A linear-to-circular polarization converter with half transmission and half reflection using a single-layered metamaterial

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A linear-to-circular polarization converter with half transmission and half reflection using a single-layered metamaterial is theoretically and numerically demonstrated. The unit cell of the metamaterial consists of two coupled split-ring resonators with identical dimensions. A theoretical analysis based on an electrical circuit model of the coupled split-ring resonators indicates that the linear-to-circular polarization converter is achieved when the magnetic coupling between the split-ring resonators is set to a certain strength. A finite-difference time-domain simulation reveals that the single-layered metamaterial behaves as the linear-to-circular polarization converter and that the polarization converter has the combined characteristics of a half mirror and a quarter-wave plate.

Polarization is one of the important characteristics of electromagnetic waves. The manipulation of polarization is essential for optical communication, sensing, and other applications.1 Fundamental optical elements such as waveplates, polarizers, and polarization rotators are used to manipulate polarization. These elements are fabricated mainly using anisotropic media, chiral media, the Brewster effect, and the Faraday effect.1

Metamaterials have been demonstrated to enable the development of useful polarization control devices that cannot be achieved using naturally occurring media. The electromagnetic response of metamaterials can be controlled by design of the shape, material, and configuration of the unit structure. In addition, metamaterials are geometrically scalable; therefore, exotic polarization control devices can be fabricated for electromagnetic waves from radio to optical frequencies using metamaterials. For example, cross polarization converters,2–7 linear-to-circular polarization converters,7–11 achromatic wave plates,12 and broadband circular polarizers13 have been realized over a broad frequency range. Metamaterials with giant optical activity,1,14–20 which can be used as thin polarization rotators, have also been studied.

In this Letter, we theoretically and numerically demonstrate that a linear-to-circular polarization converter with half transmission and half reflection can be realized using a single-layered metamaterial composed of coupled resonators. Metamaterials composed of coupled resonators have been intensively investigated for the control of electromagnetic waves.18–28 A metamaterial with characteristics similar to that employed in this work has been previously reported.10 However, no theory for the structural design of such a metamaterial has been reported. The theory is necessary for the further development of electromagnetic wave control using metamaterials. An electrical circuit model of the metamaterial unit structure is used to theoretically show how the linear-to-circular polarization converter with half transmission and half reflection can be realized. The characteristics of the designed metamaterial based on the theory are then numerically analyzed to confirm that the metamaterial does behave as the linear-to-circular polarization converter. The polarization converter consists of a single-layered metamaterial and has combined characteristics of a half mirror and a quarter-wave plate. This study is expected to lead to the development of single-layered metamaterials that have combined characteristics of multiple fundamental optical elements, which could thus contribute to the miniaturization of optical systems.

A linear-to-circular polarization converter with half transmission and half reflection can be realized using a single-layered metamaterial with the unit structure shown in Fig. 1(a). The unit structure of the metamaterial is composed of two identical split-ring resonators. The axes of the rings are orthogonal to each other and to the direction of the wavevector of the incident electromagnetic wave. The unit structure is periodically arranged in the y and z directions.

First, the theory of the linear-to-circular polarization converter is described using the electrical circuit model of the metamaterial unit structure shown in Fig. 1(b). The electrical circuit model consists of two identical inductor-
For a capacitor series resonant circuits coupled with each other via a mutual inductance. Each inductor-capacitor series resonant circuit corresponds to the split-ring resonator, and the mutual inductance represents the magnetic coupling between the two split-ring resonators. The voltage source represents the y or z polarized incident electromagnetic wave.

Applying Kirchhoff's voltage law to the electrical circuit yields \( I_1 = -ZV/(Z^2 + (\omega M)^2) \) and \( I_2 = -i\omega MV/[Z^2 + (\omega M)^2] \), where \( \omega \) is the angular frequency of the voltage source and \( Z = R - \{\omega L - [1/(\omega C)]\} \).

For \( \omega = \omega_0 = 1/\sqrt{LC} \) and \( |M| = R/\omega_0 \), we obtain \( I_1 = -V/(2R) \) and \( I_2 = -i(M/|M|)[V/(2R)] \), which is reduced to \( I_2/I_1 = iM/|M| \). This implies that the radiation from the two split-ring resonators in each unit cell have the same amplitudes and a phase difference of \( \pi/2 \).

That is, the metamaterial radiates a circularly polarized electromagnetic wave when the y or z polarized electromagnetic wave with \( \omega = \omega_0 \) is incident on the metamaterial with \( |M| = R/\omega_0 \). This phenomenon yields the linear-to-circular polarization converter with half transmission and half reflection, as described in the next paragraph.

