Finite Element Analysis and Calculation Method of Residual Flexural Capacity of Post-fire RC Beams

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Abstract

Fire tests and subsequent bending tests of four reinforced concrete (RC) beams were performed. Based on these tests, the post-fire performance of RC beams was further studied using finite-element simulation through reasonable selection of suitable thermal and thermodynamic parameters of steel and concrete materials. A thermodynamic model of RC beams with three sides under fire was built using finite-element analysis (FEA) software ABAQUS. The FEA model was validated with the results of fire tests. Different factors were taken into account for further parametric studies in fire using the proposed FE model. The results show that the main factors affecting the fire resistance of the beams are the thickness of the concrete cover, reinforcement ratio of longitudinal steel, the fire exposure time and the fire exposure sides. Based on the strength reduction formula at high temperature of steel and concrete and four test results, an improved section method was proposed to develop a calculation formula to calculate the flexural capacity of RC beams after fire. The theoretical calculation method proposed in this paper shows good agreement with FEA results, which can be used to calculate the flexural capacity of RC beams after fire.

Keywords: Post-fire, reinforced concrete beams, finite element, ABAQUS, theoretical calculation

1 Introduction

Reinforced concrete (RC), as one of the major building materials of modern architectural structures, will form nonuniform temperature fields after exposure to fire (Shao et al. 2003; Cheng 2007). The thermal stress and expansion (Zhao 2014) due to fire will lead to stress redistribution of overall structures and reduction of structural flexural capacity or even the local structural destabilization, which further induce the collapse of overall structures and bring about massive losses to the possessions and life of humans (Ban 2008). Thus, the temperature fields should be determined first for subsequent analysis of RC structures to understand the degradation in the mechanical properties and flexural capacity after exposure to fire, which are the keys to evaluate whether the RC structures can be used anymore (Tian 2002; Zeng 2006). So far, there is considerable research in the aspect of test and finite-element analysis (FEA). Fu et al. (2015) studied how the fire surfaces of RC beams affected the temperature fields. Liao et al. (2013) designed the shear tests and a FEA method was proposed to calculate the shear flexural capacity among one room temperature and seven post-fire RC beams, which has the accepted accuracy. Sun et al. (2002) investigated the mechanical performances in the tensile region and the compressive region of beam sections at elevating temperature and after cooling. Shi and Guo (2000) tested the structural performances of RC beams at high temperature and presented the temperature distributions of beam sections, a coupling temperature–stress constitutive law of concrete and the main performances at different surfaces after fire exposure. Lu et al. (1993) studied the 1-, 2- and 3-surface fire exposure of 12 RC simply-supported beams, presented the bending moment–curvature relationships in different temperature fields and computationally...
scholars’ research primarily focusses on the constitutive reinforcement, and proposed a method to calculate the flexural capacity of theoretical calculation of the post-fire RC beams is barely investigated. In addition, the major factor affecting the behaviour of the RC beams in fire is not well investigated. The 500 °C isotherm method specified in Eurocode (BSI 2004) is the most widely used theoretical calculation method for the residual bearing capacity of RC beams after fire. The method is based on the assumption that concrete above 500 °C does not provide load-bearing capacity to members, while concrete within 500 °C is considered to maintain its initial values of strength and elastic modulus. However, this method is a rough calculation method cannot accurately evaluate the structures after fire. BS 5950: Part 8 (2003) specifies a calculation method of the fire resistance of members in bending which is called 'moment capacity method' or called "section method". It is used primarily for steel sections but concrete members. Therefore, RC beams fire tests and subsequent bending test were performed in this paper, based on the test results and the existing moment capacity method, a new theoretical calculation model using improved section method to calculate the flexural capacity of the post-fire beams. The model first conducts heat transfer analysis to obtain the temperature field of the beam section, and then divides the beam section to small divisions and obtain the average temperature of each divisions. Finally, based on the material strength reduction formulas in fire, the flexural capacity can be accurately obtained for RC beam after fire. Compared with the 500 °C isotherm method, the advantage of this method is that it fully considers the changes of the temperature field and accurately considers the changes of the material strength caused by the temperature changes in each micro area, and the results obtained are more accurate. The accuracy of the theoretical calculation is validated through the comparison with simulation and test results, which offers a new way for the evaluation of post-fire RC structures. In addition, a thermodynamic model of RC beams with three sides under fire was built using finite-element analysis (FEA) software ABAQUS. The FEA model was validated with the results of fire tests. Different factors were taken into account for further parametric studies in fire using the proposed FE model.

