Enhancing entanglement, local and non-local information of accelerated two-qubit and two-qutrit systems via weak-reverse measurements

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Abstract – Weak-reverse measurements (WRM) are employed to recover and protect the entanglement losses of entangled two-qubit and two-qutrit systems that accelerate. It is shown that the upper bounds of entanglement for a partially entangled two-qutrit state are higher than that for a partially entangled two-qubit state. The possibility of protecting local information encoded in partially entangled states using the WRM measurements is much better than that encoded in maximum entangled states. It is shown that the possibility of protecting the local encoded information on an accelerated 2-qutrit state, using the WRM is higher than that for a 2-qubit state.

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Introduction. – Entanglement represents an essential resource for quantum information [1]. Due to the interaction with environments, the entangled systems lose some of their entanglement. There are some techniques that have been introduced to recover the losses of entanglement and protect it from decoherence. One of these techniques is quantum purification [2–5]. Moreno et al. [6] showed that it is possible to improve the coherence by using nested environments. Xiao et al. [7] showed that the teleportation of Fisher information can be enhanced by using partial measurements. The possibility of purifying and concentrating entanglement by using local filtering is discussed by Yashodamma et al. [8].

It is well known that accelerated systems lose some of their correlations, and consequently their efficiencies to perform quantum information tasks decrease [9–11]. This loss of entanglement depends on the value of the acceleration and the dimensions of the accelerated system [12]. Guo et al. [13] showed that the local weak-reverse measurements (WRM) can be used as a technique to protect and improve the correlation in qubit-qutrit systems. Xiao et al. [14] showed that the entanglement losses caused by amplitude damping coherence can be retrieved by using WRM.

In this manuscript we will investigate the possibility of protecting and improving the entanglement, local and the non-local information of accelerated entangled qubit-qubit and qutrit-qutrit systems. In our treatment, we assume that only one particle is accelerated.

The outline of this manuscript is the following: the suggested protocol is described in the next section. The systems and their analytical solutions are given in the third section. The effect of the weak-reverse strengths on the degree of entanglement is discussed in the fourth section, where different initial state settings are considered. The dynamics of the local and non-local information is investigated in the fifth section. Finally, a conclusion is given in the last section.

Model. – Our idea is to consider two users, Alice and Bob, who share a two-qubit or two-qutrit system. These systems are initially prepared in maximum or partial entangled states. Here, we consider that only Alice’s particle is accelerated, which may be a qubit or a qutrit. The users decided in advance to use the weak-reverse measurements to protect their quantum communication channel. In the following steps, we describe the proposed protocol:

1) Weak measurements.

Here, both users Alice and Bob perform the weak measurements on their own particles, either qubit or qutrit. We assume that the users share initially a
state defined by $\rho_{ab}^\text{in}$. After performing the weak measurements, the output state $\rho_{ab}^\text{out}$ between the users becomes

$$\rho_{ab}^\text{out} = W_i^{(1)} W_i^{(2)} (\rho_{ab}^\text{in}) W_i^{(2)\dagger} W_i^{(1)\dagger}, \quad i = q \text{ or } t,$$

where $W_i$ are the weak measurements which are defined by the following operators [15,16]:

$$W_q = |0\rangle\langle 0| + \sqrt{1 - \alpha_q^{(1)}} |1\rangle\langle 1| \quad \text{(for qubit)},$$

$$W_t = |0\rangle\langle 0| + \sqrt{1 - \alpha_t^{(1)}} |1\rangle\langle 1| + \sqrt{1 - \alpha_t^{(2)}} |2\rangle\langle 2| \quad \text{(for qutrit)},$$

where $\alpha_q^{(\ell)}, \alpha_t^{(\ell)}, \ell = 1, 2$ are the strengths of the weak measurements of the qubit and the qutrit, respectively.

2) Acceleration step.

