Flipped SU(5) Predicts $\delta T/T$

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Abstract

We discuss hybrid inflation in supersymmetric flipped SU(5) model such that the cosmic microwave anisotropy $\delta T/T$ is essentially proportional to $(M/M_P)^2$, where $M$ denotes the symmetry breaking scale and $M_P (= 2.4 \times 10^{18}$ GeV) is the reduced Planck mass. The magnitude of $M$ determined from $\delta T/T$ measurements can be consistent with the value inferred from the evolution of SU(3) and SU(2) gauge couplings. In other words, one could state that flipped SU(5) predicts (more precisely ‘postdicts’) $\delta T/T$. The scalar spectral index $n_s = 0.993 \pm 0.007$, the scalar to tensor ratio satisfies $r \lesssim 10^{-6}$, while $dn_s/d\ln k \lesssim 4 \times 10^{-4}$.

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In a class of realistic supersymmetric (SUSY) models, inflation is associated with the breaking of some gauge symmetry $G$, such that the cosmic microwave anisotropy is essentially proportional to $(M/M_P)^2$, where $M \sim M_{\text{GUT}} = 2 - 3 \times 10^{16}$ GeV denotes the symmetry breaking scale $[1]$. The simplest example of $G$ is provided by $U(1)_{B-L}$, and more complicated examples based on $SU(5)$ $[2]$ and $SO(10)$ $[3]$ have also been presented. The Higgs sector in these grand unified models is typically rather complicated, so that strictly speaking, the scale $g M$ cannot be identified with the gauge coupling unification scale (here $g$ denotes the gauge coupling associated with $G$). For instance, in the $SO(10)$ example inflation is associated with the breaking of $U(1)_{B-L}$ rather than its $SU(5)$ subgroup. In this letter, we hope to overcome this hurdle by identifying $G$ with $SU(5) \times U(1)_X$, the so-called flipped $SU(5)$ model $[4]$. This model is known to possess several advantages over standard grand unified models such as $SU(5)$ and $SO(10)$ itself that have often been discussed in the literature. A particularly compelling case is provided by the ease with which the doublet-triplet (D-T) splitting can be achieved in models based on $G$. Another potential advantage is the absence of topological defects especially monopoles that could create cosmological difficulties.

In this letter, we wish to highlight yet another advantage of models based on $G$, namely the ease with which a predictive hybrid inflation scenario $[1]$ can be realized which is consistent with D-T splitting and works within a minimal Higgs framework. This is in contrast with recent attempts to construct analogous models based on $SU(5)$ $[2]$ and $SO(10)$ $[3]$, which turn out to require non-minimal Higgs sectors including gauge singlet fields. Although the symmetry breaking scale determined from $\delta T/T$ in the latter case turns out to be comparable to the scale $M_{\text{GUT}}$ determined from the evolutions of the three low energy gauge couplings, an identification of the two scales is not quite possible, partly arising from the fact that there are extra Higgs fields, and also the fact that one has to resort to ‘shifted’ hybrid inflation $[5]$ to avoid the monopole problem. These two issues it appears can be nicely evaded in the
flipped model, so that one could argue that $\delta T/T$ is predicted and turns out to be in excellent agreement with the observations [3]. This is the first model of inflation we are aware of in which this claim can be made. Other testable predictions include $n_s = 0.993 \pm 0.007$, $dn_s/d\ln k < 4 \times 10^{-4}$, and the scalar to tensor ratio $r \lesssim 10^{-6}$. A $U(1)_R$ symmetry plays an essential role in the construction of this predictive inflationary scenario [1]. We will find that its presence implies a “double seesaw” mechanism for realizing suitable masses both for ‘right’ handed and the light neutrinos. Inflation is followed by (non-thermal) leptogenesis [8] with the reheat temperature consistent with the gravitino constraint [9].

