Status of Supersymmetry in the Light of Recent Experiments

Utpal Chattopadhyay\textsuperscript{1}, Achille Corsetti\textsuperscript{2}, and Pran Nath\textsuperscript{2}

\textsuperscript{1} Harish-Chandra Research Institute, Chhatnag Road, Jhusi, Allahabad 211019, India
\textsuperscript{2} Department of Physics, Northeastern University, Boston, MA 02115, USA

Abstract. In this talk we discuss the constraints on supersymmetry arising from data from a number of recent experiments. There appears to be good cumulative evidence from experiment in favor of positivity of the sign of the Higgs mixing parameter $\mu$. Implications of this result for Yukawa unification are discussed since Yukawa unification is sensitive to the $\mu$ sign. An analysis of dark matter under the constraints of Yukawa unification is also given. It is shown that the simultaneous imposition of all existing constraints sharply defines the parameter space of models. Specifically models with nonuniversality of gaugino masses provide a simple resolution to the positivity of the $\mu$ parameter and Yukawa unification. Implications of these results for colliders and for the next generation of dark matter searches are also discussed.

1 Introduction

Over the recent past accumulation of data from experiments that probe physics beyond the standard model has begun to constrain new physics. In this talk we discuss the constraints on supersymmetry arising from these experiments. If 'recent experiments' is interpreted to include the experimental results over the past decade then there is an impressive body of data which taken together can put important limits on the parameter space of supersymmetric models. Perhaps at the top of the list here is the precision data on the gauge coupling constants\textsuperscript{[1]} which looks very favorable for supersymmetry in that the minimal supersymmetric standard model with a low lying sparticle spectrum is in excellent accord with the data within $1\sigma$-2$\sigma$ (for a review see\textsuperscript{[2]}). A small discrepancy that might exist can be easily taken account of with the help of Planck scale corrections\textsuperscript{[3]}. However, the gauge coupling unification does not put very stringent limits on the sparticle masses or on the sign of the Higgs mixing parameter $\mu$. Constraints on the sparticle masses and on the $\mu$ parameter arise from the experiment on the flavor changing neutral current process $b \rightarrow s + \gamma$\textsuperscript{[4,5,6]}. Here one finds that the sparticle spectrum is constrained and one sign of $\mu$ is preferred (the positive sign in the standard convention\textsuperscript{[7]}) and for the other $\mu$ sign most of the parameter space of a class of supersymmetric models is eliminated\textsuperscript{[8,9]}. Interestingly the central value of the $b \rightarrow s + \gamma$ experiment\textsuperscript{[4,5,6]} is somewhat lower than the central value of its standard model prediction\textsuperscript{[10]} providing a slight hint of a supersymmetric contribution. This is so because a supersymmetric loop correction from the chargino exchange can provide a contribution with a negative
sign. Another important constraint comes from the recent lower limit on the LEP experiment on the Higgs mass \[1\]. Thus before the LEP experiment closed down, it was able to place a lower limit on the Higgs mass of 115 GeV for the Standard Model. For the supersymmetric case the lower limit is a function of \(\tan \beta\) and can be as low as about 90 GeV for large \(\tan \beta\). Again these lower limits impose important constraints on the parameter space of supersymmetric models. Additional constraints on supersymmetric models emerge from the \(g - 2\) Brookhaven experiment \[2\]. We will discuss the \(g - 2\) experiment and its implications in detail in Sec. 2. One of the main results that emerges from the BNL data is the positivity of the \(\mu\) parameter which is in accord with the sign preferred by the \(b \rightarrow s + \gamma\) constraint. Further, the positivity of \(\mu\) has important implications for Yukawa coupling unification. This topic will be discussed in detail in Sec. 3. One of the important features of supersymmetry is that it provides a candidate for cold dark matter under the assumption of R parity conservation. Thus using renormalization group a class of supergravity models lead naturally to the lightest neutralino to be the lightest supersymmetric particle (LSP) and hence a candidate for dark matter \[3\]. Further, this class of models can produce just the right amount of dark matter that is indicated by the current astrophysical observations. The price one pays for generating the right amount of dark matter is to further constrain the parameter space of supersymmetric models. This topic will be discussed in Sec. 4. Finally, we note that the SuperKamiokande experiment has now reached its maximum sensitivity \[4\] for the detection of the mode \(p \rightarrow \bar{\nu} K^+\) preferred by supersymmetric models and has placed a new limit of \(\tau(p \rightarrow \bar{\nu} K^+) > 1.9 \times 10^{33}\text{yr}\). This result puts the minimal supersymmetric grand unified models under considerable stress. We will briefly discuss this topic in the conclusion.

## 2 \(g - 2\) Experiment

The anomalous magnetic moment \(a = (g - 2)/2\) is an important probe of new physics beyond the standard model. Typically new physics contributions to the anomalous moment obey \(a_l(\text{new physics}) \sim m_l^2/\Lambda^2\) where \(\Lambda\) is the scale of the new physics. Because of this \(a_\mu\) is a much more sensitive probe of new physics than \(a_e\). Last February the BNL experiment gave a new determination of \(a_\mu\) with a substantially lower error \[12\] than previous measurements. The current evaluation is \(a_\mu^{exp} = 11659203(15) \times 10^{-10}\). As of a few months ago the Standard Model prediction consisting of the qed, electro-weak and hadronic corrections amounted to \(a_\mu^{SM} = 11659159.7(6.7) \times 10^{-10}\) \[15\]. This gives \(a_\mu^{exp} - a_\mu^{SM} = 43(16) \times 10^{-10}\) which is a 2.6\(\sigma\) difference between theory and experiment. However, over the past few months there has been a reevaluation of the Standard Model prediction. This revision arises from a change in sign of the light by light hadronic correction. Thus previous analyses gave for \(a_\mu^{had}(LbL)\) the value \[16,17\] \(a_\mu^{had}(LbL) = -8.5(2.5) \times 10^{-10}\). More recent evaluations \[18,19\] however, find the sign of \(a_\mu^{had}(LbL)\) to be opposite to that of the previous determinations. Thus the analysis of Ketch \textit{et. al} gives \[18\] \(a_\mu^{had}(LbL) = 8.3(1.2) \times 10^{-10}\) and a follow up
analysis by Hayakawa and Kinoshita gives \[ a_{\mu}^{\text{had}}(LbL) = 8.9(1.5) \times 10^{-10} \]. Support for the change in sign also comes from partial analysis by Bijnens et al. \[23\] which gives \( a_{\mu}^{\text{had}}(LbL) = 8.3(3.2) \times 10^{-10} \) and by Blockland et al. \[24\] which gives \( a_{\mu}^{\text{had}}(LbL) = 5.6 \times 10^{-10} \). Taking the average of the first two and correcting the standard model prediction one gets \( a_{\mu}^{\text{SM}} = 11659176.8(6.7) \times 10^{-10} \). This leads to the 1.6\(\sigma\) deviation between experiment and the standard model result, i.e.,

\[
a_{\mu}^{\exp} - a_{\mu}^{\text{SM}} = 26(16) \times 10^{-10}
\]  

