Transient heat transfer for GPMG's barrel 7.62x51mm

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Abstract. In this paper, a modified analytical solution for time-dependent one dimension heat diffusion equation in cylindrical coordinate had been mathematically solved to calculate the temperature distribution through thick-walled weapon barrel subjected to successive shots. The analytical solution has been compared with experimental measurements for the temperature at the outer surface of the barrel at two different positions. The General Purpose Machine Gun (GPMG) 7.62X51 mm was considered as the case of study. The results show a good agreement between the analytical model and the experimental work with a difference less than 3 %. The main difference between them mainly as a result of the very short time of the heating phenomena as it takes about 6 (ms) only, although the used Data Acquisition System was able to take a high sampling rate and a high sensitive thermocouple type k with only 0.5 mm thickness had been used.

1. Introduction
Not only the weapon's barrel receives bulky amount of heat due to successive shots but also it has a very short time to cool down which leads to heat accumulation on the barrel wall thickness. Consequently, the effect of the thermal pulses with rapid heating and cooling to the barrel wall leads to a dramatic change in temperature distribution through the gun barrel wall. A time-dependent heat diffusion equation should be mathematically solved in order to estimate the temperature variation through the barrel wall thickness. The chromium layer at the inner surface of the barrel is very brittle, it is responsible to resist the cracks and crack extension under the action due to repeated of the fire. The repeated fire makes the barrel bore geometry changes and this leads to decrease the projectile muzzle velocity, projectile stability, the firing accuracy and the tactical technical performance of the machine gun declining, [1].The solution of time dependant heat diffusion equation with nonhomogeneous boundary conditions is solved numerically [2,3], or algebraically solved [4-7] and using advanced software like MATLAB and Mathematica in one dimension [8] or two dimensions, [9].In this paper, a modified model for a time-dependent heat diffusion model, [4] had been solved. The modification in the analytical model was on convection coefficient of gases at the inner surface of the barrel; it had been taken to exponentially decay at the discharge period instead of linear decreasing. On the other hand, the analytical model this time was validated using an experimental measurement not simulation model. The experimental work was carried on 7.62X51 mm General Purpose Machine Gun’s barrel using a highly sensitive thermocouple type k with only 0.5 mm thickness.

2. Modified Analytical Model
The heat diffusion equation in radial direction with no heat generated from the barrel elements and constant heat conductivity is as shown in equation (1), [10]:
\[ \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} = \frac{1}{\alpha} \frac{\partial T}{\partial t} \]  

(1)

Where; \( T \) is the barrel temperature in \([k]\), \( r \) is the barrel radius in \([m]\), \( \alpha \) is the thermal diffusivity of the weapon barrel material in \([m^2/s]\) and \( t \) is time in \([s]\).

The inner surface boundary condition, according to Newton’s law of cooling is taken as follows, [11]

\[ k \left[ \frac{\partial T}{\partial r} \right]_{r=R_i} = h_f (T - T_f) \]  

(2)

Where; \( k \) is the thermal conductivity of the weapon barrel material in \([W/mk]\), \( R_i \) is the inner surface of the barrel, \( h_f \) is the flow- may be gases or air- convection coefficient in \([W/m^2k]\) and \( T_f \) is the flow temperature in \([k]\).

Similarly, the outer surface boundary condition is as in equation (3), [11]

\[ k \left[ \frac{\partial T}{\partial r} \right]_{r=R_o} = h_\infty (T_\infty - T) \]  

(3)

Where; \( R_o \) is the outer surface of the barrel, \( h_\infty \) is the ambient convection coefficient of air in \([W/m^2k]\) and \( T_\infty \) is the ambient temperature in \([k]\).

