There exist consistent temporal logics admitting changes of History

Gavriel Segre

Introducing his Chronology Protection Conjecture Stephen Hawking said that it seems that there exists a Chronology Protection Agency making the Universe safe for historians.

Without taking sides about such a conjecture we show that the Chronology Protection Agency is not necessary in order to make the Universe unsafe for historians but safe for logicians.
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I. INTRODUCTION

Given a time-orientable space-time \((M, g_{ab})\) let us recall that \([1], [2]\):

Definition I.1

chronology violating set of \((M, g_{ab})\):

\[
V_{\text{chronology}}(M, g_{ab}) := \cup_{p \in M} I^+(p) \cap I^-(p) \tag{1.1}
\]

Definition I.2

chronology horizon of \((M, g_{ab})\):

\[
H_{\text{chronology}}(M, g_{ab}) := \partial [I^+(V_{\text{chronology}}(M, g_{ab}))] \tag{1.2}
\]

A curious feature of General Relativity is that there exist solutions \((M, g_{ab})\) of Einstein’s equation such that 

\[ V_{\text{chronology}}(M, g_{ab}) \neq \emptyset. \]

The so called time travel paradoxes then occur.

These paradoxes can be divided in two classes:

- consistency paradoxes involving the effects of the changes of the past (epitomized by the celebrated Grandfather Paradox in which a time-traveller goes back in the past and prevents the meeting of his grandfather and his grandmother)

- bootstrap paradoxes involving the presence of loops in which the source of the production of some information disappears (as an example let us suppose that Einstein learnt Relativity Theory from \([1], [2]\) given to him by a time-traveller gone back to 1904).

They has been faced by the scientific community in different ways (see the fourth part ”Time Travel” of \([3]\) as well as \([4]\)):

1. adding to General Relativity some ad hoc axiom precluding the physical possibility of causal loops (such as the strong form of Penrose’s Cosmic Censorship Conjecture)

2. appealing to consistency conditions (such as in Novikov’s Consistency Conjecture) requiring that causal loops, though allowing causal influence on the past, don’t allow alteration of the past

3. arguing that the problem is removed at a quantum level (such as in Hawking’s Chronology Protection Conjecture \([5]\) stating that the classical possibilities to implement time-travels are destroyed by quantum effects)

4. arguing that the so called time-travel paradoxes are only apparent and may be bypassed in a mathematical consistent way

As to General Relativity we think that discarding tout court non globally-hyperbolic solutions of Einstein’s equation considering them ”unphysical” is a conceptually dangerous operation:

in presence of ”unphysical” solutions of physical equations we have always to remember that our intuition is not a neutral quality but is affected by the Physics to which we are used.

Consequentially, in presence of new Physics, it is natural that it appears to us as counter-intuitive.

If Dirac had discarded as ”unphysical” the negative-energy solutions of his equation he would have never predicted the existence of anti-matter \([6]\).

While we agree with Visser’s \([4]\) idea that solutions with ”non-localized” chronology violating set may be seen as produced by a sort of garbage in-garbage out phenomenon (where the garbage in are perverse initial-value conditions and the garbage-out are the resulting perverse solutions) we think that solutions with ”localized” chronology violating set should be taken seriously.

Of course, depending on the precise mathematical definition that we adopt for the term ”localized”, we may arrive to different conclusions.

A minimal definition of the term ”localized” would consist in imposing that \(V_{\text{chronology}}(M, g_{ab}) \neq M\).

This is sufficient to discard G"odel’s solution, the Van Stockum - Tipler time machine, some spinning cosmic string time machine but not Gott time machine that may be excluded only assuming a more restrictive definition of ”localized” as ”suitably bounded”. 
An other argument often used in the literature consists in the refutation as "unphysical" of any non asymptotically-flat space-time; this is (at least) curious: the fact that asymptotically flatness is a condition required in order to be able to define a black-hole (as the complement of the causal past of future null infinity $B := M - J^- (I^+)$) is not a good reason to assume as "physicality"'s criterion one incompatible with the assumption of homogeneity and isotropy under which the Friedmann-Robertson-Walker solutions of Classical Cosmology are derived.

