Nucleon Transversity and Hyperon Polarization

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Abstract

We calculate the transverse polarizations of the produced hyperons in the semi-inclusive deep inelastic scattering of unpolarized lepton beam on transversely polarized nucleon, since these polarizations provide a potential method for extracting the transversity distribution $h_1(x)$ of the nucleon. In this calculation we use the SU(6) wavefunctions of the octet baryons and the spectator model for the distribution functions of nucleons and the fragmentation functions of hyperons. We find that that $H_1(z)$ of the $\Sigma$ hyperons are much larger than that of the $\Lambda$ hyperon. Therefore, when one tries to extract the transversity distribution $h_1(x)$ from the hyperon polarizations, measuring the polarizations of the $\Sigma$ hyperons is more efficient than $\Lambda$.

Keywords: Transversity, Hyperons, Transverse polarization

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1 Introduction

The unpolarized distribution \( f_1(x) \) and the helicity distribution \( g_1(x) \) of nucleon have been extensively investigated. However, the transversity distribution \( h_1(x) \) is less known since it can not be measured in fully deep inelastic scattering since it is chiral-odd. When the spins of two hadron beams in the hadron collider are transversely polarized, \( h_1(x) \) can be extracted by measuring the double-spin transverse asymmetry \( [1] \). Besides this, an important approach is using the Collins mechanism \( [2] \) in semi-inclusive deep inelastic scattering (SIDIS). By using this method, HERMES, COMPASS and CLAS have been making important progresses \( [3] \). In applying the Collins mechanism to the SIDIS process \( lp^\uparrow \rightarrow l\pi X \) (where \( p \) is proton or neutron), one needs to separate the contributions from the Collins and Sivers mechanisms \( [2, 4] \).

In this paper we study another method which can be used in extracting the transversity distribution \( h_1(x) \), which was presented in Refs. \( [5, 6, 7] \). This method considers the process \( lp^\uparrow \rightarrow l\Lambda^\uparrow X \) with an unpolarized lepton beam, a transversely polarized proton or neutron target \( (S_N) \) and the measurement of the transverse polarization \( P_N \) of a produced \( \Lambda \) hyperon; transverse means orthogonal to the \( \gamma^* - \Lambda \) plane \( [5, 6, 7] \):

\[
P_N = \frac{2(1 - y)}{1 + (1 - y)^2} \frac{\Sigma_q e_q^2 h_{1q}(x) H_{1q}(z)}{\Sigma_q e_q^2 f_{1q}(x) D_{1q}(z)}.
\]

When we neglect the sea quark contributions (which should be safe in the large \( x \) and \( z \) regions), (1) becomes

\[
P_N = \frac{2(1 - y)}{1 + (1 - y)^2} \frac{4h_{1u}(x) H_{1u}(z) + h_{1d}(x) H_{1d}(z)}{4f_{1u}(x) D_{1u}(z) + f_{1d}(x) D_{1d}(z)}.
\]

The polarizations of \( \Lambda, \Sigma^+, \Sigma^0, \Sigma^-, \Xi^0 \) and \( \Xi^- \) can be measured by measuring the angular distributions of the following two-body decays \( [8] \):

\[
\begin{align*}
\Lambda(1116 \text{ MeV}) & \longrightarrow p\pi^- (63.9\%) , \quad n\pi^0 (35.8\%) , \\
\Sigma^+(1189 \text{ MeV}) & \longrightarrow p\pi^0 (51.6\%) , \quad n\pi^+ (48.3\%) , \\
\Sigma^0(1193 \text{ MeV}) & \longrightarrow \Lambda\gamma \ (100\%) , \\
\Sigma^-(1197 \text{ MeV}) & \longrightarrow n\pi^- (99.8\%) , \\
\Xi^0(1315 \text{ MeV}) & \longrightarrow \Lambda\pi^0 \ (99.5\%) , \\
\Xi^-(1322 \text{ MeV}) & \longrightarrow \Lambda\pi^- \ (99.9\%).
\end{align*}
\]

The COMPASS group performed experiment \( [9, 10, 11] \) measuring the transverse polarizations of \( \Lambda \) and \( \overline{\Lambda} \) produced in the SIDIS process in which 160 GeV longitudinally polarized muon beam was incident on the transversely polarized \( \text{NH}_3 \) target, and obtained the result that the transverse polarizations of \( \Lambda \) and \( \overline{\Lambda} \) are compatible with zero within their error bars and they have no dependence on \( x \) or \( z \). Eq. (1) represents the transfer of transverse spins from an initial nucleon to a produced hyperon,
and it is satisfied for a polarized lepton beam as well as for an unpolarized lepton beam. This COMPASS experiment selected events at $Q^2 > 1$ GeV and $0.1 < y < 0.9$, and in the kinematic range $8 \cdot 10^{-3} < x < 0.1$ and $0.1 < z < 0.5$. So, it seems that it is difficult to investigate the nucleon transversity through the measurement of the $\Lambda$ polarization using Eq. (2). There were various speculations about the smallness of the measured transverse polarizations of $\Lambda$ and $\overline{\Lambda}$ such as: It could be attributed to the cancellation of the $u$ and $d$ quark contributions [9]. It might be due to the smallness of the transversity distribution in the available $x$-range or to the fact that the polarized fragmentation function into a transverse lambda is small in the COMPASS kinematic range [11]. In this paper we try to understand this question.

