Research on Takagi-Sugeno Fuzzy-Model-Based Vehicle Stability Control for Autonomous Vehicles

Zeyu Jiao 1, Jian Wu 2,*, Zhengfeng Chen 1, Fengbo Wang 1, Lijun Li 3, Qingfeng Kong 1 and Fen Lin 4,*

1 College of Mechanical and Automotive Engineering, Liaocheng University, Liaocheng 252000, China; jiaozeyu1997@163.com (Z.J.); chenzhengfeng_01@163.com (Z.C.); wangfengbo@lcu.edu.cn (F.W.); wzksfqkong@163.com (Q.K.)
2 School of Mechanical and Automotive Engineering, Liaocheng University, Liaocheng 252000, China
3 School of Physics Science and Information Engineering, Liaocheng University, Liaocheng 252059, China; llijun@lcu.edu.cn
4 College of Energy and Power Engineering, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, China
* Correspondence: wujian@lcu.edu.cn (J.W.); flin@nuaa.edu.cn (F.L.)

Abstract: Human–machine cooperative driving is an important stage in the development of autonomous driving technology. However, in emergencies, the problem of vehicle stability control for human–computer cooperative autonomous vehicles is still worthy of attention. This paper mainly realizes the stability control of the human–machine cooperative driving vehicle through active steering and considers the influence of the change of vehicle speed on the vehicle stability control performance. Therefore, a vehicle stability control method based on the superposition of steering torque is proposed, in which the Takagi-Sugeno fuzzy model is used to solve the variable parameter problem. Firstly, a vehicle system model with steering moment as input is established to ensure that the driver can participate in the steering control. Secondly, the nonlinear T-S fuzzy model is established by fuzzifying the local linear model. Then, the parallel-distributed-compensation (PDC) method is used to design the vehicle stability controller, and the asymptotic stability of the system in the range of variable parameters is proved by using the Lyapunov stability principle. Finally, the simulation and experimental results show that the control method can improve the handling stability of the human–machine cooperative driving vehicle under the condition of vehicle speed variation.

Keywords: autonomous driving; T-S fuzzy model; stability control

1. Introduction

Human–machine cooperative driving is an important stage in the development of autonomous driving technology. Among them, the vehicle stability control method of human–machine cooperative driving vehicles has gradually become the focus of researchers [1,2]. The existing stability control methods are mainly realized by active steering or braking, but for the current stability control, the driver cannot participate in it, which easily reduces the driver’s “situational awareness” and greatly affects the driver’s sense of participation and security. Therefore, this paper focuses on vehicle stability control for human–machine co-driving autonomous vehicles.

These existing stability control methods include PID control, model predictive control, and fuzzy control. PID control method has a good control effect, and the structure is simple. Many scholars used the PID method to study vehicle stability and achieved a good control effect. An adaptive PID control algorithm that takes the front wheel rotation angle of the vehicle as the control input was proposed to improve the stability of the vehicle and the accuracy of path tracking under unknown scenes [3]. A human tuning method based on improved PID parameters was proposed to improve the control effect of PID and the stability of vehicles [4]. A fuzzy PID feedback control system was established,
which improves the vehicle trajectory tracking ability and ensures the vehicle’s handling stability [5]. However, PID control algorithm is susceptible to interference, and when the system is affected by interference, the system will become unstable.

Scholars have studied other advanced control methods. An adaptive control strategy based on the reference model was proposed in the literature [6]. An optimal linear quadratic regulator (LQR) was proposed in the literature [7], which adopted steering and braking coordination control to improve vehicle stability on a preset path. A kind of human–machine co-driving controller was proposed with a hierarchical structure, which can effectively achieve vehicle stability control [8]. A control algorithm based on LMI was proposed in [9], and experiments showed that this method can ensure the stability and path tracking accuracy of the system. At present, the active steering stability control is mainly realized through the steering angle control, but for the current stability control, the driver cannot participate in the steering control of the vehicle [10–14]. Therefore, the angle control-based active steering stability control cannot meet the requirements of human–machine co-driving. The human–machine co-driving can be realized by torque superposition. The steering torque is generated by the intelligent driving assistance system to guide the driver to drive correctly. The use of the steering torque superposition scheme can ensure that the driver has the ultimate control authority over the vehicle and improve the safety and comfort of driving [15–17].

Therefore, many scholars have studied the autonomous driving control method with steering torque as the control input. In [18], the human–machine cooperative drive mode was proposed, which realized the vehicle’s automatic steering control through the torque, and it made up for the driver’s inability to participate in steering control. In [19], the author designed a torque-based steering assist system to improve the control stability of the vehicle. In [20], a vehicle slip Angle algorithm for yaw stability control is designed to ensure vehicle lateral stability control. The results show that this strategy can effectively track the yaw velocity and lateral acceleration of vehicles.

However, the above vehicle stability control methods are designed under the premise of constant speed. In actual driving, the speed of the car is always in the process of changing. The model of vehicle stability control has time-varying characteristics and nonlinear characteristics if the change in vehicle speed is taken into account. The fuzzy control method has good performance and is relatively simple to control nonlinear systems [21–23]. In addition, many scholars have studied nonlinear control and observation based on Takagi-Sugeno (T-S) fuzzy modeling [24]. T-S fuzzy control provides a design idea for the system considering parameter variation [25]. Literature [26] proved that the T-S fuzzy control method can be used to deal with the varying parameter problem of the vehicle systems. T-S fuzzy system uses linear mapping as an output function. Within the permitted state range, the prior membership function of each rule can be used to quantitatively describe the validity of a posteriori linear system [27]. Different fuzzy rules are used to obtain the combination of linear models of the system; then, the parallel distributed compensation (PDC) method was adopted [28]. Based on the Takagi-Sugeno fuzzy model, the static output feedback $H\infty$ control problem of the vehicle lateral dynamics system was studied in the literature [29].

