Nonlinear vibrations of a circular plate reinforced by ribs

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Abstract
The behavior of a circular elastoplastic plate, consisting of a shell and reinforced ribs of a quadrangular cross-section and under the action of dynamic loads, has been investigated. The rib sections are considered to be constant. The relationship between deformations and displacements is taken geometrically nonlinear, and between stresses and deformations in the form of Hooke's law. Refined equations of the Timoshenko type are taken as resolving ones. It is believed that the vibrations of the plate are excited by a dynamic load acting on the surface of the plate. The solution of the differential equations for the vibrations of the plate, taking into account the elastoplastic properties of the shell materials and the reinforced ribs, is carried out by the finite difference method. The elastic and elastoplastic models are used to calculate the deflections of the central point and forces depending on the location of the ribs. In particular, it has been established that: when the plate is reinforced with one rib, the smallest deflection of the center point can be achieved when the rib is located in the middle of the radius; the proposed model and calculation method allow achieving the desired deflection value by varying the number of ribs.

1. Introduction
In many areas of modern technology and construction, plates and shells reinforced with ribs are widely used [1, 2]. In this case, it is important to study the dynamic behavior of such systems under the influence of dynamic loads of various characters. The complexities of solving the problems of dynamics of ribbed plates and shells by analytical methods force researchers to confine themselves to their relatively simple configurations [3]. When solving dynamic problems of ribbed plates and shells, the determination of natural frequencies and vibration modes plays an important role. A lot of work has been done in this direction. These works are adjoined by articles [4,5], where natural vibrations of reinforced circular and annular plates are investigated. This also includes the work [6] devoted to the study of the propagation of multimode waves in ribbed plates. Some issues of vibrations of rigid orthotropic shell structures under dynamic bending are discussed in [7].
Among the dynamic problems of plates and shells, a special place is occupied by problems of unsteady vibrations [8]. The classical theories of vibration of plates and shells give good results in the region of the lowest frequencies. With the transition to higher-frequency oscillations, the refined oscillation theories [9]
of such systems should be applied. In this case, there are also questions of the correct formulation of the boundary and contact conditions [10]. At the same time, refined theories also do not have a significant advantage when using analytical methods for solving problems. Therefore, in recent years, more and more numerical methods have been used to solve problems of the dynamics of plates and shells [11-13]. Because a relatively new and promising direction of research into the behavior of ribbed structural elements and, in particular, plates, are numerical studies of the required parameters [14]. Wave finite element methods and spectral finite elements underlie some numerical developments [15-17]. The subject of numerical calculations of various aspects of ribbed structures is quite extensive. The works [18,19] are devoted to the analysis of some scientific works in which the research of this direction was carried out. Let us note the methods of creating mathematical models of unsteady vibrations of circular cylindrical shells and rods [20, 21], the distribution of which for calculating the dynamics of ribbed plates allows developing new refined equations of vibration taking into account more complex physical and mechanical properties of their materials.

In connection with the increasing number of applications of reinforced structures and their elements, and despite the increasing number of publications in this area, the problem of creating refined models is very urgent. These models are designed to describe the dynamic processes in these systems and the search for effective analytical methods for solving the corresponding initial-boundary value problems of mathematical physics, taking into account the nonlinear properties of both the skin and the ribs [22,23]. Based on this, this article is devoted to the analytical and numerical calculation of a circular plate, reinforced with annular ribs, taking into account geometric nonlinearity, elastoplastic properties of the material and a discrete number of reinforcing ribs.

