UNSTABLE ROTATIONAL STATES
OF CLOSED STRING WITH MASSIVE POINTS

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Abstract

For the closed string carrying 2 or 3 point-like masses the stability problem for central and linear rotational states is considered. This problem is important for applications of these model to describing baryons, glueballs or other exotic hadrons. The linear rotational state correspond to an uniform rotation of the system with rectilinear string segments, connecting massive points. The state is named “central” one, if there is a massive point at the rotational center.

It is shown that the linear rotational states with 2 massive points are stable with respect to small disturbances. But the central rotational states with 3 masses are not stable, if the central mass it less than energy of the string with other massive points. This effect may change properties of excited hadron states, in particular, increase their width.

1. Introduction

Various string models of mesons, baryons and exotic hadrons include a system of massive points connected by strings [1] – [11]. In these models the massive points describe quarks or constituent gluons and the Nambu-Goto string simulates strong interaction between them and describes the QCD confinement mechanism.

The mentioned models includes the string with massive ends as the meson string model $q\bar{q}$ [1] or the quark-diquark baryon model $q\bar{q}q$ [5]; the linear string baryon model $q\bar{q}qq$ [3, 4]; the Y baryon model $qqqY$ [2, 6]; the “triangle” (or $\Delta$) baryon configuration $qqq\Delta_q$ [7], and (generalizing the last model) the closed string with $n$ massive ends [10, 11].

If we use a set of rotational states (planar uniform rotations of a system), all the listed string hadron models generate linear or quasilinear Regge trajectories $J \sim \alpha_0 + \alpha'E^2$, and may be applied for describing excited hadron states with high angular momenta $J$ are energies (masses) $E$ [4] – [11].

In these applications the problem of stability for rotational states with respect to small disturbances is very important for choosing the most adequate string model for baryons [4], for glueballs and other exotic hadrons [10] – [17]. Another reason for interest to this stability problem is the existence of quasirotational states [18, 19] in the linear vicinity of stable rotational states describing radial excitations or daughter Regge trajectories for hadrons.

The stability problem for rotational states is solved for the string with massive ends [18, 19], for the linear string baryon model $q\bar{q}qq$ [20]; and for the Y baryon configuration [21]. Analytical investigations and numerical simulations of small disturbances demonstrated that rotational
states of the string with massive ends are stable [18, 19], but they are unstable for string baryon models \(q-q-q\) and \(Y\) [20, 21]. For the last two models there are exponentially growing modes in spectra of small disturbances.

In this paper the stability problem is solved for the certain class of rotational states of the closed string carrying \(n\) point-like masses. We consider the case \(n = 3\) and the rotational states (named central states) with a massive point at the rotational center. In the particular case, if the central mass equals zero, this state is named linear rotational state with \(n = 2\).

The considered model describes baryons [7] or glueballs [10, 11].

Note that previously the stability problem for rotational states was solved for the closed string with \(n = 1\) massive point [22].

2. Dynamics and central rotational states

The classical dynamical equations for the closed string with tension \(\gamma\) carrying \(n\) point-like masses \(m_1, m_2, \ldots, m_n\) result from the action [7, 10]

\[
A = -\gamma \int \sqrt{(\dot{X}, X')^2 - \dot{X}^2 X'^2} \ d\tau d\sigma - \sum_{i=1}^{n} m_i \int \sqrt{\dot{x}^2_i(\tau)} \ d\tau,
\]

and have the form

\[
\frac{\partial^2 X^\mu}{\partial \tau^2} - \frac{\partial^2 X^\mu}{\partial \sigma^2} = 0
\]

and

\[
m_j \frac{d}{d\tau} \frac{\dot{x}_j^\mu(\tau)}{\sqrt{\dot{x}_j^2(\tau)}} + \gamma \left[ X^\mu + \dot{\sigma}_j(\tau) \dot{X}^\mu \right]_{\sigma=\sigma_j-0} - \gamma \left[ X^\mu + \dot{\sigma}_j(\tau) \dot{X}^\mu \right]_{\sigma=\sigma_j+0} = 0,
\]

if the orthonormality conditions on the world surface \(X^\mu(\tau, \sigma)\)

\[
(\partial_\tau X \pm \partial_\sigma X)^2 = 0,
\]

and the conditions

\[
\sigma_0(\tau) = 0, \quad \sigma_n(\tau) = 2\pi,
\]

are fulfilled [7, 11]. Here \(\dot{X}^\mu \equiv \partial_\tau X^\mu, X^\mu \equiv \partial_\sigma X^\mu\), Scalar product in Minkowski space \(R^{1,3}\) is \((a, b) = \eta_{\mu\nu} a^\mu b^\nu, \eta_{\mu\nu} = \text{diag}(1, -1, -1, -1)\); speed of light \(c = 1\), the parameter \(\sigma\) varies in the limits \(\sigma_0 \leq \sigma \leq \sigma_n\), that is \(\sigma \in [0, 2\pi]\), equations \(\sigma = \sigma_j(\tau)\) and

\[
x^\mu = x^\mu_j(\tau) \equiv X^\mu(\tau, \sigma_j(\tau)), \quad j = 0, 1, \ldots, n
\]

determine world lines of the massive points, for the case \(j = 0\) and \(j = n\) they describe the same trajectory of the \(n\)-th point, and their equality forms the closure condition

\[
X^\mu(\tau^*, 2\pi) = X^\mu(\tau, 0)
\]

on the tube-like world surface [7, 22]. These equations may contain two different parameters \(\tau\) and \(\tau^*\), connected via the relation \(\tau^* = \tau^*(\tau)\). This relation should be included in the closure condition [6] of the world surface.

Conditions (11, 15) always may be fixed without loss of generality, if we choose the relevant coordinates \(\tau, \sigma\) [7].
Eqs. (2), (3) are equations of motion for the massive points resulting from the action. They may be interpreted as boundary conditions for Eq. (1).

The system of equations (11)–(6) describe dynamics of the closed string carrying \( n \) point-like masses without loss of generality. One also should add that a tube-like world surface of the closed string is continuous one, but its derivatives may have discontinuities at world lines of the massive points (except for derivatives along these lines) \([7]\). These discontinuities are taken into account in Eqs. (2), (3).

Exact solutions of the system (11)–(6) describing rotational states of the closed string with masses we obtained and classified in Refs. \([10, 11]\). These states are divided into three classes: hypocycloidal, linear and central rotational states. In hypocycloidal states the rotating string is composed of segments of a hypocycloid. For linear states these segments are rectilinear and all masses \( m_j \) move at nonzero velocities \( v_j \). Central states are states with a massive point (or some of them) placed at the rotational center.

