Tracking quintessential inflation from brane worlds

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We analyse the mechanism of quintessential inflation in brane world scenarios for a number of particle physics inspired scalar potentials. We constrain the parameter space of those scalar potentials and comment on the likelihood that we could discriminate these models from standard inflation, based on upcoming large scale structure and cosmic microwave background observations.

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I. INTRODUCTION

There has been interest, recently, on the cosmological implications arising in a certain class of brane world scenarios where the Friedmann equation is modified at very high energies [1]. The key feature is that the modification makes it easier to obtain inflation in the early universe, by contributing extra friction to the scalar field equation of motion [2–4]. Analysing this situation, it has been explicitly demonstrated how inflation naturally occurs in such models even when the potential is too steep to allow conventional inflation to occur [5,6]. Inflation ends when the brane world corrections begin to lose their dominance, and reheating takes place as a result of gravitational particle production, rather than the usual inflaton decay mechanism. In this case the inflaton energy density subsequently redshifts sufficiently quickly that the produced radiation comes to dominate. One of the nice features this allows for is that the inflaton potential does not have to have a feature in it, and the field does not have to decay completely as we do not require a minimum to the potential about which the inflaton normally oscillates as it decays. Instead, we can consider the intriguing possibility that the inflaton actually survives as inflation ends. As the universe evolves through radiation to matter domination the inflaton potential does not play much of a role, however, at a recent redshift of \( z \sim 1 \) it once again comes to dominate driving the universe to an accelerated expansion. This mechanism has been named “quintessential inflation” scenario [7,8]. In this paper we investigate this possibility in more depth. Following Ref. [12] the density of particles produced after inflation is

\[
\rho_{\text{R}}^{\text{end}} = 0.01 g_p H_{\text{end}}^4 ,
\]

where \( g_p \) is the number of scalar fields involved in particle production, likely to be \( 10 \lesssim g_p \lesssim 100 \), and \( H_{\text{end}} \) is the value of the Hubble parameter at the end of inflation.

In our brane world scenario, the ratio of the energy densities of the scalar field to the relativistic particles is well determined from Eq. (5),

\[
\frac{\rho_\phi^{\text{end}}}{\rho_{\text{R}}^{\text{end}}} \approx \frac{36 \nu_0^2}{0.01 g_p \lambda_b^4 V_{\text{end}}^{1/3}} .
\]

Hence, the ratio of the energy densities is inversely proportional to the number of fields involved in gravitational...
production $g_p$. Moreover, it decreases as the time between the end of inflation $a_{\text{end}}$ and the maximum of the energy density of the produced relativistic particles $a_p$ increases. Defining $\Delta \equiv \log(a_p/a_{\text{end}})$, we find from our numerical simulations that this value is weakly dependent on the parameters of the scalar potential or the number of $e$-foldings of inflation, and spans the interval $0.29 \lesssim \Delta \lesssim 0.4$. Therefore, when constraining the parameter space of scalar potentials, we will consider both the average case of $\Delta = 0$ with $g_p = 100$, and the case of instant reheating $\Delta = 0$ with $g_p = 10$, which corresponds to the situation when the quoted ratio is its maximum.

II. INFALATIONARY PARAMETERS

It proves useful to define the slow–roll parameters, analogously to the usual inflationary case. Following Refs. [3,9] we define $\epsilon$, $\eta$ and $\xi^2$, generalising the usual ones [13], by

$$\epsilon = \frac{1}{2\kappa^2} \left( \frac{V''}{V} \right)^2 \frac{1 + V/\lambda_b}{(1 + V/2\lambda_b)^2},$$

$$\eta = \frac{1}{\kappa^2} \frac{V''}{V} \frac{1}{1 + V/2\lambda_b},$$

$$\xi^2 = \frac{1}{\kappa^4} \frac{V'''}{V^2} \frac{1}{1 + V/2\lambda_b^2},$$

where prime indicates a $\phi$–derivative and the slow-roll approximation has been employed.

