DEFORMED DISTANCE DUALITY RELATIONS AND SUPERNOVA DIMMING

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ABSTRACT

The basic cosmological distances are linked by the Etherington cosmic distance duality relation, $\eta(z) = D_L(z)/(1 + z)^{-2}/D_A(z) \equiv 1$, where $D_L$ and $D_A$ are, respectively, the luminosity and angular diameter distances. In order to test its validity, some authors have proposed phenomenological expressions for $\eta(z)$, thereby deforming the original Etherington’s relation and comparing the resulting expressions with the available and future cosmological data. The relevance of such studies is unquestionable since any violation of the cosmic distance duality relation could be a signal of new physics or non-negligible astrophysical effects in the usually assumed perfectly transparent universe. In this Letter, we show that under certain conditions such expressions can be derived from a more fundamental approach with the parameters appearing in the $\eta(z)$ expression defining the cosmic absorption parameter as recently discussed by Chen and Kantowski. Explicit examples involving four different parameterizations of the deformation function are given. Based on such an approach, it is also found that the latest supernova data can also be explained in the framework of a pure cold dark matter model (Einstein–de Sitter). Two different scenarios with cosmic absorption are discussed. Only if the cosmic opacity is fully negligible, the description of an accelerating universe powered by dark energy or some alternative gravity theory must be invoked.

Key words: cosmological parameters – distance scale – opacity – supernovae: general

Online-only material: color figures

1. INTRODUCTION

The cosmic distance duality relation (CDDR) is a mathematical identity relating the luminosity distance $D_L$ with the angular diameter distance $D_A$ by the expression

$$\frac{D_L}{D_A}(1 + z)^{-2} = 1.$$  \hspace{1cm} (1)

The validity of this constraint uniting the two basic distances in cosmology depends neither on the Einstein field equations nor on the nature of the matter-energy content. It only requires the phase space conservation of photons and that sources and observers are connected by null geodesics in a Riemannian spacetime. Therefore, it remains valid for spatially homogeneous and isotropic (anisotropic) cosmologies, as well as for inhomogeneous cosmological models (Etherington 1933; Basset & Kunz 2004).

The above relation is usually taken for granted when relativistic models of the universe are confronted with the existing cosmological observations. Despite that, the distance duality (DD) relation is in principle testable by means of astronomical observations. The basic idea is to find cosmological sources whose intrinsic luminosities are known (standard candles) as well as their intrinsic sizes (standard rulers). After determining both $D_L$ and $D_A$ at the same redshift, it should be possible to directly test the Etherington result. Naturally, by cosmological sources with known $D_L$ and $D_A$ at the same redshift, it should be possible to directly test the Etherington result. Naturally, by cosmological sources with known $D_L$ and $D_A$ we are not necessarily referring to the same class of objects. Under certain conditions, as recently discussed by Holanda et al. (2010), one may consider two different classes of objects as, for instance, supernovae and galaxy clusters for which $D_L$ and $D_A$ are separately determined. Note also that ideally both quantities must be measured in such a way that any relationship coming from a specific cosmological model is not used, that is, it must be determined by means of intrinsic astrophysically measured quantities. In practice, the validity of the CDDR has been tested by assuming a phenomenologically deformed expression of the form (Holanda et al. 2010; Meng et al. 2011; Nair et al. 2011; Khedekar & Chakaborti 2011):

$$\frac{D_L}{D_A}(1 + z)^{-2} = \eta(z),$$  \hspace{1cm} (2)

where $\eta(z)$ is the deformation function which quantifies a possible epoch-dependent departure from the standard photon conserving scenario ($\eta = 1$).

Naturally, a deformed CDDR can also be adopted to test the possibility of a new physics. In this line, Basset & Kunz (2004) used Type Ia supernova (SN Ia) data as measurements of the luminosity distance and the estimated $D_A$ from FR\textsuperscript{IIb} radio galaxies (Daly & Djorgovski 2003) and ultracompact radio sources (Gurvitz et al. 1999; Lima & Alcaniz 2000, 2002; Santos & Lima 2008) in order to test possible new physics signatures based on the following expression:

$$I. \quad \eta(z) = (1 + z)^{\beta-1} \exp \left[ \gamma \int_0^z \frac{dz'}{E(z')(1 + z')^\delta} \right],$$  \hspace{1cm} (3)

where $E(z') \equiv H(z')/H_0$ is the dimensionless Hubble parameter ($H_0$ is the Hubble constant). Note that for arbitrary values of $\delta$, the strict validity of the DD relation corresponds to $(\beta, \gamma) \equiv (1, 0)$. By marginalizing on $\Omega_M$, $\Omega_{\Lambda}$, and Hubble parameters, they found a $2\sigma$ violation caused by excess brightening of SN Ia at $z > 0.5$. It was also argued that such an effect would be associated with the lensing magnification bias.

