More Superstrings from Supergravity

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Abstract

The four six-dimensional “little string” theories are all described in the infinite momentum frame (IMF) as matrix theories by non-trivial 1+1 dimensional infra-red fixed points. We characterize these fixed points using supergravity. Starting from the matrix theory definition of M5–branes, we derive an associated dual supergravity description of the fixed point theories, arising as the near horizon geometry of certain brane configurations. These supergravity solutions are all smooth, and involve three dimensional Anti–de Sitter space $AdS_3$. They therefore provide a complete description of the fixed point theories, and hence the IMF little string theories, if the AdS/CFT correspondence holds.

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1. Introduction and Summary

1.1. Motivations

Recently[1], it has been shown that all of the ten dimensional superstring theories, described in the infinite momentum frame (IMF) by the “matrix string” description, have a similar qualitative structure in the region of weak string coupling:

• At weak coupling they are all described by 1+1 dimensional infra–red fixed point theories which are essentially trivial orbifold conformal field theories. These theories may be described as the flow from an effective 1+1 dimensional field theory: the obvious matrix extension of the relevant Green–Schwarz action, whose prototype was discussed in this framework in ref.[2].

• In the same limit, there is an approximate supergravity description, dual (or nearly so) in the sense of ref.[3,4] which is simply the near horizon geometry of the fundamental string solution of a species T–dual to the matrix string in question.

• The neighbourhood of the core of the supergravity solution corresponds, via the duality map, to the weak matrix string coupling limit. In the limit, the flow to the trivial fixed point (describing the free matrix string) moves one to the center of the supergravity solution, where the curvature diverges, and the dual description breaks down, as it should.

Far away from weak coupling, the matrix descriptions of the strings cease to all resemble one another, and become either 0+1 dimensional (for the type IIA or $E_8 \times E_8$ heterotic systems) or 2+1 dimensional (for the type IIB or $SO(32)$ type I/heterotic systems). This is of course consistent with the fact that the very strong coupling limits of all of the strings are somewhat different from each other, according to string duality: The first two are dual to eleven dimensional supergravity, while the latter are dual to ten dimensional string theories.

In the case of the latter class, the natural description of the theory at intermediate coupling is a 2+1 dimensional interacting fixed point theory. The theory has a supergravity dual described as eleven dimensional supergravity compactified on $AdS_4 \times S^7$, for the type IIB system, or an orbifold $AdS_4 / \mathbb{Z}_2 \times S^7$ for the $SO(32)$ system. The isometries of the compactification translate into the superconformal symmetries and R–symmetries of the 2+1 dimensional conformal field theory living at the boundary of $AdS_4$.

Matrix string theory is a useful alternative way of defining and characterizing string theories. In the case of ten dimensions we now have a complete understanding of the overall structure of these theories, and a good understanding of when we can expect a dual supergravity description to help in studying the defining field theory.

There arises the obvious question: What is the analogous story for the more newly discovered class of superstring[5,6,2,7,8] theories, the ones which live in six dimensions? These theories have certain properties which make it interesting to begin answering the question.
In the light of what was learned for the ten dimensional theories, we can anticipate some of the structure of the matrix–string–via–supergravity description for the six dimensional strings:

• The strings all seem to be most naturally defined at intermediate coupling. This is believed to follow from the fact that they are self-dual objects, naturally coupled electrically and magnetically to a three–form field strength $H^{(3)}$. For this to be true, their coupling is frozen at some value of the coupling of order one.

• This means that the strings are always interacting, and therefore we should not expect that the matrix string theory will involve a trivial orbifold conformal field theory. Instead, there will be some non–trivial interacting theory. This is already known to be true for the $(0,2)$ or “type iia” theory. We will see that it is indeed true for all of the theories.

• We should expect further that there should exist a supergravity dual description of the theories. This dual will be complete in the sense that there will be no curvature singularities in the solution, giving us a complete dual theory. For the $(0,2)$ theory, the relevant fixed point is conjectured\[3\] to be dual to type IIB supergravity compactified on $AdS_3 \times S^3 \times T^4$. We will see that in every case, the $AdS_3 \times S^3$ space will arise as the dual, although (of course) the supergravity will be different in each case.

We see therefore that the structure of the matrix string definition, or equivalently, the supergravity origin of all of the (IMF) six dimensional string theories is rather simple compared to the ten dimensional theories, precisely because they prefer not to be defined at weak coupling.

1.2. Summary of Results

• Using the defining matrix theory of longitudinal M5–branes\[3\], and following the appropriate limits, we observe that the matrix strings are all defined in terms of 1+1 dimensional interacting fixed points. (This was already observed for the $(0,2)$ little string\[7,10,11\].)

