Hadron structure and elastic scattering

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Prelude.

It was in September 1954 when I met Pomeranchuk. He lectured Theory of Relativity. Next year he became a tutor of our "theoretical group". In some time he proposed me to learn the book of Flügge on nuclear physics. After struggling with difficult subject written in German I asked him some questions. He liked it and one by one proposed several problems to be solved. In parallel, he insisted on passing through the series of famous Landau exams. It was a good school. Its lessons I described in the book of reminiscences about Pomeranchuk. As a supervisor, he advised my diploma paper to be published in JETP Letters and recommended me to Prof. Tamm as a PhD student. Soon I proposed the one-pion exchange model which was later extended to multiperipheral and multireggeon models. Pomeranchuk got interested in it and asked me to come for discussion. He was deeply interested in properties of hadron collisions. Our contacts lasted till his death.

In this paper I describe some new findings about elastic scattering of hadrons studied now up to LHC energies. I briefly reviewed this at the Pomeranchuk centennial seminar at ITEP. This would be extremely interesting subject for him. Let me just mention Pomeron and famous Pomeranchuk theorem to remind you his basic contributions in this field. I dedicate the paper to the memory of my teacher Isaak Pomeranchuk.

Abstract

When colliding, the high energy hadrons can either produce new particles or scatter elastically without change of their quantum numbers and other particles produced. Namely elastic scatterings of hadrons are considered in this paper. The general machinery of their theoretical treatment is described. Some new experimental data are presented and confronted to phenomenological approaches. The internal structure of hadrons is the main subject of these studies. Its impact on properties of their interactions is reviewed. It is shown that protons become larger and darker with increase of their collision energy and

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reveal some substructure. The violation of the geometric scaling in the diffraction cone and new problems of description of differential cross sections outside it are described.

1 Introduction

Hadron interactions are strong and, in principle, should be described by quantum chromodynamics (QCD). However, experimental data show that their main features originate from the non-perturbative sector of QCD. Only the comparatively rare processes with high transferred momenta can be treated theoretically rather successfully by the perturbative methods due to the well-known property of the asymptotical freedom of QCD. Thus, in absence of methods for rigorous solution of QCD equations, our understanding of the dynamics of the main bulk of strong interactions is severely limited by the model building or some rare rigorous relations. In fact, our approach to high energy hadronic processes at present is at best still in its infancy.

From experiment we have learned, at least, about five subregions of the elastic scattering differential cross section. Here, we discuss only two of them: the diffraction cone and the Orear regime. The diffraction cone at small angles reminds the quasiclassical effects with Gaussian decrease in angles. The region at larger angles with exponential decrease of the cross section called Orear region became noticeable only at energies of colliding particles above several GeV. It persists till present LHC energies of 7 and 8 TeV.

The extended review was earlier published in [1]. The sections 2 and 3 are the abbreviated versions of its corresponding parts. Others present some results obtained later.

2 The main relations

The measurement of the differential cross section is the only source of the experimental information about their elastic scattering. Herefrom, the main characteristics of hadron interactions directly related to the elastic scattering amplitude such as the total cross section, the elastic scattering cross section, the ratio of the real to imaginary part of the amplitude, the slope of the diffraction cone etc are obtained. The first two of them are functions of the total energy only, while others depend on two variables - the total energy and the transferred momentum (or the scattering angle).
The dimensionless elastic scattering amplitude $A$ defines the differential cross section in a following way:

$$
\frac{d\sigma(s)}{dt} = \frac{1}{16\pi s^2}|A|^2 = \frac{1}{16\pi s^2}(\text{Im}A(s, t))^2(1 + \rho^2(s, t)) \tag{1}
$$

Here, the ratio of the real and imaginary parts of the amplitude has been defined

$$
\rho(s, t) = \frac{\text{Re}A(s, t)}{\text{Im}A(s, t)} \tag{2}
$$

In what follows, we consider the very high energy processes. Therefore, the masses of the colliding particles may be neglected, and one uses the expression $s = 4E^2 \approx 4p^2$, where $E$ and $p$ are the energy and the momentum in the center of mass system. The four-momentum transfer squared is

$$
- t = 2p^2(1 - \cos \theta) \approx p^2\theta^2 \approx p_t^2 \quad (\theta \ll 1) \tag{3}
$$

with $\theta$ denoting the scattering angle in the center of mass system and $p_t$ the transverse momentum.

