Dynamics of binary black holes in low and high-mass young star clusters: the impact of long-term star cluster evolution

Stefano Torniamenti\textsuperscript{1,2,3,*}, Sara Rastello\textsuperscript{1,2}, Michela Mapelli\textsuperscript{1,2,3†}, Ugo N. Di Carlo\textsuperscript{4,3}, Alessandro Ballone\textsuperscript{1,2,3}, Mario Pasquato\textsuperscript{1,2,5}

\textsuperscript{1}Physics and Astronomy Department Galileo Galilei, University of Padova, Vicolo dell’Osservatorio 3, I–35122, Padova, Italy
\textsuperscript{2}INFN - Osservatorio Astronomico di Padova, Vicolo dell’Osservatorio 5, I–35122 Padova, Italy
\textsuperscript{3}INAF - Osservatorio Astronomico di Padova, Vicolo dell’Osservatorio 3, I–35122 Padova, Italy
\textsuperscript{4}McWilliams Center for Cosmology and Department of Physics, Carnegie Mellon University, Pittsburgh, PA, 15213, USA
\textsuperscript{5}Physics Department, Montreal University, Montreal, Quebec H3T 1J4, Canada

Accepted XXX. Received YYY; in original form ZZZ

ABSTRACT
Dynamical interactions in dense star clusters are considered one of the most effective formation channels of binary black holes (BBHs). Here, we present direct N-body simulations of two different star cluster families: low-mass (\(\sim 500 - 800 \, M_{\odot}\)) and high-mass star clusters (\(\geq 5000 \, M_{\odot}\)). We show that the formation channels of BBHs in low- and high-mass star clusters are extremely different and lead to two completely distinct populations of BBH mergers. Low-mass clusters host mainly low-mass BBHs born from binary evolution, while BBHs in high-mass clusters are relatively massive (chirp mass up to \(\sim 50 \, M_{\odot}\)) and driven by dynamical exchanges. Tidal disruption dramatically quenches the formation and dynamical evolution of BBHs in low-mass clusters on a very short timescale (\(\lesssim 100 \, \text{Myr}\)), while BBHs in high-mass clusters undergo effective dynamical hardening until the end of our simulations (1.5 Gyr). In high-mass clusters we find 4% of BBHs with primary mass in the pair-instability mass gap, all of them born via stellar collisions, while none of them forms in low-mass clusters. These differences are crucial for the interpretation of the formation channels of gravitational-wave sources.

Key words: black hole physics – binaries: general – galaxies: star clusters: general – stars: kinematics and dynamics - gravitational waves - methods: numerical

1 INTRODUCTION
Over the last six years, the LIGO (Aasi et al. 2015) and Virgo (Acernese et al. 2015) interferometers detected an increasing number of gravitational wave (GW) events (e.g., Abbott et al. 2016a; Abbott et al. 2016b; Abbott et al. 2019, 2021a). At the end of the third observing run, the third GW transient catalog (GWTC-3) consists of 90 GW candidates (Abbott et al. 2021b,c). Most of these events are produced by the inspiral of two black holes (BHs). Among the most peculiar events, the merger remnant of GW190521 (Abbott et al. 2020b; Abbott et al. 2020d) is the first intermediate-mass black hole (IMBH) ever detected in the mass range 100 – 1000 \(M_{\odot}\), with a remnant mass of 142\(^{+28}_{-16}\) \(M_{\odot}\). Also, GW190412 (Abbott et al. 2020a) represents the first observation of a BBH with asymmetric masses. The population of mergers in GWTC-3 also includes two binary neutron star (BNS) mergers, GW170817 and GW190425 (Abbott et al. 2017a,b, 2020c), and two black hole-neutron star (BHNS) candidates, GW200105_162426 and GW200115_042309 (Abbott et al. 2021d). Furthermore, Nitz et al. (2021) and Olsen et al. (2022) reported several additional GW candidates (see also Venumadhav et al. 2019, 2020; Nitz et al. 2020, 2021).

The abundance of detected GW sources allows us to attempt to reconstruct their formation channels. In fact, thanks to the distinctive features that different formation channels imprint on the merging progenitors, even a few hundreds detections may be sufficient to identify their main formation pathways (Fishbach et al. 2017; Zevin et al. 2017; Stevenson et al. 2017; Farr et al. 2017; Vitale et al. 2017; Gerosa & Berti 2017; Gerosa et al. 2018; Bouffanais et al. 2019, 2021a,b; Wong & Gerosa 2019; Wong et al. 2021; Zevin et al. 2021; Doctor et al. 2020; Kimball et al. 2020; Ng et al. 2021).

The isolated formation scenario, for example, predicts the formation of BBHs with primary masses up to 40 – 50 \(M_{\odot}\), mostly equal-mass systems, with preferentially aligned spins and vanishingly small eccentricity in the LIGO–Virgo band (Mandel & de Mink 2016; Gerosa et al. 2018). According to this scenario, the formation of tight enough binary black holes (BBHs) can take place through evolutionary processes like common envelope (Tutukov & Yungelson 1973; Bethe &...
The dynamical formation scenario, instead, involves dynamical processes in dense stellar environments, like young star clusters (YSCs, e.g., Portegies Zwart & McMillan 2002; Banerjee et al. 2010; Mapelli et al. 2013; Ziosi et al. 2014; Goswami et al. 2014; Mapelli 2016; Banerjee 2017, 2018; Rastello et al. 2019; Perna et al. 2019; Di Carlo et al. 2019, 2020a; Kumamoto et al. 2019, 2020; Banerjee 2021; Rastello et al. 2020, 2021; Dall’Amico et al. 2021; Chattopadhyay et al. 2022), globular clusters (e.g., Downing et al. 2010; Benacquista & Downing 2013; Rodriguez et al. 2015, 2016a; Antonini & Rasio 2016; Asker et al. 2017; Fujii et al. 2017; Askar et al. 2018; Fragione & Kocsis 2018; Rodriguez et al. 2019) and nuclear star clusters (e.g., O’Leary et al. 2009; Miller & Lamb 2009; McKernan et al. 2012; Arca-Sedda & Capuzzo-Dolcetta 2018; McKernan et al. 2018; VanLundingham et al. 2016; Stone et al. 2017; Hoang et al. 2018; Arca-Sedda & Gualandris 2018; Antonini et al. 2019; Arca Sedda & Benacquista 2019; Arca Sedda et al. 2020; Mapelli et al. 2021b).

