Purchasing power parity is one of the most important equilibrium conditions in international macroeconomics. Empirically, it is also one of the most hotly contested. Numerous recent studies, for example, have sought to determine the validity of purchasing power parity using data from the post–Bretton Woods float and have reached different conclusions. We assert that most such studies are flawed for two reasons. First, the post-1973 data contain, by definition, only a very limited amount of the low-frequency information relevant for examination of long-run parity. Second, the dynamic econometric techniques used to model deviations from parity are typically quite crude with respect to admissible low-frequency dynamics. Both de-

Helpful comments were received from seminar participants at the Sixth World Congress of the Econometric Society, New York University, the University of British Columbia, the University of Southern California, and the Penn Macro Lunch Group, as well as from Richard Baillie, Bill Bomberger, Larry Christiano, David Denslow, Bernard Dumas, Hali Edison, Jeff Frankel, Don McCloskey, Glenn Rudebusch, and Peter Schotman. An anonymous referee also made valuable comments that helped sharpen our results. We gratefully acknowledge financial support from the National Science Foundation (grant SES-892715), the University of Pennsylvania Research Foundation (grant 3-71441), the Institute for Empirical Macroeconomics (Diebold), the Australian National University (Husted), and the Financial Institutions Center at the University of Florida (Rush). Yin-Wong Cheung provided expert research assistance.

[Journal of Political Economy, 1991, vol. 99, no. 6]
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ficiencies are rectified in the present paper, with dramatic results. We construct a new data set of 16 real exchange rates covering more than a century of the classic gold standard period, and we study deviations from parity using long-memory models that allow for subtle forms of mean reversion. For each real exchange rate, we find that purchasing power parity holds in the long run.

I. Introduction

The doctrine of purchasing power parity is more than four centuries old (Bernholz 1982). It remains a key ingredient in modern models of exchange rate dynamics (e.g., Dornbusch 1976) and is also widely used in policy deliberations (e.g., in the determination of target zones). The idea of purchasing power parity is simply that, when measured in the same units, the monies of different countries should command the same basket of goods. Otherwise, international arbitrage should bring about adjustments in prices, exchange rates, or both, which ultimately restore parity.

It is well known, however, that strict parity obtains only under strict conditions. Many real-world complications—including transactions costs, nontradables, trade restrictions, exchange market intervention, and taxation—may interfere with the workings of purchasing power parity. Moreover, the use of aggregate price indexes (with potentially different and changing market baskets) further complicates empirical investigation. It is perhaps not surprising, then, that empirical tests of purchasing power parity as a short-run proposition have failed to produce a consensus (cf. McCloskey and Zecher 1976, 1983; Frenkel 1981; Hakkio 1984; Rush and Husted 1985).

Nevertheless, most macroeconomists (including ourselves) would agree with Dornbusch and Krugman (1976, p. 540), who remark that "under the skin of any international economist lies a deep-seated belief in some variant of the purchasing power parity theory of the exchange rate." In particular, the hypothesis of long-run parity, that is, a tendency for the real exchange rate to revert (albeit perhaps slowly) to its parity value, is attractive. The data remain discomfiting, however: deviations from parity appear highly persistent. In fact, a number of authors (e.g., Roll 1979; Adler and Lehmann 1983; Darby 1983; Mussa 1986; Diebold 1988; Meese and Rogoff 1988; Baillie and McMahon 1989) have argued that real exchange rates are well approximated by martingales, so that shocks have a completely permanent effect on the levels, while changes are unpredictable. That is, they argue that there is little or no tendency for nominal exchange rates and prices to adjust in such a way as to promote purchasing power parity.
We find such behavior of the real exchange rate to be economically implausible. The real exchange rate is a relative price. Accepting the hypothesis of nonstationarity of the real exchange rate implies that it can, and will, take on any value in finite time. The potential for such wide-ranging behavior of the relative price of one nation’s goods in terms of the other nation’s goods seems unlikely.

We shall suggest a reconciliation of the economically appealing view of purchasing power parity as a long-run equilibrium and its apparent empirical rejection. Our approach has two main parts.

1. We study the behavior of real exchange rates during the gold standard era, a high point of international cooperation. The gold standard era lends itself to study because it affords us long spans of data, which are precisely what is required to test hypotheses about long-run reversion of the real exchange rate to its parity value (and precisely what is lacking in studies using data only from the post-1973 float). Indeed, our shortest sample spans 74 years, while our longest spans 123 years.

