A new understanding of fermion masses from the unified theory of spins and charges

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Abstract

In this letter we try to answer those of the open questions of the Standard model which concern the appearance of families, mass protection mechanism and the Yukawa couplings - by using the approach (proposed by one of us [1, 2, 3, 4, 5, 6, 7, 8]), which suggests a new way beyond the Standard model. The approach has in the starting action for fermions, which carry in \(d = 1 + 3\)–dimensional space only the spin (two kinds of the spin) and interact with only spin connection and vielbein fields, the term manifesting as a mass term in \(d = 1 + 3\). (After making several approximations and assumptions) we connect free parameters of the approach with the experimental data and investigate a possibility that the fourth family appears at low enough energies to be observable in the new generation of accelerators.

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Introduction: The Standard model of the electroweak and strong interactions leaves unanswered many open questions, among which are also the questions about the origin of the families, the Yukawa couplings of quarks and leptons and the corresponding Higgs mechanism and the weak scale. Understanding the mechanism for generating families, their masses and mixing matrices might be one of the most promising ways to physics beyond the Standard model. The approach, unifying spins and charges\[1, 2, 3, 4, 5, 6, 7, 8\], might by offering a new way of describing families and mass matrices, give an explanation about the origin of the Yukawa couplings and show a way beyond the Standard Model. It was demonstrated in the refs.\[5, 6, 7\] that a left handed SO(1, 13) Weyl spinor multiplet includes, if the representation is analyzed in terms of the subgroups SO(1, 3), SU(2), SU(3) and the sum of the two U(1)’s, all the spinors of the Standard model - that is the left handed SU(2) doublets and the right handed SU(2) singlets of (with the group SU(3) charged) quarks and (chargeless) leptons - answering the question where does originate the connection between the weak charge and the handedness (which concerns only the spin in \(d = 1 + 3\)).

The approach assumes two kinds of the spin connection fields - the gauge fields of the two kinds of the Clifford algebra objects - and a vielbein field in \(d = (1+13)\)–dimensional space, which might\[19\] manifest - after some appropriate compactifications (or some other kind of making the rest of \(d - 4\) space unobservable at low energies) - in the four dimensional space as all the gauge fields of the known charges, as well as the Yukawa couplings, determining mass matrices of families of quarks and leptons and accordingly their masses and mixing matrices. This letter is a short review of the two papers\[7, 8\], which analyze how do terms, which lead to masses of quarks and leptons, appear in the approach unifying spins and charges as a part of the (vacuum expectation values of) spin connection and vielbein fields. No Higgs is needed in this approach to “dress” right handed spinors with the weak charge, since the terms of the starting Lagrangean, which include \(\gamma^0\gamma^s\), with \(s = 7, 8\), do the job of a Higgs field of the Standard model.

Two kinds of the Clifford algebra objects: We assume two kinds of the Clifford algebra objects defining two kinds of the generators of the Lorentz algebra\[1, 2, 11, 12\]. One kind are the ordinary Dirac \(\gamma^a\) operators defining the generators of the Poincaré algebra for spinors \(S^{ab}\) (\(S^{ab} = \frac{1}{4}(\gamma^a\gamma^b - \gamma^b\gamma^a)\)). The second kind\[20\] of the Clifford objects \(\tilde{\gamma}^a\) commutes with \(\gamma^a\).
(\{\tilde{\gamma}^a, \gamma^a\}_+ = 0) and defines accordingly \(\tilde{S}^{ab}(\tilde{S}^{ab} = \frac{1}{2}(\tilde{\gamma}^a\gamma^b - \gamma^b\tilde{\gamma}^a))\), with \(\{\tilde{S}^{ab}, S^{cd}\}_- = 0\). They are responsible for the generation of families.