The initial condition at time \( t=0 \) defines the temperature of the barrel before the firing:

\[ T(r, t)_{t=0} = T_o \]  

(4)

As the solution of equations (1-4) is too difficult, the solution is performed in two steps. The first step starts from the imitation of the cartridge till the end of gases discharge from the barrel and this the heating period. On the other hand, the second step ends before the initiation of the next cartridge and this period named as cooling period. Moreover, the coefficient of convection of gases and temperature of gases at the first stage usually is taken as an average value. The modification was on the convection coefficient of the heating period. The convection coefficient of gases\( (h_{gas}) \) from the initiation of the cartridge till the point of the projectile leaves the muzzle is determined by, [12]:

\[ h_{gas} = h_\infty + \lambda_n \rho C_p \bar{v} \]  

(5)

Where; \( h_{gas} \) is the convection coefficient of gases, \( \lambda_n \) is a dimension less constant which is approximated by \( \lambda_n = [13.2 + 4 \log(200 \cdot R_i)]^2 \), \( \rho \) is the mean gases density and it simply calculated by dividing the propellant burnt mass by the inner volume of the barrel till the projectile base, \( C_p \) is the specific heat and it is considered to equals 1050 \([J/kgk]\) and \( \bar{v} \) is the mean velocity of gases and it assumed to be linearly distributed inside the barrel. It equals the projectile velocity\( (\bar{v} = v_p) \) from one end and equals the breech velocity\( (\bar{v} = v_B = 0) \) from the other end. \( \bar{v} = 0.5 \cdot v_p \).

The convection coefficient of gases in eq (5) is valid till the projectile leaves the barrel but, after the projectile leaves the barrel, the convection coefficient of gases\( (h_{gas}) \) is not considered to linearly decay as [4] but it considered to exponentially decay according to Bravin’s law, [6] as follows:
\[ h_{\text{gas}} = h_{\text{gas}_M} e^{-At} \]  

(6)

Where; \( h_{\text{gas}_M} \) is the convection coefficient of gases at muzzle section, \( A \) is the exponent of AAPG or the discharge period of powder gases from the barrel.

Figure 1. Gases convection coefficient using equation (6) for a single shot at the inner surface, 402 mm from the muzzle section

Figure 2. Gases convection coefficient using previous model, [4] for a single shot at the inner surface, 402 mm from the muzzle section

Figure 1 shows the convection coefficient of gases \( (h_{\text{gas}}) \) using equation (6) during the firing time interval due to single shot at the inner surface, 402 mm away from the muzzle section and the exponentially decrease of it at the discharge period is clearly represented by the shaded area. It had a maximum value about \( 7 \times 10^5 \text{[W/m}^2\text{k]} \) and the average value about \( 4.5 \times 10^4 \text{[W/m}^2\text{k]} \). On the other hand, figure 2 shows the gases convection coefficient using previous model, [4] for a single shot at the same section of figure 1, it had an average value about \( 2.6 \times 10^5 \text{[W/m}^2\text{k]} \). Using the algebraic solution of the system of equations (1-6) using the same technique of [4].

3. Experimental measurements

The experimental work was carried on General Purpose Machine Gun’s barrel. The GPMG was the best choice for measurement as it has a high rate of fire besides the outer surface of its barrel is almost totally uncovered with weapon casing and it is easy to measure at different stations. The measurement was carried using a highly sensitive thermocouple type k with only 0.5 mm thickness. The
measurement had been performed at 2 stations at the outer surface. The two stations of measurement were selected at 140 (mm) and 402 (mm) from the muzzle. The selection of the two stations related to show the effect of increasing of the heating period and the effect of barrel thickness. As one station is near to the muzzle which had only 6 mm thickness and the other station is near to the chamber which had a 15 mm thickness.

3.1. Description of experimental setup
The experimental setup as in figure 3 consists of:
- Test rig (1).
- GPMG 7.62x51 (mm) (2).
- Two thermocouples Type k made of Chrome-Aluminium with 0.5mm diameter (3).
- Data Acquisition system with a card with 64 ports and sampling rate 200 kHz maximum (4)
- PC-computer.

![Figure 3. GPMG and its fixation.](image)

3.2. Test procedure
The two thermocouples (2) are firmly fixed at two different stations at the outer surface of the GPMG's barrel which is elastically mounted on the test rig (1). Although the DAC was able to take a highly sampling rate, the phenomena of heating takes a very short times about 6 (ms) only. So, after each experiment a 60 (s) period of time is taken as a relaxation time. The first experiment was firing 3 successive shots in automatic trigger and 60 (s) as a relaxation time. For another continuous shot, experiment is repeated after the barrel was reached the room temperature. The second experiment was firing 10 successive shots and 60 (s) as a relaxation time. Before performing the third experiment the barrel is left till it reached the room temperature. The third experiment was firing 15 successive shots and 60 (s) as a relaxation time after the barrel was reached the room temperature.

