A Luenberger-style Observer for Robot Manipulators with Position Measurements

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Abstract—This paper presents a novel Luenberger-style observer for robot manipulators with position measurements. Under the assumption that the state evolutions that are to be observed have bounded velocities, it is shown that the origin of the observation error dynamics is globally exponentially stable and that the corresponding convergence rate can be made arbitrarily high by increasing a gain of the observer.

Comparisons and relations between the proposed observer and existing observers are discussed.

The effectiveness of the result here presented is illustrated by a simulation of the observer for the Pendubot, an underactuated two-joint manipulator.

I. INTRODUCTION

Observers for nonlinear systems have been extensively studied in the last decades (for example, see [1] and [2]). In the specific case of robotic manipulators it has been of great interest to design observers that estimate velocities from position measurements. In fact, many commercially-available robotic manipulators are not equipped with velocity sensors; as a result, observers that estimate velocities from position measurements have been instrumental in designing feedback controllers that use only position measurements.

The literature on this topic is vast; refer to [3] for a literature review. In some articles it is proved explicitly that, in certain conditions, the state of the observer that is used to do feedback from position measurements converges to the state of the robotic manipulator even when the observer is in open-loop. Examples of such articles are [4], [5], [6], [7], and [3]. In addition, [8] and [9] present observers for robotic manipulators without employing them in a position feedback loop. Moreover, rigid robotic systems are a subclass of Euler-Lagrange systems, and intrinsic observers for Euler-Lagrange systems with position measurements are proposed in [10] and [11].

This paper introduces a novel asymptotic observer for rigid robotic manipulators with position measurements. The proposed observer is Luenberger-style and is very simple in structure. Under the standard and realistic assumption that the state evolutions that are to be observed have bounded velocities with bounds known a priori, it is shown that the origin of the observation error dynamics is globally exponentially stable; in addition, it is demonstrated that the corresponding convergence rate can be made arbitrarily high by increasing a gain of the observer.

The observer here presented is compared with three similar existing observers for robotic manipulators, and it is shown that in several aspects the comparison is favorable to the proposed observer.

The rest of the paper is organized as follows; in section II the observer is presented; section III discusses comparisons and relations between the proposed observer and some existing results; in section IV a simulation of the observer for the Pendubot is shown.

In what follows ||·|| denotes the Euclidean norm of a vector or matrix; moreover, given $v \in \mathbb{R}^n$, $v_i$ denotes its $i$-th component; $\mathbb{R}^n_{>0}$ denotes the open interval $(0, \infty)^n$.

II. MAIN RESULT

The dynamic equation of a $n$-joint rigid robot can be expressed as

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + F\dot{q} + g(q) = Hu \tag{1}$$

(see [12]). In (1), $q \in \mathbb{R}^n$ is the vector of joint positions, $u \in \mathbb{R}^m$ is the vector of control inputs, $M(q) \in \mathbb{R}^{n \times n}$ is the inertia matrix, $C(q, \dot{q})$ is the Coriolis and centrifugal matrix, $F$ is the diagonal matrix of viscous friction coefficients, $g(q)$ is the gravity vector, and $H$ is the input matrix that differs from the identity if the system is not fully actuated. $M$, $C$, and $g$ are assumed to be continuously differentiable.

Denote by $Q \subseteq \mathbb{R}^n$ the set of all feasible values of $q$. $Q$ is typically determined by the mechanic structure of the robot and/or by the way the robot operates. Note that if $q_i$ is the joint position of a prismatic joint, then $q_i$ ranges on a bounded set because for physical reasons the relative linear displacement of two links connected by a prismatic joint cannot grow indefinitely. However, if $q_i$ is the joint position of a revolute joint, then $q_i$ could range on an unbounded set since it can occur that the links connected by a revolute joint can rotate indefinitely with respect to each other. Thus, in the general case $Q$ is unbounded.

