QCD Sum Rule for Heavy Baryons

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Abstract

We construct the heavy baryonic currents by using the Bethe-Salpeter wave functions in the heavy quark limit. We discuss the one-loop renormalization of these heavy baryonic currents as well as their two-point correlators up to the order $1/M_h$. For a special case, we do the QCD sum rule for masses of the doublet $(3/2,5/2)$. Many baryons with spins larger than $3/2$ have been found in the experiments. A successful description of these particles in theory will further confirm the quark model. However, up to now there is few systemic methods to study such kind of particles. On the one hand, any relativistic model for a three-body system will be very difficult to be dealt with. On the other hand, there are also some problems to apply the non-relativistic models to baryons, not only because it is still difficult to get a solution of a three-body system but also because of susceptible validity of the non-relativistic approximation. In order to get more knowledge of the excited baryons, it may be useful to start our study from the simplest system. The heavy baryons, which contain only one heavy quark, probably are what we are looking for. In such systems, the spin of the heavy quark decouples with the light freedoms when the mass of the heavy quark goes to infinity. This is so called the heavy quark symmetry (HQS) \cite{1}. Then the total spin of the heavy baryons can be constructed in the frame of the diquark picture. The effects of HQS breaking can be taken into account perturbatively in the expansion of $1/M_h$ in the frame of the heavy quark effective theory (HQET) \cite{2}. Conversely, the property of the excited heavy baryons could check whether HQET works well in such a system (it is still amazing why the life time of $\Lambda_b$ is so smaller than that of $B$). Studying excited heavy baryons also provides useful information for semileptonic decays of heavy baryons which can serve to determine Kobayashi-Maskawa matrix elements. The reliable estimate of the widths of $\Lambda_b \to \Lambda_c^{excited} + l \nu$, which probably are small, could give a more tight constraint on $\Lambda_b \to \Lambda_c + l \nu$ \cite{3}.

Based on HQET, a non-perturbative method is still needed to deal with the heavy baryons. QCD sum rule as a model-independent method is very powerful and has wide applications \cite{4}. Our discussion in this manuscript will be restricted in its framework. In the following we firstly discuss the choice of the baryonic currents in section 1. Then the general formulas of two-point correlators are discussed in section 2. In section 3 we give an example of the calculation for the masses of the heavy baryon doublet $(3/2,5/2)$.

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1
1 Baryonic currents

In the heavy quark limit, it is possible to choose a baryonic current that only annihilates the heavy baryon state with the same quantum number. This unique character is also valid up to order $1/M_h$ in QCD sum rule. In the practical application, this character is good enough at least for $b$ heavy baryons.

There are many ways to construct such baryonic currents. In this paper, we would like to deduce them from the Bethe-Salpeter wave functions. Let us consider the heavy baryon wave function

$$\langle 0| Th(x_1) q_1(x_2) q_2(x_3) | B \rangle = e^{-im_B v \cdot x_1} \int \frac{d^4 p}{(2\pi)^4} \frac{d^4 k}{(2\pi)^4} e^{-ip x} e^{-ik x'} \chi(v, p, k),$$  \hspace{0.5cm} (1)

where $x = x_2 - x_3$, $x' = x_1 - \frac{p_2 + p_3}{2}$, $v$ is the velocity of the baryon. We have already set $m_h \to \infty$ and suppressed the color index for convenience. For an arbitrary spin $n + \frac{1}{2}$, $\chi(v, p, k)$ has two independent structures \[^3\]. For the parity $P = (-1)^n$,

$$\chi(v, p, k) = \begin{cases} u^{\mu_1 \cdots \mu_n} \Phi_{\mu_1 \cdots \mu_n} \gamma_5 C, \\ \sum_{S(\mu_1 \cdots \mu_{n+1})} \gamma_5 \gamma_5^{\mu_1} u^{\mu_2 \cdots \mu_{n+1}} \Phi_{\mu_1 \cdots \mu_{n+1}} C, \end{cases}$$

and for $P = (-1)^{n+1}$,

$$\chi(v, p, k) = \begin{cases} u^{\mu_1 \cdots \mu_n} \Phi_{\mu_1 \cdots \mu_n} C, \\ \sum_{S(\mu_1 \cdots \mu_{n+1})} \gamma_5 \gamma_5^{\mu_1} u^{\mu_2 \cdots \mu_{n+1}} \Phi_{\mu_1 \cdots \mu_{n+1}} \gamma_5 C, \end{cases}$$

where $u^{\mu_1 \cdots \mu_n}$ is a general Rarita-Schwinger tensor spinor, $\sum S(\mu_1 \cdots \mu_n)$ makes indices $\mu_1 \cdots \mu_n$ symmetric and $\Phi_{\mu_1 \cdots \mu_n}$ is a transversely symmetric tensor function of $\gamma_\perp, p_\perp$ and $k_\perp$. The $\gamma_\perp$ (so as $p_\perp$ and $k_\perp$) is defined as

$$\gamma_\perp^\mu = \gamma^\mu - \not{x} \gamma^\mu.$$

The first and second state of \[^2\] degenerate with the second and first state of \[^3\] resp. $n = 0$ is a special case, in which the first state of \[^2\] is a singlet.

Then the baryonic currents can be constructed straightforwardly. For instance, we can choose

$$j_{\nu_1 \cdots \nu_n}^\mu (x) = \Gamma_{\nu_1 \cdots \nu_n, \nu_{1} \cdots \nu_n}^\mu h(x) j_{\nu_1 \cdots \nu_n} (x)$$

for the first state of \[^2\], where

$$\Gamma_{\nu_1 \cdots \nu_n, \nu_{1} \cdots \nu_n}^\mu = g_{\nu_1}^\mu \cdots g_{\nu_n}^\mu - \frac{1}{2n + 1} \sum_{S(1 \cdots n)} \gamma_\perp^{\mu_1} \gamma_\perp^{\nu_1} g_{\nu_2}^\mu \cdots g_{\nu_n}^\mu,$$

and where

$$j_{\nu_1 \cdots \nu_n} = \sum_{\nu_{1} \cdots \nu_n} g_{\nu_1 \cdots \nu_n} T_{\nu_1 \cdots \nu_n} \sum_{\nu_{1} \cdots \nu_n} g_{\nu_1 \cdots \nu_n} T_{\nu_{1} \cdots \nu_{n}} + \cdots$$

