Semi-Analytical Method for Accurate Calculation of Well Injectivity During Hot Water Injection for Heavy Oil Recovery

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Abstract. Representation of wells in numerical simulation of petroleum reservoirs is a challenging task due to large difference in typical scales of grid blocks (tens to hundreds meters) and wells (~0.1 m), with high pressure and saturation gradients around wells. Although a variety of grid refinement techniques can be used for local simulations, they have limited application in field-scale problems due to huge model dimensions. Thus, auxiliary quasi-stationary local solutions (so-called inflow performance relations) are used to relate well flow rate with well and grid block pressures. These auxiliary solutions are strictly derived for linear cases and generalized to non-linear problems by using grid-block averaged values of fluid and reservoir properties. In the case of hot water injection for heavy oil recovery, this results in significant errors in well injectivity calculations due to large temperature and saturation gradients dynamically influencing viscosity and relative permeability distributions around the well. In this paper we propose a method which combines a semi-analytical solution of the hyperbolic Entov-Zazovsky problem for non-isothermal oil displacement with integration for pressure distribution taking into account nonlinear dependencies of fluid viscosities and relative permeabilities on temperature and saturations. Both constant injection rate and constant well pressure cases are considered. Example calculations demonstrate that the method helps to avoid underestimation of well injectivity in non-isothermal problems caused by grid-block averaging of fluid and reservoir properties in conventional inflow performance relations.

1. Introduction
The world's reserves of heavy (high-viscosity) oils are almost twice as large as those of light oils. Significant concentrations of asphaltenes, heavy metals, sulfur, as well as the presence of salts in oil, lead to increased viscosity and greatly complicate oil recovery, production and transportation processes. Thermal methods of enhanced oil recovery (thermal EOR) are among the most suitable and widespread for heavy oils.

In this paper we consider a typical case when hot water is injected through wells into the reservoir characterized by low temperature and high oil viscosity. High specific heat capacity of hot water makes it possible to exert an intense thermal effect on the formation. With an increase in temperature, the viscosity of oil is rapidly decreased. As a result, oil mobility is increased, as well as the flow rates of production wells. However, calculation of such processes requires considering many factors related
to heat propagation in the reservoir, heat exchange between the fluids and the rock, and heat loss. Numerical simulations of non-isothermal multiphase fluid flows in natural petroleum reservoirs (porous media) are used for this purpose.

For reservoir simulation models, large difference in scales of grid blocks (tens to hundreds meters) and wells (~0.1 m) is typical. At the same time, the vicinity of wells is characterized by high pressure, saturation and temperature gradients. Thus, representation of wells in numerical simulation of petroleum reservoirs is a challenging problem. For local simulations, a variety of grid refinement techniques can be used. However, they are not widely applied in field-scale problems due to huge model dimensions. Instead, auxiliary quasi-stationary local solutions – the so-called inflow performance relations – are used to relate well flow rate with well and grid block pressures [1-3]. These auxiliary solutions are strictly derived for linear cases. For non-linear problems, they're usually generalized by using grid-block averaged values of fluid and reservoir properties. However, more accurate semianalytical procedures were presented for some special cases [4].

In the case of thermal EOR, the use of grid-block averaged values of temperature and saturations mostly affects injectivity (specific fluid injection rate) calculations for injection wells. This is due to large and dynamically changing temperature and saturation gradients influencing viscosity and relative permeability distributions around the well. Thus, calculation of changing injectivity during hot water injection is one of the difficulties in thermal EOR simulations. Significant errors in injectivity calculations result in large deviations of temperature, saturation and oil viscosity distributions, and hence highly affect overall flow dynamics. Usually this problem is solved by refining the mesh near the well, and it was shown that thermal simulation results are very sensitive to grid block sizes [5,6]. But with the required degree of refinement, computational costs for large-scale simulations become unacceptably high, so only local simulations can provide accurate results.

