Topological Black Holes of Gauss-Bonnet-Yang-Mills Gravity

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Abstract

We present the asymptotically AdS solutions of Gauss-Bonnet gravity with hyperbolic horizon in the presence of a non-Abelian Yang-Mills field with the gauge semisimple group $\text{So}(n(n-1)/2-1,1)$. We investigate the properties of these solutions and find that the non-negative mass solutions in 6 and higher dimensions are real everywhere with spacelike singularities. They present black holes with one horizon and have the same causal structure as the Schwarzschild spacetime. The solutions in 5 dimensions or the solutions in higher dimensions with negative mass are not real everywhere. In these cases, one needs a transformation to make the solutions real. These solutions may present a naked singularity, an extreme black hole, a black hole with two horizons, or a black hole with one horizon.

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I. INTRODUCTION

The field equations which describe gravitation in classical interaction with the Yang-Mills fields are highly complicated nonlinear equations. However, considering the static spherically symmetric spacetime and specific gauge group such as $SU(N)$, the field equations will reduce to a system of ordinary differential equations. In particular, Bartnik and McKinnon \[1\] were the first who discovered a class of asymptotically flat spherically symmetric solitonic solutions of Einstein gravity in the presence of a non-Abelian $SU(2)$ Yang-Mills field in four dimensions, numerically. This attracted a great deal of attentions in the properties of the Einstein-Yang-Mills (EYM) gauge theories for their rich geometrical and physical structure. Indeed, it was a surprising discovery since for a long time it had been conjectured that there are no solitonic solutions in EYM models, based on the fact that there are no solitons in pure gravity \[2\], pure YM theories \[3\] nor EYM in three dimensional spacetime \[4\].

Historically, Yasskin \[5\] found the first black hole solutions of the EYM equations and he conjectured that these solutions were the only possible ones. However, about two decades later, it was shown that this conjecture would be false. In fact, soon after the discovery of solitons for the $SU(2)$ EYM theory \[1\], static spherically symmetric black hole solutions of this model were found numerically \[6, 7\]. It is well-known now that the EYM theory contains hairy black hole solutions (see \[8\] for a detailed review). Among EYM black hole solutions, those in asymptotically AdS space seem to be more interesting. This is due to the fact that all the asymptotically flat solutions of pure EYM gravity, discovered to date, are unstable \[9\]. This intimates that, although the no-hair theorem is violated in this case, its spirit is preserved: stable black hole are still simple objects characterized by a few parameters. But the situation is different in the case of asymptotically AdS spacetime for which it is possible to find at least some stable black holes with hair \[10, 11\]. The non-linear nature of field equations of the EYM theory make them so intricate, and most of the work in this context have been done numerically. Nevertheless, some analytical solutions have been found by considering some particular ansatz to make the field equations simpler. One of this ansatz is the familiar Wu-Yang ansatz which was originally introduced in $N = 4$ field theory \[12\] and was applied by Yasskin \[5\] to find an exact black hole solution. Recently, this ansatz has been used to find the EYM solutions in higher dimensions \[13\].

Accepting a nonlinear theory in the matter side of the gravitational field equations invites
nonlinear terms in the geometrical side of the field equations too, and therefore we extend
the geometry side of the field equations from Einstein tensor to the so-called Lovelock
tensor \[14\]. Lovelock gravity is the most general classical theory of gravity, for which the
field equations are generally covariant and contain at most second order derivatives of the
metric. In this paper, we investigate the topological black holes of second order Lovelock
(Gauss-Bonnet) gravity coupled with the non-Abelian gauge field with \(S\!o(n(n-1)/1-1,1)\)
gauge symmetry. These kinds of black holes in Gauss-Bonnet-Yang-Mills (GBYM) gravity
with spherical horizon have been introduced in \[15\].

