A ROBUST AND EFFICIENT MULTI-SCALE SEASONAL-TREND DECOMPOSITION

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ABSTRACT

Many real-world time series exhibit multiple seasonality with different lengths. The removal of seasonal components is crucial in numerous applications of time series, including forecasting and anomaly detection. However, many seasonal-trend decomposition algorithms suffer from high computational cost and require a large amount of data when multiple seasonal components exist, especially when the periodic length is long. In this paper, we propose a general and efficient multi-scale seasonal-trend decomposition algorithm for time series with multiple seasonality. We first down-sample the original time series onto a lower resolution, and then convert it to a time series with single seasonality. Thus, existing seasonal-trend decomposition algorithms can be applied directly to obtain the rough estimates of trend and the seasonal component corresponding to the longer periodic length. By considering the relationship between different resolutions, we formulate the recovery of different components on the high resolution as an optimization problem, which is solved efficiently by our alternative direction multiplier method (ADMM) based algorithm. Our experimental results demonstrate the accurate decomposition results with significantly improved efficiency.

Index Terms—Time series, seasonal-trend decomposition, multi-scale decomposition, multiple seasonality

1. INTRODUCTION

Recently, the explosive growth of the Internet of Things (IoT), Artificial Intelligence for IT Operations (AIOps) and many other applications lead to huge amounts of time series signals. Therefore, signal processing and mining for time series have received lots of research interests \cite{1,2,3}. Seasonality including multiple seasonality is commonly observed in time series data. For example, the traffic data typically exhibit the daily and weekly periodicities \cite{4}. Compared with single seasonality, multiple seasonality makes the seasonal-trend decomposition more challenging.

The seasonal-trend decomposition is an important procedure in the analysis of periodic time series as it is the basis for seasonal adjustment in many applications including forecasting and anomaly detection \cite{5,6,7,8}. Recently, some seasonal-trend decomposition algorithms have been proposed, including MSTL \cite{9}, STR \cite{10}, TBATS \cite{11}, and RobustSTL \cite{12,13}. These algorithms usually estimate different components iteratively using the original data. Thus, their computational cost is high, especially when the seasonality lengths become longer. Meanwhile, some multi-scale methods are proposed recently for time series storage, query processing \cite{14}, and local pattern discovery \cite{15} for improved efficiency.

In this paper, we propose a multi-scale seasonal-trend decomposition algorithm for time series with multiple seasonality. To the best of our knowledge, this work is the first one which applies the multi-scale approach in seasonal-trend decomposition. Let us consider the seasonal-trend decomposition of the traffic data again. Suppose we observe the traffic data every one minute. Note that we need sufficient data to estimate the seasonal components. For a single traffic time series, in order to accurately estimate both the daily and weekly seasonal components, the input time series should cover three weeks, i.e., 60 \times 24 \times 7 \times 3 points. The resulting storage and computation cost is a burden in many applications as lots of time series are required to be processed in parallel. In our proposed framework, we model different components in different scales. Intuitively, the daily seasonal component is represented using the data recorded every minute, but the weekly seasonal component can be approximated using data on a lower resolution, e.g., hourly data. By considering different components at different resolutions, our seasonal-trend decomposition requires significantly less data and produce accurate components much more efficiently. For example, typically we can only use hourly data in the last three weeks and per-minute data in the last three days to accurately perform decomposition, which only requires 24 \times 18 + 1440 \times 3 points (about 15.7\% of original data).

Specifically, in our framework we first down-sample the time series onto a lower resolution. And then we compute the seasonal difference on the lower resolution using the shorter periodic length to reduce the shorter periodicity, based on which the rough estimates of trend and the other seasonal component can be obtained by applying existing seasonal-trend decomposition algorithms directly. Next we build the relationship between time series on different resolutions, and then decompose the time series on the high resolution by solving an optimization problem efficiently using our proposed ADMM \cite{16} based algorithm.
2. PROBLEM STATEMENT

