Three-Dimensional Dynamic Modeling and Motion Analysis for an Active-Tail-Actuated Robotic Fish with Barycentre Regulating Mechanism

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Abstract—Dynamic modeling has been capturing attention for its fundamentality in precise locomotion analyses and control of underwater robots. However, the existing researches have mainly focused on investigating two-dimensional motion of underwater robots, and little attention has been paid to three-dimensional dynamic modeling, which is just what we focus on. In this article, a three-dimensional dynamic model of an active-tail-actuated robotic fish with a barycentre regulating mechanism is built by combining Newton’s second law for linear motion and Euler’s equation for angular motion. The model parameters are determined by three-dimensional computer-aided design (CAD) software SolidWorks, HyperFlow-based computational fluid dynamics (CFD) simulation, and grey-box model estimation method. Both kinematic experiments with a prototype and numerical simulations are applied to validate the accuracy of the dynamic model mutually. Based on the dynamic model, multiple three-dimensional motions, including rectilinear motion, turning motion, gliding motion, and spiral motion, are analyzed. The experimental and simulation results demonstrate the effectiveness of the proposed model in evaluating the trajectory, attitude, and motion parameters, including the velocity, turning radius, angular velocity, etc., of the robotic fish.

Index Terms—Three-dimensional dynamic modeling, Newton-Euler method, computational fluid dynamics (CFD), grey-box model estimation, robotic fish.

I. INTRODUCTION

In recent years, underwater robots including varieties of underwater remotely operated vehicles (ROV), autonomous underwater vehicles (AUV), and bio-inspired aquatic systems have been developed and shown great potentials in promoting marine resource exploitation [1], [2], marine economy development [3], [4], and marine ecological environment protection [5], [6]. The research topics of underwater robots cover locomotion control and optimization [6], [7], underwater navigation and localization [8], [9], environment perception and object recognition [10], [11], underwater communication [12], [13], etc. In particular, mechanism investigation of dynamic performance of underwater robots is fundamental and critical for the above-mentioned researches. Besides, precise dynamic modeling of underwater robots has always been focus and difficulty in underwater robot research.

For dynamic modeling, the typical modeling methods include Lagrangian dynamics method, Newton-Euler method, Lighthill’s elongated-body theory, Schiehlen method, etc. Basing on the Newton-Euler method, Y. Shi’s group has built a dynamic model of an AUV, and then investigated dynamic model-based trajectory tracking control of planar motions of the AUV [16]–[18]. Without consideration of three-dimensional motions, J. Yu’s group has formulated a robotic fish dynamics using Schiehlen method [19] and Lagrangian dynamics method [20]. It has been demonstrated that the proposed dynamic model is efficient for seeking backward swimming pattern of the robotic fish [20]. They have also proposed a data-driven dynamic modeling method in which the Newton-Euler formulation is applied to analyze the robotic fish dynamics, and parameters in the dynamic model are identified using experimental data of rectilinear motion and turning motion of the robotic fish, also without investigating three-dimensional motions. F. Zhang’s group has established an analytical model for spiral motion of an underwater glider steered by an internal movable mass block, and experiments in the South China Sea have validated the accuracy of the model for achieving desired spiral motion [21]. They have also explored a dynamic model for a blade-driven glider with gliding motion [22]. However, the motion of glider is different from rhythmic motion of the fin-actuated underwater robot. X. Tan’s group has explored dynamic analyses of a tail-actuated robotic fish [23]–[25] and a fish-like glider [26], [27]. For the tail-actuated robotic fish, Lighthill’s large-amplitude elongated-body theory has been combined with rigid-body dynamics and hybrid tail dynamics for building a dynamic model [23]–[25]. However, only surface motion of the robotic fish has been explored. For the fish-like glider, they have built a Newton-Euler method based dynamic model for investigating spiraling maneuver [26] and gliding motion [27]. However, the fish-like glider is just driven by displacing an internal movable mass and pumping fluids, while its tail is not active, without a continuously varied tail angle.

The above-mentioned studies have demonstrated that dynamic modeling is fundamental and essential for locomotion analysis of underwater robots. However, most of the researches have only focused on investigating two-dimensional motions...
in horizontal plane or vertical plane. Especially for fin-actuated underwater robots, though there are a few preliminary works that have considered dynamic modeling in three-dimensional space [19], [20], [28], the proposed models are typically validated by limited experiments, without validation in a large-scale parameter space. Besides, for three species of underwater robots including active-fin-actuated underwater robot with barycentre regulating mechanism, blade-driven underwater robot [22], and internal movable mass block-driven underwater robot [24], all of which can adjust their centers of mass, there exist significant differences among their dynamics, because an active-fin-actuated underwater robot with barycentre regulating mechanism is able to generate extra rhythmic oscillation of robot body. However, dynamic modelling for such an underwater robot has been rarely investigated.

