The Interaction of Frictional Slip and Adhesion for a Stiff Sphere on a Compliant Substrate

How friction affects adhesion is addressed. The problem is considered in the context of a very stiff sphere adhering to a compliant, isotropic, linear elastic substrate and experiencing adhesion and frictional slip relative to each other. The adhesion is considered to be driven by very large attractive tractions between the sphere and the substrate that can act only at very small distances between them. As a consequence, the adhesion behavior can be represented by the Johnson–Kendall–Roberts model, and this is assumed to prevail also when frictional slip is occurring. Frictional slip is considered to be resisted by a uniform, constant shear traction at the slipping interface, a model that is considered to be valid for small asperities and for compliant elastomers in contact with stiff material. A simple model for the interaction of friction and adhesion is utilized, in which some of the work done against frictional resistance is assumed to be stored reversibly. This behavior is considered to arise from surface microstructures associated with frictional slip such as interface dislocations, where these microstructures store some elastic strain energy in a reversible manner. When it is assumed that a fixed fraction of the work done against friction is stored reversibly, we obtain good agreement with data. [DOI: 10.1115/1.4045794]

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1 Introduction

There is significant interest in the question of how frictional slip affects adhesion between solid objects. An exemplar for this topic is a stiff sphere pressed by a force \( P \) into a compliant substrate and then, due to a tangential force \( T \), moved parallel to the flat surface of the substrate, as shown in Fig. 1. It is assumed that there is adhesion between the sphere and the substrate while friction resists the tangential motion. We consider the adhesion to be due to very large attractive interaction forces that arise only when the surface of the sphere is very close to the surface of the substrate. In addition, we assume that the radius, \( a \), of the contact is much smaller than the radius, \( R \), of the sphere. As a result of these two assumptions, we can utilize the Johnson–Kendall–Roberts (JKR) [1] model of adhesion. During tangential motion of the sphere relative to the substrate, there can be slip between the surface of the sphere and that of the substrate. As a result, an interface shear traction of magnitude \( \tau \) is associated with a work rate \( \tau d\delta/\delta t \), where \( d\delta/\delta t \) is the projection of the slip rate vector on the direction of the shear traction. Following Johnson [2], we assume that some of this work rate, designated \( \tau d\delta/\delta t \), is stored reversibly and can thus be recovered. Clearly, the remainder, \( \tau d\delta/\delta t \), of the work rate \( \tau d\delta/\delta t \) is dissipated irreversibly as heat. The work stored reversibly, if released, can contribute to overcoming the adhesion between the sphere and the substrate. In such circumstances, slip would diminish the effective strength of adhesion and lead to a reduction in the size of the contact at a fixed value of \( P \) [3,4]. We comment that it may be possible that the processes involved cause slip to eliminate spontaneous elastic surface microstructural deformations, and that, therefore, slip may

Fig. 1 A rigid sphere adhering to a linear elastic isotropic substrate and subject to applied loads \( P \) and \( T \) and experiencing displacements \( \Delta \) and \( D \). The contact between the sphere and the substrate has radius \( a \).
contribute to an effective increase in adhesion and an increase of the size of the contact at a fixed value of $P$ [5]. However, such a situation seems less likely.

We note that Johnson [2] presented a model for the interaction of friction and adhesion that relied on concepts of mixed-mode fracture mechanics [6] but with nonlinear features included that allowed for the presence of slip. As indicated above, the mechanism assumed by Johnson [2] for the effect that slip has on adhesion is the reversible storage and release of energy when slip occurs at the interface. Furthermore, Kim et al. [7] addressed the meaning of fracture mechanics energy release rates when frictional slip is present in the interaction of the adherent sphere and the flat substrate. By doing so, Kim et al. [7] put the model of Johnson [2] into context, proving it to be meaningful over a range of conditions that can be encountered in adhesion and friction, including large degrees of slip.

The current paper serves to clarify and illustrate further some ideas from Refs. [2, 7] that apply to how frictional slip can influence adhesion. Specifically, by invoking JKR adhesion, along with a reliance on the explicit generation and loss of elastic surface microstructural deformations due to slip, the present paper attempts to provide clearer insights into how slip affects adhesion.

