WORKSHOP QUEUE SYSTEM MODIFICATION THROUGH MULTI PRIORITY STRATEGY

U. C. Okonkwo 1, I. P. Okokpujie 2*, B. N. Odo 3, O. S. I. Fayomi 4,5

1,3Department of Mechanical Engineering, Nnamdi Azikiwe University, Awka, Nigeria
2,4Department of Mechanical Engineering, Covenant University, Ota, Ogun State, Nigeria
5Department of Chemical, Metallurgical and Materials Engineering, Tshwane University of Technology, P.M.B. X680, Pretoria, South Africa.

*Corresponding author: imhade.okokpujie@covenantuniversity.edu.ng; cu.okonkwo@unizik.edu.ng;

Abstract

This research focused on the modification of a queuing system in a workshop practice that repairs crankshafts using multi priority strategy. The random distribution of the inter arrival time of the crankshafts as well as the service time were statistically determined using Chi square goodness of fit test. The results obtained show that all the classes conform to Poisson distribution. For the non-preemptive priority, the mean waiting time in queue results for the first, second and third classes are 0.066, 0.09 and 0.224 day, respectively, while for the preemptive priority the three classes show 0.007, 0.036 and 0.258. Besides, the mean waiting time in queue for no priority system is 0.17 day. Arrivals that are of higher priority classes in preemptive priority systems enjoy huge improvement when compared with non-preemptive system. On the other hand, the improvement gained in higher priority classes has detrimental effect on the low priority class. Similar scenario plays out when the results of the mean waiting time in the system was analyzed. It is therefore advocated that priority strategy should be adopted for a system that may have urgent or semi-urgent jobs among the pool of jobs that needs repair to avoid possible losses arising from frequent reneging or balking of such jobs.

Keywords: Queue system, Priority Class, Preemptive, mean time in system, Non-Preemptive, mean waiting time

1. Introduction

Queuing models is a dominant tool for determining the performance of queuing systems [1]. In queuing system customers who have to wait in line can be either animate beings or inanimate objects, examples of objects that can wait in line include machine waiting to be repaired, an order waiting to be processed, vehicles waiting to be refueled, etc. Sometimes, different jobs are given different treatment. There are jobs that is more significant than others, and should be given priority over other jobs at the operation process of the workshop or system. If it is not required to determine specific queue performance physiognomies for each type of job, such a system may still be analyzed as a standard queuing system with one type of job only. However, if it is required to know performance physiognomies for each type of job, the model describing this system gets far more complex. Usually, they are two imaginable modifications in priority situation, which are preemption and non-preemption. In the preemption case; a client with an extraordinary priority is permitted to enter service instantaneously even if th lower priority one is currently in service. In most cases, a priority self-control is said to be non-preventive if there is no disturbance. A client with upper priority waits on the queue for service to be rendered appropriately.
Of recent, many of the queueing theory models are enthusiastic to examining priority queues, where jobs are branded and aided the following priority system: high-priority jobs anticipate average-priority jobs, which will lead to anticipate of the queue low-priority jobs. Priority queue structures come up in countless areas. Occasionally the priority of a job is determined by the jobs owner through a facility level agreement, whereby certain customers have selected to pay extra so as to get high-priority access to some high-demand supply. For priority queues with Poisson arrivals numerous exact results are accessible like the models of Jaiswal [2], Takagi [3], Okonkwo and Omenyi [4].

However, the development of the multi-server priority queues with Poisson influxes will lead to the approximation formation, such as Bondi and Buzen [5] and Harchol-Balter et al [6] and Okonkwo and Omenyi [7]. For single-server priority queues with general influxes, few priority class models exist, like Zhao et al [8] and Horvath [9]. Nearly all examination of dual priority M/M/k systems includes the use of Markov chains, which have two priority classes produces enormously in double dimensions, single dimension for each priority class. In other to avoid this, many scientists such as Kao and Narayan [10]; Nishida [11] and Kapadia et al [12] has carried out different study on how to easy the chain using various process.

Another stream of research exploits on the exponential job sizes by clearly writing out the stability equations and then definition roots through producing functions. Generally, these yield difficult mathematical terminologies vulnerable to mathematical variabilities at higher loads [13] and [14]. In this research, unlike Davis [15], Kella and Yechiali [16] (for non-preemptive priorities), and Buzen and Bondi [17] that used same mean, it uses different mean for each of the priority class. Therefore, the need for futher research work for various aspect of modelling is required for production out improvement [18-21].

