Acoustic topological devices based on emulating and multiplexing of pseudospin and valley indices

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Abstract
We present a design paradigm for acoustic devices in which robust and controllable transport of wave signals can be realized. These devices are based on a simple acoustic platform, where different topological phases such as acoustic quantum spin Hall and quantum valley Hall insulators are emulated by engineering the spatial symmetries of the structure. Edge states along interfaces between different topological phases are shown to be promising information channels, where the multiplexing of pseudospin and/or valley degrees of freedom is unambiguously demonstrated in various devices including a multiport valve for acoustic power dividing and feeding. The information capacity in the input channel is substantially enhanced due to the creating of an extra dimension for the data carriers. The topological devices proposed here, when integrated with other state-of-the-art communication techniques, may suggest a significant step towards acoustic communication circuits with complex functionalities.

1. Introduction

Topology, originally an abstract mathematical concept, has become a research hotspot in recent years in the community of condensed matter physics. Many interesting phenomena of electron transport behavior, such as the quantum spin Hall (QSH) and quantum valley Hall (QVH) effects, were extensively studied and explored, and topology plays a crucial role in these effects [1, 2]. Soon, people realized that similar phenomena are not restricted to quantum systems, and nontrivial topology can also bring exciting phenomena and results in classical wave systems. Recently, classical analogs of QSH and QVH insulators were predicted and verified both theoretically and experimentally in electromagnetic [3–18], acoustic [19–33], elastic [34–43], and even water surface wave systems [44]. Very recently, robust and high-capacity phononic communications were realized in the context of Lamb waves through topological edge states by multiplexing the pseudospin and valley indices [43].

Acoustic wave, one of the most common waves in our daily life, has substantial advantages in numerous application scenarios such as underwater engineering and human health monitoring, where other types of waves are not competent. Furthermore, compared to its photonic and electronic counterparts, acoustic devices usually work at the kHz ~ MHz frequency range, corresponding to wavelengths on the order of millimeter ~ meter, which uniquely bridges frequency and wavelength gaps between optical and electrical devices, thus play an irreplaceable role in various scenarios. For many industrial applications for acoustic devices, robust and controllable transport of acoustic signals along preferred paths is highly demanded. It is also desired that these signal channels are immune to ambient disorders and defects, and it would be even better if an enhanced data transmission rate can be realized at the same time [45].

In this article, we systematically study the robust transport of acoustic edge state signals along interfaces between distinct topological phases. It is well known that elastic and electromagnetic waves can have different polarizations, which is naturally beneficial to the construction of pseudospin degrees of freedom (DOF). In contrast, acoustic wave in fluid media is a purely scalar wave, and the polarization DOF is not available. To
overcome this difficulty, we design a bilayer structure with a honeycomb lattice of connected cavities, where the layer index becomes a new DOF that can be utilized for the realization of different topological phases. The spatial symmetries of the cavity lattice are intentionally engineered so that acoustic analogs of QSH and QVH insulators are effectively emulated. Interestingly, we find that the edge states at the QSH/QVH boundaries are simultaneously protected by the pseudospin and valley indices, and thus naturally provide doubled information carriers in a single channel. The multiplexing of pseudospin and/or valley DOFs are unambiguously demonstrated in a multiport valve for acoustic power dividing and feeding, where the information capacity of acoustic signal in the input channel is effectively doubled due to the creation of an extra dimension for the data carriers. Other related acoustic devices are also demonstrated and studied, such as a sorting and routing platform and strongly asymmetric propagation of edge states. All these interesting devices are designed based on the same platform and under the principle of topology, when integrated with other state-of-the-art communication techniques [32, 33, 45–47], may boost the performance of existing acoustic devices to an unprecedented level.

