Supporting Middle Grades Mathematics Teachers and Students: A Curricular Activity System Used in an Urban School District

George J. Roy
University of South Carolina
Columbia, SC
roygj@mailbox.sc.edu

Vivian Fueyo
University of South Florida
St. Petersburg, FL

Philip Vahey
Center for Technology in Learning, SRI International
Menlo Park, CA

Abstract

The exploration of proportional relationships is foundational to the mathematics studied in the middle grades and beyond. Research has shown that an early emphasis on procedures often leaves students with a shallow understanding of the important underlying mathematical concepts of proportional relationships. One approach that addresses the needs of both teachers and students when it comes to mathematics that middle grades students find challenging (e.g., ratios and proportions) is a unified curricular activity system (i.e., a system of curricular materials, technology, and teacher professional development). The mixed-method study presented in this article outlines a curricular activity system that was designed to underpin middle grades mathematics teachers’ implementation of a learning module addressing proportional relationships. The results of the study provide evidence that a curricular activity system in which teacher professional development is coherently aligned to technology-embedded curriculum impacts teachers’ ability to teach as well as students’ learning of proportional relationships.

Keywords: technology, curriculum, mathematics

Introduction

The study of proportional relationships in the middle grades has been described as the capstone of mathematics studied in the elementary grades and also the foundation to higher mathematics including...
algebra, geometry, and statistics studied in high school and beyond (National Research Council, 2001). It is during the middle grades that the emphasis of mathematics shifts from being primarily additive in nature to more multiplicative in nature. Consequently, the understanding of proportional relationships is central to a wide range of mathematics topics studied in the middle grades. For instance, proportional relationships underpin but are not limited to the following topics: measurement, percent, scale, rate of change, similarity, and estimates of a population based upon a sample. Current college- and career-ready mathematics standards (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) stress proportional relationships as a domain to be explored during the middle grades. As far back as 1989, the National Council of Teachers of Mathematics (NCTM) stressed its importance in the middle grades, stating that “the ability to reason proportionally develops in students throughout grades 5–8. It is of such great importance that it merits whatever time and effort that must be expended to assure its careful development” (NCTM, 1989, p. 82).

In their seminal research, Cramer and Post (1993) identified three fundamental proportional reasoning types: missing-value problems, numerical comparison problems, and qualitative prediction and comparison problems. In a missing-value problem, there are four quantities, three of which are provided, while the fourth is unknown. To solve the problem, the reader must reason multiplicatively to determine the missing value. While being able to solve such problems is important, the procedural ability to solve a missing value problem does not itself indicate that a student has a deep understanding of proportionality. For instance, solving missing-value problems is not related to being able to determine a proportional from a non-proportional relationship (Tjoe & De La Torre, 2014). In the second problem type, numerical comparison problems, four quantities are provided, and a student must make a determination (i.e., greater than, less than, or equal to) based on the multiplicative relationships given. This type of problem is often challenging for middle grades students if they solely “solve” proportions procedurally and are asked to interpret results (Kastberg, D’Ambrosio, Lynch-Davis, Mintos, & Krawczyk, 2013/2014). The final type Cramer and Post identified is a qualitative problem in which the situation is described without numbers so the relationship is determined from the context. A variety of strategic approaches are used by students based upon problem type in each of these proportional situations (Fernández, Llinares, & Valls, 2008).

One approach is to emphasize procedures to solve the presented proportional situations. An early emphasis on computational procedures can leave students with a shallow understanding of the important underlying mathematical concepts of proportional relationships (National Research Council, 2001). Instead, research suggests that students must have a solid understanding of proportional situations and be able to reason about proportional relationships informally before they use computational procedures (Langrall & Swafford, 2000). For instance, Langrall and Swafford proposed that teachers allow students to explore proportional relationships through the use of contexts that can be visually modeled prior to introducing computational strategies. Research shows that ultimately, students who have the ability to reason proportionally will compare multiple pieces of mathematical information both quantitatively and qualitatively in order to determine the co-variation between two or more quantities (Cramer & Post, 1993; Lamon, 1993; Langrall & Swafford, 2000). Moreover, students who reason proportionally are able to make connections among a variety of mathematical representations including graphical, tabular, pictorial, narrative, and algebraic (Lesh, Post, & Behr, 1987).

