Measurement induced dynamics for quantum communication in strongly correlated systems

Sima Pouyandeh,1 Farhad Shahbazi,1 and Abolfazl Bayat2
1Department of Physics, Isfahan University of Technology, Isfahan 84156-83111, Iran
2Department of Physics and Astronomy, University College London, Gower St., London WC1E 6BT, United Kingdom

(Dated: March 18, 2014)

We show that a single qubit measurement followed by a unitary operation which is determined by the random outcome of the measurement can be used for both encoding the information and performing a quantum quench in a strongly correlated anti-ferromagnetic many-body system. The proposed mechanism, does not need any local control over the interaction of the system and instead exploits local operations on the quantum state of the system and harnesses the inherent entanglement in the ground state of the strongly correlated system. The performance remains always better than the classical threshold for chains up to length 50. Cold atoms in optical lattices are ideal physical setup for realizing our proposal and all the ingredients needed for local measurements and unitary operations are already available in the experiments.

PACS numbers: 03.67.-a, 03.67.Hk, 37.10.Jk, 32.80.Hd

I. INTRODUCTION

Strongly correlated many-body systems often have highly entangled nontrivial ground states. The dynamics of such systems can be used for propagating information across distant sites and has been studied intensively in the last decade. Very recently, experimental realization of quantum state transfer through the natural dynamics of many-body systems has been achieved in NMR and coupled optical fibers in linear optics. Most of the proposals so far (see and the references therein), with very few exceptions like, are based on attaching an extra qubit, which encodes an “unknown” quantum state, to a chain of strongly interacting particles which is normally assumed to be in its ground state. This mode of transmission does not seek to harness the intrinsic entanglement of many-body systems and the symmetries of the Hamiltonian seems to be more important. Moreover, attaching and detaching a single qubit to a many-body system is practically hard and needs a very fine control over the interaction of particles which is missing in many physical systems such as cold atoms.

Quantum measurement is one of the mysteries of physics which has been hardly understood since the birth of quantum mechanics. According to quantum theory, measuring any observable results in a random output which is one of the eigenvalues of a Hermitian operator that is associated to that particular observable. The probability of such an outcome is determined by the overlap of the initial wave function and the corresponding eigenvector of the observable operator. In fact, after the measurement the wave function of the system goes under an abrupt change and collapses to that particular eigenstate of the observable operator. Since the outcome of the measurement is fully random it seems to be impossible to incorporate it to quantum communication in which deterministic encoding is absolutely essential. On the other hand, since in quantum measurement the state of the system collapses instantaneously it can be used to induce dynamics in the system by changing its state and thus may be used as an alternative approach to attaching scenarios for quantum communication in strongly correlated systems.

Cold atoms in an optical lattice are excellent test bed for many-body experiments. Both bosons and fermions have been realized in the Mott insulator phase, where there is exactly one atom per site, and by properly controlling the intensity of laser beams one can tune the interaction between neutral atoms to behave as an effective spin Hamiltonian. Local addressability of atoms with the resolution of single sites has opened a totally new window for exploring many-body systems. Single site unitary operations and measurements are in fact becoming viable and accessible with high fidelities. Thanks to these new advancements, the correlated particle-hole pairs and string orders together with their time evolution have been explored experimentally. Furthermore, in recent experiments the propagation of a single impurity spin and magnon bound states in a ferromagnetic spin chain have been investigated. New cooling techniques have enabled, reaching for the first time, the temperatures required for observing quantum magnetic phases emerged due to spin interactions. In view of these, it is very timely to put forward new proposals which are doable with current achievements in cold atom experiments. In particular, one may think of new ways for quantum communication across a strongly correlated many-body interacting systems.

In this paper, we introduce a mechanism for exploiting the inherent entanglement of many-body systems for quantum communication across a spin chain. The encoding of information is done through a single qubit measurement followed by the operation of a unitary gate which is determined by the random outcome of the measurement. The following measurement induced dynamics propagates the quantum state through the chain till it reaches the other side in which the information is captured by switching off the interaction couplings. Our protocol can also be interpreted as remote state preparation as a “known” quantum state is prepared on one side of the chain and then is transferred to the other side via the natural time evolution of the system.

