Gravitational redshift in quantum-clock interferometry

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Relativistic effects in macroscopically delocalized quantum superpositions
• **Macroscopically delocalized quantum superpositions:**
  coherent superposition of atomic wave packets

Kovachy et al., *Nature* (2015)

• Differences in *dynamics* of superposition components
  → entirely *Newtonian*

• **Same relativistic effects** on superposition components
  (e.g. atomic clocks)

★ **Goal** (QM + GR): experiment with *general relativistic effects* acting *non-trivially* on the *quantum superposition*
Proper time as *which-way* information

- Quantum **superposition of clocks** (*COM* + *internal state*) experiencing **different proper times**

\[
|\psi_a\rangle + |\psi_b\rangle
\]

→ **reduced visibility** of interference signal

*Zych et al., Nat. Comm.* (2011)
Proper time as *which-way* information

- Quantum **superposition of clocks** (*COM + internal state*) experiencing **different proper times**

\[ |\psi_a\rangle \otimes |\Phi(\tau_a)\rangle + |\psi_b\rangle \otimes |\Phi(\tau_b)\rangle \]
\[ |\langle \Phi(\tau_b)|\Phi(\tau_a)\rangle| < 1 \]

\[ \rightarrow \text{reduced \textit{visibility} of interference signal} \]

*Zych et al., Nat. Comm. (2011)*
Outline

1. Relativistic effects in macroscopically delocalized quantum superpositions

2. Key elements of *quantum-clock interferometry*

3. Major challenges in quantum-clock interferometry

4. *Doubly differential* scheme for *gravitational-redshift* measurements

5. Feasibility and extensions
Key elements of quantum-clock interferometry
Quantum-clock model

- **Initialization pulse:**

\[
|g\rangle \rightarrow |\Phi(0)\rangle = \frac{1}{\sqrt{2}} \left( |g\rangle + ie^{i\varphi} |e\rangle \right)
\]

- **Evolution:**

\[
|\Phi(\tau)\rangle \propto \frac{1}{\sqrt{2}} \left( |g\rangle + ie^{i\varphi} e^{-i\Delta E \tau/\hbar} |e\rangle \right)
\]

- **Quantum overlap:**

\[
|\langle \Phi(\tau_b) | \Phi(\tau_a) \rangle| = \cos \left( \frac{\Delta E}{2\hbar} (\tau_b - \tau_a) \right)
\]
• Comparison of independent clocks (after read-out pulse):

\[
\Delta \tau_b - \Delta \tau_a \approx \left( \frac{g L_z}{c^2} \right) \Delta t
\]

for optical atomic clocks

\[
\Delta E \sim 1 \text{ eV} \quad L_z \sim 1 \text{ cm}
\]

• Instead of independent clocks we pursue a quantum superposition at different heights.
Comparison of independent clocks (after read-out pulse):

\[ \Delta \tau_b - \Delta \tau_a \approx \left( g L_z / c^2 \right) \Delta t \]

for optical atomic clocks

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Instead of independent clocks we pursue a quantum superposition at different heights.
Theoretical description of the clock

- two-level atom (internal state):

\[ \hat{H} = \hat{H}_1 \otimes |g\rangle \langle g| + \hat{H}_2 \otimes |e\rangle \langle e| \]

- classical action for COM motion:

\[
S_n [x^\mu (\lambda)] = -m_n c^2 \int d\tau = -m_n c \int d\lambda \sqrt{-g_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda}} \\
(n = 1, 2)
\]

\[ m_1 = m_g \]

\[ m_2 = m_g + \Delta m \]

\[ \Delta m = \Delta E/c^2 \]

- free fall

- including external forces
Atom interferometry in curved spacetime (including relativistic effects)

- **Wave-packet evolution** in terms of
  - *central trajectory* (satisfies *classical e.o.m.*) \( X^\mu(\lambda) \)
  - *centered wave packet* \( |\psi_c^{(n)}(\tau_c)\rangle \)

