Is the Up Quark Massless?

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Several lattice calculations of a combination of the low energy constants of the chiral Lagrangian, \(2\alpha_8 - \alpha_5\), are presented. This combination is critical for the preclusion of a massless up quark. The result found is \(2\alpha_8 - \alpha_5 = 0.115 \pm 0.051^{\text{stat}} \pm 0.25^{\text{syst}}\), which is well outside of the range allowed by a massless up quark.

1. Introduction

The symmetries of QCD allow for a CP breaking term in the QCD Lagrangian. However, measurements of the neutron dipole moment have demonstrated that the coefficient of this term is very small, if not zero. The unnatural smallness of this coefficient is known as the strong CP problem. A massless up quark has long been proposed as a potential elegant solution to the problem. Without the up quark mass term, one is free to remove the CP violating term through a field redefinition.

At tree level Chiral Perturbation Theory (ChPT) appears to answer the question of a massless up quark. The quark mass ratios dictate the form of the Lagrangian’s mass matrix, which at lowest order fully determines the light meson mass ratios.

However, the NLO terms in the chiral Lagrangian contribute to these ratios as well. If this contribution is strong enough, it would mimic the effects of a massive up quark and allow a massless up quark to be consistent with experimental results. This is known as the Kaplan-Monohar ambiguity.

Distinguishing between a light and a massless up quark requires knowledge of the coefficients of the NLO terms in the chiral Lagrangian, the Gasser-Leutwyler (GL) coefficients. Specifically, it is the combination of constants \(2\alpha_8 - \alpha_5\) which corrects the relevant ratio. If this combination falls within a certain range, \(-3.3 < 2\alpha_8 - \alpha_5 < -1.5\), current experimental results can not rule out a zero up quark mass.

The full set of GL coefficients, including the combination of interest, can not be determined by experiment. Various phenomenological arguments have been used in the past to assemble a standard set of values for the coefficients. However, these coefficients are determined by the low energy non-perturbative behavior of QCD, and thus the lattice offers the best opportunity for calculating them directly.

2. Partially Quenched Chiral Perturbation Theory (PQChPT)

PQChPT is the tool with which one can calculate the GL coefficients on the lattice. PQChPT is distinct from standard ChPT in that it is constructed from the symmetry of a graded group. This graded group follows from the pre-
Table 1
Simulation details.

| $L/T$ | $\beta$ | $m_S$ | $N_f$ | $a^{-1}$ (MeV) | hyp$^1$ | $2\alpha_8 - \alpha_5$ |
|-------|---------|-------|-------|----------------|---------|-----------------|
| 16/32 | 5.3     | 0.01  | 3     | 1347.9(31)     | $\sqrt{}$| 0.115(51)       |
| 16/32 | 5.3     | 0.01  | 3     | 1260(28)       |         | 0.071(14)       |
| 8/32  | 5.3     | 0.01  | 3     | 1366(49)       |         | 0.47(12)        |
| 8/32  | 5.115   | 0.015 | 3     | 730(130)       |         | 0.310(19)       |
| 8/32  | 5.1235  | 0.02  | 3     | 703(97)        |         | 0.314(17)       |
| 8/32  | 5.132   | 0.025 | 3     | 780(120)       |         | 0.375(13)       |
| 8/32  | 5.151   | 0.035 | 3     | 800(130)       |         | 0.437(18)       |

assumed quark content of Partially Quenched QCD (PQQCD), separate valence and sea quark flavors in addition to ghost quark flavors, which in perturbation theory cancel loop corrections due to valence quarks.

The Lagrangian of PQChPT up to $O(p^4)$ follows, with only relevant NLO terms shown.

$$\mathcal{L} = \frac{f^2}{4} \text{sTr} \left[ \partial_\mu U \partial^\mu U^\dagger \right] + \frac{f^2}{4} \text{sTr} \left[ \chi U^\dagger + U \chi \right]$$

$$+ L_4 \text{sTr} \left[ \partial_\mu U \partial^\mu U^\dagger \right] \text{sTr} \left[ \chi U^\dagger + U \chi \right]$$

$$+ L_5 \text{sTr} \left[ \partial_\mu U \partial^\nu U^\dagger \chi \right] \left( U^\dagger + U \chi \right)$$

$$+ L_6 \text{sTr} \left[ \chi U^\dagger + U \chi \right]^2$$

$$+ L_8 \text{sTr} \left[ \chi U^\dagger \chi U^\dagger + U \chi U \chi \right] + \cdots$$

