Performance Analysis of $M$-QAM Multihop Relaying over mmWave Weibull Fading Channels

Abdelaziz Soulimani, Student Member, IEEE, Mustapha Benjillali, Senior Member, IEEE,
Hatim Chergui, Member, IEEE, and Daniel B. da Costa, Senior Member, IEEE

Abstract

The paper presents a comprehensive closed-form performance analysis framework for multihop millimeter wave (mmWave) communications as a potential scheme in the next fifth generation (5G) systems and Internet of things (IoT) applications. To take into consideration the channel fading in the mmWave range, we adopt the advocated Weibull model for its flexible ability to cover different channel conditions, as has been supported by many recent measurement campaigns in various emerging 5G and IoT scenarios. The analyzed scheme consists basically of multiple “Detect-and-Forward” relays with generalized high-order quadrature amplitude modulation ($M$-QAM) transmissions. The end-to-end performance is evaluated in terms of outage probability, ergodic capacity, bit error rate, and symbol error rate. For all metrics, we present exact closed-form expressions along with their asymptotic behavior, some in terms of generalized hypergeometric functions. The exactness of our analysis is illustrated by numerical examples, and assessed via Monte-Carlo simulations for different system and channel parameters. Finally, as a secondary contribution, noting the increasing popularity of Fox’s $H$ and bivariate $H$ functions, and the fact that they are not yet available in Matlab, we provide new and generalized codes for computing these functions which will be of utility for different contexts.

Index Terms

5G, BER, Capacity, Fox $H$-function, IoT, mmWave,
Multihop Relaying, Outage Probability, QAM, SER, Weibull Fading.

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A. Soulimani and M. Benjillali are with the National Institute of Telecommunications (INPT), Rabat, Morocco. [e-mails: {abdelaziz.soulimani.ma, benjillali}@ieee.org].

H. Chergui [email: chergui@ieee.org].

D. B. da Costa is with Federal University of Ceará, CE, Brazil [email: danielbcosta@ieee.org].
I. Introduction

Millimeter wave (mmWave) communications [1] (and references therein) have gained great interest as a key enabler technology for the fifth generation (5G) of mobile communications. The mmWave band offers large unlicensed bandwidths to answer the huge demand for increased capacity, and higher spectral-efficiency. In addition, it presents interesting anti-interference abilities, and the short wavelength allows new spatial processing techniques. While this band was underutilized in the previous wireless communications—mainly due to practical implementation limitations, cost, and stability—it is nowadays very attractive thanks to cost-effective hardware technologies, and novel directional high-gain antennas. Given their short range and weak penetration over different materials, mmWaves offer also efficient spectrum utilization and secure transmissions.

Several measurement campaigns (for example [2]) were conducted to better understand and model the wireless channel in the mmWave band. The double-parameterized Weibull distribution has been shown to fit very well with the small scale variations [3], [4], with a shape parameter that can be modeled as a lognormal distribution.

Cooperative and multihop communications [5] a common solution to increase the coverage, while preserving high throughput and reliability, with low transmission powers. It is a potential approach to overcome the severe channel conditions that usually impact mmWave signals. On the other hand, the drawbacks of multihop relaying (in terms of increased channel use, coordination overhead, and delay) may be dealt with using optimized transmission parameters and network protocols in a comprehensive and cross-layer framework.

In general, two main relaying classes are available for cooperative communications, namely, the non-regenerative and the regenerative strategies [6]. In this work, we adopt the popular regenerative “detect-and-forward” (DetF) relaying scheme [7], where the relays re-transmit (without decoding and re-encoding) the demodulated binary sequences. The adoption of DetF is motivated by its simplicity, reduced delay, and interesting performance with lower processing complexity and channel state information constraints [8].

There is a significant amount of work on the performance analysis of multihop relaying communications; we only list a few here that are in line with the perspective of this work. For instance, in [9], the authors have analyzed the performance of multihop relaying systems over Weibull fading channels in terms of bit error rate (BER) and outage probability. However, the authors considered the amplify-and-forward (AF) strategy, and based their analysis on a simple approximation of the end-to-end signal-to-noise ratio (SNR) resulting in a lower-bound discussion, which reduced the accuracy and the generality of the results. A similar analysis of BER and outage probability over multihop Weibull fading channels was also conducted in [10] but in the context of free space optical communications with only binary pulse position modulations. The end-to-end performance of multihop regenerative relaying communications was discussed in [11]. However, the investigation was in the context of underlay, interference-limited, cognitive networks, and assumed Nakagami-\(m\) fading channels. To the best of our
knowledge, the performance of regenerative multihop schemes, over the mmWave Weibull-modeled fading channels, remains an open problem, especially in terms of error rates with high-order \( M \)-ary quadrature amplitude modulation (QAM) that is of interest in modern and emerging communication systems. This paper completes and extends our effort in this direction [12].

