Calculation Method of Aerodynamic Performance of Small Propeller with Serrated Trailing Edge

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Abstract. The small propeller with serrated trailing edge of the UAV has the characteristics of low aerodynamic noise. In this paper, based on the momentum-leaf theory and modified the airfoil parameters, the aerodynamic performance calculation method of the propeller with a serrated tail structure is put forward. Combined with the CDF technology of three-dimensional aerodynamic performance analysis of small propeller blades at low Reynolds number, this method was used to calculate the aerodynamic performance of the small propeller with serrated tail structure, and the calculation results were experimentally verified. The experimental results show that, using the modified airfoil parameters, the established aerodynamic performance calculation method of the tail serrated structure propeller hover state has a good effect.

1. Introduction
Because of its unique vertical take-off and landing, hovering in the air and low-speed flight capabilities, small multi-propeller UAVs are particularly suitable for completing missions in special environments such as tight spaces, and have been paid attention by research institutions in various countries in recent years. Small multi-propeller drones are commonly used in military low-altitude reconnaissance and observation tasks, and their noise has become one of the key issues to be solved urgently. The research results show that the low-Reynolds number small propeller designed with serrated features on the trailing edge can effectively reduce broadband noise [1], and its quantitative control of noise has become a hot topic in current research.

The height, density, shape, etc. of the teeth of a small propeller with serrated trailing edge affect the aerodynamic performance of the propeller. The shape of this type of airfoil with special structure is significantly different from that of the traditional airfoil. The definition of its characteristic parameters is lack of research, such as chord length and reference area, which makes the traditional aerodynamic performance calculation model not directly applicable to the small propeller with serrated trailing edge. Therefore, in order to establish a model for calculating the hover state aerodynamic performance of the tail sawtooth structure small propeller, the tail sawtooth structure small propeller characteristic parameters need to be modified to fit the aerodynamic characterization of the small propeller with serrated trailing edge.

In this paper, the small propeller with serrated trailing edge is taken as the object. based on the momentum-leaf theory and modified the airfoil parameters, the aerodynamic performance calculation method of the propeller with a serrated tail structure is put forward. The aerodynamic performance of
the small propeller with serrated tail structure is calculated by this method, and the calculation results are verified by experiment.

2. Aerodynamic performance calculation model of the small propeller with serrated trailing edge

A validated tail sawtooth structural wing model was first built as the object of study. The wing model needs to have good aerodynamic performance and noise reduction. The wing profile parameters are then corrected according to the profile characteristics of the tail sawtooth structure wing profile. The rotor aerodynamic performance calculation model is based on the corrected parameters.

2.1. Design of airfoil serrated tail structure

There are generally two ways to process the serrated structure: lengthen the tail of the original airfoil to add sawtooth and remove material at the tail to form the sawtooth. Considering that the way of removing material at the tail can maintain the original shape and excellent aerodynamic characteristics of the airfoil to a greater extent, this paper selects the propeller processed in this way as the research object.

As the main source of aerodynamic noise of various types of airfoil, the trailing edge noise has a certain research foundation on the role and mechanism of applying serrated trailing edge sawtooth to the wing, propeller and wind turbine for noise reduction. Based on past research results, we can draw two important parameters related to the geometric features of the trailing edge zigzag. First, the ratio of the half height of the saw tooth to the thickness of the boundary layer at the trailing edge cannot be less than 0.25, because if the tooth height is too small, the tail turbulence will not interact with it. Figure 1 shows the main design parameters of sawtooth. Second, in order to reduce the trailing edge noise, the aspect ratio (λ/h) of the sawtooth cannot be greater than 4, which means that the angle of the trailing edge sawtooth must be less than 45°. Existing research cannot prove that the above conditions are the optimal design criteria for the low-Reynolds number small propeller tail serration design, but studies have shown that the trailing edge serration design that meets the above conditions can achieve good aerodynamic and aeroacoustic characteristics [1].

