Confinement, Vacuum Structure: from QCD to Quantum Gravity

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Abstract

A minimal Lorentz gauge gravity model with $R^2$-type Lagrangian is proposed. In the absence of torsion the model admits a topological phase with unfixed metric. The model possesses a minimal set of dynamical degrees of freedom for the torsion. Remarkably, the torsion has the same number of dynamical off-shell degrees of freedom as the metric tensor. We trace an analogy between the structure of the quantum chromodynamics and the structure of possible theory of quantum gravity.

Key words: QCD, confinement, quantum gravity, torsion, effective theory.

1 Introduction

The gauge approach to gravity based on [1] gauging the Lorentz and Poincare groups [1,2,3] was proposed as a possible way to construct a consistent quantum theory of gravity. The extension of gravity models to the case of non-Riemannian space-time geometry reveals new possibilities towards construction of renormalizable quantum gravity with torsion [4,5]. Recently, a Lorentz gauge model of gravity with Yang-Mills type Lagrangian including torsion has been developed further in [6]. It has been proposed that the Einstein gravity can be induced as an effective theory via mechanism similar to the dual Meissner effect of color confinement in quantum chromodynamics (QCD). In that model the space-time metric is treated as a fixed classical field while the contortion supposed to be a quantum field. Such a treatment of the metric is not satisfactory from the conceptual point of view since one has to assume the existence of the classical space-time with a metric given a priori. In the present paper we propose a model which admits the existence of a pure topological...
phase with an arbitrary metric from the start. We conjecture that the torsion can be confined in analogy to confined gluons in QCD.

2 Abelian projection in \(SU(3)\) QCD

Let us start from the concept of the Abelian projection in QCD [7]. The principal role in this construction belongs to the scalar fields \(\hat{n}_a^i, a = 1, 2, 3; i = 1, 2\) which parameterize the coset \(SU(3)/U(1) \times U(1)\). In the general construction of the Abelian projection [8,9] the scalar field \(\hat{n}\) is given by a set of over determined variables which is not convenient for description of the effective theory like Faddeev-Niemi-Skyrme model [10]. We give an explicit construction of the Abelian projection for the group \(SU(3)\) with a minimal set of degrees of freedom for \(\hat{n}^i\).

The Cartan algebra of \(SU(3)\) Lie algebra is generated by two vectors \(n_3 = \hat{n}_3^a t^3, n_8 = \hat{n}_8^a t^8\) with \(t_{3,8}\) as generators of \(SU(3)\). Let us parametrize the lowest weight vector \(\hat{n}_8^a\) in terms of complex triplet field \(\Psi\) which parameterized the coset \(CP^2 \simeq SU(3)/SU(2) \times U(1)\)

\[
\hat{n}_8^a = N_1 \bar{\Psi} \lambda^a \Psi, \quad \bar{\Psi} \Psi = 1, \quad (1)
\]

where the normalization factor \(N_1 = -3/2\) provides the conditions

\[
\hat{n}_8^a \hat{n}_8^b = 1, \quad d^{abc} \hat{n}_8^a \hat{n}_8^b = -\frac{1}{\sqrt{3}} \hat{n}_8. \quad (2)
\]

To construct the second Cartan vector \(\hat{n}_3\), which is orthogonal to \(\hat{n}_8\), it is convenient to define projectional operators

\[
P^{ab} = \hat{n}_8^a \hat{n}_8^b, \quad P^a = \delta^{ab} - \hat{n}_8^a \hat{n}_8^b \quad (3)
\]

and introduce another independent complex triplet field \(\Phi\) \((\Phi\Phi = 1)\) orthogonal to \(\Psi\). With this the vector \(\hat{n}_3\) can be parameterized as follows

\[
\hat{n}_3^a = P^a \bar{\Phi} \lambda^b \Phi = \bar{\Phi} \lambda^a \Phi + \frac{1}{2} \bar{\Psi} \lambda^a \Psi. \quad (4)
\]

The parametrization defined by (1, 4) is invariant under dual local \(\tilde{U}(1) \times \tilde{U}'(1)\) group transformation. The Abelian projection of \(SU(3)\) gauge connection is similar to the decomposition of \(SU(2)\) gauge potential [7].
\[ \vec{A}_\mu = \hat{A}_\mu + \vec{C}_\mu + \vec{X}_\mu, \quad \hat{A}_\mu = A_{\mu i} \hat{n}_i + \vec{C}_\mu \]
\[ \vec{C}_\mu^a = -f^{abc} \hat{n}^b \partial_\mu \hat{n}^c i \equiv -\hat{n}_i \times \partial_\mu \hat{n}_i, \tag{5} \]

where \( \hat{A}_\mu \) is a restricted potential, \( \vec{C}_\mu \) is a magnetic potential, and \( \vec{X}_\mu \) represents the off-diagonal (valence) gluon. One can verify that the vectors \( \hat{n}_i \) are covariantly constant

\[ \hat{D}_\mu \hat{n}_i \equiv (\partial_\mu + \hat{A}_\mu) \hat{n}_i = 0. \tag{6} \]