Here, we propose a more general, closed-form, performance framework, and we summarize our contributions in the following points:

- We generalize the derived metrics in [12] (i.e., outage probability, BER, and ergodic capacity) to real-valued Weibull shape parameters instead of rational ones.
- We here derive the end-to-end symbol error rate (SER) expression of the analyzed system model.
- To further simplify the analysis, and provide insight into the system behavior, we also obtain the asymptotic expressions for the presented metrics, and we assess the usefulness and accuracy of this approach.
- Finally, and as a secondary yet important contribution, we implement the Fox \( H \) and bivariate \( H \)-functions in Matlab with generalized contours that are independent of the function parameters. Our codes, unlike a few existing versions that are very dependent on the numerical examples of the context where they were developed, would be of interest and may be readily used by a broader community.

The remainder of this paper is organized as follows. Section II introduces the proposed system model and adopted notations. Next the analytical expressions of the adopted performance metrics are derived in section III. Specifically, exact and asymptotic expressions of outage probability, ergodic capacity, bit error rate, and symbol error rate are derived. Numerical examples along with simulation results, using our implementation of the hypergeometric functions, are presented in section IV and section V concludes the paper. Finally, appendices describe the new Matlab implementation codes for Fox \( H \) and bivariate \( H \)-functions.

II. SYSTEM MODEL

In this work, we consider a cooperative transmission from a source node (S) to a destination (D) through \( N - 1 \) DetF relay nodes \( R_i, i = 1, \ldots, N - 1 \), as shown in Fig. 1. We assume that all nodes are operating in the half-duplex mode, with the same modulation order \( M \), and that all transmissions are orthogonal, e.g., over different time or frequency resources. Each receiving node considers only the previous adjacent transmitter, and the direct link between (S) and (D) is not taken into consideration (i.e., no signal is received at the destination directly from the source because of considerable path loss).

The transmitted signal from the source node is denoted by \( x \). \( y_i \) and \( \tilde{x}_i \) are, respectively, the received and transmitted signals at the \( i \)-th node, and \( y \) is the received signal at the destination. Thus, we can express the

\[^1\] Although this is obviously not the optimal serial transmission protocol, it is adopted to simplify the analysis and avoid all the considerations that are out of the scope of the contribution in this paper.
communication over the \(i\)-th link, \(i = 1, \ldots, N\), under the form

\[ y_i = h_i \tilde{x}_{i-1} + n_i, \quad (1) \]

where \( \tilde{x}_0 = x \), \( y_N = y \), and \( n_i \) denotes the zero-mean additive white Gaussian noise (AWGN) at the \(i\)-th, with the same variance \(N_0\) over all links. Channel coefficients \( h_i \) are assumed to follow independent but not necessarily identically distributed (i.n.i.d.) flat Weibull fading profiles with parameters \((\beta_i, \Omega_i)\), and their probability density function (PDF) can be expressed as

\[ p_{h_i}(h) = \frac{\beta_i}{\Omega_i^{\beta_i}} h^{\beta_i-1} \exp \left( -\frac{h}{\Omega_i} \right). \quad (2) \]

In this case, note that \(|h_i|^2\) also follows a Weibull distribution with parameters \((\beta_i/2, \Omega_i^2)\) [13]. All channel coefficients are assumed to be known perfectly at the receivers, since channel estimation is out of scope here. The instantaneous SNR over each hop may then be written as \(\gamma_i = |h_i|^2 \! \bar{\gamma}\), where \(\bar{\gamma}\) is the average SNR; supposed to be the same for all links without any loss of generality. The PDF of the instantaneous SNR over the \(i\)-th hop is thus given by

\[ p_{\gamma_i}(\gamma) = \frac{\alpha_i}{\phi_i} \gamma^{\alpha_i-1} \exp \left( -\frac{\gamma}{\phi_i} \right), \quad (3) \]

where, \(\alpha_i = \beta_i/2\) and \(\phi_i = (\bar{\gamma} \Omega_i^2)^{\alpha_i}\).

