The Third way to 3D Supersymmetric Massive Yang-Mills Theory

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Abstract

We construct three-dimensional, $\mathcal{N} = 1$ off-shell supersymmetric massive Yang-Mills (YM) theory whose YM equation is "third way" consistent. This means that the field equations of this model do not come from variation of a local action without additional fields, yet the gauge covariant divergence of the YM equation still vanishes on-shell. To achieve this, we modify the massive Majorana spinor equation so that its supersymmetry variation gives the modified YM equation whose bosonic part coincides with the third way consistent YM model constructed earlier in [1].
1 Introduction

"Third way consistency" is a novel mechanism discovered in the context of three-dimensional (3D) pure gravity theories in [2] where Einstein's field equation was modified with an interaction term that does not come from variation of a local action for the metric. Nevertheless, the modified field equation still makes sense since the divergence of this gravitational source term vanishes if one uses the field equation again, see [3] for a review. Additional such 3D gravity models were obtained in [4, 5]. Meanwhile, a 3D gauge theory example was found in [1]. Interacting p-form theories with this property in general dimensions were constructed in [6]. An open question regarding this class of models is whether it is possible to incorporate supersymmetry in them. In this paper we answer this question in the affirmative by explicitly constructing a supersymmetric version of [1].

The third way consistent field equation of [1] can be viewed as a deformation of the well-known 3D topologically massive Yang-Mills (TMYM) theory. TMYM itself was constructed a long time ago [7, 8, 9] and its spin-2 counterparts led to many important developments in the area of 3D massive gravity theories. The model of [1] is then obtained by adding an extra term to the field equation of TMYM. This additional term is quadratic in the Yang-Mills (YM) field strength and can not be derived from an action with the YM field alone. While supersymmetric generalizations of TMYM have been formulated, see for example [10, 11], no supersymmetric version of its third way consistent deformation has been constructed yet. The purpose of our paper is to fill this gap, i.e., to see whether the third way and supersymmetric extensions of TMYM can be combined and whether theories in the lower-right corner of the following diagram exist:

Due to the lack of an action without auxiliary fields for [1], we will work with equations of motion. We will in particular attempt to construct a deformation of the equations of motion of supersymmetric TMYM, such that the resulting bosonic and fermionic equations are mapped to each other under supersymmetry. It will then be most convenient for us to work with an off-shell supersymmetric YM multiplet, for which the supersymmetry algebra closes without using the field equations. In this way we avoid having to consider modifications of the supersymmetry transformation rules of the YM multiplet. In this paper, we will study the simplest possible choice, namely a single off-shell $\mathcal{N} = 1$ supersymmetric YM multiplet which only has a Majorana spinor in addition to the YM gauge field.

We start with a brief description of the $\mathcal{N} = 1$ off-shell supersymmetric TMYM in the next section. In section 3, we first review the model of [1]. Then, we make a modification of the spinor field equation of the $\mathcal{N} = 1$ supersymmetric TMYM theory so that its supersymmetry variation gives a modified YM-equation whose bosonic part is the model found in [1]. The full description of our model is given in equation (17). It has one extra mass parameter compared to TMYM [7, 8, 9] like [1] and when fermions are set to zero it reduces to [1]. We finish this section by showing that our modified YM-equation is third way consistent. We conclude with some comments in section 4.
2 A Review of 3D, \( \mathcal{N} = 1 \) Topologically Massive super-Yang-Mills

In this paper we would like to construct the \( \mathcal{N} = 1 \) off-shell supersymmetric version of the third way consistent massive 3D YM theory of \cite{1}. Since this theory will correspond to a deformation of the \( \mathcal{N} = 1 \) supersymmetric TMYM theory, we will first review the latter here. This section also serves to introduce the notation and conventions used in the rest of this letter.

We will consider YM theory for an arbitrary non-abelian gauge group \( G \) with structure constants \( f^I_{JK} \) (with \( I, J, K = 1, \cdots, \dim(G) \)). The off-shell \( \mathcal{N} = 1 \) super-YM multiplet then consists of the gauge field \( A^I_\mu \) and a Majorana spinor \( \chi^I \), both transforming in the adjoint of \( G \). In our conventions, a gauge transformation with parameter \( \Lambda^I \) of these fields is given by

\[
\delta A^I_\mu = \partial_\mu \Lambda^I - f^I_{JK} A^K_\mu , \quad \delta \chi^I = - f^I_{JK} \Lambda^K \chi^J .
\]