The string world surface for the central or linear states can be presented as \([10]\)

\[
X^\mu(\tau, \sigma) = x_0^\mu + e_0^\mu a_0 \tau + u(\sigma) \cdot e^\mu(\omega \tau). \tag{7}
\]

Here

\[
e^\mu(\omega \tau) = e_1^\mu \cos \omega \tau + e_2^\mu \sin \omega \tau, \tag{8}
\]

the unit vectors

\[e_0^\mu, e_1^\mu, e_2^\mu, e_3^\mu,\]

associated with coordinates \( x^\mu \), form the orthonormal basis in \( R^{1,3} \). In the closure condition \( (6) \ \tau^* = \tau \). For the solution (7) of Eq. (11) the following values are the constants:

\[
\sigma_j(\tau) = \sigma_j = \text{const}, \quad j = 1, 2, \tag{9}
\]

\[
Q_j = \frac{\gamma}{m_j} \sqrt{\dot{x}_j^2(\tau)}, \quad j = 1, 2, 3. \tag{10}
\]

The function

\[
u(\sigma) = \frac{a_0}{\omega} \cdot \begin{cases} \sin \omega \sigma, & \sigma \in [0, \sigma_1], \\ 2\dot{s}_1 \dot{c}_1 \cos \omega \sigma + (\dot{s}_1^2 - \dot{c}_1^2) \sin \omega \sigma, & \sigma \in [\sigma_1, \sigma_2], \\ -\dot{S} \cos \omega \sigma + \dot{C} \sin \omega \sigma, & \sigma \in [\sigma_2, 2\pi] \end{cases} \tag{11}
\]

is determined from Eq. (11), continuity of the function \( X^\mu(\tau, \sigma) \) on the lines \( \sigma = \sigma_j \) and conditions \( (2)–(6) \ [10] \). Here and below we use the following notations for constants:

\[
c_1 = \cos \omega \sigma_1, \quad \dot{s}_1 = \sin \omega \sigma_1, \quad \dot{c}_1 = \cos 2\pi \omega, \quad \dot{S} = \sin 2\pi \omega, \\
c_3 = \cos \omega (2\pi - \sigma_2), \quad \dot{s}_3 = \sin \omega (2\pi - \sigma_2), \quad \dot{c}_2 = \cos \omega \sigma_2, \quad \dot{S}_2 = \sin \omega \sigma_2.
\]

Expression (7) describes a rotation of two coinciding string segments, connecting two points with masses \( m_1, m_2 \). These points move along circles at velocities \( v_1 \) and \( v_2 \), the system rotates in the plane \( e_1, e_2 \) at angular velocity \( \Omega = \omega/a_0 \). The point with mass \( m_3 \) is at rest at the rotational center \( (v_3 = 0) \). Values of parameters \( \sigma_j, Q_j, m_j, v_j, a_0 \) are related by the following equations, resulting from Eqs. (2)–(6) \([10] \):

\[
\sigma_2 - \sigma_1 = \pi, \tag{12}
\]

\[
v_1 = \dot{s}_1 = \sin \omega \sigma_1, \quad v_2 = \dot{s}_3 = \sin \omega (\pi - \sigma_1), \tag{13}
\]

3
\[ 2 \frac{Q_1}{\omega} = \frac{s_1}{c_1} = \frac{v_1}{\sqrt{1 - v_1^2}}, \quad 2 \frac{Q_2}{\omega} = \frac{s_3}{c_3} = \frac{v_2}{\sqrt{1 - v_2^2}}, \quad (14) \]

\[ a_0 = \frac{m_1 Q_1}{\gamma \sqrt{1 - v_1^2}} = \frac{m_2 Q_2}{\gamma \sqrt{1 - v_2^2}} = \frac{m_3 Q_3}{\gamma}, \quad (15) \]

\[ \frac{m_1 v_1}{1 - v_1^2} = \frac{m_2 v_2}{1 - v_2^2}. \quad (16) \]

If the values \( v_1 \) and \( v_2 \) are given, one can calculate from Eqs. (13) the following values:

\[ \omega = \frac{(1)k_1 \arcsin v_1 + (1)k_2 \arcsin v_2}{\pi} + 2(k_1 + k_2), \quad \sigma_1 = \frac{(1)k_1 \arcsin v_1 + 2\pi k_1}{\omega}. \quad (17) \]

Here \( k_1 \) and \( k_2 \) are arbitrary integers resulting in the inequalities \( 0 < \sigma_1 < \pi, \ s_1 > 0, \ s_2 > 0 \). The simplest case \( k_1 = k_2 = 0 \) corresponds to 2 coinciding rectilinear string segments, connecting the points \( m_1, m_2 \). For other permissible values \( k_1 \) and \( k_2 \) these segments are kinked curves, folded on one (rotating) straight line. The fold points move along circles at the speed of light.

3. Stability problem for central rotational states

Possible applications of solutions (7) in hadron spectroscopy essentially depend on stability or instability of these states with respect to small disturbances. In this section we study spectrum of these disturbances for the central rotational states.

This problem has been recently solved for the closed string with \( n = 1 \) massive point in Ref. [7]. In Ref. [10] central rotational states with \( n = 3 \) are considered, but some simplifying assumptions in this paper appeared to be incorrect, and the results need refinement.

To solve the stability problem for the central rotational states (7) we consider the general solution of Eq. (1) for the string with 3 masses

\[ X^\mu(\tau, \sigma) = \frac{1}{2}[\Psi^\mu_+(\tau + \sigma) + \Psi^\mu_-(\tau - \sigma)], \quad \sigma \in [\sigma_{j-1}, \sigma_j], \quad j = 1, 2, 3. \quad (18) \]

Here the functions \( \Psi^\mu_{j\pm}(\tau \pm \sigma) \) are smooth, the world surface (18) is smooth between world lines of massive points.

We denote \( \Psi^{(r)\mu}_{j\pm} \) the functions in the expression (18) for the rotational states (7). Their derivatives \( \dot{\Psi}^{(r)\mu}_{j\pm} \equiv \frac{d}{d\tau} \Psi^{(r)\mu}_{j\pm} \) in accordance with Eq. (11) are

\[ \dot{\Psi}^{(r)\mu}_{1\pm}(\tau) = a_0 [e_0^\mu \pm e_\mu(\omega \tau)], \]
\[ \dot{\Psi}^{(r)\mu}_{2\pm}(\tau) = a_0 [e_0^\mu + 2v_1 c_1 e_\mu(\omega \tau) \pm (2v_1^2 - 1) e_\mu(\omega \tau)], \quad (19) \]
\[ \dot{\Psi}^{(r)\mu}_{3\pm}(\tau) = a_0 [e_0^\mu - \dot{S}e_\mu(\omega \tau) \pm \dot{C} e_\mu(\omega \tau)], \]

Here the rotating vector

\[ \dot{e}_\mu(\omega \tau) = -e_1^\mu \sin \omega \tau + e_2^\mu \cos \omega \tau, \]

is orthogonal to the vector \( e_\mu(\omega \tau) \) (8).