Most of the previous research has focused on mathematical modeling of a variety of queuing scenario. They often assume a given distribution of arrivals and service and use that for modeling. Much work has not been done on empirical determination of the actual distribution of relevant queuing parameters. In this work the distributions of the arrivals of the case study will actually be determined. Besides, not much work has been done on comparative analysis of queuing system performance measures with and without priority as well as with preemption and without preemption and its impact on the system. In this study such comparative analysis will be carried out.

2. **Queue Set Up Challenge of the Workshop (Case Study)**

What actually triggered off this research work was an experience with a leading maintenance workshop specializing in the regrinding of crankshaft. The company is Ezenwa Brothers Limited with Headquarters at Onitsha, Nigeria. The Company usually receives variety of crankshafts (i.e. crankshafts from various vehicles). These vehicles come from different locations and were used for different purposes.

These differences notwithstanding, the company has one stream of queue and two servers, the two servers being the two machines used for the servicing. Ezenwa Brothers Limited workshop receives their customers in single queue form and services them with first come first serve (FCFS) basis. They follow the register which stipulates the arrival time of each crankshaft needing repair. It was observed unfortunately that the high number of crankshafts waiting for service are usually many and this has made many customers to abandon their crankshaft and go for a new one, take their crankshaft in search of other workshop that can offer similar services (reneging) and even in
some situations not register their crankshaft for service considering the number already in queue (balking).

These problems have resulted in many customers not being satisfied with their services. Having considered the complexity of the matter, it becomes necessary that a new queue model should be applied to enhance service delivery. Therefore, considering the varieties of customers requiring service from the Company and with the obvious difference in service level requirement for each class, a priority class queue model will enhance service delivery hence was applied.

2.1 New Model Formulation

The adopted queuing system priority, the jobs are separated into classes priority, from 1 to n, due to the increase of the priority class number with course reduction on the priority operations. In comparison the clients in the priority class i and priority class j, the I class is being given high consideration than clients in class j, that means I < j. In some cases the first-come-first-serve method is applied to the queuing discipline. There are two elementary regulator approaches to determine the condition whereby a client of class i reaches to find a client of class j in check, where i < j, is known to be preemptive and non-preemptive systems. In a preventive priority queuing system, service is episodic and the newly arrived client begins service. The customer whose service was disturb and returns to the head of the jth class. Further refinement, in a preventive resume priority queuing structure, the customer whose service was episodic begins service at the point of disruption on the next surplus to the service facility.

In this study, the use of multiple channel queuing models with priority of three classes was applied. The first class is for crankshafts of heavy duty vehicles. The second class is for crankshafts of vehicles that came from distant places and the third class is for others. Arrivals are served on a first-in, first-out (FIFO) basis for a given priority. Preemption and non-preemption were evaluated.

It was assumed that all the servers work at the same average rate. The crankshafts form a single line to be serviced by two (machines) servers as shown in Fig 1.

![Illustration of the Formulated Priority Queue Process](image)

2.2 Arrival and Service Rate Determination per Day

After studying the queue scenario of the Company for six (6) months, the month of July 2015 was selected and critically analysed. The number of daily arrivals of crankshafts and services made were collected. Thereafter, the mean arrival and service rates of the distribution were calculated using the equations 1 and 2, respectively.
\[
\lambda = \frac{\sum_{n=0}^{\infty} nF_n}{\sum_{n=0}^{\infty} F_n}_{AR}
\]

while

\[
\mu = \frac{\sum_{n=0}^{\infty} nF_n}{\sum_{n=0}^{\infty} F_n}_{SR}
\]

Where \(n\) is the number of crankshafts that arrived for servicing or received services for arrival rate or service rate determination, respectively. \(F_n\) is the frequency for arrival or for service as the case may be. Stability is assumed when the arrival rate is less than the service rate during busy period.

