Quantum State Diffusion: from Foundations to Applications

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Abstract

Deeper insight leads to better practice. We show how the study of the foundations of quantum mechanics has led to new pictures of open systems and to a method of computation which is practical and can be used where others cannot. We illustrate the power of the new method by a series of pictures that show the emergence of classical features in a quantum world. We compare the development of quantum mechanics and of the theory of (biological) evolution.
1. Introduction

Stochastic modifications of the Schrödinger equation have been proven to be useful in a variety of contexts, ranging from fundamental new theories to practical computations. In this contribution dedicated to Abner Shimony we recall the motivations which led us to work in this field and present some thoughts about what such modifications could mean for future theories. As we shall see, both of these topics are at the border between physics and philosophy, what Abner likes to call ”experimental methaphysics”.

For Albert Einstein, John Bell and Abner Shimony a quantum theory which can be used to analyse experiments, but which does not present a consistent and complete representation of the world in which we live, is not enough. In recent years their ranks have been joined by many others, including those who have proposed alternative quantum theories to remedy this defect. One such theory is quantum state diffusion (QSD) [1,2,3,4], which has led to new ways of looking at laboratory experiments, and correspondingly new numerical solutions for open quantum systems. The philosophy of physics has led to useful practical physics. That such barely physical motivation has led to new concepts and practical tools is refreshing for physicist like Abner Shimony (and probably most of the readers of this book)!

In the next two sections we present our understanding of the quantum measurement and quantum nonlocality problems and their connections to the QSD model that developed from these considerations. Then, section 4 presents the QSD model and section 5 illustrates this model with several examples. This is followed in section 6 by an illustration of the classical limit of the dynamics of an open quantum system through the emergence of a strange attractor out of quantum indeterminism. Finally, in section 7 we introduce an analogy between the theory of biological evolution and quantum theory, and suggest that the quantum equivalent of the double helix has still to be found! Actually, in order to take into account the different views of the authors about the nature of probabilities, this section is divided in two subsections entitled ”God does not play dice (NG)” and ”We can’t know whether God plays dice (ICP)”, respectively. Hence the title of this contribution could have been ”From Foundations to Applications and back to Foundations”!

2. The quantum measurement problem

For realists like Abner Shimony and us there is a Quantum measurement problem. A simple quantum measurement results in a single event, usually labeled by a real number $\alpha_{out} \in R$. It starts with one quantum system in an initial state $\psi_{in}$ in its Hilbert space $\mathcal{H}$. The measurement consists of the emergence of $\alpha_{out}$ under the influence of the quantum system, and since it is a quantum measurement, a corresponding change in $\psi$ is usually unavoidable. In ideal cases, the output quantum
state of the system for a given value of $\alpha_{\text{out}}$ is given by [5]:

$$\psi_{\text{out}} = \frac{P_{\alpha_{\text{out}}} \psi_{\text{in}}}{|P_{\alpha_{\text{out}}} \psi_{\text{in}}|},$$  \hspace{1cm} (2.1)

where $P_{\alpha_{\text{out}}}$ is a projection operator. The probability of this single event happening is given by the weight:

$$\text{Probability}(\alpha) = \text{Weight}(\alpha) = |P_{\alpha} \psi_{\text{in}}|^2.$$  \hspace{1cm} (2.2)

In all cases in the real world there is one single event with one single result, that is, one single output. But the standard description implies many outputs, each with its own weight. Next, these weights are identified with the probability of single events or with the ”weights” of world components.

This is the first aspect of quantum measurement. A realist seeks a dynamics which describes the single event and is consistent with these probabilities. In the terminology familiar to Abner, one seeks a dynamics which describes (how and when) the actualization of potential properties.

**Nonlinear stochastic dynamics.**

Since (2.1) is nonlinear and since (2.2) is a probability, the dynamics should be nonlinear and stochastic. Since the Schrödinger equation is linear and determinate, it cannot define the state of a quantum system in such a realist theory. Additional terms are needed. This idea appeared in [6], see also [7]. Abner did also contribute to this program by listing some desiderata for modifications of the Schrödinger equation [8].

