A simple model for explaining muon-related anomalies and dark matter

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We propose a model to explain several muon-related experimental anomalies and the abundance of dark matter. For each SM lepton family, we introduce vector-like exotic leptons that form an iso-doublet and a right-handed Majorana fermion as an iso-singlet. A real/complex scalar field is added as a dark matter candidate. We impose a global $U(1)_\mu$ symmetry under which fields associated with the SM muon are charged. To stabilize the dark matter, we impose a $Z_2$ (or $Z_3$) symmetry under which the exotic lepton doublets and the real (or complex) scalar field are charged. We find that the model can simultaneously explain the muon anomalous magnetic dipole moment and the dark matter relic density under the constraints of various lepton flavor-violating observables, with some details depending upon whether the scalar field is real or complex. Besides, we extend the framework to the quark sector in a way similar to the lepton sector, and find that the recent anomalies associated with the $b \to s \mu^+\mu^-$ transition can also be accommodated while satisfying constraints such as the $B_{s(d)} \to \mu^+\mu^-$ decays and neutral meson mixings.

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I. INTRODUCTION

In search of new physics, most results from the Large Hadron Collider (LHC) at the energy frontier are consistent with the Standard Model (SM) predictions and only push the existence of new particles to higher scales. On the other hand, we have encountered over the past few years a few observables in low-energy flavor physics that show evidence of deviations from the SM expectations. Interestingly, many of these processes involve the muon.

A long-standing puzzle is the muon anomalous magnetic dipole moment, \( a_\mu \) or \((g - 2)_\mu\). With advances in theory and inputs from various experiments, \( a_\mu \) has been calculated to a high precision. In comparison with experimental data, we observe a discrepancy at the 3.3σ level: \( \Delta a_\mu = (26.1 \pm 8.0) \times 10^{-10} \) [3].

Recently, some evidence of deviations seemed to occur in decays involving the \( b \to s\mu^+\mu^- \) transition, such as the binned angular distribution of the \( B \to K^*\mu^+\mu^- \) decay [4–7] and the decay rate deficit of the \( B_s \to K^{(*)}\mu^+\mu^- \) and \( B_s \to \phi\mu^+\mu^- \) decays [8, 9]. More recently, the LHCb Collaboration reported anomalies in a set of related observables, \( R_K = BR(B \to K\mu^+\mu^-)/BR(B \to Ke^+e^-) \) and \( R_{K^*} = BR(B \to K^*\mu^+\mu^-)/BR(B \to K^*e^+e^-) \). The former was found to be \( 0.745^{+0.090}_{-0.074} \pm 0.036 \) for the dilepton invariant mass-squared range \( q^2 \in (1, 6) \text{ GeV}^2 \) [10], showing a 2.6σ deviation. The latter was determined in two dilepton \( q^2 \) bins:

\[
R_{K^*} = \begin{cases} 
0.66^{+0.11}_{-0.07} \pm 0.03 & \text{for } q^2 \in [0.045, 1.1] \text{ GeV}^2 , \\
0.69^{+0.11}_{-0.07} \pm 0.05 & \text{for } q^2 \in [1.1, 6] \text{ GeV}^2 .
\end{cases}
\]

These two observables point to lepton non-universality in the \( B \to K^{(*)}\ell^+\ell^- \) (\( \ell = e, \mu \)) decays. Depending on scenarios [11], global fits to the data reveal deviations in the Wilson coefficients in the related weak decay Hamiltonian, most notably in \( C_9 \) associated with the operator \( O_9^\ell = \frac{2}{3v} (\bar{s}_L \gamma^\mu b_L)(\bar{\ell} \gamma_\mu \ell) \).

Motivated by the above-mentioned flavor anomalies, we propose a simple model with interactions specific to the muon. In addition to the SM particles, we introduce an exotic

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1 There are other analyses giving slightly different estimates of the discrepancy. For example, Ref. [1] gives \( \Delta a_\mu = (33.5 \pm 8.2) \times 10^{-10} \) showing a discrepancy at the 4.1σ level, while Ref. [2] quotes \( \Delta a_\mu = 288(63)(49) \times 10^{-11} \) indicating a 3.5σ deviation. In our numerical analysis, we use the result given by Ref. [3].
vector-like lepton doublet and a right-handed neutrino for each SM lepton family and an inert scalar boson that can be either real or complex. A global $U(1)_\mu \times Z_N$ symmetry is imposed on the model, with the muon-related fields (including the left-handed and right-handed muons and the associated exotic muon) charged under the $U(1)_\mu$ and the exotic lepton doublets and the inert scalar carrying nontrivial charges under the $Z_N$. Here $N = 2$ if the scalar field is real and $N = 3$ as a minimal choice if it is complex. With the imposed $Z_2$ or $Z_3$ symmetry, we make a connection to the observed dark matter (DM) relic abundance in the Universe \( \text{[12]} \), with the inert scalar particle serving as a bosonic weakly-interacting massive particle (WIMP) candidate. Due to the muon-specific interactions, there is a strong correlation between $a_\mu$ and DM parameters. By extending the model to the quark sector in a way analogous to the lepton sector except for no additional global $U(1)$ symmetry, we find that the $b \to s\ell^+\ell^-$ anomalies can be accommodated without conflict with various constraints such as the $B_{s(d)} \to \mu^+\mu^-$ decays, neutral meson mixings, and lepton flavor-violating (LFV) observables.

