Method Article

A single-step protocol for closing experimental atom balances

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A B S T R A C T

Molar balances are considered to be closed if they are within 95–105%. It was shown in the companion paper “https://doi.org/10.1016/j.cej.2018.12.113; Chem. Eng. J., 361, 805–811 (2019)” that even this condition can give rise to pronounced deviations in conversion or selectivity data (Heynderickx, 2019). This manuscript offers a very simple a posteriori calculation procedure to address these deviations via simple linear algebra. The specific details of this procedure, called 'CLOBAL', after 'closing the balances', are shared (1) by showing the mathematics behind-the-scene and (2) by showing the specific programming code with an itemized guideline through the code.

Key benefits of proposed procedure GLOBAL script are:

- Physical quantities such as molar flow rates, concentrations or absolute number of moles are updated via a one-step linear procedure to close the corresponding atom balances;
- The presented GLOBAL procedure, is executed in Excel\textsuperscript{\textregistered}, which is accessible and practical for every user – no need for special license and the code is provided; and
- Parameter estimation, using treated data, results in smaller confidence intervals and lower residual sum of squares (RSSQ).

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A R T I C L E   I N F O

Method name: GLOBAL – after ‘closing the atom balances’, which is exactly what the presented procedure does

Keywords: Closing atom balances, Accuracy, Excel\textsuperscript{\textregistered} data sheet

Article history: Received 25 December 2018; Accepted 20 December 2019; Available online 10 January 2020

DOI of original article: http://dx.doi.org/10.1016/j.cej.2018.12.113

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http://dx.doi.org/10.1016/j.mex.2020.100781

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Speciation Table

| Subject Area: | Chemical Engineering |
|-------------|---------------------|
| More specific subject area: | Fields with experimental outcomes such as molar flow rates, concentrations, moles in organic chemistry experiments, catalysis . . . |
| Method name: | CLOBAL – after ‘closing the atom balances’, which is exactly what the presented procedure does |
| Name and reference of original method: | P. M. Heynderickx, Closing the balance by the CLOBAL procedure: towards more accurate concentration, conversion and selectivity values, https://doi.org/10.1016/j.cej.2018.12.113 |
| Resource availability: | Example of customized procedure is given in file clobal_01.xlsm |

Method details

When chemical reactions are performed the corresponding element or atom balances should be always closed [1–5]. For example, if the carbon balance is envisaged in a non-nuclear reaction, the initial number of carbon moles should equal the carbon in the reaction products. Typical acceptable ranges for an atom balance are between 90 % and 110 %. Experimental error is logically invoked to explain why atom balances are not exactly equal to 100 %.

This manuscript describes a very simple and elegant method to set atom balances equal to 100 %. Striking consequence of the given CLOBAL procedure is a more accurate calculation of conversion and selectivity values and a lower residual sum of squares during parameter estimation, accompanied by smaller confidence intervals for the parameters [1].

Consider n measurements of n physical quantities, which ‘true’ values are called \( \varphi_{j,i} \), \( i = 1 \ldots n \). For the sake of example, these quantities are the outlet molar flow rates in a mixture of n compounds, \( A_j \). Each of these compounds \( A_j \) has \( a_{ij} \) atoms of type \( e_i \), \( i = 1 \ldots m \). Normally the number of compounds exceeds the number of elements taken into account, i.e., \( m < n \). Since there are no nuclear reactions or transformations included, Eq. (1) holds for the true values with \( \varphi_{j,0} \), the initial value for quantity \( \varphi_j \):

\[
\sum_{j=1}^{n} a_{ij} \varphi_{j,0} = \sum_{j=1}^{n} a_{ij} \varphi_j \quad i = 1 \ldots m
\]  

(1)

Eq. (1) is an ideal representation, i.e., all the balances for atom type \( e_i \), \( i = 1 \ldots m \), are 100 % closed.