The individual results and their probabilities appear as a result of a dynamical process, but these conditions are not enough to define the process. For that we have to consider the strange and apparently contradictory locality properties of quantum systems. A first aspect of this is the fact that quantum nonlocality cannot be used to send classical information faster than light. This is the ”peaceful coexistence” between quantum nonlocality and relativity, using again Abner’s illuminating terminology [9]. This leads to a strong constraint on the nonlinear stochastic dynamics, that respects this peaceful coexistence: the mean quantum state should follow a closed evolution equation. If not, then ”parameter independence” [10,11] would be violated. This constraint was first met in [12] and Abner was among the very first (and few) who realized this and encouraged this line of research.
3. Quantum nonlocality and localization.

The theoretical paper of Einstein, Podolsky and Rosen on elements of reality was followed by Bell [13] with his inequality for local realistic theories. In [14] it was shown that the Bell inequality is violated by almost all quantum states. The papers of Bell led to series of experiments, culminating in the experiment of Aspect et al [15], which showed that classical events linked by a quantum system have nonlocal properties that cannot be reproduced in any purely local dynamics. This experimental result does not depend on any theory of quantum mechanics, whether it be the usual theory, or an alternative stochastic or deterministic realistic theory. The experiment shows that the physics is nonlocal, so every quantum theory must be nonlocal. The difference between the usual theory and alternative theories such as QSD is that in the latter the nonlocality is forced to be explicit.

The nonlocality is accompanied by a reduction or localization of the wave packet. For example when a spherical photon wave packet (electromagnetic wave) is absorbed by a surface, an electron may be emitted from a very small region of that surface. In a realistic theory like quantum state diffusion the wave packet is localized by the interaction with the surface to the region occupied by the emitted electron.

This is the same localization process that leads to measurement, or the selection of a particular state \( \alpha_{\text{out}} \) for a dynamical variable from a spectrum of possible \( \alpha \), and that leads to the localization of the centre of mass of the Moon, a stone or a Brownian particle. Without this localization the Schrödinger equation disperses the wave function into larger and larger regions of position space, and through interactions with other systems, into larger and larger regions of phase space. This is the second aspect of the quantum measurement.

The phase space localization which we see with our classical eyes [16] is reproduced in the dynamics of quantum state diffusion, and this dynamical localization also makes the equations relatively easy to solve in practice, by a numerical method that simulates the physical processes seen in the laboratory and the rest of the world.

In the foundations of quantum theory the destruction of coherence, which is necessary to produce localization or reduction and make the transition from quantum to classical mechanics, must be distinguished from the dissipation of coherence. As part of a fundamental theory, the destruction cannot depend on any division between system and environment.

The localization is due to destruction of coherence. Schrödinger dynamics dissipates coherence, as when a small system like a molecule interacts with a thermal environment and gets entangled with it. This is represented mathematically by the change in the reduced density operator for the small system. But it cannot and does not
destroy coherence, because the entanglement between system and environment is retained if a sufficiently large part of the environment is included as part of the system. And because the Schrödinger dynamics does not destroy coherence, it cannot produce the localization which is necessary to make the transition from quantum to classical mechanics. In quantum state diffusion this coherence is destroyed, thus allowing the localization which is seen in measurement and the dynamics of the classical world.

4. Nonlinear stochastic dynamics

It follows that the dynamics of the quantum state should be stochastic and nonlinear [17]. This can be achieved by means of deterministic equations interrupted by stochastic quantum jumps [18,19,20], or by a continuous diffusion process, that is, quantum state diffusion [1,3,4].

There are now several alternative quantum theories based on quantum state diffusion. Following earlier pioneering work of Bohm and Bub [21] and Pearle [6,22], Gisin [12] introduced a simple example of quantum state diffusion with real fluctuations that was generalized by Diósi [23] and Gisin [17]. The complex Itô form of quantum state diffusion described here was introduced by us in 1992 and satisfies a condition of unitary invariance [23,24] and symmetry [25]. The detailed theory and its applications are given in [1,3,4,16]. The mathematics of quantum state diffusion also appears in connection with the theory of continuous quantum measurements in ordinary quantum theory, as shown, for example, in Barchielli & Belavkin [26], Carmichael [20] and references therein.

The original form of the GRW theory of Ghirardi, Rimini and Weber was based on quantum jumps [27], but it has been reformulated as an intrinsic quantum state diffusion theory by Diósi [28] (first publication of a stochastic Schrodinger equation related to the GRW model), Pearle [29] (first stochastic Schrodinger equation for the ”raw” ensemble [30]), and Gisin [17] and Ghirardi-Pearle-Rimini [30] (first stochastic Schrodinger equation for the ”physical” ensemble).

