Optical measurement of the instantaneous wavy surface structure on a water plane jet

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Abstract. Laser beam refraction technique for measurement of free surface waves on a high-speed water jet has been developed in the present work. The new method employs two pulse laser diodes and the single light-spot detector. Through detecting the two dimensional trajectory of laser beams refracted at free surface, the local slope-angle fluctuation at two measuring locations 1.2 mm apart on a jet free surface is evaluated. The experiments are conducted for several locations along the jet center axis within the range of average velocity up to 5 m/s. The wave speed is evaluated from dominant time lag of cross-correlation coefficient for individual waves, which are discriminated by applying the zero-up-crossing method to the slope-angle time series data. The shape of waves is also calculated by integrating the free-surface slope angle data numerically. The ensemble averaged wavelength increases with the distance from nozzle exit almost monotonically. On the other hand, the mean wave height first increase with distance, reaches a maximum and then decreases. The maximum value of wave height is less than 0.23 mm for the tested velocity range. The steepness of free surface wave, that is the ratio of wave height and wavelength, takes a maximum at 20 ~ 30 mm downstream nozzle exit. This suggests that the wave breaking occurs on the jet free surface for higher velocity case.

1. INTRODUCTION

The occurrence of waves on a liquid free surface has been known to affect the absorbing and diffusion phenomena through the interface between different materials and phases. Then, many researches have been performed to measure the free surface waves during the last several ten years. In the recent proposals for neutron source facilities (Katsuta et al., 1996), the evaluation of the wave height on the high-speed (~15 m/s) liquid lithium jet becomes important design issue because the occurrence of free-surface fluctuation may induce no only an increase in vaporization and dispersion of lithium into high vacuum environment but also an insufficient absorption of high deuteron beam energy (40 MeV, 125 mA x 2).

Several measurements have been conducted on jet free-surface waves on lithium jet (Horiike et al., 2003; Kondo et al., 2006) and water as stimulant of lithium (Nakamura et al., 1998; Itoh et al., 2004); however only restricted data have been obtained because of the difficulties in measuring waves on high-speed liquid surface. Non-intrusive, high-response methods have been strongly preferred for such measurement.

Then, the aim of present study is to develop the measurement technique for the waves on a high-speed liquid jet. Present optical technique makes it possible to measure the local slope angle...
fluctuations at adjoined two locations on a wavy surface. High-speed light spot detector is used to
detect the trajectories of laser beams refracted at jet free surface. The wave speed is derived from
the dominant time lag of cross correlation coefficients between slope-angle fluctuation data at
two measuring location. The spatial shape of waves is also reconstructed by integrating the free-
surface slope angle data.
Since this study focuses the development of measurement technique for liquid free surface, the
experiments is concerned with a lower velocity case (≤ 5 m/s) rather than of proposed beam
target flow. However, the present method will be applied for higher velocity case with adjusting
the geometry of two lasers (that is, spatial interval between two lasers, and their diameters) and
with improving the time response of the laser blinking. Furthermore, the measured data in the
present work will be useful in verifying the wave generation mechanism, such as free surface
instability theory, even if the data obtained from water jet in simulating lithium flow (Kondo et
al., 2005).

2. EXPERIMENT

2.1 Experimental Facility
Figure 1 shows a schematic view of test section, which is made of transparent acrylic resin. The
facility uses water as the working fluid at room temperature and atmospheric pressure. Water is
used since its kinematic viscosity is not too much different from those of lithium at the operating
temperature (523K) of the proposed beam target (Itoh et al., 1999). The nozzle-exit height is 10
mm and the width is 100 mm. A plane water jet is generated by a two-dimensional convergent
nozzle and flows along a horizontal flat wall. The jet width is fixed by sidewalls and the jet free
surface is open to atmosphere to allow optical measurement and visual observation.

