An updating method for structural dynamics models with unknown excitations

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Abstract.
This paper presents an extension of the Constitutive Relation Error (CRE) updating method to complex industrial structures, such as space launchers, for which tests carried out in the functional context can provide significant amounts of information. Indeed, since several sources of excitation are involved simultaneously, a flight test can be viewed as a multiple test. However, there is a serious difficulty in that these sources of excitation are partially unknown. The CRE updating method enables one to obtain an estimate of these excitations.

We present a first application of the method using a very simple finite element model of the Ariane V launcher along with measurements performed at the end of an atmospheric flight.

1. Introduction

Modeling tools have been a major accomplishment of the last 25 years considering how radically they transformed, and continue to transform, industry and research. This is a true revolution, which is far from complete. Of course, mastering the models (by which we mean controlling the calculation process itself as well as validating the results against a set of experimental data) is an essential requirement in using these tools. Until now, model validation and verification were carried out mainly in a deterministic sense. Even for deterministic models, there are a number of open issues which still require research, but one must recognize that most of the scientific obstacles have been eliminated.

The method detailed in the following pages is a general method which can be applied to any type of structural dynamics model. In this paper, the method will be implemented for the Ariane V aerospace launcher. In fact, registration or model updating is one of the main problems regarding margin diminution, especially in the case of structures such as launchers. The models are more and more effective, but in spite of all improvements some degree of ignorance remains, especially concerning damping and joints. The excitation also is poorly understood: its level, its
time of occurrence and its duration are still unknown. Thus, flight tests equipped with proper 
instrumentation can provide a broad source of information for registration or model updating. 
Flight tests are very complex in the sense that the excitations comes from different sources. The 
main problem is, of course, that these excitations are unknown.

Concerning registration or model updating, one can choose between two strategies:
in the first approach, very few assumptions are made concerning the model and the 
excitations; the information is provided directly by the measurements. The advantage of this 
strategy is the ease with which it can be implemented and adapted to a wide range of problems. 
With such a method, every constitutive parameter is considered to be a candidate for updating. 
The second approach (which is the one we will follow in this paper) uses the information and 
mode of behavior previously established for the model. Only part of the model and its excitation 
are considered as unknowns, the rest being assumed to be reliably known. This means that part 
of the model is used as a reference. Therefore, a first step consists in separating the equations 
which are considered reliable from those which are considered unreliable (i.e. the constitutive 
relations of the joints about to be updated). This strategy was first introduced at LMT-Cachan 
[2, 5], and has been the subject of extensive development, especially regarding the updating of 
probabilistic models [9]. Its main drawback is its numerical cost, but it enables one to gather 
information which would be inaccessible by any other method.

This paper presents an extension of this approach to complex industrial structures, such as 
space launchers, for which flight tests can provide significant amounts of information. Indeed, 
a flight test can be viewed as a multiple test during which several sources of excitations are 
involved simultaneously. However, there is a serious difficulty in that these sources of excitation 
are partially unknown.

We present a first application of the method using a very simple finite element model of 
the Ariane V launcher along with measurements performed at the end of an atmospheric flight. 
First, we show how the proposed method leads to an estimation of initially unknown excitations. 
Then, we update the finite element model by optimizing the stiffnesses of two joints.

2. Hypothesis

An important assumption which we will make throughout this study is that the behavior of 
the structure is vibratory and slowly time-dependent:

\[ X(t) = A(t) \sin(\omega t + \phi_t) \]  

(1)

The modulus \( A(t) \), frequency \( \omega \) and angle \( \phi_t \) vary slowly with \( t \). However, at each time \( t \) several 
modes may occur, each defined by \( (A, \omega, \phi) \).

The model’s unknown quantities are the stiffnesses; first, the system will be considered 
undamped.

3. The reference problem

3.1. Description of the problem
3.1.1. Substructuring

The complete structure is divided into several substructures denoted \( E \in E \), where \( E \) is the set of the substructures. A joint connecting two substructure \( E \) and \( E' \) is denoted \( L_{EE'} \). The excitation itself is unknown, but the substructure to which it is applied is known and denoted \( E \).

