Resolving Conflict in Decision-Making for Autonomous Driving

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Abstract

Recent work on decision making and planning for autonomous driving has made use of game theoretic methods to model interaction between agents. We demonstrate that methods based on the Stackelberg game formulation of this problem are susceptible to an issue that we refer to as conflict. Our results show that when conflict occurs, it causes suboptimal and potentially dangerous behaviour. In response, we develop a theoretical framework for analysing the extent to which such methods are impacted by conflict, and apply this framework to several existing approaches modelling interaction between agents. Moreover, we propose Augmented Altruism, a novel approach to modelling interaction between players in a Stackelberg game, and show that it is less prone to conflict than previous techniques.

1 Introduction

A wealth of previous research on autonomous driving has employed overly simplistic models of how other agents will behave, such as assuming constant velocity (Eiras et al. 2020; Sadigh and Kapoor 2015; Bender et al. 2015) or that vehicles only execute a small number of fixed trajectories for a given type of manoeuvre (Hermes et al. 2009; Ward et al. 2017). These assumptions lead to unexpected and undesirable behaviour, like failing to merge onto a highway or change lanes because the autonomous vehicle (AV) is not capable of anticipating that starting such a manoeuvre will result in the human drivers making space for the AV to finish completing the manoeuvre. The emerging field of interactive planning and decision-making for autonomous driving aims to address these problems by building interactive models that allow an AV to influence the actions taken by human drivers (Sadigh et al. 2016, 2018). For example, the scenario presented in Figure 1a; the AV (orange) is approaching a parked fire truck with a vehicle (purple) occupying the adjacent lane. Operating under the assumption that the purple car will continue to move at a constant velocity, the AV has no choice but to slow down and wait until the right lane is empty in order to continue. In contrast, an AV that is capable of anticipating how an interactive driver would respond to its actions might decide on a different course of action. For example, Sadigh et al. (2018) demonstrate an interactive planner that enables the AV to begin accelerating in order to determine whether the purple car will slow down to allow it to change lanes in front.

It is common in recent work to on autonomous driving to develop hierarchical systems, where a discrete decision-maker decides on intents or approximate trajectories for the AV, and a lower level motion planning system chooses continuous actions that realise these decisions (Fisac et al. 2019; Albrecht et al. 2020; Eiras et al. 2020; Cunningham
Incorporating models of interaction based on Stackelberg games into AV planning and decision-making systems has become a popular way of improving upon the naïve models used by older systems that do not attempt to account for the interactive behaviour of other drivers. However, while solving Stackelberg games is computationally easier than alternative game theoretic formulations, the solution method presumes that players (i.e., the AV and other driver) take the role of either leader of follower, with the leader selecting their action first and the follower selecting their best response. In practice, without a means of direct communication and agreed upon protocol, it is impractical for two vehicles to dynamically allocate the roles of leader and follower. Given that the roles of leader and follower are ambiguous, the assumptions required for a Stackelberg game do not always hold in autonomous driving scenarios. This begs the question: will the violated assumptions result in undesirable behaviour?

We use the term conflict to refer to a class of situations where ambiguity over who takes the roles of leader and follower leads to unexpected behaviour. We demonstrate that such conflicts can result in suboptimal or unsafe behaviour, such as both vehicles coming to a stop in an attempt to give way to each other, or both vehicles aggressively trying to move into the same space. In addition, we propose a metric—the Area of Conflict (AoC)—for quantifying the extent to which a given Stackelberg game-based decision-maker will result in conflict. This metric can be seen as a measure of robustness to the level of aggressiveness or passiveness exhibited by other agents in the driving environment. Using the insight provided by our conflict analysis we design the Augmented Altruism method, a technique for interactive decision-making that is less susceptible to conflict than previous approaches. Our approach is based on the concept of altruism (Andreoni and Miller 1993) from game theory, which we extend to account for reciprocal altruistic considerations of other agents.

The key contributions in this work are as follows:

- Identification of a shortcoming in previous work on interactive planning and decision-making for autonomous driving that we refer to as conflict.
- The introduction of a metric, Area of Conflict (AoC), for measuring the incidence of conflict for a pair of decision-makers in a Stackelberg game formulation.
- We propose Augmented Altruism, a novel method for decision-making that is provably less prone to conflict than other methods in literature for reasonable reward values.

## 2 Related Work

Our method is an extension of the concept of Altruism, as found in Game Theory literature. Andreoni and Miller (1993) presents the idea of altruism being a scalar value, $\alpha$, that multiplies or adds with the rewards of the interacting agents to influence an agent’s decision-making by the potential payoffs to the other agents. In the work, Andreoni and Miller provide three distinct models for altruism: pure, duty and reciprocal. The definition used in this work most aligns with the definition for pure altruism. (Bansal et al. 2018) introduce selfishness, an equivalent, but opposite concept to our proposed definition, which is used for collaborative automotive planning. (Düring and Pascheka 2014) presents a method for decision-making for autonomous vehicles that corresponds to the pure altruism model, with an altruism coefficient of 1.

