Federated and Meta Learning over Non-Wireless and Wireless Networks: A Tutorial

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Abstract—In recent years, various machine learning (ML) solutions have been developed to solve resource management, interference management, autonomy, and decision-making problems in non-wireless and wireless networks. Standard ML approaches require collecting data at a central server for training, which cannot preserve the data privacy of devices. To address this issue, federated learning (FL) is an effective method to allow edge devices to collaboratively train ML models without sharing local datasets for data privacy. Typically, FL focuses on learning a global model for a given task and all devices and hence cannot adapt the model to devices with different data distributions. In such cases, meta learning can be employed to adapt learning models to different data distributions using a few data samples. In this tutorial, we conduct a comprehensive review on FL, meta learning, and federated meta learning (FedMeta). Compared to other tutorial papers, our objective is to leverage how FL/meta-learning/FedMeta can be designed, optimized, and evolved over non-wireless and wireless networks. Furthermore, we analyze not only the relationship among these learning algorithms but also their advantages and disadvantages in real-world applications.

Index Terms—Distributed learning, federated learning, meta learning, federated meta learning, non-wireless and wireless networks.

I. INTRODUCTION

The rapid development of new technologies and the increasing number of devices can lead to unprecedented growth in data collection [1], [2]. According to a report issued by the International Data Corporation (IDC), the data generated globally in 2018 was 36.3 TB, with an average of about 10 billion GB per day [3]. Machine learning (ML) algorithms are used to analyze these datasets and facilitate dynamic decision-making for specific tasks [4], [5], such as controlling self-driving cars [6], pattern recognition [7], or prediction of user behavior [8].

In traditional cloud computing systems [9], datasets, including pictures, videos, audio, and location information, collected by the internet of thing (IoT) devices or mobile devices are uploaded and analyzed centrally at a cloud-based server or data center [10], where centralized ML models are trained. However, this centralized training method may not be sustainable for increasingly complex and heterogeneous non-wireless/wireless networks due to the following reasons:

- Limited bandwidth, dynamic wireless channels, and high interference can cause high transmission latency, which can undermine real-time applications, such as wireless virtual reality (VR) [11]–[13] and self-driving car systems [14].
- Delivering a large amount of data to the cloud results in high communication overhead, and high storage and computational costs [15], [16].
- The collected data can be private and sensitive, such as users’ medical and financial information. Transmitting such data to the cloud is not desirable as it may put users’ privacy at high risk, violating governments’ data privacy legislations such as the European Commissions General Data Protection Regulation (GDPR) [17] and the Consumer Privacy Bill of Rights in the U.S. [18].

To address the above issues, distributed learning has been introduced to learn models from distributed data in a more effective and efficient manner. Distributed learning refers to multi-node ML algorithms and systems that are designed to resolve the computational problems of complex centralized algorithms over large-scale datasets [19], [20]. Compared to the centralized approaches, applying distributed learning algorithms to train several learning models based on distributed datasets increases the possibility of achieving lower biases especially when the number of datasets is large [21]–[23].

Conventional distributed learning algorithms, such as k-nearest neighbor [24], support vector machine [25], [26], Bayesian networks [27], [28], and decision trees [29], [30], are trained by exchanging raw data, which can hardly protect the user data privacy [31]. To guarantee data privacy and facilitate collaborative ML of complex models from distributed IoT or mobile devices, a distributed ML method, namely, federated learning (FL), was first introduced in [32]. The standard steps of FL are: 1) each device uses its own dataset to train a local model; 2) devices send their local models to the server for model aggregation; 3) the server transmits the updated global model back to the devices. These steps are repeated across multiple iterations until convergence. Compared to traditional centralized learning and distributed learning relying on the exchange of raw data, FL has the following advantages:

- **Data Privacy:** In FL, only model weights are exchanged between the server and the devices rather than the datasets, which protects user privacy.
- **Data Diversity:** FL facilitates access to heterogeneous
data, and models from different devices may be aggregated to build a better global model.

Apart from the aforementioned advantages, the effectiveness of FL has been demonstrated in many applications, including training predictive models for human trajectory/behavior via mobile devices [33], automatically learning users’ behavior patterns via smart home IoT [34], and diagnosis in health artificial intelligence (AI) for cooperation among multiple hospitals and government agencies [35]–[37]. However, applications usually suffer from data heterogeneity (i.e., data with different statistical characteristics) and system heterogeneity (i.e., systems with different types of computation units), which severely affect the convergence and accuracy of FL algorithms. It is noted that FL mainly focuses on learning one ML task across multiple devices [38]. To overcome this limitation, meta learning, also known as learning to learn, can be integrated with FL, leading to so-called federated meta learning (FedMeta), to solve multi-task learning problems. Especially, initialization-based meta-learning algorithms are well known for fast adaptation and good generalization to new tasks [39]. Compared to FL, FedMeta has the following advantages:

- FedMeta achieves a reduction in the required communication cost due to the faster convergence, and an increase in learning accuracy. Also, it can be adapted to arbitrary tasks across multiple devices.
- FedMeta enables model sharing and local model training without significant expansion in model size. Thus, it does not occupy a large amount of memory and the resulting global model can be personalized for each device.

A. Related Works

In this section, related surveys and tutorials on FL and meta learning are briefly reviewed and the novel aspects of this paper are elaborated.

1) FL: Over the past 5 years, a number of surveys and tutorials on FL have appeared regarding non-wireless [40]–[50] and wireless [51]–[63] networks, respectively. To differentiate the scope of our tutorial from other existing surveys and tutorials, in Table I, we classify the existing FL surveys and tutorials into different categories based on their focus, and compare and summarize the content of these existing surveys and tutorials according to the structure of our tutorial in Tables II and III. Table I reveals that FL surveys and tutorials for non-wireless networks mainly focus on either fundamental definitions, architectures, and applications of FL [40], [42], [43], or a specific subfield of FL, including emerging trends of FL [44], communication efficiency of FL [45], fairness-aware FL [46], federated reinforcement learning (FRL) [47], FL for natural language processing (NLP) [48], incentive schemes for FL [49], [50], and unlabeled data mining in FL [41]. FL surveys and tutorials for wireless networks mainly focus on either fundamental theories, key techniques, challenges, and future directions [52]–[55], [62], or a specific subfield of FL applied in wireless networks, including FL for IoT [56], [59], [60], communication-efficient FL [51], [61], FL for mobile edge computing (MEC) [57], and collaborative FL with less reliance on a central server [58].

2) Meta Learning: There have been several meta learning surveys and tutorials in non-wireless networks over the past 20 years [64]–[70], and to the best of the authors’ knowledge, there is no surveys and tutorials in meta learning in wireless networks. To differentiate the scope of our tutorial from others, we classify the existing meta learning surveys and tutorials into different categories based on their focus in Table I, and compare and summarize the content of the existing surveys and tutorials according to the structure of our tutorial in Table IV. Table I shows that the existing meta learning surveys and tutorials focused either on the general advancement of meta learning [64], [65], [68], [69], or the application of meta learning in a specific meta learning field, including data mining [66], NLP [67], and multi-modal meta learning [70]. To the best of the authors’ knowledge, there are no surveys and tutorials covering the application of FedMeta in wireless networks.

As shown in Tables II, III, and IV, the existing surveys and tutorials only cover a limited number of subtopics of our tutorial and just briefly described the corresponding learning algorithms. In our tutorial, we introduce the underlying design concepts of the relevant algorithms in detail. Most importantly, we also analyze the relationship and evolution of these algorithms in non-wireless and wireless networks, which have not been investigated in other surveys/tutorials before.

B. Summary of Contributions

Because of the privacy-preserving nature of FL and the fast adaptation to different tasks of meta learning, researchers from both academia and industry have started to work on the joint design of FL/meta learning/FedMeta and wireless networks. This paper provides the first detailed tutorial outlining the research areas, relationships, evolutions, challenges, and opportunities regarding these three learning concepts in non-wireless and wireless networks. The main contributions of this article can be summarized as follows:

- Based on the canonical FL algorithm, namely, federated averaging (FedAvg), we summarize six research areas in FL over non-wireless networks, including model aggregation, gradient descent, communication efficiency, fairness, Bayesian learning, and clustering, and we present the relationship and evolution of the corresponding learning algorithms in detail. Then, we introduce two research areas, including digital and analog over-the-air computation schemes, of how FL and wireless factors affect each other over wireless networks.
- Three research areas in meta-learning, including metric-based, model-based, and gradient-based meta-learning, are summarized. Based on the fundamental gradient-based meta-learning scheme, namely, model-agnostic meta-learning (MAML), we discuss how MAML has evolved in non-wireless networks in detail. Then, we analyze how meta-learning can be exploited to solve wireless communication problems.
- The fundamental principle of FedMeta and its evolution are summarized in detail. Then, two specific research areas, including device selection and energy efficiency, in FedMeta over wireless networks are introduced.
TABLE I
AN OVERVIEW OF SELECTED SURVEYS AND TUTORIALS ON FL AND META LEARNING

| Subject | Ref. | Contributions |
|---------|------|---------------|
| FL in Non-Wireless Networks | 40, 42, 43 | Definitions, architectures, and applications of the FL framework |
| | | Survey on emerging trends in FL |
| | | Survey on communication efficiency of FL |
| | | Survey on the fairness of FL |
| | | Survey on federated reinforcement learning (FRL), including horizontal FRL and vertical FRL |
| | | Survey on FL for natural language processing (NLP) |
| | | Surveys on incentive schemes for FL |
| | | Survey on unlabeled data in FL |
| FL in Wireless Networks | 52, 55, 62 | Fundamental theories, key techniques, challenges, and future directions for FL over wireless networks |
| | | Surveys on FL in resource-constrained IoT, and recent advances in FL and IoT |
| | | Surveys on communication-efficient FL over wireless networks |
| | | Survey on FL in MEC |
| | | Survey on collaborative FL over wireless networks |
| | | Survey on asynchronous FL |
| Meta Learning in Non-Wireless Networks | 64, 65, 68, 69 | Introductory tutorial on meta learning, e.g., meta learning models and applications |
| | | Tutorial on meta learning on algorithm selection for data mining |
| | | Survey on meta learning for NLP |
| | | Tutorial on multimodality-based meta-learning in terms of the methodologies and applications |

TABLE II
COMPARISON OF SURVEYS AND TUTORIALS ON FL OVER NON-WIRELESS NETWORKS

| Reference | 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 | This paper |
|-----------|----|----|----|----|----|----|----|----|----|----|----|------------|
| Year | 2019 | 2020 | 2021 | 2022 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | - |

Section III: FL in Non-Wireless Networks

- (HL.A): Model Aggregation
  - (HL.B): Communication Efficiency
  - (HL.C): Fairness
  - (HL.D): Bayesian Machinery
  - (HL.F): Clustering

TABLE III
COMPARISON OF SURVEYS AND TUTORIALS ON FL OVER WIRELESS NETWORKS

| Reference | 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 | 61 | 62 | 63 | This paper |
|-----------|----|----|----|----|----|----|----|----|----|----|----|----|----|------------|
| Year | 2019 | 2020 | 2020 | 2020 | 2020 | 2020 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | - |

Section IV: FL in Wireless Networks

- (IVA.1): Device Selection
  - (IVA.2): Communication Efficiency
  - (IVA.3): Energy Efficiency
  - (IVA.4): Asynchronous
  - (IVA.B): Over the Air Communication

- Finally, open research problems that can lead to new FL/meta-learning/FedMeta research directions are introduced.

The rest of this paper is organized as follows. In Section II, we introduce the background and fundamentals of distributed learning. In Sections III and IV, we present important research fields in FL over non-wireless and wireless networks, respectively. In Sections V and VI, we introduce research areas in meta learning over non-wireless and wireless networks, respectively. Section VII presents FedMeta and Section VIII introduces open research problems and future directions in FL, meta learning, and FedMeta. A graphical illustration of the content of Sections II to VII is provided in Fig. 1. Finally, conclusions are drawn in Section IX.

II. BACKGROUND AND FUNDAMENTALS OF DISTRIBUTED LEARNING

In distributed learning, there are two approaches for learning across servers or devices, namely, data parallelizing and model parallelizing. For data parallelizing, data is divided into several datasets, and servers or devices apply the same ML algorithm to different datasets to train a single ML model that is available to all servers or devices. In contrast, for the model parallel approach, the model is segmented into different sub-models and these sub-models are updated in different devices. The global model is therefore the aggregation of all sub-models. However, not all ML algorithms are available for the model
parallel approach. Note that data parallelizing and model parallelizing can also be employed simultaneously [71]. In this section, we describe the background and fundamentals of distributed learning, including offline and online learning, types of ML algorithms and topologies, and challenges of distributed learning in non-wireless/wireless networks.

A. Offline and Online Learning

Based on whether the learning model is updated with newly arriving data [72], distributed learning algorithms can be categorized into offline learning and online learning. The goal of both offline and online learning is to maximize the accuracy or long-term reward for prediction or decision-making tasks given the knowledge of previous observations or rewards observed from the environment, respectively.

Offline learning can be divided into two phases, namely, the training phase and the testing phase. In the training phase, the learning model is first trained with the training dataset until a set of hyperparameters achieving the highest accuracy or reward is found. In the testing phase, the trained learning model is employed for prediction or decision-making without performing any further model update. The offline training method suffers from high re-training costs when dealing with new training data/environments, and thus, has poor scalability for real-world applications, especially when the amount of data grows and the environment evolves rapidly [73].

Online learning can be applied to most ML algorithms, where the learning models are trained to handle prediction or any type of decision-making task by learning from a sequence of data samples one by one in each time slot. Online learning overcomes the drawbacks of offline learning in that the learning models are updated constantly and efficiently when new training data arrives. Thus, online learning models are much more efficient and scalable for large-scale ML tasks in real-world applications [74].

To efficiently solve prediction/decision-making problems, the servers and devices need to be connected and communicate with each other for information exchange, which can be represented via the topology of the distributed exchange, which can be introduced in detail.

B. Machine Learning Algorithms

ML algorithms learn to make predictions or decisions based on datasets or observed states from the environment. To train a learning model, feedback is required to gradually improve the learning model. Depending on the type of feedback, ML algorithms can be categorized into supervised learning, unsupervised learning, semi-supervised learning, and reinforcement learning algorithms.

1) Supervised Learning: Supervised learning, as one of the most basic types of ML, learns a function that maps an input to an output based on labeled data. Each training data sample is a pair consisting of an input object (typically a vector) and a desired output value. The goal of the function is to minimize the error between the true data label and the output. Through analyzing the training data, the learning model can map new datasets to their corresponding labels. Unfortunately, the more training data, the more complex the model that needs to be trained to achieve a good prediction result [75], [76].

2) Unsupervised Learning: Unsupervised learning is applied for learning based on unlabeled data samples, and employs ML algorithms to analyze and cluster unlabeled datasets, which can discover hidden patterns or data groupings without the need for human intervention. Common applications of unsupervised learning include clustering and principal component analysis (PCA). In clustering, the learning model automatically groups the training data into groups with similar features. In PCA, the learning model compresses the training datasets by identifying which features are the most useful for discriminating among different training datasets, and discards the other non-important features, to reduce the dimensionality of the datasets [77].
Section II
Background and Fundamentals of Distributed Learning

- Offline and Online Learning
- Machine Learning Algorithms
- Learning Topologies
- Challenges over Wireless Networks
  - Supervised Learning
  - Unsupervised Learning
  - Semi-supervised Learning
  - Reinforcement Learning
  - Communication Limitation
  - Computation Limitation
  - Data Privacy
  - Centralized Topology
  - Tree-like Distributed Topology
  - Parameter Server Topology
  - Fully Distributed Topology

Supervised Learning
Unsupervised Learning
Semi-supervised Learning
Reinforcement Learning

Section III
FL in Non-Wireless Networks

- Model Aggregation
- Gradient Descent
- Communication Efficiency
- Fairness
- Bayesian Machine
- Clustering

- FedAvg
- Adaptive Aggregation
- Client Drift
- SET
- Adaptive Optimizer
- Structured and Sketched Update
- Adaptive Optimizer
- FedProx
- FOLB
- SCAFFOLD
- FedMA
- TAT-Fed

Section IV
FL in Wireless Networks

- Digital
- Analog
- Model Aggregation
- Resource Allocation
- Communication, Energy, and Computation Efficiency Optimization
- Asynchronism
- Device Selection
- Packet Transmission Error
- Communication Protocol-based
- Importance-based

Section V
Meta Learning in Non-Wireless Networks

- Meta-based
- Model-based
- Gradient-based

- Bayesian
- Machine
- Relation
- Prototypical
- MAML
- RAL
- MetaLearner
- SNAM
- MAML
- Meta-SGD
- Batch
- MAML
- LLAM
- LEI
- MAML
- Meta-MAML

Section VI
Meta Learning in Wireless Networks

- Traffic Prediction
- Transmission Rate Maximiization
- MIMO Detectors

Section VII
Federated Meta Learning

- FedMeta in Non-Wireless Networks
- FedMeta in Wireless Networks

FedMeta in Non-Wireless Networks
- MAML/Meta-SGD-based FedMeta
- Device Selection
- Energy Efficiency

FedMeta in Wireless Networks
- MAML/Meta-SGD-based FedMeta
- Device Selection
- Energy Efficiency

Fig. 1. Organization and content of Sections II to VII.
3) **Semi-supervised Learning:** Semi-supervised learning uses labeled as well as unlabeled data to perform learning tasks, and is conceptually sitting between supervised and unsupervised learning. It uses a large amount of unlabeled data in combination with small sets of labeled data. With the help of the labeled data, the learning model can accurately classify the unlabeled data, and can lead to a substantial improvement in learning accuracy [78].

4) **Reinforcement Learning:** In reinforcement learning (RL), the agents select actions based on the observations obtained from the environment to maximize the long-term reward [79]. Different from supervised learning, RL does not need labeled input and output data. Instead, it focuses on finding a balance between exploration (of unknown territory) and exploitation (of current knowledge), and trains the ML models to make a sequence of decisions [80]. However, there are still many problems in the implementation of RL in practical scenarios. For example, it is impossible to explore all state-action spaces. Although distributed RL algorithms can be used to solve the problem, they usually need to collect data from each agent, which cannot prevent agent information leakage and protect agent privacy. Thus, federated reinforcement learning (FRL) has been proposed to solve the privacy issues [47].

### C. Learning Topologies

One of the considerations for the design of distributed ML systems is the topology in which the servers and devices are organized. In this subsection, several learning topologies are introduced. One of the factors determining the topology is the degree of distribution that the learning system employs to transmit learning models. Four general learning topologies in distributed ML are shown in Fig. 2, as proposed in [81] and [82].

1) **Centralized Topology:** In a centralized topology, a central server is connected to all edge servers to obtain learning models, as shown in Fig. 2 (a). The central server first obtains learning models from all edge servers, and the model aggregation is executed at the central server. Then, the central server transmits the aggregated learning model to all edge servers. Each edge server then broadcasts the learning model to the devices it is connected to, and each device updates the transmitted learning model based on its local dataset.

