Evaluation of Lubricant Film Thickness for Ball Bearings 6207 & 6307 with Elliptical & Circular Contact Area

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Abstract - Rolling element bearings are widely used in the industry, aviation, marine and automobiles. Failure of bearing may cause the damage of machineries and also impose threat to the life of human beings. Therefore it is essential to monitor the performance of bearing continuously. The major monitoring techniques used are based on Vibration Signature, Acoustic Emission Measurement, Interferometry, Chemical Analysis of Lubricant, Measurement of Lubricant Film Thickness by various methods (Resist ance, Capacitance, Impedance Measurement, etc.). The present work is based on measurement of lubricant film thickness by Resistance Method using Hertz Contact Theory. The contact area between the races and balls plays a vital role for the formation of elastohydrodynamic lubricant film. Formation of elastohydrodynamic lubricant film is the deciding factor for the life of the bearing. To prolong the life it is important to maintain the lubricant film between the ball and races. Aim of this present paper is to calculate the Resistive lubricant film thickness based on elliptical and circular contact area for the deep groove ball bearing 6207 & 6307 for two lubricants, namely A and B. A comparison has also been made for Resistive elastohydrodynamic lubricant film thickness for elliptical and circular contact area. The results indicate that the method used is capable to predict the failure of Rolling Element Bearing. It is also observed that, the developed formula along with inexpensive measuring instruments can be effectively used for online condition monitoring of rolling element bearings with ease.

Keyword-K Rolling element bearing, Elastohydrodynamic lubricant film, Elastohydrodynamic lubricant film thickness, Resistive film thickness (RFT), Hertz contact area, Circular contact area, Elliptical contact area, Elliptical Parameter, Complete elliptical integral of second kind.

I. INTRODUCTION

Lubricant film is maintained between the ball and races of the rolling element bearing when it is in operating condition. Lubricant film depends upon the load and speed of the shaft on which the bearing is mounted. Matharu et. al. [1] have developed a new monitoring technique to measure the Resistive lubricant film thickness of Rolling Element Bearings. The formula developed was based the circular contact area between the ball and race using the Hertz contact theory. According the Hertz contact theory, when the two isotropic, homogeneous, linear elastic solids with smooth surfaces solids are pressed together with a force Q directed normal to the surfaces, an approximately elliptic or circular contact area is formed. In this paper the Resistive lubricant film thickness has been calculated by the analogy given by Matharu et. al.[2] assuming the contact area Elliptical and circular both. Both the contact area has been calculated here for the bearing 6207 & 6307 by Hertz contact theory. Correspondingly the lubricant film thickness has been calculated for lubricants A & B. And the variation of Resistive lubricant film thickness with load and speed is analyzed here. Number of researcher have also developed the techniques to monitor the lubricant film thickness, based on some parameters, by various methods [3]-[5]. Jie Zhang [6] described a lubricant film monitoring system for a conventional deep groove ball bearing using high frequency ultrasonic transducer mounted in a hole drilled on the static outer raceway of the bearing. The film thickness is calculated using the reflection coefficient characterized by lubricant film. Bruce [7] measured the lubricant-film thickness in a rolling element bearing using a piezoelectric thin film transducer to excite and receive ultrasonic signals. High frequency (200 MHz) ultrasound is generated using a piezoelectric aluminum nitride film deposited in the form of a very thin layer onto the outer bearing raceway of a deep groove 6016 ball bearing. The reflection coefficient from the lubricant layer is then measured from within the lubricated contact and the oil-film thickness extracted via a quasi static spring model. The above monitoring methods are
II. METHODOLOGY

For a ball bearing, the Resistive lubricant film thickness (RFT) can be estimated by the formula given by Matharu et. al [1], [2] expressed below.

\[(h_0)_T = \frac{a_1 a_2}{(a_1 + a_2)} \frac{R_T}{\rho}\]

Where,

\[R_T = R_{IR} + R_{OR}, \quad R_{IR} = \frac{\rho h_o}{a_1}, \quad R_{OR} = \frac{\rho h_o}{a_2}\]

The contact area between the inner & outer race with the balls are assuming to be elliptical & circular, which has been calculated by the formula mentioned below.

