Scientific paper

Shear Capacity of RC Beams Based on Beam and Arch Actions

Devin Gunawan¹*, Kazumasa Okubo², Takuro Nakamura³ and Junichiro Niwa⁴

Received 27 December 2019, accepted 25 April 2020 doi:10.3151/jact.18.241

Abstract

This study aims to propose a calculation method of shear capacity for RC beams based on the shear resisting mechanisms known as the beam and arch actions. Static four-point bending tests were conducted on 12 RC beams with various stirrup ratio and shear span ratio, and the contribution of each mechanism was calculated based on the strain distribution of tensile steel bars. The contribution of concrete in the beam action at the peak load tends to decrease with higher stirrups amount, while the contribution of the arch action tends to increase with higher stirrups amount and smaller shear span ratio. Based on these results, a new method to calculate the shear capacity of RC beams was proposed. The shear capacity carried by the beam action was obtained from deterministic equations. The shear capacity carried by the arch action was obtained through iteration process, which indirectly takes the compatibility condition into account. From the comparison of a total of 86 experimental and analytical results, the proposed method showed better accuracy to estimate the shear capacity of RC beams with stirrups.

1. Introduction

In the calculation of shear capacity of RC beams with stirrups, such as in the current JSCE Standard Specification (JSCE 2017), the Modified Truss Theory is often applied. In this theory, the shear capacity of RC beams is calculated as the sum of the contribution of the stirrups and the contribution of other mechanisms such as the interlocking of aggregates in the crack plane and the dowel action.

While it is known to be simple and subjective, it has been mentioned in several studies that this theory also has several problems. In this theory, the shear failure of a beam is assumed to occur at the timing of stirrup yield, while usually RC beams still can resist to shear force even after the stirrups yield. It also has been reported that the shear failure might happen while some stirrups are still in elastic condition, especially in the cases of RC beams with excessive amount of stirrups (Nakata et al. 2018; Sakaguchi et al. 2013). In addition, the contribution of the other mechanisms is assumed to be equal to the shear capacity of RC beams without stirrups, and remains constant from the initiation of diagonal crack until the failure. However, as the diagonal crack propagates and the beam deforms, it is natural to consider that this contribution by mechanisms such as the interlocking of aggregates and dowel action will fall (Niwa et al. 1995).

Considering the shear resisting mechanism may solve these problems. The shear resistance of RC beams is generally divided into beam action and arch action. Beam action includes the truss mechanism of the stirrups, and other mechanisms such as the interlocking of aggregates and the dowel action. Arch action is formed by the concrete compressive strut and the tensile steel bar as tie material (Park et al. 1975).

Previous research (Nakamura et al. 2008, 2018; Iwamoto et al. 2017) has shown that the resisting mechanism against the shear force would change as the beam deformation progressed. It was confirmed that as the diagonal crack propagates, the resistance by the other mechanisms in the beam action would fall, and the shear resistance at this state would be mainly carried by the resistance of stirrups in the beam action, as well as the resistance of the arch action. After the stirrup yield until the failure, the increase in the applied shear force is mainly resisted by the arch action.

The above results indicate that it is important to calculate the shear capacity based on the behaviors of the beam and arch actions. One main problem of the conventional shear capacity calculation is regarding the compatibility condition, which is usually ignored. A shear capacity calculation based on the beam and arch actions might solve this problem, since the behaviors of these actions correspond to the beam deformation.

Even though the importance of the beam and arch actions has been presented in previous research works, the calculation method of shear capacity, which is based on the beam and arch actions, has not been established. Therefore, in this research, after reorganizing the ex-
Experimental data that the authors conducted in the past regarding RC beams (Gunawan et al. 2018), the effects of parameters such as the stirrup ratio and shear span ratio were discussed, then a new method to calculate the shear capacity of rectangular RC beams based on the beam and arch actions was proposed.

2. Experimental program

2.1 Test specimens and materials
The details of 12 tested specimens, including the arrangement of stirrups are summarized in Table 1, and illustrated in Fig. 1. Table 2 summarizes the mechanical properties of the used steel bars. The yield strength of steel bars was determined using the offset method, except for D10 stirrups. All specimens were designed to fail in shear in the test shear span.

All specimens had the cross sections with the width of 150 mm and the height of 350 mm. The shear span was 750 and 1050 mm for the shear span ratio \(a/d\) of 2.5 and 3.5, respectively. The effective depth was 300 mm. Stirrups were arranged with the stirrup ratio \(r_w\) ranging 0.00\% to 0.65\% in the test shear span, and 0.76\% in the non-test shear span. Concrete with design cylinder compressive strength of 40 N/mm\(^2\) was used. The water cement ratio was 47\% and the maximum size of coarse aggregate was 20 mm.

2.2 Loading method and measurements
A four-point bending test with simply-supported condition was provided to all specimens. Steel plates of 75 mm width were placed on the supports. Teflon sheets and

| Table 1 Details of specimens. |
|--------------------------------|
| **Name** | **d (mm)** | **b (mm)** | **a (mm)** | **a/d** | **p_s (%)** | **r_w (%)** | **s (mm)** |
|---------|-------------|-------------|-------------|----------|-------------|-------------|------------|
| 2.5-0.00 | 300         | 150         | 750         | 2.5      | 1.69        | 0.00        | -          |
| 2.5-0.17 |             |             |             |          |             | 0.17        | 250        |
| 2.5-0.28 |             |             |             |          |             | 0.28        | 150        |
| 2.5-0.38 |             |             |             |          |             | 0.38        | 110        |
| 3.5-0.00 |             |             |             |          |             | 0.00        | -          |
| 3.5-0.17 |             |             |             |          |             | 0.17        | 250        |
| 3.5-0.28 |             |             |             |          |             | 0.28        | 150        |
| 3.5-0.38 |             |             |             |          |             | 0.38        | 110        |
| 3.5-0.53 |             |             |             |          |             | 0.33        | 80         |
| 3.5-0.65 |             |             |             |          |             | 0.65        | 65         |
| 3.5-0.38-D10 |             |             |             |          |             | 0.38        | 250        |
| 3.5-0.53-D10 |             |             |             |          |             | 0.53        | 180        |