2. Acoustic analogs of QSH and QVH insulators

2.1. A honeycomb lattice with a bilayer structure

To realize a topologically nontrivial acoustic crystal, we construct a honeycomb lattice of connected cavities, where \( L = \sqrt{3} a \) is the lattice constant in the \( xy \)-plane, and \( a = 1 \) m is the distance between nearest cavities. Figures 1(a) and (b) show the unit-cell structure, which has two layers and each layer consists of two cavities, with \( A_1 \) and \( B_1 \) on the upper-layer, and \( A_2 \) and \( B_2 \) on the lower-layer, respectively. All cavities have the same height, i.e. \( h_c = 1.2a \). The radius of each cavity can be different so that the spatial symmetries can be intentionally engineered. Throughout our study the total volume of cavities on the upper layer is kept the same as that on the lower layer, i.e. \( r_{A_1}^2 + r_{B_1}^2 = r_{A_2}^2 + r_{B_2}^2 = 2a^2 = \text{const.} \) Along the \( z \)-direction, upper- and lower-layer cavities are connected by necked waveguides with height \( h_{\text{neck}} = 0.2a \) and radius \( r_{\text{neck}} = 0.072a \). To make the cavity structure as simple as possible, we ensure that all unit-cells have the same height, i.e.
2\ell_c + h_{\text{neck}} = 2.6a. Within the xy-plane, neighboring cavities are connected by waveguides with radii \( r_{\text{arm}} = 0.12a \) which are located \( h_{\text{arm}} = 0.8a \) from the necked waveguides.

Let us start with a unit-cell structure with \( r_{\text{A1}} = r_{\text{B1}} = r_{\text{A2}} = r_{\text{B2}} = 0.4a \), where both inversion symmetry within the xy-plane and mirror symmetry along the z-direction are preserved. The corresponding band structure is shown in figure 1(c), where two linear crossings of bands are observed at \( K \) point. If there is no inter-layer coupling between the upper- and lower-layer (as can be realized by setting \( h_{\text{neck}} \to \infty \) or \( r_{\text{neck}} \to 0 \)), a doubly degenerate linear dispersion will be found at \( K \) point, which is known as a double Dirac cone. Obviously, the existence of necked waveguides introduces nonzero inter-layer coupling, which splits the double degeneracy into two separate bands, in a similar way to the Zeeman effect in condensed matter physics. The effective Hamiltonian around \( K \) point can be written as \( H_0 = V_0 (\delta k_x \sigma_x + \delta k_y \sigma_y) \sigma_z - Z_0 s_z \), where \( \delta k_x = (\delta k_x, \delta k_y) = \vec{k} - \vec{K} \) is the deviation of Bloch wavevector \( \vec{k} \) from the \( \vec{K} \) point, and \( v_0 = 12.7 \text{ m s}^{-1} \) denotes the group velocity of the linear bands around \( K \) valley. \( \sigma_i \) are Pauli matrices acting in the orbital and layer subspaces, respectively. \( Z = 3.8 \text{ Hz} \) represents the inter-layer coupling strength, and is kept unchanged throughout our study. In figure 1(c), the band structure predicted by \( H_0 \) is plotted as red lines, which agrees very well with the rigorous numerical calculation denoted by blue lines. Actually, the doubly degeneracy points around \( K \) point form a ring in the \( k \)-space, known as a nodal-ring. This nodal ring is protected by the mirror symmetry of the unit-cell structure. In figure 1(d), the pressure fields of four Bloch states at \( K \) point are illustrated, and we observe that two eigenstates at lower frequency are symmetric with respect to the mirror symmetry along the \( z \)-direction, and the other two at higher frequency are anti-symmetric. Throughout this paper, full-wave simulation results are obtained by using COMSOL Multiphysics, a finite-element based soft package.

### 2.2. Acoustic QSH insulator

If we increase \( r_{\text{A1}} \) and \( r_{\text{B2}} \) and reduce \( r_{\text{A2}} \) and \( r_{\text{B1}} \) accordingly, the mirror symmetry along the \( z \)-direction is broken, as a result the nodal ring degeneracy in the \( k \)-space is lifted. Figure 2(a) shows the unit-cell configuration with \( r_{\text{A1}} = r_{\text{B2}} = 0.45a \) and \( r_{\text{B1}} = r_{\text{A2}} = 0.3428a \), where an inversion symmetry in 3D space \((x', y', z') \to (-x, -y, -z)\) is preserved. The corresponding band structure is shown in figure 2(d), where a gap is formed between the 2nd and 3rd bands.