A. Elliptical Area

The elliptical contact area at inner race and outer race for the calculation of Resistive film thickness can be calculated by Hertz contact theory [8-9]:

\[a_1 = \pi x a_i x b_i \quad & \quad a_2 = \pi x a_i x b_i\]

where

\[a_i = \left[\frac{6\epsilon QR}{\pi k^*}\right]^{1/3} \quad b_i = \left[\frac{6k^2\epsilon QR}{\pi E^*}\right]^{1/3}\]

The parameters involved in the above can be calculated as follows

\[\frac{1}{R} = \frac{1}{R_x} + \frac{1}{R_y}, \quad \frac{1}{R_x} = \frac{1}{r_{ax}} + \frac{1}{r_{bx}} \quad & \quad \frac{1}{R_y} = \frac{1}{r_{ay}} + \frac{1}{r_{by}}\]

\[\epsilon = 1 + \left(\frac{q_a}{a}\right), \quad q_a = \left(\frac{\pi}{2}\right) - 1, \quad a = \frac{R_y}{R_x} \quad & \quad k = (a)^{(2/\pi)}\]

\[\frac{2}{E^*} = \frac{1 - v_1^2}{E_1} + \frac{1 - v_2^2}{E_2}\]

B. Circular Area

The circular contact area at inner race and outer race for the calculation of Resistive film thickness can be calculated by Hertz contact theory [8], [9]:

\[a_{1c} = \pi(a_{1c})^2 \quad & \quad a_{2c} = \pi(a_{oc})^2\]

where

\[a_i = \left[\frac{3Qr_{ic}r}{4E^*(r_{ic} + r)}\right]^{1/3} \quad & \quad a_o = \left[\frac{3Qr_{oc}r}{4E^*(r_{oc} - r)}\right]^{1/3}\]

The parameters involved in the above can be calculated as follows (Matharu et. al (8)):

\[\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}, \quad \frac{1}{E^*} = \frac{1 - v_2^2}{E_1} + \frac{1 - v_2^2}{E_2} = k_1 + k_2 \quad & \quad a = \left[\frac{3QR}{4E^*}\right]^{1/3}\]

In Table 1 useful dimensions are given, which has been used for the calculation of elliptical & circular contact area.

| TABLE 1: Useful Dimensions of Bearing for Calculation of Elliptical & Circular Contact Area |
|---------------------------------|---------------------------------|
| BEARING | INNER RACE | OUTER RACE |
|         | \(r = r_{ax} = r_{ay}\) (mm) | \(r = r_{ax} = r_{ay}\) (mm) | \(r = r_{bx} = r_{by}\) (mm) | \(r = r_{bx} = r_{by}\) (mm) |
| 6207    | 5.75       | 21        | -5.98          | 5.75       | -32.5      | -5.98       |
| 6307    | 6.75       | 22        | -7.02          | 6.75       | -35.5      | -7.02       |

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On the basis of above formula, different parameter for elliptical area has been calculated for the Bearing 6207 and it is tabulated in the given Table 2.

### III. Calculation

Calculation for elliptical & circular contact area is done for the bearing 6207 & 6307 and then Resistive lubricant film thickness is calculated for lubricant A & B. The calculation is done for different speed of the bearing. Different parameters for the calculation of Resistive lubricant film thickness assuming the elliptical contact area is tabulated in Table 2.

| TABLE 2: Parameters for Calculation of Elliptical Contact Area |
|---------------------------------------------------------------|
| **PARAMETERS** | **BEARING 6207** | **BEARING 6307** |
| **INNER RACE** | **OUTER RACE** | **INNER RACE** | **OUTER RACE** |
| --- | --- | --- | --- |
| \( \frac{1}{R_x} \) | 0.22 | 0.14 | 0.19 | 0.12 |
| \( \frac{1}{R_y} \) | 0.0067 | 0.0067 | 0.0057 | 0.0057 |
| \( R_x \) | 4.51 | 6.99 | 5.17 | 8.33 |
| \( R_y \) | 149.5 | 149.5 | 175.5 | 175.5 |
| \( \frac{1}{R} = \left( \frac{1}{R_x} \right)^{1/2} \) | 0.2282 | 0.1498 | 0.1993 | 0.1257 |
| \( R \) | 4.38 | 6.67 | 5.02 | 7.96 |
| \( a = \frac{R_y}{R_x} \) | 33.12 | 21.40 | 33.98 | 21.06 |
| \( k = \frac{a^2}{\pi} \) | 9.28 | 7.03 | 9.44 | 6.96 |
| \( q_e = (\pi/2) - 1 \) | 0.57 | 0.57 | 0.57 | 0.57 |
| \( \varepsilon = 1 + \frac{q_e}{a} \) | 1.02 | 1.03 | 1.02 | 1.03 |
| \( E^*(N/mm^2) \) | 227363 | 227363 | 227363 | 227363 |

