Exploring the onset of collective intelligence in self-organised trails of social organisms

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Abstract.
We investigate the emergence of self-organised trails in collective motion of social organisms by means of an agent-based model. We present numerical evidences that an increase in the efficiency of navigation between the target areas, in dependence of the colony size, exists. Moreover, the shift, from the maladaptive to the adaptive behaviour, can be quantitative characterised, identifying and measuring a well defined crossover point. This point corresponds to the minimal number of individuals necessary for the onset of collective intelligence. Its scaling behaviour, as a function of the environment size, is clearly characterised.

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1. Introduction

Collective motion of social organisms is an elegant example of an emergent phenomenon which can produce adaptive and efficient behaviours based on a distributed cooperative intelligence. Among the different structures that this phenomenon can generate \([1, 2]\), one of the more amazing is the self-organisation of trails, both in ants colonies \([4]\), and in human communities \([3]\). Ant trail formation can be generated by means of the deposition of chemical pheromones which enables indirect ants communication through an environmental marking procedure. By means of this mechanism, the ants implement a reinforcement rule which allows to select the shortest path to connect food source with nest location. In this way, an adaptive behaviour based only on local information and interaction is achieved.

This astonishing behaviour has inspired important technological applications. One of the most notorious is a technique for general purpose optimisation \([5]\). A more recent one is the experimental implementation of a navigation strategy for swarms of robots challenged to find a path between two target areas in an unknown environment \([6]\). The solution of this practical problem opened new questions in relation to the scalability of this approach to larger groups and larger environment size. In other words, the characterisation of the behaviour of this strategy of navigation in dependence of the group size and its scaling in dependence of the environment size became a central topic of investigation. An earlier result, related to the problem of the dependence on the community size, was obtained collecting field data from real ant colonies \([7]\). That study shown a general increase in the number of ants walking to the feeder along the trail in relation to colony size and a very simplified mean-field model suggested that a minimum number of ants is required for trail formation to become effective. Inspired by these previous results, the purpose of our work is to study and clearly characterise the nature of these specific aspects of trail formation throughout an accurate numerical analysis of the results produced by a microscopic model which directly generates the trails.

Continuum microscopic models which are able to describe the self-organisation of ants trail formation are well known in the literature \([8]\). These models are related to the general class of active Brownian particle models. The motion of this random particles is determined by a field which is directly influenced by the movements of the particles themselves. This non-linear feedback, which operates between the particles and the generated field at the microscopic level, results in the self-organisation of the trails at the macroscopic level. The use of this formalism allows the introduction of analytical approximations for attaining general results. Otherwise, explicit solutions and simulations of the process are obtained throughout the discretisation and numerical solution of the continuous model. Another approach consider a discretised space and time where the motion of each individual is described by some transition rules. These agent based simulations \([9]\) describe the same process of the continuum models, by means
of these rules, which implement the movements and the deposition of the pheromone.

The use of agent based models is very important, since they allow to understand the role of fluctuations and noise, as well as the limitations and validity of the continuous and the mean field descriptions. In fact, the intrinsic stochasticity produced at the individual level generates an internal noise which, in general, can cause impacting consequences [10, 11]. Moreover, as the central aim of our work is to understand which is the minimum number of ants required for trail formation to become effective, the description of the discrete nature of individuals is essential to characterise threshold and finite size effects. These effects can not be characterised by a continuum description where every small amount of the density of population is acceptable, even if unrealistically small [12].

In the following (Sec. 2), we introduce the details of the agent based model. In order to realise an in deep numerical analysis of the phenomenon, and not just some specific examples of trails formation, we consider a very simplified model. In fact, it counts on a single pheromone and it depends on only two variables. Even so, trail formation is obtained based only on local information and interaction. In Sec. 3 we report the numerical analysis for the characterisation of the efficiency of navigation and for the quantitative description of the shift from the maladaptive to the adaptive behaviour. We would like to stress that the aim of the work is not to identify a classical phase transition, but we are interested in the scaling behaviour for finite-size systems. A discussion of these important points can be found at the end of the paper.

