AGN jet-driven stochastic cold accretion in cluster cores

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1 INTRODUCTION

There is a general consensus that the intracluster medium (ICM) in low-entropy galaxy cluster cores is able to remain in rough global thermal equilibrium because the powerful jets from the central AGN inject sufficient energy to compensate for radiative losses (for a review, see McNamara \& Nulsen 2007, 2012; Fabian 2012; Soker 2016). There are, however, a number of key issues associated with this “radio feedback” schema that have yet to be firmly pinned down, one of which is: how is the central supermassive black hole (SMBH) able to transport sufficient energy to the cluster core (\(\sim 10s \) of kpc) at a timescale shorter than the core cooling time (\(\lesssim 0.5 \) Gyr). There are two scenarios proposed for AGN feedback: (1) hot/Bondi accretion which invokes spherical accretion with the mass flow rate set by the hot gas density and temperature on the cluster core scales (Bondi 1952); and (2) the competing cold mode accretion model, in which a fraction of the gas in the cluster cores condenses into cold clouds and filaments (when the ratio of the cooling time to the free-fall time \(t_{cool}/t_{ff}\) falls below a threshold value in the range 5-20; e.g., McCourt et al. 2012; Voit et al. 2015; but see Hogan et al. 2016), which then decouple from surrounding hot intracluster medium (ICM), rain down upon the central SMBH, and power AGN feedback strong enough to prevent catastrophic cooling in the core.

Of the two, the Bondi accretion model, though more frequently invoked in the literature, has a number of shortcomings. For one, Bondi accretion is only applicable if the accretion flow is spherically symmetric and single phase, both conditions that are at odds with the multiphase gas with angular momentum observed in several cool cores (e.g., see Macchetto et al. 1997; McDonald et al. 2010; McNamara, Rohanizadegan, \& Nulsen 2011; Tremblay et al. 2016). Moreover, for a polytropic hot flow \(p \propto \rho^\gamma\), the mass accretion rate is given by (Shu 1992; Frank, King, \& Raine 2002)

\[
M_B = 4\pi \lambda(\gamma) G^2 M_{BH}^2 \frac{\rho_\infty}{c_{s,\infty}^2} \approx 0.0015M_5 \text{yr}^{-1} M_{BH,9}\rho_{\infty,0.1} T_{\infty,3}\nu_{3}^{-1/2},
\]

where \(c_{s,\infty} \equiv (\gamma k_B T_{\infty}/\mu m_p)^{1/2}\) is the sound speed in the...
ambient medium far from the black hole; the mean particle mass \( \mu = 0.62 \); \( M_{BH,0} \) is the black hole mass in units of \( 10^9 M_\odot \); \( T_{icw,0.1} \) and \( T_{keV} \) are the ambient particle number density and temperature scaled to 0.1 \( \text{cm}^{-3} \) and 1 keV, respectively; and since \( \lambda(\gamma) \) varies weakly with \( \gamma \) (\( \lambda \) varies from 1/4 for \( \gamma = 5/3 \) to 1.1 for \( \gamma = 1 \)) we adopt \( \gamma = 5/3 \) for convenience. In three nearby systems where both the ambient conditions governing the Bondi flow rate and the mass accretion rate onto the black hole can be deduced, the former is at least two orders of magnitude too large. These nearby examples are Sgr A*, the Galactic centre BH \( \text{[Bondi rate of } 10^{-2} - 10^{-3} M_\odot \text{yr}^{-1}] \) (Baganoff et al. 2003) vs. the accretion rate at \( \sim 10 \dot{M}_{\text{Edd}} \) of \( \lesssim 10^{-3} M_\odot \text{yr}^{-1} \) (Marrone et al. 2007 and references therein); NGC 3115 \( \text{[Bondi rate of } 2.2 \times 10^{-2} M_\odot \text{yr}^{-1}] \) vs. the accretion rate onto the BH at least two orders of magnitude smaller (Wong et al. 2011, 2014); and M87 \( \text{[Bondi rate of } 0.1 - 0.2 M_\odot \text{yr}^{-1}] \) vs. the accretion rate onto the BH of \( \lesssim 10^{-3} M_\odot \text{yr}^{-1} \) (Russell et al. 2015).

Nemmen & Tchekhovskoy (2015) find that the median efficiency (defined as the ratio of the observed jet power and \( M_{BH} \dot{M} \)), \( M_{BH} \dot{M} \) is the SMBH accretion rate) required to supply the cavity power of nearby radio galaxies is \( \sim 300 \% \), assuming \( M_{BH} \sim 0.01 M_B \) typical of similar nearby hot accretion flows. This implies that hot accretion, with reasonable assumptions, is insufficient for powering most of the cavities observed in cluster cores. At the extreme end of the spectrum, the estimated jet/cavity power in some of the galaxy clusters (e.g., MS0735, Cen A) is found to be much larger than \( 0.1 M_B \dot{M} \); i.e., the Bondi mass flow rate is insufficient to power the observed X-ray cavities even if all the hot gas in the Bondi flow is accreted by the SMBH (McNamara, Romanzadejan, & Nulsen 2011). And finally, the accretion rate in the Bondi regime takes a long time to adjust to the conditions \((n, T)\) changing in the cluster cores (\( \sim 1 \text{kpc} \)) at the core cooling timescale \( \sim 100 \text{Myr} \) (e.g., Soker et al. 2009).

The cold mode feedback model sidesteps many of the challenges associated with the hot/Bondi model. Very briefly (we refer the readers to Sharma et al. 2012a, Voit et al. 2015, and Prasad et al. 2015 [hereafter PSB15] for a more expansive discussion), the model is based on the realization that even if the ICM in the cluster core exists in rough global thermal balance, if the ratio of the cooling time and the gravitational free-fall time \( (t_{cool}/t_f) \) falls below a critical value \( (\sim 10 \text{ or so}; Sharma et al. 2012a; Gaspari et al. 2012; Voit et al. 2015; Prasad et al. 2015; Li et al. 2015; Choudhury & Sharma 2016), the gas will become susceptible to local thermal instabilities and fragment, leading to the formation of a multiphase medium consisting of cold dense clouds condensing from the hot diffuse ICM itself. The clouds then fall towards central AGN, resulting in increased SMBH accretion and feedback that, in turn, quenches runaway cooling in the cluster core. The main question with this model is: will the infalling cold gas, which has non-zero angular momentum, end up forming a viscosity-mediated standard accretion flow (SAF) in which the gas will flow inwards on a (long) viscous accretion timescale, or will the gas circularise sufficiently close to the SMBH and accrete on a timescale shorter than the core cooling time.

A particularly promising accretion mechanism, overcoming the angular momentum barrier by stochastically feeding cold gas clumps and fueling AGN activity, was invoked by Pizzolato & Soker (2005, 2010); Hopkins & Hernquist (2006); Nayakshin & King (2007). The AGN are stochastically fed by cloud clumps which cancel each others’ angular momentum through inelastic collisions, leading to the formation of a small transient disk accreting onto the black hole (Nayakshin & King 2007). This turbulent accretion mechanism of cold gas clouds onto SMBH was later called cold chaotic accretion (Nayakshin et al. 2012). Using isothermal simulations of idealised turbulent accretion flow over central 100 pc, Hobbs et al. (2011) show that dense gas accretes ballistically onto the central SMBH at a rate few order of magnitude larger than the case without turbulence.

Gaspari et al. (2013) carried out simulations of cluster cores (although only run for 40 Myr, a duration much shorter than the core cooling time) with idealised turbulent driving (which is fixed in time), and showed that the condensation of cold clumps from the hot ICM can boost the mass accretion rate on to the SMBH by \( \sim 100 \) compared to the Bondi accretion rate. These simulations were later generalised to include an initially rotating ICM (Gaspari et al. 2015) and cooling down to much lower temperatures allowing for three phases (hot, atomic and molecular; Gaspari et al. 2017), although still using idealised turbulence and running for much less than a core cooling time. The process of colliding cold gas clouds losing angular momentum and boosting SMBH accretion has been called chaotic cold accretion (CCA). Other terminology has also been in past (ballistic, stochastic, forced accretion, etc.). In this paper we use a related but somewhat different term stochastic cold accretion (SCA) to highlight that a turbulent system such as the ICM should be described statistically (Monin & Yaglom 1971).