The quantum state diffusion theory replaces the deterministic evolution of the density operator $\rho$ representing an ensemble of open systems,

$$\dot{\rho} = -(i/\hbar)[H, \rho] + \sum_j \left(L_j \rho L_j^\dagger - \frac{i}{2} L_j^\dagger L_j \rho - \frac{i}{2} \rho L_j^\dagger L_j \right), \quad (4.1)$$

by a unique stochastic diffusion of a quantum state, representing an individual system of the ensemble in interaction with its environment. $H$ is the Hamiltonian, and the $L_j$ are environment operators which represent the interaction with the environment. The corresponding quantum state diffusion equation is a stochastic
differential equation for the normalised state vector $|\psi\rangle$, whose differential Itô form is

$$
|d\psi\rangle = -(i/\hbar)H|\psi\rangle dt - \frac{1}{2} \sum_j \left( L_j^\dagger L_j + \ell_j^* \ell_j - 2 \ell_j^* L_j \right) |\psi\rangle dt + \sum_j (L_j - \ell_j) |\psi\rangle d\xi_j, \quad (4.2)
$$

where the $\ell_j$ are defined by

$$
\ell_j = \langle L_j \rangle = \langle \psi | L_j | \psi \rangle. \quad (4.3)
$$

The stochastic fluctuation or noise of the diffusion process is all contained in the standard normalised Wiener fluctuation terms $d\xi_j$, which are of order $(dt)^{\frac{1}{2}}$ and which satisfy the relations

$$
d\xi_j d\xi_k = 0, \quad d\xi_j^* d\xi_k = \delta_{jk} dt, \quad (4.4)
$$

$$
M d\xi_j = 0. \quad (4.5)
$$

The state vector executes a Brownian motion on the unit sphere in Hilbert space. Each environment operator $L_j$ contributes to the Brownian motion in just two real directions, and the diffusion is isotropic in this two-dimensional space. The latter property makes the process (4.2) unique, as illustrated by examples in ref. [25,31].

If the density matrix evolution (4.1) and the stochastic pure state evolution (4.2) have same initial conditions, then for all times the latter defines a decomposition of $\rho_t$ in terms of a classical mixture of pure states $\psi_t$:

$$
\rho_t = M(|\psi_t\rangle \langle \psi_t|),
$$

where $M$ denotes the mean over the classical noises $\xi_j$. This relation is basic for the QSD model. It guarantees that all means of quantum expectation values $M(\langle A \rangle_{\psi_t})$ agree with standard quantum mechanics. In particular it guarantees that QSD cannot be used to signal faster than ordinary quantum mechanics, i.e. not faster than light. Furthermore, the distribution of pure states determined by (4.2) may provide deeper insight into the physics of the open system; for instance it allows us to distinguish dissipation of decoherence from destruction of coherence.

As a practical method of computation, QSD gains over the direct solution of the master equation, because for a basis of $N$ states, QSD needs a computer store with
$N$ elements, and the time of computation is also proportional to $N$. For the direct solution these are proportional to $N^2$. However practical QSD is run as a Monte Carlo method, and the computation time increases as the size of the sample. We have found, however, that a lot can often be learned from a single run!

The localization by the environment has been demonstrated by many theorems [3] and numerical examples. For the measurement of a dynamical variable, the $L_j$ is the operator corresponding to that dynamical variable, and the individual states localize on one of the eigenspaces of that dynamical variable. Dissipation and thermal interactions are represented by nonselfadjoint operators, and the states tend to localize to wave packets, which are localized in phase space. Localization in phase space is typical, whereas localization to an eigenspace (reduction) is very special.

The phase space localization can be analysed in detail near the semiclassical limit, where a typical state localizes in three stages, corresponds to three levels of dispersion in phase space [16]. In the first stage the density is so much dispersed that it is not confined to a region in which the dynamical variables can be approximated by dynamically linear variables, that is by linear combinations of the canonical coordinates and momenta. In the second stage it is so confined, but the region is large compared to $\hbar/m$ so the dynamically linear theory applies in its classical version. In the third stage the localization has effectively confined the system to a region of phase space comparable to $\hbar/m$, near to the limit imposed by Heisenberg indeterminacy. It is important to notice that in the early stages there is usually simultaneous localization in conjugate dynamical variables. There is nothing wrong with this, since it does not take place near the Heisenberg limit.