Without loss of generality, in this paper we assume the input time series are with two different seasonal components. The method proposed in this paper can be extended to time series with more seasonal components by applying our algorithm repeatedly. Formally, we consider the time series \( \{y_t\} \) can be decomposed as the sum of trend, seasonal and remainder components:

\[
y_t = \tau_t + s_{d,t} + s_{w,t} + r_t, \quad t = -T_0 + 1, \ldots, 0,
\]

where \( \tau_t \) and \( r_t \) denote the trend and remainder components, respectively, \( s_{d,t} \) and \( s_{w,t} \) denote two seasonal components with period length \( T_d \) and \( T_w \) (\( T_w > T_d \)), respectively. Here we denote current as \( t = 0 \) for convenience. In the following, we call \( s_{w,t} \), i.e., the seasonal component with longer period length \( T_w \), “the long seasonal component”, and \( s_{d,t} \) “the short seasonal component”. We assume that the trend is smooth locally and the remainder is composed of Gaussian noise \( n_t \) and possible outliers \( o_t \), i.e. \( r_t = n_t + o_t \). \( T_d \) and \( T_w \) are assumed to be known, as these parameters can be generally estimated accurately using well-developed multiple periodicity detection and estimation methods, e.g. RobustPeriod \( [12] \).

To reduce the storage cost, in this paper we propose to store the time series data in a multi-scale manner. Specifically, for recent periods we store \( y_t \) with high resolution, as we may need detailed information to identify the anomalies or the change of trend; for the relatively older data, the low resolution is used for storage to reduce the memory cost. Mathematically, with \( \{\hat{y}_t^h\} \) and \( \{\hat{y}_t^l\} \) denotes the stored high-resolution and low-resolution time series, respectively, we have

\[
\hat{y}_t^h = y_t \quad -T_r < t \leq 0
\]
\[
\hat{y}_t^l = \frac{1}{N} \sum_{i=(t-1)N+1}^{tN} y_i \quad -T_0 < tN \leq -T_r
\]

where \( T_r \) denotes the number of high-resolution data points we stored, and \( N \) denotes the down-sampling factor when store the low-resolution data. Here we also assume \( T_r^l = T_r^h / N, T_d^l = T_d^h / N \) and \( T_w^l = T_w^h / N \) are integer.

3. PROPOSED MULTI-SCALE DECOMPOSITION

In our multi-scale decomposition framework, we first down-sample the high-resolution data and obtain a full low-resolution time series. Then by exploiting the periodic nature of the short seasonality, we reduce the double periodic into a single periodic time series which is decomposed by applying existing decomposition algorithm. In this paper we select RobustSTL \( [12] \) due to its ability to handle abrupt trend change and seasonality shift. Finally, we build a relationship between the differentiation of the low-resolution time series and high-resolution time series, and decompose the high-resolution time-series by solving an optimization problem which utilizes the local smoothness of the trend.

3.1. Decomposition of Low-Resolution Time Series

In this subsection, we discuss how to decompose the low-resolution time series using RobustSTL \( [12] \). We augment \( \hat{y}_t^l \) for \(-T_r < t \leq 0\). According to \( [2] \) and \( [3] \), we define

\[
\hat{y}_t^l = \frac{1}{N} \sum_{i=(t-1)N+1}^{tN} \hat{y}_i^h, \quad -T_r < tN \leq 0
\]

Let \( \tau^l, s_{d,t}^l, s_{w,t}^l \) and \( r^l_t \) denote the corresponding low-resolution of trend, two seasonal and remainder components of the time series, respectively. Then we have \( \hat{y}_t^l = \tau^l_t + s_{d,t}^l + s_{w,t}^l + r^l_t \).

Note the down-sampled time series still contains two seasonal components. To further simplify the problem, we compute the seasonal difference of \( \{\hat{y}_t^l\} \) using the short periodic length. Specifically, the difference of the time-series is

\[
\hat{g}_t^l = \nabla T_d^l \hat{y}_t^l = \hat{y}_{t+T_d^l}^l - \hat{y}_t^l = \nabla T_d^l \tau^l_t + \nabla T_d^l s_{w,t}^l + r^l_t \quad (5)
\]

where \( r^l_t = \nabla T_d^l s_{d,t}^l + \nabla T_d^l r^l_t \). Here we use \( \nabla T \) to denote the difference of two points in a time series with a time difference of \( T \) points. As \( \tau_t \) is a local smooth signal with suddenly change, it is easy to see that \( \nabla T_d^l \tau^l_t \) is also smooth locally. Another observation is \( \nabla T_d^l s_{w,t}^l = s_{w,t+T_d^l}^l - s_{w,t}^l \) is periodic and zero-mean time series with period length equals to \( T_w^l \) since \( s_{w,t}^l \) is seasonal and period length equals to \( T_w^l \). Moreover, as the long seasonal component are assumed to change slowly across the nearby periods, we conclude that \( r^l_t \) can be modeled as a combination of Gaussian noise and outliers. Based on the above discussion, we can assume that \( \hat{g}_t^l \) is a time series with single seasonality. Thus, \( \nabla T_d^l s_{w,t}^l, \nabla T_d^l \tau^l_t \) and \( r^l_t \) can be computed by using RobustSTL.

