Phase-space Distribution Functions of Feebly Particles and Their Signatures

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The history of Higgs-sourced feebly models starts long ago, e.g. [6]; however, the list of the corresponding constraints tends to not include all relevant ones inferred from the small-scale structure measurements. For a thorough review of such theories, see [6] and [7]. To the best of our knowledge, our results and our mass intervals yield, for the first time, much more concrete predictions about the Higgs-sourced DM minimal extensions of the SM than found in [5]: in a sense, we concentrate more on the small-scale issues rather than the general model building of feebly theories. Furthermore, we claim that the theoretical prototypes presented in this work satisfy all experimental and theoretical constraints, and at the same time provide clear solutions to the missing satellite and high values of rotation curves problems, together with the cuspy simulated profiles and the diversity problem (a pedagogical review of the small-scale problems and their potential solutions can be found in [8] and [4]). Non-thermal Higgs-sourced DM theories are discussed in [9].

Our approach, although respecting the same constraints and solving the same problems as the minimal flavored dark QED theory [10] or the neutrinoophilic framework presented in [11], is very different, since the DM modes are the FIMPs sourced entirely from the Higgs field. On the other hand, the feeble theories could, in principle, admit also a leptophilic or neutronophilic virtue, discussed in [12], accommodating the recent DAMPE results, and in [13], explaining the long standing neutron anomaly, respectively.

Recently, the XENON collaboration in the XENON1T experiment observed an excess in the electron recoil energy in the region $E \lesssim 7$keV compared to the known background [14]. This is a strong motivation, independent from the small-scale problems, to investigate the possibility of dark matter which admits masses of a few keV. Nevertheless, such extensions are extremely model-dependent and are beyond the scope of this work.

This paper is organized as follows. We start by deriving the general phase-distribution function of the feebly particles, then we continue with a sufficient approximation for the cut-off scales of the linear power spectrum generated by the feebly interacting particles at the phase-space distribution function level and we present our list

I. INTRODUCTION

Until now, no direct nor indirect detection of dark matter (DM) modes has been observed, although various signals hint towards their particle nature. The lack of clear experimental evidence may possibly hint to the existence of feebly interacting dark matter (FIMP) candidates as DM. The main aim of this work is to provide a proof of existence of theories with FIMPs sourced by the Higgs field, which provide solutions to the enduring $\Lambda$CDM issues. Such theoretical scenarios are subject to several constraints, enlarging the predictive power of the allowed theories. Therefore, in this work, we are interested in theories of FIMPs extending the standard model of particle physics (SM) minimally: feebly in the sense of the extremely weak and out-of-equilibrium interactions between the FIMPs and the SM-modes. We achieve our goal by studying these theories for the first time at the level of the phase-space distribution function of the participating FIMPs. In other words, we derive the general phase-space distribution function of FIMPs providing us with unprecedented predictive power regarding the properties and signatures of such particles.

The list of the latest constraints on the feeble theories derived in this work arises mostly from the corresponding signatures of the small-scale structures generated by the proposed DM population sourced solely by the Higgs field. Although these bounds are much tighter than the ones inferred by collider measurements, they lead to acceptable theories alleviating the small-scale crisis of $\Lambda$CDM cosmology. In particular, we show that the recent Lyman-$\alpha$ bounds [1] together with the positive observations regarding the rotation curves, the missing dwarf satellites, and the cored profiles of galaxies discussed in [2-4], indicate the presence of new physics well below the weak scale. This secluded dark sector turns out to be strongly interacting with itself, departing from the common idea of weakly interacting particles (WIMP) [4].
of constraints arising from the study of the small-scale structures in Higgs-sourced theories. Finally, we show that the allowed spectra are tight and we conclude by entertaining the existence of two different paradigms as prototype classes of such theories.

II. THE GENERAL PHASE-SPACE DISTRIBUTION FUNCTION

In this section, we present an analytical solution of the Boltzmann equation for the general Higgs-sourced case. The defining equation of the DM distribution function

\[ C[f](p) := \frac{1}{2\sqrt{\pi}} \sum_{\text{in/out}} \left( \prod_{\mathbf{p} \not= \mathbf{p}} \int d\omega_i \right) |\langle \text{out} \rangle (S - 1) |\langle \text{in} \rangle|^2 \]

with \( V^{(4)} \) the appropriate space-time normalization volume for all d.o.f. of the underlying field. We used the abbreviation in/out for the initial/final, identically free states and \( d\omega_i \) for the Lorentz-invariant measure; the \( \pm \) stands for Bose-Einstein-/Fermi-statistics respectively.