De Bernardis et al. (2006) also searched for deviations of the standard CDDR by using the angular diameter distances from galaxy clusters provided by the sample of Bonamente et al. (2006). By assuming $\eta(z) = \text{constant}$, they obtained...
a non-violation of CDDR in the framework of the cosmic concordance ΛCDM model. Later on, Avgoustidis et al. (2009, 2010), working in the context of a flat ΛCDM model, also adopted an extended CDDR expressed as

\[ \eta(z) = (1 + z)^\beta. \]  

The above deformation function is a particular case (\( \beta = 1 + \epsilon, \gamma = 0 \)) of the general expression adopted by Basset & Kunz (2004). In their analysis, the recent SN Ia data as compiled by Kowalski et al. (2008) were combined with the latest measurements of the Hubble expansion at redshifts in the range 0 < z < 2 (Stern et al. 2010), and the free parameter was constrained to be \( \epsilon = -0.04^{+0.08}_{-0.07} \) (2σ). More recently, such a parameterization has also been adopted by Khdekar & Chakaborti (2011) in their studies connecting the Tolman test and CDDR using the redshifted 21 cm wavelength from disk galaxies. It was also argued that future data from the planned Square Kilometer Array may provide the best test to detect any violation of the CDDR.

In a series of papers, Holanda et al. (2010, 2011a, 2011b) also explored a different route to test the CDDR based on the following deformation functions:

\[ \eta(z) = 1 + \eta_0 z, \quad \text{IV. } \eta(z) = 1 + \eta_0 z/(1 + z). \]  

A basic difference between the first and the second parameterization is that the latter includes a possible epoch-dependent correction which avoids the divergence at extremely high z. At low redshifts, when second-order terms are neglected, the second parameterization reduces to the first one, that is, \( \eta(z) \simeq 1 + \eta_0 z \). Such one-parametric formulae are also interesting because in the limit of extremely low redshifts (\( z \ll 1 \)), one finds \( \eta(z) = 1 \) as should be expected since \( D_L = D_L \) (see also parameterization II). In addition, for a given data set, the likelihood of \( \eta_0 \) must be peaked at \( \eta_0 = 0 \), in order to satisfy the standard duality relation.

In this context, by taking the SNe Ia from constitution data (Hicken et al. 2009) and galaxy cluster samples compiled by De Filippis et al. (2005) and Bonamente et al. (2006), a direct test of the CDDR was accomplished (Holanda et al. 2010). As an extra bonus, the consistency between the strict validity of CDDR and the assumptions about the geometry based on elliptical and spherical \( \beta \) models was discussed in detail. The sphericity assumption for the cluster geometry resulted in a larger incompatibility with the validity of the duality relation in comparison with an isothermal non-spherical cluster geometry.

In this connection, it is also worth mentioning that Li et al. (2011) re-discussed this independent cosmological test by using the latest Union2 SN Ia data and the angular diameter distances from galaxy clusters, thereby obtaining a less serious violation of the standard duality expression. In a simultaneous but independent work, Nair et al. (2011) also investigated the strict validity of the CDDR by using the latest Union2 SN Ia data and the angular diameter distances from galaxy clusters, FRIIb radio galaxies, and mock data. As an attempt to determine a possible redshift variation of the CDDR relation, they proposed six different (one and two indices) parameterizations including, as particular cases, the ones adopted by Holanda et al. (2010, 2011a). As physically expected, their results depend on both the specific parameterization and the considered data sample. In particular, they conclude that the one index parameterization, namely, \( \eta_D = \eta_S/(1 + z) \) and \( \eta_D = \eta_S \exp[(z/(1 + z))/(1 + z)] \), does not support the CDDR relation for the given data set. Meng et al. (2011) also reinvestigated the model independent test by comparing two different methods and several parameterizations (one and two indices) for \( \eta(z) \). Their basic conclusion was that the triaxial ellipsoidal model is suggested by the model independent test at 1σ, while the spherical \( \beta \) model can only be accommodated at 3σ confidence level, thereby agreeing with the results earlier derived by Holanda et al. (2010).

It should be stressed that all the above described attempts to test the CDDR have been carried out based on a phenomenological approach. Usually, it is also not clear whether the non-standard relation is the result of a modified luminosity distance, or whether it should be associated with an extended angular distance relation, or both (see, however, Basset & Kunz 2004).

