Astrophysical $S$-factor of the direct $\alpha(d, \gamma)^6\text{Li}$ capture reaction in a three-body model

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Abstract

At the long-wavelength approximation, electric dipole transitions are forbidden between isospin-zero states. In an $\alpha+n+p$ model with $T=1$ contributions, the $\alpha(d, \gamma)^6\text{Li}$ astrophysical $S$-factor is in agreement with the experimental data of the LUNA collaboration, without adjustable parameter. The exact-masses prescription used to avoid the disappearance of $E1$ transitions in potential models is not founded at the microscopic level.

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I. INTRODUCTION

A radiative-capture reaction is an electromagnetic transition between an initial scattering state and a final bound state. Astrophysical collision energies can be very low with respect to the Coulomb barrier and cross sections are then tiny. The dominant multipolarity is $E1$ in general. In the special case of reactions between $N = Z$ nuclei however, $E1$ transitions are forbidden by an isospin selection rule at the long-wavelength approximation (LWA) and $E2$ transitions become crucial. Nevertheless, $E1$ transitions are not exactly forbidden since isospin is an approximate symmetry. The analysis of the recent LUNA data \cite{1,2} for the $\alpha(d,\gamma)^6\text{Li}$ reaction indicates that $E1$ cross sections dominate the $E2$ cross sections below about 0.1 MeV. Since $E1$ transitions vanish in many models, recent calculations use the exact-masses prescription to avoid their disappearance \cite{3}. Here we present results of the simplest model allowing $E1$ transitions thanks to small $T=1$ components, i.e. the $\alpha+n+p$ three-body model.

II. ISOSPIN-FORBIDDEN $E1$ TRANSITIONS

The electric multipole operators read at the LWA,
\begin{equation}
\mathcal{M}_{\mu}^{E\lambda} = e \sum_{j=1}^{A} (\frac{1}{2} - t_{j3}) r'_{j} Y_{\lambda\mu}(\Omega'_j),
\end{equation}
where $A$ is the number of nucleons, $r'_{j} = (r_j',\Omega'_j)$ is the coordinate of nucleon $j$ with respect to the centre of mass of the system, and $t_{j3}$ is the third component of its isospin operator $t_j$. The isoscalar (IS) part of the $E1$ operator vanishes at the LWA since $\sum_{j=1}^{A} r'_{j} Y_{1\mu}(\Omega'_j) = 0$ and this operator becomes an isovector (IV),
\begin{equation}
\mathcal{M}_{\mu}^{E1} = -e \sum_{j=1}^{A} t_{j3} r'_{j} Y_{1\mu}(\Omega'_j) = \mathcal{M}_{\mu}^{E1,IV}.
\end{equation}

At the LWA, $E1$ matrix elements thus vanish between isospin-zero states. This leads to the total isospin $T$ selection rule in $N = Z$ nuclei and reactions: $T_i = 0 \rightarrow T_f = 0$ is forbidden. But $E1$ transitions are not exactly forbidden in these $N = Z$ systems because isospin is not an exact quantum number. Small $T=1$ admixtures appear in the wave functions. The main isovector $E1$ contributions are due to $T_f = 1$ admixtures in the final state or to $T_i = 1$
admixtures in the initial state. Moreover, the isoscalar $E1$ operator reads beyond the LWA,

$$\mathcal{M}_{\mu}^{E1,IS} \approx -\frac{1}{60} e\kappa_{\gamma}^{2} \sum_{j=1}^{A} r_{j}^{3} Y_{1\mu}(\Omega_{j}),$$  \hspace{1cm} (3)$$

up to terms that should give only a small contribution [4], contrary to other expressions often used in the literature. The isoscalar $E1$ contribution to the capture involves the $T = 0$ parts of the wave functions.

III. THREE-BODY MODEL OF $\alpha(d,\gamma)^6LI$ REACTION

The present wave functions [5] are adapted from the $\alpha + n + p$ model of Ref. [3]. The $J_f = 1^+$ final bound state is described in hyperspherical coordinates and the initial scattering states are described in Jacobi coordinates. Three-body effective $E1$ and $E2$ operators are constructed which assume that the $\alpha$ particle or cluster is in its $0^+$ ground state. For example, the isovector part of the effective three-body $E1$ operator reads at the LWA,

$$\tilde{\mathcal{M}}_{\mu}^{E1,IV} = \frac{1}{2} e r Y_{1\mu}(\Omega_{r}),$$  \hspace{1cm} (4)$$

where $r$ is the Jacobi coordinate between $n$ and $p$. The expressions of the isoscalar part of the $E1$ operator beyond the LWA and of the $E2$ operator can be found in Ref. [3].