(1)

We note in passing that there is no unanimity yet on the size of the light by light hadronic correction. Indeed, very recently another evaluation of \( a_{\mu}^{\text{had}}(LbL) \) based on chiral perturbation theory was given in Ref.\[22\]. This analysis finds \( a_{\mu}^{\text{had}}(LbL) = (5.5^{+5}_{-6} + 3.1\tilde{C}) \times 10^{-10} \) where \( \tilde{C} \) stands for corrections arising from the subleading contributions. The result of Eq.(1) corresponds to a value of \( \tilde{C} \sim 1 \). However, the authors of Ref.\[22\] indicate that a \( \tilde{C} \) range of -3 to 3 or even larger is not unreasonable. In addition, to the uncertainties associated with the light by light contribution, one also has the errors associated with the evaluation of the \( a^2 \) correction to the vacuum polarization. In the evaluation of Eq.(1) we used the \( a^2 \) correction to the vacuum polarization as given by the analysis of Ref.\[23\] which gives \( a_{\mu}^{\text{had}}(\alpha^2\text{vac.pol.}) = (692.4 \pm 6.2) \times 10^{-10} \). However, there are several other evaluations of this quantity\[24\] and this subject is still a topic of further investigation.

We discuss next the SUSY contribution to \( a_{\mu} \). The first analysis of the supersymmetric correction to \( a_{\mu} \) was given soon after the development of SUGRA unified models\[25\]. At the one loop level one has \( a_{\mu}^{\text{SUSY}} = a_{\mu}^{\tilde{\chi}^\pm} + a_{\mu}^{\tilde{\chi}^0} \). For the CP conserving case one has that the chargino contribution is the larger one. It is given by\[26\]

\[
a_{\mu}^{\tilde{\chi}^\pm} = \frac{m_{\tilde{\chi}^\pm}^2}{48\pi^2} \frac{A_R^{(a)} A_R^{(a)}}{m_{\tilde{\chi}^\pm}^2} F_1 \left( \frac{m_{\tilde{\chi}^\pm}}{m_{\tilde{\chi}^\pm}} \right)^2 + \frac{m_{\mu}}{8\pi^2} \frac{A_R^{(a)} A_R^{(a)}}{m_{\tilde{\chi}^\pm}^2} F_2 \left( \frac{m_{\tilde{\chi}^\pm}}{m_{\tilde{\chi}^\pm}} \right)^2
\]

(2)

where \( A_L(A_R) \) are the left(right) chiral amplitudes. Now the chiral interference term proportional to \( A_L A_R \), particularly the contribution from the lighter chargino term typically dominates the chargino exchange contribution. The above amplitude contains some very interesting properties\[27\]. One finds that typically \( A_L \sim 1/\cos\beta \) and because of this \( a_{\mu}^{\text{SUSY}} \sim \tan\beta \). Further, it is easy to show that \( A_L \) depends on the sign of \( \mu \tilde{m}_2 \) and thus effectively the sign of \( a_{\mu}^{\text{SUSY}} \) is controlled by the sign of \( \mu \tilde{m}_2 \) (where we use the sign convention of Ref.\[7\]). Thus one finds that typically\[27\]

\[
a_{\mu}^{\text{SUSY}} > 0, \mu \tilde{m}_2 m > 0; \quad a_{\mu}^{\text{SUSY}} < 0, \mu \tilde{m}_2 m < 0
\]

(3)

More recently the absolute signs of the supersymmetric contribution was checked by taking the supersymmetric limit and it was shown that, as expected, in this limit \( a_{\mu}(\text{SM}) + a_{\mu}(\text{SUSY}) = 0 \)\[28\]. Soon after the BNL result of Ref.\[12\] was announced, several analyzes on its implications for the spartic spectrum
were carried out\cite{30,31} using the result $a_{\mu}^{exp} - a_{\mu}^{SM} = (43 \pm 16) \times 10^{-10}$ which found upper limits on sparticle masses well within reach of the LHC. Specifically in the work of Ref.\cite{30} using a 2\(\sigma\) error corridor around the announced difference $a_{\mu}^{exp} - a_{\mu}^{SM}$ so that $10.6 \times 10^{-10} < a_{\mu}^{SUSY} < 76.2 \times 10^{-10}$ it is found that the BNL data implies that in mSUGRA the following upper bounds on sparticle masses hold: $m_{\tilde{z}} \leq 650 GeV$, $m_{\tilde{\nu}} \leq 1.5 TeV$ ($\tan \beta \leq 55$), $m_{1/2} \leq 800 GeV$, $m_{0} \leq 1.5 TeV$ ($\tan \beta \leq 55$). In view of the revised difference of Eq.(1) a new analysis of the allowed parameter space in mSUGRA was carried out in Ref.\cite{32}. Results are displayed in Fig.1 with 1\(\sigma\) and 1.5\(\sigma\) error corridors. One finds that the 1\(\sigma\) upper limits with the revised result are exactly the same as the 2\(\sigma\) upper limits of the previous analysis\cite{30}. Specifically, in this case the squarks and gluinos upper limits lie below 2 TeV. Now the LHC can explore squarks/gluinos up to 2 TeV\cite{33}. Thus the revised 1\(\sigma\) upper limits gotten from the $g - 2$ result imply that sparticles should become visible at the LHC. The upper limits for the 1.5\(\sigma\) case, however, are significantly larger as may be seen from Fig.1 and one finds that part of the allowed parameter space lies outside the reach of the LHC.