(1)

$$U = \exp \left( 2i \Phi/f \right)$$

(2)

$$\chi = 2\mu \text{ diag} \left( \{m_S, m_V\} \right)$$

(3)

where $\Phi$ contains the pseudo-Goldstone “mesons” of the broken $SU(N_f + N_V)_{L} \otimes SU(N_f + N_V)_{R}$ symmetry and $U$ is an element of that group. In our calculations three degenerate sea quarks were used, $N_f = 3$, while the number of valence quarks $N_V$ cancels in all expressions, affecting only the counting of external states. The constants $f$, $\mu$, and $L_i$ are unknown, determined by the low energy dynamics of PQQCD.

It was recently realized that, because the valence and sea quark mass dependence of the PQChPT Lagrangian is explicit and because full QCD is within the parameter space of PQQCD ($m_V = m_S$), the values obtained for the GL coefficients in a PQQCD calculation are the exact values for the coefficients in full QCD $^2$.[4]. Furthermore, the independent variation of valence and sea quark masses allows additional lever arms in the determination the these coefficients. Because the $N_f$ dependence of the Lagrangian is not explicit, the GL coefficients are functions of $N_f$. Thus, it is important to use a physical number of sea quarks, as we have, when extracting physical results.

3. Predicted Forms

PQChPT predicts forms for the dependence of the pseudoscalar mass and decay constant on the valence quark mass, here assuming degenerate sea quarks and degenerate valence quarks, and cutting off loops at $\Lambda_P = 4\pi f$.

$$m_\pi^2 = (4\pi f)^2 z m_V \left\{ 1 + z m_V \left( 2\alpha_8 - \alpha_5 + \frac{1}{N_f} \right) \right. $$

$$\left. + \frac{z}{N_f} (2m_V - m_S) \ln z m_V \right\}$$

(4)

$$f_\pi = f \left\{ 1 + \frac{\alpha_5}{2} z m_V \right. $$

$$\left. + \frac{z N_f}{4} (m_V + m_S) \ln \frac{z}{2} (m_V + m_S) \right\}$$

(5)

$$z = \frac{2\mu}{(4\pi f)^2}$$

(6)

These forms differ slightly from those in $^6$.[4], as the NLO dependence in the sea quark mass has been absorbed into $\mu$ and $f$. This is allowed as the error due to this change manifests when $z$ appears in the NLO terms, pushing the discrepancy up to NNLO. Accounting for these absorbed terms would require a systematic study at several sea quark masses.

The forms above are derived assuming degenerate light mesons. However, our use of staggered fermions on rather coarse lattices is likely to generate significant flavor symmetry breaking, and thus a splitting of the light meson masses. Although there is no simple quantitative method to account for this split, one effect might be a reduction in the importance of meson loops, and thus a weakening of the chiral log terms. In order to study the systematic errors due to flavor symmetry breaking, we applied hypercubic blocking $^1$.[4] to our primary ensemble.

$^1$Lattice spacing determined via $r_0$.

$^4$Denotes a hypercubic blocked ensemble.
4. Results

The meson mass equals the physical kaon mass. The figures is the valence quark mass at which cubic blocking shifted the result, suggesting that yet larger volumes may be required. Additionally, the application of hypercubic blocking shifted the result, suggesting that flavor symmetry breaking is indeed affecting the value. Despite systematic shifts in the calculated value between ensembles, it remained well outside the allowed range for a zero up quark mass.

The quoted result of $2\alpha_8 - \alpha_5 = 0.115 \pm 0.051^{\text{stat}} \pm 0.25^{\text{syst}}$ comes from our hypercubic blocked $8^3 \times 32$ ensemble, with generous systematic errors due to the significant fluctuations in the result between ensembles. This falls outside the range allowed by a zero up quark mass, $-3.3 < 2\alpha_8 - \alpha_5 < -1.5$.

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