A reason for pursuing this topic in a festschrift dedicated to John Hutchinson is that there are aspects of this area that are reminiscent of mixed-mode fracture mechanics [6], a subject in which John did much pioneering work. Therefore, this paper serves to remind R.M.M. of the happy days of some time ago when John and R.M.M. worked within a broader collaborative group addressing research on mixed-mode fracture in the context of advanced composite materials. K.S.K. also has fond memories of learning mixed-mode fracture mechanics from John to solve a thin film detachment problem and is very pleased to contribute to this paper in his honor. We add that the co-authors of this paper are thrilled that John is ranked number one in mechanical engineering research in the world according to recent findings by Ioannidis et al. (PLOS Biology, 17(8): e3000384) and note that it comes as no surprise.

2 Background

Experiments that combine measurements of adhesion parameters during sliding suggest that frictional slip weakens adhesion and leads to a reduction in the size of the contact when the applied compression is held constant. Savkoor and Briggs [3] measured the contact radius for a rubber hemisphere pressed against clean glass at constant applied compressive load and found that the size of the contact diminished as the tangential force was increased. Carpick et al. [4] carried out AFM experiments with a platinum-coated tip pressed against a mica surface. They were unable to measure the contact size but comment that the pull-off force during lateral scanning was smaller in magnitude than when no lateral scanning was implemented. Johnson [2] notes that this reduction was by 11%. On the other hand, Menga et al. [8] cite some experiments in which frictional slip enhances adhesion. For example, Krick et al. [5] identify an increase in the contact area during sliding of nitride rubber spheres on borosilicate float glass; we note, however, that Krick et al. [5] attribute the observed increase in adhesion to a viscoelastic effect.

With this background, we consider the geometry illustrated in Fig. 1, where a rigid sphere of radius $R$ is adhered, through a contact circle of radius $a$, to a compliant, linear elastic, isotropic substrate. The substrate has elastic modulus $E$, Poisson’s ratio $\nu$, and reduced modulus

$$E' = \frac{E}{1 - \nu^2} \tag{1}$$

The sphere is subjected to an applied load $P$ in compression and another, $T$, parallel to the substrate surface, with the sphere experiencing a displacement $\Delta$ toward the substrate and $D$ parallel to it. Note that the force $T$ parallel to the substrate is shown in Fig. 1 applied to the sphere at its lowest point; the reason for this will become clear below. However, the reader is reminded that elementary considerations of statics for a rigid body allows us to apply a combination of moments and forces anywhere to the sphere that is statically equivalent to the force, $T$, shown in Fig. 1, thereby avoiding the difficulty of actually applying the force at the bottom of the sphere.

It is assumed that $a \ll R$ so that Hertzian contact [9], JKR adhesion theory [1], the Maugis–Dugdale adhesion model [10], and the shearing model of Keer and Goodman [11] can be exploited in the appropriate regimes. We take the adhesion energy in the absence of motion and loading parallel to the substrate to be $w_o$. It is equal to $\gamma_1 + \gamma_2 - \gamma_{12}$, where $\gamma_1$ and $\gamma_2$ are the effective specific surface energies of, respectively, the sphere and the substrate when exposed to air, and $\gamma_{12}$ is the effective specific energy of the interface between the sphere and the substrate when touching. When the material of the sphere slips relative to the substrate, it is assumed that the slipped surface experiences a constant, uniform shear traction, $\tau_o$. As discussed by Johnson [9], a friction model with uniform, constant shear traction is valid for a singleasperity in a rough surface. In addition, experimental results of Carpick et al. [4] for an AFM tip demonstrate a constant, uniform shear traction during sliding. A friction model with uniform, constant shear traction is also appropriate for a compliant elastomer in contact with a stiff material [8].

3 Reversible Energy Released During Contact Area Shrinkage

As noted above, we consider the possibility of the existence of elastic surface microstructural deformations that are generated and relaxed by slip between the sphere and the substrate. We postulate that these features could be interface dislocations or similar defects as discussed by Johnson [2]. In our analysis, we assume these to be associated with the surface of the substrate as it is deformable whereas the sphere is not. However, in general, such features could occur in both of the objects in contact. We note that such defects generally have stored elastic energy associated with them. Therefore, if such defects are generated by slip, they will release elastic energy when eliminated upon shrinkage of an adhesive contact area that has experienced sliding. This implies that, as noted above, their released energy can weaken the effect of adhesion.