Similar to altruism in Game Theory, there is Social Value Orientation (SVO) in the fields of psychology and behavioural economics (McClintock and Allison 1989). SVO can be used as an indicator for a person’s reward allocation preferences over. Unlike altruism, which is restricted to depicting egoistical or prosocial behaviours, SVO can also identify malicious or masochistic behaviours in decision-makers. Schwarting et al. (2019) implement a version of SVO that is similar to our proposed altruism implementation in that they weight the planning agent’s reward, and the rewards of the other agents according to the planning agent’s SVO value. As in (Sadigh et al. 2016), the authors model the interaction as a Stackelberg game and demonstrate that SVO can be used to augment lane merge prediction. In contrast, in this work we use our proposed method as a decision-making model. In (Schwarting et al. 2019) the SVO model requires access to a single, accurate reward function to model the behaviour of any other agent, which is learnt offline. In our work we weaken this requirement, using Game Theoretic methods to choose the most appropriate reward function to model the behaviour of other vehicles.

Sadigh et al. (2016) present an approach to interactive planning, in which a reward function is learnt from human data, and then the structure of the reward function is exploited in order to identify optimal interactive actions (Sadigh et al. 2016, 2018). Fisac et al. (2019) proposes a similar approach, although they do not rely on knowing the structure of the reward function in their planning, only that the reward function is known and accurate. These works formulate the interaction problem as a two-person Stackelberg game (Von Stackelberg 2010) with a dense reward, assuming that the other agents in the environment abide by the leader-follower hierarchy specified.

All of these approaches to modelling interaction with Stackelberg games require the roles of leader and follower to be agreed upon beforehand in order to avoid undesirable behaviour. Our theoretical analysis shows that the method we propose is less vulnerable to this ambiguity than these previous approaches.

## 3 Conflict in the Stackelberg Game

In a two-person Stackelberg game one player takes the role of the leader and the other the role of the follower. The leader chooses the action that maximises their reward under the assumption that the follower will behave optimally with respect to the leader’s choice. For example, using the reward matrix given in Figure 1b, if $C_1$ were the leader then they would choose to lane change ahead of $C_2$ (and get reward of 1) anticipating that the follower, $C_2$ will choose to give way in response (and get a reward of 0). However, if $C_2$ were the leader instead, they would choose to continue (and get reward 1) and $C_1$ would be forced to decelerate. Thus, if
both cars independently assume the role of leader, C1 will attempt a lane change and C2 will not give way. This example shows that a conflict can emerge if it has not been agreed in advance which agent is the leader and which agent is the follower. In the case of autonomous driving, without any means of direct communication, no such agreement can be reached. We define conflict as follows:

**Definition.** Conflict occurs in a Stackelberg game when a lack of agreement about who takes the role of leader and who takes the role of follower leads to the players arriving at different pure strategy equilibria.

Conflicts can be resolved in one of two ways: (i) enabling players to negotiate the roles of leader and follower before playing the game, or (ii) designing the reward such that the players will arrive at the same pure strategy equilibrium irrespective of who takes the role of leader or follower.

Conflicts can be problematic as they can result in unforeseen catastrophic situations, as shown in the previous example. Therefore it is important that the decision-making method used by an AV has as low an incidence of conflict as possible, so that it is robust to the aggressiveness of other agents. In our example it is clear from the reward matrix that the players are in conflict, and there is no clear way to resolve it.

### 4 Impact of Conflict

In this section we show how conflict can impact the ability of an agent to accomplish its objective in a safe and timely fashion. We consider the four possibilities; both agents plan assuming they are the leader, both agents agree Car 1 is the leader, both agents agree Car 2 is the leader, and both agents think the other is the leader. When the agents do not agree, which can occur if there is conflict in the game, they will compute different equilibria to the game. We demonstrate in various simulated driving scenarios, that this conflict can result in longer objective completion times.

#### 4.1 Experimental Setup

Each agent $i$ has a finite set of intentions, $A_i$, with each intention associated with a cost function, $J_a : X^T \to \mathbb{R}, a \in A_i$, where $X$ is the state space and $T$ is the trajectory duration. An equilibrium to a game, $(a_i, a_{−i})$, defines a cost function, $J_i = J_{a_i} + J_{a_{−i}}$. Each agent $i$ uses the cost function $J_i$ to compute an optimal trajectory to achieve the joint intentions. We treat the problem as a receding horizon optimal control problem, and use a Model Predictive Control (MPC) planning scheme to solve it.

The vehicle’s state is defined by its $x$ and $y$ position as well as its velocity and heading; $\vec{x} = [x, y, v, \theta] \in X$. Vehicles can control their linear acceleration, $a$, and angular acceleration, $\omega$. Control inputs are of the form $\vec{u} = (a, \omega)$. The standard bicycle model, $F : X \times \mathbb{R}^2 \to X$ (Kong et al. 2015) is used to define the vehicle dynamics. Vehicles are rectangular with length $L = 4.6$ metres and width $W = 2$ metres. Each lane is 4 metres wide.