The structure of the paper is as following. In section II the model is introduced, in section III the unrestricted measurement induced dynamics is introduced, in section IV the proposal for restricted measurement is discussed and in section V entanglement distribution is analyzed. Then in section...
allows us to write the ground state reduced density matrix of each spin is maximally mixed. This unique and lies in the subspace that half of the spins are up.

In the rest of the paper we try to exploit the random measurement induced dynamics for the purpose of quantum communication.
outcome of the measurement the quantum state of the system initialized to one of the following states

\[ |\Psi^+(0)\rangle = \sqrt{2}P^{(+1)}(GS) \]
\[ |\Psi^-(0)\rangle = \sqrt{2}R_uP^{(-1)}(GS) \]  

(6)

where \( P^{(\pm 1)} = |\psi^{(\pm 1)}\rangle\langle\psi^{(\pm 1)}| \) are the projecting operators and \( \sqrt{2} \) is the normalization factor. Each of these states are obtained by probability of 1/2 and as it is clear the unitary operation \( R_u \) acts only when the outcome of the measurement is \( |\psi^{(-1)}\rangle \).

Since neither of these states are the eigenvector of the Hamiltonian they evolve as

\[ |\Psi^\pm(t)\rangle = e^{-iHt}|\Psi^\pm(0)\rangle. \]

(7)

By tracing out all spins except the receiver, which is taken to be the last spin \( N \), one can get the density matrix of received state

\[ \rho_N^\pm(t) = Tr_\mathcal{R}|\Psi^\pm(t)\rangle\langle\Psi^\pm(t)|. \]

(8)

To quantify the quality of state transfer one can compute the fidelity as

\[ F_\pm^u(t) = \langle \psi^{(+1)}|\rho_N^\pm(t)|\psi^{(+1)}\rangle. \]

(9)

Thanks to the SU(2) symmetry of the system \( F_\pm^u(t) \) is independent of \( \theta \) and \( \phi \) which means that all quantum states are transferred by the same fidelity.

In Fig. 2(a) the fidelity \( F_+^u(t) \) and \( F_-^u(t) \) are both plotted as functions of time. As it is clear from the figures the fidelity starts evolving after a certain time that information reaches the last site. Then due to constructive quantum interferences at a particular time \( t = t_{opt} \) the information reaches the receiver site and fidelity peaks for the first time. Though the later peaks might be larger it is physically unwise to wait for such long times as in practical cases the interaction with environment and its induced decoherence deteriorates the quality of transmission. So that we focus on the first peak at which the fidelity takes its maximal value, i.e. \( F_{max}^\pm = F_\pm^u(t_{opt}) \).

In Figs. 2(b) and (c) the maximal fidelities \( F_{max}^+ \) and \( F_{max}^- \) are plotted versus length \( N \). As it is clear from these figures the fidelities are both high and go down almost linearly with very small slopes. A linear fit to data shows that \( F_{max}^+ = -0.007N + 1.024 \) and \( F_{max}^- = -0.005N + 1.016 \). One can use these linear fits to extrapolate the fidelities in longer chains which shows that for chains up to \( N \sim 50 \) the fidelities are still above the classical threshold 2/3. This indeed shows the very high potential of this strategy for quantum state transfer across a many-body system. In Fig. 2(d) the optimal time \( t_{opt} \) is plotted versus \( N \) which also shows a linear dependence on \( N \).

IV. QUANTUM STATE TRANSFER: RESTRICTED BASIS

Very often due to practical issues it is not possible to accomplish quantum measurement in any arbitrary basis on a single spin as needed in the encoding of the previous section. Instead quantum projecting measurement may be possible only for a certain basis, let say \( \sigma_z \). The outcome of the measurement is thus either \( |\uparrow\rangle \) or \( |\downarrow\rangle \) and the quantum state of the whole system collapses to \( |\uparrow\psi\rangle \) or \( |\downarrow\psi\rangle \) respectively. To initialize the spin into a general superposition like Eq. (3) a further unitary operation on first site is needed. Depending on the outcome of the measurement we apply one of the following unitary operators to the first spin

\[ R_\uparrow = |\psi^{(+1)}\rangle\langle\uparrow| + |\psi^{(-1)}\rangle\langle\downarrow| \]
\[ R_\downarrow = |\psi^{(-1)}\rangle\langle\uparrow| + |\psi^{(+1)}\rangle\langle\downarrow| \]  