2 Research Methodology

For evaluating post-fire residual flexural capacity of RC beams, three approaches are adopted: experiment investigate (fire tests and post-fire static load tests) of RC beams (stage 1), numerical analysis of RC beams on the basis of Finite-Element software (stage 2) and finally, theoretical analysis of residual flexural capacity on the basis of improved section method (stage 3). A flow chart in Fig. 1 illustrates the three stages for evaluating residual strength of fire-exposed RC beams.
3 Fire and Bending Tests of RC Beams

3.1 Fire Tests

3.1.1 Test Preparations

The experiments were conducted in the Structure Laboratory of Jilin Jianzhu University. 4 RC beams were fabricated with dimensions and reinforcement details shown in Fig. 2. The mix proportion of concrete (Table 1) is W:C:S:G = 0.38:1:1.11:2.72. The basic parameters of the reinforcement are shown in Table 2. Two of them were exposed to 60 min high temperature in a furnace (Table 3). The temperature distribution of the beams subjected to fire were, therefore, obtained, which provides a comparison for the subsequent numerical analysis.

To record the temperature changes during the experiments, thermocouples were embedded into the pre-drilled holes of the beams F1 and F2 and filled with cement mortar. Then the beams were covered with fireproofing cotton at locations shown in Fig. 3 to simulate 3-sided fire exposure.

3.1.2 Test Observations

The two beams were placed into a furnace (Fig. 4) for 60 min fire exposure. During the heating stage of the members, when the exposure time is 12 min, a small amount of water vapor can be seen on the beam surface from the observation hole in the furnace. When the exposure time is 21 min, a large

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**Table 1 Mix proportion of concrete**

| Water (Kg) | Cement (Kg) | Sand (Kg) | Gravel (Kg) |
|------------|-------------|-----------|-------------|
| 175        | 461         | 512       | 1252        |

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**Fig. 1** Flow chart illustrating the three stages of RC beams exposed to fire

**Fig. 2** Dimensions and reinforcement of beams

**Fig. 3** Dimensions and reinforcement of beams

**Fig. 4** Flow chart illustrating the three stages of RC beams exposed to fire.
amount of water vapor volatilizes from the furnace, and the water vapor is the most vigorous when it is 27 min. As the exposure time increases, the water vapor in the furnace gradually decreases. When the exposure time is 42 min, the water vapor in the furnace basically disappears. The main reason for this phenomenon is that some of the free water inside the beam and the small amount of water gel evaporated at the beginning of the fire. As the temperature in the furnace increases, a large amount of free water and water gel begin to volatilize. At the same time, the beam is dehydrated in a large amount, and when the water vapor in the furnace is substantially disappeared, the beam is completely dehydrated.

When the RC beams are dehydrated at high temperature, the sound of slight cracking as well as a frequent slight popping can be heard. A large number of diagonal cracks are noticed on the surface of the beam when it is taken out from the furnace after tests, and the longer the exposure time is, the more larger the crack is. From the overall point of view, there are phenomena such as peeling, outer drum, and crispness, and the surface color changes from normal color to light yellow or even dark yellow. The reason for this phenomenon is that during the process of the fire, some complicated chemical reactions occur inside the beam, such as the decomposition of Ca(OH)$_2$ and minerals of aggregate, which cause certain damage to the interior of the beam. The internal expansion of the beam causes surface cracking.