On the basis of the above analyses, this article mainly focuses on investigating three-dimensional dynamic modeling in a large-scale parameter space for an active-tail-actuated robotic fish with a barycentre regulating mechanism, which has been rarely investigated. Multiple swimming patterns including rectilinear motion, turning motion, gliding motion, and spiral motion are investigated. Firstly, a mathematical description of the dynamic model is proposed basing on Newton-Euler method. Then multiple methods, including SolidWorks software, computational fluid dynamics (CFD) simulation, and grey-box model estimation method, are used for determining model parameters. Finally, numerical simulations and massive kinematic experiments with a robotic fish prototype in a large-scale parameter space are applied to mutually validate the accuracy of the dynamic model in predicting key features, including trajectory, attitude, velocity, etc., of the robotic fish.

The remainder of this article is organized as follows. Section II introduces the bio-inspired robotic fish. Section III establishes a Newton-Euler dynamic model for the robotic fish and determines the model parameters. Section IV presents simulation and experiment results. Section V concludes this article with an outline of future work.

II. THE ROBOTIC FISH

Figure 1 (a) shows the hardware configurations of the robotic fish. Its size (Length×Width×Height) is about 29.1 cm×11.6 cm×13.4 cm. It is composed of a 3D-printed shell, a tail, and three compartments, including a control compartment, an engine compartment, a battery compartment, and a pressure acquisition system compartment. Figure 1 (b) shows the interior of the engine compartment. Three motors, which serve different functions, are wrapped in the engine compartment. Specifically, motor 1 is connected with the tail. It is used to generate propulsive force. Motor 2 is used for driving a rotating bracket. The bracket is connected to motor 3 and a crank-slider mechanism. Motor 3 is used to drive the crank-slider mechanism mentioned above to which a weight block is connected. Through controlling motor 2 and 3, the weight block can move along the direction parallel to principal axis of the robotic fish and rotate about output shaft of motor 2. By controlling the three motors using given frequency, amplitude, and offset parameters, the robotic fish can realize rectilinear motion, turning motion, gliding motion, and spiral motion, as shown in Figure 2. More about motions of the robotic fish can be in the supplementary video.

III. DYNAMIC ANALYSIS FOR THE ROBOTIC FISH

A. Definition of the Coordinate Systems

Figure 3 shows the coordinate systems of the robotic fish. $O_{1}x_{1}y_{1}z_{1}$, $O_{b}x_{b}y_{b}z_{b}$, $O_{h}x_{h}y_{h}z_{h}$, and $O_{L}x_{L}y_{L}z_{L}$ indicate the global inertial coordinate system, the body-fixed coordinate system, the rotating-bracket-fixed coordinate system, and the tail-fixed coordinate system, respectively. The origin $O_{b}$ is fixed at the intersection of horizontal section and longitudinal section of the robotic fish, above center of mass $C_{m}$ of the robotic fish. The longitudinal section is the symmetrical plane of the shell. The horizontal section coincides with the symmetrical plane of the tail and is perpendicular to the longitudinal plane. The origin $O_{h}$ is fixed at the connection point of motor 2 and the rotating bracket in Figure 1 (c), and expressed as $[a_{rb}, b_{rb}, c_{rb}]$ in $O_{h}x_{h}y_{h}z_{h}$. The origin $O_{L}$ is fixed at the connection point of the tail and the engine compartment, and expressed as $[a_{t}, b_{t}, c_{t}]$ in $O_{b}x_{b}y_{b}z_{b}$. $O_{1}x_{1}y_{1}z_{1}$ coincides with the initial $O_{b}x_{b}y_{b}z_{b}$.
where $\theta$ indicates pitch angle of the robotic fish.

Fig. 2. Multiple three-dimensional swimming patterns of the robotic fish. (a) Rectilinear motion. (b) Gliding motion. (c) Turning motion. (d) Spiral motion. (e) The red point on fish shell means center of mass. It moves backward/forward when the weight block moves backward/forward with a distance of $\Delta s$ in gliding motion and spiral motion (lower), comparing with rectilinear motion and turning motion (upper). The tails in turning motion and spiral motion have non-zero offsets compared to those in rectilinear motion and gliding motion. $O_1X_1Y_1Z_1$ indicates the global inertial coordinate system. $F$ indicates the net propulsive force. $U_{k}$ is the resultant velocity of the robotic fish. $U_s$ is the resultant velocity of the velocity $V_{sZ_1}$ along the axis $O_1Z_1$ and the velocity $V_{sX_1}$ along the axis $O_1X_1$. $U_s$ is the resultant velocity of the velocity $V_{sZ_1}$ along the axis $O_1Z_1$ and the velocity $V_{sX_1}$ on $X_1 - Y_1$ plane. $R_t$ and $R_s$ indicates the radius in turning motion and spiral motion, respectively. $\Delta h$ indicates depth variation of the robotic fish.

