A first-class approach of higher derivative
Maxwell-Chern-Simons Proca model

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Abstract

The equivalence from the point of view of the Hamiltonian path integral quantization between a higher derivative extension of Maxwell-Chern-Simons Proca model and some gauge invariant theories is investigated in the framework of gauge unfixing approach. The Hamiltonian path integrals of the first-class systems take manifestly Lorentz-covariant forms.

1 Introduction

The quantization of a second-class constrained system can be achieved by the reformulation of the original theory as a first-class one and then quantizing the resulting first-class theory. This quantization procedure was applied to various models [1–16] using a variety of methods to replace the original second-class model to an equivalent model in which only first-class constraints appear. The conversion of the original second-class system into equivalent theories which have gauge invariance can be accomplished without enlarging the phase space starting from the possibility of interpreting a second-class constraints set as resulting from a gauge-fixing procedure of a first-class constraints one and "undo" gauge-fixing [17–21]. Another method to construct the equivalent first-class theory relies on an appropriate extension of the original phase space through the introduction of some new variables. The first-class constraint set and the first-class Hamiltonian are constructed as power series in the new variables [22–24].

Various aspects of the equivalence [25] between self-dual model [26] and Maxwell-Chern-Simons (MCS) [27] have been studied using one of the two methods mentioned in the above [16,28,29]. In [30,31] one finds generalizations that involve both the Maxwell and Chern-Simons terms [25–27]. In the first part of this paper we investigate the MCS-Proca model [31–34].

\[ S = \int d^3x \left( -\frac{1}{4} \partial_{[\mu} A_{\nu]} \partial^{[\mu} A^{\nu]} - b \varepsilon_{\mu \nu \rho} A^\mu \partial^\nu A^\rho - \frac{m^2}{2} A_\mu A^\mu \right), \]  

(1)

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from the point of view of the Hamiltonian quantization using the gauge-unfixing (GU) approach. A generalization of the Proca action for a massive vector field with derivative self-interactions in \( D = 4 \) has been constructed in \([35]\). In the second part we consider a higher order extension of the MCS-Proca model (which contains higher-order derivatives involving CS term) described by Lagrangian action

\[
S = \int d^3x \left[ -\frac{1}{4} \partial_{\mu} A_{\nu} \partial^{\mu} A^{\nu} + \frac{1}{2} \varepsilon_{\mu\nu\rho} \left( \partial_{\lambda} \partial^{\lambda} A^{\mu} \right) \partial^{\nu} A^{\rho} - \frac{1}{2} A_{\mu} A^{\mu} \right].
\]

and we apply the same quantization procedure.

The paper is organized in four sections. In section 2 starting from MCS-Proca model we construct an equivalent first-class model using gauge-unfixing method and meanwhile we obtain the path integral corresponding to the first-class system associated with this model. The gauge-unfixing method relies on separating the second-class constraints into two subsets, one of them being first-class and the other one providing some canonical gauge conditions for the first-class subset. Starting from the canonical Hamiltonian of the original second-class system, we construct a first-class Hamiltonian with respect to the first-class subset through an operator that projects any smooth function defined on the phase space into an application that is in strong involution with the first-class subset. In section 3 we exemplify in detail the gauge-unfixing method on extended MCS-Proca (MECS-Proca) model and construct the path integral of the equivalent first-class systems associated with this second-class theory. Section 4 ends the paper with the main conclusions.

2 The MCS-Proca model

The MCS-Proca model is described by the Lagrangian action \((1)\) where \(a\) and \(b\) are some real constants. We work with the Minkowski metric tensor of ‘mostly minus’ signature \(\sigma_{\mu\nu} = \sigma^{\mu\nu} = diag(+−−)\). The canonical analysis \([36, 37]\) of the model described by the Lagrangian action \((1)\) displays the second-class constraints (scc)

\[
\chi^{(1)} \equiv p^0 \approx 0, \quad \chi^{(2)} \equiv \partial^i p_i - b \varepsilon_{0ij} \partial^i A^j - m^2 A_0 \approx 0,
\]

and the canonical Hamiltonian

\[
H_c = \int d^2x \left( -\frac{\partial^i p^i}{2} - A_0 \partial_i \pi^i + \frac{\partial^i A_j}{2} \partial^j A^i \right)
+ b \varepsilon_{0ij} \partial^i A^j + \frac{b}{2} \varepsilon_{0ij} \partial^i A^j p^j - \frac{m^2}{2} A_i A^i + \frac{m^2}{2} A_{\mu} A^{\mu},
\]

where \(p^\mu\) are the canonical momenta conjugated with the fields \(A_\mu\). The number of physical degrees of freedom \([19]\) of the orginal system is equal to

\[
\mathcal{N}_O = (6 \text{ canonical variables} − 2 \text{ scc}) / 2
\]
The same result with respect to the number of degrees of freedom is obtained in [33]. Moreover, in [33] it is shown that the MCS-Proca model describes a topological mass mix with two massive degrees of freedom, with masses $\sqrt{b^2 + m^2} \pm |b|$.

According to the GU method we consider (4) as the first-class constraint (fcc) set and the remaining constraints (3) as the corresponding canonical gauge conditions. Further we redefine the first-class constraint as

$$G \equiv -\frac{1}{m^2} \left( \partial^i p_i - b \varepsilon_{0ij} \partial^i A^j - m^2 A_0 \right) \approx 0.$$  (7)

The other choice, considering (3) as the first-class constraint set and the remaining constraints (4) as the corresponding canonical gauge conditions, yields a path integral that cannot be written (after integrating out auxiliary variables) in a manifestly covariant form [10, 13]. The next step of the GU approach is represented by the construction of a first-class Hamiltonian with respect to (7). The first-class Hamiltonian $H_{GU}$ is given by

$$H_{GU} = H_c - \chi(1) [G, H_c] + \frac{1}{2} \chi(1) \chi(1) [G, [G, H_c]] - \cdots.$$  (8)

Performing the canonical transformation

$$A_0 \to \frac{1}{m} p, \quad p^0 \to -m \phi,$$  (9)

the constraint (10) becomes

$$G \equiv -\frac{1}{m^2} \left( \partial^i p_i - b \varepsilon_{0ij} \partial^i A^j - m^2 A_0 \right) \approx 0,$$  (10)

and the first-class Hamiltonian (8) takes the form

$$H_{GU} = \int d^2 x \left[ -\frac{1}{2a} p_ip^i + \frac{1}{2} \partial_i A_j \partial^i A^j + \frac{b}{a} \varepsilon_{0ij} A^i p^j 
- \frac{b^2}{2a} A_i A^i + \frac{1}{2} (\partial_i \varphi - mA_i) (\partial^i \varphi - mA^i) 
- \frac{1}{m} \left( \partial_i p^i - b \varepsilon_{0ij} \partial^j A^i - m^2 A_0 \right) \right].$$  (11)

The number of physical degrees of freedom of the GU system is equal to

$$N_{GU} = \frac{(6 \text{ canonical variables} - 2 \times 1 \text{ fcc})}{2} = \frac{2}{2} = N_O.$$  (12)

The original second-class theory and respectively the gauge-unfixed system are classically equivalent since they possess the same number of physical and, moreover, the corresponding algebras of classical observables are isomorphic. Consequently, the two systems become equivalent at the level of the path integral.
quantization, which allows us to replace the Hamiltonian path integral of the MCS-Proca model with that of the gauge-unfixed first-class system.

\[
Z_{GU} = \int D(A, \varphi, p^i, p, \lambda) \mu([A], [\varphi]) \exp \left\{ \frac{1}{2} \int d^3x \left[ \left( \partial_0 A_i - \partial_i A_0 - \frac{1}{2} \partial_0^2 A^i - \partial^i \varphi \right) p^i \right. \right.
\]
\[
- \frac{1}{2} \lambda \left( \partial^i p_i - b \varepsilon_{0ij} \partial^j A^i \right) \right\} ,
\]
where the integration measure \( \mu([A], [\varphi]) \) associated with the model subject to the first-class constraint \( [10] \) includes some suitable canonical gauge conditions and it is chosen such that \( [13] \) is convergent \( [38] \).

