Abstract—Based on the ideas of cyclotomic cosets, idempotents and Mattson-Solomon polynomials, we present a new method to construct GF($2^m$), where $m > 0$ cyclic low-density parity-check codes. The construction method produces the dual code idempotent which is used to define the parity-check matrix of the low-density parity-check code. An interesting feature of this construction method is the ability to increment the code dimension by adding more idempotents and so steadily decrease the sparseness of the parity-check matrix. We show that the constructed codes can achieve performance very close to the sphere-packing-bound constrained for binary transmission.

Keywords—Coding, idempotent, non binary LDPC, Mattson-Solomon polynomial

I. INTRODUCTION

Since the recent rediscovery of low-density parity-check (LDPC) codes, a great deal of effort has been devoted to constructing LDPC codes that can work well with the belief-propagation iterative decoder. The studies of long block-length LDPC codes are very much established. The recent works of [1], [2] have shown that, for long block-lengths, the best performing LDPC codes are irregular codes and these codes can outperform turbo codes of the same block-length and code-rate. These long LDPC codes have degree distributions which are derived from differential evolution [1] or Gaussian Approximation [3]. It can be shown that, using the concentration theorem [4], the performance of infinitely long LDPC codes of a given degree distribution can be characterised by the average performance of the ensemble based on cycle-free assumption. This assumption, however, does not work for short and moderate block-length LDPC codes due to the inevitable existence of cycles in the underlying Tanner Graphs. Consequently, for a given degree distribution, the performance of short block-length LDPC codes varies considerably from the ensemble performance. Various methods exist for the construction of finite block-length irregular codes [5],[6],[7]. In addition to irregular LDPC codes, algebraic constructions exist and the resulting codes are regular and usually cyclic in nature. Some examples of algebraic LDPC codes are the Euclidean and Projective Geometry codes [8].

It has been noticed by the authors that, in general, there is a performance association between the code minimum distance ($d_{\text{min}}$) and decoding convergence. The irregular LDPC codes converge very well with iterative decoding, but their $d_{\text{min}}$ are reasonably low. On the other hand, the algebraically constructed LDPC codes, which have high $d_{\text{min}}$, tend not to converge well with the iterative decoder. It is not surprising that algebraically constructed codes may outperform the irregular codes. The latter have error-floor which is caused by the $d_{\text{min}}$ error-events. On the encoding side, the existence of algebraic structure in the codes is of benefit. Rather than depending on the parity-check or generator matrices for encoding, as in the case of irregular codes, a low-complexity encoder can be built for the algebraic LDPC codes. One such example is the linear shift-register encoder for cyclic LDPC codes. Assuming that $n$ and $k$ denote the codeword and information length respectively, algebraic codes that are cyclic offer another decoding advantage. The iterative decoder has $n$ parity-check equations to iterate with instead of $n - k$ equations, as in the case of non-cyclic LDPC codes, and this leads to improved performance.

It has been shown that the performance of LDPC codes can be improved by going beyond the binary field [9], [10], Hu et al. showed that, under iterative decoding, the non binary LDPC codes have better convergence properties than the binary codes [10]. They also demonstrated that a coding gain of 0.25dB is achieved by moving from GF(2) to GF($2^m$). Non binary LDPC codes in which each symbol takes values from GF($2^m$) offer an attractive scheme for higher-order modulation. The complexity of the symbol-based iterative decoder can be simplified as the extrinsic information from the component codes can be evaluated using the frequency domain dual codes decoder based on the Fast-Walsh-Hadamard transform.

Based on the pioneering works of MacWilliams [11],[12] on the idempotents and the Mattson-Solomon polynomials, we present a generalised construction method for algebraic GF($2^m$) codes that are applicable as LDPC codes. The construction for binary codes using idempotents has been investigated by Shibuya and Sakaniwa [13], however, their investigation was mainly focused on half-rate codes. In this paper, we construct some higher code-rate non binary LDPC codes with good convergence properties. We focus on the design of short block-length LDPC codes in view of the benefits for thin data-storage, wireless, command/control data reporting and watermarking applications. One of the desirable features in any code construction technique is an effective method of determining the $d_{\text{min}}$, and this feature is not present...
II. CYCLOTOMIC COSETS, IDEMPOTENTS AND MATTRSON-SOLOMON POLYNOMIALS