We note that it is convenient to write the Liouville operator in terms of the comoving momentum \( \pi \equiv a(t) p_{\mu} / a(t(M)) M \) at some reference temperature \( M \) introducing the reduced Higgs-mass \( \mu \equiv m_h / T \)

\[ L[f](\pi, \mu) = \frac{3 p^0 H}{\mu - \partial_\mu \log[g_\pi^2(\mu)]} \partial_\mu f(\pi, \mu). \]  (3)

Here \( \partial_\mu \) is the partial derivative with respect to the reduced source mass. In other words, it is straightforward to start our approach by modeling the DM distribution function \( f(\pi, \mu) \) via direct solving of the Boltzmann equation; this way allows us to obtain all relevant observables directly by integration of the now known phase-space distribution function.

We start by assuming that the relativistic degrees of freedom vary slowly with respect to the temperature, \( \partial_\mu g_\pi^2(\mu) \approx 0 \), during the DM production epoch, which is after the electroweak phase-transition and before the bottom quark annihilation. Without loss of generality, we take

\[ \pi = \frac{p}{T} \bigg|_{t_1} \]  (4)

for convenience, where \( t_1 \) is the time of the DM production.

After some algebra and numerics, we determine the form of the DM distribution function \( f(\pi, \mu) \) per feeble mode. For example, for the Higgs decay into two feeble particles, where the Higgs boson is in local thermal equilibrium (LTE), we obtain

\[ f(\pi, \mu) = f_0(\mu) \frac{e^{-\pi}}{\sqrt{\pi}} \left( \frac{\mu}{2\sqrt{\pi}} \right) - \frac{\mu}{\sqrt{\pi}} e^{-\mu^2/4\pi} \right). \]  (5)

Here, \( f_0(\mu) \) is the model dependent amplitude, which varies very slowly as \( \mu \) changes, we choose the reference frame \( M = T_1 \) as before and \( \bar{\pi} := \left( \int d\mathbf{p} \exp \left[ -\mathbf{p}^2/2 \right] \right)^2 \).

One notices that the feebly distribution function is almost thermal at large values of \( \mu \), admitting a first moment of \( \langle \pi \rangle \approx 2.45 \). Moreover, the phase-space distribution function reduces to

\[ f_{DM}(\pi) \approx \lim_{\mu \to \infty} f(\pi, \mu) = \frac{f_0}{\sqrt{\pi}} e^{-\pi} \]  (6)

at later times, which trivially solves the collisionless Boltzmann equation, encapsulating the notion of feebly interactions between the produced particles and the Higgs boson. It turns out to be a good first approximation of studied observables, since the coefficient of variation (CV) is \( \sim 0.6 \), the same as in the case of ultra-relativistic fermions or bosons in LTE. We note that the DM is populated as long as the hot DM bound of 1% [10] is achieved, namely around \( \mu = 9.1 \equiv \mu_1 \); this is the proper definition of \( t_1 \): the time when the DM relic abundance has been populated.

The above result is process-dependent, therefore, we take the effort to compute the DM distribution function in a more general case of the Higgs (or even other standard model particles) annihilation into DM via a dimension \( \ell \) operator of massless modes. This is actually the high energy limit of the scenario studied above: the Higgs boson (or other standard model particles) is still ultra-relativistic and produces DM modes via feeble interactions. Without loss of generality, we consider the case of \( 2 \to 2 \) processes; technically, the source modes
are assumed to achieve LTE at a temperature \( T_3 < \Lambda \), where \( \Lambda \) is the energy suppression of the corresponding operator. We obtain the analytic solution

\[
f_\text{DM}^\ell(\pi, \mu) = \frac{f_0(\mu, \mu_3)}{\pi^{5-\ell}} \ e^{-\pi}. \tag{7}
\]

Again, here \( f_0(\mu, T_3, \Lambda) \) varies very slowly with respect to \( \mu \) and depends polynomially on \( \mu_3 = T_3/\Lambda \), which makes the DM distribution function independent of \( \mu \) at the lowest order.

