A hyperbolic Embedding Model for Directed Networks

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Abstract. Network embedding is a fervid topic in current networks science and observes that most real complex systems can be embedded in hidden metrics space and emerge as the geometrical property, where the geometric distance between nodes determines the likelihood of links connected. Among those, hyperbolic space associated with the structural organization of many real complex systems, it has thus received extensive attention. However, the majority of methods and measurements, recently developed, less take these features into consideration for the asymmetry of links. Here, we discuss how to multiplex node information as an embedding foundation through identifying the bipartite structure of directed networks; and we proposed the generally mapping framework which hybrids the topological structure of complex networks, directed links and the hidden metrics space. By splitting the different properties of a node, possibilities between different types of nodes can be modeled. In addition to that, we apply this model to some real systems, including international trade networks and C.elegans neural networks. Results confirm that directed networks enable mapping into metrics space as well, and network embedding information can improve the scope of application of existing models.

Keywords: Hyperbolic Space, Network Embedding, Geometry Metrics, Directed Networks, Bipartite Structures,

1. Introduction

Complex networks can largely simplify real systems and preserve the essential information of the interaction structure, and thus become an ideal tool for investigating complex systems. However, complex networks are the non-geometric properties model, which makes a large set of tools and methodologies developed in geometry cannot be applied to complex networks. In this context, there is a wave of exploring geometric properties problems in complex networks, aiming at mapping complex networks into latent or low-dimensional metric space, which is summarized as network embedding, including a number of issues: brain science[1], international trade[2 3], route transfer[4 5 6], and protein formation mechanism[7 8].
Advances in network geometry pointed out that structural properties observed in networks derived from real complex systems can emerge as the geometrical property in the hyperbolic space. Hyperbolic geometry is a branch of non-Euclidean geometry, and it has many applications in practical engineering techniques. More importantly, the random geometric model and growth hyperbolic model\cite{12, 11} (that is, network embedding hyperbolic space) proposed later can easily explain the heterogeneity of networks and high clustering, and even gives the clear meaning of each coordinate. The model not only can simulate the growth of networks, but also cover the dynamic process of the classic BA networks model in complex networks. As a consequence, related studies are becoming increasingly popular and important.

When this research framework was proposed, it attracted wide attention of scholars, and has developed some models\cite{12, 11, 13, 14, 15}. These models have a great performance in studying the potential structure and function of networks, but it also can’t describe real systems completely. One drawback of these models is that it ignores the direction of links. A relationship between nodes may be in inequality in most real networks, which is so-called the asymmetry property of links. Although asymmetry property may bring many challenges to detected link prediction in the latent space, ignoring directed networks space mapping will lose a lot of important information, and can not fully represent structure and function of real systems.

Another branch of studies is network representation learning \cite{16, 17}(NRL, encoding and representing each node in a unified low-dimensional space.), which is based on machine learning methods and matrix analysis. The HOPE algorithm \cite{18} combines four metrics, Katz Index, Rooted PageRank, Common Neighbors, and Adamic-Adar, to preserve asymmetric transfer properties in directed networks. LINE model \cite{19} preserves the local and global network structure by optimized the objective function. A link sampling algorithm is proposed to improve the efficiency and efficiency of the algorithm. A Popularity Scaled Latent Space Model\cite{20} assumes, for each node, a popularity parameter and different types of degree heterogeneity can be modeled by assuming different distributions for popularity parameters. NRL has provided many useful modeling inspirations for us, such as motif, random walk, potential theory, common neighbors, etc. Unfortunately, an obvious limitation is coordinates of each node are not given the actual meaning, compared to the meaning of the space coordinates with hyperbolic (popularity and similarity), after vector representation of nodes.

In order to further solve this problem, we should explore the intrinsic relationship between directed links and the network topology. Interestingly, in a non-trivial way, the asymmetry of links (directed links) is reflected on the topology and play an important role in the function and evolution of systems. It is worth paying attention that directed networks exist the hidden the bipartite structure\cite{21}. We checked that all kinds of directed networks have such structure and this phenomenon is the universal law of directed networks. The contribution of our work is that we offer a directed network embedding scheme based on node information multiplexing and identifying potential topology structure (bipartite structure) and a new idea for the dimensionality reduction
of directed networks data. For another thing, we, using the visualization technology, provide the new snapshot of complex networks in the hyperbolic space, which enables us to show the nodes status and macro-level structure and features.

