Virtual Full-Duplex Buffer-Aided Relaying in the Presence of Inter-Relay Interference

Su Min Kim, Member, IEEE and Mats Bengtsson, Senior Member, IEEE

Abstract—In this paper, we study virtual full-duplex (FD) buffer-aided relaying to recover the loss of multiplexing gain caused by half-duplex (HD) relaying in a multiple relay network, where each relay is equipped with a buffer and multiple antennas, through joint opportunistic relay selection (RS) and beamforming (BF) design. The main idea of virtual FD buffer-aided relaying is that the source and one of the relays simultaneously transmit their own information to another relay and the destination, respectively. In such networks, inter-relay interference (IRI) is a crucial problem which has to be resolved like self-interference in the FD relaying. In contrast to previous work that neglected IRI, we propose joint RS and BF schemes taking IRI into consideration by using multiple antennas at the relays. In order to maximize average end-to-end rate, we propose a weighted sum-rate maximization strategy assuming that adaptive rate transmission is employed in both the source to relay and relay to destination links. Then, we propose several BF schemes cancelling or suppressing IRI in order to maximize the weighted sum-rate. Numerical results show that our proposed optimal, zero-forcing, and minimum mean square error BF-based RS schemes asymptotically approach the ideal FD relaying upper bound when increasing the number of antennas and/or the number of relays.

Keywords—Full-duplex, buffer-aided relaying, inter-relay interference, relay selection, beamforming

I. INTRODUCTION

Since cooperative relaying can improve both spectral efficiency and spatial diversity, it is a promising core technology for next-generation wireless communication networks. So far, most studies have considered half-duplex (HD) relaying based on two-phase operation where a source transmits data to relays at the first time slot and the relays forward it to a destination at the second time slot [1], [2]. However, such HD relaying causes a loss of multiplexing gain expressed as an one-half pre-log factor. To overcome the loss of multiplexing gain, several practical full-duplex (FD) relaying solutions have been studied [3], [4], [5], [6], [7], [8], [9], [10], [11], [12]. Since strong self-interference is a main problem which has to be resolved in FD relaying, the previous work primarily focused on self-interference cancellation based on antenna separation techniques in the wireless propagation domain and signal cancellation techniques in the analog circuit and digital domains. Although these studies showed feasibility of FD relaying using small-scale wireless communication devices such as WiFi and IEEE 802.15.4, the technology is still premature for cellular communications, which require additional cancellation gains due to practical limitations such as varying center frequencies, bandwidth, and circuit imperfections.

In order to mitigate the loss of multiplexing gain in HD relaying, successive relaying protocols have been proposed for a two-relay network [13], [14], [15], [16], [17] and multiple-relay networks [18], [19], [20]. In these protocols, two relays take turns acting as receivers and transmitters successively and a source and a transmitting relay transmit their own information simultaneously. Here, the source transmits new information and the relay transmits previously received information. The main issue for such successive relaying protocols is to efficiently handle inter-relay interference (IRI) from the transmitting relay to the receiving relay. Towards this end, successive interference cancellation (SIC) and/or sophisticated coding and joint decoding techniques have been employed in the literature. However, the SIC requires strong interference scenarios and the joint decoding requires high computational complexity. Furthermore, although the successive relaying asymptotically achieves the spectral efficiency of the FD relaying with respect to the number of channel uses, it requires a sufficiently long block length (equivalently, coherence time) over slow fading channels.

Employing a buffer at the relay, such long block length constraints can be relaxed. Focusing on these advantages, buffer-aided relaying has been proposed in a three-node network [21], [22], [23], [24]. The key idea is an opportunistic relaying mode selection (buffering or forwarding) according to channel conditions. HD buffer-aided relaying can achieve up to two-fold spectral efficiency under asymmetric channel conditions between \{S→R\} and \{R→D\} links, compared to HD relaying without buffer. Additionally, bidirectional buffer-aided relaying with two-way traffic [25], [26], [27], [28], buffer-aided relaying over dual-hop broadcast channels [29] and a shared relay channel with two source-destination pairs [30] have been studied. By extending to multiple-relay networks, several opportunistic relaying schemes, which exploit the best HD buffer-aided relay, have been proposed [31], [32], [33]. Ikhlef et al. [34] have proposed a max−max relay selection (MMRS) scheme, which selects the best \{S→R\} and


{\mathcal{R}\rightarrow \mathcal{D}} \) relays with the maximum channel gains. However, the MMR\$ scheme does not fully take advantage of the benefits of buffer-aided relaying since it maintains the two-phase operation. Therefore, Krikidis et al. \cite{32} have proposed a max–link relay selection (MLRS) scheme, which selects the best relaying mode as well as the maximum channel gain.

Most recently, Ikle\$ et al. \cite{33} have proposed a space full-duplex max–max relay selection (SFD-MMR\$) scheme, which mimics the FD relaying by utilizing the best receiving and transmitting relays operating simultaneously. In this scheme, they did not consider IRI by assuming fixed-relays with highly directional antennas. However, this assumption does not always hold and it is hard to be practically realized as the number of relays increases. With consideration of IRI, Kim and Bengtsson \cite{34} proposed a virtual FD buffer-aided successive opportunistic relaying (BA-SOR) rate transmission scheme, which has been originally devised for FD-rate transmission. However, a fixed low SIC threshold, with zero-forcing beamforming (BF) for IRI cancellation in the source and relays, the main objective was to minimize the total energy expenditure. In \cite{35}, the average end-to-end rate of the BA-SOR scheme, which has been originally devised for fixed rate transmission, is numerically shown for adaptive rate transmission. Nomikos et al. \cite{35, 36} have proposed a buffer-aided successive opportunistic relaying (BA-SOR) scheme employing SIC at the receiving relay for fixed rate transmission. In \cite{35}, even if it partially overcome the strong interference requirement of SIC through power allocation at the source and relays, the main objective was to minimize the total energy expenditure. In \cite{36}, the average end-to-end rate of the BA-SOR scheme, which has been originally devised for fixed rate transmission, is numerically shown for adaptive rate transmission. However, a fixed low SIC threshold, \( r_0 \) (e.g., 2 bps/Hz), has been applied even for adaptive rate transmission whereas the threshold value should be set to the information rate of the \( \{\mathcal{R}\rightarrow \mathcal{D}\} \) link. Thus, this yields an optimistic result in terms of the average throughput.

In this paper, our main goal is to approach the average end-to-end rate of ideal FD relaying even in the presence of IRI. To this end, we propose transmission schemes based on a joint RS and BF design utilizing multiple buffer-aided relays and multiple antennas at the relays. For the joint RS and BF design, we first propose a weighted sum-rate maximization using instantaneous channel and buffer states for achieving the average end-to-end rate maximization. Then, we separately design linear BF for each (receiving and transmitting) relay pair, which cancels or suppresses IRI, and optimal RS for maximizing the weighted sum-rate based on the beamformers found for each relay pair. To focus on maximizing the average end-to-end rate, we employ adaptive rate transmission at the source and relays (i.e., channel state information at transmitter for both nodes) and consider delay-tolerant applications. Our main contributions in this work are summarized as follows:

- A new RS criterion based on a weighted sum of instantaneous rates is proposed to maximize the average end-to-end rate in a virtual FD buffer-aided relaying network with adaptive rate transmission.
- Various transmit and receive BF design strategies at the multiple antenna relays are proposed in order to cancel or suppress IRI.
- We show that joint RS and BF schemes achieve the ideal FD relaying bound in terms of the average end-to-end rate asymptotically with increasing the numbers of antennas and/or the number of relays.

- Compared to our previous work \cite{34}, we propose new joint RS and BF schemes. Moreover, we provide extensive numerical results including average end-to-end rate, average delay, effect of IRI intensity, behavior of optimal weight factor, and effect of finite buffer size.

The rest of this paper is organized as follows. In Section II, the system model is presented. The instantaneous rates and average end-to-end rate of a buffer-aided relaying network are described in Section III. Buffer-aided joint RS and BF schemes considering IRI are proposed in Section IV. In Section V, the performance of the proposed schemes are evaluated through simulations. Finally, conclusive remarks and future work are provided in Section VI.

