FAST MAGNETIC FIELD AMPLIFICATION IN THE EARLY UNIVERSE: GROWTH OF COLLISIONLESS PLASMA INSTABILITIES IN TURBULENT MEDIA

D. FALCETA-GONÇALVES1,2 AND G. KOWAL2

1 SUPA, School of Physics & Astronomy, University of St Andrews, North Haugh, St Andrews, Fife KY16 9SS, UK
2 Escola de Artes, Ciências e Humanidades, Universidade de São Paulo, Rua Arlindo Bettio, 1000, São Paulo, SP 03828-000, Brazil

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ABSTRACT

In this work we report on a numerical study of the cosmic magnetic field amplification due to collisionless plasma instabilities. The collisionless magnetohydrodynamic equations derived account for the pressure anisotropy that leads, in specific conditions, to the firehose and mirror instabilities. We study the time evolution of seed fields in turbulence under the influence of such instabilities. An approximate analytical time evolution of the magnetic field is provided. The numerical simulations and the analytical predictions are compared. We found that (i) amplification of the magnetic field was efficient in firehose-unstable turbulent regimes, but not in the mirror-unstable models; (ii) the growth rate of the magnetic energy density is much faster than the turbulent dynamo; and (iii) the efficient amplification occurs at small scales. The analytical prediction for the correlation between the growth timescales and pressure anisotropy is confirmed by the numerical simulations. These results reinforce the idea that pressure anisotropies—driven naturally in a turbulent collisionless medium, e.g., the intergalactic medium, could efficiently amplify the magnetic field in the early universe (post-recombination era), previous to the collapse of the first large-scale gravitational structures. This mechanism, though fast for the small-scale fields (∼kpc scales), is unable to provide relatively strong magnetic fields at large scales. Other mechanisms that were not accounted for here (e.g., collisional turbulence once instabilities are quenched, velocity shear, or gravitationally induced inflows of gas into galaxies and clusters) could operate afterward to build up large-scale coherent field structures in the long time evolution.

Key words: early universe – intergalactic medium – magnetic fields – magnetohydrodynamics (MHD) – plasmas – turbulence

1. INTRODUCTION

The baryonic fraction of the intergalactic medium (IGM) is composed of dilute warm/hot plasmas. Depending on the local environment, the plasma properties of the IGM may be different. For instance, the most diffuse components of the IGM, known as intergalactic “voids,” are vast regions between clusters of galaxies in the universe as seen today. Filaments of somewhat compressed and warmer (∼10^5–10^7 K) gas form what is broadly understood as the IGM, while the denser (∼10^3–10^5 cm^-3) and shock-heated (∼10^7–10^8 K) plasmas in clusters of galaxies are known as the intracluster medium (ICM). For many decades the ICM has been known to be magnetized (see reviews by Kronberg 1994; Enßlin et al. 2005; Widrow et al. 2012; Durrive & Neronov 2013, and references therein), but unfortunately any constraints on the magnetization of the IGM and intergalactic voids lack more substantial observational confirmation. The magnetic intensity and its respective correlation length must naturally be related to the processes from which they originate and/or amplify, which depend on local dynamical and physical properties. However, the origin and/or amplification of the magnetic fields at cosmological scales are not completely understood yet and represent a long-standing major issue in modern astrophysics (Widrow 2002).

Seed fields may have been generated at the pre-equipartition epoch by subatomic scale processes, such as hadron phase transitions, or at later (post-equipartition) and pregalactic epochs by large-scale MHD processes, e.g., the ∇p × ∇ρ battery term from Ohm’s law (Biermann 1950), currents induced by electronic scattering of anisotropic radiation fields (Langer et al. 2005; Durrive & Langer 2014), evolution of turbulent fields (e.g., Banerjee & Jedamzik 2004, and many others), and even shock-excited Weibel instabilities (Schlickeiser & Shukla 2003; Schlickeiser 2005). Jedamzik et al. (1998) suggest, however, that photon diffusion damping would affect post-equipartition, but pre-recombination, magnetic fluctuations. Therefore, most of the amplification processes mentioned above would only be important after the recombination era. These could, in principle, generate magnetic fields with intensities of the order of 10^−30–10^−11 G. A strong magnetic field has several implications for the cosmological evolution of the universe, from primordial nucleosynthesis to the statistics of the cosmic microwave background (CMB). Current data limit the primordial (comoving) magnetic field intensity to B_{1 Mpc} < 5 nG at 1 Mpc length scales (see Trivedi et al. 2010; Planck Collaboration XVI 2014, for references on WMAP and Planck data analysis, respectively). Such field intensities are below the values inferred for the ICM (B ∼ 0.1–10 μG; see Govoni & Feretti 2004, and references therein), as well as for more diffuse groups of galaxies (e.g., Nikiel-Wroczyński et al. 2013). Recently, arguable lower limits for the magnetization of the diffuse IGM have been also provided based on gamma-ray observations of blazars (see Neronov & Vovk 2010). These evidences combined together suggest that the universe was magnetized even before the structure formation. Therefore, the problem seems to be twofold: the first being the actual origin of the magnetic fields (seed fields), which may have occurred very early in the universe history (pre-equipartition), and the second being the amplification of such seed fields to the values observed in the diffuse gas of clusters of galaxies.
Observations of the Faraday rotation (FR) effect currently provide the best estimates for the IGM magnetic fields. In the FR effect the position angle of linearly polarized radiation shifts on the plane of sky as a function of wavelength $\lambda$ as $\Delta \phi = RM \times \lambda^2$, with the rotation measure (RM) being $RM (\text{rad} \, \text{m}^{-2}) \simeq 812 \int_{0}^{L_{\text{I}}} n_e (\text{cm}^{-3}) B_l (\mu \text{G}) d l$, where $l$ denotes the direction parallel to the line of sight.

It is well known that RM strongly depends on the spatial distribution of the electron density ($n_e$) and the magnetic field geometry along the line of sight ($B_l$), and basic geometrical assumptions with respect to them are usually made. It is also natural to presume that the IGM magnetic field is not uniform. FR maps present ordered magnetic fluctuations at scales from ~1 kpc, associated with cooling flows, up to ~1 Mpc, associated with powerful active galactic nucleus (AGN) jets and cluster radio relics (e.g., Feretti et al. 2012). These length scales, however, cannot be directly understood as correlation scales for the magnetic field. Observational estimates of typical length scales usually assume an ad hoc correlation between the magnetic field and local plasma density in the range of 1–30 kpc (e.g., Bonafede et al. 2010; Vacca et al. 2012). Unfortunately, given the turbulent nature of the IGM, the actual correlation length ($l_{\text{corr}}$) of the IGM/ICM magnetic field depends also on the spatial distribution of the velocity field (e.g., Burkhart et al. 2009; Xu et al. 2009; Falcke-Onoalves et al. 2014), and $l_{\text{corr}}$ for the magnetic field of the IGM cannot be observationally estimated yet. Still, even though around equipartition level with the thermal and kinetic counterparts, the presence of strong magnetic fields ($B\sim \mu \text{G}$) is surprising, given the general understanding that $\mu \text{G}$ scale fields would be the result of galactic dynamo amplification of much less intense pregalactic seed fields and the related timescales.

During the past decade, several authors have employed numerical simulations and analytical approximate solutions of collisional plasmas to study the magnetic amplification due to turbulent dynamo, galactic fields diffused into the ICM by outflows, and AGNs (see, e.g., Dolag et al. 2002; Dubois et al. 2009; Donnert et al. 2009; Falcke-Onoalves et al. 2010; Schober et al. 2012; Xu et al. 2012; Schober et al. 2013; Cho 2014; Federrath et al. 2014, and references therein). It was found that all these processes could, in principle, account for the amplification of a pre-existing seed field to observable values. Such degeneracy among different mechanisms could be removed if the correlation lengths of density, velocity, and magnetic fields were directly and independently obtained observationally. This is because the different processes suggested so far operate either at different scales or with different cross-correlations between the physical parameters ($B$, $n$, $p$, $v$). However, the possibility that $\mu \text{G}$ magnetic fields could be present in vast volumes of the more diffuse IGM, where the density of galaxies and the impact of AGNs are relatively small, points to the turbulent dynamo as a viable and ubiquitous process.

