Development of research network on Quantum Annealing Computation and Information using Google Scholar data

Antika Sinha

Department of Computer Science, Asutosh College, Kolkata, West Bengal 700026, India

We build and analyse the network of 100 top-cited nodes (research papers and books from Google Scholar; the strength or citation of the nodes range from about 44 000 up to 100) starting in early 1980 until last year. These searched publications (papers and books) are based on Quantum Annealing Computation and Information categorized into four different sets: (A) Quantum/Transverse Field Spin Glass Model, (B) Quantum Annealing, (C) Quantum Adiabatic Computation and (D) Quantum Computation Information in the title or abstract of the searched publications. We fitted the growth in the annual number of publication \( n_p \) in each of these four categories, A–D, to the form \( n_p \sim \exp(t/\tau) \), where \( t \) denotes the time in years. We found the scaling time \( \tau \) to be of the order of about 10 years for categories A and C, whereas \( \tau \) is of the order of about 5 years for categories B and D.

This article is part of the theme issue ‘Quantum annealing and computation: challenges and perspectives’.

1. Introduction

Although the classical gate-based digital computation has been extremely successful, by the early 1980s, serious limitations of classical gate-based computers started to become obvious for the so-called (computationally)
hard problems. Some of the prominent examples were that of the $N$-city Travelling Salesman Problem (of searching the minimum travel distance path from $N!$ order candidates) or the ground states in randomly frustrated $N$ Ising-spin glasses (search for the minimum energy state out of the $2^N$ states), where search time is not bounded by a polynomial in $N$.

Inspired by the metallurgical annealing technique, Kirkpatrick et al. [1] in 1983 proposed a novel stochastic or statistical physical technique, post which researchers attempted to achieve stochastic practical computational solutions of such hard multi-variable optimization problems. Although some practical solutions could be obtained using this technique very quickly, the search time still could not be bounded by any polynomial (in $N$).

In parallel, Feynman [2] in 1982 proposed quantum gate-based computing architectures for solving the computationally hard problems. So far, such gate-based quantum computers are not very implementable or achieved any practical level. In 1989, Ray et al. [3] proposed a quantum tunnelling-induced stochastic search algorithm for the search of the ground state of the Sherrington–Kirkpatrick spin glass model. Although there were many criticisms of this idea, Aeppli and Rosenbaum with their co-workers [4] in 1991 experimentally demonstrated its feasibility. In 1998, Kadwaki & Nishimori [5] numerically showed the superiority of the Quantum Annealing technique over the classical annealing technique, and finally, Johnson et al. [6] developed and marketed the first quantum annealing machine (D-Wave computers) in 2010. Since then a revolution in quantum computing and information processing has taken place.

Here, we study the growth of research publications (papers, books) of the well-studied problem-area ‘Quantum Annealing Computation and Information’ from 1980 up to 2021 using annual number of publications ($n_p$) and a research network containing 100 top-cited nodes as publications and the corresponding edges as citations. All data are collected between 8 and 10 April 2022, using the popular and publicly available search engine Google Scholar, where the citation strength of the searched nodes are seen to vary nearly between 44 000 to 100. The set of 100 top-cited publications are categorized into four different categories (A to D) considering 25 publications from each category as: (A) Quantum/Transverse Field Spin Glass Model (QISG), (B) Quantum Annealing (QA), (C) Quantum Adiabatic Computation (QAC) and (D) Quantum Computation Information (QCI). $n_p$ from each categories are observed to follow exponential distribution against time in years and best fits as $n_p \sim \exp (t/\tau)$. We find that the ‘similarity classes’ of growth time constant (similar value of $\tau$) exists for categories A and C and for categories B and D that we discuss further in the paper.

2. Research network and study of its growth

In this section we study the research network (in §2a) containing the nodes (publications), their edges (citations) and also analyse the network growth (in §2b). The dataset is obtained using Google Scholar (a public, popular search engine for scholarly literature across a wide variety of disciplines and sources) and the access period of the dataset is 8 to 10 April 2022. The year of publications of these research documents (nodes/publications) in the dataset range from 1980 to 2021.

