Aharonov-Bohm interferences from local deformations in graphene

Fernando de Juan, Alberto Cortijo, Maria Vozmediano and Andres Cano

Nature Physics (2011)
DOI: 10.1038/NPHYS2034
Graphene

- Two dimensional crystal
- One of the strongest materials ever measured: Young modulus of TPa
- Yet, corrugations (and 3D structure in general) ubiquitous
- It supports large values of strain (20%)
- Interplay of electronics and structure!

Castro Neto, Guinea, Peres, Physics World (Nov 2006)
Substrate induced curvature

AFM experiments:
Ripples correlate with substrate morphology

Lui et al., Nature 462 339 (2009)
“Observation of Graphene Bubbles and Effective Mass Transport under Graphene Films” Stolyarova et al., Nanolett. 9 332 (2009).

“Scanning Tunneling Microscopy Characterization of the Electrical Properties of Wrinkles in Exfoliated Graphene Monolayers”, Xu et al., Nanolett. 9 4446 (2009)
Controlling strain

“Controlled ripple texturing of suspended graphene and ultrathin graphite membranes” Bao et al., Nat. Nanotech. 4, 562 (2009)

“Introducing Nonuniform Strain to Graphene Using Dielectric Nanopillars” (cond-mat/1106.1507 Tomori et al.)

“Graphene bubbles with controllable curvature” (Cond-mat/1108.1701, Manchester group)

“Topological properties of artificial graphene assembled by atom manipulation”, where they produced atomically engineered strains. (Manoharan group, APS 2011)

And the list goes on...

“Impermeable atomic membranes from graphene sheets” Scott Bunch et al., Nanolett. 8, 2458 (2008)
Electrons in graphene

Tight binding band structure:

- Nearest neighbour hopping: $t \sim 2.7$ eV
- Two atoms per unit cell
- 2x2 Hamiltonian

$$H = -t \sum_{<ij>} a_i^\dagger b_j + cc.$$
Strain and gauge fields

\[ H = -\sum_{n=1}^{3} (t + \delta t_n) \begin{pmatrix} 0 & e^{-i(\vec{K} + \vec{q}) \cdot \vec{d}_n} \\ e^{i(\vec{K} + \vec{q}) \cdot \vec{d}_n} & 0 \end{pmatrix} \]

\[ \begin{pmatrix} 0 & e^{-i\vec{K} \cdot \vec{d}_n} \\ e^{i\vec{K} \cdot \vec{d}_n} & 0 \end{pmatrix} \equiv \vec{\sigma} \times \vec{d}_n \]

Dirac fermions: expand \( H \) in \( q \)

\[ \approx -t \sum_{n=1}^{3} \begin{pmatrix} 0 & e^{-i\vec{K} \cdot \vec{d}_n} (1 - i\vec{d}_n \vec{q}) \\ e^{i\vec{K} \cdot \vec{d}_n} (1 + i\vec{d}_n \vec{q}) & 0 \end{pmatrix} \]

\[ = -t \sum_{n=1}^{3} (\vec{\sigma} \times \vec{d}_n) i\sigma_z (\vec{q} \cdot \vec{d}_n) \]

\[ = -t \sum_{n=1}^{3} (\vec{\sigma} \cdot \vec{d}_n)(\vec{q} \cdot \vec{d}_n) = -t \vec{\sigma} \cdot \vec{q} \]

Gauge field: expand \( H \) in \( \delta t_n \)

\[ H = \sum_{n=1}^{3} \delta t_n \vec{\sigma} \times \vec{d}_n \]

\[ A_1 = \frac{\sqrt{3}}{2} (\delta t_1 - \delta t_2) \]

\[ A_2 = \frac{1}{2} (\delta t_1 + \delta t_2 - 2\delta t_3) \]

\[ H = \vec{A} \cdot \vec{\sigma} \]

