Reach-averaged flow resistance in gravel-bed streams
Liguo Zhang and Wenguang Luo

ABSTRACT

Previous studies about flow resistance in gravel-bed streams mostly use the log-law form and establish the relationship between the friction factor and the relative flow depth based on field data. However, most established relations do not perform very well when applied to shallow water zones with relatively large roughness. In order to clarify the hydraulic variables defined in the single cross-section, and find the reasons that reflect the instability of flow and uneven boundaries of the river, the concepts of hydraulic variables, such as hydraulic radius, are re-defined in the river reach in the paper. The form drag in the river reach is solved based on a reach-averaged flow resistance model which is developed by force balance analyzing of the water body in the given river reach. The reach-averaged form drag relation is then formulated by incorporating the Einstein flow parameter and a newly derived roughness parameter defined in the river reach. A large number of field data (12 datasets, 780 field measurements) is applied to calibrate and validate the form drag relation. The relation is found to give better agreement with the field data in predicting flow velocity in comparison with existing flow resistance equations. A unique feature of the reach-averaged resistance relation is that it can apply to both deep and shallow water zones, which can be treated as a bridge to link the flow hydraulics in plain rivers and mountain streams.

Key words | Einstein flow parameter, form drag, gravel-bed streams, reach-averaged

HIGHLIGHTS

● For river sections, the variables of each river section are defined first. Skin resistance is solvable (refer to the relevant literature, its formation mechanism, etc.), so the solution obtains the morphological resistance value.
● Probing morphological resistance influence factors.
● With the new method, we have improved the calculation accuracy of the surface resistance of complex riverbeds.

INTRODUCTION

Flow resistance is important for predicting floods and sediment transport in plain and mountain streams. It also plays an increasingly significant role in bank stability, protective engineering design, and aquatic ecosystems (Ferguson 2007). In essence, flow resistance consists of skin friction, which is caused by the fluid viscosity and is proportional to the contact area of the interface between the flow and the boundaries, and form drag, which is mechanically the pressure differences due to flow separation around bed forms or bed structures and is related to the scales of the obstacles (Giménez-Curto & Lera 1996; Nikora et al. 2001, 2004).

Early studies focused on deep water zones with relatively small roughnesses, such as plain rivers. The Manning formula (Manning 1891) is an empirical relation
that accounts for the flow velocity, hydraulic radius, and channel slope in plain rivers. Keulegan (1938) derived the famous log-type resistance formulas for open channel flow based on the theoretical investigations of Prandtl and von Karman and the experimental data of Nikuradse. The resistance relation can be explicitly related to the characteristic roughness height in the log-type formula for fully developed turbulent boundary layers under the condition of high submergence (Katul et al. 2002). Subsequent studies mostly used the log-law formula structure. Considering plain rivers with large bed forms, van Rijn (1984) divided the bed roughness into the grain roughness and the bed form roughness. He related the bed form roughness to the scales of the bed forms (e.g., wavelengths and heights of dunes) and developed resistance relations for this case. However, theoretical solutions of bed forms are still limited due to the complex characteristics of the bed surface and the difficulty of making precise measurements. Einstein & Barbarossa (1952) addressed the idea that bed forms are a function of sediment transport along the river bed. Sediment transport is itself a function of the skin friction $f'$ on the sediment grains and of the representative grain diameter $d$. He then established the relation between the form drag coefficient $f''$ and a dimensionless Einstein flow parameter $d/R'$. Although Einstein’s form drag relation was initially developed for plain rivers, their thinking on form drag production is also applicable to the mountain streams.

Quantifying flow resistance becomes more difficult under the conditions of shallow water zones with relatively large roughnesses, such as gravel-bedded streams, which feature low submergence, steep slopes, wide grain size distributions, different kinds of bed structures (e.g., step-pool system, cascades) and sharp variations in the streamwise direction. Under these conditions, the flow is extremely unsteady and is affected by wakes caused by sediment particles (Nowell & Church 1979; Schmeecke & Nelson 2003) within the boundary layer, which is underdeveloped. To our knowledge, no unified theory for the velocity distribution in the roughness layer has been developed (Smart et al. 2002; Lamb et al. 2008). Several studies (Byrd & Furbish 2000; Wohl & Thompson 2000; Mclean & Nikora 2006; Ferguson 2007) have demonstrated that the velocity distribution deviates from the logarithmic form in the gravel-bed streams. Katul et al. (2002) developed a flow resistance model for shallow water using a mixing layer analogy for the inflectional velocity profile within and just above the roughness layer. The mixing layer theory can apply to the prediction of the roughness of gravel-bed streams. However, the hyperbolic tangent velocity profile is not appropriate for steep slopes. Many researchers (Hey 1979; Smart 1984; Bathurst 1985; Maxwell & Papanicolaou 2001) have used the logarithmic form of the Keulegan formula (1938) to derive semi-empirical resistance relations based on field data. Ferguson (2007) proposed a variable power resistance equation that is asymptotic to the Manning–Strickler formula for large relative flow depth and roughness-layer formulations for small relative flow depth. This equation can be applied to both deep and shallow water zones.

Previous studies have mainly focused on empirically quantifying the form drag in the shallow gravel-bed streams as the skin friction is nearly negligible, and the form drag makes up almost all of the total flow resistance (MacFarlane & Wohl 2003; Rickenmann & Recking 2011). Several researchers (Raju et al. 1983; Shields & Gippel 1995; Green 2006) have applied the ratio of the frontal area of an object to the cross-sectional flow area as the blockage ratio to measure form drag. Aberle et al. (1999) used the standard deviation of the bed surface elevation to predict the friction factor. Pagliara & Chiavaccini (2006) incorporated the effect of the arrangement, shape, and distribution density of boulders into a parameter to measure form drag. Qin & Ng (2012) used a semivariogram approach to derive a steepness parameter to express form drag. Other researchers (Wohl & Thompson 2000; Maxwell & Papanicolaou 2001; Curran & Wohl 2003; Church & Zimmermann 2007) have focused on step-pool channels and established resistance relations based on the step geometry. In addition, Wilcox et al. (2006) established a resistance relation for large woody debris using the cylinder drag method.

The boundaries are always non-uniform in plain and mountain streams. The hydraulic variables defined in the cross-sections can not reflect the non-uniformity of the boundaries on the flow. Moreover, in shallow water zones with relatively large roughnesses, the flow is unsteady, and the boundary varies dramatically. The concept of cross-sectional resistance is actually unreal. In addition, the pressure differences in the streamwise direction are uneven.
Therefore, it is needed to define the hydraulic radius, friction factor, and roughness parameter at the river reach scale to quantify the reach-averaged flow resistance.