While AGN jet-ICM simulations have been run successfully on cosmological timescales for some time now (e.g., Gaspari et al. 2012; Li et al. 2015; Prasad et al. 2015), we investigate the detailed angular momentum distribution of cold gas and its implications on SMBH accretion in realistic jet-ICM simulations over cosmological time scales for the first time. Here is a brief outline of the paper. Section 2 briefly presents our numerical setup. In section 3, we calculate the circulation radius and viscous time of the cold gas based on the standard accretion physics. We show that the angular momentum distribution of cold gas crossing \( \sim 1 \text{kpc} \) is close to isotropic which will lead to angular momentum cancellation and the formation of an efficient accretion flow. We use the mass distribution of angular momentum to estimate the accretion rate onto the SMBH. In section 4, we compare some of the results from our simulations against key observational. In section 5 we discuss the implications of our results, highlighting the need for better understanding of accretion. We summarise our paper in section 6.

2 NUMERICAL SIMULATIONS

In this paper we discuss three 3-D AGN jet-ICM simulations carried out in spherical \((r, \theta, \phi)\) coordinates. The details of the numerical set up are given in PSB15. The three runs are: (i) an initially hydrostatic ICM in a fixed NFW potential (this is the fiducial run in PSB15 with the halo mass of \( 7 \times 10^{14} M_\odot \)); (ii) the same run but with the inner and outer

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Table 1. List of runs

| Run       | \(r_{\text{in}}\) (kpc) | \(r_{\text{out}}\) (kpc) | \(\epsilon\) | Run Time (Gyr) | \(M_{\text{in,cold/hot}}\) \((M_{\odot} \text{yr}^{-1})\) | \(M_{\text{cold}}\) \((10^{41} M_{\odot})\) | \(M_{\text{BH}}\) \((M_{\odot} \text{yr}^{-1})\) PL, \(\beta\) | \(M_{\text{BH,SAP}}\) \((M_{\odot} \text{yr}^{-1})\) | \(M_{\text{BH,SCA}}\) \((M_{\odot} \text{yr}^{-1})\) |
|-----------|--------------------------|--------------------------|--------------|----------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| NFW\(a\)  | 1                        | 200                      | 6 \times 10^{-5} | 4              | 18/1/3/5                          | 1                               | 0.2                             | 0.007                           | 0.015                           | 0.2                             |
| NFW       | 0.5                      | 100                      | 6 \times 10^{-5} | 5              | 10/7/0.9                           | 5                               | 0.54                           | 0.01                           | 0.1                             | 1.27                            |
| NFW+BCG   | 0.5                      | 500                      | 5 \times 10^{-4} | 3              | 6/4/2/8                            | 0.1                             | 0.008                          | 0.003                           | 0.001                           | 0.03                            |

The resolution of all runs, done in spherical \((r_{\text{min}} \leq r \leq r_{\text{max}}, 0 \leq \theta \leq \pi, 0 \leq \phi \leq 2\pi)\) coordinates, is 256 \times 128 \times 32. Logarithmic grid is used in the \(r\)-direction, and a uniform one in others. \(M_{\text{in,cold/hot}}\) is the average cold/hot mass flow rate across \(r_{\text{in}}\). \(M_{\text{BH}}\) is the estimate of average Bondi accretion rate using power-law (PL) and isothermal beta model (\(\beta\)) extrapolations; \(M_{\text{cold}}\) is the total cold \((T < 0.1 \text{ keV})\) gas mass in the simulation domain by the end. \(M_{\text{in,cold/hot}}, M_{\text{BH,SAP}}, M_{\text{BH,SCA}} \& M_{\text{BH}}\) are averaged from 1 Gyr till the end of the run.

\(a\) the fiducial run.

\(\dagger\) jet efficiency relative to \(\dot{M}_\text{in}\) (see Eq. 6 in PSB15).

radii reduced by half; (iii) and a NFW+BCG ( the latter is modeled as a singular isothermal potential with circular velocity \(V_c = 350 \text{ km s}^{-1}\) potential run. The NFW+BCG run is carried out with PLUTO (Mignone et al. 2007) code, whereas the NFW runs are done with ZEUS-MP (Hayes et al. 2006). The primary purpose of the NFW+BCG runs is to facilitate a detailed analysis of how the addition of the BCG potential impacts the various aspects of the cold mode accretion model, a topic of considerable interest (c.f. Hogan et al. 2016, 2017). We will present this analysis in a follow-up paper. Most of the results and analysis in this paper are based on the NFW runs although we occasionally call upon the NFW+BCG simulations to highlight that our key results are robust.

The initial entropy profile for both NFW and NFW+BCG runs are identical (see Eq. 7 in PSB15). The initial ICM density for the NFW+BCG run is almost twice of the NFW runs (which have identical initial density profiles). The different runs have different inner \((r_{\text{in}})\) and outer \((r_{\text{out}})\) radial boundaries (see Table 1). The density at the outer radial boundary is fixed to their initial values. The total mass (including both hot and cold phases) crossing the inner boundary is calculated \((\dot{M}_\text{in})\), and a fraction \(\epsilon \dot{M}_\text{in} c^2\) is put back into the ICM in the form of kinetic power \((P_{\text{kin}})\) of AGN jets.

All our runs behave in a very similar manner. There are cooling and heating cycles, and the feedback driven heating events are less frequent for a higher feedback efficiency. Since the feedback efficiency, \(\epsilon\), is ten times higher for the NFW+BCG run, the amount of cold gas and accretion rate through the inner boundary are suppressed relative to NFW runs. The NFW run with \(r_{\text{in}} = 0.5 \text{ kpc}\) uses the same efficiency parameter \(\epsilon\) as the fiducial run with \(r_{\text{in}} = 1 \text{ kpc}\). However, because of a smaller inner radius and non-zero angular momentum of the condensing gas, the mass accretion rate across the inner boundary is lower as some of the gas circularises at 0.5 kpc \(< r < 1 \text{ kpc}\). Consequently, the feedback energy input is smaller and the run shows a higher accumulation of cold gas in the central regions (see \(M_{\text{cold}}\) in Table 1). Density and temperature profiles, and other X-ray and jet properties are discussed in detail in PSB15, and will not be repeated in this paper. Here we focus on the angular momentum distribution of cold gas in our simulations and the plausibility of stochastic accretion onto SMBHs (section 3), and comparison of simulation results with observations (section 4).

3 ANGULAR MOMENTUM OF COLD GAS & SMBH ACCRETION

Our simulations, like Gaspari et al. 2012; Li et al. 2015; Yang & Reynolds 2016, use the total (dominated by cold gas) accretion rate at \(\sim 1 \text{ kpc}\) to estimate the feedback energy deposited by jets. One of the major open problems with cold mode feedback is: how does the cold gas originating at \(\gtrsim 1 \text{ kpc}\) lose angular momentum and get rapidly accreted onto the SMBH. This is needed for feedback to prevent a cooling flow. Although it is numerically formidable to resolve both the cluster core and accretion flow onto the SMBH, we attempt to estimate the accretion rate onto the SMBH based on the angular momentum distribution of cold gas within our simulation domain. While the large angular momentum gas forms a massive disk at \(\gtrsim 1 \text{ kpc}\), the smaller angular momentum gas crossing the inner boundary has stochastic angular angular momentum changing over short timescales. Rapid angular momentum cancellation in this stochastic cold gas can allow substantial cold gas to be channeled to the SMBH sufficiently fast to prevent a cooling flow (e.g., see Pizzolato & Soker 2010).

