Dielectric metamaterials with quasicrystal structure

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Abstract. Three types of quasicrystal lattices based on the Penrose tiling are considered. We analyze the maximum filling fraction of these structures and find a design with the filling fraction corresponding to the case of periodic lattices. By using simulations of a Gaussian beam propagation through the quasicrystal structure we obtain a homogeneous field distribution that is a hallmark of near-zero regimes in metamaterials.

1. Introduction

Photonic crystals and metamaterials are artificial periodic structures which have unique abilities to control the propagation of electromagnetic waves. In the last decade metamaterials in optical domain is mostly associated with dielectric structures having weak absorption unlike metal based metamaterials [1]. Since both metamaterials and photonic crystals can be composed of dielectric scatterers of similar shapes it is challenging to unravel how a photonic crystal becomes a metamaterial when the structure parameters are changed. Recently, a photonic crystal to metamaterial transition in structures consisting of dielectric rods arrange in a square lattice was studied [2] for the case of TE polarization (the magnetic field oscillates along the axis of the rods). The metamaterial phase was determined by appearance of a polariton-type feature in the band structure (Figure 1b). Such metamaterials operate due to magnetic Mie resonances in the structural elements, and a resonant effective magnetic permeability appears. The polariton-type feature is formed by the first and second dispersion branches: the second branch has a minimum at the center of the Brillouin zone, while the first one has the maximum at its boundary [3]. Besides, negative values of effective parameters near-zero regimes catch a remarkable attention [4]. TE polarization allows magnetic Mie resonances and effective mu-near-zero regime. We notice that similar effects exist for TM polarization, when the structure acquires a resonant effective dielectric permittivity owing to electric Mie resonances on the rods [5, 6].

The concept of band structures is a consequence of translational symmetry, i.e. perfect periodicity of the structure. Thus, the polariton-type features (which determine metamaterial phase) are related to the perfect crystal structures. On the other hand, the metamaterial behavior appears due to Mie resonances in structural elements, i.e. it is not associated with a periodicity explicitly. Is it possible to observe metamaterial effects in quasicrystals? We remind that quasicrystals are non-periodic, but strong ordered structures [7]. In particular, despite a translational symmetry in quasicrystals is absent, they have relatively clear diffraction patterns indicating a long-range order. We notice to review papers on photonic quasicrystals [8, 9].

In this paper, we are looking for a two-dimensional quasicrystal metamaterial with a Penrose lattice, which has a fifth-order rotation axis that does not permit a periodic structure. We
Figure 1. (a) Typical bandgap diagram of a photonic crystal: the interaction between two light cones (gray dashed lines) results in a frequency gap (shaded in green) at the boundary of the Brillouin zone (for example, in the X point). (b) Typical bandgap diagram of a metamaterial: a coupling of the almost flat Mie-resonance branch (red dashed line) and the light cone (gray dashed line) leads to a polariton-type feature and the Mie gap (shaded in red). Figure reproduced with permission from ref. [10]. Copyright (2018) American Chemical Society.

Figure 2. Dependence of maximum filling fraction on symmetry index. The blue solid circles correspond to maximum filling fraction for structures, black line is a linear approximation. Insets: (a) hexagonal lattice, (b) square lattice, (c) honeycomb lattice, (d) Penrose lattice, (e) Delaunay triangulation, (f) extended Penrose lattice.

analyze the maximum filling fraction of several periodic and quasiperiodic lattices to choose the optimal one. By simulation of a Gaussian beam propagation through samples we find near-zero index regimes which uncover metamaterial behavior in periodic and quasiperiodic structures.

2. Results
First, let us consider perfect periodic lattices. It is straightforward to evaluate the maximum filling fraction $f$ for three types of structures composed of similar non-overlapped dielectric rods arranged at the sites of hexagonal, square and honeycomb lattices (Figure 2). The filling fraction demonstrates almost a linear dependence on a symmetry index $n$ that is $n = 3$ for the hexagonal lattice, $n = 4$ for the square lattice and $n = 6$ for the honeycomb lattice.

Next, we discuss quasicrystals structures. A convenient way to generate a quasicrystal of a
high dimension is to use the projection and cut method [7]. However, an application of this method for generating a Fibonacci chain which is a one-dimensional quasicrystal is illustrative. We consider an auxiliary two-dimensional square lattice and project it onto a pair of one-dimensional subspaces named as a projection subspace and a physical subspace. If a projection of a square lattice site onto the projection subspace are contained in a window determined by a square lattice unit cell, then the projection of this site onto the physical subspace is a site of the one-dimensional quasicrystal lattice.

Here we consider a two-dimensional quasicrystal with the structure of the Penrose tiling with \( n = 5 \). In this case, we project a five-dimensional hypercubic lattice onto a three-dimensional projection subspace and a two-dimensional physical subspace. The hypercubic sites in the projection subspace being contained in the window (the projection of a hypercubic unit cell onto the three-dimensional subspace has the form of an icosahedron) generate the Penrose lattice in the two-dimensional physical subspace.

