Dynamical construction of Kähler-Einstein metrics on bounded pseudoconvex domains

Hajime Tsuji

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Abstract
We shall prove that the complete Kähler-Einstein metric on a bounded strongly pseudoconvex domain with $C^\infty$-boundary is the normalized limit of a sequence of Bergman metrics. This is a noncompact version of [T, p.110, Theorem 1.2].

1 Introduction
Let $\Omega$ be a bounded strongly pseudoconvex domain in $\mathbb{C}^n$ with $C^\infty$-boundary $\partial \Omega$. There has been constructed several canonical metrics on $\Omega$.

Among them, the Bergman metric and the Kähler-Einstein metric are important. First the Bergman metric is constructed as follows. Let $A^2(\Omega)$ be the space of $L^2$-holomorphic $n$-forms on $\Omega$, i.e.,

$$A^2(\Omega) := \left\{ \eta \in H^0(\Omega, \mathcal{O}_\Omega(K_\Omega)) \left| \int_{\Omega} |\eta|^2 < \infty \right. \right\},$$

where $|\eta|^2 := (\sqrt{-1})^n \eta \wedge \bar{\eta}$. $A^2(\Omega)$ has a structure of a Hilbert space with respect to the inner product:

$$(\sigma, \tau) := (\sqrt{-1})^n \int_{\Omega} \sigma \wedge \bar{\tau} \quad (\sigma, \tau \in A^2(\Omega)).$$

Let $\{\sigma_j\}_{j=1}^\infty$ be a complete orthonormal basis of $A^2(\Omega)$. We set

$$K(\Omega)(z) := \sum_{j=1}^\infty |\sigma_j(z)|^2 \quad (z \in \Omega)$$

and call it (the diagonal part of) the Bergman kernel or the Bergman volume form on $\Omega$. Then

$$\omega_B := (\sqrt{-1})\partial \bar{\partial} \log K(\Omega)$$

is a complete Kähler form on $\Omega$ ([Ko]), $K(\Omega)$ has the following extremal property:

$$K(\Omega)(z) = \sup \{|\sigma|^2(z) | \sigma \in A^2(\Omega), \| \sigma \| = 1 \}.$$

Another important complete Kähler metric on $\Omega$ is the complete Kähler-Einstein metric. In [C-Y], Cheng and Yau constructed a complete Kähler-Einstein metric on a bounded strongly pseudoconvex domain in $\mathbb{C}^n$ with $C^\infty$-boundary $\partial \Omega$. More precisely they proved the following theorem:
Theorem 1.1 (C-Y) Let \( \Omega \) be a bounded strongly pseudoconvex domain in \( \mathbb{C}^n \) with \( C^\infty \)-boundary.

Then there exists a complete \( C^\infty \)-Kähler-Einstein form \( \omega_E \) on \( \Omega \) such that

\[ -\text{Ric}(\omega_E) = \omega_E \]

holds on \( \Omega \), where

\[ \text{Ric}(\omega_E) := -\sqrt{-1} \partial \bar{\partial} \log \det(g_{ij}). \]

Here \((g_{ij})\) is the hermitian matrix defined by

\[ \omega_E = \frac{\sqrt{-1}}{2} \sum_{i,j=1}^{n} g_{ij} \, dz_i \wedge dz_j. \]

We call the volume form \( dV_E := \frac{1}{n!} \omega^n_E \) associated with \( \omega_E \) the Kähler-Einstein volume form on \( \Omega \).

The purpose of this paper is to relate the Bergman volume form \( K(\Omega) \) and the Kähler-Einstein volume form \( dV_E \) in terms of a dynamical system of Bergman kernels as in [T].

First we set \( K_1 := K(\Omega) \) and let \( h_1 := K_1^{-1} \). Then \( h_1 \) is a \( C^\infty \)-hermitian metric on the canonical bundle \( K_\Omega \) of \( K_\Omega \) with strictly positive curvature.

