Entanglement manipulation for mixed states in a multilevel atom interacting with a cavity field

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Abstract. We derive an explicit formula for an entanglement measure of mixed quantum states in a multi-level atom interacting with a cavity field within the framework of the quantum mutual entropy. We describe its theoretical basis and discuss its practical relevance (especially in comparison with already known pure state results). The effect of the number of levels involved on the entanglement is demonstrated via examples of three-, four- and five-level atom. Numerical calculations under current experimental conditions are performed and it is found that the number of levels present changes the general features of entanglement dramatically.

PACS numbers: 32.80.-t, 42.50.Ct, 03.65.Ud, 03.65.Yz

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1. Overview

Compared to the long history of the theoretical understanding of entanglement of atom-field systems extending over many decades [1,2], intensive experimental investigations started only recently involving different systems [3]. Entanglement lies at the heart of quantum mechanics, and is profoundly important in quantum information. To identify the fundamentally inequivalent ways quantum systems can be entangled is a major goal of quantum information theory [3]. It might be thought that there is nothing new to be said about bipartite entanglement if the shared state is pure, but in a recent paper [4] it has been shown that exact coherence of the atom is in general never regained for a two-level model with a general initial pure quantum state of the radiation field. Also, it has been shown that the purification of the atomic state is actually independent of the nature of the initial pure state of the radiation field.

From the viewpoint of the Phoenix-Knight [5] entropy formalism, the quantum field entropy and entanglement of a coherent field interacting with a three-level systems have been investigated [6]. However, the method used in those papers cannot be applied when the system is taken to be initially in a mixed state. A method using quantum mutual entropy to measure the degree of entanglement in the time development of the two-level system model has been adopted in [7]. The question of how mixed a two-level system and a field mode may be such that free entanglement arises in the course of the time evolution according to a Jaynes-Cummings type interaction has been considered [8-11].

It is important to point out that further insights into the dynamics of the multi-level systems may be helpful in developing quantum information theory [12]. Recently, there is much interest in multi-level quantum systems to represent information [13-15]. It was demonstrated that key distributions based on multi-level quantum systems are more secure against eavesdropping than those based on two-level systems [13]. Key distribution protocols based on entangled three-level systems were also proposed [16]. The security of these protocols is related to the violation of the Bell inequality. The multi-level system provides in this context a much smaller level of noise [17,18]. Rydberg atoms crossing superconductive cavities are an almost ideal system to generate entangled states, and to perform small scale quantum information processing [19]. In this context entanglement generation of multi-level quantum systems was also reported [12,20-23].

Our motivation is to suggest the use of quasi-mutual entropy for measuring quantum entanglement in the multi-level systems (further elaborated below). This is because the quasi-mutual entropy can be thought of as the original entanglement measure of mixed (rather than simply pure) input states. Motivated by recent experiments we analyze the entanglement degree of a multi-level system. Using an appropriate representation and without using the diagonal approximation, an explicit expression for a mixed state entanglement is derived. Although various special aspects of mixed states entanglement have been investigated previously, the general features of the dynamics, when a multi-level system is considered, have not been treated before and the present paper therefore fills a gap in the literature. In the present work we consider the situation for which the multi-level system is initially in a mixed
Entanglement manipulation for mixed state. We essentially generalize the entanglement degree due to the quasi-mutual entropy, usually employed in the two-level system, to the multi-level system interacting with a cavity field. The physical situation which we shall refer to, belongs to the experimental domains of cavity quantum electrodynamics.

The plan of the remainder of the paper as follows. In Section 2, we go through a more rigorous set of definitions leading up to the exact solution of the multi-level system and give exact expression for the unitary operator $U_t$ involved. In Section 3, we consider one of the intermediate definitions namely the accessible entanglement degree and develop several results related to this quantity. These include a more convenient expression that automatically takes into account an arbitrarily number of atomic levels, where the atom is initially in the mixed state, depending on both the value of the accessible entanglement and on the measurements required for its definition. In Section 4, we apply the theory to study a few examples in detail. In particular, we show how difficult it is to derive rigorously the entanglement involving more than three-levels. The final part of this article is devoted to some important developments of entanglement measures and we close the paper with a list of open questions.

