THE POSITIVE FEEDBACK OF POPULATION III OBJECTS ON GALAXY FORMATION

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ABSTRACT

We study the formation of molecular hydrogen in cooling gas behind shocks produced during the blow-away process thought to occur in the first collapsed, luminous (Population III) objects in the early universe. We find that for a wide range of physical parameters, the H2 fraction is \( f \approx 6 \times 10^{-3} \). The H2 mass produced in such explosions can exceed the amount of relic H2 destroyed inside the photodissociation region surrounding a given Population III object. We conclude, differently from the suggestion of Haiman et al., that these first objects might have a net positive feedback on subsequent galactic formation. We discuss the effects of radiation and the implications of our results for the soft-UV background.

Subject headings: cosmology: theory — galaxies: formation — intergalactic medium

1. INTRODUCTION

Current models of cosmic structure formation based on cold dark matter (CDM) scenarios predict that the first collapsed, luminous (hereafter Population III) objects should form at redshift \( z \approx 30 \) and have a total mass \( M \approx 10^8 M_{\odot} \) or a baryonic mass \( M_b \approx 10^5 M_{\odot} \) (Couchman & Rees 1986; Haiman, Rees, & Loeb 1997, hereafter HRL; Tegmark et al. 1997). This conclusion is reached by requiring that the cooling time, \( t_c \), of the gas be shorter than the Hubble time, \( t_H \), at the formation epoch.

The appearance of Population III objects is now thought to cause a partial destruction of the available molecular hydrogen either in the intergalactic medium (IGM) and/or in collapsing structures; the result is a negative feedback on galaxy formation. This effect has been pointed out by HRL, and it works as follows. As stars form in the very first generation of objects, the emitted photons in the energy band 11.2–13.6 eV are able to penetrate the gas and photodissociate H2 molecules both in the intergalactic medium (IGM) and in collapsing structures, if they can propagate that far from their source. This negative feedback and its possible limitations are discussed by Ciardi, Abel, & Ferrara (1998, hereafter CAF). Here we propose and investigate instead a possible positive feedback based on supernova (SN) explosions, which, under many aspects, is reminiscent of a scaled version of the explosive galaxy formation scenario introduced by Ostriker & Cowie (1981) and put forward by many others. Population III objects are very fragile owing to their low mass and small gravitational potential: only a few SNe are sufficient to blow away (Ferrara 1998) their baryonic content and drive an expanding blast wave into the IGM, which eventually becomes radiative and allows the swept gas to cool in a dense shell. The cooling transient, as we will see, is characterized by a strong nonequilibrium condition in which recombination lags behind the temperature decrease. As already pointed out by Shapiro & Kang (1987) and Kang & Shapiro (1992), this is a favorable condition for H2 formation. Our conclusions are that the amount of molecular hydrogen thus formed can exceed the amount destroyed via photodissociation, yielding a net increase of the H2 in the universe. As a consequence, the galaxy formation process is not halted but instead is favored by the effects of Population III formation.

Section 2 describes the main properties of multi-SN explosions in the early universe; § 3 is devoted to the calculation of H2 formation in their cooling shell. In § 4, we compare the magnitude of positive and negative feedbacks; some discussion of the results is given in § 5.

2. PROPERTIES OF POPULATION III REMNANTS

The mechanical luminosity of an OB association in a Population III object at redshift \( z \) can be written as

\[
L = \frac{\epsilon_0 \rho \Omega_0 f_b}{\tau_{ff}} M \approx 4 \times 10^{36} \Omega_{0.5, f_b, 8}(1+z)^{1/2} M_{\odot} \text{ ergs s}^{-1} \tag{1}
\]

(Ciardi & Ferrara 1997, hereafter CF), where \((1+z)_{30} = (1+z)/30 \text{ and } \epsilon_0 = 10^{51} \text{ ergs is the energy of a SN explosion}; \text{we assume a Salpeter initial mass function (IMF), according to which one supernova is produced for each 56 } M_{\odot} \text{ of stars formed. The baryon density parameter is } \Omega_0 = 0.05 \Omega_{0.5, z}, \text{ of which a fraction, } f_b \approx 0.08 f_{b, 8} \text{ (Abel et al. 1997a), is able to cool and become available to form stars. The dark matter halo mass and density are } M_b = M/10^5 M_{\odot} \text{ and } \rho = 200 \rho_c = 200[1.88 \times 10^{-27} h^2(1+z)^2] \text{ g cm}^{-3} \text{; the corresponding free-fall time is } \tau_{ff} = (4\pi G \rho)^{-1/2}; \tau^{-1} = 0.6 \% \text{ is the star formation efficiency, calibrated on the Milky Way. The correlated SN explosions drive a blast wave in the surrounding gas, which eventually propagates into the IGM; because of their low mass, the effects of the Population III interstellar medium on the blast-wave expansion can be neglected (Mac Low & Ferrara 1998). The evolution of the shock radius, } R_s, \text{ can be obtained in the thin shell approximation by solving numerically equations (5) and (6) of CF. For the sake of simplicity, we use the following (Sedov) analytical solution of the same equations, which holds for the adiabatic case when the external confining pressure and...
the gravitational pull of the DM halo are neglected:

