A summary is presented of the more theoretical aspects of the presentations made at Hyperon 99. In addition, some material is covered which was not presented at the symposium but which I feel is pertinent to the main theme of hyperons and/or, more in particular, to discussions conducted during the symposium.

1. INTRODUCTION

In this summary talk I shall attempt not only to highlight some of the issues touched upon by the speakers at this symposium but also to cover some of those topics which, for one reason or another, were left uncovered. All or most of the topics mentioned by Holstein in his opening overview talk were indeed dealt with during the last few days. However, some deserving issues, notably hypernuclei, were left untouched as too were other aspects of hyperon physics not considered by Holstein. One of these is the large-$N_c$ expansion, which has been promoted in this context by the San Diego group and which should have been aired by Liz Jenkins who unfortunately was unable to attend, another is the possibility of using perturbative QCD to describe large-$p_T$ semi-inclusive hyperon production and the associated surprisingly large measured transverse hyperon polarisations. Last but not least, is the case of hypernuclei, which was only very briefly mentioned by Holstein.

While I hope to do justice to the speakers and, in particular, to the missing subjects, time and space clearly do not permit as complete a job as I might have liked. Thus, I shall only attempt to give a flavour of what was discussed here at Fermilab and its relevance to the future of hyperon physics programmes but leave the details to the speakers contributions; and also to fill in what I feel were important gaps, with at least a hint of what might have been said. Of course, for the full details the reader is referred to the original talks.

Before turning to the more serious part of the talk, the use of the expression “perturbative QCD” reminds me of something that struck me on the flight from Italy to Chicago. Along with the pre-packed lunch, came a salad and a small container of what purported to be “Creamy Italian Dressing”. Now, it might even be that the long list of exotic ingredients adds to its appeal in the eyes of some, and, certainly, hidden in there were the three prime ingredients I have been taught to use in Italy: namely, oil (though it should be of the olive variety and not soybean), vinegar and salt. However, I strongly doubt that any Italian in the audience will ever have put such a concoction onto his or her salad at home. That is to say: we have on occasions (albeit only a very few) at this symposium heard the words QCD mentioned, but it would be very hard indeed, given the usual trimmings or non-perturbative model input, to extract anything about the presumed fundamental theory of hadronic interactions itself from the sort of phenomenology discussed. On the one hand, this is refreshing for those of us who are a little weary of hearing about the latest $n$-loop or next-to-next-to...-leading-order calculation. On the other, there should be a wariness that much of the model building that goes on in hadronic physics, with the ever-comforting benefit of hindsight, often risks being little more than a patching-up job on a rather cloudy situation.

Let me now turn to the task in hand, I have divided the summary talk into sections describing: the static properties of hyperons; semi-leptonic, radiative and non-leptonic hyperon decays; hyperon polarisation and hypernuclei; with a little space dedicated to some concluding remarks.

2. STATIC PROPERTIES

I shall consider here the description of masses and magnetic moments (in particular, those of the baryon octet). Lipkin reminded us of a remarkable series of predictions of the naive quark
model: e.g., the relation between the baryon and pseudoscalar and spin-one meson octet masses,
\[ m_\Lambda - m_N \simeq \frac{3(m_{K^+} - m_K) + (m_K - m_\pi)}{4}; \] (1)
experimentally, the left-hand side is 177 MeV and the right-hand side, 180 MeV. A similarly successful relation between the baryon octet and decuplet and pseudoscalar and spin-one meson masses is provided by
\[ \frac{m_\Delta - m_N}{m_\Sigma^* - m_\Sigma} \simeq \frac{m_p - m_\pi}{m_{K^*} - m_K}; \] (2)
where experimentally the two sides are 1.53 and 1.61 respectively. The simple but nevertheless important conclusion to be drawn is that quarks bound inside mesons behave just like quarks inside baryons.

With regard to the magnetic moments, there exist further simple relations:
\[ \mu_p + \mu_n \simeq \frac{2m_p}{m_N + m_\Delta}, \]
where the experimental values are 0.880 and 0.865 respectively, and
\[ \mu_\Lambda \simeq \frac{1}{3} \frac{m_{\Sigma^*} - m_\Sigma}{m_\Delta - m_N}, \]
here the experimental values are both -0.61!