We briefly review the theory of cyclotomic cosets, idempotents and Mattson-Solomon polynomials to make this paper relatively self-contained. Let us first introduce some notations that will be used throughout this paper. Let $m$ and $m'$ be positive integers with $m|m'$, so that $\text{GF}(2^m)$ is a subfield of $\text{GF}(2^{m'})$. Let $n$ be a positive odd integer and $\text{GF}(2^n)$ be the splitting field for $\text{GF}(2^{m'})$, so that $n|2^{m'}-1$. Let $r = (2^{m'}-1)/n$, $l = (2^{m'}-1)/(2^m-1)$, $\alpha$ be a generator for $\text{GF}(2^{m'})$ and $\beta$ be a generator for $\text{GF}(2^n)$, where $\beta = \alpha^l$. Let $T_a(x)$ be the set of polynomials of degree at most $n-1$ with coefficients in $\text{GF}(2^n)$.

**Definition 2.1:** If $a(x) \in T_{m'}(x)$, then the finite-field transform of $a(x)$ is:

$$A(z) = \text{MS}(a(x)) = \sum_{j=0}^{n-1} a(\alpha^{-rj}) z^j$$  \hspace{1cm} (1)

where $A(z) \in T_{m'}(z)$. This transform is widely known as the Mattson-Solomon polynomial. The inverse transform is:

$$a(x) = \text{MS}^{-1}(A(z)) = \frac{1}{n} \sum_{i=0}^{n-1} A(\alpha^i) x^i$$  \hspace{1cm} (2)

**Definition 2.2:** Consider $e(x) \in T_{m'}(x)$, $e(x)$ is an idempotent if the property of $e(x) = e(x)^2 \mod (1 + x^n)$ is satisfied. In the case of $m = 1$, the property of $e(x) = e(x^2) \mod (1 + x^n)$ is also satisfied.

An $(n, k)$ cyclic code $C$ can be described by the generator polynomial $g(x) \in T_m(x)$ of degree $n-k$ and the parity-check polynomials $h(x) \in T_m(x)$ of degree $k$ such that $g(x)h(x) = 1 + x^n$. It is widely known that idempotents can be used to generate $C$. Any $\text{GF}(2^n)$ cyclic code can also be described by a unique idempotent $e_g(x) \in T_m(x)$ which consists of a sum of primitive idempotents. This unique idempotent is known as the generating idempotent and, as the name implies, $g(x)$ is a divisor of this idempotent, i.e. $e_g(x) = m(x)g(x)$, where $m(x)$ contains the repeated factors or non-factors of $1 + x^n$.

**Lemma 2.1:** If $e(x) \in T_m(x)$ is an idempotent, $E(z) = \text{MS}(e(x)) \in T_1(z)$.

**Proof:** (cf. [11, Ch 8]) Since $e(x) = e(x)^2 \mod (1 + x^n)$, from equation (1) it follows that $e(\alpha^{-rj}) = e(\alpha^{-rj})^2$, $\forall j \in \{0, 1, \ldots, n-1\}$ for some integers $r$ and $l$. Clearly, $e(\alpha^{-rj}) \in \{0, 1\}$ implying that $E(z)$ is a binary polynomial.

**Definition 2.3:** If $s$ is a positive integer, the binary cyclotomic coset of $s \mod n$ is:

$$C_s = \{2^i s \mod n \mid 0 \leq i \leq t\},$$

where we shall always assume that the subscript, $s$, is the smallest element in the set $C_s$ and $t$ is the smallest positive integer with the property that $2^{t+1}s = s \mod n$. If $\mathcal{N}$ is the set consisting of the smallest elements of all possible cyclotomic cosets then

$$C = \bigcup_{s \in \mathcal{N}} C_s = \{0, 1, 2, \ldots, n-1\}.$$

**Lemma 2.2:** Let $s \in \mathcal{N}$ and let $C_{s_i}$ represents the $i$th element of $C_s$. Let the polynomial $e_s(x) \in T_m(x)$ be given by

$$e_s(x) = \sum_{0 \leq i \leq \left| C_s \right| - 1} e_{C_{s_i}} x^{e_{C_{s_i}}},$$  \hspace{1cm} (3)

where $|C_s|$ is the number of elements in $C_s$ and $e_{C_{s_i}}$ is defined below.

i) if $m = 1$, $e_{C_{s_i}} = 1$.

ii) if $m > 1$, $e_{C_{s_i}}$ is defined recursively as follows:

$$\begin{align*}
&\text{for } i = 0, \quad e_{C_{s_i}} \in \{1, \beta, \beta^2, \ldots, \beta^{2^n-2}\}, \\
&\text{for } i > 0, \quad e_{C_{s_i}} = e_{C_{s_{i-1}}}^2,
\end{align*}$$

The polynomial so defined, $e_s(x)$, is an idempotent. We term $e_s(x)$ a **cyclotomic idempotent**.

