Critical Saturation Function Based Sliding Mode Control for Path Tracking of Mobile Robot

Hong-Bin Ning1,2, Xiao-Fei Deng1,*, Jun-Wei Zhu1, Kai-Qing Zhou1

1 School of Information Science and Engineering, Jishou University, Jishou 416000, Hunan China.
2 Shaoyang Branch, China United Network Communications Group Co., Ltd., Shaoyang, Hunan 422000, China.

E-mail: xiaofei0228@163.com

Abstract. This paper proposes a new sliding mode controller, as to enhance the performance of path tracking of three-wheeled mobile robot system. To begin with, backstepping design is used to construct a new sliding mode control law for better dynamic response and robustness. Next, in order to reduce chattering, the symbolic function is replaced with the critical saturation function in the conventional reaching law. At last, the simulation results show that backstepping design combined with saturation function can not only improve the response rate and robustness of the system, but also effectively reduce the chattering.

1. Introduction

Wheeled mobile robot has broad application prospects in many fields, such as intelligent industrial manufacturing. However, the control of wheeled robot- a nonlinear uncertain system, it is difficult to apply in practice. Therefore, more and more scholars have invested in the research of sliding mode control.

As a strategy of variable structure control system, sliding mode control was proposed by former Soviet scientists in 1950s, and has been widely developed and applied since then. It is a kind of nonlinear control, which is realized by switching function, and controls the structure (control law or controller parameter) of the controller according to the degree of deviation of the system state from the sliding mode, so that the system operates according to the law prescribed by the sliding mode. A stable discrete sliding mode control insensitive to the choice of sampling interval and not yielding chattering [1]. Then, a new terminal sliding mode manifold for the second-order system to enable the elimination of the singularity problem associated with conventional terminal sliding mode control, and then presents a global non-singular terminal sliding mode controller for rigid manipulators [2]. After that, people raised a new design approach of an adaptive fuzzy terminal sliding mode controller for linear systems with mismatched time-varying uncertainties [3]. The design method they proposed does not need to know the bounds of uncertainty in advance, and the stability of the fuzzy control system is also guaranteed.

With the rapid development of robot, sliding mode control technology is rapidly applied to robot system. Scholars combine sliding mode technology with robot technology, and improve the sliding mode control algorithm based on the characteristics of the robot they studied. Various robot control schemes have been developed, such as linear schemes, passivity-based schemes, Lyapunov-based schemes, sliding mode control schemes, nonlinear H-infinity schemes and robust adaptive control schemes [4]. Later, a new approach for the design of variable structure control (VSC) of nonlinear...
systems [5]. The approach is based on a new method called the reaching law method, and is complemented by a sliding mode equivalence technique. A design method based on sliding mode adaptive fuzzy control system for trajectory tracking control of unknown nonlinear dynamic robots [6]. This technique has good system stability and tracking error convergence. Above the previous research results, present a sliding mode control law for a nonholonomic wheeled mobile robot using reverse step technology, and for solving the speed jump problem of the traditional sliding mode tracking controller, a sliding mode control law based on the neural dynamics model is proposed [7].

Referring to the results and methods of previous researchers, on the one hand, backstepping design can improve the robustness of the system and improve the tracking accuracy, but it cannot reduce the chattering. On the other hand, the saturation function can effectively reduce the flutter, but due to the fixed boundary layer thickness, the system trajectory cannot converge asymptotically to the given switching plane, so there is no sliding mode on the switching plane, which reduces the robustness of the system on the switching plane. In order to solve this problem, this paper combines the backstepping idea with bounded saturation function, and designs a sliding mode controller which can not only improve the robustness, but also reduce the chattering.

The structure of this paper is as follows. The model of three-wheeled mobile robot concerned is described in Section II. Traditional design of sliding mode control and the design steps are introduced in Section III. Sliding mode controller based on improved backstepping design which has adopted Critical Saturation Function is put forward in Section IV. In the end, some experiments show the benefits of the proposed method in Section V and the conclusion is given in Section VI.

2. Kinematic Model of Three-wheeled Mobile Robot

In this paper, the three-wheeled mobile robot is studied, which is made of two driving wheels and a steering wheel. In the course of the study, assumed that the robot under studying is an ideal system, namely: the robot is rigid body; wheels do purely roll motion on the horizontal plane; and the wheels not slipping phenomenon when it is running. On this basis, a wheeled mobile robot model is established with the point C of the geometric center of the mobile robot as the reference point. The model of the studied mobile robot is shown in figure 1.