4. Comparison of results
In order to compare the results of the analytical model with the experimental work, the analytical model was carried out at the same two sections of the experimental work 402 mm and 140 mm from
the muzzle section. Also, the initial temperature of the barrel is considered as the same initial temperature of the experiment and changed from one experiment to another one.

In figure 4 and 5 show the outer surface temperature of the barrel for firing 3 successive shots and take 60 (s) as a relaxation time at 402 mm and 140 mm far from the muzzle section respectively. It is shown that the temperature in the two figures had the same trend as it increase till it reach about 301 [K] and 300 [K] respectively almost for the analytical model and the experimental measurement, then, it decay with time till the end of the relaxation period.

**Figure 4.** Transient temperature-time history of at the outer surface of the barrel for 3 shots, 402 mm from the muzzle section.

**Figure 5.** Transient temperature-time history of at the outer surface of the barrel for 3 shots, 140 mm from the muzzle section.
Figure 6. Transient temperature-time history of at the outer surface of the barrel for 10 shots, 402 mm from the muzzle section.

Figure 7. Transient temperature-time history of at the outer surface of the barrel for 10 shots, 140 mm from the muzzle section.

Figure 8. Transient temperature-time history of at the outer surface of the barrel for 15 shots, 402 mm from the muzzle section.
Figure 9. Transient temperature-time history of at the outer surface of the barrel for 15 shots, 140 mm from the muzzle section.

Figure 4 and 5 show the outer surface temperature of the barrel for firing 3 successive shots and take 60 (s) as a relaxation time at 402 mm and 140 mm from the muzzle section respectively. The temperature at the outer surface increases to 302 [K] and 300 [K], respectively. Figure 6 and 7 show the outer surface temperature of the barrel for firing 10 successive shots and take 60 (s) as a relaxation time at 402 mm and 140 mm from the muzzle section respectively. The temperature at the outer surface increases to 313 [K] and 311 [K], respectively. Figure 8 and 9 show the outer surface temperature of the barrel for firing 15 successive shots and take 60 (s) as a relaxation time at 402 mm and 140 mm from the muzzle section respectively. The temperature at the outer surface increases to 302 [K] and 300 [K], respectively. It is clear that the temperature in the two figures increase till it reach about 324 [K] and 321 [K] respectively. In each figure at the beginning of firing there is a difference between the analytical model and the experimental measurement, then, it show a good agreement between the two models at the relaxation period. The difference between the two models may be as a result of the very short time of the heating phenomena as it takes about 6 (ms) only as mentioned before. Moreover, although, the DAC was able to take a highly sampling rate, was not a real time results. But, when the DAC take some time with almost constant phenomena no heating impulse gives a good agreement with the analytical model.

5. Conclusion

In this paper, a modified analytical model had been used to investigate the transient temperature of the barrel thickness due to firing. The modification was on the convection coefficient of gases at the heating period especially after the projectile leaves the barrel till the end of discharge of powder gases. The convection coefficient of gases at this period is assumed to exponentially decay according to Bravin’s law. On the other hand, the measurement had been performed at 2 stations at the outer surface of PMG’s barrel. The two stations of measurement were selected at 140 (mm) and 402 (mm) from the muzzle. The selection of the two stations related to show the effect of increasing of the heating period and the effect of barrel thickness. As one station is near to the muzzle which had only 6 mm thickness and the other station is near to the chamber which had a 15 mm thickness. The analytical model show a good agreement with the experimental work, but there are a difference between them at the beginning. The difference mainly as a result of the very short time of the heating phenomena as it takes about 6 (ms) only, although the DAC was able to take a highly sampling rate and a highly sensitive thermocouple had been used. The temperature difference between the analytical solution and the experimental work is less than 3%. But, due to relaxation time the analytical model gives a good agreement with the experimental work and that declares the necessity of the relaxation period to enhance the measurement results due to the short period of the phenomena. Moreover, that declares the positive effect of the change of inner surface boundary conditions during heating period.
6. References

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