LSAP: Rethinking Inversion Fidelity, Perception and Editability in GAN Latent Space

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Abstract

As the research progresses, inversion is mainly divided into two steps. The first step is Image Embedding, in which an encoder or optimization process embeds images to get the corresponding latent codes. Afterward, the second step aims to refine the inversion and editing results, which we named Result Refinement. Although the second step significantly improves fidelity, perception and editability are almost unchanged and deeply depend on inverse latent codes from first step. Therefore, a crucial problem is gaining the latent codes with better perception and editability while retaining the reconstruction fidelity. In this work, we first point out that these two characteristics are related to the degree of alignment (or disalignment) of the inverse codes with the synthetic distribution. Then, we propose Latent Space Alignment Inversion Paradigm (LSAP), which consists of an evaluation metric and solutions for inversion. Specifically, we introduce Normalized Style Space (SN space) and Normalized Style Space Cosine Distance (NSCD) to measure disalignment of inversion methods. Meanwhile, it can be optimized in both encoder-based and optimization-based embedding methods to conduct a uniform alignment solution. Extensive experiments in various domains demonstrate that NSCD effectively reflects perception and editability, and our alignment paradigm archives the state-of-the-art in both two stages. Code is available at https://github.com/caopulan/GANInverter/tree/main/configs/lsap.

1. Introduction

In recent years, Generative Adversarial Networks (GANs) [15] are used in various tasks [28, 51] and have dramatically improved image synthesis ability. Style-based generative models [23, 24, 22] further enhance the realism and resolution of image generation, achieving state-of-the-
The intermediate latent space \( \mathcal{W} \) space in StyleGAN encodes high-semantic information. As a strong prior, well-trained generator has demonstrated powerful capabilities and improved multiple tasks from traditional approaches, e.g., neural talking head [35, 53], face parsing [50, 57], and style transfer [29, 52].

These applications require latent codes, which are inherently available for synthetic images but cannot be applied directly to real images. To this end, inversion methods are designed to embed images into GAN’s latent space via various approaches. Existing works can be mainly divided into two stages. The first stage aims to attain latent codes, usually achieved by training an encoder or optimizing the reconstruction error, which we named \textit{Image Embedding}. In the second stage, researchers employ diversiform strategies to improve inversion and editing results, e.g., predicting generator weights [4, 10], predicting intermediate feature [45, 32], and finetuning the generator [37, 11], which we named \textit{Result Refinement}. Previous works [42] illustrate that fidelity, perception, and editability are three essential characteristics of inversion. However, in \textit{Result Refinement}, more attention has been paid to improve fidelity, maintaining visual details like the background, hat, and eyeglasses while inheriting editability and perception by the inverse codes from first step. Hence, in order to achieve superior performance in these three characteristics simultaneously, a robust latent code embedding technique is still necessary.

The latent space from random sampling and transformation possesses a particular distribution, which we named synthetic distribution. Intuitively, latent codes from this distribution have better performance. Supervision from the discriminator constrains the sampled latent codes to generate photorealistic images. Moreover, editing directions are gained by sampling [38] and analyzing [18] in synthetic latent space. Hence, the key point of perception and editability is the alignment between inverse codes and synthetic distribution. An existing method [42] solves this problem by latent code discriminator and achieves more reasonable perception and editability. However, there are two significant shortcomings. Firstly, it limits the reconstruction performance since introducing a discriminator makes training unstable. Secondly, this approach cannot be applied to the optimization-based inversion methods. Therefore, our key motivation is the idea of constructing an alignment paradigm between embedding latent space and synthetic latent space which can be applied to both encoder-based and optimization-based inversion methods and retains reconstruction ability.

In this work, we thoroughly analyze the disalignment in inversion and propose the Latent Space Alignment Inversion Paradigm (LSAP). Specifically, we first introduce the Normalized Style Space (\( S^N \) space). We prove that \( S^N \) space is more suitable and efficient for measuring disalignment than \( Z/\mathcal{W}/S \) space. Moreover, we introduce a metric Normalized Style Space Cosine Distance (NSCD) to evaluate the inversion methods at latent code level, which have shown experientially reflecting perception and editability. Then, we conduct the alignment solution in encoder-based and optimization-based methods, employing an alignment loss based on NSCD. We present extensive experiments to demonstrate the effects and generality of our alignment paradigm. We achieve the best trade-offs in encoder-based methods and drastically improve the perception and editability in the optimization-based method. Besides, we reach the state-of-the-art with HFGI [45], SAM [32], and PTI [37], which further demonstrates the potentiality and generality of our method. As shown in Figure 1, our visual results are natural and faithful. The key contributions of this work are summarized as follows:

- We rethink the fidelity, perception and editability in inversion task. As dividing inversion process into \textit{Image Embedding} and \textit{Result Refinement}, we point out that fidelity is enhanced in the second step while perception and editability are related to alignment between inverse codes and synthetic distribution.

- We propose an effective and generalized Latent Space Alignment Inversion Paradigm, including measurement and alignment solutions to improve perception and editability.

- To demonstrate the effect of our aligning paradigm, we take extensive experiments in various domains. NSCD reflects the perception and editability in a numerical way. Our alignment paradigm reaches better trade-offs between fidelity-perception and fidelity-editability. Applying to \textit{Result Refinement} methods, LSAP\(_E\) achieves state-of-the-art.