Consider the slipping segment of the contact, as depicted in plan view in Fig. 2. The slipping segment occupies an annulus $b < r \leq a$, where $r$ and $\theta$ constitute a polar coordinate system with origin at the center of the contact. A Cartesian coordinate system is also shown in Fig. 2, with the direction of motion of the sphere being along the
x-axis in the positive direction. The contact within the circle $r \leq b$ experiences no slip so that its displacement in the sliding direction is the same as that of the sphere. Per unit area of the slipping interface, the total slip work rate is divided up according to

$$\tau_o \frac{d\delta}{dt} = \tau_o \frac{d\delta^v}{dt} + \tau_o \frac{d\delta^{irr}}{dt}$$  \hspace{1cm} (2a)$$

where, as noted above, $\delta^v$ is the reversible part of $\delta$ associated with interface defects and features as described above, $\delta^{irr}$ is the irreversible part, and $r$ is the time. The reversible and irreversible slip add up to the total slip

$$\delta = \delta^v + \delta^{irr}$$  \hspace{1cm} (2b)$$

Since $\delta^{irr}$ is reversible, the reversible shear work stored in the interface is

$$E_S = \tau_o \int_A \delta^v dA$$  \hspace{1cm} (3)$$

where $A$ is the area of the contact. Note that the integrand is zero in the non-slipping segment of the interface within $0 < r \leq b$. Since the irreversible slip rate generates heat, we conclude that the rate of heat generation is

$$\frac{dQ}{dt} = \tau_o \int_A \frac{d\delta^{irr}}{dt} dA = \tau_o \int_A \left( \frac{dA}{dt} - \frac{d\delta^{irr}}{dt} \right) dA$$  \hspace{1cm} (4)$$

From this result, we see that if all slip work is dissipated as heat, there can be no slip work that is stored reversibly.

In addition, energy balance requires that

$$\frac{dQ}{dt} = P \frac{dA}{dt} + \tau_o \int_A \frac{d\delta^v}{dt} dA = \tau_o \int_A \frac{d\delta^{irr}}{dt} dA$$  \hspace{1cm} (5)$$

where $E_A + E_D$ is the strain energy stored in the substrate and $E_S$ is the energy stored reversibly in elastic surface microstructural deformation. Note that we keep $E_A$ separate from $E_D + E_D$, although both are strain energy. In addition, we distinguish between the strain energy, $E_A$, associated with displacement of the sphere normal to the surface, and $E_d$, associated with sphere motion parallel to the substrate surface. From JKR [1], we obtain

$$P = 2aE^* \left( \Delta - \frac{a^3}{3R} \right)$$  \hspace{1cm} (6)$$

and

$$E_A = E^* \left( a\Delta^2 - \frac{2a^3\Delta}{3R} + \frac{a^5}{5R^2} \right)$$  \hspace{1cm} (7)$$

so that Eqs. (4) through (7) together lead to

$$\tau_o \int_A \frac{d\delta^v}{dt} dA = T \frac{dD}{dt} - E^* \left( \frac{a^2}{R} - \Delta \right) \frac{2}{dt} d\frac{da}{dt} - \frac{dE_D}{dt} d\frac{dE_S}{dt} + 2\pi a \omega d\frac{da}{dt}$$  \hspace{1cm} (8)$$

We first consider limited slip so that $b > 0$. We assume such limited slip to have occurred during monotonic tangential motion of the sphere. The $x$ component of displacement within the contact circle at the surface of the substrate is

$$u_x = D - 0 \leq r \leq b$$  \hspace{1cm} (9a)$$

$$u_y = D - \frac{2\tau_o}{\pi(1-\nu)E^*} \int_0^r \rho \left[ \left( 2-\nu \right) \cos^{-1} \left( \frac{\rho}{r} \right) - \frac{\nu\rho}{r} \sqrt{r^2 - \rho^2} \cos \theta \right] d\rho$$ \hspace{1cm} \text{where} \hspace{1cm} b \leq r \leq a$$  \hspace{1cm} (9b)$$

a result that is due to Keer and Goodman [11] but rededuced by Savkoor [12]. In limited slip, the extent of slip that takes place during monotonic motion is given by

$$\delta = D - u_s = \frac{2\tau_o}{\pi(1-\nu)E^*} \int_0^r \rho \left[ \left( 2-\nu \right) \cos^{-1} \left( \frac{\rho}{r} \right) - \frac{\nu\rho}{r} \sqrt{r^2 - \rho^2} \cos \theta \right] d\rho$$ \hspace{1cm} \text{where} \hspace{1cm} b \leq r \leq a$$  \hspace{1cm} (10)$$

while it is zero within $0 \leq r \leq b$.

