Boundary States of D-Branes and Dy-Strings

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Polchinski’s recent construction of Dirichlet-branes of R-R charges, together with Witten’s mechanism for forming bound states of both NS-NS charges and R-R charges, provides a rigorous method to treat these dy-branes. We construct the massless sector of boundary states of D-branes, as well as of dy-strings of charge \((p,1)\). As a consequence, the string tension formula predicted by duality in the type IIB theory is obtained.
1. Introduction

One of the most fruitful dualities in string theory is the string-string duality in the type IIB string in ten dimensions \[1\], it implies U-duality in various dimensions below ten. Using U-duality, Witten shows that the weakly coupled type II string, together with weakly coupled eleven dimensional supergravity, controls dynamics at strong coupling in various regions of the moduli space of the type II strings \[2\]. The fundamental string corresponds to a singular string solution to the low energy effective action \[3\], carrying charge corresponding to the NS-NS second rank antisymmetric field. Given the conjectured \(SL(2, \mathbb{Z})\) duality, one expects infinitely many string solitons carrying both NS-NS charges and R-R charges. This is indeed true in the context of the low energy effective action, solutions are recently constructed by Schwarz \[4\], for pairs of coprime integers \((p, q)\), where \(p\) is the NS-NS charge and \(q\) is the R-R charge. In analogy with dyons, one might call these strings as dy-strings, since the discrete duality group \(SL(2, \mathbb{Z})\) acts on them exactly the same way as it acts on dyons in some four dimensional quantum field theories. In addition to dy-strings, there are solitonic solutions of p-branes of NS-NS charges and R-R charges, for a review see \[5\]. There are important implications of the existence of such p-branes. Some applications can be found for example in \[6\] \[7\] \[8\]. Given the importance of these objects in nonperturbative string theory, one would like to ask whether it is possible to describe these p-branes beyond the low energy limit. In a couple of insightful papers, Polchinski points out that Dirichlet-branes are nonperturbative objects and give rise to the characteristic nonperturbative contribution of order \(\exp(-O(1/g_s))\) \[9\], and these branes indeed carry R-R charges and the minimal Dirac quantization condition is satisfied \[10\]. With this exact conformal field theory formulation at hand, we take a minor step in this paper to construct the boundary states for the D-branes, as well as for \((p, 1)\) strings based on a construction of Witten \[11\].

In the type IIB theory, in addition to the metric \(G_{\mu\nu}\), antisymmetric tensor field \(B^{1}_{\mu\nu}\) and the dilaton \(\phi\) in the NS-NS sector, there are a second antisymmetric tensor field \(B^{2}_{\mu\nu}\) and a second scalar \(\chi\), and a self-dual fourth rank antisymmetric tensor field \(A_{\mu_1...\mu_4}\), all arise in the R-R sector. Einstein metric \(g_{\mu\nu} = G_{\mu\nu}e^{-\phi/2}\) is the natural metric to discuss duality, since it is invariant under a duality transformation. Form a complex field \(\lambda = \chi + ie^{-\phi}\). Under the group element \(\Lambda = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\), \(\lambda\) transforms according to \[12\]

\[
\lambda 
\rightarrow \frac{a\lambda - b}{-c\lambda + d},
\]

and accordingly the doublet of antisymmetric tensor fields transforms as \(B \rightarrow \Lambda B\). The low energy effective action remains invariant. The singular string solution of \[3\] can be
used to generate a string soliton solution carrying charges \((p, q)\), as in [4]. Existence of five-brane solutions carrying corresponding “magnetic” charges of the B fields implies that \(p\) and \(q\) are integers. Furthermore, for a stable such solution, \(p\) and \(q\) must be coprime, as the string tension formula dictates [4]. Let \(\lambda_0\) be the asymptotic value of the complex field \(\lambda\), the \(SL(2, \mathbb{Z})\) invariant string tension is

\[
T_{p,q} = \frac{1}{\sqrt{\text{Im}\lambda_0}} |p + q\lambda_0| T,
\]

as expected on a general ground, where \(T\) is the fundamental string tension. It should be mentioned that the above tension is measured against the Einstein metric. In a very recent paper, Schwarz generalizes this tension formula to include various p-branes [13].