2. We model the behavior of real exchange rates using a class of long-memory models substantially more general (with respect to the low-frequency dynamics of interest) than standard time-series representations. Such generality is particularly important in the present application because it potentially enhances our ability to discriminate slow parity reversion from nonreverting martingale behavior.

II. Historical Background

A. Monetary History

In order to understand the subtleties of data construction and the subsequent empirical analysis, it is important to understand the monetary history of the nineteenth century. We study data from six countries: the United States, United Kingdom, Sweden, Belgium, France, and Germany. For each, the nineteenth century was a time of gradual movement from a silver or bimetallic monetary standard to a monometallic gold standard. Often countries were on a de facto monometallic standard while legally on a bimetallic standard. Table 1 lists the dates of de facto gold standard adoption.¹

The monetary history of the United Kingdom is relatively simple. With the exception of 1798–1820, the so-called restriction during which the Bank of England was legally permitted to issue irredeem-

¹ It took some countries (e.g., France and the United States) longer to return to de jure. In addition, countries frequently had been on the gold standard at times previous to those listed but subsequently left it. After the year given in table 1, none of the countries left the gold standard until World War I.
Real Exchange Rates

Table 1

Year of Adoption of the Gold Standard

| Country           | Year |
|-------------------|------|
| Belgium           | 1874 |
| France            | 1875 |
| Germany           | 1872 |
| Sweden            | 1873 |
| United Kingdom    | 1821 |
| United States     | 1879 |

able paper currency, the United Kingdom was on a gold standard from 1750 to 1913, longer than any other country in our sample.

Sweden's monetary history is also straightforward. From 1830 to 1872, it was on a monometallic silver standard. In 1873, it converted to a monometallic gold standard.

The financial histories of the other countries are more complicated. Those of Belgium and France are bound together: for most of the time Belgium was a monetary satellite of France. French coins commonly circulated in Belgium and were legal tender. In fact, from 1851 to 1859, Belgium issued no new coinage, relying instead on French coinage. In 1860, France and Belgium, together with Switzerland and Italy, formed the Latin Monetary Union, which made each country's coinage legal tender in the others.

For many years, Belgium and France were legally on bimetallic standards that valued silver to gold at 15.5:1. Depending on the market price, the mint ratio overvalued one metal, which then circulated as coinage. Gold was overvalued from 1851 to 1866, so Belgium and France were effectively on a gold standard. At other times silver was overvalued and an effective silver standard prevailed. From 1867 to 1873, silver became increasingly overvalued at the mint, and the countries were faced with the prospect of minting an ever increasing amount of silver. First Belgium (in 1874) and then France (in 1875) placed severe restrictions on the mintage of silver and stepped up the mintage of gold. Given the small quantities of silver minted thereafter (and the very large quantities of gold), both countries had moved de facto to a gold standard.

From 1764 to 1857, the states that would become Germany were on a bimetallic standard that overvalued silver. In fact, in 1857 only silver circulated, and Germany formally adopted a silver standard.

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2 Belgium made an abortive attempt to gain gold between 1848 and 1850 by changing the mint ratio between gold and silver. Only minor amounts of gold were minted, so we assume that this effort failed.
Following the Franco-Prussian War, however, Germany used the reparations received from France to buy gold and thus switched, both legally and effectively, from silver to gold in 1872.

The financial history of the United States is perhaps the most complex of all. From 1791 to 1861, it was on various bimetallic standards. From 1791 to 1834, the mint ratio overvalued silver, so the country was effectively on a silver standard. In 1834, the mint ratio was adjusted and gold was overvalued; the United States was then effectively on a gold standard until 1862. In that year, to help finance the Civil War, large amounts of inconvertible paper currency (greenbacks) were issued. The United States remained on a paper standard until the Resumption Act of 1873 committed it to return to a gold standard in 1879. The return was accomplished smoothly and the United States operated on a de facto gold standard, which was made de jure as well in 1900.

