Linear voltage regulation in DC-to-DC converters

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Abstract. The article deals with the most dynamically developing branch of power electronics related to solving tasks of voltage regulation in DC-to-DC converters. Linear regulation of the output voltage is carried out using pulse-width modulation. A linear regulatory characteristic is obtained over the entire control range from zero to maximum values. The formula of the output voltage regulation characteristic is obtained through a mathematical model of a pulse-width control method with a variable pulse width.

1. Introduction

The wide variety of control systems can be classified by the most important distinguishing features. At the heart of the functions performed by control systems are the requirements for the processes, implemented by technological installations with specified parameters of power sources, electric drives, or power sources of radio-technical devices. Modern factory produced converters include local and remote control, alphanumeric indicators for displaying input and output voltages, output current, frequency, accuracy of maintaining various parameters and other data.

These installations require particularly accurate regulation in open and closed control systems. And using energy-efficient, controlled electric drive with static electricity converters can improve efficiency, equipment payback, and production profitability [1,2].

2. Principles of converting DC voltage

To improve the consumer properties of products, we can optimize parameters, increase the working frequency of conversion, reduce power losses on power elements, and reduce dynamic loads in the power part of the circuit. Regulation of AC and DC voltages uses pulse-width modulating methods with varying duty cycle [3–5].

Pulse-width DC-to-DC converters convert DC voltage to pulse one, the average value of which needs to be adjusted. The output voltage of these converters (before the output filter), as a rule, has the form of unipolar pulses (Figure 1).
Figure 1. Pulse-width modulation of converters with unipolar pulses of equal duration.

The frequency of conversion depends on the dynamic properties of the switches, of which the converter is made. Due to the constant value of the supplied voltage at the input of the converter, natural commutation of the switches (thyristors) is impossible, which requires using fully-controllable elements (lockable thyristors, transistors). GTO-thyristors allow switching up to 1 kHz, IGBT-transistors – up to about 10 kHz, field-effect transistors – up to 1 MHz and more [1].

The equation of the regulatory characteristic of the pulse-width converter with unipolar and equal in duration pulses (unipolar modulation), is determined by the degree $C$ of regulation of the output voltage:

$$ C = \frac{U_{out}}{U_{in}} = \frac{1}{T U_{in}} \int_0^T U_{in} dt = \frac{t_u}{T} . $$

An essential point in the converters of DC voltage is the desired linear dependence of output voltage on the control effect. The peculiarity of dependence $U_{out} = f(U_{control})$ in pulse-width modulation (PWM) of voltage is the non-linearity of the output characteristic [6–16]. Regulatory performance in this mode of regulation is steeply falling, making it difficult to develop regulators when using microprocessors. Linearity of the characteristic is a huge advantage of the converter, ensuring optimal construction of devices of automatic process control in output circuits of rectifiers.

We have developed the method of modulated pulse-width control of the power elements of the converter with changes in the duration of power pulses, allowing to obtain a linear dependence of output voltage on the control sinusoidal voltage.

Partial linearity of the regulatory characteristics of a controlled rectifier can be obtained using PWM when changing the control angle $\alpha$ according to the arccosine dependence [17].

The authors proposed a method of regulating the output voltage of PWM, allowing to obtain linear regulatory characteristic in the range of 0 to 1.

3. Mathematical description of the proposed linear pulse-width regulation method

The control scheme contains a sinusoidal voltage generator and a triangular pulse generator. Positive half-sinusoid $u(x) = U_c \sin x$ ($U_c$ is the amplitude) over the range of 0 to $\pi$ corresponds to $m$ triangular pulses. The limits of pulses are lines $l_k$, the equations of which are described by the general formula:

$$ u_k(x) = (-1)^{k-1} 2U \left( \frac{m}{\pi} x - k + \frac{1 + (-1)^{k-1}}{2} \right), \ x \in [x_{k-1}, x_k], \ k = 1, 2, \ldots, 2m, $$

