Delta baryons in the separation geometry model.

Abstract

Extension of the separation geometry model of baryon structure from [physics/0109024] and [hep-ph/0201270] to the spin 3/2 Delta baryons. Theoretically derived masses in MeV; $\Delta^{++} = 1240.0$, $\Delta^{-} = 1243.4$, $\Delta^{0} = 1233.9$, $\Delta^{+} = 1232.6$ with the first one differing considerably from the quoted empirical value. Mass difference values are discussed.

G Filewood
Research Centre for High Energy Physics,
School of Physics, University of Melbourne,
Parkville, Victoria 3052 Australia.

March 25, 2022

1 Motivation

In two previous papers [2] the background ideas and methodologies of separation geometry were described in some detail. The purpose of this paper is to outline a simple method of extension of the calculation technique to the spin 3/2 baryon resonances $\Delta^{-}$, $\Delta^{0}$, $\Delta^{+}$, $\Delta^{++}$ as an extension of the proton and neutron mass calculations presented previously and to test the result against the recent calculation of Capstick et. al [1] for the mass differences of these objects.

2 Brief background concepts.

Separation geometry approaches the issue of physical structure from quite a different perspective to standard QFT. Instead of superimposing fields satisfying local gauge invariance on a background four-dimensional space-time continuum, separation geometry works with models of particles as geometry based on explicitly local gauge dependent dimensional decomposition of the four-dimensional space-time continuum. This decomposition is well defined and is isomorphic to the cardinality structure of the real number continuum; i.e. it presupposes that space-time is a real continuum. The dimensional decomposition reduces fields to a local-gauge dependent form which is found to be suitable for the calculation of masses of fundamental fermions and the vector gauge bosons; in large measure because the problem
of infinities associated with renormalisation of QFT’s and running gauge couplings are eliminated in such a local gauge dependent approach; which fixes the coupling scale as a result of fixing the local-gauge (local phase) of the objects but in a way which allows for a natural transition to local-gauge independent geometry in the continuum limit and in which the geometry representation theory is independent of the actual local ‘gauge’ selected so that the calculated masses are likewise ultimately independent of local phase information. Separation geometry appears to be complementary to QFT; it has strengths in precisely the areas that QFT/standard model is weak (fermion masses, free parameters, no logical underpinning of the origin of fermion generation structure, raison d’etre for gauge structure etc.) but is weak in precisely the areas that QFT is strong (calculation of dynamical parameters; decay times, cross-sections etc).

The geometries that define local-gauge dependent dimensionally decomposed fields were called affine geometries in the previous papers and their properties were defined and studied. The peculiar property of affine geometry is that, whilst it allows us to define a quantum object explicitly in terms of a local phase, it renders that local phase unmeasurable. The geometric invariants of these objects are assumed to manifest as physical observables in the continuum limit; intrinsic spin, charge, mass etc. For each invariant there is a physical observable and for each intrinsic quantum physical observable there is a geometric invariant (properties such as momentum of a lepton are ‘extrinsic’ variables and independent of the geometry). The local phase (gauge) of the object is never an observable (the structure of the continuum in the theory prevents this). The geometries define bounded spaces which in the limit of continuous geometry generators must, because of the geometrical construction of the theory, define compact group symmetries with the exception of the foundation geometry (which is a one-dimensional interval whose length is the gauge property) which evolves in the continuum limit to a non-compact symmetry associated with translations in space. It is an unproven supposition that these local-gauge-dependent features lead, with dimensional ‘reconstitution’, to local-gauge independent, i.e. physical, fields although significant data has been retrodicted (along with some precision predictions) which lend support to this supposition.

As should be expected for a theory of fundamental structure the theory has extreme economy; there are essentially only two affine geometries of interest. These are the affine cubic and tetrahedral geometries (and their associated sub-geometries). In the calculation process the cube is reduced to tetrahedral equivalent sub-groups so the geometry of the tetrahedron and its’ associated sub-geometries, along with the geometry of the real number continuum, constitute the essential geometric elements of the theory. All the physical observable structure is abstracted from just the geometry, ultimately, of a tetrahedron in various incarnations.