The elastic scattering cross section is given by the integral of the differential cross section (1) over all transferred momenta:

$$
\sigma_{el}(s) = \int_{t_{\text{min}}}^{0} dt \frac{d\sigma(s)}{dt} \tag{4}
$$

The total cross section $\sigma_t$ is related by the optical theorem with the imaginary part of the forward scattering amplitude at high energy $s$ as

$$
\sigma_t(s) = \frac{\text{Im}A(p, \theta = 0)}{s} \tag{5}
$$

Elastically scattered hadrons escape from the interaction region declining mostly at quite small angles within the so-called diffraction cone\footnote{The tiny region of the interference of the Coulomb and nuclear amplitudes at extremely small angles does not contribute to the elastic scattering cross section and we discard it.}. Therefore the main attention has been paid to this region. As known from experiment, the diffraction peak has a Gaussian shape in the scattering angles or exponentially decreasing as the function of the transferred momentum squared:

$$
\left\{ \frac{d\sigma}{dt} \right\}_{t=0} = e^{Bt} \approx e^{-Bp^2\theta^2}. \tag{6}
$$
In view of the relations (4), (5), (6), any successful theoretical description of the differential distribution must succeed in fits of the energy dependence of the total and elastic cross sections as well.

The diffraction cone slope $B$ is given by

$$B(s,t) \approx \frac{d}{dt} \left[ \ln \frac{d\sigma(s,t)}{dt} \right].$$

(7)

In experiment, the slope $B$ depends slightly on $t$ at the given energy $s$. E.g., at the LHC, its value changes by about 10% within the cone for $|\Delta t| \approx 0.3$ GeV$^2$. We neglect it in a first approximation.

The normalization factor in Eq. (6) is

$$\left( \frac{d\sigma}{dt} \right)_{t=0} = \frac{\sigma_t^2(s)(1 + \rho_0^2(s))}{16\pi},$$

(8)

where $\rho_0 = \rho(s,0)$. Eq. (8) follows from Eqs (1) and (5) at $t = 0$.

According to the dispersion relations which connect the real and imaginary parts of the amplitude and the optical theorem Eq. (5), the value $\rho_0$ may be expressed as an integral of the total cross section over the whole energy range. In practice $\rho_0$ is mainly sensitive to the local derivative of the total cross section. Then to a first approximation the result of the dispersion relation may be written in a form [2, 3, 4]

$$\rho_0(s) \approx \frac{1}{\sigma_t} \left[ \tan \left( \frac{\pi}{2} \frac{d}{d \ln s} \right) \right] \sigma_t = \frac{1}{\sigma_t} \left[ \frac{\pi}{2} \frac{d}{d \ln s} + \frac{1}{3} \left( \frac{\pi}{2} \right)^3 \frac{d^3}{d \ln s^3} + ... \right] \sigma_t.$$

(9)

At high energies $\rho_0(s)$ is mainly determined by the derivative of the logarithm of the total cross section with respect to the logarithm of energy.

The bold extension of the first term in this series to non-zero transferred momenta would look like

$$\rho(s,t) \approx \frac{\pi}{2} \left[ \frac{d\ln \text{Im}A(s,t)}{d \ln s} - 1 \right].$$

(10)

If one neglects the high-$|t|$ tail of the differential cross section, which is several decades lower than the optical point, and integrates in Eq. (4) using Eq. (6) with constant $B$, then one gets the approximate relation between the total cross section, the elastic cross section and the slope

$$\frac{\sigma_t^2(1 + \rho_0^2)}{16\pi B \sigma_{el}} \approx 1.$$

(11)
The elastic scattering amplitude must satisfy the general properties of analiticity, crossing symmetry and unitarity. The unitarity of the $S$-matrix $SS^+=1$ imposes definite requirements on it. In the $s$-channel it looks like

$$
\text{Im}A(p, \theta) = I_2(p, \theta) + F(p, \theta) = \frac{1}{32\pi^2} \int \int d\theta_1 d\theta_2 \frac{\sin \theta_1 \sin \theta_2 A(p, \theta_1) A^*(p, \theta_2)}{\sqrt{|\cos \theta - \cos(\theta_1 + \theta_2)|[\cos(\theta_1 - \theta_2) - \cos \theta]}} + F(p, \theta). \quad (12)
$$

The region of integration in (12) is given by the conditions

$$
|\theta_1 - \theta_2| \leq \theta, \quad \theta \leq \theta_1 + \theta_2 \leq 2\pi - \theta. \quad (13)
$$

The integral term represents the two-particle intermediate states of the incoming particles. The function $F(p, \theta)$ represents the shadowing contribution of the inelastic processes to the elastic scattering amplitude. Following [5] it is called the overlap function. It determines the shape of the diffraction peak and is completely non-perturbative. Only some phenomenological models pretend to describe it.