Since Liz Jenkins was unable to be present at the symposium and despite Lipkin’s bold attempt at an impersonation, we did not learn anything about the 1/Nc-expansion approach and the work of the San Diego group. It is impossible here to do justice to this field and the interested reader is referred to the comprehensive review article by Jenkins. Let me simply try to give a flavour of what is involved and the results achieved.

A spin-flavour symmetry is found to emerge for baryons in the large-Nc limit; large-Nc baryons form irreducible representations of the spin-flavour algebra, and their static properties may be computed in a systematic expansion in 1/Nc. Symmetry relations for static baryon matrix elements may then be obtained at various orders in the 1/Nc expansion by neglecting sub-leading 1/Nc corrections. These symmetry relations (such as those already mentioned) may then be arranged according to a 1/Nc hierarchy, i.e., the higher the order in 1/Nc is the sub-leading correction, the better one expects the relation to be satisfied. For QCD baryons with Nc = 3 one then naturally expects such a hierarchy to be based on steps of roughly 1/3.

Thus, for example, the celebrated Coleman-Glashow mass relation,
\[ (p - n) - (\Sigma^+ - \Sigma^-) + (\Xi^0 - \Xi^-) = 0, \]
is O(1/Nc) in the 1/Nc expansion, so that the mass relation should be more accurate than would be predicted by mere flavour-symmetry breaking arguments alone. In fig. 1 the diagram, taken from Jenkins and Lebed, shows a hierarchy of baryon-mass relations in both 1/Nc and an SU(3) flavour-symmetry breaking parameter, \( \epsilon \sim 0.3 \), as predicted by the theoretical analysis. The accuracy with which the magnitude of the deviations follows the expected pattern is striking.

![Figure 1](image)

Figure 1. Isospin-averaged baryon-mass (normalised) combinations from [J]. The error bars are experimental and the horizontal scale is merely a label for the given combinations. The open circle is an \( O(1/N_c^2) \) mass combination; the three solid triangles are \( O(\epsilon/N_c) \), \( O(\epsilon/N_c^2) \), and \( O(\epsilon^2/N_c^2) \) mass relations; the open squares are \( O(\epsilon/N_c^2) \), \( O(\epsilon^2/N_c^2) \), and \( O(\epsilon^2/N_c^3) \) relations; and the cross is an \( O(\epsilon^3/N_c^3) \) relation.

Analogously, the baryon magnetic moments may also be studied. Results show that in the large-Nc limit the isovector baryon magnetic moments are determined up to a correction of relative order 1/Nc^2, so that the ratios of the isovector magnetic moments are determined for Nf = 2 flavours up to a correction of relative order 1/Nc^2.
And again one finds that the general $1/N_c$ hierarchy is respected.

3. SEMI-LEPTONIC DECAYS

Another problem to which the $1/N_c$ expansion has been applied is that of hyperon semi-leptonic decay (HSD). Moreover at this symposium members of the KTeV collaboration have presented their results for the hitherto completely unexplored $\Xi^0 \to \Sigma^+ e\bar{\nu}$. Let us first examine the experimental situation: in fig. 2 the measured decay modes and nature of the data available are indicated, and in table 1 the values obtained for the decay widths and angular asymmetries are displayed.

![Figure 2. The SU(3) scheme of the measured baryon-octet $\beta$-decays: the solid lines represent decays for which both rates and asymmetry measurements are available; the long dash, only rates; the short dash, $f_1 = 0$ decays; and the dotted line, the recent KTeV data.](image)

Noticeably missing from the data table is the recently published preliminary KTeV measurement of the decay $\Xi^0 \to \Sigma^+ e\bar{\nu}$, the interested reader is referred to the talks presented here by Alavi-Harati [7] and Bright [8]. The interesting point here is that the various approaches to dealing with SU(3) breaking in this sector provide well-defined and strongly bound predictions for both the decay rate and axial form factor; the data are now tantalisingly close to differentiating between the various predictions. It must be said (to the authors chagrin), however, that the present world data actually still marginally favour the original SU(3)-based prediction of Cabibbo theory [9] (more than a third of a century old).

A discussion of the theoretical problems involved was presented by García [10]. While pointing out that the only severe discrepancy (i.e., larger than three standard deviations) with respect to Cabibbo theory lies with the rate for $\Sigma^- \to \Lambda^0 e\nu$, García also stressed that the to exploit the experimental information to the full, one should fit to the asymmetry parameters ($\alpha_e\nu$, $\alpha_c\nu$, and $\alpha_B\nu$) and not merely to the extracted value of $g_1$ alone.