\[ R_s(t) \approx 0.76 \left( \frac{E_t}{\rho_b} \right)^{1/5} \text{ ,} \quad (2) \]

where \( E = L_{\text{tot}} \), \( t_{\text{tot}} \approx 10^7 \text{ yr} \) is the average lifetime of massive stars, \( E \) is the total energy of the explosion, and \( \rho_b = \Omega_c \rho \) is the IGM density at redshift \( z \). In equation (2), we have neglected the effects of Hubble expansion, since \( R_s \ll R_s/H \) (see eq. (3)); for a comparison, see Tegmark, Silk, & Evrard (1993) and Nath & Trentham (1997).

The corresponding temperature of the postshock gas is \( \approx 27.6(R/\text{km s}^{-1})^2 \text{ K} \). Such hot gas will not be able to cool until its cooling time, \( t_c \), becomes smaller than the Hubble time, \( t_H \). The energy losses, \( \Lambda(T) \), due to gas-radiative processes in a primordial gas are of order \( 10^{-23} \text{ ergs cm}^{-3} \text{ s}^{-1} \) in the range \( 10^3 \leq T \leq 10^4 \text{ K} \) (Schmutzler & Tschamnter 1993); the other relevant cooling agent is inverse Compton cooling off CMB photons (Ikeuchi & Ostriker 1986). Comparing the two rates, it is easily seen that the inverse Compton process dominates the cooling for \( z \geq 850, t_c \leq t_H \), for \( T = 10^5 \text{ K} \). The condition \( t_c \ll t_H \) requires that \( z \geq 8 \times 10^5 \text{ K} \); shocked gas produced at epochs earlier than this will be able to cool and condense in a dense shell behind the shock front. Note that as the temperature drops to lower values with subsequent gas recombination, cooling via hydrogen Ly\( \alpha \) line excitation will start to become important. At \( t = t_c = 1.2 \times 10^5(1+z)^{-4/3} \text{ Gyr} \), the shock radius and postshock gas temperature are, respectively,

\[ R_s(t_c) \approx 0.161^{1/5} h_b^{3/5} (1+z)^{-19/30} M_6^{1/5} h^{-1/5} \text{ kpc} \text{ ,} \quad (3) \]

\[ T_s(t_c) \approx 4.9 \times 10^2 f_{2/5}^{1/5} (1+z)^{-21/6} M_6^{1/5} h^{-1/5} \text{ K} \text{ .} \quad (4) \]

These relations hold if the Mach number of the shock with respect to the IGM sound speed is \( M = 2.3 (T/T_s)^{1/2} \gg 1 \). This condition is verified even if the IGM has been preionized and \( T \approx 10^4 \text{ K} \).

3. MOLECULAR HYDROGEN FORMATION

In the following, we calculate the time-dependent, non-equilibrium ionization and molecule abundance evolution behind the multi-SN shock as a function of the object mass and the epoch of the explosion. The postshock cooling gas is assumed to be initially at temperature \( T_c \) and density \( 4 \rho_b \). In hydrogen of density \( n = n_{H} + n_{H^2} + n_{H_2} \) (we neglect He), the evolution of the ionization fraction, \( x = n_{H^+}/n \), molecular fraction, \( f = n_{H_2}/n \), and temperature, \( T \), is determined by the following rate equations:

\[ \dot{x} = k_i n_x (1-x) - k_e x^2 \text{ ,} \quad (5) \]

\[ \dot{f} = k_s n_{H_2} (1-x) - k_{11} x - [k_{12} x + k_{13} (1-x)] n_f \text{ ,} \quad (6) \]

\[ \dot{T} = -T \left[ \frac{(y-1) \Lambda(T)}{\gamma} + \frac{\dot{x}}{1+x} \right] \text{ .} \quad (7) \]

The various rate coefficients \( k_i \) are labeled according to the nomenclature given in Abel et al. (1997b, Appendix A); \( n_{H_2} \)

is the H\(^-\) species number density, \( y = 5/3 \) is the specific heat ratio, and \( \rho = (1+x)n_0 T \) is the gas pressure. In addition to the standard atomic cooling processes, \( \Lambda(T) \) includes Compton and H\(_{2}\) cooling; the latter has been adopted from Martin, Schwarz, & Mandy (1996). We assume that no heating is provided to the gas after it has been heated by the shock to the initial temperature, and we neglect adiabatic cooling due to Hubble expansion for reasons similar to those given in the previous section.