3.1.2. Equations of the problem

Let \( X_E \) denote the vector of the DOFs of Substructure \( E \). \( X_L \) is the restriction of \( X_E \) related to the joint \( L \) connecting \( E \) to another substructure. The dynamic equations of any substructure \( E \in E \) can be written as follows:

\[
(K_E - \omega^2 M_E)X_E = \sum_{L \in L, E' \in E} F_{E'E}^L + F_{E'}^{\bar{E}} \delta_{E'}(2)
\]

with \( \delta_E = 1 \) if \( E = E' \) and \( \delta_E = 0 \) else.

\( M_E \) and \( K_E \) denote the stiffness and mass matrices associated with Substructure \( E \). \( F_{E'E}^{L} \) is the load applied by Substructure \( E' \) onto Substructure \( E \) through Joint \( L_{E'E} \in L \).

All joints \( L \in L \) are assumed to be elastic and without mass. The behavior of a joint can be expressed as:

\[
F_{E'E} = k_{E'E}(X_L^E - X_L^{E'})(3)
\]

where \( k_{E'E} = k_{EE'}^{L} \) is the stiffness of the joint connecting \( E \) and \( E' \). Finally, let us remember that the excitation, denoted \( F_{E'}^{\bar{E}} \), is unknown.

The problem defined by (2) and (3) has only one solution, unless the frequency \( \omega \) is an eigenfrequency of the structure and \( F_{E'}^{\bar{E}} = 0 \). If \( \omega \) is an eigenfrequency, the solution is the \( \omega \)-eigenmode with \( F_{E'} = 0 \). (If we had considered a damped model, even with a small damping value, a solution could have been obtained for all \( \omega \neq 0 \).)

3.2. Updating method and modified definition of the CRE

3.2.1. Separation of the quantities

For a flight test viewed as a multiple test, the excitation is unknown except for the substructure where it is applied. At each \( t \), one has access to:

\[
(\omega, \tilde{X}(\omega)) \quad \text{for} \quad \omega \in \Omega, \quad t \in [0, T]
\]

(4)

The most important thing to do at this stage is to separate the equations and data considered to be "reliable" from those considered to be "unreliable":

- the behavior of the joints;
- the measured values \( \tilde{X} \).

The remaining equations and quantities are assumed to be reliable. Again let us recall that the excitation is unknown.

3.2.2. Admissible fields

A solution \( s = \{X_E \text{ with } E \in E; F_{E'E}^L, F_{E'E}^{L'} \text{ with } L \in E; F_{E'}^{\bar{E}} \} \) is considered to be admissible if:

\[
(K_E - \omega^2 M_E)X_E = \sum_{L \in L, E' \in E} F_{E'E}^L + F_{E'}^{\bar{E}} \delta_E(5)
\]

\[
F_{E'E}^L + F_{E'E}^{L'} = 0 \quad \forall E \in E(6)
\]

Let \( S_{ad} \) denote the space of the admissible solutions.
3.2.3. The modified Constitutive Relation Error (modified CRE)

The modified CRE for the frequency $\omega$ and for the parameters in $S_{ad}$ is defined as:

$$e^2(s) = \frac{1}{2} \sum_{L \in E, E' \in E} [F_{E'E}^L - k_{E'E}^L (X_{E'}^L - X_E^L)]^T (k_{E'E}^L)^{-1} [F_{E'E}^L - k_{E'E}^L (X_{E'}^L - X_E^L)] + \frac{r}{1 - r k} \sum \frac{1}{(X - \tilde{X}(\omega)), K(X - \tilde{X}(\omega))}$$

where:

- the scalar quantity $r \in [0, 1]$ represents one’s degree of confidence which in the measurements contained in $\tilde{X}(\omega)$;
- the scalar quantity $k$ is the ratio of the elastic energy of the full structure to the energy of a joint calculated with $\tilde{X}(\omega)$: $k = \sum_{L \in E, E' \in E} (\tilde{X}_{E'}^L - \tilde{X}_E^L) \frac{k_{E'E}^L (\tilde{X}_{E'}^L - \tilde{X}_E^L)}{	ilde{K}^E - \tilde{K}_E^E}$
- $K$ is the stiffness matrix calculated with the initial material parameters;
- $\tilde{X}$ is the vector of the model’s DOFs;
- $\tilde{X}(\omega)$ is the vector of the measured quantities at frequency $\omega$.

