Lyα Line Formation in an Expanding H I Superbubble in the Primeval Galaxy DLA 2233+131

Sang-Hyeon Ahn and Hee-Won Lee

1. Dept. of Astronomy, Seoul National University
2. Research Institute for Basic Sciences, Seoul National University

Seoul, Korea

e-mail : sha@astro.snu.ac.kr, hwlee@astro.snu.ac.kr

Abstract

Various types of galaxies observed in the cosmological scales show P-Cygni type profiles in the Lyα emission lines. The main underlying mechanism for the profile formation in these systems is thought to be the frequency redistribution of the line photons in an expanding H I component surrounding the emission source.

A Monte Carlo code is developed to investigate the Lyα line transfer in an optically thick and moving medium with a careful consideration of the scattering in the damping wing. Typical column densities and the expansion velocities investigated in this study are $N_{HI} \sim 10^{17-20} \, \text{cm}^{-2}$ and $\Delta V \sim 100 \, \text{km s}^{-1}$.

The main features in emergent line profiles include a primary emission peak in the red part and a much weaker secondary peak in the blue part. We investigate the dependence of the profile on the kinematics and on the column density. The primary peak recedes to the red and the width of the feature increases as $N_{HI}$ increases. It is also found that symmetric double peak profiles are obtained in the static limit, where bulk motion is negligible.

The P-Cygni type profile in the Lyα emission line of DLA 2233+131 ($z = 3.15$) is noted to be similar to those found in the nearby galaxies and distant ($3 < z < 3.5$) H II galaxies. Our numerical results are applied to show that the DLA system may possess an expanding H I supershell with bulk flow of $\sim 200 \, \text{km s}^{-1}$ and that the H I column density $N_{HI}$ is approximately $10^{20} \, \text{cm}^{-2}$.

From the observed Lyα flux and adopting a typical size of the emission region $\sim 1 \, \text{kpc}$, we estimate the electron density of the H II region to be $\sim 1 \, \text{cm}^{-3}$ and the mass of H II region $\sim 10^8 \, \text{M}_\odot$. We also conclude that it requires $\sim 10^3$ O5 stars for photoionization, which is comparable to first-ranked H II regions found in nearby spiral and irregular galaxies.

We briefly review the physical quantities of several astronomical objects which may possess similar kinematic properties to those of DLA 2233+131. We point out the fact that this kind of outflowing media are often accompanied by the sites associated with active star formation.

Key words : Radiative transfer - Monte Carlo - QSO:damped Lyman alpha system:individual:DLA 2233+131 - cosmology:galaxy formation
A typical high-redshift quasar spectrum shows a variety of absorption features blue-ward of the broad Lyα emission line, most of which are not associated with the quasar itself. These absorption systems are thought to be related with the neutral hydrogen and metal elements in the intervening medium along the line of sight to the quasar, and therefore constitute an ideal subject for studying the cosmological structure formation processes. Depending on the neutral hydrogen column density $N_{HI}$, the absorption systems are usually classified into three classes, i.e. damped Lyα (hereafter DLA) systems, Lyman limit systems, and Lyα forest systems.

With the advent of the Hubble Space Telescope and 10 m class telescopes such as Keck I, II, detailed analyses of absorption line profiles have been performed, and provided very important kinematic information about the absorption systems.

Recently Prochaska & Wolfe (1997) analyzed the absorption line profiles of the 17 DLAs obtained with the HIRES echelle spectrograph on the Keck telescope and concluded that the kinematics of the DLAs supports the hypothesis that they are rapidly rotating “cold” disks (see also Haehnelt et al. 1997).

There have been a number of DLA galaxy candidates possibly exhibiting Lyα emission features in their spectra (e.g. Hunstead et al. 1990, Möller & Warren 1993, Pettini et al. 1995 etc.). However, the first clear Lyα emission feature was detected in the spectra of the DLA 2233+131 (Djorgovski et al. 1996).

Djorgovski et al. (1996) reported an isolated galaxy (or a protogalaxy) 2.3” (or 17 kpc) away from the line of sight to the quasar, QSO 2233+131 by detecting the Lyα emission having the same redshift as DLA 2233+131. They also noted that the emission profile is asymmetric to the red and that there is a possible absorption feature to the blue. The width of the absorption feature is measured to be $\sim 200 \text{ km s}^{-1}$. It is not certain that the absorption is related to a very near Lyα forest, as was pointed out by several researchers (Djorgovski et al. 1996, Lu et al. 1997). However, the similarity of the profile to the P-Cygni profile leads to an interesting possibility that an expanding medium envelops the emission source, which can be thought to be a supershell seen in nearby galaxies (see section 4. of Irwin (1995) for a review of outflowing structures). Since the first firm identification of DLA candidate DLA 2233+131, Djorgovski et al. (1997) found several new candidates.

In addition to this direction, there are at least two other groups adopting efficient programs to find out primeval galaxies.

Firstly, Giavalisco et al. (1996) found that there are a number of star-forming galaxies at $3.0 < z < 3.5$ possessing compact spheroidal cores often surrounded by low surface brightness nebulosities. Morphologically similar objects are also found in the Hubble Deep Field (Steidel et al. 1996a). Moreover, Steidel et al. (1996b) argue that the spectra of galaxies with $z > 3$ are remarkably similar to those of nearby star-forming galaxies and often show Lyα lines with P-Cygni profiles characteristic of expanding HI media. The HI column density is inferred to be $10^{20} \text{ cm}^{-2}$ in these systems.

One excellent example of nearby star-forming galaxies is provided by Legrand et al. (1997) and Lequeux (1995) who performed a detailed analysis of the emission profiles of the star-forming galaxy Haro 2. They suggest that these P-Cygni type profiles are
formed by the expanding H I shell surrounding an H II region. At least three more galaxies showing such profiles are found in the literature (Kunth et al. 1996).

Recently Steidel et al. (1997) used the Lyman break criterion to obtain ∼ 80 new candidates that are similar to those found previously. What is interesting is that ∼ 75% of them exhibit P-Cygni type Lyα profiles (Pettini et al. 1997). Lowenthal et al. (1997) also observed the spectra of 24 remote galaxies selected from the Hubble Deep Field.

Secondly, using the flux magnification via gravitational lensing by foreground clusters of galaxies, very faint and remote galaxies can be found. Trager et al. (1997) found a multiply-imaged galaxy at z ∼ 4 gravitationally lensed by the rich cluster CL0939+4713, which are the same kind of those discovered by Lowenthal et al. (1997). Using the same tactics, Franx et al. (1997) discovered the currently remotest galaxy at z = 4.92. Recently Frye et al. (1997) found the remote galaxy at z = 4.04 which is lensed by the cluster of galaxies, A2390 (z = 0.23). Surprisingly, ∼ 75% of the primeval galaxies described above exhibit P-Cygni type Lyα emission lines, which implies the ubiquity of an expanding HI envelope around H II region.