The decomposition (5) allows two types of gauge transformation: (I) the background gauge transformation described by

\[ \delta \hat{A}_\mu = \frac{1}{g} \hat{D}_\mu \bar{\alpha}, \quad \delta \vec{X}_\mu = -\bar{\alpha} \times \vec{X}_\mu, \tag{7} \]

and, (II) the quantum gauge transformation described by

\[ \delta \hat{A}_\mu = 0, \quad \delta \vec{X}_\mu = \frac{1}{g} \hat{D}_\mu \bar{\alpha}. \tag{8} \]

The background gauge transformation shows that \( \hat{A}_\mu \) by itself satisfies the full \( SU(3) \) gauge degrees of freedom, even though it describes the Abelian part of the potential. Furthermore \( \vec{X}_\mu \) transforms covariantly like a vector.

3 Parallels between QCD and Quantum Gravity

The basic geometric objects in approaches to formulation of gravity as a gauge theory of the Poincare group \([1,2,3]\) are the vielbein \( e^m_a \) and the general Lorentz affine connection \( A_m^{\alpha d} \). The covariant derivative with respect to Lorentz gauge transformation is defined in a standard manner

\[ D_a = e^m_a (\partial_m + A_m), \tag{9} \]

where \( A_m \equiv A_{mcd} \Omega^d \) is a general affine connection taking values in the Lorentz Lie algebra. The affine connection \( A_{mcd} \) can be rewritten as a sum of Levi-Civita spin connection \( \varphi_{mc}^d(e) \) and contortion \( K_{mc}^d \)

\[ A_{mc}^d = \varphi_{mc}^d(e) + K_{mc}^d, \tag{10} \]
In analogy with QCD we can define two types of Lorentz gauge transformations consistent with the original Lorentz gauge transformation:

(I) the classical, or background, gauge transformation

\[
\begin{align*}
\delta e^m_a &= \Lambda^b_a \epsilon^m_b, \\
\delta \varphi_m(e) &= -\partial_m \Lambda - [\varphi_m, \Lambda], \\
\delta K_m &= -[K_m, \Lambda], 
\end{align*}
\]

(11)

(II) the quantum gauge transformation

\[
\begin{align*}
\delta e^m_a &= \delta \varphi_m(e) = 0, \\
\delta K_m &= -\hat{D}_m \Lambda - [K_m, \Lambda],
\end{align*}
\]

(12)

where \( \varphi_m \equiv \varphi_{med} \Omega^{ed} \), and the restricted covariant derivative \( \hat{D}_m \) is defined by means of the Levi-Civita connection only. Under the decomposition (10) the Riemann-Cartan curvature is splitted into two parts respectively

\[
R_{abcd} = \hat{R}_{abcd} + \tilde{R}_{abcd}.
\]

(13)

From the comparison of the Abelian decomposition in QCD with the decomposition of the Lorentz spin connection one can find the analogy between the restricted potential \( \hat{A}_\mu \) and valence gluon \( \hat{X}_\mu \) in QCD on the one hand, and the Levi-Civita connection \( \omega_{\mu cd} \) and contortion \( K_{\mu cd} \) in Lorentz gauge gravity on the other. Obviously, in QCD we can not treat the off-diagonal component \( \hat{X}_\mu \) as a true vector. The reason is that if we introduce, for instance, a mass term for the off-diagonal gluon into the Lagrangian then the renormalizability will be lost. For the same reason we can not treat the contortion \( K_{\mu cd} \) as a true tensor in gravity models in attempts to formulate a quantum renormalizable theory in the case if we wish to keep two types of Lorentz gauge symmetries.

Let us consider the following aspect of the confinement problem in QCD regarding the fact that quarks and gluons are not observable single particles. One heuristic argument why we can not observe the color single states is the following \cite{1}: quarks and gluons are not gauge invariant and we have no a conserved color charge like the electric charge in Maxwell theory. So that quarks and gluons can not be observable as single physical particles. If we accept the hypothesis that a Lorentz gauge model of gravity with torsion possesses two types of gauge symmetry (11,12) then we will be forced to accept the confinement of torsion.

