A Ramond-Neveu-Schwarz string with one end fixed

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Abstract

We study an RNS string with one end fixed on a $D0$-brane and the other end free as a qualitative guide to the spectrum of hadrons containing one very heavy quark. The mixed boundary conditions break half of the world-sheet supersymmetry. Boson-fermion masses can still be matched if space-time is 9 dimensional; thus $SO(8)$ triality still plays a role in the spectrum, although full space-time supersymmetry does not survive. We quantize the system in a temporal-like gauge where $X^0 \sim \tau$. Only odd $a$ and even $d$ R modes remain, while the NS oscillators $b$ become odd-integer moded. Although the gauge choice eliminates negative-norm states at the outset, there are still even-moded Virasoro and even(odd) moded super-Virasoro constraints to be imposed in the NS(R) sectors. The Casimir energy is now positive in both sectors; there are no tachyons. States for $\alpha'M^2 \leq 3$ are explicitly constructed and found to be organized into $SO(8)$ irreps by (super)constraints, which include a novel $\sqrt{E_0}$ operator in the NS and $\Gamma^0 \pm I$ in the R sectors. GSO projections are not allowed. The pre-constraint states above the ground state have matching multiplicities, indicating spacetime supersymmetry is broken by the (super)constraints. A distinctive physical signature of the system is a slope twice that of the open RNS string. When both ends are fixed, all leading and subleading trajectories are eliminated, resulting in a spectrum qualitatively similar to the $J/\psi$ and $Y$ particles.
1 Introduction

Bosonic String theory was originally discovered \[1, 2\] while trying to account for observed properties of hadron dynamics, and it was indeed qualitatively successful in reproducing features of hadron physics such as linear Regge trajectories for mesons, amplitudes with Dolen-Horn-Schmid duality \[3\] and desired high-s and t behaviors and poles \[4\]. The nearly-massless up and down quarks and antiquarks were incorporated into the string picture by being placed at open string endpoints, where Neumann boundary conditions ensured their moving at the speed of light. Harari-Rosner diagrams \[5, 6\] were useful for keeping track of internal quantum numbers and visualising duality between Regge pole exchanges and resonances in the s-channel.

There was an obvious need to find an extension of the string model that could encompass baryons, which had masses similar to those of the mesons (except for the very light pion), and also lay on Regge trajectories with approximately the same slope. These similarities suggested a new kind of symmetry (albeit partially broken, in view of the non-vanishing mass differences) between bosons and fermions, and string models featuring such a symmetry were constructed by Ramond \[7\] and Neveu and Schwarz \[8\]. It gradually became clear, however, that hadron dynamics was governed by a gauge theory, and superstring theory was elevated to the status of a promising candidate for describing quantum gravity and other fundamental physical phenomena at distances of \(10^{-34}\) m.

The partial success of strings in modelling hadrons was attributed to gluons forming string-like flux tubes between quarks.

It may be instructive to return to this oldest use of string theory as a phenomenological guide for the study of new generations of baryons and mesons containing at least one heavy (of the top, bottom or charmed variety) and one light quark. In an earlier work \[9\] we modeled such mesons as excitations of an open string with one end fixed on a D0-brane and the other end free. This gave rise to testable predictions such as a doubled Regge slope and a reduction in states due to the restriction to odd oscillators. Whether the description is of any merit will become apparent when higher spin excitations of heavy quark bearing mesons are found and studied. Strings with mixed boundary conditions have also been considered in \[10, 11, 12, 13\].

In this note, we examine the Ramond-Neveu-Schwarz (RNS) version of \[9\]. Our main motivation and hope is that this may provide qualitative hints about the spectrum of baryons and mesons with a single c, b or a t quark, although we of course do not expect it to provide a heavy-hadron model that is realistic in all its details.

Apart from its possible phenomenological usefulness, the system is also of some intrinsic string-theoretic interest in a number of respects. One of these is that the system we are studying can be thought of as an open RNS string viewed from a frame where its mid-point is at rest if one disregards one half of the string. Exactly the same “half a Neumann-Neumann string” picture with
even modes in the R and odd modes in the NS sector is encountered in String Field Theory [17], where the interaction of two strings is effected by joining them up to the mid-point and leaving the other halves free. Another point of interest is the use of a gauge halfway between the light-cone and the old covariant treatments. This gauge naturally suggests itself in the mixed ND string. The old covariant method of quantization of even the standard RNS string is described only very sketchily in standard reference works such as [14, 15, 16]; the detailed implementation of the constraints peculiar to our problem and the subsequent emergence of the physical spectrum turns out to be surprisingly intricate.

The plan of the paper is as follows: In section 2 we obtain the mode expansions and the restricted SUSY transformations allowed by our novel boundary conditions. We then impose a gauge we might call “the rest-frame gauge”. Section 3 is devoted to the Virasoro and super-Virasoro constraints in the NS and R sectors. Some of the new features that emerge include the limitation of the Virasoro algebra to even modes, while the modes of the R and NS constraints can only be odd and even respectively. The Virasoro and super-Virasoro algebras turn out to have non-vanishing anomalies in all dimensions for reasons we will discuss. In section 4, we work out the low-lying members of the NS and R spectra. The organization and projection of physical states into specific $SO(D-1)$ irreps via the constraints involves novelties such as an NS constraint that may be regarded as “a bosonic-sector square root of $L_0$” and an R constraint which is a truncated form of the usual Ramond-Dirac operator $F_0$. The spectrum does not exhibit full space-time supersymmetry, but it is at least possible to match the masses of bosons and fermions if $D - 1 = 8$, which means aspects of $SO(8)$ triality are still reflected in the states. Because of changes in the modes and their Casimir energies, both the NS the R ground states become massive. Both the absence of purely transverse states and a spacetime dimension less than 10 suggest are features of consistent string theories in sub-critical dimensions. For higher states, we present a level multiplicity formula which counts the number of states before the constraints are imposed. Remarkably, the number of unconstrained bosonic and fermionic states (except for the bosonic ground state) are equal up to all the levels we have calculated. This indicates that the partial breaking of world-sheet supersymmetry through mixed boundary conditions is reflected in the space-time picture in a surprisingly indirect way through the superconstraints. The paper ends with concluding remarks in section 5.