So far, many investigations about the P-Cygni profiles have been concentrated on the outflowing systems possessing rather moderate optical depth. The Sobolev approximation is regarded as a powerful method to understand the behavior of resonantly scattered photons in an expanding medium with a bulk flow much larger than the thermal speed (Sobolev 1960, Mihalas 1978). However, when the medium has high optical thickness, the scattering in the damping wing is not negligible any more and the validity of the Sobolev approximation becomes questionable.

In this paper, we develop a Monte Carlo code for the radiative transfer of Lyα photons in an optically thick and expanding medium that is expected in the H II superbubbles in star-forming galaxies. Extrapolating the kinematical information from our line profile study, we try to find a possible relation of the P-Cygni type absorption profile to the existence of superbubble in the primeval starburst galaxies.

The paper is composed as follows. In section 2 we describe the Monte Carlo method to deal with the Lyα transfer in a thick moving medium. In section 3 we present the main results. We apply our results to the absorption features in the Lyα emission of DLA 2233+131 in the following section. Discussions and the implications for the identity of DLA 2233+131 are given in the final section.

2. Monte Carlo Simulations

2.1 Model Description

In this subsection we give a brief model description and a basic atomic physics for the Monte Carlo code, which computes the profile of the emergent Lyα photons scattered inside an optically thick and expanding medium.

The photon source is assumed to be located at the center of the spherical scattering medium which is truncated at r = r_{max}. We assume that the constant density distribution n(r) = n_0 throughout the scattering region. We only consider an isotropic and point-like
line emission source in this work, which is not a bad approximation for emission sources such as H II regions surrounded by H I regions.

The velocity field of the scattering medium is assumed to be given by a Hubble-type flow, i.e.,

$$\mathbf{v} = H_v \mathbf{r},$$

where $H_v$ is the bulk flow velocity gradient, $\mathbf{v}$ is the bulk velocity, and $\mathbf{r}$ is the distance from the center of the velocity field. In this type of velocity field, the distance is more conveniently measured by a parameter $s$ defined by

$$s \equiv H_v r / v_{th},$$

where $v_{th}$ is the thermal velocity.

If we introduce the Doppler width $\Delta \nu_D$ defined by

$$\Delta \nu_D \equiv \nu_0 v_{th} / c,$$

then the frequency of a line photon is also conveniently described by the normalized frequency shift $x$ defined by

$$x \equiv (\nu - \nu_0) / \Delta \nu_D.$$  

Because the scattering medium is assumed to be optically thick, scattering in the damping wing should be considered with much caution. Therefore we introduce the damping coefficient, $a$, normalized by $\Delta \nu_D$, i.e.,

$$a = \Gamma / 4\pi \Delta \nu_D,$$

where $\Gamma$ is the damping constant. The scattering cross section in the rest frame of the scatterer is then given by

$$\sigma_\nu = \sigma(x) = \frac{\pi e^2}{m_e c f_{osc}} \frac{\Gamma / 4\pi^2}{(\nu - \nu_0)^2 + (\Gamma / 4\pi)^2}$$

$$= \frac{\pi e^2}{m_e c f_{osc}} \frac{1}{\pi \Delta \nu_D} \frac{a}{a^2 + x^2},$$

where $f_{osc}$ is the oscillator strength, $m_e$ is the electron mass (Rybicki & Lightman 1979).

The scattering optical depth $\tau_{12}(x)$ of a line photon with frequency shift $x$ and wave vector $\mathbf{k}_i$ corresponding to the distance from the position $\mathbf{r}_1$ to $\mathbf{r}_2$ is computed to determine the subsequent scattering position in the Monte Carlo calculation. Due to the Hubble-type velocity field and a constant density field inside the scattering medium, we get the same Hubble-type flow if we make a translation so that $\mathbf{r}_1$ coincides with the new origin. The line center frequency $\nu'_0(\mathbf{r})$ in the observer’s frame differs from that in the scatterer’s rest frame by the Doppler shift, which is given by

$$\nu'_0(\mathbf{r}) = \nu_0(1 - H_v r / c + v_{loc} / c),$$
where \( r \) is the distance from the new origin \( r_1 \) and \( v_{loc} \) is the local velocity of the scatterer due to the thermal motion.

The normalized frequency shift \( x'(r) \) is now simply given by

\[
x'(r) = x - s + u,
\]

where \( u \equiv v_{loc}/v_{th} \). Therefore the scattering cross section \( \sigma(x) \) becomes a function of the position due to the coupling of the line center frequency and the position determined by the expansion of the medium.

The thermal distribution of the scatterers is assumed to be given by a Maxwellian distribution, that is,

\[
n = n_0 \int dv_{loc} f(v_{loc}),
\]

where \( f(v_{loc}) = \frac{1}{\sqrt{\pi}v_{th}} e^{-(v_{loc}/v_{th})^2} = \frac{1}{\sqrt{\pi}v_{th}} e^{-u^2} \) is the Maxwellian distribution function.

Combining Eqs. 1-9, we obtain

\[
\tau_{12}(s) = \int_{-\infty}^{\infty} du \int_0^s ds' n_0 f(u)\sigma[x'(s')] \\
= \tau_0 \int_{-\infty}^{\infty} du e^{-u^2} \left[ \tan^{-1}\left( \frac{u + x}{a} \right) - \tan^{-1}\left( \frac{u + x - s}{a} \right) \right].
\]

Here, \( \tau_0 \) is the Sobolev-type optical depth defined by

\[
\tau_0 = \frac{\pi e^2}{mc f_{osc} n_o} \frac{\lambda_0}{\pi^{3/2} H_v}
\]

(Sobolev 1960).

The inverse transformation of Eq. 10 is obtained to find the normalized path length \( s \) corresponding to a given scattering optical depth \( \tau_{12} \). The escape optical depth \( \tau_{esc} \) is also obtained by setting \( s = s_{max} \), where

\[
s_{max} = H_v r_{max}/v_{th} = \Delta V_{bulk}/v_{th}.
\]

Here, \( \Delta V_{bulk} \) is the difference of the bulk velocities at the center and the boundary of the scattering medium.

The conversion of the H I column density and the Sobolev type optical depth \( \tau_0 \) is then given by

\[
\tau_0 = 10^4 \left( \frac{N_{HI}}{4.1 \times 10^{18} \text{ cm}^{-2}} \right) \left( \frac{H_v \cdot r_{max}}{100 \text{ km s}^{-1}} \right)^{-1} \left( \frac{f_{osc}}{0.4162} \right).
\]

### 2.2 Monte Carlo Approach
In this subsection, we present a detailed description of the Monte Carlo procedure. There are a few approaches to the resonance line transfer in an optically thick and static medium (Osterbrock 1962, Adams 1972, Harrington 1973, Sengupta 1994, Gould & Weinberg 1996). However, there are also studies on the line formation in a thick expanding medium (e.g. Rybicki & Hummer 1978).

