THE EVOLUTION OF SUPERNOVAE IN CIRCUMSTELLAR WIND-BLOWN BUBBLES. I.
INTRODUCTION AND ONE-DIMENSIONAL CALCULATIONS

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ABSTRACT

Mass loss from massive stars ($\geq 8 M_{\odot}$) can result in the formation of circumstellar wind-blown cavities surrounding the star, bordered by a thin, dense, cold shell. When the star explodes as a core-collapse supernova (SN), the resulting shock wave will interact with this modified medium around the star, rather than the interstellar medium. In this work we first explore the nature of the circumstellar medium around massive stars in various evolutionary stages. This is followed by a study of the evolution of SNe within these wind-blown bubbles. The evolution depends primarily on a single parameter \( \Lambda \), the ratio of the mass of the dense shell to that of the ejected material. We investigate the evolution for different values of this parameter. We also plot approximate X-ray surface brightness plots from the simulations. For very small values \( \Lambda \ll 1 \) the effect of the shell is negligible, as one would expect. Values of \( \Lambda \leq 1 \) affect the SN evolution, but the SN “forgets” about the existence of the shell in about 10 doubling times or so. The remnant density profile changes, and consequently the X-ray emission from the remnant will also change. The initial X-ray luminosity of the remnant is quite low, but interaction of the shock wave with the dense circumstellar shell can increase the luminosity by 2–3 orders of magnitude. As the reflected shock begins to move inward, X-ray images will show the presence of a double-shelled structure. Larger values result in more SN energy being expended to the shell. The resulting reflected shock moves quickly back to the origin, and the ejecta are thermalized rapidly. The evolution of the remnant is speeded up, and the entire remnant may appear bright in X-rays. If \( \Lambda > 1 \), then a substantial amount of energy may be expended in the shell. In the extreme case the SN may go directly from the free expansion to the adiabatic stage, bypassing the Sedov stage. Our results show that in many cases the SNR spends a significant amount of time within the bubble. The low density within the bubble can delay the onset of the Sedov stage and may end up reducing the amount of time spent in the Sedov stage. The complicated density profile within the bubble makes it difficult to infer the mass-loss properties of the pre-SN star by studying the evolution of the resulting SNR.

Subject headings: circumstellar matter — hydrodynamics — shock waves — supernova remnants — supernovae: general — X-rays: ISM

1. INTRODUCTION

Core-collapse supernovae (SNe) arise from the collapse of massive stars with mass \( M \gtrsim 8 M_{\odot} \). As these stars evolve along the main sequence (MS) and beyond it, they lose a considerable amount of mass in the form of winds (Kudritzki & Puls 2000). The collective action of the winds from different evolutionary stages may sweep up the material in the ambient medium to form a wind-blown cavity, bordered by a thin, dense, cold shell of material, which contains most of the swept-up material. When the star explodes as an SN, the shock wave resulting from the explosion evolves in this highly modified circumstellar (CS) medium, and not in the pristine interstellar medium (ISM; McKee 1988). The evolution of the supernova remnant (SNR) is therefore very different from the classical evolution in a constant-density ISM, where the remnant successively advances through the phases of free expansion, adiabatic or Sedov stage, radiative stage, and dispersal into the surrounding medium (see, e.g., Woltjer 1972).

Evidence has built up over the last few years that the medium around a pre-SN star is considerably modified by the action of winds, radiation, and stellar outbursts. This is highlighted by the famous Hubble Space Telescope (HST) picture of the three-ring nebula surrounding the SN 1987A. The combined action of winds and radiation in sculpting this medium is undeniable (Blondin & Lundqvist 1993; Chevalier & Dwarkadas 1995). Other known cases of SNe exploding in wind-driven shells are less spectacular. There are indications that the Cygnus Loop is the remnant of an old SN explosion that went off in a wind-blown bubble (Levenson et al. 1997). CS interaction models were earlier proposed for N49 (Shull et al. 1985) and N132D (Hughes 1987). Borkowski et al. (1996) have modeled the X-ray emission from Cas A as arising from an SN explosion within a wind-blown cavity.

Examples of wind-blown bubbles around massive stars are far more numerous (Chu 2003; Cappa et al. 2003). Ring nebulae are seen around MS O and B stars, Ofpe/WN9 stars (Nota 1999), and many Wolf-Rayet (W-R) stars in general. Bipolar nebulae are observed around many luminous blue variable (LBV) stars (Weis 2001), the best-known and best-studied one being \( \eta \) Carinae (Davidson & Humphreys 1997). CS nebulae are observed around red supergiant stars such as VY CMa (Smith et al. 2001; Humphreys et al. 2002) and around blue supergiant (BSG) stars such as NGC 6164/6165 (Leitherer & Chavarría-K. 1987). The progenitor of SN 1987A is thought to be a BSG. A similar structure is seen around the star Sher 25, suggesting that it may be an analog of SN 1987A (Chu 2003).

The evolution of remnants within wind-blown cavities has merited some attention in the past. The problem was discussed numerically by Ciotti & D’Ercole (1989). It was placed on a firm theoretical footing with analytic calculations by Chevalier & Liang (1989, hereafter CL89). Early numerical calculations are described in a series of papers by Tenorio-Tagle and collaborators, wherein they discussed the one-dimensional evolution...
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Core-collapse SNe (which includes classes Type II and Type Ib/c) arise from the gravitational collapse of massive stars. The line dividing massive versus nonmassive stars is usually drawn at about \(8 \ M_\odot\), which is the mass at which a star will not eventually turn into a white dwarf (Langer 1994). This, however, still leaves a large range of stars, with initial MS mass between 8 and 200 \(M_\odot\), that can explode as SNe. The evolution and mass-loss characteristics of these stars are quite different over this large range. Thus, the environment into which a core-collapse SN evolves will be considerably different depending on the initial mass of the star.

The velocity of the wind emitted from a star is generally comparable to or a few times larger than the escape velocity of the star, as would be expected. The mass-loss rate depends on what drives the material from the stellar surface (Lamers & Cassinelli 1999). Stars like the Sun have coronal, pressure-driven winds with a very low mass-loss rate. Hot, luminous stars such as O, B, and W-R stars have radiatively driven winds. The mass-loss mechanism in cool evolved stars, such as red supergiants (RSGs), is not well understood, although it is suspected to be due to radiation pressure on dust grains.

The two parameters, mass-loss rate and wind velocity, determine the density of the wind around the star. For a wind with constant properties, the density \(\rho\) at a radius \(r\) is given by \(\rho = M/(4\pi r^2 v_w)\), where \(M\) is the mass-loss rate and \(v_w\) is the wind velocity. In general, we can differentiate two clear regimes, discussed in the next two subsections.

2. STELLAR MASS LOSS

Most stars larger than about 11 \(M_\odot\) will become red supergiant stars (A. Heger 2004, private communication). The lack of RSGs above 50–60 \(M_\odot\) appeared to indicate the existence of an upper limit to the RSG stage. Conflicting reports about this result have been recently clarified by Lamers et al. (2001). More importantly for this paper, most single stars below about 35 \(M_\odot\) will explode as SNe while in the RSG stage (A. Heger 2004, private communication). These results are quoted only for solar metallicities; for lower metallicities the number of RSGs reduces considerably. Plus rotational mixing, semiconvection, and overshoot will change these numbers. Nevertheless, since the number of stars decreases sharply with increasing mass, the result generally implies that a large fraction of core-collapse SNe arise from RSG stars.

While the mass-loss process in red supergiants is not generally well understood, they are observed to have very slow winds, on the order of 20–100 km s\(^{-1}\), with high mass-loss rates on the order of a few times \(10^{-8}\) to \(10^{-4}\) \(M_\odot\) yr\(^{-1}\). The mass loss is not uniform but could vary over time periods of a few thousand years. One of the best-observed post-RSG stars, IRC +10420, shows evidence for an increasingly complex environment around the star, with multiple shells, several individual condensations, and a generally asymmetric structure (Humphreys et al. 1997). These authors infer a mass-loss rate about \(10^{-6}\) \(M_\odot\) yr\(^{-1}\) in the MS phase, increasing to a few times \(10^{-4}\) to \(10^{-2}\) \(M_\odot\) yr\(^{-1}\) during various stages of evolution. Another extreme RSG star VY CMa also shows evidence of an asymmetric structure, reflection arcs and knots, and several bright condensations (Smith et al. 2001). The deduced mass-loss rate is about \(3 \times 10^{-3}\) \(M_\odot\) yr\(^{-1}\).

The high mass-loss rate and low wind velocity imply a high density for the CS material around the star. In general, the density can be written as

\[\rho_{\text{RSG}} \sim 5 \times 10^{-20} \dot{M}_{\text{CS}} r_{\text{CS}}^{-2} v_{\text{CS}}^{-1},\]
where $\dot{M}_{-4}$ is the mass-loss rate in units of $10^{-4} M_\odot$, $r_{17}$ is the radius in units of $10^{17}$ cm, and $v_1$ is the wind velocity in units of 10 km s$^{-1}$. Assuming that the RSG stage lasts for $10^3$ yr, this RSG wind can alter the medium to a distance of a few parsecs.

