Solitary massive excitation of quantum gravity as a dark particle

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Abstract

We construct a static and spherical excited state without singularities in renormalizable quantum gravity with background-free nature asymptotically. Its diameter is given by a correlation length of the quantum gravity, and it has a Schwarzschild tail. The inside where nonperturbative dynamics are activated and the tail outside are smoothly connected by introducing a running coupling given by a function of the radial coordinate. If the mass is several times the Planck mass, we can obtain it as a solution of linearized equations of motion for the gravitational potentials incorporating the running effect. It may be a candidate for dark matter, and will give a new perspective on black hole physics.
1 Introduction

In Einstein’s theory of gravity, a point-like particle with mass beyond the Planck scale is a black hole. In fact, the Compton wavelength of such a particle is shorter than the horizon size of the mass, and thus its information is confined and lost. Hence, describing the particle as a point is no longer justified. This is the reason why the Planck scale has been recognized as a wall that cannot be exceeded. Here, to overcome such a problem, we argue that spacetime will transition to a new phase before reaching the wall, and a quantum world without singularities will come out. Renormalizable asymptotically background-free quantum gravity \[1, 2, 3, 4, 5, 6\] formulated based on a certain conformal field theory \[7, 8, 9, 10, 11, 12, 13, 6\] suggests the existence of a dynamical energy scale that clearly separates quantum spacetime from classical spacetime.

The action is given by the sum of a conformally invariant gravity part
\[I_4 = \int d^4x \sqrt{-g} \left[ -C_{\mu\nu\lambda\sigma}^2 / t^2 - bG_4 \right] \] and other lower derivative terms \[I_L = \hbar^{-1} \int d^4x \sqrt{-g} \left[ M^2 R / 2 - \lambda + \cdots \right] \], where \(C_{\mu\nu\lambda\sigma}\) is the Weyl tensor and \(G_4\) is the Euler density. The dots denote matter actions that become conformally invariant in the ultraviolet (UV) limit. For simplicity, bare and renormalized quantities are not distinguished here. \(t\) is a dimensionless coupling constant which represents a deviation from conformally flat configurations. The perturbation by \(t\) is justified because the beta function becomes negative. \(b\) is introduced to remove UV divergences proportional to the Euler term, and is not an independent one, but expanded by \(t\). Since the gravitational field is exactly dimensionless, \(I_4\) is a dimensionless action, thus has no \(\hbar\).

One of the reasons why considering the positive definite action involving the square of the Riemann curvature tensor is that singularities become unphysical because the action diverges, unlike the Einstein-Hilbert action that is not even bounded below. Nevertheless, simply applying perturbation theory to such a fourth derivative gravity causes the problem of ghosts. In order to solve it, we need to apply a nonperturbative method developed from two-dimensional quantum gravity \[14, 15, 16, 17\].

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The metric field is decomposed as \( g_{\mu\nu} = e^{2\phi} \bar{g}_{\mu\nu} \) and \( \bar{g}_{\mu\nu} = (\hat{g} e^h)_{\mu\nu} = \hat{g}_{\mu\lambda}(\delta^\lambda_{\nu} + h^\lambda_\nu + \cdots) \), where \( h^\mu_\nu \) is the traceless tensor field that is controlled by \( t \). The method is that a diffeomorphism invariant measure \([dg]_g[df]_g\) is rewritten using a practical measure defined on the background metric \( \hat{g}_{\mu\nu} \) as \([dg]_g[df]_g = [d\phi]_g[dh]_g[df]_g e^{iS} \), where matter fields are symbolically represented by \( f \). \( S \) is the Wess-Zumino action for conformal anomalies [18, 19, 20], that is Jacobian to preserve diffeomorphism invariance. In particular, a zeroth order term of \( t \) exists in \( S \) and is given by the Riegert action [7],

\[
S_R = - (b_c/16\pi^2) \int d^4x \sqrt{-\bar{g}} [2\phi \Delta_4 \phi + (\bar{G}_4 - 2\nabla^2 \bar{R}/3)\phi],
\]

where \( \sqrt{-\bar{g}} \Delta_4 \) is a conformally invariant fourth derivative differential operator. The quantities with the bar are the ones defined by \( \bar{g}_{\mu\nu} \). This action gives a kinetic term of the conformal factor field \( \phi \). The coefficient \( b_c \) has a right sign and is about 10 in ordinary particle models including GUT models, here \( b_c = 10 \) is employed.

