An external-shock origin of the $E_p \propto \mathcal{E}_\gamma^{1/2}$ relation for Gamma-Ray Bursts

A. Panaitescu

Space Science and Applications, MS D466, Los Alamos National Laboratory, Los Alamos, NM 87545, USA

ABSTRACT

We investigate the possibility that the $E_p \propto \mathcal{E}_\gamma^{1/2}$ relation between the peak energy $E_p$ of the $\nu F_\nu$ spectrum and energy output $\mathcal{E}_\gamma$ for long-duration GRBs arises from the external shock produced by the interaction of a relativistic outflow with the ambient medium. To that aim, we take into account the dependence of all parameters which determine $E_p$ and $\mathcal{E}_\gamma$ on the radial distribution of the ambient medium density and find that the $E_p \propto \mathcal{E}_\gamma^{1/2}$ relation can be explained if the medium around GRBs has a universal radial stratification. For various combinations of GRB radiative process (synchrotron or inverse-Compton) and dissipation mechanism (reverse or forward shock), we find that the circumburst medium must have a particle density with a radial distribution different than the $R^{-2}$ expected for constant mass-loss rate and terminal speed.

Key words: radiation mechanisms: non-thermal - shock waves - gamma-rays: bursts

1 INTRODUCTION

Lloyd, Petrof & Mallozzi (2000) have established that the 25–1000 MeV fluence $\Phi$ of bright BATSE Gamma-Ray Bursts (GRBs) is strongly correlated with the photon energy $E_p^{(obs)}$ at which peaks the burst $\nu F_\nu$ spectral energy distribution. More recently, Sakamoto et al (2008) have shown that 83 Swift-BAT and HETE-2 bursts display a $E_p^{(obs)} \propto \Phi^{0.52\pm0.11}$ correlation, with the burst fluence measured at 15–150 keV, while Ghirlanda et al (2008) report $E_p^{(obs)} \propto \Phi^{0.32\pm0.05}$ for 76 bursts (with known redshifts), the burst fluence being calculated in the 1 keV–10 MeV range (i.e. bolometric).

Lloyd et al (2000) found that the 8 GRBs with redshifts known at that time are not standard candles and, thus, the $E_p^{(obs)} - \Phi$ correlation is not due to cosmo logical effects but is, most likely, intrinsic. In that venue, Amati et al (2000) have shown that the intrinsic peak energy $E_p$ and the isotropic-equivalent burst output $\mathcal{E}_\gamma$ at 1 – $10^4$ keV are correlated, $E_p \propto \mathcal{E}_\gamma^{0.52\pm0.06}$, for a set of 9 bursts with known redshifts (most of which are among those used by Lloyd et al 2000). Later, Amati (2006) found that $E_p \propto \mathcal{E}_\gamma^{0.54\pm0.06}$ for a set of 41 GRBs, while Ghirlanda et al (2008) arrive at $E_p \propto \mathcal{E}_\gamma^{0.54\pm0.01}$ for 76 bursts.

The lack of bursts with a high fluence and average/low peak energy bursts in the $E_p^{(obs)} - \Phi$ correlation is, evidently, not due to selection effects (i.e. at least half of that correlation is real), with the thresholds for burst triggering and measuring the peak energy possibly affecting only bursts with a high peak energy and low/average fluence. Ghirlanda et al (2008) and Nava et al (2008) investigate this possibility and conclude that selection effects are negligible for pre-Swift bursts but do truncate the distribution of Swift bursts in the $E_p^{(obs)} - \Phi$ plane. However, as the range of peak energies of Swift bursts is much narrower than that of the entire sample, they conclude that the $E_p^{(obs)} - \Phi$ is not an artifact of selection effects.

2 POSSIBLE ORIGINS FOR THE $E_p \propto \mathcal{E}_\gamma^{1/2}$ RELATION

The simplest explanation of the $E_p \propto \mathcal{E}_\gamma^{1/2}$ relation is that it arises from viewing geometry and/or relativistic effects. Such an explanation is generic, i.e. it does not make use of a certain mechanism for the production of the GRB.

In the former framework, GRBs arise from narrow jets seen at various angles $\theta$, the intrinsic burst emission being relativistically boosted by a factor $D = [\Gamma(1-\beta \cos \theta)]^{-1} \simeq 2/(\Gamma \theta^2)$, with $\Gamma$ being the jet Lorentz factor and the viewing angle $\theta$ being larger than the both the jet opening and the relativistic beaming angle $\Gamma^{-1}$. Relativistic beaming of the comoving frame emission (denoted with primed quantities) implies that the observed burst peak energy is $E_p = D E_p'$ and the inferred isotropic-equivalent GRB output is $\mathcal{E}_\gamma = D^3 \mathcal{E}_\gamma'$ (the factor $D^3$ arising from $D^2$ for angular beaming and $D$ for boost of photon energy). Hence, in this scenario, the simplest expectation is that $E_p \propto \mathcal{E}_\gamma^{1/3}$, assuming that the comoving-frame peak energy $E_p'$ and GRB output $\mathcal{E}_\gamma'$ are universal (i.e. they have same values for all bursts) or, at least, uncorrelated. Toma, Yamazaki & Nakamura (2005) obtain the $E_p \propto \mathcal{E}_\gamma^{1/3}$ analytical expectation in a more sophisticated way (for a typical GRB spectrum) but their numerical integration of the Doppler-boosted emis-
sion yields $E_p \propto \mathcal{E}_\gamma^{0.4}$ for observer offsets that are comparable to (but larger than) the jet opening, which corresponds to higher energies $E_p$ and $\mathcal{E}_\gamma$. Using an annulus geometry for the GRB outflow (i.e., a hollow jet), Eichler & Levinson (2004) obtain a relation between the apparent $E_p$ and $\mathcal{E}_\gamma$ consistent with or shallower than the $E_p \propto \mathcal{E}_\gamma^{1/2}$ relation.