• The limits which define the matrix string theories also define certain supergravity backgrounds, which can be interpreted as “dual” descriptions in the sense of ref.\[3\]. The dual descriptions are all smooth:
  
  ◦ The $(0,2)$ theory is given\[3\] by type IIB supergravity on $AdS_3 \times S^3 \times T^4$.
  ◦ The $(1,1)$ theory comes from type IIA supergravity on $AdS_3 \times S^3 \times T^4$.
  ◦ The $(0,1)$ $E_8 \times E_8$ theory is defined by $SO(32)$ heterotic supergravity on $AdS_3 \times S^3 \times T^4$, or alternatively, type IIA supergravity on $AdS_3 \times S^3 \times K3$.
  ◦ The $(0,1)$ $SO(32)$ theory is defined by $E_8 \times E_8$ heterotic supergravity on $AdS_3 \times S^3 \times T^4$.

• In all cases therefore, there is the appropriate $SO(2,2)$ bosonic component of the superconformal symmetry and $SO(4)$ R–symmetry. The supersymmetry of the relevant
supergravity supplies the appropriate fermionic extension. The R–symmetry has the dual interpretation as the Lorentz group in this light–cone definition of the little strings.

- In the two (0,1) cases, the extra $SO(32)$ or $E_8 \times E_8$ global symmetries of the little heterotic string theories arise as global symmetries of their defining fixed point theories. These in turn come from the fact that the supergravity compactification will produce a gauge symmetry in $AdS_3$ in each case. The AdS/CFT correspondence then demotes this gauge symmetry to a global symmetry of the boundary theory in a similar way to what happens for the Kaluza–Klein gauge symmetries arising from isometries of the $S^3$.

2. The case of type iia

We start with the matrix theory definition of M–theory in the infinite momentum frame (IMF). It is given by the $\mathcal{N}=16$ supersymmetric $U(N)$ quantum mechanics arising from $N$ coincident D0–branes’ world–volume, in the limit $\ell_s \rightarrow 0$ and $N \rightarrow \infty$. The special longitudinal direction, $x^{10}$, (initially compactified on a circle of radius $R_{10}$), is decompactified in the limit also. The type IIA string theory used to define this theory has parameters:

$$g_{\text{IIA}} = R_{10}^{3/2} \ell_p^{-3/2}, \quad \ell_s = \ell_p^{3/2} R_{10}^{-1/2},$$

(2.1)

where $\ell_s$ is the string length and $\ell_p$ is the eleven dimensional Planck length.

Our ultimate goal is to construct the six dimensional (0,2) interacting string theory living on the world volume of a collection of NS–fivebranes of the type IIA theory. Such branes originate from M–theory as M5–branes, transverse to the circle which shrinks to give the type IIA string. Such branes are placed in the matrix theory by adding hypermultiplets to the quantum mechanics, a procedure which is really adding D4–branes to the $N$ D0–brane system in the defining type IIA theory. Let us add $M$ such D4–branes, oriented along the directions $x^1, \ldots, x^4$. We need to tune this hypermultiplet theory into its Higgs branch, which is to say we dissolve the D0–branes into the D4–branes, endowing them with $N$ units of D0–brane charge.

This system therefore defines $M$ M5–branes oriented along $x^1, \ldots, x^4, x^{10}$, with $N$ units of momentum in the $x^{10}$ direction. Following the usual matrix string procedure, we may now imagine that the momentum is actually along the $x^5$ direction and shrink that direction to get a definition of the resulting type IIA system. In doing so, we arrive at an economical description of the system by $T_5$–dualizing the defining type IIA system, giving a type IIB string theory configuration consisting of $M$ D5–branes with $N$ D1–branes (or a single D1–brane wound $N$ times on $\hat{x}^5$, the dual direction).

The 1+1 dimensional Yang–Mills coupling on the D1–branes’ world–volume is given by: $1/g^2_{\text{YM}} = \ell_s^2/g_{\text{IIB}} = \ell_p^2 R_5/R_{10}$. As the radius $R_5$ shrinks to zero, the ten dimensional type IIB string coupling gets very large. We have a weakly coupled description in terms of the
S–dual system of $M$ F5–branes (a shorter term for NS–fivebranes) with $N$ F1–branes (fundamental type IIB strings) inside their world volume. We shall sometimes think of this as one F1–brane with $N$ units of winding in the $\hat{x}^5$ direction.