One approach identified by research that supports the teaching of mathematics (e.g., proportional relationships) identified in current college- and career-ready standards for mathematics is a curricular activity system (Roschelle, Knudsen, & Hedges, 2010; Vahey, Knudsen, Rafanan, & Lara-Meloy, 2013). A curricular activity system coherently aligns curricular materials (e.g., paper-based curriculum material including student workbooks and teacher guides), technology, and teacher professional development (PD), recognizing that these are all situated in a larger educational school, district, and state context. By embedding technology into a progression of key learning activities, a variety of factors, including teachers’ professional needs as well as the curricular resources they utilize with their students, are addressed. In the present study, a curricular activity system was designed to investigate middle grades mathematics teachers’ implementation of learning module with the learning focus on proportional relationships.

The SunBay Mathematics Project (SunBay) is collaboration among faculty from a college of education, educational researchers from a non-profit
research institute, and school district leaders from a large, urban K–12 school district. Since its inception, SunBay has been aimed at investigating middle grades mathematics teachers’ teaching of key mathematics concepts identified by the college- and career-ready standards. The current study was informed by two prior research studies. In the first study, researchers conducted a randomized control experiment with 825 seventh-grade students to investigate their learning of rate and proportionality in five distinct geographic regions in Texas (Roschelle, Scechtman, et al., 2010). The second study was a replication study with 246 seventh-grade students in a single, large, urban school district in Florida (Vahey, Roy, & Fueyo, 2013). Results of both studies indicated that students learn proportional reasoning concepts when the three components of curricular activity system are coherently aligned.

Curricular Activity System Approach

Elements of the curricular activity system emphasized in this project include (a) a learning module emphasizing proportional relationships that was aligned to the state standards and served as replacement for state adopted textbook lessons; (b) dynamic technology emphasizing multiple mathematical representations; and (c) focused teacher PD exploring the coordination of technology within the learning module. Each element of the curricular activity system will be described in the following sections.

The Learning Module: Managing the Soccer Team: A Unit on Rate and Proportionality

The Managing the Soccer Team (SRI International, 2009) learning module used in this research study is a three-week curricular unit that incorporates technology to support middle grades students in cultivating coherent and meaningful understandings of proportional relationships. The learning module is a consumable, paper-based unit that was used instead of the textbook chapters emphasizing ratios and proportions found in the district’s adopted textbook series. The mathematics progression in Managing the Soccer Team begins with an investigation emphasizing the mathematical relationship of speed, time, and distance. As students progress through the learning module, they engage in a series of investigations that connect graphs of time and position to a simulation of real-world motion to develop an informal understanding of steepness of a line. By the end of the learning module, students explore slope and explain their understanding of direct variation through the algebraic equation, \( y = kx \). This is accomplished by incorporating dynamic technology tasks that provide students with the mathematical foundation for exploring proportional relationships at a developmentally appropriate level for middle grades students.

Dynamic Technology

Technology can be a powerful vehicle for mathematical sense making and reasoning (e.g., Dick & Hollebrands, 2011; NCTM, 2000). Accordingly, current college- and career-readiness standards indicate that mathematically proficient students should use technological tools when solving mathematical problems (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010), but technology infusion into the classroom setting alone is insufficient to improve teaching and learning (Dynarski et al., 2007). Roschelle, Knudsen, and Hagedus (2010) cautioned against the over-estimation of the potency of technology, positing that only a small number of teachers can successfully realize technology’s potential without substantial guidance. Furthermore, expecting that technology alone will transform a teacher’s educational practice ignores the critical classroom relationship among teachers, their students, and the mathematical content. The type of conceptual learning envisioned in current mathematics standards will not occur without addressing the teacher-learner dynamic or the relationship teachers and students have with the mathematics content. To this point, Earle (2002, p.8) declared, “Integrating technology is not about technology—it is primarily about content and effective instructional practices. Technology involves the tools with which we [teachers] deliver content and implement practices in better ways.” Consequently, it is through the theoretical lens of a unified curricular activity system (Roschelle, Knudsen, & Hagedus, 2010; Vahey, Knudsen, et al., 2013) that learning modules integrate technology and are aligned to teacher PD with the singular focus of meeting the needs of a wide variety of teachers and students during classroom instruction.