A potential problem with the off-aperture jet model is the expected distribution of GRB peak photon fluxes. The average photon flux (taken as a measure for the peak photon flux) of bursts seen at an offset angle less than $\theta$ is larger than $C(\theta) \propto \mathcal{E}_{\gamma}^{\ast}/(E_{p,\gamma}(\ell_{\gamma}) \propto D^2 \gamma_{\ast}^{\ast} / (DE_{\gamma}^{\ast} / D) \propto D_2 \times \theta^{-6}$, where $t_\gamma$ is the burst duration. The number of such bursts is $N(\theta) \propto \theta^2$. Thus, the cumulative peak-flux distribution in this model is $N(>C) \propto C^{-1/3}$ (for a volume-limited sample), which is flatter than that measured by BATSE (e.g. Pendleton et al. 1996), showing $N(>C) \propto C^{-1}$ at peak fluxes between 1 and 10 photons/cm$^2$/s and $N(>C) \propto C^{-3/2}$ at peak fluxes above 10 photons/cm$^2$/s.

Relativistic beaming of the emission from a jet wider than $\Gamma^{-1}$ and seen from a location within its aperture may also be a possible origin of the $E_p \propto \mathcal{E}_\gamma^{1/2}$ relation, as both quantities of interest, $E_p$ and $\mathcal{E}_\gamma$, are affected by the source relativistic motion. In this case, $D \propto \Gamma$, $E_p = \Gamma E_{\gamma}^{\ast}$, and $\mathcal{E}_\gamma = \Gamma^2 \mathcal{E}_{\gamma}^{\ast}$ (only one power of $D$ because relativistic angular beaming also reduces the source observed angular size by a factor $D_2$ relative to that of the entire source), hence $E_p \propto \mathcal{E}_\gamma$ is expected if comoving-frame burst properties were universal or uncorrelated. Thus, the $E_p \propto \mathcal{E}_\gamma^{1/2}$ relation cannot be explained with just relativistic effects and requires a correlation of the comoving-frame peak energy $E_p^{\ast}$ and GRB output $\mathcal{E}_\gamma$ or a correlation of at least one of these quantities with the source Lorentz factor $\Gamma$ (Scheuer 2003). Evidently, progress in this direction requires that a specific mechanism for the GRB emission generation is adopted (as done below).

In that venue, Zhang & Mészáros (2002) showed that the $E_p \propto \mathcal{E}_\gamma^{1/2}$ relation may be accommodated with internal shocks, by noting that the comoving-frame magnetic field of a Poynting outflow (or that generated through shock dissipation) satisfies $B \propto L_{p,int}^{\ast}/Rt_\gamma$, where $L_{p,int}^{\ast}$ is the outflow’s Poynting flux luminosity (or that of the dissipated, internal energy) and $R$ is the radius at which the burst emission is produced. The GRB synchrotron emission peaks at $E_p \propto \gamma^2 B \propto \gamma^2 L_{p,int}^{\ast}/R$, where $\gamma m_e c^2$ is the typical electron energy in the GRB source. Thus, one obtains the $E_p \propto \mathcal{E}_\gamma^{1/2}$ relations if (1) the outflow’s Poynting (or internal energy) luminosity is a good measure of the GRB output (in the sense that the $L_{p,int}^{\ast}/\mathcal{E}_\gamma$ ratio is universal) and if (2) $\gamma$ and $R$ are universal (or not correlated with $L_{p,int}^{\ast}$).

Note that the above argument applies to any dissipation mechanism. For internal shocks, the first condition above would lead to a constraint between the history of ejecta Lorentz factors ($\Gamma(t)$) and the distribution of ejecta mass with the Lorentz factor, while the second requirement for the GRB radius would constrain only $\Gamma(t)$. As for the condition on the electron Lorentz factor, we note that, if electrons acquire a fraction $\epsilon_e$ of the outflow’s internal energy, i.e. $N\gamma^2 m_e c^2 = \epsilon_e U'$, where $N \propto \mathcal{E}_\gamma / \Gamma$ is the electron number ($\mathcal{E}_\gamma$ being the outflow isotropic-equivalent kinetic energy) and $U' \propto V'U' \propto V'L(RT)^{-2}$ is the internal energy (with $V' = 4\pi R^2 c L'$, being the volume of the GRB source), then $\gamma \propto \epsilon_e L_{p,int}^{\ast}/L_h$, where $L_h = \mathcal{E}_\gamma / t_\gamma$ is the outflow kinetic luminosity. The requirement that $\gamma$ is not correlated with $L_{p,int}^{\ast}$ (leading to the $E_p \propto \mathcal{E}_\gamma^{1/2}$ relation) implies that either $\epsilon_e \propto L_{p,int}^{\ast-1}$ or that $L_h \propto L_{p,int}^{\ast}$, otherwise one would obtain that $E_p \propto L_{p,int}^{5/2} \propto \mathcal{E}_\gamma^{5/2}$.

Similarly, constraints on some model properties are required to explain the $E_p \propto \mathcal{E}_\gamma^{1/2}$ relation if the burst emission results from Comptonization of the thermal radiation produced by magnetic reconnection or shock dissipation below the baryonic and/or pair photospheres. In this model (Mészáros & Rees 2000, Ryde 2004, Ramirez-Ruiz 2005), the GRB peak energy and luminosity are correlated because both depend on the photospheric temperature. Rees & Mészáros (2005) have shown that, if dissipation occurs above the saturation radius, then $E_p \propto \Gamma^2 L_\gamma^{1/4}$, where $L_\gamma$ is the GRB luminosity. Then the $E_p \propto \mathcal{E}_\gamma^{1/2}$ relation requires a certain correlation of the burst luminosity with the photosphere’s Lorentz factor. Within the same model for the burst emission, Thompson (2006) has shown that the $E_p \propto \mathcal{E}_\gamma^{1/2}$ relation is obtained if the burst thermal radiation is produced at the stellar progenitor’s photosphere, for which the rest-frame temperature of the photons is $T_{\theta} \propto (L_\gamma / \Gamma^2)^{1/4}$, and assuming that the outflow opening is set by its lateral expansion at the sound speed ($\theta \propto \Gamma^{-1}$) and that the collimation-corrected GRB output ($\propto \mathcal{E}_\gamma^{1/2}$) is universal (as was first indicated by the analysis of Frail et al. 2001 and later shown to be incorrect by the assumption that the GRB collimated output ranges over 2 decades — e.g. figure 1 of Ghirlanda, Ghisellini & Lazzati 2004).