Figure 1. Definition of sawtooth geometry parameters

As a classic symmetrical airfoil, NACA0012 airfoil has good performance under low Reynolds number and is widely used in the research of airfoil performance under low Reynolds number. In this study, the trailing edge sawtooth design was realized by removing the tail material on the basis of NACA0012 airfoil. As shown in Figure 2, the original chord length c is 20 mm. The propeller tail sawtooth height 2h and width λ are both 3 mm, which satisfies the height condition at low Reynolds number and small angle of attack [2]. In addition, the aspect ratio (λ/h) of 2 also satisfies the constraint condition of less than 4.
2.2. Calculation model

The aerodynamic performance calculation model of the small propeller with serrated tail structure is based on the traditional momentum-leaf theory. The calculation method of this method is mature, high precision and small amount of calculation. Only the aerodynamic characteristics of the airfoil need to be calculated, which avoids the difficulty of selecting the turbulence model and the poor stability of the numerical calculation for the numerical solution of the N-S equation of the propeller's full flow field. It has been widely used in the research of the propeller [3-6].

According to the momentum theory in the propeller hovering state [7], the following formula is used to calculate the tension increment $\Delta T$ of the ring-shaped propeller disc with the distance $r$ from the center of the propeller disc and the width $dr$.

$$\Delta T = 4\rho\pi v_1^2 r \Delta r$$

(1)

In the formula: $\rho$ is the air density, and $v_1$ is the induced velocity there.

Similarly, according to the leaf element theory [7], the increment of tension is as follows:

$$m = \frac{\lambda h}{(s + \lambda) h}$$

(2)

$$c' = c - 2hm$$

(3)

$$\Delta T = b\frac{D}{2} \left(Kr \right)^2 \left(\theta - \frac{v_1}{Kr}\right) c' \Delta r$$

(4)

The chord length of a propeller with a tail serration has been redefined in the formula. Considering the effect of the void created by the tail sawtooth on the lift, this paper corrects for string length based on duty cycle $m$. Since the saw teeth in this study were densely arranged, the duty cycle was one-half. In the upper equation: $s$ is the sawtooth spacing, $b$ is the number of blades, $a$ is the slope of the wing lift line, $K$ is the propeller angle velocity and $\theta$ is the blade mounting angle. By making the two equations equal, the induction velocity equation at any radius $r$ can be obtained as follow:

$$v_1 = \frac{-K}{2ac'\rho} + \sqrt{\frac{K}{2ac'\rho}} + \frac{8\pi \rho K^2 a^2 \theta c'}{8r}$$

(5)

Using the induced velocity $v_1$ of the paddle disc plane at the paddle $r$ position, it is further possible to find the angle $T$ at that position.
According to the angle of the wing type, combined with the results of the lift resistance characteristics of the wing type, we can find out the tensile force and torque of the propeller from the unit length of the propeller and the torque load. The ideal pull factor at hover is:

\[ T_{\text{ideal}} = \int_{x_0}^1 \frac{dC_r}{d} \frac{r}{R} \]  

(7)

Where R is the propeller radius. Due to the three-dimensional flow effect of the small propeller tip and the strong tip vortex in the low Reynolds number state, the blade load is zero for a distance near the tip. If this loss is ignored, the pull of the small propeller is significantly overestimated. After taking into account the paddle tip loss factor B, the paddle plate tension factor can be obtained.

\[ B = 1 - \frac{\sqrt{2C_{T_{\text{ideal}}}}}{b} \]  

(8)

\[ C_r = C_{T_{\text{ideal}}} - \int_{\beta}^{1} \frac{dC_r}{d} \frac{r}{R} \]  

(9)

The torque coefficient of the propeller blade is as follows:

\[ m_{K_0} = \int_{0}^{1} \frac{dm_{K_0}}{d} \frac{r}{R} \]  

(10)

\[ m_{K_i} = \int_{0}^{\beta} \frac{dm_{K_i}}{d} \frac{r}{R} \]  

