Quarkyonic Matter and Chiral Spirals

Toru Kojo
RIKEN BNL Research Center, Brookhaven National Laboratory, Upton, NY-11973, USA
E-mail: toru@quark.phy.bnl.gov

Abstract. The nuclear matter, deconfined quark matter, and Quarkyonic matter in low temperature region are classified based on the $1/N_c$ expansion. The chiral symmetry in the Quarkyonic matter is investigated by taking into account condensations of chiral particle-hole pairs. It is argued that the chiral symmetry and parity are locally violated by the formation of chiral spirals, $\langle \psi \exp(2i\mu_q z^\gamma_5 \tau^i) \psi \rangle$. An extension to multiple chiral spirals is also briefly discussed.

1. Quarkyonic Matter
Quantum Chromodynamics (QCD) in high baryon density and low temperature has attracted continuous interests [1]. Recently large $N_c$ [2] arguments raised conceptual questions in dense QCD [3]. It turns out that a transition scale from the nuclear to the quark matter ($\mu_q \sim \Lambda_{QCD}$) and that of deconfinement ($\mu_q \sim \sqrt{N_c} \Lambda_{QCD}$) are, at least conceptually, different. The phase where a quark density is sufficiently high to form the Fermi sea nevertheless excitations are confined, is called Quarkyonic phase.

To characterize the Quarkyonic matter, below we will first distinguish the nuclear matter and quark matter, and then classify the quark matter into the deconfined quark matter and the Quarkyonic matter. The $1/N_c$ expansion will be used for theoretical classifications. The following arguments based on the $1/N_c$ expansion are only approximate, nevertheless may be useful, in a similar way as the chiral limit for the chiral transition, or the heavy quark limit for the confinement-deconfinement transition.

The nuclear matter starts to appear slightly below the threshold of nucleons, $\mu_q = M_N/N_c \sim \Lambda_{QCD}$, and changes into the quark matter within a small variation $\sim 1/N_c^2$ in a quark chemical potential. This picture is based on the following arguments. A change in $\mu_q$ requires a large change of the nucleon Fermi momentum $k_F$, since $k_F$ is divided by a large mass in kinetic energy $\sim k_F^2/M$. Accordingly a number density of nucleons $\sim k_F^2$ increases rapidly, and nucleons start to overlap, then strong nuclear forces in short distance becomes relevant. In such a region, it is natural to change effective degrees of freedom from nucleons to quarks for which interactions are weaker in shorter distance. Here quarks need not belong to particular nucleons. A 1st order like rapid growth of a quark density characterizes transitions from the hadronic-nuclear to the quark matter. The large $N_c$ limit provides a clear distinction between nuclear and quark matters.

Now let us discuss properties of the quark Fermi sea. Quarks deep inside of the Fermi sea are perturbative, since Pauli blocking prevents them from being affected by small momentum transfer processes. Such quarks share a large fraction of the Fermi sea, so perturbative estimates should reasonably work for bulk quantities such as pressure to which all quarks contribute. On the other hand, small momentum transfer processes can affect quarks near the Fermi surface.
Although such quarks have large momenta $\sim \mu_q$, they find comoving quarks with which they exchange small momenta, generating nonperturbative phenomena. They play deterministic roles for the phase structure (since perturbative contributions are common to different phases), transport processes, which are sensitive to excitation properties near the Fermi surface.

Now a question is how such a soft interaction looks like in finite density. In the QCD vacuum, quantum fluctuations of gluons provide the confinement, while those of quarks screen gluons reducing confining effects. The strength of fluctuations can be roughly characterized by a number of degrees of freedom: $O(N_c^2)$ for gluons, and $O(N_c)$ for quarks. In finite density, an allowed phase space for low momentum $q\bar{q}$ excitations increases, so do screening effects. The perturbative estimate of the screening scale is $\sim g_s^2 \mu_q^2 \sim \mu_q^2/N_c$, which becomes comparable with $\Lambda_{\text{QCD}}$ when $\mu_q \sim \sqrt{N_c} \Lambda_{\text{QCD}}$. This is the scale where the deconfinement takes place.