B. Data Construction

Having briefly discussed relevant aspects of nineteenth-century monetary history, we now discuss the construction of our real exchange rate series. We study annual real exchange rates constructed from nominal exchange rates and both consumer price indexes (CPIs) and wholesale price indexes (WPIs) for six countries: Belgium, France, Germany, Sweden, the United Kingdom, and the United States. The starting date of each real exchange rate series is constrained by the availability of CPI or WPI data; for most countries, the data extend back from 1913 more than 100 years to the beginning of the nineteenth century. Table 2 details the sample periods for each of the price indexes (and the associated real exchange rates). Our final data set contains 16 real exchange rates, beginning from the date corresponding to WPI or CPI availability and ending in 1913. The WPI rates pertain to Belgium/France, Belgium/Germany, Belgium/United States, Belgium/United Kingdom, France/Germany, France/United States, France/United Kingdom, United States/Germany, United States/United Kingdom, and Germany/United Kingdom; the CPI rates pertain to France/Belgium, France/Germany, France/Sweden, Belgium/Germany, Belgium/Sweden, and Sweden/Germany.

With the exception of about 20 years for both the United Kingdom and the United States, all the countries in our sample were on a metallic standard (gold or silver). If two countries were on the same effective standard, the exchange rate between their currencies was

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3 Officer (1983) contains a complete discussion of the mint ratios used during 1791–1834.
firmly fixed; if they were on different standards, the exchange rate floated.

We started by converting all the national price levels into terms of gold, consistently using the appropriate de facto rather than de jure monetary standard of the country. Then we calculated the real exchange rates by taking the ratio between the countries' price levels. Finally, each real exchange rate was normalized to 1.0 in 1870.

In order to express all price levels in terms of gold, we needed to calculate the "exchange rate" between each country's circulating currency and gold, taking account of periods in which countries were on a de facto or de jure silver standard and periods in which they were on a fiat money standard. This would have been easy had data been available on the exchange rate between a country on a gold standard (e.g., the United Kingdom) and a country off gold. With the few exceptions noted below, such data apparently do not exist. Thus we were forced to assume that (1) the exchange ratio between a country's money and gold did not vary when the country was on a gold standard and that (2) when the country was on a silver standard, the (silver) money/gold "exchange rate" was determined by the market silver/gold price ratio.

We took price levels given in terms of domestic currency from B. Mitchell (1980, tables I1, I2), with the exception of the United States,

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4 Because the United Kingdom was on a gold standard for most of the sample period, this roughly amounts to converting all prices to pounds sterling.

5 A general reference for all the countries is Willis and Beckhart (1929); a specific reference for Belgium and France is Willis (1968).

6 That is, exchange rates between countries both on a gold standard do not vary. This assumption is reasonable because any movements of the exchange rate between the gold import and gold export points would necessarily be small. Moreover, Clark (1984), Officer (1986), and Spiller and Wood (1988) indicate that variations beyond the gold points, if any, are also small. Such small changes in otherwise fixed exchange rates are dominated empirically by price-level movements.

7 That is, the exchange rate between a gold standard and a silver standard country is determined by the market price of gold relative to that of silver. With the few actual exchange rate data found, this assumption appears not to be seriously violated.
which we took from the U.S. Department of Commerce (1975, ser. E40, E51). Then these series were multiplied by the price of domestic currency in terms of gold to convert to price levels measured in terms of gold. The "exchange rates" between domestic currency and gold, that is, the gold prices of domestic currency, were calculated as follows.

*Belgium.*—From 1832 to 1850, silver had a mint value of 15.5. The market value of silver in terms of gold (from the most important world market, the London market, given in Del Mar [1880]) was used to compute the ratio between currency and gold. From 1851 to 1866, gold was overvalued at the mint, so Belgium was on a de facto gold standard. Thus no change was necessary to express price levels in terms of gold. From 1867 to 1873, silver circulated, so the market value of silver was again used to calculate the gold/currency ratio. Finally, from 1874 to 1913, Belgium remained on a gold standard.

*France.*—From 1806 to 1874, silver had a mint ratio of 15.5:1. Silver was overvalued from 1806 to 1850 and from 1867 to 1874, so it circulated. For the periods 1806–11 and 1814–29, an exchange rate between France and the United Kingdom was given on page 643 of House of Representatives (1886). We used this exchange rate together with the ratio between the pound sterling and gold to calculate a franc/gold ratio. For 1812, 1813, 1830–59, and 1867–74, we used the market value of silver relative to that of gold to compute the franc/gold ratio. During the remaining years, gold circulated, and France was on a gold standard.

*Germany.*—From 1792 to 1871, Germany was on either a de facto or de jure silver standard, so the market ratio of silver to gold was used to compute the ratio between the mark and gold. From 1872 to 1913, Germany was on a gold standard.

