Annihilation-Type Charmless Radiative Decays of $B$ Meson in Non-universal $Z'$ Model

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Abstract

We study charmless pure annihilation type radiative $B$ decays within the QCD factorization approach. After adding the vertex corrections to the naive factorization approach, we find that the branching ratios of $B_0^d \to \phi \gamma$, $B_s^0 \to \rho^0 \gamma$ and $B_s^0 \to \omega \gamma$ within the standard model are at the order of $\mathcal{O}(10^{-12})$, $\mathcal{O}(10^{-10})$ and $\mathcal{O}(10^{-11})$, respectively. The smallness of these decays in the standard model makes them sensitive probes of flavor physics beyond the standard model. To explore their physics potential, we have estimated the contribution of $Z'$ boson in the decays. Within the allowed parameter space, the branching ratios of these decay modes can be enhanced remarkably in the non-universal $Z'$ model: The branching ratios can reach to $\mathcal{O}(10^{-8})$ for $B_s^0 \to \rho^0 (\omega) \gamma$ and $\mathcal{O}(10^{-10})$ for the $B_d^0 \to \phi \gamma$, which are large enough for LHC-b and/or Super B-factories to detect those channels in near future. Moreover, we also predict large CP asymmetries in suitable parameter space. The observation of these modes could in turn help us to constrain the $Z'$ mass within the model.
1 Introduction

Rare $B$ decays induced by flavor changing neutral currents (FCNC) play important roles in particle physics, where they are always regarded as ideal places for probing signals of new physics. The GIM suppression of FCNC amplitude is absent in many new physics scenarios beyond the standard model (SM), which could give large enhancement of FCNC contributions over the SM predictions. However, due to our poor knowledge of non-perturbative QCD, predictions for many interesting exclusive decays are polluted by large hadronic uncertainties. Therefore, it would be of great interest to explore rare $B$ decays, which are induced with few hadronic parameters as well as only by FCNC currents. Two body radiative $B$ decays involve simple hadronic dynamics with only one hadron in the final states, so they suffer much less pollution than non-leptonic decays.

In studying the radiative decays such as $B \rightarrow K^{*+} \gamma$, $\rho(\omega) \gamma$, the isospin breaking effects between the charged $B^{\pm}$ and neutral $B^{0}$ in these modes are mainly from the annihilation type diagrams $[1, 2, 3, 4]$. Many of pure annihilation type radiative decays, such as $B \rightarrow \phi \gamma$ and $B \rightarrow J/\psi \gamma$, have been analyzed in the QCD factorization approach $[5, 6]$ and in the perturbative QCD approach $[7]$. We find the branching ratio of $B \rightarrow \phi \gamma$ is at the order of $\mathcal{O}(10^{-11} \sim 10^{-12})$ in the SM. The decay rate is too small to be observed at presently running $B$ factories, BaBar and Belle. Any measurements of the decays at BaBar and Belle would be direct evidences of new physics. In this work, we explore the decay $B \rightarrow \phi \gamma$ and similarly the decay $B^{0}_{s} \rightarrow \rho^{0}(\omega) \gamma$ in the non-universal $Z'$ model $[8]$, which could be naturally derived in certain string constructions $[9]$, E6 models $[10]$ and so on. Generally speaking, within the such model a flavor mixing can be induced at the tree level in the up-type and/or down-type quark sector after diagonalizing their mass matrices. In some new physics model, FCNCs due to $Z'$ exchange can be induced by mixing among the SM quarks and the exotic quarks, which have been predicted to have different $Z'$ quantum numbers. Here we will consider the model in which the interaction between the $Z'$ boson and fermions are flavor non-universal for left handed couplings and flavor diagonal for right handed couplings. The effects of the $Z'$ on other processes of the interest have been investigated in a number of papers such as $[11, 12]$, especially in $B$ physics $[13, 14, 15, 16]$. The recent review about $Z'$ in detail is referred to Ref. $[17]$.

To keep completeness, we first calculate these decays in the naive factorization approach. Then we add the vertex corrections to the four quark operators, which have been performed in the so called QCD factorization approach $[18]$ in the SM, utilizing the light-cone wave functions of the light vector mesons. A similar work within the R-parity violating SUSY can be also found in Ref. $[5]$. However, in this work we will revisit these processes with the updated parameters in the non-universal $Z'$ model.

