Abstract  Tidal analysis provides a cost-effective way of estimating aquifer properties. Tidal response models that link aquifer properties with tidal signal characteristics, such as phase and amplitude, have been established in previous studies, but none of the previous models incorporate the skin effect. It is found in this study that the skin effect and the wellbore storage effect can have significant influence on the results of tidal analysis and should be included in tidal response models. New models are proposed with skin and wellbore storage effects fully incorporated so that aquifer information can be assessed more accurately based on tidal analysis. The models can be applied to confined aquifers with only horizontal flow or semiconfined aquifers with both horizontal flow and vertical flow. For confined aquifers, the new model indicates that positive skin leads to larger phase lag between the tidal response and the theoretical tide, and negative skin can reduce the phase lag or even cause a phase advance. For semiconfined aquifers, both the skin effect and the vertical flow affect the phase difference between the tidal response and the theoretical tide, and with the proposed model, contribution from these two sources can be separated and analyzed independently, making it feasible to evaluate semiconfined aquifer properties considering both factors. Increasing wellbore storage causes larger phase lag or smaller phase advance for both types of aquifers. Real-world examples for confined and semiconfined aquifers are analyzed, respectively, to demonstrate practical applications of the proposed models.

1. Introduction

The Earth tides, which are caused by the gravitational forces exerted on the Earth by celestial bodies including the Moon and the Sun, result in aquifer deformations, and the response of the aquifer can be reflected by the change in wellbore pressure in a closed well or the fluid flow into and out of the wellbore in an open well. The effects of Earth tide on confined and semiconfined aquifers have long been studied by hydrologists for the purpose of extracting useful aquifer information from cyclic tidal responses (Dean et al., 1994; Hsieh et al., 1987; Jacob, 1940; Merritt, 2004; Morland & Donaldson, 1984; Robinson & Bell, 1971; Roeloffs, 1996; Wang et al., 2018). The comparison between actual tidal responses and the theoretical tides can reveal aquifer properties such as transmissivity and compressibility, which in practice has many applications in hydrogeological studies, including long-term monitoring of permeability changes after earthquakes (Elkhoury et al., 2006; Lai et al., 2014; Shi & Wang, 2014; Shi et al., 2015; Xue et al., 2013), fracture characterization (Hanson & Owen, 1982; Faidouzi, 2017), monitoring of compaction, and compressibility changes (Dean et al., 1994; Netland et al., 1996), etc.

Typically, tidal data can be analyzed from two perspectives—amplitude and phase. The ratio of tidal data amplitude to theoretical tide amplitude is associated with poroelastic properties of the formation including Skempton’s coefficient, bulk modulus, and Poisson’s ratio (Arditty et al., 1978; Bredehoeft, 1967; Burbey, 2010; Dean et al., 1994; Jacob, 1940; Sato, 2006; Sato & Horne, 2018; Van der Kamp & Gale, 1983). Aquifer flow properties, such as permeability, are more closely related with the phase difference between tidal data and the theoretical tide. Cooper et al. (1967) found that water level changes in open wells due to harmonic disturbances have the same frequency as the disturbance, although phases and amplitudes of the water level response are generally different from those of the disturbance, depending on transmissivity and storage coefficient of the aquifer and period of the disturbance. Hsieh et al. (1987) applied the diffusion equation to model the phase lag between the tidal response induced by the Earth tide and the theoretical tide as a function of transmissivity and storativity of the aquifer, wellbore radius, and period of the Earth
tidal response, assuming that the formation is confined, laterally extensive, homogeneous, and isotropic. According to Hsieh et al. (1987), there is always a phase lag under those assumptions because time is needed for the fluid in the aquifer to respond to tidal forces and flow into or out of the well. Other researchers suggested that phase advance, with measured tidal response leading theoretical tides, could exist under some different scenarios. For example, in a composite system, when a compressible aquifer is surrounded by more rigid aquifers, tidal responses measured in the well tapping in the compressible aquifer can lead the theoretical tide due to the tidal strain differentiation between adjoining aquifers (Gieske & De Vries, 1985). Maas and De Lange (1987) explained a similar phenomenon of phase advance using the principle of superposition. Another case in which a phase advance can happen is when the aquifer is not perfectly confined (i.e., semiconfined) and there is a vertical leakage from the target aquifer to the overlaying aquifer. Allègre et al. (2016) applied a vertical flow model to explain the phase advance and infer permeability from water level variations without considering the horizontal flow. Wang et al. (2018) considered both the horizontal flow and the vertical leakage and extended Hsieh’s model (Hsieh et al., 1987) using the specific leakage, which is defined as the ratio of hydraulic conductivity to thickness of the leaking aquitard. Based on the model presented by Wang et al. (2018), when aquifer transmissivity and storativity are known, the magnitude of the vertical leakage can be estimated.

However, none of the models discussed above considered the skin effect, and the wellbore storage effect is not explicitly incorporated, even though the flow transient in the wellbore can be strongly affected by the wellbore storage effect and the skin effect. Wellbore storage effect is commonly caused by fluid expansion or changing liquid level and is characterized by the wellbore storage coefficient. The skin effect can be due to a zone surrounding the well that is penetrated by cement or mud filtrate generated during the drilling or completion processes, and can cause significant permeability change in the affected zone. It is found in this study that the wellbore storage coefficient and the skin factor are important parameters influencing the results of tidal analysis and should be incorporated in tidal response models to improve the accuracy and reliability of the estimation of aquifer properties based on tidal analysis. Therefore, new tidal response models for both confined aquifers and semiconfined aquifers with skin effect and wellbore storage effect taken into consideration are proposed in this study. Analytical solutions and applications of the models are illustrated in detail, and the change in tidal analysis results due to various skin factors are explained and demonstrated with real-world examples on the basis of previous work by Hsieh et al. (1987) and Wang et al. (2018) for confined aquifers and semiconfined aquifers, respectively.

Purposes and potential applications of this study include the following:

1. Incorporation of wellbore storage and skin effect into tidal analysis and its application, considering both confined and semiconfined aquifer.
2. If the aquifer transmissivity is known, wellbore storage or skin can be quantified using the Earth tide analysis without the need for well flow test or pumping test. Determination of wellbore storage and skin is important for pressure transient analysis and wellbore and aquifer property estimation. The conventional way of evaluating wellbore storage and skin involves interpretation of well test data or pumping test data (Chen & Chang, 2002, 2003; Cinco-Ley & Samaniego, 1977; Gringarten et al., 1979; Novakowski, 1989; Ramey, 1970). However, when well test data or pumping test data are not available (e.g., when the well is closed and static), conventional methods cannot be applied. As proposed in this study, an alternative method to determine wellbore storage or skin takes advantage of the Earth tide analysis. The Earth tide effect influences pressures in closed wells or water levels in open wells regardless of whether the well is active or not. By modeling the pressure change or fluid flow induced by tidal forces, we can estimate wellbore storage or skin based on tidal responses measured at the wellbore. Even though aquifer tests can generally provide more aquifer information than the Earth tide analysis using a single tidal constituent, the tidal analysis allows for continuous monitoring of the wellbore and the aquifer with less costs even when the well is not flowing, which is helpful especially when long-term monitoring is the purpose. Furthermore, the tidal analysis based on our model can be used in combination with aquifer tests during the same period of time or in between tests conducted at different times so that tidal analysis and aquifer test analysis can be complementary to each other and together provide more information on aquifer properties over various time periods.
3. For a confined aquifer, if the wellbore storage coefficient and the skin factor are known, the aquifer transmissivity can be estimated based on tidal analysis. The main difference between our method and the method originally proposed by Hsieh et al. (1987) is that the skin effect is fully considered in our model.
Table 1