Following a procedure used by Menga et al. [8], with the use of Eq. (9b), we compute the strain energy in the substrate that is associated with shear as

$$\varepsilon_b = \frac{1}{2} \int_A \tau_o u_x dA = \frac{1}{2} T \delta D - \frac{2(2-\nu)\tau_o^2}{(1-\nu)E^*} \int_b^r \frac{\rho}{r} \sqrt{r^2 - \rho^2} \cos \theta \frac{d\rho}{dr}$$  \hspace{1cm} (11)$$

where $\tau_o$ is the Cartesian component of the traction applied by the sphere to the substrate within the contact circle.

Substitution of Eqs. (3), (10), and (11) into Eq. (8) provides

$$\frac{4(2-\nu)\tau_o^2}{2(1-\nu)E^*} \int_b^r \frac{\rho}{r} \sqrt{r^2 - \rho^2} \cos \theta \frac{d\rho}{dr}$$  \hspace{1cm} (12)$$

where we have recognized that, after completion of the integrals, the term in Eq. (10) containing $\cos \theta$ will cancel. Also, we note that differentiation of $r$ with respect to time at a given material point in the substrate is omitted in the procedures used as the consequential result will also cancel upon completion of the integrals.

We note that Kim et al. [7] demonstrated that

$$\frac{d}{dt} \frac{d}{dt} d\left( \phi^v \right) dA + \frac{d^2}{dt^2} d\left( \phi^{irr} \right) dA = \int_A \frac{d\delta^v}{dt} dA + 2\pi a \omega \frac{da}{dt}$$  \hspace{1cm} (13)$$

where $D^v$ is the average of the reversible slip around the perimeter of contact. We thus obtain a simplification of Eq. (12) in the form

$$\frac{4(2-\nu)\tau_o^2}{2(1-\nu)E^*} \int_b^r \frac{\rho}{r} \sqrt{r^2 - \rho^2} \cos \theta \frac{d\rho}{dr}$$  \hspace{1cm} (14)$$

We note that Savkoor [12] provides

$$D = \frac{2(2-\nu)\tau_o}{(1-\nu)E^*} \sqrt{a^2 - b^2}$$ \hspace{1cm} D \leq \frac{2(2-\nu)\tau_o}{(1-\nu)E^*}$$  \hspace{1cm} (15a)$$
\[ T = 2\tau_r \left( b\sqrt{a^2 - b^2} + a^2\cos^{-1} \frac{b}{a} \right), \quad D \leq \frac{(2 - \nu)\tau_{r,a}}{(1 - \nu)E} \]  
(15b)

showing that they are functions of \( a \) and \( b \) together with material parameters. The limit on \( D \) is where gross slip commences.

To obtain the next step, we consider the case where slip does not affect adhesion, in which situation JKR [1] leads to the usual result that

\[ E' \left( \frac{a^2}{R} - \Delta \right)^2 = 2\pi a w_0 \]  
(16)

It can be further shown that the independence of adhesion from slip requires that \( \varepsilon_s = 0 \) and that

\[ \frac{dQ}{dt} = T \frac{dD}{dt} - \frac{d\delta'}{dt} \]  
(17)

because otherwise slip, given its role in generating \( \varepsilon_D \) and \( \varepsilon_s \), would be able to contribute to adhesion. Together, these observations lead to the conclusion that, in the situation where slip does not influence adhesion, \( \delta'' = 0 \). This result, together with the condition in Eq. (16), leads to

\[ \frac{4(2 - \nu)\tau_r^2}{(1 - \nu)E} \int_r^D \frac{d}{dr} \left[ \frac{\rho \cos^{-1} \left( \rho / \rho_0 \right)}{\sqrt{\rho^2 - \rho_0^2}} \right] \right\} \]  
(18)

