Exploration of primary school teacher students’ understanding in fraction concept

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Abstract. Fraction is a very complex mathematical material which must be mastered by every teacher who teaches in elementary school. Fraction concept understanding is very prominent because it is a prerequisite for comprehension in advanced mathematical concepts. In contrast with the other mathematical subjects in general, the fraction concept is considered as difficult material because it has many definitions. It creates a sustainable pedagogical challenge among the mathematics education community. Therefore, it must be ensured that all students of primary school teacher (PGSD students) have understood the concept of fraction before they teach in school. This research was conducted to explore the comprehension of the fraction concept among PGSD students. This research was in a form of qualitative research by employing test questions related to the concept of fractions toward 23 PGSD students. In this research, comprehension related to the aspect was tested through an analysis of non-standard problem solving processes that were verified by interviews. The results of the analysis showed that none of the students were completely correct in solving the problem. The main cause of the occurrence of the many errors in respondents was because they experienced false convictions or misconception related to the concept of fraction. For further research, we believe that PGSD students would have a proper comprehension of the fraction concept if at the beginning of the learning they receive reinforcement related to the concept.

1. Introduction

Mathematics is a subject that has been taught starting from elementary school to college in which one of the materials is fraction. Fraction is very difficult and complex mathematics material [1, 5] which must be mastered by every primary school teacher. Fraction plays a central role in mathematics learning [1, 2], but many students even teachers have difficulty, false conviction, or misconception in learning the topic [3, 4, 6]. This is because the fractional nature does not have single meaning [7]. Comprehension of the fraction concept is very prominent because it is a prerequisite for comprehending advanced mathematical concepts both at higher level of education and in the context of everyday problems [1]. However, the concept of fraction is not a simple concept; it has its own uniqueness that is different from natural numbers and integers [8]. The concept of fraction is considered as a difficult-to-learn and difficult-to-teach concept which creates a sustainable pedagogical challenge among the mathematics education community, including elementary school teachers [2]. It is supported by the previous research result that shows students of primary school teacher (PGSD students) in Indonesia experience misconception in fraction concept as part whole congruent part [9]. Therefore, every elementary school teacher must understand fraction well before teaching in school because they are the first to introduce fraction material to the students. This is what underlies the researchers, which is to know how fraction concept understanding among PGSD students as prospective elementary school teachers.
Fraction has seven meanings or sub-constructs which are sorted according to their level of difficulty [10], namely: (i) Part group congruent part. In this category, students associate fraction by paying attention to "a" of the set object. Fraction of a/b is interpreted as an area which is divided into b congruent part with regard to the part; (ii) Part whole congruent part. In this category, learners associate fraction of a/b with geometric region is divided into b congruent part and pay attention to the “a” section. Fraction of a/b is interpreted as a set consisting of b objects that are congruent with regard to the object; (iii) non-congruent part group part. In this category, learners associate fraction a/b with a set consisting of b incongruent objects and pay attention to the object in the set or a/b fraction is simply interpreted as an area that is divided into b part and that is not congruent with regard to their parts; (iv) Part group comparison. In this category, students associate the set of a/b with a relative ratio of two sets which are A and B in which the number of objects in set A is a and set B is all congruent objects or fractions of a/b interpreted as relative comparisons of two regions A and B. Number of regions that is congruent at A is expressed as a while B is b; (v) Number line. It is a number line that contains line segments with a length of one unit. The line segment is divided into b that has equal parts. Students associate fraction a/b by observing a point (representing “a” section) on the number line; (vi) Parts whole comparison. Learners associate fraction a/b with a relative ratio of two geometric areas A and B. The number of congruent parts in geometric area A is a, while in geometric area B is b in which all areas in A and B are congruent; and (vii) whole non-congruent parts. Learners associate fraction of a/b with geometric regions that have been divided into b which has equal parts in broad, but not congruent and pay attention to “a” section. Thus, the seven sub-constructs can be grouped into three models [11], namely: Parts group model, consisting of sub-constructs (i), (ii), and (iv); Parts whole model, consisting of sub-constructs (ii), (vi), and (vii); and the Number line model, consisting of sub-constructs (v). These three model groups will be analyzed in this article to explore the comprehension of PGSD students regarding fraction materials.

2. Methodology

This research was conducted in one of universities in Makassar, Indonesia. The subjects of this research were 23 students of primary school teacher in the fourth semester who were taking concentration in mathematics and had studied fraction material. They were selected by convenience sampling; that is, the subjects were contacted by the first author and were recruited on the basis of their willingness to participate in the study [12]. All subjects were asked to solve problems related to the fraction concept consisting of 3 questions. The first question described the concept of fraction as part of an area, the second question described the concept of fraction as part of a set, and the third problem described the concept of fraction as a number line. The problem was adopted from some literature with little modification. The problem was in Figure 1 below:

1. Look at the triangle figure below!

Which figure shows \( \frac{1}{2} \) ?

