Enhanced fluctuations of the tunneling density of states near bottoms of Landau bands measured by a local spectrometer.

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We have found that the local density of states fluctuations (LDOSF) in a disordered metal, detected using an impurity in the barrier as a spectrometer, undergo enhanced (with respect to SdH and dHvA effects) oscillations in strong magnetic fields, \( \omega_c \tau \geq 1 \). We attribute this to the dominant role of the states near bottoms of Landau bands which give the major contribution to the LDOSF and are most strongly affected by disorder. We also demonstrate that in intermediate fields the LDOSF increase with \( B \) in accordance with the results obtained in the diffusion approximation.

Resonant tunneling through individual impurities has been identified and studied in vertical [3,7,8] and lateral [3,7,8] mesoscopic structures. When an impurity level in a potential barrier passes through the Fermi level in the emitter, it manifests itself as a step in the current-voltage (IV) characteristic, with the magnitude determined by the impurity coupling to the reservoirs and the onset smeared due to the coupling or the thermal distribution of carriers in the contact. Upon increasing bias, the current onset is followed by a plateau where temperature-independent and magnetic-field-sensitive reproducible features have been observed in several experiments on small-area vertical structures [3,7,8]. The latter were attributed to the fluctuations in the local density of single-particle states in a disordered emitter. It has then been suggested [3,7,8] that the impurity carrying the current (spectrometer) can act as a probe of the local density of states fluctuations (LDOSF) in the bulk of metallic contacts. When shifted with a bias, the spectrometer detects a 'fingerprint' of the LDOSF as a function of energy.

In the present paper, we study the evolution with magnetic field of the LDOSF in a 3D disordered metal, a heavily doped semiconductor, and discuss the results from the point of view of the fluctuation and correlation properties of single-particle wave functions in disordered media. We have measured the fingerprint of the LDOSF, \( \delta \nu (\varepsilon) \), in the differential conductance \( G(V) = \frac{dI}{dV} \) in a broad range of magnetic fields, \( B \), and analyzed its variance, \( \langle \delta G^2 \rangle \), and correlation parameters. In intermediate fields, \( \omega_c \tau \sim 1 \), we have detected an increase of the fluctuation magnitude, in agreement with the theoretically predicted behavior [3,7,8]: \( \langle \delta G^2 \rangle_B / \langle \delta G^2 \rangle_{B=0} \approx 1 + (\omega_c \tau)^2 \). At higher fields, \( \omega_c \tau \geq 1 \), we have observed large \( 1/B \)–periodic oscillations in \( \langle \delta G^2 \rangle \). We conclude that the LDOSF in strong fields are dominated by the states near the bottoms of the Landau bands which have a distinguished role relative to the rest of the spectrum.

The investigated structure consists of a 50 Å GaAs well imbedded between two 81 Å Al0.33Ga0.67As barriers. Each Si doped GaAs contact consists of three layers: 4800 Å with nominal doping \( 10^{18} \text{cm}^{-3} \) is followed by 4800 Å with \( 2 \times 10^{17} \text{cm}^{-3} \) and the latter is separated from the barrier by undoped spacer of 300 Å and 200 Å, for top and bottom contact respectively. The lateral area of the nominally undoped quantum well is reduced to a 700 Å diameter disk using the ion bombardment technique [1]. This decreases the number of active impurities in the barrier, thus avoiding overlapping spectra of the LDOSF produced by individual spectrometers. A schematic band diagram of the resonant tunneling device with an impurity level \( S \) in the quantum well is shown in Fig. 1, inset. By testing several samples, we have selected one with a distinct impurity level, which is also well separated from the states of the quantum well which lie about 10 meV above. This energy range determines the interval where the LDOS in the contact can be studied.

At zero bias, the spectrometer level \( S \) is above the Fermi level \( \mu \) of the emitter with 3D metallic conduction. The alignment of \( S \) and \( \mu \) with increasing bias is registered as a step in IV. In the differential conductance \( G(V) \) shown in Fig. 1, this current threshold corresponds to a peak at 0.05 V. At low temperatures, its height is \( G_T \approx \frac{4e^2}{h} \Gamma_{\text{max}} \), and its width is related to the energetic width of the spectrometer \( \Gamma \approx \Gamma_{\text{max}} \approx 120 \mu \text{eV} \) determined by the tunneling coupling between the impurity and the contacts [12]. The values of \( \Gamma_{\text{min,max}} \) depend on the transparencies of the two barriers, so that \( \Gamma_{\text{max}} \) corresponds to the lower (collector) barrier and \( \Gamma_{\text{min}} \) to the higher (emitter) barrier, \( \Gamma_{\text{min}} \approx 5 \times 10^{-3} \Gamma_{\text{max}} \) as estimated from the value of \( G_T \). The relation between bias \( V \) and the energy scale of the spectrometer is established by the coefficient \( \beta = \frac{dV}{d(\varepsilon)} = 0.24 \), found for the selected structure from the analysis of the temperature smearing of the threshold peak.

