Intercomponent correlations in attractive one-dimensional mass-imbalanced few-body mixtures

Daniel Pęcak and Tomasz Sowiński
Institute of Physics, Polish Academy of Sciences, Aleja Lotnikow 32/46, PL-02668 Warsaw, Poland

(Received 28 September 2018; published 11 April 2019)

Ground-state properties of a few attractively interacting ultracold atoms of different mass confined in a one-dimensional harmonic trap are studied. The analysis is performed in terms of the noise correlation, which captures the two-particle correlations induced by the mutual interactions. Depending on the mass ratio between the components’ atoms, the interparticle correlations change their properties significantly from a strong pairlike correlation to an almost uncorrelated phase. This change is accompanied by a simultaneous change in the structure of the many-body ground state. A crucial role of the quantum statistics is emphasized by comparing properties of the Fermi-Fermi mixture with a corresponding Fermi-Bose system.

DOI: 10.1103/PhysRevA.99.043612

I. INTRODUCTION

Recent years have brought many examples that the systems consisting of a few ultracold atoms can be prepared and well controlled experimentally with extreme precision [1–7]. It became possible to measure (as functions of mutual interactions) not only single-particle properties of the system but also higher multiparticle correlations. The latter are fundamentally important, since they directly reflect different nonclassical multiparticle properties of the system, being direct manifestations of indistinguishability and entanglement forced by interactions. It is quite obvious that these higher correlations cannot be neglected if one needs to characterize an obtained quantum state appropriately. As shown recently, the two-body position and momentum correlation functions can be measured and they are indeed a very powerful tool to characterize quantum states [8]. In principle, one can have experimental access also to higher-order correlations between particles. For example, it can be done by using atomic microscopes which allow one to measure positions of all particles at the same time [9–17]. All this means that on an experimental level, the ultracold physics starts to explore much more complicated features of many-body systems than simple single-particle densities.

Theoretical studies of one-dimensional few-body mixtures are very rich in the literature. In a great majority, due to the experimental motivation from the Heidelberg group [3,7], these considerations concern two-component mixtures of repelling particles with equal mass (see, for example, Refs. [18–26]). Accordingly, less attention is given for attractive systems for which some precursors of the Cooper-like pairing were observed [7], theoretically explained [27–29], and explored [30–33].

Recent years have brought tremendous progress in experimental studies of fermionic (Li-K, Dy-K) [34–37] as well as bosonic-fermionic (Li-Na, Rb-K, Cs-Li, Li-K) [38–43] mixtures of a large number of ultracold atoms of different mass. Although for such mixtures the few-body regime has not been achieved yet, first theoretical predictions show that such systems may have essentially different properties [44–49] than systems with equal-mass atoms.

In this general context, the question of properties of mass-imbalanced few-body mixtures in the attractive regime seems to be very relevant and important. In the following, we perform the first step in this direction and we analyze a destructive effect of a mass difference on intercomponent pairing correlations emerging in a strong attractive regime. We identify, describe, and quantify these highly nonclassical correlations as functions of mass ratio between particles forming opposite components and their number. We also emphasize the role of the quantum statistics in this destructive process.

The work is organized as follows. In Sec. II we introduce the theoretical model of the few-body ultracold system studied and we briefly discuss a numerical method of treatment used. Next in Sec. III, we refresh the concept of noise correlation and we introduce a natural measure quantifying an amount of intercomponent correlations in the system. In Sec. IV we analyze the simplest situation of two atoms for repulsive and attractive interactions and different masses. Importantly, in Sec. V we broadly discuss intercomponent correlations induced by attractions for different strengths of interactions and different mass ratios. For completeness, in Sec. VI we examine the consequences of the quantum statistics by studying Bose-Fermi mixtures. Finally, in Sec. VII we conclude.

II. THE MODEL

In the following we consider a two-component mixture of ultracold fermions of masses $m_\downarrow$ and $m_\uparrow$ confined in a one-dimensional harmonic trap of a frequency $\omega$. We assume that the particles belonging to different species interact dominantly in the $s$-wave channel, and we model this interaction with the $\delta$-like potential. In contrast, for fermions of the same kind (for which the $s$-wave channel is closed due to the Pauli exclusion principle), mutual interactions are negligible and we ignore them. Under these assumptions the many-body Hamiltonian of the system reads

$$\hat{\mathcal{H}} = \sum_{\sigma \in \{\uparrow, \downarrow\}} \int dx \, \hat{\Psi}_\sigma^\dagger(x) H_\sigma \hat{\Psi}_\sigma(x)$$
$$+ g \int dx \, \hat{\Psi}_\uparrow^\dagger(x) \hat{\Psi}_\downarrow^\dagger(x) \hat{\Psi}_\downarrow(x) \hat{\Psi}_\uparrow(x),$$

(1)
where the single-particle Hamiltonians $H_\tau$ are given by

$$H_\uparrow = -\frac{\hbar^2}{2m_\uparrow} \frac{d^2}{dx^2} + \frac{m_\uparrow \omega^2}{2} x^2, \quad (2a)$$
$$H_\downarrow = -\frac{\hbar^2}{2m_\downarrow} \frac{d^2}{dx^2} + \frac{m_\downarrow \omega^2}{2} x^2. \quad (2b)$$

