$CP$ violation for $B^0 \rightarrow \rho^0(\omega)\rho^0(\omega) \rightarrow \pi^+\pi^-\pi^+\pi^-$ in QCD factorization

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Abstract. In the QCD factorization (QCDF) approach we study the direct $CP$ violation in $B^0 \rightarrow \rho^0(\omega)\rho^0(\omega) \rightarrow \pi^+\pi^-\pi^+\pi^-$ via the $\rho - \omega$ mixing mechanism. We find that the $CP$ violation can be enhanced by double $\rho - \omega$ mixing when the masses of the $\pi^+\pi^-$ pairs are in the vicinity of the $\omega$ resonance, and the maximum $CP$ violation can reach 28%. We also compare the results from the naive factorization and the QCD factorization.

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1 Introduction

$CP$ violation is an extensive research topics in recent years. In Standard Model (SM), $CP$ violation is related to the weak complex phase in the Cabibbo-Kobayashi-Maskawa (CKM) matrix. In the past few years more attention has been focused on the decays of $B$ meson system both theoretically and experimentally. Recently, the large $CP$ violation was found by the LHCb Collaboration in $B^0 \rightarrow \pi^+\pi^-\pi^+\pi^-$ and $B^\pm \rightarrow K^{\mp}\pi^+\pi^-$. Hence, the theoretical mechanism for the three or four-body decays become more and more interesting. In this paper, we focus on the interference from intermediate $\rho$ and $\omega$ mesons in the four-body decay.

It is known that the naive factorization [5,6], the QCD factorization (QCDF) [7,8,9], the perturbative QCD (PQCD) [10,11,12], and the soft-collinear effective theory (SCET) [13,14] are the most extensive approaches for calculating the hadronic matrix elements. These factorization approaches present different methods for dealing with the hadronic matrix elements in the leading power of $1/m_b$ ($m_b$ is the b-quark mass). Direct $CP$ violation occurs through the interference of two amplitudes with different weak phases and strong phases. The weak phase difference is directly determined by the CKM matrix elements, while the strong phase is usually difficult to control. However, and not well determined from a theoretical approach. The B meson decay amplitude involves the hadronic matrix elements which computation is not trivial. Different methods may present different strong phases. Meanwhile, we can also obtain a large strong phase difference by some phenomenological mechanism. $\rho - \omega$ mixing has been used for this purpose in the past few years [15,16,17,18,19,20,21,22,23,24,25]. In this paper, we will investigate the $CP$ violation via double $\rho - \omega$ mixing in the QCDF approach.

In the QCDF approach, at the rest frame of the heavy B meson, B meson can decay into two light mesons with large momenta. In the heavy-quark limit, $CP$ corrections can be calculated for the non-leptonic two-body B-meson decays. The decay amplitude can be obtained at the next-to-leading power in $\alpha_s$ and the leading power in $A_{QCD}/m_b$. In the QCD factorization, there is cancellation of the scale and renormalization scheme dependence between the Wilson coefficients and the hadronic matrix elements. However, this does not happen in the naive factorization. The hadronic matrix elements can be expressed in terms of form factors and meson light-cone distribution amplitudes including strong interaction corrections.

The remainder of this paper is organized as follows. In Sec. 2 we present the form of the effective Hamiltonian. In Sec. 3 we give the calculating formalism of $CP$ violation from $\rho - \omega$ mixing in $B^0 \rightarrow \rho^0(\omega)\rho^0(\omega) \rightarrow \pi^+\pi^-\pi^+\pi^-$. Input parameters are presented in Sec. 4. We present the numerical results in Sec. 5. Summary and discussion are included in Sec. 6.

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2 The effective hamiltonian

With the operator product expansion, the effective weak Hamiltonian can be written as [20]

\[
\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{n=0, c} \sum_{a=d, s} V_{ph} V_{ph}^* c_1 O_1^p + c_2 O_2^p + \sum_{i=3}^{10} c_i O_i + c_{T\gamma} O_{T\gamma} + c_{SG} O_{SG} + H.e.c.,
\]

\[
(1)
\]

where \(G_F\) represents the Fermi constant, \(c_i (i = 1, \ldots, 10, 7\gamma, 8g)\) are the Wilson coefficients, \(V_{ph}, V_{ph}^*\) are the CKM matrix elements. The operators \(O_i\) have the following forms:

\[
\begin{align*}
O_1^p &= \bar{p} \gamma_{\mu}(1 - \gamma_5) b_{\mu} \gamma_{\rho}(1 - \gamma_5)p,
O_2^p &= \bar{p} a_{\mu}(1 - \gamma_5) b_{\rho} \gamma_{\rho}(1 - \gamma_5)p a_{\mu},
O_3 &= \bar{q} \gamma_{\mu}(1 - \gamma_5) b_{\mu} \gamma_{\rho}(1 - \gamma_5) q',
O_4 &= \bar{q} \gamma_{\mu}(1 - \gamma_5) b_{\rho} \sum_{q'} q' \gamma_{\mu}(1 - \gamma_5) q'_{\rho},
O_5 &= \bar{q} \gamma_{\mu}(1 - \gamma_5) b_{\mu} \gamma_{\rho}(1 - \gamma_5) q',
O_6 &= \bar{q} \gamma_{\mu}(1 - \gamma_5) b_{\rho} \sum_{q'} q' \gamma_{\mu}(1 - \gamma_5) q'_{\rho},
O_7 &= \bar{q} \gamma_{\mu}(1 - \gamma_5) b_{\mu} \gamma_{\rho}(1 - \gamma_5) q',
O_8 &= \bar{q} \gamma_{\mu}(1 - \gamma_5) b_{\rho} \sum_{q'} q' q' \gamma_{\mu}(1 - \gamma_5) q'_{\rho},
O_9 &= \bar{q} \gamma_{\mu}(1 - \gamma_5) b_{\mu} \gamma_{\rho}(1 - \gamma_5) q',
O_{10} &= \bar{q} \gamma_{\mu}(1 - \gamma_5) b_{\rho} \sum_{q'} q' q' \gamma_{\mu}(1 - \gamma_5) q'_{\rho},
O_{T\gamma} &= \frac{-2m_b}{m_u c}\bar{\gamma}_5 m_b \bar{\gamma}_5 \sigma_{\mu\nu}(1 + \gamma_5) T^{\mu\nu} b,
O_{SG} &= \frac{-2m_b}{m_u c}\bar{\gamma}_5 m_b \bar{\gamma}_5 \sigma_{\mu\nu}(1 + \gamma_5) G^{\mu\nu} b,
\end{align*}
\]

