Overcomplete quantum tomography of a path-entangled two-photon state

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Path-entangled N-photon states can be obtained through the coalescence of indistinguishable photons inside linear networks. They are key resources for quantum enhanced metrology, quantum imaging, as well as quantum computation based on quantum walks. However, the quantum tomography of path-entangled indistinguishable photons is still in its infancy as it requires multiple phase estimations increasing rapidly with N. Here, we propose and implement a method to measure the quantum tomography of path-entangled two-photon states. A two-photon state is generated through the Hong-Ou-Mandel interference of highly indistinguishable single photons emitted by a semiconductor quantum dot-cavity device. To access both the populations and the coherences of the path-encoded density matrix, we introduce an ancilla spatial mode and perform photon correlations as a function of a single phase in a split Mach-Zehnder interferometer. We discuss the accuracy of standard quantum tomography techniques and show that an overcomplete data set can reveal spatial coherences that could be otherwise hidden due to limited or noisy statistics. Finally, we extend our analysis to extract the truly indistinguishable part of the density matrix, which allows us to identify the main origin for the imperfect fidelity to the maximally entangled state.

Introduction

Path-entanglement is an important resource in the field of precision measurements, where the use of entangled particles provides accuracy beyond the standard quantum limit. A textbook example is the quantum enhanced optical phase measurement [1], that has already shown important applications in the field of microscopy [2–4], lithography [5, 6], biology sensing [7, 8] as well as gravitational-wave detection [9]. The quantum advantage arises from the use of path entanglement in interferometric protocols. For instance, a path-entangled N-photon state in the form of $|N\rangle = e^{i\phi} |00\ldots0\rangle$, referred to as a N00N state, enables an N-fold enhancement in the phase resolution with a measurement sensitivity of $\Delta \phi = \frac{\pi}{4N}$, beyond the standard quantum limit of $\Delta \phi = \frac{\pi}{\sqrt{N}}$ [10]. Path-entanglement has also been proposed as a resource for quantum computing, both for intermediate—i.e., non-universal—tasks like Boson sampling [11], as well as for universal quantum computation using quantum walks of indistinguishable particles [12–14].

Various schemes are proposed to generate N00N states using beam-splitters, ancillary photons and post-selection for path-entangled states [15, 16], or through mixing quantum and classical light for polarization entangled states [17, 18]. Today’s state of the art consists of N = 5 photon N00N states [19] with most demonstrations using polarization encoding protocols [10, 20]. Indeed, while path-encoding offers great potential, it requires a phase control that is challenging to implement with bulk optics. Recent integrated photonics architectures have enabled the generation of on-chip path-entanglement [21–24], thus benefiting from robust and precise phase control and reconfigurability [25]. However, the quantum tomography of multi-photon path-entangled states has been scarcely addressed so far. The tomography of a path-entangled single photon can be achieved using quantum homodyne tomography [26] and entanglement witnesses have been derived for two paths [27, 28] and were recently extended to multiple paths [29]. Path-entanglement of two photons has been demonstrated on chip, making use of a path-encoded C-NOT gate [21] or the equally low probability of generating a photon pair in two non-linear crystals [22]. Yet, in both cases the quantum tomography was achieved for distinguishable two-photon states and was mostly intended to quantify the chip performance rather than in-depth characterization of the produced state.

The most natural way of obtaining a two-photon path-entangled N00N state is to perform the Hong-Ou-Mandel (HOM) experiment [30] with perfectly indistinguishable single photons: by impinging on the two inputs of a balanced beam splitter, they interfere and leave the beam splitter in a maximally-entangled state—a textbook experiment that has been realized with both heralded [10, 31] and on-demand single photon sources [32]. To date, the creation of a two-photon N00N state has been supported through the observation of the expected phase dependence for coincidences measured at the output of a Mach-Zehnder interferometer. Yet, to the best of our knowledge, the full tomography of path-entangled indistinguishable photon states has not been performed, even at the level of two photons.

Here, we propose a novel method to derive the density matrix of indistinguishable two-photon two-path state. We discuss the accuracy of standard tomography tech-
niques and show how an overcomplete set of measurements enables us to confidently extract all coherences that could be otherwise hidden because of poor statistics. Finally, by exploiting the bosonic nature of photons as proposed by Adamson and coworkers [33], we extend our approach to assess the contribution of partially distinguishable photons to the density matrix, which brings insight into the cause for non-maximal entanglement.

