An effective theory of Feshbach resonances and many-body properties of Fermi gases

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For calculating low-energy properties of a dilute gas of atoms interacting via a Feshbach resonance, we develop an effective theory in which the parameters that enter are an atom-molecule coupling strength and the magnetic moment of the molecular resonance. We demonstrate that for resonances in the fermionic systems \textsuperscript{6}Li and \textsuperscript{40}K that are under experimental investigation, the coupling is so strong that many-body effects are appreciable even when the resonance lies at an energy large compared with the Fermi energy. We calculate a number of many-body effects, including the effective mass and the lifetime of atomic quasiparticles in the gas.

Due to the enormous progress made in trapping and cooling atomic gases, it is now possible to create degenerate Fermi gases\textsuperscript{1,2}. The fact that interactions between atoms can be varied essentially at will by exploiting Feshbach gases\textsuperscript{1,2}, the fact that interactions between atoms can be varied essentially at will by exploiting Feshbach resonances makes these systems candidates for exploring experimentally strongly correlated Fermi systems, in which the magnitude of the scattering length \(a\) is larger than the inter-particle separation. Many aspects of such systems have been considered, among them the equation of state\textsuperscript{3} and the cross-over between the BCS state and a Bose-Einstein condensate\textsuperscript{4,5}.

In this paper we begin by showing how, in the spirit of Landau Fermi-liquid theory and effective field theories, low energy Feshbach resonances may be characterized by a small number of phenomenological parameters. Related ideas have been proposed in Refs.\textsuperscript{4,6}, and here we provide a compact quantitative formulation. To do this we give a field-theoretic account of a Feshbach resonance in a two-atom system which may readily be generalized to the many-body problem. Next we examine the conditions under which the Feshbach resonance induces strong correlations in a gas of fermionic atoms. Finally, we calculate a number of many-body properties.

The first step in formulating an effective low-energy theory is to identify the relevant degrees of freedom. These we shall take to be two species \((\sigma = \uparrow, \downarrow)\) of atoms both of mass \(m\), and a single low-lying molecular state whose energy relative to that of two stationary atoms in the open channel will be denoted by \(2\nu\). It is important to remark that this molecular state consists partly of a "bare" molecule in a closed channel (or channels), together with a dressing cloud of high-energy atom pairs in the open channel. Dressing of the molecular state by low-momentum pairs, which gives rise to threshold behavior, will be calculated explicitly. In addition, the matrix element \(g\) describing coupling of atoms to the molecular state is different from the bare coupling. As an example, we calculate the width parameter \(\Delta B\) which enters the standard expression \(\alpha(B) = a_{\text{bg}}[1 - \Delta B/(B - B_0)]\) for the scattering length \(a\) as a function of magnetic field \(B\) near a Feshbach resonance. Here \(a_{\text{bg}}\) is the background scattering length due to non-resonant processes. The resonant contribution to the effective atom-atom interaction at zero energy is \(-g^2/(2\nu)\). This in turn is equal to \(4\pi\hbar^2a_{\text{res}}/m\), where \(a_{\text{res}}\) is the resonant contribution to the scattering length, from which it follows that

\[
\alpha(B) = a_{\text{bg}}\Delta B = \frac{m}{4\pi\hbar^2} \frac{g^2}{\Delta\mu}.
\]

Here \(\Delta\mu = 2\partial\nu/\partial B\) is the difference in magnetic moments between two atoms and the molecule. The result\textsuperscript{6} is independent of the detailed microscopic model, provided there is only one low-lying molecular state.

We now demonstrate how the parameters \(2\nu\) and \(g\) entering the low-energy theory are related to bare quantities for an explicit microscopic model, with the Hamiltonian

\[
\hat{H} = \sum_{\mathbf{k},\sigma} \epsilon_{\mathbf{k}} \hat{a}_{\mathbf{k}\sigma}^{\dagger} \hat{a}_{\mathbf{k}\sigma} + \sum_{\mathbf{p},\mathbf{k}} g_{\text{bare}}(\mathbf{k}) \sqrt{V} \hat{a}_{\mathbf{p}/2+k\uparrow}^{\dagger} \hat{a}_{\mathbf{p}/2-k\downarrow} + h.c
\]

\[+ \sum_{\mathbf{p}} E_{\text{bare}} \hat{b}_{\mathbf{p}} \hat{b}_{\mathbf{p}} + \sum_{\mathbf{k},\mathbf{q},\mathbf{q}'} V(\mathbf{q},\mathbf{q}') \hat{a}_{\mathbf{k}+\mathbf{q}}^{\dagger} \hat{a}_{\mathbf{k}-\mathbf{q}} \hat{a}_{\mathbf{k}+\mathbf{q}'} \hat{a}_{\mathbf{k}}.\]