It is assumed that only Alice’s particle is moving with a uniform acceleration meanwhile Bob’s particle is assumed to be inertial. If the shared particles are fermions, then in the Minkowski frame the qubit state is transformed in the Rindler frame as [17-19],

$$|0_M\rangle = \cos r |0\rangle |0\rangle_{II} + \sin r |1\rangle |1\rangle_{II},$$

$$|1_M\rangle = |1\rangle |0\rangle_{II},$$

while, for the qutrit system the vacuum, the spin-up and spin-down states are transformed into Rindler space as

$$|0_M\rangle = \cos^2 r |0\rangle |0\rangle_{II} + \cos r \sin r \cos (|U|_I |D\rangle_{II} + |D\rangle_1 |U\rangle_{II} + e^{-i\phi} \sin^2 r |P\rangle_{II},$$

$$|U_M\rangle = \cos r |U\rangle |0\rangle_{II} + \sin r |P\rangle |1\rangle_{II},$$

$$|D_M\rangle = \cos r |D\rangle |0\rangle_{II} - \sin r |P\rangle |1\rangle_{II},$$

where $|U\rangle, |D\rangle$ and $|P\rangle$ are the spin-up, spin-down and pair states, respectively. The acceleration $r$ is defined such that $\tan r = \exp[-\pi \omega^2 t^2/2], \ 0 \leq r \leq \pi/4, -\infty \leq \omega \leq \infty$. The frequency, $\omega$ is the speed of light and $\phi$ is the phase which can be absorbed in the definition of the operators [20,21]. After tracing out the particle in the second region (II), the final state represents the accelerated quantum channel $\rho_{ab}^{\text{acc}}$ between Alice and Bob.

3) Reverse measurement step.

In this step, the users apply the reverse measurement operations on the accelerated state $\rho_{ab}^{\text{acc}}$ to obtain the final state as

$$\rho_{ab}^{\text{Final}} = \mathcal{R}_t \mathcal{R}_q (\rho_{ab}^{\text{acc}}) \mathcal{R}_q^\dagger \mathcal{R}_t^\dagger, \quad i = q \text{ or } t,$$

where

$$\mathcal{R}_q = \sqrt{1 - \beta^{(2)}} |0\rangle\langle 0| + |1\rangle\langle 1|,$$

$$\mathcal{R}_t = \sqrt{1 - \beta^{(1)}} (|1\rangle\langle 1| + \sqrt{1 - \beta^{(2)}} |2\rangle\langle 2|)$$

and $\beta^{(\ell)}, \ell = 1, 2$ are the strengths of the reverse measurement operations. Some properties of the final state, eq. (5), such as the behavior of entanglement, local non-local information, are examined.

The suggested systems.

1) Two-qubit state.

A qubit is a unit of quantum information that is described by a state in a 2-level quantum system. It can be written as a superposition of two orthogonal basis as

$$|\psi\rangle = b_1 |0\rangle + b_2 |1\rangle,$$

where $|b_1|^2 + |b_2|^2 = 1$. A general 2-qubits system can be described by 15 parameters, 3 for each qubit and 9 parameters describe the correlation between the two qubits [22].

In this section, it is assumed that the users Alice and Bob, share a partially entangled system of the X-state type. This state can be written by using the computational basis as

$$\rho_x = B_1 \left( |00\rangle\langle 00| + |11\rangle\langle 11| \right)$$

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where
\[ \tilde{B}_1 = c_1^2 B_1 (1 - \beta_{q_1}^{(1)}) (1 - \beta_{q_1}^{(2)}), \]
\[ \tilde{B}_2 = c_2 B_2 \sqrt{1 - \beta_{q_1}^{(1)}}, \]
\[ \tilde{B}_3 = c_1^2 B_3 (1 - \beta_{q_1}^{(1)}) (1 - \beta_{q_1}^{(2)}), \]
\[ \tilde{B}_4 = c_2 B_4 \sqrt{1 - \beta_{q_1}^{(1)}}, \]
\[ \tilde{B}_5 = (1 - \beta_{q_1}^{(2)}) \alpha_{0}^{(1)}, \]
\[ \tilde{B}_6 = (1 - \alpha_{q_1}^{(1)})(1 - \alpha_{q_1}^{(2)}) B_1, \]
\[ \tilde{B}_7 = (1 - \alpha_{q_1}^{(1)})(1 - \alpha_{q_1}^{(2)}) B_1, \]
\[ \tilde{B}_8 = \tilde{B}_2. \]

and \( N_q = \tilde{B}_1 + \tilde{B}_3 + \tilde{B}_5 + \tilde{B}_6 \) is the normalization factor and \( c = \cos r, s = \sin r \).