2 Calculation in the Standard Model

In the SM, the common starting point is the effective weak Hamiltonian which mediates flavor-changing neutral current transitions of the type $b \rightarrow D$ ($D = d, s$):  

$$
\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left[ \sum_{p = u, c} V_{pb} V_{pD}^\ast \left( C_1 O_1^b + C_2 O_2^b \right) - V_{tb} V_{tD}^\ast \sum_{i = 3}^{10, 7, 8} C_i O_i \right].
$$

(1)
The explicit forms of the operators $O_i$ and the corresponding Wilson coefficients $C_i$ at the scale of $\mu = m_b$ can be found in Ref. [12]. $V_{u1}(J)$, $V_{u2}(J)$ are the Cabibbo-Kabayashi-Maskawa (CKM) matrix elements. According to the effective Hamiltonian [1], we can draw the lowest order diagram of this channel, as shown in Fig. 1, which is dominated by the photon radiated from light quark in $B_s^0$ meson. When the photon is radiated from heavy $b$ quark, the energetic up or down quark will suppress the $\rho^0$ production by a power of $L^2_{QCD}/m_b^2$, therefore, here we neglect its contribution. This point have been clearly discussed in $B \to K^*\gamma$ decays [2,3].

For convenience, we denote that $\eta^*_+ \otimes \epsilon^*_+$ are transverse polarization vectors of the final vector meson and photon, respectively. The photon energy and momentum are defined as $E_\gamma$ and $q$, and the momentum of $B_s^0$ meson is $p_B = m_B$. In the rest frame of $B_s^0$ meson, we take the photon and the vector meson moving along the $n_- = (1, 0, 0, -1)$ and $n_+ = (1, 0, 0, 1)$ directions, respectively. Within the effective Hamiltonian and naive factorization hypothesis, we can write down the amplitudes as follows:

$$A(B_s^0 \to \rho^0\gamma) = \frac{G_F}{2} \left[ V_{ub} V^*_{ts} a_2 - V_{ub} V^*_{td} \left( \frac{3}{2} a_7 + \frac{3}{2} a_9 \right) \right] \times \sqrt{4 \pi \alpha_w f_{\rho} m_\rho F_V} \left\{ -\epsilon_{\mu\nu\rho\sigma} \eta^*_+ \epsilon^*_+ \nu^\rho q^\sigma + i \{ [(\eta^*_+ \cdot \epsilon^*_+)(v \cdot q) - (\eta^*_- \cdot q)(v \cdot \epsilon^*_+)] \} \right\}, \quad (2)$$

$$A(B_s^0 \to \omega\gamma) = \frac{G_F}{2} \left[ V_{ub} V^*_{ts} a_2 - V_{ub} V^*_{td} \left( 2 a_3 + 2 a_5 + \frac{1}{2} C_7 + \frac{1}{2} a_9 \right) \right] \times \sqrt{4 \pi \alpha_w f_{\rho} m_\rho F_V} \left\{ -\epsilon_{\mu\nu\rho\sigma} \eta^*_+ \epsilon^*_+ \nu^\rho q^\sigma + i \{ [(\eta^*_+ \cdot \epsilon^*_+)(v \cdot q) - (\eta^*_- \cdot q)(v \cdot \epsilon^*_+)] \} \right\}, \quad (3)$$

$$A(B_s^0 \to \phi\gamma) = \frac{G_F}{\sqrt{2}} \left[ -V_{ub} V^*_{td} \left( a_3 + a_5 - \frac{1}{2} a_7 - \frac{1}{2} a_9 \right) \right] \times \sqrt{4 \pi \alpha_w f_{\rho} m_\rho F_V} \left\{ -\epsilon_{\mu\nu\rho\sigma} \eta^*_+ \epsilon^*_+ \nu^\rho q^\sigma + i \{ [(\eta^*_+ \cdot \epsilon^*_+)(v \cdot q) - (\eta^*_- \cdot q)(v \cdot \epsilon^*_+)] \} \right\}, \quad (4)$$

where $a_i$ is defined as the combination of the Wilson coefficients,

$$a_i = C_i + \frac{C_{i+1}}{N_c}. \quad (5)$$
In order to calculate the form factor the distribution of the heavy meson, the integral in above formulae is often parameterized as:

\[ \Phi \]

for an odd (even) value of \( Q \). Within the naive factorization hypothesis, because of no strong phases entering into these processes, there should not exist any CP asymmetry for the processes.

