Hints for a Low $B_s \to \mu^+\mu^-$ Rate and the Fourth Generation

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With full 2011 LHC data analyzed, there is no indication for deviation from Standard Model (SM) in CP violating phase for $B_s \to J/\psi \phi$, nor in the forward–backward asymmetry for $B^0 \to K^{*0}\mu^+\mu^-$. SM sensitivity, however, has been reached for $B_s \to \mu^+\mu^-$ rate, and there may be some hint for a suppression. We illustrate that, if a suppressed $\mathcal{B}(B_s \to \mu^+\mu^-)$ bears out with 2012 data, it would imply a lower bound on the fourth generation quark mixing product $|V^*_{ts} V_{tb}|$.

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I. INTRODUCTION

The Winter conferences have brought forth a host of new experimental results from the LHC. Continuing the 2011 trend, the Standard Model (SM) stands tall, and there are no strong hints of new physics beyond SM (BSM). On the flavor front, a fit to $B_s \to J/\psi \phi$ events by LHCb with 1 fb$^{-1}$ data yields $\Delta \Gamma_s$ that is in good agreement with SM, while combining the $\phi_s \equiv 2\Phi_B$ (the CP violating phase of $B_s \to B_s$ mixing) measurement with the result from $B_s \to J/\psi \pi\pi$ gives

$$\phi_s = -0.002 \pm 0.083 \pm 0.027 \text{ rad} \quad (\text{LHCb 1 fb}^{-1})$$

$$= -0.002 \pm 0.087 \text{ rad},$$

which is fully consistent with the result of $0.03 \pm 0.16 \pm 0.07$ with 1/3 the dataset. Again with 1 fb$^{-1}$ data, LHCb has advanced the measurement of forward-backward asymmetry in $B^0 \to K^{*0}\mu^+\mu^-$, giving a first measurement [2] of the zero-crossing point

$$q^2_0 = (4.9^{+1.1}_{-1.3}) \text{ GeV}^2, \quad (\text{LHCb 1 fb}^{-1})$$

which is consistent with SM expectation of 4.0–4.3 GeV$^2$ [3].

It is interesting then, that more apparent progress has been made on the quest for the $B_s \to \mu^+\mu^-$ rare decay mode; SM sensitivity has genuinely been reached, and data [4, 5] might be suggestive of a rate below SM expectations. Given that a decade long search for $B_s \to \mu^+\mu^-$ was motivated by the possible enhancement up to factors of hundreds to thousands, by powers [6] of $\tan \beta$ in the settings of supersymmetry or two Higgs doublet models, we are now at the juncture of a mindset change, switching from possible huge enhancements of old, to SM-like or even sub-SM values as it might emerge. It is in this context that we wish to explore in this Brief Report the implications on relevant flavor parameters involving a fourth generation of quarks, $t'$ and $b'$ (SM4).

It should be noted that bounds on $t'$ and $b'$ masses have reached the 600 GeV level by direct search at the LHC, hence we have nominally crossed the threshold of the unitarity bound (UB) of 500–550 GeV [8]. In the following, we will proceed naively, extending our previous work [8], and return to comment on UB and other issues towards the end of our discussion.

II. LOW VERSUS SM-LIKE $B_s \to \mu^+\mu^-$ RATE

It is difficult to enhance $B_s \to \mu^+\mu^-$ in SM4 by more than a factor of two or three, because it is constrained by $B \to X_s \ell^+\ell^-$ (together with $B \to X_s \gamma$), which is consistent with SM. Hence, this mode appeared less relevant for SM4, until recently. In contrast, the aforementioned $\tan \beta$ enhancement effect feeds scalar operators that do not enter $b \to s \gamma$ and $b \to s t^\ell \ell^-$ processes, hence were far less constrained. However, the scalar operators are now muted by the prowess of the LHC (and previous searches at the Tevatron).

A dramatic turn of events were already played out in 2011, where the combined result [10] of LHCb and CMS, $\mathcal{B}(B_s \to \mu^+\mu^-) < 1 \times 10^{-9}$ at 95% Confidence Level (CL), refuted the CDF result [11] of $(18^{+11}_{-9}) \times 10^{-9}$, which was at the time itself hot-off-the-press. Adding close to 3 fb$^{-1}$ data to the previous 7 fb$^{-1}$ analysis, the CDF value dropped a bit to $(13^{+9}_{-7}) \times 10^{-9}$, but the Tevatron has ran out of steam. ATLAS has also turned out a bound of $22 \times 10^{-9}$ based on 2.4 fb$^{-1}$ data, which is not yet competitive even with summer 2011 results from LHCb or CMS. The highlight this Winter was therefore the $B_s \to \mu^+\mu^-$ results from CMS and LHCb.

