Abstract. Studies of the observed characteristics of black-hole (BH) X-ray binaries can provide us with valuable information about the process of BH formation. In this paper I address some of the aspects of our current understanding of BH formation in binaries and point out some of the existing problems of current theoretical models. In particular, the measured orbital periods and donor-star properties indicate that a common-envelope phase appears to be a necessary ingredient of the evolutionary history of observed BH X-ray transients, and that it must be associated only with a modest orbital contraction. The timing of this common-envelope phase is crucial in determining the final BH masses and current evolutionary models of mass-losing massive stars place strong constraints on the possible masses for immediate BH progenitors and wind mass loss from helium stars. Last, it is interesting that, even in the absence of any source of mass loss, the highest helium-star masses predicted by current evolutionary models are still not high enough to account for the measured BH mass in V404 Cyg ($>10 \, M_\odot$). An alternative for the formation of relatively massive BH may be provided by the evolutionary sequence proposed by Eggleton & Verbunt (1986), which invokes hierarchical triples as progenitors of BH X-ray binaries with low-mass companions.

1. Introduction

Radial velocity measurements of the non-degenerate donors in X-ray transients at quiescence combined with information about donor spectra and optical light curves allow us to measure the masses of accreting compact objects (e.g., Charles 1998), as well as other binary properties (e.g., orbital periods, donor masses and spectral types, kinematic properties). At present, measured masses for nine X-ray transients exceed the optimum maximum neutron mass of $3 \, M_\odot$ (e.g., Kalogera & Baym 1996) and the binaries are thought to harbor black holes (BH). These observed BH X-ray transients seem to form a rather homogeneous sample and studies of its properties as a whole can shed a light to their evolutionary history and the process of BH formation.

Black-hole X-ray transients are similar to low-mass X-ray binaries with neutron stars in that mass transfer is driven by Roche-lobe overflow and the donors are less massive than the BH. This maximum mass ratio of about unity (see Kalogera & Webbink 1996) allows the donor to transfer mass stably to the
compact object. However, for the majority of the BH X–ray transients (including six BH candidate systems based on their spectral properties, see Chen, Shrader, & Livio 1997) the donors are less massive than \( \sim 1 \text{M}_\odot \), much less massive than the typical BH masses. Only two systems, J1655-40 and 4U1543-47, have intermediate–mass donors (more massive than \( \simeq 2 \text{M}_\odot \)) of \( \sim 2.3 \text{M}_\odot \) and \( \sim 2 - 3 \text{M}_\odot \), respectively\(^1\). Further, orbital periods are found in the range of a few hours to days, much smaller than orbital periods of binary progenitors that could accommodate the radial expansion of evolved massive stars (BH progenitors).

In what follows, we consider a number of the observed characteristics of BH X–ray transients and use them as clues and windows to the evolutionary history of these binaries, BH formation, and helium–star evolution.

2. Binary Orbital Periods and Common–Envelope Evolution

Currently observed orbital periods vary from just a few hours to a couple of days typically (with the exception of V404 Cyg, which has an orbital period of about 6 days). These orbital periods along with the measured BH masses and the estimated donor masses (based on spectral type classifications) clearly indicate that the orbital separations of the X-ray transients are of order 10 R\(_\odot\) (even for V404 Cyg where the donor is on the giant branch, we derive \( \simeq 35 \text{R}_\odot \)). Since evolved massive stars typically expand to \( \sim 1000 \text{R}_\odot \) (even for stars that lose their hydrogen envelopes though winds, see Schaller et al. 1992), the progenitors of the observed binaries must have experienced a drastic orbital contraction. Based on this simple observation it is generally accepted that a common-envelope phase is necessary to explain the current short orbital periods.