(10)

where \( R_\uparrow (R_\downarrow) \) is applied if the outcome of the measurement in the \( \sigma_z \) basis is \( |\uparrow\rangle \) (\( |\downarrow\rangle \)) to rotate it to \( |\psi^{(+1)}\rangle \). The resulted states are not eigenstates of the Hamiltonian \( H \) and thus system evolves accordingly. At any time \( t \) one can see that the quantum state of the system is one of the following states depending on the measurement result

\[ |\Psi^\uparrow(t)\rangle = e^{-iHt}R_\uparrow \otimes I |\uparrow\psi\rangle, \]
\[ |\Psi^\downarrow(t)\rangle = e^{-iHt}R_\downarrow \otimes I |\downarrow\psi\rangle. \]

(11)

As before we compute the density matrix of the last spin by tracing out the rest

\[ \rho_N^\alpha(t) = Tr_\mathcal{R}|\Psi^\alpha(t)\rangle\langle\Psi^\alpha(t)| \quad \text{for} \ \alpha = \uparrow, \downarrow. \]

(12)

To quantify the quality of the state transfer we compute the fidelity as

\[ F_\alpha^u(t) = \langle \psi^{(+1)}|\rho_N^\alpha(t)|\psi^{(+1)}\rangle. \]

(13)

Unlike the fidelity \( F_\pm^u(t) \) for unrestricted measurement basis the \( F_\alpha^u(t) \) depends on input parameters \( \theta \) and \( \phi \). To have an
input independent quantity one may compute the average fidelity for all possible pure input states on the surface of the Bloch sphere

\[ F_{\text{av}}(t) = \frac{1}{4\pi} \int F_r^\alpha(t) \sin(\theta) d\theta d\phi. \] (14)

Using a little bit of maths one can show that the quantum state of the system is \(|GS\rangle_L \otimes |GS\rangle_R\). A Bell measurement is performed on the first spins of both chains which projects them on one of the following four possible maximally entangled Bell states

\[ |B_0\rangle = \sqrt{2} |\uparrow\downarrow \rangle - |\downarrow\uparrow \rangle, \]
\[ |B_1\rangle = \sqrt{2} |\uparrow\downarrow \rangle + |\downarrow\uparrow \rangle, \]
\[ |B_2\rangle = \sqrt{2} |\uparrow\uparrow \rangle - |\downarrow\downarrow \rangle, \]
\[ |B_3\rangle = \sqrt{2} |\uparrow\uparrow \rangle + |\downarrow\downarrow \rangle. \] (16)

Since the two chains do not interact any of these four possible outcomes will occur with the probability of 1/4. The symmetry of the system implies that the final entanglement is the same for all of them and thus we assume that the outcome of the measurement is the singlet \(|B_0\rangle\). After measurement the first sites of the two chains get entangled and hence at any time \(t\) the quantum state of the system can be written as

\[ |\psi(t)\rangle = 2e^{-iH_Tt} P_{B_0}^{B_0}\langle GS\rangle_L \otimes |GS\rangle_R. \] (17)

where \(P_{B_0}^{B_0} = |B_0\rangle \langle B_0|\) projects the first sites of the two chains (i.e. spins at sites \(1_L\) and \(1_R\) as depicted in Fig. 2(b)) into a singlet state \(|B_0\rangle\), the factor 2 at the beginning of the formula is for normalization and \(H_T = H \otimes I + I \otimes H\) is the total Hamiltonian of the system. One can compute the reduced density matrix of the last two sites by tracing out all the rest. The special symmetries of the system and conservation of parity during the evolution implies that

\[ \rho_{N_L,N_R}(t) = \frac{1}{2} \begin{pmatrix} a(t) & 0 & 0 & 0 \\ 0 & 1-a(t) & b(t) & 0 \\ 0 & b(t) & 1-a(t) & 0 \\ 0 & 0 & 0 & a(t) \end{pmatrix}. \] (18)

where both \(a\) and \(b\) are real numbers and can be written as

FIG. 3: (Color online) (a) The average fidelity \(F_{\text{av}}(t)\) as a function of \(Jt\) for a chain of length \(N = 20\) in a restricted basis protocol. (b) The maximal average fidelity \(F_{\text{av}}(t_{opt})\) in terms of length \(N\).

