Singlet–triplet relaxation induced by confined phonons in nanowire-based quantum dots

Y Yin

Hefei National Laboratory for Physical Sciences at Microscale, Department of Physics, University of Science and Technology of China, Hefei, Anhui 230026, People’s Republic of China
E-mail: yin80@ustc.edu.cn

Received 21 July 2010, in final form 27 September 2010
Published 3 November 2010
Online at stacks.iop.org/SST/25/125004

Abstract

The singlet–triplet relaxation in nanowire-based quantum dots induced by confined phonons is investigated theoretically. Due to the quasi-one-dimensional nature of the confined phonons, the singlet–triplet relaxation rates exhibit multi-peaks as a function of magnetic field, and the relaxation rate between the singlet and the spin-up triplet state is found to be enhanced in the vicinity of the singlet–triplet anti-crossing. We compare the effect of the deformation-potential coupling and the piezoelectric coupling and find that the deformation-potential coupling dominates the relaxation rates in most cases.

1. Introduction

As one of the most promising candidates for qubits, spins in semiconductor quantum dots (QDs) have attracted much attention in the last two decades [1–3]. Although much progress has been made in single-electron QDs, an attractive alternative is to base the qubits on singlet–triplet (ST) states in two-electron QDs [4–9]. This is mainly due to two important features of the ST states. One is the Zeeman-driven ST transition with anti-crossing due to the spin–orbit coupling (SOC), which enables coherent manipulation of spin states [9–11]. The other one is the low ST relaxation rate which makes the spin polarization remain for a sufficiently long time [12–14]. As the QDs are either self-assembled ones sitting on the surface of the substrate or fabricated by confining electrons in quantum wells by electrodes, bulk phonons in the substrate play an important role. In a low-temperature regime, acoustic bulk phonons in conjunction with the SOC serve as the main source of the ST relaxation due to the suppression of the effect of the hyperfine interaction in the presence of large magnetic field. Previous studies show that in the vicinity of the ST anti-crossing, the relaxation rate is greatly suppressed due to the mismatch of the phonon emission wavelength to the dot size. Furthermore, the spin-up triplet states which couple to the singlet ground state through the SOC have a much shorter lifetime compared to the other two triplet states due to the strong ST mixing between them [15–20].

Despite the progress in two-electron QDs, the complicated fabrication process makes them difficult to be scaled up, which limits their application from an industrial point of view. Recently, self-assembled two-electron QDs embedded in InAs nanowires were fabricated [21–23]. The well-controlled growth process enables parallel production in massive number with similar properties, which makes these nanowire-based QDs more suitable for integration into silicon and scaling up for large hierarchical systems. Although a similar Zeeman-driven ST transition has been obtained in nanowire-based two-electron InAs QDs [24–26], the ST relaxation can be quite different due to the properties of the confined phonons in the nanowires. The nanowires are perpendicular to the substrate, making the bulk phonons in the substrate less important than the confined phonons in nanowires [27, 28]. The regular structure of the nanowires results in the quasi-one-dimensional confined phonons, which lead to novel properties in optical absorption and transport for the nanowire-based QDs [29, 30]. Therefore, these confined phonons are expected to have a pronounced impact on the ST relaxation, which has not been studied to the best of our knowledge. In this work we will investigate the ST relaxation induced by the confined phonons.
2. Model and formalism

Consider a two-electron elongate QD embedded in an InAs [0 0 0 1] cylindrical nanowire with a radius $R$ in the presence of external magnetic field $B$ along the wire. We model the QD by an anisotropic harmonic potential $V_c(r, z) = \frac{1}{2} m^* \omega_0^2 r^2 + \frac{1}{2} m^* \omega_0^2 z^2$ and the QD by an anisotropic harmonic potential $H(B)$ cylindrical nanowire with a radius $R$ along the wire. We model $V_c(r, z)$ being the effective dot length $d_c = \sqrt{\hbar \pi / m^* \omega_0}$ and dot diameter $d_0 = \sqrt{\hbar \pi / m^* \omega_0}$ with $z$-axis along the wire. $m^*$ is the effective mass of the electron.

