CRITICAL CORE MASSES FOR GAS GIANT FORMATION WITH GRAIN-FREE ENVELOPES

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ABSTRACT

We investigate the critical core mass and the envelope growth timescale, assuming grain-free envelopes, to examine how small cores are allowed to form gas giants in the framework of the core-accretion model. This is motivated by a theoretical dilemma concerning Jupiter formation: modelings of Jupiter’s interior suggest that it contains a small core of $<10 \, M_⊕$, while many core-accretion models of Jupiter formation require a large core of $>10 \, M_⊕$ to finish its formation by the time of disk dissipation. Reduction of opacity in the envelope makes it difficult to maintain the envelope’s hydrostatic structure without gravitational energy released by contraction of the envelope (Mizuno 1980; Stevenson 1982; Ikoma et al. 2000). Since the opacity sources are dust grains and gaseous components in the envelope, a minimum critical core mass is achieved in the case of grain-free envelopes. All the previous studies except one calculation done by Mizuno (1980; see Section 2) assumed grain-dominated opacity in the outer envelope. Thus, in this paper, we consider grain-free envelopes and make an extensive investigation of the critical core mass and timescale for gas accretion.

Results from the recent works by Podolak and his colleagues (Podolak 2003; Movshovitz & Podolak 2008) are encouraging. They have directly simulated the dynamical behavior of dust grains to determine their size distribution and then calculated grain opacity in the accreting envelope. Their numerical simulations revealed that grain opacities in the envelope can be much lower than those in the protoplanetary disk. The reason is that small grains initially suspended in the outer envelope quickly grow large in size and then settle down into the deep envelope where temperature is high enough so that grains evaporate.

In the following section, the details of the gas opacity used in this study are described. Our numerical calculations and results for the critical core mass and the timescale of gas accretion are shown in Sections 3 and 4, respectively. We discuss a possibility of Jupiter formation with a small core in Section 5.

2. GAS OPACITY

The opacity of the envelope gas is lowest when the envelope contains only hydrogen and helium. In this study, we first consider such a metal-free case.¹ We compute chemical equilibrium

¹ In the community of astrophysics, the word “metal” represents elements heavier than helium.
between H$_2$, H, H$^+$, H$_2^+$, H$_3^+$, and e$^-$, and then calculate the opacity that includes the bound–free and free–free absorptions by H$^+$, Rayleigh scatterings by H$_2$, H, and He, Thomson scattering by e$^-$, and the collision-induced absorptions (CIAs) due to H$_2$–H$_2$, H$_2$–H, H$_2$–He, and H–He. Values of quantities relevant to those calculations are given in Lenzuni et al. (1991), and references therein. The CIA opacities are computed by using the latest programs and tables available on Borysow’s Web page$^2$ (Borysow et al. 1985, 1997, 2000; Borysow & Frommhold 1990; Borysow 1991, 1992, 2002; Zheng & Borysow 1995; Birnbaum et al. 1996).

Metal-containing envelopes with no grains are also considered in this study. It is uncertain what fraction of metals in protoplanetary disks are incorporated into dust grains and what fraction remains in their gaseous forms. For example, the amount of water adsorbed onto grain surfaces depends strongly on the thermal state of the protoplanetary disk. Water is adsorbed onto grain surfaces in a relatively cold disk, while it remains in the gas phase in a relatively hot disk (Markwick et al. 2002; Woods & Willacy 2009). Adsorption rates of water are also sensitive to the abundances and sizes of dust grains in the disk (Aikawa & Nomura 2006), but such dust properties remain poorly known. Alkali atoms may be important: they are known to be one of the dominant gas opacity sources near 1500 K in the outer atmospheres of brown dwarfs and giant planets (Guillot et al. 1994; Burrows et al. 2000), though alkali atoms have a tendency to reside onto grain surfaces in protoplanetary disks (Hasegawa & Herbst 1993). Therefore, we consider two cases in addition to the metal-free case: in one case, called the alkali case hereafter, alkali atoms are all in the gas phase; and in another case, called the no-alkali case, they are absent in the gas.

In the alkali case, we use the gas-opacity data provided by Freedman et al. (2008), which incorporate the revised solar abundances and alkali atoms. The solar abundances have been recently revised by re-analyses of two forbidden lines of neutral oxygen and carbon from the solar photosphere under the assumption of the local thermodynamic equilibrium, although their values should still require scrutiny because of uncertainties in three-dimensional (3D) hydrodynamical models for the solar atmosphere (Allende Prieto et al. 2001, 2002; Lodders 2003). Opacity data without alkali atoms are kindly provided by R. S. Freedman (2009, private communication). In practice, we use the up-to-date opacity data of Freedman et al. (2008), which cover the wider ranges of temperature and pressure compared to the published ones and include the latest HITRAN spectroscopic data and an additional opacity source, CO$_2$ (in R. S. Freedman 2009, private communication).

