Comments on Unitarity in the Antifield Formalism

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Abstract

It is shown that the local completeness condition introduced in the analysis of the locality of the gauge fixed action in the antifield formalism plays also a key role in the proof of unitarity.
1 Introduction

The antifield formalism is a method to deal with the unphysical degrees of freedom of a gauge theory in a manifestly covariant way. However, the unitarity of the formalism is not obvious without further investigation. One indirect unitarity proof exists and is based on the following argument:

(i) under quite general regularity conditions the antifield formalism is equivalent to the Hamiltonian formulation of the BRST symmetry [1, 2];
(ii) if the dynamics can be asymptotically linearized, the Kugo - Ojima quartet mechanism [3] can be applied and the state cohomology of the Hamiltonian BRST charge is isomorphic to the space of states in a physical gauge. [4, 5, 6, 7]. It is thus endowed with a positive definite inner product 1.

More recently, a different approach has been taken, which investigates unitarity directly at the Lagrangian level without going through the Hamiltonian BRST formulation [8, 5, 6, 7, 9]. This approach also proceeds within the framework of perturbation theory and considers the asymptotic, (i.e., linearized) L-th order reducible gauge theory described by

- a quadratic action
\[ S^{(0)} = \int dt \, L(\dot{\phi}^i, \dot{\phi}^j) \] (1)
depending on the fields and their first order derivatives
- reducible gauge generators of the form
\[ \delta \psi^i = R^{i}_{(0)0} \epsilon^\alpha + R^{i}_{(0)1} \dot{\epsilon}^\alpha \] (2)
- reducibility coefficients of the form
\[ R^{\alpha_k}_{(k)\alpha_{k+1}}(t, t') = R^{\alpha}_{(k)0 \alpha_{k+1}} \delta(t - t') + R^{\alpha}_{(k)1 \alpha_{k+1}} \frac{d}{dt} \delta(t - t') \quad k = 1, ..., L \] (3)
where the \( R^{\alpha_k}_{(k)\alpha_{k+1}}'s \) (\( l = 0, 1 \quad k = 0, ..., L \quad \alpha_0 = i \quad \alpha_1 = \alpha \)) are all constant c-number matrices.
The authors of [8] conclude that unitarity holds if and only if \( R_{(L)0} \) is of maximum rank
\[ \text{rank} \, R^{\alpha}_{(L)0 \alpha_{L+1}} = m_{L+1}, \quad \alpha_{L+1} = 1, ..., m_{L+1} \] (4)

1We assume, of course, that the physical (i.e., gauge invariant) degrees of freedom are quantized with a positive metric.
and the \( R_{(k)0} \)'s "form an exact sequence" meaning that

\[
\text{Im } R_{(k)0} \equiv \{ A^\alpha_k = R_{(k)0\alpha_{k+1}}^\alpha \lambda_{\alpha_{k+1}}, \lambda_{\alpha_{k+1}} \text{ arbitrary} \} \\
= \text{Ker } R_{(k-1)0} \equiv \{ A^\alpha_k | R_{(k-1)0\alpha_k}^\alpha A^\alpha_k = 0 \}.
\]  

(5)

[The extra conditions on the \( R_{(k)1} \)'s imposed in [8] are actually simplifying assumptions to prevent the appearance of second class constraints in the passage from the gauge fixed Lagrangian action to the canonical formalism and are thus not essential for unitarity.]

But unitarity of the theory is guaranteed to hold since the general analysis based on the Hamiltonian formalism applies. Indeed, the regularity conditions are fulfilled for a quadratic action, because all relevant matrices are constant and have thus constant ranks. This means that one should be able to prove (4) and (5) by using properties of the Hamiltonian formulation.

It is however of interest to see how the conditions (4) and (5) hold independently of Hamiltonian arguments. This letter provides a direct Lagrangian proof that the conditions (4) and (5) are automatically fulfilled - and so need not be imposed as extra conditions. This proof relies on the fact that the gauge transformations and the reducibility coefficients should be chosen so as to be not only complete but also locally complete [10]. Hence, the local completeness requirement, which was first introduced in order to control locality of the gauge fixed action, plays also a key role in the proof of unitarity. This is not surprising since locality (in time) and unitarity are known to be intimately connected.