An interesting aspect of the BNL data is that it determines the sign of $\mu$ given the sign of $m_{\tilde{\nu}}$. Thus assuming CP conservation one finds that the BNL data determines the sign($\mu m_{\tilde{\nu}}$) to be positive. Thus $\mu$ is determined to be positive for a wide class of supersymmetric models where $m_{\tilde{\nu}}$ is positive. Now $\mu$ positive is favored by the $b \rightarrow s + \gamma$ constraint\cite{8} and a positive $\mu$ is also favorable for the satisfaction of the relic density constraints. However, a positive $\mu$ is typically not favored by Yukawa coupling unification. We will discuss this issue in detail in Sec.3.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{Regions corresponding to the 1.5\(\sigma\) and the 1\(\sigma\) constraints of $a_{\mu}^{SUSY}$ for $\tan \beta = 45$ from Ref.\cite{32}. The top left gray regions do not satisfy the radiative electroweak symmetry breaking requirement or the lighter chargino mass limit, whereas the bottom patterned regions are typically discarded by stau becoming the LSP. The bottom patterned region near the higher $m_{1/2}$ side and on the border of the white allowed regions are discarded because of CP-odd Higgs boson turning tachyonic at the tree level.}
\end{figure}
Fig. 2. Allowed regions corresponding to 1.5σ and 1σ constraints on SUSY for nonuniversal gaugino mass scenario of the SU(5) 24 plet case from Ref. [32]. The top gray regions correspond to disallowed areas via radiative electroweak symmetry breaking constraint. The bottom patterned regions for tan β = 5, 10 and 30 are typically eliminated via the stability requirement of the Higgs potential at the GUT scale. Part of the region with large |c_{24}m_{1/2}| and large m_0 bordering the allowed (white) region for tan β = 30 is eliminated via the limitation of the CP-odd Higgs boson mass turning tachyonic at the tree level. For tan β = 40 most of the region (patterned and shaded) to the right of the allowed white small region is eliminated because of λ_b going to the non-perturbative domain due to a large supersymmetric correction to the b quark mass.

An interesting question is the possible effect of extra dimensions on the analysis of g − 2 and whether such contributions can be large enough to produce a significant background for the supersymmetric effects. This question has been examined in a variety of extra dimension models [34,35]. The simplest possibility considered is a model with one extra dimension with the fifth dimension compactified [34]. Specifically we consider a five dimensional model with the large extra dimension compactified on S^1/Z_2 with radius R (M_R = 1/R = O(TeV)). The spectrum of this theory contains massless modes with N=1 SUSY in 4D, which precisely form the spectrum of MSSM in 4D. In addition one has massive Kaluza-Klein modes which fall in N=2 multiplets. One of the interesting things that happen in the above scenario is that the mechanism that generates corrections to g − 2 also generates corrections to the Fermi constant [34]. Now the Standard Model prediction on G^SM_F is in fairly good accord with the experimental value. Thus the contribution arising from extra dimensions must be accommodated within the error corridor between the experimental value of G_F and its standard model prediction. This constrains the size of the extra dimension so that M_R ≥ 3 TeV. Now large extra dimensions affect the value of a_µ from contributions via the excitations of the W, Z, γ. However, because of the fact that M_R ≥ 3 TeV one finds that the effect of the extra dimension is numerically small, i.e., one finds that the corrections of the extra dimensions to a_µ is up to two or
more orders of magnitude smaller than the supersymmetric correction. Similar results hold for the case of strong gravity extra dimension models when one includes the constraints on extra dimensions from the current experiment. Thus for practical purposes one finds that the contribution from extra dimensions does not produce a strong background to the supersymmetric contribution. The above analysis also shows that $g - 2$ is not a sensitive probe of extra dimensions. Perhaps the most effective way to probe extra dimensions is via direct production of the Kaluza-Klein states at colliders, where Kaluza-Klein masses up to 6 TeV can be probed. Another important phenomenon is the effect of CP violation of $g - 2$. This was investigated in Refs. and it was found that $g - 2$ is a sensitive function of the phases. Indeed the constraints on $g - 2$ can be utilized to constrain the phases themselves. Further, if the new physics effect in the BNL experiment turns out to be of size $\sim 10^{-9}$ then this would also be encouraging for the possible observation of the muon electric dipole moment (EDM) $d_\mu$. This is so because the muon anomaly is related to the real part and the EDM to the imaginary part of the same diagrams exchanging charginos and neutralinos. These results are very interesting in view of a recent proposal to probe $d_\mu$ with a sensitivity which is six orders of magnitude better than the current sensitivity [31].

3 $b - \tau$ Unification and $\mu$ Sign

$b - \tau$ unification prefers $\mu < 0$ [42, 43]. This issue is tied closely with SUSY correction to the $b$ quark mass. 

$$m_b(M_Z) = \lambda_b(M_Z) \frac{v}{\sqrt{2}} \cos \beta (1 + \Delta_b)$$ (4)
where $\Delta_b$ is the loop correction to $m_b$. The largest contributions to $\Delta_b$ arise from the gluino and the chargino exchanges. The loop correction to $m_b$ is given by $\Delta_b^\mu = \frac{2\alpha_3 \mu M_{\tilde{g}}}{3\pi} \tan \beta I(m_{\tilde{b}_1}^2, m_{\tilde{b}_2}^2, M_{\tilde{g}}^2)$, $\Delta_b^{\chi^+} = \frac{Y_{\tilde{t}} \mu A_t}{4\pi} \tan \beta I(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2, \mu^2)$ (5)

where $Y_{\tilde{t}} = \frac{\lambda_{t}}{4\pi}$ and $I(a, b, c) > 0$. A useful criterion for $b - \tau$ unification and more generally for Yukawa unification are the parameters $\delta_{ij} = |\lambda_i - \lambda_j|/\lambda_{ij}$ where $\lambda_{ij} = (\lambda_i + \lambda_j)/2$. It is well known that $b - \tau$ unification requires a negative contribution to the $b$ quark mass. However, from Eq.(5) one finds that the dominant gluino exchange contribution to $\Delta_b$ is positive for a positive $\mu$ which is not what is preferred by the $b - \tau$ unification constraint. How to reconcile the positivity of $\mu$ with the Yukawa unification has been discussed in several recent works [45,46,47,48]. We discuss here in some detail the scenarios where such a phenomenon arises naturally from gaugino mass nonuniversality. The basic mechanism here is rather simple. With nonuniversalities one can arrange the gaugino masses $\tilde{m}_2$ and $m_{\tilde{g}}$ to have opposite signs which allows for consistency with $g - 2$ and $b - \tau$ unification with a positive $\mu$ sign. Further such opposite sign correlations between $\tilde{m}_2$ and $m_{\tilde{g}}$ arise naturally for certain group structures. We discuss below SU(5) and SO(10) examples where the above phenomenon manifests [48]. For the case of SU(5) unification the gaugino mass matrix in general transforms like the symmetric product of two adjoint representations of SU(5) and this product can be expanded as the sum of the representations 1, 24, 75 and 200. Of these one finds that the 24 plet component yields opposite signs for the SU(2) and SU(3) gauginos. Thus one has [49]

$$(24 \times 24)_{sym} = 1 + 24 + 75 + 200$$

$M_3 : M_2 : M_1 = 2 : -3 : -1, \quad 24 - plet$$

(6)

