Gauge Unification in Nonminimal Models with Extra Dimensions

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Abstract

We consider gauge unification in nonminimal models with extra spacetime dimensions above the TeV scale. We study the possibility that only a subset of the supersymmetric standard model gauge and Higgs fields live in the higher dimensional ‘bulk’. In two of the models we present, a choice for bulk MSSM matter fields can be found that preserves approximate gauge unification. This is true without the addition of any exotic matter multiplets, beyond the chiral conjugate mirror fields required to make the Kaluza-Klein excitations of the matter fields vector-like. In a third model, only a small sector of additional matter fields is introduced. In each of these examples we show that gauge unification can be obtained without the necessity of large string scale threshold corrections. We comment briefly on the phenomenology of these models in the case where the compactification scale is as low as possible.
1 Introduction

Recently, Dienes, Dudas and Gherghetta (DDG) [1, 2] have suggested the intriguing possibility that grand unification may occur at intermediate or even low energy scales in models with extra spacetime dimensions compactified on orbifolds of radii \( R \). Above the compactification scale \( \mu_0 = 1/R \), the vacuum polarization tensors for the gauge fields of the minimal supersymmetric standard model (MSSM) receive finite corrections from a tower of Kaluza-Klein (KK) excitations which contribute at one loop. As a consequence, the gauge couplings develop a power-law rather than logarithmic dependence on the ultraviolet cut off of the theory, \( \Lambda \) [1, 3]. While this is not running in the conventional sense, the gauge couplings nonetheless evolve rapidly as a function of \( \Lambda \), so that it is possible to achieve an accelerated unification.

In the minimal scenario proposed by DDG, all the non-chiral MSSM fields, the two Higgs doublets and the gauge multiplets, live in a \( 4 + \delta \) dimensional spacetime, and have an associated tower of KK excitations. The chiral MSSM fields are assumed to lie at fixed points of the orbifolds, and thus have no KK towers. This is the simplest way of avoiding the difficulties associated with giving mass to chiral KK states. DDG demonstrated that an approximate gauge unification could be achieved in this scenario, at a grand unification scale \( M_{\text{GUT}} \) that is much smaller than its usual value in supersymmetric theories, \( 2 \times 10^{16} \) GeV. However, as pointed out in Ref. [2, 4], strict comparison to the low energy data reveals that for TeV-scale compactifications, the DDG model predicts a value for \( \alpha_3(m_z) \) that is higher than the prediction in conventional unified theories, which is already ~5 standard deviations higher than the experimental value†. Assuming that the unification point coincides with the string scale, then a specific model of string scale threshold corrections is required before one can claim that unification in the DDG model is actually achieved.

It is the purpose of this paper to point out that there are a number of simple variations on the DDG proposal that achieve gauge unification much more precisely than the minimal scenario described above. To begin, however, let us consider the variations that are as successful as the minimal case. As pointed out by DDG, one possibility is to allow \( \eta \) generations of matter fields to experience extra dimensions, and to add to the theory their chiral conjugate mirror fields, so that suitable KK mass terms may be formed. Assuming that the orbifold is \( S^1/Z_2 \), then one may take the mirror fields to be \( Z_2 \) odd, so that unwanted zero modes are not present in the low-energy theory. Given that the KK excitations of the matter fields form complete SU(5) multiplets, it is perhaps not surprising that if unification is achieved for \( \eta = 0 \), it will also be preserved for \( \eta \neq 0 \), at least for some range of \( \mu_0 \).

†Using the same two loop code and input values described later in this paper, we find that \( \alpha_3(m_z) \approx 0.1276 \) in conventional supersymmetric unified theories, compared to the world average, \( 0.1191 \pm 0.0018 \).
What is less obvious is that the KK excitations of the matter fields may be chosen to form *incomplete* SU(5) multiplets, and an approximate unification may still be preserved if only *some* of the MSSM gauge and Higgs fields experience extra dimensions. In Section 2, we will present three models that demonstrate (i) that it is not necessary for all gauge groups to live in the higher dimensional bulk in order to achieve unification, and (ii) a precise unification may sometimes be obtained without introducing any exotic matter, beyond the mirror fields described above. In Section 3 we study these cases quantitatively, taking into account weak-scale threshold corrections, and two-loop running up to the compactification scale. For TeV scale compactifications, we will see that all of the nonminimal scenarios we present in this paper unify more precisely than the minimal DDG scenario, and do not require large threshold corrections at the unification scale. In Section 4 we summarize our conclusions, and make some brief comments on the phenomenological implications of these models when the compactification scale is low.

