The jet-disk symbiosis model for Gamma Ray Bursts: cosmic ray and neutrino background contribution

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Abstract. The relation between the cosmological evolution of the jet-disk symbiosis model for GRBs and the cosmic ray energy distribution is presented. We used two different Star Formation Rates (SFR) as a function of redshift and a Luminosity Function (LF) distribution to obtain the distribution in fluence of GRBs in our model and compare it with the data. We show a good agreement between the fluence distribution we obtain and the corrected data for the 4B BATSE catalogue. The results we obtain are generally valid for models that use jet physics to explain GRB properties.

The fluence in the gamma ray band has been used to calculate the energy in cosmic rays both in our Galaxy and at extragalactic distances as a function of the redshift. This energy input has been compared with the Galactic and extragalactic spectrum of cosmic rays and neutrinos. Using our jet disk symbiosis model, we found that in both cases GRBs cannot give any significant contribution to cosmic rays. We also estimate the neutrino background, obtaining a very low predicted flux. We also show that the fit of our model with the corrected fluence distribution of GRBs gives strong constraints of the star formation rate as a function of the redshift.

Key words: Gamma ray : bursts – Stars : statistics – ISM : cosmic rays – Cosmology : distance scale

1. Introduction

More than 30 years after their discovery, thanks to the Burst and Transient Source Experiment (BATSE) and the Italian-Dutch satellite BeppoSAX, the scientific community now knows that Gamma Ray Bursts (GRBs) are isotropically distributed in the sky (Fishman & Meegan 1995) and that at least some of them are at cosmological distances (GRB970228: Djorgovski et al. 1999b, GRB970508: Metzger et al. 1997, GRB971214: Kulkarni et al. 1998, GRB980613: Djorgovski et al. 1999a, GRB980703: Djorgovski et al. 1998, GRB990123: Hjorth et al. 1999, GRB990510: Vreeswijk et al. 1999, GRB990712: Galama et al. 1999). But the present data available for redshift position and host galaxy localization are still too few to give us good statistics to study the evolution of GRBs and their redshift distribution. Before the discovery of GRB afterglows by BeppoSAX, the only way to study their distributions was to compare some GRB properties (like for example the intensity), with some parametric models (Fenimore & Bloom 1995, Cohen & Piran 1995, Kommers et al. 1999). Because of this lack of information, it is still necessary to assume that GRBs follow the statistical distribution of some other better known objects to obtain the GRBs fluence or flux distribution itself.

The origin of GRBs is still controversial. According to different models, their progenitor can be identified with the merging of two neutron stars, or with the collapse of a massive star. In the model presented by Pugliese et al. (1999), GRBs are created inside a pre-existing jet in a binary system formed by a neutron star and an O/B/WR companion, where the input energy comes from the collapse of the neutron star into a black hole and the emission is due to synchrotron radiation from the ultrarelativistic shock waves that propagate along the jet with a low-energy cut-off in the electron distribution. Following this scenario, the birth of GRBs cannot happen too far from the region where the progenitor formed, and this implies that their rate should be connected with the Star Formation Rate (SFR). Already other authors studied the connection between the SFR and GRBs flux distribution. For example, Wijers et al. (1998) showed that the assumption that the GRB rate is proportional to the SFR in the universe is consistent with the GRB flux distribution.

In Sect. 2 we calculate the cumulative distribution of GRB fluences using two SFR distributions as a function of redshift, the one by Miyaji (Miyaji et al. 1998), and the other by Madau (Madau et al. 1996). We compared it with the data from the BATSE catalogue. In Sect. 3 we...
calculate the maximum energy available in our model to obtain high energy cosmic rays. In Sect. 4 we present our results for the contribution of GRBs to the cosmic rays distribution, both Galactic and extragalactic and in Sect. 5 the eventual contribution from GRBs to the neutrino flux.

2. The rate of GRBs

The aim of this section is the calculation of the number of GRBs per year and per 100 Mpc$^3$ as a function of redshift, assuming beamed emission in a jet, (e.g. Pugliese et al., 1999). The following calculations are quite general and we emphasize that their validity does not depend on the particular model used.

To obtain a result as close as possible to the data, we chose the fluence as the quantity that can well represent the characteristics of GRBs. We follow Petrosian & Lloyd (1997), who showed that the fluence is the most appropriate parameter to study the cosmological evolution of GRBs. The relation between the redshift $z$ and the fluence $f$ is given by (Petrosian & Lloyd 1997) as a function of the specific luminosity $L$ expressed in [erg s$^{-1}$ Hz$^{-1}$]:

$$f = \frac{L}{4\pi d_L^2(1+z)^{\alpha-3}} \text{ erg/cm}^2. \quad (1)$$

Here $\alpha$ is the photon flux spectral index, $d_L = (2c/h)(1 + z - \sqrt{1 + z})$ is the cosmological luminosity distance (Weinberg, 1972), and $H_0 = h (100 \text{ km s}^{-1} \text{ Mpc}^{-1})$ is the Hubble constant. In the transformation from the emitter frame to the observer frame the two corresponding redshift contributions from the frequency and the time dependence are cancelled.

