Phenomenological analysis of the double pion production in nucleon-nucleon collisions up to 2.2 GeV

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Abstract

With an effective Lagrangian approach, we analyze several $NN \rightarrow NN\pi\pi$ channels by including various resonances with mass up to 1.72 GeV. For the channels with the pion pair of isospin zero, we confirm the dominance of $N^*(1440) \rightarrow N\sigma$ in the near threshold region. At higher energies and for channels with the final pion pair of isospin one, we find large contributions from $N^*(1440) \rightarrow \Delta\pi$, double-$\Delta$, $\Delta(1600) \rightarrow N^*(1440)\pi$, $\Delta(1600) \rightarrow \Delta\pi$ and $\Delta(1620) \rightarrow \Delta\pi$. There are also sizeable contributions from $\Delta \rightarrow \Delta\pi$, $\Delta \rightarrow N\pi$, $N \rightarrow \Delta\pi$ and nucleon pole at energies close to the threshold. We well reproduce the total cross sections up to beam energies of 2.2 GeV except for the $pp \rightarrow pp\pi^0\pi^0$ channel at energies around 1.1 GeV and our results agree with the existing data of differential cross sections of $pp \rightarrow pp\pi^+\pi^-$, $pp \rightarrow nn\pi^+\pi^+$ and $pp \rightarrow pp\pi^0\pi^0$ which are measured at CELSIUS and COSY.

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I. INTRODUCTION

Double pion production in both pion- and photo-induced reactions has been an intriguing field to study baryon spectrum and given insight to the properties of strong interaction \[1, 2\]. These reactions close to threshold are also an interesting area to test chiral symmetry and have been extensively explored experimentally \[3\] and theoretically \[4\]. Recently the double pion production in the electro-production off protons has advanced a important step \[5\]. All essential contributions are identified from the data and the major isobar channels are well determined. On the other hand, as another fascinating platform for studying resonances properties, double pion production in nucleon-nucleon collisions has been accurately measured at the facilities of CELSIUS and COSY in the past few years, and the comprehensive data of various differential cross sections are obtained up to beam energies 1.3 GeV \[6–11\]. However, on the theoretical side, the study of this reaction is scarce. The state-of-art one is still the Valencia model calculation \[12\] of more than ten years ago after some much earlier calculations of one pion exchange (OPE) model \[13\] of more than 45 years ago. Thus a more comprehensive analysis matching the modern data is very necessary.

The early OPE model, which mainly focused on the old data at beam energies of 2.0 GeV and 2.85 GeV \[14\], included two types of diagrams with the final two pions produced from a single and two baryon line(s), respectively. It used the amplitudes of $\pi N \rightarrow \pi \pi N$ and $\pi N \rightarrow \pi N$ extracted from limited data and the off-shell corrections were considered under several assumptions. It did not account for the explicit production mechanisms of double pion and other exchanged meson besides $\pi$-meson. The Valencia model is characteristic by the the dominance of $N^\ast(1440) \rightarrow N\sigma$ in the near threshold region in the isospin allowed channels while the double-$\Delta$ and $N^\ast(1440) \rightarrow \Delta\pi$ rise up at higher energies and in channels where $N^\ast(1440) \rightarrow N\sigma$ is forbidden by the isospin conservation. Recently, the experimental data \[6, 9\] confirm the predicted behavior close to threshold and this makes $pp \rightarrow pp\pi^+\pi^-$ and $pp \rightarrow pp\pi^0\pi^0$ good places to study $N^\ast(1440)$ whose structure is still controversial. Current data seem to show a weaker $N^\ast(1440) \rightarrow \Delta\pi$ than that listed in Particle Data Group \[15\] and support the explanation of $N^\ast(1440)$ as the monopole excitation of the nucleon. Contrarily, the case is much more complicated at higher energies. The dominance of the double-$\Delta$ mechanism in the Valencia model results in that the total cross section of $pp \rightarrow pp\pi^0\pi^0$ is about a factor of four larger than that of $pp \rightarrow nn\pi^+\pi^+$, while the new
exclusive and the old bubble-chamber data are consistent to conclude an approximate equal value of these two channels. The isospin decomposition unambiguously reveal that more isospin 3/2 resonances besides $\Delta$ is required to explain the data $[6, 8]$, and this is also the reason that the Valencia model including simply the $N^*(1440)$ and $\Delta$ achieved merely a rough agreement in most channels. Indeed, at higher energies the contribution from higher lying resonances, especially those having large double pion decay channels, should become relevant. The recent detailed measurements performed by CELSIUS and COSY make the further exploration of these problems possible.

In the present work, we try to incorporate the resonances with mass up to 1.72 GeV in an effective Lagrangian model with the motivation to give a reasonable explanation to the six isospin channels of $NN \to NN\pi\pi$ simultaneously and get better understanding of dynamics for this kind of reactions. Our paper is organized as follows. In Sect. III we present the formalism and ingredients in our computation. The numerical results and discussion are demonstrated in Sect. III and a brief summary is given in Sect. IV.

II. FORMALISM AND INGREDIENTS

We consider nearly all possible Feynman diagrams as depicted in Fig. 1, and exchanged diagrams are also included. We use the commonly used interaction Lagrangians for $\pi NN$, $\pi\Delta\Delta$, $\eta NN$, $\sigma NN$ and $\rho NN$ couplings,

$$L_{\pi NN} = -\frac{f_{\pi NN}}{m_\pi} N \gamma_5 \gamma_\mu \vec{\tau} \cdot \partial^\mu \vec{\pi} N,$$

$$L_{\pi\Delta\Delta} = \frac{f_{\pi\Delta\Delta}}{m_\pi} N \gamma_5 \gamma_\mu \vec{\tau} \cdot \partial^\mu \vec{\pi} \Delta_\nu + h.c.,$$

$$L_{\eta NN} = -ig_{\eta NN} N \gamma_5 \eta N,$$

$$L_{\sigma NN} = g_{\sigma NN} N \sigma N,$$

$$L_{\rho NN} = -g_{\rho NN} N (\gamma_\mu + \frac{k}{2m_N} \sigma_{\mu\nu} \partial^\nu) \vec{\tau} \cdot \vec{\rho} N. \quad (5)$$