Water flow in gravel-bed streams is extremely non-uniform due to various river bed forms/structures, such as step-pools, riffle-pools, cascades, and isolated large particles. It is critical to explore the effects of the spatial heterogeneity and boundary non-uniformity of a given river reach on the three-dimensional hydraulics. The extension of the hydraulic variables from cross-sectional scale to reach scale can reflect the flow unsteadiness and boundary non-uniformity. McLean & Nikora (2006) accounted for the spatial heterogeneity in the near-bed region of sand- or gravel-bed rivers by double-averaging (in space and in time) the continuity and momentum equations, but there are still several difficulties when the double-averaged hydrodynamic equations are applied in the field. Smart et al. (2002) defined the ratio of the overlying water volume to the plan area of the bed as the volumetric hydraulic radius within a reach. Yager et al. (2007) used the volumetric hydraulic radius to calculate bed load transport. Yang (2013) defined the hydraulic radius by the ratio of the water volume to the wetted area within a reach to describe the three-dimensional hydraulics. He argued that the form drag is related to the separation zone or dead zone after the bed forms or structures and proposed the ratio of the volume of the separation zone to the water volume as the new form drag roughness parameter in three-dimensional hydraulics.

In the paper, we develop an alternative resistance relation to predict the mean flow velocity for both deep and shallow water zones in gravel-bed streams. The river reach is taken as the research object and the concepts of hydraulic elements and flow resistance which were defined in cross-sections are extended to river reach. A reach-averaged resistance model is established in consideration of boundary non-uniformity and flow heterogeneity. The cross-sectional hydraulic elements are re-defined in river reach, such as hydraulic radius, friction factor, and roughness parameter. Flow resistance is partitioned into skin friction and form drag according to their different production mechanism by force balance analysis of the water body in the river reach. The skin friction in the reach is estimated by the Manning–Strickler formula and the form drag is modeled by a modified Einstein form drag parameter. 780 field data measurements are applied for calibration and validation of the new proposed resistance relation. The multiple regression analysis method is used to derive the coefficients of the reach-averaged resistance relation. Validation results show the reach-averaged resistance relation fit the data well and behave best among the existing widely used flow resistance relations.

### DATA AND THEORY

#### Dataset

The datasets used in this study, which are collected from 12 different sources, are summarized in Table 1. Parts of these data have been used by Rickenmann & Recking (2011).
Details about data sources are described below. Data of Church & Rood (1985) were from essentially sinuous and meandering channels. Colosimo et al. (1988) measured the data in 43 reaches with quasi-uniform flow conditions. Bathurst (1978) obtained the data from three straight and uniform boulder reaches. Bathurst (1985) measured the data in straight riffles-pool reaches or over plane beds without pools. Thorne & Zevenbergen (1985) monitored the data in two uniform boulder streams. Jarrett (1984) got the data from straight and uniform channel reaches with minimal vegetation. Orlandini et al. (2006) obtained the data in step-pool streams. Griffiths (1981) measured the data in 72 straight wide trapezoidal reaches with little vegetation in New Zealand rivers. Hey & Thorne (1986) obtained the data in the self-formed channel in erodible material with riffle-pools and bank vegetation cover. Wohl & Wilcox (2005) obtained the data in field data from British mountain rivers with slopes of 0.4–4%. Among these data, flow velocity was measured directly by salt dilution methods or a current meter, and could also be concluded from the continuity equation. The sediment grain size distribution was obtained by the Wolman method or sieving of several representative samples. Actually, the existing general approaches for observing and processing field data (Bathurst 1985; MacFarlane & Wohl 2003; Comiti et al. 2007; Nitsche et al. 2012; Zhang et al. 2013) are to use the average values of the measured hydraulic elements of several cross-sections that are preselected in the reach to represent the reach-averaged values. In other words, most of the collected flow data are reach-scaled by averaging several cross-sectional values.

The paper divides the data into two parts, including 512 data measurements for Part A, which cover slopes of 0.004–28.7% and \( R/d_{90} \) values of 0.17–17,935.92, and 268 for Part B, which cover slopes of 0.009–28.7% and \( R/d_{90} \) values of 0.16–74.94. Part A is used for calibration of the new-derived form drag relation in the following paragraph, and Part B is used for validation of the new-derived form drag relation in the following paragraph.

### Representative flow resistance formulae

Great efforts have been made to extend the Manning and Keulegan formula to shallow water zones, including representative studies from Bathurst (1985), Ferguson (2007), and Rickenmann & Recking (2011).

Details about the studies of the representative flow resistance formulae are as follows:

Bathurst (1985) proposed an empirical equation based on field data from British mountain rivers with slopes of 0.4–4%:

\[
\sqrt{f} = \frac{U}{U_*} = 4 + 5.62 \log \left( \frac{R}{d_{84}} \right)
\]  

(1)

where \( U \) is the average flow velocity, \( U_* \) is the bed shear velocity, \( R \) is the hydraulic radius, and \( d_{84} \) is the grain diameter for which 84% of the sediment sample is finer.

Ferguson (2007) developed a variable equation for gravel and boulder bed streams by merging the Manning–Strickler formula for deep water zones and roughness-layer theory for shallow water zones together based on field data at relative submergences from 0.1 to more than 30:

\[
\sqrt{f} = \frac{U}{U_*} = \frac{a_1 a_2 R/d_{84}}{\sqrt{a_1^2 + a_2^2 (R/d_{84})^{5/3}}}
\]  

(2)

where \( a_1 = 6.5 \) and \( a_2 = 2.5 \).
Rickenmann & Recking (2011) also proposed a variable flow resistance equation with a large field dataset using a dimensionless variables analysis approach:

$$\sqrt{\frac{8}{f}} \frac{U}{U_s} = 4.416 \left( \frac{h}{d_{90}} \right)^{1.904} \left[ 1 + \left( \frac{h}{1.283d_{90}} \right)^{3} \right]^{-1.083}$$  \hspace{1cm} (3)

where $h$ is the water depth.

In addition, Einstein & Barbarossa (1952) put forward a dimensionless Einstein flow parameter $\psi$ and proposed a form drag relation:

$$\psi = \frac{\rho_s - \rho_l}{\rho_s} \frac{d}{R S}$$  \hspace{1cm} (4a)

$$\sqrt{\frac{8}{f}} \frac{U}{U_s} = F(\psi) = F\left( \frac{d}{RS} \right)$$  \hspace{1cm} (4b)

where $\rho_s$ and $\rho_l$ are the density of sediment particles and water, $d$ is the sediment grain diameter and taken as $d_{90}$ here as large particles affecting the formation of bed forms or structures and the energy dissipation more, $R'$ is the hydraulic radius related to skin friction, $S$ is the river slope.

