REFINED HEINZ OPERATOR INEQUALITIES AND NORM INEQUALITIES

A. G. GHAZANFARI

Abstract. In this article we study the Heinz and Hermite-Hadamard inequalities. We derive the whole series of refinements of these inequalities involving unitarily invariant norms, which improve some recent results, known from the literature. We also prove that if $A, B, X \in M_n(\mathbb{C})$ such that $A$ and $B$ are positive definite and $f$ is an operator monotone function on $(0, \infty)$. Then

$$|||f(A)X - Xf(B)||| \leq \max\{|||f'(A)|||, |||f'(B)|||\}|||AX - XB|||.$$ 

Finally we obtain a series of refinements of the Heinz operator inequalities, which were proved by Kittaneh and Krnić.

Mathematics subject classification (2010): 47A30, 47A63, 15A45.

Keywords and phrases: Norm inequality, operator inequality, Heinz mean.

REFERENCES

[1] I. ALI, H. YANG AND A. SHAKOOR, Refinements of the Heron and Heinz means inequalities for matrices, J. Math. Inequal. 1 (2014), 107–112.
[2] R. BHATIA, Interpolating the arithmetic mean inequality and its operator version, Linear Algebra Appl. 413 (2006), 355–363.
[3] R. BHATIA, Matrix analysis, Springer-Verlag, New York, 1997.
[4] R. BHATIA, C. DAVIS, A Cauchy-Schwarz inequality for operators with applications, Linear Algebra Appl. 223/224 (1995) 119–129.
[5] D. DRISSI, Sharp inequalities for some operator means, J. Matrix Anal. Appl. 28 (2006), 822–828.
[6] Y. KAPIL, C. CONDE, M. S. MOSLEHIAN, M. SINGH, M. SABABHEH, Norm Inequalities Related to the Heron and Heinz Means, Mediterr. J. Math. (2017), 14:213.
[7] Y. KAPIL, M. SINGH, Contractive maps on operator ideals and norm inequalities, Linear Algebra Appl. 459 (2014), 475–492.
[8] R. KAUR, M. SINGH, Complete interpolation of matrix version of Heron and Heinz means, J. Math. Inequal., 16, 1 (2013), 93–99.
[9] R. KAUR, M. SAL MOSLEHIAN, M. SINGH, C. CONDE, Further refinements of the Heinz inequality, Linear Algebra and its Applications 447 (2014) 26–37.
[10] F. KITTANEH, On the convexity of the Heinz means, Integr. Equ. Oper. Theory 68 (2010), 519–527.
[11] F. KITTANEH AND M. KRNIĆ, Refined Heinz operator inequalities, Linear and Multilinear Algebra, 61 (8) (2013), 1148–1157.
[12] F. KUBO, T. ANDO, Means of positive linear operators, Math. Ann. 246 (1980) 205–224.
[13] G. LAROTONDA, Norm inequalities in operator ideals, J. Funct. Anal. 255 (2008) 3208–3228.
[14] J. E. PEČARIĆ, T. FURUTA, J. MIČIĆ HOT, Y. SEO, Mond-Pečarić Method in operator inequalities, Element, Zagreb, 2005.
[15] J. G. ZHAO, J. L. WU, Operator inequalities involving improved Young and its reverse inequalities, J. Math. Anal. Appl. 421, 2 (2015), 1779–1789.