2. Related Work

\textbf{GAN Inversion.} As we mentioned above, the inversion process can be divided into two steps. Firstly, an initial latent code is gained by optimization or encoder from a given image. Optimizing reconstruction error typically reach better fidelity, while it requires several minutes per image [24, 7, 1, 2]. Training an encoder [42, 36, 46, 16, 7] to invert images is efficient during inference but achieves inferior reconstruction results. The second step aims to refine the inversion and editing results, using various strategies. Some methods [4, 10] adjust the convolution weights of the generator by hypernetwork [17]. ReStyle [3] introduces an iterative refinement mechanism, refining the latent code by a residual-based encoder. HFGI [45] proposes a distortion consultation approach for high-fidelity reconstruction. SAM [32] inverses the different segments of image into different intermediate layer by predicting "invertibility". Gen-
GAN-based Manipulation. Thanks to the rich semantic information of GAN’s \cite{23, 24, 22} latent space, many works have proposed various methods to control generated results by manipulating latent representation. Some methods \cite{9, 14, 40, 38} find editing directions of attributes (e.g., smile, gender, age, and pose) by semantic labels. Others find meaningful directions in an unsupervised \cite{18, 39, 43, 44} or self-supervised \cite{20, 34} way. Moreover, language-image models are explored to edit images by back-propagating the gradient of objective text \cite{33}. Some works \cite{41, 25} further introduce segmentation information to gain better performance, which may be extended to the human body by body GANs \cite{12, 13} and human parsing techniques \cite{49, 48, 50, 47} in the future. Since those manipulation approaches are almost built on latent codes, editability is also a crucial characteristic of inversion.

3. Latent Space Disalignment

In this section, we first rethink the source of fidelity, perception, and editability, and we point out that the latter two are deeply related to the alignment (or disalignment) between inverse codes and synthetic distribution. To illustrate and address this problem in inversion task, we formulate the disalignment degree of inversion process.

3.1. Fidelity, Perception and editability

As first proposed by Tov et al. \cite{42}, fidelity\footnote{Image distortion is originally used in e4e \cite{42}. To represents the ability of inversion methods, we use fidelity instead of it.}, perception and editability are three vital characteristics of inversion methods. Fidelity measures the reconstruction ability, requiring methods embedding image into latent space which can reconstruct images faithfully. Perception evaluates the reconstructed images’ perceptual quality, consisting of sharpness and naturalness in practice. Besides, editability represents editing capability of inverse codes, which is a comprehensive measurement, including editing effects, attributes disentanglement, etc.

Source. We first trace these three characteristics. Minimizing image distortion is a significant objective function applied in all inversion methods. It gives the algorithm the ability to reconstruct given images faithfully. Perception is gained by the powerful generating capability of GANs, since generator is trained to generate the photorealistic results with high resolution by a discriminator. Editability benefits from the highly semantic latent space of GANs. Given the editing direction, we can modify the latent codes to edit corresponding attribute. However, the ability of perception and editability is conditional. Specifically, under the constraint from the discriminator, latent space in trained generator tuning \cite{37, 11} can get the best inversion performance but is considerably time-consuming.

GAN is required to fit dataset distribution, from which latent code can generate high-quality images. The generator may not generate good results from out-of-distribution latent code. That is also verified by latent code truncation, that codes near to mean code can generate high-quality results. Moreover, editing directions are obtained by sampling latent codes \cite{38} or analyzing generator weights \cite{39}. That is also built on a specific latent space in GAN. We name the latent space distribution in GAN as synthetic distribution, which is converted by pre-trained networks from multivariate normal standard distribution.

Impacts from two inversion stages. Inversion process can be divided into two stages: Image Embedding and Result Refinement. The latent codes are first attained by an encoder or optimized by minimizing image distortion. In this phase, reconstruction error is slightly large. In Result Refinement step, methods focus on recovering visual details (e.g., background, cloths) by adjusting weights \cite{4} or intermediate features \cite{45} of generator. This stage further improves the fidelity and even can invert the out-of-distribution images \cite{37, 11}. However, perception and editability are inherited from the first step. In practice, if the inverse codes cannot be edited or generate images with good perceptual quality, the refined results still show the same effect. Hence, an essential problem is obtaining latent codes with better performance. In this work, we focus on the first step to study fidelity, perception, and editability of latent codes.

3.2. Disalignment Formulation

To illustrate disalignment between synthetic and inverse latent space, we firstly define \( \mathcal{P} \) space as a reference space, denoting \( \mathcal{P}_{inv} \) and \( \mathcal{P}_{syn} \) as inverse and synthetic latent space, respectively. \( G_\mathcal{P} \) is defined as the generator from \( \mathcal{P} \) space to image space. Suppose that \( \mathcal{Z} \) is the multivariate standard normal distribution and \( \mathcal{X} \) is the real image distribution. We establish two mapping functions \( F : \mathcal{Z} \rightarrow \mathcal{P}_{syn} \) and \( G_\mathcal{P} : \mathcal{P}_{inv} \rightarrow \mathcal{X} \).
and $I : \mathcal{X} \rightarrow \mathcal{P}_{inv}$. In practice, $I$ serves as an embedding method, used to convert images into $\mathcal{P}$ space latent code. Moreover, $F$ is a mapping function consisting of multiple parts in the front of generator. It is worth noting that aligning dose not mean making inverse distribution identical to synthetic distribution in this circumstance. It requires inverse codes located at high probability area in synthetic distribution. Hence we can define disalignment $\mathcal{D}$ between these two spaces as follow:

$$\mathcal{D} = -\mathbb{E}_{x_{inv} \sim \mathcal{P}_{inv}} p_{syn}(x_{inv})$$

(1)

**Compared to Kullback-Leibler divergence.** Another way to measure and optimize the disalignment is Kullback-Leibler divergence (KL divergence). In practice, latent code discriminator in e4e [42] can be considered as a method to minimize $D_{KL}(\mathcal{P}_{syn} || \mathcal{P}_{inv})$. However, as we analyzed above, aligning aims to let inverse codes have high probability in synthetic distribution, not to let these two distributions identical. Moreover, KL divergence can not be directly measured in inversion task. Hence it can not be used in optimization-based methods and served as a metric to indicate the characteristics of inversion methods. The difference also is also illustrated in Figure 2.