This paper is organized as follows. In Section \( \text{III} \) we describe the proposed model with the extended lepton sector and the inert scalar, and show the contributions to $a_\mu$, DM relic density, and such LFV processes as $\tau \to e\gamma$, $Z \to e\tau$ and $\tau \to e\mu\bar{\mu}$ decays. In Section \( \text{III} \) we extend the model to the quark sector to include the exotic quark fields and formulate the effective weak Hamiltonian for the $b \to s\ell^+\ell^-$ transitions. It is then used to explain the above-mentioned $B$ anomalies subject to various constraints. Section \( \text{IV} \) combines the analysis in the previous two sections and shows the result of a global fit. Section \( \text{IV} \) summarizes our findings.

II. MODEL SETUP

In this section, we concentrate on the lepton and scalar sectors (all assumed to be colorless) of the model, and will discuss the quark sector in the next section. In addition to the SM $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge group, we impose on our model an additional global $U(1)_\mu \times Z_2$ or $U(1)_\mu \times Z_3$ symmetry \(^2\), depending upon the choice of a new inert

\(^2\) We note that the $Z_3$ symmetry can be generalized to $Z_N$ with $N > 3$. In this case, the $Z_N$ charges of $L'_{e,\mu,\tau}$ is $\tilde{\omega} \equiv e^{2\pi i/N}$ and that of $S$ is $\tilde{\omega}^{-1}$. What this affects is the allowed interactions of the $S$ field in the scalar potential.
Lepton Fields & Scalar Fields

|                | $L_{L_{e,\tau}}$ | $L_{L_{\mu}}$ | $\ell_{R_{e,\tau}}$ | $\ell_{R_{\mu}}$ | $L_{e,\tau}'$ | $L_{\mu}'$ | $N_{R_i}$ | $H$ | $S$ |
|----------------|------------------|---------------|---------------------|------------------|--------------|-----------|----------|-----|-----|
| $SU(2)_L$     | 2                | 2             | 1                   | 1                | 2            | 2         | 1        | 2   | 1   |
| $U(1)_Y$      | $-1/2$           | $-1/2$        | $-1$                | $-1$             | $-1/2$       | $-1/2$    | 0        | $1/2$| 0   |
| $U(1)_\mu$    | 0                | $Q_\mu$       | 0                   | $Q_\mu$          | 0            | 0         | 0        | 0   | 0   |
| $Z_2$         | 1                | 1             | 1                   | 1                | $-1$         | $-1$      | 1        | 1   | $-1$|
| $Z_3$         | 1                | 1             | 1                   | 1                | $\omega$    | $\omega$ | 1        | 1   | $\omega^2$|

TABLE I: Contents of colorless fermion and scalar fields in the model, with their charge assignments under the SM $SU(2)_L \times U(1)_Y$ and global $U(1)_\mu \times Z_{2,3}$ symmetries. The row of $Z_2$ ($Z_3$) symmetry is for the scenario when $S$ is a real (complex) field.

For each distinct SM lepton family, we introduce corresponding $SU(2)_L$-doublet vector-like fermions $L'_i = [N'_i, E'_iT]$ and a $SU(2)_L$-singlet right-handed Majorana fermions $N_i$ ($i = e, \mu, \tau$). These exotic leptons are assumed to be heavier than their SM counterparts. Moreover, the fermions in the second families of SM and exotic leptons carry a $U(1)_\mu$ charge, denoted by $Q_\mu$. This serves the purpose of evading the $\mu \rightarrow e\gamma$ constraint, as to be discussed later. We also introduce an $SU(2)_L$-singlet scalar boson $S$, which does not carry any gauge or $U(1)_\mu$ charge. We will consider both possibilities of $S$ being real or complex. In this set up, only the exotic lepton doublets $L'_{e,\mu,\tau}$ and the inert scalar boson $S$ carry nontrivial $Z_2$ or $Z_3$ charges. In the case of a real $S$, these fields all have the $Z_2$ charge of $-1$. In the case of a complex $S$, $L'_{e,\mu,\tau}$ and $S$ have respective charges of $\omega \equiv e^{2\pi i/3}$ and $\omega^2$ under the $Z_3$ symmetry. This provides a mechanism to prevent mixing between the SM fields and the exotic fields as well as to maintain the stability of the DM candidate $S$.\footnote{The neutral components of $L'_i$ cannot be DM candidates, as they would be ruled out by direct detection via the $Z$ boson portal.}