In the forward direction $\theta=0$ this relation in combination with the optical theorem (5) reduces to the general statement that the total cross section is the sum of cross sections of elastic and inelastic processes:

$$
\sigma_t = \sigma_{el} + \sigma_{inel}. \quad (14)
$$

The unitarity relation (12) has been successfully used [6, 7, 8, 9] for the model-independent description of the Oraer region between the diffraction cone and hard parton scattering which became the crucial test for phenomenological models.

Experimentally, all characteristics of elastic scattering are measured as functions of energy $s$ and transferred momentum $t$. However, it is appealing to get knowledge about the geometrical structure of scattered particles and the role of different space regions in the scattering process. Then one should use the Fourier-Bessel transform to get correspondence between the transferred momenta and these space regions. The transverse distance between the centers of colliding particles called as the impact parameter $b$ determines the effective transferred momenta $t$. The amplitudes in the corresponding representations are related as

$$
h(s, b) = \frac{1}{16\pi s} \int_{t_{\text{min}}=-s}^{0} dt A(s, t) J_0(b\sqrt{-t}). \quad (15)
$$
Peripheral collisions at large $b$ lead to small transferred momenta $|t|$.

The amplitude $A(s, t)$ may be connected to the eikonal phase $\delta(s, b)$ and to the opaqueness (or blackness) $\Omega(s, b)$ at the impact parameter $b$ by the Fourier-Bessel transformation

$$A(s, t = -q^2) = \frac{2s}{i} \int d^2b e^{iqb}(e^{2i\delta(s, b)} - 1) = 2is \int d^2b e^{iqb}(1 - e^{-\Omega(s, b)}). \quad (16)$$

Assuming $\Omega(s, b)$ to be real and using Eq. (5) one gets

$$\sigma_t = 4\pi \int_0^\infty (1 - e^{-\Omega(s, b)}) bdb. \quad (17)$$

Also

$$\sigma_{el} = 2\pi \int_0^\infty (1 - e^{-\Omega(s, b)})^2 bdb, \quad (18)$$

and

$$B = \frac{\int_0^\infty (1 - e^{-\Omega(s, b)}) b^3 db}{2 \int_0^\infty (1 - e^{-\Omega(s, b)}) bdb}. \quad (19)$$

To apply the inverse transformation one must know the amplitude $A(s, t)$ at all transferred momenta. Therefore, it is necessary to continue it analytically to the unphysical region of $t$ [10]. This may be done [11]. Correspondingly, the mathematically consistent inverse formulae contain, in general, the sum of contributions from the physical and unphysical parts of the amplitude $A(s, t)$. The amplitude in (12) enters only in the physical region. Only this part of its Fourier-Bessel transform is important in the unitarity relation for the impact parameter representation as well. It is written as

$$\text{Im} h(s, b) = |h(s, b)|^2 + F(s, b), \quad (20)$$

where $h(s, b)$ and $F(s, b)$ are obtained by the direct transformation of $A(s, t)$ and $F(s, t)$ integrated only over the physical transferred momenta from $t_{\min}$ to 0. They show the dependence of the intensity of elastic and inelastic interactions on the mutual impact parameter of the colliding particles. The integrals over all impact parameter values in this relation represent analogously to the relation (14) the total, elastic and inelastic cross-sections, respectively.

However, the accuracy of the unitarity condition in $b$-representation (20) is still under discussion (see, e.g., [12]) since some corrections due to unphysical region enter there even though their role may be negligible.
3 Where do we stand now?

First, let us discuss what we can say about asymptotic properties of such fundamental characteristics as the total cross section $\sigma_t$, the elastic cross section $\sigma_{el}$, the ratio of the real part to the imaginary part of the elastic amplitude $\rho$ and the width of diffraction peak $B$ at infinite energies. Then we compare this with some trends of present experimental data.