Finally we agree with the Headrick-Gott’s refutation of the claim that the observed absence of time-travellers in the present and in the past would be an empirical datum supporting the physical impossibility of time-travel: no experimental fact on $M - H_{\text{chronology}}(M, g_{ab})$ can be used as an argument in favour or against the hypothesis that $V_{\text{chronology}}(M, g_{ab}) \neq \emptyset$.

When also Quantum Mechanics is taken into account we again agree with Visser’s viewpoint according to which the Kay-Radzikowski-Wald singularity theorems (stating that in presence of a non-empty chronology-violating set there are points of the chronology horizon where the two-point function is not of Hadamard form) has to be interpreted not as a support of Hawking’s Chronology Protection Conjecture but as an evidence of the fact that in such a situation Semi-classical Quantum Gravity (defined as Quantum Field Theory on a fixed curved background augmented with the semi-classical Einstein equation $R_{ab} - \frac{1}{2}Rg_{ab} = 8\pi \langle \hat{T}_{ab}\rangle$ taking into account the back-reaction of the quantum fields on the spacetime’s geometry) is not a good approximation of Quantum Gravity.

Hence we think that the status of Hawking’s Chronology Protection Conjecture may be decided only at the Quantum Gravity level.

Since most of the phenomenology of Quantum Gravity is detectable only at the Planck scale (lengths of the magnitude of the Planck length $l_{\text{Planck}} \sim 10^{-33} \text{ cm}$, energies of the magnitude of the Planck energy $E_{\text{Planck}} \sim 10^{19} \text{ GeV}$) that is enormously far from our present possibility of experimental investigation, the quantum status of Hawking’s Chronology-Protection Conjecture may be at present investigated only on a theoretical basis, all the rival alternative proposals (String Theory, Loop Quantum Gravity, Connes’ Quantum Gravity, Simplicial Quantum Gravity, Prugovecki’s Quantum Gravity, Finkelstein’s Quantum Gravity, · · ·) being far from giving, on this issue, clear and univocal answers.

Without taking sides about the physical possibility of chronology violations, in this paper we will show that, from a logical viewpoint, the so-called time travel paradoxes may be seen as a consequence of the fact that, in presence of chronology violations, we insist on the adoption of the usual Temporal Logic.

The adoption of suitable unusual Temporal Logics allows to get rid of any mathematical inconsistency.

Indeed not only the affection of the past but even its change can be formalized by suitable consistent Temporal Logics.
II. TEMPORAL LOGICS

Temporal Logics are the particular Modal Logic in which the following temporal operators are introduced \[19\]:

- \( P := "it has been true that" \)
- \( H := "it has always been true that" \)
- \( F := "it will be true that" \)
- \( G := "it will always be true that" \)

The absence of any temporary operator in a formula in Temporary Logic means that it is temporally referred to the present time:

\[ x := "x is true at the present time" \] (2.1)

To avoid too many parenthesis we will assume that the temporal operators \( P, H, F, G \), are at the top of the strength’s hierarchy of Propositional Logic introduced in the appendix A.

Remark II.1

We adhere to Arthur Prior’s traditional notation.

From a conceptual viewpoint is anyway important to remark that:

1. \( P \) is the past existential quantificator. This fact may be remarked introducing the notation:

\[ \exists_- := P \] (2.2)

2. \( H \) is the past universal quantificator. This fact may be remarked introducing the notation:

\[ \forall_- := H \] (2.3)

3. \( F \) is the future existential quantificator. This fact may be remarked introducing the notation:

\[ \exists_+ := F \] (2.4)

4. \( G \) is the future universal quantificator. This fact may be remarked introducing the notation:

\[ \forall_+ := G \] (2.5)

In the case in which one assume a nonrelativistic notion of absolute time taking values on \( \mathbb{R} \) and identifying conventionally the present time with \( t = 0 \) the temporal operators may be defined considering time-dependent propositions of the form:

\[ x(t) := "The proposition x is true at time t" \] (2.6)

and defining them as:

**Definition II.1**

\[ Px := \exists t \in (-\infty, 0) : x(t) \] (2.7)

\[ Hx := x(t) \forall t \in (-\infty, 0) \] (2.8)

\[ Fx := \exists t \in (0, +\infty) : x(t) \] (2.9)

\[ Gx := x(t) \forall t \in (0, +\infty) \] (2.10)

\[ x := x(0) \] (2.11)
Remark II.2

Assuming a nonrelativistic notion of time with a nontrivial topological structure (such as, for instance, those discussed in [20]) or a relativistic notion of time one has to give up the definition II.1

Remark II.3

Given a formula containing a composed temporal operator of the form $xy \ x, y \in \{P, H, F, G\}$ let us observe that $x$ alters the present time for $y$.

In the case in which the the definition II.1 may be adopted this fact can be expressed by the following:

Proposition II.1

HP:

The definition II.1 is assumed

TH:

1. $PPx = \exists t_1 \in (-\infty, 0) : (\exists t \in (-\infty, t_1) : x(t))$ (2.12)
2. $PHx = \exists t_1 \in (-\infty, 0) : (x(t) \forall t \in (-\infty, t_1))$ (2.13)
3. $PFx = \exists t_1 \in (-\infty, 0) : (\exists t \in (t_1, +\infty) : x(t))$ (2.14)
4. $PGx = \exists t_1 \in (-\infty, 0) : (x(t) \forall t \in (t_1, +\infty))$ (2.15)
5. $HPx = (\exists t \in (-\infty, t_1) : x(t)) \forall t_1 \in (-\infty, 0)$ (2.16)
6. $HHx = (x(t) \forall t \in (-\infty, t_1)) \forall t_1 \in (-\infty, 0)$ (2.17)
7. $HFx = (\exists t \in (t_1, +\infty) : x(t)) \forall t_1 \in (-\infty, 0)$ (2.18)
8. $HGx = (x(t) \forall t \in (t_1, +\infty)) \forall t_1 \in (-\infty, 0)$ (2.19)
9. $FPx = \exists t_1 \in (0, +\infty) : (\exists t \in (-\infty, t_1) : x(t))$ (2.20)
PROOF:
The thesis follows by multiple application of the definition II.1.

Let us observe that the temporal operators H and G may be defined in the following way:

**Definition II.2**

\[ Hx := \neg Px \]  
\[ Gx := \neg Fx \]

Let us now introduce the following basic:

**Definition II.3**

*Temporal Logic*:

A modal logic obtained adding to Classical Propositional Logic (see appendix A) the temporal operators P, H, F, G and suitable axioms and inference rules governing their behavior among which there are the following:

\[ AXIOM_- := Hx \rightarrow Px \]  
\[ AXIOM_+ := Gx \rightarrow Fx \]

**Remark II.4**

Given a temporal logic it is important to keep attention about the meaning given to a logical variable x appearing in a formula.

We will assume that x will continue to have the same meaning it had in Propositional Calculus (defined by the definition A.7).

Consequently x will not be allowed to contain temporal operators.

**Remark II.5**

Let us remark that if the definition II.1 is assumed the axiom \( AXIOM_- \) and \( AXIOM_+ \) become trivially provable statements.
III. CHANGING THE PAST

Given a temporal logic $T$:

**Definition III.1**

$T$ admits changes of History:

1. "The fact that $x$ is now true doesn’t imply that it will be always true that $x$ has been true”
   \[ \neg (x \rightarrow GPx) \]  
   (3.1)

2. "The fact that $x$ has been true doesn’t imply that it will be always true that $x$ has been true”
   \[ \neg (Px \rightarrow GPx) \]  
   (3.2)

3. "The fact that $x$ has always been true doesn’t imply that it will be always true that $x$ has been true”
   \[ \neg (Hx \rightarrow GPx) \]  
   (3.3)

4. "The fact that $x$ is now true doesn’t imply that it will be true that $x$ has been true”
   \[ \neg (x \rightarrow FPx) \]  
   (3.4)