Great effort on the understanding of turbulent dynamos has been employed by means of analytical and numerical studies (see Schekochihin et al. 2007; Brandenburg et al. 2012; Schober et al. 2012; de Gouveia Dal Pino et al. 2013, and references therein). The turbulent dynamo is the process by which kinetic energy of turbulent motions is converted into magnetic energy. Turbulent cells stretch and fold field lines in a weakly magnetized plasma, increasing the total magnetic energy. Such a dynamo can, in principle, operate at the large range of scales where there is turbulence, depending on the properties of the flow. This was first pointed out by Batchelor (1950) and further developed into the Kraichnan–Kazantsev theory (Kazantsev 1967; Kraichnan 1968). Here, and throughout this paper, we assume the intergalactic plasma to be a highly conducting medium in the sense that the resistivity ($\eta$) is much smaller than the viscosity ($\nu$), i.e., the Prandtl number $Pm \equiv \nu/\eta \gg 1$. In this case, the small-scale dynamo (SSD) theory predicts an exponential growth of magnetic energy, up to equipartition at the viscous scale. The power spectrum of the magnetic fluctuations is then proportional to $k^{3/2}$, resulting in a concentration of energy at small scales. Once saturated at small scales, the dynamo enters in a nonlinear phase showing slower amplification rates (linear with time). The scale at which turbulence is dissipated regulates the efficiency of the nonlinear phase for the dynamo process. If we consider the standard collisionality of the ICM, the nonlinear phase of the turbulent dynamo is possibly of little interest for cosmological magnetic field studies. It has been suggested, though, that the enhanced effective collisionality due to plasma instabilities could even completely suppress the kinetic phase, and the nonlinear dynamo would be all that is left (see Mogavero & Schekochihin 2014). The full evolution of the magnetic fluctuations in homogeneous and isotropic turbulence, integrated over a Hubble time, has been presented in a semianalytic approach by Saveliev et al. (2012). These authors showed that, as we discuss below, for the purpose of this work, the exact determination of the transition between kinetic and nonlinear regimes is not relevant. Owing to the little understanding of the anomalous collisionality in diffuse media, and the fact that the nonlinear regime is even slower than the kinetic one, we will concentrate only on the early kinetic phase.

We may consider that during the kinetic phase the magnetic field intensity increases as $B(t) = B_0 \exp (t/\tau_\text{d})$, with the timescale $\tau_\text{d} = l_d/\delta v L$, with $\delta v L$ the turbulent amplitude at the viscous scale $l_d$. The saturation occurs when $B(t_{\text{sat}}) \simeq \delta v L (4 \pi \rho)^{1/2} \simeq \delta v L \text{ Re}^{-1/4} (4 \pi \rho)^{1/2}$, which results in $t_{\text{sat}} \sim 2 L^{1/2} \text{ Re}^{-1/2} \ln \left( \text{M}_{\lambda, L}^{0.8} \text{ Re}^{-1/4} \right)$, (1)

where $\rho$ stands for the mass density of the fluid and $\text{M}_{\lambda, L}^{0.8} = \delta v L (4 \pi \rho)^{1/2}/B_0$ the Alfvén Mach number at the large scale $L$ with respect to the seed field intensity $B_0$. Large turbulent Reynolds numbers result in faster SSDs; however, the saturation occurs at lower amplification levels, reducing its effectiveness. In the opposite trend, relatively smaller values of $\text{Re}$ result in slower SSDs, but with larger $B(t_{\text{sat}})$ and fast amplification. However, in order to explain the cosmological amplification of magnetic fields after the recombination era, we need large $B(t_{\text{sat}})$ and fast amplification. During the formation of clusters of galaxies, starting at at $z \sim 1.0$ (which represents a lookback time of <8 Gyr), velocities of the order of hundreds of $^{8}\text{When the magnetic energy density is small compared to the kinetic energy density of the smallest turbulent scale.}$  

$^{4}\text{Here we define the Reynolds number as Re} = \delta v L/\nu$, where $\nu$ is the fluid viscosity and $L$ is the largest turbulent scale. We also make use of the Kolmogorov scaling relation for the velocity $\delta v \sim L^{1/3}$.}$  

$^{5}\text{For a standard $\Lambda$CDM cosmological model, assuming a dark energy density} \ \Lambda = 0.714, \text{ matter density (baryons + dark matter)} \ \Omega_M = 0.286, \text{ and a Hubble parameter} \ h = 0.7.}$
kilometers per second\(^6\) are driven at scales as large as \(L \sim 1\) Mpc. For typical intergalactic gas densities of \(n \sim 10^{-3} \text{ cm}^{-3}\) one finds \(B(\text{sat}) \sim \mu G\) for \(Re \sim 10^{10} - 1000\), which resembles the values expected for the IGM using the Spitzer viscosity. For a seed field \(B_0 \lesssim nG\), using the same parameters, one also finds \(t_{\text{sat}} > 10^5\) Gyr. It is clear, at least from these crude estimates, that only superestimated values of the magnetic seed field and a fine-tuned Reynolds number could explain the magnetic field intensities observed in the local universe (see, e.g., Cho 2014).

Given the inefficient amplification of the field by the turbulent dynamo, another mechanism must be considered. It has been recently pointed out that the IGM may not be well described by a standard MHD theory. The intergalactic plasma, extremely rarefied, behaves as a gyrotropic collisionless plasma, i.e., the Larmor radius of the ions \((\lambda_i \sim eB/mc\nu_{\text{th},i})\) is smaller than the mean free path \(\lambda_{\text{mfp}} \sim (n\sigma_{\text{ii}})^{1/2}\), where \(\sigma_{\text{ii}}\) is the ion–ion collision cross section (Schekochihin et al. 2005; Santos-Lima et al. 2014). This relation can be rewritten as the condition for gyrotropic \((\lambda_i \ll \lambda_{\text{mfp}}\) section 6 As estimated from equipartition with the thermal component, i.e., mildly transonic turbulence.

2. MAGNETOHYDRODYNAMIC DESCRIPTION OF COLLISIONLESS PLASMAS

The proper description of collisionless plasmas relies on the full calculation of the particle dynamics, including both electromagnetic and collisional forces. Unfortunately, such an approach is of little practical use, with no analytical model available yet. One of the alternatives for this problem relies on numerically integrating the equation of motion of an ensemble of charged test particles (ions), coupled to a fluid (electrons). Such an approach is also called particle-in-cell (PIC) numerical simulation. PIC simulations provide the dynamical evolution of momentum distributions of particles as they interact with magnetic fields and due to collisions, which allow us to directly study, e.g., the effects of collisionless plasma instabilities on the isotropization of pressures, or on the rise of pressure anisotropies. However, the spatial coverage of the computational domain in PIC simulations is limited to a finite number of Larmor radii. For this reason it is not possible to study both the evolution of the distributions of particle momenta and the system dynamics at large scales, such as in the case of the IGM, simultaneously. At first approximation, we may consider the distribution of momenta of particles to be Maxwellian (or bi-Maxwellian in the case of magnetized collisionless plasmas), for which a fluid description of a pressure-anisotropic plasma is available (Chew et al. 1956).

2.1. Single-fluid Approximation of a Plasma with Pressure Anisotropy

The derivation of the CGL-MHD equations from the Vlasov–Maxwell equations is provided, for example, in Kulsrud (1983). By neglecting heat conduction and other possible heating/cooling sources, the one-fluid CGL-MHD set of equations for a plasma with pressure anisotropy can be rewritten, in conservative form, as

\[
\begin{align*}
\frac{\partial}{\partial t} \begin{bmatrix} \rho \\ \rho \mathbf{u} \\ \rho u_\perp \\ \rho u_\parallel \\ \rho u_\perp B \\ \rho u_\parallel B \\ e \\ \mathbf{u} \\ \mathbf{B} \\ \mathbf{f} \\ \mathbf{f}_\nu \end{bmatrix} &= \\
\mathbf{J} &= \\
\mathbf{J} &= \\
\mathbf{J} &= \begin{bmatrix} 0 \\ f \\ 0 \\ f_v \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} \\
\mathbf{J} &= \begin{bmatrix} 0 \\ f \\ 0 \\ f_v \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}
\end{align*}
\]

where \(\rho, \mathbf{u}, \mathbf{B},\) and \(p_{\perp,\parallel}\) represent the plasma density, velocity, magnetic field, and perpendicular/parallel pressures with respect to orientation of the local magnetic field, respectively, and \(e = (p_{\perp} + p_{\parallel}/2 + \rho u^2/2 + B^2/2)\) is the total energy density. The anisotropy in pressure for a gyrotropic plasma is then defined as \(\Delta \equiv (p_{\perp} - p_{\parallel})/p_{\parallel}\). \(\Pi_B\) and \(\Pi_B\) are the pressure and the magnetic stress tensors, respectively, given as

\[
\Pi_P = p_{\perp} I + \left(p_{\parallel} - p_{\perp}\right) \mathbf{b} \mathbf{b},
\]

and

\[
\Pi_B = \left(B^2/8\pi\right) I - \mathbf{B} \mathbf{B}/4\pi.
\]

where \(I\) is the unitary dyadic tensor and \(b = B/B\). The source term \(f\) in the equations above, represents the external force responsible for driving the turbulence.