### III. Performance Analysis

In this section, we investigate the end-to-end performance of the scheme presented in Sec. III in terms of outage probability, ergodic capacity, BER, and SER. To focus the presentation, we present the exact and the asymptotic expressions for each performance metric as we progress. We note that the asymptotic behavior of the resulting hypergeometric functions may be obtained using a direct expansion [14, Th. 1.7 and 1.11], but this requires the satisfaction of restricted conditions. Consequently, we adopt here a general method using the residue theorem as explained in [15].
A. Outage Probability

An end-to-end outage event occurs when the transmission rate $\rho$ is higher than the mutual information $\iota$ over the equivalent channel between the end nodes. The outage probability can be written as

$$P_{\text{out}} = \Pr \left[ \bigcap_{i=1}^{N} \iota_i \leq \rho \right],$$

(4)

or, equivalently, in our case since the links are independent

$$P_{\text{out}} = \Pr \left[ \min (\gamma_1, \ldots, \gamma_N) \leq \gamma_{\text{th}} \right],$$

(5)

where $\gamma_{\text{th}} = 2^\rho - 1$ is the outage threshold. The expression in (5) can be rewritten as

$$P_{\text{out}} = 1 - \Pr \left[ \min (\gamma_1, \ldots, \gamma_N) \geq \gamma_{\text{th}} \right] = 1 - \prod_{i=1}^{N} (1 - \mathcal{F}_{\gamma_i}(\gamma_{\text{th}})), $$

(6)

where $\mathcal{F}_{\gamma_i}(\cdot)$ is the cumulative distribution function (CDF) of $\gamma_i$ given, considering our channel model, by

$$\mathcal{F}_{\gamma_i}(\gamma) = 1 - \exp \left( -\alpha_i^\phi \gamma \right).$$

(7)

Substituting (7) in (6) yields the following compact closed-form expression for the end-to-end outage probability

$$P_{\text{out}} = 1 - \exp \left( -\sum_{i=1}^{N} \gamma_{\text{th}}^\alpha_i / \phi_i \right).$$

(8)

For low $\gamma_{\text{th}}$ values, (8) can be simplified as

$$P_{\text{out}} \simeq \sum_{i=1}^{N} \gamma_{\text{th}}^\alpha_i / \phi_i.$$

(9)

Assuming further identical channel parameters ($\phi_i = \phi, \alpha_i = \alpha$ for all $i = 1..N$), the outage probability can be simplified to

$$P_{\text{out}} \simeq N \gamma_{\text{th}}^\alpha / \phi.$$

(10)

One direct application of this expression, if a given outage probability target $\Pi$ is fixed, would be for example the maximum number of hops (i.e., maximum distance) between the source and the destination which is obtained as

$$N \simeq \phi \Pi / \gamma_{\text{th}}^\alpha.$$

(11)

B. Ergodic Capacity

The ergodic capacity corresponds to the maximum long-term achievable rate averaged over all states of the time-varying channel. In the present context of multihop communication with regenerative relays, the average ergodic capacity can be expressed as

$$\overline{C} = \min_{i=1,\ldots,N} \overline{C}_i,$$

(12)

where $\overline{C}_i$ is the bandwidth-normalized average ergodic capacity of the $i$-th hop, given by

$$\overline{C}_i = \int_{0}^{+\infty} \log_2 (1 + \gamma) \ p_{\gamma_i}(\gamma) \ d\gamma.$$

(13)
**Exact analysis:** In order to evaluate the integral in (13), we express the \( \exp(\cdot) \) and the \( \log(\cdot) \) functions through the generalized Fox \( H \)-function representation,

\[
\exp(-x) = H_{0,1}^{1,0}\left(x \left| \begin{array}{c}
(0,1)
\end{array} \right. \right), \quad \log (1 + x) = H_{2,2}^{1,2}\left(x \left| \begin{array}{c}
(1,1) , (1,1)
\end{array} \right. \right);
\]

and with the help of [14, Theorem 2.9], (13) can be derived in closed-form as

\[
\bar{C}_i = \frac{\alpha_i}{\ln 2 \phi_i} H_{3,1}^{3,1}\left(1 \left| \begin{array}{c}
(-\alpha_i , \alpha_i) , (1 - \alpha_i , \alpha_i) \\
(0,1) , (-\alpha_i , \alpha_i) , (-\alpha_i , \alpha_i)
\end{array} \right. \right),
\]

which is needed to compute the end-to-end ergodic capacity in (12). A general Matlab code to implement Fox’s \( H \)-function is provided in Appendix A.

**Asymptotic expression:** For high values of \( \gamma \), and recalling that \( \phi_i = (\gamma \Omega_i^2)^{\alpha_i} \), the asymptotic behavior of (15) can be analysis by using the same approach in [15, sec. IV], yielding

\[
\bar{C}_i \approx \frac{\Psi_0 (1)}{\ln(2) \alpha_i} + \log_2 (\gamma \Omega_i^2),
\]

where \( \Psi_0 (\cdot) \) denotes the digamma function [16].