5. "The fact that $x$ has been true doesn’t imply that it will be true that $x$ has been true”
   \[ \neg (Px \rightarrow FPx) \]  
   (3.5)

6. "The fact that $x$ has always been true doesn’t imply that it will be true that $x$ has been true”
   \[ \neg (Hx \rightarrow FPx) \]  
   (3.6)

Let us remark that:

**Proposition III.1**

HP:

1. $T$ temporal logic
2. the definition II.1 is assumed

TH:

$T$ doesn’t admit changes of History

PROOF:

1. 
   \[ x = x(0) \]  
   (3.7)

while:

\[ GPx = (\exists t \in (-\infty, t_1) : x(t)) \forall t_1 \in (0, +\infty) \]  
(3.8)

that considering in the right hand side the value $t = 0$ implies that:

\[ x \rightarrow GPx \]  
(3.9)

that applying the $AXIOM_+$ of definition II.3 and the theorem A.7 implies that:

\[ x \rightarrow FPx \]  
(3.10)
2. \[ Px = \exists t \in (-\infty, 0) : x(t) \] (3.11)

while:

\[ GPx = (\exists t \in (-\infty, t_1) : x(t)) \forall t_1 \in (0, +\infty) \] (3.12)

that considering as particular value of \( t \) in the right-hand side of eq. 3.12 the one whose existence is stated by the right-hand side of eq. 3.11 implies that:

\[ Px \rightarrow GPx \] (3.13)

that applying the AXIOM of definition II.3 and the theorem A.7 implies that:

\[ Px \rightarrow FPx \] (3.14)

3. \[ Hx = x(t) \forall t \in (-\infty, 0) \] (3.15)

so that considering any value \( t \in (-\infty, 0) \) as the \( t \) in the right-hand side of eq. 3.12 implies that:

\[ Hx \rightarrow GPx \] (3.16)

that applying the AXIOM of definition II.3 and the theorem A.7 implies that:

\[ Hx \rightarrow FPx \] (3.17)

Let us now prove two propositions that will be our key ingredients in analyzing the consistency of temporal histories admitting changes of History.

**Proposition III.2**

**HP:**

1. T temporal logic
2. T admits changes of History

**TH:**

"The fact that the Principle of Contradiction now holds doesn’t imply that it will always be true that the Principle of Contradiction held ":

\[ \neg[\neg(x \land \neg x) \rightarrow GP\neg(x \land \neg x)] \] (3.18)

**PROOF:**

The thesis immediately follows applying the definition III.1 to the Principle of Noncontradiction (the theorem A.3) holding at the present time ■

The conceptual reason why the Principle of Noncontradiction is so important in Propositional Logic (as well as in Predicative Logic of 1\(^{st}\) and 2\(^{nd}\) order) derives by the Scotos Theorem (the theorem A.5) asserting that "ex absurdum quodlibet sequitur", i.e. that every proposition can be inferred starting from a contradiction.

The situation is deeper in a temporal logic admitting changes of History:
Proposition III.3

"The fact that everything may be proved from a contradiction doesn’t imply that it will always be true that everything could be proved from a contradiction"

HP:

1. T temporal logic
2. T admits changes of History

TH:

\[ \neg[(x_1 \land \lnot x_1 \rightarrow x_2) \rightarrow GP(x_1 \land \lnot x_1 \rightarrow x_2)] \quad (3.19) \]

PROOF:
The thesis immediately follows applying the definition III.1 to the Scotus Theorem (the theorem A.3) holding at the present time.

Let us now analyze the notion of consistency of a temporal logic.

Given a temporal logic T:

Definition III.2

*T is consistent:*  
\[ \neg(x \land \lnot x) \quad (3.20) \]

Then:

Proposition III.4

every temporal logic is consistent

PROOF:
The thesis is nothing but theorem A.3 of Propositional Logic.

Definition III.3

*T is always-consistent:*  
\[ \neg[x \land \lnot x] \land [H\neg(x \land \lnot x)] \land [G\neg(x \land \lnot x)] \quad (3.21) \]

Proposition III.3 shows that always-consistency is not necessary to exorcize the "ex absurdo quodlibet sequitur" phenomenon.