2 Three scenarios

We assume 4 + δ spacetime dimensions, with δ dimensions each compactified on a $Z_2$ orbifold of radius $1/\mu_0$. The fields that experience extra dimensions are periodic in the δ new spacetime coordinates $y_1 \ldots y_\delta$, and are either even or odd under $\vec{y} \rightarrow -\vec{y}$. For example, in the case where $\delta = 1$, these ‘bulk’ fields have expansions of the form

$$\Phi_+ = \sum_{n=0}^{\infty} \cos\left(\frac{n\pi}{R}\right)\Phi^{(n)}(x^\mu) ,$$

$$\Phi_- = \sum_{n=1}^{\infty} \sin\left(\frac{n\pi}{R}\right)\Phi^{(n)}(x^\mu) \quad (2.1)$$

where $n$ indicates the KK mode. The only other fields in the theory are those which live at the orbifold fixed points $y = 0$ or $y = \pi R$, and have no KK excitations.

The effect of a tower of KK states on the running of the MSSM gauge couplings was computed by DDG, and is given in a useful approximate form by

$$\alpha_i^{-1}(\Lambda) = \alpha_i^{-1}(m_z) - \frac{b_i}{2\pi} \ln\left(\frac{\Lambda}{m_z}\right) + \frac{\tilde{b}_i}{2\pi} \ln\left(\frac{\Lambda}{\mu_0}\right)$$

$$- \frac{\tilde{b}_i X_\delta}{2\pi\delta} \left[ \left(\frac{\Lambda}{\mu_0}\right)^\delta - 1 \right] . \quad (2.2)$$

Here the $\tilde{b}_i$ are the beta function contributions of a single KK level, and $X_\delta$ is given by

$$X_\delta = \frac{2\pi^{\delta/2}}{\delta \Gamma(\delta/2)} . \quad (2.3)$$

\[\text{For other possibilities, see Ref. [8].}\]
For the scenarios considered by DDG, these beta functions are

\[ b_i = \left( \frac{33}{5}, 1, -3 \right) \quad \tilde{b}_i = \left( \frac{3}{5}, -3, -6 \right) + \eta(4, 4, 4) \]  

where \( \eta \) is the number of generations of matter fields that experience extra dimensions. DDG observe that a sufficient condition for gauge unification to be preserved is that the ratios

\[ B_{ij} = \frac{\tilde{b}_i - \tilde{b}_j}{b_i - b_j} \]  

be independent of \( i \) and \( j \). Thus, they point out that in the scenario above

\[ \frac{B_{12}}{B_{13}} = \frac{72}{177} \approx 0.94 \quad \text{and} \quad \frac{B_{13}}{B_{23}} = \frac{11}{12} \approx 0.92 \]  

We will now show that there are a variety of other models, each with a different set of MSSM fields living at the orbifold fixed points, that lead to \( B_{12}/B_{13} \approx B_{13}/B_{23} \approx 1 \). First, notice that the \( \tilde{b}_i \) of the minimal scenario can be decomposed into the contributions from the KK excitations of each MSSM field, as shown in Table 1. An overlined field required so that the given KK tower is vector-like. This table is useful in that it allows us to mix and match. We will do so taking into account the string constraint that bulk matter may only transform under bulk gauge groups. For example, consider a model with all leptons and gauge fields living in the bulk, but with Higgs fields and quarks at the fixed points. The \( \tilde{b}_i \) are given by

\[ \tilde{b}_i = (0, -4, -6) + 3 \cdot (9/5, 1, 0) = (27/5, -1, -6) \]  

In this case we find

\[ \frac{B_{12}}{B_{13}} = \frac{128}{133} \approx 0.96 \quad \text{and} \quad \frac{B_{13}}{B_{23}} = \frac{19}{20} \approx 0.95 \]
### Table 2: Three Scenarios. For explanation of the notation, see the text.

| Scenario | Bulk MSSM Fields | Exotic Fields | $B_{12}/B_{13}$ | $B_{13}/B_{23}$ |
|----------|------------------|---------------|----------------|----------------|
| Minimal  | SU(3), SU(2), U(1), H | none          | 0.94           | 0.92           |
| 1        | SU(3), SU(2), U(1), 3E, 3L | none          | 0.96           | 0.95           |
| 2        | SU(3), U(1), U, D, 3E | none          | 1.00           | 1.00           |
| 3        | SU(2), U(1), H, 3L, E | Two $5 + \overline{5}$ w/ blk leptons | 1.00 | 1.00 |

As we will confirm explicitly in the next section, this scenario achieves unification more precisely than the minimal one.