The Sobolev approximation has been one of the most favored methods in dealing with the line transfer in an expanding medium. However, the validity of the Sobolev method is limited to the cases where the bulk flow is much larger than the thermal velocity and the optical depth of the medium does not greatly exceed unit. This is because the scattering in the damping wing is no more negligible in an optically thick medium where the typical line center optical depth \( \tau_c \gtrsim 10^4 \). Therefore we develop a Monte Carlo code in which the scattering in the damping wing is carefully considered in a thick moving medium.

The Monte Carlo code begins with the choice of the frequency and the propagation direction \( \mathbf{k}_i \) of the incident photon from an assumed Ly\( \alpha \) profile.

Then we determine the next scattering site separated from the initial point by the normalized propagation length \( s \) defined by Eq. 2, which corresponds to the optical depth of \( \tau = -\ln(R) \), where \( R \) is a uniform random number in the interval \((0, 1)\). Due to the unwieldiness of the inverse relation of Eq. 10, we tabulate the normalized path length \( s(x, \tau) \) in advance and look up the table to find \( s \) by the linear interpolation. For our numerical computation, the range of \( x \) is taken to be from \(-4s_{\text{max}} \) to \( 4s_{\text{max}} \) with a step of \( \Delta x = 0.5 \), and \( \tau \) runs from 0 to \( 4s_{\text{max}} \) with a step of 0.002, where \( s_{\text{max}} = 10 \) is assumed.

In Fig. 1 we present the \( s(x, \tau) \) table as a surface plot. We restrict the propagating length \( s \) to be \( 0 < s < 2s_{\text{max}} \), and set \( s = 2s_{\text{max}} + 1 \) if the true value of \( s(x, \tau) \) exceeds \( 2s_{\text{max}} \). A U-shaped distribution of \( s \) is seen in the figure, which is naturally explained by the fact that the scattering medium is transparent for extreme blue and red photons.

The emitted photon traverses a distance \( s \) found by the above procedure and is scattered off if \( s < 2s_{\text{max}} \). In this scattering event the frequencies of the absorbed photon and the re-emitted one in the rest frame of the scatterer should be matched. For the case of an optically thin and expanding medium, this frequency matching is combined with the Sobolev approximation to yield the absorption profile in the form of the Dirac delta function.

As mentioned earlier, in contrast with the case of a thin medium, for a very thick medium the scattering in the damping wing is not negligible. A good care need to be exercised to distinguish the scattering in the damping wing from the resonance scattering, because they show quite different behaviors in the properties including the scattering phase function and the polarization (Lee 1997, Lee & Blandford 1997, Blandford & Lee 1997).

Because the natural line width is much smaller than the Doppler width, the local velocity of the scatterers that can resonantly scatter the incident photon is practically a single value. However, when the scattering occurs in the damping wing, the local velocity of the scatterer may run a rather large range. Therefore, in order to enhance the efficiency of the Monte Carlo method, it is desirable to determine the scattering type before we determine the local velocity \( \mathbf{u} \) of the scatterer.
We present a more quantitative argument about the preceding remarks. Under the condition that a given photon is scattered off by an atom located at a position \( s \), the local velocity component \( u \) along the direction \( k_i \) is simply given by

\[
 f(u) \propto \frac{e^{-u^2}}{(u-b)^2 + a^2}, \tag{14}
\]

where \( b = s - x \). The normalization condition is used to get

\[
 f(u) = \frac{e^{-u^2}}{(u-b)^2 + a^2} \left[ \frac{\pi}{a} H(a, b) \right]^{-1}. \tag{15}
\]

Here, the Voigt function \( H(a, b) \) is evaluated by a series expansion in \( a \), i.e.,

\[
 H(a, b) = H_0(a, b) + aH_1(a, b) + a^2H_2(a, b) + a^3H_3(a, b) + \cdots, \tag{16}
\]

where \( H_n(a, b), n = 0, 1, 2, 3 \) are tabulated by Gray (1992).

Because of the smallness of \( a \), the function \( f \) has a sharp peak around \( u \approx b \), for which the scattering is resonant. Therefore, the probability \( P_r \) that a given scattering is resonant is approximately given by

\[
 P_r \approx \int_{-\infty}^{\infty} d(\Delta u) \frac{e^{-b^2}}{(\Delta u)^2 + a^2} \left[ \frac{\pi}{a} H(a, b) \right]^{-1} = \frac{e^{-b^2}}{H(a, b)}. \tag{17}
\]

The probability that a scattering occurs in the damping wing is

\[
 P_{nr} = 1 - P_r. \tag{18}
\]

In the code we determine the scattering type in accordance with the scattering type probabilities \( P_r \) and \( P_{nr} \). If a scattering is chosen to be resonant, then we set \( u = b \). Otherwise, the scattering occurs in the damping wing, and \( u \) is chosen according to the velocity probability distribution given by Eq. 15.

We give the propagation direction \( k_f \) of a scattered photon in accordance with the isotropic phase function. The scattered velocity component \( v_{\perp} \) perpendicular to the initial direction \( k_i \) on the plane spanned by \( k_i \) and \( k_f \) is also governed by a Maxwell-Boltzmann velocity distribution, which is numerically obtained using the subroutine \textit{gasdev()} suggested by Press \textit{et al.} (1989). The contribution \( \Delta x \) of the perpendicular velocity component \( v_{\perp} \) to the frequency shift along the direction of \( k_f \) is obviously

\[
 \Delta x = v_{\perp} [1 - (k_i \cdot k_f)^2]^{1/2}/c. \tag{19}
\]

Therefore, the frequency shift \( x_f \) of the scattered photon is given by

\[
 x_f = x_i - u + u(k_i \cdot k_f) + v_{\perp} [1 - (k_i \cdot k_f)^2]^{1/2}, \tag{20}
\]

where \( x_i \) is the frequency shift of the incident photon.
In each scattering event the position of the scattered photon is checked and if it is out of the medium we collect this photon according to its frequency and escaping direction. The whole procedure is repeated to collect typically about $10^3$ photons in each frequency bin.

3. Results

3.1 Escape Probability

In Fig. 2 we show the emergent Ly\(\alpha\) profile from a thick expanding medium. The horizontal axis represents $\Delta \lambda/\Delta \lambda_D = -x$, and the vertical axis stands for the relative flux. Note that the wavelength deviation is the negative of the frequency deviation and $\Delta \lambda_D$ is the Doppler wavelength. The emission source lying in the center of the scattering medium is assumed to be given by a Gaussian profile proportional to $e^{-(x/2\sigma_x)^2}$, where the width $\sigma_x$ is set to be 5. The use of a Gaussian profile may be justified by the observed Gaussian line profiles for other emission lines such as H\(\alpha\) or [O III] (Forbes et al. 1996, Legrand et al. 1997). We choose $v_{th} = 10$ km s\(^{-1}\), $s_{max} = 10$, which corresponds to $\Delta V_{bulk} = 100$ km s\(^{-1}\). We also choose $\tau_0 = 10^4$ that leads to $N_{HI} = 4.1 \times 10^{18}$ cm\(^{-2}\) given the above choice of $s_{max}$.

