Influence of shear on plastic bending strength of I cross-sections steel beams

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Abstract. In general, steel beams are stressed by bending and shear. Due to the loading a normal stresses $\sigma$ and shear stresses $\tau$ arise in individual cross-sections of steel beams. But they are usually stressed mainly by bending. This follows both from several theoretical analyzes and especially from the corresponding experiments. Therefore, current standards for the design of steel structures allow the impact of shear to be neglected to a certain level of shear stress of the most stressed - decisive cross-sections of beams (EN 1993-1-1, respectively CSN EN 1993-1-1 and others). However, in the case of shorter beams, especially in places of support and in places of application of concentrated forces, the shear effects of the load can also be significant. In the case of more significant shear effects of the load, the plastic flexural strength of the decisive cross-sections and also the overall load-bearing capacity of the steel beams must be adequately reduced. However, the degree of flexural strength reduction with a significant effect of shear in the case of I cross-sections is still unambiguous. It is therefore important to experimentally investigate steel beams in the area of significant shear effects. The presented article contains selected results of experimental investigation of the significant effect of shear on the flexural strength of steel beams I cross-sections.

1. Introduction
Due to the loading and adequate normal stresses $\sigma$, and shear stresses $\tau$ the steel beams can be loaded in the elastic, elastic-plastic and plastic stress stage. The strength theory of constant deformation work by Huber-Misses-Hencky, is generally used to define the individual stages of stress in steel structures. According to this theory, the resulting stress under the assumed combination stress

$$\sigma_{\pi} = \sqrt{\sigma^2 + 3\tau^2}$$

(1)

and the plasticity condition has a shape

$$\sigma_{\pi} = \sqrt{\sigma^2 + 3\tau^2} = f_y$$

(2)

where $f_y$ is the yield stress.

In the elastic stress stage the stress $\sigma_{\pi} < f_y$ should be, in the limiting elastic stress stage $\sigma_{\pi} = f_y$ is in the only most stressed place of the beam. The gradual plasticization occurs in the most stressed cross-
sections and areas of the beam, when the limit elastic stage exceed. In the elastic-plastic stage of stress, the condition \( \sigma_{se} = f_y \) is fulfilled in the part of the most stressed cross-section or cross-sections. There is a condition in the plastic stage of stress \( \sigma_{pl} = f_y \) fulfilled in at least one of the most stressed cross-sections of the beam [1, 2].

2. Theoretical analysis

Assessing the effect of shear on the load-bearing capacity of steel beams at their elastic stress stage is not a problem. It follows from the assumed distribution of normal and shear stresses along the height of the decisive cross-section and the strength condition (1), resp. (2). The distribution of normal stresses \( \sigma \) varies linearly along the section depth. The distribution of shear stresses \( \tau \) in the case of I cross-section is usually considered constant along the web depth. In the elastic-plastic stage of steel beams, the stress distribution \( \sigma \) and \( \tau \) is not clear along the depth of the cross-sections, therefore the assessment of the influence of shear on their bending strength is problematic.

At present, there are a number of theoretical solutions for plastic bending-shear strength of steel beam cross-sections. They are usually based on the assumption of a certain stress distribution \( \sigma \) and \( \tau \) in the limiting plastic stage. For the considered stress distribution \( \sigma \) and \( \tau \), the respective interaction dependences of the bending and shear strengths are then derived [1, 2]. The simplest case of stress distribution \( \sigma \) and \( \tau \) in the ultimate stress stage of the assumed I cross-section is illustrated in Figure 1.

**Figure 1.** The simplest case of stress distribution \( \sigma \) and \( \tau \) in the plastic stage of stress

The distribution of the corresponding shear stresses \( \tau \) over the whole web depth is assumed. Therefore, normal stresses in the web \( \sigma < f_y \). The presumed stress distribution \( \sigma \) and \( \tau \) in the ultimate plastic stage of the stresses correspond the interaction equation (3). This interaction equation is also applied in the standards EN 1993-1-1, CSN EN 1993-1-1 and others.

\[
\frac{M}{M_{pl}} + \frac{M_{plw}}{M_{pl}} \left( \frac{2V}{V_{pl}} - 1 \right)^2 = 1. \tag{3}
\]
The intention of several previous theoretical solutions was to take into account to a greater extent the stress distribution $\sigma$ and $\tau$ corresponding to the initial elastic stage of cross-sectional stress. Such a theoretical solution is e.g. solution of B. M. Broude [3]. The original theoretical solution was also elaborated in the previous period by the first author of this paper [4, 5]. The peculiarity of this solution was also that I considered a cross section combined from two steels, while the steel of the flanges is of higher strength than the steel of the web ($f_{yf} > f_{yw}$). It should be noted that more consistent theoretical solutions lead to more complex expressions of the dependence of bending-shear strength I cross-sections stressed in the elastic-plastic region. Corresponding experimental results therefore appear to be particularly important for the practical interpretation of these dependences.

