INVESTIGATION OF $W^+W^-\gamma$ COUPLINGS

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Abstract

We reviewed how measurements of weak boson production at high energy $ep$ and $e\gamma$ collisions can provide important information on anomalous $W^+W^-\gamma$ couplings. We also considered the single muon production through the virtual $W$-decay at the Pohang Light Source (PLS) facility, and found this process is not adequate to be detected at the PLS until a large luminosity ($\sim 10^{33}/\text{sec/cm}^2$) Free Electron Laser is installed.

1 Introduction

Despite impressive experimental confirmation of the correctness of the Standard Model (SM), the most direct consequence of the $SU(2) \times U(1)$ gauge symmetry, the nonabelian self-couplings of $W$, $Z$, and photon remains poorly measured to date. Furthermore, gauge boson coupling strengths are strongly constrained by gauge invariance, and are sensitive to deviations from the SM. Hence, experimental bounds on these couplings might shed light on new physics beyond the SM.

In order to parametrize non-standard effects, it is important to know what sort of additional couplings can arise once the restrictions due to gauge invariance are lifted. As has been previously shown\(^*\), there can be 14 or more non-standard couplings.

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in the most general case. To keep the analysis manageable, we restrict ourselves to C, P and $U(1)_{\text{em}}$ conserving couplings. This restriction leads to just two anomalous form factors for the $WW\gamma$ couplings, traditionally denoted by $\lambda_\gamma$ and $\kappa_\gamma$ in the $WW\gamma$ sector of the SM, which can be related to the anomalous electric quadrupole and the anomalous magnetic dipole moment of the $W$. In the SM at tree level, $\lambda_\gamma = 0$ and $\kappa_\gamma = 1$. At present the best experimental limits, $-3.6 < \lambda_\gamma < 3.5$ and $-3.5 < \kappa_\gamma < 5.9$, are from a recent analysis of the $W\gamma$ production at $sp\bar{p}$s by UA(2) collaboration [3]. While these bounds are compatible with the SM, they are still too weak to really be considered as a precision test of the SM. Furthermore, in the absence of beam polarization, it is unlikely that there will be a significant improvement from the study of $W$ pair production at LEP-II [4].

At future high energy $e^+e^-$ and $e\gamma$ colliders, probing of the $WW\gamma$ vertex can be performed more precisely [5, 6]. One can consider several processes at those colliders. Among them the process $e + \gamma \to W + \nu$ has been preferred. This process has several advantage over the others such as $e^+e^- \to W^+W^-$ which also has $WWZ$ vertex. If we restrict the decay products of $W$ as $\mu + \bar{\nu}_\mu$, we have a very clean, virtually background-free, events. There are no final particles detected other than $\mu$ and missing $p_T$ is attributed to the two neutrinos($\nu_e, \bar{\nu}_\mu$). In view of the detected particle, we must take into account the process, $e + \gamma \to W^* (\to \mu + \bar{\nu}_\mu) + \nu_e$.

We can also consider photoproduction of a single $W$ boson at $ep$ colliders. In $ep$ collision, hadronic jets are produced due to the subprocess $\gamma + q \to W + q'$ and it will provide a precise test of the structure of the Standard Model $WW\gamma$ vertex. And the situation there is much cleaner, for example, than in $pp$ or $p\bar{p}$ colliders, where a $W$ and a photon have to be identified in the final state [3].

Theoretical studies of the $WW\gamma$ vertex at $ep$ colliders have been performed [7, 8, 9]. The measurement of $\kappa_\gamma$ at $ep$ colliders using the shape of the $p_T$ distribution of $W$ production at large $p_T$ has been previously investigated in [7]. However, this method suffers from the disadvantage of being sensitive to uncalculated higher-order QCD corrections, uncertainties in the parton distribution of the photon, experimental systematic uncertainties, etc [11]. We have previously found [8, 9] that a measurement of the anomalous coupling in the $WW\gamma$ vertex at $ep$ colliders can best be achieved by considering the ratio of the $W$ and $Z$ production cross sections. The advantage of using a cross section ratio is that uncertainties from the luminosity, structure functions, higher-order corrections, QCD scale, etc. tend to cancel [8]. Recently, we investigated the possibility of measuring both $\kappa_\gamma$ and $\lambda_\gamma$ at the same time by considering the total cross sections of massive gauge bosons $W$ and $Z$ at $ep$ colliders [9].

In this paper, we study the anomalous $WW\gamma$ vertex by using $ep$ and $e\gamma$ colliders. In section 2, the most general $WWV (= \gamma, Z)$ vertex, which is C and P even, is reviewed. We discuss possible processes which produce single $W$ in $e\gamma$ and $ep$ collision. In $ep$ collision, we review the techniques to deal with the single $W$ production process.
in detail. We show that the ratio of $W$ and $Z$ production cross sections is particu-
larly well suited to an experimental determination of the anomalous $WW\gamma$ coupling
parameters $\kappa_\gamma$ and $\lambda_\gamma$, being relatively insensitive to uncertainties in the theoretical
and experimental parameters. Section 3 is devoted to the discussion of the process
$\gamma + e \rightarrow \mu + \bar{\nu}_\mu + \nu_e$. In view of the decay products, we emphasize the advantages of
the process and derive the simplified squared amplitude of this process, where $W$ is
virtual, in a factorized form. Discussion is given in section 4.