The following properties of (1) are inherent to robot dynamics (see [13, p. 139]) and they will be useful in the sequel

$P1$: $\det(M(q)) \neq 0 \quad \forall q \in Q$

$P2$: $\|M^{-1}(q)\| \leq M_0 \quad \forall q \in Q$

$P3$: $\forall i \in \{1, \ldots, n\}$, the $i$-th element of the vector $C(q, \dot{q})\dot{q}$ is equal to $\dot{q}^T N_i(q)\dot{q}$ with $N_i$ symmetric, continuously differentiable, and such that $\exists \bar{N}_i > 0$ that satisfies

$$\|N_i(q)\| \leq \bar{N}_i \quad \forall q \in Q .$$
It is assumed that the vector of joint positions \( q \) is measured, but the vector of joint velocities \( \dot{q} \) is not measured; then, \( 1 \) has the following state space representation

\[
\begin{align*}
\dot{q} &= v \\
\dot{v} &= -M^{-1}(q)(C(q,v)v + Fv + g(q)) - Hu \\
y &= q
\end{align*}
\]  
(2)

This paper presents an observer for systems of the form \( 2 \); clearly, such observer is useful for estimating the joint velocities \( v \).

The state evolution of \( 2 \) that we want to observe \((q(t), v(t))\) is assumed to be defined \( \forall t \geq 0 \) and with bounded joint velocities, that is there exist \( V_i \geq 0 \) for \( i = 1, \ldots, n \) such that

\[
|v_i(t)| \leq V_i \quad \forall t \geq 0 \quad \forall i \in \{1, \ldots, n\},
\]

and it is assumed that the \( V_i \)'s are known a priori.

This assumption is definitively realistic. In fact, it is reasonable to expect that the joint velocities of a robot will not exceed certain a priori bounds that come from the mechanic limitations of the robot and/or from the way the robot operates. Moreover, this assumption is recurrent in the literature on observers for robotic manipulators; for example, an equivalent assumption is made for proving the convergence of the observers presented in [3], [5], [6], [7], [9] and [11].

Denote by \( \hat{q} \) and \( \hat{v} \) the estimates of \( q \) and \( v \) respectively; then, the proposed Luenberger-style observer is defined by the following equations

\[
\begin{align*}
\dot{\hat{q}} &= \hat{v} - \theta\alpha(\hat{q} - q) \\
\dot{\hat{v}} &= -M^{-1}(q)(C(q, \sigma_V(\hat{v}))(\sigma_V(\hat{v}) + F\hat{v} + g(q) - Hu) - \theta^2\beta(\hat{q} - q)
\end{align*}
\]  
(4)

In \( 4 \), \( \alpha, \beta \), and \( \theta \) are positive scalar design parameters, \( V = (V_1, \ldots, V_n) \) is the vector of the velocities bounds, and \( \sigma_V \) is a component-wise saturation function with vector saturation level \( V \); specifically, given \( Y \in \mathbb{R}^n \) such that \( Y_i \geq 0 \) for \( i = 1, \ldots, n \), \( \sigma_V : \mathbb{R}^n \to \mathbb{R}^n \) is defined as follows

\[
\sigma_V(x)_i = \begin{cases} 
  x_i & \text{if } |x_i| \leq Y_i \\
  Y_i & \text{if } x_i > Y_i \\
  -Y_i & \text{if } x_i < -Y_i 
\end{cases}
\]

\( i = 1, \ldots, n \).

Observer \( 4 \) is obtained as follows. Make a copy of the system \( 2 \) to be observed; add innovation terms to that copy; saturate \( \hat{v} \) in the Coriolis terms of the \( \hat{v} \) equation.

Note that the saturation level on each component of \( \hat{v} \) is chosen so that if the initial states of system \( 2 \) and observer \( 4 \) are identical, then observer \( 4 \) tracks exactly system \( 2 \). In fact, if \((q(t), v(t))\) is a state evolution of \( 2 \) corresponding to a certain input \( u(t) \), and it satisfies the bounds \(|v_i(t)| \leq V_i \quad \forall t \geq 0 \quad \forall i \in \{1, \ldots, n\} \), then \((\hat{q}(t), \hat{v}(t))\) is also the state evolution of the observer \( 4 \) corresponding to the same input and to the initial state \((\hat{q}(0), \hat{v}(0)) = (q(0), v(0))\).

The insertion of the saturation \( \sigma_V \) in equations \( 4 \) was inspired by [14]. However, the observer presented in [14] applies to a class of systems that does not include systems of the type \( 2 \).

The following theorem states that observer \( 4 \) is globally exponentially convergent with convergence rate arbitrarily fast.

