ORIGINAL ARTICLE

Numerical simulation of the non-Newtonian fracturing fluid influences on the fracture propagation

Min Wen | Hui Huang | Zening Hou | Fei Wang | Hao Qiu | Nan Ma | Shengtian Zhou

Abstract

Hydraulic fracturing fracture propagation is the main factor affecting the fracturing effect. In view of the more diversified selection of fracturing fluid, the problem of fracture propagation in non-Newtonian fluid fracturing is often encountered in fracturing. In this paper, the influencing factors of fracture propagation of non-Newtonian fluid fracturing fluid are analyzed. Based on PKN model, a three-dimensional fracture propagation model considering the rheology and filtration of fracturing fluid is established, the better fracture shape simulation results are given, and the effects of fluid properties, rock physical properties, and injection parameters on fracture propagation are analyzed in detail. The research shows that (1) in the process of fracture propagation, the fracture length and width gradually increase, and the corresponding pressure in the fracture also increases. The overall increase trend is that the growth is rapid in the initial stage of fracturing and gradually slows down in the later stage. (2) The greater the consistency coefficient and rheological index, the greater the fracture width and pressure in the fracture, and the easier it is to form wide fracture. The larger the filtration coefficient is, the weaker the fracture forming ability is, and the fracture width, length, and pressure in the fracture are reduced. (3) Rock physical properties have a certain influence on fracture propagation, Young's modulus has a great influence on fracture propagation, while Poisson's ratio has little influence on it. However, the larger the Young's modulus, the more difficult the fracture is to expand, and the smaller the corresponding fracture width. (4) The larger the pump injection rate, the larger the fracture size and the higher the pressure in the fracture. The model can provide a theoretical basis for fracturing design and fracture shape prediction.

KEYWORDS

filtration, fracture model, fracture propagation, influencing factors, non-Newtonian fluid
INTRODUCTION

The geometry and trend of hydraulic fracturing fracture is one of the main factors affecting the fracturing effect. Therefore, it is of great significance to describe the geometry of hydraulic fracturing fracture as much as possible for hydraulic fracturing design. The most basic form of hydraulic fracturing simulation is also a complex process, which is coupled with at least the following three processes: (1) mechanical deformation caused by flow pressure on the fracture surface; (2) fluid flow in fractures; (3) fracture propagation. In this regard, scholars at home and abroad have carried out a lot of research: Rock mass deformation is simulated by linear elasticity, and the relationship between fracture width and fluid pressure is expressed by integral equation. The fluid flow is simulated by lubrication theory, and the relationship between fluid velocity, fracture width, and pressure gradient is expressed by nonlinear partial differential equation. The criterion of fracture propagation is usually described by the energy release rate method based on linear elastic fracture mechanics. For example, when the tip stress intensity factor is greater than the rock toughness, the fracture will expand.

At present, hydraulic fracture models are divided into two-dimensional, quasi three-dimensional, and full three-dimensional models. The general two-dimensional model assumes that the fracture height is fixed, the fracture extends only along the fracture length, and there is no fluid flow in the vertical direction of the fracture, that is, the fluid flows only along the fracture length. The earliest two-dimensional model began in the 1950s, in which pioneering work includes KDG model\(^1-3\) and PKN model\(^4-6\).

Scholars have simplified the actual situation to varying degrees and developed various models to describe the geometric shape and extension law of hydraulic fractures.\(^7-42\) The research of these follow-up models has greatly expanded the fracture simulation theory. However, most of the results are numerical solutions, which greatly depends on the development of numerical algorithms such as difference and finite element. Most of the model architectures are based on the results of PKN or KDG models, or compared with them, because although they have conditional assumptions, they give analytical solutions. The calculation amount of three-dimensional model is large, and generally, only theoretical calculation is done. The pseudo three-dimensional model is essentially the treatment of two-dimensional model. Therefore, the commonly used two-dimensional models PKN and KGD can be used as the basis for fracture propagation research. When these models are used for simulation, the filtration loss is usually ignored, and the fracturing fluid is often considered as Newtonian fluid, resulting in great differences between the calculation of fracture width and fracturing fluid consumption, which is not applicable to field injection. Based on PKN model, considering power-law fluid and filtration factors, this paper establishes an approximate three-dimensional propagation model, simulates the geometric changes in fracture propagation, and focuses on the analysis and comparison of the parameters affecting fracture propagation, which has important guiding significance for field application.