The optical arrangement for measurement of free surface waves is also shown in Fig. 1. Two
laser beams are shot vertically on the water surface. We employ pulse laser diodes (Global laser,
Premier LC, 655 nm, 1 mW) for the light sources. This diode is possible to blink by 300 kHz.
The control of blinking of lasers will be mentioned in the next section with a topic of detection of
two laser’s loci by using an optical sensor. A focusing lens (150 mm focal length) and cylindrical
collimator (1 mm dia., 3 mm length) reduces the beam diameter to be ≤ 24 µm at the water
surface. The interval of two lasers \( L \) was adjusted to be 1.2 mm. The control signals for blinking
of lasers were synchronized carefully with the data acquisition of output signals of the optical
sensor. The lasers were alternately blinked at the frequency of 40kHz, and the output signals
including laser control signals were sampled at the rate of 800 kHz in the present experiment.

The experiments are conducted for cross-sectional average velocity at the nozzle exit \( U = 3 \sim 5 
\) m/s. Optical measurement of free surface is achieved for the distance \( x = 80 \) mm from nozzle exit
as the optical system and sensor are moved along the center axis of the test section by using fine
motion stages.

Fig. 1 Schematic view of test section and optical system.
2.2 Measurement of Free-surface Slope Angle

Present approach estimates the fluctuation of free surface slope angle from measurement of refracted laser beam loci. The relationship between the displacement of the beam spot on the sensor and the local slope angle $\theta$ of free surface is shown in Fig. 2.

The beam is refracted by the local slope of the water surface and passes through the water jet, the transparent back wall and air before hitting the optical sensor (Hamamatsu Photonics, S1881). The displacement $r_x$ in streamwise $x$-direction is represented by local slope angle

$$r_x = D_w \tan(\theta_w - \theta_a) + D_b \tan(\theta_b) - D_a \tan(\theta_a), \quad (1)$$

where $D_w$ and $D_b$ are thickness of jet and transparent back wall (10 mm), and $D_a$ is distance from backwall to the sensor (20 mm). $\theta_w$, $\theta_b$ and $\theta_a$ are the angle of refracted beam at each interface, which is obeyed the Snell’s refraction law,

$$\frac{\sin \theta_w}{\sin \theta_a} = \frac{n_w}{n_a}, \quad \frac{\sin(\theta_w - \theta_a)}{\sin \theta_b} = \frac{n_b}{n_a}, \quad \frac{\sin \theta_b}{\sin \theta_a} = \frac{n_a}{n_b}, \quad (2)$$

where $n_w$, $n_a$ and $n_b$ are refractive indexes of water, acrylic resin and air respectively.

![Fig. 2 Relation between free surface slope angle and displacement on optical sensor.](image)

The instantaneous free surface slope angle $\theta_x$ is calculated by substituting the detected displacement $r_x$ into eqs. (1) and (2). Although the solution $\theta_x$ of eqs. (1) and (2) can be obtained by using numerical iteration such as bisection method, a lot of time will be needed for data processing. Then, preliminary evaluated relation between $r_x$ and $\theta_x$ (least-square fitted with 5th order algebraic equation) are used for the instantaneous calculation of $\theta_x$ in the present work.

The theoretical curve calculated from eqs. (1) and (2) is shown in Fig. 3 in comparison with the calibration data. The experimental slope-angle data is obtained by inclining the thin cover glass on the filled water surface in the present test section. The deviation of theoretical curve from the calibration data is less than $\pm 0.02$ rad. We also checked the fluctuation in the jet thickness $D_w$ affects only little the measured value of $\theta_x$. For example, a 10% (= 1 mm) increase in the jet thickness (that is quadruple as large as the typical mean amplitudes of waves) would shift the theoretical curve by only 0.01 rad for present experimental condition. Then, the overall uncertainty in this measurement is evaluated to be less than $\pm 0.03$ rad. Moreover, the maximum
slope angle measurable with this method is determined by the thickness of light path, the refraction indexes of materials and the size of the optical sensor. It is $\pm 0.80$ rad for the present geometry.

![Graph showing the relationship between displacement on optical sensor and free-surface slope angle.](image)

**Fig. 3** Relation between displacement on optical sensor and free-surface slope angle.

The optical sensor S1881 consists of single p-i-n photo diode and can detect the two dimensional displacement of the beam spot on the sensitive area. As shown in Fig. 4, the beam spot induces photocurrent signals $X_1$ and $X_2$ in streamwise, and $Y_1$ and $Y_2$ in spanwise direction by photovoltaic effect. These signals are amplified by high responsive op-amp and are sampled independently. Then, the instantaneous displacements $r_x$ and $r_y$ is calculated from

$$
  r_x = \frac{L_x}{2} \frac{(X_2 + Y_1) - (X_1 + Y_2)}{X_1 + X_2 + Y_1 + Y_2},
$$

$$
  r_y = \frac{L_y}{2} \frac{(X_2 + Y_1) - (X_1 + Y_2)}{X_1 + X_2 + Y_1 + Y_2}.
$$

(3)

where $L_x = L_y = 26$ mm are the length of the each side of sensitive area (including non-active area) of S1881.