Flipped $SU(5) (= SU(5) \times U(1)_X)$ is a maximal subgroup of $SO(10)$, and contains sixteen chiral superfields per family:

$$10_1 = \begin{pmatrix} d^c & Q \\ \nu^c & \end{pmatrix}, \quad \mathbf{5}_{-3} = \begin{pmatrix} u^c \\ L \end{pmatrix}, \quad 1_5 = e^c. \quad (1)$$

Here the subscript refers to the $U(1)_X$ charge in the unit of $\frac{1}{\sqrt{40}}$. The MSSM hypercharge is given by a linear combination of a diagonal $SU(5)$ generator and $U(1)_X$ charge operator:

$$\frac{1}{2} Y = -\frac{1}{5} Z + \frac{1}{5} X, \quad (2)$$

where $Z = \text{diag.}(-1/3, -1/3, -1/3, 1/2, 1/2)$ is the MSSM hypercharge operator in the standard $SU(5)$ model. Comparison with standard $SU(5)$ reveals the interchanges,

$$u^c \leftrightarrow d^c, \quad e^c \leftrightarrow \nu^c. \quad (3)$$

Since $\nu^c$ has replaced $e^c$ in the 10-plet, the latter now belongs to the $SU(5)$ singlet representation. The MSSM electroweak Higgs doublets reside in two five dimensional representations as follows:

$$5_{-2} = \begin{pmatrix} D^c \\ H_d \end{pmatrix}, \quad \mathbf{5}_2 = \begin{pmatrix} D^c \\ H_u \end{pmatrix}. \quad (4)$$

\[3\]We normalize $SU(5)$ and $SO(10)$ generators such that $Y_{SU(5)}^2 = \frac{1}{2}$, and $Y_{SO(10)}^2 = 1$, which is consistent with the $U(1)_X$ charge normalization.
Comparing to the standard $SU(5)$, $H_u$ and $H_d$ are replaced each other;

$$H_u \longleftrightarrow H_d .$$

(5)

The breaking of $SU(5) \times U(1)_X$ to the MSSM gauge group is achieved by providing superlarge VEVs to Higgs of 10 dimensional representations, namely $10_H$ and $\overline{10}_H$ along the $\nu^c$, $\overline{\nu}^c$ directions. We will provide a very simple superpotential shortly showing how this is achieved. But first let us briefly recall how the D-T splitting problem is elegantly solved in this framework. Consider the superpotential couplings,

$$10_H 10_H 5_{-2} \quad \text{and} \quad \overline{10}_H \overline{10}_H \overline{5}_2 .$$

(6)

With superlarge VEVs of $10_H$ and $\overline{10}_H$ along the $\nu^c$, $\overline{\nu}^c$ directions respectively, we see that $\overline{\nu}^c$ and $D^c$ in the $5_{-2}$ and $\overline{5}_2$ pair up to be superheavy with their corresponding partners, $d^c_H$ and $\overline{d}^c_H$ from $10_H$ and $\overline{10}_H$. [$Q_H$ and $\overline{Q}_H$ contained in $10_H$ and $\overline{10}_H$ are, on the other hand, absorbed by the gauge sector, when flipped $SU(5)$ is broken to the MSSM gauge group.] Thus, the electroweak Higgs doublets remain unpaired as desired, as long as a bare mass term $5_{-2} \overline{5}_2$ with coefficient of order $M_{GUT}$, permitted by the gauge symmetry can be avoided. Indeed, if this coefficient can be of order $M_W$ rather than $M_{GUT}$, we would achieve two worthwhile goals. Namely, the MSSM $\mu$ problem would be resolved and dimension five nucleon decay would be essentially eliminated. The need for some additional symmetry is also mandated by our desire to implement a predictive inflationary scenario along the lines discussed in earlier papers [1, 2, 3, 5, 7]. We have found that a $U(1)_R$ symmetry is particularly potent in constraining the inflationary superpotential and will therefore exploit it here.

Disregarding the pure right handed neutrino sector for the moment, the superpotential responsible for breaking the $SU(5) \times U(1)_X$ gauge symmetry, resolving the D-T splitting problem, and generating Dirac mass terms for the charged fermions and neutrinos is as follows:
\[ W = \kappa S \left[ 10_H \overline{10}_H - M^2 \right] + \lambda_1 10_H 10_H f_h + \lambda_2 \overline{10}_H \overline{10}_H \overline{f}_h \]
\[ + y_{ij}^{(d)} 10,10,5_h + y_{ij}^{(u,\nu)} 10,\overline{5}_j \overline{f}_h + y_{ij}^{(e)} 1,\overline{5}_j \overline{5}_h. \] (7)