2.2. Mass Loss from Hot Massive Stars

Stars above about 35 $M_\odot$ suffer from considerable mass loss, which tends to strip them of their H envelopes. These W-R stars are presumed to be the progenitors of Type Ib/c SNe, which do not show H lines in their spectrum. These stars are in general hot, massive stars, which lose mass via radiatively driven winds from the stellar surface. Their progenitors are mainly early-type O and B stars, with mass-loss rates on the order of $10^{-8}$ to $10^{-5} M_\odot$ yr$^{-1}$ and terminal wind velocities about 1000–3000 km s$^{-1}$ (de Jager et al. 1988). When the star evolves off the MS stage and into the W-R phase (with perhaps some intermediary stages depending on the initial mass), the mass-loss rate can increase to a few times $10^{-3} M_\odot$ yr$^{-1}$ (van der Hucht 1997). Thus, these stars can lose a large fraction of their stellar mass during their lifetime. For example, in one of Norbert Langer’s models, a 35 $M_\odot$ star has only about 9 $M_\odot$ remaining when it explodes as a SN, with 26 $M_\odot$ being deposited in the surrounding medium. This material can form large wind-blown cavities stretching for tens of parsecs. The various evolutionary stages may lead to the formation of multiple wind-blown shells, which may not always be visible.

Given the large wind velocities of MS and W-R hot stars, the density of the CS medium will be considerably reduced as compared to the RSG stars mentioned above. The density from a W-R wind at a radius $r$ from the star is given by

$$\rho_{W-R} \sim 2.5 \times 10^{-23} \dot{M}_{-4} r_{17}^2 v_1^{-1},$$

where $\dot{M}_{-5}$ is the mass-loss rate in units of $10^{-5} M_\odot$, $r_{17}$ is the radius in units of $10^{17}$ cm, and $v_1$ is the wind velocity in units of 10 km s$^{-1}$.

It is clear that the density of the wind around a W-R star is about 2–3 orders of magnitude less than that around an RSG star. This has important consequences for the SN evolution, since much of the emission from the young remnant is due to the interaction with the CS medium (Chevalier & Fransson 1994). In most cases this depends on the density of the surrounding medium and will therefore be considerably reduced in a lower density medium.

The above discussion shows that while SNe arise from both RSG and W-R progenitors, the immediate medium into which the shock wave evolves will be quite different. Both of the stages are post-MS stages, and the W-R stage may sometimes even be a post-RSG stage. In general, the wind from the star in each case will evolve inside a previously evacuated wind-blown cavity carved out by the MS stage. An important distinction, however, is that the wind from the RSG star is usually slower than the wind from the preceding stage and will not shock it but will result in a new pressure equilibrium. The fast W-R wind will drive a strong shock wave into the surrounding medium. These variations can lead to very different structures around the star.

What is clear is that in many cases a SN will expand into a low-density environment during its initial evolution, with density lower than the surrounding ISM. In the case of SNe from hot massive stars, this low-density medium will start right from the star itself. In the case of RSG progenitors, there may exist a very high density medium around the star, in which the remnant may evolve for a few tens to hundreds of years. Following this, the remnant will continue to evolve for some time in a low-density medium formed by the wind from the MS star, usually an O or B star. In this work, therefore, we have concentrated on remnants evolving in a low-density CS wind-blown bubble.

2.3. Other Considerations

The evolution of single, massive stars to the SN stage has received considerable attention. However, many stars have one or more companions, the presence of which can considerably alter the evolution of the star. Mass transfer from or to a companion star can slow down or speed up the evolution of the star, alter its evolutionary track, and even result in a stellar explosion as in the case of Type Ia SNe. The best-studied SN, SN 1987A, is known to have a BSG progenitor, a B3 Ia star when it exploded. A BSG progenitor was unexpected, as it indicated that the star on the H-R diagram went from the blue to the red and back to the blue region before it exploded. Although some single-star scenarios have managed to produce a BSG before the star explodes (see, e.g., Woosley et al. 1997), they require very low metallicities. Rotation changes the dynamics somewhat and may also help to explain the origin of the bipolar CS shell surrounding SN 1987A, but it still appears to require special conditions. A BSG progenitor has consequently been suggested as indicative of binary evolution (Podsiadlowski 1992). While conclusive evidence is lacking, it does imply that stars in binary systems evolve differently from single stars, and SN progenitors in binary systems may differ from single-star progenitors. Not surprisingly, the medium around the star will also be correspondingly different. SN 1987A clearly shows the presence of a beautiful bipolar CS shell structure, surrounded by a thin dense shell. However, the density within the bubble is neither as large as that in RSGs nor as low as that in W-R stars, but somewhat intermediate between the two, on the order of 1 particle cm$^{-3}$. A region of ionized wind material, with a density of about 100 particles cm$^{-3}$, also exists inner to the CS nebula (Chevalier & Dwarkadas 1995).

3. CIRCUMSTELLAR WIND-BLOWN BUBBLES

The interaction between the wind from a star and the surrounding medium, be it the ISM or the wind from a previous epoch, can lead to the formation of wind-blown cavities surrounded by a thin, dense, cool shell. The structure of these wind-driven bubbles, in the adiabatic case, was first elucidated by Weaver et al. (1977) and further explored by Koo & McKee (1992a, 1992b). In the simplest case, that of mass loss from a fast wind with constant parameters interacting with a slower wind (or the ISM) with constant parameters, a self-similar solution can be obtained. Figure 1 shows the pressure and density structure within a wind-blown bubble in this case. Going from left to right, we identify four regions: (1) the freely flowing fast wind, (2) the shocked fast wind, (3) the shocked ambient medium, and (4) the unshocked ambient medium. An outer or forward shock ($R_o$) separates the shocked and unshocked ambient medium, and an inner shock, alternatively referred to as a reverse or wind termination shock ($R_i$), separates the shocked and unshocked fast wind. The shocked ambient medium is separated from the shocked fast wind by a contact discontinuity ($R_{CD}$). The example shown is that of an “energy-conserving” structure. The interior of this structure consists mainly of a low-density, high-pressure and therefore high-temperature region. The isotropic pressure of this region is responsible for driving the outer shock wave. The temperature could be high enough to be visible in X-rays, but the emission measure is usually low, so not many bubbles have been observed with present-day X-ray telescopes.

For a wind with constant properties expanding into a constant-density ISM, Weaver et al. (1977) showed that a self-similar solution could be obtained for the evolution of the contact
discontinuity with time, namely, $R_{CD} \propto (L_w/\rho)^{1/5} t^{3/5}$. Here $L_w$ is the mechanical wind luminosity $L_w = 0.5 M_{\odot} v_w^2$. If the fast wind expands into a slower wind from a previous epoch, the result becomes $R_{CD} \propto (L_w/\rho)^{1/3} t$. Thus, the shell expands with constant velocity.

The above description has ignored many other contributing factors. The wind properties are unlikely to be constant throughout the evolution, thus giving rise to multiple shells. UV photons from the hot stars will ionize the medium around the star, thus altering the dynamics. Factors such as conduction (Weaver et al. 1977) and/or magnetic fields may change the interior dynamics. Heat conduction in particular has been cited as one of the reasons for lowering the temperature in the interior of the bubble. Multi-dimensional effects such as hydrodynamic instabilities can alter the evolution (Dwarkadas & Balick 1998), as we shall see in a future paper. The presence of instabilities can result in considerable turbulence within the interior of the bubble, breaking the approximations of isotropic pressure and a radial velocity field.

The results above refer to a homogeneous medium. In general, the ISM is known to be inhomogeneous and clumpy. However, McKee et al. (1984) have shown that an early-type MS star may homogenize the medium up to a radius

$$R_h(t_{ms}) = 56 n_m^{-0.3} \text{pc},$$

where $t_{ms} = 4.4 \times 10^6 S_{49}^{-0.25}$ yr is the MS lifetime of stars of spectral type B0–O4, $S_{49}$ is the ionizing flux in units of $10^{49}$ photons s$^{-1}$, and $n_m$ is the mean density the cloud gas would have if it were homogenized. Therefore, we expect that the medium around an early-type star can be treated as a smooth medium at least for distances up to about 50 pc from the star. In later papers we will consider the effects of a clumpy medium.

Thus far we have considered only spherical CS bubbles. The CS structures around many massive stars, especially W-R stars and/or Ofpe/W9 stars, do fall in this category. However, many nebulae around LBV stars show distinct evidence of bipolarity (Weis 2001). The bipolar structure around SN 1987A has been clearly mapped using light echoes (Crotts et al. 1995). It is unclear what additional ingredients contribute to the formation of these aspherical nebulae. A common explanation tends to invoke an asymmetry in the surrounding medium, such as an equatorial disk, which inhibits the expansion of these nebulae along the equator but allows for free expansion in the polar regions, leading to bipolarity. An alternative explanation attributes the asphericity not to the external medium but to the wind from the star itself. For example, a rotating star may give rise to a wind that is faster in the polar regions and slower in the equatorial regions (see, e.g., Dwarkadas & Owocki 2002). In this paper we consider the interaction of SN shock waves with spherical nebulae. In future papers we will consider how the interaction changes due to deviations from sphericity.

