A significant fraction of multi-planet planetary systems contain a pair of giant planets engaged in a mean motion resonance (MMR). These planets are mainly in the low order 2:1 MMR (as GL 876, HD 128311, and HD 73526), but in a few systems higher order resonances have been suggested as well; two planets around 55 Cancri may be in 3:1 MMR, or HD 202206 may host a pair of planets in 5:1 MMR. In the recently discovered system, HD 45364, the giant planets are revolving in 3:2 MMR (Correia et al. 2009).

The observed orbital solutions and the formation of the majority of resonant systems have been studied thoroughly by many authors. It has been shown, for instance, by Kley et al. (2004) that a sufficiently slow migration process of two giant planets embedded in an ambient protoplanetary accretion disk ends with either a 3:1 or 2:1 resonant configuration depending on the speed of migration. The formation of resonant systems in 2:1 MMR has been modelled exhaustively by hydrodynamical and N-body simulations as well. The system GJ 876 has been investigated by Lee & Peale (2002). Kley et al. (2005), Crida et al. (2008), and the systems HD 128311 and HD 73526 by Sándor & Kley (2006) and Sándor et al. (2007), respectively. Most recently, the formation of the system HD 45364 with planets in the 3:2 MMR has been investigated by Rein et al. (2010). Analytical studies related to the stationary solutions of the 2:1 and 3:1 MMR have been done by Beaugé et al. (2003) and Beaugé et al. (2006), for instance.

Regarding the 3:1 MMR case, the presence of the planet 55 Cancri-c has already been questioned (Naef et al. 2004). Additionally, recent orbital fits indicate that the planets 55 Cancri-c and 55 Cancri-d are not in a resonant configuration (Fischer et al. 2008). Thus, until the recent discovery of the two planets in a 3:1 MMR around the F-type star HD 60532 by Desort et al. (2008), there has been a serious lack of knowledge about the observed behaviour of 3:1 resonant systems. The results of the dynamical study performed by Desort et al. (2008) did not prove without any doubt that the giant planets are in fact in a 3:1 MMR.

The final confirmation of the 3:1 MMR between the giant planets in HD 60532 is given in a recent paper by Laskar & Correia (2009), in which two new orbital fits are provided, slightly improving on the previous fit of Desort et al. (2008) with $i = 90^\circ$. Through a detailed stability analysis of different orbit integrations, Laskar & Correia (2009) suggest that an inclination of $i = 20^\circ$ is the most likely configuration for a co-planar model. By this assumption, the planetary masses are increased by a factor of $1/\sin(i) \approx 3$ in comparison to Desort et al. (2008). In a recent paper, Libert & Tsiganis (2009) analyse the excitation of mutual inclination for planetary systems driven into resonance by planet-disk interaction. They use a damped N-body evolution and find that for stronger eccentricity damping, the mutual inclination is less excited. However, they could not place any constraints on the observed inclination of the system HD 60532.

The very large difference in planetary masses between the two orbital solutions having $i = 20^\circ$ or $90^\circ$ also raises the impor-
tant question of which orbital solutions are preferred by formation based on the planetary migration scenario. In the present paper we aim at answering this question by performing fully hydrodynamical evolution of planets embedded in the disk. Through this procedure, we obtain realistic migration and eccentricity damping rates that will allow us to determine the most probable final state of the system.

The paper is organized as follows. First, we numerically integrate the orbits of giant planets using the two sets of orbital solutions given by Laskar & Correia (2009) as initial conditions. Then by considering different planetary masses, we investigate the possible capture into the 3:1 MMR between the giant planets. After having formed a resonant system, we compare the results of our simulations to the orbital behaviour of giant planets obtained from numerical integrations of Sect. 2. Finally, by performing gravitational three-body numerical integration with dissipative forces for migration, we study how an inner disk influences the behaviour of the system toward its stationary solutions. We demonstrate that the presence of the inner disk determines the final resonant configuration of the system.

