Effects of chirality of a helical magnetic field on a superconductor

Saoto Fukui\textsuperscript{1}, Masaru Kato\textsuperscript{1} and Yoshihiko Togawa\textsuperscript{2}, Osamu Sato\textsuperscript{3}

\textsuperscript{1} Department of Mathematical Sciences, Osaka Prefecture University, 1-1, Gakuencho, Nakaku, Sakai, Osaka 599-8531, Japan
\textsuperscript{2} Department of Physics and Electronics, Osaka Prefecture University, 1-1, Gakuencho, Nakaku, Sakai, Osaka 599-8531, Japan
\textsuperscript{3} Osaka Prefecture University College of Technology, 26-12, Saiwaicho, Neyagawa, Osaka 572-8572, Japan

E-mail: st110035@edu.osakafu-u.ac.jp

Abstract. We have investigated vortex states in a superconductor under a helical magnetic field from a chiral helimagnet. We solve the Ginzburg-Landau equations with three-dimensional finite element method. In this model, two antiparallel vortices appear under the helical magnetic field. There directions are normal to the largest surface of the superconductor.

1. Introduction

A chiral helimagnet has a long periodic helical magnetic structure [1]. The magnetic structure is shown in figure 1 (a). Under an applied magnetic field, the chiral helimagnetic structure transforms into a periodic soliton structure, which is called a chiral soliton lattice shown in figure 1 (b). An origin of this chiral helimagnetic structure is a competition of two interactions between nearest neighbor spins; the Dzyaloshinsky-Moriya (DM) interaction [2, 3] and a ferromagnetic exchange interaction.

![Figure 1](image-url)  

**Figure 1.** Spin configurations of (a) a chiral helimagnetic structure, and (b) a chiral soliton lattice under an applied magnetic field $H_{\text{appl}}$.  

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It is known that the chiral helimagnet gives various physical properties, for example, a giant magnetoresistance and discrete changes of the resistance [4]. We focus on effects of the chiral helimagnet on a superconductor. There are many investigations of effects of a ferromagnet on the superconductor [5, 6, 7, 8]. In the ferromagnet / superconductor bilayer system, vortices appear in the superconductor spontaneously under an uniform magnetic field from a magnetic domain of the ferromagnet. Also, the pinning effect occurs due to ferromagnetic dots on the superconductor. On the other hand, the chiral helimagnet creates the helical magnetic field. We consider the possibility of novel influences of the chiral helimagnet on vortex structures in the superconductor. In our previous study [9, 10, 11, 12, 13], we investigated vortex structures in a two-dimensional superconductor under a helical magnetic field instead of the chiral helimagnet / superconductor bilayer system (figure 2(a)). Then, the effect of the helical magnetic structure in the chiral helimagnet was replaced by the oscillating magnetic field. Solving the Ginzburg-Landau equations with the two-dimensional finite element method (FEM), we obtained various vortex structures.

So far, we have restricted ourselves to the two-dimensional superconductor. However, in order to consider effects of chirality of the helical magnetic field on the vortex structure, we must consider a three-dimensional superconductor. In this report, we show the three-dimensional vortex structures in a three-dimensional superconductor under the three-dimensional helical magnetic field from the chiral helimagnet.

\[ F(\psi, A) = \int_V \left( f_n + \alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4 \right) dV + \int_V \left\{ \frac{1}{2m_e} \left| -i \hbar \nabla - \frac{e_s A}{c} \right| \psi \right|^2 + \frac{|\mathbf{h}|^2}{8\pi} - \frac{\mathbf{h} \cdot \mathbf{H}}{4\pi} \right\} dV, \tag{1} \]

where $\psi$ is a superconducting order parameter and $f_n$ is a free energy density of the normal state. $\alpha(T)$ is a function of temperature, $\alpha(T) = \alpha'(T - T_c)$. $\alpha'$ and $\beta$ is a positive constant, $T_c$ is a critical temperature of the superconductor. $m_e$ and $e_s$ is an effective mass and an effective charge of electrons in the superconductor, respectively. $\mathbf{H}$ is an external magnetic field, $\mathbf{h} = \nabla \times \mathbf{A}$ is a local magnetic field, and $\mathbf{A}$ is a magnetic vector potential. $\psi$ and $\mathbf{A}$ are normalized as,