The quantum numbers of the superfields appearing in Eq. (7) are listed in Table I.

| \( S \) | \( \Sigma \) | \( 10_H \) | \( \overline{10}_H \) | \( 5_h \) | \( \overline{5}_h \) | \( 10_i \) | \( \overline{5}_i \) | \( 1_i \) |
|---|---|---|---|---|---|---|---|---|
| \( X \) | 0 | 0 | 1 | -1 | -2 | 2 | -3 | 5 |
| \( R \) | 1 | 2 | 0 | 0 | 1 | 1 | 0 | 0 |
| \( Z_2 \) | + | + | + | + | + | + | - | - | - |

Table I

The \( U(1)_R \) symmetry eliminates terms such as \( S^2 \) and \( S^3 \) from the superpotential, which yields a predictive inflationary scenario \[\text{II}\]. Higher dimensional baryon number violating operators such as \( 10_i 10_j 10_k \langle S \rangle / M^2 \), \( 10_i \overline{5}_j \overline{5}_k \langle S \rangle / M^2 \), etc. are heavily suppressed as a consequence of \( U(1)_R \). Thus, we expect proton decay to proceed via dimension six operators mediated by the superheavy gauge bosons. The dominant decay mode is \( p \to e^+ / \mu^+ , \pi^0 \) and the estimated lifetime is of order \( 10^{36} \) yrs \[\text{III}\].

We note that the ‘matter’ superfields (and \( 10_H \)) are neutral under \( U(1)_R \), so that the \( Z_2 \) subgroup of the latter does not play the role of ‘matter’ parity. An additional \( Z_2 \) symmetry (‘matter’ parity) has been introduced to avoid undesirable couplings such as \( 10_H 10_i 5_h, 10_H \overline{5}_i \overline{5}_h, 10_H 10_i \overline{5}_j S, 10_H \overline{5}_i \overline{5}_j 1_k S, \) etc. \[4\] This \( Z_2 \) ensures that the LSP is absolutely stable and consequently a desirable candidate for CDM.

The superpotential in Eq. (7) and the “D-term” potential, in the global SUSY limit, possesses a ground state in which the scalar components (labelled by the same notation as the corresponding superfield) acquire the following VEVs:

\[ |\langle 10_H \rangle| = |\langle \overline{10}_H \rangle| = M, \quad \text{and} \quad \langle S \rangle = 0. \] (8)

\[4\] 10_H 10_i 5_h and 10_H \overline{5}_i \overline{5}_h (\supset \langle \nu_L^c \rangle d_i \nu^c L_i H_u) induce superheavy mass terms of \( d_i^c \), \( L_i \), and \( H_u \). 10_H 10_i \overline{5}_k S and 10_H \overline{5}_i \overline{5}_j 1_k S (\supset \langle \nu_L^c \rangle Q_i L_j d_k^c \langle S \rangle / M^2; \langle \nu_L^c \rangle d_i^c \nu^c \langle S \rangle / M^2; \langle \nu_L^c \rangle L_i L_j e_k^c \langle S \rangle / M^2) lead to proton as well as LSP decays.
Thus, the gauge symmetry $SU(5) \times U(1)_X$ is broken at the scale $M$, while SUSY remains unbroken. In a $N = 1$ supergravity framework, after including the soft SUSY breaking terms, the $S$ field acquires a VEV proportional to the gravitino mass $m_{3/2}$. As explained above, unwanted triplets in $10$-plet and $5$-plet Higgs become superheavy by the $\lambda$ couplings in Eq. (7). From the $y^{(d)}_{ij}$ and $y^{(e)}_{ij}$ terms in Eq. (4), d-type quarks and charged leptons acquire masses after electroweak symmetry broken. $y^{(u,\nu)}_{ij}$ term provides u-type quarks and neutrino with (Dirac) masses. Since $u^c_i$ and $L_i$ are contained in a single multiplet $\mathcal{F}_i$, the mass matrices for the u-type quarks and Dirac neutrinos are related in flipped $SU(5)$:

$$M^u_{ij} = M^\nu_{ji} = y^{(u,\nu)}_{ij} \langle H_u \rangle .$$

We point out here that $U(1)_R$ forbids the bare mass term $5_h \mathcal{F}_h$, so that the electroweak Higgs doublets do not acquire superheavy masses. To resolve the MSSM $\mu$ problem we invoke, following [12], the following term in the Kähler potential:

$$K \supset y_\mu \frac{\Sigma}{M_P} 5_h \mathcal{F}_h + h.c. .$$

Intermediate scale SUSY breaking triggered by the hidden sector superfield $\Sigma$ via $\langle F_\Sigma \rangle \sim m_{3/2} M_P$ yields the MSSM $\mu$ term, $\mu \equiv y_\mu \frac{F_\mu}{M_P} \sim m_{3/2}$. The quantum numbers of $\Sigma$ are listed in Table I.

To realize the simplest inflationary scenario, the scalars must be displaced from their present day minima. Thus, for $\langle S \rangle >> M$, the scalars $10_H$ and $\overline{10}_H \rightarrow 0$, so that the gauge symmetry is restored but SUSY is broken. This generates a tree level scalar potential $V_{tree} = \kappa^2 M^4$, which will drive inflation. In practice, in addition to the supergravity corrections and the soft SUSY breaking terms, we also must include one loop radiative corrections arising from the fact that SUSY is broken by $\langle S \rangle \neq 0$ during inflation. This causes a split between the masses of the scalar and fermionic components in $10_H$, $\overline{10}_H$. For completeness, following Refs. [14] [15], we provide here the inflationary potential that is employed to compute the CMB anisotropy $\delta T/T$. 


the scalar spectral index $n_s$, and the tensor to scalar ratio $r$:

$$V = V_{\text{tree}} \times \left[ 1 + \frac{\kappa^2 N}{32\pi^2} \left( 2\ln\frac{\kappa^2 |S|^2}{\Lambda^2} + (1 + z)^2 \ln(1 + z^{-1}) 
+ (1 - z^{-1})^2 \ln(1 - z^{-1}) \right) + \frac{|S|^4}{2M_P^4} + am_{3/2}\kappa M^2|S|^2, \right]$$

(11)

where $z \equiv |S^2|/M^2$, $N (= 10)$ denotes the dimensionality of $10_H$, $\overline{10}_H$, and $\Lambda$ is a renormalization mass. We have employed a minimal Kähler potential and $a = 2|2 - A| \cos[\text{arg}S + \text{arg}(2 - A)]$, where $A$ denotes the “A-paramater” associated with the soft terms. Note that during the last 60 or so e-foldings the value of the $S$ field is well below $M_P$, especially for $\kappa \lesssim 10^{-2}$, which means that the supergravity correction proportional to $|S|^4/M_P^2$ is adequately suppressed (see Fig. 1). In addition, the soft term also can be safely ignored for $\kappa \gtrsim 10^{-3}$.

Neglecting the supergravity correction and the soft term in Eq. (11), $\delta T/T$ is given by

$$\frac{\delta T}{T} \approx \sqrt{\frac{N}{45N}} \times \left( \frac{M}{M_P} \right)^2,$$

(12)

where we took $z << M$. $N$ indicates the number of e-foldings (=50–60) and $N = 10$ as mentioned above. Substituting $\delta T/T \sim 6 \times 10^{-6}$ (corresponding to the comoving wave number $k_0 = 0.002$ Mpc$^{-1}$), one estimates $M$ to be of order $10^{16}$ GeV. Alternatively, one could insert in Eq. (12) the magnitude of $M$ determined from the symmetry breaking of flipped $SU(5)$ and thereby ‘predict’ $\delta T/T$. There is good agreement one finds with the observations.

To make a more precise comparison between $M$ determined from $\delta T/T$ and its value determined from the evolution of the $SU(3)$ and $SU(2)$ gauge couplings, we should include the supergravity corrections as well as the corrections coming from the soft terms. The results are exhibited in Fig. 1.