![Diagram of optical sensor, S1881.](image)

**Fig. 4** Output signals of optical sensor, S1881.

The present single-photodiode sensor is suitable for a measurement of fluctuation on high-speed jet free surface since its maximum response frequency is 300 kHz for continuous beam. However, if more than two beams are hit on the sensitive area at the same time, it only detects the median
point of each beam because of the overlap of induced photocurrents. Therefore, we have the two laser-diodes switched by turns in order to prevent the two beams from existing on sensitive area simultaneously. Although this switching operation fragments the obtained cross correlation coefficient, the dominant time lag can be determined by examining appropriate blinking frequency and spatial interval of two lasers.

2.3 Reconstruction of Wave Shape

The cross correlation coefficient $R(\tau)$ is represented by the slope angle fluctuation $\theta_{x1}$ and $\theta_{x2}$,

$$R(\tau) = \frac{\theta_{x1}(t)\theta_{x2}(t+\tau)}{\sqrt{\theta_{x1}^2 \theta_{x2}^2}}, \quad (4)$$

where $\tau$ is time lag at spatial interval of measuring location. The wave speed $c_i$ is evaluated from the dominant time lag $\tau_a$, corresponding with the maximum cross-correlation coefficient, as $c_i = L/\tau_a$. The waves with different wavelengths are considered to propagate with different wave speed. Then, in the present work, $\tau_a$ is calculated for individual waves, which are separated from time series data of $\theta_{x1}$ by using zero-up-crossing method (Bazargan et al., 2007).

In what follows, we explain the reconstruction of spatial wave shape from measured slope angle data. When the shape of free surface is denoted as $z = \eta_x(x)$, the local slope angle $\theta_x$ in streamwise direction can be written as

$$\tan \theta_x = \frac{\partial \eta_x}{\partial x} = \frac{\partial \eta_x}{\partial t} \frac{\partial t}{\partial x}, \quad (5)$$

where $\partial x/\partial t$ is the speed of free surface wave which passes through the measuring location. If waves are assumed to be traveling with frozen profiles across the measurement location, the wave shape $\eta_x$ is obtained by integrating eq. (5) with respect to time as

$$\eta_x = \int_{\text{wave}} \left( \frac{\partial x}{\partial t} \right) \tan \theta_x \, dt = \int_{\text{wave}} c_x \cdot \tan \theta_x \, dt. \quad (6)$$

Thus, $\eta_x$ is obtained from numerical integration of eq. (6) by substituting the evaluated wave speed $c_x$ for individual waves. The wave height $h$ is calculated from the deviation between maximum and minimum value of $\eta_x$. The wavelength $\lambda$ is also calculated as the product of $c_x$ and wave period, which is separated from time series data of $\theta_x$ by using zero-up-crossing method.

3. RESULTS AND DISCUSSIONS

3.1 Typical Slope Angle Data

Typical data of displacement $r_x$ of beam spots are compared with the control signals in Fig. 5. While control signal takes positive value, laser 1 (LD1) is switched on (and laser 2 (LD2) is switched off). The free surface fluctuation at two measuring location is well captured, alternatively. However, it is found that the transitional signals are included in the moment that the laser interchanges. The transitional motion of temporal signals is clearly quicker than the wave period. And such signals were also observed in the preliminary test when still water (no wave) surface were measured. Therefore, these temporal outputs are considered to be related to change of beam intensity during the switching operation. Then, only one signal at the intermediate time while each laser is switched on is employed as responsible data for slope angle analysis. The
responsible data are indicated by red or blue symbols in Fig. 5.