To evaluate the performance of the representative equations listed above and give a visualized display, comparison of these equations is plotted against the dataset in Part B.

Figure 1 shows that these equations fit the data well in the deep water zone but badly in the shallow water zone. The theoretical foundations and structures of these equations are basically based on the logarithmic law of the Keulegan formula (1938) for open channel rough flows. Hence, there is no doubt that these classical equations approach each other in the range of large relative depth $R/d$. But with $R/d$ smaller than 10, they diverge as the hydraulic mechanics change in mountain streams and the boundaries are more complex.

The datasets in Figure 2(a) are scattered and it illustrates that the Einstein form drag relation cannot apply in gravel-bed streams very well. Considering slope $S$ may have a significant correlation with $d$, the paper did the step-wise regression analysis on $d/R'$ and $S$ by the SPSS Statistics software. The result shows that the slope $S$ can be excluded and the relative roughness height $d/R'$ is reserved. Figure 2(b) shows that the performance of the Einstein form drag relation improves especially in the range of $d/R'$ larger than 1, which is nearly the shallow water zone. While in the range of $d/R'$ smaller than 1, which is nearly the deep water zone, the data points are still scattered. This implies that the form drag relation proposed by Einstein & Barbarossa (1952) is more suitable for the mountain streams than the plain rivers. This may be ascribed to the fact that the boundary conditions of plain and mountain streams are different. In mountain rivers with small relative flow depth, the sizes of the large particles are similar to the dimensions of the bed structures (Chin 1989, 1999; Yager et al. 2007). Large particles are the dominant nodes in the channel morphology process and have significant impacts on the development of bed structures, such as step-pool systems, cascades, and riffle-pools (Church & Zimmermann 2007). These bed structures are the primary sources of flow energy dissipation. However, in plain rivers with relatively high submergences, the sizes of individual particles are much smaller than the dimensions of the bed forms, which are responsible for a large proportion of the energy dissipation. Consequently, the sediment grains cannot dominate the dimensions and development of bed forms and the energy dissipation in plain rivers. Therefore, Einstein’s form drag relation behaves better in mountain streams than in plain rivers.

On account of this, this paper attempts to propose an alternate resistance relation covering from the lower values to the higher values of $R/d$ in gravel-bed streams.
METHODOLOGY

Figure 3 illustrates the procedure of the methodology of deriving the form drag relation in gravel-bed streams. The paper defines the hydraulic variables in the river reach scale and solves the reach-averaged form drag. Considering the mechanism of the formation of bed forms or structures, the Einstein flow parameter and a relative roughness parameter by Yang (2013) are taken as the main impact factor of the reach-averaged form drag. Then, a reach-averaged form drag relation is obtained by applying the flow data.

Figure 4 shows water flow in a non-uniform river reach with bed forms or structures. Figure 5 shows water separation after bed forms or structures. \( U \) is the average flow velocity in the river reach, \( V_w \) is the volume of water within the reach, and \( A_w \) is the wetted area of the interface between water and the boundary. \( V_{dc} \) is the maximum

Definition of reach-averaged hydraulic variables

In order to describe the effect of boundary non-uniformity on flow resistance, the paper defines the reach-averaged hydraulic variables in a given river reach.
volume of the separation zone after the irregularities, and $L_{dz}$ is the length of the separation zone from the irregularities to the reattachment point.

For a uniform flow, the hydraulic radius $R$ equals the ratio of cross-sectional area $A_{cs}$ to the wetted perimeter of one cross-section $\chi_{cs}$. For a non-uniform flow, we obtain a volumetric hydraulic radius in the following form by extending the concept of cross-sectional hydraulic radius:

$$ R = \frac{V_w}{A_w} \tag{5} $$
Further $R$ can be written in integral and discrete form as follows:

$$R = \frac{V_w}{A_w} = \frac{\int_0^L A_{cs} dx}{\int_0^L \chi_{cs} dx} = \frac{\sum_{i=1}^n A_{csi} l_i}{\sum_{i=1}^n \chi_{csi} l_i} = \frac{\overline{A_{cs}} L}{\overline{\chi_{cs}}} = \frac{A_{cs}}{\chi_{cs}} \quad (6)$$

where $A_{csi}$ is the area of the $i$th cross-section, $\chi_{csi}$ is the wetted perimeter of the $i$th cross-section, $l_i$ is the length of the $i$th micro-reach, $L$ is the length of the reach, $\overline{A_{cs}}$ and $\overline{\chi_{cs}}$ are the mean values of the cross-sectional area and wetted perimeter along a given river reach, respectively.

Some emerging field measurement techniques, such as LIDAR (Feurer et al. 2008), stereo imaging systems (Asada et al. 1999), and underwater vehicles (Galceran et al. 2015), can be applied to obtain three-dimensional boundary information on a given region or the river-immersed topography to obtain the volumetric hydraulic radius $R$. For the reach-averaged flow velocity $U$, salt dilution methods (Lee & Ferguson 2002) are always applied to measure it. If traditional cross-sectional flow hydraulic variables are measured, their averaged values can be taken as reasonable approximations of reach-defined hydraulic variables as long as the measured cross-sections in a river reach are sufficient.

In quasi-uniform flow, the relative flow depth $R/d$ is always taken as a measurement of bed roughness. Whereas for the non-uniform flow, the sediment grain size can not reflect the non-uniformity of the river bed surface. The paper addressed the idea of Yang (2013) and defined a relative roughness parameter in the river reach which is related to the volume of the separation water zone after the bed forms/structures.

The extension of the relative roughness to the river reach scale can be expressed as follows:

$$r_y = \frac{V_{dc}}{V_w} \quad (7)$$

Obviously, the relative roughness parameter $r_y$ means the energy dissipation caused by the irregularities and is an indication of form drag. The modeling of $r_y$ can be seen in the ‘Impact factor’ section.

### Quantifying of reach-averaged form drag

For a given river reach with bed forms/structures, the force balance and resistance partitioning between two cross-sections separated by a distance $L$ can be expressed as follows:

$$\rho V_w g S = \int_{A_w} \tau' dA + F_D = \overline{\tau} A_w + F_D \quad (8)$$

where $\rho$ is the water density, $g$ is the gravitational acceleration, $S$ is the river slope, $\tau'$ is the skin friction, $F_D$ is the form drag resistance, and $\overline{\tau}$ is the averaged skin friction in the river reach.