The result in (16) captures the effect of the link parameters on the ergodic capacity. In particular, for given values of \( \gamma \) and \( \Omega \), the ergodic capacity is an increasing function of \( \alpha_i \) (since \( \Psi_0 (1) \approx -0.577 < 0 \)). Also, \( \bar{C} \) is a linear function of \( \gamma \)(dB). Moreover, in the case of similar scale parameters (\( \Omega_i = \Omega \)), the end-to-end ergodic capacity reduces in the high SNR regime to

\[
\bar{C} \approx \frac{\Psi_0 (1)}{\ln(2) \min (\alpha_i)} + \log_2 (\gamma \Omega^2).
\]

**C. Bit Error Rate**

For regenerative relays, it was shown in [17] that BER can be expressed as

\[
\overline{\text{BER}} = \sum_{i=1}^{N} \overline{\text{BER}}_i \prod_{j=i+1}^{N} (1 - 2\overline{\text{BER}}_j),
\]

where \( \overline{\text{BER}}_i \) stands for the average BER of the individual \( i \)-th hop.

**Exact analysis:** Over fading channels, \( \overline{\text{BER}}_i \) can be expressed as

\[
\overline{\text{BER}}_i = \int_{0}^{+\infty} P_b(\gamma) p_{\gamma_i}(\gamma) d\gamma,
\]

\[\text{The adoption of generalized functions (like the hypergeometric functions family, Meijer’s G-function, Fox’s H-function) is gaining in popularity both in software computation tools (like Mathematica and Matlab) and among the performance analysis community. We also adopt this efficient and accurate computational approach in the present work.}\]
where, in our case, \( p_{\gamma}(\cdot) \) is given by (3), and \( P_b(\cdot) \) denotes the exact instantaneous BER of an \( M\)-QAM transmission over a Gaussian channel, which was derived in [18] for an arbitrary order \( M \) under the form

\[
P_b(\gamma) = \frac{1}{\log_2 \sqrt{M}} \sum_{m=1}^{\log_2 \sqrt{M}} P_b(\gamma, m),
\]

(20)

with

\[
P_b(\gamma, m) = \frac{1}{\sqrt{M}} \sum_{n=1}^{\nu_m} \Phi_{m,n} \text{erfc} \left( \sqrt{\omega_n} \gamma \right),
\]

(21)

and \( \nu_m = (1 - 2^{-m}) \sqrt{M} - 1, \omega_n = \frac{3(2n+1)^2 \log_2 M}{2M-2} \), and \( \Phi_{m,n} = (-1)^{\alpha n-1} \left( 2^{m-1} - \left[ \frac{n2^{m-1}}{\sqrt{M}} + \frac{1}{2} \right] \right) \). Hence, the \( i \)-th hop BER can be written as

\[
\text{BER}_i = \frac{1}{\sqrt{M} \log_2 \sqrt{M}} \sum_{m=1}^{\log_2 \sqrt{M}} \sum_{n=0}^{\nu_m} \Phi_{m,n} \zeta_{n,i},
\]

(22)

with

\[
\zeta_{n,i} = \int_0^{+\infty} \text{erfc} \left( \sqrt{\omega_n} \gamma \right) p_{\gamma}(\gamma) d\gamma = \frac{\alpha_i}{\phi_i} \int_0^{+\infty} \gamma^{\alpha_i-1} \text{erfc} \left( \sqrt{\omega_n} \gamma \right) \exp \left( -\frac{\gamma}{\phi_i} \right) d\gamma.
\]

(23)

Following the same approach explained in [11,13], in addition to \( \exp(\cdot) \) function, we express the erfc(\( \cdot \)) function under its generalized Fox H-function representation as well

\[
\text{erfc}(\sqrt{x}) = \frac{1}{\sqrt{\pi}} H_{1,2}^{2,0} \left( x \left| \begin{array}{c} (1,1) \, (0,1), (1/2,1) \end{array} \right. \right),
\]

(24)

and hence (23) becomes

\[
\zeta_{n,i} = \frac{\alpha_i}{\phi_i} \sqrt{\pi} \int_0^{+\infty} \gamma^{\alpha_i-1} \cdot H_{1,1}^{1,0} \left( x \left| \begin{array}{c} (1,1) \, (0,1) \end{array} \right. \right) \cdot H_{1,2}^{2,0} \left( \omega_n \gamma \left| \begin{array}{c} (1,1) \, (0,1), (1/2,1) \end{array} \right. \right) d\gamma,
\]

(25)

which can be expressed in closed-form using the Fox H-function for real values of the shape parameter under the form [14 Theorem 2.9]

\[
\zeta_{n,i} = \frac{\alpha_i \omega_n^{-\alpha_i}}{\sqrt{\pi} \phi_i} H_{2,2}^{1,2} \left( \phi_i \omega_n^{-\alpha_i} \left| \begin{array}{c} (1 - \alpha_i; \alpha_i), (1/2 - \alpha_i; \alpha_i) \end{array} \right. \right) \left( 0,1 \right). (-\alpha_i, \alpha_i),
\]

(26)

By substituting (26) in (22), and then in (18), we obtain the closed-form expression of the end-to-end average BER.