As the Universe inflates, the associated number of $e$–foldings of the scale factor $a$, is given by

$$N = \int_{a}^{a_{\text{end}}} d\ln a \simeq -\kappa^2 \int_{\phi}^{\phi_{\text{end}}} \frac{V}{V'} \left( 1 + \frac{V}{2\lambda_b} \right) d\phi,$$

where $a_{\text{end}}$ corresponds to the end of inflation, given by the condition $\epsilon = 1$. In this model, the number of $e$–foldings corresponding to scales entering the Hubble radius today can be unambiguously determined [11]. The scale that leaves the Hubble radius during inflation and re–enters today is $k = a_N H_N = a_0 H_0$, i.e.

$$1 = \frac{k}{a_0 H_0} = \frac{a_N}{a_0} \frac{a_{\text{end}}}{a_p} \frac{H_N}{H_0}.$$

The subscript $N$ corresponds to the value of the quantity $N$ $e$–foldings from the end of inflation. Noting that $a_N/a_{\text{end}} = \exp(-N)$ and $\phi_{\text{end}}^0 = \phi_{\text{end}}^0 (a_0/a_p)^4$ we find

$$N = \ln \left[ \left( \frac{3\Omega}{0.01 g_p} \right)^{1/4} (\kappa H_0)^{-1/2} a_{\text{end}}^{1/2} \frac{H_N}{a_p H_{\text{end}}} \right].$$

where $\Omega_R = 2.471h^{-2} \times 10^{-5}$ is the fractional energy density in radiation today.

The amplitude of the scalar and tensor perturbations produced during inflation are given by [3,9]

$$A_S^2 = \frac{\kappa^4}{150\pi^2} \frac{V}{\epsilon} \left( 1 + \frac{V}{2\lambda_b} \right) \left( 1 + \frac{V}{\lambda_b} \right),$$

$$A_T^2 = \frac{\kappa^4}{150\pi^2} V \left( 1 + \frac{V}{2\lambda_b} \right) F^2(\lambda_b),$$

where

$$F^2(x) \equiv \left[ \sqrt{1 + x^2} - x^2 \sinh^{-1}\left( \frac{1}{x} \right) \right]^{-1}$$

and

$$x \equiv \left( \frac{3}{4\pi\lambda_b} \right)^{1/2} H M_4 = \left[ \frac{2V}{\lambda_b} \left( 1 + \frac{V}{2\lambda_b} \right) \right]^{1/2}.$$

In the low energy limit, $\rho \ll \lambda_b$ ($x \ll 1$), $F^2 \approx 1$, whereas $F^2 \to 3V/2\lambda_b$ in the high energy limit.

Having obtained the amplitude of the scalar and tensor perturbations the corresponding scalar and tensor spectral indices can be obtained. Using

$$\frac{d}{d\ln k} \approx -\frac{V'}{3H^2} \frac{d\phi}{d\phi},$$

we find [9]

$$n_S - 1 = \frac{d \ln A_S^2}{d \ln k} = -6\epsilon + 2\eta,$$

$$n_T = \frac{d \ln A_T^2}{d \ln k} = -2\epsilon \left( 1 + 2 \frac{F' V}{F^2 V'} \frac{1 + V/2\lambda_b}{1 + V/\lambda_b} \right).$$

If we define the quantity $\delta$ to be 0 for the standard inflationary scenario (without the brane corrections) and $\delta = 1$ in the brane inspired case, then we can simplify Eq. (19) for the low and high energy limits as,

$$n_T = -(2 + \delta) \epsilon.$$

We also obtain that the consistency equation is the same in either limit [9],

$$n_T = -2 \frac{A_T^2}{A_S^2}. $$

The derivatives of the scalar and tensor spectral indices give in the limits of high and low energies

$$\frac{dn_S}{d \ln k} = -(24 - 6\delta)\epsilon^2 + 16\epsilon \eta - 2\xi^2,$$

$$\frac{dn_T}{d \ln k} = -(8 + \delta)\epsilon^2 + (4 + 2\delta)\epsilon \eta.$$

In the next section we turn our attention to a number of particle physics inspired potentials, to determine how the brane corrections modify the dynamics of the associated scalar field, leading to inflation in a number of scenarios where inflation would not have normally been present. In the following section we will proceed to investigate the class of models where the inflaton field then becomes the quintessence field by today.
III. BRANE INFLATION