In Table 2, we present three scenarios with $B_{ij}$ ratios that are significantly better than in the minimal scenario. We indicate the gauge group when the corresponding gauge multiplet is a bulk field, $H$ for both MSSM Higgs fields, and $n\Phi$ for $n$ generations of an MSSM matter field $\Phi \equiv (Q, U, D, L, E)$. Note that it is possible in scenario 1 to exchange an $L$ for an $H$; the vector-like tower of KK excitations associated with a zero-mode left-handed lepton field have the same effect on the $\tilde{b}$ as the tower associated with the MSSM Higgs fields. Scenarios 2 and 3 demonstrate that it is not necessary to assume that both SU(2) and SU(3) gauge multiplets live in the higher dimensional bulk in order to obtain a successful unification. As far as we are aware, this point has not been made in the literature. Note that only the third scenario involves extra matter, two SU(5) $5 + \overline{5}$ pairs in which only the leptons live in the bulk. We assume that the exotic matter zero modes have a mass of $\sim m_{\text{top}}$ for the purpose of our subsequent analysis. More strikingly, Scenarios 1 and 2 demonstrate that it is possible to achieve an improved unification in nonminimal models without the addition of any exotic matter multiplets, beyond the mirror fields required to render the KK towers of the matter fields vector-like. We will now consider all three scenarios quantitatively, and show that none require large threshold corrections at the unification scale.

### 3 Numerical Results

Our numerical analysis of gauge unification in the scenarios listed in Table 2 is quite conventional. We adopt the $\overline{MS}$ values for the (GUT normalized) gauge couplings $\alpha_1(m_z) = 58.99 \pm 0.04$ and $\alpha_2(m_z) = 29.57 \pm 0.03$ that follow from data in the 1998 Review of Particle Physics [3]. We run these up to the top quark mass, where we then assume the beta functions of the supersymmetric standard model, and where we convert the gauge couplings to the $\overline{DR}$ scheme. We take into account threshold effects, due to varying superparticle masses, at the one-loop level, and running between $m_{\text{top}}$ and the compactification scale $\mu_0$. 

\[4\]
at the two loop level. We then use Eq. (2.3) above the scale $\mu_0$ to determine the unification point. Thus, our procedure is similar to Ref. [4], except that we allow for greater freedom in our choice of weak scale threshold corrections. This procedure is iterated with trial values of $\alpha_3(m_z)$ until a suitable three coupling unification is achieved. For each of the given scenarios we obtain a prediction for $\alpha_3(m_z)$ assuming no threshold corrections at the unification scale. While such high scale threshold corrections should be present generically, our approach allows us to test the assumption that these need not be large.

In Fig. 1, we show the qualitative behavior of unification in scenarios 1, 2 and 3 by plotting the running couplings above a compactification scale of 2 TeV, assuming the experimental value of $\alpha_3(m_z) = 0.1191 \pm 0.0018$ [5]. Table 3 presents predictions for $\alpha_3(m_z)$.
assuming either an intermediate or low unification scale. We display results for $\delta = 1$ and 2, and for $\mu_0 = 2$ TeV and $10^8$ GeV. These choices are sufficient to understand the qualitative behavior of the results: as $\delta$ increases, the predictions for $\alpha_3(m_z)$ increase monotonically, while for increasing values of $\mu_0$, the predictions approach that of the MSSM without extra dimensions. Table 4 provides the predictions for $\alpha_3(m_z)$ including representative weak scale threshold effects, in which we’ve either placed all the non-colored MSSM superpartners at 1 TeV, with the rest at $m_{\text{top}}$, or vice versa. Let us consider the results for each of the scenarios in turn:

- **Minimal scenario:** This is the $\eta = 0$ scenario of DDG, which we include as a point of reference. The beta functions for this scenario are given in Eq. (2.5). Note that for $\delta = 1$ and $\mu_0 = 2$ TeV the low energy value of $\alpha_3(m_z)$ is $\sim 30$ standard deviations above the experimental central value, $0.1191 \pm 0.0018$ [5], and improves to $\sim 16$ standard deviations if one assumes that colored MSSM superpartners at the weak threshold are all at $m_{\text{top}}$ while noncolored sparticles are at $\sim 1$ TeV. These results agree qualitatively with those in Ref. [4], where a different approximation for weak scale threshold effects was used.

