Numerical simulations of the accretion-ejection instability

S. E. Caunt and M. Tagger

DSM/DAPNIA/Service d’Astrophysique (CNRS URA 2052), C.E.A. Saclay,
91191 Gif sur Yvette, France

Abstract.
The Accretion-Ejection Instability (AEI) is explored numerically using a global 2d model of the inner region of a magnetised accretion disk. The disk is initially currentless but threaded by an external vertical magnetic field created by external currents, and frozen in the flow. In agreement with the theory a spiral instability, similar in many ways to those observed in self-gravitating disks, but driven by magnetic stresses, develops when the magnetic field is close to equipartition with the disk thermal pressure. The present non-linear simulations give good evidence that such an instability can occur in the inner region of accretion disks.

Keywords: magnetohydrodynamics, instabilities, accretion disks

1. Introduction

The accretion-ejection instability (AEI) (Tagger & Pellat (1999); see also Varnière et al., Rodriguez et al., and Tagger et al., this workshop) can occur close to the inner edge of a magnetised accretion disk when the plasma beta (the ratio of gas to magnetic pressures) is around unity. The instability appears, if the magnetic field decreases outwards sufficiently fast, as a spiral wave of low azimuthal wavenumber, made unstable through the interaction with a Rossby vortex (associated with the gradient of vorticity) which it generates at its corotation radius.

We have undertaken numerical simulations, with a setup optimised according to our previous knowledge of the AEI. We consider an infinitely thin disk threaded by a moderate vertical magnetic field: this is justified by the properties of the AEI, which are essentially constant vertically across the disk (unlike the magneto-rotational instability (Balbus & Hawley, 1991) for which vertical modes are required). The disk is embedded in vacuum and, in order to separate different physical effects, we consider here only configurations where initially the equilibrium magnetic field is due to external currents. Starting with small scale initial random fluctuations in the fluid velocity, the simulations show large-scale spiral perturbations evolving in the disk, and corresponding to the expected properties of the AEI.

The following sections present the essentials of the model, basic results and conclusions. More detailed discussions of these are covered elsewhere (Caunt & Tagger, 2000).
2. The model

Under the assumption, valid for the physics of the AEI, that perturbations are essentially independent of \( z \) in the disk, we integrate vertically the standard MHD equations (momentum, continuity, induction) describing the fluid motion in a disk around a central object. We thus solve for the 2d \( (r, \phi) \) evolution of the fluid properties (surface density, velocity, magnetic field) averaged over \( z \). A conservative scheme of the type described by Stone & Norman (1992) is used to ensure that mass, angular momentum and magnetic flux are conserved to numerical precision throughout the simulation.

We assume that the disk is threaded by a poloidal magnetic field which is symmetric about the midplane, hence purely vertical at \( z = 0 \). In the vacuum surrounding the disk the magnetic field can be described by a magnetic potential which, in turn, results from perturbed currents in the disk. It can thus be calculated from a Poisson equation, similar to the one describing the gravitational potential of a self-gravitating disk, but whose source is the vertical component of the magnetic field at the disk surface (rather than density in the gravitational case). Currents within the disk can be derived from the jump in the horizontal component of the field across the disk. Hence magnetic stresses can be included in an otherwise purely hydrodynamic model.

We assume that the central object has a mass of \( M = 10M_\odot \) and the disk extends from between \( r = 1,000 \) km to \( r = 50,000 \) km with a constant aspect ratio of \( \epsilon = 0.1 \). We use a logarithmic radial grid which allows us to get a better resolution in the inner region of the grid, where it is necessary, and to model a very large radial extent of the disk. It also has the added advantage that unwanted effects of the boundary condition at the outer radius of the simulation are avoided (waves are damped more effectively as they approach the outer radius).

3. Results

The results presented here are aimed at illustrating certain properties of the instability as determined by linear theory, specifically the growth rate/magnitude of instability for different field strengths and development of the spiral wave and Rossby vortex.

Figure 1 shows the evolution of the rms velocity within the disk for values of \( \beta = 0.1, 0.5, 1.0, 5.0, 10.0 \). As is shown, the perturbed velocity grows rapidly for \( \beta = 0.5 \) and \( \beta = 1.0 \), exceeding the initial random noise after approximately 10 orbits. The growth is approximately exponential during this time. For the other values of \( \beta \) we see
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Figure 1. Comparison of the evolution of the instability for different values of the initial $\beta$. Values of $\beta = 0.1, 0.5, 1.0, 5.0$ and 10.0 are shown as dotted, dot-dashed, dashed, triple-dot-dashed and long-dashed lines respectively. Time is given in units of the orbital time at the inner radius. The plots show the evolution of (top) the rms velocity, and (bottom) the rms magnetic field.

Figure 2. Plots of the spiral structure in the inner region of the disk at times $t = 60, 120$ and 180 orbits. Starting from initially random fluctuations at $t = 0$, the disk has clearly developed 3-armed followed by 2-armed and finally 1-armed spirals.

very little action in comparison. The $\beta = 0.1$ and $\beta = 10.0$ cases are virtually stable with the $\beta = 5.0$ being somewhat more active but failing to achieve the same amplitude as the most unstable cases. This run is still interesting to observe as the growth of the instability is slower and the evolution of the spiral wave is somewhat cleaner.

The evolution of the instability into a well developed spiral wave is best seen for the $\beta = 5.0$ case. Figure 2 shows the different number of spiral arms that occur over time (plotted is the radial velocity): initial random perturbations develop to $m = 3$, $m = 2$ and $m = 1$ spiral structures, as the magnetic flux is advected toward the central region. The transition is generally through a mixture of these for all values.
of unstable $\beta$. For the more active values ($\beta = 0.5$ and $\beta = 1.0$) the transition is much quicker and always ends up with $m = 1$.

The AEI relies on the existence of a Rossby vortex generated at corotation. Figure 3a shows the radial variation of vorticity in the disk. Clearly shown is a discontinuity close to the inner radius. The velocity field, shown in Figure 3b clearly shows that the fluid flow contains a vortex at the same radius. This is a good indication that a Rossby vortex has indeed been generated at corotation.

### 4. Conclusions

Using this model we have shown the essential features predicted by linear theory of the AEI. We have seen that the instability is clearly strongest for field strengths of the order of equipartition, and leads to the development of low azimuthal wavenumber spiral waves and a Rossby vortex.

Future work will link in the third type of wave (namely the vertical emission of Alfvén waves) from which the formation of winds and jets is believed to be possible.

### References

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