2) **Tree-like Distributed Topology:** In a tree-like distributed topology, many connected nodes are arranged like branches of a tree, as shown in Fig. 2 (b). There is only one connection between any two connected nodes, which creates a natural parent and child hierarchy. Also, the tree-like distributed
topology is easy to scale and manage. This is because each edge server or device only needs to communicate with its parent and child servers [83]. The edge servers in the tree accumulate the local gradients of their children and pass them to their parents to calculate a global gradient.

3) Parameter Server Topology: The parameter server topology features a distributed set of edge servers and a centralized set of parameter servers to maintain the shared models [84], as shown in Fig. 2 (c). All learning models are stored in the parameter servers with a global shared memory, and all edge servers can obtain and update models in any time slot. The advantage of this topology is that all edge servers have direct access to the models stored at the parameter servers. However, the parameter servers need to handle all communication, which can result in a high overhead [85].

4) Fully Distributed Topology: In a fully distributed topology, there are no central servers, as shown in Fig. 2 (d). The fully distributed learning topology consists of a set of independent edge servers, and each edge server independently updates its learning model and directly exchanges its learning model with other edge servers. The fully distributed topology can achieve higher scalability compared to the centralized topology, and is robust to failures of individual servers due to power outages or malfunction [86]. However, when a large number of edge servers exist in the network, the overhead is extremely high.

To solve the high overhead problem in distributed topologies, a fast, and communication-efficient distributed framework, so-called group alternating direction method of multipliers (GADMM), was proposed in [87]–[89]. In GADMM, at most half of the edge servers are competing for the limited communication resources in each time slot. Meanwhile, each edge server exchanges the trained model only with two neighboring edge servers, thereby training a global model with a lower amount of communication overhead in each exchange.

D. Challenges of Distributed Learning

In non-wireless/wireless networks, it is obvious that the overall communication and computation costs grow with an increasing number of servers and devices. To design efficient distributed ML algorithms for non-wireless/wireless networks, the communication cost, computation cost, and data privacy are three main aspects to be considered apart from the testing accuracy and convergence time.

1) Communication Limitation: In wireless networks, a number of servers and devices may share the same spectrum resource due to the limited bandwidth [90]. Therefore, the communication among the devices and edge servers may suffer from high interference, poor channel conditions, and noise, which lead to low reliability, high transmission latency, and low learning accuracy [91].

2) Computation Limitation: Training and operating ML algorithms usually require computation units with high processing capability, especially when the ML models are complex. However, the devices have limited computation and energy capabilities. To minimize the computation latency at the device side, the use of edge servers with high processing capability using graphics processing units (GPUs) have been recently proposed to move the computations from the device to the edge.

3) Data Privacy: Transmitting datasets of edge devices to edge servers can cause data breach if the datasets have privacy-sensitive information, one potential solution is to only exchange the weights of ML models. Nevertheless, it is possible that the transmitted model parameters can be reversely traced, so that the privacy is still not preserved [92].

To address the communication, computation, and data privacy issues, FL has emerged as an efficient approach to exploit the distributed devices to collaboratively train a ML model. FL was first introduced by Google in 2016, where multiple devices collaboratively train a ML model without sharing their private data under the supervision of a central server, this ensures the privacy of the training data of all devices. FL has two entities: a centralized server that owns the global model and a set of devices that store the local models and training datasets. Meanwhile, FL consists of four procedures [93]: 1) training the local model based on the local dataset at the local device; 2) transmitting the local models from the devices to the central server; 3) aggregating the local models to a global model at the central server; and 4) updating the received global model from the central server at devices. The original data of each device is stored locally and do not need to be exchanged with or migrated to each other, which guarantees the privacy of each device. Thus, devices can enjoy the advantages of shared models trained by other devices, without data sharing.

In the following two sections, we introduce several main research areas in FL over non-wireless and wireless networks. In particular, we first present six FL research areas in non-wireless networks. Then, we present two FL research areas in wireless networks.

III. FL IN NON-WIRELESS NETWORKS

In this section, we introduce six main FL research areas in non-wireless networks. For ML algorithms, given a labeled set of inputs and their corresponding outputs, the learning models are trained and tested, as shown in Fig. 3. The goal of ML algorithms is to minimize the loss function designed based on a specific learning problem. The loss function plays an important role in training ML algorithms and improvement of their performance [94], and is expressed as

$$L(w) = \frac{1}{N} \sum_{i=1}^{N} l_i(w),$$

where $N$ is the number of data samples, and $l_i(w)$ is the loss of the $i$th input data based on learning weights $w$. To find the minimum value of the loss function, gradient descent is introduced to calculate the derivative of the loss function via $\frac{\partial L(w)}{\partial w}$. Then, the weights of the learning models are updated as

$$w = w - \eta \frac{\partial L(w)}{\partial w},$$

where $\eta$ is the learning rate. After a sufficient number of iterations, the loss function can achieve its minimum value,
Algorithm 1 Gradient Descent Algorithm

1: Initialize learning weights \( w \) and learning rate \( \eta \).
2: for Iteration = 1,...,I do
3:     for \( i = 1,...,N \) do
4:         Input the data \( x_i \).
5:         Obtain the loss \( l_i(w) \).
6:     end for
7:     Calculate the loss function \( L(w) \) for the input data based on (1).
8:     Update the learning weights \( w \) based on (2).
9: end for

\[
L(w) = \sum_{i=1}^{N} l_i(w)
\]

A. Model Aggregation

Model aggregation is the process of integrating models from multiple devices in order to create a new model [96]–[98]. There are several ways for model aggregation as explained in the following.

Federated Averaging: The most basic model aggregation method is the Federated Averaging (FebAvg) algorithm proposed in [32], where the weights of local models are averaged at the central server to update the global model [99]. The training objective of FedAvg is given as follows [32] Eq. (1)

\[
\min_{w} L(w_G) = \sum_{k=1}^{K} p_k f_k(w_k),
\]

where

\[
p_k = \frac{n_k}{N} \quad \text{and} \quad f_k(w_k) = \frac{1}{n_k} \sum_{i \in D_k} l_i(w_k).
\]

In (3) and (4), \( p_k \) is the percentage of the number of data samples at the \( k \)th device over the total number of data samples at all devices, \( n_k \) is the number of data samples of dataset \( D_k \), and \( n \) is the total number of data samples, which is calculated as \( n = \sum_{k=1}^{K} n_k \). In (4), \( l_i(w_k) \) is the loss of the FL model due to the \( i \)th data sample calculated by the local model weights \( w_k \) of the \( k \)th device, and \( w_G \) is the weight of the global model.

There are two approaches to update the global model. The first approach is to compute the gradient of each device. Then, the central server aggregates these gradients from \( K \) devices and updates the global model using

\[
w_{G}^{t+1} = w_{G}^{t} - \eta \sum_{k=1}^{K} p_k g_k^t, \quad \text{with} \quad g_k^t = \nabla f_k(w_k^t),
\]

where \( \nabla f_k(w_k^t) \) is the gradient computed at the \( k \)th device. The second approach is to update the weights of the local model at each device using

\[
w_{k}^{t+1} = w_{k}^{t} - \eta g_k^t,
\]

where \( g_k^t \) is the gradient calculated using (5). Then, the global model at the central server is updated as

\[
w_{G}^{t+1} = \sum_{k=1}^{K} p_k w_k^{t+1}.
\]

In the second approach, each device first performs the gradient descent for its local model using the local datasets, and the central server averages these local models. In this way, each device can iterate the local update in (6) multiple times before uploading local models, which can accelerate the convergence speed.

Although FedAvg has achieved great success and is one of the most well-known algorithms in FL, the statistical heterogeneity challenges in the data are still difficult to overcome.

Although FedAvg has achieved great success and is one of the most well-known algorithms in FL, the statistical heterogeneity challenges in the data are still difficult to overcome. That is to say, the training data follow a non-independent and non-identical distribution (non-IID), which negatively affects
the convergence behavior. To address this issue, adaptive aggregation is introduced.

**Adaptive Aggregation**: Different from the FedAvg, adaptive aggregation uses a different way to update the global model. To improve the accuracy and convergence performance of the global model, a temporally weighted aggregation method utilizing the previously trained local models was proposed in [100]. The authors in [100] assumed that the local models updated in the \((t-i)\)th time slot \((i = 1, ..., t - 1)\) are less important than those updated in the \(t\)th time slot. In practice, the training data at each device changes over time, and the local models that are more recently updated have a higher importance during model aggregation. In order to account for the freshness of the local models, the global model is updated using [100] Eq. (1)

\[
\mathbf{w}_{G}^{t+1} = \sum_{k=1}^{K} p_k \left( e^{-\frac{1}{2} \left( t - t^k \right)} \right)^{(t - t^k)} \mathbf{w}_k, \quad (8)
\]

where \(p_k\) is given in (4), \(e\) is the scalar constant used to represent the time effect, and \(t^k\) is the time slot in which the newest \(\mathbf{w}_k\) is updated. By introducing parameters \(\left( \frac{G}{2} \right)^{(t - t^k)}\), the impact of the local models updated in previous time slots reduces, and the global model updated in the current time slot is weighted to be more important for the new data, which results in higher learning accuracy.

In [101], an adaptive weighting approach, namely, Inverse Distance Aggregation (IDA), was proposed. The global model updating still follows (5) or (7). Compared with the FedAvg, the main difference of the IDA is the manner in which the weighting coefficient \(p_k\) is calculated, which is based on the inverse distance of the local model weights to the global model weights. To realize this, the \(l_1\)-norm is used as a metric to measure the distance between the weights of the local model of the \(k\)th device \(\mathbf{w}_k\) and that of the global model \(\mathbf{w}_G\), and \(p_k\) is calculated as

\[
p_k = \frac{\| \mathbf{w}_G - \mathbf{w}_k^t \|_1}{\sum_{k=1}^{K} \| \mathbf{w}_G - \mathbf{w}_k^t \|_1}. \quad (9)
\]

Calculating \(p_k\) via (9) allows us to give higher weight to devices whose distance between the weights of the local model and the weights of the global model are higher. It is important to note that the IDA approach is based on the assumption that devices with more data should have greater contribution in updating the weights to the local model.

A novel layer-wise adaptive aggregation scheme was proposed in [102] to iteratively update weights while attempting to reduce the distance between the global model and local models. Although the global model is still updated using either (5) or (7), \(p_k\) in each layer in [102] is calculated to minimize the distance between each layer of the local model and each layer of the global model. For the \(l\)th layer, the weighting coefficient \(p_{k,l}\) is calculated as

\[
p_{k,l} = \text{softmax}(s_{k,l}^t) = \frac{e^{s_{k,l}^t}}{\sum_{i=1}^{N} e^{s_{i,l}^t}}, \quad (10)
\]

where

\[
s_{k,l}^t = \| \mathbf{w}_{G,l}^t - \mathbf{w}_{k,l}^t \|_d. \quad (11)
\]

Here, \(d\) refers to the \(l_d\)-norm, which is used to calculate the distance \(s_{k,l}^t\) between the \(l\)th layer of the global model and the \(l\)th layer of the local model of the \(k\)th device. The softmax function in (10) is applied to guarantee \(p_{k,l}\) in the range of 0 to 1. This is because the softmax function is a function that converts a vector of \(N\) real values into a vector of \(N\) real values that sum to 1 [103]. The advantage of the layer-wise adaptive FL is that it can minimize the distance between the global model and the local models.

### B. Gradient Descent

Standard federated optimization methods, such as FedAvg [32], may show unfavorable convergence performance, especially in heterogeneous networks. This is mainly caused by two factors: 1) client drift, where the local models move away from the optimal global model, which can lead to unstable and slow convergence; and 2) lack of adaptivity, where the FedAvg may be unsuitable for datasets with heavy-tailed stochastic gradient noise distributions, this often happens in NLP research [104]. Heavy-tailed distributions are probability distributions whose tails are not exponentially bounded, that is to say, they have heavier tails than the exponential distribution [105]. Several novel gradient descent methods have been proposed to solve the client drift and lack of adaptivity problems, which are introduced in the following:

1) **Client Drift**: To mitigate the problem of client drift, a new Stochastic Controlled Averaging algorithm (SCAFFOLD) was proposed in [106], where control variates for the \(k\)th device \(\mathbf{c}_k\) and the variate for the server \(\mathbf{c}_G = \frac{1}{K} \sum_{i=1}^{K} \mathbf{c}_i\) were used in the gradient descent to update the local and global models, respectively. Unlike the FedAvg, the gradient descent of the \(k\)th device of the SCAFFOLD algorithm is given by

\[
\mathbf{w}_k^{t+1} = \mathbf{w}_k^t - \eta(g_k^t + \mathbf{c}_G - \mathbf{c}_k^t), \quad (12)
\]

where \(\mathbf{c}_G - \mathbf{c}_k^t\) guarantees the gradient descent moving towards the right direction, and \(\mathbf{c}_k^{t+1}\) is calculated using

\[
\mathbf{c}_k^{t+1} = \mathbf{c}_k^t - \frac{1}{N_k \eta}(\mathbf{w}_G^t - \mathbf{w}_k^t). \quad (13)
\]

In (13), SCAFFOLD uses the previous computed gradients to update the control variate, and \(N_k\) is the number of updating iteration of the \(k\)th device with its local data in the \(t\)th time slot. Then, the global control variate \(\mathbf{c}_G\) is aggregated as

\[
\mathbf{c}_G^{t+1} = \mathbf{c}_G^t + \frac{1}{K} \sum_{i=1}^{K}(\mathbf{c}_i^{t+1} - \mathbf{c}_i^t). \quad (14)
\]

The correction term \((\mathbf{c}_G^t - \mathbf{c}_k^t)\) in (12) ensures that the local model updates move towards the optimal direction, so as to address the client drift issue of FedAvg.

2) **Adaptivity**: An adaptive learning algorithm has adaptive learning parameters, such as learning rate, which can automatically adjust the statistics of the received data, available computational resources, or other information related to the environment in which it operates. Adaptive variants can help
to learn algorithms to improve the convergence performance and learning accuracy \cite{107}. To improve the convergence performance of FedAvg, three methods have been proposed, namely, Adaptive Optimizer, Federated Proximal (FedProx), and Fast-convergent FL (FOLB).

a) Adaptive Optimizer: The SGD in FedAvg may be unsuitable for settings with heavy-tailed stochastic gradient noise distributions. To address this issue, traditional adaptive optimization algorithms, such as Adagrad, Adam, and Yogi have been integrated into FL to update the global model in optimization algorithms, such as Adagrad, Adam, and Yogi noise distributions. To address this issue, traditional adaptive unsuitable for settings with heavy-tailed stochastic gradient adaptivity. EMA gives a higher weight to the most recent data where \( \beta \) denotes higher degrees of adaptivity. EMA gives a higher weight to the most recent data past gradients are forgotten in a fairly fast manner, which can be especially problematic in sparse settings, where gradients are rarely non-zero \cite{109}. To solve the problem caused by FedAdam, a simple adaptive method called FedYogi was proposed.

For the FedAdagrad optimizer, \( v^t_{\text{Adagrad}} \) is given by

\[
v^t_{\text{Adagrad}} = v^{t-1}_{\text{Adagrad}} + \frac{\eta}{\sqrt{v^{t-1}_{\text{Adagrad}}} + \tau} \nabla f_i.
\]

For the FedAdam optimizer, \( v^t_{\text{FedAdam}} \) is written as

\[
v^t_{\text{FedAdam}} = \beta_2 v^{t-1}_{\text{FedAdam}} + (1 - \beta_2) \| \tilde{w} \|^2.
\]

For the FedYogi optimizer, \( v^t_{\text{FedYogi}} \) is obtained as

\[
v^t_{\text{FedYogi}} = v^{t-1}_{\text{FedYogi}} - (1 - \beta_2) \| \tilde{w} \|^2 \text{sign}(v^{t-1}_{\text{FedYogi}} - \| \tilde{w} \|^2).
\]

These three adaptive optimizers have the same learning steps, including initialization, sampling subsets, and computing estimates, and they have been proved to achieve higher accuracy than FedAvg. However, these three adaptive optimizers have some differences. Unlike the FedAdagrad optimizer mainly well-suited for dealing with sparse data, both the FedAdam and FedYogi optimizers are suitable for sparse and non-sparse data. In addition, the FedAdam optimizer can rapidly increase the learning rate, while the FedYogi optimizer increases it in a controlled fashion, for which a detailed proof is provided in \cite{110}.

b) FedProx: FL has two key challenges that need to be addressed: 1) Significant variability in terms of the system characteristics of each device, which is referred to as system heterogeneity. For example, the storage, computation, and communication capabilities of each device in federated networks may be different due to variability in hardware (CPU, memory), network connectivity (3G, 4G, 5G, THz, WiFi), and power (battery level) \cite{111, 112}. 2) Non-identically distributed data across networks, which is referred to as statistical heterogeneity \cite{113}. Fortunately, to address these issues, FedProx has been proposed in \cite{114} based on a federated optimization algorithm that can deal with heterogeneity both theoretically and empirically. Similar to FedAvg, FedProx selects a subset of devices and averages them to form a global model. Different from FedAvg, in the local model updating of FedProx, a proximal term \( \frac{\mu}{2} \| w_k - w^*_k \|^2 \) is added to effectively limit the impact of variable local updates, where the local training objective of the \( k \)th device is determined by

\[
\min_{w_k} f_k(w_k, w^*_k) = \frac{1}{n_k} \sum_{i \in D_k} l_i(w_k) + \frac{\mu}{2} \| w^*_k - w^*_k \|^2.
\]

In (21), \( \mu \) is the regularization parameter. There are two advantages of the proximal term purpose: 1) it addresses the issue of statistical heterogeneity by restricting the local model updating to be closer to the global model without any need to set the number of local epochs manually, and 2) it allows for the aggregation of a large number of local models resulting from system heterogeneity.

c) FOLB: Following the idea of the proximal term in FedProx, FOLB was proposed in \cite{115}. FOLB aims at maximizing the training loss reduction in each iteration. The main difference between FedProx and FOLB is the way they update the global model. Also, FOLB can achieve higher model accuracy, training stability, and higher convergence speed over FedAvg and FedProx. Different from FedAvg and FedProx, in FOLB, in the \( t \)th round, the server selects two multisets of devices \( S^t_1 \) and \( S^t_2 \) with \( K \) randomly selected devices in each set, and transmits \( w^t_1 \) to the \( k \)th device from set \( S^t_1 \) and the \( k \)th device from set \( S^t_2 \). For the \( k \)th device in \( S^t_1 \), it computes the local update \( w^t_{k,1} \) and delivers both \( w^t_{k,1} \) and loss \( f_k(w^t_{k,1}) \) to the server. For the \( k \)th device in \( S^t_2 \), it only calculates and transmits its loss \( f_k(w^t_{k,1}) \) to the server. Then, rather than performing simple averaging, the server calculates the global model via

\[
w^t_{G,1} = w^t + \sum_{k \in S^t_1} \frac{\nabla f_k(w^t_G), \nabla S_i, L(w^t_i)}{\sum_{k \in S^t_i} \nabla f_k(w^t_G), L(w^t_i)} \Delta w^t_{k,1}.
\]

(22)

where

\[
\nabla S_i, L(w^t_i) = \frac{1}{K} \sum_{k \in S^t_i} f_k(w^t_G), i \in \{1, 2\}
\]

is the gradient of the global loss \( L(w^t_G) \) obtained from the local loss of the devices in set \( S^t_i \), and \( \Delta w^t_{k,1} \) is calculated as

\[
\Delta w^t_{k,1} = w^t_{k,1} - w^t.
\]

(24)
In (22), the intuition is that the local update of the $k$th device is weighted by a measure of how correlated its gradient $f_k(w_{t}^{k})$ is with the global gradient $\nabla L(w_{t}^{G})$. This correlation is assessed relative to $\nabla S_{t}^{k} L(w_{t}^{G})$, which is an unbiased estimate of $\nabla L(w_{t}^{G})$ using gradient information obtained from $S_{t}^{k}$. The weights are normalized relative to a second unbiased estimate of the total correlation among $K$ devices, obtained from $S_{t}$. Using the inner product term $< f_k(w_{t}^{k}), \nabla S_{t}^{k} L(w_{t}^{G}) >$ can help connect the local model to the global model, and decrease the distance between them [116].

