Coherent Baryogenesis

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We propose a new baryogenesis scenario based on coherent production and mixing of different fermionic species. The mechanism is operative during phase transitions, at which the fermions acquire masses via Yukawa couplings to scalar fields. Baryon production is efficient when the mass matrix is nonadiabatically varying, nonsymmetric and when it violates CP and B – L directly, or some other charges that are eventually converted to B – L. We first consider a toy model, which involves two mixing fermionic species, and then a hybrid inflationary scenario embedded in a supersymmetric Pati-Salam GUT. We show that, quite generically, a baryon excess in accordance with observation can result.

PACS numbers: 05.60.Gg, 12.60-i, 98.80.Cq

1. Introduction. Baryogenesis scenarios beyond the standard model often invoke the out-of-equilibrium decay of heavy particles [1, 2]. Well known notable exceptions are the mechanisms proposed by Affleck and Dine [3] and by Cohen and Kaplan [4], both being operative in the presence of scalar condensates.

In this letter, we present a novel baryogenesis scenario involving nonadiabatic evolution of classical scalar fields during phase transitions, at which the baryon-lepton symmetry of the gauge group of a grand-unified theory (GUT) is broken. Yukawa couplings give rise to time dependent mass terms for matter fields and thereby the resonant production of particles [3]. Furthermore, the mass matrices are temporarily nondiagonal, inducing thus flavor oscillations between species with baryonic and leptonic charge. At the end of the phase transition, when the mass matrix becomes diagonal, the charges get frozen-in. Since this mechanism relies on the interplay of coherent particle production and (B – L)-violating flavor oscillations we call it coherent baryogenesis. We emphasise the conceptual simplicity of coherent baryogenesis, since it involves the tree-level dynamics of quantum fields only.

2. Formalism. For our calculations we adapt the kinetic approach (cf. Refs. [5, 6, 7]), which makes use of the dynamics of Wigner functions, which are closely related to classical phase space distributions.

To compute them, we extend the formalism of Ref. [7]. We define the Wigner function,

\[ iS^<(k, x)_{ab} = -\int d^4r e^{ik\cdot r} \langle 0 | \bar{\psi}_b(x - r/2)\psi_a(x + r/2) | 0 \rangle, \]

where \( a, b = 1, ..., N \) are species indices and \( i\gamma^0S^<(\cdot)^\dagger = i\gamma^0S^< \) is hermitian. Our derivation will go through for a 2-point function with general density matrix, but in view of our applications in inflation we prefer to write it with respect to the vacuum from the outset. Introducing an \( N \times N \) matrix \( M \) and its hermitian and antihermitian parts, respectively,

\[ M_H = \frac{1}{2} (M + M^\dagger), \quad M_A = \frac{1}{2i} (M - M^\dagger), \]

we find that \( iS^< \) obeys the Wigner space Dirac equation

\[ \left( \frac{i}{2} \gamma^0 \partial_t - (M_H + i\gamma^5 M_A)e^{-\frac{1}{2}\gamma_5 \partial_t} \partial_0 \right)_{ab} iS^<_{ab} = 0. \quad (1) \]

The mass matrix \( M \) emerges generically from Yukawa couplings to scalar field condensates, \( \mathcal{L}_{Yu} = -y\phi \bar{\psi}_R \psi_L + h.c., \) which can induce CP-violation in the fermionic equations [5, 10, 12, 13]. It is notable, that in coherent baryogenesis the condensate does not have to carry a charge, while this is necessary for the Affleck-Dine mechanism [3], therefore being conceptually different. This can be seen explicitly from the two examples in this letter, where the scalar condensates \( \phi \) are always real, and therefore \( q_\phi = i\langle \phi^\dagger \partial_0 \phi \rangle = 0. \)

However, the time dependence of \( M \) plays here another important rôle. For \( N = 1 \), the Dirac equation consists of two first order differential equations due to the two degrees of freedom of the fermionic field. Thus, also \( dM/dt \) can contribute CP-violation, and in general, both sources from \( M \) and \( dM/dt \) cannot be simultaneously removed by local phase reparametrizations of the fermionic fields, cf. Ref. [3] for an example. When \( N > 1 \), even higher derivatives of \( M \) are involved, allowing, in principle, multiple sources of CP-violation. We stress that this is a very different situation from the Standard Model quark mixing Cabibbo-Kobayashi-Maskawa matrix, or lepton mixing Maki-Nakagawa-Sakata matrix, where at least three generations of quarks or leptons are required for one CP-violating phase.

