Weak-localization type description of conduction in the “anomalous” metallic state

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This paper is devoted to the temperature dependence of the resistivity in high mobility Si-MOSFET samples over the wide range of densities in the “metallic phase” ($n > n_c$) but not too close to the critical density $n_c$. Three domains of qualitatively different behavior in $\rho(T)$ are identified. These are: [i] “quantum domain” ($T < T_q$), where a logarithmic temperature dependence of the resistivity (with $d\rho/dT < 0$) dominates; [ii] “semi-classical domain” ($T_{\text{cr}} < T < E_F$), in which a strong variation of Drude resistivity $\rho(T)$ is observed (with $d\rho/dT > 0$); and [iii] “crossover domain” between the former two ($T_q < T < T_{\text{cr}}$), where approximately linear temperature dependence dominates (with $d\rho/dT > 0$). For high mobility Si-MOS samples we find empirically $T_q \sim 0.007E_F$ and $T_{\text{cr}} \sim 0.07E_F$. In the crossover regime and at higher densities ($\geq 2 \times 10^{11} \text{cm}^{-2}$), $\rho(T)$ goes through a minimum at temperature $T_{\text{min}}$. Both the absolute value of $T_{\text{min}}$ and its dependence on the carrier concentration are found to be in a reasonable agreement with the conventional weak-localization theory. For densities smaller than $\sim 2 \times 10^{11} \text{cm}^{-2}$, the theoretical estimate for $T_{\text{min}}$ falls outside the experimentally accessible temperature range. This explains the absence of the minimum at these densities in the data. In total, over the two decades in the temperature (domains [ii] and [iii]), the two semiclassical effects mentioned above mimic the metallic like transport properties. Our analysis shows that the behaviour of resistivity $\rho(T)$ in the region of $\rho \ll h/e^2$ can be described phenomenologically in terms of the conventional weak-localization theory.

I. INTRODUCTION

The striking metallic-like (with $d\rho/dT > 0$) temperature dependence of the resistivity observed by now in many two-dimensional (2D) electron and hole systems\textsuperscript{6} remains a subject of active interest. The major question to be addressed in this area is whether this effect manifests the existence of a metallic ground state (contrary to the conventional localization theory\textsuperscript{[9]}, which predicts that $\rho \to \infty$ for $T \to 0$ in 2D) or it is a finite-temperature effect (not necessarily of a single-particle origin).

The analysis and interpretation of the transport experiments on various 2D systems are complicated by the fact that the measurements are performed at finite though low temperatures. It is shown in this paper that the rapid metallic-like $\rho(T)$-dependence is not associated with quantum-interference effects. The origin of the $\rho(T)$-dependence is thus a semi-classical effect (in a sense that electron transport can be described by the classical Boltzmann equation whereas electron statistics may be degenerate or non-degenerate and individual scattering events may or may not require quantum-mechanical description). The microscopic nature of this semi-classical effect is not known at the moment and it is not a subject of the present paper. (Although a number of scenarios have been proposed\textsuperscript{[8][9][10][11]}, it is not clear which one or which combination of the above is capable of explaining all the experimental findings.) We also demonstrate that the semi-classical $T$-dependence is present down to to temperatures as low as 100 times less than the Fermi energy $E_F$, whereas the question on the nature of the ground state is obviously related to the “quantum” regime of even lower temperatures, where all semi-classical effects freeze out. At finite temperatures, the semi-classical $T$-dependent scattering in high mobility samples \textit{may (and does) mimic metallic-like transport behavior}. This rather complex and confusing picture explains why numerous controversial conclusions were reported based on different experimental data. This paper is concerned with the region of the ‘metallic’ state not too close to the apparent metal-insulator transition, in which $\rho \ll h/e^2$. The behaviour of the 2D carrier system in the critical region in the vicinity of this transition was considered earlier by Altshuler et al.\textsuperscript{[8]} and by Abrahams et al.\textsuperscript{[9]} from different points of view.

We present here a “phase diagram” in variables of temperature, density and resistivity and classify the domains of densities and temperatures where quantum-interference or semi-classical effects of different origin manifest themselves. We analyze the transport data over a wide range of temperatures and densities and show that different semi-classical temperature dependences dominate in different domains, down to temperatures as low as $\sim 0.01E_F$, and mimic “metallic-like” conduction. In particular, we analyze in detail the crossover from semi-classical to quantum-coherent conduction domains and show that the data can be described reasonably well with in a phenomenological model\textsuperscript{[9]}, based on the conventional weak-localization theory. The present analy-
sies deals mainly with the data obtained on Si-MOSFET samples in which the metallic conduction is most strongly pronounced and in which disorder effects can be revealed by comparing the samples produced by the same method but with different mobilities. However, we believe that our conclusions may have a wider applicability, and remain valid, at least qualitatively, for other material systems.