In Figure 1, two vital parts of disalignment measurement are which latent space is adequate to measure and how to measure $p_{syn}(x_{inv})$ for given sample. We will respectively answer these two questions in the following parts.

**4. Latent Space Alignment Inversion Paradigm**

In this section, we construct the Latent Space Alignment Inversion Paradigm (LSAP) to measure and improve perception and editability of inversion methods. Specifically, we introduce a new latent space Normalized Style Space ($S^N$) and propose the Normalized Style Space Cosine Distance (NSCD) as a measurement. Moreover, we present generalized alignment solutions in Image Embedding phase, including LSAP$_E$ and LSAP$_O$ for encoder-based and optimization-based methods, respectively.

**4.1. Normalized Style Space**

Although $Z/W/W^+$ spaces are primarily popular in previous research, in this work, we propose a new latent space, Normalized Style Space ($S^N$), and we will prove that it is better to measure disalignment.

Let us revisit the existing latent spaces first. Given the sampled random variable $z$ from $Z$ space, the mapping network first converts it into $w$ in $W$ space. Then affine modules are applied on $w$ at each resolution level, which output is $s = \{s_1, s_2, \ldots, s_k\}$, where $s_i = A_i w + b_i$ and the output space is named Style Space ($S$ space).

**Property 4.1.** Suppose that $s = \{s_1, s_2, \ldots, s_k\}$ is a set of $S$ space latent codes and corresponding to image $x = G_S(s)$. For $\forall a \in \mathbb{R}$ and $\forall i \in \{1, \ldots, k\}$, if $s' = \{s'_1, s'_2, \ldots, s'_k\}$ follows:

$$s'_i = \begin{cases} s_i, & i \neq l \\ a \times s_i, & i = l \end{cases}$$

we have $x = G_S(s) = G_S(s')$.\(^2\)

Property 4.1 illustrates that $S$ space latent codes are scaled-independent in every component. Converting codes in the unit hyper-sphere, those codes with the same angles
will generate the same results. Based on this property, we construct a new latent space, Normalized Style Space \((S^N)\), in which codes are normalized from \(S\) space by the euclidean norm. It follows:

\[
s^N_i = \frac{s_i}{\|s_i\|_2} = \frac{A_iw + b_i}{\|A_iw + b_i\|_2} \tag{2}
\]

To demonstrate differences between each latent space in measuring disalignment, we take extensive analyses:

**Property 4.2.** Given a set of \(S\) space latent codes \(s = \{s_1, \ldots, s_k\} \neq \emptyset\), \(\exists s' = \{s'_1, \ldots, s'_k\} \neq s\) such that \(G_S(s) = G_S(s')\).

**Proof.** According to Property 4.1, for \(\forall l \in \{1, \ldots, k\}\) when \(s'_i = a \times s (a \in \mathbb{R})\) and \(s'_i = s_i (i \neq l)\), we have \(G_S(s) = G_S(s')\). Since \(s_1 \neq \emptyset\), \(s'_1 \neq s_1\).

**Property 4.3.** For \(lth\) layer \((\forall l \in \{1, \ldots, k\}\), define \(F_l : \mathcal{Z}/\mathcal{W} \rightarrow \mathcal{S}\) as the mapping function between \(S\) and \(\mathcal{Z}/\mathcal{W}\) space. For all \(p_l \in \mathcal{Z}/\mathcal{W}\) \((F_l(p_l) \neq 0)\), \(\exists p'_l \neq p_l\) such that the corresponding \(S\) space latent codes satisfy: \(s'_l = a \times s_l (a \in \mathbb{R})\), where \(s_l = F_l(p_l)\) and \(s'_l = F_l(p'_l)\).

**Corollary 4.1.** Given a set of latent codes \(p = \{p_1, \ldots, p_k\}\) in \(\mathcal{Z}/\mathcal{W}/S\) space and \(p \neq 0\), \(\exists p'_l \neq p_l\) such that \(G_P(p) = G_P(p'_l)\).

According to Corollary 4.1, different latent codes in \(\mathcal{Z}/\mathcal{W}/S\) space can generate the same images, which implies the disalignment degree of these latent codes can not reflect discrepancies in generated results. Hence, we choose \(S^N\) as reference space to measure disalignment in inversion.

### 4.2. Normalized Style Space Cosine Distance

Illustrated in Figure 1, the probability of inverse latent codes in synthetic distribution is required to measure. However, it is difficult to calculate the \(p_{\text{syn}}(x_{\text{inv}})\) straightforwardly, for the formula of \(p_{\text{syn}}\) is unknown. Motivated by latent code truncation, we find that using distance between inverse code and mean code instead of \(p_{\text{syn}}(x_{\text{inv}})\) is a simple but efficient way. Code near mean code has a high probability practically. When we denote \(S^N\) space as reference space, we can use cosine distance to measure disalignment and define NSCD as follow:

\[
\text{NSCD} = 1 - \mathbb{E}_{x_{\text{inv}} \sim S^N_{\text{inv}}}[\cos(s^N_{\text{inv}}, \mu_{\text{syn}})] = 1 - \mathbb{E}_{x_{\text{inv}} \sim S^N_{\text{inv}}}[s^N_{\text{inv}} \cdot \mu_{\text{syn}}^T] \tag{3}
\]

Notably, as \(s^N_{\text{inv}}\) is the inversion result, cosine distance is differential for it to minimize in inversion process. The small value of NSCD means that \(S^N_{\text{inv}}\) space aligns with \(S^N_{\text{syn}}\) space. NSCD can reflect the perception and editability in image level, which will show in qualitative and quantitative experiments.