- **Scenario 1:** In this scenario, the gauge fields and leptons live in the bulk, while the Higgs and quarks live at orbifold fixed points. The beta functions for this scenario were given in Eq. (2.8). Notice that our previous observation that this scenario satisfies the relation $B_{12}/B_{13} = B_{13}/B_{23} = 1$ more accurately than the minimal case does translate into a better predictions for $\alpha_3(m_z)$. For $\delta = 1$ and $\mu_0 = 2$ TeV, and assuming the same choice for weak scale threshold corrections applied to the minimal scenario above, we find agreement with the experimental value of $\alpha_3(m_z)$ at the 4 standard deviation level.

- **Scenario 2:** In this scenario, the SU(2) gauge multiplet and Higgs fields are confined to the fixed point, while precisely one generation of right-handed up and down quarks, and three generations of right-handed leptons live in the bulk. The KK beta function

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Table 3: Predictions for $\alpha_3(m_z)$, assuming no weak scale threshold corrections.

| Scenario | $\delta = 1$ | $\delta = 2$ | $\delta = 1$ | $\delta = 2$ |
|----------|--------------|--------------|--------------|--------------|
|           | $\mu_0 = 2$ TeV | $\mu_0 = 2$ TeV | $\mu_0 = 10^8$ GeV | $\mu_0 = 10^8$ GeV |
| minimal  | 0.1734       | 0.1778       | 0.1520       | 0.1552       |
| 1        | 0.1438       | 0.1453       | 0.1377       | 0.1388       |
| 2        | 0.1159       | 0.1162       | 0.1199       | 0.1199       |
| 3        | 0.1155       | 0.1157       | 0.1169       | 0.1169       |

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$^\S$Had we used value of $\alpha_3(m_z)$ in the 1996 Review of Particle Physics, we would obtain results too high by about 9.8 standard deviations. The experimental determination of $\alpha_3$ has since improved.
Table 4: Predictions for $\alpha_3(m_z)$, assuming (A) all noncolored MSSM superpartners have 1 TeV masses while the rest have masses of $m_{\text{top}}$, and (B) all colored MSSM superpartners have 1 TeV masses while the rest have masses of $m_{\text{top}}$.

contributions are given by

$$\tilde{b}_i = (28/5, 0, -4).$$

(3.10)

This scenario achieves unification much more precisely than the minimal one. In the case where $\mu_0 = 2$ TeV and $\delta = 1$, the predicted value of $\alpha_3(m_z)$ is only 1.8 standard deviations below the experimental central value, ignoring all threshold corrections. Allowing the superparticle spectrum to vary as in Table 4, we find this prediction varies between

$$0.105 < \alpha_3(m_z) < 0.126.$$

(3.11)

Thus, unification can be achieved in this model without any string scale threshold corrections.

- Scenario 3: In this example, the SU(3) gauge multiplet is confined to the orbifold fixed point, and hence we are only allowed to place leptons and Higgs fields in the bulk. If we let $\eta_E$ and $\eta_L$ represent the number of right-handed and left-handed KK lepton excitations (including their chiral conjugate partners), then the constraint $B_{12} = B_{13} = B_{23}$ implies that $\eta_E = (3\eta_L - 13)/2$, which has no solution for only three generations of left-handed lepton fields. However, there is a simple way to circumvent this problem. Notice that if we add SU(5) 5+\(\bar{5}\) pairs in which only the lepton components live in the bulk, then the condition on $\eta_E$ and $\eta_L$ given above will hold, since differences in zero mode beta function pairs will remain unchanged. A solution may then be obtained by choosing $\eta_L = 5$ and
where \( \eta_E = 1 \), which implies the existence of two such \( \bar{5} + \bar{5} \) pairs, when one generation of right-handed and three generations of left-handed MSSM lepton fields are assigned to the bulk. The beta functions for this scenario are then given by

\[
b_i = \left( \frac{43}{5}, 3, -1 \right) \quad \tilde{b}_i = \left( \frac{24}{5}, 2, 0 \right).
\]

Notice in this case that the SU(3) gauge coupling only evolves logarithmically, since \( \tilde{b}_3 = 0 \). Unification may be achieved at a low scale by virtue of the power-law evolution of \( \alpha_1^{-1} \) and \( \alpha_2^{-1} \), as can be seen from Fig. 1. This scenario is about as successful as scenario 2, predicting \( \alpha_3(m_z) \) only 2.0 standard deviations below the experimental central value, ignoring all threshold corrections, and assuming that the exotic matter fields have masses of \( m_{\text{top}} \).