C. Communication Efficiency

Because of the asymmetric property of the internet connection and the large number of servers and devices, one of the major challenges in FL is the high communication overhead. There are mainly two ways to improve communication efficiency, which are decreasing and compressing the size of the learning model, respectively.

In [117], the update of the local model of the $k$th device is calculated as (5), and the $k$th device transmits $w_{k}^{t+1} \in \mathbb{R}^{d_{1} \times d_{2}}$ to the server, where $w_{k}^{t+1}$ has $d_{1}$ rows and $d_{2}$ columns. The authors in [117] proposed two ways to reduce the cost of transmitting local models to the server, which are structured updates and sketched updates. For structured updates, $w_{k}^{t+1}$ is limited to having a pre-specified structure, either a low-rank or random-mask structure, where these two approaches are independent of each other. In the low-rank structure, $w_{k}^{t+1}$ is the product of two matrices, written as

$$w_{k}^{t+1} = A_{k}^{t+1}B_{k}^{t+1},$$

where $A_{k}^{t+1} \in \mathbb{R}^{d_{1} \times d_{1}}$ and $B_{k}^{t+1} \in \mathbb{R}^{d_{2} \times d_{2}}$. In (25), $A_{k}^{t+1}$ is a randomly generated matrix during the local updating, and only $B_{k}^{t+1}$ can be optimized. Thus, $A_{k}^{t+1}$ is presented in the form of a random seed and directly saved in the server, where the random seed means that it can generate the same random number in each time slot, and the $k$th device only needs to send the trained $B_{k}^{t+1}$ to the server, which can save a factor of $d_{1} / d_{2}$ during the uplink transmission. While in the random-mask structure, $w_{k}^{t+1}$ is restricted to be a sparse matrix, where the sparse matrix consists of mostly zero values. Thus, the $k$th device only needs to send the non-zero values of $w_{k}^{t+1}$.

The sketched update is used to reduce communication costs. In this case, first the updated local model is computed without any constraints, and then the update is approximated or encoded in a compressed form before transmission to the server. Two approaches are used for the sketched update, which are subsampling and probabilistic quantizations. In subsampling quantization, rather than transmitting $w_{k}^{t+1}$, the $k$th device only needs to transmit $\tilde{w}_{k}^{t+1}$ to the server, which is created from a random subset of values from $w_{k}^{t+1}$. The server then averages the subsampled update and calculates the global model as (5). In probabilistic quantization, the way of compressing the local update is through quantizing each scalar of the local weights into one bit. Let $w = (w_1, ..., w_{d_{1} \times d_{2}}) = \text{vec}(w_{t}^{G})$, and let $w_{\text{max}} = \max(w_{i})$ and $w_{\text{min}} = \min(w_{i})$ ($i = 1, 2, ..., d_{1} \times d_{2}$), the compressed update of $w$, denoted by $\tilde{w}$, is generated using

$$\tilde{w}_{i} = \begin{cases} w_{\text{max}} & \text{with probability } \frac{w_{i} - w_{\text{min}}}{w_{\text{max}} - w_{\text{min}}} \frac{w_{\text{max}} - w_{\text{min}}}{w_{\text{max}} - w_{\text{min}}} \\ w_{\text{min}} & \text{with probability } \frac{w_{\text{max}} - w_{i}}{w_{\text{max}} - w_{\text{min}}} \frac{w_{\text{max}} - w_{\text{min}}}{w_{\text{max}} - w_{\text{min}}} \end{cases}$$

(26)

According to (26), $\tilde{w}$ is an unbiased estimator of $w$, and this method provides $32$ times compression explained in [118].

A multi-objective evolutionary algorithm was designed in [119] to minimize communication costs and improve learning accuracy simultaneously. To achieve these two objectives, modified sparse evolutionary training (SET) was proposed, which can decrease the number of connections between two layers in deep neural networks (DNNs) [120]. In the modified SET algorithm, the connection probability between two layers is computed as

$$p(\tilde{w}_{ij}^{k}) = \frac{\varepsilon(n_{k} + n_{k-1})}{n_{k} n_{k-1}},$$

(27)

and the total number of connections between two layers is calculated as

$$n_{w_{ij}^{k}} = n_{k} n_{k-1} p(\tilde{w}_{ij}^{k}),$$

(28)

where $n_{k-1}$ and $n_{k}$ are the number of neurons of the $(k-1)$th and the $k$th layer, respectively, $\tilde{w}_{ij}^{k}$ is the sparse weight matrix between two layers, and $\varepsilon \in (0, 1)$ is the parameter of SET to determine the connection sparsity. By setting $\varepsilon$, a fraction of the weights with small updates will be removed in each training epoch. In this way, the number of weights of the learning model is decreased, and thus the size of the learning model is reduced. The local and global updates still follow (6) and (7), respectively.

However, if the size of the FL model is large, decreasing or compressing the FL model via the methods in [117] and [119] still has low communication efficiency. To address this issue, quantization-based SGD has been widely adopted in FL and sign-SGD FL, recently proposed in [121], guarantees high robustness and communication efficiency. Rather than delivering gradient $g_{k}^{t}$ calculated by (5), each device quantizes the gradient with a stochastic 1-bit compressor $q(.) \in \{-1, 1\}$ and sends $q(g_{k}^{t})$ to the central server. Then, the central server calculates

$$g_{k}^{t} = \text{sign}(\frac{1}{K} \sum_{k=1}^{K} q(g_{k}^{t})), $$

(29)

and delivers $\tilde{g}_{k}^{t}$ to the devices. The local model of each device is updated as

$$w_{k}^{t+1} = w_{k}^{t} - \eta \tilde{g}_{k}^{t}.$$
with FedAvg [123]. Thus, its communication overhead is much smaller than that of FL. In FD, each device only exchanges the output of the model, the dimension of which is much smaller than that of the local model. Each device treats itself as a student, and treats the averaged model output from all other devices as its teachers output. The model output of each device is a set of logit values normalized through a softmax function, and is denoted as a logit vector whose size is determined by the number of labels of all data samples. The teacher-student output difference is measured periodically by cross entropy and becomes the loss regularizer of the student, namely, the distillation regularizer.

To guarantee communication efficiency, each device stores mean logit vectors, and periodically uploads these local-average logit vectors to a server. For each label, the uploaded local-average logit vectors from all devices are averaged, resulting in a global-average logit vector per label, which will be further downloaded to each device. When each device computes the distillation regularizer, its teacher’s output is selected as the global-average logit vector associated with the same label as the current training sample’s label.

In the $t$th time slot, the global-average logit vector $\hat{F}_{k,l}^t$ is calculated as

$$\hat{F}_{k,l}^t = \frac{1}{K} \sum_{i=1}^{K} \hat{F}_{k,l}^t,$$  

(31)

where $\hat{F}_{k,l}^t$ is the local-average logit vector of the $l$th label of the $k$th device, and is updated as

$$\hat{F}_{k,l}^t = \frac{F_l^t}{N_l},$$  

(32)

In (32), $N_l$ is the number of samples, whose learning output is the $l$th label, and $F_{k,l}^t$ is a logit vector of the $l$th label calculated as

$$F_{k,l}^t = \sum_{x \in S_k} F_l(w_k, x),$$  

(33)

where $F_l(w_k, x)$ is the logit vector of the $l$th label given input $x$ and local model weights $w_k$. In (33), $w_k$ is still updated using (6), $S_k$ is the set containing all data samples of the $k$th device, when the number of data samples of the $k$th device is larger than $N_l$. In the server, the local-average logit vector of the $l$th label $\hat{F}_l^t$ of all devices is updated using

$$\hat{F}_l^t = \frac{1}{K} \sum_{k=1}^{K} \hat{F}_{k,l}^t.$$  

(34)

In the $(t+1)$th time slot, $\hat{F}_{k,l}^{t+1}$ is updated as

$$\hat{F}_{k,l}^{t+1} = \frac{F_l^{t+1} - \hat{F}_{k,l}^t}{K - 1}. $$  

(35)

Then, $\hat{F}_{k,l}^{t+1}$ is transmitted to the $k$th device. As only the logit vector is sent, the transmission size is much smaller than the learning model, allowing on-device ML to adopt large-sized local models.

D. Fairness

Most of the current FL works assume all devices contribute equally to the global model in each communication round, rather than prioritizing them based on their contributions. However, in practice, not all devices contribute equally due to various reasons, such as the different quality and quantity of the data owned by each device. Therefore, the local model from some devices may result in better global model updates, whereas those of others may impair the performance of the global model. To address this issue, fairness needs to be considered in the FL.

One possible learning scenario for FL in large-scale applications is that it is trained based on data originating from a large number of devices in large-scale applications, and the FL model may become biased towards certain devices. To address this issue, agnostic FL (AFL) was proposed in [124], where the global model was optimized for any target distribution formed by a mixture of device distributions, with the aim to minimize the loss function of all devices and guarantee fairness among devices. Different from FedAvg, the training objective of AFL is given by

$$\min_{w_G} \max_{\lambda_G} L_G(w_G, \lambda_G) = \sum_{k=1}^{K} \lambda_k f_k(w_k),$$  

(36)

where $\lambda_k$ is the mixture weight of the $k$th device. To solve the problem, each device needs to optimize $w_k$ and $\lambda_k$ simultaneously. Using SGD, in the $(t+1)$th time slot, $w_k^{t+1}$ and $\lambda_k^{t+1}$ are calculated as

$$w_k^{t+1} = w_k^t - \gamma_w \delta w_k f_k(w_k^t),$$  

(37)

and

$$\lambda_k^{t+1} = \lambda_k^t + \gamma_{\lambda_k} \delta \lambda_k f_k(w_k^t),$$  

(38)

respectively, where $\delta w_k f_k(w_k^t)$ and $\delta \lambda_k f_k(w_k^t)$ are the unbiased estimates of the gradient, and $\gamma_w$ and $\gamma_{\lambda_k}$ are the respective learning rates. Then, the global model update can still be written as in (7). Using AFL, accuracy and fairness in applications with unknown mixture of device distributions can be guaranteed, thus, it can be used in large-scale networks.

A $q$-Fair FL ($q$-FFL) was proposed in [125] to encourage a fairer accuracy distribution across all devices, where $q (0 < q < 1)$ controls the tradeoff between fairness and accuracy. If $q = 0$, fairness in the FL is not encouraged. A larger $q$ means imposing more uniformity in the training accuracy distribution and potentially inducing fairness. The objective of $q$-FFL is given by

$$\min_{w_G} L_q(w_G) = \sum_{k=1}^{K} \frac{p_k}{q + 1} f_k^{(q+1)}(w_k).$$  

(39)

To solve $q$-FFL in (39), it is important to first determine how to set $q$. In practice, $q$ can be tuned based on the desired amount of fairness. Also, it is possible that a family of objectives with different $q$ values has to be trained so that the algorithm can explore the trade-off between accuracy and fairness for different applications. However, one concern with addressing such a family of objectives is that it requires step-size tuning for every value of $q$ and can cause the search
space of $q$ to explode. To solve the problem, the authors in [125] considered estimating the local Lipschitz constant, which could prevent the function value from skipping the optimal value, and dynamically adjust the step-size of the gradient-based optimization method for the $q$-FFL objective, avoiding manual tuning for each $q$ [126].

The local model updates of the $k$th device are calculated as in (6). The global model updates are given by the sum of the first-order derivatives of $f_k^q(w_k)$ divided by the sum of second-order derivatives of $f_k^q(w_k)$. Thus, the $k$th device computes

\[
\Delta w_k^t = L(w_G^t - w_k^{t+1}),
\]

\[
\Delta t_k^q = f_k^q(w_k^t)\Delta w_k^t,
\]

\[
h_k^t = qf_k^q(w_k^t)\|\Delta w_k^t\|_2 + Lf_k^q(w_k^t),
\]

where $L$ is the Lipschitz constant, $\Delta t_k^q$ is the first-order derivative of $f_k^q(w_k)$, and $h_k^t$ is the second-order derivative of $f_k^q(w_k)$. Then, the update of the global model in $q$-FFL is given by

\[
w_G^{t+1} = w_G^t - \sum_{k=1}^{K} h_k^t.
\]

where $\Delta t_k^q$ and $h_k^t$ are given in (41) and (42), respectively.

Unlike using the same version of the global model in [124] and [125], collaborative Fair FL (CFFL) was proposed in [127] to utilize reputation to update the local model of each device to converge to different models, which can achieve collaborative fairness by adjusting the performance of the models allocated to each participant based on their contributions. The reputation is applied to quantify the contribution of each device, and the reputation of the $k$th device is represented as

\[
c_k = \sinh(\alpha \frac{vacc_k}{\sum_{i=1}^{K} vacc_i}),
\]

where $vacc_k$ is given as

\[
vacc_k = w_k + \eta g_k.
\]

In (44), $\sinh(\alpha)$ serves as a punishment function, and $\alpha$ denotes the punishment factor, that is used to distinguish the reputations of different devices based on how informative their uploaded gradients are. The larger the variation of the gradient of the local model of the $k$th device, the higher $vacc_k$, and the higher contribution of the $k$th device. The local and global models in the CFFL are updated using (6) and (7), respectively. However, the global model allocated to the $k$th device needs to be calculated according to its contribution using

\[
w_k^{t+1} = \frac{c_k}{\sum_{i=1}^{K} c_i}w_G^{t+1},
\]

where $c_k$ is given in (44).

Due to the fact that CFFL enables devices to converge to different final models, the most contributive device receives the most accurate model. CFFL not only can achieve comparable accuracy to FedAvg, but also can guarantee higher fairness than FedAvg.

E. Bayesian Learning

FedAvg requires access to locally stored data for learning. However, it is possible that the local model cannot be trained by the local data. Such situations may be caused by catastrophic data loss or by regulations such as the general data protection regulation [128], which places severe restrictions on the storage and access of personal data. Thus, the transmitted local model cannot be updated by the local data in the current time slot. To solve this problem, Bayesian machinery can be deployed to estimate the local model weights via probabilistic neural matching based on the pre-trained local models. Then, the local models estimated by the Bayesian machinery and the updated local models trained by local data are delivered to the server for model aggregation.

The authors in [129] proposed a probabilistic federated neural matching (PFNM) algorithm, which used a Beta Bernoulli Process (BBP) [130] to model the multi-layer perceptron (MLP) weights. Through the permutation invariance of a fully-connected neural network, the proposed FGNM algorithm first matches the weights of the transmitted local model from each device to the weights of the global model. Then, it aggregates these local models by maximizing the posterior estimation of the global weights. As a result, the FGNM algorithm can achieve higher accuracy and communication efficiency than FedAvg. However, the FGNM algorithm can only be effective for simple architectures of the neural network, such as fully-connected feedforward neural networks.

To deal with this issue, Federated Matched Averaging (FedMA) was proposed in [131], which constructed the shared global model in a layer-wise manner by matching and averaging hidden layers, including channels for convolutional neural networks (CNNs), hidden units for recurrent neural networks (RNNs), and weights for fully connected layers. The training objective of the FedMA algorithm is represented as

\[
\min_{\pi_k^l} \sum_{i=1}^{L} \sum_{k,l} \min_{c} \pi_k^l c(w_{k,l}, w_{G,i}),
\]

where $L$ is the number of hidden layers, $w_{k,l}$ denotes the weights of the $l$th layer learned based on the dataset of the $k$th device, $w_{G,i}$ denotes the weights of the $i$th layer of the global model, $c(\cdot, \cdot)$ is an appropriate similarity function between a pair of weights, and $\pi_k^l$ is the permutation, which determines the contribution of the weights of the $l$th layer of the local model of the $k$th device to the neurons of the $i$th layer of the global model. From (47), we observe that the objective is to minimize the weight distance between the local model and the global model, and $c(\cdot, \cdot)$ is the squared Euclidean distance. Through optimizing the permutation $\pi_k^l$, the total weight distance between the layers of the local model of all $K$ devices and the layer of the global model can be minimized.

In the FedMA algorithm, first, the server gathers only the weights of the first layers from devices and performs one-layer matching to obtain the weights of the first layer of the
global model. Then, the server broadcasts these weights to the devices, and each device updates the first layer of its local model to train all consecutive layers with its own dataset, keeping the matched layer frozen. The procedure continues until all layers have finished matching. Thus, FedMA requires the number of communication rounds to be equal to the number of layers in the neural network. We assume that there are \( N \) layers in the neural network. Through computing a posterior estimate (MAP) of the Bayesian non-parametric model based on the BBP, for the \( n \)th layer, if \( n < N \), the global model in FedMA is updated as

\[
\mathbf{w}_{G,n} = \frac{1}{K} \sum_{k=1}^{K} \mathbf{w}_{k,n} \left( \prod_{k=1}^{K} \mathbf{w}_{k,n} \right),
\]

(48)

where

\[
\left( \prod_{k=1}^{K} \right) = \text{BBP-MAP}(\{ \mathbf{w}_{k,n} \}_{k=1}^{K}).
\]

(49)

In (49), \( \prod_{k=1}^{K} \) is a permutation matrix. The permutation matrix is an orthogonal matrix and is usually used to match the layers of neural networks in the weight space. If \( n = N \), the global model is updated as

\[
\mathbf{w}_{G,N} = \sum_{k=1}^{K} p_k \mathbf{w}_{k,N},
\]

(50)

where \( p_k \) is calculated as (4). For the \( k \)th device, the local model is updated as

\[
\mathbf{w}_{k,n+1} = \left( \prod_{k=1}^{K} \right) \mathbf{w}_{k,n+1}.
\]

(51)

Compared to FedAvg, FedMA not only improves communication efficiency, as the number of communication rounds is equal to the number of layers of the neural network, but also guarantees learning accuracy.

