Neutrinos: recent developments and origin of neutrino mass matrix

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Abstract

Certainly one of the most exciting areas of research at present is neutrino physics. The neutrinos are fantastically numerous in the universe and as such they have bearing on our understanding of the universe. Therefore, we must understand the neutrinos, particularly their mass. There is compelling evidence from solar and atmospheric neutrinos and those from reactors for neutrino oscillations implying that neutrinos mix and have nonzero mass but without pinning down their absolute mass. This is reviewed. The implications of neutrino oscillations and mass squared splitting between neutrinos of different flavor on pattern of neutrino mass matrix is discussed. In particular, a neutrino mass matrix, which shows approximate flavor symmetry where the neutrino mass differences arise from flavor violation in off-diagonal Yukawa couplings is elaborated on. The implications in double beta decay are also discussed.

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1 Introduction

Certainly one of the most exciting areas of research at present is neutrino physics. Neutrinos are fantastically numerous in the universe and as such to understand the universe we must understand neutrinos. It is fair to say that the results of the last decade on neutrinos from the sun, from the atmospheric interaction of cosmic rays, and from reactors provide a compelling evidence that the neutrinos have nonzero mass and mix.

In 1930’s protons, neutrons and electrons were considered as elementary particles. Such a picture was confronted with two fundamental problems: conservation of energy and angular momentum (A.M.) in $\beta$-decay

$$n \rightarrow p + e^-$$

This is because experimentally seen continuous $\beta$ spectrum can not be explained for 2-body final state if energy is conserved, since in that case $E_e$ would have an unique energy. Further the final state would necessarily have integral A.M. while initial state has half integral A.M.

To solve these problems Pauli assumed that there exists a new electrically neutral elementary particle, with spin 1/2, mass less than electron mass and an interaction much weaker than photon interaction. Thus

$$n \rightarrow p + e^- + \bar{\nu}_e$$

leading to continuous $\beta$ spectrum and conservation of A.M. This was the first particle postulated by a theoretician.

Direct observation of $\bar{\nu}_e$ was made much later in 1950’s, when high flux fission reactors as source of neutrons become available. $\bar{\nu}_e$’s, electron-type antineutrinos are produced in the decay of pile neutrons in a fission reactor. These can be captured in hydrogen giving the reaction

$$\bar{\nu}_e + p \rightarrow n + e^-$$

whose cross-section was measured by Reines and Cowan:

$$\sigma_{exp} = (11 \pm 2.5) \times 10^{-44} cm^2$$
to be compared with the theoretical value

\[ \sigma_{th} = (11 \pm 1.6) \times 10^{-44} \text{cm}^2 \]

Note the extreme smallness of the cross-section (nuclear cross sections are of order \(10^{-24}\text{cm}^2\)). It is a reflection of the fact that neutrino has only weak interaction. It is remarkable that neutrinos which have almost no interaction with matter have contributed to some of the most important discoveries in physics given below:

- **1950's** \( \bar{\nu}_e \): electron type anti-neutrino discovered in experiments of Reines and Cowan (1995 Nobel Prize).

- **1956**: Parity non-conservation in \( \beta \) decays was discovered (Wu et al.) after its conservation in weak interaction was questioned by Lee and Yang (1957 Nobel Prize).

- **1957**: It was proved that neutrino (antineutrino) is left handed (right handed) particle (Goldhaber et al), after Salam, Landau and Lee and Yang proposed the 2-component neutrino theory.

- **1962**: \( \nu_\mu \): The muon neutrino was discovered (Lederman, Steinberger, Schwartz et al.) (1988 Nobel Prize).

- **1970**: Solar neutrinos were detected in pioneering experiments by R. Davis.

- **1973**: A new class of weak interactions (neutral currents) was discovered in a neutrino experiments by Garamelle Collaboration at CERN, as predicted by electroweak unification (1979 Nobel Prize: Glashow, Salam, Weinberg).

- **1980's**: In experiments on neutrino beams at CERN and at Fermilab, the quark structure of nucleon was established and investigated J. I. Friedman, H. W. Kendall, R. E. Taylor (1990 Nobel Prize).

- **1983**: The intermediate \( W \) and \( Z \) bosons were discovered at CERN at masses predicted by electro-weak unification (1984 Nobel Prize: C. Rubbia and Simon Van Der Meer).
neutrinos from supernova 1987 A were detected (Kamiokanda, IMB, Bakson).

1990’s: It was found in LEP experiments that only three types of light flavor neutrinos exist in nature: \( \nu_e \), \( \nu_\mu \), \( \nu_\tau \).

2000: \( \nu_\tau \): Direct observation of nu tau was made (Fermi Lab’s Tevatron).

2001: Solar neutrino Oscillations are established; solar model is verified: Super Kamiokande (SK), Sadbury Neutrino Observatory (SNO), (2002 Nobel Prize: Davis and Koshiba).

This is an impressive list of discoveries.

