A (Simplified) Supreme Being Necessarily Exists — Says the Computer!

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Abstract

A simplified variant of Kurt Gödel’s modal ontological argument is presented. Some of Gödel’s, resp. Scott’s, premises are modified, others are dropped, and modal collapse is avoided. The emended argument is shown valid already in quantified modal logic $K$.

The presented simplifications have been computationally explored utilising latest knowledge representation and reasoning technology based on higher-order logic. The paper thus illustrates how modern symbolic AI technology can contribute new knowledge to formal philosophy and theology.

1 Introduction

Variants of Kurt Gödel’s [1970], resp. Dana Scott’s [1972], modal ontological argument have previously been analysed, studied and verified on the computer by Benzmüller and Woltzenlogel [2016] and Benzmüller and Fuenmayor [2020], and even some unknown flaws were revealed in these works.

In this paper a simplified and improved variant of Gödel’s modal ontological argument is presented. This simplification has been explored in collaboration with the proof assistant Isabelle/HOL [Nipkow et al., 2002], while employing Benzmüller’s [2019; 2013] shallow semantical embedding (SSE) approach as enabling technology. This technology supports the reuse of theorem proving (ATP) systems for classical higher-order logic (HOL) to represent and reason with a wide range of non-classical logics and theories, including higher-order modal logic (HOML) and Gödel’s modal ontological argument, which are in the focus of this paper.

The new, simplified modal argument is as follows. Gödel definition of a Godlike entity remains unchanged ($P$ is an uninterpreted constant denoting positive properties):

$P \equiv \forall x.(\lambda y. P y \rightarrow y x)$

A Godlike entity thus possesses all positive properties.

The three single axioms of the new theory are²

A1: $\forall x.(P(\lambda y. x = y) \land \neg P(\lambda y. x \neq y))$ and

A2: $\forall X. (\forall Y. (P X \land (\forall X Y. (Y \Rightarrow Y))) \rightarrow P Y)$ and

A3: $(\forall Z. P \land \neg \forall Z. \neg P \rightarrow (\forall Y. (Y \Rightarrow Y) \rightarrow (\forall Z. X \Rightarrow Y u))$ and

where the following defined terms are used ($\forall$ is a possibilist quantifier and $\forall^E$ is an actualist quantifier for individuals):

$X \equiv \forall z. X z \rightarrow Y z$

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$P \equiv \forall \forall^E u. X u \leftrightarrow (\forall Y. (Z \rightarrow Y u))$

Informally we have:

A1’ Self-identity is a positive property, self-difference is not.

A2’ A property entailed or necessarily entailed by a positive property is positive.

A3 The conjunction of any collection of positive properties is positive. (Technical reading: if $Z$ is any set of positive properties, then the property $X$ obtained by taking the conjunction of the properties in $Z$ is positive.)

From these premises it follows, in a few argumentations steps in base modal logic $K$, that a Godlike entity possibly and necessarily exists. Modal collapse, which expresses that there are no contingent truths and which thus eliminates the possibility of alternative possible worlds, does not follow from these axioms. These observations should render the new theory interesting to formal philosophy and theology.

Compare the above with Gödel’s premises of the modal ontological argument (we give the consistent variant of Scott):

A1: $\forall X. (\neg (\exists Z. \neg X Z) \leftrightarrow (\exists Z. X Z))$ and

A2: $\forall X Y. (P X \land (\forall Y. (P Y))) \rightarrow P Y)$ and

A3: $(\forall Z. P \land \neg \forall Z. \neg P \rightarrow (\forall Y. (Y \Rightarrow Y) \rightarrow (\forall Z. X \Rightarrow Y u))$ and

A4: $\forall X. (P X \rightarrow (\exists Z. X Z) \rightarrow (\forall X. X Z) \rightarrow (\forall Y. (Y \Rightarrow Y) \rightarrow (\forall Z. X \Rightarrow Y u))$ and

A5: $(\exists Z. \neg P \land \neg \forall Z. \neg P \rightarrow (\forall X. (Y \Rightarrow Y) \rightarrow (\forall Z. X \Rightarrow Y u))$ and

B: $\forall x y. (y x y) \rightarrow (y x y)$ (*Logic KB*)

²An alternative to A1’ would be: The universal property ($\lambda x. T$) is a positive property, and the empty property ($\lambda x. \bot$) is not.