The set of equations given in Equation (2) is not closed. Since the perpendicular and parallel pressures can evolve differently from each other, another equation relating the time evolution of \(p_{\perp}\) and \(p_{\parallel}\) is needed. The original double-adiabatic
condition proposed in Chew et al. (1956), based on the conservation of first and second magnetic invariants, leads to

\[
\frac{d}{dt} \left( \frac{p_B}{\rho B} \right) = 0, \quad \frac{d}{dt} \left( \frac{p_B B^2}{\rho^2} \right) = 0.
\]

The CGL-MHD double-adiabatic closure equations above may not be exactly correct. It is known that several processes, e.g., collisions or enhanced particle scattering by magnetic mirrors (or firehoses (FHs); e.g., Kunz et al. 2014), break these invariants. Another possible closure for Equation (2) is obtained by simply assuming that the system reaches a pressure anisotropy equilibrium fast enough, so \( \Delta \approx \text{const.} \) during the whole evolution. The closure problem is discussed in more detail in the following sections.

2.2. Wave Modes and Stability Condition in Gyrotropic Plasmas

A linear perturbation analysis of Equation (2) results in the known modified dispersion relations in gyrotrropic plasmas given below (see, e.g., Hau & Wang 2007; Kowal et al. 2011):

\[
\left( \frac{\omega}{k} \right)_{a}^2 = \left( \frac{B^2}{4\pi \rho} + \frac{p_\perp}{\rho} - \frac{p_\parallel}{\rho} \right) \cos^2 \theta,
\]

where \( k \) is the wavevector of the perturbation, and

\[
\left( \frac{\omega}{k} \right)_{f,s}^2 = \frac{b \pm \sqrt{b^2 - 4c}}{2},
\]

corresponding to the Alfvén transversal mode, with \( \cos \theta = k \cdot B / (kB) \) (where \( k \) is the wavevector of the perturbation), and

\[
\left( \frac{\omega}{k} \right)_{f,s}^2 = \frac{b \pm \sqrt{b^2 - 4c}}{2},
\]

where \( b = \frac{B^2}{4\pi \rho} + \frac{2p_\perp}{\rho} + \left( \frac{2p_\parallel}{\rho} - \frac{p_\perp}{\rho} \right) \cos^2 \theta, \)

corresponding to the fast ("+" or \( \pm \)) and slow ("−") magnetosonic waves, where

\[
c = -\cos^2 \theta \left[ \frac{3p_\parallel}{\rho} \right] \cos \theta - \frac{3p_\parallel}{b} + \left( \frac{p_\perp}{\rho} \right) \sin^2 \theta.
\]

The equations above result in the following stability conditions:

\[
|\Delta| \equiv \frac{|p_\perp - p_\parallel|}{p_\perp} > 2\beta_{-1}, \quad \text{for } (p_\parallel > p_\perp), \quad \text{and}
\]

\[
\frac{1}{6} \frac{p_\perp}{p_\parallel} > 1 + \beta_{-1}, \quad \text{for } (p_\parallel < p_\perp)
\]

which correspond to the FH and fluid mirror instabilities, respectively. We must point out that there is a factor of 1/6 for the latter, which is not present in the threshold of the actual mirror instability obtained from the kinetic theory. This offset is a well-known issue in the CGL-MHD closure, but this is not relevant for this work as we focus on the evolution of the FH mode only.

3. AN APPROXIMATE ANALYTICAL SOLUTION FOR THE AMPLIFICATION OF \( B \) IN THE UNSTABLE REGIME

By differentiating the pressure anisotropy \( \Delta \) in time, one obtains

\[
\frac{d\Delta}{dt} = \frac{1}{p_\perp} \left( 1 - \Delta \right) \left( \frac{dp_\perp}{dt} - \frac{dp_\parallel}{dt} \right).
\]

The first term of the right-hand side of Equation (9) can be evaluated in terms of the magnetic field intensity. The magnetic moment, or first adiabatic invariant, \( \mu = m_i v_i^2 / 2B \) is constant, of a charged particle subject to the magnetic force is also quasi-invariant for a plasma. Therefore, by approximating \( \nu \) to the thermal velocity \( v_\text{th} \), associated with the Maxwellian distribution of velocity components perpendicular to the magnetic field, and assuming \( dp_\parallel / p_\perp \approx dB / B \), and Equation (9) results in

\[
\frac{d\Delta}{dt} \approx (1 - \Delta) \frac{dB}{B} dt + F_{\text{iso}},
\]

where

\[
F_{\text{iso}} = -\frac{1}{p_\perp} \frac{dp_\perp}{dt} - v_\text{th} |\Delta|,
\]

represents an effective isotropization rate \( (\nu_{\text{eff}} \approx F_{\text{iso}} \Delta^{-1}) \). Notice that the first term on the right-hand side of the equation above corresponds to the time evolution of \( p_\parallel \) as a consequence of the changes in \( B \) and \( p_\perp \). From here we describe the interchange of parallel and perpendicular pressures, mediated by magnetic fluctuations, as the net magnetic scattering of the particle distributions \( (\nu_{\text{scatt}} \sim p_\perp^{-1} \Delta^{-1} dp_\parallel / dt) \). The last term of the equation above has been deliberately added to account for the effect of collisions, by means of a Braginskii collision frequency \( \nu_{\text{coll}} \). From these two terms it is possible to consider two different regimes with (i) collision-dominated isotropization, for \( \nu_{\text{coll}} > \nu_{\text{scatt}} \); and (ii) scattering-dominated isotropization, for \( \nu_{\text{coll}} \ll \nu_{\text{scatt}} \). The effect of collisionless instabilities on the pitch angle (magnetic) scattering is not fully understood. If we take the FH instability as an example, perturbations (e.g., in the velocity field) may result in local growth of magnetic field intensity \( \delta B \). This, in turn, results in an increase of the perpendicular velocities of the ions, i.e., an increase of \( p_\perp \) at the expense of a decrease of \( p_\parallel \). This could, in principle, be used to estimate \( dp_\parallel / dt \). However, the total internal energy may not be conserved if particles are lost during scattering, or if other energy loss processes are included, making the problem not practical without a full kinetic description. An alternative, as pointed by Schekochihin & Cowley (2006a), is to assume this term to saturate the amplification of the magnetic field perturbations quasi-linearly at the wavelength of fastest growth, which occurs around the Larmor radius \( (\omega \sim \Omega_i) \), where \( \Omega_{\text{sci}} = eB/m_i c \) is the cyclotron frequency of the ions; Gary et al. 1998); therefore, \( \delta B / B \propto \frac{\Delta}{\Delta \Omega_i^{-1}} \), given the

\( ^7 \) As, for example, with the use of the second adiabatic invariant.
maximum growth rate (see Equations (6) and (7))

\[ \gamma_{\text{max}} \sim \left| \Delta \right| - \frac{2}{\beta} \right)^{\alpha} \Omega_t, \]

with \( \alpha = 1 \) and \( 1/2 \) for mirror and FH instabilities, respectively. As we show further in this paper, the FH instability is of major interest in the amplification of \( B \), and we use \( \alpha = 1/2 \) from here. The scattering frequency can then be estimated as \( \nu_{\text{scatt}} \sim b B^2 / B^2 \gamma_{\text{max}} \sim (|\Delta| - 2/\beta)^{3/2} \Omega_t \), and Equation (10) becomes

\[ \frac{d \Delta}{dt} = (1 - \Delta) \frac{1}{B} \frac{dB}{dt} + \left( |\Delta| - \frac{2}{\beta} \right)^{3/2} \Omega_t - \nu_{\text{ii}} |\Delta|. \]  

### 3.1. Magnetic Field Amplification with Constant \( \Delta \)

This problem is tractable analytically if we assume that the evolution of the FH-unstable regions in the plasma occur with constant \( \Delta \). This approximation is justified by the fact that any process that may amplify the pressure anisotropy (e.g., turbulence, anisotropic cosmic-ray scattering, and others) is precisely counterbalanced by the ones leading to isotropization. This is actually a probable scenario given that the increase of the magnetic field by external sources is independent of \( \Delta \), while the isotropization processes are a function of the anisotropy itself. Therefore, the system should evolve around an “equilibrium” anisotropy value \( \Delta_0 \). If the relaxation timescale is short compared to the dynamical timescales of the system, we may assume \( \Delta(t) \approx \Delta_0 \). We must stress that the validity of such a conjecture depends on the level of anisotropy itself, i.e., it is probably correct for \( \Delta_0 \rightarrow 0 \) and certainly does not stand for \( \Delta_0 \gg 0 \).