*Sweden.*—From 1830 to 1872, Sweden was on a silver standard. In 1873, it converted to a gold standard and valued its previous silver coinage at 15.813:1. Thus for 1830–72 we used the market price of silver in terms of gold to calculate the currency to gold exchange ratio.

*United Kingdom.*—The United Kingdom was off a gold standard only from 1798 to 1820. Andréadès (1966, pp. 212, 242) reports the depreciation of the currency in terms of gold, which we used to calculate our pound/gold series.

*United States.*—Perkins (1978) and Officer (1983) report a series of United States/United Kingdom exchange rates for 1791–1900. We used these with the pound/gold ratio calculated for the United Kingdom to compute the dollar/gold price. We corrected for the depreciation of the (fiat) dollar during the greenback era, 1862–78, using data from W. Mitchell (1908).
III. Long-Memory Models of Real Exchange Rate Dynamics

Before considering dynamic models for the real exchange rate, we must define it. The real exchange rate is given by

\[ r_t = \frac{p_t^*}{p_t}, \]

where \( r_t \) is the time \( t \) real exchange rate, and \( p_t^* \) and \( p_t \) are time \( t \) foreign and domestic price levels, denominated in terms of gold.\(^8\)

When the price levels are interpreted as measuring the foreign and domestic prices of an “average good,” the real exchange rate is simply the relative price of one country’s good in terms of the other country’s good.\(^9\)

The hypothesis of instantaneous purchasing power parity has an immediate interpretation in terms of real exchange rate behavior, requiring that \( r_t \) be constant. The data clearly do not satisfy this instantaneous parity hypothesis; of greater interest is whether parity holds in an appropriate long-run sense. The hypothesis of long-run parity is usefully couched in terms of the time-series properties of \( r_t \).

In particular, if the effects of shocks to \( r_t \) vanish in the long run (in a sense to be defined precisely), then we shall say that long-run parity holds.

Now let us consider dynamic models for the real exchange rate.\(^10\) A conventional ARIMA\((p, d, q)\) representation is

\[ \Phi(L)(1 - L)^d r_t = \Theta(L) \epsilon_t, \quad \epsilon_t \sim (0, \sigma^2), \]

where \( \Phi(L) = 1 - \phi_1 L - \ldots - \phi_p L^p \), \( \Theta(L) = 1 + \theta_1 L + \ldots + \theta_q L^q \), all roots of \( \Phi(L) \) and \( \Theta(L) \) lie outside the unit circle, and \( d \) is an integer (typically zero or one).

The ARIMA\((p, d, q)\) representation is restrictive with respect to admissible low-frequency dynamics, however, which motivates our use of the more general ARFIMA\((p, d, q)\) (autoregressive fractionally integrated moving average) representation. In the ARFIMA\((p, d, q)\)

\(^8\) Recall that in constructing the domestic and foreign price indices measured in terms of gold, we multiplied each country’s price index by its gold/currency exchange ratio. Thus the nominal exchange rate that normally appears in the definition of the real exchange rate is already embedded in the price series.

\(^9\) The type of “average good” depends on the price index used. If CPIs are used, the real exchange rate is the relative price of consumption baskets. If WPIs are used, the real exchange rate is the relative price of production baskets.

\(^10\) In accordance with the literature, \( r_t \) should be interpreted as the natural log of the real exchange rate. In all the empirical work reported subsequently, the log real exchange rate is the object of analysis.
representation, \( d \) is not required to be an integer.\(^{11}\) The operator \((1 - L)^d\) is defined through its binomial expansion

\[
(1 - L)^d = 1 - dL + \frac{d(d - 1)}{2!} L^2 - \frac{d(d - 1)(d - 2)}{3!} L^3 + \ldots
\]

For \( d = 1 \), \((1 - L)^d\) is the usual first-differencing filter; for noninteger \( d \), however, it is an infinite-order lag operator polynomial with slowly declining coefficients.\(^{12}\)

The ARFIMA(\( p, d, q \)) model belongs to the class of long-memory processes, so named for their ability to display significant dependence between observations widely separated in time. Standard ARIMA(\( p, 0, q \)) processes are short-memory because the autocorrelation (or dependence) between observations \( \tau \) periods apart \((\rho(\tau))\) decays rapidly as \( \tau \) increases. Indeed, ARIMA(\( p, 0, q \)) autocorrelations decay exponentially:

\[
\rho(\tau) \sim k^\tau, \quad 0 < k < 1, \tau \to \infty.
\]

