Recursion method for the quasiparticle structure of a single vortex with induced magnetic order

Linda Udby\(^1\), Brian M. Andersen\(^2\), and Per Hedegård\(^3\)

\(^1\)Materials Science Dept., Risø National Lab., Frederiksbergvej 399, DK-4000 Roskilde, Denmark
\(^2\)Department of Physics, University of Florida, Gainesville, Florida 32611-8440, USA
\(^3\)Ørsted Laboratory, Niels Bohr Institute, Universitetsparken 5, DK-2100 Copenhagen Ø, Denmark

(Dated: March 23, 2022)

We use a real-space recursion method to calculate the local density of states (LDOS) within a model that contains both \(d\)-wave superconducting and antiferromagnetic order. We focus on the LDOS in the superconducting phase near single vortices with either normal or antiferromagnetic cores. Furthermore, we study the low-energy quasiparticle structure when magnetic vortices operate as pinning centers for surrounding unidirectional spin density waves (stripes). We calculate the Fourier transformed LDOS and show how the energy dependence of relevant Fourier components can be used to determine the nature of the magnetic field-induced order, and predict field-induced LDOS features that can be tested by future scanning tunneling microscopy (STM) experiments.

PACS numbers: 74.20.-z, 74.72.-h, 74.25.Jb, 74.25.Ha

INTRODUCTION

It is becoming evident that competing phases cause many of the anomalous properties of doped Mott insulators. An example is given by the vortex state of underdoped high-\(T_c\) superconductors where antiferromagnetism (AF) ’pops up’ near the vortices\(^1\)\(^2\). Initial experimental evidence for this claim came from STM experiments on YBa\(_2\)Cu\(_3\)O\(_y\) (YBCO) and Bi\(_2\)Sr\(_2\)CaCu\(_2\)O\(_{8+y}\) (BSCCO) observing weak low-energy quasiparticle peaks around 5-7 meV\(^3\)\(^4\). This strongly contradicts the expected LDOS in the vortex center of a pure BCS \(d\)-wave superconductor (\(dSC\)) which is dominated by the so-called zero-energy state (ZES), a single broad resonance centered at the Fermi level\(^5\). Further evidence for AF cores has come from both nuclear magnetic resonance measurements\(^6\) and muon spin rotation experiments\(^7\). The field-induced magnetization is not necessarily restricted to the core regions as determined by the coherence length \(\xi\). For instance, elastic neutron scattering on underdoped La\(_{2-x}\)Sr\(_x\)CuO\(_2\) (LSCO) showed that the intensity of the incommensurate peaks in the superconducting phase is considerably increased when a magnetic field is applied perpendicular to the CuO\(_2\) planes\(^8\) or when Zn is doped into the samples\(^9\). Similar results have been found in the oxygen doped sample La\(_{2}\)CuO\(_{3+y}\)\(^10\). The momentum position and field-enhanced sharpening of this elastic signal corresponds to a spin density wave period of roughly eight lattice constants \(8a\) extending far outside the vortex cores, suggesting that the magnetic cores operate as pinning centers for surrounding spin density waves\(^11\)\(^12\). This unusual behavior agrees with in-field STM measurements on optimally doped BSCCO which found local field-induced checkerboard LDOS patterns with a period close to \(4a\)\(^13\). Similar structure has been reported in zero field STM experiments\(^14\). Pronounced checkerboard ordering has also been detected in Na\(_2\)Ca\(_{2-x}\)CuO\(_2\)Cl\(_2\)\(^15\).

More recently, Levy et al.\(^16\) confirmed the results of Ref.\(^13\) and found that the checkerboard modulation does not disperse with energy, and mapped out the energy dependence of the amplitude of the Fourier component corresponding to the ordering vector of the modulation.

