Jet confinement by magneto-torsional oscillations

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Abstract Many quasars and active galactic nuclei (AGN) appear in radio, optical, and X-ray maps, as a bright nuclear sources from which emerge single or double long, thin jets. When observed with high angular resolution these jets show structure with bright knots separated by relatively dark regions. Nonthermal nature of a jet radiation is well explained as the synchrotron radiation of the relativistic electrons in an ordered magnetic field. We consider magnetic collimation, connected with torsional oscillations of a cylinder with elongated magnetic field, and periodically distributed initial rotation around the cylinder axis. The stabilizing azimuthal magnetic field is created here by torsional oscillations, where charge separation is not necessary. Approximate simplified model is developed. Ordinary differential equation is derived, and solved numerically, what gives a possibility to estimate quantitatively the range of parameters where jets may be stabilized by torsional oscillations.

1 Introduction

Objects of different scale and nature in the universe: from young and very old stars to galactic nuclei show existence of collimated outbursts - jets. Geometrical sizes of jets lay between parsecs and megaparsecs. The origin of jets is not well understood and only several qualitative mechanisms are proposed which are not justified by calculations. Theory of jets must give answers to three main questions: how jets are formed? how are they stabilized? how do they radiate? The last question is related to the problem of the origin if relativistic particles in outbursts from AGN, where synchrotron emission is observed. Relativistic particles, ejected from the central machine rapidly lose their energy so the problem arises of particle acceleration inside the jet, see review of Bisnovatyi-Kogan (1993).

It is convenient sometimes to investigate jets in a simple model of infinitely long circular cylinder, Chandrasekhar & Fermi (1953). The magnetic field in the collimated jets determines its direction, and the axial current may stabilize the jet’s elongated form at large distances from the source (e.g. in AGNs), (Bisnovatyi-Kogan et al., 1969). When observed with high angular resolution these jets show a structure with bright knots separated by relatively dark regions (Thomson et al., 1993). High percentages of polarization, sometimes exceeding 50%, indicate the nonthermal nature of the radiation, explained as synchrotron emission of the relativistic electrons in an ordered magnetic field.

Here we consider stabilization of a jet by magnetohydrodynamic mechanism, connected with torsional oscillations. We suggest that the matter in the jet is rotating, and different parts of the jet rotate in different directions. Such distribution of the rotational velocity produces azimuthal magnetic field, which prevents a disruption of the jet. The jet remains to be in a dynamical equilibrium, when it is representing a periodical, or quasiperiodical structure along the axis, and its radius is oscillating with time all along the axis. The space and time period of oscillations depend on the conditions at jet formation: the length scale, the amplitude of the rotational velocity, and the strength of the magnetic field. The time period of oscillations should be obtained during construction of the dynamical model, what also should show at which input parameters may exist a long jet, stabilized by torsional oscillations.
Here we construct a simplified model of this phenomena, which confirms the reality of such stabilization, see also Bisnovatyi-Kogan (2007).

2 General picture

Consider a long cylinder with a magnetic field directed along its axis (Fig.1). This cylinder will expand unlimitedly under the action of pressure and magnetic forces. The limitation of the radius of this cylinder could be possible in dynamic state, when the whole cylinder undergoes magneto-torsional oscillations. Such oscillations produce toroidal field, which prevent a radial expansion. There is a competition between the induced toroidal field, compressing the cylinder in radial direction, and gas pressure, together with the field along the cylinder axis (poloidal), tending to increase its radius. During magneto-torsional oscillations there are phases, when either compression or expansion forces prevail, and, depending on the input parameters, we may expect three kinds of a behavior of such cylinder.

1. The oscillation amplitude is low, so the cylinder suffers unlimited expansion (no confinement)

2. The oscillation amplitude is too high, so the pinch action of the toroidal field destroys the cylinder, and leads to formation of separated blobs.

3. The oscillation amplitude is moderate, so the cylinder survives for an unlimited time, and its parameters (radius, density, magnetic field etc.) change periodically, or quasi-periodically in time. Here we try to find a simple approximate way for obtaining a qualitative answer, and to make a rough estimation of parameters leading to different regimes.

