Nonstrange and other flavor partners of the exotic $\Theta^+$ baryon

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Given presently known empirical information about the exotic $\Theta^+$ baryon, we analyze possible properties of its $SU(3)_F$-partners, paying special attention to the nonstrange member of the antidecuplet $N^*$. The modified PWA analysis presents two candidate masses, 1680 MeV and 1730 MeV. In both cases the $N^*$ should be highly inelastic. The theoretical analysis, based on the soliton picture and assumption of $\Gamma_{\Theta^+} < 5$ MeV, shows that most probably $\Gamma_{N^*} < 30$ MeV. Similar analysis for $\Xi_{3/2}$ predicts its width to be not more than about 10 MeV. Our results suggest several directions for experimental studies that may clarify properties of the antidecuplet baryons, and structure of their mixing with other baryons.

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I. INTRODUCTION

The observation of narrow peaks in the invariant mass spectra of $nK^+$[1, 2, 3, 4, 5] and $pK_S$[6, 7, 8, 9, 10] events, verified by independent groups and laboratories worldwide, has solidified the evidence for an exotic baryon $\Theta^+$ with strangeness +1, a mass of about 1540 MeV, and a narrow width. The existence of such a particle implies a whole new family of $SU(3)_F$-partners, beyond the familiar octets and decuplets. This state was predicted (both the mass and the narrow width) in [11] on the basis of a chiral soliton approach to hadron dynamics. In this approach it should be a member of a flavor antidecuplet with $J^P = 1/2^+$. The antidecuplet had emerged even earlier in various versions of the soliton approach to 3-flavor QCD (for a brief review of the history and earlier references see, e.g., [11, 12, 13]). However, the expected masses of its members had always been rather uncertain, at least up to $\sim 100$ MeV. Until Ref. [11], the question of width, also essential for experimental searches, had not been addressed at all. To make the $\Theta^+$-mass prediction more definite, it was suggested in [11] to identify the non-strange member of the antidecuplet with the $N(1710)$, the only nucleon state listed in the PDG-tables[17] having $J^P = 1/2^+$ and being in the expected mass interval. Clearly, this assignment resulted in a good agreement with later experimental findings for the $\Theta^+$ mass. Given the present more detailed knowledge of $\Theta^+$-properties, and with higher-statistics experiments under preparation, it is important to reconsider the antidecuplet nature of the $N(1710)$.

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Let us first summarize our present knowledge of the $\Theta^+$-width. The theoretical prediction, simultaneous with its mass, was $\Gamma_{\Theta^+} < 15$ MeV \[11\], unexpectedly narrow for strong decays. Existing measurements, instead of determining $\Gamma_{\Theta^+}$, have only shown it to be smaller than experimental resolution (see Table I). Most experimental publications have given an upper bound of about $\sim 20$ MeV. Xenon bubble chamber data, corresponding in essence to the charge exchange reaction $K^+n \rightarrow K^0p$, have provided the slightly lower bound of 9 MeV \[7\].

Less direct determinations \[19\], using previously measured $K^+d$ total cross sections, have led to a stronger limitation $\Gamma_{\Theta^+} < 6$ MeV. A similar bound, $< 5$ MeV, was obtained in Ref. \[20\] within a more elaborated theoretical description. The partial-wave analysis (PWA) of available $KN$ (elastic and charge exchange) scattering data similarly claims to exclude widths above $1–2$ MeV \[22\].

A more detailed reexamination of the approach in Ref. \[19\] provides nearly the same result of $\Gamma_{\Theta^+} < 1.5$ MeV \[23\]. A similar method applied to the Xenon data \[4\] has allowed even the tentative claim of a lower limit $\Gamma_{\Theta^+} = 0.9 \pm 0.3$ MeV \[22\] (with additional assumptions and an unknown systematic uncertainty). One should emphasize, however, that all of these indirect treatments assume the existence of a $\Theta^+$, which they can not confirm. Moreover, they are based mainly on rather old data, which may be shifted by the next generation of higher precision measurements. Nevertheless, we should take these results into account when discussing the $\Theta^+$ as given by the present data.

Evidently, all of the above estimates for $\Gamma_{\Theta^+}$ are in sharp contrast with the width $\sim 100$ MeV ascribed \[13\] to the $N(1710)$, initially considered to be a unitary partner of the $\Theta^+$ \[11\]. Of course, members of the same unitary multiplet can have different widths, but in the absence of a special reason (say, mixing with members of another multiplet) it would be more natural for them to have comparable widths.

Additional information related to the assignment of unitary partners is due to a recent experimental result \[24\] giving evidence for one further explicitly exotic particle $\Xi^{-\ast}_{3/2}$, with a mass $1862 \pm 2$ MeV and width $\sim 18$ MeV (i.e. less than resolution). Such a particle had been expected to exist as a member of the same antidecuplet containing the $\Theta^+$, but its mass was predicted to be about $2070$ MeV \[11\], essentially different from the experimental value. This has posed similar problems for the masses of other unitary partners of the $\Theta^+$, nucleon-like and $\Sigma$-like. The supposed antidecuplet looks today as shown on Fig. I, with $\Sigma$- and $N$-masses determined by the Gell-Mann–Okubo rule.