3.1.3 Test Results and Analysis

Three temperature measuring points are set on each specimen (Fig. 5). The time–temperature curves (Fig. 6) of the three temperature measuring points of each beam section after 60 min of fire are obtained from the above beam tests. Due to the thermal inertia of concrete, the temperature distribution of the section is uneven after fire, and the temperature gradient is larger near the furnace. The temperature is generally rising, but it can be seen from the figure that there is a temperature platform at about 120 °C, which may due to the evaporation of water in the beam causing partial heat loss.

3.2 Static Tests

After fire tests, the two beams were cooled to room temperature, then, static tests were performed. The effects of fire exposure time on the flexural capacity of the beam were, therefore, investigated. In brief, 2 none-fire-exposed beams and 2 post-fire beams were tested using a 4-point bending test rigs, as it is shown in Fig. 7. The instrumentations are shown in Fig. 7.

3.2.1 Test Phenomena

Under the vertical load, the displacement of the RC beams gradually increases with the increase of the load. After a period of loading, tiny hairline cracks can be seen first in the mid-span of the beam. The crack developed slowly and upward, and new vertical cracks appeared

| Specimen No. | Concrete strength grade | Test conditions          | Fire exposure time (min) |
|--------------|-------------------------|--------------------------|--------------------------|
| N1           | C30                     | Room-temperature static test | –                        |
| N2           | C30                     | Room-temperature static test | –                        |
| F1           | C30                     | Post-fire static test     | 60                       |
| F2           | C30                     | Post-fire static test     | 60                       |

### Table 2 Basic parameters of the reinforcement

| Type of reinforcement | Specification of steels | Yield strength (N/mm$^2$) | Ultimate strength (N/mm$^2$) | Elastic modulus ($\times 10^5$ N/mm$^2$) |
|-----------------------|-------------------------|---------------------------|------------------------------|------------------------------------------|
| Pulled                | HRB400                  | 469                       | 583                          | 2.05                                     |
| Compressed            | HRB400                  | 469                       | 583                          | 2.05                                     |
| Stirrup               | HPB300                  | 346                       | 431                          | 2.13                                     |

### Table 3 Basic parameters of specimens

| Specimen No. | Concrete strength grade | Test conditions          | Fire exposure time (min) |
|--------------|-------------------------|--------------------------|--------------------------|
| N1           | C30                     | Room-temperature static test | –                        |
| N2           | C30                     | Room-temperature static test | –                        |
| F1           | C30                     | Post-fire static test     | 60                       |
| F2           | C30                     | Post-fire static test     | 60                       |
remained basically unchanged. After a while, the load drop rapidly, until the failure of the beam.

**3.2.2 Test Results and Discussion**

The load–deflection curves of the 4 beams are plotted in Fig. 8. Clearly, during the loading of the RC beams, the mid-span load–deflection relation first increased proportionally. When the load reached certain value, it did not increase any more, and the curve reached yield stage. Then the load dropped abruptly, and the beams failed. The curves showed the flexural capacity was weakened by about 30% after the fire exposure.

**4 Finite-Element Modelling**

To be able to simulate the response of the RC beams after fire, a sequential Thermal–Mechanical Coupling approach is implemented in ABAQUS modelling. The heat transferring analysis was performed first to get the temperature distribution. The thermal result then input into the mechanical model for further mechanical analysis.

**4.1 Basic Assumptions**

Some basic assumptions were proposed to simplify the simulation:

1. The concrete strength of different mesh in the compression zone is different, but the strength in each mesh is regarded as a constant;
2. The equivalent reduction coefficient of the concrete strength of the entire compression zone can be obtained by weighting the average of the cross-sectional compression zone area;
3. The heat loss due to moisture evaporation during the fire was not considered;
4. There was no bonding slip between the reinforcement and concrete.