A. Exponential potentials.

The simplest case to investigate involves the dynamics associated with a pure exponential potential [5],

$$V(\phi) = V_0 \exp(\alpha \kappa \phi).$$  \hspace{1cm} (24)

For completeness we recall the key results. During slow–roll inflation, from Eq. (7) we have

$$\epsilon \simeq \frac{2 \alpha^2 \lambda_b}{V_0}.$$  \hspace{1cm} (25)

Inflation ends when $\epsilon = 1$, giving

$$V_{\text{end}} \simeq 2 \alpha^2 \lambda_b,$$  \hspace{1cm} (26)

which implies that typically the term quadratic in the density still dominates the linear term at the end of inflation.

The potential $V_N$, $N$ e–foldings from the end of inflation can be obtained from Eq. (10) and is given by the simple formula

$$V_N = V_{\text{end}} (N + 1).$$  \hspace{1cm} (27)

The amplitude of density perturbations generated during the inflationary period fixes the brane tension

$$\lambda_b = \left[ \frac{2 \kappa^4}{75 \pi^2} \alpha^6 (1 + N)^4 \right]^{-1} A_S^2,$$  \hspace{1cm} (28)

which significantly is independent of the mass parameter $V_0$.

From Eqs. (3) and (27), we have, $H_N = (N + 1) H_{\text{end}}$; and substituting into Eq. (12) we find that $N = 70.7$ for $\Delta = 0.35$, $g_\text{p} = 100$ and $N = 72.1$ for $\Delta = 0$, $g_\text{p} = 10$.

Inserting this into Eq. (28), and using the observed value from COBE of $A_S = 2 \times 10^{-5}$ [14], we find that

$$\lambda_b \simeq \frac{1 \times 10^{-17}}{\alpha^6} M_4^4 = \left( \frac{10^{15} \text{GeV}}{\alpha^{3/2}} \right)^4,$$  \hspace{1cm} (29)

indicating a brane tension of order the Grand Unified Theories scale.

For the scalar and tensor spectral indices and their derivatives we find,

$$n_S - 1 = (1 + N) \frac{d n_S}{d \ln k} = - \frac{4}{1 + N},$$  \hspace{1cm} (30)

$$n_T = (1 + N) \frac{d n_T}{d \ln k} = - \frac{3}{1 + N}.$$  \hspace{1cm} (31)

B. Power law potentials

For power law potentials,

$$V(\phi) = V_0 (\kappa \phi)^n,$$  \hspace{1cm} (32)

during slow–roll inflation we have

$$\epsilon \simeq \frac{2 n^2 \lambda_b}{V_0} (\kappa \phi)^{-n-2}. $$  \hspace{1cm} (33)

For the particular case of $n = -2$ the condition for inflation is $\lambda_b < V_0/8$, i.e. when this condition is satisfied, inflation ends in the low energy limit. When $n \neq 2$, we obtain, at the end of inflation,

$$V_{\text{end}} = V_0 \left( \frac{2 n^2 \lambda_b}{V_0} \right)^{n/(n+2)}.$$  \hspace{1cm} (34)

and $N$ e–foldings prior to the end,

$$V_N = V_{\text{end}} \left[ n + (n + 2) N \right]^{n/(n+2)}.$$  \hspace{1cm} (35)