3.1. $F$ and $D$ or $g_A$'s (a rose by any other name . . .)

Besides the obvious possibility as a measurement of the Cabibbo-Kobayashi-Maskawa (CKM) matrix element $V_{us}$, the solidity of which has been cast into doubt (see, e.g., [8]), the study of HSD provides unique access to the matrix element $g_1(x, Q^2)$, whose integral in $x$ (the Bjorken scaling variable or partonic momentum fraction) is given in terms of quark spin contributions to the nucleon:

$$\Gamma_1^p = \int_0^1 dx g_1(x, Q^2)$$

$$= \frac{1}{2} \left[ \frac{4}{3} \Delta u + \frac{1}{3} \Delta d + \frac{1}{3} \Delta s \right] \times \left[ 1 + \delta_{PQCD} + \ldots \right]$$

(note that here and in what follows the representation of the radiative corrections etc. is only intended to be schematic). The SMC experiment, for example, measures [11]

$$\Gamma_1^p = 0.120 \pm 0.005 \pm 0.006,$$

which, combined with the prediction that

$$\Gamma_1^p = \left[ F - \frac{4}{3} D + \frac{2}{3} \Delta s \right] \left[ 1 + \delta_{PQCD} + \ldots \right],$$

leads to an extracted value for the strange-quark spin $\Delta s \simeq -0.1$ (using $g_A^p = 1.267$ and $F/D = 0.58$), which is a surprisingly large value for a sea contribution and constitutes the variously denominated spin “crisis”, “problem” or “puzzle”. The point is that if $F/D$ were to shift to 0.5 say, then the extracted value would become $\Delta s \simeq 0$, neatly resolving all conflict.

Given the obviously important rôle that HSD plays in this analysis, it is clearly vital to understand to what extent the values of $F$ and $D$
extracted from HSD are to be considered reliable and, perhaps more to the point, just what they are. Thus, I would respond to Lipkin’s earlier comments by saying that “playing” with parametrisations (of SU(3) breaking) is legitimate in the context of attempting to understand what may be happening, in order to place (reliable) bounds on other predictions. And certainly, it is only a cosmetic question whether to parametrise using $F$ and $D$ or $g_A$ or any other description that may have physical meaning in a given analysis.

3.2. $V_{us}$ or $\sin \theta_C$

In the context of this symposium probably the more interesting aspect of HSD is the possibility of measuring $V_{us}$. It was noted during one of the talks that the Particle Data Group no longer considers HSD as a reliable source of this Standard Model parameter, preferring the so-called $K_{e3}$-decay data [3]. Let me note in passing that the proton-spin analysis does not yet require the same level of precision.

There are several difficulties that render the extraction of $V_{us}$ from HSD data a delicate process. First, but not foremost, in the discussion of hadronic physics, is the continuing saga of neutron $\beta$-decay; the discrepancies present in this sector cloud the issue of CKM unitarity and therefore need to be resolved before real progress can be made with regard to $V_{us}$. An oft neglected question is that of the rôle of so-called second-class currents. These have not yet been investigated experimentally to any real extent, except to show that their presence could have a profound effect on the extracted value of $g_A$, possibly even shifting the ratio $F/D$ back to its original SU(6) value.

The area where most theoretical effort has been made, and using a number of approaches, is that of SU(3) breaking. The problem here is that most of the analyses presented in the literature to date are highly model dependent (indeed, often the main aim is to test the model and not necessarily provide a reliable analysis of parameters at all). Moreover, a severe failing of many published analyses is that they are highly selective of the data used. While it may make sense to examine the effect of neglecting this or that data set, if data are discarded on the basis of apparent discrepancy with SU(3) symmetry predictions, then the resulting bias is as obvious as it is unacceptable.

It is evident then that to resolve these difficulties, an improved experimental database is required: the presently available data do not sufficiently over-constrain the system, which needs different combinations both of the $F$ and $D$ parameters and of $|\Delta S| = 0$ and $|\Delta S| = 1$ decays, and also of both rates and angular correlations for the same decay modes. The KTeV data will go some way to meeting this request, providing as it does an evaluation of $F + D$, the same combination as found in neutron $\beta$-decay. However, there are several modes that have been measured but not yet with sufficient precision to be of real use; attempts should be made to improve these, not forgetting an eye towards the possibility of second-class current contributions.