Performing in the path integral the transformation
\[
p \rightarrow p, \quad \lambda \rightarrow \bar{\lambda} = \frac{1}{m} \lambda + \frac{1}{m} p
\]
and partial integrations over all momenta \( \{p^i, p\} \), the argument of the exponential becomes
\[
S_{GU} = \int d^3x \left[ -\frac{1}{2} \partial_i A_j \partial^i A^j - \frac{1}{2} (\partial_0 A_i - \partial_i A_0) (\partial^0 A^i - \partial^i \varphi) + \varphi - b \varepsilon_{0ij} \partial^j A^i - b \varepsilon_{0ij} A^i \partial^j A^0 - \frac{1}{2} \partial_0 (\partial^i \varphi - m A^i) - \frac{1}{2} \partial_0 (\partial^0 \varphi - m \bar{A}) \right]
\]
In terms of the notations \( \bar{A}_\mu \equiv \{ \bar{A}_0, A_i \} \), the last functional reads as
\[
S_{GU} = \int d^3x \left[ -\frac{1}{2} \partial_\mu A_\nu \partial^\mu A^\nu - b \varepsilon_{\mu\nu\rho} A^\rho \partial^\nu A^\mu - \frac{1}{4} \partial_\mu (\partial^\nu \varphi - m A^\nu) \right]
\]
The functional \( [18] \) associated with the first-class system takes a manifestly Lorentz covariant form and describes the (Lagrangian) Stückelberg coupling between the scalar field \( \varphi \) and the 1-form \( A_\mu \) \( [39] \).

The MCS-Proca model can be correlated to another first-class theory whose field spectrum comprise two types of 1-form gauge fields. For this purpose we consider the following fields/momenta combinations
\[
P_i \equiv p_i + b \varepsilon_{0ij} A^j, \quad \mathcal{P}_i \equiv A_i + \frac{1}{m} \partial_i p_0
\]
\[
F_i \equiv A_i + \frac{1}{m} \partial_i p_0
\]
\[ F_0 \equiv A_0, \tag{21} \]

which are in (strong) involution with the first-class constraint (7)
\[ [P_i, G] = [F_i, G] = [F_0, G] = 0. \tag{22} \]

We observe that the first-class Hamiltonian (8) can be written in terms of these gauge invariant quantities
\[
H_{GU} = \int d^2x \left[ -\frac{1}{m^2} P_i P^i + \frac{a}{2} \partial_i [F_j] \partial^j [F^i] \\
+ \frac{m^2}{2} F_i F^i - \frac{m^2}{2} F_0 F^0 + m^2 F_0 G \right]. \tag{23} \]

By direct computation we find that 
\[ F_\mu \equiv \{F_0, F_i\} \] is divergenceless
\[ \partial_\mu F_\mu = 0, \tag{24} \]

and satisfy the equations
\[
\partial^\nu \partial_\nu F_0 = \frac{1}{a} \partial^i P_i + O(G), \tag{25} \\
\partial^\nu \partial_\nu F_i = \frac{m^2}{a} F_i + \frac{2b}{a} \varepsilon_{0ij} P^j + O(G). \tag{26} \]

Enlarging the phase space by adding the bosonic fields/momenta \( \{V^\mu, P_\mu\} \), we can write the solution to (24) as
\[ F_\mu = -\frac{1}{m} \varepsilon_{\mu \nu \rho} \partial^\nu V^\rho. \tag{27} \]

When we replace (27) in (7) the constraint takes the form
\[
-\frac{1}{m^2} \left( \partial^i p_i - b \varepsilon_{0ij} \partial^j A^i + m \varepsilon_{0ij} \partial^i V^j \right) \approx 0, \tag{28} \]

and remains first-class. From the gauge transformation of the quantity \( \partial_i p_0 \) we obtain that
\[ \partial_i p_0 = m \varepsilon_{0ij} P^j. \tag{29} \]

In this moment we have a dynamical system with phase space locally parametrized by \( \{A_i, p^i, V^\mu, P_\mu\} \), subject to the first-class constraints (28) and too many degrees of freedom
\[
\mathcal{N}_{GU}^\prime = (10 \text{ canonical variables} - 2 \times 1 \text{ fcc}) / 2 \\
= 4 \neq \mathcal{N}_0. \tag{30} \]

In order to cut the two extra degrees of freedom, we add to the first-class constraint set (28) two new first-class constraints
\[ P^0 \approx 0, \quad -\partial_i P^i \approx 0, \tag{31} \]

and we obtain a first-class system with a right number of physical degrees of freedom
\[
\mathcal{N}_{GU}^\prime = (10 \text{ canonical variables} - 2 \times 3 \text{ fcc}) / 2 \\
= 4 = \mathcal{N}_0. \tag{32} \]
\[ = 2 = N_O. \] (32)

Using (27) and (29) in (8), we obtain for the first-class Hamiltonian the following form

\[
H'_{GU} = \int d^2x \left[ -\frac{\mu}{2} (p_i + b\varepsilon_{0ij} A^j) (p^i + b\varepsilon^{0ik} A_k) + \frac{\mu}{2} \partial_i A_j \partial^i A^j \right] + \frac{m^2}{2} \left( A_i + \frac{1}{m} \varepsilon_{0ij} P^j \right) \left( A_i + \frac{1}{m} \varepsilon^{0ik} P_k \right) - \frac{1}{\mu^2} \partial_i V_j \partial^i V^j + \frac{1}{2m} \varepsilon_{0ij} \partial^i V_j \left( \partial_k p^k - b \varepsilon_{0kl} \partial^k A^l + m \varepsilon_{0kl} \partial^k V^l \right). \] (33)

For the first-class system endorsed with the phase space parameterized by \{A_i, p^\mu\} and subject to the first-class constraints (4), the fundamental classical observables read as \{A_i + \frac{1}{m} \varepsilon_{0ij} \partial_j \phi_0, A_0, p_i + b \varepsilon_{0ij} A^j\}, and for that with the phase space parameterized by \{A_i, p^\mu, V^\mu, P_\mu\} and subject to the first-class constraints (28) and (31), the fundamental classical observables are \{A_i + \frac{1}{m} \varepsilon_{0ij} P^j, -\frac{1}{m} \varepsilon_{0ij} \partial_j V^j, p_i + b \varepsilon_{0ij} A^j\}. As the number of physical degrees of freedom is the same for both theories and we can identify a set of fundamental classical observables (such that they are in one to one correspondence and they possess the same Poisson brackets) the first-class theories are equivalent. As a result, the GU and the first-class systems remain equivalent also at the level of the Hamiltonian path integral quantization. This further implies that the first-class system is completely equivalent with the original second-class theory. Due to this equivalence we can replace the Hamiltonian path integral of MCS-Proca model with that one associated with the first-class system

\[
Z' = \int D \left( A_i, V^\mu, p^\mu, P_\mu, \lambda, \lambda^{(1)}, \lambda^{(2)} \right) \mu' \left( [A_i], [V^\mu] \right) \exp \left[ i \int d^3x \left( \partial_0 A_i \right) p^i + \left( \partial_0 V^\mu \right) P_\mu - H'_{GU} - \lambda^{(1)} P^0 + \lambda^{(2)} \partial_i P^i + \frac{m}{2} \lambda \left( \partial_i p^i - b \varepsilon_{0ij} \partial^i A^j + m \varepsilon_{0ij} \partial^i V^j \right) \right]. \] (34)