**Theorem 6**: Let \((q(t), v(t))\) be the state evolution of \( 2 \) corresponding to the input \( u(t) \). Assume that \((q(t), v(t))\) is defined \( \forall t \geq 0 \) and there exist \( V_i \geq 0 \) for \( i = 1, \ldots, n \) such that

\[
|v_i(t)| \leq V_i \quad \forall t \geq 0 \quad \forall i \in \{1, \ldots, n\}.
\]

Then, \( \forall (\alpha, \beta, \gamma) \in \mathbb{R}_{>0} \exists \theta^* > 0 \) such that if \( \theta \geq \theta^* \) the following property holds. \( \exists k > 0 \) such that the state evolution \((\hat{q}(t), \hat{v}(t))\) of \( 4 \) corresponding to the same input \( u(t) \) and to any initial state \((\hat{q}(0), \hat{v}(0)) \in \mathbb{R}^n \times \mathbb{R}^n \) is defined \( \forall t \geq 0 \) and satisfies

\[
\left\| \begin{pmatrix} \hat{q}(t) - q(t) \\ \hat{v}(t) - v(t) \end{pmatrix} \right\| \leq k \left\| \begin{pmatrix} \hat{q}(0) - q(0) \\ \hat{v}(0) - v(0) \end{pmatrix} \right\| e^{-\gamma t} \quad \forall t \geq 0.
\]

**Proof**: To simplify the notation, let

\[
A(q, v) = C(q, v)v.
\]

Since \( 7 \) holds, in the rest of the proof regard

\[
\begin{align*}
\dot{\hat{q}} &= v \\
\dot{\hat{v}} &= -M^{-1}(q)(A(q, \sigma_V(v)) + Fv + g(q) - Hu) \\
y &= q
\end{align*}
\]  
(9)

as the given system instead of \( 2 \).

Fix \( \alpha > 0 \) and \( \beta > 0 \), and assume that \( \theta > 0 \). Similarly to [1, p. 100] set

\[
\xi(t) = \frac{1}{\theta} (\hat{q}(t) - q(t)) \quad \zeta(t) = \frac{1}{\theta^2} (\hat{v}(t) - v(t))
\]

Then,

\[
\begin{pmatrix} \xi(t) \\ \zeta(t) \end{pmatrix} = \theta G \begin{pmatrix} \xi(t) \\ \zeta(t) \end{pmatrix} + \begin{pmatrix} 0 \\ f(q(t), v(t), \zeta(t), \theta) \end{pmatrix}
\]

where

\[
G = \begin{pmatrix} -\alpha I & I \\ -\beta I & 0 \end{pmatrix}
\]

and

\[
f(q, v, \zeta, \theta) = -M^{-1}(q) \cdot \left\{ \frac{1}{\theta^2}(A(q, \sigma_V(v + \theta^2 \zeta)) - A(q, \sigma_V(v))) \right\} + F\zeta.
\]

(11)

Note that \( G \) is Hurwitz since \( \alpha \) and \( \beta \) are positive. Let \( S \) be the solution of the Lyapunov equation \( C^T S + SG = -I \), and consider the candidate Lyapunov function for system \( 10 \)

\[
V(\xi, \zeta) = \begin{pmatrix} \xi \\ \zeta \end{pmatrix}^T S \begin{pmatrix} \xi \\ \zeta \end{pmatrix}.
\]
Then
\begin{align*}
\dot{V}(\xi, \zeta) & \leq -\theta \left\| \begin{bmatrix} \xi \\ \zeta \end{bmatrix} \right\|^2 \\
& \quad + 2\|S\| \left\| \begin{bmatrix} \xi \\ \zeta \end{bmatrix} \right\| \left\| f(q(t), v(t), \zeta, \theta) \right\|.
\end{align*}
(12)

In order to find a proper upper bound for \(\|f(q(t), v(t), \zeta, \theta)\|\), proceed as follows.

Using (11) it follows that
\[
\frac{\partial A}{\partial v}(q, v) = 2 \begin{pmatrix} v^T N_1(q) \\ \vdots \\ v^T N_n(q) \end{pmatrix}.
\]

Let \(\hat{V} = \{v \in \mathbb{R}^n \mid |v_i| \leq V_i, i = 1, \ldots, n\}\).

