Analysis of autoparametric oscillations in three-phase electro-ferromagnetic circuits

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Abstract: The paper presents mathematical models for the analysis of autoparametric oscillations at the frequency of subharmonic oscillations in three-phase electro-ferromagnetic circuits, which are developed taking into account the main factors and conditions for obtaining subharmonic oscillations in which the process under consideration occurs. With an increase in the degree of adequacy of the mathematical model, its form and algorithmization of problems on PC become more complicated.

A decrease in the quality of electric energy in electric networks can lead to a violation of the nominal operating modes of the main technological equipment, which, in turn, is a cause of reduced productivity, reduced service life and increased level of probability of accidents. In this regard, the development of measures and devices that can perform automated monitoring of the quality of electric energy is very relevant.

In the process of exploitation of electric networks of mining often fails to provide high quality electric energy at the same time, the growth capacity of consumers, a large number of conversion devices used to regulate the speed of electric drives of various mechanisms that lead to the need to improve the quality of electricity. Currently, research is underway to identify the influence of higher harmonics on the quality of electricity in the electric networks of mines and mines [1-3]. However, these studies are often reduced to analyzing the power coefficients of special equipment and increasing it to acceptable values or studying the harmonic composition of the network [4, 5]. Currently, a number of studies are being conducted to reduce the negative impact of converter technology on the power supply network using active filters of higher harmonics [5-8], as well as to increase the parameters of electromagnetic compatibility in electric machines. A number of publications present the results of research on the quality parameters of electric energy for mining enterprises [6-8]. The modern development of converting equipment and, as a rule, its introduction into the composition of electromechanical systems, has significantly expanded the range of operating modes of technological equipment in mine workings. However, the use of a frequency-controlled electric drive and DC motors requires the use of converter...
and rectifier devices, which, being a non-linear load, lead to an increase in the consumption of reactive energy, and this significantly worsens the quality parameters of electric energy in electric networks. Losses in transformers, electric motors and power lines occur, among other things, due to the operation of energy-saving equipment. In this role, for example, are the so-called frequency-controlled AC electric drives, the speed of which is regulated by a change in the supply network, and other types of equipment. They get energy from semiconductor converters, which, in contrast to the network, form a non-sinusoidal voltage form (higher harmonics) – it is deviations from the sinusoid that cause an increase in energy loss. In this connection, the problem arises of revealing the influence of higher harmonics on the stability of the electric network, the qualitative and quantitative composition of these harmonics.

For the most in-depth study and control of the emerging excitations of nonlinear oscillations in electromagnetic circuits and systems of current fluctuations in electric circuits, it is necessary to develop a tool (mathematical complexes) that allows taking into account the main factors acting on the process as a whole.

One of the effective and acceptable methods of research and management of a complex self-oscillating process is the creation of a mathematical model, algorithmization of problem solving and computational experiment (CE) on a personal computer (PC).

The main algorithmic means of implementing mathematical models that allow us to study the behavior of the process are numerical methods. On the basis of mathematical models and numerical methods, it is possible to create software and information software, which in combination with a PC is a tool for scientific knowledge and control of the above process as a whole.

Studies of the occurrence of auto-parametric oscillations using the conducted CE on a PC allowed us to determine acceptable ranges of changes in the main parameters of this process and identify the conditions for the most complete suppression of undesirable ferroresonance overvoltages in power lines in practice.

An approximate solution of the nonlinear differential equations of the EFMC made it possible to analyze subharmonic oscillations (SHO) and their stability in a stationary mode depending on the input action and circuit parameters. However, determining the influence of the initial conditions on the process of excitation and establishment of the SHO by approximate methods does not solve differential equations. Therefore, to clarify these issues, a numerical method was applied to solve the three-phase EFMC equation.

For this purpose, a mathematical model of the above process based on the law of conservation of energy was developed. The modeling of the process of excitation of third-order subharmonic oscillations in three-phase electro-ferromagnetic circuits is considered.