**4.2 Heat Transfer Analysis**

**4.2.1 Thermal Properties of the Materials**

To obtain temperature fields of RC beams, heat transfer analysis was performed using ABAQUS. The specific heat capacity, thermal conductivity, and density of the materials should be first determined, which are the basis for the analysis. The parameters were selected based on Wang and He (2009).

The thermal conductivity of concrete \( \lambda_{CT} \) (W/(m \cdot °C)) is

\[
\lambda_{CT} = 2 - 0.24 \left( \frac{T}{120} \right) + 0.012 \left( \frac{T}{120} \right)^2 \quad (20 °C \leq T \leq 1200 °C),
\]
where $T$ is the temperature, °C.

The specific heat capacity of concrete $C_{cT}$ (J/(Kg °C)) is

$$C_{cT} = 900 + 80 \left( \frac{T}{120} \right) - 4 \left( \frac{T}{120} \right)^2 \quad (20^\circ C \leq T \leq 1200^\circ C). \quad (2)$$

The thermal conductivity of reinforcement steel $\lambda_{sT}$ (W/(m °C)) is

$$\lambda_{sT} = \begin{cases} 54 - 3.33 \times 10^{-2} T \quad (20^\circ C \leq T < 800^\circ C) \\ 27.3 \quad (800^\circ C \leq T \leq 1200^\circ C) \end{cases}. \quad (3)$$

The specific heat capacity of reinforcement $C_{sT}$ (J/(Kg °C)) is

$$C_{sT} = \begin{cases} 425 + 7.73 \times 10^{-1} T - 1.69 \times 10^{-3} T^2 + 2.22 \times 10^{-6} T^3 \quad (20^\circ C \leq T < 600^\circ C) \\ 666 + \frac{13002}{17820 - T} \quad (600^\circ C \leq T < 735^\circ C) \\ 545 + \frac{17820}{731 - T} \quad (735^\circ C \leq T < 900^\circ C) \\ 650 \quad (900^\circ C \leq T \leq 1200^\circ C) \end{cases}. \quad (4)$$
4.2.2 Fire Temperature Curve

The heat transfer analysis was performed on the basis of a heating curve. Here the ISO-834 fire curve from the International Organization (ISO 1999) is adopted (Fig. 9). The curve can be expressed as (Fu 2016b, 2018)

\[ T = T_0 + 345 \log (8t + 1), \]

where \( t \) is the heating time (min); \( T_0 \) is the initial room temperature (°C, which is 20 °C here).

4.2.3 FE Model Setup

A 3D finite-element model is built, as shown in Fig. 10. The concrete was simulated as DC3D8 8-node (Fu et al. 2010) linear heat transfer units, and the reinforcement steel as DC1D2 2-node heat transfer link units. The contact between reinforcement and concrete was simulated using Embedded Region. The beams were exposed to fire on three sides, and the modes of heat transfer in the fire-exposed faces were surface radiation and surface film condition, and the unfired faces were surface film condition. The thermal parameters of the materials were calculated according to Eqs. (1) to (4). After the heat transfer analysis steps and boundary conditions were set up, the models were meshed into cells in size of 10 mm. The temperature distribution of RC beam exposed to fire is shown in Fig. 10, the temperature field nephogram.

Fig. 8 Load-deflection curves of different beams

a  Beam N1  

b  Beam N2  

c  Beam F1  

d  Beam F2
(Fig. 11) and the simulated results (Fig. 12) were shown as follows.

4.2.4 FE Model Validation

The temperature fields of post-fire RC beams obtained using ABAQUS is compared with test results, which can be visually observed. The simulated results were basically consistent with the test results.

4.3 Static Analysis Procedure

The degradation of the compressive strength of concrete, the yield strength of steel at high temperature were related to the maximum temperature. To be consistent with the test conditions, the temperature field simulation was performed first to determine the strength degradation of the materials, then the static load simulation was carried out. The mechanical performances of reinforcement or concrete varied with the change of maximum temperature, so the relations with temperature should be considered to determine the post-fire mechanical performances. They are determined according to the formula introduced in the following section.