Note that for simplicity the frequencies $\omega$ are equal for both components, and that implies only one energy scale $\hbar \omega$ in the system. The field operator $\hat{\Psi}_\tau(x)$ annihilates a particle of type $\tau$ at given point $x$. The quantum statistics is reflected in the natural anticommutation relations, $\{ \hat{\Psi}_\sigma(x), \hat{\Psi}_\tau^\dagger(x') \} = \delta(x-x')$ and $\{ \hat{\Psi}_\sigma(x), \hat{\Psi}_\tau(x') \} = 0$. Note that particles of different types are fundamentally distinguishable. Therefore any appropriate relations between fields $\hat{\Psi}_\uparrow(x)$ and $\hat{\Psi}_\downarrow(x)$ are equivalent to the commutation relations $\{ \hat{\Psi}_\uparrow(x), \hat{\Psi}_\downarrow^\dagger(x') \} = \{ \hat{\Psi}_\downarrow(x), \hat{\Psi}_\uparrow^\dagger(x') \} = 0$. Evidently, the Hamiltonian (1) does commute with the particle number operators in a given component $\hat{N}_\tau = \int dx \hat{\Psi}^\dagger(\tau) \hat{\Psi}(\tau)$. Therefore, the properties of the system can be examined independently in the subspaces of given $N_\uparrow$ and $N_\downarrow$. To make a whole analysis as clear as possible, in this work we focus on balanced systems, i.e., the systems with equal number of particles in both components, $N_\uparrow = N_\downarrow$. The effective one-dimensional interaction strength $g$ between fermions from opposite components can be derived from the full three-dimensional theory of scattering by integrating out the perpendicular degrees of motion [50]. In the following we express all quantities in the natural harmonic oscillator basis and we diagonalize the matrix obtained via the exact diagonalization method converge slowly with the size of the Hilbert space. However, it is worth noting that the many-body eigenenergies obtained via the exact diagonalization method converge slowly with the size of the Hilbert space. In the following article we focus on the two-body correlations, therefore a poor convergence of the ground-state energy does not reduce the credibility of the results. We checked that in the cases studied it is sufficient to use the first ten single-particle states in the decomposition of the field operator $\hat{\Psi}_\sigma(x)$ to obtain reliable results. The corresponding sizes of the many-body Hilbert space is presented in Table I.

### III. NOISE CORRELATION

The simplest observable that characterizes an interacting few-body system is the single-particle density profile being the diagonal part of the single-particle density matrix:

$$\rho_\sigma^{(1)}(x) = \langle G_0 | \hat{\Psi}_\sigma^\dagger(x) \hat{\Psi}_\sigma(x) | G_0 \rangle. \quad (5)$$

It can be simply understood as the probability density of finding a single particle from the component $\sigma$ at position $x$. Similarly, the probability density of finding a single particle with the momentum $p$ reads

$$\tau_\sigma^{(1)}(p) = \langle G_0 | \hat{\Psi}_\sigma^\dagger(p) \hat{\Psi}_\sigma(p) | G_0 \rangle. \quad (6)$$

Note that in the latter definition the field operator is expressed in the momentum domain. These two single-particle quantities are the simplest (apart from the energy of the state) measurable observables which characterize the many-body quantum system. However, they do not possess any information about correlations between simultaneously measured particles belonging to opposite components. These features are captured by the two-body correlations which are encoded complementarily in the two-body densities in position and momentum domains:

$$\rho^{(2)}(x,y) = \langle G_0 | \hat{\Psi}_\uparrow^\dagger(x) \hat{\Psi}_\downarrow^\dagger(y) \hat{\Psi}_\downarrow(x) \hat{\Psi}_\uparrow(y) | G_0 \rangle, \quad (7a)$$
$$\tau^{(2)}(p,k) = \langle G_0 | \hat{\Psi}_\uparrow^\dagger(p) \hat{\Psi}_\downarrow^\dagger(k) \hat{\Psi}_\downarrow(p) \hat{\Psi}_\uparrow(k) | G_0 \rangle. \quad (7b)$$

In the noninteracting case ($g = 0$), the many-body wave function of the ground state is a simple product of two antisymmetric wave functions, one for each component. Consequently, two-particle densities are the products of the corresponding single-particle densities, $\rho^{(2)}(x,y) = \rho_\uparrow^{(1)}(x) \rho_\downarrow^{(1)}(y)$ and $\tau^{(2)}(p,k) = \tau_\uparrow^{(1)}(p) \tau_\downarrow^{(1)}(k)$. When the interactions are turned on ($g \neq 0$), these relations do not hold anymore, since intercomponent correlations emerge in the system. It turns out that these additional correlations forced by interactions are
well captured by the so-called noise correlations introduced in [53–55] and exploited recently in the context of repulsive few-body systems [56,57]. These quantities are defined as the following:

\[
G_{\rho}(x; y) = \rho^{(2)}(x; y) - \rho^{(1)}_i(x)\rho^{(1)}_i(y), \quad (8a)
\]

\[
G_{\tau}(p; k) = \tau^{(2)}(p; k) - \tau^{(1)}_i(p)\tau^{(1)}_i(k), \quad (8b)
\]

and they show just the differences between the exact two-particle densities and one predicted by the single-particle picture. It is worth noting that the noise correlation can be measured experimentally as well as in the position domain [8,58].