$$+ \frac{(-1)^{n+1}}{(2n - 1)(2n - 3) \cdots (n + 1)} \sum_{S(\nu_1, \cdots, \nu_n)} g_{\nu_1 \cdots \nu_n} T_{\nu_1 \cdots \nu_n}^\mu \cdot$$
for an odd \( n \) and

\[
\begin{align*}
\hat{j}_{\nu_1 \cdots \nu_n} &= T_{\nu_1 \cdots \nu_n} - \frac{1}{2n - 1} \sum_{S(\nu_1 \cdots \nu_n)} g_{\perp \nu_1 \nu_2} T_{\nu_3 \cdots \nu_n} + \cdots \\
&+ \frac{(-1)^{\frac{n-1}{2}}}{(2n - 1)(2n - 3) \cdots (n + 2)} \sum_{S(\nu_1 \cdots \nu_n)} g_{\perp \nu_1 \nu_2} \cdots g_{\perp \nu_{n-2} \nu_{n-1}} T_{\nu_1 \cdots \nu_{n-1}}^u T_{u \nu_n}^{u \nu_n}
\end{align*}
\]

(8) for an odd \( n \). In (7) \( T_{\nu_1 \cdots \nu_n} \) is defined as

\[
q_1(x)^T C \gamma_5 \sum_{S(\nu_1 \cdots \nu_n)} D_{\perp \nu_1} \cdots D_{\perp \nu_{n-1}} \gamma_{\perp \nu_n} \rho_2(x),
\]

where \( g_{\perp \mu} = g_{\mu \nu} - v^\mu v^\nu \) and \( D_\mu = \partial_\mu + igA_\mu \). It is not difficult to check (7) is orthogonal to other states in (3) and (4). For instance, (7) is orthogonal to the second state of (3) and the first state of (4), because \( \int d^4p d^4k Tr \{ C \gamma_5 \Phi_{\mu_1 \cdots \mu_n} C \} = 0 \). For the second state of (4), we can write its matrix element of the current (7) as

\[
\langle 0 | J_{\mu_1 \cdots \mu_n}^\perp(x) | B \rangle = \Gamma_{\mu_1 \cdots \mu_n \nu_1 \cdots \nu_n} \left( \sum_{S(\rho_1 \cdots \rho_n)} \gamma_5 \gamma_1^{\rho_1} \cdots \gamma_1^{\rho_n} \right) \sum_{S(\nu_1 \cdots \nu_n)} \sum_{S(\rho_1 \cdots \rho_n)} (A g_{\perp \mu_1} \cdots g_{\perp \mu_n} + B g_{\perp \mu_1} \cdots g_{\perp \mu_n} \cdots g_{\perp \mu_n}),
\]

(9) where the term proportional to \( A \) vanishes, the rest are also zero because \( \gamma_5 \gamma_1^{\rho_1} u^{\rho_2 \cdots \rho_n} \) vanishes after contraction of any two indices \( \rho_i \).

Similarly we choose

\[
J_{\mu_1 \cdots \mu_n}^\perp(x) = \Gamma_{\mu_1 \cdots \mu_n \nu_1 \cdots \nu_n} h(x) \hat{j}_{\nu_1 \cdots \nu_n}(x),
\]

(10) as the baryonic current of the second state of (3), where

\[
\Gamma_{\mu_1 \cdots \mu_n \nu_1 \cdots \nu_n} = \sum_{S(\nu_1 \cdots \nu_n)} \gamma_1^{\mu_1} \gamma_1^{\nu_1} \gamma_1^{\mu_2} \gamma_1^{\nu_2} \cdots \gamma_1^{\mu_n \nu_n}.
\]

(11) The currents for the rest states can be obtained by omitting the \( \gamma_5 \) inserted in the two light quarks. It does not change the quantum number (except isospin) of the currents to insert \( \hat{j} \) and \( \hat{D} \) into \( q T q \), or replace insertion of \( \gamma_1^\mu \) by \( D_\perp^\mu \), or let \( D_\perp^\mu \) act on \( q T \). But these insertions probably affects the isospin character. For instance, in the case of \( n = 1 \), currents (4) and (10) only have \( I = 0 \) state. After the insertion of \( \hat{j} \), they only have \( I = 1 \) state. We will go back to discuss this later. Giving a definite quantum number, we only consider the currents with the lowest dimension, because higher dimension currents make QCDSR more unstable. So we will not discuss the currents with insertion of \( \hat{D} \) later.

The baryonic currents also can be obtained by using the wave functions proposed in (3). The result is the same as above.

Now we are going on the renormalization of these baryonic currents, which must be used in the QCD sum rule beyond \( \alpha_s \) correction as well as matching the matrix elements of these currents from HQET to the full QCD. The one-loop renormalization of these baryonic currents without the covariant derivative \( D_\mu \) has been considered in (4). When we consider the renormalization of the currents with one or more derivative \( D_\mu \), the currents with the
same quantum number and dimension will mix with each other. For convenience we firstly consider a symmetric tensor current, which is transverse, traceless and only has one covariant derivative $D_{\mu}$, i.e.

$$j_1^{\mu \nu} = q_1^T CT(D_{\perp}^{\mu} \gamma_{\perp}^{\nu} + D_{\perp}^{\nu} \gamma_{\perp}^{\mu} - \frac{2}{3} g_{\perp}^{\mu \nu} \partial)q_2 h.$$  \hspace{1cm} (12)