The third-order formalization of A3 given here has been proposed by Anderson and Gettings [1996], see also Fitting [2002]. Axiom A3, together with the definition of G, implies that being Godlike is a positive property. Since supporting this inference is the only role this axiom plays in the argument, $P G$ could be (and has been; cf. Scott [1972]) taken as an alternative to our A3.
Necessary existence (NE) and essence (E) are defined as (the other definitions are as above):

\[
\# E \chi x \equiv \chi x \land (\forall z. \, z x \rightarrow (z \equiv z))
\]

\[
\# \neg E \chi x \equiv \lambda w. \, (\forall y. \, E \chi y \rightarrow \Box \exists y) w
\]

Informally: a property \( Y \) is the essence of an entity \( x \) if (i) \( x \) has property \( Y \) and (ii) \( Y \) necessarily entails every property of \( x \). Moreover, an entity \( x \) has the property of necessary existence if the essence of \( x \) is necessarily instantiuated.

We also give informal readings of G"odel's axioms: A1 says that one of a property or its complement is positive. A2 states that a property necessarily entailed by a positive property is positive, and A3 is as before. A4 expresses that any positive property is necessarily so. A5 postulates that necessary existence is a positive property. Axiom B (symmetry of the accessibility relation associated with modal \( \Box \) operator) is added to ensure that we are in modal logic KB instead of just K.\(^3\)

Using G"odel's premises as stated it can be proved that a Godlike entity possibly and necessarily exists, and this proof can be verified with the computer. However, as is well known [Sobel, 1987], modal collapse is implied; see also Fitting [2002] and Sobel [2004] for further background information.

Recent results of Benzmüller and Fuenmayor [2020] show that different modal ultrafilter properties can be deduced from G"odel's premises. These insights are key to the new argument presented in this paper: If G"odel's premises entail that positive properties form a modal ultrafilter, then why not turning things around, and start with an axiom U1 postulating that positive properties are an ultrafilter? Then use U1 instead of A1 for proving that a Godlike entity necessarily exists, and on the fly explore what further simplifications of the argument are triggered. This research plan worked out and it led to the new modal ontological argument presented above, where U1 has been replaced by A1' and where A2 has been strengthened into A2' accordingly.

The proof assistant Isabelle/HOL and its integrated ATP systems have supported our exploration work surprisingly well, despite the undecidability and high complexity of the underlying logic setting. As usual, we here only present the main steps of the exploration process, and various interesting eureka and frustration steps in between are dropped.

Paper structure: An SSE of HOML in HOL is introduced in Sect. 2. The foundations outlined there ensure that the paper is sufficiently self-contained; readers familiar with the SSE approach may simply skip it. Modal ultrafilter are defined in Sect. 3. Section 4 recap the G"odel/Scott variant, and then an ultrafilter-based modal ontological argument is presented in Sect. 5. This new argument is further simplified in Sect. 6, leading to our new proposal based on axioms A1', A2' and A3 as presented above. Related work is mentioned in Sect. 7.

Since we develop, explain and discuss our formal encodings directly in Isabelle/HOL [Nipkow et al., 2002], some familiarity with this proof assistant and its underlying logic HOL [Andrews, 2002] is assumed. The entire sources of our formalization are presented and explained.

Sections 2 and 3 have been adapted from [Benzmüller and Fuenmayor, 2020] and [Kirchner et al., 2019].

2 Modeling HOML in HOL

Related work focused on the development of various SSEs, cf. [Benzmüller, 2019; Kirchner et al., 2019] and the references therein. These contributions, among others, show that the standard translation from propositional modal logic to first-order (FO) logic can be concisely modeled (i.e., embedded) within HOL theorem provers, so that the modal operator \( \Box \), for example, can be explicitly defined by the \( \lambda \) term \( \lambda \varphi.\lambda w.\, (Rwv \rightarrow \varphi w) \), where \( R \) denotes the accessibility relation associated with \( \Box \). Then one can construct FO formulas involving \( \Box \varphi \) and use them to represent and prove theorems. Thus, in an SSE, the target logic is internally represented using higher-order (HO) constructs in a theorem proving system such as Isabelle/HOL. Benzmüller and Paulson [2013] developed an SSE that captures quantified extensions of modal logic. For example, if \( \forall x.\, \varphi x \) is short-hand in HOL for \( \Pi(\lambda x.\, \varphi x) \), then \( \Box \forall x.\, \varphi x \) would be represented as \( \Box \Pi'(\lambda x.\, \varphi x, P x w) \), where \( \Pi' \) stands for the \( \lambda \)-term \( \lambda \Phi.\, \lambda w.\, \Pi(\lambda x.\, \Phi x w) \), and the \( \Box \) gets resolved as above.