At every iteration of the MPC, each agent generates a trajectory $x^+_i = \{\vec{x}^+_i, u^+_i\}^T_{t=0}$ such that $x^+_i, u^+_i, x^-_{−i}, u^-_{−i} = \arg \min_{x_i, u_i, x_{−i}, u_{−i}} J_i(x_i, u_i, x_{−i}, u_{−i})$ s.t. $\vec{x}^{i+1} = F(\vec{x}^+_i, \vec{u}^+_i) \quad \forall 0 \leq t \leq T, j \in \{i, −i\}$

$$g(x_j) \leq 0, j \in \{i, −i\}$$

$$h(u_j) \leq 0, j \in \{i, −i\}$$

$$\frac{(\vec{x}^+_i[0] − \vec{x}^-_{i−1}[0])^2}{a^2} + \frac{(\vec{x}^+_i[1] − \vec{x}^-_{i−1}[1])^2}{b^2} \geq 1.$$ 

For each agent $i$ this method also produces $x^-_{i−1}, u^-_{i−1}$ which is the presumed trajectory agent $−i$ will follow, presuming their trajectory is also optimal with respect to $J_i$ (this will be the case if the agents are not in conflict as they will have computed the same equilibrium). These values are discarded by agent $i$. The vehicles then follow their respective optimal trajectories for 2 timesteps before replanning. We presume that at the time of planning the agents have perfect awareness of each other’s states. A lookahead horizon of $T = 4$ seconds is used, with a timestep, $dt = 0.2$ seconds. The experiment finishes when both agents have completed their objective, or if the trajectory duration exceeds 10 seconds.

Constraints, $g$, are applied to the physical state of each agent; these ensure the vehicles stay on the road, and that each vehicle’s velocity does not exceed 15 m/s. Action constraints, $h$, enforce that acceleration is in the range of $[-9 m/s^2, 3 m/s^2]$, and the angular acceleration is in the range $[-1degs^2, 1degs^2]$. The final constraint is an obstacle avoidance constraint, to ensure that the vehicles do not construct plans that in which collisions occur. An ellipse with semi-major and semi-minor axes $a$ and $b$ is fitted around agent $i$’s position, preventing the vehicles from getting too close. In all experiments $a = W + \delta$ and $b = L + \epsilon$ where $\epsilon$ and $\delta$ are small constants. An Interior Point Optimizer (IPOPT) method from the CasADi optimisation library (Andersson et al. 2019) is used to solve the non-linear programming problem.

We record the time taken for both agents to complete their objectives, as measured by a boolean check based on the intention specified in the computed equilibrium. Trajectories that fail to complete both agent’s objectives (e.g., due to a collision or frozen robot problem (Trautman and Krause 2010)) are given a maximal score. The hypothesis is that conflict interferes with the ability to interact effectively, hence the time for the objectives to be completed will be longer for those sets of leader-follower assumptions that are in conflict. We evaluate this claim by staggering the starting positions of each vehicle and recording the completion times. The vehicles are displaced to a maximum displacement of 1.5 vehicle lengths (6.9 metres). Empirically we observe that outside this range the cars cease to be interacting and can generally accomplish their objectives without influencing the other’s behaviour.

#### 4.2 Experiment 1: Lane Change

Figure 1a demonstrates the default setup for the lane change experiment; both cars start next to each other in adjacent lanes, travelling at the speed limit. Car 1 (Orange) can ei-
they choose to change lanes ahead (LCA) of Car 2 (Purple), or change lanes behind (LCB). Car 2 can either Give Way (GW) to allow Car 1 to perform the lane change ahead, or else they can continue (C). Car 1 would prefer to merge ahead of Car 2, but Car 2 would prefer not to give way, which would prevent Car 1 from merging ahead (this is captured in the reward matrix defined in Figure 1b). If Car 1 believes they are the leader, they will optimise for the equilibrium \((LCA, GW)\), whereas, using the same assumption, C2 will optimise \((LCB, C)\). As these equilibria do not match, we conclude the game is in conflict.

All \(J_i, a \in A_i \cup A_j\), in this experiment have common features incentivising the vehicles to get into the right hand lane as early as possible, and to drive at the speed limit as much as possible. These features all have the same weight, so that neither agent’s cost function gives them more of an incentive to reach their objective than the other. The remaining feature penalises an agent \(i\) for being ahead of agent \(-i\) if agent \(i\) is expected to give way. This is given as

\[
f(\vec{x}_i, \vec{x}_{-i}) = \max(\vec{x}_i[1] - \vec{x}_{-i}[1], 0)
\]

The weights of all features are fit experimentally, and all features have positive non-zero weight.

