Research Article

Multiple Attribute Group Decision Making Based on Simplified Neutrosophic Integrated Weighted Distance Measure and Entropy Method

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Simplified neutrosophic set (SNS) is a popular tool in modelling potential, imprecise, and uncertain information within complex environments. In this paper, a method based on the integrated weighted distance measure and entropy weight is proposed for handling SNS multiple attribute group decision-making (MAGDM) problems. To this end, the simplified neutrosophic (SN) integrated weighted distance (SNIWD) measure is first developed for overcoming the limitations of the existing methods. Afterward, the proposed SNIWD’s several properties and particular status are studied. Moreover, a flexible and useful MAGDM approach that combines the strengths of the SNIWD and the SNS is proposed, wherein the SN entropy measure is applied to calculate the unknown weight information regarding attributes. Finally, a numerical case of investment evaluation and subsequent comparative analysis are conducted to prove the superiority of the proposed framework.

1. Introduction

The aim of the multiple attribute group decision-making (MAGDM) problem is to determine suitable alternatives with respect to multiple attributes according to the judgement provided by various decision makers. It is impossible for a decision maker to always express an accurate preference because of the increasing uncertainties of the assessed problems. To solve the difficulties, many effective mathematical tools are introduced during the decision process. The fuzzy set (FS) firstly developed by Zadeh [1] is widely used to model imprecise and vague information in MAGDM. An element’s membership value in fuzzy theory lies in the range [0, 1], while the value of its complement is called the nonmembership. To provide a more effective method, the conception of intuitionistic FS (IFS) was proposed by Atanassov [2] which is described by membership and nonmembership functions, and their sum cannot exceed 1. Later, Yager [3] presented the Pythagorean FS (PFS), whose special merit is that the square sum of the membership and nonmembership shall lie in interval [0, 1]. Thus, the PFS is a more powerful tool to describe uncertainties than the IFS and FS. Up to now, the PFS has gained more and more attention and has been widely used in decision making as well as other areas [4–15].

Recently, Smarandache [16] defined the idea of the neutrosophic set (NS) utilizing three parameters: the degrees of truth, indeterminacy, and false for the first time. These three components in the NS are entirely irrelevant from each other, which help people present their preference more flexibly and accurately compared with the previous IFS and PFS. To enhance the computational efficiency of the NS, Wang et al. [17] and Ye [18] put forward the concept of simplified neutrosophic set (SNS). The SNS has gained increasing attention from researchers in these years because of its preponderance in describing uncertainties. For example, Ye [19] extended the TOPSIS method to handle simplified neutrosophic (SN) environments and studied its application in selecting suppliers. Peng et al. [20] introduced an outranking method for SN MAGDM problems. Peng et al. [21]
developed some aggregation methods for SN information. Kucuk and Sahin [22] provided a hybrid method for SN decision-making in which the weight information is unknown. Ye [23] gave a netting approach to cluster SN information based on new associated coefficients. Sahin and Liu [24] developed several SN aggregation operators utilizing the possibility information. Liu and Luo [25] proposed a power aggregation to induce the SNS and explored its usefulness in MAGDM. Ye [26] introduced a generalized ordered weighted SN cosine similarity measure and applied it to solve MAGDM problems. Zeng et al. [27] presented a novel TOPSIS approach for SN decision-making considering the high-efficiency correlation coefficient. Ye [26] introduced a generalized decision-making in which the weight information is unknown. Moreover, it generalizes a wide kind of existing SN distance measures, including the SNOWD and the SNWD measures. We also verify the merits of the proposed SNIWD measure by exploring its application to MAGDM problems, in which the weight information of attributes is unknown.

The reminder of this paper is carried out as follows: Section 2 gives the backgrounds of the SNS and the OWD measure. Section 3 defines the SNIWD measure and studies its main properties and various cases. Section 4 constructs a MAGDM model based on the SNIWD measure and entropy measure, and a mathematical example is provided in Section 5. Finally, Section 6 draws some valuable conclusions.

2. Preliminaries

2.1. The Simplified Neutrosophic Set (SNS)

Definition 1 (see [16]). A neutrosophic set (NS) \( P \) in a finite set \( X \) is denoted as

\[
P = \{ \langle x, T_p(x), I_p(x), F_p(x) \rangle \mid x \in X \},
\]

where \( T_p(x), I_p(x), \) and \( F_p(x) \) are called the truth, the indeterminacy, and the falsity-membership functions, respectively. Moreover, \( T_p(x), I_p(x) \) and \( F_p(x) \) are the standard and nonstandard subsets of real numbers \( ]0^{-}, 1^{+}[ \) and satisfy

\[
0^{-} \leq \sup T_p(x) + \sup I_p(x) + \sup F_p(x) \leq 3^{+}.
\]

To extend the application of the NS in engineering and science areas, Ye [18] defined the simplified neutrosophic set (SNS).