The modified CRE at frequency $\omega$ and time $t$ is equal to the minimum of $e^2(s)$ with $s \in S_{ad}$

The calculation of $s \in S_{ad}$ which minimizes $e^2(s)$ for a given set of material parameters is described in the following section. The modified CRE at time $t$ is expressed as follows, then integrated with time:

$$\bar{e}^2 = \frac{1}{m_{est}(\Omega_t)} \int_{\Omega_t} e^2(\omega) d\omega$$

The global error can be viewed as the sum of a model error and a measurement error:

$$\bar{e}^2 = e^2_{CRE} + r \bar{e}^2_{exp}$$

The second term enables one to correct the experimental results. If this scalar term is significant and the error comes from only a few sensors, these sensors can be considered to be defective and excluded from the calculation of the modified CRE. A new search for the minimum of the error can then be performed.

The calculated value of $\bar{e}^2$ enables one to decide whether model updating (in this case, updating of the joints’ stiffnesses) is necessary. In any case, even if no updating is carried out, the search for the minimum of the error leads to an estimate of the unknown excitation $F_E$.

3.2.4. Search for $s \in S_{ad}$ which minimizes $e^2(s)$

Searching for $s \in S_{ad}$ which minimizes $e^2(s)$ consists in minimizing $e^2(s)$ under the following constraints:

- the dynamic equations of each substructure $E \in E$,

$$\begin{align*}
(K_E - \omega^2 M)X_E = & \sum_{L \in E, E' \in E} F_{E'E}^L + F_E^E \delta_{E,E'} \quad \forall E \in E
\end{align*}$$

- the action-reaction principle at each joint $L \in L$,

$$F_{E'E}^L + F_{E'E'}^L = 0 \quad \forall L \in L$$
4. Validation of the model

4.1. The updating process

Having corrected the measurements, i.e. having eliminated the defective sensors from the calculation, we need to detect whether the model needs to be updated. Let us denote $\epsilon_2$ the relative CRE, defined as follows:

$$
\epsilon_2^{CRE} = \frac{1}{T} \int_0^T \frac{\sum_{L \in \mathbf{L}, E, E' \in \mathbf{E}} \left\| F_{E'E}^L - k_{E'E'}^L (X_{E'}^L - X_E^L) \right\|_{CRE}^2}{\sum_{L \in \mathbf{L}, E, E' \in \mathbf{E}} \left\| k_{E'E'}^L (X_{E'}^L - X_E^L) \right\|_{CRE}^2} dt
$$

(10)

with

$$
\| \cdot \|^2_{CRE} = \frac{1}{\text{mes}(\Omega_t)} \int_{\Omega_t} \left[ (k_{E'E'}^L)^{-1} \right] d\omega
$$

(11)

So the model needs to be updated if:

$$
\epsilon_2^{CRE} \geq \epsilon_0^2
$$

(12)

where $\epsilon_0^2$ is a given threshold value.

This value depends on the model that is chosen to stand for the structure. For a complex and rich model able to represent the main phenomenons involved, the value can be low (typically 5%). For more easier and poorer models, having a low relative error has no sense; what is worse, the model may no be able to provide a low error. For such models, the threshold value of criterion (12) will be set to 10-20%.

The updating procedure is iterative and stops as soon as Criterion (12) becomes false.

