Quasi two-dimensional antiferromagnet on a triangular lattice RbFe(MoO$_4$)$_2$

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RbFe(MoO$_4$)$_2$ is a rare example of a nearly two-dimensional Heisenberg antiferromagnet on a triangular lattice. Magnetic resonance spectra and magnetization curves reveal that the system has a layered spin structure with six magnetic sublattices. The sublattices within a layer are arranged in a triangular manner with the magnetization vectors 120° apart. The $H - T$ phase diagram, containing at least five different magnetic phases is constructed. In zero field, RbFe(MoO$_4$)$_2$ undergoes a phase transition at $T_N = 3.8$ K into a non-collinear triangular spin structure with all the spins confined in the basal plane. The application of an in-plane magnetic field induces a collinear spin state between the fields $H_{c1} = 47$ kOe and $H_{c2} = 71$ kOe and produces a magnetization plateau at one-third of the saturation moment. Both the ESR and the magnetization measurements also clearly indicate an additional first-order phase transition in a field of 35 kOe. The exact nature of this phase transition is uncertain.

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I. INTRODUCTION

The problem of an antiferromagnet on a triangular planar lattice (AFMT) has been intensively studied theoretically. The ground state in the Heisenberg and XY-models is a “triangular” planar spin structure with the three magnetic sublattices arranged 120° apart. The orientation of the spin plane is not fixed in the exchange approximation.

A non-zero magnetization appears in the presence of a magnetic field due to the canting of the sublattices. Possible field-induced structures are shown on Fig. 1. All configurations with equal magnetization vectors but with different sublattice orientations have the same energy in the molecular field approximation. The umbrella-like structure “a” with the sublattices tilted from the spin plane towards the field, and the planar structures “b” and “b′” are among these degenerate configurations. In this approximation the b-configuration becomes collinear (c-configuration) at a particular field $H_c = H_{sat}/3$, where $H_{sat}$ is the saturation field. In magnetic fields above $H_c$, the structure is again noncollinear, with two parallel sublattices tilted with respect to the third, forming the “canted” d-phase. Finally, at the saturation field $H = H_{sat}$, a spin-flip transition to the phase “f” occurs.

Because of the degeneracy of the classical spin configurations, both quantum and thermal fluctuations play an important role in the formation of the equilibrium state of the AFMT. They result in a free-energy gain of the planar structure with respect to the umbrella-like structure. Due to the contribution of fluctuations to the free energy the more symmetric configuration “b” is preferred to “b′”. Further, the fluctuations stabilize the collinear spin configuration “c” in a range of magnetic fields $H_{c1} \leq H \leq H_{c2}$ around the special point $H = \frac{1}{3}H_{sat}$. Thus, a magnetization plateau should be observed over a relatively wide field range.

In the case of an easy-plane magnetic anisotropy, an analogous evolution of the sublattice orientations is expected when a magnetic field is applied in the easy plane. The umbrella-like structure should be realized for magnetic fields applied along the six-fold axis $C_6$, provided the easy-plane anisotropy is sufficiently strong.

The magnetic resonance spectrum of a 120°-spin structure differs from the spectrum of a usual antiferromagnet both in the number of normal modes and in the dependencies of frequencies on the applied magnetic field. There are three eigenmodes for a 2D triangular structure. For the case of an easy-plane anisotropy, two of them are degenerate in zero field, but have nonzero frequency; the third mode has zero-frequency in zero field. In a magnetic field there are three different resonance frequencies of uniform spin precession.

The presence of the interplanar antiferromagnetic exchange requires one to consider at least a six-sublattice 3D magnetic structures. For a weak interplanar exchange however, the main features of the 2D-system should remain unchanged. The interpbrane exchange may cause additional field-induced phase transitions, where the mutual orientation of spins in neighboring planes changes. Possible magnetic structures of the 3D XY AFMT in a magnetic field were analyzed in Ref., while the Heisenberg AFMT was considered in Ref. A two-fold period along the $C_6$ (or $C_3$) axis is assumed here. The structures under consideration are shown schematically in Fig. 1 and denoted as B1, B2, B3, C, D1 and D2. For the
FIG. 1: Schematic representation of the proposed spin structures of a Heisenberg antiferromagnet on a triangular lattice. Structures a, b, b', c, d and f are related to the 2D system \( J' = 0 \). Structures B0, B1, B2, B3, C and D represent the triangular antiferromagnet with a weak antiferromagnetic interlayer exchange in the six-sublattice model. Solid and dashed arrows with the same numbers correspond to magnetic moments of neighboring spins from neighboring layers.