In the most general renormalizable Lagrangian consistent with the symmetries of the
model, the lepton sector and the Higgs potential are given respectively by

\[-\mathcal{L}_L = y_{\ell_i} \bar{L}_i H P_R \ell_i + f_{\ell} \bar{L}_i P_R L'_i S + f_{\tau e} \bar{L}_e P_R L'_\tau S + f_{\tau e} \bar{L}_\tau P_R L'_e S + y_{N_i} \bar{L}_i \tilde{H} P_R N_i + y_{N_i} \bar{L}_\tau \tilde{H} P_R N_i + M_{L_i} \bar{L}_i P_R L'_i + M_{\ell} \bar{L}_\ell P_R L'_\ell + M_{\tau} \bar{L}_\tau P_R L'_\tau + M_{N_{ij}} \bar{N}_i P_R N_j + \text{H.c.} , \quad (\text{II.1})\]

\[V = \mu_H^2 |H|^2 + \mu_S^2 |S|^2 + \lambda_H |H|^4 + \lambda_S |S|^4 + \lambda_H S |H|^2 |S|^2 + \mu (S^3 + S^* 3) , \quad (\text{II.2})\]

where \(i, j \in \{e, \mu, \tau\}\) are to be summed over when repeated, the charged-lepton mass is assumed diagonal without loss of generality, and \(\tilde{H} \equiv (i \sigma_2) H^*\) with \(\sigma_2\) being the second Pauli matrix. The Higgs potential given above is the one with a \(Z_3\) symmetry. The one with a \(Z_2\) symmetry can be readily obtained by taking \(\mu = 0\). As in the SM, the first term of the Yukawa Lagrangian, Eq. (II.1), provides mass for SM charged leptons when \(H\) develops a nonzero vacuum expectation value (VEV), \(\langle H \rangle \equiv v/\sqrt{2}\). The Yukawa interactions with the \(f\) coefficients mediate interactions among \(S, \text{SM leptons and exotic leptons}\). In particular, the \(f_{\mu}\) term contributes to the muon anomalous magnetic dipole moment at the level. The electron anomalous magnetic dipole moment \(a_e\) is at most \(10^{-13}\) even if similar numerical values of the relevant parameters are used, while the experimental upper bound is of the order \(3.7 \times 10^{-13}\) [13]. Thus our model always satisfies the constraint of \(a_e\). It is straightforward to find that the \(\mu \to e\gamma, \tau \to e\gamma\) and \(\tau \to \mu\gamma\) decays do not impose a stringent constraint on the model, as there is no mixing between muon and electron (or tau) as seen in Eq. (II.1).

### A. Neutrino sector

Mass of the active neutrinos can be induced via the canonical seesaw mechanism, and the mass matrix is given by

\[(\mathcal{M}_\nu)_{\alpha\beta} \approx \sum_{i,j=e,\mu,\tau} (m_D)_{\alpha i} (M_{N}^{-1})_{ij} (m_T^D)^{j\beta}, \quad (\text{II.3})\]

where \(m_D \equiv y_N v/\sqrt{2}\) and \(\alpha, \beta \in \{e, \tau\}\). The mass matrix \(\mathcal{M}_\nu\) is then diagonalized as \(D_\nu \equiv U_{\text{PMNS}} M_\nu U_{\text{PMNS}}^T\), where \(U_{\text{PMNS}}\) and \(D_\nu\) can be determined using the current neutrino oscillation data [14]. Note the here the lightest active neutrino is predicted to be massless because \(m_D\) is a \(2 \times 3\) matrix.

Without loss of generality, we work in the basis where all the coefficients in the scalar
potential (II.2) are real, and parameterize the SM scalar doublet as

$$H = \begin{pmatrix} w^+ \\ \frac{1}{\sqrt{2}}(v + h + iz) \end{pmatrix} \quad \text{with } v = 246 \text{ GeV},$$

(II.4)

where $w^+$ and $z$ are to be absorbed by the SM $W$ and $Z$ bosons, respectively. Moreover, we assume that the $S$ field does not develop a nonzero VEV. To stabilize the scalar potential and to have a global minimum given by Eq. (II.4), the quartic couplings should satisfy the following conditions [15]:

$$\lambda_S, \lambda_H, \lambda_{HS} > 0, \quad \text{and} \quad 2\sqrt{\lambda_H \lambda_S} < \lambda_{HS}. \quad \text{(II.5)}$$

B. Muon anomalous magnetic dipole moment

The interaction relevant to the muon $g - 2$ is

$$f_\mu \bar{\mu} \slashed{P} E'_\mu S + \text{H.c.} \quad \text{(II.6)}$$

With $S$ of mass $m_S$ and $E'_\mu$ of mass $M_{\mu'}$ running in the loop, we obtain

$$\Delta a_\mu = \frac{|f_\mu|^2}{8\pi^2} \int_0^1 dx \frac{x^2(1-x)}{x(x-1) + r_{\mu'} x + (1-x)r_S}, \quad \text{(II.7)}$$

where $r_{\mu'} \equiv (M_{\mu'}/m_\mu)^2$ and $r_S \equiv (m_S/m_\mu)^2$. To explain the current $3.3\sigma$ deviation [3]

$$\Delta a_\mu = (26.1 \pm 8.0) \times 10^{-10}, \quad \text{(II.8)}$$

the model has three degrees of freedom: $|f_\mu|$, $m_S$, and $M_{\mu'}$.  

C. Bosonic dark matter candidate

Stabilized by the $Z_2$ or $Z_3$ symmetry, the $S$ boson serves as a DM candidate. We first discuss the bounds coming from the spin independent scattering cross section reported by several direct detection experiments such as LUX [17], XENON1T [18], and PandaX-II [19], as in our model there is a Higgs portal contribution. We have checked that as long as

\footnote{For a comprehensive review on new physics models for the muon $g - 2$ anomaly as well as lepton flavor violation, please see Ref. [16].}
\(\lambda_{HS} + \lambda'_{HS} \lesssim \mathcal{O}(0.01)\), there is no constraint from direction detection. Therefore, we assume in this work that these quartic couplings are sufficiently small but satisfy Eq. (II.5).