4.3.1 Constitutive Model of Concrete Under High Temperature

The post-fire mechanical properties of concrete were related to the fire exposure time, way of cooling, and type of skeletal materials. Here the model of post-fire concrete proposed by Yu et al. (2005a, b) was used. The compression stress–strain model (Fig. 13) of post-fire concrete can be determined according to formulas 7 and 8. The reduction factor of axial compressive strength of concrete with temperature change is shown in Fig. 14 according to formula 6.

The strength reduction factor of post-fire concrete \( \Psi_{cT} \) at \( T \)°C is

\[
\Psi_{cT} = \frac{f_c(T)}{f_c} = \frac{1}{1 + 9 \times \left(\frac{(T-20)}{800}\right)^{c_1}},
\]

where \( f_c(T) \) is the axial compressive strength of concrete at high temperature, N/mm²; \( f_c \) is the axial compressive strength of concrete at room temperature, N/mm²; \( c_1 \) is the parameter. (which is 3.55 here for ordinary concrete), for high-performance concrete (HPC) concrete \( c_1 = 6.70 \).
The compression peak strain of post-fire concrete $\varepsilon_0(T)$ at $T$ °C is

$$
\varepsilon_0(T) = \left( 1 + c_4 \frac{(T - 20)}{100} \right)^2 \varepsilon_0,
$$

(7)

where $\varepsilon_0$ is the compressive strain of concrete at room temperature; $c_4$ is the parameter. (which is 0.037 here for ordinary concrete), for high-performance concrete (HPC) concrete $c_4 = 0.017$.

The compression stress–strain relationship of post-fire concrete can be expressed as
where 

\[ \sigma_c / f_{cu} \]

is the compressive stress of post-fire concrete, N/mm²; \( \varepsilon_c \) is the compressive strain of post-fire concrete; \( f_{cu} \) is the cube crushing strength of concrete at room temperature, N/mm².

The elastic modulus of post-fire concrete \( E_c(T) \) (N/mm²) at \( T \) °C is

\[
E_c(T) = E_c + 2.15 \times 10^{-3} \left[ \frac{(T-20)}{800} \right]^{4.33} + 3.7 \times 10^{-2} \left[ \frac{(T-20)}{100} \right]^2,
\]

where \( E_c \) is the elastic modulus of concrete at room temperature, N/mm².

The yield strength reduction factor of reinforcement \( \Psi_y(T) \) at \( T \) °C proposed by Miao et al. (2013) is as follows:

\[
\Psi_y(T) = \begin{cases} 
1 & 0 \degree C < T \leq 200 \degree C \\
1.33 - 1.64 \times 10^{-3} T & 200 \degree C < T \leq 700 \degree C
\end{cases}
\]

where \( f_y(T) \) is the post-fire yield strength of reinforcement, N/mm²; \( f_y \) is the yield strength of reinforcement at room temperature, N/mm².

The stress–strain model of reinforcement at high temperature is shown as follows (Fig. 15).

\[
\frac{f_t(T)}{f_t} = 0.976 + \left[ 1.56 \times \left( \frac{T}{100} \right) - 4.35 \times \left( \frac{T}{100} \right)^2 + 0.345 \times \left( \frac{T}{100} \right)^3 \right] \times 10^{-2} \quad 20 \degree C \leq T \leq 800 \degree C.
\]
4.3.3 Modelling Process

When the beams were simulated using ABAQUS, the constitutive relations of concrete and reinforcement under high temperature (as shown in Figs. 13 and 15) were input into the model. The contact between reinforcement and concrete was simulated through Embedded Region (Fu 2012, Fu et al. 2008). The heat transfer analysis was first performed. Then the result file (.odb) of the heat transfer simulation results was imported into the predefined fields, the loads and analytical steps were followed. The mesh size was 10 mm, in which the concrete was set as C3D8R 8-node linear hexahedron units, while the reinforcement as T3D2 2-node linear 3-dimensional truss units. After submitted the job, the results are provided in Fig. 16. Finally, based on the post-processing function of the software, the post-fire load–deflection curves of the beams were plotted (Fig. 17).