We note that all terms appearing in Eq. (18), other than material properties, are functions of only \( a, b, \) and \( t \). From this, we conclude that Eq. (18) is an identity and not a result unique to adhesion being independent of slip. From this conclusion, it follows that Eq. (14) simplifies to

\[ \frac{E'}{2\pi a} \left( \frac{a^2}{R} - \Delta \right)^2 + \tau_r \delta'' = w_o \]  
(19)

demonstrating that the reversible energy release rate upon shrinkage of the contact area, in this case, is

\[ \delta'' = \frac{E'}{2\pi a} \left( \frac{a^2}{R} - \Delta \right)^2 + \tau_r \delta'' \]  
(20a)

Note the analogy to the Mauis–Dugdale model for adhesion [10] where, in the absence of slip, the reversible energy release rate is \( \sigma_r H \), where \( \sigma_r \) is the adhesive traction and \( H \) is the gap between the two material surfaces at the perimeter of the contact.

We can also write the reversible energy release rate in terms of the applied load, in which case

\[ G'' = \frac{E'}{2\pi a} \left( \frac{a^2}{3R} - \frac{P}{2Ea} \right)^2 + \tau_r \delta'' \]  
(20b)

Now consider gross slip, in which case \( b = 0 \). The results developed above can be converted to account for gross slip. However, we find it more insightful to use additional analytical results to tackle this case. Furthermore, the explicit manipulations that are made possible through use of the analytical results will provide confirmation of the conclusions drawn earlier.

Results developed by Menga et al. [8] show that slip is given by

\[ \delta = D - u_s = D - \frac{2\tau_r a}{(1 - \nu)E} \right\} \]  
(21)

where, in this case, \( D \) is an arbitrary amount of slip; i.e., it is no longer given by Eq. (15a) but is an independent parameter. In Eq. (21), \( K(r/a) \) is the complete elliptic integral of the first kind and \( L(r/a) \) is the complete elliptic integral of the second kind. Note that we have generalized the result slightly compared to that of Menga et al. [8] to account for an arbitrary Poisson’s ratio of the substrate. The result in Eq. (21) is obtained from analysis provided in Refs. [8,9,12], along with results to be found in Ref. [13].

We note that the result in Eq. (21) has a deficiency in regard to the direction of the shear traction at the leading edge of contact. This issue is described in Appendix.

When \( P = 0 \), the displacement within the contact circle in the vertical direction is, in gross slip,

\[ u_c = -\frac{(1 - 2\nu)\tau_r x}{2(1 - \nu)E} \]  
(22)

with the positive sense of this displacement being toward the substrate. By defining \( T \) to be applied at the bottom of the sphere, we avoid any constraint that will limit the displacement given by Eq. (22). Menga et al. [8] used the equivalent of Eq. (11) to compute the strain energy of distortion of the substrate during gross slip due to monotonic motion of the sphere as

\[ \varepsilon_D = \frac{4(2 - \nu)\tau_r^2 a^3}{3(1 - \nu)E} \]  
(23)

Again, we have generalized the result to allow for any value of Poisson’s ratio.

Note that during gross slip, the strain energy, \( \varepsilon_D \), is independent of \( D \) and depends only on the size of the contact. The force, \( T \), in this case is given by

\[ T = \tau_r \pi a^2 \]  
(24)

with \( \varepsilon_s \) still given by Eq. (3). Since Eq. (13) is still valid, Eq. (8) leads to

\[ \tau_r \pi a^2 \frac{dD}{dt} = \frac{2(2 - \nu)\tau_r^2}{(1 - \nu)E} \int_A \frac{d}{dr} \left[ a L \left( \frac{r}{a} \right) \right] \right\} \]  
(25)

where, as previously, the term in Eq. (21) containing \( \cos \theta \) contributes zero upon integration and the differentiation with respect to time in the integral on the left hand side of Eq. (25) is applied only to \( a \) and not to \( r \). We note that [13]

\[ \int_A \frac{d}{dr} \left[ a L \left( \frac{r}{a} \right) \right] \right\} \]  
(26)

and Eq. (25) becomes Eq. (19), showing that Eq. (20) is still the reversible energy release rate upon shrinkage of the contact area.

These results together show that slip can only influence adhesion if \( \delta'' \neq 0 \), i.e., if \( \varepsilon_s \neq 0 \). Conversely, if all slip is dissipated as heat, \( \delta'' = 0 \) and slip cannot affect adhesion.