As far as the magnetic resonance spectrum is concerned, the interplanar exchange should result in a splitting of the main eigenfrequencies and in the appearance of new modes due to the increased number of sublattices.

FIG. 2: Temperature dependence of the magnetic susceptibility of RbFe(MoO\(_4\))\(_2\) for two directions of the magnetic field.

FIG. 1: Temperature dependence of the magnetic susceptibility of RbFe(MoO\(_4\))\(_2\) for two directions of the magnetic field.

In the present paper, we describe a study of the magnetic and resonant properties of RbFe(MoO\(_4\))\(_2\), a material which can be considered as a rare example of a nearly two-dimensional Heisenberg antiferromagnet on a triangular lattice and that can be prepared in single-crystal form. At room temperature the crystal structure of RbFe(MoO\(_4\))\(_2\) has the space group \( P\bar{3}m1 \). The magnetic Fe\(^{3+} \) ions with spin \( S = \frac{5}{2} \) are placed on the hexagonal lattice with lattice parameters \( a = 5.69 \) Å and \( c = 7.48 \) Å. The MoO\(_4\)-tetrahedra are placed between the layers of Fe\(^{3+} \)-ions and form the structure with the three-fold axis. The exchange integral \( J \) representing the interaction within the planes should be much larger than the exchange integral, \( J' \), of the nearest neighbor ions in adjacent planes. The large difference in these exchange integrals is due to the different exchange paths of the indirect exchange interactions: via two oxygen ions within the planes and via three or even more oxygen ions be-
The structure of RbFe(MoO$_4$)$_2$ may be considered (see, e.g., Ref. 14) as an ensemble of layers with a triangular lattice occupied by $S = 5/2$ Fe$^{3+}$ ions, and these magnetic layers are separated by layers of MoO$_4$-Rb-MoO$_4$. Magnetic ions in neighboring layers are placed one above another.

Evidence for a structural transformation at $T=180$ K was recently reported in Ref. 15. Small changes of lattice constants, Raman spectra and ESR linewidth indicate a structural transformation, identified as a second-order phase transition from a highly symmetric $P3m1$ room-temperature structure into a very similar but less symmetric $P3c1$ low-temperature structure. This transformation corresponds to rotations of the MoO$_4$ tetrahedra. The "triangular" spin structure of Fe$^{3+}$-layers in crystals of RbFe(MoO$_4$)$_2$ was recently confirmed by the elastic neutron scattering experiment in zero magnetic field. At the same time, according to the observed neutron diffraction, an incommensurate modulation of the ordered spin structure in the C$^3$-direction is present there: the mutual orientation of spins from neighboring planes is close to the antiparallel one but is slightly tilted at an angle of $17^\circ$. The low-temperature magnetization curves for powder samples of RbFe(MoO$_4$)$_2$ were reported earlier. The magnetization saturated at a field of $H_{sat} = 186$ kOe and a magnetization plateau marking the collinear phase was observed.

Neutron scattering experiments of Ref. 17 confirming the triangular magnetic structure were performed for powder samples of the related compounds CsFe(SO$_4$)$_2$ and RbFe(SO$_4$)$_2$.

We have verified experimentally the theoretical concepts outlined above by taking advantage of single-crystal samples of RbFe(MoO$_4$)$_2$. The choice of a molybdate instead of a sulphate allowed us to avoid the hydration and to obtain single crystals. Field-induced phase transitions and low frequency spin dynamics in the different phases were studied by means of magnetization measurements and ESR spectroscopy. In the present paper we describe several field-induced phase transitions including transitions not detected earlier. Each phase is found to possess a characteristic set of spin-resonance modes.

**II. SAMPLES AND EXPERIMENTAL TECHNIQUES**

Single-crystal samples of RbFe(MoO$_4$)$_2$ were synthesized by means of the spontaneous crystallization from a flux melt. The mixture of RbFe(MoO$_4$)$_2$ and of K$_2$Mo$_2$O$_7$ in the molar ratio of 1:2 was heated in a platinum crucible up to a temperature of 1300 K and held at this temperature for 12 hours then cooled to 1000 K at a rate of 3 K/h. The nucleation of crystals was localized due to the temperature gradient near a platinum rod which was inserted into the melt in the precristallization state. The platinum rod was withdrawn from the solution after the crystallization. The K$_2$Mo$_2$O$_7$ flux was removed by dissolving in water. A much slower dilution of the crystals of RbFe(MoO$_4$)$_2$ takes place at the same time. The crystals have the shape of thin hexagonal plates with the size of 3-4 mm along each edge. The lattice parameters are in accordance with those reported for powder samples. The magnetization curves and the temperature dependencies of the magnetic susceptibility were measured using a vibrating sample magnetometer with the field range 0 - 120 kOe. Magnetic resonance spectra were taken by a set of transmission-type magnetic resonance spectrometers with resonators covering the range 9-120 GHz.
III. EXPERIMENTAL RESULTS