\[ \tilde{\psi} = \frac{\psi}{\sqrt{\alpha/\beta}}, \quad \tilde{\mathbf{A}} = \frac{2\pi}{\Phi_0} \mathbf{A}, \tag{2} \]
where $\Phi_0 = ch/2e$ is the quantum flux. In the three-dimensional FEM, we use a tetrahedron as a finite element, which is shown in figure 3. There are four volume coordinates for $e$–th element,
\[ N^e_i = (a_i + b_i x + c_i y + d_i z)/6V_e \quad (i = 1, 2, 3, 4), \]  
where $V_e$ is a volume of the tetrahedron for $e$–th element and $a_i$, $b_i$, $c_i$, and $d_i$ are given in the Appendix. Using these volume coordinates, the order parameter $\tilde{\psi}^e$ and the vector potential $\tilde{A}^e$ in $e$–th element are expanded as,
\[ \tilde{\psi}^e(x, y, z) = \sum_i \tilde{\psi}^e_i N^e_i, \quad \tilde{A}^e(x, y, z) = \sum_i \tilde{A}^e_i N^e_i \quad (i = 1, 2, 3, 4), \]
where $\tilde{\psi}^e_i$ and $\tilde{A}^e_i$ are values of the order parameter and the magnetic vector potential at $i$–th vertex in $e$–th element. After Frechét derivatives of the free energy (equation (1)) about $\tilde{\psi}$ and $\tilde{A}$, then substitute equation (4). Therefore, we obtain these equations,
\[ \sum_j \left[ P_{ij}(\{\tilde{A}\}) + P^{2R}_{ij}(\{\tilde{\psi}\}) \right] \text{Re} \tilde{\psi}^e_j + \sum_j \left[ Q_{ij}(\{\tilde{A}\}) + Q^2_{ij}(\{\tilde{\psi}\}) \right] \text{Im} \tilde{\psi}^e_j = V^R_i(\{\tilde{\psi}\}), \]  
\[ \sum_j \left[ -Q_{ij}(\{\tilde{A}\}) + Q^2_{ij}(\{\tilde{\psi}\}) \right] \text{Re} \tilde{\psi}^e_j + \sum_j \left[ P_{ij}(\{\tilde{A}\}) + P^{2L}_{ij}(\{\tilde{\psi}\}) \right] \text{Im} \tilde{\psi}^e_j = V^L_i(\{\tilde{\psi}\}), \]  
\[ \sum_j R_{ij}(\{\tilde{\psi}\}) \tilde{A}_{jx} + \sum_j S_{ijx}^{xy} \tilde{A}_{jy} + \sum_j S_{ijx}^{xz} \tilde{A}_{jz} = T^x_i - U^x_i, \]  
\[ \sum_j R_{ij}(\{\tilde{\psi}\}) \tilde{A}_{jy} + \sum_j S_{ijy}^{xy} \tilde{A}_{jx} + \sum_j S_{ijy}^{yz} \tilde{A}_{jz} = T^y_i - U^y_i, \]  
\[ \sum_j R_{ij}(\{\tilde{\psi}\}) \tilde{A}_{jz} + \sum_j S_{ijz}^{xz} \tilde{A}_{jx} + \sum_j S_{ijz}^{yz} \tilde{A}_{jy} = T^z_i - U^z_i. \]

Coefficients are also given in Appendix. We solve these equations self-consistently including nonlinear terms.

\[ \begin{align*}
\text{Figure 3.} & & \text{A tetrahedron as a three-dimensional finite element. 1, 2, 3, and 4 are volume coordinates.} \\
& & \text{We assume that the effect of the chiral helimagnet on the superconductor is taken as a helical magnetic field. The magnetic field from the chiral helimagnet is obtained from the Hamiltonian for the chiral helimagnet,} \\
& & \mathcal{H} = -J \sum_n \mathbf{S}_n \cdot \mathbf{S}_{n+1} + \mathbf{D} \cdot \sum_n \mathbf{S}_n \times \mathbf{S}_{n+1} + 2\mu_B \mathbf{H}_{\text{appl}} \cdot \sum_n \mathbf{S}_n,
\end{align*} \]
where $J(>0)$ is a coefficient of the ferromagnetic exchange interaction and $D = (D, 0, 0)$ is a DM vector $^{[14]}$. $S_n$ is the spin of the $n$-th site. $\mu_B$ is the Bohr magneton and $H_{\text{appl}} = (0, 0, H_{\text{appl}})$ is the homogeneous applied magnetic field. $S_n$ is expressed by the polar coordinate as

$$S_n = S(\sin \theta_n \cos \phi_n, \sin \theta_n \sin \phi_n, \cos \theta_n). \tag{11}$$

