Towards the physical point hadronic vacuum polarisation from Möbius DWF

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Hadronic vacuum polarisation

Can be computed in Euclidean space-time [Blum ’02]

\[ \Pi_{\mu\nu} = a^4 \sum_x e^{iQx} \langle J_{\mu}^{em}(x)J_{\nu}^{em}(0) \rangle \]

- \( \Pi_{\mu\nu}(Q) = (Q^2 \delta_{\mu\nu} - Q_{\mu}Q_{\nu})\Pi(Q^2) \)
- \( \hat{\Pi}(Q^2) = \Pi(Q^2) - \Pi(0) \)
- \( a_{HLO}^{\mu} = (\frac{\alpha}{\pi})^2 \int_0^\infty dQ^2 f(Q^2) \times \hat{\Pi}(Q^2) \)

Systematic uncertainties to be controlled - general

1. Simulations at physical \( m_\pi \)
2. Controlled continuum limit, FV effects
3.Disconnected diagrams [V. Gülpers, Mon, 14.55] [Della Morte et al. ’10]
4. Obtaining a real world result: charm quark, isospin effects . . .
Systematic uncertainties to be controlled - HVP related

- Conventional simulations do not allow access to sufficiently low Fourier momenta
- Integral is dominated in the region where relative errors are enhanced
- Structure of HVP tensor is such that $\Pi(0)$ is not directly accessible
- Systematic uncertainty introduced by extrapolation

Conventional procedure

- $\Pi(Q^2) = \frac{\Pi_{\mu\nu}(Q^2)}{Q_\mu Q_\nu - \delta_{\mu\nu} Q^2}$
- Transverse projection: $Q_\mu = 0$
- Take only diagonal components $\Pi_{\mu\mu}$
- $a^{HLO}_\mu = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dQ^2 f(Q^2) \times \hat{\Pi}(Q^2)$
Improving the systematics of connected HVP

Several new methods on the market

- R123 procedure ($\Pi(Q^2 = 0)$, utilising twisted BC formalism) [de Divitiis et al. '12]
- Padé approximants [Aubin et al. '12]
- Dispersive model study [Golterman et al. '13]
- Hybrid strategy [Golterman et al. '14] [Mon, 14.15, Sess 1D]
- HPQCD time moments [Chakraborty et al. '14] [Mon, 15.15, Sess 1D]
- ...

Challenge: Apply the optimal procedure to physical point data

This work: Fitting Padé approximants on the fresh DWF physical point data

inspired by [Aubin et al. '13]
Several new methods on the market

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**Challenge:** Apply the optimal procedure to physical point data

**This work:** Fitting Padé approximants on the fresh DWF physical point data

inspired by [Aubin et al. ’13]
Non physical $m_\pi$, $a^{-1} \approx 1.3, 1.7, 2.3$ GeV

- Local current at source, conserved at sink
- DWF (Möbius scale=1.0), Iwasaki/DSDR gauge action
- Fitting $Q^2$- dependence of $\Pi(Q^2)$ up to $Q^2_C \approx 2.5 - 9$ GeV$^2$

- Strong $m_\pi$ dependence
- Eliminate the systematics of chiral extrapolation: computing HVP at $m^\text{phys}_\pi$
$N_f = 2 + 1$ Domain Wall ensembles

Boyle et al. '11

$m_{\pi}^{\text{phys}}$

$a^2 [\text{fm}^2]$ vs $m_{\pi} [\text{MeV}]$

- $a^\mu_{\text{HLO}}$ from DWF for non-physical $m_{\pi}$ [Boyle et al '11]
- physical point HVP (●) recently measured → preliminary results!
$a_{\mu}^{HLO}$ from DWF at physical pion mass

**Physical point lattice parameters:**

- Möbius DWF, Iwasaki gauge action
  - $48^3 \times 96 \times 24$, $a^{-1} = 1.73$ GeV - measurements underway
  - $64^3 \times 128 \times 12$, $a^{-1} = 2.31$ GeV