The brane tension is no longer independent of the mass scale $V_0$ but is related to it by,

$$\lambda_b = V_0 \left( \frac{10^{15} \text{GeV}}{\alpha^{3/2}} \right)^{n/2} [2n (n + (n + 2) N)]^{\frac{n+2}{4}}.$$  \hspace{1cm} (36)

when $n \neq 4$, otherwise $\lambda_b$ is completely free and only $V_0$ is fixed, yielding

$$V_0 = \frac{75 \pi^2}{256 \kappa^4} \left( \frac{1}{1 + N} \right)^2 A_S^2.$$  \hspace{1cm} (37)

For the scalar and tensor spectral indices and their derivatives we find,

$$n_S - 1 = - \frac{2 + 4n}{n + (2 + n) N},$$  \hspace{1cm} (38)

$$n_T = - \frac{3n}{n + (2 + n) N},$$  \hspace{1cm} (39)

$$\frac{d n_S}{d \ln k} = -2 \frac{2 + 5n + 2n^2}{(n + (2 + n) N)^2},$$  \hspace{1cm} (40)

$$\frac{d n_T}{d \ln k} = -3 \frac{2n + n^2}{(n + (2 + n) N)^2}.$$  \hspace{1cm} (41)

which reduce to Eqs. (30) in the limit $n \to \infty$.

For negative powers of $n$, the precise number of e–foldings corresponding to the scale entering the Hubble radius today is easily determined from Eqs. (12) and (35).

In Fig. 1 we show the dependence of the number of e–foldings $N$ with $n$ for four different combinations of the time delay between the production of particles and the end of inflation and the number of fields experiencing particle production.
\[ \delta r = 0 \]

hence we take the number of \( e \)-foldings corresponding to the scale entering the Hubble radius today of \( N = 50 \). The filled areas correspond to estimated error bars on these observables based on upcoming Planck observations, according to Ref. [15]. We quote, \( \delta n_S = 0.006 \), \( \delta r = 0.04 \) and \( \delta dn_S/d\ln k = 0.01 \). The error bars are centred on a \( \phi^4 \) standard inflationary cosmology for the sake of illustration. There are several pieces of information one can extract from this figure. Firstly, a \( \phi^4 \) theory in brane worlds is degenerate with the same theory in standard inflation. The case becomes worse for a \( \phi^4 \) theory as the exponential potential presents similar values for all the observables. Secondly, in brane worlds, an exponential and power law with \( n < -10 \) are degenerate potentials. Thirdly, if we could rule out the exponential potential, the remaining models would be truly non-degenerate only if \( |n| > 20 \). And finally, it appears that the derivative of the spectral index will be unable to give us any information about the underlying inflationary cosmology. In summary, based on this small sample, it appears that upcoming satellite experiments won’t be able to conclusively discriminate between the various models of inflation.

For completeness we review the main features of power law potentials in the case of standard inflation. In this case, there is no freedom for the mass scale \( V_0 \) which is fixed by the density perturbations as

\[ V_0 = \left[ \frac{k^4}{75\pi^2} \frac{1}{n^2} (2n(n/4+N))^{n/2} \right]^{-1} A_S^2. \] (42)

The corresponding spectral indices are

\[ n_S - 1 = (n/4+N) \frac{dn_S}{d\ln k} = \frac{1}{2} n + \frac{2}{n/4+N}, \] (43)

\[ n_T = (n/4+N) \frac{dn_T}{d\ln k} = -\frac{1}{2} \frac{n}{n/4+N}. \] (44)