We can look into this question in some more detail. Let us for a moment assume that the primordial binary was wide enough to avoid common–envelope evolution. Then the only possible way to achieve the required orbital contraction is through natal kicks imparted to black holes. The magnitude of these kicks may be quite modest, since the effects of kicks on binary characteristics are more important when their magnitude is comparable to orbital velocity of the binary. For massive binaries with orbital separations well in excess of 1000 R\(_\odot\), orbital velocities lie in the range \( \simeq 10 - 50 \text{km s}^{-1} \). However, the required degree of orbital contraction is rather large, with a ratio of circularized post–supernova to pre–supernova orbital separations:

\[
\alpha_C \equiv \frac{A_C}{A_{in}} < \frac{10}{1000}. \tag{1}
\]

Under the assumption of angular momentum conservation during circularization, it is:

\[
\alpha_C = \alpha (1 - e^2), \tag{2}
\]

where \(\alpha \equiv A/A_{in}\), \(A\) is the immediate (non–circularized) post–supernova separation, and \(e\) is the post–supernova eccentricity. Further, a strict lower limit on

\(^1\)One more X-ray transient, V4641 Sgr, with an intermediate–mass donor of 5–8 M\(_\odot\) has been reported since the time this talk was presented; Orosz et al. (2000).
\(\alpha\) can be derived analytically (e.g., Flannery & van den Heuvel 1975; Kalogera 1996):

\[
\alpha \equiv \frac{A}{A_{\text{in}}} \geq \frac{1}{2}.
\]  

(3)

Combining equations (1), (2), and (3) we derive that the required degree of post-supernova orbital contraction can occur only for binaries with post-supernova eccentricities in excess of 0.99\(^{2}\) (!). Analysis of post-supernova binary characteristics, for cases of significant mass loss and kicks comparable to the pre-supernova orbital velocity shows that only a negligible fraction (less than 0.1\%) of surviving (bound) systems acquire eccentricities that high (see Figures 4 and 10 in Kalogera 1996). Such a low survival probability in the absence of a common-envelope phase would lead to a negligible formation rate for BH X-ray transients (see also Portegies-Zwart, Verbunt, & Ergma 1997). Therefore, we conclude that a common-envelope phase is necessary for the formation of all of the observed BH X-ray transients.

3. Evolutionary Constraints

Given that the progenitors of BH binaries must experience a common-envelope (CE) phase, which leads to both orbital contraction and loss of hydrogen envelope of the BH progenitor, it is quite possible that the evolutionary history is very similar to that of low-mass X-ray binaries with neutron stars (e.g., van den Heuvel 1983). In what follows, we consider structural and evolutionary requirements for such a formation path. In the next section we examine how these requirements determine the donor properties of the BH binaries formed (for more details, see Kalogera 1999).

The binary primary must be massive enough so that its helium core exposed at the end of the CE phase collapses into a BH. The X-ray phase is initiated when the donor fills its Roche lobe because of orbital shrinkage through magnetic braking (for low-mass donors) or of radial expansion through nuclear evolution on the main sequence (for intermediate-mass donors).

Black-hole binary progenitors evolve through this path provided that the following constraints are satisfied:

- The orbit is small enough that the primary fills its Roche lobe and the binary enters a CE phase.
- At the end of the CE phase the orbit is wide enough so that both the helium-rich primary and its companion fit within their Roche lobes. The constraint for the companion turns out to be stricter.
- The system remains bound after the collapse of the helium star. In the case of small or zero kicks imparted to the BH, this sets an upper limit on the mass of the BH progenitor.
- After the collapse, the orbit must be small enough so that mass transfer from the donor starts before it leaves the main sequence and within \(10^{10}\) yr.

\(^{2}\)For V404 Cyg, the corresponding lower limit on eccentricity is 0.96.
Figure 1. Limits on the parameter space of the final (pre–collapse) helium–star mass, $M_{\text{He,f}}$, and the ratio, $M_{\text{He,f}}/M_{\text{He}}$, for six values of the $\alpha_{\text{CE}} = 0.6, 1.0, 1.4, 1.6, 2.0, 3.0$, and for a 7 $M_\odot$ BH. Conditions in the unshaded areas do not allow the formation of BH binaries with main–sequence Roche–lobe filling donors; conditions in the light–gray, dark–gray, and black areas allow the formation of systems with only low–mass, only intermediate–mass, and both types of donors, respectively.

