Rate-Memory Trade-Off for the Cache-Aided MISO Broadcast Channel with Hybrid CSIT

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Abstract—One of the famous problems in communications was the so-called “PN” problem in the Broadcast Channel, which refers to the setting where a fixed set of users provide perfect Channel State Information (CSI) to a multi-antenna transmitter, whereas the remaining users only provide finite precision CSI or no CSI. The Degrees-of-Freedom (DoF) of that setting were recently derived by means of the Aligned Image Set approach. In this work, we resolve the cache-aided variant of this problem (i.e., the “PN” setting with side information) in the regime where the number of users providing perfect CSI is smaller than or equal to the number of transmit antennas. In particular, we derive the optimal rate-memory trade-off under the assumption of uncoded placement, and characterize the same trade-off within a factor of 2.01 for general placement. The result proves that the “PN” impact remains similar even in the presence of side information, but also that the optimal trade-off is not achievable through serving independently the two sets of users.

I. INTRODUCTION

Coded caching has emerged as a promising tool for coping with the challenging increase of content demand in wireless networks. Initially, the benefit of coded caching was identified for the setting in which a server communicates to $K$ users through an error-free single-stream shared link of fixed capacity [1]–[3]. In this setting, the server has access to a library of $N$ files, and each user has access to a local memory (cache) of size equal to the size of $M$ files, where it can store content from the library. In the aforementioned scenario, it was shown in [1]–[3] that coded caching provides a speed-up factor (or coded-caching gain) of $K\frac{M}{N}+1$ as compared to the uncoded caching case, since it allows us to serve $K\frac{M}{N}+1$ users simultaneously.

The promising gains of coded caching in this initial setting fostered the interest in understanding how these gains could be translated into wireless networks, which was analyzed e.g. by considering uneven link capacities [4], [5] or multi-antenna transmissions [6], [7]. The context of multi-antenna coded caching has recently received considerable attention, especially after the latest results in [8], which reveal that multi-antenna coding gains exist even in the presence of side information. In particular, we show in this work that the optimal rate-memory trade-off is not achievable through separated transmission to the two sets of users.

II. SYSTEM MODEL AND PROBLEM DEFINITION

A. Communication Channel

We consider the $L \times K$ MISO BC in which a transmitter (TX) with $L$ transmit antennas serves $K$ single-antenna cache-aided users. The received signal at user $i$ is written as

$$Y_i(t) = \sum_{j=1}^{L} h_{i,j}(t)X_j(t) + \zeta_i(t),$$ (1)

where $X_j(t)$ is the transmit signal from the $j$-th antenna of the transmitter, normalized such that $E|X_j(t)|^2 \leq P$, and $P$ is the nominal SNR value which in the DoF framework is allowed to approach infinity [11]. Furthermore, $\zeta_i(t)$ is the i.i.d. additive white Gaussian noise (AWGN), and $h_{i,j}$ denotes the fading coefficient between the $j$-th antenna of the TX and user $i$. We assume that $|h_{i,j}|$ is bounded away from zero and infinity. The channel vector from the $L$ transmit antennas to user $i$ is denoted by $H_i$, and the global channel matrix by $H$. We will make use of the notation $|n| = \{1, 2, \ldots, n\}$. We further define the vectors $X(t) = (X_1(t), \ldots, X_L(t), X^{[r]}_t = \{X_j(t)\}_{t \in [r]}, X^{[r]}_t = \{X(t)\}_{t \in [r]}$, and $Y^{[r]}_t = \{Y_i(t)\}_{t \in [r]}$.

For the DoF metric, the results obtained assuming finite precision CSI (see [11], [12] for a rigorous definition) also hold if there was no CSI for the users with finite precision CSI [11]. Thus, we omit the latter for conciseness.

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B. Cached-aided Network

We consider a cached-aided scenario in which the TX has access to a content library of $N$ files, each one of size equal to $B$ bits. We assume that $N \geq K$, and we denote the $n$-th file of the library as $W_n$, $n \in [N]$. Each user has a local memory cache of size $MB$ bits, with $M \in \{0,1,\ldots,N\}$, in which it stores (coded or uncoded) data from the library. We denote the normalized cache size with respect to the library size as $\gamma = \frac{M}{N}$, and the cache available at user $i$ as $Z_i$, $|Z_i| = MB$.

