Abstract

The $DD\rho$ form factor is evaluated in a QCD sum rule calculation for both $D$ and $\rho$ off-shell mesons. We study the double Borel sum rule for the three point function of two pseudoscalar and one vector meson currents. We find that the momentum dependence of the form factors is very different if the $D$ or the $\rho$ meson is off-shell, but they lead to the same coupling constant in the $DD\rho$ vertex. We discuss two different approaches to extract the $DD\rho$ coupling constant.

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Among the basic things that we want to know about hadrons are their sizes, or, equivalently, their form factors [1,2]. The size of a hadron depends on how we “look” at it. The most extensively studied particle is the nucleon, which has been probed mainly by photons. In lower energy experiments, where also the four momentum transfer ($q^2$) is low, it was possible to determine the electromagnetic form factor (and the charge radius) of the nucleon. In higher energy experiments and very large values of $q^2$ a very different picture of the nucleon emerged, in which it is made of pointlike particles, the quarks. From these observations one may conclude that, when probing the nucleon, nearly on-shell photons ($q^2 \sim 0$) recognize sizes whereas highly off-shell photons ($q^2 << 0$) do not. This statement is supported by the phenomenologically very successful vector meson dominance hypothesis, according to which real photons are with a large probability converted to vector mesons (which are extended objects) and then interact with the nucleon. Recent HERA data on electron-proton reactions can be well understood introducing a “transverse radius of the photon”, parametrized as [3]

$$r_\gamma \simeq \frac{1}{\sqrt{Q^2 + m^2}}$$  \hspace{1cm} (1)

where $Q^2 = -q^2$ and $m$ is the mass of the vector meson considered. This empirical formula tells us that for $Q^2 \to \infty$ the photon is pointlike and “resolves” the nucleon target, i.e.,
identifies its pointlike constituents and does not “see” the size of the nucleon. Moreover this formula indicates that for \( Q^2 \sim 0 \) and for light mesons (like the \( \rho^0 \)) the photon has appreciable transverse radius and therefore also identifies the global nucleon extension. Finally, in the above formula we may have a heavy vector meson (\( J/\psi \) or \( \Upsilon \)) which will, either real or virtual, resolve the nucleon into pointlike constituents. This feature nicely explains why the \( J/\psi \) photoproduction cross section grows pronouncedly with the (photon-proton) energy whereas the \( \rho \) cross section grows very slowly \([3]\). In the former case the compact \( J/\psi \) interacts with the small \( x \) gluons in the protons, which have a fastly growing population. In the latter case, the \( \rho \) identifies the global and slowly growing geometrical size of the proton.

Along this line of reasoning one might ask i) how is the nucleon form factor when probed by heavy mesons (\( J/\psi, D, D^*, ... \) ? and also ii) which are the form factors of these heavy mesons when probed by light particles such as photons, pions and \( \rho \) mesons? Apart from their intrinsic value as fundamental knowledge about nature, the answers to these questions have immediate applications to the physics of quark gluon plasma (QGP). Indeed, form factors and coupling constants of all vertices of the type \( BB' \Lambda_c \) and \( MM' \Lambda_c \), where \( B(B') \), \( M(M') \) and \( \Lambda_c \) are respectively baryons, light mesons and charmed mesons, are relevant for the calculation of interaction cross sections of charmed mesons in nuclear matter \([4, 12]\).

A couple of years ago we started our program of computing the above mentioned quantities in the framework of QCD sum rules (QCDSR). Concerning question i) , in \([13]\) we calculated the coupling constant in the vertex \( NDA_c \) with the \( D \) meson off-shell. In \([14]\) we extended the calculation performed in \([13]\) and computed the \( Q^2 \) dependent form factors of the \( NDA_c \) and \( ND^* \Lambda_c \) vertices. We have also studied the \( D^*D\pi \) and \( B^*B\pi \) vertices \([15]\). One of the conclusions of these works is that when the off-shell particle in the vertex is heavy, the form factor tends to be broader (or harder) as a function of \( Q^2 \), which means larger cut-off parameters and smaller associated sizes.