Suppose that we have already constructed \( \{K_1, \ldots, K_m\} \) and \( \{h_1, \ldots, h_m\} \) \((h_k = K_k^{-1})\). Then we define

\[
K_{m+1} := K(\Omega, (m+1)K_\Omega, h_m),
\]

i.e.,

\[
K_{m+1} = \sum_{j=1}^{\infty} |\sigma_j^{(m+1)}|^2,
\]

where \( \{\sigma_1^{(m+1)}, \ldots, \sigma_k^{(m+1)}, \ldots\} \) is a complete orthonormal basis of the Hilbert space:

\[
A^2(\Omega, (m+1)K_\Omega, h_m) := \left\{ \sigma \in H^0(\Omega, \mathcal{O}_\Omega((m+1)K_\Omega)) \mid \int_{\Omega} |\sigma|^2 \cdot h_m < +\infty \right\},
\]

with respect to the inner product:

\[
(\sigma, \tau)_{(m+1)} = (\sqrt{-1})^{n^2} \int_{\Omega} \sigma \wedge \bar{\tau} : h_m.
\]

And we set

\[
h_{m+1} := K_{m+1}^{-1}.
\]

Since \( A^2(\Omega, (m+1)K_\Omega, h_m) \) is very ample by Hörmander's \( L^2 \)-estimate of \( \bar{\partial} \)-operators, we see that the dynamical system \( \{K_m\}_{m=1}^{\infty} \) is well defined. The dynamical system is essentially the same as in [T]. The following is the main theorem in this paper.
Theorem 1.2 Let $\Omega$ be a bounded strongly pseudoconvex domain in $\mathbb{C}^n$ with $C^\infty$-boundary. Let $\{K_m\}_{m=1}^\infty$ be the dynamical system of Bergman kernels constructed as above.

Then $\lim_{m \to \infty} \sqrt{(ml)^{-n}K_m}$ exists in compact uniform topology and

$$\lim_{m \to \infty} \sqrt{(ml)^{-n}K_m} = (2\pi)^{-n}dV_E$$

holds. □

The proof of Theorem 1.2 is very similar to the proof of [T, p.110, Theorem 1.2]. We just need to take care of the uniformity of the estimates, since $\Omega$ is noncompact.

2 Proof of Theorem 1.2

To ensure the uniformity of the estimate, we shall introduce the following notion.

Definition 2.1 Let $M$ be a complete Kähler manifold. We say that $M$ has bounded geometry of order $\ell$, if $M$ admits a covering of holomorphic coordinate charts $\{(V, (v^1, \cdots, v^n))\}$ and positive numbers $R, c, A_1, \cdots, A_\ell$ such that

1. For any $x_0 \in M$, there is a coordinate chart $(V, (v^1, \cdots, v^n))$ with $x_0 \in V$ and with respect to the Euclidean distance $d$ defined by $v^i$-coordinates, $d(x_0, \partial V) > \sqrt{n} \cdot R$ holds.

2. If $(g_{i\bar{j}})$ denotes the metric tensor with respect to $(V, (v^1, \cdots, v^n))$, then $(g_{i\bar{j}})$ is $C^\ell$ and

$$\frac{1}{C}(\delta_{i\bar{j}}) < (g_{i\bar{j}}) < C(\delta_{i\bar{j}})$$

holds and for any multi index $\alpha, \beta$ with $|\alpha| + |\beta| \leq \ell$, we have

$$|\partial^{|\alpha|+|\beta|} g_{i\bar{j}}| \leq A_{|\alpha|+|\beta|}$$

holds. □

Theorem 2.2 ([C-Y]) Let $\Omega$ be a bounded strongly pseudoconvex domain in $\mathbb{C}^n$ with $C^\infty$-boundary. Let $\omega_E$ be the complete Kähler-Einstein form $\omega_E$ such that $-\text{Ric}(\omega_E) = \omega_E$ on $\Omega$ as in Theorem 1.1.

Then $(\Omega, \omega_E)$ has bounded geometry of $\infty$-order. □

Let $(\Omega, \omega_E)$ be a bounded strongly pseudoconvex domain with $C^\infty$-boundary. Let $\{K_m\}_{m=1}^\infty$ be the dynamical system of Bergman kernels as in Section 1. Hereafter we shall estimate $\{K_m\}_{m=1}^\infty$.