2 The classical RNS string with one end fixed

2.1 Equation of motion:

We start with standard material to set our notation and conventions. In the superconformal gauge the action reads

$$S = -\frac{1}{2\pi} \int d^2\sigma \left\{ \partial_{\alpha} X_{\mu} \partial^{\alpha} X^{\mu} - i \Psi^\rho \rho^{\alpha} \partial_{\alpha} \Psi_{\mu} \right\}, \quad (1)$$
where $\rho^\alpha$ represents the two dimensional Dirac matrices and a convenient basis satisfying $\{\rho^\alpha, \rho^\beta\} = -2\eta^{\alpha\beta}$ is

$$\rho^0 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \quad \rho^1 = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}.$$ We will denote the upper and lower components of $\Psi$ as $\Psi_-$ and $\Psi_+$, respectively. We should note that the Dirac operator, $i\rho^\alpha \partial_\alpha$, is real in this representation; thus, the components of the Majorana world sheet spinor may be taken as real. In this paper, our focus will be on these spinor degrees of freedom since the bosonic fields have already been treated in [9].

The fermion equation of motion derived from (1) is simply the two dimensional Dirac equation and in the chosen basis it splits up into the following equations:

$$\frac{1}{2} \partial_\sigma \Psi_\mu^- = \left( \frac{\partial}{\partial \sigma} + \frac{\partial}{\partial \tau} \right) \Psi_\mu^- = 0, \quad (2)$$

$$\frac{1}{2} \partial_\tau \Psi_\mu^+ = \left( \frac{\partial}{\partial \sigma} - \frac{\partial}{\partial \tau} \right) \Psi_\mu^+ = 0. \quad (3)$$

The equation of motion for the bosonic field in the same basis is

$$\partial_\tau (\partial_\sigma X^\mu) = \partial_\sigma (\partial_\tau X^\mu) = 0. \quad (4)$$

### 2.2 Boundary conditions and mode expansions:

The boundary conditions for the the bosonic fields are as in [9]: We locate the infinitely massive “quark” at the $\sigma = 0$ end and identify this point with the origin of target space coordinates through the Dirichlet boundary condition

$$X^1(0, \tau) = 0, \quad (5)$$

while we adopt the Neumann boundary condition

$$\partial_\sigma X^\mu(\sigma_{\text{max}}, \tau) = 0 \quad (6)$$

for the massless end. If we use the usual integrally moded oscillators to expand $X^\mu$, these two conditions lead to the requirement that $\alpha^\mu_n = 0$ for even $n$ and $\sigma_{\text{max}} = \pi/2$ (another option, adopted in [10] among others, is to maintain the usual range $[0, \pi]$ for $\sigma$, in which case the $\alpha^\mu_n$ oscillators become half-integrally moded; our choice is to work with the odd-numbered subset of the original integral modes rather than to introduce an entirely new set of oscillators). As mentioned in [9], the disappearance of the even modes is quite similar to the same phenomenon in the elementary quantum mechanics problem of a half-oscillator potential $V = \frac{1}{2}kx^2$ for $x \geq 0$, $V = \infty$ for $x < 0$. As in the usual RNS string, vanishing of the surface term requires $\Psi_\mu^+ = \pm \Psi_\mu^-$ at each end,
and without loss of generality, we set $\Psi^\mu_+(0, \tau) = \Psi^\mu_-(0, \tau)$ for $\sigma = 0$ and then consider the two cases depending on the relative sign at $\sigma_{max} = \pi/2$. Leaving the question of how to reconcile the choice at $\sigma = 0$ with world-sheet supersymmetry to the next section, the two cases arising from the sign choices are the following:

(i) The Ramond (R) boundary condition

$$\Psi^i_+(\frac{\pi}{2}, \tau) = \Psi^i_-(\frac{\pi}{2}, \tau)$$

leads to the exclusively even-moded expansion

$$\Psi^i_-(\sigma, \tau) = \sum_{n:even} d_n^i e^{-in(\tau - \sigma)},$$

$$\Psi^i_+(\sigma, \tau) = \sum_{n:even} d_n^i e^{-in(\tau + \sigma)}.$$  

(ii) The Neveu-Schwarz (NS) boundary condition

$$\Psi^i_+(\frac{\pi}{2}, \tau) = -\Psi^i_-(\frac{\pi}{2}, \tau)$$

results in the odd-integer modes

$$\Psi^i_-(\sigma, \tau) = \sum_{r:odd} b_r^i e^{-ir(\tau - \sigma)},$$

$$\Psi^i_+(\sigma, \tau) = \sum_{r:odd} b_r^i e^{-ir(\tau + \sigma)}.$$  

We note that while the boundary conditions merely restrict the Ramond oscillators $d_n^i$ to even and the $\alpha_n^i$ to odd modes, the Neveu-Schwarz sector actually changes from half-integral to odd integral oscillators. This will lead to a massless NS and a massive R ground state, reversing the usual RNS results.

### 2.3 Broken global world-sheet SUSY:

The action is invariant under the infinitesimal world-sheet supersymmetry transformations

$$\delta X^\mu = \bar{\epsilon} \Psi^\mu,$$  

$$\delta \Psi^\mu = -i\rho^\alpha \partial_\alpha X^\mu \epsilon.$$  

with $\epsilon$ a constant anticommuting spinor. Let us examine the space components of (13) and (14) first. For fermionic fields (13) combined with (5) gives

$$\delta X^i(0, \tau)_{\sigma=0} = (\epsilon^i_+ \Psi^i_- - \epsilon^i_- \Psi^i_+)_{\sigma=0} = 0.$$  

This would normally be satisfied either by
and
\[ \epsilon_+ = \epsilon_-, \]  
or by
\[ (ii) \]
\[ \Psi_+^i(0, \tau) = -\Psi_-^i(0, \tau), \]  
and
\[ \epsilon_+ = -\epsilon-. \]  

However, having ruled out (ii) by our choice of boundary conditions in the previous subsection, we are restricted to (i), which eliminates half of the worldsheet supersymmetry. We could have reversed the fermion boundary conditions at the two ends and consequently chosen (ii), but the obvious conclusion is that our mixed Dirichlet-Neumann conditions are compatible with only half of the usual supersymmetry transformations. This breaking of worldsheet SUSY will have to manifest itself in the space-time spectrum, as we will see later. Since Poincaré invariance has been explicitly broken, we do not expect spacetime supersymmetry operators, which are essentially square-roots of the Poincaré generators, to be effective.