At each vertex a relevant off-shell form factor is used. In our caculation, we take the same form factors as that used in the Bonn potential model $[16],$

$$F_M^{NN}(k_M^2) = \left( \frac{\Lambda_M^2 - m_M^2}{\Lambda_M^2 - k_M^2} \right)^n, \quad (6)$$
FIG. 1: Feynman diagrams for \( NN \rightarrow NN\pi\pi \). The solid, dashed and dotted lines stand for the nucleon, mesons and intermediate \( \sigma \) (or \( \rho \))-meson. The shading histograms represent the intermediate resonances or nucleon poles. In the text, we use \( R \rightarrow NM, R1 \rightarrow R2M \) and double-\( R \) to label (1)(2), (3)(4) and (5)(6), respectively.

with \( n=1 \) for \( \pi \)- and \( \eta \)-meson and \( n=2 \) for \( \rho \)-meson. \( k_M, m_M \) and \( \Lambda_M \) are the 4-momentum, mass and cut-off parameters for the exchanged meson, respectively. The coupling constants and the cutoff parameters are taken as: \( f_{\pi NN}^2/4\pi = 0.078, g_{\eta NN}^2/4\pi = 0.4, g_{\sigma NN}^2/4\pi = 5.69, g_{\rho NN}^2/4\pi = 0.9, \Lambda_\pi = \Lambda_\eta = 1.0 \text{ GeV}, \Lambda_\sigma = 1.3 \text{ GeV}, \Lambda_\rho = 1.6 \text{ GeV}, \) and \( \kappa = 6.1 \). We use \( f_{\pi\Delta\Delta} = 4f_{\pi NN}/5 \) from the quark model \([1, 12]\). The mass and width of \( \sigma \)-meson are adopted as 550 MeV and 500 MeV, respectively.

We include all \( N^* \) and \( \Delta^* \) resonances with spin-parity \( 1/2^\pm, 3/2^\pm, 5/2^\pm \) and mass up to 1.72 GeV listed in Particle Data Group (PDG) tables \([15]\). The resonances with further higher masses are expected to give negligible contributions in the energy region considered here and their two pion branching ratios have large uncertainties, so we do not include them at present. The effective Lagrangians for the relevant resonance couplings are \([19, 20]\),

\[
L_{\pi NR}^{1/2^+} = g_{\pi NR} \bar{N} \gamma_5 \gamma_\mu \vec{\tau} \cdot \partial^\mu \pi R + h.c.,
\]

\[
L_{\eta NR}^{1/2^+} = g_{\eta NR} \bar{N} \gamma_3 \eta R + h.c.,
\]

\[
L_{\sigma NR}^{1/2^+} = g_{\sigma NR} \bar{N} \sigma R + h.c.,
\]
\( \mathcal{L}^{1/2^+}_{\rho NR} = g_{\rho NR} \gamma_\mu \gamma_5 \tau^\mu \bar{\rho}^\nu R + h.c. \), \( (10) \)

\( \mathcal{L}^{1/2^+}_{\pi DR} = g_{\pi DR} \gamma_\mu \gamma_5 \tau^\mu \partial^\nu \bar{\pi} R + h.c. \), \( (11) \)

\( \mathcal{L}^{1/2^-}_{\pi NR} = g_{\pi NR} \gamma_\mu \gamma_5 \tau^\mu \bar{\pi} R + h.c. \), \( (12) \)

\( \mathcal{L}^{1/2^-}_{\eta NR} = g_{\eta NR} \gamma_\mu \gamma_5 \tau^\mu \bar{\eta} R + h.c. \), \( (13) \)

\( \mathcal{L}^{1/2^-}_{\rho NR} = g_{\rho NR} \gamma_\mu \gamma_5 \tau^\mu \bar{\rho}^\nu R + h.c. \), \( (14) \)

\( \mathcal{L}^{3/2^+}_{\pi DR} = g_{\pi DR} \gamma_\mu \gamma_5 \tau^\mu \partial^\nu \bar{\pi} R + h.c. \), \( (15) \)

\( \mathcal{L}^{3/2+}_{\pi NR} = g_{\pi NR} \gamma_\mu \gamma_5 \tau^\mu \bar{\pi} R + h.c. \), \( (16) \)

\( \mathcal{L}^{3/2+}_{\eta NR} = g_{\eta NR} \gamma_\mu \gamma_5 \tau^\mu \bar{\eta} R + h.c. \), \( (17) \)

\( \mathcal{L}^{3/2+}_{\rho NR} = g_{\rho NR} \gamma_\mu \gamma_5 \tau^\mu \bar{\rho}^\nu R + h.c. \), \( (18) \)

\( \mathcal{L}^{3/2+}_{\pi DR} = g_{\pi DR} \gamma_\mu \gamma_5 \tau^\mu \partial^\nu \bar{\pi} R + h.c. \), \( (19) \)

\( \mathcal{L}^{3/2^-}_{\pi NR} = g_{\pi NR} \gamma_\mu \gamma_5 \tau^\mu \bar{\pi} R + h.c. \), \( (20) \)

\( \mathcal{L}^{3/2^-}_{\rho NR} = g_{\rho NR} \gamma_\mu \gamma_5 \tau^\mu \bar{\rho}^\nu R + h.c. \), \( (21) \)

\( \mathcal{L}^{3/2^-}_{\pi DR} = g_{\pi DR} \gamma_\mu \gamma_5 \tau^\mu \partial^\nu \bar{\pi} R + h.c. \), \( (22) \)

\( \mathcal{L}^{3/2^-}_{\pi NR} = g_{\pi NR} \gamma_\mu \gamma_5 \tau^\mu \bar{\pi} R + h.c. \), \( (23) \)

\( \mathcal{L}^{5/2^-}_{\pi NR} = g_{\pi NR} \gamma_\mu \gamma_5 \tau^\mu \gamma_5 \gamma_\mu \gamma_5 \gamma_\nu \gamma_\nu \gamma_\lambda \bar{\pi} R_{\nu \lambda} + h.c. \), \( (24) \)

\( \mathcal{L}^{5/2^-}_{\rho NR} = g_{\rho NR} \gamma_\mu \gamma_5 \tau^\mu \gamma_5 \gamma_\nu \gamma_\nu \gamma_\lambda \bar{\rho} R_{\nu \lambda} + h.c. \), \( (25) \)

\( \mathcal{L}^{5/2^-}_{\sigma NR} = g_{\sigma NR} \gamma_\mu \gamma_5 \gamma_\nu \gamma_\nu \gamma_\lambda \gamma_\lambda \bar{\sigma} R_{\nu \lambda} + h.c. \), \( (26) \)

\( \mathcal{L}^{5/2^-}_{\pi DR} = g_{\pi DR} \gamma_\mu \gamma_5 \tau^\mu \gamma_5 \gamma_\nu \gamma_\nu \gamma_\lambda \bar{\pi} R_{\nu \lambda} + h.c. \), \( (27) \)