### 2.3 Use of Poisson Distribution for Arrival of Parts

Okonkwo [22] observed that in most academic and commercial offering, it is often assumed that the probability of spare parts inventory demands is given by a Poisson distribution with a mean such as annual demand rate, without confirming the reasonableness of such assumption. Although in many instances, the distributions of stochastic demands are Poisson, however, in few occasions, distribution may not be Poisson. These may result in significant errors in the results that will be generated from these models, as statistical distribution used in a model is a major contributing factor in determining the validity of the generated results.

It is in line with this understanding that in this study, the arrivals and services of the crankshafts that came for servicing in the workshop will be tested to confirm the reasonableness of the use of Poisson distribution. The following steps were followed in using the chi-square goodness of fit test technique.

1. Calculation of the Poisson distribution \(P_n\) using the probability distribution of Poisson random variable.

\[
P_n = \frac{(\lambda^n e^{-\lambda})}{n!}
\]

\(n = 0, 1, 2, 3...\infty \quad \text{where } \lambda > 0\)

\(\lambda\) is the average number of successes occurring in the given interval or specified region (Taha, 1987; Burghardt, 1991).

2. Calculation of the expected or theoretical frequency \(T_n\) for each interval under the assumption that the hypothesized distribution is correct.

\[
T_n = P_n(\sum F_n)
\]

3. Calculation of the chi-square statistics using the formula

\[
\chi^2 = \sum_{n=0} \frac{(F_n - T_n)^2}{T_n}
\]

4. Comparison of \(\chi^2_{\text{observed}}\) with \(\chi^2_{\alpha,\nu}\) distribution.
This is at a certain level of statistical significance ($\alpha$) and degree of freedom ($Df$). $Df = c-k-1$
Where $c$ = number of cells and $k$ = number of parameters estimated for the hypothesized distribution.

For the distribution to be Poisson $\chi^2_{\text{observed}} < \chi^2_{Df, \alpha}$

If $\chi^2_{\text{observed}} > \chi^2_{Df, \alpha}$ the hypothesis is rejected because the distribution will not be Poisson [23-26]

In other words, a decision is made. If the value of the $\chi^2_{\text{observed}} < \chi^2_{Df, \alpha}$, the distribution is accepted as following Poisson distribution. Otherwise, it is rejected as not following Poisson distribution.

The degrees of freedom (Df) for each of the spare parts were found using the formula: $Df = c – k – 1$, where $c$ is the number of cells (intervals) and $k$ is the number of parameter estimated, in this case 1. The statistically significant level ($\alpha$) that was used for each of the spare parts was 0.05.

### 2.4 Determination of System Performances without Priorities

System performance was determined without using priority by using the conventional equation model parameters;

$$P_0 = \left[\sum_{n=0}^{M-1} \frac{(\lambda \mu)^n}{n!} + \frac{(\lambda \mu)^M}{M! [1 - \frac{\lambda \mu}{\mu}]} \right]^{-1}$$  \hspace{3cm} (6)

$$L_q = \frac{\lambda \mu (\frac{\lambda \mu}{\mu})^M}{(M-1)(M\mu - \lambda)^2 P_0}$$  \hspace{3cm} (7)

$$W_q = \frac{L_q}{\lambda}$$  \hspace{3cm} (8)

$$L_S = L_q + \frac{\lambda}{\mu}$$  \hspace{3cm} (9)

$$W_S = W_q + \frac{1}{\mu} = \frac{L_s}{\lambda}$$  \hspace{3cm} (10)

### 2.5 Determination of System Performance Using Priorities

System performance was determined assuming priority classes by adopting Wang [25] priority models:

The steady-state expected waiting time in the system $W_k$ as well as waiting time in queue $W_{kq}$ (including service time) is taken as for a member of priority class $k$.

For non-pre-emptive priorities

$$W_k = \frac{1}{AB_{k-1}B_k} + \frac{1}{\mu}, \text{ for } k = 1, 2... N$$  \hspace{3cm} (11)

$$W_{kq} = \frac{1}{AB_{k-1}B_k}, \text{ for } k = 1, 2... N$$  \hspace{3cm} (12)

If $M = 1$, then $A = \frac{\mu^2}{\lambda}$  \hspace{3cm} (13)

If $M > 1$ then $A = M\mu + \frac{M(M\mu - \lambda)}{(\frac{\lambda}{\mu})^M} \sum_{i=1}^{M-1} \frac{(\frac{\lambda}{\mu})^i}{i!}$  \hspace{3cm} (14)