(2)

where \(\alpha\) and \(\beta\) are color indices, \(O_1^p\) and \(O_2^p\) are the tree operators, \(O_{3-10}\) are QCD penguin operators which are isosinglets, \(O_{T\gamma}\) and \(O_{SG}\) are the electromagnetic and chromomagnetic dipole operators, \(e_{q'}\) are the electric charges of the quarks and \(q' = u, d, s, c, b\) is implied.

For the decay channel \(B^0 \rightarrow \rho(0)(\omega)\rho(0)(\omega)\), neglecting power corrections of order \(A_{\text{QCD}}/m_b\), the transition matrix element of an operator \(O_i\) in the weak effective Hamiltonian is given by [39]

\[
\langle V_1 V_2 | O_i | B \rangle = \sum_j F_{j B \rightarrow V_1}^0 (m_{V_2}^2) \int_{0}^{1} du T_{1j}^{f}(u) \Phi_{V_2}(u) + (V_1 \leftrightarrow V_2)
\]

\[
\int_{0}^{1} d\xi d\eta d\varphi T_{11}(\xi, u, v) \Phi_{B}(\xi) \Phi_{V_2}(v) \Phi_{V_2}(u)
\]

(3)

Here \(F_{j B \rightarrow V_1}(m_{V_2}^2)\) denotes \(B \rightarrow V_{1,2}\) (\(V_{1,2}\) represent \(\rho^0\) and \(\omega\) mesons) form factor, and \(\Phi_{V}(u)\) is the light-cone distribution amplitude for the quark-antiquark Fock state of mesons \(\rho^0\) and \(\omega\). \(T_{1j}^{f}(u)\) and \(T_{11}(\xi, u, v)\) are hard-scattering functions, which are perturbatively calculable.

The hard-scattering kernels and light-cone distribution amplitudes (LCDA) depend on the factorization scale and the renormalization scheme. \(m_{V_{1,2}}\) denote the \(\rho^0\) and \(\omega\) masses, respectively.

We match the effective weak Hamiltonian onto a transition operator, the matrix element is given by (\(\lambda^{(D)}_{p} = V_{ph} V_{ph}^*\), \(D = d\)) [39]

\[
\langle V_1 V_2 | \mathcal{H}_{\text{eff}} | B \rangle = \sum_{p = u, c}^{10} \lambda^{(D)}_p \langle V_1 V_2 | T_{p}^{h} | B \rangle + \langle T_{p}^{h} | B \rangle.
\]

(4)

where \(T_{p}^{h}\) denotes the contribution from vertex correction, penguin amplitude and spectator scattering in terms of the operators \(a_{p}^{h}\), \(T_{p}^{h}\) refers to annihilation terms contribution by operators \(b_{p}^{h}\). \(h\) is the helicity of the final state.

The flavor operators \(a_{p}^{h}\) are defined in [39] as follows:

\[
a_{p}^{h}(V_1 V_2) = \left( c_i + \frac{C_i}{N_c} \right) N_{h}^{i}(V_2)
\]

\[
+ \frac{C_{i j}}{N_{c}} \bar{C}_{i j} \alpha_{u, c} \left[ V_{h}^{i}(V_2) + \frac{4\pi^{2}}{N_{c}} H_{1}^{h}(V_1 V_2) \right]
\]

\[
+ P_{P}^{h}(V_2),
\]

(5)

where \(N_c\) is the number of colors, the upper (lower) signs apply when \(i\) is odd (even), and \(C_p = N_{c}^{2} - 1\). It is understood that the superscript ‘\(p\)’ is to be omitted for \(i = 1, 2\). The quantities \(V_{h}^{i}(V_2)\) account for one-loop vertex corrections, \(H_{1}^{h}(V_1 V_2)\) for hard spectator interactions, and \(P_{P}^{h}(V_2)\) for penguin contributions. \(N_{h}^{i}(V_2)\) is given by

\[
N_{h}^{i}(V_2) = \begin{cases} 0 & i = 6, 8, 10; \text{all cases.} \\ 1 & i = 1, 2; \text{all cases.} \\ \end{cases}
\]

(6)

The coefficients of the flavor operators \(a_{p}^{h}\) can be expressed in terms of the coefficients \(\alpha_{p}^{h}\). We will present the form in the following section. Using the unitarity relation

\[
\lambda^{(D)}_{u} + \lambda^{(D)}_{\bar{u}} + \lambda^{(D)}_{c} = 0,
\]

(7)

we can get

\[
\sum_{p = u, c} \lambda^{(D)}_{p} T_{1}^{h}
\]

\[
= \sum_{p = u, c} \lambda^{(D)}_{p} \left[ \delta_{p u} \alpha_{1}(V_1 V_2) A(\vert q_{u} \rangle \langle \bar{u} D) \right]
\]

\[
+ \delta_{p u} \alpha_{2}(V_1 V_2) A(\vert q_{u} \rangle \langle \bar{u} D) \right] + \lambda^{(D)}_{u} \left[ \alpha_{4}^{u}(V_1 V_2) \right]
\]

\[
- \alpha_{4}^{c}(V_1 V_2) \sum_{q} A(\vert q_{c} \rangle \langle q D) \right] + \lambda^{(D)}_{c} \left[ \alpha_{4}^{\text{q,EW}}(V_1 V_2) \right]
\]