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**4.3. Alignment Inversion**

Inversion methods in Image Embedding phase aim to embed images into latent space in encoder-based or optimization-based way, and the process is as follows:

\[
p^* = \arg \min_p [\mathcal{L}(x, G_P(p))] \tag{4}
\]

\[
E^* = \arg \min_E [\mathbb{E}_{x \sim \mathcal{X}}(\mathcal{L}(x, G_P(E(x))))] \tag{5}
\]

where \(x\) is given image, \(\mathcal{X}\) is image dataset, \(\mathcal{L}\) is image level loss function (e.g., MSE, LPIPS [56], identity loss [8]) and \(E\) is an encoder. Since inversion methods are mainly supervised at the image level, there is a lack of limitation of inverse latent space distribution. To construct a uniform solution to train encoder or optimize the latent codes, we can constrain the disalignment degree by adding an alignment term in \(\mathcal{L}\).

Thanks to NSCD’s differentiable property, we can apply it to inversion methods to construct a direct and efficient alignment solution, shown in Figure 3. According to Equation 3, we first sample \(k\) \((k = 50,000\) in our experiments\) latent codes from the multivariate normal distribution and convert them into \(S^N\) by pre-trained generator to get mean code \(\mu_{\text{syn}}\). Then, we define an alignment loss as follows:

\[
\mathcal{L}_{\text{NSCD}}(x) = 1 - (F(I(x)) \cdot \mu_{\text{syn}}^T) \tag{6}
\]

where \(I\) is a Image Embedding method. \(L_{\text{NSCD}}\) is calculated by given images \(x\) (i.e., a batch of images in encoder training or one image in optimization) in each iteration. Moreover, we present the details of our encoder and optimization methods separately.

**Encoder.** The pipeline of encoder-based alignment inversion method is shown in Figure 3(b). Given real images, encoder is optimized by minimizing multiple losses at image level and latent code level. Following [36] and [42], \(L_{\text{img}}\) consists of distortion loss, perception loss, and identity loss. Besides, delta-regulation loss [42] is also applied to inverse codes, minimizing the deviation of \(\mathcal{W}^+\) codes among each level. The whole training object is defined by:

\[
\mathcal{L} = \lambda_1 \mathcal{L}_{\text{lpips}} + \lambda_2 \mathcal{L}_{\text{sim}} + \lambda_3 \mathcal{L}_{\text{d-reg}} + \lambda \mathcal{L}_{\text{NSCD}} \tag{7}
\]

where \(\lambda_1, \lambda_2, \lambda_3, \lambda\) are hyper-parameters to adjust the weight of each component in loss function. In encoder-based method, \(L_{\text{NSCD}}\) aims to align the encoder’s output space with synthetic latent space.

**Optimization.** Optimization-based inversion method updates latent code iteratively. Compared to encoder-based approach, \(L_{\text{NSCD}}\) is used to minimize the distance between a certain latent code with synthetic latent space. Following [24], we apply two losses in image level and latent code

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3Proof can be found in Appendix.
level, respectively:
\[
L = L_{\text{lpips}} + \lambda L_{\text{NSCD}} \tag{8}
\]

The encoder-based and optimization-based method are denoted as LSAP\textsubscript{E} and LSAP\textsubscript{O} respectively.

5. Experiments

In this section, we conduct extensive experiments to demonstrate the effects of LSAP in various domains, including face, object (cars), scene (churches), and animal (wild animals). Implementation details can be found in Appendix.

**Baselines.** We take comprehensive comparisons in both two inversion stages. For Image Embedding methods, we compare our LSAP\textsubscript{E} to pSp [36] and e4e [42] for encoder-based approaches and our LSAP\textsubscript{O} to StyleGAN2 [24] projection for optimization-based approaches. Moreover, we evaluate e4e and LSAP\textsubscript{E}’s performance with four Result Refinement methods: ReStyle [3], HFGI [45], SAM [32] and PTI [37], where HFGI and SAM can be considered as intermediate feature prediction methods, and PTI is a generator tuning method.

**Evaluation.** We use MSE and LPIPS [56] to evaluate fidelity in all fields, and calculate identity similarity by ArcFace [8] between input and reconstructed images in face domain. To evaluate perception and editability, we use NSCD and latent editing consistency (LEC) [42]. Furthermore, we calculate the identity preservation between origin images and edited images under the same editing effects by each inversion method to demonstrate identity impacts during manipulation.

### Table 1: Fidelity results on face domain.

We show the reconstruction results of encoder-based (E), optimization-based (O), and two-stage methods including latent codes refinement (E+L), feature prediction (E+F) and generator tuning (E+T), respectively. Gain refers to our improvement of MSE over the X+e4e baselines.