Anyway even imposing always-consistency changes of History are not banned:

Proposition III.5

\[ \exists T \text{ always-consistent temporal logic admitting changes of History} \quad (3.22) \]

PROOF:
It is sufficient to take the axioms of T such that:

"the fact that the fact that Principle of Contradiction now holds doesn’t imply that it will always be true that the Principle of Contradiction held doesn’t imply that the Principle of Contradiction didn’t hold":

\[ \neg\{\neg[n(x \land \lnot x) \rightarrow GP\neg(x \land \lnot x)] \rightarrow P(x \land \lnot x)\} \quad (3.23) \]
APPENDIX A: CLASSICAL PROPOSITIONAL LOGIC

In this section we will briefly review the basic notions of Propositional Calculus in its boolean formulation [21]:

Given the binary alphabet $\Sigma := \{0, 1\}$ (where 1 is identified with "true" and 0 is identified with "false") let us introduce the following logical operators:

**Definition A.1**

*negation:*

$\neg : \Sigma \mapsto \Sigma$:

$$
\neg 0 := 1 \quad (A1) \\
\neg 1 := 0 \quad (A2)
$$

**Definition A.2**

*conjunction:*

$\land : \Sigma \times \Sigma \mapsto \Sigma$:

$$
0 \land 0 := 0 \quad (A3) \\
0 \land 1 := 0 \quad (A4) \\
1 \land 0 := 0 \quad (A5) \\
1 \land 1 := 1 \quad (A6)
$$

**Definition A.3**

*disjunction:*

$\lor : \Sigma \times \Sigma \mapsto \Sigma$:

$$
0 \lor 0 := 0 \quad (A7) \\
0 \lor 1 := 1 \quad (A8) \\
1 \lor 0 := 1 \quad (A9) \\
1 \lor 1 := 1 \quad (A10)
$$

**Definition A.4**

*implication:*

$\to : \Sigma \times \Sigma \mapsto \Sigma$:

$$
0 \to 0 := 1 \quad (A11) \\
0 \to 1 := 1 \quad (A12) \\
1 \to 0 := 0 \quad (A13) \\
1 \to 1 := 1 \quad (A14)
$$
Definition A.5

biimplication:
\[ \leftrightarrow : \Sigma \times \Sigma \rightarrow \Sigma : \]

\[ 0 \leftrightarrow 0 := 1 \quad \text{(A15)} \]
\[ 0 \leftrightarrow 1 := 0 \quad \text{(A16)} \]
\[ 1 \leftrightarrow 0 := 0 \quad \text{(A17)} \]
\[ 1 \leftrightarrow 1 := 1 \quad \text{(A18)} \]

In order of avoiding too many parentheses we will assume the following (traditional) hierarchy of strength for the logical connectives:

1. \( \neg \)
2. \( \land \) and \( \lor \)
3. \( \rightarrow \) and \( \leftrightarrow \)

Clearly:

**Theorem A.1**

\[(x_1 \leftrightarrow x_2) \leftrightarrow ((x_1 \rightarrow x_2) \land (x_2 \rightarrow x_1)) \quad \text{(A19)}\]

Given a number \( n \in \mathbb{N}_+ \) and a map \( f : \Sigma^n \rightarrow \Sigma : \)

**Definition A.6**

\( f \) is a tautology:

\[ f(x_1, \ldots, x_n) = 1 \; \forall x_1, \ldots, x_n \in \Sigma \quad \text{(A20)} \]

We will adopt the following:

**Definition A.7**

logical notation:

\[ f(x_1, \ldots, x_n) := f \text{ is a tautology} \quad \text{(A21)} \]

It can be easily verified that:

**Theorem A.2**

Double negation affirms:

\[ \neg \neg x \leftrightarrow x \quad \text{(A22)} \]

**Theorem A.3**

Principle of Noncontradiction:

\[ \neg (x \land \neg x) \quad \text{(A23)} \]