It is clear that the CS bubble results from the impact of a fast wind on a slower ambient medium. However, in many cases the situation may be reversed, i.e., it is a slower wind that impacts on a faster wind from a previous epoch. This may very well be the general case for RSG winds, whose velocity is about 20 km s$^{-1}$, and which may follow a much faster MS wind. Such a situation is illustrated in Figure 2, which illustrates a snapshot from a simulation in which a MS wind is followed by a RSG wind. The figure shows the density and pressure profiles close to the end of the RSG stage, as the RSG wind creates a new pressure equilibrium. Unlike the case illustrated above, no wind-driven bubble is created, but the wind is almost freely expanding all the way to the interaction region with the MS wind, from which it is separated by a contact discontinuity. The evolution of the SN ejecta in this medium would be described by the self-similar solutions for power-law ejecta evolving in a power-law ambient medium (see below), up until the time the ejecta collided with the shell. Given the low wind velocity ($v_{RSG}$) and a maximum RSG lifetime ($t_{RSG}$) of about 100,000 yr, the interaction of SN ejecta with a RSG wind can last for about

$$t_{int} = 400 \left( \frac{t_{RSG}}{100,000 \text{ yr}} \right) \left( \frac{v_{RSG}}{20 \text{ km s}^{-1}} \right) \left( \frac{v_{SN}}{5000 \text{ km s}^{-1}} \right)^{-1} \text{yr},$$

where we have used an average velocity of 5000 km s$^{-1}$ to take deceleration of the ejecta by the dense RSG wind into account. For higher mass stars the RSG phase may be more short lived, and the interaction time will decrease. After this period the remnant will continue to evolve in a low-density wind-blown bubble formed by the MS star, until it collides with the dense shell.

4. THE SN EJECTA

We are dealing with young SNe, which are still in an ejecta-dominated stage. To describe the SN evolution from the early
stages, a prescription for the profile of the ejected material is required. This profile is established very early on, in the first few days after the explosion. The velocity of the ejected material tends toward free expansion, with \( v = \dot{r}/t \) as expected. The density in free expansion at a particular velocity decreases as \( r^{-3} \) (Chevalier & Fransson 1994).

The density distribution of the ejected material depends on the structure of the pre-SN star. In core-collapse SNe where the explosion energy is all contained in the center, the distribution of ejected material depends on whether the envelope is convective or radiative (MM99), and in the former case on the composition. The radiative case is more typical of W-R stars and other very massive stars, whereas RSGs are better described by the convective case. Analytical and numerical calculations (Chevalier & Soker 1989; MM99) have shown that the ejecta density can be reasonably well represented as a power law in the outer parts of the ejecta \( (\rho_{SN} \propto r^{-n}) \), with a flat or shallower distribution \( (n < 3) \) below a certain velocity \( \dot{v}_C \). MM99 find that the RSG profile distribution can be described by a power law with an exponent \( n = 11.7 \), whereas the convective case is better described by a slightly shallower power law \( n = 10.2 \). We note that these are approximations, although good ones; in general, the power law may vary over the ejecta structure. In fact, Dwarkadas & Chevalier (1998) showed by inspection of the numerical models that the ejecta in Type Ia SNe could best be described by an exponential profile.

The interaction of power-law ejecta \( (\rho_{SN} \propto r^{-n}) \) with a power-law surrounding medium \( (\rho_{CS} \propto r^{-x}) \) can be described by a self-similar solution (Chevalier 1982), wherein the contact discontinuity evolves with time as \( R_{CD} \propto t^{(n-3)/(n-x)} \). Such a formulation has often been used in the past to describe SN evolution in stellar winds.

In this work we assume that the ejecta are described by a power law, with a power-law exponent of 9. This is close enough to the values suggested by MM99, while also being the value suggested for the ejecta of SN 1987A. The mass of material ejected in the explosion is taken as \( 10^{5} \) \( M_{\odot} \). Our values are different from those used by Tenorio-Tagle and collaborators in all their publications (Tenorio-Tagle et al. 1990). They used an SN density profile decreasing as \( r^{-2} \) and a total mass of ejected material of \( 4 \times 10^{5} \) \( M_{\odot} \). Their ejecta density profile is too shallow compared to the work of MM99. Besides, a value of \( n > 5 \) ensures a finite mass and energy for the ejecta. The difference between the density profile used herein and that employed in Tenorio-Tagle et al. (1990) is that their shallower profile results in a considerable amount of the ejected mass being present at large velocities. We have used a higher value for the ejected mass on the assumption that many massive stars, especially in the middle-mass range of \( 10 \sim 30 \) \( M_{\odot} \), would leave behind a compact remnant and eject most of their remaining mass in the explosion. This is an arbitrary number, and in future papers we will compute models with a smaller value of the ejected material. As discussed below, the remnant evolution is not so much a function of the ejected mass itself as of the ratio of the mass of the dense CS shell to the ejected mass. The evolution to the Sedov stage, however, will depend on the amount of material ejected (see § 8).

5. THIN-SHELL MODEL

In this section we develop an analytical thin-shell model for the evolution of the SN–CS shell interaction. Although some approximations are necessary, the model shows the dependence on the parameters and gives general insight into the evolution.

There are two main approximations. First, the CS shell plus swept-up external gas can be regarded as a thin shell with thickness \( \Delta R \) much less than radius \( R \). This approximation should be adequate for the early evolution but will break down as the outer shock front moves away from the shell position. Second, the SN energy interior to the shell can be regarded as giving a region of constant internal energy. This approximation is inaccurate during the early evolution, when there are pressure waves and shock waves moving through the interior, but should become accurate later in the evolution once complete thermalization of the ejecta energy has occurred. A final approximation is that the expansion is spherically symmetric; that is, instabilities do not have a major effect on the evolution.

We consider the CS shell to have a mass \( M_{sh} \) and to be surrounded by a cold medium with density distribution \( \rho = \rho_a(R/R_i)^{-s} \), where \( \rho_a \) is a constant, \( R_i \) is the initial shell radius, and \( s \) is a constant < 3. The cases of most interest are \( s = 0 \) (constant-density ISM) and \( s = 2 \) (CS gas from steady mass loss). The mass conservation equation is

\[
\frac{dM}{dt} = 4\pi R^2 \rho_a V,
\]

where \( M \) is the shell mass including the swept-up mass and \( V \) is the shell velocity. The mass equation can be integrated to yield

\[
M = M_{sh} [1 + m(y^{3-s} - 1)],
\]

where \( m = M_i/M_{sh} = (4\pi R_i^2 \rho_a)/(3-s)M_{sh} \) is a constant, \( M_i \) is the mass that the surrounding medium would have if it extended from \( r = 0 \) to \( R_i \), and \( y = R/R_i \).

The shell initially receives an impulse when it is hit by the SN shock front. CL89 have described some of the details of this interaction, allowing for a steep outer density profile in the SN ejecta. Here we make the approximation that the initial shell velocity from this interaction is \( V_i = (\alpha p_{SN}/\rho_{sh})^{1/2} \), where \( \alpha \) is a constant (\( \sim 6 \); see CL89), \( p_{SN} = E/2\pi R_i^2 \), \( E \) is the SN energy, and \( \rho_{sh} \) is the initial density in the shell. The initial pressure in the shell, \( p_i \), for the shell evolution is then given by \( E = 2\pi R_i^2 p_i + M_{sh}V_i^2/2 \). We find it convenient to use a dimensionless velocity variable

\[
w^2 = \frac{V^2}{m_p / \rho_a}.
\]

With this definition, the initial shell velocity is

\[
w_i = \left[ \frac{m_p \rho_{sh}}{\alpha \rho_a} - \frac{1}{(3-s)} \right]^{-1/2} = \frac{1}{(3-s)} \left( \frac{R_i}{\alpha \Delta R} - 1 \right).
\]

The equation of motion of the shell is

\[
M \frac{dV}{dt} = 4\pi R^2 \left( p_{in} - \rho_s V^2 \right),
\]

where \( p_{in} \) is the interior pressure and evolves as \( \rho_s V^{-3} \) due to adiabatic expansion. From the above discussion and substitutions, the equation can be written as

\[
\frac{dw^2}{dy} = \frac{2(3-s)w^2 (y^{-3} - my^{-2}w^2)}{[1 + m(y^{3-s} - 1)]}.
\]
This equation can be solved using standard techniques to give

\[
  w^2 = \left[ w_i^2 + (3 - s)(1 - m)(1 - y^{-2}) + 2m(3 - s) \times (1 - s)^{-1} (y^{1-s} - 1) \right] \left[ 1 + m(y^{1-s} - 1) \right]^{-2},
\]

where \( s \neq 1 \).

