A method for solving the open-pit mine production scheduling problem using the constraint programming paradigm

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Abstract. A method is developed to solve the problem of open-pit mine production scheduling, which is based on Constraint Programming and is used in working out an optimal plan of mining operations, taking into account heterogeneous economical, technological and some others requirements imposed by the customer. An original model is proposed to represent the problem considered as a constraint satisfaction problem. As the customer may verify the requirements, the domain model should enable addition of the new constraints and/or eliminate the existing ones. With the domain model being developed, the method proposed makes it possible to reach accurate solutions at the expense of highly effective reduction algorithms for each type of constraints and at the expense of specialized heuristics for non-perspective alternatives reduction in a search tree. The approach proposed enables addition of different constraints, even those for which it is difficult to find an analytical expression.

1. Introduction

Open-pit mining is a surface mining technique of extracting minerals from the earth by their removal from an open-air pit. In ore body modeling, it is usually represented in a discrete form as the set of blocks. Over the last thirty years various approaches were proposed for its solution, such as classical linear and mathematical programming [1–6], as well as metaheuristic programming, particularly, genetic algorithms [7–9].

The most developed approach to the solution of the open pit mine production scheduling problem is the approach based on the methods of integer linear programming [1, 4]. One of its advantages is that it enables effective searching for the global optimum for the open pit mine production scheduling problem. If metaheuristic programming is used, one just has to settle for searching the local optimum. The principal disadvantage of integer linear programming is that all the constraints must be presented in a form of linear equations and inequalities. Unfortunately, in solving practical problems, this disadvantage is an absolute obstacle in applying this approach. The choice of a mining technology depends on a type of the deposit and on the useful mineral to be recovered, as well as on the heterogeneous economical, technological and other requirements, which, according to the customer’s plan, should be taken into consideration. In so doing, the customer requirements may be updated; therefore, the model of the subject domain is a constantly evolving model and should be open for addition of new constraints or for removing the existing ones. Under these conditions it is not easy to apply classical methods of the Operations Research theory.
In the course of these studies a new method of intelligent search of the exact solutions of the open pit mine production scheduling problem was developed. The method is demonstrated on the example of the problem concerning the search of the intermediate pit wall position, taking into account a priori specified performance on useful mineral (UM) and on overburden rocks (OR). To reach the exact solution of the open pit mine production scheduling problem, the present studies propose the Constraint Programming technology [10–12]. The idea to apply the Constraint Programming technology in solving the problems of the class considered was first recognized in [13], but, to the authors’ opinion, it has not been further developed. The constraint programming system architecture enables the co-processing of the heterogeneous constraints with new types of constraints being smoothly introduced, if necessary. In doing so, a general algorithm of the solution search is not changed: it is necessary only to formulate specialized methods-propagators for new types of constraints, which implement inference on the constraints of this type.

2. The problem formulation
The initial (at the start of scheduling) and the final pit wall position are the bounds of the geological environment. The initial data of the problem is a model of a geological environment presented as a set of blocks of the same size (Fig. 1). Each block has specified parameters such as size, position in space, the useful mineral (UM) percentage in a block, and the block value, i.e. the difference between the cost of operations on UM-recovery from block material (block mining and block material processing) and the benefit of the UM realization.

![Figure 1. An open-pit block model](image)

The formulation of the scheduling problem to determine the position of the pit walls at each stage of mining so that the amounts of useful mineral (UM) and overburden rock (OR) between the sequential positions would correspond to the specified amounts accurate to the admissible error. In so doing, the profit (the overall block value) from pit mining should be maximal. The pit mining, i.e. the pit deepening and extension, is carried out observing both all the technical (the width of the catch bench) and the technological constraints on what blocks should be mined before the specified block is mined. The initial value of each block changes, depending on the period of its mining. Let the final position of the open pit wall be referred to as an ultimate pit and the current boundary of excavation - as a working pit or simply a pit.

3. Constraint programming technology
The Constraint Programming (CP) technology known at present as a powerful tool is used in solving the problems of combinatorial search and combinatorial optimization. For the technology to be used, any problem should be formulated as a Constraint Satisfaction Problem (CSP) consisting of three components [15]: \(<X, D, C>\). \(X\) – is the set of variables \(\{X_1, X_2, ..., X_n\}\). \(D\) – is the set of domains \(\{D_1, ..., D_n\}\), where \(D_i\) is the domain of variable \(X_i\). \(C\) – is the set of constraints \(\{C_1, ..., C_m\}\), which prescribe legal combinations of variable values. Each domain \(D_i\) describes the set of legal values \(v_1, ..., v_k\) for variable \(X_i\).
Constraints may be arithmetic expressions; logical formulae; tables; expressions formulated by the language of specialized theories. The solution of the CSP is a complete assignment which satisfies all the constraints. Sometimes it is necessary to reach such a solution in which the values of the variables would optimize a functional which was specified.

