Stückelberg model and Composite $Z'$

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Based on Ref. [1], we study a composite $Z'$ model which effectively induces the Stückelberg model in low energy. It turns out that the mass of the composite $Z'$ boson contains the Stückelberg mass term in sharp contrast to the conventional $Z'$ model. We also find that the masses of the composite scalar and the right-handed neutrinos are determined by the infrared fixed points. If future experiments confirm that the gauge coupling $g$ of $Z'$ is sufficiently large, say, $g^2/(4\pi) \gtrsim 0.015$ for the $U(1)_{B-L}$ model, and also establish the existence of the Stückelberg mass term for $Z'$, it might be evidence of the compositeness of $Z'$.

I. INTRODUCTION

The standard model (SM) was almost confirmed by the discovery of the Higgs boson [2]. It turns out that the perturbation theory works up to high energy near the Planck scale. Is there a room for strongly interacting theories such as (walking) Technicolor, top condensate and other models [3, 4]? We here explore a possibility of a composite $Z'$ which effectively induces the Stückelberg model in low energy [5, 6]. If the strong coupling region is around the Planck or the GUT scale, a big $U(1)$ gauge coupling $g$ is not necessarily needed in low energy. For the $U(1)_{B-L}$ model, $g^2/(4\pi) \gtrsim 0.015$ is sufficient. We find that the masses of the extra scalar and the right-handed neutrino are controlled by the infrared fixed points. In sharp contrast to the conventional $U(1)_{B-L}$ model, the $Z'$ mass inevitably has the contribution of the Stückelberg mass term in the composite $Z'$ model. This extra contribution to the $Z'$ mass might be the remnant of the strong dynamics in high energy.

II. STÜCKELBERG MODEL AND COMPOSITE VECTOR BOSON

Let us start from a model with a Majorana-type scalar four-fermion coupling and a vector one:

$$\mathcal{L} = \bar{\eta} \gamma_i \eta \eta c + G_{S}(\eta c\eta c) - G_{V}(\eta c\eta c)\eta c,$$

where $\eta$ is a two-component fermion, for example, a right-handed neutrino, and $\eta c$ is the charge conjugation. By introducing composite scalar and vector fields, $\phi \sim \eta c\eta c$, $\phi^\dagger \sim \eta c\eta c$, and $A_\mu \sim \eta c\eta c$, we can rewrite the theory in terms of the system of the fermion, and the composite scalar and vector bosons.

In low energy, the composite scalar and vector fields acquire the kinetic terms via the bubble diagrams. Then the induced effective theory in a low energy scale $\mu$ is

$$\mathcal{L}_{\text{eff}} = \bar{\eta} i \gamma_i \eta c + D_\mu \phi |D_\mu \phi|^2 - M_\phi^2 \phi^\dagger \phi - \lambda_\phi (\phi^3)^2 - \frac{Z_\phi}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} f^2 A_\mu^2,$$

where $D_\mu \eta \equiv \partial_\mu \eta - i A_\mu \eta$, $D_\mu \phi \equiv \partial_\mu \phi + 2i A_\mu \phi$, and the scalar quartic coupling $\lambda_\phi$ is also induced by the bubble diagram. The wave function renormalization constants are

$$Z_\phi = \frac{1}{16\pi^2} \log \Lambda^2/\mu^2, \quad Z_A = \frac{1}{24\pi^2} \log \Lambda^2/\mu^2,$$

where we used the proper time regularization. Introducing $g \equiv Z_A^{-1/2}$ and $y \equiv Z_\phi^{-1/2}$, and rescaling $A_\mu$ and $\phi$ as $A_\mu \rightarrow g A_\mu$ and $\phi \rightarrow y \phi$, respectively, the effective theory has the canonical kinetic terms,

$$\mathcal{L}_{\text{eff}} = \bar{\eta} i \gamma_i \eta c + |D_\mu \phi|^2 - M_\phi^2 \phi^\dagger \phi - \lambda_\phi (\phi^3)^2 - \frac{y f}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} g^2 f^2 A_\mu^2.$$

The field-dependent rotations for the fermion and scalar variables,

$$\varphi \equiv e^{i \frac{B(x)}{\sqrt{2}} \eta}, \quad \bar{\varphi} \equiv e^{-i \frac{B(x)}{\sqrt{2}} \eta c}, \quad \chi \equiv e^{-2i \frac{B(x)}{\sqrt{2}} \phi}, \quad \chi^\dagger \equiv e^{2i \frac{B(x)}{\sqrt{2}} \phi^\dagger},$$

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and the redefinition of the gauge field \( \tilde{A}_\mu \equiv A_\mu + \frac{1}{2g} \partial_\mu B \) yield

\[
\mathcal{L}_{\text{eff}} = \bar{\varphi} (i \partial + g \tilde{A}) \varphi + |(\partial_\mu + 2i g \tilde{A}_\mu) \chi|^2 - \tilde{M}^2 \chi^+ \chi - \lambda (\chi^+ \chi)^2 - y \varphi \varphi \chi - y \varphi \varphi \chi^+ \\
- \frac{1}{4} F_{\mu \nu} F^{\mu \nu} + \frac{1}{2} g^2 f^2 \left( \tilde{A}_\mu - \frac{1}{g} \partial_\mu B \right)^2 .
\]

(6)

It is nothing but the Stückelberg model with the complex scalar field \( \tilde{A} \). Since we introduced the redundant field \( B(x) \), we should add a delta function \( \delta(\xi_B - 1) \) with \( \xi_B \equiv e^{\frac{B(x)}{m_B}} \) in the path integral, which is connected with the gauge fixing term. Note that we cannot avoid quadratically fine-tuning to the mass terms of the composite scalar and vector fields in this Nambu-Jona-Lasinio (NJL) picture.

