Asymptotic freedom in strong magnetic field

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Abstract

Perturbative gluon exchange interaction between quark and antiquark, or in a 3q system, is enhanced in magnetic field and may cause vanishing of the total $q\bar{q}$ or 3q mass, and even unlimited decrease of it – recently called the magnetic collapse of QCD. The analysis of the one-loop correction below shows a considerable softening of this phenomenon due to $q\bar{q}$ loop contribution, similarly to the Coulomb case of QED, leading to approximately logarithmic dumping of gluon exchange interaction ($\langle V \rangle \approx \mathcal{O}(1/\ln|eB|)$) at large magnetic field.

1

Analysis of the hydrogen atom or positronium in strong magnetic field shows a considerable enhancement of the Coulomb interaction, leading to the increase of binding energy [1,2,3,4]. This fact is due to reduction of the system size in the plane perpendicular to the direction of the magnetic field (MF) $B$, making it closer to the one-dimensional Coulomb system. As was shown in [5,2], the binding energy in the leading order in $\alpha$ grows as $\ln^2 \left( \frac{B}{m_e} \right)$. It was shown later, that the one-loop corrections to the one-photon exchange seriously change the situation: in the hydrogen atom the binding energy tends to the finite limit [6,7], while it shows an unbounded growth in positronium [8]. One should note that the absolute value of binding energy in both cases is not large and the upper limit of binding energy in hydrogen atom is 1.74 keV [7,9] while in positronium the collapse (vanishing) of the total mass occurs at very strong fields: $B_{cr} \sim 10^{40}$ Gauss [8].
Recently the dynamics of $q\bar{q}$ system in strong magnetic field was studied in the framework of the relativistic Hamiltonian, derived from the path integral for the corresponding Green’s function [10]. The relevant technic in the case of no MF was extensively developed in [11]. It was shown, that the one-gluon-exchange (OGE) interaction, or color Coulomb, becomes increasingly important for large MF, when OGE is taken in the leading (no quark loop) approximation. In particular, the mass of the $(q\bar{q})$ meson vanishes at $\sqrt{|e_qB|} \sim O(1 \text{ GeV})$, i.e. for $B \approx 10^{19} - 10^{20} \text{ Gauss}$. This fact would imply a radical reconstruction of the vacuum, a proposal made in a different context in [12, 13].

A similar situation occurs in the case of baryons in strong MF: the baryon (e.g. the neutron) mass vanishes at approximately the same $B_{\text{crit}}$, as for mesons [13].

It is therefore very important to check whether the quark loop corrections may stabilize the hadron mass at high MF, similarly to the case of the hydrogen atom. As for gluon loop corrections, ensuring asymptotic freedom (AF), they are neutral to MF, and AF only decreases the growth of binding energy (b.e.) [10] (b.e. grows as $\ln \ln \frac{eB}{\sigma}$ instead of $\ln^2 eB$ in atoms), but does not prevent the collapse. But those are fermion loop contributions which stabilized hydrogen atom, and we shall below study the quark-antiquark loops in the case of the $q\bar{q}$ mesons, taking into account both confinement and OGE interaction.

2

We start with the standard one-loop expression for the gluon self-energy part, which contributes to the gluon propagator as [15]

$$D(q) = \frac{4\pi}{q^2 - \frac{g^2(\mu_0^2)}{16\pi^2}\tilde{\Pi}(q)}$$

where $\tilde{\Pi}(q)$ contains the sum of gluon and quark loop terms,

$$\tilde{\Pi}(q) = q^2\Pi_{gl}(q) - \Pi_{q\bar{q}}(q).$$

In absence of MF and neglecting strong interaction between gluons, one has

$$\Pi_{gl}(q) = -\frac{11}{3}N_c\ln \frac{|q^2|}{\mu_0^2}, \quad \Pi_{q\bar{q}}(q) = -\frac{2}{3}n_f q^2 \ln \frac{|q^2|}{\mu_0^2},$$