\cite{1} author acknowledges Y.M. Cho for elucidating this argument.
4 Minimal model of quantum gravity with torsion

We are interested in such a Lagrangian in Riemann-Cartan space-time which is reduced to Gauss-Bonnet topological invariant in the limit of Riemannian geometry. So that, we will consider the following Lagrangian

\[ \mathcal{L} = -\frac{1}{4} (\alpha R^2_{abcd} + (1 - \alpha) R_{abcd} R^{cd\alpha \beta} - 4\beta R^2_{bd} - 4(1 - \beta) R_{bd} R^{db} + R^2 + 6\gamma A^2_{abcd}), \]  

(14)

where the irreducible tensor \( A_{abcd} \) is defined as follows [11]

\[ A_{abcd} = \frac{1}{6} (R_{abcd} + R_{acdb} + R_{adbc} + R_{bcad} + R_{bdca} + R_{cdab}). \]  

(15)

It turns out that the model described by the Lagrangian (14) admits dynamical degrees of torsion (contortion) for the special values of the parameters \( \beta = 0, \gamma = -3\alpha \). The parameter \( \alpha \) provides unimportant overall factor, so that one can set \( \alpha = 1 \) without loss of generality. For convenience, we will keep the parameters \( \alpha, \gamma \) arbitrary and later we will show that propagating torsion requires a unique value for the parameter \( \gamma = -3 \). The Lagrangian (14) takes the form (omitting total divergence terms)

\[ \mathcal{L} = \frac{1}{2} \left[ R^2_{abcd} + 2R_{abcd} R^{cd\alpha \beta} + 6R_{abcd} R^{acbd} \right]. \]  

(16)

In addition to the equations of motion \( \delta \mathcal{L} / \delta K_{bcd} = 0 \) one should impose gauge fixing conditions. To fix Lorentz gauge symmetry we choose the following constraints which are compatible with equations of motion

\[ \partial^i (K_{i\delta} - K_{\delta i}) = 0, \]  

(17)

\[ (\alpha + \gamma) \partial^i K_{i\gamma} = \gamma (\partial^i K_{i\delta} - \partial^i K_{\delta i}), \]  

(18)

\[ \partial^i \partial^j K_{i0j} = 0. \]  

(19)

For simplicity we choose the covariant constant background space-time as a Riemannian space-time of constant curvature \( \hat{R}_{abcd} = \rho(\eta_{ac} \eta_{bd} - \eta_{ad} \eta_{bc}) \). We will use the following decomposition of the space components of the contortion
\[ K_{\mu\gamma\delta} = \varepsilon_{\gamma\delta\rho} \vec{K}_{\mu\rho}, \]
\[ \dot{K}_{\mu\rho} = S_{\mu\rho} + \frac{1}{2}(\delta_{\mu\rho} \Delta - \partial_{\mu} \partial_{\rho}) S + (\partial_{\mu} S_{\rho} + \partial_{\rho} S_{\mu}) + \varepsilon_{\mu\rho\sigma} A_{\sigma}, \]
\[ K_{\mu0\rho} = R_{\mu\rho} + \frac{1}{2}(\delta_{\mu\rho} \Delta - \partial_{\mu} \partial_{\rho}) R + (\partial_{\mu} R_{\rho} + \partial_{\rho} R_{\mu}) + \varepsilon_{\mu\rho\sigma} Q_{\sigma}, \] (20)

Some of the equations of motion and gauge conditions represent constraints for components of \( K_{bcd} \). One can solve all constraints and gauge conditions in linearized approximation and substitute solution into the initial Lagrangian. After lengthy calculations one can find a final effective Lagrangian (quadratic in fields \( K_{bcd} \)) which contains only independent physical dynamical degrees of freedom

\[ \frac{1}{2} \mathcal{L}_{\text{eff}}^{(2)} = \frac{3}{8} S^{\alpha\beta} \rho S_{\alpha\beta} + A^{tr,\alpha}(\Box + 2\rho)A^{tr}_{\alpha} \]
\[ - (\varphi + \partial_{i}\psi)^{2} - \varphi \frac{\Box + 6\rho}{\Delta} \varphi + \psi(\Box + 6\rho)\psi, \] (21)
\[ \varphi = \partial^{i}Q_{i}, \quad \psi = -\frac{2}{3} \partial^{i}S_{i}. \] (22)

In conclusion, we propose a model of quantum gravity with dynamical torsion. The model has a number of advantages to compare with Yang-Mills type Lorentz gauge gravity. In the absence of torsion the model reduces to a pure topological gravity, i.e., one has a topological phase where the metric is not specified a priori. The metric can obtain dynamical content after dynamical symmetry breaking in the phase of effective Einstein gravity which is induced by quantum torsion corrections. Remarkably, the contortion in our model has the same number of degrees of freedom as the metric in Einstein gravity. This could be an additional hint that torsion can be interpreted as a quantum counter-part to the classical graviton.

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