More than half a century ago it was claimed [13, 14] that according to the general principles of the field theory and ideas about hadron interactions the total cross section can not increase with energy faster than $\ln s$. The upper bound was recently improved [15] with the coefficient in front of the logarithm shown to be twice smaller than in the earlier limit:

$$\sigma_t \leq \frac{\pi}{2m^2} \ln^2(s/s_0),$$  \hspace{1cm} (21)

where $m$ is a pion mass.

If estimated at present energies, this bound is still much higher than the experimentally measured values of the cross sections with $s_0=1$ GeV$^2$ chosen as a ”natural” scale. Therefore it is of the functional significance. It forbids extremely fast growth of the total cross section exceeding asymptotically the above limits. Both the coefficient in front of logarithm in (21) and the constancy of $s_0$ are often questioned. In particular, some possible dependence of $s_0$ on energy $s$ is proclaimed (see, e.g., [16]).

The Heisenberg uncertainty relation points out that such a regime favors the exponentially bounded space profile of the distribution of matter density $D(r)$ in colliding particles of the type $D(r) \propto \exp(-mr)$. Since the energy density is $ED(r)$ and there should be at least one created particle with mass $m$ in the overlap region, then the condition $ED(r) = m$ gives rise to $r \leq \frac{1}{m} \ln(s/m^2)$ and, consequently, to the functional dependence of (21).

It was namely Heisenberg who first proposed earlier just such a behavior of total cross sections [17]. He considered the pion production processes in proton-proton collisions as a shock wave problem governed by some non-linear field-theoretical equations.

To study asymptotics, some theoretical arguments based on general principles of field theory and analogy of strong interactions to massive quantum electrodynamics [18] were promoted. The property that the limits $s \to \infty$ and $M \to 0$ (where $M$ denotes the photon mass) commute has been used [19], i.e. the asymptotics of strong interactions coincides with the massless
limit of quantum electrodynamics. These studies led to the general geometrical picture of the two hadrons interacting as Lorentz-contracted black disks at asymptotically high energies (see also the review paper [20]). In what follows, we discuss some other possibilities as well. However, as a starting point for further reference, we describe the predictions of this proposal.

The main conclusions are:
1. For black ($\Omega(s, b) \to \infty$) and logarithmically expanding disks with finite radii $R = R_0 \ln s$, $R_0=\text{const}$ one gets from (17) that $\sigma_t$ approaches infinity at asymptotics as

$$\sigma_t(s) = 2\pi R^2 + O(\ln s); \quad R = R_0 \ln s; \quad R_0 = \text{const}. \quad (22)$$

2. The elastic and inelastic processes should contribute on equal footing

$$\frac{\sigma_{et}(s)}{\sigma_t(s)} = \frac{\sigma_{in}(s)}{\sigma_t(s)} = \frac{1}{2} \mp O(\ln^{-1} s). \quad (23)$$

This quantum-mechanical result differs from "intuitive" classical predictions.

3. The width of the diffraction peak $B^{-1}(s)$ should shrink because its slope increases as

$$B(s) = \frac{R^2}{4} + O(\ln s) \quad (\text{see also [21]}) \quad (24)$$

4. The forward ratio of the real part to the imaginary part of the amplitude $\rho_0$ must vanish asymptotically as

$$\rho_0 = \frac{\pi}{\ln s} + O(\ln^{-2} s). \quad (25)$$

This result follows directly from Eq. (9) for $\sigma_t \propto \ln^2 s$.

5. The differential cross section has the shape reminding the classical diffraction of light on the disk

$$\frac{d\sigma}{dt} = \pi R^4 \left[ \frac{J_1(qR)}{qR} \right]^2, \quad (26)$$

where $q^2 = -t$.

6. The product of $\sigma_t$ with the value $\gamma$ of $|t|$ at which the first dip in the differential elastic cross section occurs is a constant independent of the energy

$$\gamma \sigma_t = 2\pi^3 \beta_t^2 + O(\ln^{-1} s) = 35.92 \text{ mb} \cdot \text{GeV}^2, \quad (27)$$
Table 1. The gray and Gaussian disks models \( (X = \sigma_{el}/\sigma_t, \ Z = 4\pi B/\sigma_t) \)