The system of nonlinear differential equations of a symmetric three-phase electro-ferromagnetic circuit with a series-connected nonlinear inductance and a capacitance without a neutral wire after changing the variables are described by the following equations:

\[
\frac{d\phi_1}{d\tau} = v_1 
\]

\[
\frac{dv_1}{d\tau} = -\frac{9\alpha}{C\omega^2} \phi_1 - \frac{3R\alpha}{\omega} v_1 - \frac{3R\beta}{\omega} \phi_1 \cdot v_1 - \frac{9\beta}{C\omega} \phi_1 + \]

\[
+ \frac{1}{3} (v_1 + v_2 + v_3) + \frac{9U_m}{\omega} \cos(3\tau + \varphi_0), 
\]

\[
\frac{d\phi_2}{d\tau} = v_2. 
\]
\[
\frac{dv_1}{d\tau} = -\frac{9\alpha}{C\omega} \phi_2 - \frac{3R\alpha}{\omega} v_1 - \frac{3R\beta}{\omega} \phi_1^2 v_2 - \frac{9\beta}{C\omega^2} \phi_1^3 + \\
+ \frac{1}{3} (v_1 + v_2 + v_3) + \frac{9Um}{\omega} \cos(3\tau - 120^\circ + \phi_0),
\]

(4)

\[
\frac{d\phi_1}{d\tau} = v_1,
\]

(5)

\[
\frac{dv_2}{d\tau} = -\frac{9\alpha}{C\omega} \phi_1 - \frac{3R\alpha}{\omega} v_1 - \frac{3R\beta}{\omega} \phi_1^2 v_2 - \frac{9\beta}{C\omega^2} \phi_1^3 + \\
+ \frac{1}{3} (v_1 + v_2 + v_3) + \frac{9Um}{\omega} \cos(3\tau - 240^\circ + \phi_0),
\]

(6)

For the numerical solution of equations (1)-(6), we use the forecast and correction method [10], which has an advantage over other methods, for example, over the Runge-Kutta, Kutta-Merson, and other methods. Here, the amount of computation and the cost of computer time for implementation tasks are much smaller than in other methods and quadratic convergence of the iterative method is ensured.

The values of the desired variable at the initial time were calculated using the modified Euler method [9]. For this, the right-hand sides of equations (1) ÷ (6) are denoted by \(F_1, F_2, F_3, F_4, F_5, F_6\). As a result, we obtained formulas for computing the desired variable functions \(U_1, U_2, U_3, \phi_1, \phi_2, \phi_3\):

\[
U_1(\tau) = U_1(0) + \frac{\Delta\tau}{2} \left[ F_1(U_1(0), U_2(0), U_3(0), \phi_1(0), \phi_2(0), \phi_3(0)) + \\
+ F_1(U_1(0) + \Delta\tau F_1(U_1(0), U_2(0), U_3(0), \phi_1(0), \phi_2(0), \phi_3(0)), \\
U_2(0) + \Delta\tau F_2(U_1(0), U_2(0), U_3(0), \phi_1(0), \phi_2(0), \phi_3(0)), \\
U_3(0) + \Delta\tau F_3(U_1(0), U_2(0), U_3(0), \phi_1(0), \phi_2(0), \phi_3(0)), \\
\phi_1(0) + \Delta\tau F_4(U_1(0), U_2(0), U_3(0), \phi_1(0), \phi_2(0), \phi_3(0)), \\
\phi_2(0) + \Delta\tau F_5(U_1(0), U_2(0), U_3(0), \phi_1(0), \phi_2(0), \phi_3(0)), \\
\phi_3(0) + \Delta\tau F_6(U_1(0), U_2(0), U_3(0), \phi_1(0), \phi_2(0), \phi_3(0))) \right]
\]

(7)

and similarly for other calculations of the desired function variables.