Obviously the renormalization of $j_1^{\mu \nu}$ is the same as $J_+^{\mu \nu}$ in (5) and (10) in the heavy quark limit. The complete set of operators which may mix with (12) is

$$
\begin{align*}
J_1^{\mu \nu} &= q_1^T CT(D_{\perp}^{\mu} \gamma_{\perp}^{\nu} + D_{\perp}^{\nu} \gamma_{\perp}^{\mu} - \frac{2}{3} g_{\perp}^{\mu \nu} \partial)q_2 h \\
J_2^{\mu \nu} &= q_1^T CT(D_{\perp}^{\mu} \gamma_{\perp}^{\nu} + D_{\perp}^{\nu} \gamma_{\perp}^{\mu} - \frac{2}{3} g_{\perp}^{\mu \nu} \partial)q_2 h \\
J_3^{\mu \nu} &= q_1^T CT(\gamma_{\perp}^{\mu} q_2 D_{\perp}^{\nu} h + \gamma_{\perp}^{\nu} q_2 D_{\perp}^{\mu} h - \frac{2}{3} g_{\perp}^{\mu \nu} \gamma_{\perp}^{\rho} q_2 D_{\perp}^{\rho} h),
\end{align*}
$$

where $\Gamma$ could be $1, \gamma_5, \bar{\gamma}$ or $\gamma_5 \bar{\gamma}$ and $q^T \partial \gamma_{\perp}^{\mu} = (D_{\perp}^{\mu} q)^T$. $j_1^{\mu \nu}$ is trivial because it differs from $j_1^{\mu \nu} + j_2^{\mu \nu}$ by a total derivative. The nuisance operators such as

$$q_1^T CT(A_{\perp}^{\mu} \gamma_{\perp}^{\nu} + A_{\perp}^{\nu} \gamma_{\perp}^{\mu} - \frac{2}{3} g_{\perp}^{\mu \nu} A)q_2 h$$

do not appear because they do not vanish by the equations of the motion.

Then the bare current $[j_1^{\mu \nu}]$ can be expressed as

$$[j_1^{\mu \nu}] = Z_q Z_h^2 (Z_1 j_1^{\mu \nu} + Z_2 j_2^{\mu \nu} + Z_3 j_3^{\mu \nu}),$$

where $Z_q = 1 - C_{F} \frac{\alpha_{s}}{4 \pi \epsilon}$ and $Z_h = 1 + 2 C_{F} \frac{\alpha_{s}}{4 \pi \epsilon}$ are the renormalization coefficients of the light quark and the heavy quark resp. and $\bar{j}_i^{\mu \nu}$ ($i = 1, 3$) are the renormalized currents. In order to determine the renormalization coefficients $Z_i$ ($i = 1, 3$), we need to insert the current into 1PI diagrams and extract the ultraviolet divergence. We find that the divergence associated with the three-quark vertex are sufficient to determine all counterterms. The relevant Feynman diagrams are shown in Fig.1. In the Feynman gauge, we obtain

$$
\begin{align*}
Z_1 &= 1 + [((H^2/6) + 3/2) CB - C_{F}] \frac{\alpha_{s}}{4 \pi \epsilon} \\
Z_2 &= (H^2/12 + 1) CB \frac{\alpha_{s}}{4 \pi \epsilon} \\
Z_3 &= 0,
\end{align*}
$$

where $\gamma_{\mu} \Gamma \gamma_{\nu} = H \Gamma \gamma_{\nu}$, $C_B = \frac{N_c+1}{2 N_c}$, $C_F = \frac{N_c^2-1}{2 N_c}$ and $\epsilon$ is defined as $D = 4 - 2 \epsilon$ in the MS-scheme. The renormalization of $J_2^{\mu \nu}$ is similar as that of $J_1^{\mu \nu}$. Therefore $j_1^{\mu \nu}$ and $j_2^{\mu \nu}$ can construct a renormalization-invariant space. The one-loop renormalization invariant currents can be chosen as

$$J_{\pm}^{\mu \nu} = j_{1}^{\mu \nu} \pm j_{2}^{\mu \nu}.$$

They satisfy the equation

$$[J_{\pm}^{\mu \nu}] = Z_q Z_h^2 (Z_1 \pm Z_2) j_{\pm}^{\mu \nu}.$$ 

The current $J_{\pm}^{\mu \nu}$ corresponds to the state $I = 1$ for $\Gamma = 1$, $\psi$ or $\gamma_5 \bar{\psi}$ or $I = 0$ for $\Gamma = \gamma_5$. $J_{-}^{\mu \nu}$ corresponds to $I = 0$ state for $\Gamma = 1$, $\bar{\psi}$, or $\gamma_5 \bar{\psi}$ or $I = 1$ for $\Gamma = \gamma_5$.

The anomalous dimensions are obtained by

$$\gamma_{\pm} = \frac{d \log(Z_1 \pm Z_2)}{d \log \mu} = - \left[ \frac{H^2}{6} + 3/2) CB - C_{F} \pm \frac{H^2}{12} + 1) CB \right] \frac{\alpha_{s}}{2 \pi},$$

\hspace{1cm} (17)
where $\mu$ is the renormalization point. The relation between HQET's currents and the corresponding ones in the full QCD in the leading logarithmic approximation is

$$J_{QCD} = C(\mu)J_{HQET}(\mu),$$  \hspace{1cm} (18)

where the matching condition is

$$C_\pm(\mu) = C_\pm(M_h)e^{\int_{\mu}^{(\mu)} \frac{g_3^2(\mu)}{\beta(\mu)}}$$  \hspace{1cm} (19)

and where $C_\pm(M_h) = 1 + \mathcal{O}(\alpha_s(M_h))$. It is more complicated to get the full matching condition. The relevant references could be seen in [8][9]. We do not discuss this case.

(15) can be extended to a general case. For the rank $n$ tensor current with $n-1$ covariant derivative $D$, the complete set of operators is

$$j_{\nu_1...\nu_n}^i = (T_{\nu_1...\nu_n}^i - \frac{1}{2n-1} \sum_{S(\nu_1...\nu_n)} g_{\nu_1\nu_2} T_{\nu_3...\nu_n}^i + \cdots$$

$$+ \frac{(-1)^{\frac{n-1}{2}}}{(2n-1)(2n-3)\cdots(n+1)} \sum_{S(\nu_1...\nu_n)} g_{\nu_1\nu_2} \cdots g_{\nu_{n-1}\nu_n} T_{\nu_1...\nu_n}^i \frac{i}{\frac{i}{2}}, h(i = 1, n)$$

for an even $n$ and

$$j_{\nu_1...\nu_n}^i = (T_{\nu_1...\nu_n}^i - \frac{1}{2n-1} \sum_{S(\nu_1...\nu_n)} g_{\nu_1\nu_2} T_{\nu_3...\nu_n}^i + \cdots$$

$$+ \frac{(-1)^{n+1}}{(2n-1)(2n-3)\cdots(n+2)} \sum_{S(\nu_1...\nu_n)} g_{\nu_1\nu_2} \cdots g_{\nu_{n-2}\nu_{n-1}} T_{\nu_1...\nu_{n-1}}^i \frac{i}{\frac{i}{2}}, h(i = 1, n)$$

for an odd $n$, where

$$T_{\mu_1...\mu_n}^i = q_1(x)^T C_T \sum_{S(\nu_1...\nu_n)} \tilde{D}_{\nu_1} \cdots \tilde{D}_{\nu_{n-1}} D_{\mu_1} \cdots D_{\mu_{n-1}} \gamma_{\nu_n} q_2(x).$$  \hspace{1cm} (22)