| Method          | Type | MSE ↓  | Gain | LPIPS ↓  | Similarity ↑ |
|-----------------|------|--------|------|----------|---------------|
| pSp [36]        | E    | 0.0351 |      | 0.1628   | 0.5591        |
| e4e [42]        | E    | 0.0475 |      | 0.1991   | 0.4966        |
| LSAP\textsubscript{E} | E    | 0.0397 |      | 0.1766   | 0.5305        |
| StyleGAN2-W [24]| O    | 0.0696 |      | 0.1987   | 0.3066        |
| LSAP\textsubscript{O}-W | O   | 0.0690 |      | 0.1986   | 0.2989        |
| StyleGAN2-W+ [24]| O   | 0.0279 |      | 0.1179   | 0.7463        |
| LSAP\textsubscript{O}-W+ | O  | 0.0359 |      | 0.1376   | 0.6587        |
| ReStyle\textsubscript{e4e} [3] | E+L | 0.0429 |      | 0.1904   | 0.5062        |
| ReStyle\textsubscript{LSAP}  | E+L  | 0.0296 | -31.1% | 0.1506   | 0.6148        |
| HFGI\textsubscript{LSAP} [45] | E+F  | 0.0296 |      | 0.1172   | 0.6816        |
| HFGI\textsubscript{LSAP} [45] | E+F  | 0.0210 | -29.0% | 0.0945   | 0.7405        |
| SAM\textsubscript{LSAP} [32]| E+F  | 0.0143 |      | 0.1104   | 0.5568        |
| SAM\textsubscript{LSAP} [32]| E+F  | 0.0117 | -18.1% | 0.0939   | 0.6184        |
| PTI\textsubscript{e4e} [37]  | E+T  | 0.0074 |      | 0.0750   | 0.8633        |
| PTI\textsubscript{LSAP}      | E+T  | 0.0067 | -9.4%  | 0.0666   | 0.8696        |

### Table 2: Perception and editability results on face domain.

We compare NSCD, LEC, and identity similarity [8] between three encoder-based inversion methods.

| Domain       | Method  | MSE ↓ | LPIPS ↓ | NSCD ↓ |
|--------------|---------|-------|---------|--------|
| Car          | e4e     | 0.1201| 0.3252  | 0.0646 |
|              | LSAP\textsubscript{E}| 0.1049| 0.3106  | 0.0492 |
| Church       | e4e     | 0.1505| 0.4307  | 0.0761 |
|              | LSAP\textsubscript{E}| 0.1144| 0.3426  | 0.0588 |
| Wild Animal  | e4e     | 0.0882†| 0.2658†| 0.0379†|

Table 3: Quantitative results on other domains. † means the model is unavailable and we train the encoder by official code.

### Quantitative Results.

We provide the reconstruction results in Table 1 to evaluate fidelity on face domain. Our LSAP-based demonstrates outstanding performance in every baseline. Compared to pSp and StyleGAN2 projection, our additional alignment loss marginally sacrifices fidelity but improves perception and editability a lot which will be shown below. Meanwhile, LSAP\textsubscript{E} attains superior performance than e4e. Applied to Result Refinement methods, LSAP\textsubscript{E} outperforms e4e in all three methods. With ReStyle and HFGI, which use model to refine result and inference rapidly, LSAP\textsubscript{E} gains about 30% improvement of MSE. PTI\textsubscript{LSAP} gains best results in inversion.

In other domains, we compare LSAP\textsubscript{E} to the most commonly used encoder e4e to illustrate the generality of our approach. Results are shown in Table 3. As can be seen, LSAP\textsubscript{E} attains better performance in all three domains, which indicates our alignment solution is robust in GAN inversion task.

Moreover, we evaluate the perception and editability by NSCD, LEC and identity preservation during manipulation, which can be found in Table 2. LSAP\textsubscript{E} achieves the best NSCD and LEC in three editing attributes and identity similarity in two attributes. It is worth mentioning that although e4e has decent editability, it gets worse identity preservation than pSp in ”pose” and ”smile”, which is caused by their reconstruction gap. Nevertheless, LSAP\textsubscript{E} reaches a higher similarity in ”pose” and ”age”, which indicates that our approach can preserve portrait identity well during manipulation.

### Qualitative Results.

We perform the qualitative comparisons in Figure 4. For reconstruction ability of Image Em-
Figure 4: **Inversion and editing results of encoder-based and two-stage inversion methods on face domain.** We show the comparison of encoder-based, optimization-based and two-stage method respectively. LSAP\(_E\) improves the perception and editability while retaining the fidelity, and HFGI\(_{LSAP}\), SAM\(_{LSAP}\) and PTI\(_{LSAP}\) further reduce image distortion.

| Input | pSp | e4e | LSAP\(_E\) (ours) | HFGI\(_{LSAP}\) | SAM\(_{LSAP}\) | SAM\(_{LSAP}\) | PTI \(_{LSAP}\) | PTI \(_{LSAP}\) |
|-------|-----|-----|-----------------|----------------|--------------|--------------|----------------|----------------|
| **Smile** | ![Input](image1) | ![e4e](image2) | ![LSAP\(_E\) (ours)](image3) | ![HFGI\(_{LSAP}\)](image4) | ![SAM\(_{LSAP}\)](image5) | ![SAM\(_{LSAP}\)](image6) | ![PTI \(_{LSAP}\)](image7) | ![PTI \(_{LSAP}\)](image8) |
| **Pose** | ![Input](image9) | ![e4e](image10) | ![LSAP\(_E\) (ours)](image11) | ![HFGI\(_{LSAP}\)](image12) | ![SAM\(_{LSAP}\)](image13) | ![SAM\(_{LSAP}\)](image14) | ![PTI \(_{LSAP}\)](image15) | ![PTI \(_{LSAP}\)](image16) |
| **Age** | ![Input](image17) | ![e4e](image18) | ![LSAP\(_E\) (ours)](image19) | ![HFGI\(_{LSAP}\)](image20) | ![SAM\(_{LSAP}\)](image21) | ![SAM\(_{LSAP}\)](image22) | ![PTI \(_{LSAP}\)](image23) | ![PTI \(_{LSAP}\)](image24) |

Figure 5: **Editability effects of LSAP for optimization-based methods.** LSAP makes optimized latent codes more editable and image quality is improved in both \(\mathcal{W}\) and \(\mathcal{W}^+\) spaces.

