Calculation the Cross Sections for $^6\text{Li}(\alpha, p)^9\text{Be}$ Reaction by Reverse Reaction

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ABSTRACT:
In this study light elements $^6\text{Li}$, $^9\text{Be}$, $^{10}\text{Be}$ for $^6\text{Li}(\alpha, p)^9\text{Be}$ reaction with proton energy from (27.5) MeV to (67.5) MeV with threshold energy (2.3626) MeV are used according to the available data of reaction cross sections. The Q-value is equal (2.125MeV) and parity of ($^9\text{Be}=3/2^+ \quad (B=3^+ \quad (\text{Li} = 1^-)$ for the ground state. The more recent cross sections data of $^6\text{Li}(\alpha, p)^9\text{Be}$ reaction is reproduced in fine steps and by using (Matlab-7.6-2008a) program and get the equation from 3-degree for plotted. We deduced that the high probability to produced $^9\text{Be}$ by bombard $^6\text{Li}$ by alpha particle.

Keywords: The cross sections, nuclear reactions, reverse reaction, compound nucleus, $^6\text{Li}(\alpha, p)^9\text{Be}$ reaction, $^9\text{Be}(p, \alpha)^6\text{Li}$ reaction.
1-THEORY:

The cross section of compound nucleus is given by [1]:

$$\sigma = \frac{\pi}{k^2 g} \frac{\Gamma^2}{(E - E_R)^2 + \Gamma^2 / 4}$$ \hspace{1cm} (1)

Where \( g \) is a statistical factors.

\( E \) is the kinetic energy of an incident particle.

\( E_R \) is a single isolated resonance energy.

\( \Gamma \) is the width of the state.

\( k \) is the wave number which is given by:

$$k = \frac{1}{\lambda} = \frac{p}{\hbar} = \frac{mv}{\hbar} = \frac{\sqrt{2mE}}{\hbar}$$ \hspace{1cm} (2)

Where \( \lambda \) is the de-Broglie wavelength divided by 2\( \pi \) of incident particle.

\( \hbar \) is the Plank constant divided by 2\( \pi \).

\( p \) is the momentum of an incident particle.

\( m \) and \( v \) are the mass and velocity of an incident particle.

The statistical g-factors is given by [1]:

$$g = \frac{2I_a + 1}{(2S_a + 1)(2S_X + 1)}$$ \hspace{1cm} (3)

Where \( I \) is the total angular momentum of the resonance (compound nucleus) which is given by:

$$I_c = S_a + S_X + \ell_a$$ \hspace{1cm} (4)

Where \( S_a \) is the spin of the incident particle.

\( S_X \) is the spin of the target.

\( \ell_a \) is the orbital angular momentum of incident particle.

The total width of the state is the sum of the partial widths [2,3]:

$$\Gamma = \sum_i \Gamma_i$$ \hspace{1cm} (5)

or

$$\Gamma = \frac{\hbar}{\tau}$$ \hspace{1cm} (6)

Where \( \tau \) is the lifetime of any decay state;

At resonance

The \( \Gamma^2 \) in equation (1) is directly related to the formation of the resonance and to its probability to decay into a particular exit channel. That is, for the reaction \( a+X = b+Y \), a different exit width must be used [4]:

$$\sigma = \frac{\pi}{k^2 g} \frac{\Gamma_{aX} \Gamma_{bY}}{(E - E_R)^2 + \Gamma_{aX} \Gamma_{bY} / 4}$$ \hspace{1cm} (7)
Where $\Gamma_{aX}$ is the partial width for decay into $a+X$.

$\Gamma_{bY}$ is the partial width for a different exit.

Equation (7) is the Breit-Wigner formula for the shape of a single, isolated resonance.