2. The multi-level system

We start by devoting this section to a brief discussion on the multi-level atom [24,25] being it the model describing the interaction between a single multi-level atom and a quantized cavity field. To set the stage, we first begin by describing the multilevel-atom model. Therefore, the physical system on which we focus is an $m$-level. The atom interacts with a high Q-cavity which sustain a number of modes of the field with frequencies $\Omega_j, j = 1, 2, \ldots, m - 1$. We denote by $\hat{a}_j$ and $\hat{a}_j^\dagger$ the annihilation and creation operators for the field mode $j$, and $\omega_j$ is the frequency associated with the level of the atom. We assume that the mode $i$ affects the transition between the upper atomic level and the level $(i+1)$. Therefore in the rotating wave approximation we can cast the Hamiltonian of the system in the form [24] ($\hbar = 1$)

$$\hat{H} = \hat{H}_0 + \hat{H}_1,$$

where the Hamiltonian for the interacting system $\hat{H}_0$ is given by

$$\hat{H}_0 = \sum_{j=1}^{m-1} \Omega_j \hat{a}_j^\dagger \hat{a}_j + \sum_{i=1,2,\ldots,m} \omega_i \left| i \right\rangle \left\langle i \right|.$$

The interaction Hamiltonian between the atomic system and the cavity field is given by

$$\hat{H}_1 = \sum_{j=1}^{m-1} (\lambda_j (\hat{S}_{1,j+1} \hat{a}_j + h.c.).$$

The transition in the $m$-level atom is characterized by the coupling $\lambda_i$. The operator $\hat{S}_{ii}$ describes the atomic population of level $|i\rangle_A$ with energy $\omega_i, (i = 1, 2, \ldots, m)$ and the operator $\hat{S}_{ij} = |i\rangle\langle j|, (i \neq j)$ describes the transition from level $|i\rangle_A$ to level $|j\rangle_A$. 
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We have applied the rotating wave approximation discarding the rapidly oscillating terms and selecting the terms that oscillate with minimum frequency \[26\]. The resulting effective Hamiltonian may be written as

\[\hat{H}_0 = (\omega_1 - \Delta) I + \sum_{j=1}^{m-1} \Omega_j \left( \hat{a}_j^\dagger \hat{a}_j - \hat{S}_{j+1,j+1} \right),\]

\[\hat{H}_1 = \Delta \hat{S}_{11} + \sum_{j=1}^{m-1} \lambda_i \left( \hat{S}_{1,j+1} \hat{a}_i + \hat{S}_{j+1,i} \hat{a}_i^\dagger \right).\]

We have used \[\sum_{i=1}^{m} = I\]. Here we assume that the detuning parameter \(\Delta\) is given by

\[\Delta = \omega_1 - \omega_{j+1} - \Omega_j, \quad j = 1, 2, \ldots, m - 1.\]

It can be shown that \(\hat{H}_0\) and \(\hat{H}_1\) are constants of motion,

\[[\hat{H}_0, \hat{H}_1] = [\hat{H}, \hat{H}_0] = 0.\]

We assume that, before entering the cavity, the atom is prepared in a mixed state. Mixed states arise when there is some ignorance with respect to the system, so that consideration has to be given to the possibility that the system is in any one of several possible states, \(S_{ii}\), each with some probability, \(\gamma_i\), of being realized. To this end, the initial state of the atom can be written in the following form

\[\rho = \left( \gamma_1 \hat{S}_{11} + \gamma_2 \hat{S}_{22} + \gamma_3 \hat{S}_{33} + \ldots + \gamma_m \hat{S}_{mm} \right) \in S_A,\]

where \(\gamma_i \geq 0\), and \[\sum_{i=1}^{m} \gamma_i = 1\]. In terms of quantum information processes, an understanding of mixed states is essential, as it is almost inevitable that the ideal pure states will interact with the environment at some stage.