**Generation of the two-photon state**

We use a recently developed semiconductor single-photon source [34] to generate a two-photon path-entangled state. The device consists of an electrically-controlled single InGaAs quantum dot (QD) inserted in an optical cavity and placed in a cryostat at 8 K, see Fig. 1.a. The QD exciton transition is resonantly excited with 15 ps laser pulses at 82 MHz repetition rate. The transition is driven to its excited state using a \( \pi \)-pulse controlled through the laser intensity. The resonant fluorescence photons are collected in a crossed polarization scheme so as to separate them from the excitation laser, and are subsequently coupled to a single mode optical fiber. Fig. 1(d) shows the coincidence counts obtained when measuring the second order auto-correlation function \( g^{(2)}(t) \) with two single photon detectors at the outputs of a fibered beam splitter. The very small area of the peak at zero delay gives \( g^{(2)}(0) = 0.03 \pm 0.01 \), evidencing the excellent single-photon purity of the source. Note that this residual signal arises mostly from scattered laser light since no spectral filtering was used in contrast to Ref. 34. To create the two-photon path-entangled state in a HOM configuration, two photons successively generated 12.2 ns apart are first probabilistically routed on both outputs of a free space polarizing beam splitter (PBS), see Fig. 1.a. A 12.2 ns fibered delay line is added to one of the arm in order to temporally overlap both photons on the fibered HOM beam splitter (BS_{HOM}), which provides an excellent spatial-mode overlap of 0.997 and well balanced reflection and transmission coefficients \( R=0.508 \) and \( T=0.492 \). For perfectly indistinguishable photons, the two-photon should exit the beam splitter in the maximally entangled two-photon state \( |\psi_{2002}\rangle = \frac{1}{\sqrt{2}}(|2,0\rangle - |0,2\rangle) \) where the first (second) number refers to the photon number in the path 0 (resp. 1), see Fig. 1.a. Directing the signal of the two output path modes 0 and 1 towards single-photon detectors leads to the standard experimental configuration used to measure the mean wavepacket overlap of the two photons. The corresponding coincidence histogram is shown in Fig. 1.e from which a HOM visibility of 0.945 is deduced, corresponding to a mean wavepacket overlap of 0.975 when correcting for the imperfect \( g^{(2)}(0) \).

**Two-photon two-path state quantum tomography**

The state of two photons distributed over two paths, where the two photons cannot be distinguished in any degrees-of-freedom other than their spatial mode, is described by a \( 3 \times 3 \) density matrix \( \rho^{in} \) in the \( |2,0\rangle, |1,1\rangle, \) and \( |0,2\rangle \) basis [33].

Tomographical reconstruction of N00N states has been addressed for two orthogonal polarization modes of one spatial mode [35, 36], where all coherences can be derived using N-fold coincidences and SU(2) transformations via phase retarders and wave plates. Such scheme can in principle be transposed to path encoding, yet at the cost of stabilizing two independent optical phases: one phase in one path, and the other in an additional Mach-Zender interferometer needed to mimic a tunable beam-splitter. Here, we propose an alternative approach based on a single phase and an ancillary spatial mode.

Fig. 1.b presents the proposed experimental setup and Fig. 1.c the corresponding mode diagram. Photons in path 0 and 1, corresponding to the creation operators \( a_0^\dagger \) and \( a_1^\dagger \) are sent to a final fibered beam splitter labelled BS2 in a Mach-Zehnder configuration. Path 0 is directly coupled to one of the inputs of BS2. A free space beam splitter BS1 is inserted on the other arm of the Mach-Zehnder to entangle path 1 with the ancillary mode, the
path labelled 2. The free space part between BS1 and BS2 is not optically stabilized, generating a slowly varying optical phase \( \phi \) which is periodically measured. As shown below, a set of 9 photon correlations measurements, from a proper combination of paths \( i \) and \( j \), and for two different phases, rendering the correlation rates \( R_{i,j}(\phi) \) for \( 0 \leq i, j \leq 5 \), allows performing the quantum state tomography in the spatial mode basis.

