The Visual Lightcurve of Comet C/1995 O1 (Hale–Bopp) from 1995 to 1999

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Abstract

The great comet C/1995 O1 (Hale–Bopp) presented a remarkable opportunity to study its long-term brightness over four years. We used 2240 observations published in the International Comet Quarterly from 17 observers during 1995 July to 1999 September to create a secular lightcurve. In order to account for observer differences, we present a novel algorithm to reduce scatter and increase precision in a lightcurve compiled from many sources. It is implemented in a publicly available code, ICQSPLITTER, which uses a self-consistent statistical approach. To first order, the comet’s lightcurve approximates an $r^{-1.5}$ response for both pre- and postperihelion distances. The preperihelion data are better fit with a fifth-order polynomial with inflection points at 4.0, 2.6, 2.1, and 1.1 au, some of which are associated with physical changes in the coma. Outbursts may have occurred a few days before perihelion and at ~2.2 and 7.4 au postperihelion. The A$\rho$ values derived from the final magnitudes are consistent with an $r^{-1.5}$ dependence on heliocentric distance and are within a factor of 2–4 of those derived from spectroscopy and narrowband photometry. We present correlation equations for visual magnitudes and CO and H$_2$O production rates that are consistent with the preperihelion brightness increasing due to CO outgassing until about 2.6–3.0 au from the Sun and then are strongly correlated with H$_2$O production rates. We also present two generalized correlation equations that may be useful for observation planning and data analysis with the James Webb Space Telescope and other observatories.

Unified Astronomy Thesaurus concepts: Comets (280); Long period comets (933); Coma dust (2159); Astrostatistics (1882); Light curves (918); Visual observation (1778); Comae (271)

1. Introduction

Comet nuclei are among the best-preserved icy–rocky remnants from the solar system’s formation. Far from the Sun, when comets are inactive, the heliocentric dependence of their bare nuclei’s brightness will follow an $r^{-2}$ response due to the reflection of sunlight off the surface, assuming a constant phase angle, $\theta$, surface area, and albedo.

When comets get close enough to the Sun, the ices incorporated in the nucleus sublime and generate coma of gas and dust. Once they become active, the heliocentric dependence of their brightness changes due to a variety of parameters about which we know very little, including dust particle size distribution, relative contributions of gas and dust, and possibly other sources of energy (Fulle 2004). Coma brightness may change when the mass-loss rate changes, or large particles or grains in the coma fragment into smaller particles without changing the actual mass-loss rate. Moreover, the nucleus’s spin pole orientation will affect a comet’s heating from the Sun (Whipple 1980).

Over human history, most reports of comets are from visual observations of their comae, including some reports of comet 1P/Halley as long as 800 yr ago (e.g., Choi et al. 2018). Estimates of a comet’s visual magnitude are often used to assess its activity level. For comets that produce high amounts of dust, lightcurves can also provide useful information about the bulk behavior of the dust coma, which, when coupled with more rigorously derived values of the dust and images from infrared, will provide observational constraints for nucleus composition models. Changes in slope may pinpoint a significant transition in the comet nucleus’s processing, such as the initiation of distant activity, ramp-up of water-ice sublimation, breakup events, or outbursts (Meech & Svoren 2004a).
One of the most continuous data sets on any comet was the visual secular lightcurve of Halley’s coma during its 1981–1986 apparition compiled from hundreds of CCD “V” magnitude and total visual magnitude estimates and corrected for geocentric distance of the comet (Green & Morris 1987). The lightcurve showed many significant changes in slope, and peaked at ~60 days past perihelion.

A similar opportunity arose with the “great” comet C/1995 O1 (Hale–Bopp), because it was exceptionally bright upon its discovery at ~7 au and easily observed for ~7–8 months with the naked eye, and much longer with binoculars and small telescopes. From 1995 to 1999, dozens of observers dedicated nearly every clear night to recording the visual magnitude of Hale–Bopp and submitted brightness estimates to the *International Comet Quarterly (ICQ)*17 to make them available for further analysis by others.

From these ~15,000 observations of Hale–Bopp archived with the ICQ, we studied a subset of 2240 measurements from 17 highly experienced observers. We ultimately created a final lightcurve using a reduced number of 795 mag from 13 observers. In this paper, we describe our procedures for removing points, correcting for geocentric distance and phase angle, and our new self-consistent statistical approach for combining different observers and its efficacy. This reduction process increases the accuracy and precision of a comet lightcurve.

We present this algorithm through the analysis of the comet Hale–Bopp, which was the subject of the largest observational campaign in history. We also highlight some of the major features and implications of the refined lightcurve. We present our open source software package ICQSplitter (Curtis et al. 2020), which was used to develop these results. This serves as the companion paper to our archive publication with the NASA Planetary Data Systems Small Bodies Node archive (Womack et al. 2018).

### 2. Observations and Reductions

#### 2.1. Pruning Data Points from the Original Data Set

Obtaining an accurate total visual magnitude estimate of a comet is a much more difficult task than obtaining one of a nondiffuse object, such as a variable star. A detailed explanation and summary of recommended methods are found at the ICQ Magnitude-estimation Methods website18 and are also discussed in Green (1997). The original total visual magnitudes, $m_{tot}$, used in this study were taken from ICQ issues 96–100 and provided by D. W. E. Green (2020, personal communication), who selected observers with good coverage for both hemispheres (see Table 1). Only about 65% of the original 2240 observation reports remained after we removed 789 points following the best-practice guidelines from Green & Morris (1987), Green (1997), and D. W. E. Green (2010, personal communication), which included removing values when any of the following occurred:

1. No magnitude was provided (some observations only reported astrometry)
2. Methods of obtaining magnitudes that were not the Vsekhsvyatskij–Stevenson–Sidgwick (S), Van Biesbroeck–Bobrovnikoff–Meisel (B), Modified Out–Out, In-focus, or Extrafocal–Extinction methods
3. Poor observing conditions were noted
4. Under 20° elevation with no extinction correction applied
5. Telescopes used for $m_{tot} = 5.5$ and brighter
6. Binoculars used for $m_{tot} = 1.5$ and brighter
7. CCD or photoelectric detectors used
8. Poor-quality star comparison catalogs used
9. More than one magnitude per day submitted by an observer (if this occurred, we kept the measurement made with the smallest aperture instrument, which is believed to be most accurate).