**Definition 2.4:** Let $\mathcal{M} \subseteq \mathcal{N}$ and let $u(x) \in T_m(x)$ be

$$u(x) = \sum_{s \in \mathcal{M}} e_s(x).$$  \hspace{1cm} (4)

Then (refer to lemma 2.2) $u(x)$ is an idempotent and we call $u(x)$ a **parity-check idempotent**.

The parity-check idempotent $u(x)$ can be used to describe the code $C$, the parity check matrix being made up of the $n$ cyclic shifts of the polynomial $x^d\deg(u(x))u(x^{-1})$. If $(u(x), 1 + x^n) = h(x)^t$ then, in general, $\text{wt}(u(x))$ is much lower than $\text{wt}(h(x))^2$. Based on this observation and the fact that $u(x)$ contains all the roots of $h(x)$, we can construct cyclic codes that have a low-density parity-check matrix.

**Definition 2.5:** Let the polynomial $f(x) \in T_1(x)$. The difference enumerator of $f(x)$, denoted as $D(f(x))$, is defined as follows:

$$D(f(x)) = f(x)f(x^{-1}) = d_0 + d_1x + \ldots + d_{n-1}x^{n-1}.$$  \hspace{1cm} (5)

where we assume that $D(f(x))$ is a modulo $1-x^n$ polynomial with real coefficients.

**Lemma 2.3:** Let $m = 1$ and let $d_i$ for $0 \leq i \leq n-1$ denote the coefficients of $D(u(x))$. If $d_i \in \{0, 1\}, \forall i \in \{1, 2, \ldots, n-1\}$, the parity-check polynomial derived from $u(x)$ is orthogonal on each position in the $n$-tuple. Consequently (i) the $d_{min}$ of the resulting $C$ is $1 + \text{wt}(u(x))$ and (ii) the underlying Tanner Graph has girth of at least 6.

**Proof:** (i) (cf. [14, Theorem 10.11]) Let a codeword $c(x) = c_0 + c_1x + \ldots + c_{n-1}x^{n-1}$ and $c(x) \in T_1(x)$. For each non zero bit position $c_j$ of $c(x)$ where $j \in \{0, 1, \ldots, n-1\}$, $G(a, b)$ denotes the greatest common divisor of $a$ and $b$. $\text{wt}(f(x))$ denotes the weight of polynomial $f(x)$. 


there exists three integers \(a\), \(b\), and \(c\) such that \(2(b-a) \equiv (c-b)\) for \(a < b < c\). If these three integers are associated to the variable nodes in the Tanner Graphs, a cycle of length 6 can be formed between these variable nodes and some check nodes.

From Lemma 2.4 we can deduce that \(u(x)\) is the parity-check polynomial for One-Step Majority-Logic Decodable codes if \(d_i \in \{0, 1\}, \forall i \in \{1, 2, \ldots, n-1\}\) or the parity-check polynomial for Difference-Set Cyclic codes if \(d_i = 1, \forall i \in \{1, 2, \ldots, n-1\}\).

**Lemma 2.4:** For the non binary GF\((2^m)\) cyclic codes, the \(d_{\text{min}}\) is bounded by:

\[
d_0 < d_{\text{min}} \leq \min \{\text{wt}(g(x)), 1 + \text{wt}(u(x))\}
\]

where \(d_0\) denotes the maximum run of consecutive ones in \(U(z)\) taken cyclically modulo \(n\).

**Proof:** The lower-bound of the \(d_{\text{min}}\) of a cyclic code, BCH bound is determined from the number of consecutive roots of \(e_g(x)\) and from lemma 2.1 it is equivalent to the run of consecutive ones in \(U(z)\).