The relevant terms for the relic density of the S boson are

\[
(\bar{\ell}_{\mu} P_R E'_{\mu} + \bar{\nu}_{\mu} P_R N'_{\mu}) S + \text{H.c.} , \tag{II.9}
\]

where the other \(f\) terms are assumed to be negligible in comparison with \(f_{\mu}\), as a larger value of \(f_{\mu}\) is required to obtain a sizable \(\Delta a_{\mu}\). Such interactions will lead to pair annihilation of the S bosons in the SM muons and muon neutrinos. To explicitly evaluate the relic abundance of S, one has to specify whether the S field is real or complex. For a real S we have both \(t\)- and \(u\)-channel annihilation processes that lead to a more suppressed \(d\)-wave cross section, while for a complex S, on the other hand, there is only the \(t\) channel that leads to a \(p\)-wave dominant cross section. The cross sections for the two scenarios are approximately given by

\[
(\sigma_{v_{\text{rel}}})(2S \rightarrow \mu \bar{\mu}) \approx \frac{|f_{\mu}|^4}{60\pi} \left(\frac{m_S^2 + M_{\mu'}^2}{m_S^2 + M_{\mu'}^2}\right)^4 v_{\text{rel}}^4, \quad \text{(Real S)} , \tag{II.10}
\]

\[
(\sigma_{v_{\text{rel}}})(SS^* \rightarrow \mu \bar{\mu}) \approx \frac{|f_{\mu}|^4}{96\pi} \left(\frac{m_S^2 + M_{\mu'}^2}{m_S^2 + M_{\mu'}^2}\right)^2 v_{\text{rel}}^2, \quad \text{(Complex S)} , \tag{II.11}
\]

in the limit of massless final-state leptons. Here the approximate formulas are obtained by expanding the cross sections in powers of the relative velocity \(v_{\text{rel}}\): \(\sigma_{v_{\text{rel}}} \approx a_{\text{eff}} + b_{\text{eff}} v_{\text{rel}}^2 + d_{\text{eff}} v_{\text{rel}}^4\). The resulting relic densities of the two scenarios are found to be

\[
\Omega h^2 \approx \frac{5.35 \times 10^7 x_f^3}{\sqrt{g_s(x_f) M_{\text{PL}} d_{\text{eff}}}}, \quad \text{(Real S)} , \tag{II.12}
\]

\[
\Omega h^2 \approx \frac{1.78 \times 10^8 x_f^2}{\sqrt{g_s(x_f) M_{\text{PL}} b_{\text{eff}}}}, \quad \text{(Complex S)} , \tag{II.13}
\]

respectively, where the present relic density is 0.1199 \pm 0.0054 at the 2\(\sigma\) confidential level (CL) \[12\], \(g_s(x_f \approx 25) \approx 100\) counts the degrees of freedom for relativistic particles, and \(M_{\text{PL}} \approx 1.22 \times 10^{19}\) GeV is the Planck mass.

Taking the central value of the relic density as an explicit example, the above formulas can be simplified to give:

\[
|f_{\mu}| \approx 0.057 \times \frac{m_S^2 + M_{\mu'}^2}{m_S^{3/2} \cdot \text{GeV}^{1/2}} \quad \text{(Real S)} \tag{II.14}
\]

\[
|f_{\mu}| \approx 0.033 \times \sqrt{\frac{m_S^2 + M_{\mu'}^2}{m_S \cdot \text{GeV}}} \quad \text{(Complex S)} \tag{II.15}
\]
for which one still has to impose the perturbativity upper bound of $\sqrt{4\pi}$. It is then straightforward to search for viable parameter space in the $(m_S, M_{\mu'})$ plane by combining Eq. (II.14) or (II.15) with Eq. (II.7).

### D. Lepton Flavor-Violating Processes

In this subsection, we consider LFV processes at one-loop level, as they can arise from the mixing between the electron and tau flavor eigenstates in this model. First, the mass matrix of the exotic charged lepton masses is given by

$$M_L' = \begin{pmatrix} M_e & M_{e\tau} \\ M_{\tau e} & M_{\tau} \end{pmatrix}. \tag{II.16}$$