Dynamic technology (Moreno-Armella, Hagedus, & Kaput, 2008) allows for simultaneous connections among linked representations (i.e., pictorial, graphical, narrative, tabular, and algebraic representations) and provides a real-life context to engage in conceptually based mathematics (Kaput & Roschelle, 1997). An example of this representation, Figure 1, shows two runners, one in an orange uniform and one in a purple uniform, at the starting line of a 50-meter race.

To help students investigate which of the runners will win a race, the instructional routine of “predict, check,
“and explain” (Roy et al., 2016) is embedded throughout the curriculum. First, students are prompted to predict who will win the race by looking at the World. As students quickly note, because both runners are at the starting line (see Figure 1), it is difficult to determine which of the runners wins the race solely by looking at the World. As a result, the Time-Position Graph, shown in Figure 2, becomes a focal mathematical representation in helping the students make sense of this race scenario.

At this point, some students predict who will win the race based upon correct interpretations of the graph, whereas some students will make their predictions based upon incorrect interpretations of the graph. For example, some students incorrectly conclude that the runner in purple wins the race because her line (purple) on the position graph is longer, whereas other students identify the runner in orange as the winner because she finished the race in nine seconds, and the runner in purple finished it in 11 seconds. To check their predictions, the students run the digital simulations as many times as they need by pressing the play arrow while guided by task-specific questions. Contrary to solely viewing the start of the race in the World, students are able to determine the winner of the race through a task that requires them to make connections between the two representations. Simultaneously, teachers are provided the opportunity to question the students and have them explain similarities and differences between their predictions and actual results when using the time-position graph and the computer simulation of the race. As the students continue through the learning module, they build more sophisticated mathematical understandings as they explore proportional relationships using the World, graphs, tables, algebraic equations, and narratives. Eventually, the students explore slope and develop an understanding of direct variation at a developmentally appropriate level for middle grades students.

Teacher PD. An important priority for middle level education research is generating knowledge about educator development (Middle Level Education Research Special Interest Group, 2016). One challenge identified in the research is that much of the PD received by teachers does not directly relate to
their classroom experiences (Ball & Cohen, 1999). Our curricular activity system seeks to counteract this, developing PD built on the following principles identified in research: (a) an emphasis on instruction; (b) exploring classroom instruction through planning, implementation, and reflection; (c) a collective expertise; (d) clear expectations; and (e) collegiality, caring, and respect during PD (Ball & Cohen, 1999; Elmore & Burney, 1999; House, 1994; Little, 2001).

Here, we describe our efforts to examine a curricular activity system approach in mathematics classes in the middle grades. Key findings will be detailed in response to the following questions:

(1) What is the impact of a curricular activity system on middle grades teachers’ perceptions of aligned teacher PD?
(2) What is the impact of a curricular activity system on middle grades students’ understanding of ratio and proportionality?

Methods

Setting/Participants
An urban school district in the southeastern United States with more than 101,000 K–12 students was the setting of this research study. The district’s student population is 0.3% Native American, 3.8% Multiracial, 4.3% Asian, 13.9% Hispanic, 18.8% Black, and 58.7% White. The district operates 22 middle schools and is one of the top 25 most attended school districts in the United States (Sable, Plotts, & Mitchell, 2010).

All the mathematics teachers from nine middle schools in the district were invited by school-based and district-based administrators to participate in the research project. Twenty-six middle grades mathematics teachers from nine middle schools either volunteered or were nominated by school-based leadership to participate in the study. Their teaching experience ranged from 2 to 35 years, and their academic preparation included undergraduate degrees that were not in middle-level education but rather in finance, engineering, business administration, accounting, mathematics, history, health education, and special education. All but four of the teachers had earned a bachelor’s degree as their highest level of education; three of these four teachers had earned master’s degrees, and one teacher had a doctoral degree. As Ingersoll and Perda (2010) identified in their research, several teachers who participated in the study entered teaching mid-career. Eight participating teachers had careers outside of education prior to teaching mathematics in the middle grades. Their previous careers included accounting, engineering, finance, marketing, and technology. The participating teachers taught students from all three middle grades (i.e., sixth, seventh, and eighth), and a range of courses across student prior achievement and mathematical content areas including remedial mathematics courses in grades seven and eight, regular mathematics courses in grades six and seven, advanced mathematics courses in grades six and seven, Algebra IA, Algebra I, and Algebra I honors.