Polchinski argues in [10] that one should identify the solitonic brane solutions carrying R-R charge [14] with Dirichlet branes, as were first introduced in [13]. The following significant features of D-branes support this identification. First, due to Dirichlet boundary conditions obeyed by open strings moving along D-branes, the ground state of such open string (explained as excitation of the D-brane) carries R-R charge. Second, the amplitude between two parallel D-branes separated by a transverse distance \(y\) scales as \(y^{p-7}\), where \(p\) is the dimension of D-branes. When \(p = 1\), we have strings, and the amplitude behaves exactly as predicted by string solution [3]. Finally, the minimal Dirac quantization condition is satisfied by the charge of the p-brane and the charge of its dual, the \((6-p)\)-brane. A general p-brane carrying both NS-NS charge as well as R-R charge can be viewed as a bound state of p-branes of NS-NS charge and D-branes. This is similar to bound states of electrons and monopoles in \(N = 4\) supersymmetric Yang-Mills theory [16]. This view indeed is proven in a very recent paper of Witten [11], where some intriguing mechanism is explained for forming a bound state of charges \((p, q)\) with \(p\) and \(q\) coprime, for strings and fivebranes. We shall refer to these branes as dy-branes.

That the fundamental string and the D-string can form bound states is easy to see. The fundamental string carries a unit of NS-NS charge. A macroscopic string solution shows that a test string parallel to it does not feel any force, since the exchange of graviton cancels exchange of \(B^{1}_{\mu\nu}\) states. Imagine however a test D-string parallel to this macroscopic string. Now since D-string does not carry NS-NS charge, it does not feel the antisymmetric tensor field. It still feels the gravitational force, and this force can be shown to be attractive. (The solution of [3] takes the form \(G_{00} = \exp(2\phi)\), where \(G_{00}\) is the sigma-model metric for the fundamental string, then \(G'_{00} = \exp(\phi)\) is the sigma model metric for the D-string. It is easy to see that this gives rise to an attractive force on the D-string.) This is the reason for forming bound states of \(n\) fundamental strings of the same orientation with a
D-string. This string carries charges \((p, 1)\), therefore is expected to be stable. Witten’s recent paper [11] provides an exact treatment of such bound states.

To study interaction of fundamental strings in the background of a dy-brane, one convenient way is to construct a boundary state associated with this dy-brane. The boundary state certainly encodes properties of the dy-brane. We shall in sect.2 construct the massless sector of the boundary state for a D-brane, after briefly reviewing Polchinski’s construction. Lacking a manifestly supersymmetric covariant formulation, a full state is hard to construct. It may be useful to explore Green-Schwarz light-cone formulation, although we shall not do it here. We see directly a R-R field strength arise in the R-R sector of the boundary state. In sect.3, we proceed to employ Witten’s construction of \((p, 1)\) strings to compute the boundary state. This state is manifestly supersymmetric, therefore confirms that the string state is a BPS-saturated state. The string tension formula is readily read off from the boundary state. The agreement of this formula with the one predicted by duality shows that Witten’s construction is correct. Or turn it around, it provides another evidence for \(SL(2, Z)\) duality in the type IIB theory. The final section is devoted to a discussion.

2. Boundary States of the D-branes

First, let us explain the idea about D-branes of [10]. Consider two D-branes, one locates at the transverse position \(X^i = 0\), another at \(X^i = y^i\), \(i = p + 1, \ldots, 9\). The amplitude between these two D-branes is caused by exchange of closed string states. To compute it, one need to define a boundary state for each D-brane, say \(|B, 0\rangle\) and \(|B, y\rangle\). These boundary states encode information as how closed string states are produced in the presence of D-branes. The amplitude is then given by