**B. Three-Dimensional Kinematic Analysis**

1) Translational Motion of the Robotic Fish: The position of the robotic fish is denoted as $C_I = \begin{bmatrix} X_I, Y_I, Z_I \end{bmatrix}^T$ in $O_1X_1Y_1Z_1$. The velocity of robotic fish is denoted as $V_I = \begin{bmatrix} V_{x_I}, V_{y_I}, V_{z_I} \end{bmatrix}^T$ in $O_1X_1Y_1Z_1$ and $V_b = \begin{bmatrix} V_{x_b}, V_{y_b}, V_{z_b} \end{bmatrix}^T$ in $O_bX_bY_bZ_b$, respectively. The relationship between $V_I$ and $V_b$ is expressed as

$$V_I = \dot{C}_I = R_{bl} \cdot V_b$$

where $R_{bl}$ is the transformation matrix from $O_bX_bY_bZ_b$ to $O_1X_1Y_1Z_1$, taking the form as

$$R_{bl} = \begin{bmatrix} c_\psi c_\theta - s_\psi s_\theta s_\varphi + c_\psi s_\theta c_\varphi \\ s_\varphi c_\psi + c_\varphi s_\theta s_\psi \\ s_\varphi s_\theta + c_\varphi s_\theta c_\psi \\ -c_\varphi s_\psi s_\theta - c_\theta s_\varphi \end{bmatrix}$$

where $\psi$, $\theta$, and $\varphi$ indicate roll, pitch, and yaw angle of the robotic fish, respectively.

2) Rotational Motion of the Robotic Fish: The angular velocity of the robotic fish is denoted as $\omega_b = \begin{bmatrix} \omega_{b_x}, \omega_{b_y}, \omega_{b_z} \end{bmatrix}^T$ in $O_bX_bY_bZ_b$ and $\omega_I = \begin{bmatrix} \dot{\varphi}, \dot{\theta}, \dot{\psi} \end{bmatrix}^T$ in $O_1X_1Y_1Z_1$. The relationship between $\omega_b$ and $\omega_I$ is expressed as

$$\omega_I = \begin{bmatrix} \sin \varphi \tan \theta \\ \cos \varphi \tan \theta \\ \sin \varphi / \cos \theta \\ \cos \varphi / \cos \theta \end{bmatrix} \cdot \omega_b$$

3) Motion analysis of the weight block: As shown in Figure 4 (c), the weight block is able to rotate through controlling output angle $\xi_2$ of motor 2. Thus roll angle $\varphi$ of the robotic fish is able to be adjusted. On the other hand, as shown in Figure 4 the distance $s_w$ is able to be adjusted through controlling output angle $\xi_3$ of motor 3. Thus the weight block
is able to move along the guideway, and pitch angle $\theta$ of the robotic fish is able to be adjusted. $s_w$ takes the form as

$$s_w = s_{w_0} + \Delta d$$  \hspace{1cm} (4)

where $s_{w_0}$ indicates the initial value of $s_w$, with which pitch angle and roll angle of the robotic fish are 0. $\Delta d$ is the distance between the weight block's current position and its initial position in the kinematics experiments.

The coordinate of center of mass of the weight block $C_w[x_{C_w}, y_{C_w}, z_{C_w}]$ is expressed in $O_b x_b y_b z_b$, taking the form as

$$x_{C_w} = a_{rb} + d_1 - (s_w - d_3)$$
$$y_{C_w} = b_{rb} + d_2 \cdot \sin \xi_2$$
$$z_{C_w} = c_{rb} + d_2 \cdot \cos \xi_2$$ \hspace{1cm} (5)

The coordinate of center of mass of the robotic fish $C_m[x_{C_m}, y_{C_m}, z_{C_m}]$ takes the form as

$$j_{C_m} = \frac{(M_{ewj} + M_{wj})}{m_{total}}$$ \hspace{1cm} (6)

where $j = x, y, z$, $M_{ewj}$ is static moment about the $O_b x_b$ axis for the part apart from the weight block. $M_{wj}$ is static moment about the $O_b y_b$ axis for the weight block, taking the form as

$$M_{wj} = m_w \cdot j_{C_w}$$ \hspace{1cm} (7)

where $m_w$ is the mass of the weight block.

The initial coordinate of center of mass of the robotic fish is expressed as $[x_{C_{m0}}, y_{C_{m0}}, z_{C_{m0}}]$. Besides, both pitch angle $\theta$ and roll angle $\varphi$ of the robotic fish are zero when the weight block is at its initial position.

**C. Three-Dimensional Force Analysis**

In this part, the forces and torques acting on the tail and fish body of the robotic fish are analyzed. For the tail, the lift and drag are considered. For the fish body, we respectively consider lift force and drag force in $x_b - y_b$ plane and $x_b - y_b$ plane, gravity, buoyancy, and impact of water flow.