Definition 2 (see [18]). A simplified neutrosophic set (SNS) \( Q \) in a finite set \( X \) is described in the following form:

\[
Q = \{ \langle x, T_q(x), I_q(x), F_q(x) \rangle \mid x \in X \},
\]

where \( T_q(x), I_q(x), \) and \( F_q(x) \) represent the truth, the indeterminacy, and the falsity-membership functions, respectively, and satisfy

\[
0^{-} \leq T_q(x) + I_q(x) + F_q(x) \leq 3^{+}.
\]

For convenience, element \( q = (T_q, I_q, F_q) \) is generally named as a simplified neutrosophic number (SNN), and the complement of \( q = (T_q, I_q, F_q) \) is defined as \( q^c = (F_q, 1 - I_q, T_q) \).

Let \( q = (T_q, I_q, F_q) \) and \( s = (T_s, I_s, F_s) \) be two SNNs; some of mathematical operations are provided by Ye [18]:

1. \( q \oplus s = (T_q + T_s - T_q \cdot T_s, I_q \cdot T_s + I_s \cdot F_q + F_s + F_q) \)
2. \( \lambda q = (1 - (1 - T_q)^\lambda, (I_q)^\lambda, (F_q)^\lambda) \) \( \lambda > 0 \)

Definition 3 (see [19]). Let \( x_i = (T_{x_i}, I_{x_i}, F_{x_i}) \) \( (i = 1, 2) \) be two SNNs; then, the Hamming distance measure between \( x_1 \) and \( x_2 \) is presented as follows:

\[
d_{SNW}(x_1, x_2) = \frac{1}{3} \left| T_{x_1} - T_{x_2} \right| + \left| I_{x_1} - I_{x_2} \right| + \left| F_{x_1} - F_{x_2} \right|.
\]
2.2. **The SNOWD Measure**

**Definition 4** (see [47]). Let \( Q = \{ q_1, q_2, \ldots, q_n \} \) and \( S = \{ s_1, s_2, \ldots, s_n \} \) be two collections of SNNs, and \( d_{\text{SNWD}}(q_i, s_i) \) is the distance between SNNs \( q_i \) and \( s_i \); then, the simplified neutrosophic ordered weighted distance (SNWD) measure can be defined as follows:

\[
\text{SNWD}(Q, S) = \left( \sum_{i=1}^{n} w_i \left( d_{\text{SNWD}}(q_i, s_i) \right)^k \right)^{1/k},
\]

where \( k > 0 \) and \( w = (w_1, w_2, \ldots, w_n) \) is the weighted vector of \( d_{\text{SNWD}}(q_i, s_i) \) such that \( w_i \in [0, 1] \) and \( \sum_{i=1}^{n} w_i = 1 \).

Motivated by the OWD measure [31], Sahin and Kucuk [47] proposed the conception of the SNOWD measure, whose significance property is the ordered mechanism for the aggregated information.

**Definition 5** (see [47]). Let \( Q = \{ q_1, q_2, \ldots, q_n \} \) and \( S = \{ s_1, s_2, \ldots, s_n \} \) be two sets of SNNs, and \( d_{\text{SNOWD}}(q_i, s_i) \) is the distance between SNNs \( q_i \) and \( s_i \); then, the simplified neutrosophic ordered weighted distance (SNOWD) measure is defined as

\[
\text{SNOWD}(Q, S) = \left( \sum_{i=1}^{n} w_i \left( d_{\text{SNOWD}}(q_i, s_i) \right)^k \right)^{1/k},
\]

where \( d_{\text{SNOWD}}(q_{\sigma(i)}, s_{\sigma(i)}) \) is the reorder values such that \( d_{\text{SNOWD}}(q_{1}, s_{1}) \geq d_{\text{SNOWD}}(q_{2}, s_{2}) \geq \cdots \geq d_{\text{SNOWD}}(q_{n}, s_{n}) \). The associated weight vector of the SNOWD is \( w = (w_1, w_2, \ldots, w_n) \) with \( \sum_{i=1}^{n} w_i = 1 \) and \( w_i \in [0, 1] \).