$B_{k}$ to $B_k$ is calculated thus:

$$B_0 = 1$$

$$B_k = 1 - \frac{1}{M\mu} \sum_{i=1}^{k} \hat{\lambda}_i$$  \hspace{3cm} (15)

Similarly, for pre-emptive priorities, the average times spent in the system and in the queue are:
\[ W_k = \frac{1}{\mu B_{k-1} B_k} + \frac{1}{\mu}, \text{ for } k = 1, 2, ..., N \]  
(16)

\[ W_{kq} = W_k - \frac{1}{\mu}, \text{ for } k = 1, 2, ..., N \]  
(17)

For both non preemptive and preemptive priorities, the average number of persons in the queue and in the system follows the Little’s formula and thus we have:

\[ L_k = \lambda_k W_k \]  
(18)

\[ L_{kq} = \lambda_k W_{kq} \]  
(19)

3. Results

3.1 Analysis of Randomness of Arrivals without Priority

Randomness of arrivals without priorities was tested using chi square goodness of fit method. The summary of distributions for the arrivals per day extracted from daily arrivals is shown in Table 1. The number of observations which are arrivals per day is \( n \) while \( F_n \) is the observations frequency or arrivals frequency per day.

| \( n \) | \( n \leq 7 \) | 8 | 9 | 10 | 11 | 12 | 13 | \( n \geq 14 \) |
|---|---|---|---|---|---|---|---|---|
| \( F_n \) | 1 | 3 | 4 | 7 | 5 | 4 | 2 | 1 |

Once the observers frequency \( (F_n) \) is known, the next is to calculate the arrival rate \( (\lambda) \) using equation 1 which was found to be 10.2963.

Table 2: Chi Square Goodness of Fit Test Arrival Data and Calculated Variables without Priority

| \( n \) | \( F_n \) | \( P_n \) | \( T_n \) | \( \chi^2_{\text{observed}} \) |
|---|---|---|---|---|
| 5 | 1 | 0.0326 | 0.8789 | 0.0167 |
| 8 | 3 | 0.1058 | 2.8554 | 0.0073 |
| 9 | 4 | 0.1210 | 3.2667 | 0.1646 |
| 10 | 7 | 0.1246 | 3.3635 | 3.9318 |
| 11 | 5 | 0.1166 | 3.1483 | 1.0891 |
| 12 | 4 | 0.1000 | 2.7013 | 0.6244 |
| 13 | 2 | 0.0792 | 2.1395 | 0.0091 |
| 14 | 1 | 0.0583 | 1.5735 | 0.2090 |
| \( \sum F_n \) | 27 | | | |
| \( \sum nF_n \) | 278 | | | |
| \( \chi^2_{\text{observed}} \) | 6.0520 | | | |

Arrival Rate \( (\lambda) \) 10.2963

With the arrival rate information, the Poisson distribution at a given \( (n) \) was determined using equation 3. Similarly, the expected/theoretical frequency \( T_n \) as well as the observed chi square value obtained using equations 4and 5. The results obtained are shown in Table 2.

The degree of freedom \( (df) \) considering the eight cells arrival frequency distribution is 6 (i.e. 8-2). The level of significance \( (\alpha) \) throughout is taken to be 0.05. With the above information, the critical value of \( \chi^2 \) extracted from the chi squares table shows that a value of 12.592 or more is required for significance to occur for arrival distribution without priority. Having an observed chi square of 6.052 as against 12.592 shows that it conforms to Poisson distribution and the variation in distribution as shown in
3.2 Analysis of Randomness of Arrivals with Priority

Tables 3, 4 and 5 show the randomness of arrival results for high, medium and low priority classes. The calculations of the relevant parameters without priorities were done following the steps used in doing the one without priority already handled. From the arrival rates results obtained, it will be noted that the summation of the arrival rates for the various priority classes (i.e. high = 1.40741, medium = 2.48148 and low = 6.40741) gave the arrival rate when there was no priority (i.e. 10.2963). There is need to also determine whether these three classes of arrival rates based on priority follow Poisson distribution. It was against that background that $\chi^2$ test was applied to test whether or not a significant difference existed between the observed and theoretical frequencies.

Fig. 2 is the composite graph of the theoretical frequency compared with the observed frequency with the results obtained from the company. It can be easily observed that they followed Poisson distribution.