The single-electron Hamiltonian can be expressed as

$$H_e = \frac{p^2}{2m^*} + V_c(r, z) + H_B + H_{SO},$$

where $H_B = \frac{1}{2} g \mu_B B \cdot \sigma$ is the Zeeman splitting with $g$, $\mu_B$ and $\sigma$ being the $g$ factor of electron, Bohr magneton and Pauli matrix respectively. $H_{SO}$ represents the SOC term. In this work, we concentrate on the Rashba coupling which is dominant in InAs nanowires, which has the form $H_{SO} = \frac{\hbar}{2} \sigma^\gamma \cdot p\gamma$ with $\gamma$ being the Rashba coupling strength [24, 31]. We assume the dot diameter $d_0 \ll d_c$ so that only the lowest electron subband in the radial direction is needed. We also assume that the external magnetic field is weak enough so that its orbital effect can be neglected.

The confined phonons are calculated with an isotropic elastic continuum model which is widely used in the study of nanowires, carbon nanotubes and nanoparticles [32–37]. The nanowire is modeled as an infinite cylinder with the stress vanishing at the surface of the wire. The displacement field of nanowires, carbon nanotubes and nanoparticles [32–37]. The elastic continuum model which is widely used in the study of nanowires, carbon nanotubes and nanoparticles [32–37]. The elastic continuum model which is widely used in the study of nanowires, carbon nanotubes and nanoparticles [32–37].

Given the displacement field, both the deformation-potential coupling and the piezoelectric coupling can be calculated. The deformation-potential coupling is given by the divergence of the displacement field, which reads

$$H_{DP}^{R}(r) = -\nabla \cdot {u(r)},$$

where $\nabla$ is the deformation-potential coupling strength. Note that only the longitudinal component of the confined phonon mode has contribution to the deformation-potential coupling. The piezoelectric coupling is given by [33]

$$H_{PZ}^{R}(r) = \frac{e}{\kappa} \int d r_e \frac{\nabla \cdot \mathbf{P}_{PZ}}{|r - r_e|},$$

where $\mathbf{P}_{PZ}$ is the polarization induced by the displacement field and $\kappa$ is the dielectric constant. For InAs [0 0 0 1] nanowires with wurztite structure, $\mathbf{P}_{PZ}$ can be expressed as

$$\mathbf{P}_{PZ}(r) = e_{15}(\partial_t u_x(r) + \partial_r u_r(r))e_r + [e_{31}(\partial_t u_x(r) + \partial_r u_r(r)) + e_{13}\partial_t u_z(r)]e_z,$$

with $e_{15}$, $e_{31}$ and $e_{33}$ being the piezoelectric constants. Note that due to the anisotropy of the polarization, the piezoelectric coupling can be more sensitive to the confined phonon mode than the deformation-potential coupling.

In the following discussion, we restrict to the dilatation modes (which have angular momentum quantum number $l = 0$ and angular component $\partial_{\theta} \phi_l^m(r) = 0$ in equation (2)) since only these modes can couple to the electrons in the QDs in the lowest subband of radial direction\(^1\) [30, 41].

For two-electron QDs, the total Hamiltonian of the system is given by

$$H = (H_e^1 + H_{e}^2 + H_C) + H_{PZ} + H_{DP},$$

where $H_C = \frac{\hbar^2}{2} \sigma^\gamma \cdot \mathbf{P} + \mathbf{P} \cdot \mathbf{B}$ represents the Coulomb interaction between the two electrons. $H_{DP} = \sum_{i<j} \hbar \omega_{0q} a_{i,q} a_{j,q}$ is the Hamiltonian for the confined phonons and $H_{PZ} = H_{PZ}^1 + H_{PZ}^2$ is the electron–phonon coupling. The superscripts ‘1’ and ‘2’ label the two electrons.