**Theorem A.4**
De Morgan Laws:

\[ x_1 \land x_2 \leftrightarrow \neg(\neg x_1 \lor \neg x_2) \quad \text{(A24)} \]
\[ x_1 \lor x_2 \leftrightarrow \neg(\neg x_1 \land \neg x_2) \quad \text{(A25)} \]

Corollary A.1

Tertium non datur:

\[ x \lor \neg x \quad \text{(A26)} \]

PROOF:

The thesis follows applying the theorem A.4 and the theorem A.3 □

Theorem A.5

Scotus Theorem (ex absurdo quodlibet sequitur):

\[ x_1 \land \neg x_1 \rightarrow x_2 \quad \text{(A27)} \]

PROOF:

\[ x_1 \land \neg x_1 \rightarrow x_1 \quad \text{(A28)} \]
\[ x_1 \land \neg x_1 \rightarrow \neg x_1 \quad \text{(A29)} \]
\[ \neg x_1 \rightarrow \neg x_1 \lor x_2 \quad \text{(A30)} \]
\[ x_1 \land (\neg x_1 \lor x_2) \rightarrow x_2 \quad \text{(A31)} \]

Combining equation A28, equation A29, equation A30 and equation A31 the thesis follows □

It may be also easily verified that:

Theorem A.6

Reflectivity of implication:

\[ x \rightarrow x \quad \text{(A32)} \]

Theorem A.7

Transitivity of implication:

\[ ((x_1 \rightarrow x_2) \land (x_2 \rightarrow x_3)) \rightarrow (x_1 \rightarrow x_3) \quad \text{(A33)} \]

from which it follows that:

Corollary A.2

Modus ponens:

\[ x_1 \land (x_1 \rightarrow x_2) \rightarrow x_2 \quad \text{(A34)} \]

Furthermore it may be easily verified that:

Theorem A.8
Principle of Contraposition:

\[(x_1 \to x_2) \leftrightarrow (\neg x_2 \to \neg x_1)\]  \hspace{1cm} (A35)

from which it follows that:

**Corollary A.3**

*Modus tollens:*

\[\neg x_2 \land (x_1 \to x_2) \to \neg x_1\]  \hspace{1cm} (A36)

Furthermore:

**Theorem A.9**

*Reductio ad absurdum:*

\[ (\neg x_1 \to x_2 \land \neg x_2) \to x_1 \]  \hspace{1cm} (A37)

**PROOF:**

The thesis immediately follows combining theorem [A.3] and theorem [A.8] \(\blacksquare\)

Classical Propositional Calculus may be formalized as a formal systems in many ways, for instance choosing as axiom the theorem [A.3] and as rules of inference the theorem [A.6] theorem [A.7] and the theorem [A.8]
### APPENDIX B: NOTATION

| i.e. | id est |
|------|--------|
| ∀    | for all (universal quantifier) |
| ∃    | exists (existential quantifier) |
| $G$, $\forall_+$ | future universal quantifier |
| $H$, $\forall_-$ | past universal quantifier |
| $F$, $\exists_+$ | future existential quantifier |
| $P$, $\exists_-$ | past existential quantifier |
| $x = y$ | $x$ is equal to $y$ |
| $x := y$ | $x$ is defined as $y$ |
| $\partial S$ | boundary of $S$ |
| $I^+(S)$ | chronological future of the set $S$ |
| $I^-(S)$ | chronological past of the set $S$ |
| $J^+(S)$ | causal future of the set $S$ |
| $J^-(S)$ | causal past of the set $S$ |
| $\mathcal{I}^+$ | future null infinity |
| $\mathcal{I}^-$ | past null infinity |
| $V_{\text{chronology}}(M, g_{ab})$ | chronology violating set of the spacetime $(M, g_{ab})$ |
| $H_{\text{chronology}}(M, g_{ab})$ | chronology horizon of the spacetime $(M, g_{ab})$ |
| $\Sigma$ | binary alphabet |
| $\land$ | conjunction |
| $\lor$ | disjunction |
| $\neg$ | negation |
| $\rightarrow$ | implication |
| $\leftrightarrow$ | biimplication |
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