\[
- \alpha_{4}^{\text{EW}}(V_1 V_2) \sum_{q} \frac{3}{2} e_{q} A(\vert q_{q} \rangle \langle q D) \right]
\]
where

\[ \sum_{p=u,c} \lambda_p^{(D)} b_p^{(D), Tp,h} = \sum_{p=u,c} \lambda_p^{(D)} \times \left[ \delta_{pu} b_1(V_1 V_2) \sum_{q,q'} B([iuq'][iq'u][\bar{D}b]) + \delta_{pu} b_2(V_1 V_2) \sum_{q,q'} B([i\bar{u}q'][iq'D][\bar{u}b]) - \lambda_1^{(D)} \left[ b_3(V_1 V_2) \sum_{q,q'} B([i\bar{u}q'][iq'D][\bar{q}b]) + b_4(V_1 V_2) \sum_{q,q'} B([iqq'][iq'D][\bar{q}b]) + b_{3,EW}(V_1 V_2) \sum_{q,q'} \frac{3}{2} e_q B([iqq'][iq'D][\bar{q}b]) + b_{4,EW}(V_1 V_2) \sum_{q,q'} \frac{3}{2} e_q B([iqq'][iqq'][\bar{q}b]) \right] \right], \]

where \( b_p^{(D), h} \) and \( B \) will be given in the following section.

### 3 CP violation in

\( B^0 \to \rho^0(\omega) \rho^0(\omega) \to \pi^+\pi^-\pi^+\pi^- \)

#### 3.1 Formalism

The \( B \to V_1(\epsilon_1, P_1) V_2(\epsilon_2, P_2) \) (\( \epsilon_1(P_1) \) and \( \epsilon_2(P_2) \) are the polarization vectors (momenta) of \( V_1 \) and \( V_2 \), respectively) decay rate is written as

\[ \Gamma = \frac{G_F^2 P_c}{64 \pi m_B^2} \sum_{\sigma} A^{(\sigma)} + A^{(\sigma)}, \]

where \( P_c \) refers to the c.m. momentum. \( A^{(\sigma)} \) is the helicity amplitude for each helicity of the final state. The decay amplitude, \( A \), can be decomposed into three components \( H_0, H_+, H_- \) according to the helicity of the final state. With the helicity summation, we can get

\[ \sum_{\sigma} A^{(\sigma)} + A^{(\sigma)} = |H_0|^2 + |H_+|^2 + |H_-|^2. \]

In the vector meson dominance model \( \text{[27]} \), the photon propagator is dressed by coupling to vector mesons. Based on the same mechanism, \( \rho - \omega \) mixing was proposed \( \text{[28]} \). The formalism for \( \text{CP} \) violation in the decay of a bottom hadron, \( B \), will be reviewed in the following. The amplitude for \( B \to V\pi^+\pi^- \), \( A \), can be written as

\[ A = \langle \pi^+\pi^- V|H_T^\perp|B \rangle + \langle \pi^+\pi^- V|H_P^\parallel|B \rangle, \]

where \( H_T^\perp \) and \( H_P^\parallel \) are the Hamiltonians for the tree and penguin operators, respectively. We define the relative magnitude and phases between these two contributions as follows:

\[ A = \langle \pi^+\pi^- V|H_T^\perp|B \rangle [1 + re^{i\delta} \sin \phi], \]

where \( \delta \) and \( \phi \) are strong and weak phase differences, respectively. The weak phase difference \( \phi \) arises from the appropriate combination of the CKM matrix elements: \( \phi = \text{arg}([V_{tb} V_{td}]/[V_{tb} V_{td}]) \). The parameter \( r \) is the absolute value of the ratio of tree and penguin amplitudes,

\[ r = \left| \frac{\langle \pi^+\pi^- V|H_P^\parallel|B \rangle}{\langle \pi^+\pi^- V|H_T^\perp|B \rangle} \right|. \]

The amplitude for \( B \to V\pi^+\pi^- \) is

\[ \tilde{A} = \langle \pi^+\pi^- V|H_T^\perp|B \rangle + \langle \pi^+\pi^- V|H_P^\parallel|B \rangle. \]

Then, the CP violating asymmetry, \( A_{CP} \), can be written as

\[ A_{CP} = \frac{|A|^2 - |\tilde{A}|^2}{|A|^2 + |\tilde{A}|^2} = \frac{-2(T_0^2 r_0 \sin \delta_0 + T_2^2 r_+ \sin \delta_+ + T_2^2 r_- \sin \delta_- \cos \phi)}{\sum_{i=0,\pm} T_i^2 (1 + r_i^2 + 2r_i \cos \delta_i \cos \phi)}, \]

where

\[ |A|^2 = \sum_{\sigma} A^{(\sigma)} + A^{(\sigma)} = |H_0|^2 + |H_+|^2 + |H_-|^2 \]

and \( T_i (i = 0, +, -) \) represent the tree-level helicity amplitudes. We can see explicitly from Eq. \( \text{15} \) that both weak and strong phase differences are needed to produce \( \text{CP} \) violation. \( \rho - \omega \) mixing has the dual advantages that the strong phase difference is large and well known \( \text{[15]} \). In this scenario one has

\[ \langle \pi^+\pi^-\pi^+\pi^-|H_T^\perp|B \rangle = \frac{2g_2^2}{s_T} \bar{P}_{\mu} (t_\omega + t_\omega^a) + \frac{g_2^2}{s_T} (t_\rho + t_\rho^a), \]
\[
\langle \pi^+ \pi^- | H | \bar{B} \rangle = \frac{2g^2}{s_{\rho \omega}^2} \bar{\Pi}_{\rho \omega}(p_\rho + p_\omega^a) - \frac{g^2}{s_{\rho}^2} (p_\rho + p_\omega^a),
\]

