The no core shell model in an effective field theory framework

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Abstract. One of the outstanding problems in nuclear-structure theory is the construction of two-body (and higher-body) effective interactions in a model (or basis) space. In this presentation we discuss a recently developed approach to this problem, where one starts with an effective field theory (EFT), which contains only nucleonic fields and is formulated directly in a No-Core-Shell-Model (NCSM) space. Such an approach helps us to understand the gross features of nuclear systems from a QCD perspective. It also leads to a new method for the construction of effective interactions suitable for NCSM calculations. We then present applications to light nuclei within the pionless EFT and obtain reasonable results. Finally, we discuss future applications and extensions, such as testing the limits of the pionless EFT and extending the formalism to the pionfull EFT.

1. Introduction
A number of many-body techniques, such as the Green’s Function Monte Carlo [1, 2, 3, 4], the Effective-Interaction Hyperspherical Harmonics [5] and the No Core Shell Model [6, 7, 8], now exist for calculating microscopically the structure of nuclei up to around mass $A = 16$. These approaches start with free realistic nucleon-nucleon (NN) potentials and, in many cases, also theoretically motivated three-nucleon (NNN) forces. These interactions are then employed in the different nuclear many-body formalisms to yield essentially parameter free results, which explain very well the properties of light nuclei, $i.e.$, $A \leq 16$. There is also the Coupled Cluster (CC) method [9, 10, 11], which can handle heavier nuclei, but generally for closed-shell nuclei or near closed-shell nuclei.

In general, these bare NN and NNN interactions need to be renormalized and/or truncated from the infinite Hilbert space to a finite model space, in which the many-nucleon calculations are performed. Thus, it is the effective NN and NNN interactions in these truncated model spaces that are necessary for carrying out most of the different many-body approaches for calculating microscopically nuclear properties. Because, in principle, all interactions are effective, their construction in free space, empirically and/or theoretically, is not necessary, especially if one can develop a formalism for directly constructing the effective interactions in the truncated model space. Not only would such a procedure be more efficient, but it would also have the possibility of yielding effective interactions that would converge more rapidly in the different many-body techniques, thereby allowing calculations to be performed for heavier-mass nuclei.

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2. Motivation and goal

Why do we want to put the NCSM in an EFT framework? First, we want to have a formalism, which will provide an understanding of the gross features of nuclear systems from a QCD perspective. Second, as discussed above, we want to develop a new approach for the construction of effective interactions suitable for NCSM calculations. Consequently, our final goal is the formulation of an EFT with only nucleonic fields directly in the NCSM model space [12, 13, 14].

3. Formalism

3.1. Brief overview of Effective Field Theory (EFT)

EFT [15, 16, 17, 18] is based on a separation of scales, such as the nucleon mass or the pion mass, where the details of the physics at energies greater than the separation energy (representing the short-distance physics) are irrelevant for understanding the physics below the separation energy (representing the long-distance physics). In EFT the low-energy degrees of freedom are explicitly included, while the high-energy degrees of freedom are integrated out.

One then constructs a potential, in this case in the NCSM model space, which is consistent with the symmetries of the QCD Lagrangian and is expanded as a Taylor series in the momenta or as a contact gradient expansion in coordinate space. Each term in this potential expansion contains a constant. These constants are known as the low-energy constants (LECs). For now, the LECs are determined by fitting to experimental data.

