Using the $K^+ p \rightarrow \pi^+ K^+ n$ reaction to determine the $\Theta^+$ quantum numbers

E. Oset, a T. Hyodo b and A. Hosaka b

a Departmento de Física Teórica and IFIC, Centro Mixto Universidad de Valencia-CSIC, Institutos de Investigación de Paterna, Aptd. 22085, 46071 Valencia, Spain

b Research Center for Nuclear Physics (RCNP), Ibaraki, Osaka 567-0047, Japan

We study the $K^+ p \rightarrow \pi^+ K^+ n$ reaction with some kinematics suited to the production of the $\Theta^+$ resonance recently observed and show that the measurement of cross sections and polarization observables can shed light on the spin, isospin and parity of the $\Theta^+$ state.

1. The $K^+ p \rightarrow \pi^+ K^+ n$ reaction and the $\Theta^+$ quantum numbers

A recent experiment by LEPS collaboration at SPring-8/Osaka [1] has found a clear signal for an $S = +1$ positive charge resonance around 1540 MeV. The finding, also confirmed by DIANA at ITEP [2], CLAS at Jefferson Lab. [3] and SAPHIR at ELSA [4], might correspond to the exotic state predicted by Diakonov et al. in Ref. [5].

The challenge now is to determine the $\Theta^+$ quantum numbers through some reaction. We present one particularly suited reaction [6] with the process

$$K^+ p \rightarrow \pi^+ K^+ n .$$

(1)

which has some peculiar features since there are no resonances in the initial state and by choosing small momenta of the $\pi^+$, we shall be also far away from the $\Delta^+$ resonance and hence the $K^+ n(\Theta^+)$ resonance signal can be more clearly seen.

A successful model for the reaction Eq. [7] was considered in Ref. [7], consisting of the mechanisms depicted in terms of Feynman diagrams in Fig. 1. The term (a) (pion pole) and (b) (contact term) which are easily obtained from the chiral Lagrangians involving meson-meson [8] and meson-baryon interaction [9] are spin flip terms (proportional to $\sigma$), while the $\rho$ exchange term (diagram (c)) contains both a spin flip and a non spin flip

![Feynman diagrams](image)

Figure 1. Feynman diagrams of the reaction $K^+ p \rightarrow \pi^+ K^+ n$ in the model of Ref. [7].
part. Having an amplitude proportional to \( \sigma \) is important in the present context in order to have a test of the parity of the resonance. Hence we choose a situation, with the final pion momentum \( p_{\pi^+} \) small compared to the momentum of the initial kaon, such that the diagram (c), which contains the \( S \cdot p_{\pi^+} \) operator can be safely neglected. The terms of Fig. 1 (a) and (b) will provide the bulk for this reaction. If there is a resonant state for \( K^+n \) then this will be seen in the final state interaction of this system. This means that in addition to the diagrams (a) and (b) of Fig. 1 we shall have those in Fig. 2. If the resonance is an \( s \)-wave \( K^+n \) resonance then \( J^P = 1/2^- \). If it is a \( p \)-wave resonance, we can have \( J^P = 1/2^+, 3/2^+ \).

The restriction to have small pion momenta eliminates also other possible mechanisms involving crossed resonance exchange.

We write the couplings of the resonance to \( K^+n \) as \( g_{K^+n} \), \( \bar{g}_{K^+n} \) and \( \tilde{g}_{K^+n} \) for \( s \)-wave and \( p \)-wave with \( J^P = 1/2^+, 3/2^+ \) respectively, and relate them to the \( \Theta^+ \) width via

\[
\begin{align*}
g_{K^+n}^2 & = \frac{\pi M_R \Gamma}{Mq}, \\
\bar{g}_{K^+n}^2 & = \frac{\pi M_R \Gamma}{Mq^3}, \\
\tilde{g}_{K^+n}^2 & = \frac{3\pi M_R \Gamma}{Mq^3}.
\end{align*}
\]

(2)

A straightforward evaluation of the meson pole and contact terms leads to the \( K^+n \rightarrow \pi^+K^0 \) amplitudes

\[
- \tilde{t}_i = a_i \sigma \cdot k_{in} + b_i \sigma \cdot q',
\]

(3)

where \( i = 1, 2 \) stands for the final state \( K^+n, K^0p \) respectively and \( k_{in} \) and \( q' \) are the initial and final \( K^+ \) momenta.