In [30], studies on vehicle stability control methods focus on the condition of uniform speed, but in practice, the vehicle speed is constantly changing, and vehicle stability control methods without considering speed have certain limitations. In this paper, the T-S fuzzy model considers the influence of vehicle velocity nonlinear variation on vehicle stability control. In this paper, the speed is taken as the variable parameter to design fuzzy rules [31], and different fuzzy rules are used to obtain the linear model combination of the system, thus reducing the difficulty of solving nonlinear problems.

In terms of system anti-interference, in literature [32], the author designed a fuzzy logic controller based on steering control to achieve the portability of the vehicle’s path tracking effect and improve the robustness of the control system. In reference [33], the author proposed a fault-tolerant control strategy for the possible faults and uncertainties of autonomous driving vehicles, ensuring that the vehicle can also realize the path tracking
function after a fault occurs. In reference [34], the author designed top-ten feedback robust controller, which not only realized the vehicle’s emergency obstacle avoidance but also improved the anti-interference ability of the system and verified the effectiveness of the controller through experiments. In reference [35], the author proposed a sliding mode control system to solve the uncertainty of the system and proved the stability of the system by using the Lyapunov function.

Inspired by the literature mentioned above, this paper mainly realizes the stability control of the human–machine cooperative driving vehicle through active steering and considers the influence of the change of the vehicle speed on the vehicle stability control performance. The steering torque is used as the control input, and the current loop control method is adopted to ensure the driver’s final control authority over the vehicle. Additionally, the T-S fuzzy model proposed takes into account the influence of the change of vehicle speed on vehicle stability control. The main innovation points are:

1. A vehicle stability control method based on Takagi-Sugeno fuzzy model is proposed. This method uses vehicle speed as a variable parameter to design fuzzy rules, which solves the problem of vehicle stability control under the condition of changes in vehicle speed.

2. The vehicle stability control method proposed in this paper takes the steering moment as the control input, which ensures the driver has the final control authority of the vehicle and improves the safety and comfort of driving.

The rest of this paper is organized as follows: In Section 2, the vehicle dynamics model is established. Section 3 introduces the steering torque control strategy based on the T-S fuzzy model. The simulation results are given in Section 4. Section 5 introduces the results of the T-S controller in the semi-experiment. Section 6 is the summary.

2. Establishment of Vehicle Dynamics Model

2.1. Modelling of Steering System

For the convenience of modeling, the force of the steering system (see Figure 1) is equivalent to the knuckle.

![Figure 1. Schematic diagram of the steering system.](image)

The steering model takes steering torque as input and is described as follows:

\[ J_{eq} \ddot{\delta}_f + B_{eq} \dot{\delta}_f + K_{t_2} \delta_f = N_s N_m \tau_m \]  

(1)

where \( N_s \) and \( N_m \) are the transmission ratios of the steering system and the motor, respectively, \( \tau_m \) stands for the steering torque of the auxiliary motor, \( K_{t_2} \) represents the equivalent torsional stiffness, and \( \delta_f \) is the front wheel rotation angle.

The equivalent moment of inertia \( J_{eq} \) and the equivalent damping \( B_{eq} \) of the steering system are shown in Formulas (2) and (3), respectively:

\[ J_{eq} = J_{fw} + (N_m N_s)^2 J_m + N_s^2 (J_c + m_tr_p^2) \]  

(2)

\[ B_{eq} = B_{fw} + (N_m N_s)^2 B_m + N_s^2 (B_c + B_r r_p^2) \]  

(3)
where the moment of inertia of the motor is $I_m$, the rotational inertia of the steering shaft is $J_c$, the moment of inertia of the steering rod is $J_{fw}$, the damping of the steering column is $B_m$, $B_{fw}$ is the damping of the steering rod, $B_c$ is the damping of the steering shaft, $B_r$ is the damping of the rack and gear, $m_r$ is the mass of the rack and gear, and $r_p$ is the radius of the pinion.

If the state variable $x_1 = [\delta_f \delta_i]^T$ and input variable $u_1 = \tau_m$ are set, the dynamic model (1) can be expressed as:

$$\begin{align*}
\dot{x}_1 &= A_1 x_1 + B_1 u_1 \\
y_1 &= C_1 x_1
\end{align*}$$

$$A_1 = \begin{bmatrix}
\frac{-B_{eq} J_{eq}}{I_{eq}} & 0 \\
0 & 1
\end{bmatrix}, \quad B_1 = \begin{bmatrix}
\frac{N_m N_c}{I_{eq}} & 0 \\
0 & 1
\end{bmatrix}, \quad C_1 = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

Equation of state (4) expresses the relationship between $\delta_f$ and $\tau_m$.

2.2. DOF Vehicle Dynamics Model

The dynamic characteristics of the vehicle can be represented by a 2-DOF vehicle model. The model in Figure 2 can be described as:

$$\begin{align*}
(D_f + D_r)v_y + \left(\frac{l_f D_r - l_r D_f}{l_y^2 + l_x^2} \right) & - D_f \delta_f = m \left(\dot{v}_x + v_y \dot{\psi} \right) \\
\left(\frac{l_l D_f - l_r D_r}{l_y^2} \right) & + \left(\frac{l_f^2 D_f + l_r^2 D_r}{l_y^2} \right) \dot{\psi} - l_D \delta_f = I_y \dot{\psi}
\end{align*}$$

where $\psi$ represents the yaw angle, $\delta_f$ represents the front wheel rotation angle, $v_x$ and $v_y$ represents the longitudinal and transverse speed of the vehicle, and $D_f$ and $D_r$ are the front and rear wheel cornering stiffness, respectively.

Figure 2. DOF vehicle dynamics model.