As explained above, the noise correlations (8) are appropriate quantifiers of intercomponent correlations. However, having two different noise correlations for two different experimental parameters, it is very hard to select one having higher correlations. Therefore, it is very convenient to introduce some geometric distance between an actual two-particle density profile and that obtained as a product of a single-particle density profile. Fortunately, single- and two-particle density profiles all have mathematical properties of density distributions. Therefore, the natural metric in their space exists. The Frobenius distance [59,60] (known also as Hilbert-Schmidt norm) can be extracted directly from the noise correlations:

\[
||G_{\rho}|| = \left(\int dx dy |G_{\rho}(x; y)|^2\right)^{1/2}, \quad (9a)
\]

\[
||G_{\tau}|| = \left(\int dp dk |G_{\tau}(p; k)|^2\right)^{1/2}. \quad (9b)
\]

It is quite obvious that the distance vanishes for the noninteracting system and it grows when an average magnitude of the intercomponent correlations increases. In the following, we will quantify correlations mainly in the language of this quantity.

### IV. TWO-ATOM SYSTEM

Before we analyze intercomponent correlations for a larger number of attractively interacting particles, let us start from the simplest situation of two particles of equal mass (\(\mu = 1\)) for which the exact analytical expression for the ground-state wave function and its energy is known [61]. In the context of the noise correlation, the ground state of a few repelling fermions (\(g > 0\)) was considered recently in [56], where the interactions were modeled by the Gaussian-shaped interparticle potential. The width of the Gaussian was much smaller than the natural harmonic oscillator length; hence we reproduce the results by using the pure \(\delta\) potential. As seen in the upper row of Fig. 1(a), for strong repulsions (\(g = 5\)) the noise correlation \(G_{\rho}\) become negative on the diagonal. As noticed in [56], it is a direct manifestation of the fact that due to repulsions it is almost not possible to find two particles in the same position. Importantly, this effect cannot be captured by a simple product of single-particle densities. For the same interaction, also some nontrivial behavior of the noise correlation in the momentum domain \(G_{\tau}\) is present [right panel in Fig. 1(a)].

The situation changes qualitatively for the attractive scenario (\(g = -5\)). In this case, the probability of finding two particles at the same position is highly enhanced when compared to quite poor predictions of single-particle distributions. The most prominent difference between repulsive and attractive systems is, however, visible in the momentum domain. For an attractive system, one finds a very strong anticorrelation between interacting particles signified as a high positive value of the noise correlation along the line \(p = -k\) [see the bottom-right plot in Fig. 1(a)]. This means that the probability of finding two particles having exactly opposite momenta is significantly larger than that predicted by the single-particle picture. Importantly, in the case of the two particles studied, the situation does not change significantly when different masses of particles are considered. As it is seen in Fig. 1(b), even for a large mass ratio (\(\mu = 4\)), the strong correlation in positions and anticorrelation in momenta are present in the system.

### V. MANY-BODY SYSTEM

The two-particle system described above is trivial from a quantum statistics point of view. Therefore, in this section we...
focus on attractive systems with a larger number of particles ($N_1 = N'_1 = 4$). First, we calculate the noise correlation for the balanced system of equal-mass particles (upper row in Fig. 2). As can be seen, the intercomponent correlations for the attractive scenario ($g < 0$) qualitatively resemble main features observed in the two-body scenario—strong correlations in positions and anticorrelations in momenta are clearly visible. It should be noted, however, that the noise distribution in the momentum domain is much flatter along line $p = -k$ than the corresponding one obtained for a smaller number of particles. This effect is forced by an inherent indistinguishability of fermions, and it can be viewed as one of the indicators of the Cooper-like pairing in the system [29].

The situation changes when some factor lifting the balance in the system is present. In principle, in our case, there are two distinct mechanisms leading to the imbalance. The first originates in a direct difference of the number of particles in each component. The second is forced by different masses of the atoms forming opposite components ($\mu \neq 1$). In the case of harmonic confinement there exists a quite important difference between these two scenarios. It is clearly visible in the noninteracting limit. When a difference in numbers of particles is considered, contributions to the total energy of the system of both components are different (they have different Fermi energies). In contrast, the Fermi energy is insensitive to any change in the mass of particles, since in the case of harmonic confinement the single-particle energies $E_i$ do not depend on mass (due to the same frequency $\omega$). All this suggests that these two different mechanisms may have a different impact on the properties of the system. In this work, we focus only on the imbalance forced by the mass difference ($\mu \neq 1$), assuming always a balance in the particle number $N_1 = N'_1$.