In this paper we calculate the transverse polarizations of the produced hyperons in the SIDIS process of unpolarized lepton beam on transversely polarized nucleon, with the SU(6) wavefunctions of the octet baryons and the spectator model for the distribution functions of nucleons and the fragmentation functions of hyperons. Ref. [12] performed the similar calculation. However, since the purpose of this paper is to understand why the COMPASS group obtained the polarization of $\Lambda$ compatible with zero and how we can overcome this difficulty in investigating the nucleon transversity through the hyperon polarization measurement, we present the results in detail and show clearly that the sizes of the polarizations of $\Lambda$ and $\Sigma$ are expected to be very different, and the polarization of $\Sigma$ is much larger than that of $\Lambda$.

We find that that $H_1(z)$ of the $\Lambda$ hyperon is very small for both initial $u$ and $d$ quarks, and therefore the polarization of the produced $\Lambda$ is very small for both transversely polarized proton and neutron targets. On the other hand, $H_1(z)$ of $\Sigma^+$ is large for initial $u$ quark and $H_1(z)$ of $\Sigma^0$ is large for both initial $u$ and $d$ quarks, and $H_1(z)$ of $\Sigma^-$ is large for initial $d$ quark. Then, we find that the polarization of the produced $\Sigma^+$ is large for proton target, that of $\Sigma^0$ is large for both proton and neutron targets, and that of $\Sigma^-$ large for neutron target. Therefore, when one tries to extract the transversity distribution $h_1(x)$ of nucleon from the hyperon polarization in the SIDIS process $lp^+ \rightarrow lH^+X$ (where $p$ is proton or neutron), measuring the polarizations of the $\Sigma^+$ and $\Sigma^0$ hyperons is more efficient than $\Lambda$ for the proton target, and measuring the polarizations of the $\Sigma^-$ and $\Sigma^0$ hyperons is more efficient than $\Lambda$ for the neutron target. When we consider that the polarization of $\Sigma^0$ is measured through the daughter $\Lambda$ polarization, measuring the polarization of $\Sigma^+$ is most efficient for the proton target, and $\Sigma^-$ for the neutron target.
2 SU(6) Wavefunction

For the octet baryons, we have the following relations for fragmentation functions from the SU(6) wavefunctions given in Appendix A [13, 14]:

\[
D(u \rightarrow p) = 2 \left( \frac{3}{4}D^s + \frac{1}{4}D^a \right), \quad D(d \rightarrow p) = D^a. \tag{9}
\]

\[
D(u \rightarrow n) = D^a, \quad D(d \rightarrow n) = 2 \left( \frac{3}{4}D^s + \frac{1}{4}D^a \right). \tag{10}
\]

\[
D(u \rightarrow \Lambda) = \frac{1}{4}D^s + \frac{3}{4}D^a, \quad D(d \rightarrow \Lambda) = \frac{1}{4}D^s + \frac{3}{4}D^a, \quad D(s \rightarrow \Lambda) = D^s. \tag{11}
\]

\[
D(u \rightarrow \Sigma^+) = 2 \left( \frac{3}{4}D^s + \frac{1}{4}D^a \right), \quad D(s \rightarrow \Sigma^+) = D^a. \tag{12}
\]

\[
D(u \rightarrow \Sigma^0) = \frac{3}{4}D^s + \frac{1}{4}D^a, \quad D(d \rightarrow \Sigma^0) = \frac{3}{4}D^s + \frac{1}{4}D^a, \quad D(s \rightarrow \Sigma^0) = D^a. \tag{13}
\]

\[
D(d \rightarrow \Sigma^-) = 2 \left( \frac{3}{4}D^s + \frac{1}{4}D^a \right), \quad D(s \rightarrow \Sigma^-) = D^a. \tag{14}
\]

\[
D(u \rightarrow \Xi^0) = D^a, \quad D(s \rightarrow \Xi^0) = 2 \left( \frac{3}{4}D^s + \frac{1}{4}D^a \right). \tag{15}
\]

\[
D(d \rightarrow \Xi^-) = D^a, \quad D(s \rightarrow \Xi^-) = 2 \left( \frac{3}{4}D^s + \frac{1}{4}D^a \right). \tag{16}
\]

In the above the superscript \( s(a) \) denotes that the accompanying diquark is a scalar (axial-vector) diquark. The same relations are also satisfied for the distribution functions.