\[
A(y) = \int_0^\infty dt \langle B, 0 | \frac{1}{4} (1 + (1) F)(1 + (-1) \tilde{F}) e^{-t \Delta} | B, y\rangle,
\]

where the GSO projection is inserted, and \(\Delta\) is the closed string propagator. One alternative way to compute \(A\) is to view the above cylindric amplitude as the one-loop amplitude of the open string with two ends attached to D-branes. The formula corresponding to this picture is

\[
A(y) = V_{p+1} \int \frac{dp^{p+1}}{(2\pi)^{p+1}} \int dt \sum e^{-t(p^2 + m_i^2)},
\]

where the factor \(V_{p+1}\) is the p-brane world-volume, its origin in \((2.1)\) is the delta-function arising from the inner product which enforces conservation of longitudinal momentum.
The sum in the above formula runs over all possible open string states, with all possible periodic boundary conditions in the loop channel. Polchinski’s result is

\[
A = V_{p+1} \int \frac{dt}{t} (2\pi t)^{(p+1)/2} e^{-ty^2/8\pi\alpha'^2} \left[ \frac{1}{2q} \left( \prod_{n=1}^{\infty} \left( \frac{1 + q^{2n-1}}{1 - q^{2n}} \right)^8 - \prod_{n=1}^{\infty} \left( \frac{1 - q^{2n-1}}{1 - q^{2n}} \right)^8 \right) - 8 \prod_{n=1}^{\infty} \left( \frac{1 + q^{2n}}{1 - q^{2n}} \right)^8 \right],
\]

(2.3)

where the first two terms come from the NS sector, and the third from the Ramond sector. The sum is zero, reflecting the fact that with the presence of the D-brane, some supersymmetry is unbroken. This point will be made explicit later. Interpreted in the closed string tree channel, the second term in (2.3) comes from the R-R sector. Taking the infrared limit in this channel, the contribution of the massless fields is picked up [10]:

\[
A = (1 - 1) V_{p+1} 2\pi (4\pi^2 \alpha')^{3-p} G_{9-p}(y),
\]

(2.4)

where \(G_{9-p}(y)\) is the massless scalar Green function in the \(9-p\) transverse dimensions. Let \(T_p\) be the D-brane tension. The force strength, due to the coupling of the D-brane world-volume to the \(p+1\) antisymmetric tensor, is read off from (2.4),

\[
T_p^2 = 2\pi (4\pi^2 \alpha')^{3-p}.
\]

(2.5)

The remarkable consequence of this formula is the minimal Dirac quantization condition satisfied by the \(p\)-brane and its dual, the \((6-p)\)-brane.

Our interest in this paper lies in constructing corresponding boundary state for a D-brane as well as its generalizations. For the usual open string with Neumann boundary conditions, boundary states are discussed in [17] [18] [19]. For the D-instanton, where Dirichlet boundary condition is enforced in all coordinates, the boundary state is constructed in [20]. For our purpose, though, a generalization of that construction is necessary for the following reasons. First, we wish to see the explicit form of the anti-symmetric tensor contained in the boundary state; second, supersymmetry is an important ingredient in generalizing the D-branes to dy-branes. The second reason requires a boundary state containing operators in all pictures. An exact such boundary state including all oscillators is formidable. Fortunately for our purpose, we need to know only the massless sector, and a useful algebraic strategy for constructing this part is presented in [18].

Let us start with the D-instanton. (Throughout this paper we work in Euclidean spacetime. Results for Minkowski spacetime are obtained by Wick-rotation.) The boundary state \(|B\rangle\) satisfies the boundary conditions [20],

\[
(\alpha_\mu_n - \bar{\alpha}_\mu_{-n}) |B\rangle = (\psi^\mu_m + i\bar{\psi}^\mu_{-m}) |B\rangle = 0,
\]