**F. Clustering**

For the aforementioned FL techniques, the central server updates only a single global model. In contrast, for clustered FL, the central server updates multiple global models, where the number of global models is equal to that of the clusters. Clustered FL partitions devices into different groups as in [132], where \( K \) devices were partitioned into \( M \) disjoint clusters. This method captures settings where different groups of devices have their own learning tasks. It is assumed that all devices do not have any knowledge of the other device's cluster identity. To minimize the loss function while estimating the cluster identities, an iterative Federated Clustering Algorithm (IFCA) was proposed in [132]. In the \( t \)th time slot, the central server transmits \( M \) updated global models to these \( M \) clusters. Then, a random subset of devices is selected to update their local models with their corresponding data samples and delivers the local models to the central server. As the central server does not know the cluster identities of the selected devices, it estimates the identity of the \( k \)th device using

\[
\hat{j} = \arg \min_{j \in M} f_k(\mathbf{w}_{G,j}),
\]

(52)

where \( M \) is the set including all clusters, \( f_k(\mathbf{w}_{G,j}) \) is the loss of the \( k \)th device in the \( j \)th cluster, and \( \mathbf{w}_{G,j} \) is the global model of the \( j \)th cluster. From (52), we observe that the \( k \)th device belongs to the \( j \)th cluster that achieves the minimum loss. Given the estimated clusters, the global model of each cluster is updated using (7), and the local model of each device is still updated using (6).

However, in [132], all selected devices need to communicate with the central server, thus, there is a large overhead when a large number of devices transmit. To address this issue, a hierarchical clustering (HC) algorithm for local model updating was proposed in [133], where the clusters with the highest similarity were merged. In the \( t \)th time slot, the \( d \)-norm distances between all clusters are calculated to judge
their similarity. The distance \( \hat{d}_{i,j} \) between the \( i \)th cluster and the \( j \)th cluster in the global model is calculated as
\[
\hat{d}_{i,j} = \| w_{G,i}^t - w_{G,j}^t \|_d.
\] (53)

If \( \hat{d}_{i,j} \) is smaller than the threshold \( \hat{d}_{th} \), the \( i \)th and \( j \)th clusters can be merged together. This procedure continues until all clusters with similarity are merged into a single cluster. Then, these merged clusters select a portion of their devices to aggregate the local models in the server via (7), and the local models in the devices are still updated using (6). By selecting a portion of the devices in the merged clusters to aggregate the local models, the communication overhead is reduced.

Although authors in [132] and [133] considered clustered FL, both works adopted a single central server to capture the global models of all devices by aggregating their local models. In [134], a multi-center aggregation mechanism for multiple global models in clustered FL was proposed, where devices belong to a specific cluster, and the cluster updates its own global model with its corresponding updated local models, as shown in Fig. 5. The learning objective of multi-center clustered FL is to minimize the total weighted distance between the global model and the local models, and the multi-center weight distance-based loss (MD-Loss) is represented as
\[
\mathcal{L} = \frac{1}{K} \sum_{k=1}^{K} \sum_{i=1}^{m} r_{k,i} \text{Dist}(w_k, w_{G,i}),
\] (54)
where
\[
\text{Dist}(w_k, w_{G,i}) = \| w_k - w_{G,i} \|_2,
\] (55)

\( r_{k,i} = \{0,1\} \) is the cluster assignment, \( r_{k,i} = 1 \) indicates that the \( k \)th device belongs to the \( i \)th cluster, vice versa. In (54), \( K \) and \( m \) are the number of devices and clusters, respectively. In (55), \( w_{G,i} \) is the global model of the \( i \)th cluster.

To solve (54), a federated stochastic expectation maximization (FeSEM) algorithm is deployed in the following three steps:

First, the cluster assignment \( r_{k,i} \) is updated using
\[
r_{k,i} = \begin{cases} 1, & \text{if } i = \arg \min_j \text{Dist}(w_k, w_{G,j}); \\ 0, & \text{otherwise}. \end{cases}
\] (56)

From (56), we observe that the \( k \)th device belongs to the \( i \)th cluster that can achieve the minimum weight distance.

Second, the global model of the \( k \)th cluster is aggregated as
\[
w_{G,i} = \frac{1}{m} \sum_{k=1}^{K} r_{k,i} w_k.
\] (57)

Finally, the local model of the \( k \)th device is still updated using (6). The multi-center aggregation mechanism can better capture the heterogeneity of data distributions across devices, and simultaneously facilitates the optimal matching between devices and servers.

These six research areas of FL mentioned above are mainly focused on non-wireless networks. The differences between FedAvg and other FL algorithms mentioned are presented in Fig. 6. Based on Fig. 6, FedAvg includes four components, which are the learning function, \( p_k \) calculation, local model updating, and global model aggregation, corresponding to equations (3), (4), (6), and (7), respectively. Through jointly or separately optimizing these four equations, the learning efficiency, accuracy, and fairness can be improved.

IV. FL IN WIRELESS NETWORKS

Recently, there is a growing interest in optimizing wireless networks with data-driven ML-based methods. In this section, we present the research areas of FL for wireless networks. Considering FL in wireless networks, all weights of local or global models are delivered via wireless links instead of non-wireless ones. Thus, the model aggregation, learning accuracy, and learning efficiency of FL can be influenced by wireless factors, such as the set of devices that participate in FL, computational capacity, transmission power and wireless channel, and spectrum resource allocation. The impact of these factors on FL performance is introduced as follows:

1. With the increasing number of devices participating in model aggregation, the generalization of the global model increases. However, more devices lead to high interference and a low transmission rate.
2. Computational capacity of each device affects the learning latency. High computational capability leads to high learning efficiency and low learning latency.
3. Transmission power and wireless channel determine transmission rate and reliability. Low transmission power and dynamic wireless channel result in low transmission rates and high transmission errors.
4. Spectrum resource allocated to each device affects transmission rate. When more spectrum is allocated to the device, its transmission rate increases.

To exchange a large number of model weights over time-varying channels, there are two types of solutions, which are “digital” and “analog” approaches that convert all global or local models into bits and modulated signals, respectively. In this section, we introduce these key research areas of FL over wireless networks. For the digital approach, we consider model aggregation, communication, energy, and computation efficiency optimization, resource allocation, and asynchronous FL. For the analog approach, we consider over-the-air computation.

A. Digital Approach of FL over Wireless Networks

The digital approach needs to guarantee a high transmission rate, low transmission error, and high communication, energy, and computation efficiency of FL over wireless networks. In this subsection, we mainly introduce 1) Model Aggregation, 2) Communication, Energy, and Computation Efficiency Optimization, 3) Resource Allocation, and 4) Asynchronous FL.

1) Model Aggregation: The model uploading and downloading of FL can be affected by dynamic wireless channels. Due to the limited bandwidth of the wireless network, not all devices can transmit local models to the central server. Also, poor channel state results in high transmission error. To select optimal devices to upload the local models and minimize transmission error, device selection and packet transmission error minimization schemes need to be considered.
Device Selection: Three main device selection schemes have been studied, which are probabilistic updating, importance-based updating, and novel communication protocol-based updating.

a) Probabilistic Updating: A traditional probabilistic scheduling policy was developed in [135] to characterize the convergence performance of FL in wireless networks. In particular, the effectiveness of three different scheduling policies, i.e., random scheduling (RS), round robin (RR), and proportional fair (PF) were considered to select a portion of devices for local model aggregation under limited bandwidth constraints. In RS, the access point (AP) randomly selects associated devices in each time slot for local model updating, and each device is allocated with a sub-channel to deliver the associated devices in each time slot for local model updating, and assigns each group to access the radio channels and update their weights in each time slot. While in PF, the AP selects K out of \( K \leq \hat{K} \) associated devices in each time slot according to the following policy

\[
\mathbf{m}^* = \arg \max_{\mathbf{m} \in \{1, 2, \ldots, K\}} \left\{ \frac{\hat{P}_{1,t}}{\hat{P}_{1,t}}, \ldots, \frac{\hat{P}_{K,t}}{\hat{P}_{K,t}} \right\},
\]

where \( \mathbf{m} = \{m_1, \ldots, m_K\} \) is a length-K vector and \( \mathbf{m}^* = \{m_1^*, \ldots, m_K^*\} \) represents the indices of the selected K devices, \( \hat{P}_{k,t} \) and \( \hat{P}_{k,t} \) are the instantaneous and time average signal-to-noise ratio (SNR) of the kth device in the tth time slot, respectively. Thus, the device with a higher SNR is selected [136]. The updating of the local and global models still follows (6) and (7), respectively.

Based on the probabilistic analysis of the scheduling policies in [135], it shows that PF outperforms RS and RR in terms of convergence rate under a high SNR threshold. This is because a high SNR threshold reduces the chance of successful transmission from an arbitrary device, while PF improves the convergence rate by selecting devices with better channel qualities in order to increase their transmission success probabilities. However, RR is preferable when the SNR threshold is low, this is because low SNR threshold results in a high success probability.

b) Importance-based Updating: To exploit the importance of devices, a novel probabilistic scheduling framework was developed to apply unbiased update aggregation for the federated edge learning (FEEL) in [137], where the importance of a local model update was measured by its gradient divergence. In the tth time slot, K devices are selected for model aggregation according to a scheduling distribution \( \mathbf{p}^t = (p_{1,t}^1, p_{2,t}^2, \ldots, p_{K,t}^K) \), where \( p_{k,t}^k \) is the probability that the kth device is selected, and can also indicate the level of importance that the kth device can contribute to the global model convergence. The local model is still updated using (6), and the global model is updated using

\[
w_{G}^{t} = \frac{1}{n} \sum_{k=1}^{K} n_{k} \frac{1}{p_{k}^{t}} w_{k}^{t},
\]

where \( n_k \) is the number of data at the kth device, and \( n = \sum_{i=1}^{K} n_i \). The selected local model \( w_{k}^{t} \) needs to be scaled by a coefficient \( n_k / n \) at the edge server. This is because this coefficient well quantifies the unbalanced property in global data distribution and thus makes the global model unbiased. Based on the probabilistic scheduling framework,
the importance indicator of each local model is defined as
\[ I_k^* = \frac{n_k}{n^p_k} \| \mathbf{w}_k - \mathbf{w}_G \|_2 . \] (60)

The model divergence reflects the deviation between the local model and the global model, and the smaller the model divergence, the more it can contribute to the global model convergence. In other words, the smaller the \( p_k^t \), the less contribution of the \( k \)th device to the global model. The optimization problem is to achieve a trade-off between gradient divergence and latency, where \( \rho \in [0,1] \) is defined as the weight coefficient that balances the gradient divergence and latency. Then, the objective function is represented as
\[
\min_{(p_1^t, \ldots, p_K^t)} \sum_{k=1}^{K} p_k^t \left[ \rho I_k + (1 - \rho)T_k^t \right],
\] (61)
\[
\text{s.t.} \quad \sum_{k=1}^{K} p_k^t = 1,
\] (62)
\[
\sum_{k=1}^{K} p_k^t \geq 0,
\] (63)
where \( I_k^* \) is given by (60), and the optimal probability \( p_k^t \) is given by
\[
p_k^t \ast = \frac{n_k}{n} \| \mathbf{w}_k \| \sqrt{\frac{\rho}{(1-\rho)T_k^t + \lambda^t}},
\] (64)
where \( \lambda^t \) is the lagrangian multiplier that satisfies (62), and \( T_k^t \) is the uplink transmission latency. From (64), the optimal scheduling decision is mainly determined by data unbalanced indicator \( \frac{n_k}{n} \), the norm of local model \( \| \mathbf{w}_k \| \), and the uplink latency \( T_k^t \). Through the importance-aware device scheduling strategy, it can achieve less than half of the convergence time and up to 2\% higher final accuracy.

c) Novel Communication Protocol-based Updating:
Apart from deploying probabilistic scheduling to select the optimal devices for model aggregation, a communication protocol designed for FL over wireless networks, called federated learning with client selection (FedCS), was proposed in [138]. First, the server requests \([K \times C]\) random devices to participate in the current training task, where \( K \) is the total number of all devices, \( C \in (0, 1] \) is the fraction of devices that participating in training in each time slot, and \( \lfloor . \rfloor \) is the ceiling function. The devices that receive the request will inform the server of their resource information, such as wireless channel states, computational capacities, and size of data resources, which are relevant to the current training task. After receiving the resource information, the server selects as many devices as possible to transmit their local models for model aggregation within a certain deadline. The optimization problem is to maximize the number of devices for model aggregation as
\[
\max \quad S,
\] (65)
\[
\text{s.t.} \quad T_{CS} + T_{Agg} + T_{up} + T_{down} \leq T_{th},
\] (66)
where \( S \) is the number of the selected devices for model aggregation, and \( T_{CS}, T_{Agg}, \) and \( T_{down} \) are the time required for device selection, model aggregation, and downlink transmission, respectively. In (66), \( T_{up} \) is the time required for local model updating and uplink transmission. To solve the optimization problem in (65), a heuristic algorithm based on the greedy algorithm was proposed. Although the proposed FedCS algorithm can achieve a higher accuracy than that of FedAvg, the computation complexity is extremely high with a large number of devices. Also, the local and global updating still follow (6) and (7), respectively.

However, authors in [135], [137], [138] considered perfect wireless channels, and in the uplink transmission, there will be transmission errors caused by unstable wireless channels. To solve the problem, the impact of the packet transmission error on FL is considered.

Packet Transmission Error: The authors in [139] considered packet transmission errors, which could affect the local model aggregation of FL. In [139], a closed-form solution for the convergence rate of FL was derived as a function of packet error rates. Based on this solution, the BS optimizes the resource allocation and the device optimizes its transmission power to decrease packet error rates. The optimization problem in [139] is to minimize the training loss of FL over wireless networks. The expression of the model aggregation with packet transmission error is denoted as
\[
\mathbf{w}_g(a^t, P^t, \mathbf{R}^t) = \frac{\sum_{k=1}^{K} N_k a_k^t \mathbf{w}_k^t C(\mathbf{w}_k^t)}{\sum_{k=1}^{K} N_k a_k^t C(\mathbf{w}_k^t)}.
\] (67)

In (67), \( \sum_{k=1}^{K} N_k a_k^t C(\mathbf{w}_k^t) \) is the total number of training data samples, which is determined by \( a_k^t \) and \( C(\mathbf{w}_k^t) \). \( N_k \) is the number of training data samples of the \( k \)th device, \( a_k^t \in \{0, 1\} \) is the device association index of the \( k \)th device. If \( a_k^t = 1 \), the \( k \)th device is selected to update the local model to the BS, vice versa. Also, in (67), \( \mathbf{P}^t = [P_1^t, \ldots, P_K^t] \) is the transmit power matrix, \( C(\mathbf{w}_k^t) \) is the packet transmission index of the \( k \)th device, which is presented as
\[
C(\mathbf{w}_k^t) = \begin{cases} 1, & \text{with probability } 1-q_k(r_{k,n}^t, P_k^t), \\ 0, & \text{with probability } q_k(r_{k,n}^t, P_k^t), \end{cases}
\] (68)
and \( q_k(r_{k,n}^t, P_k^t) \) is the packet error rate of the local model of the \( k \)th device. If \( C(\mathbf{w}_k^t) = 0 \), the local model of the \( k \)th device contains data error, and the BS will not use it to update the global model. In (68), \( r_{k,n}^t \) is the resource block (RB) allocation index, and \( r_{k,n}^t = 1 \) means that the \( n \)th RB is allocated to the \( k \)th device in the \( t \)th time slot. Meanwhile, in (68), \( q_k(r_{k,n}^t, P_k^t) \) is expressed as
\[
q_k(r_{k,n}^t, P_k^t) = \sum_{n=1}^{R} r_{k,n}^t q_{k,n}^t,
\] (69)
where \( q_{k,n}^t \) is the packet error rate over the \( n \)th RB with \( m \) being a waterfall threshold and is defined as
\[
q_{k,n}^t = E_{h_k} \left( 1 - \exp \left( -\frac{m(\bar{I}_n + B N_0)}{P_k^t} \right) \right).
\] (70)

In (70), \( h_k \) is the channel gain between the BS and the \( k \)th device, \( N_0 \) is the noise power spectral density, and \( \bar{I}_n \) is the interference caused by the other devices. The local model updating is still written as (7). Through optimizing the power and RB allocation via the Hungarian algorithm [140], both the packet transmission error and the training loss can be minimized.
2) Communication, Energy and Computation Efficiency Optimization: For FL over wireless networks, one of the challenges is to maximize the communication, energy, and computation efficiency, which can be influenced by the bit rate, energy, and computation capability. In this subsection, the research on the optimization of communication, energy, and computation efficiency is introduced.

a) Communication Efficiency: Authors in [141] introduced momentum gradient and sparse communication to increase the communication efficiency of FL over wireless networks. To optimize the transmission rate of each device, the optimal sub-carrier is allocated to each device. The modified momentum method is used to accelerate the performance of SGD. Then, based on the sparse communication, the global model is updated using

\[ w_{G}^{t+1} = \sum_{k=1}^{K} p_{k} \text{sparse}(w_{k}^{t}), \quad (71) \]

where \( p_{k} \) is calculated by (4), and sparse function \( \text{sparse}() \) in (71) converts \( w_{k}^{t} \) to sparse form by squeezing out any zero elements [142]. Using sparse communication, the server and devices only transmit a fraction of the weights that considerably reduce the communication latency. With the help of momentum and sparse communication, the convergence speed and latency of FL over wireless networks can be guaranteed, however, the accuracy decreases.

It is important to know that training and transmitting weights during FL may consume a large amount of energy. To deal with this issue, the energy consumption minimization problem was studied.

b) Energy Efficiency: To minimize the total energy consumption for local computation and wireless transmission, an iterative algorithm was proposed in [143]. The total energy consumption of all devices at each step is calculated as

\[ E = \sum_{k=1}^{K} (E_{k}^{C} + E_{k}^{T}), \quad (72) \]

where \( E_{k}^{C} \) and \( E_{k}^{T} \) are the local computation energy and wireless transmission energy of the \( k \)-th device, respectively. To minimize the energy, closed-form solutions for the time allocation \( t_{k} \), bandwidth allocation \( b_{k} \), power control \( p_{k} \), computation frequency \( f_{k} \), and learning accuracy \( \delta \) are derived. At each iteration, to optimize \( (t_{k}, b_{k}, p_{k}, f_{k}, \delta) \), the authors first optimized \( (t_{k}, \delta) \) under fixed \( (b_{k}, p_{k}, f_{k}) \). Then, \( (b_{k}, p_{k}, f_{k}) \) are updated based on the obtained \( (t_{k}, \delta) \). Thus, the optimal solution of \( (b_{k}, p_{k}, f_{k}) \) or \( (t_{k}, \delta) \) can be obtained at each time slot. Also, the local and global model updatings still follow (6) and (7), respectively.