The bare current $[j_i^{\mu_1...\mu_n}]$ could be written as

$$[j_i^{\mu_1...\mu_n}] = Z_2 Z_3^\frac{1}{2} \sum_{k=1,n} Z_{ikj} Z_{k,n}^{\mu_1...\mu_n}.$$  \hspace{1cm} (23)

After calculating the Feynman diagrams similar to Fig.1, we obtain

$$Z_{ik} = 1 + \left\{ \left( \frac{1}{m-k+1} + \frac{1}{k} \right) C_B - \left( \sum_{m=0}^{n-k-1} \frac{m+1}{(n-k+1)(n-k-m)} + \sum_{m=0}^{k-2} \frac{m+1}{k(m-k-1)} \right) 2C_F \right\}$$

$$+ \sum_{m=0}^{k} \sum_{l=0}^{m} C_{k-1}^m C_{n-k}^m C_{n-k}^l (m-l)!l!(n-k-1)!k!(n-l+1)! (m-l+1)! \left( \frac{H^2}{2} \right) C_B \right\} \frac{\alpha_s}{4\pi\epsilon}$$

for $i = k$,

$$Z_{ik} = \left[ \sum_{m=0}^{l} \sum_{l=0}^{m} C_{k-1}^m C_{n-k}^l C_{n-k}^l \frac{(m-l)!l!(n-k+1)!(k-l-1)!H^2}{2} \frac{C_B}{2} \right] \frac{\alpha_s}{4\pi\epsilon}$$

(25)
for $i < k$ and

$$Z_{ik} = \left[ \sum_{m=0}^{k-1} \sum_{l=0}^{m} C_{k-1}^m C_{n-k}^{m-l-1} C_{m}^{l} \frac{(m-l)!(i-l-1)!(n-k+1)!(k-l-1)!}{l!(n-l+1)!} \frac{H^2}{2} C_B \right] \frac{\alpha_s}{4\pi\epsilon}$$

for $i > k$. The renormalization-invariant currents can be obtained through the diagonalization of this matrix. We do not find a simple way to diagonalize it. However, it should not be difficult to get the numerical solution for a special $n$.

## 2 Two-point correlator

The basic idea of the QCD sum rule is to consider the two-point correlator

$$\Pi_{\mu_1 \cdots \mu_n, \nu_1 \cdots \nu_n}(k) = \int d^3x e^{ik \cdot x} i\langle 0 | J_{\mu_1 \cdots \mu_n}(x), \bar{J}_{\nu_1 \cdots \nu_n} | 0 \rangle$$

for a region of $k$ in which one can incorporate the asymptotic freedom property of QCD via the operator product expansion (OPE), and then relate it to the hardonic matrix elements via the dispersion relation. The currents $J_{\mu_1 \cdots \mu_n}(x)$ and $J_{\mu_1 \cdots \mu_n}^\dagger(x)$ were defined in (5) or (10). They have the same quantum number and the same $\Gamma$ defined in (22). We do not discuss the case that these two currents have different $\Gamma$, because the perturbation part of such correlators is zero. The extra indices $i, j$ were defined in (24).

In order to analyze the construction of the correlator (27) up to the order $1/M_h$, we write down the effect Lagrangian of the heavy quark up to the order $1/M_h$

$$L_{eff} = \bar{h}_v i v \cdot Dh_v + \frac{1}{2M_h}(\bar{h}_v i D h_v - \frac{1}{2} \bar{h}_v g G_{\mu\nu} \sigma^{\mu\nu} h_v).$$

In the leading order of HQET, the the spin of the heavy quark decouples with the light freedom, therefore (27) can be expressed as

$$\Pi_{\mu_1 \cdots \mu_n, \nu_1 \cdots \nu_n}(\omega) = \Pi_0(\omega) I_{\alpha_1 \cdots \alpha_n, \beta_1 \cdots \beta_n} \Gamma_{\mu_1 \cdots \mu_n, \alpha_1 \cdots \alpha_n} \frac{1 + \frac{i}{2} \Gamma_{\nu_1 \cdots \nu_n, \beta_1 \cdots \beta_n}}{2},$$

where $I_{\alpha_1 \cdots \alpha_n, \beta_1 \cdots \beta_n}$ is transverse, symmetric and traceless for indices $\alpha_1 \cdots \alpha_n$ and $\beta_1 \cdots \beta_n$ resp.. Since $x_\mu$ is proportional to $v_\mu$ via the heavy quark propagator $\frac{1 + i}{2} \int dt \delta(x - vt)$, only one scalar argument $\omega = k \cdot v$ survives in (29). $I_{\alpha_1 \cdots \alpha_n, \beta_1 \cdots \beta_n}$ is only composed of $v_\mu$ and $g_{\mu\nu}$ and must be written as

$$I_{\alpha_1 \cdots \alpha_n, \beta_1 \cdots \beta_n} = \sum_{S(\alpha_1 \cdots \alpha_n)} \sum_{S(\beta_1 \cdots \beta_n)} \left( g_{\alpha_1 \beta_1} \cdots g_{\alpha_n \beta_n} - \frac{2}{2n-1} g_{\alpha_1 \alpha_2} g_{\beta_1 \beta_2} g_{\alpha_3 \beta_3} \cdots g_{\alpha_n \beta_n} ight.$$

$$+ \cdots + \frac{(-2)^{\frac{2}{n-1}(2n-3) \cdots (n+1)}}{(2n-1)(2n-3) \cdots (n+1)} g_{\alpha_1 \alpha_2} g_{\beta_1 \beta_2} \cdots g_{\alpha_{n-1} \alpha_n} g_{\beta_{n-1} \beta_n} \left. \right)$$