2) Two-qutrit system.
A qutrit is the unit of quantum information [1] that can be written as a superposition of three orthogonal bases, \( |0\rangle, |1\rangle \) and \( |2\rangle \) as
\[ |\psi_{i}\rangle = t_1 |0\rangle + t_2 |1\rangle + t_3 |2\rangle, \]
where \( |t_1|^2 + |t_2|^2 + |t_3|^2 = 1 \). In this section, we assume that the users Alice and Bob share initially a two-qutrit system in the form
\[ |\psi_{i}\rangle = \frac{1}{\sqrt{2 + \gamma^2}} \left( |00\rangle + |11\rangle + \gamma |22\rangle \right), \]
where it turns into a maximum entangled state at \( \gamma = 1 \) [24]. The users apply the protocol which is described in the previous section. At the end the users Alice and Bob share the following state:
\[ \rho_{ab}^{E} = \frac{1}{N_q} \left( D_1 |00\rangle \langle 00| + D_2 |00\rangle \langle 11| + D_3 |11\rangle \langle 00| + D_3 |11\rangle \langle 11| + D_6 |20\rangle \langle 20| + D_7 |22\rangle \langle 20| + D_8 |22\rangle \langle 22| + D_9 |22\rangle \langle 22| + D_{10} |00\rangle \langle 22| + D_{11} |11\rangle \langle 22| \right), \]

where
\[ D_1 = c_2^2 R_{00}^2 A_1, \]
\[ D_2 = c_3^3 R_{00} R_{11} A_2, \]
\[ D_3 = c_2^2 s^2 R_{10}^2 A_1, \]
\[ D_4 = c_3^3 R_{00} R_{11} A_4, \]
\[ D_6 = c_2^2 R_{11}^2 A_3, \]
\[ D_7 = c_3^3 R_{11}^2 R_{00} A_4, \]
\[ D_8 = c_2^2 s^2 R_{00}^2 R_{11} A_8, \]
\[ D_9 = c_3^3 R_{22}^2 A_9, \]
\[ D_{10} = R_{00} R_{11} A_3 c^3, \]
\[ D_{11} = R_{11} R_{22} A_6 c^2. \]

The normalization factor is given by \( N_q = D_1 + D_3 + D_5 + D_6 + D_8 \) and the coefficients \( A_i, i = 1, \ldots, 9, \) are given by
\[ A_1 = \frac{1}{2 + \gamma^2}, \]
\[ A_2 = \frac{1}{2 + \gamma^2}, \]
\[ A_3 = 2 + \gamma^2, \]
\[ A_4 = A_2, \]
\[ A_5 = (2 + \gamma^2) A_7, \]
\[ A_6 = \gamma A_2 \sqrt{1 - \alpha_{q_1}^{(1)}}, \]
\[ A_7 = \frac{1}{2 + \gamma^2} \sqrt{1 - \alpha_{q_1}^{(1)}}, \]
\[ A_8 = (1 - \alpha_{q_1}^{(1)}) A_2, \]
\[ A_9 = \gamma^2 A_1 (1 - \alpha_{q_1}^{(1)}), \]
and \( R_{ij}, i, j = 0, 1, 2, \) are given by
\[ R_{00} = \sqrt{1 - \beta_{q_1}^{(1)}}, \]
\[ R_{01} = \sqrt{1 - \beta_{q_1}^{(2)}}, \]
\[ R_{02} = \sqrt{1 - \beta_{q_1}^{(1)}}, \]
\[ R_{10} = \sqrt{1 - \beta_{q_1}^{(2)}}, \]
\[ R_{11} = \sqrt{1 - \beta_{q_1}^{(1)}}, \]
\[ R_{12} = \sqrt{1 - \beta_{q_1}^{(2)}}, \]
\[ R_{20} = \sqrt{1 - \beta_{q_1}^{(2)}}, \]
\[ R_{22} = \sqrt{1 - \beta_{q_1}^{(1)}}, \]