The results in Figure 2 demonstrate that when both agents decide on the same equilibrium (e.g. in the case of \((LCA, GW)\) and \((LCB, C)\)) the cars are consistently able to resolve the scenario cooperatively in less time than in conflict. If both cars presume the other car is the leader in the Stackelberg game, they both attempt to accommodate the other, effectively achieving the equilibrium \((LCB, GW)\), and they fail to complete their objectives. Similarly, when both cars presume they are the leader, they effectively execute the \((LCA, C)\) solution. In this case the cars almost always fail to complete the task safely. Even when the cars are not in conflict, if Car 2 started ahead of Car 1, and Car 1 is attempting to merge ahead, this can result in a collision, so conflict does not entirely eliminate the risk in the manoeuvre.

### 4.3 Experiment 2: Intersection

Another scenario where conflict can emerge is at an unsignalled intersection, where two cars wish to pass through an intersection safely but also as quickly as possible. We omit social conventions encouraging an agreed upon hierarchy, and instead consider the case where each agent would prefer to be first through the intersection. This scenario is depicted in Figure 3.

In this scenario each vehicle has the option to continue through the intersection (C), or to give way to the other car allowing them to proceed. In the case both cars assume they are the leader in the Stackelberg game, both will expect the other to give way ((C, GW) and (GW, C), respectively). Since these equilibria do not match, there is conflict in this situation. If both agents presume they are the leader the effective solution executed is \((C, C)\), which will result in them proceeding through the intersection at the same time, and each will interfere with the other’s ability to achieve their objective. In the case both presume the other is the leader, they will both wait for the other to proceed, \((GW, GW)\), resulting in neither achieving their objective.

In the default setup for this experiment each car starts with 0 velocity, equidistant from the intersection on perpendicular roads, as shown in Figure 3a. The speed limit and acceleration constraints are the same as in the previous experiment. In the optimal planning problem we constrain both vehicles’ ability to turn as, while trajectories that weave around other vehicles would be valid solutions to the problem, such trajectories would be considered dangerous in this setting in real life.

In order to motivate the giving way behaviour, we define the feature

\[
f(\vec{x}_i, \vec{x}_{-i}, \theta_{L_i}) = \cos(\theta_{L_i}) \ast \max(\vec{x}_{-i}[0] - \vec{x}_i[0], 0) + \sin(\theta_{L_i}) \ast \max(\vec{x}_{-i}[1] - \vec{x}_i[1], 0),
\]

where \(\theta_{L_i}\) is the angle of the lane that agent \(i\) is on. This feature has positive non-zero weight in \(J_{0_i}\) if agent \(i\) plans on proceeding ahead of agent \(-i\) (C), and it penalises agent \(i\) for being behind \(-i\) along the direction to their objective. The remaining features correspond to staying in the lane and motivating the agent to accelerate to the speed limit if posse-
Figure 3: (a): Experimental setup for intersection experiment; Car 1 (Orange) and Car 2 (Purple) wish to cross straight through the intersection. Each has the option to proceed through the intersection promptly (*C*), or to give way and let the other car proceed first (*GW*). (b) Reward matrix associated with the experiment; each car would prefer to get through the intersection promptly. Diagonal entries in table represent states where either both vehicles collide or neither agent’s objective is satisfied, which is mutually undesirable.

Figure 4 gives the results of our experiments. As in our previous experiment, we observe that when the agents agree on the role of the leader and follower in the game, the average objective completion time is lower than in conflict. We also observe that, in general, these execution times are consistently lower. Unlike in our previous experiment, we observe that in the aggressive conflict case (bottom left cell in Figure 4a) catastrophic failures (i.e. where the objective was not achieved within the allotted time) occur close to the diagonal entries. This is due to the fact that in this case the cars can achieve their objectives, for certain initial states, without accounting for the other vehicle significantly; if one car is relatively much further away from the intersection than the other, then the other car can proceed to achieve their objective without interacting with the distant car. We observe that, paradoxically, in this case shorter durations were achieved when one car was as far away as possible, with the other as close as possible, with other combinations resulting in longer durations as the cars interact more significantly.

5 Altruism and Area of Conflict

Altruism-based techniques for decision making in game theory can be thought of as methods for transforming the reward matrix of a game. In this section we will define our variant of altruism (Andreoni and Miller 1993), as well as an augmentation to the definition that accounts for the reciprocal considerations of other altruistic agents. We will also provide a definition for Area of Conflict.

5.1 Altruism

We model the interactive driving decision-making problem as a static game played on a reward matrix, indexed by intentions, where each cell in the matrix contains the rewards received by each player if they each chose the corresponding intention combination. Figure 5 presents a general reward matrix where if the row player, *R*, and column player *C*, chose intentions A1,B1 respectively, *R* would receive a reward of *r*111 and *C* would receive a reward of *r*112. The grid is 2 × 2 for demonstrative purposes and, in general, the grid can be of any size *M* × *N* where *M* is the number of actions available to *R* and *N* is the number of actions available to *C*, and each cell contains a reward pair (*r*mn1, *r*mn2). Unless the full index is required, we will refer to the row player’s reward for a particular intention combination as *r*1 and, correspondingly, *r*2 for the column player’s reward.