where \( t_V(V = \rho \text{ or } \omega) \) is the tree amplitude and \( p_V \) is the penguin amplitude for producing a vector meson, \( V \). \( t_V^\pi(V = \rho \text{ or } \omega) \) is the tree annihilation amplitude and \( p_V^\pi \) is the penguin annihilation amplitude. \( g_\rho \) is the coupling for \( \rho \to \pi^+ \pi^- \), \( \bar{\Pi}_{\rho \omega} \) is the effective \( \rho \to \omega \) mixing amplitude, and \( s_V \) is from the inverse propagator of the vector meson \( V \),

\[
s_V = s - m_V^2 + i m_V \Gamma_V,
\]

with \( s \) being the invariant mass of the \( \pi^+ \pi^- \) pair. The direct \( \omega \to \pi^+ \pi^- \) is effectively absorbed into \( \bar{\Pi}_{\rho \omega} \), leading to the explicit \( s \) dependence of \( \bar{\Pi}_{\rho \omega} \). Making the expansion \( \bar{\Pi}_{\rho \omega}(s) = \bar{\Pi}_{\rho \omega}(m_V^2) + (s - m_V^2) \bar{\Pi}_{\rho \omega}'(m_V^2) \), the \( \rho - \omega \) mixing parameters were determined in the fit of Gardner and O’Connell [32]; \( \text{Re}\bar{\Pi}_{\rho \omega}(m_V^2) = -3500 \pm 300 \) MeV\(^2\), \( \text{Im}\bar{\Pi}_{\rho \omega}(m_V^2) = -300 \pm 300 \) MeV\(^2\), and \( \bar{\Pi}_{\rho \omega}'(m_V^2) = 0.03 \pm 0.04 \). In practice, the effect of the derivative term is negligible. From Eqs. (10)–(13), one has

\[
\sum = \frac{2\bar{\Pi}_{\rho \omega}(p_\rho + p_\omega^a) + s_\omega(p_\rho + p_\omega^a)}{2\bar{\Pi}_{\rho \omega}(t_\omega + t_\omega^a) + s_\omega(t_\omega + t_\omega^a)},
\]

Defining

\[
t_\omega + t_\omega^a = \alpha \epsilon^i \delta_\alpha, \quad p_\rho + p_\omega^a = \beta \epsilon^i \delta_\beta, \quad p_\omega + p_\omega^a = r \epsilon^i \delta_\eta,
\]

where \( \delta_\alpha, \delta_\beta, \) and \( \delta_\eta \) are strong phases, one finds the following expression from Eq. (21):

\[
\sum = \frac{2\bar{\Pi}_{\rho \omega} + \beta \epsilon^i \delta_\beta s_\omega}{s_\omega + 2\bar{\Pi}_{\rho \omega} \alpha \epsilon^i \delta_\alpha}.
\]

3.2 The calculation details

In the QCD factorization approach, \( \alpha_i \) associated with the coefficients \( \alpha_i \) can be written as follows (helicity indices are neglected) [8,9]:

\[
\begin{align*}
\alpha_1 &= a_1, \\
\alpha_2 &= a_2, \\
\alpha_3^\rho &= a_3^\rho + a_3^\rho, \\
\alpha_3^\phi &= a_3^\rho + a_3^\rho, \\
\alpha_4^\rho &= a_4^\rho - r_X a_4^\rho, \\
\alpha_4^\phi &= a_4^\rho - r_X a_4^\rho,
\end{align*}
\]

where we have used the notation

\[
r_X = \frac{2m_V f_V}{m_B f_V},
\]

with \( f_V, f_V \) referring to the transverse decay constant and decay constant of the vector meson, respectively.

The flavor operators \( \rho^{i,k} \) include short-distance non-factorizable corrections such as vertex corrections and hard spectator interactions. \( V_2 \) is the emitted meson and \( V_1 \) shares the same spectator quark with the \( B \) meson.

The vertex corrections are given by [8,9]:

\[
V_0^0(V_2) = \begin{cases}
\int_0^1 dx \Phi_\parallel^\omega(x) \left[ 12 \ln \frac{m_B}{\mu} - 18 + g(x) \right], & \text{if } i = 1, 4, 9, 10 \\
\int_0^1 dx \Phi_\parallel^\omega(x) \left[ -12 \ln \frac{m_B}{\mu} + 6 - g(1 - x) \right], & \text{if } i = 5, 7 \\
\int_0^1 dx \Phi_\parallel^\omega(x) \left[ -6 + h(x) \right], & \text{if } i = 6, 8
\end{cases}
\]

\[
V_0^\pm(V_2) = \begin{cases}
\int_0^1 dx \Phi_\parallel^\pm(x) \left[ 12 \ln \frac{m_B}{\mu} - 18 + g_T(x) \right], & \text{if } i = 1, 4, 9, 10 \\
\int_0^1 dx \Phi_\parallel^\pm(x) \left[ -12 \ln \frac{m_B}{\mu} + 6 - g_T(1 - x) \right], & \text{if } i = 5, 7 \\
0, & \text{if } i = 6, 8
\end{cases}
\]

with

\[
g(x) = 3 \left( 1 - 2x \ln x - i\pi \right)
\]

[8,9]
for $i = 5, 7, \text{and}$

$$H^{-}_i(V_1 V_2) = - \frac{i f_{iB} f_{iV_1 f_{iV_2 m_{B} m_{V_1} m_{B}}}}{m_{B} X_{B}^{(0, V_1 V_2)} m_{V_2}} \lambda_{B} \int_{0}^{1} du dv \frac{\Phi^V_{-}(u) \Phi^V_{-}(v)}{uv},$$

(37)

for $i = 6, 8$. One can find that the expressions for $H^+_i(V_1 V_2)$ are independent of the choice for transverse polarization vectors.