In the present work we will further investigate form factors involving heavy mesons in order to extend our previous conclusions. We also want to better estimate the uncertainties in the procedure of determining coupling constants with our techniques. In particular, we want to check whether the coupling is the same regardless of which particle is off-shell. For these purposes we consider the vertex \( DD\rho \) and compute form factors and coupling constants for the cases where the \( D \) is off-shell, the \( \rho \) is off-shell and then compare the results. Following the QCDSR formalism described in our previous works we write the three-point function associated with a \( DD\rho \) vertex, which is given by

\[
\Gamma^{(D)}_{\mu}(p, p') = \int d^4x \, d^4y \, \langle 0 | T\{ j_+(x) \bar{j}_0(y) j_\mu^+(0) \} | 0 \rangle \, e^{ip'.x} \, e^{-ip\cdot y}, \tag{2}
\]

for a \( D \) off-shell meson, and by

\[
\Gamma^{(\rho)}_{\mu}(p, p') = \int d^4x \, d^4y \, \langle 0 | T\{ j_+(x) \bar{j}_\mu^+(y) j_0(0) \} | 0 \rangle \, e^{ip'.x} \, e^{-ip\cdot y}, \tag{3}
\]

for a \( \rho \) off-shell meson, where \( j_+ = i\bar{d}\gamma_5 c, \, j_0 = i\bar{c}\gamma_5 u \) and \( j_\mu^+ = \bar{u}\gamma_\mu d \) are the interpolating fields for \( D^+, D^0 \) and \( \rho^+ \) respectively with \( u, d \) and \( c \) being the up, down, and charm quark fields.

The general expression for the vertex function in Eqs. (2) and (3) has two independent structures. Writing \( \Gamma_{\mu} \) in terms of the invariant amplitudes associated with these two structures:
\[
\Gamma(p, p') = \Gamma_1(p^2, q^2) \rho + \Gamma_2(p^2, q^2) \rho',
\]
we can write a double dispersion relation for each one of the invariant amplitudes \(\Gamma_i\) \((i = 1, 2)\), over the virtualities \(p^2\) and \(p'^2\) holding \(Q^2 = -q^2\) fixed:
\[
\Gamma_i(p^2, p'^2, Q^2) = -\frac{1}{4\pi^2} \int_{s_{\text{min}}}^{\infty} ds \int_{m_i^2}^{\infty} du \frac{\rho_i(s, u, Q^2)}{(s - p^2)(u - p'^2)},
\]
where \(\rho_i(s, u, Q^2)\) equals the double discontinuity of the amplitude \(\Gamma_i(p^2, p'^2, Q^2)\) on the cuts \(s_{\text{min}} \leq s \leq \infty, m_i^2 \leq u \leq \infty\), and where \(s_{\text{min}} = m_i^2\) in the case of the \(D\) off-shell, where the dispersion relation is written in terms of the two \(D\) mesons' momenta, and \(s_{\text{min}} = 0\) in the case of \(\rho\) off-shell, the dispersion relation being now written in terms of the \(\rho\) and the \(D\) meson momenta.

The double discontinuity of the perturbative contribution reads:
\[
\rho_{i}^{\text{pert}(D)}(s, u, Q^2) = \frac{6(s + Q^2 + u)[m_i^2(m_i^2 + s + Q^2 - u) - Q^2 u]}{[(s + u + Q^2)^2 - 4su]^3/2},
\]
\[
\rho_{2}^{\text{pert}(D)}(s, u, Q^2) = \frac{12s[m_i^2(m_i^2 + s + Q^2 - u) - Q^2 u]}{[(s + u + Q^2)^2 - 4su]^3/2},
\]
for a \(D\) off-shell meson, with the integration limit condition:
\[
(m_i^2 + Q^2)(u - m_i^2) \geq sm_i^2,
\]
and
\[
\rho_{1}^{\text{pert}(\rho)}(s, u, Q^2) = \frac{3(s + Q^2 - u)[m_i^2(m_i^2 - s - Q^2 - u) + su]}{[(s + u + Q^2)^2 - 4su]^3/2},
\]
\[
\rho_{2}^{\text{pert}(\rho)}(s, u, Q^2) = \frac{-3(s - Q^2 - u)[m_i^2(m_i^2 - s - Q^2 - u) + su]}{[(s + u + Q^2)^2 - 4su]^3/2},
\]
for a \(\rho\) off-shell meson, with the integration limit condition:
\[
(s - m_i^2)(u - m_i^2) \geq Q^2 m_i^2.
\]

