Is quantum decoherence reality or appearance?

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Abstract. It has been experimentally demonstrated that quantum coherence can persist in macroscopic phenomena [J.R. Friedman et al., Nature, 406 (2000) 43]. To face the challenge of this new fact, in this article QM in its standard form is assumed to be extended by one beable (hidden variable), i.e., a quantum observable with always definite values in nature (but usually only statistically given in the quantum state). Localization is taken as the most plausible beable. The paradoxical aspects of conventional QM take now a different form. Suitably defining the notion of "subject" fully within the QM formalism, proving the quantum conditional subsystem-state theorem, and choosing the relative-decoherence interpretation of QM, the paradoxes formally disappear, leaving one with decoherence relative to the definite values of the beable; thus being only appearance, not absolute reality in QM. Relative to a different subject one has perseverance of coherence. Hence, in this approach it is claimed that decoherence and coherence, both exist in reality, but are not "seen" by the same subject, and "subject" is, in this interpretation, indispensable. The two mentioned, apparently contradictory phenomena, are in this way decoupled from each other, contradiction is avoided, and any one of the two can be treated in the way that is usual in QM.

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1. INTRODUCTION

As it is well known, the linear dynamical law of standard quantum mechanics (QM) does not imply, in general, definite measurement results in disagreement with experience. Namely, spontaneous evolution preserves coherence, and definite measurement results require decoherence. Therefore, the basic question is the one in the title of this article. It can be put also as follows: Is decoherence within the quantum-mechanical formalism absolute, i. e., a part of absolutely objective reality, or, perhaps, it appears only relative to a suitable subject, i. e., isn’t it actually only appearance with restricted quantum-mechanical reality?

Putting it in yet another way, the question is whether coherence (or superposition of states) perseveres in macroscopic phenomena. Affirmative experimental answers to this question have been obtained recently [1], [2].

Various interpretations of QM resolve the mentioned fundamental difficulty by going beyond QM in a number of different manners [3]. The Ghirardi-Rimini-Weber (GRW) theory [4] is, perhaps, the most popular present-day approach to QM.

Hidden variables (HVs) [5] attempt to do this in another way with the advantage that standard QM is left unchanged (in contrast to what happens in the GRW theory). Bell’s theorem [6] claims that QM precludes local hidden variables.

The well known fact that Bell’s theorem and its numerous proofs arose an enormous interest in the literature shows that local HVs were a great hope. Nobody seems to like what appears, at first sight, to be the only idea that remains: nonlocal HVs.

Standard HV theories assume definite values (in nature) for all quantum observables. Within what remains possible there is an alternative to nonlocality: Assuming that only one quantum observable has always definite values in nature irrespective of the quantum state. This observable is, naturally, the position (or localization) observable. We have the case of definite-position extended QM in the renowned approach of Bohm [7], [5], and in some speculations of Bell [8], following whom we call this privileged observable a beable.

In a recent article [9] Ghirardi has suggested an experimentum crucis to distinguish between the GRW theory and any approach that preserves the whole of standard QM (like the mentioned beable extension). Since the suggested experiment should soon be feasible, and since it is conceivable that the outcome will be unfavorable for the GRW theory, it is important to consider a serious alternative. To present such an alternative is the basic aim of this article.

The approach presented below defines ”subject” in a simple way within QM, and then treats it as indispensable. To some extent this might fulfill the expectation of Wilczek [10] though he envisages ”subject” as a formidably complex notion.

It is shown below that in the suggested one-beable-extended QM the mentioned fundamental difficulty disappears, or, to be more precise, it takes a form that may be acceptable within QM.
Relative decoherence

2. THE COHERENCE WHICH IS THE STUMBLING BLOCK

Let subsystem 1 be the object on which some dichotomic observable is measured, and let subsystem 2 be the measuring instrument. Let, further, \( Q'_2 \) and \( Q''_2 \), be two orthocomplementary projectors for the measuring apparatus such that their occurrence (as events) means that the "pointer observable" takes up the respective "pointer positions" (meant symbolically). Let, finally, the state into which the system \( 1 + 2 \) evolves due to the measurement interaction be in terms of state vectors (written as Dirac kets):

\[
|\Psi\rangle_{12} = (w')^{1/2} |\Psi\rangle'_{12} + (w'')^{1/2} |\Psi\rangle''_{12},
\]

where

\[
|\Psi\rangle'_{12} \equiv (w')^{-1/2}Q'_2 |\Psi\rangle_{12}, \quad |\Psi\rangle''_{12} \equiv (w'')^{-1/2}Q''_2 |\Psi\rangle_{12}
\]

\( \text{(2a, b)} \)

(whether one, actually, has \( 1 \otimes Q'_2 \) instead of just \( Q'_2 \), is, of course, seen from the context). The expansion coefficients, which are assumed to be positive, are given by the usual expressions:

\[
w' \equiv \langle \Psi |_{12} Q'_2 |\Psi\rangle_{12}, \quad w'' \equiv \langle \Psi |_{12} Q''_2 |\Psi\rangle_{12}.
\]

\( \text{(2c, d)} \)

The measurement paradox consists in the lack of definite measurement results in (1), which has "inherited" the, in general existing, coherence in the initial state of the object subsystem. However, if we assume that QM is extended by the localization beable, and that the two "pointer positions" have a literal meaning (they do not overlap spatially, and their sum gives the entire space), then ipso facto, as an immediate consequence of the beable postulate, for each individual \( 1 + 2 \) system the "pointer" will take up one position, i.e., either \( Q'_2 \) or \( Q''_2 \) will occur in nature, but, in our theory, this is on the beable level.