C. Future observational constraints

In Fig. 2 we show how a selection of different observational parameters depend on the precise value of \( n \) comparing standard inflation with brane world inflation. In particular, we consider here the spectral index \( n_S \), the ratio of tensor to scalar perturbations \( r \equiv 4\pi A_T^2/A_S^2 = -2\pi n_T \) and the derivative of the spectral index \( dn_S/d\ln k \). For the positive power law case, reheating can happen through conventional mechanisms, and hence we take the number of \( e \)-foldings corresponding to the scale entering the Hubble radius today of \( N = 50 \). The filled areas correspond to estimated error bars on these observables based on upcoming Planck observations, according to Ref. [15]. We quote, \( \delta n_S = 0.006 \), \( \delta r = 0.04 \) and \( \delta dn_S/d\ln k = 0.01 \). The error bars are centred on a \( \phi^4 \) standard inflationary cosmology for the sake of illustration. There are several pieces of information one can extract from this figure. Firstly, a \( \phi^4 \) theory in brane worlds is degenerate with the same theory in standard inflation. The case becomes worse for a \( \phi^4 \) theory as the exponential potential presents similar values for all the observables. Secondly, in brane worlds, an exponential and power law with \( n < -10 \) are degenerate potentials. Thirdly, if we could rule out the exponential potential, the remaining models would be truly non-degenerate only if \( |n| > 20 \). And finally, it appears that the derivative of the spectral index will be unable to give us any information about the underlying inflationary cosmology. In summary, based on this small sample, it appears that upcoming satellite experiments won’t be able to conclusively discriminate between the various models of inflation.

\[ d n_S / d \ln k = \]
IV. QUINTESSENCE

In the case of reheating by means of gravitational production of relativistic particles, the scalar field does not totally decay. Many authors [7,8] have suggested that the field could then dominate the energy density at late times, providing the dark energy that appears to be driving the observed acceleration of the universe today [16].

In the brane world picture the potential does not have to have a shallow slope to satisfy the slow–roll conditions followed by a steep fall where the latter break, as in standard inflation. In fact, we have seen that the potential can be exponentially steep and still inflate at early times and at the point where the brane correction becomes negligible inflation comes gracefully to an end. This feature broadens the range of potentials one can consider for “quintessential inflation” provided the brane effects become negligible before nucleosynthesis [9–11].

Using the modified Friedmann equation (3), Huey and Lidsey [9] showed that for a pure inverse power law potential, the requirement that the field was both sub–dominant prior to nucleosynthesis, and tracked the matter and radiation in the universe before today implied that the value of the power n in Eq. (32), is so large (and negative) that the corresponding equation of state ($w_\phi = p_\phi/\rho_\phi$) is too large to give an accelerating universe at this epoch. They concluded that within the class of inverse power law potentials, the only successful model would correspond to a kind of “creeping quintessence”, in which the energy density of the field is kinetic energy dominated from its value at the end of inflation to its present value. The field effectively remains frozen (with equation of state $w_\phi = -1$) until the energy density of matter matches its energy density, which occurs only around a redshift $z \approx 1$. At this point the field starts rolling down its potential again, and dominates the total energy density slowly raising the equation of state from $-1$, thereby explaining the present day acceleration of the universe. Although an interesting scenario, its similarity to a pure cosmological constant at late times, makes the evolution of this quintessence field very difficult to confirm on the basis of upcoming SN Ia or CMB observations, especially if the equation of state of dark energy is very close to $-1$ as observations suggest [17]. However, if the contribution of the quintessence field to the total energy density is significant, it can increase the expansion rate resulting in a shift in the positions and an increase in the amplitude of the peaks of the angular power spectrum [18], hence opening up the possibility that we may be able to discriminate a quintessence field from a bare cosmological constant type behaviour.

In what follows we analyse a class of scalar potentials in which the energy density of the quintessence field tracks the background energy density during the epoch of decoupling. We are able to constrain the parameters of the potentials by employing the values allowed on the fractional density of a scalar field, both at the epoch of nucleosynthesis and at the time of recombination [19]:

$$\Omega_{\phi}^{\text{nuc}} < 0.045, \quad \Omega_{\phi}^{\text{rec}} < 0.39.$$  \hspace{1cm} (45)

In the next subsections we will consider the possible cases of evolution of the scalar field between the end of inflation and the epoch the attractor is reached.