Coded caching systems operate in two phases: the placement phase, in which the users fill their cache with content from the library, and the delivery phase, in which each user requests a library file to be delivered by the TX. The file requested by user $k$ is denoted as $W_{d_k}$, $d_k \in [N]$, and the vector of requested file indexes is denoted by $d \triangleq (d_1,\ldots,d_K) \in [N]^K$. We allow coded cache placement, but we also analyze the particular case in which only uncoded placement is allowed.\(^2\)

C. Hybrid CSIT

We focus on the MISO BC scenario\(^3\) in which the TX only obtains perfect CSI for the channel of a set of $K_F$ users, whereas the CSI of the other $K_P = K - K_F$ users is available only up to finite precision. We assume that $K_P \leq L$.\(^4\)

We remark that this setting is fundamentally different from that in [7], where it was shown that, in the $L \times K$ cache-aided MISO BC, we can achieve the optimal coded-caching gain under one-shot linear schemes ($K\gamma + L$) even if the TX has CSI for only $L$ served users at a time. The difference resides in the fact that, in [7], the TX needs CSI from only $L$ users at a time, but the users who are providing the CSI can (and do) change when the subset of served users changes. Hence, [7] derives the minimum instantaneous CSI requirements for a setting where all users must eventually provide CSI. In contrast, we consider in this work that a fixed subset of users provide CSI, and this subset does not change throughout the whole transmission. Thus, there are two classes of users according to the quality of the provided CSI.

D. Problem Definition

We present the formal definition of the rate-memory trade-off considered. Let $\{W_n\}_{n \in [N]}$ be $N$ i.i.d. random variables, each uniformly distributed over $[2^B]$, and let us introduce the notation $W_{k|}\{W_n\}_{n \in [k]}, k \leq N$. We scale the size of the files proportionally to the number of channel uses $\tau$ as $B = \tau R$, where $R$ is the transmission rate in bits per channel use.

A code $C(\tau,R,M)$ consists of a prefetching strategy, an encoding scheme and $K$ decoding functions, which are explained in the following. We characterize a prefetching strategy $\phi$ by its $K$ caching functions $\phi_k, k \triangleq (\phi_1,\ldots,\phi_K)$, each of which is a function that maps the library content into the cache content of one user during the placement phase. Thus, $\phi_k : [2^{[B]}]^N \rightarrow [2^{(MB)}]$, and $Z_k \triangleq \phi_k(W_1,\ldots,W_N)$. The encoding scheme $\psi : [N]^K \times [2^B]^N \rightarrow C^L \times \tau$ maps a demand vector $d$ and $N$ files into a codeword vector $X[d] \triangleq \psi(d,W_{[N]})$ satisfying the average power constraint $E[\|X[t]\|^2] \leq P$. Finally, a decoding function $\mu_k : [N]^K \times C^L \times [2^{(MB)}] \rightarrow [2^B]$ maps a requested demand, the signal received $Y_{k|}$, and the cache content $Z_k$ into an estimate $\hat{W}_k \triangleq \mu_k(d,Y_{k|},Z_k)$ of $W_{d_k}$. The probability of error is defined as

$$P_{e,\tau} \triangleq \max_{d \in [N]^K} \max_{k \in [K]} \Pr(\hat{W}_k \neq W_k).$$

Note that (2) reflects a worst-case metric over all possible demands. Thus, we consider that each user requests a distinct file.

We want to characterize the channel uses required to transmit a single bit of content to each user [1], [5]. This delay will be referred to as the delivery time, and it is given by $T = \frac{\tau}{R}$. More rigorously, for a given prefetching strategy $\phi$, a delivery time $T_{\tau,\phi}(M)$ is said to be $\varepsilon$-achievable if and only if, for every $\varepsilon > 0$ and big enough file size $B$, there exists a code $C(\tau,R,M)$ with probability of error less than $\varepsilon$.

