PHENOMENOLOGY OF NEUTRINO OSCILLATIONS AT A NEUTRINO FACTORY

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It is shown in the three flavor framework that neutrino factories enable us to measure some of the oscillation parameters, such as the sign of $\Delta m^2_{32}$, $\theta_{13}$, $\delta$.

Some efforts are made to determine the parameters (the muon energy and the neutrino path length) of a neutrino factory to optimize the signals.

1 Introduction

There have been several experiments which suggest neutrino oscillations. It has been known in the two flavor framework that the solar neutrino deficit can be explained by neutrino oscillation with the set of parameters $(\Delta m^2_{\odot}, \sin^2 2\theta_{\odot}) \approx (O(10^{-5} \text{eV}^2), O(10^{-2}))$ (small angle MSW solution), $(O(10^{-5} \text{eV}^2), O(1))$ (large angle MSW solution), or $(O(10^{-10} \text{eV}^2), O(1))$ (vacuum oscillation solution), and the atmospheric neutrino anomaly can be accounted for by $(\Delta m^2_{\text{atm}}, \sin^2 2\theta_{\text{atm}}) \approx (10^{-2.5} \text{eV}^2, 1.0)$. In the three flavor framework there are two independent mass squared differences and it is usually assumed that these two mass differences correspond to $\Delta m^2_{\odot}$ and $\Delta m^2_{\text{atm}}$. Throughout this talk I will assume three neutrino species which can account for only the solar neutrino deficit and the atmospheric neutrino anomaly. Without loss of generality I assume $|\Delta m^2_{21}| < |\Delta m^2_{32}| < |\Delta m^2_{31}|$ where $\Delta m^2_{ij} \equiv m_i^2 - m_j^2$. The flavor eigenstates are related to the mass eigenstates by $U_{\alpha j}$ ($\alpha = e, \mu, \tau$), where $U_{\alpha j}$ are the elements of the MNS mixing matrix $U$:

$$
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix}
= U
\begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3
\end{pmatrix},
$$

$$
U \equiv
\begin{pmatrix}
U_{e1} & U_{e2} & U_{e3} \\
U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\
U_{\tau 1} & U_{\tau 2} & U_{\tau 3}
\end{pmatrix}
= 
\begin{pmatrix}
c_{12}c_{13} & s_{12}c_{23}s_{13}e^{i\delta} & s_{12}c_{13} \\
-s_{12}c_{23} & c_{12}s_{23}s_{13}e^{i\delta} & s_{12}c_{13} \\
-\frac{s_{12}s_{23}}{c_{23}} & -\frac{c_{12}s_{23}}{c_{23}} & 0
\end{pmatrix}.
$$

With the mass hierarchy $|\Delta m^2_{31}| \ll |\Delta m^2_{32}|$ there are two possible mass patterns which are depicted in Fig. 1a and 1b, depending on whether $\Delta m^2_{32}$ is

To explain the LSND anomaly, one needs at least four neutrino species.
positive or negative. If the matter effect is relevant then the sign of \( \Delta m_{32}^2 \) can be determined by distinguishing neutrinos and anti-neutrinos. The Superkamiokande experiment uses water Cherenkov detectors and events of neutrinos and anti-neutrinos are unfortunately indistinguishable, so the sign of \( \Delta m_{32}^2 \) is unknown to date.

It has been shown in the three flavor framework that combination of the CHOOZ reactor data and the atmospheric neutrino data of the Kamiokande and the Superkamiokande implies very small \( \theta_{13} \), i.e., \( \sin^2 2\theta_{13} < 0.1 \) which is essentially the result of the CHOOZ data. When \( |\theta_{13}| \) is small, the MNS matrix looks like

\[
U \simeq \begin{pmatrix}
\cos \theta_\odot & \sin \theta_\odot & \epsilon \\
-\sin \theta_{\odot} \cos \theta_{\text{atm}} & \cos \theta_{\odot} \cos \theta_{\text{atm}} & \sin \theta_{\text{atm}} \\
\sin \theta_{\odot} \sin \theta_{\text{atm}} & -\cos \theta_{\odot} \sin \theta_{\text{atm}} & \cos \theta_{\text{atm}}
\end{pmatrix},
\]

where \( \theta_{12}, \theta_{23} \) have been replaced by \( \theta_\odot \) and \( \theta_{\text{atm}} \), respectively.