The works in [141] and [143] mainly focused on accelerating the training tasks from the communication and energy perspective, i.e., minimizing the communication overhead and energy consumption. However, computation efficiency is also one of the major characteristics of FL over wireless networks, which may greatly affect learning performance.

c) Computation Efficiency: There are mainly two ways to increase computation efficiency for FL in wireless networks, including deploying high computation units and efficient gradient descent methods.

High Computation Unit: In recent years, GPU has been proposed to accelerate the training latency and efficiency of FL. Authors in [144] considered the training acceleration problem from the CPU to GPU under communication and computation resource constraints. Using the Karush-Kuhn-Tucker (KKT) conditions, the closed-form solutions of joint batch size selection and communication resource allocation were derived, and the relationship between training latency and training batch size was analyzed. The local and global updatings in [144] still follow (6) and (7), respectively. Although using GPU for FL training improved the learning efficiency, authors in [144] relied on an impractical assumption that each device was equipped with a GPU. The mobile device with GPU for training can cost a large amount of energy, especially for battery-limited devices.

Gradient Descent: To improve the training performance of FL over wireless networks, several novel gradient descent methods have been studied. For the gradient descent methods in FL over wireless networks, the authors in [145] and [141] mainly deployed SGD and momentum gradient descent to update the local model, respectively.

(1) SGD: Different from the gradient descent in (6), that calculated from the entire dataset, SGD randomly selects one data sample from the whole dataset at each time slot to reduce computation complexity [146]. For the SGD in [145], the local model of the \( k \)-th device in the \( t \)-th time slot is updated as

\[ w_{k}^{t+1} = w_{k}^{t} - \eta g_{k,i}^{t}, \quad (73) \]

where \( i \) is the \( i \)-th data sample of the \( k \)-th device.

(2) Momentum Gradient Descent: Momentum is a method that helps to accelerate the gradient descent in the relevant direction and dampens oscillations. This is achieved by adding a momentum term \( \sigma \) of the update vector of the past time slots to the current update vector [147]. For the gradient descent with momentum in [141], the local model is updated using

\[ w_{k}^{t} = w_{k}^{t-1} + u_{k}^{t}, \quad (74) \]

where

\[ u_{k}^{t} = \sigma u_{k}^{t-1} + g_{k}^{t}. \quad (75) \]

Also, the model aggregation of both gradient descent methods follows (7). Using SGD and momentum, a faster convergence rate can be achieved. Note that the gradient descent methods of FL in non-wireless networks mentioned in Section III can still be deployed in FL over wireless networks.

Authors in [141], [143]–[145] optimized the communication, energy, and computation efficiency, separately. In fact, the communication, energy, and computation efficiencies are correlated with each other, which requires joint design and optimization among them.

d) Joint Design of Communication, Energy, and Computation Efficiency: In FL over wireless networks, there are two trade-offs, between computation and communication latencies, and between learning latency and device energy consumption. Authors in [148] decomposed the problem into two sub-problems, which were the learning latency versus device...
energy consumption problem solved by the Pareto efficiency model, and the computation versus communication latencies problem, solved via finding the optimal learning accuracy with KKT conditions. The Pareto efficiency model minimizes the learning latency and does not increase the energy costs of each device [149]. Through iteratively obtaining the closed-form solutions of these two sub-problems, the authors characterized how the computation and communication latencies of mobile devices affect trade-offs between energy consumption, learning time, and learning accuracy. However, authors in [148] relied on an impractical assumption that the channel state information (CSI) remained unchanged during the whole FL process. In [150], the authors considered imperfect CSI. Under imperfect CSI, channels between the server and devices over each resource block (RB) are predicted using their past observations. Then, based on Lyapunov optimization, joint device scheduling and RB allocation policy were proposed to minimize the loss function in FL over wireless networks. The local and global model updatings in [148], [150] still follow (6) and (7), respectively.

In summary, authors in [135], [137]–[139], [141], [143]–[145], [148], [150] independently considered the model aggregation and communication, energy, and computation efficiency optimization. However, in practical scenarios, they are correlated with each other, and in the next section, we introduce the resource allocation of FL over wireless networks.

3) Resource Allocation: For the research on resource allocation of FL over wireless networks, it jointly designs the spectrum resource allocation and device selection. Due to the limited number of RBs in the uplink transmission, only a fraction of devices can transmit local models to the base station (BS). To solve this problem, a probabilistic device selection scheme was proposed in [151] to select the devices whose local models have significant effects on the global model. The update of the global model in the $t$th time slot is still updated as (7), while $p_k$ in (7) is calculated as

$$ p_k = \frac{a_k^t \bar{N}_k}{\sum_{i=1}^{K} a_k^t \bar{N}_i}, $$

where $a_k^t \in \{0, 1\}$ is the device association index of the $k$th device, and $\bar{N}_k$ is the number of training data samples of the $k$th device. If $a_k^t = 1$, the $k$th device is selected to update the local model to the BS, vice versa. To select the optimal set of devices to upload the local models and minimize the uplink and downlink transmission latency, the optimization problem of a joint RB allocation and device selection scheme was proposed and written as

$$ \min_{\mathbf{A}, \mathbf{R}} \sum_{t=1}^{\mathcal{C}} T' (\mathbf{a}', \mathbf{R}') I^t $$

s.t. $a_{k, t}^t, r'_{k,n}^t, I^t \in \{0, 1\},$

$$ \sum_{i=1}^{K} r'_{k,n}^t \leq 1, $$

$$ \sum_{n=1}^{R} r'_{k,n}^t = a_{k}^t, $$

where $T' (\mathbf{a}', \mathbf{R}')$ is the transmission latency, $\mathbf{A} = [a_1, ..., a_n]$ is the device selection matrix of all iterations, $\mathbf{R} = [R_1, ..., R_C]$ is the RB allocation matrix for all devices of all iterations, $\bar{R}$ is the number of RBs, and $\bar{C}$ is a constant, which is large enough for the proposed FL to converge. In (78), $I^t = 0$ means that the proposed FL converges, vice versa. In (79), $r_{k,n}^t = 1$ means that the $n$th RB is allocated to the $k$th device in the $t$th time slot, and (79) implies that at most one RB is allocated to the $k$th device. In (80), it means that all RBs should be allocated to the devices associated with the BS. To increase the convergence speed of FL, DNNs are used to predict the local models of devices that cannot transmit their local model weights. To enable the BS to predict the local model, each device should have a chance of connecting to the BS to provide local model weights for training DNNs. Thus, a probabilistic device association scheme was proposed as

$$ P_k^t = \begin{cases} \frac{||e_k^t||}{\sum_{k=1}^{K} ||e_k^t||}, & \text{if } k \neq k^*, \\ 1, & \text{if } k = k^*, \end{cases} $$

where $P_k^t$ is the probability of the $k$th device connecting to the BS in the $t$th time slot, $e_k^t = \mathbf{w}_G^t - \mathbf{w}_k^t$ is the variation between the global model and the local model of the $i$th device, and $||e_k^t||$ is the norm of $e_k^t$. In (81), the association probability between the BS and the $k$th device increases as $||e_k^t||$ increases. Thus, the probability that the BS deploys the local model of the $k$th device to aggregate the global model increases. Through using the device association scheme in (81), the BS has a higher probability of selecting devices whose local models significantly affect the global model. In addition, the $k^*$th device is always connected to the BS to provide a local model for the prediction of other devices’ local models. With the predicted local models, the global model is updated as

$$ \mathbf{w}_G^{t+1} = \sum_{k=1}^{K} \bar{N}_k a_k^t \mathbf{w}_k^t + \sum_{k=1}^{K} \bar{N}_k (1-a_k^t) \hat{w}_k^t \mathbb{I} (e_k^t \leq \gamma), $$

where $\hat{w}_k^t$ is the predicted local model of the $k$th device, $\sum_{k=1}^{K} \bar{N}_k a_k^t \mathbf{w}_k^t$ is the sum of local models of the devices connected to the BS, $\sum_{k=1}^{K} \bar{N}_k (1-a_k^t) \hat{w}_k^t \mathbb{I} (e_k^t \leq \gamma)$ is the sum of the predicted local models of the devices are not connected to the BS, $E_k^t = ||\hat{w}_k^t - \mathbf{w}_k^t||^2$ is the prediction error, and $\gamma$ is the error threshold. If $E_k^t \leq \gamma$, the BS uses the predicted local model $\hat{w}_k^t$ to update the global model, otherwise, not. From (82), we can obtain that the BS uses the predicted local models together with the transmitted local models to update the global model to decrease the FL training loss and improve the convergence speed.

4) Asynchronous FL: Authors in [135], [137]–[139], [141], [143]–[145], [148], [150], [151] mainly considered synchronous FL over wireless networks. One common problem of FL systems is the straggler issue. This problem originates from the fact that the time duration of each training round is strictly limited by the slowest participating device [152]. Two asynchronous FL policies are introduced in [153] and...
Asynchronous FL with periodic aggregation

Device 1
Device 2
Device 3
Device 4

Synchronous FL

Device 1
Device 2
Device 3
Device 4

Fully asynchronous FL

Device 1
Device 2
Device 3
Device 4

Asynchronous FL with periodic aggregation

Device 1
Device 2
Device 3
Device 4

Time-triggered FL

Device 1
Device 2
Device 3
Device 4

Device 4
Device 3
Device 2
Device 1

Fig. 7. Illustration of synchronous FL, fully asynchronous FL, asynchronous FL with periodic aggregation, and time-triggered FL.

which are fully asynchronous FL and asynchronous FL with periodic aggregation, as shown in Fig. 7. For the fully asynchronous FL in [153], in each time slot, the server receives a locally trained model \( w_{\text{new}} \) from an arbitrary device and updates the global model \( w_G \) by weight averaging, which is denoted as

\[
w_G^t = (1 - \alpha)w_G^{t-1} + \alpha w_{\text{new}},
\]

where \( \alpha \in (0, 1) \) is a mixing hyperparameter to determine the contribution of the local model to the global model. Intuitively, a larger bias of the local model results in a higher error when updating the global model. The local model updating is still written as (6). Based on [153], the fully asynchronous FL was used in [155] for edge devices with non-iid data, so that the server does not need to wait for the devices with high communication delays.

However, fully asynchronous FL with sequential updating has the problem of high communication costs caused by frequent local model updating and transmission. To address this issue, an asynchronous FL with periodic aggregation and an adaptive asynchronous FL (AAFL) were proposed in [154] and [156], respectively.

For the asynchronous FL with periodic aggregation in [154], the edge server periodically collects local models to update the global model from devices that have completed local training. While other devices continue their local training without being interrupted or dropped. Particularly, after each device updates its local model by (6), it transmits a signal to the server indicating its completion of local model training. After each time duration \( T \), the server schedules a subset of ready-to-update devices to upload their local models. The received local models are aggregated at the server by (7), and then the updated global model is distributed to these devices, and continue their local training based on the newly received global model.

The AAFL algorithm in [156] is an experience-driven algorithm based on DRL, which can adaptively determine the optimal fraction value \( \alpha \) in each time slot. Given the completion time of the learning task, local model parameters, loss function, the difference between the current loss value and target loss value, bandwidth consumption, and remaining resource budget in each time slot, the DRL agent selects the value of \( \alpha \) for model aggregation. Integrating AAFL with DRL reduces the training time and improves learning accuracy compared to fully asynchronous FL.

Based on the proposed synchronous and asynchronous FL, authors in [157] proposed a time-triggered FL (TT-Fed) over wireless networks, which was a generalized form of classic synchronous and asynchronous FL and achieved a good balance between training and communication efficiencies. The global model aggregation in TT-Fed is triggered in each fixed global model aggregation round duration \( \Delta T \). Assuming that \( T \) is the time required for the slowest device to complete one single local updating round. Thus, all devices are partitioned into \( M = \lceil \frac{T}{\Delta T} \rceil \) tiers \((\lceil . \rceil \) is the ceiling function), where the first tier is the fastest tier and the \( M \)th tier is the slowest tier.

As shown in Fig. 7, assuming that 4 devices are partitioned into 3 tiers according to the global model aggregation round duration partitioning. Device 1 and device 2 in the first tier need a single global model aggregation \( \Delta T \) to complete their local updating, while device 4 in the third tier needs three \( \Delta T \). Thus, the server has new updates from different tiers in each global model aggregation round. By using TT-Fed, it is possible for the global model to be broadcast to users in different tiers for communication overhead reduction.

Authors in [135], [137]–[139], [141], [143]–[145], [151], [153], [154], [156], [157] considered OFDMA and TDMA to transmit the local or global models. However, when a large number of devices uploading high-dimensional local models, the classic orthogonal-access schemes, such as OFDMA and TDMA, are not able to scale well with an increasing number of devices. To deal with this issue, over-the-air computation (OAC) has been proposed, it is a disruptive technology for fast data aggregation in wireless networks through exploiting the waveform superposition property of multi-access channels. In particular, the transmitted signal in the uplink transmission is superimposed over-the-air and their weighted sums, so-called the aggregated signal, are processed at the edge [158], [159].

### B. Analog Approach of FL over Wireless Networks

In the digital approach, one challenge is to overcome the communication bottleneck, which is caused by many devices uploading high-dimensional models to a central server. A promising approach is to design new multiple access schemes, and a recently emerged approach, the so-called OAC, can provide the required scalability for FL over wireless networks. OAC is a promising approach for fast wireless data aggregation via computing a nomographic function of distributed data from multiple devices, and it can accomplish the computation of target function by concurrent transmission, thereby significantly improving the communication efficiency compared to orthogonal transmission [160]. The OAC-based approach for fast global model aggregation was proposed in [161] to explore the superposition property of a wireless multi-access channel via the joint device selection and beamforming design. In the
of "learning how to learn" is written as

\[
T = \{D, \mathcal{L}\}. \quad \text{The objective of meta learning is to minimize}
\]

\[
\min_w \mathbb{E}_{T \sim p(T)} \mathcal{L}(D; w),
\]

where \( \mathcal{L}(D; w) \) measures the performance of a model trained using weights \( w \) on dataset \( D \). To solve the problem in (88), a set of \( M \) source tasks are sampled from \( p(T) \) and used in the meta-training stage as \( D_{\text{source}} = \{ (D_{\text{source}}^{i}, \mathcal{L}_{\text{source}}^{i}) \}_{i=1}^{M} \), where each task has both training and testing data. Also, the source training and testing datasets are usually called support and query sets, respectively. The meta-training step of "learning how to learn" is written as

\[
w^* = \arg \max_w \log p(w | D_{\text{source}}). \quad (89)
\]

Then, a set of \( Q \) target tasks used in the meta-testing stage is denoted as \( D_{\text{target}} = \{ (D_{\text{target}}^{i}, \mathcal{L}_{\text{target}}^{i}) \}_{i=1}^{Q} \), where each

\[
\begin{align*}
&\text{th time slot, the } k\text{th device transmits the signal } s_k^{t} \text{ to the BS, and the received signal at the BS is expressed as} \\
&\quad y = \sum_{k=1}^{K} h_k b_k s_k + n, \quad (84) \\
&\text{where } h_k \text{ is the channel vector between the } k\text{th device and the BS, } b_k \text{ is the transmitter scalar, and } n \text{ is the noise vector. Through designing the receiver beamforming vector } m, \text{ the estimated global model at the BS is calculated as} \\
&\quad \hat{w}_G = \frac{1}{\sqrt{\eta}} m^H y, \quad (85) \\
&\text{where } \eta \text{ is a normalizing factor. For the global model updating in the OAC, the difference between the estimated global model } \hat{w}_G \text{ and the target function } w_G \text{ should be minimized, and a mean-square-error (MSE) is used to quantify the performance, which is defined as} \\
&\quad \text{MSE}(\hat{w}_g, w_g) = E(\| \hat{w}_g - w_g \|_2^2). \quad (86) \\
&\text{Motivated by } [162], \text{ given the receiver beamforming vector } m, \text{ the MSE is minimized by using the zero-forcing transmitter} \\
&\quad b_k = \sqrt{\eta p_k (m^H h_k)^H} \frac{m^H h_k}{\| m^H h_k \|_2^2}. \quad (87)
\end{align*}
\]
task has both training and testing datasets. In the meta-testing stage, we use the learned weights \( w \) to train the weights of each new target task \( i \), which is denoted as

\[
\theta^*(i) = \arg\max_\theta \log p(\theta|w^*, D_{\text{target}}^{\text{train}(i)}).
\]

According to (90), we can obtain that learning on the training set of a target task \( i \) benefits from meta-knowledge \( w^* \), and evaluate the accuracy of meta-learner by the performance of \( \theta^*(i) \) on the testing dataset of each target task \( D_{\text{target}}^{\text{test}(i)} \). Meta-learning algorithms can be categorized into three main directions: (1) metric-based, (2) model-based, and (3) gradient-based optimization methods, which are introduced in detail in the following subsections.

**A. Metric-based Meta Learning**

Metric-based methods learn the meta knowledge \( w \) through a feature space that is used for various new tasks. The feature space is integrated with the weights \( \theta \) of the neural networks. Then, new tasks are learned by comparing new inputs with example inputs in the meta-learned feature space. The higher the similarity between the new input and the example input, the more likely that the new input has the same label as the example input. Thus, metric-based meta-learning aims to learn a similarity kernel that takes two inputs, and outputs their similarity score. Larger similarity scores present larger similarities. In this subsection, we introduce four key metric-based meta-learning methods, including siamese networks, matching networks, prototypical networks, and relation networks. The relationship among these four methods is presented in Fig. 9.

1) **Siamese Networks:** Authors in [165] used a siamese network to compare the distance between data samples. A siamese network consists of two neural networks that share the same weights \( \theta \). It takes two inputs \( x_1 \) and \( x_2 \), and computes two hidden states \( f_\theta(x_1) \) and \( f_\theta(x_2) \). Then, these two hidden states are input into a distance layer to calculate a distance vector, which is given by

\[
d = |f_\theta(x_1) - f_\theta(x_2)|.
\]

According to the distance vector \( d \), we can obtain whether two inputs \( x_1 \) and \( x_2 \) belong to the same class. The siamese network is a simple approach in metric-based meta-learning, and can only be deployed to supervised learning scenarios.

2) **Matching Networks:** Based on the distance comparison idea in siamese networks, authors in [166] proposed a matching network to learn the similarity between support sets and new inputs from query sets. The matching networks use a weighted combination of all example labels in the support set and an attention kernel to compute the similarity of inputs \( x_i \) and new input \( x \). The attention kernel uses the cosine distance \[167\] to calculate the similarity of the input representations, rather than using the distance vector in (91) to quantify the similarity of two inputs. The matching network is still a simple approach in metric-based meta-learning, and is not applicable outside of the supervised learning scenarios. Furthermore, it suffers from performance degradation when label distributions are biased.