To construct the two-electron basis functions, it is convenient to use the separation of variables in terms of the center of mass and relative motion respectively.

$$\langle R, r|n\eta n\eta\rangle = R^R(R) R^L(r) \phi_{n\eta}(Z) \phi_{n\eta}(z) \chi_\eta,$$

where $\chi_\eta$ represents the spin states of the two electrons, which can be expressed as

$$\chi_\eta = \left| \begin{array}{c} S \mid \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle), \\ T_{-} \mid \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle), \\ T_{0} \mid \uparrow \uparrow, \\ T_{z} \mid \uparrow \downarrow \end{array} \right|, \eta = 0, 1, 2, 3$$

In equation (7), $R^R(R)/R^L(r)$ is the wavefunction for the lowest subband in the radial direction and $\phi_{n\eta}(Z)/\phi_{n\eta}(z)$ is the wavefunction in the axial direction. The subscripts ‘$R’ and ‘$’ label the center of mass and relative motion respectively. To make the basis function to be anti-symmetric, the singlet state $|S\rangle$ always corresponds to $\phi_{0\eta}(z)$ with even parity, while the triplet states $|T_{\pm}\rangle$ correspond to $\phi_{2\eta}(z)$ with odd parity. The energy levels and eigenstates of the two electrons can be obtained by diagonalizing the full two-electron Hamiltonian ($H_{e}^1 + H_{e}^2 + H_C$) in this basis. We identify an eigenstate as singlet and/or triplet by its expectation value $\langle \sigma_1 + \sigma_2 \rangle$.

Treating $|i\rangle$ and $|f\rangle$ as the initial and final states, we can calculate the phonon-induced relaxation rate using Fermi’s golden rule. At zero temperature, the relaxation rate induced by the confined phonons reads [42]

$$\Gamma_{conf}^{ij} = \sum_{j,mv} \left| \frac{M_{j,mv}^{ij}}{\partial_t \phi_{l,mv}^{ij}} \right|^2 \theta(E) \delta_{l,mv} = E,$$

\(^1\) Note that this is due to the fact that the InAs [0 0 0 1] nanowire is with wurztite structure. For a zinc-blende nanowire, piezoelectric coupling can also couple to phonon modes with $l = \pm 2$.\)}}
where $E = |E_j - E_i|$ is the energy splitting and $\theta(E)$ is the step function. $V_{ij} = W_{ij}(r)e^{i q m_1} + W_{ij}(r)e^{i q m_2}$ comes from the total electron–phonon interaction Hamiltonian $H_{ij}^e + H_{ij}^p$ with $j = D$ for the deformation-potential coupling and $j = P$ for the piezoelectric coupling. The corresponding coefficients are $|M_{ij}^D|^2 = \hbar \Xi^2/(2\pi \rho \nu q R^2)$ and $|M_{ij}^p|^2 = 16\pi^2 e^2 \epsilon_{1s}^2/(k\nu R^2 \rho \nu q)$. The quantity $W_{ij}^p(r)$ is given in detail in the appendix.

To compare the spin relaxation rate induced by the confined phonons to the one induced by bulk phonons, we calculate the latter following [46]. For a nanostructure with confined phonons to the one induced by bulk phonons, we relaxion rate, is a direct consequence of the large DOS at the van Hove singularities in the confined phonon DOS. We employ the exact diagonalization method with the lowest 48 basis functions to converge the energy levels and ST relaxation rates [46, 47]. The magnetic field dependence of the first four levels and the corresponding expectation value of $S^z = (\sigma^+_z + \sigma^-_z)/2$ are plotted in figures 1(a) and (b) respectively. An anti-crossing at $B = 2.36$ T can be identified in the energy levels. The expectation values ($S^z$) show a crossing at the corresponding magnetic field, indicating a strong mixing between the singlet and the spin-up triplet state (ST mixing). As the orbital effect of the magnetic field is neglected, the levels show linear field dependence, which is different from the typical disk-shaped QDs [24, 31]. We emphasize that the mixing between the singlet and triplet states in our elongate QDs is more stronger than the QDs studied before [15, 17, 20]. This can be seen in the large level splitting, which is 0.14 meV at the anti-crossing. This value is much larger than the typical value for the disk-shaped GaAs QDs, which is usually a few $\mu$eV. The strong mixing is not only due to the strong SOC in InAs but also due to the large Coulomb interaction in the elongate QDs, which can enhance the relative strength of SOC [17, 24].

We start our discussion with the ST$_s$ relaxation. The magnetic field dependence of the ST$_s$ relaxation is shown in figure 2(a). It can be seen that the relaxation rates induced by bulk and confined phonons are very different from each other. For bulk phonons, the relaxation rate varies smoothly as a function of magnetic field. A local minimum can be found at the anti-crossing. For the confined phonons, the relaxation rate exhibits multi-peaks as a function of magnetic field. The two ‘sharp’ peaks located at $B = 1.11$ T and $B = 3.62$ T are actually the two divergent peaks. Moreover, the relaxation rate exhibits a local maximum at the anti-crossing in contrast to the local minimum for the bulk phonons. We attribute the multi-peak structure and the local maximum at the anti-crossing as the two main features of the ST$_s$ relaxation rate induced by confined phonons.