**HVP with Möbius DWF**

- Möbius scale $= 2.0$
- Möbius conserved current [see talk by P. Boyle, Mon 6.10 p.m., 2.B]
- Local current at source, conserved at sink
- Point source, 12 source positions
Point vs. stochastic source

- Point source, 12 source positions
- $Z(2)$ wall source, 48 source positions
- (one-end trick) [McNeile et al. '06 ]

\[ \Pi(Q^2) \]

$Q^2 [\text{GeV}^2]$
Point vs. stochastic source

- Point source, 12 source positions
- Z(2) wall source, 48 source positions
- (one-end trick) [McNeile et al. ’06]
- Comparison (12 src. positions each, log scale on y-axis)
- Point src. better in low-$Q^2$ region ($Q^2 \lesssim 0.2$ GeV$^2$)
Physical point HVP from $N_f = 2 + 1$ DWF

**Physical point data:**

- $L/a = 48^3 \times 94 \times 24, \quad a^{-1} = 1.73\,\text{GeV}$
- $\Pi(Q^2)$ convergent sequence of PAs\[^{\text{[Aubin et al,'13]}}\]
  - VMD is unreliable
- Padé approximants $[N,D]$

\[
\Pi_{[N,D]}(Q^2) = \frac{\sum_{n=0}^{N-1} a_n Q^{2n}}{1 + \sum_{m=1}^{D} b_m Q^{2m}}
\]
L/a = 48, a−1 = 1.73 GeV, mπ = 138 MeV

Q^2_C = 1.5 GeV^2
Physical point HVP from $N_f = 2 + 1$ DWF

$L/a = 48$, $a^{-1} = 1.73$ GeV, $m_{\pi} = 138$ MeV

$Q^2_C = 1.5$ GeV$^2$
Left: Physical point data (Möbius DWF)

Right: Dispersive model study [Golterman et al. ’13]

Same qualitative behaviour - Padé [2,2] looks acceptable

Nevertheless, even for Padé [2,2]

- Removing correlations
- Results for different choice of $Q_C^2$ not compatible

Quoting the value for $a_{\mu}^{HLO}$ would be premature
Physical point HVP from $N_f = 2 + 1$ DWF

Light and strange contributions separated

![Graph showing light and strange contributions]

Limited statistics (28 meas. config.) with physical $m_\pi$ already gives:

- $\frac{\delta a_\mu^{stat.}}{a_\mu}$ for light contribution is $O(10)$ larger than for strange HVP
Summary

- Current status with DWF:
  - physical point data with $\sim 10\%$ stat. errors, measurements underway
  - in addition to the previous non-phys. point computation

- Significant increase signal/noise ratio near $Q^2 = 0$ coming from the light sector

- Large systematics with conventional procedure anticipated

Outlook

- Add another lattice spacing with $m^{phys}_\pi$

- Hybrid method [See talks: K. Maltman (Mon, 14.15, 1D)]

- HPQCD time-moment approach [See talks: B. Chakraborty (Mon, 15.15, 1D)] and possible improvements:
  - Discrete moments [See talks: K. Maltman (Mon, 14.15, 1D)]
  - Large volume limit [See talks: C. Lehner (Fri, 15.35, 8D)]

- Ultimate goal: $a^{HLO}_\mu$ with full control over syst. and stat. uncertainties ($< 1\%$)
Acknowledgements

- RBC-UKQCD collab. members
- T. Blum, L. Del Debbio, R. J. Hudspith, T. Izubuchi, C. Lehner, R. Lewis, K. Maltman, for useful discussions

- The research leading to these results has received funding from the European Research Council under the European Communitys Seventh Framework Programme (FP7/2007-2013) ERC grant agreement No 279757

- The calculations reported here have been done on DIRAC Bluegene/Q computer at the University of Edinburgh’s Advanced Computing Facility
[2, 2] Padé fits for different $Q_C^2$

Take correlations into account

Reference $a_{\mu}^{HLO}(Q_C^2_{\text{ref}})$ subtracted under bootstrap [$Q_C^2_{\text{ref}} = 1.5\text{GeV}^2$]

Results for different choice of $Q_C^2$ not combatible $\rightarrow$ uncontrolled systematics