The state $N(1710)$, though listed in the PDG Baryon Summary Table \[15\] as a 3-star resonance, is not seen in a recent analysis of pion-nucleon elastic scattering data \[25\]. Studies which have claimed to see this state have given widely varying estimates of its mass and width (from $\sim 1680$ MeV to $\sim 1740$ MeV for the mass and from $\sim 90$ MeV to $\sim 500$ MeV for the width). Branching ratios have also been given with large uncertainties ($10–20\%$ for $N\pi\pi$, $40–90\%$ for $N\pi\pi$, and so on), apart from one which has been presented with greater precision ($6 \pm 1\%$ for $N\eta$).

Of course the non-observation of a broad $N(1710)$ state in pion-nucleon elastic analyses could be due to a very small $\pi N$ branching ratio. Standard procedures used in partial-wave analysis (PWA) may also miss narrow resonances with $\Gamma < 30$ MeV (a similar situation below inelastic thresholds has been discussed in \[26\]). Therefore, the true unitary partner of the $\Theta^+$ (if it is different from $N(1710)$ and sufficiently narrow) could have eluded detection.

Here we reconsider the identity of $N^\ast$, the nucleon-like partner of $\Theta^+$, and investigate the possible existence and properties of narrow non-strange state(s) near $1700$ MeV. We first consider modifications of a PWA with narrow resonances, and apply the results to $\pi N$ elastic scattering at $W \sim 1700$ MeV (Section III). Section IV presents a discussion of possible properties of the $N^\ast$ in the soliton picture with baryon mixing and for a small $\Theta^+$ width. Some expected properties of the $\Xi^{-\ast}_{3/2}$ are also considered. Our results are briefly discussed and conclusions formulated in Section V.

II. NARROW RESONANCES IN PARTIAL-WAVE ANALYSES

We have emphasized earlier \[26\] that standard methods of PWA are insensitive to very narrow resonances. Therefore, a modified approach is required to search for the presence of a narrow resonance with particular values of mass and width \[26\] (see also \[21\]). We consider the situation in more detail, separately for elastic and inelastic cases.
A. Elastic case

Interaction in the elastic case may transform a state \( a \) only to a similar state \( a' \) (changing, for example, particle momenta without changing particle identity). One can then choose physical states, so as to diagonalize the \( S \) matrix, (e.g., for the \( \pi N \) scattering, take states with definite values of energy, isospin, parity, and angular momentum), and have only diagonal transitions \( a \to a \) with \( S \) matrix elements

\[
\langle a | S | a \rangle = e^{2i\delta_a}.
\]

(1)

Standard methods employ some parametrization of the interaction phase \( \delta_a \), fitting these parameters to describe experimental data. Instead, we will split the phase as

\[
\delta_a = \delta_a^B + \delta_R.
\]

(2)

The background part \( \exp(2i\delta_a^B) \) may be parametrized as before, while the resonance part has the canonical Breit-Wigner form

\[
e^{2i\delta_R} = \frac{M_R - W + i\Gamma_R/2}{M_R - W - i\Gamma_R/2}.
\]

(3)

If refitting (over the whole database) with some fixed values of \( M_R \) and \( \Gamma_R \) provides a worse description (higher \( \chi^2 \)) than without the resonance, then a resonance \( R \) with the corresponding mass and width is unsupported. If the new description is better (has lower \( \chi^2 \)), then the resonance may exist.

At first sight, we have increased the number of parameters and, therefore, should always have a better description. This is not necessarily so, due to the specific form used to introduce these two additional parameters, and the fixed values assigned to them. Moreover, we have demonstrated in Ref. 26 that a better description (lower \( \chi^2 \)) may result for various reasons not associated with the presence of a resonance. Nevertheless, searching for a better description allows us to restrict the region in \( (M_R, \Gamma_R) \) space, where a resonance may be assumed. This is the approach that was used earlier as a basis for numerical procedures which restricted admissible widths of light \( \pi N \) resonances 26 and of the \( \Theta^+ \) 21.

B. Inelastic case

For energies near \( W \approx 1700 \text{ MeV} \), we may be sensitive to thresholds for the production of additional (or different) mesons, and one should take the inelasticity into account even when investigating purely elastic \( \pi N \) scattering.

Because of inelastic transitions, nondiagonal \( S \) matrix elements, generally, do not vanish. However, the \( S \) matrix is a unitary operator and, hence, can be diagonalized by a unitary transformation:

\[
S = U S^{(0)} U^{-1},
\]

(4)

with \( S^{(0)} \) having a diagonalized form and

\[
\sum_{n \geq 0} |U_{an}|^2 = \sum_a |U_{an}|^2 = 1,
\]

(5)

due to unitarity. One can present an elastic amplitude (for example, a partial-wave amplitude) as

\[
\langle a | S | a \rangle = \sum_{n \geq 0} |U_{an}|^2 e^{2i\delta_n},
\]

(6)
where the unitarity of $U$ has been used. The form (6) implies the well-known relation

$$|\langle a|S|a \rangle| \leq \sum_{n \geq 0} |U_{an}|^2 = 1. \quad (7)$$

If the resonance candidate $R$ is narrow enough to avoid overlap with other resonances, the explicit resonance behavior can be inserted in only one of $\delta_n$, say, to $\delta_0$:

$$\delta_0 = \delta_0^0 + \delta_R \quad (8)$$

(compare to Eq. (2)). Then,

$$\langle a|S|a \rangle = |U_{a0}|^2 e^{2i\delta_0} e^{2i\delta_R} + \sum_{n \geq 1} |U_{an}|^2 e^{2i\delta_n}. \quad (9)$$

All quantities on the right hand side of Eq. (9) depend on energy, but only $\delta_R$ has a sharp energy dependence near the resonance.