In conclusion, a few comments are in order regarding the decay mode $\Xi^0 \rightarrow \Sigma^+ e\bar{\nu}$. The two predictions that have been compared at this symposium to the KTeV results are those of Flores-Mendieta, Jenkins and Manohar [12]. Using the

| Decay | Rate ($10^6 \text{s}^{-1}$) | $g_1/f_1$ | $g_1/f_1$ |
|-------|----------------------------|-----------|-----------|
| $n \rightarrow p$ | $1.1274 \pm 0.0025^a$ | $1.2601 \pm 0.0025$ | $F + D$ |
| $\Lambda^0 \rightarrow p$ | $3.161 \pm 0.058$ | $0.60 \pm 0.13$ | $0.718 \pm 0.015$ | $F + D/3$ |
| $\Sigma^- \rightarrow n$ | $6.88 \pm 0.23$ | $3.04 \pm 0.27$ | $-0.340 \pm 0.017$ | $F - D$ |
| $\Sigma^- \rightarrow \Lambda^0$ | $0.387 \pm 0.018$ | $-\sqrt{2}/3 D$ | $-\sqrt{2}/3 D$ |
| $\Sigma^+ \rightarrow \Lambda^0$ | $0.250 \pm 0.063$ | $-\sqrt{2}/3 D$ |
| $\Xi^- \rightarrow \Sigma^0$ | $3.35 \pm 0.37^c$ | $2.1 \pm 2.1$ | $0.25 \pm 0.05$ | $F - D/3$ |
| $\Xi^- \rightarrow \Sigma^0$ | $0.53 \pm 0.10$ | $F + D$ |

$^a$ Rate given in units of $10^{-3} \text{s}^{-1}$. $^b$ Absolute expression for $g_1$ given ($f_1 = 0$). $^c$ Scale factor 2 included in error (PDG practice for discrepant data). $^d$ Data not used in these fits.
above-mentioned $1/N_c$ expansion, and mine \cite{13}, using the centre-of-mass corrections as proposed in \cite{14}. I should remark that the difference between the results of these last two papers is due in part to the publication of new data between the two, but mainly to the large strange-quark wave-function mismatch correction applied in the latter and not in the former (owing to its incompatibility with the later data). Thus, for this type of approach one finds relatively small deviations (at most a few percent) and an overall good description of the data. As for the $1/N_c$ approach, it should be noted that there a much larger fit was performed, including data on the weak nonleptonic decuplet decays, which apparently have a very strong influence and lead to very large corrections in both sectors.

3.3. Isospin Violation

However, before moving on to the next section, I should like to recall Karl’s talk on isospin violation in semi-leptonic decays \cite{15}, indeed his comments could have a wider impact than just on these decays. The question regards the possible mixing between $\Lambda^0$ and $\Sigma^0$. If isospin is conserved, then these two particles should simply correspond to the standard SU(3) states. If, on the other hand, the isospin SU(2) is broken (as evidently it is, slightly), then the physically observed particles will be mixture of the naive SU(3) states. The related mixing angle is typically taken to be $\sin \phi \simeq -0.015$.

Clearly the decays in which the effects should be most felt are those involving both $\Lambda^0$ and $\Sigma$ hyperons. Thus, for example, the ratio of decay widths:

$$R(\phi) = \frac{\Gamma(\Sigma^+ \to \Lambda^0 e^+ \nu)}{\Gamma(\Sigma^+ \to \Lambda^0 e^- \bar{\nu})}$$

should be shifted by about 6% owing to the mixing \cite{14}. Unfortunately, present experimental precision is too poor (for the $\Sigma^+$ decay) to detect such a shift. There would also be consequences for the vector coupling in these decays, which should vanish in pure Cabibbo theory but will be non-zero if there is $\Lambda^0$-$\Sigma^0$ mixing.

