Maximizing First-Order Approximate Mean of SINR Under Imperfect Channel State Information for Throughput Enhancement of MIMO Interference Networks

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Abstract
In this research paper, approximate mean of signal-to-interference-plus-noise ratio (SINR) under imperfect channel state information (CSI) is computed and maximized for throughput enhancement of MIMO interference networks. Each transmitter and receiver has, respectively, $M$ and $N$ antennas and network operates in a time-division duplex mode. Each transceiver adjusts its filter to maximize the expected value of SINR. The proposed new approach for throughput enhancement under imperfect CSI utilizes the reciprocity of wireless networks to maximize the estimated mean. The sum rate performance of the proposed algorithm is verified using Monte Carlo simulations.

Keywords Channel state information · Interference channel · Robust · Throughput · Transceiver

1 Introduction

Normally, wireless network scenarios such as interference channel (IC) share the channel among the users, resulting in multiuser interference. Among different medium access control (MAC), such as TDMA, FDMA, CDMA, a new method termed interference alignment (IA) leads to the efficient use of communication resources, since it successfully achieves the theoretical bound on the multiplexing gain. This scheme fits the unwanted signals from other users into a small part of the signal space observed by each receiver (interference subspace) while the other signal subspace is left free of interference for the desirable signal. The alignment solution has been provided in different papers (Gou and Jafar 2010; Jafar and Fakhereddin 2007; Cadambe and Jafar 2008; Gomadam et al. 2011; Peters and Heath Jr 2009; Schmidt et al. 2009; Kumar and Xue 2010).

In Cadambe and Jafar (2008), every transmitter uses a precoder which should be designed to accommodate all the interference signals into one half of the received signal space dimensions and leaves the other half without interference for the desired signal. The long precoder sizes at transmitters show a barrier to implementing such a scheme. Since such assumptions are too hard to materialize, it is very complicated to design a system based on such an elegant scheme. In Choi et al. (2009), Ma et al. (2011), Johnny and Aref (2017), the authors propose methods based on the designed scheme of Cadambe and Jafar (2008) to reduce precoder sizes. Another barrier in Cadambe and Jafar (2008) is the assumption of global channel knowledge. Johnny and Aref (2015) show that when the direct links have different characteristic functions (channel permutation or memory), in the absence of half part of CSI (cross links), one can achieve full degrees-of-freedom.

In practice, CSI is far from being perfect due to a variety of reasons, such as channel estimation error, quantization error, feedback error/delay. In Razavi and Ratnaraiah (2014, Guiazon et al. 2014), the performance of IA under CSI error was quantified. Mean loss in sum rate compared to perfect CSI case increases unboundedly as SNR increases.

The reliability of IA is little known, which is the subject of Xie et al. (2013). Authors study the error performance of IA. Since most IA algorithms require extensive channel state information (CSI), authors also investigate the impact
of CSI imperfection (uncertainty) on the error performance. Xie et al. (2013) design bit loading algorithms that significantly improve error performance of the existing IA schemes. Furthermore, Xie et al. (2013) propose an adaptive transmission scheme which produces robustness to CSI uncertainty to reduce error probability.

Beamforming Strategy Based on the Interference Alignment: In order to maximize sum rate of the MIMO interference network, beamforming strategy based on the interference alignment is used. Progressive minimization of the leakage interference is the basis for such algorithms [Gomadam et al. (2011), Algorithm 1], (Peters and Heath Jr 2009; Kumar and Xue 2010). Other algorithms include Max-SINR algorithm (Gomadam et al. 2011, Algorithm 2) and minimum mean square error (Schmidt et al. 2009). These schemes are established based on the availability of perfect CSI. The performance of transceivers is sensitive to CSI inaccuracies. Different algorithms are proposed to improve the throughput of the IC, under imperfect CSI.

Beamforming Strategy with Imperfect CSI Researchers have tried to improve sum rate of the MIMO interference network under imperfect CSI via robust transceiver design. In order to maximize system throughput, beamforming strategy based on the interference alignment is used. In Shen et al. (2010), authors applied a minimum mean square error criterion to improve robustness of the MIMO IC for a channel with uncertainty. The authors in Ma et al. (2013) proposed a robust distributed joint signal and interference alignment algorithm for the MIMO cognitive radio networks. Interference alignment is evaluated as a technique to mitigate inter-cell interference in the downlink of a cellular network for the case of imperfect channel knowledge (Tresch and Guillaud 2009).