In order to illustrate the meaning of Eqs. (9) to (16), let us write the relations for proton and \( \Lambda \) in detail. Eq. (9) shows the following relations of distribution functions \( f_1 \) and fragmentation functions \( D_1 \) for proton:

\[
f_{1u} = \frac{3}{2}f^s_1 + \frac{1}{2}f^a_1, \quad f_{1d} = f^a_1, \tag{17}
\]

\[
D_1^{u \rightarrow p} = \frac{3}{2}D^s_1 + \frac{1}{2}D^a_1, \quad D_1^{d \rightarrow p} = D^a_1, \tag{18}
\]

where the subscript \( u \) and \( d \) denote up and down quarks, respectively, and Eq. (11) shows the following relations for \( \Lambda \):

\[
f_{1u} = \frac{1}{4}f^s_1 + \frac{3}{4}f^a_1, \quad f_{1d} = \frac{1}{4}f^s_1 + \frac{3}{4}f^a_1, \quad f_{1(s)} = f^s_1, \tag{19}
\]

\[
D_1^{u \rightarrow \Lambda} = \frac{1}{4}D^s_1 + \frac{3}{4}D^a_1, \quad D_1^{d \rightarrow \Lambda} = \frac{1}{4}D^s_1 + \frac{3}{4}D^a_1, \quad D_1^{(s) \rightarrow \Lambda} = D^s_1, \tag{20}
\]

where \( (s) \) denotes the strange quark.
3 Spectator Model

3.1 Distribution Functions of Nucleon

In this paper we use the spectator model of Jakob et al. [15], which takes the following baryon-quark-diquark vertex for the scalar \((s)\) and axial-vector \((a)\) diquark,

\[ \Upsilon^s = 1 g_s(p^2) \quad \text{and} \quad \Upsilon^{a\mu} = \frac{g_a(p^2)}{\sqrt{3}} \gamma_5 \left( \gamma^\mu + \frac{P^\mu}{M} \right), \tag{21} \]

with the following form factors:

\[ g_R(p^2) = N_R \frac{p^2 - m^2}{|p^2 - \Lambda^2|^2}, \tag{22} \]

where we take \(\alpha = 2\) for the parameter \(\alpha\) in Ref. [15]. In Eq. (21) \(P^\mu\) and \(M\) are the momentum and mass of the baryon and in Eq. (22) \(\Lambda\) is a parameter for the cut off and \(p\) is the quark momentum which satisfies

\[-p^2(x, p^2_\perp) = \frac{p^2_\perp}{1-x} + \frac{x}{1-x} M_R^2 - xM^2 = \frac{p^2_\perp + \lambda^2_R(x)}{1-x} - \Lambda^2 \]

where \(\lambda^2_R(x)\) is given by

\[ \lambda^2_R(x) = (1-x)\Lambda^2 + xM_R^2 - x(1-x)M^2, \tag{23} \]

in which \(R = s\) or \(a\).

The distribution functions \(f_1\) and \(h_1\) are given by [15]

\[ f_1^R(x, p^2_\perp) = \frac{N^2_R(1-x)^3}{48\pi^2} (xM + m)^2 + \frac{1}{4} \lambda^2_R(x) \],

\[ h_1^R(x, p^2_\perp) = a_R \frac{N^2_R(1-x)^3}{48\pi^2} (xM + m)^2 \left( \lambda^2_R(x) \right)^3 \],

where \(M, m\) and \(M_R\) the baryon, quark and diquark mass, respectively, and

\[ f_1^R(x) = \frac{N^2_R(1-x)^3}{48\pi^2} (xM + m)^2 + \frac{1}{2} \lambda^2_R(x) \left( \lambda^2_R(x) \right)^3 \],

\[ h_1^R(x) = a_R \frac{N^2_R(1-x)^3}{48\pi^2} (xM + m)^2 \left( \lambda^2_R(x) \right)^3 \],

where \(a_R\) for scalar and axial-vector diquark models are given by [15]

\[ a_s = 1, \quad a_a = -\frac{1}{3}. \tag{28} \]

The normalization constant \(N_R\) is fixed by

\[ \int_0^1 dx \ f_1^R(x) = 1. \tag{29} \]
From the SU(6) wavefunctions given in Appendix A, we get the following relations for proton:

\[ f_{1u} = \frac{3}{2} f_1^s + \frac{1}{2} f_1^a, \quad f_{1d} = f_1^a. \] (30)

and for neutron:

\[ f_{1u} = f_1^a, \quad f_{1d} = \frac{3}{2} f_1^s + \frac{1}{2} f_1^a. \] (31)

The distribution function \( h_1 \) also satisfy the same relations as the above (30) and (31).

