Generation and detection of high-frequency gravitational waves in dielectrics, excited by the intense ultrashort laser pulse

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Abstract. The generation and detection of scalar or pseudoscalar high-frequency gravitational waves in some dielectrics, excited by ultrashort intense laser pulses are proposed. Hidden photons and axion particles are elemental excitations of such gravitational waves. The properties of longitudinal electromagnetic waves in dielectric media have been described from the point of view of photon-hidden photon conversion processes in these crystals. Scalar bound photonic states in dielectric media are analyzed. The experimental schemes for photon-hidden photon conversion with taking part in longitudinal electromagnetic excitations in media have been proposed. Hertz gravitational laboratory experiments with the illumination of the dielectric media by intense ultrashort laser pulses and the "Light Sching Trough Wall" scheme using are analyzed.

1. Introduction

The task of the generation and detection of gravitational waves became actual after classical Einstein’s work predicted such opportunity on the base of the general relativity theory. The interest in this problem has increased significantly after recently direct detection of gravitational waves from cosmic object [1]. The experimental results, presented in [1], are the detection of gravitational waves at relatively low frequencies (10–100 Hz), emitted by stars. In this work the laser interferometer technique based on the Michelson interferometer scheme, first proposed and theoretically justified in [2,3], has been used. In accordance with the gravitational wave emission theory, the energy loss of the object, emitting gravitational waves, is proportional to the frequency of the sixth powered degree:

$$\frac{dE}{dt} = -\frac{G}{45c^3} \mu \left(\frac{\delta^4}{ct} D_{\alpha\beta}\right)^2.$$  

(1)

Here $D_{\alpha\beta} = \int \mu(3x^\alpha x^\beta - r^2 \delta_{\alpha\beta})dV$ – tensor of quadrupole mass momentum. Thus, the intensity of gravitational waves emission sharply increases with the frequency enhancing:

$$I(\omega) = A \omega^6 (\omega = 2\pi f).$$  

(2)

So, it is interesting to investigate the possibility of the generation and detection of the high-frequency gravitational waves. The theory, developed in [4–7], predicts the opportunity of high-frequency gravitational waves emission from some stars at the presence of the strong magnetic field inside of this star as the result of the photon-graviton conversion. Elemental excitations of the space, emerging as a
result of star gravitational emission in the strong magnetic field, corresponding to axions (pseudoscalar particles), predicted by high energy elemental particle theory as the result of parity law violation [8]. The theory [9–11] predicts also the existence of hidden photons–scalar photon–like particles with nonzero rest mass. Hidden photons are bound states of two photons with opposite polarization vectors directions. The conversion of electromagnetic emission with frequency $\omega_0$ into hidden photons waves with frequency $2\omega_0$ is predicted at the presence of intensive electromagnetic emission in the space, without application of the additional constant magnetic field. In this work Hertz laboratory, gravitational experiments in different dielectric media are proposed for high-frequency gravitational wave generation and detection. The efficiency of photon–axion and photon–hidden photon conversion processes in some condensed dielectric media are analyzed.

2. To the theory of gravitational waves in the empty space

The Einstein equations of gravitational waves in the empty space may be written as [6,7]:

$$R_{ik} = \frac{8\pi G}{c^4} \left( T_{ik} - \frac{1}{2} g_{ik} T \right).$$  (3)

At the presence of electromagnetic waves $T=0$, and we have:

$$R_{ik} = \frac{8\pi G}{c^4} T_{ik}. \quad (4)$$

After changing of energy–momentum tensor $T_{ik}$ by Maxwell stress tensor $\sigma_{\alpha\beta}$ the relation for spatial components may be presented as:

$$(\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2}) h_{\alpha\beta} = -\frac{16\pi G}{c^4} \sigma_{\alpha\beta}; \alpha, \beta = 1, 2, 3.$$ (5)