The mass eigenvalues are obtained through a bi-unitary transformation on the left-handed and right-handed fields: $\text{diag}(M_{\ell'_L}, M_{\ell'_R}) = V^\dagger_{\ell'_L} M_L' V_{\ell'_R}$. Therefore, $\text{diag}(\left| M_{\ell'_L e} \right|^2, \left| M_{\ell'_L \mu} \right|^2) \equiv V^\dagger_{\ell'_L} M_L' M^\dagger_L V_{\ell'_L}$. Finally we find the following relation between the flavor and mass eigenstates:

$$\begin{pmatrix} \ell'_e \\ \ell'_\tau \end{pmatrix}_f \quad \begin{pmatrix} c_\alpha & -s_\alpha \\ s_\alpha & c_\alpha \end{pmatrix} \begin{pmatrix} \ell'_e \\ \ell'_\tau \end{pmatrix}_m, \tag{II.17}$$

where $s_\alpha \equiv \sin \alpha$ and $c_\alpha \equiv \cos \alpha$. In the following discussions, we will always refer to the mass eigenstates. In the mass eigenbasis, the relevant interactions are

$$f'_{ee} \bar{\ell}_e P R \ell'_e S + f'_{e\tau} \bar{\ell}_e P R \ell'_\tau S + f'_{\tau e} \bar{\ell}_\tau P R \ell'_e S + f'_{\tau\tau} \bar{\ell}_\tau P R \ell'_\tau S + \text{H.c.}, \tag{II.18}$$

where $f'_{ee} = s_\alpha f_{e\tau} + c_\alpha f_e$, $f'_{e\tau} = c_\alpha f_{e\tau} - s_\alpha f_e$, $f'_{\tau e} = c_\alpha f_{\tau e} + s_\alpha f_\tau$, and $f'_{\tau\tau} = c_\alpha f_{\tau\tau} - s_\alpha f_{\tau e}$.

**Two-body decays:** Because of the feature that the mixing does not involve the muon, we here consider the constraints from the $\tau \to e\gamma$ and $Z \to \tau e$ decays. The relevant branching ratio formulas for the two modes can be lifted from Ref. [20]. First, we have

$$\text{BR}(\tau \to e\gamma) \approx \frac{0.1784 \alpha_{em}}{768\pi G_F^2} \left| \sum_{a=e,\tau} f'_{ea} f'^\dagger_{a\tau} \frac{2m_S^6 + 3m_S^4 M_{e'a}^2 - 6m_S^2 M_{e'a}^4 + M_{e'a}^6 + 6m_S^4 M_{e'a}^2 \ln \frac{M_{e'a}^2}{m_S^2}}{(m_S^2 - M_{e'a}^2)^4} \right|^2, \tag{II.19}$$
where \( \alpha_{em} \approx 1/134 \) \([21]\) is the fine structure constant at the \( m_\tau \) scale, and \( G_F \approx 1.17 \times 10^{-5} \text{ GeV}^{-2} \) is the Fermi decay constant \([22]\). We also obtain

\[
\text{BR}(Z \to \tau e) = \frac{G_F}{3\sqrt{2\pi}} \frac{m_Z^3}{16\pi^2 \Gamma_Z} \left( s_w^2 - \frac{1}{2} \right)^2 \left| \sum_{a=e,\tau} f_{ea} f_{at}^\dagger \left[ F_2(\ell'_i; S) + F_3(\ell'_i; S) \right] \right|^2 ,
\]

(II.20)

where

\[
F_2(a, b) = \int_0^1 dx (1 - x) \ln \left[ (1 - x)M^2_a + xm^2_b \right] ,
\]

\[
F_3(a, b) = \int_0^1 dx \int_0^{1-x} dy (xy - 1)m^2_\Delta + (M^2_a - m^2_b)(1 - x - y) - \Delta \ln \Delta ,
\]

with \( \Delta \equiv -xym^2_Z + (x + y)(M^2_a - m^2_b) + m^2_b \) and the total \( Z \) decay width \( \Gamma^\text{tot}_Z = (2.4952 \pm 0.0023) \text{ GeV} \) \([23]\). It is noted that the combination \( \sum_{a=e,\tau} f_{ea} f_{at}^\dagger \) appear in both Eqs. (II.19) and (II.20), showing the correlation between the two observables in this model. The current upper bounds on \( \text{BR}(\tau \to e\gamma) \) and \( \text{BR}(Z \to \tau e) \) are found to be \([23]\):

\[
\text{BR}(\tau \to e\gamma) \lesssim 3.3 \times 10^{-8} \quad \text{and} \quad \text{BR}(Z \to \tau e) < 9.8 \times 10^{-6}
\]

(II.21)

at 90% CL and 95% CL, respectively.

**Three-body decays:** In our case, we consider the \( \tau \to e\mu\bar{\mu} \) decay due to the muon-specific interaction structure. \(^5\) In the approximation of heavy exotic leptons, the effective Hamiltonian for the decay is obtained from a box diagram to be

\[
\mathcal{H}_{\text{eff}}(\tau \to e\mu\bar{\mu}) = \frac{|f_{\mu\bar{\mu}}|^2}{(4\pi)^2} \sum_{i=e,\tau} (f^f_{ei} f^f_{it}) G_{\text{box}}(m_{S}, M_{\ell'_i}, M_{\ell'})(\bar{\ell}_\tau \gamma^\rho P_L \ell_e)(\bar{\ell}_\mu \gamma_\mu P_L \ell_\mu) + \text{c.c.}
\]

\[
\equiv C_{\tau \to e\mu\bar{\mu}}(\bar{\ell}_\tau \gamma^\rho P_L \ell_e)(\bar{\ell}_\mu \gamma_\mu P_L \ell_\mu) + \text{H.c.} ,
\]

(II.22)

where \( C_{\tau \to e\mu\bar{\mu}} \) has the dimension of mass squared. The branching ratio is then found to be \([24]\)

\[
\text{BR}(\tau \to e\mu\bar{\mu}) \approx \frac{m_\tau^5}{1526\pi^3 \Gamma_\tau} |C_{\tau \to e\mu\bar{\mu}}|^2 ,
\]

(II.23)

where \( \Gamma_\tau \approx 2.27 \times 10^{-12} \text{ GeV} \) is the total decay rate of the tau lepton, and should be smaller than the upper bound of \( 2.7 \times 10^{-8} \) at the 90% CL \([23]\).