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17 [http://www.icq.eps.harvard.edu/]

18 [http://www.icq.eps.harvard.edu/ICQMM.html]
The most common reasons for removing points were numbers 9, 2, 5, and 3: submitting more than one observation in a night, not using an approved magnitude method, using a telescope when the comet was brighter than 5.5 mag, or reporting observations made in poor conditions. Some observers, such as Herman Mikuz, submitted a mixture of visual and CCD magnitude estimates. When used properly, CCD and photometric detectors can provide very accurate values. However, they do not always agree well with visual magnitudes determined by eye, and it is not straightforward to combine data obtained with the two techniques (see Kidger 2002; de Almeida et al. 2009; Sosa & Fernández 2009). Thus, we removed all CCD measurements from this data set. Since CCDs were not widely used by the amateur community during the passage of Hale–Bopp, this affected relatively few points.

The remaining 1451 total visual magnitudes are the starting point for our analysis and are shown in Figure 1(a), which plots magnitude versus heliocentric distance. The other panels, (b), (c), and (d), of these figures show the lightcurve after correcting for geocentric distance, Δ, phase angle, θ, and observer bias, respectively, as we describe in the next sections. To facilitate comparison with other published studies, the data are also shown in slightly different format in Figure 2(a), which plots magnitudes as a function of “days from perihelion.”

2.2. Orbital Parameters That Affect the Apparent Brightness of a Comet

The observed brightness of the comet is a function of the constantly changing heliocentric and geocentric distances, as
well as of the phase angle of the comet. These must be accounted for to determine the true luminosity of the comet. A comet’s total visual luminosity, $L_{\text{tot}}$, is often written in terms of its absolute luminosity at 1 au from the Sun and Earth, $L_0$, scaled with a power law based on its distance to the Earth ($\Delta$ in au) and Sun ($r$ in au),

$$ L_{\text{tot}} = \frac{L_0}{r^n \Delta^k}, $$

where $k$ describes the geocentric distance dependence, which is usually assumed to be “2” due to the inverse-square-law behavior of reflected sunlight. The variable $n$ is called the “activity index” and is meant to quantify the comet’s behavior with respect to heliocentric distance. For a newly discovered comet for which little is known, $n = 4$ is often used, but the value may change during a comet’s orbit and it can vary between 1 and 8 (Meisel & Morris 1976; Meech & Svoren 2004a). In this

Figure 2. Comet Hale–Bopp’s lightcurve at every step plotted against days from perihelion. (a) Step 1: $m_{\text{tot}}$, the original total visual magnitudes after pruning; (b) Step 2: $m_{\text{helio}}$, after correcting $m_{\text{tot}}$ for geocentric distance; (c) Step 3: $m_{\text{phase}}$, after correcting $m_{\text{helio}}$ for phase angle; and (d) Step 4: $m_{\text{shift}}$, the final values after applying the statistical shifts to the $m_{\text{phase}}$ values.
paper we assume \( k = 2 \), and we fit models to the lightcurve data to obtain \( n \).

Using the luminosity–magnitude equivalency equation,
\[
m_{\text{tot}} - m_0 = 2.5 \log_{10}(L_0/L_{\text{tot}}),
\]
we can rewrite Equation (1) in terms of magnitudes:
\[
m_{\text{tot}} = m_0 + 5 \log_{10} \Delta + 2.5 n \log_{10} r,
\]
where \( m_0 \) is referred to as the absolute magnitude, or how bright a comet would appear if at 1 au from the Earth and Sun, for the case where the comet’s phase angle, \( \theta \), is constant.

The visible light from the dust coma includes both reflection and scattering of sunlight off of dust particles in the coma, as well as a much smaller contribution from the comet nucleus. Light scattering is a complicated process that depends on the physical properties of the particles in the coma and is wavelength-dependent (Kolokolova et al. 2004). When observers do correct visual magnitudes for scattering, they often assume a linear response, which is appropriate for comets with small phase angles (less than \( \sim 30^\circ \); Meech & Jewitt 1987; Kolokolova et al. 2004; Meech & Svoron 2004b). We used the phase correction function, \( \phi(\theta) \), normalized to \( 0^\circ \), from Schleicher & Bair (2011),\(^{19} \) which is a composite of a previous fit for comet Halley photometry (Schleicher et al. 1998), for smaller angles, and the phase function derived in Marcus (2007a, 2007b) for mid- and large-phase angles for C/2006 P1, which is appropriate for the large heliocentric range of the Hale–Bopp lightcurve. Including the phase angle correction term leads to
\[
m_{\text{tot}} = m_0 + 5 \log_{10} \Delta + 2.5 n \log_{10} r + 2.5 \log_{10}(\phi(\theta)).
\]

Magnitudes corrected to a geocentric distance, \( \Delta \), of 1 au are referred to as \( m_{\text{helio}} \), and are calculated with
\[
m_{\text{helio}} = m_{\text{tot}} - 5 \log_{10} \Delta
\]
using the geocentric distance of the comet at the time the observation is made. Magnitudes are further corrected to a phase angle, \( \theta \), of \( 0^\circ \), which we call \( m_{\text{phase}} \), using
\[
m_{\text{phase}} = m_{\text{helio}} - 2.5 \log(\phi(\theta)).
\]

We plot the lightcurve for each step in individual panels of Figures 1 and 2.

2.3. Statistical Analysis of Differences among the Observers

In analyzing visual data from multiple observers, questions inevitably arise of which data to reject, and under what justification, and whether combining data from observers, each with his or her own systematic errors, leads to a biased result. Without instrumental calibration, there is no certain answer to these questions, but as discussed by Kidger (2002) and Mousis et al. (2014), such calibration is itself problematic, and in any case is not available for the observations discussed here.

We offer a systematic approach to combining data from multiple observers, yielding a self-consistent consensus fit. In application to comet Hale–Bopp, the procedure does not significantly affect the gross measure, the activity index \( n \), but, applied to data already corrected for geocentric distance and for phase, it does reduce the statistical error bars.

We assume three categories of errors:

1. Every observer reports the brightness of an object on a scale that is shifted up or down from other observers, but by the same number of magnitudes, \( \delta_{\text{obs}} \), independent of distance or brightness. Without instrumental calibration, we can best estimate \( \delta_{\text{obs}} \) as that observer’s mean deviation from a consensus fit to the data (that is, an average).

2. Some observers may have a slope bias, underestimating the brightness of dimmer objects and overestimating those of brighter ones, or vice versa. While it is difficult to correct for such an error without calibration, the bias can be detected (relative to the consensus fit), and that observer’s data can be discarded.

3. Finally, some observers may have a great deal of scatter in their data but no bias. We can weight these observations less in the fits.