![Figure 1. Mobile robot kinematics model](image)

According to the above figure, the kinematic model equation (1) is obtained:

\[
\begin{align*}
\dot{x} &= v \cos \theta \\
\dot{y} &= v \sin \theta \\
\dot{\theta} &= \omega
\end{align*}
\]  

(1)

Primarily, achieve \( x_b \), \( y_b \), \( \theta_b \) tracking through designing control law, then according to the basic transformation formula of coordinates and Figure 1, the posture error equation of mobile robot can be obtained as follow:

\[
e_1 = \cos \theta (x_b - x) + \sin \theta (y_b - y)
\]  

(2)
\[ e_2 = -\sin\theta (x_b - x) + \cos\theta (y_b - y) \]  
\[ e_3 = \theta_b - \theta \]  
\( (3) \)

Get the simplified equation:
\[ \begin{cases}  
\dot{e}_1 = e_2 \omega - v + v_b \cos e_3 \\
\dot{e}_2 = -e_1 \omega + v_b \sin e_3 \\
\dot{e}_3 = \omega_b - \omega 
\end{cases} \]  
\( (5) \)

Where, \( v_b \) and \( \omega_b \) are expressed as the linear velocity and angular velocity of expected mobile robot.

3. Traditional Design of Sliding Mode Control of Path Tracking

The design of the sliding mode controller is mainly divided into two steps. The switching function is constructed so that the sliding mode determined by it has good quality.

In this step, the switching function is designed as:
\[ s = \begin{pmatrix} s_1 \\ s_2 \end{pmatrix} = \begin{pmatrix} e_1 \\ c_1 e_2 + c_2 e_3 \end{pmatrix} (c_1, c_2 > 0) \]  
\( (6) \)

If \( s_1, s_2 \) tend to 0, then \( e_1 \) converges to 0 and \( e_2 \to 0, \quad e_3 \to 0 \).

Select the constant velocity reaching law and let
\[ \begin{pmatrix} s'_1 \\ s'_2 \end{pmatrix} = \begin{pmatrix} -k_1 \text{sgn} s_1 \\ -k_2 \text{sgn} s_2 \end{pmatrix} \]  
\( (7) \)

Substitute formulas (6) and (7) into formula (5), and then set \( v \) and \( w \) aside

Get the control law is
\[ \begin{pmatrix} v \\ \omega \end{pmatrix} = \begin{pmatrix} e_2 \omega + v_b \cos e_3 + k_1 \text{sign}(s_1) \\ c_1 v_b \sin e_3 + c_2 \omega_b + k_2 \text{sign}(s_2) \end{pmatrix} \]  
\( c_1 e_1 + c_2 \)  
\( (8) \)

After the controller is built, it is necessary to judge whether the controller satisfies the stability of the system according to Lyapunov stability determination principle.

Constructing the Lyapunov function as
\[ V = \frac{1}{2} s_1^2 + \frac{1}{2} s_2^2 \]  
\( (9) \)

Let's differentiate both sides
\[ \dot{V} = s_1 s'_1 + s_2 s'_2 \]  
\( (10) \)

Design the switching function so that \( s_1, s_2 \) tend to 0, elect the constant velocity reaching law, substitute the formula (7) into the formula (10) and combine to get
\[ \dot{V} = -k_1 s_1 \text{sgn} s_1 - k_2 s_2 \text{sgn} s_2 \leq 0 \]  
\( (11) \)

\( V \geq 0 \) and it is continuously differentiable, \( \dot{V} \leq 0 \) is also a continuous function, therefore, the stability of the system can be determined.

4. Sliding Model Controller Based On Improved Backstepping Design

Backstepping design can improve the robustness of the system and improve the tracking accuracy, but it cannot reduce the chattering. The saturation function can effectively reduce the chattering, but it reduces the robustness of the system on the switching plane. On the basis of backstepping design, a new sliding mode controller added bounded saturation function is designed which can not only improve the robustness, but also reduce the chattering.

When \( e_1 = 0 \), the Lyapunov function is assumed as
\[ V_z = \frac{1}{2} e_z^2 \geq 0 \]  
(12)

\[ V_z = e_z e_2 = -e_1 e_2 \omega + v_b e_2 \sin e_3 \]  
(13)

to make \( V_z \leq 0 \), let \( e_3 = -\arcsin e_2 \), so

\[ \sin(-\arcsin e_2) = -\sin(\arcsin e_2) = -e_2 \]  
(14)

Then get

\[ \dot{V}_z = e_2 e_z = -e_1 e_2 \omega - v_b e_2^2 \leq 0 \]  
(15)

\( V \geq 0 \) and it is continuously differentiable, \( \dot{V} \leq 0 \) is also a continuous function, therefore, the stability of the system can be determined by the Lyapunov stable decision principle.

Therefore, as \( e_1 \) converges to 0 and \( e_3 \) converges to \(-\arcsin e_2, e_2, e_3 \to 0 \) is implemented. Based on the above conclusions, the switching function is designed as:

\[ s = \begin{pmatrix} s_1 \\ s_2 \end{pmatrix} = \begin{pmatrix} e_1 \\ e_3 + \arcsin e_2 \end{pmatrix} \]  
(16)

Compared with the traditional sliding mode control, the reaching law is improved through replacing symbolic function with the critical saturation function:

\[ \text{sat}(s) = \begin{cases} 
1 & s > \Delta \\
ks & |s| \leq \Delta \\
-1 & s < -\Delta 
\end{cases} \]  
(17)

Where \( \Delta \) is the critical level. The significance of using saturation function is that it provides feedback control in the critical layer to reduce chattering generated by sliding mode surface switching. Switching control is carried out outside the critical layer to make the moving point of the system approach to the sliding mode surface quickly, so as to reduce the error of system control quantity.