where in the above formula it is assumed that the outcome of the measurement is spin up and to have the formula for the outcome spin down one has to only replace \(\downarrow\) with \(\uparrow\) in Eq. (15). In fact, due to the symmetries of the system \(F_{\text{av}}(t)\) is identical for both \(\alpha = \uparrow, \downarrow\) and thus we drop the index \(\alpha\).

In Fig. 3(a) we plot \(F_{\text{av}}(t)\) as a function of time. At \(t = t_{opt}\) the average fidelity peaks for the first time. In Fig. 3(b) the maximum of average fidelity is depicted in terms of \(N\) which can be well fitted by a linear function as \(F_{\text{av}}(t_{opt}) = -0.006N + 1.020\). This shows that for chains up to length \(N \approx 60\) the average fidelity is above the classical threshold 2/3.

V. ENTANGLEMENT DISTRIBUTION

The proposed measurement induced dynamics for state transfer can also be used for entanglement distribution. To fulfill such task we consider two independent chains which do not interact with each other as shown in Fig. 1(b). Initially both chains are prepared in their ground states and hence the quantum state of the system is \(|GS\rangle_L \otimes |GS\rangle_R\). A Bell measurement is performed on the first spins of both chains which projects them on one of the following four possible maximally entangled Bell states

\[ |B_0\rangle = |\uparrow\downarrow \rangle - |\downarrow\uparrow \rangle, \]
\[ |B_1\rangle = |\uparrow\downarrow \rangle + |\downarrow\uparrow \rangle, \]
\[ |B_2\rangle = |\uparrow\uparrow \rangle - |\downarrow\downarrow \rangle, \]
\[ |B_3\rangle = |\uparrow\uparrow \rangle + |\downarrow\downarrow \rangle. \] (16)

Since the two chains do not interact any of these four possible outcomes will occur with the probability of 1/4. The symmetry of the system implies that the final entanglement is the same for all of them and thus we assume that the outcome of the measurement is the singlet \(|B_0\rangle\). After measurement the first sites of the two chains get entangled and hence at any time \(t\) the quantum state of the system can be written as

\[ |\psi(t)\rangle = 2e^{-iH_Tt} P_{B_0}^{B_0}\langle GS\rangle_L \otimes |GS\rangle_R. \] (17)

where \(P_{B_0}^{B_0} = |B_0\rangle \langle B_0|\) projects the first sites of the two chains (i.e. spins at sites \(1_L\) and \(1_R\) as depicted in Fig. 2(b)) into a singlet state \(|B_0\rangle\), the factor 2 at the beginning of the formula is for normalization and \(H_T = H \otimes I + I \otimes H\) is the total Hamiltonian of the system. One can compute the reduced density matrix of the last two sites by tracing out all the rest. The special symmetries of the system and conservation of parity during the evolution implies that

\[ \rho_{N_L,N_R}(t) = \frac{1}{2} \begin{pmatrix} a(t) & 0 & 0 & 0 \\ 0 & 1-a(t) & b(t) & 0 \\ 0 & b(t) & 1-a(t) & 0 \\ 0 & 0 & 0 & a(t) \end{pmatrix}. \] (18)