2 Local completeness

We will consider the general case in which the free action can depend on the fields and their derivatives up to some finite order that may be higher than one:

\[
S^{(0)} = \int dt \ L(\phi^i, \dot{\phi}^i, ..., \phi^{i(k)}).
\]

(6)

For notational simplicity we assume all the classical fields to be bosonic. Since the action is quadratic, gauge transformations do not depend on the fields and take the general form

\[
\delta \phi^i(t) = \int dt' \ R_{(0)\alpha}^i(t, t') \epsilon^\alpha(t')
\]

(7)
with
\[ R_{(0)\alpha}(t, t') = \sum_{l=0}^{m} R_{(0)\alpha}^i l \frac{d^l}{dt^l} \delta(t - t') \] (8)
for some finite \( m \). The \( R_{(0)\alpha}^i \)'s are c-number matrices.

As explained in [10], the gauge generators and the reducibility functions should be taken to be locally complete. Let us briefly review what this means in the context of the linear theory based on (6).

### 2.1 Definition

The gauge generators (7) are locally complete in time if any transformation
\[
\delta \phi^i(t) = \int dt' a^i(t, t') u(t'), \quad a^i(t, t') = \sum_{l=0}^{m} a^i_l \frac{d^l}{dt^l} \delta(t - t')
\] (9)
leaving the action invariant (up to a boundary term) for any choice of the arbitrary function \( u(t) \), corresponds to (7) (up to gauge transformations vanishing on-shell) with a special choice of the gauge parameter ("completeness") involving the arbitrary function \( u(t) \) and a finite number of its time derivatives ("local completeness", i.e., "locality in time of the gauge transformations"). That is, one must be able to reproduce (9) by choosing \( \epsilon^\alpha \) in (7) to be of the following form
\[
\epsilon^\alpha(t) = \int dt' w^\alpha(t, t') u(t'), \quad w^\alpha(t, t') = \sum_{l=0}^{m_w} w^\alpha_l \frac{d^l}{dt^l} \delta(t - t') .
\] (10)

So there is no gauge transformation that is not included among (7).

The fact that (9) is included among (7) implies
\[
a^i(t, t') = \int dt'' R_{(0)\alpha}^i(t, t'') w^\alpha(t'', t') \]
\[ \Longleftrightarrow a^i_l = \sum_{j=0}^{l} R_{(0)\alpha}^i l - j w^\alpha_j , \quad l = 0, \ldots, m_a + m_w \] (11)
with the convention : \( a^i_k = 0 \quad k > m_a, \quad R_{(0)\alpha}^i l = 0 \quad k > m, \quad w^\alpha_k = 0 \quad k > m_w. \)
2.2 Equivalent formulation of local completeness

Because (7) are gauge transformations, one has the Noether identities

\[ \sum_{k=0}^{m} \frac{d^k}{dt^k} (R_{(0)k\alpha}^i \frac{\delta S}{\delta \phi^i}) = 0 . \] (12)

Now, the transformation (9) leaves the action invariant if and only if

\[ \sum_{k=0}^{m_a} \frac{d^k}{dt^k} (a^i_k \frac{\delta S}{\delta \phi^i}) = 0 . \] (13)

Hence, the gauge transformations are locally complete in time if and only if the holding of (13) for \( a^i_k \) implies that \( a^i_k \) can be written as in (11). That is (12) ”should exhaust all the identities”.

2.3 Higher order reducibility coefficients

The concept of local completeness for the higher order reducibility coefficients is defined in a similar manner. At each stage, the reducibility coefficients

\[ R^{\alpha_k}_{(k)\alpha_{k+1}} (t, t') = \sum_{i=0}^{m_k} R^{\alpha_k}_{(k)i\alpha_{k+1}} \frac{d^i}{dt^i} \delta (t - t') \] (14)

must be such that

\[ \int dt' R^{\alpha_{k-1}}_{(k-1)\alpha_k} (t, t') R^{\alpha_k}_{(k)\alpha_{k+1}} (t', t'') = 0 \] (15)

and if

\[ \int dt' R^{\alpha_{k-1}}_{(k-1)\alpha_k} (t, t') v^{\alpha_k} (t') = 0 \] (16)

where \( v^{\alpha_k} (t) \) is of the form

\[ v^{\alpha_k} (t) = \sum_{i=0}^{m} v^{\alpha_k}_i \frac{d^i}{dt^i} u(t) \] (17)

and involves the arbitrary function \( u(t) \) and a finite number of its time derivatives, then

\[ v^{\alpha_k} (t) = \int dt' R^{\alpha_k}_{(k)\alpha_{k+1}} (t, t') w^{\alpha_{k+1}} (t') \] (18)
for some $w^{\alpha k+1}(t)$ (completeness), where the $w^{\alpha k+1}(t)$ are functions of $u(t)$ and a finite number of its time derivatives (local completeness, i.e., “locality in time of reducibility”)

$$w^{\alpha k+1}(t) = \sum_{l=0}^{m_w} w^{\alpha k+1}_l \frac{d^l}{dt^l} u(t).$$  

(19)

