Risk Assessment of Distribution Network Based on Random set Theory and Sensitivity Analysis

Sh Zhang¹, C X Bai¹, J Liang¹, L Jiao¹, Z Hou² and B Zh Liu²

¹ Electric Power Research Institute, State Grid Ningxia Electric Power Company, Yinchuan, China
² School of Electrical and Electronic Engineering, North China Electric Power University, Beijing, China

0zhangsh0nx@163.com, baicunxi@126.com, liangjian@dky.nx.sgcc.com.cn, 32599767@qq.com, zan_hou@163.com, bzliu@ncepu.edu.cn

Abstract. Considering the complexity and uncertainty of operating information in distribution network, this paper introduces the use of random set for risk assessment. The proposed method is based on the operating conditions defined in the random set framework to obtain the upper and lower cumulative probability functions of risk indices. Moreover, the sensitivity of risk indices can effectually reflect information about system reliability and operating conditions, and by use of these information the bottlenecks that suppress system reliability can be found. The analysis about a typical radial distribution network shows that the proposed method is reasonable and effective.

1. Introduction
Distribution network takes on the task for supplying power to the user directly. There is a large number of equipment dispersed around the system. Complex external and internal factors can bring risks to the safe and stable operation of distribution network. Therefore, it is necessary to carry out the risk assessment work [1-2]. The purpose of risk assessment is to obtain online operating indices when considering the operating environment and system conditions and other related factors.

The external environment and operating conditions of system components are changing over time, and failure rates are dependence on these dynamic factors. Thus, how to realize the unified representation and amalgamation of multisource information is urgent to be solved. Random set theory [3] is expected to solve this problem. Random set theory is an important new branch of mathematics which combines the traditional probability and set theory, and it is an extension of random variables. The remote sensing image, rainfall and other multivariate information are analysed and integrated through the calculation of random set in [4]. Then the cumulative probability distribution function of a single risk level can be obtained. The flood risk level of a certain area can also be evaluated according to certain judgment criterion, and the probability of the risk can be derived more precisely.

Parameters of system components include capacity of components, failure rates, repair rates, system load (treated as a generalized system component) and so on. The contribution of each component to the system reliability depends on the location of the component in the network and its own parameter values, so the system reliability’s sensitivity which related to the variation of each component’s parameters is different. The traditional reliability indices can only reflect the reliability level of whole system or each load point. If we can find the component parameters affecting the
system reliability significantly, then important guiding opinions can be offered for the planning and operation of distribution system. The system reliability’s sensitivity analysis provides a better way to solve this problem. In [5], Patton and Tram proposed a sensitivity analysis method for generation system reliability regarding to failure and repair rates of equipment.

In this paper, a representation method of multisource information based on random set theory is proposed. Random variables describing equipment parameters are converted to their random set form, and the belief function and plausibility function of random set are used to obtain the upper and lower cumulative probability distributions of risk indices. And we have assumed that terminal active power and reactive power are treated as input random variables to demonstrate the effect of reactive power compensation on the terminal voltage. Then the bottlenecks that suppress system reliability can be found according to the sensitivity analysis. The correctness and effectiveness of methods mentioned above are verified by the reliability assessment for a test system.

2. Random set theory

2.1. Concept of random set

Let us consider such a probability space \((\Omega,F,P)\) and a measurable space \((U,B_U)\), where \(\Omega\) is a basic event set, \(F\) is a set class composed by subset of \(\Omega\) (often called the event set), the empty set is an impossible event and \(\Omega \in F\) is an inevitable event, \(P\) is a probability measure defined in \(F\), \(U\) is an observation set, \(B_U\) is the corresponding Borel \(\sigma\)-field, where \(\sigma\)-field is also called set system which is a set formed by some elements belonging to the sample space \(\Omega\). The random variable can be defined as follows [6]. \(x: \Omega \rightarrow U\).