\( \mathcal{L}^{5/2^-}_{\pi NR} = g_{\pi NR} \gamma_\mu \gamma_5 \tau^\mu \gamma_5 \gamma_\nu \gamma_\nu \gamma_\lambda \bar{\pi} R_{\nu \lambda} + h.c. \), \( (28) \)

\( \mathcal{L}^{5/2^-}_{\pi DR} = g_{\pi DR} \gamma_\mu \gamma_5 \tau^\mu \gamma_5 \gamma_\nu \gamma_\nu \gamma_\lambda \bar{\pi} R_{\nu \lambda} + h.c. \), \( (29) \)

For the Resonance-Nucleon-Meson vertices, form factors with the following form are used:

\[ F_{R^M}^{MN} (k_M^2) = \left( \frac{\Lambda_M^2 - m_M^2}{\Lambda_M^2 - k_M^2} \right)^n, \] \( (30) \)
with \( n = 1 \) for \( N^* \) resonances and \( n = 2 \) for \( \Delta \) resonances. We employ \( \Lambda^*_\pi = \Lambda^*_\sigma = \Lambda^*_\eta = \Lambda^*_\rho = 1.0 \) for all resonances except \( \Lambda^*_\pi = 0.8 \) for \( \Delta^*(1600) \). We also use Blatt-Weisskopf barrier factors \( B(Q_{N^*\Delta\pi}) \) in the \( N^*(1440) - \Delta - \pi \) vertices \[21],

\[
B(Q_{N^*\Delta\pi}) = \sqrt{\frac{P^2_{N^*\Delta\pi} + Q_0^2}{Q^2_{N^*\Delta\pi} + Q_0^2}},
\]

Here \( Q_0 \) is the hadron scale parameter \( Q_0 = 0.197327/R \text{ GeV/c} \), where \( R \) is the radius of the centrifugal barrier in the unit of fm and is tuned to be 1.5 fm to fit the data. \( Q_{N^*\Delta\pi} \) and \( P_{N^*\Delta\pi} \) is defined as,

\[
Q_{N^*\Delta\pi}^2 = \frac{(s^*_N + s_\Delta - s_\pi)^2}{4s^*_N} - s_\Delta,
\]

\[
P_{N^*\Delta\pi}^2 = \frac{(m^2_N + m^2_\Delta - m^2_\pi)^2}{4m^2_N} - m^2_\Delta,
\]

with \( s_x \) being the invariant energy squared of \( x \) particle. Because the mass of \( \sigma \)-meson is near the two-\( \pi \) threshold, the following Lagrangians and form factor are employed for the \( \sigma - \pi - \pi \) vertex \[1, 2, 22],

\[
\mathcal{L}_{\sigma\pi\pi} = g_{\sigma\pi\pi} \partial^\mu \vec{\pi} \cdot \partial_\mu \vec{\sigma},
\]

\[
\mathcal{L}_{\rho\pi\pi} = g_{\rho\pi\pi} \vec{\pi} \times \partial_\mu \vec{\pi} \cdot \vec{\rho}_\mu,
\]

\[
F_{\sigma}(q^2) = \left( \frac{\Lambda^2 + \Lambda_0^2}{\Lambda^2 + q^2} \right)^2,
\]

where \( \vec{q} \) is the relative momentum of the emitted \( \pi \)-mesons. We use \( \Lambda = 0.8 \text{ GeV} \) and \( \Lambda_0^2 = 0.12 \text{ GeV}^2 \) to normalize this form factor to unity when \( \pi^- \) and \( \sigma^- \) meson are all on-shell. The decay width of \( \sigma \rightarrow \pi\pi \) and \( \rho \rightarrow \pi\pi \) yield \( g_{\sigma\pi\pi}^2 = 6.06 \) and \( g_{\rho\pi\pi}^2 = 2.91 \).

The form factor for the resonance, \( F_R(q^2) \), is taken as,

\[
F_R(q^2) = \frac{\Lambda^4_R}{\Lambda^4_R + (q^2 - M^2_R)^2},
\]

with \( \Lambda_R = 1.0 \text{ GeV} \). The same type of form factors are also applied to the nucleon pole with \( \Lambda_N = 0.8 \text{ GeV} \). The propagators of the exchanged meson, nucleon pole and resonance can be written as \[16, 17],

\[
G_{\pi/\eta}(k_{\pi/\eta}) = \frac{i}{k^2_{\pi/\eta} - m^2_{\pi/\eta}},
\]

\[
G_{\sigma}(k_\sigma) = \frac{i}{k^2_\sigma - m^2_\sigma + im_\sigma \Gamma_\sigma},
\]

\[
G^{\mu\nu}(k_\rho) = -\frac{i}{k^2_\rho - m^2_\rho} \left( g^{\mu\nu} - k^\mu k^\nu / k^2_\rho \right).
\]
TABLE I: Relevant parameters used in our calculation. The masses, widths and branching ratios (BR) are taken from central values of PDG [15] except the BR for $N^*(1440) \to \Delta \pi$.

| Resonance      | Pole Position | BW Width | Decay Mode | Decay Ratio | $g^2/4\pi$ |
|---------------|--------------|---------|------------|-------------|------------|
| $\Delta^*(1232)P_{33}$ | (1210, 100)  | 118     | $N\pi$    | 1.0         | 19.54      |
| $N^*(1440)P_{11}$ | (1365, 190)  | 300     | $N\pi$    | 0.65        | 0.51       |
|               |              |         | $N\sigma$ | 0.075       | 3.20       |
|               |              |         | $\Delta \pi$ | 0.135    | 4.30       |
| $N^*(1520)D_{13}$ | (1510, 110)  | 115     | $N\pi$    | 0.6         | 1.73       |
|               |              |         | $N\rho$   | 0.09        | 1.32       |
|               |              |         | $\Delta \pi$ | 0.2    | 0.01       |
| $N^*(1535)S_{11}$ | (1510, 170)  | 150     | $N\pi$    | 0.45        | 0.037      |
|               |              |         | $N\eta$   | 0.525       | 0.34       |
|               |              |         | $N\rho$   | 0.02        | 0.15       |
| $\Delta^*(1600)P_{33}$ | (1600, 300)  | 350     | $N\pi$    | 0.175       | 1.09       |
|               |              |         | $\Delta \pi$ | 0.55    | 59.9       |
| $N^*(1440)\pi$ |              |         |           | 0.225    | 289.1      |
| $\Delta^*(1620)S_{31}$ | (1600, 118) | 145     | $N\pi$    | 0.25        | 0.06       |
|               |              |         | $N\rho$   | 0.14        | 0.37       |
|               |              |         | $\Delta \pi$ | 0.45    | 83.7       |
| $N^*(1650)S_{11}$ | (1655, 165)  | 165     | $N\pi$    | 0.775       | 0.06       |
|               |              |         | $N\eta$   | 0.065       | 0.026      |
|               |              |         | $N\rho$   | 0.08        | 0.011      |
|               |              |         | $\Delta \pi$ | 0.04    | 0.063      |
| $N^*(1675)D_{15}$ | (1660, 135)  | 150     | $N\pi$    | 0.4         | 2.16       |
|               |              |         | $\Delta \pi$ | 0.55    | 3077.5     |
| $N^*(1680)F_{15}$ | (1675, 120)  | 130     | $N\pi$    | 0.675       | 5.53       |
|               |              |         | $N\sigma$ | 0.125       | 4.45       |
|               |              |         | $N\rho$   | 0.09        | 0.32       |
|               |              |         | $\Delta \pi$ | 0.1    | 9.39       |
TABLE II: Table I continued.