II. System Model

In this paper, we consider a source, \( S \), and a destination, \( D \), which have a single antenna, and \( K \) buffer-aided relays with \( M \) antennas each (e.g., in Fig. 1, \( M = 2 \)). Denote the set of HD buffer-aided decode-and-forward relays by \( \mathcal{K} = \{1, \ldots, K\} \). We assume that there is no direct path between the source and destination as in the related literature \cite{18, 19, 20, 21, 22, 23, 24, 31, 32, 33, 34, 35, 36}. This system model can be regarded as an example of relay-assisted device-to-device communications where the source and destination are low-cost devices with some limitations such as a single antenna. The source is supposed to always have data traffic to transmit. In addition, let \( h_{\mathcal{S}i} \), \( h_{\mathcal{D}j} \), and \( h_{ji} \), \( i, j \in \mathcal{K} \) denote the channel coefficient vectors and matrices of \( \{\mathcal{S}\rightarrow i\} \), \( \{j\rightarrow D\} \), and \( \{j\rightarrow i\} \) links, respectively. The derivation of the proposed algorithms does not rely on any specific assumptions about the channel conditions, but in the asymptotic analysis and in the numerical examples, we assume that all the channel coefficients follow circular symmetric complex Gaussian distributions such as \( h_{\mathcal{S}i} \sim \mathcal{CN}(0, \sigma_{\mathcal{S}i}^2 \mathbf{I}) \), \( h_{\mathcal{D}j} \sim \mathcal{CN}(0, \sigma_{\mathcal{D}j}^2 \mathbf{I}) \), and \( \text{vec}[h_{ji}] \sim \mathcal{CN}(0, \sigma_{ji}^2 \mathbf{I}) \) where \( \text{vec}[\cdot] \) denotes the vectorization of a matrix.

In order to mimic FD relaying, \( \{\mathcal{S}\rightarrow \mathcal{R}\} \) and \( \{\mathcal{R}\rightarrow \mathcal{D}\} \) transmissions are performed simultaneously by using the best pair of receiving and transmitting relays as in \cite{33, 35, 36, 40}. To this end, the receiving relay decodes the data received from the source and stores it in its buffer, while the transmitting relay encodes data from its buffer and sends it to the destination. For a given selected relay pair \( (i, j) \), \( i \neq j \), the received signal vector at the receiving relay \( i \) is

\[
y_{i}^{(i,j)} = h_{\mathcal{S}i} x_S + H_{ji} x_j + n_i = h_{\mathcal{S}i} x_S + H_{ji} w_j x_j + n_i,
\]

where \( h_{\mathcal{S}i} \in \mathbb{C}^{M \times 1} \) denotes the channel vector from the source to the \( i \)-th relay, \( H_{ji} \in \mathbb{C}^{M \times M} \) denotes the inter-relay channel matrix from the \( j \)-th relay to the \( i \)-th relay, \( x_S \) denotes the transmitted data symbol from the source, and \( x_j = w_j x_j \) denotes the transmitted data symbol vector from the \( j \)-th relay where \( w_j = [w_j^1, \ldots, w_j^M]^T \) and \( x_j \) represent the transmit BF vector of the \( j \)-th relay and the transmitted data symbol of the \( j \)-th relay, respectively. Here, \( \mathbb{E}[|x_S|^2] \leq P_S \) and \( \mathbb{E}[|x_j|^2] \leq P_R \) where \( P_S \) and \( P_R \) denote the maximum
transmit powers of the source and the relay, respectively, and \( \|w_j\| = 1 \) for \( i, j \in K \) where \( \| \cdot \| \) denotes the 2-norm. \( \mathbf{n}_i \) denotes an additive white Gaussian noise (AWGN) vector with zero mean and covariance \( \sigma_n^2 \mathbf{I} \), i.e., \( \mathbf{n}_i \sim \mathcal{CN}(0, \sigma_n^2 \mathbf{I}) \). For a given selected relay pair \((i, j)\), \( i \neq j \), the received signal at the destination is

\[
y^{(i,j)}_D = \mathbf{h}_{i,j}^H \mathbf{x}_j + n_D = \mathbf{h}_{i,D}^H \mathbf{w}_j x_j + n_D, \tag{2}
\]

where \( \mathbf{h}_{i,D} \in \mathbb{C}^{M \times 1} \) denotes the channel vector from the \( j \)-th relay to the destination, \( \mathbf{h}_{i,D}^H \) denotes the Hermitian transpose, and \( n_D \) denotes AWGN with zero mean and variance \( \sigma_n^2 \), i.e., \( n_D \sim \mathcal{CN}(0, \sigma_n^2) \).

III. INSTANTANEOUS RATES AND AVERAGE END-TO-END RATE OF A BUFFER-AIDED RELAYING NETWORK

If we employ a linear receive BF vector at the \( i \)-th receiving relay as \( \mathbf{u}_i = [u_i^1, \ldots, u_i^M]^T \), \( \|\mathbf{u}_i\| = 1 \), for given relay pair \((i, j)\), the received signal after the receive BF becomes

\[
r^{(i,j)}_i = \mathbf{u}_i^H y^{(i,j)}_i = \mathbf{u}_i^H \mathbf{h}_{S_i} x_S + \mathbf{u}_i^H \mathbf{H}_{j,i} \mathbf{w}_j x_j + \tilde{n}_i, \tag{3}
\]

where \( \tilde{n}_i = \mathbf{u}_i^H \mathbf{n}_i \sim \mathcal{CN}(0, \sigma_n^2) \). From (3) and (2), the instantaneous received SINR/SNR for the \( \{S \rightarrow i\} \) and \( \{j \rightarrow D\} \) links at time slot \( t \) are expressed, respectively, as:

\[
\gamma^{(i,j)}_{S_i}(t) = \frac{\rho_S |\mathbf{u}_i^H \mathbf{h}_{S_i}|^2}{1 + \rho_R |\mathbf{u}_i^H \mathbf{H}_{j,i} \mathbf{w}_j|^2}, \tag{4}
\]
\[
\gamma^{(i,j)}_{j,D}(t) = \frac{\rho_R |\mathbf{h}_{i,j}^H \mathbf{w}_j|^2}{\rho_S |\mathbf{u}_i^H \mathbf{h}_{S_i}|^2}, \tag{5}
\]

where \( \rho_S = \frac{P_S}{\sigma_n^2} \) and \( \rho_R = \frac{P_R}{\sigma_n^2} \).

Let \( B_i(t) \) denote the number of bits in the buffer normalized by the number of channel uses of the \( i \)-th relay at the end of time slot \( t \). Assuming a Gaussian codebook and information theoretic capacity achieving coding scheme, and taking the buffer state at the receiving relay \( i \) into consideration, the instantaneous rate of the \( \{S \rightarrow i\} \) link is

\[
C^{(i,j)}_{S_i}(t) = \min \left\{ \log_2 \left( 1 + \gamma^{(i,j)}_{S_i}(t) \right), B_{max} - B_i(t-1) \right\}, \tag{6}
\]

where \( \min \{\cdot, \cdot\} \) denotes the minimum value of arguments, \( B_{max} \) denotes the maximum buffer size, and the buffer of the \( i \)-th relay is updated by

\[
B_i(t) = B_i(t-1) + C^{(i,j)}_{S_i}(t). \tag{7}
\]

For the transmitting relay \( j \), the instantaneous rate of link \( \{j \rightarrow D\} \) in time slot \( t \) is

\[
C^{(i,j)}_{j,D}(t) = \min \left\{ \log_2 \left( 1 + \gamma^{(i,j)}_{j,D}(t) \right), B_j(t-1) \right\}, \tag{7}
\]

where the buffer of the \( j \)-th relay is updated by

\[
B_j(t) = B_j(t-1) - C^{(i,j)}_{j,D}(t). \tag{8}
\]

Assuming stationarity of the buffers, the average end-to-end rate of a buffer-aided relaying network is given by the minimum of the average \( \{S \rightarrow R\} \) and \( \{R \rightarrow D\} \) link rates (see [21], [33] for detail), i.e.,

\[
\tilde{C} = \min \left\{ \mathbb{E} \left[ C^{(i(t),j(t))}_{S_i(t)} \right], \mathbb{E} \left[ C^{(i(t),j(t))}_{j,D(t)} \right] \right\}, \tag{9}
\]

where \( \mathbb{E}[\cdot] \) denotes the expectation operation, \( i(t) \) and \( j(t) \) denote the selected receiving and transmitting relay indices in time slot \( t \), respectively, \( C^{(i(t),j(t))}_{S_i(t)} \) and \( C^{(i(t),j(t))}_{j,D(t)} \) denote the instantaneous rates of the \( \{S \rightarrow i(t)\} \) link and the \( \{j(t) \rightarrow D\} \) link in time slot \( t \), respectively. Note that the best relay pair \((i(t), j(t))\) is dynamically selected at each transmission instance for maximizing the average end-to-end rate according to relay selection schemes.