For self-scattering processes, the integrated expression of the collision term is proportional to

\[
\left( \prod_{i=1}^{4} \int d\omega_i \right) |A|^2 \delta^{(4)} \left( \sum p_i \right) \left[ f_3 f_4 - f_1 f_2 \right], \tag{8}
\]

using the shortcut notation \( f_i \equiv f(p_i(t)) \), and \( A \) denoting the scattering amplitude. This collision term gives the change of the DM number of particle 1, which is conserved due to symmetry. It is of great importance, since it implies that the DM relic abundance can not be modified via self-interactions at the lowest interaction-level; higher interaction-levels are required, which are much more suppressed. Furthermore, one could exemplary examine perturbations of the peculiar velocity of the DM fluid [15] in the context of a theory with self-interactions as in [8], given by

\[
\left( \prod_{i=1}^{4} \int d\omega_i \right) |A|^2 \delta^{(4)} \left( \sum p_i \right) \frac{n}{m} \frac{p_i}{m} \left[ f_3 f_4 - f_1 f_2 \right], \tag{9}
\]

with \( n \) the wave vector of the linear decoupled perturbation Fourier modes in a spatially flat background [17]. Here, we assumed that the DM modes are non-relativistic, taking \( p_i^0 \approx m \). One notices that also this collision integral vanishes due to the zero contributions of the \( p_1 \pm p_2 \) moments. This means that the lowest interaction-level of the self-interactions of non-relativistic DM particles does not contribute to the evolution of DM fluctuations [8] allowing us to use the results of [15] even in the context of a theory with self-interactions.

III. LINEAR POWER SPECTRUM CUT-OFF

One of the main purposes of this work is to constrain the feebly theories using present bounds arising from the observed small-scale structures. In this section we show how feebly theories could in fact admit interesting signatures at small scales.

A. Efficiency of momentum exchange

We start by reviewing the general assumption in WIMP theories, stated in [19] and [12], that the cold DM modes are in local kinetic equilibrium with some abundant (usually ultra-relativistic) species. This mechanism is described by the efficient momentum exchange between the participating particles; however, as described in [8], the elastic scattering processes between the DM modes and the plasma lose their efficiency at times around the kinetic decoupling temperature \( T_1 \). In [18], it is thoroughly explained that the remaining elastic interactions can be described effectively as sources of entropy production in an imperfect DM fluid; in other words, the momentum exchange continues to damp the perturbations that would otherwise grow to form the first gravitational bound DM objects (protohalos) [17]. Therefore, the DM particles stream free of damping and efficiently erase the small-scale structures. This entropy production is found to be related to the dissipative evolution of DM fluctuations, which are exponentially damped with a characteristic mass scale \( M_\text{d} \); in other words, small-scale structures of DM cannot sustain their form within the Hubble volume at the time of kinetic decoupling, since the plasma-particles within this volume are much more abundant and their interactions suffice to keep the former in approximate local thermal equilibrium. Furthermore, this damping mass scales with \( T_1^{-3} \) [10], the exact relation is given in [2] and we cite it here,

\[
M_\text{d} \approx 7 \times 10^{10} \left( \frac{0.1 \text{keV}}{T_1} \right)^3 M_\odot, \tag{10}
\]

where \( T_1 \) is SM-photon temperature. One notices that the above expression is independent of the mass of the DM particles, since at the time of kinetic decoupling they are almost at rest (cold). It is important to note that this problematic is valid only if the DM modes are in local kinetic equilibrium with the abundant plasma, and, therefore, it is not present in a theory with feeble interactions of a single DM species, even if the DM particles are in local thermal equilibrium with themselves via self-interactions. This is due to the symmetries of the collision term describing the self-scattering between the DM particles.

Summarizing the previous results and recalling that the lowest interaction-level of the self-interactions of non-relativistic DM particles [9] does not contribute to the evolution of DM fluctuations, in a feebly theory of a single DM species, no notion of kinetic decoupling is present and self-interactions do not allow us to extract information about the cut-off masses of the linear matter power spectrum. Therefore, we search for a different damping mechanism: the free-streaming of the feebly modes.

B. The free-streaming of feebly modes

When a particle at time \( t_1 \) starts moving freely through the expanding universe, its free-streaming length also provides a different mechanism leading to a cut-off of the linear matter power spectrum [17]. The length of the free propagation of the DM modes after the \( t_1 \)-surface,
moving freely along geodesics, up to the matter-radiation equality time, \( t_{\text{eq}} \), is defined as

\[
\ell = \int_{t_1}^{t_{\text{eq}}} \frac{dr(t)}{a(t)}. \tag{11}
\]

The approximate limit until \( t_{\text{eq}} \) is due to the rapid structure formation after that time. Moreover, the corresponding cut-off mass is estimated after taking \( \ell/2 \) as the radius of the underlying homogeneous matter sphere.