Define a set-valued mapping. \(X: \Omega \rightarrow P(U)\), where \(P(U)\) is the power set constructed by all subset of \(U\). With regard to any \(A \in B_U\), if \(X^+A \equiv \{\omega \in \Omega: X(\omega) \cap A \neq \emptyset\} \in F\), then \(X\) is strongly measurable which can be called random set.

2.2. Upper and lower inverse

With regard to a given set and any \(A \in P(U)\), the upper inverse can be defined as

\[
A^+ = X^+(A) \equiv \{\omega \in \Omega: X(\omega) \cap A \neq \emptyset\}.
\]

The lower inverse is written as

\[
A_\varepsilon = X_\varepsilon(A) \equiv \{\omega \in \Omega: \emptyset \neq X(\omega) \subseteq A\}.
\]

The inverse is

\[
A^{-1} = X^{-1}(A) \equiv \{\omega \in \Omega: X(\omega) = A\}.
\]

2.3. Mass function

Suppose \(U\) denotes a finite set, and \(P(U)\) is the power set constructed by all subset of \(U\). The mapping \(m: P(U) \rightarrow [0,1]\) satisfies \(m(A) \geq 0\).

With regard to any \(A \in P(U)\), there exists \(\sum_{A \in P(U)} m(A) = 1\), and then \(m\) is called a mass function of \(U\), where \(m(A)\) only represents the mass function of set \(A\), not including any subset of \(A\). If \(A \in P(U)\) and \(m(A) > 0\), then \(A\) is a focal element of \(m\). The focal set composed by focal elements is expressed as \(M = \{A \in P(U): m(A) > 0\}\). Its properties are described as follows [7]:

- \(m(U) = 1\).
- There not necessarily exists \(m(A) \leq m(B)\) when \(A \subseteq B\).
- There is no connection between \(m(A)\) and \(m(A^c)\), where \(A^c\) is the complement to \(A\).

2.4. Upper and lower probability
Let \( X : \Omega \rightarrow P(U) \) denotes a random set. For any \( A \in P(U) \), the upper probability is defined as

\[
P^u_X(A) \triangleq P(A^c) / P(U^c).
\]

The lower probability is

\[
P^l_X(A) \triangleq P(A) / P(U^c).
\]

If we have assumed that \( X(\omega) \neq \emptyset \) for any \( \omega \in \Omega \), then \( U^c = \Omega \), \( P(U^c) = 1 \), and what we can get is

\[
P^l(A) = P^u_X(A) = P(A^c) = P(A).
\]

That is the upper and lower probability of the random set \( A \).

### 2.5. Belief measure and plausibility measure

If the set function \( Bel : P(U) \rightarrow [0,1] \) satisfies the conditions expressed below:

- \( Bel(\emptyset) = 0, Bel(U) = 1. \)
- With regard to any \( A_1, A_2, \ldots, A_n \in P(U) \), \( Bel(\bigcup_{i=1}^{n} A_i) \geq \sum [(-1)^{|I|+1} Bel(\bigcap_{i \in I} A_i) : \emptyset \neq I \subseteq \{1, 2, \ldots, n\}] \).

then \( Bel \) is called a belief measure of \( U \) [8]. The specific representation is written as

\[
Bel(A) = P\{X(\omega) \mid A\} = P^l(A) = P(A) = \sum_{A_i \subseteq A} m(A_i).
\]

If the set function \( pl : P(U) \rightarrow [0,1] \) satisfies the conditions expressed below:

- \( pl(\emptyset) = 0, pl(U) = 1. \)
- With regard to any \( A_1, A_2, \ldots, A_n \in P(U) \), \( pl(\bigcup_{i=1}^{n} A_i) \leq \sum [(-1)^{|I|+1} pl(\bigcap_{i \in I} A_i) : \emptyset \neq I \subseteq \{1, 2, \ldots, n\}] \).