$S$ is a magnitude of the spin. In the monoaxial chiral helimagnet, $\phi = \pi/2$. From the Hamiltonian $^{}^{[\text{equation (10)}]}$ in the continuum limit, we obtain a distribution of angles of spins $\theta(x)$ $^{[14]}$:

$$\theta(x) = 2 \sin^{-1} \left[ \sin \left( \frac{\sqrt{H^*}}{k} x \right) \right] - \frac{1}{2} \pi, \tag{12}$$

where $H^* = 2\mu_B H_{\text{appl}}/(a^2 S^2 \sqrt{J^2 + D^2})$ is a normalized applied magnetic field and $k$ is the modulus of the Jacobi’s elliptic function $\sn(u/k)$. $k$ is determined by,

$$\frac{\pi \tan^{-1} (D/J)}{4\sqrt{H^*}} = \frac{E(k)}{k}, \tag{13}$$

where $E(k)$ is the complete elliptic integral of the second kind. We assume a lattice constant $a \sim \xi_0$ ($\xi_0$ is a coherence length at zero temperature). Finally, from equations (11) and (12), the external magnetic field $H = ((H_{\text{ext}})_x, (H_{\text{ext}})_y, (H_{\text{ext}})_z)$ in equation (1) is given by,

$$\begin{align*}
(H_{\text{ext}})_x(x) &= 0, \tag{14} \\
(H_{\text{ext}})_y(x) &= H_0 \sin \left( 2 \sin^{-1} \left[ \sin \left( \frac{\sqrt{H^*}}{k} x \right) \right] - \frac{1}{2} \pi \right), \tag{15} \\
(H_{\text{ext}})_z(x) &= H_0 \cos \left( 2 \sin^{-1} \left[ \sin \left( \frac{\sqrt{H^*}}{k} x \right) \right] - \frac{1}{2} \pi \right) + H_{\text{appl}}. \tag{16}
\end{align*}$$

$H_0$ is a magnitude of the magnetic field from the chiral helimagnet. In our previous study, we use only $(H_{\text{ext}})_z(x)$ (equation (16)) in the two-dimensional superconductor$^{[5, 6, 7, 8]}$. In the present study, we consider three components of the external magnetic field for the three-dimensional superconductor. We calculate the Ginzburg-Landau equations (5)-(9) using the magnetic field in equations (14)-(16).

3. Results and Discussions

We show vortex configurations in the superconductor under the helical magnetic field in equations (14)-(16) with $H_{\text{appl}} = 0$. We take the temperature $T = 0.3T_c$ and the Ginzburg-Landau parameter $\kappa = \lambda_0/\xi_0 = 5$, where $\lambda_0$ is a penetration length at zero temperature. The ratio between the ferromagnetic exchange interaction and the DM interaction is taken as $D/J = 0.16$, which is adequate for CrNb$_3$S$_6$.$^{[15]}$ The system size are $1.0L'\xi_0 \times 15\xi_0 \times 13\xi_0$. Here, $L'\xi_0 = (2\pi/\tan^{-1}(D/J))\xi_0$ is a helical period of the helical magnetic field for $H_{\text{appl}} = 0$. When $D/J = 0.16$, $L'$ is approximately 39.2699. In the system, the superconducting region is surrounded by the vacuum region shown in figure 4. The size of the superconductor is $(1.0L' - 3.0)\xi_0 \times 12\xi_0 \times 10\xi_0$. At first, we show the distribution of the magnetic field in figure 5. In equations (14)-(16), we consider the case of $|H_0/(\Phi_0/\xi_0^2)| = 0.15$. The vortex configuration under this helical magnetic field is shown in figure 6. These figures represent configurations of vortices in $xy-$planes at $z = 1.5\xi_0, 6.5\xi_0$, and $11.5\xi_0$. Vortices appear in the region $|(H_{\text{ext}})_y/(\Phi_0/\xi_0^2)| \sim 0.00$ and $|(H_{\text{ext}})_z/(\Phi_0/\xi_0^2)| \sim 0.15$, where the external magnetic field are parallel to the $z-$direction (see figures 5 (c) and (d)). There are antiparallel with each other. In the finite system, a vortex appear at the center in the system under the homogeneous
Figure 4. (a) Three-dimensional superconductor model. The system size is $1.0L/\xi_0 \times 13\xi_0 \times 15\xi_0$. The distance between the normal region and the superconducting region is $1.5\xi_0$. $xy$–plane, $yz$–plane, and $zx$–plane are (b), (c), and (d).