(30)
for an even $n$ and
\[ I_{\alpha_1 \cdots \alpha_n, \beta_1 \cdots \beta_n} = \sum_{S(\alpha_1 \cdots \alpha_n), S(\beta_1 \cdots \beta_n)} \left( g \perp \alpha_1 \beta_1 \cdots g \perp \alpha_n \beta_n - \frac{2}{2n-1} g \perp \alpha_1 \alpha_2 g \perp \beta_1 \beta_2 g \perp \alpha_3 \beta_3 \cdots g \perp \alpha_n \beta_n \right) \]
\[ + \cdots + \frac{(-2)^{n-2}}{(2n-1)(2n-3) \cdots (n+2)} g \perp \alpha_1 \alpha_2 g \perp \beta_1 \beta_2 \cdots g \perp \alpha_{n-2} \alpha_{n-1} g \perp \beta_{n-2} \beta_{n-1} g \perp \alpha_n \beta_n \]
(31)

for an odd $n$.

To the order of $1/M_h$, the term $\frac{1}{2M_h^2} h_v (iD)^2 h_v$ does not break the spin symmetry but could produce a new scalar argument $k^2$ in the correlator (27). For convenience, we choose $k$ paralleling to $v$. Then we also can write its contribution as
\[ \Pi_{\mu_1 \cdots \mu_n, \nu_1 \cdots \nu_n}(\omega) = \frac{1}{2M_h} \Pi_{1a}(\omega) I_{\alpha_1 \cdots \alpha_n, \beta_1 \cdots \beta_n} \Gamma_{\pm}^{\mu_1 \cdots \mu_n, \alpha_1 \cdots \alpha_n} \frac{1 + \gamma^\mu_1 \cdot \nu_1, \beta_1 \cdots \beta_n}{2}. \] (32)

The chromo-magnetic term $\frac{1}{2M_h^2} h_v g_{\mu \nu} \sigma_{\parallel}^{\mu \nu} h_v$ breaks spin symmetry and causes the mass-split of the heavy baryon doublets. We can write its contribution as
\[ \Pi_{1b}^{\mu_1 \cdots \mu_n, \nu_1 \cdots \nu_n}(\omega) = \frac{1}{2M_h} i \Pi_{1b}(\omega) \Gamma_{\pm}^{\mu_1 \cdots \mu_n, \alpha_1 \cdots \alpha_n} \frac{1 + \gamma^\mu_1 \cdot \nu_1, \beta_1 \cdots \beta_n}{2}, \] (33)

where $F_{ij, \alpha_1 \cdots \alpha_n, \beta_1 \cdots \beta_n}$ is transverse, symmetric and traceless for indices $\alpha_1 \cdots \alpha_n$ and $\beta_1 \cdots \beta_n$ resp.. Since $F_{ij, \alpha_1 \cdots \alpha_n, \beta_1 \cdots \beta_n}$ also only depends on $v_\mu$ and $g_{\mu \nu}$, we can write down
\[ \sigma_{\parallel}^{ij} F_{ij, \alpha_1 \cdots \alpha_n, \beta_1 \cdots \beta_n} = \sum_{S(\alpha_1 \cdots \alpha_n), S(\beta_1 \cdots \beta_n)} \left( g \perp \alpha_1 \beta_1 \cdots g \perp \alpha_{n-1} \beta_{n-1} \right) \]
\[ - \frac{2}{2n-1} g \perp \alpha_1 \alpha_2 g \perp \beta_1 \beta_2 g \perp \alpha_3 \beta_3 \cdots g \perp \alpha_{n-1} \beta_{n-1} + \cdots + \]
\[ \frac{(-2)^{n-2}}{(2n-1)(2n-3) \cdots (n+2)} g \perp \alpha_1 \alpha_2 g \perp \beta_1 \beta_2 \cdots g \perp \alpha_{n-2} \alpha_{n-1} g \perp \beta_{n-2} \beta_{n-1} \right) \sigma_{\perp} \alpha_n \beta_n \] (34)

for an odd $n$ and
\[ \sigma_{\parallel}^{ij} F_{ij, \alpha_1 \cdots \alpha_n, \beta_1 \cdots \beta_n} = \sum_{S(\alpha_1 \cdots \alpha_n), S(\beta_1 \cdots \beta_n)} \left( g \perp \alpha_1 \beta_1 \cdots g \perp \alpha_{n-2} \beta_{n-2} \right) \]
\[ - \frac{2}{2n-1} g \perp \alpha_1 \alpha_2 g \perp \beta_1 \beta_2 g \perp \alpha_3 \beta_3 \cdots g \perp \alpha_{n-2} \beta_{n-2} + \cdots + \]
\[ \frac{(-2)^{n-2}}{(2n-1)(2n-3) \cdots (n+3)} g \perp \alpha_1 \alpha_2 g \perp \beta_1 \beta_2 \cdots g \perp \alpha_{n-3} \alpha_{n-2} \beta_{n-3} \beta_{n-2} \right) \]
\[ g \perp \alpha_{n-1} \beta_{n-1} \sigma_{\perp} \alpha_n \beta_n \] (35)

for an even $n$.

It is not difficult to check that
\[ \Gamma_{+}^{\mu_1 \cdots \mu_n, \alpha_1 \cdots \alpha_n} \frac{1 + \gamma^\mu_1 \cdot \nu_1, \beta_1 \cdots \beta_n}{2} F_{ij, \alpha_1 \cdots \alpha_n, \beta_1 \cdots \beta_n} = - i(n+1) I_{\alpha_1 \cdots \alpha_n, \beta_1 \cdots \beta_n} \Gamma_{-}^{\mu_1 \cdots \mu_n, \alpha_1 \cdots \alpha_n} \frac{1 + \gamma^\mu_1 \cdot \nu_1, \beta_1 \cdots \beta_n}{2}, \]
(36)
This is consistent with the naive expectation based on the quantum mechanics.