where $U_T$ is the amplitude of triangular pulses (see Figure 2a), and abscissas of the splitting points of the $[0, \pi]$ range are defined by the formula:

$$ x_k = \frac{1}{2} \left( \frac{k}{m} \pi - 1 \right). $$
Pulse-width modulation is carried out as follows: the DC signal of duration $\pi$ is broken into rectangular pulses in accordance with the condition: constant voltage $U_0$ is broken into a number of ranges with the presence or absence of voltage by points of intersection $\xi_k$ ($k = 1, 2, \ldots, 2m$) of triangular pulses with a sinusoidal curve. Meanwhile, voltage will be present in the ranges where the sine graph lies above (Figure 2b) or, conversely, below (Figure 2c) the triangular pulses.

Rectangular pulses, therefore, are determined in the case of $u_T(x) < U_c \sin x$ by the formula:

$$x_k = \frac{\pi k}{2m}.$$  \hspace{1cm} (1)

Figure 2. Intersections of the sinusoidal voltage curve with $m$ triangular pulses (a); a sequence of rectangular pulses satisfying the condition $u_T(x) < U_c \sin x$ (b); $u_T(x) > U_c \sin x$ (c)
\[ u_m^-(x) = \begin{cases} U_0, & x \in \bigcup_{k=1}^{m}[\xi_{2k}, \xi_{2k+1}], \\ 0, & x \notin \bigcup_{k=1}^{m}[\xi_{2k}, \xi_{2k+1}] \end{cases} \tag{2} \]

and in the case of \( u_T(x) > U_c \sin x \) – by the formula:

\[ u_m^+(x) = \begin{cases} U_0, & x \in \bigcup_{k=1}^{m}[\xi_{2k-1}, \xi_{2k}], \\ 0, & x \notin \bigcup_{k=1}^{m}[\xi_{2k-1}, \xi_{2k}] \end{cases} \tag{3} \]

It is not possible to analytically obtain the abscissas \( \xi_k \) of the points \( M_k \) where straight lines \( l_k \) cross the sinusoidal curve \( u(x) = U_c \sin x \), so we will do the following:

As a point of crossing of the sinusoidal curve \( u(x) = U_c \sin x \) and straight line \( l_k \), we will take the point lying on this straight line (we will call it \( N_{k_1} \)) the abscissa of which will be \( \delta_k \), and its ordinate \( y(\delta_k) \) we will set as the arithmetic average of the values of the function \( u(x) = U_c \sin x \) on the edges of where the straight line \( l_k \) is defined, i.e. at \( x_{k-1} \) and \( x_k \):

\[ y(\delta_k) = \frac{u(x_{k-1}) + u(x_k)}{2} = \frac{1}{2}(U_c \sin x_{k-1} + U_c \sin x_k). \]

Substituting expressions for \( x_{k-1} \) and \( x_k \) from (1), we obtain:

\[ y(\delta_k) = U_c \cos \frac{\pi}{4m} \cdot \sin \frac{\pi(2k-1)}{4m}. \tag{4} \]

Coordinates of the point \( N_k(\delta_k, y(\delta_k)) \) satisfy the equation of the straight line \( l_k \):

\[ y(\delta_k) = (-1)^{k-1}2U_c \left( \frac{m}{\pi} \delta_k - k + \frac{1 + (-1)^{k-1}}{2} \right). \tag{5} \]

By equating right parts of the formulas (4) and (5), we will get the expression for \( \delta_k \):

\[ \delta_k = \left( (-1)^{k-1} \frac{U_c}{2U_i} \cos \frac{\pi}{4m} \sin \frac{\pi(2k-1)}{4m} + k - \frac{1 + (-1)^{k-1}}{2} \right) \frac{\pi}{m}. \tag{6} \]