Take new sliding mode control law is

\[ \begin{pmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \end{pmatrix} = \begin{pmatrix} e_2 \omega + v_b \cos e_3 + \text{sat}(e_1) \\ \omega_b + \frac{2 v_b \sin e_3 + \text{sat}(s_2)}{\sqrt{1 - e_2^2}} \\ 1 + \frac{e_1}{\sqrt{1 - e_2^2}} \end{pmatrix} \]  
(18)

5. Simulation Analysis

In the simulation section, the traditional sliding mode controller compared with the improved in terms of error convergence speed, tracking effect and chattering.

When the traditional sliding mode controller is used to track the sinusoidal path, assumed \( c_1 = c_2 = 0.5, k_1 = k_2 = 2.0, v_b = 1.0, \omega_b = \sin t \), and the initial posture is \([3 \ 0 \ 0] \). When using the improved sliding mode controller to track the sinusoidal path, the parameters are set as \( k_1 = k_2 = 12.0, v_b = 1.0, \omega_b = \sin t \), and the initial posture is selected as \([3 \ 0 \ 0] \).

Rate of change of posture command as follow:

\[ \begin{cases} 
\dot{e}_1 = v_b \cos \theta_t \\
\dot{e}_2 = v_b \sin \theta_t \\
\dot{e}_3 = \omega_b = \sin t
\end{cases} \]  
(19)
Figure 2 shows comparisons of error convergence. As can be seen from the figure, the improved controller in this paper and traditional sliding mode controller can both make the errors of $e_1$, $e_2$ and $e_3$ converge to 0 at $t=5.2s$. But the improved controller's sliding mode posture error convergence efficiency is faster than the traditional one. When $t=0.3s$, the errors of $e_1$, $e_2$ and $e_3$ of the improved one converge to 0, the dynamic response has been significantly improved.

Figure 3 shows comparisons of sinusoid path tracking effect. From the figure, it is obvious that the tracking path of improved sliding mode basically coincides with the target path, but there is a certain error between the tracking path the traditional sliding mode and the target path. The error of traditional sliding mode control lies in that it belongs to switching control and automatically converges to the equilibrium origin according to the properties of switching surface, but it will generate chattering.

In short, the sinusoidal path is tracked more accurately under the control of the improved sliding mode controller based on backstepping design.

Figure 4 shows the comparisons of chattering between having taken critical saturation function and no. It can be seen from the figure that the controller using the critical saturation function improving algorithm significantly eliminates chattering regardless of the input angular velocity or the linear velocity, thus the stability of the system has been improved. The above results satisfy the expected requirement and verify the effectiveness of the robot model and the new control law.

6. Conclusions
In this paper, the path tracking of mobile robot based on sliding mode control is studied. In the first place, a backstepping sliding mode controller is designed based on nonlinear kinematics model of three-
wheeled mobile robot, and the controller has a good dynamic response and robustness. Moreover, the reaching law is improved by replacing the symbol function with the critical saturation function to accomplish the expectation of which chattering could be reduced effectively. Last but not least, it has been verified through simulation experiments and get the results that the improved controller not only can more quickly and accurately track the path of the mobile robot, but also significantly eliminates chattering and improves the stability of the robot system. The idea of combining backstepping design with bounded saturation function proposed in this paper can avoid the defects caused by single principle, thus ensuring the reduction of chattering on the basis of enhanced robustness of the system. It is of great significance to reduce the chattering caused by many mechanical controls, especially switching control.

References
[1] Furuta K 1990 J. Systems & Control Letters 14(2) pp 145-52.
[2] Feng Y, Yu X and Man Z 2002 J. Automatica, 38(12) pp 2159-167.
[3] Tao C W, Taur J S and Chan M L 2004 J. Systems Man & Cybernetics Part B Cybernetics 34(1) pp 255-62.
[4] Sage H G, De Mathelin M F and Ostertag E 1999 J. International Journal of Control 72(16) pp 1498-522.
[5] Gao W and Hung J C 1993 IEE Transactions On Industrial Electronics 40(1) pp 45-55.
[6] Sun F C, Sun Z Q and Feng G 1999 IEEE Transactions on Systems Man & Cybernetics Society 29(5) pp 1-667.
[7] Wang Z P, Yang W R and Ding G X 2010 Second WRI Global Congress on Intelligent Systems.
[8] Bai Jile 2016 Research on trajectory tracking of mobile robots [D] (in Chinese). Shenyang: Shenyang University of Technology.

Acknowledgments
This research was financially supported by the Jishou University School-level University Student Innovation Project Funding (No. JDCX2018040); National College Student Innovation Training Project Funding (No. 201810531017, 201108531019); Jishou University “13th Five-Year” Communication Engineering Specialty Comprehensive Reform Pilot Construction Project; Hunan Province first-class undergraduate communication engineering professional construction project.