Pure Altruism, as defined in Andreoni and Miller (1993),
makes use of an altruism coefficient $\alpha$ to define the altruistic reward,
\[ r_i^* = r_i + \alpha r_{-i} \quad 0 \leq \alpha \leq 1, \]
where the $-i$ index corresponds to the agent that is not indexed by $i$, and $r_i^*$ is the effective reward agent $i$ uses to perform decision-making. If $\alpha = 0$ then the agents are indifferent to one another and if the value is 1 then the agents are cooperating in order to maximise the same reward, $r_i^* = r_{-i}^* = r_i + r_{-i}$.

As an alternative to Pure Altruism we propose an alternative definition for the altruistic reward,
\[ r_i^* = (1 - \alpha_i) r_i + \alpha_i r_{-i} \quad 0 \leq \alpha_i \leq 1. \]

In this work each agent $i$ has its own individual altruism coefficient, $\alpha_i$. Scaling agent $i$’s reward in parallel with agent $-i$’s allows for more flexible behaviours as compared with Pure Altruism; if $\alpha_i = 0$ then the agent is wholly egoistic, if $\alpha_i = 1$ then the agent is wholly altruistic. To avoid confusion we will refer to the Andreoni and Miller (1993) altruism as “pure altruism” and our proposed definition as “altruism”.

Extensive previous and ongoing work has been dedicated to estimating reward functions and interactive parameters (Albrecht, Crandall, and Ramamurthy 2016; Albrecht and Stone 2019; Albrecht et al. 2020; Schwarting et al. 2019). In this work we propose that the “true” reward matrix \{(r_{mn1}, r_{mn2})\}_{0 < m < M, 0 < n < N}, and altruism values $\alpha_1, \alpha_2$ are known to both agents. Each agent can then, independently, construct the reward matrix \{(r_i^*, r_{-i}^*)\}_{0 < m < M, 0 < n < N}, which they will use to choose which intention to follow.

### 5.2 Augmented Altruism

When attempting to identify equilibria in Game Theoretic problem formulations it is not uncommon to use iterative best response methods to compute the Nash Equilibrium (Vorobeychik and Wellman 2008). In practise this involves each agent choosing an optimal action based on the optimal actions for the other agents in the previous iteration. This allows for the fact that an agent’s choice of action can be affected by the choices made by other agents. If this process is repeated indefinitely, and it converges to a solution, then the solution achieved is a Nash Equilibrium (Başar and Olsder 1998).

The altruism definition presented in Equation 2 neglects from consideration that $r_i^*$ is the reward an altruistic agent $-i$ would receive from the interaction, not $r_{-i}$, and that awareness of this value could affect agent $i$’s preferences. But, by the same assumption, the value for $r_{-i}$ depends on the value of $r_i^*$. By treating the equations in Equation 2 as a system of equations, we can determine the steady state of the system, yielding what we refer to as augmented altruism,
\[ r_i^* = \frac{(1 - \alpha_i) r_i + \alpha_i (1 - \alpha_{-i}) r_{-i}}{1 - \alpha_i \alpha_{-i}} \quad i \in \{1, 2\} \]

This is an improvement on our base altruism definition, as it is a computationally tractable method for accounting for both players altruism values when evaluating options, whereas the original definition only accounted for the agent’s own $\alpha$. We refer the reader to the Appendix A.1 for a more detailed explanation and complete derivation.

### 5.3 Area of Conflict

Altruism can be used to resolve conflict scenarios; in the example in Figure 1b, if $\alpha_1 = 1$, for instance, then $C1$ would get an effective reward of 0 for attempting to cut ahead, and a reward of 1 for decelerating and allowing $C2$ to proceed. However, altruism does not entirely eliminate conflict since $\alpha_1 = 1$ and $\alpha_2 = 1$ also results in conflict.

Let each agent choose an action according to $f_i : \mathbb{R}^{M \times N} \times [0, 1] \times [0, 1] \rightarrow \{A[1], A[2]\} \times \{B[1], B[2]\}$, a function parameterised by the altruism coefficients that maps from the reward matrix to the equilibrium of the corresponding Stackelberg game for some reward matrix $I$. The previous observation indicates that, for a given reward matrix, there are potentially regions in the parameter-space $[0, 1] \times [0, 1]$ that will always result in conflict. We call the total size of these regions the Area of Conflict (AoC). It is desirable to choose a decision-making method that minimises the AoC for a given reward matrix.

In the following derivations we will refer to the reward matrix defined in Figure 5. Without loss of generality we will assume the cell $(A[2], B[1])$ is optimal for $R$ and $(A[1], B[2])$ is optimal for $C$. We further assume that there are no ambiguities in each agents’ rewards, i.e.
\[ r_{211} > r_{111}, r_{221} \]
\[ r_{122} > r_{212}, r_{112}, r_{222}. \]

It is clear that decision-making on the reward matrix with these constraints will result in conflict, regardless of the value of the parameters. Therefore it is vacuously true that if $I$ is the unchanged reward matrix, then the AoC of the resulting Stackelberg game is 1 (Von Stackelberg 2010). We will use this value as a baseline.