There are now definite measurement results. But the coherence of the two terms (two results) in (1) persists, and, as it is well known in QM, one must apply the entire superposition to the individual system. Hence, at first sight, we have no less a paradox than without the localization beable.

Besides, having both coherence and decoherence at the same time makes things confusing because they seem to contradict each other.

It is shown in this article that QM can be so understood that only one (coherence or decoherence) at a time is taken into account. Thus, decoupling one from the other, one can make use of the quantum-mechanical formalism in the usual way.

Returning to the above paradox, there is, in addition, no way to decompose the state \( |\Psi\rangle_{12}\langle\Psi |_{12} \), which is a pure state in QM, into the two substates (empirically subensembles) corresponding to the two different results.

One has a similarly paradoxical situation in the rest of the known variations on the measurement paradox: in the case of "Schrödinger’s cat" [11], where, e.g., subsystem 2 is the "cat" and 1 consists of the rest of the deadly contraption; or in the Zurek environment theory [12], where 1 is a classical system, and 2 is the environment.
Relative decoherence

In a further variation on the measurement paradox, in the paradox of "Wigner’s friend" subsystem 2 includes the consciousness of "Wigner’s friend”, who looks at the ”pointer” and sees its ”position”. The coherent state (1) is what ”Wigner”, or rather his consciousness, ”sees”.

"Wigner” has the same difficulty as we had in the measurement paradox: the ”friend” sees a definite result, and ”Wigner” himself knows, reading (1), that both results are coherently present in each individual case.

3. THE EXPERIMENTAL SITUATION

I have tacitly applied, what is called, QM of extended validity in the preceding section, viz. I have applied QM to macroscopic phenomena, which involve millions (up to $10^{23}$) microscopic constituents. Otherwise, there is no paradox. It is a well established fact that QM is fully applicable to purely microscopic phenomena.

Macroscopic objects and phenomena are characterized by so-called macrorealism, which, in our case described by (1), claims that $|\Psi\rangle_{12}$ has to be replaced by the mixed state

$$
\rho_{12} \equiv w' |\Psi\rangle'_{12} \langle \Psi|'_{12} + w'' |\Psi\rangle''_{12} \langle \Psi|''_{12},
$$

if the occurrences of the events $Q'_2$ and $Q''_2$ are classically distinguishable because it is an empirical fact that in such a case there is no coherence. But is this really so?

Many experimental and theoretical physicists put in much effort to find an answer, especially Leggett (see e. g. [4] and the references therein). Finally, an answer has been obtained that has confirmed coherence (against macrorealism) by two research teams [1] and [2] (cf also [3]). This came after a number of efforts with much progress in demonstrating the macroscopic quantum behaviour of various systems such as superconductors, nanoscale magnets, lasercooled trapped ions, photons in a microwave cavity and $C_{60}$ molecules (see references in [4]).

In the two mentioned conclusive experiments [1], [2] $Q'_2$ and $Q''_2$ in (1) are SQUID currents in the two opposite directions, and the existence of (1) has been ingeniously demonstrated. Thus, millions of electrons can flow in both directions simultaneously in a coherent way. They do not behave macrorealistically.

Still, one must be aware that a SQUID is a macroscopic device with a quantum phenomenon: the quantized magnetic flux. (Phenomena of this type occur in superconductors, superfluids etc.) We are far away from an experimental demonstration of coherent mixing of classically distinguishable properties in a purely classical object. "Measuring instrument”, "Schrödinger’s cat”, ”Wigner’s friend” etc. are still in the realm of speculations. But experimental techniques are improving with miraculous speed. Hence, I believe that the approach of this article is presented none too soon.
4. THE "MONSTROUS" SUBENSEMBLES AND THE CORRESPONDING QUANTUM MECHANICAL SUBSYSTEM STATES

Let us return to the first paradox (that of measurement). Let us take the empirical view and imagine the (laboratory) ensemble described by \(|\Psi\rangle_{12}\). We denote it by \(E\). On account of the definite-localization postulate, by which QM is extended in the advocated approach, we can imagine \(E\) as consisting of two subensembles: the one made up of those individual systems on which \(Q'_2\) occurs on the beable level, and the one containing the rest (on which \(Q''_2\) occurs on the same level). There is no way to describe these subensembles in QM, i.e., one cannot associate quantum-mechanical states with them. We call them "monstrous" subensembles, and we denote them by \(E'\) and \(E''\) respectively.