4.3.4 FE Model Validation

After RC beams was simulated in ABAQUS, the simulated and tested results of the post-fire flexural capacity of RC beams were compared, the simulated results were basically consistent with the test results.

5 Parametric Analysis

The effects of concrete cover thickness (c), fire exposure time (t), the ratio of longitudinal reinforcement (ρ), the fire sides (n), and the ratio of shear span to effective depth (λ) on the post-fire load-bearing ability were investigated using the validated model. Among them, the treatment of the cover thickness is achieved by fine-tuning (increasing 10 mm and decreasing 10 mm) the height of the beam section (h) on the basis...
Fig. 18 Variation of load–flexural capacity of beams under different parameters
of ensuring that the effective height ($h_0$) of the beam section remains unchanged. Thereby, the finite-element analysis of these factors on the flexural capacity of RC beams can be clarified. The concrete cover thickness was 15, 25 or 35 mm; the fire exposure time was 60, 90 or 120 min; the longitudinal reinforcement ratio was 0.91%, 1.25% or 1.52%, the fire sides was 3 or 4, the ratio of shear span to effective depth was 2.2, 2.6, or 3.0. The analytical results are presented in Fig. 18. To study the effect of the thickness of the concrete cover, the height of the section is adjusted by fine-tuning (increasing and decreasing 10 mm) the cross-sectional dimensions of the beam to ensure that the effective height of the beam section remains unchanged.

Clearly, the flexural capacity of RC beams was significantly weakened after the fire exposure, and the weakening was more obvious with the prolonging of fire exposure time. After the fire exposure, the cover layer thickness directly decided the temperature at the bottom of the tensile reinforcement, and with the larger cover layer thickness, the temperature at the bottom of the steels was lower and the mechanical performance was higher, leading to higher bearing capacity. The ratio of longitudinal reinforcement also affected the bending bearing capacity, and a larger reinforcement ratio led to a higher bearing capacity. Similarly, the difference in fire sides has a great influence on the bearing capacity of the beam. Compared with the three-sided fire beam, the four-sided fire beam will damage the concrete in the compression zone, resulting in a reduction in the bearing capacity. The ratio of shear span to effective depth has little effect on the residual flexural capacity of the beam after fire.

6 Calculation Method for Residual Flexural Capacity of the RC Beams

During the fire, the section temperature is unevenly distributed along the section height, so the distribution of concrete compressive strength after fire is not uniform throughout the section. To calculate the residual flexural capacity of the component after fire, the 500 °C isotherm method is adopted in Eurocode (BSI 2004). Han et al. (2007), Xiang et al. (2010) and Yang et al. (2002) used the equivalent section method to determine the ultimate flexural capacity of the beam under fire. This paper proposes a new calculation model based on the FEA results and the improved section method to calculate the residual flexural capacity of the post-fire RC beams aiming at refining the flexural capacity calculation with the accurate inclusion of defects of strength reduction of the components caused by fire.

6.1 Basic Assumptions

Before the calculation, the following basic assumptions are made for the RC beams after fire:

1. The concrete strength of each grid in the compression zone was different, but the strength within each grid was considered as the same;
2. The contribution of concrete in the tension zone to the residual flexural capacity was ignored;
3. There was no relative slip between steel and concrete during the fire;
4. The effect of temperature–stress on the strength of the concrete at high temperature was not considered.

6.2 The Limitation of this Method

1. Ignore the effect of fine-tuning the cross-sectional size of the beam on the bending capacity of the beam.
2. It may require longer time period to predict flexural capacity than the existing 500 °C isotherm method from Eurocode.