It is easy to see that

$$|U_{a0}|^2|_{W=M_R} = r_a \quad (10)$$

is the branching ratio for a particular decay mode $R \to a$. Then, Eq. (5) provides the expected relation

$$\sum_a r_a = 1. \quad (11)$$

Now, we can rewrite Eq. (9) as

$$\langle a|S|a \rangle = r_a A(W) e^{2i\delta_R} + (1-r_a)B(W), \quad (12)$$

where

$$r_a |A(W)| + (1-r_a)|B(W)| \leq 1, \quad |A(M_R)| = 1. \quad (13)$$

The expressions (12) and (13) can be used to construct a parametrization and numerical procedure, similar to one described in Ref. [25], to test for the existence of a possible resonance $R$.

Note that, in the inelastic case, description of the resonance contribution to the elastic amplitude $a \to a$ contains three parameters ($M_R, \Gamma_R, r_a$), instead of two in the elastic case. However, far from the resonance, at $|W-M_R| \gg \Gamma_R$, the contribution takes the form $\propto \Gamma_a/(M_R - W)$, sensitive only to the partial decay width

$$\Gamma_a = r_a \Gamma_R. \quad (14)$$

This is trivially true for a purely elastic resonance, for which partial and total widths coincide.

C. Fitting the data

Nucleon-like states may be revealed in various processes, with various initial and final states. But most convincing are their manifestations in $\pi N$ elastic scattering. As a result, we consider here only elastic (and charge exchange) data.

We begin by considering the $\pi N$ partial wave $P_{11}$, as this amplitude is associated with resonances having $J^P = 1/2^+$. The character of $\chi^2$-changes, $\Delta \chi^2$, after inserting a narrow resonance with a range of masses, widths, and branching fractions is illustrated in Fig. 2. The resonance mass has been allowed to vary from 1620 to 1760 MeV in 10 MeV
steps. For the total width, we have used five values in the intervals 0.1–0.9 MeV (step 0.2 MeV) and 1–9 MeV (step 2 MeV). For easier tracing, we have connected points having consecutive values of the mass and identical values for the other parameters.

Negative values of $\Delta \chi^2$ emerge most readily near $M_R = 1680$ MeV and 1730 MeV. We see that $\Delta \chi^2$ becomes negative only for $\Gamma_{el} = (\Gamma_{el}/\Gamma_{tot}) \cdot \Gamma_{tot}$ within the bounds

$$\Gamma_{el} \leq 0.5 \ [0.3] \text{ MeV}$$

for $M_R = 1680 \ [1730]$ MeV. The available data can not reliably discriminate values of $\Gamma_{el}$ below these bounds. Neither can they discriminate the particular values of $\Gamma_{tot}$. Note that, for higher values of $\Gamma_{tot}$, such states could presumably be seen in a standard PWA; however with the above restrictions for $\Gamma_{el}$ these resonances would be extremely inelastic and have little effect on the elastic scattering process. Thus, for $J^P = 1/2^+$, we see two possible mass values for the nucleon-type resonance state(s), both having rather small elastic (i.e. $\pi N$) partial widths.

It was demonstrated, however, in Ref. [26] that $\Delta \chi^2 < 0$ does not necessarily mean the real existence of a resonance. The “resonance” may be only an effective mechanism to introduce corrections, e.g., in the presence of unaccounted (or badly accounted) for singularities (say, thresholds), or insufficient and/or poor quality of data.

A true resonance should provide an effect only when being inserted into a particular partial amplitude, while non-resonant sources may show sensitivity in various partial-wave amplitudes. To check this possibility, we have repeated the insertion-refitting procedure for the partial-wave amplitudes $S_{11}$ and $P_{13}$, having the $J^P$ quantum numbers $1/2^-$ and $3/2^+$, respectively. Using the same values as before for the mass, width, and branching ratio of the assumed resonance, produces the results for $S_{11}$ and $P_{13}$ illustrated in Fig. 3. No effect emerges at $M_R = 1680$ MeV, enhancing the expectation of a true $P_{11}$ resonance effect at this mass.

In the 1700–1740 MeV region, variations for $S_{11}$ and $P_{13}$ show shallow dips, somewhat similar to one for $P_{11}$. This may cast doubt on the existence of a true narrow resonance in this interval. However, the dips for $S_{11}$ and $P_{13}$ are qualitatively different than for $P_{11}$; as a result, we do not consider a narrow resonance with $M_R = 1730$ MeV to be excluded.

