Interpolating Equation of Precision Platinum Resistance Thermometers in the Temperature Range between -80°C and 300 °C

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Abstract. This paper compares two interpolation equations for the calibration of precision platinum resistance thermometers (PPRTs) in the temperature range between -80°C and 300°C. They are the deviation function specified in the International Temperature Scale of 1990 (ITS-90), and the quadratic Callendar–van Dusen (CVD) equation. It was found that when the calibration range was between -80°C and 300 °C, ITS-90 deviation function resulted in a measurement error-of-fit of ±23mK, and the CVD equation resulted in ±24mK. In the temperature range of (-80~0)°C, ITS-90 deviation function showed distinctly better performance than the CVD equation. When the temperature range was between 0°C and 100°C, the difference among the two equations was insignificant. In the temperature range of (100~300)°C, the measurement error-of-fit of ITS-90 deviation function increased with temperature, and the maximum error-of fit under the CVD equation appeared at 200°C. If the temperature range is wider, the ITS-90 deviation function had better calibration effect. While the CVD equation worked better in a narrow temperature range.

1. Introduction

The precision platinum resistance thermometers (PPRTs)\(^1\) are used for precise temperature measurement by exploiting the change of the resistance \(R\) with the temperature \(t\). Its sensitive element is a kind of high purity platinum wire. The temperature measuring accuracy of PPRTs is between the standard platinum resistance thermometers (SPRTs) and the industrial platinum resistance thermometers (IPRTs). And anti-interference ability of PPRTs is better than the standard platinum resistance thermometer, so it is more widely used in the pharmaceutical production, on-site detection and measurement industry. At the same time, with the Minamata convention taking effect on August 16, 2017, the product that contains mercury was completely banned from 2020, which resulted in a sharp decrease in the number of standard mercury thermometers. In consideration of the accuracy, measurement range and stability, PPRTs were fully qualified and replace the standard mercury thermometer, so many calibration institutions are now using precision platinum resistance thermometer as the standard for the corresponding detection work\(^2\).

However, the method of converting the resistance value of PPRTs to temperature has not been unified. There are two widely used methods: one is the International Temperature Scale of 1990 (ITS-90)\(^3\), and the other is the Callendar-van Dusen (CVD) equation\(^4\). The effects of these two interpolation methods on the measurement error-of-fit of PPRTs will be studied.
2. Background

2.1. The precision platinum resistance thermometers

PPRTs usually use the platinum wire as sensitive element, and the resistance of the platinum wire is 100 Ω at the triple point of water temperature (0.01℃) and its resistance ratio at 100 ℃ $W_{100℃}$ is more than 1.39250. There are many kinds of PPRTs on the market with different models. The longest one is 500mm, and the shortest one is 100mm. In diameter, there are 6.4mm, 6mm, 4.8mm, 3.2mm, etc. The upper limit of temperature measurement is 300℃, 400℃, 500℃, 600℃ and other specifications.

As the temperature sensing elements of PPRTs are close to that of SPRTs, the stress caused by mechanical vibration and the temperature drift caused by platinum wire oxidation can be eliminated by annealing\(^5\). According to the research \(^6\), the annealing effect of PRTs with the upper limit of no more than 450℃ is small, which not only can not eliminate the stress, but also worsen the oxidation degree of platinum wire. Therefore, it is not necessary to conduct annealing for the thermometer with the upper limit of less than 450℃. However, for thermometers whose upper limit temperature is higher than 450℃, annealing should be carried out before calibration. The annealing temperature can refer to the requirements in the thermometer operation manual. The annealing time is generally 2 hours.

2.2. The interpolation methods

2.2.1. The International Temperature Scale of 1990

This method is based on that the sensing elements of PRTs are made of high purity platinum wire. The resistance ratios $W_t$ between the PRTs that meet the requirements are very close, and it is expected that the difference $\Delta W(t)$ between the resistance ratios of the two thermometers can be expressed by a simple polynomial. $\Delta W(t)$ is usually called the deviation function\(^7\), which is related to the temperature range used. The deviation function in the range of (0 ~ 660.323) ℃ is shown below:

$$\Delta W(t) = W_i - W_j(t) = a[W_i - 1] + b[W_i - 1]^2 + c[W_i - 1]^3$$  \(1\)

In the formula, $a$, $b$ and $c$ are the coefficients of the thermometer, which are related to the fixed point chosen when the thermometer is graded. $W_j(t)$\(^8\) is a reference function in the temperature range of (0 ~ 660.323) ℃, which is defined as below:

$$t / ℃ = D_0 + \sum_{i=1}^{9} D_i ([W_i(t) - 2.64]/1.64)^i$$  \(2\)

Where $t$ is the temperature value, and the constants $D_0$, $D_i$ and $W_i(t)$ are listed in JJG160-2007 standard platinum resistance thermometer verification regulation. The deviation function in the range of (-189.3442 ~ 0) ℃ is shown below:

$$\Delta W(t) = W_i - W_j(T_90) = a[W_i - 1] + b[W_i - 1] \ln W_i$$  \(3\)