Whereas interaction with the environment always tends to localize, the Schrödinger evolution tends to disperse or delocalize the wave function, as in the example of a free particle wave packet, in which there is a dispersion in position, and even more strongly in collisions, for which there are outgoing spherical waves. For nonlinear systems like molecules this dispersion leads to complicated wave functions that are difficult to compute. In the next section we demonstrate a computing method which takes advantage of this localization.

So for real open systems there is generally a competition between the localizing state diffusion and the dispersing Schrödinger evolution. Fortunately the localization is a rapid process, and it often overcomes the Schrödinger dispersion, particularly for large systems, so there is a classical world for us to live in!
5. Examples and practical applications

Now we give some examples, mostly from the original QSD papers. Figure 1 shows an interaction with a measuring apparatus. The full representation would take into account the detailed physics of this interaction, but just as the important features of a resistor can be represented by a single real variable, its resistance \[4\], so the important features of a quantum measurement can be represented by a single environment operator, which is proportional to the dynamical variable being measured. Notice from the QSD equation that for an environment operator \(L_j = L\), the fluctuation term and the corresponding drift term containing \(L\) are both zero when the state is in an eigenstate of \(L\).

In Figure 1 it is the energy or photon number of a quantized electromagnetic field that is being measured, represented by \(L = a^\dagger a\), and the field starts in a linear combination of the lowest five odd states. The mean energy is plotted against time for 9 different runs, which can be thought of as 9 different experiments: measurements of the system in the same initial state. The energy starts by fluctuating wildly, and then settles down to one of the energy eigenstates, with a probability given by the usual quantum formulae of Section 1. As we see, the probability of the usual ‘interpretation’ of quantum mechanics, becomes a probability derived from the dynamics. This equality follows from the fact that the ensemble of pure states gives the same probabilities as the usual density operator.

Figure 2 shows an example from quantum optics. It represents a damped oscillator with an applied sinusoidal resonant force, in interaction representation. In this representation the hamiltonian and the single non-selfadjoint annihilation environment operator are

\[
H = 2i(a^\dagger - a), \quad L = a, \quad (5.1)
\]

and the system starts in the state \(n = 8\). The first the damping and the fluctuations dominate, but then the system settles down to a state with negligible oscillation and constant mean energy. This state is a coherent state, which is a moving Gaussian wave pact. This is an eigenstate of the annihilation operator, so the fluctuation term is zero, which is why it does not fluctuate. This is a good example of the localization in phase space which, according to QSD, gives us our classical world.

Figure 3 illustrates an oscillator in a thermal heat bath [32] in units for which \(\hbar = 1\). The four graphs represent the position and momentum, that continue to fluctuate, as one would expect, and they also illustrate the standard deviations as a function of time, and these reduce towards 1/2, giving the smallest product permitted by Heisenberg indeterminacy.
Figure 4 shows a single run for a double well potential in a thermal bath at zero temperature. This is an example of a ‘superselection rule’ such as occurs for symmetric molecules. In QSD the system settles down into one well or the other, as observed by experiment.

One of the surprising things about QSD is that it can represent physical situations in which there are ‘quantum jumps’. In QSD they do not happen instantaneously, but the process is very fast. This is illustrated in Figure 5.

\[ H = 0, \quad L_1 = 6a^\dagger a, \quad L_2 = 0.1a \]  \hspace{1cm} (5.2)

There is simultaneously a damping process represented by the annihilation operator, and a relatively strong interaction corresponding to a measurement process represented by \( L_1 \). Because of the damping process the mean over the ensemble of the energy or photon number decreases exponentially, but because of the measurement, the state tends to ‘try to’ stay in the neighborhood of a particular eigenstate of the energy. The result is that for each run, there is an almost constant energy for extended intervals of time, interrupted by a sequence of jumps. However, the simulation of this process is numerical very inefficient, so as a practical method QSD does not work well in simulating jumps.

QSD owes a lot of its success as a practical method because of localization, and is at its best when the localization is strong. This is because the localization confines the state vectors so that the variance of dynamical variables becomes smaller, and, in effect, the state vector is confined to a smaller region of phase space. The quantum state can then be represented in a moving basis (MQSD) [33,34], which follows this region of phase space. In practical problems of optics, like second harmonic generation, this can save many orders of magnitude in space and time. This gain is over and above the factor of \( N \) gained by representing a state vector instead of a density operator, as mentioned in the previous section.