The velocity of $C_{pt}$, in Figure 5 is expressed as

$$\mathbf{v}_t = \mathbf{v}_b + \omega_b \times O_b C_{pt} + \omega_t \times O_t C_{pt}$$ \hspace{1cm} (9)

where $O_b C_{pt}$ is the vector from $O_b$ to $C_{pt}$. It is expressed as

$$O_b C_{pt} = (a_1 - r_c \cdot \cos \xi_1) \cdot \mathbf{x}_b + (b_1 - r_c \cdot \sin \xi_1) \cdot \mathbf{y}_b + c_1 \cdot \mathbf{z}_b$$ \hspace{1cm} (10)

where $\mathbf{x}_b$, $\mathbf{y}_b$, and $\mathbf{z}_b$ are unit vector along the $O_b x_b$ axis, $O_b y_b$ axis, and $O_b z_b$ axis in $O_b x_b y_b z_b$, respectively. $O_t C_{pt}$ is the vector from $O_t$ to $C_{pt}$, and it is expressed as

$$O_t C_{pt} = -r_c \cdot \cos \xi_1 \cdot \mathbf{x}_b - r_c \cdot \sin \xi_1 \cdot \mathbf{y}_b + 0 \cdot \mathbf{z}_b$$ \hspace{1cm} (11)

$\omega_t$ is the oscillating angular velocity of the tail, and it is expressed as

$$\omega_t = \dot{\xi}_1 \cdot \mathbf{z}_b = 2\pi f_1 A_1 \cos (2\pi f_1 t) \cdot \mathbf{z}_b$$ \hspace{1cm} (12)

The tail of the robotic fish is regarded as a rigid plate without spanwise wave motion, which is different from fins in [29]. There are various forms of tail-generated force and torque [25, 28, 30–33] for different of types of tails. Here, we have adopted forms as in [25, 28, 33], which are typically applied to express torque and force caused by a rigid plate-like tail. Specifically, the lift $F_L^t$ and drag $F_D^t$ of the tail are expressed as

$$F_L^t = \frac{1}{2} \rho |\mathbf{v}_t|^2 S \lambda C_{L_\alpha}(|\alpha_t|)$$ \hspace{1cm} (13)

where $\lambda = L, D$, $\rho$ is the density of water. $S_t$ is the surface area of the tail. $C_{L_\alpha}$ and $C_{L_\lambda}$ are force coefficients which will be determined in section III. E. $\alpha_t$ is the angle of attack of the tail, which is expressed as

$$\alpha_t = \arcsin (\mathbf{n}_t \cdot \hat{\mathbf{v}}_t)$$ \hspace{1cm} (14)

where $\mathbf{n}_t$ is the normal vector of the tail, which is expressed as

$$\mathbf{n}_t = -\sin \xi_1 \cdot \mathbf{x}_b + \cos \xi_1 \cdot \mathbf{y}_b + 0 \cdot \mathbf{z}_b$$ \hspace{1cm} (15)

Basing on the above analyses, the three-dimensional drag $F_D^t$ acting on the tail is expressed as

$$F_D^t = -F_D^t \hat{\mathbf{v}}_t$$ \hspace{1cm} (16)

The three-dimensional lift $F_L^t$ acting on the tail is expressed as

$$F_L^t = \begin{cases} |\mathbf{v}_t| \sin \alpha_t - \mathbf{n}_t \cdot \hat{\mathbf{v}}_t & \text{if } \mathbf{n}_t \cdot \hat{\mathbf{v}}_t > 0 \\ |\mathbf{v}_t| \sin \alpha_t + |\mathbf{v}_t| \hat{\mathbf{v}}_t & \text{if } \mathbf{n}_t \cdot \hat{\mathbf{v}}_t \leq 0 \end{cases}$$ \hspace{1cm} (17)

Then, the tail-generated torque $M_b^t$ acting on the robotic fish is expressed as

$$M_b^t = O_b C_{pt} \times (F_L^t + F_D^t)$$ \hspace{1cm} (18)
2) Force Analysis for Fish Body: Figure 6 shows the drag \( F_{D_1}^b (i = 1, 2) \) and lift \( F_{L_1}^b (i = 1, 2) \) acting on the fish body, of which the values are expressed as

\[
F_{D_1}^b = \frac{1}{2} \rho |V_{b1}|^2 S_b C_{D_{b1}} (|\alpha_{b1}|) \\
F_{L_1}^b = \frac{1}{2} \rho |V_{b1}|^2 S_b C_{L_{b1}} (|\alpha_{b1}|)
\]

(19)

where \( C_{D_{b1}} \) and \( C_{L_{b1}} \) are force coefficients which will be determined in section III. E.

\[
V_{b1} = V_{bx} \cdot \hat{x}_b + V_{bz} \cdot \hat{z}_b \\
V_{b2} = V_{bx} \cdot \hat{x}_b + V_{by} \cdot \hat{y}_b
\]

(20)

\( S_b (i = 1, 2) \) is the surface area tensor of the robotic fish. It is defined as

\[
S_b = V_{b1}^T \cdot A_1 \cdot V_{b1}, (i = 1, 2)
\]

