Electric field induced by magnetic flux motion in superconductor containing fractal clusters of a normal phase

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The influence of the fractal clusters of a normal phase, which act as pinning centers, on the dynamics of magnetic flux in percolative type-II superconductor is considered. The voltage-current characteristics of such a superconductor are obtained taking into account the effect of fractal properties of cluster boundaries on the magnetic flux trapping. It is revealed that the fractality reduces the electric field arising from magnetic flux motion and thereby raises the critical current of a superconductor.

One of the most promising ways of increasing the current-carrying capability of superconductors is to make the artificial pinning centers in their bulk. Specifically, the normal phase clusters created in the course of the film growth at the sites of defects on the boundary with the substrate can act as such centers [1]-[4]. New possibilities for pinning enhancement are opening in the case that the normal phase clusters have fractal boundaries [5], [6]. It is the influence of such fractal clusters on the critical currents and on the voltage-current (V-I) characteristics of the superconductor in the resistive state that will interest us.

The problem setting is quite similar to that considered in Ref. [5]. We deal with the superconductor containing inclusions of a normal phase of columnar shape. In the course of the cooling below the critical temperature in the magnetic field (“field-cooling” regime) the two-dimensional distribution of the trapped magnetic flux is created in such a superconducting structure. As this takes place, the magnetic flux is locked in the isolated normal phase clusters that gives rise to effective pinning. Then the transport current is passed through the sample transversely to the orientation of the frozen magnetic field. Suppose that there is a superconducting percolation cluster in the plane carrying the electric current. When the transport current is rising, the trapped flux remains unchanged until the vortices start to break away from the clusters of pinning force weaker than the Lorentz force created by the current. Thus, the vortices will cross the superconducting space through the weak links, which connect the normal phase clusters between themselves. Such weak links form especially readily at the sites of various structural defects in high-temperature superconductors (HTS), which are characterized by an extremely short coherence length [2], [7]-[10]. In conventional low-temperature superconductors weak links can be formed due to the proximity effect in sites of minimum distance between the adjacent normal phase clusters.

Thus, whatever the microscopic nature of weak links may be, they form the channels for vortex transport. In accordance with their configuration each normal phase cluster contributes to the overall critical current distribution. When a transport current is gradually increased, the vortices will break away first from the clusters of small pinning force, and therefore, of small critical current. Thus the decrease in the trapped magnetic flux \( \Delta \Phi \) is proportional to the number of all the normal phase clusters of critical currents less than a preset value \( I \). Hence, the relative decrease in the trapped flux can be expressed with the cumulative probability function \( F = F(I) \) for the distribution of the critical currents of clusters:

\[
\frac{\Delta \Phi}{\Phi} = F(I) \quad , \quad \text{where} \quad F(I) = \Pr \{ \forall I_j < I \} \tag{1}
\]

The right-hand side of Eq. (1) is the probability that any \( j \)th cluster has the critical current \( I_j \) less than a given upper bound \( I \).

On the other hand, the magnetic flux trapped into a single cluster is proportional to its area \( A \), so the decrease in the total trapped flux can be represented by the cumulative probability function \( W = W(A) \) for the distribution of the areas of the normal phase clusters, which is a measure of the number of the clusters of area smaller than a given value of \( A \):

\[
\frac{\Delta \Phi}{\Phi} = 1 - W(A) \quad , \quad \text{where} \quad W(A) = \Pr \{ \forall A_j < A \} \tag{2}
\]

In order to clear up how the transport current acts on the trapped magnetic flux, it is necessary to find out the relationship between the distribution of the critical currents of the clusters (Eq. (1)) and the distribution of their
areas (Eq. (2)). This problem was solved in Ref. 3 for the case of the exponential distribution of the areas of the normal phase clusters with fractal boundary, which occurs in HTS structures based on YBCO films. The exponential distribution is the special case of gamma distribution, which describes the cluster area distribution in the most general way:

$$W(A) = \frac{\gamma(g + 1, \frac{A}{A_0})}{\Gamma(g + 1)}$$  \hspace{1cm} (3)

where $\gamma(\nu, z)$ is the incomplete gamma function, $\Gamma(\nu)$ is Euler gamma function, $A_0$ and $g$ are the parameters of gamma distribution which set the mean area of the cluster $\bar{A} = A_0(g+1)$ and its standard deviation $\sigma_A = A_0\sqrt{g+1}$. Gamma distribution of Eq. (3) is reduced to the exponential one at $g = 0$.

