Mathematical modeling of the high-rise buildings deformation development process in Moscow (Vosstania square)

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Abstract. The article presents the results of the initial data preparation for the construction of a predictive mathematical model in the study of the deformation process of a high-rise building. The possibilities of forming statistically homogeneous groups of realizations using graphs constructed in the software product Surfer and analytical calculations using the coefficients of variation, calculation of Mahalanobis distances and Hotteling criterion when grouping statistically uniform observation cycles have been shown. The forecast of possible precipitations with the use of the constructed predictive model was performed.

Introduction
Prediction of deformations is one of the most difficult tasks arising in the study of the engineering structures operation patterns, which is solved in order to guarantee the reliability of the structure, rational planning of repair work, clarify the frequency and accuracy of geodetic observations [1]. In this case, modeling the deformation processes of engineering structures can be implemented on the basis of the correlation theory of random functions when the normal conditions or the normalization of the distribution and linearity or linearization of the simulated processes are fulfilled [2]. In this case, based on the deterministic-probabilistic nature of the development of the structures’ deformation, the statistical homogeneity of the process implementations used to build the model should be ensured, replacing the formal statistical criterion for the sample size display.

The statistical homogeneity of geodetic data on the movements of the observed points of a structure is ensured by the fact that the foundation and the structures of the construction have rigid connections and work together as a whole; in addition, there is a common ground conditions of the base and uniformity of the same design elements. For these reasons, the general pattern prevails in the interaction of the foundation and structures of the construction, which leads to uniformity of the structure observed points movement in certain directions and for specific structures [3].

The starting material for the considered predictive modeling was building No. 1 on Kudrinskaya Square (until 1992 - Vosstania Square). The project of the building was developed by the trunk workshop No. 9 of the Institute Mosproject [4]. Figure 1a shows the appearance of the structure.

The results of the field observations were performed in the period 1951-1964 (materials of observations of the building foundation slab fallout were at one time kindly provided by Professor P.I. Bright). In total, for the period indicated, 12 measurement cycles were carried out on the deformation marks laid at the base of the base plate. The total number of the pledged strain marks 62: of them - 12...
marks in the central part; and 25 marks in the left and right wing. Figure 1b shows the layout of sedimentary deformation marks over the entire base plate of the building.

The available results of only geodetic observations of the subsidence of structures on Kudrinskaya Square (completed over 12 years) and limited information on changes in the forecast background allowed us to construct only a kinematic model.

**Methods of Research**

In order to ensure the correctness of the random functions correlation theory application for modeling the deformation process and choosing the period for forecasting and anticipating, using the timeline for the development of the deformation marks sediment in time, based on the results of geodetic observations, the foundation period for the forecast was chosen and also tested by the criterion of asymmetry and the precipitation values normal distribution excess in all observation cycles.

The process of separating statistically homogeneous groups for the implementation of a simulated deformation process can be accomplished in two ways: on the basis of visual selection of fragments and by calculation.

The visual selection of fragments (individual sections of the base plate) with a statistically uniform nature of the precipitation process development was based on the assumption that the degree of proximity of sedimentary marks to identical isolines in one cycle and to similar isolines in other cycles reflects the level of statistical precipitation uniformity in the fragmentary section under consideration [3]. A visual assessment of the precipitation process development was performed in two ways:

- by plotting the development process of the foundation plate precipitation using the standard MS Excel program;
- by constructing schemes of isolines sediment deformation brands using the software Surfer.

Using these two methods, it is possible to obtain preliminary results and select statistically homogeneous cycles, which can then be used to construct a predictive mathematical model.

**Research Results**

Plotting the process of a structure foundation slab precipitation development using the standard MS Excel program. Figure 2 presents a graph of the structure foundation plate precipitation development process of 62 strain marks according to the results of geodetic observations. The plotted graph visually reflects the linear development of the precipitation process from the 1st to the 7th cycle, inclusive, for the entire time of 12.3 years with slight fluctuations in the implementation of the process within the framework of statistical homogeneity. After analyzing the results, we can initially conclude that cycles 3, 4, 5, 6 are statistically homogeneous, which means that all these cycles can be processed together. As a result, for the construction of a predictive mathematical model, the forecast base period was preselected for the period 2.9-6.7 years.
Figure 2. Graph of the precipitation base plate development process

Construction of contour isolate contours of stamps, made in the software package Surfer. The software package Surfer is a standard program for constructing graphic images of functions of two variables, with which you can draw in the three-dimensional shape of the surface of maps [5]. And since the interpolation algorithms are incorporated into the software package, they can be used to create a digital model of the surface from non-uniformly distributed data in high quality: process and convert data into a contour, surface, frame, vector, image, shaded area. In our case, when building a digital model, its surface was represented as values at the nodes of a rectangular regular grid, the discreteness of which was determined depending on the problem to be solved. To convert this data, we used our own GRD Surfer files, which are standard for mathematical modeling packages. The contour maps constructed using the Surfer program for 6 different observation cycles are presented in Figure 3. A comparison of the contours of the contour isolines over cycles 1 through 6 showed that the spatial-temporal development of the process stretched horizontally from left to right. The greatest deformation occurred in 1, 2 and 3 measurement cycles, since these changes occurred at the beginning of the construction period.
Figure 3. The diagram of isolines of deformation marks:

a) 1st measurement cycle \( t = 0.9 \) years; b) 2nd measurement cycle \( t = 1.9 \) years; c) 3rd measurement cycle \( t = 2.9 \) years; d) 4th measurement cycle \( t = 4.0 \) years; e) 5th measurement cycle \( t = 5.5 \) years; f) 6th measurement cycle \( t = 6.7 \) years

In the first cycle of measurements, the grades with the numbers 64, 65, 61, 60, 56, 39, 34, 33 received the greatest draft. In the second cycle, the precipitations of the grades located in the central part of the building increase. We can partially observe the process of damping the deformation only starting from the 4th measurement cycle. Analyzing the final results, one can visually identify the following homogeneous measurement cycles: all considered cycles 3 through 6 are homogeneous, which means that they can be processed together.