The above dips in $\Delta \chi^2$ could be induced by the nearby thresholds, $N\omega$ with $W_{th} \approx 1720$ MeV, and $N\rho$ with $W_{th} \approx 1710$ MeV (note, however, its larger distance from the physical region due to the width of the $\rho$). It is interesting that $M_R = 1680$ MeV appears also near the $K\Sigma$ threshold with $W_{th} \approx 1685$ MeV. There are many-particle thresholds as well, but their contributions are expected to be less important than two-particle ones, due to much smaller phase-space near the thresholds, and we do not consider them here. None of these thresholds have been accounted for in the PWA parametrizations of $\pi N$ data.

Concluding this section, we emphasize that our results suggest two possible masses for a narrow nucleon-like resonance(s) having $J^P = 1/2^+$ and a mass near 1700 MeV. One of these, near 1680 MeV, looks more promising. Though our approach can give candidate values of mass and width for narrow resonance(s), it does not prove the existence of a resonance. Therefore, all our candidates need further direct and detailed experimental checks.

### III. THEORETICAL ANALYSIS

Let us discuss the above results as compared to expected properties of the antidecuplet members in the soliton picture.

The antidecuplet mass differences (say, between $\Theta^+$ and $N^*$, its non-strange partner), based on this picture and presented in Ref. [11], is about 180 MeV. Using the measured value $M_{\Theta} = 1540$ MeV, we should obtain $M_{N^*} = 1720$ MeV, which is close to the heavier candidate mass, 1730 MeV, of the preceding Section.

However, the soliton calculation of this mass difference requires some assumptions. In particular, it depends on the value of the $\sigma$-term, which is the subject of controversy. Its value, taken according to the latest data analysis [27], leads to an antidecuplet mass difference of about 110 MeV [28].

Moreover, today one is able to use another, more phenomenological approach. If the states $\Xi_{3/2}$ [24] and $\Theta$ are indeed members of the same antidecuplet, then, according to the Gell-Mann-Okubo rule, the mass difference of any
two neighboring isospin multiplets in the antidecuplet should be constant and equal

\[(M_{3/2} - M_{3})/3 \approx 107 \text{ MeV}.\]

This gives \(M_{N} \approx 1650 \text{ MeV}\), near but lower than our lighter candidate mass, 1680 MeV. Due to \(SU(3)_{F}\)-violating mixing with lower-lying nucleon-like octet states, \(M_{N}\) may shift upward, and reach about 1680 MeV \(\cite{28}\). Mixing with higher-lying nucleon-like members of exotic 27- and 35-plets may also play a role (see Refs. \(\cite{12, 13, 14}\)). Note that the effects of multiplet mixing on masses is parametrically of order \(O(m_{s}^{2})\) which is beyond approximations used in Ref. \(\cite{11}\).

In discussing the expected decays for the antidecuplet candidates, we mainly follow Ref. \(\cite{11}\). Though expressions for the widths will be taken in a somewhat more general form \(\cite{29}\), mixing of states will be taken in the same simplified form as in Ref. \(\cite{11}\). We further assume that only one nucleon-like resonance exists near 1700 MeV.

A. Decays of \(\Theta^{+}\)

We can write a partial width for the decay \(B_{1} \rightarrow M + B_{2}\) as \(\cite{30}\)

\[\Gamma (B_{1} \rightarrow MB_{2}) = g_{B_{1}B_{2}M}^{2} \frac{|\vec{p}|^{3}}{2\pi(M_{1} + M_{2})^{2}} \frac{M_{2}}{M_{1}},\]  

(16)

where \(|\vec{p}|\) is the c.m. momentum of the final meson. In terms of the baryon masses \(M_{1}, M_{2}\), and the meson mass \(m\), we have

\[|\vec{p}| = \sqrt{[M_{1}^{2} - (M_{2} + m)^{2}] \cdot [M_{1}^{2} - (M_{2} - m)^{2}]/2M_{1}}.\]  

(17)

Then, in the framework of the chiral soliton approach, for the total width of \(\Theta^{+}\) (summed over two decay modes), we obtain \(\cite{31}\)

\[\Gamma (\Theta^{+} \rightarrow KN) = \frac{3}{5} \cdot \left(\cos \phi \cdot G_{10}^{0} + \sin \phi \cdot H_{10}^{0} \sqrt{\frac{5}{4}}\right)^{2} \cdot \frac{|\vec{p}_{KN}|^{3}}{2\pi(M_{\Theta} + M_{N})^{2}} \frac{M_{N}}{M_{\Theta}}.\]  

(18)

Here, we admit the possibility of \(SU(3)_{F}\)-symmetry breaking, which allows mixing of the baryon antidecuplet and octet states. For simplicity, we mix here only a pair of soliton rotational states, the ground state \(N\) and its rotational excitation \(N^{*}\), which is a member of the same antidecuplet as \(\Theta^{+}\). As a result, instead of one coupling parameter \(G_{10}^{0}\), we obtain the set of three parameters \((H_{10}^{0}, \sin \phi, G_{10}^{0})\). The chiral soliton approach allows these to be determined, but with different levels of reliability.