The helicity dependent factorizable amplitudes defined by

$$X^{(BV_1, V_2)} = \langle V_2(p_2, c^2) | J_{\mu} | 0 \rangle \langle V_1(p_1, c^1) | J_{\mu} | B \rangle$$

(45)

have the expressions

$$X^{(BV_1, V_2)} = \frac{i f_{iV_1 f_{iV_2 m_{B} m_{V_1} m_{B}}}}{2m_{V_1}} \left[ (m_{B} - m_{V_1}^2 - m_{V_2}^2) (m_{B} + m_{V_1}) A_{1V_1}(q^2) - \frac{4m_{B} P_c^{H}}{m_{B} + m_{V_1}} A_{2V_1}(q^2) \right],$$

(46)

$$X^{_(BV_1, V_2)} = -i f_{iV_2 m_{B} m_{V_1}} \left[ \left( 1 + \frac{m_{V_1}}{m_{B}} \right) A_{1V_1}(q^2) \right]$$

(47)

where $A_{iV_1}(i = 1, 2)$ and $V_{B}^{V_1}$ are weak form factors.

**Penguin terms**

At order $\alpha_s$, corrections from penguin contractions are present only for $i = 4, 6.$ For $i = 4$ we have [39]

$$P_a^{h,p}(V_2) = \frac{C_{p} \alpha_{s}}{4 \pi N_c} \left[ c_1 G_{s}^{h}(s_p) + g_{V_2} \right]$$

$$+ c_3 [ G_{V_2}^{h}(s_1) + G_{V_2}^{h}(1 + 2 g_{V_2})$$

$$+ (c_4 + c_6) \sum_{i=1}^{b} \left[ G_{V_2}^{h}(s_i) + g_{V_2}^{h} \right]$$

$$- 2 \epsilon_{G_{s}} G_{s}^{h} \right],$$

(48)

where $s_i = m_i^2/m_b^2$ and the function $G_{V_2}^{h}(s)$ is given by

$$G_{V_2}^{h}(s) = 4 \int_{0}^{1} du \Phi^{V_2,h}(u) \int_{0}^{1} dx x \bar{x} \ln[s - \bar{x} \bar{x} - i \epsilon],$$

$$g_{V_2} = \left( \frac{4}{3} \ln \frac{m_b}{\mu} + \frac{2}{3} \right) \int_{0}^{1} \Phi^{V_2,h}(x) dx,$$

$$g_{V_2}' = \left( \frac{4}{3} \ln \frac{m_b}{\mu} - \frac{2}{3} \right) \int_{0}^{1} \Phi^{V_2,h}(x) dx,$$

(49)
with $\Phi^{i,0}_\pm = \Phi^{V_i}_\pm$, $\Phi^{i,0}_\pm = \Phi^{V_i}_\pm$. For $i = 6$, the result for the penguin contribution is

$$P^{R,P}_{6}(V_2) = \frac{C_F a_6}{4\pi N_c} \left\{ c_4 G_{V_2}(s_p) + c_1 \left[ G_{V_2}^\theta(s_s) + G_{V_2}^\theta(1) \right] + (c_4 + c_6) \sum_{i=7}^b G^b_{V_2}(s_i) \right\},$$

(50)

where the function $G_{V_2}(s)$ is defined as

$$G_{V_2}(s) = 4 \int_0^1 du \Phi_{V_2}(u) \int_0^1 dx \bar{x}x \ln[s - \bar{u}xx - i\epsilon],$$

$$G_{V_2}^\theta(s) = 0.$$  

(51)

The transverse penguin contributions vanish for $i = 6, 8$: $P^{R,P}_{6,8} = 0$. The $r^{V_2}_{V_2}$ term in Eq. (50) is factorized out so that when the vertex correction $V_{6,8}$ is neglected, $a_6^0$ contributes to the decay amplitude in the product $r^{V_2}_{V_2} P^{R}_{6,8}$ [30]. For $i = 8, 10,$

$$P^{R,P}_{8}(V_2) = \frac{\alpha_{em}}{9\pi N_c} (c_1 + N_c c_2) G_{V_2}(s_p),$$

(52)

$$P^{R,P}_{10}(V_2) = \frac{\alpha_{em}}{9\pi N_c} \left\{ (c_1 + N_c c_2) [G_{V_2}^h(s_p) + 2g_{V_2}] - 3c_6^h G_{V_2}^\theta \right\}.$$  

(53)

For $i = 7, 9,$

$$P^{R,P}_{7,9}(V_2) = \frac{\alpha_{em}}{3\pi} C_F \frac{m_{b,m_b}}{m_{V_2}} + \frac{2\alpha_{em}}{27\pi} (c_1 + N_c c_2) \left[ \delta_{pc} \ln \frac{m^2}{\mu^2} + \delta_{ps} \ln \frac{\nu^2}{\mu^2} + 1 \right].$$

(54)

The relevant integrals for the dipole operators $O_{g,\gamma}$ are [30]

$$G^\theta_g = \int_0^1 \Phi^{V_2}(u) \frac{du}{u},$$

$$G^\theta_{\theta g} = 0.$$  

(55)

The dipole operators $Q_{sg}$ and $Q_{\gamma g}$ do not contribute to the transverse penguin amplitudes at $O(\alpha_s)$ due to angular momentum conservation [35].

**Annihilation contributions**

The annihilation contributions to the decay $\bar{B} \to V_1 V_2$ can be described in terms of $b^{P,H}_i$ and $b^{P,H}_{i,EW}$

$$G_F \sqrt{2} \sum_{p=u,c} \lambda_p \langle V_1 V_2 | T_b^{P,H} | \bar{B} \rangle$$

$$= i G_F \sqrt{2} \sum_{p=u,c} \lambda_p f_B f_{V_1} f_{V_2} \sum_i (b^{P,H}_i + b^{P,H}_{i,EW}).$$

(56)

