Logarithmic Fingerprints of Virtual Supersymmetry †

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Abstract

We consider the high energy behaviour of the amplitudes for production of leptons, quarks, Higgs bosons, sleptons, squarks, gauge bosons, charginos and neutralinos at lepton colliders. We concentrate our discussion on the terms arising at one loop which grow logarithmically with the energy, typically $[a \ln \frac{s}{m^2} - ln^2 \frac{s}{m^2}]$. We show that in each of the above processes the coefficient "a" reflects in a remarkable way the basic gauge and Higgs structure of the underlying interactions. A comparison with experiments at future colliders should thus provide a clean way to test the validity of the MSSM structure.

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It is by now well-known that the electroweak radiative corrections to standard processes increase strongly with the center of mass energy $\sqrt{s}$. This arises already at the one loop level due to the presence of large double (DL) and single (SL) logarithms $\frac{2}{\pi} \ln \frac{s}{m^2}$, $\frac{2}{\pi} \ln \frac{s}{m^2}$. In the TeV range such terms reach the several tens of percent and should be easily measurable (and analyzable) at future lepton colliders [4] whose experimental accuracy should be at the few permille level.

The relevance of these large logarithmic effects at high energy colliders has been stressed recently in the process $e^+e^- \rightarrow f\bar{f}$ for both the SM [3] and the Minimal Supersymmetric Standard Model (MSSM) cases [5, 6], and in the process of production of scalar pairs in the MSSM [7, 8]. The processes $e^+e^- \rightarrow \gamma\gamma, \gamma Z, ZZ$ [9] has been analyzed in the same spirit as well as $\gamma\gamma \rightarrow f\bar{f}$ [10] measurable at photon-photon colliders. The logarithmic structure of the processes $e^+e^- \rightarrow W^+W^-, H^+H^-, H^0H^0, \chi^+\chi^-, \chi^0\chi^0$ has also been analyzed and we extract some results from a publication in preparation [11].

In order for these analyses of the MSSM cases to be already meaningful in the one TeV range, a necessary condition is that of a light SUSY scenario in which all the SUSY masses relevant for the considered process are supposed to be not heavier than a few hundred GeV. This condition can be rescaled for higher energies provided that $M_{SUSY} << \sqrt{s}$. However, the large size of the effects in the several TeV range would require a treatment (even approximate) of the higher order effects in order to obtain a good theoretical prediction (at the one percent level). Such attempts have already started [12] and it has been claimed that DL as well as universal and angular dependent SL can be exponentiated, see also [7] for an application to the MSSM case. This means that the one loop structure is the basic ingredient on which we can concentrate our discussion.

Comparing the SM and the MSSM logarithmic effects in these various processes, we were impressed by a number of recurrent, impressively simple differences. The purpose of this short note is that of presenting in a systematic way these differences and of discussing their intuitive, deep physical origins.

The logarithmic terms at the one loop level

At the one loop level the logarithmic terms appearing in $e^+e^-$ processes can be separated into three categories, Parameter Renomalization (PR) terms, Universal terms and Angular dependent terms:

(1) the PR terms

These are the well-known terms generated by the gauge boson or gaugino self-energy contributions. They are process dependent, but can be computed in a straightforward way from the Born amplitude i.e.,
\[
\frac{1}{4\pi^2} \left[ g_1^4 \beta_1 \frac{dA^{Born}}{dg_1^2} + g_2^4 \beta_2 \frac{dA^{Born}}{dg_2^2} \right] \left[ -\ln \frac{s}{\mu^2} \right]
\]

where \(\beta_1 = -\frac{5}{9} N_{fam} - \frac{1}{24}, \ (\ -\frac{5}{6} N_{fam} - \frac{1}{4})\) and \(\beta_2 = -\frac{1}{3} N_{fam} + \frac{43}{24}, \ (\ -\frac{1}{2} N_{fam} + \frac{5}{4})\) are the SM, (MSSM) values of the usual RG functions associated to the gauge couplings \(g_{1,2}\).

(2) Universal terms

Also called "Sudakov" terms, these terms appear to be typically of the form \([a \ln \frac{s}{m^2} - \ln^2 \frac{s}{m^2}]\). They factorize the Born amplitude in a process independent and angular independent fashion. They are specific of the quantum numbers of each external particle. In the high energy range they are the dominant terms.

The single logs correspond to collinear singularities and the double logs to a coincidence of collinear and soft singularities in the one loop diagrams.

