Numerical Analysis on the High-Strength Concrete Beams Ultimate Behaviour

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Abstract. Development of technologies of high-strength concrete (HSC) beams production, with the aim of creating a secure and durable material, is closely linked with the numerical models of real objects. The three-dimensional nonlinear finite element models of reinforced high-strength concrete beams with a complex geometry has been investigated in this study. The numerical analysis is performed using the ANSYS finite element package. The arc-length (A-L) parameters and the adaptive descent (AD) parameters are used with Newton-Raphson method to trace the complete load-deflection curves. Experimental and finite element modelling results are compared graphically and numerically. Comparison of these results indicates the correctness of failure criteria assumed for the high-strength concrete and the steel reinforcement. The results of numerical simulation are sensitive to the modulus of elasticity and the shear transfer coefficient for an open crack assigned to high-strength concrete. The full nonlinear load-deflection curves at mid-span of the beams, the development of strain in compressive concrete and the development of strain in tensile bar are in good agreement with the experimental results. Numerical results for smeared crack patterns are qualitatively agreeable as to the location, direction, and distribution with the test data. The model was capable of predicting the introduction and propagation of flexural and diagonal cracks. It was concluded that the finite element model captured successfully the inelastic flexural behaviour of the beams to failure.

1. Introduction

Improved performance computing systems and the possibility of their use in the design of engineering structures encourages intensive development of numerical methods for the analysis of static and dynamic structural behaviour. Numerical methods are the only way to achieve useful solutions to complex spatial structures made with materials behaving nonlinearly like concrete.

Concrete is a commonly used building material throughout the world. Material is brittle under low confining pressure and weak in tension. Development of modern civil engineering construction has caused demand for new types of concrete which are required to possess improved strength, toughness, and durability [1]. High-strength concrete can be designed to have a higher workability, and a higher mechanical properties compared to the traditional concrete [2–4]. The use of high-strength concrete in the building industry will continue to grow. The effect of concrete compressive strength and flexural tensile reinforcement ratio on load-deflection behaviour and ductility of reinforced high-strength concrete beams has been reported by several previous investigators [5–8]. Reinforced HSC members required not only experimental testing, but also finite element modelling of the failure behaviour. Few researchers studied the finite element modelling of reinforced concrete beams [9–11] by using the
ANSYS finite element package. However, the available publications on the finite element investigation of reinforced high-strength concrete beams are still limited.

The subject of the paper is the reinforced concrete beams consisting of reinforcement steel bars distributed discretely in the high-strength concrete. The aim of the work is to model the deformation processes and the failure behaviour of reinforced concrete beams loaded statically taking into account the physical and mechanical nonlinearity of structural materials: concrete and reinforcement steel. Finite element solutions for HSC beams were obtained by using ANSYS [12]. Experimental results of the work described in [6] and finite element modelling results are compared graphically and numerically.

2. Modelling of materials
2.1. Modelling of high-strength concrete

The description of limit state of concrete under static loading has been the subject of numerous publications. The equations of the limit surface for concrete are described in the paper [13]. The proposed limit surface equations are depended on the first invariant of the stress tensor and the second, and the third invariant of the stress deviator. This description allows the most faithful approximation of concrete experimental results in complex stress state. In this paper, the limit surface equation is depended on five stress parameters, in accordance with [14]. Moreover, the limit surface evolution law is introduced. The failure criterion of concrete in a complex stress state is described in the following equation

\[ F_\alpha / f_c = S_\alpha, \quad \alpha = 1, \ldots, 4 \]

in which: \( F_\alpha \) – the function of \( \sigma_{xp}, \sigma_{yp}, \sigma_{zp} \) normal stresses conditions in the direction of the Cartesian coordinate system xyz, \( S_\alpha \) – failure surface dependent on the principal stresses \( \sigma_1, \sigma_2, \sigma_3 \), where: \( \sigma_1 = \max(\sigma_{xp}, \sigma_{yp}, \sigma_{zp}) \), \( \sigma_3 = \min(\sigma_{xp}, \sigma_{yp}, \sigma_{zp}) \), \( \sigma_1 \geq \sigma_2 \geq \sigma_3 \), and strength parameters: \( f_c \) – uniaxial compressive strength causing crushing, \( f_i \) – uniaxial tensile strength, \( f_{cb} \) – ultimate biaxial compressive strength, \( f_1 \) – ultimate compressive strength for a biaxial compression state superimposed in hydrostatic stress state \( \sigma_{ha} \), \( f_2 \) – ultimate compressive strength for a uniaxial compression state superimposed in hydrostatic stress state \( \sigma_{ha} \).