Parameters used for the calculation of circular contact area and Resistive lubricant film thickness is tabulated in the Table 3.

| TABLE 3: Parameters for Calculation of Circular Contact Area |
|-------------------------------------------------------------|
| **PARAMETERS** | **BEARING 6207** | **BEARING 6307** |
| **D** | 8.50 | 13.50 |
| \( r = \frac{d}{2} \) | 4.25 | 6.75 |
| \( r_i = r_{bx} \) | 20.00 | 22.00 |
| \( r_o = r_{bx} \) | -28.50 | -35.50 |
| \( k_1 = k_2 \) | 0.000004398 | 0.000004398 |

Calculations for ball bearing 6207 are being shown below for a single lubricant. Values of parameters for ball bearing 6307 are also calculated similar to the calculations for 6207, which are not shown in the paper.

#### A. Determination of Elliptical Contact Area for Bearing 6207 with Lubricant A

1) **For Inner Race**

\[
\begin{align*}
\alpha_1 &= \left[ \frac{6\varepsilon QR}{\pi k E^*} \right]^{1/3} = \left[ \frac{6 \cdot 1.02 \cdot 4.38 Q}{\pi \cdot 9.28 \cdot 227363} \right]^{1/3} = 0.0159 \left[ \frac{Q}{N} \right] \frac{1}{3} \\
\beta_1 &= \left[ \frac{6k^2 \varepsilon QR}{\pi E^*} \right]^{1/3} = \left[ \frac{6 \cdot 9.28^2 \cdot 1.02 \cdot 4.38 Q}{\pi \cdot 227363} \right]^{1/3} = 0.1478 \left[ \frac{Q}{N} \right] \frac{1}{3} \\
\alpha_2 &= \pi \alpha_1 \beta_1 = \pi \cdot 0.0159 \cdot 0.1478 \left[ \frac{Q}{N} \right] \frac{2}{3} = 0.0074 \left[ \frac{Q}{N} \right] \frac{2}{3}
\end{align*}
\]
2) For Outer Race

\[ a_i = \left( \frac{6\varepsilon Q R}{\pi k E^*} \right)^{1/3} = \left[ \frac{6 \times 1.02 \times 6.67Q}{\pi \times 7.03 \times 227363} \right]^{1/3} = 0.0202 [Q]^{1/3} \]

\[ b_i = \left( \frac{6k^2\varepsilon Q R}{\pi E^*} \right)^{1/3} = \left[ \frac{6 \times 7.03^2 \times 1.02 \times 6.67Q}{\pi \times 227363} \right]^{1/3} = 0.1417 [Q]^{1/3} \]

\[ a_2 = \pi a_i b_i = \pi \times 0.0202 \times 0.1417 [Q]^{2/3} = 0.0090 [Q]^{2/3} \]

\[ (h_o)_T = \frac{a_1 a_2}{(a_1 + a_2)} \frac{R_T}{\rho} = \frac{0.0074 [Q]^{2/3} \times 0.0090 [Q]^{2/3} \times R_T}{0.0074 [Q]^{2/3} + 0.0090 [Q]^{2/3}} \frac{R_T}{\rho} = 0.00405 \]

B. Determination of Circular Contact Area for Bearing 6207 with Lubricant A

1) For Inner Race

\[ a_{ic} = \left( \frac{3Q(k_i + k_2)r_{ic}r_i}{4(r_{ic} + r)} \right)^{1/3} = \left[ \frac{3Q(4.398 \times 10^{-6} + 4.398 \times 10^{-6})21 \times 5.75}{4(21 + 5.75)} \right]^{1/3} = 0.0310 [Q]^{1/3} \]

\[ a_1 = \pi (0.0285)^2 [Q]^{2/3} = 0.00302 [Q]^{2/3} \]

