Design of far field beamforming problem by window function

Dan Li¹,a, Xiaofen Tan¹,b, Zhiguo Feng²,c

¹School of Mathematical Sciences, Chongqing Normal University, Chongqing, China
Chongqing Normal University, Chongqing, China
²Faculty of Mathematics and Computer Science, Guangdong Ocean University, Zhanjiang, Guangdong, China

a253443281@qq.com, b1134748902@qq.com, c18281102@qq.com

Abstract. The design of the broadband beam former problem is to find the coefficients vector for all FIR filters such that output fits to a given target spatial directivity pattern in a specified domain. This problem is always very complicated. In this paper, we propose a fast window function method to solve this problem. First, we formulate a simplified optimization problem, where the frequency response function is the decision function. Then an optimal frequency response vector can be obtained by solving this problem. Next we apply the window function to cut off the infinite length of the coefficient. The window function method is efficient and can be applied in real time applications. Numerical example is illustrated to show the efficiency of the method.

1. Introduction
Microphone array model [1, 2] play an important role in the acoustic signal processing, which have many applications such as hearing aid, video conferencing, mobile phone. The beam former design [3] in far filed has been widely used to speech enhancement [4], localization problem [5], blind signal separation problem [6, 7]. Among many beam former design criteria, minimax design is more robust in practice. However, when the number of filter lengths is large and discrete points of specified domain increases, the computation of minimax beam former design problem becomes very complicated. Hence, efficient methods should be developed such that it can be applied in real applications.

In digital signal processing, window function is an efficient tool to design FIR filter. Compared with the other beam former design methods, the computational complexity of window function method is very small. Furthermore, for real time processing, window function method can also be used to solve the problem, and the solution meet the requirement of real time processing. Window function method has not been applied to design far-field broadband beamformer. Hence, we propose the window function method to solve the broadband beam former design problem efficiently. There exist many methods in the literature which can solve the beam former design problem. In [8], the near field beam former design problem was solved by a linear semi-infinite programming method, where a rotation technique was applied to transform the quadratic constraints into linear constraints. In [9], a two stage method was applied to solve the beam former design in near field. To improve the performance of beam former, a placement design was proposed in [10] to find the positions of the microphones together with the coefficients of the filters, and the optimal value need been improved. Furthermore, there are some robust beam former design problems, such as [11] and [12]. The performance can be remained in the optimal level, even if the uncertain parameters exist.
The rest of this article is organized as follows. In Section 2, the design of broadband beam former is formulated as a semi-infinite programming problem. In Section 3, we propose the window function method to solve the beam former design problem. A numerical example is given in Section 4 to illustrate the efficiency and effectiveness of the proposed method and conclusion is summarized in Section 5.

2. Beam Former Design Problem

The structure of microphone array in far field is illustrated in Figure 1. For a linear microphone array with \( N \) elements, the transfer function is given by

\[
A_n(f, \Theta) = e^{-j2\pi f r_n \cos \Theta / c}, \quad \Theta \in [0, \pi], \quad n = 1, \ldots, N,
\]

(1)

Where \( r_n \) is the distance from the \( n \)-th microphone to a reference point; \( c \) is the propagation speed of acoustic signal in the air; \( \Theta \) is the angle between the direction of the signal and the microphone array.

![Figure 1. Structure of far field microphone array.](image)

After the speech signal are received by the microphones, there are \( L \)-taps FIR (Finite impulse response) filters behind such that the signal data is filtered and sum to an output function given by

\[
G(f, \Theta) = \sum_{n=1}^{N} W_n(f) A_n(f, \Theta) = \sum_{n=1}^{N} W_n(f) e^{-j2\pi f r_n \cos \Theta / c}
\]

(2)

\[
= Y^T(f, \Theta) W(f), \quad \Theta \in [0, \pi],
\]

Where

\[
Y(f, \Theta) = [Y_1(f, \Theta), \ldots, Y_N(f, \Theta)]^T,
\]

\[
W(f) = [W_1(f), \ldots, W_N(f)]^T
\]

Is the filter frequency response vector given by

\[
W_n(f) = w_n^T d_0(f), \quad n = 1, \ldots, N
\]