After consulting with the district middle grades mathematics coordinator, it was determined that the Managing the Soccer Team learning module on proportional relationships would be appropriate for 16 of the 26 middle grades mathematics teachers to implement with their students. More specifically, the unit was determined appropriate by district leadership for teachers of seventh grade and related courses (e.g., students in a sixth grade advanced course or students in an eighth grade remedial course); the other eight teachers taught other learning modules that were more closely aligned to the mathematics topics in the courses they were teaching.

To ensure that teachers could effectively use the Managing the Soccer Team learning module and technology in their unique classroom settings, during our PD sessions we emphasized the types of knowledge that research has identified as being crucial to teachers’ effectiveness in incorporating technology in the classroom (Koehler & Mishra, 2009; Mishra & Koehler, 2006; Niess, 2005). Among the foci of the PD were the knowledge of students’ understanding, thinking, and learning of proportional relationships using dynamic technology, and knowledge of instructional strategies and representations for teaching and learning proportional relationships with technology. Moreover, the foci were coordinated to the teachers’ goals for student learning as recommended (see, e.g., Penuel, Fishman, Yamaguci, & Gallagher, 2007).

These foci were explored through a two-phase PD experience during the first three months of the school year. In August prior to the beginning of the school year, the teachers participated in three consecutive full days of PD. The second phase of PD consisted of three half-day sessions after school in September, and one day-long PD session at the end of October. In total, the teachers met for 41 hours of PD to learn to teach the learning module.

During each PD session in August, the participating teachers first explored the learning module as learners.
Collaborative group norms for the sessions included “do the math” and “make bold conjectures.” The norm of “do the math” was introduced to give teachers permission to behave not as though they were learners who did not already know the correct answers but instead as students who were learning the materials. The norm of “make bold conjectures” was introduced, and bold was emphasized, so that teachers would move from making conjectures that they knew were true (e.g., when two functions are plotted on the same time-distance graph, the steeper one is faster) to conjectures that may be false but have intuitive appeal (e.g., a longer line on a position-time graph indicates faster motion than a shorter line). When such conjectures were found to be false, teachers were then asked to further analyze them to consider why they had intuitive appeal and to consider why students might make such conjectures (e.g., in the case of two lines on a time-position graph, if both represent travel for the same amount of time, the longer line will represent the faster motion). After investigating the learning module through the perspective of a learner, the focus of the PD sessions shifted so that teachers would consider “ways of representing and formulating the subject that make it comprehensible to others” (Shulman, 1986, p.9). Consequently, another PD norm, “think about your classroom,” emphasized engaging the participating teachers to begin to make instructional connections to their diverse settings and individual teaching approaches. Once established, these three norms were maintained for each of the four follow-up PD sessions held in September and October. During these sessions, the emphasis was on continuing to explore the learning module in a collaborative environment in which participants were supported and simultaneously challenged to think about their classroom implementation of the learning module. For a more detailed discussion regarding a previous iteration of PD, see Roy, Vanover, Fueyo, and Vahey (2012).

At the end of the PD sessions, the 16 teachers implemented the three-week learning module with 64 different classes, ranging from remedial middle grades mathematics to Algebra I Honors, consisting of 983 total students.

**Data Collection**

The two primary foci of the research study were participating teachers’ perceptions of the aligned teacher PD and middle grades students’ understanding proportional relationships. To investigate these research emphases, we collected the following data.