We seek a consensus fit to the data. The comet exhibits small but noticeable deviations from a power law on time (distance) scales larger than any outbursts. In particular, the preperihelion data between 2 and 1 au suggest a positive curvature in the graph of (minus) magnitude against log distance (see Figure 4). A straight-line fit would penalize observations that report this feature accurately. Based on the number of apparent features between the closest (0.91 au) and farthest (9.4 au postperihelion) observations, we fit to fifth-order polynomials. Fifth order was chosen as the simplest polynomial that could fit the apparent inflection points in the observations, serving as a low-pass filter that attenuates features smaller than about 3 au (full wave). A fourth-order polynomial resulted in 11% larger deviations, while sixth order improved the fit by less than 1%. The effect is to reflect the smoothed consensus.

The deviations between an observer’s measurements and the consensus fit at the same distance are considered noise. The set of all such deviations by one observer defines the noise distribution for that observer, characterized by a mean \( \delta_{\text{obs}} \) variance \( \sigma^2_{\text{obs}} \) skewness, kurtosis, and so on.

To compute the self-consistent fit, we iterate the following until convergence to a fractional tolerance of 0.0001\(^{20} \) (absolute tolerance if any fitting parameter is less than 0.0001) of all six polynomial fitting parameters:

(i) We fit a fifth-order polynomial through all the data of the (possibly shifted) magnitudes against log distance by the method of least squares (Legendre 1805), weighting each observation inversely as the observer’s variance of deviations, \( \sigma^2_{\text{obs}} \). Initially, the magnitudes have been corrected for geocentric distance and for phase angle, but have not been shifted. Also initially, the weights are all equal because we do not know the distributions of deviations.

(ii) Using the same data as in (i), we can also fit a straight line, recovering the activity index \( n \) and statistical error bars on that fit based on the shifted, weighted data.

(iii) For each observer, we consider the distribution of deviations between the observations and the fit at the given distance (i.e., in the graph, the vertical vectors between data points and the polynomial fit). For each

\(^{19}\) Tabulated values of the composite phase function are at http://asteroid.lowell.edu/comet/dustphase.html and are stored at NASA PDS (Womack et al. 2018).

\(^{20}\) In the present work, statistical error bars limit us to two to three significant digits, so a smaller convergence tolerance would be meaningless.
Table 2

| Correction Step                      | Preperihelion | Postperihelion |
|--------------------------------------|---------------|----------------|
| 1. Raw data ($m_{\text{tot}}$)      | 7.0 ± 0.9 au  | 0.9 ± 8.0 au   |
| 2. Geocentric distance correction only ($m_{\text{phase}}$) | 3.22 ± 0.02 | 3.66 ± 0.03 |
| 3. Geocentric distance and phase correction ($m_{\text{phase}}$) | 3.66 ± 0.02 | 3.97 ± 0.03 |
| 4a. Self-consistent shifts           | 3.66 ± 0.01  | 3.94 ± 0.01   |
| 4b. Drop observers, self-consistent shifts ($m_{\text{obs}}$) | 3.63 ± 0.01 | 3.92 ± 0.02 |

Notes. The slope of magnitude against $\log_{10}(r)$ is equal to $2.5\alpha$; see Equation (3). Steps 4a and 4b use self-consistently shifted magnitudes and weight each observer’s data inversely as $\sigma_{\text{obs}}^2$, the variance of the noise. Error bars are standard errors of fit (Press et al. 2007) and would be considerably smaller, preperihelion, if slow variations (Figure 4) were subtracted. The fitting method reduces statistical error but does not significantly alter the activity index estimates.

For the present data set, these iterations converge to the specified tolerance after between 10 and 12 iterations. We now have a self-consistent fit and set of shifted data. Row 4a of Table 2 shows small changes in best-fit slopes after the procedure, with the statistical error of the straight-line fit cut roughly in half. Note that non-straight-line features in the consensus polynomial fit limit how far the statistical error bars in a straight-line fit can shrink.

The self-consistent procedure has eliminated the need to discard data arbitrarily (e.g., points more than some number of standard deviations above or below a consensus fit, which otherwise would throw off least-squares fits) by weighting points inversely with the observer’s variance. However, as noted above, an observer whose systematic error changes with brightness would still affect the activity index adversely. We can detect such a systematic nonstationarity by applying Student’s $t$-test to each observer’s data set, comparing the mean deviation from the consensus fit in the first half of the observer’s data (sorted by distance) to the mean deviation in the second half. Since the variances of the two halves may not be equal, we normalize by the “pooled variance” to get an approximate $t$-statistic (Press et al. 2007). Assuming approximately Gaussian noise, we calculate the $p$ (probability) value that the $t$-statistic would be as large as observed or larger under the null hypothesis that the first and second halves of the data were drawn from the same distribution, that is, that the observer did not contribute bias to the slope relative to the consensus (stationary).

For a $p$ value less than 0.05, we reject the null hypothesis and conclude, on the basis of the $t$-test, that the observer’s data bias the slope relative to the consensus. We then discard such observers from the data set and repeat the self-consistent iteration ((i)–(iv)), starting from the original (phase-corrected, unshifted) magnitudes but without the discarded observations. If repeating the $t$-test at the end of the second set of iterations results in more observers biasing the slope (relative to the consensus) based on the $t$-test, we then repeat the procedure until no new observers are discarded. For the preperihelion data, one additional observer failed the $t$-test after the second set of iterations, so we iterated a third time without that observer’s data. At the end of the third set of iterations, no new observers failed the $t$-test. For the $t$-test component to run successfully, a minimum of four points per observer is required to evaluate the $t$-test; however, the results will improve with more data, and we recommend using more points. Considering that ~35% of the original data in this study were removed for technical reasons in Section 2.1, we recommend that users start with total visual magnitude data sets containing at least 50–100 points summed over all observers to increase the likelihood of a high-quality final lightcurve.

As shown by Line 4b of Table 2, throwing out observers results in small changes in the slopes. For the preperihelion data, seven observers out of 17 were discarded, and for postperihelion data, four out of the remaining 12 observers were discarded because they failed the stationarity tests. At the conclusion of Step 4b in Table 2, an additional 656 points were removed for having failed the stationarity test and only 795 points remained, or 35% of the original amount.

For comet Hale–Bopp, the self-consistent procedure resulted in adjustments to the slope roughly comparable to the original statistical error bars while cutting the error bars approximately in half, as reflected in the visibly smaller scatter in the lightcurve. (See Table 2 between Lines 3 and 4b and Figure 1 between panels (c) and (d).) These changes are far smaller than those associated with correcting for geocentric distance and phase. The observations in the present work were taken by people using generally reliable star comparison catalogs. Future work will likely rely on less homogeneous amateur networks, in which case a self-consistent method for combining and weighting magnitudes and discarding subsets with systematic slope bias could prove useful.