In general we observe that conflict will occur if:
\[ (r_{211} > r_{112}^* \land r_{122}^* > r_{212}^*) \]
\[ \lor (r_{211}^* < r_{112}^* \land r_{122}^* < r_{212}^*) \]

The definition of the AoC for the following decision-making models follows from Equation 5 (In order to save space we will let $A = r_{211} - r_{112}$ and $B = r_{122} - r_{212}$):

- **Pure Altruism:** ($\alpha_1 > \frac{A}{B} \land \alpha_2 > \frac{B}{A} \lor (\alpha_1 < \frac{A}{B} \land \alpha_2 < \frac{B}{A})$

- **Altruism:** ($\alpha_1 > \frac{A}{B + A} \land \alpha_2 > \frac{B}{B + A} \lor (\alpha_1 < \frac{A}{B + A} \land \alpha_2 < \frac{B}{B + A})$

- **Augmented Altruism:**
\[ (1 - \frac{1}{\alpha_1} \frac{A}{B} < \alpha_2 < \frac{B}{B + 1 - \alpha_1 A} \land 0 < \alpha_1 < 1) \]
Table 1: Calculated AoC values for various interactive decision-making models based on the lane change reward matrix given in Figure 1b, and general expressions for computing the AoC for each of the methods considered.

| Method                        | Lane Change AoC | General AoC                                                                 |
|-------------------------------|-----------------|-----------------------------------------------------------------------------|
| Baseline (Von Stackelberg 2010) | 1               | 1                                                                           |
| Pure Altruism (Andreoni and Miller 1993) | 1       | $\min\left(\frac{A}{B}, \frac{B}{A}\right)$                              |
| SVO (Schwarting et al. 2019)  | 0.5             | $\frac{\pi_2}{p_1p_2} + \left(\frac{\pi_2}{p_1} - p_2\right)\left(\frac{\pi_2}{p_2} - p_1\right)$ |
| Altruism (Ours)               | 0.5             | $2 \left(\frac{AB}{(A+B)^2}\right)$                                      |
| Aug-Altruism (Ours)           | 0.38623         | $\ln(A+B)(\frac{A}{B} + \frac{B}{A}) - \left(\frac{A}{B}\ln(A) + \frac{B}{A}\ln(B)\right) - 1$ |

Figure 6: Conflict regions for various reward models corresponding to the reward matrix in Figure 1b. Red regions designate regions in the parameter space that result in conflict. Plots correspond to: (a) Baseline, (b) Pure Altruism, (c) SVO, (d) Altruism, (e) Augmented Altruism.

In the above it also holds that $0 \leq \alpha_i \leq 1$, except in the case of augmented altruism, where there is the extra constraint that $0 < \alpha_1 < 1$. Each of the logical conjunctions ($\land$) specifies a bounded region of parameter space which will result in conflict, and the logical disjunctions ($\lor$) define pairs of non-overlapping regions (see Figure 6 for a graphical depiction of these regions). Therefore we can define the AoC as the sum of the areas of these regions in parameter space. For Pure Altruism, and our proposed Altruism, these are straightforward computations. For the remaining derivations we refer the reader to Appendix B. The general definitions for AoC of the standard Stackelberg Game, the Pure Altruism, SVO, Altruism, and Augmented Altruism variants are provided in Table 1 as well as evaluations corresponding to the reward matrix in Figure 1b.

We observe that the AoC for the Augmented Altruism significantly outperforms the other considered models. This means that, in repeated pairings of agents with altruism scores sampled uniformly from $[0,1]$, the incidence of conflict would be lowest when using this model. In general we empirically observe that, for reasonable magnitudes of $\frac{A}{B}$, Augmented Altruism consistently outperforms the other models. Figure 7a shows the AoC plotted against $A$, when $B = 1$. We observe that for $0.33 < A < 3$ Augmented Altruism achieves minimal values. From Figure 7b we see that when $B = 3.5$ this range is $1.6 < A < 10.4$. This demonstrates the effectiveness of the proposed model for minimising conflict.

6 Conclusion

In this work we identified conflict, a vulnerability in popular decision methods for autonomous driving. We experimentally demonstrated the significant negative consequences conflict can have in common interactive driving scenarios. We proposed a metric, Area of Conflict, that measures the potential incidence of conflict for a decision-making method. We derived a novel method for adjusting the values used in decision-making in such a way to reduce the methods vulnerability, and provided theoretical guarantees that the methods reduce the incidence of conflict for reasonable reward values.
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References

Albrecht, S. V.; Brewitt, C.; Wilhelm, J.; Eiras, F.; Dobre, M.; and Ramamoorthy, S. 2020. Integrating Planning and Interpretable Goal Recognition for Autonomous Driving. arXiv preprint arXiv:2002.02277.