3. Look at the number line figure below!

What fraction is shown by the shaded area on the number line ?

2. Look at the pictures and statements below!

Will be taken 3 green lid jars from 8 jars

Do the figures (A) and (B) show \( \frac{3}{8} \) ?

Figure 1. Test questions
The research method used was direct observation by researchers on the research subject during the administration of problem solving questions, which was then followed by semi-structured interviews. Interviews were conducted to explore more deeply the respondents’ comprehension of the fraction concept and to equate perceptions between respondents and researchers regarding the answers they had written. In order to facilitate the research process, mobile phones were used as audio recording devices.

In the interview, the subject was given a paper written on 3 mathematics questions about fraction. Subjects were asked to answer the question as well as the reasons and then followed by interviews. If the subject’s answer was correct and accompanied by the right reasons, then the observation was declared complete. However, if the subject's reason was not clear, the researcher would clarify it. If the subject gave an answer without reason, the researcher would ask for confirmation. When the solution was wrong, the researcher intervened by directing the subject to get the right answer.

In this research, the comprehension of the fraction concept was tested through an analysis of non-standard problem solving processes. Schoenfeld (1985) distinguished four categories of knowledge and behavior needed to understand a person's process in solving mathematical problems, including: resources (prior knowledge of individuals to solve problems), heuristics (strategies and techniques for solving non-standard problems), control (global decisions the selection and implementation of resources strategies, and the belief system (one's mathematical perspective) [13]. In this research, special attention was done to the resources - the subjects' knowledge about fraction revealed in the problem solving process.

3. Results and Discussion

According to the results of the research that has been collected, none of the questions could be answered correctly by all respondents. In general, the distribution of respondents' answers to the three questions given was presented in table 1 below:

| The Problem | True Answer (%) | False Answer (%) |
|-------------|-----------------|------------------|
| 1           | 0 (0%)          | 23 (100%)        |
| 2           | 17 (73.91%)     | 6 (26.09%)       |
| 3           | 4 (17.39%)      | 19 (82.61%)      |

A more detailed description of the respondent's answers to the three problems given was presented in the following table:

3.1. Respondents' Answer to Problem Number 1

Based on Table 1, 100% of respondents gave wrong answers related to problems 1. All respondents considered that only image c showed fractions $\frac{1}{2}$. Figure 2 below presented some examples of answers given by respondents:

- The answer is figure C because the image divides triangle into 2 equal parts.
- Figure C is the correct answer because based on the definition of fractions, an area is said to be $\frac{1}{2}$ if the area is divided into two equal areas.
- Answer: figure C
  Reason: Figure C shows a comparison of black areas as large as white areas
- Picture C. Because the shaded area is as large as the unshaded area.

Figure 2. Respondents' answers to problem 1
Based on Figure 2, it could be concluded that the respondents claimed the images a and b which did not show a fraction of $\frac{1}{2}$ because the two images did not divide the two triangles into the same size. According to them, the shaded part must be as large as the unshaded part. That was what causes the correct answer (according to respondents) to problem 1 was image c. The reason was explained by all respondents, even though the editor of the word written was different, but the intention was the same.

In regards of responding to the answer given, the researcher conducted an interview and obtained an interview quote which was presented in Table 2 below:

| Interview Script |
|------------------|
| **Problem 1** Interviewer : Why the figures a and b don’t show $\frac{1}{2}$?  |
| Respondent: Because the shaded area and unshaded area are not equal. |
| Interviewer: How do you share the cake to 3 people if you have an irregular shape cake? (The interviewer tries to ask a different case with the same concept) |
| Respondent: I cut the cake into 3 parts, then I give it to the person |
| Interviewer: Are the cakes which they get same? |
| Respondent: No. Because it’s difficult to share them equally. |
| Interviewer: How many parts are accepted by each person, if the cake is divided into 3 parts? |
| Respondent: One-third |
| Interviewer: Even though the size is unequally? |
| Respondent: In the real world it can be said as a third, but in mathematics it is not one third because the size is not the same |
| Interviewer: Ok. In your opinion, what is fraction according to the problem 1? |
| Respondent: The triangle is considered one half when it is divided into two equal parts (point to the triangle image in problem 1). |

From the results of interviews, researchers tried to provide a different problem with the same concept. It aimed to find out how far the respondent's comprehension was related to the concept of fractions as part of the whole. The problem given was: "if you have a cake that has an irregular shape, then asked to share the cake with three people, can every part that the person receive can be expressed as $\frac{1}{3}$?". An interesting response given by one respondent was presented in table 2. He said that in everyday life, the portion of the cake was $\frac{1}{3}$ but mathematically it was not true. According to him, in mathematics part of an area could be said to be fraction if the area was divided equally. That was the concept of fraction according to the respondent. All respondents seemed so confident about the answers that none of the respondents changed their answers at the time of the interview.