The shifts, $-\delta_{\text{obs}}$, to the data resulting from this self-consistent procedure are listed in Table 1 and are equivalent to the last step in the equation

$$m_{\text{tot}} = m_0 + 5 \log_{10}(r) + 2.5n \log_{10}(\Delta) + 2.5 \log_{10}(\phi[\theta]) + \delta_{\text{obs}},$$

and we define

$$m_{\text{shift}} = m_{\text{phase}} - \delta_{\text{obs}},$$

or as it is written in complete form:

$$m_{\text{shift}} = m_{\text{tot}} - 5 \log_{10}(\Delta) - 2.5 \log_{10}(\phi[\theta]) + \delta_{\text{obs}}.$$  (9)

Using Equation (9), we can rewrite Equation (7) as

$$m_{\text{shift}} = m_0 + 2.5n \log_{10}(r),$$

which we use to measure $n$.

The $m_{\text{shift}}$ values represent the final corrected magnitudes of the lightcurve and are shown in Figures 1(d) and 2(d), as well as a close-up version in Figure 4.

Correcting for geocentric distance and phase angle makes noticeable changes to the lightcurve shape. By design, making self-consistent shifts to the data does not change the
lightcurve’s shape but reduces the uncertainty. For ease of comparison, fifth-order polynomial fits to the data at each correction step are shown in Figure 3.

The original total visual magnitudes and those at all steps outlined in this paper underwent full panel merit review with the NASA Planetary Data Systems Small Bodies Node and are publicly archived (Womack et al. 2018). All data reduction was performed using the ICQSplitter program, which is available for download on GitHub. This paper describes the functionality of ICQSplitter Version 3.0 as of 2020 January 28.

3. Discussion

3.1. Comparison with Other Hale–Bopp Lightcurves

Several other groups produced visual lightcurves of Hale–Bopp that spanned many years; however, they were not always derived the same way. For example, some did not account for the effect of geocentric distance, even though this had up to a 42% impact on the activity index, \( n \), for Hale–Bopp. Figure 1 shows an illustration of this effect, where a “shoulder” feature, seen in panel 1(a) at \( \sim 3.5–4.0 \) au preperihelion, is effectively removed once we account for the geocentric distance in panel 1(b). This type of artifact might invite unwarranted physical interpretation.

Table 3 lists some of these long-term lightcurve data sets for Hale–Bopp. Our lightcurve is the only one in Table 3 that corrected for light scattering in the coma (see Section 2.2). Our calculations show that scattering can be significant for Hale–Bopp’s lightcurve and at times contributed up to a 92% change to the calculated brightness (e.g., compare Figures 1(b) and (c) and Figure 3). This result underscores our motivation in sharing the ICQSplitter code, so that this potentially important correction step may be more accessible to other researchers.

The comet magnitude websites from the Comet Observation Database\(^{23}\) (COBS) and Seiichi Yoshida\(^{24}\) provide data and interactive tools to analyze visual and CCD magnitudes submitted to the International Comet Quarterly (ICQ) and the Minor Planet Center (MPC). Both Yoshida and COBS present only total visual magnitudes \( m_{\text{tot}} \), but include correction for geocentric distance. However, when fitting slopes to their lightcurves, they do not correct for phase angle variations or include any adjustments for observer differences (Filomenko & Churyumov 2001; Zakrjasek & Mikuz 2018; Womack et al. 2018).

\[^{22}\text{https://github.com/curtisa1/ICQSplitter}\]

\[^{23}\text{https://cobs.si/}\]

\[^{24}\text{http://www.aerith.net/comet/catalog/1995O1/1995O1.html}\]
considering all four teams started with ICQ measurements. Yoshida and COBS give activity indices that are within \(\sim 10\%\) of each other’s values, Liller’s “fit by eye” value, and our measurements. Our lightcurve is also within 2\%–6\% agreement with Kidger et al. (1997), except for 2.6–2.1 au (1996 October 25 to December 4), when their values are \(\sim 30\%\) higher than ours, and during 1.1–0.9 au (1997 March 1 to April 1), when their \(n\) is \(\sim 60\%\) of our index. However, the data have high dispersion in both of these regions in Kidger et al. (1997), and they overlap with our values within their uncertainties. Ferrín (2010) presents two possible activity indices within 6.3 au of the Sun that are \(\sim 20\%–30\%\) flatter than everyone else’s value for this long range. Their lower index may be due to the fact that they only fit to the 5\% brightest points and combine visual and CCD-derived magnitudes. As we have shown in this paper, some observers submitting to the ICQ are not self-consistent, and some submit systematically brighter, or lower, magnitudes than the average. Thus, it may be that preferentially fitting the brightest points when multiple observers are involved contributed to biases in the Hale–Bopp results by Ferrín (2010).

### 3.2. Preperihelion Changes in the Lightcurve Slope

Given the significant effect that the phase angle correction can have on coma brightness and the benefits of using a self-consistent method for adding data from different observers, the rest of our analysis is completed using our final lightcurve data (Step 4).

Comet Hale–Bopp’s lightcurve provides information about the long-term response of the nucleus to solar heating and continuous thermal processing at different levels. Additional information about the nucleus, such as shape, rotation, or active regions, is needed to constrain models in detail, especially when making accurate predictions of the heliocentric dependence of water outgassing (Marshall et al. 2019). However, long-term lightcurves with higher resolution and accuracy may be useful in constraining the parameter space for nucleus thermal models and provide an important way to connect models of coma activity and nucleus processing, especially over multiple orbits or larger arcs of a given orbit. A change in the lightcurve’s shape may also indicate the onset of sublimation or outgassing of a new volatile or pinpoint the timing of prominent outbursts.

To first order, the activity index of the entire preperihelion lightcurve is \(n = 3.63 \pm 0.01\) (Figure 4). This is within \(\sim 10\%\) of the \(r^{-4}\) value that is often assumed for inbound, long-period comets, especially when little else is known (Meech & Svoren 2004b). Hale–Bopp was not a dynamically new comet and likely experienced at least a few heating cycles over its lifetime (Bailey et al. 1996; Marsden 1997; Sekanina & Kracht 2017;
Vokrouhlický et al. 2019). The lightcurve’s approximate \( r^{-4} \) form is consistent with nucleus models that are under the influence of a CO-dominated outgassing regime with a possible water-ice phase change from the amorphous to crystalline state (Prialnik 1997; Enzian 1999).