2. Orbital behaviour of the system HD 60532

In this section we present the orbital behaviour of the giant planets around the star HD 60532. To integrate the equations of the three-body problem, we used a Bulirsch-Stoer numerical integrator. The initial conditions for our simulations were calculated from the orbital solutions given by Laskar & Correia (2009), see Fits I and II in Table 1.

A resonant system can be characterized by studying the behaviour of the resonant angles. According to Murray & Dermott (1999), in the planar case of the 3:1 MMR, there are three different resonant angles: $\theta_1 = 3\lambda_2 - \lambda_1 - \omega_1 - \omega_2$, $\theta_2 = 3\lambda_2 - \lambda_1 - 2\omega_1$, and $\theta_3 = 3\lambda_2 - \lambda_1 - 2\omega_1$, where $\lambda_i$ denotes the mean longitude of the $i$-th planet, and $\omega_i$ its periastron. Here index “1” stands for the inner and index “2” for the outer planet. By introducing the corotational angle $\Delta\omega = \omega_2 - \omega_1$ and arbitrarily choosing a $\theta_i$, all the remaining resonant angles can be expressed with $\theta_i$ and $\Delta\omega$. If one of the $\theta_i$ ($i = 1, 2, 3$) librates around a constant value, the system is in a 3:1 MMR. If $\Delta\omega$ also librates, the system is said to be in apsidal corotation.

By studying Fig. 1 one can follow the behaviour of the eccentricities and resonant angles as functions of time for Fit II. The eccentricities show quite large oscillations, the inner planet’s eccentricity varies between $e_1 \sim 0.1 - 0.33$, while the outer planet’s eccentricity is between $e_2 \sim 0 - 0.15$. All resonant angles $\theta_i$ are librating, $\theta_1$ around $0^\circ$, $\theta_2$, $\theta_3$, and $\Delta\omega$ around $180^\circ$. This means that the system is in a 3:1 MMR and also in apsidal corotation, and furthermore the orbits of the giant planets are anti-aligned. In Fig. 1 we display the behaviour of the resonant angle $\theta_3$, (as used by Laskar & Correia 2009) and $\Delta\omega$. By using Fit I, we found the same dynamical behaviour of the giant planets as Laskar & Correia (2009), therefore we do not display these results here.

Table 1. Orbital solutions provided by Laskar & Correia (2009)

| Fit | Planet | Mass [M$_J$] | $a$ [AU] | $e$ | $M$ [deg] | $\omega$ [deg] |
|-----|--------|-------------|---------|----|----------|-------------|
| I   | inner  | 1.0484      | 0.7597  | 0.279 | 22.33    | 352.15    |
|     | outer  | 2.4866      | 1.5822  | 0.027 | 179.4    | 136.81    |
| II  | inner  | 3.1548      | 0.7606  | 0.278 | 21.95    | 352.83    |
|     | outer  | 7.4634      | 1.5854  | 0.038 | 197.53   | 119.49    |

Regarding the behaviour of the resonant angles and the possible apsidal corotation, a comprehensive analytical study of the 3:1 MMR has been done by Beaugé et al. (2003). The authors find that the resonant angle $\theta_3$ and $\Delta\omega$ both librate around $180^\circ$ if the eccentricity of the inner planet is below a certain limit.
by Sándor et al. (2007) and Crida et al. (2008), the presence of free environment. On the other hand, as demonstrated recently such a case, the outer planet feels the negative torques of only summed to be cleared by the accretion of the gas to the star. In this section we show the results of our hydrodynamical simulations obtained by using different planetary masses.

First we investigated the behaviour of the giant planets having low masses, corresponding to the Fit I, where $i = 90^\circ$. In this case $m_1 = 1.048M_J$, $m_2 = 2.487M_J$ corresponding to $q_1 = 0.0007$ and $q_2 = 0.00165$ dimensionless mass units (the stellar mass is unity). We used index “1” for the inner and index “2” for the outer planet. After releasing them, however, the planets did not show a convergent migration, and no resonant capture took place. This was also true for the intermediate value of the inclination $i = 30^\circ$, where the planetary masses are a factor of two higher than in Fit I.