We prove now that, nevertheless, subsystem 1 is described in the "monstrous" subensembles by the respective quantum-mechanical conditional states (statistical operators), which have the form as if \(Q'_2\) or \(Q''_2\) occurred in ideal quantum measurement on subsystem 2 (on the quantum-mechanical level):

\[
\rho'_1 \equiv (w')^{-1} \text{Tr}_2 \left( |\Psi\rangle_{12} \langle \Psi|_{12} Q'_2 \right), \quad \rho''_1 \equiv (w'')^{-1} \text{Tr}_2 \left( |\Psi\rangle_{12} \langle \Psi|_{12} Q''_2 \right),
\]

(4a, b)

where "\(\text{Tr}_2\)" denotes the partial trace over subsystem 2. The probabilities of the two results, and, at the same time, the statistical weights of the two "monstrous" subensembles \(E'\) and \(E''\) in \(E\), i.e., \(w'\) and \(w''\), are given by (2c,d) above.

To prove the stated quantum conditional subsystem-state theorem, which is the basis of the approach advocated in this article, we must give a precise definition of a beable, which we assume to have a purely discrete spectrum. (Since we are concerned with position, one should think of intervals, cf subsection 12.C.) Let us take a general quantum state described by a statistical operator \(\rho\).

(i) Each particle always has a definite value from the spectrum of the beable (quantum-mechanical observable) in nature irrespective of the quantum state \(\rho\). (If the quantum state does not predict the mentioned beable value, then the latter is called "hidden").

(ii) When the beable observable is measured, and the quantum state does not predict a definite value of the beable observable, then it gives on each individual system the "hidden" value of the beable. If the quantum state does predict a sharp value, it is the same as the value on the beable level.

(iii) If the laboratory ensemble that gives empirical representation to \(\rho\) has \(N\) systems, then \(N_i\) of these have a given beable value, where \(\sum_i N_i = N\) (the summation is over all beable values), and the relative frequencies \(N_i/N\) reproduce the probabilities predicted by QM in the usual way:

\[
N \to \infty, \quad \Rightarrow \quad N_i/N \Rightarrow w_i = \text{Tr} \rho Q_i,
\]

(5)

where \(Q_i\) are the characteristic projectors of the beable observable, and \(w_i\) are the statistical weights of the "monstrous" subensembles in the entire ensemble that gives empirical representation to \(\rho\). (This is a generalization of \(w'\) and \(w''\), cf (2c,d).)
A proof of this theorem is given in the Appendix. In previous work the second requirement in the above definition of a beable observable was put more broadly, and the quantum conditional subsystem-state theorem was proved in a more complicated way.

Thus, as far as subsystem 1 is concerned, the "monstrous" subensembles \( E' \) and \( E'' \) are seen to be described by statistical operators, and thus, they are normal quantum-mechanical states (with no monstrosity whatsoever). The subensembles \( E' \) and \( E'' \) are still monstrous regarding subsystem 2 and with respect to the whole (in general, entangled) 1 + 2 system.

Now it is clear that, if we want to rid ourselves of the monstrosities, we must find a way to restrict consistent quantum-mechanical description to subsystem 1, and, nevertheless, we must be able to recover the description of subsystem 2 and of the entire system.

5. SHIFT OF THE SPLIT AND CONVERSION OF THE ENVIRONMENT INTO SUBJECT

The object \((O)\) of quantum-mechanical description is, as a rule, a restricted part of the universe, and the rest, usually called the environment, is left out. In the decoupling of decoherence from coherence, that I am striving to expound, I want only the former to belong to the object. But I do not want coherence to be quite left out. It should be present in the omitted environment, converting it into, what I call, the subject \((S)\) (playing the role of a kind of "observer" that is defined strictly quantum mechanically). Between the object and the subject there is an imaginary cut (/); altogether we have a split: \(O/S\). If the environment is not yet converted into subject, we write \(O/\ldots\).

The cut is due to a subjective choice on our part, and it can be shifted to broaden the object. This is important for the introduction of the beable observable in order to convert the environment into subject. Afterwards, the cut is shifted back.

Let us elaborate this on the mentioned paradox of measurement theory.

6. DEFINITE BUT RELATIVE MEASUREMENT RESULTS

Relation (1) applies to the composite system object plus measuring apparatus at the end of the measurement interaction. Calling this system \(1 + 2\), we can start with the split \(O/\ldots \equiv 1/(2+\text{the rest of the universe})\). To convert the environment, which is now \((2+\text{the rest of the universe})\), into a subject, we shift the cut broadening the object, i.e., we go over to \(O/\ldots \equiv (1 + 2)/\text{(the rest)}\). Then the localization beable observable can be normally introduced as a suitable second-subsystem observable (with only two characteristic projectors \(Q'_2\) and \(Q''_2\) - corresponding to two intervals on the pointer scale - in our simplified discussion). We can then shift the cut back to obtain a description
of subsystem 1 with the environment converted into a subject: $O/S \equiv 1/(2+\text{the rest})$.