A similar situation arises for the SO(10) case. Here the gaugino mass matrix transforms like the symmetric product of two adjoint representations of SO(10) and this product can be expanded as the sum of the representations 1, 54, 210 and 770. Of these one finds that the 54 plet component yields opposite signs for the SU(2) and SU(3) gauginos. In fact in this case there are more than one possible ways to achieve the opposite signs for the SU(2) and SU(3) gauginos depending on the pattern of symmetry breaking. Thus one has [50]

$$(45 \times 45)_{sym} = 1 + 54 + 210 + 770$$

$M_3 : M_2 : M_1 = 1 : -3/2 : -1 \quad 54 - plet$$

$M_3 : M_2 : M_1 = 1 : -7/3 : 1 \quad 54' - plet$$

(7)

where the 54-plet and the 54'-plet correspond to two different patterns of symmetry breaking. In general the gaugino masses could be a linear combination of the different representations such that $\tilde{m}_i(M_G) = m_{1/2} \sum_r C_r n_r^i$ where $n_r^i$ are characteristic of the representation $r$ and $C_r$ are the relative weights. In the analysis below we will consider only one representation in the sum. Thus for SU(5) we will consider the gaugino masses arising from the 24 plet representation and
for the SO(10) case we will consider the cases where the gaugino masses arise
either from the 54 plet representation or from a 54’ representation. More
generally, of course, one may have linear combinations of different representations
and the analysis given here can be extended easily to these more general cases.

We consider first the 24 plet case of SU(5). Here an analysis similar to that of
Fig.1 is given in Fig.2 where the regions of the parameter space allowed by the 1σ
and by the 1.5σ $a_\mu$ constraint is exhibited. We note that the allowed regions
are significantly modified by the presence of nonuniversalities. An interesting
phenomenon here is that $\tan\beta = 40$ as the $b$ quark Yukawa coupling gets into a nonperturbative domain
because of large supersymmetric corrections to the $b$ quark mass. A similar
analysis for the 54 case of SO(10) is given in Fig.3 where we give an analysis of the
allowed parameter space for 1σ and for the 1.5σ $a_\mu$ constrains. As is well
known for the large $\tan\beta$ case of SO(10) one needs nonuniversalities of the Higgs
scalar mass parameters at the GUT scale to accomplish radiative breaking of
the electroweak symmetry. In the analysis of Fig.3 we used $m_{H_1}^2 = 1.5m_\mu^2$ and
$m_{H_2}^2 = 0.5m_\mu^2$. The GUT boundary conditions for the above soft parameters were varied (up to 50%)
to test the stability of the results. It was found that there was no significant change in the results as a consequence of these modifications.
The above scenarios produce a negative contribution to the $b$ quark mass for $\mu$
positive and thus lead to Yukawa unification for a positive $\mu$ consistent with the
$g - 2$ and the $b \to s + \gamma$ constraints.

4 Implications for Dark Matter

Supersymmetric dark matter and its detection has been a topic of considerable
theoretical investigation over the years (see Ref. [51] for an overview of dark matter
and Ref. [52] for a sample of recent works on the analysis of supersymmetric
dark matter). However, there are very few works on the study of supersymmetric
dark matter coupled with Yukawa unification [53]. Here we investigate the implications of the positivity of the $\mu$ sign and Yukawa unification for the detection rates in the direct detection of neutralino dark matter [32]. We focus on the scalar
cross section

$$\sigma_{\chi p}(\text{scalar}) = \frac{4\mu^2}{\pi} \left( \sum_{i=u,d,s} f_i^p C_i + \frac{2}{27} (1 - \sum_{i=u,d,s} f_i^p) \sum_{a=c,b,t} C_a \right)^2 \quad (8)$$

where $f_i^p$ (i=u,d,s quarks) are the quark densities defined by $m_p f_i^p = < p | m_{q_i} \bar{q}_i | p >$.
There are significant uncertainties associated with the determination of $f_i^{p,n}$. An
analytical solution for $f_i^{p,n}$ can be gotten by using [54] $\sigma_{\pi N}$, $x$, and $\xi$ as inputs
where $< p | 2^{-1}(m_u + m_d) (\bar{u} u + \bar{d} d) | p > = \sigma_{\pi N}$, $x = x_0 / \sigma_{\pi N} = < p | \bar{u} u + \bar{d} d - 2 s s | p >$,
and $\xi = < p | \bar{u} u - \bar{d} d | p > / < p | \bar{u} u + \bar{d} d | p >$. In terms of the above variables $f_i^p$ are given by $f_i^u = (1 + \xi) m_u \sigma_{\pi N} / m_p (m_u + m_d)$,
$f_i^d = (1 - \xi) m_d \sigma_{\pi N} / m_p (m_u + m_d)$, and similar relations hold for $f_i^{n,t}$. One also finds that $f_i^{p,n}$ satisfy the relation $f_i^p f_i^n = f_i^u f_i^d$. We use the above to give
a numerical evaluation of $f_{p,n}^n$ and its uncertainties. Using the most recent evaluation of $\sigma_{\pi N}$ of $\sigma_{\pi N} = (64 \pm 9)\text{GeV}$ given by the SAID pion-nucleon database\cite{55}, $x = 0.55 \pm 0.12$, the value of $\xi$ given by the baryon mass splittings, i.e., by the formula $\xi = (\Xi^- + \Xi^0 - \Sigma^+ - \Sigma^-)/x/(\Xi^- + \Xi^0 + \Sigma^+ + \Sigma^- - 2m_p - 2m_n)$ and using $\xi = 0.196x$ one finds $\xi = 0.108 \pm 0.024$. Additionally using the quark mass ratios $m_u/m_d = 0.553 \pm 0.043$, $m_s/m_d = 18.9 \pm 0.8$ one finds $f_u^p = 0.027 \pm 0.005$, $f_d^p = 0.038 \pm 0.006$, $f_n^u = 0.022 \pm 0.004$, $f_n^d = 0.049 \pm 0.007$, and $f_s^n = f_s^p = 0.37 \pm 0.11$. We use these quark densities to discuss the implications of the scenarios discussed in Sec.3 for dark matter.