Considering the $\psi$ is small, the $v_x$ and $v_y$ can be expressed by Formula (6):

$$\begin{align*}
\dot{X} &= v_x - v_y \psi \\
\dot{Y} &= v_x \psi + v_y
\end{align*}$$

where $\psi$ represents the yaw angle

$$\begin{align*}
\dot{x}_2 &= A_2 x_2 + B_2 u_2 \\
y_2 &= C_2 x_2
\end{align*}$$

Among them, $x_2 = \begin{bmatrix} v_y & \psi & Y & \psi \end{bmatrix}^T$, $u_2 = \delta_f$. The coefficient matrices are as follows:

$$A_2 = \begin{bmatrix}
\frac{D_f + D_r}{l_y^2 + l_x^2} & \frac{l_f D_r - l_r D_f}{l_y^2} - v_x & 0 & 0 \\
\frac{l_l D_f - l_r D_r}{l_y^2} & \frac{l_f^2 D_f + l_r^2 D_r}{l_y^2} - \frac{l_y^2}{l_x^2} & 0 & 0 \\
0 & 1 & 0 & 0
\end{bmatrix}$$
2.3. Combinatorial Dynamics Model

In combination with the above two sections, this section combines the steering system model (4) with the dynamics model (7) to obtain the overall dynamics model:

\[
\dot{x} = A_c x + B_u u \\
y = A_c x + B_u u
\]  

(8)

Among them, \(x = [\delta f, \delta v, y, \psi, Y, \psi]^T\), \(u = \tau_m\), \(A_c = \begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix}\), \(B_u = \begin{bmatrix} B_1 \\ 0 \end{bmatrix}\), \(C_c = \begin{bmatrix} C_1 & 0 \\ 0 & C_2 \end{bmatrix}\), \(D_c = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}\), \(D_3 = [0, 1]\).

All parameters used in the combinatorial dynamic model are listed in Table 1.

Table 1. Parameters of the combinatorial dynamic model.

| Symbol | Description                           | Value (Unit) |
|--------|--------------------------------------|--------------|
| \(J_{eq}\) | Steering moment of inertia          | 0.1 kg m\(^2\) |
| \(B_{eq}\) | Steering moment of damping          | 0.8 Nm s/rad  |
| \(K_{iz}\) | Steering resistance coefficient      | 10 Nm/rad     |
| \(D_f\)  | Front-wheel cornering stiffness      | −140,000 N/rad |
| \(D_r\)  | Rear-wheel cornering stiffness       | −100,000 N/rad |
| \(m\)    | Vehicle mass                         | 1405 kg       |
| \(l_z\)  | Moment of inertia of the vehicle     | 1802 kg m\(^2\) |
| \(l_f\)  | Distance from CG to the front axle   | 1.016 m       |
| \(l_r\)  | Distance from CG to the rear axle    | 1.562 m       |

Remark 1. The vehicle steering system model (4) is combined with the dynamics model (7), and the vehicle overall dynamics model (8) is obtained. Model (8) takes the front wheel rotation angle as the system state and the motor torque as the control input. At this time, the steering wheel is not locked, which solves the problem that the driver cannot participate in the steering control.

2.4. Establishment of Reference Model

The purpose of the controller design is to ensure vehicle stability. Therefore, tracking the ideal yaw rate is one of the control objectives. The yaw rate of a vehicle in a constant circular motion is the ideal yaw rate. It can be expressed as [36]:

\[
\dot{\psi}_n = \frac{v_x}{L(1 + K_u v_x^2)} \delta_f
\]

(9)

where \(L\) is the distance between the front and rear axles, \(K_u\) is the stability factor, and its calculation formula is as follows:

\[
K_u = \frac{m}{L} \left( \frac{I_f}{D_r} - \frac{I_r}{D_f} \right)
\]

(10)

Since the ground attachment will also affect the lateral dynamic state of the vehicle, the ideal yaw rate velocity of the vehicle is modified to obtain Equation (11), where \(\dot{\psi}_d\) is the corrected yaw rate, and \(\mu\) is the ground adhesion coefficient.

\[
\dot{\psi}_d = \min \left\{ \dot{\psi}_n, 0.9 \frac{\mu g v_x}{v_x} \right\} \text{sgn} \delta_f
\]

(11)
2.5. Augment Model for Stability Control

By designing reasonable control input, the difference between the actual value and ideal value can be minimized. That is, the yaw rate deviation $\dot{\psi} - \dot{\psi}_d$, road deviation $Y - Y_d$, and yaw angle deviation $\dot{\psi} - \dot{\psi}_d$ should be minimized, and the energy consumption of control input $u$ should be minimized as far as possible. That is, by obtaining the control law $K$ and applying it to the system, the system performance is optimized.

Therefore, the control system should be designed to make the system state $y_1(t)$ track the ideal value $r(t)$. Integrate the error between $y_1(t)$ and $r(t)$, we get:

$$\eta(t) = \int_0^t (y_1(\tau) - r(\tau)) \, d\tau$$

where $y_1(t) = [\dot{\psi} \ Y \ \dot{\psi}]^T$ and $r(t) = [\dot{\psi}_d \ Y_d \ \dot{\psi}_d]^T$.

Taking $\eta(t)$ as an additional state vector and combining Equation (12) with Equation (8), the state equation of the augmented system is:

$$\begin{aligned}
\dot{x}(t) &= A_x x(t) + B_u u(t) \\
\eta(t) &= A_d x(t) + B_d u(t) - r(t) \\
y(t) &= C_x x(t) + D_u u(t)
\end{aligned}$$

where $A_d = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$, $B_d = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$.

Then:

$$\begin{aligned}
\dot{x}_c(t) &= A_c x_c(t) + B_c u_c(t) - U_e(t) \\
y_c(t) &= C_c x_c(t) + D_c u_c(t)
\end{aligned}$$

where $x_c(t) = \begin{bmatrix} x(t) \\ \eta(t) \end{bmatrix}$, $B_c = \begin{bmatrix} B_a \\ B_d \end{bmatrix}$, $C_c = \begin{bmatrix} C_c & 0 \\ 0 & 0 \end{bmatrix}$, $D_c = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $u_c(t) = \tau_m(t)$, $U_e(t) = \begin{bmatrix} 0 \\ r(t) \end{bmatrix}$.