Notes: \(d\): effective depth, \(b\): web width, \(a\): shear span length, \(p_s\): tensile steel bar ratio, \(r_w\): stirrup ratio, \(s\): stirrup spacing.

| Table 2 Mechanical properties of steel bars. |
|---------------------------------------------|
| **Name** | **Tensile steel bar** | **Compression steel bar** | **Stirrups in test shear span** |
|---------|------------------------|---------------------------|-------------------------------|
|         | Size | \(f_y\) (N/mm\(^2\)) | \(E\) (kN/mm\(^2\)) | Size | \(f_c\) (N/mm\(^2\)) | \(E\) (kN/mm\(^2\)) | Size | \(f_y\) (N/mm\(^2\)) | \(E\) (kN/mm\(^2\)) |
|---------|------|----------------------|-----------------|------|----------------------|-----------------|------|----------------------|-----------------|
| 2.5-0.00 |      | 1197                 | 350             |      | 199                  | 339             | D6   | 395                  | 174             |
| 2.5-0.17 |      | 1170                 | 349             |      | 184                  | 339             | D10  | 395                  | 174             |
| 2.5-0.28 |      | 1197                 | 350             |      | 199                  | 339             | D6   | 395                  | 174             |
| 2.5-0.38 | D22  | 1152                 | 348             |      | 197                  | 339             | D6   | 395                  | 174             |
| 3.5-0.00 |      | 1170                 | 349             |      | 184                  | 339             | D10  | 395                  | 174             |
| 3.5-0.17 |      | 1170                 | 349             |      | 184                  | 339             | D10  | 395                  | 174             |
| 3.5-0.28 |      | 1152                 | 348             |      | 197                  | 339             | D6   | 395                  | 174             |
| 3.5-0.38 |      | 1152                 | 343             |      | 196                  | 339             | D10  | 395                  | 174             |
| 3.5-0.53 |      | 1152                 | 343             |      | 196                  | 339             | D10  | 395                  | 174             |
| 3.5-0.65 |      | 1152                 | 343             |      | 196                  | 339             | D10  | 395                  | 174             |

Notes: \(f_y\): yield strength by offset method except for D10 stirrups, \(E\): elastic modulus.
grease were inserted between the specimen and the supports to prevent the horizontal friction. Steel plates of 65 mm width were placed at the loading points. During loading tests, applied load, mid span displacement, and strain of steel bars were measured. The strain of tensile steel bars was measured at the middle span, support points, and 4 points within the test shear span. The strain of each stirrup at the test shear span was measured at 1 to 3 points. The measuring points of strain of steel bars are also illustrated in Fig. 1. The data was recorded by a data logger. Two digital cameras were set to take pictures for the observation of crack propagation.

2.3 Results

Table 3 shows the calculation and experimental results. The calculated shear capacity \( V_{y,cal} \) is the sum of the capacity carried by other mechanisms \( V_{c,cal} \) and the capacity carried by stirrups \( V_{s,cal} \) (JSCE 2017). \( V_{s,cal} \) was calculated by the equation proposed by Niwa et al. (1986). \( V_{s,cal} \) is based on the truss theory with the angle of diagonal compression of 45°. These are shown in Eqs. (1), (2) and (3).

\[
V_{s,cal} = A_w f_{yw} \frac{z}{s} \quad (3)
\]

where

\( f_{c'} \) : cylinder compressive strength of concrete (N/mm²),
\( E_c \) : elastic modulus of concrete,
\( V_{c,cal} \) : shear capacity carried by other mechanisms,
\( V_{s,cal} \) : shear capacity carried by stirrups,
\( V_{y,cal} \) : calculated shear capacity,
\( V_{u,exp} \) : experimental shear capacity (half of peak load).

Table 3 Mechanical properties of concrete, calculation and experimental results of shear capacity.

| No        | \( f_{c'} \) (N/mm²) | \( E_c \) (kN/mm²) | \( V_{c,cal} \) (kN) | \( V_{s,cal} \) (kN) | \( V_{y,cal} \) (kN) | \( V_{u,exp} \) (kN) | \( V_{u,exp} / V_{y,cal} \) |
|-----------|----------------------|--------------------|----------------------|----------------------|----------------------|----------------------|-----------------------------|
| 2.5-0.00  | 46.0                 | 33.0               | 68.0                 | 0.0                  | 68.0                 | 68.1                 | 1.00                        |
| 2.5-0.17  | 42.7                 | 32.2               | 66.3                 | 22.4                 | 88.7                 | 122.3                | 1.38                        |
| 2.5-0.28  | 40.8                 | 32.2               | 65.3                 | 43.5                 | 108.8                | 130.7                | 1.20                        |
| 2.5-0.38  | 45.0                 | 31.2               | 67.5                 | 50.9                 | 118.4                | 169.1                | 1.43                        |
| 3.5-0.00  | 41.7                 | 31.7               | 57.8                 | 0.0                  | 57.8                 | 56.1                 | 0.97                        |
| 3.5-0.17  | 42.3                 | 30.1               | 58.0                 | 22.4                 | 80.4                 | 78.9                 | 0.98                        |
| 3.5-0.28  | 42.4                 | 31.3               | 58.1                 | 43.5                 | 101.6                | 139.9                | 1.38                        |
| 3.5-0.38  | 41.7                 | 32.0               | 57.7                 | 51.0                 | 108.7                | 145.8                | 1.34                        |
| 3.5-0.53  | 41.4                 | 30.6               | 57.6                 | 70.0                 | 127.6                | 160.0                | 1.25                        |
| 3.5-0.65  | 42.4                 | 31.2               | 58.1                 | 86.2                 | 144.3                | 178.6                | 1.24                        |
| 3.5-0.38-D10 | 41.3               | 31.7               | 57.6                 | 51.7                 | 109.3                | 139.1                | 1.27                        |
| 3.5-0.53-D10 | 49.1               | 34.6               | 61.0                 | 71.9                 | 132.9                | 169.0                | 1.27                        |

Notes: \( f_{c'} \) : cylinder compressive strength of concrete, \( E_c \) : elastic modulus of concrete, \( V_{c,cal} \) : shear capacity carried by other mechanisms, \( V_{s,cal} \) : shear capacity carried by stirrups, \( V_{y,cal} \) : calculated shear capacity, \( V_{u,exp} \) : experimental shear capacity (half of peak load).