2) For Outer Race

\[ a_{oc} = \left( \frac{3Q(k_1 + k_2)r_{oc}r_o}{4(r_{oc} - r)} \right)^{1/3} = \left[ \frac{3Q(4.398 \times 10^{-6} + 4.398 \times 10^{-6})32.5 \times 5.75}{4(32.5 - 5.75)} \right]^{1/3} = 0.0359 [Q]^{1/3} \]

\[ a_1 = \pi (0.0321)^2 [Q]^{2/3} = 0.00404 [Q]^{2/3} \]

\[ (h_o)_T = \frac{a_1 a_2}{(a_1 + a_2)} \frac{R_T}{\rho} = \frac{0.00302 [Q]^{2/3} \times 0.00404 [Q]^{2/3} \times R_T}{0.00302 [Q]^{2/3} + 0.00404 [Q]^{2/3}} \frac{R_T}{\rho} = 0.00173 \]

Both circular areas above comply with area determination by Matharu et.al. [2]. Now, the Resistive film thickness for lubricant A & B based on elliptical and circular contact area is given in the Table 4, 5, 6 & 7.

| Load (kg) | \( a_i = \frac{\pi}{a_1 \times b_1} \) | \( a_2 = \frac{\pi}{a_2 \times b_2} \) |
|----------|--------------------------------|--------------------------------|
| 4        | 0.0528                         | 0.0641                         |
| 10       | 0.0973                         | 0.1181                         |
| 20       | 0.1544                         | 0.1875                         |
| 30       | 0.2024                         | 0.2457                         |
| 40       | 0.2452                         | 0.2977                         |

| Load (kg) | \( a_i = \frac{\pi}{a_1 \times b_1} \) | \( a_2 = \frac{\pi}{a_2 \times b_2} \) |
|----------|--------------------------------|--------------------------------|
| 4        | 0.0528                         | 0.0641                         |
| 10       | 0.0973                         | 0.1181                         |
| 20       | 0.1544                         | 0.1875                         |
| 30       | 0.2024                         | 0.2457                         |
| 40       | 0.2452                         | 0.2977                         |

### TABLE 4: Resistive Film Thickness Based on Elliptical Contact Area for Bearing 6207

| Load (kg) | 400 rpm | 600 rpm | 800 rpm | 1000 rpm |
|----------|---------|---------|---------|----------|
| Lubricant- A | 69.1501 | 51.6524 | 38.0546 | 26.5567 |
| Lubricant- B | 69.1501 | 51.6524 | 38.0546 | 26.5567 |
| Lubricant- A | 69.1501 | 51.6524 | 38.0546 | 26.5567 |
| Lubricant- B | 69.1501 | 51.6524 | 38.0546 | 26.5567 |
| Lubricant- A | 69.1501 | 51.6524 | 38.0546 | 26.5567 |
| Lubricant- B | 69.1501 | 51.6524 | 38.0546 | 26.5567 |
| Lubricant- A | 69.1501 | 51.6524 | 38.0546 | 26.5567 |
| Lubricant- B | 69.1501 | 51.6524 | 38.0546 | 26.5567 |

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TABLE 5: Resistive Film Thickness Based on Circular Contact Area for Bearing 6207

| Load (kg) | Resistive Film Thickness (RFT) \((h_{RT})\) (Picometer) |
|-----------|--------------------------------------------------|
|           | 400 rpm | 600 rpm | 800 rpm | 1000 rpm |
|           | Lubricant-A | Lubricant-B | Lubricant-A | Lubricant-B | Lubricant-A | Lubricant-B | Lubricant-A | Lubricant-B |
| 4         | 0.02157 | 0.02886 | 29.4766 | 6.4475 | 36.4214 | 7.0354 | 41.9772 | 22.6348 | 49.2306 | 61.8683 |
| 10        | 0.03974 | 0.05317 | 42.9254 | 9.2773 | 55.1492 | 11.1183 | 59.6976 | 47.6498 | 66.5202 | 98.0792 |
| 20        | 0.06308 | 0.0844 | 49.1871 | 12.7212 | 60.4685 | 19.7694 | 78.0675 | 59.0216 | 84.8364 | 141.1934 |
| 30        | 0.08266 | 0.1106 | 59.7228 | 16.6694 | 71.5492 | 21.3249 | 96.9756 | 62.6230 | 107.6192 | 157.5337 |
| 40        | 0.10104 | 0.13398 | 60.8877 | 16.0093 | 75.9306 | 21.3249 | 106.0163 | 126.7102 | 111.0306 | 214.9434 |