In contrast, the defining characteristic (in the time domain) of ARFIMA(\( p, d, q \)) processes is a slower, hyperbolic, autocorrelation decay:

\[
\rho(\tau) \sim \tau^{2d-1}, \quad d < \frac{1}{2}, d \neq 0, \tau \to \infty.
\]

The intuition of long memory also emerges clearly in the frequency domain. A real exchange rate displays long memory if its spectral density, \( f_r \), increases without limit as angular frequency tends to zero: \( \lim_{\lambda \to 0} f_r(\lambda) = \infty \). For an ARFIMA(\( p, d, q \)) series, \( f_r(\lambda) \) behaves like \( \lambda^{-2d} \) as \( \lambda \to 0 \), so \( d \) parameterizes low-frequency behavior. This contrasts with the usual ARIMA(\( p, 1, q \)) model, in which the spectral density is forced to behave like \( \lambda^{-2} \) (corresponding to \( d = 1 \)) as \( \lambda \to 0 \). A richer range of spectral behavior near the origin becomes possible when the integer \( d \) restriction is relaxed.

In short, the ARFIMA representation is a parsimonious low-frequency generalization of the popular ARIMA class. The ARIMA representations emerge, however, as potentially restrictive special cases that can capture only a narrow type of long-memory behavior. Specifically, all ARIMA(\( p, 0, q \)) models are short-memory, while the long memory associated with ARIMA(\( p, 1, q \)) models is of a very special type. The generality afforded by ARFIMA representations in approximating Wold representations is valuable in the context of real

\(^{11}\) We present here only a cursory review as necessary for the subsequent empirical analysis. For extended discussion and references, see Diebold and Nerlove (1990).

\(^{12}\) Covariance stationarity corresponds to \( d < \frac{1}{2} \). Thus, as with integer-integrated series, one can always transform a fractionally integrated series to covariance stationarity by taking a suitable number of integer differences.
exchange rate dynamics because of the crucial importance of low-frequency components.

The ARFIMA model can be put into moving average form. First, write the ARFIMA model (2) as \((1 - L)^d r_t = B(L)\epsilon_t\), where \(B(L) = \Phi^{-1}(L)\Theta(L)\). Extracting the factor \(1 - L\) gives

\[(1 - L)r_t = A(L)\epsilon_t = \sum_{i=0}^{\infty} a_i \epsilon_{t-i},\]

where \(A(L) = (1 - L)^{1-d}B(L)\) and \(a_0 = 1\). The moving average parameters \(a_i, i = 0, 1, 2, \ldots\), are called the impulse responses; they track the response of future real exchange rate changes to a unit innovation. The cumulative impulse responses, \(c_j = \sum_{i=0}^{j} a_i, j = 0, 1, 2, \ldots\), track the response of future real exchange rate levels to the same unit innovation. Parity reversion occurs (i.e., \(c_\infty = 0\)) when \(d < 1\). Conversely, \(c_\infty = \infty\) when \(d > 1\); \(c_\infty\) is finite and nonzero only in the unit root case, \(d = 1\). More important, examination of the sequence of cumulative impulse response coefficients at horizons of economic interest (say, 1–10 years) provides important information regarding the pattern and speed with which shocks to parity are propagated.

**IV. Empirical Analysis**

Examination of time-series plots of the various real exchange rates makes clear the need for a class of models enabling flexible parameterization of low-frequency behavior. A representative real exchange rate, that for United States/Germany, is shown in figure 1. Deviations from purchasing power parity appear pronounced and prolonged, yet there appears to be a tendency toward mean reversion.

It is possible, however, that standard unit root tests would have low power against such slow mean reversion, in spite of the large spans of data available. In fact, a battery of standard unit root tests applied to the real exchange rates yielded results that were mixed and hard to interpret; overall, they provided little evidence of long-run parity. (Detailed results are available on request.) The possibility remains open, however, that subtle forms of reversion to parity, against which standard unit root tests may have low power, are operative. We therefore proceed to estimate long-memory time-series models.

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13 Notice that parity reversion occurs when \(d < 1\), whereas covariance stationarity requires \(d \leq \frac{1}{2}\). Thus parity reversion can occur even in the absence of covariance stationarity.