In this research paper, approximate mean of signal-to-interference-plus-noise ratio (SINR) under imperfect channel state information (CSI) is computed and maximized for throughput enhancement of MIMO interference networks. Each transceiver adjusts its filter by maximizing the expected value of SINR.

The contribution of this paper or the presented novelty compared to the previous work in Gomadam et al. (2011) is that approximate mean of SINR over CSI error is used for maximization to enhance throughput of MIMO interference networks. Numerical results demonstrate when approximate mean is used for maximization, the proposed transceivers will lead to sum rate improvement.

The convergence of the proposed algorithm is demonstrated. Accuracy of approximation is studied via Monte Carlo simulations. Monte Carlo simulations demonstrate that more accurate approximation can be achieved with less SNR.

2 System Model

In a K-user MIMO IC, transmitter j and receiver k have M and N antennas, respectively. Independent symbols $D^j$ with power $P$ are sent by the jth transmitter. True and estimated channel matrices between transmitter j and receiver k are denoted by $G^{kj}$ and $H^{kj}$, respectively. Then, the error model is described by:

$$ G^{kj} = H^{kj} + E^{kj}. $$

The elements of $E^{kj}$, error matrix, are independent and identically distributed Gaussian with zero mean and variance $\sigma^2$. The received signal at receiver k is expressed by

$$ y^k = \sum_{j=1}^{K} (H^{kj} + E^{kj})X^j + Z^k, $$

where $X^j$ is the $M \times 1$ signal vector transmitted by the transmitter j and $Z^k \sim CN(0, N_0 I)$ is additive white Gaussian noise (AWGN) vector. Transmitter j precodes symbol vector by using the precoder matrix. $V^j$ is the $M \times D^j$ precoder matrix. Columns of $V^j$, $v^j_d$ are unit norm vectors. Receiver k estimates the transmitted symbol vector by using the interference suppression matrix $U^k$. The received signal is filtered by $U^k$ as $\overline{y}^k = U^k \hat{y}^k$. Each node works in a time-division duplex (TDD) mode. At two consecutive time slots, first, nodes on the left-hand side send the data to the nodes on the right-hand side. Then, the role of nodes is switched and the nodes on the left-hand side receive the data, as illustrated in Fig. 1.

The relation between the original and reciprocal channel matrices is $\overline{G}^{kj} = G^{kj \dagger}$ (Gomadam et al. 2011). Since the receivers of the reciprocal channel play the roles of original network’s transmitters and vice versa, then $\overline{y}^k = U^k$ and $U^j = V^j$.

3 New Approach for Throughput Enhancement Under Imperfect CSI

In this section, the proposed algorithm is formulated.

3.1 SINR$^k_d$ Under Imperfect CSI

According to the system model, the SINR value for the $d$th data stream at $k$th receiver is expressed by

$$ \text{SINR}^k_d = \frac{\| P_u^k G^{k_d} v^k_d \|^2}{P \sum_{i=1}^{K} \sum_{m=1}^{D^i} \| u^k_d G^{i_m} v^i_m \|^2 - P \| u^k_d G^{k_d} v^k_d \|^2\| u + N_0 \| u^k_d \|^2}. $$

(3)
SINR is used to distinguish two working modes:

\[ \text{original channel} (\text{top network}) \]

Original and reciprocal channels obtained by switching the roles of transmitters and receivers in the original channel (top network). Reciprocal network (below channel) is obtained by the same process as the original network.

\[ \text{Reciprocal network} (\text{below channel}) \]

In this case:

\[ \text{Original and reciprocal channels} \]

\[ \text{Fig. 1 System model. Reciprocal network (below channel) is obtained by switching the roles of transmitters and receivers in the original channel (top network). Original and reciprocal channels distinguish two working modes} \]

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\[ \text{Statistical Linearization Argument} \]