3) **Prototypical Networks:** Similar to matching networks, prototypical networks proposed in [168] also used samples in the support set. However, rather than calculating the similarity between samples in the support set and new inputs, prototypical networks map inputs to a dimensional vector space such that inputs of a given output class are close together. Since the number of class prototypes is smaller than that of samples in the support set, the amount of comparisons decreases, which further reduces computational costs. However, prototypical networks can only be used in supervised learning scenarios.

4) **Relation Networks:** Different from the pre-defined similarity metric in siamese and matching networks, relation networks (RN) proposed in [169] used a trainable similarity metric. RN consists of two modules, which are an embedding module responsible for embedding inputs, and a relation module computing similarity scores between new inputs \( x \) and example inputs \( x_i \) from support sets. Then, a classification decision is made by selecting the class of the example input which outputs the highest similarity score. RN uses the Mean-Squared Error (MSE) as a similarity score, and the MSE is then propagated backward through the entire network to update the weights in embedding and relation modules. Because of the trainable similarity metric, the accuracy performance of RN is better than that of siamese and matching networks with a fixed similarity metric.

**B. Model-based Meta Learning**

In contrast to the metric-based approaches deploying fixed neural networks at the testing phase, model-based meta-learning algorithms depend on the internal state of each task. Specifically, model-based approaches process the support set in a sequential fashion. In each time slot, the internal
state captures relevant task-specific information with the given inputs, which can be used to make predictions for new inputs. Meanwhile, task information from previous inputs should be remembered, so that model-based methods have a memory component. In this subsection, we introduce four key model-based meta-learning methods, including memory-augmented neural networks (MANNs), meta networks (MetaNets), recurrent meta-learners (RMLs), and simple neural attentive meta-learner (SNAIL). The relationship of these four networks are shown in Fig. 10.

1) Memory-augmented Neural Networks: MANNs were proposed in [170] to allow for quick task-specific adaptation with the help of a neural turing machine (NTM) [171] and an external memory. The learning procedure of MANNs is that the data of a task is processed as a sequence. First, the support set is input to MANN. Then, the query set is evaluated. The interaction between NTM and external memory is that NTM gradually accumulates meta knowledge across tasks, and the external memory helps to store the obtained knowledge. Given new inputs, NTM leverages the previously obtained meta knowledge stored in the external memory to make predictions. MANNs integrate the external memory and a neural network to achieve meta learning. Different from metric-based meta-learning, MANN can be used for both classification and regression problems. However, it has higher architectural complexity.

2) Meta Networks: Similar to MANN, MetaNets proposed in [172] also leveraged an external memory to store the meta knowledge. However, different from MANN, MetaNets are divided into two distinct subsystems, which are base-learner and meta-learner, as shown in Fig. 11. The base-learner is used to perform tasks, and provide meta knowledge for the meta-learner. Then, the meta-learner calculates fast task-specific weights for itself and the base-learner. The training of MetaNet consists of three main procedures: (1) Acquisition of meta knowledge; (2) Generation of fast weights; (3) Optimization of slow weights. MetaNets depend on base-learner and meta-learner for each task. Although it can be used for both supervised and reinforcement learning scenarios, the learning architecture is quite complex and leads to a high burden on memory usage and computation time.

3) Recurrent Meta-learner: RMLs proposed in [173] and [174] were meta-learners based on recurrent neural networks (RNNs), and were specifically proposed for reinforcement learning scenarios. The internal learning architecture of the selected RNN allows for fast adaptation to new tasks. Similar to MANN, RML still uses memory to store the meta knowledge and the task data is sequentially input into the learning model. However, RMLs have simple learning architectures, mainly perform well on simple reinforcement learning tasks, and cannot be adapt to complex learning scenarios.

4) Simple Neural Attentive Meta-learner: Similar to MANN, SNAIL proposed in [175] still processes task data in sequence. However, rather than using external memory, SNAIL deploys a special model architecture to serve as memory. The special model consists of 1D temporal convolutions [176] and a soft attention mechanism [177]. The 1D convolutions are used for memory access, and the attention mechanism allows SNAIL to pinpoint specific experiences. Furthermore, SNAIL contains three building blocks, which are DenseBlock, TCBlock, and AttentionBlock. The DenseBlock deploys a single 1D convolution to the input and connects to the result, the TCBlock consists of a series of DenseBlocks, and the AttentionBlock learns the important parts of prior experience. A key advantage of SNAIL is that it can be used for both supervised and reinforcement learning scenarios, and it achieves better learning accuracy performance than that of the other three model-based meta-learning algorithms.

C. Gradient-based Meta Learning

Different from the metric-based and model-based meta learning approaches, gradient-based meta-learning is mainly...
trained with an interleaved training procedure, including inner loop of task-specific adaptation and outer loop of meta initialization training \[178, 179\]. The traditional gradient-based meta learning is model-agnostic meta learning (MAML). Based on MAML, several other advanced meta learning models have been proposed, which are meta-SGD, reptile, Bayesian MAML (BMAML), Laplace approximation for meta adaptation (LLAMA), latent embedding optimization (LEO), MAML with Implicit Gradients (iMAML), and online MAML, as shown in Fig. 12.

1) MAML: MAML is a model and task-agnostic algorithm for meta-learning that trains model weights with a small number of gradient steps and leads to fast learning on a new task \[180\]. Thus, MAML has two advantages: 1) it can be fine-tuned, which means that it quickly adapts to new tasks, and 2) it requires fewer training samples \[181\]. The learning trend of MAML is shown in Fig. 13. From the learned initialization weights \( \theta \), MAML can quickly move to the optimal set of weights \( \theta^* \) for the task \( T_i \) (\( i = 1, 2, 3 \)).

The meta learning model is represented by a function \( f_\theta \) with weights \( \theta \). When it adapts to a new task \( T_i \), the model weights \( \theta \) become \( \theta_i \). The updated weights \( \theta_i \) is computed by using gradient descent on task \( T_i \), which is denoted as

\[
\theta'_i = \theta - \alpha \nabla_\theta \mathcal{L}_{T_i}(f_\theta).
\]

(92)

The step size \( \alpha \in (0, 1) \) can be fixed as a hyperparameter or dynamically meta-learned. The learning model weights are trained by optimizing the performance of \( f_{\theta_i} \) with respect to \( \theta \) across the tasks sampled from \( p(T) \). More concretely, the meta-objective is to minimize the loss function of all tasks with the learned initialization weights \( \theta \), which is presented as

\[
\min_\theta \sum_{T_i \sim p(T)} \mathcal{L}_{T_i}(f_{\theta_i}) = \sum_{T_i \sim p(T)} \mathcal{L}_{T_i}(f_\theta - \alpha \nabla_\theta \mathcal{L}_{T_i}(f_\theta)),
\]

(93)

where \( \mathcal{L}_{T_i}(f_{\theta_i}) \) is the loss function of the \( i \)th task \( T_i \) with its model weights \( \theta_i \), and \( \theta_i \) is updated by (92). According to (92) and (93), the optimization of meta learning is performed over the initialized model weights \( \theta \), and the objective is achieved by the updated model weights \( \theta' \). The initialized model weights \( \theta \) are updated as

\[
\theta = \theta - \beta \nabla_\theta \sum_{T_i \sim p(T)} \mathcal{L}_{T_i}(f_{\theta_i}),
\]

(94)

where \( \beta \) is the meta step size, and \( \nabla_\theta \sum_{T_i \sim p(T)} \mathcal{L}_{T_i}(f_{\theta_i}) \) is calculated as

\[
\nabla_\theta \sum_{T_i \sim p(T)} \mathcal{L}_{T_i}(f_{\theta_i}) = \sum_{T_i \sim p(T)} (1 - \alpha \nabla^2 f_i(\theta)) \nabla f_i(\theta - \alpha \nabla f_i(\theta)).
\]

(95)

In (95), \( I \) is an identity matrix. The detailed procedures of MAML are introduced in Algorithm 2.

**Algorithm 2 Model-Agnostic Meta-Learning**

1: \( p(T) \): distribution over tasks.
2: Initialize step size hyperparameters \( \alpha \) and \( \beta \), randomly initialize learning weights \( \theta \).
3: while not done do
4: Sample batch of tasks \( T_i \sim p(T) \).
5: for all \( T_i \) do
6: Evaluate \( \nabla_\theta \mathcal{L}_{T_i}(f_\theta) \) with respect to \( K \) data samples.
7: Calculate adapted weights with gradient descent:
\[
\theta_i' = \theta - \alpha \nabla_\theta \sum_{T_i \sim p(T)} \mathcal{L}_{T_i}(f_{\theta_i'}).
\]
8: end for
9: Update \( \theta = \theta - \beta \nabla_\theta \sum_{T_i \sim p(T)} \mathcal{L}_{T_i}(f_{\theta_i'}) \).
10: end while

Second derivatives, suggesting that most of the performance improvement in MAML comes from the gradients of the objective at the first-order update, rather than the second updates from differentiating through the gradient update. Previous research works observed that ReLU neural networks were almost locally linear, which suggested that the second derivatives may be close to zero in most cases, partially explaining the performance improvement of the first-order approximation \[184\]. In addition, the first-order approximation can achieve 33% speed-up in terms of network computation.

To further reduce the computation time, authors in \[183\] introduced Hessian-free MAML (HF-MAML), which did not require computation of the Hessian vectors, and its computation complexity was the same as that of FO-MAML, but it achieved better convergence rate than FO-MAML. It is because Hessian-free is a method to avoid the vanishing gradient problem while using backpropagation in DNNs \[186\]– \[189\]. The idea behind HF-MAML is that for any function \( \phi \), the product of its Hessian \( \nabla^2 \phi(\theta) \) and any vector \( v \) can be approximated as

\[
\nabla^2 \phi(\theta)(v) \approx \left\lfloor \nabla \phi(\theta + \delta v) - \nabla \phi(\theta - \delta v) \right\rfloor \over 2\delta,
\]

(96)

with an error of at most \( \rho\delta ||v||^2 \), where \( \rho \) is the parameter for Lipschitz continuity of the Hessian of \( \phi(\theta) \). By integrating (96) into (95), we can obtain that

\[
\phi(\theta) = f_i(\theta), v = \nabla f_i(\theta - \alpha \nabla f_i(\theta)).
\]

(97)

Thus, \( \nabla^2 f_i(\theta) \nabla f_i(\theta - \alpha \nabla f_i(\theta)) \) in (95) is calculated by (97), which decreases the computation complexity.

2) Meta-SGD: Meta-SGD was proposed in \[190\], which is an easily trainable meta-learner that could initialize and adapt any learner in just one step, for both supervised learning and reinforcement learning. Different from MAML, Meta-SGD can achieve a much higher accuracy not only by the learner initialization, but also by the learner update direction and learning rate, all in a single meta learning process. The main difference is the way it updates \( \theta_i' \), different from MAML that updates \( \theta_i \) based on (92), Meta-SGD updates \( \theta_i' \) using

\[
\theta'_i = \theta - \alpha \circ \nabla_\theta \mathcal{L}_{T_i}(f_\theta),
\]

(98)
where \( \alpha \) is a vector of the same size as \( \theta \) that determines both the update direction and learning rate, and \( \circ \) denotes the element-wise product. The adaptation term \( \alpha \circ \nabla_{\theta} \mathcal{L}_{T_i}(f_{\theta}) \) is a vector whose direction denotes the update direction. Note that learning rate \( \alpha \) in (98) is a vector rather than a scalar in (92), and (98) allows for a higher flexibility in the sense that each weight has its own learning rate. Compared with MAML, Meta-SGD is easier to implement and can learn more efficiently due to that both of its update direction and learning rate can be optimized.

![Fig. 14. Diagram of Reptile.](image)

3) **Reptile**: Like MAML, Reptile learns a weight initialization that can be fine-tuned quickly on a new task. However, the way in which Reptile tries to obtain the optimal weights is quite different from MAML. Given the initialized model weight \( \theta \), it works by repeatedly sampling only one task in each time slot, training on it, and moving the initialized weights towards the trained weights on that task [191]. As shown in Fig. 14, the initialized weights \( \theta \) are moving towards the optimal weights for tasks 1 and 2. Because in each time slot, only one task is selected to train the learning weights, the initialized weights \( \theta \) oscillate between tasks 1 and 2. Thus, Reptile is a very simple meta-learning algorithm, and does not require updating model weights through the optimization process like MAML, making it more suitable for optimization problems where a limited number of update steps are required, and saving time and memory costs. In Reptile, \( \theta'_i \) for the \( i \)th task is updated using (98). However, for the initialization weights \( \theta \), it moves toward to the trained weights, which is updated as

\[
\theta = \theta - \beta (\theta'_i - \theta) \tag{99}
\]

In (99), \( \theta'_i - \theta \) is the distance between initialization weights \( \theta \) and learning weights for the \( i \)th task \( \theta'_i \). The Reptile algorithm is shown in Algorithm 3. Although the Reptile is an extremely simple meta learning technique, the convergence and accuracy performance may be a bit worse than that of MAML because of its simple learning procedure.

4) **BMAML**: Unlike MAML that learns a distribution over potential solutions, Bayesian MAML (BMAML) in [192] learns \( M \) possible weights \( \Theta = \{ \theta^i \}_{i=1}^M \) and jointly optimizes them in parallel. To update these weights, authors in [192] deployed Stein Variational Gradient Descent (SVGD) [193]. SVGD is a non-parametric variational inference method, which leverages the advantages of Markov chain Monte Carlo (MCMC) [194] and variational inference. Also, it converges faster than MCMC because its update rule is deterministic. Specifically, SVGD maintains \( M \) instances of model weights, called particles. In the \( t \)th time slot, each model weight vector \( \theta_t \in \Theta \) is updated using

\[
\theta_{t+1} = \theta_t + \alpha \phi(\theta_t), \tag{100}
\]

where \( \alpha \) is learning rate, and \( \phi(\theta_t) \) is given as

\[
\phi(\theta_t) = \frac{1}{M} \sum_{m=1}^{M} \left[ k(\theta^m_t, \theta_t) + \nabla_{\theta^m_t} \log p(\theta^m_t) + \nabla_{\theta^m_t} k(\theta^m_t, \theta_t) \right]. \tag{101}
\]

Here, \( k(\theta^m_t, \theta_t) \) in (101) is a similarity kernel between \( \theta^m_t \) and \( \theta_t \). In (101), the update of one particle relies on the other gradients of particles, \( k(\theta^m_t, \theta_t) \) moves the particle in the direction of gradients of other particles based on particle similarity, and \( \nabla_{\theta^m_t} k(\theta^m_t, \theta_t) \) enforces repulsive force between particles so that they do not collide to a same point. Then, these particles are used to approximate the probability distribution of labels in testing datasets, which is denoted as

\[
p(y_j^{test} | \theta^m_j) = \frac{1}{M} \sum_{m=1}^{M} p(y_j^{test} | \theta^m_j), \tag{102}
\]

where \( \theta^m_{T_j} \) is the \( m \)th particle calculated by training the support dataset (training dataset) \( D^s_{T_j} \) of the task \( T_j \), and \( p(y_j^{test} | \theta^m_{T_j}) \) is the data likelihood of the task \( T_j \).

To train BMAML, authors in [192] proposed a novel meta loss, called Chaser Loss. This loss aims to minimize the distance between the approximated parameter distribution achieved from the support set \( p^{n+s}_{T_j}(\theta_{T_j}^s, \Theta_0) \) and true distribution \( p^{n+s}_{T_j}(\Theta_0 | D^s \cup D^q) \). Here, \( n \) is the number of SVGD steps, and \( \Theta_0 \) is the set of initial particles. Because the true distribution is unknown, we need to approximate it by running SVGD for \( s \) additional steps to obtain \( \Theta_0^{n+s} \), where \( s \) additional steps are executed on both the support and query sets. The proposed meta-loss is written as

\[
\mathcal{L}_{BMAML}(\Theta_0) = \sum_{T_j \in B} \sum_{m=1}^{M} \left\| \theta^m_{T_j} - \theta^{n+s,m}_{T_j} \right\|_2^2, \tag{103}
\]

where \( B \) is the number of sampled tasks. The BMAML algorithm is shown in Algorithm 4, where \( d_{T_j}^1(\Theta_0, \Theta_0^{n+s}) \) is the dissimilarity between two distributions \( \Theta_0^{n+s}(\Theta_0) \) and \( \Theta_0^{n+s}(\Theta_0) \).
BMAML is a robust optimization-based meta learning that can generate $M$ potential solutions for a task. However, it has to store $M$ parameter sets in memory over time, which results in substantial memory costs.

5) LLAMA: Authors in [195] reformulated MAML as a method for probabilistic inference in a hierarchical Bayesian model. Through integrating MAML into a probabilistic framework, a probability distribution over task-specific weights $\theta_j'$ is learned, and multiple potential solutions can be obtained for a task. This extended MAML is called Laplace approximation for meta adaptation (LLAMA). To minimize the error on the query set $D_{T_j}$, the model must output large probability scores for true classes. The log-likelihood loss function is denoted as

$$
\mathcal{L}_{D_{T_j}}(\theta_j') = -\sum_{(x_i, y_i) \in D_{T_j}} \log P(y_i|x_i, \theta_j').
$$

(104)

To predict the correct label $y_i$, authors in [195] deployed ML-Laplace to compute task-specific weights $\hat{\theta}_j$ updated from the initialization weights $\theta_j$ and estimated the negative log-likelihood. The detailed LLAMA algorithm is shown in Algorithms 5 and 6.

LLMAMA extends MAML in a probabilistic style, which means that there are multiple potential solutions for a single task. However, it can only be deployed for supervised learning with high computational costs, and the Laplace approximation in ML-LAPLACE may be inaccurate, which further decreases the accuracy.

6) LEO: MAML operates in a high-dimensional parameter space using gradient information from only a few data samples from the support set, which can result in poor generalization. To deal with this issue, authors in [197] proposed a latent embedding optimization (LEO) to learn a lower dimensional latent embedding space, which indirectly updates a set of initialized weights $\theta$. The detailed procedures of LEO are shown in Fig. 15. Given a task $T_i$, the data samples from the support set pass through a stochastic encoder to produce $(N_k)^2$ pairs of hidden codes, where $N$ is the number of classes in the support set, and $k$ is the number of data samples per class. Then, these paired codes are input into a relation network [169]. The outputs are grouped by class, and parameterized by a probability distribution over latent codes $z_n$ for class $n$ in a low dimensional space. The decoder further generates a task-specific model weights $\theta_n$ for class $n$. The loss from the generated weights is propagated backward to update the model weights. In practice, generating a such high-dimensional set of parameters from a low-dimensional space is quite problematic. Thus, LEO uses pre-trained models, and only generates weights for the final layer, which limits the dimension of the model.

The key advantage of LEO is that it optimizes model weights in a lower dimensional latent embedding space, which improves generalization performance. However, it is more complex than that of MAML, because it needs to encode and decode the data samples.