Finally, we used planetary masses corresponding to Fit II, where $i = 20^\circ$, and the planetary masses are $m_1 = 3.15M_J$ and $m_2 = 7.46M_J$ or, in dimensionless mass units, $q_1 = 0.0021$ and $q_2 = 0.005$. (These values are quite close to the planetary masses used by Kley et al. 2004.) In this case the giant planets also be-
inward with the same speed, or if the inner planet migrates even faster than the outer one, no resonant capture will occur. For example, if the inner planet does not open a sufficient gap, it may migrate faster than a usual type II migration. Consequently, if the planets migrate faster than the outer one, no resonant capture will occur. For example, if the inner planet does not open a sufficiently empty gap, it may migrate faster than a usual type II migration. It is also true that the inner planet’s inward migration is mainly governed by the torques coming from the middle part of the disk, which is between the planets. (The outer disk does not really exert torques to the inner planet because it is separated by the wide gap opened by the outer planet.) On the other hand, the inward migration of the inner planet can be stopped and even reversed by a sufficiently massive inner disk (Sándor et al. 2007 Crida et al. 2008). From our hydro simulations one can see that only the massive inner planet of Fit II can open a sufficiently deep gap for a slower (type II) migration, and could also create a massive inner disk as pushing the disk’s material toward the star. The planets with masses from Fit I and the intermediate case (with i = 30°) open a shallower gap, the middle disk is more massive, and the inner disk is less massive for planets having masses from Fit II. This can be checked in Fig. 3 where we display the azimuthally averaged surface density profiles of the disk for the different planetary masses. Thus, as can be seen in the upper panel of Fig. 3 the migration of the inner planet (of Fit II) is slow in the beginning and is stopped after a certain time, and right before the resonant capture (at t = 2300) its inward migration is even slightly reversed, which is not seen in the figure. The action of the inner disk means that we might expect a slow outward migration of the inner planet (Crida et al. 2008). However, this does not play any role in the resonant capture in our case, as it is the outer planet that migrates towards the inner one.

**Fig. 3.** The azimuthally averaged surface density profile of the protoplanetary disk after 500 periods of the inner giant planet for different planetary masses. Dashed line corresponds to planetary masses of Fit I, dotted line for the intermediate values, which are twice the masses of Fit I, and solid line for masses of Fit II.

Before describing the dynamical behaviour of the system after the capture into the resonance, we should comment on why this phenomenon does not occur in the cases of lower mass planets. We recall that for a resonant capture the planets should exhibit convergent migration. Consequently, if the planets migrate inward with the same speed, or if the inner planet migrates even faster than the outer one, no resonant capture will occur. For example, if the inner planet does not open a sufficiently empty gap, it may migrate faster than a usual type II migration. It is also true that the inner planet’s inward migration is mainly governed by the torques coming from the middle part of the disk, which is between the planets. (The outer disk does not really exert torques to the inner planet because it is separated by the wide gap opened by the outer planet.) On the other hand, the inward migration of the inner planet can be stopped and even reversed by a sufficiently massive inner disk (Sándor et al. 2007 Crida et al. 2008). From our hydro simulations one can see that only the massive inner planet of Fit II can open a sufficiently deep gap for a slower (type II) migration, and could also create a massive inner disk as pushing the disk’s material toward the star. The planets with masses from Fit I and the intermediate case (with i = 30°) open a shallower gap, the middle disk is more massive, and the inner disk is less massive for planets having masses from Fit II. This can be checked in Fig. 3 where we display the azimuthally averaged surface density profiles of the disk for the different planetary masses. Thus, as can be seen in the upper panel of Fig. 3 the migration of the inner planet (of Fit II) is slow in the beginning and is stopped after a certain time, and right before the resonant capture (at t = 2300) its inward migration is even slightly reversed, which is not seen in the figure. The action of the inner disk means that we might expect a slow outward migration of the inner planet (Crida et al. 2008). However, this does not play any role in the resonant capture in our case, as it is the outer planet that migrates towards the inner one.