In terms of the matrices $R^{\alpha k-1}_{(k-1)l_{\alpha k}}$, $R^{\alpha k}_{(k)l_{\alpha k+1}}$, $v^\alpha_k$, $w^{\alpha k+1}_l$ the equation (15) becomes

$$\sum_{j=0}^{n} R^{\alpha k-1}_{(k-1)n-j\alpha_k} R^{\alpha k}_{(k)j\alpha_k+1} = 0, \quad n = 0, \ldots, m_{k-1} + m_k$$  

(20)

where $R^{\alpha k-1}_{(k-1)n-j\alpha_k} = 0 \quad n - j > m_{k-1}$, $R^{\alpha k}_{(k)j\alpha_k+1} = 0 \quad j > m_k$. Furthermore, if

$$\sum_{j=0}^{n'} R^{\alpha k-1}_{(k-1)n'-j\alpha_k} v^\alpha_j = 0, \quad n' = 0, \ldots, m_{k-1} + m_v$$  

(21)

where $v^\alpha_j = 0 \quad j > m_v$, then one must have

$$v^\alpha_l = \sum_{j=0}^{l} R^{\alpha k}_{(k)l-j\alpha_k+1} w^{\alpha k+1}_j, \quad l = 0, \ldots, m_k + m_w$$  

(22)

with $w^{\alpha k+1}_j = 0 \quad j > m_w$ and $m_k + m_w \geq m_v$.

2.4 Remarks

(i) The gauge transformations and the reducibility coefficients should always be taken that way. This can always be done. However, the reducibility of the theory may be infinite if further requirements (such as Lorentz covariance) are added. An example where this situation arises is given by the N=1 superparticle [11].

(ii) The equations (21) are symmetrical between $n' = 0$ and $n' = m_{k-1} + m_v$. This symmetry is only superficial, however, because one can introduce higher order coefficients $v^\alpha_j (j > m_v)$ by adding to $v^\alpha_j(t)$ time derivatives of the reducibility identities. This modifies the equation (21) with $n' = m_{k-1} + m_v$, which is no longer the last equation, but leaves (21) with $n' = 0$ unchanged.
3 Example

Let us consider the pure gauge theory with action $S(\phi) = 0$. The transformation $\delta \phi = \dot{\epsilon}, \epsilon = \epsilon(t)$ leaves the action invariant. But it is not locally complete because the transformation $\delta \phi = \eta, \eta = \eta(t)$ also leaves the action invariant, but one cannot obtain $\delta \phi = \eta$ from $\delta \phi = \dot{\epsilon}$ by a choice of $\epsilon = k_0 \eta + k_1 \dot{\eta} + .. + k_m \eta^{(m)}$. (To eliminate the derivative, one needs to integrate and consider the boundary conditions, which is not a local, algebraic manipulation.) By contrast, the transformation $\delta \phi = \eta$ is locally complete. So, $\delta \phi = \dot{\epsilon}$ alone is an inappropriate starting point, but $\delta \phi = \eta$ is all right.

A solution to the master equation is given in this simple example by

$$S = \int dt \phi^* R C + C^* \bar{\pi}$$

(23)

where $\bar{C}$ and $\bar{\pi}$ are variables of the non-minimal sector. Choosing the gauge-fixing fermion as

$$\psi = \int dt \ C(\omega \phi + \frac{1}{2} \bar{\pi})$$

(24)

with $\omega = \alpha + \beta \frac{d}{dt} + ... + \nu \frac{d^k}{dt^k}, \alpha, \beta, ..., \nu$ constant, the gauged fixed action is

$$S_\psi = \int dt \ \bar{C} \omega R C + \bar{\pi} \omega \phi + \frac{1}{2} \bar{\pi}^2.$$  

(25)

- Taking $\omega = 1$ and eliminating the auxiliary variable $\bar{\pi}$ by means of its equation of motion, we get, for the locally complete form of the gauge transformation ($R = \delta(t - t')$)

$$S_{\psi_R} = \int dt \ \bar{C} C - \frac{1}{2} \phi^2.$$  

(26)

The primary constraint $p_C \approx 0, p_{\phi} \approx 0$ induce the secondary constraints $C \approx 0, \bar{C} \approx 0, \phi \approx 0$ and all the constraints are second class. The Dirac bracket method then eliminates the pure gauge variable $\phi$, the ghost and antighost as well as their respective momenta. The theory is trivially unitary since there is only one physical state. The occurrence of second class constraints folows from the fact that $\bar{R}_{(0)1} = 0$.