3 Profiling in axially symmetric MHD equations

We simplify the system of MHD equations with axial symmetry, $\frac{\partial}{\partial \phi}=0$, for the perfect gas at infinite conductivity, written in cylindric coordinates $(r, \phi, z)$ (Landau and Lifshits, 1982). We use for this purpose a profiling procedure. There is no gravity in the direction of the cylinder axis $(z)$, and we approximate density by a function $\rho(t, z)$, suggesting a uniform density along the radius. The components of the velocity and magnetic field are approximated as

$$v_r = r a(t, z), \quad v_\phi = r \Omega(t, z), \quad v_z = 0;$$

$$B_r = r h_r(t, z), \quad B_\phi = r h_\phi(t, z), B_z = B_z(t, z).$$

In this case the current components are written as

$$j_r = -\frac{ce r h_\phi}{4\pi} \frac{\partial h_r}{\partial z}, \quad j_\phi = \frac{ce r h_r}{4\pi} \frac{\partial h_\phi}{\partial z}, \quad j_z = \frac{c h_\phi}{2\pi}. \quad (3)$$

After neglecting velocity $v_z$ along the axis, we should omit the corresponding Euler equation, and the radial pressure gradient is approximated by the linear function, what gives, when using appropriate dimensions

$$\frac{\partial P}{\partial r} = \lambda \frac{P}{R^2} r; \quad (4)$$

where the constant $\lambda \sim 1$ is connected with the equation of state, $P(t, z)$ is the pressure, $R(t, z)$ is the radius of the cylinder. In the subsequent consideration we consider an adiabatic case with a polytropic equation of state $P = K \rho^\gamma$. Neglecting the $z$ derivatives in the Poisson equation, we obtain $\varphi_G = \pi G \rho r^2$. Substituting (1)-(4) into the original system of the equations, we obtain for the profiling functions the following equations

$$\frac{\partial a}{\partial t} + a^2 - \Omega^2 = \lambda \frac{P}{\rho R^2} - 2\pi G \rho + \frac{1}{4\pi \rho} \left( B_z \frac{\partial h_r}{\partial z} - 2 h_\phi^2 \right); \quad (5)$$

$$\frac{\partial \Omega}{\partial t} + 2a \Omega = \frac{1}{4\pi \rho} \left( B_z \frac{\partial h_\phi}{\partial z} + 2h_r h_\phi \right); \quad (6)$$

$$\frac{\partial h_\phi}{\partial t} = \frac{\partial (a B_z)}{\partial z} - 2(a h_\phi - \Omega h_r); \quad (7)$$

$$\frac{\partial h_r}{\partial t} = \frac{\partial (a B_z)}{\partial z}, \quad \frac{\partial B_z}{\partial t} = -2a B_z; \quad (8)$$

$$\frac{\partial \rho}{\partial t} = -2a \rho, \quad \frac{\partial R}{\partial t} = a R. \quad (9)$$

![Image](image.png)

Fig. 1.— Jet confinement by magneto-torsional oscillations (qualitative picture)
Jet confinement by magneto-torsional oscillations

It follows from (8), (9) relations, representing conservation of mass, and magnetic flux equivalent to freezing condition

$$\rho R^2 = C_m(z), \quad B_z R^2 = C_b(z), \quad B_z = \frac{C_b(z)}{C_m(z)} \rho.$$  \hfill (10)

In our subsequent consideration the arbitrary functions will be taken as constants: $C_m(z) = C_m$, $C_b(z) = C_b$.

It was shown by Bisnovatyi-Kogan (2007) that the approximate system of equations describes correctly small perturbations, connected with radial and torsional modes, in the equilibrium self-gravitating cylinder. We expect therefore that these modes will be described correctly in general nonlinear case in the jets where gravity is neglected.

4 Farther simplification: reducing the problem to ordinary differential equation

For the relativistic jet we neglect gravity, without which the equilibrium static state of the cylinder does not exist. We need to solve numerically the system of nonlinear equations (11)-(13) to check the possibility of the existence of a cylinder, which radius remains finite due to torsional oscillations. Instead we reduce the system to ordinary equations. Supposing a constant bulk motion velocity along $z$-axis, we consider axially symmetric jet in the comoving coordinate frame. If the confinement is reached due to standing magneto-torsional oscillations, there are points along $z$-axis where rotational velocity remains zero in this frame. Taking $\Omega = 0$ in the plane $z = 0$, let us consider standing wave torsional oscillations with the space period $\tilde{z}_0$ along $z$ axis. Then nodes with $\Omega = 0$ are situated at $z = n \tilde{z}_0/2$, $n = 0, 1, 2, ..., n = 0, 1, 2, ...$