In all specimens, after the peak load, diagonal cracks at the test shear span became more prominent, indicating the shear failure. Figure 2 shows the crack patterns at failure. A crack with the biggest opening was considered as the critical diagonal crack. The positions where the stirrup strain reached the yield strain are marked by blue rectangles.

In the specimens without stirrups, the load dropped and the specimens failed as diagonal cracks initiated. In the specimens with stirrups, the diagonal crack penetrated the compression fiber. In the specimens with relatively high \( r_{cw} \), concrete crush and spalling occurred in the vicinity of the loading point in the test shear span, showing that these specimens failed in shear compression failure. From Fig. 2, the crushing and spalling area...
seemed to be larger with higher $r_w$. The stirrup strain mostly reached the yield strain at the intersections with the diagonal cracks.

As shown in Table 3, $V_{s, \text{exp}}/V_{s, \text{cal}}$ was between 0.97 and 1.43, showing that the current equation tends to give a conservative estimation for shear capacity. The ratio tended to be higher in the specimens with stirrups.

3. Evaluation of shear resisting mechanisms

3.1 Evaluation of the contribution of each mechanism

The shear resisting mechanism is generally expressed by Eq. (4) (Park et al. 1975). The first term $jd(dT/dx)$ expresses the resistance due to the change of internal tension force, and is defined as the “beam action”. Beam action includes the resistance provided by the stirrups in form of the truss mechanism as well as the resistance by other mechanisms such as the aggregate interlocking and the dowel force. The second term $T(djd/dx)$ expresses the resistance due to the change of lever arm length, or in other words, the inclined internal compression along the shear span. This resistance is defined as the “arch action”.

Based on this relationship, it can be considered that the applied shear force $V$ is resisted by the contribution of beam action $V_{\text{beam}}$, and the contribution of arch action $V_{\text{arch}}$. $V_{\text{beam}}$ can further be separated into the contribution of truss mechanism by stirrups $V_{\text{sbeam}}$ and the contribution of the other mechanisms in beam action $V_{\text{beam}}$. Herein after, $V_{\text{beam}}$ is referred as the contribution of concrete in the beam action. These are expressed in Eq. (5).

In this study, $V_{\text{beam}}$ and $V_{\text{arch}}$ are the values when the angle of diagonal compression for the truss action is 45°. By the nature of the evaluation method, the values of $V_{\text{beam}}$ and $V_{\text{arch}}$ are both influenced by this angle. However, as can also be observed in Fig. 2, usually numerous diagonal cracks generate along the shear span, and these cracks are non-linear. Thus, there are difficulties in defining the angle of diagonal compression. For simplification, it was decided to assume this angle to be fixed at 45°, and the following considerations were made based on this assumption.

In the evaluation, $V_{\text{beam}}$ and $V_{\text{arch}}$ were evaluated using the strain distribution of tensile steel bars measured during the loading test. The evaluation flow is summarized in Fig. 3.

First, the tensile force $T$ at 5 points within the test shear span was calculated using the measured strain and the elastic modulus. These points are the positions of the strain gauges attached on the tensile steel bars.

Next, the lever arm length $jd$ at each point was calculated using the relationship shown in Eq. (6). In this research, a condition was set to make sure the $jd$ of one point is smaller than the $jd$ of the next point closer to the loading point. This is to prevent overestimation of the lever arm $jd$, especially near the support points when the load is low, which may happen because in Eq. (6) the tensile force of concrete is completely neglected.

Next, the contribution of beam and arch actions $V_{\text{beam}}$ and $V_{\text{arch}}$, in the shear span was calculated. $V_{\text{beam}}$ and $V_{\text{arch}}$ were calculated in each of the 4 sections from [a] to [b], [b] to [c], [c] to [d] and [d] to [e] in Fig. 3, then averaged afterwards. For each section, the representative $T$ and $jd$ were the average value between 2 points, and the differentials were calculated using the difference method.

After that, the contribution of stirrups in the beam action $V_{\text{sbeam}}$, which was evaluated by Eq. (7), was subtracted from $V_{\text{beam}}$ to obtain the contribution of concrete in the beam action $V_{\text{beam}}$. $V_{\text{sbeam}}$ was based on the Truss Theory, where the angle of diagonal compression was assumed to be 45° for all specimens. $\sigma_w$ was the average of tensile stress of stirrups in the shear span. For the case when several gauges attached on the same stirrups, the gauge with the highest value at the peak load was selected. In some specimens, stirrups in the vicinity of the supporting point were excluded since the diagonal cracks did not initiate there, thus the strain was obviously small.

By doing this series of evaluation, $V_{\text{arch}}$, $V_{\text{beam}}$, and $V_{\text{arch}}$ at every shear force level can be observed. Other than the applied shear force $V$ was small, or in other words, before flexural and diagonal cracks occur, the sum of $V_{\text{arch}}$, $V_{\text{beam}}$ and $V_{\text{arch}}$ was equal to $V$.

$$V = \frac{dM}{dx} = \frac{d(Tjd)}{dx} = jd \frac{dT}{dx} + T \frac{d(jd)}{dx}$$

$$V = V_{\text{beam}} + V_{\text{arch}} = (V_{\text{sbeam}} + V_{\text{beam}}) + V_{\text{arch}}$$

Fig. 3 Evaluation flow of contribution of each mechanism.
where
V: applied shear force,
M: flexural moment,
T: internal tension force,
\( j_d \): lever arm length,
x: longitudinal position from the support point,
\( V_{\text{beam}} \): contribution of the beam action,
\( V_{\text{arch}} \): contribution of arch action,
\( V_{\text{sbeam}} \): contribution of stirrups in the beam action,
\( V_{\text{cbeam}} \): contribution of concrete in the beam action and
\( \sigma_w \): average of tensile stress of stirrups in the shear span.