This design derives from an analogy to the tomography of a polarization-entangled two-photon state [37] for which a minimal set of measurements—i.e., enabling the linear reconstruction of the density matrix—includes photon-correlations between non-orthogonal polarizations. Mapping path 0 and 1 to the polarization modes \( H \) and \( V \), the above experimental configuration essentially mimics such correlation measurements. Detection on the output paths 3 and 4 accounts for the projection onto the \((H \pm e^{i\phi}V)/\sqrt{2}\) polarizations. Correlations such as \( R_{3,4} \)—without the additional BS1—evidence a \( \cos 2\phi \) dependence, and have previously been used to confirm the nature of a two-photon N00N state [32]. Yet a complete polarization tomography must also include correlations such as \( R_{5,i} \), with \( i = 3, 4 \), which in the polarization analogy correspond to correlation between linearly polarized photon \( V \) and diagonal or circular polarizations.

To derive the density matrix from a complete set of measurements, we first consider the case of a pure input state

\[
|\psi\rangle^n = \alpha|2,0\rangle + \beta|1,1\rangle + \gamma|0,2\rangle
\]

and the corresponding density matrix \( \rho^{\text{in}} \). The diagonal terms corresponding to the populations can be obtained from the correlation rates \( R_{0,1} \) for \( |1,1\rangle \), \( R_{0,0} \) for \( |2,0\rangle \) and \( R_{1,1} \) for \( |0,2\rangle \) where \( R_{i,j} = \langle \psi_{\text{in}} | \hat{a}^\dagger_i \hat{a}^\dagger_j \hat{a}_j \hat{a}_i | \psi_{\text{in}} \rangle \) refers to correlation counts obtained by coupling path \( i \) and \( j \) to detectors. The auto-correlation rate \( R_{i,i} \) are obtained by coupling the path \( i \) to a beam-splitter and two detectors. The population of the \( |1,1\rangle \) state ranges from 0, in case of perfectly indistinguishable photons, to 0.5 for fully distinguishable ones. By making use of the unitary transformation between modes 0,1,2 and modes 3,4,5 determined by the optical setup

\[
\begin{pmatrix}
\hat{a}_3^\dagger \\
\hat{a}_4^\dagger \\
\hat{a}_5^\dagger \\
\end{pmatrix} = U_{\text{setup}}
\begin{pmatrix}
\hat{a}_0^\dagger \\
\hat{a}_1^\dagger \\
\hat{a}_2^\dagger \\
\end{pmatrix}
\]

we calculate the output state \( |\psi_{\text{out}}\rangle \) and the corresponding correlation rates \( R_{i,j} = \langle \psi_{\text{out}} | \hat{a}^\dagger_i \hat{a}^\dagger_j \hat{a}_j \hat{a}_i | \psi_{\text{out}} \rangle \) as a function of the density matrix elements \( \rho^{\text{in}}_{k,l} \). By doing so, a minimal set of correlation measurements \( R_{\text{comp.}}(\phi_1, \phi_2) \) is obtained when measuring the following rates for two distinct phases \( \phi_1 \) and \( \phi_2 \):

\[
R_{\text{comp.}}(\phi_1, \phi_2) = \left( R_{0,0} , R_{0,1} , R_{1,1} , R_{3,3}(\phi_1) , R_{3,4}(\phi_1) , R_{4,5}(\phi_1) , R_{4,3}(\phi_2) , R_{3,4}(\phi_2) , R_{4,5}(\phi_2) \right)
\]

with \( |\phi_1 - \phi_2| \neq 0, \frac{\pi}{2}, \pi \). The corresponding linear transformation matrix \( M \) relating \( R_{\text{comp.}} \) to the vectorial form of \( \rho^{\text{in}} \) is invertible so that the density matrix of the analyzed state is deduced from correlation measurements through the linear equation:

\[
(\rho^{\text{in}}) = M^{-1} R_{\text{comp.}}(\phi_1, \phi_2)
\]

The same relation holds for any mixed input state for which the density matrix is a linear superposition of pure-state density matrices weighted by the corresponding state probability.

In practice, some optical losses on the setup, related to fiber to fiber, or free-space to fiber coupling, should be considered to model the corresponding correlations. These losses are modeled as additional beam splitters, labelled \( \eta_i \) for \( i = 0, 1, 2 \) as shown in Fig. 1.c. Such approach allows maintaining a unitary description of the experiment and keeping the same procedure as described above, at the cost of introducing more modes.