In this paper, we present a semianalytical method for injectivity calculations. It takes into account nonlinear dependencies of fluid viscosities and relative permeabilities on temperature and saturations, as well as heat losses to over- and underburden rocks. The method is based on a solution of a hyperbolic problem for propagation of the heat and saturation fronts with additional integration of the pressure gradient distribution. It can be further adopted in large-scale numerical simulations to lower the demands for grid refinement in calculations of hot water injection for thermal EOR.

2. The model

Although a variety of models with analytical solutions are known for calculation of temperature distributions during hot water injection, the majority of them neglects some process specifics essential for the purpose of this study: either multiphase nature of the displacement process, or dependencies of viscosities and relative permeabilities on temperature, or heat losses [7-9]. In the proposed method we adopt the Entov-Zazovsky model for one-dimensional non-isothermal displacement of oil by water based on the following assumptions [10]:

- Incompressible fluids and formation;
- Equal temperatures of water, oil and rock at specified position and moment of time;
- Thermal expansion of phases and formation can be neglected;
- Heat exchange between phases is due to convective transfer;
- Thermal conductivity of the reservoir along the direction of displacement can be neglected;
- Heat losses (to over- and underburden rocks) are calculated according to the Newton’s law.

The model combines mass balance (continuity) equations for water and oil and the heat balance (energy) equation, and generalized Darcy's law is assumed for oil and water phases. After transformations, it is reduced to the following set of water mass balance and heat balance equations in dimensionless variables [10]:

\[
\begin{align*}
\frac{\partial s}{\partial \tau} + F_\tau \frac{\partial s}{\partial X} + F_T \frac{\partial T}{\partial X} &= 0 \\
(s + h) \frac{\partial T}{\partial \tau} + (F + h) \frac{\partial T}{\partial X} + \alpha T &= 0
\end{align*}
\]
\[
\tau = \frac{U t}{L m}; \quad X = \frac{x}{L} ; \quad T = \frac{\tilde{T} - T_0}{T^0 - T_0}; \quad F = F(s, T)
\]
\[
\alpha = \frac{a_0 L}{U(c_w - c_o)}; \quad b = \frac{c_o + c_r}{c_w - c_o}; \quad h = \frac{c_o}{c_w - c_o}
\]
where \(t\) is the time, \(\tau\) is the dimensionless time, \(x\) is the coordinate, \(X\) is the dimensionless coordinate, \(s\) is the water saturation, \(\bar{T}\) is the temperature, \(T\) is the dimensionless temperature, \(L\) is the length of the reservoir section, \(m\) is the effective porosity; \(c_w, c_o, c_r\) are the volumetric heat capacities of water, oil and rock; \(U = u_w + u\) is the total (oil and water) Darcy velocity (flux per unit area of reservoir cross-section), \(u_w\) and \(u_o\) are the Darcy velocities of water and oil; 
\[
F(s, T) = \frac{k_w(s, T)}{\mu_w(T)} \left( \frac{k_o(s, T)}{\mu_o(T)} \right) \quad \text{is the water fractional flow function, such that} \quad u_w = UF; \quad u_o = U(1 - F), \quad k_w \text{ and } k_o \text{ are water and oil relative permeabilities, } \mu_w \text{ and } \mu_o \text{ are water and oil viscosities;}
\]
\(T_0\) is the initial reservoir temperature, \(T^0\) is the temperature of injected water; \(a_0\) is the heat loss coefficient per unit reservoir length.

To obtain the solution for saturation and temperature distributions, it is necessary to solve the set of quasilinear partial differential equations (1) with the following initial and boundary conditions:

\[
s = s_0, T = 0 |_{t=0; 0 \leq X \leq 1},
\]
\[
s = 1, T = 1 |_{t>0; X=0},
\]
where \(s_0\) is the initial water saturation.