The main features in the emergent profile in Fig. 2 include a primary peak in the red part and a much weaker secondary peak in the blue part. One may understand this behavior qualitatively by considering the escape probability $P_{esc}$, with which a given photon escapes from the region to an observer. We note that the single-scattering escape probability is given by

$$P_{esc} = e^{-\tau_x},$$

where the escaping optical depth $\tau_x$ is obtained from Eq. 10 setting $s = s_{max}$.

In Fig. 3 we show the single-scattering escaping optical depth and the escape probability. The dashed line represents $\tau_x$, and the $P_{esc}$ is shown by the dotted line. The dot-dashed line is the input emission profile at the origin given by a Gaussian and the solid line is the product of the initial emission at the origin and the escape probability. The analysis has a simple interpretation of the emergent profile without being scattered from the source to the observer.

Because the medium is not optically thin, the escape probability analysis is a very poor approximation to the emergent profile which is severely affected by photons scattered a large number of times. However, the qualitative nature of the red asymmetric profile is obvious from the figure. The Sobolev optical depth $\tau_0 \gg 1$ for the incident photons with frequency shift $x$ is satisfied in $-3 < x < s_{max} + 3$. Therefore, we expect a severe discrepancy in this regime.

In Fig. 4 we show the emergent profiles for the case where the incident profile from the emission source is given by a flat continuum. We use the same parameters as in Fig. 2. The broad absorption feature is formed in the frequency range of $-3 < x < s_{max} + 3$ with a sharp peak on the red side.

3.2 Line Formation Mechanism
When the static medium has a moderate line center optical depth $\tau_c < 10^5$, the radiative transfer is dominated by the diffusion in frequency space, and the spatial diffusion plays a rather minor role. Hence each photon is scattered many times in the vicinity of its source and gets a sufficient frequency shift just before it escapes, which Adams (1972) described as “a single longest flight.” For a thicker medium with $\tau_c \gtrsim 10^5$, the damping wing scattering becomes important, and the photons emerge via the spatial diffusion. This process was called “a single longest excursion” by Adams (1972). He investigated these processes using a Monte Carlo method and showed that the mean scattering number $< N >$ before escape is given by

$$< N > \approx \begin{cases} \tau_c \sqrt{\pi \ln \tau_c} & \text{for } \tau_c < 10^5 \\ 1.57^{1/2} \tau_c & \text{for } \tau_c > 10^5. \end{cases}$$

In the case of an expanding medium, using the Sobolev theory in a resonance shell having a width of velocity difference of order $v_{th}$, the effective optical depth is the Sobolev scattering optical depth $\tau_0$ that is the total scattering optical depth in the limit $\Delta V \to 0$. However, this hypothesis works only when $\tau_0 \lesssim 10^3$, in which case a frequency diffusion into the red wing leads to a direct escape from the scattering medium without further scattering in the damping wing.

When $\tau_0 \geq 10^4$, the damping wing scattering optical depth is not negligible and the line photon can be captured with significant probability despite the frequency diffusion into the red wing. The subsequent scatterings tend to reduce the frequency deviation of the line photon as the restoring force in the frequency space operates (see Adams 1972). Therefore, the line photon is again resonantly scattered after several wing scatterings and severe spatial movement. In this case, the total scattering number is dominantly affected by the total column density of the scattering medium, which is proportional to the product of $\tau_0$ and $s_{max}$ according to Eq. 13.

$$\tau_0 \leq 10^4$$ and $s_{max} = 10$, the scattering number before escape is expected to be of order $10^5$ by regarding the resonance shell as a static medium with a line center optical depth $\tau_c \sim \tau_0$ (Adams 1972). It is checked that the scattering number increases almost linearly with $\tau_0 s_{max}$.

In Fig. 5a is shown a typical scattering behavior of a photon with $x \sim -5$. The horizontal axis represents the scattering number. In the bottom panel is shown the frequency variation of the photon. In the middle panel, we show the radial distance of the photon from the center of the sphere with radius $s = 10$. The scattering type is shown in the top panel in which we denote a wing scattering by 0 and a resonance scattering by 1.

Near the 360th scattering, the frequency of the photon deviates much from the center frequency, and several scatterings in the damping wing follow. During this process, the photon propagates a large distance in real space and becomes resonant again by the effect of the restoring force (in frequency space) stated by Adams (1972). When the photon escapes from the medium, it experiences several wing scatterings, and the frequency of the photon falls on the red part.

In Fig. 5b we show a typical scattering behavior of a blue photon with $x \sim 5$. Initially it experiences several damping wing scatterings and propagates easily with a large free path until it hits the resonance zone. Once it enters the resonance regime, the photon experiences the similar processes to those of the red photons described above.
On the other hand, majority of the extremely blue and the extremely red photons escape the medium suffering from at most several damping wing scatterings.

### 3.3 Dependence on Kinematics and Column Density

In this subsection we investigate the dependence of the emergent Lyα profile on the kinematics and the column density of the scattering medium. Our numerical results are summarized in Figs. 6 and 7a,b.

The kinematics of the scattering medium is characterized by the parameter $s_{\text{max}}$ that measures the bulk velocity scale. In Fig. 6 we show the emergent profiles for $s_{\text{max}} = 0.1, 1, 2, 5, 10$ with the Sobolev optical depth $\tau_0 = 10^4$ and the width of the initial profile $\sigma_x = 5$. Here all the profiles have the same number of incident photons. It is particularly notable that the slope at the blue edge of the primary peak does not change as $s_{\text{max}}$, which strongly implies that this quantity can be very useful to put a strong constraint on $\tau_0$. This point is discussed in more detail in the following section.

As $s_{\text{max}}$ gets smaller, the strength of the secondary peak approaches that of the primary peak. In the limit of $s_{\text{max}} \to 0$, we obtain a symmetric double peak profile, and this is in agreement with the result obtained for the case of a static medium by Adams(1972). A very turbulent medium or a static medium may provide a negligible $s_{\text{max}}$.

In Fig. 7a, we show the emergent profiles for moderate column densities, $\tau_0 = 10^2, 10^3$ and $10^4$. Here, we fix $s_{\text{max}} = 10$ and use an initial Gaussian profile having a width of $\sigma_x = 5$. Because all the profiles have the same number of incident photons, the profiles are normalized to have the same area. The thick solid line indicates the emergent Lyα profile for $\tau_0 = 10^4$ which corresponds to $N_{HI} = 4.1 \times 10^{18}$ cm$^{-2}$, the dotted line for $\tau_0 = 10^3$, and the dashed line for $\tau_0 = 10^2$.