As a consequence, the gauge covariant field strength \( F^I_{\mu\nu} \) and covariant derivative \( D_\mu X^I \) of any object \( X^I \) in the adjoint representation of \( G \) are explicitly given by

\[
F^I_{\mu\nu} = 2 \partial_\mu A^I_\nu + f^I_{JK} A^K_\mu A^K_\nu , \quad D_\mu X^I = \partial_\mu X^I + f^I_{JK} A^K_\mu X^K .
\]

Note that the following properties for bilinears involving Majorana spinors then hold: \( \bar{\gamma} \epsilon \)

\[\chi \epsilon = \bar{\chi} \epsilon , \quad \bar{\chi} \gamma^\nu \epsilon = - \epsilon \gamma^\nu \chi .\]

The Fierz identity reads:

\[
\chi \epsilon = - \frac{1}{2} \bar{\epsilon} \chi + \frac{1}{2} \bar{\gamma}_\mu \chi \gamma^\mu \epsilon .
\]

Some useful \( \gamma \)-matrix relations are:

\[\gamma^\nu \gamma^\mu \gamma^\rho = - \gamma^\rho , \quad \gamma^\mu = \epsilon^{\mu\nu\rho} \gamma^\nu \gamma^\rho , \quad \gamma_\mu = - \frac{1}{2} \epsilon^{\mu\nu\rho} \gamma_\nu \gamma^\rho .\]

\[\gamma^{\mu\nu} = \epsilon^{\mu\nu\rho} \gamma^\rho .\]

\[\epsilon^{\mu\nu\rho} = \epsilon_{\rho\mu\nu} .\]

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\[\gamma \]
on the right-hand-side of (5) should then also vanish and it is easy to see that this is true on-shell.

For what follows, we note that the supersymmetry transformation of the fermionic equation of motion (6) leads to the bosonic one (5), as follows:

$$\delta \left( \gamma^\mu D_\mu \chi^I + 2 \mu \chi^I \right) = \frac{1}{4} \left( D^\nu F^I_{\nu \mu} + \mu \epsilon_\mu \epsilon^\nu F^I_{\nu \rho} + 2 f^I_{JK} \gamma^J \chi^K \right) \gamma^\mu \epsilon \ .$$  \hspace{1cm} (8)

The fact that the supersymmetry variation of the spinor field equation is of the form $Y_\mu \gamma^\mu \epsilon$ where $Y_\mu = 0$ is the bosonic field equation, will be our guiding principle in constructing the supersymmetric version of the model of [1] in the next section.

3 Third Way Consistent 3D, $\mathcal{N} = 1$ Massive super-Yang-Mills

We first briefly review the third way consistent massive YM theory constructed in [1]. Its source free field equation for an arbitrary gauge group $G$ is obtained by adding an extra term to the equation of motion of TMYM (itself obtained by setting $\chi^I$ to zero in (5)):

$$\epsilon_\mu \epsilon^\rho F^I_{\mu \rho} + 2 \mu \tilde{F}^I_\mu + \frac{2}{m} \epsilon_\mu \epsilon^\rho F^I_{\mu \rho} = 0 \ ,$$  \hspace{1cm} (9)

where, $m$ is another mass parameter and we introduced the dual field strength $\tilde{F}^I_\mu$ notation as

$$\tilde{F}^I_\mu = \frac{1}{2} \epsilon_\mu \epsilon^\rho F^I_{\mu \rho} \quad \Leftrightarrow \quad F^I_{\mu \rho} = -\epsilon_\mu \epsilon^\rho F^I_\rho \ .$$  \hspace{1cm} (10)

It is easy to see that this equation can not be the Euler-Lagrange equation of a gauge-invariant local action for the YM vector field alone [1] and hence its consistency is not automatic. Indeed, if we hit (9) with the covariant derivative $D^\mu$ the first two terms are identically zero whereas the interaction term is not. However, if we use (9) again it vanishes due to the Jacobi identity, which is the essence of the third way consistency mechanism. The special cases $\mu = 0$ and $\mu = 2m$ of this model were studied earlier in [12] and [13] respectively.