We note that Eq. (20) is a special case of a more general result derived by Kim et al. [7], but it has now been obtained specifically for the case where JKR adhesion prevails in the absence of slip.

### 4 A Simple Model for Reversible Slip

At this stage, we have simply postulated that some of the slip between the sphere and the substrate generates reversible work
and the remainder dissipates heat. We now introduce a very simple model for the reversible part of the slip by assuming that it is a fixed fraction of the total slip up to a saturation level. This approach is inspired by a model for the effect of friction on adhesion that is due to Johnson [2]. Furthermore, we can justify the model by pointing to the fact that the density of many defects increases with the magnitude of deformation, where, in this case, the increasing deformation is tied to the magnitude of slip.

Since it is the average of the reversible slip around the contact perimeter that controls the reversible energy release rate of the system upon axisymmetric shrinkage of the contact area, we cast our simple model in the form

\[ \delta^\text{re} = \alpha \delta_0, \quad 0 \leq \delta_0 \leq \delta^m \]

\[ \delta^\text{m} = \alpha \delta^m, \quad \delta^m \geq \delta^0 \]

(27a)

(27b)

where the subscript \( o \) indicates that the parameter is being evaluated at the perimeter of the contact and the bar overhead indicates that the average around the perimeter is being used. The parameter \( \delta^m \) is the magnitude of slip at which reversible energy storage saturates. We note that if \( \alpha = 0 \), there is no reversible sliding and adhesion is unaffected by slip.

Consider limited slip. In that case, Savkoor’s [12] result in Eq. (10) may be averaged around the perimeter to obtain [2,7]

\[ \delta_o = \frac{2(2 - \nu)\nu a}{\pi(1 - \nu)E\alpha} \left( \frac{1}{1 - \frac{b^2}{a^2}} \cos^{-1} \frac{b}{a} + \frac{1}{a} \right) \]

(28)

\[ D = \frac{(2 - \nu)\tau_o a}{4(1 - \nu)E\alpha} \]

and \( D \) is given by Eq. (15a), while \( T \) is determined by Eq. (15b). We assume that the magnitude of slip experienced in a particular case is small, as is the situation for the data of Savkoor and Briggs [3], where we estimate that it was less than 0.1 \( \mu \)m in their experiments. Thus, the parameter \( \delta^m \) does not shrink much below \( \delta^0 \). We make use of this observation to obtain a linearized relationship from Eq. (15) in the form

\[ D = \frac{(2 - \nu)T}{4(1 - \nu)E\alpha} \]

(29)

Similarly, we expand the result in Eq. (28) asymptotically for values of \( \delta^m \) close to \( \delta^0 \) and make use of Eq. (29) to obtain

\[ \delta_o = \frac{(2 - \nu)T^2}{16\pi(1 - \nu)\nu E\alpha} \]

(30)

As a consequence, the energy release rate from Eq. (20b) becomes

\[ \sigma^o = \frac{E}{2\pi a} \left( \frac{2a^2}{3R} - \frac{P}{2E\alpha} \right)^2 + \frac{a(2 - \nu)T^2}{16\pi(1 - \nu)E\alpha a} \]

\[ 0 \leq \sigma_o \leq \sigma^m \]

(31)

Equilibrium requires this to be equal to the adhesion energy so that

\[ \frac{E}{2\pi a} \left( \frac{2a^2}{3R} - \frac{P}{2E\alpha} \right)^2 + \frac{a(2 - \nu)T^2}{16\pi(1 - \nu)E\alpha a} = \sigma_o, \quad 0 \leq \sigma_o \leq \sigma^m \]

(32)

We note that the result in Eq. (32) coincides with one by Papangelo and Ciavarella [14] (their variant “c”) that is derived from mixed-mode fracture mechanics [6]. This convergence arises because our result in Eq. (32) is essentially a small-scale slip version of our more general result.

In Fig. 3, we compare results from Eq. (32) with the data of Savkoor and Briggs [3]. As noted above, they used a rubber hemisphere pressed against a glass plate. Our results for a rigid sphere pressed against an elastic substrate provide an equivalent model. To compare our results with their data, we use \( E = 0.1724 \text{ MPa} \), a value we have deduced from Savkoor and Briggs’ [3] adhesion data in the absence of applied loads. We also assume incompressibility of the rubber so that \( \nu = 0.5 \). The radius of the rubber hemisphere used by them is \( R = 20 \text{ mm} \). Parametric values used by us to compare our results with Savkoor and Briggs’ [3] data are given in Table 1.