The calculated coincidence rates are shown Fig. 2 for various input states in order to illustrate the sensitivity of the corresponding measurements. In the case of the ideal maximally entangled state \( |\psi_{\text{ann}}\rangle \) (solid lines), both coincidence count rates \( R_{3,4} \) and \( R_{3,3} \) are expected to vary with \( \phi \), with a maximum contrast being determined by the coherence term between \( |0,2\rangle \) and \( |2,0\rangle \) and the losses in the Mach-Zehnder. For a
fully mixed two-photon state (dotted lines), all coincidences show no dependence on \( \phi \) (overlapping dotted red and blue lines for \( R_{34,4} \) and \( R_{35,4} \)). The dashed line shows the calculated rates for the pure state \( |\psi_r^{1n} = \cos(\theta) |2, 0\rangle + \sin(\theta) e^{i\phi} |1, 1\rangle - \cos(\theta) |0, 2\rangle \), with \( \theta = 0.2 \). The population on the \( |1, 1\rangle \) component results in a cos \( \phi \) dependence of \( R_{35,4} \) and \( R_{45,4} \), shifted by its initial phase—\( \pi/4 \) in the present example—as a result of interference in the Mach-Zehnder which produces a dephasing \( \pm \phi \) of the ket \( |1, 1\rangle \) with respect to \( |0, 2\rangle \) and \( |2, 0\rangle \). The corresponding coherences also imprint a \( \phi \) dependence on top of the \( 2\phi \) modulation in \( R_{3,3} \), responsible for a small asymmetry.

**Overcomplete set of measurements**

To obtain the correlation rates \( R_{3,4}, R_{3,5} \) and \( R_{4,5} \), we use the experimental configuration sketched in Fig. 1.b. The phase \( \phi \) in the interferometer arm freely evolves over time and correlation counts are continuously acquired. The phase is measured every ten seconds by closing one input path of the HOM beam splitter \( BS_{HOM} \) using an electronically controlled shutter so that only one-photon path enters the analysis setup. The intensity signal recorded on path 3 or 4 is due to the single photon interference, and oscillates with \( \phi \) giving access to its time dependence. Fig. 3.a shows the time trace of the corresponding signal recorded over a ten-hour period. It shows large fluctuations of \( \cos(\phi) \) indicating that \( 2\pi \) variations of \( \phi \) take place over a typical 10 min timescale. Fig. 3.b shows the corresponding histogram of total acquisition times distributed over 20 phase bins, showing a reasonably flat dependence with \( \phi \).

Three detectors are used on path 3, 4 and 5 to record the three detection counts simultaneously. Time tagging of the events on the three detectors is recorded with respect to the laser trigger in order to reconstruct the correlation rates as a function of \( \phi \). To remove the errors due to fluctuations of the signal over time—arising from mechanical fluctuations in the relative laser spot-source overlap—the coincidences counts of each measurement is normalized. Normalization is achieved with the correlation peaks recorded at time delays corresponding to multiple of the laser repetition period \( (k \times 12 \text{ ns}) \) with \( |k| \geq 2 \). These peaks are due to single photon events arising from different excitation pulses and their magnitude can also be theoretically predicted from the product of single detection rates \( R_j = \langle \psi_{\text{out}} | a_j^\dagger a_j | \psi_{\text{out}} \rangle \).

**Standard quantum tomography**

We first use the standard linear tomography approach, making use of Eq.3. As discussed by Thew and co-authors, linear quantum tomography does not require that the projectors forming a complete set of measurements are orthogonal [38]. Mathematically, any couple of phases such that \( |\phi_1 - \phi_2| \neq 0, \frac{\pi}{2}, \pi \), \( R_{\text{comp}} \), \( (\phi_1, \phi_2) \) allows a reconstruction of the density matrix. Indeed, we note that \( (\phi_1, \phi_2) = \left( \frac{\pi}{4}, \frac{3\pi}{4} \right) \) does not allow discriminating between the ideal N00N state from the totally mixed state, see solid and dotted lines in Fig. 2.c. In this case, \( R_{34} \) and \( R_{44} \) turn out be only sensitive to the imaginary part of the coherence. In order to determine with the same precision the real and imaginary parts of all coherences an optimal choice providing the lowest uncertainties in the state tomography is found for a couple of phases such that \( |\phi_1 - \phi_2| \approx \frac{\pi}{2} \).