(11)

\[ m_{K} = m_{K_0} + m_{K_i} \]  

(12)

2.3. Procedure for calculating the aerodynamic performance of small propellers with a serrated tail

The criterion for direct evaluation of the aerodynamic performance of a small propeller under hover is the tensile force and torque of the propeller at different mounting angles and at different revolutions. The block diagram of the program to calculate the aerodynamic performance in the small propeller hover state is shown in Figure 3.
Based on the above procedure for calculating the aerodynamic performance of a small propeller with a tail sawtooth structure, it is possible to calculate the aerodynamic characteristics of a small propeller with known geometric parameters of the propeller, its operating state and the aerodynamic performance of the propeller blade.

3. Calculation and analysis of low Reynolds number aerodynamic characteristics of the tail sawtooth structure wing

Computational models of propeller aerodynamic performance based on momentum-foil theory require known aerodynamic performance of the propeller blade type. The aerodynamic properties of the wing are the coefficients of lift and drag of the wing with different Reynolds numbers. The aerodynamic performance of the wing type can be obtained by experiment or simulation, and this paper uses CFD simulation to obtain it.

3.1. Calculation of the aerodynamic characteristics of the wing with a serrated tail at low Reynolds number

Two-dimensional CDF calculations cannot reflect the effect of the wing tail sawtooth on the aerodynamic characteristics of the wing, so they need to be calculated by a three-dimensional grid. The turbulent flow model is modeled using the Spalart-Allmaras model and the pressure-velocity coupling using the SIMPLE algorithm [8, 9]. Momentum equations, energy equations, and turbulent viscosity are dissipated in a second-order headwind format.

The calculation uses manually separated structured grids, which generate grids with good orthogonality of the grid cells in the wall area, and can better simulate boundary layer flow and turbulence. The calculations are performed using the C-type calculation domain, as shown in Figure 4, where the entrance boundary is 10C from the wing-shaped tail and the exit boundary is 10C from the...
wing-shaped tail, and the grid plotting region is divided into six regions on the wing-shaped surface, with the highest grid density at the head and tail. A structured hexahedral mesh (see Figure 5) was used for all computational domains, and the winged surface was locally encrypted with a minimum mesh mass of 0.68.

![Figure 4. Computational domain](image)

![Figure 5. Grid around the wing type](image)

Small propellers generally operate in the \( Re < 150,000 \) range. To illustrate the validity of the numerical simulations, the aerodynamic characteristics of the original NACA0012 wing type at \( Re = 100000 \) were first calculated. As can be seen in Figure 6, the results of the calculations agree well with the experimental results of NACA0012 under the same external conditions in the literature [10], which explains the accuracy of the CDF calculations in this paper.

![Figure 6. Comparison of the calculated lift resistance characteristics of the NACA0012 wing type at \( Re = 100000 \) with the reference values in the literature](image)

3.2. Low Reynolds number aerodynamic characterization of the tail sawtooth structure wing
The results of the pneumatic simulation of the tail sawtooth wing type are shown in Figure 7, where the reference area used in the calculation of the lift and drag coefficients is the actual projected area. The reference length used in the Reynolds number calculation is \( c \).
As can be seen from Figure 7, the aerodynamic performance of the tail sawtooth construction small propeller decreases with decreasing Reynolds number. At low Reynolds number, the propeller boundary layer flow has a strong viscous effect, which leads to the creation of separation vesicles. With the reduction of the number of Renault, the separation of the bubble will gradually increase, eventually leading to a reduction in aerodynamic performance, as manifested in the lift coefficient and lift resistance ratio of the reduction. This trend exists for the original NACA0012 wing type at low Reynolds number [10]. The data in Fig. 7 show that the aerodynamic performance of the wing type still decreases with decreasing Reynolds number.