As explained in previous works [46,62], with varying $\mu$ the single-particle harmonic orbitals change their shape and they become different for different components. Although the single-particle energies remain unchanged, the mutual repulsions force the system to excite lighter particles. As a consequence, for strong enough repulsions ($g > 0$) the separation of the density profiles emerges. A similar effect of the phase separation driven by the mass imbalance was also studied in the case of homogeneous systems and nonharmonic confinements [44,63,64].

In the case of attractive interactions ($g < 0$), the situation also changes when compared to the balanced system $\mu = 1$. It is clearly visible when the noise correlations are considered (Fig. 2). Although in the position domain the main effect caused by $\mu \neq 1$ is quite trivial (the distribution of the heavier component is just narrower), in the momentum domain the change is significant. As it is seen in Fig. 2, the strong anticorrelations along the line $p = -k$ are smeared, and for a large enough mass ratio $\mu$ any traces of correlated pairs almost vanish. To show quantitatively how the intercomponent correlations change with varying parameters of the system, first, we focus on the Frobenius distance $||G_\tau||$ as a function of the attraction $g$ for the fixed mass ratio $\mu$ (Fig. 3). From this figure, one can easily deduce the general behavior of the system. First, it is clearly visible that independently of the mass ratio $\mu$ the intercomponent correlations grow with an amplitude of interactions. This fact is in full accordance with our intuition—stronger intercomponent forces lead to stronger correlations between particles. One also notices that for fixed interactions and increasing mass ratio $\mu$ correlations measured by $||G_\tau||$ decrease, i.e., particles become less correlated. We can quantify this behavior more precisely in a few different ways. The simplest one is by calculating the derivative $d||G_\tau||/dg$ close to the perturbative regime ($-0.5 \lesssim g < 0$) where a linear growth of correlations is visible. As shown in the inset of Fig. 3, in this range of interactions, the slope of the derivative (the second derivative of $||G_\tau||$) evidently depends on mass and around $\mu \gtrsim 2$ the rate $d||G_\tau||/dg$ becomes almost independent of $\mu$. To find the origin of this surprising change of the slope, we performed a direct
interaction strength $g$ that around $\mu \approx 2$ (the exact value depends on $g$), two sectors are strongly suppressed and the excitations are dominated by only one type of states with $k = 1$. The interactions are measured in the natural units of the harmonic oscillator, $(\hbar^2/\omega m_j)^{1/2}$.

numerical inspection of the many-body ground state. We find that in the case studied, around $\mu \approx 2$, a specific change of the ground-state structure is clearly visible. It can be viewed by performing a specific decomposition of the many-body ground state. Generally, the ground state of the system can be written as a superposition of Fock states belonging to four disconnected sectors of the many-body Hilbert space:

$$|G_0\rangle = \alpha_0 |F_0\rangle + \sum_{k=1}^{3} \sum_{j} \alpha_j^{(k)} |F_j^{(k)}\rangle. \quad (10)$$

The first sector contains only the ground state of the noninteracting system $|F_0\rangle$. Three other sectors spanned by vectors $\{|F_j^{(k)}\rangle\}$ have the following properties. For $k = 1$, the states $|F_j^{(1)}\rangle$ are products of the noninteracting ground state of $\uparrow$ (heavier) particles and different excited states of $\downarrow$ (lighter) ones. Conversely, for $k = 2$, the states $|F_j^{(2)}\rangle$ are products of different excited states of $\uparrow$ (heavier) particles and the noninteracting ground state of $\downarrow$ (lighter) particles. Finally, for $k = 3$ the states $|F_j^{(3)}\rangle$ are built only from the excited states in both components. Having this decomposition, one can calculate contributions from different sectors to the interacting ground state of the system. These contributions are quantified by four numbers $C_k = \sum_{j} |\alpha_j^{(k)}|^2$ and $C_0 = |\alpha_0|^2$. Intercomponent correlations are encoded in excitations of the system and therefore they are directly reflected in nonvanishing values of $C_k$ with $k = 1, 2, 3$. In Fig. 3(b) we plot these quantities as functions of the mass ratio $\mu$ for weak and strong interaction $g$. Note that around $\mu \approx 2$ the derivative has a clearly visible minimum. As it is seen, for equal-mass system $\mu = 1$, all sectors of the system’s excitations contribute to building the correlations. However, when the mass ratio increases, one of the sectors ($k = 1$) starts to dominate. At the same time the other sectors are strongly suppressed, and from around $\mu_c \approx 2$ (the exact value depends on interaction strength), the many-body ground state can be written almost perfectly as a superposition of Fock states having all heavy fermions located in the lowest harmonic oscillator orbitals. This means that for $\mu > \mu_c$, the intercomponent correlations are much less sensitive to any further variations of the mass ratio and they come only from variations of the internal structure of the lighter component.