- With $R = \frac{d}{dt} \delta(t - t')$ however (and $\omega$ still equal to 1),

$$\tilde{S}_{\psi_R} = \int dt \ \bar{C} C - \frac{1}{2} \phi^2.$$  

(27)
The primary constraints $p_C + \bar{C} \approx 0$, $p_{\bar{C}} \approx 0$, $p_\phi \approx 0$ induce the secondary constraint $\phi \approx 0$. These four constraints are second class, and using the Dirac bracket, one can set $\bar{C} = -p_C$, $p_C = p_\phi = \phi = 0$ in the action

$$S_H = \int dt \ p_C \dot{C} + p_\phi \dot{\phi} - \frac{1}{2} \dot{\phi}^2 + \lambda (p_C + \bar{C}) + \mu p_\phi + \nu p_\phi$$

getting

$$S_H = \int dt \ p_C \dot{C}.$$ (29)

The BRST symmetry for $\tilde{S}_{\psi_R}$ is given by $\delta_\epsilon \bar{C} = -\phi \epsilon$, $\delta_\epsilon \phi = \dot{C} \epsilon$ and the corresponding Noether charge expressed in phase space reads $\Omega = p_\phi \dot{C} + p_{\bar{C}} \dot{\bar{C}}$ and vanishes when enforcing the second class constraints : $\Omega' = 0$. So $\Omega'$ acts neither on $C$ nor on $p_C$. These variables remain in the BRST cohomology and the theory based on the incorrect description (27) contains extra states besides the physical state. As a result, it is not unitary.

4 Proof of (4) and (5)

We will assume from now on that the local completeness conditions are fulfilled.

4.1 Theorem

(i) $R_{(L)0}^0$ is of maximal rank

(ii) the $R_{(k)0}'$s form an exact sequence.

The proof of the theorem relies on the following lemmas :

4.2 Lemma 1

If $R_{(0)0}^i$ is not of maximum rank, the gauge transformations are reducible. The system is then an L-th stage reducible theory with $L \geq 1$, and the 0-th stage cannot be the last step.

Proof : One has

$$R_{(0)0}^i \mu^\alpha_a = 0$$ (30)
for some $\mu_a^\alpha \neq 0$ that are constant vectors. Thus from (12) one gets  
\[ \sum_{k=1}^{m} \frac{d^k}{dt^k} (R^i_{(0)k\alpha} \frac{\delta S}{\delta \phi^i} \mu_a^\alpha) = 0 \quad (identically) \]  
(31)
and so  
\[ \sum_{k=0}^{m} \frac{d^{k-1}}{dt^{k-1}} (R^i_{(0)k\alpha} \frac{\delta S}{\delta \phi^i} \mu_a^\alpha) = C \quad (identically) \]  
(32)
where $C$ is a constant independent of the fields.

But the left-hand side vanishes on-shell, which implies $C = 0$, i.e.,
\[ \sum_{k=0}^{m-1} \frac{d^k}{dt^k} (R^i_{(0)k+1\alpha} \frac{\delta S}{\delta \phi^i} \mu_a^\alpha) = 0, \]  
(33)
which is of the form (13). According to the local completeness assumption there exists $w_{ia}^\alpha$, $l = 0, ..., m_a$ such that
\[ R^i_{(0)k\alpha} \mu_a^\alpha = \sum_{j=0}^{k-1} R^i_{(0)k-1-j\alpha} w_{ja}^\alpha, \quad k = 1, ..., m + m_a + 1. \]  
(34)