The building blocks have the expressions

$$b_1 = \frac{C_F}{N_c} c_1 A_1^i,$$

$$b_2 = \frac{C_F}{N_c} c_2 A_1^i,$$

$$b_3 = \frac{C_F}{N_c} \left[ c_3 A_1^i + c_5 (A_3^i + A_2^i) + N_c c_4 A_1^i \right],$$

$$b_4 = \frac{C_F}{N_c} \left[ c_4 A_1^i + c_6 A_2^i \right],$$

$$b_{3,EW} = \frac{C_F}{N_c} \left[ c_9 A_1^i + c_7 (A_3^i + A_4^i) + N_c c_8 A_1^i \right],$$

$$b_{4,EW} = \frac{C_F}{N_c} \left[ c_{10} A_1^i + c_8 A_2^i \right],$$

(57)

where we have omitted the superscripts $p$ and $h$ in above expressions for simplicity. The subscripts $1, 2, 3$ of $A_1^i, A_2^i, A_3^i, A_4^i$ denote the annihilation amplitudes induced from $(V \to A) (V \to A)$, $(V \to A)(V \to A)$ and $(S \to P)(S \to P)$ operators, respectively, and the superscripts $i$ and $f$ refer to gluon emission from the initial and final-state quarks, respectively. $V_1$ contains an antiquark from the weak vertex and $V_2$ contains a quark from the weak vertex [39]. The explicit expressions of weak annihilation amplitudes are:

$$A_{i,0}^1 (V_1 V_2) = \pi \alpha_s \int_0^1 \frac{du}{u} \frac{dv}{v} \left\{ \Phi_{\parallel V_2}(v) \Phi_{V_2}^{V_1}(v) \left[ \frac{1}{u(1-\bar{u}v)} + \frac{1}{uv^2} \right] - r_{\chi} V_1 V_2 \Phi_{V_2}(u) \Phi_{V_2}(v) \frac{2}{uv} \right\},$$

(58)

$$A_{i,-}^1 (V_1 V_2) = -\pi \alpha_s \frac{2m_{V_1} m_{V_2}}{m_{B}^2} \int_0^1 \frac{du}{u} \frac{dv}{v} \left\{ \Phi_{\parallel V_2}(u) \Phi_{V_2}^{V_1}(v) \left[ \frac{u + v}{u^2 v^2} + \frac{1}{(1-\bar{u}v)^2} \right] \right\},$$

(59)

$$A_{i,0}^1 (V_1 V_2) = \pi \alpha_s \frac{2m_{V_1} m_{V_2}}{m_{B}^2} \int_0^1 \frac{du}{u} \frac{dv}{v} \left\{ \Phi_{\parallel V_2}(u) \Phi_{V_2}^{V_1}(v) \left[ \frac{2}{u v^2} - \frac{v}{(1-\bar{u}v)^2} \right] - \frac{\bar{u} v}{v^2 (1-\bar{u}v)} \right\},$$

(60)

$$A_{i,0}^2 (V_1 V_2) = \pi \alpha_s \int_0^1 \frac{du}{u} \frac{dv}{v} \left\{ \Phi_{\parallel V_2}(v) \Phi_{V_2}^{V_1}(v) \left[ \frac{1}{v (1-\bar{u}v)} + \frac{1}{u^2 \bar{v}} \right] - r_{\chi} V_1 V_2 \Phi_{V_2}(u) \Phi_{V_2}(v) \frac{2}{uv^2} \right\},$$

(61)
\[ A_{2}^{V}(V_{1}V_{2}) = -\pi\alpha_{s} \frac{2m_{V_{1}}m_{V_{2}}}{m_{b}^{2}} \int_{0}^{1} du dv \left\{ \phi_{V_{1}}(u) \phi_{V_{2}}(v) \times \left[ \frac{u + v}{u^{2} + v^{2} + \frac{1}{(1-uv)^{2}}} \right] \right\}, \]

\[ A_{2}^{+}(V_{1}V_{2}) = -\pi\alpha_{s} \frac{2m_{V_{1}}m_{V_{2}}}{m_{b}^{2}} \int_{0}^{1} du dv \left\{ \phi_{V_{1}}(u) \phi_{V_{2}}(v) \times \left[ \frac{2}{u^{2}v - \frac{u}{(1-uv)^{2}}} \right. \right. \right.
\left. \left. \left. - \frac{u}{u^{2}(1-uv)} \right] \right\}. \]

\[ A_{3}^{V_{1}}(V_{1}V_{2}) = \pi\alpha_{s} \int_{0}^{1} du dv \left\{ \phi_{V_{1}}^{\perp}(u) \phi_{V_{2}}^{\perp}(v) \left[ \frac{2\bar{u}}{uv(1-uv)} + r_{V_{1}} \Phi_{m_{1}}(v) \Phi_{m_{2}}(v) \right. \right. \right.
\left. \left. \left. - \frac{2u}{uv(1-uv)} \right] \right\}, \]

The logarithmic divergences in annihilation can be extract into unknown variable \( X_{A} \)
\[ \int_{0}^{1} \frac{du}{u} \to X_{A}, \quad \int_{0}^{1} \frac{ln\ u}{u} \to -\frac{1}{2}X_{A}. \] (68)

### 3.3 The calculation of CP violation

In order to obtain the CP violation of \( B \to \rho^{0}(\omega)\rho^{0}(\omega) \to \pi^{+}\pi^{-}\pi^{-} \) in Eq.(16), we calculate the amplitudes \( t_{\omega}, t_{p}, t_{o}, p_{p}, p_{o}, p_{u} \) and \( p_{r}^{p} \) in Eq.(43)\( ^{19} \) in the QCD factorization, which are tree-level and penguin-level amplitudes. The decay amplitudes for the process \( B \to \rho^{0}\rho^{0}(\omega) \) are in the QCD factorization as follows:

\[ A_{B\to \rho^{0}\rho^{0}} = A_{B\to \rho^{0}\rho^{0}}(\alpha_{p}^{0} - \delta_{p}\alpha_{2} - \frac{1}{2}\alpha_{4,EW} - \frac{3}{2}\alpha_{3,EW}) \]

\[ + \beta_{A}^{0} - \frac{1}{2}\beta_{3,EW} + \delta_{p}\beta_{1} + 2\beta_{4} + \frac{1}{2}\beta_{4,EW}, \] (69)

\[-2A_{B\to \rho^{0}\omega} = A_{B\to \rho^{0}\omega}(\delta_{p}\alpha_{2} - \delta_{p}\beta_{1} + 2\alpha_{4}^{0} + \alpha_{4}^{0}) \]
\[ + \frac{1}{2}(1 + \beta_{3,EW} - \frac{1}{2}\alpha_{4,EW} + \beta_{4}^{0} - \frac{1}{2}\beta_{3,EW}) \]
\[ - \frac{3}{2}\beta_{3,EW} - \frac{1}{2}\beta_{3,EW} + \beta_{4}^{0} - \frac{1}{2}\beta_{3,EW} \]
\[ - \frac{3}{2}\beta_{3,EW}, \] (70)

where

\[ A_{V_{1}V_{2}} = \frac{iG_{F}}{\sqrt{2}} (V_{1}|\bar{q}b)_{V-A} |B\rangle (V_{2}|\bar{q}q)|0\rangle \] (71)