After $T_5$–dualizing again, we obtain a type IIA system of $M$ F5–branes with F1–branes (fundamental type IIA strings this time) inside their world–volume with $N$ units of momentum in $x^5$.

This chain of dualities is similar to the chain of reasoning which defines the matrix (IMF) ten dimensional type IIA string[2]: There, the defining lagrangian came from a system of D1–branes with $N$ units of winding. This was S–dual to a system of wound type IIB F1–branes. A T–duality on the winding direction gave the type IIA string (F1–brane) with $N$ units of momentum. The Fock space of the IMF matrix string was built up from these winding type IIB strings, and the explicit description at all couplings was given in terms of the D1–brane system, which is a 1+1 dimensional Yang–Mills theory: a matrix–valued type IIA Green–Schwarz action. At weak coupling, the target space of the theory (moduli space of the 1+1 dimensional Yang–Mills theory) is simply $S^N(\mathbb{R}^8)\equiv(\mathbb{R}^8)^N/S_N$, where $S_N$ is the group of permutations of $N$ objects, the D1–branes, and $\mathbb{R}^8$ is the space allowed D1–brane positions, the permitted values of the (in general matrix–valued) bosonic fields of the Yang–Mills theory. This is an orbifold theory. As shown in ref.[14,13,2], the twisted sectors of this orbifold describe long type IIA strings which can survive in the large $N$ limit to define strings with finite momentum in the IMF direction[2].

The same thing happens here. There is a non–trivial interacting theory living on the type IIA F5–brane’s world–volume even as we take the limit $g_{\text{IIA}}\rightarrow 0$, as argued in ref.[3]. The “little strings” (sometimes called[7,15] “microstrings”) which carry the basic degrees of freedom of the theory are described by the 1+1 dimensional theory we have defined. It is a 1+1 dimensional theory derived from the D1–branes’ + wrapped $2$ D5–branes’ world–volume. The theory has $M$ hypermultiplets in the fundamental of $U(N)$ and it has been tuned to its Higgs branch. In other words, as instantons of the D5–brane’s $SU(M)$ gauge theory, the D1–branes are far from the point–like limit[16] and are instead fat instantons, having finite scale size. They are delocalized inside the D5–branes, in the directions $\{x^1, \ldots, x^4\}$.

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1 Ref.[2] went on to show how to switch on interactions in this second quantized definition of the type IIA string.
2 We will generically think of the five–branes as wrapped on a $T^4$, transverse to the strings. Therefore they also contribute to the stringy 1+1 dimensional model. $N$ and $M$ are now on similar footing, and our earlier identification of $N$ as the momentum of the little strings is modified by $M$, as wrapping induces some more D1–brane charge proportional to $M$. The schematics of the discussion is correct for motivational purposes, as the reasoning in this paper will not need the full discussion of the momentum[2,4,5].
Furthermore, we are interested in the zero coupling limit of the final type IIA theory and so we should take the strong coupling limit of this configuration. The 1+1 dimensional Yang–Mills theory is therefore strongly coupled in the limit that we want, and it flows to the infra–red. The resulting infra–red fixed point defines for us the matrix \((0, 2)\) “little string theory”.

The target space of this theory is a hyperKähler deformation of \(S^{NM}(T^4)\). There has been much work devoted to this theory in the literature.

2.1. The role of Type IIB Supergravity

Notice that like the ten dimensional case, we are led to describe the long strings in the theory as winding type IIB F–strings. These long strings arise in the large \(N\) limit, which we must take to properly define the original matrix M–theory, and here in order to obtain the light cone type IIA string theory.

In this large \(N\) limit, we must take seriously the supergravity fields generated by the D1–brane configuration. If we take large \(M\) also, we can fully describe the supergravity fields with a metric valid for low curvature everywhere:

\[
\begin{align*}
\text{ds}^2 &= \left(1 + \frac{g_{\text{IIB}} \ell_s^2 N}{v r^2}\right)^{-1/2} \left(1 + \frac{g_{\text{IIB}} \ell_s^2 M}{r^2}\right)^{-1/2} \left[(-dt^2 + dx_5^2) + \left(1 + \frac{g_{\text{IIB}} \ell_s^2 M}{r^2}\right) \sum_{i=1}^{4} dx_i^2\right] \\
&+ \left(1 + \frac{g_{\text{IIB}} \ell_s^2 N}{v r^2}\right)^{1/2} \left(1 + \frac{g_{\text{IIB}} \ell_s^2 M}{r^2}\right)^{1/2} (dr^2 + r^2 d\Omega_3^2).
\end{align*}
\]