2 Reviews on the Anomalous Tri-boson Vertex in
Single $W$ Production

If we restrict ourselves to C and P even couplings with electromagnetic gauge invar-
ience, the most general $WWV(V \equiv \gamma, Z)$ vertex(Fig. 1) can be parametrized in terms
of an effective Lagrangian\[1\]

\[
L_{eff}^{WWV} / g_{WWV} = i g_1^V (W_{\mu\nu} W^\mu V^\nu - W_{\mu} V_{\nu} W_{\mu\nu}) + i \kappa_\gamma W_{\mu} W_{\nu} V_{\mu} V_{\nu} + \frac{i \lambda_\gamma}{m_W^2} W_{\rho\mu} W_{\mu\nu} V^\rho V^\nu ,
\]

where $W^\mu$ and $V^\mu$ stand for the $W^-$ and the $V$ field, respectively, and $W_{\mu\nu} \equiv \partial_\mu W_\nu - 
\partial_\nu W_\mu$, $V_{\mu\nu} \equiv \partial_\mu V_\nu - \partial_\nu V_\mu$, $g_{WW\gamma} = -e$ and $g_{WWZ} = -e \cot \theta_W$. In SM, $g_1^V = \kappa_\gamma = 1$
and $\lambda_\gamma = 0$. The static properties of the $W$, the magnetic dipole moment ($\mu_w$) and
electric quadrupole moment ($Q_w$) of the $W$ are related\[2\] to these couplings as,

\[
\mu_w = \frac{e}{2 m_w} (1 + \kappa_\gamma + \lambda_\gamma) \quad \text{and} \quad Q_w = -\frac{e}{m_w^2} (\kappa_\gamma - \lambda_\gamma).
\]

Electromagnetic gauge invariance requires $g_1^Z = 1$ and five anomalous couplings in-
volved in Eq.(1) survive: $\Delta g_1^Z \equiv g_1^Z - 1, \Delta \kappa_\gamma \equiv \kappa_\gamma - 1, \Delta \kappa_Z \equiv \kappa_Z - 1, \lambda_\gamma$ and $\lambda_Z$. These five couplings are reduced to a smaller number by symmetry requirements\[10\].
If we require the global $SU(2)_L$ symmetry, then $\lambda \equiv \lambda_\gamma = \lambda_Z$ and the others are zero. Requiring an intrinsic $SU(2)_L$ symmetry, we have four independent couplings with the relation,

$$1 + \Delta g^1_Z = -\tan^2 \theta_w \frac{\Delta \kappa_\gamma}{\Delta \kappa_Z}.$$ (3)

For $WW\gamma$ coupling only, there are only two free parameters $\lambda_\gamma$ and $\Delta \kappa_\gamma$.

To investigate the $WW\gamma$ vertex, we first review single $W$ production processes in $e\gamma$ and $ep$ collision. The process $e + \gamma \rightarrow W + \nu_e$ is our main concern in $e\gamma$ collision. But at $ep$ collider, single $W$ production may also be possible via the process $\gamma + q \rightarrow q^{(l)} + W$. Fortunately, the cross sections of the two kinds of processes are closely related with each other as shown below. So we consider the two processes simultaneously. The relevant helicity amplitude may be obtained directly from Ref.\[7\]. And the hard scattering cross section of the process $\gamma + q \rightarrow q^{(l)} + W$ is given in Ref.\[7\] as

$$\left(\frac{d\hat{\sigma}^D}{dt}\right) (\gamma + q \rightarrow q^{(l)} + V) = \frac{1}{16\pi \bar{s}^2} \Sigma |V|^2,$$ (4a)

with

$$\Sigma |V = Z|^2 = -(g^2 e^2 e_q^2 g_q^2) T_0(\hat{u}, \hat{t}, \hat{s}, m_Z^2)/2,$$

$$\Sigma |V = W|^2 = -(g^2 e^2 |V_{qq'}|^2) T(\hat{u}, \hat{t}, \hat{s}, m_W^2, |e_q|, \kappa_\gamma, \lambda_\gamma)/2,$$

$$\Sigma |V = W|^2_{SM} = -(g^2 e^2 |V_{qq'}|^2) T(\hat{u}, \hat{t}, \hat{s}, m_W^2, |e_q|, 1, 0)/2$$

$$= -(g^2 e^2 |V_{qq'}|^2) \left( |e_q| - \frac{s}{s + \hat{t}} \right)^2 T_0(\hat{u}, \hat{t}, \hat{s}, m_W^2)/2,$$ (4b)

and

$$g^2_q = \frac{1}{2} (1 - 4 |e_q|^2 \sin^2 \theta_w + 8 |e_q|^2 \sin^4 \theta_w), \quad \sin^2 \theta_w = 0.23,$$

where the subscript SM denotes the Standard Model parametrization with $\kappa_\gamma = 1$, $\lambda_\gamma = 0$, and where

$$T_0 \left( \hat{s}, \hat{t}, \hat{u}, m_\nu^2 \right) = \left( \hat{t}^2 + \hat{u}^2 + 2 \hat{s} m_\nu^2 \right),$$

$$T \left( \hat{s}, \hat{t}, \hat{u}, m_w^2, |e_q|, \kappa_\gamma, \lambda_\gamma \right) = (|e_q| - 1) \frac{\hat{u}}{\hat{t}} + |e_q| \frac{\hat{t}}{\hat{u}} + 2 |e_q| (|e_q| - 1) m_w^2 \frac{\hat{u}}{\hat{t}}$$

$$- \left( (|e_q| - 1) \frac{\hat{u}}{\hat{t}} - |e_q| \frac{\hat{t}}{\hat{u}} \right) (2 \hat{s} m_w^2 - (1 + \kappa_\gamma) \hat{u} \hat{t}) \frac{1}{m_w^2 - \hat{s}} + \frac{s}{2 m_w^2}$$

$$- \left( 2 \hat{u}(\hat{u} + \hat{s}) \frac{1}{m_w^2} + (1 + \kappa_\gamma) \left[ \hat{s} - (\hat{u} + \hat{s})^2 \frac{1}{m_w^2} \right] \frac{1}{2 (m_w^2 - \hat{s})} \right) \frac{1}{\hat{t}}.$$ (4c)
\[ + \left(8\hat{u}^2 - 16\hat{s}m_w^2 - 4(1 + \kappa_\gamma)\hat{u}^2 \left[1 + \frac{\hat{s}}{m_w^2}\right]\right)\]
\[+(1 + \kappa_\gamma)^2 \left[4\hat{u}\hat{t} + (\hat{u}^2 + \hat{t}^2)\frac{\hat{s}}{m_w^2}\right]\frac{1}{8(m_w^2 - \hat{s})^2}\]
\[-\lambda\frac{\hat{s}\hat{t}\hat{u}}{2m^4_w(m_w^2 - \hat{s})} + \lambda(y(2\kappa_\gamma + \lambda_\gamma - 2)\frac{\hat{s}}{8m_w^2} \left[1 + \frac{2\hat{t}\hat{u}}{(m_w^2 - \hat{s})^2}\right].\]