A. Susceptibility and magnetization curves

The temperature dependence of the susceptibility of RbFe(MoO$_4$)$_2$ at low temperatures is shown in Fig. 2 and clearly demonstrates the transition into the magnetically ordered state at $T_N = 3.8$ K. The susceptibility measurements in the whole range 10-300 K reveal the temperature dependence of the Curie-Weiss type $(T + \Theta_c)^{-1}$ with the value of the Weiss constant $\Theta_c = 22 \pm 2$ K. A small (about 0.1%) step-like anomaly in the reverse susceptibility appears at 180 K giving an additional indication for the structural transformation mentioned in the Introduction. There is a significant deviation from the Curie-Weiss behavior below 10 K. A strong anisotropy of the susceptibility also appears in this temperature range, well above $T_N$.

Below the transition point $T_N = 3.8$ K, the magnetization curves $M(H)$ are quite different for different orientations of the magnetic field. For $H \parallel C^3$, the magnetization increases linearly with the field for all values of the applied field (see Fig. 3). For $H \perp C^3$, the field dependence of the magnetization is much more complicated, as shown in Fig. 4 for $T = 1.6$ K. There are abrupt changes in the slope of the magnetization curve at 47 kOe and 71 kOe, with the differential magnetic susceptibility being significantly reduced in the region between these fields.

An additional, first-order phase transformation with a hysteresis in $M(H)$ curve was detected at $H_{c3} = 35$ kOe. Magnetization measured between 30 and 38 kOe shows a large difference in signal obtained for the rising and falling magnetic field. A smaller, but still clearly observable difference exists down to a field of 20 kOe.

In order to detect the transitions by changing temperature we measured the temperature dependence of the magnetic moment at constant magnetic field. Several examples of these data are shown in Fig. 5. The temperatures of the abrupt changes of the magnetization curves are marked by arrows and indicate the field dependence of the Néel temperature and the temperature dependence of the fields $H_{c1}$ and $H_{c2}$. The values of critical magnetic fields and temperatures derived from the curves like those shown in Figs. 4 and 5 are collected on the $H - T$ phase diagram in Fig. 6. There are at least 4 ordered antiferromagnetic phases P1, P2, P3, P4 and a paramagnetic phase PM.

It should be noted that smeared changes in the magnetization slope were observed at the critical fields $H_{c1}$ and $H_{c2}$ for temperatures slightly above the Néel temperature. These regions are marked in Fig. 6 by shadowed ovals.

B. Antiferromagnetic resonance

On cooling the samples, the ESR line broadens markedly below 10 K. After passing through the Néel point, the ESR line shifts from the paramagnetic reso-
The field-dependencies of the microwave transmission at $T = 1.3$ K are shown in Fig. 8 and Fig. 9 for two orientations of the magnetic field. We can derive the field dependencies of the spin-resonance modes in the ordered state from the positions of the resonance-absorption lines at different frequencies. As shown in Fig. 9, the microwave absorption data are sensitive to all the phase transitions detected in the magnetization measurements. The hysteresis loop around the field $H_{c3}$ is clearly evident in the microwave absorption. The change of the microwave absorption at this transition is frequency-dependent. The curves taken at a frequency around 25 GHz are the most sensitive to this transition as they demonstrate the largest hysteresis around $H_{c3}$.

For $H \parallel C^3$, two branches of the resonance are clearly seen. The frequency of the first branch rises with the field, while the frequency of the second branch decreases (see Figs. 8 and 10). The frequencies of these two branches are monotonic functions of the applied magnetic field as expected for the umbrella-like structure.

For $H \perp C^3$, a complicated nonmonotonic dependence of the resonant frequencies $\nu_i$ with varying magnetic field was observed (see Fig. 11). The frequency-field curves contain abrupt changes at the fields of the phase transitions in accordance with the data of the magnetization measurements. The values of the phase-transition fields derived from the microwave-absorption curves are marked on the phase diagram in Fig. 6 by filled squares and are in a good agreement with the results of static magnetization measurements. The total number of the observed spin resonance modes is five.