At this point, one may also ask whether such expressions of \( \eta(z) \) can be derived from a more fundamental approach. In the affirmative case, it is also important to study their consequences for the present accelerating stage of the universe.

In this Letter we consider both questions. First, we show how any deformed CDDR can be derived by analyzing possible theoretical modifications on the luminosity distance without refraction effects. Analytical expressions for the dimensionless cosmic absorption parameter describing the above four parameterizations will be explicitly obtained in the framework of Gordon’s optical metric as developed by Chen & Kantowski (2009a, 2009b) to include cosmic absorption. It will be also explicitly assumed that the angular diameter distances are not modified because their measurements involve only standard rulers and angular scales, and, more importantly, that the possible refractive effects have been neglected. Apart from such hypotheses, the approach discussed here is quite general and can be applied for any deformation function, \( \eta(z) \). Second, we also apply our results for the latest SN Ia data. As we shall see, the modified luminosity distance can accommodate the observed supernova dimming even for a non-relativistic cold dark matter (CDM) Einstein–de Sitter model (\( \Omega_M = 1 \)). The validity of the ΛCDM description is obtained only in the extreme limit of perfect cosmic transparency (negligible cosmic absorption).

2. LUMINOSITY DISTANCE AND DUALITY RELATION

The concept of an optical metric was introduced long ago in a seminal paper by Gordon (1923). He proved the existence of a mapping between any solution of the general relativistic Maxwell’s equations for a fluid with refraction index \( n(x) \) and the vacuum solutions in a related optical spacetime. In Gordon’s treatment, only the refraction phenomenon was considered. More recently, by describing the Maxwell field as a monochromatic wave, Chen & Kantowski (2009a, 2009b) generalized such treatment in order to include the possibility of an absorption phenomenon in the universe. As it appears, the presence of such an effect naturally breaks the validity of the standard DD relation as given by Equation (1) since the light absorption violates the photon number conservation law. In this case, they prove that the luminosity distance takes the following form:

\[ D_L(z) = e^{\tau /2} D_L^S(z) = e^{\tau /2}(1 + z) \int_0^z \frac{dz'}{E(z')}, \]  

where \( \tau \) denotes the optical depth associated with the cosmic absorption of the universe and the superscript \( S \) specifies the standard luminosity distance (no absorption) for which the universe is assumed to be transparent. It is worth noticing that effects coming from a possible new physics like the interaction
between photons and dark matter as discussed by Basset & Kunz (2004) are assumed to be negligible (no new physics takes place). For a spatially homogeneous and nondispersive (gray) absorption, the quantity τ as derived by Chen & Kantowski (2009a) reads

$$\tau(z) = \int_0^z \frac{\alpha_s(z')}{(1+z')E(z')} dz'$$

where \(\alpha_s\) is the dimensionless cosmic absorption parameter \((\alpha/H_0\) in the notation of Chen & Kantowski). In the above expression the refractive index was fixed to unity \((n(z) = 1)\), and, therefore, any possible refraction effect has been neglected. This is an important point, since in this case the standard angular diameter distances are not modified.

Let us now discuss how the several functions \(\eta(z)\) defining the deformed CDDR introduced in an ad hoc way, can be related to the dimensionless cosmic absorption parameter, \(\alpha_s(z)\). As one may check, by inserting expression (6) into (2), we find that the cosmic optical depth and the deformation function must be related by the simple expression \(e^{\tau/2} = \eta(z)\). In addition, this also means that any smooth \(\eta(z)\) deformation function defines an effective cosmic absorption parameter given by

$$\alpha_s(z) = \frac{2\eta(z)(1+z)E(z)}{\eta(z)}$$

where a prime denotes a derivative with respect to the redshift. Such a relation unifying the dimensionless cosmic absorption parameter and the deformation function, usually introduced by hand in the DD relation, is one of the main results of this section. When \(\eta(z)\) is constant, the cosmic absorption \(\alpha_s(z)\) is identically null, and, therefore, \(\tau = 0\). As noted earlier, this also means that only the standard relation as determined by Etherington (1933) is possible at this limit, that is, \(\eta(z) = 1\) (note that \(\tau\) and \(\eta\) are related by an exponential function). As one may check, the four deformed DD relations can analytically be expressed in terms of the dimensionless absorption parameter as

1. Basset & Kunz (2004): \(\alpha_s(z) = 2[(\beta - 1)E(z) + \gamma(1+z)^{1-\beta}]\).
2. Avgoustidis et al. (2009): \(\alpha_s(z) = 2\epsilon E(z)\).
3. Holanda et al. (2010): \(\alpha_s(z) = \frac{2\eta_0(1+z)E(z)}{1 + \eta_0 z}\).
4. Holanda et al. (2010): \(\alpha_s(z) = \frac{2\eta_0 E(z)}{1 + \eta_0 z}\).