Writing the whole action as \( I_{4D\text{QG}} = S + I_4 + I_L \), we can rewrite the path integral to \( \int [d\phi dh df]_g e^{iI_{4D\text{QG}}} \) as a standard quantum field theory on \( \hat{g}_{\mu\nu} \). Here, neither \( I_4 \) nor \( S \) depends on \( h \), thus together they are an action describing quantum gravity states in the world beyond the Planck scale. In the following, \( \hbar = 1 \).

The most significant feature of the theory is that a conformal invariance remains as part of diffeomorphism invariance at the UV limit of \( t \to 0 \) [11, 12, 13]. Here, to emphasize it is a gauge symmetry, namely Becchi-Rouet-Stora-Tyutin (BRST) symmetry, we call it BRST conformal symmetry. It means that all of theories with different backgrounds connected each other by conformal transformations are gauge equivalent. Therefore, the flat metric can be employed as a background without affecting physics. The appearance of this gauge symmetry in the UV limit is called “asymptotic background freedom”. Although negative-metric modes are necessary components for the BRST conformal algebra to close at the quantum level, we can show that

\footnote{Normally, renormalization is done by replacing \( h^\mu_\nu \) with \( th^\mu_\nu \). On the other hand, \( \phi \) has no own coupling constant, thus the non-renormalization theorem holds for it [2, 3, 4, 5].}
they all are not gauge invariant, and thus do not appear as physical states \(^2\).

The theory has three physical scales, namely renormalization group invariants \([5]\), that must be determined experimentally. They are the Planck mass \(m_{\text{pl}} = \frac{1}{\sqrt{G}} \approx 10^{19}\text{GeV}\), the cosmological constant that is ignored here, and the dynamical scale \(\Lambda_{\text{QG}}\) originated from the negativity of the beta function. The last one is the scale that separates quantum and classical spacetimes, and is predicted to be \(\Lambda_{\text{QG}} \approx 10^{17}\text{GeV}\) from an inflation scenario driven by quantum gravity dynamics only \([21, 22, 23, 6]\).

## 2 Equations of motion

Considering spherical scalar fluctuations of gravity and assuming they are small, we write the metric as \(ds^2 = -(1+2\Psi) d\eta^2 + (1+2\Phi) dx^2\). Here, \(\Phi\) and \(\Psi\) are called the gravitational potentials. The linearized Einstein equation around a static point mass is then expressed as \(-2M_P^2 \partial^2 \Phi = 0\) and \(\Psi = -\Phi\), where \(M_P = \frac{1}{\sqrt{8\pi G}}\) is the reduced Planck mass and \(\partial^2 = \partial_r^2 + (2/r) \partial_r\) is the spatial Laplacian in which the radial derivative is written as \(\partial_r = \partial/\partial r\).

For small mass \(m\), the gravitational potential is given by \(\Phi = r_g/2r\), where \(r_g = 2Gm\) is the Schwarzschild radius.

Here we consider a solitary excited state of the quantum gravity that has a tail of the Schwarzschild solution outside. The diameter of the excited state will be a correlation length, given by the inverse of the dynamical scale denoted as \(\xi_{\Lambda} = 1/\Lambda_{\text{QG}}\), and thus the radius is \(R_h = \xi_{\Lambda}/2 \approx 10^{-31}\text{cm}\). If the magnitude of \(2\Phi\) at the edge is sufficiently smaller than 1, namely \(r_g \ll R_h\), a certain linear approximation described below becomes effective. This condition is expressed as \(m \ll m_{\text{pl}}^2/4\Lambda_{\text{QG}}\), and the right hand side is about 25 times \(m_{\text{pl}}\), but it is about several times to be valid in actual calculations.