The system (1) is hyperbolic and the problem is Riemann-type. Since the function \(F(s, T)\) is non-monotonic, discontinuities (jumps/fronsts) and continuous parts of the solution are identified. The characteristic equation is:

\[
\begin{vmatrix}
F_i' - \xi_1 & F_T' \\
0 & \frac{F + h}{s + b} - \xi_2
\end{vmatrix} = 0
\]

and characteristics are given by:

\[
\frac{dX}{d\tau} = \xi_1 = F_s'
\]

\[
\frac{dX}{d\tau} = \xi_2 = \frac{F + h}{s + b}
\]

Relations that hold on the characteristics are found by composing the extended matrix:

\[
\begin{pmatrix}
F_i' - \xi_1 & F_T' \\
0 & \frac{F + h}{s + b} - \xi_2
\end{pmatrix} = \left( \begin{array}{cc}
\frac{-\partial s}{\partial \tau} & -\frac{\partial T}{\partial \tau} + \frac{a T}{s + b}
\end{array} \right)
\]

and then

\[
\frac{ds}{d\tau} = \frac{F_i'}{(\xi_2 - \xi_1)} \left( \frac{dT}{d\tau} + \frac{a T}{s + b} \right)
\]

\[
\frac{dT}{d\tau} = -\frac{a T}{s + b}
\]

Condition (6) is held on the characteristics of the first set (s-characteristics) given by (3), and condition (7) – of the second set (T-characteristics) given by (7).
Heat loss to the surrounding rocks (last term in equation (1)) leads to continuously decreasing temperature in the heated zone and at the heat front. The trajectory of the heat front \( X_0 = X_0(\tau) \), as well as the temperature \( T^- (\tau) \) and saturations \( s^\pm (\tau) \) at the heat front, are found as follows:

\[
\frac{dx_0}{d\tau} = V_0(\tau) = \xi_1(s^-, T^-) = \xi_2(s^+, 0)
\]

\[
\frac{dT^-}{d\tau} = - \frac{aT^-}{s^+ + b}
\]

Treating \( T^- \) as an independent variable and integrating, one obtains:

\[
X_0(T^-) = \int_{T^-}^{T_0} F(s^-(\tau), T^+ + b) \frac{d\tau}{aT^-}, \quad \tau(T^-) = \int_{T^-}^{T_0} s^-(\tau) + b \frac{d\tau}{aT^-}
\]

The superscript \( ' - ' \) corresponds to the values behind the temperature jump, and \( ' + ' \) – to the values in front of the jump. It is shown that \( T^+ = 0 \) [10].

Having found the front trajectory and temperature from (9), one can compute saturation and temperature distributions behind the heat front by numerical integration of (6)-(7) along the characteristics. In front of the temperature jump (the heat front) the problem is reduced to the well-known Buckley-Leverett isothermal displacement [11], with the saturation front and continuous saturation distribution behind it found from (3) for \( T = 0 \).

For an injection well, change to the radial formulation is given by

\[
X = \frac{r^2}{R^2}, \quad \tau = \frac{V(t)}{\pi R^2 mH},
\]

where \( r \) is the radial coordinate, \( R \) is the radius of specified external contour (well drainage area), \( V(t) = \int_0^t Q(t) dt = \int_0^t 2\pi R U(t) dt \) is the cumulated volume of injected water, \( Q(t) = 2\pi H \cdot U(t) \) is the injection rate, and \( H \) is the reservoir thickness.