Although the results were first obtained by canonical computations of self-energy, triangle and box diagrams, in the covariant \(\xi = 1\) gauge, or in an axial gauge, the simplest way to obtain these log terms is through the splitting function formalism \([13]\). With the splitting functions

\[
\frac{1 + x^2}{1 - x}, \quad \frac{1 - x}{2}, \quad \frac{2x}{1 - x}, \quad \frac{1}{2}, \quad 2\left[ \frac{x}{1 - x} + \frac{1 - x}{x} + x(1 - x) \right], \quad \frac{x^2 + (1 - x)^2}{2}, \quad x(1 - x),
\]

for \(f \rightarrow gf\), \(f \rightarrow sf\), \(s \rightarrow gs\), \(s \rightarrow f\bar{f}\), \(g \rightarrow gg\), \(g \rightarrow f\bar{f}\), \(g \rightarrow s\bar{s}\) (where \(f, s, g\) represent fermions, scalars and gauge bosons), respectively, and the addition of the parameter renormalization terms, one immediately recovers the results of the diagrammatic computations.

(3) Angular dependent terms

They are just residual parts of the soft-collinear singularity arising from box diagrams involving gauge boson exchanges which generate \(\ln^2|x|\) terms \((x \equiv t \simeq -\frac{s}{2}(1 - \cos \theta)\) or \(u \simeq -\frac{s}{2}(1 + \cos \theta)\), being the Mandelstam parameters). After having extracted the universal angular independent part \(\ln^2 s\), one remains with an angular dependent terms of the type

\[
2\ln \frac{s}{m^2} \ln \frac{|x|}{s} + \ln^2 \frac{|x|}{s}
\]

whose contribution constitutes an additional process-dependent single log. There are only few such terms, which are all of pure SM origin and have been computed for all considered processes.

Among these 3 types of terms the richest structure is in the Sudakov part (2), on which we now concentrate. We write this contribution as
\[ A^{1 \, \text{loop}} = [1 + \frac{\alpha}{\pi} c] \, A^{\text{Born}} \]  \hspace{1cm} (2)

This factorization applies to each external line of the process, \( c \) being a coefficient that depends on the nature of the external particle, on the type of interaction and on the energy.

When the external particle is one member of two mixed states \((i = 1, 2; \text{see the examples below})\), the above equation has to be written in a matrix form:

\[ A_{i}^{1 \, \text{loop}} = \sum_{j} \left[ \delta_{ij} + \frac{\alpha}{\pi} c_{ij} \right] \, A_{j}^{\text{Born}} \] \hspace{1cm} (3)

We now list a number of typical examples.

**chiral lepton or quark** \((f_{L,R})\)

**in the SM**

\[
c(f_{L,R}) = \frac{1}{8} \left[ 3 \ln \frac{s}{M_{W}^{2}} - \ln^{2} \frac{s}{M_{W}^{2}} \right] \left[ I_{f}(I_{f} + 1) \, \frac{Y_{f}^{2}}{s_{W}^{2}} \right]_{L,R} + \left[\left[ m_{t}^{2} + m_{b}^{2} \right][\delta_{f,tL} + \delta_{f,tR} + 2m_{t}^{2}\delta_{f,tL} + 2m_{b}^{2}\delta_{f,tR} \right] \right]_{L,R} \]

**in the MSSM**

\[
c(f_{L,R}) = \frac{1}{8} \left[ 2 \ln \frac{s}{M_{W}^{2}} - \ln^{2} \frac{s}{M_{W}^{2}} \right] \left[ I_{f}(I_{f} + 1) \, \frac{Y_{f}^{2}}{4s_{W}^{2}} \right]_{L,R} + \left[\left[ 2m_{t}^{2}(1 + \cot^{2} \beta) + 2m_{b}^{2}(1 + \tan^{2} \beta) \right][\delta_{f,tL} + \delta_{f,tR} + 2m_{t}^{2}\delta_{f,tR} + 2m_{b}^{2}\delta_{f,tR} \right] \right]_{L,R} \]

One recognizes the Yukawa part appearing for heavy quarks, where \( \beta \) is the mixing angle between the vacuum expectation values of the up and down Higgs chiral superfield (in standard notation \( \tan \beta = v_{u}/v_{d} \)). The scale \( M \) which appears in the single logs is in principle the value of the highest mass running inside the corresponding loop. In the SM case it should be the top quark mass; in the MSSM case it will be a heavy squark or a chargino mass. We shall come back to this point in the final discussion.