For a given memory constraint $M$, the rate-memory trade-off $T^*(M)$ is defined as the minimum delivery time that can be achieved by any prefetching scheme with vanishing error probability and for sufficiently large file size. Thus, it is defined as $T^*(M) \triangleq \sup_{\phi \in \Phi} \limsup_{B \to \infty} \min_{\tau \in \Theta} T_{\tau,\phi}(M)$. We omit hereinafter the dependence on $M$ due to space constraints.

In pursuit of highlighting the impact of the multi-user interference, we consider the fundamental limit where $P \to \infty$. Therefore, we consider the optimal Normalized Delivery Time (NDT), which is defined as the ratio of the optimal delivery time $T^*$ over the time required to deliver a single file to a cache-less user, in the absence of interference, as $P$ approaches infinity [4], [15]. Consequently, it is defined as

$$\text{NDT} \triangleq \lim_{P \to \infty} \frac{T^*}{1/\log P} = \lim_{P \to \infty} T^* \log P.$$\(^3\)

Let us introduce also the Degrees-of-Freedom [11] (DoF) metric, which are defined as $\text{DoF} \triangleq \lim_{P \to \infty} C(\frac{\log P}{P})$, where $C(P)$ denotes the capacity of the setting. Hence, the NDT can also be written as $\text{NDT} = \frac{K(1-\gamma)}{\text{DoF}}$.\(^4\)

The metrics presented above are similarly defined when considering only uncoded prefetching, in which case we will just insert a sub-index $u$ (e.g. $\text{NDT}_{u,\tau,\phi,\psi}$). Furthermore, when necessary, we will use the full notation $\text{NDT}(K,L,\gamma,K_P)$ to reflect the particular network configuration. For example, the result for the well-known $K$-user SISO BC with uncoded prefetching would correspond to $\text{NDT}(K,1,\gamma,0)$.

III. MAIN RESULTS

We will henceforth make use of the term $(K,L,\gamma,K_P)$ MISO BC to denote the $L \times K$ cache-aided MISO BC in which only a fixed set of $K_P \leq L$ users provide perfect CSI, whereas the other $K_F = K - K_P$ users provide finite precision CSI. We begin with the characterization of the optimal rate-memory trade-off for the particular case of uncoded placement.
Theorem 1. Under the assumption of uncoded prefetching, the optimal normalized delivery time of the \((K, L, \gamma, K_P)\) MISO BC when \(K_P \leq L\) is given by

\[
\text{NDT}_u(K, L, \gamma, K_P) = \text{Conv}_{(K_P+1)\gamma} \left( \frac{(K_P+1)(1-\gamma)}{(K_P+1)\gamma+1} \right) = \text{NDT}_u(K_F + 1, 1, \gamma, 0), \tag{4}
\]

where \(K_F = K - K_P\) and \(\text{Conv}_A(f(A))\) denotes the lower convex envelope of the points \(\{(A, f(A)) | A \in \{0, 1, \ldots, K\}\}\).\footnote{For sake of readability and concision, we omit the fact that the NDT is naturally upper bounded by \((1-\gamma)\); for example, in (4), it holds that \(\text{NDT}_u(K, L, \gamma, K_P) = \min\{((1-\gamma)\text{Conv}_{(K_P+1)\gamma} \left( \frac{(K_P+1)(1-\gamma)}{(K_P+1)\gamma+1} \right)\} \).}

Proof. The achievable scheme and the converse are presented in Section IV and Section V-B, respectively.

Remark 1. The NDT required to serve \(K_F + K_P\) users (\(K_P \leq L\)) from a multi-antenna TX is the same as the one required to serve \(K_F + 1\) users from a single-antenna TX. Thus, starting from a cache-aided setting with \(K_P\) users and finite precision CSIT, we can add as many as \(L\) users who provide perfect CSIT at the cost of a single finite-precision-CSIT user.

Remark 2. The NDT in (4) is not achievable through separate transmission to the two classes of users, since that approach attains a NDT of \(\frac{1}{1+K_P\gamma} + (1-\gamma)\), which is strictly bigger.