Then, one can obtain

$$\frac{\rho V_w g S}{A_w} = \overline{\tau} + \frac{F_D}{A_w} \quad (9)$$

According to the Einstein–Barbarossa hydraulic radius partitioning approach (Einstein & Barbarossa 1952), the averaged skin friction can be expressed as follows:

$$\overline{\tau} = \rho g R'S \quad (10)$$

where $R'$ is the hydraulic radius of the river reach which is related to the reach-averaged skin friction.

Lots of researchers (Bray 1979; Parker & Peterson 1980; MacFarlane & Wohl 2003) argued that the Manning–Strickler formula can be used to estimate grain-induced resistance
in gravel-bed streams. Thus, we use the Manning–Strickler formula in the following form to calculate the hydraulic radius $R'$ that is related to skin friction

$$\frac{U}{U_0} = \frac{U}{\sqrt{gR'S}} = 8.1 \left(\frac{R'}{K_s}\right)^{1/6}$$

(11)

where $K_s$ is the equivalent roughness height that is related to skin friction, and $U_0$ is the shear velocity that is related to skin friction. Here, $d_{50}$ was used to represent the grain roughness $K_s$ as was suggested by Keulegan (1938) and MacFarlane & Wohl (2003).

Then, the relation of $R'$ is obtained as follows:

$$R' = \left(\frac{U \cdot d_{50}^{1/6}}{8.1 g^{1/2} S^{1/2}}\right)^{1.5}$$

(12)

We referenced the formula of the Darcy–Weisbach friction factor to define the form drag coefficient $f'$ by the ratio of the form drag to the hydraulic drag force in a river reach as follows:

$$\frac{f'}{8} = \frac{F_D}{\rho U^2 A_w}$$

(13)

According to Equations (8)–(10) and (13) it can be written as follows:

$$\sqrt{\frac{8}{F_D}} = \sqrt{\frac{\rho U^2}{\gamma R S - \gamma R'R}} = \frac{U}{\sqrt{g R S - g R'R}}$$

(14)

Then, we can quantify the reach-averaged form drag coefficient $f''$ using Equations (14) and (12) by applying the reach-averaged flow velocity, hydraulic radius, water surface gradient, and sediment grain size.

**Impact factor of reach-averaged form drag**

As we know, form drag is related to the bed forms or structures in the river reach. The paper held the view that form drag is a function of the formation and scales of bed forms or structures. Two parameters are applied to describe the two impact factors. The Einstein flow parameter $d/\alpha R'$ means skin friction acting on the sediment grains and is an indication of bed forms or structures formation. The relative roughness parameter in the river reach $r_f$ proposed by Yang (2013) means energy dissipation caused by bed forms or structures and is a function of the shape, size, and distribution of the irregularities on the river bed surface.

The parameter $r_f$ should be modeled further for application. According to Yang (2013), the volume of the separation water zone can be expressed as follows:

$$V_{dz} = k_0 A_p L_{dz}$$

(15a)

$$L_{dz} = k_1 \frac{U^2}{2g}$$

(15b)

where $k_0$ and $k_1$ are coefficients related to the shapes of irregularities, and $A_p$ is the projected area of the irregularities perpendicular to the flow direction.

For the irregularities, the form drag can be expressed as follows:

$$F_D = C_d A_p \frac{U^2}{2}$$

(16)

where $C_d$ is the drag coefficient, and can always be taken as 0.4 for the immobile particles.

By substituting (15a), (15b), (16), and (9) into (7), one can obtain

$$r_f = \frac{V_{dz}}{V_w} = \frac{k_0 k_1 S}{C_d} \frac{R - R'}{R}$$

(17)

In a word, $(R - R')/R$ is a measurement of the separation water zone and can depict the characteristics of different bed forms/structures.

Therefore, $d_{90}/R'$ and $(R - R')/R$ are the two controlling factors of form drag production. The paper held the view that the two effects function together and proposed the new form drag relation by merging the two parameters together as follows:

$$\sqrt{\frac{8}{F_D}} = \frac{F(d_{90}/R')^\alpha (R - R')^\beta}{R'^{\alpha - 1}}$$

(18)

where $\alpha$ and $\beta$ are the coefficients that need to be solved.
Combining Equations (14) and (18), the new-derived reach-averaged form drag relation can be expressed as follows:

$$\sqrt{8/\bar{f}} = \frac{U}{\sqrt{gRS - gRS'}} = F\left[\left(\frac{d_{90}}{R'}\right)^{\alpha} \cdot \left(\frac{R - R'}{R}\right)^{\beta}\right]$$  \hspace{1cm} (19)

One can obtain the new form drag relation by calibrating and validating the coefficients in Equation (19) by applying the reach-averaged flow data.

**Calibration of reach-averaged form drag relation**

According to the ‘Dataset’ Part, the whole data are divided into Part A and Part B (Table 1). Part A is used to calibrate the new form drag relation Equation (19) and Part B is for the validation.

In order to solve the coefficients $\alpha$ and $\beta$ in Equation (19), the paper firstly established the optimum correlation between $(8/\bar{f})^{0.5}$ and $(d_{90}/R')^{\alpha} \cdot [(R - R')/R]^{\beta}$. We did logarithm operation to $(8/\bar{f})^{0.5}$ and $(d_{90}/R')^{\alpha} \cdot [(R - R')/R]^{\beta}$ and then use the multiple linear regression analysis in the SPSS Statistics software by applying the dataset of Part A.

It can be obtained that $\alpha = -0.24$, $\beta = -0.70$, and the correlation coefficient $R^2$ is 0.96. The values of $\alpha$ and $\beta$ are both negative. This accords with the law of flow resistance varying with hydraulic variables.

We set $Z = (d_{90}/R)^{0.24} \cdot [(R - R')/R]^{0.70}$ and then plot $\sqrt{8/\bar{f}}$ versus $Z$ in Figure 6.

It is observed from Figure 6 that the new form drag relation is consistent with the data. The entire relation can be approximately decomposed into three zones. By applying the slope clustering method to analyze the relation, we determine that the two turning points of the three zones are at approximately $Z = 0.35$ and 1.5.