Above the threshold, the current is determined by the emitter barrier transparency \( \Gamma_{\text{min}} \) and the emitter den-
sity of states $\nu$ at the energy $E_S$ below the Fermi level. As the barrier height does not change significantly over a small $V$-range, the current becomes a measure of the LDOS in the emitter: $I(V) \propto \nu(E_S)$. Fluctuations with energy of the LDOS give rise to the reproducible, temperature independent fine structure in IV. It is seen on top of a smooth decrease in the current reflecting the averaged 3D density of states.

In some samples, we have detected Zeeman splitting of the spectrometer level in magnetic fields parallel and perpendicular to the current. In such cases it is seen that the upper spin level generates a replicated image of the LDOS which is shifted along the upper spin level generates a replicated image of the LDOS. The shift of the images corresponds to the difference between the spin-splitting of an impurity level in the quantum well and that of free electrons in the bulk. To avoid the overlap, a sample has been chosen with no significant splitting of the images in the magnetic field.

Fig. 2 shows the dependence $G(V)$ measured in magnetic fields $0 < B < 10.5 T$ applied parallel to the current and changed with a step of 20 mT. Fluctuations $\delta G(V)$ have a correlation voltage of $\Delta V_c \approx 0.5 mV$, which is comparable to the width $\Gamma/e\beta$ of the threshold conductance peak. With increasing magnetic field up to $B \approx 4 T$ the fingerprint in $G(V,B)$ changes randomly, with a correlation field $\Delta B_c \approx 0.05 T$. At high fields, the fluctuations transform into a more regular pattern where individual features, assigned to Landau bands, tend to move with increasing field towards the threshold peak, similar to the observation by Schmidt et al. [8].

To interpret fluctuations $\delta G(V)$ as an image of the LDOS, we employ a picture based on the properties of single-electron wave functions, $\psi_i(r)$ in a disordered metal [12-15]. We want to stress that the LDOS measured by the resonant tunneling spectroscopy reflect not only the randomness in the structure of chaotic wave functions obeying the Porter-Thomas statistics [10,11]:

$$\langle |\psi(r)|^2 \rangle = f(n) L^{-nd} \quad \text{(1)}$$

The sum in Eq. (1) includes a large number of eigenstates, $N(\Gamma, L) \sim \nu_0 L^d$, each of which typically contributing as little as $|\psi(r)|^2 \sim L^{-d}$, with the mean value of the LDOS, $\nu_0$, independent of the sample size $L$. As far as fluctuations $\delta \nu$ are concerned, one might naively expect that these fluctuations should vanish upon enlarging the sample, since for the sum of $N(\Gamma, L)$ independently fluctuating values $\delta |\psi(r)|^2 \sim L^{-d}$ the variance $\langle \delta \nu^2 \rangle$ can be estimated as $N(\Gamma, L) \langle \delta L |\psi(r)|^2 \rangle^2$, which is equivalent to $\Gamma^{-2} \nu_0 \Gamma L^{d-2} \sim \nu_0 \Gamma^{1-1} \to 0$ when $L \to \infty$. However, the correlations between wave functions at close energies make the LDOSF in a large sample finite and independent of its size. This statement can be explained using Thouless’s scaling picture of quantum diffusion [17]. We construct the electron states in a large sample by representing them as linear combinations of wave functions defined in its smaller parts, one of which contains the observation point $r$, and by gradually combining the smaller parts up to the actual size $L$ of the sample. For an intermediate length scale $\xi$ of the constituent part containing the observation point, its states (which we call ‘mother’ states of generation $\xi$) are spaced by $\Delta(\xi) \sim (\nu_0 L^d)^{-1}$. Diffusive spreading of these states into a larger part, when it is combined of several blocks, leads to their random mixing with the states from the neighboring $\xi$-size blocks within the Thouless energy $\gamma \sim hD/\xi^2$ [17, D is the classical diffusion coefficient.