This practical success is a direct consequence of the very property of localization that was introduced in the first place to represent quantum measurement and classical dynamics in quantum state diffusion as a theory for the foundations of quantum theory. So the study of the foundations of quantum mechanics has led to new pictures of open systems and to a method of computation which is practical and can be used where others cannot.
6. Emergence of a classical strange attractor out of a quantum fog.

In this section we apply the quantum state diffusion model to an open quantum system whose classical counterpart is chaotic. This provides a nice illustration of how QSD describes with equations and with figures the appearance of classical features in a quantum theory. The system, first introduced in this context by Spiller and Ralph [35], is a damped, driven, non-linear oscillator with Hamiltonian (in the interaction picture, $\hbar = 1$):

$$
H = \frac{i}{2} \chi a^\dagger a^2 + iF(t)(a^\dagger - a)
$$

and one environment operator $L = \sqrt{\gamma}a$. The coefficient $\chi$ represents the anharmonicity and $\gamma$ the friction. The function $F(t)$ is a periodic string of rectangular pulses defined as $F(t) = 0$ if $t \mod \tau < \tau_1$ and $F(t) = F_0$ if $t \mod \tau > \tau_1$, where $\tau = \tau_1 + \tau_2$ is the period. We shall use the following values: $\chi = 0.004$, $F_0 = 2$, $\gamma = 0.1$, $\tau_1 = 5$ and $\tau_2 = 4.9$.

The corresponding classical dynamical equation reads

$$
\frac{d\xi}{dt} = -\frac{i}{2} \gamma \xi + F(t) - i\chi \xi^2 \xi^* \quad (6.1)
$$

where $\xi$ is a complex number whose real and imaginary part represent position and momentum, respectively. An interesting invariance property of this system under scaling allows one to enlarge the portion of phase space explored by the system during evolution. More specifically, there is a one-parameter $\beta$-scaling transformation $\bar{\xi} = \xi/\beta$, $\bar{\gamma} = \beta \gamma$, $\bar{t} = t/\beta$, $\bar{F_0} = F_0$, $\bar{\chi} = \beta^3 \chi$ that does not change the classical equation (6.1), except for a global scaling of the coordinates $\xi$. This is relevant for our purpose since in the quantum case enlarging the explored phase space ($\beta \to 0$) corresponds in a natural way to the classical limit. Indeed, since the localization produced by QSD can not violate the Heisenberg uncertainty relations, the dimension of the characteristic dimension of the anharmonic potential relative to $\hbar$ is crucial. If the wavepacket is localized on the size of $\hbar$ and if this is relatively small (compared to the potential), then the wavepacket remains localized and follows more or less the classical trajectory. If, on the contrary, the wavepacket remains relatively extended, the classical dynamics is smeared and purely quantum dynamical features dominate.

This is illustrated on figure 6 which represents four QSD trajectories for different values of the scaling parameters $\beta$. These trajectories are represented in phase space at times integer multiple of the period $\tau$. $\text{Re}\langle a \rangle$ and $\text{Im}\langle a \rangle$ are proportional to position and momentum, respectively. In the upper figure $\beta = 1$ and the explored phase space is small with respect to $\hbar$ (recall $\hbar = 1$). Not much structure appears, as in Wheeler’s smoky dragon. In the second and third figures $\beta = 2$ and $\beta = 5$,
respectively. The size of the relevant phase space is larger compared to $\hbar$ and to the size of the wavepacket (the latter is closed to the limit set by Heisenberg’s relations). Already some clear structure appears. This structure looks familiar to people experienced with chaotic classical systems (6.1) [36]. But, actually, there is no need to study the classical equation (6.1). Simply look at the bottom of figure 6 where $\beta = 10$ and the QSD trajectory is almost identical to the classical one. It corresponds to a strange attractor, a typical feature for an open classical system. Clearly this classical feature continuously emerges from the quantum world when the relative dimension of the wavepacket and the characteristic dimension of the potential get smaller, as can be seen in figure 6 from the top to the bottom. Note that the classical system (6.1) has also a fixed point close to (-5,5) which appears in the two central figures together with random transitions between this (classically) regular region and the (classically) chaotic region. For $\beta = 10$ these transitions happen only rarely, in particular none are displayed in the bottom figure 6. In reference [37] the full classical limit of the QSD equation applied to this example is presented.

7. Quantum State Diffusion, Probabilities and biological evolution.

In this section we present some views about QSD, the role of probability in physics and similarities with biological evolution. Since our views differ substantially, each of us wrote a separate subsection.