3.2. Decomposition of High-Resolution Time Series

In this subsection we discuss how to estimate different components on high resolution with the help of the decomposition results of low-resolution time series. To bridge the gap between the high-resolution and low-resolution data, we compute the seasonal difference of the high-resolution of the data:

\[
\hat{g}_t^h = \nabla T_d^h \hat{y}_t^h = \hat{y}_{t+T_d^h}^h - \hat{y}_t^h = \nabla T_d^h \tau_t^h + \nabla T_d^h s_{w,t}^h + \tilde{r}_t^h \quad (6)
\]

where \( \tilde{r}_t^h = \nabla T_d^h s_{d,t}^h + \nabla T_d^h r_t^h \). Define \( \tau^h = [\tau^h_0^l, \ldots, \tau^h_{T_r^h-1}^l]^T \), \( \tau^h = [\tau^h_0^l, \ldots, \tau^h_{T_r^h-1}^l]^T \), \( s_{w}^h = [s_{w,0}^h, \ldots, s_{w,T_r^h-1}^h]^T \), \( g^h = [g^h_0^l, \ldots, g^h_{T_r^h-1}^l]^T \), and \( \tilde{r}^h = [\tilde{r}^h_0^h, \ldots, \tilde{r}^h_{T_r^h-1}^h]^T \), we have \( \tau^h = A \tau^h \), \( s_{w}^h = A s_{w}^h \) and \( g^h = D_{T_d^l} \tau^h + D_{T_d^l} s_{w}^h + \tilde{r}^h \) where \( A \) denotes the aggregation matrix, defined as \( A = 1/N(I \otimes 1^T) \), \( I \in \mathbb{R}^{T_r^h \times T_r^h} \) is an identity matrix, \( I \in \mathbb{R}^{N \times 1} \) is a vector with all entries equal to 1, and \( \otimes \) denotes the Kronecker product, \( D_{T_d^l} \in \mathbb{R}^{(T_w^l-T_d^l) \times T_r^h} \) denotes the difference matrix. The ith row of \( D_{T_d^l} \) is defined as a full zero vector with its ith and \((i+T_d^l)\)th entries equal to 1 and -1, respectively.
Let define $\nabla \tau^T_l = [\nabla \tau^T_l 0, \nabla \tau^T_{l-1}, \ldots, \nabla \tau^T_{l-T_l+1}]$, and define $\nabla s^T_l u = [\nabla s^T_l u,0, \nabla s^T_{l-1}, \ldots, \nabla s^T_{l-T_l+1}]$, we have $\nabla \tau^T_l \tau^T_l = D_{\tau^T_l} \tau^T_l$ and $\nabla s^T_l u = D_{s^T_l} s^T_l$, where $D_{\tau^T_l} \in \mathbb{R}^{(T_l-1)\times T_l}$ denotes the difference matrix. The $i$th row of $D_{\tau^T_l}$ defined as a full zero matrix with its $i$th and $(i+T_l)$th entries equal to 1 and -1, respectively. Then we propose to minimize following objective function to extract trend and the long period seasonality.

$$\|g^h - \bar{D}_{\tau} x\|_1 + \lambda_1 \|z - Bx\|_2^2 + \lambda_2 \|Dx\|_1 + \lambda_3 \|D^2 x\|_1$$

where $\bar{D}_{\tau} = [D_{\tau}, D_{\tau}], x = [(t^h)^T, (s^h)^T]^T$, $z = [\nabla \tau^T_l \tau^T_l, \nabla \tau^T_{l} s^T_l]^T$, $B = \text{bdiag}(D_{\tau^T_l} A, D_{\tau^T_l} A)$, $\hat{D} = \text{bdiag}(D, D)$ and $D^2 = \text{bdiag}(D^2, D^2)$. Here $\text{bdiag}(A, B)$ is a block diagonal matrix with the two blocks are matrix $A$ and $B$, respectively. $\nabla \tau^T_l \tau^T_l$ and $\nabla \tau^T_{l} s^T_l$ are the values of $\nabla \tau^T_l \tau^T_l$ and $\nabla \tau^T_{l} s^T_l$ that obtained from the decomposition of the low-resolution time series, respectively. $D \in \mathbb{R}^{(T_l-1)\times T_l}$ and $D^2 \in \mathbb{R}^{(T_l-2)\times (T_l-1)}$ denote the first and second order difference matrix, respectively. The $i$th row of $D$ and $D^2$ are defined as $[0_i^T, 1, -1, 0_{i-2}^T]_T$ and $[0_i^T, -1, 2, -1, 0_{i-3}^T]_T$, where $0_i$ denotes a vector of length $i$. We note that the first term of [7] is the empirical loss, the second and third terms push the decomposition of the high-resolution time series consistent with that of the low-resolution time series. The last term of [7] forces the trend and the long component to be smooth locally. We note that in many practical applications the long seasonal component changes relatively slowly on the high resolution, thus it is reasonable to assume its local smoothness.