Since at each stage only a finite basis is involved in the construction of the new states, some correlations exist between the new eigenstates, although at small $\xi$ the spread of ‘mother’ states $\gamma$ is larger than $\Gamma$ and these correlations are small. However, the Thouless energy $\gamma$ decreases with increasing $\xi$, and once $\xi$ exceeds length $L_T = \sqrt{hD/\Gamma}$, the information carried by a set of ‘mother’ states from a generation $\xi < L_T$ will not leave the energy interval covered by the spectrometer. As a result, the sum of the densities $|\psi_i(r)|^2$ in Eq. (1) will only depend on the situation at the length scale $L_T$ and not vary with further refining of the spectrum. Thus, $L_T$ and $N(\Gamma, L_T)$ represent the largest length scale and number of states for which correlations between individual eigenfunctions could be neglected and the above naive estimate of $\langle \delta \nu^2 \rangle$ from independent fluctuators used. Then, for the random difference between two values of $\nu$ in the neighboring $\Gamma$-intervals one should take $\langle \delta \nu^2 \rangle \sim N(\Gamma, L_T) \langle (\delta L |\psi(r)|^2 \rangle^2 \sim \nu_0 \Gamma^{-1} L_T^{-d}$.

From this, we can now estimate the fluctuations in the current plateau regime of the differential conductance which is a measure of the derivative, with respect to energy, of the LDOS in Eq. (1). We normalise the variance $\langle \delta G^2 \rangle$ by the height of the threshold conductance peak which depends on the average LDOS and the spectrometer width $V_T = \Gamma/\beta \epsilon$, so that $G_T \propto N(\Gamma, L_T) / V_T$. Because $\langle \delta G^2 \rangle$ can be taken from the above uncorrelated difference in $\nu$ as $\langle \delta G^2 \rangle \propto \langle \delta \nu^2 \rangle / V_T^2$, we arrive at

$$\langle \delta G^2 \rangle / G_T^2 \approx N(\Gamma, L_T)^{-1} \approx (\Gamma / hD)^{(d-2)/2} / (\nu hD). \quad \text{(2)}$$

We also estimate the correlation voltage of fluctuations as $V_T$ and the correlation magnetic field as $\Delta B_c \approx \Phi_0 / L_T^2$, where $\Phi_0$ is the flux quantum.
In a 3D system with an anisotropic diffusion tensor \((D_{x}, D_{y}, D_{z})\), Eq. (2) transforms into \(\langle \delta G^2 \rangle / G^2 \propto (D_{x} D_{y} D_{z})^{-1/2}\). This relation also determines the classical effect on the variance \(\langle \delta G^2 \rangle\) of a magnetic field \(B = BI\). Assuming that the cyclotron motion suppresses transverse diffusion as \(D_{x,y} = D/(1 + (\omega_{c} T)^2)\), \(\text{Eqs. 13-21}\) gives \(10\):

\[
\langle \delta G^2 \rangle_B / \langle \delta G^2 \rangle_{B=0} \approx 1 + (\omega_{c} T)^2, \quad \omega_{c} T \ll 1.
\]

(3)

Fig. 3 represents the result of our statistical analysis of conductance fluctuations in small magnetic fields. The amplitude of fluctuations is found from an individual \(G(V)\) curve at a fixed \(B\) in Fig. 2 as \(\langle \delta G^2 \rangle = \|G(B, V) - \|G(B, V)\|\|^2\) (||...|| stands for the averaging over range \(\Delta V \approx 6\) mV after the threshold peak). To decrease the scatter, a further averaging over a \(B\)-range of 0.25T has been performed. The result is compared to that in Eq. (3). The increase in \(\langle \delta G^2 \rangle_B\) agrees with the expected quadratic dependence. From Fig. 3, we find the momentum relaxation time \(\tau \approx 0.9 \times 10^{-13}s\) and use it to estimate the mobility, \(\mu = 0.22\) m²/Vs, in the emitter. The obtained values agree with those expected for the emitter with the same nominal doping [21] and justify our use of the diffusion approximation since \(\tau \varepsilon /h \sim 10^{-2}\). We also use these values to estimate the zero-field diffusion coefficient, \(D \approx 40\) cm²/Vs and \(\Delta B_{c} \approx \Gamma /eD \approx 0.03\) T which is close to the experimental value.

