General arguments related to “triviality” predict that, in the broken phase of $(\lambda \Phi^4)_4$ theory, the condensate $\langle \Phi \rangle$ re-scales by a factor $Z_\phi$ different from the conventional wavefunction-renormalization factor, $Z_{\text{prop}}$. Using a lattice simulation in the Ising limit we measure $Z_\phi = m^2 \chi$ from the physical mass and susceptibility and $Z_{\text{prop}}$ from the residue of the shifted-field propagator. We find that the two $Z$’s differ, with the difference increasing rapidly as the continuum limit is approached. Since $Z_\phi$ affects the relation of $\langle \Phi \rangle$ to the Fermi constant it can sizeably affect the present bounds on the Higgs mass.
1 Introduction

Theoretical and numerical evidence \cite{1, 2, 3, 4, 5, 6, 7} strongly supports the view that $(\lambda \Phi^4)_4$ theories are “trivial”. The physical meaning of this result, however, remains controversial. The conventional interpretation is based on Renormalization-Group-Improved-Perturbation-Theory (RGIPT), while a quite different interpretation is advocated in Refs. \cite{8, 9, 10}. The two pictures predict quite different structures for $V_{\text{eff}}$, the effective potential of the theory. RGIPT organizes the various contributions according to the perturbative/loop-expansion classification (as leading, next-to-leading,...terms). Due to the lack of perturbative asymptotic freedom there are inescapable problems with this method when taking the continuum limit \cite{11}. The approach of Refs. \cite{8, 9, 10} focuses on the class of approximations to $V_{\text{eff}}$ that are consistent with the non-interacting nature of the shifted field $h(x) \equiv \Phi(x) - \langle \Phi \rangle$. The resulting predictions, unlike the RGIPT predictions, yield an excellent fit to the lattice data for $V_{\text{eff}}$ \cite{12, 13}.

A key feature of the alternative picture is the presence of “two $Z$’s”. In the class of approximations (including one-loop, gaussian, and post-gaussian calculations \cite{14}), where the $h$-field is governed by an effective quadratic Hamiltonian, the effect of bare $h-h$ self-interactions can be reabsorbed into the $h$-field mass $M_h$ and into the normalization of a physical, “renormalized” vacuum field \cite{15}

$$v_R \equiv v_B/\sqrt{Z_\phi}. \quad (1)$$

Here $v_B = \langle \Phi \rangle$ and $Z_\phi$ is defined such that

$$\frac{d^2 V_{\text{eff}}}{d\phi_R^2} \bigg|_{\phi_R = \pm v_R} = M_h^2 \quad (2)$$

The role of $Z_\phi$ is essential. It provides the key-ingredient to get a non trivial effective potential in a “trivial” theory. One finds \cite{8, 9, 10, 14, 15} that in the continuum limit (cutoff $\Lambda \to \infty$):

$$Z_\phi \sim \ln \frac{\Lambda}{M_h} \to \infty. \quad (3)$$

Therefore, although $M_h^2/v_B^2 \to 0$ in the continuum limit, in accord with the rigorous arguments of \cite{11}, one has

$$M_h^2/v_R^2 = \text{cutoff - independent}. \quad (4)$$

The divergence in Eq.(3) cannot be understood within RGIPT and is due to the peculiarity \cite{8, 10} of the $p_\mu = 0$ mode, responsible for spontaneous symmetry breaking. Physically, it represents a condensate of the symmetric-phase $\Phi$ particles \cite{8, 10}. In the $\Lambda \to \infty$ limit the infinitely increasing particle density in the condensate compensates for the vanishing strength of the elementary 2-body processes (“triviality”), thus yielding a finite, negative energy density and a finite Higgs mass. As such, $Z_\phi$ is quite different from $Z_\text{prop}$, the residue of
the shifted-field propagator, associated with the normalization of the $p_\mu \neq 0$ asymptotic one-particle states \[17\] and which is bounded by $Z_{\text{prop}} \leq 1$ from Källen-Lehmann decomposition (with “triviality” implying $Z_{\text{prop}} \rightarrow 1$). For this reason, in the presence of spontaneous symmetry breaking, field re-scaling cannot be given as an “operatorial” statement \[12\].