**Fig. 4.** Dynamical behaviour of the two giant planets embedded in the surrounding protoplanetary disk: semi-major axes and eccentricities. During the first 500 orbital periods (of the inner planet), the planets are kept fixed to obtain a steady state in the disk. The upper panel shows the behaviour of the semi-major axes, while the bottom panel shows the time evolution of the eccentricities (red line corresponds to the inner, green line to the outer giant planet). The capture into the 3:1 MMR occurs around 2300 periods of the inner planet. The planetary masses \( m_1 = 3.15 M_J \) and \( m_2 = 7.46 M_J \) are taken from Fit II.

If the effect of the inner disk is not taken into account, the inward migrating outer planet always captures the inner planet into an MMR. In this case, the results of Kley et al. (2004) clearly demonstrate that the final system is either in 3:1 or in 2:1 MMR mainly depending on the speed of the outer planet’s migration. A fast migration of the outer planet may result in its crossing the 3:1 MMR without capture and ending the migration in the more robust 2:1 MMR.

In the case of the large planetary masses, around 2300 periods of the inner planet, the planets enter into the 3:1 MMR. The evolution of the semi-major axes and eccentricities during the migration and after the resonant capture is shown in Fig. 4. After the resonant capture, the eccentricities first increase and then tend to oscillate (around their mean values). However, the oscillations of the eccentricities remain bounded approximately in the range given by the numerical integration shown in Fig. 1. The inner disk certainly does contribute to the apparently very effective damping of the inner planet’s eccentricity, as shown in
the formation scenarios of the resonant systems HD 73526 and GJ 876 by Sándor et al. (2007) and Crida et al. (2008).

According to our simulation, the resonant angles \( \theta_i \) (i = 1, 2, 3) and the corotation angle \( \Delta \omega \) show libration, \( \theta_i \) librates around 0°, and the remaining angles around 180°, see Figure 5. The dynamical behaviour of our modelled system is qualitatively the same as the observed one, displayed in Figure 1, even though in our case the libration amplitudes are somewhat smaller.

Interestingly though, the behaviour of the resonant angles does not match the theoretical results of Beaugé et al. (2003) and the numerical simulations done by Kley et al. (2004). Although the eccentricity of the inner planet exceeds the limit of \( e_1 \sim 0.13 \), the libration of \( \theta_1 \) and \( \Delta \omega \) is found to be around 180°. As already mentioned, this might be the consequence of the system not yet reaching a stationary solution. In what follows, we study this unexpected result through numerical integration of the gravitational three-body problem using properly parametrized non-conservative forces for migration.

5. Toward the stationary solutions of the 3:1 MMR in HD 60532

In the previous section we presented a reasonably well-working formation scenario for the system HD 60532. The eccentricities of the giant planets and resonant angles (including the corotation angle) of the formed system behave very similarly to the results of numerical integration using initial conditions from Fit II. In this section, we intend to explain why the observed and the formed systems do not fit the previous studies of Beaugé et al. (2003) and Kley et al. (2004). To do so, we performed two series of simulations; in the first one, only the outer planet is forced to migrate inward, while the inner planet does not feel any additional damping forces. In the second one, the inner planet feels the (accelerating) effect of the inner disk as well.