Defining $v_{0a}^\alpha = \mu_a^\alpha$, $v_{j+1a}^\alpha = -w_{ja}^\alpha$ and using (30) this gives: the systems
\[ \sum_{j=0}^{k} R^i_{(0)k-j\alpha} v_{ja}^\alpha = 0, \quad k = 1, ..., m + m_a + 1 \]  
(35)
possess non-trivial solutions for each $a$. This implies that the gauge transformations defined by $R^a_{(0)\alpha}(t, t')$ are reducible because
\[ (35) \iff \int dt' R^a_{(0)\alpha}(t', t'') v_a^\alpha(t', t'') = 0 \]  
(36)
where
\[ v_a^\alpha(t, t') = \sum_{l=0}^{m_a+1} v_{la}^\alpha \frac{dt}{dt'} \delta(t - t') \]  
(37)
or in other words, the non-vanishing gauge functions
\[ e_{a}^\alpha(t) = \int dt' v_{a}^\alpha(t, t') u(t') \]  
(38)
yield $\delta \phi^i = 0$.  

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4.3 Lemma 2

If \( R_{(k)0\alpha k+1} \) has zero (right) eigenvectors

\[
R_{(k)0\alpha k+1} \mu^\alpha_{k+1} = 0 \quad \mu^\alpha_{k+1} \neq 0
\]

then the \( R_{(k)} \)'s are reducible. More precisely, to each zero eigenvector of the \( R_{(k)0} \)'s corresponds one reducibility identity.

Proof: The proof proceeds in a similar way. Consider the vector

\[
v^\alpha_{l\alpha} = R^\alpha_{(k)l+1\alpha k+1} \mu^\alpha_{k+1}.
\]

Then

\[
\sum_{j=0}^{n} R_{(k-1)n-j\alpha k}^{\alpha k-1} v^\alpha_{j\alpha} = 0, \quad n = 0, ..., m_k - 1 + m_k - 1.
\]

Indeed using (40) and (20), we get for the left-hand side of (41)

\[
\sum_{j=0}^{n} R_{(k-1)n-j\alpha k}^{\alpha k-1} R_{(k)j+1\alpha k+1}^{\alpha k} \mu^\alpha_{k+1} = -R_{(k-1)n+1\alpha k}^{\alpha k-1} R_{(k)0\alpha k+1}^{\alpha k} \mu^\alpha_{k+1} = 0.
\]

The local completeness assumption on \( R_{(k)\alpha k+1}^{\alpha k} (t, t') \) implies the existence of \( w_{ja}^{\alpha k+1} \) such that (22) is satisfied for our choice of \( v^\alpha_{j\alpha} \):

\[
R_{(k)\alpha k+1}^{\alpha k} \mu^\alpha_{k+1} = \sum_{j=0}^{l-1} R_{(k)l-j\alpha k+1}^{\alpha k} w_{ja}^{\alpha k+1}, \quad l = 1, ..., m_k + 1 + m_w.
\]

Together with \( R_{(k)0\alpha k+1}^{\alpha k} \mu^\alpha_{k+1} = 0 \), we find

\[
\sum_{j=0}^{n} R_{(k)n-j\alpha k+1}^{\alpha k} v^\alpha_{j\alpha} = 0
\]

where \( v_{0\alpha}^{\alpha k+1} = \mu_{a}^{\alpha k+1}, \quad v_{j+1\alpha}^{\alpha k+1} = -w_{ja}^{\alpha k+1} \). This implies that the reducibility coefficients \( R_{(k)\alpha k+1}^{\alpha k} (t, t') \) are reducible,

\[
\int dt' R_{(k)\alpha k+1}^{\alpha k} (t, t') v_a^{\alpha k+1} (t', t'') = 0
\]

where

\[
v_a^{\alpha k+1} (t, t') = \sum_{l=0}^{m_w+1} v_{la}^{\alpha k+1} \frac{q^l}{dt^l} \delta(t - t').
\]

We are now in a position to prove the theorem.
4.4 Proof of the theorem

(i) If $R_{(L)}0$ had a right zero eigenvector, then, according to the lemmas, the $R_{(L)}$'s would be reducible and the theory, contrary to the initial hypothesis, cannot be a L-th stage reducible theory. Hence, $R_{(L)}0$ is of maximum rank.