If we perform in path integral the transformation

\[
\lambda \rightarrow \bar{\lambda} = \frac{1}{m^2} \left( \lambda - \frac{m}{2} \varepsilon_{0ij} \partial^i V^j \right) \] (35)

and partial integrations over \{V^0, p_i, P_0, P_i, \lambda^{(1)}\}, the argument of the exponential becomes

\[
S'_{GU} = \int d^3x \left[ -\frac{\mu}{2} \partial_i A_j \partial^i A^j - \frac{\mu}{2} \left( \partial_0 A_i - \partial_i \bar{\lambda} \right) \left( \partial^0 A^i - \partial^i \bar{\lambda} \right) - b \varepsilon_{0ij} \bar{\lambda} \partial^i A^j - b \varepsilon_{0ij} A^i \partial^j \bar{\lambda} + \frac{1}{2} \partial_i V_j \partial^i V^j + \frac{1}{2} \left( \partial_0 V_i - \partial_i \lambda^{(2)} \right) \left( \partial^0 V^i - \partial^i \lambda^{(2)} \right) + m \varepsilon_{0ij} \bar{\lambda} \partial^i V^j + m \varepsilon_{0ij} A^i \partial^0 V^j + m \varepsilon_{ij0} A^i \partial^j \lambda^{(2)} \right]. \] (36)
In terms of the notations
\[ \tilde{\lambda} \equiv \tilde{A}_0, \quad \lambda^{(2)} \equiv \tilde{V}_0, \]
the argument of the exponential takes a manifestly Lorentz covariant form
\[ S'_{GU} = \int d^3x \left( -\frac{i}{2} \partial_{[\mu} \tilde{A}_{\nu]} \partial^{[\mu} \tilde{A}^{\nu]} - b\varepsilon_{\mu\nu\rho} \tilde{A}^\rho \partial^\nu \tilde{A}^\rho \\
+ \frac{i}{4} \partial_{[\mu} \tilde{V}_{\nu]} \partial^{[\mu} \tilde{V}^{\nu]} + m\varepsilon_{\mu\nu\rho} \tilde{A}^\rho \partial^\nu \tilde{V}^\rho \right). \]

where \( \tilde{A}_\mu \equiv \{ \tilde{A}_0, A_i \} \) and \( \tilde{V}_\mu \equiv \{ \tilde{V}_0, V_i \} \). The functional (38) associated with the first-class system describes a Chern-Simons coupling between the two 1-forms \( \tilde{A}_\mu \) and \( \tilde{V}_\mu \). [40]

If we set in (1) \( a = 1 \) and \( b = 0 \), we find Proca model [10] and for \( a = 0 \) and \( b = 1 \) we obtain self-dual model [26]. It has shown [10, 13, 15] that from the point of view of Hamiltonian path integral quantization the Proca model is equivalent with a gauge theory described by

\[ S = \int d^3x \left[ -\frac{1}{4} \partial_{[\mu} \tilde{A}_{\nu]} \partial^{[\mu} \tilde{A}^{\nu]} - \frac{1}{2} (\partial_\mu \varphi - m \tilde{A}_\mu) (\partial^\mu \varphi - m \tilde{A}^\mu) \right], \]

or, for a suitable extension of the phase space, with first-class theory

\[ S = \int d^3x \left[ -\frac{1}{4} \partial_{[\mu} \tilde{A}_{\nu]} \partial^{[\mu} \tilde{A}^{\nu]} + \frac{i}{4} \partial_{[\mu} \tilde{V}_{\nu]} \partial^{[\mu} \tilde{V}^{\nu]} + m\varepsilon_{\mu\nu\rho} \tilde{A}^\rho \partial^\nu \tilde{V}^\rho \right]. \]

In [16] it has proved that the self-dual model is connected to a first-class theory whose exponent of the exponential from Hamiltonian path integral takes a manifestly Lorentz-covariant form

\[ S = \int d^3x \left[ -\varepsilon_{\mu\nu} \tilde{A}^\rho \partial^\nu \tilde{A}^\rho - \frac{1}{2} (\partial_\mu \varphi - m \tilde{A}_\mu) (\partial^\mu \varphi - m \tilde{A}^\mu) \right], \]

or

\[ S = \int d^3x \left[ -\varepsilon_{\mu\nu} \tilde{A}^\rho \partial^\nu \tilde{A}^\rho + \frac{i}{4} \partial_{[\mu} \tilde{V}_{\nu]} \partial^{[\mu} \tilde{V}^{\nu]} + m\varepsilon_{\mu\nu\rho} \tilde{A}^\rho \partial^\nu \tilde{V}^\rho \right]. \]

Setting \( a = 1 \) and \( b = 0 \) in the functionals (18) and (38) we arrive at (39) and respectively (40). In principle, the functional (41) can be obtained from (18) setting \( a = 0 \) and \( b = 1 \), but this is not a possible choice because the parameter \( a \) should be always nonzero. The same remark can be done regarding the functionals (38) and (42).

3 The MECS-Proca model

In order to construct an equivalent first-class system starting from the MECS-Proca model in the framework of the GU approach we need to know the structure of the constraints set of the model. As the second term in the action (2)
contains higher derivative terms \( \{ \partial_\lambda \partial^\lambda A_\mu \} \), the canonical analysis will be done by a variant of Ostrogradskii method [41–45] developed in [46] based on a formalism we define the variables new fields to account for higher derivative terms. In the equivalent first order formalism [47] and applied to a number of particle and field theoretic models [48–51]. The method consists in converting the original higher derivative theory to an equivalent first order theory by introducing new fields to account for higher derivative terms. In the equivalent first order formalism we define the variables \( B_\mu \) as

\[
B_\mu = \partial_0 A_\mu, \tag{43}
\]

and enforce the Lagrangian constraints

\[
B_\mu - \partial_0 A_\mu = 0, \tag{44}
\]

by Lagrange multiplier \( \xi^\mu \)

\[
\mathcal{L} = -\frac{1}{4} \partial_i A_{ij} \partial^i A^j - \frac{1}{2} (B_i - \partial_i A_0) (B^i - \partial^i A^0) \\
\quad + \frac{1}{8} \varepsilon_{0ij} (\partial_0 B^i + \partial_i \partial^k A^0) \partial^j A^0 + \frac{1}{2} \varepsilon_{ij0} (\partial_0 B^i + \partial_i \partial^k A^0) B^j \\
\quad + \frac{1}{2} \varepsilon_{ij0} (\partial_0 B^i + \partial_i \partial^k A^0) \partial^j A^0 - \frac{1}{2} A_\mu A^\mu + \xi^\mu (B_\mu - \partial_0 A_\mu). \tag{45}
\]

The canonical analysis of the model described by the first order Lagrangian [45] displays the irreducible second-class constraints

\[
\Phi(\xi) = \Pi_\mu \approx 0, \tag{46}
\]

\[
\Phi^{(A)\mu} = p^\mu + \xi^\mu \approx 0, \tag{47}
\]

\[
\Phi_1^{(B)} = \pi_i + \frac{1}{2} \varepsilon_{0ij} B^j - \frac{1}{2} \varepsilon_{0ij} \partial^i A^0 - \frac{1}{2} \varepsilon_{0ij} \partial^j \Pi^0 \approx 0, \tag{48}
\]

\[
\Phi_1^{(B)} = \pi_0 - \frac{1}{2} \varepsilon_{0ij} \partial^i A^j - \frac{1}{2} \varepsilon_{0ij} \partial^j \Pi^0 \approx 0, \tag{49}
\]

\[
\Phi_{111}^{(B)} = \epsilon_0 - \frac{1}{2} \varepsilon_{0ij} \partial^j B^i \approx 0, \tag{50}
\]

\[
\Phi_{111}^{(B)} = \partial_i \epsilon^i + A_0 - \frac{1}{2} \varepsilon_{0ij} \partial_i \partial^k (\partial^j A^j) \approx 0, \tag{51}
\]

\[
\Phi_{111}^{(B)} = \partial_i A_0 - \frac{1}{2} \varepsilon_{0ij} A^0 \partial_i A^j \approx 0, \tag{52}
\]