We use \( \Lambda = 0.5, m = 0.36, M_s = 0.6, M_a = 0.8 \) and \( M = 0.94 \) (nucleon mass) in the unit of GeV, when we calculate the distribution and fragmentation functions of the proton and neutron. We use \( \Lambda = 0.5, m = 0.36, M_s = 0.8, M_a = 1.0 \) when we calculate the fragmentation functions of \( \Lambda \) and \( \Sigma \) hyperons, and \( \Lambda = 0.6, m = 0.36, M_s = 1.0, M_a = 1.2 \) when we calculate the fragmentation functions of \( \Xi \) hyperons. For the baryon mass \( M \), we use 1.12 for \( \Lambda \), 1.19 for \( \Sigma^\pm \) and \( \Sigma^0 \), and 1.32 for \( \Xi^0 \) and \( \Xi^- \). The distribution functions of proton obtained by using the formulas in this section are presented in Fig. 1.

### 3.2 Fragmentation Functions of Hyperons

By applying to the above distribution functions \( f_1 \) and \( h_1 \) the Gribov-Lipatov reciprocity relation [15]

\[ \Delta^{[r]}(z, k_T) = \frac{1}{2z} \Phi^{[r']}\left(\frac{1}{z}, k_T\right) = \frac{1}{2z} \Phi^{[r']}\left(\frac{1}{z}, -\frac{k_T}{z}\right), \] (32)
Figure 2: (a) $D_1^{u\rightarrow \Lambda}(z)$ and (b) $H_1^{u\rightarrow \Lambda}(z)$. We note that $D_1^{u\rightarrow \Lambda}(z) = D_1^{d\rightarrow \Lambda}(z)$ and $H_1^{u\rightarrow \Lambda}(z) = H_1^{d\rightarrow \Lambda}(z)$.

Figure 3: (a) $D_1^{u\rightarrow \Sigma^+}(z)$ and (b) $H_1^{u\rightarrow \Sigma^+}(z)$. 
the following fragmentation functions $D_1$ and $H_1$ are obtained \cite{15}:

\[ D_1^R(z, z^2k^2_{\perp}) = \frac{N_R^2(1-z)^3}{16\pi^3 z^4} \frac{(M + m)^2 + k^2_{\perp}}{\left(k^2_{\perp} + \lambda^2_R(\frac{1}{z})\right)^4}, \quad (33) \]

\[ H_1^R(z, z^2k^2_{\perp}) = a_R \frac{N_R^2(1-z)^3}{16\pi^3 z^4} \frac{(M + m)^2}{\left(k^2_{\perp} + \lambda^2_R(\frac{1}{z})\right)^4}, \quad (34) \]

where

\[ \lambda^2_R(\frac{1}{z}) = (1 - \frac{1}{z})\Lambda^2 + \frac{1}{z} M^2_R - \frac{1}{z}(1 - \frac{1}{z}) M^2, \quad (35) \]

with $M$, $m$ and $M_R$ are the hyperon, quark and diquark mass, respectively, and

\[ D_1^R(z) = \frac{N_R^2 z^2(1-z)^3}{48\pi^2} \frac{(M + mz)^2 + \frac{1}{4}z^2\lambda^2_R(\frac{1}{z})}{\left(z^2\lambda^2_R(\frac{1}{z})\right)^3}, \quad (36) \]

\[ H_1^R(z) = a_R \frac{N_R^2 z^2(1-z)^3}{48\pi^2} \frac{(M + mz)^2}{\left(z^2\lambda^2_R(\frac{1}{z})\right)^3}. \quad (37) \]

Here, we fix the normalization constant $N_R$ by

\[ \int_0^1 dz \ D_1^R(z) = 1. \quad (39) \]

We normalized $D_1^R(z)$ in this way by considering that the number of hadrons produced by a quark is determined by what is the number of that quark inside the produced hadron. The normalization constant $N_R$ is cancelled in the ratio of Eq. (2) since it is an overall multiplicative constant.

From the SU(6) wavefunctions written in Appendix A, we get the relations for $D_1$ given in Eq. (20) for $\Lambda$ hyperon, and the same relations are also satisfied for $H_1$. The relations for other octet baryons can be obtained from Eqs. (9) to (16). For example, Eq. (12) gives the relations for $\Sigma^+$:

\[ D_1^{u\to\Sigma^+} = 2 \left(\frac{3}{4} D_1^u + \frac{1}{4} D_1^d\right), \quad D_1^{d\to\Sigma^+} = 0, \quad D_1^{(s)\to\Sigma^+} = D_1^s, \quad (40) \]

\[ H_1^{u\to\Sigma^+} = 2 \left(\frac{3}{4} H_1^u + \frac{1}{4} H_1^d\right), \quad H_1^{d\to\Sigma^+} = 0, \quad H_1^{(s)\to\Sigma^+} = H_1^s. \quad (41) \]
Figure 4: (a) $F(x)$ of $\Lambda$ (lower line) and $F(x)$ of $\Sigma^+$ (upper line) for the proton target. (b) $G(z)$ of $\Lambda$ (lower line) and $G(z)$ of $\Sigma^+$ (upper line). $F(x)$ and $G(z)$ are defined in Eq. (42).