3.2 Transition of shear resisting mechanisms

Figure 4 shows the transition of the shear resisting mechanisms \( V_{\text{arch}}, V_{\text{beam}} \), and \( V_{\text{sbeam}} \) during the loading test of several specimens. The applied shear force \( V \) is also shown. The overall transition was observed as follows.

Before the initiation of diagonal cracks, the shear force was resisted mainly by the concrete in the beam action. As seen in the graphs, initially only \( V_{\text{cbeam}} \) was increasing along with \( V \). When \( V \) increased up to a certain degree, \( V_{\text{beam}} \) started decreasing. As shown in the figures, this timing generally matched with the observed timing of the initiation of diagonal cracks, and roughly matched \( V_{\text{sbeam}} \).

As shown in Figs. 4(a) and 4(c), the specimens without stirrups failed at this timing.

After the initiation of diagonal cracks, the resistance of the arch action and the stirrups in the beam action became significant, while the resistance of the concrete in the beam action declined gradually. As seen in the graphs, at the same time when \( V_{\text{cbeam}} \) started decreasing, the increasing rate of \( V_{\text{arch}} \) became higher, and \( V_{\text{sbeam}} \) started increasing.

After stirrups yielded, the resistance of arch action became dominant. As seen in the graphs, after increasing the applied shear force further, \( V_{\text{sbeam}} \) gradually became flat. This indicates the yielding of most stirrups in the shear span. At this point, the resistance of concrete in the beam action was mostly lost, as \( V_{\text{cbeam}} \) became significantly smaller. In some specimens, the value of \( V_{\text{cbeam}} \) reached zero, indicating that the resistance of concrete in the beam action was totally lost. Only \( V_{\text{arch}} \) was increasing along with \( V \).

At failure, the resistance of the arch action decreased along with the applied shear force. As seen in the graphs, in the vicinity of the peak \( V \), \( V_{\text{arch}} \) in most specimens became flat, then eventually started decreasing.

The above behaviors of \( V_{\text{arch}}, V_{\text{beam}}, \) and \( V_{\text{sbeam}} \) showed good agreement to the results from previous research (Nakamura et al. 2008, 2018; Iwamoto et al. 2017).

![Fig. 4 Transition of shear resisting mechanism.](image-url)
3.3 Contribution of stirrups and concrete in beam action at the peak

The comparison of the contribution of stirrups in the beam action at the peak load $V_{c,peak}$ is shown in Fig. 5. The value $V_{c,cal}$ calculated from Eq. (3) is also shown. It can be observed that $V_{c,peak}$ was mostly equal to $V_{c,cal}$. This is a natural result, since as shown in Fig. 2, the strain value of most of the stirrups in the shear span exceeded the yield strain.

Figure 6 shows the relationship between the contribution of concrete in the beam action at the peak load $V_{c,peak}$ with the stirrups amount $r_{w,foy}$. The results of specimens without stirrups were excluded. For some specimens, $V_{c,peak}$ was below zero, which indicated that the contribution of concrete in the beam action was totally lost. Since it is unnatural for a mechanism to have a minus resistance, in these cases, it will be considered as $V_{c,peak} = 0$.

The results from previous research (Nakamura et al. 2018) are also shown. These results were obtained numerically by three dimensional Rigid-Body-Spring-Model (3-D RBSM) analysis, and the contribution of shear resisting mechanisms was calculated using the stress distribution of concrete and steel bars. Details of the model can be seen in the paper. The results of cases with the stirrup ratio $r_w$ between 0.14% to 0.56%, and shear span ratio $a/d$ of 2.35 and 3.14 were picked, as these are the closest cases to the experiments in this study. The considered beams had the effective depth $d$ of 255 mm. The materials properties were concrete compressive strength $f_c$ of 40.8 N/mm² and stirrup yield strength $f_{wy}$ of 363 N/mm². Note that since the exact values of $V_{c,peak}$ were not clarified in the paper, the values of $V_{c,peak}$ used in the below discussion were estimated from the graphs.

It was shown that with more stirrups, the contribution of concrete in the beam action at the peak load tended to be decreasing. From Fig. 6, with the increase in $r_{w,foy}$, $V_{c,peak}$ in the same series showed a decreasing tendency. This indicates that the allowable deformation in the beam increased with more stirrups. As stated in the previous chapter, after the initiation of diagonal cracks, $V_{c,peak}$ started decreasing as the applied shear force increased, or in other words, as the deformation proceeded. It can be considered that, in the specimens with less amount of stirrups, the failure happened when the deformation is still small, thus the concrete in the beam action is still able to resist to some portion of shear force at the peak. In the specimens with more stirrups, the failure happened after the deformation had proceeded further, thus at the peak, the concrete in the beam action is only able to resist to a small portion of shear force.

The specimens with small shear span ratio $a/d$ might have a smaller resistance of concrete in the beam action at peak. It can be seen from Fig. 6 that the $V_{c,peak}$ of 3.5-D6 series was mostly 0. This might be due to the contribution of the arch action, which will be shown in the next chapter to be more dominant with smaller $a/d$. Since the arch action was dominant, the beam still could resist to the shear force even though $V_{c,peak}$ had dropped to zero.

The resistance of concrete in the beam action seemed to be lost easier when the stirrup spacing was wide. This can be observed in Fig. 6 when comparing the results of 3.5-D6 series and 3.5-D10 series. There were several specimens with the similar stirrups ratio $r_w$ but achieved with different stirrup size. Specimens with D10 stirrups had the stirrup spacing much wider than $d/2$, which did not meet the regulation in the JSCE Standard Specification (JSCE 2017). The $V_{c,peak}$ of these specimens with D10 stirrups reached 0, while $V_{c,peak}$ of the specimens with D6 stirrups was maintained at some degree.