4. WEAK RADIATIVE DECAYS

The subject of weak radiative hyperon decays has been discussed in detail by Żenczykowski \cite{17}. One of the central problems here is the apparent violation of Hara’s theorem \cite{18}, again in existence for over a third of a century. The decays $B \to B'\gamma$ can be described in terms of the weak Hamiltonian matrix element:

$$\langle B' | H_W | B \rangle \propto \bar{u}(p') \gamma_{\mu} \sigma^{\mu\nu} q_{\nu} (C + D\gamma_5) u(p), (11)$$

where the term in $C$ is magnetic and $D$ is electric. Hara’s theorem is based on U-spin and states that $D = 0$ for $B$ and $B'$ belonging to the same U-spin multiplet. From the observation that U-spin is not badly broken, one expects $D$ to be small (say, of order 10%). The experimental implication is a small asymmetry parameter:

$$\alpha = \frac{2 \Re C^* D}{|C|^2 + |D|^2}, \quad (12)$$

for the decays $\Sigma^+ \to p\gamma$ and $\Xi^- \to \Sigma^- \gamma$. Experimentally the former is $-0.76 \pm 0.08$; i.e., the theorem is almost maximally violated. Such a large value is indeed very difficult to explain consistently.

Successful approaches (salvaging Hara’s theorem) may be found in the literature, due to Le Younac et al. \cite{19} and Borasov and Holstein \cite{20}. The central idea of these two groups is the insertion of additional intermediate states, from the $(70, 1^+)$ in the case of the former and $\frac{1}{2}^-$ in the latter, into the pole diagrams (see fig. 3) used in calculating the radiative decays. On the other hand, Żenczykowski has argued that there are strong indications that Hara’s theorem may indeed be violated. Such a violation would, of course, imply a failure of one or more of the fundamental input assumptions to the theorem: gauge-invariance, CP conservation and a local (hadronic) field theory. The last (in the case of finite-size hadrons) is the weakest of these assumptions.

Figure 3. The pole diagrams contributing to hyperon radiative decays; the cross indicates the intermediate-state insertions.
The point then is that one cannot infer from this asymmetry alone the violation (or otherwise) of Hara’s theorem. The key to unravelling the situation can only be found in further experimental data on the other weak radiative hyperon decays: for example, the experimental asymmetry for the decay $\Xi^0 \to \Lambda^0 \gamma$ is $0.43 \pm 0.44$, which, if confirmed as large and positive, would contradict most of the models that allow Hara’s theorem to be maintained. In any case more data are required to perform serious theoretical investigations.

5. NON-LEPTONIC DECAYS

5.1. CP Violation

The subject of CP violation in non-leptonic hyperon decays was addressed in the talk by Valencia [21]. In order to gain access to CP violation one has to measure asymmetries between hyperon and anti-hyperon decays. As pointed out by Holstein in his talk, Nature has constructed a perverse sort of hierarchy, whereby the processes that are easiest to measure are those least sensitive to CP violation (owing to small prefactors) and vice versa. One of the best candidates in the trade-off between experimental feasibility and sensitivity is the asymmetry parameter, $\alpha$ (governing the correlation between the parent polarisation and daughter momentum), in the non-leptonic hyperon decays: e.g., $\Lambda^0 \to p p\pi^-$ and $\Xi^- \to \Lambda^0 \pi^-$. One thus constructs the following asymmetry:

$$A = \frac{\alpha + \bar{\alpha}}{\alpha - \bar{\alpha}} = -\tan(\delta_P - \delta_S) \sin(\phi_P - \phi_S),$$

where $\delta_P, S$ are the strong ($\Delta I = \frac{1}{2}$) phases and $\phi_P, S$ are the weak (CP-violating) phases. The estimated size of such an asymmetry, e.g., for the mode $\Lambda^0 \to p p\pi^-$ is $O(10^{-5})$, which is doable experimentally but tough.

The ingredients that go into the calculation of such an asymmetry are clearly the two types of phases. The strong phases can be accessed theoretically via Watson’s theorem, which relates $A \to B \pi$ to $B \pi$ scattering. This is, of course, of no practical use for any decay other than $\Lambda^0 \to p p\pi^-$. Recent calculations using chiral perturbation theory suggest that the phases might be very small for all other modes. For the S and P waves in $\Lambda^0$ decay they are found to be

$$\delta_S^+ \sim 6^\circ, \quad \delta_P^+ \sim -1.1^\circ,$$

where the errors are estimated (assumed) to be of the order of $\pm 1^\circ$.