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leading to the standard AF expression for the OGE potential \( (q^2 = -Q^2 = -(q_1^2 + q_3^2), \quad \alpha_s^{(0)} = \frac{g^2(\mu_0^2)}{4\pi}) \)

\[
V(Q) = -\frac{4}{3} \frac{\alpha_s^{(0)} 4\pi}{Q^2 \left(1 + \frac{\alpha_s^{(0)} \beta_0}{4\pi \ln \frac{Q^2}{\mu_0^2}}\right)} = -\frac{16\pi}{3Q^2} \alpha_s(Q), \quad \alpha_s(Q) = \frac{4\pi}{\beta_0 \ln \frac{Q^2}{\Lambda^2}},
\]  

where \( \beta_0 = \frac{11}{3}N_c - \frac{2}{3}n_f. \)

In the case of strong MF one can retain in \( \tilde{\Pi}_{q\bar{q}}(q) \) the contribution of the lowest Landau levels (LLL), which couples only to \((q_0, 0, 0, q_3)\) polarizations and obtain the expression, known for a long time [16] for the \((e^+e^-)\) case, which is rewritten in our case by the replacement \( \alpha_{QED} \rightarrow \alpha_s^{(0)} n_f \).

\[
\frac{\alpha_s^{(0)}}{4\pi} \tilde{\pi}_{q\bar{q}}(q) = -\frac{\alpha_s^{(0)} n_f |e_qB|}{\pi} \exp \left(-\frac{q_1^2}{2|e_qB|}\right) T \left(\frac{q_3^2}{4m^2}\right),
\]  

where

\[
T(z) = -\ln \left(\frac{\sqrt{1+z} + \sqrt{z}}{\sqrt{z(1+z)}}\right) + 1 = \begin{cases} \frac{2}{3}z, & z \ll 1 \\ 1, & z \gg 1 \end{cases}.
\]

A convenient approximation with accuracy better than 10% is \( T(z) = \frac{2z}{3+2z} [7]. \)

At this point one should define the mass parameter \( m \), which in the case of QED was (renormalized) electron mass [6, 7, 8]. In our case the gluon and quark loop contributions correspond to the graphs in Fig. 1, where we have denoted gluon line as a double quark line to make clear the gauge interacting regions, and the confining regions are cross-hatched. One can see in Fig. 1,
that $q$ and $\bar{q}$ in the quark loop are not interacting by simple gluon exchange similarly to the $e^+e^-$ loop in the lowest order, but in the $q\bar{q}$ case only the exchange of white objects (mesons or glueballs) can take place in higher orders.

Moreover, quarks are moving on the borders of the confining surfaces and hence should have the typical energies of quarks at the ends of the string – they are denoted as $\omega = \sqrt{\frac{p_{q}^2 + m_q^2}{\mu_0^2}}$ in the path-integral Hamiltonian \[10\] \[11\] and are of the order of $\sqrt{\sigma}$, $\sigma$ is string tension, $\sigma = 0.18$ GeV$^2$. Thus one can replace $4m^2$ in (5) by $4\sigma$.

Finally, one should take into account the nonperturbative (confining) interaction inside the gluon loops, as shown in Fig 1. As shown in \[17\] this amounts to the replacement $\ln \frac{Q^2}{\mu_0^2} \rightarrow \ln \frac{Q^2 + M_B^2}{\mu_0^2}$, where $M_B \approx 1$ GeV and is expressed solely through $\sigma$. As a result one obtains the following form of the OGE interaction with account of gluon and quark loop effects

$$V(Q) = -\frac{16\pi\alpha_s^{(0)}}{3} Q^2 \left( 1 + \frac{\alpha_s^{(0)}}{4\pi} \frac{11}{3} N_c \ln \frac{Q^2 + M_B^2}{\mu_0^2} \right) + \frac{\alpha_s^{(0)}|e_q B|}{\pi} \exp \left( -\frac{q^2_{\perp}}{2|e_q B|} \right) T \left( \frac{q^2_3}{4\sigma} \right),$$

where $\alpha_s^{(0)} = \frac{4\pi}{\frac{11}{3} N_c \ln \frac{\mu_0^2 + M_B^2}{\Lambda^2}}$, and $Q^2 = q^2_{\perp} + q^2_3$.