(2.6)
where the first equation corresponds to $\partial_s X^\mu = 0$, and the index $m$ is a half-integer for the NS-NS sector and an integer for the R-R sector. The boundary condition for the ghost and anti-ghost remains the same as in the Neumann case. The boundary condition for the super-ghosts conform with the boundary condition for the fermions. $|B\rangle$ can carry a finite momentum due to Dirichlet boundary condition, so one can convert it into a fixed point $X$ in spacetime through Fourier transformation. We shall work with bosonized super-ghosts as in [21]. The massless sector of the NS-NS part of $|B\rangle$ turns out to be the same as in the Neumann case

$$\langle B_0 \rangle_{NS} = \sum_{s=-\infty}^{\infty} \left( V_s^\mu(p) \tilde{V}_s^\mu_{-2-s}(p) - V_s^b(p) \tilde{V}_s^c_{-2-s}(p) + V_s^c(p) \tilde{V}_s^b_{-2-s}(p) \right) \frac{1}{2} (c_0 + \tilde{c}_0) \langle \Omega \rangle, \quad (2.7)$$

where the sum is over all pictures. We follow closely notations in [18]. $\langle \Omega \rangle$ is the $SL(2,C)$ invariant state. For simplicity we shall replace $\frac{1}{2} (c_0 + \tilde{c}_0) \langle \Omega \rangle$ by $\langle \Omega \rangle$. Operators $V_s$ in picture $s$ are obtained from the operators in the base picture

$$V_{-1}^\mu(p) = \psi^\mu e^{-\phi} e^{ipX}, \quad V_{-1}^b(p) = 2\beta e^{-\phi} e^{ipX},$$

$$V_{-1}^c(p) = \frac{1}{2} \gamma e^{-\phi} e^{ipX}, \quad (2.8)$$

by applying the picture changing operator $X$

$$X = i e^{\phi} \psi \cdot \partial X + e^{2\phi} \partial \eta b + 2c \partial \xi. \quad (2.9)$$

The precise relation is $V_{s+1} = X V_s$. $V_s$ carries momentum $p$, again for simplicity we will ignore index $p$ in all formulas below. As the formula (2.7) shows, the total super-ghost number is $-2$. It can be checked that $|B_0\rangle_{NS}$ is BRST invariant, and $X \langle B_0 \rangle_{NS} = \tilde{X} \langle B_0 \rangle_{NS}$.

In order to obtain the R-R part of $|B_0\rangle$, we demand half of supersymmetry be unbroken when applied to the whole state. To this end, we introduce supersymmetry generators [21]

$$Q_{-1/2}^A = \oint \frac{dz}{2\pi i} S^A e^{-\phi/2}(z), \quad (2.10)$$

and its counterpart in the right-moving sector. SUSY operators in other pictures can be obtained by applying $X$. The R-R part of the boundary state takes the form

$$|B_0\rangle_R = \sum_s V_s L \tilde{V}_{-2-s} \langle \Omega \rangle, \quad (2.11)$$

where $V_s L \tilde{V}_{-2-s}$ stands for

$$V_s^A L_{AB} \tilde{V}_{-2-s}^B.$$
with
\[ V_{-1/2}^A = S^A e^{-\phi/2} e^{i p X}, \] (2.12)
and its picture-changed forms. Note that all operators carry momentum \( p \). In determining \(|B_0\rangle_R\) by requiring supersymmetry, one will need the following formulas
\[
\{Q_r^A, V_s^\mu\} = -\frac{1}{2} V_{r+s}^B (p^\nu \gamma^\nu \gamma^\mu)_B^A, \\
[Q_r^A, V_s^B] = -\frac{i}{\sqrt{2}} (C\gamma^\mu)^{AB} V_{r+s}^\mu.
\] (2.13)

Some irrelevant terms in the above (anti-)commutators are omitted, as in [18].