(21)

where

\[
A_1 = \begin{bmatrix} S_{xx} & S_{xy} \\ S_{xy} & S_{yy} \end{bmatrix}, A_2 = \begin{bmatrix} S_{xx} & S_{xy} \\ S_{yx} & S_{yy} \end{bmatrix}
\]

(22)

\( A_1 \) and \( A_2 \) are diagonal matrices, \( S_{xx}, S_{yy}, \) and \( S_{xy} \) indicates the maximum cross section area perpendicular to the axes \( O_b x_b, O_b y_b, \) and \( O_b z_b \). \( \alpha_{b1} (i = 1, 2) \) is angle of attack of fish body, taking the form as

\[
\alpha_{b1} = \arcsin (n_{b1} \cdot \hat{V}_{b1})
\]

(23)

\( n_{b1} (i = 1, 2) \) is the normal vector, taking the form as

\[
n_{b1} = \hat{z}_b, n_{b2} = \hat{y}_b
\]

(24)

Basing on the above-analyses, the three-dimensional drag \( F_{D_1}^b (i = 1, 2) \) is expressed as

\[
F_{D_1}^b = -F_{D_1}^b V_{b1}
\]

(25)

besides, rotations of the robotic fish cause damping torques \( M_{\omega} \) acting on fish body, and \( M_{\omega} \) is expressed as

\[
M_{\omega} = C_{\omega_b} \cdot \omega_b
\]

(27)

where \( C_{\omega_b} \) is damping torque coefficient, taking the form as

\[
C_{\omega_b} = \text{diag} \{ C_{\omega_{b1}}, C_{\omega_{b2}}, C_{\omega_{b3}} \}
\]

(28)

In addition, the robotic fish is subjected to force \( M_1 \) caused by impact of water flow, and \( M_1 \) is expressed as

\[
M_1 = M_{I_{x_b}} \cdot \hat{x}_b + M_{I_{y_b}} \cdot \hat{y}_b + M_{I_{z_b}} \cdot \hat{z}_b
\]

(29)

where

\[
M_{I_{x_b}} = 0 \\
M_{I_{y_b}} = \frac{1}{2} \rho |V_{b1}|^2 S_y C_{M_{I_{y_b}}} (\alpha_{b2}) \\
M_{I_{z_b}} = \frac{1}{2} \rho |V_{b1}|^2 S_z C_{M_{I_{z_b}}} (\alpha_{b2})
\]

\( C_{M_{I_{y_b}}} \) and \( C_{M_{I_{z_b}}} \) are torque coefficients which will be determined in section III. E.

3) The Effect of Gravity and Buoyance: The gravity \( F_g \) and buoyancy \( F_b \) of the robotic fish are expressed in \( O_b x_b y_b z_b \), taking the form as

\[
F_g = m_{total} \cdot R_{bt}^{-1} \cdot g
\]

(31)

\[
F_b = -m_b \cdot R_{bt}^{-1} \cdot g
\]

(32)

where \( m_{total} \) and \( m_b \) are total mass and buoyancy mass of the robotic fish, respectively.

The torque \( M_g \) caused by the buoyance of the robotic fish is expressed as

\[
M_g = O_b C_m \times F_g
\]

(33)

where \( O_b C_m \) is the vector from \( O_b \) to \( C_m \), taking the form as

\[
O_b C_m = x_{C_m} \hat{x}_b + y_{C_m} \hat{y}_b + z_{C_m} \hat{z}_b = \text{diag} \{ x_{C_m}, y_{C_m}, z_{C_m} \}
\]

(34)

D. Newton-Euler Dynamic Model

Basing on Newton’s second law, the total force \( F_{total} \) acting on the robotic fish is expressed as

\[
\{ F_{total} = \frac{dMV_{C_m}}{dt} \} = \frac{dV_{C_m}}{dt} + \frac{d\omega_b}{dt} \times O_b C_m + \omega_b \times V_b + \omega_b \times (\omega_b \times O_b C_m)
\]

(35)

where \( M = \text{diag} \{ m_{total}, m_{total}, m_{total} \} \). \( V_{C_m} \) indicates velocity of center of mass \( C_m \) of the robotic fish, taking the form as

\[
V_{C_m} = V_b + \omega_b \times O_b C_m
\]

(36)
Basing on Euler’s equation, the total torque $M_{\text{total}}$ about $C_m$ is expressed as
\[
M_{\text{total}} = \frac{dH_{C_m}}{dt} = \omega_b \times (\omega_b \times \omega_b) + \omega_b \times (M_b \times \omega_b) - M_b \times \omega_b
\]
(38)
where $H_{C_m}$ is the moment of momentum about $C_m$ of the robotic fish, taking the form as
\[
H_{C_m} = J \omega_b + M \cdot O_b C_m \times V_b
\]
(39)
\[
\frac{dH_{C_m}}{dt} = J \dot{\omega}_b + \omega_b \times (J \omega_b) + M \cdot (\omega_b \times O_b C_m) \times V_b + M \cdot O_b C_m \times (V_b + \omega_b \times V_b)
\]
(40)
\[
J = \text{diag}\{J_{xx}, J_{yy}, J_{zz}\}
\]
where $J_{xx}$ is the moment of inertia about $x$-axis, $J_{yy}$ is the moment of inertia about $y$-axis, and $J_{zz}$ is the moment of inertia about $z$-axis.