(a) Research network

In this subsection, we study the searched publications on ‘Quantum Annealing Computation and Information’ as a research network that consists of the 100 top-cited nodes (publications) subdivided into four categories (25 in each) as follows: (A) Quantum/Transverse Field Spin Glass Model, (B) Quantum Annealing, (C) Quantum Adiabatic Computation and (D) Quantum Computation Information; see tables 1 and 2. The network is directed, i.e. within network, a node (publication) if it receives an edge (citation) from some node does not cite or link that node back; the network is shown in figure 1. The order of the network is 100, and the size of the network is nearly about 350 considering the edges as citations from the latest (by year of
publication) 10 nodes in each of the categories to any nodes within the network. In table 3, we list the top 10 publications from the studied research network, which either received maximum citations (indegree), or cited (outdegree) maximum number of nodes of the network among other references; e.g. among the 100 top-cited publications, node C1 is cited by 22 other nodes (publications) of the studied network, and on the other hand, node C4 has cited 29 nodes from the research network among others. In figure 2, we also show the plot of correlations between cross-category versus self-category citation strength (i.e. how much citations any chosen node has given to other-category nodes versus the self-category nodes) using the references of the latest 10 publications from each of the four research categories as given in tables 1 and 2. This network gives an idea about how research among the various components of “Quantum Computing” is going on at the present time.

(b) Growth behaviour

Here, we study the growth statistics of the annual number of publications $n_p$ for all the four categories (A–D). The growth of $n_p$ with time $t$ (year) is fitted to

$$n_p = a \times \exp\left[\frac{(t - t_0)}{\tau}\right],$$

and the scaling time $\tau$ is estimated for each of the categories.

(A) QISG
We searched for annual $n_p$ with the exact phrase ‘quantum spin glass’ and the additional words ‘transverse field’ in the document title or abstract starting from 1980 to 2021. An exponential relationship is observed between $n_p$ and $t$, which fits as equation (2.1), where $t_0 = 1980$ and $\tau \approx 9.7$; see figure 3.

(B) QA
We searched for annual $n_p$ with the exact phrase ‘quantum annealing’ in the document title or abstract starting from 1988 to 2021. An exponential relationship is observed between $n_p$ and $t$, which fits as equation (2.1), where $t_0 = 1988$ and $\tau \approx 5.0$; see figure 4.

(C) QAC
We searched for annual $n_p$ with the exact phrase ‘quantum-adiabatic comput’ (where ‘comput’ stands for computer, computing or computation) in the document title or abstract starting from

Figure 1. Network of hundred top cited (in the range of about 44000–100) nodes or research publications (papers and books) from Google Scholar (date of access: 10 April 2022) on “Quantum Annealing Computation and Information” from 1980 to 2020. These nodes are classified into four different categories (A–D): (a) QISG, (b) QA, (c) QAC, and (d) QCI. These nodes for the last 40 years are further subdivided into 5 years slot according to the respective years of publication. The network links (from citing to cited nodes) are also shown. (Online version in colour.)
Figure 2. Correlation plot of cross-category versus self-category citations for each of the selected publications received from among the rest, given in tables 1 and 2. The positions of the data points with respect to the 45° diagonal line show that there are more in/self-category citations than citations from other/cross-categories, statistically justifying the broad categorization of the nodes (publications) into four groups as given in tables 1 and 2.

1998 to 2021. An exponential relationship is observed between $n_p$ and $t$, which fits as equation (2.1), where $t_0 = 1998$ and $\tau \simeq 8.7$; see figure 5.

(D) QCI

We searched for annual $n_p$ with the exact phrase ‘quantum-computing’ in the document title or abstract starting from 1980 to 2021. An exponential relationship is observed between $n_p$ and $t$, which fits as equation (2.1), where $t_0 = 1980$ and $\tau \simeq 5.3$; see figure 6.