With respect to the isolated channel, this scenario predicts the formation of merging BBHs with larger primary masses (e.g., McKernan et al. 2012; Mapelli 2016; Antonini & Rasio 2016; Gerosa & Berti 2017; Stone et al. 2017; McKernan et al. 2018; Di Carlo et al. 2019, 2020a; Rodriguez et al. 2019; Yang et al. 2019; Arca Sedda & Benacquista 2019; Arca Sedda et al. 2020; Mapelli et al. 2021a), isotropic spin distributions (e.g., Rodriguez et al. 2016b), and, in some rare cases, non-zero eccentricity in the LIGO–Virgo band (e.g., Samsing 2018; Samsing & D’Orazio 2018; Samsing et al. 2018; Rodriguez et al. 2018; Zevin et al. 2019; D’Amico et al. 2021).

In this work, we will focus on the dynamical formation of BBHs in young and open stellar clusters. These systems are of key importance to interpret the formation of BBHs, because they host the formation of the most massive stars (Lada & Lada 2003; Portegies Zwart et al. 2010; Crowther et al. 2010), which are the progenitors of massive BHs. Thanks to the high central densities of the host cluster ($\rho \gtrsim 10^5 M_{\odot} pc^{-3}$, Portegies Zwart et al. 2010), binary stars can efficiently interact with the surrounding stars since the very beginning of their life. This leaves a deep imprint on the properties of the population of BBHs and, in turn, merging BBHs. When YSCs are eventually disrupted by the tidal field of their host galaxy, their stellar content is released into the galactic field. Thus, a large fraction of BBHs which are now in the field may have formed in young stellar systems.

The dynamical formation and evolution of BBHs in YSCs are explored in a realistic way by means of direct \(N\)-body simulations, where up-to-date prescriptions for single and binary stellar evolution are implemented. In many cases, only the first few hundreds Myr of the life of the star cluster are considered. This is due, on the one hand, to their small relaxation timescales, \(t_{\text{rel}} \lesssim 100\ \text{Myr}\) (Portegies Zwart et al. 2010). In particular, the rapid decrease of their central density within the first Myr suppresses the interaction rate in later phases, making them less and less important in shaping the BBH properties. On the other hand, producing large sets of \(N\)-body simulations, which are necessary to explore the population of merging BBHs with sufficient statistics, has high computational costs. Thus, evolving large sets of YSCs for thousands of Myr would turn to be prohibitively expensive. As a consequence, many simulation of YSCs only take into account the first 100 Myr of the star cluster life (Di Carlo et al. 2019, 2020a,b; Rastello et al. 2020, 2021).

The aim of this work is to evaluate the impact of the late phases (up to 1500 Myr) of the dynamical evolution of the star cluster on the population of BBH mergers. To this purpose, we have run two sets of \(N\)-body simulations of YSCs in different mass regimes and studied the evolution of the population of BBHs. To calculate the impact of the long-term evolution of the cluster, we compared the BBH merger populations at two different snapshots, 100 Myr and 1500 Myr. The paper is organized as follows: in Section 2, we describe the details of the \(N\)-body simulations and our method. In Section 3, we report the results for the BBH populations and mergers. Finally, Section 4 summarises our conclusions.

2 METHODS

2.1 Direct \(N\)-body code

We performed our simulations with the \(N\)-body code \textsc{nbody6++GPU} (Wang et al. 2015, 2016), coupled with the population synthesis code\textsuperscript{1} \textsc{mobse} (Mapelli et al. 2017; Giacobbo et al. 2018; Giacobbo & Mapelli 2018). \textsc{nbody6++GPU} is the GPU parallel version of \textsc{nbody6} (Aarseth 2003). It implements a 4th-order Hermite integrator, individual block time-steps (Makino & Aarseth 1992) and Kustaanheimo-Stiefel regularization of close encounters and few-body systems. The force contributions at short time steps (irregular forces) are computed by a neighbour scheme (Nitasori & Aarseth 2012), and for long time steps (regular force/timesteps) the force is evaluated by considering all the particles of the system. The irregular-force calculation is performed using CPUs, while the regular forces are evaluated on GPUs using the CUDA architecture. A solar neighbourhood-like static external field (Wang et al. 2016) is included in the force integration. This choice for the tidal field is quite conservative, because the static tidal field does not take into account possible perturbations by disk- and bulge-shocking and encounters with molecular clouds, which can accelerate the star cluster disruption (Gieles et al. 2006). Orbital decay and circularization by GW emission are calculated following Peters (1964). Post-Newtonian terms are not explicitly included in this version of the code.

\textsc{mobse} is a customized and upgraded version of \textsc{bse} and includes up-to-date prescription for massive stellar winds (Giacobbo et al. 2018), core-collapse (Fryer et al. 2012) and electron-capture supernovae (Giacobbo & Mapelli 2019), natal kicks (Giacobbo & Mapelli 2020) and (pulsational) pair instability supernovae (Mapelli et al. 2020). Stellar winds are modeled by assuming that the mass loss of hot massive stars

\textsuperscript{1} \textsc{mobse} is publicly available at this link.
depends on metallicity as $M \propto Z^\beta$, where $\beta$ is modelled as in Giacobbo et al. (2018).

For this work, we adopt the delayed model for core-collapse supernovae from Fryer et al. (2012). In this model, there is no mass gap between neutron stars and BHs: we assume that compact objects more massive than 3 $M_\odot$ are BHs. Nataal kicks are modeled according to the prescription by Giacobbo & Mapelli (2020): the magnitude of the kick can be expressed as $v_{\text{kick}} \propto f_{105} m_{\odot} m_{\text{rem}}^{-1}$, where $f_{105}$ is a random number drawn from a Maxwellian distribution with a one-dimensional root mean square velocity $\sigma = 265 \text{ km s}^{-1}$, $m_{\text{rem}}$ is the mass of the remnant, and $m_{\odot}$ is the difference between the final mass of the star before the supernova explosion and the mass of the remnant. Binary evolution processes (tides, mass transfer, common envelope and GW-orbital decay) are implemented as in Hurley et al. (2002). The common envelope process is implemented by adopting the energy formalism (Webbink 1984). In this case, we assume $\alpha = 3$, while the concentration parameter $\lambda$ is calculated self-consistently as in Claey et al. (2014).