2.1 Upper estimate

By Theorem 2.2 $(\Omega, \omega_E)$ has bounded geometry of $\infty$-order. Hence there exist positive numbers $R, c, A_1, \cdots, A_\ell, \cdots$ and a covering of holomorphic coordinate charts $\{(V, (v_1, \cdots, v_n))\}$ satisfying the conditions in Definition 2.1. Let $x_0 \in \Omega$ be an arbitrary point. Then since $(\Omega, \omega_E)$ has a bounded geometry of $\infty$-order, there exists a local coordinate $(U, (z_1, \cdots, z_n))$ such that
(1) $U$ is biholomorphic to the unit open polydisc $\Delta^n(R)$ with center $O$ via $(z_1, \cdots, z_n)$,

(2) $g_{ij} = \delta_{ij} + O(\|z\|^2)$,

(3) 

\[
\det(g_{ij}) = \left(\prod_{i=1}^n \left(1 - \frac{1}{2}|z_i|^2\right)\right)^{-1} + O(\|z\|^3).
\]

First we note that 

\[K_1(x_0) \leq K(\Delta^n(R))(O) = \frac{1}{2^n(\pi R^2)^n}|dz_1 \wedge \cdots \wedge dz_n|^2\]

holds. Hence there exists a positive constant $C_{1,+}$ such that

(2.1.2) 

\[K_1 \leq C_{1,+} \cdot dV_E\]

holds on $\Omega$.

Now we proceed by induction on $m$. Suppose that for some $m \geq 1$, there exists a positive constant $C_{m,+}$ such that

(2.1.3) 

\[K_m \leq C_{m,+} \cdot (dV_E)^m\]

holds on $\Omega$. We note that by the extremal property of the Bergman kernel

(2.1.4) 

\[K_{m+1}(x) = \sup \left\{|\sigma|^2(x) \big| \sigma \in \Gamma(\Omega, \mathcal{O}_\Omega((m+1)K_\Omega)), \int_\Omega |\sigma|^2 \cdot h_m = 1\right\}\]

holds. Then by the induction hypothesis, we have that for every $r \leq R$,

(2.1.5) 

\[K_{m+1}(x_0) \leq C_m \cdot K(B(x_0, r), (m+1)K_\Omega, dV_E^{-m})\]

holds. By the Taylor expansion (2.1.1) of $\det(g_{ij})$, we have that there exists a positive constant $c < 1$ independent of $m$ and a positive function $\delta(r)$ of $r$ such that

(2.1.6) 

\[K(B(x_0, r), (m+1)K_\Omega, dV_E^{-m})(x_0) \leq \left(\frac{m+1}{2\pi}\right)^n \cdot \left(1 - e^{a+1}\right)^{-1} (1 + \delta(r)) \cdot (2^{-n}|dz_1 \wedge \cdots \wedge dz_n|^2)^{m+1}\]

and $\lim_{r \to 0} \delta(r) = 0$ hold. Here $c$ corresponds to the fact that for a positive number $\rho < 1$

\[\frac{\sqrt{-1}}{2} \int_{\Delta(\rho)} \left(1 - \frac{1}{2}|t|^2\right)^m dt \wedge d\bar{t} = \frac{2\pi}{m+1} \left(1 - \left(1 - \frac{1}{2}\rho^2\right)^{m+1}\right)\]

holds, where $\Delta(\rho) = \{t \in \mathbb{C} \big| |t| < \rho\}$. And $\delta(r)$ corresponds to the Taylor expansion (2.1.1). This implies that

(2.1.7) 

\[\lim_{m \to \infty} \sup_{m} \sqrt{(m!)^{-n} K_m} \leq (1 + \delta(r))(2\pi)^{-n} \cdot 2^{-n}|dz_1 \wedge \cdots \wedge dz_n|^2\]

holds. Since the estimate is independent of $r$, letting $r$ tend to 0, noting $\lim_{r \to 0} \delta(r) = 0$, we have the following lemma.
Lemma 2.3
\[
\limsup_{m \to \infty} \sqrt{m!}^{-n} K_m \leq (2\pi)^{-n} dV_E
\]
holds on \( \Omega \). \( \Box \)

2.2 Lower estimate

Now we shall estimate \( \{K_m\}_{m=1}^\infty \) from below. The lower estimate is also similar to the one in [T, Section 3.2]. The only difference is the use of the fact that \((\Omega, \omega_E)\) has a bounded geometry of \( \infty \)-order.