Let us first describe the LHCb result. Using a multivariate analysis (MVA), LHCb gave [7] the 95% CL bound of

$$\mathcal{B}(B_s \to \mu^+\mu^-) < 4.5 \times 10^{-9}, \quad (\text{LHCb 1 fb}^{-1})$$

which is approaching rather close to the SM value [12] of

$$\mathcal{B}(B_s \to \mu^+\mu^-) = (3.2 \pm 0.2) \times 10^{-9}. \quad (\text{SM})$$

In fact, LHCb gave a fitted number,

$$\mathcal{B}(B_s \to \mu^+\mu^-) = (0.8^{+1.8}_{-1.3}) \times 10^{-9}, \quad (\text{LHCb 1 fb}^{-1})$$

which naively implies possibly negative branching ratio! The central value is from the maximum log-likelihood, while the errors correspond to varying the log-likelihood by 0.5. The main upshot may be that LHCb does not really see any clear hint of a SM-strength signal! Either this is a downward fluctuation of the “true SM” value of Eq. [4], or Nature has a sub-SM value in store for us. We caution, of course, that statistics is still rather low.

The CMS result [12] is, at 95% CL,

$$\mathcal{B}(B_s \to \mu^+\mu^-) < 7.7 \times 10^{-9}, \quad (\text{CMS 5 fb}^{-1})$$
by a cut-based analysis. A mild deficit seems to be indicated when compared with the median expected limit of $< 8.4 \times 10^{-9}$. But the handful of events reveal some interesting pattern. In the Barrel detector region, one expects $\sim 2.7$ signal events if SM were true, together with $\sim 0.8$ events from background. Only two events were observed, which are separated by $\sim 100$ MeV, wider than the nominal detector mass resolution. This suggests the presence of background events. Whether this constitutes one event each for signal and background, or if both events are background, it seems to echo LHCb \[2\] in some “downward” fluctuation from the SM value of Eq. \[4\]. However, if both LHCb and CMS sense a downward signal fluctuation, then the likelihood that the actual signal might be lower would be enhanced!

In the Endcap detector region, the situation is a bit puzzling. Here, signal and background are both expected at 1.2 event level, while a total of 4 events were seen \[3\]. But they all cluster within 50 MeV or less, inside a signal mass window of 150 MeV, which is set at twice the detector mass resolution (poorer than in the Barrel detector). However, since the Endcap is less sensitive than the Barrel, we refrain from further comment, except that the “excess” events push up the bound of Eq. \[4\] slightly. Thus, by CMS Barrel detector alone, the “discrepancy” with median expected is a little larger.

Although anything can happen at the present statistics level, LHCb expects to add $\sim 1$ fb$^{-1}$ in 2012, while CMS would add $\sim 15$ fb$^{-1}$, both at the slightly higher collision energy of 8 TeV. We therefore like to emulate future prospects as follows. For the indication of lower than SM rate, we shall take Eq. \[5\] at face value. Projecting to full 2011-2012 data, besides the doubling of LHCb data, CMS data should increase more than four fold (an MVA approach should increase the effective luminosity). Although one cannot really project what is the combined effective reduction of errors, we take the factor $\sqrt{6} \sim 2.5$. I.e. in our subsequent numerics, besides the 1$\sigma$ allowed range for Eq. \[6\], we will show also the 1/2.5$\sigma$ range, which would give $(0.8^{+0.7}_{-0.5}) \times 10^{-9}$. While this is rather aggressive, it would illustrate a sub-SM result when LHCb combined with CMS probes genuinely below SM values. It is not impossible that, by end of 2011-2012 run, we find $B(B_s \to \mu^+ \mu^-)$ to be consistent with zero, i.e. at $10^{-9}$ or less. We note that ATLAS could also eventually contribute significantly to $B_s \to \mu^+ \mu^-$ search.