II. IDENTIFICATION OF THE QUANTUM, SEMI-CLASSICAL AND CLASSICAL DOMAINS

A. Parameter space

Figure 1 shows a typical temperature dependence of the resistivity, $\rho(T)$, measured over a wide range of densities on a high mobility Si-MOSFET sample [14]. The most striking effect is the strong (almost sixfold) change in resistivity $\rho(T)$ with temperature in the “metallic” regime of $d\rho/dT > 0$. It can be shown that the phonon interference effects are irrelevant.

The range of high temperatures $T > E_F$ corresponds to a non-degenerate carrier system; the corresponding boundary is depicted in Fig. 1 by the vertical dash-dotted line (on the right). In the opposite limit of much lower temperatures and through the whole “metallic” range of densities $n$ from $\sim 1 \times 10^{11}$ to $35 \times 10^{11}$ cm$^{-2}$ (we use units of the temperature, K, for the energy; $E_F = 7.3 \times n$ [K/(10$^{11}$ cm$^{-2}$)]). As temperature decreases, the strong variation of $\rho(T)$ seems to saturate rather definitely at $T = (0.08 - 0.1)E_F$. A closer look reveals, however, that there is still (though much weaker) temperature dependence even at lower temperatures. It is this dependence which is the main subject of this paper.

B. Phase diagram: From quantum to classical domains

The range of high temperatures $T > E_F$ corresponds to a non-degenerate carrier system; the corresponding boundary is depicted in Fig. 1 by the vertical dash-dotted line (on the right). In the opposite limit of much lower temperatures and through the whole “metallic” range of densities, a negative magnetoresistance is observed for $n-$SiMOS samples in weak perpendicular magnetic fields [13]. As is well-known, this effect is due to the suppression of quantum interference [14] and can be utilized to extract the phase breaking time, $\tau_\varphi$, from the data. In Refs. [13,14], $\tau_\varphi$ was found to fall off with temperature as $T^{-1}$, whereas the transport mean free time, $\tau$, (as is seen from $\rho(T)$ curves in Fig. 1) is almost $T$-independent for low temperatures. For that reason, $\tau_\varphi$ drops faster than $\tau$ as $T$ increases and at a certain temperature, $T_{\tau_\varphi = \tau}$, the two times become equal. Evidently, $T_{\tau_\varphi = \tau}$ defines a boundary between the quantum-interference ($T < T_{\tau_\varphi = \tau}$) and semi-classical ($T > T_{\tau_\varphi = \tau}$) domains. This boundary is depicted in Fig. 1 by the dashed line “$\tau_\varphi = \tau$”. The domain “SCI” in Fig. 1 corresponds to the semi-classical physics where quantum-interference effects are irrelevant. We emphasize that the range of temperatures where the resistivity exhibits a strong “metallic”-like $T$-dependence, belongs almost entirely to the semi-classical domain and is therefore a semi-classical effect (though not necessarily of a single-particle origin–it can be, e.g., a Fermi-liquid effect, such as $T$-dependent screening [16]).

Another important crossover temperature is $T_{T = h/\tau}$, at which $T = h/\tau$. For $T < T_{T = h/\tau}$ interaction corrections to the conductivity [14,15] are expected to set in. The corresponding boundary is denoted by “$T_T = 1$” in Fig. 1. Again, a large fraction of the $\{T, n\}$ domain of the rapid, metallic-like $\rho(T)$-dependence is to the right of the “$T_T = 1$” line, where the quantum interaction effects are irrelevant as well.

One might expect the quantum corrections to the conductivity to set in as $T$ decreases below the “$\tau_\varphi = \tau$”...
and “Tτ=1” lines. In fact, the actual temperature at which the quantum correction becomes observable may be significantly lower than the corresponding crossover temperatures. For example, in order for the weak-localization correction to be larger than, e.g., 1% of the total resistivity, the ratio τρ/τ has to be larger than exp[0.01πh/(e^2ρ)], which is much larger than one for ρ ≲ πh/100e^2. Note that even the latter estimate works only if the semi-classical (Drude) resistivity does not depend on temperature. The metallic-like temperature dependence of the Drude resistivity pushes the onset of the quantum-interference effects down to even lower temperatures.