Here \(E_{\text{bare}} = 2\nu_{\text{bare}} + p^2/4m\), where \(2\nu_{\text{bare}}\) is the energy of a bare molecule with momentum zero measured with respect to the energy of a pair of atoms at rest in the open channel, \(\epsilon_{\mathbf{k}} = k^2/2m\) is the kinetic energy of an atom, \(g_{\text{bare}}(\mathbf{k})\) is the bare molecule-atom coupling matrix element, \(V\) is the non-resonant interaction between atoms, and \(V\) is the volume of the system. The quantity \(2\nu_{\text{bare}}\) cannot be measured directly because the energy of the molecule is affected by coupling to the open channel, so we eliminate it in favor of \(\nu\), the energy of the dressed molecule. The energy of a molecule at rest in a vacuum is given by the poles of the molecule propagator

\[
D_{\nu_{\text{vac}}} = |\omega - 2\nu_{\text{bare}} - \Pi_{\text{vac}}(\omega)|^{-1},
\]

where in a compact matrix notation the vacuum molecule self energy has the form

\[
\Pi_{\text{vac}}(\omega) = \frac{i\nu_{\text{bare}}G_{\nu_{\text{vac}}}G_{\nu_{\text{vac}}} G_{\nu_{\text{vac}}} G_{\nu_{\text{vac}}}}{\omega - k^2/2m - i\delta}
\]

\[+ \sum_{\mathbf{k}} \frac{V(\nu,\nu)}{\nu} \hat{a}_{\mathbf{k}+\mathbf{q}}^{\dagger} \hat{a}_{\mathbf{k}-\mathbf{q}} \hat{a}_{\mathbf{k}+\mathbf{q}'} \hat{a}_{\mathbf{k}}.\]

We separate out the threshold behavior by writing
\( \Pi_{\text{vac}}(\omega) - \Pi_{\text{vac}}(0) = g_{\text{vac},0}^2 m^{3/2} \sqrt{-\omega} / (4\pi) + \Delta \Pi_{\text{vac}}(\omega), \)

\[ \Delta \Pi_{\text{vac}}(\omega) = m^2 \omega \int_0^{\infty} \frac{dk}{2\pi^2} \frac{g_{\text{vac}}(k,\omega)g_{\text{vac}}(k,0) - g_{\text{vac},0}^2}{m\omega - k^2}, \]

and \( g_{\text{vac},0} \equiv g_{\text{vac}}(0,0) \). The \( \sqrt{-\omega} \) term is due to the threshold in the density of states for the open channel at \( \omega = 0 \). For positive energy it gives rise to damping of the molecular resonance, while for \( \omega < 0 \) it results in the binding energy of the molecular state varying as \( 1/\omega^2 \) close to the resonance. We shall assume that frequencies of interest are small compared with the characteristic frequencies that enter the coupling matrix elements \( g_{\text{vac}}(k,\omega) \). One then finds \( D_{\text{vac}}^{-1}(\omega) = \omega/\tilde{z} - 2\nu_{\text{bare}} - \Pi_{\text{vac}}(0) - g_{\text{vac},0}^2 m^{3/2} \sqrt{-\omega(4\pi)^{-1}}, \) where \( \tilde{z}^{-1} = 1 - \partial \Delta \Pi_{\text{vac}}(\omega)/\partial \omega \). Physically, \( \tilde{z} \) is the renormalization factor for the molecular resonance when low-energy atom pairs in the open channel are neglected. This shows that the resonance energy, which is defined by \( \text{Re}[D_{\text{vac}}^{-1}(2\nu)] = 0 \), is given by

\[ 2\nu = \tilde{z}[2\nu_{\text{bare}} + \Pi_{\text{vac}}(0)] \quad (2) \]