**transverse** \( W_{\pm}^{T}, \gamma, Z_{T} \) **in the SM and in the MSSM**

\[
c(W) = \frac{1}{4s_{W}^{2}} \left[ -\ln^{2} \frac{s}{M_{W}^{2}} \right] \]

\[
c_{\gamma\gamma} = \frac{1}{4} \left[ -\ln^{2} \frac{s}{M_{W}^{2}} \right] \quad c_{ZZ} = \frac{c_{W}^{2}}{4s_{W}^{2}} \left[ -\ln^{2} \frac{s}{M_{W}^{2}} \right] \quad c_{\gamma Z} = \frac{c_{W}}{4s_{W}} \left[ -\ln^{2} \frac{s}{M_{W}^{2}} \right] \]

\(4\)
neutral Higgs and charged or neutral Goldstones in the SM

\[
c(H_{SM}) = c(G^0) = c(G^\pm) = \frac{(1 + 2c_W^2)}{32s_Wc_W^2} \left[ 4ln \frac{s}{M_W^2} - ln^2 \frac{s}{M_W^2} \right] + \frac{3}{16s_W^2M_W^2} \left[ m_t^2 + m_b^2 \right] [-ln \frac{s}{M^2}]
\]

The charged and neutral Goldstone states are equivalent at high energy to the longitudinal \( W_L^\pm \) and \( Z_L \) components.

sleptons or squarks in the MSSM \( (\tilde{f}_{L,R}) \)

Same expression as for leptons and quarks in the MSSM.

charged and neutral Higgs bosons and Goldstones in the MSSM

A first \( 2 \times 2 \) matrix describes the \( (H^\pm, G^\pm \equiv W_L^\pm) \) set, as well as the \( (A^0, G^0 \equiv Z_L) \) set:

\[
c_{11} = \frac{(1 + 2c_W^2)}{32s_W^2c_W^2} \left[ 2ln \frac{s}{M_W^2} - ln^2 \frac{s}{M_W^2} \right] + \frac{3}{16s_W^2M_W^2} \left[ m_t^2 \cot^2 \beta + m_b^2 \tan^2 \beta \right] [-ln \frac{s}{M^2}]
\]

\[
c_{22} = \frac{(1 + 2c_W^2)}{32s_W^2c_W^2} \left[ 2ln \frac{s}{M_W^2} - ln^2 \frac{s}{M_W^2} \right] + \frac{3}{16s_W^2M_W^2} \left[ m_t^2 + m_b^2 \right] [-ln \frac{s}{M^2}]
\]

\[
c_{12} = \frac{3}{16s_W^2M_W^2} \left[ m_t^2 \cot \beta - m_b^2 \tan \beta \right] [-ln \frac{s}{M^2}]
\]

A second matrix describes the \( (H^0, h^0) \) set

\[
c_{H^0H^0} = \frac{(1 + 2c_W^2)}{32s_W^2c_W^2} \left[ 2ln \frac{s}{M_W^2} - ln^2 \frac{s}{M_W^2} \right] + \frac{3}{16s_W^2M_W^2} \left[ m_t^2 \sin^2 \alpha(1 + \cot^2 \beta) + m_b^2 \cos^2 \alpha(1 + \tan^2 \beta) \right] [-ln \frac{s}{M^2}]
\]

\[
c_{h^0h^0} = \frac{(1 + 2c_W^2)}{32s_W^2c_W^2} \left[ 2ln \frac{s}{M_W^2} - ln^2 \frac{s}{M_W^2} \right] + \frac{3}{16s_W^2M_W^2} \left[ m_t^2 \cos^2 \alpha(1 + \cot^2 \beta) + m_b^2 \sin^2 \alpha(1 + \tan^2 \beta) \right] [-ln \frac{s}{M^2}]
\]

\[
c_{H^0h^0} = \frac{3 \cos \alpha \sin \alpha}{16s_W^2M_W^2} \left[ m_t^2(1 + \cot^2 \beta) + m_b^2(1 + \tan^2 \beta) \right] [-ln \frac{s}{M^2}]
\]

where \( \alpha \) is the mixing angle between the neutral CP even physical Higgs bosons; at tree level, \( \alpha \) is a simple combination of \( \tan \beta \) and the masses of the neutral (CP even and odd) physical Higgs bosons.
**Charginos $\chi^\pm_i$ in the MSSM**