3.1 Standard accretion estimates

In this section we review the standard accretion physics that we apply later to estimate the accretion rate onto the SMBH (section 3.3.1). For a given specific angular momentum \((l)\), the circularization radius of the gas is given by

\[ R_{\text{circ}} \equiv \frac{l^2}{GM_{\text{enc}}} \approx 0.24 \text{ kpc} \frac{M_{\text{enc}}^{-1}g_2^{28}}{g_{28}}, \quad (2) \]

where \(M_{\text{enc}}\) is the mass enclosed (including only SMBH + Dark Matter contributions; i.e., ignoring gas mass) within the circularization radius. The viscous accretion time at the
that the inner radius of our computational domain is well beyond the Bondi radius, and the SMBH contribution to gravity is negligible everywhere in the computational domain ($r_{in} \leq r \leq r_{out}$). The slope of $t_{\text{visc}}$ and $R_{\text{circular}}$ as a function of $l$ changes at the radius at which the enclosed mass is dominated by the extended potential rather than the SMBH. The arrows in Figure 1 mark important quantities such as Bondi radius, inner radius of the computational domain, and a viscous time of 0.2 Gyr. Accretion flow with a viscous time more than the core cooling time (here taken to be 0.2 Gyr, a conservative estimate) would not be able to respond faster than the cooling rate. A faster response of the SMBH accretion rate is required to prevent a cooling flow.

### 3.2 Stochastic cold accretion

In this section we discuss the angular momentum distribution of cold gas in our numerical simulations. We show that the time-averaged angular momentum distribution of cold gas crossing our inner simulation boundary is stochastic, with almost equal mass going around in different directions. This implies that in the region of circularization, angular momentum cancellation will take place almost at the local dynamical time. Moreover, the cold gas with $l < 10^{28}$ cm$^2$ s$^{-1}$ changes its mean angular momentum over a timescale shorter than the core cooling time; i.e., cold gas with small angular momentum is able to respond fast enough to close the feedback loop.

#### 3.2.1 Time-averaged angular momentum pdf

To start with, we emphasise that when considering gas whose specific angular momentum ($l$) distribution is stochastic, it is essential to explicitly account for the vector nature of the specific angular momentum, $l = r \times v = (−v_{\phi} R \hat{\epsilon}_{2} − r [v_{\phi} \cos \theta \cos \phi + v_{\phi} \sin \phi \hat{\epsilon}_{3}] \hat{\epsilon}_{x} + r [−v_{\phi} \cos \theta \sin \phi + v_{\phi} \cos \phi \hat{\epsilon}_{3}] \hat{\epsilon}_{y})$; where $R/r$ is the cylindrical/spherical radius). In standard accretion disk literature, $l$ is generally taken to be $l_{c}$ since the gas is considered to be coplanar. However, in our simulations the low angular momentum cold gas distribution shows almost equal importance of all the components of $l$.

To examine this, we consider the quantity $(dM/dl)\Delta l$, the incremental mass in the computational domain with angular momentum between $l$ and $l + \Delta l$ (we plot the mass distribution in a logarithmic bin $dM/d\log_{10} l = |l| \log_{10} e dM/dl$, as one can simply read off the mass from the figure). We can choose $l$ to be $|l|$, $l_{\perp}$, or $l_{\parallel}$. The left panel of Figure 2 shows the time-averaged cold mass distribution as a function of $|l_{\parallel}|$ (both clockwise and counter-clockwise rotations are shown) for our three runs. The right panel shows the time-averaged mass distribution as a function of the total specific angular momentum $|l|$. Comparing the $|l|$ and $|l_{\parallel}|$ distributions, we find a dramatic lack of cold gas with small $|l|$ but a non-negligible mass of cold gas with very low $|l_{\parallel}|$. This gas with low $|l_{\parallel}|$ has large $|l_{\perp}|$ and $|l_{\parallel}|$ components, and therefore is not expected to circularise in the $x$-$y$ plane.

Focussing on gas with $l < l_{in}$ (corresponding to the circularization radius in Eq. 2 equal to the inner radial boundary $r_{in}$), where $l_{in} \sim 10^{29}$ cm$^2$ s$^{-1}$, we find that in all our simulations the specific angular momentum, for the two gravitational potentials (NFW and NFW+BCG) used in our simulations. Note

![Figure 1](image_url)
simulations the low angular momentum gas ($l < l_{in}$) has roughly similar distribution in $l_z$ and $|l|$, with $dM/dl_z \propto l_z^2$ and $dM/d|l| \propto |l|$.

3.2.2 Angular momentum pdf of mass crossing $r_{in}$

Apart from the angular momentum pdf of cold gas mass within the computational domain, it is instructive to consider the pdf of the cold gas crossing the inner boundary, which is more relevant for SMBH accretion estimates. Figure 3 shows the time-averaged angular momentum pdf ($dM/d\log_{10} l$) with respect to $l_x$, $l_y$, $l_z$, and $|l|$ of cold gas accreting through the inner boundary of our fiducial run. The pdfs relative to all three components ($l_x$, $l_y$, $l_z$) are similar, including the clockwise and counter-clockwise components, suggesting that the cold gas crossing the inner boundary is roughly isotropic. The $dM/d\log_{10} l$ pdf with respect to $|l|$ is truncated at higher $|l|$, cutting off sharply at $|l| \approx 4 \times 10^{28} \text{ cm}^2 \text{s}^{-1}$, which corresponds to the specific angular momentum at the inner boundary ($l_{in}$; see Fig. 2); the gas with larger angular momentum cannot overcome the centrifugal barrier and fall in through the inner boundary. The accretion rate pdfs in Figure 3 and the mass pdfs in Figure 2 have similar slopes at low $l$ with respect to $l_x$, $l_y$, $l_z$, and $|l|$.

The left panel of Figure 4 represents the rotational direction ($\cos \theta = l_z/|l|$) of cold gas at all times in the fiducial NFW run. The right panel shows the corresponding rms $\cos \theta = (l_z^2/|l|^2)^{1/2}$ ($\langle \rangle$ stands for mass-weighted averaging). The blue solid (black dashed) line is the time-averaged, mass-weighted orientation of cold gas within (crossing) the computational domain. The gray shaded region shows the $1\sigma$ scatter of cold gas within the computational domain while the yellow shaded region represents the $1\sigma$ scatter of the cold gas crossing the inner boundary. The cold gas with $|l| \lesssim 10^{28} \text{ cm}^2 \text{s}^{-1}$ has equal scatter around the value expected for the mean and rms angle between $l$ and the $z$-axis for isotropic distribution ($\langle \cos \theta \rangle = 0$ and $\langle \cos^2 \theta \rangle^{1/2} = 1/\sqrt{3} \approx 0.58$), implying that the time-averaged angular momentum distribution of the cold gas with small angular momentum within the simulation domain and crossing the inner boundary is almost isotropic. The larger angular momentum gas, on the
other hand, has a clockwise bias as also seen in Figures 2 & 3.

3.2.3 Time dependence of angular momentum pdf

Now that we have shown that the time-averaged angular momentum distribution of cold gas with $|I| \lesssim 10^{28}$ erg s$^{-1}$ is roughly isotropic, we want to study its time dependence. In particular, stochastic accretion of cold gas is a viable solution of the cooling flow problem only if the low angular momentum cold gas changes direction on a time scale shorter than the core cooling time (taken to be 0.2 Gyr as a conservative estimate; for a longer cooling time, more cold gas can power SMBH accretion). If the angular momentum variability time scale is shorter, the cold gas can undergo angular momentum cancellation (and hence accretion) faster than the core cooling time.

