Flow instability in $^3$He-A as analog of generation of hypermagnetic field in early Universe.

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It is now well-recognized that the Universe may behave like a condensed matter system in which several phase transitions have taken place. Superconductors and the superfluid phases of $^3$He are condensed matter systems with useful similarities to the Universe: they both contain Bose fields (order parameter) and Fermions (quasiparticles) which interact in a way similar to the interaction of Higgs and gauge particles with fermions in particle physics\cite{1,2}. This analogy allows us to simulate many properties of the cosmologically relevant physical (particle physics) vacuum in condensed matter, while direct experiments in particle physics are still far from realization.

Recently, the anomalous generation of momentum (called “momentogenesis”) was experimentally confirmed in $^3$He: in the non-trivial background of a moving $^3$He vortex, quantum effects gave rise to the production of quasiparticles with momentum which were detected by measuring the force on the vortex\cite{3}. This phenomenon is based on the same physics as the anomalous generation of matter in particle physics and bears directly on the cosmological problem of why the Universe contains much more matter than antimatter (“baryogenesis”). Here we report the experimental observation of the effect opposite to momentogenesis: the conversion of quasiparticle momentum into a non-trivial order parameter configuration or “texture”. The corresponding process in a cosmological setting would be the creation of a primordial magnetic field due to changes in the matter content.

Processes in which magnetic fields are generated are very relevant to cosmology as magnetic fields are ubiquitous in the universe. The Milky Way, other galaxies, and clusters of galaxies are observed to have a magnetic field whose generation is still not fully understood. One possible mechanism is that a seed field was amplified by the complex motions associated with galaxies and clusters of galaxies. The seed field itself is usually assumed to be of cosmological origin.

In earlier work, it has been noted that the two genesis problems in cosmology – baryo- and magnetogenesis – may be related to each other\cite{4,5}. More recently, a stronger possible connection has been detailed in \cite{6,7} (related work may be found in \cite{8}). In this work, magnetogenesis proceeds in the following two steps:

(i) At an early stage of the universe, possibly at the Grand Unification epoch (10$^{-35}$ s after the big bang), an excess of chiral right-handed electrons, $e_R$, is produced due to parity violation. This is effectively described by introducing a chemical potential, $\mu_R > 0$, for the right-handed electrons leading to a certain number of $e_R$ in thermal equilibrium at temperature $T$. The equilibrium relativistic energy and particle number density are:

$$\epsilon_R = \frac{1}{6} T^2 \mu_R^2, \quad n_R = \frac{\partial \epsilon_R}{\partial \mu_R} = \frac{1}{3} T^2 \mu_R \quad (1)$$

(ii) A striking property of a system with chiral fermions is that fermionic charge such as $(n_R - n_L)$ – the number of right-handed minus the number of left-handed particles – is conserved at the classical level but not if quantum properties of the physical vacuum are taken into account. This charge can be transferred to the “inhomogeneity” of the vacuum via the axial anomaly as was first theoretically predicted by Adler\cite{10} and Bell and Jackiw\cite{11}, while its condensed matter analogue was recently verified experimentally in $^3$He\cite{3}. The inhomogeneity which absorbs the fermionic charge arises as a magnetic field configuration, and the charge absorbed by the magnetic field, $\nabla \times A$, can be expressed as:

$$(n_R - n_L)A = \frac{1}{2\pi^2} A \cdot (\nabla \times A) \quad (2)$$

The right-hand side is the so called Chern-Simons (or topological) charge of the magnetic field.

The transformation of particles into a magnetic field configuration opens the possibility for the cosmological origin of a magnetic field from a system of fermions and this is a key step in the scenario described by Joyce and Shaposhnikov\cite{12}. The $e_R$ excess generated in the early universe survives until the electroweak phase transition (at about 10$^{-10}$ s after the big bang) when anomalous lepton (and baryon) number violating processes become efficient and can erase the excess. In doing so, there is an instability towards the production of a hypermagnetic field. Since, a part of the hypermagnetic field is the electromagnetic field, the present universe contains a primordial (electromagnetic) magnetic field.

