SELF-SIMILAR TEMPORAL BEHAVIOR OF GAMMA-RAY BURSTS

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ABSTRACT

We apply Fourier analysis to 214 light curves of long gamma-ray bursts and study the statistical properties of their power density spectra (PDSs). The averaged PDS is found to follow a power law of index $-5/3$ over almost two decades of frequency, with a break at $\sim$2 Hz. Individual PDSs are exponentially distributed around the power law. It provides evidence that the diversity of the bursts is due to random realizations of the same process that is self-similar over the full range of timescales. The $-5/3$ slope of the average spectrum may indicate that gamma-ray bursts are related to a phenomenon well studied in hydrodynamics—fully developed turbulence.

Subject heading: gamma rays: bursts

1. INTRODUCTION

Discovered three decades ago, the phenomenon of gamma-ray bursts (GRBs) remains one of the mysteries of the universe (Fishman & Meegan 1995; Hartmann 1996). Being isotropically distributed on the sky (Meegan et al. 1996), the bursts are likely to occur at cosmological distances (Paczynski 1992; Piran 1992; Kulkarni et al. 1998). Their light curves have many random peaks, and in spite of extensive statistical studies (see, e.g., Nemiroff et al. 1994; Norris et al. 1996; Stern 1996), their temporal behavior of GRBs remains a puzzle.

In our analysis, we used light curves of GRBs observed by the Burst and Transient Source Experiment in the energy band, 50 < $h\nu$ < 300 keV, with 64 ms resolution. The background was subtracted using linear fits to the 1024 ms data. To avoid large Poisson fluctuations in the light curves, we excluded dim bursts with peak count rates less than 250 counts per 64 ms bin. (The background subtraction and the peak search are described in Stern, Poutanen, & Svensson 1997, 1999.) Long bursts are of particular interest since their internal temporal structure can be studied by spectral analysis over a larger range of timescales.

We chose bursts with durations of $T_{90} > 20$ s, where $T_{90}$ is the time it takes to accumulate from 5% to 95% of the total fluence of a burst. The resulting sample contains 214 bursts. We calculated the Fourier transform of each light curve and found the corresponding power density spectrum (PDS), which is given by the squared Fourier amplitude summed for the two frequencies, $f$ and $-f$.

Bursts have very diverse PDSs, so that it is hard to see any systematic shapes in individual PDSs (Giblin, Kouveliotou, & van Paradijs 1998). Therefore, we proceed by studying the statistical properties of PDSs for our sample of 214 bursts. In particular, we calculate the average PDS. This is meaningful if the bursts are produced by a common physical mechanism, so that their light curves can be considered as time fragments of the same stochastic process. Such a process was suggested to be responsible for the stretched exponential shape of the average time profile of GRBs (Stern 1996; Stern & Svensson 1996). We search for a signature for the underlying process in the PDS statistics.

2. AVERAGE PDS

The averaging of the PDSs requires a specification of the weight for each burst. We normalize GRB light curves by setting their peak count rates to unity. This corresponds to the PDS normalization by the squared peak count rate. We will discuss how a different normalization would affect the statistics of PDSs.

In Figure 1, the average power density spectrum, $\tilde{P}_f$, for the sample of 214 peak-normalized bursts is presented. Poisson fluctuations of the time bin counts start to become important at high frequencies, $f > 1$ Hz. The individual “Poisson level” of a burst equals its total fluence, including the background. The spectrum above this level displays the intrinsic variability of the signal. The horizontal solid line in Figure 1 shows the averaged Poisson level.

The striking feature of the average PDS is the power-law behavior that covers almost 2 orders of magnitude in frequency. Its slope, $\alpha$, is approximately equal to $\alpha = -5/3$, which is shown by the dashed line in Figure 1. The deviation from the power law at the low-frequency end is due to the finite duration of bursts (the average $T_{90}$ is about 80 s for our sample). To see the behavior of $\tilde{P}_f$ at high frequencies, we smooth it on the scale $\Delta \log f = 0.03$ and multiply by $f^{0.03}$. The result is shown by the solid curve in the top panel in Figure 2. The dotted curve displays the same spectrum after the subtraction of the Poisson level. The subtraction was performed individually in each burst before the averaging. This is equivalent to the subtraction of the effective level shown in Figure 1 from the average PDS.

One can see that $\tilde{P}_f$ is very close to a power law with index $\alpha = -5/3 \approx -1.67$ in the range $0.02 < f < 1$ Hz (the best power-law fit in this range has $\alpha = -1.67 \pm 0.02$). After the subtraction of the Poisson level, a break appears between 1 and 2 Hz. The existence of this break is better seen for bright GRBs, where we do not have to account for the Poisson noise. Therefore, we also plot the average PDS for the 27 bursts with peak count rates greater than 2000 counts bin$^{-1}$ (the bottom panel in Fig. 2). The break is again seen at $\sim$2 Hz. The break is too sharp to be explained as an artifact of 64 ms time binning, which suppresses the PDS by a factor of $|\sin (\pi f \Delta / \pi f \Delta)|^2$, where $\Delta = 64$ ms is the time bin (cf. van der Klis 1989).