Entanglement. – To quantify the residual amount of entanglement \( E \) contained in the accelerated system, we use the negativity as a measure of entanglement, which is defined as
\[ E = \max \left( 0, \sum_i \lambda_i \right), \]

where \( \lambda_i \) are the eigenvalues of the partial transpose with respect to Alice’s qubit (qutrit) of the final states (9) and (13), respectively [25].

Let us assume that the system is initially prepared in the X-state (8), then the degree of entanglement is given by (13), where \( \lambda_i \)
\[ \lambda_{1,2} = \frac{1}{2N_q} \left\{ \left( \tilde{B}_1 + \tilde{B}_7 \right) \pm \sqrt{(\tilde{B}_1 - \tilde{B}_7)^2 + 4\tilde{B}_2 \tilde{B}_8} \right\}, \]
\[ \lambda_{3,4} = \frac{1}{2N_q} \left\{ (\tilde{B}_3 + \tilde{B}_5) \pm \sqrt{(\tilde{B}_3 - \tilde{B}_5)^2 + 4\tilde{B}_4 \tilde{B}_6} \right\}. \]

Figure 1(a) displays the entanglement behavior of a system initially prepared in the singled state. It is clear that at \( r = 0 \), the entanglement is maximum, i.e., \( E = 1 \). The general behavior shows that the entanglement decreases as the acceleration parameter \( r \) increases. As one increases the strengths of the local operations, the entanglement
The entanglement decays as the local operations' strengths increase for different intervals. The length of these intervals depends on the initial state settings: two-qubit, or two-qutrit, maximum/partial entangled states. The partially entangled two-qutrit state is more robust than the partially entangled two-qubit state. The possibility of protecting and improving the entanglement of the two-qutrit state is larger than that for the two-qubit state.

Fig. 1: (Colour online) The entanglement of the final state of the two-qubit state (9) where we set \( \alpha_q^{(1)} = \alpha_q^{(2)} = \beta_q^{(1)} = \beta_q^{(2)} = \alpha_q \) and where the initial state is prepared in (a) the single state and (b) the Werner state with \( c_{11} = c_{22} = c_{33} = 0.7 \).

Fig. 2: (Colour online) The entanglement of the two-qutrit system, eq. (13), where (a) is the maximum entangled state (\( \gamma = 1 \)) and (b) the partial entangled state, i.e., \( \gamma = 0.5 \), where \( \alpha_t^{(1)} = \alpha_t^{(2)} = \beta_t^{(1)} = \beta_t^{(2)} = \alpha_t \).

Local and non-local information. --

Two-qubit state. In this subsection we quantify Alice’s information (\( I_a \)), Bob’s information (\( I_b \)) and the non-local information (\( I_{nloc} \)) between them. The three types of information are given by

\[
I_a = - \frac{\tilde{B}_1 + \tilde{B}_3}{N_q} \log \frac{\tilde{B}_1 + \tilde{B}_3}{N_q} - \frac{\tilde{B}_5 + \tilde{B}_7}{N_q} \log \frac{\tilde{B}_5 + \tilde{B}_7}{N_q},
\]

\[
I_b = - \frac{\tilde{B}_1 + \tilde{B}_3}{N_q} \log \frac{\tilde{B}_1 + \tilde{B}_3}{N_q} - \frac{\tilde{B}_5 + \tilde{B}_7}{N_q} \log \frac{\tilde{B}_5 + \tilde{B}_7}{N_q},
\]

\[
I_{nloc} = \sum_{i=1}^{4} \lambda_i \log \lambda_i,
\]

where \( \lambda_i \) are given by eq. (16).