Since the boundaries of the rectangular pulses are defined by points \( \delta_k \) (5) instead of points \( \xi_k \), the formulas (2) and (3) that determine the sequence of these impulses will also change:

for the case \( u_T(x) < U_c \sin x \)
\[ u^{-}_n(x) = \begin{cases} U_0, & x \in \bigcup_{k=1}^{m} [\delta_{2k}, \delta_{2k+1}], \\ 0, & x \not\in \bigcup_{k=1}^{m} [\delta_{2k}, \delta_{2k+1}] \end{cases} \quad (7) \]

and for the case \( u_T(x) > U_c \sin x \)

\[ u^{+}_n(x) = \begin{cases} U_0, & x \in \bigcup_{k=1}^{m} [\delta_{2k-1}, \delta_{2k}], \\ 0, & x \not\in \bigcup_{k=1}^{m} [\delta_{2k-1}, \delta_{2k}] \end{cases} \quad (8) \]

Now we will calculate the average voltage for each case:

1. \( u_T(x) < U_c \sin x \) \( \text{formula (7)}. \)

\[
U_{\text{avg}}^{-} = \frac{1}{\pi} \int_{0}^{\pi} u^{-}_n(x)dx = \frac{1}{\pi} \sum_{k=1}^{m} \int_{\delta_{2k}}^{\delta_{2k+1}} U_0 dx = \frac{U_0}{\pi} \sum_{k=1}^{m} (\delta_{2k+1} - \delta_{2k}) = \\
= \frac{U_0 U_c \cos^2 \frac{\pi}{4m} \cdot \tan^{-1} \frac{\pi}{2m}}{U_c \cdot m}.
\]

For \( m \gg 1: \cos \frac{\pi}{4m} \rightarrow 1, \tan \frac{\pi}{2m} = \frac{\pi}{2m} \) and the formula takes on the form of:

\[
U_{\text{avg}}^{-} = \frac{2U_0 U_c}{\pi U_c}.
\quad (9)
\]

2. \( u_T(x) > U_c \sin x \) \( \text{formula (8)}. \)

\[
U_{\text{avg}}^{+} = \frac{1}{\pi} \int_{0}^{\pi} u^{+}_n(x)dx = \frac{1}{\pi} \sum_{k=1}^{m} \int_{\delta_{2k-1}}^{\delta_{2k}} U_0 dx = \frac{U_0}{\pi} \sum_{k=1}^{m} (\delta_{2k} - \delta_{2k-1}),
\]

\[
U_{\text{avg}}^{+} = U_0 - \frac{2U_0 U_c}{\pi U_c}.
\quad (10)
\]

On Figure 3 we present the regulatory characteristic (9) and (10) of the converter derived from this method of regulation.
According to this regulatory characteristic, our method allows to regulate the DC-to-DC converter voltage output linearly in the entire range of 0 to 1.

**Computer simulation**
Computer simulation was done in the MatLab Simulink environment. A diagram of our model including a measuring device and a display is shown in Figure 4.

![Simulation model of a voltage regulator](Diagram)

The results of computer simulations are shown in table 1. Columns 4 and 5 show the obtained average voltage values of rectangular pulses with varying duty cycle depending on the amplitude of the sinusoidal voltage under two conditions. Columns 2 and 3 show the results of theoretical calculations performed according to formulas (9) and (10), respectively.
Comparing the results of calculation and computer simulation, we should note their minimal discrepancy (within 1...2%).

Figure 5 shows the graphs of the regulatory characteristics plotted using the simulation data.

![Figure 5. Regulatory characteristics of the DC-to-DC converter obtained through computer simulation](image)

Similarly to the calculated data, the simulation results reflect the linearity of the regulatory characteristic.

4. Conclusion

As a result of using the PWM method, the regulatory characteristics of DC voltage become linear and allow for voltage regulation from zero to maximum values. At the same time, it is much easier to use microprocessor technology to make such voltage regulators. Results of the developed algorithm of the control device utilizing the PWM method can find broad use in power electronics, electric drives, and other areas.