If \( f(A, B) \) is concentrated near its mean, then \( E \left[ f(A, B) \right] \) can be expressed in terms of \( \mu_1 = E[A] \) and \( \mu_2 = E[B] \). According to the statistical linearization argument (Kay 1993), \( \text{SINR}_\text{g} \) is approximated by a Taylor series expansion around mean value \( (\mu_1, \mu_2) \):

\[ \text{SINR}_\text{g}(A, B) \approx \text{SINR}_\text{g}(\mu_1, \mu_2) + \frac{\partial \text{SINR}_\text{g}}{\partial A}(A - \mu_1) (B - \mu_2) + \frac{\partial^2 \text{SINR}_\text{g}}{\partial A^2}(A - \mu_1)^2 \]

\[ + \text{SINR}_\text{g}(\mu_1, \mu_2) (B - \mu_2)^2. \]

Next, the mean of \( A \) and \( B \) is computed as follows:

\[ \mu_1 = \text{Mean}[A] = u_d^{\dagger} \left[ P H^{kk} v_d^{\dagger} v_d H^{kk} + P \sigma^2 I \right] u_d. \]

\[ \mu_2 = \text{Mean}[B] = u_d^{\dagger} \left[ P \sum_{j=1}^{K} \sum_{m=1}^{d_j} H^{ij} v_m^{\dagger} v_m^{\dagger} H^{ij} - P H^{kk} v_d^{\dagger} v_d^{\dagger} H^{kk} + \left( P \sigma^2 \sum_{j=1}^{K} D_j - P \sigma^2 + N_0 \right) I \right] u_d. \]

where \( \text{Mean}[A] \) is obtained as follows. \( \text{Mean}[B] \) is computed similarly.

\[ \text{Mean} \left[ (H^{ij} + E^{ij}) v_m^{\dagger} v_m (H^{ij} + E^{ij}) \right] = \text{Mean} \left[ H^{ij} v_m^{\dagger} v_m H^{ij} \right] \]

\[ + \text{Mean} \left[ E^{ij} v_m^{\dagger} v_m E^{ij} \right] + \text{Mean} \left[ E^{ij} v_m^{\dagger} v_m E^{ij} \right] \]

\[ = H^{ij} v_m^{\dagger} v_m H^{ij} + 0 + 0 + \sigma^2 \left( v_m^{\dagger} v_m \right) I = H^{ij} v_m^{\dagger} v_m H^{ij} + \sigma^2 I. \]

In simplistic way, \( A \) and \( B \) can be approximated by \( \mu_1 \) and \( \mu_2 \). Therefore, \( \text{SINR}_\text{g} \) with respect to \( \mu_1 \) and \( \mu_2 \) is given by

\[ \text{SINR}_\text{g} \approx \frac{u_d^{\dagger} \left[ P H^{kk} v_d^{\dagger} v_d H^{kk} + P \sigma^2 I \right] u_d}{u_d^{\dagger} \left[ P \sum_{j=1}^{K} \sum_{m=1}^{d_j} H^{ij} v_m^{\dagger} v_m H^{ij} - P H^{kk} v_d^{\dagger} v_d^{\dagger} H^{kk} + \left( P \sigma^2 \sum_{j=1}^{K} D_j - P \sigma^2 + N_0 \right) I \right] u_d}. \]
The algorithm starts with arbitrary transmit and receive filters and then iteratively updates these filters to provide the solution. The goal is to achieve a robust transceiver by progressively increasing Mean SINR. The iterative algorithm alternates between the original and reciprocal networks. Within each network, only the receivers update their filters. The algorithm is implemented by the following two steps:

**Step I**

In the original network, the columns of interference suppression filter are updated by each receiver as follows:

\[
E[\text{SINR}_d^k] \simeq \frac{\mu_1}{\mu_2} + \int \int \left[ \frac{\text{C \text{SINR}}_{d}^k(\mu_1, \mu_2)}{C_A} (A - \mu_1) + \frac{\text{C \text{SINR}}_{d}^k(\mu_1, \mu_2)}{C_B} (B - \mu_2) \right] f(A, B) dA \times dB
\]

\[
+ \int \int \left[ \frac{\text{C \text{SINR}}_{d}^k(\mu_1, \mu_2)}{C_A^2} (A - \mu_1)^2 + \frac{\text{C \text{SINR}}_{d}^k(\mu_1, \mu_2)}{C_B^2} (B - \mu_2)^2 \right] f(A, B) dA \times dB.
\]