The shapes of emergent profiles show a systematic change with the Sobolev optical depth. The primary red peak recedes to the red, as the column density increases. And the height of the primary peak decreases and the width increases. As $\tau_0$ increases, the range of frequency diffusion increases and therefore the width of the primary red feature increases.

In the cases of high optical depths $\tau_0 \geq 10^4$, the profile of the emergent flux changes in a similar way. In Fig. 7b we show the results for the cases $\tau_0 = 10^4, 10^5$, and $10^6$. Up to $\tau_0 = 10^5$ the blue edges of the red primary peaks appear to occur at the same location $x \simeq -2$. The slope at the blue edge of the primary red peak decreases as the Sobolev optical depth, that is, the overall profile changes from a sharp primary feature to a broad hump as $\tau_0$ increases. In contrast with the cases of Fig. 7a, the secondary blue peaks show remarkable changes. As $\tau_0$ increases, the double peak profile becomes more symmetric, which is obtained in the static limit. This shows that the kinematics plays a minor role as the optical depth increases.

For an extremely thick medium, $\tau_0 \gtrsim 10^6$, the feature appears to have two broad humps outside the absorption trough. The strength of the red part is now almost similar to that of the blue part, which highly implies that the anisotropy introduced by the kinematics has little effect on the line formation. The enhanced width of the feature can be a good measure of the column density of the scattering medium.

It is noted that the strength ratio of the primary red part to the secondary blue part is observationally important and that several sets of $(\tau_0, s_{\text{max}})$ may give the same
strength ratio. In the following section we give a further discussion about observational implications.

### 3.4 Observational Implications

We are primarily concerned with the quantities that can be observationally measured and put meaningful and strong constraints on $\tau_0$ and $s_{max}$ that specify the column density and the kinematics of the expanding H I medium.

We propose that the slope at the blue edge of the primary part and the location of the primary peak can be one important indicator of $\tau_0$ under the assumption that the spectroscopy can be performed with enough resolution and signal to noise ratio.

In our work, the bin size for collecting the emergent photons is $\Delta x = 1$, that is, the bin size corresponds to the thermal velocity $v_{th} = 10T_4^{1/2}$ km s$^{-1}$. The normalization is done by setting the total number of line photons to be 10000. Therefore, the slope at the blue edge is defined by the ratio of the photon number collected in the bin corresponding to the maximum peak point to the number of bins between the location of the blue edge and the maximum peak point of the primary red peak.

As shown in Fig. 7b, for $\tau_0 = 10^4$, $s_{max} = 10$, the slope at the blue edge of the primary part is given by

$$l = \frac{3800 \text{ photons}}{1 \text{ bin}},$$  \hspace{1cm} (23)

and similarly for $\tau_0 = 10^5$ it is given by

$$l = \frac{470 \text{ photons}}{1 \text{ bin}}.$$  \hspace{1cm} (24)

It is particularly noted that these slopes are meaningful only when the total number of Ly$\alpha$ photons throughout the feature is normalized to 10000. It is also interesting that

$$l \leq \frac{20 \text{ photons}}{1 \text{ bin}},$$  \hspace{1cm} (25)

for $\tau_0 = 10^6$.

On the other hand, $s_{max}$ may be constrained by the quantities including the peak-to-peak distance, or equivalently the width of the absorption trough and the ratio of the strengths of the primary peak and the secondary peak. However, a good caution is needed because this quantity can be a poor indicator when a huge column density is involved.

With lower quality spectroscopy we may use equivalent widths to characterize the emission profile in a similar manner which Ringwald & Naylor (1997) used in their analysis of the lines from the cataclysmic variable, BZ Camelopardalis. If we measure observationally the equivalent widths $EW_1$, $EW_2$, and $EW_{abs}$ corresponding to the primary red part, secondary blue part and the absorption trough respectively, then the profile fitting procedure will be reduced to minimize the differences of the equivalent widths obtained from numerical simulations.

### 4. Application to DLA 2233+131

11
DLAs are believed to be associated with a disk component of a normal spiral galaxy or a dwarf galaxy from the considerations of the similarity of the H I column density to a typical galactic value and the width of the accompanying metal absorption systems amounting to a typical galactic rotational speed. There have been a few reports on the detection of the Lyα emissions in the spectra of DLAs (e.g. Djorgovski et al. 1996, 1997; Möller & Warren 1993; Warren & Möller 1996).

The first firm candidate is DLA 2233+131 reported by Djorgovski et al. (1996). The emission feature is characterized by a sharp blue edge, an absorption trough in the blue part, and a small secondary peak at around 5040 Å. Djorgovski et al. (1996) ascribe the absorption feature to an ambient gas. On the other hand, Lu et al. (1997) suggest that some Lyα forest clouds at lower redshifts may be responsible for the absorption feature in the blue part. However, as Djorgovski et al. (1996) and Lu et al. (1997) point out, it is highly probable that DLA 2233+131 is a normal and isolated spiral galaxy. Therefore it will be a very rare coincidence if we have a Lyα emitting DLA system obscured by other intervening clouds at a smaller redshift. It is more probable, therefore, that the absorption feature is formed by the material surrounding the emission source. Lu et al. (1997) also comment that the absorption by either dust or the resonance scattering by neutral hydrogen in the galaxy can significantly alter the energy distribution of the escaping Lyα photons.

It is known that Lyα photons from a galaxy are mainly emitted by the recombination processes in H II regions. Out of 8 nearby H II galaxies, Kunth et al. (1996) find 4 galaxies exhibiting a P-Cygni profile in the Lyα emission. Subsequently, for one of such blue compact dwarf galaxies, Haro 2, Legrand et al. (1997) compared the observed P-Cygni Lyα profile with the profiles properly scaled from the observed Hα profile of Haro 2. In their study it was found that the Lyα profile shows a significant excess in the red part. They interpreted the red excess as the frequency redistributed flux mainly caused by the line photons back-scattered from an expanding shell. They conclude that the galaxies with a P-Cygni Lyα profile may possess superbubbles.

Furthermore, P-Cygni profiles are detected even at \(3 < z < 3.5\) in the normal star-forming galaxies (Steidel et al. 1996b). The similar P-Cygni profiles are also detected in the spectra of galaxies found in the Hubble Deep Field, where the redshifts are estimated to be \(z \sim 3\) (Lowenthal et al. 1997).

Therefore, we may draw a tentative conclusion that the P-Cygni type absorption feature appearing in the Lyα emission of DLA 2233+131 strongly implies that the DLA system also contains similar superbubbles, which are often found in nearby blue compact H II galaxies.