At low Reynolds number conditions, the wing type with a trailing edge sawtooth structure has a certain variation in the slope of the lift coefficient with the change in the angle of the headlong curve in the small headlong range at different Reynolds number conditions. There is a good linearity in the curve of the lift coefficient with respect to the change of the angle of welcome in the E region. The slope of the curve slope begins to decrease with the change in the onset angle in the F region of the lift coefficient, reflecting the effect of the viscous effect. Calculation of the propeller aerodynamic performance based on momentum-foil theory requires the calculation of a certain width of paddle plate lift by the slope of the lift line. A better linearity at a smaller angle can effectively guarantee the accuracy of the calculation.

![Figure 7. Curves of wing-type lift coefficient, drag coefficient and lift/resistance ratio of the serrated trailing edge with onward angle.](image)

4. Calculation and analysis of low Reynolds number aerodynamic characteristics of the tail sawtooth structure wing

The aerodynamic characteristics of the small propeller hover state are experimentally verified. The main parameters are the tension and torque of the propeller at different mounting angles and different revolutions. For this purpose, a small propeller test stand was designed to measure small propeller pull and torque.
4.1. Experimental device design

Brushless motor is used for propeller power and PWM is used for speed regulation. Rotation measurement is carried out using photoelectric non-contact measurement methods. To avoid ground effects, the propeller is reversed during operation, the force sensor measures pressure and the torque sensor measures normally. Brushless motor model X4108S, KV value 690, force sensor model DYZ-101, range 0-5N, torque sensor model DYJN-104, range 0-0.5N-m. The test device's tension and torque error are within 1%, propeller blade error is within 2%. The actual objects are shown in Figure 8 below.

![Image of experimental device](image1.png)

**Figure 8.** Small rotary wing aerodynamic characterization experiment.

4.2. Experimental model of a small propeller with a serrated tail

The paddle used for the experiments and calculations was the NACA0012 wing type with serrated teeth, the shape of which is shown in Figure 9. For calculation accuracy, the small propeller is designed to be twist-free. To ensure the quality of the sawing, the propellers are machined using 3D printing and the material is made of photosensitive resin. The blade diameter is 230 mm and the mounting angle $\alpha$ is 6° and 10° respectively.

![Image of paddle](image2.png)

**Figure 9.** Shape of the paddle.

4.3. Comparison of experimental and computational results

To validate the established method of pneumatic characterization of small propellers with a tail sawtooth structure. The experiment measured the relationship between the propeller pull and torque of the paddle at different revolutions and the results are shown in Figure 10. The theoretical calculation results in Figure 10 are obtained for the rotary wing based on the aerodynamic performance of the wing type in Figure 7 under the performance calculation model in this paper.
From the comparison of the curves in the figure 10, it can be seen that the small propeller pull and torque calculations are in good match with the test results, and the maximum errors of pull and torque are within 3%, which verify the feasibility of the model and calculation method.

5. Conclusion
In this paper, a set of computational methods for the aerodynamic performance of small rotors with tail sawtooth structures is established based on the corrected reference area and chord length using momentum-foil theory. The aerodynamic performance of the rotary wing is calculated based on the aerodynamic parameters obtained by CFD technology, and the results are verified by experiment. An analytical study of the aerodynamic performance of a small propeller with a sawtooth tail structure was carried out and the following conclusions were drawn.

(1) At low Reynolds numbers, the aerodynamic performance of the wing type with a serrated trailing edge structure decreases with decreasing Reynolds numbers. This is shown by the decrease of the lift coefficient and the lift-resistance ratio of the wing type as the Reynolds number decreases, reflecting the effect of increased viscosity at low Reynolds numbers.

(2) The tail sawtooth structure wing type has a good linear relationship between the coefficient of lift and the coefficient of drag with the change of the angle of the head-on curve in the small range of the head-on angle at low Reynolds number conditions. This feature makes the tail sawtooth structure small propeller more suitable for momentum-foil theory computational models.

(3) After comparison and analysis with the test data, the method established in this paper based on the modified wing type parameters can be used to solve the problem of calculating the aerodynamic performance of the small propeller of the tail sawtooth structure.

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