Now the desired decoupling of decoherence from coherence is achieved. At the end of measurement we have subsystem 1 as our object of quantum-mechanical description. We have the decoherence:

$$\rho_1 = w'\rho_1' + w''\rho_1''$$

(6)

where $\rho_1 \equiv \text{Tr}_2 |\Psi\rangle_{12} \langle \Psi |_{12}$ is the subsystem state decomposed into the two quantum conditional subsystem-states defined in (4a,b) (the respective first-subsystem states of the composite "monstrous" subensembles $E'$ and $E''$).

We have now relative decoherence, namely, for each individual-system case the first-subsystem state is either $\rho_1'$ or $\rho_1''$ (but not both, i.e., not $\rho_1$), and it is a state relative to the "pointer position", i.e., relative to the occurrence of $Q_2'$ or $Q_2''$ on the beable level. In this manner one has definite results in a quantum-mechanical description.

There is nothing wrong with the objectivity or realness of the decoherence on the quantum-mechanical level. But this objectivity is not in the sense of absolute observer independence, as one expects in the case of absolute decoherence (which is much favored in the literature [3], [4]).

The decoherence is "real" for anyone who "looks" at the "pointer", or to whom this observer might communicate the result of measurement. The mentioned "looking" or the mentioned communication amounts to an interaction that results in a new composite pure state of the type (1), in which subsystem 2 now contains, besides the "pointer positions", also the consciousness of the first and possibly that of other observers. We have now a more intricate composite subject. In case there are two or more human observers, they all see the same measurement result.

In a theory with subjects, "reality" or "objectiveness" is recognized by intersubjective agreement. Naturally, the expounded relative decoherence does have the attributes of reality because there is sufficient intersubjective agreement. Though, there is no universal intersubjective agreement as one would have in the case of absolute decoherence. Thus, one might say that decoherence is only appearance within the quantum-mechanical formalism.

Returning to the case prior to communication, when subsystem 2 contained only the "pointer positions", we can say that the coherence has not disappeared. We can take another measurement apparatus as subsystem 3, and define its "pointer observable" as a beable observable. For simplicity, we assume that we have again localization intervals as the "pointer positions". (For a more general case, cf subsection 12.G.)

We make the shift to the split $O/\ldots \equiv (1+2+3)/(\text{the rest})$, introduce the new beable observable, and shift back to $O/S \equiv (1+2)/(3+\text{the rest})$. The environment is again converted into a subject, and the second apparatus (possibly handled by a second observer, all included in subsystem 3) begins the second measurement on the coherent state given by (1).

The coherence in (1) is real, as the second observer can easily ascertain by performing a measurement of the coincidence occurrence of two suitably chosen subsystem events (projectors): $P_1P_2$. The suitability of the subsystem projectors
Relative decoherence

consists in their ability to give a different probability in the pure state $|\Psi\rangle_{12}$ given by (1) than in the corresponding decoherent quantum state $\rho_{12}$ (cf (3)). Since the former probability is

$$
\langle \Psi |_{12} P_1 P_2 | \Psi \rangle_{12} = w'\langle \Psi |'_{12} P_1 P_2 | \Psi \rangle'_{12} + w''\langle \Psi |''_{12} P_1 P_2 | \Psi \rangle''_{12} +
$$

$$(w')^{1/2}(w'')^{1/2}\langle \Psi |'_{12} P_1 P_2 | \Psi \rangle''_{12} + (w'')^{1/2}(w')^{1/2}\langle \Psi |''_{12} P_1 P_2 | \Psi \rangle'_{12},$$

and the latter, i. e., $\text{Tr}_{12}P_1 P_2$, equals the sum of the first two terms above, ”suitability” amounts to the last two terms giving a nonzero number. Such a choice of the projectors is, in principle, possible, and therefore the reality of the coherence in (1) is an experimentally establishable fact.

By decoupling we have achieved that decoherence and coherence now do not contradict each other. Namely, they are relative to different observers: the decoherence is relative to the first measuring apparatus (and, possibly, the first observer if it is included in subsystem 2) at the end of the first measurement, and the coherence is relative to the second measuring apparatus (and, possibly, the second observer) at the beginning of the second measurement.

The important point to notice is that, like in my previous article on the bubble chamber [17], I propose an interpretation of QM, which may be called the relative-decoherence interpretation, in which quantum-mechanical prediction always presupposes prior conversion of the environment into a subject, i. e., introduction of a suitable beable observable defined in a subsystem of the environment.

This is hardly a new interpretation having in mind that Niels Bohr insisted on the importance of observer and on its inseparability from object in QM, as it is well known. I only propose a very concrete way of doing this by shrinking Bohr’s well known idea of the role of a classical measuring apparatus to a simple beable observable (with a discrete spectrum), which is mainly localization in a perfectly classical sense. This approach stands opposed to the approaches without subject and with absolute decoherence, especially to the GRW theory [4].