The measurement of \( \theta_\odot \equiv \theta_{12} \) and \( \theta_{\text{atm}} \equiv \theta_{23} \) is expected to be greatly improved in the future experiments on solar and atmospheric neutrinos, so the remaining problems in the three flavor framework are to determine (1) the sign of \( \Delta m_{32}^2 \), (2) the magnitude of \( \theta_{13} \), (3) the magnitude of the CP phase \( \delta \). Recently a lot of research have been done on neutrino factories, and the three problems mentioned above may be solved at neutrino factories. In this
I would like to discuss these three issues in some detail and show which set of parameters optimizes each signal. I will assume that the volume of the detector is 10 kt, the intensity of the beam is $10^{21}$ muons/yr, and the data are taken for one year as the reference values in the following discussions. I will also assume $E_\mu \leq 50$ GeV.

2 Neutrino factories

Before discussing the three problems given at the end of the Introduction let me give a little background for neutrino factories. As has been shown in Refs. 15, 16, the information of neutrino oscillations can be obtained by looking at “wrong sign muons” which are produced in $\nu_e \rightarrow \nu_\mu \rightarrow \mu^-$ or $\bar{\nu}_e \rightarrow \bar{\nu}_\mu \rightarrow \mu^+$ and the numbers $N_{\text{wrong}}(\mu^\pm)$ of the wrong sign muons are given by

$$N_{\text{wrong}}(\mu^-) = n_T \frac{12E_\mu^2}{\pi L^2 m_\mu^2} \int d \left( \frac{E_\nu}{E_\mu} \right) \left( \frac{E_\nu}{E_\mu} \right)^2 \left( 1 - \frac{E_\nu}{E_\mu} \right) \sigma_{\nu N}(E_\nu) P(E_\nu),$$

$$N_{\text{wrong}}(\mu^+) = n_T \frac{12E_\mu^2}{\pi L^2 m_\mu^2} \int d \left( \frac{E_{\bar{\nu}}}{E_\mu} \right) \left( \frac{E_{\bar{\nu}}}{E_\mu} \right)^2 \left( 1 - \frac{E_{\bar{\nu}}}{E_\mu} \right) \sigma_{\bar{\nu} N}(E_{\bar{\nu}}) P(E_{\bar{\nu}}),$$

where $E_\mu$ is the muon energy, $L$ is the length of the neutrino path, $n_T$ is the number of the target nucleons, $\sigma_{\nu N}(E_\nu)$ and $\sigma_{\bar{\nu} N}(E_{\bar{\nu}})$ are the (anti-)neutrino nucleon cross sections given by

$$\sigma_{\nu N}(E_\nu) = \left( \frac{E_\nu}{\text{GeV}} \right) \times 0.67 \times 10^{-38} \text{cm}^2,$$

$$\sigma_{\bar{\nu} N}(E_{\bar{\nu}}) = \left( \frac{E_{\bar{\nu}}}{\text{GeV}} \right) \times 0.33 \times 10^{-38} \text{cm}^2,$$

and $P(E_\nu)$ and $P(E_{\bar{\nu}})$ are the oscillation probabilities which are given by (on the assumption of constant density of the matter)

$$P(E_\nu) \equiv P(\nu_e \rightarrow \nu_\mu) = s_{23}^2 \sin^2 2\theta_{13}^{(-)} \sin^2 \left( \frac{B^{(-)} L}{2} \right),$$

$$P(E_{\bar{\nu}}) \equiv P(\bar{\nu}_e \rightarrow \bar{\nu}_\mu) = s_{23}^2 \sin^2 2\theta_{13}^{(+)} \sin^2 \left( \frac{B^{(+)} L}{2} \right),$$

where $A \equiv \sqrt{2}G_F N_c$ stands for the matter effect of the Earth, $\theta_{13}^{(\pm)}$ is the effective mixing angle in matter given by

$$\tan 2\theta_{13}^{(\pm)} = \frac{\Delta E_{32} \sin 2\theta_{13}}{\Delta E_{32} \cos 2\theta_{13} \pm A},$$

(2)
and

\[ B^{(\pm)} = \sqrt{(\Delta E_{32} \cos 2\theta_{13} \pm A)^2 + (\Delta E_{32} \sin 2\theta_{13})^2}. \]

(3)

Using these formula, the ratio \( N_{\text{wrong}}(\mu^+)/N_{\text{correct}}(\mu^-) \) is plotted in Fig. 2a and 2b as a function of \( E_\mu \) and \( L \) for typical values of \( \theta_{13} \) with \( \sin^2 2\theta_{23} = 1.0 \) and \( \Delta m^2_{32} = 3.5 \times 10^{-3}\text{eV}^2 \). As was shown by Gomez-Cadenas, the ratio of (background events)/\( N_{\text{correct}}(\mu) \) is of order \( 10^{-5} \). In the case of smaller value of \( \theta_{13} \) (\( \theta_{13} = 1^\circ \) or \( \sin^2 2\theta_{13} = 0.001 \)), it should be possible to detect wrong sign events at neutrino factories if \( L > 1000\text{km} \) for all the muon energies \( (10 \leq E_\mu \leq 50 \text{ GeV}) \), with our reference values of the beam and the detector \( (10^{21}\mu/\text{yr} \cdot 10\text{kt} \cdot 1\text{yr}) \).