If we look at the four eigenstates of the acoustic QSH system at \( K \) point, we can observe that two of them look like \( p \) orbitals of electrons, and the other two are similar to \( d \) orbitals. We can name these eigenstates as \( p_{x}, p_{y} \),...
and $d_{32y}$, $d_{-3y}$ states, respectively, according to their mode symmetries. It follows that pseudospin-1/2 states can be achieved through hybridizing these $p$ and $d$ states: $p_h = (p_x + ip_y)/\sqrt{2}$ and $d_e = (d_{32y} \pm id_{-3y})/\sqrt{2}$, with $p_h$ and $p$ being the dipolar pseudospin up and down states, respectively, and $d_+$ and $d_-$ the corresponding quadrupolar states. This is an acoustic analog of QSH insulator, where linear combinations of the four eigenstates at $K$ point are regrouped into two copies of pseudospin up and down states [23]. After taking into account the orbital DOF, we can write the effective Hamiltonian around $K$ point as

$$H_{\text{K}} = v_f (\delta \kappa_x \sigma_x + \delta \kappa_y \sigma_y) + \Delta \sigma_z - \zeta_0 \sigma_0,$$

where $\Delta \sigma_z$ mimics electrons' spin–orbit coupling effect, and

$$\Delta = \lambda \left( \tau_{11}^2 - \tau_{22}^2 \right)$$

characterizes the strength of mirror symmetry breaking. From numerical calculations we get $\Delta = 5.775$ Hz, so that the proportional coefficient $\lambda$ can be determined as $\lambda = 67.946$ Hz m$^{-2}$. From the effective Hamiltonian $H_{\text{K}}$, we know that the two bands below the gap carry nonzero spin Chern number $C_5 = \text{sgn}(\Delta) = 1$.

2.3. Acoustic QVH insulator

In contrast, if we increase the radii of cavities on $A$-site and reduce those on $B$-site, the inversion symmetries within the $xy$-plane are broken, but the mirror symmetry along the $z$-direction is preserved, as shown in figure 2(b). Figure 2(e) shows the band structure for $r_{A1} = r_{A2} = 0.45a$ and $r_{B1} = r_{B2} = 0.3428a$, where we can see that a gap is opened at $K$ point. This is an analog of QVH insulator, where the valley DOF can be defined due to the preserved $C_5$ symmetry in the $xy$-plane [40]. Therefore, the corresponding effective Hamiltonian is

$$H_{\text{V}} = v_f (\delta \kappa_x \sigma_x + \delta \kappa_y \sigma_y) + \eta \sigma_z - \zeta_0 \sigma_0,$$

where $\eta = \lambda (r_{A1}^2 + r_{A2}^2 - 2r_{B1}^2) = 5.775$ Hz describes the strength of in-plane inversion symmetry breaking. Around $K$ point, the nontrivial topology of bands below the gap are characterized by the valley Chern number $C_{\text{V}} = \text{sgn}(\eta) = 1$.

2.4. Phase diagram

When both inversion and mirror symmetries are broken, we have $r_{A1} = r_{B1} = r_{A2} = r_{B2}$ but keeping $r_{A1}^2 + r_{B1}^2 = r_{A2}^2 + r_{B2}^2 = 2F^2 =$ const, as shown in figure 2(c). The band structure with $r_{A1} = 0.46a$, $r_{B1} = 0.3292a$, $r_{A2} = 0.3789a$, and $r_{B2} = 0.42a$ is shown in figure 2(f). The effective Hamiltonian around $K$ point is $H_{\text{K}} = v_f (\delta \kappa_x \sigma_x + \delta \kappa_y \sigma_y) + \eta \sigma_z - \zeta_0 \sigma_0$, obviously, both the band gap width and bands' topology are ultimately determined by the combination/competing effect between $\eta$, $\Delta$, and $Z$. Since we have $Z = 3.8$ Hz, $\Delta = 4.62034$ Hz, and $\eta = 2.3945$ Hz for this case, it turns out that $Z^2 + \Delta^2 > \eta^2$, which means that the QSH effect dominate over the QVH one. Thus, the topology of the bands below the gap can be described by a nonzero spin Chern number $C_5 = \text{sgn}(\Delta) = 1$.