With these formulas in our hand, we begin to construct the QCD sum rule for masses of the heavy baryons. At first, the imaginary part \( \langle 37 \rangle \) can be obtained by insertion of the physical states. On the assumption that only one resonance and continuum contribute to the correlator \( \langle 27 \rangle \), we can write

\[
Im\Pi_{\mu_1...\mu_n,\nu_1...\nu_n}(\omega) = \int d^4xe^{i\mathbf{k}\cdot\mathbf{x}}\langle 0|J_{\mu_1...\mu_n}(x)|B_{n\pm\frac{1}{2}}\rangle\langle B_{n\pm\frac{1}{2}}|J_{\nu_1...\nu_n}|0\rangle + \text{continuum}
\]

\[
= \pi\delta(\omega - \Lambda)|f_{n\pm\frac{1}{2}}|^2M_{n\pm\frac{1}{2}}^{2n+3}\Gamma_{\alpha_1...\alpha_n,\beta_1...\beta_n}^{\mu_1...\mu_n,\alpha_1...\alpha_n}e_{\alpha_1...\alpha_n}\nu, \quad \text{continuum},
\]

where \( M_{n\pm\frac{1}{2}} \) is the mass of the heavy baryon, \( \Lambda \) is defined as

\[
M_{n\pm\frac{1}{2}} = M_h + \Lambda + \mathcal{O}\left(\frac{1}{M_h^2}\right)
\]

and

\[
\langle 0|J_{\mu_1...\mu_n}|B_{n\pm\frac{1}{2}}(k)\rangle = if_{n\pm\frac{1}{2}}M_{n\pm\frac{1}{2}}^{n+3}\Gamma_{\alpha_1...\alpha_n,\beta_1...\beta_n}^{\mu_1...\mu_n,\alpha_1...\alpha_n}e_{\alpha_1...\alpha_n}\nu.
\]

In \( \langle 38 \rangle \) \( u \) is a \( \frac{1}{2} \) spinor and \( e_{\alpha_1...\alpha_n} \) is a symmetric polarization tensor and satisfies

\[
u^{\alpha_1}\epsilon_{\alpha_1...\alpha_n} = 0, \quad e_{\alpha_\alpha 3...\alpha_n} = 0. \quad \text{(39)}
\]

\( \Gamma_{\alpha_1...\alpha_n,\beta_1...\beta_n}^{\mu_1...\mu_n,\alpha_1...\alpha_n}e_{\alpha_1...\alpha_n}\nu \) can serve as a \( n \pm \frac{1}{2} \) spinor. There are \( 2n + 1 \) independent \( \epsilon_{\alpha_1...\alpha_n} \) which are normalized as

\[
\sum_{i=1}^{2n+1}\epsilon^{\dagger}_{\alpha_1...\alpha_n}\epsilon_{\beta_1...\beta_n} = I_{\alpha_1...\alpha_n,\beta_1...\beta_n}. \quad \text{(40)}
\]

Then the sum rule can be established. The correlator \( \langle 27 \rangle \) obey the dispersion relation

\[
\Pi_{\pm}(\omega) = \frac{1}{\pi}\int \frac{Im\Pi_{\pm}(\omega')d\omega'}{\omega' - \omega} = \int \frac{\rho_{\pm}(\omega')d\omega'}{\omega' - \omega}, \quad \text{(41)}
\]

where the spectral density \( \rho_{\pm}(\omega) = \frac{1}{\pi}Im\Pi_{\pm}(\omega) \) can be obtained from \( \langle 23 \rangle \), \( \langle 22 \rangle \), \( \langle 23 \rangle \) and \( \langle 30 \rangle \) (via OPE in a proper region of \( \omega \)) on the one hand,

\[
\Pi_{\pm}(\omega) = \Pi_0(\omega) + \frac{1}{2M_h}(\Pi_{i\alpha}(\omega) + a_+\Pi_{ib}(\omega)), \quad a_+ = n, \quad a_- = -(n+1). \quad \text{(42)}
\]

On the other hand,

\[
\rho_{\pm}(\omega') = \pi\delta(\omega - \Lambda)|f_{n\pm\frac{1}{2}}|^2M_{n\pm\frac{1}{2}}^{2n+3} + \text{continuum}
\]

is obtained from \( \langle 37 \rangle \). Thus, the baryonic mass as well as its matrix element can be extracted from \( \Pi_0 \). One may note that there are two sources of uncertainty in \( \Pi_0 \). At first, we have to truncate the series of OPE. Thus, the contributions from higher dimension operators raise the main uncertainty of QCDSR. Another uncertainty is from the contributions of heavier resonances and continuum. In order to suppress these uncertainties, the Borel transformation is applied on the both side of \( \Pi_0 \). Then we obtain

\[
\int_0^s d\omega e^{-\frac{\omega}{M}}\rho_{\pm}(\omega) = f_{n\pm\frac{1}{2}}f_{n\pm\frac{1}{2}}M_{n\pm\frac{1}{2}}^{2n+3}e^{-\frac{\Lambda}{M}}, \quad \text{(43)}
\]
where \(M\) is the Borel parameter and the upper bound of the integral \(s\) is used to remove the rest contributions from the continuum. The mass parameter \(\Lambda\) can be obtained by taking the partial derivative respect to \(1/M\) on both sides of (43)

\[
\Lambda_\pm = \frac{\int_0^s d\omega \rho_\pm (\omega) e^{-\frac{\omega}{M}}}{\int_0^s d\omega \rho_\pm (\omega) e^{-\frac{\omega}{M}}}.
\]

(44)

The Borel parameter should be chosen in a region that \(\Lambda_\pm\) is not sensitive to it. In graphic language, there should be a plateau in the plot of \(M-\Lambda_\pm\).