3. Stability Controller Design on T-S Fuzzy Model

3.1. Description of T-S Fuzzy Model

A nonlinear system whose input–output relationship is linear at a local operating point can be represented by the following T-S fuzzy model:

**Rule**: when $\theta_1(t)$ is $M_{i1}$ and \ldots and $\theta_p(t)$ is $M_{ip}$,

Then

$$\begin{aligned}
\dot{x}(t) &= A_i x(t) + B_i u(t) \\
y(t) &= C_i x(t) + D_i u(t)
\end{aligned}$$

where $Rule^i$ is the $i$-th rule of the T-S fuzzy model, $\theta(t) = [\theta_1(t) \ \theta_2(t) \ \ldots \ \theta_p(t)]$ is the variable parameters of the system (15), $p$ is the number of variable parameters, $\gamma$ is the total number of rules, and $M_{ip}$ is the fuzzy set. $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, and $y(t) \in \mathbb{R}^q$ are state, input, and output of the system, respectively. $A_i \in \mathbb{R}^{n \times n}$, $B_i \in \mathbb{R}^{n \times m}$, $C_i \in \mathbb{R}^{q \times n}$, and $D_i \in \mathbb{R}^{q \times m}$ are matrices of corresponding dimensions. Each linear consequent equation represented by $A_i x(t) + B_i u(t)$ is called a "subsystem".

3.2. Design of Augment Model Based on T-S Fuzzy Model

Combined model (15), considering the influence of a nonlinear change of vehicle speed on vehicle stability control and taking the speed $v_s(t)$ as the variable parameter of the model (14). According to the variation range of the vehicle in the actual situation, vehicle speed is divided into five fuzzy sets, which are as follows: $v_{1s}(t) = 30 \text{ km/h}$, $v_{2s}(t) = 50 \text{ km/h}$, $v_{3s}(t) = 70 \text{ km/h}$, $v_{4s}(t) = 90 \text{ km/h}$, $v_{5s}(t) = 110 \text{ km/h}$, which are respectively marked as "$M_1$", "$M_2$", "$M_3$", "$M_4$", "$M_5$". Therefore, $\theta(t) = \dot{\theta}(t) = v_s(t)$,
p = 1, γ = 5, and the augmented system (14) can be represented by the following T-S fuzzy model:

Rule: when \( \vartheta(t) \) is “\( M_i \)”,

Then

\[
\begin{align*}
\dot{x}_e(t) &= A_{ei}x_e(t) + B_{ei}u_e(t) - U_e(t) \\
y_e(t) &= C_{ei}x_e(t) + D_{ei}u_e(t)
\end{align*}
\]

(16)

According to the T-S fuzzy model (16), the state equation of the subsystems can be obtained. Then, the most commonly used weighted average method is used for the fuzzification solution to obtain the state equation of the fuzzy model (17):

\[
\dot{x}_e(t) = \frac{5}{\sum_{i=1}^{5} \mu_i(\vartheta(t))} (A_{ei}x_e(t) + B_{ei}u_e(t) - U_e(t)) / \sum_{i=1}^{5} \mu_i(\vartheta(t))
\]

(17)

where \( \mu_i(\vartheta(t)) = M_i(\vartheta(t)) \), \( M_i(\vartheta(t)) \) denote that \( \vartheta(t) \) belongs to the membership function of \( M_i \) species, and the membership degree function is shown in Figure 3.

![Figure 3. Degree of membership.](attachment:figure3.png)

If set:

\[
g_i(\vartheta(t)) = \frac{\mu_i(\vartheta(t))}{\sum_{i=1}^{5} \mu_i(\vartheta(t))}
\]

(18)

So:

\[
\dot{x}_e(t) = \sum_{i=1}^{5} g_i(\vartheta(t)) (A_{ei}x_e(t) + B_{ei}u_e(t) - U_e(t))
\]

(19)

For any \( t \), \( \sum_{i=1}^{5} \mu_i(\vartheta(t)) = 1 \).

Similarly, we can get the output equation of the fuzzy model (16). Therefore, the augmented system (14), considering the vehicle speed as a variable parameter, can be expressed by the following T-S fuzzy system:

\[
\begin{align*}
\dot{x}_e(t) &= \sum_{i=1}^{5} g_i(\vartheta(t)) (A_{ei}x_e(t) + B_{ei}u_e(t) - U_e(t)) \\
y_e(t) &= \sum_{i=1}^{5} g_i(\vartheta(t)) (C_{ei}x_e(t) + D_{ei}u_e(t))
\end{align*}
\]

(20)

All figures and tables should be cited in the main text as Figure 1, Table 1, etc.

According to the PDC approach, each control rule is designed from the corresponding rule of a T-S fuzzy model. The designed fuzzy controller shares the same fuzzy sets with the fuzzy model. That is, the number of fuzzy rules of the fuzzy controller is 5, the variable parameter is \( v_x(t) \), and the fuzzy sets are “\( M_4 \)”, “\( M_2 \)”, “\( M_3 \)”, “\( M_4 \)”, “\( M_5 \)”. Combined with the model (20), the fuzzy control rules are:

Control Rule: when \( \vartheta(t) \) is “\( M_i \)”,

Then

\[
u_{eij}(t) = K_{x_e}(t), j = 1, 2, \ldots, 5
\]

(21)

The feedback control rate \( u_{eij}(t) \) of the T-S fuzzy system (20) at each local working point is solved, and the global controller is obtained by combining control rules (21):
Theorem 1. The condition of the fuzzy system (25) is globally asymptotically stable if there exists a common positive definite matrix $S$ for all subsystems, and at the same time:

$$A_i^T S + S A_i < 0 \quad i = 1, 2, \ldots, \gamma$$

(26)