2 Neutrino Mass

Neutrino occurs in one helicity state (Left handed). This together with lepton number \( L \) conservation implies \( m_\nu = 0 \). However there is no deep reason that it should be so. There is no local gauge symmetry and no massless gauge boson coupled to lepton number \( L \), which therefore is expected to be violated. Thus one may expect a finite mass for neutrino. Moreover, all other known fermions, quarks and charged leptons, are massive. But the intriguing question is: why \( m(\nu_e) \ll m(e) \), which needs to be understood, even though we do not understand why e.g. electron mass is what it is and why muons and tauons are heavier than electron see Fig. 1 [1]. This is the so called flavor problem which has so far eluded us. Neutrino mass has added importance for two other reasons:

- The interesting phenomena of neutrino oscillations is possible if one or more of neutrinos have non vanishing mass.

- Non-vanishing of neutrino mass has important implications in Astrophysics and Cosmology. It is a candidate for hot dark matter.
2.1 Astrophysical Constraint on Neutrino mass

The total mass-energy of the universe is composed of several constituents, each of which is characterized by its energy density, which is expressed in terms of critical density

\[ \rho_{0i} \equiv \Omega_{oi}\rho_c \]

Critical density is the minimum density required for the expansion of the Universe to be turned around by the gravitational attraction of all the matter in it and is defined as

\[ \rho_c = \frac{3H_0^2}{8\pi G_N} \]  \hspace{1cm} (1)

where \( H_0 \) is the Hubble constant and \( G_N \) is the Newton’s gravitational constant. Using the present value of \( H_0 \) (Hubble constant), namely \( H_0 = 100h_0 \) km s\(^{-1}\)Mpc\(^{-1}\), Mpc=3 × 10\(^{19}\) km so that

\[ H_0 = h_0\left(1 \times 10^{10}\text{yr}\right)^{-1} \]

where

\[ h_0 = 0.72 \pm 0.05. \]  \hspace{1cm} (2)

This gives

\[ \rho_c = 1.879h_0^2 \times 10^{-29}\text{gcm}^{-3} \]
\[ = 1.054h_0^2 \times 10^4\text{eVcm}^{-3} \]  \hspace{1cm} (3)

What is the neutrinos contribution to hot dark matter (since relic light \( \nu \)'s had relativistic velocity) ? Neutrinos are fantastically numerous

\[ n_{\nu} = \frac{3}{10}n_{\gamma} = 112\text{cm}^{-3} \]
\[ n_{\gamma} = 400\text{cm}^{-3} \]

so if they have even a tiny mass, they can outweigh all the stars and galaxies in the universe. The neutrinos contribution to energy density is

\[ \rho_{\nu_0} = (112)\left(\sum_i m_{\nu_i} \text{eV}\right)\text{eVcm}^{-3} \]
so that

\[
\Omega_{\nu}^{HDM} = \frac{\rho_{h_0}}{\rho_{c0}} = \frac{112}{1.05} h_0^2 \sum_i m_{\nu_i} \text{(eV)} = \frac{\sum_i m_{\nu_i} \text{(eV)}}{93.8 h_0^2}
\] (4)

Unfortunately there is no direct particle physics evidence on \(\sum_i m_{\nu_i}\). We shall come back to this question later. Here we simply note that \(\rho_{h_0} \leq \rho_{c0}\), implies that

\[
\sum_i m_{\nu_i} \leq 93.8 h_0^2 \text{(eV)} = 49 \text{eV}
\] (5)

This is the astrophysical constraints on light neutrino masses.

### 2.2 Double \(\beta\)-Decay

The double \(\beta\)-decay is another way to look for a finite mass of neutrino. Two kinds of double \(\beta\)-decay can be considered:

\[
\begin{align*}
(2\nu) & \quad (A, Z) \rightarrow (A, Z + 2) + 2e^- + 2\bar{\nu}_e \\
(0\nu) & \quad \rightarrow (A, Z + 2) + 2e^-.
\end{align*}
\]

Usually the neutrinos are assumed to be Dirac particles: neutrino \(\nu\) and antineutrino \(\bar{\nu}\) are distinct. In Majorana picture, they are identical. Thus

\[
\begin{align*}
n & \rightarrow p + e^- + \bar{\nu}_L \equiv p + e^- + \nu_L \\
\nu_L + n & \rightarrow p + e^-,
\end{align*}
\]

so that neutrinoless double beta decay

\[
(2n) \rightarrow (2p) + 2e^-
\]

is possible. The important physics issues in \(0\nu\) double \(\beta\)-decay are:

(i) Lepton number must not be conserved, which is possible if neutrinos are Majorana particles: \(\nu \equiv \bar{\nu}\)
(ii) Helicity of the neutrino cannot be exactly $-1$, this can be satisfied if $m_\nu \neq 0$.