\[ \langle \gamma (\epsilon^*, q) | \bar{d} \gamma_\mu (1 - \gamma_5) b | B_0^0 (v) \rangle = \sqrt{4 \pi \alpha_e} \left[ - F_V (E_\gamma) \epsilon_\mu \epsilon_{\nu \sigma} \epsilon^{\nu \sigma} q^\nu q^\sigma + i F_A (E_\gamma) (\epsilon_\mu^* q \cdot v - q_\mu v \cdot \epsilon^*) \right]. \]  

(6)

In order to calculate the form factor \( F_V \), we need two-particle light-cone projector for an initial \( B \) meson:

\[ \mathcal{M}_{ab}^B = \frac{i}{4 N_c f_B M_B} \{ (1 + \gamma^5) \langle \Phi_{B_1} (l_+) + \Phi_{B_2} (l_+) \rangle \}_{ab}, \]  

(7)

where \( \Phi_{B_1} (l_+) \) and \( \Phi_{B_2} (l_+) \) are the leading twist light-cone distribution functions. Thus, we obtain the standard result:

\[ F_V (E_\gamma) = F_A (E_\gamma) = \frac{Q_s f_B M_B}{2 \sqrt{2} E_\gamma} \int d l_+ \frac{\Phi_{B_1} (l_+)}{l_+}, \]

(8)

where \( Q_s = -1/3 \) is the charge of the \( s \) quark in units of the proton’s charge. Because we have little knowledge about the distribution of the heavy meson, the integral in above formulae is often parameterized as:

\[ \int d l_+ \frac{\Phi_{B_1} (l_+)}{l_+} = \frac{1}{\lambda_B}. \]  

(9)

Consequently, we write down the helicity amplitudes for these channels as:

\[ \mathcal{M}_{B_d \rightarrow \rho \gamma}^{++} = \frac{G_F}{2} \sqrt{4 \pi \alpha_e F_V f_\rho m_\rho M_B} \left[ V_{ud} V_{us}^* a_2 - V_{ud} V_{us}^* \left( \frac{3}{2} a_7 + \frac{1}{2} a_9 \right) \right], \]  

(10)

\[ \mathcal{M}_{B_d \rightarrow \phi \gamma}^{++} = \frac{G_F}{2} \sqrt{\frac{4 \pi \alpha_e F_V f_\phi m_\phi M_B} {2 \sqrt{2}}} \left[ V_{ub} V_{us}^* a_2 - V_{ud} V_{us}^* \left( 2a_3 + 2a_5 + \frac{1}{2} C_7 + \frac{1}{2} a_9 \right) \right], \]  

(11)

\[ \mathcal{M}_{B_d \rightarrow \rho \gamma}^{-+} = \frac{G_F}{2} \sqrt{\frac{4 \pi \alpha_e F_V f_\rho m_\rho M_B} {2 \sqrt{2}}} \left[ -V_{ud} V_{us}^* \left( a_3 + a_5 - \frac{1}{2} C_7 - \frac{1}{2} a_9 \right) \right], \]  

(12)

\[ \mathcal{M}_{B_d \rightarrow \phi \gamma}^{-+} = \mathcal{M}_{B_d \rightarrow \omega \gamma}^{00} = \mathcal{M}_{B_d \rightarrow \rho \gamma}^{00} = 0. \]  

(13)

Depending on the parameter values listed in Table 1, one can get the averaged branching ratios as:

\[ \mathcal{B}(B_d^0 \rightarrow \rho^0 \gamma) = 1.1 \times 10^{-10}; \]
\[ \mathcal{B}(B_d^0 \rightarrow \omega \gamma) = 5.6 \times 10^{-11}; \]
\[ \mathcal{B}(B_d^0 \rightarrow \phi^0 \gamma) = 1.7 \times 10^{-13}. \]  

(14)

Within the naive factorization hypothesis, because of no strong phases entering into these processes, there should not exist any CP asymmetry for the processes.

Table 1: Summary of input parameters

| \( \lambda \) | \( A \) | \( \bar{\rho} \) | \( \bar{\eta} \) | \( \lambda_\text{MS}(f=4) \) | \( \tau_B^0 \) | \( \tau_B^0 \) | \( \lambda_B \) | \( \alpha_e \) | \( \alpha_s \) |
|------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 0.225      | 0.818           | 0.141           | 0.348           | 250 MeV         | 1.54 ps         | 1.46 ps         | 0.35            | 1/132           | 0.214           |