**Teachers’ perceptions of PD.** One of the major components of a curricular activity system is teacher PD, particularly PD that supports the curriculum and technology being used by teachers during their implementation of a curricular intervention. We used a web-based instrument to measure participating teachers’ perceptions regarding each PD session. The evaluation instrument included three open-ended response items and Likert-type items on a scale ranging from 1 (Not at all) to 5 (Very much) with 3 indicating neutrality. Each PD evaluation served three goals: (a) to evaluate the recently completed PD session, (b) to have information and feedback from the teachers with which to plan future PD sessions, and (c) to gather data for future analysis. Items queried the teachers to reflect upon the mathematical, technological, and pedagogical supports provided during the PD session. Because the Likert-type items on the PD evaluations were ordinal, a frequency distribution of each individual item was produced to evaluate each of the sessions. Then, each item on each of the sessions’ evaluations was analyzed to determine trends across the seven PD sessions. The open-ended items prompted the teachers to describe (a) what went well, (b) what needed improvement, and (c) any other general comments the teachers had about the PD session. The open-ended items on the PD evaluations were analyzed by coordinating a qualitative analysis of the data (Rossmann & Rallis, 2003). To do so, we first compiled all responses to each open-ended question on the evaluations. Two researchers then independently compared each response to all previous responses to identify emerging themes. The researchers then compared and synthesized the emerging themes they identified individually in order to establish a collective understanding of the themes. Finally, representative quotes were identified and used to summarize an overarching explanation of the open-ended items.

**Student pre- and post-learning module assessments.** To measure the mathematical concepts in the Managing the Soccer Team learning module, we employed a 30-item content-specific student assessment emphasizing ratios and proportional relationships that was used in prior research and that has been found to have an internal reliability of 0.86 (for a detailed description of the creation of the assessment, see Shechtman, Haertel, Roschelle, Knudsen, & Singleton, 2010). The assessment included 11 items that address procedural concepts (which we call M1 knowledge) and 19 items that emphasize in-depth relational concepts (which we call
M2 knowledge). M1 knowledge emphasizes missing-value problems by expecting students to solve \( \frac{a}{b} = \frac{c}{d} \) or \( y = kx \) in which all but one value is provided. Other M1 knowledge items require students to interpret basic graphs or tables. An example of an M1 item is shown in Figure 3.

M2 knowledge emphasizes more conceptual reasoning across two or more mathematical representations (i.e., graphical, tabular, narrative, and algebraic); an example of an M2 item is shown in Figure 4. For an item to be scored correct, the student must answer each part of the item correctly.

Immediately prior to and subsequent to the learning module being taught, the students took the 30-item...
content-specific student assessment emphasizing ratios and proportional relationships. Higher scores on the assessment indicate greater understanding of rate and proportionality concepts.

Results

This study sought to explore the implementation of a curricular activity system as an intervention in a large, diverse urban school district. Because a curricular activity system emphasizes aligned teacher PD with attention to students’ learning, the results are framed using both of these lenses.

Teachers’ Perceptions of Aligned PD

Prior to the teachers’ implementation of the learning module, the teachers participated in the first tier of PD. The teachers then participated in a second tier of PD that corresponded with them teaching the learning module. Immediately following each PD session, the teachers completed a web-based evaluation. We report here on three aspects of teachers’ perceptions about the PD: (a) that they felt prepared to use the technology, (b) that they were pedagogically prepared to teach the learning module, and (c) the general usefulness of the PD to each of them as a teacher.

Prepared to use the technology. In sum, the teachers reported that the PD sessions prepared them to teach Managing the Soccer Team using the dynamic software. More than 85% of the participants felt prepared to use technology during the three-day summer PD session, as well as sessions 5, 6, and 7, but during session 4, only 50% of the teachers felt prepared to use the technology used in the learning module. In the end, 100% of the teachers reported that they were prepared to use the technology after the last PD session (see Figure 5).

Prepared to teach the learning module. The teachers reported that the PD sessions prepared them to teach the Managing the Soccer Team unit. In all but one of the PD sessions (i.e., Day 4), at least 75% of the teachers felt prepared (i.e., reported a rank of 4 or 5) to utilize the technology embedded in the unit. By the final PD session, 100% of the teachers reported that they were prepared pedagogically to teach the unit (see Figure 6).