In this passage, we propose a method for mapping nodes of a directed network to hyperbolic spaces through the bipartite structure of directed networks, and our methodology is grounded in the network topology information and its characteristics. From this perspective, we have introduced the concept of the bipartite structure of directed networks firstly. Based on it, the mapping model for nodes from directed networks to hyperbolic space is discussed in section 2. Finally, we use the data of international trade network and C.eiegans network to test the rationality of our mapping method.

2. Method

2.1. Interaction between links with direction and bipartite structure in real network.

Directed links, as the important linking features of network mode, are used to concern increasingly about the dynamic of real systems evolution and node status. Meanwhile, asymmetry property also increases the difficulty of network embedding. We, actually, have long neglected is that the asymmetry of links can be identified in the topological information of complex systems, that is, a bipartite structure[22]. Nodes are divided into two non-overlapping groups in bipartite networks, and the two sides of links are from different nodes groups. Inversely, directed networks only have one type of nodes. Remodeling modeling methods of directed networks, thus, is a critical way to overcome difficulties.

Understandably, nodes in directed networks can split into two parts to obtain a directed network with bipartite structure (DNBS) in fig.1. Namely, each node $v_i$ is composed of $x_i$ and $y_i$ for each directed network, belonging to different groups (group A1 and group A2). A directed link is from $v_i$ to $v_j$ ($v_i \rightarrow v_j$) can be mapped by
$x_i \rightarrow y_j$. By doing so, the number of nodes is twice as large especially, but it does not affect subsequent research, and a directed network $A$ will be converted to a bipartite network $B$ as followed:

$$B = \begin{bmatrix} O & A1 \\ A2 & O \end{bmatrix}$$

Different from the bipartite network, the two types of node of the directed network with bipartite structure are, actually, one-to-one correspondence, that is two different attributes belonging to the same node(top features and bottom features). For directed networks, for example, in the case of international trade networks, countries with stronger export capabilities (top features) are more likely to become trading partners with ones who are in the high level of import(bottom features). It can be seen that directed networks embedding method, compared with multi-layer networks embedded, mainly focuses on the display of links between layers, while the different layers represent the export capacity and import level of the country.

Since bipartite structure has been discovered the hidden geometric property in latent metrics space[14], we also expect origin-node and end-node are able to form directed links under the multi-scale node multiplexing perspective. The relation of directed and topology yields a powerful idea to spatial mapping and directed links prediction. To doing this, we develop a new method to present the asymmetry of the adjacency matrix of networks in hyperbolic space by the hybrid of direction and topology.

2.2. A geometric model of complex network in hyperbolic space.

The DHNE model is a directed network embedding model in order to describe how generative geometric directed graphs in the hyperbolic space. The model multiplexes node information as an embedding foundation through identifying the bipartite structure of directed networks and considers the trade-off between node popularity, abstracted by the radial coordinate, and similarity, represented by the angular coordinate distance, as the definition of connection possibilities. In the binary network embedding, the popularity is higher, and the popularity of this node is higher. So the probability that the new node will link to it will be greater.

Different from binary networks, the way in which directed networks are linked should be tradeoff in four types of force, namely that the similarity and popularity of out degree and in degree direction. A node with a higher in degree directed popularity and similarity should tend to connect with a node with higher out degree directed. By doing this, each nodes $i$ has spatial coordinates($r_{1i}, r_{2i}, \theta_{1i}, \theta_{2i}$) in hyperbolic space. Radial coordinates ($r_{1i}$ and $r_{2i}$) and angular coordinates ($\theta_{1i}$ and $\theta_{2i}$) represent the popularity and similarity of nodes, respectively. Radial coordinates $r_i$ is the function of intrinsic property of node, called hidden variables $\kappa_i$. Next, we will introduce the hyperbolic space embedding process of directed networks.
The model has some input parameters:

- $N > 0$: number of nodes in the directed network.
- $\gamma > 0$: the exponent of the power-law degree distribution, $p(k) \sim k^{-\gamma}$. Similarly, in DHNE model, $\gamma_{in}$ is an exponent the power-law in-degree distribution, $p(k) \sim k^{-\gamma_{in}}$; $\gamma_{out}$ is an exponent the power-law in-degree distribution, $p(k) \sim k^{-\gamma_{out}}$.
- $\beta \in [1, +\infty]$: $\beta$ is difficult to obtain by analytical differential equations, however, it is closely related to clustering coefficients. To this end, we use the common neighbor number distribution as the method of estimating $\beta$ in our model, and we obtain thus $\beta_{in}$ and $\beta_{out}$. More details information can be seen in the model implementations section.
- $\langle k \rangle$: average degrees $\langle k \rangle$, average out-degrees $\langle k_{out} \rangle$ and average in-degrees $\langle k_{in} \rangle$.
- $\zeta = \sqrt{-K}$, where $K$ is the curvature of the hyperbolic plane. Since changing $z$ rescales the node radial coordinates and this does not affect the topological properties of networks [12], we will consider $K = -1$ in our model.

Computational implementations of the DHNE model:

(1) Data preprocessing: Hyperbolic space is suitable for scale-free networks embedding, but not all real networks possess scale-free properties. It is worth noting that most networks are heterogeneous (namely, node degrees and weights), and these heterogeneities can be used to filter out the sparse subnetworks (called backbone) which represent the relevant structure remaining after the accidental interaction covered by
the information contained in the system is removed [15 27]. Of course, if data degree distribution of real networks satisfied Power-law relationship performance is better, the data preprocessing step can be ignored, such as C.elegance networks.

(2) The transformation of directed networks into bipartite structures: multiplexing principle based on nodes, the sequences of nodes degree \( \{k_i\} \) is split into two parts of the complementary association, including the degree set \( \{k_{in,i}\} \) and the degree set \( \{k_{out,i}\} \), FIG. 1.

(3) Embedding into hyperbolic space: Each node \( v_i \) is divided into two parts (\( x_i \) and \( y_i \)) and each part will be obtained a pair of spatial coordination (that is, \( x_i (r_{1i} \) and \( \theta_{1i} \)), \( y_i (r_{2i}, \theta_{2i}) \)), and the connected probability \( (p_{ij}) \) of directed link \( (v_{ij}) \) in the hyperbolic space represented by the Poincar disk is defined by

\[
f(d_{h,1i2j}) = (1 + e^{\beta/2(d_{h,1i2j}-R)})^{-1},
\]

which is the Fermi-Dirac distribution. The distance \( d_{h,ij} \) between two nodes with coordinates is given by the hyperbolic law of cosines:

\[
cosh d_{h,1i2j} = \cosh r_{1i} \cdot \cosh r_{2j} - \sinh r_{1i} \cdot \sinh r_{2j} \cos d_{a,1i2j},
\]

where the angular distance \( d_{a,1i2j} = \min(\pi - |\theta_{1i} - \theta_{2j}|) \), and notely that hyperbolic distance can be well approximated by \( r_{1i} + r_{2j} + 2d_{a,ij} \). \( R \) is the dis of hyperbolic space and the parameter of \( \beta \) control clustering coefficient of network and the long-distance connection in networks.

According to the equivalence of the network hyperbolic geometric model and the hidden metric space model, we determine the spatial coordinates of the nodes by latter. Specifically, node obtains a pair of hidden variable (that is, \( \kappa_{1i} \) and \( \kappa_{2i} \)) based on the power law distribution. The radial coordinates \( r \) and hidden variables \( \kappa \) of the node satisfy:

\[
r_{1i} = R - \ln(\kappa_{1i}/\kappa_{10}) \text{ through mathematical derivation rigorously, and } \kappa_{1i} \text{ is the hidden variable of nodes. } r_{2i} \text{ is as the same way as } r_{1i}.
\]

The probability of a link between nodes with effective distances of \( \chi \) is defined as \( p(\chi) = (1 + \chi^\beta)^{-1} \) (The calculation method of the parameter specific value will be explained in step4).