| Resonance Pole Position | BW Width | Decay Mode | Decay Ratio | $g^2/4\pi$ |
|-------------------------|----------|------------|-------------|------------|
| $N^*(1700)D_{13}$ (1680, 100) | 100      | $N\pi$     | 0.1         | 0.075      |
|                         |          | $N\rho$    | 0.07        | 0.043      |
|                         |          | $\Delta\pi$| 0.04        | 0.003      |
| $\Delta^*(1700)D_{33}$ (1650, 200) | 300      | $N\pi$     | 0.15        | 1.02       |
|                         |          | $N\rho$    | 0.125       | 0.69       |
|                         |          | $\Delta\pi$| 0.45        | 0.072      |
| $N^*(1710)P_{11}$ (1720, 230) | 100      | $N\pi$     | 0.15        | 0.012      |
|                         |          | $N\eta$    | 0.062       | 0.042      |
|                         |          | $N\sigma$  | 0.25        | 0.085      |
|                         |          | $N\rho$    | 0.15        | 36.1       |
|                         |          | $\Delta\pi$| 0.275       | 0.12       |
| $N^*(1720)P_{13}$ (1675, 195) | 200      | $N\pi$     | 0.15        | 0.12       |
|                         |          | $N\eta$    | 0.04        | 0.28       |
|                         |          | $N\rho$    | 0.775       | 190.7      |

$$G_N(q) = -\frac{i(q + m_N)}{q^2 - m_N^2}. \quad (41)$$

$$G_{1/2}^R(q) = -\frac{i(q + M_R)}{q^2 - M_R^2 + iM_R\Gamma_R}. \quad (42)$$

$$G_{3/2}^R(q) = -\frac{i(q + M_R)G_{\mu\nu}(q)}{q^2 - M_R^2 + iM_R\Gamma_R}. \quad (43)$$

$$G_{5/2}^R(q) = -\frac{i(q + M_R)G_{\mu\nu\alpha\beta}(q)}{q^2 - M_R^2 + iM_R\Gamma_R}. \quad (44)$$

Here $\Gamma_R$ is the total width of the corresponding resonance, and $G_{\mu\nu}(q)$ and $G_{\mu\nu\alpha\beta}(q)$ is defined as,

$$G_{\mu\nu}(q) = -g_{\mu\nu} + \frac{1}{3}\gamma_{\mu}\gamma_{\nu} + \frac{1}{3M_R^2}(\gamma_{\mu}q_{\nu} - \gamma_{\nu}q_{\mu}) + \frac{2}{3M_R^2}q_{\mu}q_{\nu}, \quad (45)$$

$$G_{\mu\nu\alpha\beta}(q) = -\frac{1}{2}g_{\mu\alpha\beta\nu} + \frac{1}{5}g_{\mu\beta\nu\alpha} + \frac{1}{10}(\gamma_{\mu}\gamma_{\beta}\gamma_{\alpha}\gamma_{\nu} + \gamma_{\mu}\gamma_{\beta}\gamma_{\nu}\gamma_{\alpha} + \gamma_{\nu}\gamma_{\alpha}\gamma_{\mu}\gamma_{\beta}). \quad (46)$$

$$-\frac{1}{10}(\gamma_{\mu}\gamma_{\beta}\gamma_{\alpha}\gamma_{\nu} + \gamma_{\nu}\gamma_{\alpha}\gamma_{\mu}\gamma_{\beta}). \quad (47)$$
\[ \tilde{g}_{\mu \nu}(q) = -g_{\mu \nu} + \frac{q_{\mu} q_{\nu}}{M_{R}^2}, \quad \tilde{\gamma}_\mu = -\gamma_\mu + \frac{q_{\mu}}{M_{R}^2}. \]  

(48)

Because constant width is used in the Breit-Wigner (BW) formula, we adopt the pole positions of various resonances for parameters appearing in the propagators.

The coupling constants appearing in relevant resonances are determined by the empirical partial decay width of the resonances taken from PDG [15], and then we adjust the values of cut-off in form factors to fit the data. The relations between the branching ratios of the adopted resonances and the corresponding coupling constants squared can be calculated straightforwardly with above Lagrangians, and most of them can be found in the appendix of Ref. [17]. The detailed calculations of \( g_{\rho NR} \) and \( g_{\sigma NR} \) from the \( R \to N\rho(\sigma) \to N\pi\pi \) decay are given in Ref. [23]. The values of coupling constants used in our computation are compiled in the Table I, together with the properties of the resonances and the central value of branch ratios. It should be noted that we adopt a nearly half of the decay width of \( N^*(1440) \to \Delta \pi \) in PDG as the recent data favored [6, 9, 24].

Then the invariant amplitudes can be obtained straightforwardly by applying the Feynman rules to Fig. 1. As to the different isospin channels, isospin coefficients are considered. We do not include the interference terms among different diagrams because their relative phases are not known, and the Valencia model seems to show that such terms are very small.