Theoretically, several groups have proposed that the origin of the unexpected behavior inside the cores is related to locally nucleated AF\(^17\)\(^18\), but other scenarios have also been proposed\(^19\)\(^20\). From a computational point of view, in order to model the existence of nanoscale inhomogeneity, it is necessary to use methods that easily allow one to obtain the LDOS as a function of energy and large real-space regions. Traditionally this is done by numerical diagonalization of the Bogoliubov-de Gennes (BdG) equations, which, at present, is typically restricted to quite small lattices (\(\lesssim 40 \times 40\) sites). In this paper we use a recursion method generalized to the \(d\)-wave superconducting state to calculate the LDOS near an increasingly complex single vortex. This method is easily applied to large systems allowing for e.g. high-resolution Fourier LDOS images. First we study the pure \(dSC\) vortex for realistic band structure parameters relevant for overdoped cuprates. Second, we discuss the case of an AF vortex core in the optimally doped regime and focus on the spatial dependence of the expected LDOS. Finally, we calculate the LDOS when the vortex pins surrounding incommensurate stripe order as may be relevant for LSCO and underdoped BSCCO, and discuss the energy dependence of the resulting Fourier transform. As opposed to most earlier theoretical work on the AF vortex problem\(^17\), we focus on the final LDOS structure and the Fourier transformed LDOS maps which can be used as an STM tool to determine the nature of the field-induced order and the origin of the ZES splitting. Lastly, we compute the LDOS resulting from the recently proposed pair-density wave ordered state.
MODEL AND METHOD

In the following we study the mean-field Hamiltonian defined on a 2D lattice

\[
\hat{H} = -\sum_{\langle ij \rangle\sigma} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} - \mu \sum_{i\sigma} c_{i\sigma}^\dagger c_{i\sigma} + \sum_{\langle ij \rangle} \left( \Delta_{ij} c_{i\uparrow}^\dagger c_{j\downarrow} + H.c. \right) + \sum_{i} m_{i} \left( \hat{c}_{i\uparrow}^\dagger \hat{c}_{i\uparrow} - \hat{c}_{i\downarrow}^\dagger \hat{c}_{i\downarrow} \right),
\]

where \( \hat{c}_{i\sigma}^\dagger \) creates an electron with spin \( \sigma \) at site \( i \), \( t_{ij} \) is the hopping integral to nearest \( t \) or next-nearest \( t' \) neighbors and \( \mu \) is the chemical potential. The AF and dSC order parameters are given by \( m_{i} \) and \( \Delta_{ij} \), respectively. The Hamiltonian is the effective mean-field model obtained after performing two Hubbard-Stratonovich transformations of the extended Hubbard model with the on-site repulsion causing the AF, and the attractive nearest neighbor interaction resulting in the dSC. It has been used extensively in the past few years to gain insight into the electronic structure of phases of coexisting AF and dSC.[12, 17, 21, 22, 23]

Below, we solve the Hamiltonian using appropriate Ansätze for both \( \Delta_{ij} \) and \( m_{i} \). The lack of self-consistency can sometimes be useful in clarifying, for instance, the nature/origin of vortex core states.[30]. We restrict the discussion to the case when the applied magnetic field is much smaller than \( H_{c2} \) and consequently ignore the vector potential \( A \).

In order to obtain the LDOS near single vortices we use a recursion method.[24, 25] generalized to the superconducting state. The starting point is to generate a new orthonormal basis of states from the recursion relation

\[
\hat{H} |n\rangle = a_n |n\rangle + b_{n+1} |n+1\rangle + b_n |n-1\rangle.
\]

For each recursion the Greens function of the \( n \)'th level is generated recursively from the Lanczos coefficients \( a_n \) and \( b_n \)

\[
G_n(\omega) = \frac{1}{\omega - a_n - b_n^2 G_{n+1}(\omega)}.
\]

Hence, the local Greens function can be found if \( G_N(\omega) = 0 \) for some number \( N \), or if an appropriate analytical solution of \( G_N \) for an infinite chain can be attached.

The retarded Greens function is

\[
G_{\sigma\sigma}^{R}(\omega) = \sum_{\alpha} \left( \frac{\langle \alpha | c_{i\alpha}^\dagger | 0 \rangle^2}{\omega - E_{\alpha\sigma} + i\eta} + \frac{\langle \alpha | c_{i\sigma} | 0 \rangle^2}{\omega + E_{\alpha\sigma} + i\eta} \right),
\]

where \( \eta \) is used as an artificial smearing factor with \( \eta = 0.02t \). In general, it is not necessary to perform four recursions (I from \( c_{i\uparrow}^\dagger |0\rangle \); II from \( c_{i\downarrow} |0\rangle \); III from \( c_{i\uparrow}^\dagger |0\rangle \); IV from \( c_{i\downarrow}^\dagger |0\rangle \)) to calculate the spin-summed LDOS since \( G_{n=0}^{IV}(\omega) = G_{n=0}^{I}(\omega) \) and \( G_{n=0}^{III}(\omega) = G_{n=0}^{II}(\omega) \). Thus, it is sufficient to perform only two recursions to obtain the total LDOS, \( \rho(i, \omega) \), in the form of a continued fraction