Figure 5 shows the orientation of the average mass-weighted angular momentum distribution of cold gas crossing the inner radial boundary $|I_r/L|$, and $|I| < 10^{29}$ cm$^2$s$^{-1}$ and $|I| < 10^{28}$ cm$^2$s$^{-1}$ cold gas are shown. The $I_r/L$ ratios of the total cold gas and with $|I| < 10^{29}$ cm$^2$s$^{-1}$ show that this gas only changes its orientation (as measured by cos $\theta$ crossing zero) till only 1.5 Gyr, and attain a fixed sense of rotation thereafter. Even though this cold gas (dominated by large angular momentum gas) shows small fluctuations on short time scales (< 30 Myr), it still maintains its orientation for the rest of the simulation time. Time of 1.5 Gyr coincides with the formation of the massive cold torus in the fiducial simulation (see Fig. 3 in PSB15). The rms $I_r^{2}/|I|$ value for this cold gas remains well above 0.58, suggesting that most of it settles down in a disk-like structure. Note that both $I_r/L$ and rms $(I_r^{2}/|I|)^{1/2}$ (see the purple dashed lines with markers in Fig. 5) are much more variable for low angular momentum cold gas ($|I| < 10^{28}$ cm$^2$s$^{-1}$) as compared to the total cold gas. The mean angle between $I$ and the $z-$axis $(\langle \cos \theta \rangle)$ fluctuates around zero and the rms $(\cos^{2} \theta)$ fluctuates around the isotropic value (0.58) on a timescale shorter than 0.2 Gyr. Therefore, only the low angular momentum gas ($|I| < 10^{28}$ cm$^2$s$^{-1}$) is expected to participate in feedback heating of the ICM.

Results from our AGN jet-ICM simulations (Figure 5) show that cold gas accretion at $\lesssim 1$ kpc is stochastic, with the angular momentum of cold gas with $|I| < 10^{28}$ cm$^2$s$^{-1}$ changing on time scales shorter than the core cooling time. The mixing and inelastic cloud-cloud collisions of these randomly oriented cold gas clouds near the regularization radius will result in angular momentum cancellation. After losing angular momentum these clouds will fall ballistically to the centre, where they ought to establish a compact turbulent accretion flow with $H/R \gtrsim 0.1$ (see Hobbs et al. 2011) and viscous time shorter than 0.2 Gyr, the cooling time of the cluster core. From our simulation results, a specific angular momentum $\lesssim 10^{28}$ cm$^2$s$^{-1}$ can be taken as the limit for stochastic cold accretion (see Figs. 4 & 5).

3.3 SMBH accretion estimates

In this section we estimate the mass accretion rate onto the central SMBH, first using the standard steady accretion flow model and then using the more plausible stochastic cold gas accretion. For the standard flow model estimate, we assume that cold gas forms a thick extended disk before accreting onto the central SMBH on the local viscous time at the circularization radius. For stochastic cold accretion, the angular momentum of the randomly oriented cold gas clouds cancel each other and the clouds settle down in a compact accretion flow very close to central SMBH. In this scenario accretion happens at the dynamical time (roughly the timescale for angular momentum cancellation) because instead of relying on viscosity to transport out angular momentum, accretion is powered by angular momentum cancellation among infalling clouds with random orientations.

Typical efficiency required with respect to the mass accretion rate $\dot{M}_{\text{in}}$ at $1$ kpc to suppress the cooling rate to at least 10% of the pure cooling flow value, $\dot{M}_{\text{cf}}$, is $\sim 10^{-4}$. Our cluster mass is $M_{200} = 7 \times 10^{14} M_{\odot}$; this $\epsilon \equiv P_{\text{jet}}/M_{\text{enc}}$ is expected to be smaller for lower mass halos (see Fig. 8 in PSB15), where $P_{\text{jet}}$ is the jet power. This means that, assuming a SMBH mechanical efficiency of 10% ($P_{\text{jet}} = 0.1 M_{\odot} c^{2}$), $\dot{M}_{\text{BH}} \gtrsim 10^{-3} M_{\odot}$ is required to sufficiently suppress a cooling flow; i.e., at least 0.1% of the cold gas infalling at 1 kpc must be accreted by the SMBH on a timescale shorter than the core cooling time. Thus, for our fiducial run (for which $\dot{M}_{\text{in}} \approx 20 M_{\odot}$ yr$^{-1}$; see Table 1) average $\dot{M}_{\text{BH}} \gtrsim 0.02 M_{\odot}$ yr$^{-1}$ is required to suppress a cooling flow.

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3.3.1 Viscosity-mediated standard accretion flow estimate

The time scale on which the gas crossing the inner boundary reaches the SMBH is determined by the nature of the accretion flow. If the accreting cold gas has zero angular momentum it falls at the free-fall rate, but in presence of angular momentum, the time for matter to cross the inner boundary and reach the SMBH is the sum of the local free-fall time at the inner radius \( t_{\text{in}} \) and the viscous timescale at the circularization radius \( t_{\text{vis}} \). Using these considerations, we can estimate the viscosity-mediated standard accretion flow (SAF) rate onto the SMBH, using the mass distribution of the cold gas crossing the inner boundary \( (dM/dt) \) here \( l = |l| \), as

\[
\dot{M}_{\text{BH,SAF}} \approx \int_{\text{ISCO}}^{l_{\text{max}}} dM \frac{dM}{dt} \frac{t_{\text{in}}}{t_{\text{in}} + t_{\text{vis}}},
\]

where \( l_{\text{max}} \) is the maximum angular momentum of the cold gas that can contribute to accretion and \( l_{\text{ISCO}} \) is the angular momentum corresponding to the innermost stable circular orbit (ISCO), which for a Schwarzschild blackhole is, \( r_{\text{ISCO}} = 6GM_{\text{BH}}/c^2 \approx 2.9 \times 10^{-4} M_{\text{BH}} \, \text{pc} \). An appropriate upper angular momentum cut-off \( l_{\text{max}} \) is the angular momentum for which the viscous accretion time equals 0.2 Gyr, the core cooling time (we show later in this section that \( \dot{M}_{\text{BH,SAF}} \) is rather insensitive to \( l_{\text{max}} \)). From Eq. 3, the specific angular momentum for a viscous time of 0.2 Gyr is \( 0.48 \times 10^{28} \, \text{cm}^2\text{s}^{-1} \). Gas with a larger angular momentum accretes on a time scale longer than the core cooling time and hence cannot stop a cooling flow. Moreover, matter with angular momentum smaller than \( l_{\text{ISCO}} \) falls directly into the SMBH with no way to extract its gravitational potential energy.

The low angular momentum distribution of the gas
crossing the inner simulation boundary (with respect to |l|; see Fig. 3) can be roughly approximated as \( dM(l)/dl_{28} \approx 0.56 M_\odot \text{yr}^{-1} l_{28} (l_{28} = l/10^{28} \text{cm}^2 \text{s}^{-1}) \). Eq. 4, with this approximation, can be written as

\[
M_{BH,SAF} \approx 0.56 M_\odot \text{yr}^{-1} \int_{l_{isco,28}}^{l_{max,28}} dl_{28} \frac{l_{28}}{1 + t_{visc}(l_{28})/t_{r,in}}.
\]

From Eq. 3, we can write

\[
t_{visc}(l_{28})/t_{r,in} = 120 \left( \frac{t_{r,in}}{15 \text{Myr}} \right)^{-1} \alpha_{0.1}^{-1} \left( \frac{H}{R} \right)^{-1} \frac{M_{BH,SAF} l_{28}}{10}. \]

Taking \( l_{max,28} = 0.48 \), which corresponds to a viscous time equal to the core cooling time (Eq. 3) for the fiducial parameters,\(^2\)

\[
M_{BH,SAF} \approx 0.56 \int_{1.5 \times 10^{-3}}^{0.48} dl_{28} \frac{l_{28}}{1 + 120 l_{28}^2} \approx 0.018 M_\odot \text{yr}^{-1}.
\]

This value is uncomfortably close to the minimum value required to prevent a cooling flow (\( t_{r,in}/0.1 \approx 0.02 M_\odot \text{yr}^{-1} \)). Moreover, we have used a large \( H/R = 0.1 \) instead of the standard thin AGN disk value of \( H/R \approx 10^{-3} \). Using \( H/R = 10^{-3} \) gives \( l_{max,28} = 0.022 \) and \( M_{BH,SAF} = 1.3 \times 10^{-3} M_\odot \text{yr}^{-1} \), way too small compared to the required accretion rate. The value of the \( \alpha \) viscosity parameter is uncertain, with the variability observations giving an order of magnitude larger value (\( \gtrsim 0.1 \)) compared to MHD simulations without net flux (see King et al. 2007 and references therein). A smaller \( \alpha \) makes the case for stochastic cold accretion (SCA) even stronger. We list the actual value of the integral in Eq. 4 (rather than making a power-law approximation) in Table 1 for \( H/R = 0.1 \).