MATHEMATICAL MODELS

In this paper, based on PKN model, non-Newtonian fluid is considered, fluid filtration is added, and the section is treated as ellipse. The model is improved, and the three-dimensional fracture shape is simulated, as shown in Figure 1. In the current calculation of PKN model, Newtonian fluid is usually considered or filtration is not considered, which is meaningless for field application.

First, make some necessary assumptions:

1. The fracture has the same height everywhere.
2. In the section perpendicular to the fracture length, the liquid pressure is constant.
3. The fracture section is elliptical.
4. The fracture is a double wing fracture symmetrical to the well axis, and the length of the fracture on one side is the fracture length \(L\) in the following formula.
5. Fracturing fluid is power-law fluid.

According to Carter filtration calculation method:

\[
\frac{dq}{dx} = -\frac{2HC}{\sqrt{t - \tau(x)}}
\]

(1)

**FIGURE 1** Schematic diagram of fracture shape
where \( q \) is the flow rate, \( h \) is the height, \( C \) is the filtration coefficient, and \( t \) is the time to start filtration.

By integrating the above formula along the fracture:

\[
q = q_0 - 2HC \int_0^L \frac{dx}{\sqrt{1 - \tau(x)}}
\]

(2)

where \( q_0 \) is the pump injection rate, which is generally kept constant during fracturing process.

The flow at the end of the fracture is zero, namely \( q(L, t) = 0 \):

\[
\int_0^L \frac{dx}{\sqrt{1 - \tau(x)}} = \frac{q_0}{2HC}
\]

(3)

For the above formula, Laplace transform is used to approximate:

\[
L = \frac{q_0 t^{1/2}}{\pi CH}
\]

(4)

\[
q(x) = q_0 \left[ 1 - \frac{2}{\pi} \sin \left( \frac{x}{L} \right) \right]
\]

(5)

When the fracturing fluid is a power-law fluid, the apparent viscosity of the fluid is:

\[
\mu_a = K_a \left( \frac{6q}{HW^2} \right)^{n-1}
\]

(6)

\[
K_a = K \left( \frac{2n+1}{3n} \right)^n
\]

(7)

where \( n \) is rheological index and \( K \) is consistency coefficient, Pa s\(^n\).

The fluid flow equation in the fracture is:

\[
\frac{\partial \Delta p}{\partial x} = \frac{32}{3\pi} K_a \left( \frac{6q}{H} \right)^n W^{-(2n+1)}
\]

(8)

England-Green formula:

\[
\Delta P(0, t) = \frac{GW(0, t)}{(1 - \nu)H}
\]

(9)

\[
G = \frac{E}{2(1 + \nu)}
\]

Continuity equation:

\[
\frac{\partial^2 q}{\partial x} = - \frac{\pi H}{4} \frac{\partial W}{\partial t}
\]

(10)

**Table 1** Calculation parameters

| Parameters                  | Value  |
|-----------------------------|--------|
| Young’s modulus/GPa        | 1.8    |
| Consistency coefficient/Pa s\(^n\) | 0.211 |
| Rheological index          | 0.63   |
| Pump flow rate/m\(^3\) min\(^{-1}\) | 2.5   |
| Minimum principal stress/MPa | 15    |
| Poisson’s ratio            | 0.25   |
| Fracture length/m          | 30     |
| Filtration coefficient/m\(^3\) min\(^{-0.5}\) | 0.003 |
| Layer thickness/m          | 20     |

The maximum fracture width formula can be derived from formulas (9) and (10):

\[
W(0, t) = \left[ \frac{64}{3\pi} (n+1) \right]^{1/2} \left[ \frac{6q_0}{H} \right]^{1/n} \left[ \frac{K_a (1 - \nu) HL}{G} \right]^{1/2 (n+1)}
\]

(11)

The distribution of fracture width along the fracture propagation direction is:

\[
W(x, t) = W(0, t) \left( \frac{x}{L} \sin^{-1} \frac{x}{L} + \left[ 1 - \frac{x^2}{L^2} \right]^{1/2} - \frac{\pi x}{2L} \right)^{1/4}
\]

(12)

Assuming that the shape of fracture width is ellipse, the fracture section equation is:

\[
\frac{y}{(W/2)^2} + \frac{z}{(H/2)^2} = 1
\]

(13)

### 3 | RESULTS AND DISCUSSION

The above model is solved by programming to simulate the change of fracture geometry, and the effects on fracture propagation are analyzed.

#### 3.1 | Results analysis

Table 1 shows the calculation parameters used in the implementation of the model in this paper.

