Estimation of effective young’s modulus based on 3-D finite element model with particle-matrix interface for metal matrix composites

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Abstract. Effective Young’s Moduli of composite materials are estimated by variety of methods so far. This work attempts to estimate the Effective Young’s Modulus of aluminium356 with silicon carbide particle (SiC\textsubscript{p}) Metal Matrix Composite (MMC) with different particle aspect ratios using a simplified finite element model. Simplifications were made in terms of geometry of the particle. An improved model of the composite with an interface between the particle and the matrix was then developed and used to compare the results with the simplified model. The results were validated with experimental results, and the predicted values obtained from the Rule of Mixtures and the Halpin–Tsai (HT) model. The Young’s moduli estimated by considering the interface geometry in 3-D model is found to have better agreement with the experimental values than the simplified model at higher aspect ratios. The improvement in the results of the 3-D model with interface considerations is expected to make the model suitable for advanced studies.

1. Introduction

An MMC is a composite material with at least two constituent parts, one being a metal. Predicting the mechanical properties of MMCs has gone through a considerable challenge throughout their history. Among MMCs, aluminium alloy based composites show remarkable improvement in the mechanical, thermal and wear properties to fulfill the demand of the industries. They are substantially studied and universally used in industry fields in recent decades because of their outstanding properties, such as high specific stiffness and strengths\cite{1,2}, excellent friction and wear resistance\cite{3-6}, high electrical and thermal conductivity\cite{7}, and high temperature mechanical behaviour\cite{8,9}.

The casting technique has upgraded much over the past decade and it is used as a way to produce particle reinforced MMCs\cite{10-12}. Continuing refinement in process is being made by two inter-related approaches:

- A knowledge of the exact relationship of the microstructure of the material to its properties and
- The introduction of innovative processing to control and tailor the microstructure to give the desired properties\cite{10}.
The mechanical properties of MMCs generally depend on the microstructure of the materials [13]. The optimization of mechanical properties of these composites is based on the awareness of the relationship between the microstructure and the macroscopic response of MMC [13,11].

There are a few analytical and numerical models for predicting the mechanical behaviour of composite materials, with comparatively low errors [14-16]. The analytical models mainly include the Eshelby model, the variational principles of elasticity theory, the self-consistent model, the composite cylinder model and other unit-cell models [17-20]. These models normally deal with simplified assumptions and uniform Representative Volume Element (RVE) or unit cell, which encapsulates the major features of the microstructure. Numerical methods usually contain less simplified assumptions both in the mathematical formulation and in the selection of geometry of the RVE. These methods are also able to provide accurate predictions and mainly include the boundary element method, the finite elements method (FEM), and the finite difference method[17-20].

FEA is a viable method to analyse particle dispersed MMCs due to its modelling capability of the particle, being able to model different geometries and analyse their effect on the composite properties [21]. In this work, composite material was prepared with A356 (aluminium alloy) as the matrix and SiCₚ of average particle size 40 micron as the reinforcement material. Tensile test was carried out to estimate the bulk property of the composite. Estimation of Young’s modulus based on 3-D finite element model was carried out by using Finite Element Analysis (FEA). A simple model which assumes cylindrical shape for the particle was used to simulate the model[22]. An improved model of the composite with an interface between the particle and the matrix was then used to compare the result with the simplified model. The results were validated with experimental results, and the analytical values were obtained from the Rule of Mixtures and the HT model.

2. Methodology
Stir casting method was used to prepare A356/SiCₚ composite with fifteen volume percentage of SiCₚ reinforcement material in it. From the composite produced, a sample for tensile test with a gauge diameter of 10 mm and gauge length of 50 mm was prepared as per ASTM E8M standards. Tensile test using Universal Testing Machine (UTM) to get the tensile strength of the specimen was conducted. A 30mm thick sample for microstructure study was made which was hand polished using silicon carbide coated emery paper having grit numbers (FEPA standard) of P220, P320, P400, P600, P1000, P1200, P1500 and P2000. The final polishing was carried out using a double disc polishing machine running at 300rpm with velvet cloth attached to the discs and diamond paste (sizes 3-4 μm and 0.5-1 μm) used as abrasive and kerosene as lubricant to get a mirror like finishing. The microstructure of the composite was analysed with the help of LEICA DM 2700 M optical microscope and a microstructure analysing software, ImageJ to get the volume fraction, dimensions and thus the aspect ratios of the reinforcement material.