### 3.3.2 Stochastic cold accretion (SCA) estimate

Accretion for the stochastic cold gas (Figs. 4 & 5) show that \(|l| < 10^{26} \text{cm}^2 \text{s}^{-1}\) cold gas is stochastic in space and time over 0.2 Gyr) is expected to proceed at almost the local free-fall time till the gas settles in a thick disk close to the ISCO where the viscous time is very short compared to the free fall at the inner boundary of our simulations, \( t_{r,in} \). From Eq. 3, the viscous accretion time for \( l = 2 \times 10^{27} \text{cm}^2 \text{s}^{-1} \) is 14 Myr. We assume that the stochastic cold gas with initial \( l \leq 10^{28} \text{cm}^2 \text{ s}^{-1} \) cancels its angular momentum to \( \lesssim 2 \times 10^{27} \text{cm}^2 \text{ s}^{-1} \) on a timescale shorter than 15 Myr (the free-fall time at the inner boundary). The stochastic cold accretion (SCA) rate, therefore, can be approximated as

\[
M_{BH,SCA} \approx \int_{l_{isco}}^{l_{max}} dl M/dl,
\]

where \( l = |l| \). This estimate assumes that most of the mass accreting onto the SMBH originates as cold gas beyond \( \sim 1 \) kpc. Not much gas condenses within 1 kpc because at most times \( t_{isco}/t_{r,in} \gg 10 \) at these radii. Moreover, the available hot gas mass (out of which cold gas can condense) decreases inward. We take \( l_{max} = 10^{28} \text{cm}^2 \text{s}^{-1} \) as the upper cut-off of the integral in Eq. 8 because cold gas is stochastic only for angular momentum lower than this.

\(^2\) Eq. 7 is not sensitive to the upper cut-off as \( M_{BH,SAF} \propto l_{28}^{-1} \).

---

**Figure 6.** Mass accretion rate estimates as a function of time based on standard accretion flow (SAF in black thick solid line; Eq. 4) and stochastic cold accretion (SCA in green dashed line; Eq. 8) for our fiducial run. Also shown in magenta solid lines is the expected mass accretion rate onto the SMBH assuming that the SMBH accretion rate is \( c M_{BH}/0.1 \approx 6 \times 10^{-4} M_\odot \). Red dot-dashed line is the jet power (see section 4.3 on how we calculate the jet/cavity power) as a function of time. All quantities show large variations because of heating/cooling cycles driven by condensation/overheating. The accretion rate at \( r_{in} \) is less time variable compared to \( M_{SCA} \) and \( M_{SAF} \) but overall trends are similar.

The integral in Eq. 8 can be approximated as \( M_{BH,SCA} \approx 0.2 M_\odot \text{yr}^{-1} l_{max,28} \) (see Table 1 for the exact value) for our fiducial run for which \( dM/dl_{28} \approx 0.56 M_\odot \text{yr}^{-1} l_{28} \). Note that the SCA estimate is independent of disk parameters (\( \alpha \& H/R \)) as we assume that accretion in the SCA regime happens at the local free-fall time (which is shorter than the free-fall time at the inner boundary). Unlike viscosity-mediated standard accretion flow (SAF) integral (Eq. 4) which is rather insensitive to \( l_{max} \), SCA integral (Eq. 8) increases quadratically with \( l_{max} \). Note that the SMBH mass accretion rate in the SCA regime is comfortably larger than the required \( M_{BH} \approx 0.02 M_\odot \text{yr}^{-1} \) to quench a cooling flow. In fact, we can also comfortably accommodate the reduction of \( M_{BH} \) in the RIAF regime due to outflows.

Figure 6 shows the jet power, and the mass accretion rates estimated from standard accretion flow (SAF; Eq. 4 with \( H/R = 0.1 \)) and stochastic cold accretion (SCA; Eq. 8) as a function of time. As expected, the stochastic cold accretion (SCA) mass accretion rate is higher on average as compared to viscosity-mediated standard accretion flow (SAF). Also \( M_{SCA} \), in comparison to \( M_{SAF} \), is variable on a shorter time. Notice the spikes in \( M_{SCA} \), in the cooling phases of the core and the lack of stochastic accretion when jets overheat the core. Similar trends are also seen for \( M_{in} \); i.e., all accretion rates are higher in the cooling phases when jet power is small and suppressed in the overheating phases. We discuss the application of stochastic cold accretion estimate based on the angular momentum distribution of cold gas as the basis of a more robust feedback prescription in section 3.4.
3.4 Improving cold feedback prescription

Table 1 lists important time-averaged quantities from our numerical simulations. The accretion rate estimates based on the standard accretion flow (SAF; Eq. 4 with $H/R = 0.1$) and stochastic cold accretion (SCA; Eq. 8 with $l_{max} = 4.8 \times 10^{27}$ cm$^2$ s$^{-1}$) are included, as is the average rate of mass crossing the inner boundary. The NFW run with $r_{in} = 0.5$ kpc but the same $c$ as the fiducial NFW run (with $r_{in} = 1$ kpc) shows that the average mass accretion (hot + cold) through the inner boundary is reduced by a factor of $\approx 2$ ($11.6 \, M_\odot$ yr$^{-1}$ as opposed to $21.6 \, M_\odot$ yr$^{-1}$ for the fiducial run) but the cold gas mass accumulating within the computational domain (dominated by the high angular momentum gas; e.g., see Fig. 2) by the end of the simulation is about five times higher. In fact, the average cold gas mass accumulation rate for the $r_{in} = 0.5$ kpc run is $5 \times 10^{11} M_\odot/5$ Gyr $\sim 100 M_\odot$ yr$^{-1}$, only a factor of 2 lower than the pure cooling flow rate. A lot of cold gas crossing 1 kpc circularises before crossing the inner boundary at 0.5 kpc (compare the angular momentum pdfs and $l_{in}$ for different runs in the right panel of Fig. 2), and the mass accretion rate at $r_{in} = 0.5$ kpc ($M_{in}$) is much smaller than the average mass accretion/deposition rate. Importantly, a smaller $M_{in}$ for a fixed $\epsilon$ gives a much lower feedback heating, allowing a large amount of cold gas with sufficient angular momentum to accumulate within the computational domain. The NFW+BCG simulation also uses $r_{in} = 0.5$ kpc but with a larger feedback efficiency ($\epsilon = 5 \times 10^{-4}$), and therefore the mass of the accumulated cold gas is reasonable. These numbers illustrate the weakness of the current feedback models in which the suitable $\epsilon$ depends, rather sensitively, on the radius at which $M_{in}$ is measured.

Instead of measuring $M_{in}$ at an arbitrary inner radius and tuning $\epsilon$ to get a reasonable match to observations as we are presently doing, our simulations suggest a more reliable way to estimate the SMBH accretion rate based on the mass distribution of low angular momentum cold gas (Eq. 8 with $|l| \lesssim 10^{28}$ cm$^2$ s$^{-1}$). Table 1 shows that $M_{SCA}$ for the NFW run with $r_{in} = 0.5$ kpc is about 6.4 times larger than the NFW run with $r_{in} = 1$ kpc but $M_{in}$ is about a factor of two smaller for the former. This implies that $M_{BH}$ estimate based on the angular momentum distribution of cold gas is more robust because a larger mass accretion rate (as obtained from SCA) will prevent excessive cooling and mass deposition seen in the NFW run with $r_{in} = 0.5$ kpc. Therefore, we anticipate similar outcomes (similar cold gas mass, small scale accretion rate, etc.) for feedback simulations with identical $\epsilon$s if the SMBH accretion rate is estimated using the angular momentum distribution of the cold gas. In future we plan to carry out such simulations.

While the results in this section strongly suggest the importance of stochastic cold feedback in maintaining thermal equilibrium in cluster cores, our simulations have several limitations like the absence of star formation, magnetic fields and anisotropic thermal conduction. In the absence of stellar feedback and gas consumption by star formation we see a build up of a massive cold gas torus. The next section explores the implications of cold gas depletion due to star formation, via post-processing using a simple model. In the following section, we also compare our cold mode feedback results with Bondi/hot model and with observations.