As an example case, we use the following data: reservoir thickness \( H = 4 \) m; volumetric heat capacity of oil \( c_o = 2211 \cdot 10^3 \frac{J}{m^3 \cdot \circ C} \), of hot water \( c_w = 4274 \cdot 10^3 \frac{J}{m^3 \cdot \circ C} \), of rock \( c_r = 2400 \cdot 10^3 \frac{J}{m^3 \cdot \circ C} \); reservoir temperature \( T_0 = T_r = 10 \circ C \); hot water temperature \( T^0 = T_w = 130 \circ C \); well radius \( r_w = 0.1 \) m; effective porosity \( m = 0.2 \); external contour radius \( R = 250 \) m; reservoir effective permeability \( k = 2.2 \mu m^2 \); hot water injection rate \( Q = 50 \frac{m^3}{day} \); oil viscosity \( \mu_o(\bar{T}) = e^{0.0004117^2 - 0.108 \cdot 7^2 + 10.167} \) mPa-s, water viscosity \( \mu_w(\bar{T}) = A \cdot 10^7 \cdot e^{247.8 \circ C} \) mPa-s, where \( A = 3.15 \cdot 10^{-2} \), \( C = -148.15 \circ C \); relative permeabilities for water \( k_w(s) = c_w \frac{(s - s_{swc})^2}{(1 - s_{swc} - s_{swc})^2} \) for oil \( k_o = c_o \frac{(1 - s - s_{sw})^2}{(1 - s_{swc} - s_{swc})^2} \), \( c_w = 0.0007 \cdot \bar{T} + 0.1823 \), \( c_o = 1 \), residual oil saturation \( s_{or} = 0.75 \cdot e^{-2.554 \cdot 10^{-3} (\bar{T} - T_r)} \), critical water saturation \( s_{wc} = 0.0013 \cdot \bar{T} + 0.0923 \), initial water saturation \( s_0 = s_{wc} \); \( \alpha = 5 \).

Oil and water relative permeabilities are visualized in Figure 1 for \( T = 1 \) (\( \bar{T} = T^0 \)). Water fractional flow as function of saturation for \( T = 0 \) (\( \bar{T} = T_0 \)) and \( T = 1 \) (\( \bar{T} = T^0 \)) is presented in Figure 2.

Figures 3 and 4 show the solution for water saturation and temperature as functions of radial coordinate at \( t = 56 \) days. Due to the high ratio of oil viscosity to water viscosity at initial reservoir temperature, the saturation front moves much faster than the heat front, but its contribution to oil displacement (the height of the saturation jump) is small. The main displacement takes place at the heat front, where the temperature jump is accompanied by the second larger jump in saturation.
Figure 1. Relative permeabilities for $T = 1$: water $k_w$ (red) and oil $k_o$ (blue)

Figure 2. Water fractional flow for $T = 0$ (blue) and $T = 1$ (red)
Figure 3. Solution of the Entov-Zazovsky problem. Water saturation $s(r)$ at $t = 56$ days

Figure 4. Solution of the Entov-Zazovsky problem. Temperature $\tilde{T}(r)$ at $t = 56$ days

3. The injectivity
The injectivity is a characteristic of an injection well showing its possibility of injecting a displacement agent, in this case, the hot water, into the reservoir. It is determined as the injection rate per unit repression:

$$I(t) = \frac{Q(t)}{\Delta p(t)},$$

where the repression $\Delta p$ is the difference between the well bottomhole pressure (at $r = r_w$) and the reservoir pressure (at $r = R$).

Well injectivity primarily depends on the penetrated interval (height) of the reservoir, its permeability, relative permeabilities of the injected and reservoir fluid, and their viscosities. For a vertical well that completely penetrates the reservoir, the quasi-stationary inflow performance formula for injection flow rate is:
\[ Q = \frac{2\pi k}{B \ln(\frac{R}{r_w})} \frac{k_w(T_0, \bar{s})}{\mu_w(T_0)} \frac{k_o(T_0, \bar{s})}{\mu_o(T_0)} H \Delta p, \] (13)

where \( B \) is the water formation volume factor (\( B = 1 \) in the example case considered), \( \bar{T} \) and \( \bar{s} \) are the temperature and water saturation averaged over the considered well drainage area (from \( r_w \) to \( R \)). Skin effect can be taken into account by adding the skin-factor \( S \) to the denominator of (13) [2].

Formula (13) does not consider temperature and saturation changes along the radial coordinate, and hence the corresponding nonlinear changes in the dependent properties – relative permeabilities and viscosities. As a result, significant underestimation of the injection rate is typical for large-scale numerical simulations of thermal EOR, where formula (13) is used within the grid block penetrated by the injection well with \( \Delta p \) being the difference between the well bottomhole pressure and the grid block pressure, \( H \) being the grid block thickness and \( R \) being the so-called equivalent radius [2, 3].