The constants \(H_{10}^{0}\) and \(G_{10}^{0}\) can be expressed in terms of universal constants \(G_{0}, G_{1},\) and \(G_{2}\). The corresponding expressions are (see \(\cite{14, 52}\) for \(H_{10}^{0}\) and \(\cite{11}\) for \(G_{10}^{0}\)):

\[H_{10}^{0} = G_{0} - \frac{5}{2} G_{1} + \frac{1}{2} G_{2},\]  

(19)

\[G_{10}^{0} = G_{0} - G_{1} - \frac{1}{2} G_{2}.\]  

(20)

Analysis of Ref. \(\cite{11}\) showed that the constant \(G_{2}\) is very small, and in what follows we neglect it. The octet coupling \(G_{8} = G_{0} + G_{1}/2\) can be reliably extracted from properties of octet and decuplet baryons; in accordance with Ref. \(\cite{11}\), we take it here as \(G_{8} \approx 18\). In this way, we can relate constants \(H_{10}^{0}\) and \(G_{10}^{0}\) by

\[2G_{10}^{0} - H_{10}^{0} \approx G_{8} \approx 18.\]
Keeping this in mind, we give below only values of $G_{10}$. Note that at small $G_2$ and $G_1$, $G_{10} \approx H_{10} \approx G_8$, while at small $G_2$ and $G_{10}$, $H_{10} \approx -G_8$.

The mixing angle $\phi$ is less reliable. Basing on its estimates in Ref. [11], we use $\sin \phi \approx 0.085$ (i.e., $\phi \approx 5^\circ$).

The coupling $G_{10}$ is the least known quantity. In the soliton picture, it receives different contributions which tend to cancel each other, making a definite conclusion difficult. Nevertheless, $G_{10}$ was demonstrated to be suppressed, and the conservative upper bound of $G_{10} < 9.5$ has been given [11]; moreover, it was shown that in the non-relativistic limit for the quarks, the constant $G_{10}$ tends to zero (see also Ref. [33], where it was shown that this cancellation happens for any number of colors). All these results suggest the coupling constant to be considerably smaller than its upper limit $34$.

With expression (18) and $M_\Theta = 1540$ MeV, the restrictions $\Gamma(\Theta^+ \to KN) \leq 1$ [3; 5] MeV lead to $-1.4 [-2.9; -4.0] \leq G_{10} \leq 2.9 [4.5; 5.6]$. We do not insist on the limit $\Gamma_{\Theta^+} \sim 1$ MeV as given by Refs. [21, 22, 23], as it is based mainly on older experiments. But we consider values higher than 5 MeV to be improbable.

B. Decays of $N^*$

A large numerical value of $G_8$ (and $H_{10}$), compared to $G_{10}$, makes even a small octet–antidecuplet mixing a very important effect in decays of the $\Theta^+$. It is even more important for decays of the $N^*$, the non-strange $P_{11}$ member of the antidecuplet (assumed in Ref. [11] to be identified with $N(1710)$).

First of all, the octet–antidecuplet mixing allows the decay $N^* \to \pi \Delta$, otherwise forbidden for the pure antidecuplet member. In addition, mixing essentially influences the partial decay width $N^* \to K \Lambda$. A description of mixing effects in $N^*$ decays is most simple just for these two modes, since for them only the initial state $N^*$ has octet partner(s) to mix with. (Both the $\Delta$ and $\Lambda$ could mix with the antidecuplet members only under isospin violation. In decays of the $\Theta^+$, only the final nucleon can mix, by assumption, just with $N^*$..) These decay modes allow us to draw interesting conclusions concerning the $N^*$.

The corresponding partial widths of the $N^*$ are [33]:

$$\Gamma(N^* \to \pi \Delta) = \frac{12}{5} \cdot (\sin \phi \cdot G_8)^2 \cdot \frac{|\vec{p}_{\pi \Delta}|^3}{2\pi (M_{N^*} + M_{\Delta})^2} \cdot \frac{M_{\Delta}^2}{M_{N^*}^2},$$

$$\Gamma(N^* \to K \Lambda) = \frac{3}{20} \cdot (\cos \phi \cdot G_{10} + \sin \phi \cdot G_8 \cdot \frac{4}{\sqrt{3}})^2 \cdot \frac{|\vec{p}_{K \Lambda}|^3}{2\pi (M_{N^*} + M_{\Lambda})^2} \cdot \frac{M_{\Lambda}}{M_{N^*}}.$$  \hspace{1cm} (21)

The decay widths here are summed over possible charge states of the final hadrons.

Note that $\Gamma_{N^*} \equiv \Gamma(N^* \to \pi \Delta)$ is independent of the very uncertain antidecuplet coupling $G_{10}$. Using the above-given values of other parameters, for $M_{N^*} = 1680 [1730]$ MeV, leads to

$$\Gamma_{N^*} \approx 2.8 [3.5] \text{ MeV} \hspace{1cm} (23)$$

and (for positive $G_{10}$) $\Gamma^K_{N^*} \geq 0.17 [0.36] \text{ MeV}$. Taking $G_{10} = 2.9$ (the highest value compatible with $\Gamma_\Theta = 1$ MeV) gives

$$\Gamma^K_{N^*} \approx 0.70 [1.56] \text{ MeV}, \hspace{1cm} (24)$$

again, for $M_{N^*} = 1680 [1730]$ MeV.