3 Calculation algorithm.

A set of rules has been developed [2] which makes the calculation process for the mass of particles relatively simple. These rules have been derived from the discrete version of QCD
which is a consequence of the embedding of tetrahedral affine symmetry into cubic affine symmetry. Discretised QCD has an explicitly gauge-dependent discrete symmetry in colour space but has many features that resemble standard QCD.

The rest-mass calculation of a hadron in separation geometry is handled in pieces. Each ‘piece’, with an appropriate non-perturbative radiative correction, of mass is then added to give a total mass. The pieces are;

1; Constituent quark mass. This is due to the energy-momentum of the current quarks and is represented as the matrix order (the cardinality or number of matrix elements in a set) which, in the discrete version of QCD, is represented as the number of tetrahedral-equivalent matrix units in a six-tetrahedral-component vector object called the ‘particle vector’. This is an irreducible representation of the symmetry (whilst it is probably not an irreducible representation of discrete SU(3)_c - none other is known - it is the minimum required to express the full S_8 cubic permutation symmetry as a tetrahedral embedding; T_r is irreducible so the T_r SU(3) embedding is irreducible and the result follows).

2; Gluon energy; found in a similar way by adding up the matrix order of the analogous representations of the gluons which couple to the particle vector.

3; Current quark intrinsic mass; this is also expressed in terms of tetrahedral units and represents the effective rest-mass of the individual quarks. This is calculated from matrix ‘operators’, also formed as six-tetrahedral-component objects, which couple to the particle vector to describe the state present.

4; Current quark separation energy; rather like a potential energy of separation of the current quarks due to the strong interaction at the energy scale of the calculation which is fixed by the symmetry. These are termed U(1) components in the text because there is the suggestion that they are related to a discrete U(1) symmetry. (The electromagnetic potential energy of separation of the current quarks is automatically incorporated in the the current-quark ‘operators’ structure and associated radiative correction - which are non-perturbative and governed by a semi-empirically determined ansatz; see below).

The details in the case of the nucleons are covered in the mentioned papers [2]. One identifies the the order of the various components and then multiplies by the matrix order of the tetrahedral group(s) which is either 22 or 24 elements depending on whether the two group generators are acting as massless intrinsic fermion-spin generators (22 elements) or not (in which case you have 24 massive elements); and then one adds them all up. For second and third generation quarks, scalar components arising explicitly from the Higgs field must be added to the current quark masses calculated but these are not required for the first generation quarks (which do not acquire scalar components in the separation geometry model; at least not explicitly - analogous to treating the mass as (?!dynamical) in origin independent of the Higgs field).

All components, with the exception of 4, acquire a simple multiplicative radiative correction of the form \( R = (1 + \alpha q^2 = m_e^2 + G_f) \) where \( \alpha \) is the electro-magnetic coupling strength and \( m_e \) is \( \approx \) the electron rest mass (which is roughly equivalent to the mass of a single tetrahedral unit) and \( G_f \) is the weak coupling constant expressed as a dimensionless number to represent its’ effective strength with respect to \( \alpha_{em} \) at the low energy scale; here of order
$10^{-5}$. Here the digit ‘1’ in $\mathcal{R}$ is also functionally the strong coupling constant when applied to quarks - the scale of $\alpha_s$ is fixed by the tetrahedral symmetry at unity (this is the great advantage of calculating in an explicitly local-gauge dependent discrete environment where one does not have a running coupling to deal with but instead has a fixed point scale; all mass calculations reduce to the tetrahedral scale - roughly 0.5MeV - and the radiative correction is universal across fermion species as we have in the discrete scheme quark/lepton unification at the level of tetrahedral symmetry). In this sense then, component 4 has a multiplicative radiative correction of $\mathcal{R}^s = \alpha_s = 1$ when applied to strongly interacting particles.