### III. Construction Algorithm for the Codes

Based on the mathematical theories outlined above, we devise an algorithm to construct GF\((2^m)\) \(C\) which are applicable for iterative decoding. The construction algorithm can be described in the following procedures:

1. Given the integers \(m\) and \(n\), find the splitting field (GF\((2^m')\)) of \(1 + x^n\) over GF\((2^m)\). We can only construct GF\((2^m)\) cyclic codes of length \(n\) if and only if the condition of \(m|m'\) is satisfied.
2. Generate the cyclotomic cosets modulo \(2^m' - 1\) and denote it \(C'\).
3. Derive a polynomial \(p(x)\) from \(C'\). Let \(s \in \mathcal{N}\) be the smallest positive integer such that \(|C'_s| = m\). The polynomial \(p(x)\) is the minimal polynomial of \(\alpha^s:\)

\[
p(x) = \prod_{0 \leq i < m} \left( x + \alpha^{C'_s i} \right)
\]

(6)

Construct all elements of GF\((2^m)\) using \(p(x)\) as the primitive polynomial.

4. Let \(C\) be the cyclotomic cosets modulo \(n\) and \(\mathcal{N}\) be a set containing the smallest number in each coset of \(C\). Assume that there exists a non empty set \(\mathcal{M} \subset \mathcal{N}\) and following definition 2.4 construct the parity-check idempotent \(u(x)\). The coefficients of \(u(x)\) can be assigned following lemma 2.2.

5. Generate the parity-check matrix of \(C\) using the \(n\) cyclic shifts of \(\sum_{s \in \mathcal{M}} E_s(x)\) of \(u(x)\).

6. Compute \(r\) and \(l\), then take the Mattson-Solomon polynomial of \(u(x)\) to produce \(U(z)\). Obtain the code dimension and the lower-bound of the \(d_{\text{min}}\) from \(U(z)\).

Note that care should be taken to ensure that there is no common factor between \(n\) and all of the exponents of \(u(x)\), apart from unity, in order to avoid a degenerate code.

**Example 3.1:** Let us assume that we want to construct a GF\((64)\) \(n = 21\) cyclic idempotent code. The splitting field for \(1 + x^{21}\) over GF\((64)\) is GF\((64)\) and this implies that \(m = m' = 6, r = 3\) and \(l = 1\). Let \(C\) and \(C'\) denote the cyclotomic cosets modulo \(n\) and \(2^m' - 1\) respectively. \(|C'_1| = 6\) and therefore the primitive polynomial \(p(x)\) has roots of \(\alpha^j\), \(\forall j \in C'_{1}\), i.e., \(p(x) = 1 + x + x^6\). By letting \(1 + \beta + \beta^6 = 0\), all of the elements of GF\((64)\) can be defined.

If we let \(u(x)\) be the parity-check idempotent generated by the sum of the cyclotomic idempotents defined by \(C_s\) where \(s \in \{\mathcal{M} : 5, 7, 9\}\) and \(e_{c_{s, o}}, \forall s \in \mathcal{M}\) be \(\beta_{23}, 1\) and \(1\) respectively, \(u(x) = \beta_{23} x^5 + x^7 + x^9 + \beta_{46} x^{10} + \beta_{43} x^{13} + x^{14} + x^{15} + \beta_{53} x^{17} + x^{18} + \beta_{58} x^{19} + \beta_{29} x^{20}\) and its Mattson-Solomon polynomial \(U(z)\) tells us that it is GF\((64)\)(21, 15) cyclic code with \(d_{\text{min}} \geq 5\).

A systematic algorithm has been developed to sum up all combinations of the cyclotomic idempotents to search for all possible GF\((2^m)\) cyclic codes of a given length. The search algorithm is targeted on the following key parameters:

1. Sparseness of the resulting parity-check matrix. Since the parity-check matrix of \(C\) is directly derived from \(u(x)\) which consists of the sum of the cyclotomic idempotents, we are only interested in low-weight cyclotomic idempotents. Let us define \(W_{\text{max}}\) as the maximum \(\text{wt}(u(x))\) then the search algorithm will only choose the cyclotomic idempotents whose sum has total weight less than or equal to \(W_{\text{max}}\).

2. High code-rate. The number of roots of \(u(x)\) which are also roots of unity define the dimension of \(C\) and let us define \(k_{\text{min}}\) as the minimum information length of \(C\). We are only interested in the sum of the cyclotomic idempotents whose Mattson-Solomon polynomial has at least \(k_{\text{min}}\) zeros.