The outline of this paper is as follows. In Sec. (II) we introduce the field equations of the
\((n+1)\)-dimensional Gauss-Bonnet gravity in the presence of YM fields with a semisimple
gauge group. In Sec. (III), we obtain the 5-dimensional solution of this theory and inves-
tigate its properties. Section (IV) is devoted to the higher-dimensional solutions of GBYM
gravity and their properties. We finish our paper with some concluding remarks.

II. FIELD EQUATIONS

The model which will be discussed here is an \((n+1)\)-dimensional GBYM system for an
\(N\)-parameters gauge group \(G\), which is assumed to be at least semisimple with structure
constants \(C^a_{bc}\). The metric tensor of the gauge group is

\[
\Gamma_{ab} = C^c_{ad}C^d_{bc},
\]

where the Latin indices \(a, b, \ldots\) go from 1 to \(N\), and the repeated indices is understood to
be summed over. According to Cartan’s criteria the determinant of \(\Gamma_{ab}\) is not zero, and
therefore one may define

\[
\gamma_{ab} \equiv -\frac{\Gamma_{ab}}{|\text{det } \Gamma_{ab}|^{1/N}},
\]

where \(|\text{det } \Gamma_{ab}|\) is the positive value of determinant of \(\Gamma_{ab}\). The action of \((n+1)\)-dimensional
GBYM gravity may be written as

\[
I_{\text{GBYM}} = \int d^{n+1}x \sqrt{-g} \left[ \mathcal{L}_1 + \alpha_2 \mathcal{L}_2 - 2\Lambda - \gamma_{ab} F^{(a)}_{\mu\nu} F^{(b)\mu\nu} \right],
\]

where \(\mathcal{L}_1 = R\) is just the Einstein-Hilbert Lagrangian, \(\mathcal{L}_2 = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4 R_{\mu\nu} R^{\mu\nu} + R^2\)
is the Gauss-Bonnet Lagrangian, \(\alpha_2\) is Gauss-Bonnet coefficient with dimension \((\text{length})^2\)
which is assumed to be positive, \( \Lambda = -n(n-1)/2l^2 \) is the cosmological constant and \( F_{\mu \nu}^{(a)} \)'s are the gauge fields:
\[
F_{\mu \nu}^{(a)} = \partial_\mu A_\nu^{(a)} - \partial_\nu A_\mu^{(a)} + \frac{1}{2e} C_{bc}^{a} A_\mu^{(b)} A_\nu^{(c)}.  
\] 
In Eq. (4) \( e \) is a coupling constant and \( A_\mu^{(a)} \)'s are the gauge potentials. Variation of the action (3) with respect to the spacetime metric \( g_{\mu \nu} \) and the gauge potential \( A_\mu^{(a)} \) yields the GBYM equations as
\[
G_{\mu \nu}^{(1)} + \alpha_2 G_{\mu \nu}^{(2)} + \Lambda g_{\mu \nu} = 8\pi T_{\mu \nu},  
\]
\[
\nabla_\mu F^{\mu \nu} + \frac{1}{e} C_{bc} A_\mu^{(b)} F^{(c) \mu \nu} = 0,  
\]
where \( G_{\mu \nu}^{(1)} \) is the Einstein tensor, \( G_{\mu \nu}^{(2)} \) is
\[
G_{\mu \nu}^{(2)} = 2(R_{\mu \sigma \kappa \tau} R_{\nu}^{\sigma \kappa \tau} - 2R_{\mu \rho \sigma} R_{\nu}^{\rho \sigma} - 2R_{\nu}^{\rho \sigma} R_{\mu}^{\rho \sigma} + RR_{\mu \nu}) - \frac{1}{2} \mathcal{L}_2 g_{\mu \nu},
\]
and the stress-energy tensor of the YM field is
\[
T_{\mu \nu} = \frac{1}{4\pi} \gamma_{ab} \left( F_{\mu \nu}^{(a)} F_{\nu \lambda}^{(b)} - \frac{1}{4} F_{\lambda \mu}^{(a) \lambda} F_{\nu \mu}^{(b)} g_{\mu \nu} \right).  
\]