(Here, \(v\) is a dimensionless measure of the volume of the \(T^4\) on which the D5–brane is wrapped.) In the limit, this solution becomes simply \(AdS_3 \times S^3 \times T^4\), where the radius of the first two factors is set by the product \(MN\), and the latter by \(M/N\).

\[
\text{ds}^2 \sim \ell_s^2 \sqrt{NM} \left(u^2 (-dt^2 + dx_5^2) + \frac{du^2}{u^2} + d\Omega_3^2\right) + \sqrt{\frac{M}{N}} \sum_{i=1}^{4} dx_i^2.
\]

We have absorbed some inessential constants into \(r\) and then set \(u = r/\ell_s^2\). This theory is conjectured to be the dual of the 1+1 dimensional \((4, 4)\) superconformal field theory we are interested in the strong coupling IR limit. (The string coupling has also diverged in the limit, and we need to consider the S–dual configuration for weak coupling. This is same metric for that S–dual IIB supergravity solution, however, as the dilaton is constant in this limit. This structure will persist in the later discussions too.) The theory lives at the “boundary” of the \(AdS_3\). The required \(SO(2, 2)\) superconformal algebra with its \(SO(4)\) R–symmetry arises from the isometries of the \(AdS_3\) and the \(S^3\) respectively. This theory has been studied from this point of view recently in refs.
This AdS/CFT correspondence is conjectured to be the full description of the non-trivial conformal field theory. In this sense, the \( (0,2) \) “little string” theory (in the infinite momentum frame) has a supergravity origin.

3. The case of type iib

There is a little string theory living on \( F5 \)-branes of type IIB string theory as well. We should try to characterize it also.

Starting again with our matrix definition of \( M5 \)-branes, we may proceed to descend to type IIB theory, with its \( F5 \)-branes by compactifying on an additional circle, \( x^4 \), in addition to the one which we shrunk to get the type IIA theory. We are shrinking M–theory on a torus, and therefore should obtain the type IIB theory\(^28,29\). The extra detail of the \( M \) \( D4 \)-branes in our defining type IIA theory should be interesting.

In doing so, we obtain, after \( T_{45} \)-dualizing, a type IIA string theory again with \( M \) \( D4 \)-branes located now in \( \{x^1, x^2, x^3, \hat{x}^4, \hat{x}^5\} \) with \( N \) \( D2 \)-branes in \( \{\hat{x}^4, \hat{x}^5\} \). These \( D2 \)-branes are delocalized inside the \( D4 \)-branes, as before. They are not infinite in the \( \hat{x}^4 \) directions, as the \( D4 \)-branes are pointlike there, and so they end on them. We get the directions \( \{\hat{x}^4, \hat{x}^5\} \) directions both decompactified when we define the resulting type IIB theory at intermediate coupling. To get a weakly coupled type IIB string theory, we would let \( \hat{x}^5 \) grow faster than \( \hat{x}^4 \). In a frame where we fix \( \hat{x}^5 \) large, we see that the \( \hat{x}^4 \) direction shrinks away. We shall do this presently.

The effective gauge coupling of the 1+1 dimensional theory living on the part of the \( D2 \)-brane is given by: \( 1/g_{YM}^2=\ell_s/\tilde{g}_{IIA}=R_4R_5/R_{10} \). For small \( R_4, R_5 \), both the dual type IIA string theory coupling \( \tilde{g}_{IIA} \) and the Yang–Mills coupling \( g_{YM} \) are large. This means that we should consider our configuration as an M–theory configuration: eleven dimensional supergravity with branes. The \( M \) \( D4 \)-branes become \( M \) \( M5 \)-branes, now stretched along \( \{x^1, x^2, x^3, \hat{x}^5, x^{10}\} \), while the \( N \) \( M2 \)-branes are stretched along \( \{\hat{x}^5, \hat{x}^4\} \). They end on the \( M5 \)-branes, and are delocalized inside them.