We leave the superscript \(D\) in Eq.(4a) which stands for the direct photo-process in \(ep\) collision following Ref.[9]. By setting the quark charge \(|\hat{e}_q| = 1\), we can obtain the matrix elements for the processes, \(e + \gamma \rightarrow \nu + W\) and \(e + \gamma \rightarrow e + Z\). With the definitions of \(Y = \hat{s}/4m_w^2\), \(X = (Y - 1/4)(1 + \cos \hat{\theta})/2\) and \(\chi = 1 - \kappa_\gamma\), the differential cross section with respect to \(\hat{\theta}\), the angle between the outgoing \(W\) and the incoming photon is

\[
\frac{d\hat{\sigma}}{d\cos \hat{\theta}}(\gamma + q \rightarrow q' + W) = \frac{\pi\alpha^2(Y - 1/4)}{128m_w^2Y^2(Y - X)^2\sin^2 \hat{\theta}_w} F(|\hat{e}_q|), \tag{5a}
\]

where

\[
F(|\hat{e}_q|) = X \left[8Y - 4 + (8X^2 + 4X + 1)/Y\right]
-8\chi X(Y + X) - 32\lambda_\gamma(\lambda_\gamma - \chi)YX(Y - X) + 64\lambda_\gamma^2YX(Y - X)^2
+(\lambda_\gamma - \chi)^2 \left[(Y^2 + X^2)(4Y - 4X - 1) + 4XY\right]
+8\xi(|\hat{e}_q|) \left[-\chi(Y + X) + (\xi(|\hat{e}_q|) + 2X)f\right], \tag{5b}
\]

with

\[
\xi(|\hat{e}_q|) = (Y - X)(1 - |\hat{e}_q|) \quad \text{and} \quad f = \left[(Y - 1/4)^2 + (X + 1/4)^2\right]/(XY). \tag{5c}
\]

The function \(F(|\hat{e}_q| = 1)\) represents the matrix element for \(e + \gamma \rightarrow \nu + W\). In the Standard Model i.e. \(\lambda_\gamma = \chi = 0\), \(F(|\hat{e}_q| = 1)\) vanishes when the outgoing \(W\) and the incoming photon are antiparallel, \(X = 0\). And that is the famous radiation zero \([12]\).

It is interesting to note that the radiation zero is not a unique feature of the Standard Model. The radiation zero will be present \([13]\) whenever

\[
\lambda_\gamma + \kappa_\gamma = 1 \quad (\text{or} \quad \lambda_\gamma = \chi) \quad \text{and} \quad X = 0 \tag{6}
\]

for the process \(e + \gamma \rightarrow \nu + W\).

Next we focus on the total production of \(W\) and \(Z\) in \(ep\) collisions. In the short term these processes will be studied at HERA(\(E_e = 30\) GeV, \(E_p = 820\) GeV, \(\mathcal{L} = \))

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200 pb\(^{-1}\) yr\(^{-1}\)), while in the long term availability of LEP \(\times\) LHC\((\, E_e = 50\) GeV, \(E_p = 8000\) GeV, \(\mathcal{L} = 1000\) pb\(^{-1}\) yr\(^{-1}\)) collider will give collision energies in excess of 1 TeV. We first calculate the total cross sections for the five different processes which contribute to single \(W\) and \(Z\) production at \(ep\) colliders. From the sum of these contributions we then calculate the ratio \(\sigma_{\text{total}}(W)/\sigma_{\text{total}}(Z)\) as a function of the anomalous \(WW\gamma\) coupling parameters \(\kappa_\gamma\) and \(\lambda_\gamma\). The five processes are

\[
e^- + p \rightarrow e^- + W^\pm + X, \quad (7a)
\]

\[
\rightarrow \nu + W^- + X, \quad (7b)
\]

\[
\rightarrow e^- + Z + X \ (Z \text{ from hadronic vertex}), \quad (7c)
\]

\[
\rightarrow e^- + Z + X \ (Z \text{ from leptonic vertex}), \quad (7d)
\]

\[
\rightarrow \nu + Z + X. \quad (7e)
\]

The largest contributions for \(W\) and \(Z\) productions come from the processes (7a) and (7c) which are dominated by the real photon exchange Feynman diagrams with a photon emitted from the incoming electron, \(e^- + p \rightarrow \gamma/e^- + p \rightarrow V + X\). The dominant subprocesses for \(e + p \rightarrow V + X \) would appear to be the lowest order \(q_{/\gamma}^{(r)} + q \rightarrow V\), where \(q_{/\gamma}\) is a resolved quark inside the photon. However this may not be strictly true, even at very high energies, since quarks inside the photon \(q_{/\gamma}\) exist mainly through the evolution \(\gamma \rightarrow q\bar{q}\). Hence the direct process \(\gamma + q \rightarrow q^{(r)} + V\) could be competitive with the lowest order resolved process contribution \(q_{/\gamma}^{(r)} + q \rightarrow V\). This raises the subtle question of double counting [8, 15]. Certain kinematic regions of the direct processes contribute to the evolution of \(q_{/\gamma}\) which is already included in the lowest order process. Both double counting and the mass singularities are removed [16] if we subtract the contribution of \(\gamma + q \rightarrow q^{(r)} + V\) in which the \(t\)-channel-exchanged quark is on-shell and collinear with the parent photon. Thus the singularity subtracted lowest order contribution from the subprocesses \(q_{/\gamma}^{(r)} + q \rightarrow V\) is

\[
\sigma^L(e^- + p \rightarrow \gamma/e^- + p \rightarrow V + X) = \frac{C^L_{\gamma}}{s} \int_{m^2_{/s}}^1 \frac{dx_1}{x_1} \times \left[ \sum_{qq'} (f_{q/e} - \tilde{f}_{q/e})(x_1, m^2_{q'}) f_{q'/p}(\frac{m^2_{q'}}{x_1 s}, m^2_{q'}) + (q \leftrightarrow q') \right], \quad (8a)
\]

where

\[
C^L_w = \frac{2\pi G_F m^2_{\nu}}{3\sqrt{2}} |V_{qq'}|^2, \quad C^L_Z = \frac{2\pi G_F m^2_{\tau}}{3\sqrt{2}} g_\gamma^2. \quad (8b)
\]