\[
\rho(i, \omega) = \text{Im} \frac{-\frac{1}{\omega - a_{0} + i\eta}}{\omega - a_{1} + i\eta} \frac{(b_{1})^{2}}{\omega - a_{2} + i\eta} + \frac{(\omega - a_{1} + i\eta)}{(\omega - a_{0} + i\eta)} \frac{(b_{0})^{2}}{\omega - a_{2} + i\eta},
\]

which can be compared to the differential tunneling conductance as measured by e.g. an STM tip. Of course, when there is spin degeneracy (here: \( m_{i} = 0 \)) only one recursion is needed to produce the total LDOS.

In the cases studied, we find that the Lanczos coefficients converge nicely when increasing the number of recursions, i.e. the system size. Below we simply perform the truncation \( G_N = 0 \) where \( N \) is some number of order \( 10^3 \), and have checked that this choice does not affect the reported results.

RESULTS

In this section we use the recursion method to study \( \rho(i, \omega) \) around a single vortex in the dSC state both with and without antiferromagnetism in the core region. This is supposed to model the vortex LDOS in the overdoped and optimally doped regime, respectively. The core center is positioned at the origin \( (0,0) \) and lengths are measured in units of the lattice constant \( a \).

A single vortex without induced stripe order

As is well-known, in an s-wave superconductor the vortex generates states localized transverse to the flux line. These have been studied in great detail both theoretically[26, 27, 28] and experimentally[29]. The core states result from the opposite sign of the supercurrent term in the particle and hole part of the BdG equations[26, 27, 31]. The reduction of the pair potential near the vortex core causes only minor quantitative changes to these states. We have verified that the recursion method described above applied to s-wave superconductors successfully reproduce these Caroli-de Gennes-Matricon bound states.

To model an isolated d-wave vortex the following pairing potential is used

\[
\Delta_{ij} = \Delta \tanh(|r|/\xi) \exp(i \varphi_{ij}),
\]

where \( \Delta \) is positive (negative) on \( x \) (\( y \)) links, \( r = (r_x + r_y)/2 \) and \( \exp(i \varphi_{ij}) = (x + iy)/r \) with \( r = (x, y) \). In agreement with Ref. [30, 31], we find that the suppression of the gap in the core region results in only minor quantitative changes: in general the suppression tends to push the states slightly further toward the Fermi level.
the pure dSC state, the vortex is dominated by the well-known ZES\[5\]. However, the ZES is centered exactly at zero energy only for $\mu = t' = 0$. As opposed to the Caroli-de Gennes-Matricon states in the s-wave vortex, the ZES is made up of several states that merge to form the broad peak as the system size is increased in agreement with the extended nature of this peak\[20\]. In Fig. 1 we show $\rho(i, \omega)$ of a dSC vortex along the anti-nodal (a-b) and nodal (c-d) directions for $\Delta = 0.1t$, $\xi = 5$ and $\mu = t' = 0$ (a,c,e), and $\mu = -1.18t, t' = -0.4t$ (b,d,f). The latter parameter set provides a reasonable fit to the Fermi surface of slightly overdoped BSCCO with a van Hove singularity at $\omega_{vH} = -\sqrt{(4t' - \mu)^2 + (4\Delta)^2}$. It is the $d$-wave symmetry that causes the angular dependence (compare e.g. Fig. 1(a) and 1(c)) of the higher energy core states at $r \neq 0$. The energy and amplitude of these core states are seen to be sensitive to the band parameters. This is also true for the ZES state as seen from Fig. 1(e-f): at $t' = 0$ the low-energy spatial form of the LDOS has a star-shape due to the nodal dSC phase\[5, 32, 33\]. However, for the more realistic BSCCO band parameters, the star is rotated with small maximum intensity along the anti-nodal directions, which is our prediction for the overdoped regime of BSCCO where competing AF order is expected to be absent. For $t' = 0$, it is well-known that a similar $\pi/4$ rotation takes place at higher energies revealing the spatial form of the higher energy core states\[32, 33\].

