Analysis on the Characteristics of Camera Lens Distortion

Tae-Eun Kim*
Department of Multimedia, Namseoul University, 91 Daehak-ro Seonghwan-eup Seobuk-gu Cheonan-si Chungnam, 31020, South Korea; tekim5@empas.com

Abstract

Objectives: Recent high zoom lens as CCD (Charge Coupled Device) camera is released with the resolution to be able to easily acquire the digital image that has been variously utilized, from day-to-day utilization to take advantage of specialized domain such as computer vision (computer vision). It has a number of advantages for obtaining a normal zoom lens CCD camera video which is now commercially available, however the camera calibration in the actual image process geometrically have been the considerable difficulties due to the unstable capture and the movement of the pick-up process in a variety of zoom lens camera. Methods: The camera parameters for the zoom lens calibration will be test variables calculated over a camera lens, a variety of focal lengths whenever the zoom occur, especially if you already have zoom movement at the point the lens test is complete, recalculation of the camera calibration parameters cause the difficulties. In this research the zoom lens to correct the distortion of image obtaining the camera calibration, and a method that extracts the camera parameters through the DLT (Direct Linear Transformation) test method is proposed. Findings: DLT accuracy of the camera is black was carried out in the following two aspects. The first was to analyze the three dimensional position differences between the estimated position and the actual three-dimensional observation that is calculated by the model formula to moderate, and the second is that pixel position that is pixels accuracy projected onto the image plane in the three-dimensional space of the object the accuracy was evaluated in an absolute manner. In this study, under the assumption that the aperture condition is fixed zoom, the zoom was determined the relationship between the camera and variable focus for the condition setting. A zoom lens camera model by setting the zoom and focus conditions at regular intervals as short-focus lens model was established by the DLT method and each camera parameter is tested individually. Applications: In future research the image correction software will be develop for the 3-dimensional location information based on three-dimensional information generating process based on camera calibration method by the DLT method.

Keywords: Camera Lens, Camera Parameter, Computer Vision, Direct Linear Transformation

1. Introduction

As a base study of future 3D TV development, it is necessary to mathematize the correlations of interior/exterior camera parameters of cameras into a camera model for camera calibration. Interior parameters include the characteristics, principal point, focus distances, and distortion parameters of camera and lens, whereas exterior parameters have parameters related to the external environment of camera such as camera rotation and location movement. Regarding such parameters, the information on parameters is acquired by precision optical experiment or parameters are estimated using the geometrical relationship between camera and control points. While general zoom lens CCD camera that are used commonly have many advantages in obtaining images, they are geometrically unstable when taking images and cause considerable difficulties in camera lens calibration due to the various movements of zoom during photoshoots. Since a zoom occurs differently in camera parameters for zoom lens calibration, calibration parameters are calculated across various focus distances during camera lens
calibration. Especially, there is also a difficulty of having to calculate parameters for camera calibration again if the zoom moves after lens calibration is already complete. Therefore, this study proposes a method of extracting camera parameters through DLT (Direct Linear Transformation) calibration method.

2. Characteristics of Zoom Camera

2.1 Composition of Zoom Lens
The optical elements of zoom lens CCD camera consist of zoom, focus, aperture, etc. Zoom is used to obtain images that have different resolutions by changing focus distances; focus is used to adjust the focus of subject when the shooting distance has changed or to figure out the depth of field of an image that is out of focus; and aperture is used to adjust the amount of light ray incident through the lens. Zoom lens consist of a few combined lens groups such as focus lens group, zoom lens group, and auxiliary lens group.

2.2 Lens Distortion of Zoom Camera
Using a multi-collimator or angle observation instrument, it is difficult to apply calibration through optical experiments on general cameras in reality due to the lack of precise experiment equipment and economic aspect. Therefore, non-metric cameras to easily acquire image data and an analytical calibration technique that does not require specific equipment is used. By applying a mathematical model that can modify constant errors such as the distortion of camera lens, bundle adjustment is made and accurate results are obtained. The distortion of cameras on ideal optical system includes radial distortion error and tangential distortion errors Figure 1.