The electron structure functions \(f_{q/e}\) are obtained as usual

\[
f_{q/e}(x, Q^2) = \int_x^1 \frac{dy}{y} f_{q/\gamma}(\frac{x}{y}, Q^2) f_{\gamma/e}(y), \quad (9)
\]
where \( f_{\gamma/e} \) is the appropriate Weizäcker-Williams approximation \([14]\) of (quasi-real) photon radiation, and \( f_{q/\gamma} \) is the usual photon structure function. The part of photon structure function, \( \tilde{f}_{q/\gamma} \), which results from photon splitting at large \( x \) (with large momentum transfer), has the leading order form as

\[
\tilde{f}_{q/\gamma}^{(0)}(x, Q^2) = \frac{3\alpha e^2}{2\pi} (1 - 2x + 2x^2) \log \left( \frac{Q^2}{\Lambda^2} \right),
\]

and as before \( \tilde{f}_{q/e}^{(0)}(x, Q^2) = \int_x^1 \frac{dy}{y} \tilde{f}_{q/\gamma}^{(0)}(x y, Q^2) f_{\gamma/e}(y) \).

(10)

To obtain the total contribution from the direct subprocess, \( \gamma + q \rightarrow q'(\gamma) + V \), we must integrate Eq. (4), regularizing the \( \hat{t} \)-pole of the collinear singularity by cutting at the scale \( \Lambda^2 \) which determines the running of the photon structure functions \( f_{i/\gamma} \). This corresponds to the subtraction used to redefine the photon structure functions in Eq. (8a). Then the hard scattering cross sections from the direct subprocesses are

\[
\hat{\sigma}(\gamma + q \rightarrow q'(\gamma) + V) = \frac{C_D^{\nu}}{\hat{s}} \eta_V,
\]

(11a)

where

\[
\eta_{V=Z} = (\hat{s}, m^2_{z}, \Lambda^2) = (1 - 2\hat{z} + 2\hat{z}^2) \log \left( \frac{\hat{s} - m^2_{z}}{\Lambda^2} \right) + \frac{1}{2} (1 + 2\hat{z} - 3\hat{z}^2),
\]

\[
\eta_{V=W} = (\hat{s}, m^2_{w}, \Lambda^2, |e_q|, \kappa, \lambda) = (|e_q| - 1)^2 (1 - 2\hat{z} + 2\hat{z}^2) \log \left( \frac{\hat{s} - m^2_{w}}{\Lambda^2} \right) \quad - \quad \left[ (1 - 2\hat{z} + 2\hat{z}^2) - 2|e_q|(1 + \kappa + 2\hat{z}^2) + \frac{(1 - \kappa)^2}{4\hat{z}} - \frac{(1 + \kappa)^2}{4} \right] \log \hat{z} \quad + \quad \left[ (2\kappa + \frac{(1 - \kappa)^2}{16}) \frac{1}{\hat{z}} + \frac{3(1 + |e_q|^2)}{2} \right] \hat{z} 
\]

\[
+ \left( 1 + \kappa \right) |e_q| - \frac{(1 - \kappa)^2}{16} + \frac{|e_q|^2}{2} \left( 1 - \hat{z} \right) \quad - \quad \frac{\lambda^2}{4\hat{z}^2} (\hat{z}^2 - 2\hat{z} \log \hat{z} - 1) \quad + \quad \frac{\lambda}{16\hat{z}^2} (2\kappa + \lambda - 2) [(\hat{z} - 1)(\hat{z} - 9) + 4(\hat{z} + 1) \log \hat{z}], \quad (11b)
\]

with

\[
C^{D}_{w} = \frac{\alpha G_{\nu} m^2_{w}|V_{qq'}|^2}{\sqrt{2}}, \quad C^{D}_{z} = \frac{\alpha G_{\nu} m^2_{w}g^2_{q} e^2_{q}}{\sqrt{2}} \quad \text{and} \quad \hat{z} = \frac{m^2_{w}}{\hat{s}}. \quad (11c)
\]

The first terms in the \( \eta_{V=W,z} \) represent the collinear singularity from the \( \hat{t} \)-pole exchange, which is related to the photon structure-function of Eq. (10). This is the
singularity that has already been subtracted in Eq. (8), and so we can now add the two contributions, Eqs. (8) and (12), without double counting. The total contribution from the direct subprocess $\gamma + q \rightarrow q^{(i)} + V$ is

$$\sigma^D\left( e^- + p \rightarrow \gamma/e + p \rightarrow V + X \right) = \frac{C^D}{s} \int_{m_V^2/s}^1 \frac{dx_1}{x_1} \int_{m_V^2/x_1s}^1 \frac{dx_2}{x_2} \times \left[ \sum_q f_{\gamma/e}(x_1, Q^2) f_{q/p}(x_2, Q^2) \right] \eta_V(\hat{s} = x_1 x_2 s). \quad (12)$$

The processes (7b) and (7d), which give a substantial contribution as energy increases, are dominated by configurations where a (quasi-real) photon is emitted (either elastically or quasi-elastically) from the incoming proton and subsequently scatters off the incoming electron, i.e., $e^- + p \rightarrow e^- + \gamma/p \rightarrow e^- + Z$ (or $\rightarrow \nu + W^-$). In these processes $Z$ is produced from leptonic vertex, and as explained in Eq.(6) because of the famous radiation zero, if $\lambda_\gamma + \kappa_\gamma = 1$ the production of $W^-$ toward the direction of incoming proton will be suppressed. For the elastic photon, the cross section can be computed using the electrical and magnetic form factors of the proton. For the quasi-elastic scattering photon, the experimental information [18] on electromagnetic structure functions $W_1$ and $W_2$ can be used, following Ref. [19]. The hard scattering cross section is given from Eqs.(5) and (11) with the obvious substitution of $|e_q| = 1$,

$$\hat{\sigma}(e^- + \gamma/p \rightarrow e^- + Z \text{ or } \nu + W^-) = \frac{C^{D}_{V=W,Z}}{\hat{s}} \eta_{V=W,Z}(|e_q| = 1). \quad (13)$$

Notice that since in these processes there is no contribution from $\hat{t}$-pole quark exchange diagram, which dominates for the processes (7a) and (7c), the production cross section of $ep \rightarrow \nu W^- X$ is significantly smaller compared to $ep \rightarrow eW^\pm X$. However, due to the contribution from the diagram with $WW\gamma$ vertex the rate for $ep \rightarrow \nu W^- X$ grows more rapidly with energy than the rate for $ep \rightarrow eZX$, as shown in Table 1. For process (7e), which is a pure charged current process, we simply use the results of Bauer et. al. [13] to add to the contributions from (7c) and (7d). The contribution from this process to the total $Z$ production cross section is almost negligible even at LEP $\times$ LHC $ep$ collider energies, as can be seen in Table 2.