To investigate decays $N^* \to \pi N$, $\eta N$, and $K \Sigma$, one needs to account for mixing of both initial and final baryons. Taking for coupling constants the linear approximation in $\sin \phi$, we obtain

$$\Gamma(N^* \to \pi N) = \frac{3}{20} \cdot (G_{10} \cdot \frac{\sin \phi}{\sqrt{5}} \cdot \left[ 7G_8 - \frac{5}{4}\right]^2 \cdot \frac{|\vec{p}_{\pi N}|^3}{2\pi (M_{N^*} + M_N)^2} \cdot \frac{M_N}{M_{N^*}}) \hspace{1cm} (25)$$

and

$$\Gamma(N^* \to \eta N) = \frac{3}{20} \cdot (G_{10} \cdot \frac{\sin \phi}{\sqrt{5}} \cdot \left[ G_8 - \frac{5}{4}\right]^2 \cdot \frac{|\vec{p}_{\eta N}|^3}{2\pi (M_{N^*} + M_N)^2} \cdot \frac{M_N}{M_{N^*}}). \hspace{1cm} (26)$$
\[
\Gamma (N^* \to K\Sigma) = \frac{3}{20} \left( G_{10}^\pi + \sin \phi \sqrt{5} \left[ 2G_8 + \frac{5}{2} H_{10}^\pi \right] \right)^2 \cdot \frac{|\vec{p}_{K\Sigma}|^3}{2\pi (M_{N^*} + M_{\Sigma})^2} \frac{M_{\Sigma}}{M_{N^*}}. \tag{27}
\]

For positive \(G_{10}^\pi\), it gives a cancellation of the vertex for the \(\pi N\) decay mode and enhancement due to mixing for the \(\eta N\) mode (note in addition that the decay \(N^*(1680) \to K\Sigma\) is forbidden and \(N^*(1730) \to K\Sigma\) is suppressed by kinematics). This picture does not look contradictory. However, a detailed description of the decays by expressions \(25\), \(26\), and \(27\) may be too simplified. Indeed, Eq. \(26\) with \(G_{10}^\pi = 2.9\) (which leads to the strongest cancellation compatible with \(\Gamma_{\Theta^+} = 1\) MeV) gives

\[
\Gamma^*_{N} = 2.1 \ [2.3] \text{ MeV}
\]

for \(M_{N^*} = 1680 \ [1730] \text{ MeV}\), in contradiction with the restrictions \(15\). In other words, in such a simple picture of mixing between only two nucleon-like states, restrictions for \(\Gamma^*_{N}\) are incompatible with \(\Gamma_{\Theta^+} \sim 1\) MeV. However, a very small \(\pi N\) partial width \(\Gamma^*_{\pi N}\) could be easily accommodated by the soliton picture of the octet–antidecuplet mixing if \(\Gamma_{\Theta^+} \sim 5\) MeV.

The situation can be changed if we take into account mixing with one more nucleon-like state. Such a case was suggested, e.g., by Jaffe and Wilczek \(39\), as mixing with \((N(1440))\). However, they assumed the complete flavor separation for a colored quark-antiquark pair, which contradicts the existing understanding of how the OZI-rule works. We prefer a more phenomenological version \(28\). If the sign of the mixing angle \(\theta_N\), as introduced in Ref. \(28\), is opposite to the sign of the octet–antidecuplet mixing angle \(\phi\), then the additional contribution may provide additional cancellation, and diminish the partial width of the \(\pi N\) decay mode, making it compatible with \(\Gamma_{\Theta^+} \sim 1\) MeV. Regrettfully, at present, we can not determine the correct relative sign for mixings of \(N^*\) with \(N\) and, say, \((N(1440))\). This would require a knowledge of the nature and/or inner structure of all the involved states. Given the small value of \(G_{10}^\pi\), we should not expect a large mixing angle \(\theta_N\) of \(N^*\) with \((N(1440))\), as it would lead to rather large \(\pi N\) partial width of \(N^*\).

Thus, in the framework of the soliton picture (with mixing to additional nucleon-like states), a \(\Theta^+\)-width of about 1 MeV implies that its non-strange partner in the antidecuplet may have a partial width of about 4 MeV for the \(SU(3)_F\)-violating \(\pi \Delta\) decay mode, up to 1 MeV for the \(K \Lambda\) mode, and a couple of MeV for the \(\eta N\) channel. Decay to \(K\Sigma\) should be small, if possible at all. The total \(N^*\)-width, with all decay modes together, might achieve \(\sim 10\) MeV, so that the \(N^*\) state would be wider than the \(\Theta^+\), though still narrow.

As a resonance in \(\pi N\) collisions, the state \(N^*\) should be rather narrow and highly inelastic, with a preference to decay mainly into \(\pi \pi N\) final states. Restrictions \(16\) for \(\Gamma^*_{N}\) lead to a very small elastic branching for \(N^* \to \pi N\), not more than 5%. Such a peak can not be extracted by standard methods of PWA.

The assumption of a larger \(\Gamma_{\Theta^+}\) (up to about 5 MeV) does not influence the most intensive decay channel \(N^* \to \pi \Delta\). Therefore, it does not change the essential features of the above conclusions. Note that measurements of the ratio of \(\pi N\) and \(\eta N\) partial widths may provide us with valuable information about octet–antidecuplet mixing.