In the Neumann case, operators must carry a zero momentum, as a consequence of the Neumann boundary conditions. It turns out that in order to determine the R-R part, a nonvanishing regulator momentum is needed [18], and \( p^\mu_L = -p^\mu_R \), to be consistent with Neumann boundary conditions. As a consequence, there is a solution to \((Q_r^A \pm \tilde{Q}_r^A)(|B_0\rangle_{NS} + |B_0\rangle_R) = 0\). However, in the Dirichlet case, momenta in both sectors are the same, therefore there is a solution to \((Q_r^A \pm i\tilde{Q}_r^A)(|B_0\rangle_{NS} + |B_0\rangle_R) = 0\)

\[ |B_0\rangle_R = \pm \frac{1}{\sqrt{2}} \sum_s V_s p^\mu \gamma^\mu C\tilde{V}_{-2-s} |\Omega\rangle. \] (2.14)

One may regard the plus sign represents the D-instanton, and the minus sign represents the anti-instanton. The reason for this is that the Green function between two D-instantons is vanishing due to cancelation between the NS-NS sector and the R-R sector, while the Green function between one instanton and an anti-instanton is not vanishing. Each term in (2.14) is of the form \( p^\mu V \gamma^\mu C\tilde{V} \), therefore can be interpreted as the derivative of a scalar field. This scalar is just the one arising from the R-R sector and appearing as the real part in the complex scalar \( \lambda \). Its derivative is just its field strength. This shows that indeed the fundamental string couples to R-R bosons through their field strength. We see that the D-instanton carry a charge of \( \chi \).

The above discussion is easily generalized to the Dirichlet p-brane. Imagine that the world-volume of this p-brane coincide with the subspace \( X^\alpha, \alpha = 0, \ldots, p \). Its position can be fixed at, say, \( X^i = 0, i = p + 1, \ldots, 9 \), or it carries a finite momentum \( p^i \) in the transverse directions. The NS-NS part of the boundary operator is simply
\[ |B_0\rangle_{NS} = \sum_s \left(-V_s^\alpha \tilde{\nabla}_{-2-s}^\alpha + V_s^i \tilde{\nabla}_{2-s}^i - V_s^b \tilde{\nabla}_{-2-s}^c + V_s^c \tilde{\nabla}_{-2-s}^b\right) |\Omega\rangle, \] (2.15)
where the sign in front of the longitudinal part is different from that in front of the transverse part, since the boundary condition along the world-volume is Neumann. Now
as in the pure Neumann case, one introduces nonvanishing $p^\alpha_L = -p^\alpha_R$ as a regulator. The transverse components are the same in both the left-moving and the right-moving sectors. After doing so, one can check that the boundary state (2.15) is BRST invariant. We shall use $p$ standing for $(p^\alpha_L, p^i)$.

Again we impose half of supersymmetry to obtain the R-R part. It is easy to see that both the combinations in the pure Neumann case and the pure Dirichlet case do not work for $p \geq 0$. An ansatz should be such that when $p = -1$, the combination is the one for the D-instanton, and when $p = 9$, it is the one in the pure Neumann case. There are two choices

$$Q_r^A \pm i\tilde{Q}_r^B(\gamma^0 \ldots \gamma^p)_B^A. \quad (2.16)$$

When $p = 9$, $i\gamma^0 \ldots \gamma^9$ is just $\gamma^{11}$. Since $\tilde{Q}_r^A$ is chiral, this matrix disappears in (2.16), and one recovers $Q_r^A \pm \tilde{Q}_r^A$, the combinations in the pure Neumann case. It can be checked that the above combinations are consistent with the mixed boundary conditions. Requiring that the whole boundary state be annihilated by (2.16), we are led to

$$|B_0\rangle_R = \pm (-1)^{p(p-1)/2+1} \frac{1}{\sqrt{2}} \sum_s V_s p^\nu \gamma^\nu \gamma^0 \ldots \gamma^p C\tilde{V}_{-2-s}|\Omega\rangle. \quad (2.17)$$

Clearly, the massless R-R field is the field strength of the $(p+1)$-form $A_0 \ldots \gamma^p(X^i)$. This result agrees with that of Polchinski’s which is based on a general argument. Again, the plus sign corresponds to the D-brane, and the minus sign the Dirichlet anti-brane (with the opposite orientation of its world-volume).