In our model (Pugliese et al. 1999), we used $\alpha = 2$, and the corresponding value for the fluence is:

$$f = \frac{L}{4\pi d_L^2} (1+z) \text{ erg/cm}^2. \quad (2)$$

In this way the redshift as a function of the fluence is:

$$1 + z(f, f_*) = \left( \frac{1}{2} \sqrt{\frac{f}{f_*}} + 1 \right)^2, \quad (3)$$

where $f_* = \frac{L(\nu_0) d_L}{4\pi (1+z)^{\alpha-3} \nu_0^2} \text{ erg/cm}^2$ is the reference fluence and $L(\nu_0)$ is expressed in [erg s$^{-1}$ Hz$^{-1}$].

At this point of our calculations it is important to define the role of the parameter $f_*$. In fact a relevant question is whether GRBs are standard candles (i.e. $f_*$ is constant) or whether they are distributed with a Luminosity Function (LF) with, e.g., a power law in $f_*$. Developing our calculations, we arrived at the same results found by Komsers et al. (1999). Here the authors showed that the best model is the one in which the star formation rate is combined with the luminosity function distribution. In agreement with them, also in our model GRBs cannot be considered as standard candles. We use a distribution for the luminosity function with a power law index $-\beta$, and the corresponding law for the parameter $f_*$ is given by:

$$\phi(f_*) df_* = \phi_0 f_*^{-\beta} df_*, \quad (4)$$

where $\phi_0$ is the normalization parameter equal to $1/(f_{sa}^{-1} - f_{sb}^{-1})$. We adopt $f_{sa} = 10^{-8}$ erg/cm$^2$, and $f_{sb} = 2.2 \times 10^{-4}$ erg/cm$^2$.

Here we do not yet answer the question what the physics of this luminosity function may be. It is plausible in the context of our model, that it is directly connected to the mass flow in the pre-existing jet prior to the GRB explosion. If this were the correct interpretation, the mass accretion rate, and correspondingly, the mass flow rate in the jet may follow a power law distribution, a point which we will pursue elsewhere.

We also calculate the GRB rate using two different star formation rates and compare the corresponding results with the data.

2.1. The number count

The number count of GRBs sources is given by the expression $dN(z) = F(z)(dt/dz) dV dz$, where $F(z)$ is equal to the product of the SFR $\psi(z)$ and the luminosity function $\phi(z)$. Obviously the luminosity function may change with redshift $z$, but for simplicity and as a first step, we use this ansatz. In term of the fluence $f$ of GRBs in the gamma ray band, and of the reference fluence $f_*$, the number count is:

$$dN(f, f_*) = [\psi(f, f_*)] \left[ \frac{dt}{dz}(f, f_*) \right] \left[ 4\pi d_L^2(f, f_*) \frac{df}{dz}(f, f_*) \right] \times [dz(f, f_*)], \quad (5)$$

where:

- The first term $\psi(z(f, f_*))$ represents the star formation rate as a function of the redshift.
- The second term represents the temporal interval in which the rate is calculated. It is given by:

$$\frac{dt}{dz}(f, f_*) = \frac{1}{H_0} \frac{1}{(\frac{1}{2} \sqrt{f_*/f} + 1)^{\beta}}. \quad (6)$$

- The third term is the interval of volume as a function of the fluence. It corresponds to the value:

$$4\pi d_L^2 \frac{d}{dz}dL = 4\pi c^3 \frac{f_*}{h^2} \left( \frac{1}{2} \sqrt{\frac{f_*}{f} + 1} \right) \left( \sqrt{\frac{f_*}{f} + 1} \right). \quad (7)$$

- The fourth term represents the redshift interval and it is given by:

$$dz(f, f_*) = -\frac{1}{2\pi} \sqrt{\frac{f_*}{f} + 1} \left( \frac{1}{2} \sqrt{\frac{f_*}{f} + 1} \right) df. \quad (8)$$

The integration over the parameter $f_*$ will be shown in the Eq. 11.
2.2. The GRB rate distribution using the Miyaji SFR

Here we use the star formation rate as a function of the redshift presented by Miyaji et al. (1998), and compare the corresponding GRB rate with the data corrected for selection and calibration effects by Petrosian (priv. comm.) for the 4B BATSE catalogue.