Figure 5: The $y$-dependent front factor $f(y) = (2(1 - y))/(1 + (1 - y)^2)$ in (42).

| $h$ | $u$ | $d$ | $s$ | $F^N(x) \ G^h(z)$ |
|-----|-----|-----|-----|------------------|
| $\Lambda$ | $(\frac{1}{4}s + \frac{3}{4}a)$ | $(\frac{1}{4}s + \frac{3}{4}a)$ | $s$ | $(\frac{h_{1u}}{f_{1u} + f_{1d}})^N \ (\frac{H_{1u}}{D_{1u}})^{\Lambda}$ |
| $\Sigma^+$ | $(2)(\frac{3}{4}s + \frac{1}{4}a)$ | $0$ | $a$ | $(\frac{h_{1d}}{f_{1u} + f_{1d}})^N \ (\frac{H_{1u}}{D_{1u}})^{\Sigma^+}$ |
| $\Sigma^0$ | $(\frac{3}{4}s + \frac{1}{4}a)$ | $(\frac{3}{4}s + \frac{1}{4}a)$ | $a$ | $(\frac{h_{1s} + h_{1d}}{4f_{1u} + f_{1d}})^N \ (\frac{H_{1u}}{D_{1u}})^{\Sigma^0}$ |
| $\Sigma^-$ | $0$ | $(2)(\frac{3}{4}s + \frac{1}{4}a)$ | $a$ | $(\frac{h_{1u}}{f_{1d}})^N \ (\frac{H_{1u}}{D_{1d}})^{\Sigma^-}$ |
| $\Xi^0$ | $a$ | $0$ | $(2)(\frac{3}{4}s + \frac{1}{4}a)$ | $(\frac{h_{1u}}{f_{1d}})^N \ (\frac{H_{1u}}{D_{1d}})^{\Xi^0}$ |
| $\Xi^-$ | $0$ | $a$ | $(2)(\frac{3}{4}s + \frac{1}{4}a)$ | $(\frac{h_{1u}}{f_{1d}})^N \ (\frac{H_{1u}}{D_{1d}})^{\Xi^-}$ |

Table 1: The scalar and axial-vector diquark contents in the octet baryons ($h$) and the function $F(x)G(z)$ in (42) for the nucleon ($N$) target.
Figure 6: $F(x)G(z)$ in (42) for the hyperons produced from the proton target.
Figure 7: $F(x)G(z)$ in (42) for the hyperons produced from the neutron target.
4 Polarizations of Hyperons

Using the distribution functions of proton and the fragmentation functions of hyperons given in section 3, we calculate the polarizations given in (2) of the produced hyperons in the SIDIS process of unpolarized lepton beam on transversely polarized proton target.

For the Λ hyperon, we use Λ = 0.5, m = m_u = m_d = 0.36, M_s = 0.8, M_a = 1.0 and M = 1.116 (Λ hyperon mass) in the unit of GeV. Since D_1^{u→Λ} = D_1^{d→Λ} and H_1^{u→Λ} = H_1^{d→Λ} from (11), we find that Eq. (2) can be written as

\[ P_N = \frac{2(1-y)}{1 + (1-y)^2} \frac{4h_{1u}(x) + h_{1d}(x)}{4f_{1u}(x) + f_{1d}(x)} \frac{H_{1u}(z)}{D_{1u}(z)} \equiv \frac{2(1-y)}{1 + (1-y)^2} F(x) G(z) . \] (42)

Therefore, the x and z dependent part is factorized as a product of F(x) and G(z) for the Λ production, and this factorization also happens for other five hyperons. F(x) and G(z) for all hyperons are presented in the last column in Table 1. However, this factorization does not happen for the proton and neutron productions as presented in Appendix B. The fragmentation functions are obtained from the formulas given in section 3.2. D_1^{u→Λ} and H_1^{u→Λ} are presented in Fig. 2, and D_1^{u→Σ^+} and H_1^{u→Σ^+} in Fig. 3. For the Λ (Σ^+) production from the proton target, F(x) and G(z) are presented as the lower (upper) line in Fig. 4. The y-dependent front factor f(y) = (2(1−y))/(1 + (1−y)^2) in Eq. (42) is drawn in Fig. 5. The x and z dependent part F(x) G(z) in Eq. (42) for the Λ (Σ^+) production from the proton target is presented in Fig. 6(a) (in Fig. 6(b)), and for other hyperons in Fig. 6(c)-(f). We find in Fig. 6 that the polarization of the Λ hyperon is much smaller than the polarizations of Σ^+ and Σ^0, the reason for which is that H_1^{u→Λ} is much smaller than H_1^{u→Σ^+} and H_1^{u→Σ^0}. (Figs. 2(b) and 3(b) show that H_1^{u→Λ} is much smaller than H_1^{u→Σ^+}.) Therefore, when one tries to extract the transversity distribution h_1(x) from the SIDIS process l^+p \rightarrow l\Sigma^0 X on the proton target, investigating by measuring the polarizations of Σ^+ and Σ^0 is more efficient than the polarizations of Λ. The x and z dependent parts F(x) G(z) in the case of the neutron target are presented in Fig. 7, which shows that the polarizations of Σ^- and Σ^0 are much larger than that of Λ. When we consider that the polarization of Σ^0 is measured through the daughter Λ polarization, measuring the polarization of Σ^+ is most efficient for the proton target, and Σ^- for the neutron target.