The failure of concrete is defined by four domains of stresses – \( \alpha \): compression-compression-compression state \( \alpha = 1 \); \( 0 \geq \sigma_1 \geq \sigma_2 \geq \sigma_3 \), tension-compression-compression state \( \alpha = 2 \); \( \sigma_1 \geq 0 \geq \sigma_2 \geq \sigma_3 \), tension-tension-compression state \( \alpha = 3 \); \( \sigma_1 \geq \sigma_2 \geq 0 \geq \sigma_3 \), and tension-tension-tension state \( \alpha = 4 \); \( \sigma_1 \geq \sigma_2 \geq \sigma_3 \geq 0 \). The failure surface is depended on the radii \( r_t, r_c \) and the strength parameters: \( \xi \) and angle \( \theta \). Deviatoric sectional radius \( r_t \) is determined by the parameters \( a_0, a_1, a_2 \) selected in order that \( f_i, f_{cb}, f_1 \) lie on the limit surface. All the mentioned parameters are given in [11].

The limit surface with an evolution law is used as a criterion of concrete failure according to the following interpretation. The material is damaged if equation 1 is fulfilled. The state of failure can be distinguished as the state of crushing if any principal stress is tensile. Otherwise it can be distinguished as the state of crushing if all principal stresses are compressive. The evolution of limit surface is determined by the proposal of hardening/softening law shown in figure 1. The stress-strain relationship for the uniaxial compression is confirmed by the experimental observations showing much larger the limit strain of concrete in the structure than in the sample. We observed that the use of the stress-strain relation based on Model Code 90, leads to a significant decrease in the ultimate deflection of beam. Moreover, experimental results indicated that a low ductility for the high-strength concrete in the structural members is not justified, because the strains reached up to 6–12‰ [6,7].

Linear stress-strain relation for uniaxial compression at the level of 0.3\( f_c \) was assumed. Then begins the region of elastic-plastic strengthening with linear increase in stress up to the uniaxial compressive strength \( f_c \). After this, the stresses in concrete decrease to 0.8\( f_c \) at the limit strain \( \varepsilon_{cu} \). Strain \( \varepsilon_{cu} = 6\% \) at \( f_c \) and the limit compression strains, \( \varepsilon_{eq} = 12\% \) were assumed. Stress-strain curve for tensile concrete is linear up to the tensile strength \( f_t \) (see figure 1b). It is assumed that the tensile modulus of elasticity is equal to the compressive one. After reaching \( f_t \), the tensile stress suddenly decreases to a value \( T_c f_t \).
Parameter $T_c$ should be chosen from the range $0.6 \leq T_c \leq 1$. The stiffening effect shows a gradual decrease of tensile strength to zero, is described by the strain equal to 0.8‰ if $T_c = 0.6$, or 1.4‰ if $T_c = 1$.

![Stress-strain relation for HSC](image)

**Figure 1.** Stress-strain relation for HSC (a) uniaxial compression, (b) uniaxial tension

The hexahedral elements were applied for the concrete. Finite element is defined by the isotropic properties of the material, by eight nodes with three degrees of freedom in each of them, and by the displacements of nodes in three-dimensional orthogonal local coordinate system. In each finite element, the strain and stress is calculated at all points of numerical integration. Smear crack model provides a description of the cracking at any point of numerical integration in three directions perpendicular to the principal stresses. Crack formation is described by the model of concrete. In the graphical representation the results are presented in the form of a circular cracking outline in the direction perpendicular to the principal stress. Subsequently, tangential stresses to the plane of the first crack may produce the second, and the third crack, which may emerge at the point of numerical integration in the direction perpendicular to the causative component corresponding to principal stress. In the state of cracking and crushing the stiffness matrix of finite element is adapted to the state of the damage. The parameter $\beta_t$ is introduced for reducing shear transfer causing slip in the plane perpendicular to the crack surface. The relationship between stress and strain of the cracked concrete in one plane is described by the following stiffness matrix

$$
D^k = \frac{E_c}{1 + \nu_c} \begin{bmatrix}
H_t / (1 + \nu_c) / E_c & 0 & 0 & 0 & 0 & 0 \\
0 & 1 / (1 - \nu_c) & \nu_c / (1 - \nu_c) & 0 & 0 & 0 \\
0 & \nu_c / (1 - \nu_c) & 1 / (1 - \nu_c) & 0 & 0 & 0 \\
0 & 0 & 0 & 0.5 \beta_t & 0 & 0 \\
0 & 0 & 0 & 0 & 0.5 \beta_t & 0 \\
0 & 0 & 0 & 0 & 0 & 0.5 \beta_t
\end{bmatrix}
$$

Graphical interpretation of the softening parameter $H_t$ and the multiplier $T_c$ for tensile stress relaxation is shown in figure 1b. More details can be find in [12].