In their paper Miyaji et al. (1998), calculated the distribution of Seyfert galaxies as a function of redshift. We considered this same distribution for the SFR, and approximated it with the following function:

\[ \psi(z) = A \exp(a z), \]  

where \( z \) is the redshift, \( A \approx 10^{-6}[h^{-3} Mpc^{-3}] \), \( a = 2.5 \), and we used \( \Omega = 1, \Lambda = 0 \) as a simple reference. This law is valid up to \( z = 2 \), and then continues as a constant to high redshifts.

\[ \psi(f, f_\star) = A \exp \left[ a \left( \frac{1}{2} \sqrt{\frac{f}{f_\star}} + 1 \right)^2 - 1 \right]. \]  

(10)

Using the LF given in Eq. 4, the rate of GRB fluence is obtained from the product of the terms of the Eqs. 6-8, and Eq. 10, where we now integrate over \( f_\star \), that is:

\[ N(f) df = 2 c^3 \frac{h^3}{1} A \phi_0(f^{-\beta} df) \int_{f_{\min}}^{f_{\max}} x^{-1/2} \times \]

\[ \exp \left[ a \left( \frac{1}{2} \sqrt{x} + 1 \right)^2 - 1 \right] \times \]

\[ \left( \frac{1}{2} \sqrt{x} + 1 \right) \left( \sqrt{x} + 1 \right)^{-3} dx, \]  

(11)

where \( x = f_\star/f \), and \( x_{\max} \) and \( x_{\min} \) are the limits of integration defined as the maximum and the minimum of the intersection between the interval \([f_{a}, f_{b}]\) relative to the LF distribution and the interval in which the ratio \( f_\star/f \) is defined.

In Fig. 1 we plotted the cumulative curves corresponding to different luminosity function power law indexes (\( \beta = 1.5 \) dotted line, \( \beta = 2.0 \) dashed line, and \( \beta = 3.0 \) solid line) in Eq. 11 and compared them with the corrected data of the 4B catalogue from BATSE. It is evident that even if we change the luminosity function distribution index, we cannot obtain any better fit or calculate any GRB rate using this SFR distribution.

2.3. The GRB rate distribution using the Madau SFR

In this paragraph we calculate the GRB rate based on the same procedure as in the last section, using the SFR as a function of redshift presented by Madau et al. (1996) and the luminosity function distribution of Eq. 4.

\[ \text{Fig. 1. The corrected data (Petrosian priv. comm.) from the 4B BATSE catalogue (full circles) are compared with the distribution of GRB fluence calculated using the Miyaji SFR and a power law luminosity function index } \beta = 1.5 \text{ (dotted line), } \beta = 2.0 \text{ (dashed line), and } \beta = 3.0 \text{ (solid line).} \]

This SFR as a function of the fluence is given by:

\[ \psi(f, f_\star) = A \exp \left[ a \left( \frac{1}{2} \sqrt{\frac{f}{f_\star}} + 1 \right)^2 - 1 \right]. \]  

(10)

\[ \text{Fig. 2. Madau SFR curves at different slopes after the redshift } z^*_\star = 1.7. \text{ The full circles are the data points from Steidel (1999). In this article, Steidel considers the curve } \alpha_2 = 1 \text{ as one of the many possible curves consistent with both the current data on the far-IR background and the galaxies detected in the new submillimiter band.} \]

In this article, Steidel considers the curve \( \alpha_2 = 1 \) as one of the many possible curves consistent with both the current data on the far-IR background and the galaxies detected in the new submillimiter band.
We approximated the Madau SFR by the combination of two exponential functions. The first part of the curve is:

$$k_1(z) \simeq \exp(a_1 z),$$

(12)

where $z$ is the redshift, and $a_1 = 2.9$ is a constant given by the IR source counts. This law is valid up to $z^* = 1.7$, after which it is substituted by the function:

$$k_2(z) \simeq \exp(-a_2 z),$$

(13)

and $a_2$ is a constant.

Because of the uncertainties of the dust extinction at high redshift and also the difficulties in the redshift determination for the SCUBA sources (Sanders 1999), the star formation history beyond the redshift $z = 2$ is still unclear, therefore we considered different slopes for this part of the Madau SFR curve and plotted them in Fig. 3 together with the experimental data given by Steidel (1999).

The cumulative GRB fluence distribution is obtained from Eq. (11), where the SFR function is substituted with Eq. (12) and Eq. (13). The parameters that we can change to fit the data are the redshift $z^*$, the power law index $a_2$ in the Eq. (13) and the power law index $\beta$ and the upper limit $f_{\ast b}$ of the interval in which the luminosity function distribution is defined. Following one of the possible curves that fit the data shown by Steidel (1999), we chose $z^* = 1.7$, $\beta$ determines the slope of the cumulative distribution curve, $a_2$ defines curves with different slope, and variations in $f_{\ast b}$ correspond to little changes in the part of the curve relative to the strongest GRBs, we chose $f_{\ast b} = 2.2 \times 10^{-8}$ erg/cm$^2$. To reproduce the corrected data by Petrosian of the 4B catalogue, we probed all the different cases changing the slopes of the SFR and the power law index of the fluence distribution. The only values for the power law index of the luminosity distribution that successfully fits the data was $\beta = 1.55$.