The selection of fragments by calculation was implemented with the calculation of the coefficients of variation and the calculation of the Mahalanobis distance and the Hotteling criterion.

The calculation of the coefficients of variation must be performed firstly based on the all grades precipitation observations results embedded in the foundation of a structure in order to assess the overall state change and behavior of the structure [3]. An increase in the coefficients of variation indicates the time when an unfavorable course of the precipitation process occurred, and a decrease in the values of the coefficients of variation reflects an improvement in the behavior and condition of the deformable structure (Table 1). In our case, starting from the 3rd measurement cycle, the small values of the variation coefficients reflect a low degree of deformability of the base plate under study. Since the coefficients of variation \( \tilde{D}_X(t_j) \) and arithmetic averages \( \tilde{m}_X(t_j) \) in 3, 4, 5, 6 cycles are close to each other, these cycles can be considered as statistically homogeneous.
Table 1. The results of calculating the coefficients of variation and arithmetic means

| Cycle no. | 1   | 2   | 3   | 4   | 5   | 6   | 7   |
|-----------|-----|-----|-----|-----|-----|-----|-----|
| $\bar{m}(t_j)$ | 41.98 | 74.69 | 94.52 | 107.02 | 112.29 | 116.97 | 134.84 |
| $\bar{v}(t_j)$ | 0.195 | 0.258 | 0.274 | 0.258 | 0.253 | 0.253 | 0.245 |

Calculating the Mahalanobis distance. For the correct selection of the required statistically homogeneous starting material, studies on their grouping using the Mahalanobis distance calculation algorithm $D_m(x)$ [6] were carried out, which was calculated using expression (1):

$$D_m(X_i, X_j) = \sqrt{(X_i - X_j)^T K^{-1} (X_i - X_j)},$$

(1)

where $X_i$ is the precipitation of the marks obtained in the $i$-th cycle;

$X_j$ - sediment marks obtained in the $j$-th cycle;

$K$ - covariance matrix of vectors $X_i$ and $X_j$.

It should be noted that the covariance matrix is degenerate, therefore there is no inverse for such a matrix. In this regard, the Mahalanobis statistics was determined using the $K^+$ - pseudoinverse matrix to the original matrix $K$. As a result of the calculations, the Mahalanobis distances $D_m(x)$ between various combinations of measurement cycles were obtained. For the subsequent verification of the statistical homogeneity of the selected sections, the Hottelling criterion was used [6] (expression 2).

$$T^2 = \frac{n}{n+1} D_m(x)^2.$$  

(2)

As a result, experimental values of the Hotteling magnitude were obtained for the compared cycles of geodesic observations. Further, the value $T^2$ was associated with the $F$ distribution (Fisher distribution). The calculated value of the Hottelling criterion was calculated using the expression (3) for the significance level $\alpha = 0.01$ and the statistics $F_\alpha = 4.98$ (with 2 and $n-2$ degrees of freedom).

$$\frac{T^2}{\alpha} = \frac{2(n-1)F_\alpha}{n-2} = 10.126$$

(3)

All the calculated estimates for each of the measurement cycles, exceeding the calculated value, characterized the violation of the control condition and, therefore, were rejected. Table 2 shows the cycles selected for further processing.

Table 2. The combinations of cycles selected for the construction of the kinematic model

| Cycle no. | $D_m(x)$ | $T^2$ |
|-----------|----------|-------|
| 3-5       | 2.9      | 8.277 |
| 3-6       | 2.228    | 4.885 |
| 4-5       | 0.018    | 0.032 |

Thus, according to the results of the calculations, 3, 4, 5, and 6 measurement cycles have been chosen to build the kinematic model. Thus, it was confirmed that the prerequisites for the constructive use of
the correlation theory of random functions were fulfilled and the construction of the predictive kinematic models in the form of the first two conditional moment functions was valid.

In the process of predictive mathematical modeling, two main tasks were solved, the first of which was to check the adequacy of the predicted kinematic model by inverse verification, the second was to assess the time for a possible stabilization of the structure’s draft [2]. To solve the first predictive task of assessing the adequacy of the kinematic model by inverse verification, the period of the forecast base was chosen, covering the 3, 4, 5, 6th observation cycles, and the control prediction was performed on the 7th observation cycle, i.e. the lead period was 12.3 years. The constructed predictive mathematical model was given the form:

\[
\hat{m}_x = (t_2, t_1) = 83.52107e^{0.05258t_j} \quad \eta_{yx} = 0.958, \\
\frac{1}{\sigma_x(t_j)} = 0.0238 \frac{1}{t_j} + 0.0304 \quad \eta_{\sigma t} = 0.976, \\
\hat{r}_x = 0.783373e^{-0.03651t_j} \quad \eta_{rt} = 0.999.
\]

(4) (5) (6)

Summary

The performed prediction on the constructed model for brands with numbers 4, 5, 8, 9, 10, 11, 12, 14, 16, 17, 31, 53, 63, 66, 67, 68, 71 showed that the prediction error was within twice the standard of the predicted prediction error band; and for most of the deformation marks, these errors were less than three times the standard value in the experimental uncertainty. The results of the long-term forecasting based on the constructed model may be of practical interest for the specialists who are observing the state of the building at the present time, taking into account that the process of precipitation attenuation has not been finished yet.

References

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