#### 3.1.1. The Case of \( \nu_{\text{ii}} \gg \nu_{\text{scatt}} \)

Let us first consider the case in which collisions dominate the isotropization of pressure. This limit could well describe the initial stages of the evolution of a cosmological seed field after the recombination era, given that \( \beta \gg 1 \) and \( \nu_{\text{ii}} \gg \Omega_t \). The evolution of the magnetic field in this case depends mostly on the dynamics of the plasma, and not on instabilities driven by the pressure anisotropy. Given that the resistive dissipation is extremely small in the IGM, we may apply the “frozen-in” condition to the plasma. The evolution of the magnetic field is then dominated by the turbulent rate of strain at the viscous scales (see, e.g., Schekochihin & Cowley 2006a):

\[ \frac{1}{B} \frac{dB}{dt} \approx \frac{b b \cdot \nabla u - \nabla \cdot u}{L} \sim \frac{\nu_{\text{ii}}}{L} \nabla^{1/2}. \]  

For \( \Delta = \text{const} \) and \( \nu_{\text{ii}} \gg \nu_{\text{scatt}} \), and combined with Equation (14), Equation (13) is then reduced to

\[ |\Delta| \sim \left( \frac{v_{\text{ii}}}{\delta v_{\text{ii}}} \right)^{1/2} \left( \frac{L}{\lambda_{\text{mfp}}} \right)^{1/2} - 1. \]  

As discussed in the previous section, the kinetic phase of the turbulent dynamo (Equation (14)) provides an exponential growth of the seed field up to equipartition at the smallest turbulent scales. However, as \( B \) grows, \( \Omega_t \rightarrow \nu_{\text{ii}} \) and the plasma becomes gyrotropic. This is supposed to occur for extremely weak magnetic fields \((B_{\text{gyro}}(G) \gtrsim 10^{-20}n \text{cm}^{-3} \text{T(eV)}^{-1/2})\), possibly even earlier than the saturation of the turbulent dynamo at small scales.

Another interesting aspect of the turbulence during the early evolutionary phase is to generate the pressure anisotropy by itself (Equation (15)). This is physically understood as the consequence of a fast growth of the magnetic field not counterbalanced by collisions. Depending on the properties of the turbulence, one obtains \( |\Delta| > 0 \). Such values are not expected in highly magnetized plasmas. Once \( \nu_{\text{ii}} < \nu_{\text{scatt}} \), owing to the amplification of \( B \), the balance is reached since \( \nu_{\text{scatt}} \) strongly depends on the magnetic field intensity. Local measurements of the Earth’s magnetosphere and the solar wind reveal pressure anisotropies around its quasi-stability threshold \( |\Delta| \sim |\Delta|_{\text{crit}} \) and many signatures of its role on the dynamical evolution of these systems have been found (e.g., Winterhalter et al. 1995; Soucek et al. 2008; Bale et al. 2009). The interesting implication of Equation (15) is that, owing to its lower magnetization level (much higher \( \beta \)), \( |\Delta| \) in the IGM/ICM could have been much larger at higher redshifts, compared to the values found currently in the solar system.

#### 3.1.2. The Case of \( \nu_{\text{ii}} \ll \nu_{\text{scatt}} \)

Once the magnetic field becomes strong enough for the plasma to become gyrotropic, instabilities will take over the amplification of the magnetic field. Now, as \( \Delta = \text{const} \) and \( \nu_{\text{ii}} \ll \nu_{\text{scatt}} \), and replacing the cyclotron frequency \( \Omega_t \), Equation (13) becomes

\[ \frac{dB}{dt} \sim \frac{e}{m_e c} \left( \frac{|\Delta| - 2/\beta}{1 + |\Delta|} \right)^{3/2} B^2, \]  

which, at the \( \beta > 1 \) limit, results in a first-order nonlinear ordinary differential equation with an analytical solution:

\[ B(t) \sim \left( B_0^{-1} - A t \right)^{-1}, \]  

with \( A \approx \frac{e}{m_e c} |\Delta|^{3/2} \). The magnetic field growth is explosive around \( t \sim A^{-1} \), and the saturation occurs for \( B(t_{\text{sat}}) \approx B_0 \). The quasi-stability condition, given by the \( \beta \approx 2/|\Delta| \) level, is reached in a timescale

\[ t_{\text{sat}}(s) \sim \left( |\Delta|^{-1/2} \Omega_{t,0} \right)^{-1} \sim 10^{-4} B_0^{-1}(G)|\Delta|^{-1/2}. \]  

where \( \Omega_{t,0} \) represents the cyclotron frequency measured for the seed field \( B_0 \) and \( \psi = 1/2 \) or \( 3/2 \) for \( |\Delta| \gg 1 \) and \( |\Delta| \ll 1 \), respectively. Equation (18) then provides an approximate timescale for the explosive growth of the magnetic field due to the collisionless plasma instability. For example, if we consider a seed field of \( B_0 \sim 10^{-17} \text{G} \) subject to such instabilities, with \( |\Delta| > 10^{-2} \), one finds amplification up to equipartition with pressure anisotropy \((\beta \sim |\Delta|^{-1})\) at \( t < 400 \text{Myr} \).

As pointed out before, such values of pressure anisotropies are easily reached in turbulent weakly magnetized collisionless plasmas. The main conclusions of this section are that (i) pressure anisotropies could be naturally generated in the IGM/ICM in the early universe (post-recombination era) at timescales comparable to the eddy turnover time at viscous

\footnote{Notice that the amplification of \( B \) in our model can occur for FH instability, i.e., \( p_j > p_\perp \) (see Equation (9)), and therefore \( \Delta < 0 \) and \( 1 - \Delta > 1 \).}
4. NUMERICAL SIMULATIONS

In the previous section we demonstrated that the evolution of plasma instabilities driven by a pressure anisotropy could, in principle, explain the amplification of a magnetic field seed at the recombination era to the values observed in the local universe. These estimates, however, were obtained under strict approximations and limit cases. In this section we describe the numerical methods used for modeling of the dynamical evolution of the unstable plasmas.

4.1. Governing Equations and Methods

In order to perform the numerical modeling of the plasma evolution with the constant pressure anisotropy and turbulence injection, we used the GODUNOV code,9 which solves the CGL-MHD equations (Equation (2)) in the conserved form.

For simplicity $p_{\parallel} = a_{\parallel}^2 \rho$ and $p_{\perp} = a_{\perp}^2 \rho$ are expressed by the sound speeds $a_{\parallel}$ and $a_{\perp}$, parallel and perpendicular to $B$, respectively. Therefore, the momentum equation can be rewritten as

$$\frac{\partial \rho u}{\partial t} + \nabla \cdot \left( \rho uu + \left[ a_{\perp}^2 \rho + \frac{B^2}{8\pi} \right] I - (1 - \alpha) \frac{BB}{4\pi} \right) = f,$$

(19)

where $\alpha = (a_{\parallel}^2 - a_{\perp}^2)/V_A^2 = \frac{1}{2} (\beta_{\parallel} - \beta_{\perp}) = \frac{1}{2} \beta_{\perp} (\xi - 1)$ is the pressure anisotropy degree with $\xi \equiv p_{\parallel}/p_{\perp} = a_{\parallel}^2/a_{\perp}^2$ being the pressure ratio.

The numerical integration of the modified10 CGL-MHD equations was done in GODUNOV using the second-order shock-capturing Godunov scheme (see Kowal et al. 2011). The time integration was done using the second-order Strong Stability Preserving Runge–Kutta method (see Gottlieb et al. 2009, and references therein). Spatial reconstruction was done using the second-order total variation diminishing interpolation with the Van Leer limiter (van Leer 1974), and numerical fluxes were calculated using the general Harten–Lax–van Leer Riemann solver (see, e.g., Einfeldt 1988). We incorporated the field-interpolated constrained transport scheme (see, e.g., Tóth 2000) into the integration of the induction equation to maintain the $\nabla \cdot B = 0$ constraint numerically.

4.2. Model of Turbulence

In our numerical modeling we drive turbulence using a method described by Alvelius (1999). The forcing is implemented in spectral space and concentrated around the injection scale related to a wavevector $k_{\text{inj}}$. We perturb a finite number of discrete Fourier components of velocity in a shell extending from $k_{\text{inj}} - \Delta k_{\text{inj}}$ to $k_{\text{inj}} + \Delta k_{\text{inj}}$ with a Gaussian profile of the half width $k_c$ and the peak amplitude $\tilde{v}_f$ at the injection scale. The amplitude of driving is solely determined by its power $P_{\text{inj}}$. The parameters describing our forcing do not change during the evolution of the system.

The randomness in time makes the force neutral in the sense that it does not directly correlate with any of the timescales of the turbulent flow, and it also determines the power input solely by the force–force correlation. In the models presented in this paper we use isotropic forcing.