Then, using (13) it follows that
\[
\left\| \frac{\partial A}{\partial v}(q, v) \right\| \leq B \quad \forall (q, v) \in Q \times \hat{V}.
\]

Then, by [14, Lemma 2]
\[
\|A(q, \sigma_V(v + \theta^2 \zeta)) - A(q, \sigma_V(v))\| \leq \theta^2 B |\zeta| \quad \forall (q, v, \zeta, \theta) \in Q \times \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}.
\]

Since \(q(t) \in Q \quad \forall t \geq 0\), letting \(L = M_0(B + \|F\|)\), from (11), (15), and (16) it follows that
\[
\left\| f(q(t), v(t), \zeta, \theta) \right\| \leq \|L\| \|\zeta\| \leq \left\| \begin{bmatrix} \xi \\ \zeta \end{bmatrix} \right\|.
\]

Then, from (12)
\[
\dot{V}(\xi, \zeta) \leq -\left\| \begin{bmatrix} \xi \\ \zeta \end{bmatrix} \right\|^2 \left( \theta - 2\|S\|L \right).
\]

As a result, if \(\theta > 2\|S\|L\), the equilibrium at the origin of system (10) is globally exponentially stable. Standard calculations (see [15, p. 154]) show that the rate of the decaying exponential that bounds from above \(\|\xi(t), \zeta(t)\|\) is given by
\[
\frac{\theta - 2\|S\|L}{2\|S\|}.
\]

As a result, to guarantee that this rate is greater or equal than \(\gamma\), it suffices to take
\[
\theta \geq \theta^* = 2\|S\|(\gamma + L)
\]
(16)

Remark 17: Even though the state of the observer converges to the state of the plant for any value of the initial state of the observer \((\hat{q}(0), \hat{v}(0))\), it is enough to consider values of \((\hat{q}(0), \hat{v}(0))\) with \(\hat{v}(0) \in \hat{V}\) where \(\hat{V}\) was defined in (13); in fact, it is known a priori that the trajectory to be observed \((q(t), v(t))\) is such that \(v(t) \in \hat{V}\ \forall t \geq 0\). Moreover, since \(q\) is measurable, it should be feasible to set \(\hat{q}(0) \approx q(0)\); consequently, in practice it is enough to guarantee that the observer converges when \((\hat{q}(0), \hat{v}(0)) \in \{(\hat{q}(0), \hat{v}(0)) \mid \|\hat{q}(0) - q(0)\| < \epsilon, \hat{v}(0) \in \hat{V}\}\) where \(\epsilon > 0\) is a parameter whose value depends on the accuracy of the position sensors.

III. Comparisons and Relations with Existing Results

The proposed observer is derived under assumptions equivalent to those for the observers in [5], [6], and [7] used in open-loop; moreover, those observers and the one here proposed present similar convergence properties. However, the observer here introduced compares favorably to those in [5], [6], and [7] because it is simpler in structure and consequently easier to implement. Indeed, the proposed observer is a plain Luenberger-style observer with a saturation on some of the \(\dot{v}\) terms. Note that the observer in [7] has the advantage over the observer here proposed of being of reduced order; however, the structure of the former is quite complicated and, as pointed out in [16], the procedure to choose its parameters is quite complex.

From a mathematical point of view, the observer here introduced compares favorably with those in [5] and [6]. In fact, in the proposed observer the error dynamics have the origin that is globally asymptotically stable; on the other hand, the origin of the error dynamics is only semiglobally stabilized in the case of the observers in [5] and [6]. However, taking into account the considerations in Remark 17 it follows that achieving global rather than semiglobal convergence might not be relevant from a practical point of view. An additional point in favor of the observer here presented with respect to observers in [5], [6], and [7], is that the proof of its convergence is simpler.

The proposed observer is related to the nonlinear observer introduced in [17] as discussed in the rest of the section.

The nonlinear observer presented in [17] applies to a certain class of multi-output nonlinear systems that includes systems of the form
\begin{align*}
\dot{q}_1 &= f^1(u, q, v_1) \\
\dot{q}_2 &= f^2(u, q, v_1, v_2) \\
& \quad \vdots \\
\dot{q}_n &= f^n(u, q, v) \\
\dot{v}_1 &= f_1^2(u, q, v) \\
& \quad \vdots \\
\dot{v}_n &= f^n_2(u, q, v) \\
y &= q
\end{align*}
(18)

where \(q_i, v_i \in \mathbb{R} \quad i = 1, \ldots, n\), \(u \in \mathbb{R}^m\), and it is assumed that
\begin{itemize}
\item[A1]: \(u(t) \in U\) a compact subset of \(\mathbb{R}^m\).
\item[A2]: \(\forall (k, i) \in \{1, 2\} \times \{1, \ldots, n\}\) \(f^k_i \in C^1\) and \(f^k_i\) is globally Lipschitz with respect to \((q, v)\) uniformly with respect to \(u \in U\).
\item[A3]: \(\exists \ 0 < \alpha < \beta\) such that \(0 < \alpha \leq \frac{\partial f^1_i}{\partial v_1}(u, q, v) \leq \beta \ \forall (u, q, v) \in U \times \mathbb{R}^n \times \mathbb{R}^n \quad \forall i \in \{1, \ldots, n\}\).
\end{itemize}
Let \( \alpha, \beta, \) and \( \theta \) be scalars; then, in [17] the following Luenberger-style observer for system (18) is proposed