We turn now to the simplest AF core situation where the suppression of the dSC gap inside the core causes a concomitant increase of the competing AF order\[2\]. For simplicity, we model the AF core by

$$m_i = m(-1)^{(x+y)}(1 - \tanh(|r_i|/\xi)),$$

where $r_i = (x, y)$. Due to an associated local increase of the electron density, such vortices will in general be charged\[35\], and have been shown to remain stable when including the long-range Coulomb repulsion\[17\]. Below, we therefore use $\mu = t' = 0.0$ in order to model the close to half-filled vortex core regions as found in the self-consistent studies\[17, 18\]. In Fig. 2 we show the final LDOS as a function of energy and distance to the AF vortex core. The induced magnetization leads to a splitting of the ZES as found previously\[17, 18\]. With increased magnetic order $m$, the resonant core states are pushed to higher energies and lose spectral weight. The vortex region is fully void of apparent core states for $m \gtrsim$
we show dispersive core states in Fig. 2. For example, in Fig. 3 similarity with that of the pseudo-gap.

Spatial averaging may mask the observability of the dispersive core states in Fig. 2. For example, in Fig. 3 we show $\rho(i, \omega)$ (same parameters as in Fig. 2(a)) at $(0, 0)$ and $(0, 7.5)$ averaged over a coherence length $\xi$. As seen, the resulting LDOS appears to be that of approximately non-dispersing resonant states which rapidly lose weight when moving away from the core region. This is similar to the measured differential tunneling conductance near the vortex cores of YBCO and BSCCO. However, an unambiguous experimental determination of the type of order that induces the splitting of the ZES is important, and is related to the general discussion of time-reversal symmetry breaking for zero-energy Andreev states in $d$-wave superconductors. Of course, vortices that support AF cores would lead to a field-induced commensurate ($\pi, \pi$) signal in neutron scattering. However, purely from a tunneling point of view, it is possible to distinguish AF and e.g. $d_{x^2-y^2} + id_{xy}$ induced order by using spin polarized STM. This is also shown in Fig. 3 where the positive (negative) bias peak is seen to be related to the spin-down (spin-up) LDOS, respectively. Importantly, this bias asymmetry will alternate with site, allowing for an unambiguous experimental test of AF order by a magnetic STM tip scanned through the vortex core region.

A single vortex with induced stripe order

In this section we calculate $\rho(i, \omega)$ around a dSC vortex which operates as a pinning center for unidirectional spin- and charge density modulations (stripes), expected to be relevant for STM experiments in the underdoped regime. Such inhomogeneous stripe solutions indeed exist in a regime of intermediate AF coupling within self-consistent mean-field models that include the competition between AF and dSC order. We also briefly discuss the expected LDOS resulting from the recently proposed pair-density-wave (PDW) induced order consisting of a density wave of Cooper pairs without global phase coherence.

Whereas the previous section dealt with the details of $\rho(i, \omega)$ inside the core, the field-induced periodic order can most conveniently be studied in Fourier space

$$\rho_{\mathbf{q}}(\omega) = \frac{1}{N} \sum_{i} \rho(i, \omega) e^{-i\mathbf{q} \cdot \mathbf{r}_i}. \quad (8)$$

