POINTS ON THE SPHERE SURFACE

Purpose. The extensive use of the computer-aided design system (CAD) in education and industry puts forward new demands on the scope, content and quality of up-to-date descriptive geometry course. The purpose of the work is topicality analysis of traditional descriptive geometry methods for 3D modelling and development of methodological recommendations for its teaching as a subject together with a selected CAD. Methodology. Conclusions about effectiveness and expedience of application of descriptive geometry methods are drawn on the basis of tools analysis and comparison of modern CADs and descriptive geometry for solving problems arising during 3D modelling. Generalization of teaching experience made it possible to give recommendations about optimization of the descriptive geometry course taking into account present-day requirements to professional skills of an engineer. Findings. CADs tools and descriptive geometry methods are compared by way of specific example to give the answer what is more suitable for solving problems arising during 3D modelling. Originality. We presented the methodological recommendations about optimization of descriptive geometry teaching together with a selected CAD. We proposed the conception of a modern textbook on descriptive geometry. First of all, the textbook must describe algorithms for solving problems by means of standard CAD tools exactly in 3D, not on the plane. It is desirable to accompany these algorithms by pictorial images in order to have an opportunity to grasp an idea quickly and implement it through methods of direct modeling in CAD application. We also touched practical problems of students’ motivation to ensure high effectiveness of graphical education. Practical value. This paper may be useful mainly for educators in the field of engineering graphics because it raises a vital question ‘Descriptive Geometry versus CAD’ which now has no definite answer. Topicality and teaching approaches of different solution methods of spatial problems by means of projections is subject of a dispute, taking into account that CAD tools are continuously updated. This paper by way of specific example shows some advantages and limitations of descriptive geometry and CAD, as well as touches the issues of their efficient joint application for teaching.

Keywords: descriptive geometry; CAD; sphere; teaching methods; motivation

Introduction

The surface of a sphere is considered definite, if the position of its centre and the radius are known. The sphere is the surface of revolution. The sphere is also the surface of a full sphere. In computer-aided design systems, a three-dimensional model of a sphere can be constructed by rotating a semicircle around its diameter, i.e. by kinematic way of surface formation. In AutoCAD for construction, there is also a separate command [9], the initial data for which are the coordinates of the centre point of the sphere and the size of its radius – the analytical method.

Purpose

In some cases, there is a need to fit a sphere into the already existing geometry, i.e. to construct the sphere surface when the position of several points on its surface is known, while the centre and radius are unknown. As it is known, for the unambiguous definition of a sphere it is necessary to know the coordinates of at least four noncoplanar points lying on its surface. 3D modelling programs do not have built-in tools for such constructions, and textbooks on descriptive geometry (DG) describe only the kinematic and analytical way of determining the surface. The graphical specification of a second-order surface by points is not
a typical way of determining a surface, although it can occur when solving practical problems.

**Methodology**

As sphere is the geometric locus of points in space equidistant from a point called the sphere centre, then the solution of this problem consists in finding a point equidistant from the four given values [2]. Thus, with 3D modelling, we come across a separate class of problems, when it is necessary to go from one set of input data (4 points on the sphere surface) to the other (sphere centre and radius). Such problems can be successfully solved by DG methods, and the construction algorithms are used in computer simulation.

![Fig. 1. Construction of sphere centre by 4 points (four replacements of projection planes)](image)

This circle can be regarded as a parallel of the desired sphere. Then the sphere centre will lie on the perpendicular reconstructed from the centre of the parallel. It is necessary to build two different parallels at given points and restore perpendiculars from
their centres. The intersection point of the perpendiculars will be the sphere centre.

On the plane, by using a double replacement of the projection planes, we found the natural value of each of the two planes formed by the given points. Then we constructed the sphere parallels, perpendiculars from their centres, and found the intersection point of these perpendiculars. From the point of view of DG, this method of solving the problem on the plane is not optimal, because there is another one, which requires fewer graphic. However, this is a simple and understandable way to solve the problem straightforward.

Figures 2 and 3 show the second way of finding the desired sphere centre. We take any three points for belonging to a common parallel and find its centre. Using the fourth point, we construct another parallel and find the sphere centre. Since any of the given points can be taken as the fourth one, there are four possible solutions.