### 2.4 Gauge Choice:

For the bosonic string with one end fixed at the origin, it was natural to choose the gauge condition
\[ X^0 = a_0^0 \tau = lp^0 \tau \]  
just as the light-cone gauge \( X^+ \sim \tau \) is suited to a string with both ends moving at the speed of light. Indeed, the first excited state of such a free open string is a massless vector boson with \( D - 2 \) polarizations, which is the number of transverse oscillators used in the light-cone gauge. Our problem, in contrast, leads to a massive vector with \( D - 1 \) polarization states, which is in accord with (20). Combining this gauge choice with (13), we get
\[ \delta X^0 = \hat{e} \Psi^0 = 0. \]  

This is similar to setting all the \( \Psi^+ \) and \( X^+ \) oscillators to zero in the light-cone gauge treatment of the usual RNS string. The condition (21) kills all the \( \theta^0_n \) modes of the NS-sector. On the other hand, for the R-sector, all \( d^0_n \) modes are zero except \( d^0_0 \). To see why this mode survives, consider the explicit form of (21) with the \( \Psi^i \) in (15) replaced with \( \Psi^0 \) and \( \epsilon_+ = \epsilon_-. \) This is only compatible with \( d^0_0 \) being non-zero. This is just as well, since we would not want to lose an element of the Clifford algebra in \( (D - 1, 1) \).
3 Quantization

With the “rest-frame gauge” chosen above, the $\alpha^0_n$ modes and their associated negative norm states \[^9\] are discarded at the outset. However, unlike the situation in the light-cone gauge, this does not mean that all the states obtained by hitting the ground state with the spacelike oscillators are automatically physical. In the light-cone gauge, the constraints are implemented at the operatorial level, allowing one to solve for the $\alpha^-_n$ in terms of transverse oscillators. Our gauge choice does not involve an off-diagonal space-time metric and does not offer a similar possibility. There are still constraints which must be imposed on all possible states, and only the states annihilated by these constraints are the final physical ones. This is similar to non-covariant treatments of gauge theories where one first sets $A^0 = 0$, but then also imposes $\nabla \cdot A = 0$ to isolate the physical degrees of freedom. Since negative metric states in the Hilbert space are eliminated from the beginning, Faddeev-Popov ghosts are not needed.

3.1 Canonical Quantization:

We will follow the conventional canonical quantization procedure. The canonical anticommutation relations for the fermionic fields are

$$\{\Psi^\dagger_\alpha(\sigma, \tau), \Psi^\dagger_\beta(\sigma', \tau)\} = \pi \delta(\sigma - \sigma') \delta^{ij} \delta_{\alpha\beta}. \quad (22)$$

Substituting the known mode expansions, we obtain anticommutation relations for the modes. These are

$$\{b^i_r, b^j_s\} = \delta^{ij} \delta_{r+s}(r, s: \text{odd}) \quad (23)$$

for the NS-sector, and

$$\{d^i_m, d^j_n\} = \delta^{ij} \delta_{m+n}(m, n: \text{even}) \quad (24)$$

for the R-sector. Finally, the $d^0_0$ mode obeys

$$\{d^0_0, d^0_0\} = -I. \quad (25)$$

As in the ordinary RNS string, the $d^0_0$ and $d^0_0$ are proportional to the Dirac gamma matrices with $\Gamma^0 = i \sqrt{2} d^0_0$ in $(D-1, 1)$ dimensional Minkowski space.
3.2 Super-Virasoro constraints

The unphysical modes are eliminated by imposing \( T_{\alpha\beta}^+ | \Psi_{\text{phys}} \rangle = 0 \) and \( J_\alpha^+ | \Psi_{\text{phys}} \rangle = 0 \) on the states, where \( T_{\alpha\beta}^+ \) and \( J_\alpha^+ \) are the positive-frequency components of the energy-momentum tensor and the supercurrent, respectively.

The energy-momentum tensor for the fermionic part is

\[
T_{\alpha\beta} = \frac{i}{4} \bar{\Psi}^\mu \rho_\alpha \partial_\beta \Psi^\mu + \frac{i}{4} \bar{\Psi}^\mu \rho_\beta \partial_\alpha \Psi^\mu - \frac{i}{4} \eta_{\alpha\beta} \bar{\Psi}^\mu \rho^\gamma \partial_\gamma \Psi^\mu
\]

and, in the chosen gauge, its components in conformal coordinates are

\[
T_{++} = \frac{1}{2} (T_{00} + T_{01}) = \frac{i}{2} : \Psi^i \partial_+ \Psi_{+i} :, \quad (27)
\]

\[
T_{--} = \frac{1}{2} (T_{00} - T_{01}) = \frac{i}{2} : \Psi^i \partial_- \Psi_{-i} :. \quad (28)
\]

The supercurrent is

\[
J_\alpha = \frac{1}{2} \rho^\beta \rho_\alpha \Psi^\mu \partial_\beta X^\mu. \quad (29)
\]

The nonzero components of the supercurrent, again in conformal coordinates, are

\[
J_+ = \Psi^i \partial_+ X_i + \Psi^0 \partial_+ X_0, \quad (30)
\]

\[
J_- = \Psi^i \partial_- X_i + \Psi^0 \partial_- X_0. \quad (31)
\]

The conservation law of the supercurrent yields

\[
\partial_- J_+ = \partial_+ J_- = 0. \quad (32)
\]

We now examine the implications of these constraints in the Neveu-Schwarz and Ramond sectors.