To describe any small disturbances of the rotational motion, that is motions close to states (7) we consider vector functions \( \Psi^{\mu}_{j\pm} \) close to \( \dot{\Psi}^{(r)\mu}_{j\pm} \) in the form

\[ \Psi^\mu_{j\pm}(\tau) = \dot{\Psi}^{(r)\mu}_{j\pm}(\tau) + \varphi^\mu_{j\pm}(\tau). \quad (20) \]
The disturbance \( \varphi_{j\pm}^{\mu}(\tau) \) is supposed to be small, so we omit squares of \( \varphi_{j\pm} \) when we substitute the expression (20) into dynamical equations (2), (3) and (6). In other words, we work in the first linear vicinity of the states (7). Both functions \( \Psi_{j\pm}^{\mu} \) and \( \Psi_{j\pm}^{(r)\mu} \) in expression (20) must satisfy the condition
\[
\dot{\Psi}_{j+}^2 = \dot{\Psi}_{j-}^2 = 0,
\]
resulting from Eq. (4), hence in the first order approximation in \( \varphi_{j\pm} \) the following scalar product equals zero:
\[
(\dot{\Psi}_{j+}^{(r)}, \varphi_{j\pm}) = 0.
\]

For the disturbed motions the equalities \( \tau^* = \tau \) and \( \sigma_j(\tau) = \sigma_j = \text{const} \) [3], generally speaking, is not carried out and should be replaced with the equalities
\[
\sigma_1(\tau) = \sigma_1 + \delta_1(\tau), \quad \sigma_2(\tau) = \sigma_2 + \delta_2(\tau), \quad \tau^* = \tau + \delta(\tau), \tag{22}
\]
where \( \delta_j(\tau) \) and \( \delta(\tau) \) are small disturbances.

Expression (20) together with Eq. (13) is the solution of the string motion equation (1). Therefore we can obtain equations of evolution for small disturbances \( \varphi_{j\pm}^{\mu}(\tau) \), substituting expressions (20) and (22) with Eq. (19) into other equations of motion (2), (3), the closure condition (6) and the continuity condition
\[
X^\mu(\tau, \sigma_j(\tau) - 0) = X^\mu(\tau, \sigma_j(\tau) + 0), \quad i = 1, 2 \tag{23}
\]
in linear approximation. We are to take into account nonlinear factors \( \left\{ \left[ \frac{d}{d\tau} X(\tau, \sigma_j(\tau)) \right]^2 \right\}^{-1/2} \) and contributions from the disturbed arguments \( \tau^* \) and \( \sigma_j(\tau) \) (22), for example:
\[
\dot{\Psi}_{3\pm}^{(r)\mu}(\tau^* \pm 2\pi) \simeq \dot{\Psi}_{3\pm}^{(r)\mu}(\tau \pm 2\pi) + \delta(\tau) \ddot{\Psi}_{3\pm}^{(r)\mu}(\tau \pm 2\pi).
\]

This substitution for the central rotational state (7) with \( n = 3 \) and vector-functions \( \dot{\Psi}_{j\pm}^{(r)\mu} \) (19) after simplifying results in the following system of 6 vector equations in linear (with respect to \( \varphi_{j\pm}^{\mu}, \delta_j \) and \( \delta \)) approximation:
\[
\begin{align*}
\varphi_{1+}^{\mu}(+1) + \varphi_{1-}^{\mu}(-1) &- \varphi_{2+}^{\mu}(+1) - \varphi_{2-}^{\mu}(-1) + 4\dot{c}_1 a_0[\dot{e}^\mu(\omega\tau) \dot{\varphi}_{11}(\tau) + \omega \ddot{e}^\mu(\omega\tau) \delta_j] = 0, \\
\varphi_{2+}^{\mu}(+2) + \varphi_{2-}^{\mu}(-2) &- \varphi_{3+}^{\mu}(+2) - \varphi_{3-}^{\mu}(-2) - 4\dot{c}_3 a_0[\dot{e}^\mu(\omega\tau) \dot{\varphi}_{12}(\tau) + \omega \ddot{e}^\mu(\omega\tau) \delta_j] = 0,
\end{align*}
\]
\[
\begin{align*}
\varphi_{3+}^{\mu}(+) + \varphi_{3-}^{\mu}(-) - \varphi_{4+}^{\mu}(+) - \varphi_{4-}^{\mu}(+) &+ 2a_0 \ddot{e}^\mu \delta(\tau) = 0,
\end{align*}
\]
\[
\begin{align*}
\frac{d}{d\tau} \left\{ \varphi_{1+}^{\mu}(+1) + \varphi_{1-}^{\mu}(-1) + 2\dot{c}_1 a_0(\dot{e}^\mu \dot{\varphi}_{11} + \omega \ddot{e}^\mu \delta_j) + G_1(e_0^\mu + v_1 \dot{e}^\mu) \right\} + \\
+ Q_1 \left[ \varphi_{1+}^{\mu}(+1) - \varphi_{1-}^{\mu}(-1) - \varphi_{2+}^{\mu}(+1) + \varphi_{2-}^{\mu}(-1) \right] = 0, \tag{24}
\end{align*}
\]
\[
\begin{align*}
\frac{d}{d\tau} \left\{ \varphi_{2+}^{\mu}(+2) + \varphi_{2-}^{\mu}(-2) - 2\dot{c}_3 a_0(\dot{e}^\mu \dot{\varphi}_{12} + \omega \ddot{e}^\mu \delta_j) + G_2(e_0^\mu - v_2 \dot{e}^\mu) \right\} + \\
+ Q_2 \left[ \varphi_{2+}^{\mu}(+2) - \varphi_{2-}^{\mu}(-2) - \varphi_{3+}^{\mu}(+2) + \varphi_{3-}^{\mu}(-2) \right] = 0.
\end{align*}
\]
\[
\begin{align*}
\frac{d}{d\tau} \left\{ \varphi_{1+}^{\mu} + \varphi_{1-}^{\mu} + (\varphi_{1+} - \varphi_{1-}) e_0^\mu \right\} + Q_3 \left[ \varphi_{3+}^{\mu}(+) - \varphi_{3-}^{\mu}(-) - \varphi_{4+}^{\mu} + \varphi_{4-}^{\mu} + 2\omega a_0 \dot{e}^\mu \delta \right] = 0.
\end{align*}
\]

In the last three equations arguments \( \tau \) for \( \varphi_{1\pm}^{\mu}, \delta_j \) and \( \omega \tau \) for \( e^\mu, \dot{e}^\mu \) are omitted; we use the following notations for arguments
\[
(\pm_1) \equiv (\tau \pm \sigma_1), \quad (\pm_2) \equiv (\tau \pm \sigma_2), \quad (\pm) \equiv (\tau \pm 2\pi),
\]
for the scalar products
\[\varphi_{j+}^0 \equiv (\epsilon_0, \varphi_{j+}), \quad \varphi_{j-}^3 \equiv (\epsilon_3, \varphi_{j+}), \quad \varphi_{j+} \equiv (e, \varphi_{j+}), \quad \varphi_{j-} \equiv (\dot{e}, \varphi_{j+}) \] (25)

and
\[G_1 = \varphi_1(+) - \varphi_1(-) - v_1 \dot{e}_1^1 \left[ \dot{\varphi}_1(+) + \dot{\varphi}_1(-) - 2\omega_0 \delta_1 \right],\]
\[G_2 = \ddot{e}_1^1 \left\{ \ddot{C}_2[\varphi_2(-) - \varphi_2(+)] + \ddot{S}_2[\varphi_2(+) + \varphi_2(-)] + 2\omega v_2 a_0 \delta_2 \right\}.
\]

The first two equations (24) results from Eqs. (23), the third — from Eq. (6), other ones are consequence of Eqs. (2) and (3). Equations (24) are simplified with using Eqs. (12) – (17), (19) and equalities (21), resulting in the following relations for projections (25) of disturbances:
\[
\varphi_{1+}^0(\tau) = \mp \varphi_{1+}(\tau), \quad \varphi_{2+}^0 = \mp (2v_1^2 - 1) \varphi_{2+} - 2v_1 \ddot{e}_1 \dot{\varphi}_{2+}, \quad \varphi_{3-}^0 = \ddot{S}_3 \varphi_{3+} + \ddot{C}_3 \varphi_{3+}. \] (26)

The linearized system of equations (24), (26) describes evolution of small disturbances of the considered central rotational state (7), (19).