To see how these expressions can be resolved to produce the right representation, consider the following series of reductions:

\[
\Box \forall x. P x
\]

\[
\equiv \Box \Pi'(\lambda x.\, \lambda w.\, P x w)
\]

\[
\equiv \Box(\lambda \varphi.\, \lambda \lambda w.\, (\lambda x.\, \lambda w.\, P x w))
\]

\[
\equiv \Box(\lambda \lambda w.\, \Pi(\lambda x.\, \lambda w.\, P x w))
\]

\[
\equiv \Box(\lambda \lambda w.\, \lambda v.\, (R w v \rightarrow \varphi v))\lambda w.\, \Pi(\lambda x.\, P x w)
\]

\[
\equiv (\lambda \lambda w.\, \Pi(\lambda \lambda w.\, (R w v \rightarrow \varphi v))\lambda w.\, \Pi(\lambda x.\, P x w))
\]

\[
\equiv (\lambda w.\, \Pi(\lambda w.\, (R w v \rightarrow \varphi v))\lambda w.\, \Pi(\lambda x.\, P x w))
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\equiv (\lambda w.\, \Pi(\lambda w.\, (R w v \rightarrow \varphi v))\lambda w.\, \Pi(\lambda x.\, P x w))
\]

\[
\equiv (\lambda w.\, \Pi(\lambda w.\, (R w v \rightarrow \varphi v))\lambda w.\, \Pi(\lambda x.\, P x w))
\]

Thus, we end up with a representation of \( \Box \forall x. P x \) in HOL. Of course, types are assigned to each (sub-)term of the HOL language. We assign individual terms (such as variable \( x \) above) the type \( e \), and terms denoting worlds (such as variable \( w \) above) the type \( e \). From such base choices, all other types in the above presentation can actually be inferred.

An explicit encoding of HOML in Isabelle/HOL, following the above ideas, is presented in Fig. 1.\(^4\) In lines 4–5 the base types \( i \) and \( e \) are declared. Note that HOL [Andrews, 2002] comes with an inbuilt base type bool, the bivalent type of Booleans. No cardinality constraints are associated with types \( i \) and \( e \), except that they must be non-empty. To keep our presentation concise, useful type synonyms are introduced in lines 6–9. \( \sigma \) abbreviates the type \( i \Rightarrow e \), and terms of type \( \sigma \) can be seen to represent world-lifted propositions, i.e., truth-sets in Kripke’s modal relational semantics [Garson, 2018]. The explicit transition from modal propositions

\(^3\)B’s counterpart \( \varphi \rightarrow \Box \diamond \varphi \) is implied and could be used instead.

\(^4\)In Isabelle/HOL explicit type information can often be omitted due the system’s internal type inference mechanism. This feature is exploited in our formalization to improve readability. However, for all new abbreviations and definitions, we usually explicitly declare the types of the freshly introduced symbols. This supports a better intuitive understanding, and it also reduces the number of polymorphic terms in the formalization (heavy use of polymorphism may generally lead to decreased proof automation performance).
Further modal logic connectives, such as \( \Box \) and \( \Diamond \), are introduced in line 27. In line 28, user-friendly binder-notation for variables is additionally defined. Instead of distinguishing between \( \forall \) and \( \exists \) as in our illustrating example, these symbols are overloaded here. The introduction of the possibilist \( \exists \)-quantifier in lines 29–30 is analogous.

Further actualist quantifiers, \( \forall F \) and \( \exists E \), are introduced in lines 33–37; their definition is guarded by an explicit, possibly empty, \( \text{existsAt} \) (infix \( @ \)) predicate, which encodes whether an individual object actually “exists” at a particular possible world, or not. These additional actualist quantifiers are declared non-polymorphic, and they support quantification over individuals only. In the subsequent study of the ontological argument we will indeed apply \( \forall \text{E} \) and \( \exists \text{E} \) for different types in the type hierarchy of HOL, while we employ \( \forall F \) and \( \exists F \) for individuals only.