2. The model

The ant colony is composed by a population of \( P \) individuals. They can move on a regular 2-dimensional square lattice with \( L \times L \) sites and periodic boundary conditions. We choose \( L \) odd, with the origin of the coordinate in the centre of the lattice. The nest is located in \((0,0)\) and the food source in \((0,D)\), with \( D = (L - 1)/2 \). The time unit \( t \) is the time interval between two updatings of the positions of all the individuals of the colony.

In the initial state each ant is located at the nest. An individual at site \((x,y)\) can move only on the top site \((x,y+1)\) and on the right \((x+1,y)\) or the left one \((x-1,y)\). Steps towards the bottom site are forbidden. In this way, individuals effectively walk along paths where no loops are allowed. This rule represents the fact that ants have a general orientation mechanism which can determine the north of the direction [13]. We implemented also a model where ants diffuse in all the four lattice directions until they find the nest. In this case, results are equivalent, just a longer transition towards the quasi-stationary state is observed.

When an ant reaches the objective site, which represents the food source, the possible directions of motion change, with all the movements allowed, except the step towards the top site. Moreover, the ant starts to deposit, in the new visited sites, an amount of pheromone \( \phi = \exp[-\gamma(t - t_{\text{act}})] \). Here \( t_{\text{act}} \) is the time when the ant left the
food, and $\gamma$ controls the critical time for an effective deposition. This means that ants can effectively mark their trajectories only for a limited time after they left the food.

When an ant reaches another time the nest, $t_{act} = t$, making it effective in depositing pheromone once more. This actualisation of $t_{act}$ is only implemented when an ant reaches the food from the nest or the nest from the food. It follows that ants which get lost are ineffective in depositing the pheromone.

At each time step all the deposited pheromone evaporates with a tax equal to $\epsilon$:

$$\phi_{t+1}(x, y) = (1 - \epsilon) \times \phi_t(x, y).$$

Sites with a higher level of pheromone are more prone to be visited. In fact, ants move to site $(x, y)$ with probability $(1 + \phi_t(x, y))/(S + 3)$, where $S$ is the sum of all the pheromone presents in all the neighbour sites.

### Table 1. Parameters of the model.

| Parameter | Description                        | Typical value |
|-----------|------------------------------------|---------------|
| $P$       | colony size                        | 100           |
| $L$       | linear size of the system          | 17            |
| $\gamma$  | pheromone effective deposition constant | 0.2          |
| $\epsilon$| pheromone evaporation rate         | 0.05          |

3. Results and discussion

Fixed the values of all the parameters, we run different simulations with the aim of exploring the onset of the cooperative intelligence which allows the emergence of trails. The passage from a system where a diffusive behaviour is present, towards a system where a short path is selected, can be easily monitored measuring the efficiency of navigation between the two target areas. This is achieved counting the number of ants which realise the trajectory from the nest to the food in a time unit, normalised over the total population ($E_f$). The result is obviously the same if we measure the number of ants which realise the trajectory from the food to the nest. In the case where no pheromone is deposited ($E_0$), the efficiency depends just on the value of $L$. This efficiency value describes a pure diffusive behaviour, which corresponds to a maladaptive state. In contrast, if deposition is present, the efficiency is strongly dependent on all the variables: $\gamma$, $\epsilon$, $P$ and $L$.

A good parameter for the description of the state of the system is $E = E_f/E_0$, which measures the gain in transportation efficiency for systems with self-organised trails. As can be seen in Figure 1, its dynamics is quite simple: after a fast transient the system reaches a quasi-stationary state where the value of $E$ is maintained around a plateau. Adaptive states, where trail-based foraging emerged, present $E$ values clearly greater than the unit.
Figure 1. The time evolution of efficiency for a system with $L = 41$, $\gamma = 0.2$, $\epsilon = 0.05$, and $P = 1500$. The nest is located in (0,0), the food in (0,20). The points represent a time average over 50 time steps. The lower curve represents the case where no pheromone is deposited ($E_0$).