We note that if we take \( m = 1, s = 0, y \gg 1 \), and \( w_i = 0 \) in the above equation, we then find that \( w^2 \propto y^{-5} \). This gives \( R \propto t^{2/7} \), which is similar to the dependence obtained by McKee & Ostriker (1977) for the late phase of SNR expansion. Even the constant of equality depends on the same quantities. The solution of McKee & Ostriker (1977) is obtained when the evaporated cloud mass is negligible and the mass swept up by the SN shock wave is the mass of the surrounding medium up to that point. It is not surprising that in this limit we find a similar behavior. The wind-blown shell for \( m = 1 \) consists of all the swept-up mass up to that point. The shell in this case is the preexisting shell due to the wind-blown bubble and not the dense shell that has formed by the SN material cooling. However, once the SN shock collides with this wind-blown dense shell and pushes it sufficiently far, the evolution is essentially the same, and therefore we reproduce the solution that applies to a SNR in the radiative stages. Note that if \( m \neq 1 \), then the relation changes, as it does for \( y \) just slightly larger than 1.

For the sake of completeness we also show the solution where \( s = 1 \). This may occur, for example, in a region consisting of molecular clouds (Cernicharo et al. 1985; Stiwe 1990), although it must be noted that these are very clumpy, and a homogeneous treatment is not really applicable. In this case we get

\[
  w^2 = \frac{w_i^2 + 2(m - 1)(y^{-2} - 1) + 4m \ln y}{[1 + m(y^2 - 1)]^2}.
\]
We also show the solution for planar \((q = 1)\), cylindrical \((q = 2)\), and spherical \((q = 3)\) geometry when \(s = 0\):

\[
w^2 = \frac{w_t^2 + 3 + 6my^{q/3} - 3(1 - m)y^{-2q/3} - 9m}{[1 + m(y^q - 1)]^2},
\]

(12)

The parameters here have definitions equivalent to those for the spherical case.

The solutions for spherical expansion with \(s = 0\) and 2 are illustrated in Figure 3. The range of \(m\) is from 0 (no surrounding medium) to 1 (mass in shell equal to mass external medium would have if it extended from \(R_i\) to \(r = 0\)). The \(m = 1\) case is thus what would be obtained if the shell is just the swept-up external medium, as could be the result of a fast wind from the central star sweeping up a slower wind from a previous epoch. The \(m = 0\) case would occur, for instance, if the shell were composed of mass ejected from the central star in a low-density ambient medium. Low density in this case implies that the mass of any ejected shell is large enough, and the ambient density small enough, that the shocked shell would not sweep up a mass comparable to its own over many doubling times of the radius. Mass ejections on the order of a solar mass have been presumed to occur in LBVs such as \(\eta\) Carinae. If \(m = 0\), the shell mass remains constant with time. Strictly speaking equations (6) and (7) are not valid for \(m = 0\), although the final solution in equation (10) still holds. The \(m = 0\) cases show no deceleration, but rapid deceleration does occur for higher \(m\). For \(s = 0\), it can be clearly seen from equation (10) that for any finite, nonzero value of \(m\), the denominator goes as \(y^q\) for large \(y\). This implies that in the presence of an ambient medium, over several doubling times the shell is invariably going to slow down to a low velocity irrespective of the initial mass and velocity of the shell. It should be kept in mind, however, that with deceleration the thin-shell approximation is expected to break down if the flow is nonradiative.

One way to estimate the deceleration is to examine a “braking index” \(\Omega = RR^2/R^2\). For a self-similar flow expanding as \(R \propto t^k\), we have \(\Omega = (k - 1)/k\); the thicknesses of shocked regions for various values of \(k\) can be found in Chevalier (1982). There is a critical value of \(k\) corresponding to a point explosion blast wave for which the shell swept up by the SN shock wave is expected to become very broad. The value is \(k = 2(5 - s)\) or \(\Omega_c = -(3 - s)/2\). Once this value of \(\Omega\) is reached, the shock front can be expected to strongly separate from the shell. The braking index can also be calculated for the shell motion, with the result for the \(m = 1\) case:

\[
\Omega(s = 0) = -3 \left( \frac{w_t^2 - 6 + 5y^\lambda}{w_t^2 - 6 + 6y^\lambda} \right),
\]

\[
\Omega(s = 2) = -\frac{w_t^2 + 2 - 3y^{-1}}{w_t^2 + 2 - 2y^{-1}}.
\]

(13)

We thus find that \(\Omega = \Omega_c\) for \(y = (6 - w_t^2)/4\) (\(s = 0\)) and \(y = 4/(2 + w_t^2)\) (\(s = 2\)); the thin-shell phase is quite short lived for \(m = 1\). At the other extreme of \(m = 0\), there is no deceleration, which is expected because the ambient medium does not exert any significant pressure on the shell, and the shell tends toward a constant velocity. From equation (12), for a spherical shell this occurs when \(y^2 > 1\).

6. NUMERICAL SIMULATIONS

The numerical simulations described herein were carried out using the VH-1 code, a one-, two-, and three-dimensional numerical hydrodynamics code developed at the University of Virginia by John Hawley, John Blondin, and coworkers. It is based on the piecewise parabolic method described in Colella & Woodward (1984). The method is well suited to simulating compressible flows in astrophysics, and the code has been used previously by this author and others for astrophysical calculations. Cooling is employed in the form of a cooling function. We use the one described in Sutherland & Dopita (1993). Cooling is necessary in order to simulate the evolution of the CS bubble and the formation of the thin, dense, cooled shell, but the amount of cooling is relatively insensitive to the details of the cooling function used. Cooling is not generally significant in the evolution of the remnant, except when the shell mass significantly exceeds the ejecta mass, and the shock-shell structure begins to cool as the shock impacts with the shell (see § 7.3). No contact discontinuity steepener is used. Simulations are carried out on a spherical-polar \((\theta, \phi)\) grid. Inflow (describing SN ejecta) and outflow (describing surrounding medium) boundary conditions are used at the inner and outer boundary, respectively, while reflecting boundary conditions are used at the remaining boundaries in the case of two dimensions. One advantage of this code is that it employs an expanding grid, which tracks the outer shock front and expands along with it. All grid zones are allowed to expand, and no new grid zones are added (the code is not adaptive). This feature is very useful in simulations where the dimensions of the system change by many orders of magnitude over the evolutionary timescale.

7. SUPERNOVA–CIRCUMSTELLAR BUBBLE INTERACTION

The structure of the CS bubble as described in § 3 consists of a highly evacuated cavity surrounded by a thin, dense shell. When the star explodes as an SN, the shock wave will interact primarily with a low-density medium, with density lower than that of the surrounding ISM. Emission from the remnant arises mainly from the hot gas in the shocked interaction region between the SN shock wave and the ambient medium (Chevalier & Fransson 1994). The high temperature of the shocked swept-up gas leads to strong X-ray emission. The ability of shocks to accelerate particles to relativistic energies, as well as the magnetic field that mediates the shocks, can give rise to synchrotron emission. The low density of the ambient material results in a much lower intensity of emission, since emission at most wavelengths is a function of the square of the density. Thus, the emission from the remnant will be considerably reduced compared to an explosion in the ISM; i.e., the remnant will be muffled (McKee 1988). In some cases explosion within a CS bubble made by a hot massive star may render the remnant unobservable until the shock wave impacts the dense CS shell.

The density and pressure profiles within the bubble (Fig. 1) show a double-shocked structure separated by a contact discontinuity. When the SN ejecta collide with the freely expanding wind, they drive a strong shock into the wind. A reverse shock is also formed that decelerates the ejecta material. Due to the presence of many shocks and density discontinuities, the subsequent interaction is quite complex, as demonstrated in analytical studies carried out by CL89. In what follows we focus on numerical studies of this interaction. These initial calculations are spherically symmetric, and their intent is to introduce the basic parameters and present initial results. Therefore, only one-dimensional calculations are presented herein, and references to multidimensional calculations refer to future papers in this series.

Most of the mass of the CS bubble is contained in a thin, dense, cool shell bordering the cavity, which suggests that this is
the essential parameter influencing the dynamics. This is verified by our simulations and previous work by others (Tenorio-Tagle et al. 1990). The interaction is found to depend on one parameter, the ratio of the mass in the swept-up wind-blown shell to the mass of the ejected material, which we denote as $\Lambda$. If the bubble was formed by the interaction between two winds, then the swept-up mass, and hence $\Lambda$, will vary as the radius $r$ (and not $r^3$ as it would for a constant-density surrounding medium).

The role played by the $\Lambda$ parameter can be understood by comparing to a somewhat analogous situation of an obstruction placed in the path of a body of flowing water such as a river. A small pebble is obviously not going to make much difference. A slightly bigger stone may have an instantaneous impact, but the momentum of the flowing water will quickly overrun it and soon forget that it ever existed. A large boulder would have a much larger impact, and the water, if it is not able to move the stone, will have to flow around it. The flow will be diverted and its velocity altered. And finally, if the river is blocked by a dam, the flow will be halted or at least controlled by the obstruction.