(4) Spatial positions and parameters estimation: we infer the popularity and similarity node coordinates in the real networks have been solved by the Maximum Likelihood Estimation method, the detail methods as shown in as is shown in ref [2 5 26]. This method is to find the spatial coordinates of each node such that the likelihood that the observed topology of directed networks is generated by the model described above is maximal. The likelihood followed:

\[
L = \prod_{ij} [p(x_{ij})]^{a_{ij}} [1 - p(x_{ij})]^{1-a_{ij}},
\]

where, if two nodes have a link from node \( v_i \) to node \( v_j \), \( a_{ij}=1 \), otherwhile \( a_{ij}=0 \). The equation (3) can be processed by logarithmic linearization:

\[
\ln L = \ln \{\kappa_*, \theta_*\} | a_{ij}, \gamma_*, \beta_*, K_*
\]

\[
C - \gamma \sum_{i=1}^{N} \ln k_* + \sum a_{ij} \ln p(x_{ij}) + \sum (1-a_{ij}) \ln[1-p(x_{ij})].
\]
Figure 3. Cumulative degree distribution of indegree and outdegree in Neural network and international trade network.

where, the constant $C$ is independent of $\kappa_*$ and $\theta_*$. It should be noted that in order to facilitate the writing of the formula, variables of the out degree direction and in degree direction are uniformly represented by variables with an asterisk. Derivating of the equation (7), we can obtain parameter $\kappa_*$ (the expected degree of node $i$), namely the expected degree $\kappa_*$ as followed:

$$\kappa_* = k_\ast - \frac{\gamma_*}{\beta_*},$$  \hfill (6)

Unfortunately, the angular coordinates of the nodes have no analytical solution, so we use the standard Metropolis-Hastings (SMH) algorithm to estimate the parameters. Another issue worthy of attention is the estimation of the parameter of model $\beta \in (1, +\infty)$. $\beta$ is intractable due to the nontrivial dependence of $m$ (the number of common neighbors) \[14\]. For the classical method, beta is obtained by generating random networks to fit best parameters\[2, 5\], but this method is difficult to randomly generate a topology similar to the directed network. The new method of estimating parameters urgently needs to be solved. The clustering coefficient can imply the geometry of the network actually\[14, 25, 26\], while the clustering coefficients of a bipartite network are defined by the common neighbor. In the directed network mapping model, we employ the power exponent of the common neighbor distribution to estimate the model parameters. More importantly, in the bipartite structure, there is a common neighbor distribution power exponent of two types of nodes. To highlight the key role of common neighbor, we further compare the mapping effects of two power exponents to determine the estimated value of beta.

(5) Unique the metric of distances and coordinates: Our model is more meticulously considers the intrinsic driving in the link generation process, namely that
Table 1. The mapping results of important properties. $\gamma_{in}$, $\gamma_{out}$, $\beta_{out}$ and $\beta_{in}$ represents the power index of the in-degree (out-degree) distribution and the common neighbor distribution, respectively.

| year     | $\gamma_{in}$ | $\gamma_{out}$ | AUC$_{out}$ | AUC$_{in}$ | $\beta_{out}$ | $\beta_{in}$ |
|----------|---------------|----------------|-------------|------------|---------------|---------------|
| Trade1996| 2.76          | 3.00           | 0.92        | 0.92       | 3.50          | 3.50          |
| Trade2001| 2.76          | 3.15           | 0.90        | 0.91       | 3.50          | 3.50          |
| Trade2006| 2.45          | 3.50           | 0.91        | 0.92       | 3.50          | 3.13          |
| Trade2011| 2.39          | 3.50           | 0.91        | 0.91       | 3.50          | 3.02          |
| Trade2016| 2.27          | 3.36           | 0.92        | 0.92       | 3.50          | 2.99          |
| C.elegans| 3.00          | 3.42           | 0.68        | 0.70       | 2.64          | 2.95          |

the node with high $r_{1i}$ status tends to generate the link with the node with high $r_{1i}$, rather than simply considering the gross popularity of nodes. However, it also comes with the new problem - the asymmetry of above-mentioned distance which does not comply with the three premise of distance definition and can not be called distance, strictly speaking. In addition, some real tasks such as nodes clustering analysis and the node-centrality computation require symmetric metrics. A classical symmetric distance measure defined by random walk, which is calculated by $d_{ij} = l_{ij} + l_{ji}$. We thus call $d_h = d_{h,1i2j} + d_{h,2i1j}$ the symmetric hyperbolic distance because it is a mixture of $d_{h,1i2j}$ and $d_{h,2i1j}$.