We define a basis of spinor representations as eigen states of the chosen Cartan subalgebra of the Lorentz algebra \(SO(1, 13)\), with the operators \(S^{ab}\) and \(\tilde{S}^{ab}\) in the two Cartan subalgebra sets, with the same indices in both cases. When introducing the notation \([7, 11, 12]\)

\[
(\pm i): = \frac{1}{2}(\gamma^a \mp \gamma^b), \quad [\pm i]: = \frac{1}{2}(1 \pm \gamma^a\gamma^b), \quad \text{for } \eta^{aa}\eta^{bb} = -1,
\]

\[
(\pm): = \frac{1}{2}(\gamma^a \pm i\gamma^b), \quad [\pm]: = \frac{1}{2}(1 \pm i\gamma^a\gamma^b), \quad \text{for } \eta^{aa}\eta^{bb} = 1,
\]

it can be shown that the above binomials are all ”eigen vectors” of the generators \(S^{ab}\), as well as of \(\tilde{S}^{ab}\)

\[
S^{ab}(k) = \frac{k}{2} (k), \quad S^{ab} [k] = \frac{k}{2} [k],
\]

\[
\tilde{S}^{ab}(k) = \frac{k}{2} (k), \quad \tilde{S}^{ab} [k] = -\frac{k}{2} [k].
\]

Defining \((k) = \frac{1}{2}(\tilde{\gamma}^a + \frac{i}{k}\gamma^b)\) we find the relations

\[
\gamma^a (k) = \eta^{aa} [k], \quad \gamma^b (k) = -ik [-k],
\]

\[
\gamma^a [k] = (-k), \quad \gamma^b [k] = -ik\eta^{aa} (-k),
\]

\[
\tilde{\gamma}^a (k) = -i\eta^{aa} [k], \quad \tilde{\gamma}^b (k) = -k [k],
\]

\[
\hat{\gamma}^a [k] = i (k), \quad \hat{\gamma}^b [k] = -k\eta^{aa} (k),
\]

\[
(k)(k) = 0, \quad (-k)(-k) = \eta^{aa} [k], \quad (k)[k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k][k]
TABLE I: The 8-plet of quarks - the members of $SO(1,7)$ subgroup, belonging to one Weyl left handed ($\Gamma^{(1,3)} = -1 = \Gamma^{(1,7)} \times \Gamma^{(6)}$) spinor representation of $SO(1,13)$. It contains the left handed weak charged quarks and the right handed weak chargeless quarks of a particular colour ($(1/2, 1/(2\sqrt{3}))$). Here $\Gamma^{(1,3)}$ defines the handedness in $(1 + 3)$ space, $S^{12}$ defines the ordinary spin (which can also be read directly from the basic vector, since, in particular, $S^{12} = \frac{12}{2} (\pm)$), $\tau^{13}$ defines the third component of the weak charge, $\tau^{21}$ defines the $U(1)$ charge, $\tau^{33}$ and $\tau^{38}$ define the colour charge and $\tau^{41}$ another $U(1)$ charge, which together with the first $U(1)$ charge defines $Y = \tau^{21} + \tau^{41}$ and $Y' = -\tau^{21} + \tau^{41}$. Leptons differ from quarks only with respect to the part which concerns the indices 9, · · ·, 14. A singlet in the colour, which belongs to the same Weyl, must be of the kind · · · | · · · || $(+)[+][+]$ (one finds that $\nu_R$ looks like $(+i)(+)|(+)(+)||(+)[+]$) The reader can find the whole Weyl representation in the ref. [6].