Figure 2 shows the change of calculated fracture length with time. It can be seen that the fracture length also gradually increases during fracturing. It takes about 20.5 min for the fracture length to reach the preset fracture length of 30 m. In this process, the growth trend of fracture length gradually slows down.

Figure 3 shows the change of fracture differential pressure at the fracture mouth with time. It can be seen that the fracture differential pressure increases rapidly at the
initial stage of fracturing. With the continuous growth of fracturing time, the growth trend slows down gradually and even forms a platform. Figure 4 shows the change of pressure in the crack along the crack length. It can be seen that the pressure decreases gradually along the fracture propagation direction. From the curve slope, it can be seen that the low trend is gradually accelerated.

Figure 5A shows the change of the final width of the fracture along the fracture (from the fracture mouth to the fracture end). It can be seen that the maximum width of the fracture is about 6 mm at the fracture mouth and 0 at the fracture end. Figure 5B is a cross-sectional view of the fracture mouth. It can be seen that the fracture width changes with time. At the beginning, the fracture width increases rapidly, and the later fracture width increases slowly.

Figure 6 is a three-dimensional effect diagram during fracturing at 0.5 min and 19.5 min, which can clearly see
the morphological changes of the fracture. During the forward extension of the fracture, the fracture width gradually increases in an elliptical shape, while the fracture width at the end is kept at 0 during the whole fracture extension process.

3.2 Analysis of influencing factors

The basic parameters of influencing factor analysis adopt the values given in the Table 1. When one of the parameters is changed, the other parameters remain unchanged. It is mainly considered from three aspects: injection parameters (injection rate), rock physical properties (Young’s modulus and Poisson’s ratio), and fluid properties (filtration and rheology).

FIGURE 6 Three-dimensional dynamic fracture propagation simulation diagram ($t = 0.5\text{ min}$ and $t = 19.5\text{ min}$)

FIGURE 7 Variation of maximum fracture width along fracture length at different rheological index

FIGURE 8 Variation of differential pressure in fracture with time at different rheological index

FIGURE 9 Variation of maximum fracture width along fracture length at different consistency coefficient
3.2.1 | Rheological parameters

Considering the influence of consistency coefficient and rheological index on fracture propagation, the distribution of fracture width and the variation of differential pressure in the fracture with time are calculated and analyzed when the consistency coefficient \( k = 0.211, 0.5, 0.8 \text{ Pa s}^n \) and rheological index \( n = 0.3, 0.63, 0.9 \). Refer to Table 1 for other parameter settings. The calculation results are shown in Figures 7-10. It can be seen that the rheological parameters have a great impact on fracture propagation.

It can be seen from Figures 7 and 8 that when the power-law fluid takes different rheological indexes, the fracture width and differential pressure in the fracture are greatly different. The larger the rheological index is, the larger the fracture width is at the same time, the easier it is to obtain short width fracture. This is because the greater the rheological index, the greater the apparent viscosity of the fluid, so the greater the viscous dissipation energy of the fluid, the greater the pressure in the fracture, the greater the fracture width. When the rheological index \( n = 1 \), it is Newtonian fluid. When the rheological index \( n = 0.6 \), the fracture width is about 3/5 of Newtonian fluid. Therefore, the property of fracturing fluid has a great impact on fracture propagation, and the appropriate fracturing fluid should be selected according to actual working conditions.

It can be seen from Figures 9 and 10 that when the fluid takes different consistency coefficients, the fracture width and differential pressure in the fracture vary greatly. The greater the consistency coefficient, the greater the fracture width at the same time, the easier it is to obtain short width fracture. The greater the apparent viscosity of the fluid, the greater the fracture width. The cause of action is the same as the rheological index and is consistent with the field application.

3.2.2 | Filtration rate

For the influence of filtration on fracture propagation, the fracture width distribution, fracture length, and differential pressure in the fracture with time are calculated and analyzed when the filtration coefficient \( C = 0.0005, 0.001, 0.003 \text{ m/min}^{-0.5} \). Refer to Table 1 for other parameter settings. The calculation results are shown in Figures 11-13. It can be seen that the filtration also has a great impact on fracture propagation.

It can be seen from Figures 11-13 that when the fluid takes different filtration coefficients, the fracture size and differential pressure in the fracture vary greatly. The smaller the filtration coefficient, the greater the fracture length and width and the greater the differential pressure in the fracture at the same time. This is because the smaller the filtration coefficient is, the less fluid will leak into the formation, so the more fluid will be used for fracture making, resulting in the larger the overall size of the fracture and the greater the pressure in the fracture per unit time. Therefore, the filtration property of fracturing fluid has a great impact on fracture propagation. Low filtration and high-quality fracturing fluid in field application can save time and quantity.