The phenomenological side of the vertex function is obtained by considering the contribution of the \(\rho\) and one \(D\) meson, or the two \(D\) mesons states to the matrix element in Eqs. (2) and (3) respectively:
\[
\Gamma^{\text{phen}(D)}_{\mu}(p, p') = -\frac{m_D^2 f_D m_p^2}{m_c} \frac{m_D^2}{g_\rho} g_D(Q^2) \frac{1}{p^2 - m_p^2} \frac{1}{p'^2 - m_D^2} \times
\left(-2p'_{\mu} + \frac{m_D^2 + m_p^2 + Q^2}{m_p^2} p_{\mu}\right) + \text{higher resonances},
\]
(12)
\[ \Gamma_\mu^{\text{phen}(\rho)}(p, p') = -\frac{m_D^4 f_D^2}{m_c^2} g_\rho(Q^2) \frac{1}{p^2 - m_D^2 p'^2 - m_D^2} (p'_\mu + p_\mu) + \text{higher resonances}, \quad (13) \]

where

\[ g_D(Q^2) = \frac{m_D^2 f_D}{m_c} \frac{g^{(D)}_{DD\rho}(Q^2)}{Q^2 + m_D^2}, \quad (14) \]

and

\[ g_\rho(Q^2) = \frac{m_\rho^2 g^{(\rho)}_{DD\rho}(Q^2)}{g_\rho} \frac{Q^2}{Q^2 + m_\rho^2}, \quad (15) \]

where \( m_D, m_\rho, f_D \) and \( g_\rho \) are the masses and decay constants of the mesons \( D \) and \( \rho \) respectively, and \( g^{(M)}_{DD\rho}(Q^2) \) is the form factor at the \( DD\rho \) vertex when the meson \( M \) is off-shell. The contribution of higher resonances in Eqs. (12) and (13) will be taken into account as usual in the standard form of continuum contribution from the thresholds \( s_0 \) and \( u_0 \) [16]. Finally we perform a double Borel transformation [16] in both variables \( p^2 = -p^2 \rightarrow M^2 \) and \( p'^2 = -p'^2 \rightarrow M'^2 \), for each invariant amplitude, and identify the two representations described above.

For consistency we use in our analysis the QCDSR expressions for the decay constants appearing in Eqs. (12) and (13) up to dimension four in lowest order of \( \alpha_s \):

\[ f_D^2 = \frac{3m_c^2}{8\pi^2 m_D^2} \int_{m_c^2}^{u_0} du \frac{(u - m_c^2)^2}{u} e^{(m_D^2 - u)/M^2} - \frac{m_c^4}{m_D^4} \langle \bar{q}q \rangle e^{(m_D^2 - m_c^2)/M^2}, \quad (16) \]

\[ \frac{m_\rho^2}{g_\rho^2} = \frac{M^2}{8\pi^2} \left( 1 - e^{-s_0/M^2} \right) + \frac{m_q}{M^2} \langle \bar{q}q \rangle, \quad (17) \]

where we have omitted the numerically insignificant contribution of the gluon condensate.

In the case of the \( \rho \) off-shell, the sum rules in both structures give the same results, as can be seen by Eqs. (9), (10), (13) and the limit condition Eq. (11). However, in the case of \( D \) off-shell both structures give different results, as can be seen by Eqs. (6), (7) and (12). In particular, for the structure \( p_\mu \), the quark condensate also contributes with:

\[ \Gamma_\mu^{<\bar{q}q>(D)}(p, p') = \frac{m_c \langle \bar{q}q \rangle}{p^2(p'^2 - m_c^2)} p_\mu, \quad (18) \]

and the resulting sum rule in this structure is not very stable as a function of the Borel mass. Therefore, in this work we will concentrate in the \( p'_\mu \) structure, which we found to be the more stable one.

The parameter values used in all calculations are \( m_q = (m_u + m_d)/2 = 7 \text{ MeV}, m_c = 1.5 \text{ GeV}, m_D = 1.87 \text{ GeV}, m_\rho = 0.77 \text{ GeV}, \langle \bar{q}q \rangle = -(0.23)^3 \text{ GeV}^3 \). We parametrize the continuum thresholds as

\[ s_0 = (m_M + \Delta_s)^2, \quad (19) \]
where \( m_M = m_D(m_\rho) \) for the case that the \( \rho(D) \) meson is off-shell, and
\[
u_0 = (m_D + \Delta_u)^2. \tag{20}
\]
The values of \( u_0 \) and \( s_0 \) are, in general, extracted from the two-point function sum rules for \( f_D \) and \( g_\rho \) in Eqs. (16) and (17). Using the Borel region \( 3 \leq M^2 \leq 5 \text{GeV}^2 \) (for the \( D \) meson) and \( 0.5 \leq M^2 \leq 0.9 \text{GeV}^2 \) (for the \( \rho \) meson) we found a good stability for \( f_D \) and \( g_\rho \) with \( \Delta_s = \Delta_u \sim 0.5 \text{GeV} \). In our study we will allow for a small variation in \( \Delta_s \) and \( \Delta_u \) to test the sensitivity of our results to the continuum contribution.