Theorem 2. The optimal normalized delivery time of the \((K, L, \gamma, K_P)\) MISO BC when \(K_P \leq L\) satisfies

\[
\text{NDT}_u(K_F + 1, 1, \gamma, 0) \geq \text{NDT}(K, L, \gamma, K_P) \geq \frac{1}{2.00884} \text{NDT}_u(K_F + 1, 1, \gamma, 0). \tag{5}
\]

Proof. The achievable scheme is the same as for Theorem 1, whereas the converse is presented in Section V-C.

While the achievable scheme can be directly generalized for the case where \(K_P > L\), the generalization of the bounds is not straightforward. Nevertheless, it is expected that the insight in Remark 1 will hold (i.e., that we can add \(K_P\) perfect-CSIT users at the cost of \(K_P/L\) finite-precision-CSIT users).

IV. ACHIEVABILITY OF THEOREM 1 AND THEOREM 2

We denote the set of users providing perfect CSI as \(K_P \subseteq [K]\). In the scheme, our aim will be to provide the users in \(K_P\) with both full spatial multiplexing gains and full coded caching gains, which will naturally surpass the performance of a simple separate transmission to the two user sets. As it turns out, the way to achieve these gains is allocating identical caches at all users in \(K_P\), and thus the scheme assumes only \(\Lambda = K - K_P + 1\) different cache states. This is clarified below.

A. Placement

We consider \(\Lambda\) cache states of size \(MB\) bits, and denote them as \(Z_i^\gamma, i \in [\Lambda]\). Let \(\Lambda_\gamma\) be an integer and \(T\) the set of \((\Lambda_\gamma)\) subsets in \([\Lambda]\) of size \(\Lambda_\gamma\), i.e., \(T \triangleq \{r \subseteq [\Lambda] : |r| = \Lambda_\gamma\}\). First, we split each message \(W_n, n \in [N]\), into \(|T| \triangleq (\Lambda_\gamma)_\Lambda\) non-overlapping subfiles of equal size, such that \(W_n \rightarrow \{W_n^\tau : \tau \in T\}\), and we assign the content of each of the \(\Lambda\) caches as \(Z_i^\gamma = \{W_n^\tau : i \in \tau, \tau \in T\}_{\Lambda=1}^N\). Although the subpacketization is analogous to the one developed by Maddah-Ali and Niesen in [1], in our case the cache assignment is different: Let us first assume that the users providing perfect CSI are the last ones, i.e., \(K_P = \{K_P + 1, \ldots, K\}\). Then, all the users in \(K_P\) store the content of the last cache state, \(Z_i = Z_i^\gamma, \forall i \in K_P\), while the other users store each a different cache state, such that \(Z_i = Z_i^{\gamma(i)}, \forall i \in [\Lambda - 1]\). Hereinafter, we refer to the set of users that share the same cache as a cache group, even if only one of these groups has more than one user.

B. Transmit Signal

During the delivery phase, the TX serves simultaneously \(\Lambda_\gamma + 1\) cache groups. Let \(X \triangleq \{x \subseteq [\Lambda] : |x| = \Lambda_\gamma + 1\}\) be defined as the set of \(|X| = (\Lambda_\gamma + 1)\) subsets in \([\Lambda]\) of size \(\Lambda_\gamma + 1\). The delivery consists in \(|X|\) sequential transmissions, one for each \(x \in X\). Let \(W_{d_\gamma,\lambda}\) denote the part of the file requested by user \(j\) that is stored in all the caches in \(x\) except in the cache allocated at user \(j\) in \(x\). Note that \(c_j = j\) if \(j \notin K_P\), and \(c_j = \Lambda\) if \(j \in K_P\). For each such \(\lambda\), the transmit signal takes a different expression depending on whether \(\lambda \in x\) or not (recall that all users in \(K_P\) store the \(\Lambda\)-th cache state).

1) Case \(\lambda \notin x\): The transmit signal when the \(\Lambda\)-th cache group is not in \(x\) is given by \(X(t) = v_0(t) \sum_{g \in \Lambda_\gamma} W_{d_\gamma,\lambda}\), where \(v_0(t)\) is a randomly chosen \(L \times 1\) precoding vector.