| \( f_B \) | \( f_{B_1} \) | \( f_\rho \) | \( f_\phi \) | \( f_\omega \) |
|------------|-----------------|-----------------|-----------------|-----------------|
| 216 MeV    | 236 MeV         | 210 MeV         | 150 MeV         | 221 MeV         |
| 175 MeV    | 187 MeV         | 151 MeV         |

| \( m_B \) | \( m_{B_1} \) | \( m_\rho \) | \( m_\phi \) | \( m_\omega \) |
|------------|---------------|--------------|-------------|--------------|
| 5.28 GeV   | 5.36 GeV      | 1.02 GeV     | 0.77 GeV    | 0.78 GeV     |

\[ M_+^+ \] is the charge of the \( B \) meson.
Up to now in our calculation, non-factorizable contributions have been neglected. As next step, we add the vertex corrections and the leading non-factorizable diagrams, shown in Fig. 2. To achieve the goal, the QCD factorization framework [18] proposed by Beneke, Buchalla, Neubert and Sachrajda is very suitable to be applied. To calculate non-factorizable diagrams, we also need the two-particle light-cone projector of the vector mesons:

\[ \mathcal{M}_\perp^{\rho^0} = -\frac{f_\perp^{\rho^0} m_{\rho^0}}{4N_c} \left\{ \varepsilon_\sigma^{\perp} \varepsilon_\pi^{\perp} (u) + \frac{i}{8} \varepsilon_{\mu\nu\rho\sigma} \varepsilon_\sigma^{\perp} n_\pi^{\rho} n_\sigma^{\mu} \frac{\partial S_\alpha^{(a)}(u)}{\partial u} \right\}_{\perp}, \]

where \( g_\perp^{(v)}(u) \) and \( g_\perp^{(a)}(u) \) are twist-3 distribution amplitudes of vector mesons, and explicit formulae can be found in Ref. [24].

After adding the contributions, the form of amplitudes for the decay modes becomes the similar, just replacing \( a_i \) by \( a'_i \), which involve the \( \mathcal{O}(\alpha_s) \) corrections. \( a'_i \)'s are calculated to be

\[
\begin{align*}
    a'_2 &= a_2 + \frac{\alpha_s}{2\pi N_c f_\rho} C_F C_1 F_1, \\
    a'_3 &= a_3 + \frac{\alpha_s}{4\pi N_c f_\rho} C_F C_4 F_1, \\
    a'_5 &= a_5 + \frac{\alpha_s}{4\pi N_c f_\rho} C_F C_6 F_2, \\
    a'_7 &= a_7 + \frac{\alpha_s}{4\pi N_c f_\rho} C_F C_8 F_2, \\
    a'_9 &= a_9 + \frac{\alpha_s}{4\pi N_c f_\rho} C_1 F_1, \\
\end{align*}
\]
Table 2: The values of \( a_i \) in different scenario. In the LO column, \( a_i \) are defined in Eq. (5); in the NLO column, the values are \( a'_i \) defined in Eq. (10); in the \( Z' \) Model, they are values of \( a'_i + \Delta a'_i \), which are defined in Eq. (29).

| \( a'_i + (\Delta a'_i) \) | LO | NLO | \( Z' \) Model |
|-----------------------------|----|-----|----------------|
| \( a_2 \) | 0.170 | 0.149 − i0.010 | 0.149 − i0.010 |
| \( a_3 \) | 0.002 | −0.002 − i0.002 | (0.019 + i0.007)\( e^{\Delta} \) + (−0.002 − i0.002) |
| \( a_5 \) | −0.005 | 0.003 + i0.002 | (0.009 − i0.008)\( e^{\Delta} \) + (0.003 + i0.002) |
| \( a_7 \) | 0.000 | −0.000 + i0.000 | (3.798 − i0.067)\( e^{\Delta} \) + (−0.000 + i0.000) |
| \( a_9 \) | −0.008 | −0.008 − i0.000 | (3.932 − i0.050)\( e^{\Delta} \) + (−0.008 − i0.000) |

where \( F_{1,2} \) arise from one gluon exchange between the two currents of color-octet operators as shown in Fig. 2,

\[
F_1 = \int_0^1 du \left( g^{(a)}_4(u) - g^{(v)}_1(u) \right) \left[ -14 - 3i\pi - 12\ln \frac{\mu}{m_b} \right] \\
+ \left( 5 + \frac{u}{1-u} \right) \ln u - \frac{\pi^2}{3} + 2\text{Li}_2\left( \frac{u-1}{u} \right), \quad (17)
\]

\[
F_2 = \int_0^1 du \left( g^{(v)}_1(u) + g^{(a)}_4(u) \right) \left[ -14 - 3i\pi - 12\ln \frac{\mu}{m_b} \right] \\
+ \left( 5 + \frac{u}{1-u} \right) \ln u - \frac{\pi^2}{3} + 2\text{Li}_2\left( \frac{u-1}{u} \right). \quad (18)
\]