The global controller is defined by:

$$u(t) = \sum_{j=1}^{\gamma} \mu_j(\theta(t)) K_j x(t) / \sum_{j=1}^{\gamma} \mu_j(\theta(t))$$

(27)

Substitute Equation (27) into Equation (25) to obtain the closed-loop system:

$$\dot{x}(t) = \sum_{i=1}^{\gamma} \sum_{j=1}^{\gamma} g_{ij}(\theta(t)) g_{ij}(\theta(t)) (A_i + B_i K_j) x(t)$$

(28)

The closed-loop T-S fuzzy system can also be written as:

$$\dot{x}(t) = \left[ \sum_{i=1}^{\gamma} \sum_{j=1}^{\gamma} g_{ij}(\theta(t)) g_{ij}(\theta(t)) W_{ii} x(t) + 2 \sum_{i=1}^{\gamma} \sum_{j<i}^{\gamma} g_{ij}(\theta(t)) g_{ij}(\theta(t)) \frac{W_{ii} + W_{jj}}{2} x(t) \right]$$

(29)

where $W_{ii} = A_i + B_i K_j$, $W_{jj} = A_j + B_j K_j$, and $W_{ji} = A_j + B_j K_i$.
We can derive the stability condition of the closed-loop system (29), and obtain Corollary 1:

**Corollary 1.** The condition of the fuzzy system (29) is globally asymptotically stable if there exists a common positive definite matrix $S$ for all subsystems, and at the same time:

$$W_{ii}^T S + SW_{ii} < 0 \quad i = 1, 2, \ldots, \gamma$$  \hspace{1cm} (30)

$$\left(\frac{W_{ij} + W_{ji}}{2}\right)^T S + S \left(\frac{W_{ij} + W_{ji}}{2}\right) < 0 \quad i < j$$  \hspace{1cm} (31)

**Proof of Theorem 1.** Select a function as $V(x(t)) = x(t)^T S x(t)$, where $S > 0$, then:

$$\dot{V}(x(t)) = x^T(t) S x(t) + x^T(t) S x(t)$$

$$= \sum_{i=1}^{\gamma} \sum_{j=1}^{\gamma} g_i(\theta(t)) g_j(\theta(t)) x^T(t)$$

$$\cdot \left[ (A_i + B_i K_j)^T S + S (A_i + B_i K_j) \right] x(t)$$

$$= \sum_{i=1}^{\gamma} g_i(\theta(t)) g_j(\theta(t)) x^T(t) \left( W_{ii}^T S + SW_{ii} \right) x(t)$$

$$+ 2 \sum_{i=1}^{\gamma} \sum_{j<i} g_i(\theta(t)) g_j(\theta(t)) x^T(t)$$

$$\cdot \left[ \left( \frac{W_{ij} + W_{ji}}{2} \right)^T S + S \left( \frac{W_{ij} + W_{ji}}{2} \right) \right] x(t)$$

where $g_i(\theta(t)) \geq 0, g_j(\theta(t)) \geq 0, \sum_{i=1}^{\gamma} \sum_{j=1}^{\gamma} g_i(\theta(t)) g_j(\theta(t)) > 0$, and $\sum_{i=1}^{\gamma} \sum_{j<i} g_i(\theta(t)) g_j(\theta(t)) > 0$.

$\square$

According to the known conditions (30) and (31), we can get $\dot{V}(x(t)) < 0$ at $x(t) \neq 0$. According to Lyapunov stability theory, the closed-loop system (29) is globally asymptotically stable under given conditions.

From Corollary 1, the stability condition of the system (24) can be expressed as:

$$H_{ii}^T S + SH_{ii} < 0 \quad i = 1, 2, \ldots, 5$$  \hspace{1cm} (32)

$$\left( H_{ij} + H_{ji} \right)^T S + S \left( H_{ij} + H_{ji} \right) < 0 \quad i < j$$  \hspace{1cm} (33)

By solving the inequality (32) and (33) with the Yalmip toolbox in MATLAB software, the matrix $X$ and the feedback gains $K_i$ of the global controller (22) can be derived.

**Remark 2.** Considering the influence of vehicle speed nonlinear change on vehicle stability control performance, the T-S fuzzy model with vehicle speed as a variable parameter is established. Combined with the Lyapunov stability principle, the conditions (30) and (31) to guarantee the global asymptotic stability of the T-S fuzzy system (24) are derived by using Corollary 1. Yalmip toolbox in MATLAB software was used to solve the problem, and a global stability controller (22) was obtained according to the control rules, which ensured the stability of the system.

4. Simulated Analysis

In order to verify the control effect and general applicability of the T-S model controller, two groups of simulations are designed in this paper. The two groups of simulation speed and target path are different. At the same time, an LMI controller and an MPC controller are designed for comparison. The LMI controller and MPC controller do not consider the varying parameters of the system, in which the vehicle speed of the system is a fixed value of 60 km/h.

In the first group of simulations, the vehicle speed change curve is shown in Figure 4a, and the lateral displacements of the two control schemes are shown in Figure 4b. Obviously, due to the change in the vehicle speed, the LMI controller and MPC controller cannot...
adjust the control input of the vehicle in time, resulting in a large overshot of the lateral displacement near 5 s and 7 s in Figure 4b, but compared with the LMI controller, the T-S fuzzy controller can adjust the control input in real-time according to the change in vehicle speed. By observing Figure 4c, it can be found that the LMI controller has a large tracking error, and the T-S model controller can better guarantee the accuracy of path tracking.

Figure 4d,e show the torque input and steering angle, respectively, of the three control methods. Compared with the other controllers, the torque of the T-S model controller changes more gently. The error of the vehicle tracking path can be greatly reduced, and the handling stability of the vehicle is improved. Moreover, the T-S model controller has a smaller total torque input, the maximum value is about 5 Nm, and the energy consumption is smaller. By observing Figure 4d, it can be found that at the 3 s, the torque under the LMI controller was significantly lagged.