Let us write the equations, describing the cylinder behavior in the plane $z = 0$, where $\Omega = 0$. All values in this plane we denote by ($\tilde{}$). We take also for simplicity $\lambda = 1$. We have than equations in the plane $z = 0$ as

$$\frac{d\tilde{h}_\varphi}{dt} = C_b \left( \frac{\partial(\Omega/R^2)}{\partial z} \right)_{z=0} - 2\tilde{a}\tilde{h}_\varphi, \quad \frac{d\tilde{R}}{dt} = \tilde{a}\tilde{R},$$  \hfill (14)

$$\dot{\tilde{\rho}} \tilde{R}^2 = C_m, \quad \dot{\tilde{B}}_z \tilde{R}^2 = C_b, \quad \dot{\tilde{B}}_z = \frac{C_b}{C_m} \tilde{\rho}.$$  \hfill (15)

Initial conditions for the system (11) - (15) are

$$\tilde{R} = R_0, \quad \dot{\tilde{\rho}} = \rho_0 = \frac{C_m}{R_0^2}, \quad \tilde{B}_z = \frac{C_b}{R_0},$$  \hfill (16)

$$\dot{\tilde{a}} = \tilde{h}_r = \tilde{\varphi} = 0 \text{ at } t = 0.$$

In (11)-(15) we have used relations

$$\dot{\tilde{\rho}} = R_0 \frac{\rho_0 R_0^2}{R^2}, \quad \dot{\tilde{B}}_z = \rho_0 \frac{C_b R_0^2}{R^2},$$  \hfill (17)

valid for any time. If the cylinder rotational velocity is antisymmetric relative to the plane $z = 0$, $\Omega = 0$, and cylinder density distribution is symmetric relative to this plane, then we have extremum (maximum) of the azimuthal magnetic field $h_\varphi$, with $(\frac{\partial h_\varphi}{\partial z})_{z=0} = 0$, and zero value of $\tilde{h}_r = 0$, which reaches an extremum (minimum) in this plane with $(\frac{\partial h_r}{\partial z})_{z=0} = 0$. The product $a \rho$ also reaches an extremum in the plane $z = 0$, so that $(\frac{\partial(a \rho)}{\partial z})_{z=0} = 0$. The term with $z$ derivative in the equation (14) is not equal to zero, and changes periodically during the torsional oscillations. We substitute approximately the derivative $d/dz$ by the ratio $1/\tilde{z}_0$, where $\tilde{z}_0$ is the space period of the torsional oscillations along $z$ axis. While $\Omega = 0$ in the plane $z = 0$, its derivative along $z$ is changing periodically with an amplitude $\Omega_0$, and frequency $\omega$, which should be found from the solution of the problem. We approximate therefore

$$\left( \frac{\partial(\Omega/R^2)}{\partial z} \right)_{z=0} = \frac{\Omega_0}{\tilde{z}_0 R^2} \cos \omega t.$$  \hfill (18)

Finally, we have from (11)-(14), (15) the following approximate system of equations, describing the nonlinear torsional oscillations of the cylinder at given $\tilde{z}_0$ and $\Omega_0$.

$$\frac{d\tilde{h}_r}{dt} = C_b \left( \frac{\partial(\Omega/R^2)}{\partial z} \right)_{z=0},$$  \hfill (13)

$$\frac{d\tilde{h}_\varphi}{dt} = C_b \left( \frac{\partial(\Omega/R^2)}{\partial z} \right)_{z=0},$$  \hfill (19)
The combination of the last two equations gives
\[ d(\tilde{h}R^2) = Cb\Omega_0 z_0 \cos \omega t, \] (20)
as has a solution, satisfying initial condition (16),
\[ \tilde{h}R^2 = Cb\Omega_0 z_0 \omega \sin \omega t, \] with account of which the first and third
equations in (19) are written as
\[ \tilde{R} d(\tilde{aR}) = K - (Cb\Omega_0 z_0 \omega)^2 \sin^2 \omega t, \] (21)
The problem is reduced to a system (22) with two non-
dimensional parameters:
\[ D = \frac{1}{2\pi KCm} (Cb\Omega_0 z_0 \omega)^2, \] and
\[ y(0), z = 0 \] at \( \tau = 0 \).
(22)