In the same way, $\Lambda \ll 1$ will not have a substantial impact, as the momentum of the ejected material will quickly destroy the preexisting shell. The impact of the surrounding shell begins to be felt when $\Lambda \lesssim 1$. When $\Lambda \gg 1$, the shell can significantly alter the SN evolution, speed up the remnant evolution, and perhaps even alter the various phases through which the remnant will evolve.

We illustrate here two cases: case 1 where $\Lambda \lesssim 1$, and case 2 where $\Lambda > 1$. In each case we start with the formation of the bubble by the interaction of two winds. The initial wind was assumed to have a high mass-loss rate ($M = 2.5 \times 10^{-5} M_\odot \text{yr}^{-1}$) and a low wind velocity (50 km s$^{-1}$), analogous to, say, an RSG wind. The second mass-loss phase was assumed to have a slightly lower mass-loss rate ($M = 8 \times 10^{-5} M_\odot \text{yr}^{-1}$) but a much larger wind velocity (2500 km s$^{-1}$), appropriate, say, for a W-R wind. These values are of course arbitrary and vary with each individual case, but they do serve to illustrate the basic point of evolution in a low-density medium. The two-wind interaction is simple and sufficient enough to understand the basic characteristics of the SN-bubble interaction, and the resulting structures are representative of the wind-blown bubbles obtained from simulations that include the evolutionary history of the star (to be presented in future papers). The differences between the bubble in the two cases presented here are due to the different lifetime of each wind phase.

The SN density profile is described by a power law with exponent $n = 9$. Below a certain velocity, the density is flat in the inner regions. The ejecta mass is uniformly taken to be $10 M_\odot$ and the explosion energy to be $10^{53}$ ergs. The evolution of the SN shock wave into the freely expanding wind is calculated with a high-resolution computation. These two simulations are then mapped onto the grid to form the initial conditions for the SN–CS bubble interaction study.

7.1. Case 1: $\Lambda \sim 0.14$

We first consider a case where the mass of the wind-swept shell is 14% of the ejecta mass. We assume that this bubble is formed within the ISM; i.e., although the medium surrounding the dense shell is a slow wind, we assume that the density of this wind will not fall below that of an ISM density of 1 particle cc$^{-1}$.

Figure 4 shows snapshots from the evolution of the SN shock wave within the bubble. The solid lines indicate density, and the dashed lines pressure. $R_I$ indicates the SN shock wave driven into the freely expanding wind. $R_s$ signifies the wind termination shock, and $R_{bb}$ the wind-swept shell. The simulation starts at approximately 8.36 yr, which is the time that the SN shock has been propagating in the freely expanding wind region of the CS wind-blown bubble.

The expanding shock wave collides with the wind termination shock in about 45 yr, leading to the formation of a transmitted shock wave that expands into the roughly constant density portion of the wind-blown bubble and a reflected shock wave that traverses back into the SN ejecta. The collision results in an instantaneous increase in the pressure, as is clearly visible in Figure 4 at time 53.6 yr. Note that each time the SN shock wave collides with a shock or density discontinuity, a transmitted and reflected shock wave pair will be formed. The increased pressure will result in an increased temperature, leading to a rise in X-ray emission following the impact.

The transmitted shock continues to expand in an almost constant density medium until it reaches the wind-blown shell. Meanwhile the reflected shock, which has a high velocity, moves back past the original SN reverse shock and into the SN ejecta. Although the reflected shock is moving backward in a Lagrangian sense, to an external observer the entire system is expanding outward in radius.

At about 110 yr the forward shock (the transmitted shock from the previous stage) collides with the surrounding dense shell. The collision results in a compression of the dense shell and is again marked by a sharp rise in pressure and therefore temperature and a dramatic increase in the X-ray luminosity of the remnant (see Fig. 7 below). As illustrated before, the collision results in a transmitted shock (which is not immediately apparent as it takes some time to emerge from the dense shell) and a reflected shock. The high pressure generated by the interaction gives the newly formed reflected shock a much higher velocity than the SN reverse shock. The reflected shock therefore overtakes the reverse shock in its journey back to the center.

The nature of the density profile is of significant interest at this point. As can be seen in Figure 4 at time 247 yr, the density decreases as we move outward in radius from the reflected shock. This will be reflected in the X-ray surface brightness profile. However, as the SN continues to evolve, other factors come into play: over a period of a few doubling times the remnant begins to forget that the interaction ever took place. This will happen when the mass of the swept-up material becomes much larger than that of the shell.

At about the same time, the reflected shock reaches the constant-density ejecta region. The net effect is that the SN density profile begins to change, as can be clearly seen in Figure 4 at time 1018 yr. The density, which was initially decreasing from the reflected shock to the contact discontinuity, begins to increase from the reflected shock to the contact. By 1000 yr this change is clearly visible, although it takes several thousand years before the density profile will completely change and the SN no longer displays any trace of the shell interaction.

It is important that this change be taken into account when computing the emission from the remnant. The changing density profile will substantially alter the appearance of the remnant, as described below. As seen in the bottom right panel of Figure 4, it takes some time for this change to become complete, when the remnant no longer shows any traces of the interaction. In our simulation the reflected shock converges toward the center before this occurs, but the remnant in the bottom right panel of Figure 4 is close to this stage.

As has been studied by many authors since the 1970s (see, e.g., Gull 1975) and explored in detail by Chevalier et al. (1992), the contact discontinuity between the reverse-shocked ejecta and the surrounding medium in the expanding shock wave is being decelerated by the swept-up ambient medium and therefore will...
be Rayleigh-Taylor (R-T) unstable. In our simulations, at time 1018 yr in Figure 4, there is a region just behind the reflected shock where the density is decreasing as we move outward in radius, whereas the pressure is increasing. The opposing density and pressure gradients indicate that this region will also be unstable to the R-T instability. Our multidimensional simulations (to be presented in a future paper) confirm this suggestion. In the earlier studied case dense gas is being decelerated by low-density gas, leading to the instability. In our case dense gas is being accelerated by high-pressure low-density gas. Note that although in the first situation the shock is expanding outward while in the current case it is expanding inward, the R-T fingers tend to expand outward in both cases, from the high-density to the low-density region. The instability is of particular importance because it occurs within the remnant, and as the shock moves inward, the unstable region moves in and away from the forward shock. The instability may lead to the formation of clumps and small-scale structure in the interior of the remnant.

The evolution of the radius and velocity of the forward shock is shown in Figure 5. The expansion parameter $\delta$ (where $R \propto t^\delta$) is shown in Figure 6. For an SN shock wave with power-law density ejecta evolving in a medium with a power-law density structure, the value of $\delta$ is a constant (Chevalier 1982). A constant value of 0.86 is evident in the early part of this simulation, while the shock wave is evolving in the freely expanding wind region. However, this behavior changes once the shock wave
impacts the wind termination shock, as is readily apparent in Figure 6. A much larger change occurs when the shock impacts the dense wind-blown shell. The velocity of the shock decreases considerably, its radius remains almost constant, and hence the expansion parameter drops precipitously. The velocity of the shock should drop to almost zero when the shock enters the dense shell. However, since the shell is well resolved, until the shock exits the shell in the form of a transmitted shock, we cannot estimate the precise location of the shock. Hence, the radius and therefore velocity are obtained approximately, giving a larger value for the minimum velocity than is actually the case. After the transmitted shock emerges from the shell, it is continually sweeping up interstellar material, and its velocity decreases much faster than within the bubble. As shown in Figure 6, the expansion parameter also gradually decreases. Once the SN has completely forgotten about the interaction and the reverse shock has reached the center, we would expect the SN to evolve to the Sedov solution. In this case the remnant is just about approaching this value at the end of the simulation.

The current example and others studied in this paper correspond to the $m = 1$ case in §5, wherein the wind-driven shell is fully composed of swept-up material. It is difficult to determine the precise time when the forward shock emerges from the shell and therefore the precise value of $w_b$, but it is low. As mentioned in §5, for the $m = 1, s = 2$ case, the shock will separate from the shell by the time the radius of the thin shell has doubled, if not earlier, depending on the initial value of the velocity imparted to the shell. This is clearly visible in Figure 4. The initial radius of the wind-blown shell is just less than a parsec. By the time the remnant is 1000 years old, the shell radius has more than tripled, and the forward shock can be seen to have already detached from the shell. The velocity profile of the forward shock resembles that for the $m = 1, s = 2$ case (Fig. 3), with an initial rise followed by a gradual decline, although the calculation refers to the shell, not the shock wave itself.

A key observation from Figure 6 is that the expansion parameter varies almost continuously with time once the SN shock impacts the wind termination shock. This behavior can be contrasted for evolution of the SN in a wind or constant-density medium, where the expansion parameter is a constant for power-law ejecta. The expansion parameter is a quantity that can be measured independent of the distance to an object. It is used to compute the age of the remnant, to discriminate between various models of SNR expansion, and to determine the current phase of expansion. The varying behavior of the expansion parameter within a bubble may cause a problem for such determinations because what one gets is only an instantaneous value that is not representative of the stage of expansion.