We see that the dimensionless Hubble parameter, \(E(z)\), and the free parameters appearing in the deformation functions define completely the cosmic absorption parameter as introduced by Chen & Kantowski (2009a, 2009b).

At this point some comments are in order. Initially, we note that in the limiting case \(\gamma = 0\) and \(\beta = \epsilon + 1\), the first expression for the cosmic absorption reduces to the second one. This should be expected since at such a limit the general deformation function of Basset & Kunz (2004) reduces to the one proposed by Avgoustidis et al. (2009). Note also that such expressions are satisfied for all Friedmann–Robertson–Walker (FRW) geometries (arbitrary values of \(\Omega_L\)) and energetic content (baryons, dark matter, and dark energy). Since we are describing absorption \((\alpha_s > 0)\), this means that the parameters \(\epsilon\) and \(\eta_0\) must be positive.

3. EXTENDED LUMINOSITY DISTANCE AND SUPERNOVA DIMMING

Presently, constraints based on SN Ia data are considered to be the best method for studying the cosmic expansion history at \(z < 1.5\). Let us now confront the extended luminosity distance including absorption as an optical metric phenomenon with the latest supernova data. In our subsequent analyses we consider only the flat ΛCDM model for which the dimensionless Hubble parameter, \(E(z)\), takes the following form:

$$E(z) = \sqrt{\Omega_M(1+z)^3 + \Omega_\Lambda}$$

where \(\Omega_\Lambda = 1 - \Omega_M\).

To begin with, let us consider the Union2 supernova sample which is formed by 557 measurements of distance moduli from SNe Ia as compiled by Amanullah et al. (2010). In order to avoid effects from the Hubble bubble, only 506 supernovae with redshifts greater than \(cz = 7000\) km s\(^{-1}\) were selected (Conley et al. 2007; Kessler et al. 2009; Sinclair et al. 2010). As is widely known, the SN Ia Union2 data are obtained by adding new data points (including the high-redshift SN Ia) to the original Union data (Kowalski et al. 2008). For this enlarged sample, a number of refinements to the original Union analysis chain have been done, in particular, the relative importance of systematic effects was highlighted (in this connection, see also Sullivan et al. 2011 to the Legacy Survey sample (SNLS3)).

In our statistical analysis we consider a maximum likelihood determined by \(\chi^2\) statistics

$$\chi^2(p) = \sum_{SNIa} \frac{[\mu_o(z_i, p) - \mu_{th}(z_i)]^2}{\sigma_{obs,i}^2}$$

where \(\mu_{th}(z_i) = 5\log_{10} D_L(z_i) + \mu_0\) is the theoretical distance moduli, \(\mu_0 = 25 - \log_{10} H_0\), \(D_L\) is the luminosity distance from Equation (6), \(\sigma_{obs,i}\) is the uncertainty in the individual distances, and the complete set of parameters is given by \(p \equiv (H_0, \Omega_M, \alpha_s)\). It should be stressed that for the considered SN Ia subsample we have combined the statistical + systematic errors in quadrature as compiled in Table 7 of Amanullah et al. (2010) and neglected \(\sigma_{\epsilon}^2\). We have also marginalized on the Hubble constant.

In Figure 1(a), we show the contours on the \(\Omega_M - \alpha_s\) plane corresponding to a flat ΛCDM model and by considering that the absorption \(\alpha_s\) is constant. As indicated, the shadow lines are cuts in the regions of 68.3%, 95.4%, and 99.7% of probability. The constraints on the free parameters are restricted to \(0.0 \leq \alpha_s \leq 1.55\) and \(0.21 \leq \Omega_M \leq 1.0\) at 2\(\sigma\) of statistical confidence while to the latter we have obtained \(0.0 \leq \alpha_s \leq 1.46\) and \(0.22 \leq \Omega_M \leq 1.0\) at 2\(\sigma\) of statistical confidence. In the absence of absorption \((\alpha_s = 0)\) the limit on the density parameter is \(0.21 \leq \Omega_M \leq 0.34\) (concordance model) while for \(\Omega_M = 1\) (Einstein–de Sitter), we find that the absorption parameter lies on the interval \(1.20 < \alpha_s < 1.54\) (2\(\sigma\)). Note that the positiveness of \(\alpha_s\) (absorption) implies that a pure de Sitter model \((\Omega_\Lambda = 1, \Omega_M = 0)\) is not allowed by such data. The best fits to the free parameters are \(\Omega_M = 0.27\). \(\alpha_s = 0.0\) with \(\chi^2_{\text{min}} = 330.5\).