3.4 Contribution of arch action at the peak

Figure 7 shows the relationship between the contribution of the arch action at the peak load and the stirrups amount $r_{w,foy}$. The results of specimens without stirrups were excluded. The results of previous research (Nakamura et al. 2018) are also shown, with the same cases as above.

It was shown that, with more stirrups, the contribution of the arch action at the peak load tended to increase. As shown in Fig. 7, with the increase in $r_{w,foy}$, $V_{arch,peak}$ in the same series showed an increasing tendency. $V_{arch,peak}$ in some specimen was smaller than the specimen with...
lower $r_{w,fwy}$ especially when $r_{w,fwy}$ was relatively small. This indicates that the shear behavior might be unstable when the stirrup ratio is small. However, considering the overall behavior, $V_{arch, peak}$ clearly increased due to the increase in $r_{w,fwy}$.

The contribution of the arch action at the peak load also increased with smaller shear span ratio. This can be observed in Fig. 7, since the results of 2.5-D6 series were higher compared to 3.5-D6 series. Similar tendency can also be observed with the results from the previous research. This is quite natural since the shorter arch structure will be stronger than the slenderer arch structure.

4. Calculation of shear capacity based on shear resisting mechanisms

4.1 Shear capacity carried by beam action

From the discussions in the preceding sections about the specimens with stirrups, the following points are essential to calculate the shear capacity carried by the beam action. As already mentioned, these results were obtained based on the assumption that the angle of diagonal compression is 45°. Thus, the proposed method would also follow the same assumption.

1) Most stirrups are yielded at the failure.
2) The diagonal crack generally initiates when the applied shear force $V$ reaches the calculated capacity carried by concrete $V_{c,cal}$.
3) The resistance of concrete in the beam action starts decreasing after the diagonal crack initiation.

4) Some portion of the resistance in concrete in the beam action is maintained at the failure.
5) The portion of the resistance in 4) tends to decrease with higher stirrups amount.
6) Shear span ratio $a/d$ may affect the portion of the resistance in 4).

First, based on 1), the shear capacity carried by stirrups in the beam action $V_{beam,cal}$ can be calculated using the Truss Theory, as shown in Eq. (3).

Next, based on 2), 3) and 4), the shear capacity carried by concrete in the beam action $V_{beam,peak}$ can be calculated through Eq. (9) by multiplying a decrement factor $\alpha$ to the diagonal crack initiation force, which can be estimated as $V_{c,cal}$ in Eq. (2).

Based on 5) and 6), the decrement factor $\alpha$ can be calculated as the function of either stirrups amount or shear span. In this study, this relationship is obtained empirically, using the results of experiments conducted by the authors and the results from previous research. The outline of each research is shown in Table 4.

![Fig. 7 Contribution of arch action at peak.](image)

![Fig. 8 Relationship of $\alpha$ and $r_{w,fwy}$.](image)

| Reference       | Data Number | Results from | $f'_c$ (N/mm²) | $a/d$ | $r_{w,fwy}$ (%) | $f_{w,fwy}$ (N/mm²) | Evaluation method | Remarks             |
|-----------------|-------------|--------------|----------------|-------|----------------|---------------------|-------------------|---------------------|
| This research   | 6           | Experiment   | 40.8 to 45.0   | 2.5, 3.5 | 0.28 to 0.65 | 339 to 395          | Strain distribution of tensile steel bars | -                   |
| Nakamura et al. (2018) | 20         | Numerical Analysis | 40.8 | 1.57 to 4.31 | 0.14 to 0.56 | 363                | Strain distribution of concrete and longitudinal steel bars | $V_{beam}$ estimated from graph |
| Chi et al. (2018) | 6            | by RSBM      | 40.0           | 3.51   | 0.16, 0.30    | 295                 |                   |                     |
| Nakamura et al. (2008) | 4           | Experiment   | 18.9 to 36.2   | 2.5, 4 | 0.05, 0.11    | 350 to 380          | Strain distribution of tensile steel bars | $V_{beam} = V_{beam,peak} - V_{c,cal}$ |

Notes: $f'_c$: cylinder compressive strength of concrete, $a/d$: shear span ratio, $r_{w,fwy}$: stirrup ratio, $f_{w,fwy}$: yield strength of stirrups.
dispersion on the calculated shear capacity is quite small, thus it is considered acceptable.

\[ V_{\text{beam cal}} = V_{c, \text{cal}} \times \alpha \]  
\[ \alpha = -0.053 \cdot r_w \cdot f_{yw} + 0.18, \text{ where } r_w \cdot f_{yw} < 3.3 \]  

where

\( \alpha \): decrement factor,
\( r_w \): stirrup ratio and
\( f_{yw} \): stirrup yield strength (N/mm²).

Similarly, the relationship between \( \alpha \) and \( a/d \) is shown in Fig. 9. The overall tendency shows that \( \alpha \) may increase with larger \( a/d \). However, as the concerned range only covered up to \( a/d = 4.31 \), it cannot be guaranteed whether the same tendency is kept even with the higher \( a/d \). This might result in overestimation of shear capacity. On the other hand, even though the range of \( r_w f_y \) was also limited up to \( r_w f_y = 2.2 \), an even higher \( r_w f_y \) will result in lower \( \alpha \), which has lower possibility to result in overestimation. Thus, in the context of safety design, it is better to calculate \( \alpha \) by Eq. (10), which takes into account the stirrup amount \( r_w f_y \).

When using Eq. (10), if \( r_w f_y \) exceeds the value of 3.3, then \( \alpha \) will become less than zero. This implies two points. First, the shear capacity carried by concrete in the beam action \( V_{\text{beam cal}} \) is totally lost at the peak. Second, the stirrups in the shear span are still in elastic condition. For now, \( r_w f_y < 3.3 \) can be set as the application limit of the equation.

4.2 Shear capacity carried by arch action

From the discussions in the previous sections about the specimens with stirrups, the following points are essential to calculate the shear capacity carried by the arch action.