We now make use of the fact that the helicity operator \( \hat{h} = \mathbf{k} \cdot \gamma^0 \gamma_5 \gamma^5 \), commutes with the Dirac operator in (1) and decompose the Wigner function...
\[ -i\gamma_0 S_h^\pm = \frac{1}{4} (\mathbb{1} + \hbar \mathbf{k} \cdot \mathbf{\sigma}) \otimes \rho^\mu g_{h\mu}, \]  

(2)

where we have omitted the species indices, \( \mathbf{k} = k/|k| \) and \( \sigma^\mu \), \( \rho^\mu \) \((\mu = 0, 1, 2, 3)\) are the Pauli matrices. We multiply \( \mathbb{1} \) by \( \rho^\mu \), take the trace and integrate the hermitian part over \( k_0 \). Introducing the 0th momenta of \( g_{h\mu} \), \( f_{\mu h} = \int (dk_0/2\pi) g_{h\mu} \), we obtain the system of equations

\[
\begin{align*}
\dot{f}_{0h} + i [M_H, f_{1h}] + i [M_A, f_{2h}] &= 0 \quad (3) \\
\dot{f}_{1h} + 2h|k|f_{2h} + i [M_H, f_{0h}] - \{M_A, f_{3h}\} &= 0 \\
\dot{f}_{2h} - 2h|k|f_{1h} + \{M_H, f_{3h}\} + i [M_A, f_{0h}] &= 0 \\
\dot{f}_{3h} - \{M_H, f_{2h}\} + \{M_A, f_{1h}\} &= 0.
\end{align*}
\]

The \( f_{\mu h}(x, k) \) can be interpreted as follows: \( f_{0h} \) is the charge density, \( f_{1h} \) is the axial charge density, and \( f_{2h} \) and \( f_{3h} \) correspond to the scalar and pseudoscalar density, respectively. Note that the commutators in (3), which mix particle flavors, are essential for the production of the charges \( f_{\mu h} \), and thus for our scenario.

For an originally diagonal mass matrix and adiabatic conditions, the initial Wigner functions describing zero particles are (cf. Ref. [3]):

\[
\begin{align*}
 f_{0h}^{ab} &= |\mu h|^{ab} + |R_h^{ab}|^2, \\
 f_{1h}^{ab} &= -2\Re(L_h^{ab} R_h^{*ab}), \\
 f_{2h}^{ab} &= 2\Im(L_h^{ab} R_h^{*ab}), \\
 f_{3h}^{ab} &= \delta_{ab} M_a^* \\
\end{align*}
\]

(4)

with

\[
L_h^{ab} = \delta_{ab} \sqrt{\omega_a + h_k}, \quad R_h^{ab} = \delta_{ab} \frac{M_a^*}{\sqrt{2\omega_a (\omega_a + h_k)}},
\]

where \( \omega_a = \sqrt{k^2 + |M_a|^2} \). For a nondiagonal, but hermitian, \( M \), one obtains initial conditions by an appropriate unitary transformation. If additionally \( M_A \neq 0 \), a biunitary transformation is necessary for diagonalisation.

Since \( f_{0h} \) is the zeroth component of the vector current, the charge of the species \( a \) carried by the mode with momentum \( k \) and helicity \( h \) is simply \( q_{ah}(k) = f_{0h}^{aa} \).

From eqs. [3] one can derive that, in order to generate a nonvanishing charge \( q_{ah}(k) \), i.e. \( f_{0h}^{aa} \neq 0 \), \( M \) should not be symmetric. Note also, that the Lagrangean

\[
\mathcal{L} = \bar{\psi}_a \mathcal{D} \psi_a - \bar{\psi}_h (M_H + i\gamma_5 M_A) \psi_a
\]

is \( U(1) \) symmetric, and thus \( \sum_a q_{ah}(k) \) is conserved.