The weakly coupled type IIB string limit, with \( M \) \( F5 \)-branes, is described by taking \( \hat{x}^4 \rightarrow 0 \), giving \( M \) \( F5 \)-branes in type IIA with fundamental \( \hat{x}^5 \)-wound F–strings inside; \( T_5 \)-duality completes the route to the type IIB system with F–strings possessing momentum in \( x^5 \). Requiring weakly coupled type IIB therefore focuses the discussion on \( M5 \)-brane worldvolume. The leg of the \( M2 \)-branes not inside the \( M5 \)-branes becomes less important to the discussion and the physics of the effective string inside the \( M5 \)-branes’ worldvolume dominates. This 1+1 dimensional theory therefore describes whatever interacting theory there is on the \( F5 \)-brane at weak type IIB string coupling.
3.1. The role of Eleven Dimensional Supergravity

Furthermore, the large $N, M$ limit allows us to discuss the system in terms of the supergravity solution:

$$ds^2 = \left(1 + \frac{\ell_p^2 N}{vr^2}\right)^{-1/3} \left(1 + \frac{\ell_p^2 M}{r^2}\right)^{-2/3} (-dt^2 + dx_5^2) + \left(1 + \frac{\ell_p^2 N}{vr^2}\right)^{-1/3} \left(1 + \frac{\ell_p^2 M}{r^2}\right)^{1/3} (dx_1^2 + dx_2^2 + dx_3^2 + dx_{10}^2) + \left(1 + \frac{\ell_p^2 N}{vr^2}\right)^{2/3} \left(1 + \frac{\ell_p^2 M}{r^2}\right)^{1/3} \left(du^2 + \frac{d\hat{x}_4^2}{u^2} + d\Omega_3^2\right).
$$  \hfill (3.1)

(Here, $v$ is a dimensionless measure of the volume of the $T^4$ on which the M5–brane is wrapped. $r^2 = \sum_{i=6}^{10} x_i^2$) In the limit, this solution becomes simply $AdS_3 \times S^3 \times T^4 \times S^1$. It is easy to compute that (after a rescaling) the radius of the first two factors is set by the product $(MN^2)^{1/3}$:

$$ds^2 \sim (MN^2)^{1/3} \ell_p^2 \left(u^2(-dt^2 + d\hat{x}_5^2) + \frac{du^2}{u^2} + d\Omega_3^2\right) + \left(\frac{M}{N}\right)^{1/3} (dx_1^2 + dx_2^2 + dx_3^2 + dx_{10}^2) + \left(\frac{N}{M}\right)^{1/3} d\hat{x}_4^2.
$$  \hfill (3.2)

(Again, we have absorbed some constants into $r$ and then set $u = r/\ell_p^2$. This is the dual supergravity description of the theory in the limit. Notice that we get this eleven dimensional supergravity solution at $R_4 = R_5$, which means that the type IIB coupling we are studying is not small, but at the self–dual value $g_{IIB} = R_5 / R_4 = 1$. We actually want the limit $g_{IIB} \rightarrow 0$, if we are to directly study the decoupling limit which yields the physics of the little string trapped inside the F5–brane.

3.2. The Role of Type IIA Supergravity

We therefore need to study this geometry in the ten dimensional limit that $\hat{x}_4 \rightarrow 0$. Luckily, as the $\hat{x}_4$ metric component is $(N/M)^{1/3}$, a constant, there is no resulting non–trivial dilaton dependence for the ten dimensional theory, and no need to multiply the rest of the metric in eqn. (3.2) by any functions of the radial variable.

Using the relation between the ten and eleven dimensional metrics (A is the R–R one–form potential):

$$ds_{11}^2 = e^{4\phi / 3} \left[ (d\hat{x}_4 + A^\mu dx_\mu)^2 + e^{-2\phi} ds_{10}^2 \right],
$$  \hfill (3.3)
it is easily established that our metric \((3.2)\) becomes precisely the ten dimensional solution \((2.3)\). We are therefore left with type IIA supergravity\(^3\) compactified on \(AdS_3 \times S^3 \times T^4\). It is natural to conjecture that the AdS/CFT correspondence defines for us a 1+1 dimensional superconformal field theory on the boundary with the correct superconformal algebra and R–symmetries as before. Of course, the details of the theory are different, as they should be: This is a different supergravity theory. This 1+1 dimensional superconformal field theory defines the \((1,1)\) six dimensional little string theory. This fixed point has a dual supergravity solution which is smooth everywhere. The type iib system in the infinite momentum frame therefore arises from a simple supergravity description.

4. The case of the little \(E_8 \times E_8\) heterotic string.

The next step is obvious. We may place a family of \(M\) \(M_5\)-branes into the \(E_8 \times E_8\) string theory by introducing \(M\) \(D_4\)-branes into the defining \(D_0\)-brane system, which additionally contains 16 \(D_8\)-branes and 2 \(O_8\)-planes.\(^4\) We orient the eight dimensional objects in \(\{x^1, \ldots, x^4, x^6, \ldots, x^9\}\), and the \(D_4\)-branes in \(\{x^1, \ldots, x^4\}\), as before. This defines \(M\)-theory on an interval (in \(x^5\)) defined by an \(M_9\)-plane at each end of it, with \(M\) \(M_5\)-branes located pointwise along it. Everything has momentum in the \(x^{10}\) direction.