In figure 3, we plot the phase diagram of the bilayer acoustic crystal with $Z = 3.8$ Hz, where $\Delta$ and $\eta$ are varied from 0 to 7 Hz. Actually, $Z = \lambda \left( r_{A1}^2 - r_{A2}^2 \right)$ and $\eta = \lambda (r_{A1}^2 + r_{A2}^2 - 2F^2)$ represent the mirror-symmetry-breaking and inversion-symmetry-breaking strengths, respectively. The red curve represents the boundary of phase transition, i.e. $Z^2 + \Delta^2 = \eta^2$. Above the red curve (cyan region), we have $Z^2 + \Delta^2 < \eta^2$, so that the QVH effect dominates, and the corresponding topological indices are $C_{\text{V}} = 1$ and $C_5 = 0$. Below the red curve (yellow region), we have $Z^2 + \Delta^2 > \eta^2$, so that the QSH effect dominates, and the topological invariants are $C_5 = 1$ and $C_{\text{V}} = 0$. Several points with coordinates ($\Delta$, $\eta$) are marked in the diagram, where the red point (0, 0), brown point (5.775, 0), purple point (0, 5.775), and green point (4.62034, 2.3945) correspond to figures 1, 2(a), (b), and (c), respectively. In particular, the black point (0, 3.8) stays on the phase transition boundary line, i.e. $Z^2 + \Delta^2 = \eta^2$, where the bulk band gap closes.

3. Edge states along interfaces between different topological phases

Nontrivial bulk band gaps imply the existence of topological edge states. Figure 4(a) shows the projected band structure of a ribbon containing two QSH-like systems with opposite spin Chern numbers. The ribbon has 8 unit-cells on each side of the interface, and on the $S_1$ side we have $r_{A1} = r_{B2} = 0.45a$ and $r_{B1} = r_{A2} = 0.3428a$, so that $\Delta > 0$ and $C_5 = \text{sgn}(\Delta) = 1$. On the $S_2$ side of the interface, in contrast, we have $r_{A1} = r_{B2} = 0.3428a$ and $r_{B1} = r_{A2} = 0.45a$, so that $\Delta < 0$ and $C_5 = \text{sgn}(\Delta) = -1$. The value of $C_5$ changes by 2 when crossing the $S_1/S_2$ interface, thus we have 2 edge bands according to the bulk-boundary correspondence. Interestingly, pressure fields of each edge band are distributed mainly on one layer of the acoustic crystal (i.e. either on the upper or the lower layer), because the layer index is well defined if the inter-layer coupling is not very strong. If we look at the acoustic energy flux distributions for each edge band, we can find that the local Poynting vectors rotating in opposite directions between the $k_y > 0$ section and the $k_y < 0$ section. More specifically, edge states in cyan color have counterclockwise circulating Poynting vectors and positive group velocities, corresponding to pseudospin up states. In contrast, edge states in magenta color have clockwise circulating Poynting vectors and negative group velocities, correspond to pseudospin down states. The locking of the pseudospin up and down states with counter-propagations of edge states is reminiscent of the QSH effect in electronic systems.
Similarly, figure 4(c) shows the edge states along the interface between two QVH-like systems with opposite valley Chern numbers. On V1 side of the interface, we have $r_{A1} = r_{A2} = 0.45a$ and $r_{B1} = r_{B2} = 0.3428a$, so that $\eta > 0$ and $C_V = \text{sgn}(\eta) = 1$. On V2 side, $r_{A1} = r_{A2} = 0.3428a$ and $r_{B1} = r_{B2} = 0.45a$, so that $\eta < 0$ and $C_V = -1$. There are also 2 edge bands at the V1/V2 interface, but pressure fields of each edge band are distributed in both layers, because the layer index is not a good quantum number here. In the $K$ valley, both edge states have positive group velocities (in cyan), while in the $K'$ valley they have negative group velocities (in magenta).

