The role of plasma radius as a condition for sustaining a coaxial discharge at various wave modes

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Abstract. A gas discharge can be produced and sustained by travelling electromagnetic waves in various geometries: planar, spherical, cylindrical and coaxial. An electromagnetic wave travelling along a dielectric tube can produce plasma outside the tube when a metal rod is placed along the tube axis, which is the typical arrangement of a coaxial surface-wave-sustained discharge (CSWD). The CSWD has been studied intensively both theoretically and experimentally since 1998.

In the case of a SWD in cylindrical geometry, plasma is mainly produced and sustained by the azimuthally symmetric waves. In coaxial geometry, there are both experimental and theoretical indications showing that higher wave modes may also produce and sustain plasma under certain conditions. In order to find out these conditions theoretically, we developed a one-dimensional fluid model.

The purpose of this work is to investigate theoretically the behavior of wave phase diagrams under various discharge conditions and to find the discharge conditions under which plasma can be produced, as well as those conditions when this is not possible.

1. Introduction

The coaxial structure is a relatively new type of plasma source, which was proposed recently [1,2] and has since been intensively studied [3,4]. A typical configuration of this type of gas discharges is presented in figure 1. This plasma source produces large-volume clean electrodeless plasma and is very attractive because of the widening range of possible technological applications as in UV lamps [5], in surface cleaning, material treatments, plasma sterilization and thin film deposition [6]. At the same time, fundamental research provides deeper knowledge and better understanding of the physics of the discharge. The plasma is both radially and axially inhomogeneous. It has been found both experimentally and theoretically that the cylindrical plasma column is produced by one wave mode only. Usually this is the azimuthally symmetric one ($m = 0$, $m$ being the azimuthal wave number). A travelling dipolar wave ($m = 1$) can sustain plasma only under some special conditions. The purpose of this work is to investigate the characteristics of the various wave modes, the ability of a given mode to sustain plasma in a coaxial structure, and the role of the dielectric tube. We studied theoretically four

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configurations: (i) vacuum–plasma; (ii) metal–vacuum–plasma; (iii) dielectric–plasma; (iv) metal–dielectric–plasma, varying the azimuthal wave number \( m \), the plasma parameter \( \sigma \) and the discharge conditions.

2. Basic assumptions and equations

At low pressures, the electron–neutral collisions frequency \( \nu \) is smaller than the wave angular frequency \( \omega \) and the plasma can be considered a weakly dissipative medium. The ratio \( \nu/\omega \) can than be neglected in the plasma permittivity expression, i.e., we can use the simple form \( \varepsilon_p = 1 - \omega_p^2/\omega^2 \), \( (\omega_p = (4\pi e^2 n/m)^{1/2} \) being the plasma frequency) [7]. The coaxial structure is investigated on the basis of a one-dimensional fluid model. The plasma is produced and sustained by a surface electromagnetic wave \( (f = 2.45 \text{ GHz}) \), which propagates along the interface between the plasma and the dielectric tube or vacuum.

Our model is based on Maxwell’s equations in cylindrical coordinates from which we obtain the wave equation. The solutions of the wave equation yields the wave-field components amplitude as combinations of Bessel or modified Bessel functions:

\[
F_z^v(a_v, \rho) = C_1 J_0(a_v \rho) + C_2 K_0(a_v \rho) \\
F_z^d(a_d, \rho) = \begin{cases} 
C_3 J_0(a_d \rho) + C_4 K_0(a_d \rho) & x > \sigma \sqrt{\varepsilon_d} \\
C_5 J_0(a_d \rho) + C_6 N_0(a_d \rho) & x < \sigma \sqrt{\varepsilon_d} 
\end{cases} \\
F_z^p(a_p, \rho) = C_7 J_0(a_p \rho)
\]

where

\[
a_v = \left(x^2 - \sigma^2 \varepsilon_p \right)^{1/2}, \quad a_d = \left(x^2 - \sigma^2 \varepsilon_d \right)^{1/2}, \quad a_p = \left(x^2 - \sigma^2 \right)^{1/2}
\]

\[
\rho = \frac{r}{R}, \quad x = kR, \quad \sigma = \frac{\omega R}{c}
\]

The boundary conditions are the conditions for continuity of the electromagnetic field’s tangential components at the medium boundaries and the condition for annulment of the \( E_z \)–component at the metal cylinders. By solving this system of equations, we obtain all wave-field components, as well as the local dispersion relation. It is difficult to present the dispersion equation in a compact form; we give it below in a more general form:

\[
D(m, \sigma, \eta, \gamma, k, \omega, \omega_p) = 0.
\]

In our case, the dispersion relation (2) is local because of the axial inhomogeneity of the plasma. Since the wave frequency \( \omega \) is constant and the plasma frequency \( \omega_p \) changes along the column, the dispersion equation gives the dependence of the normalized plasma density \( N = (\omega_p/\omega)^2 \) on the dimensionless wave number \( x = kR \) (usually presented as \( \omega/\omega_p \) vs. \( x \), or the so-called phase diagrams) at different wave modes. The geometry of the discharge is accounted for by the dimensionless parameters presented in table 1.
Table 1. Dimensionless parameters of the discharge geometry.