Also we suppose that the initial state of the field is given by

\[|\varpi_1\rangle = \left( \sum_{n_1,n_2,\ldots=0}^{\infty} b_{n_1} b_{n_2} \ldots b_{n_{m-1}} |n_1, n_2, \ldots, n_{m-1}\rangle \right) \in S_F,\]

where \(b_{n_i} = \langle \varpi | n_i \rangle\), \(b_{n_i}^2\) being the probability distribution of photon number for the initial state. The continuous map \(\mathcal{E}_t^*\) describing the time evolution between the atom and the field is defined by the unitary operator generated by \(\hat{H}\) such that

\[\mathcal{E}_t^* : S_A \rightarrow S_A \otimes S_F,\]

\[\mathcal{E}_t^* \rho = \hat{U}_t (\rho \otimes |\varpi\rangle \langle \varpi|) \hat{U}_t^*,\]

\[\hat{U}_t \equiv \exp \left( -\frac{i}{\hbar} \int_0^t \hat{H}(t) dt \right).\]

where \(\varpi = |\varpi_1\rangle \langle \varpi_1|\). Bearing these facts in mind we find that the evolution operator \(\hat{U}_t\) takes the next from

\[\hat{U}_t \equiv \exp \left( - (\omega_1 - \Delta) t \right) \left[ \prod_{j=1}^{m-1} \exp \left( -i\Omega_j \tilde{N}_j t \right) \right] \exp \left( -i \int_0^t \hat{H}_1 dt \right).\]
where $\hat{N}_j = \hat{a}_j^\dagger \hat{a}_j - S_{j+1,j+1}$. The first two factors in equation (11) produce phases that will not affect the results that follow, while calculations of the third factor show that it takes the following compact matrix form

$$\exp\left(-i\hat{H}_1 t\right) = \exp\left(-\frac{i}{2} \Delta t\right) \begin{bmatrix} \hat{U}_0 & \hat{U}_1 \\ \hat{U}_1^* & \hat{U}_2 \end{bmatrix},$$

(11)

where $\hat{U}_0$ is the single element matrix $\{\hat{U}_1\}$ which takes the following form

$$\hat{U}_{11} = \cos \hat{\mu}_n t - \frac{i \Delta \sin \hat{\mu}_n t}{2 \hat{\mu}_n}.$$

(12)

The matrix $\hat{U}_1^*$ is the $1 \times (m - 1)$ row matrix $\{\hat{U}_{1k}\}$, where

$$\hat{U}_{1k} = -i \frac{\sin \hat{\mu}_n t}{\hat{\mu}_n} \lambda_k \hat{a}_k, \quad k \in \{1, 2, 3, \ldots, m - 1\}$$

(13)

and $U_{BA}$ its Hermitian conjugate. Finally the matrix $\hat{U}_2$ is the $(m - 1) \times (m - 1)$ square matrix $\{\hat{U}_{ij}\}$ of which the elements can be written as

$$\hat{U}_{ij} = \delta_{ij} \exp\left(-\frac{i}{2} \Delta t\right) - \lambda_i \hat{a}_i^\dagger v^{-1} \left(\cos \hat{\mu}_n t - \exp\left(-\frac{i}{2} \Delta t\right) + \frac{i \Delta \sin \hat{\mu}_n t}{2 \hat{\mu}_n}\right) \lambda_j \hat{a}_j,$$

(14)

with $i, j = 1, 2, \ldots, m - 1$ and

$$\hat{\mu}_n = \left(\frac{\Delta^2}{4} + \sum_{i=1}^{m-1} \lambda_i^2 \hat{a}_i \hat{a}_i^\dagger\right)^{\frac{1}{2}}, \quad v^{-1} = \sum_{i=1}^{m-1} \lambda_i^2 \hat{a}_i \hat{a}_i^\dagger$$

(15)

Having obtained the explicit form of the unitary operator $U_t$, we are therefore able to discuss the entanglement of the system.

3. Derivation of the entanglement degree

Entanglement is recognized nowadays as a key ingredient for fundamental tests of quantum mechanics and as a basic resource of quantum information processing [1]. Quantifying the amount of entanglement between quantum systems is a recent pursuit that has attracted a diverse range of researchers [5-15]. When we look at the entanglement of the mixed state as a whole, can we still calculate the relative entropy of entanglement? This is in general very difficult to do for multipartite mixed states, and some partial methods for upper bounds have only been presented recently [27]. In this section, we will apply the results obtained previously to derive the entanglement degree for a single multi-level atom interacting with a cavity field without using the diagonal approximation method adapted in [8,9]. With a certain unitary operator, the final state after the interaction between the atom and the field is given by

$$\mathcal{E}_t \rho = U_t (\rho \otimes \varpi) U_t^*$$

$$= \gamma_1 U_t |a, \varpi\rangle \langle \varpi, a| U_t^* + \sum_{i=2}^{m-1} \gamma_i U_t |b_i, \varpi\rangle \langle \varpi, b_i| U_i^*.$$