\(^5\) The constraint from the \( \tau \to eee \) decay is weaker.
E. Global analysis

To perform a global analysis of the model, we require that both \( \Delta a_\mu \) and the DM relic density fall within the 1\( \sigma \) range of the measured data and that the LFV processes satisfy their respective upper bounds quoted above. In addition, we restrict ourselves to the regions that the couplings \( f' \in [0.001, 1] \) and the masses

\[
m_S \in [1, 400] \text{ GeV}, \quad M_{\mu'} \in [100, 500] \text{ GeV}, \quad (M_\ell') \in [1.2M_X, 2 \text{ TeV}], \quad (II.24)
\]

where \( 1.2m_S \leq M_{\mu'} \) is imposed to prevent the possibility of co-annihilation as well as the stability of \( S \).

Fig. I shows the allowed parameter space in the \((m_S, M_{\mu'})\) plane by scanning all the other parameters. The left (right) plot is for the scenario where \( S \) is a real (complex) scalar boson. In both plots and on top of the LFV constraints, the orange (green) dots further satisfy the current \( \Delta a_\mu \) (the current \( \Omega h^2 \)) value at the 1\( \sigma \) level. In these scatter dots, only the blue ones are allowed by all of the constraints. The left shows that the real \( S \) scenario favors the parameter space of \( 70 \text{ GeV} \lesssim m_S \lesssim 185 \text{ GeV} \) and \( 100 \text{ GeV} \lesssim M_{\mu'} \lesssim 350 \text{ GeV} \). In contrast, the right plot shows that the complex \( S \) boson is preferred to have a small mass, \( 7 \text{ GeV} \lesssim m_S \lesssim 14 \text{ GeV} \) while \( 100 \text{ GeV} \lesssim M_{\mu'} \lesssim 400 \text{ GeV} \). Such different behaviors in \( m_S \) between the two scenarios are rooted in the \( d \)- and \( p \)-wave scattering cross sections given in Eqs. (II.10) and (II.11).
In Fig. 2 we show scatter plots in the \((m_S, |f_\mu|)\) plane in the same style as in Fig. 1. The left plot shows that the allowed range of \(|f_\mu|\) is \(1.3 \lesssim |f_\mu| \lesssim \sqrt{4\pi}\), while the right plot has \(0.9 \lesssim |f_\mu| \lesssim \sqrt{4\pi}\), with \(\sqrt{4\pi}\) being the limit of perturbativity.

In Fig. 3 we show the distribution of \(\text{BR}(\tau \to e\gamma)\) and \(\text{BR}(Z \to e\tau)\) according to our global scan. Note that this result is independent of whether the \(S\) boson is real or complex. The upper bound on \(\text{BR}(Z \to e\tau)\) comes from the same structure of Yukawa combination \((f'f'^\dagger)_{e\tau}\) as shown in Eqs. (II.19) and (II.20). Since both of \(\text{BR}(\tau \to e\gamma)\) and \(\text{BR}(Z \to e\tau)\) can reach up to their current experimental bounds, they can be tested in the near future.
III. EXTENSION TO QUARK SECTOR

In view of the recent anomalies in B physics, we extend the model to have three families of vector-like exotic quarks $Q'$ that are $SU(2)_L$ doublets. However, the $S$ field has to be complex. Note also that one is not allowed to introduce an additional global symmetry similar to $U(1)_\mu$ above because the quark mixing or quark masses cannot be reproduced. The relevant Lagrangian for the quark sector is then given by

$$-L_Q = y_{u_{ij}} \bar{Q}_i \tilde{H} P_R u_R j + y_{d_{ij}} \bar{Q}_i \tilde{H} P_R d_R j + g_{ij} \bar{Q}_i P_R Q'_j S + M_{Q'_i} \bar{Q}'_i Q'_i + \text{H.c.} ,$$

where $Q' \equiv [u', d']^T$, $i, j \in \{u, c, t\}$ ($\{d, s, b\}$) for the up-quark (down-quark) sector. The first two terms are the same as the ones in the SM, while the third term is a new interaction that is important to the phenomenology discussions below. Here we take the mass matrix of $Q'$s to be flavor-diagonal. For the subsequent discussions, the relevant interactions in the mass eigenbasis are:

$$f_{\mu}\bar{\ell}_\mu P_R \ell'_\mu S + f'_{ij}\bar{u}_i P_R u'_j + \bar{d}_i P_R d'_j S + \text{H.c.} .$$