3. Results

In this section, we apply our model to map some real networks into hyperbolic space, including international trade and c. elegans neural network. And then, some evidence will be given to identifying the effectiveness of mapping hyperbolic networks by accuracy evaluation index (As the evaluation indicator of link prediction, AUC statistics measure how much our model can reproduce networks in hyperbolic space and results have been shown in Table 1, which demonstrates the topological properties of networks can reproduce the Direction of complex networks in hyperbolic space. AUC $> 0.90$ points out, especially social and economic network, that the asymmetry of the link matrix of the network can be well displayed in The hyperbolic space and the information of link direction is hidden in radial and the angular coordinates of nodes. According to the mapping method, we perform several experiment s on the embedding of the directed network, and the results are relatively stable.

As further validation of the quality of embedding, we compare several basic properties of simulated hyperbolic networks based on the hyperbolic mapping method of directed networks. We, firstly, perform the empirical results of the network degree distribution, which indicates real systems exist similar degrees distribution (in-degree distribution and out-degree distribution) between hyperbolic networks and complex networks. We also observe a remarkable match between the international trade network and directed hyperbolic network in Fig.3. Finally, a description and visualization of
mapped networks are displayed in Fig.2, and topology for clarity, we show the different attribution of nodes in a directed network using two hyperbolic discs. Core-periphery structure of the hyperbolic network, which shows only about 35% nodes in the center position of the hyperbolic space.

3.1. Complex ecosystems in hyperbolic space

*Caenorhabditis elegans* is a soil-dwelling nematode and evolutionarily rudimentary. It contains about 300 neurons and the neural interconnection via chemical synapses and gap junctions, which can be obtained from the wormatlas database [28]. This structure can be regarded as nodes and links of a complex network. Despite a century of investigation, knowledge of nematode neuronal networks is incomplete [29, 30]. Here, we use directed hyperbolic network embedded framework and the potential geometry of neurons as the entry point, trying to provide a new perspective for studying topology and visualization of neural network. Firstly, the C.elegans neural network is embedded into the hyperbolic space by the geometric features of the neurons and geometric distance (hyperbolic distance), and the topology is found to conform to the simple and powerful probability-based linking rule - hyperbolic embedding.

Secondly, we compare the embedded results with the real position of neurons and geometric distances. The position distance of neurons is the relative position between the neurons, and geometric distances are hyperbolic distance and angular distance. The results are shown in Figure 9. From the results, we can find that the angular distance is similar to position distance distribution. The difference between the hyperbolic distance and position distance, on the contrary, is larger. It not only shows that effective distance between neurons also contain distances of other dimensions rather than only position distance, but also the effective distance of the nervous system is the result of a nonlinear hybrid of topological information and spatial information.

Finally, We use the idea of community detection and disc zoning to obtain the community structure of neural networks and compare it with real neural function areas. The angular distance of the hyperbolic disc means the similarity of nodes, and the partition of the disc is the potential biochemical module defined in geometric space [7]. The partition method is as follows:

1. The theoretical framework of hyperbolic embedding shows that the angular distance randomly distributed between $[0, 360^\circ]$. We thus divide the hyperbolic disc into 10 parts with 36 degrees each, according to 10 neural functional areas. The important thing to note here is the difference in initial position selection, which may affect the effect of the partition.

2. To solve this problem, we use the concept of community structure to determine the partitions of the disc. The definition of community structure is that the links of the inside community are dense, and the links between the communities are sparse [31]. Corresponding to the hyperbolic embedded model, we define the index $\eta$ which represents the ratio of distances inside the community to the external distances, to
describe the optimal partition, $\eta$ as followed:

$$\eta = \frac{\sum_{i=1}^{n} C_i}{\sum_{j=1, i\neq j}^{n-1} C_j}$$  \hspace{1cm} (7)$$

where $n$ is the number of partition and $C_i$ represents the average distance between nodes inside or outside the community. $\eta$ value is $[0, 1]$. $\eta$ is closer to 1, the better the community division.