For cold particles that are already non-relativistic at the time of kinetic decoupling, if such regime is present, the cut-off mass yields \([17]\)

\[
M_{\text{fs}} \approx 4.2 \times 10^8 \left( \frac{1 + \log \left[ g_1^{1/4}(T_1) (T_1/5 \text{ keV}) /10.2 \right]}{(m/100 \text{ keV})^{1/2} g_1^{1/4}(T_1) (T_1/5 \text{ keV})^{1/2}} \right)^3 M_\odot. \tag{12}
\]

However, in this work we are interested in FIMP production mechanisms sourced by the Higgs field; therefore, the DM modes are born relativistic mostly at \( t_1 \) even if their mass is much larger than the plasma temperature. This surface coincides with the production time of the particles but not with the time \( t_2 > t_1 \) when the DM modes become non-relativistic. The free-streaming length of such modes can be approximated as

\[
\ell \approx \frac{z_2^2}{z_2} t_{\text{eq}} \left[ 1 + \log \left( \frac{z_2}{z_{\text{eq}}} \right) + \mathcal{O} \left( \frac{T_0}{m_{\text{e}}} \right) \right], \tag{13}
\]

assuming that \( t_2 \gg M_{\text{ph}}/m_\pi^2 \). \( 0 \) denotes the observable values at present time. We also considered the change of the expansion parameter \( a(t) \) as a function of time, but such changes do not affect the above result up to order of \( z_2T_0/m_\pi \). Here, plugging in the most recent results inferred from Planck measurements \([16]\), one obtains

\[
\ell \approx 0.048 \text{ Mpc} \left( \frac{10^7}{z_2} \right) \left[ 1 + \log \left( \frac{z_2}{3365} \right) \right], \tag{14}
\]

with the redshift at the time where the DM modes enter the non-relativistic regime denoted by \( z_2 \) and given by

\[
z_2 = \pi_1^{-1} \frac{m}{T_0}, \tag{15}
\]

assuming a 1→2 decay channel. Here, \( m \) is the DM particle mass and \( \pi_1 \) is the comoving momentum at the time of production, taking \( M = T_0 \). Exemplary, if the FIMPs become non-relativistic at \( z_2 \sim 10^7 \), they admit warm dark matter (WDM) properties regarding the small-scale structure formation \([2]\). The corresponding numerical estimate bounds the DM mass stronger from below than the naive entropy dilution calculation \([7]\). In a FIMPy theory the cut-off mass to the linear matter power spectrum is the smallest possible protohalo \([15]\) that could be formed and is given by \( M_{\text{ph}} \equiv M_{\text{fs}} \), since the damping due to acoustic oscillations is absent. However, this does not encapsulate the change of the phase-space distribution function of the DM particles over their production time. Hence we will now compute the free-streaming length directly from \( f(\pi, \mu) \). The cut-off of the linear power spectrum is estimated via

\[
M_{\text{fs}}(\mu_1) = N(\mu_1)^{-1} \int_0^\infty d\pi \pi^2 M_{\text{fs}}(\pi) f(\pi, \mu_1), \tag{16}
\]

where \( N(\mu) \) is the appropriate temperature dependent normalization of the distribution function, independent of the reference temperature \( M \).

### IV. APPLICATION ON COSMOLOGICAL CONSTRAINTS

At this point we use the phase-space distribution function of the DM particles in order to study such theories as an application on cosmological constraints. Moreover, by inverting such constraints we show that feebly theories are able to solve the enduring small-scale problems appearing in the ΛCDM paradigm.

#### A. Constraints at dwarf scales

It is suggested by the authors of \([2], [19] \) and \([20]\) that an exponential suppression of the linear matter power spectrum at dwarf scales (sub-kpc distances) alleviates the missing satellite problem of ACDM cosmology. The sufficient cut-off mass should be of order of magnitude \( 10^8 \) solar masses and not above \( 10^8 \) solar masses; such protohalos masses alleviate the small-scale abundance problem \([8]\) and \([4]\).