The first feature, e.g. the multi-peak structure of the relaxation rate, is a direct consequence of the large DOS at the van Hove singularities in the confined phonon DOS. To see this, we plot the spectrum of the relevant confined

3. Results

In the numerical calculation, we use the parameters for the InAs $[0 \ 0 \ 0 \ 1]$ wurtzite nanowires [24–26]. The deformation-potential strength $\Xi$ is chosen to be 5.8 eV, the density is $\rho = 5900$ kg m$^{-3}$, the static dielectric constant $\kappa$ is 15.15, and the longitudinal and transverse sound velocities are chosen to be $\nu_L = 4410$ m s$^{-1}$ and $\nu_T = 2130$ m s$^{-1}$ respectively [43]. Since the piezoelectric constant for the wurtzite InAs nanowire is largely missing, we use the constants calculated by transformation from the zinc-blende value $e_{14}$, which are $e_{15} = e_{31} = -e_{33}/2 = -e_{14}/\sqrt{3}$, with $e_{14} = 3.5 \times 10^6$ V m$^{-1}$ [30, 44]. The $g$ factor is set to $-9.0$ [24], and the Rashba coupling constant $\gamma$ is chosen to be $1.5 \times 10^{-11}$ eV m$^{-1}$ (corresponding to the SO length $\lambda_{SO} = 200$ nm) [45]. We set the radius of the nanowire $R$ to 15 nm. The effective QD diameter $d_0$ is set to 8 nm. The dot length $d_z$ is set to 70 nm.

Figure 1. The lowest four energy levels and the expectation value of $S^z = (\sigma^+_z + \sigma^-_z)/2$ versus the external magnetic field $B$. 
phonon modes and the corresponding DOS in figures 2(b) and (c) respectively. The ST, relaxation induced by the confined phonons via the deformation-potential coupling and the piezoelectric coupling is plotted as a function of energy splitting in figure 2(b) (Here we only plot the relaxation rate before the anti-crossing for clarification), and the relaxation after the anti-crossing has similar behavior. It is easy to see that the peaks in the relaxation rate correspond to the van Hove singularities in the DOS, indicating the strong enhancement due to the large DOS. The divergent peaks correspond to the van Hove singularity with  \( q \neq 0 \), while the non-divergent peaks are induced by the van Hove singularity with  \( q = 0 \). This is because as  \( q \to 0 \), the form factor tends to zero, which will suppress the relaxation rate [42]. Note that the form factor can also have zeros for  \( q \neq 0 \), which results in the dips in the relaxation rate. There are two types of zeros. The first one is due to the axial component (corresponding to the common peaks for both the deformation-potential coupling and the piezoelectric coupling) [48], the other one is due to the radial component, which induces different dips for different electron–phonon coupling mechanisms. Note that the deformation-potential coupling dominates the relaxation rate in most cases except in the energy region  \([0.2, 0.35] \text{meV} \). This is because in this region, the corresponding phonon mode tends to be the transverse mode which decouples the electrons via the deformation-potential coupling [42].

It can be seen from figure 2 that the second feature, e.g. the local maximum of the relaxation rate, occurs in the region far away from the van Hove singularities, where the confined phonon DOS exhibits a plateau. In this region, the behavior of the relaxation rate is decided by the combined effect of the ST mixing and the phonon emission efficiency. Both mechanisms enhance the relaxation rate at the anti-crossing, resulting in the local maximum. For bulk phonons, the DOS is a quadratic function of the energy splitting, so the relaxation rate can be suppressed as energy splitting decreased, resulting in the local minimum at the anti-crossing. Note that the energy splitting  \( E \) here (\( E = 0.14 \text{meV} \)) is much larger than the energy splitting in typical disk-shaped GaAs QDs, so the suppression of the relaxation rate due to the decreasing phonon emission efficiency is absent.