The constraint programming technology presents powerful and flexible techniques, algorithms to solve the combinatorial search problems. Any constraint satisfaction algorithm should contain two obligatory components: a) a component implementing inference (propagation, filtration); b) a component implementing search. Inference (propagation) is implemented as a reduction of the variable domains initially specified.

In real problems, the amount of constraints is great, and any negligible change of domain in any constraint at the stage of propagation induces a cascade of a great number of propagation algorithms for other constraints. In so doing, one can observe a tendency: the simpler the constraint, the less possibilities the propagation algorithm has to reduce the domains of variables which participate in the constraint. Thus, it is expedient to unite the set of similar constraints into one constraint and develop effective algorithms of propagation for the given set. Such composite constraints are referred to as global constraints. In [11], a global constraint is interpreted as a constraint describing the relation on an unlimited a priori amount of variables.

In the case of the problem presented in the study, use is made of global constraints of two basic types: a sort constraint and a bin-packing constraint. Consider these in detail.

Constraint sort [16]. According to this constraint, the values of the variables \( X_1, \ldots, X_n \) must be sorted by the rule: \( X_1 \leq X_2 \ldots \leq X_n \). Applied to the problem studied, this constraint is used to describe the technological restrictions on the order of block mining.

Constraint bin-packing [17]. The bin-packing problem is defined as follows: there is \( n \) objects and \( m \) bins, with each of the objects having its own volume and each of the bins having its own capacity. The aim is to distribute all the objects into the bins so that the capacity of each of the bins is not exceeded. The constraint is defined by:

- array of variables \( X_1, \ldots, X_n \), each having domain \([1, \ldots, m]\) and means that object numbered \( i \) is to be placed into bin numbered \( X_i \);
- array of volumes \( \text{vol}_1, \ldots, \text{vol}_n \) which states that object numbered \( i \) is of volume \( \text{vol}_i \);
- array of capacities \( \text{cap}_1, \ldots, \text{cap}_m \) which states that bin numbered \( j \) is of capacity \( \text{cap}_j \).

In solving the problem, this constraint is used to define the requirements on the UM and OR output during the period of mining.

4. Problem modelling in the constraint programming paradigm

In the constraint programming paradigm, the problem of open pit mine production scheduling is formulated as follows: each block is matched to integer variable \( X_{i,j,k} \) that can take the values from 1 to \( N \), where \( N \) is the last possible mining period of operations. At present the solution takes into account the constraints on a) the order of block material mining; b) the UM- and OR-production specified. The objective function is to maximize profits from the open-pit mining. However, the method proposed has no principal limits on processing the constraints of other types. The solution of the problem is to assign the integer values to all variables \( X_{i,j,k} \) so that to simultaneously satisfy all the constraints listed above to maximize objective function.

Consider the semantics of the constraints in detail. The constraint on the order of block material mining ensures sequential pit mining, excluding the variants of block mining that are technologically impossible. The principles of open pit deepening and are determined by the rules of block mining.

As an example, consider the simplest constraint: in order to mine one block in the current mining period, it is necessary for the five blocks on top to be mined this very mining period, or earlier. Figure 2 shows 6 blocks illustrating this constraint.

The constraint is formulated as follows: to mine block \( X_{i,j,k} \), where \( i, j, k \) – are the block coordinates, it is necessary to dig out blocks \( X_{i,j,k+1}, X_{i-1,j,k+1}, X_{i-1,j,k+1}, X_{i,j-1,k+1}, X_{i,j-1,k+1}, X_{i,j-1,k+1} \), to put it another way, it is necessary to mine five blocks located on top, if there are any.
This constraint can be specified by five inequalities:
$$X_{i,j,k} \geq X_{i,j,k+1}, \quad X_{i,j,k} \geq X_{i-1,j,k+1}, \quad X_{i,j,k} \geq X_{i+1,j,k+1}, \quad X_{i,j,k} \geq X_{j-1,k,k+1}, \quad X_{i,j,k} \geq X_{j+1,k,k+1}.$$  

Now consider an alternative way to describe this constraint, which is aimed at use of global constraint sort.

Assume that an imaginary strait line goes through block $X_{i,j,k}$ and through each of blocks $X_{i,j,k+1}$, $X_{i-1,j,k+1}$, $X_{i+1,j,k+1}$, $X_{j-1,k,k+1}$, $X_{j+1,k,k+1}$ (Figure 3). In fact, each of five lines specifies an array of variables that are consistent with the pit blocks through which this strait line goes.

One can form one constraint sort for each of five arrays mentioned above, which prescribes that, as the variables of index $k$ decreases, their values form a monotonous non-decreasing sequence.

![Figure 2. A block mining scheme.](image1)

![Figure 3. An alternative way to represent the block mining scheme.](image2)

Constraints on the useful mineral and overburden rock production specified are modeled by linear equations.