III. NUMERICAL RESULTS AND DISCUSSION

As a starting point, in Fig. 2 we demonstrate our calculated total cross sections of six isospin channels compared with the existing data [6, 9–11, 14]. Our numerical results give an overall good reproduction to all six channels. The pre-emission diagrams (see (2), (4), (6) in Fig. 1) tend to be negligibly small, consistent with the Valencia model, so we do not include them in our concrete computation. In Fig. 2 we do not show the following negligible contributions: double-\( N^* \), \( N^* \to N\rho \), \( N^* \to N\pi \), \( N^*(1520) \to \Delta \pi \), \( N^*(1650) \to \Delta \pi \), \( N^*(1675) \to \Delta \pi \), \( N^*(1680) \to \Delta \pi \), \( N^*(1680) \to N\sigma \), \( N^*(1700) \to \Delta \pi \), \( N^*(1710) \to \Delta \pi \), \( N^*(1710) \to N\sigma \), double-\( \Delta^*(1600) \), \( \Delta^*(1600) \to N\pi \), double-\( \Delta^*(1620) \), \( \Delta^*(1620) \to N\rho \), \( \Delta^*(1620) \to N\pi \), double-\( \Delta^*(1700) \), \( \Delta^*(1700) \to N\rho \), \( \Delta^*(1700) \to N\pi \), and \( \Delta^*(1700) \to \Delta \pi \). These terms are minor either because of their small branching ratios of double pion channel such as \( N^*(1535), N^*(1650) \) and \( N^*(1700) \), or belonging to higher partial waves such as
Δ*(1620) and N*(1675), or lying beyond the considered energies such as N*(1680), Δ*(1700), N*(1710) and N*(1720). It should be mentioned that ρ-meson exchange is much smaller than π-meson exchange in the available diagrams except for nucleon poles but we still include the ρ-meson exchange in the calculation for the completeness of our model.

Our results underestimate the data in the close-to-threshold region where the final state interactions (FSI) should be relevant. We do not consider the initial state interaction (ISI) either, because at present we do not have an unambiguous method at hand to simultaneously include the FSI and ISI in our model. The ISI usually has a weak energy dependence, so adjusting cut-off parameters in the form factors may partly account for it effectively [20]. We would give some qualitative observations of FSI.

Next we shall first address the pp → nnnπ+π+ channel because it has negligible N* contribution to be more clean. Then we shall discuss other channels and explore the different situation at each channel. In the following we assume the same definitions of various differential cross sections as graphically illustrated in the experimental articles [6, 8]. The $M_{ij}$ and $M_{ijk}$ are the invariant mass spectra, and the angular distributions are all defined in the overall center of mass system. The $\Theta_M$ is the scattering angle of $M$, and $\delta_{ij}$ is the opening angle between $i$ and $j$ particles. The $\Theta^{ij}_i$ (or $\vartheta^{ij}_i$ corresponding to $\hat{\Theta}^{ij}_i$ defined in Ref. [6, 8]) is the scattering angle of $i$ in the rest frame of $i$ and $j$ with respect to the beam axis (or the sum of momenta of $i$ and $j$). The values of vertical axis are all arbitrarily normalized.

A. The channel of $pp \rightarrow nnn\pi^+\pi^+$

In this channel, we find that the $\Delta \rightarrow N\pi \rightarrow N\pi\pi$ term is dominant below 1000 MeV while the $\Delta \rightarrow \Delta\pi$ and double-nucleon-pole terms are also important. The $\Delta \rightarrow N\pi \rightarrow N\pi\pi$ term is not included in the Valencia model [12]. Our model seems to overestimate the COSY-TOF upper-limit by a factor of around four. The $\Delta \rightarrow \Delta\pi$ terms in two models are consistent with each other because we use the same coupling constant of $\pi\Delta\Delta$ from quark model but our double-$\Delta$ term contributes smaller as we use a smaller cut-off parameter in $\pi N\Delta$ form factor. Between 1000 MeV and 1700 MeV, the contribution of the double-$\Delta$ term is the most important one, and the $n\pi^+$ invariant mass distribution at 1100 MeV do show a clear $\Delta$ peak as can be seen in Fig. 3. We also find that the $\Delta \rightarrow N\pi \rightarrow N\pi\pi$ and $\Delta \rightarrow \Delta\pi$ terms are crucial to get the right shape of differential cross sections at 1100 MeV. Though
the data is of poor statistics, the $\pi^+\pi^+$ invariant mass spectrum does not show obvious low-mass peak and this is realized by the inclusion of the $\Delta \rightarrow N\pi \rightarrow N\pi\pi$ and $\Delta \rightarrow \Delta\pi$ in our model. The $\delta_{\pi\pi}$ has also a significant improvement compared to the double-$\Delta$ alone. These distributions should be very useful to constrain the poorly known coupling constant of $\pi\Delta\Delta$. The particular enhancement compared to our model without FSI in the $nn$ invariant mass spectrum is probably an indication of strong $^1S_0nn$ FSI.

The contribution from $\Delta^*(1600) \rightarrow N^*(1440)\pi$ term has a steep rise and begins to take over as the largest one for $T_p$ above 1700 MeV. Besides, at large energies contributions from the $\Delta^*(1600) \rightarrow \Delta\pi$ and $\Delta^*(1620) \rightarrow \Delta\pi$ become significant. So in these energy region of $pp \rightarrow nn\pi^+\pi^+$, it is a good place to explore the properties of these $\Delta^*$ resonances. We would like to point out that these behaviors together with the dominance of $\Delta \rightarrow N\pi \rightarrow N\pi\pi$ and $\Delta \rightarrow \Delta\pi$ close to threshold alleviate the isospin problem of the $pp \rightarrow nn\pi^+\pi^+$ and $pp \rightarrow pp\pi^0\pi^0$ channels mentioned at the beginning of our article, because the isospin coefficients of these terms in $pp \rightarrow nn\pi^+\pi^+$ are bigger than that in $pp \rightarrow pp\pi^0\pi^0$ channel and this is contrary to the case of double-$\Delta$. As a result, we get an improvement on the description of all isospin channels.

It should be addressed that it is very useful to pin down the cut-off values in form factors of the relevant $\Delta$ and $\Delta^*$ contributions using the data of $pp \rightarrow nn\pi^+\pi^+$ at first, and then it makes much easier for us to determined the $N^*$ contributions in other channels. The new value of total cross section of $pp \rightarrow nn\pi^+\pi^+$ measured at CELSIUS is in line with previous data and this gives our some confidence on the extracted parameters. Further accurate measurements of the $pp \rightarrow nn\pi^+\pi^+$ channel should be very helpful for the improvement of the model.