In Fig. 3 we show the cumulative fluence distribution corresponding to the different slopes of Fig. 2 and the power law index $\beta = 1.55$. We compared these curves with the data corrected for selection and calibration effects by Petrosian for the 4B BATSE catalogue: we can fit these data only if $a_2$ is in the range $0.8 \div 1.3$. In Fig. 3 the data are fitted with $a_2 = 1.0$.

Considering the total number of GRBs in the BATSE catalogue, an observing time of 8 years, a volume scale of $h^{-3}10^{10.8}$ Mpc$^3$, a beaming factor $\frac{\pi}{2\theta_{11}^2} = \frac{\pi}{2\theta_{11}^2}$, where $\theta_{11} = \theta_1/(10^{-1}\text{rad})$ is the opening angle of the jet, and the factor 22 coming from the integral of the distribution in Eq. (13) calculated with these new SFR and LF, we obtain the following rate of GRBs:

$$10^{-5.4}(h^3\theta_{-11}^{-2}) \text{GRBs (yr)}^{-1} (100 \text{ Mpc}^{-3}).$$

(14)

This value obtained considering beaming effects is close to the number given by Piran (1999).

**Fig. 3.** Cumulative curves corresponding to the different slopes of Fig. 2

**Fig. 4.** Comparison between the distribution of GRB fluence using the power law Luminosity function $\phi(f_\ast) \propto f_\ast^{-1.55}$, and the Madau SFR (solid line) and the 4B corrected data from Petrosian (priv. comm.) (full circles).

The result depends strongly on the power index, while it is not influenced by the interval $[f_{\ast a}, f_{\ast b}]$ of the LF distribution we used. We defined a lower limit for the fluence equal to $10^{-8}$ erg/cm$^2$, and a ratio $f_{\ast b}/f_{\ast a}$ equal to $10^4$. A change in the upper limit of this interval corresponds to a small change in the tail of the fluence distribution curve, for $x > 10^{-4}$. In our model, this interval in the fluence corresponds to an interval in the initial energy deposited in the jet in the range $[10^{48}, 10^{52}]$ ergs.

### 3. Maximum energy available

Another important step is to check if it is possible to produce neutrinos and high energy cosmic rays with our
model (Pugliese et al. 1999). Therefore we calculated the maximum energy available in the emission region. Irrespective of any acceleration mechanism, the maximum energy of charged particles, here protons, is that given by the spatial limit in the comoving frame, where the gyromotion just fits the space available:

\[
E_{\text{max}}^{(ob)} = \frac{1}{2} \gamma_{\text{sh}} e (B_j \Delta z)^{(sh)}.
\]

Here \( e \) is the charge of an electron, \( B \) is the magnetic field in the shock frame given by \( B_j^{(sf)}(t) = 10.24 (E_{51}^{-1/4} \Phi_{-5/3}^{3/4} v_{0.3}^{-3/4} (\epsilon^{-1/2} \theta_{-1 j}^{-1} \gamma_{m.2}^{-1/2}) \Delta z^{-3/4} \), and \( \Delta z \) is the thickness of the emission region. For the calculation of the thickness we have two possibilities, and each of them corresponds to one of the following cases:

- For the first case we used the thickness of the emission region in the shock frame (sh), given by \( z/(4 \gamma_{sh}) \):

\[
E_{\text{max}}^{I,(ob)} = \gamma_{\text{sh}} (e B_{(sh)}^{(ob)} z_2^{-1} 4 \gamma_{sh}) \simeq 1.91 \times 10^{21} (E_{51}^{1/4} \Phi_{-5/3}^{1/4} \epsilon_{0.3}^{-1/4} \epsilon_{0}^{-1/2} \theta_{-1 j}^{-1/2} m_{2}^{-1/2}) t^{-1/4} \text{ eV.}
\]  

- In the second case we considered the width of the shell:

\[
E_{\text{max}}^{II,(ob)} = \gamma_{\text{sh}} (e B_{(sh)}^{(ob)} \theta z_2) \simeq 0.93 \times 10^{23} E_{51}^{1/2} \epsilon_0^{-1/2} \theta_{-1 j}^{-1/2} m_{2}^{-1/2} \text{ eV.}
\]

At this point it is necessary to check when the thickness of the emission region is lower than the width, which corresponds to \( E_{\text{max}}^{I,(ob)} < E_{\text{max}}^{II,(ob)} \). To do this we calculated the time at which these two quantities are equal:

\[
t_{(E_{\text{max}}^{I,(ob)} = E_{\text{max}}^{II,(ob)})} \simeq 5.62 \times 10^{6} (E_{51}^{1/4} \Phi_{-5/3}^{1/4} \epsilon_{0.3}^{-1/4} \theta_{-1 j}) \text{ s.}
\]

It means that for about two months \( E_{\text{max}}^{I,(ob)} < E_{\text{max}}^{II,(ob)} \). Obviously, only \( E_{\text{max}}^{I,(ob)} \) is valid, so after 10 seconds the upper limit of the maximum energy available in our model is about \( 1.07 \times 10^{22} \text{ eV} \), while after two months the energy to consider is \( E_{\text{max}}^{II,(ob)} \). Adiabatic losses will diminish these energies for charged particles like protons.