for \( 2\nu_{\text{bare}} + \Pi_{\text{vac}}(0) > 0 \). Thus, the energy of the resonance is shifted, and the difference between the magnetic moments of the two atoms in the open channel relative to that of the molecule is \( \Delta \mu = 2\nu/\partial B = \tilde{z} \Delta \mu_{\text{bare}}, \Delta \mu_{\text{bare}} = 2\nu_{\text{bare}}/\partial B \) being the difference in magnetic moments between two atoms in the open channel and the bare molecular state. The energy of the molecular state is given for small \( B - B_0 \) by \( 2\nu = \Delta \mu(B - B_0) \). For negative \( \omega \) one finds \( 1 - \partial \Pi_{\text{vac}}(\omega)/\partial \omega = \tilde{z}^{-1} + g_{\text{vac},0}^2 m^{3/2} / (8\pi|\omega|^{1/2}) \). The singularity for \( \omega \to 0 \) reflects the fact that the wave function of a very weakly bound molecular state consists mainly of low-energy pairs of atoms in the open channel, with only a very small closed-channel component.

According to standard theory, the resonant contribution \( a_{\text{res}} \) to the scattering length is \( 4\pi a_{\text{res}} / m = g_{\text{vac},0}^2 D_{\text{vac}}(\omega = 0) = -g_{\text{vac},0}^2 / [2\nu_{\text{bare}} + \Pi_{\text{vac}}(0)] \). With the help of Eq. 2, this may be written as \(-\tilde{z}^{-1}g_{\text{vac},0}(2\nu)/2\nu \), in agreement with Eq. 1 if the effective coupling between atoms and a dressed molecule is given by

\[ g = \sqrt{\tilde{z}} g_{\text{vac},0}. \quad (3) \]

The discussion so far applies equally well for bosons and fermions, apart from additional statistical factors if the two atomic species are identical.

As an application, we consider in the rest of the paper a two-component Fermi gas. We assume that the populations of the atomic species are equal and denote the total density by \( n \). A dimensionless measure of the importance of the many-body effects due to the resonant interaction is \( k_F a_{\text{res}} \), where \( k_F \) is the Fermi momentum, and we now estimate this quantity for systems of experimental interest. Feshbach resonances for fermionic atoms have been reported for \( ^{40}\text{K} \) and \(^{6}\text{Li} \). For \( ^{40}\text{K} \), there is a Feshbach resonance for atoms in the states \( |9/2, -9/2\rangle \) and \( |9/2, -7/2\rangle \) with parameters \( B_0 \approx 201.5 \text{ G}, \Delta B \approx 8 \text{ G}, \) and \( a_{bg} = 164a_0 \). The only other state in the ground-state manifold to which the open channel can couple by the central part of the interaction is \( |9/2, -9/2\rangle \) and \( |7/2, -7/2\rangle \). Diagonalization of the hyperfine Hamiltonian yields \( \Delta \mu_{\text{bare}} = 1.78 \mu_B \) for \( B \approx 200 \text{ G} \) whereas a coupled-channel calculation gives \( \Delta \mu = 0.118 \mu_B \) for the molecule dressed only by high-energy pairs. From this, we obtain \( \Delta \mu/\Delta \mu_{\text{bare}} = 0.067 \) for \( ^{40}\text{K} \), which demonstrates that the renormalization of the molecular energy due to the coupling to the open channels is significant even away from threshold. With these parameters we find \( k_F|a_{\text{res}}| \approx 15 n_{12}^{-1/3} \epsilon_F/ \nu_{\text{bare}} \), the many-body contribution to the effective interaction between atoms in the open channel is given by \( g^2/|\omega - 2\nu| \). Since in calculations of many-body effects typical energies are of order \( \epsilon_F \), this interaction may be treated as being independent of frequency provided \( 2\nu \gg \epsilon_F \). Our discussion shows that even when this condition is satisfied, systems can be strongly coupled with \( k_F|a_{\text{res}}| \gg 1 \).