IV. BUFFER-AIDED JOINT RELAY SELECTION AND BEAMFORMING SCHEMES IN THE PRESENCE OF INTER-RELAY INTERFERENCE

Our main objective is to maximize the average end-to-end rate given in (9) through a joint RS and BF design. This should preferably be done separately for each time instance, by only considering the instantaneous channel and buffer states. Optimizing the minimum of the instantaneous rates, which corresponds to maximizing a lower bound on (9), was proposed in [21] as a traditional HD best relay selection. Here, we propose an alternative approach based on Lagrangian relaxation in order to maximize (8).

\[1\]For notational convenience, we define \( \mathbf{h}_{i,D} \) as the complex conjugate channel vector differently from the definition of \( \mathbf{h}_{S_i} \), i.e., \( \mathbf{h}_{i,D}^H = [h_{i,D}^1, \ldots, h_{i,D}^M] \) while \( \mathbf{h}_{S_i} = [h_{S_i}^1, \ldots, h_{S_i}^M] \).
In a virtual FD buffer-aided relaying network, assuming the maximum transmit power \( P_t \) at the source and relays, the average end-to-end rate maximization is formulated as follows:

\[
\begin{align}
\max_{\{u_{(t)}, w_{j(t)}, i(t), j(t)\}} & \quad \min \left\{ \mathbb{E}\left[ C_{\text{SR}}^{(i(t), j(t))}(t) \right], \mathbb{E}\left[ C_{\text{RD}}^{(i(t), j(t))}(t) \right] \right\} \\
\text{s. t.} & \quad \|u_{i(t)}\| \leq 1, \|w_{j(t)}\| \leq 1, \quad \forall i(t) \neq j(t).
\end{align}
\]

Since the constraint (9c) is trivial in the virtual FD buffer-aided relaying network, we omit it in the following. The optimization problem can be reformulated into

\[
\begin{align}
\max_{\{u_{(t)}, w_{j(t)}, i(t), j(t)\}} & \quad \tilde{C} \\
\text{s. t.} & \quad \mathbb{E}\left[ C_{\text{SR}}^{(i(t), j(t))}(t) \right] \geq \tilde{C}, \\
& \quad \mathbb{E}\left[ C_{\text{RD}}^{(i(t), j(t))}(t) \right] \geq \tilde{C}, \\
& \quad \|u_{i(t)}\| \leq 1, \|w_{j(t)}\| \leq 1.
\end{align}
\]

Introducing Lagrange multipliers for the first two constraints (10b)–(10c), we obtain the partial Lagrangian

\[
\begin{align}
\max_{\{u_{(t)}, w_{j(t)}, i(t), j(t)\}} & \quad C + \lambda_{\text{SR}} \left( \mathbb{E}\left[ C_{\text{SR}}^{(i(t), j(t))}(t) \right] - \tilde{C} \right) \\
& \quad + \lambda_{\text{RD}} \left( \mathbb{E}\left[ C_{\text{RD}}^{(i(t), j(t))}(t) \right] - \tilde{C} \right) \\
\text{s. t.} & \quad \|u_{i(t)}\| \leq 1, \|w_{j(t)}\| \leq 1.
\end{align}
\]

Since our objective is to maximize the long-term average end-to-end rate, the Lagrange multipliers \( \lambda_{\text{SR}} \) and \( \lambda_{\text{RD}} \) are constant over time. Assuming non-negative, ergodic, stationary buffers for all the relays, and sufficiently large buffer sizes, it is equivalent to

\[
\begin{align}
\max & \quad \mathbb{E}\left[ \lambda_{\text{SR}} C_{\text{SR}}^{(i(t), j(t))}(t) + \lambda_{\text{RD}} C_{\text{RD}}^{(i(t), j(t))}(t) \right] \\
\iff & \quad \max \lambda_{\text{SR}} C_{\text{SR}}^{(i(t), j(t))}(t) + \lambda_{\text{RD}} C_{\text{RD}}^{(i(t), j(t))}(t),
\end{align}
\]

where enables a joint RS and BF based on instantaneous rates at each time slot. Rescaling by \( \lambda_{\text{SR}} + \lambda_{\text{RD}} \) and introducing a parameter \( \alpha = \lambda_{\text{SR}} / (\lambda_{\text{SR}} + \lambda_{\text{RD}}) \), we arrive at

\[
\begin{align}
\max_{\{u_{(t)}, w_{j(t)}, i(t), j(t)\}} & \quad \alpha C_{\text{SR}}^{(i(t), j(t))}(t) + (1 - \alpha) C_{\text{RD}}^{(i(t), j(t))}(t) \\
\text{s. t.} & \quad \|u_{i(t)}\| \leq 1, \|w_{j(t)}\| \leq 1.
\end{align}
\]

The conventional centralized/distributed RS approaches [37, 38, 39] can be applied for implementation of the proposed RS schemes. To reduce the amount of feedback on CSI, the distributed RS approach is more desirable than the centralized approach in practice. For example, first of all, the relays can estimate channels based on orthogonal pilot signals from the source and the destination at the same time. Then, the relays can estimate inter-relay channels in a round robin manner using orthogonal MIMO pilot signals. If BSIs and CSIs for \( \{S \rightarrow R\} \) links (i.e., \( h_{S_i} \)'s) are shared among the relays, each relay is able to select the local-best receiving relay by regarding itself as the transmitting relay. Similarly to the timer-based distributed RS in [37], each relay sets a timer based on an inverse of its local-best objective function value and sends a request-to-send message after timer expiration. Then, the destination sends back a clear-to-send message for the earliest access relay. If the relay receives the message, it broadcasts the RS information to all the nodes. Afterwards, the source and the transmitting relay start to transmit their own packets. Through this procedure, the best relay pair can be determined in a distributed manner. The detailed implementation issues are beyond the scope of this work.

The instantaneous rates in (13) are determined by effective SINR/SNRs at the receiving and transmitting relays, depending on the transmit and receive beamformers. Thus, the transmit and receive beamformers have to be determined separately for each candidate pair of relays. However, finding the optimal BF vectors for every given relay pair is non-convex and therefore we propose an iterative optimal BF-based RS scheme. Since the iterative solution requires a high computational complexity, several low-complexity suboptimal BF-based RS schemes are also proposed in the rest of this section. Note that the suboptimal schemes have less complexity in BF design than the optimal scheme but the same complexity in RS protocol. (See Section IV-V.) For simplicity, we omit the time slot index \( t \) in the relay indices \( i(t) \) and \( j(t) \) hereafter.

\[\text{in } [0, 1]. \text{ Note that it depends on system properties such as the number of relays, the number of antennas, and channel statistics. Behavior of the optimal } \alpha \text{ parameter is shown and discussed in Section V-D.}\]

Finally, using the predetermined optimal \( \alpha^* \) and the optimized BF vectors for each relay pair, the best relay pair is determined by

\[
\begin{align}
& \quad \{i^*(t), j^*(t)\} \\
& \quad = \arg \max_{i(t) \neq j(t)} \alpha^* C_{\text{SR}}^{(i(t), j(t))}(t) + (1 - \alpha^*) C_{\text{RD}}^{(i(t), j(t))}(t),
\end{align}
\]

where \( C_{\text{SR}}^{(i(t), j(t))}(t) \) and \( C_{\text{RD}}^{(i(t), j(t))}(t) \) are given in (6) and (7), respectively. Throughout this paper, we consider an exhaustive search under global channel state information (CSI) and buffer state information (BSI) for all proposed schemes to obtain the optimal performance in RS. Discussion on the complexity of our proposed schemes for both RS and BF aspects is provided in Section IV-V.
A. Proposed Optimal Beamforming-based Relay Selection Scheme