The notable feature across the board for new physics searches at the LHC, however, is that no cracks were found in SM’s armor. Thus, we offer a second case of SM-like behavior. Here, we take the central value from SM, and mimic the current error bar by satisfying the 95% CL bound from CMS. We get from Eq. \(6\):

$$B(B_s \to \mu^+ \mu^-) = (3.2 \pm 2.7) \times 10^{-9}.$$  

(7)

Again, we will discuss the 1$\sigma$ and 1/2.5$\sigma$ allowed range of Eq. \(7\) for projections into the future. Actual error reduction would likely be more than 1/2.5 for SM-like central values in Eq. \(4\).

We follow our previous paper \[9\] and combine the above scenarios for $B(B_s \to \mu^+ \mu^-)$ with measurements of $\phi_s$ and $A_{FB}(B^0 \to K^+\rho^0 \mu^+ \mu^-)$ (we shorthand as $A_{FB}$ below). Our target physics is the flavor parameters of the fourth generation for $b \to s$ transitions, namely $V_{ts}^\ast V_{tb} \equiv r_{sb} e^{i \phi_{sb}}$. If the current hint for 125 GeV SM-like Higgs boson does not get substantiated by 2012 data, a very heavy fourth generation could provide the mechanism for electroweak symmetry breaking through its strong Yukawa interaction \[13\]. We will find that a sub-SM $B(B_s \to \mu^+ \mu^-)$ value would imply a lower bound on $r_{sb} = |V_{ts}^\ast V_{tb}|$, which would be rather interesting.

We had suggested that the three measurements of $\phi_s$, $B(B_s \to \mu^+ \mu^-)$ and $A_{FB}$ would help map out the preferred $V_{ts}^\ast V_{tb}$, or ($r_{sb}$, $\phi_{sb}$) parameter space. The main measurements are $\phi_s$ and $B(B_s \to \mu^+ \mu^-)$, with $A_{FB}$ providing further discrimination, both in its shape, and now also the $q_0^2$ value \[2\]. Three cases were discussed. Case A was for large and negative $\phi_s$, where we used $\sin 2\phi_{B_s} = -0.3 \pm 0.1$, and enhanced $10^5 B(B_s \to \mu^+ \mu^-) = 5.0 \pm 1.5$. This was motivated by hints for large and negative time-dependent CPV in $B_s \to J/\psi \phi$ from Tevatron studies. Although a $-0.2 \pm 0.1$ value could still be entertained at the $2\sigma$ level, there is not more to be said beyond our previous work, while the likelihood for enhanced $B(B_s \to \mu^+ \mu^-)$ is receding. Thus, we no longer present this case. Case B and C were for $\sin 2\Phi_{B_s}$ taking SM value of $-0.04 \pm 0.01$, while $10^5 B(B_s \to \mu^+ \mu^-)$ takes the slightly enhanced or depressed values of $5.0 \pm 1.5$ and $2.0 \pm 1.5$, respectively. By design, the overlap between Case B and Case C is precisely when $B(B_s \to \mu^+ \mu^-)$ is SM-like. Thus, the three Cases of A, B and C map out the foreseen parameter space in $r_{sb}$ and $\phi_{sb}$ as data improves.

With the present experimental situation for $B(B_s \to \mu^+ \mu^-)$, which could either be sub-SM as in Eq. \(4\), or SM-like, as in Eq. \(9\), we reinvestigate the implications for the preferred region in the $r_{sb}$-$\phi_{sb}$ plane. For both cases, we impose the $\phi_s \equiv 2\Phi_{B_s}$ constraint of Eq. \(4\). The observed shape and $q_0^2$ value from $A_{FB}$ are further applied to constrain parameter space. We take $m_{\ell^\prime} = 650$ GeV for sake of illustration.

III. RESULTS

The $B_s$-$B_s$ mixing amplitude is

$$M_{12}^s = \frac{G_F M_B^2}{12\pi^2} m_{B_s} f_{B_s} B_{B_s} \eta_{B_s} \Delta_{12}^s,$$  

(8)

with

$$\Delta_{12}^s = \left(\lambda_{t q}^{SM}\right)^2 S_0(t, t) + 2\lambda_{t q}^{SM} \lambda_{t \ell} \Delta S_0^{(1)} + \lambda_{t \ell}^2 \Delta S_0^{(2)},$$  