3. Experimental verification
From the previous analysis, it seems reasonable to verify in particular plastic load-bearing capacity and failure of steel beams in cross-sections and areas with significant shear effect ($V > 0.5 V_{pl}$). The partial experimental program consisted of testing a total of 6 beams, three rolled sections I 160 of S 235 steel and three welded I sections, dimensions and materials adapted to the rolled beam. The individual beams had different lengths, resp. span $l$ in order to show a different effect of shear on their bending strength. Test program, designation and ranges of individual groups, resp. pairs of beams are obviously from Table 1.

| Beams          | Designation | Span I  |
|----------------|-------------|--------|
| Rolled I 160   |             |        |
| NV1            | 500 mm      |
| NV2            | 650 mm      |
| NV3            | 800 mm      |
| NZ1            | 500 mm      |
| NZ2            | 650 mm      |
| NZ3            | 800 mm      |
| Welded I 160   |             |        |

The actual material properties of the beams were determined by standard tensile tests. From the obtained working diagrams, the yield strength $f_y$, the yield strength $f_u$ and the elongation $A_5$ were determined, especially for rolled and welded beams (Table 2). The corresponding cross-sectional characteristics determined for the individual beams are given in Table 3.

| Table 1. Test program |
|-----------------------|
| Beams          | Designation | Span I  |
|----------------|-------------|--------|
| Rolled I 160    |             |        |
| NV1             | 500 mm      |
| NV2             | 650 mm      |
| NV3             | 800 mm      |
| NZ1             | 500 mm      |
| NZ2             | 650 mm      |
| NZ3             | 800 mm      |

| Table 2. Material properties |
|------------------------------|
| Rolled | Welded |
| $f_y$ [MPa] | 280 | 295 |
| $f_u$ [MPa] | 415 | 452 |
| $A_5$ [%]  | 28  | 25  |

All beams were loaded symmetrically with two concentrated forces $F$ with a spacing of 100 mm. They were examined during gradual loading and unloading:
- relative deformations $\varepsilon$ in the middle cross-section on the flanges and in the middle of the web by means of strain gauges (1,2, to 9),
- beam deflections in above the supports (settling) and in the middle of the beam by means of electrical resistance sensors (10,11,12).
The beams were loaded until the total failure using a computer-controlled hydro pulse device. Individual relative deformations $\varepsilon$ and deflections $v$, resp. beam settlements were measured, registered and evaluated using control panel and computer. The overall arrangement of the tests is evident from Figure 2.

**Table 3. Cross-sectional characteristics of individual test beams**

| Beams | NV1      | NV2      | NV3      | NZ1      | NZ2      | NZ3      |
|-------|----------|----------|----------|----------|----------|----------|
| $A$ [mm$^2$] | 2331.41  | 2269.79  | 2282.34  | 2438.95  | 2475.03  | 2426.83  |
| $A_w$ [mm$^2$] | 981.06   | 964.93   | 977.87   | 937.97   | 948.61   | 928.35   |
| $W_{el}$ [mm$^3$] | 117480   | 114563   | 114814   | 126755   | 128285   | 126900   |
| $W_{pl}$ [mm$^3$] | 147255   | 143210   | 143768   | 157466   | 159669   | 157149   |
| $I_y$ [mm$^4$]   | 9468900  | 9242387  | 9262664  | 10270287 | 10281450 | 10262246 |
| $W_{el,w}$ [mm$^3$] | 23333    | 23075    | 23386    | 22216    | 22385    | 22345    |
| $W_{pl,w}$ [mm$^3$] | 35000    | 34610    | 35079    | 33324    | 33578    | 33017    |

**Figure 2.** Overall arrangement of the tests

4. **Experimental knowledge and results**

The rolled beams behaved flexibly up to a load of $F_{el,exp}$. This was manifested by the rapid stabilization of the deflections in the middle of the beams. After this loading, the inelastic behavior of the beam began to manifest itself due to plasticization and local deformations of the compressed flange at the place of loading. However, the beam deflections gradually stabilized. At certain load $F$ the deflections of the beams gradually increased due to plasticization in the middle cross-section and local deformation of the loaded flange. This load can be considered as the ultimate plastic experimental load $F_{pl,exp}$. 
The welded beams were done similarly to rolled beams. Up to a certain load, they behaved flexibly, resp. quasi flexible. This load is considered to be the ultimate elastic load $F_{el,exp}$. At certain load $F$ the deflections of the beams gradually increased due to plasticization of the middle part and local deformation the loaded flange. This load can be considered as the ultimate plastic experimental load $F_{pl,exp}$. The determined experimental limit loads $F_{el,exp}$ and $F_{pl,exp}$ are given for all tested beams in Table 4.