The unification of $SU(3)$ and $SU(2)$ gauge couplings occurs at the scale $g_5M$, where $g_5 (\approx 0.7)$ denotes the $SU(5)$ gauge coupling.$^5$ In Fig. 1 we plot $M$ versus $\kappa$, $^5$In flipped $SU(5)$, $\sqrt{5/3}g_Y$ is not unified with $g_5$ at the scale $g_5M$. The $SU(5)$ and $U(1)_X$ gauge
which shows that the coupling unification scale $g_5 M$ lies in the range $3.8 \times 10^{15} - 1.4 \times 10^{16}$ GeV for $10^{-5} \lesssim \kappa \lesssim 10^{-2}$. $g_5 M$ determined from the two loop RG evolutions of the MSSM gauge couplings with the initial values $g_3^2/4\pi(M_Z) = 0.1187 \pm 0.002$ and $\sin^2 \theta_W^\text{MS}(M_Z) = 0.23120 \pm 0.00015$ turns out to be $6.1 \times 10^{15}$ GeV $\lesssim g_5 M \lesssim 1.02 \times 10^{16}$ GeV (or $8.7 \times 10^{15}$ GeV $\lesssim M \lesssim 1.46 \times 10^{16}$ GeV) [10]. Values of $\kappa$ of order $10^{-2} - 10^{-3}$ and $10^{-5}$ are in good agreement with this. To quantify this somewhat differently, in Fig. 2 we plot the predicted curvature perturbation (for $k_0 = 0.002 \text{ Mpc}^{-1}$)

$$R = \frac{1}{2\sqrt{3\pi M_p^2}} \frac{V^{3/2}}{|V'|}$$

(13)
as a function of $\kappa$, for varying values of the symmetry breaking scale $M$. Good agreement with observations is possible for $\kappa$ around $10^{-5}$, or $\kappa$ in the vicinity of $10^{-2}$ [14,15]. Note that for $\kappa$ larger than $2 \times 10^{-2}$, the scalar spectral index exceeds unity due to supergravity corrections. Thus, values of $M$ close to $10^{16}$ GeV are preferred if $\kappa$ is in the vicinity of $10^{-2}$. Note that for $\kappa \sim 10^{-2} - 10^{-3}$, the supergravity corrections and soft terms are safely neglected, in which case, from Eq. (12), $\delta T/T$ is ‘predicted.’

From Fig. 2 we note that the region $\kappa \sim (2 - 3) \times 10^{-5}$ also leads to $R$ in good agreement with the observations. While this cannot be claimed as a ‘prediction,’ it can be experimentally distinguished from the region $\kappa \sim 10^{-2} - 10^{-3}$ by a precise measurement of the scalar spectral index and other quantities which we now discuss. (Note that for $\kappa$ less than or of order few $\times 10^{-6}$, $R$ rapidly falls below the observed value for all plausible values of the symmetry breaking scale $M$).

Fig. 3 displays the dependence of the scalar spectral index $n_s$ on $\kappa$. With $\kappa \lesssim 10^{-2}$ required by the constraint $T_r \lesssim 10^9$ GeV, we find that $n_s = 0.993 \pm 0.007$. Measurement of $n_s$ to better than a percent is eagerly awaited. Fig. 4 shows that $dn_s/d\ln k \lesssim 4 \times 10^{-4}$. The tensor to scalar ratio $r \lesssim 10^{-6}$ (Fig. 5).
The end of inflation is reached when the scalar field $S$ leaves the ‘slow roll’ regime and rapidly approaches its true minimum near the origin. This is also the signal for the fields $10_H, \overline{10}_H$ to leave their positions at the origin and proceed to their minima at $M$. Since the breaking of $SU(5) \times U(1)_X$ does not produce monopoles, there are no cosmological problems to worry about, as stated earlier. The scalar fields perform damped oscillations about their minima and eventually decay, leading to reheat temperature $T_r \lesssim 10^9$ GeV, in order to the gravitino problem is avoided. See Fig. 5 for the dependence of $T_r$ on $\kappa$. It shows that $\kappa \lesssim 10^{-2}$ for $T_r \lesssim 10^9$ GeV.