| $i$ | $|\psi_i>$ | $\Gamma^{(1,3)}$ | $S^{12}$ | $\tau^{13}$ | $\tau^{21}$ | $\tau^{33}$ | $\tau^{38}$ | $\tau^{41}$ | $Y$ | $Y'$ |
|-----|------------|-----------------|---------|------------|---------|---------|---------|---------|-----|-----|
| 1   | $u_R^{1L}$ | $\frac{03}{12}$ | 56 78 9 1011 1213 14 | 1 1/2 | 1 0 1/2 | 1/2 1/(2\sqrt{3}) 1/6 | 2 3 1/3 |
| 2   | $u_R^{1R}$ | $\frac{03}{12}$ | 56 78 9 1011 1213 14 | 1 -1/2 | 1 0 1/2 | 1/2 1/(2\sqrt{3}) 1/6 | 2 3 1/3 |
| 3   | $d_R^{1L}$ | $\frac{03}{12}$ | 56 78 9 1011 1213 14 | 1 1/2 | 1 0 -1/2 | 1/2 1/(2\sqrt{3}) 1/6 | -1/3 2/3 |
| 4   | $d_R^{1R}$ | $\frac{03}{12}$ | 56 78 9 1011 1213 14 | 1 -1/2 | 1 0 -1/2 | 1/2 1/(2\sqrt{3}) 1/6 | -1/3 2/3 |
| 5   | $u_L^{1L}$ | $\frac{03}{12}$ | 56 78 9 1011 1213 14 | -1 1/2 | -1 -1/2 | 0 1/2 1/(2\sqrt{3}) 1/6 | 1/6 1/6 |
| 6   | $u_L^{1R}$ | $\frac{03}{12}$ | 56 78 9 1011 1213 14 | -1 -1/2 | -1 -1/2 | 0 1/2 1/(2\sqrt{3}) 1/6 | 1/6 1/6 |
| 7   | $u_L^{1L}$ | $\frac{03}{12}$ | 56 78 9 1011 1213 14 | -1 1/2 | -1 -1/2 | 0 1/2 1/(2\sqrt{3}) 1/6 | 1/6 1/6 |
| 8   | $u_L^{1R}$ | $\frac{03}{12}$ | 56 78 9 1011 1213 14 | -1 -1/2 | -1 -1/2 | 0 1/2 1/(2\sqrt{3}) 1/6 | 1/6 1/6 |

The reader can find the whole Weyl representation in the ref. [6].
Assuming that a kind of breaking symmetries[10] makes a starting Weyl spinor in \(d = 1 + 13\) to manifest after a break into \(SO(1,7) \times SU(3) \times U(1)\) as massless spinors - one \(SU(3)\) triplet and one \(SU(3)\) singlet - each of them a member of an \(SO(1,7)\) octet[21], there must be accordingly also eight families since one can easily notice that each member of the octet on Table II carries two indices: the index of the row and the family index. Namely, any \(\tilde{S}_{ab}; a, b \in 0,1..,8\), not belonging to the Cartan subalgebra, transforms any member of the starting family, represented on Table I, into another family, with the same spin and charges.

A Weyl spinor in \(d = (1 + 13)\) in the two kinds of spin connection fields: A spinor carries only the spin (no charges) and interacts accordingly with only the gauge gravitational fields - with vielbeins and two kinds of spin connection fields - the gauge fields of \(p^a, S^{ab}\) and \(\tilde{S}^{ab}\), respectively. One kind is the ordinary gauge field (gauging the Poincaré symmetry in \(d = 1 + 13\)). The contribution of this field to the mass matrices manifests in only the diagonal terms - connecting the right handed weak chargeless quarks or leptons to the left handed weak charged partners within one family of spinors. The second kind of gauge fields is in our approach responsible for families of spinors and couplings among families of spinors - contributing to diagonal matrix elements as well - and might explain the appearance of families of quarks and leptons and the Yukawa couplings of the Standard model. We write the action[7] for a Weyl (massless) spinor in \(d(= 1 + 13)\) - dimensional space as follows[22]

\[
S = \int d^d x \mathcal{L}
\]

\[
\mathcal{L} = \frac{1}{2} (\bar{\psi} \gamma^\alpha p_0 \psi) + h.c. = \frac{1}{2} (\bar{E} \bar{\psi} \gamma^\alpha f_\alpha p_0 \psi) + h.c.
\]

\[
= \bar{\psi} \gamma^\alpha (p_m - \sum A_i^A \omega^{Ai \alpha} + \sum_{s=7,8} \bar{\psi} \gamma^s p_m \psi) + \text{the rest},
\]

\[
p_0 = p_\alpha - \frac{1}{2} S^{ab} \omega_{aba} - \frac{1}{2} \tilde{S}^{ab} \tilde{\omega}_{aba}.
\]