The next question we address is how the molecular properties are modified by the presence of other atoms. Such effects have recently been considered by Combescot. The molecular propagator in a medium is given by

\[ D(p,\omega) = \omega - E_p + 2\mu - \Pi(p,\omega) = D_0(p,\omega)^{-1} - \Pi(p,\omega) \]

where \( \Pi(p,\omega) \) is the many-body self energy and \( 2\mu \) is the chemical potential of a molecule. As a simple approximation for \( \Pi \) we take the same processes as in the calculation of \( \Pi_{\text{vac}} \) except that the atom propagator is that in the presence of a medium, rather than that in a vacuum.

This yields

\[ \Pi(p,\omega) = \nu_{\text{bare}} G_{\text{mb}}^{(2)}(p,\omega) g_{\text{mb}}(p,\omega) \]

where \( G_{\text{mb}}^{(2)}(p,\omega) = \nu_F(\xi_F + 2k_F) + \nu_S(\xi_S + 2k_S) - 1/(\xi_S + 2k_S - \omega - i\delta) \)

with \( \xi_S = k_S - \mu \), describes the propagation of a pair of atoms with total momentum \( p \), energy \( \omega \), and relative momentum \( 2k \) in the presence of the other atoms. Here

\[ \nu_F(x) = \exp(\beta x) / [\exp(\beta x) + 1]^{-1} \]

with \( \beta = k_B T \). The many-body atom-molecule interaction in the ladder approximation is given by

\[ g_{\text{mb}}(p,\omega) = g_{\text{vac}}(\omega) [1 - \Delta G_{\text{mb}}(T_{bg}(\omega))]^{-1} \]

with \( \Delta G_{\text{mb}} = G_{\text{mb}}^{(2)}(p,\omega) - G_{\text{vac}}(\omega) \) and \( \omega = \omega + 2\mu - p^2/4m \).

To clarify, we ignore such correction terms in the following discussion of the molecular properties assuming \( k_F a_{bg} \ll 1 \) and thus \( g_{\text{mb}}(p,\omega) \approx g_{\text{vac}}(\omega) \). Expressing \( \Pi(p,\omega) \) in terms of \( \Pi_{\text{vac}}(p,\omega) \) we obtain

\[ \tilde{z} D^{-1}(p,\omega) = \omega - E_p - g^2 m^{3/2} \Theta(\omega) / 4\pi \approx -\Pi(p,\omega). \quad (4) \]

Here \( E_p = p^2/4m + 2\nu - 2\mu \) is the difference between the
molecule energy and the chemical potential when dressing by low-energy atoms is neglected. The term
\[ \tilde{\Pi}(p, \omega) = \frac{1}{g^2} \int \frac{d^3 k}{(2\pi)^3} \left[ G_{\text{mb}}^{(2)}(p, k, \omega) - G_{\text{vac}}(k, \omega) \right] \] (5)
gives the contributions to the self energy coming from the presence of the atoms. In calculating \( \tilde{\Pi} \) we have replaced \( \sqrt{z}g_{\text{vac}}(k, \omega) \) by \( g = \sqrt{z}g_{\text{vac},0} \) since contributions from high momentum states, where the difference between \( g_{\text{vac}}(k, \omega) \) and \( g_{\text{vac},0} \) is appreciable, are cut off by the Fermi functions. This approximation will be used in the rest of the paper. The importance of Eqs. (4) and (5) is that they show explicitly how molecular properties at low energies may be expressed solely in terms of experimentally measurable quantities \( g \) and \( \Delta \mu \), without arbitrary assumptions about the behavior of the bare coupling matrix elements for high momenta.