2) Case \(\lambda \in x\): In this case, we have that

\[
\mathbf{X}(t) = v_0(t) \sum_{g \in \chi_\lambda} W_{d_\gamma,\lambda} + \sum_{p \in K_P} v_{K_P \setminus \gamma}(t) W_{d_\gamma,\lambda}, \tag{6}
\]

where \(v_{K_P \setminus \gamma}(t)\) is an \(L \times 1\) precoding vector designed to belong to the null space of the \(K_P - 1\) users in \((K_P \setminus p)\).

C. Received Signal

1) Case \(\Lambda \notin x\): The received signal at user \(i\) is given by

\[
Y_i(t) = h_i(t)v_0(t) \left( \sum_{g \in \chi_\lambda} W_{d_\gamma,\lambda}\right) + \zeta_i(t), \tag{7}
\]

Note that, for any \(i \in \chi_\lambda\), user \(i\) can remove all the undesired messages because they are cached in \(Z_i\). Thus, user \(i\) obtains

\[
Y_i^\gamma(t) = h_i(t)v_0(t)W_{d_\gamma,\lambda} + \zeta_i(t), \tag{8}
\]

and it can decode the intended subfile \(W_{d_\gamma,\lambda}\).

2) Case \(\Lambda \in x\): Any user not in \(K_P\) can decode its subfile as in the previous case, since all the non-intended subfiles are available in its local cache. For users in \(K_P\), they can remove the messages intended by the finite-precision-CSI users, which are available in their local cache, thus obtaining

\[
Y_i^\gamma(t) = h_i(t) \left( \sum_{p \in K_P} v_{K_P \setminus \gamma}(t)W_{d_\gamma,\lambda}\right) + \zeta_i(t). \tag{9}
\]
By definition of $v_{K_P \setminus p}(t)$, it holds that $h_i(t)v_{K_P \setminus p}(t) = 0$ for any $i \in K_P : i \neq p$. From that, it follows that

$$Y_i^{'}(t) = h_i(t)v_{K_P \setminus p}(t)W_{d_i \setminus \Lambda} + \zeta_i(t),$$

and thus, user $i$ can decode its intended subfile $W_{d_i \setminus \Lambda}$.\[8.0ex]

**Remark 4.** The users must decide whether they will provide perfect CSIT at the placement phase to be able to assign the cache states correctly. Yet, the optimal NDT is also achieved if only a subset $K_P \subset K_P$ of users in $K_P$ appear for delivery.

For the case where $K_P > L$, the scheme is generalized by having $[K_P/L]$ cache groups with several users.

**D. Degrees-of-Freedom**

This scheme always serves $\Lambda + 1$ groups simultaneously. However, the number of served users depends on whether $\Lambda \in \chi$ or not: the TX serves $\Lambda + 1$ users if $\Lambda \notin \chi$, but $\Lambda + K_P$ users if $\Lambda \in \chi$. Let $\Lambda \triangleq \{ \chi \in \chi : \Lambda \in \chi \}$ denote the set of those $\chi \in \chi$ that include $\Lambda$. Then, it follows that

$$|\Lambda| = |\{ \chi \in \chi : \Lambda \in \chi \}| = (\Lambda - 1) \Lambda. \tag{11}$$

Since all the subfiles have the same size, the time required for decoding each of them is the same, no matter which user-set $\chi$ is being served. Thus, from (11) it follows that

$$\text{DoF} = \left( \frac{\Lambda - 1}{\Lambda^2} \right) (\Lambda + K_P) + \left( \frac{\Lambda}{\Lambda^2} + 1 \right) (\Lambda - 1) = (1 + \Lambda) (K_P - 1) - \frac{\Lambda^2 + 1}{\Lambda}. \tag{12}$$

By applying that $\Lambda = K - K_P + 1$ into (12), we obtain that

$$\text{DoF} = 1 + K + \frac{K_P - 1}{K - K_P + 1}. \tag{13}$$

Since the NDT can be written as $\text{NDT} = \frac{K(1-\gamma)}{\gamma_{\text{cond}}}$, by applying that $\Lambda = K - K_P + 1 = K_F + 1$ into (13) yields

$$\text{NDT} = \frac{\Lambda(1-\gamma)}{1 + \gamma} = \text{NDT}_{\text{a}}(K_F + 1, 1, 1, 0), \tag{14}$$

which concludes the achievability proof for the cases in which $\Lambda \gamma = (K_F + 1) \gamma$ is an integer. In other cases, the NDT can be achieved through the usual memory sharing approach [1]-[3].