Obviously, if there is no IRI, maximal ratio combining (MRC) at the receiving relay and maximal ratio transmit (MRT) BF at the transmitting relay, named IRI-free BF, are optimal. However, this idealized IRI-free BF is not optimal in the presence of IRI. In this subsection, we propose an iterative optimal BF-based RS scheme to maximize the average end-to-end rate. Denoting \( f(\gamma) = \log_2(1 + \gamma) \), the optimization problem [13] for given \( \alpha \) is reformulated into:

\[
\max_{\{u, w, \alpha\}} \alpha f\left( \rho_S |u^H_i h_{S_i}|^2 \right) + (1 - \alpha) f\left( \rho_R h_{D_i}^H w_j h_{D_j} \right) \quad \text{s.t.} \quad \|u_i\| \leq 1, \|w_j\| \leq 1. \tag{15}
\]

Since this optimization problem cannot be solved directly due to non-convexity, we propose an alternating optimization which iterates between (i) for fixed \( w_j \), optimizing \( u_i \), and (ii) for fixed \( u_i \), optimizing \( w_j \).

For given \( w_j \), the optimal receive beamformer \( u_i \) is obviously given by the MMSE solution,

\[
u_i = c_u (\rho_R H_{ji} w_j h_{D_j}^H + I)^{-1} h_{S_i}, \tag{16}\]

where the scaling factor \( c_u \) is selected such that \( \|u_i\| = 1 \).

For given \( u_i \), optimizing the transmit beamformer \( w_j \) is a non-convex problem. Denote \( g_{ji} \triangleq H_{ji}^H u_i \). The Lagrangian of the optimization problem is

\[
\mathcal{L}(w_j, \lambda) = \alpha f\left( \rho_S |u^H_i h_{S_i}|^2 \right) + (1 - \alpha) f\left( \rho_R h_{D_i}^H w_j h_{D_j} \right) + \lambda (1 - \|w_j\|), \quad \text{(17)}
\]

where the gradient with respect to \( w_j \) is obtained by

\[
\nabla_{w_j} \mathcal{L}(w_j, \lambda) = -\frac{\alpha f'((\gamma_S)\rho_S |u^H_i h_{S_i}|^2)}{(\rho_R g_{ji}^H w_j h_{D_j} + \|u_i\|^2)^2} 2\rho_S g_{ji}^H g_{ji}^H w_j + (1 - \alpha) f'((\gamma_D)\rho_R h_{D_i}^H w_j h_{D_j}) 2\rho_R h_{D_i}^H w_j w_j^* - 2\lambda w_j, \quad \text{(18)}
\]

where \( \gamma_S = \alpha \frac{\rho_S |u^H_i h_{S_i}|^2}{(\rho_R g_{ji}^H w_j h_{D_j} + \|u_i\|^2)^2} \) and \( \gamma_D = \rho_R h_{D_i}^H w_j h_{D_j}^H \).

Hence, the KKT conditions give

\[
(\lambda I + \mu g_{ji}^H g_{ji}^H) w_j = (1 - \alpha) f'((\gamma_D)\rho_R h_{D_i}^H w_j h_{D_j}) w_j, \quad \text{(19)}
\]

where \( \mu \triangleq \frac{\alpha f'((\gamma_S)\rho_S |u^H_i h_{S_i}|^2)}{(\rho_R g_{ji}^H w_j h_{D_j} + \|u_i\|^2)^2} \rho_S \). Thus, the optimal \( w_j \) has the form

\[
w_j = c_w \left( \lambda I + \mu g_{ji} g_{ji}^H \right)^{-1} h_{D_j}, \tag{20}\]

for some values of the positive real-valued parameters \( \lambda \) and \( \mu \), and scaling constant \( c_w \). Using the matrix inversion lemma

\[
(A - UD^{-1}V)^{-1} = A^{-1} + A^{-1}U (D - VA^{-1}U)^{-1} VA^{-1} \tag{21}\]

is rewritten as

\[
w_j = c_w \left( \lambda I - \lambda^{-1} g_{ji} (g_{ji}^H g_{ji} + \frac{\lambda}{\mu})^{-1} g_{ji}^H \right) h_{D_j} \]

\[
= c_w \left( h_{D_j} - \frac{g_{ji}^H h_{D_j}^*}{g_{ji}^H g_{ji} + \frac{\lambda}{\mu} g_{ji}^H} \right) \]

\[
= c_w \left( h_{D_j} - \frac{g_{ji}^H h_{D_j}^*}{g_{ji}^H g_{ji} + \frac{\lambda}{\mu} g_{ji}^H} \right) + \left( 1 - \frac{g_{ji}^H h_{D_j}^*}{g_{ji}^H g_{ji} + \frac{\lambda}{\mu} g_{ji}^H} \right) \left( g_{ji}^H h_{D_j}^* g_{ji}^H + \frac{\lambda}{\mu} g_{ji}^H g_{ji} \right), \tag{22}\]

\[\]
Proposed Relay Selection Scheme with Zero-Forcing Beamforming (ZFBF)-based IRI Cancellation

In this subsection, we propose to optimize a transmit beamformer based on zero-forcing (ZF) at the transmitting relay. First of all, we use the MRC beamformer for the receiving relay $i$, i.e., $u_i = \frac{h_{ij}}{\|h_{ij}\|}$ and then maximize the effective channel power gain of the \{R→D\} link under a ZF condition. Therefore, for a given relay pair $(i, j)$, the following optimization problem is formulated:

$$\begin{align*}
\max_{z} & \quad |z^H V_{ji}^H h_{jD}|^2 \\
\text{s.t.} & \quad u_i^H h_{jD} w_j = 0, \quad \|w_j\| = 1.
\end{align*}$$

Let $V_{ji} \in \mathbb{C}^{M \times (M-1)}$ be a matrix whose columns span the null-space of $g_{ji} \triangleq H_{ji}^H u_i$. Then, any BF vector $w_j$ fulfilling the first constraint in (26b) can be written as $w_j = V_{ji} z$, where $z \in \mathbb{C}^{(M-1)\times1}$. Hence, the optimization problem is reformulated by

$$\begin{align*}
\max_{z} & \quad z^H V_{ji}^H h_{jD} V_{ji} z \\
\text{s.t.} & \quad \|V_{ji} z\| = 1.
\end{align*}$$

The solution of this problem is $z^* = c_z (V_{ji}^H V_{ji})^{-1} V_{ji}^H h_{jD}$, resulting in $w_j^* = c_z V_{ji} (V_{ji}^H V_{ji})^{-1} V_{ji}^H h_{jD} = c_z w_j$, where the scalar $c_z$ is chosen so that $\|w_j^*\| = 1$, i.e., $w_j^* = \frac{w_j}{\|w_j\|}$ in which

$$w_j^* = V_{ji} (V_{ji}^H V_{ji})^{-1} V_{ji}^H h_{jD} = \left( I - \frac{g_{ji} g_{ji}^H}{g_{ji}^H g_{ji}} \right) h_{jD} = h_{jD} - c H_{ji}^H h_{Si},$$

where the scalar $c = \frac{h_{ji}^H h_{ji}}{g_{ji}^H g_{ji}}$ since $u_i = \frac{h_{ji}}{\|h_{ji}\|}$. This optimum solution implies a projection of $h_{jD}$ onto the null-space $V_{ji}$.

Therefore, substituting $u_i$ and $w_j$ into (4) and (5), the instantaneous SNRs for the \{S→i\} and \{j→D\} links are expressed, respectively, as:

$$\begin{align*}
\gamma_{Si}^{(i,j)} (t) & = \rho_S \|h_{Si}\|^2, \\
\gamma_{jD}^{(i,j)} (t) & = \rho_{R} \|h_{jD}\|^2 - \tilde{c}^2 \|h_{jD} - c H_{ji}^H h_{Si}\|^2,
\end{align*}$$

where the scalar values $c = \frac{h_{ji}^H h_{ji}}{\|h_{ji}\|^2}$ and $\tilde{c} = h_{ji}^H h_{ji} h_{ji}^H h_{ji} / \|h_{ji}^H h_{ji}\|^2$.