5 Conclusion

We calculated with the SU(6) wavefunctions of the octet baryons and the spectator model the transverse polarizations of the produced hyperons in the SIDIS of unpolarized lepton beam on transversely polarized nucleon target, since these polarizations provide a potential method for extracting the transversity distribution h_1(x) of the
nucleon. We find that the SU(6) wavefunctions imply that $H_1(z)$ of the $\Lambda$ hyperon is very small for both initial $u$ and $d$ quarks, and therefore the polarization of the produced $\Lambda$ is very small for both transversely polarized proton and neutron targets. On the other hand, $H_1(z)$ of $\Sigma^+$ is large for initial $u$ quark and $H_1(z)$ of $\Sigma^0$ is large for both initial $u$ and $d$ quarks, and $H_1(z)$ of $\Sigma^-$ is large for initial $d$ quark. Therefore, when one tries to extract the transversity distribution $h_1(x)$ from the SIDIS $lp^\uparrow \rightarrow lH^\uparrow X$, it is expected that the polarizations of the $\Sigma^+$ and $\Sigma^0$ hyperons are large for proton target and the polarizations of the $\Sigma^-$ and $\Sigma^0$ hyperons are large for neutron target. When we consider that the polarization of $\Sigma^0$ is measured through the daughter $\Lambda$ polarization, for extracting $h_1(x)$ from the SIDIS process $lp^\uparrow \rightarrow lH^\uparrow X$ it is most efficient to work by measuring the polarization of $\Sigma^+$ for the proton target, and the polarization of $\Sigma^-$ for the neutron target.

Since the discovery of large $\Lambda$ polarization in $p + \text{Be} \rightarrow \Lambda^0 + X$ at 300 GeV [16], the polarizations of other hyperons were measured in $p + \text{Be} \rightarrow \text{Hyperon} + X$ at 400 GeV: $\Sigma^+$ [17], $\Sigma^-$ [18], $\Xi^0$ [19] and $\Xi^-$ [20]. The polarization of $\Sigma^0$ was measured in $p + \text{Be} \rightarrow \Sigma^0 + X$ at 28.5 GeV through the daughter $\Lambda$ polarization [21]. The polarization was also measured for the $\Lambda$ hyperons which were produced by unpolarized protons incident on hydrogen and deuterium targets [22], and a lot of experiments measuring the hyperon polarizations have been performed in order to explore the nature of this single-spin asymmetry phenomenon. Therefore, it is expected that it is possible to perform experiments which measure the polarizations of all the six hyperons in the SIDIS process $lp^\uparrow \rightarrow lH^\uparrow X$. The results of these measurements would provide the information of the transversity of nucleon, and also of the hyperon structures.

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Appendix A: SU(6) wavefunctions of octet baryons

\[ p^\dagger = \frac{1}{\sqrt{2}} (ud)_{0,0} u^\dagger \]
\[ + \frac{1}{\sqrt{3}} \left( \sqrt{\frac{2}{3}} (uu)_{1,1} d^\dagger - \sqrt{\frac{1}{3}} (uu)_{1,0} d^\dagger \right) - \frac{1}{\sqrt{6}} \left( \sqrt{\frac{2}{3}} (ud)_{1,1} u^\dagger - \sqrt{\frac{1}{3}} (ud)_{1,0} u^\dagger \right) \]  
(43)

\[ n^\dagger = \frac{1}{\sqrt{2}} (ud)_{0,0} d^\dagger \]
\[ - \frac{1}{\sqrt{3}} \left( \sqrt{\frac{2}{3}} (dd)_{1,1} u^\dagger - \sqrt{\frac{1}{3}} (dd)_{1,0} u^\dagger \right) + \frac{1}{\sqrt{6}} \left( \sqrt{\frac{2}{3}} (ud)_{1,1} d^\dagger - \sqrt{\frac{1}{3}} (ud)_{1,0} d^\dagger \right) \]  
(44)