The inside of the excited state is expressed by the quantum gravity. The

\(^2\)Physical fields are given by real primary scalars only \([10, 11, 12, 13, 6]\). Their reality will be guaranteed by the positivity of the whole action.
vicinity of the boundary separating the inside and the outside will be a
strong coupling region of $t$, so its effect should be involved. Here quantum
corrections are incorporated by replacing the coefficient of the Riegert action
as $b_c \to b_c(1 - a_1 t^2 + \cdots) = b_c B(t)$. As its nonperturbative expression, we
use the form boldly summed up as $B(t) = [1 + a_1 t^2]^{-1}$.

The energy-momentum tensor consists of three terms as $T_{\mu\nu} = T^{(4)}_{\mu\nu} + T^{EH}_{\mu\nu} + T^M_{\mu\nu}$. The first is derived from the fourth derivative terms $S + I_4$, the
second is from the Einstein-Hilbert action, and the last is from matter fields.
The matter part is here represented as a relativistic perfect fluid, and its
energy density is denoted by $\rho$.

The equations of motion are given by $T_{\mu\nu} = 0$. There are two combina-
tions that $\rho$ disappears and result in equations for the gravitational potentials
only $^3$:

$$\frac{b_c}{8\pi^2} B(t) \left( -2 \partial_\eta^4 \Phi + \frac{10}{3} \partial_\eta^2 \partial^2 \Phi - \frac{4}{3} \partial^4 \Phi + \frac{2}{3} \partial_\eta^2 \partial^2 \Psi \right) + M_\rho^2 \left( 6 \partial_\eta^2 \Phi - 4 \partial^2 \Phi - 2 \partial^2 \Psi \right) = 0$$

(1)

and

$$\frac{b_c}{8\pi^2} B(t) \left( \frac{4}{3} \partial_\eta^2 \partial^2 \Phi - \frac{8}{9} \partial^4 \Phi - \frac{4}{9} \partial^4 \Psi \right) + \frac{2}{t^2} \left( 4 \partial_\eta^2 \partial^2 \Phi \right)$$

$$- \frac{4}{3} \partial^4 \Phi - 4 \partial_\eta^2 \partial^2 \Psi + \frac{4}{3} \partial^4 \Psi \right) - 2 M_\rho^2 \left( \partial^2 \Phi + \partial^2 \Psi \right) = 0.$$  (2)

The part containing $b_c$ is derived from the Riegert action, the part with $1/t^2$
is from the Weyl action, and $M_\rho^2$ is from the Einstein-Hilbert action. In
addition, from the time-time component, an energy conservation equation
containing $\rho$ is yielded as

$$\frac{b_c}{8\pi^2} B(t) \left( - \frac{2}{3} \partial_\eta^2 \partial^2 \Phi + \frac{4}{9} \partial^4 \Phi + \frac{2}{9} \partial^4 \Psi \right)$$

$$+ \frac{2}{t^2} \left( - \frac{4}{3} \partial^4 \Phi + \frac{4}{3} \partial^4 \Psi \right) + 2 M_\rho^2 \partial^2 \Phi + \rho = 0.$$  (3)

$^3$(1) and (2) are (4.16) and (4.17) multiplied by $\partial^2$ with taking $\phi(\eta) = 0$ in [22],
respectively. (3) is from (4.20), in which $\rho D (= \delta \rho)$ is rewritten as $\rho$. 5
These equations do not yet include the dynamics involving the correlation length $\xi_\Lambda$. Here we take in a running coupling constant further as a manifestation of nonperturbative and nonlinear effects. It is usually defined by $t^2 = [\beta_0 \log(Q^2 \xi^2_\Lambda)]^{-1}$, where $Q$ is a physical momentum and $\beta_0$ is a coefficient of the beta function. Now, we replace $Q$ with the inverse of $2r$, so that

$$t^2(r) = [\beta_0 \log(R_h^2/r^2)]^{-1},$$

which diverges at the edge $r = R_h$. Then we replace $t^2$ in (1), (2), and (3) with $t^2(r)$. At the same time, $B$ is replaced with $\bar{B}(r) = [1 + a_1 t^2(r)]^{-1}$.