2.3 Calibration
When the real-world space coordinate on some points is known, the coordinate reflected on the image of 3D points in the image coordinate system can be calculated using camera parameters associated with the camera model. However, since it is difficult to immediately apply fixed focus lens model, which is like pinhole camera used in a single-lens camera, for zoom lens model, calibration is performed by setting up an individual fixed focus lens model for each discrete period of zoom movement. For interior camera parameters, principal point (Cx, Cy), focus distances (fx, fy), three radial distortion parameters (k1, k2, k3), and two tangential distortion parameters (p1, p2) can be considered, and in camera exterior parameters, there is T (Tx, Ty, Tz), which is a translation vector from the origin of rotation matrix R (α, β, γ) and real-world space coordinate to the origin of camera space coordinate. To find out such camera parameters, this study applied the camera calibration technique by DLT (Direct Linear Transformation).

2.3.1 Calibration Method by DLT (Direct Linear Transformation)
To determine the interior/exterior parameters of camera, this study performed calibration through DLT (Direct Linear Transformation) technique; it is based on collinearity that the center of camera lens, image point of picture, and the target point of an actual 3D space that corresponds to this must be in line. Figure 2 shows the basic collinearity concept of ground photogrammetry. The formula that converts image coordinate system into ground coordinate is expressed.

\[
x' = m_{11}X + m_{12}Y + m_{13}Z + D_d
\]

\[
y' = m_{21}X + m_{22}Y + m_{23}Z + D_d
\]

\[
z' = m_{31}X + m_{32}Y + m_{33}Z
\]

However, m11, m12, …, m33 represents rotation matrix elements.

In Figure 2, camera lens center O can be expressed as \(X_0, Y_0, Z_0\) for ground coordinate system X, Y, and Z.
If point α on the picture of corresponding point on the actual 3D space is expressed as \( x_\alpha, y_\alpha, z_\alpha \), the rotated image coordinate system \( x', y', z' \) becomes parallel to ground coordinate system \( X, Y, Z \). Also, \( x_\alpha, y_\alpha, z_\alpha \) is expressed as the following formula.

\[
\frac{x_\alpha}{X_\alpha - X_0} = \frac{y_\alpha}{Y_\alpha - Y_0} = \frac{z_\alpha}{Z_\alpha - Z_0}
\]

Therefore,

\[
\begin{align*}
  x'_\alpha &= \left( \frac{X_\alpha - X_0}{Z_\alpha - Z_0} \right) \times Z' \\
  y'_\alpha &= \left( \frac{Y_\alpha - Y_0}{Z_\alpha - Z_0} \right) \times Z' \\
  z'_\alpha &= \left( \frac{Z_\alpha - Z_0}{Z_\alpha - Z_0} \right) \times Z'
\end{align*}
\]

(2)

If formula (2) is substituted into formula (1), it is as follows.

\[
\begin{align*}
  x_\alpha &= m_{11} \times (x'_\alpha - X_0) + m_{12} \times (y'_\alpha - Y_0) + m_{13} \times (z'_\alpha - Z_0) \\
  y_\alpha &= m_{21} \times (x'_\alpha - X_0) + m_{22} \times (y'_\alpha - Y_0) + m_{23} \times (z'_\alpha - Z_0) \\
  z_\alpha &= m_{31} \times (x'_\alpha - X_0) + m_{32} \times (y'_\alpha - Y_0) + m_{33} \times (z'_\alpha - Z_0)
\end{align*}
\]

(3a)

(3b)

(3c)

If formula (3a) and formula (3b) are divided into formula (3c) and \( z'_\alpha \) is substituted by \(-f\), a general collinearity is induced in the short-range photogrammetry.

\[
\begin{align*}
  x_\alpha &= -f \times m_{13} \times (x'_\alpha - X_0) + m_{12} \times (y'_\alpha - Y_0) + m_{13} \times (z'_\alpha - Z_0) \\
  y_\alpha &= -f \times m_{23} \times (x'_\alpha - X_0) + m_{22} \times (y'_\alpha - Y_0) + m_{23} \times (z'_\alpha - Z_0)
\end{align*}
\]