![Fig.2](image)

3 The sign of \( \Delta m^2_{32} \)

As was mentioned in the Introduction, the mass pattern corresponds to either Fig. 1a or 1b, depending on whether \( \Delta m^2_{32} \) is positive or negative. Determination of this mass pattern is important, since Figs. 1a and 1b correspond to one and two mass states, assuming that the lowest mass is almost zero.\[\text{[1]}\]

\[\text{[2]}\]

The mixed dark scenario in which neutrinos have masses of order 1 eV seems to be disfavored.
As we can see from (1), if $\Delta m_{32}^2 > 0$ then the effective mixing angle $\theta_{13}^{M(-)}$ is enhanced and $P(\nu_e \rightarrow \nu_\mu)$ increases. On the other hand, if $\Delta m_{32}^2 < 0$ then $\theta_{13}^{M(+)}$ is enhanced and $P(\bar{\nu}_e \rightarrow \bar{\nu}_\mu)$ increases. So, at neutrino factories where baseline is relatively large and therefore the matter effect plays an important role, the sign of $\Delta m_{32}^2$ may be determined by looking at the difference between neutrino and anti-neutrino events which should reflect the difference between $P(\nu_e \rightarrow \nu_\mu)$ and $P(\bar{\nu}_e \rightarrow \bar{\nu}_\mu)$. In Figs. 3 and 4 the numbers of events per each neutrino energy $E_\nu$ (and $E_{\bar{\nu}}$ is identified here) at a neutrino factory with the parameter $10^{21} \mu$/yr $\cdot 10^6$ t $\cdot$ 1 yr are given for $\Delta m_{32}^2 = 3.5 \times 10^{-3}$ eV$^2$ (Fig. 3) and $\Delta m_{32}^2 = -3.5 \times 10^{-3}$ eV$^2$ (Fig. 4), respectively (I have assumed $\sin^2 2\theta_{23} = 1.0$ and $\sin^2 2\theta_{13} = 0.095$). The black and gray lines stand for wrong sign $\mu^-$ and $\mu^+$ events, respectively, and five cases of the muon energy $E_\mu = 10, 20, \cdots, 50$ GeV are plotted. For later purposes, behaviors with respect to the CP violating phase $\delta$ are also considered and the solid, dotted and dashed lines are the numbers of events with $\delta = 0$, $\delta = -\pi/2$, $\delta = \pi/2$, respectively. The deviation of the solid lines from the dotted or dashed lines is mainly due to the matter effects, and the deviation of the dotted lines from the dashed ones is in general smaller.

Since the cross section $\sigma_{\nu N}$ and $\sigma_{\bar{\nu} N}$ are different (the ratio is 2 to 1), it is useful to look at the quantity

$$\frac{N_\nu - 2N_\bar{\nu}}{\delta (N_\nu - 2N_\bar{\nu})} = \frac{N_\nu - 2N_\bar{\nu}}{\sqrt{N_\nu + 4N_\bar{\nu}}}$$

whose absolute value should be much larger than one to demonstrate $\Delta m_{32}^2 > 0$ or $\Delta m_{32}^2 < 0$. Now let me introduce the quantity

$$R \equiv \frac{[N_{\text{wrong}}(\mu^-) - 2N_{\text{wrong}}(\mu^+)]^2}{N_{\text{wrong}}(\mu^-) + 4N_{\text{wrong}}(\mu^+)}.$$  

If $R \gg 1$ then we can deduce the sign of $\Delta m_{32}^2$. The contour plot of $R=\text{const.}$ is given in Fig. 5a and 5b for typical values of $\theta_{13}$ with $\Delta m_{32}^2 = 3.5 \times 10^{-3}$ eV$^2 > 0$, $\sin^2 2\theta_{23} = 1.0$. If $\sin^2 2\theta_{13}$ is not smaller than $10^{-3}$, it is possible to determine the sign of $\Delta m_{32}^2$. Irrespective of the value of $\theta_{13}$, $L \sim 5000$ km, $E_\mu = 50$ GeV seem to optimize the signal, as far as the quantity $R$ is concerned.
\[ \Delta m^2_{32} > 0 \]