In reality this is not the case due to experimental error and, hence, the experimental values for the molar flow rate, absolute number of moles or concentrations do not close Eq. (1). The purpose of this manuscript is to offer a method for small corrections on these physical quantities in order to close the balances 100 %. The order of magnitude of these corrections can be compared to the error related to typical calibration data, as outlined in the companion paper [1], and, if the calibration curve has a high \( R^2 \), subsequently small corrections to the concentrations, mol fractions, or derived flowrates, are to be expected with this method. The proposed correction on the physical quantity, \( \varphi_{j,c} \) with \( j = 1 \ldots n \), should result in a full closure of the m balances, so that Eq. (2) is valid:

\[
\sum_{j=1}^{n} a_{ij} \varphi_{j,0} = \sum_{j=1}^{n} a_{ij} (\varphi_j + \varphi_{j,c}) \quad i = 1 \ldots m
\]  

(2)

Eq. (2) represents m so-called ‘fundamental relations’ for the n corrections \( \varphi_{j,c} \). Hence, \( n-m \) additional relations are required to solve for all of their values. These can be found from Eq. (3), which states that the weighted sum of corrections should be minimal, with \( w_j \) the weight factor corresponding for correction \( \varphi_{j,c} \):

\[
R = \sum_{j=1}^{n} w_j \varphi_{j,c}^2 \rightarrow \min
\]  

(3)
Eqs. (2) and (3) form the basis for a so-called ‘Lagrange multiplier optimization problem’: R needs to be minimized and the solution is subjected to equality constraints, see Eq. (2). The great advantage of the Lagrange multiplier method is that it allows not to explicitly solve the constraint equations and use them to eliminate extra variables. The complete function, also called the Lagrangian function S [6], with the so-called ‘Lagrange multiplicators’, 2-$\lambda_i$ (i = 1 . . . m), which has to be minimized, reads as Eq. (4):

$$S = \sum_{j=1}^{n} w_j \varphi_{j,c}^2 + \sum_{i=1}^{m} 2\lambda_i \left( \sum_{j=1}^{n} a_{ij} \varphi_{j,0} - \sum_{j=1}^{n} a_{ij} (\varphi_j + \varphi_{j,c}) \right) \rightarrow \min$$

(4)

The prefactor ‘2’ for the equality constraint can be added for the sake of elegance, so that in further calculations the factor 2, as a result of the derivative of the quadratic function (3), can be cancelled out. Taking the derivative with respect to $\varphi_{j,c}$ gives Eq. (5):

$$\frac{\partial S}{\partial \varphi_{j,c}} = 2 w_j \varphi_{j,c} - \sum_{i=1}^{m} 2\lambda_i a_{ij} = 0 \quad j = 1 \ldots n$$

(5)

From Eq. (5) the optimized corrections for the n flow rates, $\varphi_{j,c}$, are given by Eq. (6):

$$w_j \varphi_{j,c} = \sum_{k=1}^{m} \lambda_k \ a_{kj} \quad j = 1 \ldots n$$

(6)

Eq. (6) contains n relations and m + n unknowns, hence, m additional relations are needed, which can be found in Eq. (2). The subsequent substitution of Eq. (6) in the latter gives Eq. (7):

$$\sum_{j=1}^{n} a_{ij} (\varphi_j - \varphi_{j,0}) + \sum_{k=1}^{m} \lambda_k \cdot \sum_{j=1}^{n} a_{kj} \frac{a_{ij}}{w_j} = 0 \quad i = 1 \ldots m$$

(7)

Eq. (7) represents a set of m linear relations for $\lambda_k$, i = 1 . . . m, is found and upon solving, the Lagrange multipliers are inserted into Eq. (6) to obtain the individual correction for each of the individual n molar flow rates:

$$\varphi_{j,c} = \frac{1}{w_j} \cdot \sum_{k=1}^{m} \lambda_k \ a_{kj} \quad j = 1 \ldots n$$

(8)

The corrected quantities $\varphi_j + \varphi_{j,c}$, for j = 1 . . . n, give complete balances (1). Expressions (7) and (8) are sufficiently detailed to replicate the presented CLOBAL protocol.

The given expressions (7) and (8) can be written in general matrix notation, which will form the basis of the Excel® macro that gives the corrections.

In order to validate the presented methodology, the condensation of benzaldehyde and heptanal, which is an important aldol-type reaction in the production of jasmine aldehyde [7–9], is taken as showcase in the companion paper [1]. There are 5 compounds to be considered: benzaldehyde ($C_7H_6O$), heptanal ($C_7H_{14}O$), jasmine aldehyde ($C_7H_{14}O$), as desired product, and water ($H_2O$) and the dimer 2-pentyl-2-nonenal ($C_{14}H_{26}O$) as by-product (n = 5). Three atom types are used: C, O and H (m = 3), so that the stoichiometric matrix, allocating all coefficients $a_{i,j}$ is given by Eq. (9):

$$a = \begin{pmatrix} 7 & 1 & 6 \\ 7 & 1 & 14 \\ 14 & 1 & 18 \\ 0 & 1 & 2 \\ 14 & 1 & 26 \end{pmatrix}$$