The SNOWD measure possesses some good properties that the OWD also has, including boundedness, commutativity, idempotency, and monotonicity. However, the SNOWD can only consider the weights of ordered deviations, but fail to reflect the weights (importance) of aggregated arguments that the SNWD can. Therefore, we shall propose an integrated weighted distance measure to eliminate the existing defects in the SNOWD measure.

### 3. **SN Integrated Weighted Distance (SNIWD) Measure**

It is observed from Definitions 1 and 5 that the SNWD can reflect the importance of the input argument but fails to account for the positions’ weights of the ordered distances that the SNOWD can, while the SNOWD cannot emphasize the importance of aggregated deviations that the SNWD can. To solve the limitations, we present the SN integrated weighted distance (SNIWD) measure that can combine both merits of the SNOWD and the SNWD measures.

**Definition 6.** Let \( Q = \{ q_1, q_2, \ldots, q_n \} \) and \( S = \{ s_1, s_2, \ldots, s_n \} \) be two collections of SNNs, and \( d_{\text{SNOWD}}(q_i, s_i) \) is the distance between SNNs \( q_i \) and \( s_i \); then, the SNIWD measure is defined as

\[
\text{SNIWD}(Q, S) = \left( \sum_{i=1}^{n} \psi_i \left( d_{\text{SNOWD}}(q_{\sigma(i)}, s_{\sigma(i)}) \right)^k \right)^{1/k},
\]

where the integrated weights \( \psi_i \) are defined as

\[
\psi_i = \epsilon w_i + (1 - \epsilon) \omega_{\sigma(i)},
\]

wherein \( \omega_i \) is the weight of \( d_{\text{SNOWD}}(q_i, s_i) \) \( (i = 1, 2, \ldots, n) \) such that \( \omega_i \in [0, 1] \) and \( \sum_{i=1}^{n} \omega_i = 1 \), \( \omega_i \) is the relative weight of the SNIWD satisfying \( \sum_{i=1}^{n} \omega_i = 1 \) and \( \epsilon \in [0, 1] \) is a real parameter satisfying \( \epsilon \in [0, 1] \).

Obviously, when \( \epsilon = 1 \) and \( \epsilon = 0 \), the SNIWD is generalized to the SNOWD and the SNWD measures, respectively. Therefore, the SNIWD can be viewed as a combination of the SNOWD and SNWD measures, which can be proved by the following formula:

\[
\text{SNIWD}(Q, S) = \left( \sum_{i=1}^{n} \psi_i \left( d_{\text{SNOWD}}(q_{\sigma(i)}, s_{\sigma(i)}) \right)^k \right)^{1/k} = \left( \sum_{i=1}^{n} \omega_i \left( d_{\text{SNOWD}}(q_{\sigma(i)}, s_{\sigma(i)}) \right)^k \right)^{1/k} = \left( \sum_{i=1}^{n} \omega_i \left( d_{\text{SNWO}}(q_{\sigma(i)}, s_{\sigma(i)}) \right)^k \right)^{1/k} \cdot \left( \sum_{i=1}^{n} \epsilon \omega_i \right)^{1/k}.
\]

**Example 1.** Let \( Q = \{(0.9, 0.4, 0.7), (0.6, 0.2, 0.4), (0.4, 0.8, 0.5), (0.7, 0.1, 0.6)\} \), \( S = \{(0.5, 0.5, 0.3), (0.5, 0.4, 0.1), (0.3, 0.7, 0.4), (0.4, 0.4, 0.1)\} \),

\[
\begin{align*}
\text{SNIWD}(Q, S) & = \left( \sum_{i=1}^{n} \psi_i \left( d_{\text{SNOWD}}(q_{\sigma(i)}, s_{\sigma(i)}) \right)^k \right)^{1/k} \\
& = \left( \sum_{i=1}^{n} \omega_i \left( d_{\text{SNOWD}}(q_{\sigma(i)}, s_{\sigma(i)}) \right)^k \right)^{1/k} + (1 - \epsilon) \sum_{i=1}^{n} \omega_i \left( d_{\text{SNOWD}}(q_{\sigma(i)}, s_{\sigma(i)}) \right)^k \right)^{1/k}.
\end{align*}
\]