The additional non-conservative force responsible for migration and eccentricity damping can be parametrized by the migration rate \( \dot{a}/a \) and the eccentricity damping rate \( \dot{e}/e \), or by the corresponding \( e \)-folding times \( \tau_e \) and \( \tau_\Delta \) of the semimajor axis and eccentricity of the outer planet (see Lee & Peale 2002, Beaugé et al. 2004 for two possible approaches). In our simulations, we used the force corrections as suggested by Lee & Peale (2002). The relations between the damping rates and \( e \)-folding times are \( \dot{a}/a = -1/\tau_a \) and similarly for the eccentricities \( \dot{e}/e = -1/\tau_e \). One can define the ratio between the \( e \)-folding times \( K = \tau_e/\tau_\Delta \) (or \( \dot{e}/e = -K\dot{a}/a \)), which according to Lee & Peale (2002) determines the final state of the system in the case of a sufficiently slow migration.

If the inner planet is influenced by an inner disk, it may be forced to migrate outward and its eccentricity is also damped. This effect can also be modelled by using a (repelling) non-conservative force parametrized by \( \dot{a}/a \) (having a positive sign for outward migration), and \( \dot{e}/e \) for the eccentricity damping. A detailed study of the resulting dynamical effect of these parametrizations has been presented recently by Crida et al. (2008).

In the first series of simulations, only the outer planet migrated on different timescales \( \tau_a \), while its eccentricity was damped on timescales \( \tau_e \) such that their corresponding ratios \( K = \tau_e/\tau_\Delta \) were always between \( K \sim 1...10 \), which is typically found in hydrodynamical simulations (Kley et al. 2004). In the second series of runs, besides the migration of the outer planet, the inner planet’s eccentricity was also damped, while its semi-major axis was not influenced. Typical cases of these simulations can be seen in Figs. 6 and 7. In these particular cases the following migration and eccentricity damping timescales were applied; for the outer planet \( \dot{a}_2/\dot{a}_2 = -5 \times 10^{-5} \text{year}^{-1}, \dot{e}_2/e_2 = -2 \times 10^{-4} \text{year}^{-1} \) (or \( \tau_a = 2 \times 10^3 \), \( \tau_e = 5 \times 10^5 \) years), and if the inner disk’s influence on the inner planet was also taken into account, we used for the inner planet \( \dot{a}_1/\dot{a}_1 = 0 \) (as its semi-major axis was not influenced) and \( \dot{e}_1/e_1 = -2 \times 10^{-4} \text{year}^{-1} \).

Comparing Figs. 6 and 7 it can be seen immediately that, in the first case when the inner planet was unaffected, the resonant angles behave according to the theoretical predictions of Beaugé et al. (2003) (asymmetric libration of the resonant angles), while in the second case the libration of the resonant angles is around 180° and 0°, respectively. The inclusion of the inner disk helps to keep the inner planet’s eccentricity at lower values close to those coming from the observations. In Fig. 6 the eccentricity of the inner planet tends to 0.5, while in Fig. 7 it will show only small amplitude oscillations around the value 0.3.

Finally, we can conclude that the effect of an inner disk in the three-body simulations (with dissipative forces) alters the final state of the resonant configuration (here a 3:1 MMR), keeping the resonant angles far from their mean libration, thus impeding the system for reaching the equilibrium solutions suggested by Beaugé et al. (2003). In both cases the systems will remain in their final configurations, when the damping effect of the disk is reduced and eventually non existent.

![Fig. 5. Dynamical behaviour of the two giant planets embedded in the surrounding protoplanetary disk for Fit II. The evolution of the resonant angle \( \theta_1 \) and \( \Delta \omega \) are shown in the top and bottom panels, respectively. After entering into the 3:1 MMR, the orbits of the giant planets exhibit apsidal corotation.](image-url)
Fig. 6. Behaviour of the eccentricities (red for the inner, green for the outer planet), the resonant angle $\theta_3$, and the corotation angle $\Delta\omega$ when only the outer planet migrates inward and the inner planet is not affected.

Fig. 7. Behaviour of the eccentricities (red for the inner, green for the outer planet), the resonant angle $\theta_3$, and the corotation angle $\Delta\omega$ when the outer planet experiences migration and eccentricity damping, and the inner planet only the eccentricity damping effect of an inner disk.