| Input | StyleGAN2 – \(\mathcal{W}\) | LSAP\(_{LSAP}\) – \(\mathcal{W}\) | LSAP\(_{LSAP}\) – \(\mathcal{W}^+\) | Input | StyleGAN2 – \(\mathcal{W}\) | LSAP\(_{LSAP}\) – \(\mathcal{W}\) | LSAP\(_{LSAP}\) – \(\mathcal{W}^+\) |
|-------|----------------|----------------|---------------|-------|----------------|----------------|---------------|
| **pose** | ![Input](image25) | ![StyleGAN2 – \(\mathcal{W}\)](image26) | ![LSAP\(_{LSAP}\) – \(\mathcal{W}\)](image27) | ![LSAP\(_{LSAP}\) – \(\mathcal{W}^+\)](image28) | ![Input](image29) | ![StyleGAN2 – \(\mathcal{W}\)](image30) | ![LSAP\(_{LSAP}\) – \(\mathcal{W}\)](image31) | ![LSAP\(_{LSAP}\) – \(\mathcal{W}^+\)](image32) |
| **smile** | ![Input](image33) | ![StyleGAN2 – \(\mathcal{W}\)](image34) | ![LSAP\(_{LSAP}\) – \(\mathcal{W}\)](image35) | ![LSAP\(_{LSAP}\) – \(\mathcal{W}^+\)](image36) | ![Input](image37) | ![StyleGAN2 – \(\mathcal{W}\)](image38) | ![LSAP\(_{LSAP}\) – \(\mathcal{W}\)](image39) | ![LSAP\(_{LSAP}\) – \(\mathcal{W}^+\)](image40) |

*bedding* methods, our alignment paradigm attains comparable reconstruction quality with pSp. Meanwhile, LSAP improves image perception and editability a lot. In comparison with e4e, LSAP\(_E\) achieves better fidelity and editability. For example, editing results of the man in the first image of Figure 4 from e4e have redundant glasses (smile). In two-stage methods, HFGI, SAM and PTI improve the reconstruction capacity from e4e and LSAP\(_E\). Inversion results and editing effects from those methods are similar to the corresponding results from encoders while retaining more image details. PTI\(_{LSAP}\) achieves state-of-the-art for its high fidelity perception, and editability performance.

For optimization-based methods, our approach makes optimized code editable, which can be found in Figure 5. Vanilla projection in both \(\mathcal{W}\) and \(\mathcal{W}^+\) spaces generate unnatural face details, while results significantly improve by
Figure 6: Inversion results on other domains. In car and church domains, the official e4e models are available and we train the encoder on AFHQ [6] Wild dataset.

Figure 7: Inversion and editing comparison between e4e and LSAP\textsubscript{E}. We illustrate these two encoders’ inversion and editing results and the corresponding results with SAM. LSAP significantly improves editability and retains more visual details during inversion.

LSAP. It demonstrates that our approach provides a concise and straightforward solution even in optimization-based methods.

We further visualize inversion and editing results from e4e and LSAP\textsubscript{E} in other domains, including cars, churches, and wild animals. Result are demonstrated in Figure 6 and Figure 7. For inversion, LSAP\textsubscript{E} achieves slight improvement in fidelity, since the color and reflection are reconstructed accurately. For example, in the second image of Figure 7, the reflection is represented in LSAP\textsubscript{E} result, while the result from e4e only shows the white color. During editing, LSAP\textsubscript{E} demonstrates the excellent ability to generate good editing results. With SAM technique, LSAP\textsubscript{E} also achieves a better result in both inversion and manipulation.

Perception and Editability in Two Stages. In § 3.1, we point out that fidelity is improved in the Result Refinement step, while perception and editability are inherited from Image Embedding. As can be seen in Figure 4, when editing results from encoder are weak, such as attributes entanglement, the corresponding results from two-stage methods are basically similar. For example, editing the third image with "age", results from e4e wear additional glasses, then the results from HFGI\textsubscript{e4e}, SAM\textsubscript{e4e}, and PTL\textsubscript{e4e} also have the same impacts. Hence, although Result Refinement can largely improve the fidelity, the Image Embedding still plays an important role in inversion.

NSCD. In § 3 we find these two characteristics are related to alignment between inverse codes and synthetic distribution. Then we introduce that NSCD can numerically reflect them, which is validated in our experiments. Those with smaller NSCD have better image quality and editing results (e.g., e4e and LSAP\textsubscript{E}), while the large NSCD means weak generating and manipulation results (e.g., pSp and StyleGAN2-W\textsuperscript{+}). Compared to LEC [42], NSCD is evaluated on latent codes and irrelevant to editing vector, which is more general and convenient.

6. Conclusion

Fidelity, perception and editability are three critical characteristics of inversion methods. We start by tracing the source of fidelity, perception, and editability in inversion process and find that it is significant to embed images aligning with synthetic distribution in Image Embedding step, which also greatly impacts Result Refinement results. Hence, we propose a Latent Space Alignment Inversion Paradigm (LSAP), containing the measurement and solution of latent space disalignment. Specifically, to illustrate the disalignment straightforwardly and numerically, we propose Normalized Style Space Cosine Similarity (NSCD) as metric with Normalized Style Space (S\textsuperscript{N}) latent space. Thanks to the differentiable characteristic of NSCD, we conduct a uniform solution in encoder-based and optimization-based approaches. Through extensive experiments in four domains and three types baselines, LSAP shows promising results in all three features, and two-stage methods with LSAP achieve state-of-the-art.
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A. Property Proof

In this section, we provide detailed proof of Property 4.1 and Property 4.3.