2. Methods

2.1. Problem Statement
Consider a round plate clamped along the edge. The plates are reinforced with annular ribs and referred to a cylindrical coordinate system \((r, \varphi, z)\). It is assumed that the materials of the ribs and the skin are the same. The cross-sections of the ribs are rectangular quadrangles, the dimensions of which are the same and constant. It is assumed that the external dynamic load \(P(t)\) acting on the surface excites the vibrations of the plate. The location of the ribs will be taken into account by the formula:

\[
H_i(r) = h \delta(r - r_i), \quad 0 \leq r \leq r_0,
\]

where the unit columnar function is given by the formula \([\ldots]\)

\[
\delta(r - r_i) = \begin{cases} 
0, & r < a_i, \ r > b_i, \\
1, & a_i \leq r \leq b_i,
\end{cases}
\]

\(r_0\) - radius of a plate; \(r_i\) - the radial coordinate of the midpoint of the contact area of the shell and the \(i^{th}\) rib; \(h, c\) - height and width of the rib. In this case, the coordinates of the extreme points of the \(i^{th}\) edge are determined as \(a_i = r_i - c/2; \ b_i = r_i + c/2\).

The vibration equations for a Timoshenko-type plate have the following form [35]:

\[
(N_i r)' - N_2 = r \rho \left[ \ddot{u}(h_0 + F) + \ddot{w} S \right], \quad (rQ)' + (N_i r w)' = r \rho \ddot{w}(h_0 + F) - rP, \]

\[
(rM_i)' - M_2 - rQ = r \rho \left[ \ddot{w}(h_0^3 / 12 + J) + \dot{u} S \right]
\]  

(1)
where $\rho$, $h_0$ - density and thickness of the shell, respectively.

$$F = \sum_{i=1}^{m} F_i \delta(r - r_i), \quad S = \sum_{i=1}^{m} S(r_i) \delta(r - r_i), \quad J = \sum_{i=1}^{m} J(r_i) \delta(r - r_i)$$

$F_i$ - cross-sectional area of the rib, $S(r_i)$ - static moment and $J(r_i)$ - moment of inertia of this section with respect to the axis. The boundary conditions of the problem are as follows:

$$r = r_0, \quad u = w = \psi = 0 \quad \text{and} \quad r = 0, \quad u = \frac{\partial w}{\partial r} = \psi = 0, \quad (2)$$

where $u, w$ - displacement of the points of the median surface of the plate, $\psi$ - angle of rotation of the normal segment. Initial conditions are zero.

### 2.2. Basic Equations

The deformations of the shell points have the following form:

$$\varepsilon_1^x = \varepsilon_1 + z \frac{\partial \psi}{\partial r}, \quad \varepsilon_2^x = \varepsilon_2 + z \frac{\psi}{r}.$$

Here

$$\varepsilon_1 = \frac{\partial u}{\partial r} - \frac{w}{r_0} + \frac{1}{2} \left( \frac{\partial w}{\partial r} \right)^2, \quad \varepsilon_2 = \frac{u}{r} - \frac{w}{r_0}, \quad \varepsilon_{13} = \frac{\partial w}{\partial r} + \psi$$

is a deformation of the points of the middle surface of the shell.

The problem is solved taking into account the elastoplastic properties of the sheathing materials and ribs. To describe dynamic deformation beyond the elastic limit, the theory of plastic flow is used [36]. Applying the stepwise method for solving the problem, we divide the loading time into $N$ small steps numbered in ascending order. Then, at the $n$-th step of loading, the deformations and stresses at the points of the shell are determined as follows

$$\varepsilon_\alpha = \varepsilon_\alpha^y + \sum_{i=1}^{n} \Delta_n \varepsilon_\alpha^p, \quad \sigma_\alpha = \frac{E}{1-\mu^2} \left[ \varepsilon_1 + \mu \varepsilon_2 - \sum_{i=1}^{n} (\Delta_n \varepsilon_1^p + \mu \Delta_n \varepsilon_2^p) \right],$$

$$\sigma_2 = \frac{E}{1-\mu^2} \left[ \varepsilon_2 + \mu \varepsilon_1 - \sum_{i=1}^{n} (\Delta_n \varepsilon_2^p + \mu \Delta_n \varepsilon_1^p) \right], \quad \sigma_{13} = \frac{E}{2(1+\mu)} (\varepsilon_{13} - \sum_{i=1}^{n} \Delta_n \varepsilon_{13}^p),$$