We then apply the regression analysis method without considering the few scattered points to obtain the two log-linear relations as follows:

For the zone $Z < 0.35$,

$$\sqrt{8/\bar{f}} = 6.92 \cdot Z^{-0.80}$$  \hspace{1cm} (20a)

which can be rewritten as

$$\sqrt{8/\bar{f}} = 6.92 \cdot \left(\frac{d_{90}}{R'}\right)^{0.24} \cdot \left(\frac{R - R'}{R}\right)^{0.70}^{-0.80}$$

$$= \frac{2.6 \cdot U^{0.29} \cdot R^{0.56}}{d_{90}^{0.14} \cdot S^{0.14} \cdot (R - R')^{0.56}}$$  \hspace{1cm} (20b)

![Figure 6](http://iwaponline.com/jwcc/article-pdf/12/5/1580/923523/jwc0121580.pdf)

**Figure 6 | Relationship between $(8/\bar{f})^{0.5}$ and $Z$**
For $Z > 1.5$,

$$\sqrt{\frac{8}{T^2}} = 6.85 \cdot Z^{-1.8} \quad (21a)$$

which can be rewritten as

$$\sqrt{\frac{8}{T^2}} = 6.85 \cdot \left( \frac{d_{90}}{R'} \right)^{0.24} \cdot \left( \frac{R - R'}{R} \right)^{0.70} \cdot Z^{-1.8}$$

$$= \frac{0.76 \cdot U^{0.65} \cdot R_{1.26}}{d_{90}^{0.32} \cdot S^{0.32} \cdot (R - R')^{1.26}} \quad (21b)$$

Analysis of the exponential coefficients of the hydraulic variables on both sides of the two rewritten relations (Equations (20b) and (21b)) demonstrates that the form drag relations are not auto-correlated.

To obtain a relation for the entire zone, we add the two log-linear relations together as Ferguson (2007) did

$$f'' = \frac{Z^{1.6}}{6.92^2} + \frac{Z^{3.6}}{6.85^2} \quad (22a)$$

Then,

$$\sqrt{\frac{8}{T^2}} = \frac{6.92 \cdot 6.85}{Z^{0.8} \cdot \sqrt{6.85^2 + 6.92^2 \cdot Z^2}} \quad (22b)$$

where $Z = (d_{90}/R')^{0.24} \cdot [(R - R')/R]^{0.70}$, as was described above, and the hydraulic radius that is related to the skin friction $R'$ can be calculated by Equation (12), which is written as follows:

We now obtain the generalized relation of the form drag which can be applicable to different regions.

In open channel flow, three basic hydraulic parameters dominate the flow mechanics: the flow velocity $U$, hydraulic radius $R$, and river slope $S$, under the condition that the characteristic sediment grain size distributions or other topographic information are additionally known. Then, the full flow information can be solved based on two of the three basic parameters by applying the generalized relation (Equation (22b)). Therefore, Equation (22b) is actually a type of hydraulic geometric relation of gravel-bed streams.

The parameter $Z = (d_{90}/R')^{0.24} \cdot [(R - R')/R]^{0.70}$ is not a simple and direct indicator. In order to ensure the application ranges of the two log-linear resistance relations, by considering the formulation structure of the parameter $Z$, we attempt to search for the relation between the parameter $Z$ and the relative flow depth $R/d_{90}$.

In Figure 7, the relative flow depth $R/d_{90}$ versus the parameter $Z$ is plotted by applying the datasets of Part A.

As can be seen in Figure 7, the relative flow depth $R/d_{90}$ is roughly negatively related to the parameter $Z$. From the correlation of $R/d_{90}$ against $Z$, one can know that when $Z$ is smaller than 0.55, $R/d_{90}$ is larger than 9; when $Z$ is larger than 3.5, $R/d_{90}$ is smaller than 0.8.

In other words, Equation (20) applies to the deep water zone, that is, the relative flow depth $R/d_{90}$ is larger than 9. Equation (21) applies to the shallow water zone, that is, the relative flow depth $R/d_{90}$ is smaller than 0.8.

Validation of reach-averaged form drag relation

In order to test the applicability of the form drag relation Equation (22b), the datasets of Part B are used. The parameter $Z$ can be calculated by applying Equation (12) using datasets of Part B. The form drag coefficient $f''$ can be calculated by applying Equations (14) and (12) using data Part B. Then, the relationship of $(8/f'')^{0.5}$ against the parameter $Z$ is plotted in Figure 8.

---

**Figure 7** | Relationship between $R/d_{90}$ and $Z$
Figure 8 shows that the form drag relation Equation (22b) derived from the datasets of Part A fits the datasets of Part B very well.

Now, Table 2 lists the important variables of the equations in the paper.

### RESULTS AND DISCUSSION

#### Present and existing resistance formulae evaluation results

In order to evaluate the performance of the present resistance relation and existing representative resistance formulae, we examined the applicability of predicting flow velocity by Bathurst (1985), Ferguson (2007), Rickenmann & Recking (2011), and the new-derived form drag relation by applying the datasets of Part B.

Then, the datasets of Part B were used to verify the form drag relation Equation (22b) by comparing the predicted flow velocities with the measured ones. Considering the fact that flow velocities cannot be calculated from Equation (22b) explicitly, we programmed an iteration procedure to solve flow velocities. The iteration code is provided in the paper. Figure 9 shows that the predicted flow velocities $U_p$ fit with the measured flow velocities $U_m$ well, and the correlation coefficient $R^2 = 0.87$. Most data points are distributed in the range of $R_d = 0.5–2$. The examination indicates that the generalized form drag relation derived in this study for

| Table 2 | Summary of some important variables in the equations |
|---------|------------------------------------------------------|
| Variables | Meaning |
| $U$ | The average flow velocity in the river reach |
| $V_w$ | The volume of water within the reach |
| $A_w$ | The wetted area of the interface between water and the boundary |
| $V_{dz}$ | The maximum volume of the separation zone after irregularities |
| $A_p$ | The projected area of irregularities perpendicular to the flow direction |
| $L_{dz}$ | Length of the separation zone from irregularities to the reattachment point |
| $S$ | The river slope |
| $R$ | The hydraulic radius |
| $A_{cs}$ | Cross-sectional area |
| $x_{cs}$ | Wetted perimeter of one cross-section |
| $r_y$ | The relative roughness in the river reach |
| $\dot{\tau}$ | The skin friction |
| $F_D$ | The form drag resistance |
| $\dot{\tau}$ | The averaged skin friction in the river reach |
| $R'$ | The hydraulic radius of the river reach which is related to the reach-averaged skin friction |
| $K_s'$ | The equivalent roughness height that is related to skin friction |
| $U_*$ | The shear velocity that is related to skin friction |
| $d_{xx}$ | The grain diameter for which xx% of the sediment sample is finer |
| $f''$ | The form drag coefficient |
| $\alpha$ | Coefficient in the new-derived form drag relation |
| $\beta$ | Coefficient in the new-derived form drag relation |

Figure 9 | Comparison of the predicted velocities by Equation (22b) and the measured ones.
Reach-averaged flow resistance in gravel-bed streams is convincing for predicting the flow velocity.

The other three flow resistance relations are also tested by applying the datasets of Part B as a comparison.