7.1 God does play dice (by NG)

Let us assume that Nature is nondeterministic: God plays dice. First, let us emphasize that this would not be the end of science. Quite the contrary, it was a fresh start for one of the most important of today’s sciences: biology. This creative time makes the evolution much more interesting. Moreover, instead of spreading out to infinity, or remaining in a boring stationary state, as with the Schrödinger equation, the system localizes dynamically, as in the QSD model. How would physical laws look like? in particular the laws describing the (nondeterministic) evolution of physical systems. I do not know. But it is likely that the evolution equations would incorporate random numbers. What is a random number: one does not really know. Actually it does not really matter. After all one does not know how to prove that a program is bugfree, but we use programs to compute the number that deterministic theories predict and we compare these numbers to experiments. Similarly we could use any reasonable random number generator to compute the number of the nondeterministic theory and compare the statistical predictions to experiments. (In [38] I have proposed axioms for propensities (true randomness) in such a way that they are determined by the set of definite (actual) properties. In this way randomness can be recognized (contrary to Kolmogorovian randomness) and - moreover - a significant part of the quantum mechanical Hilbert space
But then: when and where does chance happen? and what "causes" it? And what happens to the correlations that interactions between systems creates? Let us first consider the last question.

In classical as well as in quantum mechanics the correlations become more and more subtle as time and interactions increase. In realistic situations one can prove that quickly the correlations are so mixed up that it is impossible to put them into experimental evidence. Hence one can forget about them and consider only density matrices in quantum mechanics [39,40] or distribution functions in phase space in classical mechanics [41]. From a pragmatic point of view one can as well consider that the correlations are not only hidden, but really destroyed: there is a correlation sink. The distinction is particularly sharp in quantum physics. Either one assumes that the correlations (also called quantum entanglement) get only hidden and one is satisfied to prove that this assumption can not be falsified. Or one assumes the existence of a correlation sink and investigates the consequences. The first consequence is clearly that the Schrödinger equation would no longer be the ultimate (nonrelativistic) evolution equation. This is actually the main argument in favor of the first alternative, which I phrased on purpose in such a way to underline that it is not more scientific as the second alternative. In references [3,42] and [43] arguments in favor of each alternative have been discussed. In this article we clearly follow the second alternative, mainly because new physics is more likely to emerge from new theories than from old ones! A generalization of Schrödinger equation is either deterministic or stochastic. Deterministic generalizations, such as Weinberg’s [44], have however been ruled out by the requirement of keeping the "peaceful coexistence" between quantum mechanics and relativity [17,45,46]. This brings us back to the other questions mentioned above, concerning chance.

Let us now come to the question of physical chance and its mathematical description, specially in quantum mechanics. It seems that there are only two kinds of possible "causes" to chance:

1. It just happens, without any explanation. This requires a "universal random generator" (a God who plays dice) and physical laws to exploit this randomness to shape Nature.

2. It takes place at intersections between independent causal chains, like in Cournot’s thesis [47]. This requires a cut somewhere through Nature, in order to guarantee the independence of the causal chains, like the quantum/classical cut in the Copenhagen interpretation.

Following the first alternative, God plays dice. How could spontaneous chance be described? The mathematical Wiener process could mimic the universal random
generator: it is just a stupid Markov process that keeps forgetting every thing from its past and condemned to make again and again similar random choices! But it enables $\psi_t$, the state of the physical system, to acquire a shape and a localization. Accordingly, the quantum world takes advantage of random chance to evolve into one, among many possible, classical looking state of affair [16], as illustrated in section 6. Notice the similarity with biological evolution: there the randomness is provided by the accidental (another world for random) mutations and Nature takes advantage of these fluctuations to produce order, and even live. According to Darwinism the random mutations are independent from the environment. The latter intervenes only in the selection mechanism. Similarly in a stochastic version of the Schrodinger equation the fluctuations $d\xi_t$ could be independent of the environment, the latter taking advantage of the fluctuation to shape the physical system.

So far in the history of Sciences, people have always looked for deterministic theories behind apparently random phenomena. This has been extremely productive. The idea that God plays dice is that in the future scientists looking for stochastic theories behind apparently organized phenomena will be even more productive for Science.

7.2 We can’t tell whether God plays dice (by ICP)

The theory of probability and its problems are at least as important in biology as in physics. In the theory of evolution, random processes come in universally in the source of new variation, and also in the mixing of genes in the special case of sexual reproduction. We shall be concerned only with the new variation.