### 4. Cosmic ray contribution

Cosmic rays are ionized nuclei, mainly protons, that extend from low energies (few hundred MeV) up to very high energies (about \( 3 \times 10^{20} \text{ eV} \)). Their spectrum is described by a power law \( (dN/dE) = E^{-(\kappa+1)} \), and shows two breaks in the slope. From low energies up to about \( 5 \times 10^{15} \text{ eV} \), known as the knee, the spectrum follows a pure power law with \( \kappa \approx 1.7 \). The detailed shape of this break and the precise position are still unknown. Beyond the knee up to about a second break point at \( 3 \times 10^{18} \text{ eV} \), known as the ankle, the pure power law has an index \( \kappa \approx 2 \).

It has been proposed (Biermann 1993, Stanev et al. 1993), that three components contribute to the cosmic ray spectrum: a) explosion of supernovae into a homogeneous interstellar medium (ISM), accelerating particles up to energies of about \( 10^{15} \text{ GeV} \). The spectrum for these particles is a power law with an index of -2.75, after considering the leakage from our Galaxy. b) Explosion of stars into their former stellar wind (like Wolf Rayet stars), producing particles with energies up to about \( 3 \times 10^{9} \text{ GeV} \). The corresponding spectrum switches at the knee from -2.67 to -3.07 and this difference in the spectral index derives from a diminution of the particle curvature drift energy gain. In Biermann’s cosmic ray model for the Galactic component (Biermann 1997), the energetic protons are produced in the shocks of supernova explosions in the interstellar medium, while all the heavier elements are produced in the shock waves propagating in the stellar wind of the progenitor star. c) Production of particles with energies up to \( 10^{12} \text{ GeV} \) from the hot spots of Fanaroff Riley class II radio galaxies. Their spectrum has an index -2 at the source, and one needs to take the interaction with the cosmological microwave background into account.

We expect the spectrum of cosmic rays above about \( 5 \times 10^{19} \text{ GeV} \) to be strongly attenuated because of the interaction of nuclei and protons with the 2.7 K cosmic microwave background, giving rise to the so-called Greisen-Zatsepin-Kuzmin (GZK) cut-off. The extragalactic sources cannot produce all the total cosmic ray energy density observed at Earth, but they could give a contribution to the ultra-high energy (UHE) part of the spectrum. The origin of the cosmic rays above \( 3 \times 10^{18} \text{ eV} \) is not yet clear, but the common idea is that they are extragalactic and probably connected with the most powerful radio galaxies (Biermann & Strittmatter 1987, Berezinsky & Grigor’eva 1988, Rachen & Biermann 1993, Rachen et al. 1993). At the moment, our knowledge of the sources of the highest energy cosmic rays is limited by the small number of events detected by the present experiments. Considering that at \( 10^{20} \text{ eV} \) the rate of cosmic rays is about 1 event per km² per century, it is clear that to detect them it is necessary to have both large aperture detectors and a long exposure time.

In this general context, it is interesting to check the eventual Galactic and extragalactic energetic contribution given by GRBs to the cosmic ray spectrum in our model. In fact GRBs seem to be very powerful explosions and they inject a large amount of energy and elementary particles into the interstellar medium.

We used the GRB rate obtained with the SFR from Madau and two different energetic approaches to calculate the contribution from GRBs to the cosmic rays and the neutrino spectra. First we considered that each GRB gives the same contribution equal to 10% of a fixed initial
energy of $10^{51}$ ergs. Secondly we assume that each GRB contributes proportional to its own fluence, which we assume is distributed with a power law, corresponding to a range of initial energy of $[10^{48}, 10^{52}]$ ergs.

4.1. Extragalactic contribution

For GRBs it is important to identify which particles contribute to the cosmic ray flux. As Rachen & Mészáros (1998) showed, during the main burst protons lose most of their energy because of adiabatic expansion, while neutrons can be better candidates to obtain ultra high energy cosmic rays (UHECR) and neutrinos. In fact neutrons carry about 80% of the proton energy, and because they are not coupled to the magnetic field, they can escape the fireball and through the $\beta$-decay give a cosmic ray proton spectrum. We followed this same logic in our calculations below.

a) We assumed that the total energy discharged by each GRB in the ISM for hadrons as well as neutrinos is equal to 10% of the initial energy $E_{51} = E/(10^{51}$erg) deposited in the jet, that is $\eta_0 = (10\% E)/(10^{50}$erg). It means that in the Hubble time, the energetic contribution per unit volume inside the whole universe given by all the GRBs is equal to $10^{-21.0}(h^3\theta_{-1j}^2\eta_0)$ erg/cm$^3$.