A simple analytical method namely Rule of Mixtures is used to evaluate the effective elastic response of a composite with perfect particle-matrix bonding under load. This rule can be formulated for particle-reinforced MMCs as follows [23]:

\[ E_C = E_m V_m + E_p V_p \]  

Where, \( E \) and \( V \) are respectively the Young’s modulus and volume fraction, for the matrix (m) and the particles (p). As observed, the volume fraction of particles in a MMC plays a determining role, affecting the elastic properties. Nevertheless this model does not take into account the particle aspect ratio. The HT model, assumes a perfectly oriented discontinuous reinforcement in the composite, parallel to the applied load [24, 25]. According to the HT model, the Young’s modulus for composites can be determined by:

\[ E_c = \frac{E_m(1+2\pi q V_p)}{1-q V_p} \]  

Where, \( E_c \), \( E_p \) and \( E_m \) are respectively the Young’s moduli of the composite, the particle and the matrix respectively.

The geometrical parameter \( q \) can be written as,
\[ q = \frac{\left( \frac{E_p}{E_m} \right) - 1}{\left( \frac{E_p}{E_m} \right) + 2s} \]  

Equation (3)

Where, \( s \) is the aspect ratio of the particle and \( V_p \) is the volume fraction.

Modelling of the RVE of the composite using the data acquired from microstructure study was carried out using ANSYS APDL 16.0. For the 3-D model, a 10-nodes high order 3-D solid element (Solid187) was selected to create the finite element mesh. Mesh convergence study was carried out which lead to selection of tetrahedral mesh with 30 number of divisions along all the three directions. The unit cell side is taken as 17.2 µm from the reference [22] and the size of the particles used for experiments are in the same range. This cell represents the particle-matrix system, and their repetition in the space reproduces the MMC. Due to the symmetry of the problem, only an eighth part of the cell is meshed for numerical analysis [23]. That is, the RVE consists on one-eight symmetric part of the unit-cell.

Simulations were done by applying a 5 MPa tensile stress on the upper end nodes of the RVE. Nodes on the bottom of the RVE are fully constrained as they are the nodes opposite to the face on which the stress is applied. Symmetrical boundary conditions are applied on the XY and the YZ plane in order to restrict the deformation in the normal direction. Simulations were carried out to obtain the Effective Young’s Moduli \( (E_c) \) of the RVE with varying aspect ratios from 0.2, 0.6, 1.0, 1.4 and 1.8 for both the models with and without interface consideration. The mechanical properties of the interface were assumed to be the average of the particle and matrix properties [23]. Finally, the simulation results were compared with results obtained from both experimental and analytical models.

3. Results and discussions

![Figure 1](image1.png)

Figure 1. (a) Prepared composite using stir casting method (b) tensile test specimen (c) specimen for microstructure study

A356/SiC\(_p\) composite is made by stir casting method (Figure 1a). Tensile testing samples were prepared by using lathe(Figure 1b). Tensile test result revealed that the metal composite have good tensile strength than unreinforced A356 alloys. The specimen had a yield stress of 147.7 N/mm\(^2\) and an elongation of 3.05 mm at yield. The Young’s Modulus of the material is found as 87.55 GPa which is the improved value of the same from the A356 alloy. There is 2.06% increase in yield stress and 16.73% increase in Young’s modulus for the composite as compared to the base alloy. A 30 mm thick piece is cut from composite using lathe for microstructure study. The part is polished to make it as the sample(Figure 1c).

The sample was observed under optical microscope. The images are captured at suitable magnification and recorded using computer & analysed. The microstructure image is shown in Figure 2. The SiC particles appeared with irregular shapes, their minimum and maximum average lengths being \( 5.23\pm4.03 \) µm and \( 16.69\pm9.48 \) µm, respectively; volume fraction of the particle obtained was 0.1219. The obtained data from the microstructure study is used as the input for modelling the RVE. The obtained unit-cell is shown in Figure 3 a. Figure 3 b and 3 c show the RVE for the model of particles with aspect ratios of 0.2 and 1.8, respectively. Figure 3 d shows the RVE for the model of the composite considering interface between the particle and the matrix.
The effective Young’s Modulus of the AMMC (Aluminium Metal Matrix Composite) obtained from the conventional Rule of Mixtures is 112.5 GPa. By using Halpin-Tsai model, it is found that the AMMC has effective elastic response values 82.64GPa, 86.41 GPa, 89.32GPa, 91.64GPa and 93.52 GPa for aspect ratios 0.2, 0.6, 1.0, 1.4 and 1.8 respectively.

