Nonlinearity of nonradial modes in evolved stars

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Abstract. We show that in evolved stars, even at relatively low surface amplitudes, nonradial modes become strongly nonlinear in the hydrogen shell source, where the Brunt-Väisälä frequency has its absolute maximum. The measure of nonlinearity is the product of horizontal displacement times the radial wavenumber, $|\xi H k_r|$. It becomes large already in evolved $\delta$-Scuti stars. This nonlinearity presents a major problem for interpretations of amplitude modulation in RR Lyrae stars in terms of nonradial mode excitation.

Keywords: $\delta$-Scuti stars, RR Lyrae stars

1. Introduction

All recent models of Blazhko-type modulation in RR Lyrae stars postulate departures from pure radial pulsation. In the oblique pulsator model (Shibahashi, 2000) the $\ell = 2$ component is induced by magnetic field. In the resonant model (Nowakowski and Dziembowski, 2001) low-$\ell$ modes are excited due to a resonant coupling with the radial mode. Amplitudes of postulated nonradial components are quite sizable, according to Kovács (2002) they reach up to 0.7 radial mode amplitudes.

Apart of RR Lyrae stars, nonradial oscillations with quite high amplitudes are observed in some evolved $\delta$-Scuti stars. The best example is a post-MS star 4CVn, where several modes have been identified as those of $\ell = 1$ or 2 (Breger and Pamyatnykh, 2002).

The nonradial modes observed in RR Lyrae and evolved $\delta$-Scuti stars are of a mixed character. While it is known that such modes at observed amplitudes are only weakly nonlinear in the p-mode cavity, the nonlinearity in the g-mode cavity, where most of the mode energy is concentrated, has never been studied carefully.

2. A measure of nonlinearity

We study the nonlinearity of the g-waves in the asymptotic approximation, which may be applied when the oscillation frequency is much
smaller than the Brunt-Väisälä and Lamb frequencies. Then we have, approximately,

$$ \xi = A(r) \left( \cos \Phi(r) e_r - \frac{r k_r}{\sqrt{l(l+1)}} \sin \Phi(r) \nabla_H \right) Y_l^m(\theta, \phi) \exp(-i \omega t), $$  

where $\xi$ is the displacement, $A(r)$ is a slowly varying amplitude, $\Phi(r)$ is a rapidly varying phase,

$$ k_r = \frac{d \Phi}{dr} = \frac{\sqrt{l(l+1)} N(r)}{\omega} $$  

is the radial wavenumber, and $N(r)$ is the Brunt-Väisälä frequency. One can see that due to the high value of $N/\omega$, the following strong inequalities take place:

$$ k_r \gg \frac{\sqrt{l(l+1)}}{r} \equiv k_H, \quad |\xi_H| \gg |\xi_r| \quad \text{(except near } \sin \Phi = 0) \quad (3) $$

Standard estimate of nonlinearity, i.e. comparing the $\partial v / \partial t$ and $(v \cdot \nabla) v$ terms in the momentum equation, is correct providing that the curvature effect is included. Then we obtain

$$ SN \equiv \max(|\xi_H| k_r) \gtrsim 1 $$  

as the criterion for the strong nonlinearity. This is different from that given by Kumar and Goodman (1996). The most precise way to obtain our criterion is to apply the asymptotic approximation (Eqs. 1,2) to the amplitude expansion of the Hamiltonian. However, at least fourth order expansion is needed (see Van Hoolst, 1994, who considers the general case of the nonradial stellar oscillation).

3. Surface amplitude at the onset of the strong nonlinearity

As an application we considered the $\ell = 1$ and 2 modes in three models of evolved stars: a TAMS $\delta$-Scuti star, a post-MS $\delta$-Scuti star, and an RR Lyrae star. The two $\delta$-Scuti models have $\log T_{\text{eff}} = 3.86$ and the RR Lyrae model has $\log T_{\text{eff}} = 3.84$.

Fig.1 shows the ratio of the Brunt-Väisälä frequency to the fundamental radial mode frequency in our models. The differences are striking. The maximum value of this ratio in the RR Lyrae model is at least by an order of magnitude larger than those in $\delta$-Scuti models. This suggests that nonlinearity in the interior may be a greater problem in more evolved stars. To assess the problem we need an estimate of
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Figure 1. Ratio of the Brunt-Väisälä frequency to the fundamental radial mode frequency in three selected models.

$\xi_H$ from the observed surface amplitude. To this aim we performed LNA calculations for the selected three models focusing on linearly unstable modes of $\ell = 1$ and 2. In this way we derived bolometric light amplitudes corresponding to $SN = 1$ which we call the critical amplitudes.

The critical amplitudes for the TAMS star were found typically higher than 100 mmag which is well above the values observed in MS $\delta$-Scuti stars. Thus the nonlinearity is not a problem in this case. For the post-MS model the critical amplitudes are much lower, often below 10 mmag. Some of the modes in 4CVn have observed amplitudes exceeding the critical values. Thus the nonlinearity in the deep interior may be a problem in this case.

In the case of RR Lyrae model, critical amplitudes, shown in Fig.2, are even lower. The highest values are found for $\ell = 1$ modes in the vicinity of the first overtone. But even in this case, the value of 0.02 mag is significantly lower than observed. We see that the critical amplitudes are much lower near the fundamental mode frequency and for the whole $\ell = 2$ sequence. It is thus clear that the observed close peaks
Critical amplitudes of the bolometric flux for nonradial modes in the RR Lyrae model. The long tickmarks denote frequencies of the lowest radial modes.

in RR Lyrae stars cannot be interpreted in terms of linear nonradial eigenmodes.

The nonlinearity presents a problem for all models involving nonradial motion, whether it is due to a nonradial mode excitation or to a magnetically induced asphericity. A fully nonlinear treatment of the motion is needed if we want to model the observed amplitude modulation.

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