(1) Utilize equation (5) to calculate \( d_{\text{SNOWD}}(q_i, s_i) \) \( (i = 1, 2, \ldots, 4) \):

\[
\begin{align*}
d_{\text{SNOWD}}(q_1, s_1) &= 0.3, \\
d_{\text{SNOWD}}(q_2, s_2) &= 0.2, \\
d_{\text{SNOWD}}(q_3, s_3) &= 0.1, \\
d_{\text{SNOWD}}(q_4, s_4) &= 0.4.
\end{align*}
\]

(2) Rank \( d_{\text{SNOWD}}(q_i, s_i) \) \( (i = 1, 2, \ldots, 4) \) according to the decreasing order:
Remark 1. Let \( k = 1 \); then, we obtain the SN integrated weighted Hamming distance (SNIWHD) measure, and the SN integrated weighted Euclidean distance (SNIWED) measure is formed when \( k = 2 \).

\[
\begin{align*}
\psi_1 &= 0.6 \times 0.22 + (1 - 0.6) \times 0.3 = 0.252, \\
\psi_2 &= 0.6 \times 0.28 + (1 - 0.6) \times 0.2 = 0.248, \\
\psi_3 &= 0.6 \times 0.36 + (1 - 0.6) \times 0.4 = 0.376, \\
\tilde{\omega}_4 &= 0.6 \times 0.14 + (1 - 0.6) \times 0.1 = 0.124. \\
\end{align*}
\]

(4) Let \( k = 2 \); then, calculate the distance between \( Q \) and \( S \) utilizing the SNIWD measure defined in equation (9):

\[
\begin{align*}
\text{SNIWD} (Q, S) &= \left( \frac{1}{4} \psi_i \left( d_{\text{SNN}}(q_{i(\ell)}, s_{\ell}) \right)^2 \right)^{1/2} \\
&= \left( 0.252 \times 0.4^2 + 0.248 \times 0.3^2 + 0.376 \times 0.2^2 + 0.124 \times 0.1^2 \right)^{1/2} \\
&= 0.2809. \\
\end{align*}
\]

We can also illustrate the aggregation by applying the SNIWD measure given in equation (11):

\[
\begin{align*}
\text{SNIWD} (Q, S) &= \left( \frac{1}{n} \sum_{i=1}^{n} \psi_i \left( d_{\text{SNN}}(q_{i(\ell)}, s_{\ell}) \right)^2 \right)^{1/2} \\
&= \left( 0.6 \times \left( \frac{1}{n} \sum_{i=1}^{n} \omega_i \left( d_{\text{SNN}}(q_i, s_i) \right)^2 \right) + (1 - 0.6) \times \left( \frac{1}{n} \sum_{i=1}^{n} \omega_i \left( d_{\text{SNN}}(q_i, s_i) \right)^2 \right) \right)^{1/2} \\
&= (0.6 \times 0.0762 + (1 - 0.6) \times 0.083)^{1/2} \\
&= 0.2809. \\
\end{align*}
\]

Remark 2. If \( \varepsilon = 1 \), then the SNIWD is reduced to the SNOWD measure. Thus, all particular SN distance measures of the SNOWD mentioned in the result of Sahin and Kucuk [47] are the SNIWD’s special cases, for example:

(i) The SN Hamming ordered weighted distance (SNIHOWD) measure \( (k = 1) \)

(ii) The SN Euclidean ordered weighted distance (SNEOWD) measure \( (k = 2) \)

(iii) The SN geometric ordered weighted distance (SNOWGD) measure \( (k \rightarrow 0) \)

(iv) Maximum SN distance measure \( \omega = (1, 0, 0, \ldots, 0) \)

(v) Minimum SN distance measure \( \omega = (0, 0, 0 \ldots 0) \)

(vi) Normalized SN distance measure \( \omega = (1/n, 1/n, \ldots, 1/n) \)

Remark 3. If \( \varepsilon = 0 \), then the SNIWD is reduced to the SNWD measure. Then, we can achieve various families of the
SNWD that can be seen as the SNIWD's particular status, such as:

(i) The SN Hamming weighted distance (SNHWD) measure \( (k = 1) \)

(ii) The SN Euclidean weighted distance (SNEWD) measure \( (k = 2) \)

(iii) The SN geometric weighted distance (SNGWD) measure \( (k \rightarrow 0) \)

(iv) Normalized SN distance measure \( (\omega = (1/n, 1/m, q_1 \ldots h_{1/m})) \)

Remark 4. By applying similar analysis introduced in the recent literature [48–53], more other cases of the SNIWD measure can be created, such as the the centered-SNIWD, median-SNIWD, and the Olympic-SNIWD measures. The following theorems show that the SNIWD measure satisfies some desirable properties of monotonicity, boundedness, idempotency, commutativity, and reflexivity.