The transition in the ground-state structure at around $\mu_c$ can also be visualized by plotting the distance $||G_\tau||$ and its derivative $d||G_\tau||/d\mu$ as functions of the mass ratio $\mu$ for different numbers of particles and different interactions (see Fig. 4). As it is seen, for some particular value of the mass ratio $\mu_c \approx 2$ the derivative has a clearly visible minimum. Although the critical value $\mu_c$ depends on system parameters...
(interaction $g$, number of particles), the mechanism is always the same—for mass ratio larger than $\mu_c \approx 2$, the exact form of the ground state is significantly simplified and it manifests a very high probability of finding all heavy particles in their noninteracting ground state.

To lower the energy of the attractive system, it is preferred that the two kinds of particles have the same spatial distributions—then the interaction integrals $U_{ijkl}$ are the largest. In principle, spatial distributions can be adjusted by exciting particles to higher single-particle orbitals. Although the excitation cost is the same for both components, due to the different length scales for the components, adjusting the density profile of heavier particles requires much more excitations. Therefore it is energetically favorable to excite light particles keeping the heavy component almost in the noninteracting ground state. This phenomenological explanation is in full accordance with our numerical many-body calculations described above. Obviously, it cannot predict the exact value of the critical mass ratio $\mu_c$, which is surprisingly small. Let us also note that this argumentation is also in full agreement with the mechanism of the spatial separation induced by repulsions in mass-imbalanced systems described in [46].

To make the analysis as complete as possible, we also discuss the intercomponent correlations in terms of the von Neumann entropy, which has been successfully used for the bosonic system also in the context of the mass imbalance [65]. In contrast to the noise correlation, which is based on the single-particle Hamiltonians $H_F$ and $H_B$ are given by Eqs. (2), respectively [we replace spin-$\uparrow$ fermions described by the field $\Psi_\uparrow(x)$ by interacting spinless bosons described by the field $\Phi(x)$]. The bosonic field operator $\Phi(x)$ obeys the standard commutation relations $[\Phi(x), \Phi(x')] = \delta(x-x')$ and $[\Phi(x), \Phi(x')] = 0$.

The additional term in the Hamiltonian (12) which is proportional to $g'$ describes mutual interactions between bosons. To mimic the fermionic nature of bosons, in the following one assumes that $g'$ tends to infinite repulsions, i.e., according to the Bose-Fermi mapping there exists a one-to-one correspondence between bosonic and fermionic wave functions in the component with replaced statistics [66]. Consequently, the spatial densities of infinitely repelling bosons and noninteracting fermions are exactly the same. Note, however, that there is a significant difference when density distributions of momenta are compared. This theoretical prediction was recently observed experimentally for two distinguishable fermions [2] as well as for bosons confined in elongated traps [67,68]. Of course, it is not possible to set $g' \to \infty$ in the numerical approach used. However, we checked that in the case of four bosons setting $g' = 8$ appropriately mimics very strong repulsions for bosons, and we use this value as the benchmark of infinite repulsions (see Appendix B for details).

In the limit of infinite repulsions between bosons ($g' \to \infty$), the only difference between the two systems studied [modeled by Hamiltonians (1) and (12)] lies in the symmetry of the many-body wave function under exchange of two heavy particles. Despite this fact, the intercomponent correlations (in the momentum domain) forced by attractive forces have significantly different properties. As it is seen in Fig. 5, the noise correlations in the position domain for Bose-Fermi and Fermi-Fermi mixtures are very similar, independently of the mass ratio $\mu$. This observation is a direct manifestation of the mapping mentioned above. However, in the momentum domain, the correlations described by the noise $G_t$ are completely different. Even for the equal-mass case $\mu = 1$ the evident anticorrelation of momenta, previously clearly visible for fermions, is smeared and destroyed. This observation strongly suggests that the fermionic statistics present simultaneously in both components is crucial in building strong pairing (anticorrelations in momenta) in the system.

VI. ROLE OF THE QUANTUM STATISTICS

Finally, let us also discuss the role of the quantum statistics in forming intercomponent correlations in the system studied. This analysis can be done systematically by changing one of the fermionic components to the bosonic one with the same number of particles and masses. In such a case the system is described by the modified Hamiltonian of the form

$$\hat{H} = \int dx \left[ \hat{\Psi}^\dagger(x) H_F \hat{\Psi}(x) + \hat{\Phi}^\dagger(x) H_B \hat{\Phi}(x) \right] + g \int dx \hat{\Psi}^\dagger(x) \hat{\Phi}^\dagger(x) \hat{\Phi}(x) \hat{\Psi}(x) + g' \int dx \hat{\Phi}^\dagger(x) \hat{\Phi}^\dagger(x) \hat{\Phi}(x) \hat{\Phi}(x). \quad (12)$$

VII. CONCLUSION

To conclude, in this paper we discussed the properties of a two-component mixture of a few ultracold atoms in a one-dimensional harmonic trap with attractive mutual interactions. We focus on the intercomponent correlations in terms of the noise correlation, which effectively filters out single-particle features of the system from the two-body densities. In this
FIG. 5. The noise correlation $G_\rho$ and $G_\tau$ calculated for the strongly attractive Bose-Fermi system ($g = -5$) described by the Hamiltonian (12) in the regime of strong repulsions between bosons ($g' = 8$) for $N_B = 4$ bosons and $N_F = 4$ fermions. While the intercomponent correlations in the position domain are very similar to those obtained for fermionic mixtures, the anticorrelations in the momentum domain predicted previously are significantly destroyed, even in the equal-mass case ($\mu = 1$). Position and momentum are measured in natural units of the harmonic oscillator, $\sqrt{\hbar/m\omega}$ and $\sqrt{\hbar m\omega}$, respectively.