The values of the compressive load, \( P \), given in Table 1 are those used by Savkoor and Briggs [3] in their experiments. The adhesion energies, \( \sigma_o \), in the table are those measured by them. Savkoor and Briggs [3] comment that the adhesion energy at low peeling rates is extremely sensitive to surface conditions and, for this reason, was measured by them for each experiment from the contact area at zero load. This observation gives the context for the differing values of the adhesion energy in Table 1. In our comparison of the model of Eq. (32) with the experiments, we chose a value of \( \alpha \) for each compressive load that gave a good fit of the model to the data. We justify this model fitting by the same reasoning used by Savkoor and Briggs [3] that the behavior observed in the experiments is extremely sensitive to surface conditions and varied from experiment to experiment, as confirmed by the differing values of the adhesion energy. We provide these values of \( \alpha \) in Table 1.

We note also that we have assumed that the reversible slip that occurs in the experiments of Savkoor and Briggs [3] is less than the magnitude, \( \delta^m \), at which reversible energy storage saturates. It can be seen from Fig. 3 that, once an appropriate value of \( \alpha \) has been selected, the model agrees reasonably well with the data of Savkoor and Briggs [3]. The result of the model for \( P = 3.4 \text{ mN} \) implies that detachment occurs for the case where the tangential force is 23 mN, i.e., at the data point having the smallest value of the contact radius. However, Savkoor and Briggs [3] give no indication that such detachment occurred in their experiments and, indeed, comment that they terminated their experiment at arbitrary values of the shearing load, \( T \).

Table 1 Parameter values used in Eq. (32) for comparison of the adhesion/friction model with the data of Savkoor and Briggs [3]

| \( P \) (mN) | \( \sigma_o \) (J/m²) | \( \alpha \) |
|---------|--------------------|--------|
| 3.4     | 0.108              | 0.34   |
| 0       | 0.143              | 0.53   |
| 3.4     | 0.123              | 0.85   |
| 6.5     | 0.137              | 0.67   |
5 Discussion

We have developed a model for the effect of sliding on the adhesion of a spherical object to a flat surface, where one of the components is very stiff and the other is compliant. Our model is inspired by one developed by Johnson [2] who based his one on mixed-mode fracture mechanics [6], as did Savkoor and Briggs [3] whose approach was less general than Johnson’s [2]. The essence of Johnson’s [2] model was the identification of $\tau_0 \delta_0$ as an analogy for what is termed mode II in fracture mechanics. In addition, Johnson [2] utilized $\sigma_H$ from the Maugis–Dugdale [10] model to represent mode I and used the ratio $\tau_0 \delta_0 / \sigma_H$ in his model for how slip would affect adhesion. Without derivation, we note that if Johnson’s [2] model is specialized to the JKR regime, and if the degree of slip is assumed to be small, the asymptotic form of his model becomes identical to ours, as represented by our Eq. (32). We note, also, the similarities between Johnson’s [2] Fig. 9 and our results in our Fig. 3, though Johnson plots contact area in his figure, whereas we have plotted the contact radius. We conclude, therefore, that our model is essentially identical to that of Johnson [2], but is, perhaps, more transparent due to the special case we have chosen to analyze. Our treatment, we believe, also clarifies some points previously made by Kim et al. [7], an outcome brought about, again, due to the relative simplicity of the special case we have treated in the present paper.

We also comment on the unfortunate aspect of slip rate reversal that is a feature of the solutions developed by Menga et al. [8], and, by implication, also of the analysis of Keer and Goodman [11] and Savkoor [12]. This issue is discussed in Appendix. This slip rate reversal that extracts energy from the interface is unsatisfactory. A better solution would not exhibit such a deficiency, but we are unaware, at present, of an alternative analysis. It is to be hoped that our insights are not held hostage by the deficient features of the solution. What can certainly be said is that the important results in the form of Eq. (20) and (32) are correct notwithstanding the slip rate reversal occurring in the solutions of Menga et al., Keer and Goodman, and Savkoor [8,11,12]. We can assert this because the basis for Eq. (20) and (32) was originally derived by Kim et al. [7] independently of the details of the solutions in Refs. [8,11,12]. Furthermore, it can also be said that an interface that converts all of the slip into heat will have $\delta'' = 0$ even after the slip rate reversal is eliminated as a feature of a solution. Thus, an interface that converts all slip into heat will have adhesion characteristics that are unaffected by slip. This conclusion is robust because there is no mechanism in such an interface to take heat and convert it to adhesion energy.