Is the power-law behavior of the average PDS related to the type of normalization we chose? To test this, we tried other normalizations, e.g., by dividing the light curve by the burst fluence. We found the same qualitative behavior of the average PDS: a power law of index $\alpha \approx -5/3$ with superimposed fluc-

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3. PDS DISTRIBUTION

Power spectra of each individual burst are very diverse, and they show strong deviations from the average power law. The deviations, however, follow a simple statistical behavior. We constructed the corresponding histogram at each frequency and found that individual $P_f$ are distributed around $P_f$ according to the law $dN/dP_f = N \exp (-P_f/P_f)$. The histogram is not accurate since the number of bursts ($N = 214$) is modest, but the statistics increases when we sum up the histograms at adjacent frequencies. After this summation, the $P_f$ distribution remains narrow and is described by the exponential law as seen in Figure 3. Note that we get this distribution only for peak-normalized bursts. For comparison, we also plot an analogous histogram for bursts normalized by fluence.

The PDS of each individual burst can be decomposed into the power law with $\alpha = -5/3$ and superimposed exponentially distributed fluctuations. The exponential distribution, also denoted the two-dimensional $\chi^2$ distribution, just indicates that the two (sin and cos) components of the Fourier transform are normally distributed around the average value (van der Klis 1989). Similar fluctuations are present in the PDS of the well-known standard noises, such as Poisson noise (PDS slope of $\alpha = 0$), flicker noise ($\alpha = -1$), and Brownian motion ($\alpha = -2$). Note the difference of the GRB variability from the standard noises. In the case of a noise, the exponentially distributed fluctuations are suppressed when the PDS is smoothed by averaging $P_f$ over adjacent frequencies. By contrast, the smoothed power spectra of GRBs continue to show the exponential fluctuations around $P_f$, independently of the smoothing scale. This behavior makes it difficult to recognize the power law in an

Fig. 1.—The averaged PDS for 214 peak-normalized bursts with duration $T_{\text{on}} > 20$ s. The solid horizontal line shows the averaged Poisson level.

Fig. 2.—Top panel: same as Fig. 1, but now the average PDS is smoothed on a scale of $\Delta \log f = 0.03$ and multiplied by $f^{5/3}$ (solid curve). The dotted curve shows the spectrum after the subtraction of the Poisson level. The error bars show the typical uncertainties in $P_f$. Bottom panel: the average PDS for the 27 brightest bursts in the sample.

Fig. 3.—The solid histogram displays the $P_f$ distribution of 214 peak-normalized GRBs. We divided the interval $-2.4 < \log (P_f/P_f) < 1.2$ into 30 equal bins $[\Delta \log (P_f/P_f)]$ and determined how many bursts have $P_f$ within a given bin. The histogram was constructed at each frequency and then summed up over all frequencies in the range $0.03 < f < 1$ Hz. The solid curve shows the exponential distribution, $dN/d \log P_f \propto P_f \exp (-P_f/P_f)$. The dotted histogram displays the $P_f$ distribution found when normalizing the light curves by fluence. The integrals of all distributions are normalized to unity.
individual burst. Nevertheless, the standard $P_\delta$ distribution around the mean $-5/3$ slope supports the view that different GRBs are random statistical realizations of the same stochastic process described by the power law.

4. DISCUSSION

We conclude that the whole diversity of GRBs can be unified: each burst can be considered as a time fragment of the same process that is self-similar on timescales from $\sim0.5$ s up to the total duration of a burst. It follows that GRBs have a common origin, being generated by some standard emission mechanism. Note, however, that only long bursts have been analyzed here, and short bursts may belong to a distinct class, as indicated by their duration distribution (Kouveliotou et al. 1993). The typical timescale of $\sim0.5$ s that is observed in the PDSs of long GRBs may be connected with the separate class of short, $\sim1$ s, bursts.

Within the accuracy of measurement ($\sim1\%$), the slope of the average spectrum equals $-5/3$. The Kolmogorov spectrum of velocity fluctuations in a turbulent medium has the same slope. Is it just a coincidence or is it an indication that bursts are somehow related to turbulence? In the cosmological scenario, the GRB emission is generated in a relativistic outflow from a central source (Rees & Mészáros 1994), which is likely to result from the coalescence of two neutron stars or a neutron star and a black hole (Paczynski 1986; Piran, Shemi, & Narayan 1993). Turbulence develops easily in a supersonic outflow and is known to occur, e.g., in the solar wind (Coleman 1968). Fluctuations of the velocity and magnetic field in the solar wind have PDSs with slopes that equal $-5/3$, which indicates the presence of developed turbulence.

If GRBs are due to the emission from bright blobs in a relativistic outflow, and if the observed radiation is highly sensitive to the direction of the blob velocity, then a fluctuation of the velocity might “switch off” or “switch on” the blob in the observed light curve. This is only one of the possible speculations. A relation between GRB light curves and turbulence remains a conjecture until it is supported by a specific emission model. An additional piece of information, which can be used in developing a model, is the 2 Hz break in the average PDS. The break implies that pulses shorter than $\sim0.5$ s are suppressed in the light curve for some physical reason. The deficit of short pulses has also been demonstrated directly by analyzing the pulse structure of the light curves (Norris et al. 1996). Note that the sharpness of the break supports the assumption of similar conditions in different GRBs. For example, if the Lorentz factor of the emitting gas varied strongly from burst to burst, we would observe a smooth turnover instead of the sharp break.

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