3. High \(d_{\text{min}}\). Let us define \(d\) as the minimum value of the \(d_{\text{min}}\) of \(C\). The sum of the cyclotomic idempotents should have at least \(d - 1\) consecutive powers of \(\beta\) which are roots of unity but not roots of \(u(x)\).

The search algorithm can be relaxed to allow the existence of cycles of length 4 in the resulting parity-check matrix of \(C\). The condition of cycles-of-length-4 is not crucial as we will show later that there are codes that have good convergence properties when decoded using iterative decoder. Clearly, by eliminating the cycles-of-length-4 constraint, we can construct more codes.

Following definitions 2.1 and 2.4

\[
U(z) = MS \left( \sum_{s \in \mathcal{M}} E_s(x) \right) = \sum_{s \in \mathcal{M}} E_s(z)
\]

and hence it is possible to maximise the run of the consecutive ones in \(U(z)\) if the coefficients of \(e_s(x)\) are aligned appropriately. It is therefore important that all possible non zero values of \(e_{c_{s, o}}, \forall s \in \mathcal{M}\) are included in the search in order to guarantee that we can obtain codes with the highest possible \(d_{\text{min}}\) or at least to obtain a better estimate of the \(d_{\text{min}}\).
IV. CODE PERFORMANCE

As an example of the performance attainable from an iterative decoder, computer simulations have been carried out for several GF($2^m$) cyclic LDPC codes. We assume BPSK signalling and the iterative decoder used is the modified belief-propagation decoder which approximates the performance of a maximum-likelihood decoder [15],[16]. The frame-error-rate (FER) performance of the GF($2^p$)/(21, 15) cyclic LDPC code is shown in Fig. 1 and is compared with the sphere-packing-bound [17],[18] for binary codes of length 126 bits offset by the binary transmission loss\(^3\). We can see that the performance of the code is within 0.2dB away from this bound at $10^{-3}$ FER. The binary level minimum-distance of this GF(64)/(21, 15) cyclic LDPC code is 9.

\(^3\)In the rest of this paper, we assume that the sphere-packing-bound has been offset by the information theoretical loss associated with binary transmission.

![Frame error performance of the GF($2^p$)/(21, 15) cyclic LDPC code](image1)

Fig. 1. Frame error performance of the GF($2^p$)/(21, 15) cyclic LDPC code

![Frame error performance of the GF($2^p$)/(255, 175) cyclic LDPC code](image2)

Fig. 2. Frame error performance of the GF($2^p$)/(255, 175) cyclic LDPC code which is equivalent to (510, 350) binary code. At $10^{-3}$ FER, the performance of this code is approximately 0.36dB away from the sphere-packing-bound of length 510 bits. While both of the codes mentioned above are free from cycles of length 4, good convergence codes exist even if they have cycles of length 4 in the underlying Tanner Graph. One such example is the GF($2^p$)/(91, 63) cyclic code whose FER performance is shown in Fig. 3. At $10^{-3}$ FER, the code performs around 0.35dB away from the sphere-packing-bound of length 273 bits. The parameters of the codes in Fig. 2 and 3 are available in Table I. Some other examples of the non binary GF($2^m$) cyclic LDPC codes with their parameters and distance from the sphere-packing-bound are also shown in Table I