| Doodson’s number | Name      | Freq. (cpd) | Origin                  | Amplitude (m) |
|------------------|-----------|-------------|-------------------------|---------------|
| Low frequency tides |          |             |                         |               |
| 0 1 0 0 1 0 0     | M_m       | 0.0363      | Lunar elliptic          | −0.03518      |
| 0 2 0 0 0 0       | M_f       | 0.0732      | Lunar declinational     | −0.06663      |
| Diurnal tides     |          |             |                         |               |
| 1 1 0 0 0 0       | O_1       | 0.9295      | Lunar principal         | −0.26221      |
| 1 1 2 0 0 0       | P_1       | 0.9973      | Solar principal         | −0.12203      |
| 1 1 0 0 0 0       | K_1       | 1.0027      | Luni-solar declinational| 0.36878       |
| Semidiurnal tides |          |             |                         |               |
| 2 1 0 1 0 0       | N_2       | 1.8961      | Lunar major elliptic of M_2 | 0.12099     |
| 2 0 0 0 0 0       | M_2       | 1.9324      | Lunar principal         | 0.63192       |
| 2 2 2 0 0 0       | S_2       | 2.0000      | Solar principal         | 0.29400       |

4. For a semiconfined aquifer, if the wellbore storage coefficient, the skin factor, and the aquifer transmissivity are known, the magnitude of vertical leakage can be evaluated from tidal analysis. The original model for tidal responses in semiconfined aquifers was established by Wang et al. (2018), and the new model in this study considers the skin effect and adds the skin factor as an input parameter.

5. Explanation of the phenomena of phase advance from the perspective of enhanced permeability characterized by the negative skin. It is found that both negative skin and vertical leakage can lead to a phase advance. With the model proposed in this paper, it is possible to separate these two effects and evaluate skin independently even when vertical leakage exists.

2. Theory

2.1. Tidal Forces

The Moon and the Sun apply gravitational forces on the Earth, causing deformation of the aquifer and pressure change or fluid flow at the wellbore. The tidal waves are composed of various tidal constituents from a wide spectrum. Each tidal constituent is a sinusoidal function of time with an amplitude determined from the equilibrium tide and a period resulting from the cyclic motion of the Moon, the Sun, and the Earth (Bartels, 1957; Hicks & Szabados, 2006; Melchior, 1983). The theoretical tidal potential at a location with east longitude \( \varphi \) and colatitude \( \theta \) can be expressed as

\[
\eta(\theta, \varphi, t) = \sum_i H_i(\theta, \varphi) \cos(\omega_i t + \chi_i + \delta_i(\theta, \varphi))
\]  

where the subscript \( i \) corresponds to each harmonic tidal constituent. \( H \) is the amplitude, and \( \omega \) is the tidal frequency. \( \chi \) is an astronomical argument (Schwiderski, 1980). \( \delta \) is the Greenwich phase with \( t \) being Greenwich time (GMT). Based on its tidal frequency, a tidal constituent can be categorized as a diurnal component, a semidiurnal component or a long-period component. Major tidal constituents and their amplitudes and frequencies are listed in Table 1 (Cartwright & Tayler, 1971; Cartwright & Eden, 1973; Melchior, 1966).

The observed tidal responses (e.g., water level or pressure variations) follow a form similar to Equation 1 with the same tidal constituents but different amplitudes \( H' \) and phases \( \delta' \):

\[
\eta'(\theta, \varphi, t) = \sum_i H'_i(\theta, \varphi) \cos(\omega_i t + \chi_i' + \delta'_i(\theta, \varphi))
\]

Aquifer and well properties determine the amplitude ratio \( H'_i/H_i \) and the phase difference \( \delta'_i - \delta_i \). The observed tidal response can be decomposed into different harmonic constituents using Fourier analysis, and the amplitude \( H'_i \) and phase \( \delta'_i \) for a specific constituent can be retrieved from the data. Theoretical amplitudes and phases are known, so \( H'_i/H_i \) and \( \delta'_i - \delta_i \) can be obtained from the data. The models discussed in sections 2.2 and 2.3 relate aquifer and well properties to the amplitude ratio and the phase difference, considering the wellbore storage effect and the skin effect.
2.2. Confined Aquifer

When the aquifer is confined, vertical flow is prevented by impermeable layers both above and below the aquifer. A schematic of a confined aquifer system is shown in Figure 1. The well in Figure 1 is closed, and the target aquifer is penetrated by the well entirely. Tidal forces cause a cyclic pressure fluctuation at the wellbore. The damaged zone that causes the skin effect has a radius of \( r_s \), and the skin effect results in a pressure drop \( \Delta p_s \) at the wellbore. The skin factor can be defined as

\[
s = \frac{\Delta p_s}{\left( r \frac{\partial p}{\partial r} \right)_{r=r_w}} \quad (3)
\]

where \( p \) is the excess pressure in the aquifer above the initial baseline pressure.

It is assumed that outside the damaged zone the aquifer is isotropic, homogeneous and laterally extensive, which is similar to the classic aquifer model considered by Cooper et al. (1967) and Hsieh et al. (1987). In practical applications, the model actually provides a volumetric average estimation of aquifer properties within the radius of investigation of tidal analysis. Therefore, for aquifers that do not strictly meet the assumptions with heterogeneity or anisotropy on a small scale compared with the radius of investigation, the model can still be used to analyze tidal responses and compare the differences appeared in tidal behaviors on a volumetric average basis.