The three-body states contain $S = 0$ and 1 components. Because of the isospin zero of the $\alpha$ particle and the antisymmetry of the $n + p$ subsystem with orbital momentum $l$, the components with $l + S$ odd correspond to $T = 0$ and those with $l + S$ even to $T = 1$. The initial scattering state is described by the product of a frozen deuteron wave function ($l_i = 0, S_i = 1$) and $\alpha + d$ $L$ partial scattering waves. Hence, it is purely $T_i = 0$. The $J_f = 1^+$ final bound state contains a small $T_f = 1$ component (about 0.5%). The $E1$ transitions start from $L_i = 1$ and the $E2$ transitions from $L_i = 0$ and 2.

This model requires an asymptotic correction to the $E2$ matrix elements. Indeed the overlap integrals $I_L(R)$ of the deuteron and $\alpha + n + p$ final wave functions decrease too fast beyond 10 fm as shown for $L = 0$ by Fig. [1]. This is corrected by matching at 7.75 fm the overlap integrals with the exact Whittaker asymptotic function multiplied by realistic asymptotic normalization coefficients.

Total $E1 + E2$ astrophysical $S$ factors calculated in the three-body model with the $E2$ correction are compared in Fig. [2] with experimental data. The isoscalar $E1$ capture contribution is small and can be neglected in first approximation. The isovector $E1$ contribution
IV. COMMENT ON THE EXACT-MASSES PRESCRIPTION

To obtain non-vanishing $E1$ transitions in the two-body or potential model, experimental masses are used in the effective charge of $N = Z$ nuclei,

$$Z_{\text{eff}}^{(E1)} \propto \left( \frac{Z_1}{A_1} - \frac{Z_2}{A_2} \right) \rightarrow Z_{\text{eff}}^{(E1)} \propto m_N \left( \frac{Z_1}{M_1} - \frac{Z_2}{M_2} \right)$$  \hspace{1cm} (5)

where $m_N$ is the nucleon mass and $Z_{1,2}$, $A_{1,2}$ and $M_{1,2}$ are the charges, mass numbers and experimental masses of the colliding nuclei, respectively. This exact-masses prescription is unfounded.

(i) $E1$ transitions would remain exactly forbidden in the $d(d, \gamma)^4\text{He}$ reaction, in contradiction with $ab$ $initio$ calculations.

(ii) Using the mass expression $M = A m_N + (N - Z) \frac{1}{2} (m_n - m_p) - B(A, Z)/c^2$, effective charges would depend on the binding energies $B(A_{1,2}, Z_{1,2})$,

$$m_N \left( \frac{Z_1}{M_1} - \frac{Z_2}{M_2} \right) \approx \frac{1}{2m_Nc^2} \left( \frac{B(A_1, Z_1)}{A_1} - \frac{B(A_2, Z_2)}{A_2} \right).$$  \hspace{1cm} (6)

Binding energies per nucleon $B(A, Z)/A$ mostly depend on the main $T = 0$ components of the wave functions and not on the small $T = 1$ components physically responsible for the non vanishing of “forbidden” $E1$ transitions.

(iii) $E1$ matrix elements would be unphysically sensitive to the long $T_f = 0 \alpha + d$ tail of the $^6\text{Li}$ wave function.
V. CONCLUSION

Isovector $E1$ transitions with $T = 1$ admixtures in the final state and $E2$ transitions explain the order of magnitude of the LUNA data \cite{1,2}, without any adjustable parameter. Isoscalar $E1$ transitions beyond the LWA are negligible for the $\alpha(d,\gamma)^6\text{Li}$ reaction. The exact-masses prescription is not founded and should not be trusted for reactions between $N = Z$ nuclei, such as $\alpha(d,\gamma)^6\text{Li}$ and $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$. A three-body model with $T = 1$ admixtures in both initial and final states should be developed. Microscopic six-body and \textit{ab initio} calculations are difficult but possible and necessary for a deeper understanding of this reaction.

![Graph](image)

FIG. 2: Total $E1 + E2$ astrophysical $S$ factor (full and dashed lines). Experimental data from Refs. \cite{6} (triangles), \cite{7} (open circles), and \cite{2} (full circles). Adapted from Ref. \cite{5}.

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