- Mass transfer from the donor proceeds stably and at sub–Eddington rates. This sets an upper limit to the donor mass on the zero-age main sequence and to the orbital size for more evolved donors.

### 4. Donor Masses in Black–Hole X–ray Binaries

For a specific value of the BH mass, the above constraints translate into limits on the properties, circularized post–collapse orbital sizes $(A_C)$ and donor masses $(M_d)$, of BH binaries with Roche–lobe filling donors. The relative positions of these limits on the $A_C – M_d$ plane and the resulting allowed $M_d$ ranges are exactly determined by three well constrained model parameters:

- The amount of mass loss from the binary during BH formation, characterized by the ratio $M_{\text{He,f}}/M_{\text{BH}}$, where $M_{\text{He,f}}$ is the mass of the helium–rich BH progenitor at the time of the collapse. For the post–collapse system to remain bound it must be $1 \leq M_{\text{He,f}}/M_{\text{BH}} \leq 3$. 
• The amount of mass lost in the helium–star wind between the end of the CE phase and the BH formation, characterized by the ratio $M_{He,f}/M_{He}$, where $M_{He}$ is the initial helium–star mass (at the end of the CE phase). This ratio must lie in the range $0 - 1$.

• The CE efficiency, $\alpha_{CE}$, defined as the ratio of the CE binding energy to the orbital energy released during the spiral–in of the companion. Although the absolute normalization of $\alpha_{CE}$ is not well determined (see Kalogera 1999), values higher than unity imply the existence of energy sources other than the orbit (ionization or nuclear burning energy).

Note that the last two of the evolutionary constraints ($\S$3) depend only on the BH mass, while $\alpha_{CE}$ affects only the upper limit on $A_C$ (first of the constraints $\S$3). For different values of these three parameters, the positions of the limits on the $A_C - M_d$ plane change and three different outcomes with respect to the donor masses are possible: BH binaries can be formed with (i) only low–mass; (ii) only intermediate–mass; (iii) both low– and intermediate–mass donors.

The donor types as a function of the three parameters, $M_{He,f}$, $M_{He,f}/M_{He}$, and $\alpha_{CE}$, are shown in Fig. 1, for a 7 $M_\odot$ BH. For $\alpha_{CE}$ smaller than $\sim 0.5$, the orbital contraction is so high that the donor stars cannot fit in the post–CE orbits, and hence no BH X-ray binaries are formed. As $\alpha_{CE}$ increases, CE ejection without the need of strong orbital contraction becomes possible for the more massive of the donors, while formation of binaries with low–mass donors occurs only if $\alpha_{CE} > 1.5$. The results become independent of $\alpha_{CE}$ for values in excess of $\sim 2$, when the upper limit for CE evolution (first of the constraints) lies at high enough values of $A_C$ that it never interferes with the other limits.

The dependence of these results on the two mass–loss parameters (wind and collapse) are determined by their association with orbital expansion. For strong helium–star wind mass loss ($M_{He,f}/M_{He} < 0.35$), the progenitor orbits expand so much that donors less massive than the BH can never fill their Roche lobes on the main sequence. Both low– and intermediate–mass donors are formed only if less than 50% of the initial helium-star mass is lost in the wind. Mass loss at BH formation is limited to BH progenitors less massive than about twice the BH mass so that post–collapse systems with low–mass donors remain bound. Note that amounts of mass lost in helium-star winds and in BH formation are actually anti-correlated. If one is close to the maximum allowed then the other must be minimal (see Fig. 1).