A.1. Proof of Property 4.1

Property 4.1 Suppose that \( s = \{ s_1, s_2, \ldots, s_k \} \) is a set of style space latent codes and corresponds to image \( x = G_S(s) \). For \( \forall a \in \mathbb{R} \) and \( \forall l \in \{ 1, \ldots, k \} \), \( \hat{s} \) follows:

\[
\hat{s}_n = \begin{cases} 
  s_n, & \text{if } n \neq l \\
  a \times s_n, & \text{if } n = l
\end{cases}
\]

We have \( x = G_S(s) = G_S(\hat{s}) \).

Proof. According to StyleGAN2, style latent codes are used in weight demodulation way. For \( l \)th layer, convolution layer weights \( W_{i,j,k} \) are scaled by \( l \)th style latent codes \( s_l \) firstly:

\[
W'_{ijk} = s_l^i \times W_{ijk}, \quad (9)
\]

where \( i, j, k \) enumerate input feature maps, output feature maps and spatial footprint, respectively.

To integrate instance normalization in convolution layer, StyleGAN2 demodulates each output feature map by \( \sigma_j = \sqrt{\sum_{i,k} W'_{ijk}^2} \), assuming that input activations are i.i.d. random variables with unit standard deviation (ignore \( \epsilon \) used for numerical stable):

\[
W''_{ijk} = \frac{W'_{ijk}}{\sqrt{\sum_{i,k} W'_{ijk}^2}} \quad (10)
\]

Substitute formula 9 into formula 10, then we can reach:

\[
W''_{ijk} = \frac{s_l^i \times W_{ijk}}{\sqrt{\sum_{i,k} (s_l^i \times W_{ijk})^2}} \quad (11)
\]

Suppose that \( \hat{s}_l = a \times s_l \),

\[
\hat{W}_{ijk}'' = \frac{\hat{s}_l^i \times W_{ijk}}{\sqrt{\sum_{i,k} (\hat{s}_l \times W_{ijk})^2}} = \frac{a \times s_l^i \times W_{ijk}}{\sqrt{\sum_{i,k} (a \times s_l^i \times W_{ijk})^2}} = \frac{s_l^i \times W_{ijk}}{\sqrt{\sum_{i,k} (s_l^i \times W_{ijk})^2}} = W''_{ijk} \quad (12)
\]

Thus, if scale \( s \) by \( a \in \mathbb{R} \) in an arbitrary layer, convolution weights are identical, meaning generated images are the same.
A.2. Proof of Property 4.3

Property 4.3 For \( l \)th layer \((\forall l \in \{1, \cdots, k\})\), define \( F_l : Z/\mathcal{W} \rightarrow S \) as the mapping function between \( S \) and \( Z/\mathcal{W} \) space. For \( \forall p_l \in Z/\mathcal{W} \) \((F_l(p_l) \neq \mathbf{0}) \) and \( a \in \mathbb{R} \), \( \exists \hat{p}_l \neq p_l \) such that the corresponding \( S \) space latent codes satisfy: \( s'_l = a \times s_l \), where \( s_l = F_l(p_l) \) and \( s'_l = F_l(\hat{p}_l) \).

Proof. We prove this property separately under \( \mathcal{W} \) and \( Z \) spaces. Since cases under each layer level are the same without loss of generality, to express concisely, we consider the situation under an arbitrary layer and ignore \( l \) in the later formulation.

\( \mathcal{W} \) space The mapping function between \( \mathcal{W} \) and \( S \) space is established by linear projection in generator, as follows:

\[
s = F(w) = Aw + b \tag{13}
\]

If \( \exists y \), such that

\[
Ay = (a - 1)b \tag{14}
\]

and let

\[
w' = aw + y \tag{15}
\]

we have

\[
s' = Aw' + b = A(aw + y) + b = aAw + ab = as \tag{16}
\]

In StyleGAN, \( A \in \mathbb{R}^{m \times n} (m \leq n) \) may not be a square matrix in some resolution levels and the rank of \( A \) is unstable. It indicates Equation 14 can not be solved by inverse of \( A \) directly. We can obtain \( y \) by solving the least squares problem:

\[
\min_y \| Ay - (a - 1)b \| \tag{17}
\]

Hence, for \( \forall w \), when \( w' = aw + y, F(w) = a \cdot F(w') \). In addition, we can prove \( w' \neq w \) by the counterfactual method. If \( w' = w \), we have \( y = (1-a)w \) and \( A(1-a)w = (a-1)b \), so \( Aw = -b \) and \( s = 0 \). Due to \( s \neq 0 \), \( w' \neq w \) and \( w' = aw + y, F(w) = a \cdot F(w') \), we prove that property holds in \( \mathcal{W} \) space.

\( Z \) space Although we have proved in \( \mathcal{W} \) space, the mapping function between \( Z \) and \( \mathcal{W} \) or \( Z \) and \( S \) is represented by a multilayer perception, which is difficult to prove directly by formula. Fortunately, as the objective function is defined, we can obtain \( z' \) by optimization, satisfying \( s = F(z) = a \times F(z') = ks' \) and \( z' \neq z \).

B. Implementation Details

Datasets. We conduct the whole experiment on four domains: faces, cars, churches, and wild animals, corresponding to human, object, scene, and animal, respectively. In all domains, we use the official StyleGAN2 generator. For face domain, we train the LSAP\(_E\) on FFHQ [23] (70,000 face images) and evaluate on CelebA-HQ [31, 21] test dataset (2824 images). Editing directions are gained by [38]. For car domain, we use Stanford Cars [27] dataset with 8,144 images for training and randomly selected 1000 images for evaluation and edit images by [18]. For church domain, we use LSUN [54] Church dataset with 126,227 training images and 300 test images. For wild animal domain, we use AFHQ [6] Wild dataset.