Figure 5. (a) The distribution of the helical magnetic field, (b) $x$–component of the magnetic field, (c) $y$–component of the magnetic field, (d) $z$–component of the magnetic field. The magnitude of the magnetic field $|H_0/(\Phi_0/\xi_0^2)| = 0.15$.

Figure 6. Configurations of the vortices in the $xy$–plane at (a) $z = 1.5\xi_0$, (b) $z = 6.5\xi_0$, and (c) $z = 11.5\xi_0$.

Figure 7. Configurations of the vortices in the $zx$–plane at (a) $y = 1.5\xi_0$, (b) $y = 7.5\xi_0$, and (c) $y = 13.5\xi_0$. 
magnetic field [16]. In this system, the vortex doesn’t appear at the center ($x/\xi_0 \sim 20.0$ and $y/\xi_0 \sim 7.5$), where $|\langle H_{\text{ext}} \rangle_z / \langle \Phi_0 / \xi_0^2 \rangle | = 0.00$. Under the helical magnetic field, vortices appear at the largest magnetic field that is normal to the surface of the superconductor. Therefore, the configuration of vortices depends on the shape of the superconductor. This dependence of the configuration doesn’t occur under the homogeneous magnetic field.

Moreover, two vortices appear antiparallel with each other. In our previous study, we investigated vortex structure under the oscillating magnetic field normal to a two-dimensional superconductor [12]. Under the oscillating magnetic field, vortices prefer to appear in all largest magnetic field regions except for ends of the superconductor. Present results agree with previous results. Two vortices normal to $z$–direction appear in the three-dimensional system.

Finally, we focus on effects of the $y$–component magnetic field. We show distributions of the order parameter in $zx$–plane at $y = 1.5\xi_0$, $7.5\xi_0$, and $13.5\xi_0$ in figure 7. At $y = 7.5\xi_0$ (see figure 7(b)), we found that two vortices parallel to $z$–axis appear. Although $y$–component magnetic field also oscillates with the same amplitude to $z$–component magnetic field, vortices don’t appear. When the system size is small, the magnetic field avoids to penetrate into the superconductor due to the Meissner effect. The $zx$–plane is smaller than $xy$–plane, so vortices affected by $y$–component of the magnetic field don’t appear due to the Meissner effect or demagnetizing effect. Therefore, the appearance of vortices affects on the system size and when the system size becomes larger, vortices will appear in both $xy$–plane and $zx$–plane simultaneously. Moreover, vortices will appear parallel to the helical magnetic field, which doesn’t appear in the case of the homogeneous magnetic field and the two-dimensional system.

4. Conclusions
We have obtained vortex structures in the three-dimensional superconductor under the helical magnetic field from the chiral helimagnet. Solving the Ginzburg-Landau equations in the model shown in figure 4, we found that two antiparallel vortices along the $z$–direction appear. The magnetic field along $z$–direction, $(H_{\text{ext}})_z$ changes spatially and these vortices appear at the largest $(H_{\text{ext}})_z$. However, when the system size is small, vortices don’t appear due to the Meissner effect. In the future, we will investigate vortex structure under the helical magnetic field in the larger system and clarify effects of the chirality of the helical magnetic field on the superconductor.

Appendix
Coefficients in Eqs.(5)-(9) are given as,

$$P_{ij}(\{A\}) = \sum_\alpha K_{ij}^{\alpha\alpha} + \sum_{i_1,i_2} I_{i_1i_2ij} \sum_\alpha A_{i_1i_2}^\alpha A_{i_2i_j}^\alpha - \frac{1}{\xi(T)^2} J_{ij},$$

$$P_{ij}^{2R}(\{\psi\}) = \frac{1}{\xi(T)^2} \sum_{i_1,i_2} I_{i_1i_2ij} (3\text{Re} \psi_{i_1}^e \text{Re} \psi_{i_2}^e + \text{Im} \psi_{i_1}^e \text{Im} \psi_{i_2}^e),$$