The flow transient in a confined aquifer system (Figure 1) under the cubic tidal stress \( \sigma_t \) is governed by the following equation:

\[
\frac{\partial^2 p}{\partial r^2} + \frac{1}{r} \frac{\partial p}{\partial r} = \frac{\phi \mu c_i}{k} \left( \frac{\partial p}{\partial t} - B \frac{\partial \sigma_t}{\partial t} \right) \quad (4)
\]

where \( \mu \) is the fluid viscosity, \( c_i \) is the total compressibility, and \( B \) is Skempton’s coefficient. The outer boundary condition for the laterally extensive aquifer \( (r = \infty) \), as shown in Figure 1, can be expressed as

\[
p(\infty, t) = B \sigma_t(t) \quad (5)
\]

The inner boundary conditions involving the skin effect and the wellbore storage effect are

\[
P_w = \left[ p - s \left( r \frac{\partial p}{\partial r} \right) \right]_{r=r_w} \quad (6)
\]
\[
\frac{\partial p_w}{\partial t} = \frac{2\pi kh}{\mu} \left( r \frac{\partial p}{\partial r} \right)_{r = r_w}
\]  

(7)

where \(p_w\) is the pressure measured inside the wellbore. \(s\) is the skin factor, and \(C\) is the wellbore storage coefficient. This system of equations can be simplified if we define

\[
\bar{p}(r, t) = p(r, t) - B\sigma_t(t)
\]

(8)

Then the governing equation becomes

\[
\frac{\partial^2 \bar{p}}{\partial r^2} + \frac{1}{r} \frac{\partial \bar{p}}{\partial r} = \frac{\phi \mu c_i}{k} \frac{\partial \bar{p}}{\partial t}
\]

(9)

and the boundary conditions become

\[
\bar{p}(\infty, t) = 0
\]

(10)

\[
p_w = \left[ \bar{p} - s \left( r \frac{\partial \bar{p}}{\partial r} \right) \right]_{r = r_w} + B\sigma_t
\]

(11)

\[
C \frac{\partial p_w}{\partial t} = \frac{2\pi kh}{\mu} \left( r \frac{\partial p}{\partial r} \right)_{r = r_w}
\]

(12)

From Equation 1, we know that the tidal stress that causes the pressure fluctuation is a cyclic function of time. As a result, tide-induced fluid pressure and wellbore pressure oscillations also follow a cyclic pattern.

\[
\sigma_t(t) = \sigma_0(\omega) \exp(i\omega t)
\]

(13)

\[
\bar{p}(r, t) = p_0(r, \omega) \exp(i\omega t)
\]

(14)

\[
p_w(t) = p_{w0}(\omega) \exp(i\omega t)
\]

(15)

where \(i\) is the imaginary unit. \(\omega\) is the tidal frequency, and \(\sigma_0(\omega)\), \(p_0(r, \omega)\) and \(p_{w0}(\omega)\) are complex amplitudes of the cubic tidal stress, fluid pressure and wellbore pressure fluctuations, respectively. By inserting Equations 13–15 into Equations 9–12, we can reduce the governing equation to an ordinary differential equation:

\[
\frac{d^2 p_0}{dr^2} + \frac{1}{r} \frac{dp_0}{dr} = i\omega \frac{\phi \mu c_i}{k} p_0
\]

(16)

with the following boundary conditions

\[
p_0(\infty, \omega) = 0
\]

(17)

\[
p_{w0} = \left[ p_0 - s \left( r \frac{\partial p_0}{\partial r} \right) \right]_{r = r_w} + B\sigma_0
\]

(18)

\[
i\omega C p_{w0} = \frac{2\pi kh}{\mu} \left( r \frac{\partial p_0}{\partial r} \right)_{r = r_w}
\]

(19)

The general solution to Equation 16 is:

\[
p_0(r, \omega) = A_1 I_0 \left( r \sqrt{\frac{i\omega \mu c_i}{k}} \right) + A_2 K_0 \left( r \sqrt{\frac{i\omega \mu c_i}{k}} \right)
\]

(20)
where \( I_0 \) and \( K_0 \) are the zero-order modified Bessel function of the first and second kind, respectively. The outer boundary condition gives that \( A_1 = 0 \). From the boundary condition, Equation 19, the constant \( A_2 \) can be obtained as

\[
A_2 = - \frac{\text{i} \omega \mu C}{2 \pi k h r_w \sqrt{\text{i} \omega \mu C / k} (r_w \sqrt{\text{i} \omega \mu C / k})} p_w 0
\]

\[
= - \frac{\alpha_D}{2 S_D k_1 (\alpha_D)} p_w 0
\]

(21)

where \( \alpha_D \) and \( S_D \) are defined as follows:

\[
\alpha_D = r_w \sqrt{\text{i} \omega \mu C / k} = \sqrt{2 \pi S_D \tau_D}
\]

(22)

\[
S_D = \frac{\pi \psi c_i h r_w^2}{C} = \frac{1}{2 C_D}
\]

(23)

\[
T_D = \frac{2 \pi^2 k h}{C \mu \omega} = \frac{\pi \tau k h}{C \mu}
\]

(24)

where \( C_D \) is the dimensionless wellbore storage coefficient, and \( \tau = 2 \pi / \omega \) is the period of fluctuation. In this case, \( T_D \) and \( S_D \) can be comprehended as dimensionless transmissivity and storativity, and they are related with conventional aquifer transmissivity and storativity through the following equations:

\[
\frac{T_D}{T} = \frac{\pi \tau}{C \rho g}
\]

(25)

\[
\frac{S_D}{S} = \frac{\pi r_w^2}{C \rho g}
\]

(26)

where \( T \) is the conventional aquifer transmissivity \((T = k h / \mu \rho g)\), which is defined as the product of hydraulic conductivity and aquifer thickness. \( S \) is the conventional aquifer storativity \((S = \phi c_i h \rho g)\) and is the product of aquifer specific storage and thickness. \( \rho \) is water density, and \( g \) is the gravitational acceleration.

From the inner boundary condition, Equation 18, \( p_w 0 \) is obtained as

\[
p_w 0 = \left[ 1 + \frac{\alpha_D}{2 S_D k_1 (\alpha_D)} + \frac{\pi t S}{T_D} \right]^{-1} B \sigma_0
\]

(27)

We can see from Equation 27 that at the wellbore, the pressure response to tidal forces is a function of not only aquifer storativity and transmissivity but wellbore storage coefficient and skin factor as well. As a result, the solution provides a way to estimate wellbore storage coefficient and skin factor from tidal analysis given aquifer storativity and transmissivity. On the other hand, if aquifer storativity and transmissivity are unknown, they can be inferred from tidal analysis under different wellbore storage and skin scenarios. Applications of the solution are discussed in more detail in section 3.

The original partial differential equation can be solved with the Laplace transform as well. Numerical inversion of the solution in Laplace space provides the same result as Equation 27. Note that when the well is active with a flow rate other than zero, we can only use the Laplace transform method to solve this problem. Details about the solution obtained from Laplace transformation are included in the Appendix.

**2.3. Semiconfined Aquifer**

The key difference between a semiconfined aquifer and a confined aquifer is the existence of vertical flow. In a semiconfined aquifer, there can be both horizontal flow and vertical flow. The configuration of semiconfined aquifer considered in this section is illustrated in Figure 2. The target aquifer is overlain by a permeable aquitard, and above the permeable aquitard is an unconfined aquifer. Such a classical leaky aquifer model was first studied by Hantush and Jacob (1955). Wang et al. (2018) investigated a similar system and estimated the magnitude of vertical leakage with tidal analysis. The solution proposed by Wang et al. (2018) is extended with the inclusion of skin effect in this study.
Figure 2. Semiconfined aquifer system.

It is assumed that both the overlaying permeable aquitard and the target aquifer are laterally extensive and that the permeable aquitard has negligible storage and is incompressible. The governing equation is similar to Equation 4 but with a term that accounts for the vertical flow $H'p$, where $H' = \frac{K'}{\beta'}$. $K'$ and $b'$ are the vertical hydraulic conductivity and the thickness of the overlaying aquifer, respectively. $T$ is the transmissivity of the target aquifer.