The running coupling constant (4) is a function that vanishes at the origin, gradually increases as away from it, and diverges sharply at the edge, which plays the role of connecting the inside and outside of the excited state. Indeed, taking it into account, (2) shows that around the origin with $\bar{t} \simeq 0$, configurations in which the inside of the second parentheses vanishes becomes dominant, whereas at the edge where $\bar{t} \to \infty$ the fourth derivative conformal dynamics disappear, leading to Einstein gravity.

In this way, we obtain the dynamical equations incorporating nonperturbative effects while maintaining linearity. The parameters $\beta_0$ and $a_1$ are rather vague because they are depend on the traceless tensor dynamics, and hence it is adequate to determine them phenomenologically. Here, $\beta_0 = 0.5$ and $a_1 = 0.1$ are employed, which are almost the same as those used in the inflation scenario [22, 23].

3 Spherical excitations

Let us consider a static solution of the equations of motion. In this case, by introducing new variables $X = 2\Phi + \Psi$ and $Y = \Phi - \Psi$, the coupled equations, (1) and (2), can be completely divided into two as $\bar{B} \partial^4 X + 3H_D^2 \partial^2 X = 0$ and $(4/t^2) \partial^4 Y - M_P^2 \partial^2 Y = 0$, where $H_D = M_P \sqrt{8\pi^2} b_c$, which has a value between the Planck mass and the reduced Planck mass. The static solution also satisfies the energy equation (3) of $\rho = 0$, thus it represents a purely gravitational excitation.
Further rewrite the variables as
\[ X(r) = \left( \frac{r_g}{2r} \right) f(r) \] and
\[ Y(r) = \left( \frac{r_g}{r} \right) g(r), \]
where \( r_g \) is an unknown constant for the time being. The relationship between \( r_g \) and mass given before will be determined from the energy equation (3) later. Applying the spatial Laplacian to these variables gives
\[ \partial_r^2 f(r) = \left( \frac{r_g}{2r} \right) f(r) \] and
\[ \partial_r^4 f(r) = \left( \frac{r_g}{2r} \right)^2 f(r). \] The same applies to the variable \( Y(r) \). We then obtain
\[ \partial_r^4 f(r) + 3H_0^2 \left[ 1 + a_1 \tilde{r}^2(r) \right] \partial_r^2 f(r) = 0, \]
\[ \partial_r^4 g(r) - \frac{1}{4} M_P^2 \tilde{r}^2(r) \partial_r^2 g(r) = 0. \] (5)

As conditions that the variables \( X \) and \( Y \) and \( \partial_r^2 X \) and \( \partial_r^2 Y \) do not diverge at the origin, we obtain
\[ f(0) = 0, \quad g(0) = 0, \quad \partial_r^2 f(0) = 0, \quad \partial_r^2 g(0) = 0. \] (6)

As stated in Introduction, singularities are unphysical, thus solutions in which \( f \) or \( g \) is finite at the origin are excluded because they are fake ones caused by employing the linear approximation. The conditions given by the second derivative of \( r \) are requirements for each term of the energy equation (3) to be finite. Furthermore, as conditions for smoothly connecting to the Schwarzschild solution at the edge, we set
\[ f(R_h) = 1, \quad g(R_h) = 1, \quad \partial_r f(R_h) = 0, \quad \partial_r g(R_h) = 0. \] (7)

The first two show that \( \Phi(R_h) = -\Psi(R_h) \) holds as the Einstein equation does. The last two conditions are for smooth connection.

