ROBUSTNESS OF PD-PI CONTROLLER USED WITH THIRD ORDER PROCESSES

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Abstract- Robustness is one of the requisites used in controllers design. The objective of this research is to investigate the robustness of a PD-PI controller used to control a third-order process against uncertainty in the process parameters. A variation of ±20 % in process parameters is considered through simulation to study its effect on the system performance parameters using the tuned controller. The variation of the process integral gain produced a change of 0.7 % in the settling time, a change of 0.8 % in maximum percentage overshoot and a change of 0.4 % in undershoot. The variation in process time constant T₁ resulted in a change of 0.4 % in the settling time but it has no effect in both maximum percentage overshoot and undershoot. However, for all the changes in the process parameters the phase margin is from 58.1 to 73 (deg), and the gain margin is infinity (dB) at infinity (rad/s), which indicates the good robustness of the used PD-PI controller when used with third order process.

Keywords – Third order process; PD-PI controller; uncertainty in process parameters; controller robustness; control system performance

I. INTRODUCTION

During operation, processes are subject to uncertainty in their parameters. Therefore, it is important to investigate the effectiveness of the used controller in dealing with such uncertainty. Hu, Chang, Yeh and Kwatny (2000) used the H∞ approximate I/O linearization formulation and μ-synthesis to design a nonlinear controller for an aircraft longitudinal flight control problem and address tracking, regulation and robustness issues [1]. Gong and Yao (2001) generalized a neural network adaptive robust control design to synthesize performance oriented control laws for a class of nonlinear systems in semi-strict feedback forms through the incorporation of back stepping design techniques [2]. Lee and Na (2002) designed a robust controller for a nuclear power control system. They used the Kharitonov and edge theorem to determine the controller which was simpler than that obtained by the H∞ [3]. Arvanitis, Syrkos, Stellas and Sigrimis (2003) analyzed PDF controllers designed and tuned to control integrator plus dead time processes in terms of robustness. They performed the robustness analysis in terms of structured parametric uncertainty description [4]. Lhommeau, Hardouin, Cottenceau and Laulín (2004) discussed the existence and the computation of a robust controller set for uncertain systems described by parametric models with unknown parameters assumed to vary between known bounds [5]. Dechanupaprittha, Hongsombut, Watanabe, Mitani and Ngammroo (2005) introduced the design of robust superconducting magnetic energy storage controller in a multi machine power system by using hybrid tabu search and evolutionary programming. The objective function of the optimization problem considered the disturbance attenuation performance and robust stability index [6]. Chin, Lau, Low and Seet (2006) proposed a robust PID controller based on actuated dynamics and an un actuated dynamics shown to be global bounded by the Sordalen lemma giving the necessary sufficient condition to guarantee the global asymptotic stability of the URV system [7]. Vagja and Tzes (2007) introduced a robust PID controller coupled into a Feed forward compensator for set point regulation of an electrostatic micromechanical actuator. They tuned the PID

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controller using the LMI-approach for robustness against the switching nature of the linearized system dynamics [8]. Fiorentini and Bolender (2008) described the design of a nonlinear robust/adaptive controller for an air-breathing hypersonic vehicle model. They adapted a nonlinear sequential loop-closure approach to design a dynamic state-feedback control for stable tracking of velocity and altitude reference trajectories [9]. Labibi, Marquez and Chen (2009) presented a scheme to design decentralized robust PI controllers for uncertain LTI multi-variable systems. They obtained sufficient conditions for closed-loop stability of multi-variable systems and robust performance of the overall system [10]. Matusu, Vanekova, Porkop and Bakosova (2010) presented a possible approach to design simple PI robust controllers and demonstrate their applicability during control of a laboratory model with uncertain parameters through PLC [11].

Kada and Ghazzawi (2011) described the structures and design of a robust PID controller for higher order systems. They introduced a design scheme combining deadbeat response, robust control and model reduction techniques to enhance the performance and robustness of the PID controller [12]. Surjan (2012) applied the genetic algorithm for the design of the structure specified optimal robust controllers. The parameters of the chosen controller were obtained by solving the nonlinear constrained optimization problem using IAE, ISE, ITAE and ITSE performance indices. He used constraints on the frequency domain performances with robust stability and disturbance rejection [13]. Jiao, Jin and Wang (2013) analyzed the robustness of a double PID controller for a missile system by changing the aerodynamic coefficients. They viewed the dynamic characteristics as a two-loop system and designed an adaptive PID control strategy for the pitch channel linear model of supersonic missile [14]. Hassaan (2014) stated that, With the Sallen-Key compensator, the control system is stable for the whole range of the process parameters variation (±20%) [15]. Hassaan (2014) investigates the robustness of PDF, PDFF, PIDF and PID plus first-order lag controllers when used to control second-order processes with bad dynamics. He showed that the PDF, PDFF and PIDF controllers are robust [16]. Hassaan (2014) investigated the robustness of I-PD, PD-PI and PI-PD, controllers used to control second-order processes against uncertainty in the process parameters. A variation of ±20% in process parameters is considered through simulation to study its effect on the system performance parameters using the tuned controllers. He concluded that Tuned I-PD, PD-PI and PI-PD controllers are robust [17]. Pradham, Ray, Sahu and Moharana (2014) proposed a control strategy to improve the power factor and voltage regulation at a distribution supply system for more robustness [18]. Hassaan (2014) studied the robustness of a feedback PD compensator used with both second-order and third-order processes. He showed that this compensator is completely robust for process parameters variation in the range ± 20% [19].