(4)

The formula becomes as follows if many different constant errors during camera photoshoots are considered in formula (4).

\[
\begin{align*}
  x_\alpha + \Delta x - C_x &= -c \times m_{13} \times (x'_\alpha - X_0) + m_{12} \times (y'_\alpha - Y_0) + m_{13} \times (z'_\alpha - Z_0) \\
  y_\alpha + \Delta y - C_y &= -c \times m_{23} \times (x'_\alpha - X_0) + m_{22} \times (y'_\alpha - Y_0) + m_{23} \times (z'_\alpha - Z_0)
\end{align*}
\]

(5)

The least square method is mainly used as an adjustment method for accidental errors from the observation process, but photogrammetry includes constant errors by camera lens. Such constant errors are enabled with calibration by "error model" that has defined their occurrence characteristics, so they are interpreted by adding the constant error model to collinearity model formula. The bundle adjustment method by self-calibration to correct constant errors such as camera lens distortion was devised to increase the accuracy of close-range photogrammetry. After, it was adopted into aero-triangulation or satellite sensor modeling, and all kinds of models on block adjustment by bundle adjustment were suggested. Formula (5), a collinearity formula that includes the constant error model \( \Delta x, \Delta y \) is expressed as below if it is simplified.

\[
\begin{align*}
  \Delta x &= \frac{L_1 X + L_2 Y + L_3 Z + L_4}{L_6 X + L_7 Y + L_8 Z + L_5} \\
  \Delta y &= \frac{L_3 X + L_4 Y + L_5 Z + L_6}{L_6 X + L_7 Y + L_8 Z + L_5}
\end{align*}
\]

(6)

Since the coordinate measured by the precise coordinate measuring device includes accidental errors \( V_x \) and \( V_y \) constant errors, formula (6) can be expressed as.

\[
\begin{align*}
  (x + V_x + \Delta x)(L_3 X + L_4 Y + L_5 Z + L_6) &= L_3 (x + V_x + \Delta x) + L_4 y + L_5 z + L_6 \\
  (y + V_y + \Delta y)(L_3 X + L_4 Y + L_5 Z + L_6) &= L_3 x + L_4 (y + V_y + \Delta y) + L_5 z + L_6
\end{align*}
\]

(7)

If \( L_6 X + L_7 Y + L_8 Z + L_5 \) and formula (7) is rewritten,

\[
\begin{align*}
  V_x &= \frac{L_4 x + L_5 y + L_6 z + L_7}{A} \\
  V_y &= \frac{L_3 x + L_4 y + L_5 z + L_6}{A}
\end{align*}
\]

(8)

To correct constant errors here, formula (8) on each point \( i \) from photo \( j \) can be expressed as the following matrix formula,

\[
\begin{bmatrix}
  V_x \\
  V_y \\
  \vdots
\end{bmatrix} + \begin{bmatrix}
  B_{j1} & B_{j2} & \cdots & B_{j15} & K_{j1} & \cdots & K_{j15}
\end{bmatrix} \begin{bmatrix}
  D_{j1} \\
  D_{j2} \\
  \vdots
\end{bmatrix} = 0
\]

(9)
and the conditional equation can be written as.

$$V_i + B_i \Delta_i + D_i = 0$$  \hspace{1cm} (10)

Here,

$$V_i = \begin{bmatrix} V_{x_1} \\ V_{y_1} \end{bmatrix}, \quad B_i = B_x, \quad \text{matrix of } B_y,$$

and $\Delta_j = \Delta_j = B_x$, unknown value matrix of photo $j$.