**V. CONVERSE OF THEOREM 1 AND THEOREM 2**

We first present two useful lemmas that are instrumental for the converse of both theorems. This will be followed first by the converse of Theorem 1 and later by that of Theorem 2.

**A. Lower Bounding the Achievable Delivery Time**

We derive in the following a lower bound for any $\epsilon$-achievable delivery time. Before presenting the result, we introduce the notations $Z_i = \{ Z_{i|\tau} \} \in [\tau], W_i = \{ W_{i|\tau} \} \in [\tau], \text{ and } \mathcal{H} = \{ h_{i,t}(t) \} \in [\mathcal{K}], i \in [\tau], t \in [\tau]$. We start by focusing on the “PN” problem with side information. This important setting can be characterized by the following key lemma.

**Lemma 1.** Consider the $(K, L, \gamma, K_P = 1)$ MISO BC where the TX has perfect CSI only for one user. Then, it holds that $H(Y^\tau_k | Z_{[k]}, W_{[k]}, \mathcal{H}) - H(Y^\tau_{k+1} | Z_{[k]}, W_{[k]}, \mathcal{H}) \geq \frac{\tau}{P} \log(P) \forall k \in [K].$ \tag{15}

**Proof.** The proof follows from a new application of the Aligned Image Set approach. Due to space constraints, the proof is relegated to the extended version [12].

The previous lemma is instrumental in the derivation of the next lower bound on the delivery time. Before presenting the lemma, let us define $K' \triangleq \min(K, K')$.

**Lemma 2.** Consider the $(K, L, \gamma, K_P = 1)$ MISO BC where the TX has perfect CSI only for one user. For any prefetching scheme $\phi$ and any demand $d$, the $\epsilon$-achievable delivery time $T_{\epsilon, \phi}$ is lower-bounded by

$$T_{\epsilon, \phi} \geq \frac{1}{\log(1 + K^\prime P + \epsilon) \log(1 + 1/P)} \tag{16}$$

$$H(W_{d_k} | Y^\tau_{k+1}| Z_{[k]}, W_{[k-1]}, \mathcal{H}) \leq \frac{1}{K' + \epsilon} \tag{17}$$

for any $k \in [K]$.

**Note.** The LHS of (19) can be upper-bounded as $H(Y^\tau_{k+1} | Z_{[k]}, \mathcal{H}) \leq \frac{\tau}{P} \log(1 + K^\prime P)$.

Moreover, from the fact that conditioning reduces entropy, the RHS of (19) can be further lower-bounded by applying $H(Y^\tau_{k+1} | Z_{[k+1]}, W_{[k+1]}, \mathcal{H}) \leq H(Y^\tau_{k+1} | Z_{[k+1]}, W_{[k]}, \mathcal{H})$. This, together with Lemma 1, leads to

$$\sum_{k=1}^{K'} H(W_{d_k} | Z_{[k]}, W_{[k-1]}) - K' \left( \frac{1}{K' + \epsilon} \right) - \tau \log(1 + K^\prime P) \leq \tau H(Y^\tau_{k+1} | Z_{[k]}, \mathcal{H}) \leq \frac{\tau}{K' + \epsilon} \log(1 + K^\prime P), \tag{20}$$

where we have removed the condition on $\mathcal{H}$ because both messages and caches are independent of the channel $\mathcal{H}$. Since $\tau = T_{\epsilon, \phi} B$, we obtain Lemma 2 by rearranging the terms.

Lemmas 1-2 are the key contribution of this work, and they are essential for proving both Theorem 1 and Theorem 2. Lemma 2 can be seen from two perspectives: First,
it represents the non-trivial extension of [3, Lemma 2] from the single-server error-free shared-link setting of [3] to the $(K, L, \gamma, K_p)$ MISO BC with $K_p = 1$. As explained in [3], these lemmas represent an enhanced cut-set bound that improves the compound cut-set bound previously used in other works [1]. Second, it extends the results of the “PN” BC setting to the important case with receiver side information.