Figure 10 plots the predicted flow velocities calculated by the flow resistance relation of Bathurst (1985) versus the measured ones. The validation results indicate that the measured flow velocities $U_m$ are consistent with the calculated ones $U_p$. The correlation coefficient $R^2 = 0.76$.

Ferguson (2007) developed a resistance relation for both deep and shallow water zones, and Rickenmann & Recking (2011) concluded that it has the highest accuracy in predicting the average flow velocity among the many resistance equations that they reviewed. Figure 11 shows the validation results of the Ferguson (2007) resistance relation with the datasets of Part B. Graphically, the performance of Ferguson (2007) is close to that of the present form drag relation Equation (22b). The correlation coefficient $R^2 = 0.84$.

Figure 12 demonstrates the predicted flow velocities by flow resistance relation of Rickenmann & Recking (2011) versus the measured ones. The verification indicates that the measured flow velocities $U_m$ are in accordance with the predicted ones $U_p$. The correlation coefficient $R^2 = 0.79$.

It can be seen from the figures above that the validation results of the existing flow resistance formulae are similar to those of the resistance relation proposed in this study. To quantitatively compare the performance of these resistance relations, their discrepancy ratio ($R_d$), mean normalized error (MNE), and average geometric deviation (AGD) were computed and listed in Table 5.

The discrepancy ratio $R_d$ is equal to the ratio of the predicted flow velocities $U_p$ to the measured ones $U_m$. The mean normalized error MNE is defined as follows:

$$MNE = \frac{1}{N} \sum_{i=1}^{N} \frac{|U_p - U_m|}{U_m}$$

(23)

The average geometric deviation AGD is defined as follows:

$$AGD = \left[\prod_{i=1}^{N} \max \left(\frac{U_p}{U_m}, \frac{U_m}{U_p}\right)\right]^{1/N}$$

(24)
Generally, the formula performs better if the discrepancy ration $R_d$ in each range is bigger. Conversely, the formula performs better if the parameters MNE and AGD are smaller.

One may notice that the present form drag relation performs best in every index except the discrepancy ratio $R_d$ in the range of 0.8–1.25. Such is the case with the Ferguson (2007) resistance relation: it performs best among the other three existing resistance formulae, except for the indexes of $R_d$ in ranges of 0.8–1.25 and 0.67–1.5. Generally speaking, the resistance relation of Rickenmann & Recking (2011) behaves a little better than that of Bathurst (1985).

It can be seen from the figures that Bathurst (1985) behaves best enough with the datasets of Griffiths (1981), Hey & Thorne (1986), and Bathurst (1985), while worse with the datasets of Zhang et al. (2013) and Wohl & Wilcox (2005). The $R_d$ in ranges of 0.8–1.25 and 0.67–1.5 of Bathurst (1985) are also sufficiently bigger and this means the data gather tightly nearby $R_d = 1.0$.

One may figure out the reason from the formula structure of the Bathurst (1985) resistance relation and the relative flow depth $R/d_{94}$ ranges easily. The Bathurst (1985) resistance relation is basically based on the logarithmic law of the Keulegan formula (1938) for rough open channel flows and therefore a better fit for the larger relative flow depth.

Without consideration of the data of Bathurst (1985), the relative flow depth $R/d_{94}$ of the datasets of Griffiths (1981) and Hey & Thorne (1986) reaches nearly two orders of magnitude. This indicates that these data are from the deep water zone and the log-law could apply too. However, the relative flow depths $R/d_{94}$ of the datasets of Zhang et al. (2013) and Wohl & Wilcox (2005) are smaller than 10 and too shallow to apply the log-law.

For the resistance relation of Ferguson (2007), Rickenmann & Recking (2011), and the present study, they all try to cover the ranges of deep and shallow water zones. As a result, they may behave similarly for data with different ranges of relative flow depth.

Considering the relatively complicated form of the new form drag relation Equations (20b) and (21b), we attempt to simplify Equations (20b) and (21b) to explore the physical essence behind the new form drag relation.

### Discussion of the computational accuracy

As is illustrated above, the resistance relation for all datasets is clearly divided into three segments, which are applicable to the deep water zone, the transition zone, and the shallow water zone, respectively. To be specific, when $Z < 0.35$, that is $R/d_{90} > 9$, the form drag relation Equation (20b) can be rewritten into the following form according to Equation (14):

\[
\frac{U}{\sqrt{gS(R - R)}} = \frac{6.02}{(d_{90} 0.14)} (R - R) 0.06^{1/0.71}
\]

That is,

\[
\frac{U}{\sqrt{gRS}} = 6.02 \left( \frac{R}{d_{90}} \right) 0.2^{1/0.71}
\]

This can be simplified as $(R - R)^{0.06}$ as follows:

\[
\frac{U}{\sqrt{gRS}} = 6.02 \left( \frac{R}{d_{90}} \right) 0.2^{1/0.71}
\]
Similarly, when \( Z > 1.5 \), that is, \( R/d_90 < 0.8 \), the form drag relation Equation (21b) can be rewritten into the following form according to Equation (14):

\[
\frac{U}{\sqrt{gS(R - R')}} = \frac{0.76 \cdot U^{0.65} \cdot R^{1.26}}{d_90^{0.32} \cdot \left( S^{0.32} \cdot (R - R')^{1.26} \right)}
\]  

(27a)

That is,

\[
\frac{U}{\sqrt{gRS}} = 3.7 \left( \frac{R^{1.08}}{d_90^{0.32} \cdot (R - R')^{0.76}} \right)^{1/0.35}
\]  

(27b)

Because \( Z > 1.5 \) and \( R/d_90 < 0.8 \), the form drag relation is suitable for zones of large roughness and shallow water, as in the mountainous region. Skin friction is always considered to be negligible in mountain streams; thus, the hydraulic radius that is related to the skin friction \( R_0 \) is also much smaller than the hydraulic radius \( R \).

Because \( R - R' \) approximates \( R \),

\[
\frac{U}{\sqrt{gRS}} = 3.7 \left( \frac{R}{d_90} \right)^{0.91}
\]  

(28)

Now we derive two simple form resistance relations based on the relative flow depth (\( R/d_90 \)) parameter for deep and shallow water zones, respectively.

It is known that the resistance equation of Ferguson (2007) is obtained by adding the resistance relations of the deep and shallow water zones together. The resistance relations for the deep and shallow water zones are as follows:

For deep water zone,

\[
\frac{U}{\sqrt{gRS}} = 8.2 \left( \frac{R}{d_90} \right)^{1/2}
\]  

(29a)

For shallow water zone,

\[
\frac{U}{\sqrt{gRS}} = 2 \left( \frac{R}{d_90} \right)
\]  

(29b)

The two-segmented flow resistance relations that were derived in this study for deep and shallow water zones approximate to those in Ferguson (2007). This is also the reason that the results of the form drag relation of this study and the resistance equation of Ferguson (2007) are close. Figure 13 exhibits the comparison results of the segmented resistance relations for deep and shallow water zones of the present study and Ferguson (2007). It can be concluded that in the deep water zone, the flow resistance calculated by the form drag relation of this study is a little bigger than that calculated by Ferguson (2007). In the shallow water zone the opposite is the case.