Hassaan (2015) investigated the robustness of feedback first-order lag-lead, feed-forward second-order lag-lead and feed-forward first-order lag-lead compensators used to control second-order processes against uncertainty in the process parameters. A variation of ±20% in process parameters is considered through simulation to study its effect on the system performance parameters using the tuned controllers. He stated that, tuned feedback first-order lag-lead, feed-forward second-order lag-lead and feed-forward first-order lag-lead compensators used to control a second order process are robust [20]. Emma D. Welson et al (2018), introduced that evaluation of closed-loop robustness has generally relied on empirical methods. They have proved that, expressions for the H∞ norm of two commonly used PI/P control implementations, the feedback and forward path forms, are used, for the first time, to quantify closed-loop robustness [21]. Bharat Verma and Prabin Kumar Padhy (2019), focused on online PID controller tuning with the guaranteed robustness of the controller. A new single variable tuning method is developed for the online robustness and performance adjustment. They implemented that, the proposed rules only depend upon the previously optimized PID parameters.[22].

Min Zheng, Tao Huang and Guangfeng Zhang (2019), proposed that robust tuning of controller parameter is considered an effective way to deal with continuously changing end-user specs and raw product properties. They showed that, the specifications such as settling time, overshoot and robustness have a direct meaning in terms of process output and remain most popular amongst process engineers. They implemented an intuitive tuning procedure for robustness which is based on linear system tools such as frequency response and band limited specifications thereof, loop shaping remains a mature and easy to use methodology [23]. Clara M. Ionesco et al (2020), showed that successful operation in a globalization context can only be ensured by robust tuning of controller parameter as an effective way to deal with continuously changing end-user specs and raw product properties. They presented that; Recently next to these popular loop shaping methods, new tools have emerged, i.e. fractional order controller tuning rules. The key feature of the latter group is an intrinsic robustness to variations in the gain, time delay and time constant values, hence ideally suited for loop shaping purpose. They sketched and discussed both methods in terms of their advantages and disadvantages [24]. Singer, Hassaan and ElHamil (2020) studied the robustness of a PI-PD controller used to control a third order process. They proved the good robustness of the PI-PD controller when used to control the prescribed process [25].
II. PROPOSED ALGORITHM

A. The Process

The process considered in this analysis is a third order process having the following forward transfer function in a unity feedback system as shown in Fig.1:

\[ G_p(s) = \frac{K_{ip}\omega_n^2}{s^3 + 2\zeta\omega_n s^2 + \omega_n^2 s + K_{ip}} \]  

(1)

where

- \( G_p(s) \) is the close loop transfer function of the third order process.
- \( K_{ip} \) integral gain (\( K_i \)) of the process (in this prescribed third order process \( K_i = 0.5 \))
- \( \omega_n \) natural frequency (\( \omega_n = 0.447 \) rad/s)
- \( \zeta \) damping ratio (\( \zeta = 1.34 \))

B. The PD-PI Controller

The controller used in this study is a proportional + derivative (PD) - proportional + integral (PI) controller. In this controller, the PD and PI parts of the controller are connected in series. The input to the PD part is the system error, while the input of the PI part is the output of the PD part [26]. The controller transfer function, \( G_c(s) \), is:

\[ G_c(s) = (K_{pc1} + K_d) [K_{pc2} + (K_i/s)] \]

(2)

C. Control System Transfer Function

The closed loop transfer function of the control system incorporating the PD-PI controller and the third order process, \( M(s) \), is obtained from the block diagram of Fig.3 and Eqs.2 and 3.

\[ M(s) = \frac{b_2 s^2 + b_3 s + b_4}{a_0 s^4 + a_1 s^3 + a_2 s^2 + a_3 s + a_4} \]

(3)