These estimates are in accord with an empiric finding [14] that the logarithmic quantum corrections (with dp/dT < 0) become well pronounced only starting at T/E_F < 0.007 (or T/τ_ρ = τ ≲ 10); this boundary is depicted by the left vertical line “Tρ”. It will be shown later in this paper that this empirical boundary can be understood within the weak-localization theory for a given measured ρ(T)-temperature-dependence in the semi-classical region. In the crossover range, T_ρ < T < T_τ, as will be shown below, the “localizing” quantum correction to the conductivity are masked by a semi-classical T-dependence (with dp/dT > 0).

As density decreases and ρ increases approaching the insulating state, the negative magnetoresistance peak becomes broader [12,13] which makes identification of the quantum/semi-classical boundary progressively more difficult. The corresponding region in the vicinity of the MIT is encircled in Fig. 1. No quantum corrections to the conductivity have been quantified experimentally in this region so far. On the other hand, the theoretical perturbative results for quantum corrections to conductivity are not supposed to be valid in this range of r_s ≈ 8−10 (here r_s = E_c/E_F is the ratio of the Coulomb to Fermi energy) and ρ ≳ h/e^2.

III. DATA ANALYSIS IN THE QUANTUM AND SEMI-CLASSICAL DOMAINS

A. Experimental details

In the rest of this paper, we will use the experimental data on ρ(T) obtained on four Si-MOS samples, whose relevant parameters are listed in Table 1.

| sample     | μ_peak (m^2/Vs) | n_c   | p_c   |
|------------|-----------------|-------|-------|
| Si-22/9.5  | 3.3             | 0.83  | 2     |
| Si-15a     | 3.2             | 0.82  | 2.5   |
| Si-43b     | 1.96            | 1.4   | 0.7   |
| Si-4/32    | 0.9             | 2.0   | 0.58  |

TABLE I. Sample parameters. μ_peak is the peak mobility at T = 0.3 K. n_c and p_c are the critical density (in units of 10^13 cm^-2) and critical resistivity (in units of h/e^2), respectively.

B. Localizing upturn in ρ(T): weak-localization theory

Within the conventional theory of disordered Fermi liquids [14,15] which is applicable in the “metallic” domain (ρ < πh/e^2), weak localization and interaction should eventually lead to the “localizing” (dp/dT < 0) T-dependence of the resistivity for low enough temperatures. The competition between these two effects and the one(s) responsible for the “metallic” (dp/dT > 0) dependence at higher temperatures should result in a resistivity minimum. It is (partially) the lack of observation of such a minimum near the “transition” in Si MOSFET’s as so far prevented an identification of the encircled region in Fig. 1. However, at higher densities the minimum does occur [10,17] within the accessible temperature range. Also, the minimum is observed near the transition in p-GaAs [21]. It is thus of crucial importance to understand what are the predictions of the conventional theory with respect to the position of the minimum and its dependence on the carrier density and other parameters.

In this regard, three of the authors (BLA, DLM, and VMP) have recently put forward the following argument [10]. Let us assume, in accord with experimental data [10], that

i) the Drude resistivity is of the form

ρ_d(T) = ρ_1 + ρ_0(T),

where the temperature-dependent part, ρ_0(T), is metallic-like (dp_0/dT > 0) and much smaller than the T-independent one, ρ_1; the latter assumption corresponds to the crossover domain “2” in Fig. 1;

ii) the phase breaking time, τ_ρ, scales with T as [13,15]:

τ_ρ(T) ∝ T^{-p};

iii) both the Drude and observable resistivities are small compared to h/e^2, so that the weak localization (WL) theory is applicable.

It then follows immediately from the familiar expression for the WL correction in 2D [13]

1/ρ = 1/ρ_d(T) - (e^2/πh) ln[τ_ρ(T)/τ(T)],

that under these conditions the resistivity has a minimum at a temperature determined from the following equation [21]:

T_{min} = \frac{e^2 p_0^2}{\pi h \rho_1^2} \left( \frac{dp_0}{dT} \right)_{T=T_{min}}^{-1}.