For site-centered stripes with spin (charge) period of 8 (4) lattice constants, we show a typical checkerboard result for $\rho(i, \omega)$ in Fig. 4(a). The checkerboard pattern arises from including both vertical and horizontal stripes, which is a simple way to include the assumed slow fluctuation of the stripe domains. In addition, as shown recently, quenched disorder can severely smear any clear distinction between stripe and checkerboard symmetry breaking. Fig. 4(b) shows $|\rho_{\mathbf{q}}|$ as a function of $q_x$. FIG. 3: (Color online) DOS averaged over a coherence length $\xi$ within the core (solid black line), and just outside the core (dashed black line). The red dash-dotted line (blue thick dash-dotted line) show the spin-up (spin-down) resolved LDOS at the core center similar to the top scan in Fig. 2(a). FIG. 4: (Color online) (a) LDOS summed in the window $\omega \in [-0.4t, 0.4t]$ with $m/t = 1.0$. White (black) corresponds to high (low) LDOS. (b) $|\rho_{\mathbf{q}}(\omega)|$ at $q_y = 0.0$ vs $q_x$ for the energies: $\omega/t = 0.0, 0.1, 0.2, 0.3, 0.4, 0.5$. The curves are offset for clarity. (c) $\rho_{\mathbf{q}}(\omega)$ vs $\omega$ for a $\mu$-wave ($\Delta$-wave, PDW) with $t' = \mu = 0.0$, $A = 0.05t$ (dashed, black lines), and the full vortex induced stripe situation with $t' = -0.4t$, $\mu = -1.18$, $m/t = 0.4$ (red, solid line), and $m/t = 1.0$ (blue, dash-dotted line). Note that for all the curves $\rho_{\mathbf{q}}(0) = 0$ as expected for a $d$-wave superconductor at $T = 0$. (d) LDOS for the clean dSC (PDW) shown by the dashed (solid) line.
for \( q_0 = 0.0 \) for various energies \( \omega \). The non-dispersive peak at the charge ordering vector \( Q = (2\pi/4, 0) \) resulting from the stripes is seen to completely dominate other quasiparticle interference effects. It is evident from Fig. 4(b) that \( |\rho_Q(\omega)| \) displays a non-monotonic dependence on energy. In fact, as pointed out in Ref. 11 for the case of weak translational symmetry breaking, useful information about the induced order and the underlying quasiparticle structure is contained in \( \rho_Q(\omega) \), the real part of \( \rho_Q(\omega) \) at the ordering vector \( Q \). In general, for weak induced order \( \rho_Q(\omega) \) will exhibit peaks near \( \omega = \omega_{vH} \) due to the logarithmic divergence coming from the van Hove points at \((0, \pm \pi)\) and \((\pm \pi, 0)\), and near energies determined from degeneracy points \( E_k = E_{k+Q} \), where \( E_k \) is the quasiparticle spectrum for the homogeneous dSC 11.

For the simple nested Fermi surface \((t' = \mu = 0)\) and in the case a weak unidirectional \( \mu \)-wave \((\mu = A \sin(2\pi/4x))\) or a \( \Delta \)-wave \((\Delta = \Delta_0 + A \sin(2\pi/4x))\), this is illustrated in Fig. 4(c) by the black dashed lines. For the \( \Delta \)-wave \((\mu \)-wave\) \( \rho_Q(\omega) \) is symmetric (anti-symmetric) with characteristic peaks inside (outside) the bulk gap as well as weight at \( \omega = 0.4t \) which is the van Hove energy for this band structure 38, 41. In Fig. 4(c) we also show the full numerical result for \( \rho_Q(\omega) \) in the vortex state for different strengths of the magnetic order \( m \). As seen, the stripe induced features in \( \rho_Q(\omega) \) are roughly symmetric around \( \omega = 0.0 \). We find that sign changes in \( \rho_Q(\omega) \) at low energy \( \omega \ll \Delta \) are only present for weak induced order \( m/t \lesssim 0.45 \). We have checked that \( \rho_Q(\omega) \) is determined almost entirely by the stripe order: omission of the vortex flow causes only minor quantitative changes at \( \omega \ll \Delta \). We expect the qualitative results presented in Fig. 4(c) to apply primarily to LSCO and LBCO. In BSCCO, on the other hand, it is becoming clear that a strong component of the LDOS inhomogeneity is given by gap disorder 38, 41, 42.

We now turn briefly to the discussion of the LDOS near vortices with induced PDW order which, for simplicity, is modeled with a \( \Delta \)-wave. It is clear that PDW modulations cannot be the only induced order since that would not lead to a splitting of the ZES in the core center, and would not explain the enhanced spin response in the neutron experiments in the mixed state. Nevertheless, the question remains whether for certain regions of the phase diagram it coexists with or dominates the induced spin and/or charge order surrounding the cores, resulting in distinct features of the measured LDOS. As shown in Fig. 4(c), in the case of particle-hole symmetric bands \( \rho_Q(\omega) \) is a good probe of the induced order since \( \rho_Q(\omega) \) is symmetric or antisymmetric with respect to the bias voltage for periodic modulations in the \( \tau_1 \) or \( \tau_3 \) channel of Nambu space, respectively. However, realistic band parameters and possible coexistence of other symmetry breakings will strongly modify \( \rho_Q(\omega) \) making detailed fitting to various assumed order parameters necessary 41.

CONCLUSIONS

We have presented theoretical results for the quasiparticle structure near an increasingly complex vortex of a \( d \)-wave superconductor. We have discussed distinct LDOS features expected when magnetic or pair density wave order is induced by an applied magnetic field, and have suggested new tunneling experiments to test for field-induced antiferromagnetic order near the vortex cores of high-\( T_c \) materials.