Thus $(0\nu)\beta\beta$–decay is especially interesting:

$$T_{1/2} \propto Q^{-5} < m_\nu >^{-2},$$

where decay $Q$ value $\approx T_{e1} + T_{e2}$. Here

$$< m_\nu > = \sum_i \lambda_i |U_{ei}|^2 m_{\nu_i}$$

where $\lambda_i$ is a possible sign since Majorana neutrinos are CP eigenstates; as shown the expectation value is weighted by neutrino’s electron couplings. There is direct evidence of $(2\nu)\beta\beta$ decay

$$(2\nu)\beta\beta \quad ^{82}Se \rightarrow ^{82}Kr$$

$$T_{1/2} = \left(1.1^{+0.8}_{-0.3}\right) \times 10^{20} yrs$$

For neutrinoless double $\beta$ decay

$$^{76}Ge \rightarrow ^{76}Se + 2e^-$$

$$T_{1/2} \geq 1.9 \times 10^{25} yrs$$

One recent result [2] has claimed the evidence for this decay with the best value $T_{1/2} = 1.5 \times 10^{25}$ yrs. This analysis claims $\langle m_\nu \rangle = \left(0.39^{+0.17}_{-0.28}\right)$ eV which has been commented upon [3]. If the above finding were to be confirmed, it would be the first indication of lepton number violation in nature and that Majorana neutrino can exist in nature. We shall come to other implications of above value of $\langle m_\nu \rangle$ later.

### 2.3 Cosmological Constraints

We can see the universe 300,000 years after Big Bang by studying the cosmic microwave background radiation (CMB), which is a direct relic of the universe when it became transparent to electromagnetic radiation. Fluctuations in the CMB radiation (at the level of a few parts in $10^5$) have been detected with angular resolutions from $7^o$ to a few arc minutes in the sky [4]. These indicate the first clumping of matter particles into cosmic structures, which
is resisted by the repulsive pressure of photons. The net result was gravity driven acoustic–like oscillations. These oscillations left their signature in the anisotropy of the CMB. Since the amplitude and position of the primary and secondary peaks are directly determined by the sound speed (and hence the equation of state) and by the geometry and expansion of the universe, they can be used as powerful test of the density of baryons and dark matter (DM) and other cosmological parameters.

Recent measurements of the fluctuations by an orbiting observatory called the Wilkinson Microwave Anisotropy Probe (WMAP) and their analysis have settled a number of issues about the universe, its age, its expansion rate and its composition. The results are summarized below [4].

\[
\begin{align*}
\text{Age of Universe} &= 13.4 \pm 0.3 \text{ billion years.} \\
\Omega &= \rho / \rho_c = 1.02 \pm 0.02 \\
\rho_{DM} &= (2.25 \pm 0.38) \times 10^{-27} \text{ Kg/m}^3 \\
\Omega_{DM} &= 0.23 \pm 0.05 \\
\Omega_b &= 0.046 \pm 0.005 \\
\Omega_{\nu} &< 0.015, \; m_{\nu} < 0.23 \text{ eV}
\end{align*}
\]

(10)

Note that visible baryon density is only about 4.6 percent. The situation is summarized in Figure 2 [5]. On the composition of the universe there is dramatic observation that the fraction of cosmic mass-energy residing in ordinary matter is only about 4 %. Around 23 % of the universe is made up of another substance, called dark matter, proposed 25 years ago when it became clear that all the galaxies behaved as if they were more massive than they seemed to be. The remaining 72 % is a new discovery, called dark energy, that work against gravity on large scales implying that the expansion of the universe is speeding up, rather than decelerating. In essence what we have learned about the universe is largely restricted to 4 %. The nature of 96 % is essentially unknown. One thing is certain that we have to go beyond the ordinary matter and radiation we already know. For the dark matter we have a real chance of learning within the next 5 to 10 years when we might discover a new type of matter at CERN, Geneva where world’s largest accelerator is being developed. Such a matter is predicted by a new symmetry in particle physics, called supersymmetry. For dark energy, we have to wait unless or until there is a unified theory of space-time, trying to bring gravity within the same framework as other interactions.
3 Origin of Neutrino Masses

The minimal standard model involves 3 chiral neutrino states, but it does not admit renormalizable interactions that can generate neutrino masses. If there is no $SU_L(2) \times U_Y(1)$-singlet fermion in nature, then neutrino masses are necessarily Majorana

$$\mathcal{L}^{mass} = \frac{1}{2} m \psi^T C^{-1} \psi + h.c. \quad (11)$$

$$\Delta L = 2$$

However, even if such a field exist, its mass is naturally much greater than the weak scale in which case light neutrinos are Majorana fermions as we shall see. In the SM, Majorana neutrino masses are forbidden by a global Baryon–Lepton ($B - L$) symmetry but there is no reason to expect that this symmetry is fundamental. If one allows right-handed neutrinos $\nu_R$ which are $SU_L(2) \times U_Y(1)$ singlets, then one can write Yukawa interactions:

$$\mathcal{L}_Y = \tilde{\ell}_Li \phi h_{ij} e_{Rj} + \tilde{\ell}_Li \tilde{\phi} h_{ij} \nu_{Rj} - \frac{1}{2} \bar{\nu}_R^c M \nu_R + h.c. \quad (12)$$