We turn now to properties of atoms described by the propagator \( G(p, \omega) \) given in the ladder approximation by \( \Sigma(p, \omega) \) with \( G_0(p, \omega)^{-1} = \omega - \epsilon_p \). The atom self energy is given in the ladder approximation by \( \Sigma(p, \omega) \) where \( \omega_{\text{m}} = (2m + 1)\pi k_B T \) and integration over \( k \) [14]. The matrix \( \Gamma(p, \omega, k') \) obeys the equation \( \Gamma(p, \omega) = V_{\text{eff}}(p, \omega) + V_{\text{eff}}(p, \omega)G_{\text{mb}}^{(2)}(p, \omega)\Gamma(p, \omega) + V_{\text{eff}}(p, \omega) = V + g_{\text{bare}}D_{\text{h}}(p, \omega)g_{\text{bare}} \). After some manipulations we can express \( \Sigma(p, \omega) \) as
\[ \Sigma(p, \omega) = \Sigma_{\text{bg}}(p, \omega) + \int \frac{d^3 k}{(2\pi)^3} g_{\text{mb}}(k + p, -\frac{k - p}{2}, \omega)^2 \times \int \frac{d \epsilon}{2\pi} \left[ n_F(\epsilon)A_k(\epsilon, \omega)D(k + p, \epsilon + \omega) - n_B(\epsilon + \omega)A_m(k, \epsilon + \omega) \right] \] (6)
with \( n_B(x) = [\exp(\beta x) - 1]^{-1} \) and \( A_m(k, \omega) = -2\text{Im}D(k, \omega) \) and \( A_k(k, \omega) = -2\text{Im}G(k, \omega) \). Using the low energy approximations \( T_{\text{vac}} = 4\pi a_{\text{bg}}/m \) and \( g_{\text{vac}}(k, \omega) = g_{\text{vac},0} \), we obtain
\[ \frac{g_{\text{mb}}(p, k, \omega)}{g_{\text{vac},0}} = 1 + \frac{d^3 q}{(2\pi)^3} \Delta G_{\text{mb}}(p, q, \omega)T_{\text{bg}} \] (7)
For \( T \to 0 \), the self energy due to the background scattering \( V \) is given in the ladder approximation by \( \Sigma_{\text{bg}}(p, \omega) = \frac{2k_F^2}{3(3\pi^2)}|k_{\text{res}}|_{p}a_{\text{res}} + O(k_{\text{res}}a_{\text{res}})^2 \) [12]. Since \( D \) is given by Eqs. (4) and (5), one sees from Eqs. (6) and (7) that the atom self energy may be expressed in terms of \( g \) and \( \nu \) (or equivalently \( \Delta \mu \) and \( B_0 \)).

From \( \Sigma \) one can calculate the properties of the atom-like excitations given by the poles of \( G \). We begin by describing results for weak coupling, i.e. \( k_F|a_{\text{res}}| < 1 \). For \( 2\nu \gg \epsilon_F \) and \( T = 0 \), one finds that the effective mass of an excitation at the Fermi surface is given by
\[ m^* = m + \frac{g^2}{16\nu} + (21 \ln 2 - 3)(T_{\text{vac}} - g^2/2\nu)^2 n^2 \] (8)
and the wave function renormalization factor \( Z \) by
\[ Z^{-1}(p) = 1 + \frac{g^2}{8\nu^2} + \frac{9}{8} \ln \frac{T_{\text{vac}} - g^2/2\nu}{\epsilon_F^2} n^2 \] (9)
The second terms in Eqs. (8) and (9) come from the frequency dependence of the molecule propagator, while the final ones are identical with Galitskii’s results for a dilute Fermi gas with short-range interactions parametrized by a scattering length \( a = a_{\text{bg}} + a_{\text{res}} \). If one neglects \( a_{\text{bg}} \), the last two terms in the expression for \( m^* \) have the same dependence on \( \nu \), and the last term is \( \approx 2k_F|a_{\text{res}}|/\epsilon_F \) times the second one. Thus for small \( \gamma \), the second term dominates over the last one while, for the resonances in \( ^6\text{Li} \) and \( ^{40}\text{K} \) considered above, the last term is the more important at densities of experimental interest. For example, for the \( ^{40}\text{K} \) resonance, the ratio is \( \sim 30n_{T,2}^{-1/3} \), which demonstrates that the effective interaction is equivalent to a contact interaction between atoms of strength \( g^2/2\nu \), provided that \( \nu \gg \epsilon_F \). Note that for the resonances in \( ^6\text{Li} \) and \( ^{40}\text{K} \) considered in this paper, one can have \( k_F|a| \gtrsim 1 \) even when \( \nu \gg \epsilon_F \). Thus, the gas can be strongly interacting due to the Feshbach resonance while the interaction still can be regarded as effectively instantaneous with a scattering length \( a \), in agreement with the earlier order of magnitude estimates.