Figure 4. Edge states along interfaces between nontrivial topological phases. (a) shows the projected band structure for a ribbon sample containing a S1/S2 interface, with $C_S = 1$ and $C_V = -1$ on different sides. Gray dots represent bulk states, and cyan (pseudospin up) and magenta (pseudospin down) dots represent edge states. (b) Upper panel shows the configuration of the ribbon, where there are 8 unit-cells on both sides of the interface. Lower panel shows the pressure field distributions of eigenstates at $k = 0.35\pi/\sqrt{3}a$. (c) and (d), the same as (a) and (b), except that the ribbon sample contains a V1/V2 interface. (e) and (f), the same as (a) and (b), except that the ribbon sample contains a S1/V1 interface.
edge state’s pseudospin index is locked with its valley index, i.e. the $K$ valley is locked with the pseudospin down (magenta) state, and the $K'$ valley is locked with the pseudospin up (cyan) state. This property can be utilized for directional guiding and transporting of the acoustic edge modes, as will be shown later. Due to the interplay of pseudospin and valley DOF, pressure field distribution for the edge band is found in both layers of the acoustic crystal.

4. Acoustic devices and applications

4.1. A topological sorting and routing platform

The locking of pseudospin and valley DOFs can be utilized for directional transport of sound signals along preferred paths. In this regard, we design a topological sorting and routing platform, which is composed of four domains, as shown schematically in figure 5(a). S1 and S2 denote two different QSH domains with opposite spin Chern numbers, and V1 and V2 denote two different QVH domains with opposite valley Chern numbers. When an acoustic source of 46 Hz is introduced near port 2 (denoted by the yellow star), it will excite an edge state propagating downwards along the S1/V2 interface, and this edge state has a pre-determined pseudospin and valley indices due to the locking of these discrete DOFs. After passing through the S1/S2 interface, it arrives at the S1/S2/V1 junction. Then it goes straightly through the junction and continues its propagation along the yellow path and finally arrives at port 4. This is the only possible path that the edge state can take, otherwise its valley index cannot be preserved. In contrast, if a source of 46 Hz is introduced near port 3 (denoted by the purple star), it will excite an edge state at the S2/V2 interface. It turns out that this edge state will follow the purple path, and after passing through two junctions it will finally arrive at port 1. The full-wave simulations results shown in figures 5(b) and (c) completely confirm our topology-based theoretical analysis, and unambiguously illustrate the decisive role played by the valley DOF in the directional transport. Here we note that such a directional transport is robust against certain kind of defects and disorders, due to the protection of nontrivial topology. This is particularly useful for potential applications in acoustic signal processing.

Like most acoustic systems, ours is also influenced by the effect of visco-thermal losses. By introducing an imaginary part into the wavevector $k$ and defining $\beta = \text{Im}(k) / \text{Re}(k)$ as an index of loss, we can study the influence of $\beta$ on the transport behaviors of the edge states. We find that when $\beta$ is small, the edge states are still robust. But when $\beta$ is around or larger than 0.6%, the energy of the edge states is substantially attenuated after traveling a distance of ten lattice constants or less. Since all connecting waveguides have their radii $r_{\text{neck}} = 0.072a$ much smaller than the wavelength ($a \sim \lambda_0 / 8$), the substantial influence of losses on the transport behaviors is reasonable and not surprising.

4.2. A multiport valve for power dividing and feeding

Robust and controllable energy transport is always highly desired for many acoustic applications. In this regard, a topological multiport valve for acoustic power dividing and feeding can be designed and demonstrated. Figure 6(a) shows a schematic layout of the valve, which contains four domains and four ports, with domain S1 and S2 being QSH-like, and V1 and V2 being QVH-like. When an acoustic source is put near port 1 (as marked by the yellow star), an edge state propagating rightwards is excited. When countering the S1/V1/V2/S2 junction, the elastic wave energy is divided into two parts and fed into two separate signal channels, i.e. one along the S1/V1 interface, and the other along the S2/V2 interfaces. Following the yellow paths, the wave signal finally arrives at port 2 and port 4, respectively. Throughout the whole transport process, port 3 is totally isolated from the excitation. Full-wave simulation result shown in figure 6(b) agrees well with the theoretical prediction.