Finally, as explained before, we again emphasize that for the processes of $e\gamma$ collisions, $e + \gamma \rightarrow W^- + \nu$, and $e + \gamma \rightarrow Z + e$, we can get all the relevant results by setting the quark charge $|e_q| = 1$.

### 3 The Process in $e\gamma$ Collisions

Let us now consider the $W$ decay to final state fermions in $e\gamma$ collisions. The net process is represented by $e\gamma \rightarrow \nu_e f \bar{f}$. In the hadronic decay of the $W$ boson, $e + \gamma \rightarrow
Figure 2: Feynman diagrams for the process $e + \gamma \rightarrow \nu_e + \mu + \bar{\nu}_\mu$

$\nu_e + q + \bar{q}$, there are two hadronic jets and large missing transverse momentum ($\vec{p}_T$) due to the neutrino from the initial electron beam. But in leptonic decay, $e + \gamma \rightarrow \nu_e + \mu + \bar{\nu}_\mu$, we can detect only $\mu$ with the missing $p_T$ which is attributed to the neutrinos. If we consider single $\mu$ production in $e\gamma$ collision, it must be produced via the process $e + \gamma \rightarrow \nu_e + \mu + \bar{\nu}_\mu$. In this respect, the single $\mu$ production in $e\gamma$ collision has strong merit for being studied. Single $\mu$ production in $e\gamma$ collisions has recently been studied in Ref. [6, 14]. But they restricted the production of $\mu$ as a decay product of real $W$ at a future TeV energy colliders. Here we include the single $\mu$ production via virtual $W$.

The lowest order tree level Feynman diagrams are given in Fig. 2. Let us introduce invariant variables commonly used for 2→3 processes as,

$$
\begin{align*}
s &= (e + \gamma)^2, & s' &= (\nu_e + \mu)^2, \\
t &= (e - \nu_e)^2, & t' &= (\gamma - \nu_e)^2, \\
u &= (e - \mu)^2, & u' &= (\gamma - \mu)^2,
\end{align*}
$$

(14)

where we express the momentum of each particle by its name.

The matrix element $\mathcal{M}$ is given by,

$$
\mathcal{M} = \frac{g^2 e}{2} (\mathcal{M}_a + \mathcal{M}_b + \mathcal{M}_c),
$$

(15)
In Table 1 we show the total $W^\pm$ production cross section at HERA and LEP $\times$ LHC $ep$ colliders for a range of values of the anomalous $WW\gamma$ coupling parameters $\kappa_\gamma$ and $\lambda_\gamma$. The error range represents the variation in the cross section by varying the theoretical input parameters as follows: $m_t^2/10 \leq Q^2 \leq m_t^2$, and $Q^2 = p_T^2(V)$, photon structure functions $f_{q/\gamma}$ from DG [20] and DO+VMD [21], and proton structure functions $f_{q/p}$ from EHLQ1 [22], HMRS(B) [23] and GRV [24]. There are very strong $Q^2$ dependences in the total cross sections of $W, Z$ production. We find that for any values of $\kappa_\gamma$ and $\lambda_\gamma$, always $\sigma_{w,z}(Q^2 = p_T^2) \leq \sigma_{w,z}(Q^2 = m_\gamma^2/10) \leq \sigma_{w,z}(Q^2 = m_\gamma^2)$. We also find that there exists a quite strong dependence on the structure functions of $f_{q/p}$ and $f_{q/\gamma}$ for $W^+$ production, but almost no dependence for $W^-, Z$ production at HERA energies. Fortunately the ratio of $W$ and $Z$ production cross sections give much weaker dependence on the variation of the theoretical input parameters, as can be seen in Tables 1, 2 and 3.

$$M_a = \bar{u}_L(\nu_e)\gamma^\mu \frac{f^\mu + f^\tau}{s} \bar{f}_\gamma u_L(e) \frac{-g_{\alpha\beta}}{s + t + t' - m_w^2} \bar{u}_L(\mu)\gamma^\beta v_L(\bar{\nu}_e)$$

$$M_b = \bar{u}_L(\nu_e)\gamma^\mu u_L(e) \frac{\Gamma_{\alpha\beta\gamma\delta}}{t - m_w^2(s + t + t' - m_w^2)} \bar{u}_L(\mu)\gamma^\beta v_L(\bar{\nu}_e)$$

$$M_c = \bar{u}_L(\nu_e)\gamma^\mu u_L(e) \frac{-g_{\alpha\beta}}{t - m_w^2} \bar{f}_\gamma \frac{f^\tau}{u'} \gamma^\beta v_L(\bar{\nu}_e)$$

Here the $WW\gamma$ vertex factor $\Gamma_{\mu\nu\sigma}$ in SM is defined by

$$\Gamma_{\mu\nu\sigma} = (k_+ - k_-)_{\sigma}g_{\mu\nu} + (k_+ - k_\gamma)_{\mu}g_{\nu\sigma} + (k_\gamma - k_+)_{\nu}g_{\mu\sigma},$$

where $k_+, k_-, k_\gamma, \mu, \nu$ and $\sigma$ are the momentums with incoming direction and corresponding indices of $W^+, W^-$, and $\gamma$, respectively.