where both \(a\) and \(b\) are real numbers and can be written as
As it is clear from the figure, entanglement decays almost linearly with a small slope such that it reaches $E_{\text{max}}$ with a total excitation of zero. In contrast, the odd chains have doubly degenerate ground states $|GS_L\rangle$ and $|GS_R\rangle$ that each can be converted to another by applying $\prod_k \sigma_z^k$. In a chain of length $N$, the ground state $|GS_L\rangle$ ($|GS_R\rangle$) lies in the manifold of parity +1 (-1) in which $(N + 1)/2$ number of spins are up (down) and the rest are down (up). In such states there is no SU(2) symmetry and one can split their degeneracy by applying a small magnetic field in the $z$ direction to choose one the ground states. Due to the absence of the SU(2) symmetry the fidelity of state transfer in both restricted and unrestricted basis depends on input parameter $\theta$. So, to quantify the quality of state transfer we consider a system of length $N$ initially prepared in one of its ground states, let say $|GS_L\rangle$. Then a restricted measurement in $\sigma^z$ basis is performed on the first spin of the chain which projects the first qubit on either spin $\uparrow$ or spin $\downarrow$. Depending on the outcome of the measurement a further application of $R_x$ or $R_y$ rotates the first spin into $|\psi^{(+1)}\rangle$ and initialization process is accomplished. A further time evolution of the system transfers this quantum states through out the chain. Just as before one can trace out the state of all spins but the last one and get the reduced density matrix of the last site $\rho_N(t)$ from which the fidelity is computed just as in Eq. (13). To have an input independent quantity one can also average over all possible input states on the surface of the Bloch sphere just as the one in Eq. (14) to get the average fidelity $F_{av}(t)$.

Just as before we consider the first peak of the average fidelity at the optimal time $t_{\text{opt}}$. In TABLE I we give a comparison for the average fidelity of even and odd chains versus length $N$ when the outcome of the measurement is spin up. By comparing the values one can realize that the quality of transfer is slightly lower for odd chains. For instant the average fidelity in the odd chain of length $N = 19$ is 0.88 while for a longer even chain of $N = 20$ is 0.91. This means that the SU(2) symmetry of the ground state in the even chains makes the quality of transfer even higher than the slightly shorter chains but with an odd length.

VI. ODD CHAINS

So far we have only considered even chains for which the ground state is unique and supports the SU(2) symmetry with total excitation of zero. In contrast, the odd chains have doubly degenerate ground states $|GS_L\rangle$ and $|GS_R\rangle$ that each can be converted to another by applying $\prod_k \sigma_z^k$. In a chain of length $N$, the ground state $|GS_L\rangle$ ($|GS_R\rangle$) lies in the manifold of parity +1 (-1) in which $(N + 1)/2$ number of spins are up (down) and the rest are down (up). In such states there is no SU(2) symmetry and one can split their degeneracy by applying a small magnetic field in the $z$ direction to choose one the ground states. Due to the absence of the SU(2) symmetry the fidelity of state transfer in both restricted and unrestricted basis depends on input parameter $\theta$. So, to quantify the quality of state transfer we consider a system of length $N$ initially prepared in one of its ground states, let say $|GS_L\rangle$. Then a restricted measurement in $\sigma^z$ basis is performed on the first spin of the chain which projects the first qubit on either spin $\uparrow$ or spin $\downarrow$. Depending on the outcome of the measurement a further application of $R_x$ or $R_y$ rotates the first spin into $|\psi^{(+1)}\rangle$ and initialization process is accomplished. A further time evolution of the system transfers this quantum states through out the chain. Just as before one can trace out the state of all spins but the last one and get the reduced density matrix of the last site $\rho_N(t)$ from which the fidelity is computed just as in Eq. (13). To have an input independent quantity one can also average over all possible input states on the surface of the Bloch sphere just as the one in Eq. (14) to get the average fidelity $F_{av}(t)$.

Just as before we consider the first peak of the average fidelity at the optimal time $t_{\text{opt}}$. In TABLE I we give a comparison for the average fidelity of even and odd chains versus length $N$ when the outcome of the measurement is spin up. By comparing the values one can realize that the quality of transfer is slightly lower for odd chains. For instant the average fidelity in the odd chain of length $N = 19$ is 0.88 while for a longer even chain of $N = 20$ is 0.91. This means that the SU(2) symmetry of the ground state in the even chains makes the quality of transfer even higher than the slightly shorter chains but with an odd length.