3. CRITICAL CORE MASS

The spherically symmetric hydrostatic structure of a protoplanet is simulated in a manner similar to previous studies (e.g., Mizuno 1980). The protoplanet consists of a solid core and a gaseous envelope. The core has a constant density of 3.2 g cm$^{-3}$, and its structure is not computed. The envelope is assumed to be in purely hydrostatic equilibrium and has a uniform chemical composition. While the composition depends on the case, the abundance ratios of elements taken into account are always solar (Lodders 2003). All simulations are computed with spline-interpolated SCVH EOS tables for hydrogen and helium provided by Saumon et al. (1995), contributions from heavy elements being ignored. The inner boundary conditions are applied to the core surface. At the outer boundary, the temperature and density are equal to the midplane values of a protoplanetary disk at the protoplanet’s orbit, $T_{\text{disk}}$ and $\rho_{\text{disk}}$, because the envelope is assumed to be in equilibrium with the disk gas at the outer edge. The outer radius of the protoplanet is defined by the smaller of the accretion radius and the tidal radius. Note that our results are insensitive to the outer boundary conditions. We handle the accretion rate of planetesimals, $\dot{M}_c$, as a free parameter in this study. We follow the same procedure as Mizuno (1980) to determine the critical core mass, $M_{\text{crit}}$, (please see Mizuno 1980 for details). Table 1 summarizes input parameters and their values used in this study.

Figure 1 shows $M_{\text{crit}}$ as a function of $\dot{M}_c$ for the three cases. For comparison, we plot results for additional two cases in which the envelope contains both grains and gas as opacity sources. We assume that the grain opacity takes a constant value, $f$, for the three cases. Therefore, we now focus on results for $\dot{M}_c$, as a free parameter in this study. We follow the same procedure as Mizuno (1980) to determine the critical core mass, $M_{\text{crit}}$, (please see Mizuno 1980 for details). Table 1 summarizes input parameters and their values used in this study.

![Figure 1](http://www.astro.ku.dk/~aborysow/programs/index.html)

Table 1

| Parameter                  | Value                      |
|----------------------------|----------------------------|
| Semimajor axis, $a$        | 5.2 AU                     |
| Core density               | 3.2 g cm$^{-3}$            |
| Disk temperature, $T_{\text{disk}}$ | 150 K                    |
| Disk density, $\rho_{\text{disk}}$ | 5.0 $\times$ 10$^{-11}$ g cm$^{-3}$ |
| Planetesimal accretion rate, $\dot{M}_c$ | 1.0 $\times$ 10$^{-3}$ to 1.0 $\times$ 10$^{-10}$ M$_\odot$ yr$^{-1}$ |

Figure 1. Critical core masses, $M_{\text{crit}}$, as functions of core-accretion rate, $\dot{M}_c$. The solid, dashed, and dotted lines represent the results for the metal-free, alkali, and no-alkali cases (see Section 2 for the definitions). For comparison, the results of two cases in which there exist grains in the envelope are also shown with the double dot-dashed and dot-dashed lines: $f = 0.01$ and 0.001, respectively, where $f$ is the grain depletion factor (see the text for its definition). We have used the grain opacity tables derived from Pollack et al. (1985). (A color version of this figure is available in the online journal.)
have little contribution to determine $M_{\text{crit}}$. The reason is that convection governs heat transfer in deep, hot parts of the envelope where alkali atoms have a great contribution to the opacity.

Figure 1 also demonstrates that all the lines except one for the metal-free case converges, as $M_\ast$ decreases. In the case of low $M_\ast$, namely, low luminosity, the outermost isothermal layer extends deep in the envelope. In that isothermal layer, the density increases rapidly to keep the pressure gradient needed for supporting the core’s gravity. Thus, because of high densities, the opacity from H$_2$O and CO$_2$ dominates the grain opacity in the deep radiative envelope in spite of low temperature.

Finally, it is worth mentioning differences from previous studies. Mizuno (1980) calculated $M_{\text{crit}}$ in the metal-free case for only one value of core-accretion rate, $1 \times 10^{-6} M_\odot$ yr$^{-1}$, and derived 1.5 $M_\oplus$. While the CIA opacities at high temperatures have been revised thanks to the progress in quantum mechanical models and improvement of experimental data in the 1990s, Mizuno’s (1980) value of $M_{\text{crit}}$ is in good agreement with our result for the same parameters. Ikoma et al. (2000) investigated cases with low grain opacity. Our calculations for $f = 0.01$ and $f = 0.001$ yield larger $M_{\text{crit}}$ than those from Ikoma et al. (2000) because they assumed relatively small grain opacities compared to those used in this study.