(9)

where $\lambda_{q} \equiv V_{ts}^\ast V_{tb}$. With $S_0$ and $\Delta S^{(1)}$ as defined in Ref. \[14\], Eq. \(8\) manifestly respects GIM \[15\]. The CPV phase

$$\phi_s = 2\Phi_{B_s} \equiv \arg M_{12}^s = \arg \Delta_{12}^s,$$  

(10)
depends only on \( m_{\ell'} \) and \( \lambda_{t'} = V_{ts}^* V_{tb} \). Note that
\( \lambda_i^{SM} = -\lambda_c - \lambda_u \approx -0.04 - V_{us}^* V_{ub} \). Although we take PDG \[16\] values for the phase of \( V_{ub} \), it is exciting that the phase of \( V_{ud}^* V_{ub} \) is starting to be directly measured via interference of tree processes. For \( B(B_s \rightarrow \mu^+ \mu^-) \), the \( f_{B_s} \) dependence is largely removed \[17\] by taking the ratio with \( \Delta m_{B_s}/\Delta m_{B_s}^{\text{exp}} \), which works for SM4 as in SM. That is,
\[
B(B_s \rightarrow \mu \mu) = C \frac{\tau_{B_s} \eta_{tt}}{B_{B_s} \eta_B} \frac{\lambda_i^{SM} Y_0(x_{t'}) + \lambda_{t'} \Delta Y_0^2}{|\Delta s_{12}|/|\Delta m_{B_s}^{\text{exp}}}.
\]
where \( C = 3g_{tB}^2 m_{\mu}^2 / 2\pi^3 m_W^2 \), and \( \eta_Y = \eta_Y(x_{t'}) = \eta_Y(x_{t'}) \). We plot, in Fig. 1(a), the contours for \( \phi_s \) within 1\( \sigma \) and \( 1/\sqrt{2} \sigma \) range of Eq. \ref{eq:phi_s}, in the \( r_{sb} \equiv |V_{ts}^* V_{tb}|, \phi_{sb} \equiv \arg V_{ts}^* V_{tb} \) plane, for \( m_{\ell'} = 650 \) GeV. Here, LHCb holds a monopoly, and statistics is expected to only double during 2012. Similarly for \( B(B_s \rightarrow \mu^+ \mu^-) \), we plot the contours within 1\( \sigma \) and 1/2.5\( \sigma \) range of Eq. \ref{eq:phi_s}, which is sub-SM in strength. The \( m_{\ell'} \) value used is beyond the 550 GeV nominal UB bound \[8\], and one is no longer sure of the numerical accuracy of Eqs. \ref{eq:phi_s} and \ref{eq:phi_s}, i.e. the perturbative computation of the functions \( \Delta S_0^{(2)} \) and \( \Delta Y_0 \) would become questionable. However, some form such as Eq. \ref{eq:phi_s} should continue to hold even above the UB, and we shall continue to use existing formulas.

The overlap between the \( \phi_s \) and \( B(B_s \rightarrow \mu^+ \mu^-) \) contours now favor \( \phi_{sb} \) in the 4th quadrant with |\( \sin \phi_{sb} | \) small, where the darker regions are for more aggressive error projections towards the future. It should be clear that a precise determination of \( \phi_{sb} \) depends much more on the precision of \( \phi_s \) measurement.

The SM-like case of Eq. \ref{eq:phi_s} is less interesting, but given the continued success of the SM into the LHC era, should be viewed as more probable. We illustrate in Fig. 1(b) for \( m_{\ell'} = 650 \) GeV the overlap of the contours for \( \phi_s \) in Eq. \ref{eq:phi_s} and \( B(B_s \rightarrow \mu^+ \mu^-) \) in Eq. \ref{eq:phi_s}. Besides some high \( r_{sb} \) region for modest |\( \phi_s | \) values, the generic feature is relatively small \( r_{sb} \), with \( \phi_{sb} \) undetermined by the present precision of \( \phi_{sb} \) measurement. This small \( r_{sb} \) case is rather intuitive, that of subdued 4th generation effect. We shall see that the larger \( r_{sb} \) values are ruled out by the observation of SM-like behavior for \( A_{FB} \), as we have seen in our previous paper.