VII. IMPERFECTIONS

Preparing the system in its anti-ferromagnetic ground state needs cooling to zero temperature which in reality cannot be achieved. Hence, the initial state of the system is inevitably truncated.
TABLE I: A comparison between the attainable average fidelity at the optimal time, i.e. $F_{av}(t_{opt})$ between the even and odd chains for the case that the outcome of the measurement is spin up. As the number shows the even chains, with SU(2) symmetry, produce higher fidelity even for slightly longer chains.

| $N$ (even) | 4   | 6   | 8   | 10  | 12  | 14  | 16  | 18  | 20  |
|------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| $F_{av}$ (even) | 0.9991 | 0.9867 | 0.9735 | 0.9604 | 0.9482 | 0.9368 | 0.9264 | 0.9171 | 0.9082 |
| $N$ (odd)  | 3   | 5   | 7   | 9   | 11  | 13  | 15  | 17  | 19  |
| $F_{av}$ (odd) | 0.9715 | 0.9526 | 0.9367 | 0.9236 | 0.9118 | 0.9013 | 0.8915 | 0.8834 | 0.8761 |

FIG. 5: (Color online) The imperfection effects over a chain of length $N = 10$: (a) The fidelity $F_{max}$ as a function of dimensionless temperature $K_B T/J$. Thanks to the SU(2) symmetry of the thermal initial state, all projection basis give the same fidelity. The fidelity $F_{max}$ as a function of decoherence coupling $\gamma/J$ when the first qubit is projected into $|\uparrow\rangle$. (b) The fidelity $F_{max}$, averaged over 100 different realizations, in terms of randomness strength $\epsilon$ when the first qubit is projected into $|\uparrow\rangle$.

Another imperfection is randomness in the coupling of the Hamiltonian as making a uniform chain might be very challenging in some physical realizations. This means that in the Hamiltonian of Eq. (1) we have $J_k = J (1 + \delta_k)$, where $\delta_k$ is a dimensionless random number with a uniform distribution in the interval $[-\epsilon, \epsilon]$. In fact, $\epsilon$ determines the strength of randomness in the couplings. We fix the optimal time to be $t_{opt}$, determined from the uniform chain (i.e. $\epsilon = 0$), as the real time at which fidelity peaks depends on all couplings $J_k$’s. We then average the fidelity $F(t_{opt})$ over several different realizations (we did for 100) of the system for a fixed $\epsilon$. In Fig. 5(c) we depict the fidelity $\langle F(t_{opt}) \rangle$ averaged over 100 different realization as a function of $\epsilon$ when the first qubit is projected into the state $|\uparrow\rangle$. It is seen that although the average fidelity decreases by increasing the randomness the mechanism shows a relatively high resistance against this destructive effect as fidelity remains above 0.85 even for twenty percent of randomness (i.e. $\epsilon = 0.2$).
The proposed mechanism is most suitable for realization in optical lattices in which an array of cold atoms in their Mott insulator phase sit in the minimums of a periodic potential, formed by counter propagating laser beams, as shown in Fig. 6(a). In the limit of high on-site energy the double occupancy is prohibited and the interaction between atoms is effectively explained by a spin Hamiltonian [10]. Changing the intensity of the laser beams tunes the tunneling rate of the atoms and thus controls the exchange coupling of the spin chain globally. In two or three dimensional lattices by tuning the intensity of the corresponding laser beams one can independently control the coupling of the atoms in each dimension globally. Recently, local addressability of the atoms have also been possible in optical lattices [11, 12], makes local measurements and spin rotations, the two essential ingredients of our proposal accessible. Using such local operations the propagation of a single [14] and double [15] spin flips in a ferromagnetic chain have been experimentally observed.

To perform spin measurement on a single site one can use the techniques developed in Ref. [13]. In that methodology an intense perpendicular laser beam is focused to the target atom and couples one of the atomic levels which represents $|\downarrow\rangle$ to one of the excited states. This generates a strong radiation pressure which pushes the atom out of the lattice only when atom is in state $|\downarrow\rangle$ and does not affect it otherwise. This leaves the site empty if its atom is in state $|\downarrow\rangle$ and full if the atom is in state $|\uparrow\rangle$ as it is shown schematically in Fig. 6(b).