(9)

First-order estimation of the mean value can be expressed by \(E[\text{SINR}_d^k] \simeq \frac{\mu_1}{\mu_2}\). Second-order estimation of the mean value is \(E[\text{SINR}_d^k] \simeq \frac{\mu_1}{\mu_2} + 2\frac{\mu_1}{\mu_2} \text{VAR}[B] \cdot \frac{\mu_1}{\mu_2} = 0\). Therefore, \(\text{SINR}_d^k\) with respect to \(\mu_1\) and \(\mu_2\) is first-order estimation of the mean. In the context of computation, it is hard to compute \(\text{VAR}[B]\) theoretically (it is mathematically intractable).

Numerical results demonstrate when first-order approximation of the mean is used for maximization, the proposed transceivers will lead to sum rate improvement, as shown in Figs. 3, 4 and 5. Although first-order approximation is maximally within %60 of the true value, proposed scheme achieves higher sum rate compared to baseline schemes considered for comparison.

### 3.2 Algorithm Formulation

It is shown in Chong and Zak (2001) that the optimization problem in (10) is equivalent to (12)

\[
\max_{u_d^k} \quad \text{max} \, u_d^k \Psi_{Q, I} u_d^k
\]

\[
\text{s.t.} \quad u_d^k F u_d^k = 1.
\]

(12)

For the equivalent problem, i.e., constrained maximization in (12), Lagrangian function can be derived as

\[
l(u_d^k, \lambda) = u_d^k \Psi_{Q, I} u_d^k + \lambda \left( 1 - u_d^k F u_d^k \right).
\]

Lagrange conditions are \(\frac{\partial l(u_d^k, \lambda)}{\partial u_d^k} = 0\) and \(\frac{\partial l(u_d^k, \lambda)}{\partial \lambda} = 0\). The solution is denoted by \(u_d^k\) and Lagrange multiplier by \(\lambda^*\). It is also shown in Chong and Zak (2001) that \(u_d^k\) is the eigenvector corresponding to the maximal eigenvalue of \(F^{-1} Q\) and \(\lambda^*\) is \(u_d^k \Psi_{Q, I} u_d^k\). To summarize

**Step II**

Maximization of (7) over \(u_d^k\) can be stated as follows:

\[
\text{max}_{u_d^k} \quad \frac{\Psi_{Q, I} u_d^k}{u_d^k (P \sum_{j=1}^K \sum_{m=1}^{D_j} H_{k m} v_m^j v_d^k + P \sigma^2) u_d^k}
\]

\[
\forall d \in \{1, \ldots, D \},
\]

\[
(10)
\]

The unit vector that maximizes (7) is given by [solution is given in Chong and Zak (2001). Brief discussion about solution is given in next page].

\[
u_d^k = \vartheta [F^{-1} Q],
\]

(11)

Operator \(\vartheta[\cdot]\) denotes the eigenvector corresponding to the maximal eigenvalue of a matrix.

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max $u_d^\dagger Q u_d^*$.

\[ \max_{u_d} u_d^\dagger Q u_d^* + \lambda \left( 1 - u_d^\dagger F u_d^* \right) \]

s. t. $u_d^\dagger F u_d^* = 1$.

$u_d^* = \phi[F^{-1} Q]$  

operator $\phi[.]$ denotes the eigenvector corresponding to the maximal eigenvalue of a matrix

$\lambda^* = u_d^* Q u_d^*$

Transmit precoding matrices in the reciprocal network are the receive interference suppression matrices in the original network, determined in Step I. Each receiver solves the following optimization problem:

\[ \max_{u_d} \text{Mean}[\text{SINR}_d], \forall d \in \{1, \ldots, D_r\}. \]

The transmit precoding matrices, $V^k$, are the receive interference suppression matrices $U^k$ from the original network, that their columns are given by (11). The optimal $d$th unit column of $U^j$ is given by

\[ \overrightarrow{u}_d = \phi \left[ F^{-1} Q \right]. \]  

(13)

Now, receive interference suppression matrices in the reciprocal network, obtained using (13), replace the transmit precoding matrices in the original network, and then the algorithm returns to Step I. The switching between both channels continues in this manner. The steps of the algorithm are given in Fig. 2.