These findings together explain why the suggested \(N^*\) has not been observed up to now. At the same time, the above estimated width of the \(N^*\), as compared to \(\Theta^+\), looks much more reasonable for the antidecuplet member state than the range of values tabulated for the \((N(1710))\) by the PDG \(12\).

C. Decays of \(\Xi_{3/2}\)

The above approach can be applied also to other antidecuplet members. We will not discuss here decays of the \(\Sigma\)-like partner of the \(\Theta^+\), which should be more essentially influenced by mixing(s). Instead, we consider possible decays of the \(\Xi\)-like partner.

The quantum numbers and mass 1862 MeV of the \(\Xi_{3/2}\) admit 2-body decay modes \(\pi \Xi, K \Sigma\), and \(\pi \Xi(1530)\). The latter decay would be analogous to \((N^* \to \pi \Delta)\), but is stronger forbidden by the \(SU(3)_F\)-symmetry, since \(\Xi_{3/2}\), contrary to \(N^*\), can not be mixed with octet members, because of its different isospin. The decay could be allowed, nevertheless, by mixing of the decuplet \(\Xi(1530)\) with some octet \(\Xi\)'s, which appears to be negligible. Another possibility for this decay would involve the mixing of the \(\Xi_{3/2}\) with a similar state belonging to a higher \(SU(3)_F\)-multiplet, such as the
27- or 35-plet. However, at present, we have not any definite information on such states. That is why we do not discuss here the decay to $\Xi(1530)$, though its experimental study could give interesting and useful information.

For the partial widths of decay modes $\pi \Xi$ and $K\Sigma$, we obtain expressions similar to Eq. (16):

$$
\Gamma (\Xi_{3/2} \to \pi \Xi) = \frac{3}{10} \cdot G_{\Xi\pi}^2 \cdot \frac{|p_{\pi\Xi}|^3}{2\pi(M_{\Xi_{3/2}} + M_{\Xi})^2} \cdot \frac{M_{\Xi}}{M_{\Xi_{3/2}}} ,
$$

$$
\Gamma (\Xi_{3/2} \to K\Sigma) = \frac{3}{10} \left( \cos \phi \cdot G_{\Xi K} - \sin \phi \cdot H_{\Xi K} \frac{\sqrt{5}}{4} \right)^2 \frac{|p_{\pi\Xi}|^3}{2\pi(M_{\Xi_{3/2}} + M_{\Xi})^2} \cdot \frac{M_{\Sigma}}{M_{\Xi_{3/2}}} .
$$

Note that the decay $\Xi_{3/2} \to \pi \Xi$ depends only on $G_{\Xi\pi}^2$.

For positive $G_{\Xi\pi}$, consistent with $\Gamma_{\phi^+} \approx 1$ MeV (i.e., $G_{\Xi\pi} \approx 2.9$), these widths are the order of a few MeV,

$$
\Gamma (\Xi_{3/2} \to \pi \Xi) \approx 2.6 \text{ MeV} , \quad \Gamma (\Xi_{3/2} \to K\Sigma) \approx 2.0 \text{ MeV} .
$$

On the other hand, for negative $G_{\Xi\pi}$, again consistent with $\Gamma_{\phi^+} \approx 1$ MeV (this time, $G_{\Xi\pi} \approx -1.4$; note that negative values of $G_{\Xi\pi}$ seem to be allowed theoretically), we obtain an extremely narrow $\Xi_{3/2}$:

$$
\Gamma (\Xi_{3/2} \to \pi \Xi) \approx 0.6 \text{ MeV} , \quad \Gamma (\Xi_{3/2} \to K\Sigma) \approx 23 \text{ keV} ,
$$

with $\Gamma_{\Xi_{3/2}}$ perhaps somewhat smaller than $\Gamma_{\phi^+}$. In both cases $\Gamma_{\Xi_{3/2}}$ is small and would be very difficult to measure directly. Instead, there is another interesting possibility. The relative intensity of the two $\Xi_{3/2}$-decay modes, $\pi \Xi$ and $K\Sigma$, may be very different, depending on the manner of $SU(3)_F$-violating mixing. The branching ratio for the latter mode may be either negligible, or of the same order (or even larger) as compared to the former one. A measurement of this ratio should be experimentally feasible.

IV. CONCLUSION AND DISCUSSION

To summarize, given our current knowledge of the $\Theta^+$, the state commonly known as the $N(1710)$ is not the appropriate candidate to be a member of the antidecuplet together with the $\Theta^+$. Instead, we suggest candidates with nearby masses, $N(1680)$ (more promising) and/or $N(1730)$ (less promising, but not excluded). Our analysis suggests that the appropriate state should be rather narrow and very inelastic. Similar considerations have been applied to the $\Xi_{3/2}(1862)$, assumed to be also a member of the same antidecuplet. It should also be quite narrow.

One can ask how definite are our theoretical predictions. They have, indeed, essential theoretical uncertainties. For example, the mixing angle $\phi$, taken from Ref. [11], was actually determined through formulas containing the $\sigma$-term (just as the mass difference in the antidecuplet). If we use parameters corresponding to more recent information, for both the $\sigma$-term and the mass difference, we obtain larger mixing, up to $\sin \phi \approx 0.15$. With our formulas, this would most strongly influence the partial width $N^+ \to \pi \Delta$, increasing it to about 15 MeV. Other partial widths of $N^+$ change not so dramatically, and the total width appears to remain not higher than $\sim 30$ MeV. Such a width could well be measured, but not in elastic scattering, because of an expected very small elastic branching ratio. Note, however, that the above large value for $\sin \phi$ may appear problematic, since the formulas of Ref. [11] assume linearisation with respect to $SU(3)_F$-violation, and need to be reconsidered if the violation appears to be large.