As usual, we can choose to place the momentum in the \(x^5\) direction, and shrink it. The theory becomes a type IIB system with \(M\) \(D_5\)-branes, with \(N\) \(D_1\)-branes delocalized inside them. The background of 16 \(D_9\)-branes has an \(SO(16) \times SO(16)\) Wilson line.

The \((0,4)\) 1+1 dimensional theory on the world volume of the \(D_1\)-branes has \(M\) hypermultiplets from the 1–5 sector and 32 fermions from the 1–9 sector. Without the \(D_5\)-branes, this theory goes in the strong coupling limit to an IR fixed point which defines the weakly coupled \(E_8 \times E_8\) heterotic string. In the present case, the flow defines the content of the interacting \((0,1)\) six dimensional theory on the world–volume of the \(F_5\)-brane of the \(E_8 \times E_8\) heterotic string theory. This interacting theory has also a global \(E_8 \times E_8\) symmetry. (See also refs.\(^{39}\), for related models.)

4.1. The role of \(SO(32)\) Supergravity

In similar fashion to that which we described for the type iia system, the large \(N\) and

\(^3\) Yes, we could have arrived at this from a more direct starting point: T–dualizing the type IIB \(AdS_3\) compactification in the \(T^4\), therefore constructing a type IIA solution. It was nevertheless instructive to proceed by this route, seeing where the matrix prescription takes us, in the spirit of ref.\(^1\).

\(^4\) For a reminder of the details of this starting point, see ref.\(^1\). The original references are refs.\(^{34,35,36,38}\).
M limit tells us to examine the supergravity fields around the D1–D5 system, but now in type IB string theory.

The supergravity solution is precisely the same as it was for the type IIB case. The D5–branes remain small instantons of the D9–brane gauge group and so there is no modification to the supergravity solution by an expression for large instantons.

Therefore, in the limit we are led to type IB’s $SO(32)$ supergravity compactified upon $AdS_3 \times S^3 \times T^4$, and in the strong coupling limit (implied by shrinking $R_5$) we replace this with the heterotic $SO(32)$ supergravity with the same compactification (this is valid as the dilaton is constant). Formally, we still need to have winding around the $\hat{x}^5$ direction. Near the limit, we can think of it as a large circle, and the spacetime become $AdS_3$ in the limit. We can place the required $SO(16) \times SO(16)$ Wilson line around this direction, as dictated by the model.

The near–$AdS_3$ inherits a gauge symmetry $SO(16) \times SO(16)$ from ten dimensions. There are states in the adjoint $(\mathbf{120}, \mathbf{1}) + (\mathbf{1}, \mathbf{120})$. We expect that in the presence of the Wilson line, the $N \to \infty$ approach to $AdS_3$ will give masses to states in the $(\mathbf{16}, \mathbf{16})$, while states in the $(\mathbf{128}, \mathbf{1}) + (\mathbf{1}, \mathbf{128})$ become massless, performing the expected enhancement to $E_8 \times E_8$ at strong coupling.

It is natural to conjecture that there is a non–trivial 1+1 dimensional superconformal field theory living at the boundary of the $AdS_3$ space. This is the matrix theory of the $E_8 \times E_8$ little heterotic string theory, with infinite momentum frame in the $x^5$ direction. In this $N \to \infty$ limit, the correct long strings should emerge in the usual way. The gauge symmetry anticipated above gives rise to a global $E_8 \times E_8$ symmetry of the interacting conformal field theory living at the boundary, and hence of the six dimensional spacetime little string theory.

4.2. The role of Type IIA/Heterotic Duality

There is of course another route to defining the $E_8 \times E_8$ little heterotic string. This may also be viewed as a proof of the correctness of the above prescription (particularly the incomplete argument for the gauge/global symmetry) using type IIA/heterotic duality.\textsuperscript{[40,41,42]}

In section 4.1 we recovered the $SO(32)$ type IB system compactified on $AdS_3 \times S^3 \times T^4$. As a ten dimensional system, we took it to strong coupling. We used ten dimensional $SO(32)$ type IB/heterotic duality to relate this to a weakly coupled heterotic system compactified on the same space.

Next, we can consider the system as six dimensional again and replace the $T^4$ with a $K3$ while replacing the heterotic theory with the type IIA theory. As shown in ref.\textsuperscript{[41], 5}

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\textsuperscript{5} We have no direct proof of this, but we will supply an indirect one using string duality in the next section. It will be interesting to find a direct demonstration of this phenomenon.
the type IIA string recovers the $E_8 \times E_8$ symmetry of a dual heterotic string from the intersection lattice of the $K3$.