4.2. Iteration of the procedure

During an iteration, the relative Constitutive Relation Errors (CREs) are calculated at each joint $L \in \mathbf{L}$:

$$
(\epsilon_2^{CRE}_L)^2 = \left\| F_{E'E}^L - k_{E'E'}^L (X_{E'}^L - X_E^L) \right\|_{CRE}^2 (k_{E'E'}^L)^{-1} \left\| F_{E'E}^L - k_{E'E'}^L (X_{E'}^L - X_E^L) \right\|_{CRE}^2
$$

(13)

and one write $\mathbf{L}_R$ for the subspace of the joints $L$ such that:

$$
(\epsilon_2^{CRE}_L)^2 \geq 0.8 \sup_{L \in \mathbf{L}} (\epsilon_2^{CRE}_L)^2
$$

(14)

which are the joints which need to be updated because they are poorly modeled.

4.3. Search for the error minimum

The global error $\epsilon_2^{CRE}$ is a function of the joints' stiffnesses. The updating procedure consists in seeking the joint stiffness parameters which minimize $\epsilon_2^{CRE}$.

This is an optimization problem in which the joints' stiffnesses $L \in \mathbf{L}_R$ are the optimization parameters. Once the minimum of the error has been reached for the $\mathbf{L}_R$ set, a new $\mathbf{L}_R$ set can be defined.

That minimum can be reached thanks to Gradient- or Newton-methods. But the stop criterion (12) concerns the relative error ; so that, if the Newton-like method provides a local minimum, the relative error will not verify $\epsilon_2^{CRE} \leq \epsilon_0^2$, and the updating procedure will not be stopped.
4.4. Special issues

The updating procedure may never end: the constitutive relation error may not decrease. This means that the model is too simplistic and unable to match the measurements. An in-depth modification of the model is necessary.

5. An application to an industrial structure

In this section, we illustrate the method described above using a simple 5-DOF model of a portion of the Ariane V launcher.

5.1. Updating a simplistic model of the Ariane V launcher

5.1.1. The FE model

The model of the structure we considered consists of three substructures:

- the Central Body, denoted CB,
- the Booster,
- the Nozzle

and two joints:

- the Damping Device DD, which is the joint (initially considered elastic) between CB and Booster,
- the Stop is the elastic joint between the nozzle and the booster.

The model of Figure 1 includes only the vertical movements of the structure, which is a highly restrictive assumption, but it is capable of representing the main phenomena.

The loads corresponding to oscillations of the thrust occur at Nodes 1, 2 and 3, and the associated unknown quantities are denoted $\mathbf{F}_{\text{Booster}} = \{F_1, F_2, F_3\}$.

5.1.2. Time- and frequency-dependent quantities

The variable quantities are:

\[ m_{CB} = \phi_{CB}(t)\bar{m}_{CB} \quad k_{DD} = \phi_{DD}(t)\psi_{DD}(\omega)\bar{k}_{DD} \]
\[ m_{\text{Booster}} = \phi_{\text{Booster}}(t)\bar{m}_{\text{Booster}} \quad k_{\text{Stop}} = \phi_{\text{Stop}}(t)\bar{k}_{\text{Stop}} \]

$\psi_{DD}$ and the three $\phi_*$ functions are known. The remaining material quantities were assumed to be constant.
5.1.3. Eigenmodes of the model

The 5-DOF model enabled the calculation of 5 structural eigenmodes, which are presented in Table 1 and depend on the stiffness and mass distributions.

| substructure | Mode 1 (0 Hz) | Mode 2 (3-5 Hz) | Mode 3 (23-29 Hz) | Mode 4 (33-39 Hz) | Mode 5 (47-65 Hz) |
|--------------|---------------|-----------------|-------------------|------------------|------------------|
| CB           | 0%            | 2.0%            | 98.0%             | 0.0%             | 0.0%             |
| Booster      | 0%            | 1.8%            | 0.1%              | 28.3%            | 69.9%            |
| DD           | 0%            | 96.2%           | 1.9%              | 1.0%             | 1.0%             |
| Stop         | 0%            | 0.0%            | 0.1%              | 70.7%            | 29.1%            |

Table 1. 5-DOFs eigenproblem - Energetical contribution of the components

The values inside the table correspond to a deformation-energy ratio for each component: \( \frac{(X_{L}^{E} - X_{L}^{E'})^{-1} (X_{L}^{E} - X_{L}^{E'})}{X^{K}X} \).

