Students’ reasoning in analyzing temperature from $P-V$ diagram representing unfamiliar thermodynamics process

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Abstract. Thermodynamic processes discussed in textbooks are typically presented in the $P-V$ diagram and limited to familiar processes such as isobaric, isochoric, isothermal, and adiabatic. The purpose of this qualitative research was to explore students’ reasoning in analyzing temperature from given $P-V$ diagram representing thermodynamics processes that do not belong to one of those familiar processes. We interviewed 6 students as the volunteers. The interview was guided by two $P-V$ diagrams. First, students dealt with a process that was similar to but different from isothermal or adiabatic. Second, students dealt with a process which the pressure decreased linearly with volume. We also asked the students to convert the $P-V$ diagram into the $T-V$ diagram. We found that the students tended to use the surface features of the diagrams, instead of applying $PV=nRT$ appropriately. For example, they tended to think of a curved $P-V$ diagram as an isothermal or adiabatic process, and in interpreting a linear $P-V$ diagram they tended to think temperature must be constant if the pressure decreases linearly with volume. The students also failed to convert the $P-V$ diagram to the $T-V$ diagram due to the lack of idea and mathematical skill to solve the problem.

1. Introduction
A graph is a powerful tool to interpret data and represent the relationships between variables [1-7] in various disciplines such as social sciences (psychology, economics, and sociology) and natural sciences (physics, biology, and chemistry). Therefore, the ability to use a graph is an important goal in physics education.

The ability to use a graph is an important key in learning physic. Meltzer [8] and Susac, et al. [9] reported that students who are able to construct graphs could solve problems more easily than other students. However, previous studies have found that most students had difficulties in constructing and interpreting graph [2-7, 10-12].

Thermodynamic processes that discussed in physics textbooks [13-14] are typically presented in the $P-V$ diagram and limited to specific processes such as isobaric, isochoric, isothermal, and adiabatic. The diagrams are typically used to analyze process variables such as heat and work, not to represent the relationship among state variables such as $P$, $V$, and $T$. In addition, several previous research had already made simulations that help the students to understand and recognize $P-V$ diagrams on familiar processes [15-16]. Nonetheless, the students still have difficulties in constructing $P-V$ diagrams on familiar processes based on the following instruction [10, 17].

Thermodynamic processes are not limited to four familiar processes, but there are also other processes called unfamiliar processes. To enhance the students’ understanding of the $P-V$ diagrams, it is important to know whether the students comprehend the unfamiliar process in the $P-V$ diagram.
Some previous studies reported on students’ understanding of unfamiliar processes at P-V diagrams. However, the research had not revealed the students’ interpretation of the unfamiliar processes. More studies used unfamiliar processes to measure students’ understanding about work concept ($\Delta U = Q - W$) in the $P-V$ diagrams [8, 18-20], to calculate internal energy of the system based on the processes occurring in the $P-V$ diagram [8, 21], and to construct $P-V$ diagram based on following instruction [10].

Therefore, this research aimed to describe students’ reasoning in analyzing the temperature represented by $P-V$ diagram on the unfamiliar process. This study explored how students determined the temperature in unfamiliar processes through $P-V$ diagrams and how students constructed a process of a $P-V$ diagram into a $T-V$ diagram.

2. Method
The research was done to explore students’ reasoning in analyzing temperature from given $P-V$ diagram that did not belong to one of those familiar processes. We interviewed 6 graduate physics education students at Universitas Negeri Malang with initial names H, I, K, S, Y and Z as volunteering respondents. All of them were interested in thermodynamics. They were also familiar with the $P-V$ diagram and thermodynamic processes.

There were two types of problems presented in the $P-V$ diagram (figure 1 & 2). First, we assigned the students to complete the written tests, and then they were interviewed individually in order to deeply understand their reasoning. All problems required the students to determine the temperature based on the $P-V$ diagrams. The first type dealt with a process that was similar to but different from isothermal or adiabatic (figure 1). This problem was used to check whether students were affected by isothermal or adiabatic processes. The second type dealt with a process in which the pressures of gas decrease linearly with its volume (figure 2). In addition to interpreting the given $P-V$ diagram, the second problem also asked the students to convert the $P-V$ diagram into the $T-V$ diagram.