\[ \Lambda^\dagger = \frac{1}{\sqrt{3}} (ud)_{0,0} s^\dagger + \frac{1}{\sqrt{12}} (us)_{0,0} d^\dagger - \frac{1}{\sqrt{12}} (ds)_{0,0} u^\dagger \]
\[ + \frac{1}{2} \left( \sqrt{\frac{2}{3}} (us)_{1,1} d^\dagger - \sqrt{\frac{1}{3}} (us)_{1,0} d^\dagger \right) - \frac{1}{2} \left( \sqrt{\frac{2}{3}} (ds)_{1,1} u^\dagger - \sqrt{\frac{1}{3}} (ds)_{1,0} u^\dagger \right) \]  
(45)

\[ \Sigma^{\dagger \dagger} = \frac{1}{\sqrt{2}} (us)_{0,0} u^\dagger \]
\[ + \frac{1}{\sqrt{3}} \left( \sqrt{\frac{2}{3}} (uu)_{1,1} s^\dagger - \sqrt{\frac{1}{3}} (uu)_{1,0} s^\dagger \right) - \frac{1}{\sqrt{6}} \left( \sqrt{\frac{2}{3}} (us)_{1,1} u^\dagger - \sqrt{\frac{1}{3}} (us)_{1,0} u^\dagger \right) \]  
(46)

\[ \Sigma^0 \dagger = \frac{1}{2} (us)_{0,0} d^\dagger + \frac{1}{2} (ds)_{0,0} u^\dagger \]
\[ + \frac{1}{\sqrt{3}} \left( \sqrt{\frac{2}{3}} (ud)_{1,1} s^\dagger - \sqrt{\frac{1}{3}} (ud)_{1,0} s^\dagger \right) \]
\[ - \frac{1}{\sqrt{12}} \left( \sqrt{\frac{2}{3}} (us)_{1,1} d^\dagger - \sqrt{\frac{1}{3}} (us)_{1,0} d^\dagger \right) - \frac{1}{\sqrt{12}} \left( \sqrt{\frac{2}{3}} (ds)_{1,1} u^\dagger - \sqrt{\frac{1}{3}} (ds)_{1,0} u^\dagger \right) \]  
(47)

\[ \Sigma^{-\dagger} = \frac{1}{\sqrt{2}} (ds)_{0,0} d^\dagger \]
\[ + \frac{1}{\sqrt{3}} \left( \sqrt{\frac{2}{3}} (dd)_{1,1} s^\dagger - \sqrt{\frac{1}{3}} (dd)_{1,0} s^\dagger \right) - \frac{1}{\sqrt{6}} \left( \sqrt{\frac{2}{3}} (ds)_{1,1} d^\dagger - \sqrt{\frac{1}{3}} (ds)_{1,0} d^\dagger \right) \]  
(48)

\[ \Xi^0 \dagger = \frac{1}{\sqrt{2}} (us)_{0,0} s^\dagger \]
\[ - \frac{1}{\sqrt{3}} \left( \sqrt{\frac{2}{3}} (ss)_{1,1} u^\dagger - \sqrt{\frac{1}{3}} (ss)_{1,0} u^\dagger \right) + \frac{1}{\sqrt{6}} \left( \sqrt{\frac{2}{3}} (us)_{1,1} s^\dagger - \sqrt{\frac{1}{3}} (us)_{1,0} s^\dagger \right) \]  
(49)

\[ \Xi^{-\dagger} = \frac{1}{\sqrt{2}} (ds)_{0,0} s^\dagger \]
\[ - \frac{1}{\sqrt{3}} \left( \sqrt{\frac{2}{3}} (ss)_{1,1} d^\dagger - \sqrt{\frac{1}{3}} (ss)_{1,0} d^\dagger \right) + \frac{1}{\sqrt{6}} \left( \sqrt{\frac{2}{3}} (ds)_{1,1} s^\dagger - \sqrt{\frac{1}{3}} (ds)_{1,0} s^\dagger \right) \].  
(50)
The $S^z = +\frac{1}{2}$ state of proton is given in (43) and the $S^z = -\frac{1}{2}$ state is given by

$$p^\dagger = \frac{1}{\sqrt{2}}(ud)_{0,0}u^\dagger + \frac{1}{\sqrt{3}}\left(-\sqrt{\frac{2}{3}}(uu)_{1,-1}d^\dagger + \sqrt{\frac{1}{3}}(uu)_{1,0}d^\dagger\right) - \frac{1}{\sqrt{6}}\left(-\sqrt{\frac{2}{3}}(ud)_{1,-1}u^\dagger + \sqrt{\frac{1}{3}}(ud)_{1,0}u^\dagger\right).$$

That is, $(ud)_{0,0}u^\dagger$, $\left(\sqrt{\frac{2}{3}}(uu)_{1,1}d^\dagger - \sqrt{\frac{1}{3}}(uu)_{1,0}d^\dagger\right)$ and $\left(\sqrt{\frac{2}{3}}(ud)_{1,1}u^\dagger - \sqrt{\frac{1}{3}}(ud)_{1,0}u^\dagger\right)$ in (43) are replaced by $(ud)_{0,0}u^\dagger$, $\left(-\sqrt{\frac{2}{3}}(uu)_{1,-1}d^\dagger + \sqrt{\frac{1}{3}}(uu)_{1,0}d^\dagger\right)$ and $\left(-\sqrt{\frac{2}{3}}(ud)_{1,-1}u^\dagger + \sqrt{\frac{1}{3}}(ud)_{1,0}u^\dagger\right)$ in (51). The $S^z = -\frac{1}{2}$ state of other octet baryons can be obtained in the same way.