A high degree of uncertainty emerges also because our approach can not definitely establish the existence of the resonance(s). We have assumed the presence of only one state with $J^P = 1/2^+$, either $N(1680)$ or $N(1730)$. If both exist, they should essentially mix due to their nearby masses, strongly changing our estimates.

Nevertheless, even having in mind all theoretical uncertainties, we can suggest several directions for experimental studies. First of all, one should search for possible new narrow nucleon state(s) in the mass region near 1680 and/or 1730 MeV. Searches may use various initial states, (e.g., $\pi N$ collision or photoproduction). We expect the largest effect in the $\pi \pi N$ final state (though $\pi \Delta$ is forbidden by $SU(3)_F$). The final states $\eta N$ and $K\Lambda$ may also be interesting.
and useful, especially the ratio of $\eta N$ and $\pi N$ partial widths as the latter is very sensitive to the structure of the octet–antidecuplet mixing. Another interesting possibility to separate antidecuplet and octet components of $N^*$ is provided by comparison of photoexcitation amplitudes for neutral and charged isocomponents of this resonance, the point being that the antidecuplet component does not contribute to the photoexcitation of the charged component of $N^*$ (see details in Ref. [40]).

For $\Xi_{3/2}^+$, attempts to measure the total width are necessary, though it could possibly be even smaller than $\Gamma_{\Theta^+}$. Branching ratios for $K\Sigma$ and $\pi \Xi(1530)$, in relation to $\pi \Xi$, are very interesting. These may give important information on the mixing of antidecuplet baryons with octets and higher $SU(3)_F$-multiplets.

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[30] The factor $\frac{M_2}{M_1}$ is related to the nonrelativistic normalization of baryonic states in the soliton picture. Note also that degree of this factor depends on the spin of the participating baryons.

[31] We take this opportunity to correct a misprint in the corresponding expression of Ref. [11]. The coefficient in front of its second term should be $\sqrt{5}/2$ instead of $\sqrt{5}/4$.

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| Collaboration | Mass (MeV) | Width (MeV) | Ref | Collaboration | Mass (MeV) | Width (MeV) | Ref |
|--------------|-----------|------------|-----|--------------|-----------|------------|-----|
| DPP          | 1530      | <15        | [11]| DPP          | 1710      |            | [11]|
| LEPS         | 1540±10   | <25        | [1] |              |           |            |     |
| DIANA        | 1539±2    | <9         | [6] |              |           |            |     |
| CLAS/γn      | 1542±5    | <21        | [2] |              |           |            |     |
| CLAS/γp      | 1540±10   | <32        | [3] |              |           |            |     |
| ELSA         | 1540±4±2  | <25        | [4] |              |           |            |     |
| ITEP/ν       | 1533±5    | <20        | [7] |              |           |            |     |
| HERMES       | 1526±2.6±2.1 | <19    | [8] |              |           |            |     |
| CLAS/γp      | 1555±10   | <26        | [5] |              |           |            |     |
| ZEUS         | 1527±2    | <24        | [9] |              |           |            |     |
| NA49         | 1535      |            | [10]|              |           |            |     |
| USC          | 1543      | <6         | [19]| KH           | 1723±9    | 120±15     | [16]|
| GWU          | 1540–1550 | ≤1         | [21]| CMU          | 1700±50   | 90±30      | [17]|
| Jülich       | 1545      | <5         | [20]| KSU          | 1717±28   | 480±230    | [18]|
| LBNL         | 1540      | 0.9±0.3    | [22]|              |           |            |     |
FIG. 1: Tentative unitary anti-decuplet with $\Theta^+$. Isotopic multiplet (constant values of the charge) shown by solid (dashed) lines.
FIG. 2: Change of overall $\chi^2$ due to insertion of a resonance into $P_{11}$ for $M_R = 1660 - 1760$ MeV with $\Gamma_{tot} = 0.1, 0.3, 0.5, 0.7,$ and $0.9$ MeV (top panel) and $1, 3, 5, 7,$ and $9$ MeV (bottom panel), and $\Gamma_{el}/\Gamma_{tot} = 0.1$ (left column), $0.2$ (middle column), and $0.4$ (right column) using $\pi N$ PWA [25]. The curves are given to guide the eye. Vertical arrows indicate $M_R = 1680$ and $1730$ MeV.
FIG. 3: Change of overall $\chi^2$ due to insertion of resonances into $S_{11}$ (top panel) and $P_{13}$ (bottom panel) for $M_R = 1660 - 1760$ MeV with $\Gamma_{\text{tot}} = 0.1, 0.3, 0.5, 0.7, \text{ and } 0.9$ MeV and $\Gamma_{\text{el}}/\Gamma_{\text{tot}} = 0.1$ (left column) and 0.2 (right column). Notation as in Fig. 2.