Here $g_{ik} = g^{(0)}_{ik} + h_{ik}; i, k = 0, 1, 2, 3$ is the metric tensor of weak gravitational waves, described by $h_{ik}$ and $g^{(0)}_{ik}$; $G$ and $c$ are fundamental constants; $E$ and $H$ are corresponding electric and magnetic field stresses value. If the flat gravitational wave propagates along the $x$-axis, the constant magnetic field $\vec{H} = \vec{e}_z H_0$ is directed along the $y$-axis, and electromagnetic field with wave vector $k_x$ is described as:

$$\vec{E} = \vec{e}_z E_{0\alpha} \cos(\omega_0 t - k_x x); \vec{H} = \vec{e}_x (H_0 - H_{0\alpha} \cos(\omega_0 t - k_x x)).$$ (6)

In this case, takes place:

$$\sigma_{22} = \frac{1}{8\pi} \left[ H_0^2 \left( \frac{1}{2} H_{0\alpha} - 2H_0 H_{0\alpha} \cos(\omega_0 t - k_x x) + \frac{1}{2} H_{0\alpha} \cos(2\omega_0 t - 2k_x x) \right) \right]$$ (7)

Taking into account only one term in (5), oscillating with frequency $\omega_0$, we have:

$$\left( \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{c^2 \partial t^2} \right) h_{22} = \frac{4G}{c^4} H_0 H_{0\alpha} \cos(\omega_0 t - k_x x). \quad (8)$$

After the transition to the complex presentation of the fields ($h_{22} \rightarrow h_{22}^B = h_{22} \exp(\pm i(k_x x - \omega_0 t)); H_{0\alpha} \cos(\omega_0 t - k_x x) \rightarrow \hat{H}_{0\alpha}^B = H_{0\alpha} \exp(\pm i(k_x x - \omega_0 t))$) we have:

$$\left( \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{c^2 \partial t^2} \right) \hat{h}_{22}^B = g H_0 \hat{H}_{0\alpha}^B; g = \frac{4G}{c^4}.$$ (9)

The solution of (4) may be presented as:
\[ h_{22}^{+} = \exp(-i\omega_{0}t) \int dx' \frac{1}{4\pi} \frac{\exp[\pm ik_{g}(x-x')] \exp[i(k_{q}x - \omega_{q}t)]}{(x-x')^{2}} gH_{0}H_{0a}. \]  

(10)

Accordingly, we have:
\[ h_{22}^{+}(x,t) = iH_{0} \frac{gH_{0a}L}{2k_{g}} F(q) \exp(i(k_{g}x - \omega_{q}t)); F(q) = \frac{\sin qL/2}{qL/2}; q = k_{p} - k_{g}. \]  

(11)

Here \( k_{g} \) and \( k_{p} \) are the wave vectors of gravitational and electromagnetic waves correspondingly and \( L \) is the length of the applied magnetic field. Taking into account the wave vector conservation law in elemental photon-graviton conversion processes we have the note, that the relation (9) is close to the corresponding one for photon-axion conversion processes, described before in [8]. Accordingly, we have for the likelihood of \( N_{g} \) gravitons appearance as the result of \( N_{p} \) exciting photon emitted of the light source and resulting photon-graviton conversion process:
\[ P_{p \rightarrow g} = \frac{N_{g}}{N_{p}} = \frac{1}{4} (gH_{0}L)^{2}. \]  

(12)

Here \( N_{p} \) is the number of photons, falling at the unit square during 1 s. Constant \( g = \frac{4G}{c^{4}} \) is dramatically small for empty cosmic space. So, the photon-graviton conversion may have waited from the very large objects, surrounded by enormous strong constant magnetic fields. Conserving in (7) the term \( \frac{1}{2} H_{0a}^{2} \cos(2\omega_{0}t - 2k_{p}x) \), we conclude, that the emission of gravitational waves with frequency \( 2\omega_{0} \), as a result of the two-photon parametric process also in the absence of a static magnetic field may be occurred. In this case in the elemental conversion process, the two photons with frequency \( \omega_{0} \) and opposite polarization vectors convert into a hidden photon (graviton) with \( 2\omega_{0} \) frequency. As for (11) for the likelihood of \( N_{g} \) gravitons appearance as the result of \( N_{p} \) exciting photon emitted of the light source and resulting photon-graviton conversion process it takes place:
\[ P_{p \rightarrow g} = \frac{N_{g}}{N_{p}} = \frac{1}{4} (gH_{0}L)^{2} = \frac{1}{4} (gE_{0a}L)^{2}. \]  

Such probability, calculated for empty space, depends on the intensity of exciting light emission and is very small, taking into account, that \( g = \frac{4G}{c^{4}} \). So, it is useful to analyze the discussed conversion processes in the media.