Global validity of a world-lifted formula \( \psi \), denoted as \( \{ \psi \} \), is introduced in line 40 as an abbreviation for \( \forall \text{E} \psi / \psi \).

Consistency of the introduced concepts is confirmed by the model finder nitpick [Blanchette and Nipkow, 2010] in line 43. Since only abbreviations and no axioms have been introduced so far, the consistency of the Isabelle/HOL theory HOML as displayed in Fig. 1 is actually evident.

In line 44–47 its is studied whether instances of the Barcan formulae are implied. As expected, both principles are valid only for possibilist quantification, while they have counter models for actualist quantification.

**Theorem 1.** The SSE of HOML in HOL is faithful for base modal logic \( K \).

**Proof.** Follows [Benzmüller and Paulson, 2013].

Theory HOML thus successfully models base modal logic \( K \) in HOL. To arrive at logic \( KB \) the symmetry axiom \( B \) as shown earlier can be postulated.

### 3 Modal Ultrafilter

Theorem ModalUltrafilter, see Fig. 2, imports theory HOML and adapts the topological notions of filter and ultrafilter to our modal logic setting. For an introduction to filter and ultrafilter see the literature, e.g., [Burris and Sankappanavar, 1981].

Modal ultrafilter are introduced in lines 18–19 as world-lifted characteristic functions of type \( (\gamma \Rightarrow \sigma) \Rightarrow \sigma \). A modal ultrafilter is thus a world-dependent set of intensions of \( \gamma \)-type properties; in other words, a \( \sigma \)-subset of the \( \sigma \)-powerset of \( \gamma \)-type property extensions. An ultrafilter \( \mathcal{F} \) is defined as a filter satisfying an additional maximality condition: \( \forall \mathcal{F} \in \phi \vee \neg \neg \phi \in \mathcal{F} \) where \( \phi \in K \) is elementhood of \( \gamma \)-type objects in \( \sigma \)-sets of \( \gamma \)-type objects (see line 44), and where \( \neg \neg \) is the relative set complement operation on sets of entities (line 9).

A Filter \( \mathcal{F} \), see lines 12–15, is required to
1. be large: $U \in \phi$, where $U$ denotes the full set of $\gamma$-type objects we start with,
2. exclude the empty set: $\emptyset \not\in \phi$, where $\emptyset$ is the world-
   lifted empty set of $\gamma$-type objects,
3. be closed under supersets: $\forall \phi \psi. (\phi \in \phi \land \phi \subseteq \psi)$ \rightarrow $\psi \in \phi$ (world-lifted $\subseteq$-relation is defined in line 7), and
4. be closed under intersections: $\forall \phi \psi. (\phi \in \phi \land \psi \in \phi)$ \rightarrow $($*$\phi \cap \psi*)$ $\subseteq \phi$ (where $\cap$ is defined in line 8).

Benzm"uller and Fuenmayor [2020] have studied two dif-
ferent notions of modal ultrafilter (called $\gamma$- and $\delta$-
 ultrafilter) which are defined on intensions and extensions of properties,
respectively. This distinction is not needed in this paper; what
we call modal ultrafilter here corresponds to their
$\gamma$-ultrafilter.

4 Gödel’s Modal Ontological Argument

The full formalization of Scott’s variant of Gödel’s argument,
which relies on theories ModalUltrafilter and HOML,
has been presented in Fig. 3. Line 3 starts out with the declaration of
the uninterpreted constant symbol $P$, for positive properties,
which is of type $\gamma \Rightarrow \sigma$. $P$ is thus an intensional, world-
depended concept.

The premises of Gödel’s argument, as already discussed
earlier, are stated in lines 5–24.5

An abstract level “proof net” for theorem T6, the neces-
sary existence of a Godlike entity, is presented in lines 26–
34. Following the literature the proof goes as follows: From
A1 and A2 infer T1: positive properties are possibly exempli-
fied. From A3 and the defn. of $G$ obtain T2: being Godlike
is a positive property (Scott actually directly postulated
T2).

Using T1 and T2 show T3: possibly a Godlike entity exists.
Next, use A1, A4, the defns. of $G$ and $E$ to infer T4: being
Godlike is an essential property of any Godlike entity.

Next, use A5, B and the defns. of $G$ and $NE$ have
T5: the possible

Remark: whether we use actualist or possibilist quantifiers for
individuals in the definition of $\subseteq$ or $T4$ turned out irrelevant in this
paper, and we consistently use actualist quantifiers in the remainder.

existence of a Godlike entity implies its necessary existence.
$T5$ and $T3$ then imply $T6$.