A. Monotonic evolution

After the initial period of inflation and following that of the kinetic energy domination, the scalar field is known to freeze at a value given by [20]

$$\phi_{\text{ke}} = \phi_{\text{end}} + \frac{\sqrt{6}}{\kappa} \left[1 + \frac{1}{2} \ln \left(\frac{\rho_{\phi}^{\text{end}}}{\rho_R^{\text{end}}}\right)\right].$$  \hspace{1cm} (46)

Using Eq. (6) and the results of the previous section for an exponential scalar potential, we can calculate the bounds on the allowed slope of the potential. We require the field to enter the scaling regime (whereupon $\Omega_\phi = 4/\alpha^2$ when radiation is dominant) after nucleosynthesis but before radiation–matter equality, i.e.,

$$4\rho_{eq}^\alpha / \alpha^2 \lesssim V(\phi_{\text{ke}}) \lesssim 4\rho_{\text{nuc}}^\alpha / \alpha^2.$$  \hspace{1cm} (47)

These requirements impose the following bounds: $3.6 \lesssim \alpha \lesssim 4.8$ for $g_\phi = 100$, $\Delta = 0.35$ and $3.4 \lesssim \alpha \lesssim 4.5$ for $g_\phi = 10$, $\Delta = 0$, results found to be in good agreement with our numerical integrations (see row corresponding to zero oscillations in Table I). Now, in the standard Friedmann cosmology, a pure exponential potential with a slope within this range of parameters has a stable attractor solution for the energy density that scales with the background [21]. Therefore, this solution does not provide a late time accelerating universe. A number of modifications to this type of potential have been proposed and investigated in depth which do fit the observational data [22–24]. These models present a pure exponential term dominating for almost the entire history of the universe and only recently the other features of the potentials become dominant to provide the accelerating universe, hence, the above constraints also apply to the exponential part of these potentials. In Fig. 3, we show the evolution of the equation of state of the scalar field for the two exponential potentials (2EXP) [23,10]

$$V(\phi) = V_0 \left(e^{\alpha_1 \phi} + e^{\beta_1 \phi}\right).$$  \hspace{1cm} (48)

We would like to make two remarks on the pure exponential case. The first concerns the small range of the parameter $\alpha$ where this scenario can be implemented revealing some degree of fine tuning. The second, refers to the smallness of the parameter itself. In this range of parameters, the contribution of the field is large enough for the integrated Sachs–Wolf (ISW) effect to increase the power on large scales. Under the assumption that the
anisotropies we see today are primordial, the normalisation of the power spectrum to COBE at the multipole $l = 10$ therefore, results in a wrong suppression of the small scales anisotropies. Hence, for these type of potentials, the analysis of the angular power spectrum must be made with special care to take into account the ISW effect.

B. Oscillatory evolution

In the case of very steep potentials, the kinetic energy of the field at the end of inflation can be large enough to overtake the potential by a huge factor. Consequently, the field might freeze at a value corresponding to an energy density lower than the energy density of the background at equality, or even lower than today’s energy density. One then requires an additional mechanism to slow down the field’s evolution. Potentials with a minimum can provide the necessary solution.

For potentials with a minimum, the field can perform several wide damped oscillations around its minimum while energy is continually being converted between kinetic and potential. Hence, extra time for the friction term in Eq. (4) to slow down the field. In the regime of wide oscillations the equation of state of the field is close to unity as the potential is very steep and looks like $V \propto \phi^n$ for $n \gg 1$. Hence, in practice, the field energy density continues decaying as $a^{-6}$. In Fig. 3 we show the evolution of the equation of state of the field for a potential with a minimum (solid line). Fig. 4 shows the evolution of the energy densities of the scalar field and of the background fluid and the evolution of the scalar potential for the same potential with a minimum. After inflation the field quickly becomes kinetic energy dominated. The sharp peaks with $w = -1$ in Fig. 3 or the epochs when $V(\phi) = V_{\text{minimum}}$ in Fig. 4 represent the points at which the field passes through the minimum. In this example, the field becomes sub-dominant just before nucleosynthesis as $\Omega_\phi$ drops below 0.5 before $z > 10^{10}$. After a few oscillations the field has slowed down enough and freezes (for $z \approx 10^5$, in this example). From here the usual dynamics resume. When the background fluid’s energy density drops, the field reaches the attractor solution. The contribution of the field increases when it approaches the minimum of the potential, then the equation of state quickly decreases to $-1$ accelerating the universe by today.