In this work, we present a possible origin of the $E_p \propto \mathcal{E}_\gamma^{1/2}$ relation related to the dynamics of the GRB source, assuming an observer located within the opening of the relativistic outflow (i.e. viewing geometry is not at work). If the burst emission is synchrotron, then the peak of the GRB $\nu F_\nu$ spectrum is at $E_p \propto \gamma^2 B \Gamma$ and the flux density at that photon energy is $F_p \propto \mathcal{E}_\gamma^{1/2} \Gamma$, where $\gamma$ is the electron typical comoving-frame Lorentz factor and $N$ the number of electrons in the GRB source. The GRB output being $\mathcal{E}_\gamma \sim F_p \Gamma E_p \propto \epsilon_{\gamma}$, it follows that the $E_p \propto \mathcal{E}_\gamma^{1/2}$ relation requires $B\Gamma^2 \propto B \Gamma \gamma (N t_\gamma)^{1/2}$. If the burst emission were inverse-Compton scatterings of the synchrotron emission generated by some electrons, then $E_p$ picks an extra-factor $\gamma^2$ and $F_p$ a factor $\tau$, the optical thickness to electron scattering of the GRB source. Then the $E_p \propto \mathcal{E}_\gamma^{1/2}$ relation requires $B \Gamma^2 \propto B \Gamma \gamma (N t_\gamma)^{1/2}$. It is tempting to attribute the $E_p \propto \mathcal{E}_\gamma^{1/2}$ relation to (i) variations from burst to burst of the $B \Gamma$ factor, which appears both in the peak energy $E_p$ and the GRB output $\mathcal{E}_\gamma$, (ii) universality of $\gamma$, and (iii) the remaining “dummy” parameters ($N$, $t_\gamma$, $\tau$) being either universal or uncorrelated with $B \Gamma$ (so that they do not yield a different $E_p \propto \mathcal{E}_\gamma$ dependence). We note that variations in $\gamma$ (for synchrotron) or $\gamma^2$ (for inverse-Compton) from burst to burst that are larger than those of $B \Gamma$ would induce a $E_p \propto \mathcal{E}_\gamma$ correlation for either emission process.

The burst duration $t_\gamma$, which is the only observable that appears in equations (1) and (2), has a spread of 1.5–2.0 orders of magnitude among long-bursts, which is comparable to the observed spread in GRB energy $\mathcal{E}_\gamma$ at fixed peak en-
ergy $E_p$ (see figure 1 of Ghirlanda et al 2008). This suggests that the observed spread in the $E_p \propto \mathcal{E}_\gamma^{1/2}$ relation requires some correlation among the dummy parameters, although it is possible that the range of measured $\mathcal{E}_\gamma$ is smaller than the true spread because, for a fixed peak energy, bursts of a lower GRB output may fall below detection.

If the GRB emitting electrons are accelerated at shocks, then it is unlikely that the product $\gamma B^2$ can vary among bursts while $\gamma$ is universal, because acceleration of electrons at relativistic shocks is expected to yield an electron Lorentz factor $\gamma$ that depends on that of the shock. As the latter bears a connection with the GRB source Lorentz factor $\Gamma$, a universal $\gamma$ requires either universal $\Gamma$ and $\epsilon_e$ or an ad-hoc correlation of these parameters. In the former case, the $E_p \propto \mathcal{E}_\gamma^{1/2}$ relation would rest entirely on variations in the magnetic field $B$ among bursts. The nearly 3 decades spread in observed $E_p$ and that $E_p \propto B \propto \epsilon_e^{1/2}$ imply that the fraction $\epsilon_B$ of the internal energy stored in shock-generated magnetic fields has a range of 6 decades. Thus, a universal $\gamma$ requires a mechanism for electron acceleration at shocks that is completely decoupled from the generation of magnetic fields, which is an extreme requirement. For example, in the Weibel instability model of Medvedev (2006), proton current filaments created by the instability produce electric fields which accelerate electrons over distances of about the proton plasma skin-depth, leading to $\epsilon_e \approx \epsilon_B^{1/2}$, hence the 3 decades range of observed peak energies $E_p$ would be associated with an electron $\gamma$ which is far from being universal.

Thus, it seems much more likely that $E_p \propto \mathcal{E}_\gamma^{1/2}$ relation is not due just to variations in the $\gamma B^2$ term among bursts and that some or all of the other parameters appearing in equations (1) and (2) contribute as well. To include their effect in driving the $E_p \propto \mathcal{E}_\gamma^{1/2}$ relation, we assume that the outflow’s energy is dissipated by shocks which accelerates electrons and generate magnetic fields that acquire quasi-universal fractions of the dissipated energy. Some justification for the latter assumption is that, if the electron and magnetic parameters $\epsilon_e$ and $\epsilon_B$ were correlated as for the Weibel instability model, then their variations among bursts would induce a $E_p \propto \mathcal{E}_\gamma^{3/4}$ correlation for synchrotron emission and $E_p \propto \mathcal{E}_\gamma^{5/6}$ correlation for inverse-Compton.