Take a defining function \( \rho \) of \( \Omega \) and set \( \rho^\#(z_0, z) = |z_0|^2 \rho(z) \), which is a defining function of \( C^\ast \times \Omega \). The strictly pseudoconvexity of \( \Omega \) then ensures that \( g[\rho] \)
\[
= \sum_{j,k=0}^n \frac{\partial^2 \rho}{\partial z_j \partial \bar{z}_k} dz_j d\bar{z}_k
\]
is a Lorentz-Kähler metric in a neighborhood of \( C^\ast \times \partial \Omega \) in \( C^\ast \times \overline{\Omega} \). We shall recall the following theorem.

Theorem 2.4 The Bergman kernel of a strongly pseudoconvex domain \( \Omega \) with \( C^\infty \)-boundary has the following singularity at the boundary \( \partial \Omega \).

Let \( r \) be a defining function of \( \Omega \) satisfying
\[
J[r] = 1 + O^{n+1}(\partial \Omega),
\]

where
\[
J[r] = (-1)^n \det \left( \frac{\partial r/\partial z_i}{\partial r/\partial \bar{z}_k} \right) (j, k = 1, \cdots, n)
\]
and \( f = O^{n+1}(\partial \Omega) \) stands for a function such that \( f/r^n \in C^\infty(\overline{\Omega}) \). Then
\[
K(\Omega) = r^{-n-1} \left( c_n + c'_n \| R \|_2^2 + \cdots \right),
\]
where \( c_n = n!/\pi^n \), \( c'_n = (n-2)!/(24\pi^n) \) and \( R \) is the curvature of the Lorenz Kähler metric near the boundary. \( \Box \)

On the other hand, the complete Kähler-Einstein metric of \([C-Y]\) is equivalent to the model metric constructed as follows.

Let \( \Omega = \{ \phi < 0 \} \). We set
\[
\omega := \sqrt{-1} \partial \bar{\partial} \left( - \log(-\phi) \right) = \frac{\sqrt{-1}}{2} \sum g_{ij} dz_i \wedge d\bar{z}_j
\]
Then by the direct calculation, we have
\[
\det(g_{ij}) = \left( \frac{-1}{\phi} \right)^{n+1} \det(\varphi_{ij})(-\phi + |d\varphi|^2)
\]
holds (cf. [C-Y] p.510). By the construction of \( \omega_E \), we have that there exists a positive constant \( C \) such that
\[
\frac{1}{C} \omega \leq \omega_E \leq C \omega
\]
holds on $\Omega$. Hence by Theorem 2.4 and (2.2.2), we see that there exists a positive constant $C_{1,-}$ such that

$$K_1 \geq C_{1,-} \cdot dV_E$$

holds on $\Omega$.

Now we shall use the fact that $(\Omega, \omega_E)$ has bounded geometry. Let $x_0$ be an arbitrary point and let $(U, (z_1, \cdots, z_n))$ be a holomorphic local coordinates satisfying following conditions:

1. $U$ is biholomorphic to the unit open polydisc $\Delta^n(R)$ with center $O$ via $(z_1, \cdots, z_n)$,
2. $g_{ij} = \delta_{ij} + O(\|z\|^2),$
3. $\det(g_{ij}) = \left(\prod_{i=1}^{n} \left(1 - \frac{1}{2}|z_i|^2\right)\right)^{-1} + O(\|z\|^3).$

Let $\rho$ be a $C^\infty$-function on $\Omega$ such that

1. $\rho \equiv 1$ on $B(x_0, R/3)$,
2. $0 \leq \rho \leq 1,$
3. $\text{supp}\, \rho \subset B(x_0, R),$
4. $\rho$ is a function of the Euclidean distance from $x_0$ on $B(x_0, R)$ with respect to $(z_1, \cdots, z_n),$
5. $|d\rho| \leq 3/R.$

Let $\phi$ be the function defined by

$$\phi := \alpha \cdot \log \frac{dV_E}{dV_{\mathbb{C}^n}},$$

where $dV_{\mathbb{C}^n}$ denotes the Euclidean volume form on $\mathbb{C}^n$ and $\alpha > 0$ is a positive constant which will be specified later.