**B. The channel of $pp \rightarrow pp\pi^+\pi^-$**

Below 1000 MeV the $N^*(1440) \rightarrow N\sigma$ term is the largest while the $N^*(1440) \rightarrow \Delta\pi$ term is the second. Of these two terms the $\sigma$-meson exchange gives much bigger contribution than the $\pi$-meson exchange as depicted in Fig. and this shows the importance of isoscalar excitation of $N^*(1440)$. The Double-$\Delta$ term is negligible at this low energies as well as the $\Delta \rightarrow \Delta\pi$ term. Contributions from the nucleon pole and $N \rightarrow \Delta\pi$ terms are visible below 700 MeV. The proton and pion angular distributions in the center of mass system at 650
and 680 MeV are trivially isotropic and the model does agree with the measured data [6]. So we do not show them here. The differential cross sections at 750, 775, 800 and 895 MeV are given in Fig. 5, Fig. 6, Fig. 7 and Fig. 8 respectively. Our model calculations reproduce the published data well and are also compatible to the very preliminary data of CELSIUS at 895 MeV [8] as shown in Fig. 8. The role of \( N^*(1440) \rightarrow \Delta \pi \) is evident in invariant mass spectrums and the data is fitted better than that including \( N^*(1440) \rightarrow N\sigma \) alone. Most obviously, the anisotropic shape of \( \vartheta_{\pi\pi} \) is well fitted after including the \( N^*(1440) \rightarrow \Delta \pi \) while \( N^*(1440) \rightarrow N\sigma \) term is symmetric, so \( \vartheta_{\pi\pi} \) together with \( \vartheta^{p+} \) and \( \vartheta^{p-} \) is used to determined the ratio of partial decay widths of \( N^*(1440) \rightarrow \Delta \pi \) and \( N^*(1440) \rightarrow N\sigma \) [6]. The results are strongly energy dependent and give a smaller decay width of \( N^*(1440) \rightarrow \Delta \pi \) than that listed in PDG. This is believed to support the breathing mode of \( N^*(1440) \) [25].

The FSI is evident in \( pp \) invariant mass spectrum, but seems to be much weaker than in the case of \( pp \rightarrow nn\pi^+\pi^+ \) channel.

In the Valencia model the double-\( \Delta \) is dominant above 1300 MeV. However, because we use smaller cut-off parameter for the \( \pi N \Delta \) form factor in order to fit both \( nn\pi^+\pi^+ \) and \( pp\pi^0\pi^0 \) channels, our model shows that \( N^*(1440) \rightarrow \Delta \pi \) begins to take over above 1100 MeV, and double-\( \Delta \) and \( N^*(1440) \rightarrow N\sigma \) are also important and comparable. We give the differential cross sections at 1100 MeV and 1360 MeV in Fig. 9 and Fig. 10 which can be tested by the measured data of CELSIUS [8]. The prominent features are the double hump structure in \( M_{\pi^+\pi^-} \) and the upward bend in \( \delta_{\pi^+\pi^-} \) which arise from the \( N^*(1440) \rightarrow \Delta \pi \). The Valencia model give very similar results because \( M_{\pi^+\pi^-} \) and \( \delta_{\pi^+\pi^-} \) are sensitive to the appearance of \( N^*(1440) \rightarrow \Delta \pi \). These seem to somewhat incompatible to the preliminary data [7, 8] which show the phase space behavior in these two spectrums. The same phenomena also happen in the channel of \( pp \rightarrow pp\pi^0\pi^0 \) at high energies, and we will discuss them altogether later.

C. The channel of \( pp \rightarrow pp\pi^0\pi^0 \)

The \( N^*(1440) \rightarrow N\sigma \) term dominates below 1000 MeV and the nucleon pole term also gives significant contribution below 800 MeV. The \( N^*(1440) \rightarrow \Delta \pi \) and double-\( \Delta \) contributions are comparable in this energy region. So it should be cautious to use this channel to extract the ratio of the partial decay widths for the decay of \( N^*(1440) \). Indeed, the extracted
ratios from $pp \rightarrow pp\pi^0\pi^0$ are about one third of those from $pp \rightarrow pp\pi^+\pi^-$ at the same nominal mass under the assumption that $N^*(1440)$ dominates in this energy range \[6, 8\]. The significant double-$\Delta$ and nucleon pole contributions might account for this discrepancy and should be reasonably incorporated in the fit.

Above 1100 MeV, the double-$\Delta$ term dominates; the $N^*(1440) \rightarrow \Delta\pi$ and $N^*(1440) \rightarrow N\sigma$ are also important and give similar contributions. Other contributions are much smaller. The most striking feature in this energy region is that a level-off behavior happens in the total cross section between 1000 and 1200 MeV, while other channels rise smoothly when increasing the incident energy. Our model fails to describe this behavior and also overestimates the high energy data. It is possible that this shape is caused by the interference of different diagrams which are not included in our model, but this would require a peculiar energy dependence of $N^*$ as shown by the isospin decomposition \[6, 8\]. Another possible explanation is that there maybe exist a steep rise of some kind of contribution when other contributions are saturated in this energy region. This happens in the channel of $pp \rightarrow nn\pi^+\pi^+$ where a weak level-off at 1600 MeV is caused by the steep rise of $\Delta^*(1600) \rightarrow N^*(1440)\pi$. However, this is not the case for the $pp \rightarrow pp\pi^0\pi^0$ channel where $\Delta^*(1600) \rightarrow N^*(1440)\pi$ gives much smaller contribution due to the isospin factor. So this problem is left for further clarification.

In Fig. 11, Fig. 12 and Fig. 13 we show the differential cross sections of $pp \rightarrow pp\pi^0\pi^0$ at beam energies of 775, 895 and 1000 MeV, respectively. The data at 775 MeV are well reproduced and $N^*(1440) \rightarrow N\sigma$ is overwhelmingly dominant. Some of the angular distributions are sensitive to the presence of the $N^*(1440) \rightarrow \Delta\pi$ contribution, and hence can be used to determine the partial decay ratios of $N^*(1440)$, although this is somewhat complicated by the double-$\Delta$ and nucleon pole contributions as we pointed out earlier. The contribution of $N^*(1440) \rightarrow \Delta\pi$ and double-$\Delta$ terms become much clearer at 895 and 1000 MeV, though slight discrepancy in invariant mass spectrums at 895 MeV exists between our model and the measured data, which remind us that it needs a further improvement in the crossover region. The phase space shapes of $M_{\pi^0\pi^0}$ and $\delta_{\pi^0\pi^0}$ begin to appear ever since 895 MeV and up to high energies of this channel as shown in Fig. 14, Fig. 15 and Fig. 16 which are the differential cross sections of $pp \rightarrow pp\pi^0\pi^0$ at 1100, 1200 and 1300 MeV. However, because the influence of the $N^*(1440) \rightarrow \Delta\pi$ does not decrease much or disappear at these energies, our model gives a double hump structure in $M_{\pi^0\pi^0}$ and forward peak in $\delta_{\pi^0\pi^0}$ which are contradictory to the CELSIUS data. In order to explain the data, it is required that the
$N^*(1440) \rightarrow \Delta \pi$ shows up at low energies but immediately saturated at about 1000 MeV. That is a peculiar energy dependence behavior which does not supported by our model. Except for $M_{\pi\pi}$ and $\delta_{\pi\pi}$, other spectrums at high energies are well fitted by our model both in $pp \rightarrow pp\pi^+\pi^-$ and $pp \rightarrow pp\pi^0\pi^0$. So we would rather conclude that something happens in the $\pi\pi$ system which needs a more thorough investigation as the next step. The $\pi\pi$ rescattering is found to be negligible at these energies [26].