**Appendix B: Polarizations of produced proton and neutron**

When hyperons are produced from the nucleon target, the $x$ and $z$ dependent part in Eq. (2) for $P_N$ is factorized as $F(x)G(z)$ as shown in Eq. (42). However, when nucleons are produced, this factorization does not happen and we should use Eq. (2) as written in the last column in Table 2. We have the following relations for $D_1$ of proton and neutron:

$$D_1^{u\rightarrow p} = 2\left(\frac{3}{4}D_1^s + \frac{1}{4}D_1^a\right), \quad D_1^{d\rightarrow p} = D_1^a,$$

$$D_1^{u\rightarrow n} = D_1^a, \quad D_1^{d\rightarrow n} = 2\left(\frac{3}{4}D_1^s + \frac{1}{4}D_1^a\right),$$

and the same relations are also satisfied for $H_1$ of proton and neutron.

| $h$ | $u$ | $d$ | $s$ | $F^{N\hbar}(x, z)$ |
|-----|-----|-----|-----|-----------------------|
| $p$ | $(2)(\frac{3}{4}s + \frac{1}{4}a)$ | $a$ | 0 | \(4h_{1s}(x)H_{1s}^{p}(z) + h_{1d}(x)H_{1d}^{p}(z)\) |
| $n$ | $a$ | $(2)(\frac{3}{4}s + \frac{1}{4}a)$ | 0 | \(4h_{1s}(x)D_{1s}^{p}(z) + h_{1d}(x)D_{1d}^{p}(z)\) |

Table 2: The scalar and axial-vector diquark contents in the nucleons ($h$) and the function $F^{N\hbar}(x, z)$ is the $x$ and $z$ dependent part in (2) for the nucleon ($N$) target.
References

[1] V. Barone, A. Drago, and P.G. Ratcliffe, Phys. Rept, 359, 1 (2002).
[2] J. C. Collins, Nucl. Phys. B 396, 161 (1993).
[3] M. Anselmino, M. Boglione, U. D’Alesio, A. Kotzinian, F. Murgia, A. Prokudin and C. Turk, Phys. Rev. D 75, 054032 (2007).
[4] D. W. Sivers, Phys. Rev. D 41, 83 (1990); Phys. Rev. D 43, 261 (1991).
[5] R. A. Kunne et al., “Electroproduction of polarized Lambdas (a proposal for the European Electron Facility),” Italian Phys. Soc. Proc. 44, 401 (1993).
[6] D. Boer and P.J. Mulders, Phys. Rev. D 57, 5780 (1998).
[7] M. Anselmino, Transversity and Lambda polarization, Talk given at Workshop on Future Physics at COMPASS, Geneva, Switzerland, 26-27 Sep 2002, hep-ph/0302008.
[8] K. A. Olive et al. [Particle Data Group Collaboration], Chin. Phys. C 38, 090001 (2014).
[9] T. Negrini [COMPASS Collaboration], AIP Conf. Proc. 1149, 656 (2009).
[10] T. Negrini, CERN-THESIS-2009-221.
[11] C. Schill [COMPASS Collaboration], Fizika B 20, 93 (2011).
[12] J. J. Yang, Phys. Rev. D 65, 094035 (2002).
[13] R. Van Royen and V. F. Weisskopf, Nuovo Cim. A 50, 617 (1967), Erratum: [Nuovo Cim. A 51, 583 (1967)].
[14] R. Jakob, P. Kroll, M. Schurmann and W. Schweiger, Z. Phys. A 347, 109 (1993).
[15] R. Jakob, P.J. Mulders, and J. Rodrigues, Nucl. Phys. A 626, 937 (1997).
[16] G. Bunce et al., Phys. Rev. Lett. 36, 1113 (1976).
[17] C. Wilkinson et al., Phys. Rev. Lett. 46, 803 (1981).
[18] L. L. Deck et al., Phys. Rev. D 28, 1 (1983).
[19] K. J. Heller et al., Phys. Rev. Lett. 51, 2025 (1983).
[20] R. Rameika et al., Phys. Rev. D 33, 3172 (1986).
[21] E. C. Dukes et al., Phys. Lett. B 193, 135 (1987).
[22] K. Raychaudhuri et al., Phys. Lett. B 90, 319 (1980).