(9)

The difference in actual value and initial value is given by vector $\Phi$, see Eq. (10), and the correction vector is defined by Eq. (11):

$$(\Phi)_j = \varphi_{j,0} - \varphi_j \quad j = 1 \ldots n$$

(10)
Sub global()
    ' Implementation of CLOBAL procedure for closing experimental balances
    Const m_max$ = 10
    Const n_max$ = 100
    Dim m As Integer    ' number of atom types
    Dim n As Integer    ' number of experimental compounds
    Dim nData As Integer    ' number of data vectors (e.g. at different time points)
    Dim data_m(n_max$ + 1) As Double    ' initial data matrix
    Dim datam(m_max$ + 2) As Double    ' actual data matrix
    Dim atom(n_max$, m_max$) As Double    ' atom matrix
    Dim x1, x2, x3, x4, x5, x6, x7 As Variant    ' auxiliary matrices
    Dim Rng0, Rng1, Rng2, Rng3 As Range    ' variable ranges
    Dim temp1, temp2 As Double    ' auxiliary variables
    Dim atom_name(m_max$) As Variant    ' atoms in the balances
    ' Clean the previous (worksheet) data
    Worksheets("results").Range("a2:a21000").Clear
    ' Reading data from 'atom' sheet
    nData = Worksheets("data").Cells(2, 9).Value
    m = Worksheets("atom").Cells(3, 2).Value
    n = Worksheets("atom").Cells(4, 2).Value
    For j = 1 To m
        atom_name(j) = Worksheets("atom").Cells(3, 5 + j).Value
        Next
    ' Reading data from 'data' sheet
    For i = 1 To nData
        data_m(i, 1) = Worksheets("data").Cells(2, 1 + i).Value
        Next
    Set Rng0 = Sheets("data").Range(Sheets("data").Cells(2, 2), Sheets("data").Cells(2, 2 + nData + 1 - 1))
    Set Rng3 = Sheets("data").Range(Sheets("data").Cells(2, 1), Sheets("data").Cells(2 + nData, 1))
    ' Feedback of results
    Worksheets("results").Cells(3, 1) = "Original balances"
    Worksheets("results").Cells(8 + nData, 1) = "Lagrange multipliers"
    Worksheets("results").Cells(6 + nData, 3 + m) = "Corrections"
    Worksheets("results").Cells(8 + 2 * nData, 3 + m) = "Corrected data"
    For j = 1 To m
        Worksheets("results").Cells(4, 1 + j).Value = atom_name(j)
        Next
    ' Start procedure
    ReDim x3(1 To m, 1 To nData)
    For j = 1 To m
        For i = 1 To nData
            x3(j, i) = atom_m(i, j)
            Next
    ' Loop for complete data treatment
    For ii = 1 To nData
        datam(ii, 1) = Worksheets("data").Cells(2 + ii, 1 + i).Value
        Next
        datam(ii, 2) = 1 / datam(ii, 1)    ' weight factors
        Next
        ' Processing data
        ReDim x1(1 To n, 1 To 1)
        For i = 1 To n
            x1(i, 1) = data_m(i, 1) - datam(i, 1)
            Next
        ReDim x2(1 To n, 1 To m)
        For i = 1 To n
            For j = 1 To m
                Next
\[ \Phi_j = \varphi_{j,c} \quad j = 1 \ldots n \]  

The solution for the m Lagrange multipliers is given by Eq. (12) with substitution of matrix \( \nu \), see Eq. (13):

\[ \lambda = \left( a^T \nu \right)^{-1} a^T \Phi \]  

\[ \left( \nu \right)_{ij} = \frac{1}{w_j} \left( a \right)_{ij} \quad i = 1 \ldots n, j = 1 \ldots m \]  

Eq. (12) represents the solution of Eq. (7) in matrix notation with respect to the Lagrange multipliers.