At resonance $E = E_R$ and $\Gamma_{aX} \Gamma_{bY} = \Gamma^2$ since $\Gamma_{aX} = \Gamma_{bY} = \Gamma$, we call $\Gamma_{aX}$ the partial width for decay into $a+X$ and $\Gamma_{bY}$ the partial width for any other channels energetically allowed, then equation (7) becomes:

$$\sigma = \frac{4\pi}{k^2} g$$

Equation (8)

The basic assumption of the compound nucleus model is that the compound nucleus has been formed in such a complicated set of interactions that it does not remember the initial stage of formation. The cross sections for the reaction $X(a,b)Y$ can be split into a formation cross section of the compound nucleus [C.N.]* corresponding to the process:

$$a + X \rightarrow [C.N.]* \rightarrow Y + b$$

Equation (9)

And the fractional probability that [C.N.]* breaks up into particles $b+Y$. We can therefore write [5].

$$\sigma(a,b) = \sigma_{a,b}(T_0) Pb(E)$$

Equation (10)

Where $T_0$: bombarding energy in center of mass.

$E$: corresponding excitation energy of the compound nucleus.

$Pb(E)$: Fractional probability of [C.N.]* to break up into Y+b.

2-REVERSE REACTION : 

If the cross-sections of the reaction $A(\alpha,p)B$ are measured as a functions of $T_\alpha$ ($T_\alpha$ = Kinetic energy of $\alpha$-particle ) the cross-sections of the inverse reaction $B(p,\alpha)A$ can be calculated as a function of $T_p$ ($T_p = $ Kinetic energy of proton) using the reciprocity theorem [6] which states that :

$$\frac{\sigma(\alpha,p)}{g_{\alpha,p}} = \frac{\sigma(p,\alpha)}{g_{p,\alpha}}$$

Equation (11)

Where $\sigma(\alpha,p)$ and $\sigma(p,\alpha)$ represent cross-sections of $(\alpha,p)$ and $(p,\alpha)$ reactions respectively , $g$ is a statistical factor and $\lambda$ is the de-Broglie wave length divided by $2\pi$ and is given by

$$\lambda = \frac{\hbar}{MV}$$

Equation (12)

Where $\hbar$ is Dirac constant ($\hbar/2\pi$ ), $\hbar$ is plank constant , $M$ and $V$ are mass and velocity of alpha or proton.

From eq.(12),we have

$$\lambda^2 = \frac{\hbar^2}{2MT}$$

Equation (13)
The statistical g-factors are given by [6]

$$2J_c + 1$$

$$g_{\alpha, p} = \frac{2J_c + 1}{(2I_A + 1)(2I_\alpha + 1)}$$

(14)

and

$$2J_c + 1$$

$$g_{p, \alpha} = \frac{2J_c + 1}{(2I_B + 1)(2I_p + 1)}$$

(15)

The conservation law of the momentum and parity implies that:

$$I_A + I_\alpha = J_c = I_B + I_p$$

(16)

$$\Pi_A \cdot \Pi_\alpha (-1)^{\ell_\alpha} = \Pi_c = \Pi_B \cdot \Pi_p (-1)^{\ell_p}$$

(17)

$J_c$ and $\Pi_c$ are total angular momentum and parity of the compound nucleus.

$I_A$ and $\Pi_A$ are total angular momentum and parity of nucleus A.

$I_B$ and $\Pi_B$ are total angular momentum and parity of nucleus B.

$I_\alpha$ and $\Pi_\alpha$ are total angular momentum and parity of $\alpha$-particle.

$I_p$ and $\Pi_p$ are total angular momentum and parity of proton.

$$\Pi_c = \Pi_p = +1$$

(18)

$$I_\alpha = s_\alpha + \ell_\alpha$$

(19)

Where

$I_\alpha$ is the total angular momentum of alpha particle $s_\alpha$ is spin of $\alpha$-particle $= 0$

$\ell_\alpha$ is the orbital angular momentum of $\alpha$-particle

$$I_p = s_p + \ell_p$$

(20)