3. Summary and discussion

Here, we study (in §2a) the structure of the research network containing 100 top-cited nodes (publications) from 1980 up to 2021 on the broad topic ‘Quantum Annealing Computation and Information’, searched from Google Scholar. The searched publications (papers and books) are categorized into four subcategories: (A) Quantum/Transverse Field Spin Glass Model, (B) Quantum Annealing, (C) Quantum Adiabatic Computation and (D) Quantum Computation Information, each consisting of 25 publications. The citation links between the nodes of the entire network (of 100 nodes) were also found. The network is shown in figure 1, and the Google Scholar citations of 100 nodes of the network are given in tables 1 and 2. The size of the network (containing the nodes and the links) is nearly 350. We considered (see table 3) the top 10 nodes in each category having maximum links (in or out) within the network. Apart from self-connections within the sets (categories), major connections are seen to flow as AB, AC, BC, BA, CB, CD, CA and DC. Indeed, the link statistics, within each categories, and those across the categories, are given in tables 1–3. From figure 2, one can clearly see that the citation correlations within each category are more than cross-category correlations, justifying the categorization of the topics as given in tables 1 and 2.

As shown in figures 3–6, for each category, we have marked a few notable publications within these 40 plus years (1980–2022). For category A, in figure 3, we noted the publications by Chakrabarti [101] for the first study on the quantum phase transition behaviour of the Mattis-type and the Edwards–Anderson Ising Spin glasses, which, along with their further extensions [3], were questioned in the theoretical study [38] by Goldschmidt and Lai in 1990, while supporting experimental evidences were reported in the pioneering study [4] in 1991 by Wu et al. The first monograph on the quantum phase transitions in transverse Ising spin (including glass) models was published by Chakrabarti et al. in 1996 [102]. This experimental study was further
extended and analysed by Kao et al. in 2001 [103], and further in 2008 [48] by Ancona-Torres et al. Next, an extended monograph on quantum phase transition in Ising models and glasses [104] by Suzuki et al. was published in 2012. Mukherjee et al. [105] published further numerical results supporting the existence of ergodic region of the spin glass phase in the transverse Ising Sherrington-Kirkpatrick model in 2018, while the analytical study [106] in 2021 by Leschke et al. found some support for such tunnelling-induced ergodicity in the model. Two recent papers in 2022 on this problem, namely, [107] by Yaacoby et al. and [108] by Schindler et al. throw some encouraging new light on this point. For category B, in figure 4, similarly marked publications, starting with the 1988–1990 one by Apolloni et al. [109], put the term ‘quantum annealing’ in the title of the conference publication. The seminal 1989 paper by Ray et al. [3] indicated that quantum fluctuations can help explore rugged energy landscapes of the classical Ising spin glasses by escaping from local minima (having tall but thin barriers) using tunnelling. Finnila et al. [11] proposed it for a search of the ground state in large molecules in 1994. Kadowaki 

Table 1. Citation data of the nodes/research publications (papers and books) for categories A and B (date of data collection is 10 April 2022).