$$c_{\chi^+_i \chi^+_j} = \frac{1}{4 s_W^2} \left[ -\ln^2 \frac{s}{M_W^2} \right] (Z_{2i}^+ Z_{2j}^+ P_L + Z_{2i}^- Z_{2j}^- P_R) +$$

$$\frac{(1 + 2 c_W^2)}{32 s_W^2 c_W} \left[ 2 \ln \frac{s}{M_W^2} - \ln^2 \frac{s}{M_W^2} \right] (Z_{2i}^+ Z_{2j}^+ P_L + Z_{2i}^- Z_{2j}^- P_R) +$$

$$\left[ -\ln \frac{s}{M^2} \right] \left( \frac{3}{16 s_W^2 M_W^2} \right) [m_t^2 (1 + \cot^2 \beta) Z_{4i}^+ Z_{4j}^+ P_L + m_b^2 (1 + \tan^2 \beta) Z_{3i}^+ Z_{3j}^- P_R]$$ (15)

The mixing matrix elements $Z_{1i}^\pm$ correspond [14] to the charged gaugino components and $Z_{2i}^\pm$ to the charged higgsino components.

**Neutralinos $\chi^0_i$ in the MSSM**

$$c_{\chi^0_i \chi^0_j} = \frac{1}{4 s_W^2} \left[ -\ln^2 \frac{s}{M_W^2} \right] (Z_{2i}^N Z_{2j}^N) +$$

$$\left[ \frac{(1 + 2 c_W^2)}{32 s_W^2 c_W} \right] \left[ 2 \ln \frac{s}{M_W^2} - \ln^2 \frac{s}{M_W^2} \right] (Z_{4i}^N Z_{4j}^N + Z_{3i}^N Z_{3j}^N) +$$

$$\left[ -\ln \frac{s}{M^2} \right] \left( \frac{3}{16 s_W^2 M_W^2} \right) [m_t^2 (1 + \cot^2 \beta) Z_{4i}^N Z_{4j}^N + m_b^2 (1 + \tan^2 \beta) Z_{3i}^N Z_{3j}^N]$$ (16)

The mixing matrix elements $Z_{2i}^N$ correspond [14] to the neutral gaugino ($\tilde{W}_3$) components (there is no contribution from the Bino $\tilde{B}$), and $Z_{3i}^N$, $Z_{4i}^N$ to the neutral higgsino components.

**Discussion of the SM terms**

The $[-\ln^2 \frac{s}{M^2}]$ terms, in which $M = M_W$ or $M_Z$, arise from the coincidence of soft and collinear singularities. They only appear (because of helicity conservation vertices) in SM gauge terms. They correspond to the term $1/(1-x)$ in the splitting function with emission of a gauge boson. The minus sign is fixed by unitarity (positivity of the transition probability).

These SM gauge terms contain also a single $[\ln \frac{s}{M^2}]$ part arising from the remaining part of the gauge splitting functions. For a fermion line one obtains the combination $[3 \ln \frac{s}{M^2} - \ln^2 \frac{s}{M^2}]$ and for a scalar line $[4 \ln \frac{s}{M^2} - \ln^2 \frac{s}{M^2}]$. The factor 3 or 4 can be traced back to the spin nature of the gauge vertices $f \bar{f} g$, $s \bar{s} g$ (where $g$ is a gauge boson) and more technically to the Lorentz transformation from the c.m. frame to the collinear frame of the usual $1 + \cos^2 \theta$ and $\sin^2 \theta$ distributions of fermion or scalar pairs.

We have also obtained SL of Higgs origin due to Yukawa couplings to heavy quarks. They appear both in heavy quark production processes and in other final states involving Higgs and Goldstones (where the heavy quarks contribute virtually). The minus sign in front of the SL is also a consequence of unitarity.
Discussion of the SUSY terms and of the complete MSSM terms

Additional single logarithmic terms \(-ln \frac{s}{M^2}\) arise from diagrams involving scalar couplings of supersymmetric particles (sfermions, charginos, neutralinos, charged or neutral Higgses). They have also two different origins. First, a gauge origin, with the same gauge couplings as in SM terms, so that the complete MSSM combination is now \(2ln - ln^2\). Secondly, a Yukawa origin, but in this case the extended Higgs structure generates new contributions depending on the parameter \(tan\beta\), and in the case of external \(H^0, h^0\), also on the mixing angle \(\alpha\). In the MSSM (and except for the \(H^0, h^0\) case) \(tan\beta\) is the only new SUSY parameter which enters the asymptotic expressions, and leads to \(m^2\cot^2\beta\) and \(m_b^2\tan^2\beta\) terms. The minus sign in front of the SL is also a consequence of unitarity (positivity of the corresponding transition probability). As already mentioned, the scale \(M\) which appears in these single logs is in principle the value of the highest mass running inside the corresponding loop, but at logarithmic accuracy, provided that one is especially interested in the slope in \(log s\), as suggested in Ref.\[6, 7\], the choice of \(M\) is harmless.