The weak phases are calculable via an effective weak-interaction Hamiltonian:

$$H_{\text{eff}}^{\Delta S = 1} = \frac{G_F}{\sqrt{2}} V_{us}^* V_{us} \sum_{i=1}^{12} c_i(\mu) O_i(\mu) + \text{h.c.}$$

The short-distance coefficients, $c_i(\mu)$, are well-known while the matrix elements of the relevant operators are rather model dependent and are certainly not known with any precision. Using vacuum saturation, one can show that one of the operators, $O_4$, dominates: its matrix element is calculated to be $y_0 \approx -0.08$ and the uncertainty in the calculation outweighs the uncertainty from neglecting the other operators. At this point we have

$$\phi_P - \phi_S \sim -0.4 y_0 A^2 \lambda^4 \eta,$$

where the last three factors are the CKM matrix parameters of the Wolfenstein parametrisation and provide $A^2 \lambda^4 \eta = 10^{-3}$; thus, one obtains $A(\Lambda^0) \approx -3 \cdot 10^{-5}$. The errors on such an estimate are probably best set at around 100%.

A particular interest in such numbers is stimulated by the effect of possible extensions to the Standard Model, the most popular being Supersymmetry. For example, if the recently confirmed large value for $\epsilon'/\epsilon$ is to be ascribed to new Supersymmetric couplings, then the same would produced an enhanced CP-violating asymmetry in non-leptonic decays, with $A(\Lambda^0) \sim O(10^{-3})$ being possible.

5.2. $\Delta I = \frac{3}{2}$ Amplitudes

A related subject, dealt with by Tandean [22], is the study of the little-known $\Delta I = \frac{3}{2}$ amplitudes in hyperon non-leptonic decays. In view of the situation with regard to the $\Delta I = \frac{1}{2}$ amplitudes and the problem of simultaneously fitting the S- and P-wave contributions, it is instructive to study the $\frac{3}{2}$ amplitudes. The analysis presented was based on calculations performed in chiral perturbation theory [23]. On the theoretical side the situation is rather favourable: at leading order, the amplitudes can be described in terms of just one weak parameter and this can be fixed from the S-wave amplitudes measured in $\Sigma$ decays. This then allows a full set of predictions for the P-waves. Unfortunately, as is often the case, the experimental situation is less favourable.
and, despite the large corrections found in their one-loop calculations, the large errors on the measured values does not yet allow a meaningful comparison of data and theory.

6. HYPERON POLARISATION

Another long-standing puzzle in hadronic physics (although a relative youngster compared to other topics discussed above) is that of the large transverse hyperon polarisations observed in large-$p_T$ semi-inclusive hyperon production (for example, see [24] for recent data). A related phenomena is that of the left-right asymmetry in pion production of transversely polarised targets (see [25], for example). The general phenomenology was presented here by Pondrom [26] and the more theoretical aspects of the problem were discussed by Soffer in his talk [27]; I would like to enlarge on some of the points made and touch upon a few others. Let me first stress that, in the absence of parity violation, the only single-spin asymmetries allowed are those which correlate the polarisation vector to the normal of the scattering plane, just as in the examples mentioned above.

The archetypal process is $pp \rightarrow \Lambda^0\uparrow X$, where neither initial-state hadron is polarised while the final-state $\Lambda^0$ hyperon is found to emerge strongly polarised along the normal to the scattering plane. The principal characteristics of this phenomenon are as follows (see also Fig. 4): the polarisation

1. is large, reaching values of the order of tens of percent;
2. grows more-or-less linearly with $x_F$;
3. grows more-or-less linearly with $p_T$ up to $p_T \sim 1$ GeV;
4. remains large and approximately constant for $p_T \gtrsim 1$ GeV, up to the largest measured values of $p_T \sim 4 - 5$ GeV;
5. follows the expected SU(6) pattern of signs and relative magnitudes.

To the extent that it has been studied, a similar description also applies to the pion and other asymmetries where the spin vector belongs to the initial state.

It is not difficult to see (by expressing the amplitudes in a suitable helicity basis) that such single-spin asymmetries must be proportional to the imaginary part of the interference between a spin-flip and a non-flip amplitude. This poses a two-fold difficulty in gauge theories with light fermions:

- tree-level or Born amplitudes are always real,
- spin-flip amplitudes are proportional to a current fermion mass.

The first requires loop diagrams, which lead to suppression by a power of $\alpha_S$ and which typically also lead to colour-factor and kinematical mismatch. The second, naively, implies suppression by, at best, a power of the strange-quark mass divided by $p_T$. Thus, Kane, Pumplin and Repko [28] (prior to the release of any experimental results) were led to the conclusion that such large effects would spell doom for perturbative QCD. As history now tells, the effects were far from zero but perturbative QCD is still very much alive and kicking!