| Geometric factors         | Notations                          | Parameters                                      |
|---------------------------|------------------------------------|------------------------------------------------|
| Plasma radius             | \( R \) (outer radius of the dielectric tube) | \( \sigma = \omega R/c \) (\( c \)-speed of light) |
| Dielectric tube radius    | \( R_d \) (inner radius of the dielectric tube) | \( \gamma = R_d/R \)                           |
| Tube thickness            | \( d = R - R_d \)                  | \( \gamma = 1 - d/R \)                         |
| Radius of the metal rod on the axis | \( R_m \)                      | \( \eta = R_m/R \)                             |

3. Results and discussion

By solving the local dispersion relation at fixed \( \omega = 2\pi f = 2.45 \, \text{GHz} \) and varying the wave modes, we obtain the phase diagrams for the vacuum–plasma configuration (figure 2). The dotted line in figure 2 (and in the other figures) corresponds to the resonance plasma density \( N_{\text{res}} = 1 + \varepsilon_v = 2 \) (or resonance frequency \( \omega/\omega_p \)\(_{\text{res}} = 2^{1/2} \)). All phase diagrams approach this value as the dimensionless wave number \((kR)\) increases. For small values of \(kR\), the phase diagrams are divided in two groups: (i) at higher plasma radii (parameter \(\sigma\)), the phase diagrams follow the standard behavior of \(\omega/\omega_p\) increasing with \(kR\), which corresponds to the plasma density decreasing from the wave launcher to the column end (dashed lines). (ii) for smaller values of the plasma parameter \(\sigma\) and for \(m = 0\), only backward wave propagation is observed in the phase diagrams \((\omega/\omega_p\) decreasing with \(kR\), black line in figure 2), while for dipolar and quadrupolar azimuthal wave modes, a small region of forward wave propagation is also observed. For small \(\sigma\), all phase diagrams are in the region of very low plasma density \((N < N_{\text{res}})\). We, therefore, assume that the azimuthally symmetric wave mode cannot sustain the discharge at small \(\sigma\). There exists a value of \(\sigma\) (the so-called critical value \(\sigma_{cr}\)) which divides the two groups of phase diagrams [3]. For the vacuum–plasma configuration, the "critical" value of \(\sigma\) corresponds to \((fR)_{\text{cr}} \approx 7.45 \, \text{GHz cm}\) meaning that one can expect to produce plasma in such configuration by using sufficiently high enough wave frequency and plasma...
radius. The situation is similar for the dielectric–plasma configuration, as can be seen in figure 3a. In this case, the “critical” value of $\sigma$ is smaller than that for the vacuum–plasma configuration and depends on the dielectric permittivity of the tube material. For a dielectric with $\varepsilon_d = 3.8$ (quartz), it corresponds to $(fR)_{cr} \approx 3.86$ GHz.cm. Our results show that the value of $\sigma_{cr}$ decreases exponentially as the permittivity of the medium increases (figure 3b).

The next step in the study is changing the configuration by adding a metal rod on the tube axis of the previous configurations. As one can see in figures 4a and 4b, this leads to a significant change in the phase curves at $m = 0$. Even a thin metal rod (a fraction of $10^{-5}$ of the plasma radius) is sufficient to change the phase diagrams, so that a region of forward wave propagation appears at small $\sigma$. This means that now the azimuthally symmetric wave is also able to create and sustain the discharge for $\sigma < \sigma_{cr}$, while for $\sigma > \sigma_{cr}$ the plasma density is higher.

The role of the metal rod thickness is the same for both configurations, as seen in figure 5. Increasing the normalized metal rod radius $\eta$ leads to an increase in the plasma density. The maximum of the phase diagrams divides them into two regions, namely, a region of forward wave propagation (from small values of the dimensionless wave number to the maximum value) and a region of backward wave propagation. We assume that only the forward wave can produce plasma and the plasma column ends upon reaching the maximum of the phase diagram. As $\eta$ increases, the phase diagrams approach zero and the region of backward wave propagation disappears.

**Conclusions**

The investigation presented shows that the plasma radius is a key discharge condition for sustaining the plasma. At small values of the plasma radius (small values of the plasma parameter $\sigma$), the azimuthally symmetric wave cannot sustain the plasma, because in the phase diagrams of both vacuum–plasma and dielectric–plasma configurations there is only a backward wave propagation region. The higher wave mode phase diagrams are above the resonance frequency, which corresponds to a plasma density below resonance. We assume that the plasma cannot be sustained under these conditions. At high values of the plasma radius (high $\sigma$ value), all phase diagrams are below the resonance line at small
\[ k_c R, \text{ i.e., all wave modes can sustain plasma with high density in both vacuum–plasma and dielectric–plasma configurations.} \]

There exists a critical value of plasma parameter \( \sigma \) which divide the phase diagrams in two parts: at \( \sigma < \sigma_{cr} \), the wave cannot sustain plasma. The value of \( \sigma_{cr} \) decreases exponentially with the increase of the dielectric permittivity. The maximum value of \( \sigma_{cr} (=1.56) \) occurs in the plasma–vacuum configuration.

Adding even a very thin metal rod on the tube axis significantly improves the conditions for sustaining plasma.

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