Figure 1. Evolution of the half-mass radius $r_h$ (upper panels, solid lines), core radius $r_c$ (lower panels, solid lines), and tidal radius $r_t$ (dashed lines) for low-mass clusters (left) and high-mass clusters (right). Each set is divided into three subsets: for the low-mass clusters $M_{\text{SC}} \in [500, 600] M_\odot$ (violet), [600, 700] $M_\odot$ (green), [700, 800] $M_\odot$ (yellow). For the high-mass clusters $M_{\text{SC}} \in [5000, 6000] M_\odot$ (violet), [6000, 7000] $M_\odot$ (green), [7000, 8000] $M_\odot$ (yellow). Each line shows the median value over the simulated YSCs per each mass bin. The black lines (right panels) refer to the same physical quantities for the star clusters with $M_{\text{SC}} = 5 \times 10^4 M_\odot$. 
2.2 Initial conditions for star clusters

2.2.1 Stellar and binary populations

We generate the initial masses of stars (single stars, primary and secondary members of binary systems) according to a Kroupa (2001) initial mass function between 0.1 M\(_{\odot}\) and 150 M\(_{\odot}\). We assume a metallicity Z = 0.002, approximately corresponding to 0.1 Z\(_{\odot}\). For binary systems, we assume a distribution of mass ratios \( F(q) \propto q^{-1} \), with \( q \in [0.1, 1] \) (Sana et al. 2012). Also, our algorithm generates a binary fraction that matches the observational results by Moe & Di Stefano (2017). In this work, the binary fraction for the primary mass in the different mass bins (expressed in solar masses) is set to: \( f_{b,[0,1.0]} = 0.20 \), \( f_{b,[0.8,2]} = 0.40 \), \( f_{b,[2,5]} = 0.59 \), \( f_{b,[5,9]} = 0.76 \), \( f_{b,[9,16]} = 0.84 \), \( f_{b,[16,100]} = 0.94 \). We generate the orbital parameters of binary systems following the observational prescriptions by Sana et al. (2012). In particular, we randomly draw the orbital periods from: \( P(P) \propto P^{-0.5} \), with \( P = \text{log}_{10}(P/\text{days}) \in [0.15, 5.5] \), and the eccentricities from \( F(e) \propto e^{-0.45} \), with \( e \in [10^{-5}, e_{\text{max}}(P)] \). For a given orbital period, we set an upper limit for the eccentricity distribution according to Moe & Di Stefano (2017): \( e_{\text{max}}(P) = 1 - [P/(2 \text{days})]^{-2/3} \). We refer to Torniamenti et al. (2021) for more details on our binary population.

2.2.2 Stellar clusters

We initialize stellar positions and velocities in the simulated YSCs with fractal initial conditions, with a fractal dimension \( D = 1.6 \), in order to mimic the observed clumpiness of embedded star clusters (Cartwright & Whitworth 2004; Sánchez & Alfaro 2009; Kuhn et al. 2019). We generate fractal phase space distributions with MCLUSTER (Kipfer et al. 2011).

We uniformly sample the half-mass of our star clusters between 0.5 and 2 pc (e.g., Portegies Zwart et al. 2010; Krumholz et al. 2019). To evaluate the impact of long-term dynamics on the properties of BBH mergers in different dynamical regimes, we consider two sets of star clusters in different mass ranges (M\(_{SC}\)):

- **Low-mass star clusters**, with mass ranging from 500 M\(_{\odot}\) to 800 M\(_{\odot}\). These clusters present short dynamical evolution timescales at all scales: this reduces the probability of dynamical interactions and, consequently, of dynamical exchanges. Also, YSCs in this mass range typically host a few massive stars, and, consequently, BHs. This further suppresses the rate of dynamical exchanges\(^2\) (Rastello et al. 2021).

- **High-mass star clusters**, with mass ranging from 5000 M\(_{\odot}\) to 8000 M\(_{\odot}\). These clusters have a higher rate of dynamical encounters, as a consequence of the higher densities, longer dynamical timescales and larger number of massive stars. Thus, they are expected to produce a larger number of exchanged binaries and BBHs (Rastello et al. 2021). In this sample we also include two star clusters with mass 5 × 10\(^4\) M\(_{\odot}\).

In both cases, we sample the mass of star clusters from a power-law distribution \( dN/dM_{SC} \propto M_{SC}^{-2} \), following Lada & Lada (2003). The two sets consist in 35578 and 3555 star clusters, respectively. The number of star clusters in the two samples is set to obtain the same total mass. The total kinetic (\( K \)) and potential (\( W \)) energy of the cluster are set to give a virial ratio \( q = 2K/W = 1 \).

2.3 Impact of long-term evolution

The main goal of this work is to evaluate the impact of the long-term dynamical evolution, up to 1500 Myr. In particular, we compare the population of BBH mergers that form in the first 100 Myr of the evolution of the simulated YSCs with the population of BBH mergers at 1500 Myr.

We first evaluate the population of BBH mergers that we would have obtained if we had integrated the evolution of our YSCs only for the first 100 Myr. This population consists in:

- BHs that merge within the first 100 Myr, during the \( N \)-body simulations.
- BBHs that will merge within a Hubble time. To calculate them, we consider the population of existing BHs at 100 Myr and evolve their orbital eccentricity and semi-major axis by integrating the equations of Peters (1964), to calculate the energy loss due to GW emission:

\[
\begin{align*}
\frac{da}{dt} &= -\frac{64}{5} \frac{G^2 m_1 m_2 (m_1 + m_2)}{c^6 a^3 (1 - e^2)^{7/2}} f_1(e), \\
\frac{de}{dt} &= -\frac{304}{15} \frac{G^2 m_1 m_2 (m_1 + m_2)}{c^6 a^3 (1 - e^2)^{5/2}} f_2(e),
\end{align*}
\]

\(2\) Because the observed binary fraction increases with the mass of the primary star, the most massive stars are typically born in binary systems. If few or no single BHs are present, dynamical exchanges are limited because the probability of an exchange is strongly suppressed if the intruders are less massive than the two members of the binary system.
Long-term dynamics of BBHs in YSCs

Figure 3. Mass distribution of BHs in BBHs, for low-mass clusters (left) and high-mass clusters (right). Red line: original BBHs at 1500 Myr. Orange filled histogram: original BBHs at 100 Myr. Blue dashed line: exchanged BBHs at 1500 Myr. Light blue hatched histogram: exchanged BBHs at 100 Myr. Among the original BHs, we highlight in red those with mass in the PI mass gap at 1500 Myr (red dash-dotted line) and at 100 Myr (red filled histogram).

where

\[ f_1(e) = \left(1 + \frac{73}{24} e^2 + \frac{37}{96} e^4\right) \]

\[ f_2(e) = \left(1 + \frac{121}{304} e^2\right). \]

All the BBHs that merge within a Hubble time \( t_H = 14 \text{ Gyr} \) are classified as mergers. This is equivalent to assume that the YSCs dissolve at 100 Myr, and their BBHs evolve only via GW emission (no dynamical interactions) after the death of their parent star clusters.