If the proposed DM theory possesses a kinetic decoupling regime with the DM modes already being non-relativistic, a kinetic decoupling temperature \( T_1 \sim \text{keV} \) \([4]\) provides such cut-off masses. On the other hand, in DM theories of feebly interacting particles, the above regime is absent, and the protohalo masses are mainly determined through the free-streaming mechanism and correspond to the WDM mass, as mentioned before. These masses are in turn constrained by the Lyman-α bound, as explained in \([20], [21], [22] \) and more recently in \([1]\); in
other words, the maximum cut-off mass almost touches $10^6 M_\odot$, or, equivalently, a minimum of 3.5 keV WDM mass [8] for the weak bound and lies somewhat above $10^6 M_\odot$, which corresponds to a 5.3 keV WDM mass, for the strong bound. This problematic, together with the most recent Lyman-$\alpha$ constraints and the thermal approximation [16] using the DM phase-space distribution function, yields a lower bound on the DM mass after assuming 1$\to$2 decay processes of the Higgs boson and fixed DM relic density $\Omega_{DM}h^2 = 0.12$, namely, expression (16) yields $m \gtrsim 5.8 (3.1)$ keV, regarding the strong (weak) constraint. These values are very close to the ones found in the literature regarding the lowest WDM mass [4]. The above result is obviously process-dependent, due to $f_0$, and it turns out to be a good first approximation of $M_{ph}$, since the coefficient of variation (CV) is $\sim 0.6$, the same as in the case of ultra-relativistic fermions or bosons in LTE. We find that if the DM modes are stable, then the limit $\lim_{\mu \to \infty} M_{fs}(\mu)$ exists and is only about 5% larger than the production estimate at $\mu_1$, as plotted in Fig. 1. If one uses the WDM distribution due to non-relativistic decays [23, 24], the free-streaming length is smaller by 10% due to the non-constant momentum distribution. A more accurate bound is beyond the scope of this work, since we are interested in the allowed energy scales of the feebly theories sourced by the Higgs field.

We note that it is also possible for heavier DM particles to generate similar cut-off masses, while being produced through feebly interactions, as long as their production takes place later than the typical decay time. This is, however, not possible if the decaying particle is the Higgs boson, since the feebly decay channel is not the dominant one. At least one additional portal field is needed in order to achieve $\mu_1 \gg 1$ as in [1]. Analogously, even lower DM masses are accessible if at $\mu_1$ more relativistic degrees of freedom are present. Nevertheless, such modes should admit a similar temperature as the SM-modes or be in pure LTE with them. Since no lighter-than-Higgs dark particles are observed, we assume the former case, which is strongly BBN constrained.

\[ \lambda \lesssim 10^{-4} \left( \frac{m}{50 \text{ keV}} \right)^{3/2}, \]

or, for fermionic DM, they bound the effective Fermi coupling $\tilde{G}_F$ as

\[ \tilde{G}_F \gtrsim 4 \times 10^4 \text{ GeV}^{-2} \left( \frac{m}{50 \text{ keV}} \right)^{-1/2}. \]

Nevertheless, as studied in [3] and [2], the combined effects of the SIDM properties on the non-linear evolution at dwarf-galaxies using significantly smaller SIDM cross-sections and of the cut-off mass in late decoupling models (arising from a dark acoustic damping) provide simultaneous solutions to the enduring small-scale problems of the $\Lambda$CDM paradigm. It turns out that cut-off masses, which alleviate the missing satellite problem successfully, and SIDM cross-sections about an order of magnitude smaller than the proposed ones in [4], solve the too big to fail problem, together with the diversity problem; for a thorough discussion and review of all present (positive) constraints, see [3]. For pure SIDM theories, where the damping of the matter power spectrum at the linear regime is of no importance, dwarf galaxies with typical thermal velocities of order $\sim 10^{-3}$ for the DM modes prefer large self interacting cross-sections, $\langle \sigma_T/\nu_H \rangle_{\text{therm}} \sim 1 \text{ cm}^2\text{g}^{-1}$ as shown in [25] and [4]; however, at cluster scales with velocities of order $\sim 10^{-2}$, the constraints indicate that cross-sections larger than $\langle \sigma_T/\nu_H \rangle_{\text{therm}} \sim 0.1 \text{ cm}^2\text{g}^{-1}$ are strongly disfavored, which could indicate a mild a velocity dependence of the SIDM cross-section between dwarf and cluster scales, similar to the neutron-proton scattering. The cluster constraints give a clear lower bound to the effective quartic self-coupling $\lambda$, assuming scalar DM,

\[ \lambda \lesssim 10^{-4} \left( \frac{m}{50 \text{ keV}} \right)^{3/2}, \]

or, for fermionic DM, they bound the effective Fermi coupling $\tilde{G}_F$ as

\[ \tilde{G}_F \gtrsim 4 \times 10^4 \text{ GeV}^{-2} \left( \frac{m}{50 \text{ keV}} \right)^{-1/2}. \]
the SIDM cross-sections tend to over-solve the ΛCDM issues. Hence, if the free-streaming cut-off masses are able to cumulatively affect the small-scale structure, then the FIMPs should admit SIDM virtues realized through some self interaction; furthermore the desired (σT/m)_{\text{observed}} values at dwarf scales appearing in the ETHOS-4 tuned model \[2\] fix the effective quartic coupling for scalar DM and the effective Fermi coupling for fermionic DM, respectively; this is due to the recent cluster results \[20\]. The above problematic is also well motivated from the recent constraints on dwarf-scale cross-sections \[27\], which are in tension with the proposed solution interval \[4\]. However, these feebly models do not admit dark acoustic oscillations as ETHOS-4 tuned model, since no kinetic decoupling is present, meaning that further study and simulations are needed in order to examine the exact matching of observables, which is beyond the scope of this work.