Here we have neglected the small effect of box diagrams and the diagrams with photon radiating from energetic light quarks, which are further suppressed by \( \Lambda_{QCD}/M_B \). Including \( \mathcal{O}(\alpha_s) \) contributions, the averaged branching ratios in the SM are estimated to be

\[
\mathcal{B}(B_s^0 \rightarrow \rho^0\gamma) = 1.1 \times 10^{-10}, \\
\mathcal{B}(B_s^0 \rightarrow \omega\gamma) = 2.3 \times 10^{-11}, \\
\mathcal{B}(B_s^0 \rightarrow \phi\gamma) = 2.9 \times 10^{-12}. \quad (19)
\]

Comparing with the results in Eq. (14) of the naive factorization, one finds the branching ratio of \( B_s^0 \rightarrow \rho^0\gamma \) almost unchanged. To find out the reason why the correction does not take an effect, we list the values of \( a'_i \) of these decay modes in the Table 2. From the table, we find that the corrections to \( a_7, a_9 \) are very small and can be neglected. Although Wilson coefficients of QCD penguin operators changed a little, but they give no contribution because the quark component of \( \rho^0 \) is \((u\bar{d} - d\bar{u})/\sqrt{2}\). For \( a_2 \), it changes much, but the correction is suppressed by the CKM elements. So, the unchanged branching ratio is quite reasonable. As for \( B_s^0 \rightarrow \omega\gamma \), the decrease of \( a_5 \) can cause that the branching ratio becomes even smaller than that of the naive factorization. For the decay \( B_s^0 \rightarrow \phi\gamma \), the increase of the ratio mainly comes from the change of \( a_3 \) and \( a_5 \).

Because there are both weak and strong phases in the decay modes \( B_s^0 \rightarrow \rho^0\gamma \) and \( B_s^0 \rightarrow \omega\gamma \), we can get the CP
asymmetries of these two channels as follows,

\[ \mathcal{A}(B_s^0 \to \rho \gamma) = 3\%; \]
\[ \mathcal{A}(B_s^0 \to \omega \gamma) = -27\%; \]

by the definition of CP asymmetry

\[ \mathcal{A} = \frac{BR(B_s^0 \to V \gamma) - BR(B_s^0 \to V' \gamma)}{BR(B_s^0 \to V \gamma) + BR(B_s^0 \to V' \gamma)}. \]

For the decay mode \( B_d^0 \to \phi \gamma \), there is only weak phase from \( V_{tb} V_{td}^\ast \), so that the CP asymmetry in this decay disappears within the SM.

### 3 Calculation in the Non-universal \( Z' \) Model

Now we consider the effects due to an extra \( U(1)' \) gauge boson \( Z' \). Usually, the flavor mixing can be induced at the tree level in up-type and/or down-type quark sector after diagonalizing their mass matrices. In some new physics model, FCNCs due to \( Z' \) exchange can be induced by mixing among the SM quarks and the exotic quarks, which is predicted and can make different \( Z' \) quantum numbers after the mixing. Here we will consider the model in which the interaction between the \( Z' \) boson and fermions are flavor non-universal for left handed couplings and flavor diagonal for right handed couplings. For simplicity, we neglected the mixing between the \( Z^0 \) and \( Z' \) and the evolution effect from the high scale \( M_{Z'} \) to the \( M_W \) scale.

We start to set up the relevant interactions with the new \( Z' \) gauge particle. Following the convention in Ref. [8], we write the couplings of the \( Z' \)-boson to fermions as

\[ J_{Z'}^{\mu} = g' \sum_i \bar{\psi}_i \gamma^\mu \left[ \epsilon_i^{\psi_L} P_L + \epsilon_i^{\psi_R} P_R \right] \psi_i, \]

where \( i \) is the family index and \( \psi \) labels the fermions and \( P_L, R = (1 \mp \gamma^5)/2 \). According to some string construction or GUT models such as \( E_6 \), it is possible to have family non-universal \( Z' \) couplings. That is, even though \( \epsilon_i^{L,R} \) are diagonal, the couplings are not family universal. After rotating to the physical basis, FCNCs generally appear at tree level in both left handed and right handed sectors. Explicitly,

\[ B_{\psi_L} = V_{\psi_L} e^{V_{\psi_L}^\dagger}, \quad B_{\psi_R} = V_{\psi_R} e^{V_{\psi_R}^\dagger}. \]

