Light-Baryon Spectroscopy and the Electromagnetic Form Factors in the Quark Model

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Abstract

The momentum distributions of the constituent quarks inside the nucleon and the prominent electroproduced nucleon resonances are investigated in the two most sophisticated, available quark potential models, based respectively on the assumption of the valence + gluon dominance and on the exchange of the pseudoscalar Goldstone-bosons arising from the spontaneous breaking of chiral symmetry. It is shown that both models predict a large, similar content of high-momentum components, due to the short-range part of the interquark interaction, which affect the behaviour of both elastic and transition electromagnetic form factors at large values of the momentum transfer. The electromagnetic form factors are calculated within a relativistic approach.

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formulated on the light-front, adopting a one-body current with constituent quark form factors. The results suggest that soft, non-perturbative effects can play a relevant role for explaining the existing data on elastic as well as transition form factors (at least) for $Q^2 \lesssim 10 \div 20 \ (GeV/c)^2$. 
1 Introduction

The aim of this contribution is to address few relevant questions concerning the possible consistency of the predictions of the constituent quark (CQ) model with existing data on the electromagnetic (e.m.) properties of the nucleon and the most prominent electroproduced nucleon resonances at large values of the squared four-momentum transfer $Q^2$. In these kinematical regions ($Q^2 \gtrsim$ few $(GeV/c)^2$) the pQCD hard scattering mechanism appears to be able to explain qualitatively existing data (see, e.g., Ref. [1]), but results from QCD sum rules (see, e.g., Ref. [2]) seem to suggest that also the soft Feynman mechanism can account for the same data as well.

Since the high-$Q^2$ behaviour of the form factors is correlated with the high-momentum tail of the CQ momentum distribution in the nucleon and its resonances, we first investigate the light-baryon wave functions generated by the two most sophisticated, available quark potential models, based respectively on the assumption of the valence + gluon dominance [3] and on the exchange of the pseudoscalar Goldstone-bosons arising from the spontaneous breaking of chiral symmetry [4]. It will be shown that both models predict a large content of high-momentum components due to the short-range part of the interquark interaction. Moreover, despite the different behaviour of the two models at short distances, the high-momentum tails of the light-baryon wave functions turn out to be quite similar. These high-momentum components are known to affect significantly the large-$Q^2$ behaviour of both elastic and transition e.m. form factors, which are calculated adopting the light-front quark model of Refs. [5, 6]. We will point out that: i) the introduction of constituent quark form factors in the one-body e.m. current is essential in order to explain the detailed $Q^2$ behaviour of the nucleon elastic data; ii) the short-range spin-spin interaction generating the $N - \Delta(1232)$ mass splitting is also responsible for the faster-than-dipole fall-off of the $N - \Delta(1232)$ magnetic transition form factor at large $Q^2$; iii) an approximate dipole fall-off of the $N - S_{11}(1535)$ transition magnetic form factor can be obtained, provided the nucleon elastic data are reproduced. Our results suggest that soft, non-perturbative physics can yield a relevant contribution for explaining the existing data on the high-$Q^2$ behaviour of elastic and transition e.m. form factors, in accord with the findings of QCD sum rules.

2 Quark potential models and the CQ momentum distribution

The CQ model is known to be a phenomenological model able to explain the basic features of many static hadron properties, like the baryon (and meson) mass spectra. Within this model the CQ’s are the only relevant degrees of freedom in baryons, all the other degrees of freedom being frozen in the CQ mass ($m_i$) and interaction. The baryon wave function $\Psi_B$ is therefore eigenfunction of a Schroedinger-type equation, viz.

$$\hat{H}\Psi_B = [\hat{T} + \hat{V}]\Psi_B = M_B\Psi_B \quad (1)$$
where $M_B$ is the baryon mass, $\hat{T} = \Sigma_{i=1}^{3} \sqrt{|\vec{p}_i|^2 + m_i^2}$ is the kinetic term and $\hat{V} = \hat{V}_{\text{conf}} + \hat{V}_{\text{s.r.}}$ the interaction term, given by a long-range confining part $\hat{V}_{\text{conf}}$ and a short-range component $\hat{V}_{\text{s.r.}}$, responsible for the hyperfine mass splitting. The confining potential is usually derived from a Lorentz-scalar interaction and, as suggested by the spectroscopy and lattice QCD calculations, it can be taken linearly dependent on the quark-quark distance $r_{ij} \equiv |\vec{r}_i - \vec{r}_j|$, namely $\hat{V}_{\text{conf}} \rightarrow \hat{V}_s = \Sigma_{i<j} b \cdot r_{ij}$, where $b$ is the string tension. As for the short-range part of the interquark potential, the most sophisticated choices existing in the literature are based on two alternative mechanisms of boson exchange among $CQ$’s: the one-gluon-exchange (OGE) model of Ref. [3] and the pseudoscalar Goldstone-boson exchange (GBE) model of Ref. [4].