Figure 3 displays the behavior of the non-local information and Alice’s information, where we set \( \alpha_q^{(1)} = \alpha_q^{(2)} = \beta_q^{(1)} = \beta_q^{(2)} = \alpha_q \). At zero acceleration, both types of information are maximum. As Alice’s particle is accelerated, the non-local information decreases. However, for a fixed value of \( r \), the non-local information increases as the strengths of the local operations increase. Meanwhile, Alice’s local information decreases at the expense of the non-local information. For higher values of \( \alpha_q \), the local information \( I_a \) vanishes while the non-local information \( I_{nloc} \) increases. This non-local information, no longer represents quantum information but it describes a classical information, because the two particles are almost separable at larger values of \( \alpha_q \) as displayed in fig. 1.
Figure 4 displays the effect of the WRM on the local and non-local information. The behavior of Alice’s information \( \mathcal{I}_a \) and Bob’s information \( \mathcal{I}_b \) is described in fig. 4(a), where it is assumed that the system is either initially prepared in a maximum entangled state (MES) or in a partial entangled state (PES). At zero acceleration \((r = 0)\) the information which is encoded in Alice’s and Bob’s qubit is maximum, i.e., \( \mathcal{I}_a = \mathcal{I}_b = 1 \) bit. For small values of \( r \in [0, 2.5] \), \( \mathcal{I}_a \) and \( \mathcal{I}_b \) remain maximum (1 bit). For larger values of \( r \), Alice’s information \( \mathcal{I}_a \) decreases as the acceleration \( r \) increases, while Bob’s information is slightly affected by the local WRM. Moreover, the decay rate of Alice’s information for a system prepared initially in a partial entangled state is smaller than that displayed for systems that are initially prepared in MES. This shows that the possibility of protecting Alice’s local information encoded in PES by the WRM is much better than protecting that encoded in MES.

The effect of the local WRM on Alice’s local information, \( \mathcal{I}_a \), and the non-local information \( \mathcal{I}_{nloc} \) is depicted in fig. 4(b), where it is assumed that the system is initially prepared in the MES. In these calculations we consider that the strengths of the local weak measurements and the reverse measurements are equal \((\alpha_q = 0.5)\). The behavior of information shows that both types of information are maximum at \( r = 0 \). As \( r \) increases, Alice’s local information \( \mathcal{I}_a \) has maximum values for \( r \in [0, 2.5] \), while the non-local information decreases for any value of \( r > 0 \). It is clear that as the strengths of the WRM increase, both types of information decrease. However, the decay rate of \( \mathcal{I}_{nloc} \) is much larger than that for \( \mathcal{I}_a \). For larger values of \( r \) and \( \alpha_q \), the non-local information increases at the expense of Alice’s local information.

From fig. 4, one concludes that the information which is encoded in the accelerated systems can be protected by using the weak-reverse measurement. The possibility of protecting the information which is encoded in accelerated partial entangled states is higher than that encoded in maximum entangled states. The local measurements have a very slight effect on the local information which is encoded in the non-accelerated systems. For larger values of accelerations, the non-local information increases at the expense of the information that is encoded in the accelerated systems.

Two-qutrit system. Figure 5 shows the behavior of the non-local information \( \mathcal{I}_{nloc} \) and Alice’s information \( \mathcal{I}_a \) for a system initially prepared in an entangled maximum two-qutrit system. It is clear that both types of information decrease as the acceleration \( r \) increases. The decay rate is higher for the non-local information compared with that depicted for local information \( \mathcal{I}_a \). However, as the strengths of the local operations increase both types of information are improved. For \( \alpha_t \in [0.8, 1] \) the non-local information increases at the expense of Alice’s information.

The dynamics of different types of information encoded in a system initially prepared in a maximum two-qutrit entangled state is described in fig. 6. The behavior of
Alice’s information $I_a$ and Bob’s information, $I_b$ is displayed in fig. 6(a), where it is assumed that the system is initially prepared in MES or PES. It is clear that $I_a$ for a system initially prepared in a MES is almost stable and it maximum for any value of $r$. On the other hand, starting from PES, the initial values of Alice’s information are smaller than that depicted for MES. However, the upper bounds of $I_a$ increase as the acceleration $r$ increases. These upper bounds become larger than that depicted for MES for $r \in [0, 0.8]$. For Bob’s information the behavior is completely different from those shown for the two-qubit state, where it is always smaller than Alice’s information. Moreover, it decreases as $r$ increases.