14 Sowell (1990a) conjectures that conventional unit root tests may have low power against the long-memory alternatives entertained below. This conjecture is confirmed in the Monte Carlo analysis of Diebold and Rudebusch (1991b).
which provide a flexible and general parameterization of low-frequency dynamics.

The parameters of real exchange rate models allowing for fractional integration may be estimated by a variety of methods, including a two-stage semiparametric procedure (Geweke and Porter-Hudak 1983), approximate frequency domain maximum likelihood (Fox and Taqqu 1986), and exact time domain maximum likelihood (Sowell 1990b). While the semiparametric procedure has proved useful in a number of economic applications (e.g., Diebold and Rudebusch 1989, 1991a), it is inefficient relative to maximum likelihood under correct model specification, and its distributional properties are not fully understood. Thus maximum likelihood appears to be an attractive alternative, particularly in light of the reduced computational burden afforded by approximating the Gaussian likelihood in the frequency domain.\footnote{Cheung and Diebold (1991) compare the finite-sample properties of the Fox-Taqqu (1986) and Sowell (1990b) procedures and show that their performance is comparable in samples of the size available here.}
Thus, following Fox and Taqqu (1986), we exploit the fact that maximization of the Gaussian likelihood is equivalent (asymptotically) to minimization of

$$
\sigma^2_f(\xi) = \sum_{j=1}^{T-1} \frac{I_r(2\pi j/T)}{f_r(2\pi j/T, \xi)}
$$

(3)

with respect to the ARFIMA parameter vector $\xi = (d, \phi_1, \ldots, \phi_p, \theta_1, \ldots, \theta_q)'$, where $I_r(\lambda)$ is the periodogram of $r$ at frequency $\lambda$, and

$$
f_r(\lambda, \xi) = |1 - e^{-i\lambda}|^{-2d} |B(e^{-i\lambda})|^2
$$

is proportional to the spectral density of $r$ at frequency $\lambda$. As proved by Fox and Taqqu, the resulting maximum likelihood estimator is consistent and asymptotically normal.

We consider ARFIMA($p$, $d$, $q$) representations for log real exchange rates, where both $p$ and $q$ are less than or equal to three. Because the Akaike information criterion (AIC) and the Schwartz information criterion (SIC) have different optimality properties under different conditions, which cannot be ascertained a priori, we consider the models selected by each criterion. The models selected are generally close; in fact, they agree exactly for 11 of the 16 real exchange rates. In cases in which the models selected are not identical, the model selected by the SIC is generally more parsimonious because of the more stringent degrees-of-freedom penalty imposed by the SIC.

Maximum likelihood estimates of the models selected by the AIC and SIC are reported in tables 3 and 4. The estimates were obtained by minimizing (3) using the Davidson-Fletcher-Powell algorithm. Convergence was deemed to have occurred if the change in the optimized value of (3) from one iteration to the next was less than or equal to $10^{-8}$. A variety of startup parameter configurations were tried, and in each case convergence to the same vector of estimates was obtained.

The maximum likelihood estimates of $d$ are striking. For each exchange rate and for each model selected, the estimated value of $d$ is consistent with long-run parity; that is, the unit root null is consistently rejected at conventional significance levels. For some of the exchange rates, deviations from parity appear to be long-memory, as