then \( pl \) is called a plausibility measure of \( U \). The specific representation is written as

\[
pl(A) = P\{X^*(\omega) \mid A\} = P^u(A) = P(A^c) = \sum_{A_i \supseteq A^c, \emptyset \neq A} m(A_i).
\]

For any \( E \in U \), we can obtain its probability boundaries by using the properties of random set, and the result is: \( Bel(E) \leq Pr\alpha(E) \leq Pl(E) \). There exists \( Bel(E) = Pr\alpha(E) = Pl(E) \) when set \( E \) only contains a single element. The probability boundaries of set \( E \) and the cumulative distribution functions (CDF) of random set are shown in figure 1. From the figure, what we can get is the real cumulative distribution curve is surrounded by \( Pl \) and \( Bel \) which can be called the upper and lower cumulative distribution functions.

![Figure 1. Cumulative distribution functions of random set.](image)

### 3. Sensitivity analysis for risk assessment

Sensitivity method is to obtain the sensitive extend of dependent variable to independent variable, using differential relationship among physical quantities in power systems. Thus, the sensitivity index reflects the changing degree and changing trend of the system reliability caused by the tiny change of equipment parameters [9]. If the reliability index is very sensitive to the parameters of a device, it
usually means that the equipment has a great influence on the system reliability. Therefore, it can increase the system reliability by improving the corresponding parameters of the equipment.

There are two schools of sensitivity analysis, local sensitivity analysis and global sensitivity analysis [10]. The former examines the local response of the output by varying input parameters one at a time, holding other parameters to a central value; and the latter examines the global response (averaged over the variation of all the parameters) of model output by exploring a finite (or even an infinite) region. Since it is very easy to conduct local sensitivity analysis, it is very popular in sensitivity models.

Local sensitivity analysis is also called one at a time method. There are two transformation methods about local sensitivity. One is factor variation method which may increase or decrease the parameters to be analysed by 10%. The other is deviation variation method which may add or reduce a standard deviation on the parameters to be analysed. Sensitivity coefficient is often used as a measure of the sensitivity of parameters. The simplest form of the sensitivity coefficient is [11]

$$S_i = \frac{\partial v}{\partial p_i}$$  \hspace{1cm} (9)

where $S_i$ is $i^{th}$ parameter’s sensitivity, $v$ is the output parameters of the predicted model and $p_i$ is $i^{th}$ parameter.

However, local sensitivity analysis is not computationally effective, because it can only get the sensitivity of a single parameter at a time. It cannot take into consideration the effect of interaction of different parameters. Additionally, the value of other parameters will affect the sensitivity of the parameter specified. In view of this, global sensitivity analysis is increasingly preferred to local sensitivity in recent years. However, for most of the sensitive model study published in China, only local sensitivity analysis is conducted.

4. Case study
The graph of a simple distribution network with reactive power compensation is shown in figure 2. Suppose the output is the voltage $U_2$, and the voltage $U_1$ and all impedances including generator, distribution line and transformer are constant which are expressed as follows. $U_1 = 36 \text{kV}$. $R + jX = 5.01 + j18.34 \Omega$. The value ranges of the terminal output power are denoted as follows. $P_2 + jQ_2 = (6 + j4)\pm 30\% \text{ MVA}$. Suppose the parameters $P_2$ and $Q_2$ are fitted to normally distributed data. It should be noted that the parameters given above are not necessarily subject to be normal distribution and can also be other distributions which depends on the specific situation. And the transverse component of voltage drop is not considered in the calculation.

![Figure 2. A distribution network considering reactive power compensation.](image)

The detail steps of the proposed method are listed as follows.

4.1. Determine the function of the system
According to the calculation of the voltage drop, the formulation of the terminal voltage $U_2$ is
\[ U_1 = kU_2 + \frac{P_2R + (Q_2 - Q_c)X}{kU_2} \]

where \( k \) is the transformation ratio of the transformer which is computed as \( \frac{35}{10.5} \), and \( Q_c \) is the capacity of reactive power compensation.