### 4 Proof of Convergence

Now, the convergence of the proposed algorithm is demonstrated. It is similar to convergence of Max-SINR algorithm in the MIMO interference network (Dalir and Aghaeinia 2018). The metric is defined in (14). It is proved here that each step in the algorithm increases the metric. Since it cannot increase unboundedly, this implies that algorithm converges. It is important to note that the metric is the same for both original and reciprocal networks.

\[ \max_{\forall j} \text{metric} = \sum_{k=1}^{K} \sum_{d=1}^{D_r} l(u_d^k, \lambda). \]  

(14)

Accordingly:

\[ \max_{\forall j} V^j \text{ metric} = \sum_{k=1}^{K} \sum_{d=1}^{D_r} l(u_d^k, \lambda). \]

In other words, given $V^j \forall j \in K$, Step 1 increases the value of (14) over all possible choices of $U^k \forall k \in K$. The filter $U^j$ computed in Step 3, based on $V^k = U^k$, also maximizes the metric in the reciprocal channel (16).

\[ \max_{U^j} \text{metric}, \forall j \in K \]

\[ \text{metric} = \sum_{j=1}^{J} \sum_{d=1}^{D_r} l\left( \overrightarrow{u}_d^j, \lambda^j \right) = \sum_{j=1}^{J} \sum_{d=1}^{D_r} \overrightarrow{u}_d^j Q \overrightarrow{u}_d^j + \lambda^j \left( 1 - \overrightarrow{u}_d^j F \overrightarrow{u}_d^j \right). \]  

(16)

Since $\overrightarrow{V}^k = U^k$ and $\overrightarrow{U}^j = V^j$, the metric remains unchanged in the original and reciprocal networks, according to the following equation:

\[ \text{metric} = \sum_{j=1}^{J} \sum_{d=1}^{D_r} \overrightarrow{u}_d^j \left[ P H^j_0 \overrightarrow{v}_d^j \overrightarrow{v}_d^j H^j_0 \overrightarrow{v}_d^j P \sigma^2 \right] \overrightarrow{u}_d^j \]

\[ + \sum_{j=1}^{J} \sum_{d=1}^{D_r} \lambda_d^j \left( 1 + \overrightarrow{u}_d^j \right) \left[ P \sigma^2 \sum_{k=1}^{K} D_k - \sigma^2 + N_0 \right] I \overrightarrow{u}_d^j \]

\[ - P \sum_{j=1}^{J} \sum_{d=1}^{D_r} \sum_{k=1}^{K} \lambda_d^j u_m^k \overrightarrow{H}_d^j \overrightarrow{v}_d^j \overrightarrow{v}_d^j H^j_0 \overrightarrow{u}_d^j \overrightarrow{u}_d^j u_m^k = \text{metric}. \]  

(17)

Therefore, Step 3 also can increase the value of (14). Since the value of (14) is monotonically increased after every iteration, convergence of the algorithm is guaranteed.
5 Simulation Results

The proposed new approach for throughput enhancement under imperfect CSI is evaluated in this section. Channel coefficients are i.i.d. Gaussian with zero mean and unit variance. We assume quasi-static fading so the fading channels $G_{kj}$ remain unchanged during a fading block. The overall sum rate of the system is given by $R = \sum_{k=1}^{K} \sum_{d=1}^{D^k} R_d^k$ where

Throughput of $d$th data stream at $k$th receiver: $R_d^k = \log(1 + \sin r_d^k)$,

$$\sin r_d^k = \frac{\left\| P u_d^k \right\|_2^2}{P \sum_{j=1}^{K} \sum_{m=1}^{D^j} \left\| u_d^j \right\|_2^2 \left\| H^{kj} v_m^{kj} \right\|_2^2 + N_0 \left\| u_d^j \right\|_2^2},$$

Overall sum rate: $cr$

(18)

Figure 3 represents the sum rate comparison between the proposed and basic algorithms for MIMO IC with $K = 4$ user and $N = M = 3$ antennas and $D = 1$ data stream, $(3 \times 3, 1)^4$ MIMO IC. The filters are designed with the error variance of $\sigma^2 = 0.1$. It can be observed that proposed scheme achieves higher sum rate compared to all the other schemes over the entire considered SNR$^1$ range. Proposed scheme achieves 7 dB SNR gain over the Max-SINR algorithm at providing 14 b/s/Hz sum data rate.