The effect of FSI is not obvious at low energies compared to our calculated curve but enhancement seems to happen at high energies. It is possible that this is related to the behavior of $\pi^0\pi^0$ system.

D. The channels of $pp \rightarrow pn\pi^+\pi^0$, $pn \rightarrow pp\pi^-\pi^0$ and $pn \rightarrow pn\pi^+\pi^-$

The $N^*(1440) \rightarrow N\sigma$ does not present in the $pp \rightarrow pn\pi^+\pi^0$ reaction, so the double-$\Delta$ term is the most important one in a wide energy range. The $\Delta \rightarrow \Delta\pi$ and $\Delta \rightarrow N\pi \rightarrow N\pi\pi$ terms have significant contribution below 800 MeV and also have some contribution at higher energies together with the $\Delta^*(1600)$ and $\Delta^*(1620)$ terms. The agreement with the data is very good and the FSI may influence the near-threshold region since our model slightly underestimates this part.

The channel of $pn \rightarrow pp\pi^-\pi^0$ is another reaction where the $N^*(1440) \rightarrow N\sigma$ does not contribute. Since the charged meson exchange is allowed in this channel, the $N^*(1440) \rightarrow \Delta\pi$ term is very important and is of the same order as the double-$\Delta$ term in the whole energy region. The contributions from the nucleon pole and $\Delta \rightarrow N\pi \rightarrow NN\pi$ terms are also quite significant near the threshold. Our results reproduce the new bubble chamber data measured by KEK [10] very well, but underestimate the old data [14] by about a factor of 5. Since the double-$\Delta$ contribution has been well determined by the channel of $pp \rightarrow nn\pi^+\pi^+$, we think that the main ambiguity comes from the $N^*(1440) \rightarrow \Delta\pi$ term. If future experiments confirm the old data, then the isovector mesons like $\pi$- and $\rho$-meson should play more important role in the excitation of $N^*(1440)$. On the other hand, the new data of KEK support the isoscalar excitation of $N^*(1440)$ which is favored by our model.

The $pn \rightarrow pn\pi^+\pi^-$ channel is interesting because it can shed light on the low mass enhancement in $M_{\pi\pi}$, known as the ABC effect of double pion production in nuclear fusion reactions [27]. Below 900 MeV, the $N^*(1440) \rightarrow N\sigma$ is found to be dominant while the
double-Δ and $N^*(1440) \to \Delta \pi$ terms also give some contribution. The nucleon pole and $N \to \Delta \pi$ terms are also important close to threshold. Above 1000 MeV, the double-Δ term is the most important one and the $N^*(1440)$ gives sizable contribution at high energies. The total contribution gives a reasonable description to the new KEK data while the underestimation of the data close to threshold may be due to the omission of the $pn$ FSI. Similar to the $pn \to pp\pi^-\pi^0$ channel, our model does not favor the old bubble chamber data which need large isovector excitation of $N^*(1440)$. Very recently the ABC effect is experimentally established in $pn \to d\pi^0\pi^0$ at beam energies of 1.03 and 1.35 GeV, and has been interpreted as an s-channel double-Δ resonance [27]. According to the observation of our model, the $N^*(1440)$ emerges at these energies so it is necessary to take a further look at the mechanism of ABC effect in $pn \to d\pi^0\pi^0$ reaction. As a matter of fact, it has been demonstrated that at beam momentum of 1.46 GeV (corresponding to beam energies of 800 MeV) where the $N^*(1440)$ is expected to be dominant, the deuteron momentum spectra can be reasonably explained by the interference of the $N^*(1440) \to N\sigma$ and $N^*(1440) \to \Delta \pi$ [12, 28].

E. Final State Interaction

As discussed above, the effect of FSI is anticipated to influence the results close to threshold where the s-wave is expected to be dominant. Usually the Jost function is used to account for the FSI enhancement factor,

$$J(k)^{-1} = \frac{k + i\beta}{k - i\alpha},$$  \hspace{1cm} (49)

where $k$ is the relative momentum of $NN$ subsystem in the final state. The corresponding scattering length and effective range are:

$$a = \frac{\alpha + \beta}{\alpha\beta}, \quad r = \frac{2}{\alpha + \beta},$$  \hspace{1cm} (50)

with $a = -7.82\text{fm}$ and $r = 2.79\text{fm}$ for $^1S_0$ $pp$ interaction, $a = 5.42\text{fm}$ and $r = 1.76\text{fm}$ for $^3S_1$ isoscalar $pn$ interaction, and $a = -18.45\text{fm}$ and $r = 2.83\text{fm}$ for $^1S_0$ $nn$ interaction. At higher energies, the high partial waves become important and above approximate treatment would deteriorate rapidly. Fortunately, the effect of FSI should significantly decrease. So we may just ignore it above 1.4 GeV. To investigate the influence of the FSI to the energy dependence of cross sections, we assume the Jost function for the FSI and normalize this
factor to the unity at the beam energy of 1.4GeV. For the final states with the $pn$ pair, we assume it is mainly in the $^3S_1$ isoscalar state.

Though above prescription is quite rough, the agreement with the data are considerably improved. In Fig. 17 we demonstrate the total cross section below 1.4GeV. The effect of FSI can be seen in some of the differential cross sections especially $NN$ spectrums. In Fig. 18 we take $pp \to nn\pi^+\pi^+$ channel as a typical example. The $nn$ FSI gives a sharp peak in the $nn$ spectrums which agree with the $pp \to nn\pi^+\pi^+$ data. However, the data in other channels do not favor this sharp peak and this reflects the drawbacks of our formalism. The $nn$ FSI slightly improves the fit of the $n\pi^+\pi^+$ and $nn\pi^+$ spectrums but increase the slope of $\delta_{n\pi^+}$. FSI has very small influence on other angular distributions. The situations for other channels are similar.

