We consider experiments on measuring the Casimir interaction which have been performed in the last four years. The emphasis is made on measuring differences in the Casimir pressures under a transition of the plate metal from normal to superconducting state and on the Casimir metrology platform using a commercial micromechanical sensor. In both cases several problems in the comparison between experiment and theory are discussed.

Keywords: Casimir effect; superconductors; micromechanical sensor.

1. Introduction

Measurements of the Casimir force, which have been actively made in many laboratories starting in 1997 (see Refs. [1] and [2] for a review), attract much attention in connection with their importance for fundamental physics and prospective technological applications. This is particularly true with respect to the diversified experiments performed after the Third Casimir Symposium which had happened in 2015. Two of them were devoted to measurements of the Casimir force in a gaseous or liquid media ([3],[4]) (see also Ref. [5] investigating sensitivity and accuracy of Casimir force measurements in air). Several experiments needed for future applications of the obtained results in nanotechnology were devoted to measurements of the Casimir force in an optomechanical cavity ([6]), silicon carbide systems ([7]), and between silicon nanostructures ([8]). An implementation of special techniques, such as Ar-ion and UV cleaning, in the laboratory setups allowed to reduce detrimental electrostatic effects which plague the investigation of Casimir forces ([9],[10]). Some progress has been also reached in measuring the Casimir pressure between two parallel plates at separations of a few micrometers (see the first results ([11]) and proposed improvements ([12],[13]).
The greatest breakthrough, however, was reached in measuring the difference in Casimir forces between a Ni-coated sphere and either a Ni or Au strips of the plate covered with an Au overlayer. In this experiment, the difference between theoretical predictions of two competing theories, one taking into account (the Drude model approach) and other disregarding (the plasma model approach) the relaxation of free electrons, is by a factor of 1000. As a result, the plasma model approach was found to be consistent with the measurement data and the Drude model approach was conclusively excluded by the data (at least at separations below 1 µm). This finally confirmed the results of previous experiments where the difference in theoretical predictions of the two approaches was only a few percent of the measured force.

Here, we discuss problems in the comparison between experiment and theory in two more recent measurements of the Casimir force. In Sec. 2, an experimental investigation of the Casimir force under a phase transition of plate metal from normal to superconductor state is considered. Section 3 is devoted to the Casimir metrology platform using a commercial microelectromechanical sensor. In Sec. 4, the reader will find our conclusions.

2. The Casimir Force under a Phase Transition from Normal to Superconductor State

The first investigation of the Casimir pressure between two parallel plates made of superconducting metal (Al) was performed by means of on-chip optomechanical sensor. In so doing, one of the Al plates was attached to the movable mirror of an optical cavity. Due to the variation in the Casimir pressure, the separation distance between the Al plates should change resulting in the change of the cavity length and respective shift of its resonance frequency. Measurements of the expected frequency shift have been performed with decreasing temperature starting from 100 K to 0.01 K at the separation between the plates 100 ± 10 nm (larger gaps have also been tested). However, no frequency shift was observed when cooling the plates through their critical temperature $T_c \approx 1.3$ K below which Al becomes superconducting. This means that up to the measurement errors no change in the Casimir pressure $\Delta P$ was observed after a transition of the plate metal to the superconducting state.

The motivation for performing this experiment was that a phase transition of the plate metal into a superconducting state affects its reflectivity properties at frequencies below $k_B T_c / \hbar$ (where $k_B$ is the Boltzmann constant) and should also change the magnitude of the Casimir force. The hope was expressed for the possibility to distinguish between the theoretical predictions of the Drude and plasma model approaches using this effect. This hope was based on the theoretical result investigating different approaches to the description of the Casimir force between metals in a superconducting state (see also recent Ref. [19]). For $T > T_c$ both the plasma and Drude model approaches have been used to calculate the Casimir pressure. For $T < T_c$ a superconducting metal was described either by the plasma model...
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(as is suggested in the classical textbook\textsuperscript{[20]} or by the phenomenological Mattis-Bardeen dielectric permittivity\textsuperscript{[21]} which is smoothly joined with the permittivity of the Drude model at $T = T_c$. If the plasma model is used, the zero frequency shift and, respectively, $\Delta P = 0$ are predicted at $T < T_c$. When using the Mattis-Bardeen model, $\Delta P$ was shown again equal to zero when just passing through $T_c$, but being a decreasing function with further decrease in $T$.

Thus, the experimental results obtained so far\textsuperscript{[15]} are in favor of the plasma model. One should note also that the phenomenological Mattis-Bardeen dielectric permittivity at $T < T_c$ has a $\delta$-function term of the order $\delta(\omega)/\omega$ at zero frequency.\textsuperscript{[21]} Permittivities of this kind cannot be analytically continued to the upper plane of complex frequency, do not satisfy the Kramers-Kronig relation\textsuperscript{[22]} and, thus, scarcely can be used for making reliable predictions concerning the behavior of the Casimir force at $T < T_c$.