2.2. Modelling of steel

The elastic-plastic model with linear hardening, and the same stress-strain relation for the tension and compression is applied for the reinforcing bars, figure 2.
Spatial spar element, with two nodes and three degrees of freedom in each of them, was applied in the modelling of bars. Moreover, linear elastic model was assumed for the steel plates located at the support and loaded points. The hexahedral elements were applied to model steel plates. Perfect bond between materials is assumed. The reinforcement is connected to concrete mesh nodes. Therefore, the concrete and the reinforcement mesh share the same nodes and concrete occupies the same regions seized by rebars.

3. Numerical solutions of equilibrium equations

3.1. Newton-Raphson method with adaptive descent

Newton-Raphson method is an iterative process of solving nonlinear equations. The stiffness matrix and vector of restoring loads are calculated on the basis of the displacement vector. The stiffness matrix in Newton-Raphson method with adaptive descent modification is described as the sum of two matrices [15]

\[
K^T_i = \xi K^S_i + (1 - \xi) K^T_i
\]

where: \(K^S\) – secant stiffness matrix, \(K^T\) – tangent stiffness matrix, \(\xi\) – adaptive descent parameter.

The method is based on the agreement of the adaptive descent parameter \(\xi\) in the equilibrium iteration. The secant stiffness matrix is generated in the numerical method as an effect of the nonlinear plasticity, stiffness with large deformations, concrete crushing and post-cracking stress relaxation.

3.2. Arc-length method

In the arc-length method, the equation is dependent on load parameter \(\lambda\) [16]

\[
K^T_i \Delta u_i = \lambda F^u - F^w
\]

The vector of incremental displacement \(\Delta u_i\) consists of two components

\[
\Delta u_i = \Delta \lambda \Delta u^I_i + \Delta u^{II}_i
\]

where: \(\Delta u^I_i\) – displacement increment vector caused by a unit load parameter, \(\Delta u^{II}_i\) – displacement increment vector in the Newton-Raphson algorithm.

The incremental load parameter \(\Delta \lambda\) is searched from the additional constraints equation based on orthogonality insurance. It was found that the convergence of solutions for default setting of convergence tolerance limits was difficult to achieve due to the nonlinear behaviour of reinforced concrete. Therefore, tolerance limits for displacement were increased to 5% in order to obtain the convergence of the solutions.

4. Numerical results and discussion

4.1. Investigation objects
The dimensions for rectangular beams tested by [6] were assumed for the numerical spatial beam models. Dimensions, reinforcement and loading/support arrangements are shown in figure 3.

![Figure 3. Dimensions of beams with reinforcement and loading/support arrangements (unit in mm)](image)

Half of the beam was modelled with regard to the longitudinal symmetry. Steel plates at the support were modelled as nodal imparting forces on rollers. Free rotation of the beam in the plane of bending was allowed. External force is also applied through the steel plate. Uniform distribution of forces at the nodes in the direction of the transverse symmetry axis of the steel plate was assumed. Boundary conditions are shown in figure 4.

![Figure 4. Modelling of support plate and load application plate (unit in mm)](image)

4.2. Effect of elastic modulus and shear transfer coefficient on load-deflection curves

Compressive strength is necessary to determine other parameters of the nonlinear model of HSC. The computations of model beam BP-1a made of concrete with a compressive strength of 81.2 MPa were
performed to evaluate the effect of the elastic modulus \(E_c\) and shear transfer coefficient for an open crack \(\beta_t\) on the load-deflection curves. Different relationships between the compressive strength and modulus of elasticity for HPC are shown in table 1. It may be noted that the calculated values of elastic modulus have a large scatter of results.

| Equations | \(E_c\) (MPa) for \(f_c = 81.2\) MPa |
|-----------|-------------------------------------|
| CEB-FIB   | \(E_c = 10(f_c + 8)^{1/3}\)         | 44681 |
| CAN A23.3-M90 | \(E_c = 5\sqrt{f_c}\)     | 45056 |
| ACI 363   | \(E_c = 3.32\sqrt{f_c} + 6.9\)     | 36817 |
| Prop. Kikizaki [17] | \(E_c = 3.65\sqrt{f_c}\) | 32890 |
| Prop. Neville [17] | \(E_c = 57000\sqrt{f_c}\) | 42649 |