(3)
Where \( \mathbf{w}_n \) is the coefficient vector of the \( n \)-th filter, that is,
\[
\mathbf{w}_n = [w_n(0), \cdots, w_n(L-1)]^T
\]
And \( \mathbf{d}_n(f) \) is given by
\[
\mathbf{d}_0(f) = \left[ 1, e^{-j2\pi f L/s}, \cdots, e^{-j2\pi f(L-1)/s} \right]^T.
\]

According to practical applications, the output spatial directivity pattern \( G(f, \theta) \) is fitted to a given ideal spatial directivity pattern \( G_d(f, \theta) \). In general, the function \( G_d(f, \theta) \) is defined in a specified area \( \Omega = \Omega_p \cup \Omega_s \), where \( \Omega_p \) is the passband region, and \( \Omega_s \) is the stopband region. In general, the function \( G_d(f, \theta) \) is given by
\[
G_d(f, \theta) = \begin{cases} 
 e^{-j2\pi L/s}, & (f, \theta) \in \Omega_p, \\
 0, & (f, \theta) \in \Omega_s,
\end{cases}
\]

Where \( \tau_L \) is the group delay.

In minimax design, the objective function is defined as
\[
E(\mathbf{w}) = \max_{(f, \theta) \in \Omega} \rho(\mathbf{f}, \mathbf{\theta}) \left| A^T (f, \theta) \mathbf{W}(f) - G_d(f, \theta) \right|^2,
\]

(4)

Where \( \rho(f, \theta) \) is a weighting function that measures the importance or the region. Therefore, the design problem of the broadband beam former can be formulated to find the coefficient vector such that (4) is minimized, that is,
\[
\min_{\mathbf{w} \in \mathbb{R}^{NL}} E(\mathbf{w}) = \min_{\mathbf{w} \in \mathbb{R}^{NL}} \max_{(f, \theta) \in \Omega} \rho(\mathbf{f}, \mathbf{\theta}) \left| A^T (f, \theta) \mathbf{W}(f) - G_d(f, \theta) \right|^2.
\]

This problem is equivalent to a semi-infinite programming problem
\[
\begin{aligned}
\min_{z, \mathbf{w}} & \quad z \\
\text{s.t.} & \quad E(\mathbf{w}, f, \theta) \leq z, \quad (f, \theta) \in \Omega,
\end{aligned}
\]

(5)

Where
\[
E(\mathbf{w}, f, \theta) = \rho(\mathbf{f}, \mathbf{\theta}) \left| A^T (f, \theta) \mathbf{W}(f) - G_d(f, \theta) \right|^2.
\]

Denote
\[
\hat{\mathbf{W}}_n(f) = \mathbf{W}_n(f) e^{j2\pi L/f_s} = \mathbf{w}_n^T \hat{\mathbf{d}}_0(f)
\]
\[
\hat{\mathbf{d}}_0(f) = \begin{bmatrix} 
 -j2\pi f/L & -j2\pi f/L & -j2\pi f/L \\
 f/s & f/s & f/s
\end{bmatrix}^T
\]
Then, the function in (5) is equal to
\[ E(w, f, \theta) = \rho^2(f, \theta) \left| A^T(f, \theta) \hat{W}(f) - \hat{G}_d(f, \theta) \right|^2, \]

where \( \hat{G}_d(f, \theta) \) is independent of the parameter \( L \).

Since there are infinite points in \( \Omega \), discretization method should be applied to solve the problem (5). The number of discrete points is always very large, and this problem becomes a large scale optimization problem. Hence, it is very expensive to obtain the optimal solution. For this, we introduce the window function method as follows, such that a good solution can be obtained efficiently.

3. Window Function Method

First, we decompose the region into

\[ \Omega = \bigcup_{f \in [0, f_s/2]} (f, \Omega_f), \tag{6} \]

Where \( f \in [0, f_s/2], \Omega_f \) is the corresponding spatial area of \( f \) in \( \Omega \). Let \( I_f \) denote the set of \( f \) with nonempty set \( I_f \) then for each \( f \in \Omega \), we obtain a sub problem as follows:

\[
\min_{z, w} \quad z \\
\text{s.t.} \quad E(w, f, \theta) \leq z, \quad \forall (f, \theta) \in \Omega, \tag{7}
\]

Where
\[ E_f(\hat{W}, \theta) = \rho^2(f, \theta) \left| A^T(f, \theta) \hat{W}(f) - \hat{G}_d(f, \theta) \right|^2. \]

By solving the problem (7), we can obtain the optimal objective function value \( z^*_f \) and the optimal solution \( \hat{W}^*_f(f) \).