As a result, substituting (30) and (31) into (6) and (7), respectively, the best relay pair is selected by (14).

Proposition 1: The proposed ZFBF-based RS scheme asymptotically achieves the average end-to-end rate of ideal FD relaying as the number of antennas ($M$) goes to infinity.

Proof: Let $(H)_m$ denote the $m$-th column vector of the matrix $H$ and $(H)_{m,l}$ denote the $(m, l)$-th element of the matrix $H$. Denoting $g_{ji} = [g_{ji}^1, g_{ji}^2, \ldots, g_{ji}^M]^T$ in (29), $g_{ji}^m = (H)_{j/m} u_m \sim CN(0, \sigma_{IR}^2)$ since $(H)_{j/m} \sim CN(0, \sigma_{IR}^2)$ and $\|u_m\| = 1$. Assuming $\sigma_{IR}^2 = 1$ without loss of generality, $g_{ji}^m \sim CN(0, 1)$ and therefore $g_{ji}^m h_{ij} = \sum_{m=1}^{M} |g_{ji}^m|^2$ follows a chi-squared distribution with $2M$ degrees of freedom, i.e., $g_{ji}^m h_{ij} \sim \chi^2_{2M}$. Meanwhile, the diagonal elements, $(g_{ji}^m g_{ji}^H)_{m,m} = |g_{ji}^m|^2$, follow an exponential distribution with parameter one, i.e., $(g_{ji}^m g_{ji}^H)_{m,m} \sim Exp(1), \forall m \in \{1, \ldots, M\}$ and the off-diagonal elements, $(g_{ji}^m g_{ji}^H)_{m,l}, \forall m \neq l$, follow the distribution of a product of two independent Gaussians with zero mean and unit variance (see [41]). The distribution of each element in $g_{ji}^m g_{ji}^H$ is not varying with respect to $M$ while only the size of matrix grows according to $M$. As a result, $M \to \infty$, the denominator $g_{ji}^H g_{ji}$ goes to infinity, while all the elements in the numerator $g_{ji}^m g_{ji}^H$ remain as constant values with respect to $M$. Hence, $\frac{g_{ji}^m g_{ji}^H}{g_{ji}^H g_{ji}} \to 0$ as $M \to \infty$.

Accordingly, $w_j^* \to \frac{h_{jD}}{\|h_{jD}\|}$ and $\gamma_{jD}^{(i,j)} (t) \to \rho_R \|h_{jD}\|^2$ as $M \to \infty$, which completes the proof.

Remark 1: The proposed ZFBF-based RS scheme also approach the average end-to-end rate of ideal FD relaying as the number of relays ($K$) goes to infinity due to increased selection diversity. However, increasing the number of relays cannot guarantee to achieve the performance of the ideal FD relaying as in Proposition [1] since its selection diversity is always less than or equal to that of the ideal FD relaying. In other words, the best transmitting relay should meet $\frac{g_{ji}^m g_{ji}^H}{g_{ji}^H g_{ji}} \approx 0$ for achieving the same performance as the ideal FD relaying.

Although there exist certain relays satisfying $\frac{g_{ji}^m g_{ji}^H}{g_{ji}^H g_{ji}} \approx 0$ with high probability as $K$ goes to infinity, they are a subset of the set of relays while the ideal FD relaying can always take the full selection diversity due to no IRI assumption.

Remark 2: If the ZFBF solution in (23) is set to be the initial vector for $w_j$ in the first step of the proposed iterative optimal BF, it always yields a better solution than the ZFBF, regardless of the number of iterations, since $\arg\max_{u_j} |\rho_S h_{jD}^H u_j| = u_j = \frac{h_{jD}}{\|h_{jD}\|}$ which yields exactly the same BF pair of the ZFBF-based RS scheme and an additional iteration gives a better solution. We therefore propose to initialize the alternating optimization in this way, even though there are no guarantees that this will provide the global optimum.

C. Proposed Relay Selection Scheme with Minimum Mean Square Error (MMSE)-based IRI Suppression

In this subsection, we propose a receive beamformer based on minimum mean square error (MMSE) at the receiving relay. First of all, we use the MRT beamformer for the transmitting relay $j$, i.e., $w_j = \frac{h_{jD}}{\|h_{jD}\|}$, and then we find a receive beamformer for maximizing the effective SNIR at the receiving relay. Therefore, for given relay pair $(i, j)$ and $w_j$,
the following optimization problem is formulated:

$$\max_{\mathbf{u}_i} \quad \frac{\rho_S \mathbf{u}_i^H \mathbf{h}_{Si} \mathbf{h}_{Si}^H \mathbf{u}_i}{(\rho_R \mathbf{H}_{ij} \mathbf{w}_j \mathbf{w}_j^H \mathbf{H}_{ij}^H + I) \mathbf{u}_i}$$

s. t. \(\|\mathbf{u}_i\| = 1\). \hspace{1cm} (32a)

The solution of (32) is given by the scaled MMSE as \(\mathbf{u}_i^* = c_m \rho_R \mathbf{H}_{ij} \mathbf{w}_j \mathbf{w}_j^H \mathbf{H}_{ij}^H + I \mathbf{h}_{Si}\) where the scaling factor \(c_m\) is chosen such that \(\|\mathbf{u}_i\| = 1\).

Therefore, substituting \(\mathbf{u}_i\) and \(\mathbf{w}_j\) into (32) and (33), respectively, the best relay pair is selected by (14).

Substituting (33) and (34) into (6) and (7), respectively, the best relay pair is selected by (14).

**Proposition 2**: The proposed MMSE-based RS scheme asymptotically achieves the average end-to-end rate of ideal FD relaying at low SNR.

Proof: As \(\rho_R \to 0\), \(\gamma_{Si}^{(i,j)}(t) \approx \rho_S \mathbf{h}_{Si}^H \mathbf{I}^{-1} \mathbf{h}_{Si} = \rho_S \| \mathbf{h}_{Si} \|^2\), which completes the proof.

**D. Proposed Relay Selection Scheme with Orthonormal Basis (OB)-based IRI Cancellation**

In this subsection, we propose a perfect IRI cancellation scheme based on orthonormal basis vectors. To this end, we first generate two random orthonormal vectors \(\mathbf{u}\) and \(\mathbf{q}\), i.e., \(\mathbf{u}^H \mathbf{q} = 0\), \(\|\mathbf{u}\| = 1\), and \(\|\mathbf{q}\| = 1\). Then, we use \(\mathbf{u}\) as the receive beamformer at the receiving relay and \(\mathbf{w}_j = \frac{\mathbf{H}_{ij}^{-1} \mathbf{q}}{\|\mathbf{H}_{ij}^{-1} \mathbf{q}\|}\) as the transmit beamformer at the transmitting relay, respectively.

Since \(\mathbf{u}^H \mathbf{H}_{ij} \mathbf{w}_j = \mathbf{u}^H \mathbf{H}_{ij} \frac{\mathbf{H}_{ij}^{-1} \mathbf{q}}{\|\mathbf{H}_{ij}^{-1} \mathbf{q}\|} = 0\), \(\|\mathbf{u}\| = 1\), and \(\|\mathbf{w}_j\| = 1\), substituting \(\mathbf{u}\) and \(\mathbf{w}_j\) into (4) and (5), the instantaneous SNRs for the \(\{S\to i\}\) and \(\{j\to D\}\) links are expressed as:

$$\gamma_{Si}^{(i,j)}(t) = \rho_S \| \tilde{\mathbf{h}}_{Si} \|^2,$$

$$\gamma_{jD}^{(i,j)}(t) = \rho_R \| \tilde{\mathbf{h}}_{jD} \|^2,$$

where \(\tilde{\mathbf{h}}_{Si} = \mathbf{u}^H \mathbf{h}_{Si} \sim \mathcal{CN}(0, \sigma_{Si}^2)\) and \(\tilde{\mathbf{h}}_{jD} = \mathbf{H}_{ij} \mathbf{w}_j \sim \mathcal{CN}(0, \sigma_{jD}^2)\). As a result, substituting (35) and (36) into (6) and (7), respectively, the best relay pair is selected by (14).