3.2.1 Neveu-Schwarz constraints

The super-Virasoro operators are the Fourier coefficients of the energy-momentum tensor and supercurrent. To obtain these coefficients we use the fact that \( T_{++}(\sigma) = T_{--}(-\sigma) \) and the “doubling trick” of Polchinski [15], extending the region of integration to \([-\frac{\pi}{2}, \frac{\pi}{2}]\). We can then write

\[
L^{(b)}_m = \frac{1}{\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^{im\sigma} T_{++}(\sigma) d\sigma. \quad (33)
\]

Strictly speaking, for \( m = 0 \) the above definition disagrees by an additive constant with what should properly be called \( L_0 \). The constant arises from the non-tensor Schwarzian derivative term in the transformation from the \((\tau, \sigma)\) coordinates used above to the \( z \) coordinate \((z = \exp(-i\sigma + \tau))\) in the Euclideanized
world sheet) in terms of which the \( L_n \) are actually defined. We will later compute such constants. Substituting the mode expansions, we get

\[
L_m^{(b)} = \frac{1}{2} \sum_{r: \text{odd}} (r + \frac{m}{2}) : b_r^i b_{m+r}^i : + a^b \delta_{m,0}
\]  

(34)

where \( m \) is seen to be necessarily even. It is normal ordering for \( m = 0 \) that brings the constant \( a^b \). The regularization recipe for calculating such constants for all modes will be given later. The \( L_m^{(b)} \) thus complement the \( L_m^{(a)} \) which are also even. This is an indication of the consistency of the mode elimination scheme imposed by our mixed boundary conditions.

The fermionic generators of this sector are obtained similarly using \( J_+ (\sigma) = J_- (-\sigma) \), which results in

\[
G_m = \sqrt{\frac{2}{\pi}} \int_{-\pi}^{\pi} e^{im\sigma} J_+ (\sigma) d\sigma.
\]  

(35)

In terms of the mode expansions one finds

\[
G_m = \sum_{r: \text{odd}} \alpha^i_{-r} b^i_{m+r}
\]  

(36)

where \( m \) is again forced to be even.

The super-Virasoro algebra for this sector is

\[
\begin{align*}
[L_m, L_n] &= (m - n)L_{m+n} + m(\alpha_0^a)^2 \delta_{m+n} + A(m)\delta_{m+n}, \\
[L_m, G_n] &= (\frac{m}{2} - n)G_{m+n}, \\
\{G_m, G_n\} &= 2L_{m+n} - 2(a^a + a^b)\delta_{m+n} + (\alpha_0^a)^2 \delta_{m+n} + B(m)\delta_{m+n}
\end{align*}
\]  

(37, 38, 39)

where \( L_m = L_m^{(a)} + L_m^{(b)} \) and

\[
L_0^{(a)} = -\frac{1}{2} (\alpha_0^a)^2 + \frac{1}{2} \sum_{r: \text{odd}} : \alpha_i^a \alpha_{-r}^i : + a^a,
\]  

(40)

\[
L_m^{(a)} = \frac{1}{2} \sum_{r: \text{odd}} : \alpha_i^a \alpha_{m-r}^i :
\]  

(41)

where, in the second line, \( m \) is even and nonzero. The normal ordering constant \( a^a \) will be presented later.

We see that all super-Virasoro operators have their usual forms, say as in [14], except for the fact that they are written in terms of odd modes only. We must now see how this changes the central extension terms. The \( \alpha \)-mode anomaly \( A(m) \) was calculated in [9] with the result
\[ A^\alpha(m) = \frac{(D-1)}{24}(m^3 + 2m). \] (42)

Thus we will only calculate the anomaly due to the \( b \)-modes, and this can be easily done via

\[ A^{(b)}(m) = \langle 0 \mid [L_m^{(b)}, L_{-m}^{(b)}] \mid 0 \rangle \] (43)

\[ = \langle 0 \mid L_m^{(b)} L_{-m}^{(b)} \mid 0 \rangle \] (44)

\[ = \frac{1}{4} \langle 0 \mid \sum_{r,s, odd} (r + \frac{m}{2})(s - \frac{m}{2}) b^i_{-r} b^i_{m+r} b^j_{-s} b^j_{m+s} \mid 0 \rangle \] (45)

\[ = \frac{1}{4} \langle 0 \mid \sum_{r,s, odd} (r + \frac{m}{2})(s - \frac{m}{2}) b^i_{m+r} b^j_{-r} b^j_{m-s} b^j_{s} \mid 0 \rangle, \] (46)

where \( m \) is, of course, even and positive. The expectation value in the third line vanishes for \( r > 0 \) and \( s > m \), whereas the expectation value in the fourth line vanishes for \( s < 0 \) and \( m < -r \). This yields

\[ A^{(b)}(m) = \frac{1}{4} \langle 0 \mid \sum_{r=-m+1}^{m-1} \sum_{s=1}^{m-1} (r + \frac{m}{2})(s - \frac{m}{2}) b^i_{m+r} b^j_{m+s} b^j_{-s} \mid 0 \rangle \] (47)

\[ = 2(D - 1) \sum_{s=1}^{m-1} (s - \frac{m}{2})^2 \] (48)

where all the summations above are over odd integers and \( \eta_{ij} \eta^{ij} = D - 1 \) is the number of space coordinates. The summation in the second line, when taken over all integers, produces the well-known NS-sector central extension term. On the other hand, our odd integer summation gives

\[ A^{(b)}(m) = \frac{(D-1)}{48}(m^3 - 4m). \] (49)

Hence the total central extension term of the first commutator is

\[ A(m) = \frac{(D-1)}{16} m^3. \] (50)

A similar computation for the central extension term appearing in the anti-commutator of the \( G_m \) gives

\[ B(m) = \langle 0 \mid G_m G_{-m} \mid 0 \rangle, \] (51)

\[ = \frac{(D-1)}{4} m^2. \] (52)
At this point, we note that the anomaly-free $OSp(1 \mid 2)$ closed subalgebra of the NS-sector of the usual RNS string is now reduced to the Abelian super-subalgebra of $L_0$ and $G_0$.

The mass-shell condition comes from the zero-frequency part of the Virasoro constraint $T_{\alpha\beta} = 0$. At the quantum level, the masses of the NS states follow from

$$ (L^{(a)}_0 + L^{(b)}_0) \mid \Psi_{phys} \rangle = 0 \quad (53) $$

### 3.2.2 Ramond constraints

The super-Virasoro operators of this sector are

$$ L^{(d)}_m = \frac{1}{\pi} \int_{-\pi/2}^{\pi/2} e^{im\sigma} T_{++}(\sigma) d\sigma \quad (54) $$
and

$$ F_m = \frac{\sqrt{2}}{\pi} \int_{-\pi/2}^{\pi/2} e^{im\sigma} J_+(\sigma) d\sigma. \quad (55) $$