In the following section, we study the implications of equations (1) and (2), representing the $E_p \propto \mathcal{E}_\gamma^{1/2}$ relation, in the framework of external shocks. We note that this model has the drawback that the efficiency of the GRB emission should be small (below 10 percent) for those bursts with a large number (hundreds) of pulses (Sari & Piran 1997). The same can be done for internal shocks which, as discussed above, will lead to constraints on the distribution of the ejecta Lorentz factor with mass and ejection time (or variability timescale). A low GRB efficiency is also expected for this model (e.g. Kumar 1999).

### 3 EXTERNAL-SHOCK EMISSION

The external shock driven by the interaction of the relativistic ejecta with the burst ambient medium offers two possible GRB sources: the reverse shock, which energizes the ejecta, and the forward shock, which sweeps-up the circum-burst medium. Denoting by $\Gamma'$ the Lorentz factor of either shock as measured in the frame of the incoming gas (the ejecta or the ambient medium), the shock jump conditions lead to an internal energy density in the shocked gas that is $n' = (\Gamma' - 1) n m_p c^2$, where $n' = (4 \pi^2 + 3) n_0$ is the comoving-frame particle density in the shocked fluid and $n_0$ that in the unshocked gas. Thus, for a relativistic shock $\Gamma' \gg 1$, the typical electron Lorentz factor is $\gamma \propto \Gamma'$ and the magnetic field is $B \propto \Gamma n^{1/2}$, where $n$ is the ambient medium density and $\Gamma$ the laboratory-frame Lorentz factor of the shocked gas (i.e. the GRB source), the latter being valid also for the reverse shock because the contact discontinuity between the two shocked media is in hydrostatic equilibrium, (i.e. pressure and internal energy density is the same behind both shocks).

For a source moving at Lorentz factor $\Gamma$, the burst duration is $t_s = R_s / \Gamma^2$, where $R_s$ is the GRB source radius, which results from either the spread in the photon arrival time across the visible area of angular opening $\Gamma^{-1}$ or from the observer duration of the source travel-time up to radius $R$ (provided that the source is decelerating or accelerating slower than $\Gamma \propto R^{1/2}$). Adding that the optical thickness to electron scattering is $\tau \propto N/R_s^2$, the $E_p \propto \mathcal{E}_\gamma^{1/2}$ relation of equations (1) and (2) becomes

$$\Gamma'^2 \Gamma n^{1/2} \propto \Gamma' \Gamma (n N R_s)^{1/2}$$

(3)

for synchrotron emission and

$$\Gamma'^2 \Gamma n^{1/2} \propto \Gamma'^3 \Gamma N (n/N R_s)^{1/2}$$

(4)

for inverse-Compton.

Below, we investigate the conditions required for the synchrotron and inverse-Compton emissions from the reverse and forward shocks to accommodate the $E_p \propto \mathcal{E}_\gamma^{1/2}$ relation with the following simplifications:

1. The burst emission is produced before the reverse shock crosses the ejecta shell, i.e. before the deceleration of the external shock starts. One reason for this restriction is that the shock-crossing radius offers a “milestone” in the evolution of the external shock that could be the GRB radius $R_\gamma$, while no such reference point exist after deceleration sets in.
2. A second reason is that it would be unnatural for a decelerating external shock to radiate episodically, once during the burst, until 100 s, and then again staring after 1000 s, during the afterglow, as observed in the X-ray emission of a majority of Swift GRBs (O’Brien et al 2006, Willingale et al 2007).
3. The entire emitting fluid moves at the same Lorentz factor $\Gamma(R_s)$ and is filled with the same magnetic field $B(R_s)$, with the values taken at the radius were the burst emission is released. As $E_p$, $F_p$, and $\Gamma$ are power-laws in the shock radius, for a radially extended burst emission, their burst-averaged ($E_p = \int E_p dF_p/\int dF_p$) and burst-integrated ($\int F_p E_p dt \propto \mathcal{E}_\gamma$ with $dt = dR/R_s^2$) values have the same dependence on $R_s$, as their values at $R_s$.
4. The distribution with energy of the shock-accelerated electrons is softer than $dN/d\epsilon_e \propto \epsilon_e^{-3}$ above the typical $\gamma$, so that the peak of $\nu F_\nu$ is for the typical $\gamma$ electrons and not at a higher random Lorentz factor determined by electron cooling and/or acceleration.
5. The electrons with the typical Lorentz factor $\gamma$ do not cool significantly during the GRB emission. We note that only a small of the BATSE bursts (Preece et al 2000) have the $F_\nu \propto \nu^{-1/2}$ spectrum below the peak energy $E_p$ expected in the opposite case. If the $\gamma$-electrons cool during
the burst, the \( \nu F_\nu \) spectrum still peaks at the synchrotron or inverse-Compton energy corresponding to \( \gamma \) (i.e. the peak energy \( E_p \) remains unchanged), but the flux at \( E_p \) picks a multiplying factor \( \gamma^2/\gamma \propto (\Gamma_0 n R_e)^{-1} \) for synchrotron and a factor \( (\gamma_0 / \gamma)^2 \) for inverse-Compton, owing to that most electrons are at the cooling Lorentz factor \( \gamma_c \propto \Gamma/(B^2 R_v) \) for which the radiative cooling timescale is equal to the burst duration.

Thus, the treatment provided below is not sufficiently comprehensive and serves only as an illustration of the conditions required for the external shock to account for the \( E_p \propto \mathcal{E}^{1/2} \) relation.

### 3.1 Dense ejecta (semi-relativistic reverse shock)

For the evolution of the ejecta-ambient medium interaction at times before the reverse shock crosses the ejecta shell (i.e. before the standard deceleration sets-in), the shock jump conditions can be used to derive a fourth-degree equation for the Lorentz factor \( \Gamma \) of the shocked fluid (which is the same for both the shocked ejecta and the swept-up ambient medium). As shown by Panaitescu \& Kumar (2004), the solution of that equation is

\[
\Gamma \approx \Gamma_0 \left[1 + 2 \Gamma_0 \left(n/n'_\text{ej}\right)^{1/2}\right]^{-1/2}
\]

where \( \Gamma_0 \) is the Lorentz factor of the unshocked ejecta and \( n'_\text{ej} \) their comoving-frame density.