To derive the spectrum of GRBs and to compare it with the one of cosmic rays out of our Galaxy, we need to calculate the normalization factor $N$ in $N\left(\frac{E}{E_0}\right)^{-2} dE$, where $N$ is expressed in [GeV$^{-1}$ cm$^{-2}$ s$^{-1}$ sr$^{-1}$].

$N$ is obtained directly from the integration of this power law, remembering that the result of this integral is equal to the energy per volume produced by all the GRBs:

$$\int_{E_1}^{E_2} \frac{4\pi}{c} N\left(\frac{E}{E_0}\right)^{-2} E dE = 10^{-21.0}(h^3\theta_{-1j}^2\eta_0). \quad (19)$$

The corresponding value is

$$N \simeq 10^{-8.8}(h^3\theta_{-1j}^2\eta_0) \text{ GeV}^{-1} \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}. \quad (20)$$

b) We assumed that the total energy discharged by each GRB in the ISM for hadrons as well as neutrinos is proportional to its own fluence in the $\gamma$ band. To obtain the energetic contribution per unit volume inside the whole universe given by all the GRBs in the Hubble time, we integrated the Luminosity Function distribution and had $10^{-23.6}(h^3\theta_{-1j}^2\eta_0)$ erg/cm$^3$.

The corresponding normalization factor $N$ in the curve to plot $N\left(\frac{E}{E_0}\right)^{-2} dE$ is

$$N \simeq 10^{-11.4}(h^3\theta_{-1j}^2\eta_0) \text{ GeV}^{-1} \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}. \quad (21)$$

Fig. 5. Comparison between the extragalactic GRB contribution for the case a) (solid line) and the case b) (dashed line) and the all particle cosmic ray spectrum, expressed in [GeV$^{-1}$ cm$^{-2}$ s$^{-1}$ sr$^{-1}$]. The dotted lines show that it not possible to have any contribution beyond $10^{18}$ eV, because of the interactions with the microwave background. The stars represent the cosmic ray data from Akeno experiment (Nagano et al. 1984), the open squares are the Fly’s Eye data (Baltrusaitis et al. 1985) and the full circles are the AGASA data (Yoshida et al. 1995).

In Fig. 5 we plot the all particle energy spectrum as measured by different ground-based experiments (Biermann & Wiebel-Sooth 1999), the spectrum in the case in which each GRB gives the same contribution to CR’s (solid line) and the spectrum corresponding to a contribution from each GRB proportional to its own fluence (dashed line). The dotted lines represent unlikely contributions because beyond $10^{18}$ eV the interactions with the cosmological microwave background are relevant and they make the curves much flatter, and therefore much lower. From this graphic it is clear that in our model GRBs do not give any energetic contribution to the extragalactic cosmic ray spectrum.

4.2. Galactic contribution

To calculate the contribution from GRBs to the Galactic cosmic ray spectrum it is necessary to know the GRBs production rate in our Galaxy. We considered that the ratio between the in/out GRB rates is equal to the ratio of the infrared (IR) luminosity inside and outside our Galaxy. In our Galaxy this luminosity is $L_{IR} \sim 10^{10} L_\odot \sim 10^{43.9}$ erg/s at 60$\mu$m. Using the equation (1) of Malkan & Stecker (1998), we have an extragalactic IR luminosity $L_{IR} \sim$
$10^{14.6} h^3 \text{erg (100 Mpc)}^{-3}$. This means that the Galactic GRB rate is equal to $10^{-6.4} (h^3 \theta_{-1/2}^2 \eta_{10})$ GRBs per year.

a) Following the same procedure used for the extragalactic case, we calculated first the case in which each GRB gives the same contribution to the cosmic ray spectrum. We obtained a diffuse density energy of GRBs in our Galaxy equal to $10^{-16.0} (h^3 \theta_{-1/2}^2 \eta_{10})$ erg/cm$^3$.

In our Galaxy, cosmic rays have a time scale to escape with an energy dependence that goes as $E^{-1/3}$ (Biermann 1995, Biermann et al. 1995). This term has to be taken in account to calculate the total spectrum of the primary cosmic rays inside our Galaxy. It means that it is necessary to multiply the injection spectrum $E^{-2}$ times the leakage term $E^{-1/3}$ to obtain the final plot of the contribution from GRBs to the Galactic cosmic ray spectrum: $N \left( \frac{E}{E_0} \right)^{-7/3} dE$. The normalization factor is obtained using the same procedure of the last section:

$$\int_{E_1}^{E_2} \left[ \frac{4 \pi}{c} N \left( \frac{E}{E_0} \right)^{-2} EdE \right] = 10^{-16.0} (h^3 \theta_{-1/2}^2 \eta_{10}).$$  \hspace{0.5cm} (22)

The corresponding value is

$$N \simeq 10^{-3.9} (h^3 \theta_{-1/2}^2 \eta_{10}) \text{ GeV}^{-1} \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}. \hspace{0.5cm} (23)$$

b) For the second case we assume that each GRB contribution is proportional to its own fluence in the $\gamma$-band, and integrating the luminosity function distribution we had an energy density in our Galaxy equal to $10^{-18.7} (h^3 \theta_{-1/2}^2 \eta_{10})$ erg/cm$^3$.