where $\Delta_n \varepsilon_\alpha^p$ - plastic deformation increment, $n = 1, 2, ..., N; \alpha = 1, 2, 13; \varepsilon_\alpha^y$ - elastic deformation. According to the found values of $\sigma_\alpha$, $\sigma_2$, the stress intensity is computed by the formula

$$\sigma_\alpha^{(y)} = \sqrt{\sigma_1^2 + \sigma_2^2 - \sigma_1 \sigma_2}, \quad (3)$$

$\gamma = 1, 2, 3, ...$ is the order of approach to the deformation diagram. If $\sigma_\alpha^{(1)}$ at the considered point of the grid region of the plate at $t > (n + 1)\tau$ is less than the value of $\sigma_\alpha$ calculated at step $n$, then elastic unloading of the material occurs at the point and we proceed to consider the next node or layer of the grid. Otherwise, we look for the increment $\Delta_n \varepsilon_1^p$, which we write in the form $\sum_{l=1}^{N} \Delta_n \varepsilon_1^p$, after which we find plastic deformations

$$\Delta_n \varepsilon_{13}^p = \frac{\Delta_n \varepsilon_{13}^p (\sigma_1 \sigma_2 - \sigma_1 \sigma_2)}{\sigma_1^2}, \quad \Delta_n \varepsilon_1^p = \frac{\Delta_n \varepsilon_1^p (\sigma_2 - \sigma_2)}{\sigma_2^2}. \quad (4)$$

Here, the increments of plastic deformation are determined from the following relation

$$\Delta_{N+1} \varepsilon_1^p = \left( 1 - \frac{E_1}{E} \right) \sigma_1^{(y)} - \frac{E_1}{E} \left( \sum_{n=1}^{N} \Delta_n \varepsilon_1^p + \sum_{l=0}^{N-1} \Delta_{l+1} \varepsilon_1^p \right) - \varepsilon_i^{(y)}$$

where $E_1 = \frac{4}{3} (\sigma_p - \sigma_T) \frac{100}{\delta} - strengthening model; \sigma_T$ - fluidity limit; $\sigma_p$ - strength limit; $\delta$ - residual elongation; were determined by reference books. The stress intensities are found by the formula (3).
the next iteration $\sigma_i^{(v)}$ exceeds the diagrammatic values of stresses at the found level of plastic deformations, then the calculations are repeated. Calculations are performed until the condition $|\Delta_{N+1}^{(v)} e_{rj} - \Delta_{N+1}^{(v-1)} e_{rj}| < \delta$ is satisfied, which provides a sufficient approximation to the deformation diagram.

Longitudinal, lateral forces and moments related to the shell have the following forms [1]:

$$N_i = N_i^0 + N_i^R, \quad M_i = M_i^0 + M_i^R, \quad Q = Q^0 + Q^R, \quad i = 1,2.$$  

Longitudinal, lateral forces and moments related to the shell have the following forms:

$$N_i^0 = Eh[\varepsilon_i^0 + \mu \varepsilon_j^0 - \sum_{n=1}^{N} (\Delta_n^1 \varepsilon_i^p + \mu \Delta_n^1 \varepsilon_j^p)]/(1 - \mu^2),$$

$$M_i^0 = D[\varepsilon_i^1 + \mu \varepsilon_j^1 - \sum_{n=1}^{N} (\Delta_n^2 \varepsilon_i^p + \mu \Delta_n^2 \varepsilon_j^p)]/(1 - \mu^2),$$

$$Q^0 = \frac{k^2 Eh}{2 (1+\mu)} \varepsilon_{13}^0 - \frac{E}{2 (1+\mu)} \int_{-h/2}^{h/2} f(z) \sum_{n=1}^{N} \Delta_n e_{13}^p dz, \quad i = 1,2.$$  