In the \( n'_\text{ej} \gg 4 \Gamma_0^2 n \) limit (thin and dense ejecta shell), equation (5) leads to \( \Gamma \approx \Gamma_0 \), independent of the \( n/n'_\text{ej} \) ratio, and to a mildly relativistic reverse shock of constant Lorentz factor \( \Gamma' \). If \( \Gamma' \) and \( \Gamma \) do not change with radius, then equations (3) and (4) imply that the \( E_p \propto \mathcal{E}^{1/2} \) relation is induced by a certain correlation of the ejecta Lorentz factor \( \Gamma_0 \) with the radius \( R \), where the GRB emission is released. We focus on the forward shock emission because the mildly relativistic reverse shock is unlikely to yield an emission spectrum peaking in the hard X-rays. For the forward shock, \( \Gamma' = \Gamma \approx \Gamma_0 \) and \( n \propto n R_e^2 = R^3 \) for an ambient medium density stratified as \( n \propto R^{-5/3} \) with \( s < 3 \), and \( n \approx \text{const} \) if \( s > 3 \).

For synchrotron emission and \( s < 3 \), equation (3) requires \( \Gamma_0^3 \propto R_e^{-5/3} \). If \( \Gamma_0 \) were universal, this leads to an inconsistent solution \( s = 4 \). Thus \( \Gamma_0 \) should vary among bursts, in which case the requirement imposed by the \( E_p \propto \mathcal{E}^{1/2} \) relation is that the GRB emission is released at a radius that is correlated with the ejecta Lorentz factor. Further investigation can be done if \( R_e \) is determined in some way. The termination shock of the wind expelled by the GRB progenitor is the only milestone expected in the evolution of the forward shock, though it is not evident how it could set the GRB radius; even that were achieved, the location of the termination shock should not be related to the ejecta initial Lorentz factor. Instead, we speculate that the location where the forward-shock GRB emission is released is tied to the radius \( R_+ \propto (\mathcal{E}_k/\Gamma_0^2)^{1/(3-s)} \) at which the reverse shock crosses the ejecta \( (\mathcal{E}_k \propto \text{the ejecta kinetic energy}) \), and after which the blast-wave is decelerated. Then the \( E_p \propto \mathcal{E}^{1/2} \) relation can be obtained if \( s = 10/3 \), which is inconsistent with the starting assumption \( s < 3 \).

For synchrotron emission and \( s > 3 \), equation (3) requires \( \Gamma_0^3 \propto R_e^{1/2} \). Relating \( R_e \) to the shock-crossing radius \( R_+ \), a self-consistent solution \( s = 3.5 \) is found, for which \( E_p \propto \Gamma_0^{-7/4} R_e^{-5} \).

For inverse-Compton emission and \( s < 3 \), equation (4) requires \( \Gamma_0^\gamma \propto R_e^{-5} \). If \( \Gamma_0 \) were universal, the \( E_p \propto \mathcal{E}^{1/2} \) relation would be accounted for by an ambient medium with \( s = 2.5 \). In this case, \( R_+ \approx R_e \) leads to \( E_p \propto \mathcal{E}^{1/2} \) and \( E_p \propto R_e^{-11/8} \propto \mathcal{E}^{1/2} \). If \( \Gamma_0 \) is not universal, then \( E_p \propto \mathcal{E}^{1/2} \) relation is obtained for \( s = 11/4 \), leading to \( E_p \propto \Gamma_0^2 R_e^{-11/8} \propto \mathcal{E}^{1/2} \). For \( s > 3 \), the \( E_p \propto \mathcal{E}^{1/2} \) relation requires \( \Gamma_0^3 \propto R_e^{-1} \), which for \( R_+ \approx R_e \) leads to \( s = 2.5 \), i.e. an inconsistent solution.

Therefore, for a thin ejecta shell, the \( E_p \propto \mathcal{E}^{1/2} \) relation can be explained with synchrotron emission from the forward shock if GRBs are produced at the radius where the reverse shock crosses the ejecta shell and if the ambient medium around bursts has a universal \( n \propto R^{-3} \) radial structure, bursts with higher peak energy \( E_p \) and GRB output \( E_r \) resulting for lower ejecta Lorentz factors \( \Gamma_0 \) or larger ejecta kinetic energies \( \mathcal{E}_k \). The \( E_p \propto \mathcal{E}^{1/2} \) relation can also be obtained with inverse-Compton emission if \( n \propto R^{-2.5} \) and \( \Gamma_0 \) are universal, bursts with higher \( E_p \) and \( E_r \) resulting from a lower \( \mathcal{E}_k \), or if \( n \propto R^{-2.75} \) for all bursts if \( \Gamma_0 \) is not universal, a higher \( E_p \) and \( E_r \) being obtained for a higher \( \Gamma_0 \) or lower \( \mathcal{E}_k \).

### 3.2 Tenuous ejecta (relativistic reverse shock)

In the \( n'_\text{ej} \ll 4 \Gamma_0^2 n \) limit (thick and tenuous ejecta shell), equation (5) leads to \( \Gamma \propto (\Gamma_0 / 2)^{1/2} (n'_\text{ej} / n)^{1/4} \propto \Gamma_0 \) and to a relativistic reverse shock with \( \Gamma' \approx \Gamma_0 (2T) \approx 1 \). Considering that the radial width of the ejecta shell increases linearly with its radius, the comoving-frame ejecta density is \( n'_\text{ej} \propto (\mathcal{E}_k / \Gamma_0) / R^3 \). Then, for a ambient medium with radial density profile \( n \propto R^{-s} \), we obtain that the Lorentz factor of the shocked gas evolves as \( \Gamma \propto R^{(s-3)/4} \). Therefore, if \( s > 3 \), the shocked motion is accelerated by the ram pressure of the incoming ejecta, starting from a value well below \( \Gamma_0 \) (and remaining below it at all times). For \( s < 3 \), the GRB source is decelerating (but this deceleration is substantially slower than that after the reverse shock has crossed the ejecta shell).