Where $a$ and $b$ are obtained from the measured values of the thermometer at water triple point, mercury triple point and argon triple point. $W_j(T_90)$ is the reference function in the temperature range of (-189.3442 ~ 0) ℃, which is defined as below:

$$T_90 / 273.16K = B_0 + \sum_{i=1}^{15} B_i [W_j(T_90)]^i/0.35$$  \(4\)

Where $T_90$ is the Kelvin temperature value, and the constants $B_0$, $B_i$ and $W_j(T_90)$ are listed in JJG160-2007 standard platinum resistance thermometer verification regulation.
2.2.2. The Callendar-van Dusen (CVD) equation

According to the CVD equation, the functional relationship between resistance and temperature of PRTs is as follows:

\[ R_t = R_0 [1 + At + Bt^2 + C(t - 100)t^3] \]  

(5)

The temperature \( t \) is between -200°C and 850°C, and when \( t \) is greater than 0°C, \( C \) is equal to 0. \( A \) (°C⁻¹), \( B \) (°C⁻²) and \( C \) (°C⁻³) are the temperature coefficients of PRTs[9].

3. The experimental equipment and methods

3.1. The experimental equipment

In this paper, two FLUKE models 5615-9 (s/n: 1003027 and s/n: 1003021) were selected as research objects, and the temperature range was (-200 ~ 420) °C. According to the temperature range of the thermometers, two temperature zones (-189.3442 ~ 0) °C and (0 ~ 419.527) °C were selected for indexing[10]. Therefore, it is necessary to use the fixed points defined in these temperature zones, which are argon triple point (-189.3442°C), mercury triple point (-38.8344°C), water triple point (0.01°C), tin freezing point (231.928°C) and zinc freezing point (419.527°C). Liquid nitrogen comparison method is used instead of argon triple point, so liquid nitrogen comparison groove is needed.

FLUKE model 5628 was used as the standard platinum resistance thermometer (s/n: 2151). FLUKE model 1595 was used to measure the temperature of the bridge, which was 2.0×10⁻⁸ at the uncertainty. The standard resistance choices Tinsley model 5685 (100Ω), which of stability is ±5×10⁻⁶.

Since two positive temperature resistance values, one negative temperature resistance value and 0°C resistance value are needed to calculate the CVD equation, three kinds of thermostatic tanks and freezers need to be used, among which the thermostatic tanks are thermostatic oil tank, thermostatic water tank and cryostat tank[11]. Their detailed parameters are shown in the following table.

| Device name          | Manufacturer                | Model  | Temperature fluctuation | Temperature uniformity |
|----------------------|-----------------------------|--------|-------------------------|------------------------|
| thermostatic oil tank| Huzhou Weili instrument factory | CJTH-300A | ±0.005°C/30min          | 0.006°C                |
| thermostatic water tank| Huzhou Weili instrument factory | CJTH-95A | ±0.005°C/30min          | 0.005°C                |
| cryostat tank        | Huzhou Weili instrument factory | CJTH-80A | ±0.006°C/30min; 0.007°C | 0.007°C                |

3.2. The experimental methods

According to JJG160-2007, the water three-phase point was frozen, and the frozen water three-phase point was stored in a constant temperature tank at 0.006°C. After that, the water three-phase point was placed for 24 hours until its stress release was complete. The way to realize the temperature plateau of zinc freezing point is to set the temperature of the zinc fixing point device to be 2°C higher than that of the freezing point and keep it there for 6 hours. After all the zinc in the bottle is melted, the temperature will be lowered at the rate of 0.1°C/1min. Meanwhile, a thermometer will be used to detect the internal temperature of the fixed point bottle. When the temperature rises, the induction rod is inserted into the fixed-point bottle and kept for 1min before it is taken out. The temperature of the fixed-point device is set 0.8°C lower than that of the fixed-point thermometer, and the zn-fixed-point temperature plateau is realized about 1 hour later. Tin fixed point temperature plateau can be achieved in the same way. Mercury triple point temperature plateau does not need induction, and other methods are similar. After the fixed point temperature plateau is made, the measurement shall be conducted in the order from high temperature to low temperature in the sub-temperature zone: \( ZnR \), \( tpR \), \( SnR \), \( tpR \).
$R_H$, $R_p$, $R_{dr}$, $R_y$. Using the measurement results, the ITS-90 temperature coefficients of the two platinum resistors were calculated according to formula (1), as shown in the following table.

### Table 2. The ITS-90 temperature coefficient of two platinum resistances

| No.     | $R_p$ (Ω) | $a_8$   | $b_8$   | $a_4$   | $b_4$   |
|---------|-----------|---------|---------|---------|---------|
| 1003027 | 99.9789   | -0.000367 | -0.000013 | -0.000333 | -0.000018 |
| 1003021 | 100.0021  | -0.000351 | -0.000044 | -0.000315 | 0.000010  |

The resistance values $R_y$ of two platinum resistance thermometers were measured by comparison in the thermostatic tank and the freezers at 300°C, 100°C, -196°C and 0°C. In order to improve the uniformity of the thermostatic tank, a copper isothermal block was placed in the thermostatic tank. During the measurement, the block was checked and inserted into the standard isothermal block for measurement. The temperature coefficients of the CVD equation are calculated according to the measured value and formula (5), as shown in the following table.