\[
\dot{q}_1 = f_1^1(u, \dot{q}, \dot{v}_1) - \theta \alpha (\dot{q} - q) \\
\dot{q}_2 = f_1^2(u, \dot{q}, \dot{v}_1, \dot{v}_2) - \theta \alpha (\dot{q} - q) \\
\vdots \\
\dot{q}_n = f_n^1(u, \dot{q}, \dot{v}) - \theta \alpha (\dot{q} - q) \\
\dot{v}_1 = f_2^2(u, \dot{q}, \dot{v}) - \theta^2 \beta (\dot{q} - q) \\
\vdots \\
\dot{v}_n = f_n^2(u, \dot{q}, \dot{v}) - \theta^2 \beta (\dot{q} - q).
\]  

(19)

In [17] it is proved that, \( \forall \ (\alpha, \beta) \in \mathbb{R}^{2} \) \( \exists \ \theta^* > 0 \) such that if \( \theta > \theta^* \) observer (19) is globally exponentially convergent.

An alternative observer for (18) is given by

\[
\dot{q}_1 = f_1^1(u, q, \dot{v}_1) - \theta \alpha (\dot{q} - q) \\
\dot{q}_2 = f_1^2(u, q, \dot{v}_1, \dot{v}_2) - \theta \alpha (\dot{q} - q) \\
\vdots \\
\dot{q}_n = f_n^1(u, q, \dot{v}) - \theta \alpha (\dot{q} - q) \\
\dot{v}_1 = f_2^2(u, q, \dot{v}) - \theta^2 \beta (\dot{q} - q) \\
\vdots \\
\dot{v}_n = f_n^2(u, q, \dot{v}) - \theta^2 \beta (\dot{q} - q).
\]  

(20)

It can be easily proved again that \( \forall \ (\alpha, \beta) \in \mathbb{R}^{2} \) \( \exists \ \theta^* > 0 \) such that if \( \theta > \theta^* \) observer (20) is globally exponentially convergent. The advantage of (20) over (19) is that in order to prove convergence, assumption A2 can be replaced by the following weaker assumption

A4: \( \forall(k, i) \in \{1, 2\} \times \{1, \ldots, n\} \ \ f_i^k \in C^1 \) and, denoting with \( Q \subseteq \mathbb{R}^n \) the set \( q \) ranges on, \( f_i^k \) is globally Lipschitz with respect to \( v \) uniformly with respect to \( (q, u) \in Q \times U \).

Clearly, system (3) is of the type (18) and it satisfies assumption A4; however, in general, it does not satisfy assumption A2. On the other hand, it is assumed for system (3) that the velocity \( v(t) \) of the state evolutions to be observed satisfy (4). Consequently, as said before, system

\[
\begin{align*}
\dot{q} &= v \\
\dot{v} &= -M^{-1}(q)(C(q, \sigma v(v))\sigma v(v) + Fv + g(q)) - Hu \\
y &= q
\end{align*}
\]  

(21)

can be regarded as the given system instead of (3). Note that the proposed observer (7) coincides with observer (20) instanced for system (21). Convergence of (21) can be justified as follows. From what stated before, it follows that if A1, A3, and A4 hold for (21), then convergence is achieved. Note that (21) satisfies A6 and, as shown in the proof of Theorem 5 using properties (14) [13] and [14, Lemma 2], it follows that (21) satisfies A9 too. Moreover, note that assumption A1 is not needed to prove convergence because in equations (21) \( u \) enters only through the additive term \( M^{-1}(q) Hu \) which does not depend on the unmeasured variable \( v \); as a result, the dynamics of the observation error are independent of \( u \).

IV. SIMULATION OF THE OBSERVER FOR THE PENDUBOT

The effectiveness of the proposed Luenberger-style observer is here illustrated by a simulation of the observer for the Pendubot, an underactuated two-joint manipulator moving in a vertical plane (see [18]). A sketch of the Pendubot is shown in Fig. 1. The first joint (shoulder) is actuated, whereas the second joint (elbow) is not. Both joints are equipped with position sensors (encoders), but no velocity sensors are present. Consequently, it is of interest to design an observer for the Pendubot that estimates the joint-velocities so that the observer could be included in a feedback controller that uses only position measurements.