- $\nu$ ($\delta = 0$)
- $\nu$ ($\delta = -\pi/2$)
- $\nu$ ($\delta = \pi/2$)
- $\bar{\nu}$ ($\delta = 0$)
- $\bar{\nu}$ ($\delta = -\pi/2$)
- $\bar{\nu}$ ($\delta = \pi/2$)

**Fig. 3**
Fig. 4
4 The magnitude of $\theta_{13}$

As we can see in (1), it is necessary to know the precise value of $\theta_{23}$ to determine $\theta_{13}$ accurately. For this purpose, it is useful to measure the independent quantity

$$1 - P(\nu_\mu \rightarrow \nu_\mu) = s_{23}^4 \sin^2 2\theta_{13} \left[ \sin^2 \theta_{13} \sin^2 \frac{L}{4} \left( \Delta E_{32} + A - B'(-) \right) + \cos^2 \theta_{13} \sin^2 \frac{L}{4} \left( \Delta E_{32} + A + B'(-) \right) \right].$$  

(4)

Combining (1) and (4) and assuming that our knowledge on the density profile of the Earth is exact, we can determine $\theta_{13}$ and $\theta_{23}$. In practice, however, there is always uncertainty in the density in the Earth, particularly the density deep inside of the Earth (i.e., large $L$) is not very well known, so to determine $\theta_{13}$ precisely the neutrino path $L$ had better be small, say, $L < 1000$km, as long as $N_{\text{wrong}}(\mu)$ exceeds the number of the background events.
5 The magnitude of $\delta$

In the hierarchical limit, to first order in $\Delta E_{21}/\Delta E_{32}$ and $\Delta E_{21}/A$, the appearance probabilities are given by

$$
\begin{align*}
\left\{ \begin{array}{l}
P(\nu_e \rightarrow \nu_\mu) \\
P(\bar{\nu}_e \rightarrow \bar{\nu}_\mu)
\end{array} \right\} & \simeq s_{23}^2 \sin^2 2\theta_{13}^{M(\mp)} \sin^2 \left( \frac{B(\mp)L}{2} \right) \\
& \mp \frac{1}{2} \frac{\Delta E_{21}\Delta E_{32}}{\lambda_+^{(\mp)} \lambda_-^{(\mp)}} \sin \delta \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13}^{M(\mp)} \\
& \times \sin \left( \frac{\lambda_+^{(\mp)}L}{2} \right) \sin \left( \frac{\lambda_-^{(\mp)}L}{2} \right) \sin \left( \frac{B(\mp)L}{2} \right),
\end{align*}
$$

where

$$
\lambda_+^{(-)} \equiv \frac{1}{2} \left( A - \Delta E_{32} \pm B^{(-)} \right), \quad \lambda_-^{(+)} \equiv \frac{1}{2} \left( A - \Delta E_{32} \pm B^{(+)}. \right)
$$

Because of the matter effect, one of the two independent probabilities is enhanced while the other is suppressed. For the enhanced channel ($\nu_e \rightarrow \nu_\mu$ in the case of $\Delta m_{32}^2 > 0$, $\bar{\nu}_e \rightarrow \bar{\nu}_\mu$ in the case of $\Delta m_{32}^2 < 0$), the number of events is large and one might want to take advantage of this large number. Here I therefore would like to consider the following quantity

$$
R_\delta \equiv \frac{[N_{\text{wrong}}(\delta = \frac{\pi}{2}) - N_{\text{wrong}}(\delta = -\frac{\pi}{2})]^2}{2 [N_{\text{wrong}}(\delta = \frac{\pi}{2}) + N_{\text{wrong}}(\delta = -\frac{\pi}{2})]},
$$

(5)

The denominator in (5) corresponds to square of statistical fluctuations of the number of events obtained from the T-invariant probability $P(\nu_e \rightarrow \nu_\mu; \delta) + P(\nu_e \rightarrow \nu_\mu; -\delta)$ while the numerator does to square of the number of events obtained from T-violating probability $P(\nu_e \rightarrow \nu_\mu; \delta) - P(\nu_e \rightarrow \nu_\mu; -\delta)$. This $R_\delta$ is the quantity of T-violation instead of CP-violation. In fact $R_\delta$ cannot be determined by the experimental data only, but it requires knowledge on $\Delta m_{32}^2$, $\theta_{13}$ and the density profile of the Earth to deduce $R_\delta$. Nevertheless, having large value of $R_\delta$ is a necessary condition to be able to measure $\delta$ and the experiments should be designed so that $R_\delta$ be maximized.