Suppose \( \xi_1, \xi_2 \) and \( \xi_3 \) are defined as three random variables to describe the stochastic process. \( \xi_1 = P_2, \mu_{\xi_1} = 6, \sigma_{\xi_1} = 0.6 \). \( \xi_2 = Q_2, \mu_{\xi_2} = 4, \sigma_{\xi_2} = 0.4 \). The symbol \( \mu \) and \( \sigma \) are the mean value and the standard deviation of the variables.

First of all, we consider the case that there is no capacity of reactive power compensation. That is \( \xi_3 = Q_c = 0 \).

### 4.2. Calculate the probability distribution of output

Suppose the symbols \( I_1 \) and \( I_2 \) are the value intervals of the variables \( \xi_1 \) and \( \xi_2 \) which are expressed as follows. \( I_1 = [4.2, 7.8] \). \( I_2 = [2.8, 5.2] \). The two intervals are divided averagely into 4 subintervals respectively. The detail divisions are expressed as follows. \( I_{1,j} = [u_{1,j}, u_{1,j+1}] \), \( j = 1, 2, 3, 4 \). \( I_{2,k} = [u_{2,k}, u_{2,k+1}] \), \( k = 1, 2, 3, 4 \). Thus, the domain \( \Theta = I_1 \times I_2 \) is divided into 16 focus elements, and the form of set can be defined as follows. \( F = \{ I_{1,j} \times I_{2,k} \mid j = 1, 2, 3, 4; k = 1, 2, 3, 4 \} \).

The probability of each subinterval can be calculated by the corresponding probability density function of normal distribution, and the joint probability can be expressed as \( m(I_{1,j} \times I_{2,k}) = m_1(I_{1,j})m_2(I_{2,k}) \). The symbols \( m_1(I_{1,j}) \) and \( m_2(I_{2,k}) \) denote the basic probability assignment (BPA) of subintervals and they are listed in Table 1-2.

**Table 1.** The basic probability assignment of \( \xi_1 \).

| \( I_{1,j}/MW \) | \( m_1 \) |
|-----------------|---------|
| [4.2,5.1]       | 0.0655  |
| [5.1,6.0]       | 0.4332  |
| [6.0,6.9]       | 0.4332  |
| [6.9,7.8]       | 0.0655  |

**Table 2.** The basic probability assignment of \( \xi_2 \).

| \( I_{2,k}/MVar \) | \( m_2 \) |
|-------------------|---------|
| [2.8,3.4]         | 0.0655  |
| [3.4,4.0]         | 0.4332  |
| [4.0,4.6]         | 0.4332  |
| [4.6,5.2]         | 0.0655  |

It is obvious that the output \( U_2 \) and its partial derivative function are continuous on the interval \( \Theta \), and \( U_2 \) is monotone decreasing with regard to \( P_2 \) and \( Q_2 \) respectively. The focus elements of the output \( U_2 \) and the corresponding probability assignment can be derived according to the properties of random set. The result is expressed as \( \rho(f(I_{1,j} \times I_{2,k})) = m(I_{1,j} \times I_{2,k}) \). The symbol \( \rho(f(I_{1,j} \times I_{2,k})) \) is the probability assignment of the output \( U_2 \).

The curves of upper and lower cumulative probability distribution \( F_{up}(U_2) \) and \( F_{low}(U_2) \) obtained from the method of random set are in Figure 3 (a). According to the properties of cumulative distribution function, it is obvious that the probability of voltage \( U_2 \) in the interval \( [9.6, 10] \) is very large, indicating a low-voltage risk.
According to the above situation, we consider the case that the reactive power compensation changes discontinuously. The detail expression of $\xi_3$ is in table 3. When the load $P_2 + jQ_2$ is changing, we can change the value of $Q_c$ to ensure that the low-voltage bus voltage of the transformer maintains normal. The curves of upper and lower cumulative probability distribution with the reactive power compensation are in figure 3 (b). It can be concluded that the voltage $U_2$ increases significantly, and the voltage interval [10.2, 10.6] corresponds to a large probability. It indicates that reactive power compensation is of great help to maintain voltage stability.