**Asymptotic analysis:** Again, the asymptotic analysis of the BER may be done by following the same approach in [15] sec. IV. Then, (26) becomes for high \( \overline{\gamma} \) values

\[
\zeta_{n,i} = \frac{\omega_n^{-\alpha_i}}{\sqrt{\pi} \phi_i} \Gamma \left( \frac{1}{2} + \alpha_i \right),
\]

(27)

and by replacing (27) in (22), we get

\[
\text{BER}_i = \frac{\Gamma \left( \frac{1}{2} + \alpha_i \right)}{\phi_i \sqrt{\pi M \log_2 \sqrt{M}}} \sum_{m=1}^{\log_2 \sqrt{M}} \sum_{n=0}^{\nu_m} \Phi_{m,n} \omega_n^{-\alpha_i}.
\]

(28)
One direct application of this result is the diversity order of each hop which can now be easily deduced as

\[ d_i = -\lim_{\gamma \to \infty} \frac{\log \text{BER}_i(\gamma)}{\log \gamma} = \alpha_i, \tag{29} \]

and, using (18) and (22), we get the diversity order of the end-to-end system

\[ d_{e2e} = \min_{i=1}^{N} (\alpha_i). \tag{30} \]

**Impact of the number of hops:** To get a better insight on the impact of the number of hops on the BER, let us consider that all individual links have the same parameters, i.e., similar BER on individual hops \((\text{BER}_i = P_0, i = 1, \ldots, N).\) In this case, (18) becomes

\[ \text{BER} = \frac{1}{2} \left( 1 - (1 - 2P_0)^{N} \right) \]

\[ = \frac{1}{2} - \frac{1}{2} \exp \left( \frac{-N}{\hat{N}} \right), \tag{31} \]

where \(\hat{N} = -1 / \ln (1 - 2P_0)\), and \(P_0\) is given by (22) for specific parameters.

We hence obtain an exponential relationship between the end-to-end average BER and the number of hops \(N.\)

We will illustrate this observation later in Sec. IV.

\[ \text{D. Symbol Error Rate} \]

To alleviate the presentation, we refer the reader to [19] where the authors have derived the SER of \(M\)-QAM communications using regenerative relays over Nakagami-\(m\) channels. Here, we use the same approach.

**Exact analysis:** The calculations of the SER reduce to the following integral

\[ \mathcal{I}_{\text{SER}, i} = \int_{0}^{+\infty} Q(A\sqrt{\gamma}) Q(B\sqrt{\gamma}) p_{\gamma_i}(\gamma) d\gamma, \tag{33} \]

where \(A\) and \(B\) are two positive constant coefficients (note that they cannot be both zero), and \(p_{\gamma_i}(\cdot)\) is given by (3). To evaluate (33), two cases need to be differentiated:

- **\(A \times B = 0\):** we denote \(C = \max(A, B).\) This case is similar to (23), and hence

\[ \mathcal{I}_{\text{SER}, i}(C) = \frac{\alpha_i}{4\pi \phi_i} \left( \frac{2}{C} \right)^{\alpha_i} H_{1,2}^{1,0} \left( \frac{1}{\phi_i} \left( \frac{2}{C} \right)^{\alpha_i} \right) \left( \begin{array}{c} 1 - \alpha_i, \alpha_i \frac{1}{2} - \alpha_i, \alpha_i \end{array} \right. \left. \begin{array}{c} 0, 1 \end{array} \right) \tag{34} \]

- **\(A \times B > 0\):** In this case, we replace the Q-function and the exponential with there Fox H-Function representations, and we obtain

\[ \mathcal{I}_{\text{SER}, i}(A, B) = \frac{\alpha_i}{4\pi \phi_i} \int_{0}^{+\infty} \gamma^{\alpha_i - 1} H_{0,1}^{1,0} \left( \frac{-\gamma^{\alpha_i}}{\phi_i} \right) \left( \begin{array}{c} 0, 1 \end{array} \right) \left( \begin{array}{c} 1, 1 \end{array} \right. \left. \begin{array}{c} 0, 1 \end{array} \right) \left( \begin{array}{c} A \gamma \frac{2}{2} \end{array} \right) \left( \begin{array}{c} B \gamma \frac{2}{2} \end{array} \right. \left. \begin{array}{c} 0, 1 \end{array} \right) d\gamma. \tag{35} \]