where the SM places the left-handed components of charged leptons and associated neutrinos into $SU_L(2)$ doublets $\ell_L$. $\phi$ is the usual Higgs doublet under $SU_L(2)$. The lepton number violation is induced by the third term, which is allowed by the gauge symmetry. $M$ is the Majorana mass matrix while $h$ are Yukawa couplings. After spontaneous symmetry breaking the vacuum expectation value of the Higgs field $\langle \phi \rangle \equiv v = 175$ GeV generates the Dirac mass term $(m_D)_{ij} = h_{ij} v$ and 6×6 neutrino mass matrix

$$M_\nu = \begin{pmatrix} 0 & m_D \\ m_D & M \end{pmatrix} \quad (13)$$

After diagonalization $M_\nu$ has 6 mass eigenstates $\nu_k$ that represent Majorana neutrinos ($\nu_k = \bar{\nu}_k$). One can consider some useful limits:

- Dirac: $M \to 0$: there are 6 Majorana neutrinos that merge to form 3 massive Dirac neutrinos
- Majorana: $m_D \to 0$
• Seasaw $m_D << M$: there are three light active Majorana neutrinos

In the seesaw limit, the diagonalization of $M_\nu$ gives

$$M_\nu = -m_D M^{-1} m_D$$  \hspace{1cm} (14)$$

This also yields light and heavy neutrino mass eigen states

$$\nu = V_\nu^T \nu_L + \nu_L V_\nu^*$$

$$N \approx \nu_R + \nu_R^c$$

$$m_{N_i} = M_i$$  \hspace{1cm} (15)$$

where $V_\nu$ is the neutrino mixing matrix. Thus

$$m_{\nu_{\ell}} \sim \frac{m_D^2}{M} \approx \frac{v^2}{M} \ll m_{\ell},$$  \hspace{1cm} (16)$$

by requiring the existence of large scale $M$, associated with new physics. Indeed, since $v \approx 175$ GeV, $m_\nu \approx 0.03$ eV, for $M \approx 10^{15}$ GeV. Thus Neutrino masses are a probe of physics at grand unification mass scale. We shall see that neutrino oscillations might remarkably provide a mechanism to measure extremely small masses (of order of milli electron volts and less) and indirectly provide a new scale indicative of new physics.

4 Neutrino Oscillations

4.1 Oscillations in vacuum

Neutrinos are produced in weak interactions as flavor eigenstates, characterized by $e, \mu, \tau$. The flavor eigenstates $|\nu_\alpha\rangle$ need not coincide with mass (energy) eigenstate $|\nu_i\rangle$ and are generally coherent superposition of such states

$$|\nu_\ell\rangle = \sum_i U_{\ell i} |\nu_i\rangle$$  \hspace{1cm} (17)$$

where the mixing matrix is unitary. This matrix is characterized by 3 angles, $\theta_{12} = \theta_3, \theta_{13} = \theta_2, \theta_{23} = \theta_1$, one CP violating phase $\delta$ and two Majorana phases, which we put equal to zero.

$$U = \begin{pmatrix}
  c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
  -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\
  s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -s_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & c_{23}c_{13}
\end{pmatrix}$$  \hspace{1cm} (18)$$
In vacuum, the mass eigenstates propagate as plane waves

$$|\nu_i (t, x)\rangle = \exp (-i (E_i t - \mathbf{k} \cdot \mathbf{x})) |\nu_i (0)\rangle$$  \hspace{1cm} (19)$$

where $$E_i \approx k + m_i^2 / 2k^2$$, $$k \gg m_i$$. Thus flavor eigenstates propagate as

$$|\nu_\ell (t, x)\rangle = U_{\ell i} \exp (-i (E_i t - \mathbf{k} \cdot \mathbf{x})) |\nu_i (0)\rangle$$  \hspace{1cm} (20)$$

The probability at time $$t$$ that $$\nu_\ell$$ is converted into $$\nu_{\ell'}$$ is

$$P_{\nu_\ell \rightarrow \nu_{\ell'}} = |\langle \nu_{\ell'} | \nu_\ell \rangle|^2$$

For oscillations involving two neutrinos, it takes a simple form

$$P_{\nu_\ell \rightarrow \nu_{\ell'}} = |\langle \nu_{\ell'} | \nu_\ell \rangle|^2 \approx \sin^2 \theta \cos^2 \theta |1 + \exp [- (E_1 - E_2) t]|^2$$

$$= \sin^2 2\theta \sin^2 \left[ \frac{(E_1 - E_2)}{2} t \right]$$

$$= \sin^2 2\theta \sin^2 \left[ \frac{\Delta m^2}{4k} t \right]$$  \hspace{1cm} (21)$$