The corrections \( \varphi_{j,c} \) for \( j = 1 \ldots n \) are given by Eq. (14) in one single step calculation, i.e., no iterations are required:

\[ \tilde{\Phi}_j = \nu \cdot \lambda = \nu \left( a^T \nu \right)^{-1} a^T \Phi \]  

The corresponding VBA code is given in Table 1. The input requires the number of atom types, m, and the number of compounds, n. The stoichiometric information on the atom types in the individual compounds, such as given by the stoichiometric matrix via Eq. (9), is the input in worksheet ‘atom’, see Fig. 1. On the third row, the elements are given for further use in the results sheet. In this case the carbon, oxygen and hydrogen balance are evaluated (C, O and H). The code is divided in sections:

- Row 1 to 2: start of the routine;
- Row 3 to 14: declaration of variables;
- Row 15 to 16: removing previous results (avoiding erroneous overlap in data treatment);
- Row 17 to 28: reading input from ‘atom’ sheet;
Row 29 to 34: reading input from ‘data’ sheet;
Row 35 to 48: textual setting in the ‘result’ sheet in order to receive the results;
Row 49 to 55: CLOBAL procedure starts by transposing the stoichiometric matrix (9);
Row 56 to 76: all inputted data are treated (ii = 1 . . . ndata) according to Eqs. (10)–(14):
  o x1 contains the elements of vector φ, see Eq. (10);
  o x2 contains the elements for matrix v, see Eq. (13);
  o x3 is the transposed of matrix a;
  o x4 represents aᵀv;
  o x5 represents aᵀΦ;
  o x6 contains the Lagrange multiplicators, calculated via Eq. (12); and
  o x7 contains the correction on the given physical quantities (in this case, concentrations),
calculated via Eq. (14);
Row 77 to 97: allocation of all the results;
Row 98: end of the loop over all ndata; and
Row 99: End of the routine

The data vector consists of ndata+1 rows, having the initial concentration on row 2, see Fig. 2. The value of ‘ndata’ is automatically read by the program, depending on the input in the worksheet ‘data’;

Fig. 1. Input sheet ‘atom’ for CLOBAL procedure: information on atom types and input of stoichiometry.

Fig. 2. Input sheet ‘data’ for CLOBAL procedure: experimental data, corresponding to initial conditions in the companion paper [1] (CB,0 = 1 M, CH,0 = 2 M), see Fig. 3.
maximal number of data is \( n_{\text{max}} \). The actual concentration values for the \( n \) compounds occupy the rows 3 to \( n_{\text{data}}+2 \). The first column in worksheet ‘data’ contains the independent variable, e.g., in this case the minutes at sampling. This can be used for preparation of figures, but for the given procedure it is not required.

Fig. 3 gives the results of the CLOBAL procedure: worksheet ‘results’ evaluates the original atom balances and feeds this back to the user on rows 3 to \( n_{\text{data}}+4 \). The Lagrange multipliers, calculated via Eq. (12), and the individual corrections, obtained via Eq. (14), are given on rows \( n_{\text{data}}+6 \) to \( 2\cdot n_{\text{data}}+6 \). The corrected data are given from row \( 2\cdot n_{\text{data}}+8 \) to \( 3\cdot n_{\text{data}}+9 \) and they are ready for further use, i.e., they are generated as in the input form for sheet ‘data’.

As a side note for the weight factors, the author found that the best choice is the inverse of the corresponding response; as indicated on line 60 of the code, see Table 1. This can be altered by the user in case another expression should be more appropriate.

As an example, the result of the proposed procedure is given in Figs. 4–7, from which a clear overall decrease in data spread is observable. It has to be mentioned that some points might not show any improvement, such as the point (0.30 M; 0.35 M) in Fig. 5 or the point (0.035 M; 0.024 M) in Fig. 7.

**Fig. 3.** Results sheet ‘results’ for CLOBAL procedure, corresponding to initial conditions in the companion paper [1] (CB,0 = 1 M, CH,0 = 2 M), see Fig. 5.
is purely a coincidence: when the in silico random error is applied a second time [10] and the CLOBAL procedure is subsequently applied, the balances are still closed, but the small variations are somewhat different due to the different randomized error; this time resulting in a visible improvement of the point of interest. It was shown in the companion paper [1] that parameter estimation via ODRpack...
Fig. 7. Concentration with average 10 % error (top) and concentration after CLOBAL procedure (bottom) versus real concentration: zoom of Fig. 6 for concentration range 0 to 0.20 M.

[11], using treated data, results in smaller confidence intervals and lower residual sum of squares (RSSQ).

Declaration of Competing Interest

The author declares that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgements

This work has been supported by the Research and Development Program of Ghent University Global Campus (GUGC), Korea. MethodsX and the author would like to thank the reviewers of this article for taking the time to provide valuable feedback and constructive remarks making the manuscript more scientifically sound.

Appendix A. Supplementary data

Supplementary material related to this article can be found, in the online version, at doi: https://doi.org/10.1016/j.mex.2020.100781.

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