To evaluate how dynamical encounters affect the distribution of BBH mergers in the late phases of the cluster life, we repeat the aforementioned procedure after 1500 Myr, i.e.

- we count how many BBHs merge within 1500 Myr, during the \( N \)-body simulations;
- we integrate the semi-major axis and eccentricity evolution of the other BBHs that are still bound at 1500 Myr, accounting for GW emission only (Peters 1964), and we count how many of them merge within one Hubble time.

If the dynamical interactions within the cluster are still effective after 100 Myr, they can affect the population of BBHs, and, consequently, of BBH mergers. In particular, dynamical processes can form new BBHs or harden the existing ones, allowing them to merge within a Hubble time, or even before the end of the simulation, thus increasing the population of BBH mergers. In some other cases, dynamical interactions can disrupt existing BBHs, possibly removing them from the population of mergers that we estimated at 100 Myr.

3 RESULTS

3.1 Global evolution of the cluster

Figure 1 shows the evolution of the half-mass radius \( r_h \), tidal radius \( r_t \), and core radius \( r_c \) of the two sets of clusters. Each set is split into three sub-sets of different mass in order to better take into account the impact of the cluster mass on its expansion. As a comparison for high-mass clusters, we also show the evolution of the stellar clusters with \( 5 \times 10^4 \text{ M}_\odot \).

Low-mass clusters are rapidly disrupted by the tidal field of the host galaxy, as indicated by the steep decrease of the tidal radius. The typical lifetime of these clusters is 400 Myr, with no particular distinction among the three subsets. As a consequence, \( r_h \) steeply increases, and becomes comparable to the tidal radius at about 150 Myr. As for the core, its expansion accelerates at about 400 Myr, when \( r_c \sim r_t \) and the tidal effect by the host galaxy disrupts the cluster also at the smallest scales.

High-mass clusters undergo a milder expansion during the first 600 – 700 Myr, thanks to their larger initial mass. In this case, the tidal effect of the galaxy becomes significant when \( r_h \sim r_t \), and leads to a steeper global expansion. The decrease of the tidal radius depends on the mass range of the subset, and indicates that the typical lifetime of these systems spans from 1250 Myr for clusters below 6000 \text{ M}_\odot, up to more than 3

We calculate the half-mass and core radii by considering the whole distribution of stars, not only the bound ones. By this choice, we can easily evaluate when the tidal field affects the relevant scales of the cluster by simply comparing \( r_h \) and \( r_c \) to \( r_t \).
Figure 4. Distribution of formation times ($t_{\text{form}}$) of binary systems that give birth to exchanged BBHs, in low-mass clusters (left) and high-mass clusters (right). Upper panels: all exchanged BBHs. Lower panels: exchanged BBH mergers. Blue dashed line and hatched area: BBHs that formed when both components were stars. Orange dot-dashed line: BBHs that formed when one component was a star and the other was a BH. Black line: BBHs that formed when both components were BHs. Grey area: all BBHs.

1500 Myr for clusters more massive than $7000 M_\odot$. In contrast, the $5 \times 10^4 M_\odot$ clusters undergo a far slower expansion and their tidal radius is almost unchanged at the end of the simulation. As for the core, it undergoes a modest increase throughout the simulation. At the smallest scales, in fact, the effect of the host galaxy becomes important only at the very late stages, suggesting that a bound core is still present at 1500 Myr.

Figure 2 shows the distribution of the ratio between the star cluster bound mass at 1500 Myr, $M_{\text{SC,f}}$, and its initial mass, $M_{\text{SC}}$. Low-mass clusters are completely disrupted by the tidal field of the host galaxy at the end of the simulation. In only one case, a dense core of about $120 M_\odot$, corresponding to 18% of the initial mass, can survive, thanks to the presence of a BBH with total mass $80 M_\odot$. As for high-mass clusters, one third of the stellar systems is still bound at the end of the simulation. In this set, the number of surviving clusters increases with the initial mass of the cluster. Also, more massive clusters can generally retain a higher fraction of mass. Finally, stellar clusters with $5 \times 10^4 M_\odot$ preserve about half of their initial mass at 1500 Myr.

3.2 BBH populations

Figure 3 shows the mass distribution of BHs in BBHs at the two considered snapshots: 100 Myr and 1500 Myr. Among the BBH populations, we distinguish between original BBHs, whose progenitors were already present as binary stars in the initial conditions of the simulation, and exchanged BBHs, which have formed as a consequence of dynamical exchanges.

In low-mass clusters, the BH populations at the two snapshots are almost identical, suggesting that, after 100 Myr, dynamical encounters play a negligible role in the evolution of BBHs. In contrast, high-mass clusters are still active in producing BBHs at later phases, as indicated by the large increase of exchanged BBHs between $7 M_\odot$ and $50 M_\odot$. 

100
By comparing the BH populations of the two stellar cluster sets, we see two main differences. First, low-mass clusters display a larger number of original BHs with \( m_{BH} > 20 \text{M}_\odot \). In high-mass clusters, where these BHs undergo strong dynamical interactions, they tend to form exchanged BHs rather than remain members of original BBHs. Hence, original BBHs in high-mass clusters tend to have lower masses than original BBHs in low-mass clusters.

About 6% of our simulated BBHs have primary mass \( > 60 \text{M}_\odot \). The formation of such massive BHs, with mass ranging from \( \sim 60 \text{M}_\odot \) to \( \sim 120 \text{M}_\odot \), is suppressed in single stellar evolutionary processes by pair-instability (PI) and pulsational pair instability (PPI). Nonetheless, as shown by Spera et al. (2019), Di Carlo et al. (2019) and Di Carlo et al. (2020a), BHs in the mass gap can form as a consequence of stellar mergers, which produce very massive stars that eventually collapse to BHs. In our simulations, BHs in the PI mass gap form as a result of stellar mergers and generally reside in exchanged systems. However, in 6 cases (5 at 100 Myr), a BH with mass \( > 60 \text{M}_\odot \) forms in an original binary system. These are systems in which the original binary remains bound after the merger of one component with a third star, producing an original binary with a mass-gap primary BH.