1) Concrete crush and spalling occur near the loading point at the failure in specimens with relatively high \( r_w \).
2) The crushed and spalled area in 1) seems to be larger with higher \( r_w \).
3) Resistance of the arch action is dominant after stirrup yield, and it decreases along with the applied shear force after failure.
4) Contribution of the arch action at the peak load increases with higher stirrups amount.
5) Contribution of the arch action at the peak load increases with smaller shear span ratio \( a/d \).

Based on 1) and 3), the ultimate condition of the arch action seems to be related to the concrete crush and spalling near the loading point. Concrete crush and spalling indicate a high compressive stress applied in this area due to the flexural moment and the shear force. In previous research (Sato et al. 1995), it is expressed that due to the diagonal crack, the compression zone in the shear span becomes smaller compared to the pure flexural area. From this, it can be figured that the compressive stress concentrates due to the narrowed compression zone, and results in the concrete crush and spalling. In the calculation of the shear capacity carried by the arch action mentioned below, this will be the main consideration.

Regarding 2) above, Sato et al. (1995) proposed an equation that shows that the compression zone in the shear span becomes slightly larger with higher stirrup ratio \( r_w \). It can be figured that the increase in contribution of the arch action due to the higher stirrups amount is related to this, which explains 4) above.

Usually, the contribution of the arch action is calculated from experimental or analytical data based on the distribution of strain of longitudinal steel bars and concrete along the shear span. However, this distribution can only be obtained from experiment, or from numerical analysis that will require complicated calculations. Thus, in the context of designing, to calculate the shear capacity carried by the arch action, it is more rational to consider only representative sections within the shear span. Therefore, in this research, a method to calculate by only considering two representative sections is presented.

The outline of calculation of shear capacity by the arch action \( V_{\text{arch cal}} \) is shown in Fig. 10. In this calculation, the compression forces in beam axis direction of two sections are considered. The considered sections are the loading point and the supporting point. The calculation is derived from the relationship in Eqs. (4) and (5). It can be assumed that the tensile force \( T \) and compression force \( C \) are in the equilibrium state, and that the lever arm length \( jd \) at the supporting point is zero. Based on these, Eq. (11) is obtained.

\[ V_{\text{arch cal}} = C \frac{\frac{d}{dx}(jd)}{2} \left( \frac{C_{LP} + C_{SP}}{a} \right) \frac{d}{dx} \frac{1.15}{a} \]  

![Fig. 9](image-url)  
**Fig. 9** Relationship of \( \alpha \) and \( a/d \).

![Fig. 10](image-url)  
**Fig. 10** Considered situation for calculation of \( V_{\text{arch cal}} \).

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where,  
\( C \): compression force,  
\( j d \): lever arm length,  
\( x \): longitudinal position from supporting point,  
\( C_{LP} \): compression force at the loading point,  
\( C_{sp} \): compression force at the supporting point,  
\( d \): effective depth and  
\( a \): shear span.

About the loading point, it was stated above that the ultimate condition of the arch action is related to the concrete crush at the loading point. Since concrete crush occurs, it seems rational to consider that the compression stress already reached the compressive strength \( f' \).  

Composition force at the loading point \( C_{LP} \) can be calculated using Eq. (12). Here, the stress within the compression zone is assumed to be uniform. Since the actual stress might be distributed, a reduction factor \( k \) is used. \( k \) can be calculated with Eq. (13), which is from the fib model code 2010 (fib 2013). In the code, \( k \) is used in the calculation of reduced concrete compressive strength for the dimensioning of struts or stress fields.

In the proposed method, if the actual compressive stress is distributed within the compression zone, assuming it to be uniform will overestimate the shear capacity. This overestimation will be notable with higher concrete compressive strength \( f' \). The reduction factor \( k \) may cover this problem, since \( k \) will reduce the strength further in this case.

The depth of compression zone in the shear span near the loading point \( x \), can be calculated by Eq. (14), which is an equation proposed by Sato et al. (1995). \( x \) can be obtained from the calculation of equilibrium of internal forces of RC beams after the occurrence of flexural crack. As explained in Eq. (11), the arch action considers the compression force in the beam axis direction. This force generates due to the flexural moment, but in the shear span, it is affected by the shear force. Thus, it seems reasonable to estimate \( x \) based on \( x_e \), while considering the effect of shear force. Using Eq. (14), \( x \) is calculated to be smaller than \( x_e \), with the ratio differs depending on the stirrup amount and shear span ratio.

\[
C_{LP} = k f' x_e b 
\]

\[
k = \left( \frac{30}{f'} \right)^{1/3} \leq 1.0
\]

\[
\frac{x}{x_e} = \frac{1 - e^{-a/d}}{1 + 3.2(\theta_{\text{inci}} f'_c)^{1/3}}
\]

where  
\( k \): reduction factor for \( f' \),  
\( f' \): cylinder compressive strength of concrete (N/mm²),  
\( x \): depth of compression zone in the shear span near the loading point (mm),  
\( b \): web width (mm),  
\( x_e \): depth of compression zone in the pure flexural zone (mm),  
\( r_s \): stirrup ratio and  
\( E_u \): elastic modulus of stirrup (N/mm²).

About the supporting point, in theory, since there is no flexural moment applied in this area, there should be no strain. However, as shown in Fig. 11, the strain of tensile steel bar at the supporting point started increasing at some point. This timing was confirmed to be right after the initiation of diagonal crack. This behavior was somewhat similar to that of the arch action \( V_{arch} \) discussed in the previous chapter. Previous research (Nakamura et al. 2017; Gunawan et al. 2018) attempted to evaluate the contribution of the arch action based on this behavior, and the results showed good agreement with other research works. Thus, it is rational to consider that the force at the supporting point is also essential in the calculation of shear resistance carried by the arch action.