It is possible to add a matter current $J^I_\mu$ to (9) as

$$\epsilon_\mu \epsilon^\rho D_\rho \tilde{F}^I_\mu + 2 \mu \tilde{F}^I_\mu + \frac{2}{m} \epsilon_\mu \epsilon^\rho f^I_{JK} \tilde{F}^J_\mu \tilde{F}^K_\rho = J^I_\mu \ ,$$  \hspace{1cm} (11)

provided that it satisfies

$$D_\mu J^I_\mu + \frac{4}{m} f^I_{JK} \tilde{F}^J_\mu J^K_\mu = 0 \ ,$$  \hspace{1cm} (12)

to maintain the third way consistency of the model [1].

In this paper our aim is to obtain a $\mathcal{N} = 1$ off-shell supersymmetric extension of (9). Since (9) does not come from an action without extra auxiliary fields, we will do this using the Noether procedure at the level of equations of motion. The advantage of having off-shell supersymmetry is that the supersymmetry variations (3) remain the same for any extension of the field equations (5) and (6). Notice that (9) is simply a deformation of (5) when fermions are set to zero. Therefore, to achieve our goal we will start from the fermionic equation of motion (6) of the $\mathcal{N} = 1$ supersymmetric TMYM theory and modify it so that its supersymmetry variation is of the form $Y_\mu \gamma^\mu \epsilon$ with the bosonic part of $Y_\mu = 0$ given by [1]. The full $Y_\mu = 0$ equation will be the new YM equation as happened in (8) and this together with the modified spinor equation will define our supersymmetric model. In this case, the $Y_\mu = 0$ equation will be of the form (11) and therefore
the fermionic current \( J^I_\mu \) that we will find should satisfy the third way consistency condition \( \text{(12)} \). Moreover, the supersymmetry variation of the \( y_\mu = 0 \) equation should give the derivative of the new spinor equation on-shell.

Note that \( \text{(4)} \) contains \( \epsilon^{\mu \nu \rho} f^{I}_{JK} \delta^J F^K_\rho \) as an additional term to \( \text{(4)} \) and the supersymmetry variation of the spinor \( \text{(3)} \) immediately suggests adding a term of the form \( f^{I}_{JK} \delta^J \chi^K_\mu \) to the spinor field equation \( \text{(6)} \). However, the supersymmetry variation of this term gives, in addition to what we want, a term of the form \( f^{I}_{JK} \chi^J_\mu \chi^K_\nu \). Due to the modification of the spinor field equation this term is neither zero on-shell, nor of the form that we want. Hence, we need to modify the spinor field equation further to cancel this term, either identically or on-shell. The form of this unwanted piece indicates that on-shell cancellation can be achieved by adding an extra term that is cubic in \( \chi^I_\mu \). Therefore, we consider the following modification of \( \text{(4)} \) as a candidate equation of motion of \( \chi^I_\mu \):

\[
\Psi^I = \gamma^\nu D^\nu \chi^I + 2 \mu \chi^I + \frac{2}{m} f^{I}_{JK} \delta^J \chi^K_\mu + a f^{I}_{JK} f^{K}_{MN} \gamma^\nu \chi^M_\mu \gamma^\nu \chi^N = 0 ,
\]

where “\( a \)” is a constant to be determined. As mentioned above, we determine \( a \) by requiring that the supersymmetry transformation of \( \Psi^I \) is on-shell of the form \( y_\mu \gamma^\mu \). After repeatedly using Fierz, Jacobi and \( \gamma \)-matrix identities, we find that with the choice \( a = \frac{16}{3m^2} \) one gets

\[
\delta \Psi^I = \frac{1}{4} \Xi^I_\mu \gamma^\mu \epsilon - \frac{4}{m} f^{I}_{JK} \bar{\chi}^J \epsilon \Psi^K ,
\]

where

\[
\Xi^I_\mu \equiv D^\nu F^I_{\mu \nu} + \mu \epsilon^\nu \rho \epsilon F^I_{\nu \rho} - \frac{1}{m} f^{I}_{JK} \epsilon^\rho \nu \epsilon F^J_{\mu \rho \nu} + \left( 2 - \frac{16 \mu}{m} \right) f^{I}_{JK} \bar{\chi}^J \gamma^\nu \chi^K + \frac{8}{m} \epsilon^\nu \rho f^{I}_{JK} \bar{\chi}^J \chi^K \gamma^\nu D_\rho \chi^K
\]

\[
+ \frac{16}{m^2} f^{I}_{JK} f^{K}_{MN} \chi^M_\mu \chi^N - \frac{32}{m^3} \epsilon^\nu \rho f^{I}_{JK} f^{J}_{MN} \bar{\chi}^L \chi^K \gamma^\nu \chi^L \gamma^\rho \chi^N .
\]