**Fig. 4.** Plot of the neutralino-proton scalar cross section $\sigma_{\chi p}$ vs the lightest neutralino mass $m_\chi$ for the SU(5) 24 plet case with the range of the parameters given in Fig.2 satisfying all the desired constraints including the $b - \tau$ unification constraint so that $\delta_{b\tau} \leq 0.3$ from Ref.\cite{32}. The small crosses satisfy the $g_\mu - 2$ constraints, the (blue) filled squares additionally satisfy the $b \to s + \gamma$ limits and the (red) filled ovals satisfy all the constraints, i.e., the $g_\mu - 2$ constraint, the $b \to s + \gamma$ constraint, and $\delta_{b\tau} \leq 0.3$. The area enclosed by solid lines is excluded by the DAMA experiment\cite{59}, the dashed line is the lower limit from the CDMS experiment\cite{60}, the dot-dashed line is the lower limit achievable by CDMS in the future\cite{60}, and the dotted line is the lower limit expected from the proposed GENIUS experiment\cite{61}.

An interesting issue concerns the question if the parameter space consistent with the constraints discussed in Sec.3 will pass the test of neutralino relic density constraints for cold dark matter (CDM). Current estimates show that CDM satisfies the constraint $0.02 \leq \Omega_\chi h^2 \leq 0.3$ where $\Omega_\chi$ is the ratio of the CDM relic density and the critical relic density needed to close the universe, and $h$ is the hubble parameter in units of 100 km/sMpc. Additionally, we will require that for SU(5) we satisfy the $b - \tau$ unification constraint of $\delta_{b\tau} \leq 0.3$ and for SO(10) the $b - t - \tau$ unification constraint of $\delta_{ij} \leq 0.3$ where \{i, j\} = b, t, $\tau$. 
Fig. 5. Plot of the neutralino-proton scalar cross section $\sigma_{\chi p}$ vs the neutralino mass $m_{\chi}$ for the SO(10) 54-plet case from Ref.\[32\] satisfying all the desired constraints including the $b - \tau$, $b - t$ and $t - \tau$ unification constraint so that $\delta_{b\tau}, \delta_{bt}, \delta_{t\tau} \leq 0.3$. The small crosses satisfy the $g_{\mu} - 2$ constraints, the (blue) filled squares additionally satisfy the $b \rightarrow s + \gamma$ limits and the (red) filled ovals satisfy all the constraints, i.e., the $g_{\mu} - 2$ constraint, the $b \rightarrow s + \gamma$ constraint, and Yukawa unifications with $\delta_{b\tau}, \delta_{bt}, \delta_{t\tau} \leq 0.3$.

Table 1: Sparticle masses for 24, 54, 54$'$ cases from Ref.\[32\]

|        | 24 (GeV) | 54 (GeV) | 54$'$ (GeV) |
|--------|----------|----------|-------------|
| $\chi_1^0$ | 32.3 - 75.2 | 32.3 - 81.0 | 32.3 - 33.4 |
| $\chi_1^+ $ | 96.7 - 422.5 | 94.7 - 240.8 | 145.7 - 153.9 |
| $\chi_3^0 $ | 110.5 - 564.3 | 301.5 - 757.1 | 420.9 - 633.8 |
| $\chi_4^0$ | 259.2 - 575.9 | 311.5 - 759.7 | 427.6 - 636.9 |
| $\tilde{\chi}_1^0$ | 86.9 - 422.6 | 94.6 - 240.8 | 145.8 - 153.9 |
| $\tilde{\chi}_2^0$ | 259.9 - 577.2 | 315.1 - 761.6 | 430.7 - 639.2 |
| $\tilde{g}$ | 479.5 - 1077.2 | 232.5 - 580.3 | 229.8 - 237.4 |
| $\tilde{\mu}_1$ | 299.7 - 1295.9 | 480.5 - 1536.8 | 813.1 - 1196.3 |
| $\tilde{\mu}_2$ | 355.1 - 1309.3 | 489.8 - 1482.7 | 835.3 - 1237.6 |
| $\tilde{\tau}_1$ | 203.5 - 1045.1 | 294.2 - 1172.6 | 579.4 - 863.7 |
| $\tilde{\tau}_2$ | 349.6 - 1180.9 | 422.6 - 1311.7 | 704.6 - 1018.3 |
| $\tilde{u}_1$ | 533.6 - 1407.2 | 566.7 - 1506.4 | 822.9 - 1199.8 |
| $\tilde{u}_2$ | 561.1 - 1443.0 | 584.7 - 1544.6 | 849.6 - 1232.6 |
| $\tilde{d}_1$ | 535.1 - 1407.5 | 580.3 - 1546.2 | 845.1 - 1232.5 |
| $\tilde{d}_2$ | 566.7 - 1445.2 | 590.1 - 1546.7 | 853.3 - 1235.2 |
| $\tilde{t}_1$ | 369.9 - 975.2 | 271.5 - 999.6 | 513.7 - 819.9 |
| $\tilde{t}_2$ | 513.7 - 1167.6 | 429.4 - 1107.4 | 599.4 - 848.2 |
| $\tilde{b}_1$ | 488.2 - 1152.8 | 158.1 - 1042.0 | 453.2 - 749.9 |
| $\tilde{b}_2$ | 532.3 - 1207.0 | 396.6 - 1159.2 | 610.5 - 880.4 |
| $\tilde{h}$ | 104.3 - 114.3 | 103.8 - 113.3 | 108.1 - 110.9 |
| $\tilde{H}$ | 111.9 - 798.8 | 151.5 - 1227.6 | 473.4 - 831.9 |
| $\tilde{A}$ | 110.5 - 798.8 | 151.4 - 1227.6 | 473.4 - 831.9 |
| $\mu$ | 96.0 - 559.5 | 291.1 - 752.7 | 413.1 - 628.4 |
Assuming that CDM is composed entirely of neutralinos one finds that the parameter space allowed by the constraints discussed in Sec.3 indeed allow for the satisfaction of the relic density constraints. The sparticle spectrum including the relic density constraints is discussed in Table 1. The spectrum exhibited in Table 1 is consistent with the criterion of naturalness (see, e.g., Ref.[57]) and it would be interesting to discuss the possible signal such as the trileptonic signal[58] that emerge from this spectrum. One may also discuss the detection rates for the direct detection of dark matter in these scenarios. In Fig.4 an analysis is given of the maximum and the minimum scalar cross section $\sigma_{\chi-p}$ for the 24 plet case of SU(5) as a function of the the neutralino mass. The region of the parameter space where is signal is claimed by DAMA[59] is exhibited in Fig.4. Also exhibited in Fig.4 are the experimental upper limits from the CDMS experiment[60] and the limits that the CDMS experiment and the proposed GENIUS experiment[61] will be able to achieve in the future. The analysis of Fig.4 shows that the future CDMS experimental limits will probe a major part of the parameter space of the 24 plet model, while the GENIUS detector will probe the entire parameter space of the 24 model. A similar analysis for the 54 plet case of SO(10) is given in Fig.5 with the same conclusions as for the case of Fig.4. A analysis for the 54' case is given in Ref.[32].