Moreover, these couplings may contain CP-violating phases beyond that of the SM. The effective Hamiltonians describing the transition mediated by the \( Z' \) boson have the form as:

\[ \mathcal{H}_{e/f}^{Z'}(b \to sq \bar{q}) = \frac{4G_F}{\sqrt{2}} V_{tb} V_{ts} \left[ \left( \frac{g' M_Z}{g_1 M_{Z'}} \right)^2 \frac{B_{sq}^L}{V_{tb} V_{ts}} (B_{sq}^L O_q + B_{sq}^R O_7) \right], \]
\[ \mathcal{H}_{e/f}^{Z'}(b \to dq \bar{q}) = \frac{4G_F}{\sqrt{2}} V_{tb} V_{td} \left[ \left( \frac{g' M_Z}{g_1 M_{Z'}} \right)^2 \frac{B_{dq}^L}{V_{tb} V_{td}} (B_{dq}^L O_0 + B_{dq}^R O_7) \right]. \]
where \( g_1 = e/(\sin \theta_W \cos \theta_W) \) and \( B_{ij}^{L(R)} \) denote the left (right) handed effective \( Z' \) couplings of the quarks \( i \) and \( j \) at the weak scale. The diagonal elements are real due to the hermiticity of the effective Hamiltonian but the off diagonal elements may contain effective weak phases. With the definition

\[
y = \left( \frac{g'L}{g_1 M_Z} \right)^2,
\]

we can parameterize these coefficients as

\[
\Delta C_9^{L}(b \to s) = y \left( \frac{B_{tb}^{L} B_{qs}^{L}}{V_{tb} V_{ts}^*} \right) = |\xi_1^L| \, e^{i\phi},
\]

\[
\Delta C_7^{L}(b \to s) = y \left( \frac{B_{tb}^{L} B_{qs}^{R}}{V_{tb} V_{ts}^*} \right) = |\xi_1^R| \, e^{i\phi},
\]

\[
\Delta C_9^{R}(b \to d) = y \left( \frac{B_{tb}^{R} B_{qs}^{L}}{V_{tb} V_{td}^*} \right) = |\xi_2^L| \, e^{i\phi'},
\]

\[
\Delta C_7^{R}(b \to d) = y \left( \frac{B_{tb}^{R} B_{qs}^{R}}{V_{tb} V_{td}^*} \right) = |\xi_2^R| \, e^{i\phi'},
\]

where \( \phi' = \phi - \beta \) (\( \phi \) is the weak phase associated with \( \phi \)).

In the following discussion, we adopt \( B_{uu}^{L(R)} \sim -2B_{dd}^{L(R)} \sim -2B_{ss}^{L(R)} \) for convenience (note that a possible negative sign can be accounted for by shifting \( \phi \) by \( \pi \)), which has been stated in detail in Ref. [13]. In order to see the effect of \( Z' \) boson, we have to know the values of the \( \Delta C_7 \) and \( \Delta C_9 \) or equivalently \( B_{tb}^{L}, B_{tb}^{R}, B_{qs}^{L}, B_{qs}^{R} \). Generally, one expects \( g'/g_1 \sim 1 \), if both the \( U(1) \) gauge groups have the same origin from some grand unified theories, and \( M_Z/M_{Z'} \sim 0.1 \) for a TeV scale neutral \( Z' \) boson, which yields \( y \sim 10^{-2} \). However, in Ref. [13] assuming a small mixing between \( Z - Z' \) bosons the value of \( y \) is taken as \( y \sim 10^{-3} \). It has been shown in [14] that the mass difference of \( B_s - \bar{B}_s \) mixing can be explained if \( |B_{tb}^{L}| \sim |V_{tb} V_{ts}^*| \). Similarly, the CP asymmetry anomaly in \( B \to \phi K, \pi K \) can be resolved if \( |B_{tb}^{L} B_{qs}^{L(R)}| \sim |V_{tb} V_{ts}^*| \). So, we assume that

\[
|x_1| = |\xi_1^L| = |\xi_1^R| = \frac{1}{2}|\xi_1^L| \in (10^{-3}, 10^{-2}).
\]