So, the result of the measurement is revealed through a following fluorescent picture to see whether the atom is still sitting in its initial position (projecting to $|\uparrow\rangle$) or has gone (projecting to $|\downarrow\rangle$). Notice that in this technique by probability of 1/2, for which the atom is in the state $|\downarrow\rangle$ and thus leaves the lattice, the protocol fails which reduces the rate of communication by half. This means that if a two dimensional optical lattice is used to provide several equivalent parallel noninteracting spin chains (just as the one for ferromagnetic case in Refs. [14, 15]) and the measurement is performed instantaneously on all the first qubits of parallel chains only half of them can be used to extract final information as there will be no hole in those chains and the rest should be discarded.

Apart from single qubit measurement we also need to perform unitary operations (such as $R_\uparrow$ and $R_\downarrow$ in Eq. (10)) to accomplish the initialization and encoding information. To apply such unitary operators on the target atom (i.e. site 1) a focused laser beam is exploited to generate Rabi oscillation between the qubit levels as shown in Fig. 6(a). This local operation is much quicker ($\sim 10 \mu$s) [13] than the time evolution of the system ($\sim 1 - 10 \text{ms}$) [14, 15] and can be considered as a sudden action. To have a pure local gate operation and avoid affecting the neighboring qubits one may apply a weak magnetic field gradient [13], which splits the hyperfine levels of all qubits position dependently, or use a tightly focused laser beam [12] to only split the hyperfine levels of the target atom. So then a microwave pulse, tuned only for the target qubit, operates the gate locally as it has been realized in Refs. [12, 13]. For instance, a weak magnetic field gradient of 27.4 G cm$^{-1}$ is enough for applying $\sigma^-$ on a target qubit with a pulse of duration 10 µs without affecting the neighboring sites [13].

According to the proposed mechanism for entanglement distribution a Bell measurement on the first qubits of the two chains is essential for initializing the system. We consider a geometry, shown in Fig. 6(c), in which two arrays of atoms sit in two parallel rows with the first atoms recite in the neighboring sites. To perform the Bell measurement we first raise the barriers between the atoms to switch off the interactions along the chains (i.e. $J = 0$ in both spin chains). We use the fact that the energy levels for the singlet and triple pairs are different in a single well such that the singlet state is lower in energy. To operate the Bell measurement one has to tilt the lattice adiabatically such that the atoms in the right chain tunnel into the next row and sit along the left chain. Though, the atom in the first site of the right chain has to compensate an extra on site energy $U$ for its tunneling as its target site is already occupied by the first atom of the left chain. If the amount of tilting is tuned to be resonant only with the singlet state of two atoms in the doubly occupied site then the double occupancy occurs only for the singlet state as shown in Fig. 6(d). As the other Bell states are off resonant and energetically inaccessible, the double occupancy never occurs for such states. A further fluorescence picture of the system, which can be done without disturbing the internal states [22], will determine the number of atoms in the first site and reveals if the two atoms are in a singlet state or not. A backward adiabatic evolution (i.e. returning the lattice back to its normal) restores all the atoms into their initial position while the first spins are either
projected to singlet $|B_0\rangle$ or one of the three other Bell states. If the output of the projecting measurement is singlet $|B_0\rangle$ (its probability is 1/4) then initialization is complete and by decreasing the horizontal barriers along the chains the propagation begins. On the other hand if the result is not $|B_0\rangle$ then the density matrix of the two qubits is an equal mixture of all other Bell states (its probability is 3/4). One then can apply $\sigma_z$ to the atoms in site $L$ (or $1_L$) in order to convert the $|B_1\rangle$ part of the mixture into $|B_0\rangle$ and repeat the adiabatic tilting to see if the projection to singlet is accomplished or not. This time the probability of success increases to 1/3. In the case of failure the state of the two atoms become a mixture of $|B_2\rangle$ and $|B_3\rangle$ which a local unitary operation $\sigma_y$ transforms these two states into $|B_0\rangle$ and $|B_1\rangle$ respectively. An extra repeating of the adiabatic tilting either directly gives a singlet state $|B_0\rangle$ (with the probability of 1/2) for the pair or project them into $|B_1\rangle$ (again with probability of 1/2) which then can be transformed to $|B_0\rangle$ locally. Hence, at the worst case the adiabatic tilting of the lattice has to be done three times for the initialization. Then by letting the system to evolve one can generate entanglement between the distant atoms at both sides of the system.