Now let us turn to the resonance diagrams of Fig. 2 containing a loop integral, which is initiated by the tree diagrams of Figs. 1 (a) and (b). When taking into account \( KN \) scattering through the \( \Theta^+ \) resonance, as depicted in Fig. 2 the \( K^+p \rightarrow \pi^+K^+n \) amplitude is given by

\[
- \tilde{t}_i = -\tilde{t}_1 - i\tilde{t}_1 - i\tilde{t}_2
\]

(4)

where \( \tilde{t}_1 \) and \( \tilde{t}_2 \) account for the scattering terms with intermediate \( K^+n \) and \( K^0p \), respectively. They are given by

\[
- i\tilde{t}_1^{(s)} = c_i \sigma \cdot k_{in},
\]

Figure 2. Feynman diagrams of the reaction \( K^+p \rightarrow \pi^+K^+n \) with the \( \Theta^+ \) resonance.
Figure 3. The double differential cross sections $d^2\sigma/dM_I d\cos\theta$ with $\theta = 0$ (forward direction) for $I = 0, 1$ and $J^P = 1/2^-, 1/2^+, 3/2^+$. Below, detail of the lower part of the upper figure of the panel.

\[ -i\tilde{t}_i^{(p,1/2)} = d_i \sigma \cdot q', \]
\[ -i\tilde{t}_i^{(p,3/2)} = f_i \sigma \cdot k_{in} - g_i \sigma \cdot q', \]

for $s$- and $p$-wave, and $i = 1, 2$ for $K^+n$ and $K^0p$ respectively.

We take an initial three momenta of $K^+$ in the Laboratory frame $k_{in}(Lab) = 850$ MeV/c ($\sqrt{s} = 1722$ MeV), which allows us to span $K^+n$ invariant masses up to $M_I = 1580$ MeV, thus covering the peak of the $\Theta^+$, and still is small enough to have negligible $\pi^+$ momenta with respect to the one of the incoming $K^+$.

In Fig. 3 we show the invariant mass distribution $d^2\sigma/dM_I d\cos\theta$ in the $K^+$ forward direction ($\theta = 0$). Here we see that, independently of the quantum numbers of $\Theta^+$, a resonance signal is always observed. The signals for the resonance are quite clear for the case of $I, J^P = 0, 1/2^+$ (these would be the quantum numbers predicted in Ref. [5]) and $I, J^P = 0, 1/2^-$, while in the other cases the signal is weaker and the background more important, particularly for the case of $I, J^P = 0, 1/2^+$.

The calculations are done with a $\Theta^+$ width of 20 MeV, an experimental upper bound. Should the width be smaller, the strength at the peak of our calculation would be the same but the distributions would be narrower.

Let us now see what can one learn with resorting to polarization measurements. Eqs. (5) account for the resonance contribution to the process. The interesting finding there is that if the $\Theta^+$ couples to $K^+n$ in $s$-wave (hence negative parity) the amplitude goes as $\sigma \cdot k_{in}$ while if it couples in $p$-wave it has a term $\sigma \cdot q'$. Hence, a possible polarization test to determine which one of the couplings the resonances chooses is to measure the cross section for initial proton polarization $1/2$ in the direction $z$ ($k_{in}$) and final neutron polarization $-1/2$ (the experiment can be equally done with $K^0p$ in the final state, which makes the nucleon detection easier). In this spin flip amplitude $(-1/2|t| + 1/2)$, the $\sigma \cdot k_{in}$ term vanishes. With this test the resonance signal disappears for the $s$-wave case, while the $\sigma \cdot q'$ operator of the $p$-wave case would have a finite matrix element proportional to
Figure 4. The double differential cross sections of polarized amplitude with $\theta = 90$ for $I = 0, 1$ and $J^P = 1/2^-, 1/2^+, 3/2^+$. Below, detail of the lower part of the upper figure of the panel.

$q' \sin \theta$.

In Fig. 4 we show the results for the polarized cross section measured at 90 degrees as a function of the invariant mass. The two cases with s-wave do not show any resonant shape since only the background contributes. All the other cross sections are quite reduced to the point that the only sizeable resonant peak comes from the $I, J^P = 0, 1/2^+$ case. A clear experimental signal of the resonance in this observable would unequivocally indicate the quantum numbers as $I, J^P = 0, 1/2^+$.

Acknowledgments

This work is supported by the Japan-Europe (Spain) Research Cooperation Program of Japan Society for the Promotion of Science (JSPS) and Spanish Council for Scientific Research (CSIC). This work is also supported in part by DGICYT projects BFM2000-1326, and the EU network EURIDICE contract HPRN-CT-2002-00311.

REFERENCES

1. LEPS, T. Nakano et al., Phys. Rev. Lett. 91 (2003) 012002.
2. DIANA, V.V. Barmin et al., Phys. Atom. Nucl. 66 (2003) 1715
3. CLAS, S. Stepanyan, hep-ex/0307018
4. SAPHIR, J. Barth et al., hep-ex/0307083
5. D. Diakonov, V. Petrov and M.V. Polyakov, Z. Phys. A359 (1997) 305.
6. T. Hyodo, A. Hosaka and E. Oset, nucl-th/0307105 Phys. Lett. B in print.
7. E. Oset and M.J. Vicente Vacas, Phys. Lett. B386 (1996) 39.
8. J. Gasser and H. Leutwyler, Nucl. Phys. B250 (1985) 465.
9. U.G. Meissner, Rept. Prog. Phys. 56 (1993) 903.