Table 3: Randomness of Arrival Results for Priority 1 (High Priority Class)

| n | F_n | P_n  | T_n    | $\chi^2_{observed}$ |
|---|-----|------|--------|------------------|
| 0 | 5   | 0.24478 | 6.608981 | 0.391712        |
| 1 | 10  | 0.3445 | 9.301529 | 0.05245         |
| 2 | 8   | 0.24243 | 6.54552 | 0.3232          |
| 3 | 4   | 0.11373 | 3.070738 | 0.281212        |
| $\sum F_n$ | 27 |       |        |                  |
| $\sum nF_n$ | 38 |       |        | $\chi^2_{observed}$ | 1.048574 |

Arrival Rate ($\lambda_1$) 1.40741

Table 4: Randomness of Arrival Results for Priority 2 (Medium Priority Class)

| n | F_n | P_n  | T_n    | $\chi^2_{observed}$ |
|---|-----|------|--------|------------------|
| 1 | 2   | 0.2075 | 5.60249 | 2.31646         |
| 2 | 8   | 0.25745 | 6.95124 | 0.15823        |
| 3 | 7   | 0.21296 | 5.74979 | 0.27184        |
| 4 | 6   | 0.13211 | 3.567 | 1.65952        |
| 5 | 4   | 0.06557 | 1.77029 | 2.50836       |
| 6 | 0   | 0.02712 | 0.73216 | 0.73216        |
| $\sum F_n$ | 27 |       |        |                  |
| $\sum nF_n$ | 67 |       |        | $\chi^2_{observed}$ | 7.64657   |

Arrival Rate ($\lambda_2$) 2.48148

Table 5: Randomness of Arrival Results for Priority 3 (Low Priority Class)

| n | F_n | P_n  | T_n    | $\chi^2_{observed}$ |
|---|-----|------|--------|------------------|
| 5 | 4   | 0.14843 | 4.00768 | 0.00001         |
The degree of freedom (df) considering the four cells for Priority 1 (High Priority Class) is 2 (i.e. 4 – 2), that of Priority 2 (Medium Priority Class) is 4 (i.e. 6 – 2) and Priority 3 (Low Priority Class) is 6 (i.e. 8-2). The level of significance (α) throughout is taken to be 0.05. With the above information, the critical value of $\chi^2$ extracted from the chi squares table show that a value of 5.991 or more is required for significance to occur for Priority 1, that of Priority 2 is 7.779 and Priority 3 is 12.592. Thus, where $\chi^2_{df,\alpha}$ at given degree of freedom and level of significance was found significant, it was interpreted that the variation in the distribution did not occur by chance. Carrying out the comparison shows that they conform to Poisson distribution since they have expected chi
square of 1.0386 as against 5.991, 7.64657 as against 7.779 and 6.40741 as against 12.592, for high, medium and low priority classes, respectively.

3.3 Analysis of Randomness of Services
Randomness of arrivals was tested using chi square goodness of fit method.
Table 6: Summaries of the Distributions of the Service Frequencies

| n   | n ≤ 12 | 13 | 14 | 15 | 16 | 17 | 18 | n ≤ 19 |
|-----|--------|----|----|----|----|----|----|--------|
| Fn  | 4      | 5  | 6  | 5  | 3  | 2  | 1  | 1      |

The summary of distributions for the services per day extracted from daily services is shown in Table 6. The number of observations which are serviced crankshafts per day is (n) while the Fn is the observations frequency or frequency of serviced crankshafts for the month. Once the observer’s frequency (Fn) is known, the next is to calculate the service (μ) rate using equation 2 which was found to be 14.4815

Table 7: Chi Square Goodness of Fit Test for Services Data and Calculated Variables

| n   | Fn   | Pn     | Tn    | \(\chi^2_{\text{observed}}\) |
|-----|------|--------|-------|-----------------------------|
| 12  | 4    | 0.091241 | 2.463515 | 0.9583 |
| 13  | 5    | 0.101639 | 2.744258 | 1.85419 |
| 14  | 6    | 0.105135 | 2.838637 | 3.520781 |
| 15  | 5    | 0.1015   | 2.740511 | 1.862897 |
| 16  | 3    | 0.091867 | 2.480416 | 0.10884 |
| 17  | 2    | 0.078257 | 2.112947 | 0.006038 |
| 18  | 1    | 0.06296  | 1.699923 | 0.288185 |
| 19  | 1    | 0.047987 | 1.295652 | 0.067464 |
| \(\sum F_n\) | 27  |       |       |                           |
| \(\sum nF_n\) | 391 | \(\chi^2_{\text{observed}}\) | 8.666694 |