Furthermore, high-mass clusters produce a considerable (about 1.5% of the total) number of BHs with mass \( > 100 \text{M}_\odot \), with a maximum mass of \( 270 \text{M}_\odot \). In contrast, only two BHs with mass \( > 100 \text{M}_\odot \) form in low-mass clusters. Such intermediate-mass BHs form via multiple stellar collisions (Di Carlo et al. 2021).

Figure 4 displays the distribution of formation times of the binary systems that give birth to exchanged BBHs \((t_{form})\). In low-mass clusters, 8% of these systems form when both components are still stars, and about 15% when one component is a BH. Most of the BBHs form by the pairing of two BHs, with a peak between \( 10 \sim 20\text{ Myr} \). The rapid decrease of the dynamical activity and the dissolution of the stellar cluster cause a steep decrease in the distribution of \( t_{form} \), and only 12% of the BBHs forms after 100 Myr.

In high-mass clusters, about 88% of the BBHs form from the pairing of two BHs. In this case, the distribution of \( t_{form} \) shows a flatter trend, hinting at an efficient dynamical activity of the cluster at later times. In this case, in fact, more than one third of the BBHs couple after 100 Myr. In these clusters, only 5% of BBHs systems form when both components are still stars.

### 3.3 BBH mergers

#### 3.3.1 Low-mass clusters

Figure 5 shows the mass of the secondary BH \((m_2)\) versus the primary BH \((m_1)\) of merging BBHs in low-mass clusters (left) and high-mass clusters (right). Orange diamonds: original BBHs at 1500 Myr. Red crosses: original BBHs at 100 Myr. Light blue circles: exchanged BBHs at 1500 Myr. Blue plusses: exchanged BBHs at 100 Myr. Green circles: exchanged BBHs at 1500 Myr in star clusters with mass \( 5 \times 10^4 \text{M}_\odot \).

By comparing the BH populations of the two stellar cluster sets, we see two main differences. First, low-mass clusters display a larger number of original BHs with \( m_{BH} > 20 \text{M}_\odot \). In high-mass clusters, where these BHs undergo strong dynamical interactions, they tend to form exchanged BHs rather than remain members of original BBHs. Hence, original BBHs in high-mass clusters tend to have lower masses than original BBHs in low-mass clusters.

About 6% of our simulated BBHs have primary mass \( > 60 \text{M}_\odot \). The formation of such massive BHs, with mass ranging from \( \sim 60 \text{M}_\odot \) to \( \sim 120 \text{M}_\odot \), is suppressed in single stellar evolutionary processes by pair-instability (PI) and pulsational pair instability (PPI). Nonetheless, as shown by Spera et al. (2019), Di Carlo et al. (2019) and Di Carlo et al. (2020a), BHs in the mass gap can form as a consequence of stellar mergers, which produce very massive stars that eventually collapse to BHs. In our simulations, BHs in the PI mass gap form as a result of stellar mergers and generally reside in exchanged systems. However, in 6 cases (5 at 100 Myr), a BH with mass \( > 60 \text{M}_\odot \) forms in an original binary system. These are systems in which the original binary remains bound after the merger of one component with a third star, producing an original binary with a mass-gap primary BH.

Furthermore, high-mass clusters produce a considerable (about 1.5% of the total) number of BHs with mass \( > 100 \text{M}_\odot \), with a maximum mass of \( 270 \text{M}_\odot \). In contrast, only two BHs with mass \( > 100 \text{M}_\odot \) form in low-mass clusters. Such intermediate-mass BHs form via multiple stellar collisions (Di Carlo et al. 2021).

Figure 4 displays the distribution of formation times of the binary systems that give birth to exchanged BBHs \((t_{form})\). In low-mass clusters, 8% of these systems form when both components are still stars, and about 15% when one component is a BH. Most of the BBHs form by the pairing of two BHs, with a peak between \( 10 \sim 20\text{ Myr} \). The rapid decrease of the dynamical activity and the dissolution of the stellar cluster cause a steep decrease in the distribution of \( t_{form} \), and only 12% of the BBHs forms after 100 Myr.

In high-mass clusters, about 88% of the BBHs form from the pairing of two BHs. In this case, the distribution of \( t_{form} \) shows a flatter trend, hinting at an efficient dynamical activity of the cluster at later times. In this case, in fact, more than one third of the BBHs couple after 100 Myr. In these clusters, only 5% of BBHs systems form when both components are still stars.

#### 3.3.1 Low-mass clusters

Figure 5 shows the mass of the secondary BH \((m_2)\) versus the primary BH \((m_1)\) for BBH mergers, i.e. BBHs that reach coalescence in less than 14 Gyr. In low-mass clusters, the population of BBH mergers mostly consists of original BBHs, as a further proof of the poor dynamical activity of these systems. In general, dynamical exchanges do not affect the population of BBHs after 100 Myr (as already suggested by Fig. 3), with two exceptions. First, one BBH that is predicted to merge if the simulation is run only for 100 Myr, is later disrupted by dynamical interactions, and no longer exists at 1500 Myr. Second, the most massive merger (with a final remnant mass \( m_{tot} = 99 \text{M}_\odot \)) needs to dynamically harden for longer than 100 Myr to enter the regime in which the orbital decay by GWs becomes effective. As shown in Fig. 4 (bottom panel),
activity within high-mass clusters. Late dynamical activity in high-mass clusters, six BBH mergers give up to 1000 Myr. In high-mass clusters, six BBH mergers give the majority of BBH mergers.