$$P_{ij}^{2l}(\{\psi\}) = \frac{1}{\xi(T)^2} \sum_{i_1,i_2} I_{i_1i_2ij} (\text{Re} \psi_{i_1}^e \text{Re} \psi_{i_2}^e + 3\text{Im} \psi_{i_1}^e \text{Im} \psi_{i_2}^e),$$

$$Q_{ij}(\{A\}) = \sum_i \sum_\alpha (J_{ji}^{\alpha\alpha} - J_{ij}^{\alpha\alpha}) A_{ij}^\alpha,$$

$$Q_{ij}^{2l}(\{\psi\}) = \frac{2}{\xi(T)^2} \sum_{i_1,i_2} I_{i_1i_2ij} \text{Re} \psi_{i_1}^e \text{Im} \psi_{i_2}^e,$$

$$R_{ij}(\{\psi\}) = \kappa^2 \xi(T)^2 \sum_\alpha K_{ij}^{\alpha\alpha} + \sum_{i_1,i_2} I_{i_1i_2ij} \psi_{i_1}^e \psi_{i_2}^e,$$

$$S_{ij}^{\alpha\beta} = \kappa^2 \xi(T)^2 (K_{ij}^{\alpha\beta} - K_{ij}^{\beta\alpha}),$$

$$T_{ij}^\alpha = \sum_{i_1,i_2} J_{i_1i_2}^\alpha \text{Im} (\psi_{i_1}^e \psi_{i_2}^e),$$

$$U_{ij}^\alpha = \kappa^2 \xi(T)^2 \frac{2\pi}{\Phi_0} (J_{i_1}^\beta H_\gamma - J_{i_2}^\gamma H_\beta),$$

$$V_{ij}^{2R}(\{\psi\}) = \frac{2}{\xi(T)^2} \sum_{i_1,i_2,i_3} I_{i_1i_2i_3j} \text{Re} (\psi_{i_1}^e \psi_{i_2}^e) \text{Re} \psi_{i_3}^e,$$

$$V_{ij}^l(\{\psi\}) = \frac{2}{\xi(T)^2} \sum_{i_1,i_2,i_3} I_{i_1i_2i_3j} \text{Re} (\psi_{i_1}^e \psi_{i_2}^e) \text{Im} \psi_{i_3}^e,$$
where \( \alpha \) and \( \beta = x, y, z \) and \( I_{ij}, I_{i1234i}, J^x_i, J^{x_i}_{i123i}, K_{ij}^{x_i x_j} \) are integrals. \((\alpha, \beta, \gamma)\) is a cyclic permutation of \((x, y, z)\). These integrals are given by,

\[
I_{ij} = \int_{V_e} N^e_i N_j dV, \quad I_{i1234i} = \int_{V_e} N^e_i N^e_i N^e_i N^e_i dV, \quad I_{i123i} = \int_{V_e} N^e_i N^e_i N^e_i N^e_i dV, \quad J^x_i = \int_{V_e} \partial N^e_i / \partial x_i dV, \quad J^{x_i}_{i123i} = \int_{V_e} \partial N^e_i / \partial x_i \partial N^e_i / \partial x_j dV, \quad K_{ij}^{x_i x_j} = \int_{V_e} \partial N^e_i / \partial x_i \partial N^e_i / \partial x_j dV \quad (x_i, x_j = x, y, z). 
\]

Coefficients \( a_i, b_i, c_i, d_i \) in Eq.(3) are given by,

\[
a_i = \epsilon_i \{ x_i (y_m z_n - y_n z_m) + x_m (y_n z_l - y_l z_n) + x_n (y_l z_m - y_m z_l) \}, \\
b_i = \epsilon_i \{ y_i (z_n - z_m) + y_m (z_l - z_n) + y_n (z_m - z_l) \}, \\
c_i = \epsilon_i \{ z_i (x_n - x_m) + y_m (z_l - z_n) + y_n (z_m - z_l) \}, \\
d_i = \epsilon_i \{ x_i (y_n - y_m) + x_m (y_l - y_n) + x_n (y_l - y_l) \}. 
\]

Here \( \epsilon_i = 1 \) \((i = 1, 3)\) or \( \epsilon_i = -1 \) \((i = 2, 4)\), and \((i, l, m, n)\) is a cyclic permutation of \((1, 2, 3, 4)\) and \( V \) is a volume of a tetrahedron.

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