The first equation describes the usual collisional ionization (rate \( k_i \))–recombination (\( k_e \)) balance. The equation for \( f \) is slightly more complicated, and we will describe it shortly. At the low densities considered here, in the absence of dust, H\(_2\) is formed in the gas phase mainly via the channel H\(_+\) + e\(^-\) → H\(^+\) + h\(_\nu\), at rate \( k_s \) (formation via the H\(_\rm{II}\) channel, when included, is found to be negligible in our case); it is then destroyed by one of the following mechanisms: (i) it is collisionally ionized by proton impacts (\( k_{11} \)), or (ii) it is dissociated by collisions with electrons (\( k_{12} \)) and/or H atoms (\( k_{13} \)). Abel et al. (1997b) have shown that using the equilibrium value for \( n_{H_2} \) is a very good approximation since the reactions determining the H\(^-\) abundance occur on a much shorter timescale than those responsible for the H\(_2\) chemistry; therefore, we have calculated \( n_{H_2} \) from their equation (24). Also, we note that in the above form, the rate equations correspond to the so-called minimal model described in that paper. The energy equation has been solved in the isobaric limit; this approximation is known to be appropriate for the evolution of Galactic supernova remnants (Cioffi, McKee, & Bertchinger 1988), where most of the radiative phase is spent at constant pressure; the same result was found to hold by Shapiro & Kang (1987) and Kang & Shapiro (1992) for a wide range of shock velocities in a cosmological context. The numerical solution of the above rate equations are shown in Figure 1 for two different object masses (\( M_6 = 1, 10 \)) and three initial redshifts (\( z = 20, 25, 30 \)). Almost independently of the specific values of these parameters, a high H\(_2\) fraction (\( f \approx 6 \times 10^{-3} \)) is produced by redshift \( z = 10 \), with a steep increase occurring when the gas has cooled down at \( T \approx 20,000 \text{ K} \). H\(_2\) production is faster in high-\( z \), high \( T \), shocks, but the peak abundance is practically the same in all cases; the final gas temperature is in the range 300–500 K. We have also experimented with a different H\(_2\) cooling function (Hollenbach & McKee 1979); this produces a cooler final state (\( T = 80–200 \text{ K} \) but only a marginally different asymptotic value: \( f \approx 5 \times 10^{-4} \). For comparison, we have also plotted in Figure 1 the Tegmark et al. (1997) “rule-of-thumb” value at which H\(_2\) cooling becomes important: \( f = 5 \times 10^{-4} \).

4. POSITIVE OR NEGATIVE FEEDBACK?

The results obtained in § 3 show that typically at least an H\(_2\) fraction \( f = 6 \times 10^{-3} f_6 \) is formed after explosive events leading to the blow-away of Population III objects. As discussed in § 1, HRL have argued that the same objects could suppress the surrounding H\(_2\) abundance owing to their UV photodissociating radiation. In order to evaluate the impact of Population III objects on the subsequent galaxy formation, largely regulated by the availability of H\(_2\) in this mass (and redshift) range, it is useful to compare the H\(_2\) mass production versus destruction. A lower limit to the amount of H\(_2\) produced in an explosion is readily found to be equal to
The UV/ionizing radiation from massive stars in Population III previous to blow-away will produce both an H II region and a region of photodissociated intergalactic H$_2$ (radius $R_d$) in which the object is embedded. The radius $R_d$ can be defined by requiring that the photodissociation timescale ($t_d \approx \frac{2}{3} R_d^2$; notation as in eqs. [5] and [6]) be shorter than $t_{hi}$. This condition yields the definition $R_d = \frac{S_{uv}}{\Omega_{hi}} (1 + z)^{-3/4} h^{-1/2}$, where $S_{uv} = \beta S_{hi}(0)$ is the UV photon flux in the H$_2$ Lyman-Werner (LW) bands (11.2–13.6 eV), assumed here to be proportional to the flux of LyC photons, $S_{hi}(0)$, just before the massive stars explode. The value of the constant $\beta$ depends somewhat on the IMF and on the evolutionary stage of the stellar cluster, but its value should be reasonably close to unity. Paralleling equation (1), we estimate $S_{hi}(0)$ to be