Note that scalar products of Eqs. (24) onto the vector \( e_3 \) (orthogonal to the rotational plane \( e_1, e_2 \)) form the closed subsystem from 6 equations with respect to 6 functions (25) \( \varphi_{j\pm}^3 \):
\[
\varphi_{j+}^3(+) + \varphi_{j-}^3(-) = \varphi_{j+}^3(+) + \varphi_{j-}^3(-),
\]
\[
\varphi_{j+}^3(+) + \varphi_{j-}^3(-) = \varphi_{j+}^3(\tau) + \varphi_{j-}^3(\tau),
\]
\[
\varphi_{j+}^3(+) + \varphi_{j-}^3(-) + Q_2 \left[ \varphi_{j+}^3(+) - \varphi_{j-}^3(-) - \varphi_{j+}^3(+) + \varphi_{j-}^3(-) \right] = 0,
\]
\[
\varphi_{j+}^3(\tau) + \varphi_{j-}^3(\tau) + Q_3 \left[ \varphi_{j+}^3(\tau) - \varphi_{j-}^3(-) - \varphi_{j+}^3(\tau) + \varphi_{j-}^3(\tau) \right] = 0. \] (27)

Here \( j = 1, 2, j^* \equiv j + 1 \). This system is homogeneous system with deviating arguments.

We search solutions of this system in the form of harmonics
\[\varphi_{j\pm}^3 = B_{j\pm}^3 \exp(-i\xi \tau).\] (28)

This substitution results in the linear homogeneous system of 6 algebraic equations with respect to 6 amplitudes \( B_{j\pm}^3 \). The system has nontrivial solutions if and only if its determinant
\[
\begin{vmatrix}
E_{1+} & E_{1-} & -E_{1+} & -E_{1-} & 0 & 0 \\
0 & 0 & E_{2+} & E_{2-} & -E_{2+} & -E_{2-} \\
-1 & -1 & 0 & 0 & E_{3+} & E_{3-} \\
(i\xi - Q_1) E_{1+} & (i\xi + Q_1) E_{1-} & Q_1 E_{1+} & -Q_1 E_{1-} & 0 & 0 \\
0 & 0 & (i\xi - Q_2) E_{2+} & (i\xi + Q_2) E_{2-} & Q_2 E_{2+} & -Q_2 E_{2-} \\
-i\xi - Q_3 & -i\xi + Q_3 & 0 & 0 & Q_3 E_{3+} & -Q_3 E_{3-} \\
\end{vmatrix} = 0
\]
equals zero. Here \( E_{j\pm} = \exp(\mp i\xi \sigma_j) \). This equation is reduced to the form
\[2(\cos 2\pi \xi - 1) - \xi (Q_1^{-1} + Q_2^{-1} + Q_3^{-1}) \sin 2\pi \xi + \]
\[+ \xi^2 \left( \frac{\sin^2 \pi \xi}{Q_1 Q_2} + \frac{\tilde{s}_3 \sin \sigma_2 \xi}{Q_2 Q_3} + \frac{\tilde{s}_{23} \sin \sigma_1 \xi}{Q_1 Q_3} \right) - \frac{\xi^3 \tilde{s}_3 \sin \sigma_1 \xi \cdot \sin \pi \xi}{Q_1 Q_2 Q_3} = 0, \] (29)

where \( \tilde{s}_3 = \sin(\pi - \sigma_1) \xi, \tilde{s}_{23} = \sin(2\pi - \sigma_1) \xi \).

This equation describes the spectrum of frequencies \( \xi \) for transversal (with respect to the \( e_1, e_2 \) plane) small fluctuations of the string for the considered rotational state. If equation
Figure 1: Zero level lines for real part (thick) and imaginary part (thin) of Eq. (29); a) \( Q_1 = Q_2 = Q_3 = 1 \), b) \( Q_1 = 1, Q_2 = 0.4, Q_3 = 0.1 \)

(29) has a complex root \( \xi = \xi_1 + i\xi_2 \) with positive imaginary part \( \xi_2 \), the amplitude of the correspondent disturbance will grow exponentially:

\[
\varphi_k^3 = B_k^3 \exp(-i\xi_1 \tau) \cdot \exp(\xi_2 \tau). 
\]

In this case the considered rotational state is unstable one [18, 21].

Analysis of the real and imaginary parts of Eq. (29) is presented in Fig. 1, where the thick and thin lines are zero level lines correspondingly for real and imaginary part of the function \( f(\xi) = f(\xi_1 + i\xi_2) \) in Eq. (29) for given values \( Q_j \). Roots of this equation are shown as cross points of a thick line with a thin line. If the values (10) \( Q_j \) are given, one can determine values \( \omega, \sigma_1, v_j \) from Eqs. (12) – (17). For example, \( \omega \) is one of the roots of the equation

\[
\tan \pi \omega = \frac{2\omega(Q_1 + Q_3)}{\omega^2 - 4Q_1Q_2}, \tag{30}
\]

resulting from Eqs. (14). For the state in Fig. 1a the root of this equation \( \omega \approx 0.766898 \) is chosen. It corresponds to the mass relation \( m_1 : m_2 : m_3 = 1 : 1 : 2.79 \); for the state in Fig. 1b \( \omega \approx 2.331, m_1 : m_2 : m_3 = 1 : 3.12 : 13.18 \). The frequency \( \omega \) for the state in Fig. 1a is the minimal positive root of Eq. (30), corresponding to \( k_1 = k_2 = 0 \) in Eqs. (17) (the simplest rectilinear string segments, connecting the massive points). For the state in Fig. 1b the values in Eqs. (17) \( k_1 = 0, k_2 = 1 \), and the string segments are kinked lines.

The analysis of Eq. (29) for various values \( Q_j \) and \( \omega \) demonstrates that all its roots are real numbers, therefore amplitudes of such fluctuations do not grow with growth of time \( t \).