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r} \frac{\partial p}{\partial r} - H' p = \frac{\phi \mu c_i}{k} \left( \frac{\partial p}{\partial t} - B \frac{\partial \sigma_i}{\partial t} \right)$$

(28)

The inner boundary conditions with skin effect and wellbore storage are given by Equations 6 and 7. The outer boundary condition at $r = \infty$, however, is not $B\sigma_i$ any more due to the influence of the vertical flow. Instead, the outer boundary follows a cyclic function $p_\infty(t)$ with the same frequency as $\sigma_i$ but a different amplitude $p_\infty(0)$:

$$p_\infty(t) = p_\infty(0) \exp(i\omega t)$$

(29)

The relationship between $p_\infty(0)$ and $\sigma_0$ can be found by inserting Equations 13 and 29 into the governing Equation 28, which yields

$$-H' p_\infty(0) = \frac{i \omega \phi \mu c_i}{k} (p_\infty(0) - B\sigma_0)$$

(30)

and $p_\infty(0)$ is given as

$$p_\infty(0) = \frac{i \omega \phi \mu c_i}{k} \frac{B \sigma_0}{H' r_\infty^2}$$

(31)

where $H'_D = H r_\infty^2$. By defining

$$p'(r, t) = p(r, t) - p_\infty(t)$$

(32)

and using Equation 30, the governing equation in terms of $p'$ is obtained as

$$\frac{\partial^2 p'}{\partial r^2} + \frac{1}{r} \frac{\partial p'}{\partial r} - H' p' = \frac{\phi \mu c_i}{k} \frac{\partial p'}{\partial t}$$

(33)

with the homogeneous outer boundary condition

$$p'(\infty, t) = 0$$

(34)
Since \( p'(r, t) \) is also a cyclic function, we can assume
\[
p'(r, t) = p'_0(r, \omega) \exp(i\omega t)
\] (35)
and \( p_w \) is given by Equation 15. Then Equation 33 becomes an ordinary differential equation in terms of \( p'_0(r, \omega) \):
\[
\frac{d^2 p'_0}{dr^2} + \frac{1}{r} \frac{dp'_0}{dr} - H' p'_0 = \frac{i\omega \phi c_i}{k} p'_0
\] (36)
with the following boundary conditions
\[
p'_0(\infty, \omega) = 0
\] (37)
\[
p_{w0} = \left[p'_0 - s \left( r \frac{dp'_0}{dr} \right) \right]_{r=r_w} + p_{w0}
\] (38)
\[
i\omega C p_{w0} = \frac{2\pi kh}{\mu} \left( r \frac{dp'_0}{dr} \right)_{r=r_w}
\] (39)

The solution to Equation 36 considering the outer boundary condition, Equation 37, is
\[
p'_0(r, \omega) = A_3 K_0 \left( r \sqrt{H' + \frac{i\omega \phi c_i}{k}} \right)
\] (40)
From the boundary condition, Equation 39, we have
\[
A_3 = -\frac{i\omega C}{2\pi k h r_w \sqrt{H' + \frac{i\omega \phi c_i}{k}}} p_{w0}
\] (41)
\[
= -\frac{\alpha_0^2}{2S_D \beta_D K_1(\beta_D)} p_{w0}
\]
where \( \beta_D \) is defined as
\[
\beta_D = r_w \sqrt{H' + \frac{i\omega \phi c_i}{k}} = \sqrt{H_D + \alpha_0^2}
\] (42)
From the inner boundary condition, Equation 38, \( p_{w0} \) is obtained as
\[
p_{w0} = \left[1 + \frac{\alpha_D^2}{2S_D \beta_D K_1(\beta_D)} K_0(\beta_D) + \pi l_s \frac{S}{T_D} \right]^{-1} p_{w0}
\] (43)
Using Equation 31, we can express \( p_{w0} \) as
\[
p_{w0} = \left[1 + \frac{\alpha_D^2}{2S_D \beta_D K_1(\beta_D)} K_0(\beta_D) + \pi l_s \frac{S}{T_D} \right]^{-1} \left( \frac{\alpha_0}{\beta_D} \right)^2 B \sigma_0
\] (44)
With the solution, Equation 44, we can evaluate the wellbore storage coefficient and skin factor in a leaky aquifer using tidal analysis if properties of the overlying permeable aquitard are available. It is also feasible to estimate the magnitude of the vertical leakage with different levels of wellbore storage effects and skin effects. Applications of the solution as well as the comparison between the confined aquifer solution and the semiconfined aquifer solution are explained in detail in section 3.
3. Application

As explained in the introduction section (section 1), two key parameters produced by tidal analysis are the amplitude ratio and the phase difference. The amplitude ratio is the ratio of the amplitude of a tidal constituent from the data to that of the same constituent from the theoretical tide (i.e., the loading efficiency). Similarly, the phase difference refers to the difference between the phase of a tidal constituent from the data to that from the theoretical tide. It is elucidated in this section how the amplitude ratio and the phase difference are related to aquifer and wellbore properties, based on the solutions derived in section 2. Specifically, the influences of wellbore storage and skin effects are discussed in detail for both confined aquifer and semi-confined aquifer. The results for confined aquifer and semi-confined aquifer are compared to illustrate the effects of vertical leakage with different skin factors.

3.1. Confined Aquifer

On the basis of the solution for confined aquifers given by Equation 27, the amplitude ratio $A$ and phase difference $\eta$ can be expressed in terms of $S_D$, $T_D$ and the skin factor $s$.

$$A = \left| \frac{p_{\omega 0}}{B \sigma_0} \right| = \left| 1 + \frac{\alpha D}{2S_D K_0(\alpha D)} + \pi i \frac{s}{T_D} \right|^{-1}$$  \hspace{1cm} (45)

$$\eta = \arg(p_{\omega 0} / B \sigma_0) = \arg \left( 1 + \frac{\alpha D}{2S_D K_1(\alpha D)} + \pi i \frac{s}{T_D} \right)^{-1}$$  \hspace{1cm} (46)
where $|z|$ and arg($z$) are the modulus and the argument of the complex number $z$, respectively (Sato, 2015). $A$ is the ratio of amplitude of wellbore pressure fluctuation $p_{w0}$ to the theoretical tidal fluctuation $B\sigma_0$, which is also the outer boundary condition. The phase shift $\eta$ is the difference in phase angles of $p_{w0}$ and $\sigma_0$. When the wellbore pressure response lags behind the tidal stress disturbance, $\eta$ becomes negative. In contrast, positive phase shift (phase advance) indicates wellbore pressure response leads the tidal stress.

For fixed values of skin factor and $S_D$, the amplitude ratio $A$ and the phase shift $\eta$ can be plotted as a function of $T_D$, as shown in Figure 3 when the skin factor is set to zero. Note that when the skin effect does not exist (i.e., skin factor is zero), the profiles of $A$ and $\eta$ shown in Figure 3 are exactly the same as those in the paper by Hsieh et al. (1987).