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16 To ensure covariance stationarity, and following standard practice, the models are estimated in first differences and then converted back to levels. This involves no loss of generality because one can factor $(1 - L)^d$ as $(1 - L)(1 - L)^{d-1}$. It does, however, require that we do not distinguish between mean reversion and trend reversion. In other words, we implicitly allow for deviations from parity stemming from productivity differentials, factor endowment differentials, and demand effects, as in Bergstrand (1991).
| Countries       | $d$  | $\phi_1$ | $\phi_2$ | $\phi_3$ | $\theta_1$ | $\theta_2$ | $\theta_3$ |
|-----------------|------|----------|----------|----------|------------|------------|------------|
| **WPI Rates**   |      |          |          |          |            |            |            |
| Belgium/France  | .49  | .37      |          |          |            |            |            |
|                 | (.12)| (.14)    |          |          |            |            |            |
| Belgium/Germany | -.20 | .76      | .47      |          |            |            |            |
|                 | (.27)| (.16)    | (.13)    |          |            |            |            |
| Belgium/United States | -.06 | .79     |          |          |            |            |            |
|                 | (.18)| (.11)    |          |          |            |            |            |
| Belgium/United Kingdom | -.30 | .86     |          |          |            |            |            |
|                 | (.17)| (.10)    |          |          |            |            |            |
| France/Germany  | .66  | -.05     | .74      | .18      | -.21       |            |            |
|                 | (.20)| (.10)    | (.21)    | (.24)    | (.14)      |            |            |
| France/United States | -.46 | 1.57    | -.63     |          |            |            |            |
|                 | (.27)| (.27)    | (.25)    |          |            |            |            |
| France/United Kingdom | -.41 | .91     | .53      |          |            |            |            |
|                 | (.14)| (.06)    | (.10)    |          |            |            |            |
| United States/Germany | -.05 | .84     |          |          |            |            |            |
|                 | (.18)| (.11)    |          |          |            |            |            |
| United States/United Kingdom | -.38 | 1.31    | -.40     |          |            |            |            |
|                 | (.38)| (.41)    | (.26)    |          |            |            |            |
| Germany/United Kingdom | .65  | .46     | -.28     |          |            |            |            |
|                 | (.17)| (.19)    | (.05)    |          |            |            |            |
| **CPI Rates**   |      |          |          |          |            |            |            |
| France/Belgium  | .02  | .92      |          |          |            |            |            |
|                 | (.34)| (.22)    |          |          |            |            |            |
| France/Germany  | .11  | .96      | .52      |          |            |            |            |
|                 | (.12)| (.12)    | (.16)    |          |            |            |            |
| France/Sweden   | .30  | 1.03     | .71      | .28      |            |            |            |
|                 | (.15)| (.17)    | (.21)    | (.14)    |            |            |            |
| Belgium/Germany | -.13 | .98      | .42      |          |            |            |            |
|                 | (.13)| (.15)    | (.18)    |          |            |            |            |
| Belgium/Sweden  | .26  | .35      |          |          |            |            |            |
|                 | (.04)| (.11)    |          |          |            |            |            |
| Sweden/Germany  | -.12 | 1.01     | .59      |          |            |            |            |
|                 | (.12)| (.11)    | (.10)    |          |            |            |            |

Note.—Asymptotic standard errors appear in parentheses.

evidenced by $d$ estimates in the unit interval, but significantly different from both zero and one. For other exchange rates, the deviations from parity appear to be short-memory, as evidenced by $d$ estimates insignificantly different from zero.\(^{17}\)

\(^{17}\) Note that by estimating fractionally integrated models, we can directly assess the amount of uncertainty associated with low-frequency variation in real exchange rates by examining the confidence bands for the estimate of $d$. This stands in marked contrast to the common practice of conditioning on an assumption of $d = 0$ or $d = 1$ (typically after some pretesting).
### Table 4