Now we discuss the corresponding process in $^3$He-A, in which quasiparticle momentum (the relative flow of normal and superfluid components) is transformed via the chiral anomaly into the order parameter texture. This
For example, in the presence of counterflow, the normal component of the superfluid, the constant part of the \( \hat{1} \)-vector is oriented along the flow: \( \hat{1}_0 \parallel \mathbf{w} \).

The space and time dependence of \( \delta \) produces a force on quasiparticles equivalent to the force of an “hypermagnetic field” \( \mathbf{B} = k_F \nabla \times \delta \) acting on relativistic massless fermions of unit charge \( e = \pm 1 \). Thus the analog of the vector potential \( \mathbf{A}_H \) of the hypermagnetic field is played by \( k_F \delta \).

It is important for our consideration that the \(^3\)He-A liquid is anisotropic in the same manner as a nematic liquid crystal. For the relativistic fermions this means that their motion is determined by the geometry of an effective spacetime. In \(^3\)He-A this geometry is described by the following metric tensor

\[
g^{ik} = c_{\perp}^2 (\delta^{ik} - \hat{l}_0^i \hat{l}_0^k) + c_{\parallel}^2 \hat{l}_0^i \hat{l}_0^k, \quad g^{00} = -1 \quad (4)
\]

The quantities \( c_{\parallel} = pr/m \) and \( c_{\perp} = \Delta_0/pF \) correspond to velocities of “light” propagating along and transverse to \( \hat{1}_0 \), here \( m \) is the mass of the \(^3\)He atom and \( \Delta_0 \) is the amplitude of the gap.

In the presence of counterflow, \( \mathbf{w} = \mathbf{v}_n - \mathbf{v}_s \), of the normal component of \(^3\)He-A liquid with respect to the superfluid, the constant part of the \( \hat{1} \)-vector is oriented along the flow: \( \hat{1}_0 \parallel \mathbf{w} \). In the vicinity of the gap nodes, the energy of quasiparticles is Doppler shifted by the amount \( p \cdot \mathbf{w} \approx \pm pF (\hat{1}_0 \cdot \mathbf{w}) \). The counterflow therefore produces what would be an effective chemical potential in particle physics. For right-handed particles, this is \( \mu_R = pF (\hat{1}_0 \cdot \mathbf{w}) \) (Fig. (b)) and for left-handed particles it is \( \mu_L = -\mu_R \).

If the direction of the counterflow is chosen such that \( \mu_R > 0 \), the quantities in \(^3\)He-A corresponding to the energy and number density in eq. (4), are the kinetic energy of the counterflow and the \( \hat{1}_0 \)-projection of its linear momentum, \( \mathbf{P} = \hat{\rho}_n \mathbf{w} \)

\[
\epsilon_R = \frac{1}{2} \hat{\rho}_n \mathbf{w} \cdot \mathbf{w}, \quad n_R = \frac{1}{pF} \mathbf{P} \cdot \hat{1}_0. \quad (5)
\]

Here \( \hat{\rho}_n \) is the density of the normal component, which in the anisotropic \(^3\)He-A is a tensor. The normal component consists of the thermally activated quasiparticles and corresponds to the system of chiral fermions.

To make sure that eq. (3) is analogous to eq. (4), let us consider the low-temperature limit \( T \ll T_c \), where \( T_c \sim \Delta_0 \) is the superfluid transition temperature. Then using eq. (3.94) of [13] for the density of the normal component of \(^3\)He-A and the \(^3\)He-A equivalent of the chemical potential one obtains

\[
\epsilon_R \approx \frac{1}{6} mkF \frac{T^2}{\Delta_0} (\hat{1}_0 \cdot \mathbf{w})^2 \approx \frac{1}{6} \sqrt{gT^2 \mu_R^2}. \quad (6)
\]