For each \( n = 1, \ldots, N, \) \( \hat{W}^*_f(f) \) can be expressed as the Fourier series given by

\[
\hat{W}^*_n(f) = \sum_{k=-\infty}^{+\infty} \frac{-j2\pi f}{f_s} e^{j\pi(k-\tau_L)} w^*_n(k). \tag{8}
\]

Then, for any two integers \( k \) and \( l \), we have
\[
\int_{-f_s}^{f_s} w_n^*(k)e^{-j2\pi f(k-\tau_L)/f_s}e^{j2\pi f(l-\tau_L)/f_s} df
= \int_{-f_s}^{f_s} w_n^*(k) e^{-j2\pi f(k-l)/f_s} df
= \begin{cases} 
  w_n^*(k) f_s, & k = l, \\
  0, & k \neq l.
\end{cases}
\]

Hence, we have

\[
w_n^*(k) = \frac{1}{f_s} \int_{-f_s/2}^{f_s/2} \hat{W}_n^*(f)e^{j2\pi f(k-\tau_L)/f_s} df,
\]

\[n = 1, \ldots, N, k = 0, \ldots, \infty.\] (8)

Note that \(w^*\) is an infinite sequence, which is impossible to be implemented in practice. Then, we can truncate the coefficients by window function. There are many kinds of window functions, such as Hamming windows, which do not have any parameters to adjust the window function, and Kaiser Window function, Dolph-Chebyshev window, Guassian window, which have parameters to adjust the shape of the window function.

For example, Hamming window is known as an improved cosine window given by

\[
T(n) = \begin{cases} 
  \alpha - (1 - \alpha) \cos \frac{2\pi n}{L-1}, & n = 0, 1, \ldots, L-1, \\
  0, & \text{otherwise},
\end{cases}
\]

\[\alpha = 0.54.\]

Kaiser window is an adjustable window. Its expression is given by

\[
T(n) = \frac{I_0(\alpha \sqrt{(L-1)^2 - (n-L-1)^2})}{I_0(\alpha \sqrt{(L-1)^2})}
\]

(10)

Where \(I_0(\cdot)\) the zero-order first kind of Bessel is function, and \(\alpha\) is an adjustable parameter. Then, it is an adjustable window function, and \(\alpha\) can be used as the decision variable to optimize the cost function (5).

Next, if the window function is chosen, we can truncate the coefficients as

\[
w_n(k) = w_n^*(k)T(k) \quad k = 0, \cdots, L-1,
\]

(11)
Where \( T \) the window function, and \( L \) is the length of the window function. Then, an FIR filter coefficient vector has been determined.

If we choose Hamming window function, then the filter coefficient \( w \) can be obtained directly by (11). If we choose the adjustable window functions, such as Kaiser Window function. Suppose that the parameter for the window function in (11) is \( \gamma \), then the coefficient vector \( w \) can be expressed as \( w(\gamma) \). In this way, the beam former design problem is transformed into an optimization problem with respect to the parameter \( \gamma \).

Then, the beam former design problem (5) can be transformed into

\[
\min_{\gamma} \max_{(f, \theta) \in \Omega} E(w(\gamma), f, \theta),
\]

(12)

Where \( w(\gamma) \) is defined in (10), and

\[
E(w, f, \theta) = \rho^2(f, \theta) \left| A^T(f, \theta) \hat{W}(f) - \hat{G}_s(f, \theta) \right|^2.
\]

Problem (12) is a one-dimensional optimization problem. The number of variables is only one, and the problem is mostly simplified. Similar as (5), this problem is equivalent to a semi-infinite programming problem, and the optimization can be very efficient.

4. Numerical Example

For example, a linear array of equidistant microphone array with 9 elements is used. The corresponding position of the microphones are \{-0.4, -0.3, \ldots, 0.4\}, and the reference point is zero. The passband region \( \Omega_p \) is defined as

\[
\{(f, \theta) : f \in [0.3k, 2k], \theta \in [85^\circ, 95^\circ] \cup [120^\circ, 180^\circ] \},
\]

And the stopband region \( \Omega_s \) is composed of several parts given by

\[
\{(f, \theta) : f \in [2k, 4k], \theta \in [85^\circ, 95^\circ] \},
\]

\[
\{(f, \theta) : f \in [0.3k, 2k], \theta \in [0^\circ, 60^\circ] \cup [120^\circ, 180^\circ] \},
\]

\[
\{(f, \theta) : f \in [2k, 4k], \theta \in [0^\circ, 60^\circ] \cup [120^\circ, 180^\circ] \}.
\]

The weight function is \( \rho(f, \theta) = 1 \), the delay function is \( \tau_L = (L - 1)/2 \), the sound’s speed is \( c = 340.9 \text{m/s} \), and the sampling frequency is \( f_s = 8 \text{kHz} \).