After performing the appropriate summation and applying the non-perturbative radiative correction the mass of the particle can then be calculated by, for example (and this is usually the simplest way), taking the ratio with the electron rest mass which in separation geometry is defined by the order of the tetrahedral $T_r$ group which has $4! = 24$ elements in its’ matrix representation and two generators. The generators manifest as massless intrinsic spin generators in the transition to a field theory so that the remaining $22 \ T_r$ matrix elements, with radiative correction $\mathcal{R}$, defines the electron rest mass;

$$\mathcal{R} \cdot (T_r \ (\text{No. of irrep. matrix elements}) - T_r \ (\text{generators})) = \mathcal{R} \cdot (4! - 2) \equiv 0.5110000 \text{ MeV} \ (1)$$

It is then a simple matter to convert any matrix order expression, $M$ for the mass of a hadron into MeV;

$$\text{mass (MeV)} = \frac{M}{\mathcal{R}.22}.0.511$$

where $M$ includes any radiative corrections as described. The multiplicative radiative correction $\mathcal{R}$ is a dimensionless number whose value is approximately 1.0073115 and represents the sum $(1 + \alpha_s - \frac{1}{\sqrt{\alpha_s}} + m_e^2 + G_f)$. Thus matrix order expressions have the dimension of energy.

4 Modifications to calculation algorithm for $\Delta$ baryons.

The delta baryons $\Delta^+, \Delta^{++}, \Delta^0$ and $\Delta^-$ are spin 3/2 fermions with three current quarks; $I(J^P) = \frac{3}{2}(3^+)$). For mass calculations of baryons containing only first generation quarks we have the following mass components to compute;

1. Constituent quark energy.
2. Current quark mass.
3. Gluon energy.
4. Current quark (strong or U(1)) potential terms.

We expect that a shift in spin state will essentially leave 2, 3 and 4 unchanged in comparison with the proton and neutron calculations (modulo adjustments for the different current quark content in individual $\Delta$’s) but result in an increase in the value of item 1. The simplest ansatz that could be proposed is to increase the effective constituent energy by the equivalence of one unit of spin; that is two units of constituent quark energy (each unit representing...
one half-integer of spin). Since a baryon has three quarks, this is the same as multiplying the constituent energy of the nucleon baryon by a factor of $5/3$. The actual quark content of the baryon is carried in the current quark representation - not in the constituent ‘particle vector’ representation which represents energy above and beyond the current quark rest mass due to current quark momentum. This procedure seems to work well for the delta masses.

5 The calculations.

We will compute the current quark masses for each of the four species first. The $\Delta^{++}$ consists of three up quarks and the current quark representation is;

$$\text{strong component} = \begin{pmatrix} I & q^* & q^* \\ q^* & I & q^* \\ q^* & q^* & I \end{pmatrix}, \quad \text{E.M. component} = \begin{pmatrix} I & q^* & q^* \\ q^* & I & q^* \\ q^* & q^* & I \end{pmatrix} \quad (2)$$

and the matrix orders are read off the table; in the strong component each $q^*$ and each $I$ delivers $4!$ matrix elements and in the E.M. table each $q^*$ gives a $(4!-2)$ and each identity a $4!$ of elements. There are no cancellations. This gives 420 matrices. There is a parity doubling to 840.

To calculate the U(1) components for a baryon we use a triangle diagram; we place one of the current quarks at each vertex and each line of the triangle represents a potential energy of separation. Each line between two quarks has an energy determined by the quarks at either end of the line. An up-up bond has $2(4!-2)$ matrix order, and u-d line has $(4!-2)$ order and a d-d type line has matrix order 4.$(4!-2)$. We sum over the triangle so the $\Delta^{++}$ has a U(1) matrix order of 6.$(4!-2)$ or three up-up bonds. (These values are derived from the identities of the corresponding ‘strong’ components of the current quark representation coupled to massless generators with a $I$ canceling an $I$ so that an up-up interaction is for example $I + I = 2$ etc). The $\Delta^{++}$ total current quark mass by the algorithm is then;

$$\Delta^{++} = R840 + 6(4! - 2).$$

(Note that the U(1) component does not pick up a radiative correction).