(16)
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Therefore the von Neumann entropy of the total system is given by

$$S(E^*_t \rho) = - \sum_{i=1}^{m} \gamma_i \log \gamma_i.$$  \hspace{1cm} (17)

Taking the partial trace over the atomic system, we obtain

$$\rho_i^F = tr_A E^*_t \rho.$$  

Then the von Neumann entropy for the reduced state $S(\rho_i^F)$ is computed by

$$S(\rho_i^F) = - \sum_{i=1}^{m^2} \lambda_i^F(t) \log \lambda_i^F(t),$$  \hspace{1cm} (18)

where $\{\lambda_i^F(t)\}$ are the solutions of

$$\det[\hat{\rho}(t) - \lambda(t) \hat{N}(t)] = 0,$$  \hspace{1cm} (19)

where $\hat{\rho}(t)$ and $\hat{N}(t)$ are $m^2 \times m^2$ matrices having the following elements

$$\left[\hat{\rho}(t)\right]_{ij} \equiv \langle \psi_i(t) | \rho_i^F | \psi_j(t) \rangle, \quad (i, j = 1, 2, 3, ... m^2),$$

$$\left[\hat{N}(t)\right]_{ij} \equiv \langle \psi_i(t) | \psi_j(t) \rangle, \quad (i, j = 1, 2, 3, ... m^2),$$  \hspace{1cm} (20)

and $|\psi_j(t)\rangle$ are the eigenfunctions of the following eigenvalue problem $\rho_i^F | \psi_i(t) \rangle = \lambda_i^F(t) | \psi_i(t) \rangle$.

On the other hand, the final state of the atomic system is given by taking the partial trace over the field system:

$$\rho_i^A \equiv tr_F E_i^* \rho.$$  

Then the von Neumann entropy for the reduced state $S(\rho_i^A)$ is computed by

$$S(\rho_i^A) = - \sum_{i=1}^{m} \lambda_i^A(t) \log \lambda_i^A(t),$$  \hspace{1cm} (21)

where $\lambda_i^A(t)$ can be calculated by obtaining the eigenvalues of the reduced atomic state. Using the above equations, the final expression for the entanglement degree in the $m$-level system takes the following form

$$I_{E_i^* \rho} (\rho_i^A, \rho_i^F) \equiv tr E_i^* \rho (\log E_i^* \rho - \log (\rho_i^A \otimes \rho_i^F))$$

$$= \sum_{i=1}^{m} \gamma_i \log \gamma_i - \sum_{i=1}^{m^2} \lambda_i^F(t) \log \lambda_i^F(t) - \sum_{i=1}^{m} \lambda_i^A(t) \log \lambda_i^A(t).$$  \hspace{1cm} (22)

It turns out to be rather easy to derive an analytic expression for the entanglement degree for any given system, since with the help of equation (23) it is possible to study the entanglement degree of any $m$-level system when the system starts from its mixed state.

This seems significant, and one then wonders whether the trend might continue with the general multi-atom (or ions) case. That is, whether one might be able to consider more than one atom and still be able to measure the entanglement degree using quasi-mutual entropy. Also, one might wonder whether a similar effect could carry over to different field states. That would be very nice because one could contemplate different protocols for different initial states of the
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Figure 1. The evolution of the entanglement degree $I_{\gamma_i^t,\rho}(\rho_t^A,\rho_t^F)$ as a function of the scaled time. The mean photon number $\bar{n} = 5$, and the detuning parameter $\Delta$ has zero value, where, from bottom to top depicts three-, four- and five-level atom, respectively.

Figure 2. The same as in figure 1 but now $\bar{n} = 10$.

field. To go a step further towards a deterministic entanglement degree, we note a peculiar effect in the present paper: we get more entanglement with increasing $m$ (number of levels). Indeed in the limit that $\gamma_i \sim 0, \ i > 1$, (i.e. $\gamma_1 \approx 1$), the entanglement degree is only twice of the quantum field (atomic) entropy. In the general case (i.e., $\gamma_1 \neq 1$), the final state does not necessarily become a pure state, so that we need to make use of $I_{\gamma_i^t,\rho}(\rho_t^A,\rho_t^F)$ in order to measure the degree of entanglement in the present model. Thus our initial setting enables us to discuss the variation of the entanglement degree for different values of the parameter $\gamma_i$ of the initial atomic system. A related model allowing an analytic treatment of the mixed-state entanglement as well as valuable insight, namely the two-level atom ($m = 2$) has been discussed.
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in [8,9]. An example of a truly mixed state for which the entanglement manipulations have been proven to be asymptotically reversible has been reported in Ref. [28].