**Remark 3**: From (35) and (36), the proposed OB-based RS scheme achieves the average end-to-end rate of ideal FD relaying with a single antenna at the relays.

**Remark 4**: The proposed ZFBF-based RS scheme is always better than the proposed OB-based RS scheme in terms of the average end-to-end rate while both of the schemes perfectly cancel IRI. However, it is not always better in the viewpoint of instantaneous RS and BF, since the randomly chosen beamformers in the proposed OB-based RS scheme might be better at a certain RS instance for not the \(\{S\to R\}\) link rate but the weighted sum-rate.

**E. Proposed SINR-based Relay Selection Scheme with Beamforming Neglecting IRI**

Although the IRI-free BF is not optimal in the presence of IRI, we propose to use them and utilize effective SINR/SNR measures after BF in RS as the simplest joint RS and BF scheme. Accordingly, for relay pair \(i, j\), the receive BF vector is given by \(\mathbf{u}_i = \frac{\mathbf{h}_{Si}}{\| \mathbf{h}_{Si} \|}\) and the transmit BF vector is given by \(\mathbf{w}_j = \frac{\mathbf{H}_{ij}^{-1} \mathbf{q}}{\| \mathbf{H}_{ij}^{-1} \mathbf{q} \|}\). Substituting \(\mathbf{u}_i\) and \(\mathbf{w}_j\) into (32) and (33), the instantaneous SINR and SNR of both the \(\{S\to i\}\) and \(\{j\to D\}\) links are obtained, respectively, by

$$\gamma_{Si}^{(i,j)}(t) = \frac{\| \mathbf{h}_{Si} \|^2 \rho_S}{1 + \frac{\| \mathbf{h}_{Si} \|^2 \| \mathbf{h}_{jD} \|^2}{\| \mathbf{h}_{jD} \|^2 \rho_R}},$$

$$\gamma_{jD}^{(i,j)}(t) = \frac{\| \mathbf{h}_{jD} \|^2 \rho_R}{\| \mathbf{h}_{jD} \|^2 \rho_R}.$$ 

Substituting (37) and (38) into (6) and (7), respectively, the best relay pair is selected by (14).

**F. Discussion on Complexity of the Proposed Joint Relay Selection and Beamforming**

For RS protocol, an exhaustive search within \(K \times (K - 1)\) combinations with global CSI and BSI is required to obtain the optimal performance. Accordingly, the complexity of the optimal relay pair selection is \(O(K^2)\) for all the schemes. However, through the distributed RS approach, CSIs for \(\{R\to D\}\) and \(\{R\to R\}\) links can be directly estimated at each relay from pilot signals and thus the amount of feedback on CSI can be reduced from \((2KM + M^2K(K-1)/2)\) to \(KM\). It is also worth mentioning that the number of available (fixed) relays in practical network scenarios would not be so large due to geographical limitations. On the other hand, the proposed schemes can still be effective even for a two-relay network if sufficient number of antennas are available at the relays. Moreover, the performance enhancement from more than five relays would be marginal. It will be shown in Section V-E.

For BF design for given relay pair, if we denote the complexity for computing a BF vector including \(M\) dimensional matrices and/or vectors by \(O(M)\), the complexity of all the suboptimal BF schemes becomes \(O(2M)\) since they require to compute two BF vectors. However, they can have different computational times in practice according to degree of matrix and/or vector computations. For instance, the MMSE-based scheme requires to compute \(M\) dimensional matrix inversion, while the SINR-based scheme only requires vector normalization. On the other hand, the complexity of the optimal BF scheme is derived as \(O(L(2M + C))\), where \(L\) denotes the number of iterations and \(C\) denotes the complexity to find the optimal \(\beta\) including both a rough line search and some Gauss-Newton steps. Consequently, the optimal BF scheme requires additional complexity for iteration process and an additional optimization parameter, compared to the suboptimal BF schemes.

**V. PERFORMANCE EVALUATION**

In this section, we evaluate the proposed joint buffer-aided RS and BF schemes in terms of the average end-to-end rate...
and average delay through Monte-Carlo simulations, compared to conventional HD RS schemes and SFD-MMRS scheme representing state-of-the-art in the literature. For multiple-antenna extension of the conventional schemes, we suppose that they use the IRI-free beamformers. As upper bound of the average end-to-end rate, we consider the optimal joint RS and BF in [13] assuming no IRI. The optimal weight factor $\alpha^*$ is numerically found through a line search for each scheme and each setup in advance. We consider identical and independently distributed (i.i.d.) Rayleigh block fading channels with $\sigma_{SR}^2 = \sigma_{RD}^2 = \sigma_{RR}^2 = 0$ dB unless otherwise specified, 10000 packet transmissions from the source, and zero initial buffer state at all relays, and assume $P_S = P_R$ throughout all simulations.

A. Benchmarks

1) HD Best Relay Selection (BRS) Scheme [37]: In the HD-BRS scheme without buffering at relays, the best relay is determined by

$$i^* = \arg\max_{i \in K} \min \{C_{Si}(t), C_{iD}(t)\},$$

where $C_{Si}(t) = \frac{1}{2} \log_2(1 + \gamma_{Si}(t))$ and $C_{iD}(t) = \frac{1}{2} \log_2(1 + \gamma_{iD}(t))$.

2) HD max–max Relay Selection (MMRS) Scheme [37]: In the HD-MMRS scheme maintaining the two-phase operation, the best relay at first time slot, $i^*$, and the best relay at second time slot, $j^*$, are selected as follows:

$$i^* = \arg\max_{i \in K} C_{Si}(2t), \quad j^* = \arg\max_{j \in K} C_{jD}(2t+1),$$

where $C_{Si}(2t) = \min \{\frac{1}{2} \log_2(1 + \gamma_{Si}(2t)), B_{max} - B_i(2t-1)\}$ and $C_{jD}(2t+1) = \min \{\frac{1}{2} \log_2(1 + \gamma_{jD}(2t+1)), B_j(2t)\}$.

3) HD max–l link Relay Selection (MLRS) Scheme [32]: Since the HD-MLRS scheme has been developed for fixed rate transmission, we slightly modify it to adaptive rate transmission by adding a link selection parameter $d_i$ for the $i$-th relay.

In the conventional HD-MLRS scheme releasing the two-phase operation condition, the best relay $i^*$ and the best link $d_i$ (integer variable; receiving 1 or transmitting 0) at each time slot are determined by

$$(i^*, d_i) = \arg\max_{i \in K, d_i \in \{0, 1\}} \{d_i C_{Si}(t) + (1 - d_i) C_{iD}(t)\},$$

where $C_{Si}(t) = \min \{\frac{1}{2} \log_2(1 + \gamma_{Si}(t)), B_{max} - B_i(t-1)\}$ and $C_{iD}(t) = \min \{\frac{1}{2} \log_2(1 + \gamma_{iD}(t)), B_i(t)\}$.

4) SFD-MMRS Scheme [33]: In the SFD-MMRS scheme, each $\{S \rightarrow R\}$ or $\{R \rightarrow D\}$ link selects the best relay and the second best relay based on channel gains without consideration of IRI. Denote the relay indices of the best and second best relays by $i_1$ and $j_2$ for the receiving relay and $i_1$ and $j_2$ for the transmitting relay, respectively. Then they are selected as follows:

$$i_1 = \arg\max_{i \in K} C_{Si}(t), \quad i_2 = \arg\max_{i \in K \setminus \{i_1\}} C_{Si}(t),$$

$$j_1 = \arg\max_{j \in K} C_{jD}(t), \quad j_2 = \arg\max_{j \in K \setminus \{j_1\}} C_{jD}(t),$$

where $C_{Si}(t) = \min \{\log_2(1 + \gamma_{Si}(t)), B_{max} - B_i(t-1)\}$ and $C_{jD}(t) = \min \{\log_2(1 + \gamma_{jD}(t)), B_j(t-1)\}$. If the best relays for both links are same, it finds the best relay pair among combinations with the second best relays based on a minimum of achievable rates. Then the RS scheme is given by

$$(i^*, j^*) = \begin{cases} (i_1, j_1), & \text{if } i_1 \neq j_1 \\ (i_2, j_1), & \text{if } i_1 = j_1 \text{ and } \min \{C_{Si}(t), C_{jD}(t)\} > \min \{C_{Si}(t), C_{jD}(t)\} \\ (i_1, j_2), & \text{otherwise}. \end{cases}$$ (42)

Since the SFD-MMRS scheme assumes a single antenna at relays and no IRI, we extend it to multiple antennas using the IRI-free BF and assume the receiving relays suffering from IRI for non-ideal case. The performance degradation of the non-ideal SFD-MMRS scheme due to IRI is shown as numerical results in the following subsections.