6 Conclusions

We have considered the problem of an adherent rigid sphere sliding on a linear elastic, isotropic substrate in conditions where the radius of the contact is small compared to the radius of the sphere. Adhesion is assumed to obey the JKR model. We construct an energy balance, including the rate of dissipation of heat, the rate of external work, the rate of change of stored elastic strain energy, the rate of change of reversible energy stored in the interface, and the rate of change of adhesion energy. From this balance, we find that when all frictional slip is dissipated as heat, adhesion is unaffected by slip. For a frictional model in which sliding is opposed by a uniform shear traction in the segment of the interface that slips, we find that the reversible energy release rate is that associated with JKR adhesion plus a contribution from slip. The latter is equal to the average of the reversible slip around the perimeter of the contact multiplied by the frictional shear traction. The work associated with the balance of slip, the irreversible component, is dissipated as heat. We postulate that the reversible component of slip may be zero, in which case sliding has no influence on adhesion. We find that in some cases the reversible component of slip is positive, and thus, in those cases, reduces the effectiveness of the adhesion. We successfully compare the results of our model with some data for the interaction of adhesion and slip.

Appendix: The Shear Traction at the Leading Edge of the Contact During Gross Slip

Slip in Eq. (21) is such that

$$\delta = \delta(r, \theta, D, a)$$

where functional dependence on material parameters has been omitted. For a material point on the substrate surface, each of the arguments in Eq. (A1) can change with time, with all except $a$ definitely doing so during gross slip. Thus, for a material point in the contact surface, we write

$$\frac{d\delta}{dt} = \left( \frac{\delta \delta \delta \theta}{\alpha} + \frac{\delta \delta \delta \theta}{\alpha} + \frac{\delta \delta \delta \theta}{\alpha} \right) \frac{d\theta}{dt} + \frac{\delta \delta \delta \theta}{\alpha}$$

(A2)

It is insightful to consider the special case of $\nu = 0$ as this simplifies things greatly. In that case

$$\frac{d\delta}{dt} = \left( \frac{1}{\pi E \gamma} \left( \frac{L}{a} - K \left( \frac{L}{a} \right) \right) \cos \theta \right) \frac{d\theta}{dt} + \frac{4 \tau_0 \alpha}{\pi E \gamma} K \left( \frac{L}{a} \right) \frac{da}{dt}$$

(A3)

Now, assume that the system is experiencing gross slip in steady state so that $d\delta/dt = 0$. Since the complete elliptic integral of the first kind, $K(\nu)$, diverges to $\infty$ at $\nu = 1$, the slip rate reverses in sign at the leading edge of the contact, i.e., at $(r, \theta) = (a, 0)$ and nearby. It is thus strictly inconsistent there with the assumed direction of the shear traction in the slipping contact circle. This situation is analogous to the issue explored by Adams et al. [15], who found a similar reversal of the slip rate for a sliding square bottomed punch. Note that slip reversal in Eq. (A3) can only be avoided at the leading edge of the contact if $d\delta/dt = -d\delta/dt$, in which case the leading edge is not progressing over the substrate.

We assume that, in the solutions we have used above, the same difficulty of slip rate reversal occurs at the leading edge of the sphere’s contact circle for any value of Poisson’s ratio, and for the case of limited slip as well as for gross slip.

We note that the situation just described strictly violates the second law of thermodynamics locally, as work is being extracted from the interface where the slip rate reversal is occurring. However, for want of a more consistent solution, we accept the situation, with the proviso that we should avoid violating the second law of thermodynamics on a global basis and require that the total rate of heat generation must be no less than zero. It is anticipated that there will be situations where the inconsistency of slip rate reversal does not cause a big difference to the assessments subsequently made. Indeed, Adams et al. [15] comment that it is a reasonable engineering solution to ignore the slip rate reversal, and Hills et al. [16], in their discussion of Adams et al. [15], make similar points.

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