As Figure 4 shows, however, the lightcurve noticeably deviates from a straight line in several places, and a fifth-order polynomial is a better fit to the observed inflection points at \( r = 4.0, 2.6, 2.1, \) and 1.1 au. The changes in slope were also noted at the same heliocentric distances by Kidger et al. (1997).

In Table 4 we list the power-law activity index for the five regions at each of the data reduction steps. Next, we discuss the lightcurve slope changes in the context of other coma phenomena during the same time. It will be interesting to test whether the variation of \( n \) with heliocentric distance in Table 4 for Hale–Bopp applies to other long-period comets, including whether a change of slope is accompanied by a morphological transition in the inner coma, such as the appearance of multiple jets in Region A or the development of radiation pressure effects on the jets in Region D.

### 3.2.1. Region A: \( 7.2 \geq r \geq 4.0 \) au (1995 July 17 to 1996 June 23)

Initially, there were reports of outbursts soon after discovery at \( \sim 7 \) au (see discussion; Sekanina 1996; Kidger et al. 1997; Prialnik 1997), as well as near \( 4.0 \) au (Liller 2001), but we see no evidence for these in our lightcurve. The activity index for region A is \( n = 4.46 \pm 0.11 \). The comet was still far enough from the Sun that water-ice sublimation, the dominant activity mechanism for most comets, was inefficient. During this time, the coma was dominated by outgassing of CO, with minor contributions from \( \text{CO}_2 \), HCN, H$_2$CO, and other volatiles (Jewitt et al. 1996; Crovisier 1997; Womack et al. 1997; Biver et al. 2002) and possibly moderated by crystallization of amorphous water ice (Prialnik 1997).

Optical CCD images obtained with R-filters, and some without filters, show that Hale–Bopp’s dust coma at this time exhibited a single radial jet feature that persisted for several months and then transitioned to four or more jets at \( \sim 4 \) au (Braunstein et al. 1997; Mueller et al. 1997). The appearance of the jets coincides with the next change in lightcurve slope and could indicate the onset of several active areas on the nucleus (Sekanina 1997), perhaps related to triggering of subsurface volatile pockets or mass wasting events due to mechanically or thermally unstable regions on the nucleus. CO was detected shortly after Hale–Bopp was discovered (Jewitt et al. 1996) with a high production rate, and other molecules, such as HCN, CH$_3$OH, and CO$_2$, began being detected in smaller amounts around 4 au (Womack et al. 1997, 2017; Biver et al. 2002).

### 3.2.2. Region B: \( 4.0 > r > 2.6 \) au (1996 June 23 to October 25)

Here, the slope flattens with \( n = 3.06 \), which it maintained for about four months. Previous analyses by Kidger et al. (1997) reported a much flatter \( r^{-1.5} \) brightening law for this region, which was attributed to the onset of water-ice sublimation near 3 au (Bockelée-Morvan & Rickman 1997). However, our calculations show that their flattening likely did not have a physical cause, but instead was due to a prominent and uncorrected phase angle effect. This is clearly seen if we compare the indices for the geocentric-corrected \( (n = 1.62) \) and phase-corrected \( (n = 3.11) \) lightcurves of region B in Table 4.

An outburst was reported in mid-September (3.1 au), when at least a few of the jets appeared disrupted (Braunstein et al. 1997; Tao et al. 2000; McCarthy et al. 2007). This outburst did not appear in our lightcurve, nor did it cause a change in the lightcurve, as the coma maintained a slope of \( n \sim 3 \) for another month until \( \sim 2.6 \) au (mid to late 1996 October). This slope change is within the heliocentric distance, where it is commonly thought that water-ice sublimation becomes prominent. If rapid water-ice sublimation is initiated, it can easily...
surpass outgassing of all other volatile species. Due to increased thermal flux and spatial distribution throughout the nucleus, water-ice sublimation quickly becomes the primary carrier of dust grains (Prialnik 1997), which can significantly increase the reflectance of the coma.

3.2.3. Region C: 2.6 > r ≥ 2.1 au (1996 October 25 to December 4)

In this region, the lightcurve flattened even more with an $r^{-2.36}$ trend. During this time, the comet maintained at least four nearly radial jets in the inner coma, possibly from the same active areas that started at the end of region A. If indeed supported by subsurface pockets or newly excavated patches, this sustained jet activity can inform us about the volume of volatile species (mostly water ice) and how accessible it is to sublimation and distinct outward flow.

3.2.4. Region D: 2.1 > r ≥ 1.1 au (1996 December 5 to 1997 February 25)

At ~2 au, the lightcurve returns to an $r^{-4}$ brightening law. No outbursts are detected. During this time, the inner coma morphology transitioned once again. Between 1996 November 16 (2.33 au) and 1997 February 13 (1.23 au), the coma’s dominant features changed from the nearly radial jets to curved jets consistent with radiation pressure and the beginning of a “gull wing” feature (Braunstein et al. 1997). Modeling and simulations of many optical and infrared images indicated that the changes from jet-like to arc-like features were due to a changing observing perspective and increased angular resolution, and not to physical changes in the nucleus or coma (Samarasinha et al. 1997). This again can be considered consistent with a view of water-ice-rich activated pockets that are present just below the surface.

3.2.5. Region E: 1.1 > r ≥ 0.9 au (1997 March 1 to April 1)

Here we see the fastest rate of brightening of any region, with a power-law index of $r^{-7.5}$. The extreme brightening here compared to other regions has been interpreted to mean that an outburst occurred during the dates leading up to perihelion (Samarasinha et al. 1997). The brightest points in the lightcurve occurred 3 days before perihelion, in agreement with previous analysis (Kidger et al. 1997; Ferrín 2010). Despite such a high rate of brightening and outburst, the lightcurve shows no (or minimal) sign of a perihelion shift. During this time, optical images continued to show the inner coma features dominated by curved jets with shells or arcs.

3.3. Postperihelion Lightcurve

Instead of a series of changing slopes, the postperihelion data are well fit by an $r^{-3.94±0.02}$ power law superimposed with some outbursts. We did not notice the breaks claimed by Kidger et al. (1997), nor do we see any evidence that the comet faded more rapidly as water-ice sublimation turned off around 3 au, which they reported. The lack of measurable slope change over 8 au is consistent with the comet nucleus achieving a level of stability after its perihelion passage. This is slightly (~8%) steeper than the overall change measured over the preperihelion lightcurve from 7 to 0.9 au.