Substituting the known mode expansions of $T_{++}$ and $J_+$, we find

$$ L^{(d)}_m = \frac{1}{2} \sum_{n: \text{even}} (n + \frac{m}{2}) : d_n^- d_{m+n}^i : + a^d \delta_{m,0} \quad (56) $$

where $m$ must be even, and

$$ F_s = \sum_{r: \text{odd}} a^i_{-r} d_{r+s}^i \quad (57) $$

where $s$ is necessarily odd. An exception is the operator

$$ f_0 = \alpha^0_0 d^0_0 \quad (58) $$

which is all that survives here from the generalized Dirac operator $F_0$ of the standard RNS string. The super-Virasoro algebra of this sector is as follows:

$$ [L_m, L_n] = (m-n)L_{m+n} + A(m)\delta_{m+n} + m(\alpha^0_0)^2 \delta_{m+n}, \quad (59) $$

$$ [L_m, F_r] = \left( \frac{m}{2} - r \right) F_{m+r}, \quad (60) $$

$$ \{ F_r, F_s \} = 2L_{r+s} + (\alpha^0_0)^2 \delta_{r+s} - 2(a^a + a^d) \delta_{r+m} + B(r)\delta_{r+s}, \quad (61) $$

$$ [L_m, f_0] = 0, \quad (62) $$

$$ \{ f_0, f_0 \} = -(\alpha^0_0)^2 \quad (63) $$
where \( L_m = L_m^{(\alpha)} + L_m^{(d)} \) and the commutators (or anticommutators, as appropriate) of \((a_0^0)^2\) or \((f_0)\) with other operators vanish.

The central extension terms of this algebra can be calculated as in the previous section, giving

\[
A^{(d)}(m) = \frac{(D - 1)}{48} (m^3 + 8m) .
\]

Then the total central extension term of the first commutator, including the contribution of the \( \alpha \) modes becomes

\[
A(m) = \frac{(D - 1)}{16} (m^3 + 4m) .
\]

A similar treatment for the central extension term of anti-commutators of fermionic generators yields

\[
B(r) = \frac{(D - 1)}{4} (r^2 + 1) .
\]

4 The Spectrum

4.1 Normal ordering constants:

In this section we will compute the normal ordering constants \( a^\alpha, a^b \) and \( a^d \) using Polchinski’s ‘heuristic recipe’ [19]. This involves two steps: 1) One adds the oscillator zero point energies, regularizing the divergent result via an analytic continuation of the \( \zeta \)-function. This gives the normal ordering constant in the \((\tau, \sigma)\) coordinates. 2) The \( L_n \), and in particular \( L_0 \) are by contrast the modes in the expansion of \( T(z) \) in powers of the \( z \) coordinate (on the Euclideanized world sheet, \( z = \exp(-i\tau + \sigma) \)); thus one must add the non-tensor shift coming from the Schwarzian derivative of the transformation from \((\tau, \sigma)\) to \( z \). For \( L_0 \), the shift is given by \( 1/24 \), where \( c \) is the central charge. In [9], this constant contribution was omitted; the masses there should all be increased by \( D - 1 \). In our case, the total \( c \) is \( \frac{D - 1}{2} (1 + \frac{1}{2}) \), where \( D - 1 \) obviously comes from the number of space coordinates, while the 2 in the denominator can be interpreted as the result of keeping of only odd or even modes. The last 3/2 expresses the fact that each world-sheet boson contributes 1 and each world-sheet fermion 1/2. The combined shift of \( \frac{D - 1}{48} \) thus applies to both the R and the NS sectors.

We may similarly compute the normal ordering constants for each excitation separately. The zero point energy of the \( \alpha \) oscillators is \( \frac{D - 1}{2} \) times a sum over positive odd integers, which, using \( \zeta(-1) = \frac{1}{12} \), is simply \( \zeta(-1) - 2\zeta(-1) = \frac{1}{12} \). The shift is \( \frac{D - 1}{48} \), yielding \( a^\alpha = \frac{D - 1}{16} \). The zero point energy of the odd integral
moded NS \( b \) world-sheet fermion oscillators is \(-\frac{D-1}{2}\) and the shift is \( \frac{D-1}{32} \), giving \( a^b = -\frac{D-1}{32} \). The total normal ordering constant in \( L_0(NS) \) is thus \( \frac{D-1}{32} \).

For the \( d \) oscillators, the zero point energy is \( \frac{D-1}{2}(-\frac{1}{2})(-\frac{1}{6}) \), the first minus being due to the fermionic nature of the \( d \)'s and the \( (-\frac{1}{6}) \) coming from a sum of even positive integers. With the shift of \( \frac{D-1}{32} \), we find \( a^d = \frac{3(D-1)}{32} \) and the total normal ordering constant in \( L_0(R) \) becomes \( \frac{5(D-1)}{32} \).

### 4.2 Mass formulas in the NS- and R-Sectors:

With the calculated normal ordering constants, we can write down the mass-shell conditions. For the NS-sector, we have

\[
(-\alpha'_{N\bar{D}}M^2 + N^{(a)} + N^{(b)} + \frac{D-1}{32}) \vert \Psi_{phys} \rangle = 0, \quad (67)
\]

while for the R-sector

\[
(-\alpha'_{N\bar{D}}M^2 + N^{(a)} + N^{(d)} + \frac{5(D-1)}{32}) \vert \Psi_{phys} \rangle = 0, \quad (68)
\]

where \( \alpha'_{N\bar{D}} = \frac{\ell^2}{2} \), as calculated in [9]. This doubling of the slope relative to \( \alpha'_{NN} \) of the usual open string is one of the principal distinguishing features of our system. A quick way to understand this result is to consider a classical string with one end fixed in its highest angular momentum (leading Regge trajectory) mode, where it rotates rigidly. This can also be viewed as an ordinary open string in the same mode with its center of mass at rest and then throwing away one half. In order to preserve the relation \( J \sim \alpha' M^2 \) with \( J \) and \( M \) both being halved, the slope has to be doubled.

Since half of the world-sheet supersymmetry has been broken at the beginning by the restriction \((\epsilon_+ = \epsilon_-)\), we do not expect space-time supersymmetry to appear the way it does in the RNS string subjected to the GSO projection [18]. Actually, the halving of supersymmetry via \( D \)-branes is a familiar phenomenon [10], and the fixed end here is nothing but a \( D0 \)-brane. While the spectrum is thus not expected to be fully supersymmetric, we have two further options in the degree of supersymmetry breaking:

(i) \( D - 1 = 8n \): The masses of the states in the NS-sector have the the same values as those in the R-sector for \( \alpha'_{N\bar{D}}M^2 \geq n \), partially preserving SUSY in the mass spectrum.