Assuming only left handed couplings are present, the bound on FCNC \( Z' \) coupling \( B_{tb}^{L} \) from \( B^0 - \bar{B}^0 \) mass difference has been obtained in [14] as

\[
y |\text{Re}(B_{tb}^{L})^2| < 5 \times 10^{-8}, \quad y |\text{Im}(B_{tb}^{L})^2| < 5 \times 10^{-8}.
\]

Using \( y \sim 10^{-2} \), one can obtain a more stringent bound on \( |B_{tb}^{L}| < 10^{-3} \). From these two relations one can obtain \( |B_{tb}^{L}| \sim 1 \). Thus, it is expected that \( \xi_2^{L,R} \sim 10^{-3} \) with the CKM matrix elements considered. However, in our analysis here we vary their values within the range \( |\xi_2| \in (10^{-3}, 10^{-2}) \), since the major purpose of this work is searching for new physics signal rather than obtaining acute numerical results.

It is noted that the other Wilson coefficients may also receive contributions from the \( Z' \) boson through renormalization group (RG) evolution. With our assumption that no significant RG running effect between \( M_Z \) and \( M_W \) scales, the RG evolution of the modified Wilson coefficients is exactly the same as the ones in the SM [19]. Using the values of these coefficients at \( m_b \) scale we can analogously obtain the new contribution to the transition amplitude as done in
Figure 3: For the decay mode $B \rightarrow \phi \gamma$, variation of the CP averaged branching ratio (in units of $10^{-10}$) with $\xi$ (in units of $10^{-3}$) and the new weak phase $\phi$ (left panel) and the variation of direct CP asymmetry (in %) with the new weak phase $\phi$ (right panel) where the solid, dot-dashed and dashed lines correspond to $\xi = 0.001, 0.005$ and $0.01$.

Figure 4: For the decay mode $B_s \rightarrow \rho^0 \gamma$, variation of the CP averaged branching ratio (in units of $10^{-8}$) with $\xi$ (in units of $10^{-3}$) and the new weak phase $\phi$ (left panel) and the variation of direct CP asymmetry (in %) with the new weak phase $\phi$ (right panel) where the solid, dot-dashed and dashed lines correspond to $\xi = 0.001, 0.005$ and $0.01$. 
is enhanced remarkably with suitable parameter \(\xi\) (in units of \(10^{-3}\)) and the new weak phase \(\phi\) (left panel) and the variation of direct CP asymmetry (in %) with the new weak phase \(\phi\) (right panel) where the solid, dot-dashed and dashed lines correspond to \(\xi = 0.001, 0.005\) and 0.01.

In Fig. 5: For the decay mode \(B_s \to \omega\gamma\), variation of the CP averaged branching ratio (in units of \(10^{-8}\)) with \(\xi\) (in units of \(10^{-3}\)) and the new weak phase \(\phi\) (left panel) and the variation of direct CP asymmetry (in %) with the new weak phase \(\phi\) (right panel) where the solid, dot-dashed and dashed lines correspond to \(\xi = 0.001, 0.005\) and 0.01.

the case of \(Z\) boson. The \(\Delta a'\) induced by the \(Z'\) are given as:

\[
\begin{align*}
\Delta a'_2 &= \Delta C_1 + \frac{\Delta C_2}{3} + \frac{\alpha_s C_F f^\perp}{2\pi N_C f_p} \Delta C_2 F_1, \\
\Delta a'_3 &= \Delta C_3 + \frac{\Delta C_4}{3} + \frac{\alpha_s C_F f^\perp}{4\pi N_C f_p} \Delta C_4 F_1, \\
\Delta a'_4 &= \Delta C_5 + \frac{\Delta C_6}{3} + \frac{\alpha_s C_F f^\perp}{4\pi N_C f_p} \Delta C_6 F_2, \\
\Delta a'_7 &= \Delta C_8 + \frac{\Delta C_9}{3} + \frac{\alpha_s C_F f^\perp}{4\pi N_C f_p} \Delta C_9 F_2, \\
\Delta a'_9 &= \Delta C_9 + \frac{\Delta C_{10}}{3} + \frac{\alpha_s C_F f^\perp}{4\pi N_C f_p} \Delta C_{10} F_1,
\end{align*}
\]