[1] S. C. Zhang, Science 275, 1089 (1997).
[2] D. P. Arovas, A. J. Berlinsky, C. Kallin, and S. C. Zhang, Phys. Rev. Lett. 79, 2871 (1997).
[3] I. Maggio-Aprile, C. Renner, A. Erb, E. Walker, and O. Fischer, Phys. Rev. Lett. 75, 2754 (1995).
[4] S. H. Pan, E. W. Hudson, A. K. Gupta, K.-W. Ng, H. Eisaki, S. Uchida, and J. C. Davis, Phys. Rev. Lett. 85, 1536 (2000).
[5] Y. Wang and A. H. Macdonald, Phys. Rev. B 52, R3876 (1995).
[6] N. J. Curro, C. Milling, J. Haase, and C. P. Slichter, Phys. Rev. B 62, 3473 (2000); V. F. Mitrovic, E. E. Sigmund, M. Eschrig, H. N. Bachman, W. P. Halperin, A. P. Reyes, P. Kuhns, and W. G. Moulton, Nature 413, 501 (2001); K. Kakuyanagi, K. Kumagai, Y. Matsuda, and M. Hasegawa, Phys. Rev. Lett. 90, 197003 (2003).
[7] R. I. Miller, R. F. Kiefl, J. H. Brewer, J. E. Sonier, J. Chakhalian, S. Dunsiger, G. D. Morris, A. N. Price, D. A. Bonn, W. H. Hardy, and R. Liang, Phys. Rev. Lett. 88, 137002 (2002); J. E. Sonier, F. D. Callaghan, J. H. Brewer, W. N. Hardy, D. A. Bonn, and R. Liang, cond-mat/0508079.
[8] B. Lake, H. M. Ronnow, N. B. Christensen, G. Aeppli, K. Lefmann, D. F. McMorrow, P. Vorderwisch, P. Smeibidl, N. Mangkorntong, T. Sasagawa, M. Nohara, H. Takagi, and T. E. Mason, Nature 415, 299 (2002).
[9] H. Kimura, M. Kofu, Y. Matsumoto, and K. Hirota, Phys. Rev. Lett. 91, 067002 (2003); H. Kimura, K. Hirota, H. Matsushita, K. Yamada, Y. Endoh, S. H. Lee, C. F. Majkrzak, R. Erwin, G. Shirane, M. Greven, Y. S. Lee, M. A. Kastner, and R. J. Birgeneau, Phys. Rev. B 59, 6517 (1999).
[10] B. Khaykovich, Y. S. Lee, R. Erwin, S.-H. Lee, S. Wakimoto, K. J. Thomas, M. A. Kastner, and R. J. Birgeneau, Phys. Rev. B 66, 014528 (2002).

[11] E. Demler, S. Sachdev, and Y. Zhang, Phys. Rev. Lett. 87, 067202 (2001).

[12] B. M. Andersen, P. Hedegård, and H. Bruus, Phys. Rev. B 67, 134528 (2003).

[13] J. E. Hoffman, E. W. Hudson, K. M. Lang, V. Madhavan, H. Eisaki, S. Uchida, and J. C. Davis, Science 295, 466 (2002).

[14] B. M. Andersen, H. Bruus, and P. Hedegård, Phys. Rev. B 67, 134528 (2003).

[15] T. Hanaguri, C. Lupien, Y. Kohsaka, D.-H. Lee, M. Azuma, M. Takano, H. Takagi, and J. C. Davis, Nature 430, 1001 (2004).

[16] G. Levy, M. Kugler, A. A. Manuel, Ø. Fischer, and M. Li, Phys. Rev. Lett. 95, 257005 (2005).

[17] N. Ogata, Int. J. Mod. Phys. B 13, 3560 (1999); B. M. Andersen, H. Bruus, and P. Hedegård, Phys. Rev. B 61, 6298 (2000); M. Ichioka, M. Takigawa, and K. Machida, J. Phys. Soc. Jpn. 70, 33 (2001); J.-X. Zhu and C. S. Ting, Phys. Rev. Lett. 87, 147002 (2001); A. Ghosal, C. Kallin, and A. J. Berlinsky, Phys. Rev. B 66, 214502 (2002); M. Takigawa, M. Ichioka, and K. Machida, J. Phys. Soc. Jpn. 73, 450 (2004).