The aim of this Letter is to directly test the prediction that $Z_\phi$ differs from $Z_{\text{prop}}$. We present the results of a lattice simulation of the theory (in the Ising limit) where we compute the mass $M_h$ and the residue $Z_{\text{prop}}$ from a 2-parameter fit to the lattice data for the shifted-field propagator. We then compute the zero-momentum susceptibility

$$\frac{1}{\chi} = \frac{d^2 V_{\text{eff}}}{d\varphi_B^2} \bigg|_{\varphi_B = \pm v_B}$$

and hence obtain the dimensionless quantity

$$Z_\phi \equiv M_h^2 \chi.$$  

Finally, we compare $Z_\phi$ with $Z_{\text{prop}}$.

## 2 The lattice simulation

The one-component $(\lambda \Phi^4)_4$ theory

$$S = \sum_x \left[ \frac{1}{2} \sum_\mu (\Phi(x + \hat{e}_\mu) - \Phi(x))^2 + \frac{\kappa}{2} \Phi^2(x) + \frac{\lambda}{4} \Phi^4(x) \right]$$

becomes in the Ising limit

$$S_{\text{Ising}} = -\kappa \sum_x \sum_\mu [\phi(x + \hat{\kappa}_\mu)\phi(x) + \phi(x - \hat{\kappa}_\mu)\phi(x)]$$

with $\Phi(x) = \sqrt{2}\kappa\phi(x)$ and $|\phi(x)| = 1$.

The shifted field propagator, defined at $p_\mu \neq 0$, can be computed as

$$G(p) = \langle \sum_x \exp(ipx)h(x)h(0) \rangle$$

for the values $p_\mu = \frac{2\pi}{L} n_\mu$ with $n_\mu \neq 0$. An excellent fit to the lattice data is obtained (see for example Fig.1) by using the 2-parameter formula

$$G(p) = \frac{Z_{\text{prop}}}{\hat{p}^2 + m_{\text{latt}}^2}$$

where $m_{\text{latt}}$ is the dimensionless lattice mass and $\hat{p}_\mu = 2 \sin \frac{p_\mu}{2}$. Finally the susceptibility $\chi$ is measured directly as

$$\chi_{\text{latt}} = L^4 \left[ \langle \Phi^2 \rangle - \langle \Phi \rangle^2 \right]$$
with $\Phi$ the average field for each lattice configuration, and we define

$$Z_\phi \equiv m_{\text{latt}}^2 \chi_{\text{latt}}$$

(12)

To update our field configurations we used the Swendsen-Wang [18] cluster algorithm on $20^4$ and $24^4$ lattices. After discarding 10K sweeps for thermalization, we have performed 50K sweeps, measuring our observables every 5 sweeps. Statistical errors can be estimated through a direct evaluation of the integrated autocorrelation time [19], or by using the “blocking” [20] or the “grouped jackknife” [21] algorithms. We have checked that applying these three different methods we get consistent results.

We have computed at different values of the hopping parameter $\kappa$ in order to obtain a correlation length $\xi_{\text{latt}} = 1/m_{\text{latt}}$ in the range 2 to $L/4$. The upper limit of the correlation length is required in order to take under control finite-size effects [22, 23]. Our results for $Z_\phi$ and $Z_{\text{prop}}$, in the broken phase $0.0751 \leq \kappa \leq 0.0764$ are reported in Fig.2 and show a sizeable difference for $m_{\text{latt}} < 0.3$. We have performed a consistency check that no such effect is present in the symmetric phase $0.0726 \leq \kappa \leq 0.0741$, as expected. These other results are shown in Fig.3. As an additional check, we have compared with Montvay and Weisz [23], at $\kappa = 0.074$ in the symmetric phase, and got excellent agreement (see Table 1). We also compared our results with the available data of a high statistics simulation in the broken phase [24] obtaining a rather good agreement (see Table 1).