III. 5-DIMENSIONAL STATIC SOLUTIONS

The 5-dimensional solution incorporates a logarithmic term unprecedented in other dimensions, and therefore we shall treat it in some details. The 5-dimensional, static metric with hyperbolic horizon may be written as
\[
ds^2 = -f(r) \, dt^2 + \frac{dr^2}{f(r)} + r^2 \{d\theta^2 + \sinh^2 \theta \, (d\varphi^2 + \sin^2 \varphi \, d\psi^2) \}.  
\]
Introducing the coordinates
\[
x_1 = r \sinh \theta \sin \varphi \cos \psi,  
\]
\[
x_2 = r \sinh \theta \sin \varphi \sin \psi,  
\]
\[
x_3 = r \sinh \theta \cos \varphi,  
\]
\[
x_4 = r \cosh \theta,  
\]
and using the generalized Wu-Yang Ansatz

\[ A^{(1)} = \frac{e}{r^2} (x_1 dx_4 - x_4 dx_1) = -e \sin \varphi \cos \psi d\theta - e \sinh \theta \cosh \theta (\cos \varphi \cos \psi d\varphi - \sin \varphi \sin \psi d\psi), \]

\[ A^{(2)} = \frac{e}{r^2} (x_2 dx_4 - x_4 dx_2) = -e \sin \varphi \sin \psi d\theta - e \sinh \theta \cosh \theta (\cos \varphi \sin \psi d\varphi + \sin \varphi \cos \psi d\varphi), \]

\[ A^{(3)} = \frac{e}{r^2} (x_3 dx_4 - x_4 dx_3) \]

\[ A^{(4)} = \frac{e}{r^2} (x_1 dx_2 - x_2 dx_1) = e \sinh^2 \theta \sin^2 \varphi d\psi \]

\[ A^{(5)} = \frac{e}{r^2} (x_1 dx_3 - x_3 dx_1) \]

\[ A^{(6)} = \frac{e}{r^2} (x_2 dx_3 - x_3 dx_2) \]

one can show that the gauge fields (9) satisfy the YM equation (6). The non-zero structure constants of the gauge group are:

\[ C_{24}^{3} = C_{35}^{1} = C_{41}^{2} = C_{36}^{3} = C_{51}^{3} = C_{62}^{3} = 1, \]

\[ C_{56}^{4} = C_{21}^{4} = C_{64}^{5} = C_{31}^{5} = C_{45}^{6} = C_{32}^{6} = 1, \]

which show that the gauge group is isomorphic to \( So(3, 1) \). Using the definition of the metric tensor of the gauge group, one obtains:

\[ \gamma_{ab} = \text{diag}(-1, -1, -1, 1, 1, 1). \]

The non-zero components of energy-momentum tensor (7) reduce to

\[ T^t_t = T^r_r = -\frac{3e^2}{r^4} \]

\[ T^\varphi_\varphi = T^\psi_\psi = \frac{e^2}{r^4} \]

(10)

Defining \( \alpha \equiv (n - 2)(n - 3)\alpha_2 \), it is a matter of calculation to show that the solution of the field equation (5) may be written as

\[ f(r) = -1 + \frac{r^2}{2\alpha} \left( 1 \mp \sqrt{1 - \frac{4\alpha}{I^2} + \frac{12\alpha m}{r^4} + \frac{8\alpha e^2 \ln(r)}{r^4}} \right), \]

(11)
where \( m \) is the integration constant related to the mass parameter. As one can see from Eq. (11), the solution has two branches with \(-\) and \(+\) signs. The acceptable sign for which the solution reduces to the solution of EYM gravity introduced in [16] as \( \alpha \) goes to zero is the solution with minus sign.