[18] D. Knapp, C. Kallin, A. Ghosal, and S. Mansour, Phys. Rev. B 71, 064504 (2005).

[19] J. H. Han and D.-H. Lee, Phys. Rev. Lett. 85, 1100 (2000); J.-I. Kishine, P. A. Lee, and X.-G. Wen, Phys. Rev. Lett. 86, 5365 (2001); Q.-H. Wang, J. H. Han, and D.-H. Lee, Phys. Rev. Lett. 87, 167004 (2001); C. Berthod and B. Giovannini, Phys. Rev. Lett. 87, 277002 (2001).

[20] M. Franz and Z. Tesanovic, Phys. Rev. Lett. 80, 4763 (1998).

[21] B. M. Andersen, P. Hedegård, and H. Bruus, J. Low Temp. Phys. 131, 281 (2003).

[22] M. Granath, V. Oganesyan, D. Orgad, and S. A. Kivelson, Phys. Rev. B 65, 184501 (2002).

[23] B. M. Andersen, I. V. Bobkova, P. J. Hirschfeld, and Yu. S. Barash, Phys. Rev. B 72, 184510 (2005).

[24] R. Haydock, Solid State Physics, Vol. 35, eds. Ehrenreich, F. Seitz and D. Turnbull (Academic Press, New York 1980).

[25] G. Litak, P. Miller, and B. L. Györffy, Physica C 251, 263 (1995); P. Miller and B. L. Györffy, J. Phys. Cond. Mat. 7, 5579 (1995).

[26] C. Caroli, P. G. de Gennes, and J. Matricon, Phys. Lett. 9, 307 (1964).

[27] F. Gygi and M. Schlüter, Phys. Rev. Lett. 65, 1820 (1990); Phys. Rev. B 43, 7609 (1991).

[28] J. D. Shore, M. Huang, A. T. Dorsey, and J. P. Sethna, Phys. Rev. Lett. 62, 3089 (1989).

[29] H. F. Hess, R. B. Robinson, R. C. Dynes, J. M. Valles, Jr., and J. V. Waszczak, Phys. Rev. Lett. 62, 214 (1989).

[30] C. Berthod, Phys. Rev. B 71, 134513 (2005).

[31] T. Dahm, S. Graser, C. Iniotakis, and N. Schopohl, Phys. Rev. B 66, 144515 (2002).

[32] P. I. Soininen, C. Kallin, and A. J. Berlinsky, Phys. Rev. B 50, R13883 (1994).

[33] M. Ichioka, N. Hayashi, N. Enomoto, and K. Machida, Phys. Rev. B 53, 15316 (1996).

[34] J. X. Zhu, C. S. Ting, and A. V. Balatsky, cond-mat/0109503.

[35] Y. Chen, Z. D. Wang, J.-X. Zhu, and C. S. Ting, Phys. Rev. Lett. 89, 217001 (2001).

[36] M. Sigrist, Prog. Theor. Phys. 99, 899 (1998).

[37] A. Wachowiak, J. Wiebe, M. Bode, O. Pietzsch, M. Morgenstern, and R. Wiesendanger, Science 298, 577 (2002).

[38] H. D. Chen, J.-P. Hu, S. Capponi, E. Arrigoni, and S.-C. Zhang, Phys. Rev. Lett. 89, 137004 (2002); H. D. Chen, O. Vafek, A. Yazdani, and S.-C. Zhang, Phys. Rev. Lett. 93, 187002 (2004).

[39] Z. Tesanovic, Phys. Rev. Lett. 93, 217004 (2004); A. Melikyan and Z. Tesanovic, Phys. Rev. B 71, 214511 (2005).

[40] J. A. Robertson, S. A. Kivelson, E. Fradkin, A. C. Fang, A. Kapitulnik, cond-mat/0602675; A. Del Maestro, B. Rosenow, S. Sachdev, cond-mat/0603029.

[41] D. Podolsky, E. Demler, K. Damle, and B. I. Halperin, Phys. Rev. B 67, 094514 (2003).

[42] T. S. Nunner, B. M. Andersen, A. Melikyan, and P. J. Hirschfeld, Phys. Rev. Lett. 95, 177003 (2005); A. C. Fang, L. Capriotti, D. J. Scalapino, S. A. Kivelson, N. Kaneko, M. Greven, and A. Kapitulnik, Phys. Rev. Lett. 96, 017007 (2006).