In order to illustrate the changing morphology and appearance of the remnant, as well as the increase in emission due to the shock-shell interaction, we have computed the X-ray emission from the remnant. Figure 7 shows the X-ray luminosity of the system over time. The CHIANTI database (Dere et al. 1997; Young et al. 2003) was used for this purpose. The X-ray luminosity computed here consists of the free-free and free-bound luminosity, calculated over the wave band 0.2–12.4 keV. Line emission in X-rays is neglected. This is a good approximation for temperatures above about $3 \times 10^7$ K, but lines may be dominant at lower temperatures. Over much of the evolution, the temperature in the shock interaction regions is larger than $10^7$ K, and the luminosity calculated here would suffice. In the early evolution the major contribution is from behind the forward shock. After the shock-shell interaction, the reverse-shocked ejecta also begins to contribute, and its temperature remains above $10^7$ K.
even when the temperature behind the forward shock decreases to below $10^7$ K. At this time continuum emission from the reverse-shocked ejecta and line emission from the forward-shocked material (not computed) are both important. Toward the end of the evolution the temperature behind the reverse shock also falls below $10^7$ K, and line emission predominates, in which case our calculation would seriously underestimate the total luminosity.

Almost all of the X-ray emission arises from the hot gas in the interaction region between the inner and outer SN shock waves. A major approximation that is made is to assume that the electron temperature is equal to the postshock temperature computed by the ideal gas law $[T = (\mu m_t/k)P/p]$. A mean value of $\mu$ is used; i.e., no distinction is made between the mean molecular weight of the ejecta and ambient medium. Since these are collisionless shocks, the equilibrium time due to Coulomb collisions can be large (a few hundred to thousand years) and therefore the electron and ion populations are not expected to be in equilibrium. Most of the postshock energy goes into heating the ions. Electrons are subsequently heated by plasma processes that raise the temperature, until temperature equilibration between ions and electrons occurs. Thus, it is possible that, especially in the early stages of evolution, the electron temperature is much lower than the ion temperature. In the case of SN 1006, the measured high oxygen temperature and low electron temperature indicate that shock heating was responsible for only about 5% of electron-ion equilibration at the shock front (Vink et al. 2003). This is consistent with optical measurements that indicated that $T_e/T_i < 0.07$ (Ghavamian et al. 2002) and with UV measurements that showed that $T_e/T_i < 0.05$ (Laming et al. 1996), where the “$e$” and “$i$” subscripts refer to electrons and ions, respectively. In the SNR 1E 0102.2–7219, the ratio is inferred to be $<0.05$, perhaps even substantially less (Hughes et al. 2000). The ion temperature is usually hard to measure, but when such measurements are possible in young SNRs, they generally seem to follow the trend of $T_e/T_i < 1$.

In Figure 7 we therefore show two lines for the X-ray luminosity, one using the kinetic temperature (solid line) and one that assumes that the electron temperature is only 10% of the kinetic temperature (dashed line). It is likely that in the early stages the luminosity will start out closer to the lower line, while as the evolution proceeds the upper line will be more representative of the actual luminosity.

The qualitative behavior of the two lines is similar, and the main features are seen in both. In general, we note three regions, an initial low-luminosity X-ray region, a sharp increase in the X-ray luminosity when the shock-shell impact takes place, and then a roughly constant high-luminosity region following the shock-shell impact. The impact results in an increase of almost 2–3 orders of magnitude in the X-ray luminosity. Although not computed here, a similar rise in luminosity is expected at radio wavelengths. The steep rise in emission and higher luminosity subsequent to the emission are characteristic of shock-shell impact.

In order to illustrate the changing appearance of the remnant, we have also computed approximate X-ray surface brightness profiles to demonstrate the X-ray appearance of the remnant at various times. Since the emission depends on the square of the density and square root of the temperature, we have computed the integral of this product over the line of sight. This gives us a fair idea of what the X-ray surface brightness profile would look like, up to a normalization factor. All the profiles are normalized to an arbitrary factor, with the same factor being used in both cases, so that the relative intensities can be easily compared. Only emission processes have been included; no attempt has been made to take any absorption into account. Since the intra-cavity density is so low, any absorption of the emission that occurs will happen in the dense shell only.

Figure 8 shows the density profile and corresponding surface brightness profile at various times during the evolution. Initially only the SN shock wave is visible in X-rays, as expected. By the time the SN shock reaches the wind termination shock of the bubble, the X-ray emission from the postshock region in the low-density ejecta is reduced so much that the wind-blown bubble becomes the dominant X-ray source. The interaction of the SN shock with the wind termination shock leads to a small spike in the X-ray emission. A much larger jump in X-ray intensity is seen when the SN shock collides with the dense shell, and the transmitted shock emerges. The remnant then appears limb brightened in X-rays. As the reflected shock traverses the power-law ejecta region, the X-ray emission from the reflected shock also begins to play a major role, and the SN shows a double-shelled appearance in X-rays. However, soon after the profile changes due to two factors: the remnant begins to forget the interaction, and the reflected shock reaches the inner constant-density core of the ejecta. The remnant no longer shows a double-shelled appearance but shows a decreasing X-ray intensity as we move toward the center of the remnant.

Note that the timescale between frames 5 and 6 is large, a factor of 15 doubling times, which indicates that for a considerably large time the young SNR will show a double-shelled appearance in X-rays. One condition for the remnant to appear double shelled in X-rays is that the reflected shock be still interacting with the power-law part of the ejecta density profile. Once the shock begins to interact with the constant-density ejecta, the structure of the shocked interaction region will change, and therefore the remnant will no longer appear double shelled. Another condition is that the ejecta density (which decreases as $r^{-3}$) should not make the emission measure of the reverse-shocked ejecta material so low as to be undetectable. Since the ejecta density determines the postshock density, we can express this as saying that the postshock density (behind the reverse shock) should not be less than about 5% of the shell density, as is evident in this case. Remnants with a
Fig. 8.—Plots showing the evolution of the density (top) and X-ray surface brightness (bottom) at various times, for the simulation with $\Lambda = 0.14$. 
double-shelled appearance in X-rays are not uncommon, one example being Cas A.

7.2. Case 2: $\Lambda \sim 3.7$

In the second instance we consider a situation where the mass in the swept-up wind shell is $3.7$ times that of the ejecta. The ratio $\Lambda$ is about 25 times larger than in case 1. In order to produce a large mass shell, we assumed that the wind-wind interaction continues for a very long time, while the surrounding wind density continues to decrease. This implies that the interaction occurs in a lower density medium outside the dense shell, perhaps a preexisting bubble from a previous stage. The density of the slower wind was allowed to fall to a very low value of about $3 \times 10^{-3}$ in the simulation, before a floor to the density was set.

Note that although this may appear very low, the surroundings of massive stars are complex and very low density regions are not uncommon (McKee 1988). If there was a surrounding MS bubble from a massive star, its density could easily be less than $10^{-3}$ particles cm$^{-3}$.

As the mass of the shell increases relative to that of the ejecta, the kinetic energy transmitted by the ejecta to the shell also increases. Thus, the SN shock wave can lose a substantial amount of energy upon interaction with the dense shell, and the transmitted shock is correspondingly weakened and moves with a lower velocity. The shock wave may also take a larger time to emerge from the shell.

The large shell mass results in a significant increase in pressure as the SN shock impacts the shell. This high pressure behind

Fig. 9.—Plots showing the density and pressure profile at various times, for the simulation with $\Lambda = 3.7$. 
the reflected shock can result in significantly large velocities for the reflected shock wave, which subsequently expands in a low-density medium. The reflected shock can attain very high velocities and reaches the center in a short time compared to the emergence of the transmitted shock.

Figure 9 shows pressure and density snapshots at various time intervals from a simulation of the evolution of the remnant in this case. As seen from the plot at 112 yr, the size of the bubble is much larger than in case 1. Note also the extremely low density within a large section of the bubble interior, about 0.01 particles cm$^{-3}$. Such low density regions are characteristic of bubbles around MS and W-R stars. Although the SN shock wave takes about 2000 yr to reach the wind termination shock, due to the low wind density it has not swept up a very large mass of material before that time, and therefore there is hardly any deceleration. The expansion parameter (see Fig. 11) remains constant at a value of 0.86 until the shock collides with the wind termination shock, giving rise to the usual double-shocked structure with a reflected and transmitted shock. The transmitted shock then impacts the wind-blown shell (frame 3, 5119 yr), again resulting in a reflected and transmitted shock. The impact also imparts considerable momentum to the dense shell, causing the entire remnant to expand outward. The reflected shock attains high velocities, thermalizing the inner ejecta as it moves toward the center. The transmitted shock, which is expanding into a much higher density medium, moves outward with much lower velocities.

As expected from the discussion in § 5, the forward shock is expected to separate from the shell within a doubling time of the shell. This behavior is illustrated in Figure 9, where the shock has detached itself from the shell by the time it has doubled its initial radius. The velocity profile resembles that for a decelerating shock wave that is slowing down as it sweeps up more material.