The estimate provided here may be minimal, since any energy gaps of quarks are not taken into account: Mass gaps from Lorentz vector self-energy (chiral symmetric) and possible chiral symmetry breaking would suppress $q\bar{q}$ excitations with reducing screening effects.

It might sound strange to speak about the confinement in the region where quarks are already released from nucleons. To draw a qualitative picture, let us consider quarks in terms of quantum wavefunctions. Quarks occupy states in a color singlet way, forming a color white background. A quark excitation in such a background inevitably accompanies a colored quark-hole, and they are confined forming a mesonic state. Glueballs are also confined, and their spectra are discrete without multi-gluon continuum. The dominant baryonic excitations are presumably baryon number solitons which are coherent excitations of quarks and quark-holes which keep the color singletness together with the background – there is no clear separation of baryons and the background. A correction to this simple argument comes from colored fluctuations in a color white background, which are already addressed in terms of screening effects.

These arguments can be tested by studying 1+1 dimensional QCD which is a confining model. In finite density, the strength of the confinement is unchanged irrespective of how closely quarks are packed within one spatial dimensional line. This is because screening effects in finite density are always same as those in vacuum, due to the same allowed phase space for $q\bar{q}$ fluctuations [4].

Finally it should be emphasized that we have not used a notion of chiral symmetry breaking/restoration to define the Quarkyonic matter. The Quarkyonic matter is solely defined by the presence of the quark Fermi sea and confined excitations. Their consequences on the chiral symmetry are discussed in the next section.

2. Local violation of the Chiral Symmetry: Quarkyonic Chiral Spirals
As $\mu_q$ increases, the conventional particle-antiparticle type chiral condensate disappears since generations of antiparticles need large energies $\sim \mu_q$ to transfer particles in the Dirac sea to the above of the Fermi sea. Yet the chiral symmetry can be broken by chiral pairs of particles and holes near the Fermi surface without costing much energy. Chiral condensates of this sort only employ particle-holes near the Fermi surface, modifying only particle dispersions around there. This is consistent with arguments on perturbative quarks deep inside of the Fermi sea.

For simplicity, only typical two cases, exciton and density wave types, will be considered below (for detail, see [4]). In the exciton case, we pick up a particle and a hole from the same spatial momentum region. Since a hole momentum must be flipped, the pair has a total momentum $\sim 0$. For the density wave, we pick up a particle and a hole from the opposite momentum region, so the pair has a total momentum $\sim 2\mu_q$, forming nonuniform condensates. Such situations have been first discussed in [5], in the context of the deconfined quark matter.

Since we pick up a particle and a hole with similar kinetic energies in both cases, main differences in pairing energies come from potentials. The confining potential should provide a big difference. We expect that exciton wavefunctions made of pairs with opposite large momenta are widely spread in space, costing large energies due to long strings. For density wave pairs,
particles and holes comove together maintaining small sizes of $\sim 1/\Lambda_{\text{QCD}}$. Therefore we naively expect that confining forces favor density wave pairings.

We first consider the one-pair problem picking up particle-hole from $p_z \sim \pm \mu_q$, $p_T \sim 0$. This setup will turn out to be a good starting point to analyze multi-pair problems [6].

For concrete arguments, we introduce a simple model of the linear confinement in which gluon propagators take the following form (screening effects are suppressed in large $N_c$),

$$D^{AB}_{44}(k) = -\frac{8\pi}{C_F} \frac{\sigma}{(k^2)^2} \delta^{AB} ; \quad D^{ii} = D^{ij} = 0, \quad (C_F = \frac{N_c^2 - 1}{2N_c}, \quad \sigma \sim \Lambda_{\text{QCD}}^2) \quad (1)$$

which shows a linear potential in coordinate space. This model is inspired by Coulomb gauge analyses [7]. We have omitted perturbative parts for the sake of simplicity [8].

Below we illustrate this for the Schwinger-Dyson equation for the quark self-energy. It looks like

$$\Sigma(p) + \Sigma_m(p) = -\int \frac{d^4k}{(2\pi)^4} \left( \gamma_4 t_A \right) S(k; \Sigma) \left( \gamma_4 t_B \right) D^{AB}_{44}(p - k), \quad (2)$$

where the quark propagator is dressed and depends on $\Sigma(p)$. We will investigate the self-energy for quarks near the Fermi surface, with $p_z \sim \mu_q$ and $p_T \sim 0$.