It is a matter of calculation to show that the solution is asymptotically AdS with the effective cosmological constant

\[
\Lambda_{\text{eff}} = -n(n-1)\alpha \left(1 - \sqrt{1 - \frac{4\alpha}{l^2}}\right)^{-1},
\]

with \( n = 4 \). One may note that \( \alpha \) should be less than \( l^2/4 \). The metric function \( f(r) \) given by Eq. (11) is real for \( r \geq r_0 \), where \( r_0 \) is the real root of

\[
(l^2 - 4\alpha)r^4 + 12\alpha ml^2 + 8\alpha e^2 l^2 \ln(r) = 0,
\]

given as

\[
r_0 = \exp\left\{ \frac{6m}{e^2} - \frac{1}{4e^2}\text{LambertW}\left( \frac{(l^2 - 4\alpha)}{2\alpha e^2 l^2} e^{-6m/e^2} \right) \right\}.
\]

Thus, one should restrict the spacetime to the region \( r \geq r_0 \), by introducing a new radial coordinate \( \rho \) as:

\[
\rho^2 = r^2 - r_0^2 \Rightarrow dr^2 = \frac{\rho^2}{\rho^2 + r_0^2} d\rho^2
\]

With this new coordinate, the above metric becomes:

\[
ds^2 = -f(\rho)dt^2 + \frac{\rho^2 d\rho^2}{(\rho^2 + r_0^2)f(\rho)} + (\rho^2 + r_0^2) \left\{ d\theta^2 + \sin^2\theta \left( d\varphi^2 + \sin^2\varphi d\psi^2 \right) \right\},
\]

where now one should substitute \( r = \sqrt{\rho^2 + r_0^2} \) in Eq. (11).

One can show that the Kretschmann scalar \( R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} \) diverges at \( \rho = 0 \) (\( r = r_0 \)), and therefore there is a curvature singularity located at \( r = r_0 \). Seeking possible black hole solutions, we turn to looking for the existence of horizons. In the case of having black hole solution, the radius of the black hole is obtained by solving

\[
-1 + \frac{r_+^2}{2\alpha} = \frac{r_+^2}{2\alpha} \sqrt{1 - \frac{4\alpha}{l^2} + \frac{12\alpha m}{r_+^4} + \frac{8\alpha e^2 \ln(r_+)}{r_+^4}} \geq 0,
\]

which shows that \( r_+^2 \geq 2\alpha \).
A. Extreme black holes:

First, we consider the conditions of having extreme black holes, for which the temperature vanishes. The Hawking temperature may be found by analytic continuation of the metric as

\[ T = \frac{2r_+^4 - l^2r_+^2 - e^2l^2}{2\pi l^2 r_+(r_+^2 - 2\alpha)}, \quad (17) \]

and therefore the horizon radius of the extreme black hole located at the largest real root of the equation \( T = 0 \) given as

\[ r_{\text{ext}}^2 = \frac{l^2}{4} \left( 1 + \sqrt{1 + \frac{8e^2}{l^2}} \right). \quad (18) \]

One can show that \( r_{\text{ext}}^2 \) given by Eq. (18) is larger than \( 2\alpha \) for \( \alpha \leq l^2/4 \), and therefore the solution given in Eq. (11) presents an extreme black hole provided the mass parameter is chosen to be equal to:

\[ m_{\text{ext}} = \frac{r_{\text{ext}}^4 - l^2r_{\text{ext}}^2 - 2l^2 e^2 \ln r_{\text{ext}} + \alpha l^2}{3l^2}, \]

where \( r_{\text{ext}} \) is given by Eq. (18). Numerical calculations show that the value of \( m_{\text{ext}} \) is negative.