(ii) $\int dt' R^{(k-1)}_{(k)}(t, t') R^{(k)}_{(k)}(t', t'') = 0$ implies at order zero that $R^{(k-1)}_{(k)}(0) R^{(k)}_{(k)}(0) = 0$ and hence that $\text{Im} R^{(k)}_{(k)} \subset \text{Ker} R^{(k-1)}_{(k)}$. Conversely, let us assume that $R^{(k-1)}_{(k)}(0) v^{(k)}_{0} = 0$. Then, by the second lemma, there is a reducibility identity associated with $v^{(k)}_{0}$, i.e., there exists $v^{(k)}_{0} = v^{(k)}_{0} \delta(t - t') + ...$ such that

$$\int dt' R^{(k-1)}_{(k)}(t, t') v^{(k)}_{0}(t) = 0 \quad (47)$$

The local completeness assumption $(16) - (19)$ implies then

$$v^{(k)}_{0}(t) = \int dt' R^{(k)}_{(k)}(k, t') w^{(k+1)}(t')$$

which reads, at order zero,

$$v^{(k)}_{0} = R^{(k)}_{(k)}(0) w^{(k+1)}_{0} \quad (49)$$

Consequently $\text{Ker} R^{(k-1)}_{(k)} \subset \text{Im} R^{(k)}_{(k)}$.

4.5 Remarks

(i) If only $R^{(k)}_{(k)}0$ and $R^{(k)}_{(k)}1\alpha_{k+1}$ are different from zero, $\text{rank} R^{(k)}_{(k)}1\alpha_{k+1} \leq \text{rank} R^{(k)}_{(k)}0\alpha_{k+1}$. Indeed, the systems (35) or (44) have non trivial solutions for each $a$. Suppose then that $\lambda^{a}_{b} R^{(k)}_{(k)}1\alpha_{k+1} v^{(k+1)}_{0a} = 0$, $b = 1, ..., B$ define a complete and independent set of such relations. Replacing $B$ of the $v^{(k+1)}_{0a}$'s by $\tilde{v}^{(k+1)}_{0a} = \lambda^{a}_{b} v^{(k+1)}_{0a}$ in such a way that the new set $\tilde{v}^{(k+1)}_{0a}$ is still a complete set of zero eigenvectors of $R^{(k)}_{(k)}0\alpha_{k+1}$, we find that the $\tilde{v}^{(k+1)}_{0b}$ are also zero eigenvectors of $R^{(k)}_{(k)}1\alpha_{k+1}$. The remaining $\tilde{v}^{(k+1)}_{1c}$ give rise to linearly independent $\tilde{v}^{(k+1)}_{1c}$'s. The next step consists in discussing in the same way the linear dependence of $R^{(k)}_{(k)}1\alpha_{k+1} v^{(k+1)}_{1c}$. At the end of the redefinitions, one finds that to independent zero eigenvectors of $R^{(k)}_{(k)}0\alpha_{k+1}$ correspond independent zero eigenvectors of $R^{(k)}_{(k)}1\alpha_{k+1}$ which implies the stated rank condition.
(ii) The condition considered in [9] that \( R^{\alpha_k}_{(k)1\alpha_{k+1}} \neq N^{\beta_{k+1}}_{\alpha_{k+1}} R^{\alpha_k}_{(k)0\beta_{k+1}} \) with \( N^{\beta_{k+1}}_{\alpha_{k+1}} \) invertible matrices also follows from the local completeness assumption. Indeed suppose that \( R^{\alpha_k}_{(k)1\alpha_{k+1}} = N^{\beta_{k+1}}_{\alpha_{k+1}} R^{\alpha_k}_{(k)0\beta_{k+1}} \). The reducibility functions then reads

\[
R^{\alpha_k}_{(k)\alpha_{k+1}}(t, t') = R^{\alpha_k}_{(k)0\alpha_{k+1}} \delta(t - t') + N^{\beta_{k+1}}_{\alpha_{k+1}} R^{\alpha_k}_{(k)0\beta_{k+1}} \frac{d}{dt} \delta(t - t') \tag{50}
\]

and is not locally complete because

\[
\tilde{R}^{\alpha_k}_{(k)\alpha_{k+1}}(t, t') = R^{\alpha_k}_{(k)0\alpha_{k+1}} \delta(t - t') \tag{51}
\]

is also a k-th order reducibility function. Indeed, writing the system (20) for the particular reducibility function (50), implies the system (20) corresponding to the reducibility function (51). However, trying to express (51) in terms of (50) as in (18) or (22) requires an infinite number of matrices \( w^{\alpha_{k+1}}_{\beta_{k+1}} \).

5 Conclusion

The local completeness assumption on the gauge generators and the reducibility coefficients has been introduced in [10] in the study of the locality of the gauge fixed action. We have shown here that it plays also a crucial role in guaranteeing unitarity of the antifield formalism.

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