Albrecht, S. V.; Crandall, J. W.; and Ramamoorthy, S. 2016. Belief and truth in hypothesised behaviours. Artificial Intelligence 235: 63–94.

Albrecht, S. V.; and Stone, P. 2019. Reasoning about hypothetical agent behaviours and their parameters. arXiv preprint arXiv:1906.11064.

Andersson, J. A. E.; Gillis, J.; Horn, G.; Rawlings, J. B.; and Diehl, M. 2019. CasADi – A software framework for nonlinear optimization and optimal control. Mathematical Programming Computation 11(1): 1–36. doi:10.1007/s12532-018-0139-4.

Andreoni, J.; and Miller, J. H. 1993. Rational cooperation in the finitely repeated prisoner’s dilemma: Experimental evidence. The economic journal 103(418): 570–585.

Bansal, S.; Cosgun, A.; Nakhaei, A.; and Fujimura, K. 2018. Collaborative Planning for Mixed-Autonomy Lane Merging. IEEE International Conference on Intelligent Robots and Systems 4449–4455. ISSN 21530866. doi:10.1109/IROS.2018.8594197.

Başar, T.; and Olsder, G. J. 1998. Dynamic noncooperative game theory. SIAM.

Bender, P.; Tas, O. S.; Ziegler, J.; and Stiller, C. 2015. The combinatorial aspect of motion planning: Maneuver variants in structured environments. In IEEE Intelligent Vehicles Symposium, Proceedings, volume 2015-August, 1386–1392. ISBN 9781467372664. doi:10.1109/IVS.2015.7225909.

Cunningham, A. G.; Galceran, E.; Eustice, R. M.; and Olson, E. 2015. MPDM: Multipolicy decision-making in dynamic, uncertain environments for autonomous driving. Proceedings - IEEE International Conference on Robotics and Automation 2015-June(June): 1670–1677. ISSN 10504729. doi:10.1109/ICRA.2015.7139412.

Düring, M.; and Pascheka, P. 2014. Cooperative decentralized decision making for conflict resolution among autonomous agents. INISTA 2014 - IEEE International Symposium on Innovations in Intelligent Systems and Applications, Proceedings 154–161.

Eiras, F.; Hawasly, M.; Albrecht, S. V.; and Ramamoorthy, S. 2020. A Two-Stage Optimization Approach to Safe-by-Design Planning for Autonomous Driving. arXiv preprint arXiv:2002.02215.

Fisac, J. F.; Bronstein, E.; Stefansson, E.; Sadigh, D.; Satsky, S. S.; and Dragan, A. D. 2019. Hierarchical game-theoretic planning for autonomous vehicles. In 2019 International Conference on Robotics and Automation (ICRA), 9590–9596. IEEE.

Hermes, C.; Wohler, C.; Schenk, K.; and Kummert, F. 2009. Long-term vehicle motion prediction. In 2009 IEEE intelligent vehicles symposium, 652–657. IEEE.

Kong, J.; Pfeiffer, M.; Schildbach, G.; and Borrelli, F. 2015. Kinematic and dynamic vehicle models for autonomous driving control design. In 2015 IEEE Intelligent Vehicles Symposium (IV), 1094–1099. IEEE.

McClintock, C. G.; and Allison, S. T. 1989. Social value orientation and helping behavior 1. Journal of Applied Social Psychology 19(4): 353–362.

Sadigh, D.; and Kapoor, A. 2015. Safe control under uncertainty. arXiv preprint arXiv:1510.07313.

Sadigh, D.; Sastry, S. S.; and Dragan, A. D. 2016. Planning for autonomous cars that leverage effects on human actions. In Robotics: Science and Systems, volume 2.

Schwarting, W.; Pierson, A.; Alonso-mora, J.; Karaman, S.; and Rus, D. 2019. Social behavior for autonomous vehicles. Proceedings of the National Academy of Sciences of the United States of America.

Trautman, P.; and Krause, A. 2010. Unfreezing the robot: Navigation in dense, interacting crowds. IEEE/RSJ 2010 International Conference on Intelligent Robots and Systems, IROS 2010 - Conference Proceedings 797–803. ISSN 2153-0858. doi:10.1109/IROS.2010.5654369.

Von Stackelberg, H. 2010. Market structure and equilibrium. Springer Science & Business Media.

Vorobeychik, Y.; and Wellman, M. P. 2008. Stochastic search methods for Nash equilibrium approximation in simulation-based games. In Proceedings of the 7th international joint conference on Autonomous agents and multiagent systems-Volume 2, 1055–1062. International Foundation for Autonomous Agents and Multiagent Systems.