and the canonical Hamiltonian

\[
H_c = \int d^2 x \left[ \frac{1}{2} \partial_i A_{ij} \partial^i A^j + \frac{1}{2} (B_i - \partial_i A_0) (B^i - \partial^i A^0) \\
\quad - \frac{1}{2} \varepsilon_{0ij} (\partial_0 \partial^k A^0) \partial^i A^j - \frac{1}{2} \varepsilon_{0ij} (\partial_0 \partial^k A^i) B^j \\
\quad - \frac{1}{2} \varepsilon_{ij0} (\partial_0 \partial^k A^i) \partial^j A^0 - \xi^\mu B_\mu + \frac{1}{2} A_\mu A^\mu \right], \tag{53}
\]

where \( \{ p^\mu, \pi^\mu, \Pi_\mu \} \) are the canonical momenta conjugated with the fields \( \{ A_\mu, B_\mu, \xi^\mu \} \). The nonzero Poisson brackets among the constraints functions read as

\[
[\Phi(\xi)(x), \Phi^{(A)\mu}(y)]_{x_0 = \gamma_0} = -\delta_\mu^\nu \delta^2 (x - y), \tag{54}
\]

\[
[\Phi(\xi)(x), \Phi_{111}^{(B)}(y)]_{x_0 = \gamma_0} = -\delta_\mu^0 \delta^2 (x - y), \tag{55}
\]

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The nontrivial Poisson brackets between the constraints functions are listed bellow

$$\left[ \Phi^{(E)}_\mu (x) , \Phi^{(B)}_{III} (y) \right]_{x_0 = y_0} = \delta^\mu_\nu \partial_\nu \delta^2 (x - y), \quad (56)$$

$$\left[ \Phi^{(A)\mu} (x) , \Phi^{(B)}_{III} (y) \right]_{x_0 = y_0} = \left( -\delta^\mu_0 + \frac{1}{2} \delta^\mu_i \varepsilon^{0ij} \partial_k \partial_j \right) \delta^2 (x - y), \quad (57)$$

$$\left[ \Phi^{(A)\mu} (x) , \Phi^{(B)}_{IV} (y) \right]_{x_0 = y_0} = \delta^\mu_i \partial^i \delta^2 (x - y), \quad (58)$$

$$\left[ \Phi^{(B)} (x) , \Phi^{(B)}_{IV} (y) \right]_{x_0 = y_0} = -\delta^2 (x - y), \quad (59)$$

$$\left[ \Phi^{(B)}_i (x) , \Phi^{(B)}_j (y) \right]_{x_0 = y_0} = \varepsilon_{0ij} \delta^2 (x - y), \quad (60)$$

$$\left[ \Phi^{(B)}_i (x) , \Phi^{(B)}_j (y) \right]_{x_0 = y_0} = \varepsilon_{0ij} \partial^j \delta^2 (x - y). \quad (61)$$

The number of physical degrees of freedom of the orginal system is equal to

$$\tilde{N}_O = \frac{(18 \text{ canonical variables} - 12 \text{ scc})}{2} = 3. \quad (62)$$

We notice that the number of physical degrees of freedom of the extended model is higher than the number of physical degrees of freedom of the MCS-Proca model

$$\tilde{N}_O > N_O.$$ 

This result was expected due to the higher derivative nature of the MECS-Proca model.

Imposing the constraints $\Phi^{(E)}_\mu \approx 0$ and $\Phi^{(A)\mu} \approx 0$ strongly zero and eliminating the unphysical sector $\xi^\mu$ and $\Pi^\mu$ (the reduced phase space is locally parametrized by $\{A^\mu, B_\mu, \mu^\mu, \pi^\mu\}$) we arrive at a system subject to the second-class constraints

$$\chi^{(1)}_i \equiv \pi_i + \frac{1}{2} \varepsilon_{0ij} B^j - \frac{1}{2} \varepsilon_{0ij} \partial^j A^0 \approx 0, \quad (63)$$

$$\chi^{(1)} \equiv \pi_0 - \frac{1}{2} \varepsilon_{0ij} \partial^j A^i \approx 0, \quad (64)$$

$$\chi^{(2)} \equiv -p_0 - \frac{1}{2} \varepsilon_{0ij} \partial^j B^i \approx 0, \quad (65)$$

$$\chi^{(3)} \equiv -\partial_i p^i + A_0 - \frac{1}{2} \varepsilon_{0ij} \partial_k \partial^k (\partial^j A^i) \approx 0, \quad (66)$$

$$\chi^{(4)} \equiv \partial_i A^i + B_0 \approx 0. \quad (67)$$

The nontrivial Poisson brackets between the constraints functions are listed bellow

$$\left[ \chi^{(1)}_i (x) , \chi^{(1)}_j (y) \right]_{x_0 = y_0} = \varepsilon_{0ij} \delta^2 (x - y), \quad (68)$$

$$\left[ \chi^{(1)}_i (x) , \chi^{(2)}_i (y) \right]_{x_0 = y_0} = \varepsilon_{0ij} \partial^j \delta^2 (x - y), \quad (69)$$

$$\left[ \chi^{(1)}_i (x) , \chi^{(4)}_i (y) \right]_{x_0 = y_0} = -\delta^2 (x - y), \quad (70)$$

$$\left[ \chi^{(2)}_i (x) , \chi^{(3)}_i (y) \right]_{x_0 = y_0} = \delta^2 (x - y), \quad (71)$$
\[
\left[ \chi^3(x), \chi^4(y) \right]_{x_0=y_0} = -\partial_k \partial^k \delta^2(x-y), \tag{72}
\]
while the canonical Hamiltonian takes the form
\[
H_c = \int d^2 x \left[ \frac{1}{4} \partial_i A_{ij} \partial^i A^j + \frac{1}{2} (B_i - \partial_i A_0) (B^i - \partial^i A^0) - \frac{1}{2} \varepsilon_{0ij} (\partial_k \partial^k A^i) \partial^j A^j - \frac{1}{2} \varepsilon_{ij0} (\partial_k \partial^k A^i) B^j - \frac{1}{2} \varepsilon_{ij0} (\partial_k \partial^k A^i) B^j + p^\mu B_\mu + \frac{1}{2} A_\mu A^\mu \right]. \tag{73}
\]
If we make the following linear combination of the constraints \( \chi^{(2)} \approx 0 \) and \( \chi^{(1)} \approx 0 \)
\[
\chi^{(2)} \rightarrow \chi^{(2)} + \partial^i \chi^{(1)}_i, \tag{74}
\]
the matrix of the Poisson bracket among the constraint functions becomes
\[
C_{\alpha\beta} = \begin{pmatrix}
\varepsilon_{0ij} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & -1 & 0 & -\partial_k \partial^k \\
0 & 1 & 0 & \partial_k \partial^k & 0
\end{pmatrix}. \tag{75}
\]
Analyzing the above matrix we find that the constraints \( \chi^{(1)}_i \approx 0 \) generate a submatrix (of the matrix of the Poisson brackets among the constraint functions) of maximum rank, therefore they form a subset of irreducible second-class constraints. Thus in the sequel we examine from the point of view of the GU method only the constraints \( \chi_A \equiv \{ \chi^{(1)}, \chi^{(2)}, \chi^{(3)}, \chi^{(4)} \} \approx 0 \).