**Theorem 1** (monotonicity). If \( d_{\text{SNIWD}}(q_i, s_i) \geq d_{\text{SNIWD}}(q_i, s_i') \) for \( i = 1, 2, \ldots, n \), then

\[
\text{SNIWD}((q_1, s_1), \ldots, (q_n, s_n)) \geq \text{SNIWD}((q_1, s_1), \ldots, (q_n, s_n')).
\]

**Theorem 2** (idempotency). If \( d_{\text{SNIWD}}(q_i, s_i) = d \) for \( i = 1, 2, \ldots, n \), then

\[
\text{SNIWD}((q_1, s_1), \ldots, (q_n, s_n)) = d.
\]

**Theorem 3** (boundedness). Let \( d_{\text{min}} = \min_{i} d_{\text{SNIWD}}(q_i, s_i) \) and \( d_{\text{max}} = \max_{i} d_{\text{SNIWD}}(q_i, s_i) \); then,

\[
d_{\text{min}} \leq \text{SNIWD}((q_1, s_1), \ldots, (q_n, s_n)) \leq d_{\text{max}}.
\]

**Theorem 4** (commutativity). If \( ((q_1', s_1'), \ldots, (q_n', s_n')) \) is any permutation of \( ((q_1, s_1), \ldots, (q_n, s_n)) \), then

\[
\text{SNIWD}((q_1, s_1), \ldots, (q_n, s_n)) = \text{SNIWD}((q_1', s_1'), \ldots, (q_n', s_n')).
\]

**Theorem 5** (reflexivity). If \( q_i = s_i \) for \( i = 1, 2, \ldots, n \), then

\[
\text{SNIWD}((q_1, s_1), \ldots, (q_n, s_n)) = 0.
\]

### 4. Application in MAGDM

As a generalization of various distance measures, the SNIWD is applicable to many fields, such as data analysis, decision-making, social management, pattern recognition, and financial investment. In this section, an application in the MAGDM problems is studied. Suppose that a MAGDM problem has \( m \) different alternatives \( B_1, B_2, \ldots, B_m \), and some experts \( E_1, E_2, \ldots, E_r \) are consulted to assess \( n \) finite attributes \( C_1, C_2, \ldots, C_n \). Following the available information, the general procedure based on the SNIWD and entropy measures for MAGDM can be summarized as follows.

**Step 1.** Construct the SN individual decision matrix \( R^l = (r^{(l)}_{ij})_{mn} \) where \( r^{(l)}_{ij} = (T^{(l)}_{ij}, I^{(l)}_{ij}, F^{(l)}_{ij}) \) provided by expert \( e_l \) \( (l = 1, 2, \ldots, t) \) is a SN denoting the assessment of alternative \( B_i \) with respect to attribute \( C_j \).

**Step 2.** Determine the weight vector of experts (or decision makers) based on the similarity measure method [47]. In some actual problems, the weights of the experts cannot be determined beforehand. Thus, we introduce a method to derive the weights' information of experts based on the similar measures between individual opinions \( R^l = (r^{(l)}_{ij})_{mn} \) and the overall decision matrix \( R^* = (r^{(*)}_{ij})_{mn} \):

\[
\text{sm}(R^{(l)}_{ij}, R^{(*)}_{ij}) = \sum_{i=1}^{m} \sum_{j=1}^{n} \text{sm}(r^{(l)}_{ij}, r^{(*)}_{ij}),
\]

where the distance measure \( \text{sm}(r^{(l)}_{ij}, r^{(*)}_{ij}) \) between \( r^{(l)}_{ij} \) and \( r^{(*)}_{ij} \) can be calculated by equation (6) and \( r^{(*)}_{ij} = (T^{(*)}_{ij}, I^{(*)}_{ij}, F^{(*)}_{ij}) \) is the mean value of \( r^{(l)}_{ij} = (T^{(l)}_{ij}, I^{(l)}_{ij}, F^{(l)}_{ij}) \) \( (l = 1, 2, \ldots, t) \) determined by the following formula:

\[
T^{(*)}_{ij} = \frac{1}{t} \sum_{l=1}^{t} T^{(l)}_{ij}, I^{(*)}_{ij} = \frac{1}{t} \sum_{l=1}^{t} I^{(l)}_{ij}, F^{(*)}_{ij} = \frac{1}{t} \sum_{l=1}^{t} F^{(l)}_{ij}.
\]