7) iMAML: Because of the higher-order derivatives, MAML with Implicit Gradients (iMAML) was considered in [198] to deal with the issue of long optimization path in MAML caused by gradient degradation problems, such as vanishing and exploding gradients [199], [200]. Authors in [198] integrated regularization into the objective of MAML...
to guarantee appropriate learning while avoiding over-fitting. The objective of iMAML is formulated as
\[
\min_{\theta_i} \mathcal{L}(\theta_i; D_k) + \frac{\lambda}{2} \| \theta_i - \theta \|^2, \tag{105}
\]
where \( \lambda \) is a scalar hyperparameter that controls the regularization strength. In (105), the regularization term \( \frac{\lambda}{2} \| \theta_i - \theta \|^2 \) encourages \( \theta_i \) to remain close to \( \theta \). The regularization strength \( \lambda \) plays an important role similar to the learning rate \( \alpha \) in MAML, controlling the strength of prior model weights \( \theta \) relative to the dataset \( D_T \). Ideally, the objective in (105) is solved by iteratively performing gradient descent to obtain the optimal \( \theta_i \). However, authors in [198] considered an implicit Jacobian to obtain \( \theta_i \) as
\[
\frac{\partial \theta_i'}{\partial \theta} = \left(1 + \frac{1}{\lambda} \nabla^2_{\theta} \mathcal{L}_i(\theta_i)\right)^{-1}. \tag{106}
\]
According to [201] and [202], Jacobian only depends on the final result of the algorithm, and not the path taken by the algorithm, thus, it effectively decouples the meta-gradient computation from the choice of inner loop optimizer.

iMAML significantly decreases memory costs because it does not need to store Hessian matrices like MAML, allowing for a higher flexibility in the selection of the inner loop optimizer. However, the computational costs are the same as that of MAML.

8) Online MAML: MAML assumes that a large set of tasks are available for meta training. However, in a practical system, tasks are likely available sequentially, which means that tasks may reveal one after the other. To deal with this issue, online meta learning was proposed in [203]. The objective of online meta learning is to minimize the regret, where the regret is defined as the difference between the loss of the meta-learner and the best performance achievable from online learning with non-convex loss functions [204]. This objective is captured by the regret over the entire sequence, and is denoted as
\[
\text{Regret}_T = \sum_{t=1}^{T} \mathcal{L}_T(\theta_i) - \min_{\theta} \sum_{t=1}^{T} \mathcal{L}_T(\theta), \tag{107}
\]
where \( \sum_{t=1}^{T} \mathcal{L}_T(\theta_i) \) reflects the accumulative loss calculated by the updated weights, and \( \min_{\theta} \sum_{t=1}^{T} \mathcal{L}_T(\theta) \) presents the minimum obtainable loss from a fixed set of initial model weights. The goal for the meta learner in (107) is to sequentially obtain model weights \( \theta_i \) that perform well on the loss sequence. To update \( \theta_{i+1} \), one of the simplest algorithms is following the leader (FTL) [205], [206], which updates the weights using
\[
\theta'_{i+1} = \arg \min_{\theta} \sum_{k=1}^{t} \mathcal{L}_T(\theta_k). \tag{108}
\]
The gradient descent to perform meta update is given by
\[
\theta_{i+1} = \theta_i - \beta \nabla_{\theta} \mathbb{E}_{T_k \sim p_T(\theta)} \mathcal{L}_T(\theta_k), \tag{109}
\]
where \( p_T(\theta) \) is a uniform distribution over tasks in the \( t \)th time slot, and \( \beta \) is the meta learning rate. In online meta learning, memory usage keeps increasing over time. This is because in each time slot, the incoming tasks and their corresponding datasets are stored in memory, which is used to obtain the model weights.

VI. META LEARNING IN WIRELESS COMMUNICATIONS

Due to the advantages of gradient-based meta learning, such as good generalization performance on new tasks and the model being easy to fine-tune, it has been widely used for optimization problems in wireless networks, such as traffic prediction [207], transmission rate maximization [208], and multiple-input and multiple-output (MIMO) detectors [209].

A. Traffic Prediction

One of the traditional methods for network traffic prediction is a feed-forward predictor, which consists of a traffic classifier trained to recognize specific types of traffic, such as videos, web traffic, file downloading, and a predictor that takes the network traffic and classification results as inputs, as shown in Fig. 16. However, it requires a large amount of labeled datasets to train each traffic classifier, which leads to high computation complexity. To address this issue, authors in [207] proposed a feedback traffic prediction architecture based on a meta-learning scheme. The feed-back architecture is presented in Fig. 17, where the predictor is selected based on observed prediction accuracy by DRL, rather than the traffic class. The reason for using meta learning is that, it is recently employed in robust adversarial learning, which can exploit models by taking advantage of obtainable model information and using it to create malicious attacks [210]–[212]. According to Fig. 18, the meta-learning scheme used in the predictor consists of a master policy and a set of sub-policies. The master policy is responsible for selecting which sub-policy is used for prediction during the next prediction interval. The meta-learning scheme in [207] allows for the updating of sub-predictors in real-time, so that the sub-predictors have the ability to adapt to variations in traffic patterns over time. However, when a new sub-predictor is added to sub-predictors, the master policy needs to be retrained, which leads to high computational costs.
MIMO detectors, which could be used in channel-sensitive environments. In particular, an expectation propagation (EP) for signal detection is unfolded as EPNet, and damping factors are set as trainable parameters to adapt to new channels [227]–[229]. Damping factors are relevant to channel statistics. To train the damping factors, a large amount of labeled channel state information (CSI) is required. However, in a dynamic real-time wireless system, it is impractical to obtain enough CSI to train the damping factors. Thus, meta learning is deployed to update the damping factors efficiently by using a small training set so that they can quickly adapt to new environments.

VII. FEDERATED META LEARNING

Gradient-based meta learning algorithms, such as MAML, are well known for their rapid adaptation and good generalization to multiple learning tasks, which makes them particularly suitable for federated learning with non-IID and highly personalized data [185], [225]–[227]. In this section, federated meta learning over non-wireless and wireless networks is discussed.

A. FedMeta in Non-Wireless Networks

In this subsection, we introduce three main research directions of FedMeta in non-wireless networks, including MAML/Meta-SGD-based FedMeta, Collaborative FedMeta, and ADMM-FedMeta.

1) MAML/Meta-SGD-based FedMeta: The federated meta learning framework was first proposed in [228], where authors integrated MAML and Meta-SGD [190] into FL. The objective of the algorithm is to collaboratively meta-train an algorithm using datasets from distributed devices. For model aggregation in the server, as shown in Algorithm 7, the server maintains the initialization parameters $\theta$ and $\alpha$, updates them through the testing loss from the selected devices, and transmits them to the selected devices. For the local model training and testing in devices, as shown in Algorithm 8, first, the $k$th device trains the learning weights $\theta$ obtained from the server using its support dataset $D_{Q}^{k}$. Second, the $k$th device tests the trained learning model, calculates the testing loss $L_{D_{Q}^{k}}(\theta)$ based on its query set $D_{Q}^{k}$, and transmits $L_{D_{Q}^{k}}(\theta)$ to the server. For Meta-SGD, the vector $\alpha$ is also delivered to the server as part of the algorithm parameters which are used for parameter updating. The detailed FedMeta with MAML and Meta-SGD at the server, and model training of MAML or Meta-SGD at the device are introduced in Algorithm 7 and Algorithm 8, respectively.

C. MIMO Detectors

Deep neural networks (DNNs) have the potential for efficiently balancing the bit-error rate minimization and computation complexity of MIMO detectors [217]–[220]. Unfortunately, the existing DNN-based MIMO detectors are difficult to be deployed in practical systems due to their slow convergence speed and low robustness in new environments. To deal with this issue, authors in [209] proposed meta-learning-based
Algorithm 7 FedMeta with MAML and Meta-SGD at the server
1: Initialize step size hyperparameters $\alpha$ and $\beta$, randomly initialize learning weights $\theta$.
2: for each time slot $t=1,2,\ldots$ do
3: Sample $K$ devices, and distribute $\theta$ for MAML or $(\theta, \alpha)$ for Meta-SGD to these $K$ devices.
4: for the $k$th device in $K$ devices do
5: Obtain testing loss $\nabla_\theta L_{D_k}(\theta_k)$ from the model training of the MAML.
6: Obtain testing loss $\nabla(\theta, \alpha)L_{D_k}(\theta_k)$ from the model training of the Meta-SGD.
7: end for
8: Update $\theta = \theta - \frac{\beta}{\pi} \sum_{k=1}^{K} \nabla L_{D_k}(\theta_k)$ for the MAML.
9: Update $(\theta, \alpha) = (\theta - \frac{\beta}{\pi} \sum_{k=1}^{K} \nabla(\theta, \alpha)L_{D_k}(\theta_k)$ for the Meta-SGD.
10: end for

Algorithm 8 Model Training for MAML and MetaSGD
1: Sample support set $D_S^k$ and query set $D_Q^k$ of the $k$th device.
2: $L_{D_k}^S(\theta) = \frac{1}{|D_S^k|} \sum_{(x,y) \in D_S^k} l(f_\theta(x), y)$.
3: $\theta_k = \theta - \alpha \nabla L_{D_k}^S(\theta)$ for the MAML and $\theta_k = \theta - \alpha \nabla L_{D_k}^Q(\theta)$ for the MetaSGD.
4: $L_{D_k}^Q(\theta_k) = \frac{1}{|D_Q^k|} \sum_{(x', y') \in D_Q^k} l(f_\theta(x'), y')$.
5: Transmit $\nabla_\theta L_{D_k}^S(\theta_k)$ and $\nabla(\theta, \alpha)L_{D_k}^Q(\theta_k)$ to the server.

2) Collaborative FedMeta: Authors in [229] proposed a platform-based collaborative learning framework, where a model was first trained in a set of edge nodes by FedMeta, and then it was rapidly adapted to learn a new task at the target edge node with a few data samples. This can deal with the constrained computing resources and limited local data issues of each edge node. The FedMeta algorithm used in [229] is the same as that used in [228]. However, according to Fig. 19, the main differences are that: 1) Authors in [229] deployed a set of source edge nodes only with a support dataset to train the local models, rather than authors in [228] assuming that each device had both support and query datasets to train and test the local model; 2) Authors in [229] assumed that each source edge node focused on only one task to train the local model, rather than authors in [228] assuming that each device had multiple tasks; 3) Authors in [229] used a platform to aggregate local models from all source edge nodes and transmitted the aggregated model to the target edge node for new task adaptation, rather than authors in [228] assuming that each device was able to adapt to new tasks. With model training at multiple edge nodes, FedMeta can adapt to multiple tasks in parallel. However, for cases with a large number of tasks, delivering multiple learning models simultaneously can result in high transmission latency.

3) ADMM-FedMeta: Based on [229], authors in [230] proposed an ADMM-based algorithm, called ADMM-FedMeta, to decompose the initial optimization problem into several sub-problems which can be solved in parallel across edge nodes and the platform. First, authors still applied the platform-based FedMeta architecture in [229] to enable edge nodes to collaboratively learn a meta-model with the knowledge transfer of previous tasks. Then, the FedMeta problem is defined as a regularized optimization problem, where the previous knowledge is extracted as regularization, and the optimization problem is denoted as

$$\min_{\theta, \theta} \sum_{k \in \mathcal{I}} \frac{D_k}{D_k} L_k(\phi_k(\theta_k), D_k^q) + \lambda D_k(\theta, \theta_p), \quad (110)$$

s.t. $\theta_k - \theta = 0, k \in \mathcal{I},$

where $\phi_k(\theta_k)$ is written as (92), $D_k(\theta, \theta_p)$ is a regularization parameter that can extract the valuable knowledge from the prior model to facilitate fast edge training and alleviate catastrophic forgetting [231]. In (110), $\theta_i$ is the prior model weights, $\lambda$ is a penalty parameter that is used to balance the trade-off between the loss and regularization, $\mathcal{I}$ is the set of edge nodes, $D_k$ is the dataset of the $k$th ($k \in \mathcal{I}$) edge node, $D_k$ is the number of data samples in $D_k$, and $D_k$ is divided into two disjoint datasets, i.e., the support set $D_k^s$ and query set $D_k^q$. Through penalizing variations in the model via regularization, the learned model from (110) is close to the prior model for enabling collaborative edge learning without forgetting prior knowledge, so that the learned meta model can adapt to different tasks. To solve the optimization problem in (110), the augmented Lagrangian function is deployed, which is written as

$$L(\{\theta_k, w_k\}, \theta) = \sum_{k \in \mathcal{I}} \frac{D_k}{D_k} L_k(\phi_k(\theta_k), D_k^q) + \langle w_k, \theta_k - \theta \rangle + \frac{\rho_k}{2} ||\theta_k - \theta||^2 + \lambda D_k(\theta, \theta), \quad (111)$$

where $w_k$ is a dual variable and $\rho_k > 0$ is a penalty parameter. To optimize $\theta_k, \theta$, and $w_k$, ADMM method is applied [232]-[241]. The traditional ADMM decomposes the optimization in (110) into a set of sub-problems that can be solved in parallel, which means that calculating $D_k(\theta_k, \theta)$ and $L_k(\phi_k(\theta_k), D_k^q)$ separately. Thus, the vector $\theta_k, \theta$, and $w_k$ are updated alternatively as follows

$$\theta_k^{t+1} = \arg \min_{\theta} L(\{\theta_k^t, w_k^t\}, \theta), \quad (112)$$

$$\theta_k^{t+1} = \arg \min_{\theta_k} L_k(\theta_k, w_k^t, \theta_k^{t+1}), \quad (113)$$

and

$$w_k^{t+1} = w_k^t + \rho_k(\theta_k^{t+1} - \theta_k^t). \quad (114)$$

In (112), (113), and (114), the platform and each device select the weights that can achieve the minimum loss, and update the dual variable by the difference of local and global weights. Based on (112), (113), and (114), the updating strategy is 1) updating $\theta$ at the platform and 2) updating $\{\theta_k, w_k\}$. The advantage of using ADMM-FedMeta is that the decoupled sub-problems can be allocated to multiple edge nodes to solve simultaneously, which helps to alleviate the local computational costs and improve the computation efficiency.
B. FedMeta in Wireless Networks

FedMeta integrates the advantages of FL and meta learning, which enables local model sharing without privacy issues and fast adaptation to new tasks. Through learning an initial shared model, devices can quickly adapt the learned model to their local datasets via one or a few gradient descent steps. Despite its advantages, FedMeta still has several challenges: First, the number of participating devices can be enormous. When devices are randomly selected, it can lead to a low convergence rate and minimize the latency along with energy efficiency. If these factors are not optimized, high latency determined by channel gains, interference, and transmission latency, in a wireless network is highly related to its latency, which devices are randomly selected, it can lead to a low convergence rate. The number of participating devices can be enormous. When local datasets via one or a few gradient descent steps. Despite this, devices can quickly adapt the learned model to their local datasets and CPU types of devices, and transmission latency, determined by channel gains, interference, and transmission power.

In this subsection, we introduce device selection and energy efficiency of FedMeta over wireless networks.

1) Device Selection: To deal with high training delay and communication inefficiency, authors in [243] developed a FedMeta with a non-uniform device selection scheme to accelerate the convergence, and rigorously analyzed the contribution of each device to the global loss reduction in each time slot. Then, a resource allocation problem integrating FedMeta into multi-access wireless systems is proposed to jointly improve the convergence rate and minimize the latency along with energy cost. The learning structure of FedMeta proposed in [244] is the same as that in [248]. The device selection problem in the tth time slot is formulated as

\[
\max_{z_k} \sum_{k \in \mathcal{N}} z_k u_k^t \quad \text{s.t.} \quad \sum_{k \in \mathcal{N}} z_k = n_k, \quad z_k \in \{0, 1\}. \tag{115}
\]

In (115), \(z_k\) is a binary variable, if \(z_k = 1\), the kth device is selected, otherwise, not, and \(\mathcal{N}\) is the set of devices. \(u_k^t\) is the contribution of the kth device to the convergence in the tth time slot, defined as

\[
u_k^t = \frac{1}{\tau-1} \sum_{i=0}^{\tau-1} \|\nabla F_k(\theta_{i+1}^t)\|^2 - 2(\lambda_1 + \frac{\lambda_2}{\sqrt{D_k}})\|\nabla F_i(\theta_{i+1}^t)\|, \tag{116}
\]

where \(F_k(\theta_{i+1}^t)\) is a meta function of the kth device, \(\tau\) is the number of iterations of gradient descent, \(D_k\) is the number of data samples in the kth device, and \(\lambda_1\) and \(\lambda_2\) are position constants. The detailed proof of \(u_k^t\) is presented in Appendix J of the technical report [245]. To apply FedMeta over wireless networks, the authors propose a resource allocation problem, capturing the trade-offs among the convergence, computation and communication latency, and energy consumption. Then, the optimization problem is decomposed into two subproblems. The first sub-problem aims at controlling the CPU-cycle frequencies for devices to minimize energy consumption and computation latency. The second sub-problem controls transmission power and resource block allocation to maximize the convergence speed while minimizing the transmission cost and latency. Both of the sub-problems are solved by KKT conditions.

2) Energy Efficiency: In FedMeta, each task is owned by a device, and each updating iteration needs to communicate with the server. In each iteration, each device updates its local model and transmits it to the server, where the meta model is updated in a global step and then the updated meta model is feedback to all devices. Since these procedures involve local computation and communication energy consumption, it is important to minimize the computation costs and save energy for communication, especially for a device with limited computation capability and energy. To minimize energy and computation costs when performing FedMeta, authors in [246] considered an energy-efficient FedMeta framework, where a meta-backward algorithm was proposed, to learn a meta model with low computation and communication energy consumption. In the backward manner, in the kth step, \(\theta_i^k\) is computed as

\[
\theta_i^k = \theta_i^{k+1} + \alpha \nabla \psi_i(\mathcal{L}(\theta_i, D_i)). \tag{117}
\]

However, when computing in a backward manner, the term \(\nabla \psi_i(\mathcal{L}(\theta_i, D_i))\) cannot be calculated. To address this issue, \(\nabla \psi_i(\mathcal{L}(\theta_i, D_i))\) can be replaced by \(\nabla \psi_i(\mathcal{L}(\theta_i, D_i))\), because they are close to each other under smoothness assumption. In the kth backward step, to find the optimal \(\theta_i^k\), the optimization problem is formulated as

\[
\min_{\theta_i^k} \sum_{i=1}^N \left(\mathcal{L}(\theta_i) - \mathcal{L}(\theta_i^{k,0})\right)^2, \tag{118}
\]

s.t. \(\|\theta_i - \psi_i^{k+1}\|^2 \leq \delta_k\), where \(\psi_i^{k+1} = \frac{1}{N} \sum_{i=1}^N \theta_i^{k+1}\), \(N\) is the total number of tasks, \(\delta_k \rightarrow 0\), and

\[
\theta_i^{k,0} = \theta_i^{k+1} + \alpha \nabla \theta_i^{k+1} \mathcal{L}(\theta_i^{k+1}, D_i). \tag{119}
\]

Obviously, based on the constraint in (118), we need to find the optimal \(\theta_i^k\) which is close to the average weight \(\psi\) among all tasks. To do so, a projection gradient descent (PGD) is introduced to solve the problem in (118). Note that PGD is a standard way to solve constrained optimization problems [247], [248]. The projection problem for the i-th task is written as

\[
\min_{\theta_i^k} \frac{1}{2}\|\theta_i^k - \theta_i^{k,0}\|^2, \tag{120}
\]

s.t. \(\|\theta_i^k - \psi_i^{k+1}\|^2 \leq \delta_k\).