So simply replacing the $T^4$ by a $K3$ in the type IIA supergravity $AdS_3 \times S^3 \times T^4$ compactification already established in section 3.2, we get the $E_8 \times E_8$ little heterotic string theory.

This way of realizing the string sharpens our earlier discussion of the origin of the required $E_8 \times E_8$ global symmetry, while the duality to the type IB system of the previous subsection points to its correctness.

5. The case of the little $SO(32)$ heterotic string

To define the $SO(32)$ system, we start with the D0–D4–D8–O8 theory from the previous section and shrink $x^4$ as well as $x^5$. As in the case of type IIB, we arrive at the case of intermediate coupling for the parent theory, if we treat both directions the same way: $R_4 \sim R_5$.

Taking the same limits as section 3, we arrive at an M–theory configuration involving $N$ M2–branes stretched along $\hat{x}^4$, ending on $M$ M5–branes which are pointlike along the $\hat{x}^4$ interval defined by the two M9–planes.

To get the limit of weak heterotic $SO(32)$ coupling, we would send the size of the $\hat{x}^4$ interval to zero, and the resulting strongly coupled 1+1 dimensional theory of the endpoints of the M2–branes inside the M5–branes is the theory we want. In the weakly coupled heterotic limit, where the size of the interval shrinks away completely, these 1+1 dimensional endpoints become heterotic F1–strings inside heterotic F5–branes.

In taking the limit, we have not tuned things such that the M5–branes stay away from the M9–plane endpoints even in the limit. This would give expectation values to some number of tensor multiplets in the resulting six dimensional theory. Nor have we tuned things such that the F5–branes dissolve into the M9–planes, becoming fat instantons. Instead, we are just at the dividing edge between these branches of the F5–brane moduli space, so that the full $E_8 \times E_8$ gauge symmetry is present; the F5–branes are small instantons, and we are able to disappear down the infinite throat to find the decoupled theory. This is the interacting six dimensional theory we seek.

Let us take the large $N, M$ limit, as before. We can then define what the supergravity dual should be.

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6 This is also in line with the intuition (expressed to me by T. Banks and H. Verlinde) that once we have compactified to six dimensions, the heterotic string should be best understood in terms of type IIA on $K3$. See also ref.13 for how this reasoning may be pushed back up all the way to eleven dimensions.
5.1. The role of Eleven Dimensional Supergravity

At intermediate coupling for the $SO(32)$ system, it is easy to see that the large $N, M$ limit of the M2–M5–M9 system becomes eleven dimensional supergravity compactified upon $AdS_3 \times S^3 \times T^4 \times S^1 / \mathbb{Z}_2$, the boundaries of the orbifolded $\hat{\chi}^4$ direction are the M9–planes. They are infinitely far apart in the limit $R_4 = 0$.

This should be contrasted with the case of the pure $SO(32)$ system at intermediate coupling encountered in ref. [1]. There, the system was defined by $AdS_4 / \mathbb{Z}_2 \times S^7$, i.e., the orbifold acted in the $AdS_4$ giving fixed points inside the space.

Here, the system is much simpler, as the orbifold misses the $AdS$ component entirely, due to the presence of the M5–branes.

As before, we actually want the weak $SO(32)$ heterotic coupling ($g_{HB} = R_5 / R_4$) limit, which corresponds to shrinking the $\hat{\chi}^4$ direction. There is no radial dependence of the metric component in this direction, and so the resulting ten dimensional theory is also simple (and non–singular). The relevant supergravity theory is of course now the $E_8 \times E_8$ heterotic supergravity[4].

5.2. The role of $E_8 \times E_8$ Supergravity

In the limit, we have $E_8 \times E_8$ supergravity compactified on $AdS_3 \times S^3 \times T^4$. Formally, near the limit we still need to have winding around the $\hat{\chi}^5$ direction, and so we can think of it as an extremely large circle which we will asymptote to the $AdS_3$ in the limit. We then place an $SO(16) \times SO(16)$ Wilson line in this direction.

In the $N \rightarrow \infty$ limit, (and $R_5 \rightarrow 0$ limit) the long strings emerge in the usual way. We expect also that the correct $SO(16) \times SO(16)$ quantum numbers survive to this time recover $SO(32)$ symmetry instead of $E_8 \times E_8$, consistent with the known $T_5$–duality with the Wilson line. Again, we have no direct proof of this mechanism.