On the basis of the similar measures, the weight of expert \( e_l \) \( (l = 1, 2, \ldots, t) \) can be derived by the following equation:

\[
\theta_l = \frac{\text{sm}(R^{(l)}, R^{(*)})}{\sum_{i=1}^{m} \text{sm}(R^{(l)}, R^{(*)})},
\]

where \( \theta_l \in [0, 1] \), and \( \sum_{i=1}^{m} \theta_l = 1 \). Moreover, the weight of experts derived by this method has the desirable characteristic: the larger the similarity \( \text{sm}(R^{(l)}, R^*) \) is, the more closer the individual evaluation \( R^{(l)} \) to the overall evaluation \( R^* \) is and the larger the weight of expert \( e_l \) \( (l = 1, 2, \ldots, t) \) is.

**Step 3.** Calculate the collective decision matrix \( R = (r^{(*)})_{mn} \) using the SN weighted averaging (SNWA) operator [21], where \( r_{ij} = (T_{ij}, I_{ij}, F_{ij}) = \sum_{l=1}^{t} \theta_l r^{(l)}_{ij} \).

**Step 4.** Determine the weight vector of the attribute. It is often difficult to express the weight information of the attribute in advance due to time limited or experts' professional knowledge. Thus, we develop an entropy-based method to derive the importance of attribute \( C_j \) \( (j = 1, 2, \ldots, n) \):

\[
\omega_j = \frac{1 - E_j}{n - \sum_{j=1}^{n} E_j}
\]

\( \omega_j \in [0, 1] \) and \( \sum_{j=1}^{n} \omega_j = 1 \), and the entropy measure \( E_j \) introduced by Biswas et al. [54] can be calculated from the following equation 16:

\[
E_j = \frac{1}{m} \sum_{i=1}^{m} \left( 1 - 2I_{ij} - 1 \times (T_{ij} + F_{ij}) \right).
\]
Step 5. Set ideal scheme $I = (I_1, I_2, \ldots, I_n)$ utilizing the following formula:

$$I_j = (T_{ij}, I_j, F_{ij}) = \begin{cases} 
\left( \max_i T_{ij}, \min_i I_{ij}, \min_i F_{ij} \right), & \text{for the benefit attribute,} \\
\left( \min_i T_{ij}, \max_i I_{ij}, \max_i F_{ij} \right), & \text{for the cost attribute.}
\end{cases}$$

(28)

Step 6. Apply the SNIWD measure to calculate the distances between alternative $B_i (i = 1, 2, \ldots, m)$ and ideal scheme $I$:

$$\text{SNIWD}(B_i, I) = \left( \sum_{j=1}^{m} \psi_j d_{\text{SNWID}}(r_{\sigma(i)}, I_{\sigma(j)}) \right)^{1/k}, \quad i = 1, 2, \ldots, n.$$  

(29)

Step 7. Rank the alternatives in accordance with the results obtained in the previous step, and hence, select the best choice.

5. Numerical Case of Investment Selection

In this section, we give a mathematical example of the investment selection problem [21] to verify the effectiveness and applicability of the presented method. A company would like to invest a sum of money to get a good return. Four possible alternatives are considered: (1) $B_1$ is a computer company; (2) $B_2$ is a food company; (3) $B_3$ is a car company; and (4) $B_4$ is an arms company. Three experts $\{E_1, E_2, E_3\}$ are invited to assess the companies from the following attributes: $C_1$ is the risk analysis; $C_2$ is the environmental impact analysis, and $C_3$ is the growth analysis, wherein $C_1$ and $C_3$ are of the benefit types, while $C_2$ belongs to the cost type. Then, the decision procedures are illustrated as follows.

Step 1. The individual SN decision matrix provided by experts is listed in Tables 1–3.

Step 2. On the basis of the aforementioned decision matrix, the overall decision matrix $R^* = (r^{(\ast)})_{m \times n}$ is calculated by using equation (24), listed in Table 4.