**Table 3.** The basic probability assignment of $\xi_3$.

| $I_{3,q}/MVar$ | $m_3$ |
|---------------|-------|
| 1             | 0.1   |
| 2             | 0.4   |
| 3             | 0.4   |
| 4             | 0.1   |

4.3. Sensitivity calculation

In order to obtain the influence of each parameter on the output result, we conduct the sensitivity analysis. And the influence of the load $P_2 + jQ_2$ on the voltage $U_2$ is mainly discussed. The sensitivity index reflects the changing degree and changing trend of the output caused by the tiny change of input. Sensitivity coefficient is often used as a measure of the sensitivity of parameters. The forms of the sensitivity coefficient is

$$S_1 = \frac{dU_2}{dP_2}, \quad S_2 = \frac{dU_2}{dQ_2}.$$  \hfill (11)

Then we can get the influences of $\Delta P_2$ and $\Delta Q_2$ on $\Delta U_2$. The results are shown in figure 4.
Figure 4 (a) illustrates the upper and lower cumulative probability distribution of $dU_2/dP_2$. It shows that the probability of $dU_2/dP_2$ is very large in a small neighbourhood of -0.0505, if necessary, which can be used to express the influencing degree of $\Delta P_2$ on $\Delta U_2$. Similarly, -0.184 can express the influencing degree of $\Delta Q_2$ on $\Delta U_2$ from figure 4 (b).

5. Conclusion
The random set theory is a mathematical theory which can handle in a unique framework including various types of uncertainties. When the system’s operating information is known, the random set theory can be used to estimate upper and lower probability functions of system reliability. Based on our approach, we can estimate the system reliability in the case of changes of the reactive power compensation. Indeed our method reflects more adequately the uncertainties comparing to classical probabilistic approach. This has major implications for improving the efficiency of system reliability estimation.

The sensitivity index reflects the changing degree and changing trend of the system reliability caused by the tiny change of equipment parameters. Then we can estimate the system’s reliability index effectively after the changes of component parameters. These sensitivity indices applied to the probability risk assessment help us find the bottlenecks that suppress system reliability and provide specific measures to improve the reliability of the system.

References
[1] Li W Y 2005 Risk Assessment of Power Systems: Models, Methods and Applications (Beijing: Science Press) pp 11-36
[2] Feng Y Q, Wu W Ch and Zhang B M 2008 Power system operation risk assessment using credibility theory IEEE Trans. Power Syst. 23 1309
[3] Xu X B, Wen Ch L and Liu R L 2008 The unified method of describing and modelling multisource information based on random set theory Acta Electron. Sin. 26 1
[4] Xie Y J 2012 Research on multi-source information fusion and uncertainty modelling of flood risk assessment Huazhong University of Science and Technology
[5] Patton A D and Tram N H 1978 Sensitivity of generation reliability indices to generator parameter variations IEEE Trans. Power App. Syst. 97 678
[6] Han Ch Zh, Zhu H Y and Duan Zh Sh 2006 Multisource Information Fusion (Beijing: Tsinghua University Press)
[7] Klir G J and Yuan B 1995 Fuzzy Sets and Fuzzy Logic (USA: Prentice Hall)
[8] Shafer G 1976 A Mathematical Theory of Evidence (Princeton: Princeton University Press)
[9] Crosetto M and Tarantola S 2001 Uncertainty and sensitivity analysis: tools for GIS-based model implementation INT J Geogr. Inforrn. Sci. 15 415
[10] Saltelli A, Chan K and Scott M 2000 Sensitivity Analysis, Probability and Statistics Series (New York: John Wiley & Sons)
[11] Mahamah D S 1988 Simplified sensitivity analysis applied to a nutrient-biomass model Ecol. Model 42 103