7. THE ROLE OF ENVIRONMENT ACCORDING TO ZUREK

According to the Zurek theory [12], interaction with the environment can lead to absolute decoherence on account of his claim that it is position-measurement-like, and thus explanation is obtained why classical objects do have definite positions (as opposed to superpositions of them).

Nevertheless, it is hard not to agree with the obvious criticism that goes as follows: If one takes a sufficiently large system consisting of the classical objects plus environment (possibly the whole universe), then the interaction will, if it is precisely position-measurement-like, give a composite state vector as in (1), where 1 are the classical objects and 2 is the environment; where, further, not only $Q'_2$ and $Q''_2$ represent different
distributions of matter, but also the corresponding first-subsystem states $\rho'_1$ and $\rho''_1$ imply nonoverlapping different definite positions with certainty. But there is no way to obtain absolute decoherence, because the rules of QM require that each individual $1+2$ system be described by $|\Psi\rangle_{12}$ (cf (1)) (cf also D’Espagnat’s discussion [18] of improper mixtures as $\rho_1$ in (6) is).

On the other hand, if one assumes that position of every particle is a beable, then so is the matter distribution in the environment (subsystem 2), and, utilizing it, we can convert the environment into, what I have suggested to call a "subject".

Then we do have absolute decoherence of the distinct positions of subsystem 1, but only outside QM, via the "monstrous" subensembles $E'$ and $E''$ respectively (cf section 3). To bring this into the formalism of QM, one may apply a procedure as in the preceding section and obtain relative decoherence in $\rho'_1$ and $\rho''_1$, but this time relative to the different matter distributions in the environment.

One still has coherence in the larger system $1+2$, but it is irrelevant for the classical objects (subsystem 1).

8. "SCHRÖDINGER’S CAT"

Let now $|\Psi\rangle_{12}$ given by (1) describe the end of the thought experiment with Schrödinger’s cat [11], where $Q'_2$ and $Q''_2$ are associated with the dead and the live cat respectively. In full analogy with the case of end of measurement interaction (section 6), "Schrödinger" himself may be subsystem 3, who in the individual-system case "sees" a superposition of the dead and live cat described by (1). In spite of this, it can be seen in two ways that the individual cat is either dead or alive:

(i) Let subsystem 2 be the cat, and let one assume, further, that the consciousness of the cat is a beable, and $Q'_2$ and $Q''_2$ are projectors meaning "dead" and "live" respectively. Then the cat can be the subject, and all inanimate parts of the deadly contraption, i. e., subsystem 1, be the object with state decomposition (6) expressing relative decoherence of the decayed atom (and broken ampula) and the undecayed atom (and whole ampula). Again, of course, this quantum-mechanical decoherence takes place relative to the contents of the consciousness of the cat, or rather relative to "extinguished consciousness" and "active consciousness" respectively.

(ii) One can introduce the position beable observable for the material particles, and take for subject the second subsystem with the distribution of matter in the ampula. Then the dead and live states of the first subsystem, the object, which is now the cat, are decoherent relative to the distinct distribution of matter in the ampula in case of decay and nondecay of the atom respectively. This case is symmetric to the previous one: $Q'_2$ and $Q''_2$ express the mentioned distinct distributions of matter, and (6) applies to the ensemble that is a mixture of dead and live individual cats.

In the advocated approach, one must decide on one of the two preceding alternatives for choice of object and subject. One may change over from one to the other, but one
Relative decoherence

cannot have both simultaneously.

Though the choice of subject and object is arbitrary, once made, one must stick to it in one discussion. This is similar to the case of tensor product in treating composition of subsystems in the formalism of QM: one can change over to a different order of the tensor factor state spaces (the change is an isomorphism), but one must stick to one order for the whole of an argument.

Returning to any of the two possibilities of discussing Schrödinger’s cat, one must emphasize that “Schrödinger” himself “sees” coherence, because his object is \((1 + 2)\).

We see again that introduction of a beable brings in coexistence of decoherence (in the “monstrous” subensembles on the beable level) and coherence (on the QM level). To treat both within QM, one can decouple them from one another by a suitable choice of subject and object (as illustrated).

9. ”WIGNER’S FRIEND”

Let us take as a final illustration the mentioned case of ”Wigner’s friend”. Having system 1+2 in mind, we can start with the split \(O/\ldots \equiv (1+2)/(\text{the rest})\). To introduce a suitable subject, we can assume, with von Neumann, that human consciousness is always definite in the case when an observer is looking, i. e., that it is a beable (though von Neumann might not have agreed with treating his idea in this way).

This beable is, actually, an empirical fact (we know that it is so), put as a postulate into extended QM.

Thus, let us take the beable observable which is the consciousness of ”Wigner’s friend”. It is a second-subsystem observable with only two characteristic projectors \(Q_2'\) and \(Q_2''\) in our simplified discussion. We can then shift the cut to narrow down the object (and to get rid of the coherence in it), i. e., we may go over to the split \(O/S \equiv 1/(2+\text{the rest})\) (with a well-defined subject).