A more accurate calculation of the injectivity can be made using the solution of the Entov-Zazovsky problem. The injection rate is related to the oil and water flow rates expressed through the generalized Darcy’s law, considering the fluids incompressible and neglecting capillary pressure and gravity:

\[ Q = Q_w + Q_o \neq f(r), \]
\[ Q_w = -2\pi r H u_w, \]
\[ Q_o = -2\pi r H u_o, \]
\[ u_w = -k \frac{k_w}{\mu_w} \frac{\partial p}{\partial r}, \]
\[ u_o = -k \frac{k_o}{\mu_o} \frac{\partial p}{\partial r}, \]
\[ Q = 2\pi r H \left( \frac{k_w}{\mu_w} + \frac{k_o}{\mu_o} \right) \frac{\partial p}{\partial r}, \] (14)

where \( p = p(r, t) \) is the pressure distribution in the drainage area. Expressing the pressure gradient

\[ \frac{\partial p}{\partial r} = \frac{Q}{2\pi r H \left( \frac{k_w}{\mu_w} + \frac{k_o}{\mu_o} \right)}, \] (15)

one can use the solutions for temperature \( T(r, t) \) and saturation \( s(r, t) \) based on the Entov-Zazovsky model, as well as the expressions for relative permeabilities and viscosities, to calculate the repression by numerical integration:

\[ \Delta p(t) = \int_{r_w}^{R} \frac{\partial p}{\partial r} \, dr. \] (16)

In this study, the trapezoid rule was used to compute the integral (16).

The expression for dimensional time is obtained from (11), considering \( V(t) = \int_0^t Q(t) \, dt \):

\[ \tau = \int_0^t \frac{\eta}{2\pi R^2 m H} \, dt, \] (17)

In the case of constant injection rate, absolute time is linearly related to the dimensionless time:

\[ t = \frac{\tau \pi R^2 m H}{Q}. \] (18)

Figure 5 shows the dynamics of depression for the example case with \( Q = 50 \text{ m}^3/\text{day} = \text{const.} \)
Figure 5. Dynamics of the repression during constant-rate hot water injection calculated by the presented algorithm (blue graph) or by the quasi-stationary formula (13) (red graph).

The blue graph was calculated using the presented algorithm. Temperature and saturation distributions corresponding to the solution of the Entov-Zazovsky problem were used to compute distributions of relative permeabilities and viscosities. Numerical integration of (15)-(16) was used to compute the repression.

The red graph was computed with the quasi-stationary inflow performance formula (13), using the temperature and saturation value averaged over the drainage region. As expected, averaging of the temperature and saturation distributions strongly affects the result. Repression is highly overestimated, and hence the injectivity is underestimated. This would lead to significant errors in calculation of oil recovery dynamics by hot water injection.

Now let’s consider the case of constant injection pressure. If the well bottomhole pressure and the pressure at the external contour are fixed, the injection flow rate becomes a function of time and has to be calculated. Integrating (15) by radial coordinate and taking into account that $Q$ does not depend on $r$, one obtains:

$$Q(t) = \frac{2\pi h k \Delta p}{\int_{\tau_w}^{R} \left( \frac{k_w(s(r),T(r))}{\mu_w(T(r))}, \frac{k_o(s(r),T(r))}{\mu_o(T(r))} \right) dr},$$

where the integral in the denominator is calculated numerically using the temperature and saturation distributions at current time $t$.

For the dimensionless time, the following expressions are valid:

$$\tau_i = \frac{V(t_i)}{\pi R^2 m H} = \frac{\int_{t_{i-1}}^{t_i} Q(t) dt}{\pi R^2 m H} = \frac{\sum_{j=1}^{i-1} Q_j \Delta t_j + Q_i \Delta t_i}{\pi R^2 m H},$$

$$t_i = t_{i-1} + \frac{\tau_i \pi R^2 m H - \sum_{j=1}^{i-1} Q_j \Delta t_j}{Q_i},$$

where $Q_j$ is the injection rate for the time interval from $t_{j-1}$ to $t_j = t_{j-1} + \Delta t_j$, and $t_0 = 0$.