$$S_{hi}(0) = \frac{f_{uvpp} f_{esc} \Omega_{hi} f_d}{\pi c M_p} M = 10^{48} f_{uvpp,48} f_{esc,20} \Omega_{hi} f_d M_6 \text{ s}^{-1}.$$  

where $f_{uvpp} = 48.0$,$000$ is the UV photon production per collapsed proton efficiency (Tegmark, Silk, & Blanchard 1994), and $f_{esc} = 0.2 f_{esc,20}$ is the escape fraction of such photons (absorption can be caused by both interstellar neutral H and/or dust). We have checked that this simple estimate is within a factor of 2 of the value obtained from the recently revised version of the Bruzual & Charlot (1993) spectrophotometric code. The adopted reference value $f_{esc} = 0.2$ is an upper limit derived from observational (Leitherer et al. 1995; Hurwitz, Jelinsky, & Dixon 1997) and theoretical (Dove & Shull 1994) studies. It follows that

$$R_d = 2.4(\beta S_{hi})^{3/2} (1 + z)^{-3/4} h^{-1/2} \text{ kpc},$$ \hspace{1cm} (10)

where $S_{hi} = S_{hi}(0)/10^{48}$ s$^{-1}$. Thus, the ratio between the H$_2$ mass produced and destroyed by Population III objects is

$$\frac{M_{hi}^+(z)}{M_{hi}^-(z)} = \left( \frac{f_{hi}}{f_{uvpp,48}} \right) \left[ \frac{R_d}{R_{hi}} \right]^{3/2}.$$ \hspace{1cm} (11)

The postrecombination relic fraction of intergalactic H$_2$ is estimated to be $f_{hi} \approx 2 \times 10^{-6} h^{-1}$ (Palla, Galli, & Silk 1995; Anninos & Norman 1996; S. Lepp 1997, private communication). The previous relation is graphically displayed in Figure 2, along with the values of $R_d(t_d)$ and $R_{hi}$. From that plot, we see that objects of total mass $M_e = 1$ produce more H$_2$ than they destroy for $z \approx 25$; larger objects ($M_e = 10$) provide a similar positive feedback only for $z \lesssim 15$, since they are characterized by a higher $R_d/R_{hi}$ ratio. However, since in a hierarchical model larger masses form later, even for these objects the overall effect should be a net H$_2$ production.

5. DISCUSSION AND CONCLUSIONS

The results obtained in the previous section could be modified by the fact that so far we have neglected the effects of the Population III stellar cluster radiation impinging on the cooled shell. In fact, even after all the SNe have exploded, blowing away the gas, the coeval low-mass stars will continue to produce some residual flux in the energy range 0.755–13.6 eV, relevant to the H$_2$ formation network. Two processes might have some effect in this context: (i) H$^+$ photoionization and (ii) H$_2$ two-step photodissociation.

The first process occurs at a rate $k_{2+} \approx 6 \times 10^{-17} S_{uvpp}/R_d^2(t_d) \text{ s}^{-1}$ per H$^+$ atom, or $2.4 \times 10^{-10} \beta M_6^{1/5} \text{ s}^{-1}$ using

![Fig. 1.—Evolution of the H$_2$ molecular fraction $f$ (solid lines), H ionization fraction $x$ (dotted lines), and temperature $T/10^5$ K (dashed lines) in the post-shock gas for different values of the explosion redshift, $z = 20$, 25, 30, and Population III mass, $M = 10^8$ $M_\odot$ (upper panel) and $10^7$ $M_\odot$ (lower panel). The cosmological parameters are $\Omega_c = 0.05$, $h = 1$; the cooled baryon fraction is $f_b = 0.08$. The thick horizontal line shows the Tegmark et al. (1997) threshold for efficient H$_2$ cooling.