IX. CONCLUSION

In this paper we put forward a timely proposal for quantum communication in anti-ferromagnetic Heisenberg Hamiltonian using only local operations for encoding the information. This harnesses the intrinsic entanglement of the system for inducing dynamics via a single site quantum measurement. As the outcome of measurement is ultimately random a following unitary operation which is determined by the outcome of the measurement is essential for encoding the information within the intrinsically entangled ground state of the system. By finishing the encoding procedure system is left to evolve freely and after a certain time (set by the length $N$ and the strength of the exchange coupling $J$) information reaches the receiver site which can be taken for further computational process. The quality of state transfer remains above the threshold limit for chains up to length $N \sim 50$ while system is not engineered and no extra modulation is needed. The whole process can also be interpreted as remote quantum state preparation [19] in which a known quantum state is generated remotely at the output via the free evolution of a many-body strongly correlated system. In addition, we considered several imperfections which may arise in different realizations including thermal fluctuations, interaction with environment and the effect of random couplings.

Since the encoding of information and performing the quantum quench in the system is done by only local operations the proposed mechanism is most suitable to be realized in optical lattices. The recent experiments for spin wave propagation [14] and transferring magnon bound states [15] show that all the ingredients we need is already available in the laboratory. Based on these new achievements, our proposal is just timely for being pursued in experiments and indeed can be realized with current technology.

Acknowledgements: - It is a pleasure to thank Sougato Bose, Leonardo Banchi and Bedoor Alkurtas for useful discussions. AB thanks EPSRC grant EP/K004077/1.

[1] S. Bose, Phys. Rev. Lett. 91, 207901 (2003).
[2] S. Bose, Contemporary Physics 48, 13 (2007).
[3] G. M. Nikolopoulos, Igor Jex, Quantum State Transfer and Network Engineering, Springer (2013).
[4] K. R. Koteswara Rao, T. S. Mahesh, A. Kumar, arXiv:1307.5220.
[5] A. Perez-Leija, R. Keil, A. Kay, H. Moya-Cessa, S. Nolte, L. C. Kwek, B. M. Rodriguez-Lara, A. Szameit, and D. N. Christodoulides, Phys. Rev. A 87, 012309 (2013).
[6] S. Yang, A. Bayat, S. Bose, Phys. Rev. A 84, 020302(R) (2011).
[7] A. Bayat and S. Bose, Phys. Rev. A 81, 012304 (2010).
[8] W. S. Bakr, et al., Nature 462, 74 (2009); M. Greiner, et al., Nature 415, 39 (2002); W. S. Bakr, et al., Science 329, 547 (2010).
[9] R. Jördens, et al., Nature 455, 204 (2008); U. Schneider, et al., Science 322, 1520 (2008).
[10] L. Duan, E. Demler and M. D. Lukin, Phys. Rev. Let. 91, 090402 (2003).
[11] J. F. Sherson, et al., Nature 467, 68 (2010).
[12] C. Weitenberg, et al., Nature 471, 319 (2011).
[13] M. Karski, et al., New J. Phys. 12, 065027 (2010).
[14] T. Fukuhara, et al., Nature Phys. 9, 235 (2013).
[15] T. Fukuhara, et al., Nature 502, 76 (2013).
[16] M. Endres, et al., Science 334, 200 (2011); M. Endres, et al., arXiv:1305.5652.
[17] M. Cheneau, et al., Nature 481, 484 (2012).
[18] P. Medley, D. M. Weld, H. Miyake, D. E. Pritchard, W. Ketterle, Phys. Rev. Lett. 106, 195301 (2011).
[19] C. H. Bennett, et al., Phys. Rev. Lett. 87, 077902 (2001).
[20] W. K. Wootters, Phys. Rev. Lett. 80, 2245 (1998).
[21] A. Bayat, V. Karimipour, Phys. Rev. A, 71, 042330 (2005).
[22] M. J. Gibbons, C. D. Hamley, C. Y. Shih and M. S. Chapman, Phys. Rev. Lett. 106, 133002 (2011).