The six subproofs and their dependencies have been au-
tomatically explored using state-of-the-art ATP system inte-
grated with Isabelle/HOL via its sledgehammer tool; sledge-
hammer then identified and returned the abstract level proof
justifications as displayed here, e.g. “using $T1 \ T2$ by simp”. The
mentioned proof engines/tactics blast, metis, simp and
smt are trustworthy components of Isabelle/HOL’s, since they
internally reconstruct and check each subproofs the proof as-
sistants small and trusted proof kernel. Using the definitions
from Sect. 2, one can also reconstruct and formally verify all proofs with pen and paper.\footnote{Reconstruction of proofs from such proof nets within direct proof calculi for quantified modal logics, cf. Kanckos and Woltzenlogel-Paleo \cite{Kanckos17} or Fitting \cite{Fitting02}, is ongoing work.}

The presented theory is consistent, which is confirmed in line 37 by model finder \textit{nitpick}; \textit{nitpick} reports a model (not shown here) consisting of one world and one Godlike entity.

Validity of modal collapse (MC) is confirmed in lines 40–47; a proof net displaying the proofs main idea is shown.

Most relevant for this paper is that the ATP systems were able to quickly prove that Gödel’s notion of positive properties \( \mathcal{P} \) constitutes a modal ultrafilter, cf. lines 50–57. This was key to the idea of taking the modal ultrafilter property of \( \mathcal{P} \) as an axiom U1; see the next section.

5 Ultrafilter Modal Ontological Argument

Taking U1 as an axiom for Gödel’s theory in fact leads to a significant simplification of the modal ontological argument; this is shown in lines 16–28 in Fig. 4: not only Gödel’s axiom A1 can be dropped, but also axioms A4 and A5, together with defns. \( E \) and \( \Lambda_{E} \). Even logic \( \mathbf{KB} \) can be given up, since \( \mathbf{K} \) is now sufficient for verifying the proof argument.

The proof is similar to before: Use U1 and A2 to infer T1 (positive properties are possibly exemplified). From A3 and defn. of \( \mathcal{G} \) have T2 (being Godlike is a positive property). T1 and T2 imply T3 (a Godlike entity possibly exists). From U1, A2, T2 and the defn. of \( \mathcal{G} \) have T5 (possible existence of a Godlike entity implies its necessary existence). Use T5 and T3 to conclude T6 (necessary existence of a Godlike entity).

Consistency of the theory is confirmed in line 31; again a model with one world and one Godlike entity is reported.

Most interestingly, modal collapse MC now has a simple counter model as \textit{nitpick} informs us. This counter model consists of a single entity \( e_{1} \) and two worlds \( i_{1} \) and \( i_{2} \) with \( r = \{ (i_{1}, i_{1}), (i_{2}, i_{1}), (i_{2}, i_{2}) \} \) Trivially, formula \( \Phi \) is such that \( \Phi \) holds in \( i_{2} \) but not in \( i_{1} \), invalidates MC at world \( i_{2} \).

\( e_{1} \) is the Godlike entity in both worlds, i.e., \( \mathcal{G} \) is the property that holds for \( e_{1} \) in \( i_{1} \) and \( i_{2} \), which we may denote as \( \lambda e. \lambda w. e = e_{1} \land (w = i_{1} \lor w = i_{2}) \). Using tuple notation we may write \( \mathcal{G} = \{ (1, i_{1}), (i_{2}, i_{2}) \} \).

Remember that \( \mathcal{P} \), which is of type \( \gamma \rightarrow \sigma \), is an intensional, world-dependent concept. In our counter model for MC the extension of \( \mathcal{P} \) for world \( i_{1} \) has the above \( \mathcal{G} \) and \( \lambda e. \lambda w. e = e_{1} \land w = i_{1} \) as its elements, while in world \( i_{2} \) we have \( \mathcal{G} \) and \( \lambda e. \lambda w. e = e_{1} \land w = i_{2} \). Using tuple notation we note \( \mathcal{P} \) as

\[
\{ (\{ (e_{1}, i_{1}), (e_{1}, i_{2}) \}, i_{1}), (\{ (e_{1}, i_{1}), (e_{1}, i_{2}) \}, i_{2}) \}
\]