Here $E, \mu$ - modulus of elasticity and Poisson's ratio of the shell material; $f(z)$ - function characterizing the stress distribution law over the thickness of the structure;

$$D = Eh^3 (1 + \mu)^{-1}/12; \quad \Delta_n^1 \varepsilon_i = \frac{1}{h} \int_{-h/2}^{h/2} \Delta_n e_{13}^p dz; \quad \Delta_n^2 \varepsilon_i = \frac{1}{h} \int_{-h/2}^{h/2} \Delta_n e_{13}^p dz, \quad l = 1,2.$$  

Longitudinal, lateral forces and moments related to ribs have the following forms:

$$N_i^R = \int_{h/2}^{h/2+H} \sigma_i^R dz = A(\varepsilon_i^0 + \mu \varepsilon_j^0) + B \varphi_1(\psi) - G \sum_{n=1}^{N} (\Delta_n^3 \varepsilon_i^p + \mu \Delta_n^3 \varepsilon_j^p),$$

$$M_i^R = \int_{h/2}^{h/2+H} \sigma_i^R dz = B(\varepsilon_i^0 + \mu \varepsilon_j^0) + C \varphi_1(\psi) - G \sum_{n=1}^{N} (\Delta_n^4 \varepsilon_i^p + \mu \Delta_n^4 \varepsilon_j^p),$$

$$Q^R = \int_{h/2}^{h/2+H} \sigma_{13}^R dz = D_{13} (\frac{\partial \psi}{\partial r} + \psi) - G_{13} \int_{z}^{z+h/2} f_1(z) \sum_{n=1}^{N} \Delta_n e_{13}^p dz.$$  

where

$$\varphi_1(\psi) = \frac{\partial \psi}{\partial r} + \mu \frac{\partial \psi}{r}; \quad \varphi_2(\psi) = \frac{\psi}{r} + \mu \frac{\partial \psi}{r}; \quad i = 1,2; \quad j = 2, \quad i = 1; \quad j = 1, \quad i = 2.$$  

$$\Delta_n^3 \varepsilon_i = \int_{h/2}^{h/2+H} \Delta_n e_{13}^p dz; \quad \Delta_n^4 \varepsilon_i = \int_{h/2}^{h/2+H} \Delta_n e_{13}^p dz, \quad l = 1,2; \quad A = GF; \quad B = GS; \quad C = GJ; \quad G = \frac{E}{1 - \mu^2}.$$  

$$D_{13} = G_{13} H(r); \quad G_{13} = \frac{E v}{2(1+\mu)}; \quad k^2 = \frac{5}{6}; \quad E, \mu, G \quad - \quad \text{elastic constants of rib material.}$$

2.3. Approximation of equations of the vibration

The scheme for the numerical solution of problem (1) - (2) by the method of finite differences is based on determining the displacements and angles of rotation at the mesh nodes, and the deformations, moments of longitudinal and lateral forces at the center of the element. The approximation of the derivatives in the element is as follows:

$$(\frac{\partial f}{\partial r})_{i+\frac{1}{2}} = (f_{i+1} - f_{i})/\Delta r_i,$$

where $\Delta r_i = r_i - r_{i-1}; \quad f_i$ --values of function at points $r_i, 1 \leq i \leq N + 1$. To approximate the equations of motion (1), which are centered at the nodal points, the central differences are used. Non-differentiable members are reduced to a node by averaging the corresponding values in the elements. The time derivatives were approximated by expressions of the form.
where \( \tau \) – time step; \( n \)-index, defining temporal layer. The resulting finite-difference analogue of system (1) has the form

\[
\begin{align*}
\frac{\partial^2 w_i^n}{\partial t^2} & = \frac{1}{\tau^2} \left[ w_i^{n+1} - 2w_i^n + w_i^{n-1} \right] \\
\end{align*}
\]

\( \frac{\partial w_i}{\partial r} \bigg|_{r_i} = \frac{3w_i^n - 4w_i^{n-1} + w_i^{n-2}}{2\Delta r_i} \)  

The results are shown in Figures 1-4.