Fig. 2. Construction of sphere centre by 4 points (double replacement of projection planes)

To solve the problem, it is also necessary to replace the projection planes twice. We turn the points C, B and D into a plane, passing through them the intersecting lines CD and the horizontal h. We find the natural value of this plane and construct a circle around the three points C, B, D – projection of the parallel m_3. Through the fourth point A, the projection of the second parallel k_3 is constructed. On plane P_3 both parallels are projected in the form of concentric circles, and on P_4 in the form of parallel segments of straight lines. The extreme points of the projections of parallels 2 and 3 define a circle whose centre coincides with the sphere centre. Points 2 and 3 must be joined by a straight line segment. From the middle of this segment, we restore the perpendicular, which will pass through the sphere centre.

Some 3D modelling programs allow imposing parametric constraints and restrictions on geometric objects. With the help of such tools it is possible to simplify the solution of problems and in general to avoid additional constructions [12].
For example, in SolidWorks, to find the sphere centre in a three-dimensional sketch, you can select an arbitrary point and connect it to four specified points with straight line segments. Then you need to select all the lines and define the geometric «equality» relationship between them (Figure 4).

The program will automatically move an arbitrarily chosen point to such a position that all segments become the same length, i.e. it will find the sphere centre – a point equidistant from the four given values.

Next, we need to construct a plane through the centre and any two points, construct a semicircle in this plane and obtain the sphere by the rotation method.

This method cannot be used in AutoCAD and KOMPAS-3D. In KOMPAS-3D, parametric constraints can only be imposed on flat objects in a sketch, and in AutoCAD such geometric dependencies cannot be applied to objects lying in different planes.

Findings

Despite the fact that modern CAD systems are quite a powerful tool, they are not perfect, i.e. do not have tools «for all occasions». Moreover, the effectiveness of their use directly depends on the user's skills, including his geometrical-graphic training. The study of descriptive geometry is one of the most important stages of such training.

In addition to its main purpose of developing spatial thinking, descriptive geometry studies methods of solving the applied tasks. Many of these tasks can be solved by CAD, but one of the areas remaining relevant in DG so far is the solution of tasks that require a transition from one set of initial geometric data to another set of such data needed for modelling [17].

Originality and practical value

The extensive use of CAD in education and industry also puts forward new demands on the scope, content and quality of up-to-date descriptive geometry course. [1, 3, 4–8, 10, 13–16]. Now the user has the ability to solve spatial geometric problems in the virtual three-dimensional space of the program, which makes it possible to avoid cumbersome construction on the projection planes. Therefore, the ideal DG textbook should first of all give
algorithms for task solving using the typical CAD tools exactly in 3D, and not on a plane. It is desirable to accompany these algorithms by pictorial images in order to have an opportunity to grasp an idea quickly and implement it in 3D. Complex constructions on the plane go to the background.

Conclusions

The solution of the problem in the three-dimensional space of the program is a natural process for a man. DG methods for solving spatial problems on a plane were developed when there was simply no other way to obtain a result. These methods are beautiful and elegant and often startle the imagination, but are they needed now, when the result can be achieved by pressing just a few buttons? Of course, we cannot and should not completely abandon the methods of classical DG: there are situations when they are still necessary. But the number of such tasks is getting smaller every year. Modern DG course should, first of all, include algorithms based on methods of direct modelling in 3D space, avoiding projection transformations.

As it is known, DG is considered by most students as a complex subject, for the study of which one needs certain motivation. Many students having acquainted with 3D modelling programs do not understand what the meaning of studying DG is, «if everything can be built on a computer». Therefore, methodically it makes sense to divide a joint study of DG and engineering computer graphics into two stages.

The first stage: study of the basic capabilities of CAD. At this stage the proposed tasks do not require involvement of DG and are designed to teach a user to apply the main CAD tools. The task of this stage is to give the student a powerful tool and to teach him how to use it. The user should receive positive emotions from the fact that he can build 3D models and receive drawings using computer programs. It is very important that at this stage the student learns to achieve a positive result on his own.