To further justify the behavior of the relaxation rate at the anti-crossing, we give an estimation based on perturbation theory. We concentrate on the deformation-potential coupling since it dominates the ST relaxation rate in the vicinity of the anti-crossing. We restrict the discussion to the lowest two basis  \( |00S\rangle \) and  \( |01T_z\rangle \) with energy  \( \epsilon_0 \) and  \( \epsilon_1 + g \mu_B B \) since they are almost degenerated in the vicinity of the anti-crossing. By applying the degenerated perturbation theory, we get the eigenstates

\[
|\tilde{S}\rangle = \cos \theta |00S\rangle + \sin \theta |01T_z\rangle,
\]

\[
|\tilde{T}_z\rangle = -\sin \theta |00S\rangle + \cos \theta |01T_z\rangle,
\]

where  \( \tan \theta = t/(d + \sqrt{d^2 + t^2}) \) with  \( t = \gamma \alpha_z |\phi_0(z)|\beta_1 |\phi_1(z)| \) and  \( d = (\epsilon_0 - \epsilon_1 + g \mu_B B)/2 \). The energy splitting between the two eigenstates is  \( E = 2\sqrt{d^2 + t^2} \). Note that the two states are maximally mixed as  \( E \) reaches its minimum.

The relaxation between the singlet state  \( |\tilde{S}\rangle \) and triplet state  \( |\tilde{T}_z\rangle \) can be calculated by applying equation (9) and (10). For bulk phonons, we have

\[
\Gamma_{ST}^{\text{bulk}} = E |\tilde{M}_{\text{bulk}}^D| \int_0^\infty \sin \theta \, d\theta |I_{ST}^\text{bulk}(q, \cos \theta)|^2 \\
\times e^{-t/(4\pi r(q_d, \sin \theta))},
\]

where  \( |\tilde{M}_{\text{bulk}}^D| = \sqrt{2\pi} \Sigma (4d^2 \hbar^4 \epsilon_i^5 e_i) \) is a constant. \( q_i = E/(\hbar v_S) \) is the bulk phonon emission wave vector. \( I_{ST}^\text{bulk}(q) = \langle 00S|e^{i\tilde{q}z} + e^{i\tilde{q}z'}|00S\rangle - \langle 01T_z|e^{i\tilde{q}z} + e^{i\tilde{q}z'}|01T_z\rangle \) is the axial component of the form factor. In the situation we considered here,  \( I_{ST}^\text{bulk}(q) \) varies slowly, so we approximate it as a constant. For small  \( E \), one has

\[
\Gamma_{ST}^{\text{bulk}} \propto E/\hbar v_S.
\]

For confined phonons, only the lowest confined phonon mode has contribution in the vicinity of the anti-crossing, so we have

\[
\Gamma_{ST}^{\text{conf}} = E^{-1} |\tilde{M}_{\text{conf}}^D|^2 |I_{ST}^\text{conf}(q_c)|^2 \frac{\partial q}{\partial \omega_1} |_{q=q_c} \\
\times |I_{ST}^\text{conf}(q_c, \sqrt{d}d_0)|^2, 
\]

where  \( |\tilde{M}_{\text{conf}}^D|^2 = \Sigma q_c^2/(2\sqrt{d}d_0) \) is a constant. Note that the eigenenergy of the first confined phonon mode tends to be linear in  \( q \), so we can write  \( q_c = E/(\hbar v_S) \).
In summary, we have investigated the ST relaxation induced by the confined phonons in two-electron nanowire-based QDs. We find that the behavior of the relaxation rate is dominated by the large DOS in the vicinity of the van Hove singularities, while in the region where the confined phonon DOS is flat, the effects of the phonon emission efficiency and the ST mixing are more pronounced. This results in the multi-peak structure of the ST relaxation rate and the local maximum of the ST+ relaxation rate in the vicinity of the ST+ anti-crossing. These features are very different from the previous results for disk-shaped QDs, indicating the unique property of the confined phonons. The effects of the deformation-potential coupling and piezoelectric coupling are also discussed. We find that for the InAs [0 0 0 1] nanowire, the deformation-potential coupling dominates the relaxation in most cases except in the region where the longitudinal component of the confined phonons mode is suppressed. The piezoelectric coupling is also found to be important for the axial confined phonon mode at \( q \to 0 \). It is also worth noting that the relaxation rate induced by confined phonons is much smaller than the one induced by bulk phonons in most cases; this suggests that the nanowire-based QDs are preferable for the application in quantum information and computation.