Here \(f_\alpha^a\) are vielbeins (inverted to the gauge field of the generators of translations \(e_\alpha^a\), \(e_\alpha^a f_\beta^b = \delta_\beta^b\), \(e_\alpha^a f_\beta^a = \delta_\alpha^\beta\)), with \(E = \det(e_\alpha^a)\), while \(\omega_{aba}\) and \(\tilde{\omega}_{aba}\) are the two kinds of the spin connection fields, the gauge fields of \(S^{ab}\) and \(\tilde{S}^{ab}\), respectively.
index $A$ determines the charge groups ($SU(3), SU(2)$ and the two $U(1)$’s), index $i$ determines the generators within one charge group. $\tau^{Ai}$ denote the generators of the charge groups $\tau^{Ai} = \sum_{s,t} c^{Ai}_{st} S_{st}$, $\{\tau^{Ai}, \tau^{Bj}\} = i\delta^{AB} f^{Ai,jk} \tau^{Ak}$, with $s, t \in 5, 6, \ldots, 14$, while $A^m_m, m = 0, 1, 2, 3$, denote the corresponding gauge fields (expressible in terms of $\omega_{stn}$).

We rewrote the Cartan subalgebra pairs. Accordingly all the pairs $(a, b)$ in the first sum runs over all the indices, with $a, b = 0, \ldots, 8$, while the pairs in the second and the third sum $(cd), (ac), (bd)$ denote only the Cartan subalgebra pairs. The Yukawa part of the starting Lagrangean (Eq. (4)) has the diagonal terms, that is the terms manifesting the Yukawa couplings within each family, and the off diagonal terms, determining the Yukawa couplings among families, as we shall demonstrate bellow.

One notices that in Eq. (5) only the part with the factor $\gamma^0 \bar{\psi} \gamma^5 p_0 \psi$ in Eq. (4) manifests as the Yukawa couplings of the Standard model and we rewrite it as $\mathcal{L}_Y$ in the following way

$$\mathcal{L}_Y = \bar{\psi} \gamma^0 \left\{ \frac{78}{2} \left( \sum_{y = Y, Y'} + \frac{1}{2} \sum_{(ab)} \tilde{S}^{ab} \omega_{ab+} \right) + \frac{78}{2} \left( \sum_{y = Y, Y'} - \frac{1}{2} \sum_{(ab)} \tilde{S}^{ab} \omega_{ab-} \right) + \sum_{(ac)(bd), k, l} \tilde{A}^{kl}_{\pm} ((ac), (bd)) \right\} \psi, \quad (5)$$

with $k, l = \pm 1$, if $\eta^{\alpha \alpha} \eta^{\beta \beta} = 1$ and $\pm i$, if $\eta^{\alpha \alpha} \eta^{\beta \beta} = -1$, while $Y$ and $Y'$ are the two superpositions of the two $U(1)$ subgroups of the groups $SO(6)$ and $SO(1, 7)$ as defined in refs. [7, 8].

We rewrote $\sum_{(ab)} - \frac{1}{2} \frac{78}{2} \left( \pm \right) \tilde{S}^{ab} \omega_{ab \pm} = \sum_{(cd)} - \frac{1}{2} \frac{78}{2} \left( \pm \right) \tilde{S}^{ab} \omega_{ab \pm} + \sum_{(ac), (bd), k, l} \frac{78}{2} \left( \pm \right) \tilde{A}^{kl}_{\pm} ((ac), (bd))$, where the pair $(a, b)$ in the first sum runs over all the indices, with $a, b = 0, \ldots, 8$, while the pairs in the second and the third sum $(cd), (ac), (bd)$ denote only the Cartan subalgebra pairs. Accordingly all the pairs $(ab)$ in Eq. (5) run only over Cartan subalgebra pairs. The Yukawa part of the starting Lagrangean (Eq. (4)) has the diagonal terms, that is the terms manifesting the Yukawa couplings within each family, and the off diagonal terms, determining the Yukawa couplings among families, as we shall demonstrate bellow.
members of one family, they only differ in values for different families. The last two sums in Eq.(5) contribute to the non diagonal matrix elements of either the $u$-quarks and the neutrinos (the term with the factor $\gamma^0 \gamma^7 \gamma^8$) or to the $d$-quarks and the electrons (the term with the factor $\gamma^0 \gamma^7 \gamma^8$).