As an example, we derive the raw density matrix obtained for \( (\phi_1, \phi_2) = (0, \frac{\pi}{4}) \) in Fig.3.c. It exhibits small deviations from a physical density matrix with notably \( \text{Tr}(\rho^{1n}) = 1.034 > 1 \). To determine a meaningful value of fidelity, we normalize the unphysical state by the trace and obtain a fidelity to the ideal \( |\psi_{2002}\rangle \) state of \( F = 0.85 \). To avoid the issue of non-physical properties of the resulting matrix [37], we use the maximum likelihood approach and numerically determine nine parameters \( t_\nu \) defining the physical density matrix \( \rho^{1n}(t_1, ..., t_9) \), to maximize the likelihood function

\[
P(t_1, ..., t_9) = \prod_{\nu=1}^{9} \exp - \frac{R_\nu(t_1, ..., t_9) - R_\nu}{\sigma_\nu^2}
\]

where \( R_\nu(t_1, ..., t_9) \) are the expected coincidence rate for the test input state \( \rho^{1n}(t_1, ..., t_9) \), \( R_\nu \) are the measured ones. \( \sigma_\nu \) is the standard deviation of the \( \nu^{th} \) coincidence.
The measured normalized coincidence count rates $R_{3,4}$, $R_{0,3}$, $R_{4,5}$, and $R_{4,5}$ are plotted in Fig. 4.a as a function of $\phi$, together with the phase-independent count rates $R_{0,0}$, $R_{0,1}$, and $R_{1,1}$. Experimental error bars are derived taking into account the Poissonian noise on the coincidences as well as on the normalization count rates. $R_{3,3}$ shows stronger noise due to the lower statistics available for the measurement.

The dotted lines in Fig. 4.a show the correlation rates calculated for the state deduced from linear tomography presented in Fig. 3.c., evidencing the limited accuracy of the standard tomography: the corresponding correlations fail to reproduce the experimental ones on the full $\phi$ scale.

To obtain a better insight into the two-photon state, the likelihood function is now maximized for the whole set of 79 phase dependent measurements. The density matrix of the corresponding input state $\rho_{\text{over}}$ is shown in Fig. 4.b-e. It presents a fidelity to the N00N state of 0.91, in the range of the average fidelity obtained through 9 measurements within the standard deviation. The coincidence rates corresponding to this reconstructed state are superimposed to the measurements in Fig. 4.a (solid lines). It shows a very good agreement with the experimental observations. The observed small phase dependence of $R_{4,5}$ and $R_{4,5}$ is well accounted for, evidencing coherence between the $|1,1\rangle$ and the $|0,2\rangle$ and $|2,0\rangle$ terms. This analysis shows the reliability of the information that can be extracted from such an overcomplete data set. In the next section, we extend our analysis a step further to obtain a diagnosis for the deviation of the produced state from the ideal N00N state.

**Extracting the true photon indistinguishability**

The creation of a maximally entangled 2-photon N00N state depends on various parameters: the indistinguishability of the photons impinging on the HOM beam splitter, the balance of the reflection and transmission coefficients, as well as any undesired source of background light. On one hand, the interference of two perfectly indistinguishable photons on an unbalanced beam splitter, with $|R| \neq |T|$, results in a $|1,1\rangle$ population. On the other hand, two distinguishable photons create a $|1,1\rangle$ population with a perfectly balanced beam splitter. In any case, the distinguishability of the photons affects coherences between $|2,0\rangle$ and $|0,2\rangle$ only via the reduction of their populations. In the present experiment—using a semiconductor quantum dot operated without any spectral filtering of the zero-phonon line—two origins for the photon distinguishability can be expected. First, a residual phonon sideband emission certainly takes place and slightly reduces the photon indistinguishability, as recently shown [39]. Additionally, the resonant excitation scheme leads to a small fraction of residual laser light not completely suppressed in the crossed polarized collection. This residual light is also distinguishable from the single photons emitted by the quantum dot and is