In particular, the total amount of power input from the forcing can be set to balance a desired dissipation at a statistically stationary state. In order to contribute to the input power in the discrete equations from the force–force correlation only, the force is determined so that the velocity–force correlation vanishes for each Fourier mode. The procedure of reducing the velocity–force correlation is described in Alvelius (1999).

In Equation (19), the forcing is represented by a function $f = \rho a$, where $\rho$ is local density and $a$ is random acceleration calculated using the method described above.

4.3. Initial and Boundary Conditions

For our calculations, similar to our earlier studies (see Kowal et al. 2011), the initial pressures and the uniform initial field $B_0$ are the controlling parameters. We define the Alfvénic Mach number of the injected turbulence as $M_A = (\delta v/c_A)$. The angle brackets $\langle \rangle$ represent the volume average of the parameter.

The distribution of $\rho = 1.0$, $v = 0$, and $B = B_0 \hat{z}$ is uniform in the whole domain. We do not set the viscosity or the resistivity coefficients explicitly in our models, and dissipation is determined by the numerical scheme only. Therefore, the scales at which the dissipation starts to be important are defined by the numerical diffusivity of the scheme.

The simulation box is perfectly periodic in all directions.

4.4. Simulated Models

For the present work we performed a number of numerical simulations with different plasma initial parameters, as well as different stability regimes, as given in Table 1. We also...
shown that CGL-MHD models, depending on the turbulent regimes and on the degree of pressure anisotropy (and Santos-Lima et al. 2014), result in the emergence of structures much smaller than those observed in standard MHD turbulence (Kowal et al. 2011). In standard MHD, the forcing, i.e., the process of injection of turbulence, is the dominant dynamical process and therefore dominates the process of structure formation. In the collisionless plasma approximation, on the other hand, the growth rate of the instabilities rises indefinitely with the wavenumber of the fluctuation, up to $k_{\text{max}} \sim n_e$. The numerical scheme implemented in this work for solving the CGL-MHD equations does not account for finite Larmor radius effects. Therefore, the maximum growth rate of the instabilities must be related to the minimum length scales of the system, which is the size of the finite volume of the space discretization, i.e., the grid cell. This means that both the growth rate and the geometry of the amplified magnetic fields are resolution dependent.

In Figure 2 we present the apparent configuration of the magnetic field lines in the mid-slice (in the $z$-direction). The geometry of the field lines is illustrated by the line integral convolution (LIC) technique. The LIC imaging algorithm consists of rendering a map of random streamlines that follow the orientation of the local field. The values are normalized to arbitrary units in order to provide a texture map. Such a texture performed simulations varying the numerical resolutions for the purpose of convergence validation.

### 5. RESULTS

A comparison between the spatial distributions of the density and velocity fields in CGL-MHD and standard MHD turbulent models has been provided in Kowal et al. (2011) and Santos-Lima et al. (2014). In those works it has been shown that CGL-MHD models, depending on the filling factor of unstable regimes and on the degree of pressure anisotropy itself, may lead to evident differences in the statistics of turbulent fields when compared to standard MHD turbulence. Those studies were performed for relatively strong magnetic fields, i.e., for $p_B |\Delta| \gtrsim \langle B^2 \rangle$, and therefore the statistics of $B$ were weakly dependent on CGL-MHD instabilities (see Figure 6 in Santos-Lima et al. 2014). In contrast to previous works, we focused our modeling on the weakly magnetized turbulent regimes ($p_B |\Delta| \gg \langle B^2 \rangle$).

#### 5.1. Magnetic Field Structure and Statistics

As the simulations are initiated, the driving of turbulence results in fluctuations of density, and of the velocity and magnetic fields, at different scales. It has been well known for many decades that FH and mirror instabilities, given the growth rate dependence with $k$ (Equations (6) and (7)), result in the emergence of structures much smaller than those observed in standard MHD turbulence (Kowal et al. 2011). In standard MHD, the forcing, i.e., the process of injection of turbulence, is the dominant dynamical process and therefore dominates the process of structure formation. In the collisionless plasma approximation, on the other hand, the growth rate of the instabilities rises indefinitely with the wavenumber of the fluctuation, up to $k_{\text{max}} \sim n_e$. The numerical scheme implemented in this work for solving the CGL-MHD equations does not account for finite Larmor radius effects. Therefore, the maximum growth rate of the instabilities must be related to the minimum length scales of the system, which is the size of the finite volume of the space discretization, i.e., the grid cell. This means that both the growth rate and the geometry of the amplified magnetic fields are resolution dependent.

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### Table 1

| Model           | Res | $A_{\alpha}^{\alpha}$ | $\beta_0$ | $p_\perp$ | $p_\parallel$ | $|\Delta|^b$ | $(\eta^{\text{max}})$ | $\tau_{\text{ad}}(\eta)_{\text{dyn}}$ |
|-----------------|-----|-----------------------|-----------|-----------|-------------|-------------|-------------------|---------------------|
| fh512MaD1.0     | 512 | 2.0                   | 2.0       | 0.25      | 1.0         | 3.0         | 1.6               | 0.52                |
| fh512MaD0.5     | 512 | 1.0                   | 3.0       | 4.0       | 1.0         | 0.75        | 4.3               | ...                 |
| fh512MaD0.8     | 256 | 10^4                  | 6.8 x 10^7| 1.0       | 0.04        | 0.96        | 2.5 x 10^7       | ...                 |
| fh512MaD0.6     | 256 | 10^4                  | 7.5 x 10^7| 1.0       | 0.25        | 0.75        | 2.2 x 10^7       | ...                 |
| fh512MaD1.0     | 128 | 10^4                  | 2.0 x 10^8| 1.0       | 4.0         | 3.0         | 7.6               | 0.36                |
| fh512MaD4.10    | 256 | 10^4                  | 2.0 x 10^8| 1.0       | 4.0         | 3.0         | 7.6               | 0.62                |
| fh512MaD4.0     | 256 | 10^4                  | 3.4 x 10^8| 1.0       | 25.0        | 24.0        | 1.4               | 0.13                |
| fh512MaD4.90    | 256 | 10^6                  | 9.0 x 10^2 | 1.0      | 25.0        | 24.0        | 1.3               | 0.18                |
| fh512MaD4.0     | 256 | 10^8                  | 9.0 x 10^16| 1.0      | 25.0        | 24.0        | 1.4               | 0.22                |

Notes.

$A_{\alpha}^{\alpha} = \dot{\nu}_{\alpha}/V_{\alpha}$ is the Alfvén Mach number of the turbulence injected with respect to the initial magnetic field, i.e., at $t = 0$.

$\Delta = (p_\perp - p_\parallel)/p_\perp$.

#### Figure 2

Central slice line integral convolution (LIC) map of the magnetic field lines at the saturated stage for $|\Delta| = 3.0$ and $\beta_0 = 2 \times 10^8$, corresponding to FH-unstable models, with different resolutions: $128^3$ (left), $256^3$ (center), and $512^3$ (right).
map provides a visual qualitative measure for the scaling of the magnetic field lines. Figure 2 shows the LIC technique applied for the magnetic field lines at the saturated stage for $|\Delta| = 3.0$ and $\beta_0 = 2 \times 10^5$, corresponding to FH-unstable models, for the given resolutions of $128^3$ (left), $256^3$ (center), and $512^3$ (right).

It is visually clear from these plots that the finer the resolution, the more flocculent the geometry of the amplified magnetic field becomes. This result is expected since for $|\Delta| = 3.0$ and $\beta_0 = 2 \times 10^5$ models the thermal pressure, including the pressure anisotropy free energy, is much larger than the magnetic and turbulent driving energy densities. Therefore, the FH instability dominates the plasma dynamics and the structure formation, whose growth rate should peak around the smallest scales available in the system. This is also quantitatively measured by means of the normalized power spectrum of the magnetic field, shown in Figure 3 (top panel).

The three-dimensional power spectra of the magnetic field for these models reveal that the peak shifts toward larger $k$ values as the numerical resolution increases. The peak shift is linear with the resolution, occurring at rounded values of $k_{\text{peak}} \approx 16$, 33, and 65 for resolutions of $128^3$ (black), $256^3$ (red), and $512^3$ (blue), respectively. Perturbations smaller than six to eight grid cells suffer mostly the effects of numerical dissipation.