The corresponding normalization factor $N$ in the spectrum $N \left( \frac{E}{E_0} \right)^{-7/3} dE$ is

$$N \simeq 10^{-6.5} (h^3 \theta_{-1/2}^2 \eta_{10}) \text{ GeV}^{-1} \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}. \hspace{0.5cm} (24)$$

In Fig. 6 we compared the all particle energy spectrum as measured by different ground-based experiment (Biermann & Wiebel-Sooth 1999), with the spectrum from GRBs in the case that each of them gives the same contribution (solid line) and with the one in which the contribution is proportional to the fluence (dashed line). The dotted lines beyond $10^{18}$ eV show where interactions with the microwave background may become relevant. Since massive star formation is highest in the Galactic central region, any contribution is limited to energies for which the Larmor radius becomes as large as the Galactic disk, and the AGASA data suggest only a small anisotropy at the Galactic center. But even if from an energetic point of view, GRBs could give a contribution at these high energies, this is ruled out considering that the temporal interval between two GRBs in our Galaxy is larger than the leakage time of cosmic rays at these energies. Therefore, in our model also in our Galaxy GRBs cannot give any contribution to the cosmic ray spectrum.

**5. Neutrino production**

We also would like to probe the energetic contribution of GRBs to the neutrino flux. Both the very high energy (VHE) neutrinos, with energies in the range $10^{15} - 10^{17}$ eV, and the ultra high energy (UHE) neutrinos, with energies $E \geq 10^{17}$ eV originate in the interactions of protons with photons, through the reactions $p\gamma \rightarrow n \pi^+$, $\pi^+ \rightarrow \mu^+ \nu_\mu$, $\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_e$. The efficiency for the neutrino production depends on the fraction of proton energy converted into charged pions and on how much energy pions and muons keep before decaying.

Some authors (Waxman & Bahcall 1997), proposed that GRBs can be associated with neutrinos produced in the photohadronic reaction. Pions come from the interactions between accelerated protons and gamma rays in the fireball, and their decay produces neutrinos and anti-neutrinos together with other elementary particles. Waxman & Bahcall (1997) gave an upper limit for the corresponding neutrino flux.

Other authors (Rachen & Mészáros 1998), argued that the upper limit obtained in this way is optimistic. In fact, protons emitted in the earliest burst do not have enough energy to leave the expansion region because of adiabatic losses. Instead neutrons can easily escape and contribute to the neutrino flux.

![Fig. 6. Comparison between the Galactic GRB contribution for the case a) (solid line) and the case b) (dashed line) and the all particle cosmic ray spectrum, expressed in [GeV$^{-1}$ cm$^{-2}$ s$^{-1}$ sr$^{-1}$]. The dotted lines and the points are the same of Fig. 6.](image-url)
To check what is the neutrino flux in our model, we calculated the initial proton density number in the $p\gamma$ interaction from the synchrotron photons in our model. We obtained a number of photons/cm$^3$ equal to:

$$L^{(\text{sh})}/(4\pi \frac{z}{\gamma_{\text{sh}}} \frac{100 \text{ KeV}}{c}) = 6.94 \times 10^{16} (E_{51}^{-1/4} M_{-5j}^{5/4} v_{0.3}^{-5/4}) \theta_{-1j}^{-2} t^{-7/4}.$$ (25)

To obtain the number of hits in the $p\gamma$ collisions, we used a cross section $\sigma_{p\gamma} = 2.70 \times 10^{-28} \text{cm}^2$, corresponding to the maximum of the curve describing this reaction. This implies a number of hits per proton equal to $0.99 (E_{51}^{-1/4} M_{-5j}^{5/4} v_{0.3}^{-5/4}) \theta_{-1j}^{-2} t^{-3/4}$.

![Fig. 7](image_url)

**Fig. 7.** The GRB contribution to the neutrino spectrum from our model, expressed in [GeV$^{-1}$ cm$^{-2}$ s$^{-1}$ sr$^{-1}$]. The dashed line represents the measured upper limit to the neutrino flux from the Frejus experiment (Rhode et al., 1996). Beyond $10^9$ GeV, corrections from the proton and neutron interactions with the microwave background can be expected.