Using equation (25), the similar measures between individual opinions $R^{(i)} (i = 1, 2, 3)$ and the overall decision matrix $R^*$ are calculated as

$$\text{sm}(R^{(1)}, R^*) = 0.883,$$

$$\text{sm}(R^{(2)}, R^*) = 0.903,$$

$$\text{sm}(R^{(3)}, R^*) = 0.896.$$  

(30)

Thus, the weights of experts are derived as

$$\theta_1 = 0.329,$$

$$\theta_2 = 0.337,$$

$$\theta_3 = 0.334.$$  

(31)

Step 3. According to the weights of the experts, the collective SN decision matrix can be calculated by using the SNWA operator, presented in Table 5.

Step 4. Applying equations (26) and (27), the weight vector of attributes is computed as $\omega = (0.3634, 0.2862, 0.3504)$.

Step 5. The results of the ideal scheme by applying equation (28) are calculated as given in Table 6.

Step 6. Without loss of generality, let the parameter and weight vector of the SNIWD measure be $\epsilon = 0.5$ and $\omega = (0.3, 0.5, 0.2)$, respectively. Then, based on the weights of attributes obtained in Step 5, the distances between each alternative $B_i (i = 1, 2, 3, 4)$ and ideal scheme $I$ are calculated by using equation (29):

$$\text{SNIWD}(B_1, I) = 0.1526,$$

$$\text{SNIWD}(B_2, I) = 0.0922,$$

$$\text{SNIWD}(B_3, I) = 0.1070,$$

$$\text{SNIWD}(B_4, I) = 0.0945.$$  

(32)

Step 7. Rank all the alternatives in accordance with the decreasing values of SNIWD($B_i, I$). The smaller the value of SNIWD($B_i, I$), the closest $B_i$ to the ideal scheme, and thus the better alternative $B_i$. Therefore, the alternatives can be ranked as

$$B_2 \succ B_4 \succ B_3 \succ B_1.$$  

(33)

Hence, the best alternative is $B_2$.

Moreover, we can apply some special cases of the SNIWD mentioned in Section 4 to calculate the relative distances from the alternatives to the ideal scheme for obtaining a more comprehensive picture. The aggregation results are shown in Table 7, and the subsequent ranking order is listed in Table 8.

It can be seen in Table 8 that different ranking lists can be achieved from different cases of the SNIWD measures. Therefore, this method presents a more flexible mechanism for decision makers to choose different schemes according to their own needs or actual situations.

To perform the applicability of the presented method, we conduct a comparative research on some existing approaches for handling SN decision-making problems. We select the correlation coefficient method proposed by Ye [55], cross-entropy method by Ye [56], TOSIS method developed by Zeng et al., [27], SNWA method introduced by
It is noted from Table 9 that the best choice is either $B_2$ or $B_4$, and the ranking lists of all alternatives may vary depending on the decision method used. The main reasons can be summarized as follows:

1. The proposed SNIWD and the entropy model can efficiently eliminate the large deviation opinions provided by experts through the ordered weighting mechanism, which widely exists in MAGDM problems.
2. An entropy model is put forward to derive the unknown weights' information of attributes in this paper. By contrast, the weights of attributes are determined by decision makers (experts) in advance in the aforementioned methods.

### Table 1: SN decision matrix $R^{(1)}$.

|   | $C_1$          | $C_2$          | $C_3$          |
|---|----------------|----------------|----------------|
| $B_1$ | (0.6, 0.2, 0.3) | (0.5, 0.1, 0.2) | (0.5, 0.1, 0.3) |
| $B_2$ | (0.5, 0.3, 0.2) | (0.5, 0.3, 0.3) | (0.7, 0.1, 0.3) |
| $B_3$ | (0.5, 0.1, 0.2) | (0.3, 0.1, 0.3) | (0.5, 0.2, 0.2) |
| $B_4$ | (0.5, 0.3, 0.2) | (0.7, 0.2, 0.2) | (0.7, 0.2, 0.2) |

### Table 2: SN decision matrix $R^{(2)}$.

|   | $C_1$          | $C_2$          | $C_3$          |
|---|----------------|----------------|----------------|
| $B_1$ | (0.4, 0.1, 0.3) | (0.6, 0.2, 0.2) | (0.5, 0.3, 0.3) |
| $B_2$ | (0.6, 0.1, 0.2) | (0.5, 0.2, 0.3) | (0.7, 0.2, 0.3) |
| $B_3$ | (0.5, 0.2, 0.2) | (0.3, 0.2, 0.4) | (0.6, 0.2, 0.3) |
| $B_4$ | (0.7, 0.3, 0.1) | (0.5, 0.1, 0.2) | (0.6, 0.3, 0.2) |