Notice that, theoretically, the maximum growth rate should occur at $k \sim k_{\text{peak}}$, but numerical diffusion is responsible for the dissipation of coherent structures at the smallest scales. Both the numerical viscosity and resistivity act together to reduce the effect of the instabilities in the growth of velocity and magnetic field perturbations. If the numerical viscosity is considered to be Laplacian, we can, in principle, estimate the peak location as the length scale at which the numerical viscous loss rate $(4\pi^2k_{\text{peak}}^2\nu_{\text{num}})$ equals the instability growth rate, which results in $k_{\text{peak}} \approx \sqrt{|\Delta| - 2\beta^{-1}(8\pi^2\nu_{\text{num}})^{-1}}$. Given that the asymptotic values of $\beta$ depend only on $\Delta$, when saturation of instabilities occurs, the peak location becomes insensitive to $|\Delta| - 2\beta^{-1}$, resulting in $k_{\text{peak}} \propto \nu_{\text{num}}^{-1}$, i.e., approximately linearly proportional to the numerical resolution as suggested before.

Also, it is noticeable that the numerical resolution, except for the location of the peak, is not relevant for the general shape of the power spectrum of the magnetic field fluctuations. A power spectrum $P \propto k^\alpha$, with $\alpha \sim 2$, is observed in the range of scales $\sim [k_{\text{inf}}, k_{\text{peak}}]$, in agreement with the saturation estimates for $\delta B_i^2 \propto \eta_{\text{vis}}^2 \propto k^2$ (see Section 3), for all the numerical resolutions. This is a good indication of convergence for the models. The obtained power spectrum is steeper than the one expected for the kinetic phase of the SSD, which shows that the instabilities are the dominant process in the field amplification here. This also shows that the magnetic field at large scales is comparatively weaker.

The effects of $|\Delta|$ on the structure of the magnetic field are also shown in Figure 3 (center panel). Here we present the spatial spectral power of the magnetic field for the five different models with the same thermal perpendicular pressure ($p_i$) and numerical resolution ($256^3$), at $t = 2.0\,t_{\text{dyn}}$, but for different parallel pressures ($p_\parallel$), resulting in mirror- (“M”) and FH-unstable systems with different anisotropies. The mirror-unstable models, in contrast to the FH-unstable ones, present a decreasing power with wavenumber. This is because, as shown above, the amplification of the magnetic fluctuations in the FH regime is dominated by the instabilities (positive slope of the power spectra), while in the mirror-unstable regions it is basically subject to the dynamics of the flow, i.e., the turbulence (responsible for the negative slope of the power spectra). Also, as we discuss in more detail in the next subsection, the amplitude of the power spectrum is a function of the pressure anisotropy for the FH-unstable models, while it is insensitive to $|\Delta|$ in the mirror-unstable models. In the FH case, the larger the anisotropy, the larger is the total power of the magnetic fluctuations, which is related to the real values of the fluctuations and to the degree of amplification itself.

Since saturation is an important condition for the identification of the peak and total power of the magnetic fluctuations,
we studied the time evolution of the spatial power spectrum of $B$ of one of the FH-unstable models ($|\Delta| = 24$, $\beta_0 = 9 \times 10^{16}$ and $256^3$). We chose the model with the smallest magnetic seed field given its longer saturation timescale, which could eventually influence the spectral power distribution. As can be seen from Figure 3 (bottom panel), the spectra are very similar for all snapshots ($t = 0.5, 1.0, 1.5$, and $2.0\tau_{\text{dyn}}$), with exactly the same slopes and peak locations. This means that the saturation occurs at $t \ll t_{\text{dyn}}$, and that the turbulence driven—even though super-Alfvénic (with respect to the seed field)—is not able to modify the spatial distribution of the amplified field fluctuations. It is worth mentioning here that, since the nonlinear regime (as for the SSD) apparently does not exist, another mechanism is therefore required to continue the magnetic field amplification on larger scales.

5.2. Magnetic Field Amplification

As indicated by the power spectra shown in Figure 3, the FH-unstable models result in amplified magnetic field fluctuations. In the left panel of Figure 4 we present time evolution of the magnetic energy density for models with different pressure anisotropy ratios. The magnetic energy density was averaged over all grid cells of the simulated box. As mentioned above, the mirror-unstable models show moderate growth rates for the magnetic field perturbations, which occur at timescales of $\sim t_{\text{dyn}}$, in agreement with a typical slow turbulent dynamo. The FH-unstable models, on the other hand, present much larger amplification values, corresponding to $>7$ orders of magnitude for the three models shown. Also, the amplification timescales are much shorter than in the mirror-unstable case, with $\tau_{\text{fs}} \ll t_{\text{dyn}}$. For these models the saturation of magnetic energy density occurs at larger levels for larger pressure anisotropies, as expected for the saturation threshold $\beta_{\text{sat}} \propto |\Delta|$ (see Equation (16)). Also, in qualitative agreement with Equation (18), the saturation timescales for the magnetic field amplification decrease with pressure anisotropy. The timescales and amplification levels for saturation are also presented in Table 1.

Another interesting feature related to the time evolution of the magnetic energy density is the presence of a “knee” separating two different growth regimes. The energy density level of the knee is similar for all models. As pointed out in Section 3, if the turbulent rate of strain is larger than the growth rate of the instabilities, the turbulent dynamo regime dominates. This is precisely the physical motivation of the knee. At the early stages turbulence is driven at large scales, which are related to small growth rates of the instabilities. At this point Equation (14) is not yet valid, given that the turbulence is not fully evolved, but could be replaced with a reduced effective Re number instead owing to the shorter inertial range. The actual magnetic growth rate, related to the rate of strain, increases with time as the cascade develops and the inertial range grows. This behavior is also observed in the curves of magnetic energy density evolution. Once perturbations grow at a particular scale, at which the growth rate of the instabilities takes over the numerical dissipation, the explosive amplification takes place.

The right panel of Figure 4 presents the time evolution of the turbulent Alfvén Mach number for the different models. All models start with similar turbulent strengths. From this it is clear that, for the cases with $\Delta < 0$, the bulk of the magnetic field amplification is not related to the turbulence itself (though it is needed to seed the fluctuations that are subject to the instabilities at a later stage). The mirror-unstable models demonstrate very similar evolutions, while, on the other hand, the FH-unstable models show different evolution depending on the level of pressure anisotropy. Again, as pointed out before, the larger the pressure anisotropy, the faster the plasma reaches saturation.

The same study is presented in Figure 5, but for the different numerical resolutions. We can observe that the evolution of the average magnetic energy density and the Alfvénic Mach number present similar profiles for the different models. Therefore, the overall time dependency function of these quantities is well converged in our models. The convergence is good for the asymptotic value of the average magnetic energy density as well, which saturates for similar values regardless of the resolution and depends on $\Delta$ exclusively. The timescales for saturation, however, are resolution dependent, as we have pointed out previously, being shorter for higher resolutions. This is in agreement with the fact that finer resolutions allow the growth of the shorter-wavelength perturbations, which have faster growth rates.

The effect of the seed field intensity on the magnetic growth rate is shown in Figure 6, for the models with $\Delta = 24.0$ and numerical resolution of $256^3$ cells. Here the different seed field
intensities correspond to initial thermal-to-magnetic pressure ratios $\beta_0 = 9 \times 10^{12}$ (red) and $\beta_0 = 9 \times 10^9$ (black). The time evolution of the Alfvénic Mach number (right panel) shows that saturation occurs at similar levels regardless of the seed field intensity. The saturation occurs once instabilities drive the amplification to the $\sim |\Delta| p_\perp$ level, as predicted from the CGL-MHD stability condition. Weaker initial magnetic fields result in later saturation timescales. The delay, compared to the models with stronger initial fields, occurs as the road up to the saturation level must take longer for weaker seed fields.

Notice that the “knees,” i.e., the transition between the turbulent and instability-driven dynamos, occur at similar times ($\sim t \approx 0.02 - 0.03 t_{\mathrm{dyn}}$) regardless of the initial magnetic field intensity. Also, the magnetic field amplification, relative to the seed field intensity, of the knee is similar in the three models. Both features are in agreement with Equation (14). That equation reveals that, for the same timescales and similar turbulent properties, the relative amplification $\Delta B/B$ should be similar for all models.

The time evolutions of the minimum and maximum magnetic energy densities are also shown in the left panel of Figure 6, as dashed lines. These follow qualitatively the same profiles of the mean magnetic energy density, except for the short period before the “knee,” when the maximum energy increases while the minimum energy decreases with time. This process is due to the turbulence that increases the dispersion of the magnetic field distribution, before the growth of the instabilities. The second phase happens when the instability growth rate becomes more important than the turbulent rate of strain; therefore, the “knee” occurs earlier for the maximum value curve (larger rate of strain) and later for the minimum value curve (lower rate of strain).