When considering an RC beam after the initiation of diagonal cracks, the force of the steel bar will be larger compared to the case with no diagonal cracks due to the “Moment Shift”. This is usually expressed in Eq. (15), with the inclination \( \theta \) is assumed to be 45°. Using this equation, the force occurred at the supporting point \( T_{(x=0)} \) can be calculated by substituting \( x = 0 \). The compression force at the supporting point \( C_{sp} \) is assumed to be in equilibrium with \( T_{(x=0)} \), as shown in Eq. (16). Thus, in the ultimate condition, \( C_{sp} \) can be estimated to be half of the calculated shear capacity \( V_{arch} \).

\[
T = \frac{V}{z}(x + z \cot \theta / 2) = \frac{Vx}{z} + \frac{V}{2} 
\]

\[
C_{sp} = T_{(x=0)} = \frac{V}{2}
\]

where  
\( T \): force of tensile rebar within the shear span,  
\( V \): applied shear force,  
\( x \): longitudinal position from support point,  
\( z \): lever arm length,  
\( \theta \): inclination of diagonal crack (assumed to be 45°),  

![Fig. 11 Relationship between \( V_{arch} \) and strain of tensile steel bar at the support point (3.5-0.38).](image-url)
CSP: compression force at the supporting point and $V_{u,\text{cal}}$: calculated shear capacity.

Here, a value of the applied shear force $V$ needs to be assumed to calculate CSP in Eq. (16). The ideal value would be the calculated shear capacity $V_{u,\text{cal}}$ itself. Thus, it is proposed to do a convergence calculation until $V_{u,\text{cal}}$ meets the assumed $V$ (i.e., $V_{u,\text{cal}} = V$).

Considering CSP in the calculation of shear capacity indirectly takes into account the compatibility condition of shear behavior. This is because CSP is related to the deformation of the beam due to diagonal cracks. When there is no diagonal crack, CSP can be considered to be zero. As the applied shear force increases, or in other words as the deformation progresses due to the propagation of diagonal cracks, CSP increases accordingly.

Through the convergence calculation, it is ensured that higher shear capacity $V_{u,\text{cal}}$ corresponds to higher force in the supporting point CSP, which expresses more deformation of the considered beam.

4.3 Calculation flow of the proposed method

The calculation flow is illustrated in Fig. 12. The calculated shear capacity $V_{u,\text{cal}}$ will be the sum of the shear capacity carried by concrete in the beam action, stirrups in the beam action, and the arch action, as shown in Eq. (17).

The shear capacity carried by stirrups and concrete in the beam action $V_{\text{beam,cal}}$, $V_{\text{c,cal}}$ is calculated using Eqs. (2), (3), (9) and (10).

The shear capacity carried by the arch action $V_{\text{arch,cal}}$ is calculated using Eqs. (11) to (14) and Eq. (16). As stated above, it is proposed to do a convergence calculation so that the $V_{u,\text{cal}}$ meets the assumed $V$ in Eq. (16), (i.e., $V_{u,\text{cal}} = V$). In the calculation, initially $V$ can be input as 0. The values of CSP, $V_{\text{arch,cal}}$, and $V_{u,\text{cal}}$ obtained in this stage are temporary, since the assumed $V$ is 0 ($V_{u,\text{cal}} \neq V$). Then, the temporary $V_{u,\text{cal}}$ is substituted to $V$ in Eq. (16), and CSP, $V_{\text{arch,cal}}$, $V_{u,\text{cal}}$ are calculated once again. These steps are repeated until the condition of $V_{u,\text{cal}} = V$ is satisfied.

$$V_{u,\text{cal}} = V_{\text{beam,cal}} + V_{\text{c,cal}} + V_{\text{arch,cal}} \quad (17)$$

![Fig. 12 Calculation flow of the proposed method.](image-url)
calculate the shear capacity carried by each shear resisting mechanism.

4.5 Verification of the proposed method
In order to verify the proposed method, the calculated shear capacity was compared to the results of totally 86 cases, 44 cases from experiments and 42 cases from numerical analysis. The outline of the cases is shown in Table 5. As comparison, the calculation results based on the Modified Truss Theory (MTT) from Eqs. (1) to (3) are also shown. For the cases where the elastic modulus of the concrete \( E_c \) is not stated, it was calculated based on the compressive strength \( f'_c \), following the JSCE Standard Specification (JSCE 2017). For the cases where the elastic modulus of a steel bar \( E_s \) is not stated, it was assumed to be 200 kN/mm². For the cases where the information about compression rebar is not stated, it was neglected in the calculation.

The comparison results are shown in Tables 6 to 8 and Fig. 13. Regarding the shear capacity calculated by the Modified Truss Theory, the average of \( V_{u,exp}/V_{u,cal} \) and \( V_{u,ana}/V_{u,cal} \) is 1.28, and the standard deviation is 0.19. On the other hand, regarding the shear capacity calculated by the proposed method, the average of \( V_{u,exp}/V_{u,cal} \) and \( V_{u,ana}/V_{u,cal} \) is 1.05, and the standard deviation is 0.13. The proposed method clearly gives a better estimation for shear capacity.

In Fig. 13, the horizontal axis is changed between the stirrups amount \( r_{sfv} \), concrete compressive strength \( f'_c \), and shear span ratio \( a/d \), in order to observe the sensitivity of these parameters. It can be observed that overall, the proposed method shows good and stable estimation. Even though, there were a limited number of cases with relatively low stirrups amount, low concrete compressive strength, and high shear span ratio. Here, the accuracy of the proposed method may be lower than the Modified Truss Theory, showing the need of further verification and improvement regarding these limited cases.
The proposed method still tends to result in the smaller shear capacity than the experimental and analytical results. One possible reason is the shear resistance of the compression rebar. In the proposed method, the compression rebar only affects the depth of compression zone $x_c$. However, other resistance against shear, such as the dowel action might also exist, since in several cases, thick compression steel bars were used. Either way, this point has positive impact on the shear capacity, thus it is rational to neglect it in the context of design in the safety side.