To calculate maximum filling fraction we evaluate minimum distance in the quasicrystal lattice. It defines the radius of non-overlapped rods arranged in a quasicrystal lattice. Since the Penrose lattice consists of two types of rhombs of different sizes, remarkable voids are observed in the structure (Figure 2). In this case \( f = 0.38, \ l_{\text{min}} = 0.62 \) and \( r_{\text{max}} = 0.31 \). Figure 2 shows that the filling fraction does not belong to the line dependence for periodic structures and lies far away from it. Reference [11] suggested using Delaunay triangulation for improving uniformity of the rod distribution in the Penrose quasicrystal. We briefly review this procedure in Figure 3. Let us consider an arbitrary point pattern (Figure 3a). First, we perform a Delaunay tessellation of the original point pattern. This provides a triangular partitioning that minimizes the standard deviation of the triangle angles (Figure 3b). The final lattice is formed by centroidal points of each triangles (Figure 3c). Using this procedure we generate a Delaunay-Penrose lattice (DPL) (Figure 3d) and evaluate its parameters the filling fraction \( f = 0.39, \ l_{\text{min}} = 0.46 \) and \( r_{\text{max}} = 0.23 \). This filling fraction is greater than one for the Penrose lattice. However it is far from the linear dependence yet (Figure 2).

We also consider another procedure to generate a Penrose-type lattice called an extended Penrose lattice (EPL). Additional rods are placed at the centers of each thick rhombs (Figure 3e-h). Figure 2 shows that the EPL structure has \( f = 0.55 \) being match closer to the linear dependence than other quasicrystals, so this structure is a good candidate for our study of metamaterial behavior. For this lattice \( l_{\text{min}} = 0.54 \) and \( r_{\text{max}} = 0.29 \).

O’Brien and Pendry [12] obtained a polariton feature in a square lattice with a dielectric constant \( \varepsilon = 200 \) and filling ratio \( r/a = 0.4 \). We calculate the field distribution in a prism consisting of dielectric rods arrange in the square lattice at the frequency corresponding to the

Figure 3. Protocols describing generation of (a)-(d) Delaunay triangulation, (e)-(h) extended Penrose lattice.
Figure 4. Distributions of the magnetic field for the $\varepsilon$-near zero modes in magnetic metamaterials with (a) square lattice, (b) extended Penrose lattice. Green dashed lines show metamaterial structure boundaries. Black arrows indicate the Gaussian beam incident direction. Inset in panel shows the magnetic field distribution on a larger scale. $a/\lambda=0.138$, $\varepsilon=200$, $r_{\text{max}}/l_{\text{min}}=0.4$, TE polarization.

polaritonic feature value. The field distribution of the mu-near-zero mode is uniform over the entire volume of the structure (see Figure 4a). We simulate the field distribution in the EPL quasicrystal with similar parameters $\varepsilon=200$ and filling ratio $r_{\text{max}}/l_{\text{min}}=0.4$ (Figure 4b). The mode is uniform over the structure which uncovers the existence of mu-near-zero regime in the EPL quasicrystal. Therefore, quasicrystal structures do allow transitions to the metamaterial phase.

3. Conclusion

Thus, we have calculated filling fractions for several periodic and quasicrystal structures and compared them. The extended Penrose lattice has the maximum filling fraction that lies almost on the linear dependence for periodic lattices. The simulation of the Gaussian beam propagation has shown the homogeneous field distribution corresponding to the mu-near-zero regime in metamaterials.

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References

[1] Jahani S and Jacob Z 2016 All-dielectric metamaterials Nature Nano. 11 23
[2] Rybin M V et al. 2015 Phase diagram for the transition from photonic crystals to dielectric metamaterials Nature Commun. 6
[3] Kaina N, Lemoult F, Fink M and Lerosey G 2015 Negative refractive index and acoustic superlens from multiple scattering in single negative metamaterials Nature 77 77
[4] Liberal I and Engheta N 2017 Near-zero refractive index photonics Nature Photon. 11 149
[5] Maslova E E, Limonov M F and Rybin M V 2018 Dielectric metamaterials with electric response Opt. Lett. 43 5516
[6] Maslova E E, Limonov M F and Rybin M V 2019 Transition “photonic crystal–metamaterial with electric response” in dielectric structures JETP Lett. 109 347
[7] Janot C 1994 Quasicrystals: a primer (Oxford, Clarendon Press)
[8] Poddubny A N and Ivchenko E L 2010 Photonic quasicrystalline and aperiodic structures Phys. E 42 1871
[9] Ivchenko E L and Poddubny A N 2013 Resonant diffraction of electromagnetic waves from solids (a review) Phys. Solid State 5 905
[10] Li S V, Kivshar Y S and Rybin M V 2018 Toward silicon-based metamaterials ACS Photon. 5 4751
[11] Florescu M et al. 2009 Complete band gaps in two-dimensional photonic quasicrystals Phys. Rev. B 80 155112
[12] O’Brien S and Pendry J B 2002 J. Phys.: Cond. Matt. 14 4035