From the width of the absorption trough we estimate the bulk velocity of the expanding shell in DLA 2233+131 to be \(\Delta V_{\text{bulk}} \approx 200 \text{ km s}^{-1}\) (see Fig. 2 in Djorgovski et al. 1996). It is observationally challenging to measure the local thermal velocity \(v_{\text{th}}\) in the expanding H I region. The local velocity should also include the microturbulent motion, which may not be constrained observationally. Therefore, we assume that \(v_{\text{th}}\) is in the range of 1–20 km s\(^{-1}\) and adopt \(v_{\text{th}} \approx 10 \text{ km s}^{-1}\) in this study.

The slope at the blue edge of the primary peak discussed in the previous section is computed in the case of DLA 2233+131 observed by Djorgovski et al. (1996). According to them, the observed Lyα line flux \(F_{1216}\) is

\[
F_{1216} = (6.4 \pm 1.2) \times 10^{-17} \text{ erg cm}^{-2} \text{ s}^{-1}.
\]

(26)
and the bin size is given by \( \Delta \lambda \sim 0.8 \text{ Å} \). The maximum specific flux is \( F_{\nu} \sim 8.5 \mu \text{Jy} \), which is converted to \( F_{\lambda} \sim 1.0 \times 10^{-17} \text{ erg cm}^{-2} \text{ s}^{-1} \text{ Å}^{-1} \). The number of bins between the blue edge to the maximum peak point is 6. Using our normalization scheme the slope is computed as

\[
l_{2233+131} = \frac{10^4}{6 \text{ bins}} \frac{F_{\lambda} \Delta \lambda}{F_{1216}} \sim 250 \tag{27}
\]

This result highly implies that the Sobolev optical depth \( \tau_0 \) for the outflowing H I in DLA 2233+131 is a few times \( 10^5 \), which is converted to \( N_{HI} \sim 10^{20} \text{ cm}^{-2} \) assuming that \( s_{\text{max}} \) does not greatly exceed 50. From these values and the overall shape of the profile, we exclude the possibility that \( N_{HI} > 5 \times 10^{20} \text{ cm}^{-2} \) or \( \tau_0 > 10^6 \).

Assuming a standard Friedman cosmology with \( H_0 = 75 \text{ km s}^{-1} \text{ Mpc}^{-1} \) and \( \Omega_0 = 0.2 \), the angular diameter distance to the DLA is \( 1.96 \times 10^{28} \text{ cm} \), and the luminosity distance \( r_L = 8.14 \times 10^{28} \text{ cm} \). We suppose that the H II region is a sphere of radius \( R \). Then the observed Ly\( \alpha \) line flux is calculated by

\[
F_{1216} = \frac{(h\nu_0)\alpha_B(n_e n_p)}{r_L^2} V_{\text{HII}}, \tag{28}
\]

where \( n_e \) is the electron density, \( n_p \) is the proton density, \( V_{\text{HII}} \) is the volume of the H II region, and the case B recombination coefficient \( \alpha_B = 2.59 \times 10^{-13} \text{ cm}^3 \text{ s}^{-1} \) for \( T = 10^4 \text{ K} \) (Osterbrock 1989). From Eqs. 26, 28, we obtain the electron density \( n_e \),

\[
 n_e \simeq 1 \left( \frac{1 \text{ kpc}}{R} \right)^3 \text{ cm}^{-3}. \tag{29}
\]

We extrapolate the correlation between velocity dispersion and radius of giant H II regions (Terlevich et al. 1981) and use the velocity dispersion \( \sim 300 \text{ km s}^{-1} \) of the H II region in DLA 2233+131 to estimate its size \( R \sim 1 \text{ kpc} \). If we assume that the emission region is fully ionized, then we get \( n_e \simeq 1 \text{ cm}^{-3} \), which leads to the total mass of the shell

\[
M_{\text{HII}} \simeq 10^8 M_\odot. \tag{30}
\]

Because one O5 star typically generates UV photons at a rate of \( 5 \times 10^{49} \text{ photons s}^{-1} \) (Spitzer 1978), it follows that the number of O5 stars needed to account for the Ly\( \alpha \) flux in DLA 2233+131 is \( \sim 10^3 \) under the assumption that the H II region is ionized purely by these O5 stars. According to Kennicutt et al. (1989) the first-ranked H II regions found in nearby spirals and irregulars require the total mass of the ionizing stars \( (> 10 M_\odot) \) ranging \( 10^2 - 10^6 M_\odot \). From this, we may conclude that the Ly\( \alpha \) emission line from DLA 2233+131 is originated from these first-ranked H II regions.

4. Summary and Discussion

We studied the line transfer in an optically thick and moving media including the damping wing scattering using a Monte Carlo method. The emergent profiles are characterized by the asymmetric double peaks with a primary peak in the red part, a secondary peak in the blue part, and an absorption trough in the blue.
The underlying mechanisms for the transfer and the dependence of the emergent profiles on the H I column density and on the kinematics were investigated. The primary red peak recedes to the red as $N_{HI}$ increases and for thicker media with $\tau_0 \geq 10^6$, the Ly$\alpha$ line profile becomes two separated broad humps. It is also found that as $s_{\text{max}}$ gets smaller, symmetric double peak profiles are obtained, which is regarded as the static limit where bulk motion is negligible. We briefly discuss how to constrain the physical parameters $\tau_0$ and $s_{\text{max}}$ from an observed P-Cygni type profile.

We have applied this method to explain the P-Cygni type profile in the Ly$\alpha$ emission of DLA 2233+131 assuming that it is caused by an expanding medium with a high H I column density. The slope at the blue edge of the primary peak gives a strong constraint on $\tau_0$ and we find that $\tau_0 \simeq 10^5$. Noting that the expanding bulk velocity $\Delta V_{\text{bulk}} \approx 200$ km s$^{-1}$, the column density responsible for the P-Cygni type feature is deduced to be $N_{HI} \simeq 10^{20}$ cm$^{-2}$. We also present physical quantities regarding the H II region and deduced that $n_e \simeq 1$ cm$^{-3}$, $R \simeq 1$ kpc, and $M_{H II} \simeq 10^8 M_\odot$, assuming near complete ionization. About $10^3$ massive young stars are need to account for the Ly$\alpha$ line flux of DLA 2233+131, is comparable with the first-ranked H II regions found in nearby spirals and irregulars.

We briefly review the physical quantities of several astronomical objects which may possess similar kinematic properties to those of DLA 2233+131 discussed above.