In the last equality an over-all constant appears to be the square root of the determinant of the effective metric

\[
\text{FIG. 1. a) Gap nodes and chiral fermions in } ^3\text{He-A. The pink region illustrates how the gap in the quasiparticle spectrum depends on position at the Fermi surface. The gap has two nodes along the } \hat{1} \text{ directions which are shown as the North and South poles of the Fermi sphere. The nodes occur at the North pole only for those quasiparticles that have a negative projection of the Bogoliubov-Nambu spin along their momentum. Therefore they correspond to the left-handed particles. Similarly, the node at the South pole applies only to right-handed quasiparticles. In addition, the interactions of the quasiparticles near the gap nodes with the order parameter is analogous to the interaction of massless fermions with background hypermagnetic and gravitational fields. The hypercharge of the right-handed quasiparticles is } -1 \text{ and that of left-handed quasiparticles is } +1.\]

is, in essence, the counterflow instability in \(^3\)He-A, which has been intensively discussed theoretically (see [12] and Section 7.10 of the book [13]) and recently investigated experimentally in the Helsinki rotating cryostat [14]. The \(^3\)He-A analogy of the cosmological scenario described in [1] closely follows the two steps outlined above.

(i) The Cooper pairs in \(^3\)He-A have angular momentum \( \mathbf{h} \) and locally all pairs have their angular momentum aligned along a direction \( \mathbf{1} \), which also indicates (Fig. (a)) the direction to the gap nodes of Bogoliubov quasiparticles. In particle physics language, the \(^3\)He-A vacuum is described by the uniform expectation value of a unit vector field \( \hat{1} \) and the fermionic energy levels have a zero mode for momenta in the direction of \( \hat{1} \). Close to the gap nodes these quasiparticles represent chiral relativistic fermions: they are left-handed in the vicinity of the north pole \( \mathbf{p} \approx +pF \hat{1} \), where \( pF \) is the Fermi momentum, and they are right-handed in the vicinity of the south pole where \( \mathbf{p} \approx -pF \hat{1} \).

In general, the vector \( \hat{1} \) will not be absolutely uniform. For example it may oscillate around some background distribution \( \hat{1}_0 \), which we further choose as uniform:

\[
\hat{1} = \hat{1}_0 + \delta \hat{1}(\mathbf{r}, t). \quad (3)
\]

For example, in the presence of counterflow, \( \mathbf{w} = \mathbf{v}_n - \mathbf{v}_s \), of the normal component of \(^3\)He-A liquid with respect to
in $^3$He-A: $\sqrt{-g} = 1/c_l c_s^2 = m k_F/\Delta_c$. This makes eq. (3) and eq. (4) absolutely equivalent. The factor $\sqrt{-g}$ should also be included in eq. (5) as it is part of the spatial volume element. However, isotropy is assumed in the cosmological scenario and so, in Cartesian coordinates, the factor is equal to 1.

(ii) The inhomogeneity which absorbs the fermionic charge, is represented by a magnetic field configuration in real vacuum and by a $\delta l$-texture in $^3$He-A. However, eq. (2) applies in both cases, if in $^3$He we use the identification $A = k_g \delta l$.

Just as in the particle physics case, we now consider the instability towards the production of texture due to the excess of chiral particles. This instability can be seen by considering the energy of the inhomogeneous texture on the background of the superflow. In the geometry of the superflow, the gradient contribution to the free energy of the $\delta l$-texture is completely equivalent to the conventional energy of the hypermagnetic field

$$F_{\text{magn}} = \ln \left( \frac{\Delta_0^2}{T^2} \right) \frac{g_{FF} v_F}{2 \pi^2 \hbar} \hat{\delta} \cdot (\nabla \times \hat{\delta})^2$$

$$= \frac{\sqrt{-g}}{4 \pi e_{eff}^2} g^{ij} g^{kl} F_{ik} F_{jl} \ . \quad (7)$$

Here $F_{ik} = \nabla_i A_k - \nabla_k A_i$ and we have included the effective anisotropic metric in eq. (4) appropriate for $^3$He-A.