When the skin factor is nonzero, however, the profiles of $A$ and $\eta$ deviate from those when $s$ is zero. Figure 4 a and b show $A$ and $\eta$, respectively, as a function of $T_D$ for different values of $s$ and $S_D = 10^{-7}$. Figures 5 and 6 show the same plots as Figure 4 but with different $S_D$ values ($S_D = 10^{-4}$ for Figure 5, and $S_D = 10^{-1}$ for Figure 6). It can be seen from Figure 4 that when $S_D$ is relatively small (i.e., dimensionless wellbore storage coefficient is relatively large), the amplitude ratio becomes smaller and the phase shift becomes more negative as the skin factor increases from −5 to 30. This observation makes physical sense because larger skin factor indicates lower permeability around the wellbore and greater difficulty for aquifer fluid to flow into and out of the wellbore, resulting in smaller amplitude ratio and wider phase lag. However, this is only true when the wellbore storage effect is significant and dominates the behavior of pressure fluctuations in response to tidal forces. On the other hand, when the wellbore storage effect is insignificant ($S_D$ is relatively large) and when aquifer transmissivity is small, the effect of superposition caused by nonzero skin factor

**Figure 5.** (a) Amplitude ratio $A$ and (b) phase shift $\eta$ as a function of $T_D$ for different skin $s$ when $S_D = 10^{-4}$. $T_D$ and $S_D$ are the dimensionless transmissivity and storativity defined in Equations 24 and 23, respectively.

**Figure 6.** (a) Amplitude ratio $A$ and (b) phase shift $\eta$ as a function of $T_D$ for different skin $s$ when $S_D = 10^{-1}$. $T_D$ and $S_D$ are the dimensionless transmissivity and storativity defined in Equations 24 and 23, respectively.
becomes more dominant. As discussed by Gieske and De Vries (1985) and Maas and De Lange (1987), negative phase shift or phase advance can appear in a composite aquifer when the inner zone is more permeable than the outer zone, as explained by the principle of superposition. Negative skin means the permeability of the inner zone (i.e., the damaged zone) is larger than that of the outer zone, so phase lag can be narrowed or even turn to phase advance when the skin is negative.

In Figure 5, $S_D$ is $10^{-4}$, which is in between $10^{-7}$ and $10^{-1}$, so both the wellbore storage effect and the superposition effect influence the pressure response. As a result, the value of dimensionless transmissivity $T_D$ becomes more important. When $T_D$ is relatively large, the wellbore storage effect plays a more significant role, while the superposition effect is more dominant when $T_D$ is relatively small. As a result, when $T_D$ is large, increasing the skin from $-5$ to $30$ decreases the amplitude ratio and increases the magnitude of the phase lag, while the same as the observation in Figure 4. When $T_D$ is small, however, the conclusion depends on the sign of the skin factor, and a positive skin results in a decrease in the amplitude ratio and an increase in the phase lag, whereas a negative skin manifests the opposite effects in $A$ and $\eta$. In Figure 5a, when $T_D$ is greater than 1.5, increasing the skin factor from $-5$ to $30$ causes the amplitude ratio $A$ to decrease, as the wellbore storage effect is dominant. When $T_D$ is smaller than 1.5, the superposition effect is dominant, and positive and negative skin factors change the amplitude ratio in opposite directions: increasing the skin from 0 to 30 causes $A$ to decrease, while increasing the skin from $-5$ to $-3$ causes $A$ to increase. Therefore, the red curve ($s = -3$) and the blue curve ($s = -5$) cross at $T_D = 1.5$, where there is a transition from superposition effect to wellbore storage effect. The dip in the red curve ($s = -3$) at $T_D = 1.5$ in Figure 5b also corresponds to the transition.

When $S_D$ is large, as is the case in Figure 6, the wellbore storage effect is weaker than the superposition effect regardless of the level of $T_D$, so the observation is the same with that from Figure 5 when $T_D$ is relatively small, that is, a more negative skin is associated with larger phase advance and smaller amplitude ratio while a more positive skin results in larger phase lag and smaller amplitude ratio. The observed trends are summarized in Table 2 and Table 3.

### 3.2. Semiconfined Aquifer

The amplitude ratio $A$ and phase shift $\eta$ for semiconfined aquifers can be derived from the solution, Equation 44:

$$A = \left| \frac{p_{w0}}{B \sigma_0} \right| = \left| 1 + \frac{a_D^2}{2S_D \beta_D} \frac{K_0(\beta_D)}{K_1(\beta_D)} + \pi i \frac{s}{T_D} \right|^{-1} \left( \frac{a_D}{\beta_D} \right)^2 \tag{47}$$

$$\eta = \arg \left( \frac{p_{w0}}{B \sigma_0} \right) = \arg \left| 1 + \frac{a_D^2}{2S_D \beta_D} \frac{K_0(\beta_D)}{K_1(\beta_D)} + \pi i \frac{s}{T_D} \right|^{-1} \left( \frac{a_D}{\beta_D} \right)^2 \tag{48}$$

We can see from Equations 47 and 48 that for a semiconfined aquifer $A$ and $\eta$ are determined by not only $T_D$ and $S_D$ but also by $H'$, and $H'$ reflects the magnitude of the vertical leakage ($H' = \frac{K_D}{\beta_D}$).

| Wellbore storage effect dominant | Superposition Effect dominant |
|---------------------------------|-------------------------------|
| Negative skin                   | $A$ decreases, and $\eta$ becomes more negative (larger phase lag) when $s$ increases from a negative value to zero. |
| Positive skin                   | $A$ decreases, and $\eta$ becomes more negative (larger phase lag) when $s$ increases from zero to a positive value |

Table 3: Effect of Skin on $A$ and $\eta$ Under Different Scenarios
Note that based on Equation 24, $T_D$ is affected by both the tidal frequency $\omega$ and the aquifer permeability $k$. Therefore, when plotting $A$ or $\eta$ against $T_D$, we can change $T_D$ through changing $\omega$ or $k$. For a confined aquifer, whether changing $\omega$ or $k$ to change $T_D$ does not influence the results, because $A$ and $\eta$ are functions of $S_D$ and $T_D$ only. For a semiconfined aquifer, however, the results will be different, because $\omega$ affects only $T_D$, but $k$ affects both $T_D$ and $H'$. Thus, the profiles of $A$ and $\eta$ for semiconfined aquifer are discussed under two conditions. In the first condition, $T_D$ varies with changing $k$, while in the second condition $T_D$ varies with changing $\omega$. Note that in practice, $\omega$ can only take discrete values (e.g., diurnal or semidiurnal) instead of continuous values, so the purpose of plotting $A$ and $\eta$ against $T_D$ by varying $\omega$ is only to see the trends of the profiles of $A$ and $\eta$ when $\omega$ takes different values. For the following discussion in this section, $S_D$ is set to $10^{-4}$.