In Fig. 1 we show the behavior of the form factor \( g_{DD\rho}(Q^2) \) at \( Q^2 = 1 \text{ GeV} \) as a function of the Borel mass \( M^2 \), using \( \Delta_s \) and \( \Delta_u \) given in Eqs. (19) and (20) equal to 0.5 GeV. The dashed line gives the results for \( g_{DD\rho}^{(p)}(Q^2) \) at a fixed ratio \( M^2/M^2 = m_D^2/m_D^2 \), and the dot-dashed and solid lines give the results for \( g_{DD\rho}^{(D)}(Q^2) \) at a fixed ratio \( M^2/M^2 = m_D^2/m_D^2 \) (which corresponds to \( M^2 \) varying in the interval \( 0.5 \leq M^2 \leq 0.8 \text{GeV} \)). The dot-dashed line corresponds to the structure \( p_\mu \) and the solid line to the structure \( p'_\mu \). As mentioned before the sum rule in the \( p_\mu \) structure gives a much flatter “plateau” for the form factor. In obtaining the results shown in Fig. 1 we have used for \( f_D \) and \( g_\rho \) appearing in Eqs. (4) and (13) the values \( f_D = 200 \text{ MeV} \) and \( g_\rho = 5.45 \). The behavior of the curve shown in Fig. 1 for other \( Q^2 \) and continuum threshold values is similar.

In a recent calculation for the \( D^*D\pi \) and \( B^*B\pi \) form factors \([13]\) we have included, besides the perturbative contribution, the gluon condensate contribution. We have found out that the gluon condensate is small, as compared with the perturbative contribution and decreases with the Borel mass. The most important feature of the gluon condensate is the fact that it improves the stability of the result as a function of the Borel mass. Since its contribution, in the case of the \( D^*D\pi \) form factor at \( M^2 = 5 \text{GeV}^2 \), is less than 10% of the perturbative contribution, in this work we will neglect the gluon condensate. In order to be sure that the absence of the gluon condensate will not affect our results, we will extract the value of the form factor at a higher value of the Borel mass, where we expect the gluon condensate contribution to be negligible.

Fixing \( M^2 = 4.7 \text{ GeV}^2 \) (which corresponds to \( M^2 = 0.8 \text{GeV}^2 \) for the case of off-shell \( D \)) we show, in Fig. 2, the momentum dependence of the form factor (circles for \( g_{DD\rho}^{(D)}(Q^2) \) and squares for \( g_{DD\rho}^{(p)}(Q^2) \)) in the interval \( 0.1 \leq Q^2 \leq 5 \text{ GeV} \), where we expect the sum rules to be valid (since in this case the cut in the \( t \) channel starts at \( t \sim m_x^2 \) and thus the Euclidian region stretches up to that threshold). In Fig. 2 we also show that the \( Q^2 \) dependence of the form factor represented by the circles can be well reproduced by the monopole parametrization (solid line)
\[
g_{DD\rho}^{(D)}(Q^2) = \frac{37.5}{Q^2 + 12.12}, \tag{21}
\]
and that the form factor represented by the squares can be well reproduced by the exponential parametrization (solid line)
\[
g_{DD\rho}^{(p)}(Q^2) = 2.53e^{-Q^2/6\Delta^2}. \tag{22}
\]

In order to extract the coupling constant \( g_{DD\rho} \) from the form factor, we need first to define it in both cases. In refs. \([14,15]\) the coupling constant is defined as the value of the
form factor at $Q^2 = 0$. In this case the monopole and the exponential expressions should be written as:

$$g^{(D)}_{DD\rho}(Q^2) = g_{DD\rho} \frac{\Lambda_D^2}{Q^2 + \Lambda_D^2}, \quad (23)$$

$$g^{(\rho)}_{DD\rho}(Q^2) = g_{DD\rho} \frac{Q^2}{\Lambda_\rho^2}. \quad (24)$$

However, in refs. [5,17] the coupling is defined as the value of the form factor at $Q^2 = -m_M^2$, where $m_M$ is the mass of the off-shell meson. In this case the monopole and the exponential expressions should be written as:

$$g^{(D)}_{DD\rho}(Q^2) = g_{DD\rho} \frac{\Lambda_D^2 - m_D^2}{Q^2 + \Lambda_D^2}, \quad (25)$$

$$g^{(\rho)}_{DD\rho}(Q^2) = g_{DD\rho} \frac{Q^2 + m_\rho^2}{\Lambda_\rho^2}. \quad (26)$$

The coupling constants and cut-offs resulting of the parametrizations in Eqs. (23), (24), (25) and (26) are given in Table I.

|                | $g^{(M)}_{DD\rho}(Q^2 = 0)$ | $g^{(M)}_{DD\rho}(Q^2 = -m_M^2)$ | $\Lambda_M$ (GeV) |
|----------------|-----------------------------|----------------------------------|-------------------|
| $D$ off-shell  | 3.1                         | 4.4                              | 3.5               |
| $\rho$ off-shell | 2.5                         | 4.6                              | 1.0               |

**TABLE I:** Values of the coupling constants and cut-offs which reproduce the QCDSR results for $g^{(M)}_{DD\rho}(Q^2)$.