### B. Proof of Theorem 1

We now proceed to prove Theorem 1. Some steps, especially those that draw from the proof in [2, Section V], are omitted and can be found in the extended version of this work [12].

As a first step, let us restrict ourselves to the case $K_p = 1$, and let $K' \triangleq \min(N, K)$. Before applying the derivation, recall that the bits in the library are i.i.d. and uniformly distributed. Let $B_{n,b}$ denote the b-th bit of file $n$, and let $B_{n,b}^\prime$ represent the event that $B_{n,b}$ is not cached by any user in the set $\{i\}_{i=1}^K$. Then, it follows (cf. [2]) that

\[
\sum_{k=1}^{K'} H(W_{d_k} | Z_{[k]}), W_{[k-1]} \geq \sum_{k=1}^{K'} \sum_{b=1}^{B} \mathbb{I}(B_{d_k,b}^\prime) \tag{21}
\]

where \(\mathbb{I}(\cdot)\) denotes the indicator function. Thus, by applying the same steps as in [2, Appendix A], and from Lemma 2, the value of $T_{u,c,\phi}$ is lower bounded by

\[
T_{u,c,\phi}^* \log(1 + K_P) + o(\log P) \geq \text{Conv}_{K'} \left( \frac{K(1-\gamma)}{K + 1} - K' \frac{1}{B} \right). \tag{22}
\]

Since $T_{u}^* \equiv \sup_{\epsilon > 0} \limsup_{P \to \infty} \min_{\phi} T_{u,c,\phi}^*$, it follows that

\[
T_{u}^* \log(1 + K_P) = \text{Conv}_{K'} \left( \frac{K(1-\gamma)}{K + 1} \right). \tag{23}
\]

Now, applying $\text{NDT}_{u}(K, L, \gamma, 1) = \lim_{P \to \infty} T_{u}^* \log P$ yields $\text{NDT}_{u}(K, L, \gamma, 1) = \text{Conv}_{K'} \left( \frac{K(1-\gamma)}{K + 1} \right)$, which proves Theorem 1 for the case $K_P = 1$.

Let us now consider the general case in which $K_P \leq L$. We start by considering the reduced setting in which we only serve the $K - K_P$ users for which there is finite precision or no CSIT and only one additional user among the $K_P$ users providing perfect CSIT. The optimal delay on this reduced scenario will clearly lower bound the optimal delay of the whole setting. This optimal delay of the reduced scenario is given by (24) after setting the total number of users to be $K - K_P + 1$. Then, it follows from (24) that

\[
\text{NDT}_{u}(K, L, \gamma, K_P) \geq \text{NDT}_{u}(K - K_P + 1, L, \gamma, 1) \tag{25}
\]

which concludes the proof of Theorem 1.

### C. Proof of Theorem 2

From the derivation of Lemma 2, the converse of Theorem 2 follows a similar path as the proof of Theorem 2 in [3] for the error-free symmetric setting (see [3, Section V]). Consequently, for the sake of space, its details will be found in the extended version of this work in [12].

### VI. CONCLUSIONS

We have considered the cache-aided $L \times K$ MISO BC where only a fixed set of $K_P \leq L$ users provide perfect CSI to the transmitter, whereas the remaining users provide finite precision CSIT. This setting also corresponds to the side-information variant of the well-known “PN” BC setting. For this setting, we have derived the optimal rate-memory trade-off at high SNR under uncoded placement, and characterized the same trade-off within a factor of 2.01 for coded placement. The proposed scheme capitalizes on the fact that the optimal trade-off cannot be achieved through separate transmission between the users that provide different CSIT level. The derived limits clearly show that coded-caching and multi-antenna gains are synergistic and schemes should integrate both aspects, but also that the existence of users that only provide finite precision CSIT greatly reduces the performance, which is in line with the results of the “PN” setting without side information. Analyzing the case $K_P > L$ is a meaningful extension currently under investigation, and finally, considering partial or heterogeneous CSI settings are also interesting research directions.

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