The first example we consider is the H I supershells and the worms/chimneys in our Galaxy (Heiles 1979, 1984; Koo et al. 1991, 1992) and nearby galaxies. An especially large H I supershell is found in the nearby normal spiral galaxy M101 (Kamphuis et al. 1991). In M101 the measured line-of-sight expanding velocity component of the supershell amounts to $\sim 50$ km s$^{-1}$, which is smaller than that of DLA 2233+131 by a factor of $\sim 5$. The H I column density of the supershell is estimated to be $N_{HI} \simeq 2 \times 10^{20}$ cm$^{-2}$.

The second candidate is blue compact dwarf galaxies showing similar P-Cygni type profiles in their Ly$\alpha$ emission (Legrand et al. 1997, Lequeux et al. 1995, Kunth et al. 1996, Puche et al. 1992). Kunth et al. (1996) found that 4 out of 8 observed H II galaxies exhibit P-Cygni profiles. One of them is Haro 2 whose H I column density is $2 \times 10^{20}$ cm$^{-2}$ and the expanding bulk velocity $\sim 100$ km s$^{-1}$ (Legrand et al. 1997). As Legrand et al. suggested, the possibility for the H II galaxy to exhibit the Ly$\alpha$ emission or a broad absorption trough in their spectra depends on geometrical properties of the expanding shell, which include the angle between line-of-sight and the symmetry axis of the shell. It is noticeable that these blue compact galaxies are under active star formation processes.

Currently, there are a few observational programs to discover primeval galaxies at $z > 3$.

One is to search for star-forming galaxies like those found in $3 < z < 3.5$ by Lyman break characteristics (Steidel et al. 1996a, 1996b, 1997; Pettini et al. 1997). The young galaxies detected in this way exhibit the similar P-Cygni type profiles to that found in DLA 2233+131. According to a deeper and higher resolution imaging (Giavalisco et al. 1996), these objects appear to be compact spheroidal cores often surrounded by low surface brightness nebulosities. Assuming the standard cosmology of $\Omega_0 = 1$ and $H_0 = 50$ km s$^{-1}$ Mpc$^{-1}$, a typical size of the cores is a few kpc. The authors suggested
that these are bulges of the primeval galaxies which are precursors of nearby normal galaxies.

Another program is provided by gravitational lenses. This method is now becomes a powerful tool, because the magnification of gravitational lens enables us to detect more distant objects than other method does. For example, the similar objects showing P-Cygni type profiles are also found in the Hubble Deep Field at $z \simeq 3$ (Lowenthal et al. 1997). Trager et al. (1997) and Frye et al. (1997) also found a very distant objects at $z \sim 4$.

It is remarkable that these objects show P-Cygni type Ly$\alpha$ profiles. Especially the galaxy discovered by Frye et al. (1997) shows several bright spots in the lensed image. A particularly notable point is that only one of them (i.e. N4 spot) shows a P-Cygni type Ly$\alpha$ line profile whereas others show damped Ly$\alpha$ absorption profiles (Bunker et al. 1997). This may tell us that there are several well-localized H II regions in the galaxy and only part of them shows a P-Cygni type Ly$\alpha$ emission, while the Ly$\alpha$ photons of others are shielded by an optically thick medium along the line of sight. Analogous situations are provided by nearby starburst dwarf galaxies or supershells, and we suggest that these starburst dwarf galaxies are the very counterparts of the above three kinds of primeval galaxy candidates including the DLA candidates.

Such outflowing media seen in extragalactic objects are often accompanied by the sites associated with active star formation (Reach et al. 1993). Because DLA 2233+131 and other remote galaxies are believed to be prototype of present galaxies, it is very interesting that there may exist primeval galaxies at $3 < z < 5$ showing starbursts, which also gives constraints on the cosmological models for the structure formation. Moreover this speculation supports the picture that star formations are continuous processes from $z = 0$ to $z \simeq 5$.

We propose complementary observations of metal lines in the spectrum of DLA 2233+131 in order to provide independent constraints on the relevant physical quantities. We also propose the further imaging of DLA 2233+131 by the Hubble Space Telescope which may provide independent and important clues to the identity of the system.

In a subsequent paper, we will investigate in more detail the detectability of the Ly$\alpha$ emission in these star forming objects, which is thought to be related with the geometrical factors such as inclination associated with the shielding of the dust lane.

In numerical approach it will be desirable to develop a more flexible code to treat inhomogeneous media with an arbitrary velocity field and various shapes to secure the constraints on the characteristic physical quantities such as the column density and the bulk flow scale.

Acknowledgements

HWL gratefully acknowledges the support from the Research Institute for Basic Sciences, Seoul National University. We are grateful to Prof. B. C. Koo and K. T. Kim for kind and helpful discussions. We also gratefully acknowledge the kind comments of Roger Blandford.
5. References