Let $o_{i,j,k}$ – is a constant showing the ore content in a block with indices $i$, $j$, $k$; $w_{i,j,k}$ – is a constant showing the overburden rock content in a block with indices $i$, $j$, $k$. Let $O$ – is the useful mineral production specified, $\Delta O$ is a legal error $O$, $W$ – is the overburden rock production specified, $\Delta W$ – is a legal error $W$. Then constraints on the useful mineral and overburden rock production specified can be expressed as inequalities:

$$O - \Delta O \leq \sum_{i,j,k} o_{i,j,k} X_{i,j,k} = y \leq O + \Delta O, \text{ for each } y \in [1...N];$$

$$W - \Delta W \leq \sum_{i,j,k} w_{i,j,k} X_{i,j,k} = y \leq W + \Delta W, \text{ for each } y \in [1...N].$$

These inequalities can be easily transformed into bin-packing problems for $N$ beans of the corresponding possible capacity. For instance, the constraint on that, per each mining period, the UM production volume must be in the given interval, is reduced to constraint bin-packing. Array $X_1$, ..., $X_n$, consists of variables, each of which corresponds to a block of the pit containing UM (where $n$ is the number of such blocks). The domain of these variables is $[1, ..., m]$, where $m$ is the number of the last period. The period of mining can be interpreted as ‘bin’, then array $vol_1$, ..., $vol_n$ contains the information about the UM volume in each of the corresponding blocks, array $cap_1$, ..., $cap_m$ contains the variables with domains $[O - \Delta O; O + \Delta O]$.

Consider objective function (function of maximization of the benefit from the pit mining):

To describe the dependence of the block value on the period of mining, introduce a function:

$$Fr(X_{i,j,k}) = V_{i,j,k}(0.9^{x_{ijk} - 1}).$$

where $V_{i,j,k}$ – initial value of the $X_{i,j,k}$ block. Then the optimization function will be

$$\sum_{i,j,k} Fr(X_{i,j,k}) \rightarrow \text{max.}$$

According to the constraint, each new solution reached should result in a value of the optimization function, which is greater than that of the previous solutions.

Finally, consider the search strategy and the basic heuristics applied in reaching the solution of the open pit mine production scheduling problem which is taken as a CSP. A backtracking search is used as a search algorithm. Before the search procedure is started, variables are ordered: the list starts with variables consistent with ore blocks, then variables consistent only with overburden rock blocks, follow. Other things being equal, a choice is made of a variable with the smallest size of domain (the smallest domain). Making a choice of a variable, we follow the rule: choose the smallest value in a domain.
5. Analysis of non-standard user constraints

Using the approach proposed, it is possible to add into a model a great variety of different constraints, even those for which it is difficult to find a suitable analytical expression (formula). Consider, as an example, the constraint the idea of which is that the only one deepening is permitted to be present in the open pit. To say it simpler, take a two-dimensional pit. Figure 4 shows an example of the pit configuration with several local deepenings.

Figure 4. The pit configuration with several local deepenings.

All the pit blocks are in this or that horizontal level. Each horizontal layer can be presented as a one-dimension array of variables of the CSP. When solving the CSP, a time stamp is assigned to each variable, showing in what period of pit mining the corresponding block was mined. In accordance with the constraint considered, the time stamps arrangement within each one-dimensional array should meet the following requirements: all the values of the one-dimensional array, which are before the minimal value, should be a nonincreasing sequence, and those which are after the minimal value, should be a nondecreasing sequence. In so doing, the situation is assumed when the minimal value is at the beginning and/or at the end of the array. Figure 5 shows the examples of correct (left) and incorrect (right) arrangement of the time stamps for the horizontal level of the pit.

Figure 5. Correct (left) and incorrect (right) arrangement of time stamps for a horizontal layer of the pit.

As standard procedures-propagators implementing logic required have not been found, the authors have developed an original method linepropagator. For the pit’s bottom to be of the required shape, it is necessary to specify constraint linepropagator for all the horizontal pit layers.

Let’s give a brief description of the work of linepropagator, which is as follows. At the first stage, the variables placed in sequence are found, the values of which have not been determined yet. These sequences of variables form specific “intervals of uncertaiy”. In addition, variables with assigned values should be present before and after each of the intervals. The domains of variables belonging to the “interval of uncertainty” are reduced so that these variables could not take the values greater than the maximal one of the assigned variables being at the ends of this interval (see Figure 6).

Figure 6. The first stage of the work of propagator.

At the second stage, the variables with assigned values are tested on the consistence with the requirement of ordering.

Thus, not only can the constraint being of interest for the user, be expressed analytically but it also can be expressed by the description of the work of the corresponding procedure-propagator, which is convenient if it is necessary to take into account and analyze non-standard user constraints.
6. Conclusion
Thus, the proposed approach based on the constraint programming paradigm has demonstrated its applicability in solving the open pit mine production scheduling problem being of great practical significance and characterized by high dimensionality of search space. Further research is to be aimed at the development of hybrid methods, which integrate integer programming methods and constraint programming methods to reach exact solutions of the given class of problems of much greater dimensionality.

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