The imaginary part of the atom self energy may be calculated in a similar manner. There is a contribution of order \( g^2 \) coming from creation of real molecules. This is given for \( T = 0 \) by \( \text{Im}[\Sigma(p, \epsilon_F)] = 8k_F|a_{\text{res}}|\nu^{3/2}/\epsilon_F^2 \times \{1 - \sqrt{12288\nu^2 + \epsilon_F^2} - 1/(2\nu) \} \) for \( \sqrt{8\nu^2 - 2\epsilon_F} \leq \nu \leq \sqrt{8\nu^2 + 2\epsilon_F} \) and zero otherwise. For \( 2\nu \gg \epsilon_F \), this on-shell process is unimportant for atoms with \( p \lesssim O(k_F) \). For momenta outside the range for which the above process is allowed, damping is due to creation of particle-hole pairs. One finds for \( |p - k_F|/k_F \ll 1 \) and \( T \ll T_F \) to lowest non-trivial order in \( a_{\text{bg}} \) and \( g \) that
\[ \text{Im}[\Sigma(p, \epsilon_F)] = -\frac{2}{\pi} (k_F a)^2 \left[ 1 - \frac{p}{k_F} \right]^2 + \frac{\pi k_F T_F}{\epsilon_F} \] (10)
where again \( a = a_{\text{bg}} + a_{\text{res}} \). For \( T = 0 \), this result agrees with the expression derived by Galitskii for a gas of particles interacting via a short range interaction with scattering length \( a \) [12].

We turn now to stronger couplings, and we shall neglect \( a_{\text{bg}} \) compared with \( a_{\text{res}} \). The important ingredient in Eq. (6) is the molecule spectral function
\[ A_m(p, \omega) = \frac{2\text{Im} \tilde{\Pi}(p, \omega)}{|\omega - E_p - \text{Re}(p, \omega)|^2 + \text{Im}(p, \omega)^2} \] (11)
For calculating properties of the system at \( T \ll T_F \), typical energies of interest are of order \( \epsilon_F \). For such energies, \( \text{Im} \tilde{\Pi} \) is of order \( g^2 m \epsilon_F \), and therefore for large \( \nu \) (\( k_F|a_{\text{res}}| \ll 1 \)), the denominator in Eq. (11) may be replaced by \( (2\nu)^2 \), and one recovers the weak-coupling result [10]. With decreasing molecule energy \( \nu \), the imaginary term becomes increasingly important.
we show numerical results for the quasiparticle damping rate $-\text{Im}\Sigma(p,\xi_p)$ based on Eqs. (6) and (11), for $T = 0$ and a coupling strength $g$ appropriate for the $40\text{K}$ resonance discussed above. For $k_F|\epsilon_{\text{res}}| \gg 1$, the damping rate saturates because the $\Pi$ terms in the denominator dominate. For comparison, we also show the results obtained by neglecting medium effects in the denominator. In a Boltzmann equation approach, this amounts to assuming that the differential cross section for two-

atom scattering with relative momentum $k$ is given by its vacuum expression $a^2_{\epsilon_{\text{res}}}/(1 + k^2 a^2_{\epsilon_{\text{res}}})$, where effective range contributions have been neglected. As Fig. 1 (a) shows, for large $k_F|\epsilon_{\text{res}}|$, the effect of the medium on the scattering amplitude 	extit{increases} the damping rate significantly as compared to the Boltzmann results using the vacuum scattering rate. Remarkably, the scattering rates are substantially greater than one would predict if scattering cross sections were given by the unitarity limit for two particles in a vacuum. This is due primarily to the reduction by Pauli blocking of the magnitude of $\text{Im}\Pi$, and hence also of the magnitude of the scattering amplitude. Figure 1 (b) also shows that the results agree with those of Galitskii for $k_F|\epsilon_{\text{res}}| \ll 1$.

The main results of this paper are, first, that we have shown how to formulate a theory of Feshbach resonances that involves only low-energy observables. One of these quantities is the magnetic moment of the dressed molecule, which is very different from that of the bare molecule in cases of experimental interest. We have also shown that for Fermi gases of experimental interest, many-body effects are strong even when the Feshbach resonance lies at energies well above the Fermi energy, and that for many purposes the interaction induced by the resonance may be regarded as instantaneous. As applications of the theory, we calculated a number of many-body properties of a two-component Fermi gas. One important result is that, for strong interactions, scattering rates can exceed substantially predictions based on unitarity-limited two-body cross sections.

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