It is convenient to write it as

$$P_{\nu_\ell \rightarrow \nu_{\ell'}} = \sin^2 2\theta \sin^2 \left[ \frac{1.27 \Delta m^2}{E_\nu} L \right]$$  \hspace{1cm} (22)$$

where $$L$$ is the distance (measured in meters) travelled after $$\nu_\ell$$ is converted into $$\nu_{\ell'}$$. $$\Delta m^2 = m_1^2 - m_2^2$$ in units of eV$^2$ while $$E_\nu \approx k$$ is measured in MeV. Thus the oscillations in this simple case are characterized by the oscillation length in vacuum

$$L_\nu = 4\pi \frac{E_\nu}{\Delta m^2}$$  \hspace{1cm} (23)$$

and by the amplitude $$\sin^2 2\theta$$. To look for the oscillations, the above formula also shows that one needs low energy $$\nu$$'s, long path length and large flux. In the flavor basis the Hamiltonian is

$$H_\nu = U H U^{-1}$$  \hspace{1cm} (24)$$

where $$H$$ is diagonal in $$\nu_1 - \nu_2$$ base. This gives, ignoring a trivial diagonal term not relevant for oscillations,

$$H_\nu = 2\pi \begin{pmatrix} \cos \theta & \sin \theta \\ -\frac{L_\nu}{\sin 2\theta} & \frac{2L_\nu}{\sin 2\theta} \end{pmatrix}$$  \hspace{1cm} (25)$$
4.2 Oscillations in Matter

In traversing matter neutrinos interact with electrons and nucleons of intervening material and their forward coherent scattering induces an effective potential energy $\sqrt{2}G_F N_e$ modifying $H_\nu$ given in Eq. (21) to

$$H_M = \begin{pmatrix} -\cos 2\theta & 1/2L_v \\ \sin 2\theta & 0 \end{pmatrix}$$

Thus the evolution of the flavor eigenstates in matter is governed by the Schrödinger equation

$$i\frac{d}{dx} \begin{pmatrix} \nu_e \\ \nu_\alpha \end{pmatrix} = 2\pi H_M \begin{pmatrix} \nu_e \\ \nu_\alpha \end{pmatrix}$$

where $H_M$ is given in Eq. (26) and there

$$L_0 = \frac{2\pi}{\sqrt{2}G_F N_e} = 1.7 \times 10^7 (m) / \rho (g/cm^3) Y_e$$

is the corresponding matter oscillation length. Here $N_e$ denotes the number of electrons per unit volume:

$$N_e = \frac{\rho}{m_N} Y_e$$

where $Y_e$ is the number of electrons per nucleons $\approx 1/2$ in ordinary matter. The effective oscillation length in matter is

$$L_m = L_v \frac{\sin 2\theta_m}{\sin 2\theta} = L_v \left[ \left( \frac{L_v}{L_0} \right)^2 + \sin^2 2\theta \right]^{-1/2}$$

$$\tan 2\theta_m = \tan 2\theta \left( 1 - \frac{L_v}{L_0 \cos 2\theta} \right)^{-1}$$

$$P_{(\nu_e \rightarrow \nu_e)} = 1 - \sin^2 2\theta_m \sin^2 \left[ 1.27 \frac{L}{L_m} \right]$$

(30)

$\theta_m$ is new mixing angle in matter. Thus, resonance $[\sin^2 2\theta_m = 1]$ occurs when $\cos 2\theta$ is equal to

$$\frac{L_v}{L_0} = \frac{2\sqrt{2}G_F N_e E_\nu}{\Delta m^2} = 0.22 \left[ \frac{E_\nu}{1 MeV} \right] \left[ \frac{\rho Y}{100 g/cm^3} \right] \left[ \frac{7 \times 10^{-5} eV^2}{\Delta m^2} \right]$$

(31)
The transition point between the regime of vacuum and matter oscillations is determined by the ratio $L_v/L_0$. If it is greater than 1, matter oscillations dominate. If it is less than $\cos 2\theta$, vacuum oscillation dominate. Generally there is a smooth transition between these two regimes. Matter effects become maximum at resonance $L_v/L_0 = \cos 2\theta$. This is the basis of Mikheyev-Smirnov-Wolfenstein (MSW) effect. The survival probability $P(\nu_e \rightarrow \nu_e)$ averaged over the detector position $L$ (from the solar surface) [6] is

$$P(\nu_e \rightarrow \nu_e) = \frac{1}{2} \left[ 1 + (1 - 2P_x) \cos 2\theta_m (\rho_{\max}) \cos 2\theta \right]$$

(32)

where $\theta_m (\rho_{\max})$ is the initial mixing angle, usually

$$\cos 2\theta_m (\rho_{\max}) \simeq -1$$

and $P_x$ is a finite probability for jumping from one eigenstate to the other one and conversion might be incomplete. The survival probability $\langle P(\nu_e \rightarrow \nu_e) \rangle$ as a function of $E_\nu$ is displayed for various mixing angles in Fig. 3 [7].

For the parameters corresponding to preferred solution for neutrino oscillations (see below) $\sin^2 2\theta \approx 0.8$, $\Delta m^2 \approx 7 \times 10^{-5} \text{eV}^2$ and $\rho = 100 \text{g/cm}^3$ at the center of the sun, $Y_e \approx 1/2$, $L_v/L_0 = 0.11 E_\nu/1 \text{MeV}$, $\cos 2\theta = 0.45$.