Outbursts may have occurred at 2.15 ± 0.07 au and 7.36 ± 0.20 au (Figure 4), both lasting a few weeks. The 2.15 au outburst in 1997 July led to a brightness change of 0.47 ± 0.08 mag. The 7.36 au outburst in 1999 January brightened the coma by 0.68 ± 0.23 mag and was also observed by Liller (2001)’s CCD set, which measured an increase of $\Delta m$ ~ 1.1 mag. For context, Liller’s CCD measurements were obtained with a 34” square aperture, and he did not provide an estimated visual diameter of the coma.

3.4. Absolute Magnitude

Absolute magnitudes are a theoretical construct, representing what brightness the comet would have if viewed at 1 au from both the Sun and Earth. To calculate absolute magnitudes, astronomers typically correct the apparent magnitude to 1 au from the Earth with Equation (5), but the correction to 1 au from the Sun is often model-dependent, as are the values listed in Table 4. However, since Hale–Bopp was observed twice through the heliocentric distance of 1.0 au, we have the opportunity to measure the absolute magnitude directly. This is valuable for constraining models of comet activity. Using all of the $m_{\text{shift}}$ values that were within 0.05 au of 1.0 au, we calculate these average absolute magnitudes,

preperihelion: $m_0 = -1.74 ± 0.13$
postperihelion: $m_0 = -1.92 ± 0.23$
where the uncertainties are one standard deviation. Within these uncertainties, we can see that the comet was equally bright pre- and postperihelion at 1 au. These absolute magnitudes confirm that Hale–Bopp was one of the brightest comets ever recorded. Its absolute magnitude is only challenged by Comet Sarabat in 1729 (the brightest ever, at $\sim -3.0^{25}$), and by Tycho’s Comet in 1577, with just two other comets in six centuries showing brighter absolute magnitudes (Hughes 1990). This confirms the truly exceptional nature of Hale–Bopp.

3.5. $A_f \rho$ Calculations

The quantity $A_f \rho$ is often used as a proxy for dust production rate, and it can be thought of as equivalent to the size of a hypothetical disk needed to approximate the light reflected from the dust at the comet’s distance. When compared with gas production rates, this quantity is often used to derive dust-to-gas ratios, partially as a test of nucleus homogeneity, and differences in comet behavior with heliocentric distance, including identifying possible dust outbursts (Womack et al. 1994; Weiler et al. 2003). It is derived from $A$, the albedo of the dust, $f$, the filling factor, and $\rho$, the coma radius (A’Hearn et al. 1984):

$$A_f \rho = \frac{(2r \Delta)^2}{\rho} \frac{F_{\text{comet}}}{F_{\text{Sun}}},$$  \hfill (12)

where

$$F = 10^{-\pi \alpha / 2},$$  \hfill (13)

is used to convert magnitudes, $m$, to flux, $F$, and we assume the apparent visual magnitude of the Sun is $m_{\text{Sun}} = -26.7$ (Willmer 2018). We used a value of $m_{\text{Sun}}$ of half the observers’ reported coma diameter for the aperture radius, $\rho$.

$^{25}$ Sarabat had a perihelion distance of 4.05 au, so the extrapolation to its magnitude at a heliocentric 1 au is highly speculative.

The peak sensitivity of the dark-adapted human eye is at $\sim 510$ nm, which aligns with both the Sun’s peak blackbody curve wavelength, as reflected in coma dust grains, and the $C_2$ Swan emission bands that were seen when Hale–Bopp was within $\sim 3$ au of the Sun. Hale–Bopp was extraordinarily dust-dominated (see Schleicher et al. 1997; Weiler et al. 2003), and the dust continuum brightness far exceeded that of the $C_2$ emission at visible wavelengths. Visual observers probably see reflected solar continuum in most comets beyond 2.5 au (Larson et al. 1991), and considering how dust-dominated Hale–Bopp was, it is a reasonable assumption for closer distances. Thus, for the purpose of these calculations, we assumed that the total visual magnitudes were indicative of the reflected light from the dust, and not emission from $C_2$ or other gaseous species. We acknowledge that a small amount of the brightness near perihelion could be due to $C_2$ emission.

To our knowledge, these are the first reports of $A_f \rho$ calculated from visual magnitudes for Hale–Bopp or any comet. The quantity is generally not considered to be well suited for visual magnitudes since an aperture radius is usually not provided by the observers. Also, as discussed earlier, visual magnitudes recorded without a filter (which apply to all of the data in our study) could be contaminated with gaseous emission, especially within $\sim 2$ au of the Sun, when gas output increases significantly. Nonetheless, since we could not find evidence of this being done before, and since this could be a useful resource if accurate, we calculated $A_f \rho$ values from the final shifted visual magnitudes (with the geocentric correction reversed, since Equation (12) accounts for geocentric distance).

In Figure 5 we plot our magnitude-determined $A_f \rho$ values along with the values derived from spectroscopic and filter-imaging techniques, using data from Figure 4 from Weiler et al. (2003). As Figure 5 shows, the values show a very similar trend in the pre- and postperihelion data. When combined, our preperihelion and postperihelion values follow an $A_f \rho \sim 20r^{-1.5}$ km heliocentric dependence. No heliocentric dependence was provided by
Weiler et al. (2003), but both $Af_p$ curves show broadly similar shapes with a steep increase within $\sim 3$ au and are much flatter beyond that. This behavior change at $\sim 3$ au could be indicative of the water-ice sublimation turn-on at this distance. A rapid increase in sublimation-driven activity is due to both ejection of dust particles from the surface and dragging of dust particles from the subsurface (through porous flow or disruptive mass wasting). Typically, our $Af_p$ values are 2–4 times higher than those derived with spectroscopic or photometric instrumentation. This offset is likely due in part to the relatively wide bandpass that visual observations use, which admits some gaseous emission into the observer’s field of view, such as from C$_2$, especially closer to perihelion. Another reason that $Af_p$ values are consistently higher, even at larger distances when gaseous contamination is minimal, is because of inaccurate aperture size assumptions. A more thorough analysis, such as reducing all estimates to a standard equivalent aperture of 6.7 cm, would involving creating a new technique to handle the coma size estimates and is beyond the scope of this paper. Interested readers may explore this using the data published at the NASA PDS SBN archive (doi:10.26007/XBSH-X639).

As a reminder, the magnitudes used for this work were all obtained by eye, sometimes with binoculars or telescopes, but always without filters or detectors. With future observations, more accurate $Af_p$ values can be acquired by starting with CCDs with well-constrained aperture sizes and narrowband filter imaging or spectroscopy, where the dust contribution can be more accurately measured and calibrated (e.g., Milani et al. 2013). However, if only a factor of 2–4 accuracy is needed, our analysis shows that $Af_p$ calculated with $m_{shift}$ and the observer’s coma diameter estimate may work for a comet known to be rich in dust, or at least is beyond $\sim 2.5$ au from the Sun, where gas contamination is minimal in visual observations.