3. $(p, 1)$ Strings

To describe a bound state of the D-string with charges $(0, 1)$ and $p$ fundamental strings with charges $(1, 0)$, it is helpful to compactify one spatial dimension around which the D-string wraps once. Let it be $X^1$. Now we follow Witten to construct a $(p, 1)$ string.

Interaction of closed strings with the D-string is through open strings whose both ends are attached to the D-string. The tree level diagram is represented by a disk. The boundary of the disk, attached to the world-sheet of the D-string, can wrap the circle any number of times, it follows that the total winding number of closed string states is not conserved. Consider the disk diagram for $(1, 0)$ strings with a nonvanishing antisymmetric tensor $B^1_{\alpha\beta}$ (being in the NS-NS sector, it couples to $(1, 0)$ strings), this coupling part is

$$S_B = \frac{1}{2} \int d^2 \sigma \epsilon^{ab} B^1_{\alpha\beta} \partial_\alpha X^\alpha \partial_\beta X^\beta. \quad (3.1)$$
Under a gauge transformation \( B^1_{\alpha\beta} \rightarrow B^1_{\alpha\beta} + \partial_\alpha \Lambda_\beta - \partial_\alpha \Lambda_\beta \), the above action is not invariant on a disk due to a boundary term. Such a term can be absorbed in the coupling of the boundary to the U(1) gauge field \( A_\alpha \):

\[
S_A = \int d\tau A_\alpha \frac{dX^\alpha}{d\tau}.
\]

The combined action is invariant, provided \( A_\alpha \rightarrow A_\alpha + \Lambda_\alpha \) under the gauge transformation. Note that \( A_\alpha \) lives on the world-sheet of the D-string. There are also scalar fields (with respect to the world-sheet of the D-string) \( A_i \), corresponding to the transverse displacement of the D-string world-sheet, allowed by Dirichlet conditions [15]. But these modes are irrelevant for our discussion, because we are not considering oscillating modes of the D-string. The gauge invariant field strength of \( A_\alpha \) is

\[
F_{\alpha\beta} = F_{\alpha\beta} - B^1_{\alpha\beta},
\]

with an action on the world-sheet of the D-string

\[
\frac{1}{2\lambda} \int d^2 X F^2,
\]

where \( \lambda \) is the string coupling constant, its appearance is due to the fact that the above term arises from the disk amplitude. Clearly this term provides a source for field \( B^1_{\alpha\beta} \). So if \( F \neq 0 \), the D-string will carry a NS-NS charge.

A constant \( F \) is the only solution of the equation of motion, which can be generated by placing a charge for \( A_\alpha \) at infinity. The conjugate momentum of \( A \) is \( \pi = F/\lambda \). If the abelian gauge group is indeed \( U(1) \), \( \pi \) must be quantized, and so \( F_{\alpha\beta} = p\lambda \epsilon_{\alpha\beta} \). This must be the case, for otherwise annihilation of the D-string with its anti-string will result in a continuous winding number state. Now, we are ready to construct the boundary state for this \((p, 1)\) string. Our problem is to impose certain boundary condition for closed string which is emitted from the \((p, 1)\) string. This boundary condition is Neumann in longitudinal directions, and Dirichlet in transverse directions. In addition to this, the open strings move in the background of a constant field strength on the world-sheet of the \((p, 1)\) string. Luckily, this problem for purely Neumann boundary condition is already addressed in [18].

Let us recall the result for a constant background \( F \) for purely Neumann conditions. The NS-NS part of the boundary state is

\[
|F\rangle_{NS} = [\det(1 + F)]^{1/2} \sum_s \{V_s^\mu \left( \frac{1 - F}{1 + F} \right) \mu\nu \hat{V}_\nu_{-s} + \text{ghost terms} \} |\Omega\rangle.
\]
The overall factor \([\det(1 + F)]^{1/2}\) is seen to result from the path integral with a boundary term in the disk action. The ghost terms are the same as in \(F = 0\) case. The matrix \((1 - F)/(1 + F)\) in (3.4) can be understood as exercising a rotation on the right-moving vector, and indeed is an orthogonal matrix. Again in order to obtain the R-R part, one has to introduce momenta with the constraint

\[
p_L^\mu = - \left( \frac{1 - F}{1 + F} \right)_\mu^\nu p_R^\nu. \tag{3.5}
\]