Acknowledgments

The author would like to thank M W Wu for proposing the topic as well as the directions during the whole investigation. This work was supported by the Natural Science Foundation of China under grant no 10725417, the National Basic Research Program of China under grant no 2006CB922005 and partially by the China Postdoctoral Science Foundation.
Appendix. $W_{\nu q}^{D/P}(r)$ in equation (9)

The quantity $W_{\nu q}^{D/P}(r)$ is the radial component of the electron–phonon coupling $H_{e\nu q}^{D/P}$ with the superscript 'D' for the deformation-potential coupling and 'P' for the piezoelectric coupling. This can be obtained by substituting equation (2) into equation (3)/equation (4).

For the deformation-potential coupling, $W_{\nu q}^{D}(r)$ is given by

$$W_{\nu q}^{D}(r) = (k_{q r}^2 + q^2)J_0(k_q r)x_{\nu q}^{(0)},$$

(A.1)

where $k_{q r}^2 = \left(\omega_{\nu q}/v_{L r}\right)^2 - q^2$.

For the piezoelectric coupling for the wurtzite InAs $[0001]$ nanowire, $W_{\nu q}^{P}(r)$ is given by

$$W_{\nu q}^{P}(r) = \int r dr' J_0(q r)K_0(q r')
\times q^4 \left(\chi_{\nu q}^{(0)} J_0(k_q r') (2q^2 - 3k_{q r}^2)
+ \chi_{\nu q}^{(2)} J_0(k_q r') (k_{q r}^2 - 4q^2)^{3/2}/q^2\right)/\sqrt{3},$$

(A.2)

where $r_c = \min(r, r')$ and $r_\nu = \max(r, r')$. $J_0(r) (J_0(r)/K_0(r))$ is the zeroth-order (modified) Bessel function. $\chi_{\nu q}^{(0)}$ and $\chi_{\nu q}^{(2)}$ are the coefficients in the expression for the confined phonon eigenmode which are calculated numerically following [38, 39].