Here the wave signal along the input S1/S2 channel is topologically protected by the pseudospin index, where both up and down components are permitted in the input channel. After passing through the junction, the

[Figure 5. A topological sorting and routing platform. (a) The platform is composed of 4 domains (S1, S2, V1, and V2) and 4 ports, and is surrounded by perfect matched layers (PMLs, as marked by dark yellow) that can absorb outgoing waves without reflections. When acoustic sources are placed near different input ports (such as port 2 or 3), edge states with different topological index are excited, and these signals finally arrive at different output ports (port 4 or 1). (b) and (c) show the full-wave simulation results when the source is put on port 2 or 3, respectively, which is completely consistent with the theoretical analysis based on topology.]
wave signal propagating along the two output channels (i.e. $S1/V1$ and $S2/V2$) is protected by either up or down pseudospin index, depending on which channel is chosen. Thus, the input channel naturally provides doubled information carriers (i.e. both up and down components), implies a doubly enhanced transmission rate. In this way, we realize a multiplexing of the pseudospin DOF (up and down), create an extra dimension for the data carriers in the input channel, and substantially increase the information capacity of acoustic communication.

Instead, if an acoustic source is put near port 2 (as marked by the purple star), a similar transport phenomenon is observed for the edge states, and the propagation paths are marked by purple arrows. In such a situation, the wave signal finally arrives at port 1 and port 3, respectively, and port 4 is completely isolated, as verified by the simulation result shown in figure 6(c). Here the input channel along the $S1/V1$ channel is jointly protected by the pseudospin and valley indices, and the two output channels are protected by either the pseudospin index (along the $S1/S2$ channel), or the valley index (along the $V1/V2$ channel), respectively. Thus, we realize a multiplexing of the discrete DOFs (pseudospin and valley) in the input channel, and increase the information capacity of the acoustic communication in a way similar to figure 6(b).

Actually, it is also found that power dividing and guiding phenomena can be observed when the source is put near port 3 or 4. In other words, any of the 4 ports can be utilized as the input port. And any two of the 4 ports can be utilized as the output ports, depending on which port is set as the input port. In this way, we realize a topological multiport valve for acoustic power dividing and feeding, where a multiplexing of the pseudospin and/or valley DOFs can be realized in every input channel. In addition, the propagation of the edge states in every possible path is robust against defects and disorders, due to the protection of the nontrivial topology.

Furthermore, from the pressure intensity field shown in figure 6(b), we can see that the wave energy is not equally distributed between $S1/V1$ and $S2/V2$ channels. In fact, we have $I_{S2/V2} > I_{S1/V1}$. This is not due to the topological reason. It is simply because $\alpha < \pi/2$ so that it is easier for the acoustic energy in the input channel to couple into the $S2/V2$ channel. If we have $\alpha = \pi/2$ instead, as shown in figures 6(d) and (e), we will have energy equally distributed into the two channels. To be more general, the energy distribution ratio between different channels can be efficiently controlled by tuning angles $\alpha$ and $\beta$ [38].

4.3. Asymmetric propagation of edge states

Asymmetric propagation of acoustic wave energy is another interesting phenomenon that may imply practical applications. Figure 7(a) shows a schematic layout of a topological asymmetric device, which contains four domains and four ports. When an acoustic source is put near port 1 (as marked by the yellow star on the $V1/V2$ interface), an edge state propagating rightwards is excited. When countering the $V1/V2/S1/V2$ junction, the elastic wave signal is directed into ports 2 and 4. In this way, the left-hand side signal is transported into the right-hand side. When an acoustic source is put on port 3 (as marked by the purple star), an edge state propagating leftwards is excited. At the $V1/V2/S1/V2$ junction, the elastic wave signal is again directed into ports 2 and 4. In other words, the right-hand side signal is transported back to the right-hand side. In this way, strongly
asymmetric propagation of edge states with respect to the left and right directions is realized in this simple device.

Finally, we note that the nontrivial topology proposed in this paper is realized by engineering the spatial symmetries of the unit cell configurations, where both QSH and QVH insulators are linked to the C3 symmetry within the xy-plane. If the C3 symmetry is locally preserved in perturbed unit cells where spatial disorder or defects are introduced, the wave energy transport of edge states can be still robust, as shown in [43, 48]. However, if the corresponding symmetry is not respected, strongly scattering of edge states will be resulted.