Ward, E.; Evestedt, N.; Axehill, D.; and Folkesson, J. 2017. Probabilistic model for interaction aware planning in merge scenarios. IEEE Transactions on Intelligent Vehicles 2(2): 133–146. ISSN 23798858. doi:10.1109/TIV.2017.2730588.
A Appendices

A.1 Deriving Augmented Altruism

When attempting to identify equilibria in Game Theoretic problem formulations it is not uncommon to use iterative best response methods to compute the Nash Equilibrium (Vorobeychik and Wellman 2008). In practice this involves each agent choosing an optimal action based on the optimal actions for the other agents in the previous iteration. This allows for the fact that an agent’s choice of action can be affected by the choices made by other agents. If this process is repeated indefinitely, and it converges to a solution, then the solution achieved is a Nash Equilibrium ( Başar and Olsder 1998).

The altruism definition presented in Equation 2 neglects from consideration that \( r_{-i}^* \) is the reward an altruistic agent \(-i\) would receive from the interaction, not \( r_{-i} \), and that awareness of this value could affect agent \(i\)'s preferences. But, by the same assumption, the value for \( r_{-i}^* \), depends on the value of \( r_i^* \). Therefore, in order to adequately account for the altruistic inclinations of both agents, we use iterative methods over the system of equations in Equation 2. We observe that repeated iteration over the system of equations allows each agent to account for the other’s altruistic coefficient; after each agent computes their altruistic reward once, they can repeat the process using the rewards computed in the previous iteration. This gives us the following system of equations:

\[
\begin{align*}
 r_1^k &= (1 - \alpha_1) r_1 + \alpha_1 r_2^{k-1}, \\
 r_2^k &= (1 - \alpha_2) r_2 + \alpha_2 r_1^{k-1}, \\
\end{align*}
\]

where \( k \geq 0 \) gives the iteration index. Agent \( i \) does not iterate over reward \( r_i \) as the amount of reward they would get from achieving their own objective, \((1 - \alpha_i) r_i\), is known and does not need to be optimised. Since the altruism coefficients are bounded, \( 0 \leq \alpha_1 \leq 1 \), we know this system will converge (provided \( \alpha_1 \) and \( \alpha_2 \) are not both exactly 1, as this renders the computation unsolvable). We can find the steady state for this system by solving:

\[
\begin{align*}
 r_1^\infty &= (1 - \alpha_1) r_1 + \alpha_1 r_2^\infty, \\
 r_2^\infty &= (1 - \alpha_2) r_2 + \alpha_2 r_1^\infty, \\
\end{align*}
\]

This solution gives the definition of the altruistic reward presented in Equation 3.

\[
r_i^* = \frac{(1 - \alpha_i) r_i + \alpha_i (1 - \alpha_{-i}) r_{-i}}{1 - \alpha_i \alpha_{-i}} \quad i \in \{1, 2\}
\]

B Deriving Area of Conflict

We recall that conflict will occur for Augmented Altruism if:

\[
(1 - \frac{1 - \alpha_1}{\alpha_1} A < \alpha_2 < \frac{B}{B + (1 - \alpha_1) A} \wedge 0 < \alpha_1 < 1)
\]

By solving these inequalities we get the following definition for AoC (With \( A = r_{211} - r_{121} \) and \( B = r_{122} - r_{212} \)):

\[
AoC = \int_0^1 \frac{B}{B + (1 - \alpha) A} d\alpha - \int_{\frac{\pi}{2} + \pi}^1 \frac{1 - \frac{1 - \alpha}{\alpha} A}{B} d\alpha
\]

\[
= \ln(A + B)(\frac{A}{B} + \frac{B}{A}) - (\frac{A}{B} \ln(A) + \frac{B}{A} \ln(B)) - 1
\]

For comparison we can also perform the same evaluation for SVO (Schwarting et al. (2019)).

\[
r_i^* = \cos(\theta_i) r_i + \sin(\theta_i) r_{-i} \quad 0 \leq \theta_i \leq 2\pi
\]

By the same procedure as before we observe that conflict occurs with SVO when:

\[
(\theta_1 < \tan^{-1}(\frac{A}{B}) \land \theta_2 < \tan^{-1}(\frac{B}{A}))
\]

\[
\lor(\theta_1 > \tan^{-1}(\frac{A}{B}) \land \theta_2 > \tan^{-1}(\frac{B}{A}))
\]

Even though the SVO mechanism allows for masochistic and sadistic behaviours (corresponding to angles resulting in coefficients with negative magnitudes), to facilitate comparison we constrain the SVO coefficients to be between 0 and 1. This implies \( 0 < \theta_i < \frac{\pi}{2} \). We can therefore compute the AoC for SVO as:

\[
p_1 = max(0, min(\frac{\pi}{2}, \tan^{-1}(\frac{A}{B})))
\]

\[
p_2 = max(0, min(\frac{\pi}{2}, \tan^{-1}(\frac{B}{A})))
\]

\[
AoC = \frac{p_1 p_2 + (\frac{\pi}{2} - p_1)(\frac{\pi}{2} - p_2)}{(\frac{\pi}{2})^2}
\]