The properties of BBH mergers are summarized in Table 1. The populations of BBH mergers at 100 and 1500 Myr show notable differences. A number of original BBHs that are predicted to merge at 100 Myr are later disrupted (Figure 5 and Table 1). Some exchanged BBHs are also disrupted after the first 100 Myr. However, these disrupted exchanged BBHs are compensated by the late formation and/or hardening of other exchanged BBHs: we predict only 34 merging exchanged BBHs, as shown in Table 1. As a result, at 1500 Myr, the number of exchanged BBH mergers has increased by a factor of two with respect to 100 Myr. As shown in Figure 6, most of the merging BBHs at 1500 Myr form when both components have already evolved to BHs. As opposed to low-mass clusters, merging BBHs can form at very late stages, up to 1000 Myr. In high-mass clusters, six BBH mergers give birth to intermediate mass black holes (with a remnant mass \( m_{\text{tot}} > 100 M_\odot \)). In the two most massive mergers, the primary BH is itself an IMBH. The most massive remnant has a total mass \( m_{\text{tot}} = 158 M_\odot \).

Figure 6 shows the distribution of chirp masses of merging BBHs, for the two snapshots considered. The changes in the distribution are mostly due to the long-term dynamical activity within high-mass clusters. Late dynamical activity triggers a large increase of the number of mergers with high chirp mass \( m_{\text{chirp}} \approx 35 M_\odot \).

### 3.3.2 High-mass clusters

High-mass star clusters host a population of BBH mergers about three times larger than low-mass clusters, although the total initial stellar masses of the simulated star cluster samples are approximately the same. This enhancement of BBH mergers in high-mass clusters is particularly evident for the exchanged systems, which, at 1500 Myr, represent the majority of BBH mergers.

The populations of BBH mergers at 100 and 1500 Myr show notable differences. A number of original BBHs that are predicted to merge at 100 Myr are later disrupted (Figure 5 and Table 1). Some exchanged BBHs are also disrupted after the first 100 Myr. However, these disrupted exchanged BBHs are compensated by the late formation and/or hardening of other exchanged BBHs: we predict only 34 merging exchanged BBHs, as shown in Table 1. As a result, at 1500 Myr, the number of exchanged BBH mergers has increased by a factor of two with respect to 100 Myr. As shown in Figure 6, most of the merging BBHs at 1500 Myr form when both components have already evolved to BHs. As opposed to low-mass clusters, merging BBHs can form at very late stages, up to 1000 Myr. In high-mass clusters, six BBH mergers give birth to intermediate mass black holes (with a remnant mass \( m_{\text{tot}} > 100 M_\odot \)). In the two most massive mergers, the primary BH is itself an IMBH. The most massive remnant has a total mass \( m_{\text{tot}} = 158 M_\odot \).

Figure 6 shows the distribution of chirp masses of merging BBHs, for the two snapshots considered. The changes in the distribution are mostly due to the long-term dynamical activity within high-mass clusters. Late dynamical activity triggers a large increase of the number of mergers with high chirp mass \( m_{\text{chirp}} \approx 35 M_\odot \).

### 3.4 BBH orbital properties at formation

To estimate for how long a stellar cluster is dynamically active and can affect the formation of BBH mergers, we evaluated \( t_M \), defined as the time (since the beginning of the simulation) at which the semi-major axis of the BBH has become sufficiently tight to merge within a Hubble time via GW emission. Figure 7 shows \( t_M \) as a function of the orbital properties of the BBH when it forms, that is its semi-major axis \( (a_{\text{BBH}}) \) and its orbital eccentricity \( (e_{\text{BBH}}) \).

In both low-mass and high-mass clusters, the original BBH mergers show typical values of \( t_M \lesssim 10 \text{ Myr} \), \( a_{\text{BBH}} \lesssim 0.1 \text{ AU} \), and circular orbits. These properties spring from their formation pathway. These BBHs are, in fact, the result of original binaries that hardened as a consequence of a common envelope phase. When the second BH forms, the orbital properties of the BBH already allow it to merge within an Hubble time. For this class of BBH mergers, then, \( t_M \) mainly coincides with the time at which the second BH in the binary forms. Also, because the common envelope phase leads to a large mass loss, the resulting BH masses are systematically smaller than the exchanged ones.

As a confirmation of this idea, Fig. 8 shows \( t_M \) as a function of the total mass of the merging BBH, \( m_{\text{tot}} \), and the time at which the BBH forms, \( t_{\text{BBH}} \). Original BBHs have \( t_{\text{BBH}} \lesssim 10 \text{ Myr} \) and, in most cases, \( t_{\text{BBH}} = t_M \). In high-mass stellar cluster, one original BBH presents \( t_{\text{BBH}} > t_M \): in this case, dynamical hardening allows the binary system to enter the merging regime after the BBH formation. In contrast, \( t_{\text{BBH}} \) ranges from 5 Myr to 1000 Myr for exchanged BBHs. In high-mass clusters, 13 BBHs form after the first 100 Myr. Because exchanged BBHs have not undergone mass loss by a common

---

**Table 1.** We report the number of all (first row), original (second row) and exchanged (third row) BBH mergers. We also show the number of BBHs that either merge inside the cluster or are still bound to the cluster at the end of the simulation and merge within one Hubble time (fourth row). Finally, we report the number of IMBHs produced by BBH mergers (merger remnants, fifth row), and their maximum BH mass (last row).

| BBH mergers | Low-mass clusters | High-mass clusters |
|-------------|-----------------|------------------|
|             | 100 Myr | 1500 Myr | 100 Myr | 1500 Myr |
| All         | 35     | 35      | 97     | 115     |
| Original    | 27     | 27      | 63     | 54      |
| Exchanged   | 8      | 8       | 34     | 61      |
| Inside YSC  | 0      | 0       | 13     | 5       |
| IMBHs       | 0      | 0       | 1      | 6       |
| \( m_{\text{tot, max}} [M_\odot] \) | 58     | 99      | 116    | 158     |

---

**Figure 6.** Chirp mass distribution at 100 Myr (light-blue filled histogram) and at 15000 Myr (blue line), for all the simulated clusters.
Figure 7. Time at which the semi-major axis of the BBH has become sufficiently tight to merge within a Hubble time via GW emission (according to Peters 1964) versus semi-major axis of the BBH when it forms ($a_{\text{BBH}}$), for merging BBHs in low-mass clusters (left) and high-mass clusters (right). The markers are the same as in Figure 5. The colour-map encodes the information on the orbital eccentricity at the BBH formation, $e_{\text{BBH}}$. If a BBH at 1500 Myr is also present at 100 Myr, it is marked with a white cross (original) or plus (exchanged).