This suggestion is different from the one in [3] in which it is proposed to subtract $(N_\nu - 2N_{\bar{\nu}})/(N_\nu + 2N_{\bar{\nu}})$ by the matter effect term. In either case, one needs the precise knowledge on $\Delta m_{ij}^2$, $\theta_{ij}$ and the density of the Earth. To demonstrate $\delta \neq 0$ it is necessary that $R_\delta \gg 1$. The contour plot of the ratio $R_\delta$ is given in Figs. 6a and 6b for two sets of parameters. For the set of the oscillation parameters ($\Delta m_{21}^2, \sin^2 2\theta_{12} = (1.8 \times 10^{-5} \text{eV}^2, 0.76)$,
\((\Delta m^2_{32}, \sin^2 2\theta_{23}) = (3.5 \times 10^{-3}\text{eV}^2, 1.0)\) (Fig. 6a), which gives the best fit to the data of solar and atmospheric neutrinos, we have \[ \max_{L,E_\mu} R_3 \simeq 2.3 \] (6)

for \(\sin^2 2\theta_{13} = 0.09, L \simeq 3000\text{km}, \delta = \pi/2, E_\mu \simeq 40\text{ GeV} \) with \(10^{21}\mu/\text{yr}\cdot10\text{kt}\cdot1\text{yr}.\)

\[
\begin{align*}
R_\delta & \quad (\text{best fit case}) \\
\text{(a)} & \quad \text{F} \quad \text{g} \quad 6
\end{align*}
\]

If we take other set of parameters, e.g., \((\Delta m^2_{21}, \sin^2 2\theta_{12}) = (1 \times 10^{-4}\text{eV}^2, 1.0), (\Delta m^2_{32}, \sin^2 2\theta_{23}) = (6 \times 10^{-3}\text{eV}^2, 0.8)\) (Fig. 6b), which are still in the allowed region of 99 %CL of data of solar and atmospheric neutrinos, we have

\[ \max_{L,E_\mu} R_3 \simeq 200 \]

for \(\sin^2 2\theta_{13} = 0.09, L \simeq 3000\text{km}, \delta = \pi/2, E_\mu \simeq 50\text{ GeV} \) with \(10^{21}\mu/\text{yr}\cdot10\text{kt}\cdot1\text{yr}.\)

I have also calculated the ratio \(R_3\) for smaller value of \(\theta_{13}\), and found that the

\(^c\) For solar neutrinos, there are three possible sets of parameters which give a very good fit to the data, but here I take the most optimistic one (large mixing angle MSW solution) to observe CP violation. For other solar neutrino solutions, observation of CP violation is either very difficult or impossible.
signal is optimized for almost the same set of the parameters \(E_\mu \simeq 50\ \text{GeV},\ L \sim 3500\ \text{km}\), although the finite value of \(R_\delta\) is obtained as a limit of 0/0 for very small value of \(\theta_{13}\) and one has to make sure that we have a certain amount of events to be conclusive. If (6) happens to be the case, then after running the experiment for several years it may be possible to demonstrate \(\delta \neq 0\).

6 Summary

In this talk I have discussed some quantities (the sign of \(\Delta m^2_{32}\), the magnitude of \(\theta_{13}\), the magnitude of \(\delta\)) which can be measured at neutrino factories, and made efforts to optimize the signals. Of course all the measurements would be impossible if \(\sin^2 2\theta_{13} \ll 10^{-3}\), but otherwise we may be able to determine the sign of \(\Delta m^2_{32}\), the values of \(\theta_{13}, \theta_{23}\) and the value of \(\delta\) with our reference values \(10^{21}\mu/\text{yr} \cdot 10^{kt} \cdot \text{yr}\). To measure \(\delta\) it is necessary to know with great precision quantities such as the cross sections \(\sigma_{\nu N}, \sigma_{\bar{\nu} N}\), the mixing angles \(\theta_{12}, \theta_{13}, \theta_{23}\), the mass squared differences \(\Delta m^2_{21}, \Delta m^2_{32}\) as well as density profile of the Earth. It is important to estimate how much the uncertainty on \(\Delta m^2_{ij}, \theta_{ij}\) and the density profile of the Earth affects the reach of the experiment, particularly in the measurement of CP violation, and this is a subject for future study.

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