**Parameter Estimates of Models Selected by the Schwartz Information Criterion**

| Countries          | $d$   | $\phi_1$ | $\phi_2$ | $\phi_3$ | $\theta_1$ | $\theta_2$ | $\theta_3$ |
|--------------------|-------|----------|----------|----------|------------|------------|------------|
| WPI Rates          |       |          |          |          |            |            |            |
| Belgium/France*    | .49   | . . .    | . . .    | . . .    | .37        | . . .      | . . .      |
|                    | (.12) |          |          |          | (.14)      |            |            |
| Belgium/Germany*   | -.20  | .76      | . . .    | . . .    | .47        | . . .      | . . .      |
|                    | (.27) | (.16)    |          |          | (.13)      |            |            |
| Belgium/United States* | -.06  | .79      | . . .    | . . .    | . . .      | . . .      | . . .      |
|                    | (.18) | (.11)    |          |          |            |            |            |
| Belgium/United Kingdom* | -.30  | .86      | . . .    | . . .    | . . .      | . . .      | . . .      |
|                    | (.17) | (.11)    |          |          |            |            |            |
| France/Germany     | .35   | . . .    | . . .    | . . .    | .99        | .51        | - .21      |
|                    | (.09) |          |          |          | (.11)      | (.11)      | (.14)      |
| France/United States* | -.46  | 1.57     | -.63     | . . .    | . . .      | . . .      | . . .      |
|                    | (.27) | (.27)    | (.25)    |          |            |            |            |
| France/United Kingdom | .50   | . . .    | . . .    | . . .    | .56        | . . .      | . . .      |
|                    | (.10) |          |          |          | (.10)      |            |            |
| United States/Germany* | -.05  | .84      | . . .    | . . .    | . . .      | . . .      | . . .      |
|                    | (.18) | (.11)    |          |          |            |            |            |
| United States/United Kingdom* | -.38  | 1.31     | -.40     | . . .    | . . .      | . . .      | . . .      |
|                    | (.38) | (.41)    | (.26)    |          |            |            |            |
| Germany/United Kingdom | .57   | . . .    | . . .    | . . .    | .50        | . . .      | . . .      |
|                    | (.09) |          |          |          | (.08)      |            |            |
| CPI Rates          |       |          |          |          |            |            |            |
| France/Belgium     | .80   | . . .    | . . .    | . . .    | . . .      | . . .      | . . .      |
|                    | (.12) |          |          |          |            |            |            |
| France/Germany*    | .11   | . . .    | . . .    | . . .    | .96        | .52        | . . .      |
|                    | (.12) |          |          |          | (.12)      | (.16)      |            |
| France/Sweden      | .69   | .65      | -.35     | . . .    | . . .      | .28        | . . .      |
|                    | (.12) | (.11)    | (.04)    |          |            | (.14)      |            |
| Belgium/Germany*   | -.13  | . . .    | . . .    | . . .    | .98        | .42        | . . .      |
|                    | (.13) |          |          |          | (.15)      | (.18)      |            |
| Belgium/Sweden*    | .26   | . . .    | . . .    | . . .    | .35        | . . .      | . . .      |
|                    | (.04) |          |          |          | (.11)      |            |            |
| Sweden/Germany*    | -.12  | . . .    | . . .    | . . .    | 1.01       | .59        | . . .      |
|                    | (.12) |          |          |          | (.11)      | (.10)      |            |

*NOTE.*—Asymptotic standard errors appear in parentheses.  
*There is agreement between the SIC-selected and AIC-selected models.

The estimates of the remaining autoregressive and moving average parameters are also of interest. They generally imply that the persistence of deviations from parity implied by the model as a whole is moderately high. In particular, for those models for which the deviations from parity appear to be short-memory (i.e., the estimated value of $d$ is insignificantly different from zero), the configuration of the remaining autoregressive and moving average parameters nevertheless implies substantial shock persistence. Consider, for example, the
United States/Germany rate, which was discussed earlier. The models selected by the AIC and SIC are identical and insignificantly different from an AR(1) in levels. Estimation of the AR(1) model yields a parameter of .81, which implies that the half-life of a shock that moves the real exchange rate away from its parity value is approximately 3 years.

Graphical analysis of the cumulative impulse response functions

Fig. 2.—Cumulative impulse response functions
enables direct assessment of the speed and pattern with which shocks to purchasing power parity are transmitted. In figure 2 we present cumulative impulse response functions for the models selected by the AIC for each of the 16 currencies. Reversion to parity is evident in their eventual decay toward zero; in fact, most of the impulse responses are monotone decreasing. The half-life of a shock to parity, averaged across the currencies, is 2.8 years.
V. Concluding Remarks

An emerging literature, of which our paper is a part, poses a serious challenge to the view that deviations from purchasing power parity are well approximated by martingales. In two papers that foreshadow our work, Frankel (1986, 1989) argues that the martingale hypothesis for the United States/United Kingdom real exchange rate can be rejected with a sufficiently long span of data. Grilli and Kaminsky (1989), Hakkio and Joines (1990), and Lothian (1990)—when using long spans of data—agree.\(^\text{18}\) Additionally, Huizinga (1987), Kaminsky (1987), and Glenn (1989) find some evidence of reversion to parity using variance ratio tests, as do Abuaf and Jorion (1990), who make use of multivariate techniques.

Our paper represents a culmination of the emerging literature, building on Hakkio’s (1986) conjecture that the short spans of data, naive techniques, and limited data sets frequently employed in studies of real exchange rate dynamics might produce low power against alternatives of slow parity reversion. We use (1) long spans of data, (2) flexible time-series representations, and (3) a variety of currencies, with dramatic results. We find that purchasing power parity holds in the long run for each of the currencies studied and that the typical half-life of a shock to parity is approximately 3 years.

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\(^{18}\) Hakkio and Joines (1990) and Hakkio (1991) report that for shorter spans of data they remain unable to reject the martingale hypothesis.
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