A. $B \to K^*\ell\ell$ anomalies

First, the effective Hamiltonian for the $b \to s\mu^+\mu^-$ transition induced by the operators in Eq. (III.2) through box diagrams is

$$\mathcal{H}_{\text{eff}}(b \to s\bar{\mu}\mu) = \sum_{\alpha=d,s,b} \left( g_{\alpha a} g_{\alpha b}^\dagger \right) \frac{|f_{\mu}|^2}{(4\pi)^2} G_{box}(m_S, M_{Q'_a}, M_{\mu'}) (\bar{s}\gamma^\rho P_L b)(\bar{\ell}_\mu \gamma_\rho \ell_\mu - \bar{\ell}_\mu \gamma_\rho \gamma_5 \ell_\mu) + \text{H.c.} ,$$

$$\equiv -C_{SM} \left[ C^{ab}_{09}(O_9)_{ab} - C^{ab}_{10}(O_{10})_{ab} \right] + \text{H.c.} ,$$

with $G_{box}(m_S, M_{Q'_a}, M_{\mu'}) \approx \frac{1}{2} \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \frac{x_1}{x_1 m_S^2 + x_2 M_{Q'_a}^2 + (1 - x_1 - x_2) M_{\mu'}^2}$;

where $C_{SM} \equiv \frac{V_{tb} V_{ts}^* G_{F\alpha\beta}}{\sqrt{2}}$, $V_{tb} \sim 0.9991$ and $V_{ts} \sim -0.0403$ are the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements. Remarkably, we have $C^{ab}_{09} = -C^{ab}_{10}$ for the new physics contribution, which is one of the preferred schemes to explain the $B$ anomalies.

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6 With a real singlet $S$, it is impossible to explain the $b \to s\ell^+\ell^-$ anomalies because of a cancellation between diagrams.

7 Although there exist penguin diagrams, they are subdominant because of the strong constraint from the $b \to s\gamma$ decay.
We also find that bounds from BR(B\(\rightarrow Z\)) are stronger than the BR(\(\rightarrow e^+ e^-\)) constraint. Moreover, the interactions in Eq. (III.2) also lead to

\[
H_{\text{eff}}^{\alpha} = \sum_{\beta = d, s, b} \frac{(g_{d,\beta}^\alpha g_{\beta b}^\alpha)^2}{(4\pi)^2} G_{\text{box}}(m_S, M_{Q'}, M_{\mu'}) (\bar{d}_\alpha \gamma_\mu P_L b) (\bar{\ell}_\mu \gamma^\mu P_L \ell_\mu) + \text{H.c.}
\]

\[
\equiv -C_{LL}^{d\beta} (\bar{d}_\alpha \gamma_\mu P_L b) (\bar{\ell}_\mu \gamma^\mu P_L \ell_\mu) + \text{H.c.}
\] (III.4)

for \(d_\alpha \in \{d, s\}\). Therefore, the parameters have to satisfy the constraints from the data or upper bound of BR(B\(d/s \rightarrow \mu^+\mu^-\)) reported by CMS [27] and LHCb [28]. The bounds on the coefficients \(|C_{LL}^{s\beta(d\beta)}|\) in the above effective Hamiltonian are given by [29]

\[
|C_{LL}^{s\beta(d\beta)}| \lessapprox 5 (3.9) \times 10^{-9} \text{ GeV}^{-2}.
\] (III.5)

We also find that bounds from BR(B\(s(d) \rightarrow e^+ e^-\)) and BR(B\(s(d) \rightarrow \tau^+\tau^-\)), that are proportional to \(|f|\)^2, turn out to be weaker than the BR(Z \(\rightarrow e\tau\)) constraint.

### B. Neutral meson mixing

The operators in Eq. (III.2) also contribute to neutral meson mixing at low energies. Therefore, the couplings \(g'\) and masses \(M_{Q'}\) are strongly constrained by the measured data. It is straightforward to obtain the appropriate effective Hamiltonian for meson mixing by the replacements \(f' \rightarrow g\) and \(M_{\mu'} \rightarrow M_{Q'}\) in Eq. (III.3). The mass splitting between neutral mesons \(M\) and \(\bar{M}\) is then

\[
\Delta M_M = \frac{2m_M f_M^2}{3(4\pi)^2} \sum_{i, j = d, s, b} (g_{j\beta} g_{i\alpha}^\dagger) (g_{j\beta} g_{j\beta}) G_{\text{box}}(m_S, M_{Q'}, M_{Q'}) .
\] (III.6)

Here we take into account the \(K^0 - \bar{K}^0\), \(B_d - \bar{B}_d\), and \(B_s - \bar{B}_s\) mixings [30]:

\[
\Delta m_K : \sum_{i, j = d, s, b} (g_{i\beta} g_{i\beta}) (g_{j\beta} g_{j\beta}) (G_{\text{box}})_{ij} \lessapprox 3.48 \times 10^{-15} \times \frac{24\pi^2}{m_K f_K^2} \text{ GeV} ,
\] (III.7)

\[
\Delta m_{B_d} : \sum_{i, j = d, s, b} (g_{i\beta} g_{i\beta}) (g_{j\beta} g_{j\beta}) (G_{\text{box}})_{ij} \lessapprox 3.34 \times 10^{-13} \times \frac{24\pi^2}{m_{B_d} f_{B_d}^2} \text{ GeV} ,
\] (III.8)

\[
\Delta m_{B_s} : \sum_{i, j = d, s, b} (g_{i\beta} g_{i\beta}) (g_{j\beta} g_{j\beta}) (G_{\text{box}})_{ij} \lessapprox 1.17 \times 10^{-11} \times \frac{24\pi^2}{m_{B_s} f_{B_s}^2} \text{ GeV} ,
\] (III.9)