This is consistent with the rotation in (3.4). Now it is a little tricky to construct the surviving supersymmetry. Let it be

\[
Q_{A,r} + M_A B \tilde{Q}_{B,r}.
\]

It is found in [18] that the matrix \(M\) can be expressed as

\[
M = [\det(1 + F)]^{-1/2} \exp(-\frac{1}{2} \gamma^\mu \wedge \gamma^\nu F_{\mu\nu}), \tag{3.6}
\]

where a peculiar convention is introduced: Upon expanding the exponential, one assumes all gamma matrices are anti-commuting, therefore there are only a finite number of terms.

Now the solution to the supersymmetry condition is unique

\[
|\mathcal{F}\rangle_R = \frac{i}{\sqrt{2}} \sum_s V_s p_L^\mu \gamma^\mu \exp(\frac{1}{2} \gamma^\mu \wedge \gamma^\nu F_{\mu\nu}) C \tilde{V}_{-2-s} |\Omega\rangle. \tag{3.7}
\]

The obvious generalization of (3.4) to our case is

\[
|\mathcal{F}\rangle_{NS} = [\det(1 + F)]^{1/2} \sum_s \{-V_s^\alpha \left( \frac{1 - F}{1 + F} \right)^\alpha_\beta \tilde{V}_\beta^{2-s} + V_i^\beta \tilde{V}_i^{2-s} + \text{ghost terms}\} |\Omega\rangle. \tag{3.8}
\]

The overall factor \([\det(1 + F)]^{1/2}\) is expected by the path-integral argument. The above state is BRST invariant, provided that the longitudinal momenta satisfy the same relation as in (3.5), and the transverse momenta in both sectors are the same. Explicitly, we have

\[
\det(1 + F) = 1 + p^2 \lambda^2,
\]

\[
\frac{1 - F}{1 + F} = \frac{1}{1 + p^2 \lambda^2} \begin{pmatrix}
1 - p^2 \lambda^2 & -2p \lambda \\
2p \lambda & 1 - p^2 \lambda^2
\end{pmatrix}. \tag{3.9}
\]

The matrix in (3.8) has an off-diagonal term proportional to \(2p \lambda\), signaling an antisymmetric tensor \(B_{\alpha\beta}^{1}\) as we expected.
We need to guess the correct combination of the supersymmetry. This is easy, we need to recover supersymmetry for 1-brane discussed in the previous section when $F = 0$, and to recover the purely Neumann result when $p = 9$. There is a unique solution:

$$Q_{A,r} + i\tilde{Q}^B_r \left( \gamma^0 \gamma^1 \exp \left( \frac{1}{2} \gamma^\alpha \wedge \gamma^\beta F_{\alpha\beta} \right) \right)_{BA} [\det(1 + F)]^{-1/2}. \quad (3.10)$$

Requiring that the total boundary state be annihilated by the above SUSY generator yields a unique solution

$$|\mathcal{F}\rangle_R = -\frac{1}{\sqrt{2}} \sum_s V_s p^\mu L \gamma^\mu \gamma^0 \gamma^1 \exp \left( -\frac{1}{2} \gamma^\alpha \wedge \gamma^\beta F_{\alpha\beta} \right) \tilde{C} \tilde{V}_s - \frac{1}{2} |\Omega\rangle. \quad (3.11)$$

Expanding the exponential in the above equation according to that ad hoc rule, we see that there are two terms. One corresponds to $B_{01}$, another corresponds to $\chi$. The latter is proportional to $p\lambda$