### 3.1 Stückelberg-like coupling

The second-class constraints set \( \chi_A \approx 0 \) cannot be straightforwardly separated in two subsets such that one of them being first-class and the other providing some canonical gauge conditions for the first-class subset. To make this possible we write the constraints set in an equivalent form
\[
\chi'_A = E_{AB} \chi_B, \tag{76}
\]
where \( E_{AB} \) is an invertible matrix
\[
E_{AB} = \begin{pmatrix}
\partial_k \partial^k & 0 & -1 & 0 \\
0 & 1 & 0 & 0 \\
-1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}, \tag{77}
\]
The concrete form of the constraints \( \chi'_A \approx 0 \) is
\[
\chi'^{(1)}_i = \partial_i p^i - A_0 + \partial_k \partial^k \pi_0 \approx 0, \tag{78}
\]
\[
\chi'^{(2)}_i = -p_0 + \partial_i \pi^i \approx 0, \tag{79}
\]
\[ \chi^{(3)} = -\pi_0 + \frac{1}{2} \varepsilon_{0ij} \partial^j A^i \approx 0, \quad \chi^{(4)} = \partial_i A^i + B_0 \approx 0, \]

with the matrix of the Poisson brackets among the constraint functions expressed by

\[ C_{AB} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}. \]

According to the GU method we consider \( G_a \equiv \{ \chi^{(1)}, \chi^{(3)} \} \approx 0 \) as the first-class constraints set and the remaining constraints \( C_a \equiv \{ \chi^{(2)}, \chi^{(4)} \} \approx 0 \) as the corresponding canonical gauge conditions. In [18] it has proved that for a dynamical system subject to the second-class constraints \( \{ \chi_{\alpha_0} \approx 0 \}_{\alpha_0=1,2M_0} \), the subsets \( \{ \chi_1, \chi_2, \ldots, \chi_{M_0} \} \) and \( \{ \chi_1, \chi_2, \ldots, \chi_{M_0-1}, \chi_{M_0+1} \} \) of the full set of constraints are first-class sets on \( \Sigma_{2M_0} \). Our choice corresponds to the second first-class subset.

Starting from the canonical Hamiltonian of the original second-class system we construct a first-class Hamiltonian with respect to the first-class subset in two steps [20]. First, we construct the first-class Hamiltonian with respect to the constraint \( G_1 \approx 0 \)

\[ H_{GU}^1 = H_c - C_1 \{ G_1, H_c \} + \frac{1}{2} C_1 C_1 \{ G_1, H_c \} - \cdots \]

\[ = H_c + \int d^2 x \left[ \frac{1}{2} \partial_k \left( p_0 + \frac{1}{2} \varepsilon_{0ij} \partial^j B^i \right) \partial^k \left( p_0 + \frac{1}{2} \varepsilon_{0lm} \partial^l B^m \right) \right. \]

\[- \left( p_0 + \frac{1}{2} \varepsilon_{0ij} \partial^j B^i \right) \left( \partial_k A^k + B_0 \right) \],

and then with this at hand we obtain the first-class Hamiltonian with respect to the constraint \( G_2 \approx 0 \)

\[ H_{GU} = H_{GU}^1 - C_2 \{ G_2, H_{GU}^1 \} + \frac{1}{2} C_2 C_2 \{ G_2, H_{GU}^1 \} - \cdots \]

\[ = \int d^2 x \left\{ \frac{1}{2} \partial_i \left( A_j \partial^i A^j \right) + \frac{1}{2} \left( B_i - \partial_i A_0 \right) \left( B^i - \partial^i A^0 \right) \right. \]

\[- \frac{1}{2} \varepsilon_{0ij} \left( \partial_k \partial^k A^0 \right) \partial^i A^j - \frac{1}{2} \varepsilon_{0ij} \left( \partial_k \partial^k A^i \right) B^j \]

\[- \frac{1}{2} \varepsilon_{ijkl} \left( \partial_k \partial^k A^i \right) \partial^j A^0 + p^\mu B_\mu + \frac{1}{2} A_0 A^0 \]

\[+ \frac{1}{2} \left( A_i + \partial_i \left( p_0 + \frac{1}{2} \varepsilon_{0jk} \partial^j B^k \right) \right) \left( A^i + \partial^i \left( p_0 + \frac{1}{2} \varepsilon_{0lm} \partial^l B^m \right) \right) \]

\[- B_0 \left( p_0 + \frac{1}{2} \varepsilon_{0ij} \partial^j B^i \right) \].

The number of physical degrees of freedom of the dynamical system with the phase space locally parametrized by \( \{ A_\mu, B_\mu, p^\mu, \pi^\mu \} \) subject to the second-class constraints \([35]\) and first-class constraints \([18]\) and \([80]\) and whose evolution is governed by the first-class Hamiltonian \([35]\) is equal to

\[ \mathcal{N}_{GU} = \frac{(12 \text{ canonical variables} - 2 \text{ scc} - 2 \times 2 \text{ fcc})}{2} \]

\[ = 3 = \mathcal{N}_G. \]
Based on the equivalence between the first-class system and the original second-class theory, we replace the Hamiltonian path integral of the MECS-Proca model with that of the first-class system. The Hamiltonian path integral for the first-class system constructed in the above reads as

\[
Z = \int D \left( A_\mu, B_\mu, p^\mu, \pi^\mu, \lambda^{(1)}, \lambda^{(2)} \right) \mu \left( [A_\mu], [B_\mu] \right) \det^{1/2} (\varepsilon_{\alpha i j} \delta (x - y)) \\
\times \delta (\pi_i + \frac{1}{2} \varepsilon_{0 i j} B^j - \frac{1}{2} \varepsilon_{0 i j} \partial^j A^0) \exp \left\{ \int d^3 x \left[ (\partial_0 A_0) p^0 + (\partial_0 A_i) p^i + (\partial_0 B_0) \pi^0 \right. \right.
\]
\[
+ (\partial_0 A_i) p^i + (\partial_0 B_0) \pi^0 + (\partial_0 B_i) \pi^i - \mathcal{H}_{GU} \\
- \lambda^{(1)} (\partial_i p^i - A_0 + \partial_k \partial^k \pi_0) - \lambda^{(2)} (-\pi_0 + \frac{1}{2} \varepsilon_{0 i j} \partial^j A^i) \right\},
\]

(86)

where the integration measure \( \mu \left( [A_\mu], [B_\mu] \right) \) includes some suitable canonical gauge conditions.

Performing partial integration over the momenta \( \pi_i \) in the path integral we get to the argument of the exponential in the form

\[
S = \int d^3 x \left\{ (\partial_0 A_0) p^0 + (\partial_0 A_i) p^i + (\partial_0 B_0) \pi^0 \\
- \frac{1}{2} (\partial_0 B_i) \varepsilon^{0 i j} (B_j - \partial_j A_0) - \frac{1}{2} [\partial_i A_j] \partial^i A^j \\
- \frac{1}{2} (B_i - \partial_i A_0) \left( B^i - \partial^i A^0 \right) + \frac{1}{2} \varepsilon_{0 i j} (\partial_i \partial^j A^0) \partial^i A^j \\
+ \frac{1}{2} \varepsilon_{0 i j} (\partial_i \partial^j A^0) B^j + \frac{1}{2} \varepsilon_{i j k} (\partial_i \partial^j A^0) \partial^j A^0 - p^i B_i \\
- \frac{1}{2} A_0 A^0 - \frac{1}{2} \left[ A_i + \partial_i \left( p_0 + \frac{1}{2} \varepsilon_{0 j k} \partial^j B^k \right) \right] \\
\times \left[ A^i + \partial^i \left( p_0 + \frac{1}{2} \varepsilon_{0 i j} \partial^j B^k \right) \right] + \frac{1}{2} B^0 \varepsilon_{0 j k} \partial^j B^k \\
- \lambda^{(1)} (\partial_i p^i - A_0 + \partial_k \partial^k \pi_0) - \lambda^{(2)} (-\pi_0 + \frac{1}{2} \varepsilon_{0 i j} \partial^j A^i) \right\}.
\]

(87)

Integration over \( p^i \) leads to a \( \delta \) function of the form

\[
\delta \left( \partial_0 A_i - B_i + \partial_i \lambda^{(1)} \right)
\]