Figure 8. Time at which the semi-major axis of the BBH has become sufficiently tight to merge within a Hubble time via GW emission (according to Peters 1964) versus total mass of the BBH merger ($m_{\text{tot}}$), for merging BBHs in low-mass clusters (left) and high-mass clusters (right). The markers are the same as in Figure 5. The colour-map encodes the information on the time at which the BBH forms, $t_{\text{BBH}}$. If a BBH at 1500 Myr is also present at 100 Myr, it is marked with a white cross (original) or plus (exchanged).
envelope phase, their masses are systematically higher than those of original BBH mergers, with $m_{\text{tot}} \gtrsim 40 M_\odot$.

In low-mass YSCs, only three exchanged BBH mergers have $a_{BBH} \gtrsim 0.1$ AU. In two cases, these mergers correspond to the two most massive BBHs, which formed in dynamically active environments. As a further proof of their dynamical origin, these BBHs are characterized by eccentric orbits. In high-mass clusters, where dynamical interactions play a major role, the distribution of BBH mergers extends to higher values of $a_{BBH}$ and $t_M$. In particular, exchanged binaries, when they form, are generally characterized by large semi-major axes, up to $5 \times 10^4$ AU, and thus take longer times to enter the regime in which GWs efficiently shrink the semi-major axis. In some cases, $t_M$ can be as high as 1400 Myr, indicating that dynamical hardening can play a role even at the very end of the simulation.

Finally, the dynamical encounters that lead to the formation of BBHs leave a distinctive imprint on their eccentricity. The resulting binary systems are, in fact, characterized by larger eccentricities at formation, with $e_{BBH} > 0.1$. Exchanged BBHs that have values of $t_M \lesssim 100$ Myr and high eccentricities can be later disrupted by dynamical interactions, and are no longer present at 1500 Myr.

### 3.5 Formation pathway of black holes in the mass gap

In high-mass stellar clusters, 5 BBH mergers have primary mass in the PJ mass gap. As mentioned in Section 3.2, such massive objects originate from the merger of massive stars. Figure 9 shows the evolution of BHs in BBH mergers in the PJ mass gap. In all cases, the progenitor star undergoes at least one merger with another star. The merger product of such stellar collisions is an exotic star, with an undersized He core with respect to the hydrogen-rich envelope. Such star does not develop PI, because its central properties (temperature and density) do not fall within the PI regime (e.g., Renzo et al. 2020; Costa et al. 2021). At the end of its evolution, the stellar product directly collapses into a BH more massive than $60 M_\odot$.

Successive mergers between the BH and non-compact objects do not affect the mass of the BH because, consistently with Di Carlo et al. (2020a,b), we assume that the short timescale of the merger does not allow the BH to accrete mass (but see Rizzuto et al. 2021 for a different assumption). In all the BBH mergers, the binary system forms when both objects are BHs. In one case, the BBH is disrupted by dynamical interaction at 105 Myr.

### 4 SUMMARY

We have studied the formation of BBHs in young and open star clusters via direct N-body simulations, exploiting the codes nbody6++GPU (Wang et al. 2015) and MOBSE (Mapelli et al. 2017). We simulated two different classes of star clusters: low-mass (500–800 $M_\odot$) and high-mass (5000–8000 $M_\odot$) systems. We find that the properties and timescales of BBH mergers in the two star-cluster families are extremely different.

In low-mass clusters, most BBHs form in the first 100 Myr and are the result of the evolution of original binary stars, which evolve through common envelope. They do not harden significantly after $\sim 100$ Myr. In contrast, the late evolutionary stages ($> 1$ Gyr) are crucial for high-mass clusters. Exchanged BBHs (i.e., BBHs that form via dynamical exchanges) are the most common BBH mergers in high-mass clusters (Figures 3 and 5). While exchanged BBHs form preferentially in the first $\sim 100$ Myr, they keep hardening significantly until the end of the simulations (1.5 Gyr, Figure 4). This confirms the importance of integrating the evolution of relatively massive clusters ($\gtrsim 5000 M_\odot$) for $> 1$ Gyr.

This difference between the BBH population of low-mass and high-mass star clusters mostly springs from the different two-body relaxation timescale and tidal disruption timescale of the two star cluster families. Our low-mass and high-mass star clusters have a two-body relaxation timescale (Spitzer 1987) of $\sim 10$ Myr and $\sim 30$ Myr, respectively. This means that mass segregation and other dynamical processes happen earlier in low-mass clusters. Furthermore, low-mass clusters become tidally overfilling already at $\sim 150$ Myr, while the half-mass radius of our high-mass clusters becomes comparable to the tidal radius only at $\sim 500$ Myr (Figure 1). Hence, the dynamical activity of the low-mass clusters is quenched by tidal evaporation about five times earlier than that of high-mass clusters.

In both low-mass and high-mass clusters, the latest BBHs that form (exchanged BBHs) are the most massive ones (primary mass $\gtrsim 30 M_\odot$), because dynamical exchanges favour the pairing of the most massive BHs (Figure 5). The distribution of the chirp mass of BBH mergers shows two main peaks: the first at $\sim 7 - 15 M_\odot$ and the second at $\sim 30 - 40 M_\odot$. However, the high-mass peak develops only after 100 Myr (Figure 6).

BBH mergers in low-mass clusters are driven mostly by binary evolution via common envelope: they form with short semi-major axis ($\sim 0.1$ AU) and low orbital eccentricity (Fig-
In contrast, massive BBHs in high-mass clusters form with larger semi-major axis (> 10 AU) and higher orbital eccentricity (0.1 – 1).

A non-negligible percentage (4%) of our simulated BBH mergers in high-mass clusters have primary component’s mass in the pair-instability (PI) mass gap. All of them form via stellar collisions, in which a main-sequence star merges with a more evolved star (core He burning). About 40% of these massive BBHs leave a merger remnant in the intermediate-mass BH range. In contrast, we find no IMBHs in the low-mass clusters.

Furthermore, in the high-mass clusters, we find a few original BBHs with primary mass in the PI mass gap. These are systems in which one of the two components of the binary star undergoes a collision with a third star and collapses to a BH in the PI mass gap without leading to the ionization of the original binary system.

Overall, our study shows that the formation channels of BBHs in low-mass (∼ 500 – 800 M⊙) and high-mass star clusters (∝ 5000 M⊙) are extremely different and lead to two completely distinct BBH populations. Low-mass clusters host mainly low-mass BBHs born from binary evolution, while BBHs in high-mass clusters are relatively massive and driven by exchanges. This difference is crucial for the interpretation of gravitational-wave sources.