5 Conclusion

We summarize now our conclusions. First if the $a_{\mu}^{exp} - a_{\mu}^{SM}$ difference persists at a perceptible level, i.e., at the level $\sim 10^{-9}$ or larger, then direct observation of new physics is implied. Assuming new physics is SUSY, we expect that most of the sparticles such as $\tilde{g}, \tilde{q}, \tilde{\chi}^{\pm}, \chi^{0}$ etc should become visible at the LHC. In this paper we also discussed the implications of a positive $\mu$ implied by the BNL data. Now it is well known that a positive $\mu$ is preferred by the $b \to s + \gamma$ constraint in that a large part of the parameter space is allowed by this constraint for a positive $\mu$. Further, a positive $\mu$ is beneficial for the direct search for dark matter since a large part of the parameter space is available for the satisfaction of the relic density constraints. One downside to a positive $\mu$ is that Yukawa unification is more difficult. However, it appears possible to overcome this problem. One way to accomplish this is to use nonuniversality of the gaugino masses and the analysis here shows that a simultaneous satisfaction of all the constraints, i.e., the $g - 2$, $b \to s + \gamma$ and $b - \tau$ unification constraints is possible. Models of the type discussed here which satisfy these constraints typically produce a low lying Higgs boson mass which can be probed at RunII of the Tevatron. Further, the sparticle spectrum of these models is typically also low lying and can be fully probed at the LHC. Thus these models can also be fully tested via direct detection of dark matter since a detector such as GENIUS can probe the entire parameter space of these models. Finally, we wish to draw attention to proton decay in supersymmetric GUT models[62]. The current data from SuperKamiokande indicates that the minimal SUSY GUT models including the minimal SU(5) and SO(10) models are under stress[63,64]. The above situation
arises in part due to an improved value of $\beta_p$ [65] (the coefficient of the three quark operator between $p$ and the vacuum state) and due to an increase in the lower limits on the proton decay lifetime [14]. The current experimental constraint on the muon anomaly further tends to destabilize the proton. Several approaches to correct the situation have been proposed recently which mostly involve dealing with non-minimal models [66,67].

This work was supported in part by NSF grant PHY-9901057.