### Table 3: SN decision matrix $R^{(3)}$.

|   | $C_1$          | $C_2$          | $C_3$          |
|---|----------------|----------------|----------------|
| $B_1$ | (0.3, 0.2, 0.3) | (0.5, 0.3, 0.2) | (0.5, 0.2, 0.3) |
| $B_2$ | (0.6, 0.1, 0.2) | (0.5, 0.2, 0.2) | (0.6, 0.1, 0.2) |
| $B_3$ | (0.4, 0.2, 0.3) | (0.2, 0.2, 0.5) | (0.4, 0.2, 0.3) |
| $B_4$ | (0.7, 0, 0.1)   | (0.4, 0.3, 0.2) | (0.6, 0.1, 0.2) |

### Table 4: Overall decision matrix $R^*$.

|   | $C_1$          | $C_2$          | $C_3$          |
|---|----------------|----------------|----------------|
| $B_1$ | (0.448, 0.159, 0.262) | (0.552, 0.182, 0.200) | (0.500, 0.182, 0.300) |
| $B_2$ | (0.569, 0.144, 0.200) | (0.500, 0.229, 0.262) | (0.670, 0.126, 0.262) |
| $B_3$ | (0.469, 0.159, 0.229) | (0.268, 0.159, 0.391) | (0.507, 0.200, 0.262) |
| $B_4$ | (0.644, 0.000, 0.126) | (0.552, 0.182, 0.200) | (0.637, 0.182, 0.200) |

### Table 5: Collective SN decision matrix $R$.

|   | $C_1$          | $C_2$          | $C_3$          |
|---|----------------|----------------|----------------|
| $B_1$ | (0.447, 0.158, 0.263) | (0.536, 0.182, 0.200) | (0.500, 0.182, 0.300) |
| $B_2$ | (0.570, 0.144, 0.200) | (0.500, 0.229, 0.262) | (0.670, 0.126, 0.262) |
| $B_3$ | (0.469, 0.159, 0.229) | (0.268, 0.159, 0.392) | (0.507, 0.200, 0.263) |
| $B_4$ | (0.645, 0.000, 0.126) | (0.551, 0.190, 0.200) | (0.636, 0.182, 0.200) |

### Table 6: Ideal scheme.

|   | $C_1$          | $C_2$          | $C_3$          |
|---|----------------|----------------|----------------|
| $I$ | (0.645, 0.000, 0.126) | (0.268, 0.229, 0.392) | (0.670, 0.126, 0.200) |

Peng et al. [21], ordered weighted SN cosine similarity measure [26], and power aggregation model provided by Liu and Luo [25]. All the ranking lists are illustrated in Table 9.

It is noted from Table 9 that the best choice is either $B_3$ or $B_4$, and the ranking lists of all alternatives may vary depending on the decision method used.
The existing literature. However, we present an entropy measure method to derive the unknown attribute weight information, which helps to achieve a more objective result; and (3) the proposed method based on the SNIWD is more flexible as it provides a chance for the decision maker to select the appropriate parameters that are near to his or her interests or the needs of the decision-making problems.

In our subsequent study, we will consider some other applications of the proposed approach, such as education evaluation and social network. Some new extensions by using other variables are also considered in complex situations.

### 6. Conclusions

In this study, we present a new approach based on the integrated distance measure and entropy weights for MAGDM with SN information. The SNIWD measure is proposed to improve the defects of the previous methods. The main advantage of the SNIWD is that it combines the ordered weighted and arithmetic weighted functions for reflecting the SN deviations. Moreover, it generalizes a great many of SN distance measures, including the SNOWD and the SNWD. Then, we develop a MAGDM approach based on the SNIWD and the entropy method within SN situations, wherein the entropy measure is utilized to determine the unknown weight information. A case study regarding selection of a suitable investment case is given to illustrate the efficiency of the proposed framework. The results and comparative study with other existing models test the advantages and effectiveness of our method. The preponderances of the proposed method based on the SNIWD measure and the entropy weight are summed up as follows: (1) the existing approaches based on the ordered weighted distance measures in decision-making areas only pay attention to the weights of the ordered deviation. They fail to account for the importance of attributes. By contrast, the introduced method based on the SNIWD can effectively fuse both importance of the ordered deviations and attributes; (2) the attribute weight is given by decision makers in advance in

### Data Availability

No data were used to support this study.

### Conflicts of Interest

The authors declare no conflicts of interest.

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