TABLE 6: Resistive Film Thickness Based on Elliptical Contact Area for Bearing 6307

| Load (kg) | Resistive Film Thickness (RFT) \((h_{RT})\) (Picometer) |
|-----------|--------------------------------------------------|
|           | 400 rpm | 600 rpm | 800 rpm | 1000 rpm |
|           | Lubricant-A | Lubricant-B | Lubricant-A | Lubricant-B | Lubricant-A | Lubricant-B | Lubricant-A | Lubricant-B |
| 4         | 0.0625 | 0.0773 | 82.48905 | 18.04298 | 101.92364 | 19.68824 | 117.47132 | 63.34238 | 137.76967 | 173.1358 |
| 10        | 0.1151 | 0.1424 | 120.1249 | 25.96204 | 154.33272 | 31.11404 | 167.06119 | 165.1694 | 237.41098 | 274.4703 |
| 20        | 0.1827 | 0.2260 | 137.6478 | 35.59962 | 169.21846 | 55.32373 | 218.46861 | 195.2478 | 237.41098 | 395.1237 |
| 30        | 0.2394 | 0.2961 | 167.1315 | 46.64870 | 200.22693 | 59.67671 | 271.38196 | 230.16778 | 301.16778 | 539.4018 |
| 40        | 0.2900 | 0.3587 | 170.3917 | 44.80141 | 212.48852 | 118.1128 | 296.68209 | 354.5930 | 310.71435 | 551.8719 |

TABLE 7: Resistive Film Thickness Based on Circular Contact Area for Bearing 6307

| Load (kg) | Resistive Film Thickness (RFT) \((h_{RT})\) (Picometer) |
|-----------|--------------------------------------------------|
|           | 400 rpm | 600 rpm | 800 rpm | 1000 rpm |
|           | Lubricant-A | Lubricant-B | Lubricant-A | Lubricant-B | Lubricant-A | Lubricant-B | Lubricant-A | Lubricant-B |
| 4         | 0.02538 | 0.03491 | 35.08652 | 7.67454 | 43.35298 | 8.37434 | 49.96614 | 26.94253 | 58.60000 | 73.64291 |
| 10        | 0.04675 | 0.06431 | 51.09486 | 11.04289 | 65.64505 | 13.23428 | 71.05908 | 75.28073 | 79.18012 | 116.7452 |
| 20        | 0.07421 | 0.10209 | 58.54818 | 15.14221 | 71.97667 | 23.53182 | 92.92510 | 70.25441 | 100.98219 | 168.0649 |
| 30        | 0.09724 | 0.13377 | 71.08901 | 19.84191 | 85.16605 | 25.38335 | 115.43166 | 74.51424 | 128.10099 | 229.4332 |
| 40        | 0.11779 | 0.16206 | 72.47572 | 19.05617 | 90.38148 | 50.23900 | 126.19301 | 150.8252 | 132.16160 | 234.7374 |

IV. GRAPHICAL REPRESENTATION

Variation of Resistive lubricant film thickness with load and speed for elliptical and circular contact area is shown in given figure. Here two bearings 6207 & 6307 has been considered for analysis. Film thickness for both of the bearing is analyzed separately.

A. Elliptical Contact

![Fig. 1. Variation of Resistive film thickness with speed for bearing (6207) & Lubricant (A) based on elliptical contact area](image1)

![Fig. 2. Variation of Resistive film thickness with speed for bearing (6207) & Lubricant (B) based on elliptical contact area](image2)
Fig. 3. Variation of Resistive film thickness with speed for bearing (6307) & Lubricant (A) based on elliptical contact area

Fig. 4. Variation of Resistive film thickness with speed for bearing (6307) & Lubricant (B) based on elliptical contact area

Fig. 5. Variation of Resistive film thickness with load for bearing (6207) & Lubricant (A) based on elliptical contact area