The value of \(\tau\) confirms that the crossover from weak to strong fields, \(\omega_{c} T \sim 1\), takes place at \(B \approx 4\) T where the Landau band (LB) formation is seen in Fig. 2. In the \(\omega_{c} T \geq 1\) regime, the field dependence of the variance \(\langle \delta G^2 \rangle\) has a strong oscillatory character similar to de Haas-van Alphen (dHvA) effect, with a sequence of peaks periodic in \(1/B\), Fig. 4a. However, the oscillations in \(\langle \delta G^2 \rangle\) are much more pronounced than the oscillations in the threshold peak \(G_{T}\) reflecting the modulation of the average density of states at the Fermi level in the emitter caused by depopulation of LB’s, Fig.4b. Also, the observed oscillations look significantly enhanced when compared to the Shubnikov-de Haas (SdH) oscillations of conductance in a lateral GaAs MESFET structure with the same nominal doping as the emitter, Fig. 4a, inset. The positions of the peaks in the variance \(\langle \delta G^2 \rangle\) appear to be different from those in \(G_{T}(B)\) and correspond to the filling of the LB’s for the electron concentration of approximately \(3 \times 10^{13}\) cm\(^{-3}\), which is larger than that near the barrier and suggests that electrons in the emitter further from the barrier could contribute to their origin.

These enhanced dHvA-type oscillations in the fluctuation amplitude suggest that the above estimation of \(\langle \delta G^2 \rangle\) using statistical properties of typical wave functions should be modified. This can be done by considering a special role of the states with anomalously large fluctuations of a local density, by analogy with [22] where these states were ‘prelocalized’ states. In the case of a smooth random potential with suppressed inter-LB scattering, these anomalous states are the states near the bottoms of LB’s. When in strong fields Abrikosov’s dimerization of electron motion [23] takes place, the contribution to the LDOSF from the bottom of the highest filled LB becomes distinguished from typical LDOSF and dominates in the magnitude of the variance \(\langle \delta G^2 \rangle\). For energies \(E_{S}\) close to the bottom of the \(n\)-th LB, \(E_{n} = (n + 1/2)h\omega_{c}\), not only transverse but also the longitudinal diffusion coefficient related to the highest LB, \(D_{L}^{(n)} \approx u_{T}^{2}\) T \(\propto [E_{S} - E_{n}]/\), is suppressed due to the decrease in the kinetic energy of the quasi-one-dimensional electron motion along magnetic field. When the characteristic length scale \(L_{z} = \sqrt{hD_{z}^{(n)}/\nu}\) becomes smaller than the inter-LB scattering length, the states from the upper LB start providing a contribution \(\langle \delta^{(n)} G^{2} \rangle\) to the LDOSF that is enhanced compared to the typical variance \(\langle \delta^{(typ)} G^{2} \rangle\):

\[
\langle \delta^{(n)} G^{2} \rangle / \langle \delta^{(typ)} G^{2} \rangle \approx \left( \frac{\nu^{(n)}}{\nu} \right) \left( D_{0} / D_{z}^{(n)} \right)^{1/2}.
\]

(4)

The structure of Eq. (4) explains the enhancement of oscillations in the fluctuation amplitude in Fig.4 relative to oscillations of \(G_{T}(B)\) and SdH oscillations. The latter are the measure of the ratio of the LDOS in the highest \((n-\text{th})\) LB and the total LDOS, i.e. they are represented by the first factor in Eq. (4). The unusual factor \(\left( D_{0} / D_{z}^{(n)} \right)^{1/2}\), which is responsible for the enhancement of the oscillations of the fluctuation amplitude, is a specific feature of the LDOSF effect.

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FIG. 1. Differential conductance as a function of bias with the threshold peak and the 'fingerprint' of the LDOS below the Fermi level in the emitter. Inset: Band diagram of the resonant tunneling structure with a spectrometer.

FIG. 2. Conductance fluctuations $G(V,B)$ normalised to the threshold peak. Curves for different $B$ are offset upwards and multiplied by an increasing factor to compensate for the decrease of the threshold peak with field.

FIG. 3. a) Diagram of the emitter volume where the tunneling LDOS is formed, at $B = 0$ and $B > 0$. $L_T$ is the diffusion length corresponding to electron lifetime $\hbar/\Gamma$ at the impurity level. b) Increase of the conductance fluctuations in intermediate fields due to the suppression of transverse diffusion.

FIG. 4. a) Oscillations of the conductance variance in strong fields. Inset: SdH oscillations in the bulk conductivity. b) For comparison, magneto oscillations of the threshold conductance peak.
Conductance (µS)

Voltage Bias (V)

Energy (meV)

µ

T = 1.6K

T = 0.1K

eV_{SD}
\[ \frac{\delta G^2}{G_T^2} \]

\[ B(T) \]

\[ G(T) \mu S \]

\[ B^{-1}(T^{-1}) \]

\[ G_T(T) \mu S \]