In the case of the NCSM the calculations are performed in an harmonic-oscillator (HO) space for two-nucleons up to some maximum energy, $E_{\text{max}} = (N_{\text{max}} + \frac{3}{2})\hbar\omega$. Thus, the calculations are carried out in a truncated model space, so the interactions in this truncated space represent the renormalized effective interactions. In standard NCSM calculations one needs to repeat the calculations for increasing $N_{\text{max}}$ to check that the results are converging in an appropriate manner. In EFT one introduces a cutoff $\Lambda$, known as the Ultraviolet (UV) cutoff, which depends on $N_{\text{max}}\hbar\omega$. To obtain a correct EFT result, one must achieve Renormalization Group Invariance (RGI). This means that the LECs, which are redetermined as $\Lambda$ increases, become independent of $\Lambda$. In other words, for a high enough $\Lambda$, the LECs have absorbed the high-energy physics and the calculated physical observables are approximately independent of the cutoff $\Lambda$. The Taylor series expansion of the potential provides a power counting or hierarchy between the different contributions, which yield results that are improvable order by order, i.e., leading order (LO) > next-to-leading order (NLO) > next-to-next-to-leading order (NNLO) > etc.

4. Results

Our first investigation was for an EFT without pions, i.e., a separation-scale energy of $\approx 100$ MeV, which is known as the pionless EFT. In this case the Hamiltonian in the NCSM model space contains three potential terms consistent with the QCD symmetries, namely the NN potentials in the triple $S$ and single $S$ channels and a NNN potential in the $S = \frac{1}{2}$ channel. One then uses this Hamiltonian to compute the three LECs associated, respectively, with the three potential terms above. That is, the values of the LECs are adjusted until they reproduce, in our case, the binding energies of the deuteron, the triton and the $\alpha$ particle. Once these constants are determined, the Hamiltonian in the NCSM model space is completely known and can be used to compute other nuclear properties, such as the energy of the first-excited $0^+$ state of the $\alpha$ particle or the binding energy of $^6$Li. The determination of the Hamiltonian must be done for increasing values of $\Lambda$ to achieve RGI. Because the calculations are performed using HO wave functions, i.e., in an HO potential well, one must also take the limit as $\omega \to 0$ (the Infrared (IR) cutoff), since the nucleus is a self-bound system. Thus, the LECs are functions of both $\Lambda$ and $\omega$.

Figure 1 shows our results for the energy of the first-excited $0^+$ state in the $\alpha$ particle, for increasing $\Lambda$ and decreasing $\omega$, which yields $E(J^p, T, ^4\text{He}) = E(^3_{2+}, 0)_\text{theor} = 18.8$ MeV, in the
Figure 1. Energy of the first-excited $(0^+, 0)$ state in $^4$He as a function of the UV cutoff $\Lambda$ for different values of the frequency $\hbar \omega$ in MeV. The dashed curve marks the limit $\hbar \omega \to 0$. Taken from Ref. [12].

limits $\Lambda \to \infty$ and $\omega \to 0$, compared with $E(0^+, 0, ^4\text{He})_{\text{expt}} = 20.21$ MeV. These agree within 10%, which is remarkable agreement for a LO calculation and indicates that the description of this level is insensitive to details of the short-range physics.

Following the same procedure, we also evaluated for the first time in the pionless EFT the binding energy of $^6\text{Li}$. For large values of the UV cutoff $\Lambda$ and small values of the IR cutoff $\omega$, we estimate the binding energy to be about 23 MeV, to be compared with the experimental result of 31.99 MeV. While not as precise as the first-excited-0$^+$-state energy in $^4\text{He}$, this agreement between the theoretical and experimental values for the binding energy of $^6\text{Li}$ to within 30% is consistent with the expected errors in LO in the pionless EFT.

Obviously, we want to obtain better accuracy, which means that we need to include subleading contributions in the pionless EFT, i.e., NLO, NNLO, etc. However, this becomes a very difficult task, because of lack of additional bound states in the NN system. The solution was to use the trick from lattice QCD of placing the system in a trap. In our case we use the Busch, et al. formula [19], which relates the energies of particles in an HO trap to their scattering phaseshifts. In the case of two trapped particles in an HO potential, their spectrum is entirely determined by the parameters of the effective range expansion (ERE). Thus, the scattering length yields the LO result; the effective range, the NLO result; etc. The energies obtained from the Busch formula are then used to determine the LECs.