![Figure 7](image.png)

Figure 7. Cold gas mass as a function of time in the fiducial NFW run for different mass depletion time scales due to star formation. Without depletion of cold gas due to star formation ($\tau = \infty$; this is the same as the cold gas mass as a function of time in Fig. 9 of PSB15), cold gas secularly builds up with time. Depletion of cold gas due to star formation leads to cycles in cold gas mass as a function of time.

4 COLD VERSUS HOT FEEDBACK: SIMULATIONS CONFRONT OBSERVATIONS

In this section we compare the results of cold feedback simulations with salient observations.

4.1 Gas depletion due to star formation

Unlike Li et al. (2015), our numerical simulations do not allow for the depletion of cold gas due to star formation. This leads to a secular build up of cold gas to unrealistically large values. We can account for the influence of star formation on cold gas depletion using a simple post-processing model. The cold gas mass in presence of cold gas depletion due to star formation is given by

$$\frac{dM_{\text{cold}}}{dt} = \dot{M}_{\text{cond}} - \frac{M_{\text{cold}}}{\tau},$$

where $M_{\text{cold}}$ is the total cold mass in the computational domain, $\dot{M}_{\text{cond}}$ is the rate of condensation of cold gas from the hot ICM (i.e., the derivative of the $\tau = \infty$ line in Fig. 7), and $\tau$ is the cold gas depletion time scale (due to star formation). Therefore, the star formation rate $dM_{*,}/dt = M_{\text{cold}}/\tau$. While in reality, star formation and associated feedback will impact the surrounding gas distribution, we do not take this into account as stellar feedback is subdominant on global scales in cluster cores (although it may help drive local turbulence in the cold gas; e.g., Hobbs et al. 2011). For a constant $\tau$, Eq. 9 can be solved analytically to give

$$M_{\text{cold}}(t) = \int_0^t e^{-(t-t')/\tau} \dot{M}_{\text{cond}}(t')dt'.$$
4.2 Cold gas mass, SMBH and halo mass

The mass accretion rate onto the SMBH in hot/Bondi accretion is strongly dependent on the mass of the SMBH (Eq. 1). In the standard Bondi scenario, one does not normally consider the gas to cool to very low temperatures (e.g., Quataert & Narayan 2000). Cold gas in cluster cores can be naturally understood as a result of local thermal instability in the core satisfying rough global thermal balance. Under the assumption that the cold gas mass in the Bondi scenario is proportional to the SMBH accretion rate, the cold gas mass in cluster cores is expected to be $\dot{M}_{\text{BH}} T_{\text{BH}}^{-3/2}$ (i.e., strongly dependent on SMBH and halo properties). Figure 8, adapted from McNamara, Rohanizadegan, & Nulsen (2011), shows the cold gas mass as a function of the SMBH mass for different groups and clusters. The scatter in cold gas mass is very large given the rather uniform core entropies observed in cool cores, unlike the expectation from the Bondi accretion rate. Also the observations do not show a strong correlation (anticorrelation) between the SMBH mass (cluster temperature; indicated by the colors of markers) and the mass of molecular gas. In fact, the largest molecular gas mass occurs in the hottest cluster, unlike what is expected from $T_{\text{BH}}^{-3/2}$ scaling of $\dot{M}_{\text{BH}}$ (Eq. 1; this assumes that the temperature of the hot gas at the Bondi radius scales with the core X-ray temperature; X-ray spectra do not show X-ray emitting gas below 0.3 the cluster temperature; e.g., Peterson et al. 2003; see also Fig. 1a of Hogan et al. 2016). Similarly, in Figure 8 the data point with the smallest SMBH mass has a large cold gas mass. There is a large scatter in cold gas mass, irrespective of the SMBH mass and the cluster temperature, a hallmark of cold mode feedback that is stochastic and shows cooling and heating cycles (Prasad et al. 2015; Li et al. 2015).

Figure 8 in PSB15 and Figures 2 & 7 in Li et al. (2015) show that the cold gas mass, for the same SMBH and halo, are highly time variable and show a broad distribution. In fact, at times, one may not see any cold gas because star formation and stellar feedback can, in principle, consume all the cold gas before it is replenished by ICM cooling (this is seen in Fig. 2 of Li et al. 2015, who explicitly model star formation and stellar feedback in their simulations, as well as in our Fig. 7, where we attempt to infer the impact of star formation, but not stellar feedback, on the cold gas mass in our simulation during post-processing). The range of cold gas mass seen in our numerical simulations is indicated by the three vertical intervals on the right in Figure 8. Even without star formation and stellar feedback, the lower end of the cold gas mass distribution in our simulations overlaps with the observed range of cold gas mass in cluster cores. Typically, however, the cold gas mass is too high by up to two orders of magnitude. Allowing for gas depletion by star formation on timescales of 0.2–0.5 Gyr during post-processing (Fig. 7) brings our gas mass into agreement with the observations. Nonetheless, this is an issue that requires further study, and we are in the process of explicitly including a realistic treatment of star formation and stellar feedback, as well as allowing for the effects of quasar-mode feedback that may occur when the AGN occasionally switches to that mode, in our simulations.

Figure 8. Molecular gas mass in centres of cool groups and clusters, obtained using CO observations, as a function of the SMBH mass. The filled circles represent detections and the triangles are upper limits. The cluster temperature is represented by color (obtained from Cavagnolo et al. 2009). There is a large scatter in the molecular gas mass for a given SMBH mass. Three vertical lines on extreme right denote the range of cold gas mass in our three simulations (measured from 1 Gyr till the end of the run, without accounting for cold gas depletion). A vertical line and an upper limit in extreme left are obtained from the simulations of Li et al. (2015) with their feedback efficiency of 0.01 and 0.001 (bottom panels of their Figs. 2 & 7). Our model for cold gas depletion (see Eq. 9 & Fig. 7) can also reduce the cold gas mass (of course at the expense of creating more stars).

Figure 7 shows the cold gas mass as a function of time for different cold gas mass depletion time scales ($\tau$). For a gas depletion time scale $\tau \lesssim 0.5$ Gyr the cold gas mass is always $\lesssim 5 \times 10^{10} M_{\odot}$ and most of the gas is channeled into stars. For $\tau \gtrsim 0.2$ Gyr, there are times when there is no cold gas present in the cluster core. Cold gas mass in our simulations is on the higher side and would lead to an average star formation rate of $\approx 10^{11} M_{\odot}/(5 \text{ Gyr}) = 20 M_{\odot}\text{yr}^{-1}$. This rate is comparable to that seen in some of the BCGs in cool core clusters (e.g., Bildfell et al. 2008; Mittal et al. 2015; Loubser et al. 2016) but is on the higher side of the the observed distribution. This is understandable because in PSB15 we deliberately chose the accretion efficiency of $\epsilon = 6 \times 10^{-5}$, the minimum value required to bring down the cooling rate by a factor of 0.1 with respect to cooling flow in absence of AGN feedback. This suggests that the feedback efficiency (with respect to the accretion rate at 1 kpc) in typical clusters is between $10^{-4} - 10^{-3}$. Accretion efficiency much above $10^{-3}$ (as in Gaspari et al. 2012; Li et al. 2014) leads to strong feedback that wipes out cold gas at radii $> 10$ kpc at late times, and maintains the cluster in the hot state for most of the time.
4.3 Bondi power and cavity power

Allen et al. (2006) noted a tight correlation between the Bondi power and the cavity power for nearby elliptical galaxies and argued for the validity of Bondi accretion. Russell et al. (2013) recently refined the determination of Bondi and cavities powers from Allen et al.’s sample and included more data points, and found that the correlation is much weaker. Figure 9 shows the Bondi-power cavity-power relationship obtained for our fiducial NFW run where, as we have discussed, the jet is in fact triggered by cold gas condensation, not Bondi accretion. Bondi power \( P_B = 0.1 \dot{M}_B c^2 \), where \( \dot{M}_B \) is given by Eq. 1, is calculated using the hot gas density profile in our domain, extrapolated to the Bondi radius,

\[
R_B \equiv 2GM_B/c_s^2_{\infty} \approx 35 \text{ pc } M_{\odot}/T_{\infty}^{0.5} \text{ keV}. \tag{11}
\]

Emulating what Russell et al. (2013) do with the observational data, we also use two different best-fit models for the hot gas density applied from SMBH. At most times Bondi power is within \( 10^{43} - 10^{44} \text{ erg s}^{-1} \), but cavity power has a much larger variation. A similar behaviour is seen in our other runs listed in Table 1. Data points from Russell et al. (2013) and the best-fit from Allen et al. (2006) are also shown.