\(^8\) The constraint from \(\Delta m_D\) is weaker than that of \(\Delta m_K\).
FIG. 4: Global analysis in terms of $|f_\mu|$ and $-C_9^{\mu\mu}$, satisfying all the constraints discussed above. Note here that only the complex $S$ provides non-vanishing $-C_9^{\mu\mu}$. The center black horizontal line of the green region represents the best fit value of the expected anomaly of $b \to s\bar{\mu}\mu$, while the green region is at 1$\sigma$ CL, and the red one is at 3$\sigma$ CL of the current experimental bound \[11\]. The allowed region lies on whole the range of $|f_\mu|$ that explains the muon anomalous magnetic moment, since we can control another parameter $g$ in Eq. (III.3), which is not strongly restricted by experiments such as $B_d/s \to \mu^+\mu^-$, $M - \bar{M}$ mixing.

where $(G_{box})_{ij} \equiv G_{box}(m_S, M_{Q'}, M_{Q'})$. The other parameters are also found to be $f_K = 0.160$ GeV, $f_{B_d(B_s)} = 0.200$ GeV \[30\], $m_K = 0.498$ GeV, and $m_{B_d(B_s)} = 5.280$ (5.367) GeV \[23\]. One finds that these constraints are not generally so stringent. When $g_{ij} = 0.1$ is taken universally, for example, all the bounds are always satisfied with the most stringent bound coming from $\Delta m_K$.

Here we analyze whether there is any parameter space in the allowed region for a complex scalar (7 GeV $\lesssim m_S \lesssim$ 14 GeV, 100 GeV $\lesssim M_{\mu'} \lesssim$ 400 GeV, and $0.9 \lesssim |f_\mu| \lesssim \sqrt{4\pi}$ found in the previous section) that can satisfy the $B$ anomalies. In Fig. 4, we show the a scatter plot in the $(|f_\mu|, -C_9^{\mu\mu})$ plane, where we have selected the input parameters: $g \in [0.001, \sqrt{4\pi}]$ and $M_{Q'} \in [500, 2000]$ GeV. The central black horizontal line represents the best fit value of $-C_9^{\mu\mu}$, and the green (red) region is the 1$\sigma$ (3$\sigma$) range \[11\]. It is seen that through a simple extension to the quark sector in a way analogous to the lepton sector, the model can readily accommodate the $B$ anomalies as well.
IV. CONCLUSIONS

We have proposed a model with muon-specific interactions, with the intent to explain the muon anomalous magnetic dipole moment and the dark matter relic density. In the model, we impose a global $U(1)_\mu \times Z_N$ symmetry, and introduce exotic lepton iso-doublets, right-handed Majorana fermions, and an inert scalar iso-singlet in addition to the SM field contents. In the case of a real (complex) scalar boson, we take $N = 2$ ($N = 3$ as a simplest choice). Leptons in the second family and the corresponding exotic leptons are charged under the $U(1)_\mu$ symmetry. All exotic lepton doublets and the inert scalar field have nontrivial $Z_N$ charges.

As a result of such an extension, the model features a good DM candidate and the capacity to accommodate $\Delta a_\mu$. We have studied both scenarios of real and complex $S$ as the weakly interacting massive particle DM. Through a comprehensive scan by also including constraints from lepton flavor violating processes, we have obtained the following allowed parameter space:

\[ 70 \text{ GeV} \lesssim m_S \lesssim 185 \text{ GeV}, \quad 1.3 \lesssim |f_\mu| \lesssim \sqrt{4\pi}, \quad 100 \text{ GeV} \lesssim M_{\mu'} \lesssim 350 \text{ GeV}, \quad \text{(Real S)}, \]
\[ 7 \text{ GeV} \lesssim m_S \lesssim 14 \text{ GeV}, \quad 0.9 \lesssim |f_\mu| \lesssim \sqrt{4\pi}, \quad 100 \text{ GeV} \lesssim M_{\mu'} \lesssim 400 \text{ GeV}, \quad \text{(Complex S)}. \]

In the above space, we have also found some correlation between the predictions of $\text{BR}(\tau \to e\gamma)$ and $\text{BR}(Z \to e\tau)$ due to the same Yukawa structure $(f'f'^\dagger)_{e\tau}$ involved in them. Moreover, some of the predicted values are already approaching the current upper bounds, and can be tested in near future.

In view of the recent $B$ anomalies, we have also extended the model to the quark sector in a way analogous to the lepton sector, except that no additional global symmetry needs to be introduced. We have found that the preferred Wilson coefficients in the effective Hamiltonian of the $b \to s\mu^+\mu^-$ transitions can be readily obtained while being consistent with constraints from, for example, the $B_{s(d)} \to \mu^+\mu^-$ decay and neutral meson mixings. In particular, the scenario of a real $S$ cannot explain the anomalies due to a cancellation between new physics contributions. The scalar therefore has to be complex and has a mass of $\mathcal{O}(10)$ GeV.
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