**Table 4.** Experimental limit loads $F_{el,exp}$ and $F_{pl,exp}$.

| Beams | NV1  | NV2  | NV3  | NZ1  | NZ2  | NZ3  |
|-------|------|------|------|------|------|------|
| $F_{el,exp}$ | 215.00 | 195.00 | 155.00 | 230.00 | 180.00 | 155.00 |
| $F_{u,exp}$   | 264.49 | 223.04 | 197.90 | 309.21 | 256.08 | 234.46 |

The failure of both rolled and welded I beams is illustrated in Figure 3.

![Figure 3. Failure of tested beams](image)

Using the plastic strength condition (2) and the corresponding theoretical solutions, the individual theoretical solutions were determined:
- $F_{el,M}$ – elastic bending load,
- $F_{el,MV}$ – elastic bending-shear load,
- $F_{pl,M}$ – plastic bending load,
- $F_{pl,MV,1}$ – plastic bending-shear load by standard CSN EN 1993-1-1,
- $F_{pl,MV,2}$ – plastic bending-shear load by standard ISO 10721-1,
- $F_{pl,MV,3}$ – plastic bending-shear load by B. M Broude [3],
- $F_{pl,MV,4}$ – plastic bending-shear load by P. Juhas [4].

All considered theoretical loads calculated for individual test beams are given in Table 5. The comparison of individual theoretical limit loads $F_{pl,MV,1}$ with the corresponding experimental loads $F_{u,exp}$ of rolled and welded test beams is contained in Table 6.
Table 5. Limit theoretical elastic and plastic loads

| Beams   | NV1       | NV2       | NV3       | NZ1       | NZ2       | NZ3       |
|---------|-----------|-----------|-----------|-----------|-----------|-----------|
| $F_{el,M}$ | 300.82    | 239.23    | 183.30    | 373.90    | 275.20    | 213.94    |
| $F_{el,MV}$ | 210.93    | 190.99    | 158.04    | 242.91    | 209.53    | 175.79    |
| $F_{pl,M}$ | 376.67    | 299.86    | 229.14    | 464.52    | 352.56    | 264.91    |
| $F_{pl,MV,1}$ | 290.85    | 266.73    | 219.85    | 339.74    | 294.09    | 245.84    |
| $F_{pl,MV,2}$ | 288.98    | 254.12    | 205.18    | 347.94    | 286.09    | 233.87    |
| $F_{pl,MV,3}$ | 362.57    | 258.64    | 207.29    | 385.65    | 285.81    | 222.86    |
| $F_{pl,MV,4}$ | 287.37    | 255.64    | 203.72    | 332.62    | 286.10    | 233.03    |

Table 6. Mutual comparison of corresponding experimental and theoretical limit loads

| Beams   | NV1       | NV2       | NV3       | NZ1       | NZ2       | NZ3       |
|---------|-----------|-----------|-----------|-----------|-----------|-----------|
| $F_{u,exp}/F_{pl,MV,1}$ | 0.909     | 0.836     | 0.901     | 0.910     | 0.871     | 0.954     |
| $F_{u,exp}/F_{pl,MV,2}$ | 0.915     | 0.878     | 0.965     | 0.889     | 0.895     | 1.002     |
| $F_{u,exp}/F_{pl,MV,3}$ | 0.729     | 0.862     | 0.955     | 0.802     | 0.896     | 1.052     |
| $F_{u,exp}/F_{pl,MV,4}$ | 0.920     | 0.872     | 0.971     | 0.930     | 0.895     | 1.006     |

5. Conclusions
Experimental results confirm the effect of shear on the plastic flexural strength of I cross-section beams with significant transverse forces ($V > 0.5 V_{pl}$). The values of the ratios $F_{u,exp}/F_{pl,MV,i}$ indicate that the experimental load capacities of the beams are smaller than the theoretical ones, resulting from the individual solutions. The experimental loads of $F_{u,exp}$ are closest to the theoretical $F_{pl,MV,4}$ which result from the theoretical solution of Pavol Juhás ($F_{u,exp}/F_{pl,MV,4} = 0.933$). The course of the tests showed a certain influence of experimental loads due to beam imperfections and local plasticization of the flange at the site of concentrated load transfer. Therefore we can assume better agreement of experimental and theoretical plastic loads of the beams without the observed adverse effects.

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