| Model | \(1 - e^{-\Omega} = \Gamma(s, b)\) | \(\sigma_t\) | \(B\) | \(X\) | \(Z\) | \(X/Z\) | \(XZ\) |
|-------|----------------------------------|-------------|-------|-------|-------|-------|-------|
| Gray  | \(\alpha \theta(R - b); 0 \leq \alpha < 1\) | \(2\pi\alpha R^2\) | \(R^2/4\) | \(\alpha/2\) | \(1/2\alpha\) | \(\alpha^2\) | \(1/4\) |
| Gauss | \(\alpha e^{-b^2/R^2}; 0 \leq \alpha \leq 1\) | \(2\pi\alpha R^2\) | \(R^2/2\) | \(\alpha/4\) | \(1/\alpha\) | \(\alpha^2/4\) | \(1/4\) |

where \(\beta_1 = 1.2197\) is the first zero of \(J_1(\beta\pi)\).

These are merely a few conclusions among many model dependent ones.

None of these asymptotical predictions were yet observed in experiment.

Surely, there is another more realistic at present energies possibility that the black disk model is too extreme and the gray fringe always exists. It opens the way to the numerous speculations with many new parameters about the particle shape and opacity (see the list of references in [1]).

In the Table 1 we show the predictions of the gray disk model with the steep rigid edge described by the Heaviside step-function and the Gaussian disk model. \(\Gamma(s, b)\) is the diffraction profile function.

The slope \(B\) is completely determined by the size of the interaction region \(R\). Other characteristics are sensitive to the blackness of disks \(\alpha\). In particular, the ratio \(X\) is proportional to \(\alpha\). The ratio \(Z\) plays an important role for fits at larger angles. It is inverse proportional to \(\alpha\). The corresponding formulae are given by (17), (18) and (19). The black disk limit follows from the gray disk model at \(\alpha = 1\).

The parameter \(XZ\) is constant in these models and does not depend on the nucleon transparency. On the contrary, the parameter \(X/Z\) is very sensitive to it being proportional to \(\alpha^2\). Therefore, it would be extremely instructive to get some knowledge about them from experimental data.

In the Table 2 we show how the above ratios evolve with energy according to experimental data. Most primary entries there except the last two are taken from Refs [22, 23] with simple recalculation \(Z = 1/4Y\). The data at Tevatron and LHC energies are taken from Refs [24, 25, 26]. All results are for \(pp\)-scattering except those at 546 and 1800 GeV for \(pp\) processes which should be close to \(pp\) at these energies.

The most interesting feature of the experimental results is the minimum of the blackness parameter \(\alpha\) at the ISR energies. It is clearly seen in the minima of \(X\) and \(X/Z\) and in the maximum of \(Z\) at \(\sqrt{s}=62.5\) GeV. The steady decrease of ratios \(X\) proportional to \(\alpha\) and \(X/Z\) proportional to \(\alpha^2\)
The energy behavior of various characteristics of elastic scattering.

| $\sqrt{s}$, GeV | 2.70 | 4.11 | 4.74 | 6.27 | 7.62 | 13.8 | 62.5 | 546 | 1800 | 7000 |
|-----------------|------|------|------|------|------|------|------|-----|------|------|
| $X$             | 0.42 | 0.28 | 0.27 | 0.24 | 0.22 | 0.18 | 0.17 | 0.21| 0.23 | 0.25 |
| $Z$             | 0.64 | 1.02 | 1.09 | 1.26 | 1.34 | 1.45 | 1.50 | 1.20| 1.08 | 1.00 |
| $X/Z$           | 0.66 | 0.27 | 0.25 | 0.21 | 0.17 | 0.16 | 0.11 | 0.18| 0.21 | 0.25 |
| $XZ$            | 0.27 | 0.28 | 0.29 | 0.30 | 0.30 | 0.26 | 0.25 | 0.26| 0.25 | 0.25 |

till the ISR energies and their increase at $Sp\bar{p}S$, Tevatron and LHC energies means that the nucleons become more transparent till the ISR energies and more black to 7 TeV. The same conclusion follows from the behavior of $Z$. The value of $Z$ approaches fast its limit for the Gaussian distribution of matter in the disk. This shows that the above crude models are not very bad for qualitative estimates in a first approximation.

We briefly comment on some of the important general trends of high-energy data observed in experiment.

1. Total cross sections increase with energy. At present energies, the power-like approximation is the most preferable one.

