MAGNITUDE GAP STATISTICS AND THE CONDITIONAL LUMINOSITY FUNCTION

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ABSTRACT

In a recent preprint, Hearin et al. (H12) suggest that the halo mass–richness calibration of clusters can be improved by using the difference in the magnitude of the brightest and the second brightest galaxy (magnitude gap) as an additional observable. They claim that their results are at odds with the results from Paranjape & Sheth (PS12) who show that the magnitude distribution of the brightest and second brightest galaxies can be explained based on order statistics of luminosities randomly sampled from the total galaxy luminosity function. We find that a conditional luminosity function (CLF) for galaxies which varies with halo mass, in a manner which is consistent with existing observations, naturally leads to a magnitude gap distribution which changes as a function of halo mass at fixed richness, in qualitative agreement with H12. We show that, in general, the luminosity distribution of the brightest and the second brightest galaxy depends upon whether the luminosities of galaxies are drawn from the CLF or the global luminosity function. However, we also show that the difference between the two cases is small enough to evade detection in the small sample investigated by PS12. This shows that the luminosity distribution is not the appropriate statistic to distinguish between the two cases, given the small sample size. We argue in favor of the CLF (and therefore H12) based upon its consistency with other independent observations, such as the kinematics of satellite galaxies, the abundance and clustering of galaxies, and the galaxy–galaxy lensing signal from the Sloan Digital Sky Survey.

Key words: cosmology: theory – galaxies: clusters: general – galaxies: evolution – galaxies: halos – methods: numerical

Online-only material: color figures

1. INTRODUCTION

The abundance of halos at the massive end is sensitive to variations in cosmological parameters such as the matter density parameter ($\Omega_m$) and the amplitude of the power spectrum of density fluctuations in the universe (characterized by $\sigma_8$). Therefore, observations of the abundance of galaxy clusters which reside in such massive halos can be used to constrain these cosmological parameters (see, e.g., Vikhlinin et al. 2009; Mantz et al. 2010; Rozo et al. 2010). Additionally, such observations can also be used to constrain the phenomenological behavior of dark energy and test the nature of gravity through measurements of the growth of structure, questions which are of fundamental importance for cosmologists today (see, e.g., Rapetti et al. 2010, 2012). Photometric surveys such as the Dark Energy Survey (Dark energy survey collaboration 2005), the Hyper-Suprime Cam survey (Miyazaki et al. 2006) in the immediate future, and the Large Scale Synoptic Telescope Survey in the near future, will result in a large catalog of optically selected galaxy clusters, which can be used to answer these important questions.

Identifying the galaxy cluster observables in optical surveys which tightly correlate with halo mass, establishing the scaling relations between these observables and halo mass and the scatter in these relations are all crucial steps in order to achieve the scientific goals. It is well known that the number of cluster members (also called richness) correlates with halo mass (see, e.g., Becker et al. 2007; Johnston et al. 2007; Sheldon et al. 2009), and so does the luminosity (or stellar mass) of the brightest (or central) galaxy (Mandelbaum et al. 2006; More et al. 2009b, 2011; Moster et al. 2010; Behroozi et al. 2010) or the total stellar or luminosity content in the group (see, e.g., Yang et al. 2007 for results based on abundance matching). However, these scaling relations have considerable scatter, and therefore combining multiple observables, especially those which come without additional observational costs, is an important task.

Recently, Hearin et al. (2012) suggested that at fixed richness, the magnitude difference (also called magnitude gap or equivalently the luminosity ratio) between the brightest and the second brightest galaxy contains information about halo mass. By using a simple subhalo abundance matching prescription they showed that at fixed richness, the small (large) magnitude gap systems are expected to preferentially reside in less (more) massive halos. To test the proposition with real data, the authors used the galaxy group catalog of Berlind et al. (2006) and showed that at fixed velocity dispersion (proxy for halo mass), the average richness of small magnitude gap systems is significantly larger than the average richness of the large magnitude gap systems, and the difference is larger than that expected if the luminosities of galaxies in every group were drawn randomly from the galaxy luminosity function.

Observationally, systems with large magnitude gaps have often been classified as fossil systems (see, e.g., Ponman et al. 1994; Jones et al. 2003; Khosroshahi et al. 2007; Smith et al. 2010). Studies of the magnitude gap in statistically large samples of groups show a continuous distribution of magnitude gaps rather than a clear segregation of groups into fossil and non-fossil systems (see, e.g., van den Bosch et al. 2007; Yang et al. 2008). These studies also show that the magnitude gap depends upon halo mass such that more massive halos on average have smaller magnitude gaps. It is interesting that the results of Hearin et al. (2012) indicate a reversal of this trend when one considers systems with fixed richness.