3.6. Correlation of Water and CO Production Rates with Visual Magnitudes

We wanted to test whether there was evidence for a possible effect of gas production on the visual magnitudes. In addition, this serves as an independent proof of concept that our approach can be readily combined with other data sets to promote advanced analysis. We analyzed published CO and H$_2$O production rates (Biver et al. 2002), $Q$(CO) and $Q$(H$_2$O), respectively, in the context of the fifth-order polynomial fit to our $m_{shift}$ magnitudes (Figure 3). We created Figure 6 by plotting individual production rates against fifth-order fits to final shifted magnitudes.

3.6.1. Results for Hale–Bopp Using the Final Shifted Magnitudes

The preperihelion water correlation slope has a noticeable break at $m_{shift} \sim 2$ (Figure 6). Hale–Bopp was this bright mostly when it was at $r \sim 2.6$ au (see Figures 1(d), 3, and 4). We derive these correlation equations for the two preperihelion H$_2$O regimes:

\[ m_{shift} > 2 \quad \text{(beyond } r \sim 2.6 \text{ au preperihelion):} \]

\[ \log_{10}(Q(H_2O)) = 31.4 - 0.724m_{shift}; \quad (14) \]

\[ m_{shift} \leq 2 \quad \text{(within } r \sim 2.6 \text{ au preperihelion):} \]

\[ \log_{10}(Q(H_2O)) = 30.40 - 0.229m_{shift}. \quad (15) \]

The correlation data are consistent with the comet’s brightness, as measured by the shifted visual magnitudes, increasing due to CO outgassing until about 2.6 au, when the water begins to significantly contribute to the dust release. This agrees with measurements of CO and other minor species such as CO$_2$, HCN, and other volatiles at this time (Womack et al. 2017) and coincides with the approximate heliocentric distance where water-ice sublimation began to dominate Hale–Bopp’s activity, and these equations may be useful in constraining models of its activity.
A similar change in the $Q$(CO)–magnitude correlation was reported by Bockelée-Morvan & Rickman (1997); however, they saw the break at $\sim 3 \, m_{\text{helio}}$, corresponding to $r \sim 3$ au, instead of 2.6 au. We attribute the difference in location of the break to the fact that they used $m_{\text{helio}}$ values for their analysis. This point is illustrated in Figure 6, where we also show the correlation lines computed with our $m_{\text{helio}}$ values with dotted lines, which reproduces Bockelée-Morvan & Rickman (1997)’s results with the break occurring farther out. Thus, not correcting for phase angle contributions leads to the change in slope occurring $\sim 15\%$ farther from the Sun.

Postperihelion, we find no break in the H$_2$O correlation plot and derive a single equation:

$$\log_{10}(Q(H_2O)) = 30.2 - 0.44m_{\text{shift}}.$$  \hspace{1cm} (16)

For CO, we find a single correlation equation at preperihelion,

$$\log_{10}(Q(CO)) = 29.71 - 0.21m_{\text{shift}},$$  \hspace{1cm} (17)

and a nearly identical relationship postperihelion,

$$\log_{10}(Q(CO)) = 29.72 - 0.23m_{\text{shift}}.$$  \hspace{1cm} (18)

The pre- and postperihelion correlation equations for CO (Equations (17), (18)) are remarkably similar, so we recommend one equation for CO for all heliocentric ranges in comet Hale–Bopp:

$$\log_{10}(Q(CO)) = 29.71 - 0.22m_{\text{shift}}.$$  \hspace{1cm} (19)

These equations are all plotted in Figure 6 and summarized in Table 5. Curiously, as Figure 6 shows, the $Q$(CO)–magnitude relation seems to do a zigzag about where the slope of $Q$(H$_2$O) changes. This occurred at approximately 2.6–2.0 au. It is interesting to consider whether the CO outgassing and dust production were interrupted by a sudden increase in water-ice sublimation as the comet moved closer to the Sun.

### 3.6.2. Comparison with Other Comets

To get the most accurate characterization of the production rate and visual magnitude correlation, we recommend applying a phase angle correction to the magnitudes, as is done in the previous section. However, because most published papers do not include this calibration step, we provide the correlation equations in terms of $m_{\text{helio}}$ (Table 6). They are also plotted with dotted lines in Figure 6. Due to a comparatively small outgassing range for most comets, previous studies searching for possible correlations between visual magnitudes and gas production rates have primarily relied on aggregate $Q$-magnitude data sets compiled for multiple comets over a wide range of magnitudes (A’Hearn & Millis 1980; Jorda et al. 2008; Sosa & Fernández 2009, 2011). Two correlation equations for water aggregated from many other comets and one for CO using data from Centaur 29P/Schwassmann-Wachmann (SW1) are also included in Table 6 for comparison.

The Hale–Bopp preperihelion correlation equations for water are complicated by the change in slope at $\sim 2.6$ au (corresponding to $m_{\text{helio}} \sim 3$). The water–magnitude equation beyond 2.6 au preperihelion does not agree well with the other aggregate equations, and this may be because water-ice sublimation was not strongly driving the activity at these large distances, when it likely was doing so for the other comets that were much closer to the Sun for the same magnitude values.

The postperihelion Hale–Bopp water correlation equation had no break in slope and is similar to those derived for 21 long-period comets (Sosa & Fernández 2011) and 37 unnamed comets (Jorda et al. 2008). The fit uncertainties are not provided in the table, but with the exception of SW1, they are all very small (less than a few percent). SW1 is in a special class of cometary object known as the Centaurs, follows a near-circular orbit beyond Jupiter’s distance, and is well known for its frequent large outbursts, which may have occurred during the observations (Biver 2001; Wierzchos & Womack 2020). However, the fit uncertainties do not take into account observational errors of the gas production rates, which are typically 25% or higher, or the systematic errors of the magnitudes, which we estimate to be $\leq 0.1$ magnitude. Taking into account these larger observational uncertainties, with the exception of the preperihelion water–magnitude equation, we find there is good agreement between Hale–Bopp and the compiled comets. The water–magnitude correlation equation for Hale–Bopp postperihelion is steeper than what is typically observed for other comets and may be related to the strong role CO outgassing played in releasing dust in Hale–Bopp. The similarities of the remaining correlation equations are remarkable considering that the observations span a variety of comets, with a large range of production rates, dust-to-gas ratios, and heliocentric distances.