IV. DISCUSSION

A. Basic principles

The observation of the sequence of the phase transitions for the magnetic field lying in the basal plane implies that there is an easy-plane type of anisotropy (an easy-axis anisotropy would result in the umbrella-like spin structure at $H \perp C^3$ without the cascade of field-induced phase transitions).

At the moment there is no explanation for the nature of the incommensurate modulation observed in Ref. 16, but on the assumption of the small deviation from the antiparallel orientation of spins in neighboring planes, we shall describe the field-dependent spin structure in the approximation of the perfect antiferromagnetic orientation of spins in neighboring layers in the zero-field ground state.

Thus, we consider the following model spin-Hamiltonian, following the notation of Ref. 3.

$$\mathcal{H} = 2J \sum_{(ij),n} S_{in} S_{jn} + 2J' \sum_{in} S_{in} S_{in+1} + D \sum_{in} (S_{in}^z)^2 - g\mu_B H \sum_{in} S_{in}. \tag{1}$$

Here the sums are taken within the layers $(i,j)$ and along the transverse direction $(n)$, and $D > 0$ is the constant of the anisotropy of the easy-plane type.
phase diagram one should remember that our situation of the easy-plane anisotropy should be considered neglecting the umbrella-like phase derived for the isotropic approximation. For our case of $J'/S/J = 0.095$ with an easy plane anisotropy this phase diagram predicts a sequence of phase transitions B1-B2-C-D1-D2 in the magnetic field. Thus, according to the theoretical analysis of Ref. 8, we can propose that in the field $H_{c3}$ we have transition like B1-B2 with change in the mutual orientation of spins in neighboring layers, then transition B2-C at $H = H_{c1}$ and C-D1 at $H = H_{c2}$. Thus we propose the observed sequence P1-P2-P3-P4 may be treated as B1-B2-C-D1.

It should be noticed that the phase boundary observed in the temperature range 3.4-4.2 K and in fields above $H_{c2}$ cannot be smoothly extrapolated to the saturation field 186 kOe at $T=1.3$ K. Therefore, the existence of yet another phase transition at higher magnetic field cannot be ruled out. According to the analysis given in Ref. 8, the phase D2 is energetically favourable for certain values of the interplanar exchange in a field just below the saturation field. Thus we suggest the region of the phase diagram indicated in Fig. 6 as P5 may be treated as the D2 phase.

Using only the results of magnetization and ESR measurements we cannot distinguish between the phases B1, B2, B3 and between the D1 and D2 phases. Thus, an alternative sequence of phase transitions, such as B2-B3-C-D1 (Ref. 18) should, in principle, also be considered as an explanation of the observed sequence P1-P2-P3-P4. A scenario for the field-induced transitions with B2 as the starting phase is also suggested on the basis of the molecular field approximation in Ref. 8. The tiny differences between the free energies of the phases B1 and B2 may be associated with the contribution of thermal and quantum fluctuations which should be taken into account along with the anisotropy. In any case, the exact solution of this problem will only have a limited significance for RbFe(MoO$_4$)$_2$ because of the incommensurate modulation mentioned, which is still not included in the theoretical models.

Nevertheless, the observation of the $H_{c3}$-phase transition clearly marks the effect of the interplanar exchange on the field-dependent phases of the AFMT with weakly coupled layers.

The observed boundaries in the collinear phase range may be compared to the calculations performed using an isotropic ($D = 0$) and XY (infinite $D$) models at $T = 0$ K. The calculated zero-temperature field-ranges of the C-phase (using $H_E = 67$ kOe) are shown in the phase diagram in Fig. 6 by the line segments. There is a qualitative agreement with our observations.

### C. Spin-resonance modes

For a triangular system with antiferromagnetic interplane exchange interactions, there should be five reso-
mance modes with non-zero frequencies (the frequency of the sixth mode is zero in small fields in absence of any in-plane anisotropy). We have observed all five resonances for $H \perp C^3$. For weak magnetic fields, where the exchange triangular spin configuration is only slightly distorted, the resonance frequencies may be calculated following the macroscopic theory based on a classical Lagrange formalism. This method of calculation is suitable for complicated and multi-sublattice systems, however it is valid only in the field range where the exchange magnetic structure is not strongly distorted by the magnetic field. The basic principles of the calculations and the resulting formulae are given in Appendix A, while the field-dependencies of the resonant frequencies may be calculated after (11, 12), parameters are described in the text.