3. Results and Discussions

According to the results of the solutions of equation (4), one calculates the deflection of the central point, longitudinal and lateral forces, as well as moments in the sections of the ribbed plate under the action of a uniformly distributed load \( P = P_0 e^{-t/\alpha} \), at \( P_0 = 2.5 \text{ MPa} \), \( 5 \text{ MPa} \), \( \alpha = 10^{-3}s \). Geometric and elastic characteristics of the plate are as follows: \( n_0 = 0.5m; h_0 = 0.01m; E = 7.56 \cdot 10^4 \text{ MPa}; \mu = 0.3 \text{ MPa} \); \( \rho = 25940 \text{ kg/m}^3 \); \( h = 4 \cdot 10^{-2}m \); \( c = 3.33 \cdot 10^{-2}m \). Here cases are considered where the plate is reinforced by two and four annular ribs. For comparison, the above parameters were calculated at the corresponding points of the shell. The results are shown in Figures 1-4.
Figure 1. The dependencies $w \sim r$ for unsecured and reinforced two ribs of the plate

The results of calculating the deflection of a central point that is not reinforced and reinforced by two ribs of the plate are shown in Figure 1 as graphs versus time $0 \leq t \leq 0.003\,s$. From these graphs, it follows that the maximum value of the deflection of the central point of the plate, regardless of reinforcement, is achieved in the time interval $0.001 \leq t \leq 0.002$. Thus, the deflection of the central point of the unsupported plate exceeds by at least 37% the value of the deflection of the central point of the reinforced plate. In the most effective case, when the ribs are located at distances $d = 0.4$ and $d = 0.6$ from the central point, the presence of ribs lead to a decrease in the deflection value to 44% (Figure 1). In other cases of reinforcement, the decrease in deflection values is in the range from 37% to 44%.

Figure 2 shows the dependence of the radial force $N_r$ from the coordinate $r$ at a fixed value of time $t = 6 \cdot 10^{-4}\,s$ and different $d$.

Figure 2. The dependencies $N_r \sim r$ at $t = 6 \cdot 10^{-4}\,s$ and different $d$. It can be seen that for a given moment in time, the reinforcement of the plate with one rib
The amplitude of the shear force $Q$ for a smooth plate varies according to the law, close to the sinusoidal one. When the plate is reinforced with stiffening ribs, this pattern is violated. In some cases it is possible to observe about the same regularity (the plate is supported by four ribs), and in another case, the plate is reinforced with one rib, this regularity is absent.

Figures 4a and b show the dependence of the deflection $w$ on the time $t$ of the central point of the unreinforced and reinforced by four ribs of the plates. The calculations were carried out for elastic and elastoplastic models and for values of the amplitude $P_0$ of the external exponential load equal to 2.5; 5 MPa. Figure 4a shows graphs of the deflection of central points reinforced by four ribs and non-reinforced plates with an external load amplitude of 2.5MPa. From the graphs presented, it follows that for a non-reinforced plate, taking into account plastic deformation, leads to an increase in the deflection values for all time instants. For example, the maximum values of the deflection of the central point calculated by the elastic and elastoplastic models differ by 32%. In the case when the plate is reinforced by four ribs, the graphs of the dependences of the deflection on time, in the elastic and elastoplastic cases, merge. This shows that under external exponential loads, the amplitudes of which do not exceed 2.5MPa, the influence of plastic deformation can be neglected to calculate the plates supported by four edges of the plate. At an external load with an amplitude of 5MPa and higher, Figure 2b, the effect of plastic deformation on the deflection cannot be neglected. As the results shown in Figure 2b, the larger the amplitude of the external

\[ d = 0.2 \ (1) \] leads to an increase of $N_t$ up to 55% compared to the smooth plate. Reinforcement of the plate with two $d = 0.2; 0.8 \ (2)$ ribs reduces the force $N_t$ to 55% – 58%, but reinforcement with four ribs lowers the values $N_t$ to almost zero.