Arrival Rate (\(\lambda\)) 14.48148

With the service rate information, the Poisson distribution at a given (n) was determined using equation 3. Similarly, the theoretical frequency Tn as well as the observed chi square value using equations 4 and 5, all shown in Table 7.

The degree of freedom (df) considering the eight cells service frequency distribution is 6 (i.e. 8-2). The level of significance (\(\alpha\)) throughout is taken to be 0.05. With the above information, the critical value of \(\chi^2\) extracted from the chi squares table shows that a value of 12.592 or more is required for significance to occur for service distribution. Having an observed chi square of 8.667 as against 12.592 shows that it conforms to Poisson’s distribution and the variation in distribution is not adequate to alter the nature of the distribution.

3.4 Mean Waiting Time with and Without Priority
The mean waiting time results using the case study values were calculated when no priority was observed (using equations 6 to 8) as was the case previously used by the Company before the modification. The mean waiting time was also calculated when priority was introduced having three classes with preemption as well as without preemption (using equations 11 to 17) and the results obtained are shown in Fig 3. The calculation was carried out using excel add in package.
Fig 3: Mean Waiting Time in Queue for Priority and No Priority

The mean waiting time is measured per day but working period in a day called working day is not 24 hours but 9 hours being the working period by the Company. From Fig. 3, it has been clearly shown that the preemptive resume priority system offers a significant enhancement when compared with the non preemption priority as it patterns to jobs that are of higher priority. It will only take a mean time of 0.007 working day or 3.78 minutes for a high priority arrival to start receiving service but with non preemption it will require a mean waiting time of 0.066 working day or 35.64 minutes. The trend is equally similar to medium priority class in which preemption and non-preemption recorded 0.036 working day (19.44 minutes) and 0.09 day (48.6 hours).

However, the improvement has a negative effect on the jobs that are of low priority class. It can be seen that the low priority class of preemptive priority scheme suffered most as it requires 0.258 working day or 2.322 hours to wait before regrinding commences. Meanwhile for non-preemption it will take a mean waiting time of 0.224 working day or 2.016 hours. When there was no priority, it will require 0.170 working day or 1.53 hours to wait before service commences. Even though the mean waiting time of no priority takes the smallest time when compared with the lowest class for prioritized scenario, urgent work will seriously suffer and will result in reneging or balking.
It is easy to notice that the values obtained for the mean waiting time in queue has the same trend with the results obtained for mean waiting time in the system shown in Fig 4, with the mean waiting time in the system generally greater than that in the queue as a result of the time of service that is included in the time in queue. Therefore, the closer or farther the mean waiting time in system is to that in the queue essentially depends on the service rate. Whether it is the mean waiting time in queue or for that in service, the small prize paid in improving higher priorities is an increase to that of lower Priority.

4. Conclusion

A close examination of the case study showed that there were queuing challenges in Ezenwa Brothers Limited workshop. Their queuing system is single line multiple server system. Queue system modification using multi priority scheme was introduced, whereby different types of queuing schemes were used; non preemptive priority queuing scheme, preemptive priority queuing scheme, and no priority queuing scheme, using the case study collected data for a period of one month. Comparison was made between the queuing schemes. The data obtained from the schemes were evaluated using Chi Square Goodness of Fit Test. The result of Chi Square Goodness of Fit Test showed that the system conforms to Poisson distribution. From the results the preemptive resume priority system provides a very significant improvement over the non preemption priority as it concerns jobs that are of higher priority. However, the assure waiting time for the queue and the organization for no priority have the minimum time when equated to the smallest priority, due to the fact that the development of the queue system has a negative effect on the smallest priority section, in this case a pressing job will terribly suffer resulting to impossible balking or reneging. Based on the results of the analysis, we concluded that the best suitable priority discipline for this case study to minimize waiting line is preemptive priority, because it enhanced less waiting time both in the queue and in the system.

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