![Graph showing displacement on optical sensor (mm)](image1)

**Fig. 5** Typical streamwise displacement data obtained by two lasers.

The traces of streamwise slope angle $\theta_1$ and $\theta_2$ are shown in Fig. 6. Although the deletion of transitional data restricts the practical sampling rate to 40 kHz, meaningful data are obtained at two measuring location. Because of laser switching operation, the time of data acquisition of $\theta_{x2}$ lags 0.025 ms behind that of $\theta_{x1}$.

![Graph showing control voltage (V) vs time (ms)](image2)

![Graph showing fluctuation of free-surface slope angle at two measuring location](image3)

**Fig. 6** Fluctuation of free-surface slope angle at two measuring location.
After dividing time series data of $\theta_i$ into individual waves by using zero-up-crossing method, the cross correlation coefficient eq. (4) is calculated. The wave speed $c_x$ is evaluated from the dominant time lag $\tau_a$ corresponding maximum cross-correlation coefficient $R_{max}$. In the present work, the threshold value of $R_{max}$ is chosen to be $R_{max} \geq 0.90$, and the target wave is assumed to be lost at downstream measuring location when the value of $R_{max}$ is lower than the threshold value. It is confirmed that the average value of wave height and wavelength are almost independent of the threshold value through the calculations by using another threshold value, that is $R_{max} \geq 0.95$ and 0.98, whereas a number of population decreases with an increase in the threshold value.

3.2 Visual Observation of Jet Free Surface

Typical microflash (4 $\mu$s) picture of the jet free surface is shown in Fig. 7 for $U_m = 5$ m/s. Three distinct regions can be identified with respect to the wave development in the flow direction. The jet starts with a smooth region with almost no visible waves. It is followed by the second region, which indicates periodic two-dimensional (2D) waves with dominant wavelength of 0.5 to 1.0 mm and wave amplitudes becoming greater with the distance $x$ from the nozzle exit. Finally, the 2D structure of waves breaks down into less regular 3D-wave patterns.

As the jet velocity in raised, the smooth and 2D-wave regions become shorter and eventually disappear. The linear analysis of the instability modes of the velocity shear layer underneath the free surface implies that shear-mode instabilities are responsible for periodic wave generation on high-speed liquid jet with an initially-laminar shear layer (Itoh et al., 2007). Therefore, the decrease and disappear in the smooth and 2D-wave region may be related to the relaxation distance of shear layer underneath free surface. Furthermore, these structures of jet free surface were observed in the liquid lithium jet (Hassberger, 1983; Kondo et al., 2005) and water jet in simulating beam target flow (Itoh et al., 1999).

![Fig. 7 Typical structures of jet free surface ($U_m = 5$ m/s).](image-url)
3.3 Wavelength and Wave height

The ensemble average of measured wavelength $\lambda_{ave}$ are plotted in Fig. 8 against the distance $x$ from nozzle exit for various values of $U_m$. The mean wavelength increases with $x$ for the tested whole velocity range. The absolute value of wavelength decreases with increase in the jet average velocity $U_m$. The ranges of smooth, 2D and 3D-wave regions, defined from the still photograph as shown in Fig. 7, are indicated in this figure by horizontal arrows. It is found that the wavelength increases quickly in the smooth region, while they increase at a smaller rate in the 2D and 3D-wave region. Furthermore, the data of $\theta_x$ indicated the occurrence of intermittent wave packets in the smooth region.

In order to check the validity of present technique, the wavelength was also measured from the luminance profile of the photograph. Five profiles of streamwise luminance were extracted from the photograph shown in Fig.7, and the wavelength was measured. The dominant wavelength, derived by ensemble averaging the wavelength on the luminance profile, is compared with optical measurement data for average jet velocity of $U_m = 5$m/s. The error bars indicate the ranges from the observed maximum wavelength to the minimum value. It is confirmed that the mean wavelength acquired from optical measurement agrees well with the photograph observation.

![Fig. 8 Variation of mean wavelength with the distance from nozzle exit; comparison between optical measurement and photograph observations.](image)

The mean wave height is plotted against $x$ in Fig. 9. Wave height increases with the distance from nozzle exit and reaches ~ 0.2 mm at the most downstream measuring location $x = 80$ mm. For $U_m \geq 4$ m/s, the wave height takes a peak at $x = 20$ to 30 mm from the nozzle exit. The location of the peak height approaches to the nozzle exit with increase in jet velocity, and it corresponds approximately to the end of 2D wave region. On the other hand, the lowest-velocity case ($U_m = 3$ m/s), without 3D-wave region for the measured whole location, shows no clear
peak.