Figures 4 and 5 show sum rate for $(3 \times 4, 2)^2$ (MIMO IC with $K = 2$ user and $M = 3 N = 4$ antennas and $D = 2$ data stream) and $(2 \times 2, 1)^3$ (MIMO IC with $K = 3$ user and $N = M = 2$ antennas and $D = 1$ data stream). Again, proposed scheme achieves higher sum rate compared to all the other schemes. In comparison with Max-SINR, proposed scheme improves data rate better than 16 b/s/Hz, while Max-SINR cannot achieve data rate higher than 12 b/
Comparative sum rate improvement compared to Max-SINR is shown for $(3 \times 3, 1)^4$ MIMO IC. Figures 6, 7 and 8 report higher energy efficiency for the proposed algorithm compared to the other schemes.

Numerical value of $\text{Mean} [\text{SINR}_d^k]$ and theoretical approximation are depicted for $(3 \times 3, 1)^4$ MIMO IC in Fig. 9, $(3 \times 4, 2)^2$ in Fig. 10 and $(2 \times 2, 1)^3$ in Fig. 11. The network uses proposed algorithm to compute precoding and interference suppression matrices. The filters are designed with error variances $\sigma_1^2 = 0.05$ and $\sigma_2^2 = 0.1$. 

s/Hz in $(3 \times 4, 2)^2$ MIMO IC. Comparative sum rate improvement compared to Max-SINR is shown for $(2 \times 2, 1)^3$ MIMO IC. Figures 6, 7 and 8 represent the energy efficiency (= sum rate/consumed power) comparison between the proposed and basic algorithms. Figures 6, 7 and 8 report higher energy efficiency for the proposed algorithm compared to the other schemes.
SNR scales linearly with $\sigma^2$ as it is obvious from (7). It is straightforward to say as $P\sigma^2$ decreases, the impact of any error in approximating diminishes. Here, the impacts of influential parameters on the accuracy of approximation are confirmed via Monte Carlo simulations. Accuracy of approximation is measured by $\alpha = \frac{\text{Num} - \text{App}}{\text{Num}}$. Tables 1, 2, and 3 show any approximation error will be attenuated as SNR decreases.
Fig. 7  Energy efficiency versus SNR. $(3 \times 4, 2)^2$ MIMO IC.

Fig. 8  Energy efficiency versus SNR. $(2 \times 2, 1)^3$ MIMO IC.
6 Conclusion

In this paper, a robust algorithm was proposed to improve the throughput of the MIMO interference channel, under imperfect CSI. The effect of CSI imperfection on the SINR mean was approximated. In the proposed new approach for throughput enhancement of MIMO interference networks under imperfect CSI, filters were adjusted based on the problem of SINR expectation maximization. Transceivers were designed based on the reciprocity of wireless networks. Monte Carlo simulations demonstrated that the proposed algorithm improves data rate of MIMO IC under imperfect CSI.
Fig. 11 Approximation of Mean $[\text{SINR}_d]$ and numerical value is shown versus SNR. $(2 \times 2, 1)^3$ MIMO IC.

Table 1 Accuracy of approximation in Fig. 6

| $\sigma^2$ | -5 | 0  | 5  | 10 | 15 | 20 | 25 | 30 | 35 |
|------------|----|----|----|----|----|----|----|----|----|
|            |    |    |    |    |    |    |    |    |    |
|            | 3.18 | 9.49 | 25.22 | 41.1 | 51.64 | 56.5 | 59.28 | 60.37 | 60.49 |
|            | 1.53 | 6.61 | 19.03 | 34.78 | 47.97 | 55.26 | 58.75 | 60.28 | 60.42 |

Although accuracy of approximation is higher than %50 ($\times 0.5$) for SNR $>15$ dB, it was observed in Fig. 3 that proposed scheme achieved higher sum rate compared to all the other schemes over the entire SNR range.