For the $\Delta^+$ and $\Delta^0$ we have current quark masses identical the the proton and neutron respectively which have been calculated in [hep-th/0109024](http://arxiv.org/abs/hep-th/0109024) as;

$$\Delta^+ = R564 + 4(4! - 2)$$

and;

$$\Delta^0 = R576 + 6(4! - 2)$$

and finally for the $\Delta^-$ we have three down quarks as per the chart;

$$\text{strong component} = \begin{pmatrix} I & q^* & I \\ I & I & q^* \\ q^* & I & I \end{pmatrix}, \quad \text{E.M. component} = \begin{pmatrix} I & q & I \\ I & I & q \\ q & I & I \end{pmatrix} \quad (3)$$

which has the order $15.4! + 3.(4!-2)$. With parity doubling and the addition of the U(1) for three down-quarks = 12.$(4!-2)$ we obtain;
\[ \Delta^- = R \cdot 852 + 12(4! - 2). \]

The glue order for the baryon is easily calculated as \( R(6.4!)^2 \) (this is identical to the value for the nucleons) and the constituent quark energy as;

\[
R \cdot \frac{5}{3}(6(4! - 2) \cdot 6.4!)
\]

(Notice the \( \frac{5}{3} \) factor which is the boost to the constituent energy in the transition from the nucleon expression for the constituent mass to the \( \Delta \) baryons). An easy calculation then gives the following masses;

\[
\begin{align*}
\Delta^{++} &= 1240.03 \text{ MeV}, \\
\Delta^+ &= 1232.61 \text{ MeV} \\
\Delta^0 &= 1233.90 \text{ MeV} \\
\Delta^- &= 1243.36 \text{ MeV}
\end{align*}
\]

Note that \( \Delta^0 - \Delta^+ \approx 1.3 \text{MeV} \) and \( \Delta^- - \Delta^{++} \approx 3.3 \text{MeV} \) so that \( 3(\Delta^0 - \Delta^+) \approx \Delta^- - \Delta^{++} \) broadly in agreement with model expectations given by Jenkins et. al [3] and Capstick et al [1] who predict a value of \( \approx 1.5 \text{MeV} \) and \( \approx 4.5 \text{MeV} \) for these mass differences. The calculated mass of the \( \Delta^{++} \) in particular differs significantly from the standard quoted empirical value however;

\[
\begin{align*}
\Delta^{++} &= 1230.9\pm0.3, \quad \Delta^+ = 1234.9\pm1.4, \quad \Delta^0 = 1233.6\pm0.5.
\end{align*}
\]

Note that the relation [3]:

\[
\Delta_3 = \Delta^{++} - \Delta^- - 3(\Delta^+ - \Delta^0) = \frac{\epsilon'' \epsilon'}{N_c^3} \approx 10^{-3}
\]

(4)

quoted in [1] is violated with the derived masses in this study as we obtain (changing signs in accordance with the mass hierarchy derived);

\[
\Delta_3 = \Delta^- - \Delta^{++} - 3(\Delta^0 - \Delta^+) \approx 0.6 \text{MeV}.
\]

Here two \( \epsilon \)'s are isospin violating parameters for the strong and electromagnetic mass splitting respectively suggesting that in the model presented these isospin symmetries are broken. Interestingly, however, this relation is satisfied exactly for the strong interaction \( U(1) \) components. From current quark triangle diagrams one easily obtains;

\[
\Delta_{U(1)}^- - \Delta_{U(1)}^{++} = 3(\Delta_{U(1)}^0 - \Delta_{U(1)}^+) \]