Due to different reaction thresholds, solar neutrinos with energy $E_\nu > 0.814 \text{MeV}$ can be detected in $^{37}\text{Cl}$ and those with $E_\nu > 0.233 \text{MeV}$ in $^{71}\text{Ga}$. Note that for pp neutrinos ($E_\nu < 0.42 \text{MeV}$) and $^{7}\text{Be}$ neutrinos ($E_\nu \approx 0.86 \text{MeV}$), $L_v/L_0 < \cos 2\theta$ and they undergo vacuum oscillations, while the neutrinos with $E_\nu > 4.5 \text{MeV}$, ($^{8}\text{B}$ neutrinos) undergo MSW matter oscillations.

5 Evidence for Oscillations

One looks for oscillations in two types of experiments.

5.1 Appearance experiments:

Here one searches for a new neutrino flavor, absent in the initial beam, which can arise from oscillations.
5.1.1 Atmospheric neutrino anomaly:

Atmospheric neutrinos are produced in decays of pions (kaons) that are produced in the interaction of cosmic rays with the atmosphere:

\[
p + A \rightarrow \pi^\pm + A' \\
\pi^\pm \rightarrow \mu^\pm \nu_\mu (\bar{\nu}_\mu) \\
\rightarrow e^\pm \nu_e (\bar{\nu}_e) \nu_\mu (\bar{\nu}_\mu)
\]

These neutrinos are detected in and beneath underground detectors through the reactions

\[
\nu_\mu + n \rightarrow \mu^- + p \\
\bar{\nu}_\mu + p \rightarrow \mu^+ + n
\]

and

\[
\nu_e + n \rightarrow e^- + p \\
\bar{\nu}_e + p \rightarrow e^+ + n
\]

These are respectively called \(\mu\)-like and \(e\)-like events. The observed ratios of these events was found to be substantially reduced from the expected value \(\sim 2\). There is compelling evidence that atmospheric neutrinos change flavor as the Super-Kamiokande experiment clearly indicated a deficit of up-ward \(\mu\)-like events (produced about \(10^4\) km away at the opposite side of earth) relative to the downward going events (produced about 20 km above). The \(e\)-like events showed a normal zenith angle dependence. The data is described by \(\nu_\mu \rightarrow \nu_\tau\) oscillations. The conversion probability \(P_{\nu_\mu \rightarrow \nu_\tau}\) fits the data quite well for [8]

\[
\Delta m_{23}^2 = 2.0 \times 10^{-3} eV^2, \quad \sin^2 2\theta_{23} \approx 1.0
\]

5.1.2 Solar Neutrinos

Particularly compelling evidence that the solar neutrinos change flavor has been reported by the Sudbury Neutrino Observatory (SNO). SNO measures the high energy part of the solar neutrino flux (\(\nu B\) neutrinos). The reactions

\[
\nu d \rightarrow \nu np \\
\rightarrow e pp \\
\nu e \rightarrow \nu e
\]
were studied by SNO. SNO measured arriving $\nu_e + \nu_\mu + \nu_\tau$ flux, $\phi_e + \phi_{\mu\tau}$, and the $\nu_e$ flux, $\phi_e$. From the observed rates for the first two reactions, which involve respectively neutral current and charge current, SNO finds that the ratio of the two fluxes $\phi_e$ and $\phi_e + \phi_{\mu\tau}$ is $0.306 \pm 0.050$. This implies that the flux $\phi_{\mu\tau}$ is not zero. Since all the neutrinos are born in nuclear reactions that produce only electron neutrinos, it is clear that neutrinos change flavor. Corroborating information comes from the direct reaction $\nu_e \rightarrow \nu_e$, studied by both SNO and Super-Kamiokande. The strongly favored explanation of solar neutrino flavor change is the Large Mixing Angle version of the MSW effect, with the best fit parameters [9]

$$\Delta m_{12}^2 = 7.1 \times 10^{-5} eV^2, \quad \sin^2 2\theta_{12} = 0.8 \quad [\text{See Fig. 3}] \quad (34)$$

5.2 Disappearance experiments:

Reactors are source of $\bar{\nu}_e$’s through the neutron $\beta$-decay

$$n \rightarrow p + e^- + \bar{\nu}_e$$

and experiment looks for a possible decrease in the $\bar{\nu}_e$ flux as a function of distance from the reactor, $\bar{\nu}_e \rightarrow X$ [if converted to $\bar{\nu}_\mu$, say, one would see nothing, $\bar{\nu}_\mu$ could have produced $\mu^+$ but does not have sufficient energy to do so]. Kamland experiment [10] confirms that $\bar{\nu}_e$ do indeed disappear when the reactor $\bar{\nu}_e$ have travelled $\approx 200$ km. $\bar{\nu}_e$ flux is only $0.611 \pm 0.085 \pm 0.041$ of what it would be if none of it were disappearing. Interestingly this reactor $\bar{\nu}_e$ disappearance and the solar neutrino results can be described by the same neutrino mass and mixing parameters (see Fig. 4 [7]). This gives confidence that the physics of both phenomenon has been correctly identified.