By about 20,000 yr the reflected shock has converged onto the center. A weak re-reflected shock is formed that expands outward. The impact of this shock with the dense shell does not result in an appreciable increase in the X-ray luminosity. Unlike case 1, in this case the reflected shock reaches the center before the transmitted shock has separated itself significantly from the shell, and the system does not easily forget the interaction.

It can be seen that although the remnant is almost 50,000 yr old by the end of the simulation, it does not yet display the profile characteristic of the Sedov adiabatic stage. In particular, even after the reflected shock has converged on the center and a weaker re-reflected shock is present, the system is dominated by only the forward shock wave, but the density profile is still different from the Sedov solution. This illustrates an important characteristic of SNRs evolving in low-density wind-blown bubbles: the evolution may deviate from the classical description of ejecta-dominated, Sedov, and radiative stages.

When the reflected shock is moving toward the center, almost the entire interior of the remnant is very hot, about $10^8$ K. If the density in the interior is high enough, the emission measure will be large and the entire remnant will be visible in X-rays. In the current example this is not the case, as the interior density is quite low. But it is possible to envisage a situation where the shock-shell interaction occurs earlier in the remnant’s evolution (such as case 1) but the wind-blown shell still has mass larger than the ejecta. Then the reflected shock will traverse much higher density ejecta on its way to the center, and the emission from behind the reflected shock could be comparable to that from the forward shock. The surface brightness profile would then show X-rays all the way to the center. It is possible that such an effect could be one explanation for the so-called mixed-morphology remnants, which are X-ray bright all the way to the center.

**Fig. 10.**—Radius (left) and velocity (right) plots for the evolution of the SNR in case 2.

**Fig. 11.**—Evolution of the expansion parameter for case 2.
Figure 10 shows the radius and velocity evolution of the forward shock in this case, and Figure 11 is a plot of the expansion parameter evolution. In both cases they resemble closely the counterparts in case 1. The expansion parameter remains constant up to the SN–wind termination shock interaction, then a slight decrease until the shock-shell impact takes place, followed by a significant decrease in the velocity and expansion parameter. As the transmitted shock emerges, the expansion parameter increases again and then gradually decreases until it approaches the Sedov value of 0.4 in a constant-density medium. It is clear that even though the behavior of the reflected shock and the interior structure of the remnant is quite different in both cases, this is not reflected in the behavior of the outer shock. This serves as a cautionary note that studying the behavior of the outer shock is not sufficient to provide information regarding the evolutionary phase and structure of the remnant.

Figure 12 shows the X-ray luminosity of the remnant over time. The overall evolution is similar to that in case 1, although the increase in X-ray luminosity upon impact is smaller. In this case the density is generally very small, and the temperature high enough that line emission is not important over almost the entire evolution. The X-ray luminosity is much lower in the beginning, given the low density of the bubble interior. It is unlikely that such a remnant could be observed by available X-ray instruments. In order to test this, we tried to calculate the counts per second that would be observed by Chandra, using the PIMMS Web-based count rate predictions. Assuming the lower temperature luminosity curve in Figure 13, a distance to the extended source of 3 kpc, a Raymond-Smith model, and a column density of 10\(^{22}\) cm\(^{-2}\), we find that the count rate per second is almost 4 orders of magnitude lower than the background count rate, making it unlikely that such a remnant would be detected. The luminosity would need to be about 20,000 times greater for it to be detectable. Using a power-law model with an appropriate photon index, as found for some SNRs, results in similar results, with an even lower count rate. Changing the column density and other parameters makes some difference but still leaves the count rate extremely low. Upon shock-shell impact, the luminosity increases by more than 2 orders of magnitude, but even in this case the counts per second would be lower than the background. In contrast to this, if we carry out a similar analysis for the lower line shown in Figure 7 for the time just before the SNR shock collides with the dense shell, then the higher luminosity coupled with the smaller size of the remnant predicts that the counts per second will be a factor of a few above the background, allowing the remnant to be detected.

Figure 13 shows the approximate X-ray surface brightness profiles, similar to Figure 8. Again we start out with the SN shock being the most visible source of X-rays. The interaction of the SN shock wave with the wind termination shock leads to the expected rise in X-ray emission, and the interaction with the wind-blown shell results in an even larger jump in the X-ray intensity. Up to this point the X-ray evolution is similar to the previous case. Beyond this, however, it departs considerably because the reflected shock does not have much time to interact with the power-law ejecta before reaching the constant-density inner core. Therefore, the remnant never takes on a double-shelled appearance, but the major X-ray-emitting region is always the outer transmitted shock. As the reflected shock moves in, the entire remnant may be bright in X-rays. In the current case the emission measure is low enough that the remnant may not be observable. But if the shock-shell interaction happens early enough, then this is a possibility worth investigating.

The reflected shock reaches the inner boundary before the transmitted shock has advanced significantly. On reaching the center a re-reflected shock is formed. This is expected in a realistic situation when a spherical shock wave converges onto a compact object. Note that the shock reflected off the boundary is quite weak and is not readily visible in X-rays (Fig. 13, bottom right panel).

### 7.3 \( \Lambda \gg 1 \)

In some cases the shell mass may be considerably larger than the ejecta mass. This can happen, for instance, if a massive star sheds a large amount of mass during its evolution, leading to a pre-SN star with a considerably smaller mass than the MS value. Stars with initial mass greater than about 40\( M_\odot \) may end up with pre-SN masses of 10\( M_\odot \) or less. In some cases it is possible that the mass of the wind-blown shell is considerably larger than that of the ejected material, and \( \Lambda \gtrsim 25 \). In such extreme cases, when the SN shock wave collides with the shell, it will impart a considerable fraction of the explosion energy to the shell. The shock wave compresses the shell, and the entire structure becomes radiative. The shock wave velocity decreases considerably and the time taken to emerge from the dense shell becomes extremely large. In the limit that the shock loses most of its energy to the shell, it may merge completely with the shell, and only a very weak transmitted shock will emerge from the shell. The SN has then gone from the free-expansion phase to the radiative phase, completely bypassing the Sedov phase. The reflected shock will be quite strong and will thermalize the ejecta in a much shorter time. The dense shell will expand with the additional energy imparted to it, but the SN shock wave will not be visible as a separate entity ahead of the dense shell. This case has been discussed in detail by Tenorio-Tagle et al. (1990).

Such cases, although not common, are quite plausible. Many W-R nebulae have masses that exceed \( 25-30 \ M_\odot \). Cappa et al. (2003) list some W-R nebulae with masses of hundreds to thousands of solar masses, mostly made up of swept-up material. When the star explodes as a SN in these nebulae, the value of \( \Lambda \) could be very large, possibly 50–100. In these cases where \( \Lambda > 50 \) the nebula will confine the SNR, and the remnant will never emerge from the nebula. The size and dynamics of the remnant are limited by that of the nebula, whose size is essentially set by mass loss in the MS phase.
Fig. 13.—X-ray surface brightness profiles during the evolution of the remnant in case 2.
8. DISCUSSION

It is clear that the medium around the progenitor star plays a large role in shaping the further evolution of the remnant. Depending on the nature of the progenitor star, the density of the medium into which the SN shock expands may vary considerably. RSG progenitors will have a region of very high density, surrounded by a low-density bubble formed by the MS star, bordered by a dense shell. W-R progenitors will have a low-density wind-blown bubble surrounding the star, bordered by a dense CS shell. The interior of the shell may show density variations depending on the various stages of evolution prior to the SN stage. As we have seen in the case of SN 1987A, BSG progenitors will form a cavity with density somewhat midway between the above two cases. Factors such as ionization, rotation, binary evolution, and magnetic fields could all affect these results.

Quite often the SNR spends a not insignificant fraction of its life within the bubble. It is then of interest whether the SN reaches a Sedov stage while inside the bubble or not. That is, depending on the density profile, the swept-up mass must be several times that of the ejecta mass before the remnant enters the Sedov stage (see, e.g., Dwarkadas & Chevalier 1998). Whether the remnant sweeps up enough material is a function of the ejected mass and the bubble interior density. The bubble density depends on the nature of the exploding star, which is a function of its initial mass and mass-loss rate. If the density into which the SN shock evolves is very low, then the remnant will not sweep up much material and will remain in the free-expansion phase for a much longer time than it would in the ISM. Remnants expanding in a low-density bubble will probably not reach the Sedov stage before they collide with the dense shell. As mentioned in § 7.3 in cases where $\Lambda > 50$, the remnant may evolve from the free-expansion stage directly to the radiative stage, completely bypassing the Sedov stage.

As discussed in § 2, a MS O or B star will generally evolve into an RSG star. If its initial mass is below 35 $M_\odot$, it will end its life as an RSG star; if above, it will become a W-R star before going SN (for solar composition stars).