The solution of this system obtained numerically for
different \( D \) in the interval between 1.5 and 3.2, may be
divided into 3 groups.
1. At \( D \leq 2 \) there is no confinement, radius grows to
infinity after several low-amplitude oscillations (Fig. 2).
2. With growing of \( D \) the amplitude of oscillations
increase, and at \( D = 2.15 \) radius is not growing to infin-
ity, but is oscillating around some average value, form-
ing rather complicated curves (Figs. 3-5).
3. At \( D \geq 2.28 \) the radius goes to zero. At \( D = 2.28 \)
the dependence of the radius
with time may be very complicated, consisting of low-amplitude
and large-amplitude oscillations, which finally lead to
zero. The time at which radius becomes zero may hap-
pen at \( \tau \leq 100 \), like at \( D = 2.4, 2.6 \) (Bisnovatyi-Kogan,
2006), or goes through very large radius, and returned
back to zero value at very large time \( \tau \sim 10^7 \) at
\( D = 2.3, 2.55, 2.8 \) (Figs. 6-9). At \( D \geq 3 \) the radius goes to zero
at \( \tau < 2 \).

Fig. 4.— Time dependence of non-dimensional radius
\( y \) (upper curve), and non-dimensional velocity \( z \) (lower
curve), for \( D = 2.15 \) during a long time period.
Jet confinement by magneto-torsional oscillations

Fig. 5.— Time dependence of non-dimensional radius $y$ (upper curve), and non-dimensional velocity $z$ (lower curve), for $D = 2.25$.

Fig. 6.— Time dependence of non-dimensional radius $y$ (upper curve), and non-dimensional velocity $z$ (lower curve), for $D = 2.3$. during a long time period.

Fig. 7.— Time dependence of non-dimensional radius $y$ (upper curve), and non-dimensional velocity $z$ (lower curve), for $D = 2.55$. during a long time period.

Fig. 8.— Time dependence of non-dimensional radius $y$ (upper curve), and non-dimensional velocity $z$ (lower curve), for $D = 2.8$, during a long time period.

Fig. 9.— Time dependence of non-dimensional radius $y$ (upper curve), and non-dimensional velocity $z$ (lower curve), for $D = 3.1$, during a long time period.
6 Discussion

Consider isothermal equation of state $P = K\rho = v_s^2\rho$, $v_s^2 \leq c^2/3$, $v_s$ is the sound speed. For ultrarelativistic pair-plasma $K = c^2/3$. The parameter $D$, as a function of $R_0$, $z_0$, initial $\rho_0$ and $B_{z0}$, $\Omega_0$ and $\omega$ is written as

$$D = \frac{1}{2\pi\rho_0} \frac{B_{z0}^2 R_0^2 \Omega_0^2}{z_0^2 \omega^2 v_s^2}.$$  

(23)

The amplitude of oscillations $\Omega$, and $\omega$ should be found from the solution of the nonlinear system (5)-(10), together with the interval of values of $D$ at which confinement happens. In the approximate system (22) only $D$ characterizes different regimes, what for given $R_0$, $z_0$, $\rho_0$, and $B_{z0}$ determines a function $\Omega_0(D, \omega)$ for the collimated jet. To find approximately a self-consistent model with $\Omega_0, \omega(D)$ we use the the frequency linear oscillations $\omega = kV_A$. The frequency of non-linear oscillations is smaller, so we may write

$$\omega^2 = \alpha_n^2 k^2 V_A^2 = \frac{\alpha_n^2 \pi B_{z0}^2}{\rho_0 z_0^2}, \quad \alpha_n < 1, \quad k = \frac{2\pi}{z_0},$$  

(24)

therefore $\Omega^2 R_0^2 = 2\pi^2 D\alpha_n^2 v_s^2 < c^2$, $R_0^2 = \frac{K}{\omega^2} = \frac{z_0^2 \rho_0^2 c^2}{2\pi^2 D\alpha_n^2}$. On the edge of the cylinder the rotational velocity cannot exceed the light velocity, so the solution has a physical sense only at $v_s^2 < \frac{c^2}{2\pi^2 D\alpha_n^2} \approx \frac{c^2}{4\pi\alpha_n^2}$. Taking $\alpha_n^2 = 0.1$ for non-linear oscillations we obtain a restriction $v_{z0}^2 < \frac{c^2}{4}$. To have the sound velocity not exceeding $c/2$, the jet should contain baryons, which density $\rho_0$ exceeding about 30% of the total density of the jet. In a dense quasispherical stellar cluster around a supermassive black hole, the accretion disk is changing its direction of rotation, due to different sign of the angular momentum of the falling stars. In this situation the magneto-torsional oscillations should be inevitably generated in the outflowing jets. The knots in jets are observed in different objects, including a famous M87 jet. In the last one the knots are visible in radio, optics (Hiltner, 1959), and X-rays (Wilson and Yang, 2002). It is possible that they are connected with magneto-torsional oscillations.

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