Three points are relevant for the dimensional reduction: (I) Our gluon propagator behaves as $1/(q^2)^2$, so dominant contributions of the integral (3) sharply concentrates on the small domain. (II) Different Dirac structures $S(k) = \gamma_4 S_4(k) + \gamma_z S_z(k) + \gamma_T \cdot \vec{S}_T(k) + S_m(k)$, give the ratio of $|S_T|^2 / S_z \sim k_T / k_z \sim \Lambda_{\text{QCD}} / \mu_q$, so we can drop off $S_T$ and $\Sigma_T$ in the leading order of $\Lambda_{\text{QCD}} / \mu_q$. (III) The quark energy is sensitive to the change of $k_z$ while not to that of $k_T$. Changes in $k_T$ is along the constant energy surface which looks flat for $\Lambda_{\text{QCD}} / \mu_q \ll 1$, without changing the quark energy a lot.

Because of the restricted integral region and the insensitivity of the quark propagator to $k_T$ variable, we can set $\vec{k}_T \simeq \vec{0}_T$ and factorize the integral equation:

$$\int dk_4dk_zd^2\vec{k}_T S(k_4, k_z, \vec{k}_T) D_{44}(p - k) \rightarrow \int dk_4dk_z S(k_4, k_z, \vec{0}_T) \int d\vec{k}_T D_{44}(p - k), \quad (3)$$

for which we can carry out the $\vec{k}_T$ integral. The smearing of gluon propagator yields a 1+1 dimensional confining propagator, $\sim 1/(p_z - k_z)^2$. The reduced equation then becomes

$$\tilde{\Sigma}(p_4, p_2, \vec{0}_\perp) + \Sigma_m(p_4, p_2, \vec{0}_\perp) \simeq \frac{N_c g_{\text{FD}}^2}{2} \int \frac{dk_4dk_z}{(2\pi)^2} \gamma_4 \tilde{S}(k_4, k_z, \vec{0}_\perp) \gamma_4 \frac{1}{(k_z - p_z)^2}, \quad (4)$$

where $N_c g_{\text{FD}}^2 = 4\sigma$ and we denote $\tilde{\Sigma}, \tilde{S}$ for components other than transverse ones. The resulting equation is nothing but the Schwinger-Dyson equation of 't Hooft model in axial gauge $A_z = 0$. The same approximations are applicable to the Bethe-Salpeter equation.

Since self-consistent equations for our one-pair problem take the same form as those of 't Hooft model [10], we can borrow results in the existing literatures [11]. Only nontrivial issues are relationships between 3+1 and 1+1 dimensional operators and condensation channels. In particular, concepts of spins are absent in one spatial dimension. In our dimensional reduction, the tranverse part of the quark propagators are suppressed. This implies that $\gamma_T$ terms are dropped off, so that there is no spin mixing term once we quantize spins along the directions of moving particles. Introducing projection operators for moving directions, we can write spin multiplets [9],

$$\psi_\pm = \frac{1 \pm \gamma_0 \gamma_z}{2} \psi, \quad \varphi_\uparrow = \begin{bmatrix} \varphi_\uparrow^+ \\ \varphi_\uparrow^- \end{bmatrix} = \begin{bmatrix} \psi_{R+} \\ \psi_{L-} \end{bmatrix}, \quad \varphi_\downarrow = \begin{bmatrix} \varphi_\downarrow^+ \\ \varphi_\downarrow^- \end{bmatrix} = \begin{bmatrix} \psi_{L+} \\ \psi_{R-} \end{bmatrix}, \quad (5)$$
where indices $(+,−)$ for $+z,−z$ moving particles. Let us introduce 1+1 dimensional "flavor", $Φ^T ≡ (φ^+, φ^−)$, and Dirac matrices, $Γ^0 = σ^1$, $Γ^z = −iσ^2$, $Γ^5 = σ^3$. Then 3+1 dimensional quark bilinears without spin mixings are mapped onto 1+1 dimensional "flavor" singlet operators [4, 9],

$$\bar{ψ}\psi → \bar{Φ}Φ, \quad \bar{ψ}\gamma^0\psi → \bar{Φ}Γ^0Φ, \quad \bar{ψ}\gamma^z\psi → \bar{Φ}Γ^zΦ, \quad \bar{ψ}\gamma^0\gamma^z\psi → \bar{Φ}Γ^5Φ. \quad (6)$$

The last relation reflects the fact that 1+1 dimensional $Γ^5$ characterizes moving directions. All other quark bilinears include spin mixings, so they are "flavor" non-singlet in 1+1 dimensions. They do not show condensations in the 't Hooft model.