The turbulent broadening of the magnetic energy density is clearly seen in the probability distribution function (PDF) shown in Figure 7. In the left panel we present the PDFs as they depend on the pressure anisotropy. We show the models with $p_\perp = 1.0$, 256$^3$ resolution, for $p_\parallel = 100.0$ (black), 25.0 (red), 4.0 (yellow), 0.25 (green), and 0.04 (blue), all at $2.0 t_{\mathrm{dyn}}$. All models start with a delta function PDF, at $B/B_0 = 1.0$. The broadening observed in the mirror-unstable models exemplifies the role of the injected turbulence on the distribution of the magnetic energy over the simulated domain. There is clearly no net amplification of the averaged field intensity, though. The FH-unstable models present PDFs that are shifted and skewed. Not only the averaged field intensity is amplified, as discussed above, but these PDFs—as a whole—are shifted toward larger intensities, with the shift being proportional to the pressure anisotropy.

The skewness of the PDFs arises as their peaks are further shifted toward larger intensities, compared to the bulk of the
distribution. This occurs if the amplification of the magnetic field is larger/faster for stronger fields. Such behavior could be understood as a transient process due to the turbulent broadening, in which turbulence would naturally build up a strong field in more regions as time evolves, but this is not the case. The mirror-unstable models do not present similar skewness. Also, the time evolutions of the PDFs do not support such a possibility. In the right panel of Figure 7 we show the time evolution of the PDF of magnetic field intensity for a single model. We have selected the model with 128\(^3\) resolution because it presents the slower evolution of the distribution, and the PDFs obtained at different snapshots can be compared. Here we show four PDFs calculated at \(t = 0.5, 1.0, 1.5,\) and \(2.0t_{\text{dyn}}\). The general PDF profiles change slightly with time but are similarly skewed from \(t = 0.5t_{\text{dyn}}\) up to the saturation time. The apparent constant skewness of these curves shows that the changes in the distribution of magnetic field intensity occur too early to account for the turbulence and should be related to the instability itself. This is in agreement with Equation (16), which predicts faster amplification for stronger fields.

5.3. Comparison with the Analytical Approximations

Obviously, any comparison between the simulations presented here and the magnetization of the IGM needs caution. The numerical simulations presented here correspond to full three-dimensional and stochastic numerical solutions of the problem. It represents a good benchmark for the analytical estimates (and related approximations) of the simplest zero-dimension, constant-\(\Delta\), \(\beta \to \infty\) limit, derived in Section 3.

We start analyzing the dependency of the saturation timescale observed in the simulations on the pressure anisotropy ratio (\(\Delta\)). In the top panel of Figure 8 we present the values obtained from the simulations, as described in Table 1, together with a number of analytical solutions for comparison, as given by Equation (16). The triangles represent the three models with similar parameters except for the pressure anisotropy; therefore, we fit the analytical solutions to these data. The squares correspond to the models with different initial conditions and are included for the sake of comparison only. Here we also consider the fact that the simulated instabilities are slightly delayed by the fact that we do not initiate the models with fully developed turbulence. On the contrary, the initial configuration is that of a uniform magnetic field, which is disturbed as turbulence injected at

Figure 7. PDFs of the magnetic field intensity, normalized by its initial value, computed from all grid cells. Left: for the models with different pressure anisotropies, at 2.0\(t_{\text{dyn}}\). The data represent the models with \(p_\perp = 1.0, 256^3\) resolution, but \(p_\parallel = 100.0\) (black), 25.0 (red), 4.0 (yellow), 0.25 (green), and 0.04 (blue). Right: for the \(\beta_0 = 2 \times 10^8\), \(\Delta = 3.0\), and 128\(^3\) resolution model at the different times: \(t = 0.5, 1.0, 1.5,\) and \(2.0t_{\text{dyn}}\).

Figure 8. Comparison between the numerical simulations and the analytical predictions given in Section 3. Top: \(\tau_{\text{sat}}\) vs. \(\Delta\) correlation. Triangles correspond to the simulations with similar initial conditions except for the pressure anisotropy (\(\Delta = 3, 24,\) and 99), which are the ones used for the correlation analysis. The lines correspond to the solution of Equation (16) for different turbulent delays \(\delta_{\text{sat}}\) (see text). Bottom: \(\tau_{\text{sat}}\) vs. \(B_0\) correlation. Again, triangles represent the models with similar initial conditions, but now with different seed field intensities (\(\beta_0 = 9 \times 10^8, 9 \times 10^{12},\) and \(9 \times 10^{16}\)). The lines correspond to correlations \(\tau_{\text{sat}} \propto B_0^\theta\), with Equation (18) being the case with \(\theta = -1.0\).
large-scale cascades. This turbulent delay ($\tau_{turb}$) is introduced as a free constant parameter, i.e., $\tau_{gr} \sim \tau_{cat} - \tau_{turb}$. Overall there is reasonable agreement between the simulations and the analytical solutions, those with $0.01 t_{dyn} < \tau_{turb} < 0.05 t_{dyn}$ being particularly good. It is interesting to note that this range is similar to the timescales of the “knees” observed in Figure 4.

In the bottom panel of Figure 8 the same study is presented, but now for the initial values of the seed fields. Here the triangles represent the three models with similar initial conditions except for the seed field ($B_0 = 9 \times 10^8, 9 \times 10^{12}$, and $9 \times 10^{16}$). According to Equation (18), the best fit to the data should occur for $\tau_{gr} \propto B_0^\theta$, with $\theta = -1.0$. However, as clearly seen from the plot, the best fits occur for a much weaker dependence (or even no dependence) of $\tau_{gr}$ on the initial seed field, where $\theta \sim -0.05$. The apparent discrepancy between the two results is not real, though. Notice that Equation (18) is derived as the magnetic field evolution once the condition $\nu_M \ll \nu_{scat}$ is fulfilled. The term $B_0$ is therefore somewhat misleading in the sense that one must consider the magnetic field intensity once $\nu_M \ll \nu_{scat}$, i.e., a transition field intensity. For the analytical model the transition field intensity is determined by $\nu_M$, while in the simulations by the numerical dissipation, i.e., in both cases, it is insensitive to the initial magnetic field. Another major effect responsible for this discrepancy is the fact that the growth rate depends on the Larmor radius, which cannot be properly modeled in the simulations. Naturally, $\tau_{gr}$ obtained from the simulations is overestimated as $B$ increases with time.

6. DISCUSSION

Several authors have already discussed the effects of turbulence (see, e.g., Banerjee & Jedamzik 2004; Iapichino & Brüggen 2012; Saveliev et al. 2012, 2013; Cho 2014, and many others) and structure formation (see, e.g., Dolag et al. 2002; Vazza et al. 2014, and many others) on the evolution of the intergalactic magnetic field. The lack of detection of TeV radiation from blazars points toward the existence of amplified magnetic fields, with filling factors around 60%, even at the IGM voids (Neronov & Vovk 2010; Dolag et al. 2011). For this reason, turbulence—as a ubiquitous phenomenon—has been preferred as the main mechanism for field amplification. The common problem related to turbulent models of collisional plasmas is the typical timescale needed for magnetic field amplification. As explained previously, the timescales associated with turbulent amplifications are too large for the magnetic seed amplitudes suggested so far. Structure formation, on the other hand, would be dominant (owing to shear, compression, or other star-formation-related mechanisms) in further amplification of the fields.

In this work we explore a possible scenario for magnetic field amplification at high redshifts (post-recombination), based on the growth of collisionless plasma instabilities. Two similar works have dealt with the problem in a similar approach, one being analytical (Schekochihin & Cowley 2006b; see also Schekochihin & Cowley 2006a) and the other purely numerical (Santos-Lima et al. 2014). In Schekochihin & Cowley (2006b) an analytical simplification of the problem is posed, which inspired most of the analytical derivation presented here as well. In their work, however, it is assumed that both the damping at small scales and the pressure anisotropy $\Delta$ are similarly related to $\nu_{cat}$ and $\nu_{tr}$. This is in order to obtain the time evolution of $\Delta$ and $B$ in terms of an effective collision rate. In the present we do not constrain $\nu_{cat}$, which is, on the other hand, implicitly constrained to the assumption that $\Delta$ is kept constant during the evolution of the system. Interestingly, for the super-exponential phase, both works predict the magnetic field intensity growth rate as $\propto (1 - t/t_c)^{-b}$, though with different slopes and characteristic timescales. In both estimates the growth of the magnetic field by collisionless instabilities could, in principle, explain the magnetization of the local universe based on very weak seeds ($<10^{-17}$ G). Only the analytical model presented in this work has been compared with numerical simulations.