\[
J_w \text{ and } J_{ew} \text{ are the moments of inertia about } O_b \text{ for the weight block and the part apart from weight block, respectively, taking the form as}
\]
\[
J_w = J_{ew} + J_w
\]
(41)
\[
J_w \text{ and } J_{ew} \text{ are the moments of inertia about } O_b \text{ for the weight block and the part apart from weight block, respectively, taking the form as}
\]
\[
J_w = J_{ew} + m_w \cdot \text{diag}\{r_{O_b C_w, x}^2, r_{O_b C_w, y}^2, r_{O_b C_w, z}^2\}
\]
(42)
where $\gamma = \text{ew, w, 'ew' and 'w'}$ indicate the part apart from the weight block and the weight block, respectively, $m_w$ is mass of the part apart from the weight block, and $m_w = m_{\text{total}} - m_w$. $r_{O_b C_w, x}$, $r_{O_b C_w, y}$, and $r_{O_b C_w, z}$ are components of the distance between $C_w$ and $C_m$ along the $O_b x_b$ axis, $O_b y_b$ axis, and $O_b z_b$ axis, respectively, taking the form as
\[
\begin{align*}
r_{O_b C_w, x} &= y_{C_w}^2 + z_{C_w}^2, \\
r_{O_b C_w, y} &= x_{C_w}^2 + z_{C_w}^2, \\
r_{O_b C_w, z} &= x_{C_w}^2 + y_{C_w}^2
\end{align*}
\]
(43)
where $[x_{C_w}, y_{C_w}, z_{C_w}]$ is coordinate of center of mass for the part apart from the weight block. $J_{ew}(j = x, y, z)$ takes the form as
\[
J_{ew} = M_{ew}/m_w
\]
(44)
$J_{ew}'$ is the moment of inertia about $C_\gamma$ for the part apart from weight block, taking the form as
\[
J_{ew}' = \text{diag}\{J_{ew, xx}', J_{ew, yy}', J_{ew, zz}'\}
\]
(45)

Basing on the above analyses, the concrete form of the dynamic equations [35] and [38] can be finally acquired, as shown in (46) where $F_{x_0}, F_{y_0}, F_{z_0}$ are components of the total force along the $O_b x_b$ axis, $O_b y_b$ axis, and $O_b z_b$ axis, respectively. $M_{x_0}, M_{y_0}, M_{z_0}$ are components of the total torque about the $O_b x_b$ axis, $O_b y_b$ axis, and $O_b z_b$ axis, respectively.

E. Determination of Model Parameters

In this part, model parameters, which include mass, dimensions, and moment of inertia of the robotic fish, etc. are determined by three-dimensional computer-aided design (CAD) software SolidWorks, as shown in Table S1 of the supplementary materials. We have used two robotic fish to conduct the experiments. $m_{b_1}$ is buoyancy mass for the robotic fish used in rectilinear motion and turning motion, while $m_{b_2}$ is for the robotic fish used in gliding motion and spiral motion. $m_{b_1}$ and $m_{b_2}$ are both determined by actual measurement. Lift coefficients, drag coefficients, and impact torque coefficients are determined by computational fluid dynamics (CFD) simulation. Damping torque coefficients are determined by grey-box model estimation method.

1) Determining Force Coefficients and Torque Coefficients Using Computational Fluid Dynamics (CFD) Method: Specifically, computational fluid dynamics (CFD) simulation for fish body and tail of the robotic fish were respectively conducted using a CFD software called HyperFlow, which is developed by China Aerodynamics Research and Development Center (CARDC). HyperFlow is a structured/unstructured hybrid integrated fluid simulation software. It is able to run the structured solver synchronously on structured grids and unstructured solver on unstructured grids. Besides, it has been proved to have good performance in multi-purpose fluid simulation [35]. [66]. Figure 7 shows the hydrodynamic pressure variations of the tail and fish body using CFD simulation. More details about the CFD simulation can be found in Section S1 of the supplementary materials. In the CFD simulation, angles of attack $\alpha_1$, $\alpha_{b_1}$, and $\alpha_{b_2}$ changed from 0 to $\pi/6$ rad with an interval of $\pi/60$ rad. Basing on the hydrodynamic pressure variations, the lift, drag, and impact torque coefficients under certain values of $\alpha_1$, $\alpha_{b_1}$, and $\alpha_{b_2}$ are acquired, as shown in Figure 8. Basing on data fitting method, the quantitative equations which link $\alpha_1/\alpha_{b_1}/\alpha_{b_2}$ to coefficients mentioned above can be acquired, as shown in Section S1 of the supplementary materials.