In the following investigation, we drop the dependence of two quantities of interest, \( E_p \) and \( E_r \), on the ejecta Lorentz factor \( \Gamma_0 \), i.e. we assume it to be universal, and determine the stratification index \( s \) that accommodates the \( E_p \propto \mathcal{E}^{1/2} \) relation. In this case, bursts have different peak energies and GRB outputs because their emission is produced at different radii \( R_+ \). If \( R_+ \) is identified with the shock having crossed the entire ejecta shell, then the GRB radius is set by the ejecta kinetic energy and the duration of the ejecta release, which is about the same as the observer frame burst duration: \( R_+ \propto (\mathcal{E}_k t_\gamma)^{1/(4-s)} \) (Panaitescu \& Kumar 2004).

If \( \Gamma_0 \) were not universal, the \( E_p \propto \mathcal{E}^{1/2} \) relation could be explained if the GRB radius \( R_+ \) and \( \Gamma_0 \) satisfy a certain relation. Then, relating \( R_+ \) with the shock crossing radius will lead to a certain correlation among \( \Gamma_0 \), \( \mathcal{E}_k \), and \( t_\gamma \), an avenue which we will not explore any further.

The continuous injection of relativistic electrons (in the downstream region) of a Lorentz factor \( \gamma_c \propto \Gamma' \) which
changes with the outflow radius will lead to an electron population at $R_\gamma$ that has a power-law distribution with energy, $dN/d\gamma_c \propto \gamma_c^{-q}$. The effective index $q$ can be calculated by first determining the medium structure parameter $s(q)$ that accounts for the $E_p \propto E_\gamma^{1/2}$ relation, then the dynamics of the shocked fluid $\Gamma(R)$ (which sets $\gamma_c$) and the derivative $dN/d\Gamma$ of the electron number, from where $dN/d\gamma_c$ can be obtained and the loop is closed to find the exponent $q$.

Because $\gamma$ evolves with $R$, one must check that the assumed location of the peak of $\nu F_\gamma$ is consistent with the evolution of $\gamma(R)$ and the inferred effective index $q$ of the electron distribution.

### 3.2.1 Forward shock

For the forward shock, $\Gamma' = \Gamma \propto R_\gamma^{(3-s)/4}$ and $N \propto R_\gamma^{s-3}$ if $s < 3$, while $N \approx const$ if $s > 3$. For $s > 3$, most of electrons have been accelerated before the GRB radius $R_\gamma$, and we have to find a self-consistent solution considering that the peak energy $E_p$ is either at $\gamma(R_\gamma)$ (which we will denote as $\gamma_1$) or at some electron Lorentz factor $\gamma_0$ corresponding to when relativistic electrons were first produced. We will make the simplifying assumption that $\gamma_0$ is a universal quantity.

For a $dN/d\gamma_c \propto \gamma_c^{-q}$ electron distribution, with most electrons being at $\gamma_0$, the flux at photon energy $E_0 \propto \gamma_0^4 B \Gamma^4$ is $F_0 \propto \Gamma N B$, while the flux at energy $E_1 \propto \gamma_1^4 B \Gamma^4$ is $F_1 \propto \Gamma N B (\gamma_0/\gamma_1)^{q-1}$. (1) If the peak of $\nu F_\gamma$ is at $E_0$, then it can be shown that the $E_p \propto E_\gamma^{1/2}$ relation requires $s = 5$, for which $\gamma_0 \propto R^{-1/2}$, $dN/d\Gamma \propto R^{-s}$, hence $dN/d\gamma_c \propto \gamma_c^{-q}$. Given that $d\gamma_c/d\Gamma > 0$, $q = 7$ implies that the peak of $\nu F_\gamma$ is, indeed, at $E_0$, consistent with the starting assumption. For this case, $E_p \propto R^{-3/2}$.

(2) If the peak of $\nu F_\gamma$ is at $E_1$, then the $E_p \propto E_\gamma^{1/2}$ relation leads to $s = 3 + 4/(q + 3)$, $\gamma_0 \propto R^{1/(q+3)}$, $q = 5$ which, together with $d\gamma_c/d\Gamma > 0$, implies that the peak of $\nu F_\gamma$ is, in fact, at $E_0$, in contradiction with the starting assumption.

For synchrotron emission and $s < 3$, the $E_p \propto E_\gamma^{1/2}$ relation given in equation (3) is satisfied if $s = 3.5$, hence this is not a self-consistent solution.

For inverse-Compton emission and $s < 3$, the $E_p \propto E_\gamma^{1/2}$ relation of equation (4) requires $s = 19/7$, leading to $\gamma_0 \propto R^{-1/14}$, $dN/d\Gamma \propto R^{-5/7}$, and $q = 5$ which, together with $d\gamma_c/d\Gamma < 0$, implies that the peak of $\nu F_\gamma$ is, indeed, determined by the $\gamma(R_\gamma)$ electrons. For this case, $E_p \propto R_\gamma^{25/14}$.

For inverse-Compton emission and $s > 3$, the flux at photon energy $E_0 \propto \gamma_0^4 B \Gamma^4$ is $F_0 \propto \Gamma N B$, while the flux at energy $E_1 \propto \gamma_1^4 B \Gamma^4$ is $F_1 \propto \Gamma N B (\gamma_0/\gamma_1)^{q-1}$. (1) If the peak of $\nu F_\gamma$ is at $E_0$, the $E_p \propto E_\gamma^{1/2}$ relation requires $s = 1$, incompatible with the assumed $s > 3$.

