FREE EVOLUTION OF NONLINEAR SCALAR FIELD COLLAPSE
IN DOUBLE-NULL COORDINATES

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We study numerically the fully nonlinear spherically-symmetric collapse of a self-gravitating, minimally-coupled, massless scalar field. Our numerical code is based on double-null coordinates and on free evolution of the metric functions and the scalar field. The numerical code is stable and second-order accurate. We use this code to study the late-time asymptotic behavior at fixed $r$ (outside the black hole), along the event horizon, and along future null infinity. In all three asymptotic regions we find that, after the decay of the quasi-normal modes, the perturbations are dominated by inverse power-law tails. The corresponding power indices agree with the integer values predicted by linearized theory. We also study the case of a charged black hole nonlinearly perturbed by a (neutral) self-gravitating scalar field, and find the same type of behavior—i.e., quasi-normal modes followed by inverse power-law tails, with the same indices as in the uncharged case.

1 Introduction

Until recently, the late-time evolution of non-spherical gravitational collapse was investigated primarily in the context of linear theory. The late-time behavior of such perturbations has been studied for three different asymptotic regions: (a) at fixed $r$ (outside the black hole), (b) along null infinity, and (c) along the future event horizon (EH). Qualitatively, the evolution of the linearized perturbations is similar in these three asymptotic regions: During the first stage, the perturbations’ shape depends strongly on the shape of the initial data. This stage is followed by the stage of quasi-normal (QN) ringing. Finally, there are also ‘tails’, characterized by an inverse power-law decay.

Case (a) was first studied by Price, and cases (b) and (c) by Gundlach et al. It was found that after the QN ringings die out, the perturbations in (a) decay according to $t^{-(2l+\mu+1)}$, where $\mu = 1$ if there were an initial static mode, and $\mu = 2$ otherwise. Here, $l$ is the multipole moment of the mode in question, and $t$ is the Schwarzschild time coordinate. Tails in (b) decay according to $u^{-(l+\mu)}$, and in (c) according to $v^{-(2l+\mu+1)}$, where $u$ and $v$ are the outgoing and ingoing Eddington-Finkelstein coordinates, correspondingly.

The numerical simulation of the fully-nonlinear gravitational collapse of a spherical self-gravitating scalar field was recently carried out, and the QN ringing and the ‘tails’ were demonstrated for cases (a) and (c).

In what follows we shall briefly describe a recent numerical simulation of the fully-nonlinear spherical collapse of a minimally-coupled massless scalar field. We study all three cases (a), (b) and (c), and obtain values for the power-law indices significantly closer to the linear analysis predictions than all previous nonlinear simulations. Section describes our numerical method, and Section describes our
main results. Further details are given elsewhere.

2 The numerical method

Our numerical code is based on free evolution of the dynamical equations in double-null (DN) coordinates. The constraint equations are imposed only on the initial characteristic hypersurface, and just monitored during the evolution. The main advantages and properties of the DN approach are: The DN coordinates are very well adopted to the hyperbolic character of the field equations, the interpretation of the causal structure of the spacetime is trivial, and DN coordinates can be chosen such that the metric is regular at the EH. Evolution is obtained by straightforward marching along both coordinates. Because in the dynamical equations all second derivatives are mixed, a second-order code can be obtained from just two adjacent grid-points in each direction. Therefore, only three grid points are needed in each computational cell, and the second-order accuracy is obtained by standard ‘predictor-corrector’ technique. The inevitability of some sort of Dynamical Mesh Refinement for our code can be demonstrated for Schwarzschild (for any spacetime with an EH the reasonings are similar). First, the EH must be included in the integration. If it were not, then very quickly the integration domain will get very far from the EH, and none of the cases (a), (b) or (c) would be susceptible of accurate analysis. However, whenever the EH is included in the integration we face a fundamental difficulty: As one moved along an outgoing null ray near the EH, \( r_u \) grows exponentially, where \( r \) is the area coordinate. Hence, a small error in the location of the EH would quickly blow off. The solution is to make the grid dense near the EH (to allow for an accurate integration), but dilute far from the EH (to save computation time). This is done by the following algorithms. Point Splitting: We check the variation of the metric functions and the scalar field along ingoing rays between two adjacent grid points. If the variation is greater than some threshold value, we introduce an intermediate grid-point, at which we evaluate the fields by interpolation. Chopping: The domain of integration includes the EH. Consequently, (with vanishing charge) any fixed \( u_{\text{final}} \) would lead to a crash into the singularity after a finite lapse of time. We solve the problem by simply chopping ingoing rays immediately after the apparent horizon, which we find locally. Chopping enables the code to run forever while including the EH in the integration. Due to the Point Splitting procedure, we obtain a denser grid than necessary far from the EH. We introduce Point Removal to remove unnecessary grid-points, by a method which essentially is the inverse of Point Splitting. However, in any Kruskal-like gauge \( g_{uv} \) grows rapidly along ingoing rays, and therefore Point Removal would be ineffective. We cure the problem by introducing Gauge Correction, which changes the gauge to a gauge in which \( g_{uv} \) changes only slowly. The combination of Point Removal and Gauge Correction enables a very effective saving of computation time. Our code was checked by the following methods: We compared the results obtained with different grid-parameters. We monitored the discrepancy in the two constraint equations. We compared the mass function obtained from local differentiation and from evolution of the ‘wave equation’ for the mass function. We numerically reproduced known exact solutions (Schwarzschild, Reissner-Nordström (RN), the homothetic
Table 1: Values for the Local Power Indices for cases (a), (b) and (c) in configurations (1) (uncharged case) and (2) (charged case). The value in the brackets is the linearized theory prediction.

|                  | (a): along the EH | (b): along null infinity | (c): along $r =$ const |
|------------------|-------------------|--------------------------|------------------------|
| Config. (1)      | 2.98 ± 0.01 (3)   | 2.002 ± 0.003 (2)        | 2.99 ± 0.02 (3)        |
| Config. (2)      | 2.99 ± 0.01 (3)   | 1.996 ± 0.001 (2)        | 2.99 ± 0.02 (3)        |

Roberts solution \[7\]. All checks indicated stability, convergence and second-order accuracy.

3 Results for the decay of ‘tails’

We used our numerical code for the following two configurations. (1): Collapse over Minkowski, which leads to the formation of a Schwarzschild-like black hole. (2): Collapse over a unity mass pre-existing charged background (a RN geometry) with various values of the charge. We took an initially ingoing pulse of squared-sine shape of compact support, and chose the amplitude to be high, such that the final black hole mass was $M_f \approx 3.5$ in (1) and $M_f \approx 3.8$ in (2). We then evolved the fields, and probed their values for cases (a), (b), and (c) in both configurations (1) and (2). In order to approximate null infinity [for case (b)] we probed the fields on an ingoing null ray at $v_e = 10^6 M_f$. We introduce the notion of the local power index (LPI). Namely, we calculate the evolution in $p \equiv -v (\ln \Phi)_v$ for case (c) [$v$ should be replaced by $t$ and $u$ for cases (a) and (b), respectively], where $\Phi$ is the scalar field. The motivation behind the LPI is as follows: The power-law behavior is just the leading-order term in an expansion in $v^{-1}$. For any finite value of $v$ there will also be nonvanishing contributions of the higher-order terms, which will cause a deviation from the integer power index. In addition, any averaging or best-fit technique would yield a result which depends on the interval. The LPI would thus be a fractional number, but would asymptotically approach the integer value of the leading-order power index. Table 1 summarizes our results. The agreement with the linearized-theory predictions is remarkable, and better than the results of all previous nonlinear analyses.

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