### 3.3. Efficient Implementation

**Algorithm 1** Robust high-res time series Decomposition

**Input:** High-resolution time series $\{y^h\}$, $\nabla \tau^T_l$ and $\nabla \tau^T_{l} s^T_l$ obtained from low-resolution time series decomposition.

**Output:** High-resolution $x$, $s_{d,t}$ and $s_{w,t}$.

1. Denosing $\{y^h\}$ using bilateral filter;
2. Compute $g^h$ according to (6);
3. Initialize $\rho, \bar{p}, u_1, u_2$ and $u_3$;
4. **while** not converge **do**
5. Update $\bar{p}, p'$ and $p''$ according to (10);
6. Update $x$ using (9);
7. Update $u_1$, $u_2$ and $u_3$ according to (11);
8. **end while**
9. Subtract $s_{d,t}$ using non-local seasonal filtering;

Here we apply the widely used ADMM to solve the resulting optimization problem as summarized in Algorithm 1. We first apply the variable splitting trick to split the smooth and non-smooth terms by introducing some auxiliary variables, and formulate the problem as

$$\min \|g^h - \tilde{p}\|_1 + \lambda_1 \|z - Bx\|_2^2 + \lambda_2 \|p'\|_1 + \lambda_3 \|p''\|_1$$

s.t. $\tilde{p} = \hat{D}_{\tau} x$, $p' = \hat{D} x$, $p'' = \hat{D}^2 x$ (8)

Consequently, the augmented Lagrange is

$$\|g^h - \tilde{p}\|_1 + u^T_1 (\tilde{p} - \hat{D}_{\tau} x) + \rho/2 \| \tilde{p} - \hat{D}_{\tau} x \|_2^2 + \lambda_1 \|z - Bx\|_2^2 + \lambda_2 \|p'\|_1 + u^T_3 (p' - \hat{D} x) + \rho/2 \| p' - \hat{D} x \|_2^2 + \lambda_3 \|p''\|_1 + u^T_3 (p'' - \hat{D}^2 x) + \rho/2 \| p'' - \hat{D}^2 x \|_2^2,$$

where $\rho$ is a pre-defined parameter. Ignoring the terms independent with $x$, we can find the optimal $x$ is given as

$$x = Q^{-1} h$$

where $Q = \rho \hat{D}_{\tau}^T \hat{D}_{\tau} + 2\lambda_1 B^T B + \rho \hat{D}_{\tau}^T \hat{D}_{\tau} + \rho (\hat{D}^2)^T \hat{D}^2$ and $h = \hat{D}_{\tau} (u_1 + \rho \tilde{p}) + \lambda_1 B^T z + \hat{D}^T (u_3 + \rho p'') + (\hat{D}^2)^T (u_3 + \rho p'')$. We note that the function with respect to the auxiliary variables $\tilde{p}, p'$ and $p''$ can be generally formulated as the problem $\min_{x} \lambda \|x\|_1 + \|y - x\|_2^2$, and can be solved using proximal algorithms. Then the updating rule of these auxiliary variables are given as

$$\tilde{p} = g^h - S \left( g^h - \hat{D}_{\tau} x + 1/\rho u_1, 1/\rho \right),$$

$$p' = S \left( \hat{D} x - 1/\rho u_3, \lambda_2/\rho \right),$$

$$p'' = S \left( \hat{D}^2 x - 1/\rho u_3, \lambda_3/\rho \right)$$

where $S(x, \rho)$ denotes the soft threshold operator and is defined as $S(x, \rho) = \text{sign}(x) \max(|x| - \rho, 0)$. And the Lagrange multipliers can be updated using gradient ascend:

$$u_1^{(k+1)} = u_1^{(k)} + \rho (\tilde{p} - \hat{D}_{\tau} x),$$

$$u_2^{(k+1)} = u_2^{(k)} + \rho (p' - \hat{D} x),$$

$$u_3^{(k+1)} = u_3^{(k)} + \rho (p'' - \hat{D}^2 x).$$

(11)

Note that ADMM is guaranteed to converge with a sufficient large $\rho$. To ensure fast converge, we initialize $\rho$ with a small value and increase it iteratively with a factor, say 1.15. Also the inverse of matrix $Q$ can be computed offline, then the computational complexity of the proposed algorithm is dominated by the matrix-vector operation, e.g. $Q^{-1} h$, whose computational complexity is of order $O(T^2\rho)$.