We consider the transmittance and reflectance of the metamaterial for \( \omega = \omega_0 \) and \( |M| = R/\omega_0 \). The copolarized (cross-polarized) transmittance and reflectance are defined as \( t_1 \) and \( r_1 \) \((t_2 \) and \( r_2) \), respectively. The reflected wave consists of only the radiation from the metamaterial; thus, \( r_2/r_1 = I_2/I_1 = iM/|M| \). That is, the reflected wave is a circularly polarized wave. On the other hand, the transmitted wave is a superposition of the incident wave and the radiation from the metamaterial. It is necessary for the calculation of the transmittance to evaluate the relation between the amplitude of the incident wave and that of the wave radiated from the metamaterial. To evaluate the amplitude, the radiation from the metamaterial is considered for \( M = 0 \).

In the case of \( M = 0 \), currents \( I_1 \) and \( I_2 \) are reduced to \( I_1 = -V/R = I_{10} \) and \( I_2 = 0 \) for \( \omega = \omega_0 \). When an electromagnetic wave with \( \omega = \omega_0 \) is incident on a resonant metamaterial (Lorentz medium) with a sufficiently strong response, the resonant metamaterial radiates an electromagnetic wave, which results in the disappearance of the transmitted wave. This implies that the radiated wave caused by current \( I_1 = I_{10} \) has the same amplitude and opposite phase to the incident wave. For \( \omega = \omega_0 \) and \( |M| = R/\omega_0 \), the transmittances and reflectances satisfy \( t_1 = 1/2, |t_2| = |r_1| = |r_2| = 1/2, \) and \( t_2/t_1 = I_2/I_1 = -iM/|M| \), because \( I_1 = I_{10}/2, I_2/I_1 = iM/|M| \), and \( \arg(t_1) = -\arg(I_1) \). Therefore, both of the transmitted and reflected waves are circularly polarized waves with half the power of the incident wave. The transmitted and reflected circularly polarized waves have the same helicity, because \( t_2/t_1 = -r_2/r_1 = -iM/|M| \) and the propagation directions of these two waves are opposite.

The transmission and reflection characteristics of the metamaterial shown in Fig.1(a) were analyzed using a finite-difference time-domain (FDTD) method\(^2^9\) to confirm the theory based on the electrical circuit model. The geometrical parameters of the metamaterial shown in Fig.1(a) were \( l = 12\) mm, \( w_1 = 1\) mm, \( w_2 = 4\) mm, \( g = 2\) mm, and \( t = 1\) mm. The periods of the unit cell in the \( y \) and \( z \) directions were set to 48 mm. The separation \( d \) between the two split-ring resonators was set to 10 mm, 8 mm, or 6 mm. The FDTD simulation space was discretized into uniform cubes with dimensions of \( 1\) mm \( \times 1\) mm \( \times 1\) mm.

Figure 2 shows the transmission and reflection spectra for \( d = 10\) mm, 8 mm, and 6 mm. Here, \( t_{ij} \) \((r_{ij}) \) \((i, j = y, z) \) represents the complex amplitude of the \( i \) polarized component of the transmitted (reflected) wave when the \( j \) polarized wave with unity amplitude is incident on the metamaterial. The calculated results satisfy \( t_{yy} = t_{zz}, \)

FIG. 2. Transmission spectra (upper panel) and reflection spectra (lower panel) for (a) \( d = 10\) mm, (b) 8 mm, and (c) 6 mm. The horizontal dashed lines in (b) represent transmittance and reflectance of 0.5.
This implies that almost ideal circularly polarized waves are transmitted and reflected for the D and X polarizations at $f = 2.99$ GHz, and $f_0 = 2.99$ GHz, and $f_2 = 3.01$ GHz. The horizontal dashed lines represent $1/\sqrt{2}$.

As we described above, $t_{yy} = t_{zz}$, $t_{yy} = r_{yy} = r_{zz}$, and $r_{yz} = r_{xy}$ are satisfied for the metamaterial, which implies that the eigropolarizations in the metamaterial are the $\pm 45^\circ$ polarizations. This observation is confirmed from the Maxwell equation. The wavevector $k$ is in the $x$ direction; thus, only $\varepsilon_{lm}$ and $\mu_{lm}$ ($l, m = y, z$) in the permittivity and permeability tensor components of the metamaterial are necessary for the analysis. Taking the symmetry of the metamaterial into account, the components of the permittivity and permeability tensors are, respectively, written as $\varepsilon_{lm} = \varepsilon \delta_{lm}$ and $\mu_{lm} = \mu \delta_{lm} + \mu_c (1 - \delta_{lm})$, where $\delta_{lm}$ is the Kronecker delta. After substitution of these tensors into the Maxwell equation and diagonalization, we obtain $k [E_{D+} E_{X+} E_{D-} E_{X-}]^T = \omega \varepsilon \delta_{lm} (Z_D - H_D, -Z_D, -H_D)^T$, where $E_{D+} = E_y + E_z \pm Z_D (H_y + H_z)$, $E_{X+} = E_y + E_z + Z_X (H_y + H_z)$, $E_{D-} = E_y + E_z \pm Z_X (H_y + H_z)$, $Z_D = \sqrt{(\mu - \mu_c)/\varepsilon}$, $Z_X = \sqrt{(\mu + \mu_c)/\varepsilon}$, and $T$ denotes matrix transposition. This equation indicates that the eigropolarizations in the metamaterial are the $45^\circ$ polarization (D polarization) and the $-45^\circ$ polarization (X polarization), and that the difference between the wavenumbers for the $\pm 45^\circ$ polarizations is caused by the magnetic coupling $\mu_c$ between the split-ring resonators.