Now the desired decoupling of decoherence from coherence is completed. The object of quantum-mechanical description of ”Wigner’s friend” is subsystem 1. The ”friend” ”sees” the decoherence expressed in relation (6).

We have now relative decoherence, viz., for each individual-system case the first-subsystem state is either \(\rho_1'\) or \(\rho_1''\), and it is a state relative to the contents of the consciousness of ”Wigner’s friend”, who sees a definite result.

We can say that the coherence has not disappeared. We can take ”Wigner”’s mind as subsystem 3, and define ”Wigner”’s consciousness as a beable observable. We make the shift to \(O/\ldots \equiv (1 + 2 + 3)/(\text{the rest})\), introduce the new beable observable, and shift back to \(O/S \equiv (1 + 2)/(3+\text{the rest})\). The environment is again converted into a subject, and ”Wigner” ”sees” the coherent state given by (1).

By decoupling decoherence from coherence, we have achieved that they do not contradict each other. This is so because they are relative to different observers: the first is relative to ”Wigner’s friend”, and the second is relative to ”Wigner”.

10. RELATION TO EVERETT

The definition of the conditional first-subsystem state (4a) even with a general composite-system state \( \rho_{12} \) replacing \(|\Psi\rangle_{12}\langle\Psi|_{12} \) and with \( Q_2' \) as an arbitrary second-subsystem event (projector), is a part of standard QM formalism. Its physical meaning is the conditional state in the case of ideal measurement of \( Q_2' \) in \( \rho_{12} \). This is well known, because the standard Lüders formula, which describes change of state in ideal measurement, converts \( \rho_{12} \) in the ideal occurrence of \( Q_2' \) into \((w')^{-1}Q_2'\rho_{12}Q_2'\), and partial trace of this is the RHS of (4a).

That \( \rho_1' \) has the physical meaning of the conditional state in any occurrence of \( Q_2' \) is not well known, but true \[19\]. As it has been demonstrated in this article, this physical meaning extends even to the case when \( Q_2' \) occurs on the beable level. I have called \( \rho_1' \) the relative state, meaning "the state with relation to" the beable occurrence of \( Q_2' \).

My "relative state" formally generalizes that of Everett \[20\]. Namely, when \( \rho_{12} \) is a pure state \( \rho_{12} \equiv |\Psi\rangle_{12}\langle\Psi|_{12} \), and \( Q_2' \) is an elementary projector (a quantum logical atom) in the state space of the second subsystem, i.e., \( Q_2' \equiv |\phi\rangle_2\langle\phi|_2 \), my relative state reduces to Everett’s. One can choose a second-subsystem complete orthonormal basis with \(|\phi\rangle_2 \) as one of the basis vectors, expand \(|\Psi\rangle_{12} \) in this basis, and then one has Everett’s formal context in which the conditional state \( \rho_1' \), which is now also a pure state, appears. This is what he called "relative state".

Since Everett did not introduce beables, and since the choice of the mentioned basis was quite arbitrary, his relative state was physically ill founded. I believe that he himself realized this and that his construction of the "branching of the universe" was seeking a way out of this. But, I agree with Shimony \[21\] that Everett’s "branching" is only a semantic innovation, not a new idea, and therefore also Everett’s "branching of the universe" is not less ill founded conceptually. The universe is hard to be thought of as "branching" in a noncountable infinity of ways at the same time (because this is the amount of nonuniqueness in the choice of Everett’s basis).

Anyway, I consider Everett’s article as an important step towards the relative decoherence interpretation of QM advocated in this article. (I have learnt a good deal from Everett.)

11. RELATION TO THE THEORY OF GHIRARDI, RIMINI AND WEBER

The mainstream of interpretative thought on the foundations of QM goes, as well known, towards absolute decoherence within QM. In my simplified presentation in this article this would amount to transition from the pure composite system state \(|\Psi\rangle_{12} \) given by
Relative decoherence

(1) to the mixed composite state given by

$$\rho_{12} \equiv Q'_2 |\Psi_1 \rangle_{12} \langle \Psi_1| Q'_2 + Q''_2 |\Psi_1 \rangle_{12} \langle \Psi_1| Q''_2.$$ 

Since this cannot be achieved by the use of purely QM ideas, one always adds an extra-quantum-mechanical stipulation.

The Ghirardi-Rimini-Weber (GRW) theory [4] seems to me to be the climax of absolute-decoherence endeavors. As such it has a strong appeal to many leading foundationally and realistically minded physicists, who try to comprehend what QM is really about.

Basically, I have no objection to the GRW theory. If one can interpret QM in an objective way without "observer" (cf Bell’s criticism in [22]), one should do so. If it were a question of taste, perhaps I myself would be in favor of the GRW alternative, and against my advocated relative-decoherence interpretation. But this is not a matter of taste. It is a question of fact if the coherence persists in the macroscopic phenomena or not.