We present averaged amplitude squared for the process $e + \gamma \to \nu_e + \mu + \bar{\nu}_e$ in compactly factorized form. We get the spin averaged amplitude squared as,

$$|M|^2 = g^4e^2 \frac{(s + t + u)^2 + (s + t' + u')^2 + (t + u)^2 + 2(t + u)(s + t + u')}{2(t - m_w^2)(s + t + t' - m_w^2)} \times \left[ \frac{m_w^2}{u'(t - m_w^2)} + \frac{m_w^2}{s + t + t' - m_w^2} \left( \frac{1}{s} + \frac{1}{t - m_w^2} \right) - \frac{u}{su'} \right].$$

The simplified amplitude squared as a general function of $\kappa_\gamma$ and $\lambda_\gamma$ will be presented elsewhere. Here we give the result for the case of the SM only.

## 4 Numerical Results and Discussions

In Table 1 we show the total $W^\pm$ production cross section at HERA and LEP $\times$ LHC $ep$ colliders for a range of values of the anomalous $WW\gamma$ coupling parameters $\kappa_\gamma$ and $\lambda_\gamma$. The error range represents the variation in the cross section by varying the theoretical input parameters as follows: $m_t^2/10 \leq Q^2 \leq m_t^2$, and $Q^2 = p_T^2(V)$, photon structure functions $f_{q/\gamma}$ from DG [20] and DO+VMD [21], and proton structure functions $f_{q/p}$ from EHLQ1 [22], HMRS(B) [23] and GRV [24]. There are very strong $Q^2$ dependences in the total cross sections of $W, Z$ production. We find that for any values of $\kappa_\gamma$ and $\lambda_\gamma$, always $\sigma_{w,z}(Q^2 = p_T^2) \leq \sigma_{w,z}(Q^2 = m_\gamma^2/10) \leq \sigma_{w,z}(Q^2 = m_\gamma^2)$. We also find that there exists a quite strong dependence on the structure functions of $f_{q/p}$ and $f_{q/\gamma}$ for $W^+$ production, but almost no dependence for $W^-, Z$ production at HERA energies. Fortunately the ratio of $W$ and $Z$ production cross sections give much weaker dependence on the variation of the theoretical input parameters, as can be seen in Tables 1, 2 and 3.
### HERA $W$-production Cross-section (in pb)

| $\lambda_\gamma$, $\kappa_\gamma$ | $ep \rightarrow W^+X$ | $ep \rightarrow W^-X$ | $ep \rightarrow W^{\pm}X$ |
|-----------------------------------|----------------------|----------------------|----------------------|
| $\lambda_\gamma = 0, \kappa_\gamma = 0.0$ | 0.46 ± 0.04 | 0.56 ± 0.03 | 1.02 ± 0.07 |
| $\lambda_\gamma = 0, \kappa_\gamma = 0.5$ | 0.53 ± 0.04 | 0.61 ± 0.04 | 1.14 ± 0.07 |
| $\lambda_\gamma = 0, \kappa_\gamma = 1.0$ | 0.63 ± 0.05 | 0.69 ± 0.03 | 1.31 ± 0.07 |
| $\lambda_\gamma = 0, \kappa_\gamma = 1.5$ | 0.75 ± 0.03 | 0.79 ± 0.03 | 1.54 ± 0.06 |
| $\lambda_\gamma = 0, \kappa_\gamma = 2.0$ | 0.92 ± 0.06 | 0.94 ± 0.03 | 1.85 ± 0.08 |
| $\lambda_\gamma = 0.0, \kappa_\gamma = 1$ | 0.63 ± 0.05 | 0.69 ± 0.03 | 1.31 ± 0.07 |
| $\lambda_\gamma = 0.5, \kappa_\gamma = 1$ | 0.63 ± 0.04 | 0.71 ± 0.03 | 1.33 ± 0.06 |
| $\lambda_\gamma = 1.0, \kappa_\gamma = 1$ | 0.67 ± 0.03 | 0.72 ± 0.03 | 1.39 ± 0.05 |
| $\lambda_\gamma = 1.5, \kappa_\gamma = 1$ | 0.71 ± 0.04 | 0.77 ± 0.04 | 1.48 ± 0.07 |
| $\lambda_\gamma = 2.0, \kappa_\gamma = 1$ | 0.77 ± 0.04 | 0.83 ± 0.03 | 1.61 ± 0.07 |

### LEP×LHC $W$-production Cross-section (in pb)

| $\lambda_\gamma$, $\kappa_\gamma$ | $ep \rightarrow W^+X$ | $ep \rightarrow W^-X$ | $ep \rightarrow W^{\pm}X$ |
|-----------------------------------|----------------------|----------------------|----------------------|
| $\lambda_\gamma = 0, \kappa_\gamma = 0.0$ | 6.17 ± 1.17 | 7.38 ± 1.27 | 13.63 ± 2.38 |
| $\lambda_\gamma = 0, \kappa_\gamma = 0.5$ | 7.64 ± 1.29 | 8.82 ± 1.35 | 16.34 ± 2.48 |
| $\lambda_\gamma = 0, \kappa_\gamma = 1.0$ | 9.78 ± 1.37 | 11.49 ± 1.54 | 21.16 ± 2.73 |
| $\lambda_\gamma = 0, \kappa_\gamma = 1.5$ | 13.12 ± 1.64 | 15.56 ± 1.54 | 28.77 ± 2.65 |
| $\lambda_\gamma = 0, \kappa_\gamma = 2.0$ | 17.63 ± 1.58 | 20.95 ± 1.23 | 38.63 ± 2.76 |
| $\lambda_\gamma = 0.0, \kappa_\gamma = 1$ | 9.78 ± 1.37 | 11.49 ± 1.54 | 21.16 ± 2.73 |
| $\lambda_\gamma = 0.5, \kappa_\gamma = 1$ | 11.56 ± 1.44 | 13.49 ± 1.33 | 25.09 ± 2.67 |
| $\lambda_\gamma = 1.0, \kappa_\gamma = 1$ | 16.43 ± 1.67 | 19.84 ± 1.35 | 36.17 ± 2.82 |
| $\lambda_\gamma = 1.5, \kappa_\gamma = 1$ | 24.30 ± 1.74 | 31.16 ± 2.44 | 54.77 ± 3.13 |
| $\lambda_\gamma = 2.0, \kappa_\gamma = 1$ | 35.69 ± 2.36 | 44.61 ± 2.54 | 79.68 ± 4.27 |