We expect the decay to proceed via the production of right handed neutrinos and sneutrinos arising from the quartic (dimension five) superpotential couplings $10, 10, \overline{10}_H \overline{10}_H$, in combination with the coupling $S10_H \overline{10}_H$. The former coupling is permitted by the $SU(5) \times U(1)_X$ symmetry and would normally give rise to large ($\lesssim 10^{14}$ GeV) right handed neutrino masses. Assuming hierarchical right handed neutrino masses, such couplings typically give rise to a reheat temperature $T_r$ of order $(1/10 - 1/100)M_N$, where $M_N$ denotes the mass of the heaviest right handed neutrino that can be produced by the decaying inflaton [19]. Furthermore, the decaying right handed neutrinos can provide a nice explanation of the observed baryon asymmetry via leptogenesis [8]. Following Ref. [15], one can derive the $\kappa$ dependence of $T_r$ shown in Fig. 5. The quartic coupling above is, however, inconsistent with the $U(1)_R$ symmetry, and a somewhat more elaborate scenario based on the “double seesaw” [20].

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Recently it was argued that the reheat temperature $T_r \lesssim 10^6 - 7$ GeV for $m_{3/2} \sim 100$ GeV, unless the hadronic decays of the gravitino are suppressed [10]. The low temperature leptogenesis scenario ($T_r \sim 10^2$ GeV) could work in this class of inflationary models [16], if two right handed neutrinos with masses of $10^4$ GeV or so are nearly degenerate. Note that the heaviest right handed neutrino can still have a much larger mass than the inflaton (say $\sim 10^{14}$ GeV; thus it cannot be produced by the inflaton perturbative decay). The baryon asymmetry is then of order $(T_r/m_\chi) \times \epsilon$, where $m_\chi$ denotes the inflaton mass and $\epsilon$ the lepton asymmetry per neutrino decay. $\kappa \sim 10^{-5}$ (thus $m_\chi \sim 10^{11}$ GeV) and $\epsilon \sim O(1)$ give the desired baryon asymmetry ($\sim 10^{-10}$). Actually, $\epsilon$ can be as large as 1/2, provided the neutrino mass splittings are comparable to their decay widths [17]. The constraint $T_r \lesssim 10^9$ GeV remains intact if either the hadronic decay ratio of the gravitino is small enough ($\lesssim 10^{-3}$) and $m_{3/2} \gtrsim 3$ TeV, or if the gravitino happens to be the LSP. The gravitino, in principle, could have many decay channels into hidden sector fields, which would be helpful for lowering its hadronic decay ratio. Moreover, if the gravitino is the LSP and $T_r \sim 10^{10}$ GeV, the gravitino can be the dark matter in the universe [18].
is required to implement both the usual seesaw mechanism and the desired reheat scenario. The details of one such extension are as follows.

With additional $SU(5) \times U(1)_X$ singlets superfields $\Phi, \Phi', \Psi$ and a hidden (anomalous) gauge symmetry $U(1)_H$, let us consider the superpotential:

$$W_{\nu c} = \frac{\rho_i}{M_P} \Phi \Psi 10, 10_H + \frac{\rho}{M_P} \Phi' \Psi 10, 10_H,$$

where $\rho_i, \rho$ denote dimensionless coupling constants. The quantum numbers of $\Phi, \Phi', \Psi$ are shown in Table II.

|   | $\Phi$ | $\Phi'$ | $\Psi$ |
|---|---|---|---|
| $X$ | 0 | 0 | 0 |
| $R$ | 0 | $-1$ | 1 |
| $Z_2$ | $+$ | $+$ | $-$ |
| $H$ | 1 | 1 | $-1$ |

Note that the superfields appearing in Table I are neutral under $U(1)_H$. From the “D-term” scalar potential associated with $U(1)_H$,

$$V_D = \frac{g_H^2}{2} \left| |\Phi|^2 + |\Phi'|^2 - |\Psi|^2 - \xi \right|^2,$$

the scalar components of $\Phi$ and $\Phi'$ develop non-zero VEVs; $\sqrt{|\Phi|^2 + |\Phi'|^2} = \sqrt{\xi} \sim 10^{17}$ GeV, while $\langle \Psi \rangle$ vanishes by including the soft terms in the potential. Here the parameter $\xi$ comes from the “Fayet-Iliopoulos D-term” [21]. Since $\xi \gg \kappa^2 M^4/M_P^2$, $U(1)_H$ is broken even during inflation. Thus, cosmic strings associated with $U(1)_H$ breaking would be inflated away.