A lengthy computation then shows that the supersymmetry variation of \( \Xi^I_\mu \) is given by

\[
\delta \Xi^I_\mu = -\bar{\epsilon} \gamma^\mu D_\nu \Psi^I + \frac{4}{m} f^{I}_{JK} \bar{\chi}^J \epsilon \Xi^I_\nu - \frac{16}{m^3} f^{I}_{JK} f^{K}_{MN} \gamma^\nu \chi^M \epsilon \gamma^\mu \Psi^I .
\]

Together with \( \text{(14)} \), we thus see that \( \Psi^I \) and \( \Xi^I_\mu \) transform into each other under supersymmetry. We can then propose \( \Psi^I = 0 \) and \( \Xi^I_\mu = 0 \) as a supersymmetric set of equations of motion. Since the bosonic part of \( \Xi^I_\mu = 0 \), i.e. the first three terms in \( \text{(15)} \), coincides with that of the equation of motion of pure massive Yang-Mills theory \( \text{(1)} \) given in \( \text{(9)} \), we see that \( \Psi^I = 0 \) and \( \Xi^I_\mu = 0 \) can be identified as the equations of motion of the \( \mathcal{N} = 1 \) supersymmetric version of \( \text{(9)} \). Note however that in the presence of supersymmetry, the equation of motion \( \text{(9)} \) of the pure theory gets modified by extra terms that represent a coupling of the spin-1 gauge vector \( A^I_\mu \) to the spin-1/2 gaugino \( \chi^I \).

Summarizing, we propose the following equations of motion for the third way consistent 3D, \( \mathcal{N} = 1 \) massive super-Yang-Mills theory:

\[
\epsilon^{\mu \nu \rho} f^{I}_{JK} \bar{F}^{J} F^{K}_\rho = \mathcal{J}^I_\mu ,
\]

\[
\gamma^\mu D^\nu \chi^I + 2 \mu \chi^I + \frac{2}{m} f^{I}_{JK} \delta^J \chi^K_\mu + \frac{16}{3m^2} f^{I}_{JK} f^{K}_{MN} \gamma^\nu \chi^M_\nu \gamma^\nu \chi^N_\nu = 0 ,
\]

with

\[
\mathcal{J}^I_\mu = \left( \frac{16 \mu}{m} - 2 \right) J^I_\mu + \frac{4}{m} \epsilon^{\mu \nu \rho} D_\nu J^I_\rho + \frac{16}{m^2} \epsilon^{\mu \nu \rho} f^{I}_{JK} \bar{F}^{J}_\nu J^K_\rho - \frac{32}{m^3} \epsilon^{\mu \nu \rho} f^{I}_{JK} J^J_\nu J^K_\rho .
\]

We still need to verify that our YM equation is indeed third way consistent. This will be done in the next subsection.
3.1 The ‘Third way’ Consistency

Note that our matter source $J^I_\mu$ that appears in (17) is of the form
\[ J^I_\mu = c_1 j^I_\mu + c_2 \epsilon_\mu^\nu^\rho D_\nu j^I_\rho + c_3 \epsilon_\mu^\nu^\rho f^I_J K_F^J j^K_\rho + c_4 \epsilon_\mu^\nu^\rho f^I_J j^K_\rho , \tag{18} \]
where $c_1 = \frac{16\alpha}{m^2} - 2$, $c_2 = \frac{4}{m^2}$, $c_3 = \frac{64}{m^2}$, $c_4 = -\frac{32}{m^2}$. To have a third way consistent system, $J^I_\mu$ should satisfy (12) as we discussed above. The form of the current $J^I_\mu$ in terms of $j^I_\mu$ is the same as in [1], however unlike [1] we will not assume that $j^I_\mu$ is conserved. Using (18), one can derive that on-shell the following holds
\[ D_\mu J^I_\mu + \frac{4}{m} f^I_{JK} F^\mu_{\nu} J^K_\mu = c_1 D_\mu j^\mu I + \left( c_2 - 2j_3 + \frac{4}{m} c_1 \right) f^I_{JK} F^J_{\mu} j^K_\mu \]
\[ + (2c_4 + c_2c_3) \epsilon_\mu^\nu^\rho f^I_{JK} D_\nu j^K_\rho + \left( c_3 + \frac{8}{m} c_4 \right) \epsilon_\mu^\nu^\rho f^I_{JK} F^J_{I} f^I_{MN} F^J_{\mu} f^J_{\nu} j^K_\rho \]
\[ + \left( c_3 - \frac{4}{m} c_2 \right) \epsilon_\mu^\nu^\rho f^I_{JK} D_\nu j^K_\rho . \tag{19} \]
Note that in order to derive this result, we needed to use the bosonic equation of motion (11). For our $j^I_\mu$ given in (7), $D_\mu j^\mu I \neq 0$, instead, an explicit computation (using the fermionic equation of motion of (17)) gives that
\[ D_\mu j^\mu I = - \frac{2}{m} f^I_{JK} F^\mu_{\nu} j^K_\mu + 2 f^I_{JK} \chi^{J,K} . \tag{20} \]
This shows that we can replace $D_\mu j^\mu I$ on-shell by $- \frac{2}{m} f^I_{JK} F^\mu_{\nu} j^K_\mu$. Using this in (19), it is easy to see that coefficients of all terms in the right-hand-side of (19) vanish and hence the consistency condition (12) is satisfied for our model.