In solving problem 1, respondents tried to use their initial knowledge regarding the concept of fraction. This was in accordance with Schoenfeld's opinion that one of the behaviors a person showed in solving problems was resources (using initial knowledge to solve problems) [13] although there was no guarantee that the knowledge was conceptually correct. Problem 1 revealed that respondents experienced misconceptions, which is also commonly known as false belief [14] or false convictions [3], in which the solution that was generated based on a false comprehension where the individual seemed to believe the truth of the answer even though the answer was wrong. Respondents believed that if an area was partitioned into n parts of the same size, then that part could be expressed in $\frac{1}{n}$. However, if an area was partitioned into n unequal part, then that part could not be said to be $\frac{1}{n}$. This
was what was believed to be a source of mistaken comprehension for respondents. They assumed that the only way to get $\frac{1}{n}$ part of an area was to divide the area into $n$ parts of the same size.

3.2. Respondents’ Answer to Problem Number 2

For problem number 2, it would be divided into two categories, which were respondents who answered correctly and respondents who answered incorrectly. The following was the description:

3.2.1. Respondents Who Answered Correctly.

Based on table 1, 73.91% of respondents answered problem number 2 correctly. Figure 3 presented several examples of answers given by respondents.

Based on Figure 3, the respondent stated that in problem number 2, (image (a) and image (b)) represented $\frac{3}{8}$ fractions. The answer which was seen from the concept of fraction was true. However, special attention was needed to the reasons given by respondents. All respondents who answered correctly gave the reason that 3 jars (in figure a) were part of the 8 jars available or 3 cups (in figure b) were part of the 8 glasses available. Judging from the statements written by respondents, the answers had led to the concept of fraction as a whole part of a set. But interestingly, these respondents gave the wrong answer when solving problem number 1 which was discussed earlier, which was the concept of fraction as part of the whole area. Therefore, further research was needed to be carried out through interviews regarding this matter. The excerpt of the interview was presented in table 3 below:

| Interview Script |
|------------------|
| Problem 2     |
| Interviewer : What is the meaning of your answer (point to the problem 2 picture a)? |
| Respondent: There are 8 jars taken 3 jarsand it means $\frac{3}{8}$. |
| Interviewer: But the shape is unequall. How about that? |
| Respondent: Hmm… (thought for a moment)…No problem. The jars are separate. The jar does not come from one whole part. |
| Interviewer: so what is the difference with the irregular shape cake? (try to remind previous problem) |
| Respondent: Hmm…(thinking)…I think the correct answer is only picture (b) because all glasses are equal. So just picture b represents $\frac{3}{8}$. |

Figure 3. Respondents' answers to problem number 2

Table 3. Excerpt of interview regarding problem number 2
From the interview’s excerpt in table 3, the respondent was initially convinced by the truth of the answer. He said that 3 jars taken from 8 jars could show $\frac{3}{8}$ fractions. However, when the interviewer related it to problem number 1 which was previously discussed, the respondent became confused and hesitant about the answer. Therefore, he changed the answer and said that image (a) in problem 1 did not show a $\frac{3}{8}$ fraction at the end. Only image (b) showed $\frac{3}{8}$ fractions. When traced more deeply, the respondent considered image (b) to be correct because all glasses were the same size. The problem experienced by the respondent was similar to problem number 1, which only considered that part of a set was said to be fraction, if the parts were congruent. In fact, part of a set could also be said to be fractions even though the parts were not congruent [10]. Thus, even though the respondent's answer was correct at first, the concept he understood was wrong. This meant that the respondent experienced misconception, which was a mistake in assuming a concept repeatedly so that it became a habit [4].

3.2.2. Respondents Who Answered Incorrectly.
Based on table 1, 73.91% of respondents answered correctly for the problem number 2. Some examples of answers given by respondents were presented in Figure 4 below.

![Figure 4](image)

**Figure 4.** Respondents' answers to problem number 2

Based on Figure 4, respondents assumed that only image b expressed a $\frac{3}{8}$ fraction because the size of the eight glasses was equal. Meanwhile, a picture could not be expressed as a $\frac{3}{8}$ fraction because the size of the jars was not as large. The reason given by respondents regarding problem number 2 was almost the same as problem number 1. In this case, the respondent could not understand the concept of fraction as part of a set whose parts were not congruent. This further clarified the respondents' mistakes in comprehension the concept of fraction.