### 4. EXPERIMENTS AND DISCUSSION

In this section, we conduct experiments on both synthetic and public datasets to demonstrate the effectiveness of the proposed algorithm. Though out our experiments, we initialize $\rho$ with $10^{-5}$. We compare algorithm with STL [18] and RobustSTL [12]. For those algorithms cannot handle multiple seasonality directly, we report the sum of all seasonal components as the final seasonal component.
Fig. 1. Generated synthetic data. Top row from left to right: generated seasonality $s_{d,t}$ and $s_{w,t}$, respectively. Bottom row from left to right: generated noise and raw time series. The red line in the bottom right splits the high-res and low-res data.

Fig. 2. Decomposition results on synthetic data. From left to right are decomposition results of proposed method, RobustSTL and standard STL, respectively. First column (from top to bottom): decomposed trend, the short and long seasonal components, and remainder using proposed method, respectively. Second and third column (from top to bottom): decomposed trends (low-res and high-res), seasonality and remainder, respectively.

4.1. Synthetic Data

We generate a time series of length 30240 containing two seasonality with periodic length of 1440 and 10080, respectively. We then add trend component with 3 abrupt changes, outliers as anomaly and white noise into the time series. Besides, time warping is also performed to imitate the real-word data. We keep the first 4320 points as the high-resolution series and aggregate the rest as the low-resolution series. We keep the first 4320 points as the high-resolution series and aggregate the rest as the low-resolution series. Here the aggregation factor $N$ is set to 60. Thus the high-resolution time series contains 3 periods of the short seasonal component and 1 abrupt change of trend, and the low-resolution time series is of length 432, as illustrated in Fig. 1.

Fig. 2 summarizes the decomposition results produced by three methods. Note that for STL and RobustSTL we use all the high-res data to achieve the decomposition. From Fig. 2, we observe that the proposed algorithm can decompose the short seasonal component accurately. It can also capture the abrupt trend change, outliers in remainder component. We also report the mean square error (MSE) of the decomposed trend and seasonality, and the running time in seconds of all methods in Table 1. It can be observed that proposed algorithm achieves the best trend and seasonal components with the least running time.

Fig. 3. Decomposition results on Yahoo! A1 dataset. From top to bottom are raw time series, decomposed seasonality and remainder by respective methods. The red line of the top sub-image splits the high-res and low-res data. From left to right of second and third rows are results obtained using proposed method and RobustSTL and STL, respectively.

4.2. Real Data

We next compare different algorithms on the public Yahoo A1 dataset. It is a collection of real production traffic to some of the Yahoo! properties with a total length of 504 and two seasonal components (periodic lengths are 24 and 168). We set first 72 data points as high-resolution time series and aggregate the rest with a factor 4 as low-resolution data. We inject one outlier as anomaly to show the robustness of the proposed method. Three algorithms are compared in the same setting as on the synthetic data.

Fig. 3 summarizes the decomposition results. Here we only plot the short seasonal component learned by the proposed method. From Fig. 3 we can observe that both the proposed method and RobustSTL are able to learn a decent seasonality and capture the outlier in the remainder, while STL fails. We also observe that the remainder decomposed by the proposed method involves fewer noise than RobustSTL.

5. CONCLUSION

In this paper we propose a robust and efficient seasonal-trend decomposition algorithm for time series with multiple seasonality using the multi-scale approach. It can achieve accurate decomposition with significant reduced storage and computation. In the future we plan to apply it in long-term and short-term forecasting by further utilizing the decomposed components at different resolutions.

Table 1. MSE and running time on synthetic data

| Method     | Trend   | Seasonality | Running Time (s) |
|------------|---------|-------------|------------------|
| STL        | 3.1012  | 1.7916      | 275.6            |
| RobustSTL  | 0.1090  | 0.0993      | 2138.23          |
| Proposed method | 0.0017  | 0.0019      | 17.4             |

1https://webscope.sandbox.yahoo.com/catalog.php?datatype=s&did=70
6. REFERENCES

[1] Elvin Isufi, Andreas Loukas, Nathanael Perraudin, and Geert Leus, “Forecasting time series with varma recursions on graphs,” IEEE Transactions on Signal Processing, vol. 67, no. 18, pp. 4870–4885, 2019.