The semi-relativistic Hamiltonian model developed in Ref. [3] is very successful in both meson and baryon sectors: it reproduces a large amount of experimental masses and solves the so-called baryon spin-orbit puzzle. The latter consists in the apparent absence of a significant spin-orbit splitting in the light-baryon mass spectrum at variance with naive expectations. The puzzle was solved by Isgur and co-workers [3] by partially compensating the vector spin-orbit term with the Thomas-Fermi precession spin-orbit term arising from the scalar confining interaction, and by introducing (semi)relativistic corrections to the interquark potential, which yield a significant suppression of the interaction strength in case of light quarks. Nevertheless, a residual problem still remains in the generally good picture given by the OGE model: negative-parity states are below positive-parity ones, in clear contrast to the observation.

In the GBE model of Ref. [4] the short-range part of the $CQ$ interaction is generated by the exchange of the pseudoscalar Goldstone bosons arising from the spontaneous breaking of chiral symmetry. Such a potential model predicts baryon masses in quite good agreement with the experimental data and, in particular, thanks to the flavour dependence of the exchanged mesons, the GBE model is able to yield the correct ordering among positive and negative parity states. However, as pointed out in Ref. [7], the agreement with the mass spectrum is obtained only when the Thomas-Fermi precession spin-orbit term due to the scalar confining interaction is (arbitrarily) neglected. Therefore the baryon spin-orbit puzzle is still to be solved within the GBE model (see for details [7]). Despite the mentioned flaws, we stress that both the OGE and GBE models yield a quite good overall description of the light-baryon spectrum, although they remarkably differ at short interquark distances.

The wave equation (1) has been solved by expanding the wave function $\Psi_B$ onto a (truncated) set of harmonic oscillator basis states and applying to the Hamiltonian the Rayleigh-Ritz variational principle. We have explicitly checked that a sufficiently large number of basis states has been included in order to obtain the full convergence of the quantities considered in this work. The $CQ$ momentum distribution $n(p)$, defined as $n(p) = \int d\Omega d\vec{p}_2 d\vec{p}_3 \delta(\vec{p} + \vec{p}_2 + \vec{p}_3) |\Psi_B|^2$, calculated using the OGE and GBE models, is shown in Fig. 1 in case of the nucleon and compared with the results obtained adopting only the (linear) confining part of the two interactions. It can be seen that for both models the short-range part of the potential produce a remarkable content of high-momentum components, which turn out to be not very sensitive to the specific interaction model.

The $CQ$ momentum distributions $n(p)$ in the $\Delta(1232)$, $S_{11}(1535)$ and $F_{15}(1680)$ res-
Figure 1. $CQ$ momentum distribution $p^2 n(p)$ in the nucleon versus the internal $CQ$ momentum $p$. Thick and thin lines correspond to the $OGE$ and $GBE$ models of Refs. [3] and [4], respectively. The solid lines are the results obtained using the full interaction models, whereas the dashed lines correspond to the case in which only their (linear) confining parts are considered.

Resonances, obtained within the $OGE$ and $GBE$ models, are reported in Fig. 2 and compared with the ones in the nucleon. It can clearly be seen that, although the $OGE$ and $GBE$ models substantially differ at short interquark distances, the high-momentum tails of the baryon wave functions are quite similar in both models with the only (partial) exception of the $S_{11}(1535)$ resonance. Since the high-$Q^2$ behaviour of the form factors is qualitatively correlated with the high-momentum tail of the $CQ$ momentum distribution, we naively expect the same high-$Q^2$ behaviour for the elastic nucleon, $N - S_{11}(1535)$ and $N - F_{15}(1680)$ transition form factors, while a faster fall-off is expected in case of the $N - \Delta(1232)$ transition. Such features are indeed present in the existing high-$Q^2$ data, namely both the elastic nucleon, the $N - S_{11}(1535)$ and the $N - F_{15}(1680)$ transition magnetic form factors exhibit approximately the same dipole fall-off for $Q^2$ greater than few $(\text{GeV}/c)^2$, while the $N - \Delta(1232)$ transition magnetic form factor drops faster than a dipole (see [1]).

Figure 2. $CQ$ momentum distributions $p^2 n(p)$ in the $\Delta(1232)$ (a), $S_{11}(1535)$ (b) and $F_{15}(1680)$ (c) resonances (dashed lines). Thick and thin lines correspond to the $OGE$ and $GBE$ potential models, respectively. For comparison in each picture the $CQ$ momentum distributions in the nucleon (solid lines) are explicitly shown.

