Research of Coal Resources Reserves Prediction Based on GM (1, 1) Model

Jiancheng Xiao
College of Mining and Safety Engineering of Shandong University of Science and Technology, Qingdao, China, 266590

Abstract. Based on the forecast of China’s coal reserves, this paper uses the GM (1, 1) gray forecasting theory to establish the gray forecasting model of China’s coal reserves based on the data of China’s coal reserves from 2002 to 2009, and obtained the trend of coal resources reserves with the current economic and social development situation, and the residual test model is established, so the prediction model is more accurate. The results show that China’s coal reserves can ensure the use of production at least 300 years of use. And the results are similar to the mainstream forecast results, and that are in line with objective reality.

Keyword. Gray system theory; GM (1, 1) model; Coal Recourses Reserves.

1. Introduction
Since the 21st century, China’s economic development has entered a high growth period. With the rapid economic growth, China’s resources demand is also increasing. However, coal is the same as oil as a primary energy, and reserves are limited. Mankind began to use coal resources has been hundreds of years of history, shallow buried coal has long been exhausted. Along with the development of resource exploration technology, coal mining is moving deeper and deeper, and coal resources can be mined are decreasing. Coal resources as an important energy and chemical raw materials of our country, under the conditions of today’s society, the effective forecast of the consumption rate of coal resources reserves is of vital importance to the adjustment of China’s resource strategy.

2. GM (1, 1) gray prediction model

2.1. The basic of principles GM (1, 1) gray prediction model
Gray prediction is a method of forecasting the development of systems with uncertain factors. The gray theory treats the random variable as a gray quantity that changes within a certain range, and treating the random process as a gray process over a certain range and for a certain period of time. Using gray theory to analyze past and present known or unknown information, and then predict the state of the future period of time to quantify the process, the above process is called gray prediction. In the gray theoretical forecasting system, GM (1, 1) model is the most commonly used single variable series forecasting model. The mathematical model uses the original series provided by the experiment, and form a regular prediction sequence after the mathematical operation. And use the generated sequence to establish the differential equation, then get the model calculation formula, and then compare with the measured value.
can get the residual, with the residuals on the model for continuous correction, the model will have a higher accuracy. The model of differential equation established by gray theory is the gray model, the gray model established using the gray theory is the gray model, denoted as GM (n, N), where n is the order of the differential equation and N is the number of variables. GM (1, n) model is generally used for predictive analysis, GM (1, 1) is a first order unilabiate differential equation model, which is the most common model. [1]

2.2. Establish GM (1, 1) gray prediction model

Building a model requires a sequence. If \(X^{(0)}\) is the original nonnegative column, the measured value of the original characteristic sequence of the system is:

\[
X^{(0)} = (x^{(0)}(1), x^{(0)}(2), \cdots, x^{(0)}(n))
\]  

(1)

The establishment of the gray model is based on the generation of the system model sequence, and then the sequence is transformed into the time continuous model of the differential equation [2], so the original sequence is accumulated once to get a data sequence: \(X^{(1)} = (x^{(1)}(1), x^{(1)}(2), \cdots, x^{(1)}(n))\)

Among them: \(X^{(1)}(k) = \sum_{i=0}^{k} x^{(0)}(i), x^{(1)}(1) = x^{(0)}(1)\)

The simplified differential equation of GM (1, 1) and its model are established:

\[
\frac{dx^{(1)}}{dt} + ax^{(1)} = b
\]  

(2)

We can find \(a\) and \(b\) satisfy the following relation and are obtained by the least squares method:

\[
\hat{a} = |B^T B|^{-1} B^T Y_n
\]  

(3)

According to the above method, find \(a, b\) to establish a gray prediction model:

\[
\hat{x}^{(1)}(k + 1) = (x^{(0)}(1) - \frac{b}{a} e^{-ak}) + \frac{b}{a}
\]  

(4)

The gray model is a model that generates data rather than the original data model, so the gray theory predicts that the number of sequences is generated by the above formula, rather than generating data from the model, as shown in the following equation:

\[
\hat{x}^{(1)} = [\hat{x}^{(1)}(1), \hat{x}^{(1)}(2), \ldots, \hat{x}^{(1)}(n), \hat{x}^{(1)}(n + 1)]
\]

Continuous reduction of the above formula can be the correct data prediction column:

\[
\hat{x}^{(0)} = [\hat{x}^{(0)}(1), \hat{x}^{(0)}(2), \ldots, \hat{x}^{(1)}n]
\]  

(5)

2.3. Testing method

The results of the gray model prediction are based on the measured data as the basic data to calculate the model error. When the error is relatively large and cannot meet the actual needs, through the residual series to modify the model to reduce the error generated. The 0th order residual is written as:

\[
\epsilon^{(0)} = x_i^{(0)} - \hat{x}_i^{(0)} (i=1, 2, \ldots n).
\]  

Where \(\hat{x}_i^{(0)}\) is the predicted value calculated by the model, through the
residual mean, residual variance and the mean and variance of the original data, the error test ratio and the small error probability can be calculated as

\[ e = \frac{\hat{\varepsilon}}{\hat{\sigma}}, \quad p = p\left( |\varepsilon^{(0)}(K) - \bar{\varepsilon}| < 0.6745 \sigma_e \right) \] (6)

Based on the two indicators calculated in Table 1, you can find the prediction accuracy of the test level.