Fig. 6. Variation of Resistive film thickness with load for bearing (6207) & Lubricant (B) based on elliptical contact area

Fig. 7. Variation of Resistive film thickness with load for bearing (6307) & Lubricant (A) based on elliptical contact area

Fig. 8. Variation of Resistive film thickness with load for bearing (6307) & Lubricant (B) based on elliptical contact area
B. Circular Contact

Fig. 9. Variation of Resistive film thickness with speed for bearing (6207) & Lubricant (A) based on circular contact area

Fig. 10. Variation of Resistive film thickness with speed for bearing (6207) & Lubricant (B) based on circular contact area

Fig. 11. Variation of Resistive film thickness with speed for bearing (6307) & Lubricant (A) based on circular contact area

Fig. 12. Variation of Resistive film thickness with speed for bearing (6307) & Lubricant (B) based on circular contact area

Fig. 13. Variation of Resistive film thickness with load for bearing (6207) & Lubricant (A) based on circular contact area

Fig. 14. Variation of Resistive film thickness with load for bearing (6207) & Lubricant (B) based on circular contact area
V. RESULTS AND DISCUSSION

Results in the Tables 4, 5, 6 & 7 clearly indicate increase in elliptical as well as circular contact area with increase in load. The both contact areas are more at outer race than at the respective inner race. Figures 1-16 clearly depicts increase in lubricant film thickness increases with increase in load and speed. Mathematical analysis for lubricant film thickness yields more contact area for elliptical contact than circular contact.

VI. CONCLUSION

The lubricant film thickness computed, based on Hertz contact theory, have load and bearing resistance in numerator for elliptical as well as circular contact. Experimentally it is seen that the bearing resistance increases with increase in speed. This means the lubricant film thickness will increase with increase in load and speed. From the outcome of present analysis for elliptical contact, it is found that the determined lubricant film thickness is indicative and it is termed as Resistive Lubricant Film Thickness. Use of elliptical contact area, which is the actual contact area, will yield better results than the circular contact area taken earlier in similar work.

The plots show increase in the lubricant film thickness with increase in contact area as well as with increase in lubricant viscosity. This is in-line with classical theory of lubrication. Thus it is concluded that the determined lubricant film thickness for elliptical contact can also be used for online condition monitoring of rolling element bearings successfully.

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Nomenclature

\( k \) - Elliptical parameter
\( Q \) - Load on heaviest loaded ball (N)
\( a_1 \) - Radius of contact ellipse in x-direction (mm)
\( b_1 \) - Radius of contact ellipse in y-direction (mm)
\( a_2 \) - Elliptical Contact Area at inner race (mm²)
\( a_3 \) - Elliptical Contact Area at outer race (mm²)
\( a_{1c} \) - Circular Contact Area at inner race (mm²)
\( a_{2c} \) - Circular Contact Area at outer race (mm²)
\( \alpha \) - Radius ratio
\( r_{ax} \) - Radius of ball wrt x-axis
\( r_{ay} \) - Radius of ball wrt y-axis
\( r_{bx} \) - Radius of race wrt x-axis
\( r_{by} \) - Radius of race wrt y-axis
\( R_x \) - Equivalent radius of ball & race wrt x-axis
\( R_y \) - Equivalent radius of ball & race wrt y-axis
\( R \) - Overall equivalent radius
\( \mathcal{E} \) - Elliptic integral of second kind
\( \alpha \) - Radius ratio
\( q_d \) - Constant
\( v_1, v_2 \) - Poisson’s ratio for the material of ball & race \( (v_1 = v_2 = 0.3) \)
\( E^* \) - Equivalent modulus of elasticity \( (N/mm^2) \)
\( E_1, E_2 \) - Modulus of elasticity for the material of ball & race \( (E_1 = E_2 = 206900 \text{ N/mm}^2) \)
\( R_{BR} \) - Bearing Resistance (kilo \( \Omega \))
\( R_{BB} \) - Resistance between ball and inner race (\( \Omega \))
\( R_{BB} \) - Resistance between ball and outer race (\( \Omega \))
\( \rho \) - Resistivity of lubricant (\( \Omega \cdot \text{mm} \))
\( (h_o)T \) - Representative film thickness (mm)

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