In the next Section the issue of the $CQ$ model predictions for the nucleon elastic and transition e.m. form factors will be addressed and to this end the relativistic quark model
of Ref. [5], formulated on the light front, is adopted. Since the high-momentum components generated by the OGE and GBE models turn out to be quite similar, in what follows we will limit ourselves to consider explicitly the light-baryon wave functions of the OGE model only.

3 Elastic and transition e.m. form factors

The effects of the high-momentum tail of the nucleon wave function generated by the OGE model on the elastic nucleon e.m. form factors have been investigated for the first time in Ref. [5], where a relativistic one-body e.m. current was adopted. It was shown that both the relativistic effects and the high-momentum components of the wave function lead to a sizeable overestimation of the proton form factors both at low ($\lesssim 1 \text{(GeV/c)}^2$) and high ($\gtrsim 1 \text{(GeV/c)}^2$) $Q^2$. This result is not surprising, because a pure valence quark model (i.e., without any effect from sea quark pairs) is not expected to describe dynamical properties like the e.m. form factors. One could argue that, in order to keep safe the CQ picture of the hadron structure, the CQ itself can be viewed as a non-elementary object whose structure takes into account in an effective way the presence of non-valence components. Thus, in Ref. [5] a one-body e.m. current with CQ form factors was adopted. The latter ones cannot be derived directly from QCD and therefore one is limited to constrain the CQ form factors by the request of reproducing the nucleon elastic data and to ask if existing data on the transition form factors are consistent with the same one-body e.m. current. This program has been partially carried out in Refs. [5, 6]: adopting the baryon wave functions of the OGE model, the CQ form factors were firstly fixed through the reproduction of the nucleon elastic data and then used to calculate without free parameters the $N - \Delta(1232)$ transition form factors, obtaining a good overall description of the existing data.

In this contribution we want to point out that, once the elastic data are reproduced, the faster-than-dipole fall-off of the $N - \Delta(1232)$ magnetic transition form factor can be obtained only when the effects from the short-range spin-spin interaction are taken into account in the baryon wave functions. To this end we have calculated the nucleon elastic form factors adopting two different wave functions obtained from the full OGE interaction and from its (linear) confining part only (see solid and dashed lines in Fig. 1, respectively). For each wave functions the CQ form factors have been determined by the request of reproducing the nucleon (and pion) data. The results are reported in Fig. 3 and it can be clearly seen that: i) the introduction of the CQ form factors in the one-body e.m. current is essential in order to explain the detailed $Q^2$ behaviour of the nucleon elastic data, and ii) once appropriate CQ form factors are introduced, the nucleon data alone cannot distinguish between models with and without short-range interaction effects. It should be mentioned that the phenomenological CQ form factors associated to the full OGE wave function and to the much softer wave function generated by the linear confining interaction, correspond to quite different values of the CQ size, namely $\sim 0.5$ and $\sim 0.2 \text{ fm}$, respectively.

\footnote{Due to the limitations imposed by the violation of the so-called angular condition (see Ref. [5]), we will consider in this work only the predictions of our light-front model for the dominant magnetic transition form factor.}
The deviations from the data at low $Q^2$ are discussed in Ref. [6]. For the photon point see also Ref. [9].
4 Conclusions

The momentum distributions of the constituent quarks inside the nucleon and the prominent electroproduced nucleon resonances have been investigated in the two most sophisticated, available quark potential models, based respectively on the assumption of the valence + gluon dominance and on the exchange of the pseudoscalar Goldstone-bosons arising from the spontaneous breaking of chiral symmetry. It has been shown that both models predict a large, similar content of high-momentum components due to the short-range part of the interquark interaction. Elastic and transition e.m. form factors have been calculated within a relativistic approach formulated on the light-front, adopting a one-body current with constituent quark form factors. The main results are: i) the introduction of constituent quark form factors is essential in order to explain the detailed $Q^2$ behaviour of the nucleon elastic data; ii) the short-range spin-spin interaction generating the $N-\Delta(1232)$ mass splitting is also responsible for the faster-than-dipole fall off of the $N-\Delta(1232)$ magnetic transition form factor at large $Q^2$; iii) an approximate dipole fall-off of the $N-S_{11}(1535)$ transition magnetic form factor can be obtained, provided the nucleon elastic data are reproduced. Our results suggest that soft, non-perturbative physics can yield a relevant, decisive contribution for explaining the existing data on the nucleon elastic as well as transition e.m. form factors (at least) up to $Q^2 \sim 10 \div 20 \, (GeV/c)^2$. 
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