In order to verify that \( \mathcal{P} \) is a modal ultrafilter we have to verify that the respective modal ultrafilter conditions are satisfied in both worlds. \( U \in \mathcal{P} \) in \( i_{1} \) and also in \( i_{2} \), since both \( \{ (e_{1}, i_{1}), (e_{1}, i_{2}) \} \) and \( \{ (e_{1}, i_{1}), (e_{1}, i_{2}) \} \) are in \( \mathcal{P} \); \( \emptyset \notin \mathcal{P} \) in \( i_{1} \) and also in \( i_{2} \), since both \( \{ (1, i_{1}) \} \) and \( \{ (i_{2}, i_{2}) \} \) are not in \( \mathcal{P} \). Is also easy to verify that \( \mathcal{P} \) is closed under supersets and intersection in both worlds.

\footnote{Note that in this counter model for MC, also Gödel’s axiom A4 is invalidated. Consider \( X = \lambda e. \lambda w. e = e_{1} \land w = i_{2} \), i.e., \( X \) is true for \( e_{1} \) in \( i_{1} \), but false for \( e_{1} \) in \( i_{1} \). We have \( \mathcal{P} X \in i_{2} \), but we do not have \( \Box \mathcal{P} X \) in \( i_{1} \), since \( \mathcal{P} X \) does not hold in \( i_{1} \), which is reachable from \( i_{2} \). \textit{Nitpick} is e.g. capable of computing and displaying all partial modal ultrafilters in this counter model: out of 512 candidates, \textit{nitpick} identifies 32 structures of form \( \langle \mathcal{F}, i \rangle \), for \( i \in \{ 1, 2 \} \), in which \( \mathcal{F} \) satisfies the ultrafilter conditions in the specified world \( i \). An example for such an \( \langle \mathcal{F}, i \rangle \) is

\[
\{ \{ (e_{1}, i_{1}), (e_{1}, i_{2}) \}, \{ (e_{1}, i_{1}), (e_{1}, i_{2}) \}, \{ (e_{1}, i_{1}), (e_{1}, i_{2}) \}, \{ (e_{1}, i_{2}), (i_{2}) \} \}
\]

\( F \) is not a proper modal ultrafilter, since \( F \) fails to be an ultrafilter in world \( i_{1} \).}

6 Simplified Modal Ontological Argument

What modal ultrafilters properties of \( \mathcal{P} \) are actually needed to support T6? Which ones can be dropped? Experiments with the computer confirm that, in modal logic \( \mathbf{K} \), the ultrafilter conditions 1–3 from Sect. 3 must be upheld for \( \mathcal{P} \), while 4 can be dropped. However, it is possible to merge condition 3 (closed under supersets) for \( \mathcal{P} \) with Gödel’s A2 into A2’ as shown in line 18 of Fig. 5. Moreover, instead of requiring the universal set/property \( U = \lambda e. \bot \) to be a positive property, we postulate that self-identity \( \lambda x. x = x \) is which extension-
Figure 5: Simplified variant of Gödel’s ontological argument.

7 Related Work

Fitting [2002] has suggested to carefully distinguish between intensions and extensions of positive properties in the context of Gödel’s modal ontological argument, and, in order to do so within a single framework, he introduces a sufficiently expressive HOLM enhanced with means for the explicit representation of intensional terms and their extensions; see also the intensional operations formalized by Fuenmayor and Benzmüller [2017; 2020] in the course of their analysis and verification of the variants of Fitting and Anderson [1990; 1996] contributions.

The application of computational methods to philosophical problems was initially limited to first-order theorem provers. Fitelson and Zalta [2007] used PROVER9 to find a proof of the theorems about situation and world theory in [Zalta, 1993] and they found an error in a theorem about Plato’s Forms that was left as an exercise in [Pelletier and Zalta, 2000]. Oppenheimer and Zalta [2011] discovered, using Prover9, that 1 of the 3 premises used in their reconstruction of Anselm’s ontological argument (in [Oppenheimer and Zalta, 1991]) was sufficient to derive the conclusion. The first-order conversion techniques that were developed and applied in these works are outlined in some detail in [Alama et al., 2015].

More recent related work makes use of higher-order proof assistants. Besides some already given references to the work of Benzmüller and colleagues, this includes John Rushby’s [2018] study on the Anselm’s ontological argument in the PVS proof assistant and Blumson’s [2017] related study in Isabelle/HOL.
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