5 DISCUSSION

The problems with Bondi accretion (hot accretion in general) are well recognised (see section 1). Standard viscous mediated accretion flows onto SMBHs, even with a large \( H/R \sim 0.1 \), leads to the angular momentum problem; namely, the viscous accretion time scale for the accretion flow is much longer than the cooling time of the cluster core. Therefore, enough gas cannot accrete fast enough and provide sufficient feedback power to prevent a strong cooling flow.

A number of published studies have invoked the stochastic nature of the condensing cold gas and its fundamentally different behaviour compared to the standard accretion flow to overcome the angular momentum problem (Pizzolato & Soker 2010; Hobbs et al. 2011; Nayakshin et al. 2012; Gaspari et al. 2013). Through 3-D hydrodynamic simulations, we explicitly demonstrate that the angular momentum of cold gas condensing out of the hot phase indeed varies stochastically and has an isotropic distribution (especially for the low specific angular momentum, \( l \lesssim 10^{26} \text{ cm}^2\text{s}^{-1} \), cold gas). Cancellation of almost randomly directed angular momentum on short timescales leads to the formation of

Figure 9. Bondi power (estimated using two different extrapolations of the spherically angle averaged hot gas \( [T > 0.1 \text{ keV}] \) density profiles to the Bondi radius) as a function of cavity power from our fiducial NFW run. Simulation data is sampled every 10 Myr and there is a large scatter for both extrapolations. Also shown as green triangles is the instantaneous accretion power \( \dot{e} M_{\text{in}} c^2 \) supplied by the SMBH. At most times Bondi power is within \( 10^{43} - 10^{44} \text{ erg s}^{-1} \), but cavity power has a much larger variation. A similar behaviour is seen in our other runs listed in Table 1. Data points from Russell et al. (2013) and the best-fit from Allen et al. (2006) are also shown.
a compact accretion flow that accretes onto the SMBH on the much shorter dynamical time scale (compare Eqs. 4 & 8). Recent ALMA observations of infalling cold, clumpy CO clouds in the elliptical galaxy at the centre of Abell 2597 (Tremblay et al. 2016), within 100 pc of the SMBH, provides an almost direct evidence for stochastic cold accretion (SCA). Nayakshin et al. (2012) go onto argue that even in the higher $M$ regime, SCA is responsible for the emergence of the SMBH $M_{\text{BH}} \sim \sigma$ relation because accretion via a standard extended disk results in more massive SMBHs than observed since a massive extended thin disk does not couple efficiently to an isotropic feedback-powered outflow. Turbulence due to stochastic angular momentum and infall of cold gas (viscous time is much shorter for a turbulent flow with $H/R \sim 0.1$) also prevents the problem of gravitational fragmentation in stochastic accretion flow.

Most of the earlier studies of stochastic cold accretion were carried out in numerical setups with turbulence in the cold gas driven artificially (to realise the stochastic distribution of cold gas angular momentum). However, in this paper we have shown that the turbulence emerges self-consistently from jet-ICM interactions and this behaviour is sustained over timescales of several Gyr (i.e., much longer than the cluster core cooling time). We analyze the angular momentum distribution of cold gas in our AGN jet-ICM simulations (which appear consistent with most observations of cool cluster cores; see section 4) and demonstrate that stochastic cold accretion is indeed realised in cool cluster cores stirred by time-dependent AGN jets.

In cluster simulations with limited resolution, the accretion physics input necessarily has to be inserted via a sub-grid model. One such model, which is much better than what is typically used (including ours in this paper where we estimate $M$ at $\sim 1$ kpc and multiply it by an efficiency parameter $\epsilon$ to calculate the mechanical power injected by the SMBH), is to use the mass accretion rate of the low angular momentum gas (with say $l \lesssim 10^{28}$ cm$^2$s$^{-1}$; see Eq. 8 & Fig. 6) crossing the smallest resolved radius (section 3.4 for details). Another improvement is the inclusion of star formation and realistic stellar feedback (stellar feedback is increasingly important for lower mass halos such as clusters and elliptical galaxies; e.g., see Fig. 4 in Sharma et al. 2012b) that can prevent artificial accumulation of massive cold gas in cluster cores (e.g., as done by Li et al. 2015). Also, as mentioned earlier, our choice of $\epsilon$ (feedback efficiency parameter) is on the lower side, resulting in somewhat large mass deposition and star formation in the core. More 3-D simulations with larger $\epsilon$s are required to get a complete picture.

5.1 Uncertainties in accretion physics

Much more needs to be done in understanding the details of stochastic cold accretion (SCA). In our simulations, which focus on larger ($\gtrsim 1$ kpc) scales, we have to make assumptions about the underlying stochastic accretion. In particular, we need to understand how the stochastic cold gas condensing out of the ICM is eventually accreted onto the SMBH. Whether a disk forms after the cancellation of stochastic angular momentum and on what timescale, and at what radii does the transition from SCA to a viscous thin disk (if it does at all) and to a RIAF (radiatively inefficient accretion flow) occurs. While the transition radius from a thin disk to an inner hot/thick RIAF occurs at a smaller radius for a higher accretion rate, the details are not clear even for the standard (non-stochastic) accretion flows with a fixed angular momentum axis (e.g., see the vastly different estimates of disk to RIAF transition radius in Das & Sharma 2013 [Fig. 9b], Liu et al. 1999 [Fig. 1], Yuan & Narayan 2014 [Fig. 7b]).

Strong AGN jets and X-ray cavities (which are powered by jets) are expected only in the radiatively inefficient accretion regime for which $M_{\text{BH}} \lesssim \alpha^2 M_{\text{edd}}$ (Rees et al. 1982; Churazov et al. 2005; Nemmen et al. 2007). Our SCA accretion rate at most times is lower than this threshold for jet formation (see Fig. 6). Infrequent, short duration high $M_{\text{BH}}$ events, in fact, provide a potential basis for short-lived transition from radio mode to quasar mode and for changing the direction of the jet, as observed in numerous clusters (e.g., Babul et al. 2013). Generally the mass accretion rate onto the SMBH is expected to be even lower because of outflows in RIAFs (larger suppression is expected for bigger RIAFs). Therefore, advances in accretion physics are required to estimate the size of RIAFs and the mass accretion rate onto SMBHs fed by SCA.

To sum up, majority of the cold gas condensing out of the ICM due to thermal instability has large angular momentum such that the viscous accretion timescale is longer than the core cooling time for $l \gtrsim 10^{28}$ cm$^2$s$^{-1}$ (Eq. 3). Most of this gas is expected to be Toomre unstable and to form stars, instead of being accreted onto the SMBH. The lower angular momentum cold gas ($l \lesssim 10^{26}$ cm$^2$s$^{-1}$) has stochastic angular momentum, which implies that cloud collisions (which lead to rapid angular momentum cancellation) will cause the cold, turbulent gas to be accreted rapidly, without much gravitational fragmentation. At innermost radii typically a RIAF with powerful jets is expected, since $M_{\text{BH}} \ll 0.01 M_{\text{edd}}$.