3. To the theory of gravitational waves in dielectric media

Due to the very high temperature of cosmic objects as stars, the media inside are some types of plasma with spare electrons, protons, and other charged particles. The dispersion law for transversal electromagnetic waves in plasma may be written as:
\[ \omega^{2} = \omega_{p}^{2} + c^{2}k^{2}; \omega_{p}^{2} = \frac{4\pi ne^{2}}{m}. \]  

(14)

Besides transversal electromagnetic waves in plasma, the longitudinal scalar type waves exist with very following dispersion law.
\[ \omega_{l} = \omega_{p}^{2} + s^{2}k^{2}; s << c. \]  

(15)

Longitudinal electromagnetic wave, corresponding \( \varepsilon(\omega_{l}, k) = 0 \) exist also in dielectric crystals as GaP, ZnSe, LiTaO\(_{3}\), and others. Elemental excitations, corresponding to longitudinal electromagnetic waves in plasma and dielectric media we shall call lotons. From (14), (15) we conclude, that the group
velocity of longitudinal and also transversal electromagnetic waves (at small wave vector \( k \)) may be very small. Accordingly, the coefficient \( g = \frac{4G}{c^4} = \frac{4G}{(\frac{d\omega}{dk})^4} \) should essentially larger with comparing to empty space (see relation (9)). Thus photon-graviton conversion efficiency in plasma is essential higher with comparing to empty space. The opportunity of photon-hidden photons (hiton) conversion in plasma of Sun or nuclear reactors has been predicted in works [10, 11]. We present the considerations on such type of opportunity in dielectric media in the laboratory. An idealized model of interaction between longitudinal electromagnetic waves in dielectric media and hidden photons waves of vacuum is analyzed. Corresponding equations, resemble polaritons ones in dielectric media, are:

\[
\frac{\partial^2 u}{\partial t^2} = -\omega_h^2 u + c^2 \frac{\partial^2 u}{\partial x^2} + \omega_h^2 \xi; \\
\frac{\partial^2 \xi}{\partial t^2} = -\omega_h^2 \xi + c^2 \frac{\partial^2 \xi}{\partial x^2} - \frac{\partial^2 u}{\partial t^2}.
\]

(16)

Here \( u(x,t) \) are plasma oscillation coordinates and \( \xi(x,t) \) correspond to hiton waves of vacuum. For flat monochromatic waves \( u(x,t) = u_0 \exp(ikx - \omega t) \), \( \xi(x,t) = \xi_0 \exp(ikx - \omega t) \) and approximations, that \( s=0 \) (in equation (15)), \( \omega_h^2 = 0; \omega_h^2 = \omega_v^2 + \omega_0^2 \), from (16) we obtain dispersion branches of hybrid loton-hiton waves, resembled polariton ones:

\[
\omega_k^2(k) = \frac{(\omega_h^2 + c^2 k^2)}{2}(1 \pm \sqrt{1 - \frac{4c^2 k^2 \omega_0^2}{(\omega_h^2 + c^2 k^2)^2}}).
\]

(17)

At small wave vectors the dispersion \( (k \approx 0) \) for higher loton-hiton branch it takes place:

\[
\omega_0^2 = \omega_h^2 + c^2(1 - \frac{\omega_0^2}{\omega_h^2})k^2; \frac{d\omega}{dk} = \frac{c^2(1 - \frac{\omega_0^2}{\omega_h^2})k}{\omega_h^2 + c^2(1 - \frac{\omega_0^2}{\omega_h^2})k^2}.
\]

(18)

Thus, the group velocity of loton-hiton excitations is very small at \( k \approx 0 \). If longitudinal polar waves are strongly excited as a result of heating (in the case of Sun or nuclear reactor) or intensive laser illumination or Raman Scattering (SR) processes (for non-center symmetric dielectrics as GaP, ZnSe, LiNbO\(_3\), and others) the emission of a hidden photon from the media should take place. For detection of hidden photon waves, the same dielectric crystals as emitters after non-transparent wall should be used.