Set the filter length from 50 to 400, we solve this problem with Hamming window and Kaiser Window respectively. The cost function values are depicted in Figure 2, where the performances of both Hamming window and Kaiser Window become better as the filter length increases, although there exist some vibrations.
Figure 2. Optimal cost function values in the example.

The running times of Hamming window is real time, and the running times of Kaiser Window are depicted in Figure 3. It can be seen that they are roughly one or two seconds. The maximum running time is less than 3 seconds. Hence, the implementation is very fast.

Figure 3. Running time in the example with Kaiser Window.

Finally, we set the filter length as 400 and solve this problem with Kaiser Window. Then, we plot the magnitude of actual directivity pattern, which is illustrated in Figure 4. It can be seen that the design requirements in both the passband region and the stopband region are satisfied. Hence, the window function method is both efficient and effective to obtain a good solution.
5. Summary
In this paper, the design of beam former in far field has been considered. To reduce the complexity of the problem, we proposed a window function method to solve this problem. The merit of this method is the efficiency of computation, and the optimality of the solution can also be remained. As the filter length increases, the solution can approach to the optimal solution.

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References
[1] H. L. V. Trees, Optimum Array Processing, New York: Wiley, 2002.
[2] S. C. Chan, H. H. Chen. “Uniform concentric circular arrays with frequency-invariant characteristics—theory, design, adaptive beamforming and DOA estimation,” IEEE Transactions on Signal Processing, vol. 55 (1), pp. 165 - 177, 2007.
[3] B. D. V. Veen and K. M. Buckley, “Beamforming: A versatile approach to spatial filtering,” IEEE ASSP Mag., vol. 5 (2), pp. 4 - 24, 1988.
[4] S. Gannot, D. Burshtein, E. Weinstein E, “Signal enhancement using beamforming and nonstationarity with applications to speech,” IEEE Trans Signal Process, vol. 49(8), pp. 1614-1626, 2001.
[5] Z. G. Feng, K. F. C. Yiu, R. C. K. Leung, et al, “Localization of acoustic source via optimal beamformer design,” Pacific journal of optimization, 2015, 11 (1): 37 - 56.
[6] H. Sawada, S. Araki, R. Mukai, et al, “Blind extraction of dominant target sources using ICA and time-frequency masking,” IEEE Transactions on Audio Speech & Language Processing, vol. 14(6), pp. 2165 - 2173, 2006.
[7] J. Choi, C. C. Lim, “A Cholesky factorization based approach for blind FIR channel identification,” IEEE Trans. Signal Processing, vol. 56, pp. 1730 - 1735, 2008.
[8] K. F. C. Yiu, X. Yang, S. Nordholm, et al, “Near-field broadband beamformer design via multidimensional semi-infinite linear programming techniques,” IEEE Transactions on
Speech & Audio Processing, vol. 11 (6), pp. 725 - 732, 2003.

[9] Z. G. Feng, K. F. C. Yiu, S. E. Nordholm, “A two-stage method for the design of near-field broadband beamformer,” IEEE Transactions on Signal Processing, vol. 59(8), pp. 3647-3656, 2011.

[10] Z. G. Feng, K. F. C. Yiu, S. E. Nordholm, “Placement design of microphone arrays in near-field broadband beamformers,” IEEE Transactions on Signal Processing, vol. 60 (3), pp. 1195-1204, 2012.

[11] Z. G. Feng, K. F. C. Yiu, “Design of broadband beamformers with smooth actual response in transition region,” Pacific Journal of Optimization, pp. 541 - 556, 2016.

[12] R. C. Nongpiur, “Design of minimax broadband beamformers that are robust to microphone gain, phase and position errors,” IEEE Transactions on Audio Speech and Language Processing, vol. 22, pp. 1013 – 1022, 2014.