(When differentiating Eq. [8] with respect to T, we neglect the derivative dτ/dT, whose contribution to [8] is
of the order of $\rho_d e^2/h$, which is much smaller than one within the WL theory and thus negligible.) Interaction leads to another logarithmic term in Eq. (3) with a pre-factor consisting of two parts: a universal (singlet channel) and non-universal one (triplet channel) \[8\]. This amounts to a change in Eq. (3): \[ p \rightarrow p^* \]. Although \( p^* \) may itself depend on \( r_s \) (via the corresponding dependence of the triplet channel contribution), we disregard this possibility in the present analysis and view \( p^* \) as a phenomenological parameter not necessarily equal to \( p \).

In the context of experiments on the “metal-insulator transition in 2D”, the authors of Ref. [17] further observed that if \( d\rho_0/dT \) is a decreasing function of the carrier density, \( n \), then \( T_{\text{min}} \) increases with \( n \). One arrives thus at a somewhat counter-intuitive conclusion: the deeper one goes into the “metallic” regime, i.e., into the region of larger densities and smaller resistivities, the higher is the temperature at which weak localization sets in.

This part of our paper has been criticized in Ref. [22]. It was pointed out there that the residual resistivity, \( \rho_1 \), depends on \( n \) as well. In particular, at relatively small densities \( [n = (2.6 - 5.7) \times 10^{11}\text{cm}^{-2}] \), \( \rho_1 \) decreases with increasing \( n \). This decrease can compensate the decrease in \( d\rho_0/dT \) with increasing \( n \) in the denominator of Eq. (4). Extracting the \( \rho_1(n) \)- and \( d\rho_0(n,T)/dT \)-dependences from the small-\( n \) data of Ref. \[23\] and using Eq. (4), the author of Ref. [22] concluded that \( T_{\text{min}} \) decreases with increasing \( n \) in the density interval, specified above. This would mean that the “less metallic” curves (closer to the “transition”) exhibit the weak-localization minimum at higher temperatures than the “more metallic” ones (further away from the “transition”), which is not consistent with the experiment. On the basis of this analysis, the author of Ref. [22] claimed that the “trap model”, suggested by two of us earlier \[3\], is “deconstructed”.

We disagree with the conclusions of Ref. [22]. First of all, it should be pointed out that the author of Ref. [22] did not fully appreciate the fact that the derivation of Eq. (4) relies neither on the “trap model” nor on any other specific mechanism of the temperature dependence of the Drude resistivity. Thus if something was “deconstructed” in Ref. [22], it is the applicability of the WL theory to the systems that demonstrate the “metal-insulator transition in 2D”. Such a possibility can not be excluded a priori. However, a mounting number of experiments, including those presented in this paper, shows that the “anomalous metallic dependence” is a semi-classical effect and the “anomalous metal” demonstrates all the conventional features of WL-behavior \[17,19,20,23,25\], including the localization upturn at lower temperatures. This upturn was observed in Si MOSFETs at higher densities \((n \gtrsim 10 - 20n_c, \text{where } n_c \text{ is a suitably defined “critical density” of the transition}) \[19,17\], and in p-GaAs for \( n \approx n_c \) \[20\]. On the phase diagram of Fig. [1], the upturn occurs in the vicinity of the empiric boundary \( T_q \).

Assuming that the weak-localization theory still describes the “anomalous” metallic state, what density dependence of \( T_{\text{min}} \) does it predict? To answer this question, we performed a detailed analysis of the available experimental data.

It is shown in this paper that no definite conclusion can be obtained via the procedure employed in Ref. [22]. Extracting the derivative, \( d\rho/dT \), from the data is a an ambiguous procedure: different fitting functions, which approximate the data equally well, give drastically different results for \( d\rho/dT \). Disregarding this ambiguity for a moment, for some fitting functions one obtains a non-monotonic dependence of \( T_{\text{min}} \) on the carrier density. Indeed, \( T_{\text{min}} \) first decreases with \( n \) (in the density interval analyzed in Ref. [22]), but then it goes through a minimum and increases upon further increase in \( n \). Moreover, given the discrepancy between results obtained by using different fitting functions, one should not take seriously any of the conclusions obtained in this way, including those of Ref. [22]. However, it is possible to carry out a much less ambiguous analysis of the data. As it is described below, this analysis shows that for Si MOSFET samples \( T_{\text{min}} \) increases monotonically with \( n \). Over a wide density range \((2.35n_c \leq n \leq 13n_c)\), the calculated value \( T_{\text{min}} \) is smaller than the lowest temperature accessible in the experiment in question \((T_{\text{acc}} = 0.29K)\). The fact that no minimum in the resistivity was observed for these densities receives thus a natural explanation: the temperature was still too high to reach the minimum. For larger densities, \( T_{\text{min}} \) is predicted to exceed \( T_{\text{acc}} \). Indeed, for these densities the minimum in \( \rho \) is observed. Both the absolute value and the density-dependence of the experimentally determined \( T_{\text{min}} \) is in a satisfactory agreement with the theory.