In the MS phase a CS bubble is formed by the interaction of the MS wind with the ISM. The density within this bubble is quite low, of order $10^{-4}$ to $10^{-2}$ particles cm$^{-3}$. Its size is in the tens of parsecs, depending on the initial mass of the star.

When the star becomes a RSG, the dense, slow wind gives rise to a density much higher than that of the surrounding medium, presumably the MS bubble, by about 2–3 orders of magnitude. The remnant will sweep up a large mass of material in a short time. Whether it evolves to the Sedov stage then depends on the mass of the ejected material and the size of the RSG wind region, which depends primarily on the time spent in the RSG stage.

The ejecta mass depends on the initial (zero-age main sequence) mass of the star, the mass of the progenitor star just prior to the explosion, and the nature of the compact stellar remnant. All of these factors depend on the metallicity. According to recent work by Heger et al. (2003) and Woosley et al. (2002), nonrotating stars in the mass range between 10 and 22 $M_\odot$ and with solar metallicity will leave behind a neutron star rather than a neutron star. For stars between about 22 and 26 $M_\odot$, the ejected mass is still quite large, around 10 $M_\odot$, and it is unlikely that the ejecta in this case will sweep up enough material to enter the Sedov stage before shock-shock collision. However, for stars between 26 and 34 $M_\odot$, the ejected mass decreases considerably as the mass of the remnant increases. Near 34 $M_\odot$, the ejecta mass is only about a couple of solar masses. Thus, as we get closer to the upper limit in this mass range, the ejecta are likely to enter the Sedov stage while interacting with the RSG wind. When this Sedov remnant collides with the dense shell, the reflected shock will traverse an almost evacuated medium and move very quickly back to the center.

If the remnant reaches the Sedov stage while interacting with the RSG wind, then, once it exits the RSG wind and enters the MS bubble, the evolution will continue as a Sedov-type blast wave in a low-density medium. If the remnant has not reached the adiabatic stage, then it is unlikely that it will do so in the much lower density medium, and it will probably continue to be in free expansion until the shock wave collides with the CS shell.

Stars greater than about 34 $M_\odot$ will become W-R stars and explode as W-R stars. They have dense, fast winds whose density is much lower than RSG winds, but momentum is much higher. They will form a low-density W-R bubble. Here the mass-loss rates are quite uncertain, and so are the final progenitor masses. However, a general conclusion is that the remnant masses are quite low, less than a few solar masses, while the ejected mass is high. Therefore, as we go to higher and higher initial masses, the probability that the ejecta will sweep up enough material to enter the Sedov stage slowly increases. However, the shock wave will have to travel quite a large distance within the low-density bubble to sweep up sufficient material that the Sedov stage is reached.

Note that the situation in W-R bubbles can be quite complicated and will be dealt with in a future paper in this series. Briefly, the W-R stage is a highly evolved stage of a high-mass star greater than about 35 $M_\odot$. It is likely to be preceded by an RSG stage and/or perhaps even a LBV stage, if the initial mass of the star is much larger (Langer et al. 1994). LBVs can have very large mass-loss rates and therefore very high density winds, even compared to RSGs. Therefore, the W-R stage will generally follow a phase of very high density winds. The W-R phase itself has a high mass-loss rate and wind velocity. The momentum of the W-R wind is generally going to be much larger than the RSG wind and may even exceed that of the LBV wind, although this depends on the specific parameters. If the W-R wind was preceded by a low-velocity RSG wind, the W-R wind momentum will carry the RSG wind material along with it. This combined material may collide with the MS shell and reflect back, forming more complicated structures than described herein. The low-density W-R wind may end up containing higher density material from the RSG stage. The density profile will be a complicated blend of the previous stages and not representative of the wind properties of the pre-SN progenitor star. Specific examples using actual stellar evolution models will be shown in forthcoming papers.

It is apparent that the medium into which a core-collapse SN evolves cannot be easily categorized into either a constant-density medium such as the ISM or a wind with a density $\rho \propto r^{-2}$. A wind-blown bubble formed by two-wind interaction includes both elements, a density that decreases as $r^{-2}$ followed by a more or less constant density region between the wind termination shock and the contact discontinuity. However, in stars larger than 35 $M_\odot$, mass loss in the various evolutionary stages preceding the SN explosion can give rise to very complicated density profiles beyond the wind termination shock. Close in to the star the density generally follows a $1/r^2$ law resulting from the
last phase of evolution, but farther beyond the density may vary considerably depending on the evolutionary sequence. These more realistic structures will be the subject of future papers.

9. SUMMARY

In this paper we have studied the evolution of SN shock waves in the CS structures formed by pre-SN mass loss. The evolution depends mainly on one parameter $\Lambda$, the ratio of the mass of the dense surrounding shell to the mass of the ejected material. We have explored the changes in the evolution as the value of this parameter increases and found that the evolution is significantly different from that for an explosion in the ISM. In particular, as the value of $\Lambda$ increases, we find that the energy imparted to the shell increases, the evolution of the remnant is speeded up, and the appearance of the remnant may change.

In contrast to the work of Tenorio-Tagle et al. (1990), we have used a much steeper density profile for the outer ejecta and a higher ejecta mass. We have also explored more thoroughly the region around $\Lambda \sim 1$, using two different values of $\Lambda = 0.14$ and 3.7. Simulations were also carried out, although they are not explicitly reported herein, with $\Lambda \gg 1$. Our results are consistent with the work of Tenorio-Tagle et al. (1990) for $\Lambda \gg 1$, as would be expected. However, unlike Tenorio-Tagle et al. (1990), we have explored two different cases around the value $\Lambda \sim 1$ and find considerable difference between the two cases. These differences can be noted in the evolution and the X-ray appearance. In one case a double-shelled structure is visible for a considerable amount of time after the shock-shell interaction. In the other case the double shell is mostly absent, but if the interior density is high, the entire remnant may be visible in X-rays. However, differences are not so easily apparent in observations of the forward or outer shock front. Thus, we caution that inferring the properties simply by observing the outer shock parameters can often lead to erroneous conclusions.

We have presented the evolution of the expansion parameter in each case. It is worth pointing out again that the expansion parameter varies significantly over the evolution, unlike in the case of explosion within a constant-density medium or a wind. This is very important because it implies that any instantaneous value of the expansion parameter is not representative of the evolutionary stage of the remnant and will lead to an incorrect determination of properties. And in particular, assuming that the remnant is in the Sedov stage for computing the remnant parameters could lead to large errors.

The initial evolution of the bubble, within a low-density medium, can result in a large amount of time spent in the free-expansion phase, as the swept-up mass remains quite small. The presence of the CS shell, on the other hand, results in significant deceleration of the SN shock wave and decreases the time spent in the Sedov stage as compared to the radiative stage. The net result is that the Sedov stage could be significantly shortened and in some cases (see § 7.3) may be entirely bypassed. It is plausible that the forward shock, after emerging from the CS shell, may reach the Sedov stage. Alternatively, it is also possible that, given the deceleration of the shock front, which we have seen occurs even when the shell mass is about 10% of the ejecta mass, the remnant may reach the radiative stage much earlier than in the absence of the shell.

Our X-ray profiles show that for $\Lambda \leq 1$, a double-shelled structure is seen in X-rays, with emission coming mainly from the shocked shell and from behind the reverse shock. It is possible that such a situation is present in Cas A. At later times when the outer shock has separated significantly from the shell, the emission from the outer shock begins to predominate. In the case with larger $\Lambda$ the emission from the shocked shell always seems to predominate. It is clear then that as the remnant evolves and the outer shock separates, the X-ray emission from the shocked shell will appear to come from deep within, say, the radio emission arising from behind the outer shock. Precisely such a situation has been surmised for the remnant W44 (Koo & Heiles 1995).

In this paper we have not considered multidimensional effects such as hydrodynamical instabilities and turbulence. These aspects will be dealt with in upcoming papers. Thin shells are known to be unstable, due to many instabilities ranging from the well-known R-T and Kelvin-Helmholtz type to the range of instabilities collected under the heading of Vishniac-type instabilities. These instabilities may cause the shells to fragment and lead to hydrodynamic behavior that has not been explored herein. Furthermore, variations in pressure can lead to turbulent behavior within the interior of the dense shells. Also of significance is that, as the reflected shock in case 1 moves back to the center, there is a region behind the reflected shock that is R-T unstable. This instability is of importance because it occurs in the interior of the remnant, far away from the dense shell and forward shock, and can result in ejecta clumps and filaments in the inner regions of the remnant.

Finally, the real test of these results will be to compare and validate them with observed SNRs. We are indeed fortunate to be able to closely witness the formation and evolution of a SN that occurred within a wind-blown bubble, SN 1987A. The enormous body of observational data that has been collected not only should enable us to study this SN in the utmost detail, but by extension should provide considerable information on the general theory of SN evolution in wind-blown cavities. Of course, SN 1987A has its own share of complexities, and a thorough understanding of the CS medium requires three-dimensional radiation hydrodynamics modeling, which has not yet been carried out. Specific comparisons between our models and observed remnants will be described in later papers in this series.

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