There is much theoretical and experimental research done these days determined to discover the answer to this question (see section 3). I am sure that in a few years we will know with certainty if the coherence in measurement (or measurement-like processes like the interaction with the environment in the Zurek theory) persists or not in macroscopic phenomena. If it does not, i.e., if decoherence is fact (in an absolute sense), one can forget about the relative-decoherence interpretation advocated in this article. But if coherence does persist, if it is reality, then decoherence is only appearance (in QM). In this case, one will hardly find a simpler way than the expounded proposal of relative decoherence to express the mentioned persistence of coherence not going outside the formalism of standard QM.

12. CONCLUDING REMARKS

A) Why one hidden variable? The definite-position extension of QM advocated in this article is a minimal extension. Quantum mechanics allows one to do such an extension with any one observable (or any set of compatible observables) in a natural way and without no-go troubles. The advocated theory is sufficient for our purposes. It does not preclude a wider consistent extension. After all, I have myself added a second beable in the form of the consciousness of "Schrödinger’s cat" or of "Wigner’s friend", cf also subsection 12.G. (It is likely that consciousness as an observable is compatible with the localization observable, so that, formally, we have extension with one observable, which is a set of two.)

B) Why the position observable? Space (the spectrum of the position observable) does, no doubt, occupy a unique place in physics. We have relativity and
Relative decoherence

kinematics (both nonrelativistic and relativistic) in space. The spectra of other observables do not have so much physical significance.

C) The position observable has a purely continuous spectrum. The definite position stipulation, of course, assumes that each particle (or pointlike part of a particle when the latter has a finite volume) has a definite position, i.e., a point from the continuous spectrum of the position observable. But, as well known, no measurement gives as a result the point of the continuous spectrum. Hence, the subject observable is naturally defined as one with a purely discrete spectrum. Any breaking up the real axis into nonoverlapping intervals gives, essentially, such an observable, and the particle (or part of it) belongs to precisely one of the intervals. (We had only two intervals in our simplified approach.)

D) Spatial nonlocality. One wonders if definite-position QM is compatible with the known fact of the existence of spatial nonlocality like the one in Young’s two-slit interference \[23\] or in the Mach-Zehnder interferometer. (Both have been performed with massive particles \[24\], \[25\].)

If we want to imagine a ”physical mechanism” in these experiments, then standard (unextended) QM offers that of a delocalized particle that ”passes both slits” or ”goes through both branches” of the interferometer, and then, when the two latent or potential ”paths” meet, they interfere.

Adherents to the definite-position extension of QM reject such ”potential paths” as physical nonsense. To their minds the ”physical mechanism” is necessarily based on real, though unknown, paths: the individual particle goes through one slit or through one branch. But then a nontrivial price has to be paid for an explanation of the interference. There seems to be no other way than to reinstall some kind of action at a distance in physics: the particle must ”feel” if the other slit is open or if the other branch can be traversed or not. (See, e.g., the fascinating experiments on interactionfree measurement \[26\], \[27\], and \[28\].)

One way to do this is in the manner of the de Broglie-Bohm school of thought: through nonlocal potentials depending on the wave function of the particle \[3\]. This may seem not more ”palatable” than the above mentioned delocalization in the way of potential paths. But spatial nonlocality (on the one-particle level!) is a fact that must be faced either in a kinematical way as in unextended QM through delocalization, i.e., by being ”smeared out in space”, or in a dynamical way as, e.g., in the mentioned de Broglie-Bohm theory. (Or in a third way if anybody can think of it.)

E) The Role of Classical Physics. Niels Bohr, repeatedly emphasized that the validity of classical physics must be assumed on the same level as that of QM in order to be able to interpret measurement results and hence QM itself.

The definite-position stipulation can be viewed as a realization of this idea of Bohr: It introduces into the extended theory a sufficient amount of classical ideas, and, on the
other hand, in contrast to the Copenhagen approach [29], these ideas are introduced in a perfectly natural way as far as QM is concerned. Namely, any density matrix gives a probability measure on the spectrum of any observable. In view of the fact that measurements are made on ensembles of individual systems, the natural way to interpret such a probability measure is the definite-value stipulation of the observable at issue on the individual-system level. This is called ”ignorance interpretation” of the probability in QM. If all observables are encompassed, one has a standard hidden-variables theory (cf also subsection G) below).

F) Objectivity of QM. The advocated approach assumes extended QM (encompassing measuring instruments, ”cats” and ”friends”). Then in nature both coherence and decoherence are fully objective. In QM decoherence (in an absolute and universal or fully subject-independent sense) is only appearance; its reality is restricted to suitably chosen subjects, i. e., it is relative. Also coherence in QM is observer dependent in the advocated relative-decoherence interpretation.