In this way, by introducing the Stückelberg scalar field \( B(x) \) as in Eq. (6), the original global \( U(1) \) symmetry in Eq. (1) is upgraded to the local one in Eq. (6). We also find that the Stückelberg model as a low energy effective theory corresponds to the composite model in a high energy scale \( \Lambda \), when we impose the compositeness conditions in the context of the renormalization group equations (RGE’s).

\[
\frac{1}{g^2(\Lambda)} = \frac{1}{y^2(\Lambda)} = 0, \quad \frac{\lambda(\Lambda)}{y^4(\Lambda)} = 0 .
\]

(7)

III. COMPOSITE \( Z' \) MODEL

Let us study the \( U(1)_{B-L} \) extension of the SM:

\[
\mathcal{L} = \mathcal{L}_{\text{SM}} + \mathcal{L}_\nu + \mathcal{L}_\chi + \mathcal{L}_{Z'} + \mathcal{L}_{\text{gf}},
\]

where \( \mathcal{L}_{\text{SM}} \) represents the SM part, and

\[
\mathcal{L}_\nu = \sum_{\gamma=1,2,3} \bar{\nu} \gamma_i R \nu_i R
\]

(9)

\[
\mathcal{L}_\chi = |D_\mu \chi|^2 - M^2 \chi^+ \chi - \lambda \chi^+ \chi^2 - \lambda_H |H|^2 |\chi|^2 - Y_{jk} \bar{\nu}_R^j c \nu_R^k \chi - Y_{jk} \bar{\nu}_R^j c \nu_R^k \chi^+, \quad (10)
\]

\[
\mathcal{L}_{Z'} = - \frac{1}{4} F_{\mu \nu} F^{\mu \nu} + \frac{1}{2} g^2 f^2 \left( A_\mu - \frac{1}{g} \partial_\mu B \right)^2 .
\]

(11)

The SM Higgs doublet and the gauge fixing term are denoted by \( H \) and \( \mathcal{L}_{\text{gf}} \), respectively. The \( U(1) \) part of the covariant derivative is

\[
D_\mu = \partial_\mu - ig Q_Y g_Y Y_\mu + g A_\mu - ig Q_{B-L} A_\mu,
\]

(12)

where \( Q_Y \) and \( Q_{B-L} \) represent the hypercharge and the \( B - L \) charge, respectively. The \( U(1)_Y \) and \( U(1)_{B-L} \) gauge couplings are \( g_Y \) and \( g \), respectively. Although the gauge mixing coupling \( g \) appears in general, we set \( g(\Lambda) = 0 \), because there is no gauge kinetic mixing term at the compositeness scale \( \Lambda \). Noting that the operator \( |H|^2 |\chi|^2 \) has a higher dimension than six at the compositeness scale \( \Lambda \), we may neglect the scalar quartic mixing \( \lambda_H \) at \( \Lambda \); i.e., we also set \( \lambda_H(\Lambda) = 0 \).

In Eq. (12), the Stückelberg mass term is incorporated from the beginning unlike the conventional \( Z' \) model. The existence of this term is essential in our formalism of the composite vector field.

The full set of the RGE’s for the \( U(1)_{B-L} \) model is shown in Refs. [5, 6].

The key points of the RGE’s for the gauge and Yukawa couplings are

\[
\beta_g \equiv \frac{\partial}{\partial \mu} g = \frac{a}{16\pi^2} g^3 , \quad (13)
\]

\[
\beta_y \equiv \frac{\partial}{\partial \mu} y = \frac{y}{16\pi^2} \left[ by^2 - cg^2 \right] , \quad (14)
\]
with $a = 12$, $b = 10$, and $c = 6$, where we took $Y_{jk} = \text{diag}(y, y, y)$. The compositeness conditions, $1/g^2(\Lambda) = 1/y^2(\Lambda) = 0$, yield

$$\frac{1}{g^2(\mu)} = \frac{a}{8\pi^2} \ln \frac{\Lambda}{\mu}, \quad \frac{1}{y^2(\mu)} = \frac{b}{a + c} \frac{1}{g^2(\mu)},$$

where $\Lambda$ is the compositeness scale. Note that the solution (15) corresponds to the infrared fixed point. In fact, we can easily rewrite the RGE’s as follows:

$$(8\pi^2) \mu \frac{\partial}{\partial \mu} \left( \frac{y^2}{g^2} \right) = b g^2 \frac{y^2}{g^2} \left( \frac{y^2}{g^2} - \frac{a + c}{b} \right),$$

which is similar to the Pendleton–Ross type [10]. Strictly speaking, the asymptotic free theory ($a < 0$) was studied in Ref. [10]. Thus the situation $1/g^2(\Lambda) \rightarrow 0$ occurs in low energy unlike in the asymptotic nonfree theory ($a > 0$). Owing to the nature of the infrared fixed point, even if we relax the compositeness conditions to the nonvanishing ones, $1/g^2(\Lambda), 1/y^2(\Lambda) \ll 1$, the RG flows are not changed so much.