This is quite obvious considering rectangular QAM constellations and averaging over the fading distribution.
By using [20, eq. 2.3], we get the closed-form expression of (33) in terms of the bivariate Fox H-Function

$$I_{SER_i}(A, B) = \frac{\alpha_i}{4\phi_i\pi} \left( \frac{2}{A} \right)^{\alpha_i} H^{2,0;2,0;1,0}_{1,1;2,0;1,0} \left( \frac{B}{A}, 1, \phi_i \left( \frac{2}{A} \right)^{\alpha_i} \right) (1 - \alpha_i; 1, \alpha_i), \left( \frac{1}{2} - \alpha_i; 1, \alpha_i \right) \left| \begin{array}{c} (1, 1) \frac{1}{2}, 1 \end{array} \right| (0, 1), (\frac{1}{2}, 1) \left| \begin{array}{c} (0, 1) \end{array} \right| (0, 1).$$

(36)

The interested reader may find a general Matlab code to implement the bivariate Fox’s H-Function in Appendix B.

Asymptotic Analysis: To derive the asymptotic simplification of (34) and (36), we again reuse the residue method and, after a few mathematical derivations (that we omit here for space limitations), we get

$$I_{SER_i}(C) = \frac{1}{4\phi_i\sqrt{\pi}} \Gamma \left( \frac{1}{2} + \alpha_i \right) \left( \frac{2}{C} \right)^{\alpha_i},$$

(37)

and

$$I_{SER_i}(A, B) = \frac{-\alpha_i}{8\phi_i\pi^2} \left( \frac{2}{A} \right)^{\alpha_i} \left( \frac{B}{A} \right)^{\alpha_i, 1/2 + \alpha_i, 1} G^{2,2}_{3,3} \left( 0, 1, 2, 1 + \alpha_i \right),$$

(38)

where the Meijer G-Function [21] is used.

Apart from their simplicity, we emphasize the usefulness of these asymptotic expressions especially in terms of computation time for high-order modulations compared to [19, eq. 17] and (35) are time-consuming.

IV. Numerical Results

To assess the accuracy of our closed-form and asymptotic analysis, and illustrate the performance of multi-hop relaying systems in the adopted context of mmWave, we present in this section a few numerical scenarios of interest, and we compare our analytical results to Monte-Carlo simulations.

In order to facilitate the reading of the figures for the reader, we follow this same convention in all figures when necessary: solid lines represent the exact analytical results, simulations are represented with markers (only, no lines), and asymptotic expressions correspond to dashed lines. Hence, when the markers are on a line, this should be interpreted as a perfect match between simulation and analytical results.

Fig. 2 shows the end-to-end outage probability over a three-hop channel with two QAM orders $M = 16$ and 256. Three scenarios are depicted on the figure to reflect different mmWave channel conditions:

**Scenario 1:** “poor” channel conditions with parameters $\beta = 0.7 \times \left[ 1 \sqrt{2} \pi/2 \right]$.

**Scenario 2:** “mild” channels conditions with parameters $\beta = \left[ 1 \sqrt{2} \pi/2 \right]$.

**Scenario 3:** the first hop is Rayleigh (worst link), the other two hops experience better conditions with $\beta = \left[ 2\sqrt{2} \pi \right]$.

From the figure, it is clear that the exact closed-form expression [8] matches perfectly simulation results confirming the exactness of the analysis. The accuracy of the asymptotic expression can also be appreciated, the bound being very tight for low outage thresholds and the region of interest from the outage probability point of view.
Fig. 2. End-to-end outage probability $P_{\text{out}}$ in (5) and (9) versus (decreasing) SNR threshold $\gamma_{\text{th}}$. Solid lines represent the theoretical results, while simulations are represented with the markers, and asymptotic expression corresponds to the dashed lines.

Fig. 3. End-to-end average ergodic capacity $C$ in (12) and (17) versus average SNR $\bar{\gamma}$. Solid lines represent the theoretical results, while simulations are represented with the markers, and asymptotic expression corresponds to the dashed lines.

In Fig. 3 we depict the bandwidth-normalized end-to-end average ergodic capacity $\overline{C}$ as a function of the average
SNR per hop $\gamma$ for several scenarios. We show only the obtained results with a 4-QAM system (since the curves with higher modulation orders may be deduced by a simple translation with $[\log_2(M)-1]_{dB}$) over a dual-hop channel in three cases:

**Scenario 1:** Rayleigh fading links, i.e., $\beta_1 = \beta_2 = 2$.

**Scenario 2:** dissimilar (worse than Rayleigh) Weibull fading links with $\beta_1 = \pi/5$ and $\beta_2 = \sqrt{5}/9$.

**Scenario 3:** a Rayleigh fading link and a (worse than Rayleigh) Weibull fading link with $\beta_1 = 2$ and $\beta_2 = 0.17$.