The Lagrangian function of (120) is denoted as

\[
\mathcal{L}(\theta_i, \psi, \mu) = \frac{1}{2}\|\theta_i^k - \theta_i^{k,0}\|^2 + \mu(\|\theta_i^k - \psi_i^{k+1}\|^2 - \delta_k). \tag{121}
\]

Using KKT conditions, we can solve the problem in (120). The proposed meta-backward algorithm is computationally efficient, as it has a closed-form solution calculated by KKT conditions in each iteration.
A. Federated Learning

1) FL over Non-Wireless Networks:

- **Unreliable Model Upload:** In FL, devices may deliver low-quality local models trained by malicious data samples, which can adversely affect the learning accuracy. To address this issue, a reputation metric can be deployed to measure the reliability of each device, and select reliable devices for model aggregation.

- **Systematic and Model Heterogeneity:** A large number of devices and hardware specifications bring systematic heterogeneity to FL in practical systems. Also, FL has coupled with many different learning paradigms, which brings model heterogeneity. To deal with systematic heterogeneity, multi-center FL can be considered, where the devices with similar heterogeneity are clustered into a group for model aggregation. Meanwhile, to deal with model heterogeneity, one possible solution is to learn a personalized model for each device, so-called on-device personalization [249]. Its goal is to train a model for each device, based on the dataset of each device.

- **Imbalanced Data:** FL mainly focuses on IID and non-IID data. However, the data of real-world applications, such as computer vision and biomedicine, follows an imbalanced distribution [250], [251]. Imbalanced data typically refers to a problem with classification problems where the classes are not represented equally [252]. To deal with this issue, a monitor scheme is integrated into FL that can infer the composition of training data in each FL round and detect the existence of possible global imbalance [253].

- **Unsupervised FL:** Unsupervised FL enables multiple devices to share the trained model without sharing their labeled data. However, in real-world applications, the observed data may be unlabeled, which could limit the applicability of FL. To address this issue, a federation of unsupervised learning (FedUL) was proposed in [254], where the unlabeled data were transformed into surrogate labeled data for each device, a modified model was trained by supervised FL, and the unsupervised FL learning model was recovered from the modified model. However, FedUL is not suitable for non-IID data. Thus, FedUL can be further improved by integrating with advanced FL aggregation or optimization schemes for non-IID settings [255].

- **Federated Reinforcement Learning:** In FRL, rather than just uploading and downloading models, the agents need to exchange intermediate results and observations between themselves or with a central server. However, because of limited communication resources, the overhead is high, especially with an increasing number of agents. Meanwhile, some DRL algorithms, such as deep Q network (DQN) [256] and Deep Deterministic Policy Gradient (DDPG) [257], have multiple layers or networks, which contain millions of parameters, resulting in extremely high overhead. To solve these issues, several research directions should be considered. First, dynamic global model methods need to be designed to optimize the number of model exchanges. Second, devices or agents need to exchange the important parts of models or observations.

2) FL over Wireless Networks:

- **Learning Convergence Analysis:** One of the most important considerations of FL is the convergence performance. Most existing research works in [135], [143], [151], [258]–[260] deployed traditional optimization methods to optimize wireless factors, such as transmission scheduling, transmission error, and energy to analyze the convergence of FL, and assumed that the optimization problem was convex. However, FL over practical wireless networks may not satisfy these conditions, especially when the optimization problems are non-convex. Also, the performance of convergence can be affected by dynamic wireless channels and device mobility. To address these issues, one possible solution is to deploy an FL algorithm that can handle heterogeneous device datasets, and capture the trade-off between convergence and energy consumption of devices with heterogeneous computing and power resources [242].

- **Device Dropout:** For the device selection schemes proposed in [138], [261], [262], the authors assumed that the wireless connection of each device was always available. Nevertheless, in practical wireless systems, some devices may become inactive due to poor connectivity and energy constraints, namely, device dropout. Thus, they may leave the FL process and cannot participate in model aggregation, which can severely degrade the performance of FL, such as low learning accuracy and low convergence speed [263]. To deal with this issue, new FL algorithms need to be designed to be robust for the network, where only a small number of dynamic users exist for model aggregation [264]. Also, through designing the communication protocol, devices can actively deliver local models to the edge server when they are in good connectivity conditions.

- **Hierarchical FL:** Accuracy and latency are two main factors for FL over wireless networks. The dynamic wireless environment can severely affect the transmission performance, which can lead to a low transmission rate and high transmission errors, and further lead to high transmission latency and low learning accuracy, especially for hierarchical FL (HFL). HFL is an architecture that deploys FL in heterogeneous wireless networks with three levels, including devices, SBS, and MBS [265]. In each time slot, a set of devices are selected to train the allocated global model using FedSGD algorithms [266].
and then the local models are transmitted to their corresponding SBSs to be aggregated. The MBS and SBSs communicate with each other periodically to maintain a central model. HFL combines the advantages of edge FL and cloud FL. The cloud server can access more learning models, and the edge server enjoys more efficient communications with devices, leveraging edge servers as intermediaries to perform partial model aggregation in proximity, and relieve core network transmission overhead [267], [268]. However, the disadvantages of HFL are that the cloud may have excessive communication overhead and high latency, especially when a large number of devices and edge servers exist in wireless networks. One possible solution is to consider the joint design of device clustering, asynchronous FL, and communication efficiency, and use DRL algorithms to select optimal edge servers and devices for model aggregation in different time slots [269].

- **Cooperative Edge Computing for FL:** When performing FL in a network with multiple edge servers, if there is only a small number of mobile devices exist, only one edge server can be selected for model aggregation, which can save computation resources and energy for other edge servers. If the mobile device is far away from the active edge server, the learning model downloading/uploading can be done in Device-to-Device (D2D) connections. However, when a large number of mobile devices exist, selecting only one edge server for model aggregation can lead to high overhead and workload. Thus, multiple edge servers should be active for model aggregation. In this case, mobile devices need to select the optimal edge servers for model aggregation, and active edge servers should further select one edge server for global model aggregation.

### B. Meta Learning

1) **Meta Learning over Non-Wireless Networks:**

- When a new task does not exist in the experience buffer, meta learning cannot guarantee learning convergence and accuracy [270]. Meta learning also requires a large number of task datasets. Usually, authors assume that tasks are independently and identically distributed [271], and do not consider non-stationary distribution. If the task datasets change dynamically and do not have the same distribution, meta learning cannot adapt to the variation efficiently. Thus, it is difficult for meta learning to address complex datasets. One potential way to solve this issue is to propose a meta learning algorithm that is robust to tasks with a non-stationary distribution.

- Most meta learning algorithms are trained and tested using a small number of benchmark datasets, which means that the characteristics of datasets used for training are close to the datasets for testing. Thus, to the best of our knowledge, there are no meta-learning frameworks that not only can quickly adapt to new tasks with the help of prior experience, but also are robust to bad data samples, e.g. mislabeled data or outliers.

2) **Meta Learning over Wireless Networks:**

- It is possible that meta learning involves training a large number of meta learners, which requires much more computational demand than traditional learning approaches, especially when the number of datasets of each task increases exponentially. Thus, factors of the dynamic wireless environment, such as channel state information, transmission error, transmission energy, and computation capability of each device, can affect the performance of meta learning, and how these factors affect the performance of meta learning should be studied. In addition, plenty of tasks may exist in wireless networks corresponding to a large number of devices. Thus, how to effectively sample tasks to train meta learning to satisfy the requirements of all devices and adapt to new tasks from the dynamic environment are of utmost importance.

### C. Federated Meta Learning

1) **FedMeta over Non-Wireless Networks:**

- In current FL, the shared global model still includes all devices’ privacy implicitly, while in FedMeta, a meta-learner is shared. Thus, whether FedMeta has additional advantages in protecting device privacy from the model attack perspective [272]–[275] still needs to be explored.

- Incorporating the experience replay and parameter isolation approaches into the proposed ADMM-FedMeta in [230] may further mitigate the catastrophic forgetting. In addition, although ADMM-FedMeta can be directly applied to reinforcement learning, it may result in low sample efficiency. Thus, it is essential to develop efficient collaborative reinforcement learning for FedMeta.

2) **FedMeta over Wireless Networks:**

- When a large number of tasks and devices exist in wireless networks, the overhead can be extremely large, which severely affects the transmission quality, increases the transmission latency, and may decrease the learning accuracy. To deal with these issues, one potential solution is to consider multi-model FedMeta, where devices are clustered into multiple groups, aggregate meta models in advance, and transmit them to the server for further aggregation. Thus, the possibility of multi-model FedMeta needs to be further investigated. Also, how to characterize the convergence properties and communication complexity of FedMeta considering factors of wireless networks, such as channel state information and transmission error, require further study.

### IX. Conclusions

In this paper, we have provided a comprehensive tutorial of the research evolution on FL, meta-learning, and FedMeta over non-wireless/wireless networks. For these three types of learning approaches, we have introduced how they are designed, optimized, and evolved over non-wireless and wireless networks with a detailed literature review, and their future research opportunities. Such an in-depth study on these three algorithms over non-wireless/wireless networks provides the
guidelines for optimizing, designing, and operating FL, metalearning, and FedMeta in future 6G non-wireless/wireless networks.

REFERENCES

[1] K. Lueth, “State of the IoT 2018: Number of IoT devices now at 7b-market accelerating,” https://iot-analytics.com/state-of-the-iot-update-q1-q2-2018-number-of-iot-devices-now-7b/, Aug. 2019.

[2] Cisco, “Cisco annual internet report (20182023) white paper,” Mar. 2020.

[3] D. Reinsel, J. Gantz, and J. Ryding, “The digitization of the world,” Nov. 2018.

[4] H. Zhou, C. She, Y. Deng, M. Dohler, and A. Nallanathan, “Machine learning for massive industrial internet of things,” IEEE Wireless Commun., vol. 28, no. 4, pp. 81 – 87, Aug. 2021.

[5] N. Jiang, Y. Deng, and A. Nallanathan, “Traffic prediction and random access control optimization: Learning and non-learning-based approaches,” IEEE Commun. Mag., vol. 59, no. 3, pp. 16 – 22, Mar. 2021.

[6] M. Bojarski, D. D. Testa, D. Dworakowski, B. Firner, B. Flepp, P. Goyal, L. D. Jackel, M. Monfort, U. Muller, J. Zhang, X. Zhang, J. Zhao, and K. Zieba, “End to end learning for self-driving cars,” arxiv:1604.07316, 2016.

[7] C. M. Bishop, “Pattern recognition and machine learning,” Springer, 2006.

[8] L. Jin, Y. Chen, T. Wang, P. Hui, and A. V. Vasilakos, “Understanding user behavior in online social networks: a survey,” IEEE Commun. Mag., vol. 51, no. 9, pp. 144 – 150, Sept. 2013.

[9] P. Rimal, E. Choi, and I. Lomb, “A taxonomy and survey of cloud computing systems,” 2009 Fifth International Joint Conference on IMS and IDC, pp. 44 – 51, Aug. 2009.

[10] P. Li, J. Li, Z. Huang, T. Li, C. Gao, S. Yiu, and K. Chen, “Multi-key privacy-preserving deep learning in cloud computing,” Future Gener. Comput. Syst., vol. 74, pp. 76 – 85, Sept. 2017.

[11] X. Liu and Y. Deng, “Learning-based prediction, rendering and association optimization for mec-enabled wireless virtual reality (vr) networks,” IEEE Trans. Wireless Commun., vol. 20, no. 10, pp. 6356 – 6370, Oct. 2021.

[12] X. Liu, X. Li, T. Chen, and J. Jiang, “Learning-based prediction and proactive uplink retransmission for wireless virtual reality network,” IEEE Trans. Veh. Technol., vol. 70, no. 10, pp. 10723 – 10734, Oct. 2021.

[13] X. Liu, J. Deng, C. Han, and M. D. Renzo, “Learning-based prediction, rendering and transmission for interactive virtual reality in ris-assisted terahertz networks,” IEEE J. Sel. Areas Commun., vol. Early Access, pp. 1 – 1, Oct. 2021.

[14] G. Ananthanarayanan, P. Bahl, P. Bodik, K. Chintalapudi, M. Filipose, L. Ravindranath, and S. Sinha, “Real-time video analytics: The killer app for edge computing,” Computer, vol. 50, no. 10, pp. 56 – 67, Oct. 2017.

[15] G. Tsoumakas and I. Vlahavas, “Distributed data mining,” Encyclopedia of Data Warehousing and Mining, 2009.

[16] D. Liu, G. Zhu, Q. Zeng, J. Zhang, and K. Huang, “Wireless data acquisition for edge learning: Data-importance aware retransmission,” IEEE Trans. Wireless Commun., vol. 20, no. 1, pp. 406 – 420, Jan. 2021.

[17] B. Custers, A. Sears, F. Dechesne, I. Georgieva, T. Tani, and S. van der Hof, “Eu personal data protection in policy and practice,” Hof, “Eu personal data protection in policy and practice,”

[18] B. M. Gaff, H. E. Sussman, and J. Geetter, “Privacy and big data,” In Proceedings of the 20th International Conference on Artificial Intelligence and Statistics (AISTATS), pp. 1273 – 1282, Feb. 2017.

[19] K. Sozinov, V. Vlassov, and S. Girdzijauskas, “Human activity recognition using federated learning,” in Proc. IEEE Int. Conf. Parallel Distrib. Process. Appl. Ubiquitous Comput. Commun. Big Data Cloud Comput. Soc. Comput. Netw. Sustain. Comput. Commun, pp. 1103 – 1111, 2018.

[20] T. Yu, T. Li, Y. Sun, S. Nanda, V. Smith, V. Sekar, and S. Seshan, “Learning context-aware policies from multiple smart homes via federated multi-task learning,” in Proc. IEEE/ACM Int. Conf. Internet Things Design Implement. (IoTDI), pp. 104 – 115, 2020.

[21] T. S. Brisimi, R. Chen, T. Mela, A. Olshevsky, I. C. Paschalidis, and W. Shi, “Federated learning of predictive models from federated electronic health records,” Int. J. Med. Informat., vol. 112, pp. 59 – 67, Apr. 2018.

[22] K. Powell, “Nvidia clara federated learning to deliver ai to hospitals while protecting patient data.” 2019.

[23] D. Verma, S. Julier, and G. Cirincione, “Federated ai for building ai solutions across multiple agencies,” arxiv:1809.10036, 2018.

[24] M. Arambakam and J. Beel, “Federated meta-learning: Democratizing algorithm selection across disciplines and software libraries,” In ICML Workshop, 2020.

[25] N. Schweighofer and K. Doya, “Meta-learning in reinforcement learning,” Neural Netw., vol. 16, no. 1, pp. 5 – 9, 2003.

[26] Q. Yang, Y. Liu, T. Chen, and Y. Tong “Federated machine learning: Concept and applications,” ACM Trans. Intell. Syst. Technol., vol. 10, no. 2, pp. 19 – 1, Feb. 2019.

[27] Y. Jin, X. Wei, Y. Liu, and Q. Yang, “Towards utilizing unlabeled data in federated learning: A survey and prospective,” arxiv:2002.11545, 2020.

[28] P. M. Mammen, “Federated learning: Opportunities and challenges,” arxiv:2101.05428, 2021.

[29] C. Zhang, Y. Xie, H. Bai, B. Yu, W. Li, and Y. Gao, “A survey on federated learning,” Knowl. Based Syst., vol. 216, Mar. 2021.

[30] S. Ji, T. Saravir, S. Pan, G. Long, and A. Walid, “Emerging trends in federated learning: From model fusion to federated X learning,” arxiv:2102.12920, 2021.

[31] O. Shahid, S. Pouriyeh, R. M. Parizi, Q. Z. Sheng, G. Srivastava, and M. Arambakam, “Federated learning: Democratizing algorithm selection across disciplines and software libraries,” In ICML Workshop, 2020.

[32] N. Schweighofer and K. Doya, “Meta-learning in reinforcement learning,” Neural Netw., vol. 16, no. 1, pp. 5 – 9, 2003.

[33] Q. Yang, Y. Liu, T. Chen, and Y. Tong “Federated machine learning: Concept and applications,” ACM Trans. Intell. Syst. Technol., vol. 10, no. 2, pp. 19 – 1, Feb. 2019.

[34] Y. Jin, X. Wei, Y. Liu, and Q. Yang, “Towards utilizing unlabeled data in federated learning: A survey and prospective,” arxiv:2002.11545, 2020.

[35] P. M. Mammen, “Federated learning: Opportunities and challenges,” arxiv:2101.05428, 2021.

[36] C. Zhang, Y. Xie, H. Bai, B. Yu, W. Li, and Y. Gao, “A survey on federated learning,” Knowl. Based Syst., vol. 216, Mar. 2021.

[37] S. Ji, T. Saravir, S. Pan, G. Long, and A. Walid, “Emerging trends in federated learning: From model fusion to federated X learning,” arxiv:2102.12920, 2021.

[38] O. Shahid, S. Pouriyeh, R. M. Parizi, Q. Z. Sheng, G. Srivastava, and M. Arambakam, “Federated learning: Democratizing algorithm selection across disciplines and software libraries,” In ICML Workshop, 2020.
[270] M. Zinkevich, “Online convex programming and generalized infinitesimal gradient ascent,” In ICML, Aug. 2003.
[271] S. S. Shwartz, “Online learning and online convex optimization,” Found. Trends Mach. Learn., vol. 4, no. 2, pp. 107 – 194, 2012.
[272] R. Shokri, M. Stronati, C. Song, and V. Shmatikov, “Membership inference attacks against machine learning models,” In SP, 2017.
[273] C. Song, T. Ristenpart, and V. Shmatikov, “Machine learning models that remember too much,” in ACM SIGSAC CCS, Oct. 2017.
[274] C. Szegedy, W. Zaremba, I. Sutskever, J. Bruna, D. Erhan, I. Goodfellow, and R. Fergus, “Intriguing properties of neural networks,” In Proc. Int. Conf. Learn., 2014.
[275] F. Tramer, F. Zhang, A. Juels, M. K. Reiter, and T. Ristenpart, “Stealing machine learning models via prediction apis,” In USENIX Security Symposium, Aug. 2016.