Another way for hidden photon waves generation and detection is the forming of the bound photonic states in dielectric media during parametric Raman processes [12]. During elemental spontaneous Raman Scattering (see figure.1) the quantum of exciting light with frequency and wave vector \( \omega_0, \vec{k}_0 \) destructs onto Stokes quantum \( \omega', \vec{k}' \) and phonon \( \omega, \vec{k} \). For the anti-Stokes process, the colliding of exciting light quantum \( (\omega_0, \vec{k}_0) \) with phonon \( \omega, \vec{k} \) takes place. The intensity of RS is essential to lower of exciting emission intensity: about \( 10^{-6} \). Figure 2 illustrated spontaneous RS of nonlinear dielectric crystal LiNbO\(_3\). In this spectrum the intensive total symmetrical longitudinal optical A\(_1\) (LO) mode at 872 cm\(^{-1}\) is revealed. In high-density power (about \( 10^7 \) W/cm\(^2\)) of laser emission has been used, the Stimulated Raman Scattering (SRS) was observed. In this case, the intensity A\(_1\) (LO) mode in Raman spectra become comparable to exciting light. Therefore, there is the opportunity of the generation of
hidden photons as a result of photon-hiton hybridization, described before, due to the excitation of longitudinal waves of \( \text{LiNbO}_3 \) crystal.

Figure 1. Elemental spontaneous Stokes (a) and anti-Stokes (b) Raman Scattering processes.

Figure 2. Spontaneous Raman Scattering in \( \text{LiNbO}_3 \) crystal at backward scattering along the polar axis. The wavelength of the exciting line is 785nm.

At the very high intensity of ultrashort laser pulses, many frequencies parametric Stimulated Raman Scattering (Comb Raman Scattering—CRS) may be observed [13]. CRS emission corresponds to the very high-frequency region. As a result, the duration of CRS pulses is essentially shorter with comparing to the excited emission duration. Thus, after focusing on CRS emission in condensed dielectric aromatic substances the two-photon excited luminescence may be recorded. The examples of the CRS spectrum for \( \text{CaCO}_3 \) crystal and two-photon excited luminescence for stilbene powder are presented in figure 3.
Figure 3. Comb Raman Scattering in CaCO$_3$ monocrystal, excited by powerful ultrashort laser pulses of solid-state YAG: Nd$^{3+}$ laser with wavelength $\lambda=532$ nm; at left (391 and 406 nm) - two-photon excited luminescence spectrum in stilbene powders is shown.

For many frequencies parametric SRS, the bound states of photons may be created as the result of optical phonons exchanging between two photons of exciting emission. Hamiltonian of the photons (polaritons) of dielectrics, interacting with optical phonons may be presented as [13, 14]:

$$\hat{H} = \sum_k E_k \hat{a}^{\dagger}_k \hat{a}_k + \sum_k \alpha_k \hat{b}^{\dagger}_k \hat{b}_k + g_3 \sum_{k_i,k_2,k_3} \hat{b}^{\dagger}_{k_i} \hat{a}_{k_2} \hat{a}_{k_3} + g_3 \sum_{k_i,k_2,k_3} \hat{b}_{k_i} \hat{a}^{\dagger}_{k_2} \hat{a}^{\dagger}_{k_3}$$

$$+ g_4 \sum_{k_i,k_2,k_3} \hat{a}^{\dagger}_{k_i} \hat{a}^{\dagger}_{k_2} \hat{a}_{k_3} \hat{a}_{k_3} + ...$$

(19)