First, examine the behavior of the solution near the origin. Letting \( \zeta = \partial_r^2 f \) and \( \theta = \partial_r^2 g \), look for solutions that satisfy \( \zeta(0) = \theta(0) = 0 \). In addition, since the running coupling constant is so small around the origin, it is taken to be a small constant such as \( \tilde{r}(r) = t (\ll 1) \), then the equations reduce to
\[ \partial_r^2 \zeta + K^2 \zeta = 0 \] and \( \partial_r^2 \theta - L^2 \theta = 0 \), where \( K = \sqrt{3(1 + a_1 \tilde{r}^2)} \) and \( L = M_P t/2 (\ll 1) \). From these, we obtain \( \zeta \sim \sin(Kr) \) and \( \theta \sim \sinh(Lr) \). Integrating these twice with \( r \) and finding \( f \) and \( g \) satisfying the conditions
(6) reveals that they behave like

\[ f(r) \simeq c \sin(\sqrt{3} H_D r) + dr \]  

and \( g \simeq c' \sinh(L r) + d' r \) near the origin. Since \( L \ll 1 \),

\[ g(r) \simeq d' r \]  

is obtained after all. Here the coefficients \( c, d, d' \) cannot be determined unless the equations are completely solved by imposing the boundary conditions (7) at the edge. The behavior of \( g \) shows that \( \partial^2 \Phi = \partial^2 \Psi \) holds around the origin.

![f(r) and g(r)](image)

Figure 1: Numerical results of \( f \) (solid) and \( g \) (dashed) in the case of \( b_c = 10 \), \( \beta_0 = 0.5 \), \( a_1 = 0.1 \), and \( H_D/\Lambda_{QG} = 60 \). The unit is \( H_D = 1 \), then \( m_{pl} = 1.784 \) and \( M_P = 0.356 \).

Numerically solve (5) as a boundary value problem with (6) and (7), then we obtain Fig. 1, where \( H_D/\Lambda_{QG} = 60 \), and \( H_D \) is normalized to be unity. The calculation is practically performed by setting the boundary condition
Figure 2: The gravitational potentials $\Phi$ (solid) and $-\Psi$ (dotted) for an excited state with mass $m = 2m_{pl}$. The tail $r \geq 30$ is the Schwarzschild solution $\Phi = -\Psi = r_g/2r$.

at $r = R_h - \epsilon$ right inside the edge, and $\epsilon$ is brought close to zero until the result no longer changes. Here, $\epsilon = 0.0001$. The gravitational potentials $\Phi$ and $-\Psi$ are then found as shown in Fig. 2. It can be seen that the behavior around the origin ($r \lesssim 5$) of the numerical solution is $c = 0.543$, $d = 0.037$, and $d' = 0.036$.

The state mass is defined by $m = \int_{|x| \leq R_h} d^3x T_{00}^{(4)}(x)$, where $T_{00}^{(4)}$ is the first two terms in (3). Recall that the static solution satisfies (3) of $\rho = 0$, thus rewriting the mass formula using this equation and $\Phi = (X + Y)/3 = r_g(f + 2g)/6r$ yields $m = -2M_p^2 \int_0^{R_h} 4\pi r^2 dr \partial^2 \Phi = (4\pi/3)M_p^2 r_g[f(R_h) + 2g(R_h)] = 4\pi M_p^2 r_g$, where the boundary conditions (6) and (7) are used. Thus, the relationship $r_g = m/4\pi M_p^2 = 2Gm$ is derived.
4 On time evolution