There are two very important conclusions that we can draw from the results in Table I. First of all is the fact that the form factor is harder if the off-shell meson is heavy, implying that the size of the vertex depends on the exchanged meson, in agreement with our expectation, based on the the findings of refs. [14,15]. Therefore, a heavy meson will see the vertex as pointlike, whereas a light meson will see its extension. The second conclusion is that the value of the coupling constant extracted from the QCDSR results depends, of course on its definition. Having in mind that the inherent precision of the QCDSR method is of order of 20%, we can say that the results nearly coincide for the both cases considered, i.e. off-shell $D$ and $\rho$ meson. This agreement between the two values is even more spectacular in the case where the coupling is extracted at $Q^2 = -m_M^2$.

It’s also very interesting to notice that the value used in refs. [4,10] for $g_{DD\rho}$, obtained in the framework of vector meson dominance, is: $g_{DD\rho} = 2.52$, in complete agreement with our value extracted from the form factor normalized at $Q^2 = 0$. Since their form factors are also normalized at $Q^2 = 0$, we completely endorse their procedure. However, in their
analysis they vary the value of the cut-off in the range $1 \leq \Lambda_D \leq 2$ GeV. Here we obtain larger values.

Using for the continuum thresholds $\Delta_s = \Delta_u = 0.6$ GeV, in Eqs. (19) and (20), the resulting values for the coupling constants and cut-offs are given in Table II, from where we can see that a 20% variation in the continuum thresholds leads to less than 10% variation in the couplings and cut-offs, showing a good stability of the results.

$$
\begin{array}{|c|c|c|}
\hline
 & g_{DD\rho}^{(M)}(Q^2 = 0) & g_{DD\rho}^{(M)}(Q^2 = -m_M^2) & \Lambda_M \text{ (GeV)} \\
\hline
D \text{ off-shell} & 3.3 & 4.4 & 3.8 \\
\rho \text{ off-shell} & 2.6 & 4.1 & 1.2 \\
\hline
\end{array}
$$

**TABLE II**: Values of the coupling constants and cut-offs which reproduce the QCDSR results for $g_{DD\rho}^{(M)}(Q^2)$ using $\Delta_s = \Delta_u = 0.6$ GeV.

Considering the incertainties in the continuum threshold, and the difference in the values of the coupling constants extracted when the $D$ or $\rho$ mesons are off-shell, our result for the $DD\rho$ coupling constant is:

$$
g_{DD\rho} = \begin{cases} 
2.9 \pm 0.4 & \text{in } Q^2 = 0 \\
4.3 \pm 0.3 & \text{in the pole of the off-shell meson}
\end{cases} \quad (27)
$$

To summarize: we have used the method of QCD sum rules to compute form factors and coupling constants in $DD\rho$ vertices. Our results for the couplings show once more that this method is robust, yielding numbers which are approximately the same regardless of which particle we choose to be off-shell and depending weakly on the choice of the continuum threshold. As for the form factors, we obtain a harder (softer) form factor when the off-shell particle is heavier (lighter). This confirms our expectation based on previous works and answers the question in the title: heavy mesons “see” smaller sizes in the light mesons, but the latter can not resolve small structures in the former. Our program will continue and studies of other vertices are in progress.

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FIG. 1. $M'^2$ dependence of the $DD\rho$ form factors at $Q^2 = 1$ GeV$^2$ for $\Delta_s = \Delta_u = 0.5$ GeV. The dashed line gives the QCDSR result for $g^{(p)}_{DD\rho}(Q^2)$ and the dot-dashed and solid lines give the QCDSR results for $g^{(D)}_{DD\rho}(Q^2)$ in the $p_\mu$ and $p'_\mu$ structures respectively.

FIG. 2. Momentum dependence of the $DD\rho$ form factor for $\Delta_s = \Delta_u = 0.5$ GeV. The solid lines give the parametrization of the QCDSR results through Eq. (21) for the circles, and Eq. (22) for the squares.