A list of benchmark features

We now underline the benchmark features which arise from the above results.

A first feature is that, for lepton and quark production the SM "fermion-gauge" combination
\[
[3ln \frac{s}{M^2} - ln^2 \frac{s}{M^2}] \text{ becomes } [2ln \frac{s}{M^2} - ln^2 \frac{s}{M^2}]
\]
in the MSSM when gaugino terms are added.

The second feature is that, for slepton and squark production the SM "scalar-gauge" combination
\[
[4ln \frac{s}{M^2} - ln^2 \frac{s}{M^2}] \text{ also becomes } [2ln \frac{s}{M^2} - ln^2 \frac{s}{M^2}]
\]
when the corresponding gaugino terms are added.
So the combination \([2ln \frac{s}{M^2} - ln^2 \frac{s}{M^2}]\) appears to be the typical MSSM combination of the whole fermion-sfermion supermultiplet.

The transformation of \([4ln \frac{s}{M^2} - ln^2 \frac{s}{M^2}]\) into \([2ln \frac{s}{M^2} - ln^2 \frac{s}{M^2}]\) also occurs when going from SM Higgs \(H_{SM}\) or Goldstone \(G^{\pm,0} \equiv W^{\pm}, Z\) production, to MSSM charged or neutral Higgses \(H^\pm, H^0, h^0, A^0\) or Goldstones.

For transverse gauge boson lines, only the quadratic term \([-ln^2 \frac{s}{M^2}]\) appears, both in the SM and in the MSSM. This should provide a test of the assumed "minimal" gauge structure of the MSSM, i.e. of absence of additional (higher) gauge bosons.
In the case of chargino and neutralino production, the $[2\ln \frac{s}{m_{W}^{2}} - \ln^2 \frac{s}{m_{W}^{2}}]$ combination can also be found in the Higgsino components, and the $[-\ln^2 \frac{s}{m_{W}^{2}}]$ term in the gaugino components. This leads to an additional potential check of the assumed supersymmetric nature of the interactions of these particles which can be achieved by a measurement of the production rate of the two charginos and of the four neutralinos.

Finally, we consider the Yukawa terms which contribute a term $[-\ln \frac{s}{m_{T}^{2}}]$ in the production of heavy quarks, heavy squarks, Higgs bosons and Goldstones, charginos and neutralinos. The interesting feature here is that the SM parameters $m_{t}^{2}$ and $m_{b}^{2}$ are, in the MSSM, replaced by terms that also contain the products $m_{t}^{2} \cot^2 \beta$ and $m_{b}^{2} \tan^2 \beta$. For large values of $\tan^2 \beta$ this would provide a genuine possibility of measuring this fundamental parameter, as already stressed in [6, 7, 15].

The general conclusion that can be drawn from our analysis is that the genuine SUSY electroweak Sudakov logarithmic structure differs from the corresponding one met in the SM in very simple and specific ways. This suggest the following strategy.

Through the measurements of the coefficients of the DL and SL, the production of usual particles (leptons, quarks, gauge bosons) should provide global tests of the SM gauge and Higgs structure. The spirit of these tests is similar to the one which motivates the high precision tests with $g - 2$ or $Z$ peak measurements. If new particles, candidates for supersymmetry, exist, departures from SM predictions should appear and one should then compare with MSSM predictions for these log coefficients. This can be done both for the production of usual particles and for the production of the new states (sleptons, light or heavy squarks, charged and neutral Higgses, charginos, neutralinos). This should allow to check if the MSSM description is satisfactory or if modifications or extensions are needed (higher gauge bosons, more Higgses, ...., or different forms of New Physics).

We would like to conclude by saying that even through the production of usual particles, ”virtual” supersymmetry would have a ”reality” at future accelerators, since it exhibits peculiar ”logarithmic fingerprints” that would be observable . Roughly, one might be tempted to summarize these results via a rough ”thumb rule”, sounding like:

”0,1,2,3,4..... count the logs and check supersymmetry”.

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