6.3 Calculation Steps

The beam section is meshed. The temperature of each cell can be extracted by the post-processing function of the ABAQUS from thermal analysis model. As shown in Fig. 19, the temperature of concrete after the fire takes the highest temperature $T_i$ in the $i$th cell. The
compressive strength reduction factor of concrete is obtained by substituting $T_i$ into Eq. 6. The temperature of the steel bar after the fire takes the highest temperature of the cell, where the steel is located. Equation (12) for the yield strength of the steel bar after high temperature is adopted.

According to the strength reduction model of the concrete after high temperature in Fig. 14, the post-fire residual flexural capacity is determined using the improved section method. The height of the compressive zone ($x_c$) of the concrete beam section after the fire can be calculated based on Eq. (13):

$$f_c \sum \Psi_{cT_i} \Delta b \Delta x + \Psi'_{yT} f' y A'_s = \Psi_{yT} f_y A_s,$$

(13)

where $\Psi_{cT_i}$ is the compressive strength reduction factor of the $i$th cell, $\Delta b$ is the width of a single cell, $\Delta x$ is the height of a single cell, $\Psi'_{yT}$ is the yield strength reduction factor of compressive steel, $f' y$ is the yield strength of compressive steel at room temperature, $N/mm^2$, $A_s$ and $A'_s$ are the area of steel in the compressive and tensile zone, respectively, $mm^2$.

The strength reduction factor of concrete in the compressive zone ($\Psi_{cT}$) is obtained from Eq. (14):

$$\Psi_{cT} = \frac{\sum \Psi_{cT_i} \Delta b \Delta x}{b x_c},$$

(14)

where $b$ is the width of the beam section, $x_c$ is the height of the compression zone of the concrete beam section.

After obtaining the height of the compression zone ($x_c$), the residual flexural capacity ($M_u$) of the RC beams after fire can be obtained according to Eq. (15):

$$M_u = \alpha_1 \Psi_{cT} f_c b x_c \left( h_0 - \frac{x_c}{2} \right) + \Psi'_{yT} f' y A'_s \left( h_0 - a'_s \right),$$

(15)

where $\alpha_1$ is coefficient of equivalent rectangular stress figure in compression zone of concrete, which is 1 here, $h_0$ is the effective height of the beam section, $a'_s$ is the distance from the resultant force point of the compressive steel reinforcement to the margins of the compressive section.

The flexural capacity of the RC beams after 60 min of fire is verified by the above theoretical method, and the flexural capacity after 90 min and 120 min of fire is calculated and compared with the finite-element results. The results are shown in Fig. 20.

### 6.4 Verification of the Post-fire Flexural Capacity Formula

The test data in Ref. (Xu et al. 2013) of specimen L5 is chosen to validate the correctness and applicability of the theoretical calculation model, and the dimensions and reinforcement of L5 beam is illustrated in Fig. 21. The paper selects temperature measuring point 1# for verification. After the temperature field is simulated using ABAQUS, the nephogram of temperature field is shown in Fig. 22. The comparison between the simulation results and the test results of the Temperature–Time curve of measuring point 1# is shown in Fig. 23.

![Fig. 20 Comparison of theoretical calculation and finite-element calculation](image)

![ Fig. 21 Dimensions and reinforcement of L5 beam](image)
To more comprehensively compared the accuracy of the improved section method, the 500 °C isotherm method from Eurocode (BSI 2004) is now used to calculate the L5 beam. For a rectangular beam exposed to fire on three-sided, the schematic diagram of effective section of the beam is shown in Fig. 24. In fact, the 500 °C isotherm method is a rough calculation method for the residual flexural capacity of the beam after fire. It is based on the assumption that concrete outside 500 °C no longer provides the bearing capacity of the member, while the residual concrete maintains its initial values of strength and elastic modulus. On the basis of the above assumptions, after obtaining the 500 °C isotherm, then determine a new width $b_{fi}$ and a new effective height $d_{fi}$ of the section by excluding the concrete outside the 500 °C isotherm. The rounded corners of isotherms can be

| Method                     | Result (KN·m) | Error (%) |
|----------------------------|---------------|-----------|
| Test                       | 196           | ~         |
| Simulation                 | 201           | 2.55      |
| 500 °C isotherm            | 176.77        | 9.81      |
| Improved section           | 194.45        | 0.79      |
regarded by approximating the real form of the isotherm to a rectangle or a square. Then determine the reduced strength of the reinforcement according to the temperature of reinforcing bars in the tension and compression zones. Finally, the bending capacity of the beam post-fire can be obtained based on the reduction factor of materials and the formula of bending capacity.