Furthermore, the teachers’ responses shown in Figure 6 indicated that teachers felt prepared to teach the unit during the three PD sessions prior to the school year beginning. However, this sentiment changed during the first PD session (i.e., Day 4) during the school year. The teachers perceived that they were not totally prepared (i.e., a decrease in ranks of 4 and 5 in each category). By the end of the semester, all the teachers reported feeling more prepared during the subsequent PD sessions (i.e., Days 5, 6, and 7); and 100% of the teachers indicated that they were prepared mathematically, technologically, and pedagogically (i.e., rank of 4 or 5) by the final PD session.

Generally useful. Overall, the participants perceived all the sessions were useful to them as teachers. At the end of each PD session, at least 69.6% or more of the teachers indicated the session was very useful (i.e., rank 5) to them as a teacher. Furthermore, after five PD sessions (i.e., Days 1, 2, 3, 5, 7), 100% of the teachers indicated that they found the PD sessions useful (i.e., reported a rank of 4 or 5); see Figure 7.

Figure 5 Teachers’ perceptions regarding how well PD prepared them to use technology
Three distinct themes emerged from our analysis of more than 100 responses to the open-ended items on the PD evaluations: (a) collaboration with colleagues, (b) preparation to teach the Managing the Soccer Team learning module, and (c) change in practice. We will begin our description of the teachers’ perceptions of PD with a description of their perceptions regarding collaboration and preparation to teach the learning module, and we will end with a description of change in practice.

Collaboration with colleagues. In the 12 comments grouped in this theme, the teachers noted that the collaboration among them was an important element in helping them understand the mathematics of ratios and proportional relationships. James (all names are pseudonyms) responded, “The dialogue among all of us was insightful and engaging.” Antonio stated, “The discussion was elaborate and well-rounded with many people sharing ideas,” thus corroborating James’ sentiment. Daniela concluded that “[dialogue was key to] building a community of learners and making sure everyone was at ease.” To these points, it became evident to the teachers that their understanding was deepened by exploring the mathematics they would be teaching as a group. As Julia concluded, “Collaboration with colleagues helped to build a deeper understanding of the content and making connections between speed (rate) and slope and graphical representations.”

Preparation to teach the learning module. Because this would be the first time for many participants to integrate dynamic technology as a mathematics...
learning tool into the classroom setting, technology was also a focus of learning during PD. Twenty-two of the teachers’ responses identified how the hands-on experience with technology prepared them to teach the unit. During the first three PD sessions, it was clear from their responses that some of the teachers were uneasy regarding the technology implementation. For example, Rosa said, “I would like to spend more time using the software before presenting it to my students.” Fred also shared her sentiment, stating, “I am still concerned with the tech. I know this will improve with practice (I hope).” As such, the research team continued PD efforts to alleviate these technology concerns during the afterschool sessions (i.e., PD sessions 4–7) prior to the implementation of the unit. As Rosa declared during a follow-up session, “The thing that went especially well was that we had access to play with the [software] program and get familiar with it ourselves.” In addition, James remarked, “I am looking forward to using this software in my classroom.”

The PD norm emphasizing classroom implementation also allowed the teachers to provide their perceptions regarding student learning. As James remarked, “Technology allows me to get real time feedback on student understanding.” This comment is in contrast to Antonio’s concern regarding the classroom implementation of Managing the Soccer Team. He perceived that although “the PD sessions provided me with the information needed to present the information [mathematics content], this way of learning unfortunately is foreign to the students which makes them think more than they like on some days.” Consequently, the focus of the subsequent PD sessions held during the school year emphasized each teacher’s unique approach to teaching the unit. As Stephan noted, “Working in the actual student section of the workbook helps us to develop understanding of the lesson and how we need to use it with our students.” Wendi stated, “We could see our students from our classes. We could actually begin to plan in our minds what we need to do.” As Dana concluded, “The PD sessions really gave me a good idea of what I should be doing and how things should look. I feel prepared to have the in-depth conversations about math with my students.”