(29)

and the contributions of new physics can be formulated as

\[
\begin{align*}
\Delta \mathcal{M}^{+}_{B_s \to \gamma \gamma} &= \frac{i G_F}{\sqrt{2}} \sqrt{4\pi \alpha_s F_V f_q m_q M_B} \left[ V_{ub} V_{us}^* \Delta a'_2 - V_{tb} V_{ts}^* \left( \frac{3}{2} \Delta a'_3 + \frac{3}{2} \Delta a'_5 \right) \right], \\
\Delta \mathcal{M}^{+}_{B_s \to \phi \gamma} &= \frac{i G_F}{\sqrt{2}} \sqrt{4\pi \alpha_s F_V f_q m_q M_B} \left[ V_{ub} V_{us}^* \Delta a'_2 - V_{tb} V_{ts}^* \left( 2 \Delta a'_3 + 2 \Delta a'_5 + \frac{1}{2} \Delta a'_7 + \frac{1}{2} \Delta a'_9 \right) \right], \\
\Delta \mathcal{M}^{+}_{B_s \to \phi \phi} &= \frac{i G_F}{\sqrt{2}} \sqrt{4\pi \alpha_s F_V f_q m_q M_B} \left[ -V_{tb} V_{td}^* \left( \Delta a'_3 + \Delta a'_5 - \frac{1}{2} \Delta a'_7 - \frac{1}{2} \Delta a'_9 \right) \right].
\end{align*}
\]

(30)

(31)

(32)

Noted that \(\Delta a'\) involve the new weak phase, which may change the CP asymmetries remarkably. Now using \(|\xi| = |\xi_2| = \xi\) and taking the decay mode \(B_s \to \rho^0\gamma\) as an example, we also list the correction to \(a'_i\) from \(Z'\) boson in the last column of Table 2. From the table, we note that the corresponding Wilson coefficient of electro-weak penguin is enhanced remarkably with suitable parameter \(\xi\), which may affect the branching ratio and other observed values.

In Fig. 3 Fig. 4 and Fig. 5 we show the variation of the CP averaged branching ratios of \(B \to \phi \gamma\), \(B_s \to \rho^0\gamma\) and \(B_s \to \omega\gamma\) with \(\xi\) and the new weak phase \(\phi^{(t)}\) (left panel) and the corresponding direct CP violation with \(\phi^{(t)}\) (right
panel), respectively. As anticipated, if the $\xi = 0.01$, the branching ratios can be enhanced remarkably, which can reach to $\mathcal{O}(10^{-8})$ for $B_s \rightarrow \rho^0(\omega)\gamma$ and $\mathcal{O}(10^{-10})$ for the $B_d \rightarrow \phi^0\gamma$. All results are enhanced two orders of magnitude over the predictions of the SM. From the figures, we can also argue that there have been significant enhancements in the branching ratios for large $\xi$, or in other words for a lighter $Z'$ boson. In the experimental side, these results may be inaccessible at the Belle and BaBar presently. However, it is large enough for LHC-b and/or Super B-factories. Moreover, we find that these decay modes may have large CP asymmetries when $\xi = 0.001$ and suitable weak phase $\phi$. It implies that the contributions from new physics and from the SM can be comparable, and the interference between them leads to large asymmetries. Furthermore, future observations of these modes could in turn help us to constrain the mass of $Z'$ boson within the model.

4 Conclusion

In this work, we have studied pure annihilation type radiative processes $\bar{B}_d \rightarrow \phi\gamma$, $\bar{B}_s \rightarrow \rho\gamma$ and $\bar{B}_s \rightarrow \omega\gamma$ within the QCD factorization. After adding the vertex corrections to the naive factorization approach, we find that the non-factorizable contributions can enhance the branching ratio of $\bar{B}_d \rightarrow \phi\gamma$ decays, however, the branching ratio of $\bar{B}_s \rightarrow \rho\gamma$ is almost unchanged, but for $\bar{B}_s \rightarrow \omega\gamma$ the branching ratio is even lowered, because the corrections to EW penguin operators are much smaller than those to QCD penguin operators. The smallness of these decays within the SM makes them sensitive probes of flavor physics beyond the SM. To explore new physics potential, we have estimated the contribution of the non-universal $Z'$ model to the decays. If $\xi = 0.01$, the branching ratios can be enhanced remarkably, and reach to $\mathcal{O}(10^{-8})$ for $B_s \rightarrow \rho^0(\omega)\gamma$ and $\mathcal{O}(10^{-10})$ for the $B_d \rightarrow \phi^0\gamma$. Moreover, we have also predicted large CP asymmetries in suitable parameter spaces. These results can be tested at the LHC-b and/or Super B-factories in future. The observations of these modes could in turn help us to constrain the mass of $Z'$ within the model.

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