(ii) \( D - 1 \neq 8n \): Mass values in the two sectors are completely different and SUSY is completely broken.

In the following, we will work with the minimal supersymmetry breaking option \( n = 1, D = 9 \). All states being massive, we expect to see irreps of \( SO(8) \), which is clearly a remnant of the space-time SUSY enjoyed by the ten-dimensional superstring. We now examine the spectra of the two sectors separately.
4.3 Neveu-Schwarz spectrum:

A physical state in the NS-sector must satisfy

\begin{align*}
G_{2m} \vert \phi \rangle &= 0, \, m > 0 \\
L_{2n} \vert \phi \rangle &= 0, \, n > 0 \\
L_{0} \vert \phi \rangle &= 0,
\end{align*}

and it is the last condition that leads to $\alpha'_{ND} M^2 = N^{(a)} + N^{(b)} + (D-1)/32$.

There is also a $G_0$ constraint that has to be handled separately. An examination of the NS constraint algebra (37-39) shows that all the physical state conditions can be obtained from $G_2$, $G_0$ and $L_0$. $G_0$ does not in fact annihilate physical states; it instead requires them to be its eigenstates with the masses as eigenvalues. From (39) and (67), we see that

\begin{equation}
G_0 \vert \phi \rangle = \sqrt{N^{(a)} + N^{(b)}} L \vert \phi \rangle
\end{equation}

We note that this amounts to taking the square root of the Klein-Gordon-like operator $L_0$. The novelty is that this happens not in the fermionic but in the bosonic sector!

We now examine how low-lying physical states are obtained. Since negative-metric states have been barred from the beginning, it is not immediately obvious what role is left for the above constraints to play. If we use the $L_n$ directly, the answer in the bosonic NS sector turns out to be that all $\alpha_{-n}^{-1}$ and $b_{-n}^{-1}$ oscillators for $n > 1$ are ruled out, and the surviving states are automatically organized into $SO(8)$ irreps. Hence the daughter trajectories are eliminated from the spectrum. The $G_n$ constraints prune the remaining states even further; for example, a potential $N = 3$ state of the form $\alpha_{-1}^{-1} \alpha_{-1}^{j} b_{-1}^{k} \vert 0 \rangle$ is prohibited by the $G_2$ constraint. Finally, the $G_0$ constraint allows only specific linear combinations of the states that have survived that far. This is obviously different from what happens in the light-cone gauge in the usual RNS string, where all combinations of $\alpha_{-n}^{-i}, b_{-n}^{-i}, (i = 1, ..., 8)$ oscillators on the vacuum are guaranteed to produce physical states, which then combine with the others at the same mass to give $SO(9)$ irreps. There being no obvious pattern to the allowed states beyond what we have just mentioned, we limit ourselves to displaying below the contents of the first four levels.

- $N = 0, M^2 = 0$
  \begin{equation}
  \vert 0 \rangle
  \end{equation}
  
  is a massless scalar, providing a stable vacuum for this sector.

- $N = 1, \alpha'_{ND} M^2 = 1$: We start with the two massive vector states
\[ | \alpha \rangle \equiv \alpha^i_{-1} | 0 \rangle, 8_\nu \]
\[ | \beta \rangle \equiv b^i_{-1} | 0 \rangle, 8_\nu \]

allowed by the \( G_2 \) constraint. However, the only combination permitted by \( G_0 \) is \( | \alpha \rangle + | \beta \rangle \).

- **\( N = 2, \alpha'_{ND}M^2 = 2 \):** Under \( G_2 \), the massive tensor states
  \[
  | 1 \rangle \equiv \left\{ \alpha^i_{-1} \alpha^j_{-1} - \frac{\delta^{ij}}{D-1} \alpha^k_{-1} \alpha^k_{-1} \right\} | 0 \rangle, 35_\nu
  \]
  \[
  | 2 \rangle \equiv b^i_{-1} b^j_{-1} | 0 \rangle, 28
  \]
  \[
  | 3 \rangle \equiv \left\{ \alpha^i_{-1} b^j_{-1} + \frac{2\delta^{ij}}{D-1} \alpha^k_{-1} b^k_{-1} \right\} | 0 \rangle, 35_\nu
  \]
  \[
  | 4 \rangle \equiv \left\{ \alpha^i_{-1} b^j_{-1} - \frac{2\delta^{ij}}{D-1} \alpha^k_{-1} b^k_{-1} \right\} | 0 \rangle, 28
  \]

are the allowed combinations. We recall that since the even \( \alpha \) and \( b \) modes have been eliminated at the beginning, we cannot have excitations such as \( \alpha^i_{-2} | 0 \rangle \) and \( b^j_{-2} | 0 \rangle \). Out of the above four, the condition

\[ G_0 | \phi \rangle = \sqrt{2} | \phi \rangle \]

allows only

\[ \sqrt{2} | 1 \rangle + | 3 \rangle, 35_\nu \]

and

\[ \sqrt{2} | 2 \rangle + | 4 \rangle, 28_\nu. \]

- **\( N = 3, \alpha'_{ND}M^2 = 3 \)**

  \[
  | 1 \rangle \equiv \left\{ \alpha^i_{-1} \alpha^j_{-1} \alpha^k_{-1} - \frac{\delta^{ij}}{D+1} \alpha^n_{-1} \alpha^k_{-1} - \frac{\delta^{ik}}{D+1} \alpha^j_{-1} \alpha^k_{-1} - \frac{\delta^{jk}}{D+1} \alpha^i_{-1} \alpha^k_{-1} \right\} | 0 \rangle, 112_\nu
  \]
  \[
  | 2 \rangle \equiv \left\{ b^i_{-1} b^j_{-1} \alpha^k_{-1} + b^j_{-1} b^k_{-1} \alpha^i_{-1} + b^k_{-1} b^i_{-1} \alpha^j_{-1} + \frac{2\delta^{ik}}{2 - D} b^i_{-1} b^j_{-1} \alpha^k_{-1} \right\}
  \]
  \[
  \quad - \frac{2\delta^{jk}}{2 - D} b^i_{-1} b^j_{-1} \alpha^k_{-1} | 0 \rangle, 160_\nu
  \]
  \[
  | 3 \rangle \equiv \left\{ b^i_{-1} b^j_{-1} b^k_{-1} | 0 \rangle, 56_\nu
  \]
  \[
  | 4 \rangle \equiv \left\{ b^i_{-1} b^j_{-1} \alpha^k_{-1} - b^j_{-1} b^k_{-1} \alpha^i_{-1} - b^k_{-1} b^i_{-1} \alpha^j_{-1} \right\} | 0 \rangle, 56_\nu
  \]