6. Summary

After the discovery of the system HD 60532 (Desort et al. 2008), the thorough dynamical analysis of Laskar & Correia (2009) has reliably established the first resonant system that contains two giant planets in a higher order mean motion resonance, here 3:1. Until now, only one system, 55 Cancri, has been proposed as a candidate for a system where two giant planets might be in 3:1 MMR. On the other hand, for 55 Cancri, the existence of the resonant solution could not be confirmed by Naef et al. (2004), and a new non-resonant solution was later found by Fischer et al. (2008). Thus according to our present knowledge, HD 60532 is the only known system containing giant planets in the 3:1 MMR.

Recently, Laskar & Correia (2009) has improved the orbital solution found by Desort et al. (2008). In their new fit that assumes a relatively small inclination between the common orbital plane of the planets and the tangent plane of the sky, $i = 20^\circ$, ...
the giant planets have quite high masses: $m_1 = 3.15 M_J$ and $m_2 = 7.46 M_J$.

The main path to forming a resonant planetary system is thought to be by convergent migration of planets. For an alternative scenario, formation through a scattering process has been proposed by Raymond et al. (2008). Migration is the result of dissipative forces originating in an accretion disk, which act along with the mutual gravitational forces between the planets and the central star. Being a dissipative process, a sufficiently long-lasting migration process brings the system close to a stationary solution, which corresponds to a periodic solution of the system. The stationary (minimum energy) solutions in the systems yielding 3:1 MMR have been studied by a Hamiltonian approach by Beaugé et al. (2003). It has been found that, depending on the ratio of the eccentricities, the resonant angles can exhibit antisymmetric (symmetric anti-aligned) or asymmetric libration. These results have also been confirmed by numerical simulations of Kley et al. (2004).

In a first step, we integrated the system HD 60532 numerically using the (observed) initial conditions calculated from Fits I and II of Laskar & Correia (2005). We found that all resonant angles librate and that the system is in apsidal corotation. Since $\Delta \omega$ oscillates around 180°, observationally the system lies in an antisymmetric configuration. However, for the observed mean eccentricities, the Hamiltonian approach would suggest an asymmetric configuration.

To model the formation of the system HD 60532 and find an explanation for this special antisymmetric configuration, we assumed that the system formed through a planet-disk interaction process. We performed a series of fully hydrodynamical simulations with low, intermediate, and high planetary masses $m_i / \sin i$, assuming $i = 90^\circ$, $30^\circ$, and $20^\circ$, respectively. Since the eccentricity of the inner planet oscillates around a moderate mean value $e_1 \sim 0.3$, we assumed that during the migration process an inner disk (between the inner planet and the central star) was present, providing an efficient damping mechanism on the inner planet’s eccentricity. We found that the convergent migration through planet-disk interaction, which takes the planets into the 3:1 MMR only occurred for the highest planetary masses. The dynamical behaviour of the resulting resonant planetary system is then indeed very similar to the one obtained from the radial velocity observations. Through our full hydrodynamical simulations, we support the small inclination $i = 20^\circ$ of the system as suggested by Laskar & Correia (2009).

To understand why the system does not appear to be in a minimum energy configuration to the 3:1 MMR, we performed a series of dedicated 3-body simulations with additional forces emulating the effects of the protoplanetary disk. In particular, we studied situations with and without the influence of an inner disk. We found that it is exactly the effect of the inner disk that distinguishes between an antisymmetric or asymmetric final configuration. Here, its presence is responsible for the system not reaching the minimum energy asymmetric configuration. Nevertheless, we point out that the solutions obtained in the antisymmetric state are indeed stationary solutions, which are stable when the effects of the disk are reduced.

In related earlier works, we had already established that the observed smallness of the mean eccentricities in resonant planetary systems can also be attributed to the effect of an inner disk Sándor et al. (2003) Crida et al. (2008). We conclude that an inner disk during the migration process has directly observable consequences on the post-formation dynamical behaviour of resonant planetary systems in mean motion resonances.