Where

\[
\begin{align*}
b_2 &= (K_{pc2}K_dK_{ip}\omega_n^2), \\
b_3 &= (K_{pc1}K_{pc2}K_{ip}\omega_n^2 + K_cK_dK_{ip}\omega_n^2), \\
b_4 &= (K_{pc1}K_{ip}\omega_n^2), \\
K_i &= \text{Integral gain of the PI controller}
\end{align*}
\]

\[
\begin{align*}
a_0 &= 1, \\
a_1 &= (2\zeta \omega_n), \\
a_2 &= (K_{pc2}K_dK_{ip}\omega_n^2 + \omega_n^2), \\
a_3 &= (K_{pc1}K_cK_{ip}\omega_n^2 + K_{ip}\omega_n^2), \\
a_4 &= (K_{pc1}K_{ip}\omega_n^2).
\end{align*}
\]
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- $K_d$: Derivative gain of the PD controller
- $K_{pc1}$: Proportional gain of the PD controller
- $K_{pc2}$: Proportional gain of the PI controller

The controller has four parameters to be identified to control the third order process and produce the desired performance: $K_{pc1}$, $K_d$, $K_{pc2}$, and $K_i$.

D. Controller Tuning

The PD-PI controller was tuned by the authors to control this third order process [27]. The controller parameters are tuned as follows:

$$K_{pc1} = 0.0713, K_{pc2} = 8.1791, K_i = 0.01, K_d = 0.3026$$

E. Process Uncertainty

Due to the variation in the operation conditions during operation, the process is submitted to parametric changes. It is supposed that this change be as large as ±20% of the assigned process parameters.

III. Controller Robustness

The control system considered robust if it has acceptable changes in its performance due to model changes or inaccuracy [28]. Furthermore, Lee and Na add the stability requirement to the robustness definition besides the plants having uncertainty [3]. Toscano added that the controller has to be able to stabilize the control system for all the operating conditions [29]. In this research, the robustness of the controller and hence of the whole control system is assessed as follows:

- Nominal process parameters are identified.
- The controller is tuned for those process parameters.
- A variation of the process parameters is assumed within a range of ±20% of the nominal value.
- Using the same controller parameters, the step response of the system using the new process parameters is drawn and the control system performance is evaluated through the maximum percentage overshoot, maximum percentage undershoot and settling time.
- The variation in process parameters is increased and the procedure is repeated.

The effect of the variation of process parameters on the settling time, maximum percentage overshoot, maximum percentage undershoot, gain margin and phase margin of the closed loop control system using the tuned PI-PD controller parameters are shown in Figs.3, 4, 5 and 6.

Figure 3 Effect of process parameters change on system settling time
According to Ogata [30], for a control system with good performance:
- Gain margin: has to be > 6 dB.
- Phase margin: has to be in the range: 30 <= PM <= 60 degrees.

According to Lei and Man [31], the phase margin range can be widened to be: 30 <= PM <= 90

The open loop transfer function of the closed loop control system incorporating the PD-PI controller and the third order process, using the block diagram of Fig. 2, is:

$$G(s)H(s) = \frac{(k_{ip}k_{d}k_{pc2})s^2 + (k_{pc1}k_{pc2}k_{ip} + k_{ip}k_{pc1}s + k_{pc1}k_{ip})s + k_{pc1}k_{ip}}{(T_1T_2)s^4 + (T_1 + T_2)s^3 + s^2 + k_{ip}s}$$  \hspace{1cm} (4)

Where

$$K_{pc1} = 0.0731, \ K_d = 0.3026, \ K_{pc2} = 8.1791, \ K_i = 0.01, \ K_{ip} = 0.5, \ T_1 = 1 \ s, \ T_2 = 5 \ s.$$

Using the open loop transfer function of Eq.4 and the command 'margin' of the MATLAB program, the Phase Margin of the control system against the variations in the process parameters are shown in Fig. 6. The gain margin of the control system is infinity and it is unchangeable during the robustness study.
IV. CONCLUSION

- Variation in third order process parameters within ± 20% was considered.
- Tuned PD-PI controller is robust since it controlled the third order process for set-point change maintaining good performance and stable control system for the range of parameters change.
- A variation of ± 20% in process integral gain $K_{ip}$, time constant $T_1$ and Time constant $T_2$; has no effect in the maximum percentage overshoot.
- A variation of ± 20% in process integral gain $K_{ip}$ resulted in a change of (-1.23%) to (1.23%) in the maximum percentage undershoot, and (10.34%) to (2.6%) in the settling time of the control system.
- A variation of ± 20% in Time constant $T_1$ resulted in a change of (0.41%) to (-1.23%) in the maximum percentage undershoot, and (5.5%) to (6.47%) in the settling time of the control system.
- A variation of ± 20% in Time constant $T_2$ resulted in a change of (1.44%) to (2.46%) in the maximum percentage undershoot, and (31.36%) to (11.72%) in the settling time of the control system.
- The gain margin is infinity for all the process change parameters.
- The phase margin is in the range of the control system good performance according to Lei and Man [31].

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