(5)

and this is exact. From this we might assume that strong isospin symmetry is preserved. However, that this is not apparently the case is seen from the mass hierarchy conventionally expected on the isospin scale;

\[
\Delta^-(I_3 = -\frac{3}{2}) > \Delta^0(I_3 = -\frac{1}{2}) > \Delta^+(I_3 = +\frac{1}{2}) > \Delta^{++}(I_3 = +\frac{3}{2})
\]
with the masses decreasing with increasingly positive isospin values. The separation geometry calculation suggest that there is a mass difference between the $\Delta$ resonances favouring the negative isospin values but that there is also a mass scale that is dependent on the absolute value of the isospin and not dependent upon sign so that $|I_3 = \pm \frac{3}{2}|$ states are more massive than $|I_3 = \pm \frac{1}{2}|$ states.

If we ignore the strong-interaction U(1) components completely (i.e. remove them from the mass calculation) separation geometry gives another exact mass relation between the mass difference of the $|I_3 = \pm \frac{3}{2}|$ $\Delta$'s and the $|I_3 = \pm \frac{1}{2}|$ states;

$$\Delta^- - \Delta^{++} = \Delta^0 - \Delta^+$$

The existence of exact relations eq.(5) and eq.(6) suggests that there are symmetries related to isospin in the separation geometry model of the Delta resonances which are exactly preserved for the strong interaction eq.(5) and the electro-magnetic interaction eq.(6) but that the relationship is more complex than is conventionally represented.

The most important way the separation geometry model of current quarks differs from the standard model is in terms of the identities (the $I$'s and the $I'$s) in the current quark operator structure. These have no analogue in standard model. Note that if these identities are treated as scalars (although it may be that they should actually be treated as spin 1 rather than scalar which amounts to a global gauge redefinition of the intrinsic spin of the quarks uniformly and presumably no observable consequence?) then the up and down quarks become super-partner particles as composite scalar / fermion fusions with the scalar (?spin 1) part representing the ‘holes’ in the charge topology - for example the ‘missing’ 1/3rd charge in the up quark is represented by the $I$ piece in the operator which carries no electromagnetic charge but is physically realised in terms of the strong U(1) components and also appears in the mass sum of the em charged current quark operator where it is ‘camouflaged’ - which is to say its’ mass is blended into the q and q* operators (recognisable from the appearance of an $R$ radiative correction) and presumably not independently measurable or observable.

However, the current quark bosonic identity contributions to eq.(6) cancel out and mass differences here are purely based on the difference in massless generator content of the current quark q and q* operators from the EM components; i.e. the fermionic electromagnetic generators. Both the left and right hand sides of eq.(6) give 12 matrix units which geometrically is the number of generators needed to cover (‘charge’) the surface of the cubic quark / baryon analogue (they have equivalent topology); two per square surface (one ‘square’ is one $T_r$ unit equivalent) and is the analogue of a unit of electromagnetic charge on a cubic baryon. The proton and neutron have exactly this form (ignoring the strong U(1) contributions) the neutron mass is given as $R8!$ and the proton as $R(8! - 12)$ and similarly the absolute charge difference between the two $I_3 = \pm \frac{3}{2}$ $\Delta$’s is one unit of charge topology or 12 matrix units with the R.H.S. of eq.(6) being identical to the proton and neutron E.M. mass difference.

Note that relation eq.(6) does not represent the physical $\Delta$ states but states stripped of current-quark strong interaction potential energy terms.

Lastly note that the precision prediction of the $\Delta^-$ mass is testable as this object has yet
to have its’ mass identified empirically. It would be interesting to have further measurements of the $\Delta^{++}$ mass also.

References

[1] S Capstick, R Workman; Phys Rev. D59 (1999)014032 nucl-th/9807025 see also R Workman Phys. Rev. C56 (1997) 1645-1646 nucl-th/9705021

[2] G Filewood physics/0109024 and hep-ph/0201270

[3] E Jenkins R.F Lebed Phys. Rev. D52 282 (1995)