4. ENVELOPE GROWTH TIMESCALE

The critical core mass decreases as the core-accretion rate decreases, as shown in Figure 1. Therefore, the case of $M_\ast \to 0$ (i.e., isothermal envelope) yields the absolute minimum of $M_{\text{crit}}$ (Sasaki 1989; Pečnik & Wuchterl 2005). However, for smaller $M_{\text{crit}}$, it takes longer for the core to capture disk gas (Ikoma et al. 2000; Ikoma & Genda 2006). Because there are time constraints on gas giant formation such as the disk’s lifetime (Haisch et al. 2001), a practical minimum of $M_{\text{crit}}$ is determined in this respect. It is necessary to estimate how long it takes for a protoplanet with a given core mass, $M_{\text{core}}$, to capture disk gas.

We consider an isolated protoplanet, namely, the accumulation of the envelope after planetesimal accretion is halted. The isolated protoplanet always experiences contraction of its envelope and captures disk gas because there is no energy supply due to planetesimal accretion. We simulate the quasi-static evolution of the envelope with a given $M_{\text{core}}$ after planetesimal accretion is halted and evaluate the growth timescale of the envelope. As the growth timescale of the envelope, we present values of the characteristic growth time, $\tau_g$, that is defined by Ikoma et al. (2000).

Figure 2 plots $\tau_g$ as functions of $M_{\text{core}}$. For comparison, the results for $f = 0.01$ and 0.001 are also shown. The growth time $\tau_g$ is found to be much shorter in the case of the grain-free opacities than $\tau_g$ for the grain-dominated opacities. For example, when $M_\ast = 3 M_\odot$, $\tau_g = 7 \times 10^6$ yr in the metal-free case (solid line) and $7 \times 10^7$ yr in the alkali case (dashed line), while $2 \times 10^6$ yr for $f = 0.01$ (double dot-dashed line). The envelope growth is regulated by the Kelvin–Helmholtz contraction of the envelope, that is, $\tau_g$ is proportional to $1/L$ (Ikoma et al. 2000), where $L$ is the luminosity at the envelope’s outer edge. Since a change in opacity compensates that in luminosity, $\kappa$ being an averaged opacity, $\tau_g$ should be proportional to $1/L$. In the case of the grain-dominated opacities, the dependence is simple, namely, $\tau_g \propto f$, as shown in Ikoma et al. (2000) and Ikoma & Genda (2006). Figure 2, however, demonstrates that such a simple scaling is inappropriate in low $\kappa$ cases. The curves for grain-free cases are steeper than those for grain-dominated cases; the reason has already been described in the previous section.

In any case, we have found that the envelope growth time is significantly shorter in the case of the grain-free opacities. Based on this fact, we discuss the minimum critical core mass from the viewpoint of gas giant formation and the possibility of Jupiter formation with a small core in the following section.

5. DISCUSSION

There is a theoretical dilemma concerning Jupiter formation, as described in the introduction. Modelings of Jupiter’s interior suggest that Jupiter has a small core of <10 $M_\oplus$ (e.g., Saumon & Guillot 2004), while many core-accretion models of Jupiter formation require a large core of >10 $M_\oplus$ to finish its formation by the time of disk dissipation (Pollack et al. 1996; Alibert et al. 2004, 2005; Fortier et al. 2007, 2009). The disk instability scenario has been revisited as an alternative scenario of their formation (e.g., Boss 2000; Mayer et al. 2002).

In this study, we have demonstrated that reduced opacities in the protoplanet’s envelope have the potential to resolve this dilemma. From Figure 2, one finds that $M_{\text{core}} = 0.75 M_\odot$ for $\tau_g = 1$ Myr in the metal-free case; $M_{\text{core}} = 1.7 M_\odot$ even in the alkali case. Given observed lifetimes of protoplanetary disks of several megayears, the fact above indicates the reduction of opacity allows Jupiter to have a small core that is consistent with interior modelings in principle. The feasibility of such minimum $M_{\text{crit}}$ depends on opacities in the protoplanet’s envelope, while this does not change our conclusion that the minimum $M_{\text{crit}}$ obtained here provides the lowest limit to core masses of gas giants to which the core-accretion model can apply. We need more extensive investigation of gas giant formation into which sedimentation and coagulation of grains in the accreting envelope are incorporated, although reduction of opacity was already pronounced (Podolak 2003; Movshovitz & Podolak 2008).

Our results also shed light upon the growth of solid cores. Many core-accretion models assumed the presence of a single protoplanet (e.g., Pollack et al. 1996). However, a gas giant is in practice thought of as being formed in a system of multiple protoplanets embedded in a protoplanetary disk. Compared to
cases of a single protoplanet, the final mass of a core should be small in the case of a multiple-protoplanet system. According to Kokubo & Ida (1998, 2000), the isolation mass is a few $M_\oplus$ around 5 AU. Even such a small core is enough for a gas giant to capture disk gas within several megayears, as demonstrated in this study. We need to perform comprehensive simulations on gas giant formation in multiple-protoplanet systems, which incorporate adequate models of planetary accretion such as fragmentation of planetesimals and $e$-damping due to gas drag (e.g., Duncan et al. 2010). These calculations will be our future work.

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