2. The ratio $\sigma_{el}/\sigma_{t}$ decreases from low energies to those of ISR where it becomes approximately equal to 0.17 and then strongly increases up to 0.25 at the LHC energies. However, it is still pretty far from the asymptotical value 0.5 corresponding to the black disk limit.

3. The diffraction peak shrinks about twice from energies about $\sqrt{s} \approx 6$ GeV where $B \approx 10$ GeV$^{-2}$ to the LHC energy where $B \approx 20$ GeV$^{-2}$.

4. A dip and subsequent maximum appear just at the end of the diffraction cone.

5. As regards the behavior of the differential cross section in the function of the transverse momentum behind the dip, the $t$-exponential of the diffraction peak is replaced, according to experimental data, by the $-\sqrt{|t|} \approx -p_t$-exponential at the intermediate angles:

$$d\sigma/dt \propto e^{-2a\sqrt{|t|}}, \quad a \approx \sqrt{B}.$$  \hspace{1cm} (28)

The slope $2a$ in this region increases with energy and it shifts to lower $|t|$.

6. As a function of energy, the ratio $\rho_0$ increases from negative values at comparatively low energies, crosses zero in the region of hundreds GeV and becomes positive at higher energies where it passes through the maximum of about 0.14 and becomes smaller (about 0.11) at LHC energies.
Fig. 1. The differential cross section of elastic proton-proton scattering at $\sqrt{s}=7$ TeV measured by the TOTEM collaboration (Fig. 4 in [26]). The region of the diffraction cone with the $|t|$-exponential decrease is shown.

7. The product $\gamma\sigma_t$ changes from 39.5 mb·GeV$^2$ at $\sqrt{s}=6.2$ GeV to 51.9 mb·GeV$^2$ at $\sqrt{s}=7$ TeV and deviates from the predicted asymptotic value [27]. The total cross section $\sigma_t$ increases faster than $\gamma$ decreases.

From the geometrical point of view the general picture is that protons become blacker, edgier and larger (BEL) [27]. We discuss it later. Thus even though the qualitative trends may be considered as satisfactory ones, we are still pretty far from asymptotics even at LHC energies.

4 LHC data and phenomenology

Here, we limit ourselves by the latest results of the TOTEM collaboration at the highest LHC energies 7 and 8 TeV [25, 26]. The discussion of theoretical models is also concentrated near these data.

The total and elastic cross sections at 7 TeV are respectively estimated as 98.3 mb and 24.8 mb. The cross section shape in the region of the diffraction cone [25] is shown in Fig. 1. The $t$-exponential behavior with $B \approx 20.1$ GeV$^{-2}$ is clearly seen at $|t| < 0.3$ GeV$^2$. The peak steepens at the end
Fig. 2. The differential cross section of elastic proton-proton scattering at $\sqrt{s}=7$ TeV measured by the TOTEM collaboration (Fig. 4 in [25]). The region beyond the diffraction peak is shown. The predictions of five models are demonstrated.

of the diffraction cone so that in the $|t|$ interval of $(0.36 - 0.47)$ GeV$^2$ its slope becomes approximately equal to 23.6 GeV$^{-2}$. The results at somewhat larger angles [26] in the Orear region are presented in Fig. 2. The dip at $|t| \approx 0.53$ GeV$^2$ with subsequent maximum at $|t| \approx 0.7$ GeV$^2$ and the $\sqrt{|t|}$-exponential behavior are demonstrated. Some curves according to different model predictions [28, 29, 30, 31, 32] are also drawn there. All of them fail to describe the data. We conclude that namely this region becomes the Occam razor for all models.

As we see in the Figures, various theoretical approaches have been attempted for description of different regions of the differential cross section. One by one we should name: 1. purely geometrical approach with reference to the internal geometry of colliding hadrons, 2. the analogy to the Fraunhofer diffraction, 3. the appeal to the electromagnetic and matter density distributions inside hadrons, 4. the dynamical picture of Reggeon exchanges which is the most popular one with Pomeron playing the distinguished role, 6. the OCD-inspired models. Their detailed review is given in [1]. Here, we
restrict ourselves by several recent developments not included or discussed there very briefly.

5 Proton structure

Since old days, it was clear that hadrons possess some internal structure. Namely this we discussed with Pomeranchuk after I proposed the one-pion exchange model of inelastic interactions [33]. It was treated as describing the peripheral interactions of hadrons. The deeper inside the hadron, the more pions should be exchanged and more dense populated should be the proton. Since then, the principal features have not been changed with pions replaced by partons (quarks and gluons) even though pions as a chiral anomaly play the distinguished role.