This latter approach has been successfully applied [14] to the binding of the deuteron, NN phaseshifts and the $A = 3$ system for both $J^\pi = \frac{1}{2}^+$, $T = \frac{1}{2}$ and $J^\pi = \frac{3}{2}^-$, $T = \frac{1}{2}$ as well as to the neutron-deuteron scattering length. This conference contribution is too short to show all of
Figure 2. Ground-state energy of the deuteron in the HO trap as a function of the frequency \( \omega \). The energy of LO (NLO) is given by the \( \bigcirc \) (\( \square \)) connected by the - - - - (——) line. Taken from Ref. [14].

these results, so only a selected few will be given.

Figure 2 gives our result for the g.s. energy of the trapped NN system in the \( ^3S_1 \) channel (i.e., the deuteron) as a function of the frequency \( \omega \) [14]. The energy at LO (NLO) is given by the dashed (solid) line. For small values of \( \omega \), the energy converges to the value in free space, which is, at NLO, indicated by the dotted line. This figure clearly shows how results in EFT systematically improve as one goes to higher orders in the expansion, i.e., in the power counting.

Figure 3 illustrates our results for the g.s. energy in LO in the pionless EFT for three nucleons in an HO trap coupled to \( J^p = \frac{3}{2}^+ \), \( T = \frac{1}{2} \), using only the NN force; there is no NNN force in this channel [14]. Each curve gives the result for the g.s. energy for a fixed value of the NN cutoff energy, \( N_2^{\text{max}} \), as a function of increasing NNN cutoff energy, \( N_3^{\text{max}} \), which starts at \( N_3^{\text{max}} = N_2^{\text{max}} \). This figure shows a quite significant result, namely, that it is important in performing many-body calculations to have the many-body energy greater than the two-body energy. In Fig. 3 one observes that if \( N_3^{\text{max}} \) is the same as \( N_2^{\text{max}} \), then all the degrees of freedom of the NNN system are not energetically available, as seen by the sudden and steep increase in the NNN binding energy as soon as \( N_3^{\text{max}} \) is increased beyond \( N_2^{\text{max}} \), thereby significantly improving the convergence rate of the final answer. For example, looking at the results in Fig. 3, one sees that the result for \( N_2^{\text{max}} = 8 \) and \( N_3^{\text{max}} = 10 \) is essentially as good as that for \( N_2^{\text{max}} = N_3^{\text{max}} = 16 \), which is a much larger model space. Consequently, taking advantage of this effect allows for better converged results in much smaller model spaces.

5. Conclusions and outlook
We have presented our approach for formulating the NCSM in an EFT framework. After a brief description of EFT, we used the pionless EFT to calculate the first-excited-\( 0^+ \)-state of
Figure 3. The g.s. energy in LO of the NNN system for $J^\pi = \frac{3}{2}^+, T = \frac{1}{2}$ as a function of the three-body model-space size $N_3^{\text{max}}$, for $\hbar\omega = 3$ MeV. Results are shown for different values of the two-body model-space size $N_2^{\text{max}}$. Adapted from Ref. [14].

$^4$He and the g.s. energy of $^6$Li, obtaining quite reasonable results using only the leading order (LO) in the EFT. The extension to NLO in the pionless EFT was obtained by using the trick of binding the few-fermion systems in an HO trap, which allowed us to relate the energies of the fermions in the HO trap to the NN scattering phase shifts. Results for the $A = 3$ system in the $J^\pi = \frac{3}{2}^+, T = \frac{1}{2}$ channel showed the importance of having $N_3^{\text{max}}$ for the three nucleons greater than $N_2^{\text{max}}$, in performing the many-body calculations, so as to take full advantage of all the degrees of freedom of the three nucleons. Doing this also leads to faster convergence in smaller model spaces, allowing converged calculations for heavier-mass nuclei.

Future plans include more calculations using the pionless EFT in NLO and for other light nuclei and in testing the limits of this approach. Work is also underway for extending our formalism to the EFT with pions, known as the pionfull EFT.

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