Where

$I_p$ is the total angular momentum of the proton

$s_p$ is spin of proton $= 1/2$

$\ell_p$ is the orbital angular momentum of proton

From eq.(16) we have:

$$| J_c - I_A | \leq I_\alpha \leq | J_c + I_A |$$

(21)

$$| J_c - I_B | \leq I_p \leq | J_c + I_B |$$

(22)
The reactions $A(\alpha,p)B$ and $B(p, \alpha)$ can be represented with the compound nucleus $C$ as in the following schematic diagram. It is clear that there are some important and useful relations between the kinetic energies of the proton and alpha particle. One can calculate the separation energies of $\alpha$-particle ($S_\alpha$) and proton ($S_p$) using the following relations:

$$E = S_\alpha + \frac{M_A}{M_A + M_\alpha} T_\alpha \quad \text{(23a)}$$

$$E = S_p + \frac{M_B}{M_B + M_p} T_p \quad \text{(23b)}$$

$$S_\alpha = 931.5 \left[ M_A + M_\alpha - M_c \right] \quad \text{(24)}$$

$$S_p = 931.5 \left[ M_B + M_p - M_c \right] \quad \text{(25)}$$

Combining (23a), (23b), (24) and (25) and as the $Q$-value of the reaction $A(\alpha,p)B$ is given by:

$$Q = 931.5 \left[ M_A + M_\alpha - M_B - M_p \right] \quad \text{(26)}$$

$$Q = \frac{M_B}{M_B + M_p} T_p - \frac{M_A}{M_A + M_\alpha} T_\alpha \quad \text{(27)}$$

Or:

$$T_p = \frac{M_B + M_p}{M_B} \left[ \frac{M_A}{M_A + M_\alpha} T_\alpha + Q \right] \quad \text{(28)}$$

The threshold energy $E_{th}$ is given by

$$E_{th} = - \frac{Q}{M_A} \quad \text{(29a)}$$
\[ Q = - \frac{M_A}{M_A + M_\alpha} E_{th} \]  
\[ (29b) \]

Then

\[ T_p = \frac{M_B + M_p}{M_A + M_\alpha} \times \frac{M_A}{T_\alpha - E_{th}} \]  
\[ (30) \]

Thus eq. (11) can be written as follows:

\[ \sigma(p,\alpha) = \frac{g_{p,\alpha} M_\alpha T_\alpha}{\sigma(\alpha,p)} \]  
\[ (31) \]

3. RESULTS AND DISCUSSION:

By using semi empirical formula the evaluated cross sections as a function of proton energy from (27.5)MeV to (67.5)MeV of present work are listed in table (1). From these data which were plotted and we get the mathematical equation representing the cross sections distribution in the indicated range of proton energy Fig.(1) and percentage error (±0.1819) mbarn as follows:

\[ y = -6.1e^{-0.010}x^3 + 1.5e^{-0.007}x^2 - 1.3e^{-0.005}x + 0.0004 \]

\[ y=\text{cross sections of}(p,\alpha) \quad x=\text{proton energy}(T_p) \]

In fig.(1)[7] we observed that the cross sections were smoothly decreased and the maximum cross section(0.4492 mbarn) when proton energy is equal (T_p=27.5MeV).

By using the compound theory we derived the mathematical formula from \(^9\text{Be}(p,\alpha)^6\text{Li}\) reaction for ground state to get the cross sections of \(^6\text{Li}(\alpha,p)^9\text{Be}\) reaction:

\[ \sigma_{\alpha,p} = 0.2685 \frac{T_p}{T_\alpha} \sigma_{p,\alpha} \]  
\[ (32) \]

We calculated the cross sections of alpha energy with energy range between (37.6533MeV) to (96.0714MeV) are (0.8452mbarn) to (0.027mbarn) respectively. These data are plotted in fig.(2) and listed in table(2) . We observed that the high probability to produced 9Be by bombard 6Li with alpha energy is (37.6533MeV) and we get semi empirical formula with three-degree as follow:

\[ y = -6.1e^{-0.010}x^3 + 1.5e^{-0.007}x^2 - 1.3e^{-0.005}x + 0.0004 \]
Table 1: The cross sections of \(^9\)Be(p,α)\(^6\)Li reaction with threshold energy of (2.3626MeV) [7]