C. Localizing upturn in resistivity: Experimental data

A typical temperature dependence of the resistivity was shown in Fig. [1]. As the temperature decreases from \( \sim T_F \) to \( \sim 0.1T_F \) (i.e., through the semi-classical domain), the resistivity decreases rapidly. A reasonably well description of the data in this temperature range is given by the following form \[23,17,7\]:

\[
\rho(T) = \rho_1 + \rho_0 \exp \left[ T_0(n)/T \right]^{p}.
\] (5)

For lower temperatures and higher densities, \( n \geq n^* \) (where \( n^* \approx 20 \times 10^{11}\text{cm}^{-2} \) for this sample), the strong drop in \( \rho \) slows down, goes through a shallow minimum and finally transforms into the conventional WL logarithmic \( T \)-dependence. Fig. [2] shows the behavior of \( \rho \) in the crossover regime between the domains of rapid variation and WL upturn. It is evident from Fig. [2] that \( T_{\text{min}} \) increases with \( n \).
FIG. 2. Temperature dependence of $\rho$ for sample Si15a. Numbers at the curves indicate the density in units of $10^{11}$ cm$^{-2}$. Circles on the curves mark positions of the minima. Note that $dT_{\text{min}}/dn > 0$.

In order to demonstrate the minimum in $\rho(T)$ in the range of “high” densities ($n > n^*$) in more detail, the crossover region for another sample, Si-43b, is blown up in Fig. 3.

FIG. 3. Expanded low-temperature part of the $\rho(T)$-dependence for sample Si-43b at two different densities in the range $n > n^*$. Arrows mark positions of the resistivity minima. Dashed lines: fit of the upward part of $\rho(T)$ with a ln $T$-dependence.

We determined $T_{\text{min}}$ from our data (in the way demonstrated in Figs. 2, 3) for those densities at which the minimum is clearly pronounced. In Figure 3, the dependence of $T_{\text{min}}$ on the carrier density is shown for three samples. In a qualitative agreement with Ref. [9], $T_{\text{min}}$ decreases with the density down to $n = n^* \sim (20 - 25) \times 10^{11}$ cm$^{-2}$. For lower densities, $n < n^*$, the minimum in $\rho(T)$ is probably below the lowest accessible temperature of the experiment $T_{\text{acc}} = 0.27 - 0.3$ K, and thus unobservable. On the phase diagram Fig. 1, this occurs when $T_q$ becomes equal to $T_{\text{acc}} = 0.3$ K.

For densities lower than $n^*$, $\rho(T)$ decreases with $T$ down to the lowest accessible temperature (Fig. 3). This behavior was attributed earlier to a delocalizing logarithmic temperature dependence [19]. Similar results were also reported in Ref. [27,28] for $p$-GaAs/AlGaAs. However, a more detailed analysis of the data for Si-MOSFET samples enabled us to decompose the apparent “metallic” $T$-dependence of the conductivity, observed in the range $T = (0.07 - 0.02) \times E_F$ into two contributions, namely,

$$1/\rho(T) = 1/\rho_{\text{lin}}(T) + B \ln T,$$

where

$$\rho_{\text{lin}}(T) = \rho_1(n) \left[1 + A(n) \frac{T}{T_F}\right]; A > 0.$$

The first term in Eq. (6) dominates in the crossover region, $T \gtrsim (0.007 - 0.07)T_F$ in Fig. 3 whereas the second one corresponds to the conventional weak localization correction (see Eq. (3)) and becomes more important at lower temperatures ($T < T_q$ in the diagram of Fig. 1).
metallic-like behavior which masks the localization effect at not too large densities and not too small temperatures, rather than an indication of a novel “antilocalizing” behavior. Similar conclusions regarding $p$-type GaAs/AlGaAs and SiGe quantum wells were made in Refs. [20,29].