To calculate the energy rate of neutrinos in our model we assumed that about 20% of the energy of protons $E_p = 10^{50} \text{ erg}$ goes into neutrinos and that in a first approximation there are no multiple hits for the same proton in the $p\gamma$ interaction. Following the same procedure used to calculate the extragalactic cosmic ray contribution from GRBs, we obtained that the normalization factor corresponding to the spectrum $(E/E_0)^{-2}$ in the case in which each GRB gives the same contribution to the neutrino spectrum is $N \simeq 1.55 \times 10^{-10} (h^3 \theta_{-1j}^2 \eta_0) \text{ GeV}^{-1} \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}$. This spectrum is plotted in Fig. 7, and it shows that in our jet model GRBs give a contribution to the cosmological neutrino flux at a low level.

6. Discussion and conclusions

In the first part of this article we calculated the GRB rate and compared the corresponding cumulative distribution in fluence with the observational data. There were two main points to decide on: a) which luminosity function distribution and b) which star formation rate were the best to reproduce the data.

We checked if in our jet model GRBs were standard candles. But we did not obtain any good fit, therefore we tried a power law for the luminosity distribution, $\phi(f) \propto f^{-\beta}$. Together with this function we used the SFR from Miyaji et al. (1998), in which the SFR grows linearly from $z = 0$ up to about $z = 2$ and after this redshift is flat up to $z = 6$. There was not a good agreement between the distribution in fluence we obtained and the corrected data by Petrosian for the 4B BATSE catalogue. Therefore we used the SFR model from Madau et al. (1996), in which the rate follows the same behavior of Miyaji's up to a redshift $z^*$, and after this redshift the SFR begins to decrease.

There are still some uncertainties about the shape of this second part of the distribution, so we considered different curves with different slopes (depending on a constant $a_2$) for this decreasing part. We have only two free parameters that we can change to fit the data, the power law index $\beta$, and the exponential index $a_2$. The redshift $z^*$, and the upper limit $f_{\text{sh}}$ of the interval in which the LF is defined are not really free parameter because their range is limited by the observations. We can reproduce the 4B BATSE corrected data by Petrosian using the following values: redshift $z^* = 1.7$, $\beta = 1.55$, $f_{\text{sh}} = 2.2 \times 10^{-4}$ and $a_2$ in the range $[0.8, 1.3]$. These parameters are remarkably constrained. Thus, given a final model for GRBs and a cosmological model, we may be able to derive strong limits on cosmological parameters.

The key point of the second part of the article is the calculation of the GRB rate inside and outside our Galaxy. The results we obtained depends mainly on three other parameters, the value of the Hubble constant $H_o$, the opening angle of the jet $\theta_j$, and the power law index $\alpha$ we assumed for the electron distribution in our model (see Eq. 6).

The choice of $\alpha = 2$ has been done in the first version of our model, where even if we simplified in many places the physics used, we obtained a good agreement with the data. Any possible small changes in this interval will influence the results obtained in this work in a marginal way, because the dependence on $\alpha$ in the expression of $N$ is not strong. However, putting $\alpha = 3$, which is not suggested by the data, would strongly influence our results.

On the other hand, changes in $H_o$ and $\theta_j$, because of the strong dependence in $N$ will influence the results. It is
interesting to note that both, a lower value of the Hubble constant and a smaller opening angle of the jet, go into the direction of decreasing $N$. But these changes cannot influence our results because the interactions of cosmic rays with the microwave background and the large difference between the Galactic GRBs and the leakage time of CR’s rule out that in our jet model GRBs can give any contribution beyond $10^{18}$ eV.

In the calculation of the contribution from GRBs to the neutrino flux, it is important to define the parameters that characterize the neutrino production. The number of hits in the $p\gamma$ collisions has been obtained considering only the major photohadronic interaction channel, the one that gives a single pion. At higher energies it is possible to have the channels for the production of multi-pions, $p\gamma \rightarrow n^2\pi^+\pi^-$ and $p\gamma \rightarrow n^3\pi^+\pi^-$, but as the energy increases the corresponding cross sections decrease. We used only the first channel with the highest cross section, because the energies involved are not high enough to require secondary channels.

In the context of our jet-disk symbiosis model for GRBs, there is only one way to make the extragalactic contribution to CRs significant at the highest energies, and that is to drastically increase the CR energies per GRB deposited. We considered this implausible in the context of our model because of energy conservation requirements. We use 10% of the entire energy available, and so the CR contribution may be increased over our simple calculation by a factor of a few, but not more. This implies that the contributions given by each GRB to CR and neutrino flux cannot be much bigger than the energy emitted in the $\gamma$-ray band.

Using a relatively small set of parameters, the jet-disk symbiosis model applied to GRBs, a tested star formation rate and the fundamental physics of the photohadronic interactions we arrive at the conclusion that GRBs are not able to give any significant contribution to the high energy cosmic ray spectrum both inside and outside our Galaxy and predict only a low flux of neutrinos.

A main conclusion of this work is that fitting the corrected fluence distribution of GRBs with the jet-model is well possible. The fit allows strong constraints of the star formation rate as a function of redshift.

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