Using the same expression for the critical current of the fractal cluster $I = \alpha A^{-D/2}$ (where $\alpha$ is the form factor, D is the fractal dimension of the cluster perimeter, or so-called coastline dimension 11, 12) as in Ref. 3, and taking into account the initial relationship of Eq. (1), we can get the distribution of the critical currents in the general case of gamma-distributed cluster areas:

$$F(i) = \frac{\Gamma(g + 1, Gi^{-\theta})}{\Gamma(g + 1)}$$  \hspace{1cm} (4)

where $G \equiv \frac{\theta \frac{\Gamma(g+1)+1}{(\theta^g+\frac{D}{2}e^\theta\Gamma(g+1, \theta))^{\frac{1}{\theta}}}}{\theta^g+\frac{D}{2}e^\theta\Gamma(g+1, \theta)}$, $\theta \equiv \frac{D}{2} + g + 1$

$\Gamma(\nu, z) = \Gamma(\nu) - \gamma(\nu, z)$ is the complementary incomplete gamma function, $i \equiv I/I_c$ is the dimensionless transport current, and $I_c = \alpha (A_0G)^{-D/2}$ is the critical current of the resistive transition.

The found cumulative probability function of Eq. (4) provides the comprehensive description for the effect of the transport current on the trapped magnetic flux. Using this function, the probability density $f(i) \equiv dF/di$ for the critical current distribution can be readily derived:

$$f(i) = \frac{2G^{g+1}}{\frac{D}{2}(\frac{D}{2}+g+1)}i^{-\frac{\Gamma(g+1)+1}{\Gamma(g+1)}} e^{-\frac{\Gamma(g+1)+1}{\Gamma(g+1)}\frac{G(i^{-\theta})}}$$  \hspace{1cm} (5)

This function is normalized to unity over all possible positive values of the critical currents.

The resistive state comes in the range of the currents $i > 1$, when the magnetic flux motion gives rise to the voltage across a superconductor. The appearance of some finite resistance causes the energy dissipation to accompany the passage of electric current. As for any type-II superconductor with pinning centers the dissipation in the resistive state does not mean the destruction of phase coherence yet. The superconductivity does not fully collapse until a growth of dissipation becomes avalanche-like as a result of thermo-magnetic instability.

In the resistive state the superconductor is adequately specified by its $V-I$ characteristic. The critical current distribution of Eq. (3) allows us to find the electric field arising from the magnetic flux motion after the vortices have been broken away from the pinning centers. Inasmuch as each normal phase cluster contributes to the total critical current distribution, the voltage across a superconductor $V = V(i)$ is the response to the sum of effects made by each cluster:

$$V = R_f \int_0^i (i - i')f(i')di'$$  \hspace{1cm} (6)

where $R_f$ is the flux flow resistance. The similar approach is used universally to consider behavior of the clusters of pinned vortex filaments 13, to analyze the critical scaling of $V-I$ characteristics of superconductors 14; that is to say, in all the cases where the distribution of depinning currents occurs. The following consideration is primarily concentrated on the consequences of the fractal nature of the normal phase clusters specified by the distribution of Eq. (3), so all the problems related to possible dependence of the flux flow resistance $R_f$ on a transport current will not be taken up here.

After the substitution of the probability density function of Eq. (3) in Eq. (3), upon integration, the voltage across a superconductor can be written in its final form:

$$\frac{V}{R_f} = i + \frac{DG^{g+1}i^{-\frac{\Gamma(g+1)+1}{\Gamma(g+1)}}}{2(g+1-\frac{D}{2})\Gamma(g+2)} 2F_2 \left(\begin{array}{c} g + 1, \frac{g + 1 - \frac{D}{2}}{g + 2} \\ g + 2, \frac{g + 2 - \frac{D}{2}}{g + 2} \end{array}\right) - \frac{G}{\frac{\Gamma(g+1)}{\Gamma(g+1)}}$$  \hspace{1cm} (7)
where $_2F_2\left(\nu, \mu; \xi, \eta; z\right)$ is the generalized hypergeometric function.

In the special case of an exponential distribution (at $g = 0$) the expression of Eq. (7) can be simplified:

$$\frac{V}{R_f} = i \exp\left(-Ci^{-\frac{D}{2}}\right) - C\frac{\Phi}{\sqrt{2}} \Gamma \left(1 - \frac{D}{2}, Ci^{-\frac{D}{2}}\right), \quad \text{where} \quad C \equiv \left(\frac{2 + D}{2}\right)^{\frac{D}{2} + 1} \tag{8}$$

In the extreme cases of Euclidean clusters ($D = 1$) and clusters of boundary with the maximum fractality ($D = 2$) the formula of Eq. (5) for the voltage across a sample can be further transformed:

$$(D = 1, g = 0): \quad \frac{V}{R_f} = i \exp\left(-\frac{3.375}{i^2}\right) - \sqrt{\frac{3.375}{i}} \text{erfc}\left(\frac{\sqrt{3.375}}{i}\right) \tag{9}$$

$$(D = 2, g = 0): \quad \frac{V}{R_f} = i \exp\left(-\frac{4}{i}\right) + 4 \text{Ei}\left(-\frac{4}{i}\right) \tag{10}$$

where erfc$(z)$ is the complementary error function, Ei$(z)$ is the exponential integral function.