Define the joint variables \( q_1 \) and \( q_2 \) as shown Fig. 1 and let \( u \) be the voltage input of the actuator. A dynamic model of the Pendubot can be found in [19] and is given by (11) with \( q = (q_1, q_2)^T \),

\[
M(q) = \begin{pmatrix}
\pi_1 + \pi_2 + 2\pi_3 \cos(q_2) & \pi_2 + \pi_3 \cos(q_2) \\
\pi_2 + \pi_3 \cos(q_2) & \pi_2
\end{pmatrix},
\]

\[
C(q, \dot{q}) = \begin{pmatrix}
-\pi_3 \sin(q_2)\dot{q}_2 & -\pi_3 \sin(q_2)\dot{q}_2 - \pi_3 \sin(q_2)\dot{q}_1 \\
\pi_3 \sin(q_2)\dot{q}_1 & 0
\end{pmatrix},
\]

\[
g(q) = \begin{pmatrix}
\pi_4 g_0 \cos(q_1) + \pi_5 g_o \cos(q_1 + q_2) \\
\pi_5 g_0 \cos(q_1 + q_2)
\end{pmatrix},
\]

\[
F_v = \begin{pmatrix}
0 & 0 \\
0 & 0
\end{pmatrix},
\]

\[
H = \begin{pmatrix}
1 & 0 \\
0 & 0
\end{pmatrix}.
\]
where
\[
\begin{align*}
\pi_1 &= 0.0308 \text{Vs}^2/\text{rad} \\
\pi_2 &= 0.0106 \text{Vs}^2/\text{rad} \\
\pi_3 &= 0.0095 \text{Vs}^2/\text{rad} \\
\pi_4 &= 0.2086 \text{Vs}/\text{m} \\
\pi_5 &= 0.0630 \text{Vs}/\text{m} \\
g_0 &= 9.81 \text{m/s}^2.
\end{align*}
\]

Assume that the Pendubot operates so that the angular velocities \(v = \dot{q}\) do not exceed the following bounds
\[
|v_1(t)| \leq 10 \text{ rad/s} \quad |v_2(t)| \leq 10 \text{ rad/s} \quad \forall t \geq 0.
\]

The design parameters \(\alpha\) and \(\beta\) of observer are set as \(\alpha = \beta = 1\); then, referring to magnitudes introduced in the proof of Theorem it follows that \(\|S\| = 1.81\) and that \(L\) can be set equal to 54.01; consequently, the minimum value of the gain \(\theta\) that guarantees global exponential stability of the origin of the error dynamics is \(\theta = 195\). Set \(\theta = 200\) so that the norm of the observation error will converge to 0 globally, and it will be bounded by an exponential as in with \(\gamma = 1.27\).

Choose the following initial state for the Pendubot
\[
\begin{pmatrix}
q_0^0 \\
q_2^0 \\
v_1^0 \\
v_2^0
\end{pmatrix} = \begin{pmatrix}
-\pi/2 \\
0 \\
0 \\
0
\end{pmatrix},
\]
which corresponds to the lower equilibrium, and apply the control \(u = 1.5 \sin (100t)\) that maintains the Pendubot in oscillation about the lower equilibrium.

Taking into account that \(q\) is measured, choose for the observer the initial state
\[
\begin{pmatrix}
\hat{q}_0^0 \\
\hat{q}_2^0 \\
\hat{v}_1^0 \\
\hat{v}_2^0
\end{pmatrix} = \begin{pmatrix}
-\pi/2 \\
0 \\
2 \\
2
\end{pmatrix}.
\]

The corresponding state evolutions of the Pendubot and of the observer, plotted in Fig. 2 show that the observer is convergent.

V. CONCLUSIONS

In this paper a novel asymptotic Luenberger-style observer for robot systems with position measurements has been presented. The observer is very simple in structure; it has been proven that the dynamics of the observation error have a globally exponentially stable origin with convergence rate that can be made arbitrarily high by increasing a gain of the observer.

The proposed observer compares favorably with some existing observers for robot manipulators derived under equivalent assumptions; its relation to a previous nonlinear observer has been discussed.

A simulation of the proposed observer for the Pendubot has been included to illustrate its effectiveness.

It will be topic of future research to investigate if this observer can lead to interesting results in the area of control of robot manipulators via position feedback.

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