The formula on $n$ control points,

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{bmatrix} \Delta_j + \begin{bmatrix} D_1 \\ D_2 \\ \vdots \\ D_n \end{bmatrix} = 0$$ \hspace{1cm} (11)

and the conditional equation is as follows.

$$V + B \Delta_j + D = 0$$ \hspace{1cm} (12)

$$V^T W V - \sum^T W B A_j + \sum^T T W D = 0$$ \hspace{1cm} (13)

From the condition of least square method,

$$\frac{\delta V^T W V}{\delta \Delta} = B^T W B \Delta + B^T W D = 0 \hspace{1cm} (14)$$

The value is obtained as follows.

$$\Delta_j = -(B^T W B)^{-1} B^T W D$$ \hspace{1cm} or \hspace{1cm} (15)

And, here,

$$N = B^T W B, \quad D = B^T W D, \quad W$$

the relative weight ratio matrix.

If formula (14) is multiplied by $\Delta_j^T$,

$$\Delta_j^T B^T W B \Delta_j + \Delta_j^T B^T W D = 0$$ \hspace{1cm} (16)

And, if formula (13) is substituted into formula (16),

$$V^T W V = D^T W B \Delta_j + D^T W D$$ \hspace{1cm} (17)

The relative weight ratio matrix is

$$W = \begin{bmatrix} W_1 \\ W_2 \\ \vdots \\ W_n \end{bmatrix}$$ \hspace{1cm} (18)

Here,

$$m_x^2$$ and $$m_y^2$$ are the variances of $x, y$ condition equation respectively.

From formula (18) they are

$$m_x^2 = \frac{(\frac{x_{1x} - L_x}{A})^2 m_x^2 + (\frac{x_{1y} - L_y}{A})^2 m_y^2 + (\frac{x_{1z} - L_z}{A})^2 m_z^2 + m_x^2}{m_x^2}$$

But $m_x^2$, $m_y^2$ are the variances of object space coordinate and $m_x^2$, $m_y^2$ are the variances of precise coordinate measuring device.

The degree of freedom in conditional equation is

$$DF = 2n - u$$ \hspace{1cm} (21)

However, $n$ is the number of control points, $u$ is the unknown value, and the variance of unit relative weight ratio is

$$m_0^2 = \frac{V^{-1} W V}{DF}$$ \hspace{1cm} (22)

Thus, the variance-covariance matrix of unknown value is

$$m_\Delta = m_0^2 N^{-1}$$

Interior orientation parameters $C_x, C_y, f$ become as follows from formula (16).

$$C_x = (L_1 L_6 + L_2 L_{15} + L_3 L_{12}) L^3$$

$$C_y = (L_5 L_6 + L_6 L_{10} + L_7 L_{14}) L^3$$

$$f = \frac{-C_y^2 + C_y^2 + \frac{L_5}{L_2} + \frac{L_4}{L_2} + \frac{L_6}{L_2} + L_2 + L_4 + L_6 + L_2 + L_4 + L_6 + L_2 + L_4 + L_6 + L_2}{2}$$ \hspace{1cm} (23)

3. Analysis of Experimental Results

3.1 Test-field Making and Control Point Measurement

The geometric condition between the camera and calibration object have a huge influence on camera calibration. For the types of calibration object, calibration board form
that can move along the cubic or axis is mainly used, and linear, square, and circular targets are used for calibration target patterns. As Figure 3 shows the entire view of test-field, this study installed two observation instruments at 3.3m line to measure the trigonometrical leveling and allowed photoshoots at various distances (1 ~ 7m) toward depth direction from the wall.

3.2 Camera Calibration of DLT Method

To perform the calibration of zoom lens CCD camera, the distortion of image formed by spherical aberration, coma-aberration, etc. increases as the size of aperture increases among the three optical elements—zoom, focus, aper-

Figure 3. Test-field plan.