The $V$-$I$ characteristics of a superconductor containing fractal clusters of a normal phase are presented in Fig. 1. All the curves are virtually starting with the transport current value of $i = 1$ that is agreed with the onset of the resistive state found with the use of cumulative probability function [5]. At smaller current the total trapped flux remains unchanged for lack of pinning centers of such small critical currents, so the breaking of the vortices away has not started yet. The $V$-$I$ characteristic $(a)$ is drawn in Fig. 1 for the value of the fractal dimension $D = 1.44$, which was found earlier in Ref. [5] for superconducting YBCO film structures with exponentially distributed cluster areas $(g = 0)$. Two thin lines (b) and (c), calculated for extreme cases of Euclidean clusters and clusters of the most fractality, bound the region the $V$-$I$ characteristics can fall within for any possible values of fractal dimension at $g = 0$. Dotted lines (d) and (e) show the same extreme cases, but at the different value of gamma distribution parameter: $g = 0.5$. This figure demonstrates that the fractality reduces appreciably an electric field arising from the magnetic flux motion. Furthermore, this effect shows up most clearly for exponential area distribution. Figure 2 displays the behavior of pinning gain factor

$$k_D \equiv 20 \log \frac{\Delta \Phi(D = 1)}{\Delta \Phi(\text{current value of } D)}, \quad \text{dB}$$

which is equal to relative decrease in the fraction of magnetic flux broken away from fractal clusters of the boundary dimension $D$ compared to the case of Euclidean ones [5]. The decrease in the electric field with increasing fractal dimension is especially strong in the range of currents $1 < i < 3$, where the pinning gain also has a maximum. Both these effects have the same nature, inasmuch as their reason consists in the peculiarities of fractal distribution of critical currents of Eq. (4). Figure 3 shows how the fractal dimension of the cluster boundary affects the critical current distribution. It is clearly seen that this bell-shaped curve broadens out, shifting towards greater magnitudes of current as the fractal dimension increases. This re-distribution can be described by superlinear dependence of average critical current $\bar{i}$ on the fractal dimension, specified by Euler gamma function (see Fig. 3):

$$\bar{i} = \frac{\Gamma(g + 1 - \frac{D}{2})}{\sqrt{\Gamma(g + 1)}} G^{\frac{D}{2}}$$

The highest current-carrying capability of superconductor is achieved for exponential area distribution (at $g = 0$). In this case the smaller clusters, which are of greater critical current, make the most contribution to the overall distribution. Thus the mean critical current rises more steeply at $g = 0$ than it does at $g = 0.5$ with increasing the fractal dimension. The mean value of the critical current for Euclidean clusters at $g = 0$ is equal to $\bar{i}(D = 1) = (3/2)^{1/2} \sqrt{\pi} = 3.2562$, whereas for clusters of the most fractality $(D = 2)$ this value diverges. As may be seen from Fig. 3, an increase of the fractal dimension causes a significant broadening of the tail of the distribution $f = f(i)$, whereas the whole area under a curve remains constant. It means that more and more of small clusters, which can best trap the magnetic flux, are being involved in the game. Hence the density of vortices broken away from pinning centers by the Lorentz force is reducing, so the smaller part of a magnetic flux can flow, creating the smaller electric field. In turn, the smaller the electric field, the smaller is the energy dissipated when the current passes through the sample. Therefore, the decrease in heat-evolution, which could cause transition of a superconductor into a normal state, means that the current-carrying capability of the superconductor containing such fractal clusters is enhanced.

Thus, the fractal properties of the normal phase clusters exert an appreciable effect on the dynamics of the trapped magnetic flux. The crucial change of the critical current distribution caused by increasing of the fractal dimension
of the cluster boundary forms the basis of this phenomenon. The most important result is that the fractality of
the clusters strengthens the flux pinning and thereby hinders the destruction of superconductivity by the transport
current. This phenomenon provides the principally new possibility for increasing the critical current value of composite
superconductors by optimizing their geometric morphological properties.

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FIG. 1. Voltage-current characteristics of superconductor containing fractal clusters of a normal phase. The solid lines (a), (b), and (c) pertain to the exponential distribution of the cluster areas \((g = 0)\). Curve (a) corresponds to the case of fractal clusters of dimension \(D = 1.44\); curve (b) – to Euclidean clusters \((D = 1)\); curve (c) – to clusters of boundary with the maximum fractality \((D = 2)\). Two dotted lines (d) and (e) are given for the case of gamma distribution at \(g = 0.5\). Curve (d) corresponds to Euclidean clusters \((D = 1)\), curve (e) – to clusters of the most fractality \((D = 2)\).
FIG. 2. The pinning gain for the cases: (a) – $D = 1.44, g = 0$, and (c) – $D = 2, g = 0$. 
FIG. 3. Influence of the fractal dimension of the cluster boundary on the critical current distribution. Curve (a) corresponds to the case of $D = 1.44$, $g = 0$; curve (b) – to $D = 1$, $g = 0$; curve (c) – to $D = 2$, $g = 0$; curve (d) – to $D = 1$, $g = 0.5$; curve (e) – to $D = 2$, $g = 0.5$. 
FIG. 4. The dependence of the average critical current $\bar{I}$ on the fractal dimension at the different values of gamma distribution parameter $g$. 