- **Fermi-Walker frame** associated with the *central trajectory*
  - valid for *freely falling* wave packet (geodesic)
  - but also with *external forces/guiding potential* (accel. trajectory)
  - approximately *non-relativistic* dynamics for centered wave packet

\[
\Delta p/m \ll c \quad \Delta x \ll \ell \quad \text{curvature radius}
\]
• Metric in *Fermi-Walker* coordinates: \( X^\mu(\tau_c) = (c \tau_c, 0) \)

\[
dS^2 = g_{\mu\nu} dx^\mu dx^\nu = g_{00} c^2 d\tau_c^2 + 2 g_{0i} c d\tau_c dx^i + g_{ij} dx^i dx^j
\]

\[
g_{00} = -(1 + \delta_{ij} a^i(\tau_c) x^j / c^2)^2 - R_{0ij0}(\tau_c, 0) x^i x^j + O(|x|^3)
\]

\[
g_{0i} = -\frac{2}{3} R_{0ijk}(\tau_c, 0) x^j x^k + O(|x|^3)
\]

\[
g_{ij} = \delta_{ij} - \frac{1}{3} R_{ikjl}(\tau_c, 0) x^k x^l + O(|x|^3)
\]

• Expanding the action for the *centered wave packet*:

\[
S_n[x(t)] \approx \int d\tau_c \left[ -m_n c^2 - V_n(\tau_c, 0) + \frac{m_n}{2} v^2 - \frac{1}{2} x^T \left( \mathcal{V}^{(n)}(\tau_c) - m_n \Gamma(\tau_c) \right) x - V_{\text{anh.}}^{(n)}(\tau_c, x) \right]
\]
• **Hamiltonian:**
  \[
  \hat{H}_n = m_n c^2 + V_n(\tau_c, 0) + \hat{H}_c^{(n)}
  \]
  \[
  \hat{H}_c^{(n)} = \frac{1}{2m_n} \hat{p}^2 + \frac{1}{2} \hat{x}^T \left( \mathcal{V}^{(n)}(\tau_c) - m_n \Gamma(\tau_c) \right) \hat{x}
  \]

• **Wave-packet evolution:**
  \[
  |\psi^{(n)}(\tau_c)\rangle = e^{iS_n/\hbar} |\psi_c^{(n)}(\tau_c)\rangle
  \]
  - **propagation phase**
    \[
    S_n = - \int_{\tau_1}^{\tau_2} d\tau_c \left( m_n c^2 + V_n(\tau_c, 0) \right)
    \]
  - **centered wave packet**
    \[
    i\hbar \frac{d}{d\tau_c} |\psi_c^{(n)}(\tau_c)\rangle = \hat{H}_c |\psi_c^{(n)}(\tau_c)\rangle
    \]
• **Full interferometer** (including *laser kicks*):

\[
|\psi_I\rangle = \frac{1}{2} (e^{i\phi_a} + e^{i\phi_b}) |\psi_c\rangle
\]

\[
\langle \psi_1 | \psi_1 \rangle = \frac{1}{2} (1 + \cos \delta \phi)
\]

• **Detection probability at the exit port(s):**

• **Phase shift:**

\[
\delta \phi = \phi_b - \phi_a + \delta \phi_{\text{sep}}
\]
Major challenges in quantum-clock interferometry
Insensitivity to gravitational redshift (in a uniform field)

- Consider a freely falling frame:

- Proper-time difference between the two interferometer branches \( \longrightarrow \) independent of \( g \)

(small dependence due to pulse timing suppressed by \( (v_{\text{rec}}/c) \sim 10^{-10} \))
Insensitivity to gravitational redshift (in a uniform field)

- Consider a freely falling frame:

  - Proper-time difference between the two interferometer branches is independent of $g$

  (small dependence due to pulse timing suppressed by $(v_{\text{rec}}/c) \sim 10^{-10}$)
Differential recoil

- Different recoil velocities → different central trajectories

\[ v_{\text{rec}}^{(n)} = \hbar k_{\text{eff}} / m_n \]

- Implied changes of proper-time difference are comparable to signal of interest.
Small visibility changes

- Reduced interference visibility due to deceasing quantum overlap of clock states:

\[
\langle \Psi_1 | \Psi_1 \rangle = \frac{1}{2} + \frac{1}{2} | \langle \Phi(\tau_b) | \Phi(\tau_a) \rangle | \cos \delta \phi
\]

\[
| \langle \Phi(\tau_b) | \Phi(\tau_a) \rangle | = \cos \left( \frac{\Delta E}{2\hbar} (\tau_b - \tau_a) \right)
\]