References

1. Eur. Phys. J. C15, 1(2000).
2. For a review see, K. R. Dienes, Phys. Rept. 287, 447 (1997).
3. L. J. Hall and U. Sarid, Phys. Rev. Lett. 70, 2673 (1993); T. Dasgupta, P. Mamales and P. Nath, Phys. Rev. D 52, 5366 (1995); D. Ring, S. Urano and R. Arnowitt, Phys. Rev. D 52, 6623 (1995).
4. S. Chen et al. (CLEO Collaboration), Phys. Rev. Lett. 87, 251807 (2001).
5. H. Tajima, talk at the 20th International Symposium on Lepton-Photon Interactions”, Rome, July 2001.
6. R. Barate et al., Phys. Lett. B429, 169 (1998).
7. SUGRA Working Group Collaboration (S. Abel et. al.), [arXiv:hep-ph/0003154].
8. P. Nath and R. Arnowitt, Phys. Lett. B 336, 395 (1994); Phys. Rev. Lett. 74, 4592 (1995); F. Borzumati, M. Drees and M. Nojiri, Phys. Rev. D 51, 341 (1995); H. Baer, M. Brhlik, D. Castano and X. Tata, Phys. Rev. D 58, 015007 (1998).
9. M. Carena, D. Garcia, U. Nierste, C.E.M. Wagner, Phys. Lett. B499 141 (2001); G. Degrassi, P. Gambino, G.F. Giudice, JHEP 0012, 009 (2000) and references therein; W. de Boer, M. Huber, A.V. Gladyshev, D.I. Kazakov, Eur. Phys. J. C 20, 689 (2001).
10. P. Gambino and M. Misiak, Nucl. Phys. B611, 338 (2001); P. Gambino and U. Haisch, JHEP 0110, 020 (2001). See also T. Hurth, [hep-ph/0106050]. For previous analysis see, A.L. Kagan and M. Neubert, Eur. Phys. J. C7, 5(1999).
11. [LEP Higgs Working Group Collaboration], “Searches for the neutral Higgs bosons of the MSSM: Preliminary combined results using LEP data collected at energies up to 209-GeV,” [arXiv:hep-ex/0107039].
12. H.N. Brown et al., Muon $(g-2)$ Collaboration, Phys. Rev. Lett. 86, 2227 (2001).
13. R. Arnowitt and P. Nath, Phys. Rev. Lett. 69, 725 (1992).
14. Y. Totsuka, Talk at the SUSY2K conference at CERN, June 2000.
15. A. Czarnecki and W.J. Marciano, Nucl. Phys. (Proc. Suppl.) B76, 245(1999).
16. H. Hayakawa, T. Kinoshita and A. Sanda, Phys. Rev. Lett. 75, 790(1995); Phys. Rev. D54, 3137(1996); M. Hayakawa and T. Kinoshita, Phys. Rev. D57, 465(1998).
17. J. Bijnens, E. Pallante and J. Prades, Phys. Rev. Lett. 75, 1447(1995); ibid 75, 3781(1995); E. Nucl. Phys. B474, 379(1996). See also: Ref. [20].
18. M. Knecht and A. Nyffeler, [arXiv:hep-ph/0111058]; M. Knecht, A. Nyffeler, M. Perrottet and E. de Rafael, Phys. Rev. Lett. 88, 071802 (2002).
19. M. Hayakawa and T. Kinoshita, [arXiv:hep-ph/0112102].
20. J. Bijnens, E. Pallante and J. Prades, [arXiv:hep-ph/0112255].
21. I. Blokland, A. Czarnecki and K. Melnikov, Phys. Rev. Lett. 88, 071803 (2002).
22. M. Ramsey-Musolf and M. B. Wise, theory, arXiv:hep-ph/0201291.
23. M. Davier and A. Höcker, Phys. Lett. B 435, 427 (1998).
24. For other assessments of the hadronic error see, F.J. Yndurain, hep-ph/0102312, J.F. De Troconiz and F.J. Yndurain, arXiv:hep-ph/0105026; S. Narison, Phys. Lett. B 513, 53 (2001); K. Melnikov, Int. Jour. of Mod. Phys. A16, 4591, (2001) arXiv:hep-ph/0105267; G. Cvetic, T. Lee and I. Schmidt, Phys. Lett. B 520, 222 (2001). For a review of status of the hadronic error see, W.J. Marciano and B.L. Roberts, "Status of the hadronic contribution to the muon $g-2$ value", arXiv:hep-ph/0105056; J. Prades, "The Standard Model Prediction for Muon $g-2$", arXiv:hep-ph/0108192.
25. A.H. Chamseddine, R. Arnowitt and P. Nath, Phys. Rev. Lett. 49, 970 (1982); R. Barbieri, S. Ferrara and C.A. Savoy, Phys. Lett. B 119, 343 (1982); L. Hall, J. Lykken, and S. Weinberg, Phys. Rev. D 27, 2359 (1983); P. Nath, R. Arnowitt and A.H. Chamseddine, Nucl. Phys. B 227, 121 (1983). For reviews, see P. Nath, R. Arnowitt and A.H. Chamseddine, "Applied N=1 Supergravity", world scientific, 1984; H.P. Nilles, Phys. Rep. 110, 1(1984).
26. T. C. Yuan, R. Arnowitt, A. H. Chamseddine and P. Nath, Z. Phys. C 26, 407 (1984); D.A. Kosower, L.M. Krauss, N. Sakai, Phys. Lett. B 133, 305 (1983).
27. J.L. Lopez, D.V. Nanopoulos, X. Wang, Phys. Rev. D 49, 366 (1994).
28. U. Chattopadhyay and P. Nath, Phys. Rev. D 53, 1648 (1996); T. Moroi, Phys. Rev. D 53, 6565 (1996); M. Carena, M. Giudice and C.E.M. Wagner, Phys. Lett. B 390, 234 (1997); E. Gabrielli and U. Sarid, Phys. Rev. Lett. 79, 4752 (1997); K.T. Mahanthappa and S. Oh, Phys. Rev. D 62, 015012 (2000); T. Blazek, arXiv:hep-ph/9912460; U.Chattopadhyay, D. K. Ghosh and S. Roy, Phys. Rev. D 62, 115001 (2000).
29. T. Ibrahim and P. Nath, Phys. Rev. D61, 095008(2000); Phys. Rev. D62, 015004(2000); arXiv:hep-ph/0107322.
30. U. Chattopadhyay and P. Nath, in Ref. [1].
31. L. L. Everett, G. L. Kane, S. Rigolin and L. Wang, Phys. Rev. Lett. 86, 3484 (2001); J. L. Feng and K. T. Matchev, Phys. Rev. Lett. 86, 3480 (2001); E. A. Baltz and P. Gondolo, Phys. Rev. Lett. 86, 5004 (2001); U. Chattopadhyay and P. Nath, Phys. Rev. Lett. 86, 5854 (2001); S. Komine, T. Moroi, and M. Yamaguchi, Phys. Lett. B 506, 93 (2001); Phys. Lett. B 507, 224 (2001); J. Ellis, D.V. Nanopoulos, K. A. Olive, Phys. Lett. B 508, 65 (2001); R. Arnowitt, B. Dutta, B. Hu, Y. Santoso, Phys. Lett. B 505, 177 (2001); S. P. Martin, J. D. Wells, Phys. Rev. D 64, 035003 (2001); H. Baer, C. Balazs, J. Ferrandis, X. Tata, Phys.Rev.D64: 035004, (2001); M. Byrne, C. Kolda, J.E. Lennon, arXiv:hep-ph/0108122. For a more complete set of references see, U. Chattopadhyay and P. Nath, arXiv:hep-ph/0108250.
32. U. Chattopadhyay, A. Corsetti and P. Nath, arXiv:hep-ph/0201001, arXiv:hep-ph/0202273.
33. CMS Collaboration, Technical Proposal: CERN/LHCC 94-38(1994); ATLAS Collaboration, Technical Proposal, CERN/LHCC 94-43(1994); H. Baer, C-H. Chen, F. Paige and X. Tata, Phys. Rev. D52, 2746(1995); Phys. Rev. D53, 6241(1996).
34. P. Nath and M. Yamaguchi, Phys. Rev. D 60, 116004 (1999); Phys. Rev. D 60, 116006 (1999). See also K. Agashe, N. G. Deshpande and G. H. Wu, Phys. Lett. B 489, 367 (2000). For a review see, P. Nath, arXiv:hep-ph/0011175.
35. M. L. Graesser, Phys. Rev. D 61, 074019 (2000) arXiv:hep-ph/9902310.
36. C. D. Hoyle, U. Schmidt, B. R. Heckel, E. G. Adelberger, J. H. Gundlach, D. J. Kapner and H. E. Swanson, Phys. Rev. Lett. 86, 1418 (2001) arXiv:hep-ph/0011014.
37. I. Antoniadis, Phys. Lett. B 246, 377 (1990); I. Antoniadis, K. Benakli and M. Quiros, Phys. Lett. B 331, 313 (1994) [arXiv:hep-ph/9403290]; I. Antoniadis, K. Benakli and M. Quiros, Phys. Lett. B 460, 176 (1999) [arXiv:hep-ph/9905311].

38. P. Nath, Y. Yamada and M. Yamaguchi, Phys. Lett. B 466, 100 (1999) [arXiv:hep-ph/9905413]; T. G. Rizzo, colliders, Phys. Rev. D 61, 055005 (2000) [arXiv:hep-ph/9909232].

39. T. Ibrahim, U. Chattopadhyay and P. Nath, Phys. Rev. D 64, 016010 (2001) [arXiv:hep-ph/0102332].

40. T. Ibrahim and P. Nath, Phys. Rev. D 64, 093002 (2001); J.L. Feng, K.T. Matchev, and Y. Shadmi, Nucl. Phys. B613, 366(2001).