It is natural to conjecture that there is a non–trivial 1+1 dimensional superconformal field theory living at the boundary of the $AdS_3$ space. This is the matrix theory of the $SO(32)$ little heterotic string theory, with infinite momentum in the $x^5$ direction.

6. Closing Remarks and Outlook

So we see that, in the spirit of ref. [1], interacting matrix string theories are captured by smooth dual supergravity compactifications involving $AdS$.

Interestingly, a quick calculation shows that the geometry of the 9+1 dimensional fixed point in that case is (after rescaling using (3.3) to measure in ten–dimensional units) precisely the near–horizon geometry of a fundamental string.
The four little string theories are naturally interacting six dimensional theories obtained by “capturing” ten dimensional strings with F5–branes\(^8\).

This is of course why they are four in number: only four of the ten dimensional superstring theories can ensnare such descendents, as the pure type IB system does not contain the requisite F5–branes with which to do the capturing. (In some sense, it also does not contain an honest defining type IB string either: There is no NS–NS B–field.)

This is all consistent with what we observe here, because no \(AdS_3\) geometry arises in the limit of weak type IB coupling. This is because the legs of the M2–brane defining the intermediate coupling situation (see section 5.1) are not on the same footing (pun intended) in this case\(^8\). One sees that the weak type IB coupling limit (obtained by shrinking \(\hat{x}^5\)) would lead back to type IA supergravity (as in ref.\(^1\)), but the \(AdS_3\) gets spoiled. At best, this leads to a 0+1 dimensional theory with a singular supergravity limit: presumably a non–interacting theory defined by a quantum mechanics with a trivial orbifold moduli space, following the philosophy of the present paper and ref.\(^1\).

The four theories have all been shown to have supergravity defining duals which involve \(AdS_3 \times S^3\). In each case, the supergravity changes appropriately to give the correct fermionic extension to \(SO(2,2)\) to fill out the required supersymmetry algebra for the defining 1+1 dimensional fixed point theory on the boundary.

In the heterotic cases the supergravity also supplies the required gauge symmetry, although we did not supply a direct argument for how \(SO(32)\) and \(E_8 \times E_8\) get exchanged from an \(AdS\) supergravity point of view: Somehow, a more careful examination of the interplay between the Wilson line and the approach to the \(AdS_3 \times S^3\) geometry (where the circle goes away) should give the required result that the \(SO(32)\) system (plus Wilson line) gives \(E_8 \times E_8\) gauge symmetry in the limit and \textit{vice–versa}. It is an \(AdS\) supergravity analogue of T–duality with Wilson lines. The type IIA/heterotic duality argument presented in section 4.2 is so far the best direct supergravity argument presented here.

At risk of over–emphasizing the point, let us remark upon the simplicity of the overall structure we have uncovered here for all of the string theories, combining the results of ref.\(^1\) and the present paper:

- For the five (IMF) ten dimensional superstring theories near weak coupling, there is a dual description in terms of a solution of the supergravity associated to the T–dual species of string. The solution is merely the near horizon geometry of the fundamental string solution in ten dimensions.

- The theories have weak coupling limits where they become free. This is represented by the fact that the fundamental string solution is singular at the core: supergravity must break down there as it cannot describe such a trivial theory.

\(^8\) One should contrast with the type IIB matrix situation involving \(AdS_4\) in ref.\(^1\). There, both legs of the M2–brane are inside the \(AdS_4\).
• For the four (IMF) six dimensional superstring theories, there is a dual supergravity solution, again in terms of the supergravity of the T–dual species of string. This time, the solution is simply the near horizon geometry of the fundamental string inside the six dimensional world–volume of a NS–fivebrane.

• These F1–F5 solutions are perfectly smooth everywhere. This is consistent with the fact that the dual little strings seem to have no weak coupling limits.

A final remark to be made is about the current discrepancy observed between the spectrum of supergravity on $AdS_3$ and that of 1+1 dimensional conformal field theories[27]. The structure of our observations is not affected by this. We have not compared spectra here, but only supersymmetry and global symmetries. We expect that the AdS/CFT correspondence for 1+1 dimensional superconformal field theories will be at the very least a useful guide, where at least some of the structures in supergravity and conformal field theory organize one another. The extent to which the supergravity captures all of the physics, including correlators[25], etc., remains to be seen.

The structures uncovered here may be regarded as added motivation for trying to gain understanding of the AdS/2DCFT correspondence, as it will give a handle on the little string theories, which are certainly going to be important in the final story.

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