Figure 4f shows the comparison of the yaw velocity of the vehicle. Compared with the LMI controller and MPC controller, the yaw velocity of the vehicle under the T-S model controller is smaller, which is about 60% of the LMI controller effect.

To better verify the control algorithm, we designed the second group of simulations. In the second group of simulations, the vehicle speed change curve is shown in Figure 5a, and the lateral displacements of the two control schemes are shown in Figure 5b. Compared with the first group of simulations, the path changes in the second group are relatively gentle, but the LMI controller and MPC controller still overshoot greatly. Compared with the two groups of controllers, the tracking effect of the T-S model controller is better. By looking at Figure 5c, it can be found that the T-S model controller can better guarantee the accuracy of path tracking.

Figure 5d,e show the torque input and steering angle, respectively, of the two control methods. Compared with the two controllers, the torque range of the T-S model controller is smaller. Thus, energy consumption is greatly reduced.

Figure 5f shows the comparison diagram of vehicle yaw rate. Compared with the effect of the LMI controller and MPC controller, the yaw velocity of the vehicle under the T-S model controller is smaller, which is about 90% of the effect of the LMI controller, which can better guarantee the stability of the vehicle.
5. Semi-Experimental Results Analysis

In this section, the controller is embedded in a semi-experimental platform. NI-PXI, as the vehicle-road system simulator, is embedded with CarSim real-time simulation environment, vehicle dynamics model, and double lane change driving scene model. As the steering controller of the automatic driving system, MicroAutobox calculates the target steering moment by real-time operation control strategy based on the information of vehicle state feedback and road information and outputs the steering moment by the drive plate control motor. Figure 6 is a schematic diagram of the semi-experimental scheme. In this section, two groups of experiments are designed, and the velocity speed changes curve in the two groups of experiments are shown in Figures 7a and 8a, and the target path is shown in Figures 7b and 8b, respectively.

Figure 5. (a) Speed (km/h), (b) lateral displacement (m), (c) deviation (m), (d) steering torque control input (Nm), (e) steering angle (°), and (f) yaw rate (°/s).

Figure 6. Semi-experimental scheme diagram of active steering.
Figure 7. (a) Speed (km/h), (b) lateral displacement (m), (c) deviation (m), (d) steering torque control input (Nm), (e) steering angle (°), (f) yaw rate (°/s), and (g) motor current (A).

Figure 8. (a) Speed (km/h), (b) lateral displacement (m), (c) deviation (m), (d) steering torque control input (Nm), (e) steering angle (°), (f) yaw rate (°/s), and (g) motor current (A).

The correlation curve of the first group of experiments is shown in Figure 7. The vehicle speed change curve is shown in Figure 7a. Figure 7b shows the lateral displacements, and Figure 7c represents the deviation of the vehicle tracking path. By observing Figure 7b,c, the T-S model controller, LMI controller, and the MPC controller algorithm can effectively realize path tracking. However, compared with the LMI controller effect, the deviation of
the T-S model controller algorithm is smaller, which can better guarantee the stability of the vehicle.

Figure 7d,e show the vehicle’s torque input and steering angle, respectively. It can be seen that the LMI controller and MPC controller effect have obviously overshot, and the torque input amplitude changes greatly, which easily puts the vehicle in an unstable state. T-S model controller can adjust the torque output in real-time according to the change of vehicle speed, combined with control rules, so the torque output is gentler, with less fluctuation, which can reduce the burden of the driver and make the vehicle run more smoothly. Figure 7f shows the yaw rate comparison graph collected in the experiment. By observing Figure 7f, compared with the MPC controller, the T-S model controller controls the yaw rate within a smaller range, so the control effect is better, and the stability of the vehicle is better guaranteed.

Figure 7g shows the electric motor currents collected in the experiment. Compared with the LMI controller, the T-S model controller motor current amplitude is smaller, and the energy consumption is smaller.

The correlation curve of the second group of experiments is shown in Figure 8. The vehicle speed change curve shown in Figure 8a–c are lateral displacement and vehicle tracking path deviation, respectively. As can be seen from Figure 8b,c, the path tracking deviation of the T-S model controller is small, and the accuracy is higher.

Figure 8d is the torque input collected in the test, and Figure 8e is the steering angle. The T-S controller, LMI controller, and the MPC controller can complete the work smoothly, but the LMI controller has a certain lag, while the T-S controller has a faster response speed, which better ensures the stability of the vehicle. As can be seen from Figure 8d,e, the relative torque of LMI control is large, while the output torque of the T-S controller is small, about 90% of that of the LMI controller, and the control effect is gentler, which can reduce the psychological panic of the driver.

Figure 8f is the comparison diagram of yaw velocity collected. Compared with the MPC controller, the T-S fuzzy controller ensures that the yaw velocity of the vehicle is smaller.

Figure 8g shows the motor current collected in the experiment. The motor current amplitude of the T-S model controller is smaller, which is about 90% of that of the LMI controller, and it consumes less energy.

Through the above analysis, the LMI controller and the MPC controller can only be solved at a fixed speed, and its control torque cannot change in real-time according to the speed, so the control effect is poor. The T-S model controller divides the vehicle speed into fuzzy sets, which can use control rules to adjust the output torque in real-time according to the change in vehicle speed, so the T-S model controller has a faster response and better control effect.