3.2.3 | Rock physical properties

The influence of rock physical properties on fracture propagation mainly considers two parameters: Young’s modulus and Poisson’s ratio. The fracture width distribution, fracture length, and differential pressure in the fracture with time are calculated and analyzed when Young’s modulus \( E = 1.8, 7.0, 15 \text{ GPa} \) and Poisson’s ratio \( v = 0.15, 0.25, 0.35 \). Refer to Table 1 for other parameter settings. The calculation results are shown in Figures 14-17. It can be seen that rock physical properties have a certain impact on fracture propagation.

It can be seen from Figures 14 and 15 that when different Young’s modulus is taken for rock, the fracture width and differential pressure in the fracture vary greatly. The larger the Young’s modulus, the smaller the fracture width, and the easier it is to obtain narrow fractures. This is because the greater the Young’s modulus and the higher the rock hardness, the higher the fracture pressure is required for fracturing. Therefore, the greater the pressure required for fracture expansion in the width direction, and the more difficult it is to expand and widen. Therefore, it is necessary to test rock physical properties before fracturing in fracturing operation.
It can be seen from Figures 16 and 17 that when different Poisson's ratios are taken for rocks, there is little difference in fracture width and differential pressure in fractures, because the variation range of Poisson's ratio is small for formation rocks, and the transverse deformation corresponding to Poisson's ratio has little effect on fracture propagation.

### 3.2.4 Pump injection rate

The influence of injection parameters on crack propagation is mainly considered in this paper. The pump injection rate \( q_0 = Q = 1.0, 2.5, 4.0 \text{ m}^3/\text{mi}, \) fracture width distribution, fracture length, and differential pressure in the fracture with time are calculated and analyzed respectively. Refer to Table 1 for other parameter settings. The calculation results are shown in Figures 18-20. It can be seen that the pump injection rate has a great impact on fracture propagation. Obviously, the larger the pump injection rate is, the longer the fracture length is, and the larger the fracture width under the same fracture length is, the easier it is to form short and wide fracture. The larger the pump injection rate is, the faster the differential pressure in the fracture increases with time.
CONCLUSIONS

In this paper, the influencing factors of fracture propagation of non-Newtonian fluid fracturing fluid are analyzed. Based on PKN model, considering the rheology and filtration of fracturing fluid, good fracture shape simulation results are given, and the effects of fluid properties, rock physical properties, and injection parameters on fracture propagation are analyzed in detail. The main conclusions are as follows:

1. It can be seen from the simulation results that in the process of fracture propagation, the fracture length and width gradually increase, and the corresponding pressure in the fracture also gradually increases. The overall increase trend is that the growth is rapid in the initial stage of fracturing and gradually slows down in the later stage. The three-dimensional expansion diagram of the fracture clearly shows the morphological changes of the fracture.

2. Fluid properties have great influence on fracture propagation, mainly including rheological properties and filtration properties. In terms of rheology, the change of consistency coefficient and rheological index does not affect the overall trend of fracture propagation, but with the increase of value, the viscosity of fracturing fluid increases, and the fracture width and pressure in the fracture show an upward trend. In terms of
filtration, the change of filtration coefficient also does not affect the overall trend of fracture propagation, but with the increase of value, the fracture forming ability decreases, and the fracture width, length, and pressure in the fracture decrease.

3. The physical properties of rock have a certain influence on fracture propagation, mainly including two characteristic parameters: Young’s modulus and Poisson’s ratio. Young’s modulus has a great effect on fracture propagation, while Poisson’s ratio has little effect on fracture propagation. The overall trend of fracture propagation is similar, but the greater the Young’s modulus, the more difficult the fracture is to expand, and the smaller the corresponding fracture width.

4. As a parameter that can be artificially controlled in fracturing process, pump injection rate also has a great impact on fracture propagation. The main performance is that the larger the pump injection rate, the larger the fracture size and the higher the pressure in the fracture.

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NOMENCLATURE
L fracture length
q flow rates
q₀ pump injection rate
h fracture height
C filtration coefficient
τ time to start filtration
n rheological index
K consistency coefficient
G shear modulus
E Young’s modulus
ν Poisson’s ratio
W fracture width
x, y, z position along fracture length, fracture width, and fracture height

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