-Guinea, Horowitz, Le Doussal, Phys. Rev. B 77, 205421 (2008)
-Vozmediano, Katsnelson, Guinea Phys. Rep. 496, 109–148 (2010)
A general strain tensor

\[ H = \sum_{n=1}^{3} \delta t_n \vec{\sigma} \times \vec{d}_n \]

\[ \beta = -\frac{\partial \ln t}{\partial \ln a} \approx 2 \]

\[ \delta t_n = \frac{\beta t}{a^2} (\vec{u}_n - \vec{u}_0) \delta \vec{n} \]

\[ \frac{\vec{u}_n - \vec{u}_0}{a} = \left( \vec{d}_n \cdot \vec{\nabla} \right) \vec{u}(r) \]

\[ H = \dot{\vec{A}} \cdot \vec{\sigma} \]

\[ A^i = \frac{\beta t}{a} f^{ijk} u^{jk} \]

\[ A_1 = \frac{\beta t}{a} (u^{xx} - u^{yy}) \]

\[ A_2 = \frac{\beta t}{a} (-2u^{xy}) \]

\[ f^{ijk} = \sum_{n=1}^{3} \epsilon^{il} d^j_n d^k_n d^l_n \]
Strain induced magnetic fields

- These are physically real pseudo-magnetic fields!

- But time-reversal is preserved because the two valleys have opposite magnetic fields

- They inherit the trigonal symmetry of the lattice!

Morpurgo, & Guinea, “Intervalley scattering, long-range disorder, and effective time reversal symmetry breaking in graphene”. Phys. Rev. Lett. 97, 196804 (2006).

Guinea, Katsnelson, Geim, “Energy gaps, topological insulator state and zero-field quantum Hall effect in graphene by strain engineering”. Nature Phys. 6, 30–33 (2010).

“We believe that the suggested strategies to observe the pseudo-Landau gaps and QHE are completely attainable and will be realized sooner rather than later”
Direct evidence of gauge fields

Strain-Induced Pseudo–Magnetic Fields Greater Than 300 Tesla in Graphene Nanobubbles

N. Levy, et al.
Science 329, 544 (2010);
“Observation of Landau level-like quantizations at 77 K along a strained-induced graphene ridge” He et al., cond-mat/1108.1016 (2011)

See also: “Strain-induced pseudo-magnetic fields and charging effects on CVD-grown graphene” Yeh et al. Surf. Sci. 605, 1649 (2010)
Aharonov-Bohm interferences

Probability of measuring an electron in B depends on magnetic flux through the region enclosed by the path

\[ |\psi|^2 \propto |e^{ikd_1} + e^{ikd_2}|^2 \]
\[ \propto 1 + \cos(k(d_1 - d_2)) \]

Add magnetic flux through the solenoid:
\[ \propto 1 + \cos(k(d_1 - d_2) + \phi) \]
Quantum interference in the LDOS

“Aharonov Bohm oscillations in the local density of states” A. Cano and I. Paul, Phys. Rev. B 80 153401 (2009)

- Same AB physics in the LDOS
- Impurity scattering sets the path
- Semiclassical approximation required

\[ N(r, \omega) = -\frac{2}{\pi} \text{Im} \, G^R(r, r; \omega) \]

\[ G(r, r) = G_0(r, r) + \int dr' \, G_0(r, r') U(r') G_0(r', r) + \cdots \]

\[ G_0(r - r') = \exp \left( i \frac{\pi}{\Phi_0} \int_r^{r'} A(l) \cdot dl \right) G_{00}(r - r') \]
Quantum interference in the LDOS

Multiple scattering from the same impurity is resummed in the T-matrix.

\[ U_0 \rightarrow \tilde{U}_0 = \frac{U_0}{1 - U_0 G_0(0)} \]

\[ \delta G_{\text{loop}} (r, r) = W^2 G_0(r - r_1) G_0(r_1 - r_2) G_0(r_2 - r) \]

\[ W^2 = \frac{\tilde{U}_0^2}{1 - \tilde{U}_0^2 G_0(r_1 - r_2) G_0(r_2 - r_1)} \]