Table 1: Total $W$-production cross sections (in pb) at HERA and at LEP × LHC, as a function of anomalous $WW\gamma$ coupling parameters $\kappa_\gamma$ and $\lambda_\gamma$. The error range represents the uncertainties in the cross sections by varying the theoretical input parameters: $m_{\gamma^*}^2/10 \leq Q^2 \leq m_{\gamma^*}^2$, photon structure functions $f_{\gamma q\gamma}$ (DG and DO+VMD), and proton structure functions $f_{q/p}$ (EHLQ1, HMRS(B) and GRV).
Table 2: The cross sections (in pb) for the various W, Z production channels at HERA and LEP × LHC ep colliders. The errors represent the variation in cross sections obtained by varying the theoretical input parameters: $m^2/10 \leq Q^2 \leq m^2$ and $Q^2 = p^2_T(V)$, photon structure functions $f_{q/\gamma}$ (DG and DO+VMD), and proton structure functions $f_{q/p}$ (EHLQ1, HMRS(B) and GRV). For W production, $\kappa_\gamma = 1$ and $\lambda_\gamma = 0$ are assumed.

It is quite important to note that once photoproduction experiments at HERA determine $f_{q/p}$ and $f_{q/\gamma}$ more precisely, we will be able to predict the total cross sections for each process with much greater accuracy. The subtraction terms $\tilde{f}_{q/\gamma}$ of Eq. (8) have been here calculated using the leading order photon splitting function as in Eq. (10), the same prescription also used in Ref. [19]. In our previous study [6], cut-off dependent higher order terms were included in $\tilde{f}_{q/\gamma}$ to calculate the processes (7a) and (7c).

We show in Table 2 the cross sections for the various W and Z production channels at the HERA and LEP × LHC ep colliders. The errors represent the variation in cross sections obtained by varying the input parameters, as in Table 1. We find that our results of Table 2 agree quite well with the results of Ref. [19], Table 5. Here we note several comments for Table 2; (i) W production cross sections are with the Standard Model parametrization, i.e. $\kappa_\gamma = 1$ and $\lambda_\gamma = 0$. (ii) Notice that the importance of Z production from the leptonic vertex, (7d). (iii) As explained earlier, due to the contribution from the diagram with WW$\gamma$ vertex, the rate for $\nu W^\pm X$ production grows rapidly with energy. (iv) We have not included the contribution from W and Z exchange diagrams, which is very small at HERA energies [19].

With the anticipated luminosities of $\mathcal{L} = 200 \text{ pb}^{-1} \text{ yr}^{-1}$ (HERA) and $\mathcal{L} = 1000 \text{ pb}^{-1} \text{ yr}^{-1}$ (LEP × LHC), the total Z production cross section corresponds to 84 events/yr (HERA) and 5400 events/yr (LEP × LHC). After including a 6.7% leptonic branching ratio (i.e. $Z \rightarrow e^+e^-, \mu^+\mu^-$), the event numbers become about 6 events/yr (HERA) and 360 events/yr (LEP × LHC).
HERA $W/Z$-production Ratio

| $\lambda_\gamma$, $\kappa_\gamma$ | $\sigma(W^+)/\sigma(Z)$ | $\sigma(W^-)/\sigma(Z)$ | $\sigma(W^\pm)/\sigma(Z)$ |
|----------------------------------|--------------------------|--------------------------|--------------------------|
| $\lambda_\gamma = 0$, $\kappa_\gamma = 0.0$ | 0.98 ± 0.09 | 1.20 ± 0.07 | 2.18 ± 0.14 |
| $\lambda_\gamma = 0$, $\kappa_\gamma = 0.5$ | 1.12 ± 0.07 | 1.30 ± 0.09 | 2.41 ± 0.16 |
| $\lambda_\gamma = 0$, $\kappa_\gamma = 1.0$ | 1.31 ± 0.08 | 1.45 ± 0.05 | 2.76 ± 0.12 |
| $\lambda_\gamma = 0$, $\kappa_\gamma = 1.5$ | 1.58 ± 0.08 | 1.67 ± 0.09 | 3.24 ± 0.16 |
| $\lambda_\gamma = 0$, $\kappa_\gamma = 2.0$ | 1.95 ± 0.14 | 1.97 ± 0.09 | 3.91 ± 0.22 |
| $\lambda_\gamma = 0$, $\kappa_\gamma = 1$ | 1.40 ± 0.09 | 1.53 ± 0.06 | 2.93 ± 0.14 |
| $\lambda_\gamma = 1.5$, $\kappa_\gamma = 1$ | 1.65 ± 0.10 | 1.76 ± 0.07 | 3.40 ± 0.15 |

LEP×LHC $W/Z$-production Ratio

| $\lambda$, $\kappa$ | $\sigma(W^+)/\sigma(Z)$ | $\sigma(W^-)/\sigma(Z)$ | $\sigma(W^\pm)/\sigma(Z)$ |
|---------------------|--------------------------|--------------------------|--------------------------|
| $\lambda = 0$, $\kappa = 0.0$ | 1.40 ± 0.14 | 1.69 ± 0.11 | 3.10 ± 0.24 |
| $\lambda = 0$, $\kappa = 0.5$ | 1.69 ± 0.11 | 1.98 ± 0.14 | 3.68 ± 0.23 |
| $\lambda = 0$, $\kappa = 1.0$ | 2.14 ± 0.12 | 2.54 ± 0.11 | 4.66 ± 0.22 |
| $\lambda = 0$, $\kappa = 1.5$ | 2.86 ± 0.30 | 3.44 ± 0.26 | 6.35 ± 0.47 |
| $\lambda = 0$, $\kappa = 2.0$ | 3.89 ± 0.31 | 4.75 ± 0.42 | 8.60 ± 0.68 |
| $\lambda = 0$, $\kappa = 1$ | 2.14 ± 0.12 | 2.54 ± 0.11 | 4.66 ± 0.22 |
| $\lambda = 0$, $\kappa = 1.5$ | 2.57 ± 0.13 | 2.97 ± 0.22 | 5.55 ± 0.33 |
| $\lambda = 1.0$, $\kappa = 1$ | 3.68 ± 0.28 | 4.52 ± 0.36 | 8.21 ± 0.59 |
| $\lambda = 1.5$, $\kappa = 1$ | 5.42 ± 0.66 | 6.87 ± 0.69 | 12.33 ± 1.31 |
| $\lambda = 2.0$, $\kappa = 1$ | 8.09 ± 0.94 | 10.25 ± 1.43 | 18.32 ± 2.28 |