We note that the “metallic” linear $T$-dependence of the resistivity is seen best of all in most disordered samples (see e.g., Fig. 1c in Ref. [30] and Fig. 5 in Ref. [6]), in which the exponential $\rho(T)$-dependence is much weaker and does not mask the linear one. This linear $T$-dependence is similar to that observed earlier [31] [32]. At first sight, it also seems to be consistent with the one predicted theoretically for the $T$-dependence of screening [16]. In fact, whereas for high mobility samples there is a rough similarity between the measured and predicted density dependences of the prefactor $A(n)$, for lower mobility samples we found an essential disagreement even on a qualitative level. A much more detailed theoretical and experimental work is required to establish the precise nature of the linear $\rho(T)$-dependence. This information, however, is not needed for our present purpose of translating the measured $\rho(T)$-dependence into the position of the minimum via Eq. (3).

IV. COMPARISON OF EXPERIMENT TO THEORY

A. How it is not to be done

We now turn to the analysis of the data in terms of the WL theory. The evaluation of $T_{\text{min}}$ from Eq. (4) appears to be rather straightforward: one fits the measured $\rho(T)$ with a suitable function, finds the derivative of this function and then solves transcendental equation (4) for $T_{\text{min}}$. This is what was done in Ref. [22]. Here we repeat this procedure for a much wider range of densities and for two different choices of the fitting function. In Fig. 6 we demonstrate an example of the ambiguity, which is inevitable in such a procedure. The same data for $\rho(T)$ (sample Si4/32, $n = 2.97 \times 10^{11} \text{cm}^{-2}$) is fitted with two functions: exponential (solid)

$$\rho_{\text{exp}}(T) = \left(\frac{h}{e^2}\right) \{0.19 + 0.4089 \exp(-4.243/T[K])\}$$

and polynomial (dashed)

$$\rho_{\text{poly}}(T) = \left(\frac{h}{e^2}\right) \sum_{n=0}^{4} C_n (T[K])^n$$

where $C_0 = 1.610 \times 10^{-1}, C_1 = 4.715 \times 10^{-2}, C_2 = -8.321 \times 10^{-4}, C_3 = -1.342 \times 10^{-4},$ and $C_4 = 4.921 \times 10^{-6}$. As one can see from Fig. 6, both function (8) and (9) fit the data equally well. However, the temperature derivatives of fitting functions (8) and (9) differ drastically.
The calculated values of $T_{\text{min}}$ for both fitting functions (8) and (9) are shown in Fig. 8 along with the results of Ref. [22] (circles). We have chosen $\rho = 1$ for the coefficient in Eq. (4). This is roughly consistent with the measured $T$-dependence of $\tau_p$ for these samples. Triangles (squares) depict the results for the polynomial (exponential) fit. Solid (open) symbols correspond to sample Si15a (Si22/9.5). Not surprisingly, the mentioned difference in derivatives of fitting functions leads to different values of $T_{\text{min}}$, both of which differ also from the result of Ref. [22]. The tendency of $T_{\text{min}}$ to decrease with increasing $n$, which was emphasized so strongly in Ref. [22], manifests itself in our calculations as well. However, it occurs only at small densities. As the density increases further, $T_{\text{min}}$ goes through a minimum and then starts to increase with $n$. We are not aware of what kind of fitting procedure was used in Ref. [22], and how the ambiguity demonstrated above was avoided. It is also not quite clear for us why the analysis of Ref. [22] was restricted to a rather narrow density range, whereas the data over a much wider density range had been already available. We are certain though that no definite statement can be made on the basis of the results depicted in Fig. 8.

**B. How it can be done**

The separation of the measured dependence into two contributions [cf. Eq. (6)] allows one to minimize the ambiguity of the fitting procedure. The dependence of $T_{\text{min}}$ on $n$, obtained from Eq. (6) is shown in Fig. 8, where we used the experimentally determined $\rho_0(T) = \rho_{\text{min}}$ [Eq. (6)] data for the sample Si22/9.5. As one can see, $T_{\text{min}}$ increases monotonically with $n$ and crosses the lowest accessible temperature at $n \approx 2 \times 10^{11} \text{cm}^{-2}$. This is precisely the density $n^*$ above which the minimum was actually observed (see Fig. 8). For $n \leq 6 \times 10^{11} \text{cm}^{-2}$ the calculated value of $T_{\text{min}}$ is below 100 mK. For the lowest density, at which the fit with Eqs. (6) still works reasonably well ($n = 2.35 n_c$), the calculated value of $T_{\text{min}}$ is 30 mK. The experimental difficulties arising from electron heating at such low temperatures have been discussed in detail in Ref. [22].
The $T_{\text{min}}(n)$ dependence displayed in Fig. 8 is by no means universal. It is determined by a particular form of the temperature dependence of the Drude resistivity on $T$, which varies from system to system and even from sample to sample within one system class. A careful analysis of the experimental $\rho(T)$-dependence is needed to extrapolate the position of $T_{\text{min}}$. In light of this, it can be readily understood why some p-GaAs samples [20] exhibit a resistivity minimum already for $n \sim n_c$, and the temperature of the minimum decreases with increasing $n$.