As usual, there is a “get-out” clause: the typical $p_T$ of the data is not considered large enough yet for perturbative QCD to be reliable. Having said that, a great deal of progress has been made since the early perturbative calculations and it is now known that such effects are possible even within a framework of purely perturbative QCD. Before discussing these developments, I would like to briefly discuss two of the non-perturbative approaches.

Together with other semi-classical models, Soffer already mentioned the Lund string-model approach [29] and illustrated some of its shortcomings; I shall add to this by highlighting the inconsistency in the logic from which it derives such polarisation effects. The initial motivation is conser-
vation of angular momentum in the string break-up process, producing the strange anti-strange pair. Orbital angular momentum is generated by a finite length of string being consumed to produce the energy necessary to create the pair, which are then necessarily spatially separated. This separation, combined with a finite $p_T$, leads to non-zero orbital angular momentum of the pair, which are then necessarily spatially separated. Trivial considerations show that the predicted sign is correct, assuming the strange quark polarisation is correlated to that of the final-state hyperon via SU(6) type wave-functions. However, while the spin of the $s\bar{s}$ pair is limited in magnitude to a total of one unit, the orbital contribution is essentially unbounded as $p_T$ increases (roughly speaking, $|\vec{L}| \propto p_T \sqrt{p_T^2 + m^2}$). And thus the serpent bites its own tail.

Soffer also went into some detail with regard to the use of models based on Regge theory [30, 31]. While to a certain degree such models may provide better insight (they do at least contain explicit reference to imaginary phases, which the Lund model does not), they cannot expect to apply to very large $p_T$ configurations. Moreover, all such models are at a complete loss in trying to explain the large polarisations observed in anti-hyperon production.

Let me now turn to the developments in perturbative QCD over the past years. A great deal of new understanding has developed since the days of Kane et al. and there are now good reasons for believing that explanations can be constructed within the framework of perturbative QCD. Nearly fifteen years ago Efremov and Teryaev pointed out that there exist so-called twist-three contributions that can come to the rescue [32]. First of all, they note that the mass scale, as required by gauge invariance, is not that of a current quark but a typical hadronic mass, i.e., $\sim O(1\,\text{GeV})$.

The fact that twist-three contributions are invoked is not the unnecessary complication it might seem. Indeed, it was well-known beforehand that such would have to be the case owing to the spin-flip requirement: spin-flip always implies a mass proportionality and therefore higher twist (note that twist effectively counts the inverse power of $Q^2$, or in this case $p_T$, that appears in expressions for physical cross-sections). Thus, the type of diagrams one is led to contemplate are such as that shown in Fig. 5. The extra gluon leg is attached to the polarised hadron and is symptomatic of the twist-three nature. The deeper and crucial observation of Efremov and Teryaev is that when the momentum fraction, $x_g$, carried by the odd gluon goes to zero, the propagator marked with a cross in the figure encounters a pole. This is not say that it propagates freely; it is more a statement of how to perform a contour integral. Indeed, if we adopt the usual $i\epsilon$ prescription and split the propagator into its real (principal value) and imaginary parts:

$$\frac{1}{x_g s + i\epsilon} = \text{P} \frac{1}{x_g s} + i\pi \delta(x_g),$$

where $s$ is the usual (hadronic) Mandelstam variable, then one immediately sees how an imaginary part arises. Note that, although this may look like a loop diagram, once again for reasons of gauge invariance, one can show that the factor of $\alpha_s$ is absorbed into the definition of the hadronic blob itself and thus it is formally a Born-level contribution. Note also that such three-legged blobs are exactly what one encounters in the structure function $g_2$ governing transversely polarised DIS, presently under study in various high-energy experiments.

![Figure 5](image5.png)

Figure 5. An example of the twist-three diagrams that may contribute to semi-inclusive $\pi$ asymmetry. The dashed line represents the cut through the final states, the upper, cut quark line should, in fact, fragments into the detected pion. The cross indicates the propagator that reaches the pole and that thus provides the imaginary contribution.

Such diagrams have been exploited by Qiu and Sterman [33], who have shown that they can pro-
duce large asymmetries. Moreover, I have recently shown [34] that in the particular kinematic limit \(x_g \to 0\) a novel form of factorisation occurs and it is then easy to see why these diagrams give a contribution of the same order of magnitude as the normal twist-two Born diagrams.