3. Toy model. We now consider a two species model, where fundamental \( SU(2) \) fermions couple to an adjoint scalar triplet \( \mathbf{\Phi} \) and to a singlet \( \mathbf{\Phi}^0 \), such that the mass matrix is given by \( M = \mathbf{\Phi}^0 \mathbb{1} + \mathbf{\Phi}^0 \mathbf{\sigma} \). While we fix the fields associated with the diagonal generator \( \sigma^3 \) as \( \Phi^0 = \mu \) and \( \Phi^3 = \mu/2 \), with \( \mu \) being a mass scale, we let \( \Phi^1 \) and \( \Phi^2 \) move freely in a harmonic potential, starting form arbitrary initial conditions. Conjugating the fermionic sector of \([4]\) under \( CP \) while leaving the scalar condensate invariant, we find that, in spite of \( M \) being hermitian, \( CP \) is broken for the fermions, because \( M \neq M^* \). Damping is introduced through a phenomenological decay rate \( \Gamma \) and through the Hubble expansion in a matter dominated universe, e.g. the scale factor is \( a = a_m \eta^2 \), where \( \eta \) denotes the conformal time, and \( \dot{\eta} \equiv d/d\eta = ad/dt \). The equation of motion for a scalar \( \Phi(\eta) \) is given by

\[
\Phi'' + 2\frac{\dot{\phi}}{a} \Phi' + a^2 \frac{dV}{d\Phi} + a\Gamma \Phi' = 0.
\]

Writing \( \omega_3^2 = d^2 V/d\Phi^2 \) and setting \( \Gamma = 0 \), the solutions are

\[
\Phi(\eta) = c_1 \eta^3 \cos \left( \frac{1}{3} a_m \omega_3 \eta^3 \right) + c_2 \eta^3 \sin \left( \frac{1}{3} a_m \omega_3 \eta^3 \right)
\]

(6)

For \( \Phi^1 \), we employ \( c_1 = \mu \), \( c_2 = 0 \) and \( \omega_3 = \mu \), for \( \Phi^2 \), \( c_1 = 0 \), \( c_2 = \mu \) and \( \omega_3 = 1.5\mu \), and we set \( a_m = \mu^2 \).

We approximate the effect of damping by multiplying the solutions \([3]\) by \( A = \exp \left( -\frac{\Gamma}{\mu/aM} \eta^3 \right) \), where \( \Gamma = 0.1\mu \ll \omega_3 \). The equations of motions for the Wigner functions in conformal space-time are then simply obtained by replacing \( M \) by \( aM \) in \([3]\ [4]\ [5] \). We illustrate the motion of the mixing fermionic mass terms in conformal time in figure \([4]\).

![FIG. 1: Parametric plot of the motion of \( a(\eta) A \Phi_1 \) and \( a(\eta) A \Phi_2 \) for \( \eta \in [2.3\mu^{-1}, 4\mu^{-1}] \).](image-url)

Requiring that there are no particles at \( \eta = 2.1\mu^{-1} \), we choose initial conditions in accordance with \([4]\) and solve \([3]\) numerically, to find fermion number production as displayed in figure \([2]\) as a function of the conformal momentum \( k \). We sum over the helicities, \( q_a = q_{a+} + q_{a-} \), and note, that in the present case \( q_{a+} = q_{a-} \), for \( M_A = 0 \).

Integration over \( k \), \( q_a = a^{-3} \int d^3k q_a(k)/2\pi^2 \), gives the charge densities \( q_1 = -q_2 = 2.4 \times 10^{-3}(\mu/a)^3 \). If \( q_1 \) was charged under \( B \), our toy model would lead to successful baryogenesis.

4. Hybrid inflation in a supersymmetric Pati-Salam model. We shall now discuss the implementation of coherent baryogenesis in a more realistic model. In order to generate a baryon asymmetry which survives the sphaleron washout, we require the presence of \( (B-L) \)-violation. This is the case in several GUTs, e.g. \( E(6) \),
a subgroup of which is $SO(10)$, and in the Pati-Salam group, $G_{PS} = SU(4)_C \times SU(2)_L \times SU(2)_R$, which appears as an intermediate stage of breaking of $SO(10)$ down to the Standard Model. For simplicity, here we study an extension of the hybrid inflationary scenario embedded in a supersymmetric Pati-Salam model, which is considered in Ref. [13] and does not suffer from the monopole problem. The relevant terms of the superpotential are

$$W \supset \kappa S (H^c H^c - \mu^2) - \beta S \left( \frac{H^c H^c}{M_S} \right)^2 + \zeta G H^c H^c + \xi G H^c H^c.$$  