B. Average End-to-End Rate

1) i.i.d. IRI Channel Case ($\sigma_{SR}^2 = \sigma_{RD}^2 = \sigma_{RR}^2 = 0$ dB): Fig. 2 shows the average end-to-end rate for varying SNR when $K = 2$, $M = 2$, $B_{max} \rightarrow \infty$, and the average channel qualities of all the links are identical. The ideal SFD-MMRS scheme almost achieves the upper bound obtained by [13] but it is slightly less than the upper bound. This validates that our weighted sum-rate maximization based on instantaneous rates works well to maximize the average end-to-end rate. If we impose IRI into the SFD-MMRS scheme, its performance is significantly degraded with increasing SNR, i.e., in the interference-limited regime. Although the proposed SINR-based RS scheme improves the average end-to-end rate, its contribution is not significant at medium/high SNR. On the contrary, the average end-to-end rates of the other proposed schemes still increase with increasing SNR due to IRI cancellation/suppression. Since the proposed OB-based RS scheme achieves the single antenna upper bound, the proposed optimal BF-based, ZFBF-based, and MMSE-based RS schemes outperform the single antenna upper bound. While the optimal BF optimizes both transmit and receive beamformers iteratively, the ZFBF and MMSE BF optimize just one beamformer fixing the other beamformer. As a result, the ZFBF-based and MMSE-based RS schemes achieve less average end-to-end rates than the optimal BF-based RS scheme. Moreover, the MMSE-based RS scheme achieves better performance at low/medium SNR than the ZFBF-based RS scheme, since the MMSE BF can achieve the ideal upper bound at low SNR regime as proved in Proposition 2.

Regarding the conventional HD RS schemes, the HD-MLRS and HD-MMRS schemes outperform the HD-BRS scheme since they additionally utilize buffering at relays. Furthermore, the HD-MLRS scheme outperforms the HD-MMRS scheme since it obtains more diversity gain by releasing the two-phase operation condition. Compared to the HD RS schemes, the slopes of curves of the proposed optimal BF-based, ZFBF-based, and MMSE-based RS schemes are almost double. For instance, at $\text{SNR} = 30$ dB, the proposed optimal BF-based RS...
scheme exceeds twice the average end-to-end rate by the HD-BRS scheme and the proposed ZFBB-based and MMSE-based RS schemes achieve slightly less performance than it.

Fig. 3 shows the average end-to-end rates for varying the number of antennas at relays when \( K = 2, \) \( M = 2, \) \( B_{\text{max}} \to \infty, \sigma^2_{R} = \sigma^2_{RD} = \sigma^2_{RR} = 0 \) dB), except for the SINR-based RS scheme achieves greater than twice the average end-to-end rate by the HD-BRS scheme and the proposed ZFBB-based and MMSE-based RS schemes.

respect to the number of relays even if the exact convergence is not guaranteed in this case. However, it is shown that the rate improvement is much slower than that with increasing the number of antennas. When \( K = 10, \) all the proposed schemes except for the SINR-based RS scheme achieves greater than or equal to double the average end-to-end rate by the HD-BRS scheme.

2) n.i.d. IRI Channel Cases (\( \sigma^2_{R} = \pm 10 \) dB, \( \sigma^2_{RD} = \sigma^2_{RR} = 0 \) dB): To investigate the effect of average IRI intensity, we consider two different average IRI channel conditions: (i) weak IRI case (\( \sigma^2_{R} = -10 \) dB, \( \sigma^2_{RD} = \sigma^2_{RR} = 0 \) dB); (ii) strong IRI case (\( \sigma^2_{R} = 10 \) dB, \( \sigma^2_{RD} = \sigma^2_{RR} = 0 \) dB).

Fig. 5 (a) shows the average end-to-end rate for weak IRI case when \( K = 3, M = 4, \) and \( B_{\text{max}} \to \infty. \) When \( K = 3 \) and \( M = 4, \) the proposed optimal BF-based, ZFBB-based,
and MMSE-based RS schemes already almost approach the upper bound regardless of SNR. The optimal BF-based RS scheme outperforms the other schemes and the MMSE-based RS scheme slightly outperforms the ZFBF-based RS scheme at low SNR. While the OB-based RS scheme still achieves the single antenna upper bound regardless of the average IRI intensity, the SINR-based RS scheme and the non-ideal SFD-MMRS scheme always outperform the HD RS schemes and outperform the OB-based RS scheme at low/medium SNR. Especially, when SNR = 0 dB, the non-ideal SFD-MMRS scheme achieves almost the same performance as the ideal upper bound since this channel condition yields very weak interference which is negligible.

Fig. 5(b) shows the average end-to-end rate for strong IRI case in the same setup. The proposed ZFBF-based and OB-based RS schemes yield exactly the same performance as in Fig. 5(a) since they do not depend on the intensity of IRI due to perfect IRI cancellation. The proposed MMSE-based RS scheme achieves almost identical performance with the proposed ZFBF-based RS scheme even at low SNR since the intensity of IRI is already strong compared to the \( \{S\rightarrow R\} \) channel conditions. The optimal BF-based RS scheme achieves between the upper bound and the MMSE-based and ZFBF-based RS schemes regardless of the intensity of IRI and SNR. In contrast, non-ideal SFD-MMRS scheme significantly degrades the average end-to-end rate which is always worse than those of the HD RS schemes.

C. Delay Performance

Basically, the buffer-aided relaying obtains additional selection diversity gain by sacrificing delay performance. Even if we mainly focus on the average end-to-end rate of delay-tolerant applications, we evaluate the average delay performance through simulations. Due to a full-queue assumption at the source, the average delay is defined as the average queuing delay at relays, which implies the time difference between the arrival time of a single packet at a relay and the successfully received time of the packet at the destination in number of time slots (number of channel uses). A single packet is transmitted from the source at each time slot and its size is determined by the selected \( \{S\rightarrow R\} \) channel gain according to adaptive rate transmission. Therefore, a delay of one means that a packet is stored at the relay in a certain time slot and all information bits contained in the packet are successfully forwarded to the destination in the next time slot.

In Fig. 6 we show the average delay for varying numbers of antennas when \( K = 2 \), SNR = 20 dB, \( B_{\text{max}} \rightarrow \infty \), and \( \sigma_{SR}^2 = \sigma_{RD}^2 = \sigma_{RR}^2 = 0 \) dB. Except for the proposed OB-based RS scheme, the average delays of all the schemes decrease as the number of antennas increases since the effective channel gain increases with the number of antennas. In the proposed OB-based RS scheme, the average delay is varying with the values slightly less than 50 time slots because its effective channel gain is same as the single antenna case. The ideal SFD-MMRS scheme has similar average delay with the upper bound and the non-ideal SFD-MMRS scheme and the SINR-based RS scheme have very short delays approaching one. The reason is that both schemes have the bottleneck in \( \{S\rightarrow R\} \) link due to uncoordinated IRI under i.i.d. channel condition and thus the size of packets transmitted from the source to a relay is much smaller than average \( \{R\rightarrow D\} \) link rate.