(2) If the peak of $\nu F_\gamma$ is at $E_1$, then the $E_p \propto E_\gamma^{1/2}$ relation leads to $s = 3 - 2/(q + 2)$, $\gamma_0 \propto R^{-(2q+4)/(-2q+4)}$, $dN/d\Gamma \propto R^{-(q+2)/(q-2)}$, from where $q = 5$ and $s = 19/7$, inconsistent with the $s > 3$ initial assumption.

Therefore, the $E_p \propto E_\gamma^{1/2}$ relation can be accommodated with the synchrotron emission from the pre-deceleration forward shock if all GRBs occur in a $n \propto R^{-5}$ medium, but at different radii, or by with the inverse-Compton forward shock emission if the ambient medium has a universal $n \propto R^{-19/7}$ stratification. In either case, bursts of higher $E_p$ and $E_\gamma$ are those occurring at smaller radii.

### 3.2.2 Reverse shock

For the reverse shock, $\Gamma' = \Gamma \propto R_\gamma^{(3-s)/4}$ and the number of energized ejecta electrons evolves as $dN/d\Gamma \propto R^{2(\Gamma n'_\gamma)/(\beta_0 - \beta)}$, where $\beta_0$ and $\beta$ are the lab-frame velocities of the unshocked and shocked ejecta, respectively. For $\Gamma_0 \gg \Gamma$, we have $\beta_0 - \beta \approx (2T)^{-1}$. Using $n'_\gamma \propto R^{-3}$ and $\Gamma \propto R_\gamma^{(s-3)/4}$, one arrives at $dN/d\Gamma \propto R^{(1-s)/2}$, from where $N \propto R_\gamma^{(3-s)/2}$ for $s < 3$ and $N \approx const$ for $s > 3$.

For synchrotron emission and $s < 3$, the $E_p \propto E_\gamma^{1/2}$ relation leads to a contradicting $s = 5$. For $s > 3$, assuming that the $\nu F_\gamma$ spectrum peaks at $E_0 \propto \gamma_0^4 B \Gamma^4$, the $E_p \propto E_\gamma^{1/2}$ relation requires $s = 5$, implying $\gamma_0 \propto R^{-1/2}$, $dN/d\Gamma \propto R^{-s}$, thus $q = -1$ which, together with $d\gamma_c/d\Gamma < 0$, implies that the peak of $\nu F_\gamma$ is, indeed, at $E_0$. For this case, we obtain $E_p \propto R_\gamma^{3-s/2}$. For $s > 3$, assuming that the $\nu F_\gamma$ spectrum peaks at $E_1 \propto \gamma_1^4 B \Gamma^4$, the $E_p \propto E_\gamma^{1/2}$ relation requires $s = 3 - 4/(q + 1)$, from where $\gamma_0 \propto R_\gamma^{1/(1-q)}$, $dN/d\Gamma \propto R^{3(3-q)/(q-1)}$, leading to $q = 3$ and $s = 1$, inconsistent with the starting choice $s > 3$.

For inverse-Compton emission and $s < 3$, the $E_p \propto E_\gamma^{1/2}$ relation requires $s = 1$, yielding $\gamma_0 \propto R^{1/2}$, $dN/d\Gamma = const$, hence $q = -1$, thus $d\gamma_c/d\Gamma > 0$ implies that $\nu F_\gamma$ peak energy is determined by the $\gamma_1$ electrons. In this case, $E_p \propto R_\gamma^{-1/2}$. For $s > 3$, assuming that the $\nu F_\gamma$ spectrum peaks at $E_0 \propto \gamma_0^4 B \Gamma^4$, the $E_p \propto E_\gamma^{1/2}$ relation requires $s = 1$, which is incompatible with the working condition $s > 3$. For $s > 3$, assuming that the $\nu F_\gamma$ spectrum peaks at $E_1 \propto \gamma_1^4 B \Gamma^4$, the $E_p \propto E_\gamma^{1/2}$ relation leads to $s = 3 + 2/(q + 2)$, implying $\gamma_0 \propto R^{-(2q+4)/(q-2)}$, $dN/d\Gamma \propto R^{-1/2}$, from where $q = -1$ and $s = 1$, again incompatible with the starting condition.

Thus, synchrotron emission from the reverse shock can account for the $E_p \propto E_\gamma^{1/2}$ relation provided that bursts occur at various radii in a $n \propto R^{-5}$ medium, while inverse-Compton emission can explain the same relation if $n \propto R^{-1}$. For either radiation process, bursts of higher $E_p$ and $E_\gamma$ are those occurring at smaller radii.

### 4 CONCLUSIONS

We have investigated the ability of the external shock (produced by the interaction of relativistic ejecta with the ambient medium) to accommodate the $E_p \propto E_\gamma^{1/2}$ relation between the burst peak energy and isotropic-equivalent energy release. First, we noted that it seems unlikely that the $E_p \propto E_\gamma^{1/2}$ relation is due to variations of the quantity $B\Gamma$ among bursts, with the electron Lorentz factor $\gamma$ being universal and all other quantities ($N$, $t_e$, $\tau$) in the right-hand sides of equations (1) and (2) not being correlated with $E_p$.

That is because electron acceleration at relativistic shocks is likely related to the generation of magnetic fields and with the strength of the shock and because the number of radiating electrons $N$ and the burst duration $t_e$ could be related with the dynamics of the external shock.