One should note that the eigenfrequencies are presented as frequency ranges. This is due to the fact that the stiffnesses and masses vary with time.

5.1.4. Calculation of the error

For the 5-DOF model including the correction parameters \( \alpha_{\text{Stop}} \) et \( \alpha_{\text{DD}} \), one has:

\[
\varepsilon_{\omega,t}^{2} = \frac{1}{2} \left( F_{32} - \alpha_{\text{Stop}} k_{\text{Stop}} (X_{3} - X_{2}) \right)^{2} + \frac{1}{2} \left( F_{23} - \alpha_{\text{Stop}} k_{\text{Stop}} (X_{2} - X_{3}) \right)^{2} + \frac{1}{2} \left( F_{41} - \alpha_{\text{DD}} k_{\text{DD}} (X_{4} - X_{1}) \right)^{2} + \frac{1}{2} \left( F_{14} - \alpha_{\text{DD}} k_{\text{DD}} (X_{1} - X_{4}) \right)^{2} + \frac{r}{1 - r} \frac{1}{\Pi_{11}} \Pi_{12} \Pi_{22} (X - \bar{X}) \tag{15}
\]

where \( \Pi_{11} = \Pi_{22} = 1 \) and \( \Pi_{ij} = 0 \) else.

Until then, the error is calculated at each time \( t \) and each frequency \( \omega \).

5.2. Processing of the measurements

We saw in the previous section that the error was calculated at each time and each frequency. Due to the assumption that the structural parameters vary slowly with time, a simple and efficient way to process the measurements consisted in carrying out time-dependent Fourier transforms of the signal for a duration of 0.6s every 5s. (See the spectrum in Figure 2).

However, the CRE updating method was not applied to all the frequencies, but only to those that are mechanically meaningful, i.e. the structural frequencies. Therefore, only the frequencies corresponding to high energy levels are kept for the updating procedure (Figure 3).

5.3. First evaluation of the CRE

The relative error \( \varepsilon_{\text{CRE}}^{2} \) calculated for the high-energy frequencies is shown in Figure 4. The simple model and the two parameters to update allows to draw a teaching plot of the error, function of \( \alpha_{\text{DD}} \) and \( \alpha_{\text{Stop}} \).

The important thing one should note at this point is that the error is weakly \( \alpha_{\text{DD}} \)-dependent, but strongly \( \alpha_{\text{Stop}} \)-dependent. This is due to the fact that the excitation (acoustical flow mode of
the gas in the Booster) lies mainly in a frequency range which corresponds to the third structural eigenmode. For this eigenmode, 1 the deformation-energy level associated to the stop is 70%, whereas the DD is only 1%-stressed. Therefore, one must conclude that the method offers low possibilities for the DD stiffness $k_{DD}$ to be updated; the stop, however, has good updating potential.

5.4. Estimation of the excitations

We can now draw, for each time step, the estimation of the excitation, for updating parameters so that $\epsilon^2(\alpha_{DD}, \alpha_{Stop}) \leq \epsilon_0^2$.

For example the Figure 6 shows the strength level for the first frequency $\approx 20\,Hz$ : The strength values that are drawn are related to the 10%-contour in Figure 5.
6. Conclusion

In these pages, we presented a validation method on a small model with time-dependent parameters in uncertain experimental environment. The results that are provided by the method depend from the model; if the model is too poor, it will not be able to represent all the phenomena involved, and the results will of course suffer from that lack of precision.

The validation algorithm we developed is applicable to complex models. It is dedicated mainly to complex structures which have been modeled finely when the validation of the model must rely on only limited experimental data. An important characteristic of this approach, which is definitely “mechanics-oriented”, is that it leads to a true error measure which enables one to evaluate the quality of the model while providing at the same time useful information on excitations which are initially unknown. In this context, future work will consist in adapting the method to larger and more compliant models of the Ariane V launchers, which will be likely to contain probabilistic information.

7. References

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