- Small effect for feasible parameter range:

\[
| \langle \Phi(\tau_b) | \Phi(\tau_a) \rangle | = \cos \left( \frac{\omega_0 g \Delta z}{c^2} \Delta t \right) \approx 1 - (10^{-3})^2 / 2
\]

\[
\Delta E / \hbar = \omega_0 \approx 2\pi \times 4 \times 10^2 \text{ THz}
\]

\[
\Delta z = 1 \text{ cm}
\]

\[
\Delta t = 1 \text{ s}
\]

- Extremely difficult to measure such small changes of visibility, which are masked by other effects leading also to loss of visibility.
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\langle \Psi_1 | \Psi_1 \rangle = \frac{1}{2} + \frac{1}{2} \left| \langle \Phi(\tau_b) | \Phi(\tau_a) \rangle \right| \cos \delta \phi
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\[
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\]

\[
\Delta z = 1 \text{ m}
\]

\[
\Delta t = 1 \text{ s}
\]

- Extremely difficult to measure such small changes of visibility, which are masked by other effects leading also to loss of visibility.
Doubly differential scheme for gravitational-redshift measurement
Detection probability at first exit port (independent of internal state):

\[
\langle \Psi_1 | \Psi_1 \rangle = \frac{1}{2} \left( \langle \Psi_1^{(1)} | \Psi_1^{(1)} \rangle + \langle \Psi_1^{(2)} | \Psi_1^{(2)} \rangle \right)
\]

\[
= \frac{1}{4} \left( 2 + \cos \delta \phi^{(1)} + \cos \delta \phi^{(2)} \right)
\]

\[
= \frac{1}{2} + \frac{1}{2} \cos \left( \frac{\delta \phi^{(2)} - \delta \phi^{(1)}}{2} \right) \cos \left( \frac{\delta \phi^{(1)} + \delta \phi^{(2)}}{2} \right)
\]

Phase-shift difference directly related to visibility reduction.

Precise differential phase-shift measurement involving state-selective detection is much more viable.

(immune to spurious loss of contrast + common-mode rejection of phase noise)
Two-photon process for clock initialization

- **Level structure** for group-II-type atoms (e.g. Sr, Yb) employed in optical atomic clocks:

  ![Level Structure Diagram](image)

  - **Two-photon process** resonantly connecting the two clock states.
  - **Equal-frequency** counter-propagating laser beams in lab frame:
    - constant effective phase → *simultaneity hypersurfaces* in lab frame
Two-photon pulse for clock initialization

- **Level structure** for group-II-type atoms (e.g. Sr, Yb) employed in optical atomic clocks:

  ![Atomic Level Structure Diagram](Image)

  - **Two-photon process** resonantly connecting the two clock states.

  - **Equal-frequency** counter-propagating laser beams in lab frame:

    constant effective phase  →  simultaneity hypersurfaces in lab frame

    $$ e^{-i \omega t} e^{i k \cdot x} \times e^{-i \omega t} e^{-i k \cdot x} = e^{-i 2 \omega t} $$
Compare differential phase-shift measurements for different initialization times:

\[
\begin{align*}
(\delta \phi^{(2)}(t'_i) - \delta \phi^{(1)}(t'_i)) - (\delta \phi^{(2)}(t_i) - \delta \phi^{(1)}(t_i)) &= \frac{\Delta E}{2\hbar} (\Delta \tau_b - \Delta \tau_a) = \Delta m g \Delta z (t'_i - t_i)/\hbar
\end{align*}
\]
Compare differential phase-shift measurements for different initialization times:

\[
(\delta \phi^{(2)}(t'_i) - \delta \phi^{(1)}(t'_i)) - (\delta \phi^{(2)}(t_i) - \delta \phi^{(1)}(t_i)) = \frac{\Delta E}{2\hbar} (\Delta \tau_b - \Delta \tau_a) = \Delta m g \Delta z (t'_i - t_i)/\hbar
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Compare differential phase-shift measurements for different initialization times:

\[ (\delta \phi^{(2)}(t'_i) - \delta \phi^{(1)}(t'_i)) - (\delta \phi^{(2)}(t_i) - \delta \phi^{(1)}(t_i)) = \frac{\Delta E}{2\hbar} (\Delta \tau_b - \Delta \tau_a) = \Delta m g \Delta z (t'_i - t_i)/\hbar \]
• Relativity of simultaneity: $\Delta \tau_c \approx -v(t) \Delta z/c^2 = g \left( t - t_{ap} \right) \Delta z/c^2$

\[
\left( \delta \phi^{(2)}(t'_i) - \delta \phi^{(1)}(t'_i) \right) - \left( \delta \phi^{(2)}(t_i) - \delta \phi^{(1)}(t_i) \right) = \frac{\Delta E}{2\hbar} (\Delta \tau_b - \Delta \tau_a) = \Delta m \, g \, \Delta z \left( t'_i - t_i \right)/\hbar
\]
Challenges addressed