**Change in practice.** After comparing the teachers’ responses to the open-ended questions, one final category emerged; the 30 responses in this category highlighted the teachers’ changes in practice. For example, James declared, “I am thrilled to be part of this [project]. I have no doubt that it will provide me with a learning opportunity to become a better educator, and my students the opportunity to be more enthusiastic about mathematics and mathematics learning.” Daniela resonated the sentiment of change when she wrote, “It [learning module] allows students to explore and discover concepts when used properly. My role is more of a facilitator instead of the almighty keeper of the math knowledge.” Ultimately, the comprehensive PD sessions resulted in the teachers identifying important changes in both their mathematical practices and their approaches to students’ learning mathematics. As members of a PD learning community, the teachers collaborated and engaged in rich dialogue with colleagues regarding a common purpose for incorporating technology into their mathematics teaching. They explored ways to teach pivotal mathematics concepts at the appropriate level, and as a result their pedagogical content knowledge deepened. Further, the PD allowed them to collaborate on supporting their middle grades students’ understanding of rate and proportionality with and without the use of dynamic technology, and as a result, provided the teachers with the impetus for changing their practice.

**Student Achievement**

To determine statistical difference, we conducted a paired-samples t test (see Table 1) to compare the 983 middle grades students’ understanding of ratio and proportional relationships prior to and subsequent to the implementation of the three-week learning module that integrated dynamic technology.

There was a statistically significant difference between the pre-learning module (\(\bar{x} = 13.14, s = 5.65\)) and the post-learning module assessment scores (\(\bar{x} = 19.24,\))

| Pair 1 | Mean | N  | Std. deviation | Std. error mean |
|--------|------|----|----------------|-----------------|
| Pre    | 13.14| 983| 5.65           | .18             |
| Post   | 19.24| 983| 6.15           | .19             |
The mean gain on the assessment was 6.10 questions (see Table 2). The effect size for this analysis, $d = 1.03$, indicates a large effect size. These results suggest that the learning module does have an impact on middle grades students’ understanding of ratios and proportional relationships.

Because the assessment used in this study was developed to measure a student’s understanding of both foundational rate and proportionality and more conceptual concepts, we were also interested in describing how the students performed on the two problem types, M1 (i.e., procedural content) and M2 (i.e., conceptual content). The mean gain was statistically significant for each problem type (see Table 3). The mean gain for M1 knowledge was a 0.83 question increase, and the mean gain for M2 knowledge was a 5.27 question increase from pre- to post-learning module administrations of the assessment.

Because the district leadership was concerned with the participating teachers being compared to their colleagues in the district, the mean gain scores from pre- to posttest administrations were compared to mean gain scores in previous studies (i.e., randomized-control experiment in Texas and replication study in Florida) instead of being compared to a control group. There was no statistical difference in students’ pretest scores in the current study and the previous studies, meaning that the students’ pretest scores in the studies were comparable. After the learning module was implemented by all the teachers, the mean gain scores were consistent with a randomized control experiment in Texas and a replication study in Florida (see Figure 8).

As a measure of student learning at grades six, seven, and eight, the research team explored the mean gain scores of the pre- to post-learning modules’ administrations of the assessment at each grade level. The learning module was effective in all three grades of middle grades mathematics. There was a significant difference mean gain in the pre-learning module and post-learning module test scores at all three grade levels. In addition, there was a large effect size at each grade level (see Table 4).

In summary, these results indicate that students at all grade levels learn the mathematics of ratios and proportional relationships explored in the Managing the Soccer learning module regardless of the unique teaching settings or pedagogical approaches of their teachers. Both the sixth- and seventh-grade students’ mean gain scores increased more than six questions

### Table 2
Mean Gain on Assessment

| Paired differences | Mean | Std. deviation | Std. error mean | 99.5% confidence interval of the difference | $T$ | $df$ | Sig.(2-tailed) |
|--------------------|------|----------------|-----------------|--------------------------------------------|-----|-----|---------------|
| Pair 1 Pre – Post  | −6.10| 4.29           | .14             | −6.49 to −5.72                             | −44.53| 982 | <.001         |

### Table 3
Mean Gain on Problem Types

| Paired differences | Mean | Std. deviation | Std. error mean | 99.5% confidence interval of the difference | $T$ | $df$ | Sig.(2-tailed) |
|--------------------|------|----------------|-----------------|--------------------------------------------|-----|-----|---------------|
| Pair 2 M1Pre – M1Post | −0.83| 1.70           | .05             | −0.98 to −0.68                             | −15.29| 982 | <.001         |
| Pair 3 M2Pre – M2Post | −5.27| 3.74           | .12             | −5.61 to −4.94                             | −44.17| 982 | <.001         |
from pre- to post-learning module administrations, and those in eighth grade increased more than four questions from pre- to post-learning module administrations. The results also suggest that student learning is impacted when teachers participate in focused PD that prepares them to implement a learning module that integrates dynamic technology.