For further forecasting and correction of the desired variable values, the “forecast” and “correction” method was used, the essence of which is as follows:

1) variable values are pre-calculated

\[
U_{1,p+1}^{(0)} = U_{1,p} + 2\Delta t F_1(U_{1,p}, U_{2,p}, U_{3,p}, \phi_1^{(p)}, \phi_2^{(p)}, \phi_3^{(p)}),
\]

and similarly for other variables. Where \(p = 1, 2, 3, \ldots, P - 1\);

2) correction of the values of the desired variables is performed using expressions

\[
U_{1,p+1}^{(1)} = U_{1,p} + \frac{\Delta t}{2} (F_1(U_{1,p}, U_{2,p}, U_{3,p}, \phi_1^{(p)}, \phi_2^{(p)}, \phi_3^{(p)}) + \\
+ F_1(U_{1,p}^{(s-1)}, U_{2,p}^{(s-1)}, U_{3,p}^{(s-1)}, \phi_1^{(s-1)}, \phi_2^{(s-1)}, \phi_3^{(s-1)})),
\]

(9)

and similarly for other variables being searched for.
Where: \( S = 1,2,3, \ldots, S, \) and \( S \) is the iteration number.

3) the error of the forecast and correction method is calculated using the expression:

\[
E\left( u_{i,p+1}^{(S)} \right) = 0.5 \left( u_{i,p}^{(S)} - u_{i,p}^{(S-1)} \right),
\]

\[
E\left( \phi_{i,p+1}^{(S)} \right) = 0.5 \left( \phi_{i,p}^{(S)} - \phi_{i,p}^{(S-1)} \right),
\]

(10)

To calculate the error of the method of correction and correction formulas:

\[
u_{i,p+1}^{(S+1)} - u_{i,p+1}^{(S)} = \frac{\Delta t}{2} \frac{\partial F}{\partial t} \left[ u_{i,p+1}^{(S)} - u_{i,p+1}^{(S-1)} \right],
\]

\[
\phi_{i,p+1}^{(S+1)} - \phi_{i,p+1}^{(S)} = \frac{\Delta t}{2} \frac{\partial F}{\partial t} \left[ \phi_{i,p+1}^{(S)} - \phi_{i,p+1}^{(S-1)} \right],
\]

(11)

where \( i = 1,2,3. \)

As a result of the joint solution of equations (8) - (9) with (10) - (12), we obtained a tendency to change the desired technological variables over time.

In the numerical integration of problems, an error is allowed when replacing differential operators with difference ones. Moreover, in the problems, the right-hand sides and parameters of the equations, boundary and initial data, which will be called hereinafter one general term - input data - are set with a certain error. In the process of the most numerical solution of the system produced during the finite-difference approximation of the original problem, errors associated with rounding are also unavoidable. A necessary requirement for a difference scheme is that small errors made in the input data do not increase during the calculation process and do not lead to a distortion of the solution.

Schemes that enhance the initial errors during the counting process are unstable and cannot be used to solve a particular mathematics problem, in particular, in problems of nonlinear oscillations arising in three-phase electromagnetic circuits and systems. Therefore, we will focus on the convergence, stability, and approximation order used in difference schemes. Using the following notation

\[
\tilde{U} = \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}, \quad \tilde{V} = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}
\]

U and V in a neighborhood of the point \( t = t_p \) in a series. The result was

\[
U(t) = U_p + U_p' \left( t-t_p \right) + \frac{U_p''}{2} \left( t-t_p \right)^2 + \frac{1}{6} \left( t-t_p \right)^3 U_p' \left( \tau_1 \right),
\]

\[
V(t) = V_p + V_p' \left( t-t_p \right) + \frac{V_p''}{2} \left( t-t_p \right)^2 + \frac{1}{6} \left( t-t_p \right)^3 V_p' \left( \tau_1 \right),
\]

(12)

where the point \( \left( \tau_1 \right) \) lies between \( t \) and \( t_p \).

Similarly, assuming that the point \( \tau_2 \) is located between \( \tau_{p-1} \) and \( \tau_p \), we get

\[
U_{p-1} = U_p - \Delta t U_p' + \frac{\Delta t^2}{2} U_p'' - \frac{\Delta t^3}{6} U_p' \left( \tau_2 \right),
\]

\[
V_{p-1} = V_p - \Delta t V_p' + \frac{\Delta t^2}{2} V_p'' - \frac{\Delta t^3}{6} V_p' \left( \tau_2 \right),
\]

(13)