We note that both couplings are also constrained by perturbation theory and unitarity. This sets an upper bound on the DM mass of a scalar FIMP, hence \(m \lesssim O(10)\) MeV. Analogously, in the case of fermionic DM, the ETHOS-4 results and the cluster bounds yield multi-eV masses for the force mediators of the theory for multi-keV DM masses. This result may indicate the presence of new physics well below the weak scale.

\section*{C. Big bang nucleosynthesis}

Now we try to track the footprints of a feebly theory in the period of the big bang nucleosynthesis (BBN). To do

\[\Delta N_{\text{eff}}|_{t/N_{\nu}} \approx 4\pi g \int_0^\infty \frac{d\pi}{(2\pi)^3} \left(\frac{m\sqrt{\pi}}{\mu_m}\right)^4 \epsilon(\pi, \mu_m) f\left(\frac{\pi}{\sqrt{\varepsilon}}, \mu\right) |t/\rho_\nu|,\]

where \(\epsilon(\pi, \mu_m) = \sqrt{\pi^2 + \mu_m^2} - \mu_m\), \(\mu_m = m/T\) and \(\varepsilon = g_1^\text{f}(\mu)/g_1^\text{f}(\mu_1)\). As usual, \(T\) is the photon temperature and \(f_0\), the amplitude hidden in \(f(\pi, \mu)\) per degree of freedom of the DM distribution function, encapsulates the incomplete thermalization of the FIMPs.

The existence of a single DM species, fixing \(\Omega_{\text{DM}} = 0.26\) \[10\], yields, during the BBN period, a 1σ lower bound on the DM mass, namely \(m \gtrsim 13\text{eV}\), less tight than the one inferred from the protohalo mass. This is a much more precise estimate of the deviation of the effective neutrino degrees of freedom than the one using a WDM distribution for non-relativistic decays \[23, 24\]. However, for the case of fermionic DM, the lowest allowed masses arise from the Tremaine-Gunn bound \[29\], which is tighter, \(m \gtrsim 0.5\) keV. One notices that FIMPs at the keV-scale sourced by the Higgs field are compatible with BBN \[39\] and CMB \[16\] 1σ measure-

so, we approximate the deviation of the effective neutrino degrees of freedom during the BBN period. With the neutrinos already decoupled at \(T_{\nu D} = 2.3\) MeV \[28\], the corresponding deviation is given by the ratio

\[\Delta N_{\text{eff}} = N_{\nu} \rho_{\text{eff}} / \rho_\nu,\]

with \(\rho_{\text{eff}}\) the energy density of the relativistic particle species besides the photons and neutrinos with initial conditions those at neutrino decoupling temperature \(T_{\nu D}\). This deviation works as a parametrization of the cosmic energy budget and could affect the BBN processes significantly, therefore it can be probed at high precision.

If \(m \gtrsim T_{\nu D}\), then no change in \(N_{\text{eff}}\) can be seen. However, if \(m < T_{\nu D}\), then at time \(t\) and temperature \(T_t\), we obtain

\[\Delta N_{\text{eff}}|_{t/N_{\nu}} \approx g \int d^3p (2\pi)^3 \epsilon(p) f(p, T)|_t/\rho_\nu,\]

with \(g\) the degrees of freedom of the DM field and \(\epsilon(p) = p^0 - m\) the kinetic energy of the DM modes; their distribution is not thermal, therefore, one should ignore the rest energy of the particles. The above expression can be easily written, taking \(M = T_1\), as

\[\Delta N_{\text{eff}}|_{\text{BBN}} < 0.01.\]

Moreover, assuming that the recombination takes place instantaneously at 0.3 eV, the DM modes are non-relativistic in this period, otherwise the Lyman-\(\alpha\) bounds would be violated. This outcome could also explain the tension about the decrease of the deviation of the effective neutrino number from BBN to CMB-based measurements, such a difference \(\Delta N_{\text{eff}}|_{\text{CMB}} - \Delta N_{\text{eff}}|_{\text{BBN}} < 0\) can be delivered from the effective theory parameter space.