**Discussion**

For teachers in the SunBay Mathematics Project, the focused PD resulted in important changes in both their mathematical practices and their approaches to student learning of mathematics. Although one limitation to the study is the fact that many of the teachers volunteered to participate, as members of a PD community committed to integrating technology into their individual mathematics teaching contexts, the teachers engaged in rich dialogue about their own learning of teaching and mathematics. As a group engaged in learning to teach, the teachers and the research partners created a safe environment in which all members could be learners. We found this type of supportive PD learning environment particularly valuable with the wide range of teachers in the group.

Having the opportunity to engage with peer teachers who were also vested in the successful implementation of the learning module in their own classrooms was reassuring to all teachers in the project. In the end, the important role of the teacher as learner, particularly in a supported environment, is crucial if we are to achieve the goals of mathematical standards that teachers are supposed to teach.

There was one finding that we did not anticipate. Upon analysis of our summer PD, we did not expect the teachers to describe their uneasiness to teach the unit during the first PD session of the school year (i.e., Day 4), especially because the teachers indicated they were prepared to teach the learning module at the end of each summer PD session. The teachers expressed their unease in pedagogical preparation and ability to use the technology in the learning module (see Figures 5 and 6). One reason we believe this shift in perceptions occurred was the participants’ realization regarding teaching a cognitively challenging learning module to their students was closer to implementation than during the summer sessions, and as a result it prompted concern regarding their preparation to teach the learning module. Because of continued

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**Table 4**

| Grade Level Gain Scores |
|-------------------------|
| Number of students | Pretest | Posttest | Mean gain | M1 gain | M2 gain | D |
| 6th grade       | 240     | 12.51    | 18.66    | 6.15    | 0.96    | 5.19 | 1.54 |
| 7th grade       | 667     | 13.85    | 20.13    | 6.29    | 0.74    | 5.55 | 1.44 |
| 8th grade       | 76      | 8.93     | 13.29    | 4.36    | 1.24    | 3.12 | 1.01 |

Note. All mean gain scores were significant at p < .001.
professional support, all the teachers reported feeling more prepared during the subsequent PD sessions (i.e., Days 5, 6, and 7), and 100% of the teachers indicated that they were prepared mathematically, technologically, and pedagogically (i.e., rank of 4 or 5) by the final PD session.

The findings of this study are consistent with elements of effective Phase 1 PD programs outlined in Borko (2004). By having the participating teachers explore the mathematics in the learning module as learners prior to teaching it, there was “an explicit focus on subject matter” by the research partners as a means to provide experiences that “engage teachers as learners” (Borko, 2004 p. 5). This allowed the teachers to leverage their mathematical knowledge as a means to explore their teaching practices, provide insight into students’ mathematical thinking, and as way to transform their teaching.

Conclusion

Current college- and career-readiness standards (e.g., Common Core State Standards for Mathematics) provide both the curricular foci and the mathematical practices for the next generation of students in K–12 education. As teachers implement these standards, they may not have the ability to create their own curricular materials that address the mathematics rigor in standards. In addition, the PD they experience may not address their needs (Ball & Cohen, 1999). The implementation of a comprehensive curricular activity system does offer a promising means to address the learning and teaching of key mathematics through coherent PD. The results of this study provide evidence that a curricular activity system in which PD is coherently aligned to technology-embedded curriculum impacts the teaching of pivotal mathematics content. Moreover, it provides the evidence that middle grades students develop a deep understanding of crucial mathematics when their teachers use a learning module that integrates technology and is supported by coherent PD.

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