The wave steepness as defined to be the ratio of the wave height to the wavelength is plotted in Fig. 10. The ensemble average of the ratio $h/\lambda$ for individual waves is indicated. For higher velocity case $U_m \geq 4$ m/s, mean steepness first increases with $x$, reaches a maximum at 2D-wave region and then decreases with further increase in the distance from nozzle exit. The peak is higher and clearer for higher exit velocities. The non-linear water wave theory suggests that the shape of wave collapses when the wave develops sharper and its steepness exceeds a limit value. For example, the breaking limit of Stokes waves is known to be 0.142 (Whitham, 1974). Then, the changes of steepness may indicate that the periodic 2D-wave becomes unstable at the end of growth and breaks down into less regular 3D-waves.

Fig. 9 Growth of mean wave height with the distance from nozzle exit.
4. CONCLUSIONS

Optical measurements of free surface waves on a water jet have been conducted for the jet average velocity of $U_m \leq 5\text{ m/s}$. A new technique employs high-speed optical sensor to detect the displacements of two pulse laser beams refracted at neighboring locations on jet free surface. The shape of free surface wave is reconstructed by integrating slope angle data by substituting wave speed evaluated from cross-correlation of neighboring free-surface fluctuation.

The high-speed photograph demonstrates that the periodic 2D-wave occurs and subsequently grows into 3D-wave pattern on the jet free surface. The mean wavelength measured by using present optical technique agrees with the value evaluated from the luminance profile of photograph. It is found that the wave amplitude increases with the distance from the nozzle exit and takes a maximum at the 2D-wave region for higher velocity case $U_m \geq 4\text{ m/s}$. In the present experimental range, the maximum value of mean wave height is less than 0.23 mm. The wave steepness also indicates a local peak in the 2D-wave region. This suggests that the change of periodic 2D-wave into less regular 3D structure may relate to the wave breaking on the jet free surface.

REFERENCES

Katsuta, H., et al. (1996), “Conceptual design study of IFMIF target facility”, Fusion Technol., 30(3), pp.1152-1160.

Horiike, H., et al. (2003), “Lithium Free Surface Flow Experiment for IFMIF”, Fusion Eng. Des., 66-68, pp.199-204.
Kondo, H., et al. (2006), “Experimental Study of Lithium Free-Surface Flow for IFMIF Target Design,” Fusion Eng. Des., 81(1-7), pp.687-693.

Nakamura, et al. (1998), “Experimental and Analytical Studies on High-Speed Plane Jet along Concave Wall Simulating IFMIF Li Target Flow,” J. Nucl. Mater., 258-263(1), pp.440-445.

Itoh, K. et al. (2004), “Internal-Shear Mode Instabilities on High-Speed Liquid Jet, (II) Experimental Analysis of Curved Target Flow,” J. Nucl. Sci. Technol., 41-8, pp.809-816.

Kondo, H., et al. (2005), “Surface wave on high speed liquid lithium flow for IFMIF,” Fusion Eng. Des., 75-79, pp. 865-869.

Itoh, K. et al. (1999), “Initial Free Surface Instabilities on a High-Speed Water Jet Simulating a Liquid-Metal Target,” Fusion Technol., 36, pp.69-84.

Bazargan, H. et al. (2007), “Simulation of the mean zero-up-crossing wave period using artificial neural networks trained with a simulated annealing algorithm,” J. Marine Sci. Technol. 12(1), pp. 22-33.

Itoh, K. et al. (2007), “Linear Stability Analysis on Free-Surface Liquid Jet with Different Simplification of Velocity Profile,” J. Fluid Sci. Technol., 2(2), pp.417-428.

Hassberger, J. A. (1983), “Stability of the FMIT High Speed Free Surface Liquid Jet Flowing Along a Curved Back Wall,” Proc. 10th Symp. Fusion Eng., Philadelphia, USA, December 5-9, Vol. 2, p. 1849.

Whitham, G. B. (1974). Linear and Nonlinear Waves, John Wiley & Sons, New York, p.471.