(88)

which permits calculation of the integral over \( B_i \). Performing partial integration over Lagrange multiplier \( \lambda^{(2)} \) and momentum \( \pi_0 \), the argument of the exponential becomes

\[
S = \int d^3 x \left\{ (\partial_0 A_0) \left( p^0 + \frac{1}{2} \varepsilon^{0 i j} \partial_i \partial_0 A_j \right) \\
- \frac{1}{2} \partial_i A_j \partial^i A^j - \frac{1}{2} \left[ \partial_0 A_i - \partial_i \left( A_0 - \lambda^{(1)} \right) \right] \left[ \partial^0 A^i - \partial^i \left( A^0 - \lambda^{(1)} \right) \right] \\
+ \frac{1}{2} \varepsilon_{0 i j} \partial_0 \partial^i A^0 - \lambda^{(1)} \right\} + \frac{1}{4} \varepsilon_{i j k} \partial_i \partial^j A^0 \partial^k A^0 \\
+ \frac{1}{2} \varepsilon_{i j k} \left[ \partial_i \partial^j A^0 \right] \partial^0 \left( A^0 - \lambda^{(1)} \right) - \frac{1}{2} \left[ A_i + \partial_i \left( p_0 + \frac{1}{2} \varepsilon_{0 j k} \partial^j \partial^k A^0 \right) \right] \\
\times \left[ A^i + \partial^i \left( p_0 + \frac{1}{2} \varepsilon_{0 i j} \partial^j B^k \right) \right] - \frac{1}{2} A_0 A^0 + \lambda^{(1)} A_0 \right\}.
\]

(89)
Making the notation

\[ p^0 + \frac{1}{2} \varepsilon^{0ij} \partial_i \partial_0 A_j \equiv -\varphi, \quad A_0 - \chi^{(1)} \equiv \tilde{A}_0, \]

and integrating over Lagrange multiplier \( \chi^{(1)} \), the argument of the exponential from the Hamiltonian path integral reads as

\[ S = \int d^3x \left[ -\frac{1}{2} \partial_i A_j \partial^{[i} A^{j]} + \frac{1}{2} \partial^0 A^i - \partial^0 \tilde{A}^0 \right] \]

\[ + \frac{1}{2} \varepsilon_{0ij} (\partial_\lambda \partial^\lambda \tilde{A}^0) \partial^i A^j + \frac{1}{2} \varepsilon_{0ij} (\partial_\lambda \partial^\lambda A^i) \partial^0 A^j \]

\[ + \frac{1}{2} \varepsilon_{ij0} (\partial_\lambda \partial^\lambda A^i) \partial^j \tilde{A}^0 \]

\[ - \frac{1}{2} (\partial_i \varphi - A_i) (\partial^i \varphi - A^i) - \frac{1}{2} (\partial_0 \varphi - \tilde{A}_0) (\partial^0 \varphi - \tilde{A}^0) \].

The functional (91) takes a manifestly Lorentz-covariant form

\[ S = \int d^3x \left[ -\frac{1}{2} \partial_\mu \tilde{A}_\nu \partial^{[\mu} \tilde{A}^{\nu]} + \frac{1}{2} \varepsilon_{\mu\nu\rho} (\partial_\lambda \partial^\lambda \tilde{A}^{\mu}) \partial^\nu \tilde{A}^{\rho} \right] \]

\[ - \frac{1}{2} (\partial_\mu \varphi - \tilde{A}_\mu) (\partial^\mu \varphi - \tilde{A}^{\mu}) \].

where \( \tilde{A}_\mu = \{ A_0, A_i \} \) and describes a (Lagrangian) Stückelberg-like coupling between the scalar field \( \varphi \) and the 1-form \( \tilde{A}_\mu \).

### 3.2 Chern-Simons-like coupling

In the sequel we show that the MECS-Proca model may be related to another first-class theory. Starting from the GU system constructed in the above, subject to the second-class constraints (83) and the first-class constraints (78) and (80) and whose evolution is governed by the first-class Hamiltonian (84) we consider the following field combinations

\[ \mathcal{F}_0 \equiv A_0, \quad \mathcal{F}_i \equiv A_i + \partial_i \left( p_0 + \frac{1}{2} \varepsilon_{0jk} \partial^j B^k \right), \]

\[ \mathcal{P}_i \equiv p_i - \frac{1}{2} \varepsilon_{0ij} \partial_k \partial^k A^i - \frac{1}{2} \varepsilon_{0ij} \partial^j B^0, \quad B_i \equiv B_i, \]

which are in (strong) involution with first-class constraints \( G_a \approx 0 \)

\[ [\mathcal{F}_0, G_a] = [\mathcal{F}_i, G_a] = [\mathcal{P}_i, G_a] = [B_i, G_a] = 0, \]

and moreover \( \mathcal{F}_\mu \equiv \{ \mathcal{F}_0, \mathcal{F}_i \} \) is divergenceless

\[ \partial^\mu \mathcal{F}_\mu = 0. \]

Similarly to the case of the MCS-Proca model, the first-class Hamiltonian (84) can be written in terms of these quantities

\[ H_{GU} = \int d^3x \left[ \frac{1}{2} \partial_{[i} \mathcal{F}_{j]} \partial^{[i} \mathcal{F}^{j]} + \frac{1}{2} (B_i - \partial_i \mathcal{F}_0) (B^i - \partial^i \mathcal{F}^0) \right] \]

\[ - \varepsilon_{0ij} (\partial_k \partial^k \mathcal{F}^0) \partial^i \mathcal{F}^j + \frac{1}{2} \mathcal{F}_i \mathcal{F}^i + \frac{1}{2} \mathcal{F}_0 \mathcal{F}^0 + B_i \mathcal{P}_i. \]
In order to write the solution to equation (96) we enlarge the phase space by adding the bosonic fields/momenta \( \{ V_\mu, P_\mu \} \)

\[
\mathcal{F}_\mu = -\varepsilon_{\mu\nu\rho} \partial^\nu V^\rho. \tag{98}
\]

When we replace (98) in (78) the constraint takes the form

\[
\partial_i p^i + \varepsilon_{0ij} \partial^i V^j + \partial_k \partial^k \pi_0 \approx 0, \tag{99}
\]

and remains first-class. Computing the Poisson bracket among the quantity \( \partial_i p^i \) and first-class constraint (78) and the Poisson bracket between \( P_i \) and (99) we obtain that these two quantities are correlated through the relation

\[
\partial_i p^i = \varepsilon_{0ij} P^j. \tag{100}
\]

Using (98) and (100) we write the first-class Hamiltonian as

\[
H'_{GU} = \int d^2x \left\{ \frac{1}{4} \partial_i A_{ij} \partial^i A^j \right\} + \frac{1}{2} \left[ B_i + \partial_i \left( \varepsilon_{0ij} \partial^j V^k \right) \right] \times \left[ B_i + \partial^i \left( \varepsilon^{0lm} \partial_l V_m \right) \right] \partial^j A^j \\
- \frac{1}{2} \varepsilon_{0ij} \left( \partial_i \partial^k A^j \right) B^j + \frac{1}{2} \varepsilon_{0ij} \left( \partial_i \partial^k A^j \right) \partial^j (\varepsilon^{0lm} \partial_l V_m) + p^i B_i \\
+ \frac{1}{2} \varepsilon^{i[V^j]} \partial_i V^j - \frac{1}{2} \varepsilon_{0jk} B^0 \partial^i B^k + \frac{1}{2} \left( A_i + \varepsilon_{0ij} P^j + \frac{1}{2} \varepsilon_{0jk} \partial_i \partial^j B^k \right) \times \left( A^i + \varepsilon^{0il} P_l + \frac{1}{2} \varepsilon^{0lm} \partial^i \partial^j B^m \right) \right\}. \tag{101}
\]

If we count the number of physical degrees of freedom of the system with the phase space locally parametrized by \( \{ A_i, B_\mu, V_\mu, p^i, \pi^\mu, P_\mu \} \) subject to the second-class constraints (63) and first-class constraints (80) and (99) and whose evolution is governed by the first-class Hamiltonian (101), we obtain

\[
\tilde{\mathcal{N}}'_{GU} = (16 \text{ canonical variables} - 2 \text{ scc} - 2 \times 2 \text{ fcc}) / 2 \\
= 5 \neq \mathcal{N}_O. \tag{102}
\]