#### Table I

| $C$      | $u(x)$                                           | $d_{min}$ | $d'_b$ | Comment | SPB $^1$  |
|----------|--------------------------------------------------|-----------|--------|---------|-----------|
| GF(4) (51, 29) | $\beta^2x^3 + \beta^3x^2 + \beta^4x_1 + \beta^5x_0 + x_{17} + \beta x_{24} + \beta x_{27} + x_{54} + \beta x_{30} + \beta x_{45} + \beta^2 x_{48}$ | $5$       | $10$   | m = 2, $m' = 8$, r = 5 and l = 85 | 0.25dB    |
| GF(4)/(255, 175) | $\beta x_7 + \beta^2 x_{14} + \beta x_{29} + \beta^3 x_{50} + x_{111} + \beta x_{112} + \beta x_{123} + \beta x_{131} + \beta_{x_{183}} + \beta x_{189} + \beta x_{193} + \beta x_{219} + \beta x_{222} + \beta x_{224} + \beta x_{237} + \beta x_{246}$ | $\geq 17$ | $20$   | m = 2, $m' = 8$, r = 1 and l = 85 | 0.36dB    |
| GF(4)/(274, 191) | $\beta x_{13}^2 + \beta x_{17}^3 + \beta x_{60} + \beta^2 x_{27} + \beta x_{91} + \beta^3 x_{92} + \beta x_{105} + \beta x_{107} + \beta_{x_{117}} + \beta x_{148} + \beta^2 x_{155} + \beta x_{182} + \beta x_{184} + \beta x_{190} + \beta x_{219} + \beta x_{241} + \beta x_{234}$ | $\geq 18$ | $20$   | m = 2, $m' = 12$, r = 15 and l = 1365 | 0.4dB     |
| GF(8)/(63, 40)   | $1 + \beta x_{9} + \beta x_{13} + \beta x_{18} + \beta x_{19} + \beta x_{20} + \beta x_{30} + \beta x_{34} + \beta x_{38} + \beta x_{41} + \beta x_{52}$ | $\geq 6$  | $10$   | m = 3, $m' = 6$, r = 1 and l = 9 | 0.3dB     |
| GF(8)/(63, 43)   | $\beta x_{30} + \beta x_{11} + \beta x_{18} + \beta_{x_{21}} + \beta x_{22} + \beta x_{25} + x_{27} + \beta x_{36} + \beta x_{37} + \beta x_{42} + \beta x_{44} + \beta x_{45} + \beta x_{60} + \beta x_{54}$ | $\geq 8$  | $12$   | m = 3, $m' = 6$, r = 1 and l = 9 | 0.45dB    |
| GF(8)/(91, 63)   | $\beta x_{20} + \beta x_{17}^2 + \beta x_{18} + \beta x_{30} + \beta x_{13} + \beta x_{16} + \beta x_{23} + \beta x_{26} + \beta x_{32} + \beta x_{37} + \beta x_{40} + \beta x_{42} + \beta x_{57} + \beta x_{64} + \beta x_{74}$ | $\geq 8$  | $10$   | m = 3, $m' = 12$, r = 45 and l = 585 | 0.35dB    |
| GF(32)/(31, 20)  | $1 + \beta x_{5} + \beta x_{6} + \beta x_{9} + x_{10} + x_{11} + x_{13} + \beta x_{14} + \beta x_{19} + x_{20} + x_{21} + x_{22} + x_{26}$ | $\geq 7$  | $12$   | m = 5, $m' = 5$, r = 1 and l = 1 | 0.4dB     |
| GF(32)/(31, 21)  | $\beta x_{3} + \beta x_{9} + \beta x_{10} + \beta x_{11} + \beta x_{14} + \beta x_{18} + \beta x_{20} + \beta x_{21} + \beta x_{22} + \beta x_{26}$ | $\geq 4$  | $8$    | m = 5, $m' = 5$, r = 1 and l = 1 | 0.25dB    |

\(^1\)The code minimum distance in binary level.

\(^2\)Distance to the sphere-packing-bound constrained for binary transmission.
V. CONCLUSIONS

An algebraic construction technique for GF$(2^m)$ $(m > 1)$ LDPC codes based on summing the cyclotomic idempotents to define the parity-check polynomial is able to produce a large number of cyclic codes. The fact that we consider step-by-step summation of the cyclotomic idempotents, we are able to control the sparseness of the resulting parity-check matrix. The lower-bound of the $d_{\text{min}}$ and the dimension of the codes can be easily determined from the Mattson-Solomon polynomial of the resulting idempotent. For GF(2) case where the parity-check polynomials are orthogonal on each bit position, we can even determine the true $d_{\text{min}}$ of the codes regardless of the code length. In fact, this special class of binary cyclic codes are the Difference-Set Cyclic and the One-Step Majority-Logic Decodable codes which can be easily constructed using our method. For non-binary cases, if the constructed code has low $d_{\text{min}}$, we can concatenate this code with an inner binary code to trade improvement in $d_{\text{min}}$ with loss in code-rate.

Simulation results have shown that these codes can converge well under iterative decoding and their performance is very close to the sphere-packing-bound of binary codes for the same code length and rate. The excellent performance of these codes coupled with their low-complexity encoder offers an attractive coding scheme for applications that required short block-lengths such as thin data-storage, wireless, command/control data reporting and watermarking.