Table 3: Production cross section ratio of $W/Z$ as a function of $\kappa_\gamma$ and $\lambda_\gamma$ at HERA and at LEP × LHC. We first set $\lambda_\gamma$ to its Standard Model values ($\lambda_\gamma = 0$) and then vary $\lambda_\gamma$ and vice versa.
In Table 3 we show the ratio \( \sigma(W^+)/\sigma(Z) \), \( \sigma(W^-)/\sigma(Z) \) and \( \sigma(W^++W^-)/\sigma(Z) \) for the various values of \( \kappa_\gamma \) and \( \lambda_\gamma \). The input parameters have been varied as in Table 1. Note also that we have not included the uncertainties due to higher order perturbative QCD corrections. While these are expected to have non-negligible effect on the absolute \( W \) and \( Z \) cross sections - as in \( pp \) and \( p\bar{p} \) collisions - the ratio of \( W \) to \( Z \) cross sections is one of the most reliable predictions of QCD, as every diagram, except for the diagrams with \( WW\gamma \) vertex, producing a \( W \) also produces \( Z \) up to \( O(\alpha_s^2) \) where additional diagrams produce \( Z \) via a triangular quark loop\(^{25}\). Even this contribution would vanish for equal-mass up- and down-type quarks. These \( O(\alpha_s^2) \) contributions have also been calculated\(^{26}\), and are less than 1% in \( pp(\bar{p}) \) colliders even for a very heavy top quark. Henceforth we ignore higher order QCD corrections, and investigate the uncertainties due to the theoretical input parameters as explained.

To obtain an experimentally measurable ratio \( \sigma(ep \rightarrow W^\pm \rightarrow l\nu)/\sigma(ep \rightarrow Z \rightarrow l^+l^-) \) we must multiply the cross section ratio \( \sigma(W)/\sigma(Z) \) by the leptonic branching ratio factor

\[
R_{BR}(m_t > m_W - m_b, N_\nu = 3) \equiv \frac{BR(W^\pm \rightarrow l\nu)}{BR(Z^\pm \rightarrow l^+l^-)} = 3.23. \tag{19}
\]

After 5 years of running, HERA will produce about 30 events of \( e + p \rightarrow Z + X \rightarrow l^+l^- + X \), and this will enable us to determine \( \kappa_\gamma \) and \( \lambda_\gamma \) with a precision of order

\[
\Delta \kappa_\gamma \approx \pm 0.3 \quad \text{for} \quad \lambda_\gamma = 0, \\
\Delta \lambda_\gamma \approx \pm 0.8 \quad \text{for} \quad \kappa_\gamma = 1, \tag{20}
\]

which are comparable with the expected constraints from the future LEP-II \( e^+e^- \) experiment. At LEP \( \times \) LHC, one year’s running will give

\[
\Delta \kappa_\gamma \approx \pm 0.2 \quad \text{for} \quad \lambda_\gamma = 0, \\
\Delta \lambda_\gamma \approx \pm 0.3 \quad \text{for} \quad \kappa_\gamma = 1. \tag{21}
\]

### 5 Conclusion

We have shown how measurements of weak boson production at high energy electron-proton and electron-photon colliders can provide important information on anomalous \( WW\gamma \) couplings.

In \( ep \) collisions, we have analyzed the production of massive gauge bosons – \( W \) and \( Z \). We have included both direct and indirect processes, involving the parton structure of the photon, taking careful account of the double counting problem for the latter. We have also argued that the ratio of \( W \) and \( Z \) production cross sections is particularly suited to an experimental determination of the anomalous \( WW\gamma \) coupling.
parameters $\kappa_\gamma$ and $\lambda_\gamma$, being relatively insensitive to uncertainties in the theoretical input parameters. In fact, with more precise measurements of the input parameters in the next few years - in particular the photon structure functions - the errors in the measured $\kappa_\gamma$ and $\lambda_\gamma$ values will ultimately be obtained by the statistical error from the small number of $Z$ events at HERA. In this respect, the higher energy LEP $\times$ LHC collider offers a significant improvement. Finally we note that our estimated precision on $\kappa_\gamma$ and $\lambda_\gamma$ for both $ep$ colliders, Eqs. (20) and (21), is an order of magnitude greater than existing measurements from $W\gamma$ production at $p\bar{p}$ collider $^3$.

The Pohang Light Source (PLS) facility can produce a high flux ($10^7$/sec) of background-free $\gamma$-rays after installation of a laser backscattering system. The maximum energy of the $\gamma$-rays will reach 300 MeV when the facility is upgraded to an electron energy of 2.5 GeV. In a later stage of the project, a Free Electron Laser can be expected to extend the $\gamma$-ray energy up to 1 GeV $^2$. If we consider the $e\gamma$ collision at PLS($E_e \sim 2.5$GeV,$E_\gamma \sim 1$ GeV), the total cross section of the process $e+\gamma \to W^* (\to +\mu +\bar{\nu}_\mu) +\nu_e$ is about $10^{-4}$pb. To get 1 event/yr, very large luminosity must be obtained($L \sim 10^{33}$/cm$^2$/sec). Therefore, this process is not adequate to be detected at PLS for the time being.

Attempts are at present under way by many authors to constrain the parameter space of $\lambda_\gamma$ and $\kappa_\gamma$ by considering various experimental results; production of $W+\gamma$ at $p\bar{p}$ collider $^2$, process $e\gamma \to W\nu$ at future $e^+e^-$ and $e\gamma$ colliders$^{13,5}$, and also from present low energy data$^{29}$. We are now studying this process at HERA via $e+\gamma/p \to \mu +\bar{\nu}_\mu +\nu_e$ considering both $\kappa_\gamma$ and $\lambda_\gamma$. And those approaches should be regarded as complementary in the efforts to find new physics beyond the Standard Model.

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