The dependence on $M_{He,f}$ of the orbital expansion during helium–star wind mass loss and BH formation is such that the ratio of circularized post–collapse over post–CE orbital separations becomes independent of $M_{He,f}$. This means that, for a specific BH mass, the position of the limits on the $A_C - M_d$ plane depend only on the initial helium–star mass and the CE efficiency. Indeed, in Fig. 1, the change of donor types occurs along straight lines in the $M_{He,f}/M_{He} - M_{He,f}$ plane, or else along lines of constant $M_{He}$. This simplifying property allows us to combine the panels in Fig. 1 into one plot (Fig. 2). It is evident that formation of 7 $M_\odot$ BH X–ray binaries with both low– and intermediate–mass donors (as required by the observed sample) constrains the common–envelope efficiency to relatively high values and the initial helium-star progenitors at most
Figure 2. Limits on the parameter space of the initial (post–CE) helium–star mass, $M_{\text{He}}$, and the common–envelope efficiency, $\alpha_{\text{CE}}$, properly normalized (by the maximum stellar radii of massive stars (Schaller et al. 1992) and the central–concentration parameter, $\lambda$), for a 7 M$_{\odot}$ BH. Shade coding is as in Figure 1.

twice as massive as the BH (corresponding to initial primaries in the range 25–45 M$_{\odot}$).

Additional constraints can be obtained by examining the relative numbers of systems with low– and intermediate–mass donors formed for the parameters in the black-shaded areas in Figs. 1 and 2. The lifetimes for the two different types are determined by the process that drives mass transfer. The magnetic–braking time scale, for low–mass donors is comparable to the nuclear evolution time scale of intermediate–mass stars (Kalogera 1999). The number ratio then becomes equal to the ratio of birth rates. The latter can be calculated using the derived limits on $A_C$ and $M_f$ and assumed distributions of mass ratios and orbital separations of primordial binaries. The results indicate that even when low–mass companions in primordial binaries are strongly favored, BH binaries with intermediate–mass donors are much more easily formed because of the larger range of orbital separations allowed to their progenitors (see Figure 3). Models predict a small fraction of intermediate–mass donors (as seen in the current observed sample) only for rather high $\alpha_{\text{CE}}$ values (> 3) or for moderate (but still higher than unity) $\alpha_{\text{CE}}$ values (1.5 – 2) and BH progenitors either slightly more massive or twice as massive as the BH.

We note that these results are quite robust and do not depend on the assumed BH mass nor the properties of primordial binaries (see Kalogera 1999).

5. Stellar Evolution Models and Common–Envelope Phase

More careful consideration of the above analysis shows that the constraints derived for helium–star wind mass loss depend on the timing of the CE phase
with respect to the evolution of the massive BH progenitor. Current single-star evolutionary models (Schaller et al. 1992; Hurley, Pols, & Tout 2000) for massive stars (> 10 \( M_\odot \)) of solar metallicity with wind mass loss imply that a Roche-lobe overflow (and hence a CE phase) can occur only either before or after the star’s core helium burning phase. During this phase wind mass loss accelerates, leading to significant orbital expansion (Jeans mode of mass loss), while the stellar radius remains almost constant. Also, for stars more massive than about 25 \( M_\odot \) (about 35 \( M_\odot \) for Z=0.002), the evolutionary models show a radial contraction until the star reaches core collapse (see also Kalogera 1999).