**Overcomplete quantum tomography**
also responsible for the measured non-zero $g^{(2)}(0)$ shown Fig. 1.d.

Even though the physical origin of each detected photon cannot be determined by our apparatus used for tomography, Adamson and coworkers have demonstrated that in such situation it is still possible to get more information on the two-photon state [33]. The contribution of the truly distinguishable photons to the $|1,1\rangle$ population can be separated from that due to an imperfect set up via a more refined analysis. In practice, one introduces the 4-state basis $|2,0\rangle$, $|0,2\rangle$, $\psi^+$, $\psi^-$, corresponding to the visible degree of freedom, where the $\psi^\pm$ are now the symmetric and antisymmetric states of two possibly distinguishable photons "a" and "b" on each path: $\psi^\pm = \frac{|1_a,1_b\rangle \pm |1_b,1_a\rangle}{\sqrt{2}}$. Truly indistinguishable photons can only occupy the symmetric state $\psi^+$, thus any population in the antisymmetric state $\psi^-$ reveals the presence of distinguishing information. The 4x4 density matrix $\rho^{\text{vis}}$ reads in this basis [31, 33]:

$$
\rho^{\text{vis}} = \left(\begin{array}{cccc}
\rho_{20,20} & \rho_{20,02} & \rho_{20,02} & 0 \\
\rho_{02,02} & \rho_{02,02} & 0 & 0 \\
0 & 0 & 0 & \rho_{0,-,-}
\end{array}\right) \quad (4)
$$

where the coherences between the 3x3 symmetric $\rho^+$ and 1x1 antisymmetric $\rho^-$ subspaces are zero. By considering a pure input state in the form

$$
|\psi\rangle_{\text{in}} = \left(\begin{array}{c}
a_{\alpha,a}^\dagger a_{\alpha,b}^\dagger + \beta a_{\alpha,b}^\dagger a_{\alpha,a}^\dagger + \gamma a_{\beta,b}^\dagger a_{\beta,a}^\dagger + \delta a_{\gamma,b}^\dagger a_{\gamma,a}^\dagger \end{array}\right) |0\rangle
$$

and calculating the corresponding coincidences

$$
R_{i,j} = \langle \psi_{\text{out}} | a_{i,a}^\dagger a_{i,b}^\dagger b_{j,b} a_{j,a} a_{i,b} a_{i,b} a_{i,b} \rangle_{\psi_{\text{out}}},
$$

can we determine new relations between $(\rho_{k,l}^+, \rho^-)$ and the $R_{i,j}$ terms. We observed that, as expected, the calculated $R_{i,j}$ do not formally depend on the coherence terms between the symmetric and antisymmetric part of the density matrix, even if they are considered as non-zero. We then carry out the maximum likelihood method using the overcomplete set of measurements to obtain the ten parameters defining the physical density matrix in the form of $\rho^{\text{vis}}$, see Fig. 5. Notably, most of the $|1,1\rangle$ population now appears on the antisymmetric part $\rho_{\psi^-,\psi^-}$ of the density matrix, with a negligible population on the symmetric $\rho_{\psi^+,\psi^+}$ population. This approach allows us to ascribe most of the N00N state imperfection to a partial distinguishability of the photons and not to imperfections in the HOM beam-splitter. Furthermore, knowing that the lower bound of $\rho_{\psi^+,\psi^+} + \rho_{\psi^-,\psi^-}$ is given by $g^{(2)}(0) = 0.03$, we ascribe most of the extracted distinguishability to the residual laser.

**Conclusions**

In the present work, we have proposed a simple experimental method to perform quantum state tomography of a two-photon path-entangled state. Although unavoidable experimental noise lead to uncertainties in a standard quantum tomography approach, we have shown that an overcomplete data set allows extracting highly reliable information. Moreover, accessing the indistinguishable and distinguishable parts of the density matrix, we can provide a precise diagnosis as for the deviation from the ideal state, separating the limitations arising from the photon source to those coming from the imperfections of the optical network.

High-photon number path-entangled N00N states are foreseen as important resources for many applications ranging from quantum imaging to quantum sensing and lithography and yet, the possibility to universally detect entanglement without performing a full state tomography is still debated [40]. The quantum tomography of polarization-encoded N00N state has been extended to high $N$ by making use of photon-number resolving detectors [35, 36]. Applying a similar extension to access all the required $N$th order photon correlations, we expect that our approach offers a viable method for the quantum tomography of path-encoded N00N states for any $N > 2$.

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