Figures 7a, 7b, and 7c show the effect of $K'/b'$ on $A$ for $s = -5$, $s = 0$, and $s = 5$, respectively, under the first condition where $T_D$ varies with $k$. $K'$ and $b'$ are the vertical hydraulic conductivity and the thickness of the overlying aquifer, respectively. In Figures 7 and 8, the unit of $K'/b'$ is $s^{-1}$. It can be seen from Figure 7 that $A$ tends to decrease with increasing $K'/b'$, which makes physical sense because the amplitude of the pressure response measured at the wellbore is less sensitive to the tidal stress with greater vertical leakage. When $K'/b'$ reduces, the profile of $A$ converges to that from the confined aquifer solution. In addition, when $s$ increases from $-5$ to $5$, the curve moves from the left to the right, which is the same as our observation from Figure 4a, so in this case, increasing the skin factor decreases $A$ for fixed $T_D$.

The profiles of $\eta$ with various $K'/b'$ for $s = -5$, $s = 0$ and $s = 5$ are shown in Figures 8a, 8b, and 8c, respectively, under the first condition where $T_D$ varies with $k$. In Figure 8, when $s$ is negative, the phase advance becomes larger with increasing $K'/b'$ (greater vertical leakage). When $s$ is zero or positive, the phase lag reduces and becomes less negative with increasing $K'/b'$. Thus, greater vertical leakage causes $\eta$ to change in the same upward direction regardless of the sign of the skin factor. Furthermore, with our solution, the effects of vertical leakage can be separated from the effects of enhanced horizontal permeability, making it feasible to evaluate skin factor from tidal analysis even in the presence of vertical flow. For example, when $s = -5$ and $T_D = 2.854$, the phase advance caused solely by negative skin with $K'/b' = 0 \, s^{-1}$ is $12.05^\circ$. When
Figure 9. Amplitude ratio $A$ versus $T_D$ for different skin factors. $T_D$ is the dimensionless transmissivity defined in Equation 24. $K'/b'$ is kept at $10^{-3}$ s$^{-1}$. The solid and dashed curves represent results for confined and semiconfined aquifers, respectively.

$K'/b' = 10^{-3}$ s$^{-1}$, the total phase advance increases to 17.71°, and vertical leakage accounts for 32% of the phase advance. When $K'/b'$ further increases to $10^{-2.5}$ s$^{-1}$, the total phase advance becomes 30.1°, 60% of which is due to vertical leakage.

Under the second condition, $T_D$ varies with $\omega$ and $k$ is kept constant. When $K'/b'$ is $10^{-5}$, which is relatively small in this case, the profile of $A$ is almost the same as that shown in Figure 4a. When $K'/b'$ increases to $10^{-3}$, however, the curves become bell-shaped. In Figure 9, the solid curves and dashed curves represent results for confined aquifer and semiconfined aquifer, respectively, and each color represents a skin factor value. It can be seen from Figure 9 that the difference between $A$ for confined aquifer and $A$ for semiconfined aquifer increases with $T_D$. At larger $T_D$, the amplitude ratio for a semiconfined aquifer is lower than that for a confined aquifer. The profiles of $\eta$ under the second condition are shown in Figure 10. Similarly, when $K'/b'$ is relatively small, $\eta$ for a semiconfined aquifer is very close to that for a confined aquifer, but when $K'/b'$ rises up, the difference is enlarged, especially when $T_D$ is large. The phase shift is larger (less negative or more positive) with vertical flow. One possible explanation for the observation that $A$ and $\eta$ for the semiconfined aquifer are more different than those for the confined aquifer when $T_D$ is larger is that the effect of vertical

Figure 10. Phase shift $\eta$ versus $T_D$ for different skin factors when (a) $K'/b' = 10^{-5}$ s$^{-1}$ and (b) $K'/b' = 10^{-3}$ s$^{-1}$. $T_D$ is the dimensionless transmissivity defined in Equation 24. The solid and dashed curves represent results for confined and semiconfined aquifers, respectively.
leakage is more dominant for larger $T_D$, while the permeability change due to the skin effect plays a more significant role when $T_D$ is smaller.

In general, vertical leakage tends to decrease the amplitude ratio and increase the phase advance or decrease the phase lag. The effects of vertical flow and the effects of skin can be quantified separately, so we can still infer the skin factor using tidal analysis even in a semiconfined aquifer.

4. Examples

4.1. Example for Confined Aquifer

In previous studies, tidal analysis was applied to estimate transmissivity or permeability of confined aquifers, which provides a cost-effective monitoring approach (Allègre et al., 2016; Elkhoury et al., 2006; Hsieh et al., 1987; Narasimhan et al., 1984). The skin effect, however, was not considered in these studies. Our theoretical solution indicates that the skin effect has a significant impact on the interpretation of the phase difference obtained from tidal analysis and the estimation of aquifer transmissivity. In this section, a case study originally discussed by Hsieh et al. (1987) is reanalyzed to illustrate the potential impact of the skin effect on the estimation of aquifer transmissivity. In the paper by Hsieh et al. (1987), water level data measured during 24 February to 23 June in 1985 from an open well at a site about 11 km from Parkfield, California (the Gold Hill site) were analyzed and compared to the dilatation record collected by dilatometers installed at the same site to obtain the phase difference corresponding to the $M_2$ tidal constituent. Key wellbore and aquifer parameters used for the analysis are listed in Table 4. More details about the data and the geological background can be found in the paper by Hsieh et al. (1987).

In the paper by Hsieh et al. (1987), the smoothed water fluctuation data and dilatation data were analyzed with the Fourier transform to compute the phase shift of $M_2$ tidal constituent, which is the dominant tidal constituent. In this case, the mean $M_2$ phase shift was found to be $-11.6^\circ$. Based on the mean $M_2$ phase shift and the range of storativity $S$ ($10^{-6} - 10^{-4}$), assuming the skin factor is zero, Hsieh et al. (1987) estimated the range of aquifer transmissivity to be around $8 \times 10^{-6} - 2 \times 10^{-5}$ m$^2$/s. Hsieh et al. (1987) did not mention if the skin effect was present at the studied well, so our discussion here is around the question of how the estimation of aquifer transmissivity would change if the skin effect had existed in this case. In this study, it is found that with the same input parameters, nonzero skin factor can result in a significantly different estimation of aquifer transmissivity.

In Figure 11, the solid blue curve and the solid red curve represent the relation between the aquifer transmissivity $T$ and the phase difference $\eta$ when skin is assumed to be zero, and the dashed curves indicate nonzero skin factors and are based on the solutions proposed in this study. The estimated range of $T$ given the range of $S$ ($10^{-6} - 10^{-4}$) can be found by measuring the space between the blue curves and the red curves. It can be seen from Figure 11 that positive skin factors shift the curves to the right, and negative skin factors shift the curves to the left, which means the estimation of aquifer transmissivity should increase with positive skin factor and decrease with negative skin factor. Figure 11b is a zoomed-in version of Figure 11a, and the estimated range of $T$ can be better seen in Figure 11b when the $M_2$ phase difference is $-11.6^\circ$, which is shown by the black horizontal line. In this case, a positive skin factor of 5 would change the estimated range of $T$ to $2 \times 10^{-5} - 2.4 \times 10^{-5}$ m$^2$/s, and a negative skin factor of $-3$ would change the estimation to $3 \times 10^{-6} - 8 \times 10^{-6}$ m$^2$/s. If we compare the mean values of the estimated transmissivities, we can see that the estimation in aquifer transmissivity would increase by $\sim 57$% if the skin factor is increased to 5 from 0 and decrease by $\sim 60$% if the skin factor is decreased to $-3$ from 0, so the change in aquifer transmissivity estimation is significant when the skin effect is included.