6. Conclusions

A vehicle stability control method based on the superposition of steering torque was proposed, in which the Takagi-Sugeno fuzzy model was used to solve the effect of vehicle speed changes on the stability control performance. In order to ensure that the driver can participate in steering control and realize human–machine cooperative driving, this paper took the torque as the control input of the system. The nonlinear T-S fuzzy model was established by fuzzifying the local linear model, and the influence of the nonlinear change of vehicle speed on the vehicle control performance was solved. The global asymptotic stability of the system was guaranteed by using the Lyapunov stability method. To verify the control effect of the T-S model controller, a simulation and experiment were designed. Simulation and semi-experimental results showed that the control method could improve the handling stability and the tracking accuracy of the human–machine cooperative driving vehicle under the condition of vehicle speed variation. Summary of the design of the system contribution as follows:
An algorithm is proposed based on the Takagi-Sugeno fuzzy model and design fuzzy rules with speed as the premise variable to solve the problem of vehicle stability control under variable speed.

A vehicle stability control method of steering moment is proposed, which enables the driver to participate in the control of the vehicle and solves the problem that the driver cannot participate in the steering wheel angle control.

The global asymptotic stability of the system is guaranteed by using the LMI solution and Lyapunov stability principle.

A hardware-in-the-loop test rig based on the LabVIEW-RT system was built to verify the actual control effect of the T-S controller. Simulation and semi-experimental results show that the controller can control the vehicle tracking path, effectively realize the vehicle stability control, and improve the performance and stability of the system.

In future research, we will control the vehicle stability while realizing the path tracking according to the real-time state of the vehicle and construct parameters similar to penalty factors, so that the control system can reasonably allocate the weight of vehicle path tracking accuracy and lateral stability control. In addition, we will take more variable parameters of vehicles into consideration and use THE TS fuzzy principle to build a more perfect vehicle model so that the built vehicle model is more consistent with the actual situation. Finally, we will consider the influence of driver characteristic parameters on vehicle stability control performance and use advanced sensors, such as EMG, EEG, smart eye, etc., to collect driver data and obtain characteristic driver information. By means of reinforcement learning and other methods, the driving characteristics when the driver feels comfortable are learned from the feature information so as to build a more perfect human–machine co-frame assistance system, improve the anthropoid ability of the autonomous driving system of the vehicle, and improve the driving comfort of the autonomous driving vehicle.

Author Contributions: Writing—original draft, Z.J.; conceptualization, J.W.; supervision, Z.C.; supervision, F.W.; investigation, L.L.; software, Q.K.; formal analysis, F.L. All authors have read and agreed to the published version of the manuscript.

Funding: Open Fund of Key Laboratory of Transportation Vehicle Detection, Diagnosis and Maintenance Technology (Grant No. JTZL2001).

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: No external data were used in this study.

Conflicts of Interest: The authors declare no conflict of interest.

References
1. Kirli, A.; Chen, Y.; Okwudire, C.E.; Ulsoy, A.G. Torque-Vectoring-Based Backup Steering Strategy for Steer-by-Wire Autonomous Vehicles with Vehicle Stability Control. IEEE Trans. Veh. Technol. 2019, 99, 7319–7328. [CrossRef]
2. Chatzikomis, C.; Sorniotti, A.; Gruber, P.; Zanchetta, M.; Willans, D.; Balcombe, B. Comparison of Path Tracking and Torque-Vectoring Controllers for Autonomous Electric Vehicles. IEEE Trans. Intell. Veh. 2018, 3, 559–570. [CrossRef]
3. Zhao, P.; Chen, J.; Song, Y.; Tao, X.; Xu, T.; Mei, T. Design of a control system for an autonomous vehicle based on adaptive-pid. Int. J. Adv. Robot. Syst. 2012, 9, 44. [CrossRef]
4. Cheng, Z.; Lu, Z. Research on the PID control of the ESP system of tractor based on improved AFSA and improved SA. Comput. Electron. Agric. 2018, 148, 142–147. [CrossRef]
5. Yan, S.; Yan, Z. Research on Bus Handling and Stability Control Based on Fuzzy PID Control. Mach. Build. Autom. 2015, 6, 47.
6. Wu, J.; Tian, Y.; Walker, P.; Li, Y. Attenuation reference model based adaptive speed control tactic for automatic steering system. Mech. Syst. Signal Process. 2021, 156, 107631. [CrossRef]
7. Tavan, N.; Tavan, M.; Hosseini, R. An optimal integrated longitudinal and lateral dynamic controller development for vehicle path tracking. Lat. Am. J. Solids Struct. 2015, 12, 1006–1023. [CrossRef]
8. Wu, J.; Cheng, S.; Liu, B.; Liu, C. A human-machine-cooperative-driving controller based on AFS and DYC for vehicle dynamic stability. Energies 2017, 10, 1737. [CrossRef]
9. Hu, C.; Jing, H.; Wang, R.; Yan, F.; Chadli, M. Robust Hoo output-feedback control for path following of autonomous ground vehicles. Mech. Syst. Signal Process. 2016, 70, 414–427. [CrossRef]
10. Park, M.; Lee, S.; Han, W. Development of Steering Control System for Autonomous Vehicle Using Geometry-Based Path Tracking Algorithm. *ETRI J.* 2015, 37, 617–625. [CrossRef]

11. Feng, Y.; Du, R.; Xu, Y. Steering angle balance control method for rider-less bicycle based on ADAMS. In Proceedings of the 19th International Conference on Intelligent Transportation, Rio de Janeiro, Brazil, 1–4 November 2016; Springer: Berlin/Heidelberg, Germany; pp. 15–31.