Table 1. Comparison of the 3D positioning results of 18 DLT coefficients and observed value (Unit: cm)

| Measuring Point | Observation Point X | Observation Point Y | Observation Point Z | Modeling Result Point X | Modeling Result Point Y | Modeling Result Point Z | Residual ΔX | Residual ΔY | Residual ΔZ |
|-----------------|---------------------|---------------------|---------------------|-------------------------|-------------------------|-------------------------|--------------|--------------|--------------|
| 1               | 60.02               | 610.603             | 56.497              | 57.725                  | 617.216                 | 57.051                  | 2.295        | -6.613       | -0.554       |
| 2               | 89.845              | 620.181             | 56.096              | 91.375                  | 613.415                 | 54.802                  | -1.53        | 6.766        | 1.294        |
| 3               | 119.626             | 615.424             | 56.306              | 118.453                 | 625.012                 | 57.32                   | 1.173        | -9.588       | -1.014       |
| 4               | 164.595             | 624.99              | 55.471              | 164.444                 | 632.679                 | 56.622                  | 0.151        | -7.689       | -1.151       |
| 5               | 209.18              | 615.195             | 55.332              | 210.711                 | 618.264                 | 55.772                  | -1.531       | -3.069       | -0.24        |
| 6               | 238.38              | 617.552             | 55.183              | 240.79                  | 620.745                 | 55.239                  | -2.41        | -3.193       | -0.056       |
| 7               | 269.584             | 610.227             | 55.271              | 273.397                 | 618.998                 | 56.461                  | -3.813       | -8.771       | -1.19        |
| 8               | 89.845              | 620.181             | 31.105              | 89.35                   | 620.012                 | 30.642                  | 0.495        | 0.169        | 0.463        |
| 9               | 239.462             | 620.088             | 30.459              | 240.292                 | 618.692                 | 29.944                  | -0.83        | 1.396        | 0.515        |
| 10              | 59.953              | 610.233             | 6.511               | 61.679                  | 605.385                 | 5.915                   | -1.726       | 4.848        | 0.596        |
| 11              | 89.899              | 620.131             | 6.122               | 87.914                  | 626.725                 | 5.8                     | 1.985        | -6.594       | 0.322        |
| 12              | 119.685             | 615.01              | 6.318               | 119.864                 | 618.681                 | 5.945                   | -0.179       | -3.671       | 0.373        |
| 13              | 164.627             | 624.929             | 5.687               | 164.48                  | 636.894                 | 5.451                   | 0.147        | -11.965      | 0.236        |
| 14              | 209.296             | 614.998             | 5.591               | 211.764                 | 624.17                  | 5.451                   | -2.468       | -9.172       | 0.14         |
| 15              | 239.508             | 619.984             | 5.478               | 240.418                 | 615.526                 | 4.816                   | -0.91        | 4.458        | 0.662        |
| 16              | 269.585             | 610.069             | 5.368               | 267.101                 | 592.547                 | 4.365                   | 2.484        | 17.522       | 1.003        |
| 17              | 59.901              | 610.311             | -43.48              | 65.761                  | 592.39                  | -42.796                 | -5.86        | 17.921       | -0.684       |
| 18              | 89.965              | 620.052             | -43.889             | 91.93                   | 611.621                 | -43.943                 | -1.965       | 8.431        | 0.054        |
| 19              | 119.786             | 615.054             | -43.649             | 121.986                 | 606.875                 | -43.592                 | -2.2         | 8.179        | -0.057       |
| 20              | 164.635             | 624.96              | -44.374             | 165.386                 | 625.047                 | -45.033                 | -0.751       | -0.087       | 0.659        |
| 21              | 209.416             | 614.861             | -44.392             | 211.493                 | 621.305                 | -45.794                 | -2.077       | -6.444       | 1.402        |
| 22              | 239.586             | 620.186             | -44.548             | 240.946                 | 620.301                 | -45.142                 | -1.36        | -0.115       | 0.594        |
| 23              | 266.947             | 604.138             | -44.223             | 269.262                 | 604.215                 | -44.395                 | -2.315       | -0.077       | 0.172        |
| 24              | 89.996              | 620.059             | -68.865             | 93.44                   | 606.159                 | -67.842                 | -3.444       | 13.9         | -1.023       |
| 25              | 239.708             | 619.875             | -69.544             | 240.214                 | 615.225                 | -69.321                 | -0.506       | 4.65         | -0.223       |
| Average         |                     |                     |                     |                         |                         |                         | 1.487        | 6.688        | 0.572        |
ture—of zoom lens camera. Also, while there is a change of focus on the optical axis when the aperture condition changes, aperture change does not have a huge change on the focus distance change in general. Therefore, under the assumption that aperture is in the fixed condition, this study determined the relationship between camera parameters and parameters on zoom and zoom-focus condition setting. By setting the zoom and focus conditions at a regular interval, zoom lens camera model was established as a single focus lens model by DLT technique and each camera parameters were calibrated individually. Using DLT, the study sought to evaluate its accuracy through the comparison between the method that does not consider lens distortion in various photographing conditions and the method that considers radial distortion and tangential distortion. Among exterior camera parameters, the value of $\kappa$ was close to $180^\circ$, $\phi$ to $0^\circ$, and $\omega$ to $90^\circ$, and camera translation parameters ($X_0$, $Y_0$, $Z_0$) mostly showed values similar to the camera conditions at the time of photoshoot. Among interior camera parameters, the location of principal point shows a tendency to increase, and focus distance shows a tendency to decrease and then increase again for zoom out images. This can be analyzed as due the inclusion of observation errors and the correlations among the parameters of model formula in the calibration process by geometric principle, and requires a further study. Figure 4 is the calibration results of 3m distance from the photo shoot, showing the changes of interior and exterior camera parameters on the left image by each zoom stage. At a 3m distance from the calibration board, calibration considering 11 basic DLT coefficients as well as radial and tangential distortion coefficients was performed for the images shot at 60mm zoom setting. Table 1 shows the results after determining the 3D location. Calibration was performed by considering three radial correction factors and two tangential distortion coefficients, and 3D location was calculated. As a result, the error of actual 3D observed value was 1.487cm toward X, 6.688cm toward Y, and 0.572cm toward Z. Therefore, the recovery accuracy of image that added radial distortion coefficients and tangential distortion coefficients is better than the 3D location accuracy when the image is recovered using the basic DLT coefficients.