Figure 6 shows the dynamics of injection rate for the example case with $\Delta p = 2$ MPa = const. As in the Figure 5, the blue graph was calculated using the presented algorithm, and the red graph corresponds to the quasi-stationary inflow performance formula (13) with averaged temperature and saturation values. Again, the injectivity is significantly underestimated when averaging the temperature and saturation distributions.
Figure 6. Dynamics of the injection rate during constant-pressure hot water injection calculated by the presented algorithm (blue graph) or by the quasi-stationary formula (13) (red graph).

The continuously increasing graph of the injection rate in Figure 6 is explained by increasing total mobility of the fluids in the drainage region with heat front propagation. This case is only illustrative and not very practical, since for a real well the repress would be decreased to meet the technologically specified limits on the injection rate.

Therefore, both in the case of constant injection rate and constant bottomhole pressure, the calculation using the quasi-stationary inflow performance formula significantly underestimates the injectivity of the well and leads to errors in the forecast of oil displacement by hot water injection.

Figures 7-10 also demonstrate the effect of smaller values of $R$: $R = 100$ m for Figures 7-8 and $R = 50$ m for Figures 9-10. Also, the value of $\Delta p$ was changed to 0.5 MPa for Figures 8 and 10. Other data in the calculations were the same as for Figures 5-6.

In the constant-pressure case (Figures 8 and 10), smaller value of $R$ results in proportionally larger values of $Q$, which is explained by increased pressure gradients.

In the constant-rate case (Figures 7 and 9), the initial value of $\Delta p$ is almost independent of $R$. This is due to the vanishing effect of distant points with low pressure gradients and very low fluid mobility on the integral (16). However, smaller values of $R$ result in faster ($\sim R^2$) decrease in $\Delta p$, as dictated by (18). Physically this is explained by relatively faster propagation of the heat front towards the external contour, with the corresponding increase in oil mobility.

Smaller values of $R$ also result in faster decrease of the difference between the blue and red graphs. This means that the period of largest errors in injectivity calculations with the quasi-stationary inflow performance formula is shortened by grid refinement almost proportionally to the squared refinement factor.
Figure 7. Dynamics of the repression during constant-rate hot water injection calculated by the presented algorithm (blue graph) or by the quasi-stationary formula (13) (red graph) for $R = 100$ m.

Figure 8. Dynamics of the injection rate during constant-pressure hot water injection calculated by the presented algorithm (blue graph) or by the quasi-stationary formula (13) (red graph) for $R = 100$ m and $\Delta p = 0.5$ MPa.
Figure 9. Dynamics of the repression during constant-rate hot water injection calculated by the presented algorithm (blue graph) or by the quasi-stationary formula (13) (red graph) for $R = 50$ m

Figure 10. Dynamics of the injection rate during constant-pressure hot water injection calculated by the presented algorithm (blue graph) or by the quasi-stationary formula (13) (red graph) for $R = 50$ m and $\Delta p = 0.5$ MPa

4. Conclusions
In this paper a semi-analytical method was presented for calculation of vertical well injectivity for hot water injection into a heavy oil reservoir. The method is based on the solution of a hyperbolic Entov-Zazovsky problem for temperature and saturation distributions, and integration of the pressure gradient taking into account nonlinear dependencies of fluid viscosities and relative permeabilities on temperature and saturation.

The cases of constant injection rate and constant injection pressure were examined. For both it was shown that the method helps to overcome the significant underestimation of well injectivity caused by the use of quasi-stationary inflow performance formula with averaged values of temperature and saturation. Grid refinement can shorten the period of largest errors in injectivity calculations with the quasi-stationary formula almost proportionally to the squared refinement factor.

The proposed method can be useful for large-scale numerical simulations of the thermal EOR to reduce requirements for grid refinement.
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