### Table 3. The CVD temperature coefficient of two platinum resistances

| No.     | $R_y$ (Ω) | $A$ (°C⁻¹)    | $B$ (°C⁻²)    | $C$ (°C⁻⁴)   |
|---------|-----------|---------------|---------------|--------------|
| 1003027 | 99.9782   | 3.9859e⁻³     | -5.9208e⁻⁷   | -3.9413e⁻¹² |
| 1003021 | 100.0014  | 3.9856e⁻³     | -5.9157e⁻⁷   | -3.9621e⁻¹² |

In the range of (-80 ~ 300) °C and at an interval of 10°C, the temperature of the two thermometers under the temperature coefficients of the two groups was measured, and the error between the two groups of temperature and the standard was analyzed.

### 4. Experimental results and analysis

#### 4.1. The influence of ITS-90 deviation function on measurement error-of-fit

Under the ITS-90 deviation function, the measurement errors of the two thermometers can be obtained according to the experimental results, as shown in figure 1. It can be concluded from the figure 1 that the measurement error of the two thermometers within the range of (-80 ~ 300) °C is within ±23mK, among which the error within the range of (-40 ~ 100) °C is within ±10mK. The measurement error of the two thermometers was in the range of -80 ~ 300 °C, and the trend was basically the same, which indicated that the main reason affecting the error trend was caused by interpolation. The temperature is higher in the range of (100 ~ 300) °C, and the error is larger. The error of s/n 1003027 increases from 100°C 7mK to 280°C 23mK, and the error of s/n 1003021 increases from 100°C 2mK to 280°C 21mK. Under the negative temperature, the trend is not obvious that the temperature is lower, the error is larger. There was no significant trend in the error of the two thermometers within the range of (0 ~ 100) °C.

![Figure 1. The measurement error of the ITS-90 deviation function.](image)
4.2. The influence of CVD equation on measurement error-of-fit
Under the CVD equation, the measurement error of two thermometers is shown in figure 2. The measurement error trends of the two thermometers were basically the same, and presented sinusoidal distribution. In the range of (-80 ~ 300) °C, the measurement error is within ±24mK. And in the range of (0 ~ 100) °C, the measurement error is within ±5mK. The measurement error calculated by CVD equation is the smallest at the measurement point and the largest between two measurement points. For example, the measurement error of s/n 2003027 at 0℃ is -1mK, and the measurement error at 100℃ is -1mK, while the measurement error at 50℃ is 5mK.

![Figure 2. The measurement error of the CVD equation.](image)

4.3. Comparison of the two methods
Figure 3 shows the measurement error comparison diagram of thermometer s/n 1003021 with the ITS-90 deviation function and the CVD equation, and figure 4 shows the measurement error comparison diagram of thermometer s/n 1003027. It can be seen from the two figures that the measurement error of the ITS-90 deviation function is smaller within the range of (-80 ~ 0) °C than the CVD equation, especially the measurement error of the two thermometers is within ±8mK within the range of (-40 ~ 0) °C. The reason why the measurement error of the ITS-90 deviation function is smaller in this temperature range is that the measurement uncertainty of water triple point and mercury triple point is within 4mK, which improves the accuracy of temperature calculation by interpolation method. In the range of (0 ~ 100) °C, the measurement error of the two methods is very small. The measurement error of the ITS-90 deviation function is within ±8mK, and the measurement error of the CVD equation is within ±5mK. In the range of (100 ~ 300) °C, the measurement error of the two methods is the same large, the measurement error of the ITS-90 deviation function is within ±23mK, and the measurement error of the CVD equation is within ±20mK.
5. Conclusion

The influence of the measurement temperature calculated by the ITS-90 deviation function and the CVD equation on the accuracy of precision platinum resistance thermometers was compared. The experimental results show that the measurement error of the two thermometers using the ITS-90 deviation function is within ±23mK within the range of (-80 ~ 300) °C, and the measurement error increases with the increase of the temperature within the range of (100 ~ 300) °C. And the CVD equation was used to embed two thermometers within the range of (-80 ~ 300) °C, and the measurement error was within ±24mK, and the measurement error was sinusoidal. The measurement error was within ±5mK at the measurement point calculated by CVD interpolation method. The measurement error of the intermediate value between the two measurement points was relatively large, with the largest being 24mK. The ITS-90 deviation function is recommended for temperature calibration in the range of (-80 ~ 0) °C. And in the range of (0 ~ 100) °C, there is little difference in the calibration error between the two interpolation methods. If the temperature range is wider, the ITS-90 deviation function has better calibration effect in the range of (100 ~ 300) °C. And CVD equation works better in a narrow temperature range in the same range. Because the CVD equation can arbitrarily select the measurement temperature point to modify the temperature coefficient within a narrow temperature range, so the measurement uncertainty is optimized. But the ITS-90 deviation function can only use the defined fixed point to correct the thermometer coefficient.
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