Table 2 Accuracy of approximation in Fig. 7

| $\sigma^2$ | -5 | 0  | 5  | 10 | 15 | 20 | 25 | 30 | 35 |
|------------|----|----|----|----|----|----|----|----|----|
|            |    |    |    |    |    |    |    |    |    |
|            | 4.46 | 8.37 | 23.76 | 41 | 52.65 | 57.96 | 61.61 | 62.69 |
|            | 2.07 | 4.97 | 17.53 | 34.44 | 48.16 | 57.37 | 62.69 |

Although accuracy of approximation extends to %63 in the considered SNR range of Table 2, it was observed in Fig. 4 that proposed scheme achieved higher sum rate compared to all the other schemes.

Table 3 Accuracy of approximation in Fig. 8

| $\sigma^2$ | -5 | 0  | 5  | 10 | 15 | 20 | 25 | 30 | 35 |
|------------|----|----|----|----|----|----|----|----|----|
|            |    |    |    |    |    |    |    |    |    |
|            | 7.25 | 3.93 | 17.4 | 34.95 | 45.72 | 56.95 | 63.02 | 63.13 |
|            | 3.54 | 1.36 | 10.32 | 26.44 | 41.71 | 52.84 | 59.13 |

Although accuracy of approximation extends to %63 in the considered SNR range of Table 3, it was observed in Fig. 5 that proposed scheme achieved higher sum rate compared to all the other schemes.
References

Cadambe V, Jafar SA (2008) Interference alignment and the degrees of freedom of the K user interference channel. IEEE Trans Inf Theory 54(8):3425–3441

Choi SW, Jafar SA, Chung SY (2009) On the beamforming design for efficient interference alignment. IEEE Commun Lett 13(11):847–849

Chong EKP, Zak SH (2001) An introduction to optimization. Wiley, Hoboken, pp 382–383

Dalir A, Aghaeinia H (2018) Theoretical investigation of convergence of max-SINR algorithm in the MIMO interference network. J Comput Commun 6:31–37

Gomadam K, Cadambe VR, Jafar SA (2011) A distributed numerical approach to interference alignment and applications to wireless interference networks. IEEE Trans Inf Theory 57(6):3309–3322

Gou T, Jafar SA (2010) Degrees of freedom of the K user M × N MIMO interference channel. IEEE Trans Inf Theory 56(12):6040–6057

Guiazon RF, Wong K, Wisely D (2014) Capacity analysis of interference alignment with bounded CSI uncertainty. IEEE Wirel Commun Lett 3(5):505–508

Jafar S, Fakhereddin M (2007) Degrees of Freedom for the MIMO Interference Channel. IEEE Trans Inf Theory 53(7):2637–2642

Johnny M, Aref MR (2017) An efficient precoder size for interference alignment of the K user interference channel. IEEE Commun Lett 21(9):1941–1944

Johnny M, Aref MR (2015) Interference alignment for the K-user interference channel with imperfect CSI. arXiv preprint arXiv:1512.01751

Kay SM (1993) Fundamentals of statistical signal processing: estimation theory. Prentice Hall, Upper Saddle River, pp 294–299

Kumar K, Xue F (2010) An iterative algorithm for joint signal and interference alignment. In: Proceedings of ISIT, 2010, pp 2293–2297

Ma Y, Li J, Liu Q, Chen R (2011) Group based interference alignment. IEEE Commun Lett 15(4):383–385

Ma S, Du H, Ratnaraiah T, Dong L (2013) Robust joint signal and interference alignment in cognitive radio networks with ellipsoidal channel state information uncertainties. IET Commun 7(13):1360–1366

Peters SW, Heath Jr RW (2009) Interference alignment via alternating minimization. In Proceedings of ICASSP, 2009, pp 2445–2448

Razavi SM, Ratnaraiah T (2014) Asymptotic performance analysis of interference alignment under imperfect CSI. In: Proceedings of WCNC, 2014, pp 532–537

Shen H, Li B, Tao M, Luo Y (2010) The new interference alignment scheme for the MIMO interference channel. In: Proceedings of Asilomar, 2009, pp 1106–1110

Tresch R, Guillaud M (2009) Cellular interference alignment with imperfect channel knowledge. In: IEEE international conference on communications workshops, 2009

Xie B, Li Y, Minn H, Nosratinia A (2013) Adaptive interference alignment with CSI uncertainty. IEEE Trans Commun 61(2):792–801