We give two examples corresponding to models presented before in the literature. The 2EXP potential with $\alpha/\beta < 0$, satisfies the above requirements. For the particular case of symmetric slopes i.e. $\beta = -\alpha$ [25], we find that to satisfy the nucleosynthesis bound and for the field to scale after this period and no later than radiation–matter equality, the slopes must satisfy the constraints presented in Table I.

| oscillations | $g_0 = 100$, $\Delta = 0.35$ | $g_0 = 10$, $\Delta = 0$ |
|--------------|-------------------------------|--------------------------|
| 0            | $3.6 \lesssim |\alpha| \lesssim 4.8$               | $3.1 \lesssim |\alpha| \lesssim 4.1$ |
| 1            | $5.9 \lesssim |\alpha| \lesssim 7.0$               | $5.1 \lesssim |\alpha| \lesssim 6.1$ |
| 1            | $10.0 \lesssim |\alpha| \lesssim 11.6$              | $7.9 \lesssim |\alpha| \lesssim 9.4$ |
| 2            | $12.9 \lesssim |\alpha| \lesssim 19.0$              | $10.7 \lesssim |\alpha| \lesssim 15.3$ |
| 3            | $20.3 \lesssim |\alpha| \lesssim 26.5$              | $16.6 \lesssim |\alpha| \lesssim 21.3$ |

TABLE I. Constraints on tracking quintessential inflation for 2EXP with symmetric slopes. $1^-$ refers to the case in which the field freezes when rolling up the potential and $1^+$ to the case in which the field is rolling down before it stops.

The second example is for the SUGRA type potentials [26],
\[ V(\phi) = V_0 (\kappa \phi)^n \exp \left[ \frac{1}{2} (\kappa \phi)^2 \right]. \] (49)

For initial conditions of \( \phi \ll \phi_{\text{minimum}} \), the bounds in Eq. (45) above place a restriction of \( -78 \lesssim n \lesssim -23 \) for \( g_\phi = 100, \Delta = 0.35 \) and \( n \lesssim -86 \) for \( g_\phi = 10, \Delta = 0 \), corresponding to two oscillations of the field (see Fig. 3) and \( w_\phi \approx -0.8 \), today.

To obtain the quoted results, the initial conditions of the scalar field in our simulations were such that they gave a number of \( e \)-foldings of inflation consistent with the number of \( e \)-foldings corresponding to scales entering the Hubble radius today in Eq. (12). We have taken \( \Omega_0 \approx 0.7 \), today.

V. CONCLUSIONS

We have demonstrated, giving examples, of a number of classes of scalar potentials that provide realistic models of “tracking quintessential inflation”. We have seen that the parameter space of potentials described by a pure exponential at early times is highly contrived on the basis of nucleosynthesis bounds and under the requirement that the scalar field had reached the attractor by radiation–matter equality. However, potentials with a minimum offer a larger range of parameter values for which these requirements are satisfied.

For the potentials we have considered, successful realisation of “quintessential inflation” requires potentials with very steep slopes in order to satisfy the nucleosynthesis bound, therefore we expect for all of these models similar values for the observables, i.e. \( n_S \approx 0.94 \) and \( r \approx 0.27 \) (using Eqs. (38) for large \( n \) with \( N \approx 70 \) or from Fig. 2).

We have seen in Sec. III and Fig. 2 that these models can be degenerate to a \( \phi^4 \) theory either in standard or in brane world inflation. It is important to know then, if the type of potentials considered here fit both the reconstruction of the quintessence potential from SN Ia observations [27] and the reconstruction of the inflaton potential from CMB observations [28] giving indications of a unified theory of “quintessential inflation”.

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