3 Sum rule for \((3/2,5/2)\)

For a special case, let us consider the heavy baryon doublet \((3/2,5/2)\). As a preliminary calculation, the radiative corrections are not taken in account. So the heavy quark propagator for a special case, let us consider the heavy baryon doublet \((3/2,5/2)\). As a preliminary calculation, the radiative corrections are not taken in account. So the heavy quark propagator to the order \(1/M_h\) in the coordinate space can be expressed as

\[
S_h(x) = \frac{1 + \frac{t}{2}}{(2\pi i)^d} \int_0^\infty dt \left[ 1 + \frac{it}{2M_h} (-\partial^2 - 2gG_{\mu\nu} x^\mu \partial^\nu + \frac{\pi x^2}{24} (\alpha_s G^2) - \frac{gG_{\mu\nu}}{2} \sigma_{\mu\nu} - \frac{1}{2}) \right] \delta(x-vt),
\]

(45)

where we have used the fixed point gauge \(x^\mu A_\mu (x) = 0\). The two-point correlators then are

\[
\Pi_\pm^{\mu_1 \mu_2 \nu_1 \nu_2} (x) = -i \left[ Tr \{ \Gamma \Delta_{\alpha_1 \alpha_2} (x) S_l (x) \sum_{\beta_1 \beta_2} \Pi (0)^\beta_1 \beta_2 (0) \Gamma S_l^\dagger (x) \} + (-1)^{I+1} Tr \{ S_l^\dagger (x) \sum_{\alpha_1 \alpha_2} \Gamma \Delta S_l (x) \sum_{\beta_1 \beta_2} \Pi (0) \Gamma \} \right] \Gamma_\pm^{\mu_1 \mu_2 \alpha_1 \alpha_2} S_h (x) \Gamma_\pm^{\nu_1 \nu_2 \beta_1 \beta_2},
\]

(46)

where

\[
\Delta_{\alpha_1 \alpha_2} (x) = D_{\alpha_1}^\dagger (x) \gamma_1^\alpha_2 + D_{\alpha_2}^\dagger (x) \gamma_1^\alpha_1 - \frac{2}{3} g_{\alpha_1 \alpha_2} \partial_\perp (x),
\]

(47)

\[
\Gamma = \gamma_0 \Gamma^\dagger \gamma_0, \quad \Gamma^c = C^{-1} \Gamma C, \quad S_l^\dagger (x) = C^{-1} S_l^\dagger (x) C, \quad I \text{ is the isospin of the current and the light propagator } S_l (x) \text{ in external gluonic field was given in } [4].
\]

The correlators in momentum space related to (46) via Fourier Transformation

\[
\Pi_\pm^{\mu_1 \mu_2 \nu_1 \nu_2} (k^2, \omega) = \int d^4 x e^{ikx} \Pi_\pm^{\mu_1 \mu_2 \nu_1 \nu_2} (x)
\]

(47)

Because we set \(k_\mu\) paralleling to \(v_\mu\), by using the technique of the partial integral, we can write (47) in the form

\[
\Pi_\pm^{\mu_1 \mu_2 \nu_1 \nu_2} (\omega) = \int d^4 x e^{i\omega t} \Pi_\pm^{\mu_1 \mu_2 \nu_1 \nu_2} (t)
\]

(48)

\[
= \Gamma_\pm^{\mu_1 \mu_2 \alpha_1 \alpha_2} \Gamma_\pm^{\nu_1 \nu_2 \beta_1 \beta_2} \int d^4 x e^{i\omega t} \Pi_\pm (t)
\]

We analytically continue correlators from \(t > 0\) to imaginary \(t = -i\tau\). Then the spectral density \(\rho_\pm (\omega)\) are related to \(\Pi_\pm (\tau)\) by the Laplace transformation

\[
\rho_\pm (\omega) = \frac{i}{2\pi} \int_{a-i\infty}^{a+i\infty} d\tau e^{\omega \tau} \Pi_\pm (\tau)
\]

(49)
Fig. 2 shows the Feynman diagrams that we calculate. The corresponding results are read as
\[ \rho_0(\omega) = \frac{\omega^7}{630 \pi^4} - \frac{17 \omega^3}{864 \pi^3} \langle \alpha_s G^2 \rangle, \]
\[ \rho_{1a}(\omega) = -\frac{\omega^3}{315 \pi^4} + \frac{18 \omega}{18 \pi^3} \langle \alpha_s G^2 \rangle, \] (50)
\[ \rho_{1b}(\omega) = -\frac{\omega^3}{216 \pi^3} \langle \alpha_s G^2 \rangle \]
for \( \Gamma = 1 \) with \( I = 0 \) and \( \Gamma = \gamma_5 \) with \( I=1 \),
\[ \rho_0(\omega) = \frac{\omega^7}{630 \pi^4} - \frac{7 \omega^3}{432 \pi^3} \langle \alpha_s G^2 \rangle, \]
\[ \rho_{1a}(\omega) = -\frac{\omega^3}{140 \pi^4} + \frac{25 \omega^3}{432 \pi^3} \langle \alpha_s G^2 \rangle, \] (51)
\[ \rho_{1b}(\omega) = -\frac{\omega^3}{216 \pi^3} \langle \alpha_s G^2 \rangle \]
for \( \Gamma = \bar{\rho} \) and \( \Gamma = \gamma_5 \bar{\rho} \) with \( I=0 \),
\[ \rho_0(\omega) = \frac{\omega^7}{630 \pi^4} + \frac{\omega^3}{432 \pi^3} \langle \alpha_s G^2 \rangle, \]
\[ \rho_{1a}(\omega) = -\frac{\omega^3}{105 \pi^4} + \frac{\omega^3}{24 \pi^3} \langle \alpha_s G^2 \rangle, \] (52)
\[ \rho_{1b}(\omega) = -\frac{\omega^3}{216 \pi^3} \langle \alpha_s G^2 \rangle \]
for \( \Gamma = \gamma_5 \) with \( I=0 \) and \( \Gamma = 1 \) with \( I = 1 \),
\[ \rho_0(\omega) = \frac{\omega^7}{630 \pi^4} + \frac{\omega^3}{432 \pi^3} \langle \alpha_s G^2 \rangle, \]
\[ \rho_{1a}(\omega) = -\frac{\omega^3}{13 \omega^4} + \frac{54 \omega^3}{54 \pi^3} \langle \alpha_s G^2 \rangle, \] (53)
\[ \rho_{1b}(\omega) = -\frac{\omega^3}{216 \pi^3} \langle \alpha_s G^2 \rangle \]
for \( \Gamma = \bar{\rho}, \gamma_5 \bar{\rho} \) with \( I=1 \).