In both the first and second type problems, students could compare temperatures in every state by using $PV = nRT$. By doing so, they would know the process if they compared the temperature in each process. Students could transform the $P-V$ diagram to the $T-V$ diagram if they could interpret diagram into the $P(V)$ equation and then transform it into the $T(V)$ equation which is a quadratic equation. Therefore, all problems could be solved by using the equation of state for an ideal gas ($PV = nRT$).

Consider the $P-V$ diagram of ideal gas below.

Based on the diagram, which of the following is true about the temperature of the gas?

A. $T_A = \frac{1}{3}T_B$
B. $T_A = T_B$
C. $T_A = 3T_B$

Reason:__________________________________________________________________________

Figure 1. The first-type problem comparing $T_A$ and $T_B$. 
Consider the $P$-$V$ diagram of ideal gas below.

1. Comparison of $T_A: T_B: T_C: T_D$ is ...
   A. $3 : 8 : 7 : 3$
   B. $1 : 1 : 1 : 1$
   C. $9 : 6 : 3 : 1$
   D. $1 : 4 : 7 : 9$
   Reason: ________________________________

2. The process that occurs in the diagram is a ... process.
   A. isobaric
   B. isothermal
   C. isochoric
   D. adiabatic
   E. besides isobaric, isothermal, isochoric and adiabatic process
   Reason: ________________________________

3. The $T$-$V$ diagram to be formed is ...
   A. $T'(\text{K})$
   B. $T'(\text{K})$
   C. $T'(\text{K})$
   Reason: ________________________________

**Figure 2.** The second-type problems consisting of three items with different representations.

### 3. Results and Discussion

In order to solve the problems, students tended to use the surface features of the $P$-$V$ diagrams. They did not used $PV=nRT$ to retrieve the information from the $P$-$V$ diagram. Students’ reasoning about the first and second type problems will be explained as follows.

#### 3.1 The first-type problems

In the first-type problem, each answer choice was chosen by two students. Student K and S chose, the answer A which was the correct answer, while the other students chose the wrong answer. Students H and I chose the answer B. Meanwhile, students Y and Z chose the answer C.

Student S did not say that the process was represented by the diagram, while student K said that the diagram represented an adiabatic process. Student K knew the temperature at state B was greater than
the temperature at state A. However, she considered the curve in the $P-V$ diagram as an adiabatic process due to the form of the diagram. Her reasoning was correct but she did not consider that an unfamiliar process can have a curve form.

Student K: *The process on a $P-V$ graph that has curved form is neither an isothermal process nor an adiabatic process. Since the temperature at state B is greater than state A, it is not isothermal process*

On the other hand, Student H and I stated explicitly that the process of AB is an isothermal process. Both of them tended to use the surface features of the diagram. They used their previous the knowledge about the form of the isothermal process in the $P-V$ diagram, instead of applying the state equation of ideal gas $PV=nRT$ to comparing the temperature in state A and B.

Student H: *Adiabatic process has a steeper form than isothermal process’ curve. So I think the graph is an isothermal graph, then $T_A=T_B$*

Student I: *The form is curved. The Isothermal process has a curved graph*

Another student, Student Z, thought that $T_A > T_B$. He thought that if the volume was small and the pressure was large then the temperature would increase. Meanwhile, if the volume was large and the pressure was small then the temperature would decrease. The analogy given by the student was an analogy related to the compression process. If the gas was placed in a small container then the pressure was enlarged by compressing the gas, then the gas temperature would increase. Whereas if the gas was placed in a large container and then compressed, the temperature of the gas would be smaller than the temperature of the gas placed in a small container.

Student Z: *State A has greater pressure and smaller volume, and then the temperature will increase. While state B has smaller pressure and larger volume, then the temperature will decrease.*

Option C is the only option that states $T_A > T_B$.

3.2 The second-type problems

In the second type of problem, there are 3 problems. In the first problem, five students (student H, I, S, Y and Z) chose the correct answer A while student K chose answer B. In the second problem, student Z was the only student who answered correctly (answer E). The other student thought that the diagram represented an isothermal process (answer B). In the third problem, student Z chose the correct answer C, while the other student chose the wrong answer.

Student H, I, S and Y were not consistent in understanding the diagram. They already used $PV=nRT$ to find the $T_A$, $T_B$, $T_C$, and $T_D$ (figure 3a). So, they knew that $T_A$, $T_B$, $T_C$, and $T_D$ had a different temperature. When the researcher asked about what processes occurred in the $P-V$ diagram, they thought it was an isothermal process because they only used a comparison of $T_A$ and $T_D$ (figure 3b). They also thought when pressure decrease linearly with volume, then the temperature must be constant. It was because they only thought about the familiar processes (isothermal, isobaric, isochoric and adiabatic) and they did not think about the existence of the unfamiliar processes.