Since $(\Omega, \omega_E)$ has a bounded geometry, we see that if we take $\alpha$ sufficiently large, we may assume that for every $x_0 \in \Omega$,

$$\phi_{x_0} := \phi + n\rho \log |z|^2$$

is plurisubharmonic on $\Omega$.

Now we proceed by induction on $m$. Suppose that there exists a positive constant $C_{m,-}$ such that

$$K_m \geq C_{m,-} \cdot (dV_E)^m \cdot e^{-\phi}$$

holds on $\Omega$. For $m = 1$, this inequality certainly holds by (2.2.4), if we take $C_{1,-}$ sufficiently small. Suppose that for some $m \geq 1$, (2.2.7) holds for some positive constant $C_{m,-}$.

Let $\sigma_0$ be a local holomorphic section of $(m + 1)K_\Omega$ on $U$ defined by

$$\sigma_0 := (dz_1 \wedge \cdots \wedge dz_n)^{\otimes m+1}. $$
We shall solve the $\bar{\partial}$-equation
\begin{equation}
\bar{\partial} u = \bar{\partial}(\rho \cdot \sigma_0).
\end{equation}

Then by the $L^2$-estimate for $\bar{\partial}$-operators, there exists a $C^\infty$-solution $u$ such that
\begin{equation}
\int_{\Omega} |u|^2 \cdot e^{-\phi_0} \cdot (dV_E)^{-m} \leq \frac{1}{m} \int_{\Omega} |\bar{\partial}(\rho \cdot \sigma_0)|^2 \cdot e^{-\phi_0} (dV_E)^{-m}.
\end{equation}

Then
\[\sigma := \rho \cdot \sigma_0 - u\]
is a global holomorphic section of $(m+1)K_{\Omega}$ such that
\begin{equation}
\int_{\Omega} |\sigma|^2 \cdot e^{-\phi} \cdot (dV_E)^{-m} \leq \left(1 - \frac{1}{\sqrt{m}}\right) \left(\int_{\Omega} |\rho \cdot \sigma_0|^2 \cdot (dV_E)^{-m}\right)^{\frac{1}{2}}.
\end{equation}

By the Taylor expansion
\[\det(g_{ij}) = \left(\prod_{i=1}^{n} \left(1 - \frac{1}{2} |z_i|^2\right)^{-1}\right) + O(\|z\|^3),\]
we see that
\[\int_{\Omega} |\rho \cdot \sigma_0|^2 (dV_E)^{-m} \sim \left(\frac{2\pi}{m+1}\right)^n\]
holds, in the sense that the ratio of the both sides tend to 1 as $m$ goes to infinity. This implies that if we take a sufficiently large
\begin{equation}
K_{m+1}(x_0) \geq C_m, \quad \left(1 - \frac{1}{\sqrt{m}}\right) \cdot (2\pi)^{-n} \cdot (m+1)^n \cdot (dV_E)^{m+1}(x_0) e^{-\phi(x_0)}
\end{equation}
holds. Hence we obtain the following lemma.

\textbf{Lemma 2.5}
\[\liminf_{m \to \infty} \sqrt{(n!)^{-n} K_m} \geq (2\pi)^{-n} dV_E\]
holds on $\Omega$. \hfill \square

Combining Lemmas 2.3 and 2.5 we complete the proof of Theorem 1.2.

\section{Applications}

Theorem 1.2 has been generalized as follows.

\textbf{Theorem 3.1} \textbf{([M-Y])} Let $\Omega$ be a bounded pseudoconvex domain in $\mathbb{C}^n$. Then there exists a unique complete $C^\infty$-Kähler form $\omega_E$ on $\Omega$ such that $-\text{Ric}(\omega_E) = \omega_E$ holds on $\Omega$. \hfill \square

The construction of $\omega_E$ is as follows. By the assumption, there exists a $C^\infty$-strictly plurisubharmonic exhaustion function $\varphi : \Omega \to \mathbb{R}$.