The RGE for $\lambda_\chi$ is a bit complicated:

$$\beta_{\lambda_\chi} \equiv \mu \frac{\partial}{\partial \mu} \lambda_\chi = \frac{1}{16\pi^2} \left[ 20\lambda_\chi^2 + \lambda_\chi (24y^2 - 48g^2) - 48y^4 + 96g^4 \right],$$

where we ignored the numerically irrelevant $\lambda_\chi^2 n_t$ term. Substituting the solutions (15) for $g$ and $y$, we obtain

$$(16\pi^2) \mu \frac{\partial}{\partial \mu} \left( \frac{\lambda_\chi}{g^2} \right) = 20g^2 \left( \frac{\lambda_\chi}{g^2} - k_+ \right) \left( \frac{\lambda_\chi}{g^2} - k_- \right),$$

where $k_+ \equiv \frac{2}{25} (9 + \sqrt{546}) \simeq 2.589$ and $k_- \equiv \frac{2}{25} (9 - \sqrt{546}) \simeq -1.149$. Thus the solution $\lambda_\chi/g^2 = k_+$ is an infrared fixed point. We can confirm that the analytical expression of the solution for $\lambda_\chi$ with the compositeness condition, $\lambda_\chi(\Lambda)/g^4(\Lambda) = 0$, is actually

$$\lambda_\chi(\mu) = \frac{2}{25} (9 + \sqrt{546}) g^2(\mu),$$

where we assumed positivity of $\lambda_\chi$ in any scale.

The Majorana Yukawa couplings and the quartic coupling of the extra composite scalar are proportional to the $U(1)$ gauge coupling and the coefficients are determined through the infrared fixed points. As a result, the mass ratio of $\nu_R$ and $\chi$ is controlled by the nature of the infrared fixed point.

Let us take the vacuum expectation value (VEV) of $\chi$ as $\langle v_\chi \rangle = v_\chi/\sqrt{2}$. Then the square of the masses of $\nu_R$, $\chi$ and $Z'$ are

$$M_{\nu_R}^2 \simeq 2g^2 v_\chi^2, \quad M_\chi^2 \simeq 2\lambda_\chi v_\chi, \quad M_{Z'}^2 \simeq 4g^2 v_\chi^2 + g^2 f^2.$$

We thereby find the mass relation between $\nu_R$ and $\chi$ as

$$\frac{M_\chi}{M_{\nu_R}} = \frac{\sqrt{\lambda_\chi}}{g} = \frac{\sqrt[5]{2(9 + \sqrt{546})}}{\sqrt[5]{5}} \approx 1.2,$$

owing to the nature of the infrared fixed points. In sharp contrast to the conventional approach for $Z'$, we have the contribution of the St"uckelberg mass to $M_{Z'}$,

$$\Delta \equiv \frac{M_{Z'}^2}{g^2} - 4v_\chi^2 = f^2 > 0.$$

If the experiments such as LHC and ILC observe $\Delta > 0$ and confirm $g^2/(4\pi) \gtrsim 0.015$, it implies the compositeness of $Z'$. 
IV. SUMMARY AND DISCUSSIONS

We investigated the possibility of the composite $Z'$. We showed that the NJL model effectively induces the St"uckelberg model in low energy via the fermion bubble diagrams. In terms of the RGE’s, this correspondence is realized by the compositeness conditions. We also showed that the RG flows are essentially controlled by the infrared fixed points. The nature of the infrared fixed points gives the mass ratio, $M_\chi/M_{\nu_R} = \sqrt{\lambda_\chi/y} \approx 1.2$.

In the composite $Z'$ model, there are two contributions to the $Z'$ mass: First is the VEV of $\chi$, which is the conventional one, and the second is the St"uckelberg mass term. If $g^2/(4\pi) \gtrsim 0.015$ is confirmed and also the existence of this extra mass term, $\Delta \equiv M_{Z'}^2/g^2 - 4v_\chi^2 > 0$, is established in future experiments [11], it will be an evidence of the strong dynamics in high energy.

The scenario that the composite $Z'$ boson generated around the Planck or the GUT scale survives in low energy, of course, suffers from the naturalness problem. On the other hand, we may consider a scenario that the masses of $Z'$, $\chi$, and $\nu_R$ are not so far below the compositeness scale $\Lambda$. In this case, the seesaw mechanism [12] can work. We here point out that the SM Higgs potential can be stabilized by the tree level shift of the Higgs quartic coupling essentially generated by the $Z'$ loop contribution [13]. We will study such a scenario elsewhere.

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