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References

[1] Semenov B Y 2011 Power electronics: Professional solutions (Moscow: Solon-Press)
[2] Sokolovsky G G 2006 AC electric drives with frequency regulation (Moscow: Academia)
[3] Veselovskiy A P, Budko P A, Vinogradenko A M and Kosareva L I 2018 Implementation of the way of converting AC voltage Problems of technical support of troops in modern conditions: theses of reports III intercollegiate NPC (Petersburg, Military Academy of Communications) pp 172-176
[4] Budko P A, Veselovskiy A P, Vinogradenko A M and Kosareva L I 2018 Voltage regulation in converters of high-frequency pulses with varying duty cycles Mechatronics, automation, control 8 (19) 516–522 DOI: 10.17587/mau.19.516-522
[5] Voytyuk I N, Zamyatina E N and Zamyatin E O 2019 Increasing the energy efficiency of an enterprise by point compensating of power quality distortions Proceedings of the International Scientific Conference on Energy, Environmental and Construction Engineering (EECE-2019)
[6] Romash E M, Drabovich Y I, Yurchenko N N and Shevchenko P N 1988 High-frequency transistor converters (Moscow: Radio and communication)
[7] Abraham L, Heumann K, Koppelmann F 1964 Wechselrichter fur Drehzahlsteuerung von Kaghettiemenmotoren AEG–Mitt. 2 89-106
[8] Volkov A G 2014 Mathematical model of AC-AC converter without passive elements in DC-link Source of the Document International Conference of Young Specialists on Micro/Nanotechnologies and Electron Devices (EDM 2014) 403-407
[9] Shklyarskiy Y E, Shklyarskiy A Y and Zamyatin E O 2019 Analysis of distortion-related electric power losses in aluminum industry Tsvetye Metally 4 84-91
[10] Vinogradenko A M, Veselovskiy A P, Vzesniewski S V and Galvas A V 2018 The way and the synchronization of voltage control systems Practical power electronics 2 (70) 53-55
[11] Skamyin A N and Dobush V S 2018 Analysis of nonlinear load influence on operation of compensating devices IOP Conference Series: Earth and Environmental Science 194 (5) 052023 DOI: 10.1088/1755-1315/194/5/052023.
[12] Kopteva A V, Starshaya V V, Malarev V I and Koptev V Yu 2019 Improving the eficiency of petroleum transport systems by operative monitoring of oil flows and detection of ille-gal incuts Topical Issues of Rational Use of Natural Resources Proceedings of the XV International Forum-Contest of Students and Young Researchers under the auspices of UNESCO (1) (Petersburg Mining University, Russia) pp 406-415
[13] Batueva D and Shklyarskiy J 2019 Increasing Efficiency of Using Wind Diesel Complexes through Intellectual Forecasting Power Consumption IEEE Conference of Russian Young Researchers in Electrical and Electronic Engineering (EIConRus) pp 434-436 Doi: 10.1109/EIConRus.2019.8657158
[14] Savard C and Iakovleva E V 2019 A suggested improvement for small autonomous energy system reliability by reducing heat and excess charges Batteries 5(1) 29 Doi: 10.1109/EIConRus.2019.8657097.
[15] Chmilenko F V and Rastvorova I I 2018 Improvement of quality of aluminum ingots at electromagnetic processing Journal of Physics: Conference Series 1118(1) 012030
[16] Dobush V, Belsky A and Skamyin A 2020 Electrical Complex for Autonomous Power Supply of Oil Leakage Detection Systems in Pipelines Journal of Physics: Conference Series 1441 Doi: 012021. 10.1088/1742-6596/1441/1/012021
[17] Veselovskiy A P, Budko P A, Buryanov O N and Vinogradenko A M 2017 Features of the systems of control of ventral converters Problems of technical support of troops in modern conditions: theses of reports II intercollegiate NPC (Petersburg, Military Academy of Communications, 2017) pp 150–154