Early attempts to consider elastic scattering of hadrons also stemmed from the analogous simple geometrical treatment of their internal structure [34, 35, 36, 37]. Starting from simplified pictures, one tried to fit the elastic differential cross section. However, recent fits of LHC data failed outside the diffraction cone as demonstrated above. Thus these hypotheses were not precise enough.

At the same time, one can get the direct knowledge about the proton structure important for inelastic collisions using elastic scattering data. The impact of the proton structure on inelastic processes can be viewed from the overlap function defined by the unitarity condition in $b$-representation for elastic scattering amplitudes. It has been directly computed from experimental data of the TOTEM collaboration for pp-scattering at 7 TeV and shown in Fig. 3. The similar shape was obtained (see Fig. 4 in [39]) assuming the Gaussian profile of the elastic contribution $h(s, b)$. Both shapes show the pattern with rather flat shoulder close to 1 (i.e. to complete blackness) at small impact parameters $b$ and subsequent quite steep fall-off. Attempts to fit it by a single Gaussian fail.

In view of the supposed proton substructure with a darker and stepwise kernel surrounded by a more transparent cloud of partons it is reasonable to attempt the fit with a stepwise behavior of the Gaussian exponential like

$$\ln \frac{F(s, 0)}{F(s, b)} = \frac{b^2}{a[1 - \frac{2}{\pi} \arctan \frac{b-b_0}{\lambda}]}.$$  \hspace{1cm} (29)

The fit reveals quite strong separation of the two regions at $b_0 \approx 0.3$ fm
Fig. 3. The overlap function $G(s, b) = 4F(s, b) \leq 1$ at 7 TeV (upper curve) compared to those at ISR energies 23.5 GeV and 62.5 GeV.

with a width of the transition region $\lambda \approx 0.1$ fm. According to Eq. (5) the exponential becomes three times larger in the narrow strip between the borders of the transition region $b_0 - \lambda$ and $b_0 + \lambda$. The region $b < b_0$ is completely black while at $b > b_0$ it becomes more transparent. However when compared to ISR results [38] its blackness increases with energy, especially at rather large impact parameters about 1 fm as seen in Fig. 4. These peculiar features have been used for description [40] of the CMS data [41] about jet production at high multiplicities. The increased role of central pp-collisions with small impact parameters for events triggered by jets was demonstrated. The important conclusion of this analysis is that the perturbative QCD can be applied to these processes only at high enough transverse momenta of jets exceeding 7-8 GeV.

6 Scaling laws

Since long ago, it was discussed [42, 43] a possibility that the differential cross sections might be described as functions of a single scaling variable representing a definite combination of energy and transferred momentum. No rigorous proof of this assumption has been proposed. This property was recently obtained [44] from the solution of the partial differential equation for the imaginary part $\text{Im}A(s, t)$ of the elastic scattering amplitude. The
equation has been derived by equating the two expressions for the ratio of the real to imaginary parts of the amplitude $\rho(s,t)$. They were known from the local dispersion relations (10) [2, 3, 4] with the $s$-derivative and from the linear $t$-approximation [42, 45] with the $t$-derivative (for more details see [44]).

Therefrom the following partial differential equation is valid

$$p - f(x)q = 1 + f(x),$$

(30)

where $p = \partial u/\partial x;\ q = \partial u/\partial y;\ u = \ln \text{Im} A(s,t);\ f(x) = 2\rho(s,0)/\pi \approx d\ln \sigma_t/dx;\ x = \ln s;\ y = \ln |t|;\ \sigma_t$ is the total cross section. The variables $s$ and $|t|$ should be considered as scaled by the corresponding constant factors $s_0^{-1}$ and $|t_0|^{-1}$.

The general solution of Eq. (30) reveals the scaling law

$$\frac{t}{s}\text{Im} A(s,t) = \phi(t\sigma_t).$$

(31)

For the differential cross section it looks like

$$t^2d\sigma/dt = \phi^2(t\sigma_t),$$

(32)
if the real part of the amplitude is neglected compared to the imaginary part. Thus the scaling law is predicted not for the differential cross section itself but for its product to \( t^2 \). Let us note that the often used ratio (see, e.g., [46]) of \( d\sigma/dt \) to \( d\sigma/dt|_{t=0} \propto \sigma_t^2 \) is also a scaling function. However, the expression [42] is more suitable for comparison with experiment.