41. Y.K. Semertzidis et.al., hep-ph/0012087.

42. D. Pierce, J. Bagger, K. Matchev and R. Zhang, Nucl. Phys. B491, 3(1997); H. Baer, H. Diaz, J. Ferrandis and X. Tata, Phys. Rev. D61, 111701(2000).

43. W. de Boer, M. Huber, A.V. Gladyshhev, D.I. Kazakov, Eur. Phys. J. C 20, 689 (2001); W. de Boer, M. Huber, C. Sander, and D.I. Kazakov, [arXiv:hep-ph/0106311].

44. L.J. Hall, R. Rattazzi and U. Sarid, Phys. Rev D50, 7048 (1994); R. Hempfling, Phys. Rev D49, 6168 (1994); M. Carena, M. Olechowski, S. Pokorski and C. Wagner, Nucl. Phys. B426, 269 (1994); D. Pierce et. al. of Ref.[42].

45. H. Baer and J. Ferrandis, Phys. Rev. Lett.87, 211803 (2001).

46. T. Blazek, R. Dermisek and S. Raby, Phys. Rev. Lett. 88, 111804 (2002); T. Blazek, R. Dermisek and S. Raby, [arXiv:hep-ph/0107097]; R. Dermisek, [arXiv:hep-ph/0108249]; S. Raby, [arXiv:hep-ph/0110203].

47. S. Komine and M. Yamaguchi, [arXiv:hep-ph/0110032].

48. U. Chattopadhyay and P. Nath, Phys. Rev. D 65, 075009 (2002).

49. G. Anderson, C.H. Chen, J.F. Gunion, J. Lykken, T. Moroi, and Y. Yamada, [arXiv:hep-ph/0009457]; G. Anderson, H. Baer, C-H Chen and X. Tata, Phys. Rev. D 61, 095005 (2000).

50. N. Chamoun, C-S Huang, C Liu, and X-H Wu, Nucl. Phys. B624, 81 (2002).

51. J. Ellis, astro-ph/0204059.

52. R. Arnowitt and P. Nath, Phys. Rev. D 60, 044002 (1999); A. Corsetti and P. Nath, Int. J. Mod. Phys. A 15, 905 (2000); P. Belli, R. Bernhein, A. Bottino, F. Donato, N. Fornengo, D. Prosperi, and S. Scopel, Phys. Rev. D61, 023512(2000); J. Ellis, T. Falk, K. A. Olive, M. Srednicki, Astropart. Phys. 13, 181(2000); M.E. Gomez, G. Lazarides, and C. Pallis, Phys. Lett. B487, 313(2000); J.L. Feng, K.T. Matchev, F. Wilczek, Phys. Lett. B482, 388(2000); Phys. Rev. D 63, 045024 (2001); M. Brhlik and G.L. Kane, hep-ph/0005158; R. Arnowitt, B. Dutta, and Y. Santoso, Nucl. Phys. B606, 59(2001); J. D. Vergados, Phys. Rev. D 63, 063511 (2001); T. Nihei, L. Roszkowski and R. Ruiz de Austri, JHEP 0202, 031 (2002); V. A. Bednyakov, H. V. Klapdor-Kleingrothaus and E. Zaiti, arXiv:hep-ph/0203108.

53. M.E. Gomez, G. Lazarides and C. Pallis, Phys. Rev. D61, 123512(2000).

54. A. Corsetti and P. Nath, Phys. Rev. D 64, 125010 (2001); hep-ph/0011313.

55. M. M. Pavan, I. I. Strakovsky, R. L. Workman and R. A. Arndt, arXiv:hep-ph/0110666; SAID pion-nucleon database, http://gwdac-phys.gwu.edu; A. Bottino, F. Donato, N. Fornengo and S. Scopel, hep-ph/0111223.

56. B. Ananthanarayanan, G. Lazarides and Q. Shafi, Phys. Rev. D 44, 1613 (1991).

57. K.L. Chan, U. Chattopadhyay and P. Nath, Phys. Rev. D 58, 096004 (1998).

58. P. Nath and R. Arnowitt, Mod. Phys.Lett.A2, 331(1987); H. Baer and X. Tata, Phys. Rev. D47, 2739(1993); V. Barger and C. Kao, Phys. Rev. D60, 115015(1999).

59. R. Belli et.al., Phys. Lett.B480, 23(2000), "Search for WIMP annual modulation signature: results from DAMA/NAI-3 and DAMA/NAI-4 and the global combined analysis", DAMA collaboration preprint INFN/AE-00/01, 1 February, 2000.
60. R. Abusaidi et al., Phys. Rev. Lett. **84**, 5699 (2000), "Exclusion Limits on WIMP-Nucleon Cross-Section from the Cryogenic Dark Matter Search", CDMS Collaboration preprint CWRU-P5-00/UCSB-HEP-00-01 and astro-ph/0002471.

61. H.V. Klapor-Kleingrothaus, et al., "GENIUS, A Supersensitive Germanium Detector System for Rare Events: Proposal", MPI-H-V26-1999, arXiv:hep-ph/9910205.

62. J. Ellis, D.V. Nanopoulos and S. Rudaz, Nucl. Phys. **B202**, 43 (1982); P. Nath, R. Arnowitt and A.H. Chamseddine, Phys. Rev. **D32**, 2348 (1985); Phys. Lett. **B156**, 215 (1985); J. Hisano, H. Murayama and T. Yanagida, Nucl. Phys. **B402**, 46 (1993); T. Goto and T. Nihei, Phys. Rev. **D59**, 115009 (1999); V. Lucas and S. Raby, Phys. Rev. **D55**, 6986 (1997); K.S. Babu, J.C. Pati and F. Wilczek, Nucl. Phys. **B566**, 33 (2000).

63. R. Dermisek, A. Mafi and S. Raby, Phys. Rev. D **63**, 035001 (2001)

64. H. Murayama and A. Pierce, Phys. Rev. D **65**, 055009 (2002)

65. S. Aoki et al., Phys. Rev. **D62**, 014506 (2000).

66. G. Altarelli, F. Feruglio and I. Masina, JHEP **0011**, 040 (2000)

67. P. Nath and R. M. Syed, Phys. Lett. B **506**, 68 (2001); Nucl. Phys. B **618**, 138 (2001).