The tensorial form of the first state is dictated by \( L_2 \), which is obtained from the anticommutator of \( G_2 \) and \( G_0 \). As mentioned earlier, the same \( L_2 \) prohibits states with higher oscillators of the form \( \alpha^i_{-3} | 0 \rangle \). The forms of the second and fourth states are determined by the action of \( G_2 \). Finally, only the combination

\[ \sqrt{3} | 3 \rangle + | 4 \rangle, 56_\nu \]

obeys \( G_0 | \phi \rangle = \sqrt{3} | \phi \rangle \).
4.4 Ramond spectrum:

The physical states in the Ramond sector must satisfy

\[ F_{2m+1} | \psi \rangle = 0, \ m > 0 \]  \hspace{1cm} (73)

\[ L_{2n} | \psi \rangle = 0, \ n > 0 \]  \hspace{1cm} (74)

\[ (L_0) | \psi \rangle = 0. \]  \hspace{1cm} (75)

However, using the superalgebra (59-63), one can see that these infinite set of conditions can be reduced to just the \( F_1, F_3 \) and \( L_0 \) constraints. In addition, taking the square root of (63), we have an \( f_0 \) constraint which simplifies to

\[ (\Gamma^0 + I) | \psi \rangle = 0. \]  \hspace{1cm} (76)

This is what remains of the Dirac equation. We must now discuss the properties of the Ramond ground state \( | \psi \rangle \) and the meaning of (76).

Majorana spinors with 16 components (and depending on 16 real parameters) are allowed in our \((8,1)\) Minkowski space-time. These 16 components consist of linear combinations of the two independent \( SO(8) \) spinors, which we will denote by \( 8_s \) and \( 8_c \). These are projected out of a 16-component spinor by the operators \( (\Gamma^0 + I) \) and \( (\Gamma^0 - I) \), respectively. Thus (76) indeed serves as a Dirac equation in halving the number of independent components.

We now apply combinations of creation operators with \( N = 0,1,2 \) on ground states \( | \psi_0 \rangle \), and then obtain the physically allowed combinations by imposing the \( F_1, F_3, L_0 \) and \( f_0 \) conditions:

- \( N = 0, \alpha'_{ND}M^2 = 1 \)

  \[ | 0 \rangle \equiv | \psi_\beta \rangle, \psi_\beta \sim 8_s \]

  Note the \( f_0 \) constraint (76) has eliminated \( 8_c \) and kept \( 8_s \).

- \( N = 1, \alpha'_{ND}M^2 = 2 \)

  \[ | 1 \rangle \equiv | \psi^{ij}_\beta \rangle \equiv (\alpha^{ij}_1 d^0_0 + \alpha^{ij}_2 d^0_0 - \frac{\delta^{ij}}{D-1} \alpha^{ij}_0 d^0_0) | \psi_\beta \rangle \sim 35_v \otimes 8_c \]

  is the only permissible combination. Imposing \( (\Gamma^0 + I) | 1 \rangle = 0 \) forces \( | \psi_\beta \rangle \) to be \( 8_c \) since \( \Gamma^0 \) anticommutes with \( d^k_0 \).

- \( N = 2, \alpha'_{ND}M^2 = 3 \)

  \[ | \psi^{ij}_\beta (\alpha\alpha) \rangle + | \psi^{ij}_\beta (dd) \rangle \equiv (\alpha^{ij}_1 \alpha^{ij}_2 - \frac{\delta^{ij}}{D-1} \alpha^{ij}_0 \alpha^{ij}_0) 
  
  \begin{align*}
  &- d^i_2 d^j_0 - d^j_2 d^i_0 + \frac{2\delta^{ij}}{D-1} d^k_2 d^k_0) | \psi_\beta \rangle, \sim 35_v \otimes 8_s
  
  \end{align*} \]
The $\alpha\alpha$ and $dd$ parts of this separately satisfy the Virasoro constraints (74) and (75), but the superconstraints (73) force them into this particular combination. Because of the even number of $d$ modes, the basic spinor is $8_s$.

A clarification about these states is in order. The boldface numbers refer only to the $SO(8)$ transformation properties of the states above. This is because of the fact that the R-sector $SO(8)$ generators are built out of $\alpha_i$’s and $d_i$’s, and thus transform the R-states exactly as indicated as in boldface. However, one must be careful in distinguishing between these numbers and the actual physical degrees of freedom. The NS spectrum, where the boldface numbers are identical with the number of physical states, is free of this complication. We must now count the true number of physical states in the R sector.

Staring with the $N = 1$ state, we see that we cannot be dealing with $35 \times 8$ physical states: the $d_i^0$ merely shuffle the 8 components of the ground state spinor. Thus we have at most $8 \times 8 = 64$ states, but since the tensor is traceless, 8 spinor components corresponding to $\psi^{ij}_{\alpha}$ are absent, and the true physical content is $56_s$. This is not as unfamiliar a situation as it might first appear: consider a scalar field $\theta$ and its gradient $\partial_\mu \theta$. The latter transforms as a $D$-vector, but the physical degree of freedom is still just $\theta$, which is a $D$-scalar.

Examining the physical content of the $N = 2$ states, we see that $| \psi^{ij}_{\alpha} (\alpha\alpha) \rangle$ contains $35 \times 8 = 280$ states, but these do not belong to a single irrep. Among the 280 states there is a $56_c$ of the form $d_i^0 \setminus \psi^{ij}_{\alpha} (\alpha\alpha)$, and the rest is $224_c$.