Next, examine how the static solution evolves with time. In this case, we have to solve the partial differential equations (1) and (2), but unfortunately the coupled equations cannot be separated as we did when finding the static solution. Here, we will see the behavior around the origin where the coupling constant is small. Letting $t = 0$ and $B = 1$ and rewriting the equations with the variables being $X(\eta, r) = (r_g/2r)F(\eta, r)$ and $Y(\eta, r) = (r_g/r)G(\eta, r)$, we obtain

\[
(\partial_\eta^2 - \partial_r^2)^2 F(\eta, r) + 2\partial_\eta^2(\partial_\eta^2 - \partial_r^2)G(\eta, r)
- 3H_D^2[(\partial_\eta^2 - \partial_r^2)F(\eta, r) + 2\partial_\eta^2G(\eta, r)] = 0 \tag{10}
\]

and

\[
(3\partial_\eta^2 - \partial_r^2) \partial_r^2 G(\eta, r) = 0. \tag{11}
\]

Let these equations be solved under the initial conditions $F(0, r) = f(r)$ and $G(0, r) = g(r)$, where $f$ and $g$ are the static solutions obtained above, and each behavior near the origin is given by (8) and (9). Since the gravitational potentials and each term of the energy equation (3) do not diverge at the origin, the boundary conditions of $F(\eta, 0) = G(\eta, 0) = 0$ and $\partial_r^2 F(\eta, 0) = \partial_r^2 G(\eta, 0) = 0$ are imposed.

A solution of (11) allowed under these conditions is $G(\eta, r) = \tilde{d}(\eta) r$, where $\tilde{d}$ is an arbitrary function that satisfies the initial condition $\tilde{d}(0) = d'$. Substitute this solution into (10) to find a solution of $F$. Further putting $F(\eta, r) = \tilde{F}(\eta, r) - 2\tilde{d}(\eta) r$ gives $(\partial_\eta^2 - \partial_r^2)(\partial_\eta^2 - \partial_r^2 - 3H_D^2)\tilde{F}(\eta, r) = 0$, where $\tilde{F}$ also satisfies the same boundary conditions as $F$. From this, a general form of the solution satisfying the initial condition is $F(\eta, r) = \{c + b\sin(\sqrt{3}H_D\eta) + b'[\cos(\sqrt{3}H_D\eta) - 1]\} \sin(\sqrt{3}H_Dr)\tilde{e}(\eta) - 2\tilde{d}(\eta)r$, where $b$ and $b'$ are arbitrary constants, and $\tilde{e}(\eta)$ is a function satisfying $\partial_\eta^2(\partial_\eta^2 - 3H_D^2)\tilde{e}(\eta) = 0$ and $\tilde{e}(0) = d + 2d'$.

Now, $F$ and $G$ are not monotonic functions of time, except the terms involving $\tilde{d}$ and $\tilde{e}$. The behavior of these two terms cannot be determined by
examining only near the origin. However, since all gravitational terms in (3) that determines the matter energy density $\rho$ contain the spatial Laplacian, $\tilde{d}$ and $\tilde{e}$ do not affect changes in the energy density. In this way, we can see that $\rho$ does not monotonically increase at least near the origin. Thus the excited state appears to be kept stable without the gravitational energy changing into matters.

5 Discussion

The dynamics of the asymptotically background-free quantum gravity begin to work at the energy scale $\Lambda_{\text{QG}}$ of $10^{17}\text{GeV}$ below the Planck scale. The excited state examined here has a mass sufficiently larger than $\Lambda_{\text{QG}}$ so that it can be considered that the quantum gravity is activated inside. It can be regarded as a particle when viewed from the outside because its radius $R_h$ is larger than the horizon size $r_g$.

If the mass is smaller than $\Lambda_{\text{QG}}$, the quantum gravity will not be activated and no state will be excited. The coupling constant will remain large everywhere and the assumption that it is running will not be valid. On the other hand, a macroscopic object with a semi-classical horizon whose size is larger than $R_h$ looks like a black hole. It will undergo black hole evaporation, and may eventually leave the small excited state as a remnant.

It is thought that many excited states were generated in the early universe. Primordial black holes could be formed using them as seeds. If the state is actually stable or long-lived, it can be a candidate for dark matter as a purely gravitational object.

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