By including soft SUSY breaking terms and supergravity effects from the higher order Kähler potential term,

$$K \supset \frac{h}{M_P^2} \Phi \Phi^\dagger \Phi' \Phi'^\dagger,$$

where $h$ is real and $-1 \lesssim h < 0$, $\langle \Phi \rangle$ and $\langle \Phi' \rangle$ could be determined such that $|\langle \Phi \rangle| = |\langle \Phi' \rangle|$. We have assumed here that terms proportional to $(\Phi \Phi^\dagger)^2$ and $(\Phi' \Phi'^\dagger)^2$ in the
Kähler potential are suppressed relative to the one given in Eq. (16). It turns out that $|\langle \Phi \rangle| = |\langle \Phi' \rangle|$ also holds during inflation. The VEV of the lighter mass eigenstate $[\equiv (\Phi - \Phi')/\sqrt{2}]$ vanishes both during and after inflation, while the superheavy mass eigenstate $[\equiv (\Phi + \Phi')/\sqrt{2}]$ develops a VEV of order $\sqrt{\xi}$.

With $\langle \Phi \rangle \sim \langle \Phi' \rangle \sim 10^{17}$ GeV, $\Psi$ has a superheavy Majorana mass of order $\rho \langle \Phi \Phi' \rangle / M_P \sim 10^{16}$ GeV, while $\Psi$ and $\nu^c$ obtain (pseudo-) Dirac masses of order $\rho_i \langle \Phi \rangle / M_P \lesssim 10^{15}$ GeV. Hence, the “seesaw masses” $(\sim [\rho_i^2 \langle \Phi \rangle / \rho \langle \Phi' \rangle] \times [\langle \mathcal{P}^c_H \rangle^2 / M_P])$ of the lighter mass eigenstates, which are indeed the “physical” right handed neutrinos, should be of order $10^{14}$ GeV or smaller, as desired.

In summary, it is tempting to think that inflation is somehow linked to grand unification, especially since the scale associated with the vacuum energy that drives inflation should be less than or of order $10^{16}$ GeV. Models in which $\delta T/T$ is proportional to $(M/M_P)^2$, with $M$ comparable to $M_{GUT}$ are especially interesting in this regard [1]. Supersymmetric flipped $SU(5)$ provides a particularly compelling example in which the magnitude of $\delta T/T$ can be ‘predicted’ by exploiting the gauge coupling unification scale that has been known for several years. Precise measurement of the scalar spectral index will provide an important test of this class of models.

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Alternatively, $\langle \Phi \rangle$ and $\langle \Phi' \rangle$ could be determined by introduction of $\Phi$ with proper quantum numbers assigned and the nonrenormalizable terms in the superpotential, $W \supset S(\kappa' \Phi \Phi - \rho' (\Phi \Phi)^2 / M_P^2)$. By including soft terms, it turns out $\langle \Phi \rangle = \langle \Phi' \rangle = \sqrt{\kappa' M_P^2 / \rho'}$ at a local minimum [3]. From the “D-term” potential, we have $|\langle \Phi' \rangle|^2 = \xi$. We should assume $\kappa' / \rho' << 1$ to keep intact the inflationary scenario discussed so far. In this case also the VEV of the lighter mass eigenstate vanishes both during and after inflation.
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Figure 1: The value of the symmetry breaking scale $M$ (solid) and the magnitude of the inflaton $|S|$ (dashed) vs. $\kappa$. We take $m_{3/2} = 10^3$ GeV and $a > 0$.

Figure 2: $\mathcal{R}$ vs. $\kappa$. $M = 1.24 \times 10^{16}$ GeV (solid), $M = 1 \times 10^{16}$ GeV (dashed), and $M = 8.7 \times 10^{15}$ GeV (dot-dashed). The dotted horizontal line corresponds to $\mathcal{R} = (4.7 \pm 0.3) \times 10^{-5}$. We take $m_{3/2} = 10^3$ GeV and $a > 0$. 
Figure 3: The spectral index $n_s$ vs. $\kappa$.

Figure 4: $dn_s/d\ln k$ vs. $\kappa$. 
Figure 5: The tensor to scalar ratio $r$ vs. $\kappa$.

Figure 6: The lower bound on the reheat temperature $T_r$ (solid) and the inflaton mass $m_\chi$ (dashed) vs. $\kappa$. 