ACKNOWLEDGEMENTS

MM, AB and SR knowledge financial support from the European Research Council for the ERC Consolidator grant DEMOBLACK, under contract no. 770017. We thank Nicola Giacobbo and the members of the DEMOBLACK team for useful discussions. We acknowledge that the results of this research have been achieved using the DECI resource Snellius based in the Netherlands at SURFsara, with support from the PRACE aisbl. MP acknowledges financial support from the European Union’s Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No. 896248. ST thanks Mark Gieles and the ICCUB Virgo team for useful comments and discussions.

DATA AVAILABILITY

The data underlying this article will be shared on reasonable request to the corresponding authors. The latest public version of MOBSE can be downloaded from this repository.

REFERENCES

Aarseth S. J., 2003, Gravitational N-Body Simulations. Cambridge University Press
Aasi J., et al., 2015, Classical and Quantum Gravity, 32, 115012
Abbott B. P., et al., 2016a, Physical Review X, 6, 041015
Abbott B. P., et al., 2016b, Phys. Rev. Lett., 116, 061102
Abbott B. P., et al., 2017a, ApJ, 848, L12
Abbott B. P., et al., 2017b, ApJ, 848, L13
Abbott B. P., et al., 2019, Physical Review X, 9, 031040
Abbott B. P., et al., 2020a, Phys. Rev. D, 102, 043015
Abbott R., et al., 2020b, Phys. Rev. Lett., 125, 101102
Abbott B. P., et al., 2020c, ApJ, 892, L3
Abbott R., et al., 2020d, ApJ, 900, L13
Abbott R., et al., 2021a, arXiv e-prints, p. arXiv:2108.01045
Abbott R., et al., 2021b, arXiv e-prints, p. arXiv:2111.03606
Abbott R., et al., 2021c, arXiv e-prints, p. arXiv:2111.03634
Abbott R., et al., 2021d, ApJ, 915, L5
Acernese F., et al., 2015, Classical and Quantum Gravity, 32, 024001
Antonini F., Rasio F. A., 2016, ApJ, 831, 187
Antonini F., Gieles M., Gualandris A., 2019, MNRAS, 486, 5008
Arca Sedda M., Benacquista M., 2019, MNRAS, 482, 2991
Arca-Sedda M., Capuzzo-Dolcetta R., 2018, Monthly Notices of the Royal Astronomical Society, 483, 152
Arca-Sedda M., Gualandris A., 2018, MNRAS, 477, 4423
Arca Sedda M., Mapelli M., Spera M., Benacquista M., Giacobbo N., 2020, ApJ, 894, 133
Askar A., Szkudlarek M., Gondek-Rosińska D., Giersz M., Bulik T., 2017, MNRAS, 464, L36
Askar A., Arca Sedda M., Giersz M., 2018, MNRAS, 478, 1844
Banerjee S., 2017, MNRAS, 467, 524
Banerjee S., 2018, MNRAS, 473, 909
Banerjee S., 2021, MNRAS, 500, 3002
Banerjee S., Baumgardt H., Koumpa P., 2010, MNRAS, 402, 371
Bavera S., et al., 2021, A&A, 647, A153
Belczynski K., Kalogera V., Bulik T., 2002, ApJ, 572, 407
Belczynski K., Kalogera V., Rasio F. A., Taam R. E., Zezas A., Bulik T., Maccarone T. J., Ivanova N., 2008, ApJS, 174, 223
Belczynski K., Bulik T., Fryer C. L., Ruiter A., Valsecchi F., Vink J. S., Hurley J. R., 2010, ApJ, 714, 1217
Belczynski K., Holz D. E., Bulik T., O’Shaughnessy R., 2016, Nature, 534, 512
Belczynski K., et al., 2020, AKA, 636, A104
Benacquista M. J., Downing J. M. B., 2013, Living Reviews in Relativity, 16, 4
Bethe H. A., Brown G. E., 1998, ApJ, 506, 780
Bouffanais Y., Mapelli M., Gerosa D., Di Carlo U. N., Giacobbo N., Berti E., Baibhav V., 2019, ApJ, 886, 25
Bouffanais Y., Mapelli M., Santoliquido F., Giacobbo N., Iorio G., Costa G., 2021a, MNRAS, 505, 3873
Bouffanais Y., Mapelli M., Santoliquido F., Giacobbo N., Di Carlo U. N., Rastello S., Artale M. C., Iorio G., 2021b, MNRAS, 507, 5224
Cartwright A., Whitworth A. P., 2004, MNRAS, 348, 589
Chattopadhyay D., Hurley J., Stevenson S., Raidani A., 2022, arXiv e-prints, p. arXiv:2202.08924
Claeys J. S. W., Pols O. R., Izzard R. G., Vink J., Verbunt F. W. M., 2014, A&A, 563, A83
Costa G., Bressan A., Mapelli M., Marigo P., Iorio G., Spera M., 2021, MNRAS, 501, 4514
Crowther P. A., Schnurr O., Hirschi R., Yusof N., Parker R. J., Goodwin S. P., Kassim H. A., 2010, MNRAS, 408, 731
Dall’Amico M., Mapelli M., Di Carlo U. N., Bouffanais Y., Rastello S., Santoliquido F., Bollane A., Arca Sedda M., 2021, MNRAS, 508, 3045
Di Carlo U. N., Giacobbo N., Mapelli M., Pasquato M., Spera M., Wang L., Haardt F., 2019, MNRAS, 487, 2947
Di Carlo U. N., Mapelli M., Bouffanais Y., Giacobbo N., Santoliquido F., Bressan A., Spera M., Haardt F., 2020a, MNRAS, 497, 1043
Di Carlo U. N., et al., 2020b, MNRAS, 498, 495
Di Carlo U. N., et al., 2021, MNRAS, 507, 5132
Doctor Z., Wysocki D., O’Shaughnessy R., Holz D. E., Farr B., 2020, ApJ, 893, 35
Dominik M., Belczynski K., Fryer C., Holz D. E., Berti E., Bulik T., Mandel I., O’Shaughnessy R., 2012, ApJ, 759, 52
Dominik M., Belczynski K., Fryer C., Holz D. E., Berti E., Bulik T., Mandel I., O’Shaughnessy R., 2013, ApJ, 779, 72
Downing J. M. B., Benacquista M. J., Giersz M., Spurzem R., 2010, MNRAS, 407, 1946