6 SUMMARY

We finally summarise our major findings in the following points:

- Stochastic cold accretion (SCA), which was anticipated to play a key role in accretion onto SMBHs from idealised simulations, is realised naturally in our realistic AGN jet-ICM simulations due to the turbulence induced by jets in a non-uniform ICM over several Gyr time scale.
- We find that the low angular momentum cold gas ($|l| \lesssim 10^{28}$ cm$^2$s$^{-1}$) condensing out of the ICM has an isotropic distribution of angular momentum, which we expect will result in the cancellation of angular momentum on almost a dynamical time. This is unlike a viscosity-mediated standard accretion flow, in which an extended thin disk with a long accretion time is expected to form. Another advantage of a stochastic accretion flow is that the net angular momentum of the low angular momentum cold gas varies on a timescale shorter than the core cooling time, implying that angular momentum cancellation and feedback driven by cold gas accretion can respond fast enough. Furthermore, SCA is independent of disk parameters like $\alpha$ and $H/R$. Elimination of these parameter (usually with a large range) makes SCA a simpler and more robust model.
Our work suggests an improved feedback prescription based on the angular momentum distribution of cold gas (section 3.4), which may be more robust than the usual models in which the mass accretion rate is estimated at ~ kpc scales. In the latter approach one has to fine tune the feedback efficiency parameter (ε) for different radii at which the mass accretion rate is estimated (compare our NFW runs with the inner radius of 1 and 0.5 kpc).

Most of the features of cold accretion simulations, which show multiphase cooling and heating cycles driven by accretion of cold gas, match observations such as a large scatter of cold gas mass when compared to the SMBH and halo masses (this is not expected for hot/Bondi accretion; see section 4.2). A large scatter observed between Bondi power and jet/cavity power is also consistent with our cold feedback simulations (see section 4.3).

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REFERENCES

Allen, S. W., Dunn, R. J. H., Fabian A. C., Taylor G. B., & Reynolds C. S. 2006, MNRAS, 372, 21
Babul, A., Sharma, P. & Reynolds, C. S. 2013, ApJ, 768, 11
Baganoff, F. K., Maeda, Y., Morris, M., et al. 2003, ApJ, 591, 891
Bildfell, C., Hoekstra, H., Babul, A., & Mahdavi, A. 2008, MNRAS, 389, 1637
Bondi, H. 1952, MNRAS, 112, 195
Cavagnolo, K. W., Donahue, M., Voit, G. M., & Sun, M. 2009, ApJS, 182, 12
Churazov, E., Sazonov, S., Sunyaev, R., Forman, W., Jones, C., & Böhringer, F. 2005, MNRAS, 363, L91
Choudhury, P. P. & Sharma, P. 2016, MNRAS, 457, 2554
Das, U. & Sharma, P. 2013, MNRAS, 435, 2431
Dubus, G., Hameury, J.-M., & Lasota, J.-P. 2005, A&A, 435, 251
Fabian, A. C. 2012, ARA&A, 50, 455
Frank, J., King, A., & Raine, D. 2002, “Accretion Power in Astrophysics: Third Edition,” Cambridge University Press
Gaspari, M., Ruszkowski, M., & Sharma, P., 2012, ApJ, 746, 94
Gaspari, M., Ruszkowski M., & Oh, S. P., 2013, MNRAS, 432, 3401
Gaspari, M., Brighenti, F., & Temi, P., 2015, Astron. & Astrophys., 579, A62
Gaspari, M., Temi, P., & Brighenti, F. 2017, MNRAS, 466, 677
Goodman, J. 2003, MNRAS, 339, 937
Hayes, J. C. et al. 2006, ApJS, 165, 188
Hobbs, A., Nayakshin, S., Power, C., & King, A. 2011, MNRAS, 413, 2633
Hogan, M. T. et al. 2016, preprint arXiv:1610.04617
Hogan, M. T. et al. 2017, preprint arXiv:1704.00011
Hopkins, P. F., & Hernquist, L. 2006, ApJ, 666, 1
King, A. R., Pringle, J. E., & Livio, M. 2007, MNRAS, 376, 1740
Li, Y. & Bryan, G. L. 2014, ApJ, 789, 54
Li, Y., Bryan, G., Ruszkowski, M., Voit, G. M., O’Shea, B. W., & Donahue, M. 2015, ApJ, 811, 73
Liu, B. F., Yuan, W., Meyer, F., Meyer-Hofmeister, E., & Xie, G. Z. 1999, ApJ, L17
Loubser, S. I. et al. 2016, MNRAS, 456, 1565
Macchetto, F. et al. 1997, ApJ, 489, 579
Marrone, D. P., Moran, J. M., Zhao, J., & Rao, R. 2007, ApJ, 654, L57
McCourt, M., Sharma, P., Quataert, E., & Parrish, I. J. 2012, 419, 3319
McDonald, M., Veilleux, S., Rupe, D. S. N., & Mushotzky, R. 2010, ApJ, 721, 1262
McNamara, B. R. & Nulsen, P. E. J. 2007, ARA&A, 45, 117
McNamara, B. R., Rohanizadeh, M., & Nulsen, P. E. J. 2011, ApJ, 727, 39
McNamara, B. R., Nulsen, P. E. J. 2012, New J. Phys., 14, 055023
Mignone, A., Bodo, G., Massaglia, S., Matsuoka, T., Tesileanu, O., Zanni, C., & Ferrari, A. 2007, ApJS, 170, 228
Miller, K. A., & Stone, J. M. 2000, ApJ, 534, 398
Mittal, R., Whelan, J. T., & Combes, F. 2016, 450, 2564
Monin, A. S., & Yaglom, A. M., Statistical Fluid Mechanics, Dover Ed Edition (1971)
Nayakshin, S. & King, A. 2007, arxiv.org/abs/0705.1686
Nayakshin, S., Power, P. & King, A. 2012, ApJ, 753, 15
Nemmen, R. S., Bower, R. G., Babul, A & Storchi-Bergmann, T., 2007, MNRAS, 377, 1652
Nemmen, R. S. & Tchekhovskoy, A. 2015, MNRAS, 449, 316
Peterson, J. R., Kahn, S. M., Faerels, F. B. S. et al. 2003, ApJ, 590, 207
Pizzolato, F. & Soker, N. 2005, ApJ, 632, 821
Pizzolato, F. & Soker, N. 2010, MNRAS, 408, 961
Prasad, A., Sharma P., & Babul A 2015, ApJ, 811, 108 (PSB15)
Quataert, E. & Narayan, R. 2000, ApJ, 528, 236
Rees, M. J., Begelman, M. C., Blandford, R. D., & Phinney, E. S. 1982, Nature, 295, 17
Russell, H. R., McNamara, B. R., Edge, A. C., Hogan, M. T., Main, R. A., & Vantyghem, A. N. 2013, MNRAS, 432, 530
Russell, H. R., Fabian, A. C., McNamara, B. R., & Broderick, E. A. 2015, MNRAS, 451, 588
Shakura, N. I. & Sunyaev, R. A. 1973, Astron. & Astrophys., 24, 337
Sharma P., McCourt M., Quataert E., & Parrish I. J., 2012a, MNRAS, 420, 3174
Sharma P., McCourt M., Parrish I. J., & Quataert E. 2012b, MNRAS, 427, 1219
Soker, N., Sternberg, A., & Pizzolato, F. 2009, in Heinz S., Wilcots E., eds, AIP Conf. Ser. Vol. 1201, The Moderate Cooling Flow Model and Feedback in Galaxy Formation. Am. Inst. Phys., Melville, NY, p. 321
Soker, N. 2016, New Astronomy Reviews, in press, eprint arXiv:1605.02672
Shu, F. H. 1992, The Physics of Astrophysics, Volume II, Gas Dynamics (Sausalito: University Science Books)
Tremblay, G. R. et al. 2016, nature, 534, 218
Voit, G. M., Donahue, M., Bryan, G. L., & McDonald, M. 2015, Nature, 519, 203
Wong, K.-W., Irwin, J. A., Yukita, M., et al. 2011, ApJ, 736, L23
Wong, K.-W., Irwin, J. A., Shcherbakov, R. V., et al. 2014, ApJ, 780, 9
Yang, H.-Y. K. & Reynolds, C. S. 2016, ApJ, 829, 90
Yuan, F. & Narayan, R. 2014, ARA&A, 52, 529