How black holes store information in high-order correlations
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Abstract
We explain how Hawking radiation stores significant amount of information in high-order
correlations of quantum fields. This information can be retrieved by multi-time measurements
on the quantum fields close to the black hole horizon. This result requires no assumptions about
quantum gravity, it takes into account the differences between Gibbs’s and Boltzmann’s accounts
of thermodynamics, and it clarifies misconceptions about key aspects of Hawking radiation and
about informational notions in QFT.

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Stephen Hawking’s discovery that black holes radiate thermally implies that black holes eventually evaporate. The semiclassical analysis of the evaporation process leads to the conclusion that the final state should be maximally mixed. This suggests an evolution law that takes pure states to mixed states, i.e., non-unitary time evolution. Most quantum gravity research programs presuppose unitary time evolution, hence, many researchers refer to the prediction above as the paradox of black-hole information loss.

In this essay, we will explain how Hawking radiation can store significant amount of information in higher-order correlations of the quantum field. This result follows from Quantum Field Theory (QFT) in curved spacetime, without any assumptions about quantum gravity. It is fully compatible with the semiclassical analysis of black hole evaporation, and it takes into account the differences between Gibbs’s and Boltzmann’s accounts of thermodynamics. It requires the resolution of common misconceptions about key results on Hawking radiation and about informational notions in QFT. Non-unitarity arises from the misguided attempt to define evolving single-time quantum states in a non-globally hyperbolic spacetime, i.e., in a system that lacks a global notion of time. But it does not necessitate quantum information loss.

A crucial fact in the analysis of black hole information loss is that the reduced quantum state of the field far from the horizon is asymptotically Gibbsian at the Hawking temperature $T_H$. This is the content of the Hawking-Wald (HW) theorem \[1, 2\]. It implies that the values of all physical observables outside the black hole are distributed thermally, at late times. There are no correlations between quanta of different field modes. This conclusion is important because it implies that when the black hole totally evaporates, it leaves only Hawking radiation behind, which is described by a Gibbsian quantum state. This is a maximally mixed state, hence, the process of black hole formation and evaporation ostensibly involves tremendous information loss \[3\].

For macroscopic black holes, the rate of Hawking radiation is very small, so we expect that the black hole geometry changes in a quasi-stationary way. Therefore, the semiclassical approximation will be good until the black hole shrinks near the Planck mass. Quantum gravity effects at this late stage cannot affect the radiation that has already been emitted. It follows that the final state cannot be very different from what the HW theorem predicts, i.e., a maximally mixed state with no correlations and no capacity to carry information. Hence, the conclusion of information loss is not affected by the consideration of backreaction.

Note that the above analysis presupposes a Gibbsian rather than a Boltzmannian description of Hawking radiation thermodynamics. In Gibbs’s description, equilibrium is defined at the level of microstates. In Boltzmann’s approach, equilibrium refers only to the behavior of macrostates—see, Table 1. The crucial difference is that Boltzmann’s theory does not constrain observables that are not distinguishable at the macrostate level.

The HW theorem is much less powerful than what has been commonly assumed, in the sense that it precludes only one type of correlations, namely, correlations that refer to a single moment of time. It makes no statement about multi-time correlations, i.e., correlations associated to measurements at different moments of time. In Ref. \[4\], we showed that, at the level of QFT in a background black hole spacetime, multi-time correlations in Hawking radiation are not thermal and they preserve significant memory from the history of Hawking quanta. We conclude therefore that they can carry non-trivial amounts of information.

The HW theorem treats the quantum field as a bipartite system: one part consists of field states at the horizon $\mathcal{H}^+$, and one part consists of states at the future null infinity $\mathcal{I}^+$—see, Fig. 1.a. When we trace out the states at $\mathcal{H}^+$, we are left with the reduced density
FIG. 1. Penrose diagrams for (a) a black hole formed from gravitational collapse and (b) a black hole that is formed from collapse and then evaporates.

| Gibbs          | Boltzmann          |
|----------------|--------------------|
| **definition** | statistical ensemble | individual systems |
| **description** | microstates        | macrostates         |
| **gas of N particles** | probability distribution (canonical) on state space $\Gamma = R^{6N}$ | probability distribution (Maxwell-Boltzmann) on single-particle state space $\mu = R^6$ |
| **QFT**        | fixes complete hierarchy of correlation functions | fixes two-point functions |

TABLE I. Gibbsian vs. Boltzmannian approach to statistical mechanics.

matrix for the states at $\mathcal{I}^+$. The latter density matrix is mixed—in fact, thermal—because of strong entanglement between the two subsystems.

In general, a time-evolving reduced density matrix misrepresents the probabilities for multi-time measurements in the associated subsystem. This is a well-known fact from the theory of open quantum systems that has not been taken into account in past discussions of information loss. A simple extension of the HW theorem suffices to show that multi-time correlations cannot be expressed solely in terms of the degrees of freedom at $\mathcal{I}^+$. They also involve field states from $\mathcal{H}^+$, hence, they cannot be thermal [4].

Past analyses of information loss ignored multi-time QFT measurements, as they focused mainly on the S-matrix description of the field. By contrast, multi-time field measurements are ubiquitous in quantum optics. They are described by Glauber’s photo-detection theory [5] that defines the higher-order coherences of the quantum electromagnetic field. They account for phenomena like the Hanbury-Brown-Twiss effect, photon bunching and anti-bunching.