References

[1] Loss D and DiVincenzo D P 1998 Phys. Rev. A 57 120
[2] Elzerman J M, Hanson R, Willems van Beveren L H, Witkamp B, Vandersypen L M K and Kouwenhoven L P 2004 Nature 430 431
[3] Han son R, Kouwenhoven L P, Petta J R, Tarucha S and Vandersypen L M K 2007 Rev. Mod. Phys. 79 1217
[4] Petta J R, Johnson A C, Taylor J M, Laird E A, Yacoby A, Lukin M D, Marcus C M, Hanson M P and Gossard A C 2005 Science 309 2180
[5] Petta J R, Johnson A C, Yacoby A, Marcus C M, Hanson M P and Gossard A C 2005 Phys. Rev. B 72 161301
[6] Koppens F H L, Buizert C, Tielrooij K J, Vink I T, Nowack K C, Meunier T, Kouwenhoven L P and Vandersypen L M K 2006 Nature 442 767
[7] Koppens F H L, Folk J A, Elzerman J M, Hanson R, Willems van Beveren L H, Vink I T, Tranitz H P, Wegscheider W, Kouwenhoven L P and Vandersypen L M K 2005 Science 309 1346
[8] Barthel C, Reilly D J, Marcus C M, Hanson M P and Gossard A C 2009 Phys. Rev. Lett. 103 160503
[9] Petta J R, Lu H and Gossard A C 2010 Science 327 669
[10] Johnson A C, Petta J R, Taylor J M, Yacoby A, Lukin M D, Marcus C M, and M P Hanson and Gossard A C 2005 Nature 435 925
[11] Han son R, Willems van Beveren L H, Vink I T, Elzerman J M, Naber W J M, Koppens F H L, Kouwenhoven L P and Vandersypen L M K 2005 Phys. Rev. Lett. 94 196802
[12] Fujisawa T, Austing D G, Tokura Y, Hirayama Y and Tarucha S 2002 Nature 419 278
[13] Sasaki S, Fujisawa T, Hayashi T and Hirayama Y 2005 Phys. Rev. Lett. 95 056803
[14] Meunier T, Vink I T, Willems van Beveren L H, Tielrooij K J, Hanson R, Koppens F H L, Tranitz H P, Wegscheider W, Kouwenhoven L P and Vandersypen L M K 2007 Phys. Rev. Lett. 98 126601
[15] Shen K and Wu M W 2007 Phys. Rev. B 76 235313
[16] Wang L, Shen K, Sun B Y and Wu M W 2010 Phys. Rev. B 81 235326
[17] Golovach V N, Khatskii A and Loss D 2008 Phys. Rev. B 77 045328
[18] Florescu M and Hawrylak P 2006 Phys. Rev. B 73 045304
[19] Chane y D and Maksym P A 2007 Phys. Rev. B 75 035323
[20] Climente J I, Bertoni A, Goldoni G, Rontani M and Molinari E 2007 Phys. Rev. B 75 081303
[21] Björk M T, Ohlsson B J, Sass T, Persson A I, Thrander C, Magnusson M H, Deppert K, Wallenberg L R and Samuelson L 2002 Appl. Phys. Lett. 80 1058
[22] Björk M T, Thrander C, Hansen A E, Jensen L E, Larsson M W, Wallenberg L R and Samuelson L 2004 Nano Lett. 4 1621
[23] Nilsson H A, Thrander C, Fröberg L E, Wagner J B and Samuelson L 2006 Appl. Phys. Lett. 89 163101
[24] Fas th C, Fuhrer A, Samuelson L, Golovach V N and Loss D 2007 Phys. Rev. Lett. 98 266801
[25] Pfund A, Shorubalko I, Ensslin K and Leturcq R 2007 Phys. Rev. Lett. 99 036801
[26] Pfund A, Shorubalko I, Ensslin K and Leturcq R 2009 Phys. Rev. B 79 121306
[27] Ohlsson B J, Björk M T, Magnusson M H, Deppert K, Samuelson L and Wallenberg L R 2001 Appl. Phys. Lett. 79 3335
[28] Shtrikman H, Popovitz-Biro R, Kretinin A and Heiblum M 2009 Nano Lett. 9 215
[29] Lindwall G, Wacker A, Weber C and Knorr A 2007 Phys. Rev. Lett. 99 087401
[30] Weber C, Fuhrer A, Fas th C, Lindwall G, Samuelson L and Wacker A 2010 Phys. Rev. Lett. 104 036801
[31] Romanov C L, Tamborenea P I and Ulloa S E 2009 Physica E 41 1577
[32] Cleland A N 2003 Foundation of Nanomechanics (Berlin: Springer)
[33] Takagahara T 1993 Phys. Rev. Lett. 71 3577
[34] Suzuura H and Ando T 2002 Phys. Rev. B 65 235412
[35] Chassaing P, Deme geot F, Combe N, Saint-Macary L, Kahn M L and Chaudet B 2009 Phys. Rev. B 79 155314
[36] Yu S, Kim K W, Stros c M A and Iafrate G J 1995 Phys. Rev. B 51 4695
[37] Komireno S M, Kim K W, Stros c M A and Kochelap V A 1998 Phys. Rev. B 58 16360
[38] Aub B A 1973 Acoustic Fields and Waves in Solids (New York: Wiley)
[39] Stros c M A, Kim K W, Yu S and Ballato A 1994 J. Appl. Phys. 76 4670
[40] Weber C and Wacker A 2009 Phys. Status Solidi b 246 337
[41] Nishiguchi N 1996 Phys. Rev. B 54 1494
[42] Yin Y and Wu M W 2010 J. Appl. Phys. 108 063711
[43] Madelung O (ed.) 1987 Semiconductors Landolt-Börnstein vol 17a (Berlin: Springer)
[44] Bykhovski A D, Kaminski V V, Shur M S, Chen Q C and Khan M A 1996 Appl. Phys. Lett. 68 818
[45] Hansen A E, Björk M T, Fas th C, Thrander C and Samuelson L 2005 Phys. Rev. B 71 205324
[46] Cheng J L, Wu M W and Lü C 2004 Phys. Rev. B 69 115318
[47] Jiang H, Wang Y Y and Wu M W 2008 Phys. Rev. B 77 035323
[48] Bulac D V, Trauzettel B and Loss D 2008 Phys. Rev. B 77 235301