Subtracting the value of the desired variables at points \( (p + 1) \) and \( (p-1) \), and using the expression
\[ U''_p(t_1) - U''_p(t_2) \frac{1}{2} = U''_p(t); \quad t_{p-1} \leq t \leq t_{p+1}, \]  

get

\[ U_{p+1} = U_p + 2\Delta t U'_{p} + \frac{\Delta t}{3} U''_{p}(t); \]

\[ V_{p+1} = V_p + 2\Delta t V'_{p} + \frac{\Delta t}{3} V''_{p}(t). \]

It follows that the error of the forecast method limitation is

\[ \epsilon_{u,j} = \frac{\Delta t^3}{3} U_p^{''}(t), \]

\[ \epsilon_{v,j} = \frac{\Delta t^3}{3} V_p^{''}(t). \]  

If the third derivative is constant, then the constraint error is equal \( K \cdot \Delta t^3 \), and the approximation order of this method is \( O(\Delta t)^2 \).

Since the correction is a generalization of the trapezoid method, the restriction error was determined as follows:

\[ \epsilon_{u,k} = -\frac{\Delta t^3}{12} U_p^{''}(t), \]

\[ \epsilon_{v,k} = -\frac{\Delta t^3}{12} V_p^{''}(t). \]

where: \( t_{p-1} \leq \tau \leq t_{p+1}. \)

Therefore, it is equal \( K \cdot \Delta t^3 \), and to the approximation order \( -O(\Delta t)^3 \).

It follows that the constraint error and the approximation order of the forecast and correction method are the same.

To synthesize the main parameters and analyze this process, an object-oriented software package was developed in a dialogue mode on the language, Visual Basic, and a series of computing experiments on a PC was carried out.

The solutions of the system of nonlinear differential equations (1-6) with a change in the initial phase \( \phi \nu = 0 \div 120^0 \), input voltage \( U_{m
u} = 40 \div 160 \) V, circuit parameters \( R = 0 \div 15 \) Ohm, \( C = 40 \div 160 \mu F \) on a PC received graphs of magnetic inductance flux and voltage in time, characterizing the process of SHO excitation of third-order in three-phase EFMCs without a fourth wire (figure 1).

The calculated data show that the nature of the transition process and the SHO amplitude are significantly influenced by the initial phase of inclusion \( \phi \nu \), the circuit parameters \( R, C \), the degree of non-linearity of the FE, and also the input three-phase voltage. The sphere of SHO existence of third-order is limited by the critical values of the circuit parameters \( R_{kp}, C_{kp} \), the degree of nonlinearity of the ferromagnetic element \( \mu \), and the magnitude of the input voltage.

The developed program for the numerical calculation of autoparametric oscillations in multiphase EFMCs makes it possible to identify the cause of ferroresonance overvoltages in power lines and ways to eliminate them by conducting numerical studies using a personal computer. In particular, a computer experiment on a PC shows that autoparametric oscillations at the SHO frequency in a three-phase EFMC arise with a weak nonlinearity of the photomultiplier with different variants of the phase shift of the SHO.
The duration of the transition process, and the alternation of the phases of the SHO, and phase shifts mainly depend on the initial phase of the inclusion $\varphi$ of the three-phase voltage.

The probability of occurrence of SHO in three-phase circuits without a fourth wire with a phase shift ($0^\circ, 40^\circ, 160^\circ$) is 75% cases, and the other two options ($0^\circ, 40^\circ, 40^\circ$), ($0^\circ, 80^\circ, 80^\circ$) is 30 ÷ 40%, as confirmed by the results of an experimental study [10].

The introduction of active resistance into the circuit of a neutral or linear wire limits the region of existence of the SHO, and at some critical values $R_0 = R_{ikp}$ and $R = R_{kp}$ they do not arise at all.

From the analysis of the conducted CEs, it follows that the main parameters that affect the nature of the process as a whole are $R$, $C$, $U$, $\varphi$ and the degree of non-linearity of the FE. Any change in these parameters leads to a significant change in the oscillatory process as a whole.

The results can be used in the design process, when choosing shields based on microprocessor panels, creating analysis conditions after emergency conditions of power systems and power lines, and also when eliminating ferroresonant overvoltages of power line compensation.

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