2) Determining the damping torque coefficients using grey-box model estimation method: The damping torque coefficients are determined by grey-box model estimation method [57]. In the grey-box model estimation, we recorded the rectilinear velocity of the robotic fish with given oscillating parameters, including amplitude and frequency of the tail in 28 s. The input data for grey-box model were the oscillating parameters, while the output data were the rectilinear velocity. As shown in Table S2 of the supplementary materials, we restricted ranges of the three coefficients for avoiding drift of the solution. The final values of the damping coefficients are shown in Table S2 of the supplementary materials. Figure 9
shows the measured velocity and simulated velocity obtained using the estimated coefficients. The measured velocity and simulated velocity of the robotic fish match with a 61.45% fit.

\[
\begin{align*}
\text{F}_{x1} &= m_{\text{total}} \cdot \left[ V_{b} - V_{b} \cdot \omega_{b} + V_{b} \cdot \omega_{b} - xC_{m} \left( \omega_{b}^{2} + \omega_{y}^{2} \right) + yC_{m} \left( \omega_{p} \omega_{b} - \omega_{y} \right) + zC_{m} \left( \omega_{b} \omega_{b} + \omega_{b} \right) \right] \\
\text{F}_{y1} &= m_{\text{total}} \cdot \left[ V_{b} - V_{b} \cdot \omega_{b} + V_{b} \cdot \omega_{b} - yC_{m} \left( \omega_{b}^{2} + \omega_{y}^{2} \right) + xC_{m} \left( \omega_{p} \omega_{b} - \omega_{y} \right) + zC_{m} \left( \omega_{b} \omega_{b} + \omega_{b} \right) \right] \\
\text{F}_{z1} &= m_{\text{total}} \cdot \left[ V_{b} - V_{b} \cdot \omega_{b} + V_{b} \cdot \omega_{b} - zC_{m} \left( \omega_{b}^{2} + \omega_{y}^{2} \right) + xC_{m} \left( \omega_{p} \omega_{b} - \omega_{y} \right) + yC_{m} \left( \omega_{b} \omega_{b} + \omega_{b} \right) \right] \\
\text{M}_{x1} &= J_{xx} \omega_{b} + \left( J_{xx} - J_{yy} \right) \omega_{b} \omega_{b} + m_{\text{total}} \cdot \left[ yC_{m} \left( V_{b} + V_{b} \omega_{b} - V_{b} \omega_{b} \right) - xC_{m} \left( V_{b} + V_{b} \omega_{b} - V_{b} \omega_{b} \right) - zC_{m} \left( V_{b} + V_{b} \omega_{b} - V_{b} \omega_{b} \right) \right] \\
\text{M}_{y1} &= J_{yy} \omega_{b} + \left( J_{xx} - J_{yy} \right) \omega_{b} \omega_{b} + m_{\text{total}} \cdot \left[ zC_{m} \left( V_{b} + V_{b} \omega_{b} - V_{b} \omega_{b} \right) - yC_{m} \left( V_{b} + V_{b} \omega_{b} - V_{b} \omega_{b} \right) - xC_{m} \left( V_{b} + V_{b} \omega_{b} - V_{b} \omega_{b} \right) \right] \\
\text{M}_{z1} &= J_{zz} \omega_{b} + \left( J_{yy} - J_{xx} \right) \omega_{b} \omega_{b} + m_{\text{total}} \cdot \left[ xC_{m} \left( V_{b} + V_{b} \omega_{b} - V_{b} \omega_{b} \right) - yC_{m} \left( V_{b} + V_{b} \omega_{b} - V_{b} \omega_{b} \right) - zC_{m} \left( V_{b} + V_{b} \omega_{b} - V_{b} \omega_{b} \right) \right]
\end{align*}
\]
robin fish swims in a straight line. Because of the periodical oscillation of the tail, the robotic fish body oscillates while swimming. Thus the yaw angle, pitch angle, and roll angle of the robotic fish oscillate periodically with the time. It can be seen that the simulated and measured attitudes match well in the oscillatory feature and value. A more careful inspection revealed that the yaw amplitude increases with the increasing $A_1$ while the yaw rate increases with the increasing $f_1$. For pitch angle and roll angle, the biggest errors between the estimated values and the measured values are both less than 3°, which are small enough. The errors are results of the wave motion of water which caused the roll motion and pitch motion of the robotic fish. The final trajectory of the robotic fish is shown in Figure 12 with a maximum error between the simulated trajectory and measured trajectory of 0.2407 m.