Load-deflection curves at mid-span for beam BP-1a obtained in experimental test and in FEM analysis for different values of the elastic modulus are shown in figure 5a. It was found that the proposal of elastic modulus calculation according with ACI 363 gives the best fit curve. Results of numerical analyses on impact of the shear transfer coefficient for an open crack are presented in figure 5b. These values were chosen from the range of 0 to 0.5. Numerical calculations were performed using Newton-Raphson method with adaptive descent. For the coefficient \(\beta_t\) equal to 0.5 were obtained results similar to experimental data.

4.3. Material properties

Uniaxial compressive and tensile strengths of HSC were carried out on cube and cylindrical specimens. Modulus of elasticity are calculated on the basis of the experimentally determined uniaxial compressive strength according to ACI 363. Shear transfer coefficient for open crack is estimated on the basis numerical analysis, see figure 5b. Properties of the steel bars were computed on the basis of the axial tensile tests on steel bars \(\phi 16, \phi 10, \phi 6\) mm [6]. The properties of HSC and steel bars are given for the beam models BP-1a/BP-2a, respectively.

High-strength concrete is defined by the uniaxial compressive strength \(f_c = 81.2/78.8\) MPa, modulus of elasticity \(E_c = 36.8/36.4\) GPa, uniaxial tensile strength \(f_t = 5.23/4.57\) MPa, Poisson’s ratio \(\nu_c = 0.15\),
density $\rho_c = 2600 \text{ kg/m}^3$, compressive strain at the strength stress level $\varepsilon_{c1} = 6\%$, ultimate compressive strain $\varepsilon_{cu} = 12\%$, shear transfer coefficient for an open crack $\beta_t = 0.3$, and shear transfer coefficient for the closed crack $\beta_c = 0.9$.

The appropriate material properties for the steel bars of $\phi 16/\phi 10/\phi 6$ mm diameters are as follows: modulus of elasticity $E_s = 196/194/201 \text{ GPa}$, yield stress $f_y = 437/420/353 \text{ MPa}$, uniaxial tensile strength $f_{st} = 713/624/466 \text{ MPa}$, ultimate strain at the yield stress $\varepsilon_{su} = 106/116/75\%$, modulus of plastic deformation $E_T = 2659.7/1792.1/1542.8 \text{ MPa}$, Poisson’s ratio $\nu_s = 0.3$ and density $\rho_s = 7800 \text{ kg/m}^3$.

Supporting and load transferring steel plates are defined by modulus of elasticity $E_s = 210 \text{ GPa}$, Poisson’s ratio $\nu_s = 0.3$ and density $\rho_s = 7800 \text{ kg/m}^3$.

4.4. Load capacity and deflections

Nonlinear load-deflection curves at mid-span for beams obtained in FEM analysis and test results are shown in figure 6.

![Load-deflection curves at mid-span for beam: (a) BP-1a, (b) BP-2a](image)

*Figure 6. Load-deflection curves at mid-span for beam: (a) BP-1a, (b) BP-2a*

The stiffness calculated at the region of the elastic behaviour and post-cracking was close to test result. In the modelling of high-strength concrete beams less micro-cracks were formed. Small differences in the response of beams were identified during the crack propagation and yielding of reinforcement steel. The load-deflection curves in the region of plastic strains show decrease in stiffness. Both incremental-iterative methods give the numerical results qualitatively agreement with the tests results. Test results for reinforced high-strength concrete beams show that the effects of fracture in the tensile regions are not compensated by the elastic steel bars and compressive concrete [5, 8]. Therefore, softening effects on the load-deflection curve for beam BP-1a, as a sudden load capacity drop, are observed. These results can be achieved by arc-length algorithm which enables to generate a complete load-deflection path with local and global softening stiffness. In addition, the arc-length method provides significant reduction in computation time.

4.5. Strains and crack distributions

Load-strain curves for compressive concrete at the mid-span of BP-beams were analyzed. The development of compressive strain of the concrete is shown in figure 7. Curves obtained from the arc-length solutions for the beam BP-1a with a low reinforcement ratio showed a slight decrease in the force due to the appearance of cracks in concrete.
Strains of the tensile bar under the load at the mid-span of the beam are recorded and shown in figure 8. In the case of experimental beam BP-1a we can see the descending branches of the curves. This reflects that the experimental beams were unloaded.