Overall, the proposed method shows better accuracy to estimate the shear capacity of rectangular cross section RC beams with of stirrups, while still maintaining safety design to some extent. The proposed method also covers quite a wide range of concrete strength, shear span ratio, stirrup ratio, and stirrup yield strength.

| Reference | Specimen name | $V_{u,exp}$ (kN) | $V_{c,cal}$ (kN) | $V_{s,cal}$ (kN) | $V_{sbeam,cal}$ (kN) | $V_{arch,cal}$ (kN) | $V_{u,cal}$ (kN) | $V_{c,exp}/V_{u,cal}$ | $V_{s,cal}/V_{u,cal}$ | $V_{sbeam,cal}/V_{u,cal}$ | $V_{arch,cal}/V_{u,cal}$ |
|-----------|---------------|------------------|------------------|-----------------|---------------------|-------------------|-----------------|---------------------|---------------------|---------------------|---------------------|
| D. Gunawan, K. Okubo, T. Nakamura and J. Niwa (2018) | 2.5-0.17 | 122.6 | 66.3 | 22.4 | 9.6 | 88.7 | 88.7 | 1.38 | 120.7 | 1.01 |
|         | 2.5-0.28 | 130.7 | 65.3 | 43.5 | 7.7 | 91.8 | 108.8 | 1.20 | 143.0 | 0.91 |
|         | 2.5-0.38 | 169.1 | 67.5 | 50.9 | 7.3 | 103.3 | 118.4 | 1.43 | 161.5 | 1.05 |
|         | 3.5-0.17 | 78.9 | 58.0 | 22.4 | 8.5 | 66.1 | 80.4 | 0.98 | 97.0 | 0.81 |
|         | 3.5-0.28 | 139.9 | 58.1 | 43.5 | 6.8 | 69.6 | 101.6 | 1.38 | 119.9 | 1.17 |
|         | 3.5-0.38 | 145.8 | 57.8 | 50.9 | 6.2 | 71.4 | 108.7 | 1.34 | 128.5 | 1.13 |
|         | 3.5-0.53 | 160.0 | 57.6 | 70.0 | 4.7 | 75.4 | 127.6 | 1.25 | 150.1 | 1.07 |
|         | 3.5-0.65 | 178.6 | 58.1 | 86.2 | 3.4 | 78.3 | 144.3 | 1.24 | 167.9 | 1.06 |
|         | 3.5-0.38D10 | 139.1 | 57.6 | 51.7 | 6.1 | 71.4 | 109.3 | 1.27 | 129.2 | 1.08 |

Table 6 Shear capacity comparison for verification of the proposed method (44 experimental cases).

The proposed method still tends to result in the smaller shear capacity than the experimental and analytical results. One possible reason is the shear resistance of the compression rebar. In the proposed method, the compression rebar only affects the depth of compression zone $x_c$. However, other resistance against shear, such as the dowel action might also exist, since in several cases, thick compression steel bars were used. Either way, this point has positive impact on the shear capacity, thus it is rational to neglect it in the context of design in the safety side.

Overall, the proposed method shows better accuracy to estimate the shear capacity of rectangular cross section RC beams with of stirrups, while still maintaining safety design to some extent. The proposed method also covers quite a wide range of concrete strength, shear span ratio, stirrup ratio, and stirrup yield strength.
5. Conclusions

In this study, in order to evaluate the shear capacity of rectangular RC beams based on the shear resisting mechanisms, the effects of stirrup ratio and shear span ratio on the beam and arch actions were discussed based on experimental results. The results are as follows.

(1) The contribution of concrete in the beam action at the peak tends to decrease with higher stirrup ratio. This indicates that bigger deformation can be allowed with more stirrups.

Table 7 Shear capacity comparison for verification of the proposed method (42 analytical cases).

| Reference | Specimen name | $V_{u,ana}$ (kN)* | $V_{u,cal}$ (kN) | $V_{u,cal}$ beam (kN) | $V_{u,arch}$ (kN) | $V_{u,cal}$/Average | $V_{u,ana}$/Average |
|-----------|---------------|--------------------|-----------------|----------------------|-----------------|-------------------|-------------------|
| Nakamura et al. (2018) | H30S30-C-0.2 | 118.3 | 61.4 | 51.0 | 5.8 | 68.9 | 112.4 | 1.05 |
| | | | | | | | | |
| | H30S30-C-0.4 | 193.2 | 61.4 | 68.0 | 4.2 | 71.5 | 129.4 | 1.09 |
| | H30S30-C-0.6 | 240.4 | 61.4 | 84.0 | 5.5 | 89.4 | 154.9 | 1.08 |

Table 8 Shear capacity comparison.

| $V_{u,ana}/V_{u,cal}$ and $V_{u,ana}/V_{u,cal}$ Modified Truss Theory Proposed method |
|---------------------------------|--------------------------------------------------|------------------|-------------------|
| Average | Standard deviation | Data number |
| 1.28 | 0.19 | 86 |
| 1.05 | 0.13 | |

Notes: $V_{u,ana}$: Analytical shear capacity, $V_{u,cal}$: Calculated shear capacity carried by other mechanisms (MTT), $V_{u,cal}$ beam: Calculated shear capacity carried by stirrups in the beam action (proposed), $V_{u,cal}$ arch: Calculated shear capacity carried by the arch action (proposed), $V_{u,ana}$/Average: Calculated shear capacity.

* Values were estimated from graphs.
The contribution of the arch action at the peak tends to increase with higher stirrup ratio and smaller shear span ratio.

Within the considered range, the shear capacity carried by stirrups in the beam action is proposed to be calculated using the truss theory.

The shear capacity carried by concrete in the beam action is proposed to be calculated by applying a decrement factor, which is decided from the amount of stirrups, to the diagonal crack initiation force.

The shear capacity carried by the arch action is proposed to be calculated by considering the compressive force at the loading and supporting points. The force at the loading point is calculated from the compressive strength of concrete. The force at the supporting point is calculated through iteration process, which indirectly takes the compatibility condition into account.

The proposed method showed better accuracy to estimate the shear capacity of RC beams with stirrups.

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