G) What about nonspatial ”pointer observables” in measurement? It is known that the different ”pointer positions” can be distinct values of some observable that is not even compatible with position and that these ”positions” of the ”pointer observable” can take place at one and the same location. To cover this case, the theory should be further extended by the ”pointer observable” in question, so that, at the end of the measurement interaction, we have a two-hidden-variables extension. This is possible, as one easily concludes from the fact that QM allows extension by all hidden variables, i. e., all observables can be considered to have definite values from their spectra at the price of introducing contextuality (I, 30, 31). As a consequence of contextuality, the definite value is attributed to observable plus context. In our case, we have only two contexts: the one defined by the position, and the one defined by the ”pointer observable”, both in the sense of minimal measurement [32]. (We forget about all the rest of hidden variables.) These two will raise no intuitive paradox of nonlocality.

One wonders how the definite values of the ”pointer observable” may come about, since, in our one-hidden-variable theory, these values are not always definite. There seem to be two possibilities:

(a) The very dynamics on the subquantum level that brings about some classical systems, like the measuring instrument, may be responsible for the definite values of the ”pointer observable”.

(b) It may be that some measurement interactions give rise to them on the beable level.

H) Subquantum dynamical law. If a particle has a definite position, as assumed in this approach, one must find a law how it changes spontaneously in time (a subquantum dynamical law). The theory of Bohm et al [4] does give an answer to this question. But there are also other ideas [8].
I) **Relation to the universe.** In conventional QM [29] it is hard to make sense of the idea of a wave function of the universe. In the advocated relative-decoherence interpretation of QM all reality lies in the mutual statistical correlations between the different parts of the universe, and the mentioned wave function need not have more meaning than that.

In conventional QM one may think of obtaining the quantum state (statistical operator) of a part of the universe by tracing out all other subsystems in the mentioned wave function (written as a statistical operator). In the relative-decoherence interpretation, one must allow for the definition of the subject observable in the environment (as explained), and hence it cannot be irrevocably traced out.

J) **From conditional probabilities to first-person description.** Careful analysis reveals that one requires collapse only when a second measurement is to be performed on the basis of a fixed result in the first measurement. Adherents to the no-collapse approach repeatedly point out that one can do without the idea of collapse by utilizing conditional probabilities.

The weak point of this view is that this approach lacks the very idea of occurrence of an event, and hence there is no way to understand what the "condition" of the conditional probability is. Besides this conceptual incompleteness, there is also the fact that this approach is a purely third-person description, as if God were viewing what is going on in this world. The approach does not allow one to introduce a first-person description, and it is a phenomenological fact that, in principle, I can do the experiment and it is natural that I want a first-person description. The relative-decoherence approach advocated in this article provides one both with the idea of occurrence (possession of a certain position on the beable level), and with a first-person description.

K) **Relation to my previous "relative-collapse interpretation"** [17]. "Relative collapse" and "relative decoherence" are, of course, one and the same thing. They differ only in emphasis whether it is on the definite-result subensemble (so-called selective measurement) or on the entire ensemble (so-called nonselective measurement). So I am advocating one and the same interpretation. But there is a difference between the position I took in my preceding article [17] and that in the present one. Namely, in the former, the relative-collapse postulate, as I called it, assumed that each individual quantum system possesses a definite value of the subject observable, which was any chosen fixed observable with a purely discrete spectrum. The assumed definite values were a kind of self-collapse. This was a part of the definition of the subject. It was difficult to accept all this. In the present paper there is mainly one subject observable: the position observable. The mysterious self-collapse is replaced by the stipulation that each particle always has a definite position in nature. This is easier to accept.

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13. APPENDIX

Proof of the the quantum conditional subsystem-state theorem.

We return to (1), and assume that $Q'_2$ and $Q''_2$ are the two characteristic projectors of our dichotomic beable observable for subsystem 2. Let, further, $C_1$ be an arbitrary first-subsystem observable with a purely discrete spectrum, and let $\langle C_1 \otimes 1 \rangle_{E'}$ be the average of $C_1 \otimes 1$ in the "monstrous" subensemble $E'$, determined, in principle, by the occurrence of $Q'_2$ on the beable level. (I do not assume that the average is a finite number.) Finally, let the $N'$ individual systems in the subensemble $E'$ be ordered in some arbitrary but fixed way and enumerated by $j = 1, 2, \ldots, N'$. Let then measurement of $C_1$ produce the results $\{c_j : j = 1, 2, \ldots, N'\}$. One can write:

$$\langle C_1 \otimes 1 \rangle_{E'} = \sum_{j=1}^{N'} c_j / N' = (N / N') \sum_{j} c_j / N \rightarrow (w')^{-1} \text{Tr}_{12} |\Psi\rangle_{12} \langle \Psi|_{12} (C_1 \otimes Q'_2) = \text{Tr}_1 C_1 \rho'_1,$$

when $N \rightarrow \infty$

(cf (4a)). (I have here tacitly assumed that the ideal quantum-mechanical occurrence of $Q'_2$ does not change any of the values $c_j$ on subsystem 1. It only reveals the hidden beable value of $Q'_2$. This is, actually, a tacit locality assumption on the beable observable.)

The proof for the other value of the beable (where $N'', E'' Q''_2$ appear) runs, of course, symmetrically. This ends the proof.

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