Here we focus on the time development of the entanglement degree for some special cases such as three-, four- and five-level atoms. In figure 1, we plot the function $I_{\varepsilon_{1}\rho}(\rho_{A}^{t},\rho_{F}^{t})$ which describes the entanglement degree in the case when the field is initially in a coherent state squeezed state with a mean photon number $\bar{\pi} = 5$, and the mixed state parameters $\gamma_{1} = 0.99$. In this case we see that, the entanglement degree function oscillates around values nearly equals the maximum values $\left(2\ln(m)\right)$. Let us remark that, in the pure state case, the von Neumann entropy is limited by $\ln(m)$, and then $I_{\varepsilon_{1}\rho}(\rho_{A}^{t},\rho_{F}^{t})$ reduces to $2\ln(m)$. From this figure we can say that the maximum value of entanglement degree $I_{\varepsilon_{1}\rho}(\rho_{A}^{t},\rho_{F}^{t})$ is increased as the number of levels is increased.

Nevertheless, the minimum values lie within the region between the two maximum values occurring in a similar way for different number of levels, such that with higher $m$ the minimum values of the entanglement degree occur at earlier times. In fact, for some higher values of $m$ there were no persisting periods found to lie between the maximum and minimum values. These results strongly indicate that the higher number of levels give higher entanglement as well as more oscillations. Figure 2, indicates that when the mean photon number is increased further the minimum values of the entanglement occur at later times.

![Figure 3](image-url)

Figure 3. The evolution of the entanglement degree $I_{\varepsilon_{1}\rho}(\rho_{A}^{t},\rho_{F}^{t})$ as a function of the scaled time. In this figure we consider the Fock state with $n = 5$, where, from bottom to top depicts three-, four- and five-level atom, respectively.

In figure 3, we consider the entanglement degree as a function of the scaled time with the field initially in a Fock state. The Fock state of the electromagnetic field is very difficult to produce in experiments. Nevertheless, these states are very important in quantum optics because of their intrinsic quantum nature. This case is quite interesting because the entanglement degree function oscillates around the maximum and minimum values in time. We have shown here a new phenomena where the periodic oscillations occur irrespective of
number of atomic levels involved. This reflects the various influences of the initial states of the field. A slight change in $n$ therefore, dramatically alters the entanglement. It should be noted that for a special choice of the initial state setting, the situation becomes interesting where we find that a higher multi-level atom interacting with an initially coherent field exhibits superstructures instead of the usual first-order revivals.

It is worth mentioning that the dynamics of quantum multi-level systems has always been of interest, but has recently attracted even more attention because of application in quantum computing. Several systems have been suggested as physical realizations of quantum bits allowing for the needed control manipulations, and for some of them the first elementary steps have been demonstrated in experiments [29].

4. Conclusion

Summarizing, we have shown how to determine the maximum and minimum possible values of the entanglement degree for multi-level atoms interacting with a cavity field. The forms of states that achieve these maximum and minimum values are the same as those for the case of the von Neumann entropy if we consider the pure state case. These results are applicable for measuring the entanglement for mixed states in any multi-level systems. We have identified the relation between the entanglement measures that is necessary for these mixed states setting. This relation holds between the quasi-mutual entropy and von Neumann entropy. The general formula we have derived may carry over to any multi-level system. For the examples we have examined the entanglement degree for three-, four- and five-level atoms, we have found that as the number of levels increases the maximum values of the entanglement degree also increases, but these values are achieved for earlier times when the number of levels is increased accordingly.

An open and very interesting question is whether the quasi-mutual entropy technique that we have described here can be transferred to other systems in which atomic and cavity decays are present. In those systems it may be possible to augment or simplify the definition of the quasi-mutual entropy making its applications more accessible.

Acknowledgments

M. Abdel-Aty wishes to express his thanks for the financial support and the hospitality extended to him at the IIU University Malaysia. M. R. B. Wahiddin acknowledges the support of Malaysia IRPA research grant 09-02-08-0203-EA002.

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