According to the simulation method of the residual flexural capacity of the RC beam after fire in Sect. 4.3, after the simulation of the temperature field of the L5 beam subjected to fire for 60 min, the residual flexural capacity of the L5 beam is now simulated and compared with source test data. The results are shown in Fig. 25. The residual flexural capacity of the L5 beam of the test results of the improved section method results of the simulation results of the 500 °C isotherm method results are summarized in Table 4.

The compressive strength reduction factor and the yield strength reduction factor of the section of beam after fire is determined according to Figs. 14 and 15. The flexural capacity of specimen L5 was calculated with Eqs. (13–15) to be 194.45 kN m, with error 0.79% from that (196 kN m) of specimen L5 in Ref. (Xu et al. 2013) and was calculated with 500 °C isotherm method to be 176.77 kN m with error 9.81%. The flexural capacity of the theoretical calculation model proposed in this paper is well fitted with the Ref. (Xu et al. 2013), which indicates that the method can be applied accurately and effectively to the calculation of the flexural capacity of RC beams after fire.

7 Conclusions
The flexural capability of reinforced concrete after fire exposure was simulated on the finite-element analysis software. The proposed theoretical calculation model based on improved section method and EFA was calculated and verified. The following conclusions can be drawn from the above research results.

1. The RC beams can be accurately simulated using the sequential thermo-mechanical coupling model developed in ABAQUS.
2. The fire exposure significantly weakened the flexural capacity and increase the deflection after a longer time of fire exposure.
3. Among the influence factors, the cover thickness, time of fire exposure, the ratio of longitudinal reinforcement, and the ratio of shear span to effective depth all largely affected the flexural capacity of RC beams after fire exposure. The flexural capacity was enhanced with larger cover layer thickness or higher reinforcement ratio but reduced after longer time of fire exposure. The flexural capacity was severely weakened after four-sided fire. The ratio of shear span to effective depth has little effect on the residual flexural capacity of the beam after fire.
4. The flexural capacity of the post-fire beam obtained by the improved section method was consistent with the results obtained by the FEA software, and it was consistent with the experimental data, and the accuracy was high compared to other methods, which provided reference for the calculation of flexural capacity of RC beams after fire.

Acknowledgements
This research was financially supported by the Foundation of China Scholarship Council (No. 201805975002), National Natural Science Foundation of China (Grant NO. 51378238), Science and Technological Planning Project of Ministry of Housing and Urban–Rural Development of the People’s Republic of China (No. 2017-K9-047). The authors wish to acknowledge the sponsors. However, any opinions, findings, conclusions and recommendations presented in this paper are those of the authors and do not necessarily reflect the views of the sponsors.

Authors’ contributions
BC and FF designed the research methodology; BL performed the analysis, BL and FF draft the manuscript; BC and FF reviewed the manuscript. All authors read and approved the final manuscript.

Funding
Founder: Foundation of China Scholarship Council, Award Number No. 201805975002; Founder: National Natural Science Foundation of China, Award Number. 51678274; Founder: Science and Technological Planning Project of Ministry of Housing and Urban–Rural Development of the People’s Republic of China, Award Number 2017-K9-047

Availability of data and materials
The data used to support the findings of this study are available from the authors upon request.

Competing interests
The authors declare that they have no competing interests.

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Received: 26 December 2019  Accepted: 5 August 2020
Published online: 30 September 2020

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