Under $G_{PS}$, the fields transform as $H^c = (\bar{4}, 1, 2)$, $H^c = (4, 1, 2)$, $S = (1, 1, 1)$, $G = (6, 1, 1)$ [14]. We adopt the notation

$$H^c = \left( \begin{array}{cccc} u_H^c1 & u_H^c2 & u_H^c3 & \nu_H^c \\ d_H^c1 & d_H^c2 & d_H^c3 & \bar{e}_H^c \end{array} \right),$$

and likewise for $\bar{H}^c$. With $SU(4)_C$ broken to the Standard Model, $G = D + \bar{D}$, with $D = 3$ and $\bar{D} = \bar{3}$ of $SU(3)_C$. Vanishing of the D-terms requires $H^c = \bar{H}^c$, which we assume throughout this discussion. During inflation, the neutrino-like component $\nu_H^c$ of $H^c$ has the vacuum expectation value

$$\langle |\nu_H^c| \rangle = M_S |\kappa/(2\beta)|^{\frac{1}{2}},$$

which evolves during the waterfall regime to the supersymmetric vacuum, where $S = 0$ and

$$\langle |\nu_H^c| \rangle = \left( \frac{\kappa M_S^2 - \kappa M_S^2 (\kappa M_S^2 - 4\beta \mu^2)}{2\beta} \right)^{\frac{1}{2}}.$$  

All other components of $H^c$ as well as the fields $G$ vanish at all times. Thus, $|H^c|^2 = H^c H^c = |\nu_H^c|^2$, and eq. (7) implies the following scalar potential:

$$V = 2 \left| S \nu_H^c \left( \kappa - 2\beta |\nu_H^c|^2 \right) \right|^2 + \kappa \left( |\nu_H^c|^2 - \mu^2 \right)^2 - \beta \left| \nu_H^c \right|^4.$$
$F^c \tilde{H}^c$ form a gauge singlet, and $F^c = (\bar{4}, 1, 2)$ are the superfields containing the right handed quarks and leptons,

$$F^c = \begin{pmatrix} u_1^c & u_2^c & u_3^c & \nu^c \\ d_1^c & d_2^c & d_3^c & \ell^c \end{pmatrix}.$$  

This also gives rise to the Lagrangean terms

$$\gamma \langle (\bar{\psi}_H^c) / M_S \rangle [\psi_{d}^c \phi_{d}^c + \psi_{e}^c \phi_{e}^c + c.c.,$$

allowing the decay of the $d_{H1}$-component of $\chi_{1j}$ in $10^4$ to $d^c + \nu^c$, where one of the latter particles is fermionic, the other scalar. The Majorana neutrinos $\nu^c$ are their own antiparticles; neglecting the small effects induced by possible mixing angles and a CP-violating phase in the Majorana mass matrix familiar from leptogenesis scenarios [12], their decay leaves behind no net charge. Note that in turn, our mechanism does not require any lower bound on these angles and the phase as in leptogenesis.

The coupling $\gamma_2 F^c H^c F^c H^c / M_S$ allows the decay of the $H^c$-fields through the reaction from the $d_{H1}^c$-component of $\chi_{2j}$ in $10^4$ to $d^c + \nu^c$. The charges hence get transmuted to $B - L = \frac{1}{2} (\bar{q}_1 - \bar{q}_2)$ This number is promoted by sphaleron processes to $B = (10/31) (B - L)$ [17], where we assumed two complex Higgs doublets.

Hence, the final value of $B - L$ arises here due to the transformation of other charges in decay processes. However, we point out, that models are conceivable where coherent particle production directly leads to standard model particles, the $(B - L)$-charge of which is conserved in the subsequent history of the universe.

To obtain a lower bound on the baryon-to-photon ratio, we need an upper bound for the entropy, which can be obtained by assuming a complete and instantaneous conversion of the vacuum energy $g = [N^2 M_Z^2 (4\beta) - \kappa M^2]^2$ into a thermal bath of highly relativistic particles, with the energy density $\rho = g^* T^4 / 30$, where $g^* = 221.5$ is the number of degrees of freedom in the MSSM. The entropy per unit volume is then $s = 2\pi^2 g^* T^3 / 45$, and we thus estimate the generated baryon-to-photon ratio to be $B / n_\gamma \simeq 9.4 \times 10^{-10}$. Also, a contribution from boson production may arise, which we do not consider here. Note that our result was obtained by choosing a small CP-violating phase (cf. table I), indicating that there is an ample phase space of couplings, which leads to baryon production consistent with the observed $B / n_\gamma = 6.1 \pm 0.3 \times 10^{-10}$ [19]. In conclusion, we have demonstrated that coherent baryogenesis is a viable, efficient and natural candidate for the creation of the baryon asymmetry of the universe.

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FIG. 4: The produced charges of the Dirac fermions $\chi_{1j}, \chi_{2j}$, summed over both helicities.