The dependence of $E$ as a function of $\gamma$ and $\epsilon$ is plain. In the top of Figure 2 the behaviour of $E$ is depicted for a fixed value of $\epsilon$. Changing the value of the pheromone effective deposition, $E$ grows from small values, when the deposition is really fable (small $\gamma$), towards a maximal value for an optimal $\gamma$. Then, it returns to smaller values, when the deposition remains active also for the lost ants, increasing the noise level in the reinforced paths.

In the bottom of figure 2 we can appreciate the behaviour of $E$ for a fixed value of $\gamma$. As for the previous case, a maximum value of $E$ exists in correspondence with an intermediate value of $\epsilon$. For larger value of evaporation the system, quite obviously, looses efficiency. Diminishing the value of $\epsilon$, the same behaviour is obtained.

Note that if the evaporation is absent, $E$ grows sensibly. This is followed by an impressive growth in the value of the variance of the ensamble average. In fact, for $\epsilon = 0$, if the system selects an optimal path in the first period of the simulation, high levels of efficiency are registered, otherwise low levels are reached. In this state the system is not effectively maximising its efficiency through a collective mechanism of exploration and signalisation, and efficiency strongly depends on the random configurations determined by the first paths.

Fixing the value of $\epsilon$ and $\gamma$, we turn to the most important part of our analysis. Our
Figure 2. Top: $E = E(\gamma)$, $\epsilon = 0.05$. Bottom: $E = E(\epsilon)$, $\gamma = 0.2$. For all simulations $L = 17$, $P = 100$ and the food is located in $(0,8)$. Points are obtained from a temporal average over an interval of 50000 time steps and realising an ensemble average over 100 simulations. Bars represent the value of the standard deviation of the ensemble average.

The goal is to describe the onset of the cooperative intelligence which allows the emergence of trails. This corresponds to estimate the minimal number of ants necessary for reaching the adaptive state. For this reason, we must characterise the behaviour of the efficiency in dependence of the colony size. Then, we analyse its scaling behaviour as a function of the environment size.

Figure 3 shows the clear increase in the efficiency of navigation between the two target areas in relation to colony size $P$. The depicted function has a typical sigmoidal shape. For small colony size a low value of $E$ is present, which corresponds to a maladaptive state. Increasing the colony size, a plateau with a high value of $E$ is reached, which corresponds to the adaptive state. For higher values of $P$, $E$ starts to slowly decrease. This is probably due to the fact that very large populations increase noise and generate saturation effects.

In general, our results are in accordance with the field study results presented in [7]. To our knowledge, this is the first time this fact is clearly reported by means of a microscopic model which describes the emergence of trails.

Now we estimate the minimal number of ants necessary to reach the adaptive state ($P_m$). Surprisingly, the behaviour of $E$ is fairly analogous to that of the order parameter.
of some equilibrium systems, as we can describe the region close to the point $P_m$ with the approximation: $E \propto (P - P_m)^\alpha$ (see the continuous line in Figure 3). We determine $\alpha$ and $P_m$ from plotting $\ln E$, as a function of $\ln((P - P_m)/P_m)$ and using the value of $P_m$ for which the plot is the straightest \cite{14}. In this way, we can give a systematic estimation of $P_m$. For the specific parameters used in the simulation of Figure 3, the adaptive state is reached for $P > P_m$, with $P_m = 130$. Also the existence of a $P_m$ is in accordance with the experimental results of \cite{7}, which found that a minimum number of ants is needed for the emergence of trails. More general relations between recruitment strategies and colony size \cite{15} are in agreement with these findings. In fact, species with small colony size predominantly use solitary foraging. For increasing sizes other alternative methods are used \cite{16}, and large colonies commonly use scent trails analogous to the ones we model \cite{17}.

![Figure 3. $E$ in dependence of $P$ for $L = 25$, $\epsilon = 0.05$, and $\gamma = 0.2$. The continuous line is the fitted power law: $y \propto (x - 130)^{0.52}$. Results have been averaged over 100 simulations.](image)

Now we turn our attention to the description of the scaling of these results in dependence of the environment size. Changing the L value we rescale the entire system, in fact we choose $D = (L - 1)/2$.

