Chiral behavior of the heavy meson mixing amplitudes in the standard model and beyond

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Abstract. We have computed the chiral logarithmic corrections to the heavy meson mixing amplitudes in the Standard Model and beyond. The impact of the inclusion of lowest-lying scalar heavy-light states to the decay constants and bag-parameters has been investigated and shown that this does not modify the pion chiral logarithms, but it does produce corrections which are competitive in size with the K- and eta-meson chiral logarithms. The result is highly relevant for the precise determination of the heavy meson bag-parameters from lattice studies since the pion chiral logarithms represent the most important effect in guiding the chiral extrapolations of the lattice data for these quantities.

1. Introduction

A major obstacle in the current lattice calculations of the heavy meson decay constants and bag parameters required by \(B_d; s \rightarrow B_d; s\) oscillation studies is that the \(d\)-quark cannot be reached directly. Instead an extrapolation of the results obtained by working with larger light quark masses down to the physical \(d\)-quark mass is required which induces systematic uncertainties due to spontaneous chiral symmetry breaking effects. Heavy meson chiral perturbation theory (HMChPT) [1] allows us to gain some control over these uncertainties because it predicts the chiral behavior of the hadronic quantities relevant to the heavy-light quark phenomenology which then can be implemented to guide the extrapolation of the lattice results.

HMChPT combines heavy quark effective theory (HQET) with the common pattern of spontaneous breaking of the chiral symmetry, \(SU(3)_L \otimes SU(3)_R \rightarrow SU(3)_V\). Like in the standard ChPT, in HMChPT one computes the chiral logarithmic corrections (the so-called non-analytic terms) which are expected to be relevant to the very low energy region, i.e., \(m_q \ll \Lambda_{\text{QCD}}\). In the heavy-light quark systems the situation becomes more complicated because the first orbital excitations \(\tilde{f}_P^J = 1/2^+\) are not far away from the lowest lying states \(\tilde{f}_L^J = 1/2^-\) as indicated by the the recent experimental evidence for the scalar \(D_{0s}\) and axial \(D_{1s}\) mesons [2, 3, 4] and also as suggested by results of the lattice QCD study in the static heavy quark limit [5]. Since both \(\Delta_{S_s} = m_{D_{0s}} - m_{D_s} = m_{D_{1s}} - m_{D_1}^s\) and \(\Delta_{S_{u,d}}\) are smaller than \(\Lambda_{\chi}, m_\eta,\) and even \(m_K\), the predictions based on HMChPT need to be reevaluated and their range of validity reassessed. We report on the investigation [6] of this issue on the specific examples of the decay constants \(f_{B_{d,s}}\)
and the bag parameters which enter the investigation of the SM and supersymmetric (SUSY) effects in the \( B_{d,s}^0 - \overline{B}_{d,s}^0 \) mixing amplitudes.

2. Chiral logarithmic corrections to decay constants and bag parameters

The SUSY contributions to the \( B_q^0 - \overline{B}_q^0 \) mixing amplitude, where \( q \) stands for either \( d \)- or \( s \)-quark, are usually discussed in the so called SUSY basis of \( \Delta B = 2 \) operators \( O_{1-5} \) [7]. Within SM, only \( O_1 \) (left-left) operator is relevant in describing the \( B_{d,s}^0 - \overline{B}_{d,s}^0 \) mixing amplitude. The matrix elements of these operators are conventionally parameterized in terms of bag-parameters, \( B_{1-5} \), as a measure of the discrepancy with respect to the estimate obtained by using the vacuum saturation approximation (VSA)

\[
\frac{\langle B_q^0 | O_{1-5}(\nu) | B_q^0 \rangle}{\langle B_q^0 | O_{1-5}(\nu) | B_q^0 \rangle_{\text{VSA}}} = B_{1-5}(\nu),
\]

where \( \nu \) is the renormalization scale of the logarithmically divergent operators, \( O_i \), at which the separation between the long-distance (matrix elements) and short-distance (Wilson coefficients) physics is made.