For $c \in \mathbb{R}$, we set $\Omega_c := \{ \varphi < c \}$. Then there exists a set $E$ of measure 0 in $\mathbb{R}$ such that for every $c \in \mathbb{R} \setminus E$, $\Omega_c$ is a bounded strongly pseudoconvex domain in $\mathbb{C}^n$ with $C^\infty$-boundary.

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Then by Theorem 1.1, we have the canonical complete Kähler-Einstein form \( \omega_{E,c} \) on \( \Omega_c \) such that
\[
-\text{Ric}(\omega_{E,c}) = \omega_{E,c}
\]
holds on \( \Omega_c \). Hence \( \{\omega_{E,c}|c \in \mathbb{R}\setminus E\} \) is a monotone decreasing sequence in \( c \in \mathbb{R}\setminus E \). Hence the limit
\[
dV_E := \frac{1}{n!} \lim_{c \to \infty} \omega^n_{E,c}
\]
exists on \( \Omega \). It is easy to see that \( dV_E = C^\infty \) and \( \omega_E := -\text{Ric}dV_E \) satisfies the equality: \( -\text{Ric} (\omega_E) = \omega_E \).

This construction has already been considered in [C-Y]. But the completeness of \( \omega_E \) was first proved in [M-Y].

**Theorem 3.2** Let \( \Omega \) be a bounded pseudoconvex domain in \( \mathbb{C}^n \times \Delta \). Let \( dV_s (s \in \Delta) \) denote the Kähler-Einstein volume form on \( \Omega_s := \Omega \cap (\mathbb{C}^n \times \{s\}) \).

Let \( dV_{\Omega/\Delta} \) be the relative volume form on \( \Omega \) defined by
\[
dV_{\Omega/\Delta}|_{\Omega_s} = dV_s.
\]
Then
\[
\sqrt{-1} \partial \bar{\partial} \log dV_{\Omega/\Delta} \geq 0
\]
holds on \( \Omega \), i.e., \( dV_{\Omega/\Delta}^{-1} \) is a hermitian metric on \( K_{\Omega/\Delta} \) with semipositive curvature.

**Remark 3.3** By the construction of the complete Kähler-Einstein metric in [C-Y] and the implicit function theorem, we see that \( dV_{\Omega/\Delta} \) is a \( C^\infty \)-relative volume form.

**Proof of Theorem 3.2** First we shall assume that \( \Omega \) is a strongly pseudoconvex domain with \( C^\infty \)-boundary. Let \( \{K_{m,s}\}_{s \in \Delta} \) be the dynamical system of Bergman kernels on \( \Omega_s \) constructed as in the previous section. Let \( h_m \) be the hermitian metric on \( mK_{X/S} \) defined by
\[
h_m|_{\Omega_s} = K_{m,s}^{-1}.
\]
Then by Berndtsson’s theorem ([Ber1], [Ber2]), we see that
\[
\sqrt{-1} \Theta(h_m) > 0
\]
holds on \( \Omega \) by induction on \( m \). Then by Theorem 1.1, we see that \( dV_{\Omega/\Delta}^{-1} \) has semipositive curvature.

If \( \Omega \) is a general bounded pseudoconvex domain in \( \mathbb{C}^n \times \Delta \), there exists a \( C^\infty \)-strictly plurisubharmonic exhaustion function \( \varphi \) on \( \Omega \). Then for every \( c \in \mathbb{R}, \Omega_c := \{\varphi < c\} \) is a bounded strongly pseudoconvex domain in \( \mathbb{C}^n \times \Delta \). Then the metric \( dV_{\Omega_c/\Delta}^{-1} \) on \( K_{\Omega_c/\Delta} \) has semipositive curvature. Let \( dV_{c,s} \) denote the Kähler-Einstein volume form on \( \Omega_{c,s} \). Then \( dV_{c,s} \) is monotone decreasing in \( c \in \mathbb{R} \). Since the limit of plurisubharmonic function is also plurisubharmonic, we complete the proof of Theorem 3.2. \( \Box \)
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Author’s address
Hajime Tsuji
Department of Mathematics
Sophia University
7-1 Kioicho, Chiyoda-ku 102-8554
Japan
h-tsuji@h03.itscom.net