Figure 3’ curves for the variation in the lateral force $Q$ are plotted as a function of the radial coordinate at $t = 6 \cdot 10^{-4} \ s$ and different values of the coefficient $d$.

The transverse force at the attachment points of the ribs ($d = 0.2; 0.6; 0.8$) takes values close to zero. For $r = 0.3m$ all curves Q, regardless of the presence and number of reinforcing ribs, or their absence change their sign to the opposite. This indicates that the points of the plate with the coordinate $r \geq 0.3/m$ receive a deflection in the direction opposite to the deflection direction at $r \leq 0.3/m$.
load the higher is the deflection obtained by the elastoplastic model as compared to the elastic model, for example, the maximum value of the elastoplastic deformation of a smooth plate at $P_0 = 2.5\text{MPa}$

![Figure 4](image.png)

Figure 4. According to the deflection of the central point reinforced by four ribs and non-reinforced plates at $a) P_0 = 2.5\text{MPa}$; $b) P_0 = 5\text{MPa}$;

is $\approx 0.068\text{m}$; at $P_0 = 5\text{MPa}$ it is equal $t_0 \approx 0.12\text{m}$; at $P_0 = 7.5\text{MPa}$ it is equal $t_0 \approx 0.151\text{m}$ and at $P_0 = 10\text{MPa}$ it is equal $t_0 \approx 0.190\text{m}$. With an increase in amplitude of the external load, the difference between the values of deflection of the central point calculated by the elastic and elastoplastic models increases. For example, for a point in time equal to $0.003\text{s}$ the specified difference reaches: $a) at P_0 = 5\text{MPa} - \approx 0.015\text{m}; b) at P_0 = 7.5\text{MPa} - \approx 0.07\text{m}; c) at P_0 = 10\text{MPa} - \approx 0.098\text{m}$.

The graphs of the deflection of the central point of both smooth and reinforced plates, obtained on the basis of the elastic model, for all values of the external load, are pronouncedly sinusoidal in nature. At the same time, the graphs obtained on the basis of the elastoplastic model, having reached a relative maximum under the action of average load (Figure 2$b$), have a straightforward character, with a transition to a slowly descending curve with increasing external load.

4. Conclusions

The conclusions made in this paper are the following:
- an elastoplastic model for calculating a round discretely finned plate is proposed, which allows one to determine the deflection, forces and moments at the points of the plate;
- a numerical method for calculating a reinforced plate based on an elastoplastic model is proposed;
- the maximum value of the deflection of the central point of the plate, regardless of reinforcement, is achieved in the time interval $0.001 \leq t \leq 0.002$. At the same time, the deflection of the central point of the unsupported plate exceeds by at least $37\%$ the value of the deflection of the central point of the reinforced plate;
- there is an abrupt change in the tangential force and its value is the smaller, the greater the number of ribs. The reinforcement of such a plate with four annular ribs practically nullifies the action of the force except for the points of attachment of the ribs, where there are abrupt changes in the values of this force;
- the amplitude of the lateral force for a smooth plate varies according to a law close to sinusoidal. When reinforcing the plate with stiffeners, this pattern is violated. In one case, one can observe about the same picture (the plate is supported by four edges), and in the other case (the plate is supported by one edge), this regularity is completely absent;

- the maximum deflection values of the central point of the plate, calculated by elastic and elastoplastic models, differ by 32%. In the case when the plate is supported by four ribs, the graphs of the dependences of the deflection on time in the elastic and elastoplastic cases merge. This shows that under external exponential loads, the amplitudes of which do not exceed 2.5 MPa, the influence of plastic deformation can be neglected for calculating the plate, backed by a large number of stiffeners.

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