4. Disturbances in the rotational plane

One can not solve the stability problem for the central rotational states [7] only on the base of the above studied behavior of disturbances \( \varphi_j^{\pm} \) (orthogonal to the rotational plane). We are to consider small disturbances concerning to the \( e_1, e_2 \) plane. Projections (scalar products) of equations (24) onto 3 vectors \( e_0, e(\tau), \dot{e}(\tau) \) with using relations

\[
e^{\mu}(\tau) = \tilde{c}_1 e^{\mu}(\pm_1) \mp \tilde{s}_1 \hat{e}^{\mu}(\pm_1) = \tilde{C}_2 e^{\mu}(\pm_2) \mp \tilde{S}_2 \hat{e}^{\mu}(\pm_2) = \tilde{C} e^{\mu}(\pm) \mp \tilde{S} \hat{e}^{\mu}(\pm)
\]

and their analogs for \( \hat{e}^{\mu} \) (for multiplying by vector-functions \( \varphi_j^{\mu} \) with different arguments) result in the system of 18 equations. Three of them are linear combinations of the other ones. The rest equations form the system of 15 differential equations with deviating arguments with
various values of thick and thin lines for two types of rotational states: for $\omega_1$, $\omega_2$ with respect to 15 unknown functions of $\tau$. Here we present 8 equations from this system, resulting from the 1-st, 3-rd and 6-th equations (21):

\[
\begin{align*}
\varphi_{1+}(+1) - \varphi_{1-}(-1) + (\tilde{c}_{2} - \tilde{s}_{2})[\varphi_{2+}(+1) - \varphi_{2-}(-1)] - 2\delta_{1}[\varphi_{2+}(+1) + \varphi_{2-}(-1)] &= 0, \\
\tilde{c}_{1}[\varphi_{1+}(+1) + \varphi_{1-}(-1) - \varphi_{2+}(+1) - \varphi_{2-}(-1)] - 4a_{0}\delta_{1}(\tau) + \tilde{s}_{1}[\varphi_{1+}(-1) - \varphi_{1+}(+1) + \varphi_{2+}(+1) - \varphi_{2-}(-1)] &= 0, \\
\end{align*}
\]

In the last equations the arguments ($\tau$) for $\varphi_{1+}$, $\varphi_{1-}$, $\delta$, and ($\pm$) for $\varphi_{3+}$, $\varphi_{3-}$ are omitted.

When we search solutions of this system in the form of harmonics (28)

\[
\varphi_{j\pm} = B_{j\pm}e^{-i\xi_{j}}, \quad \dot{\varphi}_{j\pm} = \dot{B}_{j\pm}e^{-i\xi_{j}}, \quad 2a_{0}\delta_{j} = \Delta_{j}e^{-i\xi_{j}}, \quad 2a_{0}\delta = \Delta e^{-i\xi_{j}},
\]

we obtain the homogeneous system of 15 algebraic equations with respect to 15 amplitudes $B_{j\pm}$, $\dot{B}_{j\pm}$, $\Delta_{1}$, $\Delta_{2}$, $\Delta$. The mentioned above condition of existence of nontrivial solutions for this system is vanishing the corresponding determinant.

This cumbersome determinant was calculated at the first stage in the particular symmetric case of equal masses

\[
m_{1} = m_{2}, \quad \sigma_{1} = \frac{\pi}{2}.
\]

with using symbolic calculations in the package MATLAB and numerical verification.

In the case (32), resulting in equalities $\tilde{s}_{1} = \tilde{s}_{3}$, $\tilde{c}_{1} = \tilde{c}_{3}$, $v_{1} = v_{2}$, $Q_{1} = Q_{2}$, the obtained equation for small disturbances is equivalent to the set of equations (is factorized):

\[
\left(\xi \tan \frac{\pi \xi}{2} - \omega \tan \frac{\pi \omega}{2}\right)\left(\tilde{c}_{1}^{2} - 2\tilde{s}_{1}\tilde{c}_{1}\omega\xi \cot \frac{\pi \xi}{2} - Z\omega^{2}\right) = 0,
\]

\[
\tilde{c}_{1}(\xi^{2} - \omega^{2})[\tilde{s}_{1}^{3}\xi^{3} - 3\tilde{c}_{1}\tilde{s}_{1}\omega^{2}\xi^{2} - \tilde{s}_{1}(1 + 3\tilde{s}_{1}^{2})\tilde{c}_{1}\omega^{2}\xi^{2} + \tilde{s}_{1}Z\omega^{2}] = (\tilde{c}_{1}^{2}\xi^{2} + 2\tilde{s}_{1}\tilde{c}_{1}\omega\xi - \tilde{c}_{1}Z\omega^{2}) \times \\
\times 4Q_{3}\left[\tilde{c}_{1}\xi^{3} \cos \frac{3\pi \xi}{2} + 2(\tilde{c}_{1}\tilde{s}_{1}\omega + \tilde{s}_{1}\tilde{c}_{1}Q_{3})\xi^{2} + (\tilde{c}_{1}\tilde{c}_{1}\omega + 2\tilde{s}_{1}\tilde{s}_{1}Q_{3})\omega \xi + \tilde{s}_{1}\tilde{s}_{1}\omega^{3}\right].
\]

Here

\[
Z_{1} = 1 + \tilde{s}_{1}^{2}, \quad \tilde{c} = \cos \pi \xi, \quad \tilde{s} = \sin \pi \xi, \quad \tilde{c}_{1} = \cos \sigma_{1} \xi, \quad \tilde{s}_{1} = \sin \sigma_{1} \xi.
\]

Analysis of equation (33) (decomposing into two factors) for complex $\xi = \xi_{1} + i\xi_{2}$ shows, that for all values $\omega$ and $Q_{1} = \frac{1}{2}\omega^{2}\tilde{s}_{1}/\tilde{c}_{1}$ all roots of this equation are real numbers and form a countable set. Their behavior is similar to that for roots of Eq. (20).

But roots of equation (34) have another properties. These roots are shown in Fig. 2 as cross points of thick and thin lines for two types of rotational states: for $Q_{1} = Q_{2} = 1/4$, $\omega = 1/2$, $v_{1} = v_{2} \approx 0.707$ on the left and $Q_{1} = Q_{2} = 1$, $\omega \approx 0.766898$, $v_{j} \approx 0.934$ on the right for various values $Q_{3}$ and corresponding $m_{3} = \gamma a_{0}/Q_{3}$ (15).
Figure 2: Zero level lines for real part (thick) and imaginary part (thin) of Eq. (34) for specified values $Q_j$ and $m_3$.

Fig. 2 demonstrates, that Eq. (34) has complex roots $\xi = \xi_1 + i\xi_2$ with positive imaginary parts $\xi_2 > 0$, if the value $Q_3$ (and the corresponding mass $m_3$) is not too small or too large. These roots generate exponentially growing modes of disturbances: $|\varphi| \sim \exp(\xi_2 \tau)$.

The same picture takes place for other values $\omega$. So we may conclude, that the central rotational states (7) of the type (32) are unstable with respect to small disturbances, if the central mass is restricted by

$$0 < m_3 < m_{3\text{cr}}. \quad (35)$$

The critical value $m_{3\text{cr}}$ (and the corresponding value $Q_{3\text{cr}}$) is determined from the condition of vanishing all complex roots of Eq. (34) (with $\text{Im} \xi > 0$) for $m_3 > m_{3\text{cr}}$. Thus, we obtain the threshold effect in stability properties: if $m_3 \geq m_{3\text{cr}}$, all roots of Eq. (34) are real ones and the state is stable. But in the case $m_3 < m_{3\text{cr}}$ the state is unstable.

