What lattice calculations tell us about the glueball spectrum

Michael Teper*

Theoretical Physics
University of Oxford
Oxford, OX1 3NP, U.K.

Abstract

I review what lattice QCD simulations have to tell us about the glueball spectrum. We see that the various lattice calculations are in good agreement with each other. They predict that prior to mixing with nearby flavour singlet quarkonia the lightest glueball states are the scalar at $1.61 \pm 0.15$ GeV, the tensor at $2.26 \pm 0.22$ GeV, and the pseudoscalar at $2.19 \pm 0.32$ GeV.

*Talk at HEP97, Jerusalem, August 1997
# 1 Introduction

My topic here concerns the glueball spectrum. The physics question is: where, in the experimentally determined hadron spectrum, are the glueballs hiding? Ideally I should be telling you what happens when you simulate QCD with realistically light quarks. But it is going to be a few years yet before I can do that. What I will focus on here are lattice glueball calculations in the SU(3) gauge theory without quarks. We now know what are the lightest states in the continuum (rather than lattice) theory; and I will tell you what they are. The next step, if we want to make contact with the real world, is to introduce the physical GeV mass scale. This introduces uncertainties which I will try to estimate for you. The final step is to discuss possible mixing scenarios with nearby flavour singlet quarkonia. At this stage we can look at the experimental spectrum and pinpoint the experimental states most likely to have large glueball components. I will not have time to say much about the latter topics and refer you instead to ref \[1\] and ref \[2\] where you will also find a more complete set of references.

The states in the pure SU(3) gauge theory are glueballs by definition - we only have gluons in the theory. If you want hadrons with quarks then you can propagate quarks in this gluonic vacuum and tie such propagators together so that the object propagating has the appropriate hadronic quantum numbers. That is to say, you calculate hadron masses in the relativistic valence quark approximation (the ‘quenched approximation’) to QCD. The spectrum one obtains this way is a remarkably good approximation to the observed light hadron spectrum. This is not too surprising: one reason we were able to learn of the existence of quarks in the first place is because the low-lying hadrons are in fact well described by a simple valence quark picture.

Of course in this theory, with no vacuum quark loops, we don’t have mixing between quarkonia and glueballs. There is however reason to believe that this mixing is weak in the real world – the Zweig (OZI) rule: hadron decays where the initial quarks all have to annihilate are strongly suppressed, e.g. in $\phi$ decays. Such a decay may be thought of as $quarks \rightarrow glue \rightarrow quarks$. Glueball mixing with quarks should therefore be $\sqrt{OZI}$ suppressed. As should glueball decays into hadrons composed of quarks. The existence of such a suppression is supported by a recent lattice calculation ref \[3\].

The picture we have in mind is therefore as follows. The glueballs will only be mildly affected by the presence of light quarks. They will, of course, decay into pions etc. but their decay width will be relatively small; and there will be a correspondingly small mass shift. Only if there happens to be a flavour singlet quarkonium state close by in mass will things be very different, because of the mixing of these nearly degenerate states. In this context we expect ‘close by’ to mean within $\sim 100$ MeV. So we view the glueballs in the pure SU(3) gauge theory as being the ‘bare’ glueballs of QCD which may mix with nearby quarkonia to produce the hadrons that are observed in experiments. All this is an assumption of course, albeit well motivated. If true it tells us that the glueballs, whether mixed with quarkonia or not, should lie close to the masses they have in the gauge theory. So we now turn to the calculation of those masses.
2 Glueballs in the SU(3) gauge theory

In lattice calculations (Euclidean) space-time is discretised to a hypercubic lattice, and the volume is made finite and (anti)periodic. So the first step is to be able to calculate masses reliably on such a lattice hypertorus. The second is to make sure the volume is large enough. Assuming this has been done we obtain the mass spectrum, \( a_m(a) \), of the discretised theory in units of the lattice spacing \( a \). What we actually want is the corresponding spectrum of the continuum theory, \( a = 0 \). To obtain this we proceed as follows. Theoretically we know that in this theory the leading lattice spacing corrections to dimensionless ratios of physical masses are \( O(a^2) \) where \( a \) is the lattice spacing ref [4]. So for small enough \( a \) we can extrapolate our calculated mass ratios to \( a = 0 \) using

\[
\frac{am_1(a)}{am_2(a)} \equiv \frac{m_1(a)}{m_2(a)} = \frac{m_1(a = 0)}{m_2(a = 0)} + c(am)^2
\]

where \( m \) may be chosen to be \( m_1 \) or \( m_2 \) or some other physical mass: the difference between these choices is clearly higher order in \( a^2 \) - which we are neglecting. In practice I shall use \( am_2 = am = a\sqrt{\sigma} \), where \( a^2\sigma \) is the confining string tension as calculated in lattice units, and \( am_1 = am_G \) will be a glueball mass.