6 Neutrino Mass Matrix

As discussed the data from solar and atmospheric neutrino and reactor antineutrinos experiments provide evidence for neutrino mass and mixing with two different mass scales and large mixing angles:

$$\Delta m_{atm}^2 \equiv \Delta m_{23}^2 = (2.0 \pm 0.5) \times 10^{-3} eV^2$$

$$\sin^2 \theta_{23} \equiv \sin^2 \theta_1 = 1.00 \pm 0.4$$
\[
\Delta m^2_{\text{solar}} \equiv \Delta m^2_{12} = (7.1 \pm 0.6) \times 10^{-5} \text{eV}^2
\]
\[
\tan^2 \theta_{12} \equiv \tan^2 \theta_3 = 0.45 \pm 0.06 \quad (35)
\]

Further the CHOOZ experiment [11] gives
\[
|U_{e3}|^2 \equiv \sin^2 \theta_2 < 4 \times 10^{-2} \quad (36)
\]

We would interpret these results in terms of small off-diagonal perturbations of a degenerate diagonal mass matrix in flavor basis for light Majorana neutrinos [12]. In this approach there is no fundamental distinction between masses of neutrinos of different flavors; the mass differences arise from small flavor violation of off-diagonal Yukawa coupling constants. Further the neutrino mass differences do not in anyway constraint the absolute value of neutrino mass. The constraint on it will come from neutrinoless double \(\beta\)-decay experiments, cosmology and direct laboratory experiments, e.g. tritium \(\beta\)-decay.

Let us consider a Majorana mass matrix in \((e, \mu, \tau)\) basis
\[
m_\nu = m_0 \begin{pmatrix}
a_{ee} & a_{e\mu} & a_{e\tau} \\
a_{e\mu} & a_{\mu\mu} & a_{\mu\tau} \\
a_{e\tau} & a_{\mu\tau} & a_{\tau\tau}
\end{pmatrix} \quad (37)
\]

It is convenient to define the neutrino mixing angles as follows
\[
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix} = U 
\begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3
\end{pmatrix} \quad (38)
\]

where \(U\) is given in Eq. (18). We shall put \(\delta\) as well as Majorana phases to be zero. In view of mixing angles given above, we shall take \(s_{13} \equiv s_2 = 0\), \(c_{13} \equiv c_2 = 1\) and \(c_1 = 1/\sqrt{2}, s_1 = \mp 1/\sqrt{2}\). The diagonalization gives
\[
a_{e\mu} = \pm a_{e\tau} = \frac{1}{\sqrt{2}} s_3 c_3 (-m_1 + m_2)
\]
\[
a_{\mu\tau} = \pm \frac{1}{2} \left[ (m_1 s_3^2 + m_2 c_3^2) - m_3 \right]
\]
\[
a_{\mu\mu} = a_{\tau\tau} = \frac{1}{2} \left[ m_1 s_3^2 + m_2 c_3^2 + m_3 \right]
\]
\[
a_{ee} = m_1 s_3^2 + m_2 c_3^2 \quad (39)
\]
In view of

\[ \Delta m_{12}^2 = m_2^2 - m_1^2 \ll \Delta m_{23}^2 = m_3^2 - m_2^2, \]  

we can take

\[ m_1 \simeq \pm m_2. \]

Thus we have two possiblities for mass matrix \( m_1 = m_2 = m_0; m_1 = -m_2 = m_0 \):

\[ m_\nu = m_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} (1 + a) & \pm \frac{1}{2} (1 - a) \\ 0 & \pm \frac{1}{2} (1 - a) & \frac{1}{2} (1 + a) \end{pmatrix} \]  

\[ m_\nu = m_0 \begin{pmatrix} \cos 2 \theta_3 & -\frac{1}{\sqrt{2}} \sin 2 \theta_3 & \mp \frac{1}{\sqrt{2}} \sin 2 \theta_3 \\ -\frac{1}{\sqrt{2}} \sin 2 \theta_3 & \frac{1}{2} (\cos 2 \theta_3 + a) & \pm \frac{1}{2} (\cos 2 \theta_3 - a) \\ \mp \frac{1}{\sqrt{2}} \sin 2 \theta_3 & \pm \frac{1}{2} (\cos 2 \theta_3 - a) & \frac{1}{2} (\cos 2 \theta_3 + a) \end{pmatrix} \]

where \( a = m_3/m_0 \). If we do not want to commit to any particular value of \( \theta_3 \), then we have the first case with the following subcases corresponding to \( m_0 = 0, a = -1, 1, -2, 2, 0 \) respectively [13]

\( i \) \( \begin{pmatrix} m_3/2 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} \)

\( ii \) \( \begin{pmatrix} m_0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \)

\( iii \) \( \begin{pmatrix} m_0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \)

\( iv \) \( \begin{pmatrix} m_3/2 & 2 & 0 \\ 0 & -1 & 3 \\ 0 & 3 & -1 \end{pmatrix} \)

\( v \) \( \begin{pmatrix} m_3/2 & 2 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{pmatrix} \)