![Fig. 2.—Ratio between the H$_2$ mass formed and destroyed by a Population III object as a function of the multi-SN explosion redshift (solid lines); values larger than unity for the ratio define the epochs where Population III objects have a positive feedback on galaxy formation. Also shown are the shell (proper) radius at cooling, $R_d(t_d)$ (dotted lines), and the photodissociation (proper) radius, $R_{hi}$ (dashed lines). The upper set of curves refers to objects of mass $M = 10^8$ $M_\odot$, whereas the bottom set corresponds to larger objects, $M = 10^7$ $M_\odot$. The cosmological parameters are as in Fig. 1.]}
We can calculate the effects of two-step photodissociation on the shell as follows. The shell H$_2$ density is 

$$N_{H_2} = \frac{f M_s}{4 \pi R_s^2 n_{H_2}} \simeq 1.5 \times 10^{17} \left(1 + \frac{1}{30}\right)^{1/2} \text{cm}^{-2}$$

(we have assumed that all the swept IGM mass is in the cool shell). Since the critical column density for self-shielding is 

$$N_{H_2}^{crit} = 10^{18} \text{cm}^{-2} \quad (\text{Draine & Bertoldi 1996}),$$

the shell is optically thick to dissociating radiation. Nevertheless, the flux in the LW bands, $J_d = S_{\text{LW}} h_{\nu}/4\pi R_s^2$, where $h_{\nu}$ is the Planck constant, will induce a dissociation front in the shell; $J_d$ is very likely dominating on a possible background radiation. The propagation speed will be 

$$v_d = \frac{\xi \lambda_{\text{hot}}}{t_d},$$

where $\lambda_{\text{hot}} = N_{H_2}^{crit}/n_{H_2}$, is the LW photon mean free path, and $\xi \approx 3-5$ is a constant obtained by comparison with numerical results. Thus, the time required to dissociate the entire shell is 

$$t_d = \frac{3 \times 10^3 (J_{d,2})^{-1} N_{H_2,17}}{J_d,2} \text{yr},$$

where $J_{d,2} = J_d/10^{-2}$ ergs cm$^{-2}$ s$^{-1}$ Hz$^{-1}$ sr$^{-1}$ and $N_{H_2,17} = N_{H_2}/10^{17}$ cm$^{-2}$. Since theoretical work (Bruzual & Charlot 1993) suggests that $\beta$ drops to $\approx 3 \times 10^{-3} \ll 1$ after all the massive stars (the main contributors to the UV flux) have died, dissociating the entire shell will require a very long time, certainly longer than the Hubble time. Finally, Kang & Shapiro (1992) pointed out that any external ionizing radiation field (which could be provided by the hot postshock gas and/or by the residual intermediate-mass stars in the cluster) tends to increase the final value of $f$, although it introduces a time delay due to a temperature plateau in the evolution that is characteristic of photoionization heating. Thus, our estimate seems to provide a robust lower limit to the molecular hydrogen abundance.

How can this H$_2$-enhanced gas be used for galaxy formation? First, regions in which Population III objects are clustered enough for their shells to interact might become sites of active star/galaxy formation. Shell interactions appear to be necessary to make them gravitationally unstable, since the temperature required for isolated fragmentation (Couchman & Rees 1986) is well below the values (≥100 K) found here. Next, these intergalactic shells can be accreted by neighbor objects, possibly increasing their H$_2$ abundance above the threshold $f \gtrsim 5 \times 10^{-4}$ required for efficient cooling (Tegmark et al. 1997).

We conclude that Population III objects can produce regions of considerably high molecular hydrogen abundance because of multi-SN shocks propagating in the IGM. We have also seen that the H$_2$ thus produced can exceed the amount of relic H$_2$ destroyed inside the photodissociation region surrounding a given Population III object. This occurrence suggests that these first objects might have a positive feedback on subsequent galactic formation. Such a conclusion is certainly valid on a local scale, as defined by the radii of the two influence spheres, $R_d$ and $R_s$, of a single object. However, our definition of $R_d$ (see § 4) does not exclude the possibility that some photodissociating flux is present outside $R_d$. If many of these objects are present in the universe at the same epoch, they might contribute to an early soft-UV background that, illuminating an isolated, collapsing cloud, could photodissociate and depress its H$_2$ content; this possibility has been discussed by HRL. However, once the surviving relic IGM fraction between photodissociated spheres is taken into account, the universe opacity to LW photons before reionization becomes order unity on scales larger than $R_d$ but smaller than the typical interdistance between Population III objects at $z = 20-30$ (CAF). This implies that the soft-UV background is very weak because of intergalactic absorption, and, as a consequence, the proposed negative feedback is considerably inhibited. Furthermore, most of the dissociating flux is absorbed by the thick H$_2$ shells discussed above. Given that in general $R_d > R_s$, where $R$ is the Hi region radius of a Population III object, the clearing of intergalactic H$_2$ by overlapping photodissociated spheres will occur before complete reionization. It is not clear at this stage whether that event occurs before the typical mass scale of the collapsing objects is such that cooling can proceed via Lyα only, thus considerably weakening the arguments in favor of a temporary halt of galaxy formation at high redshift.

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