The five spin resonance branches shown in the low-field range in Fig. 11 and the resonance frequencies for $H \parallel C^3$ (Fig. 10) may be reasonably described by the equations (8-12) with only two fitting parameters: $a=90.8$ GHz and $c=32$ GHz. The dashed curves in Fig. 11 and curves in Fig. 10 are calculated in this way with the anisotropy parameter of the susceptibility, $\eta = 0.05$, derived from the susceptibility measurements.

Using the value of the susceptibility described above and the molecular-field relations (2, 3) we can evaluate the molecular fields and parameters of the Hamiltonian: $H_E = 67$ kOe, $H_A = 5.2$ kOe, $J'/J = 0.039$. These values of $H_E$ and $H_A$ are in reasonable agreement with the values derived from the susceptibility and saturation field.

For $H \perp C^3$, the triangular spin structure of RbFe(MoO$_4$)$_2$ is already strongly distorted in a field of about 10 kOe. Therefore the macroscopic theory of magnetic dynamics given in Ref. [3] cannot be used. To have at least an approximate description of the resonant frequencies over wider range of applied fields, we used the calculations for the three-sublattice 2D model (i.e. for...
The lowest resonance frequencies are given in Appendix B, and the results of these calculations (relations 13, 18) are presented in Fig. 11 by solid lines. The values of \( D \) and \( J \) used to calculate these curves are taken from the values of \( H_E \) and \( H_A \) given above and thus agree with the parameters used for fitting the data in low fields according to relations (8-12).

For qualitative description of the resonance frequencies within the field range of the collinear phase \( H_{c1} < H < H_{c2} \), one should take into account the zero-point fluctuations stabilizing this phase in that field range. The appropriate calculation was made in the \( J' = 0 \) approximation with the assumptions of the XY-model in the same paper of Chubukov and Golosov).

V. CONCLUSION

A sequence of field-induced phase transitions, governed by the intraplane exchange interaction and by the weak interplane exchange was found in RbFe(MoO\(_4\))\(_2\), a quasi-2D antiferromagnet on a triangular lattice. The magnetic properties may, in part, be explained in terms of a 2D antiferromagnet on a triangular lattice. However, a phase transition governed by weak interplane exchange was found. This phase transition is accompanied by the changes in spin-resonance spectra. A self-consistent description of the magnetization curves, phase transitions and resonance modes in a wide field range is given.

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VI. APPENDIX A. SPIN RESONANCE MODES IN THE MACROSCOPIC APPROACH

We consider 120°-triangular structures in planes 1 and 2, which can be represented by normalized spin densities

\[
\begin{align*}
\text{S}_1(\mathbf{r}) &= l_{11} \cos q_1 r + l_{12} \sin q_1 r, \\
\text{S}_2(\mathbf{r}) &= l_{21} \cos q_2 r + l_{22} \sin q_2 r.
\end{align*}
\]

Here \( l_{11}, l_{12} \) and \( l_{21}, l_{22} \) are two pairs of orthogonal unit vectors of antiferromagnetism, \( q_x = 4\pi/3a, q_y = 0 \) - crystal period within the plane; vector \( \mathbf{r} \) takes discrete values within the basal plane pointing at the magnetic sites on a triangular lattice. This structure is due to the weak exchange interaction of spin triangles in neighboring planes for weak interplane exchange corresponds to the minimum of the Heisenberg exchange energy of nearest neighbors from different planes:

\[
l_1(111l_{21} + l_{12}l_{22})
\]

For \( \alpha > 0 \): \( l_{11} = -l_{21}, l_{12} = -l_{22} \).

For \( \alpha < 0 \): \( l_{11} = l_{21}, l_{12} = l_{22} \).

Note, that the low-frequency spectrum, which depends on the quadratic expansion on the angles of the mutual rotation of the spin triangles, is the same for the antiferromagnetic (\( \alpha > 0 \)) and ferromagnetic (\( \alpha < 0 \)) exchange.