For the study, the particles are modelled as cylinders of 612 $\mu m^3$, while the matrix that surrounds the cylindrical particle was considered as a cube of 5019 $\mu m^3$. The thickness of the interface between the particle and aluminium matrix is assumed to be 0.2 $\mu m$, which is a minimum comparable size with the dimension of the RVE. Figure 4 shows the meshed model of particle having 0.2 aspect ratio. It is found that there are 395918 elements in the RVE.
Effective Young’s Moduli of the RVE with different aspect ratios of the particle is estimated. This is carried out by finding deformations of the RVE in the direction of load applied along nine lines. The nine lines are depicted in the Figure 5. From the deformations the average strain along the direction of load applied is found and thus the effective Young’s modulus. The process is repeated for the RVE with interface consideration also.

The numerical simulations are done for the models having different aspect ratio of the particle varying from 0.2 to 1.8 for the same volume fraction of the particle, which is 0.1219. Simulations are done for models without interface consideration and for models with interface consideration. The contour plot obtained from simulation results of deformation of model having aspect ratio 0.2 and 1.8 without interface consideration is depicted in Figure 6.

**Figure 6.** Contour plot of deformation (in µm) in models having different aspect ratio

Simulations are done without considering the interface between the particle and the metal matrix. The Effective Young’s Modulus value varied from 81.94 to 86.22 with respect to the variation in aspect ratio. For the simulations done with considering the interface between the particle and the metal matrix, the Effective Young’s Modulus value varied from 81.89 GPa to 86.45 GPa with respect to the...
variation in aspect ratio. The curves comparing the values of effective Young’s Moduli for models without interface, with interface and that of Halpin-Tsai model are shown in Figure 7.

| Table 1. Values of effective Young’s Modulus in models with and without interface |
|---------------------------------------------------------------|
| Aspect ratio | Without interface | With interface | Halpin-Tsai model |
|--------------|-------------------|----------------|------------------|
| 0.2          | 81.94             | 81.89          | 82.64            |
| 0.6          | 81.99             | 81.99          | 86.41            |
| 1.0          | 83.41             | 83.20          | 89.32            |
| 1.4          | 84.67             | 84.60          | 91.64            |
| 1.8          | 86.22             | 86.45          | 93.52            |

The variation of Effective Young’s Modulus for models with and without interface consideration as well as using HT model with respect to change in aspect ratio is listed in Table 1.

![Figure 8. Variation in maximum axial stress](image8)

![Figure 9. Variation in maximum axial strain](image9)

The variations in maximum axial stress and maximum axial strain are compared between the models without interface and models with interface which are depicted in Figure 8 and Figure 9 respectively. The maximum axial stress is higher for the model with interface than for the model without interface although, this difference decreases for higher aspect ratios. The increment in the maximum stress value for higher aspect ratios can be explained with stress concentration phenomenon at the interface between particle and matrix. The particles with higher aspect ratios have higher amount of sharp corners and the stress is higher. That is particle with higher aspect ratio are under high stresses, being concentrated at the sharp corners of the particles [26].But, at higher aspect ratios as the relative amount of sharp corners or edges decreases in the interface, the value converges to that of the model without interface.

In the case of axial strain, the value goes on increasing in both the cases. The deviations between the values of the models keep increasing on moving from aspect ratio 0.2 to 1.8. That is, as the aspect ratio increases the effect of load applied is increased in the model having interface. This can be due to the phenomenon called interface de-bonding where the interface tend to detach from the particle as the relative position of the interface is nearer to the load applied surface[27].
4. Conclusions
In this work, a three-dimensional model was taken in order to study the behaviour of a particulate reinforced MMC with different aspect ratios, subjected to an axial load. Another model having interface between particle and the matrix was also used for checking the possibility of including interface within the basic model. The results of the calculated Young’s modulus using the proposed 3-D FEA model with interface was in better agreement with the experimental results than those obtained by the Halpin–Tsai model and the model without interface in higher aspect ratio. Deformation, stress and strain distributions showed an excellent agreement in both the FE models. Maximum axial stresses and strains were higher for the model with interface at all the aspect ratios. This behaviour could be explained due to relative amount of sharp edges and relative position of the interface respectively. From the results, it can be suggested that the 3D model with one eight part of unit cell as the representative volume element with interface consideration can be successfully used to model the mechanical behaviour of metal matrix composites.

In the present work the experimental study is done by tensile test. Using resonant frequency method, Young’s Modulus of the prepared material can be found more accurately and can be compared with the finite element results. The work can be extended to a higher level of accuracy by modelling the composite material using a more accurate image analysing software other than ‘ImageJ’. As a future work, parametric studies can be done with the finite element model by changing the interface thickness and volume fraction.

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