3 Conclusions

Our numerical simulation of $(\lambda\Phi^4)_4$, in the Ising limit, shows a clear difference between two measured quantities: the rescaling of the “condensate” $Z_\phi$ and the more conventional quantity $Z_{\text{prop}}$ associated with the residue of the shifted field propagator. As discussed in the introduction, this is not unexpected on the basis of the alternative description of spontaneous symmetry breaking of Refs. [8, 9, 10]. The effect shows up when increasing the correlation length and should become more and more important by approaching the continuum limit of quantum field theory $m_{\text{latt}} \to 0$. Therefore, the relation of the lattice vacuum field $\langle \Phi \rangle$ to the Fermi constant and the same limits on the Higgs mass can sizeably be affected. Indeed, these have been based on the quantity [25]

$$R_{\text{prop}} = \frac{m_{\text{latt}}^2}{\langle \Phi \rangle^2} Z_{\text{prop}}$$

(13)

rather than

$$R_\phi = \frac{m_{\text{latt}}^2}{\langle \Phi \rangle^2} Z_\phi = \frac{m_{\text{latt}}^4 \chi_{\text{latt}}}{\langle \Phi \rangle^2} Z_{\phi} = \frac{m_{\text{latt}}^4 \chi_{\text{latt}}}{\langle \Phi \rangle^2} Z_{\phi}.$$

(14)

In this sense, the discovery of $Z_\phi$ requires a “second generation” of lattice simulations to re-check the scaling behaviour of the various quantities and compare with all available theoretical descriptions of the continuum limit.
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Table 1: Our lattice data compared with results available in the literature (Refs.[23, 24]). The values for $Z_{\text{prop}}$ in Ref. [24] have been evaluated by means of renormalized perturbation theory.

| Ref.      | $L_{\text{size}}$ | # sweeps | $\kappa$ | $m_{\text{latt}}$ | $\chi$    | $Z_{\varphi}$ | $Z_{\text{prop}}$ |
|-----------|--------------------|----------|----------|-----------------|------------|----------------|-------------------|
| Our data  | $20^1$             | $6 \times 10^4$ | 0.074    | 0.2124(60)     | 142.7 ± 2.1 | 0.953(55)     | 0.969(6)         |
| Ref. 23   | $20^3 \times 24$   | $1.6 \times 10^6$ | 0.074    | 0.2125(10)     | 142.6(8)   | 0.953(15)     | -                |
| Our data  | $20^1$             | $6 \times 10^4$ | 0.076    | 0.4060(61)     | 38.26(74)  | 0.959(34)     | 0.918(6)         |
| Ref. 24   | $20^1$             | $7.5 \times 10^6$ | 0.076    | 0.395(1)       | 37.85(6)   | 0.898(5)      | 0.918(9)         |
| Our data  | $20^1$             | $6 \times 10^4$ | 0.077    | 0.5718(70)     | 18.20(32)  | 0.916(55)     | 0.899(8)         |
| Ref. 24   | $16^1$             | $10^7$    | 0.077    | 0.563(1)       | 18.18(2)   | 0.887(3)      | 0.886(3)         |
FIGURE CAPTIONS

Figure 1. The lattice data for the propagator (Eq.(9)) (circles) at $\kappa = 0.07518$ on a $24^4$ lattice with superimposed the fit Eq.(10) (dotted line).

Figure 2. $Z_\varphi$ (circles) and $Z_{prop}$ (squares) in the broken phase versus $m_{\text{lat}}$.

Figure 3. $Z_\varphi$ (circles) and $Z_{prop}$ (squares) versus $m_{\text{lat}}$ in the symmetric phase. Symbols as in Fig.2.
PROPAGATOR

○ broken phase (k=0.07518) $24^4$ lattice
FIGURE 2

$Z = m_{\text{latt}}^2 \chi_{\text{latt}}$ (20$^4$ lattice)

$Z = Z_{\text{prop}}$ (20$^4$ lattice)

$Z = m_{\text{latt}} \chi_{\text{latt}}$ (24$^4$ lattice)

$Z = Z_{\text{prop}}$ (24$^4$ lattice)

BROKEN PHASE
$Z = m_{latt}^2 \chi_{latt}$ (20$^4$ lattice)

$Z = Z_{prop}$ (20$^4$ lattice)