For that reason, we have identified the conditions that

$$E_p \propto E_\gamma^{1/2} \text{ for external-shock emission}$$
A. Panaitescu

lead to the $E_p \propto \xi_{1/2}$ relation by taking into account all the quantities that determine $E_p$ and $\xi$. After making some simplifications (uniform magnetic field, single Lorentz factor in the shocked fluid, negligible electron cooling), and considering only the external shock emission before the reverse shock crosses the ejecta shell (as an interrupted burst–afterglow emission from the same decelerating outflow seems too contrived), we have determined the dependence of $E_p$ and $\xi$ on the radius where (or up to which) the burst emission is produced, with allowance for both the reverse and forward shock, synchrotron and inverse-Compton emissions, and relativistic or semi-relativistic reverse shock. In the latter case, only the forward shock is expected to produce the high-energy prompt emission, the $E_p \propto \xi_{1/2}$ relation requiring a correlation between the ejecta Lorentz factor and the GRB radius which does not have a plausible justification. For that reason, we favour an explanation of the $E_p \propto \xi_{1/2}$ relation where the reverse shock is relativistic.

In our treatment of that case, the burst emission is assumed to arise over a small range of source radii or up to a certain radius, the $E_p \propto \xi_{1/2}$ relation resulting from variations in that radius from burst to burst. The reverse shock crossing the ejecta or the external shock encountering the termination shock of the progenitor’s freely expanding wind are the milestones in the dynamical evolution of the reverse and forward shock, respectively, that could set the location where the burst radius is produced. This implies that the variations from burst to burst in the radius at which the prompt emission is released is due to either (1) variations among bursts in the kinetic energy of the ejecta or in the duration of ejecta release (for a reverse shock origin of the GRB), or (2) to the history of the mass-loss of the GRB progenitor shortly before its core collapse (for GRBs produced by the forward shock).

Within the external-shock model for GRBs, the $E_p \propto \xi_{1/2}$ relation can be accounted for by just the power-law radial stratification of the burst ambient medium density. For the four possible combinations of dissipation shock and radiation process, we find the following density profiles: $n \propto R^{-1}$ (inverse-Compton from reverse shock), $R^{-19/17}$ (inverse-Compton from forward shock), and $R^{-5}$ (synchrotron from either shock). In general, a steep ambient profile is required to explain the slope of the $E_p \propto \xi_{1/2}$ relation because of the weak dependence of the source Lorentz factor on the density of the ambient medium ($\Gamma \propto n^{-1/4}$ — equation 5).

None of the ambient medium stratification required by the $E_p \propto \xi_{1/2}$ relation is the $n \propto R^{-2}$ profile expected for a massive stellar GRB progenitor expelling a constant speed wind a steady mass-loss rate. Considering that the burst emission occurs at $\lesssim 10^{16.5}$ cm and that the wind termination shock moves at $\sim 10$ km s$^{-1}$, this implies that, in the last $\lesssim 1000$ years before core collapse, the Wolf-Rayet progenitor of long-bursts had a varying mass-loss rate or wind terminal velocity. However, we do rule out that, by relaxing the simplifying assumptions made here, the ambient medium density profile required to explain the $E_p \propto \xi_{1/2}$ relation with the external-shock emission becomes consistent with $n \propto R^{-2}$.

As the burst model employed here is that of the external shock before the reverse shock crosses the ejecta (i.e. before deceleration begins), the ensuing afterglow emission could be attributed to the emission from the reverse or forward shocks after deceleration, with allowance for injection of ejecta and energy after the burst, to account for the extended afterglow emission (if it is from the reverse shock) and the X-ray light-curve plateaus (if it is from the forward shock). Then, the general lack of continuity of burst-to-afterglow emissions, shown by the steep fall-off of the X-ray flux by 2–3 dex at the end of the burst, would lead to a rather contrived model, where the discontinuous burst-to-afterglow emission requires a temporary switch-off of the external-shock emission, followed by a much softer emission (the afterglow). A simpler is that where the two emission phases, prompt and delayed, are attributed to different outflows, with the burst arising from a narrower jet whose bright, high-energy emission is produced only before the reverse shock crosses the ejecta or the external shock reaches the wind termination shock, but having a sufficiently low, collimated kinetic energy, so that its post-burst (forward shock) emission is dimmer than that from a wider, more energetic outflow producing the afterglow emission.

ACKNOWLEDGMENTS

The author acknowledges the support of the US Department of Energy through the LANL/LDRD 20080039DR program.

REFERENCES

Amati L. et al, 2002, A&A, 390, 81
Amati L., 2006, MNRAS, 372, 233
Eichler D., Levinson A., 2004, ApJ, 614, L13
Frail D. et al, 2001, ApJ, 562, L55
Ghirlanda G., Ghisellini G., Lazzati D., 2004, ApJ, 616, 331
Ghirlanda G., Nava L., Ghisellini G., Firmani C., Cabrera J., 2008, MNRAS, 387, 319
Kumar P., 1999, ApJ, 523, L113
Lloyd N., Petrovian V., Mallozzi R., 2000, ApJ, 534, 227
Medvedev M., 2006, ApJ, 651, L9
Mészáros P., Rees M., 2000, ApJ, 530, 292
Nava L., Ghirlanda G., Ghisellini G., Firmani C., 2008, preprint (arXiv:0807.4931)
O’Brien P. et al, 2006, ApJ, 647, 1213
Panaitescu A., Kumar P., 2004, MNRAS, 353, 511
Pendleton G. et al, 1996, ApJ, 464, 606
Preece R. et al, 2000, ApJS, 126, 19
Ramirez-Ruiz E., 2005, MNRAS, 363, L61
Rees M., Mészáros P., 2005, ApJ, 628, 847
Ryde F., 2004, ApJ, 614, 827
Sakamoto T. et al, 2008, ApJ, 679, 570
Sari R., Piran T., 1997, ApJ, 485, 270
Schaefer B., 2003, ApJ, 583, L71
Toma K., Yamazaki R., Nakamura T., 2005, ApJ, 635, 481
Thompson C., 2006, ApJ, 651, 333
Willingale R. et al, 2007, ApJ, 662, 1093
Zhang B., Mészáros P., 2002, ApJ, 581, 1230