3

We can now estimate the average value of $V(Q)$ in the meson state with the wave function, which takes into account magnetic field and confinement, $V_{\text{conf}} = \sigma \eta$. The latter is convenient to replace by the quadratic form $V_{\text{conf}} \rightarrow \tilde{V}_{\text{conf}} = \frac{\sigma}{2} \left( \frac{\eta^2_{\perp}}{\gamma} + \gamma \right)$, with $\gamma$ to be found from the stationary point condition, $\frac{\partial M_{\text{mes}}}{\partial \gamma} \bigg|_{\gamma=\gamma_0} = 0$. This replacement has accuracy of the order of 5%, which is enough for our purposes. Then the LLL wave functions can be easily written

$$\psi(\eta_1, \eta_3) = \frac{1}{\sqrt{\pi^{3/2} r_{\perp}^2 r_3}} \exp \left( -\frac{\eta_1^2}{2r_{\perp}^2} - \frac{\eta_3^2}{2r_3^2} \right),$$

where $r_{\perp}$ and $r_3$ are some functions of MF (see \[10\] for details), for large fields $r_{\perp} \approx \sqrt{\frac{2}{e B}}, r_3 \approx \sqrt{\frac{\eta}{\sigma}}$ and we can compute the OGE contribution to
the meson mass $\langle V(Q) \rangle_{mes}$,

$$\langle V(Q) \rangle_{mes} = \int V(Q) \psi^2(q_1, q_3) \frac{d^2q_1 dq_3}{(2\pi)^3},$$

(8)

where $\psi^2(q_1, q_3)$ is the Fourier transform of squared wave function $\psi^2(\eta_1, \eta_3)$. Insertion of (7) and (6) in (8) yields

$$\langle V(Q) \rangle_{mes} = -C \int \frac{e^{-\frac{q_1^2}{4} - \frac{q_3^2}{4}} d^2q_1 dq_3}{Q^2 A(q_1^2 + q_3^2) + B(q_1^2, q_3^2)},$$

(9)

where

$$A = 1 + \frac{\alpha_s^{(0)}}{4\pi} \frac{11}{3} N_c \ln \left( \frac{q_1^2 + q_3^2 + M_B^2}{\mu^2_0} \right),$$

(10)

$$B = \frac{\alpha_s^{(0)} n_f |e_q B|}{\pi} e^{-\frac{q_1^2}{8|e_q B|^2} T} \left( \frac{q_3^2}{4\sigma} \right), \quad C = \frac{16\pi \alpha_s^{(0)}}{3(2\pi)^3}.$$

(11)

Figure 2: Coulomb correction to the meson mass $\langle V(Q) \rangle_{mes}$ in GeV as a function of magnetic field with (solid line) and without (broken line) account of quark loops contributions.
Results of calculations for $\langle V(Q) \rangle_{mes}$ as a function of MF are shown on Fig.2 for asymptotically large fields and on Fig.3 for relatively small fields. The values of parameters $\alpha_s^{(0)}$ and $\mu_0$ are connected by the relation $\alpha_s^{(0)} = \frac{4\pi}{\frac{11}{3} N_c \ln \frac{\mu_0^2 + M^2}{\Lambda_V^2}}$, and we have chosen $n_f = 3$, $\mu_0 = 1.1$ GeV, $\Lambda_V = 0.385$ GeV, so $\alpha_s^{(0)} = 0.42$. As one can see from Fig.2 the account of quark loops contributions leads to the prevention of the so called magnetic collapse of QCD, – the resulting correction vanishes at large MF (roughly as $-\frac{1}{\ln |eB|}$), so the meson mass is always finite.

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