In addition, by studying two-component Bose-Fermi mixtures, we show that the quantum statistics play a crucial role in forming intercomponent correlations. In this kind of a system, the anticorrelation of particles is strongly disturbed, even in the system of equal-mass components. Since the noise correlation can be measured in current experiments [8], our results may shed some light on incoming experiments with attractively interacting few-body systems.

Our analysis is quite general, and it might also be important for building our understanding of different condensed-matter problems related to the “few” to “many” crossover or unconventional superconductivity which originates in a pairing of different-mass fermions [69–71]. For the same-mass fermions, the Fermi surfaces of both components match each other. Whenever the Fermi surfaces do not match perfectly, a nonzero net momentum of pairs together with unconventional correlations may appear in the system. A similar mechanism can occur for equal-mass systems with different trapping frequencies.

FIG. 6. The successive fidelity $F_K$ of the many-body ground state of $N_\uparrow = N_\downarrow = 4$ equal-mass fermions obtained by exact diagonalization for different values of the cutoff $K$ and two interaction strengths (a) $g = -1$ and (b) $g = -5$. For a large-enough cutoff the fidelity stabilizes (horizontal dashed line). Note that for better visibility, we use a nonlinear scaling on the horizontal axis.

The ratio for a K-Li mixture, $\mu = 40/6$, and Dy-K mixture, $\mu = 161/40$ at which the transition occurs. It is much smaller than for an impurity problem in the bosonic system studied recently [65].
ACKNOWLEDGMENTS

We would like to thank Remigiusz Augusiak for valuable comments and Jacek Dobrzyńiecki for a thorough reading of the manuscript. This work was supported by the (Polish) National Science Center through Grants No. 2016/21/N/ST2/03315 (D.P.) and 2016/22/E/ST2/00555 (T.S.). Numerical calculations were partially carried out in the Interdisciplinary Centre for Mathematical and Computational Modelling, University of Warsaw (ICM), under Computational Grant No. G75-6.

APPENDIX A: NUMERICAL CONVERGENCE

The convergence of the numerical method is ascertained by checking the successive fidelity of the many-body ground state \[\langle G_0 | G_1 \rangle \]. The many-body Fock basis \([|\phi_i\rangle]\) is built from the lowest \(K\) single-particle orbitals \(\phi_i(x), i \in \{0, \ldots, K - 1\}\). After numerical diagonalization of the many-body Hamiltonian (3) one obtains the many-body ground state \(|G_0\rangle\) and its energy \(E_0^{[K]}\). By performing calculations for successive values of the cutoff \(K\), we calculate the successive fidelity, defined as

\[
F_K = \frac{\langle G_0^{[K-1]} | G_0^{[K]} \rangle}{\langle G_0^{[K]} | G_0^{[K]} \rangle}.
\]

In Fig. 6 we plot the difference \(1 - F_K\) as a function of the cutoff \(K\) for the system of \(N = N_1 = N_2 = 4\) fermions (\(\mu = 1\)) and two different interaction strengths \(g = -1\) and \(g = -5\) [Figs. 6(a) and 6(b), respectively]. For convenience, we also display the corresponding sizes of the cropped many-body Hilbert space. We assume that the ground state is found with sufficient accuracy if the changes of the fidelity are stabilized with the increasing cutoff (horizontal lines in Fig. 6). Note that we use a nonlinear scaling on the horizontal axis, i.e., the fidelity changes very slowly (in the stabilization region) with
an increasing dimension of the Hilbert space. In these cases no significant changes of single- and two-particle densities with increasing cutoff are visible. The differences are also not significant when other quantities discussed are considered. In Fig. 7(a) we show the Frobenius distance $|\|G_F^\mu|\|$ for $\mu = 1$ and $\mu = 6$ calculated with different cutoffs $K$. As it is seen, in the range of ranges considered ($|g| \approx 5$) the final result is well converged for $K \approx 10$. The situation is less obvious in the case of a derivative of the Frobenius distance $d_1|\|G_F^\mu|\|/d\mu$ (middle row in Fig. 4), which is much more sensitive to any changes of the Fock basis [see example for $N_F = N_s = 5$ and $g = −5$ in Fig. 7(b)]. However, all the curves with $K \approx 10$ unambiguously support the observation that the ground state of the system undergoes a specific transition at around $\mu_c \approx 2$.