The 't Hooft model in finite density is known to show the chiral spirals rotating in chiral space as $⟨\bar{Φ}e^{2iμ_qz}Φ⟩$. Mapping this, we get the chiral spiral in 3+1 dimensions,

$$⟨\bar{ψ}\psi⟩ = Δ \cos(2μ_qz), \quad ⟨\bar{ψ}\gamma^0\gamma^z\psi⟩ = Δ \sin(2μ_qz). \quad (7)$$

An origin of this chiral rotation is that $\bar{ψ}_+ψ_−$ and $\bar{ψ}_−ψ_+$ moving to opposite directions each other, thus have opposite phase $e^{±2iμ_qz}$; this phase mismatch makes $⟨\bar{ψ}\gamma^0\gamma^zψ⟩ = ⟨\bar{ψ}_+ψ_−⟩ − ⟨\bar{ψ}_−ψ_+⟩$ nonvanishing. So the chiral density wave accompanies its partner, forming spiral structures.

It is clear that a spatial average of chiral condensates vanishes because of spatial modulations. So the symmetry apparently looks restored for probes with large wavelength. Note also that our chiral tensor type condensate locally breaks parity.

Finally we briefly mention an extension of the present results to those of multi-pairs. This problem was investigated in [6]. A key observation is that in confining models, chiral spirals evolved in different spatial directions interact only weakly. Therefore our results for one-patch problem may be applied with slight modifications. Such a picture indicates that a number of directions of chiral spiral formations increases as $μ_q$, with a series of phase transitions.

3. Summary and Outlook

In the Quarkyonic phase, excitations are confined despite of a large quark number density. They drive the formation of the chiral spirals with large mass gaps $∼ Λ_{QCD}$. This is in sharp contrast to the situation in the deconfined quark matter where gaps are too small to overtake screening effects [9]. So chiral spirals can be regarded as good signatures of the Quarkyonic phase. Transport processes in stars are good candidates to see consequences of chiral spirals through characteristic Nambu-Goldstone modes. A local parity violation may also provide important implications through neutrino physics. These issues deserve further investigations.

Acknowledgments

The author acknowledges his collaborators, Y. Hidaka, L. McLerran, R.D. Pisarski, and A.M. Tsvelik from whom the present ideas have been developed. This research is supported under DOE Contract No. DE-AC02-98CH10886 and Special Posdoctoral Research Program of RIKEN. He also thanks organizers and participants of the workshop Hot Quark 10.

References

[1] For a recent review, K. Fukushima and T. Hatsuda, arXiv:1005.4814 [hep-ph].
[2] G. 't Hooft, Nucl. Phys. B 72 (1974) 461.
[3] L. McLerran and R. D. Pisarski, Nucl. Phys. A 796 (2007) 83.
[4] T. Kojo, Y. Hidaka, L. McLerran and R. D. Pisarski, Nucl. Phys. A 843 (2007) 37, and references therein.
[5] D. V. Deryagin, D. Y. Grigoriev and V. A. Rubakov, Intl. Jour. Mod. Phys. A 7 (1992) 659.
[6] T. Kojo, R. D. Pisarski, and A. M. Tsvelik, arXiv:1007.0248 [hep-ph], to be published in PRD.
[7] D. Zwanziger, Phys. Rev. Lett. 90 (2003) 102001; Phys. Rev. D 69 (2004) 016002.
[8] L. Y. Glozman and R. F. Wagenbrunn, Phys. Rev. D 69 (2004) 016002.
[9] E. Shuster and D. T. Son, Nucl. Phys. B 573 (2000) 434.
[10] G. 't Hooft, Nucl. Phys. B 75 (1974) 461.
[11] For instance, B. Bringoltz, Phys. Rev. D 79 (2009) 125006.