The interaction strength between two $(p, 1)$ strings can be read off from the boundary state. The contribution from the NS-NS part determines the string tension $T_{p,1}$

$$T_{p,1}^2 = \frac{1}{8} \left[ \det(1 + F) \left( \frac{1}{1 + F} \right)^2_{\alpha\beta} + 8\det(1 + F) - 2\det(1 + F) \right], \quad (3.12)$$

where the first term comes from the longitudinal modes, the second term comes from the transverse modes, and the third from ghosts. An overall factor $1/8$ is used to normalize the result. Of course a constant factor as a function of $\alpha'$ is omitted. A little calculation then shows

$$T_{p,1}^2 = 1 + p^2 \lambda^2. \quad (3.13)$$

This is exactly the result one expects from Duality \[4\]. It is also interesting to compute the interaction strength coming from the R-R sector. It is proportional to the square of the charge in terms of the string’s own antisymmetric tensor

$$\alpha_{p,1}^2 = \frac{1}{8 \times 2} \text{tr} \left( \frac{1}{2} (1 + \gamma^{11}) \exp \left( \frac{1}{2} \gamma^\alpha \wedge \gamma^\beta F_{\alpha\beta} \right) \exp \left( -\frac{1}{2} \gamma^\alpha \wedge \gamma^\beta F_{\alpha\beta} \right) \right) \quad (3.14)$$

$$= \frac{1}{32} \text{tr} \left( 1 + \gamma^{11} \right)(1 + \frac{1}{2} \gamma^\alpha \gamma^\beta F_{\alpha\beta})(1 - \frac{1}{2} \gamma^\alpha \gamma^\beta F_{\alpha\beta}) = 1 + p^2 \lambda^2,$$

where the factor $1/8$ in the first equality is the same as in \[3.12\], an additional factor $1/2$ comes from the factor $1/\sqrt{2}$ in \[3.11\], and the chirality projector is inserted because the $V^A$’s are chiral. The fact that the above result is equal to $T_{p,1}^2$ should come as no surprise, as expected for a BPS-saturated state.
Eq. (3.13) is the \((p, 1)\) string tension measured in its own sigma model metric, and in which the \((1, 0)\) string tension is \(\lambda\). When measured in the \((1, 0)\) string metric, one has

\[
T_{p,1}^2 = \frac{1}{\lambda^2} + p^2.
\]

In this metric, the \((1, 0)\) string tension is 1. One surprising point about (3.15), pointed out in [11], is that the difference \(T_{p,1} - T_{1,0}\) is of order \(\lambda\). So the binding energy in the weak coupling limit is almost equal to energy of all \((1, 0)\) strings. The origin of this property becomes obvious by taking a look at the boundary state formula (3.8): That the effect of gauge field \(A_\alpha\) is nonlinear, makes it possible for a D-string to carry a NS-NS charge with little cost of energy.

Finally, the integer charge \(p\) can be shifted by an arbitrary amount \(\chi_0\), if one switches on a constant background of the R-R scalar \(\chi\). This effect is analogous to Witten’s effect about the shift of electric charge of a monopole when the theta-angle \(\theta\) is switched on, whose form is \(m + \theta/2\pi\) [22]. With such a shift, our previous discussion still goes through.

4. Discussion

The next logical step is to extend our discussion to include \((p, q)\) strings with \(q > 1\), and to include dy-branes. Witten in [11] suggests a ansatz for such a string: That a \(N = 8\) \(U(q)\) supersymmetric Yang-Mills theory governs properties of \((p, q)\) strings. The gauge group \(U(q)\) is generated from Chan-Paton factors associated to \(q\) D-strings. To generate charge \(p\) in the NS-NS sector, one places a quark in the \(p\)-fold tensor of the fundamental representation at infinity. The most intriguing of this scenario is the existence of adjoint matter \(X^i\), coming from transverse oscillations of the D-strings. Its vacuum expectation value determines the separations of \(q\) D-strings, yet they are noncommuting variables. A further understanding of such system is necessary in order to extend our discussion to construction of the boundary state of \((p, q)\) strings. In the case of p-branes, we need understand more about the p-brane solutions of NS-NS charges. Thus, construction of the boundary states associated to dy-branes is also for the future.

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