Figure 4 shows the transmission and reflection spectra for the $\pm 45^\circ$ polarized waves for $d = 8$ mm. First, the physical meaning of the resonant response of the metamaterial is discussed based on the spectra. The splitting of the resonance line in Fig. 2 is derived from reso-
nances at \( f = f_1 = 2.97 \text{ GHz} \) for the D polarization and \( f = f_2 = 3.01 \text{ GHz} \) for the X polarization in Fig. 4. The direction of the magnetic field is in the \(-45^\circ\) direction (45° direction) for the D polarization (X polarization). Thus, the resonance mode at \( f = f_1 \) (\( f = f_2 \)) is the symmetric (antisymmetric) mode, the current flow of which in the coupled split-ring resonator is symmetric (antisymmetric) with respect to the symmetry plane of the unit structure. For the vertical or horizontal polarization incidence, both of the two kinds of the resonance modes can be excited. Therefore, when \( f_2 - f_1 \) is much larger (smaller) than the resonance linewidth of the split-ring resonator, which is proportional to \( R \), the two kinds of the resonance modes are observed as distinct resonance lines (one resonance line) in the transmission and reflection spectra for the vertical or horizontal incidence. Next, the characteristics of the metamaterial at \( f = f_0 \) are described on the basis of the \( \pm 45^\circ \) polarizations. The absolute values of transmittances and reflectances for the \( \pm 45^\circ \) polarized waves are \( 1/\sqrt{2} \) at \( f = f_0 \). That is, the power transmittances and reflectances for \( \pm 45^\circ \) polarized waves are \( 1/2 \) at \( f = f_0 \). The difference between the arguments of the transmittances (reflectances) for the \( \pm 45^\circ \) polarizations are \( \pi/2 \) at \( f = f_0 \). These characteristics imply that the metamaterial behaves as a half mirror with quarter-wave retardation at \( f = f_0 \). Therefore, when the \( y \) or \( z \) polarized wave, of which the polarization axis is \( 45^\circ \) different from the fast and slow axes (D and X axes) of the quarter-wave plate, is incident on the metamaterial, circularly polarized waves with half the power of the incident wave are transmitted and reflected. If the coupling strength between the two split-ring resonators becomes stronger (weaker), the difference between \( f_1 \) and \( f_2 \) increases (decreases). In this case, the phase difference between the eigenpolarizations is varied from \( \pi/2 \), so that the transmitted and reflected waves become elliptically polarized waves. In addition, the amplitude of the transmitted wave becomes different from that of the reflected wave. This observation is consistent with the theory based on the electrical circuit model that describes the phenomenon on the basis of the 0° and 90° polarizations.

We have demonstrated that a linear-to-circular polarization converter with half transmission and half reflection can be realized using a single-layered metamaterial composed of coupled split-ring resonators. A theoretical analysis based on the electrical circuit model revealed that the linear-to-circular polarization converter is achieved for \( \omega = \omega_0 \) and \( |M| = R/\omega_0 \). An FDTD simulation demonstrated that \( |M| = R/\omega_0 \) can be realized for coupled split-ring resonators and that circularly polarized electromagnetic waves with the same power and helicity are transmitted and reflected for the vertical or horizontal polarization incidence. The eigenpolarizations in the metamaterial were found to be the \( \pm 45^\circ \) polarizations. The difference between the transmittance (reflectance) arguments for the eigenpolarizations are \( \pi/2 \), and the power transmittances and power reflectances for the eigenpolarizations are \( 1/2 \) at the resonance frequency of the split-ring resonators, which implies that the metamaterial has the combined characteristics of a half mirror and a quarter-wave plate. If meta-atoms with a low quality factor and sufficiently strong response are used as elements for the present metamaterial, then the frequency dependence of the metamaterial characteristics at around \( \omega = \omega_0 \) becomes small, and the linear-to-circular polarization converter with small deviations of the transmittance and reflectance from the ideal can be achieved for a relatively broad frequency range. The result of the present study suggests that compact circular polarization beam splitters and half beam splitters may be fabricated using metamaterials composed of coupled resonators.

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