The numerical results are almost equal to the experimental results in the linear-elastic region. Some incompatibilities of strain development in compressive concrete and in tensile reinforcement are observed in elastic-plastic region.

The numerical smeared cracks patterns for the BP-beams left side against the experimental cracks distributions, are shown in figure 9. Numerical results are consistent with experimental results as regards the location, direction and concentration. Arc-length method provides improved crack patterns obtained in the numerical solutions. Both the numerical and test results indicate that the crack distributions correspond to the arrangement of stirrups.
5. Conclusions
The full path of load-displacement with a local and a global softening can be obtained by using the incremental-iterative arc-length method. Moreover, this method is highly efficient. Changeable load step and properly set up the arc-length parameters provide faster numerical computation, and additionally guarantee accurate solutions. Additionally, steel plates placed at the support and at the loaded point are very important for mapping the boundary conditions during the experimental tests. In this study the usefulness of the arc-length method was verified on spatial models of the reinforced high-strength concrete beams. The results obtained from the numerical and experimental analysis are compared to each other. It is seen that the finite element failure behaviour indicates a good agreement with the test failure behaviour.

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References
[1] V. Afroughsabet, T. and Ozbakkaloglu, “Mechanical and durability properties of high-strength concrete containing steel and polypropylene fibers,” Constr. Build. Mater., vol. 94, pp. 73–82, 2015.

[2] L. Biolzi, G. L. Guerrini, and G. Rosati, “Overall structural behavior of high strength concrete specimens,” Constr. Build. Mater., vol. 11, pp. 57–63, 1997.
[3] S. Teng, T. Y. D. Lim and B. S. Divsholi, “Durability and mechanical properties of high strength concrete incorporating ultra fine ground granulated blast-furnace slag,” Constr. Build. Mater., vol. 40, pp. 875–881, 2013.

[4] M. Mazloum, A. A. Ramezaniampour, and J. J. Brooks, “Effect of silica fume on mechanical properties of high-strength concrete,” Cem. Concr. Compos., vol. 26, pp. 347–357, 2004.

[5] S. A. Ashour, “Effect of Compressive Strength and Tensile Reinforcement Ratio on Flexural Behaviour of High-Strength Concrete Beams,” Eng. Struct., vol. 22, pp. 413–423, 2000.

[6] M. E. Kamińska, “High-strength concrete and steel interaction in RC members,” Cem. Concr. Compos., vol. 24, pp. 281–295, 2002.

[7] H. J. Pam, A. K. H. Kwan and J. C. M Ho, “Post-peak behavior and flexural ductility of doubly reinforced normal- and high-strength concrete beams,” Struct. Eng. Mech., vol. 12, pp. 459–474, 2001.

[8] M. A. Rashid, and M. A. Mansur, “Reinforced high-strength concrete beams in flexure,” ACI Struct. J., vol. 102, pp. 462–471, 2005.

[9] D. M Özcan, A. Bayraktar, A. Şahin, T. Haktanir, and T. Türker, “Experimental and finite element analysis on the steel fiber-reinforced concrete (SFRC) beams ultimate behavior,” Constr. Build. Mater., vol. 23, pp. 1064–1077, 2009.

[10] M. Słowik, and P. Smarzewski, “Study of the scale effect on diagonal crack propagation in concrete beams,” Comp. Mater. Sci., vol. 64, pp. 216–220, 2012.

[11] M. Słowik, and P. Smarzewski, “Numerical modeling of diagonal cracks in concrete beams,” Archives of Civil Engineering, vol. 60, pp. 307–322, 2014.

[12] ANSYS, ANSYS 10 Documentation. Swanson Analysis System, US, 2008.

[13] A. Stolarski, “Dynamic strength criterion for concrete,” J. Eng. Mech.-ASCE, vol. 130, pp. 1428–1435, 2004.

[14] K. J. William, “Constitutive model for the triaxial behaviour of concrete,” IABSE reports of the working commissions, 19, 1974.

[15] G. M. Eggert, P. R. Dawson, and K. K. Mathur, “An adaptive descent method for nonlinear viscoplasticity,” Int. J. Numer. Meth. Eng., vol. 31, pp. 1031–1054, 1991.

[16] M. A. Crisfield, “An arc-length method including line searches and accelerations,” Int. J. Numer. Meth. Eng., vol. 19, pp. 1269–1289, 1983.

[17] A. M. Neville, “Properties of Concrete,” 4th ed., Longman, England, p. 868, 2000.