In Figure 4 we can see the behaviour of $E$, as a function of $P$, for different $L$ values. As expected, for finite sizes, $P_m$ is L dependent: $P_m(L)$. We estimate the different values of $P_m(L)$ and rescaled the data points using the rescaled parameter $[P - P_m(L)]/P_m(L)$. Efficiency is rescaled using the relation $E/L^{0.75}$, which was found looking at the scaling
of the maxima of $E$ for different $L$. As shown in detail in Figure 4, it is possible to obtain a reasonable collapse of all the curves. Data roughly collapse presenting a common rate of vanishing close to $(P - P_m(L))^{0.5}$, strongly supporting the validity of our approach. In fact, this result hardly can be considered a mere coincidence and it further legitimates the use of the power-law approximation for determining $P_m$.

The shift from a maladaptive to an adaptive behaviour occurs at a critical value that depends on the system size as: $P_m \propto L^{3.1}$. Note that the same conclusion can be obtained identifying the transition points with the $P$ values where the standard deviation of the distribution of $E$ reaches the maximum. This scaling indicates that, in the thermodynamic limit, the transition point goes to infinite. Strictly speaking, this means that the system does not display a classical phase transition, which is rigorously defined at the thermodynamic limit in which the number of constituents tends to infinity. However, it is also true that every finite system presents a well defined transition point. Even if this point is not a genuine critical point, it has a clear physical meaning as it is the value of the $P$ parameter which identifies the shift from the maladaptive to the adaptive behaviour, corresponding to the minimal number of individuals necessary to reach the adaptive state. Other models, where the transition is only observed for finite size systems, disappearing in the thermodynamic limit, and which present system size scaling, are well known in the literature [11, 18].

We must observe that, in general, when we transfer tools of statistical physics to problems of social or biological sciences, the population size is always considerably smaller than the Avogadro number, and so, not so large to justify the thermodynamic limit and its results. In fact, we are interested in the behaviour of finite-size systems, where important phenomena can appear in dependence of the number of individuals [11]. In particular, in our case, we expect that the relevant scaling is limited to populations ranging from a dozen of agents, as for a swarm of mobile robots, reaching less than half a billion of individuals, as can happen in supercolony of some ant species [19].

This finite-size results can be interpreted considering the density of the colony population ($P/L^2$): it is not sufficient to maintain a constant density to obtain the trail formation. This fact highlights the non-linearity of the phenomenon and it poses important constraints for real robot implementations. Moreover, our results can be explained in term of a simple geometric scaling. In fact, the minimal value of density, which generates a trail-based foraging, scales with the linear size of the environment, which is proportional to the size of the trail.

Also the scaling of $E$ with $L^{0.75}$ points out how the dynamics is far from merely depending on individual density. Remembering that the definition of $E$ is already normalised for the colony size, we can suppose that larger environments, postponing the effects of noise and saturation, allow the active interaction of larger community of individuals, which are able to maximise more efficiently the navigation problem.

The depicted behaviour of the finite scaling of the system is the most important result of our work. In fact, in analogy with classical models of phase transition, we can expect that the minimal number of individuals necessary to reach the adaptive state
**Figure 4.** Top: $E$ in dependence of $P$ for different values of $L$; $\epsilon = 0.05$ and $\gamma = 0.2$ (on the left). Finite-size transition points $P_m(L)$. The continuous line represents the best fitting function: $y \propto x^{3.1}$ (on the right). Bottom: Rescaled logarithmic plot of $E$. The continuous line has slope 0.5. Results have been averaged over 100 samples.

can depends on the details of the model. This would not be the case of the scaling laws, which could show some universal character, not depending on model details. In this perspective, we hope that our simplified results, perhaps refined using a model implementing more specific conditions, can be successfully used to interpret empirical observations of the scaling behaviour of ant colonies or of artificial swarms built up by cooperative mobile robots.

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