| Proton Energy(MeV) | Cross Sections(mbarn) | Proton Energy(MeV) | Cross Sections(mbarn) |
|--------------------|-----------------------|--------------------|-----------------------|
| 27.5               | 0.4492                | 49.5               | 0.0674                |
| 28.5               | 0.4159                | 50.5               | 0.0622                |
| 29.5               | 0.3825                | 51.5               | 0.057                 |
| 30.5               | 0.3491                | 52.5               | 0.0519                |
| 31.5               | 0.3157                | 53.5               | 0.048                 |
| 32.5               | 0.2824                | 54.5               | 0.0442                |
| 33.5               | 0.2646                | 55.5               | 0.0403                |
| 34.5               | 0.2468                | 56.5               | 0.0365                |
| 35.5               | 0.2291                | 57.5               | 0.0326                |
| 36.5               | 0.2113                | 58.5               | 0.0305                |
| 37.5               | 0.1935                | 59.5               | 0.0284                |
| 38.5               | 0.1776                | 60.5               | 0.0263                |
| 39.5               | 0.1617                | 61.5               | 0.0242                |
| 40.5               | 0.1458                | 62.5               | 0.0221                |
| 41.5               | 0.1299                | 63.5               | 0.0202                |
| 42.5               | 0.114                 | 64.5               | 0.0182                |
| 43.5               | 0.1068                | 65.5               | 0.0163                |
| 44.5               | 0.0995                | 66.5               | 0.0144                |
| 45.5               | 0.0923                | 67.5               | 0.0124                |
| 46.5               | 0.085                 | 68.5               | 0.0105                |
| 47.5               | 0.0777                | 69.5               | 0.0087                |
| 48.5               | 0.0726                | 70.5               | 0.0069                |

Table 2: The cross sections \(^6\)Li(α,p)\(^9\)Be reaction (the present work)

| Alpha Energy(MeV) | Cross Sections(mbarn) | Alpha Energy(MeV) | Cross Sections(mbarn) |
|-------------------|-----------------------|-------------------|-----------------------|
| 37.6533           | 0.8452                | 78.0966           | 0.0831                |
| 39.1512           | 0.7824                | 79.5945           | 0.0758                |
| 40.6491           | 0.7196                | 81.0924           | 0.0686                |
| 42.147            | 0.6568                | 82.5903           | 0.0614                |
| 43.6449           | 0.5941                | 84.0882           | 0.0574                |
| 45.1428           | 0.5313                | 85.5861           | 0.0535                |
| 46.6407           | 0.4978                | 87.084            | 0.0495                |
| 48.1386           | 0.4644                | 88.5819           | 0.0456                |
| 49.6365           | 0.431                 | 90.0798           | 0.0416                |
| 51.1344           | 0.3975                | 91.5777           | 0.038                 |
| 52.6323           | 0.3641                | 93.0756           | 0.0343                |
| 54.1302           | 0.3342                | 94.5735           | 0.0307                |
| 55.6281           | 0.3043                | 96.0714           | 0.027                 |
| 57.126            | 0.2744                | 97.5688           | 0.023                 |
| Proton Energy (MeV) | Cross Sections (mbarn) |
|--------------------|------------------------|
| 58.6239            | 0.2445                 |
| 60.1218            | 0.2146                 |
| 61.6197            | 0.2009                 |
| 63.1176            | 0.1872                 |
| 64.6155            | 0.1736                 |
| 66.1134            | 0.1599                 |
| 67.6113            | 0.1463                 |
| 69.1092            | 0.1365                 |
| 70.6071            | 0.1268                 |
| 72.105             | 0.117                  |
| 73.6029            | 0.1073                 |
| 75.1008            | 0.0976                 |
| 76.5987            | 0.0903                 |

Fig 1: The cross sections of $^9$Be(p, α)$^6$Li [7].

The cross sections are given by the equation

\[ y = 2.2e^{-0.10}x^4 - 5.3e^{-0.008}x^3 + 4.9e^{-0.006}x^2 - 0.0002x + 0.0034 \]
Fig 2: The cross sections of $^6\text{Li}(\alpha,p)^9\text{Be}$ reaction P.W.

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I am researcher in field of nuclear physics and have experience in teaching, researching in subjects of both applied and theoretical atomic physics and nuclear physics for more than 29 years.