The part $| \psi^{ij}_{\beta} (dd) \rangle$ represents another $56_c$, because the $d_i^0$ does not increase the number of states (bringing the number down from 280 to 64) and tracelessness in $ij$ takes off another 8. Thus the $N = 2$ content is $224_c + 56_c + 56_c$. Counting the $\alpha$ and $d$ modes separately and adding the numbers may seem surprising, but it is again not new: in the Higgs phenomenon, the massless photon field $A_\mu$ and $\partial_\mu \theta$ are added to form the massive vector field $B_\mu$. Although all three formally transform as 4-vectors, the final $B_\mu$ has $2+1 = 3$ degrees of freedom.

The GSO projection in its original form turns out to be inapplicable to the mixed boundary condition superstring. Recall that the GSO operation projects out one of the two chiralities of fermionic states; this projection is impossible in principle in our odd-dimensional spacetime (in the R-sector we have the remnant Dirac equation operators $\Gamma^0 \pm I$, but not in the role they play in the GSO projection). A consequence of the the absence of the GSO procedure is the fact that low-lying states do not exhibit space-time supersymmetry beyond matching masses and Regge slopes in the two sectors, but there are more basic manifestations of the inapplicability. For example, our final physical states are not homogeneous in the number of space-time fermions. Since the mode structure here is different from the standard RNS string (for example, our $b$ operators add an integer rather than half an integer to the squared mass), one could not in any case have expected the GSO procedure to work in the usual way.
4.5 Density of states:

Upper bounds for the number of bosonic and fermionic excitations at a given level can be calculated by slightly modifying standard techniques. The most important difference between the standard light-cone calculation and the one here is that our calculation gives the number of states before Virasoro and super-Virasoro constraints are applied. Thus the total number of pre-constraint string states at each level is given by $\text{tr} \omega^N$. Now let us calculate this trace for the two sectors separately.

(i) Neveu-Schwarz Sector:

Counting only odd $\alpha$ and $b$ modes leads to the result

$$g_{NS}(\omega) = \text{tr} \omega^N = \text{tr} \omega^{N_a + N_b} = \prod_{r=1,3,5,...} \frac{1 + \omega^r}{1 - \omega^r}(D-1).$$  \hfill (77)

Note that the GSO projection operator found in the usual light-cone formula is absent here.

(ii) Ramond Sector:

Even moded $d$ and odd moded $\alpha$ oscillators give

$$g_R(\omega) = \lambda \times \text{tr} \omega^N = \lambda \times \text{tr} \omega^{N_a + N_d} = \lambda \times \omega^{D-1} \prod_{r=1,3,5,...} \frac{1 + \omega^r+1}{1 - \omega^r}(D-1).$$  \hfill (78)

The factor $\lambda$ represents the degeneracy of the spinor ground state. Prior to the $f_0$ constraint, $\lambda = 16$. $\omega^{D-1}8$ is due to the fact that the ground state of this sector has $\frac{D-1}{8}$ unit higher mass than that of the Neveu-Schwarz sector.

For $D = 9$ the expressions have the following expansions:

$$g_{NS}(\omega) = 1 + 16\omega + 128\omega^2 + 704\omega^3 + 3072\omega^4 + 11488\omega^5 + 38400\omega^6 + 117632\omega^7 + ..., \quad g_R(\omega) = 16\omega + 128\omega^2 + 704\omega^3 + 3072\omega^4 + 11488\omega^5 + 38400\omega^6 + 117632\omega^7 ....$$

We have unfortunately not been able to find an analog of Jacobi’s “abstruse identity” showing $g_{NS}(\omega) - 1 = g_R(\omega)$ to all orders. Assuming the multiplicities continue to be equal, the surprising conclusion seems to be that the halving of world-sheet supersymmetry shows its effects on the spacetime spectrum not directly, but through the (super)constraints.
5 Discussion

Our concern in this note has been twofold: The first was to see whether an RNS string with one fixed end would provide qualitative phenomenological hints and distinctive signatures about the spectrum of hadrons with one very heavy quark. The second concern was the more formal one of working out the novel consequences of the unusual mixed Dirichlet-Neumann boundary conditions and seeing whether the resulting system can be quantized in a consistent way. Just as in the original attempts to apply string theory to hadrons, these two aims are partially in conflict, most notably in the dimension of space-time.

Ignoring this conflict as was customarily done in the old string-based hadrodynamics, we can summarize some basic features of our model that may actually be observed in hadrons with one very heavy quark as follows: (i) There is no space-time supersymmetry except for the fact that meson and baryon Regge trajectories have the same slope, leading to equal meson and baryon masses for the higher states. (ii) The Regge slope is twice that of the one observed for lighter hadrons.

An interesting point here is that if we consider an interaction where the free ends of two of our strings join while the heavy quark ends are kept fixed, we are led to a model in which \( X^\mu \sim \sum \alpha_n e^{-im\sigma} \sin(n\sigma) \) with \( \sigma \) in the range \([0, \pi]\). For such strings with both ends fixed on D0-branes, all \( \alpha_n \) modes and, consequently, all \( L_n \) are allowed. In this sector, there will be “daughter excitations”, with the same spin but equally spaced masses. But not in the sense that there are any leading linear Regge trajectories above them (the latter cannot be present because the balancing of centrifugal force against tension in a rigid rotation mode that defines leading trajectories is impossible when both ends are fixed). This is in qualitative, and one hopes, not entirely fortuitous accord with the multiplicity of \( b\bar{b} \) and \( c\bar{c} \) states of spin one. In any case, the Coulomb part of the QCD potential is known to play an important role in the dynamics of heavy quark-antiquark systems, ensuring deviations from a mass spectrum based on the purely string-based picture.

Turning to string-theoretic issues, we start with the question of whether using the more conventional \( \sigma \) range \([0, \pi]\) would have made a physical difference. One may anticipate that the simultaneous doubling of the \( \sigma \) range and the halving of the mode index \( n \) will result in an equivalent physical system, and this is indeed what happens. The mixed boundary conditions now produce half-integral \( \alpha \) and \( b \) modes and integral Ramond \( d \) modes, but the allowed physical spectrum is exactly as the one above except for an overall scaling. In particular, the preference for a \((8, 1)\) space-time and \( SO(8) \) symmetry remain. We prefer working with odd \( \alpha \) and \( b \) and even \( d \) modes because this leads to the disappearance of odd \( L_n \)'s. We have broken Poincaré invariance by fixing one end of the string at a special point in space; this is consistent with discarding the operators \( L_1 \) and \( L_{-1} \) which involve the generator of space translations.
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