Imposing the first-class constraints

\[
P_0 \approx 0, \quad -\partial^0 P_i \approx 0, \tag{103}
\]

the number of physical degrees of freedom is conserved

\[
\tilde{\mathcal{N}}'_{GU} = (16 \text{ canonical variables} - 2 \text{ scc} - 2 \times 4 \text{ fcc}) / 2 \\
= 3 = \mathcal{N}_O. \tag{104}
\]

For each first-class theory, derived in the above, we are able to identify a set of fundamental classical observables such that they are in one to one correspondence and they possess the same Poisson brackets. The previously exposed procedure preserves the equivalence between two first-class theories since the number of physical degrees of freedom is the same for both theories and the corresponding algebras of classical observables are isomorphic. As a result, the GU
and the first-class system remain equivalent also at the level of the Hamiltonian path integral quantization. This further implies that the first-class system is completely equivalent with the MECS-Proca model. Due to this equivalence we can replace the Hamiltonian path integral of the MECS-Proca model with that associated with the first-class system.

\[
Z' = \int \mathcal{D} \left( A_i, B_\mu, V^\mu, p^i, \pi^\mu, P_\mu, \lambda^{(1)}, \lambda^{(2)}, \lambda^{(3)}, \lambda^{(4)} \right) \mu ([A_i], [B_\mu], [V^\mu])
\]

\[
\times \det^{1/2} (\varepsilon_{ijkl} (x - y)) \delta \left[ \pi_i + \frac{1}{2} \varepsilon_{0ij} B^j + \frac{1}{2} \varepsilon_{0ij} \partial \bar{\delta}^j \left( \varepsilon^{0kl} \partial_k V_l \right) \right]
\]

\[
\times \exp \left\{ i \int d^3 x \left[ (\partial_0 A_i) p^i + (\partial_0 B_0) \pi^0 + (\partial_0 B_i) \pi^i + (\partial_0 V^0) P_0 \right.ight.
\]

\[
+ \left. (\partial_0 V^i) P_i - \mathcal{H}'_{\text{GU}} - \lambda^{(1)} \left( \partial_i p^i + \varepsilon_{0ij} \partial^j V^i + \partial_0 \partial \bar{\delta}^i \pi_0 \right) \right) - \lambda^{(2)} \left( -\pi_0 + \frac{1}{2} \varepsilon_{0ij} \partial^j A^i \right) - \lambda^{(3)} P_0 + \lambda^{(4)} \partial A_i \right\}. \tag{105}
\]

After a partial integration over the momenta \( \pi_i \) in the path integral the argument of the exponential read as

\[
S'_{\text{GU}} = \int d^3 x \left\{ (\partial_0 A_i) p^i + (\partial_0 B_0) \pi^0 + (\partial_0 V^0) P_0 + (\partial_0 V^i) P_i \right.
\]

\[
+ \frac{1}{2} \left( (\partial_0 B_i) \varepsilon^{0ij} \left[ -B_j - \frac{1}{2} \partial_j \left( \varepsilon^{0kl} \partial_k V_l \right) \right] - \frac{1}{2} \varepsilon_{ijkl} \partial^j \partial^k \right)
\]

\[
\left[ B^i + \partial^i \left( \varepsilon^{0lm} \partial_l V_m \right) \right]
\]

\[
- \frac{1}{2} \varepsilon_{ij0} \partial_0 \partial^j \partial A^i \partial^j \left( \varepsilon^{0lm} \partial_l V_m \right) - \frac{1}{2} \varepsilon_{ij0} \partial_0 \partial^j \partial A^i \partial^j \left( \varepsilon^{0lm} \partial_l V_m \right)
\]

\[
+ \left. \frac{1}{2} \left[ A_i + \varepsilon_{0ij} P^j + \frac{1}{2} \varepsilon_{0jk} \partial_0 \partial^j B^k \right] \right\} \right. \left. \frac{1}{5} \varepsilon_{i0j} \partial^j \right) + \lambda^{(1)} \left( \partial_i p^i + \varepsilon_{0ij} \partial^j V^i + \partial_0 \partial \bar{\delta}^i \pi_0 \right)
\]

\[
- \lambda^{(2)} \left( -\pi_0 + \frac{1}{2} \varepsilon_{0ij} \partial^j A^i \right) - \lambda^{(3)} P_0 + \lambda^{(4)} \partial A_i \right\}. \tag{106}
\]

Integration over \( p^i \) leads to a \( \delta \) function of the form

\[
\delta \left( \partial_0 A_i - B_i + \partial_0 \lambda^{(1)} \right) \tag{107}
\]

which permits calculation of the integral over \( B_i \). Performing partial integration over the field \( V_0 \), momenta \( \{ \pi_0, P_0, P_i \} \) and Lagrange multipliers \( \{ \lambda^{(2)}, \lambda^{(3)} \} \), the argument of the exponential from the Hamiltonian path integral takes the form

\[
S'_{\text{GU}} = \int d^3 x \left\{ -\frac{1}{2} \partial_0 A_j \partial^j A^i \right. - \frac{1}{2} \left[ \partial_0 A_i - \partial_i \left( -\lambda^{(1)} - \varepsilon_{0ij} \partial^j V^k \right) \right]
\]

\[
\times \left[ \partial^0 A^i - \partial^i \left( -\lambda^{(1)} - \varepsilon^{0lm} \partial_l V_m \right) \right]
\]

\[
+ \frac{1}{2} \varepsilon_{0ij} \partial_0 \partial^i \left( -\lambda^{(1)} - \varepsilon^{0kl} \partial_k V_l \right) \right) \partial^j A^i + \frac{1}{2} \varepsilon_{i0j} \partial_0 \partial^i \partial^j A^i \partial^0 A^i
\]

\[
+ \frac{1}{2} \varepsilon_{ij0} \partial_0 \partial \bar{\delta}^i \partial^j \partial^0 A^i \right) + \left( -\lambda^{(1)} - \varepsilon^{0kl} \partial_k V_l \right)
\]
Using the notation

\[-\lambda^{(1)} - \varepsilon_{0jk} \partial^j V^k \equiv \bar{A}_0, \quad \lambda^{(4)} \equiv \bar{V}_0,\]

the argument of the exponential from the Hamiltonian path integral reads as

\[
S'_{GU} = \int d^3 x \left[ -\frac{1}{4} \varepsilon_{ijkl} \partial^i A^j \partial^k \lambda \partial^l \lambda - \frac{1}{2} \left( \partial_0 A_i - \partial_i \bar{A}_0 \right) \left( \partial^0 A^i - \partial^i \bar{A}^0 \right) \right.
\]
\[+ \frac{1}{2} \varepsilon_{0ij} \left( \partial_\lambda \partial^\lambda A^i \right) \partial^0 A^j + \frac{1}{2} \varepsilon_{0ij} \left( \partial_\lambda \partial^\lambda \bar{A}^0 \right) \partial^i A^j \]
\[+ \frac{1}{2} \varepsilon_{ij0} \left( \partial_\lambda \partial^\lambda A^i \right) \partial^0 \bar{A}^0 + \frac{1}{4} \partial_{\lambda i} \partial^{\lambda i} \bar{V}^j \]
\[+ \frac{1}{2} \left( \partial_0 V_i - \partial_i \bar{V}_0 \right) \left( \partial^0 V^i - \partial^i \bar{V}^0 \right) \]
\[+ \varepsilon_{0ij} \left( \partial^0 V^i - \partial^i \bar{V}^0 \right) A^j + \varepsilon_{ij0} \left( \partial^j \bar{V}^i \right) \bar{A}^0].\]

The functional (110) takes a manifestly Lorentz-covariant form

\[
S''_{GU} = \int d^3 x \left[ -\frac{1}{4} \partial_{[\mu} A_{\nu]} \partial^{[\mu} \bar{A}^{\nu]} + \frac{1}{2} \varepsilon_{\mu \nu \rho} \left( \partial_\lambda \partial^\lambda \bar{A}^{\mu} \right) \partial^\nu \bar{A}^\rho \right.
\]
\[+ \frac{1}{4} \partial_{[\mu} \bar{V}_{\nu]} \partial^{[\mu} \bar{V}^{\nu]} + \varepsilon_{\mu \nu \rho} \left( \partial^\mu \bar{V}^\nu \right) \bar{A}^\rho],\]

where \(\bar{A}_\mu = \{\bar{A}_0, A_i\}\) and \(\bar{V}_\mu = \{\bar{V}_0, V_i\}\). The above functional describes a (Lagrangian) Chern-Simons-like coupling between the 1-form \(\bar{A}_\mu\) and the 1-form \(\bar{V}_\mu\).