The second stage: the tasks proposed for modelling to the student should require additional constructions, involvement of DG or analytical geometry. For example, such tasks may require a transition from a geometry task with one set of input data to another set that can be implemented...
in the relevant CAD. The tasks should contain some of the tasks of DG, which is skillfully woven into the existing geometry. Herewith by the total volume of work and visually the models at this stage can be simpler than the previous ones. A situation is created when a user who knows how to use CAD cannot build a «simpler geometry» [11]. It is necessary to draw from the user an emotional state of excitement, which is characterized by a very strong interest in what is happening and a persistent desire to continue. It is very important to give the student time to think independently before explaining the solution. Perhaps this is the most important point of learning. It makes sense to give clues that will indicate the desired direction of the solution search. The task of the teacher is to show the student DG place and possibilities for solving the problems that arise during 3D modelling.

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ТОЧКИ НА ПОВЕРХНІ СФЕРИ

Мета. Широке застосування системи автоматизированого проектування (САПР) в навчальному процесі та на виробництві висуває нові вимоги до об’єму, змісту й якості викладання сучасного курсу нарисної геометрії. Метою роботи є аналіз актуальності методів класичної нарисної геометрії для задач геометричного моделювання та розробка методичних рекомендацій її викладання як навчальної дисципліни спільно з обраною САПР.

Методика. Висновки про ефективність та доцільність використання методів нарисної геометрії зроблені на основі аналізу й порівняння можливостей сучасних САПР та нарисної геометрії для вирішення питань, які виникають у процесі тривимірного моделювання. Узагальнення досвіду викладання графічних дисциплін дозволило дати рекомендації щодо оптимізації викладання курсу нарисної геометрії з урахуванням сучасних вимог до кваліфікації інженера.

Результати. На конкретному прикладі проведено порівняння можливостей САПР та методів нарисної геометрії для вирішення проблем, які виникають при геометричному моделюванні. Розглянути практичні питання мотивації студентів при вивченні нарисної геометрії.

Наукова новизна. Надані методичні рекомендації відносно організації та оптимізації викладання курсу нарисної геометрії з урахуванням сучасних вимог до кваліфікації інженера.

Практична значимість. Дана робота, насамперед, буде цікава викладачам графічних дисциплін, так як порушує актуальне питання “нарисна геометрія проти САПР”, на яке зараз немає однозначної відповіді. Актуальність та спосіб викладання таких питань для вирішення постійно зростаючих можливостей широкого просторового моделювання, є предметом обговорення та суперечок. Дана робота на прикладі демонструє можливості і обмеження нарисної геометрії та САПР, а також торкається питань ефективності їх спільного викладання.

Ключові слова: нарисна геометрія; САПР; сфера; методика викладання; мотивація

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рекомендаций её преподавания как учебной дисциплины совместно с выбранной САПР. Методика. Выводы об эффективности и целесообразности применения методов начертательной геометрии сделаны на основе анализа, сравнения возможностей современных САПР и начертательной геометрии для решения вопросов, возникающих в процессе трехмерного моделирования. Обобщение опыта преподавания графических дисциплин позволило дать рекомендации по оптимизации преподавания курса начертательной геометрии с учётом современных требований к квалификации инженера. Результаты. На конкретном примере проведено сравнение возможностей САПР и методов начертательной геометрии для решения проблем, возникающих при геометрическом моделировании. Научная новизна. Даны методические рекомендации относительно организации и оптимизации преподавания курса начертательной геометрии совместно с выбранной САПР. Предложена концепция современного учебника по начертательной геометрии, который, прежде всего, должен давать алгоритмы для решения задачи типовыми инструментами САПР именно в 3D, а не на плоскости. Желательно, чтобы эти алгоритмы сопровождались наглядными изображениями, позволяющими быстро уловить идею и реализовать её методами прямого моделирования в среде программы трехмерного моделирования. Затронуты практические вопросы мотивации студентов при изучении начертательной геометрии.

Практическая значимость. Данная работа, прежде всего, будет интересна преподавателям графических дисциплин, т. к. поднимает актуальный вопрос «начертательная геометрия против САПР», на который сегодня нет однозначного ответа. Актуальность и способ преподавания тех или иных методов решения про странственных задач с помощью проекций на плоскости, при постоянно растущих возможностях прямого 3D моделирования средствами САПР, является предметом обсуждения и споров. Данная работа на примере показывает возможности и ограничения начертательной геометрии и САПР, а также касается вопросов эффективности их совместного преподавания.

Ключевые слова: начертательная геометрия; САПР; сфера; методика преподавания; мотивация

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