In Figure 1(b), we display the likelihood distribution functions, \(e^{-\chi^2/2}\) of \(\alpha_s\), for the Einstein–de Sitter model \((\Omega_M = 1)\) with constant absorption. The blue curve includes statistical + systematic errors, but, for comparison, we have also shown the red curve with only statistical errors. The upper
and lower horizontal lines correspond to 68.3% and 95.4% c.l., respectively. By marginalizing on the Hubble parameter we obtain \( \alpha_s = 1.38 \pm 0.08(0.15) \) with 1σ (2σ) of probability and \( \chi^2_{\text{min}} = 331.3 \). In this model the universe is always decelerating since \( q(z) = q_0 = 1/2 \). Instead of dark energy we have a cosmic medium whose absorbing properties are quantified by the dimensionless parameter \( \alpha_s \simeq 1.4 \), which is responsible for the SN Ia dimming. The dimensional absorption parameter, \( \alpha = \alpha_s H_0 \sim 10^{-4} \text{Mpc}^{-1} \), is nearly the same one previously obtained by Chen & Kantowski (2009a).

Let us now consider the deformation function, \( \eta(z) = (1+z)^{\alpha} \), as adopted by Avgoustidis et al. (2009). As shown in the previous section, the associated cosmic absorption parameter in this case is \( \alpha_s(z) = 2 \epsilon E(z) \). In order to simplify the notation, in what follows we consider the parameter \( \alpha_0 = 2 \epsilon \).

In Figure 2(a) we display the corresponding plots for the \( \Omega_M-\alpha_0 \) plane in the case of variable absorption, \( \alpha_s = \alpha_0 E(z) \). As shown in Figure 1(a), the analysis includes statistical + systematic errors. The confidence region (2σ) in this plane is defined by \( 0.0 \leq \alpha_0 \leq 0.92 \) and \( 0.20 \leq \Omega_M \leq 1.0 \). Again, the best fit is the \( \Lambda \)CDM with best fit \( \Omega_M = 0.27 \), \( \alpha_0 = 0.0 \) with \( \chi^2_{\text{min}} = 330.5 \).

In Figure 2(b), we display the likelihood distribution functions for \( \alpha_0 \). Blue and red curves correspond to the same analysis of Figure 1(b), that is, with and without systematics. For the latter case, we obtain that \( \alpha_0 = 0.88 \pm 0.05(0.10) \) with 1σ (2σ) of confidence level. Once again we have \( q(z) = q_0 = 1/2 \) in the presence of the absorption, but with \( \chi^2_{\text{min}} = 335.9 \).

It should be stressed that the above results for the \( \Omega_M-\alpha_s \) and \( \Omega_M-\alpha_0 \) planes, based on a subsample of the Union2 SN Ia data, strongly suggest that one of the following possibilities must be true:

1. The universe has an extra dark energy component (\( \Lambda \)) which provides the fuel for the observed accelerating stage \( (q_0 < 0, \alpha_s = 0) \).
2. We live in a decelerating Einstein–de Sitter universe \( (\Omega_M = 1) \) endowed with a cosmic absorption mechanism which is responsible for the dimming of the distant SN Ia \( (q_0 = 1/2, \alpha_s \neq 0) \).
3. We are not living in the extreme cases mentioned above, that is, both phenomena (cosmic opacity and \( \Lambda \)) are partially operating in the observed universe.

In conclusion, by adopting the Gordon–Chen–Kantowski description we have shown how to obtain analytical...
expressions for any deformed DD relation (parameterized by $\eta(z)$ function) in terms of the cosmic absorption parameter and vice versa. Four specific examples of $\eta(z)$ functions, recently proposed in the literature, were explicitly considered. The associated absorption parameter for each one was explicitly determined and some of its physical implications also were discussed in detail. Two different scenarios with cosmic absorption were proposed and their free parameters constrained by using the SN Ia data (Amanullah et al. 2010). Working within the framework of a flat $\Lambda$CDM model, we have found that both scenarios ($\alpha_0 = \alpha_0 E(z)$ and $\alpha_0 = \text{constant}$) are also compatible with a decelerating Einstein–de Sitter universe (see Figures 1(a) and 2(a)).

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