The SM-like shape for \( A_{FB} \) as observed by LHCb provides a powerful discriminant against larger \( r_{sb} \) values. Note that data prior to summer 2011 had suggested a deviation from SM behavior \[10\], which, besides a hint for sizable deviation in \( \sin 2\Phi_{B_s} \), was part of the motivation for Case A in our previous paper. The SM-like shape for \( A_{FB} \) is further affirmed with 1 fb\(^{-1} \) data from LHCb \[2\], while the first measurement for zero-crossing point, Eq. \ref{eq:phi_s}, is offered. We have checked the allowed parameter space of Fig. 1 and find generically that \( r_{sb} \gtrsim 0.004 \) would generate significant deviations in shape for \( A_{FB} \). The drop from roughly 0.008 \[2\] to 0.004 is due to the higher \( m_{\ell'} = 650 \) GeV taken to satisfy direct search bounds \[2\], as well as the tighter experimental constraints towards SM. We note with interest that, for the sub-SM \( B(B_s \rightarrow \mu^+ \mu^-) \) case, the slightly larger than SM central value of \( \sigma_0^2 = 4.9 \) GeV\(^2 \) in Eq. \ref{eq:phi_s} also prefers \( \phi_{sb} \) in the 4th quadrant.
IV. DISCUSSION AND CONCLUSION

After some hints for BSM physics for some years, both in $A_{FB}(B^0 \to K^{*0}\mu^+\mu^-)$ and in $\sin2\Phi_B$, [13] SM is reaffirmed by 2011 data from LHC. Interestingly, now there might be a hint for $B(B_s \to \mu^+\mu^-)$ below SM expectations. It is of course too early to tell. However, this mode has always been looked upon as possibly greatly enhanced by the less constrained scalar operators. We are at least at the turning point, where no large enhancement is observed, but now whether it is SM-like, or sub-SM, can be distinguished with full 2011-2012 data. The 4th generation $t'$ quark offers the natural toolbox in this domain, as it is constrained to be subdominant by $b \to s\gamma$ and $b \to s\ell^+\ell^-$ data since a decade, while providing a destructive mechanism in the unknown phase of $V_{tb}^*V_{td}$. In contrast, adjusting the scalar interactions to the SM strength is like training a big hammer on a small nail.

We note that, to have $B(B_s \to \mu^+\mu^-)$ near the central value of Eq. (3), the $C_{10}$ Wilson coefficient would be considerably smaller than SM value, such that one would worry about $B(B \to X, \ell^+\ell^-)$. It is then interesting to note that LHCb data does seem to indicate that the differential rate could be a little lower than the SM expectations [2]. Although a vanishing $B(B_s \to \mu^+\mu^-)$ is unlikely (more probably within $(1-2)\times10^{-9}$), it would be interesting to watch this mutually supporting trend of somewhat lower $B(B_s \to \mu^+\mu^-)$ and $B(B \to K^{*}\ell^+\ell^-)$ (or $B(B \to X_s \ell^+\ell^-)$). The darker region in Fig. 1(a) is just to stress the point.

We have used $m_{t'} = 650$ GeV to satisfy direct search bounds, which is now beyond the nominal unitarity bound. To probe much further, the 13-14 TeV run would be necessary. However, with the Yukawa coupling turning nonperturbative, the phenomenology may change [19]. Fortunately, the leading production mode of $gg \to QQ$ is not affected. The usage of such large $m_{t'}$ values is becoming dubious, and nonperturbative studies should be performed. The nonperturbative, strong Yukawa coupling could actually be the source of electroweak symmetry breaking [13]. It is interesting that, with full 2011-2012 data, we would learn whether a 125 GeV Higgs boson is substantiated, as well as whether $B_s \to \mu^+\mu^-$ is below SM expectations.

In conclusion, we illustrate what LHC data might tell us about 4th generation flavor parameters. Assuming 2011-2012 data would give $\phi_s = -0.002\pm0.062$ and taking $m_{t'} = 650$ GeV, we mocked the low $B_s \to \mu^+\mu^-$ rate case with $(0.8_{-0.5}^{+0.7})\times10^{-9}$, which would imply $|V_{ts}^*V_{tb}| \sim 0.0015-0.004$, with $-40^\circ \lesssim \arg V_{ts}^*V_{tb} \lesssim 15^\circ$. On the other hand, if a SM-like $B_s \to \mu^+\mu^-$ rate emerges, it would imply small $|V_{ts}^*V_{tb}|$ at a couple per mile, while $\arg V_{ts}^*V_{tb}$ would require more precise measurement of $\phi_s$ to determine. The $B^0 \to K^{*0}\mu^+\mu^-$ forward-backward asymmetry provides a further discriminant that rules out $|V_{ts}^*V_{tb}| \gtrsim 0.004$ for the discussed allowed regions.

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