The scaling law with the \( t\sigma_t \)-scale is known as the geometrical scaling.

\[ t^2 \frac{d\sigma}{dt} \]

Fig. 5. Violation of geometrical scaling at LHC energies.

In Fig. 5, we plot \( t^2 d\sigma/dt \) for pp-scattering at energies \( \sqrt{s} \) from 4.4 GeV to 7 TeV as functions of \( t\sigma_t \) with \( \sigma_t \) provided by the corresponding experiment. Rather reasonable scaling is observed in the diffraction cone except the TOTEM data at 7 TeV. Thus the simple geometrical scaling is not fulfilled at high energies even at low transferred momenta. To restore some approximate scaling one should replace the variable \( t\sigma_t \) by \( t^a \sigma_t \) and plot \( t^2 a d\sigma/dt \) with \( a \approx 1.2 \) as shown in [47]. However, this is not a simple geometric scaling anymore.

### 7 Orear region and real part at any \( t \)

The theoretical explanation of the new regime of exponential decrease of the differential cross section beyond the diffraction cone with angles based on consequences of the unitarity condition in the \( s \)-channel has been proposed in Refs [6, 7]. The careful fit to experimental data showed good quantitative agreement with experiment [8]. Nowadays the same approach helped explain the TOTEM findings [9].

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We consider the lefthand side and the integral term \( I_2 \) in the unitarity condition \([12]\) at the angles \( \theta \) outside the diffraction peak. Because of the sharp fall off of the amplitude with angle, the principal contribution to the integral arises from a narrow region around the line \( \theta_1 + \theta_2 \approx \theta \). Therefore one of the amplitudes should be inserted at small angles within the cone as a Gaussian while another one is kept at angles outside it. Integrating over one of the angles the linear integral equation is obtained:

\[
\text{Im} A(p, \theta) = \frac{p\sigma_t}{4\pi\sqrt{2\pi B}} \int_{-\infty}^{+\infty} d\theta_1 e^{-Bp^2(\theta-\theta_1)^2/2} f_\rho \text{Im} A(p, \theta_1) + F(p, \theta),
\]

where \( f_\rho = 1 + \rho_0 \rho(\theta_1) \).

It can be solved analytically (for more details see \([6, 7]\)) with two assumptions that the role of the overlap function \( F(p, \theta) \) is negligible outside the diffraction cone and the function \( f_\rho \) may be approximated by a constant, i.e. \( \rho(\theta_1) = \rho_l = \text{const} \).

It is easy to check that the eigensolution of this equation is

\[
\text{Im} A(p, \theta) = C_0 \exp \left( -\sqrt{2B \ln \frac{Z}{f_\rho}} \right) + \sum_{n=1}^{\infty} C_n e^{-(\Re b_n)p\theta} \cos(|\text{Im} b_n|p\theta - \phi_n)
\]

with

\[
b_n \approx \sqrt{2\pi B}|n|(1 + i \text{sign} n) \quad n = \pm 1, \pm 2, \ldots
\]

This expression contains the exponentially decreasing with \( \theta \) (or \( \sqrt{|t|} \)) term (Orear regime) with imposed on it oscillations strongly damped by their own exponential factors. These oscillating terms are responsible for the dip. The exponential in Eq. (34) is well defined. It contains the value of \( Z \) in the logarithm becoming very sensitive to \( \rho \) when \( Z \) approaches 1 (see Table 2).

The comparison with LHC data has shown that this ratio must be negative and quite large (about -2) in this region. Most of the widely used models do not predict such values. Moreover many of them get it positive. This follows from the equal numbers of zeros of real and imaginary parts. Only those models with odd sum of this number can succeed in getting negative \( \rho(s, t) \). The unitarity condition does not ask for a zero of the imaginary part to fit the dip as the models do but ascribes it to the damped oscillations contained in the solution of the equation. This discrepancy is not resolved yet. No correspondance between the two approaches has yet been established. Some equations for the ratio have been derived and solved. They favor the variable sign of it. However the problem asks for further investigation.
8 Conclusions

There are new exciting findings at LHC. They pose serious problems for theoreticians. I am sure that all of them would be of great interest to I.Ya. Pomeranchuk.

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