Allowing the sparticle mass spectrum to vary between \( m_{\text{top}} \) and 1 TeV, as in Table 4, we find that the scenario 3 prediction for \( \alpha_3(m_z) \) varies between

\[
0.104 < \alpha_3(m_z) < 0.124,
\]

for \( \delta = 1 \) and \( \mu_0 = 2 \) TeV. Again, unification is achieved without the need for threshold corrections at the high scale. Although this scenario does indeed involve some new matter fields, the choice is relatively minimal, and may be completely natural from the point of view of string theory.

## 4 Discussion

What is interesting about the scenarios we’ve presented is that gauge unification can be achieved in so many different ways. Each of our scenarios unifies more precisely than the minimal DDG model, and none requires large (or in two cases any) threshold corrections at the unification scale. These models illustrate two other interesting points as well: (1) One can achieve unification when some of the standard model gauge groups are confined to a brane. (2) There are some models that unify more precisely than DDG that do not require any additional matter fields with exotic quantum numbers, beyond the vector-like KK towers of certain MSSM fields that are chosen to live in the bulk. Before concluding, we comment briefly on some of the other phenomenological implications of these scenarios when the compactification scale is low. For a more complete discussion of the phenomenology of standard model KK excitations in models with TeV scale compactification \[7\], we refer the reader to Refs. \[8, 10\].

In scenarios 1 and 2 the gluon has a tower of KK excitations, which leads to a significant bound on the compactification scale. The KK gluon excitations are massive color octet vector mesons, with couplings both to zero mode gluons and to all the quarks. Thus, the KK gluons are in every way identical to flavor universal colorons, and are subject to
the same bounds. Recall that in coloron models, one obtains a massive color octet from the spontaneous breaking of $SU(3) \times SU(3)$ down to the diagonal color $SU(3)$. In the case where the two $SU(3)$ gauge couplings are equal, the coloron couples to quarks exactly like a gluon, or a KK gluon. The couplings of colorons or KK gluons to zero mode gluons are completely determined by $SU(3)$ gauge invariance, and hence are also the same. Thus the relevant bound on the lowest KK gluon excitation is given by $M_c > 759$ GeV at the 95% confidence level [7], which follows from consideration of the dijet spectrum at the Tevatron. This constraint places a lower bound on the scale for all the KK excitations in scenarios 1 and 2. We can obtain a similar bound on the compactification scale in scenario 3 from the production and hadronic decay of $W$ boson KK excitations, which have standard model couplings to the quarks: $M_{W'} > 600$ GeV [5]. Other direct collider bounds on the compactification scale require a more detailed analysis, given the nonstandard $W'$ and $Z'$ couplings in our models. This issue will be considered elsewhere [11]. Scenario 1 is particularly interesting when one takes into account that interaction vertices involving fields that all live in the higher dimensional bulk respect a conservation of KK number. (One can think of this as arising from the conservation of KK momentum following from translational invariance in the extra dimensions.) Hence, in scenario 1, the KK excitations of the electroweak gauge fields cannot couple to the lepton zero modes, and we obtain both $Z'$ and $W'$ bosons with otherwise standard couplings, that are naturally leptophobic! These states would likely be within the reach of the LHC for TeV scale compactifications. The other two scenarios present a more complicated phenomenology, since some generations of a given MSSM matter field live in the bulk while others live on the brane. It follows that the KK excitations of a standard model gauge field would have generation-dependent couplings to the zero mode matter fields, and may contribute to a variety of quark and lepton flavor-changing processes. Finally it is worth pointing out that in scenario 3, the fact that the KK $W$ boson excitations can’t couple to zero mode left-handed lepton fields, also leads to a leptophobic $W'$. While the purpose of the present work was to focus on gauge unification in these nonminimal scenarios, a more quantitative discussion of the TeV scale phenomenology of the scenarios described here will be presented in a separate publication [11].

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