3.3. Respondents’ Answer to Problem Number 3
Just like the previous discussion, the respondent's answer to problem number 3 would be divided into two categories, the following was the explanation.

3.3.1. Respondents Who Answer Correctly.
Based on table 1, respondents (17.39%) answered correctly for the problem number 3. The entire correct answer given by the respondent was explained in Figure 5 below.
Based on Figure 5, the answer given by the respondent was correct. The reason given by the respondent was in accordance with the concept of the number line. Although the question given was related to fraction, the placement of the fractional position in the number line was the same as an integer. In order to show that the respondent's comprehension was correct, the researcher gave a different question with the same concept. The result of the answers showed that the four respondents already had a proper comprehension of the fraction concept as number lines. This was clearly different from problem number 2 where respondents did not understand the concept even though the answers given were correct. However, it still needed to be a concern because out of the 23 respondents, only 4 people comprehended the concept of fraction as a number line so that they could give the correct answer.

3.3.2. Respondents Who Answer Incorrectly.

Based on table 1, 82.61% of respondents gave incorrect answers related to problem number 3 regarding fraction as number lines. When it was seen from the problem given, all respondent should give the correct answer because the problem was adapted from the fifth grade elementary textbook. However, this was not in accordance with the expectation of the researchers. Many of them gave quite surprising answers. Some examples of answers were presented in figure 6 below:

- Answer and reason: 4/5 because 0 and 1 countless.
- Answer and reason: 5/8 because the shaded area is in fifth position
- Answer and reason: -1 because each left-hand number line is negative
- 1/3 because before 1 there were 0
- Answer: Fraction from shaded areas is 5/7
  Reason: there are 7 boxes and shaded boxes in the fifth box

Figure 6. Respondents' answers to problem number 3
Figure 6 was the respondent's different answers related to problem number 3 in which there were 5 answers (wrong) from different respondents. In order to trace the purpose of the answer, the researcher conducted an interview and obtained the results as shown in Table 4 below:

**Table 4. Description of respondents' answers to problem number 3**

| Answer | Description |
|--------|-------------|
| 4/5    | Respondent who answer it considered 0 and 1 were not counted, so there were 5 remaining areas. He divided the 5 areas so that the shaded part was 4/5. It means that the respondent experience false convictions. |
| 5/8    | The respondent who answered it initially thought that many of the boxes in the question were 8, but it turned out that he had miscalculated because the number of boxes in the question was only 7. Even so, he changed the answer to 5/7. It means that the respondent initially had a mistake then false convictions. |
| -1     | The respondent who answered it focused on the last box containing the number 1. He did not pay attention to the first box containing the number 0. Therefore, he assumed that the two left-hand units of the number 1 in the number line were -1. What's interesting is that he considers the number 0 as just a deceiver of the problem |
| 1/3    | Respondent who answered it wrote the following reasons: |
|        | ![Diagram showing the number line with 1/3 highlighted] |
|        | The above reasons clearly indicate that respondents experience false convictions. |
| 5/7    | Respondent who answered it wrote the following reasons: |
|        | ![Diagram showing the shaded part of 5 boxes] |
|        | It shows that respondents experience false convictions. |

**4. Conclusions**

The results showed that the fraction concept was grouped into 3 models [11], namely fraction as part of an area, fraction as part of a set, and fraction as number lines, were not comprehended correctly by PGSD students. Their comprehension of the fraction concept was wrong so that none of the students were completely correct in solving the given questions. Problems experienced by students for the concept of fraction as part of an area and as part of a common set, which was in their comprehension that part of an area or set could be said to be fraction only if the part was equal or congruent. If an area was divided into n equal parts, then that part could be expressed as 1/n. However, if the area was not equal, then none of the parts could be expressed as 1/n. This also applied to fraction as part of a set. From this, it could be concluded that PGSD students experienced false convictions [3] related to the concept of fraction. For the concept of fraction as number lines, 82.61% of respondents gave the wrong answer. In this case, the respondent also experienced misconceptions or false convictions [3]. Thus, it could be concluded that PGSD students experienced misconceptions or false convictions related to the concept of fractions.
According to the opinion of the author, misconceptions or false convictions experienced by respondents occurred because they were not introduced to the concept of fraction at the beginning of learning. Therefore, as to overcome these problems, it was necessary to make a learning design that provided a strengthening attempt of the concept of fraction. This was expected to minimize problems that occurred as revealed in this research.

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