Theoretically, it is expected that dynamical friction causes brighter satellites in massive halos to merge with the central
galaxy, increasing the magnitude gap between the brightest satellite and the central galaxy. Analysis of numerical simulations reveals that larger magnitude gaps can be loosely correlated with an earlier assembly of mass in a halo (see, e.g., D’Onghia et al. 2005; Dariush et al. 2010). This suggests that central galaxies are expected to be special, they occupy the deepest portion of the potential well, where they grow by feeding on the satellites that are dragged to the center of halos by dynamical friction. Whether this is indeed the case, or whether the luminosity of the central galaxies is just a matter of chance, is a matter which can be settled with observations.

In Paranjape & Sheth (2012, see also Skibba et al. 2007), the authors examine this question by looking at the luminosity distribution of the brightest and the second brightest galaxy and the magnitude gap between the two using the group catalog of Berlind et al. (2006). They find that the luminosity distributions are consistent with the distribution of the brightest and the second brightest of a given group, and that it is just a matter of chance that any galaxy becomes the brightest in a given group. These results imply that the magnitude gap should not contain any more information about the halo mass than that contained in the richness, in apparent contrast with the results from Hearin et al. (2012), which are based on the same group catalog.

In this paper, we attempt to clarify this issue, by predicting the magnitude gap based on the conditional luminosity function (CLF), which describes the halo occupation distribution of galaxies in a halo of given mass. The CLF and its variation with halo mass has been calibrated using a wide variety of observations such as the abundance of galaxies, their clustering, and the galaxy–galaxy lensing signal measured from the Sloan Digital Sky Survey (SDSS; York et al. 2000). We show that if galaxies occupy halos according to the CLF, it is natural to expect that the magnitude gap depends on the halo mass at fixed richness. We also show that the luminosity distributions of the brightest and the second brightest galaxies are predicted to be different from those obtained by random draws from the luminosity function. However, detecting this small difference just using the luminosity distribution will require sample sizes which are larger than the one used by Paranjape & Sheth (2012).

We also note that in their paper, Paranjape & Sheth (2012) investigate the luminosity-weighted marked correlation function and show that its radial dependence implies that the luminosities of the brightest galaxies are not a matter of chance. They conclude that their results falsify the hypothesis that the luminosities of the brightest and the second brightest galaxy are drawn from the global luminosity function (i.e., without any dependence on halo mass or environment) and that the luminosity distribution alone is not an appropriate discriminant to investigate this issue. In this paper, our results based on the CLF will strengthen this argument.

This paper is organized as follows. In Section 2, we describe the CLF framework and give analytical expressions for the magnitude gap distribution based upon the CLF. In Section 3, we show the magnitude gap distribution from Monte Carlo simulations based on the CLF and compare the results to the analytical expression presented in Section 2. We also investigate the dependence of the magnitude gap upon the richness in a group and the assumed CLF parameterization. In Section 4, we construct mock galaxy catalogs based upon galaxy luminosities sampled from (1) the CLF and (2) the overall galaxy luminosity function and compare the luminosity distributions of the brightest and the second brightest galaxy in these two catalogs. Finally, we summarize our results in Section 5.

For the purposes of this paper, we adopt the following convention. We refer to galaxies as centrals (satellites), if they are drawn from the CLF which is specific to the central (satellite) galaxies (see Equations (2) and (4)). As our fiducial model, we assume that central galaxies are also the brightest galaxies in the halo. Therefore, in the fiducial case, the magnitude gap is the difference in magnitudes between the central galaxy and the brightest satellite. However, we will also investigate cases when the satellites are allowed to be brighter than the central galaxy (see Skibba et al. 2011 for observational evidence of such a possibility).