12. Tan, D.; Chen, W.; Wang, H.; Gao, Z. Shared control for lane departure prevention based on the safe envelope of steering wheel angle. *Control. Eng. Pract.* 2017, 64, 15–26. [CrossRef]

13. Ji, X.; Wu, J.; Zhao, Y.; Liu, Y.; Zhao, X. A new robust control method for active front steering considering the intention of the driver. *Proc. Inst. Mech. Eng. Part D J. Automob. Eng.* 2015, 229, 518–531. [CrossRef]

14. Wu, J.; Wang, X.; Li, L.; Du, Y. Hierarchical control strategy with battery aging consideration for hybrid electric vehicle regenerative braking control. *Energy* 2018, 145, 301–312. [CrossRef]

15. Benloucif, A.; Nguyen, A.-T.; Sentouh, C.; Popieul, J.-C. Cooperative trajectory planning for haptic shared control between driver and automation in highway driving. *IEEE Trans. Ind. Electron.* 2019, 66, 9846–9857. [CrossRef]

16. Ercan, Z.; Carvalho, A.; Tseng, H.E.; Gökşan, M.; Borrelli, F. A predictive control framework for torque-based steering assistance to improve safety in highway driving. *Veh. Syst. Dyn.* 2018, 56, 810–831. [CrossRef]

17. Wu, J.; Zhang, J.; Tian, Y.; Li, L. A Novel Adaptive Steering Torque Control Approach for Human-Machine-Cooperation Autonomous Vehicles. *IEEE Trans. Transp. Electrif.* 2021, 7, 2516–2529. [CrossRef]

18. Wang, H.; Kong, H.; Man, Z.; Cao, Z.; Shen, W. Sliding mode control for steer-by-wire systems with AC motors in road vehicles. *IEEE Trans. Ind. Electron.* 2013, 61, 1596–1611. [CrossRef]

19. Zafeiropoulos, S.; Di Cairano, S. Vehicle yaw dynamics control by torque-based assist systems enforcing driver’s steering feel constraints. In Proceedings of the 2013 American Control Conference, Washington, DC, USA, 17–19 June 2013; pp. 6746–6751. [CrossRef]

20. Piyabongkarn, D.; Rajamani, R.; Grogg, J.A.; Lew, J.Y. Development and Experimental Evaluation of a Slip Angle Estimator for Vehicle Stability Control. *IEEE Trans. Control. Syst. Technol.* 2009, 17, 78–88. [CrossRef]

21. Boada, B.; Boada, M.; Diaz, V. Fuzzy-logic applied to yaw moment control for vehicle stability. *Veh. Syst. Dyn.* 2005, 43, 753–770. [CrossRef]

22. Niasar, A.H.; Moghbeli, H.; Kazemi, R. Yaw moment control via emotional adaptive neuro-fuzzy controller for independent rear wheel drives of an electric vehicle. In Proceedings of the 2003 IEEE Conference on Control Applications, Istanbul, Turkey, 25 June 2003; Volume 1, pp. 380–385.

23. Selma, B.; Chouraqui, S.; Abouaissa, H. Fuzzy swarm trajectory tracking control of unmanned aerial vehicle. *J. Comput. Des. Eng.* 2020, 7, 435–447. [CrossRef]

24. Nguyen, A.-T.; Taniguchi, T.; Eciolaza, L.; Campos, V.; Palhares, R.; Sugeno, M. Fuzzy control systems: Past, present and future. *IEEE Comput. Intell. Mag.* 2019, 14, 56–68. [CrossRef]

25. Tanaka, K.; Yamauchi, K.; Ohtake, H.; Wang, H.O. Sensor reduction for backing-up control of a vehicle with triple trailers. *IEEE Trans. Ind. Electron.* 2008, 56, 497–509. [CrossRef]

26. Nguyen, A.-T.; Sentouh, C.; Popieul, J.-C. Driver-automation cooperative approach for shared steering control under multiple system constraints: Design and experiments. *IEEE Trans. Ind. Electron.* 2016, 64, 3819–3830. [CrossRef]

27. Takagi, T.; Sugeno, M. Fuzzy identification of systems and its applications to modeling and control. *IEEE Trans. Syst. Man Cybern.* 1985, 15, 116–132. [CrossRef]

28. Tanaka, K.; Wang, H.O. Fuzzy Control Systems Design and Analysis: A Linear Matrix Inequality Approach; John Wiley & Sons: Hoboken, NJ, USA, 2004.

29. El Youssfi, N.; El Bachiti, R.; Chabi, R.; Tissir, E.H. Static output-feedback H∞ control for T–S fuzzy vehicle lateral dynamics. *SN Appl. Sci.* 2020, 2, 101. [CrossRef]

30. He, Z.; Nie, L.; Yin, Z.; Huang, S. A Two-Layer Controller for Lateral Path Tracking Control of Autonomous Vehicles. *Sensors* 2020, 20, 3689. [CrossRef]

31. Nguyen, A.T.; Coutinho, P.; Guerra, T.M.; Palhares, R.; Pan, J. Constrained output-feedback control for discrete-time fuzzy systems with local nonlinear models subject to state and input constraints. *IEEE Trans. Cybern.* 2020, 51, 4673–4684. [CrossRef]

32. Shukla, S.; Tiwari, M. Fuzzy Logic of Speed and Steering Control System for Three Dimensional Line Following of an Autonomous Vehicle. *Int. J. Comput. Sci. Inf. Secur.* 2010, 7, 14–24.

33. Haddad, A.; Aitouche, A.; Coqquemot, V. Fault Tolerant Control Strategy for an Overactuated Autonomous Vehicle Path Tracking. *IFAC Proc. Vol.* 2014, 47, 8576–8582. [CrossRef]

34. Corno, M.; Panzani, G.; Roselli, F.; Giorelli, M.; Azzolini, D.; Savaresi, S.M. An LPV Approach to Autonomous Vehicle Path Tracking in the Presence of Steering Actuation Nonlinearities. *IEEE Trans. Control. Syst. Technol.* 2020, 29, 1766–1774. [CrossRef]

35. Hwang, C.L.; Yang, C.C.; Hung, J.Y. Path Tracking of an Autonomous Ground Vehicle with Different Payloads by Hierarchical Improved Fuzzy Dynamic Sliding-Mode Control. *IEEE Trans. Fuzzy Syst.* 2017, 26, 899–914. [CrossRef]

36. Zhao, W.; Zhang, H.; Li, Y. Displacement and force coupling control design for automotive active front steering system. *Mech. Syst. Signal Process.* 2018, 106, 76–93. [CrossRef]