In Fig 1 I show how the calculated mass ratios, for the lightest scalar glueball, vary with \( a^2\sigma \). As you see, the behaviour is linear - not surprising given the fact that \( a^2\sigma \) is indeed small for the values plotted. One fits a straight line and obtains the continuum limit as the intercept at \( a = 0 \), i.e. at \( a^2\sigma = 0 \). We obtain in this way the following lightest three masses in the continuum limit:

\[
\frac{m_{0^{++}}}{\sqrt{\sigma}} = 3.65 \pm 0.11
\]

\[
\frac{m_{2^{++}}}{\sqrt{\sigma}} = 5.15 \pm 0.21 \tag{3}
\]

\[
\frac{m_{0^{--}}}{\sqrt{\sigma}} = 4.97 \pm 0.58 \tag{4}
\]

Although we do not have continuum extrapolations for other masses, the UKQCD lattice results strongly suggest that glueballs with other \( J^{PC} \) are heavier ref [5].

The values of the glueball masses that we have used are from refs [2, 3, 4, 8] and for our sources of the string tension see refs [1, 2]. You may recall that a couple of years ago some publicity was given to an apparent discrepancy between the GF11 and UKQCD predictions for the \( 0^{++} \) glueball mass. However, as you can explicitly see in Fig 1, the various calculations are entirely in agreement with each other. The discrepancy was an illusion: it arose largely from different ways of setting the MeV scale. Such differences should be part of the final systematic error on the mass: as below.

3 Glueballs masses in GeV units

To introduce MeV units we need the string tension in these units. We can do this by calculating the mass of the \( \rho \) or nucleon or ... in the quenched approximation and extrapolate
Figure 1: The scalar glueball mass: the GF11 values (×) and the rest (●). The best linear extrapolation to the continuum limit is shown.

\[ m_\rho/\sqrt{\sigma} \text{ or } m_N/\sqrt{\sigma} \text{ or ...} \] to the continuum limit as we did for the glueballs. We then set \( m_\rho = 770 \text{ MeV} \) or \( m_N = 930 \text{ MeV} \) or ... to obtain \( a\sqrt{\sigma} \) in MeV units. Because the quenched spectrum differs slightly from the real world, these estimates differ slightly. This forms part of the error. One can estimate the error \textit{ad nauseam}, as in refs [1, 2], and this leads to an estimate

\[ \sqrt{\sigma} = 440 \pm 15 \pm 35 \text{ MeV} \tag{5} \]

where the first error is statistical and the second is systematic.

We can now use this value in eqns [3, 4] to express our glueball masses in GeV units. We obtain

\[ m_{0^{++}} = 1.61 \pm 0.07 \pm 0.13 \text{ GeV} \tag{6} \]
\[ m_{2^{++}} = 2.26 \pm 0.12 \pm 0.18 \text{ GeV} \tag{7} \]
\[ m_{0^{--}} = 2.19 \pm 0.26 \pm 0.18 \text{ GeV} \tag{8} \]
These, then, are our best lattice predictions for the lightest glueballs prior to any mixing with nearby quarkonium states. In the case of the 0++ the focus is naturally on the $f_0(1500)$ and any scalar lurking in the $f_J(1700)$. The tensor focus is naturally on the $f_2(1900)$ and the $G(2150)$. With the pseudoscalar things are murky: the lattice calculation has huge errors and is far from the obvious $\iota(1490)$ candidate; but in this case topology is important and that is a quantity that is sensitive to light quarks in the vacuum.

References

[1] F. E. Close and M. Teper, in preparation.

[2] M. Teper, Lectures at the Newton Institute, NATO-ASI School, June 1997 (hep-lat 9711011).

[3] J. Sexton et al., Phys Rev. Letters, 75 (1995) 4563.

[4] K. Symanzik, Nucl. Phys. B226 (1983) 187.

[5] G. Bali et al, Phys. Lett. B309 (1993) 378.

[6] H. Chen et al, Nucl. Phys. Proc. Suppl. B34 (1994) 357.

[7] C. Michael and M. Teper, Nucl. Phys. B314 (1989) 347.

[8] Ph. de Forcrand et al, Phys. Lett. B152 (1985) 107.