In recent years, we developed a new formalism for QFT measurements [6] that greatly
FIG. 2. QFT correlation hierarchy, Boltzmann-coarse graining and information.

generalizes Glauber’s theory. We call this method the Quantum Temporal Probabilities (QTP) method, as its original motivation was to provide a general framework for temporally extended quantum observables. In QTP, the probability density \( W_n(X_1, X_2, \ldots, X_n) \) for \( n \) particle detection events at spacetime points \( X_i \) is a linear functional of the field 2\( n \)-point function

\[
G^{(2n)}(X_1, X_2, \ldots, X_n; X'_1, X'_2, \ldots, X'_n) := \text{Tr} \left\{ T \left[ \hat{O}(X_n) \ldots \hat{O}(X_2) \hat{O}(X_1) \right] \hat{\rho}_0 \right. \\
\times \left. \bar{T} \left[ \hat{O}(X'_n) \hat{O}(X'_2) \ldots \hat{O}(X'_1) \right] \right\},
\]

where \( T \) stands for time-ordering, \( \bar{T} \) for reverse-time-ordering, and \( \hat{\rho}_0 \) is the field initial state. The local composite operator \( \hat{O}(X) \) defines the channel of interaction between the quantum field and the detector. These correlation functions are not used in S-matrix theory but they appear in the Schwinger-Keldysh formulation of QFT [7].

Full information about a quantum field is contained in the hierarchy of \( N \)-point correlation functions, for all \( N = 1, 2, \ldots, \infty \). The main achievement of QTP is that it defines a hierarchy of probability functions for multi-time measurements from the QFT correlation hierarchy [6]—see, Fig. 2.

A field description that considers anything less than the full hierarchy is coarse-grained. Coarse-graining is usually associated with loss of information. It is essential for deriving the second law of thermodynamics from microscopic theories. In particular, Boltzmann’s equation in QFT follows from a truncation of the correlation hierarchy at the level of the two-point functions [7]. The truncated hierarchy defines the thermodynamic level of description, i.e., Boltzmann’s macrostates. The second law of thermodynamics is essentially the statement that information is typically lost from this level, as it is transferred to higher-order
correlation functions.

The analysis of Hawking radiation through multi-time measurements outside the black hole revealed the following pattern: (i) Single-time measurements of the field outside the black hole ‘capture’ information from the two-point correlation functions. The associated probabilities are thermal, in accordance with the HW theorem. (ii) Multi-time measurements of the field outside the black hole ‘capture’ information from the $N > 2$ correlation functions. The associated probabilities do not behave thermally, and they support significant correlations.

Therefore, Hawking radiation behaves thermally only with respect to Boltzmann’s coarse-graining. This strongly contrasts the usual understanding of Hawking-radiation thermodynamics in terms of a Gibbsian quantum state. In the QFT context, a Gibbs state specifies the full hierarchy of correlation functions, while Boltzmann’s theory specifies only two-point correlation functions. It is widely agreed that Boltzmann’s description is more fundamental, because it applies to individual closed systems and not statistical ensembles. Furthermore, Gibbs’s theory does not naturally account for non-equilibrium thermodynamics and the second law [8].

Hence, the loss of information in black hole evaporation is identical to that in non-equilibrium statistical mechanics, in the sense that information is transferred to thermodynamically inaccessible non-local degrees of freedom. This information can only be retrieved by multi-time measurements.

Our results are consistent with the semi-classical analysis of black hole evaporation. The quantum backreaction on the spacetime metric is driven by the expectation value $\langle \hat{T}_{\mu\nu} \rangle$ of the quantum stress energy tensor $\hat{T}_{\mu\nu}$, defined in terms of the field two-point function. Backreaction transfers information to inaccessible degrees of freedom through other channels—including higher-order correlations of $\hat{T}_{\mu\nu}$. The key point here is that this information transfer takes place independently of the strength of $\langle \hat{T}_{\mu\nu} \rangle$, i.e., even if backreaction has little effect on spacetime geometry.

We proved that pre-collapse information is not necessarily lost. We believe that it is stored into higher-order correlations through backreaction, and it is accessible by multi-time measurements close to the horizon. For example, correlations at $2M < r < 3M$, in a Schwarzschild spacetime of mass $M$, carry the strongest memories about the past history of Hawking quanta [4]. Multi-time measurements after the collapse allow us to retrodict pre-collapse properties of the system. In this sense, black holes have quantum informational hair.

Finding exactly how much information is stored in Hawking radiation correlations requires a first-principles extension of quantum information theory to relativistic QFTs that is currently unavailable. However, even with maximal retrieval of pre-collapse information, unitarity will not be restored. Non-unitarity originates from the fact that the notion of an instant of ‘time’ does not exist after evaporation, i.e., the spacelike surface $S$ of Fig. 1.b is not a Cauchy surface. What breaks down is the notion of the single-time quantum state; it is a conceptual mistake to focus on unitarity. We believe that generalizations of quantum theory that are based on the notion of history [9]—treating single-time quantum states as derived concepts—are more appropriate for the physics of black hole evaporation [10] and for quantum gravity [11] [12].
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