V. SUMMARY

To summarize, we analyzed the available experimental data for the resistivity of high mobility Si-inversion layers, over a wide range of temperatures and densities. Using recent experimental data on the temperature dependence of the quantum coherence time [17] we classified the domains of temperatures and densities where quantum interference, classical, and semiclassical effects govern the conduction. We clarified experimentally the various temperature dependences dominating over these domains and analyzed them theoretically. We found, in particular, (i) the anomalous strong metallic-like temperature dependence (with $d\rho(T)/dT > 0$) belongs almost entirely to the 'high temperature' semiclassical domain $E_F > T > T_q$, an approximately linear metallic-like dependence (most likely of the semiclassical origin), $\delta \rho(T) \propto T/E_F$, dominates, and (iii) over the quantum domain $T < T_q$, the logarithmic $T$-dependence (with $d\rho(T)/dT < 0$) prevails. Through a limited temperature (crossover) range, the interplay of the two latter dependences mimics the metallic-like behavior. As the temperature is decreased further, the resistivity passes through a minimum and turns upward. Using a phenomenological theory we described the interplay of the two above effects and made a prediction on the temperature $T_{\text{min}}$ where the resistivity shows an upturn. We found a reasonably good agreement between the experimentally determined positions of these resistivity minima vs carrier density and the calculated ones. In order to calculate reliably the position of these minima, it is necessary to know precisely not only $\rho(T)$ in the low-temperature (crossover) region, but also its temperature derivative $d\rho(T)/dT$; the latter one requires highly precise data and their careful analysis. Finally, we conclude that the existing data on the temperature dependence of the resistivity of high mobility Si inversion layers over the range of $\rho \ll h/e^2$ can be successfully described in terms of a temperature dependent semiclassical Drude resistivity in combination with conventional weak localization theory.

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[1] S. V. Kravchenko, G. V. Kravchenko, J. E. Furneaux, V. M. Pudalov, and M. D’orio, Phys. Rev. B 50, 8039 (1994); S. V. Kravchenko, G. E. Bowler, J. E. Furneaux, V. M. Pudalov, and M. D’orio, Phys. Rev. B 51, 7038 (1995). V. M. Pudalov, G. Brunthaler, A. Prinz, and G. Bauer, JETP Lett., 68, 442 (1998). Y. Hanein, U. Meirav, D. Shahrar, C. C. Li, D. C. Tsui, H. Shtrikman, Phys. Rev. Lett. 80, 1288 (1998). Y. Hanein, D. Shahrar, J. Yoon, C. C. Li, D. C. Tsui, and Hadas Shtrikman, Phys. Rev. B 58 R13338 (1998). PRB 58, R7520 (1998).

[2] E. Abrahams, P. W. Anderson, D. C.Licciardello, and T. V. Ramakrishnan, Phys. Rev. Lett. 42 673 (1979). P. Lee and T. D. Ramakrishnan, Phys. Rev. B 26, 4009 (1982).

[3] B. L. Altshuler, D. L. Maslov, Phys. Rev. Lett. 83, 2092 (1999).

[4] S. Das Sarma, E. H. Hwang, Phys. Rev. Lett. 83, 164 (1999).

[5] Y. Meir, Phys. Rev. Lett. 83, 3506 (1999); cond-mat/9912423.

[6] Y. Yaish, O. Prus, E. Buchstab, S. Shapiro, G. Ben Yosef, U. Sivan, and A. Stern, cond-mat/9904324.

[7] B. L. Altshuler, D. L. Maslov, V. M. Pudalov, cond-mat/0003032 Physica E (2000), in press.

[8] E. Abrahams, S. V. Kravchenko, M. P. Sarachik, cond-mat/0006053.

[9] B. L. Altshuler, D. L. Maslov, V. M. Pudalov, Phys. Stat. Sol. (b) 218, 193 (2000); cond-mat/9900353.