\( vi \) \( \begin{pmatrix} m_3/2 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} \)
In order to generate $\Delta m_{12}^2$ and $\Delta m_{23}^2$, we will now concentrate on choice (iii), which preserves flavor and add to it a small perturbation which violates flavor in off-diagonal matrix elements:

$$m_\nu = m_0 \begin{pmatrix} 1 & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{12} & 1 & \varepsilon_{23} \\ \varepsilon_{13} & \varepsilon_{23} & 1 \end{pmatrix}$$  \hspace{1cm} (43)

where $\varepsilon_{ij} \ll 1$. The diagonalization gives

$$m_i = m_0 (1 - x_i)$$  \hspace{1cm} (44)

where $x_i (i = 1, 2, 3)$ are roots of cubic equation

$$x^3 - \left(\varepsilon_{12}^2 + \varepsilon_{13}^2 + \varepsilon_{23}^2\right)x + 2\varepsilon_{12}\varepsilon_{13}\varepsilon_{23} = 0.$$  \hspace{1cm} (45)

The choice $\varepsilon_{12} = \varepsilon_{13} = \varepsilon_{23} = \varepsilon$ will give the roots $(\varepsilon, \varepsilon, -2\varepsilon)$ and thus will not lift the degeneracy between $m_1$ and $m_2$. To lift this degeneracy we take $\varepsilon_{12} = \varepsilon_{13} = \varepsilon + \delta, \varepsilon_{23} = \varepsilon$ with $\delta/\varepsilon \ll 1$. Then the roots to the first order in $\delta/\varepsilon$ are $\varepsilon \left(1 + \frac{4\delta}{3\varepsilon}\right), \varepsilon, -2\varepsilon \left(1 + \frac{4\delta}{3\varepsilon}\right)$ so that

$$m_1 = m_0 \left[1 - \varepsilon - \frac{4\delta}{3}\right]$$
$$m_2 = m_0 \left[1 - \varepsilon\right]$$
$$m_3 = m_0 \left[1 + 2\varepsilon + \frac{4\delta}{3}\right]$$

$$\Delta m_{12}^2 \approx \frac{8}{3} m_0^2 \delta (1 - \varepsilon) \approx \frac{8}{3} m_0^2 \delta$$
$$\Delta m_{23}^2 \approx 6\varepsilon m_0^2.$$  \hspace{1cm} (46)

This gives

$$\frac{\delta}{\varepsilon} = \frac{9}{4} \frac{\Delta m_{12}^2}{\Delta m_{23}^2} \approx 5.9 \times 10^{-2}$$
$$\sqrt{\varepsilon m_0} \approx 2.1 \times 10^{-2} eV$$  \hspace{1cm} (47)

Thus $m_0$ is not constrained. However, $m_0$ is constrained by WMAP data, $3m_0 < 0.71$ eV. When analyzed in conjunction with neutrino oscillation, it
is found that mass eigenvalues are essentially degenerate with $3m_0 > 0.4$ eV. The above limits put limits on $\varepsilon$: $7.9 \times 10^{-3} < \varepsilon < 2.5 \times 10^{-2}$.

For the degenerate neutrino mass pattern $m_1 \sim m_2 \sim m_3 \gg \sqrt{\Delta m^2_{32}} = 0.045$, the effective mass in neutrinoless double $\beta$-decay is larger than $\sim 0.05$ eV, constrained from above by the mass limit from tritium $\beta$-decay. If the effective Majorana mass is confirmed to be $(0.39^{+0.17}_{-0.28})$ eV [2], it would strongly indicate that neutrinos follow degenerate mass pattern (see Fig. 5 [7]), when

$$\frac{\Delta m^2}{m^2} \ll 1.$$ 

Finally for two modest extensions of the standard model in which the neutrino mass matrix advocated in this section can be embedded, see Ref. [12].

7 Conclusion

To conclude various neutrino mass patterns and corresponding neutrino mass matrix types are possible. Further the absolute value of neutrino mass is not yet determined. However, one thing is certain that neutrinos are providing an evidence for new physics but the scale of new physics is not yet pinned down. The heavy right handed neutrinos at new physics scale may provide an explanation for baryogenesis through leptogenesis. If past is of any guide, neutrinos will enrich physics still further.

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8 Figure Captions

1. The mass spectrum of quarks and leptons we do not understand.

2. Composition of the Universe.

3. Schematic illustration of the survival probability of $\nu_e$ created at the solar center. The curves are labelled by $\sin^2 2\theta$ values.

4. Ratio of observed to expected rates (without neutrino oscillations) for reactor neutrino experiments as a function of distance, including the recent result from the Kamland experiment. The shaded region is that expected due to neutrino oscillations with large mixing parameters as determined from solar neutrino data.

5. Dependence of effective Majorana mass $\langle m_\nu \rangle$ derived from the rate of neutrinoless double $\beta$-decay on the absolute mass of the lightest neutrino. The stripes region indicates the range related to the unknown Majorana phases, while the cross hatched region is covered if one $\sigma$ errors on the oscillation parameters also included. The arrows indicate the three possible neutrino mass patterns.
Figure 2
Figure 5