It was realized in [3] that the model constructed in [1] can be found starting from the flat connection equation $F^\mu_{\nu} = 0$ and then shifting the connection $A^M_\mu$ with an arbitrary linear combination of $\tilde{F}^M_\mu$ and $j^M_\mu$. After this, it is possible to add a multiple of $\tilde{F}^M_\mu$ and $j^M_\mu$ to this equation provided that either $D_\mu j^M_\mu = 0$ as in [1] or $D_\mu j^M_\mu$ is on-shell proportional to one of the terms on the right hand side of (19) as in (20) without spoiling the third way consistency. This also shows that $J^I_\mu$ will always have the structure given in (18) in terms of $j^I_\mu$. It is remarkable that this mechanism works for the spinor equation in our model as well. Indeed a comparison of the spinor field equation of the Topologically Massive super-Yang-Mills theory [3] with ours (13) shows that the latter can be obtained from the former by shifting $A^M_\mu$ as
\[ A^M_\mu \rightarrow A^M_\mu + \alpha \tilde{F}^M_\mu + \beta j^M_\mu , \tag{21} \]
where constants are fixed uniquely as $\alpha = \frac{2}{m}$ and $\beta = -\frac{16\alpha}{3m^2}$ by requiring the supersymmetry.

4 Conclusion

The theory presented in (17) is the main result of this paper which is the first example of a supersymmetric third way consistent model. It is clear that its equations of motion can not be derived from a local action, with the field content considered in this letter. It is however conceivable that an action with auxiliary fields exists as in [1] and it would be interesting to investigate this further. In this regard, it is useful to note that such an auxiliary field formulation likely includes a fermionic auxiliary field, as is suggested by how the fermionic equation of motion appears in its own supersymmetry transformation (see (14)).\(^2\) It would also be interesting to see whether

\(^2\)We are grateful to Paul Townsend for making this suggestion.

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a superspace formulation of the results presented here can be obtained. There are also various extensions of our model to consider such as coupling with $\mathcal{N} = 1$ supersymmetric scalar or gravity multiplets and constructing $\mathcal{N} > 1$ supersymmetric versions. In [6] it was shown that one can obtain higher derivative extensions of [1] by shifting the connection (21) with further terms which are not necessarily conserved. It would be interesting to see whether a supersymmetric extension would still be possible for such deformations.

We hope that our construction will provide some insight for finding supersymmetric versions of the third way consistent gravity [2] [4] [5] and $p$-form theories [6]. As we saw in (21) the extra terms that appear in our model can be understood as coming from shifting the gauge connection $A_\mu$ with bosonic and fermionic current 1-forms of the initial theory. We expect this to be a key feature of all such models. In 3D gravity examples [2] [4] [5] the shift occurs in the spin connection in their first order formulation and we anticipate this to be supplemented with appropriate fermionic current terms in the supersymmetric case. Observe that the supersymmetry variation of our spinor field equation involves not only the YM field equation but also contains a term proportional to itself (see (14)) and a similar conclusion holds for the YM field equation (see (16)). This could be a generic feature of third way consistent supersymmetric theories.

The model of [1] exhibits a Higgs mechanism [13] [14] and is also related to multi M2-branes of 11D supergravity [15]. Moreover, presence of higher derivative terms in the bosonic third way consistent models can be understood as spontaneous breaking of a local symmetry as was illustrated for [2] in [16]. It would be nice to clarify these connections for our supersymmetric model too.

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