B. Non-extreme black holes:

Second, we consider the conditions of having non-extreme black holes. In order to do this, we introduce a critical value for the mass parameter, \( m_c \), which is the real root of \( f(r_0 = \sqrt{2\alpha}) = 0 \) given as:

\[ m_c = -\frac{1}{3} \left\{ \alpha \left( 1 - \frac{4\alpha}{l^2} \right) + e^2 \ln(2\alpha) \right\} = 0, \quad (19) \]

which is negative. The solution (11) presents a naked singularity if \( m < m_{\text{ext}} \), an extreme black hole if \( m = m_{\text{ext}} \), a black hole with two horizons if \( m_{\text{ext}} < m \leq m_c \), and a black hole with one horizon provided \( m \geq m_c \). Since \( m_c < 0 \), the solution with positive mass parameter always presents a black hole with one horizon. Note that the singularity of the spacetime with \( m > m_c \) is spacelike, a property which does not happen for the 5-dimensional solution of Gauss-Bonnet-Maxwell (GBM) gravity. Figure 1 shows the metric function versus the mass parameter for various values of \( m \).
FIG. 1: $f(r)$ versus $r$ for $n = 4, l = 1, \alpha = .1, e = .2, m < m_{\text{ext}} < 0, m = m_{\text{ext}} < 0, m_{\text{ext}} < m < m_c$
m = m_c < 0 and $m > m_c$ from up to down, respectively.

IV. $(n + 1)$-DIMENSIONAL STATIC SOLUTIONS

The static metric of an $(n + 1)$-dimensional black hole with hyperbolic horizon may be written as:

$$ds^2 = -f(r)\, dt^2 + \frac{dr^2}{f(r)} + r^2\left(d\theta^2 + \sinh^2\theta\, d\Omega_{n-2}^2\right)$$

(20)

where $d\Omega_{n-2}^2$ is the line element of $(n - 2)$-sphere. In order to obtain the gauge fields, we use the coordinates

$$x_1 = r \sinh \theta \prod_{j=1}^{n-2} \sin \varphi_i,$$

$$x_i = r \sinh \theta \cos \varphi \prod_{j=1}^{n-i-1} \sin \varphi_j; \quad i = 2, \ldots, n - 1,$$

$$x_n = r \cosh \theta,$$

and the ansatz

$$A^{(a)} = \frac{e}{r^2} (x_i dx_n - x_n dx_i); \quad a = i = 1, \ldots, n - 1,$$

$$A^{(b)} = \frac{e}{r^2} (x_i dx_j - x_j dx_i); \quad i < j,$$

(21)

where $b$ runs from $n$ to $n(n - 1)/2$. It is a matter of calculation to show that the gauge fields (21) satisfy the YM equation, and the Lie algebra of the gauge group is $SO(n(n - 1)/2 - 1, 1)$ with the following $\gamma_{ab}$:

$$\gamma_{ab} = \epsilon_a \delta_{ab}; \quad \text{no sum on } a,$$
where $\epsilon_a$ is

$$
\epsilon_a = \begin{cases} 
-1 & 1 \leq a \leq n-1 \\
1 & n \leq (a) \leq \frac{n(n-1)}{2} 
\end{cases}.
$$

Using the definition of the energy-momentum tensor (7), one obtains:

$$
T^t_t = T^r_r = -\frac{(n-2)(n-1)\epsilon^2}{2r^4},
$$

$$
T^\theta_\theta = T^\phi_\phi = T^\psi_\psi = -\frac{(n-2)(n-5)\epsilon^2}{2r^4}.
$$

(23)

Solving the $(n+1)$-dimensional field equation (5) and using the fact that the solution should be reduced to the $(n+1)$-dimensional solution of EYM equation as $\alpha$ goes to zero, one obtains:

$$
f(r) = -1 + \frac{r^2}{2\alpha} \left( 1 - \sqrt{1 - \frac{4\alpha}{l^2} + \frac{4(n-1)\alpha m}{r^n} + \frac{4(n-2)\alpha e^2}{(n-4)r^4}} \right),
$$

(24)

where $m$ is the integration constant related to the mass parameter of the spacetime. As in the case of 5-dimensional solution, if the following equation

$$
(n-4)(l^2 - 4\alpha)r^n + 4(n-2)\alpha e^2 r^{n-4} + 4(n-1)\alpha m = 0,
$$

(25)

has no real solution (non-negative mass), then the metric function (24) is real. For the case that Eq. (25) has real roots (negative mass), one should apply the transformation (14) in order to have a real spacetime, where $r_0$ is now the largest real root of Eq. (25).