In his ‘Origin of Species’ in 1859 Darwin was unable to specify the mechanism for the origin of variation in species, and was not clear as to the causes of variability. The process of selection, whether natural or under domestication, acted on whatever variation there was, to produce new varieties and species.

The mechanism became clearer with Mendel’s 1865 rules of heredity, in terms of factors which we now call genes, which gradually became accepted after 1900. The genetic theory was only successfully incorporated into a general mathematical theory of evolution by natural selection during the 1940s. In this theory the new variation is produced by the random mutation of genes, which must be very small over a few generations for effective natural selection to take place.

It was not until it was discovered in 1953 that the DNA double helix contained the genes, that the physical basis of genetic variation could be understood.

Compare this with the quantum theories of measurement.

Although the mathematical laws of probability in the usual Copenhagen theory are clear, the theory is no more clear than Darwin about the mechanism. The ‘shifty split’ between the quantum and classical domains is no better, and no worse, than
Darwin’s vagueness about the mechanism of variation in species. Both the physical
and biological theories were powerful and convincing for the purposes for which they
were introduced, and came to dominate their fields. Both of them were incomplete.

The stochastic theories of quantum mechanics, like quantum state diffusion, are
analogous to the mathematical theories of biological evolution of the 1940s. In each
case the mechanism is clear, but the cause of the stochastic fluctuations is not. Just
as genetic mutation must be slow on the time scale of the generations, and produces
fitter species by natural selection, which cannot be produced by Mendelian rules
alone, so the process of state diffusion is slow on the time scales of the Schrödinger
equation, and gives rise to classical mechanics by localization, which cannot be
produced by Schrödinger dynamics alone. Just as the process of natural selection
led to the development of the enormous and beautiful variety of modern species
from the simplest beginnings, so process of state diffusion produces the classical
world from the very different quantum world.

The quantum equivalent of the DNA double helix would be the experimental de-
tection of the elusive quantum-classical boundary. Primary state diffusion [48,49]
is a development from QSD that suggests possible experiments to detect it. It is
much more difficult to find than the double helix of DNA, but there have been such
enormous advances in experiments on individual quantum systems, particularly in
atom interferometry [50], that it may just be possible.

What appears to be random at one level of experimental sophistication may look
deterministic at another, and vice versa. We cannot tell whether God plays dice.

7.3 Conclusions (by NG and ICP)

We have shown that it is possible for those who disagree about the philosophy to work
together on the physics! We are both very happy to dedicate both the physics and
the philosophy to Abner Shimony, who has helped us both.

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**Figure captions.**

1. Mean photon number as a function of time for the measurement process. The stochastic convergence to the eigenstates can be clearly seen.

2. The forced damped linear oscillator, showing reduction towards a coherent state which has no stochastic fluctuations.

3. Illustration of the approach to thermal equilibrium of a harmonic oscillator. The initial state is the pure number state $|3\rangle$. The two lines with large oscillations represent the mean position and momentum, $\langle p \rangle$ and $\langle q \rangle$. The two lines with smaller oscillations represent the standard deviations $(\Delta p)^2$ and $(\Delta q)^2$. This example illustrates how an arbitrary initial state tends asymptotically to a coherent state. The center of these coherent states follow then a classical stochastic process [37].

4. Symmetry breaking for a double well (at $x = +8$ and $x = -8$) potential with two dissipative environment operators acting independently on each well. The plots for a single run show the mean position $\langle q \rangle$ and the RMS deviation in position $\Delta q$. The localization in the $x = +8$ well, the reduction in the variation and the damping of the stochastic fluctuations are clearly shown.

5. A quantum cascade with emission only. The continuous state diffusion automatically produces sudden transitions between quantum states in a single run, but these are not instantaneous jumps.

6. Emergence of a classical strange attractor out of a quantum fog for a Kicked, Damped, Anharmonic Oscillator (KAOS). Poincaré sections at the period of the driving force are displayed. In the upper figure the relevant dimension of the potential is large with respect to $\hbar = 1$, hence quantum indeterminacy dominates. In the lower figure, on the contrary, the potential and damping are scaled such that the typical dimension in phase space are large with respect to $\hbar = 1$, hence the strange attractor of the classical KAOS is clearly shown. The two medium figures correspond to intermediate cases, in which random transitions between a fixed point and the strange attractor can also be seen.
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