\section*{D. The relic abundance of FIMPs}

In this work, we assume that the dominant DM population is produced through processes involving the Higgs modes. Now we show that such channels can be responsible for the present DM relic density. Moreover, we
consider, without loss of generality, only decays of the Higgs modes after the electroweak phase transition as production mechanism and not Higgs scatterings at high energies. During this scenario, one should not consider annihilations, since they are suppressed by multiples of $2\pi$. Instead of solving the Boltzmann equation approximately, following the methods discussed in [11], we approach the problem in a more accurate way regarding the needs of this work. The natural way to compute the DM relic abundance is to evaluate the DM particle number at the present time $t_0$ using $n_{\text{DM}} = g \int \frac{d^3p}{(2\pi)^3} f(p, T)|_{t_0}$. This yields

$$\Omega_{\text{DM}} h^2 \approx 0.12 \left( \frac{86.8}{g^*(\mu_1)} \right) \left( \frac{\lambda_h}{2.3 \times 10^{-9}} \right)^2 \left( \frac{m}{50 \text{ keV}} \right),$$

where $\lambda_h$ is the dimensionless feebly coupling between the Higgs field and the DM fields (if dimensionful, it is given in units of $m_h$) appearing in the distribution amplitude $f_0(\mu_1) = \mu_1^{-2} \times 10^{-10} H(\mu_1)$. Therefore, the DM modes should not freeze-in before the usual feebly freeze-in due to the Higgs decays; otherwise the DM density would be reprocessed, destroying the FIMPs assumption. Hence, the mass of the DM particle is bounded from below, namely we can infer $m \gtrsim \mathcal{O}(10) \text{ eV}$, which is less tight than the corresponding bound due to the cut-off masses. In addition, since we are interested in purely feebly theories, the produced DM particles should not freeze-in at least for processes of $\mathcal{O}(\lambda^4)$, which modify their abundance [31]. This constraint yields, for the case of the scalar field $m \lesssim \mathcal{O}(10) \text{ keV}$, otherwise the $2 \leftrightarrow 4$ processes would modify $\Omega_{\text{DM}}$ as noticed in [7].

V. DISCUSSION ABOUT THE PARAMETER SPACE

The constraints derived previously for the DM masses of the scalar FIMPs in a Higgs-sourced theory are either upper or lower bounds on the masses, since the (self-)couplings depend on the DM mass either due to the cluster constraints of the SIDM or the relic density of the cold DM. One notices that the allowed parameter space, which also fulfils the requirements of the ETHOS-4 tuned model [2] and provides solutions at the same time to the enduring small-scale problems of the $\Lambda$CDM cosmology, is extremely narrow. In the case of fermionic DM, the corresponding constraints differentiate themselves from the ones of the scalar case due to the Born-regime of the SIDM scattering and the more involved thermal evolution towards the measured value of the DM relic abundance $\Omega_{\text{DM}}$. In this section, we provide information about possible realizations presenting the most simple theories which are compatible with all present constraints. More precisely, we examine the existence of a minimal bosonic and fermionic extension of the SM Higgs sector. For simplicity, we do not consider theories with mass-scales larger than the reheating temperature $T_{\text{RH}}$, where the Higgs particles are ultra-relativistic while producing the DM modes.

A. Single field theory

The simplest paradigm of a single field theory is a singlet extension of the Higgs sector; namely, we assume the presence of an uncharged scalar field $X$ [2]. In order to assume stability of DM structures, some symmetry should be imposed, e.g. a $\mathbb{Z}_2$ symmetry of the singlet field. Since the cluster scale constraints dictate $m \ll m_h$ for the case where the thermal production of the DM modes takes place after the electroweak phase-transition, the dominant channels are the feebly Higgs decays. The portal coupling is dimensionful and it is parametrized as $\lambda_X m_h$; the quartic self-coupling is denoted by $\lambda$ and the tachyonic mass parameter $m$. We examine our parameter space and we find that a DM pure scalar field with mass of $3.55 \text{ keV}$ and self-coupling of order $\mathcal{O}(10^{-7})$ realizes the ETHOS-4 tuned model providing also solutions at the cluster scales [20] due to the constant $\mathcal{O}(0.1) \text{ cm}^2 \text{ g}^{-1}$ SIDM cross-section. At dwarf-scales, the $\Lambda$CDM anomalies are alleviated due to sufficiently large cut-off masses and the SIDM virtue of the theory. However, there exist indications that a velocity dependence of the self-interaction cross-sections should be present [4], making the single field theory unable to provide such results. Moreover, such DM masses cannot be solely generated from the Higgs field and additional fine-tuning is necessary.

