the students found that not all of the eight sets could form triangles and the only three sets could form triangles, namely \{3, 5, 7\}, \{2, 7, 6\}, and \{4, 5, 6\}. Not each student found it in the same way. After they share information about how to found it, they got a more efficient and sensible way. This activity is in line with the idea of cognitive conflict. This conflict begins the process of
cognitive or intellectual reconstruction in individuals [10]. At the formulation stage, the researcher posed a mathematical problem that triggered the occurrence of discussion. The filing of such problems is one of the strategies of developing mathematical communication [11]. This fact is in line with a statement that problem-based learning has been identified to improve mathematical communication skills [12]. At the validation stage, it appeared that students were encouraged to validate the settlement and provided reasons, and to find other strategies to resolve the problem. This is consistent with the results of the study [13, 14] which found that the planning, monitoring, and evaluation activities undertaken by the students were able to improve the problem-solving process.

4. Conclusion
Based on the research results and the discussion, it can be concluded to achieve students’ understanding of the concept of the triangle, it need to make a lesson design that take into account the existing of students’ learning obstacles, either ontogenical, didactical, or epistimological obstacles. Implementation of lesson design on lecturing is done through three stages, namely action, formulation, and validation.

This research was only done on triangular lesson and a specific aspect of mathematical ability. For answering weather the design can be applied to other group of students, other aspects of mathematical ability, or other lesson topics , it was recommended that it coube investigated by doing an advanced research.

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