For any values $Q_1 = Q_2$ and any (arbitrarily small) value $m_3$ from the interval (35) there are exists the pure imaginary root $\xi^* = i\xi_2^*$ ($\xi_2^* > 0$). It tends to 0 at $m_3 \to 0$. For the states with $\omega \lesssim 0.5$, $Q_1 \leq 0.25$ equation (34) has no other complex roots except $\xi^* = i\xi_2^*$. If mass $m_3$ increases, the value $\xi_2^*$ (increment of disturbances’ growth) increases too, reaching the maximal value for masses $m_3$, close to $m_1$ in order of magnitude. If $m_3$ increases further ($Q_3$ diminishes) the increment $\xi_2^*$ decreases and vanishes at the critical value $m_{3\text{cr}} \equiv m_{3\text{cr}}^* \equiv \gamma a_0/Q_{3\text{cr}}^*$, in the (35). For the case $Q_1 = Q_2 = 1/4$ the value $m_{3\text{cr}}^* \simeq 5.05 m_1$.

The critical value $Q_{3\text{cr}}^*$, corresponding to vanishing the root $\xi^*$, may be calculated, if we
substitute $\xi = i\xi_2$ into Eq. (34) and analyze its behavior at $\xi_2 \to 0$:

$$\hat{s}_1 Z_1 \omega^5 \xi_2 - 2Q_3 Z_1 \omega^4 \hat{c}_1 (1 + \cosh \pi \xi_2) + \hat{s}_1 \omega \sinh \pi \xi_2 + \phi(\xi_2) = 0, \quad \phi(\xi_2) = \mathcal{O}(\xi_2^3).$$

The function $\phi(\xi_2) = \phi_3 \xi_2^3 + \phi_5 \xi_2^5 + \ldots$ is positive for $\xi_2 > 0$ (contains only positive summands), so the root $\xi_2 = \xi_2^*$ of this equation exists only under the condition $2Q_3(2\hat{c}_1 + \pi \hat{s}_1 \omega) > \hat{s}_1 \omega$. From this condition and equalities (14) and (32) one can find the critical value $Q_{3cr} = Q_{3c}$ for the root $i\xi_2^*$:

$$Q_{3cr} = \frac{1}{2\pi + 4\hat{c}_1(\omega \hat{s}_1)^{-1}} = \frac{1}{2\pi + 2Q_1^{-1}}. \quad (36)$$

In the limit $m_1 \to 0$, $(Q_1 \to \infty)$ expression (36) takes the form, coinciding with the critical value in the condition of instability $Q > Q_{cr} = (2\pi)^{-1}$ [?] for the central rotational states of the closed string with $n = 1$ massive point. The equation of the small disturbances spectrum for this string [?]

$$\xi^3 + 2Q(2\xi^2 + \omega^2) \tan \pi \xi + 4Q^2 \xi \tan^2 \pi \xi - \omega^2 \xi = 0$$

results from Eq. (34) ($Q \equiv Q_3$) in the mentioned limit $m_j \to 0$, $j = 1, 2$.

For values $\omega > 1/2$ the structure of complex roots of Eq. (34) is more complicated. Because of the symmetry we shall count only roots in the quadrant $\xi_1 \geq 0$, $\xi_2 > 0$. For $Q_1 > 1/4$ in certain interval of values $m_3$ the second complex root of Eq. (34) exists (denoted below by $\xi^c$); for larger values $Q_1$ corresponding to $v_1 \to 1$ the third and other roots appear. In particular, in Fig. 2 for $Q_1 = 1$ the root $\xi^c$ appears, if $Q_3 < 0.667$; for $Q_3 = 1$ the complex roots are: $\xi^* \simeq 0.262i$; $\xi^c \simeq 3.64 + 0.137i$; the third root $\xi^0 \simeq 5.49 + 0.017i$ (it exists in the narrow interval $0.86 < Q_3 < 1.65$).

For different values $Q_1$ the critical value $Q_{3cr}$ is determined by vanishing the root $\xi^c$ or $\xi^0$. For the states in Fig. 2 for $Q_1 = 1/4$ the first variant takes place and $Q_{3cr} = Q_{3cr}^c \simeq 0.07$; for $Q_1 = 1$ we see the second variant: $Q_{3cr} = Q_{3cr}^c \simeq 0.05$.

In the case $m_3 = 0$, corresponding to $Q_3 \to \infty$, there is no massive point at the center, and we have the linear rotational state with $n = 2$. In this case equation (34) takes the form

$$\left(\xi + \omega \tan \frac{\pi \omega}{2} \tan \frac{\pi \xi}{2}\right) \xi \sin \pi \xi = 0. \quad (37)$$

For all values $\omega$ it has only real roots. So the linear rotational state with $n = 2$ of the type (32) are stable.

Stability also takes place for the case $Q_3 = 0$ ($m_3 \to \infty$).

Generalization of the above analysis for the case of arbitrary masses, in particular, $m_1 \neq m_2$ results in rather complicated equation of the small disturbances spectrum. In the particular case $Q_3 = 0$ ($m_3 \to \infty$) this equation have the form

$$\left\{\hat{s}_1 \hat{c}_3 \hat{s}_3^3 \hat{c}_3^3 \xi^6 - 3\hat{s}_1 (\hat{c}_3 \hat{s}_3 + \hat{c}_3 \hat{s}_3) \hat{c}_3 \hat{s}_3^3 \hat{c}_3^2 \omega \xi^5 + \hat{s}_1 [\hat{c}_3 \hat{s}_3 (4\hat{c}_1 \hat{s}_3 + 9\hat{s}_1 \hat{c}_3) - 2\hat{s}_3 \hat{c}_3 Z_3] \hat{c}_1 \hat{c}_3 \omega^2 \xi^4 + + \hat{c} (3\hat{c}_3 \hat{s}_3 + 4\hat{c}_1 \hat{s}_3 \hat{c}_3) Z_3 \hat{s}_1 + 2\hat{s}_1 \hat{c}_3 (4\hat{c}_1 \hat{c}_3 \hat{s}_3^2 \hat{c}_3 - \hat{s}_3 Z_3) \hat{c}_3 \hat{c}_1 \omega^2 \xi^3 - - \hat{c} (3\hat{c}_3 \hat{s}_3 + 4\hat{c}_1 \hat{s}_3 \hat{c}_3) Z_3 \hat{s}_1 + 2\hat{s}_1 \hat{c}_3 (4\hat{c}_1 \hat{c}_3 \hat{s}_3^2 \hat{c}_3 - \hat{s}_3 Z_3) \hat{c}_3 \hat{c}_1 \omega^2 \xi^2 + + \hat{c} (2\hat{c}_1 \hat{c}_3 \hat{s}_3^2 - \hat{s}_3 Z_3) Z_3 \hat{c}_1 \hat{s}_3 - \hat{s}_3 \hat{c}_3 \hat{s}_3 Z_3 (2\hat{c}_1 \hat{s}_3 + \hat{c}_3 Z_3) \hat{c}_1 \omega^2 \xi + + (1 \leftrightarrow 3) \right\} \xi (\xi^2 - \omega^2) = 0. \quad (38)$$
Here $Z_3 = 1 + s_3^2$, $\tilde{c}_3 = \cos(\pi - \sigma_1) \xi$, $\tilde{s}_3 = \sin(\pi - \sigma_1) \xi$, the symbol $(1 \leftrightarrow 3)$ means repeating the same terms with transposed indices "1" and "3".