Figure 6(b) shows the behavior of the non-local information and Alice’s information for different values of the local operation strengths, with the initial system prepared in a MES of the two-qutrit system. It is clear that $I_{nloc}$ decreases as $\alpha_q$ and $r$ increase. For smaller values of $\alpha_q$, the non-local information decreases faster than that displayed for larger values of $\alpha_q$. On the other hand, for larger values of $\alpha_q$, the non-local information increases to reach its upper bounds as $r \to \infty$. Also, the general behavior of the accelerate information shows that $I_a$ decreases as the strengths of the local operations increase. However, the decay rate is smaller than the non-local information. This explains the decay of Bob’s information as displayed in fig. 6(a).

From these figures one can extract some important facts. The local information which is encoded in the accelerated part of the composite system can be protected by using the WRM. The local information which is encoded in an accelerated partial entangled state can be improved even for larger accelerations. The local information which is encoded in non-accelerated systems is very sensitive to the local operations (WRM), compared with that shown for the two-qubit state. The decay rate of information in accelerated systems is much smaller than that shown for the non-local information. For larger values of accelerations, the non-local information increases at the expense of the local information which is encoded in non-accelerated systems.

**Conclusion.** – In this paper, we assume that two users Alice and Bob share 2-qubit or 2-qutrit states as communication channels to perform some quantum information tasks. Alice is allowed to accelerate her qubit/qutrit. Subsequently, the coherence of the communication channel between the partners decreases. Here, we employed the weak-reverse technique to improve the quantum correlations between the users. It is shown that the initial state settings, the values of the acceleration and the strengths of the weak-reverse operations represent control keys to the behavior of entanglement, local and non-local information.

It is demonstrated that for small values of acceleration, the improving rate of the quantum correlation of the communication channels increases as the strength of the weak-reverse operation increases. However, as one increases the local operations’ strengths further, the correlation decreases. Our results show that starting from a partially entangled state, the improving rate of correlations is better than that depicted for the maximum entangled states. The increasing rate of the correlation of the qutrit-qutrit system is higher than that of the qubit-qubit system.
Additionally, the behavior of the local information which includes is assayed. It is shown that for the two-qubit state, the information which is encoded in non-accelerated systems is almost stable and slightly decreases for systems initially prepared in a maximum entangled state. On the other hand, the information which is encoded in accelerated systems decreases as the acceleration increases. Moreover, the encoded information in a system initially prepared in a partial entangled state is more robust than that encoded in a system initially prepared in maximum entangled states. These results are dramatically changed for the two-qutrit systems, where the encoded information in non-accelerated systems is more fragile than that encoded in accelerated systems. Also, the encoded information in the partial entangled state of the two-qutrit system is improved as the acceleration increases under weak-reverse measurements.

Generally, the behavior of the non-local information decreases as the acceleration increases. For a fixed value of acceleration, the non-local information increases as the strength of the weak-reverse measurement increases. The larger values of these strengths cause a sudden decay of the non-local information for small values of the accelerations and gradually decay for larger values of these accelerations. For the two-qutrit system, the non-local information slightly increases at larger values of the weak-reverse measurement strengths at the expense of the local information.

In conclusion, it is possible to improve and protect the accelerated communication channel by using weak-reverse measurements. The possibility of protecting the local encoded information on an accelerated 2-qutrit system is higher than that displayed for a 2-qubit state. The restrained decay rate of the coded information on an accelerated partial entangled state is better than that displayed for a maximum entangled state. The encoded information in the accelerated part of the two-qutrit system can be improved by using the local operations, while it decays for the two-qubit state. Therefore, to perform quantum key distribution or quantum coding protocols using an accelerated system, it is recommended to use partial entangled 2-qutrit systems with higher accelerations.

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