**Breaks of symmetries and observable properties:** We made the assumption that a break of symmetries leads from one Weyl representation in $d = 1 + 13$ to four massless octets (the representation of $SO(1,7)$) with the charges $SU(3), Y$ and $Y'$ as presented above, leaving us with eight equivalent representations - that is with eight families. In order to be in agreement with what we observe, we must break further the symmetry of the octet in the charge sector. Namely, looking at Table I and recognizing that $Q = \tau^{33} + Y = S^{56} + \tau^{41}$ must appear as a conserved quantity representing the electromagnetic charge, we assume that no terms of the types $S^{5a}_a \omega^{5a}_5$ and $S^{6a}_a \omega^{6a}_6$, with $a \neq 5, 6$ may appear in our $\mathcal{L}_Y$. Assuming that the break influences both sectors - $S^{ab}$ and $\tilde{S}^{ab}$ - in a similar way, we let also all the terms $\tilde{S}^{5a}_a \tilde{\omega}^{5a}_5$ and $\tilde{S}^{6a}_a \tilde{\omega}^{6a}_6$, with $a \neq 5, 6$, contribute nothing, which means that we assume the break of $SO(1,7)$ into $SO(1,5) \times U(1)$, which further means that eight families decouple entirely into two times four families. We shall not try to justify better this assumption in this letter. Instead, we shall study, what can we learn from our approach (after all these assumptions about the appropriate breaks of the starting symmetry). The first row on Table I, representing the $u_R$ quark, then appears in the following four families (while all the other members of a particular family follow from the one for $u_R$ by the application of $S^{ab}; a, b \in \{0, 8\}$ and equivalently for the quarks of other two colours and for the colourless leptons)

$$\begin{align*}
I. & \quad \begin{array}{c}
03 \\
12 \\
56 \\
78
\end{array} \\
& \quad (+i)(+) \mid (+)(+) ||\ldots \\
\hline
II. & \quad \begin{array}{c}
03 \\
12 \\
56 \\
78
\end{array} \\
& \quad [+i][+] \mid (+)(+) ||\ldots \\
\hline
III. & \quad \begin{array}{c}
03 \\
12 \\
56 \\
78
\end{array} \\
& \quad [+i](+) \mid (+)[+] ||\ldots \\
\hline
IV. & \quad \begin{array}{c}
03 \\
12 \\
56 \\
78
\end{array} \\
& \quad (+i)[+] \mid (+)[+] ||\ldots .
\end{align*}$$

(6)

Obviously, starting from one Weyl in $d$ we only can have an even number of families. We have measured up to now three families. If we further break the symmetry, like $SO(1,5)$ into $SU(2) \times SU(2) \times U(1)$, by letting, for example, all the terms $\tilde{S}^{7a}_a \tilde{\omega}^{7a}_7$ and $\tilde{S}^{8a}_a \tilde{\omega}^{8a}_8$, $a \neq 7, 8$, contribute nothing, we shall end up with twice two completely decoupled families. If we
TABLE II: The mass matrix of four families of quarks and leptons, obtained within the approach unifying spins and charges under the assumptions[7] that an appropriate break of the starting symmetry to \(SO(1, 7) \times SU(3) \times U(1)\) leads to massless quarks and leptons, while in further breaks the electromagnetic charge is conserved - and equivalently in the \(\tilde{\omega}_{abc}\) sector - that an approximate break occurs further from \(SO(1, 5)\) to \(SU(2) \times SU(2) \times U(1)\) leading to the symmetry of Eq.(7) (ref.[8]), that the mass matrices are real and symmetric and that evaluation can be done on "a tree level".

break instead \(SO(1, 5)\) into \(SU(3) \times U(1)\), one family decouples from the rest three. The experimental data seems at least for quarks to be closer to an assumption of an approximate break of \(SO(1, 5)\) to \(SU(2) \times SU(2) \times U(1)\), since the first two families are much lighter than the third and also the quark mixing matrix seems not to disagree with such an assumption.