This paper uses the idea of form drag production that was proposed by Einstein & Barbarossa (1952) and then develops a new form drag relation by incorporating the Einstein flow parameter and the relative roughness parameter in a river reach by Yang (2013). The validation demonstrates that the new relation fits the field data well. In contrast to previous resistance equations, this study mainly contributes an alternate resistance equation that makes improvements in predicting the flow velocity. The idea of modeling form drag that is used in this paper is significant. The paper shows a new river reach-scaled perspective to investigate the flow resistance within a reach and propose integral and discrete forms of the hydraulic variables in a river reach.

In addition, this paper extends the understanding of the production of form drag by Einstein & Barbarossa (1952) to both plain and mountain rivers. The scale relationship between the sediment grain sizes and the bed forms or bed
structures in plain rivers is different from that in mountain rivers. This difference makes the form drag impact factors of plain and mountain rivers a little different. In essence, the difference in the relative flow depth $R/d$ is the underlying reason for the different flow regimes in plain and mountain rivers. We developed a unified form drag relation that can be applied to both plain and mountain rivers and the new form drag relation can be considered an attempt to link the flow hydraulics in mountain streams with those in plain rivers.

**CONCLUSIONS**

(1) This paper uses large sets of field data with slopes ranging from 0.004 to 28.7% to develop a new alternate form drag relation that is appropriate for both plain and mountain rivers. The concepts of traditional hydraulic elements are extended from cross-sectional to river reach scale. The hydraulic radius, relative roughness height, flow resistance, and form drag resistance coefficients are defined in a given river reach to consider the non-uniformity of the boundary. The reach-averaged flow resistance model is founded by force balance analysis of the water body in the river reach.

(2) Skin friction and form drag were solved separately based on their different production mechanisms. Reach-averaged form drag is formulated by applying the Einstein flow parameter and the relative roughness parameter in a river reach by Yang (2013). The paper analyzed the relation between the form drag production and river bed topography in plain rivers and mountain streams based on the idea of Einstein & Barbarossa (1952). The characteristics of the flow and river bed surface differ remarkably in plain rivers and mountain streams. This distinguishes the impact factors of flow resistance in plain rivers from those in mountain streams. It is concluded that the shapes, sizes, and distributions of the bed forms/structures have significant effects on fluid hydraulics and should be considered.

(3) A new form drag relation is derived from a multivariable regression analysis by applying a large set of field data. The validation results show that the new obtained form drag relation is consistent with the field data and has good agreement in both deep and shallow water zones, which demonstrates the inherent uniformity in the resistance mechanism of plain and mountain rivers.

**DATA AVAILABILITY STATEMENT**

All relevant data are available from an online repository or repositories (https://github.com/luowengg2020/Reach-averaged-flow-resistance-in-gravel-bed-streams-data).

**REFERENCES**

Aberle, J., Dittrich, A. & Nestmann, F. 1999 Description of steep stream roughness with the standard deviation $S$. In Proc., XXVIII IAHR Biennial Congress, Graz, Austria. on CDROM. Asada, M., Tanaka, T. & Hosoda, K. 1999 Visual tracking of unknown moving object by adaptive binocular visual servoing. In: Multisensor Fusion and Integration for Intelligent Systems, IEEE/SICE/RSJ International Conference on. IEEE, Taipei, Taiwan, pp. 249–254. Bathurst, J. C. 1978 Flow resistance of large-scale roughness. J. Hydraul. Div. Am. Soc. Civ. Eng. 104 (HY4), 1587–1603. Bathurst, J. C. 1985 Flow resistance estimation in mountain rivers. J. Hydraul. Eng. 111 (4), 625–643. Bray, D. I. 1979 Estimating average velocity in gravel-bed rivers. J. Hydraul. Div. 105 (9), 1103–1122. Byrd, T. C. & Furbish, D. J. 2000 Magnitude of deviatoric terms in vertically averaged flow equations. Earth Surf. Process. Landforms 25, 319–328. Chin, A. 1989 Step pools in stream channels. Prog. Phys. Geogr. 13 (3), 391–407. Chin, A. 1999 On the origin of step-pool sequences in mountain streams. Geophys. Res. Lett. 26 (2), 231–234. Church, M. & Rood, K. 1985 Catalogue of Plain River Channel Regime Data. Dept. of Geogr., Univ. of B. C., Vancouver, BC, Canada, p. 99. Church, M. & Zimmermann, A. 2007 Form and stability of step-pool channels: research progress. Water Resour. Res. 43 (3), 10–1029. Colosimo, C., Copertino, V. & Veltri, M. 1988 Friction factor evaluation in gravel-bed rivers. J. Hydraul. Eng. 114, 861–876. Comiti, F., Mao, L., Wilcox, A., Wohl, E. E. & Lenzi, M. A. 2007 Field derived relationships for flow velocity and resistance in step-pool streams. J. Hydrol. 340, 48–62. Curran, J. H. & Wohl, E. E. 2005 Large woody debris and flow resistance in step-pool channels, cascade range, Washington. Geomorphology 51 (1–3), 141–157. Einstein, H. A. & Barbarossa, N. L. 1952 River channel roughness. Trans. Am. Soc. Civ. Eng. 117, 1121–1146. Ferguson, R. 2007 Flow resistance equations for gravel- and boulder-bed streams. Water Resour. Res. 43 (5), 687–696.
Feurer, D., Bailly, J. S., Puech, C., Lecoarer, Y. & Viala, A. A. 2008 Very-high-resolution mapping of river-immersed topography by remote sensing. *Prog. Phys. Geogr.* 32 (4), 403–419.

Galceran, E., Campos, R., Palomeras, N., Ribas, D., Carreras, M. & Rida, P. 2015 Coverage path planning with real-time replanning and surface reconstruction for inspection of three-dimensional underwater structures using autonomous underwater vehicles. *J. Field Robotics* 32, 952–983.

Giménez-Curto, L. A. & Lera, M. A. C. 1996 Oscillating turbulent flow over very rough beds. *J. Geophys. Res. Oceans* 101 (C9), 20,745–20,758.

Green, J. C. 2006 Effect of macrophyte spatial variability on channel resistance. *Adv. Water Resour.* 29 (3), 426–438.