We use HMChPT to describe the low energy behavior of the matrix elements of \( O_{1-5} \). The details of the calculation are described in [6]. We first note that in the exact heavy quark symmetry limit in which we are working, the operator \( O_3 \) can be eliminated from further discussion via relation \( \langle B_q^0 | \tilde{O}_3 + \tilde{O}_2 + \frac{1}{2} \tilde{O}_1 | B_q^0 \rangle = 0 \), where the tilde is used to stress that the operators are now being considered in the static limit \((|\nu| = 0)\). Furthermore, in the same limit \( \langle 0 | [\tilde{c}_\mu \gamma_5 q] | B_q^0 (\nu) \rangle_{\text{HQET}} = \hat{f}_q v_{\mu} \), where \( \hat{f}_q \) is the decay constant of the static \( 1/2^- \) heavy-light meson, and the states are normalized as \( \langle B_q^0 (\nu) | B_q^0 (\nu') \rangle = \delta (\nu - \nu') \). In a similar way we define the \( \hat{f}_q^+ \) decay constant of the static \( 1/2^+ \) heavy-light meson. Finally we notice that the operators \( \tilde{O}_4 \) and \( \tilde{O}_5 \) differ only in the color indices, i.e., by a gluon exchange, which is a local effect that cannot influence the long distance behavior described by ChPT [8]. Thus, in the static heavy quark limit \((m_Q \to \infty)\), we are left with only three operators \( \tilde{O}_1, \tilde{O}_2, \tilde{O}_4 \) whose bosonised versions for \( 1/2^- \) external states have also been determined in [9]. We first consider chiral loop corrections to the decay constants of pseudoscalar \( (1/2^-) \) heavy-light mesons. Scalar \( (1/2^+) \) mesons propagating in the loops induce \( \Delta_S \) dependent contributions to the corresponding loop integrals. Calculation of these contributions shows, that in the chiral limit \( m_\pi/\Delta_S \to 0 \), all leading order corrections due to \( 1/2^+ \) mesons are analytic in \( m_\pi \). On the other hand, kaon and eta logarithms are competitive in size with new terms proportional to \( \Delta_S^2 \log(4\Delta_S^2/\mu^2) \), where \( \mu \) is the renormalization scale. The relevant chiral logarithmic corrections are then those coming from the \( SU(2)_L \otimes SU(2)_R \to SU(2)_V \) theory below the \( \Delta_S \) scale:

\[
\hat{f}_q = \alpha \left[ 1 - \frac{1 + 3g^2}{2(4\pi f)^2} \frac{3}{2} m_\pi^2 \log \frac{m_\pi^2}{\mu^2} + c_f(\mu) m_\pi^2 \right],
\]

where \( g \) is the effective HMChPT coupling, \( f \) is the pion decay constant, while \( c_f(\mu) \) stands for the relevant counterterm contributions. At this point we also note that we checked that the chiral logarithms in the scalar heavy-light meson decay constant \( \hat{f}_q^+ \) are the same as for the pseudoscalar meson, with the coupling \( g \) being replaced by \( \tilde{g} \). We then calculate chiral loop corrections to the bag parameters \( B_{1q}, B_{2q} \) and \( B_{1q} \). In our calculation, all factorisable contributions can be absorbed into the HMChPT bag-parameter and decay constant definitions, leaving us with the non-factorisable loops. Again scalar \( 1/2^+ \) mesons propagating in the loops induce \( \Delta_S \) contributions to the corresponding loop integrals. Here however, in the chiral limit the two scales \((m_\pi \text{ and } \Delta_S)\) do not decouple as in the case of the heavy meson decay constants.
Therefore we attempt an expansion of the loop integrals in powers of $1/\Delta_S$, as described in [10]. This corresponds to an expansion around the decoupling limit of the positive parity states. One obtains a series of local operators with $\Delta_{SH}$ dependent prefactors – effective counterterms of a theory with no positive parity mesons. Like for the decay constants, the relevant chiral expansion of the bag-parameters turns out to be the one derived in the $SU(2)_L \otimes SU(2)_R \rightarrow SU(2)_V$ theory:

$$\widetilde{B}_{1q} = \widetilde{B}_{1q}^{\text{Tree}} - \frac{1 - 3g^2}{2(4\pi f)^2} m^2 \log \frac{m^2}{\mu^2} + c_{B1}(\mu)m^2,$$

and

$$\widetilde{B}_{24q} = \widetilde{B}_{24q}^{\text{Tree}} + \frac{3g^2Y + 1}{2(4\pi f)^2} m^2 \log \frac{m^2}{\mu^2} + c_{B24}(\mu)m^2.$$

Here $Y$ is the ratio of $\widetilde{B}_{24}$ bag parameters with pseudoscalar or vector external states. It can differ from unity even in the exact heavy quark symmetry limit as explained in [9].

3. Conclusions and Perspectives

We have revisited the computation of the $B_{s,d} - \overline{B}_{s,d}$ mixing amplitudes in the framework of HMChPT. We have considered chiral logarithmic corrections to the SM as well as SUSY bag parameters and have studied the impact of the nearest scalar mesons to the predictions derived in HMChPT in which these contributions were previously ignored. We find that their contributions are competitive in size and thus they cannot be ignored nor separated from the discussion of the kaon and/or $\eta$-meson loops. However, they do not spoil the pion logarithmic corrections to the decay constants and bag-parameters. The formulae derived in HMChPT can thus still (and should) be used to guide the chiral extrapolations of the lattice results, albeit for the pion masses lighter than $\Delta_S$. As a side-result we have confirmed that the chiral logarithmic corrections to the scalar meson decay constants are the same as for the pseudoscalar ones, modulo replacement $g \rightarrow \tilde{g}$. Similar conclusions regarding the impact of the $1/2^+$-mesons on leading chiral logarithms have been reached in other processes such as the effective HMChPT couplings between heavy and light mesons [10] as well as Isgur-Wise functions in semileptonic $B$ to $D$ meson decays [11].

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