• Differential phase-shift measurement → precise measurement, common-mode rejection (of noise & systematics)

• Comparing measurements with different initialization times → sensitive to gravitational redshift + further immunity

• Almost no recoil from initialization pulse, small residual recoil with no impact on gravitational redshift measurement,

  effect of differential recoil from second pair of Bragg pulses cancels out in doubly differential measurement.
- Residual recoil with no influence on the phase-shift for the excited state:
Feasibility and extensions
Feasible implementation

- **10-m atomic fountains** operating with Sr, Yb in Stanford & Hannover respectively.
- More than 2 s of free evolution time.
- **Doubly differential phase shift** of 1 mrad for
  \[ \frac{\Delta E}{h} = \omega_0 \approx 2\pi \times 4 \times 10^2 \text{ THz} \]
  \[ \Delta z = 1 \text{ cm} \]
  \[ \Delta t_i = 1 \text{ s} \]
- **Resolvable** in a single shot for atomic clouds with \( N = 10^6 \) atoms (shot-noise limited)
- More compact set-ups possible with guided or hybrid interferometry (less mature).
• Measurement of relativistic effects in macroscopically delocalized quantum superpositions with quantum-clock interferometry.

• Important challenges in quantum-clock interferometry and its application to gravitational-redshift measurement.

• Promising doubly differential scheme that overcomes them.

• Feasible implementation in facilities soon to become operational.

• Applicable also to more compact set-ups based on guided or hybrid interferometry.
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• Related theoretical work at Ulm University:

Proper time in atom interferometers: Diffractive vs. specular mirrors

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Stephan Kleinert       Christian Ufrecht
Thank you for your attention.

Q-SENSE
European Union H2020 RISE Project
Diffraction of atoms in internal-state superpositions

• **Bragg diffraction** at the magic wavelength → same Rabi frequency for both internal states.

    BUT *high laser power* required due to large detuning.

• **Alternative diffraction mechanism** based on *simultaneous pair* of single-photon transitions.
  
  ▸ Applicable to *fermionic* isotopes such as $^{87}\text{Sr}$ and $^{171}\text{Yb}$.
  
  ▸ Required lasers *already available* in (some of) those facilities.
• **Sequence** of simultaneous pairs of pulses:

![Diagram showing π/2 pulse and π pulse](image)

- **Net result:**
  - *internal state* unchanged
  - momentum transfer: *twice single-photon* momentum
  - equal-amplitude superposition: *undiffracted* + *diffracted* wave packet
• **Sequence** of simultaneous pairs of pulses:

- Same ac Stark shifts for both internal states, contributions to differential phase shift cancel out.

- Any light shifts cancel out in the doubly differential measurement (provided that the laser intensities are stable).
Extension to guided interferometry

• In principle, guided interferometry can be sensitive to the gravitational redshift:

• Nevertheless, the doubly differential measurement scheme has many advantages.
Extension to guided interferometry

- In principle, guided interferometry can be **sensitive** to the gravitational redshift:

- Nevertheless, the **doubly differential** measurement scheme has many advantages.
Extension to *hybrid* interferometers

- Intermediate stage with atoms held in an optical lattice, where they undergo *Bloch oscillations*:

- Similarly to pure *light-pulse* atom interferometers, they are insensitive to the gravitational redshift.

Charriere et al., Phys. Rev. A 85 013639 (2012)
Zhang et al., Phys. Rev. A 94 043608 (2016)
Extension to *hybrid* interferometers

- Intermediate stage with atoms held in an optical lattice, where they undergo Bloch oscillations:

- The *doubly differential* scheme can also be employed for measuring the gravitational redshift.
Other aspects
Proper-time difference in open interferometers
Proper-time difference in open interferometers
Proper-time difference and gravity gradients