We shall accordingly assume the following approximate shape of any of the mass matrices for the quarks and the leptons

\[
\begin{pmatrix}
A & B \\
B & C = A + kB
\end{pmatrix}
\]  

(7)

with \(A, B, C\) which are two by two matrices and where \(k\) is a constant. All these matrices ought to be calculable in terms of the fields in Eq.(5). We shall assume that after the suggested breaks of symmetries the fields appearing in Eq.(5) take a vacuum expectation values and (after the integration over all but 1 + 3 dimensions) manifest in \(d = 1 + 3\) as parameters. We shall further simplify our mass matrices by assuming that all the matrix elements are real and that the matrices are symmetric. We then end up with the mass matrices expressed in terms of the parameters \(a_\alpha, \tilde{\omega}_{abc}\) as one finds on Table II.
The parameters on Table II are related as follows

\[ k_u = -k_d, \ k_\nu = -k_e, \ \tilde{\omega}_{387u} = -b_{387u} \tilde{\omega}_{387d}, \]
\[ \tilde{\omega}_{127u} = b_{127u} \tilde{\omega}_{127d}, \ \tilde{\omega}_{078u} = b_{127u} \tilde{\omega}_{078d}, \]
\[ \tilde{\omega}_{018u} = -b_{018u} \tilde{\omega}_{018d}, \ \tilde{\omega}_{187u} = b_{018u} \tilde{\omega}_{187d}, \]

(8)

and similarly for leptons, when \( u \) is replaced by \( \nu \) and \( d \) by \( e \). It is the Lagrange density \( L_Y \) of Eq.(5) which determines the relations among the parameters - after making all the discussed assumptions (for which we only have a kind of explanation, no justification since we have not yet treated breaking of symmetries and nonperturbative effects). If we assume that breaks of symmetries and nonperturbative effects influence all the quarks and the leptons in the same way, factors \( b_{abc\alpha} \) are 1 and \( \tilde{\omega}_{abc\alpha} \) are the same for quarks and leptons. Since we do not know how breaks of symmetries and nonperturbative effects influence the parameters of mass matrices, we let \( b_{abc\alpha} \) as well as \( \tilde{\omega}_{abc\alpha} \) be different for quarks and for leptons. We study what the fit to the experimental data can tell about the parameters.\[7, 8\].

The parameter \( a_\alpha \) distinguishes among the members of one family, since it is a sum of the diagonal contributions of \(-\frac{1}{2} S^{ab}_{\tilde{\omega}_{ab\pm}} \) and the contributions of \( \tau^y A_{7,8}^y, y = Y, Y' \).

Connecting free parameters of the approach with the experimental data: It is easy to see that any \( 4 \times 4 \) matrix of the form of Eq.(7) is diagonalizable with three (rather than with six) angles and that the angles of rotations are related as follows

\[ \tan(\varphi_{\alpha} - \varphi_{\beta}) = \pm \frac{k_{\alpha}}{2}, \ (\text{or } \pm \frac{2}{k_{\alpha}}), \]
\[ \tan(a,b \varphi_{\alpha} - a,b \varphi_{\beta}) = \pm \frac{a,b \eta_{\alpha}}{2}, \ (\text{or } \pm \frac{2}{a,b \eta_{\alpha}}), \]

(9)

with \( \alpha = u, \nu, \beta = d, e \), \( a, \eta_{\alpha} = \frac{\tilde{\omega}_{127a} + \frac{1}{1+(k_{\alpha}/2)^2} \tilde{\omega}_{078a}}{\tilde{\omega}_{018a} - \frac{1}{1+(k_{\alpha}/2)^2} \tilde{\omega}_{187a}} = -a \eta_{\beta} \) and \( b, \eta_{\alpha} = \frac{\tilde{\omega}_{127a} - \frac{1}{1+(k_{\alpha}/2)^2} \tilde{\omega}_{078a}}{\tilde{\omega}_{018a} + \frac{1}{1+(k_{\alpha}/2)^2} \tilde{\omega}_{187a}} = -b \eta_{\beta} \), while the index \( a \) determines the first two by two mass matrix and \( b \) determines the second two by two mass matrix after the first step diagonalization of \( 4 \times 4 \) mass matrix (determined by the angle \( \varphi_{\alpha} \)) is performed. One further finds that \( \varphi_{\alpha} = \frac{\pi}{2} - \varphi_{\beta}, \varphi = \varphi_{\alpha} - \varphi_{\beta} \), and \( a,b \varphi_{\alpha} = \frac{\pi}{2} - a,b \varphi_{\beta}, a,b \varphi = a,b \varphi_{\alpha} - a,b \varphi_{\beta} \).