Fig. 6. Effect of average IRI intensity \((K = 3, M = 4, B_{\text{max}} \rightarrow \infty, \sigma_{SR}^2, \sigma_{RD}^2, \sigma_{RR}^2 = 0 \) dB) (a) Weak IRI \((\sigma_{RR}^2 = -10 \) dB) (b) Strong IRI \((\sigma_{RR}^2 = 10 \) dB)
significantly reduced when \( M \geq 4 \). Similarly, in the optimal BF-based RS scheme, both average \( \{ S \rightarrow R \} \) and \( \{ R \rightarrow D \} \) link rates are more balanced than those in the MMSE-based RS scheme. This results in a larger average \( \{ S \rightarrow R \} \) link rate and a smaller average \( \{ R \rightarrow D \} \) link rate for the optimal BF-based RS scheme, compared to the MMSE-based RS scheme. Accordingly, the average delay of the optimal BF-based RS scheme becomes worse than that of the MMSE-based RS scheme, since its average packet size is larger but its average \( \{ R \rightarrow D \} \) link rate is smaller than those of the MMSE-based RS scheme.

Fig. 7 shows the average delay for varying number of relays in the same setup. The basic trend of the upper bound and the proposed schemes optimizing beamformers between those \( \{ S \rightarrow R \} \) and \( \{ R \rightarrow D \} \) link rates are more balanced than those in the MMSE-based RS scheme. This results in a larger average \( \{ S \rightarrow R \} \) link rate and a smaller average \( \{ R \rightarrow D \} \) link rate for the optimal BF-based RS scheme, compared to the MMSE-based RS scheme. Accordingly, the average delay of the optimal BF-based RS scheme becomes worse than that of the MMSE-based RS scheme, since its average packet size is larger but its average \( \{ R \rightarrow D \} \) link rate is smaller than those of the MMSE-based RS scheme.

Fig. 7 shows the average delay for varying number of relays \((M = 2, \text{SNR} = 20 \text{ dB}, B_{\text{max}} \rightarrow \infty, \sigma_S^2 = \sigma_R^2 = \sigma_K^2 = 0 \text{ dB})\) 

![Fig. 6: Average delay for varying number of antennas (K = 2, SNR = 20 dB, B_{max} \rightarrow \infty, \sigma_S^2 = \sigma_R^2 = \sigma_K^2 = 0 \text{ dB})](image)

![Fig. 7: Average delay for varying number of relays (M = 2, SNR = 20 dB, B_{max} \rightarrow \infty, \sigma_S^2 = \sigma_R^2 = \sigma_K^2 = 0 \text{ dB})](image)

D. Behavior of the Optimal \( \alpha \) Parameter

In this subsection, we numerically investigate the behavior of the optimal \( \alpha \) parameter for the proposed schemes in different system setups. Fig. 8 shows the trend of optimal \( \alpha \) parameter at (a) low SNR (e.g., 5 dB) and (b) high SNR (e.g., 25 dB). The marker on each curve represents the optimal point. If \( \alpha \) is close to one, the \( \{ S \rightarrow R \} \) link rate dominates the cost function and if \( \alpha \) is close to zero, the \( \{ R \rightarrow D \} \) link rate dominates the cost function in the relay pair selection. The upper bound, the optimal BF-based RS scheme, and the OB-based RS scheme balance both link rates as \( \alpha^* = 0.5 \) since the optimum is achieved when \( E[C_{SR}(t)] = E[C_{RD}(t)] \). In contrast, the optimal \( \alpha \) values of the proposed MMSE-based and SINR-based RS schemes are close to one, while the proposed ZFBF-based RS scheme yields \( \alpha^* = 0 \). This is because the bottleneck of the MMSE-based and SINR-based RS schemes is the \( \{ S \rightarrow R \} \) link while the bottleneck of the ZFBF-based RS scheme is the \( \{ R \rightarrow D \} \) link. In particular, the ZFBF-based RS scheme only requires the effective SNRs for the \( \{ R \rightarrow D \} \) links in RS for this two-relay and two-antenna case. Comparing Fig. 8 (a) with (b), SINR values are not sensitive in optimal \( \alpha \) values. Although the optimal values of the proposed MMSE-based and SINR-based RS schemes are slightly changed, differences in the average end-to-end rate between those \( \alpha \) values are marginal.

Fig. 9 shows the effect of the number of relays on the optimal \( \alpha \) parameter. The optimal \( \alpha \) values of the upper bound, the optimal BF-based RS scheme, and the OB-based RS scheme are identical as \( \alpha^* = 0.5 \) and invariant with increasing the number of antennas. The reason is that the upper bound and the optimal BF-based RS scheme perfectly balance both average link rates and the OB-based RS scheme always achieves the single antenna bound regardless of the number of antennas. The optimal \( \alpha \) value of the proposed SINR-based RS scheme is also invariant but \( \alpha^* = 1 \) since the \( \{ S \rightarrow R \} \) link is the bottleneck. On the contrary, the optimal \( \alpha \) values of the proposed ZFBF-based and MMSE-based RS schemes approach 0.5 as the number of antennas increases since the bottleneck link rates are gradually enhanced.

Fig. 10 shows the effect of the number of relays in optimal
As the selection diversity gain increases with the number of relays, the proposed ZFBF-based and MMSE-based RS schemes have the same trend as shown in Fig. 9. The optimal BF-based RS scheme maintains the balance resulting in $\alpha^\star = 0$.5. However, in case of the upper bound, the optimal $\alpha$ values decrease as the number of relays increases but differences in the achieved end-to-end rate between the optimal points and ones given by $\alpha = 0.5$ are marginal although it is not shown in this figure.

E. Effects of Finite Buffer Size

In practice, the buffer size at relays is finite and thus it restricts the performance since a full-buffer relay cannot be selected as the receiving relay. We investigate the performance in terms of average end-to-end rate and average delay according to the finite buffer size. Fig. 11 shows the average end-to-end rate for varying buffer size when $K = 3$, $M = 2$, SNR = 20 dB, and $\sigma^2_{SR} = \sigma^2_{RD} = \sigma^2_{RR} = 0$ dB. All the schemes rapidly converge to their own performance upper limits with infinite buffer size as the buffer size increases. Accordingly, a buffer size $B_{\text{max}} \geq 50$ bits/Hz is sufficient to obtain the performance upper limits with infinite buffer size. For instance, when the bandwidth is 10 MHz, the buffer size is required to be about 60 MB in order to achieve the average end-to-end rate with infinite buffer size. This value is allowable at relays in the viewpoint of present memory size.

Fig. 12 shows the average delay performance with finite buffer size. For all the schemes, the average delays converge to their own limits as the buffer size increases although the convergence speed is different between the schemes. Interestingly,
the finite buffer size can be helpful to reduce the average delay without a loss in the average end-to-end rate. For example, even if the proposed ZFBF-based RS scheme has the longest delay, the average delay can be less than 15 time slots with achieving the average end-to-end rate limit if the buffer size is set to 50 bits/Hz. Therefore, if the buffer size is set to an appropriate value, it is enough to achieve near-optimal average end-to-end rate under reasonable average delay.

VI. Conclusion

In this paper, we proposed virtual FD buffer-aided joint RS and BF schemes taking IRI into account in a buffer-aided multiple relays network, where each relay is equipped with multiple antennas. We first formulated a weighted sum-rate maximization based on instantaneous rates maximizing the average end-to-end rate. Based on the alternative objective function, we proposed various RS schemes based on optimal and suboptimal BF designs to cancel or suppress IRI taking a trade-off between computational complexity and performance into consideration. Through simulations, the proposed joint RS and BF schemes were evaluated in terms of the average end-to-end rate and average delay, compared to several conventional HD RS and SFD-MMRS schemes. In numerical results, asymptotic trends were investigated with respect to the number of relays and the number of antennas at relays. The proposed joint RS and BF schemes recover the loss of multiplexing gain in the HD relaying even in the presence of IRI as the number of antennas and/or the number of relays increase. Moreover, the behavior of the optimal weight factor and the effects of finite buffer size were shown in various different network setups. Basic trend in the optimal weight factor is moving toward to reduce the link rate gap between the bottleneck link and the other link. Although the finite buffer size limits the average end-to-end rate, it can help to bound the average delay with achieving near-optimal average end-to-end rate if it is set to an appropriate value. For future studies, it is possible to extend to multiple source-destination pairs with multiple antennas, to apply for non-full queue traffic at the source, to consider imperfect CSI and BSI, to develop a low complexity and limited feedback RS scheme, and to apply for other applications such as cognitive radio and physical layer security.

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