Adams T. 1972. ApJ, 174, 439.
Blandford R. & Lee H.-W 1997. MNRAS submitted.
Bunker A. J., Moustakas L. A., Davis M., Frye B. L., Broadhurst T. J., & Spinrad H., 1997, ”The Young Universe: Galaxy Formation and Evolution at Intermediate and High Redshift” (Rome Observatory, 29 Sept - 3 Oct 1997) ed. S. D’Odorico, A. Fontana and E. Giallongo, A.S.P. Conf. Ser
Djorgovski S.G., Pahre M.A., Bechtold J., & Elston R. 1996. Nature, 382, 234.
Djorgovski S.G., 1997, Structure and Evolution of the IGM from QSO Absorption Line Systems, IAP Colloquium, eds. P. Petitjean and S. Charlot, in press
Forbes D. A., Phillips A. C., Koo D. C., & Illingworth G. D. 1996. ApJ, 462, 89.
Franx M., Illingworth G. D., Kelson D., van Dokkum P., & Tran, K. 1997. ApJ, 486, 75.
Frye B. & Broadhurst T. 1997. submitted to ApJL.
Giavalisco M., Steidel C. C., & Macchetto F. D. 1996. ApJ, 470, 189.
Gould A. & Weinberg D. H. 1996. ApJ, 468, 462.
Gray D. F. 1992, in The Observation and Analysis of Stellar Photospheres 2nd ed. Cambridge Press, New York
Haehnelt M., Steinmetz M., & Rauch M. 1997, ApJ submitted, astro-ph/9706201
Harrington J. P. 1973. ApJ, 135, 195.
Heiles C. 1979. ApJ, 229, 533.
Heiles C. 1984. ApJS, 55, 585.
Hunstead R.W., Pettini M., & Fletcher A.B. 1990. ApJ, 356, 23.
Irwin J. A. 1995. PASP, 107, 715.
Kamphuis J., Sancisi R., & van der Hulst T. 1991. AAL, 244, 29.
Kennicutt R. C. JR., Edgar B. K., Hodge P. W. 1989. ApJ, 337, 761.
Koo B.-C, Heiles C., & Reach W. T. 1991, in The Interstellar Disk-Halo Connection in Galaxies, IAU Symp. 144, ed. H. Bloemen, Kluwer, Dordrecht, p. 165
Koo B.-C, Heiles C., & Reach W. T. 1992. ApJ, 390, 108.
Kunth D., Lequeux J., Mas-Hesse J.M.,Terlevich E., & Terlevich R. 1996, in Starburst Activity in Galaxies proceedings by the RevMexAstroAstrofis. (ConfSeries), astro-ph/9612043
Lee H.-W & Blandford R. 1997. MNRAS, 288, 19.
Legrand F., Kunth D., Mas-Hesse J.M.,& Lequeux J. 1997, AA, in press, astro-ph/9706109
Lequeux J., Kunth D., Mas-Hesse J.M., & Sargent W.L.W 1995. AA, 301, 18.
Lowenthal J. D. et al. 1997. ApJ, 481, 673.
Lu L., Sargent W. L. W. & Barlow T. A. 1997. ApJ, 484, 131. Mihalas, D. 1978, Stellar Atmospheres, W. H. Freeman and Company, San Fransisco
Möller P. & Warren S. J. 1993. AA, 270, 43.
Osterbrock D. E. 1962. ApJ, 135, 195.
Osterbrock D. E. 1989, Astrophysics of Gaseous Nebulae and Active Galactic Nuclei, University Science Books, California
Pettini M., Hunstead R.W., King D.L., & Smith L.J., 1995. astro-ph/9502076.
Pettini M., Steidel C. C., Adelberger K. L., Kellogg M., Dickinson M., & Giavalisco M., 1997, astro-ph/9708117.
Press W. H., Flannery B. P., Teukolsky S. A., & Vetterling W. T. 1989, Numerical Recipes, Cambridge Press, New York
Prochaska J. X. & Wolfe A. M. 1997. AAS, 190, 4704.
Puche D., Westpfahl D., Brinks E., & Roy J.-R. 1992. AJ, 103, 1841.
Reach W. T., Heiles C., & Koo B.-C. 1993, in AIP Conf. Proc. 278, Back to Galaxy, ed. S. S. Holt and F. Verter (New York, AIP), p.67.
Ringwald F. A. & Naylor T., 1997, AJ accepted.
Rybicki, G. B. & Hummer, D. G. 1978. ApJ, 219, 654. Rybicki G. B & Lightman A. P. 1979, Radiative Processes in Astrophysics, John Wiley & Sons, New York
Sengupta S. 1994. MNRAS, 269, 265.
Sobolev, V. 1960, Moving Envelopes of Stars. Harvard Univ. Press, Cambridge, Mass.
Spitzer L., 1978, Physical Processes in the Interstellar Medium, John Wiley & Sons, New York
Steidel C. C., Giavalisco M., Pettini M., Dickinson M., & Adelberger K. L. 1996b. ApJL, 462, 17.
Steidel C. C., Giavalisco M., Dickinson M., & Adelberger K. L. 1996a. AJ, 112, 352.
Steidel C. C., Adelberger K. L., Dickinson M., Giavalisco M., Pettini M., & Kellogg M. 1997. ApJ in press.
Terlevich R. & Melnick J. 1981. MNRAS, 195, 839.
Trager S. C., Faber S. M., Dressler A., & Oemler A. 1997. ApJ, 485, 92.
Warren S. J. & Möller P. 1996. AA, 311, 25.
FIGURE CAPTION

Fig. 1
A surface plot of the $s(x, \tau)$ table. $x \equiv \Delta \nu / \Delta \nu_D$ and $TAU = \tau$ represents the optical depth. The plateau on the right top side corresponds to $s = 2s_{max} + 1$.

Fig. 2
The emergent Ly$\alpha$ profile from a thick expanding medium. The horizontal axis represents $\Delta \lambda / \Delta \lambda_D$ and the vertical axis for the relative flux. The emission source is assumed to be given by a Gaussian profile $\propto e^{-(x/2\sigma_x)^2}$, where the width $\sigma_x$ is set to be 5. We choose $s_{max} = 10$ and $\tau_0 = 10^4$. The emergent profile is represented by the solid line and the dotted line shows the initial Gaussian profile.

Fig. 3
The escaping optical depth and the single-scattering escape probability. The dashed line represents the optical depth $\tau_x$, and the escape probability $P_{esc} \equiv e^{-\tau_x}$ is shown by the dotted line. The dot-dashed line is the input emission profile at the origin given by a Gaussian with the width $\sigma_x = 10$. The solid line is the product of the initial emission at the origin and the escape probability.

Fig. 4
The emergent profiles for a flat incident continuum. The parameters are the same as in Fig. 2.

Fig. 5a
A typical scattering behavior of a photon with $x \sim -5$. The horizontal axis stands for the scattering number. In the bottom frequency deviation is shown, and in the top panel the scattering type is given, where 0 stands for a wing scattering and 1 for a resonant scattering. In the middle panel the radial distance of the photon from the center of the scattering sphere is shown.

Fig. 5b
Another typical scattering behavior of a photon with $x \sim 5$. See the text for more detail.

Fig. 6
The emergent Ly$\alpha$ profiles for various bulk velocity scales $s_{max}$ of the scattering medium. We fix $\tau_0 = 10^4$ and the initial Gaussian profile having a width of $\sigma_x = 5$ is used. The thick solid line is for the case of $s_{max} = 0.5$, the dotted line for $s_{max} = 2$, the long dashed line for $s_{max} = 1$, the dashed line for $s_{max} = 5$, and the dot-dashed line for $s_{max} = 10$.

Fig. 7a
The emergent Ly$\alpha$ profiles for various column densities of the scattering medium. $s_{max} = 10$ and the initial Gaussian profile having a width of $\sigma_x = 5$ is used. The thick solid line indicates the emergent Ly$\alpha$ profile for $\tau_0 = 10^4$ which corresponds to $N_{HI} = 4.1 \times$
$10^{18}$ cm$^{-2}$, the dotted line for $\tau_0 = 10^3$, and the dashed line for $\tau_0 = 10^2$. The small dips in the secondary peaks are numerical artifacts caused by the truncated matter distribution.

**Fig. 7b**
The emergent Ly$\alpha$ profiles for various column densities of the scattering medium. The thick solid line indicates the emergent Ly$\alpha$ profile for $\tau_0 = 10^4$ which corresponds to $N_{HI} = 4.1 \times 10^{18}$ cm$^{-2}$, the dotted line for $\tau_0 = 10^5$, and the dashed line for $\tau_0 = 10^6$. 
\( \tau_0 = 10^4 \)

\( S_{\text{max}} = 10 \)
\( s_{\text{max}} = 10 \)

- \( \tau_0 = 10^2 \)
- \( \tau_0 = 10^3 \)
- \( \tau_0 = 10^4 \)
$S_{\text{max}} = 10$

- $\tau_0 = 10^6$
- $\tau_0 = 10^5$
- $\tau_0 = 10^4$