Based on the above considerations, BH progenitors can fill their Roche lobe only before core helium ignition (Case B mass transfer). The exposed helium cores evolve through the complete core helium burning phase before they reach core collapse. Taking into account this implicit constraint, we are able to derive the associated constraints on helium–star wind mass loss. Current models of helium–star evolution through core helium burning (Woosley 1995) predict amounts of mass lost in the wind significantly larger than the maxima allowed for BH X-ray transient formation (< 50\%). In fact, the final helium-star masses in these models are \( \sim 4 M_\odot \), far too small to explain the BH mass measurements. Therefore, if the CE phase is initiated early in the core helium burning phase of the primary, then helium-star winds must be much weaker than thought until now. It is worth noting that more recent empirical estimates of wind mass loss rates show a downward trend (Hamann & Koesterke 1998).
However, if the models for massive star evolution with winds do not represent reality, and radial expansion (and hence Roche-lobe overflow and CE evolution) is possible after the end of core helium burning, then it is possible that the progenitors of BH binaries evolve through Case C mass transfer (see Wellstein & Langer 1999). In this case, the strength of helium-star winds becomes irrelevant to the process of BH X-ray binary formation. The reason is that the helium core is exposed only through its core carbon and later burning phases, and the total duration of these phases is so short that the wind mass loss is insignificant and the helium-star mass remains essentially constant (Woosley 1995).

Given the uncertainties in models of massive star evolution and the calculation of stellar radii, such a modification cannot be excluded at present. In any case, it becomes clear that the existence of BH X-ray transients requires that either the hydrogen-rich massive star models or the strength of helium-star winds be modified.

6. Discussion

We have shown that the observed properties of the current sample of BH X-ray transients provide us with clues to their evolutionary history and BH formation. Specifically, we find that (i) common-envelope evolution is necessary to account for the present tight orbits, (ii) orbital contraction during the common-envelope phase must be moderate and therefore CE efficiencies must be relatively high (depending on the exact radii of massive stars and their density profiles, significant contributions from energy sources other than the orbit may be required), (iii) helium stars that form black holes are at most twice more massive than the black holes at the time of collapse. All these constraints do not depend on the details of the radial evolution of massive stars. We further find that current evolutionary models for massive stars losing mass in winds appear to be in conflict with models of wind mass loss from helium stars. Assuming that models massive star evolution are more accurate than estimates of helium-star winds, we find that wind mass-loss from helium stars must be limited so that these stars lose at most half of their mass at the beginning of core helium burning.

Current stellar evolution models face one additional challenge posed by the BH mass of V404 Cyg, if the upper end of the measured range is confirmed ($10 - 14\ M_\odot$; see, e.g., Bailyn et al. 1998). Even if we ignore any mass loss from helium stars, predicted masses for helium cores of massive stars cannot account for such a high BH mass (see Wellstein & Langer 1999; Hurley et al. 2000).

An alternative model for the formation of BH X-ray binaries that invokes the evolution of hierarchical triples has actually been suggested by Eggleton & Verbunt (1986). The basic idea is that the progenitor consists of a high-mass inner binary with an outer low-mass companion. The inner binary evolves to a high-mass X-ray binary, where Roche-lobe overflow from the companion to the neutron star leads to a complete merger and possibly the formation of

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3 Moderate orbital contraction could also be achieved if the hydrogen envelope masses are reduced through binary-enhanced wind mass loss as proposed by P.P. Eggleton.
a “Thorne–Zytkow” object. During the merger the neutron star is expected to collapse into a BH, which can continue to grow through accretion. If the massive envelope around the BH expands as giants do then it is possible that this envelope will eventually engulf the outer companion. The resulting spiral-in and envelope ejection would then lead to the formation of a tight binary with a BH and a Roche-lobe filling low-mass companion.

Such an alternative formation path can very easily overcome difficulties with (initial and final) helium-star masses and requirements for large (possibly unphysical) common-envelope efficiencies. Furthermore, a significant fraction of stars appear to be members of multiple systems. These considerations could motivate a detailed analysis of this triple-star formation path for BH binaries. Such an analysis would eventually include the study of a number of very interesting problems: orbital stability of the triple system throughout the long-term phases of its evolution, the effects on the outer orbit of wind mass-loss and supernova explosions occurring in the inner binary, the unsettled question of the stability of “Thorne–Zytkow” objects (see, e.g., Cannon et al. 1992, but also Fryer, Benz, & Herant 1996) and their subsequent evolution, the frequency and orbital properties of triple systems.

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