### 3.6.3. Predicting CO and H$_2$O Production Rates for Other Comets

Hale–Bopp is the only comet for which CO and H$_2$O production rates are available for such a large range of heliocentric distances. Total visual magnitudes, however, are more easily obtained, even at large distances from the Sun. For Hale–Bopp, they appear to be correlated with the behavior of the most productive volatiles: CO and H$_2$O.

Next, we take production rate–magnitude correlation equations derived for Hale–Bopp and modify them so they can be applied to other comets to assist with observation planning. To avoid complications associated with the mysterious bend at $\sim 2.6–3.0$ au preperihelion, we use the postperihelion correlation equation for water and the average of both heliocentric regions for CO. Following this approach may be useful for planning observations of H$_2$O, CO, and even other volatile species (OH, CN, HCN, CH$_3$, H$_2$CO, and CH$_3$OH) if we assume a range of typical cometary ratios for these species relative to water or CO.

Hence, we present another correlation equation for CO and $m_{\text{tot}}$, which we derived from our $Q$(CO) and $m_{\text{helio}}$ correlation analysis for both postperihelion and preperihelion Hale–Bopp.
Table 6  
Correlation Equations for Gas Production Rate and $m_{\text{helio}}$ for Hale–Bopp and Other Comets

| Correlation | Comet(s) | $m_{\text{helio}}$ | References |
|-------------|---------|------------------|------------|
| log(QCO) = 29.9 − 0.24$m_{\text{helio}}$ | Hale–Bopp | 7 to −2 | This paper |
| log(QCO) ~ 30.2 − 0.2$m_{\text{helio}}$ | SW1 | 10 to 8 | Biver (2001) |
| log(OH-O) = 32.62 − 0.97$m_{\text{helio}}$ | Hale–Bopp pre | 5 to 3 | This paper |
| log(OH-O) = 30.63 − 0.30$m_{\text{helio}}$ | Hale–Bopp pre | 3 to −2 | This paper |
| log(OH-O) = 30.51 − 0.46$m_{\text{helio}}$ | Hale–Bopp post | 5 to −2 | This paper |
| log(OH) = 30.53 − 0.23$m_{\text{helio}}$ | 21 comets | 10 to −2 | Sosa & Fernández (2011) |
| log(OH2O) = 30.68 − 0.25$m_{\text{helio}}$ | 37 comets | 10 to −2 | Jorda et al. (2008) |

4. Summary and Conclusions

We present a detailed analysis of total visual magnitudes for comet Hale–Bopp that were submitted to the International Comet Quarterly from more than a dozen observers, resulting in a lightcurve for each of these steps: (1) total visual magnitudes with poor-quality data removed, (2) magnitudes corrected for geocentric distance, (3) magnitudes corrected for both geocentric distance and phase angle, and (4) magnitudes corrected for geocentric distance, phase angle, and systematic observer variation.

Correcting Hale–Bopp’s apparent magnitudes for geocentric distance changes the lightcurve’s shape, especially around 3–4 au preperihelion. Correcting for light scattering, as measured by the comet’s phase angle, changes the lightcurve slope by an average of ~30%. Not correcting for phase angle leads to a flattened region in the lightcurve from 4 to 2.6 au preperihelion, which was reported by other groups and erroneously attributed to the onset of water-ice sublimation near 3 au. However, we show that most of the flattening in this region was an artifact of not correcting for phase angle, which we reproduce with our model.

To first order, the preperihelion and postperihelion light-curves follow an $r^{-4}$ trend, which is consistent with typical trends often assumed for long-period comets. A closer look at the preperihelion data shows that the lightcurve deviates from a linear fit at these locations: 4.0, 2.6, 2.1, and 1.1 au. The 4.0 au inflection point coincided with the reported turn-on of four or more jets, or active areas, on the comet nucleus. The second change in slope at 2.6 au may coincide with the onset of vigorous sublimation of water ice from the nucleus. We are not aware of any other significant developments in Hale–Bopp’s coma for the other changes in slope.

The postperihelion lightcurve was more linear than the preperihelion data and the slope was remarkably constant out to at least 9 au.

Hale–Bopp appeared to undergo an outburst a few days before perihelion and two outbursts at $r = 2.15$ and 7.36 au postperihelion.

The final absolute magnitudes derived from the lightcurve are $m_a = -1.74$ and $-1.92$, which confirm that Hale–Bopp was one of the intrinsically brightest comets in the last 600 years.

The $Af$ values derived from visual magnitudes in the final lightcurve show a power-law dependence on heliocentric distance, $r^{-1.5}$, similar to those derived using spectroscopic and imaging techniques, but the visually derived $Af$ values are 2–4 times larger.

We calculated and analyzed correlation laws between the published gas production rates (for H$_2$O and CO) and final shifted magnitudes. Considering the high production rate ratio
of CO/CO2 reported for Hale–Bopp (Crovisier 1997; Womack et al. 2017), the data are consistent with the visual magnitudes increasing largely due to CO outgassing, until ~2.6 au, when the water begins to significantly contribute to the dust release. This coincides with the approximate heliocentric distance where water-ice sublimation began to dominate Hale–Bopp’s activity.

We present two equations to use for predicting water and CO production rates in comets using their apparent magnitude reports:

\[
\log_{10}(Q_{\text{H}_2\text{O}}) = 30.5 - 0.46(m_{\text{tot}} - 5 \log_{10} \Delta)
\]

\[
\log_{10}(Q_{\text{CO}}) = 29.9 - 0.24(m_{\text{tot}} - 5 \log_{10} \Delta).
\]

These relations have been validated for Hale–Bopp, which is a comet with prolonged and sustained activity that has been observed over a large heliocentric range. Further validation and improvement on the accuracy of such magnitude–production rate correlations will have to wait for future opportunities with similarly active objects. We suggest these relations be used as a quick reference for guiding modeling work and proposal planning.

The self-consistent procedure resulted in adjustments to the slope roughly comparable to the original statistical error bars, while cutting the error bars approximately in half. The code used to derive the lightcurves, ICQSplitter, is publicly available at the Archive and is a script. This material is part of the “Cometwatchers Lightcurve Project,” carried out with undergraduate researchers at St. Cloud State University and the University of South Florida, and was supported in part by the National Science Foundation under grant Nos. AST-1615917 and AST-1945950. Thanks also to Daniel W. E. Green, Harvard University, for providing photometric data ICQSplitter: Python Tool for Handling Visual or CCD Magnitudes of Comets, v3.0, Zenodo, doi:10.5281/zenodo.3628044.

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