The results provided by (15) are confirmed by simulations, and are very well approximated by (17). A notable remark is the considerable performance difference between Scenarios 1 and 3 (only the second hop makes the difference between both scenarios), confirming the sensitivity of the scheme to channel conditions on individual links.

In Fig. 4 we represent the BER from both simulation and analytical results. Three cases are investigated with different network configurations, and two modulation orders $M = 4$ and 16 with various channel conditions:

**Case 1:** dual-hop and mild channel conditions ($\beta_1 = \beta_2 = \sqrt{2}$), used as a reference.

**Case 2:** multihop with identical fading parameters $\beta_i = \sqrt{2}$, $i = 1, \ldots, 6$.

**Case 3:** dual-hop with distinct shape parameters $\beta_1 = 2$ and $\beta_2 = 3\pi/5$.

As discussed in (30), the diversity order is defined by minimum shape parameter of the individual links of the equivalent end-to-end channel.

In Fig. 5 the evolution of the BER with the number of hops is presented. The figure confirms the exponential relationship as discussed in (32). The figure again confirms the accuracy of the proposed implementation of the Fox $H$-function (18), and the accuracy of the asymptotic expression in (28). Note that the cross markers representing the asymptotic expression (28) are not visible on the figure for $\gamma = 0$ dB since the gap is still considerable at this SNR. Another remark is concerning $\hat{N}$ which increases rapidly with the average SNR $\gamma$, hence decreasing the effect of the number of hops on the overall end-to-end average BER.

Finally, in Fig. 6 we analyze the average SER performance of the proposed system through three scenarios:

**Case 1:** dual-hop communication with:

- 4-QAM: $\beta_1 = 2.2$ and $\beta_2 = 2$.
- 16-QAM: $\beta_1 = \sqrt{\pi}$ and $\beta_2 = 2$.

**Case 2:** three-hop communication with:

- 4-QAM: $\beta_1 = 2.2$, $\beta_2 = 2$ and $\beta_3 = \pi/2$.
- 16-QAM: $\beta_1 = \sqrt{\pi}$, $\beta_2 = 2$ and $\beta_3 = \pi/2$.
- 64-QAM: $\beta_1 = \sqrt{\pi}$, $\beta_2 = 2$ and $\beta_3 = 2$.

**Case 3:** 64-QAM over a 4-hop communication: $\beta_1 = \sqrt{\pi}$, $\beta_2 = \sqrt{\pi}$, $\beta_3 = 2$ and $\beta_4 = 2$. 
Fig. 4. End-to-end average bit error rate $\overline{BER}$ versus average SNR per hop $\bar{\gamma}$. Solid lines represent the theoretical results, while simulations are represented with the markers, and asymptotic expression corresponds to the dashed lines.

Fig. 5. End-to-end average BER versus the number of hops $N$, with $\beta_i = 2$ and $\Omega_i = 1$. Solid lines represent the theoretical results using the expression in (18), the theoretical results using the expression in (32) are represented with the circular markers, and the asymptotic expression (28) corresponds to the cross markers.
Again, the perfect match between analytical and simulated results, and the tight correspondence between exact and asymptotic analysis, can be appreciated from the figure. Note that, for example, adding a third hop ($\beta_3 = \pi/2$) with a 16-QAM transmission would require around 3 dB increase in power to achieve the same performance; with 4-QAM, it would require more than 4 dB. Similarly, considering a fourth hop with 64-QAM ($\beta = \sqrt{\pi}$) should be matched with only about 2 dB increase in power. These conclusions encourage the adoption of higher-order modulations in the mmWave regime even for increasing numbers of hops.

![Graph showing SER vs SNR with 4-QAM, 16-QAM, and 64-QAM modulations.]

**Fig. 6.** End-to-end average symbol error rate SER versus average SNR $\gamma$. Solid lines represent the theoretical results, while simulations are represented with the markers, and asymptotic expression corresponds to the dashed lines.

### V. Conclusion

In this paper, we have analyzed the performance of multihop detect-and-forward relaying in the context of mmWave communications as a key enabler for the next generation of mobile communication systems. Considering a general $M$-QAM modulation order, closed-form and asymptotic physical-layer level end-to-end performance metrics (outage probability, ergodic capacity, average BER, and average SER) were derived for Weibull fading links under the form of single- and bivariate Fox’s $H$-functions-based expressions. Simulation results confirmed the accuracy of our analysis for a large selection of channel and system parameters. As a secondary contribution, we proposed new and generalized implementation of Fox’s $H$ and bivariate $H$ functions in Matlab.

The extension of the system model setup to more realistic and complicated scenarios, a non-orthogonal cooperative transmission protocol, a cross-layer analysis of the interaction with other MAC-level parameters, are just a few
examples of some interesting directions that could be considered in future works.