It is interesting that the logarithmic factor in the gradient energy plays the part of the running coupling $e_{eff}^2 = (1/3\pi \hbar c) \ln(\Delta_0/T)$ in particle physics, where $e_{eff}$ is the effective hyperelectric charge; while the gap amplitude $\Delta_0$, plays the part of an ultraviolet cutoff energy scale. Now if one has the counterflow in $^3$He-A, or its equivalent – an excess of chiral charge produced by the chemical potential $\mu_R$ – the anomaly gives rise to an additional effective term in the magnetic energy, corresponding to the interaction of the charge absorbed by the magnetic field with the chemical potential. This effective energy term is:

$$F_{CS} = (n_R - n_L) \mu_R = \frac{1}{2 \pi^2} \mu_R A \cdot (\nabla \times A)$$

$$= \frac{3 \hbar}{2 m \rho} (\hat{\delta} \cdot w)(\hat{\delta} \cdot \nabla \times \hat{\delta}) \ , \quad (8)$$

The right-hand side corresponds to the well known anomalous interaction of the counterflow with the $l$-texture in $^3$He-A, where $\rho$ is the mass density of $^3$He (the additional factor of $3/2$ enters due to nonlinear effects).

For us the most important property of this term is that it is linear in the derivatives of $\hat{\delta}$. Its sign can thus be negative, while its magnitude can exceed the positive quadratic term in eq. (5). This leads to the helical instability towards formation of the inhomogeneous $\delta l$-field. During this instability the kinetic energy of the quasiparticles in the counterflow (analog of the energy sitting in the fermionic degrees of freedom) is converted into the energy of inhomogeneity $\nabla \times \hat{\delta}$, which is the analog of the magnetic energy of the hypercharge field.

Until now we had a strong analogy between particle physics and the $^3$He system. An important difference, however, arises from the “mass of the hyperphoton”, which stabilizes the counterflow in $^3$He-A. In the electroweak theory the hyperphoton is massless at high tem...
A field provides a mass to the “hypercharge gauge field” $H$ giving rise to a characteristic satellite peak in the (ATC) continuous vortex (see Fig. 3(a)). ATC vortices in the cell represents a so-called Anderson-Toulouse-Chechetkin (ATC) vortex. Initially no vortices are present in the vessel. When the velocity of the counterflow $\mathbf{v}$ in the $l$-vector forms a regular periodic structure. Each elementary cell of the structure represents the ATC vortex with a quantum of circulation of the superfluid velocity about the cell boundary $C$: $\oint_C \mathbf{v} \cdot d\mathbf{r} = h/m$. (b) The NMR signal from the array of ATC vortices in the container [17]. The position of the satellite peak indicates the type of the vortex, while the intensity is proportional to the number of vortices in the cell.

The threshold value $\Omega_c$ of the rotation velocity $\Omega$, at which the helical instability occurs, determines the critical value of the chemical potential $\mu_R$ of the chiral electrons exceeds a critical value, an instability takes place, and the container becomes filled with the $l$-texture (analogue of a hypermagnetic field) forming the vortex array.

The threshold value $\Omega_c$ is the radius of the vessel. The magnitude of $\mu_R$, which we find from the measurements [14], is in good quantitative agreement with the theoretical estimation of the mass of the “hyperphoton”, determined by the spin-orbit interaction in $^3$He-A. Thus we have modeled the formation of the hypermagnetic field for different masses of the “hyperphoton”. We have also observed the flow instability in the limit when the “hyperphoton” has zero mass: first the field $H$ was reduced to zero, then the container was accelerated to rotation at some velocity $\Omega$, and finally the rotating state, which had been obtained in this way, was analyzed in the previous manner in the NMR conditions. Since the critical velocity decreases with decreasing magnetic field, the field dependence of the instability could be worked out by this technique and the “hypermagnetic field” could be measured. In this case $\mu_R$ was essentially reduced.

Our result together with that in [3] show that the chiral anomaly is an important mechanism for the interaction of textures (the analogue of the hypercharge magnetic fields and cosmic strings) with fermionic excitations (analogue of quarks and leptons). We have thus experimentally checked both processes which are induced by the anomaly: the nucleation of fermionic charge from the vacuum in [3] and the inverse process of the nucleation of an effective magnetic field from the fermion current.
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