### 2. CONDITIONAL LUMINOSITY FUNCTION

The CLF, denoted by \( \Phi(L|M) \), is defined to be the average number of galaxies of luminosities \( L \pm dL/2 \) that reside in a halo of mass \( M \) (Yang et al. 2003). The average number of galaxies in a given halo of mass \( M \) can be found by simply integrating the CLF over the luminosities of interest, e.g., the average number of galaxies with luminosities between \( L_{\text{min}} \) and \( L_{\text{max}} \) that reside in a halo of mass \( M \) is given by

\[
\langle N \rangle_M(L_{\text{min}}, L_{\text{max}}) = \int_{L_{\text{min}}}^{L_{\text{max}}} \Phi(L|M) dL. \tag{1}
\]

For convenience, the CLF is divided into a central galaxy component (\( \Phi_c(L|M) \)) and a satellite galaxy component (\( \Phi_s(L|M) \)).

We assume that the distribution \( \Phi_c(L|M) \) is described by a log-normal distribution with a scatter, \( \sigma_c \), that is independent of halo mass, consistent with the findings from studies of satellite kinematics (More et al. 2009a, 2009b, 2011) and galaxy group catalogs (Yang et al. 2009),

\[
\Phi_c(L|M) dL = \frac{\log e}{\sqrt{2\pi} \sigma_c} \exp \left[ -\frac{(\log L - \log L_c)^2}{2 \sigma_c^2} \right] dL/L. \tag{2}
\]

The dependence of the logarithmic mean luminosity, log \( \bar{L}_c \), on halo mass is given by

\[
\log \bar{L}_c(M) = \log \left[ L_0 \left( \frac{M/M_1}{1 + (M/M_1)^{\gamma_1}} \right)^\gamma_2 \right]. \tag{3}
\]

Four parameters are required to describe this dependence: two normalization parameters, \( L_0 \) and \( M_1 \), and two parameters \( \gamma_1 \) and \( \gamma_2 \) that describe the slope of the \( \bar{L}_c(M) \) relation at the low-mass end and the high-mass end, respectively.

The satellite CLF, \( \Phi_s(L|M) \), is assumed to be a Schechter-like function,

\[
\Phi_s(L|M) dL = \Phi_0 \left( \frac{L}{L_\ast} \right)^{\alpha_s} \exp \left[ -\left( \frac{L}{L_\ast} \right)^p \right] \frac{dL}{L_\ast}. \tag{4}
\]

Here \( L_\ast(M) \) determines the knee of the satellite CLF and is assumed to be a factor \( f_s \) times fainter than \( L_c(M) \). Motivated by results from the SDSS group catalog of Yang et al. (2008), we set \( f_s = 0.562 \) (see also Reddick et al. 2012), \( p = 2 \), and assume that the faint-end slope of the satellite CLF is independent of halo mass. The logarithm of the normalization, \( \Phi_0 \), is assumed to have a quadratic dependence on log \( M \) described by three free parameters, \( b_0 \), \( b_1 \), and \( b_2 \):

\[
\log \Phi_0 = b_0 + b_1 (\log M - 12) + b_2 (\log M - 12)^2. \tag{5}
\]
Note that this functional form does not have a physical motivation; it merely provides an adequate description of the results obtained by Yang et al. (2008) from the SDSS galaxy group catalog. The parameters of the CLF and their variation with halo mass can be constrained by using observations of the abundance, the clustering, and the galaxy–galaxy lensing signal measured from the SDSS (More et al. 2012a, 2012b; Cacciato et al. 2012). In what follows, we will use the following values for the CLF parameters: \( L_0 = 10^{9.95} h^{-2} \, L_\odot \), \( M_1 = 10^{11.27} h^{-1} \, M_\odot \), \( \sigma_c = 0.156 \), \( \gamma_1 = 2.94 \), \( \gamma_2 = 0.244 \), \( \alpha_c = -1.17 \), \( b_0 = -1.42 \), \( b_1 = 1.82 \), and \( b_2 = -0.30 \), consistent with the results presented in Cacciato et al. (2012).

If the luminosities of galaxies in a halo are drawn in an uncorrelated fashion, the probability that a halo of mass \( M \) and richness \( N \) has a magnitude gap, \( \Delta m \), or equivalently the luminosity ratio, \( f_L \), between the brightest satellite galaxy and the central galaxy in a halo of mass \( M \) is then given by:

\[
P(f_L | N, M) = \int_{L_{min}}^{\infty} dL' \, p_x(L'|M) \, P_x(L'/f_L|M) \times [p_x(L'|M)]^{N-2}.
\]

where the symbol \( x \) can either stand for central (c) or satellite (s). The quantities \( \langle N_x \rangle_M \) in the relevant luminosity intervals can be obtained by replacing \( \Phi(L|M) \) by \( \Phi_x(L|M) \) inside the integral in Equation (1). For central galaxies we choose \( L_{max} = \infty \). In our model, we assume that the central galaxies are always the brightest in the halo. Therefore, in the case of satellites, we use the luminosity of the central under consideration as the upper limit, i.e., \( L_{max} = L'/f_L \).