Through this example, it is demonstrated that the consideration of skin effect is important in tidal analysis, and the solution proposed in this study should be applied to give an accurate transmissivity estimation when the wellbore is damaged or stimulated, which occurs frequently in practice.

4.2. Example for Semiconfined Aquifer

For semiconfined aquifers, factors influencing the phase difference between the tidal signal and the theoretical tide include aquifer transmissivity, wellbore storage, skin factor, and the level of vertical leakage. If three out of these four factors and the phase difference are known, the remaining one factor can be assessed.
Figure 11. Change in transmissivity estimation due to the skin effect. $S$ is the conventional aquifer storativity ($S = \phi \rho g$). $s$ is the skin factor. The horizontal black line represents the $M_2$ phase difference. The horizontal distance between the solid blue and solid red curves indicates the estimated transmissivity by Hsieh et al. (1987) assuming the skin is zero. The horizontal distance between the dashed blue and dashed red curves indicate the new transmissivity estimation when the skin is 5. Similarly, the horizontal distance between the dash-dotted curves indicate the transmissivity estimation when the skin is −3.

Table 5
Wellbore and Aquifer Properties for the Semiconfined Aquifer Example (Wang et al., 2018)

| Parameter                  | Value/range |
|----------------------------|-------------|
| Well depth                 | 960 m       |
| Wellbore radius            | 11 cm       |
| Casing radius              | 3.65 cm     |
| Aquifer transmissivity $T$ | $9.6 \times 10^{-6} - 1.4 \times 10^{-3} \text{ m}^2/\text{s}$ |
| Storativity $S$            | $2.6 \times 10^{-6} - 2.7 \times 10^{-5}$ |

Wang et al. (2018) found that the water level tidal response leads the theoretical tide, and the phase advance is estimated to be $12.5^\circ$. Based on the range of transmissivity and storativity in Table 5, the range of specific leakage $K'/b'$ (i.e., the ratio of the vertical hydraulic conductivity of the leaky aquitard to the thickness of the aquitard), was estimated to be $10^{-10} - 10^{-9} \text{ s}^{-1}$ if the skin factor was zero.

Based on our analytical solution and the illustration in section 3, we know that the effect of skin factor on the amplitude ratio $A$ and the phase difference $\gamma$ is tapered when transmissivity is relatively large. For instance, in Figure 9 and Figure 10, curves corresponding to different skin factors converge as $T_D$ becomes larger. Our finding in this example of a semiconfined aquifer coincides with this theoretical conclusion, because when the aquifer transmissivity is as large as $1.4 \times 10^{-3} \text{ m}^2/\text{s}$ (the upper bound of transmissivity provided by Wang et al. (2018), see Table 5), various skin factors result in similar estimation of $K'/b'$ (i.e., $10^{-10} - 10^{-9} \text{ s}^{-1}$). When transmissivity is $9.6 \times 10^{-6} \text{ m}^2/\text{s}$, however, the range of $K'/b'$ deviates from the original estimation depending on the value of skin factor. Figure 12 shows the change in $K'/b'$ estimation due to the skin effect when $T = 9.6 \times 10^{-6} \text{ m}^2/\text{s}$. The solid lines represent the cases when skin factor is zero, and the space between the red curve and the blue curve indicates the estimated range of $K'/b'$. Results when $s = -5$ and $s = 5$ are shown by the dash-dot lines and the
5. Discussion

After taking the skin effect and the wellbore storage effect into consideration, our solution for confined aquifers has three model parameters: $T_D$, $S_W(=1/2C_D)$, and $s$, which are defined by four physical parameters: the transmissivity $T$, the aquifer storativity $S$, the wellbore storage coefficient $C$, and the skin factor $s$. Our solution for semiconfined aquifers has an additional parameter corresponding to the level of vertical leakage. From the analytical results, the amplitude ratio and the phase shift, which can be obtained from the analysis of tidal data, are expressed as functions of those model parameters (Equations 45–48). The new tidal response models reflect more realistic wellbore and aquifer conditions by incorporating the wellbore storage coefficient and the skin factor. As for practical applications, the confined aquifer model can be used to estimate any one of the three parameters knowing the other two, and similarly, we can estimate any one of the four parameters using the other three with the semiconfined aquifer model. Nevertheless, if our knowledge of the target aquifer is limited prior to the tidal analysis, the parameter space needs to be narrowed down to relax the requirements on model inputs. Potential ways to reduce the parameter space include adding more tidal constituents with various frequencies to the analysis or analyzing the amplitude ratio and the phase shift at the same time. Introducing more tidal constituents allows Equations 45–48 to be applied with different $\omega$, but additional tidal constituents need to be selected carefully and preprocessing of the tidal data might be required (e.g., removing atmospheric effects before analyzing diurnal tidal constituents). The other option is to analyze the phase shift and the amplitude ratio at the same time. The phase shift can be directly observed from the tidal signals and the theoretical tides. The amplitude ratio, however, involves elastic properties of the aquifer ($A = \frac{\rho_s}{\rho_B}$, where $B$ is the Skempton’s coefficient). Therefore, simultaneous analyses of the amplitude ratio and the phase shift can be done to infer aquifer flow properties with less inputs if information on the Skempton’s coefficient or the tidal efficiency of the target aquifer is available. In addition, tidal analysis can be used together with aquifer tests to narrow down the parameter space and at the same time obtain a more comprehensive understanding of the aquifer over time. In the short term, tidal signals can be extracted from aquifer test data by separating the underlying trend and the tidal variations, so that tidal analysis and aquifer test analysis can be conducted at the same time, allowing for comparison and verification. In the long term, tidal analysis can be applied in between aquifer tests conducted at different times to fill in the gap and monitor the changes in aquifer properties.

Another question regarding the practical uses of the proposed models involves determining whether the aquifer is confined or semiconfined and which tidal response model should be applied. If the level of confinement of the aquifer is unknown, the tidal response model for semiconfined aquifers can be used to determine whether a vertical leakage exists and quantify the level of leakage. A phase advance with a positive skin factor indicates that there is likely to be a vertical leakage in the aquifer (i.e., the aquifer is semiconfined). Also, the tidal models can be used in combination with aquifer tests or other geological studies to evaluate the level of confinement.

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Figure 12. Change in vertical leakage estimation due to the skin effect. $S$ is the conventional aquifer storativity ($S = \rho_s \rho_B g$). $s$ is the skin factor. The horizontal black line represents the $M_c$ phase advance. The horizontal distance between the solid blue and red curves with triangle markers indicates the original vertical leakage $K'/b'$ by Wang et al. (2018) assuming the skin is zero. The horizontal distance between the blue and red curves with circle markers indicate that the new estimation of $K'/b'$ if the skin is $-5$, and the horizontal distance between the blue and red curves with circle markers indicate that when the skin is 5.
6. Summary

Our work can be summarized as follows.