| Category A (QISG) | Category B (QA) |
|-------------------|------------------|
| paper id | year | citation | reference | paper id | year | citation | reference |
| A1  | 2002 | 641 | [7] | B1  | 1998 | 1570 | [5] |
| A2  | 2015 | 316 | [8] | B2  | 2011 | 1431 | [6] |
| A3  | 1991 | 256 | [4] | B3  | 2014 | 711 | [9] |
| A4  | 1982 | 213 | [10] | B4  | 2002 | 641 | [7] |
| A5  | 1989 | 207 | [3] | B5  | 1994 | 630 | [11] |
| A6  | 2011 | 199 | [12] | B6  | 2008 | 594 | [13] |
| A7  | 2015 | 196 | [14] | B7  | 1999 | 533 | [15] |
| A8  | 1994 | 185 | [16] | B8  | 2006 | 333 | [17] |
| A9  | 1993 | 179 | [18] | B9  | 2014 | 327 | [19] |
| A10 | 1995 | 177 | [20] | B10 | 2014 | 309 | [21] |
| A11 | 2018 | 174 | [22] | B11 | 2012 | 305 | [23] |
| A12 | 1994 | 172 | [24] | B12 | 2013 | 282 | [25] |
| A13 | 1993 | 164 | [26] | B13 | 2005 | 268 | [27] |
| A14 | 1995 | 161 | [28] | B14 | 2013 | 227 | [29] |
| A15 | 2000 | 159 | [30] | B15 | 2008 | 225 | [31] |
| A16 | 1996 | 154 | [32] | B16 | 2015 | 202 | [33] |
| A17 | 2015 | 147 | [34] | B17 | 2014 | 187 | [35] |
| A18 | 2014 | 138 | [36] | B18 | 2004 | 185 | [37] |
| A19 | 1990 | 134 | [38] | B19 | 2015 | 202 | [39] |
| A20 | 1996 | 107 | [40] | B20 | 2014 | 169 | [41] |
| A21 | 1990 | 105 | [42] | B21 | 2014 | 169 | [43] |
| A22 | 1985 | 99  | [44] | B22 | 2002 | 151 | [45] |
| A23 | 1987 | 93  | [46] | B23 | 2017 | 149 | [47] |
| A24 | 2008 | 91  | [48] | B24 | 2005 | 126 | [49] |
| A25 | 1989 | 91  | [50] | B25 | 2012 | 123 | [51] |
Nishimori [5] in 1998 first formulated and numerically demonstrated its advantages in Ising glasslike systems. Brooke et al. [15] demonstrated experimentally the advantages in mixed magnets in 1999. Santoro et al. [7] demonstrated the same in 2002. The first major review of quantum annealing was published in 2008 by Das & Chakrabarti [13]. Its mathematical structure was reviewed by Morita & Nishimori in 2008 [31]. Following a major development and publication [6] by Johnson and co-workers in 2011, the D-wave quantum annealing computers became available in the market. Hauke et al. and Oliver reviewed the latest developments. Starchl & Ritsch [111] in 2022 first reviewed briefly the developments (notably identifying [3] by Ray et al. as ‘the earliest work in laying the foundation of quantum annealing’) and demonstrated theoretically the efficiencies of photonic annealers. For category C, in figure 5, we highlight the 1998 publication by Averin [64] for proposing first the adiabatic quantum computation with Cooper pairs. Next, in two successive pioneering publications in 2001 and 2000 by Farhi et al. [54,52], respectively, the Quantum adiabatic computation was precisely formulated. Santoro & Tosatti reviewed the
Table 3. Top 10 nodes (publication) having more outdegree than indegree within the network.

| Top 10 indegree (self-category citation) |
|----------------------------------------|
| paper id | C1 | B2 | B1 | D1 | B7 | B5 | B3, B12, C3 | A1, A5, C2 | B14, C15 |
| indegree | 22 | 16 | 15 | 14 | 12 | 11 | 10 | 9 | 8 | 7 |

| Top 10 outdegree (cross-category citation) |
|------------------------------------------|
| paper id | C4 | C21 | B21 | C22 | A2 | B17 | A17 | B3 | B20 | B14 |
| outdegree | 29 | 28 | 27 | 26 | 25 | 20 | 19 | 14 | 13 | 12 |

Figure 3. Annual number of publications ($n_p$) searched from Google Scholar (accessed date: 8 April 2022) with the exact phrase ‘quantum spin glass’ and the additional words ‘transverse field’ in the document title or abstract starting from 1980 ($t = t_0$) up to 2021. An exponential growth in $n_p$ is observed over these years. The data fits to $n_p \sim \exp \left( \frac{(t - t_0)}{\tau} \right)$ with $\tau \approx 9.7$ years. We have marked some of the notable events: BKC denotes Chakrabarti [101]; GL denotes Goldschmidt and Lai [38]; WER denotes Wu et al. [4]; CDS denotes Chakrabarti et al. [102]; KGL denotes Kao et al. [103]; ASA denotes Ancona-Torres et al. [48]; SIC denotes Suzuki et al. [104]; MRC denotes Mukherjee et al. [105]; LMR denotes Leschke et al. [106]; YSK denotes Yaacoby et al. [107]; SGS denotes Schindler et al. [108]. The inset shows direct functional relationship between $n_p$ and $t$ for category A from 1980 to 2021. (Online version in colour.)

Developments in 2006 [17] and Albash & Lidar [58] in 2018. For category D, in figure 6, we have marked the classic publications [112,2,113] by Benioff in 1980, Feynman in 1982 and Deutsch in 1985, respectively. The classic book [53] on quantum computation and information by Nielsen & Chuang appeared in 2002. The 2012 Physics Nobel Prize went to Haroche and Winerland for their work, which ‘hold promise for the creation of practical quantum computers’ [114]. A theoretical compendium [115] by Tanaka et al. appeared in 2017.