It is interesting to compare (50) with (51). (50) shows that \( I = 0 \) state of the current with \( \Gamma = \bar{\rho} \) and \( \Gamma = \gamma_5 \bar{\rho} \) are degenerate up to the order we consider. However, from (51), \( I = 0 \) state of the current with \( \Gamma = 1 \) is degenerate with \( I = 1 \) state of the current with \( \Gamma = \gamma_5 \) (not naively expected \( I=0 \) state). It is another example that the insertion of \( \bar{\rho} \) affects isospin. The degeneration of (50)-(53) will disappear when the light quark mass or the quark condensate is taken into account, i.e., the chiral symmetry is broken. Since our calculation in this paper is very preliminary, we leave the calculation of the quark condensates in our next paper.

Now let us proceed to the determination of the heavy baryon masses. The mass of the heavy baryon can be obtained by (44). In order to distinguish the mass split for the doublet(3/2,5/2), we expand the masses as
\[ \Lambda_\pm = \Lambda_0 + \frac{\Lambda_K}{2M_h} + a_\pm \frac{\Lambda_M}{2M_h} \] (54)
The sum rules for $\Lambda_0$, $\Lambda_K$, and $\Lambda_M$ are shown in Fig.3 for the case (50), where we use the threshold $s = 2\,\text{GeV}$ and the gluonic condensate $\langle \alpha_s G^2 \rangle = 0.07\,\text{GeV}^4$. Fig.4 shows the dependence of the $\Lambda_0$ on $s$, where it is shown that $s < \Lambda_0$ when $s = 1.7\,\text{GeV}$. The $\Lambda_0$ is not very sensitive to $s$ around $s = 1.8\,\text{GeV}$. It goes down when $\langle \alpha_s G^2 \rangle$ becomes smaller.

The sum rule for (51) is very similar to the case (50). It gives the same mass prediction within the uncertainty of these two sum rules. The sum rule of (52) is much different from that of (50) and (51). Fig.5 shows that the kinetic term $\Lambda_K$ is negative when $s \geq 1.6\,\text{GeV}$. We do not find a stable region for the threshold $s$. Fig.6 shows the sum rule of (52) at $s = 1.4\,\text{GeV}$. The situation of (53) is similar to (52). Thus we obtain

$$\Lambda_0 = 1.8 \pm 0.2\,\text{GeV}, \quad \Lambda_K = 0.5 \pm 0.5\,\text{GeV}^2, \quad \Lambda_M = 0.05 \pm 0.05\,\text{GeV}^2.$$  

(55)

for the sum rules (50) and (51) and

$$\Lambda_0 = 1.2 \pm 0.2\,\text{GeV}, \quad \Lambda_K = 0.06 \pm 0.05\,\text{GeV}^2, \quad \Lambda_M = 0.03 \pm 0.005\,\text{GeV}^2$$  

(56)

for the sum rules (52) and (53). The error bars are from varying the parameters in a proper region. $\Lambda_M$ is very sensitive to $\langle \alpha_s G^2 \rangle$. When $\langle \alpha_s G^2 \rangle = 0.04\,\text{GeV}$, it becomes unbelievably small. This is similar to the case in [12]. The reason may be that the mass split mainly is from the internal gluon exchange which we did not consider here. $\Lambda_K/1\,\text{GeV}$ is much smaller than the masses of $b$ and $c$ quarks. Therefore the expansion of $1/M_h$ stands well. Using the quark masses [7][12]

$$m_b = 4.8\,\text{GeV}, \quad m_c = 1.4\,\text{GeV},$$

(57)

we predict the masses of the baryon doublet (3/2,5/2) are

$$M_b = 6.6 \pm 0.2\,\text{GeV}, \quad M_c = 3.4 \pm 0.4\,\text{GeV}.$$  

(58)

for (50) and (51) and

$$M_b = 6.0 \pm 0.2\,\text{GeV}, \quad M_c = 2.8 \pm 0.3\,\text{GeV}.$$  

(59)

for (52) and (53).

Finally, let us give a brief summary. We have discussed the baryonic currents with arbitrary quantum numbers as well as their one-loop renormalization. Using the obtained currents, we have analyzed their two-point correlators. For a special case, we did the QCD sum rule for the masses of the heavy baryon doublet (3/2,5/2) up to the order $1/M_h$. Because we did not take account of the light quark mass and the quark condensates, we could not distinguish the two-point correlators of the currents with different parity. Also because we did not take account of the internal gluon exchange, the mass split of the heavy baryon doublet is too small. We will include these discussions in our next paper.

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Figure captions

Fig.1 Feynman diagrams for the renormalization of the baryonic currents.

Fig.2 Feynman diagrams for the two-point correlator. The smaller dots stand for the current vertices, the bigger dots stand for the $1/M_h$-order interaction vertices.

Fig.3 The Sum rule for (50). Dashed line gives $\Lambda_0$ versus Borel variable $M$, solid line is $\Lambda_K/1GeV$ versus Borel variable $M$ and dotted line is $\Lambda_M/1GeV$ versus Borel variable $M$.

Fig.4 The dependence of $\Lambda_0$ on $s$. Dashed line gives the result for $s = 1.8GeV$, solid line is for $s = 1.7GeV$ and dotted line is for $s = 2GeV$.

Fig.5 The dependence of $\Lambda_K/1GeV$ on $s$ for (52). Dashed line gives the result for $s = 1.4GeV$, solid line is for $s = 1.2GeV$ and dotted line is for $s = 1.6GeV$.

Fig.6 The Sum rule for (52). Dashed line gives $\Lambda_0$ versus Borel variable $M$, solid line is $\Lambda_K/1GeV$ versus Borel variable $M$ and dotted line $\Lambda_M/1GeV$ versus Borel variable $M$. 
$\Lambda_K (\text{GeV})$

0.12
0.1
0.08
0.06
0.04
0.02

0.8 --- 1.2 1.4 1.6 1.8 2

$M (\text{GeV})$