**APPENDIX B: FERMIORIZATION LIMIT**

In the case of mixed statistics mixtures (bosons and fermions) discussed in Sec. VI, we assumed that the infinite repulsion limit between bosons ($g' \to \infty$) is appropriately mimicked by the finite value of interactions $g' = 8$. To clarify this assumption, we compare the ground-state spatial properties of $N_B = 4$ noninteracting fermions (having the same spatial properties as infinitely repelling bosons) to $N_B = 4$ bosons with different mutual interactions, $g' \in \{0, 1, 8\}$. Assuming that the ground state of $N_B$ interacting bosons ($N_F$ fermions) is $|G_B\rangle (|G_F\rangle)$, one defines the single-particle density as

$$\rho^{(1)}_B(x) = \langle G_B|\hat{\Phi}^\dagger(x)\hat{\Phi}(x)|G_B\rangle, \quad (B1a)$$

$$\rho^{(1)}_F(x) = \langle G_F|\hat{\Psi}^\dagger(x)\hat{\Psi}(x)|G_F\rangle, \quad (B1b)$$

for bosons and fermions, respectively. These distributions are shown in Fig. 8(a). Similarly, the two-particle density distributions for the same systems of bosons and fermions are shown in Fig. 8(b) and are defined as

$$\rho^{(2)}_B(x,y) = \langle G_B|\hat{\Phi}^\dagger(x)\hat{\Phi}^\dagger(y)\hat{\Phi}(y)\hat{\Phi}(x)|G_B\rangle, \quad (B2a)$$

$$\rho^{(2)}_F(x,y) = \langle G_F|\hat{\Psi}^\dagger(x)\hat{\Psi}^\dagger(y)\hat{\Psi}(y)\hat{\Psi}(x)|G_F\rangle, \quad (B2b)$$

respectively. It is clearly seen that the distributions of noninteracting fermions and noninteracting or weakly interacting bosons are essentially different. However, for strong repulsions the bosonic system undergoes fermionization, and for $g' = 8$ its spatial distributions become very close to corresponding distributions of the noninteracting fermionic system. This observation supports our assumption that the system of $N_B = 4$ bosons interacting with the strength $g' = 8$ can be safely treated as a benchmark of infinite repulsion.