IV. SUMMARY

In this article, we present a simultaneous analysis of various isospin channels of double pion production in nucleon-nucleon collisions up to 2.2 GeV within an effective Lagrangian approach. We study the contributions of various resonances with mass up to 1.72 GeV and demonstrate that $N^*(1440)$, $\Delta$, $\Delta^*(1600)$, $\Delta^*(1620)$ and nucleon pole constitute the main ingredients to reasonably explain the measured data while the contribution of other resonances are negligible. We suggest that it is necessary to consider other influence such as the double-$\Delta$ and nucleon pole contributions when one studies the properties of $N^*(1440)$ in the channels of $pp \to pp\pi^+\pi^-$ and $pp \to pp\pi^0\pi^0$. Our model well describe the measured differential cross sections except some $\pi\pi$ spectra which are left as an open problem. Compared with the Valencia model, the main differences are: (1) Among 3 major ingredients, double-$\Delta$, $N^*(1440) \to \Delta\pi$ and $N^*(1440) \to N\sigma$ terms, considered in the Valencia model, our model increases significantly the relative contribution from the $N^*(1440) \to N\sigma$ term by reducing the relative branching ratio of $N^*(1440) \to \Delta\pi$ and assuming a smaller cut-off parameter for the $\pi N\Delta$ coupling; (2) In addition, our model introduces significant contributions from $\Delta \to N\pi \to N\pi\pi$ at energies near threshold and from $\Delta^*(1600)$ and $\Delta^*(1620)$ at energies above 1.5 GeV. Though the model should be improved to reasonably incorporate the ISI and FSI, the conclusions reached from our model should be helpful to the future experiments to be performed at COSY and HIRFL-CSR as well as further theoretical study.
on related problems. Our results also give hints to the ABC effect in the $pn \rightarrow d\pi^0\pi^0$ and $pd \rightarrow ^3\text{He}\pi^0\pi^0$ reactions which need to be further explored.

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FIG. 2: (color online) Total cross sections of $NN \to NN\pi\pi$. The black solid, red short-dash-dotted, blue dashed, orange dotted, green dotted, cyan short-dashed, green dash-dotted, royal short-dotted, magenta dash-dot-dotted, pink dotted and bold solid curves correspond to contribution from double-$\Delta$, $N^*(1440) \to N\sigma$, $N^*(1440) \to \Delta\pi$, $\Delta \to \Delta\pi$, $\Delta \to N\pi$, $\Delta^*(1600) \to \Delta\pi$, $\Delta^*(1600) \to N^*(1440)\pi$, $\Delta^*(1620) \to \Delta\pi$, nucleon pole, $N \to \Delta\pi$ and the full contributions, respectively. The solid circles and triangles represent the data from Ref. [6, 9–11]. The open circles
FIG. 3: Differential cross sections of $pp \rightarrow nn\pi^+\pi^+$ at beam energies 1100 MeV. The dashed, dotted and solid curves correspond to the phase space, double-$\Delta$ and full model distributions, respectively. The data are from Ref. [7].
FIG. 4: The $N^*(1440) \rightarrow \Delta\pi$, $N^*(1440) \rightarrow N\sigma$ terms of $pp \rightarrow pp\pi^+\pi^-$. The dashed, dotted and solid curves correspond to $\pi$-meson exchange, $\sigma$-meson exchange and total contribution.
FIG. 5: Differential cross sections of $pp \rightarrow pp\pi^+\pi^-$ at beam energies 750 MeV. The dashed, dotted and solid curves correspond to the phase space, $N^*(1440) \rightarrow N\sigma$ and full model distributions, respectively. The data are from Ref. [6].
FIG. 6: Differential cross sections of $pp \rightarrow pp\pi^+\pi^-$ at beam energies 775 MeV. The meaning of curves is the same as Fig. 5. The data are from Ref. [6].
FIG. 7: Differential cross sections of $pp \rightarrow pp\pi^+\pi^-$ at beam energies 800 MeV. The meaning of curves is the same as Fig. 5. The data are from Ref. [9].
FIG. 8: Differential cross sections of $pp \rightarrow ppp^{+}\pi^{-}$ at beam energies 895 MeV. The meaning of curves is the same as Fig. 5. The preliminary data are from Ref. [8].
FIG. 9: Differential cross sections of $pp \rightarrow pp\pi^+\pi^-$ at beam energies 1100 MeV. The dashed, dotted and solid curves correspond to the phase space, $N^*(1440) \rightarrow \Delta\pi$ and full model distributions, respectively. The preliminary data are from Ref. [7].
FIG. 10: Differential cross sections of $pp \rightarrow pp\pi^+\pi^-$ at beam energies 1360 MeV. The meaning of curves is the same as Fig. 9. The data are from Ref. [8].
FIG. 11: Differential cross sections of $pp \rightarrow pp\pi^0\pi^0$ at beam energies 775 MeV. The meaning of curves is the same as Fig. 5. The data are from Ref. [8].
FIG. 12: Differential cross sections of $pp \rightarrow pp\pi^0\pi^0$ at beam energies 895 MeV. The meaning of curves is the same as Fig. 5. The data are from Ref. [8].
FIG. 13: Differential cross sections of $pp \rightarrow pp\pi^0\pi^0$ at beam energies 1000 MeV. The dashed, dotted and solid curves correspond to the phase space, double- $\Delta$ and full model distributions, respectively. The data are from Ref. [8].
FIG. 14: Differential cross sections of $pp \rightarrow pp\pi^0\pi^0$ at beam energies 1100 MeV. The meaning of curves is the same as Fig. 13. The data are from Ref. [8].
FIG. 15: Differential cross sections of $pp \rightarrow pp\pi^0\pi^0$ at beam energies 1200 MeV. The meaning of curves is the same as Fig. 13. The data are from Ref. [8].
FIG. 16: Differential cross sections of $pp \rightarrow pp\pi^0\pi^0$ at beam energies 1300 MeV. The meaning of curves is the same as Fig. 13. The data are from Ref. [8].
FIG. 17: (color online) Total cross sections of $NN \to NN\pi\pi$. The black solid and red dashed curves correspond to the full contributions without and with final state interactions, respectively. The data are the same as the Fig. 2.
FIG. 18: Differential cross sections of $pp \rightarrow nn\pi^+\pi^+$ at beam energies 1100 MeV. The dashed, dotted and solid curves correspond to the distributions of the phase space, the full model with and without FSI, respectively. The data are from Ref. [7].