4. Conclusion

This study compared the stereo image correction technique through the camera calibration method of DLT technique, and could obtain reliable interior/exterior camera parameters under the experiment environment. While camera calibration by control point on the plane could not be conducted for DLT, stable camera calibration could be performed at high speed considering the arrangement of control points and general environment such as 3D TV or modeling.

5. Acknowledgement

Funding for this paper was provided by Namseoul University.

6. References

1. Linear infrastructure mapping using airborne video imagery and subsequent integration into a GIS [Internet], [Cited 2000 Jul 24]. Available from: http://ieeexplore.ieee.org/document/858388/?reload=true&arnumber=858388.
2. Chen Y-S, Shih S-W, Hung Y-P, Fuh C-S. Simple and efficient method of calibrating a motorized zoom lens. Image and Vision Computing. 2001 Jun; 19:1099–110.
3. Zitnick CL, Kanade T. A cooperative algorithm for stereo matching and occlusion detection. IEEE Transactions on Pattern Analysis Machine Intelligence. 2000 Jul; 22(7):675–84.
4. Salvi J, Armangue X, Batle J. A comparative review of camera calibrating methods with accuracy evaluation. Pattern Recognition. 2002 Jul; 35(7):1617–35.
5. Willey AG, Wong KW. Geometric calibration of zoom lenses for computer vision metrology. Photogrammetric Engineering and Remote Sensing. 1995 Jan; 61(1):587–93.
6. Weng J, Cohen P, Herniou M. Camera calibration with distortion models and accuracy evaluation. IEEE Pattern Analysis and Machine Intelligence. 1992 Oct; 14(10):965–80.
7. Camera calibration with a viewfinder [Internet]. [Cited 2002 May]. Available from: http://citeseerx.ist.psu.edu/
viewdoc/download?doi=10.1.1.5.5296&rep=rep1&type=pdf.
8. Willson RG. Modeling and calibration of automated zoom lenses, Doctoral Dissertation; 1994.
9. Horn BKP, Brooks MJ. Shape from shading. M.I.T. Press; 1989.
10. Oren M, Nayar SK. Generalization of the Lambertian model and implications for machine vision. International Journal of Computer Vision. 1995 Apr; 14(3):227–51.