The advantage of such an approach is that one is clearly free of the usual model dependence (providing the necessary information on input structure and fragmentation functions is available). Moreover, through the common hard-scattering diagrams, such an approach automatically links the many different possible types of processes in which single-spin asymmetries may be observed. Note also that it will naturally be applicable to the case of \(\Lambda_c\) polarisation, discussed here by Goldstein [35], which could then provide a key to the transition from the non-perturbative to perturbative regimes. On the down side, there are a large number of possible contributions (e.g., twist-three fragmentation functions) and there may also be a shortage of information, although many phenomenological analyses are now being performed to identify the origins of the effects and experiments are continuing to gather information on polarised hadronic structure.

7. HYPERNUCLEI

Very little was said at this symposium about hypernuclei. As this is another long-standing, as-yet unsettled problem and there are experiments planned to examine it in detail, I decided it would be useful to redress the balance a little here. The main observation is that a \(\Lambda^0\) (or even a \(\Sigma^0\)) can move freely within a nucleus (and indeed nuclear matter in general) without the usual hindrance of the Pauli exclusion principle, which applies to the standard nuclear contents: namely, neutrons and protons. Apart from its slightly larger mass it is very much like (but not identical to) a neutron and therefore is an ideal probe of the nuclear potential and the forces at work inside a nucleus.

An old question, still debated, is whether or not \(\Sigma\)-hypernuclei actually exist. Early reports have never been corroborated although it is hard to completely rule out the possibility and theoretically there is no solid argument against their existence.

A more recent and certainly pressing problem regards the observation of an apparent violation of the age-old \(\Delta I = \frac{1}{2}\) rule in the decays of \(\Lambda\)-hypernuclei [36]. The situation is, on the face of it, rather simple. A \(\Lambda^0\) bound inside a nucleus does not have access to the standard decay channels \(\Lambda^0 \to N\pi\) for reasons of energy. However, it may decay via an exchange reaction with another nucleon inside the nucleus:

\[
\begin{align*}
\Lambda^0 + p &\to n + p, \quad (19) \\
\Lambda^0 + n &\to n + n. \quad (20)
\end{align*}
\]

From the data on decays of \(\Lambda^4\), it appears that one has the following ratio for the \(\Delta I = \frac{1}{2}\) and \(\frac{3}{2}\) amplitudes:

\[
\frac{A\left(\frac{1}{2}\right)}{A\left(\frac{3}{2}\right)} \sim 1. \quad (21)
\]

It is rather difficult to believe that a nuclear environment with its typical binding energies of a few MeV could have such a profound effect on hadronic interactions that have typically much higher energy scales. Indeed, an interesting approach pioneered by Preparata and co-workers [37], in which a coherent dynamic pion background field plays an important role, arrives at results compatible with the experimental data while avoiding explicit violation of the \(\Delta I = \frac{1}{2}\) rule at the hadronic level.

These questions and the nature of the hypernuclei system will come under close scrutiny in the near future in the FINUDA experiment planned for DAΦNE the Frascati \(\phi\) factory. This is an \(e^+e^-\) facility designed to operate at a centre-of-mass energy corresponding to the \(\phi\) mass (1020 MeV). The dominant \(K^-\bar{K}\) decay mode means that this machine will provide a copious source of \(K^-\) and \(\bar{K}\) mesons. In the CHLOE experiment this will be exploited to study, e.g., CP violation and the \(K\)-dependence of the \(K^0\)-\(\bar{K}\) system itself, and in FINUDA to produce large clean samples of hypernuclei.

8. CONCLUSIONS

There is little point in trying to summarise a summary. However, it is worth making the overall observation that, at least in those areas of hyperon physics discussed at this symposium, the main stumbling block to progress at the present is the lack of precision data. Or to put it another way, in many cases the precision of experiment and theory are roughly equivalent. Indeed, in some cases the precision of the theory is limited by the lack of precision input data. This means that it is often impossible to distinguish between the different models found in the literature and, with no hint as to the way forward, the theorist is
left floundering. During the symposium we have thus heard various pleas from the theorists for the production of data of better quality, greater quantity or wider variety, as the case may be, and we have also heard from the experimentalists that new experiments are planned or in progress and that new data will be forthcoming.

What is also clear is that many of the problems connected to hyperon physics are of a very general hadronic nature and thus the answers to the questions posed here could have far-reaching repercussions. Let us hope that at the next edition of this symposium some of the models will be swept away, allowing theorists and experimentalists alike to concentrate their efforts and resources on the more promising routes.

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