The Lagrange function of the spin dynamics has three contributions: two “ordinary” terms of the triangular structures of the two systems of spin planes \((l_{11}, l_{12})\) and \((l_{21}, l_{22})\):

\[
\frac{1}{4} \left\{ \frac{X^2}{\gamma^2} (\Omega_1 + \gamma \mathbf{H})^2 + \frac{Y^2}{\gamma^2} (\Omega_1 + \gamma \mathbf{H}, n_1)^2 + \beta n_{1z}^2 \right\}
\]

and \((l_{21}, l_{22})\):

\[
\frac{1}{4} \left\{ \frac{X^2}{\gamma^2} (\Omega_2 + \gamma \mathbf{H})^2 + \frac{Y^2}{\gamma^2} (\Omega_2 + \gamma \mathbf{H}, n_2)^2 + \beta n_{2z}^2 \right\}
\]

while the third term, the energy of the interplane exchange, is given above. Here \( n_1 = [l_{11}l_{22}], n_2 = [l_{21}l_{22}] \) are unit vectors in the spin space, which are normal to the spin planes, \( \beta \) is the constant of uniaxial anisotropy, \( \Omega_1 \) and \( \Omega_2 \) are angular velocities of rotation of the spin triangles. The values \( \chi_\parallel \) and \( \chi_\perp \) determine the components of the tensor of the magnetic susceptibility in the ground state, along and perpendicular to the vector \( n_1 \) respectively.

For the in-plane field, the frequencies of the in-phase oscillations of spins in neighboring planes are given by:

\[
\begin{align*}
\nu_1 &= a, \\
\nu_2 &= \sqrt{a^2 + H^2}, \\
\nu_3 &= 0
\end{align*}
\]

The additional frequencies due to the interplane exchange (spins of different planes oscillate "out of phase") are

\[
\begin{align*}
\nu_4 &= b = \sqrt{a^2 + A^2}, \\
2\nu_5, 6 &= b^2 + c^2 + H^2 \pm \sqrt{(b^2 + c^2 + H^2)^2 - 4c^2(b^2 - \eta H^2)}
\end{align*}
\]

Here \( A^2 = \gamma^2 a/\chi_\perp, a^2 = \gamma^2 \beta/\chi_\perp, \eta = \chi_\perp/\chi_\parallel \), \( c^2 = 2A^2/(1 + \eta) \). Parameters \( a \), \( b \) and \( c \) are coupled by the relation

\[
(2b^2 - a^2) = (1 + \eta)c^2
\]
For the magnetic field oriented parallel to the $C_3$-axis, ordinary resonances (in-phase motion of triangles) are:

$$\nu_{1,2} = \sqrt{a^2 + \left(\frac{1+\eta}{2}\gamma H\right)^2 \pm \frac{1-\eta}{2}\gamma H},$$
$$\nu_3 = 0,$$  \hspace{1cm} \text{(11)}

while the additional frequencies (the triangles rotate in opposite directions) are:

$$\nu_{4,5} = \sqrt{b^2 + \left(\frac{1+\eta}{2}\gamma H\right)^2 \pm \frac{1-\eta}{2}\gamma H},$$
$$\nu_6 = c$$ \hspace{1cm} \text{(12)}

VII. APPENDIX B. SPIN-RESONANCE MODES IN 2D MODEL

1. Molecular field approximation

Introducing the normalized field

$$h = \frac{2\mu_0 H}{6JS} = \frac{3H}{H_{\text{sat}}}$$

we have the following resonance frequencies, for the magnetic field lying in the easy plane of 2D-AFMT:

1) below the transition to the collinear phase ($0 < h < 1$)

$$\nu_1 = \frac{6JS}{2\pi h} \left[ \frac{D}{6J} (3 - 2h - h^2) \right]^{\frac{1}{2}}$$ \hspace{1cm} \text{(13)}

2) above the transition to the canted phase ($1 < h < 3$)

$$\nu_1 = 0$$ \hspace{1cm} \text{(16)}

$$\nu_2 = \frac{6JS}{2\pi h} \left[ \frac{D}{6J} \frac{(h^6 - 3h^4 + 35h^2 + 63)}{16h^2} \right]^{\frac{1}{2}}$$ \hspace{1cm} \text{(17)}

$$\nu_3 = \frac{6JS}{2\pi h} \left[ \frac{D}{6J} \frac{(9 - h^2)(h^2 - 1)(h^2 + 7)}{16h^2} \right]^{\frac{1}{2}}$$ \hspace{1cm} \text{(18)}

2. XY-model including fluctuations

The two lowest resonant frequencies in the collinear phase are

$$\nu_1 = \frac{6JS}{2\pi h} \left( h - h_{c1} \right)^{\frac{1}{2}},$$ \hspace{1cm} \text{(19)}

$$\nu_3 = \frac{6JS}{2\pi h} \left( \frac{h_{c2} - h}{3} \right)^{\frac{1}{2}}.$$ \hspace{1cm} \text{(20)}

Here $h_{c1}$ and $h_{c2}$ are the normalized values of the critical fields $H_{c1}$ and $H_{c2}$.

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