To doing this, we obtain the optimal partition of hyperbolic space by selecting different starting partition positions, the minimum distance between node as the reference for the starting partition span, and maximizing $\eta$ as the optimal. We also further considered hyperbolic distance and angular distance as the basis for calculating $\eta$ results as shown in Figure 4. Unfortunately, due to the strong spatial positions dependence of neuron function, the common information entropy of community structure and functional partition is very low. An interesting finding is that some neurons with higher rankings of out-directed popularity belong to the same neurological functional area -lateral ganglion-such as RIAL, RIAR, SAAVR, RMDR, and SMDVR, but the partitions in the hyperbolic space are more scattered. It indicates that there are a large number of long-range links among neurons of C.elegans, for example, the neuron RIAL as in the head and the VD12 in the trailing regions have similar angular distances in the hyperbolic disc, which is also the reason why the topology partition is not consistent with the functional partition. Also, the functional classification of hyperbolic embedding is a complement to functional partitioning.

### 3.2. Economic systems in hyperbolic space

Through statistical inference techniques and structural information of networks, a directed international trade network can be embedded in hyperbolic space, and each
country obtains the two dimensions of national positions information. As the "resultant force" in physics, the national status, actually, should be determined by both import and export information, which has been neglected by the undirected network mapping model. The gap of import and export status of countries is likely to affect the trade surplus or deficit directly, which is also the vital cause of the China-US trade wars. More importantly, extracting a more dimensional national status measure helps to analyze how changes in local trade relations change the world, even reshapes the world trading system.

To study this issue further, we apply our model to visualize the international trade systems and infer the popularity and similarity properties \((r,\theta)\) of nodes according to the data of 20-year (1996-2016) of international trade. Recently studies have been discussed the correlation of radial coordinates are equivalent to correlations among node degrees and the national economic scale\([3, 32, 33, 34]\). Noting that the correlation between import radial correlations (or export radial correlations) and GDP. Radial coordinates and GDP has a significant negative correlation (about -0.5), which indicates that the spatial position of countries can identify the national economic size, especially the country’s export capacity (the correlation coefficient of imports and GDP is higher than that of exports). That is, export status can be used as a coarse-grained measure of the national economy size. While the correlation coefficient between hyperbolic distance and geographical distance is not significant (about 0.1), indicating that the international trade network does not have the geographical clustering in the hyperbolic space.

In order to take into consideration the practicability of our model, on the other hand, we analyze the long-term evolution of the international trading system based on networks hyperbolic embedding method. From the perspective view of exporting, and it presents world energy commodity trade as an imbalanced, diversified and multi-polar development. The United States and Russia have always occupied a central position, reflecting the fact that energy resources are the decisive factor in exporting capacity. Especially in the UK, with the depletion of the North Sea oil field, the UK’s export position in energy trade has been gradually marginalized; Interestingly, Saudi Arabia has also found that Saudi Arabias energy export status has gradually been marginalized due to the change in the direction of Saudi energy policy. Asia, Africa, and the European continent have become active areas and injected new vitality into the energy trade market. The European Community, China, and India have moved to central positions following the increase of their importing dominance. While India has moved towards a more central position, as a new region to superpower status during last few years; Noting that the United States has been at the core regions of the world trade, which is still a leader in the trading superpower.

4. Discussion

In this work, we developed a mapping model of directed networks in hyperbolic space and highlighted the bipartite structure of directed networks. We pay special
attention to three main issues, including: (1) how to identify the asymmetry of links from the topology information; (2) how to embed directed networks into the hyperbolic space, and the feasibility of using the empirical data to test the model; (3) New hyperbolic distances metrics hybrid Multi-dimensional informations are given. Results show that the directed link is hidden in the topology information in a non-trivial way. Based on empirical data, we map some real directed networks into hyperbolic space, including economic systems and biological ecosystems. We found that our method can demonstrate the topological features of directed networks, such as the degree of node, the degree distribution, clustering coefficient, core-periphery structure and so on. Furthermore, we also analyzed the importance of nodes and evolutionary rules through the visual technology. Results show that spatial positions of nodes in the hyperbolic space can be used to quantify the importance of nodes. Position change of nodes in space are consistent with evolution of state status in the economic system.

Although we propose a spatial embedding model for directed networks, the algorithm estimates the angular coordinates of nodes based on the maximum likelihood, and its algorithm complexity is $O(N^3)$. How to design a directed network mapping method with lower time complexity is a problem that should pay an attention in future research. More importantly, it is important to comprehensively consider the network embedding technology of the node’s real location and network topology information. It can not only contribute to the link prediction of the directional network, but also can be used to better take care of practical issues, such as the cost of nematode wiring.

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