1. The effects of Earth tide on aquifers are modeled theoretically with wellbore storage effects and skin effects taken into consideration. Models are developed for both confined aquifers and semiconfined aquifers.

2. For confined aquifers, amplitude ratio and phase shift are determined by aquifer transmissivity, wellbore storage, and the skin factor. It is found that higher positive skin factor can lead to greater phase lag due to greater difficulty for aquifer fluid to flow into and out of the wellbore. In addition, the phase shift in a confined aquifer is not always negative—a phase advance can occur when the skin factor is negative with enhanced permeability around the wellbore, and the phase advance with more negative skin is larger.

3. In practice, the target aquifer is sometimes not perfectly confined (semiconfined), and there exists vertical leakage in semiconfined aquifers. In semiconfined aquifers, the magnitude of the vertical leakage also affects the amplitude ratio and phase shift besides transmissivity, wellbore storage, and skin factor. Our solution for semiconfined aquifers indicates that greater vertical leakage tends to result in smaller amplitude ratio and larger phase advance or smaller phase lag. The effect of vertical leakage is stronger when aquifer permeability is higher and/or tidal frequency is lower. Based on our solution, we can separate the effect of vertical leakage and the effect of enhanced permeability in semiconfined aquifers and evaluate the phase shift contributed by each of the two effects independently.

4. Real-world examples based on previous work by Hsieh et al. (1987) and Wang et al. (2018) are analyzed to demonstrate the effects of nonzero skin factors on transmissivity estimation for confined aquifers and vertical leakage estimation for semiconfined aquifers. It is found that the skin effect has significant influence on the final results from tidal analysis and should be included in tidal response models.

Appendix A: Solving the governing equation with Laplace transform

The governing partial differential equations discussed in section 2 can be solved with Laplace transform.

The dimensionless form of the governing equation for confined aquifer (Equation 4) is

\[ \frac{\partial^2 p_D}{\partial r_D^2} + \frac{1}{r_D} \frac{\partial p_D}{\partial r_D} = \frac{\partial p_D}{\partial t_D} - \frac{B}{t_D} \frac{\partial \sigma_D}{\partial t_D}. \]  

(A1)

The dimensionless forms are

\[ p_D = \frac{2\pi kh}{q\mu} (p_i - p) \]  

(A2)

\[ t_D = \frac{kt}{\phi\mu c_r r_w^2} \]  

(A3)

\[ r_D = r/r_w \]  

(A4)

\[ \sigma_D = \frac{2\pi kh}{q\mu} \sigma \]  

(A5)

where \( p_i \) is the initial aquifer pressure, and the rest of the variable are the same as those previously defined in section 2. The dimensionless boundary conditions are

\[ p_D(\infty, t_D) = B \sigma_D(t_D) \]  

(A6)

\[ p_{\text{wd}} = \left[p_D - s \left(r_D \frac{\partial p_D}{\partial r_D} \right)_{r_D=1}\right] \]  

(A7)

\[ C_D \frac{\partial p_{\text{wd}}}{\partial t_D} - \left(r_D \frac{\partial p_D}{\partial r_D} \right)_{r_D=1} \]  

(A8)

Define \( y_D = p_D - B \sigma_D \), then the system of equations become

\[ \frac{\partial^2 y_D}{\partial r_D^2} + \frac{1}{r_D} \frac{\partial y_D}{\partial r_D} = \frac{\partial y_D}{\partial t_D}. \]  

(A9)
Verification of the results with Laplace transform. $T_D$ is the dimensionless transmissivity defined in Equation 24. $s$ is the skin factor. The solid lines represent the results from Equation 45 and 46, and the dots represent results from numerical inversion of the Laplace transform.

\[
y_D(\infty, T_D) = 0 \tag{A10}
\]

\[
p_{wD} = \left[ y_D - s \left( r_D \frac{\partial y_D}{\partial r_D} \right) \right]_{r_D=1} + B\sigma_D(T_D) \tag{A11}
\]

\[
C_D \frac{\partial p_{wD}}{\partial T_D} - \left( r_D \frac{\partial y_D}{\partial r_D} \right)_{r_D=1} = 1 \tag{A12}
\]

The governing equation in the Laplace space can be obtained by applying the Laplace transform to Equation A9.

\[
\lambda \overline{y_D} = \frac{1}{r_D} \frac{\partial}{\partial r_D} \left( r_D \frac{\partial \overline{y_D}}{\partial r_D} \right) \tag{A13}
\]

where $\lambda$ is the Laplace variable, and the overline indicates corresponding variables in the Laplace space. Note that Equation A13 is an ordinary differential equation, and the general solution considering boundary condition (A10) is $\overline{y_D} = A_4 K_0(\sqrt{\lambda} r_D)$. $A_4$ is a function of $\lambda$ only. The boundary conditions, Equations A11 and A12, give that

\[
A_4 = \frac{1 - C_D \lambda^2 B\sigma_D(\lambda)}{C_D \lambda^2 K_0(\sqrt{\lambda}) + s \sqrt{\lambda} K_1(\sqrt{\lambda}) + \lambda \sqrt{\lambda} K_1(\sqrt{\lambda})} \tag{A14}
\]

The solution in the Laplace space is

\[
\overline{p_{wD}} = \frac{K_0(\sqrt{\lambda}) + \sqrt{\lambda} K_1(\sqrt{\lambda})(s + B\sigma_D)}{C_D \lambda^2 K_0(\sqrt{\lambda}) + s \sqrt{\lambda} K_1(\sqrt{\lambda}) + \lambda \sqrt{\lambda} K_1(\sqrt{\lambda})} \tag{A15}
\]

Equation A15 is the general solution when the flow rate is nonzero. When the flow rate is zero, however, we cannot transform the variables into the dimensionless form and need to solve the system of equations in its dimensional form using the Laplace transform, which process is the same as the steps illustrated in this appendix. The solution for closed well in the dimensional form is

\[
\overline{p_w} = \frac{\lambda \sqrt{\lambda} K_1(\sqrt{\lambda}) B\sigma}{C_D \lambda^2 K_0(\sqrt{\lambda}) + s \sqrt{\lambda} K_1(\sqrt{\lambda}) + \lambda \sqrt{\lambda} K_1(\sqrt{\lambda})} \tag{A16}
\]

\[
= \left[ 1 + \frac{C_D \sqrt{\lambda} K_0(\sqrt{\lambda})}{K_1(\sqrt{\lambda})} \right]^{-1} B\sigma
\]
Equation A16 can be inverted numerically to attain the profiles of $A$ and $\eta$. The numerical inversion is based on the method introduced by Talbot (1979). In Figure A1, the dots represent results from the numerical inversion of the Laplace transform, and the solid lines represent the results from section 2. It can be seen that the Laplace transform gives results consistent with those from section 2, thus verifying our solution.

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