[1] A. Wenz, G. Zürn, S. Murmann, I. Brouzos, T. Lompe, and S. Jochim, Science 342, 457 (2013).
[2] G. Zürn, F. Serwane, T. Lompe, A. N. Wenz, M. G. Ries, J. E. Bohn, and S. Jochim, Phys. Rev. Lett. 108, 075303 (2012).
[3] F. Serwane, G. Zürn, T. Lompe, T. Ottenstein, A. Wenz, and S. Jochim, Science 332, 336 (2011).
[4] S. Murmann, F. Deuretzbacher, G. Zürn, J. Bjørnson, S. M. Reimann, L. Santos, T. Lompe, and S. Jochim, Phys. Rev. Lett. 115, 215301 (2015).
[5] A. M. Kaufman, B. J. Lester, M. Foss-Feig, M. L. Wall, A. M. Rey, and C. A. Regal, Nature (London) 527, 208 (2015).
[6] A. Bergschneider, V. M. Klinkhamer, J. H. Becher, R. Klemt, G. Zürn, P. M. Preiss, and S. Jochim, Phys. Rev. A 97, 063613 (2018).
[7] G. Zürn, A. N. Wenz, S. Murmann, A. Bergschneider, T. Lompe, and S. Jochim, Phys. Rev. Lett. 111, 175302 (2013).
[8] A. Bergschneider, V. M. Klinkhamer, J. H. Becher, R. Klemt, L. Palm, G. Zürn, S. Jochim, and P. M. Preiss, arXiv:1807.06405.
[9] W. S. Bakr, A. Peng, M. E. Tai, R. Ma, J. Simon, J. I. Gillen, S. Foelling, L. Pollet, and M. Greiner, Science 329, 547 (2010).
[10] J. F. Sherson, C. Weitenberg, M. Endres, M. Cheneau, I. Bloch, and S. Kuhr, Nature (London) 467, 68 (2010).
[11] A. Omran, M. Boll, T. A. Hikler, K. Kleinlein, G. Salomon, I. Bloch, and C. Gross, Phys. Rev. Lett. 115, 263001 (2015).
[12] L. W. Cheuk, M. A. Nichols, M. Okan, T. Gersdorf, V. V. Ramasesh, W. S. Bakr, T. Lompe, and M. W. Zwierlein, Phys. Rev. Lett. 114, 193001 (2015).
[13] L. W. Cheuk, M. A. Nichols, K. R. Lawrence, M. Okan, H. Zhang, and M. W. Zwierlein, Phys. Rev. Lett. 116, 235301 (2016).
[14] L. W. Cheuk, M. A. Nichols, K. R. Lawrence, M. Okan, H. Zhang, E. Khatami, N. Trivedi, T. Paiva, M. Rigol, and M. W. Zwierlein, Science 353, 1260 (2016).
[15] M. F. Parsons, F. Huber, A. Mazurenko, C. S. Chiu, W. Seitaawan, K. Woolley-Brown, S. Blatt, and M. Greiner, Phys. Rev. Lett. 114, 213002 (2015).
[16] G. J. A. Edge, R. Anderson, D. Jervis, D. C. McKay, R. Day, S. Trotzky, and J. H. Thywissen, Phys. Rev. A 92, 063406 (2015).
[17] E. Haller, J. Hudson, A. Kelly, D. A. Cotta, B. Peaudecerf, G. D. Bruce, and S. Kuhr, Nat. Phys. 11, 738 (2015).
[18] I. Brouzos and P. Schmelcher, Phys. Rev. A 87, 023605 (2013).
[19] T. Sowiński, T. Grass, O. Dutta, and M. Lewenstein, Phys. Rev. A 88, 033607 (2013).
[20] S. E. Gharsa and D. Blume, Phys. Rev. Lett. 111, 045302 (2013).
[21] M. A. García-March, B. Juliá-Díaz, G. E. Astrakharchik, T. Busch, J. Boronat, and A. Polls, New J. Phys. 16, 103004 (2014).
[22] T. Fogarty, L. Ruks, J. Li, and T. Busch, SciPost Phys. 6, 021 (2018).
[23] M. A. García-March, B. Juliá-Díaz, G. E. Astrakharchik, J. Boronat, and A. Polls, Phys. Rev. A 90, 063605 (2014).
[24] A. G. Volosniev, D. V. Fedorov, A. S. Jensen, N. T. Zinner, and M. Valiente, Few-Body Syst. 55, 839 (2014).
[25] A. G. Volosniev, D. V. Fedorov, A. S. Jensen, and N. T. Zinner, Eur. Phys. J.: Spec. Top. 224, 585 (2015).
[26] P. Kosić, Few-Body Syst. 52, 49 (2012).
[27] N. T. Zinner and A. S. Jensen, J. Phys. G: Nucl. Part. Phys. 40, 053101 (2013).
[28] P. D’Amico and M. Rontani, Phys. Rev. A 91, 043610 (2015).
[29] T. Sowiński, M. Gajda, and K. Rzążewski, Europhys. Lett. 109, 26005 (2015).
[30] P. O. Bugnion, J. A. Lothhouse, and G. J. Conduit, Phys. Rev. Lett. 111, 045301 (2013).
[31] L. Rammelmüller, W. J. Porter, and J. E. Drut, Phys. Rev. A 93, 033639 (2016).
[32] J. McKenney, C. Shill, W. Porter, and J. Drut, J. Phys. B: At., Mol. Opt. Phys. 49, 225001 (2016).
[33] J. Bjerlin, S. M. Reimann, and G. M. Bruun, Phys. Rev. Lett. 116, 155302 (2016).
[34] E. Wille, F. M. Spiegelhalder, G. Kernner, D. Naik, A. Trenkwalder, G. Hendl, F. Schreck, R. Grimm, T. G. Tiecke, J. T. M. Walraven, S. J. J. M. F. Kokkelmans, E. Tiesinga, and P. S. Julienne, Phys. Rev. Lett. 100, 053201 (2008).
[35] T. G. Tiecke, M. R. Goosen, A. Ludewig, S. D. Gensemer, S. Kraft, S. J. J. M. F. Kokkelmans, and J. T. M. Walraven, Phys. Rev. Lett. 104, 053202 (2010).
[36] M. Cetina, M. Jag, R. S. Lous, I. Fritsche, J. T. Walraven, R. Grimm, J. Levinsen, M. M. Parish, R. Schmidt, M. Knap et al., Science 354, 96 (2016).
[37] C. Ravensbergen, V. Corre, E. Soave, M. Kreyer, E. Kirilov, and R. Grimm, Phys. Rev. A 98, 063624 (2018).
[38] Z. Hadzibabic, C. A. Stan, K. Dieckmann, S. Gupta, M. W. Zwierlein, A. Görlitz, and W. Ketterle, Phys. Rev. Lett. 88, 160401 (2002).
[39] K. Günter, T. Stöferle, H. Moritz, M. Köhl, and T. Esslinger, Phys. Rev. Lett. 96, 180402 (2006).
[40] T. Best, S. Will, U. Schneider, L. Hackermüller, D. van Oosten, I. Bloch, and D.-S. Lühmann, Phys. Rev. Lett. 102, 030408 (2009).
[41] X. Cui and T. L. Ho, Phys. Rev. Lett. 110, 165302 (2013).
[42] N. J. S. Loft, A. S. Dehkharghani, A. G. Vołosniev, and N. T. Zinner, Eur. Phys. J. D 69, 65 (2015).
[43] D. Pęcak, M. Gajda, and T. Sowiński, New J. Phys. 18, 013030 (2016).
[44] D. Pęcak, A. S. Dehkharghani, N. T. Zinner, and T. Sowiński, Phys. Rev. A 95, 053632 (2017).
[45] N. L. Harshman, M. Olshani, A. S. Dehkharghani, A. G. Vołosniev, S. G. Jackson, and N. T. Zinner, Phys. Rev. X 7, 041001 (2017).
[46] S. Mistakidis, G. Katsimiga, G. Koutentakis, and P. Schmelcher, arXiv:1808.00040.