Equation (38) has only real roots, so the rotational states (7) with the infinitely heavy mass $m_3 \to \infty$ at the center are stable.

Linear rotational states with $n = 2$ ($m_3 = 0$) are also stable for any values $\omega, m_1, m_2$.

But for intermediate values $m_3$ from the interval (35) the spectrum of small disturbances contains complex frequencies $\xi = \xi_1 + i\xi_2$. Their behavior is presented in Fig. 3.

Figure 3: Frequencies of small disturbances for the state with $Q_1 = 1, Q_2 = \frac{1}{4} (m_2 \simeq 10.5 m_1)$ and specified $Q_3, m_3$

For this state $\omega \simeq 0.62025$. Behavior of complex roots is similar to the case (32): there is the range (35) $0 < m_3 < m_{3cr}$ of existence of complex roots, hence the rotational state is unstable in this range.

There are three types of complex roots in Fig. 3 (similarly to Fig. 2): the imaginary root $\xi^*$, the complex roots $\xi^c$ and $\xi^d$. If we suppose $m_1 = 1$ (it is equivalent to $\gamma a_0 \simeq 3.376$), that the range of existence $\xi^*$ is $0 < m_3 < 38.1$, for $\xi^c$ it is $0.42 < m_3 < 30.7$, and for $\xi^d$ this range is $1.64 < m_3 < m_{3cr} \simeq 67.5$. For other parameters of the state the critical mass $m_{3cr}$ is determined from vanishing the root $\xi^*$ or other roots.

Generalization of the expression (36) for the critical value $Q_{3cr}^*$, determined from vanishing the root $\xi^*$ takes the form

$$ (Q_{3cr}^*)^{-1} = 2\pi + Q_1^{-1} + Q_2^{-1}. $$

Taking into account Eq. (15) $m_3 = \gamma a_0/Q_3$ we obtain the critical value of the central mass

$$ m_{3cr}^* = 2\pi \gamma a_0 + \frac{m_1}{\sqrt{1 - v_1^2}} + \frac{m_2}{\sqrt{1 - v_2^2}} \equiv E - m_3. \quad (39) $$

It coincides with energy of this state of the string without contribution of the mass $m_3$ \[7, 10\].

We may conclude, the central rotational state is unstable if the central mass $m_3$ is nonzero and less than energy of the string with other massive points.
5. Numerical experiments

In this section the above results connected with stability or instability of rotational states are verified in numerical experiments. Slightly disturbed rotational states (7) of the closed string with \( n = 3 \) masses are simulated numerically in comparison with the similar states of the linear string baryon model \( q-q-q \) [3]. We use the approach, suggested in Refs. [18, 20, 21]. It includes solving the initial-boundary value problem, by other words, calculation of classical motion of the system on the base of given initial position of the string \( X^\mu|_{ini} = \rho^\mu(\tilde{\sigma}) \) and initial velocities of its points \( \dot{X}^\mu|_{ini} = v^\mu(\tilde{\sigma}) \), \( \tilde{\sigma} = \tilde{\sigma}(\sigma) \).

![Numerical simulation of disturbed rotational motions](image)

If this initial conditions correspond to a central rotational state (7) with a small disturbance, for example \( \rho^\mu(\tilde{\sigma}) = \rho^\mu_{rot}(\tilde{\sigma}) + \delta \rho^\mu(\tilde{\sigma}) \), \( v^\mu(\tilde{\sigma}) = v^\mu_{rot}(\tilde{\sigma}) + \delta v^\mu(\tilde{\sigma}) \), we calculate slightly disturbed rotational motion of the system. In Fig. 5 the examples of such a motion for the closed string and the linear model \( q-q-q \) are represented as a set of “photographs” or positions of the string in \( e_1, e_2 \)-plane. These positions (sections \( t = \text{const} \) of the world surface) are numbered in order of increasing \( t \) with spacing in time \( \Delta t = 0.25 \); the numbers are shown near the first massive point marked by the asterisk. The second and the third (central) masses are marked by the point and the pentagram correspondingly. Parameters of the states are pointed out, they correspond to \( Q_1 = Q_2 = \frac{1}{4} \) (compare with Fig. 2).
In three cases in Fig. 4 the small disturbance in initial data is in the form \( \delta \rho^\mu = 0 \) and \( \delta v^\mu(\tilde{\sigma}) = 0.01 \sin(\tilde{\sigma} - \tilde{\sigma}_2) \) in the interval \( \tilde{\sigma}_2 < \tilde{\sigma} < \tilde{\sigma}_3 \) (\( \tilde{\sigma}_3 - \tilde{\sigma}_2 = \pi \)) between the points \( m_2 \) and \( m_3 \), but \( \delta v^\mu \) equals zero on all other points.

As one can see in Fig. 4, if the value \( m_3 \) is in the interval \( (55) \), and the increment \( \xi_2^* \) of disturbances’ growth is large enough (the case with \( m_3 = 1 \)), the evolution of growing disturbances results in going away the central mass. The string changes into rotating curvilinear triangle and massive points change their positions.

In the case \( m_3 \geq m_{3cr} \) the motion is stable and the massive point \( m_3 \) remains near the rotational center. Similar picture takes place, if \( m_3 \simeq m_{3cr} \), in particular \( m_3 = 5 \) is close to \( m_{3cr} \simeq 5.05 \). In this case the increment \( \xi_2^* \) is very small and the disturbed motion looks like a stable one.

It is interesting to compare this picture with slightly disturbed rotational states for the linear string baryon model \( q-q-q \). If tension \( \gamma \) is twice large than for the closed string and masses \( m_j \) are the same, parameters of a central rotational state \( \omega, Q_j \) and \( v_j \) coincide for both models. It was shown in Ref. [7] that the spectrum of small disturbances \( (20) \) for these states of the system \( q-q-q \) is described by the equation

\[
\frac{m_3(1 - v_j^2) \xi(\xi^2 - \omega^2)}{m_1 v_1 \omega(\xi^2 + \omega^2)} = \frac{\tilde{c}_1 (Q_1^2 \kappa_1 - \xi^2)}{\tilde{s}_1 (Q_1^2 \kappa_1 - \xi^2)} - 2 \tilde{s}_1 Q_1 \xi + \frac{\tilde{c}_3 (Q_2^2 \kappa_2 - \xi^2)}{\tilde{s}_3 (Q_2^2 \kappa_2 - \xi^2)} + 2 \tilde{s}_3 Q_2 \xi,
\]

where \( \kappa_j = 1 + v_j^{-2} \). For any value \( m_3 > 0 \) the imaginary root \( \xi = i \xi_2^* \) of this equation exists (there is no critical maximal value), so the picture of stability differs from that for the closed string: central rotational states of the linear string baryon model \( q-q-q \) are unstable for any masses \( m_3 > 0 \).

### 6. Regge trajectories for unstable states

Rotational states of the closed string with \( n \) massive points are applied for describing orbitally excited baryons \( [4, 8] \) and the Pomeron trajectory \( [10, 11] \), corresponding to possible glueball states.