LSAP\(_E\). The input image resolution is 192 × 256 in car domain and 256 × 256 for the others. For data augmentation, we only employ random horizontal flips. We adopt the Ranger optimizer, combining the Rectified Adam [30] and the Lookahead technique [55], with 0.001 learning rate. We take all experiments on a single GPU with batch size of 8. Besides, we follow the progressive training from e4e [42]. In LSAP\(_E\), perceptual loss weight \( \lambda_1 \) is 0.8, delta-regulation loss \( \lambda_2 \) is 2e - 5, and NSCD loss \( \lambda \) is 0.5 for all domains. Similarity loss weight \( \lambda_2 \) is 0.1 for face domain over pre-trained ArcFace [8] and 0.5 for others with MOCOv2 [5] and ResNet-50 [19].

Optimization-based Method. Following [24], we adopt Adam [26] optimizer to minimize perceptual loss and NSCD loss with noise regularization. \( \lambda \) is set to 20 for \( \mathcal{W} \) space and 5 for \( \mathcal{W} \) space.

Result Refinement Method. We apply e4e and LSAP\(_E\) to three Result Refinement methods, HFGI [45], SAM [32], and PTI [37] to illustrate the effects of Image Embedding step. For HFGI, we use official weight to evaluate HFGI\(_{LSAP}\) and follow its training script to train HFGI\(_{LSAP}\). In practice, we only replace the encoder weight from e4e to LSAP\(_E\) with the same architecture. Since SAM only releases the optimization codes, we first embed images into latent codes by encoders, and then optimize the latent codes with intermediate feature for 500 iterations, with threshold \( \tau = 0.225 \). For PTI, we use inverse codes by encoders as pivotal latent codes and tune the generator for 350 steps.

Evaluation Pipeline. Since inversion and editing results are gained by multiple codebases, we conduct all image level evaluations on saved image files. MSE, LPIPS and identity similarity are calculated on 256 × 256 resolution by script from pSp\(^4\) [36]. TFor LEC and identity similarity, we use different editing factor to ensure the same editing effect for all inversion methods, which can be found in qualitative results.

C. Ablation Study

We study the hyper-parameter \( \lambda \) of \( \mathcal{L}_{NSCD} \) on face domain with LSAP\(_E\) as exmaple, and the quantitative results

\(^4\)https://github.com/eladrich/pixel2style2pixel
Table 4: **Ablation study on hyper-parameter of LSAP\(_E\).** We set \(\lambda = 0.5\) in our experiments by default.

| \(\lambda\) | Fidelity | Perceptual & Editability |
|-------------|----------|-------------------------|
|             | MSE ↓ | LPIPS ↓ | Similarity ↑ | NSCD ↓ | LEC\(_{pose}\) ↓ | LEC\(_{smile}\) ↓ | LEC\(_{age}\) ↓ |
| 0           | 0.0369 | 0.1657 | 0.5512 | 0.0736 | 24.8245 | 22.5007 | 24.8069 |
| 0.1         | 0.0382 | 0.1703 | 0.5438 | 0.0416 | 19.1594 | 14.0133 | 15.2246 |
| 0.25        | 0.0391 | 0.1737 | 0.5410 | 0.0395 | 19.1345 | 14.1382 | 15.1599 |
| **0.5**     | 0.0397 | 0.1766 | 0.5305 | 0.0385 | 19.0211 | 14.0360 | 14.6715 |
| 0.75        | 0.0406 | 0.1792 | 0.5222 | 0.0381 | 19.0949 | 14.0128 | 14.3198 |
| 1.0         | 0.0413 | 0.1809 | 0.5168 | 0.0378 | 15.8013 | 13.8433 | 14.6084 |

Figure 8: **Ablation study of image perception.** We show the inversion result from LSAP\(_E\) and the same encoder without \(\mathcal{L}_{NSCD}\) to illustrate the effect. LSAP\(_E\) significantly improves the image quality and solves the unnatural generation.

are shown in Table 4. A higher value of \(\lambda\) makes image distortion increase. This result is in line with our expectations since \(\lambda\) controls the contributions of alignment loss. Conversely, perception and editability are improved as \(\lambda\) increased. We visualize the inversion results in Figure 8 with \(\lambda = 0\) and 0.5. In the first row, quality of teeth, eyes and lip’s texture is weak. For example, in the left image in first row, the end of the left eyelid (right in the figure) is located too far from the left eye. Besides, teeth is misaligned with adhesions and lips are too smooth without normal texture. These problems are solved by LSAP, as we can see in the second row. To demonstrate change in editability, we fur-
Figure 9: **Ablation study of image editability.** We show the manipulation result from LSAP_E with different hyper-parameter \( \lambda \).

Further compare the manipulation results with \( \lambda = 0, 0.5, 1.0 \), which is shown in Figure 9. The first two images are edited with "smile" while the third is edited with "pose". When \( \lambda = 0 \), the edited images are unphotorealistic, and glasses occur with editing "smile". Results of \( \lambda = 0.5 \) and \( 1.0 \) show the similar results with excellent editability. The inversion and editing results show the superiority of our alignment paradigm.

**D. Image Perception**

We illustrate the discrepancy of image perception from each inversion method by high-resolution inverse images. As can be seen in Figure 10, the inverse results from each approach are marginally different in high resolution, especially in hair, teeth, lip, and skin area. This is not obvious in low resolution or thumbnails, as can be seen the first row. However, it makes image unnatural and fake in high resolution. We recommend comparing the visual quality at a higher resolution (e.g., \( 1024 \times 1024 \)). Our alignment paradigm improves the image quality well, as can be seen in Figure 10, our results have natural visual details.
Figure 10: **Illustrate image perception in high resolution results.** We show the inversion results from pSp, e4e, and LSAP$_E$ in high resolution to demonstrate the details of images. We also provide the low-resolution results to compare in the first row. The difference of perception is not obvious in low-resolution images.