With respect to the numerical solutions, in Kowal et al. (2011) the authors performed simulations with strongly magnetized plasmas and could not address the amplification problem. In this sense, the numerical study presented here stands as an extension of that particular work but focused on the statistical properties of the magnetic field and the role of collisionless plasma instabilities on the amplification of the magnetic field. The scalings found for density and velocity fields in the FH-unstable models of that work resemble those obtained here for the magnetic field, with a power spectrum peak at the smallest scales available in the system. Similar statistical properties were obtained by Santos-Lima et al. (2014), who performed collisionless plasma simulation with variable pressure anisotropy and studied the statistical properties of the magnetic fields as well. The plasma properties were studied in a variable-$\Delta$ framework, though, with isotropization mechanisms being introduced ad hoc. Still, it was shown that even in the cases where isotropization is strong enough to wash out the imprints of instabilities in the statistical properties of the plasma, their dynamical effects are important locally. The amplification of the magnetic field was studied and, as a consequence of small volume coverage of unstable regions, their general conclusion was that the instabilities were not efficient. In contrast to the assumption made in this work, their models do not include an external source term to excite the pressure anisotropy, which could—in principle—change the system to a quasi constant-$\Delta$ case, similar to ours.

Possible observational evidence (or constraints) for the process described in this work could be obtained from the CMB. If the collisionality condition was satisfied at the post-equipartition era, the magnetic field at small scales could have been amplified prior to the recombination era, and the ionization structure of the universe would be different, with effects on Silk damping (see Jedamzik & Abel 2013). On the other hand, if the plasma becomes gyrotropic at $z < z_{rec}$, such an effect would not be relevant.

The main problem with the models mentioned above is the lack of a proper physical description of the isotropization of the particle momenta as the system evolves. In contrast to previous works, we consider the balance between pressure anisotropy sources and isotropization effects, which simplifies dramatically the solution of the problem and results in a problem that can be addressed both numerically and analytically. Nature is obviously far more complex than a constant-$\Delta$ model, but still any approximated model that can be studied by means of both analytical and numerical methods may be extremely useful for the understanding of the basic dynamical properties related to the dynamics of collisionless plasma fluids.
6.1. Magnetic Field Characteristic Length Scales

Even if the amplification of the magnetic field, from seed fields up to equipartition levels, could be explained by the collisionless plasma instabilities, one must still consider its characteristic length scale. The zero-dimensional analytical models cannot account for this properly. Still, it is reasonable to consider the typical length scale to be that of maximum growth rate. Indeed, from the numerical simulations presented in this work, it has been shown that the turbulent flows cannot modify the magnetic field structure in timescales at which the amplification occurs. The spectral distribution of the magnetic field obtained from the simulations peaks at the smallest scales possible, those at which the numerical dissipation overtaxes the field growth. The finer the resolution, the smaller is the characteristic length scale.

In the IGM this would be given by the natural process that can overtake the pressure anisotropy instabilities, particle collisions. With a typical mean free path of roughly a few kiloparsecs, one should expect the magnetic field energy to be concentrated in fluctuations with approximately this length scale. However, magnetic field correlation lengths as large as ~1 Mpc have been inferred from observations (e.g., Feretti et al. 2012), posing the challenge not only of how to amplify the magnetic field but also of how to distribute the energy from the small to the large scales. In our FH-unstable CGL-MHD models, we obtained power spectra that are steeper than expected for the kinetic phase of the SSD. This demonstrates that the instabilities are indeed the dominant process in place in these models. On the other hand, depending on the level of pressure anisotropy, it also results in weaker large-scale fields, if compared with the SSD estimates. Given that most of the energy of the magnetic fluctuations is placed at the smallest scale in the gyrotropic fluid approximation (i.e., $\lambda_{\text{mfp}}$), one could estimate (see Section 3) the magnetic field intensity at large scales as

$$B_L \sim \left( |\Delta| P_{\mu} \right)^{1/2} \left( \lambda_{\text{mfp}} L^{-1} \right).$$

For $n \simeq 10^{-3} \text{ cm}^{-3}$, $kT \simeq 1 \text{ eV}$, one finds $B_{\text{Mpc}} \sim \sqrt{\Delta} \times 10^{-10} \text{ G}$.

The answer to this problem may be found in the properties of the turbulence itself. As discussed earlier in this paper, turbulence stretches and folds the magnetic field in a process that drives the turbulent dynamo. This process is dominant at small scales and is quenched in a given scale when the magnetic energy reaches equipartition with the turbulent one. In the collisionless plasma the instabilities are responsible for the growth of the magnetic field at the smallest scales. Even though turbulence cannot stretch the field lines at the smallest scales, it could, in principle, do it at the largest ones, depending on its strength. This is clearly shown in Figures 4-6, where the evolution of the turbulent Alfvénic Mach number is always larger than unity, even after the saturation of the magnetic field. This is because the velocity dispersion is dominated by the large-scale eddies, where equipartition was not reached. For these particular models, the turbulence is responsible for transferring the magnetic field energy from small to large scales. This process should be even more important if the pressure anisotropy is quenched (this is not the case for the simulations, in which we have kept it constant). The quenching of the pressure anisotropy, followed by the transfer of magnetic energy from small to large scales by super-Alfvénic turbulence, could therefore explain the observations. In other words, as shown in the numerical simulations, the CGL-MHD instabilities are presumably dominant over the kinematic phase of the SSD, but possibly not for the later nonlinear phase (once the instabilities are quenched).

Another possible solution for this problem would be related to the formation of large structures of matter (e.g., massive galaxies, groups, and clusters). It is quite clear now that the large-scale magnetic field is not primordial but has constantly evolved with the dynamical evolution of the universe (Jedamzik & Sigl 2011). This was also pointed out by Schlickeiser (2005, and references therein), in the context of small-scale fluctuations of Weibel instability. The collapse of gas into galaxies or clusters (Vazza et al. 2014), as well as large-scale shear, could, in principle, amplify coherent components of the magnetic field. If correct, in such a scenario magnetic energy would have been provided dominantly at small scales, in the post-recombination era, but reassembled into large-scale coherent structures ($>100 \text{ Kpc}$) following the recent structure formation of the dark and baryonic matter ($z < 10$).

7. SUMMARY

In this work we studied the magnetic field amplification process in turbulent collisionless plasmas. In such plasmas, parallel and perpendicular pressures (with respect to the local magnetic field) may become so anisotropic that instabilities may become dynamically important. It has been pointed out in previous analytical models that such instabilities, acting together with a turbulent background, may accelerate the amplification of the magnetic field up to near-equilibration levels. Such models would then have dramatic implications on our understanding of the magnetization of the early universe (post-recombination), given that the IGM then becomes collisionless. These models provide timescales and saturation estimates for the magnetic energy density but have never been confronted by full three-dimensional numerical simulations. The aims of this work were to revisit the analytical zero-dimensional models of the magnetic field evolution in turbulent collisionless plasmas and to provide a number of full three-dimensional numerical simulations that could be directly compared with these analytical estimates.

As long as pressure anisotropy is kept constant during the evolution of the system, a novel analytical solution is provided for the magnetic field evolution in systems subject to FH instability. As discussed before, such an assumption is not too far from reality, given that different processes act together to increase and decrease pressure anisotropy, and some sort of equilibrium value for $\Delta$, in the unstable regime, may exist (especially for weakly magnetized plasmas). We find that:

1. for very weak initial seed fields, the early evolution of the magnetic field is dominated by collisions, and the turbulent dynamo should be responsible for the initial amplification of the field;
2. once the field grows and $\Omega_{\mu} > \nu_{\mu}$, a transition from an isotropic to a gyrotropic plasma occurs and instabilities would then be driven. A fraction of the “free energy” available ($|\Delta| P_{\mu}$) would then be transferred to the magnetic field as the system evolves, up to saturation;
3. saturation occurs at $\beta \sim 2|\Delta|$, which is reached in a timescale $t_{\text{sat}} \propto B_0^{-1}|\Delta|^{3/2} \psi^{-1/2}$, with $\psi = 1/2$ or 3/2, for $|\Delta| \gg 1$ and $<1$, respectively.
This analytical model was then compared with a number of CGL-MHD numerical simulations, with different initial and turbulent conditions, from which the evolution of the magnetic field from the seed field, up to saturation, was followed. The main results obtained are as follows:

1. the explosive growth of magnetic field energy density is observed in the FH-unstable models, with the amplification time-scales being approximately proportional to $|\Delta |^{\nu}$, as was predicted in the simple analytical model;
2. there is no clear dependency between $\tau_{fr}$ and $B_0$ in the simulations, in contradiction to the linear anticorrelation derived in the analytical model. This is explained in terms of $B_0$ being the field intensity at the $\Omega \sim \nu_\Omega$ transition, rather than the initial seed value, and by the fact that the code is unable to evolve the Larmor frequency as a function of $B$;
3. the power spectrum of the magnetic field peaks at the smallest scales not dominated by the numerical diffusion, and the transfer of energy to large scales must occur afterward, by turbulence itself, once saturation occurs.

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