The experimental mixing matrices for quarks and for leptons are not in contradiction with our (rough) assumption that they are symmetric and real (up to charge parity symmetry breaking, which in this letter is not treated) and it also turns out that both can within the
TABLE III: The Monte-Carlo fit to the experimental data [13, 14] for the three parameters $k$, $a_\eta$ and $b_\eta$ determining the mixing matrices for the four families of quarks and leptons.

|       | $u$  | $d$  | $\nu$ | $e$  |
|-------|------|------|-------|------|
| $k$   | -0.085 | 0.085 | -1.254 | 1.254 |
| $a_\eta$ | -0.229 | 0.229 | 1.584 | -1.584 |
| $b_\eta$ | 0.420 | -0.440 | -0.162 | 0.162 |

TABLE IV: Values for the parameters $\tilde{\omega}_{abc}$ (entering into the mass matrices for the $u$—quarks, the $d$—quarks, the neutrinos and the electrons suggested by the approach - Table II, Eq.8) as following after the Monte-Carlo fit relating the parameters and the experimental data.

|       | $u$ | $d$ | $b_u$ | $\nu$ | $e$ | $b_\nu$ |
|-------|-----|-----|-------|------|-----|--------|
| $|\tilde{\omega}_{018}|$ | 21205 | 42547 | 0.498 | 10729 | 21343 | 0.503 |
| $|\tilde{\omega}_{078}|$ | 49536 | 101042 | 0.490 | 31846 | 63201 | 0.504 |
| $|\tilde{\omega}_{127}|$ | 50700 | 101239 | 0.501 | 37489 | 74461 | 0.503 |
| $|\tilde{\omega}_{187}|$ | 20930 | 42485 | 0.493 | 9113 | 18075 | 0.505 |
| $|\tilde{\omega}_{387}|$ | 230055 | 114042 | 2.017 | 33124 | 67229 | 0.493 |
| $a^a$ | 94174 | 6237 | | 1149 | 1142 | |

experimental accuracy be fitted with three angles [13, 14]. Due to the refs. [15, 16, 17] the fourth family is experimentally not excluded. We proceed by fitting the experimental data of the two mixing matrices and of the masses of the known three families of quarks and leptons by the requirement that the ratios among the $\tilde{\omega}_{abc}$ (that is $b_{abc\alpha}$) are as close to the rational numbers of small integers as possible, having in mind that a kind of breaking symmetries or nonperturbative effects or both reflect the influence of charges. We use the Monte-Carlo simulation program. What we obtained [8] is represented in Table III and Table IV and in Eqs. (10, 11, 12). We see on Table IV that all the parameters $b_{abc\alpha}$ are either very close to $\frac{1}{2}$ or to 2. We find for the masses of the four families the values

$m_{ui}/GeV = (0.0034, 1.15, 176.5, 285.2)$,

$m_{di}/GeV = (0.0046, 0.11, 4.4, 224.0)$,

$m_{vi}/GeV = (1 \times 10^{-12}, 1 \times 10^{-11}, 5 \times 10^{-11}, 84.0)$,
For the quark mixing matrix we found
\[
\begin{pmatrix}
0.974 & 0.223 & 0.004 & 0.042 \\
0.223 & 0.974 & 0.042 & 0.004 \\
0.004 & 0.042 & 0.921 & 0.387 \\
0.042 & 0.004 & 0.387 & 0.921
\end{pmatrix}
\] (11)

and for the leptons we found
\[
\begin{pmatrix}
0.697 & 0.486 & 0.177 & 0.497 \\
0.486 & 0.697 & 0.497 & 0.177 \\
0.177 & 0.497 & 0.817 & 0.234 \\
0.497 & 0.177 & 0.234 & 0.817
\end{pmatrix}
\] (12)