For linear and central rotational states \( (17) \) the energy \( E \) and angular momentum \( J \) are used in Refs. [4, 10, 11] in the following form:

\[
E = 2\pi \gamma a_0 + \sum_{j=1}^{n} \frac{m_j}{\sqrt{1 - v_j^2}} + \Delta E_{SL}, \quad (40)
\]

\[
J = L + S = \frac{\gamma a_0^2}{2\omega} \left( 2\pi + \sum_{j=1}^{n} \frac{v_j^2}{Q_j} \right) + \sum_{j=1}^{n} s_j. \quad (41)
\]

Here \( s_j \) are spin projections of massive points (quarks or valent gluons), \( \Delta E_{SL} \) is the spin-orbit contribution to the energy in the following form [4]:

\[
\Delta E_{SL} = \sum_{j=1}^{n} \left[ 1 - \left( 1 - v_j^2 \right)^{1/2} \right] (\Omega \cdot s_j).
\]

If the string tension \( \gamma \), values \( m_j \) and the type of rotational state are fixed, we obtain the one-parameter set of motions with different values \( E \) and \( J \). The parameter of this set is any of the values: \( \omega, a_0, E, J \); other values are expressed via relations \( (12) - (17) \). These states lay at quasilinear Regge trajectories. If a central (or linear) rotational state \( (7) \) is the simplest
one, that is \( k_1 = k_2 = 0 \) in Eq. (17), the asymptotic behavior of the corresponding Regge trajectory in the limit \( E \to \infty \) is \([10][11]\):

\[
J \simeq \alpha'(E - m_3)^2, \quad \alpha' = \frac{1}{4\pi\gamma}.
\]

Below we apply the central rotational states to describing the Pomeron trajectory \([11][23]\), corresponding to glueball states. We suppose that the value \( S \) in Eq. (41) corresponds to the maximal total momentum \((41)\), that is \( S = 2 \) for 2-gluon glueballs and \( S = 3 \) for 3-gluon glueballs \([15][16]\). Other values of model parameters are \([10][11]\):

\[
\gamma = 0.175 \text{ GeV}^2, \quad m_1 = m_2 = m_3 = 750 \text{ MeV}.
\]

This tension \( \gamma \) is 0.9 GeV\(^{-2}\). Estimates of gluon masses on the base of gluon propagator, in particular, in lattice calculations \([24]\) yield value

\[
\begin{align*}
\text{Figure 5: Regge trajectories for rotational states} \\
\text{Regge trajectories or graphs } J = J(E^2) \text{ of expressions (40) and (41) for linear, central and hypocycloidal rotational states with parameters (43) are presented in Fig. 5. These trajectories lie close to the pomeron trajectory } \{23, 25\}.
\end{align*}
\]

\[
J \simeq 1.08 + 0.25E^2;
\]

it is shown as the dashed line. The dash-dotted line corresponds to the linear state with \( n = 2 \), the hypocycloidal state is “triangle” type state with \( n = 2 \) (the third vertex of the triangle is the massless point), considered in Refs. \([10][11]\). For the central rotational state \( n = 3 \).

These Regge trajectories are nonlinear for small \( E \) and tend to linear if \( E \to \infty \). Their slope \( \alpha' \) in this limit tends to the limit (42) for linear and central rotational states and to the value \( \alpha' = \frac{3}{8}(2\pi\gamma)^{-1} \) for “triangle” hypocycloidal states \([11]\).

We have shown in Sect. 4 that the central rotational states \([7]\) with equal masses \([13]\) are unstable for all energies on the classic level. But this does not mean disappearance or terminating corresponding Regge trajectories in Fig. 5. The straight consequence of this instability is the contribution to width of a hadron state.

String models describe only excited hadron states with large orbital momenta \( L \). These states are unstable with respect to strong interactions and have rather large width \( \Gamma \). In string interpretation of excited hadron this width is connected with probability of string breaking;
this probability is proportional to the string length $\ell$ \cite{26,27}. The value $\ell$ is proportional to the string contribution $E_{str}$ to energy $E$ of a hadron state, in particular, for rotational states (7) this contribution to the expression (40) is

$$E_{str} = 2\pi\gamma a_0.$$  

Therefore, the component of width $\Gamma_{br}$, connected with string breaking, is proportional to $E_{str}$ with the factor 0.1 resulting from particle data \cite{27,28}:

$$\Gamma_{br} \simeq 0.1 \cdot E_{str} = 0.2 \cdot \pi\gamma a_0. \tag{45}$$

The contribution $\Gamma_{inst}$ to width $\Gamma$ due to instability of string central rotational states (7) with respect to small disturbances is determined from the increment $\xi_2$ of exponential growth

$$|\varphi| \sim \exp(\xi_2\tau) = \exp(\xi_2 a_0^{-1}t).$$

For the central state in Fig. 5 with parameters (43) the increment $\xi_2 = \xi_2^*$, so its width is

$$\Gamma_{inst} \simeq \frac{\xi_2^*}{a_0}. \tag{46}$$

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure6.png}
\caption{Width $\Gamma(E)$ (47) (solid line) as the sum of $\Gamma_{br}$ (45) (dash-dotted line) and $\Gamma_{inst}$ (46) (dashed line) for central states (a) with parameters (43); (b) with $m_1 = m_2 = 200$ MeV.}
\end{figure}

In Fig. 6a we see that the central state with equal masses (43) is unstable for all energies $E$. The corresponding width component $\Gamma_{inst}(E)$ (46) tends to zero at large $E$ (because $a_0$ increases) and tends to infinity in the limit $E \to E_{min} = \sum m_j$. In this limit both values $\xi_2^* \to 0$, $a_0 \to 0$ but their ratio $\Gamma_{inst} \to \infty$. So in the limit $E \to E_{min}$ the central states with $m_3 < m_1 + m_2$ are unstable (note that string models are not applicable in this limit).

Behavior of width $\Gamma(E)$ (47) for central rotational states of the system with $m_3 > m_1 + m_2$ is presented in Fig. 6b with the same notations. Here $m_3$ and $\gamma$ are the same values (43). In this case the threshold effect (39) of instability exists, so the “instability” width $\Gamma_{inst}(E)$ equals zero for energies $E < E_{cr} = 2m_3$ (here it is 1.5 GeV). For $E > E_{cr}$ the value $\Gamma_{inst}(E)$ exceeds $\Gamma_{br}(E)$ in the certain interval, but if $E$ increases, $\Gamma_{inst}(E)$ tends to zero and $\Gamma_{br}(E)$ increases linearly.
Conclusion

For linear rotational states (7) of a closed string with 2 massive points and central rotational states (with the mass $m_3$ at the rotational center) the stability problem is solved on the classical level. It is shown that the linear states are stable with respect to small disturbances in linear approximation, but the central states are unstable, if the central mass is less than the critical value (39). This value equals energy of the string with other massive points.

This threshold effect was investigated both analytically (instability is connected with exponentially growing modes in the spectrum of small disturbances of a rotational state) and in numerical experiments.

Instability of these central states results in some manifestations, in particular, in additional width of excited hadron states. It is shown that this contribution makes essential changes in linear dependence $\Gamma \sim E$.

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