[2] Qingsong Wen, Zhengzhi Ma, and Liang Sun, “On robust variance filtering and change of variance detection,” in ICASSP 2020-2020 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP). IEEE, 2020, pp. 3012–3016.

[3] Philippe Esling and Carlos Agon, “Time-series data mining,” ACM Computing Surveys (CSUR), vol. 45, no. 1, pp. 1–34, 2012.

[4] Phillip G Gould, Anne B Koehler, J Keith Ord, Ralph D Snyder, Rob J Hyndman, and Farshid Vahid-Araghi, “Forecasting time series with multiple seasonal patterns,” European Journal of Operational Research, vol. 191, no. 1, pp. 207–222, 2008.

[5] Samaneh Aminikhanghahi and Diane J Cook, “A survey of methods for time series change point detection,” Knowledge and Information Systems, vol. 51, no. 2, pp. 339–367, 2017.

[6] Jordan Hochenbaum, Owen S Vallis, and Arun Kejariwal, “Automatic anomaly detection in the cloud via statistical learning,” arXiv preprint arXiv:1704.07706, 2017.

[7] Nikolay Laptev, Saeed Amizadeh, and Ian Flint, “Generic and scalable framework for automated time-series anomaly detection,” in Proceedings of the 21th ACM SIGKDD international conference on knowledge discovery and data mining, 2015, pp. 1939–1947.

[8] Marina Theodosiou, “Forecasting monthly and quarterly time series using stl decomposition,” International Journal of Forecasting, vol. 27, no. 4, pp. 1178–1195, 2011.

[9] Rob J Hyndman, George Athanasopoulos, Christoph Bergmeir, Gabriel Caceres, Leanne Chhay, Mitchell O’Hara-Wild, Fotios Petropoulos, Slava Razbash, Earo Wang, and Farah Yasmeen, “Package ‘forecast’,” [Online] https://cran.r-project.org/web/packages/forecast/forecast.pdf, 2018.

[10] Alexander Dokumentov, Rob J Hyndman, et al., “STR: A seasonal-trend decomposition procedure based on regression,” Tech. Rep., Monash University, Department of Econometrics and Business Statistics, 2015.

[11] Alysha M De Livera, Rob J Hyndman, and Ralph D Snyder, “Forecasting time series with complex seasonal patterns using exponential smoothing,” Journal of the American Statistical Association, vol. 106, no. 496, pp. 1513–1527, 2011.

[12] Qingsong Wen, Jingkun Gao, Xiaomin Song, Liang Sun, Huan Xu, and Shenghuo Zhu, “RobustSTL: A robust seasonal-trend decomposition algorithm for long time series,” in Proceedings of the 33rd AAAI Conference on Artificial Intelligence (AAAI), 2019, pp. 1501–1509.

[13] Qingsong Wen, Zhe Zhang, Yan Li, and Liang Sun, “Fast RobustSTL: Efficient and robust seasonal-trend decomposition for time series with complex patterns,” in Proceedings of the 26th ACM SIGKDD International Conference on Knowledge Discovery & Data Mining (KDD), 2020, pp. 2203–2213.

[14] Galen Reeves, Jie Liu, Suman Nath, and Feng Zhao, “Managing massive time series streams with multi-scale compressed trickles,” Proceedings of the VLDB Endowment, vol. 2, no. 1, pp. 97–108, 2009.

[15] Spiros Papadimitriou and Philip Yu, “Optimal multi-scale patterns in time series streams,” in Proceedings of the 2006 ACM SIGMOD international conference on Management of data, 2006, pp. 647–658.

[16] Stephen Boyd, Neal Parikh, and Eric Chu, Distributed optimization and statistical learning via the alternating direction method of multipliers, Now Publishers Inc, 2011.

[17] Qingsong Wen, Kai He, Liang Sun, Yingying Zhang, Min Ke, and Huan Xu, “RobustPeriod: Time-frequency mining for robust multiple periodicities detection,” arXiv preprint arXiv:2002.09535, 2020.

[18] Robert B Cleveland, William S Cleveland, Jean E McRae, and Irma Terpenning, “STL: A seasonal-trend decomposition,” Journal of Official Statistics, vol. 6, no. 1, pp. 3–73, 1990.