Concluding remarks: We have studied in this letters what the approach unifying spins and charges - offering a new way beyond the Standard model of the electroweak and colour interactions - can say about the origin of families and the Yukawa couplings. The letter is a short review of the two papers [7, 8]. We have started with the action for a Weyl spinor in \( d = 1 + 13 \), which carries two kinds of spins and no charges and interacts with only the gravity. We have assumed breaks of symmetries which end up with a nonzero part of the starting Lagrange density \( \psi^t \gamma^0 (\pm) p_{0\pm} \psi \) (Eq.(5)) manifesting as Yukawa couplings - the mass term transforming a right handed weak chargeless quark or lepton into the corresponding left handed weak charged one - in \((1 + 3)-\)dimensional space. No Higgs is needed. The approach predicts an even number of families with the mass matrices determined with the two kinds of gauge fields. One kind determines only the matrix elements within a family and distinguishes among the members of a family, the second kind determines diagonal and of diagonal matrix elements. We made several assumptions and approximations to come to simple expressions, which enable us to fit parameters of the approach with the existing experimental data. Not knowing how possible breaks of symmetries influence the starting mass matrix with 10 free parameters (5 \( \bar{\omega}_{abc} \) and 4 times \( a_\alpha \) with one relation among \( a_\alpha \) and 2 times \( k_\alpha \)) we allow 22 parameters (4 times 5 \( \bar{\omega}_{abea} \) + 4 times \( a_\alpha \) plus 2 times \( k_\alpha \), which are related by 4 times \( a^{ab}_\alpha \eta_\alpha \)), which we further relate by the requirement that the ratios
\( b_{abca} \) (Eq. (8)) are as close to small rational numbers as possible (2 times 5 requirements), to be fitted using a Monte-Carlo procedure with \( 2 \times 3 \) angles and \( 4 \times 3 \) masses within the experimental accuracy.

Our rough estimate predicts that masses of the fourth family might be measurable with new accelerators. Correspondingly also mixing matrices for quarks and leptons are predicted. All these predictions must be taken with caution.

We treat quarks and leptons equivalently, assuming that breaking symmetries causes equal effects on quarks and leptons. (Assuming, for example, that quarks feel the break of \( SO(1, 5) \) approximately into \( SU(2) \times SU(2) \times U(1) \), while leptons follow the approximate break to \( SU(3) \times U(1) \), would change the prediction for leptons.) We do not take a possibility of the Majorana spinors into account.

If our approach unifying spins and charges is showing the right way beyond the Standard model, it will answer the open questions about the origin of families and the Yukawa couplings. To try to predict the properties without all the assumptions and approximations we did and without fitting parameters of the approach with the experimental data (and accordingly also predicting the weak scale) is a huge project, to which this letter (and the refs. 7, 8) is a small first step. In any case this first step might open a new window into understanding the reasons for the assumptions of the Standard model.

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[19] The approach seems to have, like all the Kaluza-Klein-like theories, a very serious disadvantage, namely that there might not exist any massless, mass protected spinors, which are, after the break of symmetries, chirally coupled to the desired (Kaluza-Klein) gauge fields.[9]. This would mean that there are no observable spinors at low energies. We have tried hard to find an example[10], a toy model, which gives a hope to Kaluza-Klein-like theories by demonstrating that a kind of a break of symmetries does lead to massless, mass protected spinors, chirally coupled to the Kaluza-Klein gauge fields, observable at low energies.

[20] The operators $\tilde{\gamma}^a$ are introduced formally as operating on any Clifford algebra object $B$ from the left hand side, but they also can be expressed in terms of the ordinary $\gamma^a$ as operating from the right hand side as follows $\tilde{\gamma}^a B := i(-)^{n_B} B_{\gamma^a}$, with $(-)^{n_B} = +1$ or $-1$, when the object $B$ has a Clifford even or odd character, respectively.

[21] Antiquarks and antileptons appear in the second quantized procedure as their charged conjugate partners.

[22] Latin indices $a, b, ..., m, n, ..., s, t, ..$ denote a tangent space (a flat index), while Greek indices $\alpha, \beta, ..., \mu, \nu, .., \sigma, \tau, ..$ denote an Einstein index (a curved index). Letters from the beginning of both the alphabets indicate a general index ($a, b, c, ..$ and $\alpha, \beta, \gamma, ..$), from the middle of both
the alphabets the observed dimensions 0, 1, 2, 3 (m, n, .. and µ, ν, ..), indices from the bottom of the alphabets indicate the compactified dimensions (s, t, .. and σ, τ, ..). We assume the signature \( \eta^{ab} = diag\{1, -1, -1, \cdots, -1\} \).