Multiple back and forth processes in the loop contribution are resummed in an analog function W.
Add a magnetic field

\[ N_{A=0} = N_{\text{return}} + N_{\text{loop}} \]

A magnetic flux will modify the loop terms:

\[ N = N_{\text{return}} + N_{\text{loop}} \cos\left( \frac{\pi \Phi}{\Phi_0} \right) \]

\[
\delta G^{(2)}(r, r) = U_0^2 \left[ G_0(r, r_1) G_0(r_1, r_2) G_0(r_2, r) e^{i \frac{\pi \Phi}{\Phi_0}} + G_0(r, r_2) G_0(r_2, r_1) G_0(r_1, r) e^{-i \frac{\pi \Phi}{\Phi_0}} \right]
\]

\[
\delta G^{(2)}(r, r) = U_0^2 \left[ G_0(r, r_1) G_0(r_1, r_2) G_0(r_2, r) 2 \cos \frac{\pi \Phi}{\Phi_0} \right]
\]

\[
N(\omega, r) = N_{A=0}(\omega, r) + N_{\text{loop}}(\omega, r) \left[ \cos\left( \frac{\pi \Phi(r)}{\Phi_0} \right) - 1 \right]
\]
Dirac fermions and interference

- Dirac fermions have a matrix Green's function

\[ G_0(r_1, r_2; \omega) = -\frac{i\omega}{4v_F} \left[ H_0(\omega |r_1 - r_2|) + i\frac{\sigma(r_1 - r_2)}{|r_1 - r_2|} H_1(\omega |r_1 - r_2|) \right] \]

- The previous manipulations require to commute them: non trivial. But all commutators proportional to \( \sigma_3 \) and vanish after the trace (note this is spoiled for gapped graphene!).

- Valley degree of freedom: very short range impurities may induce intervalley scattering. Pick longer ranged ones.

\[ N(\omega, r) = N_{A=0}(\omega, r) + N_{\text{loop}}(\omega, r) \left[ \cos\left(\frac{\pi \Phi(r)}{\Phi_0}\right) - 1 \right] \]
An experimental proposal

- In the flat sample, the STM tip measures the usual standing wave patterns.

\[ N_{A=0}(\omega, r) \]
Induce controlled strain
Circular perturbation

\[ h(r) = A \exp(-r^2 / 2\sigma^2) \]

- In the curved sample, and after subtraction of \( N_{A=0} \), we see a new standing wave pattern (Nloop) modulated by the cosine of the flux through the triangle.

\[
N_{loop}(\omega, r) \left[ \cos\left( \frac{\pi \Phi(r)}{\Phi_0} \right) - 1 \right]
\]
Strain-induced interference

- Circular perturbation
  \[ h(r) = A \exp\left(-\frac{r^2}{2\sigma^2}\right) \]

- Three-fold symmetric perturbation
  \[ u_r = u_0 r^2 \sin 3\theta \]
  \[ u_\theta = u_0 r^2 \cos 3\theta \]
  \[ u_0 = \left(u_{00}/\sigma^2\right) \exp\left(-\frac{r^2}{2\sigma^2}\right) \]

Guinea et al., Nat. Phys. 6 30 (2010).
Conclusions and future

- Strain induces effective pseudo-magnetic fields which are physically very real!

- These produce Aharonov-Bohm interferences in the LDOS which can be observed with STM

- The required strain is low but the measurement may still be challenging.

- The effect could be potentially used to measure strain locally by interferometry.

Thanks for your attention!
Aharonov-Bohm oscillations in the local density of topological surface states

Zhen-Guo Fu,1,2 Ping Zhang,2,3,* and Shu-Shen Li1,†

1 State Key Laboratory for Superlattices and Microstructures, Institute of Semiconductors, Chinese Academy of Sciences, P. O. Box 912, Beijing 100083, People’s Republic of China
2 LCP, Institute of Applied Physics and Computational Mathematics, P.O. Box 8009, Beijing 100088, People’s Republic of China
3 Center for Applied Physics and Technology, Peking University, Beijing 100871, People’s Republic of China

arXiv:1103.1710v1