### 4 Conclusion

In this paper the MCS-Proca model and a higher order derivative extension of it have been analyzed from the point of view of the Hamiltonian path integral quantization, in the framework of gauge-unfixing approach. The first step of this approach is represented by the construction of an equivalent first-class system. In order to construct the equivalent first-class system with MECS-Proca model we realize a partial gauge-unfixing (we maintain the second-class constraints (103)), meanwhile in the case of the MCS-Proca model we accomplish a total gauge-unfixing. The second step involves the construction of the Hamiltonian path integral corresponding to the equivalent first-class system for each model. The construction of the equivalent first-class system does not require an extension of the original phase space. The Hamiltonian path integral of the first-class systems takes a manifestly Lorentz-covariant form after integrating out the auxiliary fields and performing some field redefinitions. In order to obtain a manifestly covariant path integral it is not mandatory to enlarge the
phase space adding extra fields. When we work in the original phase space, we obtain the Lagrangian path integral corresponding to Stückelberg-like coupling between a scalar field and 1-form, while the extension of the phase space leads to the Lagrangian path integral for two kinds of 1-forms with Chern-Simons-like coupling.

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References

[1] T. J. Allen, M. J. Bowick and A. Lahiri, Mod. Phys. Lett. A 6, 559 (1991)
[2] A. S. Vytheeswaran, Ann. Phys. (NY) 236, 297 (1994)
[3] E. B. Park, Y. W. Kim, Y. J. Park, Y. Kim and W. T. Kim, Mod. Phys. Lett. A 10, 1119 (1995)
[4] N. Banerjee, R. Banerjee and S. Ghosh, Ann. Phys. (NY) 241, 237 (1995)
[5] H. Sawayanagi, Mod. Phys. Lett. A 10, 813 (1995)
[6] C. Bizdadea and S. O. Saliu, Phys. Lett. B 368, 202 (1996)
[7] C. Bizdadea, Phys. Rev. D 53, 7138 (1996); J. Phys. A: Math. Gen. 29, 3985 (1996)
[8] N. Banerjee and R. Banerjee, Mod. Phys. Lett. A 11, 1919 (1996)
[9] Y. W. Kim, M. I. Park, Y. J. Park and S. J. Yoon, Int. J. Mod. Phys. A 12, 4217 (1997)
[10] A. S. Vytheeswaran, Int. J. Mod. Phys. A 13, 765 (1998)
[11] S. T. Hong, Y. W. Kim, Y. J. Park and K. D. Rothe, Mod. Phys. Lett. A 17, 435 (2002)
[12] R. Ruegg and M. Ruiz-Altaba, Int. J. Mod. Phys. A 19, 3265 (2004)
[13] E. M. Cioroianu, S. C. Sararu and O. Balus, Int. J. Mod. Phys. A 25, 185 (2010)
[14] E. M. Cioroianu, Mod. Phys. Lett. A 26, 589 (2011)
[15] S. C. Sararu, Int. J. Mod. Phys. A 27, 1250119 (2012)
[16] S. C. Sararu, Int. J. Theor. Phys. 51, 2613 (2012); Cent. Eur. J. Phys. 11, 59 (2013)

[17] K. Harada and H. Mukaida, Z. Phys. C 48, 151 (1990)

[18] P. Mitra and R. Rajaraman, Ann. Phys. (NY) 203, 137 (1990)

[19] M. Henneaux and C. Teitelboim, Quantization of Gauge Systems (Princeton University Press, Princeton, 1992)

[20] R. Anishettyt and A. S. Vytheeswaran, J. Phys. A: Math. Gen. 26, 5613 (1993)

[21] C. Bizdadea and S. O. Saliu, Nucl. Phys. B 456, 473 (1995)

[22] L. D. Faddeev and S. L. Shatashvili, Phys.Lett. B 167, 225 (1986)

[23] I. A. Batalin and E. S. Fradkin, Phys. Lett. B 180, 157 (1986); Nucl. Phys. B 279, 514 (1987); Phys. Lett. B 236, 528 (1990)

[24] I. A. Batalin and I. V. Tyutin, Int. J. Mod. Phys. A 6, 3255 (1991)

[25] S. Deser and R. Jackiw, Phys. Lett. B 139, 371 (1984)

[26] P. K. Townsend, K. Pilch and P. van Nieuwenhuizen, Phys. Lett. B 136, 38 (1984)

[27] S. Deser, R. Jackiw and S. Templeton, Phys. Rev. Lett. 48, 975 (1982); Ann. Phys. (NY). 140, 372 (1982)

[28] R. Banerjee, H. J. Rothe, and K. D. Rothe, Phys. Rev. D 52, 3750 (1995); Phys. Rev. D 55, 6339 (1997)

[29] R. Banerjee and H. J. Rothe, Nucl. Phys. B 447, 183 (1995)

[30] S. Deser and R. Jackiw, Phys. Lett. B 451, 73 (1999)

[31] A. de Souza Dutra and C. P. Natividade, Phys. Rev. D 61, 027701 (1999)

[32] R. Banerjee, B. Chakraborty and T. Scaria, Int. J. Mod. Phys. A 16, 3967 (2001)

[33] S. Deser and B. Tekin, Class. Quant. Grav. 19, L97 (2002)

[34] D. Bazeia, R. Menezes, J. R. Nascimento, R.F. Ribeiro and C. Wotzasek, J. Phys. A 36, 9943 (2003)

[35] L. Heisenberg, JCAP 05 (2014) 015

[36] P. A. M. Dirac, Can. J. Math. 2, 129 (1950)

[37] P. A. M. Dirac, Lectures on Quantum Mechanics (Academic Press, New York, 1967)
[38] R. Ferraro, M. Henneaux and M. Puchin, J. Math. Phys. 34, 2757 (1993)
[39] E. C. G. Stückelberg, Helv. Phys. Acta 11, 225 (1938)
[40] C. Bizdadea, E. M. Cioroianu and S. O. Saliu, Phys. Scr. 60, 120 (1999)
[41] M. Ostrogradsky, Mem. Ac. St. Petersbourg VI 4, 385 (1850)
[42] D. M. Gitman and I. V. Tyutin, Quantization of Fields with Constraints (Springer–Verlag, Berlin, Heidelberg, 1990)
[43] V. V. Nesterenko, J. Phys. A 22, 1673 (1989)
[44] S. Kumar, Int. J. Mod. Phys. A 18, 1613 (2003)
[45] C. M. Reyes, Phys. Rev. D 80, 105008 (2009)
[46] R. Banerjee, P. Mukherjee and B. Paul, JHEP 08 (2011) 085
[47] M. S. Plyushchay, Int. J. Mod. Phys. A 4, 3851 (1989); Nucl. Phys. B 362, 54 (1991)
[48] P. Mukherjee and B. Paul, Phys. Rev. D 85, 045028 (2012)
[49] B. Paul, Phys. Rev. D 87, 045003 (2013)
[50] R. Banerjee, P. Mukherjee and B. Paul, Phys. Rev. D 89, 043508 (2014)