**Table 1.** Prediction accuracy test level

| Prediction accuracy level | P   | C    |
|---------------------------|-----|------|
| Good                      | 0.95–1 | 0–0.35 |
| Eligible                  | 0.8–0.95 | 0.35–0.5 |
| Barely qualified          | 0.7–0.8 | 0.5–0.65 |
| Unqualified               | 0–0.7 | 0.65–1 |

2.4. Application of Coal Resource Market Demand Forecast

There are many factors that affect the reserves of coal resources, such as the development of mining technology, the state’s macro-control policies, and the adjustment of the socialist market economy, foreign coal imports, and new exploration of coal resources. These factors are uncertain and not random, but rather a state of fuzzy system. However, the factors that affect the reserves of coal resources are continuously changing at different time coordinates. If the domestic coal reserves are known for a period of time, the gray forecasting model can be used to infer the trend of future reserves. The statistics of the total domestic coal consumption from 2002 to 2011 is made, and the original data sequence is formed. The GM (1, 1) gray prediction model is used to predict the calculation, as shown in Table 2.

**Table 2.** China’s coal reserves from 2002 to 2009

| Year | Coal reserves (in Billion tons) | Year | Coal reserves (in Billion tons) |
|------|--------------------------------|------|--------------------------------|
| 2002 | 3317.6                         | 2006 | 3334.8                         |
| 2003 | 3342.0                         | 2007 | 3261.3                         |
| 2004 | 3373.4                         | 2008 | 3261.4                         |
| 2005 | 3326.4                         | 2009 | 3189.6                         |

2.5. Gray generation of coal market demand statistics

The design of this model has several assumptions:

① China’s economy grew at a rate from 2002 to 2009; ② there is no range of great and highly affected economic crisis in the future; ③ the future does not occur in the world war.

Based on the above assumptions, the model can be established:

(1) The original discrete data for Table 2 for the 2002 - 2011 China coal reserves (unit: 100 million tons)

\[ x^{(0)} = [3317.6, 3342.3, 3373.4, 3326.4, 3334.8, 3261.3, 3261.4, 3189.6] \]

The original data \( x^{(0)}(i) \) cumulative generation:

\[ x^{(1)} = [3317.60, 6659.60, 10033.00, 13359.40, 16694.20, 19955.50, 23216.90, 26406.50] \]

2.6. Establishment of Coal Resource Market Demand Forecasting System

(1) The cumulative sequence can establish the data matrix \( B \) and the data vector \( y \).
Calculate the parameters of the model:

\[ \frac{1}{2} \begin{bmatrix} x^{(2)}(2) + x^{(1)}(1) & 1 \\ x^{(3)}(2) + x^{(2)}(2) & 1 \\ \vdots \\ x^{(n)}(n) + x^{(n)}(n-1) & 1 \end{bmatrix} = \begin{bmatrix} -4988.60 \\ -8346.30 \\ \vdots \\ -24811.7 \end{bmatrix} \]

(2) Calculate the parameters of the model:
\[ \hat{a} = [a, b]^T = (B^T B)^{-1} B^T y = [0.008042, 2.18785568] \]

(3) Then we build the model and get the time response equation:
\[ x^{(1)}(k+1) = (x^{(0)}(1) - \frac{b}{a})e^{-ak} + \frac{b}{a} = -3405.782451e^{-0.008042k} + 425115.6428 \]

(4) According to the established time response equation to get the model calculated value, see Table 3.

Table 3. Gray Prediction Results of China’s Coal Reserves from 2002 to 2009

| Year | Actual value | Predictive value | Residual value | Relative error (%) |
|------|--------------|------------------|----------------|--------------------|
| 2002 | 3317.6       | 3317.6           | 0.00           | 0.00               |
| 2003 | 3342         | 3378.5           | 36.50          | 1.09               |
| 2004 | 3373.4       | 3351.44          | -21.96         | -0.65              |
| 2005 | 3326.4       | 3324.6           | -1.80          | -0.05              |
| 2006 | 3334.8       | 3297.97          | -36.83         | -1.10              |
| 2007 | 3261.3       | 3271.56          | 10.26          | 0.31               |
| 2008 | 3261.4       | 3245.35          | -16.05         | -0.49              |
| 2009 | 3189.6       | 3219.36          | 29.76          | 0.93               |

(5) Verify the accuracy of the model
According to the calculation can be obtained, the variance \( S_1 = 53.47969 \) of the predicted value, the variance of the residual \( S_2 = 25.14 \), the relative error is small. According to the formula to calculate the probability of small error, the calculation accuracy is qualified. The model is established only in order to estimate the length of time of China’s coal resources can be mined, the model does not require very high accuracy. So the model can be used to predict the future changes in China’s coal reserves.

(6) Calculation results
After the calculation can be obtained after 153 years the China’s coal reserves will be remaining 100.316 billion tons. After 239 years, China's coal reserves will be remaining 49.8 billion tons. In summary, China’s coal resources in accordance with the current economic and social development conditions can also be used for about 300 years, after 300 years of China’s coal resources will be running low. At present, China's economic development emphasizes sustainable, based on the data calculated by the model, the need for a sustainable development concept is growing.

3. Conclusion
(1) GM (1, 1) gray model in the prediction of China's coal reserves, with its multiple uncertainties in the dynamic performance of the system in the performance of the superior, obtained a more realistic results.
To a certain extent, the forecast results can reflect the future trend of resource reserves in China, and provide some reference for China’s future development of resource development strategy.

(2) The GM (1, 1) model is used to forecast the reserves of coal resources in China. It has the advantages of low data, simple method and high precision of forecasting.

(3) Through the successful prediction of coal reserves, the model can be extended to other resource reserves forecast, such as metal minerals, oil, natural gas and the resources explored and mined at the same time.

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