3. Casimir Metrology Platform

In Ref.\textsuperscript{[16]} the capacitive microelectromechanical inertial sensor was used for measuring the Casimir force between an Ag-coated microsphere and an Au-coated silicon plate in ambient conditions at room temperature. Measurements were performed within the separation region from $50 \text{ nm}$ to $1 \mu \text{m}$. The electrostatic calibration has been done as described in previous literature.\textsuperscript{[1]} It turned out that the values of residual potential difference depend on separation which means that, in addition to the Casimir force and well understood electric forces due to the applied potentials, there were some uncontrolled electrostatic forces due to surface patches.\textsuperscript{[5, 9, 10]}

Contrary to many experiments on measuring the Casimir force,\textsuperscript{[1, 7, 9, 10, 14]} the Casimir metrology platform\textsuperscript{[16]} does not provide the means for an independent measurement of the sphere-plate separations. The absolute separations are determined from the fit of the measurement data to two versions of the theory, i.e., to the zero-temperature Casimir force between the ideal metal sphere and plate and to the perturbation expansion of the Casimir force in two small parameters\textsuperscript{[23]} (the relative temperature and relative penetration depth). Surprisingly, it was found that an ideal metal model at zero temperature leads to better agreement with the measured data than the perturbation expansion taking into account corrections due to nonzero temperature and nonideality of metals (the root-mean-square deviations equal to $7.4 \text{ pN}$ and $10.5 \text{ pN}$, respectively). This result is in contradiction with all previous precision measurements of the Casimir force including the first experiment of this kind\textsuperscript{[24]} performed in 1998.

It should be taken into account, however, that the perturbation expansion used is applicable only at separations exceeding several hundred nanometers\textsuperscript{[23]} and cannot be compared with the measurement data of Ref.\textsuperscript{[10]} which are taken at separations down to $65 \text{ nm}$. At such short separations one should perform numerical computations by substituting the optical data for the complex index of refraction of boundary metals into the Lifshitz formula.
In Fig. 1 we present the computational results for the Casimir force between the Ag sphere of \( R = 55 \mu m \) radius (as in Ref. 16) and Au plate as functions of separation obtained using the Lifshitz formula at \( T = 300 \text{K} \) (the bottom solid line), using the perturbation expansion\(^{23}\) (the dashed line) and assuming the ideal-metal sphere and plate at zero temperature (the top solid line). Computations are performed taking into account the roughness of the plate and sphere surfaces\(^{16}\) (with the root-mean-square amplitudes equal to 2 and 8 nm, respectively). We note that at separations below 200 nm the computational results obtained using the extrapolations of the optical data to zero frequency by means of the plasma and Drude models are rather close to each other and cannot be discriminated in this experiment. As is seen in Fig. 1, the ideal-metal Casimir force deviates significantly from the accurate theory at all separations below 200 nm, whereas the perturbation expansion is in a rather good agreement with it already at \( a > 130 \text{ nm} \). From Fig. 1 we conclude that the largest measured force point\(^{16}\) \( (F_C = 635.5 \text{ pN}) \) was obtained not at the absolute separations of \( a = 65 \) or 63 nm (as claimed in Ref. 16 from the fit to ideal metal or perturbation theories, respectively), but at some separation below 50 nm.

One should note also that a minimization of the root-mean-square deviation between the data points and theoretical predictions is not an appropriate method when measuring the strongly nonlinear quantities\(^{25}\) such as the Casimir force. As was shown long ago\(^{27}\) this method leads to quite different results when it is used within different separation intervals. It has been known that Sparnaay\(^{28}\) spelled out three fundamental requirements necessary for performing precise and reproducible
measurements of the Casimir force. According to one of these requirements, precise independent and reproducible determination of the separation between the test bodies must be performed in any Casimir experiment.

4. Conclusions

As is seen from the foregoing, in the last four years great interest has been expressed to measuring the Casimir force in different configurations and to applications of this force in nanotechnology. Here we discussed only two experiments which have problems in the comparison between experiment and theory. Until the present time the Casimir interaction between superconductors has not been measured. Because of this first measurement of the differences in Casimir pressures when decreasing temperature from above to below $T_c$ is undeniably interesting. In future it is desirable to measure the absolute Casimir pressures between superconductors both above and below the critical temperature and compare the obtained results with different theoretical predictions.

The use of a commercial capacitive sensor for demonstration of the Casimir force in ambient conditions is also promising for various applications. However, to take a status of the Casimir metrology platform, this work should be supplemented with an independent measurement of the sphere-plate separations and compared with the proper theory.

In the near future one could expect also the realization of proposed experiments on measuring the Casimir forces between parallel plates and superconductors, as well as the universal experiments aimed to find out how free charge carriers influence the Casimir force between metallic and semiconductor materials.

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