5. Methods

5.1. Simulation

Full-wave simulations are performed by using the acoustic module of COMSOL Multiphysics, a commercial finite-element solver. For the bulk band structures shown in figures 1 and 2, Bloch boundary conditions are specified at the boundaries of the unit cell. For the edge band structures shown in figure 4, a rectangular supercell containing 16 unit cells along the x-direction is constructed, with 8 unit cells on each side of the domain interface. Bloch boundary condition is specified along the y-direction, along which direction the supercell has only 1 unit cell. For pressure field distributions shown in figures 5–7, the finite simulation domain is surrounded by perfect matched layers that can absorb outgoing waves without reflections.

5.2. Effective Hamiltonian

The effective Hamiltonian can be derived by using the k.p perturbation method [29]. For a monolayer acoustic crystal consisting of a honeycomb lattice of connected cavities, when both in-plane inversion and out-of-plane mirror symmetries are reserved, there is a Dirac cone at K point, and the effective Hamiltonian around K can be written as

\[
H_{\text{eff}} = \begin{pmatrix}
\lambda\tau_u & v_D(\delta k_x - i\delta k_y) & -Z & 0 \\
v_D(\delta k_x + i\delta k_y) & -\lambda\tau_u & 0 & -Z \\
-Z & 0 & \lambda\gamma & v_D(\delta k_x - i\delta k_y) \\
0 & -Z & v_D(\delta k_x + i\delta k_y) & -\lambda\gamma
\end{pmatrix}
\]

(1)

where \( Z \) represents the inter-layer coupling strength, \( \lambda\tau_{u,l} \) denotes the intra-layer coupling within the upper or lower layer. \( \tau_{u,l} \) depends on the radii of cavities within the same layer, i.e. \( \tau_u = r_{A1}^2 - r_{A2}^2, \gamma = r_{A1}^2 - r_{A2}^2 \), and \( \lambda \) is the proportional coefficient.

Considering that \( r_{A1}^2 + r_{A2}^2 = r_{A0}^2 + r_{B1}^2 = 2R^2 \), we can define \( \Delta = \lambda(\tau_u - \gamma)/2 = \lambda(r_{A1}^2 - r_{A2}^2) \) and \( \eta = \lambda(\tau_u + \gamma)/2 = \lambda(r_{A1}^2 + r_{A2}^2 - 2R^2) \) so that rewrite equation (1) as

\[
H_{\text{eff}} = \begin{pmatrix}
\eta + \Delta & v_D(\delta k_x - i\delta k_y) & -Z & 0 \\
v_D(\delta k_x + i\delta k_y) & -\eta - \Delta & 0 & -Z \\
-Z & 0 & \eta - \Delta & v_D(\delta k_x - i\delta k_y) \\
0 & -Z & v_D(\delta k_x + i\delta k_y) & -\eta + \Delta
\end{pmatrix}
\]

\[
= v_D(\delta k_x \sigma_z + \delta k_y \sigma_\tau) s_0 + \eta s_0 + \Delta s_z - Z s_0 s_\tau
\]

(2)
where $s_\tau$ are Pauli matrices acting in the layer subspace, $\Delta s_\tau s_\tau$ represents spin–orbit coupling effect, and $Zs_\theta s_x$ mimics Zeeman effect in condensed matter physics.

6. Conclusions

To conclude, we have presented a design paradigm for acoustic devices in which robust and controllable transport of wave signals can be realized in different application scenarios. By setting nontrivial edge states as signal channels, we utilize the discrete DOFs (such as pseudospin and valley) of edge states along QSH/QVH interfaces to implement robust transport of wave signals along preferred paths. The multiplexing of pseudospin and/or valley DOFs are unambiguously demonstrated in acoustic devices for power dividing and feeding, where the information capacity in the input channel is substantially enhanced due to the creating of an extra dimension for the data carriers.

We noticed that very recently people proposed a different and novel strategy to realize robust analog signal processing for acoustic waves [32, 33]. An important signal processing task, i.e. resolution of linear differential equations, was demonstrated both theoretically and experimentally in an acoustic system that is protected by nontrivial topology against disorders and perturbations. When integrated with this and other state-of-the-art communication techniques [45–47], the building blocks proposed in our work may suggest a significant step towards acoustic communication circuits with complex functionalities.

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