Again, the solution is asymptotically AdS with the cosmological constant given in Eq. (12). Demanding the absence of conical singularity at the horizon in the Euclidian sector of the black hole solution, the Hawking temperature is

$$
T = \frac{f'(r_+)}{4\pi} = \frac{nr_+^4 - (n-2)l^2r_+^2 + (n-4)\alpha l^2 - (n-2)e^2 l^2}{4\pi l^2 r_+(r_+^2 - 2\alpha)},
$$

(26)

which vanishes for the extreme black hole with the horizon radius

$$
r_{ext}^2 = \frac{(n-2)l^2}{2n} \left( 1 + \sqrt{1 - \frac{4n(n-4)\alpha}{(n-2)^2 l^2} + \frac{4ne^2}{(n-2)l^2}} \right).
$$

(27)

One can show that $r_{ext}^2$ given by Eq. (27) is larger than $2\alpha$ for $\alpha \leq l^2/4$, and therefore the solution given in Eq. (24) presents an extreme black hole provided the mass parameter is chosen to be:

$$
m_{ext} = -\frac{(n-2)l^2 r_{ext}^{n-4}}{n^2} \left( 1 - \frac{4n\alpha}{(n-2)l^2} - \frac{4ne^2}{(n-4)l^2} + \sqrt{1 - \frac{4n(n-4)\alpha}{(n-2)^2 l^2} + \frac{4ne^2}{(n-2)l^2}} \right),
$$

(28)
FIG. 2: \( f(r) \) versus \( r \) for \( n = 6, l = 1, \alpha = .1, e = .2, m < m_{\text{ext}} < 0, m = m_{\text{ext}} < 0, m_{\text{ext}} < m < m_c \), where \( r_{\text{ext}} \) is given by Eq. (27).

Before discussing the non-extreme black holes, we introduce the critical mass \( m_c \), which is the real root of \( f(r_0 = \sqrt{2\alpha}) = 0 \) give as:

\[
m_c = -\frac{(2\alpha)^{(n-4)/2}}{n - 1} \left\{ \alpha \left( 1 - \frac{4\alpha}{\ell^2} \right) + \frac{(n - 2)e^2}{n - 4} \right\}, \tag{29}
\]

which is negative. Then the solution given by Eq. (24) presents a naked singularity if \( m < m_{\text{ext}} \), an extreme black hole provide \( m = m_{\text{ext}} \), a black hole with two horizon if \( m_{\text{ext}} < m < m_c \) and a black hole with one horizon and spacelike singularity provided \( m \geq m_c \). Since \( m_c < 0 \), the solution with positive mass parameter always presents a black hole with one horizon and spacelike singularity. Figure 2 shows the metric function versus \( r \) for various values of mass parameter. Also note that \( r_0 \) for non-negative mass is zero, and the singularity of the solution is spacelike.

V. CONCLUDING REMARKS

In this paper, we presented the topological solutions of the second order Lovelock gravity with hyperbolic horizon coupled to a Yang Mills field in the AdS background and investigated their properties. To obtain the solutions analytically, we used a spherically symmetric gauge potential with \( SO(n(n - 1)/2 - 1, 1) \) gauge group introduced in Eq. (21). Although we have a matter field, we don’t have a new hair. This is due to the special form of the potential, which causes the matter charge to be zero when one looks from infinity. We discussed the
causal structure of the spacetimes for different values of the parameters of the solutions such as the Lovelock coefficient, charge and mass. In 5 dimensions, the metric function is not real everywhere and therefore we performed a coordinate transformation to make it real in all the spacetime. However, in higher dimensions the non-negative solution was real everywhere, and it respects the cosmic censorship hypothesis. That is, the singularity of non-negative solution is spacelike and it is not naked. Moreover, the higher-dimensional solutions with negative mass need the transformation (14), and may present a naked singularity, an extreme black hole, a black hole with two horizons or a black hole with one horizon.

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