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function out = Fox_H(an, An, ap, Ap, 
    bm, Bm ,bq, Bq, z)

    %% Integrand definition
    F = @(s) (GammaProd(bm,Bm,s) 
      .* GammaProd(1-an,-An,s).* z.^-s ) 
      ./ (GammaProd(1-bq,-Bq,s) 
      .* GammaProd(ap,Ap,s));

    %% Contour preparation:
    epsilon = 10^-1.2;

    Sups = min((1-an)./An); Infs = max(-bm./Bm);
    if(isempty(Sups) && isempty(Infs))
        WPx=1;
    elseif(isempty(Sups) && ~isempty(Infs))
        WPx = Infs +epsilon;
    elseif(~isempty(Sups) && isempty(Infs))
        WPx = Sups -epsilon;
    else
        WPx = (Sups + Infs)/2; % s between Sups and Infs
    end

    %% integration:
    infity = 10;
    out = (1/(2i*pi))*integral(F,WPx-1i*infity, WPx+1i*infity);
    return

    % ***** GammaProd subfunction *****
    function output = GammaProd(p,x,X)
        [pp, XX] = meshgrid(p,X);
        xx = meshgrid(x,X);
        if (isempty(p))
            output = ones(size(X));
        else output = reshape(prod(double( 
            gamma2(pp+xx.*XX)),2),size(X));
        end
    end
end
APPENDIX B

BIVARIATEFOXII-FUNCTION’SMATLABCODE

function out = Bivariate_Fox_H(an1,alphan1,An1,ap1,alphap1,Ap1,
  bql,betaql,Bql,Cn2,Cn2,cp2, Cp2, dm2, Dm2, dq2, Dq2, en3, En3,
  ep3, Ep3, fm3, Fm3, fq3, Fq3, x, y)
  %note there is no bm since m=0
  %*****Integrand definiton *****
  F=@(s,t)(GammaProd(1-an1,alphan1,s,An1,t) .* GammaProd(dm2,-Dm2, s)...
    .* GammaProd(1-cn2,Cn2,s) .* GammaProd(fm3,-Fm3,t)...
    .* GammaProd(1-en3,En3,t) .* (x.ˆs) .* (y.ˆt))...
    ./ (GammaProd(1-bql,betaql,s,Bql,t) .* GammaProd(ap1,-alphap1,s ,-Ap1,t)...
    .* GammaProd(1-dq2,Dq2,s) .* GammaProd(cp2,-Cp2,s)...
    .* GammaProd(1-fq3,Fq3,t) .* GammaProd(ep3,-Ep3,t) );
  %*****Contour definiton *****
  % cs
  css = 0.1;
  Sups = min(dm2./Dm2);
  Infs = max((cn2-1)./Cn2);
  if(isempty(Sups) && isempty(Infs))
    cs=1;
  elseif(isempty(Sups) && ~isempty(Infs))
    cs = Infs +css;
  elseif(~isempty(Sups) && isempty(Infs))
    cs = Sups -css;
  else
    cs = (Sups + Infs)/2;% Sups< s <Infs
  end
  % ct
  Supt = min(fm3./Fm3);
  Inft = max([(1-an1-alphan1.*cs)/An1
    (en3-1)/En3)];
  if(isempty(Supt) && isempty(Inft))
    ct=1;
  elseif(isempty(Supt) && ~isempty(Inft))
    ct = Inft +css;
  elseif(~isempty(Supt) && isempty(Inft))
    ct = Sups -css;
  else
    ct = (5*Supt + Inft)/6;% Supt< t <Inft
end

W = 10; %~infinity

out = real(((1/pi/2i)ˆ2)*quad2d(F,cs-li*W,cs+li*W,ct-li*W,ct+li*W,'Singular',true));

%***** GammaProd subfunction *****

function output = GammaProd(p,x,X,y,Y)
    if(nargin==3)
        [pp, XX] = meshgrid(p,X);
        xx = meshgrid(x,X);
        if (isempty(p))
            output = ones(size(X));
        else
            output = reshape(prod(double(gammaZ(pp+xx.*XX)),2),size(X));
        end
    elseif(nargin==5)
        [~, XX] = meshgrid(p,X);
        xx = meshgrid(x,X);
        yy = meshgrid(y,X);
        [pp, YY] = meshgrid(p,Y);
        if (isempty(p))
            output = ones(size(X));
        else
            output = reshape(prod(double(gammaZ(pp+xx.*XX+yy.*YY)),2),size(X));
        end
    end
end

The gammaZ function is the complex gamma, available in
www.mathworks.com/matlabcentral/fileexchange/3572-gamma