Griffiths, G. A. 1981 Flow resistance in coarse gravel bed rivers. *J. Hydraul. Div., ASCE* 107 (HY7), 899–918.

Hey, R. D. 1979 Flow resistance in gravel-bed rivers. *J. Hydraul. Div.* 105 (HY9), 365–379.

Hey, R. D. & Thorne, C. R. 1986 Stable channels with mobile gravel beds. *J. Hydraul. Eng.* 112, 671–689.

Jarrett, R. D. 1984 Hydraulics of high-gradient streams. *J. Hydraul. Eng.* 110, 1519–1539.

Katul, G., Wiberg, P., Albertson, J. & Hornberger, G. 2002 A mixing layer theory for flow resistance in shallow streams. *Water Resour. Res.* 38 (11), 32–31.

Keulegan, G. H. 1938 Laws of Turbulent Flow in Open Channels, Vol. 21. National Bureau of Standards, USA, pp. 707–741.

Lamb, M. P., Dietrich, W. E. & Venditti, J. G. 2008 Is the critical shields stress for incipient sediment motion dependent on channel-bed slope? *J. Geophys. Res.* 113F (2), F02008.

Lee, A. J. & Ferguson, R. I. 2002 Velocity and flow resistance in step geometry. *Geomorphology* 46, 59–71.

Macfarlane, W. A. & Wohl, E. 2003 Influence of step composition on step geometry and flow resistance in step-pool streams of the Washington cascades. *Water Resour. Res.* 39 (2), 237–245.

Manning, R. 1891 On the flow of water in open channels and pipes. *Transactions of the Institution of Civil Engineers of Ireland* 20, 161–207.

Maxwell, A. R. & Papanicolaou, A. N. 2001 Step-pool morphology in high-gradient streams. *Int. J. Sediment. Res.* 16, 380–390.

McLean, S. R. & Nikora, V. I. 2006 Characteristics of turbulent unidirectional flow over rough beds: double-averaging perspective with particular focus on sand dunes and gravel beds. *Water Resour. Res.* 42 (10), 277–305.

Nikora, V., Goring, D., MacEwan, I. & Griffiths, G. 2001 Spatially averaged open-channel flow over rough bed. *J. Hydraul. Eng.* 127 (2), 123–135.

Nikora, V., Koll, K., MacEwan, I., McLean, S. & Dittrich, A. 2004 Velocity distribution in the roughness layer of rough-bed flows. *J. Hydraul. Eng.* 130 (1), 1036–1042.

Nitsche, M., Rickenmann, D., Kirchner, J. W., Turowski, J. M. & Badoux, A. 2012 Macroroughness and variations in reach-averaged flow resistance in steep mountain streams. *Water Resour. Res.* 48 (12), 2211–2240.

Nowell, A. R. M. & Church, M. 1979 Turbulent flow in a depth-limited boundary layer. *J. Geophys. Res.* 88 (C8), 4816–4824.

Orlandini, S., Boaretto, C., Guidi, V. & Sfondrini, G. 2006 Field determination of the spatial variation of resistance to flow along a steep Alpine stream. *Hydrol. Process.* 20, 3897–3913.

Pagliara, S. & Chiavaccini, P. 2006 Flow resistance of rock chutes with protruding boulders. *J. Hydraul. Eng.* 132 (6), 545–552.

Parker, G. & Peterson, A. W. 1980 Bar resistance of gravel bed rivers. *J. Hydraul. Div., ASCE* 106, 1559–1575.

Qin, J. & Ng, S. L. 2012 Estimation of effective roughness for water-worked gravel surfaces. *J. Hydraul. Eng.* 138 (11), 923–934.

Raju, K. G. R., Rana, O. P. S., Asawa, G. L. & Pillai, A. S. N. 1983 Rational assessment of blockage effect in channel flow past smooth circular cylinders. *J. Hydraul. Eng.* 21, 289–302.

Recking, A., Frey, P., Paquier, A., Belleudy, P. & Champagne, J. Y. 2008 Feedback between bed load transport and flow resistance in gravel and cobble bed rivers. *Water Resour. Res.* 44 (5), 50–50.

Rickenmann, D. & Recking, A. 2011 Evaluation of flow resistance in gravel-bed rivers through a large field data set. *Water Resour. Res.* 47 (7), 209–216.

Schmeckle, M. W. & Nelson, J. M. 2003 Direct numerical simulation of bedload transport using a local, dynamic boundary condition. *Sedimentology* 50, 279–301.

Shields, F. D. & Gippel, C. J. 1995 Prediction of effects of woody debris removal on flow resistance. *J. Hydraul. Eng.* 121, 341–354.

Smart, G. M. 1984 Sediment transport formula for steep channels. *J. Hydraul. Eng.* 110 (3), 267–276.

Smart, G. M., Duncan, M. J. & Walsh, J. M. 2002 Relatively rough flow resistance equations. *J. Hydraul. Eng.* 128 (6), 568–578.

Thorne, C. R. & Zevenbergen, L. W. 1985 Estimating mean velocity in mountain rivers. *J. Hydraul. Eng.* 111 (4), 612–624.

van Rijn, L. C. 1984 Sediment transport, part III: bed forms and plain roughness. *J. Hydraul. Eng.* 110 (12), 1733–1754.

Wilcox, A. C., Nelson, J. M. & Wohl, E. E. 2006 Flow resistance dynamics in step-pool channels: 2. partitioning between grain, spill, and woody debris resistance. *Water Resour. Res.* 42 (5), 387–403.

Wohl, E. E. & Thompson, D. M. 2000 Velocity characteristics along a small step-pool channel. *Earth Surf. Process. Landforms* 25 (4), 353–367.

Wohl, E. E. & Wilcox, A. 2005 Channel geometry of mountain streams in New Zealand. *J. Hydrol.* 300 (1), 252–266.

Yager, E. M., Kirchner, J. W. & Dietrich, W. E. 2007 Calculating bed load transport in steep boulder bed channels. *Water Resour. Res.* 43 (7), 256–260.

Yang, S. Q. 2013 A simple model to extend 1-D hydraulics to 3-D hydraulics. In *35th IAHR World Congress*, Chengdu, China, pp. 1–12.
Zhang, K. 2012 *Research on Streambed Structure and Bed Load Transport and Their Influence on Fluvial Morphology* (in Chinese). Tsinghua University.

Zhang, L. G., Fu, X. D., Guo, D. W. & Li, T. J. 2013 Flow resistance in gravel-boulder-bed streams. *J. Hydraul. Eng.* **44** (006), 680–686 (in Chinese).

First received 4 March 2020; accepted in revised form 20 August 2020. Available online 12 October 2020