ACKNOWLEDGEMENT

This research is partially funded by the UK Overseas Research Students Award Scheme.

REFERENCES

[1] T. J. Richardson, M. A. Shokrollahi, and R. L. Urbanke, “Design of Capacity-Approaching Irregular Low-Density Parity-Check Codes,” IEEE Trans. Inform. Theory, vol. 47, pp. 619–637, Feb. 2001.
[2] S. Y. Chung, G. D. Forney, Jr., T. J. Richardson, and R. L. Urbanke, “On the Design of Low-Density Parity Check Codes within 0.0045 dB of the Shannon Limit,” IEEE Comm. Letters, vol. 3, pp. 58–60, Feb. 2001.
[3] S. Y. Chung, T. J. Richardson, and R. L. Urbanke, “Analysis of Sum-Product Decoding of Low-Density Parity-Check Codes Using a Gaussian Approximation,” IEEE Trans. Inform. Theory, vol. 47, pp. 657–670, Feb. 2001.
[4] T. J. Richardson and R. L. Urbanke, “The Capacity of Low-Density Parity-Check Codes Under Message-Passing Decoding,” IEEE Trans. Inform. Theory, vol. 47, pp. 599–618, Feb. 2001.
[5] J. Campello and D. S. Modha, “Extended Bit-Filling and LDPC Code Design,” Proc. of the IEEE Globecom Conf., pp. 25–29, Nov. 2001.
[6] X. Y. Hu, E. Eleftheriou, and D. M. Arnold, “Irregular Progressive Edge-Growth Tanner Graphs,” Proc. of IEEE Intl. Symp. Inform. Theory (ISIT), Lausanne, Switzerland, July 2002.
[7] A. Ramamoorthy and R. D. Wesel, “Construction of Short Block Length Irregular Low-Density Parity-Check Codes,” IEEE Int. Conf. Comm., June 2004.
[8] Y. Kou, S. Lin, and M. Fossorier, “Low density parity check codes based on finite geometries: A rediscovery and new results,” IEEE Trans. Inform. Theory, vol. 47, pp. 2711–2736, Nov. 2001.
[9] M. C. Davey and D. J. C. MacKay, “Low-Density-Parity-Check Codes over GF(q),” IEEE Comm. Letters, vol. 2, pp. 165–167, June 1998.
[10] X. Y. Hu, E. Eleftheriou, and D. M. Arnold, “Regular and Irregular Progressive Edge-Growth Tanner Graphs,” IEEE Trans. Inform. Theory, vol. 51, pp. 386–398, Jan. 2005.
[11] F. J. MacWilliams and N. J. A. Sloane, The Theory of Error-Correcting Codes. North-Holland, 1977. ISBN 0 444 85193 3.
[12] F. J. MacWilliams, “A Table of Primitive Binary Idempotents of Odd Length $n$, $7 \leq n \leq 511$, IEEE Trans. Inform. Theory, vol. IT-25, pp. 118–123, Jan. 1979.
[13] T. Shibuya and K. Sakaniwa, “Construction of Cyclic Codes Suitable for Iterative Decoding via Generating Idempotents,” IEICE Trans. Fundamentals, vol. E86-A, no. 4, 2003.
[14] W. Peterson and E. J. Weldon, Jr., Error-Correcting Codes. MIT Press., 1972.
[15] C. J. Tjahai, E. Papagiannis, M. Tomlinson, M. A. Ambroze, and M. Z. Ahmed, “Improved iterative decoder for LDPC codes with performance approximating to a maximum likelihood decoder.” UK Patent Application 0409306.8, Apr. 2004.
[16] E. Papagiannis, M. Ambroze, and M. Tomlinson, “Improved Decoding of Low-Density Parity-Check Codes with Low, Linearly Increased Added Complexity.” Submitted to 4th IASTED International Conference on Communication Systems and Networks, 2005.
[17] C. E. Shannon, “Probability of error for optimal codes in a gaussian channel,” Bell Syst. Tech. J., vol. 38, pp. 611–656, May 1959.
[18] S. Dolinar, D. Divsalar, and F. Pollara, “Code performance as a function of block size,” TMO Progress Report, pp. 42–133, May 1998.