B. Turning motion

In turning motion experiment, varieties of turning angular velocities $\omega_t$ and turning radii $R_t$ were obtained by various combinations of oscillating offset $\xi_1$ and frequency $f_1$ of the tail. As shown in Figure 13 and Figure 14, the measured value and simulated value of $\omega_t$ match well with $R^2=0.7462$ and MAE=0.0409 rad/s, while the measured $R_t$ matches the simulated $R_t$ with a MAE=0.0657 m and an average percentage error of 18.5913%. The $\omega_t$ increases with the increasing $\xi_1$ and $f_1$. The $R_t$ decreases with the increasing $\xi_1$ and it is nearly constant with the $f_1$. Figure 15 shows the real-time yaw/pitch/roll rate of the robotic fish. It can be seen that both the roll rate $\omega_{1z}$ and pitch rate $\omega_{1y}$ of the robotic fish oscillate around zero. The yaw rate $\omega_{1x}$ oscillates around a positive value when the value of $\xi_1$ is negative, in which case the robotic fish turns left. While the $\omega_{1z}$ oscillates around a negative value when the value of $\xi_1$ is positive, in which case the robotic fish turns right. A more careful inspection reveals that the amplitude of the $\omega_{1z}$ increases with the $\xi_1$ while decreases with the $f_1$, while the rate of $\omega_{1y}$ increases with the $f_1$. For the amplitudes of $\omega_{1x}$ and $\omega_{1y}$, they decrease with the $f_1$. 

Fig. 11. Real-time attitudes of the robotic fish in rectilinear motion.

Fig. 12. Trajectory of the robotic fish under five combinations of $A_1$ and $f_1$.

Fig. 13. Measured and simulated turning angular velocity of the robotic fish. (a) Measured value. (b) Simulated value.

Fig. 14. Measured and simulated turning radius of the robotic fish. (a) Measured value. (b) Simulated value.

Fig. 15. Real-time yaw/pitch/roll rate of the robotic fish in turning motion under six combinations of $\xi_1$ and $f_1$. $\omega_{1x}$ and $\omega_{1y}$ (j = x, y, z) indicate simulated and measured value of the yaw/pitch/roll rate, respectively.
C. Gliding motion

![Graph of gliding velocity of the robotic fish](image)

Fig. 16. Measured and simulated gliding velocity of the robotic fish.

In gliding motion experiment, varieties of gliding velocities $U_g$ were obtained by changing $\Delta d$ of the weight block. $A_1$, $f_1$, and $\xi_1$ of the tail are 20°, 2.0 Hz, and 0, respectively. Figure 16 shows the measured and simulated $U_g$ of the robotic fish. The maximum and average percentage errors between the measured and simulated $U_g$ are 14.0507% and 3.5340%, respectively. It is noteworthy that because of the depth limitation of the water tank (only 0.8 m), the robotic fish reached the surface of the water before it reached the state of uniform motion. So the $U_g$ of the robotic fish for $\Delta d$=2.0 cm, -1.9 cm, -1.8 cm, and -1.7 cm is average gliding velocity of the robotic fish in its acceleration process. While the $U_g$ for $\Delta d$ varied from -1.4 cm to 0 are velocities when the robotic fish was in uniform motion state. Comparing the $U_g$ for $\Delta d$ from -1.4 cm to 0, it can be seen that $U_g$ of the robotic fish decreases with the increasing $|\Delta d|$.

D. Spiral motion

![Graph of spiral angular velocity of the robotic fish](image)

Fig. 19. Measured and simulated spiral angular velocity of the robotic fish.

The spiral motion was the result of a combination of non-zero $\Delta d$ and non-zero oscillating offset $\xi_1$ of the tail. $A_1$ and $f_1$ of the tail are 20° and 3.0 Hz, respectively. As shown in Figure 17 yaw angle of the robotic fish oscillates around varied values with the time, while pitch angle and the roll angle oscillate around constant values. It can be seen that the simulated attitudes closely track the measured attitudes. The velocity $V_{I_x}$ along the axis $O_1X_I$ and the velocity $V_{I_y}$ along the axis $O_1Z_I$ exhibit sine-like characteristics. The velocity $V_{I_z}$ along the axis $O_1Z_I$ gradually researches a negative value, which means the robotic fish is spiralling up. Figure 18 and Figure 20 shows the measured and simulated spiral angular velocity $\omega_s$ and spiral velocity $U_s$ of the robotic fish, respectively. It can be seen that both the $\omega_s$ and the $U_s$ barely change with the $\Delta d$. The maximum and average percentage error of the $\omega_s$ are 3.6004% and 1.9323%, respectively. The maximum and average percentage error of the $U_s$ are 11.4808% and 5.8953%, respectively. Figure 21 shows the measured and simulated spiral trajectory of the robotic fish in spiral motion. The measured trajectory tracks the simulated trajectory well with a maximum error of 0.3974 m.

V. CONCLUSION AND FUTURE WORK

In this article, a dynamic model that accounts for multiple three-dimensional motions, including rectilinear motion, turn-
Fig. 20. Measured and simulated spiral velocity of the robotic fish.

Fig. 21. Spiral trajectory of the robotic fish in spiral motion.

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