B. Extended field theories

The absence of velocity dependent SIDM cross-sections in the single field theory hints to a further extension of the dark sector by adding two more fields; however, at low energies, only one field is dynamical, as in the case of the single field theory. Fermionic singlets $\bar{F}$ can only couple to the Higgs field through higher-dimensional operators, as in [7]. Nevertheless, the FIMP production mechanism requires one to take a closer look at such interactions. A simple realization of a Higgs-sourced fermionic theory is the one where the uncharged scalar field $X$ is much heavier than the Higgs field, with the corresponding interaction being in LTE at previous times. Upon this, the $X$ interacts feebly with the fermionic DM fields and, as long as the heavy scalar decays, lighter modes are produced in the same manner as the scalar fields in the single field theory; however, the DM particles are sourced by the heavy scalar and not by the Higgs field directly as in [11]. The corresponding cut-off masses are almost 10% smaller than those appearing in the single field theory due to the larger values of $g^2_\ast$ at the time of the DM production.
A different version of an extended field theory is one where the DM particles are actually sourced directly by the Higgs field, but at the same time the FIMP particles are the heavy scalar $X$ themselves. Therefore, the production of the fermionic DM particles $F$ takes place as long as the density of the heavy scalars is feebly populated, assuming w.l.o.g. $\text{Br}[X \rightarrow FF]/\text{Br}[X \rightarrow hh] \gg 1$ and that $\tau_X \ll t_{1,X}$, otherwise the bound on the lowest DM mass would move towards higher values as $\mu_1 \gg 1$.

The SIDM virtue of these theories arises naturally via dark forces from some 4-Fermi interaction mediated by a hidden boson $B$, which is never thermalized and does not become physical at any time. This type of interaction imposes a further constraint on the parameters of the theory, namely the SIDM scattering rate should not freeze-in, otherwise the DM relic density would be modified through DM annihilations to $B$'s leading to a second WIMP miracle [3]. In addition, the masses are not free to choose, in general $m_B \ll m_F$ and, therefore, theories like the minimal scalar sourced sterile neutrinos [32, 33] are unable to reproduce the SIDM paradigm.

VI. CONCLUSION

In this work, we explored the possibility of the indirect observation of feebly dark matter through its signatures regarding the implied small-scale structure formation. We derived and studied their phase-space distribution functions and showed that such DM particles and their interactions are severely constrained if the generating feebly source is the Higgs field. More precisely, we presented a list of the most recent constraints on such theories, which arise from the small-scale structures of the proposed DM population. The recent Lyman-α bounds [1], together with the positive observations regarding the rotation curves, the missing dwarf satellites and the cored profiles of galaxies discussed in [2], [3], [4], indicate the possible presence of new physics well below the weak scale: multi-keV DM masses and multi-eV masses for the force mediators in the extended field theory case. Furthermore, the feebly theories favor keV values for the mediator mass, i.e. a mass around $M \sim O(1)$ keV is able to provide a solution to the enduring small scale problems of $\Lambda$CDM: annihilations of such modes could correspond to the 3.55 keV mysterious photon-ray [34]. The derived protohalo masses alleviate the missing satellite problem and the SIDM cross-sections, together with the cut-off the linear matter power spectrum, are able to alleviate the cuspy profile and massive subhalos issues of CDM simultaneously and are compatible with all present constraints at the dwarf and cluster scales. The result of this approach is even more interesting in view of the recent observations by the XENON collaboration [14], finding an excess of the electron recoil energy below 7 keV, notably between 2-3 keV, as the focus on FIMPs alleviating the small-scale problems naturally leads to dark matter with masses in this region. It would be interesting to investigate the possible signals of these models in the XENON1T detector in more detail for specific examples.

This family of Higgs-sourced theories could enable a minimalistic scenario of viable thermal evolution solving the enduring small-scale challenges appearing in the $\Lambda$CDM cosmology. However, these toy models do not admit dark acoustic oscillations as ETHOS-4 tuned model, since no kinetic decoupling is present; therefore, further study and simulations are needed in order to examine the exact impact of the free-streaming cut-off on the dwarf scale. The ultra-relativistic limit of such theories could also be interesting to study, as long as new mass-scales above the reheating temperature are introduced.

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