From Eq.(42), one can get
\[ ae^{i\sigma_{0}} = \frac{t_{\omega} + t_{p}^{a}}{t_{p} + t_{p}^{a}} = Q_{2} \frac{Q_{1}}{Q_{1}}, \] (72)

where

\[ Q_{1} = t_{p} + t_{p}^{a} \]
\[ = A_{p^{0}}^{\rho,\rho} - \alpha_{a}^{c,h} - \delta_{p}\alpha_{2} \]
\[ - \frac{1}{2}(\alpha_{a}^{u,h} - \alpha_{a}^{c,h} + \delta_{p}\beta_{1}) \]
\[ = -\frac{1}{2}A_{p^{0}}^{\rho,\omega}[\delta_{p}\alpha_{2} - \delta_{p}\beta_{1} + \alpha_{a}^{u,h}] \]
\[ - \frac{1}{2}A_{p^{0}}^{\rho,\omega}[-\delta_{p}\alpha_{2} - \delta_{p}\beta_{1} + \alpha_{a}^{u,h}] \]
\[ - \frac{1}{2}(\alpha_{a}^{u,h} - \alpha_{a}^{c,h} + \delta_{p}\beta_{1}) \]
\[ = -\frac{1}{2}A_{p^{0}}^{\rho,\nu}[\delta_{p}\alpha_{2} - \delta_{p}\beta_{1} + \alpha_{a}^{u,h}] \]
\[ - \frac{1}{2}(\alpha_{a}^{u,h} - \alpha_{a}^{c,h} + \delta_{p}\beta_{1}) \]. (73)

\[ Q_{2} = t_{\omega} + t_{p}^{a} \]
\[ = -\frac{1}{2}A_{p^{0}}^{\rho,\omega}[\delta_{p}\alpha_{2} - \delta_{p}\beta_{1} + \alpha_{a}^{u,h}] \]
\[ - \frac{1}{2}(\alpha_{a}^{u,h} - \alpha_{a}^{c,h} + \delta_{p}\beta_{1}) \]
\[ = -\frac{1}{2}A_{p^{0}}^{\rho,\omega}[\delta_{p}\alpha_{2} - \delta_{p}\beta_{1} + \alpha_{a}^{u,h}] \]
\[ - \frac{1}{2}(\alpha_{a}^{u,h} - \alpha_{a}^{c,h} + \delta_{p}\beta_{1}) \]. (74)
In a similar way, with the aid of the Fierz identities, we can evaluate the penguin operator contributions \( p_\rho \) and \( p_\omega \). From Eq. (23) we have

\[
\beta e^{i\delta_3} = \frac{p_\rho + p_\omega}{p_\rho + p_\omega} = Q_4
\]

where

\[
Q_4 = p_\rho + p_\omega^a
\]

\[
= A_\rho p_\omega^a \bigg[ 3 \alpha_4^{c,h} + \frac{3}{2} \alpha_3^{c,h} \bigg] + \beta_4^a - \frac{1}{2} \beta_3^{c,h} - \frac{1}{2} \beta_4^{c,h} \bigg] + \rho_4^a
\]

\[
Q_4 = p_\omega + p_\omega^a
\]

\[
= -\frac{1}{2} A_\omega p_\omega^a \bigg[ 2 \alpha_4^{a,h} + \frac{1}{2} \beta_4^{a,h} \bigg] - \frac{1}{2} \beta_3^{a,h} + \frac{1}{2} \beta_4^{a,h} \bigg] + \rho_4^a
\]

Form Eq. (24) we have

\[
\rho' e^{i(\delta_0 + \phi)} = \rho_\omega + p_\omega = Q_4
\]

\[
\rho' e^{i\delta_N} = Q_4
\]

where

\[
\frac{\rho_0 V_{td}^*}{\rho_{ub} V_{ud}^*} = \left( 1 - \frac{\rho^2}{4} \right)^2 + \eta^2 = \left( 1 - \frac{\rho^2}{4} \right)^2 + \eta^2
\]

**4 Input parameters**

In the numerical calculations, we should input distribution amplitudes and the CKM matrix elements in the Wolfenstein parametrization. For the CKM matrix elements, which are determined from experiments, we use the results in Ref. [36]:

\[
\rho = 0.132^{+0.022}_{-0.014}, \quad \eta = 0.341 \pm 0.013,
\]

\[
\lambda = 0.2253 \pm 0.0007, \quad A = 0.808^{+0.022}_{-0.015},
\]

where

\[
\rho = \rho(1 - \frac{\lambda^2}{2}), \quad \eta = \eta(1 - \frac{\lambda^2}{2}).
\]

The general expressions of the helicity-dependent amplitudes can be simplified by considering the asymptotic distribution amplitudes for \( \Phi_V, \Phi VH \):

\[
\Phi_V^u(u) = 6u \bar{u}, \quad \Phi_V^u(u) = 3(2u - 1),
\]

\[
\Phi_V^V(u) = 6u \bar{u}, \quad \Phi_V^V(u) = \int_u^1 dv \frac{\Phi_V^V(v)}{v},
\]

\[
\Phi_M = \int_0^u dv \frac{\Phi_V^V(v)}{v}.
\]

**5 Numerical results**

In the numerical results, we find that for the decay channel we are considering the \( CP \) violation can be enhanced
via $\rho - \omega$ mixing when the invariant mass of $\pi^+\pi^-$ is in the vicinity of the $\omega$ resonance. The uncertainties of the CKM matrix elements mainly come from $\rho$ and $\eta$. In our numerical results, we let $\rho$ and $\eta$ vary between the limiting values. We find the results are not sensitive to the values of $\rho$ and $\eta$. Hence, the numerical results are shown in Fig.1, Fig.2 and Fig.3 with the central parameter values of CKM matrix elements. From the numerical results, it is found that there is a maximum $CP$ violating parameter value, $\rho_{\max}$, when the masses of the $\pi^+\pi^-$ pairs are in the vicinity of the $\omega$ resonance. In Fig.1, one can find that the maximum $CP$ violating parameter reaches 28% in the case of $(\eta_{central}, \eta_{central})$.