Student H: *I assume this is an isothermal diagram because the pressure decreases linearly with the volume*

Student I: *It is an isothermal diagram because the form of the diagram is similar with the form of isothermal one and $T_A = T_D$*

Student S: *I only used the temperature comparison of state A and state D*

When we asked them to convert the $P-V$ diagram into the $T-V$ diagram, they converted it based on the characteristic of the isothermal process.

Student H: *Because it is an isothermal process then the temperature will be constant. Option B represented that the temperature is constant.*

Student Y: *I just use $T_A$ and $T_D$. So, I choose T-V diagram that represents the isothermal process*

On the other hand, student K considered the second-type problem diagram as an adiabatic process (figure 4). Although, in the last calculation she assumed that the process was an isothermal process. She underestimated the equation of state for ideal gas. She did not believe if the problem could be solved merely using $PV=nRT$. 

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Student K: *I do not know why. But I think it can’t solve by ideal gas law \((PV=nRT)\). So I use the Adiabatic process equation*

![Image](image_url)

**Figure 3.** The student S’ answer of the second-type problems, (a) the answer to the first problem (b) the answer to the second problem

However, at the end of the calculation, student K only used the comparison of temperature in state A and state D. That’s because she could not calculate gamma \((\gamma)\) quantity through the comparison of state AB, even though her calculation was correct. Then she only used a comparison of state A and D. After that, she interpreted gamma in state B and C equal to gamma in state A and D. Therefore, she concluded that all temperatures in four states were equal (isothermal process). When the researcher asked her to convert the process from \(P-V\) diagram into the \(T-V\) diagram, she was consistent in interpreting the AD process as an isothermal process.

![Image](image_url)

**Figure 4.** A student reasoning in solving the second-type problem

At the end of the interview, the researcher asked whether all students knew the problems were about the unfamiliar process. They agreed that the process was the unfamiliar one. However, when the researcher asked about the process in the \(P-V\) diagrams, they only thought about the diagram in familiar processes such as isothermal, isobaric, isochoric and adiabatic processes.

**Researcher:** *Is there any processes besides isothermal, isobaric, isochoric and adiabatic process?*

**Student K:** *Hmm ... I think there are adiabatic processes because it has a different temperature in one state compared to the other state.*

**Student I:** *I think there are isothermal processes*
**Researcher:** Do you think they belong to an isothermal or an adiabatic diagram?

**Student I:** I don’t think so. I think both diagrams are an unfamiliar process.

**Student H:** Yeah, I think both diagrams belong to the unfamiliar process. Early, I only think about the isothermal process because the problems that used the P-V diagrams always discuss about the isothermal, isochoric, isobaric and adiabatic processes.

The results of this study indicated that the students’ reasoning was mainly caused by students’ lacking of unfamiliar processes experience and the tendency to apply familiar processes to the unfamiliar one. They also tended to avoid $PV=nRT$ as the primary resource for interpreting the P-V diagrams. Thus, it was suggested that teachers need to use some strategies to improve students’ graphic skill such as training students to solve problems to retrieve information from a diagram. The use of media can help improve students' understanding of the P-V diagram [15-16]. In addition, students must become accustomed to construct a graph based on the given instruction [21]. If students have mastered the P-V diagrams, it will be a great help in learning another concept [8].

4. Conclusion
Most students had difficulties in analyzing the unfamiliar processes in the P-V diagram. In the curve-shaped context, the students tended to think of the curve as either isothermal or adiabatic process. Students who interpreted the process as isothermal did not apply the state equation $PV=nRT$ to justify whether the temperature was indeed constant during the process. Students who thought of the process as adiabatic used the state equation to calculate the temperature and found that the temperature converted from one state to another. Therefore, the student concluded that the process was adiabatic, instead of isothermal. In case of a linear P-V diagram, the students also claimed that the process was either isothermal or adiabatic. They tended to use a qualitative reasoning such as if the pressure decreases linearly with volume then the temperature must be constant. They always could not recognize unfamiliar processes automatically because they always thought about the familiar one.

In order to interpret the process on the P-V diagram, students needed to use $PV=nRT$ on every presented process. Students also need more experiences to analyze the unfamiliar P-V diagrams.

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