Here $E_k$ and $\alpha_k$ are the energies of photons (polaritons) and hitons, $\hat{a}^{\dagger}_k \hat{a}_k$ and $\hat{b}^{\dagger}_k \hat{b}_k$ are creation and annihilation operators of photons (polaritons) and hitons, $g_3$ and $g_4$ are the corresponding coupling parameters. Photon-photon scattering in dielectrics may be illustrated by diagrams, taking into account photon-phonon (Raman) processes. Illustrates four photon scattering (last term in equation 19) and photon-phonon (RS) scattering. In view of the approximations used ($g_3=0$), the Bethe–Salpeter equation for four photons (polaritons) with momentum $Q$ scattering has the solution ($h=1$):

$$G_2(Q,\omega) = \frac{G_{02}(Q,\omega)}{1 - g_3 G_{02}(Q,\omega)}; G_{02}(Q,\omega) = \int \frac{\rho_{02}(Q,\omega) d\omega}{\omega - E + 2i\gamma}\omega.$$  

(20)

Here $\rho_{02}(Q,\omega)$ is the density of two-photonic states. Bound photonic states appear providing that $G_2(Q,\omega)$ function has the pole. We can see that the bound state is true may be created for large enough $g_3$ constant. Such a situation may be realized, for example, if SRS, observed when a high power laser for RS is used excitation. The spectrum of bound states of two photons (polaritons) is described by the relation:

$$I(Q,\omega) = \frac{\text{Im} G_2(Q,\omega)}{\pi}.$$  

(21)

In more common case three particle processes should be taken into account (see equation 19). Taking into account the Dyson equation for interacting photons at $g_3=0$ and $g_3 \neq 0$, we arrive the equation:

$$G_1(\omega) = \frac{G_0^0}{1 - g_3^0 G_0^0 G_1^0(Q,\omega)}.$$  

(22)
4. Experimental setups for photon-hiton conversion as a result of laser excitation of dielectric media

Under high-intensity laser excitation in dielectric media, some stimulated scattering processes as SRS and CRS processes may be observed. Experimental setups for registration SRS and CRS spectra in crystals or liquids are illustrated in figure 4.

Intensive laser emission in dielectric media excites the bound photonic states of two photons, having the same symmetry as hitons due to strong photon-photon interaction. Additional enhance of bound photonic states forming is the Fermi resonance, taking place if some scalar exciton states of dielectric media have the energy level, close to the bound photonic state, having the same symmetry properties as hiton. For the detection of hiton waves, the same sample should be used after the non-transparent wall (see figures 5,6).

Figure 4. Experimental setup for anti-Stokes SRS and CRS spectra recording in crystals and liquids with using of powerful picosecond solid-state YAG: Nd³⁺ laser with wavelength $\lambda = 1064$ nm; 1—laser, 2—plate, 3—lens, 4—spectrometers, 5—computers, 6—sample, 7—fiber tip.

Figure 5. Setup for “Light Shining Through Wall” experiments with monocrystals or liquids with using of powerful picosecond solid-state YAG: Nd³⁺ laser with wavelength $\lambda = 532$ nm; 1—mirrors, 2—active media, 3—nonlinear crystal, 4,6—lens, 5—plate, 7—sample, 8—nontransparent wall, 9—spectrometers, 10—computers.
Figure 6. Setup for “Light Shining Through Wall” experiments with powders or photonic crystals with using of powerful picosecond solid-state YAG:Nd³⁺ laser with wavelength λ=532 nm; 1—mirrors, 2—active media, 3—nonlinear crystal, 4,5,9—lens, 6—holder, 7—sample, 8—nontransparent wall, 10—receiver, 11—computer.

5. Summary
Thus, we have presented the considerations on realizing of conditions for generation of high-frequency scalar or pseudoscalar gravitational waves in media. The symmetry properties of such waves coincide with hidden photon or axion ones. As a result of illumination of some crystals, having longitudinal electromagnetic modes, by powerful ultrashort laser pulses, photon-hidden photon conversion is predicted. Bound photonic states as a result of SRS and CRS processes in dielectric media are analyzed. The experimental schemes as "Light Shining Through Wall" for photon-hidden conversion and vice versa with using of some dielectric media, having longitudinal electromagnetic modes, is proposed. Thus, the conditions for the Gravitational Laboratory Hertz experiment in dielectric media have been found.

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