From the Eq. (15) one can find that the $CP$ violating parameter is related to $\sin \delta$ and $r$. In Fig.2, we show the plot of $\sin \delta_0$ ($\sin \delta_+$ and $\sin \delta_-$) as a function of $\sqrt{s}$. We can see that the $\rho - \omega$ mixing mechanism produces a large $\sin \delta_0$ ($\sin \delta_+$ and $\sin \delta_-$) at the $\omega$ resonance. As can be seen from Fig.2, the plots vary sharply in the cases of $\sin \delta_0$ and $\sin \delta_-$. Meanwhile, $\sin \delta_+$ changes weakly comparing with the $\sin \delta_0$ and $\sin \delta_-$. It can be seen from the Fig.3 that $r_0$ and $r_-^+$ change more rapidly than $r_+$ when the masses of the $\pi^+\pi^-$ pairs are in the vicinity of the $\omega$ resonance.

In the paper [23], we studied the enhanced $CP$ violation for the decay channel $B^0 \to \pi^+\pi^-\pi^+\pi^-$ in the naive factorization. Since non-factorizable contribution can not be calculated in the naive factorization, $N_c$ was treated as an effective parameter. We found that the $CP$ violating asymmetry was large and ranges from $-82\%$ to $-98\%$ via the $\rho - \omega$ mixing mechanism strongly depending on the value $N_c$ when the invariant mass of the $\pi^+\pi^-$ pair is in the vicinity of the $\omega$ resonance. However, the maximum $CP$ violation only can reach 28% via double $\rho - \omega$ mixing in the QCD factorization. The naive factorization scheme has been shown to be the leading order result in the framework of QCD factorization when the radiative QCD corrections $O(\alpha_s(m_b))$ and the order $O(1/m_b)$ effects are neglected. The QCD factorization can evaluate systematically corrections to the results from the naive factorization. The distinction between the naive factorization and the QCDF mainly come from the strong phases of the QCD corrections. In the calculating process, we find that the annihilation contributions in QCDF which introduce the unknown parameters are small. Hence, the uncertainties of the results from the QCDF become small.

6 Summary and conclusions

In this paper, we studied the $CP$ violation for the decay process $B^0 \to \rho^0(\omega)\rho^0(\omega)\to \pi^+\pi^-\pi^+\pi^-$ due to the interference of $\rho - \omega$ mixing in the QCDF approach. This process induces two $\rho - \omega$ interference. It was found the $CP$ violation can be enhanced at the region of $\rho - \omega$ resonance. As a result, the maximum $CP$ violation could reach 28%, $\rho - \omega$ mixing is small due to the isospin violation. However, the mixing can produce a large strong phase, $\delta$, in Eq. (21). This is because when the invariant masses of the $\pi^+\pi^-$ pairs are in the vicinity of $\omega$, $\sin \delta \sim m_{\omega}/m_{\omega}$, and it becomes comparable with $H_{\omega\omega}$ in Eq. (21). In other words, $\rho - \omega$ mixing becomes important in the vicinity of $\omega$. This is the reason why we can see large $CP$ violation in the vicinity of $\omega$. Beyond the $\rho - \omega$ interference region, the noticeable values of $CP$ violation are caused by the strong phases provided by the Wilson coefficients.

The LHC experiments are designed with the center-of-mass energy 14 TeV and the luminosity $L = 10^{34}$ $m^{-2}$ $s^{-1}$. The heavy quark physics is one of the main topics of LHC experiments. Especially, LHCb detector is designed to make precise studies on $CP$ asymmetries and rare decays of b-hadron systems. Recently, the LHCb Collaboration found clear evidence for direct $CP$ violation in some three-body decay channels in charmless decays of $B$ meson. Large $CP$ violation is observed in $B^+ \to K^+K^-\pi^+$, $B^+ \to \pi^+\pi^-\pi^-$ in the region $m_{\pi^+\pi^-\pi^0} < 0.4$ GeV$^2$ and $m_{\pi^+\pi^-\eta_{low}} > 15$ GeV$^2$ [3]. LHCb experiment may collect data in the region of the invariant masses of $\pi^+\pi^-$ associated the $\omega$ resonance for detecting our prediction of $CP$ violation.

In our calculations there are some uncertainties. The QCD factorization scheme provides a framework in which we can evaluate systematically corrections to the results obtained in the naive factorization scheme. However, when we take into account the nonfactorizable and chirally enhanced hard-scattering spectator and annihilation contributions which appear at order $O(\alpha_s(m_b))$ and $O(1/m_b)$, respectively, the involvement of the twist-3 hadronic distribution amplitudes leads to logarithmical divergence coming from the endpoint integrals. This brings large uncertainties in the predictions of the $CP$ violating asymmetries in the QCD factorization scheme. Furthermore, in addition to the model dependence appearing in the factorized hadronic matrix elements just as in the naive factorization scheme, we cannot avoid the model dependence and process dependence of the hard-scattering spectator and annihilation contributions due to their dependence on the
hadronic distribution amplitudes and dependence on different processes. Such dependence will also appear if one tries to include other $1/m_b$ corrections and even higher order corrections. This leads to uncertain of our results.

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