For the study of the growth of the research literature in the general topic “Quantum Annealing Computation and Information”, we again studied (in §2b) the annual growth rate $n_p$ of publications in all the four categories (A–D) with time $t$ (in years) within the period 1980–2021. All the data were collected during 8 to 10 April 2022, using Google Scholar. The annual number of publications ($n_p$) in each of these categories were fitted to equation (2.1), namely, $n_p = a \exp \left( \frac{(t - t_0)}{\tau} \right)$ and the estimated growth scaling time $\tau$ for each of the categories was estimated (see figures 3–6); category-wise fitting parameters are given in table 4. The fitting process shows the initial number of publications ($a = n_p$ at $t = t_0$ in equation (2.1)) to vary in the range 1–20 and the estimated value of $t_0$ compares well with the first notable publication of the topic or category (see the indicated years of the notable publication, Nobel Prize related to quantum computing), as shown in figures 3–6.
Figure 4. Annual number of publications ($n_p$) searched from Google Scholar (accessed date: 8 April 2022) with the exact phrase ‘quantum annealing’ in the document title or abstract starting from 1988 ($t = t_0$) up to 2021. An exponential growth in $n_p$ is observed over these years. The data are fitted to $n_p \sim \exp\left[(t - t_0)/\tau\right]$ with $\tau \simeq 5.0$ years. We have marked some of the notable events: ACD denotes Apolloni et al. (conference in 1988) [109]; RCC denotes Ray et al. [3]; FGS denotes Finnila et al. [11]; KN denotes Kadowaki & Nishimori [5]; BBR denotes Brooke et al. [15]; SMT denotes Santoro et al. [7]; DC denotes Das & Chakrabarti [13]; MN denotes Morita & Nishimori [31]; JAG denotes Johnson et al. [6]; HKL denotes Hauke et al. [110]; SR denotes Starchl & Ritsch [111]. The inset shows the direct functional relationship between $n_p$ and $t$ for category B from 1988 to 2021. (Online version in colour.)

Overall, the literature on quantum annealing and computing is growing exponentially (from about $10^1$ at $t = t_0 \sim 1980$ to the order of $10^2$–$10^4$ at $t = 2021$) with time, and while for categories like quantum spin glass (A) and quantum adiabatic computation (C), the typical growth time $\tau$ (during which the number of publications grow by a factor $e \simeq 2.7$) is about 10 years, and that for the categories quantum annealing (B) and quantum computing (D) is seen to be about 5 years (see figures 3–6). This clearly indicates remarkable activity in the research fields quantum
Figure 6. Annual number of publications ($n_p$) searched from Google Scholar (accessed date: 8 April 2022) with the exact phrase ‘quantum computing’ in the document title or abstract starting from 1980 ($t = t_0$) up to 2021. An exponential growth in $n_p$ is observed over these years. The data are fitted to $n_p \sim \exp\left(\frac{(t - t_0)}{\tau}\right)$ with $\tau \simeq 5.3$ years. We have marked some of the events: PB denotes Benioff [112]; RPF denotes Feynman [2]; DD denotes Deutsch [113]; NC denotes Nielsen & Chuang [53]; NP denotes the Nobel Prize (physics) in October 2012 that Haroche & Winerland received together for their work which ‘hold promise for the creation of practical quantum computers’ [114]; TTC denotes Tanaka et al. [115]. The inset shows the direct functional relationship between $n_p$ and $t$ for category D from 1980 to 2021. (Online version in colour.)

Table 4. Fitting parameters for equation (2.1)

| Category (research topic) | $a$ | $t_0$ (calendar year) | $\tau$ (years) |
|---------------------------|-----|-----------------------|-----------------|
| A (QISG)                  | 1.4 | 1980                  | 9.7             |
| B (QA)                    | 1.2 | 1988                  | 5.0             |
| C (QAC)                   | 25.0| 1998                  | 8.7             |
| D (QCI)                   | 7.5 | 1980                  | 5.3             |
| all categories together (A, B, C, and D) | 8.5 | 1980                  | 5.3             |

annealing (category B) and quantum computation (category D). It may be mentioned that the growth behaviour of the number of publications for all these four categories together also fits equation (2.1) with $t_0 = 1980$ and $\tau \simeq 5.3$. This growth may be compared with that of a very active contemporary field, namely, quantitative economics including econophysics (see [116]), having a growth scaling time $\tau$ of about 8 years.

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