[10] A. Prinz, V. M. Pudalov, G. Brunthaler, G. Bauer, in: Proc. Intern. Meeting SIMD-99, Mauri, (1999), edited by K.Hess. Superlattices and Microstructures, (2000), in press.

[11] Phonon scattering is weak in Si due to the weak deformation potential coupling. In GaAs/AlGaAs electron-phonon scattering is much stronger because of the piezo-coupling and phonon contribution to the ‘metallic’ temperature dependence of $\rho$ is rather essential down to sub-
K range of temperatures (A. P. Mills, Jr., A. P. Ramirez, L N. Pfeiffer, and K. W. West, Phys. Rev. Lett. 83, p. 2805 (1999).)

[12] V. Pudalov, G. Brunthaler, A. Prinz, G. Bauer, JETP Lett. 65, 932 (1997); Physica B, 249-251, 697 (1998).

[13] G. Brunthaler, A. Prinz, G. Bauer, V. Pudalov, E. Dizhur, J. Jaroszynski, P. Glod, and T. Dietl, Ann. der Phys. (Leipzig), 8, 579 (1999).

[14] P. A. Lee, T. V. Ramakrishnan, Rev. Mod. Phys 57, 287 (1985).

[15] G. Brunthaler, A. Prinz, G. Bauer, V. M. Pudalov, cond-mat/0007230.

[16] a) F. Stern, Phys. Rev. Lett. 44, 1469 (1980); b) A. Gold and V. T. Dolgopolov, J.Phys. C, 18; L463 (1985); c) A. Gold and V. T. Dolgopolov, Phys. Rev. B 33, 1076 (1986). d) S. Das Sarma, Phys. Rev. B 33, 5401 (1986).

[17] V. M. Pudalov, G. Brunthaler, A. Prinz, G. Bauer, Phys. Rev. B 60, R2154 (1999).

[18] B. L. Altshuler and A. G. Aronov, in Electron-electron interactions in disordered systems, ed. by A. L. Efros and M. Pollak (Elsevier, Amsterdam, 1985), p. 1.

[19] V. M. Pudalov, G. Brunthaler, A. Prinz, and G. Bauer, JETP Lett. 68, 534 (1998).

[20] M. Y. Simmons, A. R. Hamilton, M. Pepper, E. H. Linfield, P. D. Rose, D. A. Ritchie, cond-mat/9910368.

[21] As we learned later equation for $T_{\text{min}}$ was derived by Gold and Dolgopolov [16]b and, very likely, in some other work, given the utter simplicity of this consideration. It is also similar to the formula for $T_{\text{min}}$ in the Kondo effect theory.

[22] P. Phillips, cond-mat/9910122.

[23] V. M. Pudalov, G. Brunthaler, A. Prinz, and G. Bauer, JETP Lett. 70, 48 (1999).

[24] V. Senz, T. Heinzel, T. Ihn, K. Ensslin, G. Dehlinger, D. Grützmacher, U. Gennser, cond-mat/9910228.

[25] A. R. Hamilton, M. Y. Simmons, M. Pepper, D. A. Ritchie, cond-mat/0003293.

[26] V. M. Pudalov, JETP Lett. 66, 175 (1997).

[27] M. Y. Simmons, A. R. Hamilton, M. Pepper, E. H. Linfield, P. Rose, D. A. Ritchi, A. K. Savchenko, T. G. Griffiths, Phys. Rev. Lett. 80, 1292 (1998).

[28] A. R. Hamilton, M. Y. Simmons, M. Pepper, E. H. Linfield, P. D. Rose, D. A. Ritchie, Phys. Rev. Lett. 82, 1542 (1999), cond-mat/9808108.

[29] V. Senz, T. Ihn, T. Heinzel, K. Ensslin, G. Dehlinger, D. Grützmacher, U. Gennser, cond-mat/0004312.

[30] V. M. Pudalov, G. Brunthaler, A. Prinz, and G. Bauer, Physica E 3, 79 (1998).

[31] K. M. Cham, R. G. Wheeler, Phys. Rev. Lett. 44, 1472 (1980).

[32] E. Vyrodov, V. T. Dolgopolov, C. I. Dorozhkin, and N. B. Zhitenev, Zh. Eksp. Teor. Fiz.94, 234 (1988)– Sov. Phys. JETP 67, 998 (1988).

[33] T. E. Whall, A. D. Plews, N. L. Mattey, and E. H. C. Parker, Appl. Phys. Lett. 65, 3362 (1994).