The integrals for \( \langle N_c \rangle_M \) and \( \langle N_s \rangle_M \) can be written in terms of the complementary error function and the incomplete gamma function, respectively, such that

\[
\langle N_c \rangle_M(L_1, L_2) = \frac{1}{2} \left[ \text{erfc} \left( \frac{\log L_1 - \log L_c}{\sqrt{2} \sigma_c} \right) \right. \\
- \left. \text{erfc} \left( \frac{\log L_2 - \log L_c}{\sqrt{2} \sigma_c} \right) \right]
\]

\[
\langle N_s \rangle_M(L_1, L_2) = \frac{\Phi_c}{p} \left[ \Gamma \left[ \frac{\alpha_c + 1}{p}, \left( \frac{L_1}{L_c} \right)^p \right] \right. \\
- \left. \Gamma \left[ \frac{\alpha_c + 1}{p}, \left( \frac{L_2}{L_c} \right)^p \right] \right].
\]

1 Note that our expression differs from Paranjape & Sheth (2012) because in our case the central galaxy luminosity is assumed to be sampled from a probability distribution which differs from the distribution from which the satellites are sampled.

The probability of a halo to have a certain mass, given its richness and the magnitude gap, can be obtained from Equation (6) and Bayes’ theorem,

\[
P(M|f_L, N) = \frac{P(f_L|M, N) P(M|N)}{P(f_L|N)},
\]

and as expected it depends upon the mass–richness relation via the probability distribution \( P(M|N) \). The probability distribution within a given bin of richness \([N_1, N_2]\) is given by

\[
P(M|f_L, N_1 < N < N_2) = \sum_{N=N_1}^{N_2} \frac{P(f_L|M, N) P(M|N)}{P(f_L|N)}.
\]

Finally, the distribution of the magnitude gap at fixed halo mass (without regard to the richness) is given by

\[
P(f_L | M) = \sum_{N=2}^{\infty} P(f_L | N, M) P(N|M).
\]

In what follows, we will also investigate the effect of allowing satellite galaxies to be brighter than the central galaxies in their halo. The analytical expressions for predicting the magnitude gap distribution in this case are presented in the Appendix.

3. RESULTS FROM SIMULATED SAMPLE

We now demonstrate explicitly that for fixed richness, the CLF predicts that the magnitude gap in a given group of galaxies depends upon the mass of the halo in which these galaxies reside. The CLF varies with halo mass and therefore it is not surprising that this indeed is the case. For this purpose, we generate Monte Carlo samples of galaxies that populate halos according to the CLF in the following manner.

For a halo of given mass, we first draw the luminosity of its central galaxy from \( \Phi_{cen}(L|M) \), given by Equation (2). In order to avoid the existing correlation between halo mass and richness affecting our conclusions, we fix the number of satellites \( N_{sat} = 20 \). For each of the \( N_{sat} \) satellites, we then draw a luminosity from the satellite CLF \( \Phi_{sat}(L|M) \), given by Equation (4). While drawing the satellite luminosities, we adopt a luminosity threshold, \( L_{min} \), corresponding to \( 0.5 \, M_\odot - 5 \log h = -19 \) (here \( 0.5 \, M_\odot \) indicates the SDSS r-band magnitude, \( K \)-corrected to \( z = 0.1 \); see Blanton et al. 2003). As mentioned before, we also assume that the satellites are always fainter than the central galaxy drawn for a given halo.

The resultant distribution of the magnitude gaps is shown in Figure 1 using a solid histogram for a wide range of halo masses. Note that we do not expect a halo with a given number of satellites, e.g., \( N_s = 20 \), to have such a large distribution in halo mass. The typical scatter in halo masses at fixed richness is of the order of \( \sim 45\% \) (Rozo et al. 2009). The wide range in halo mass has been chosen only for the purpose of highlighting the trend. It can be seen that the distribution of the magnitude gaps depends upon halo mass. We use Equation (6) to predict this distribution analytically and compare it to the results from our simulations. The result of this analytical calculation is shown as solid curves in Figure 1 and agrees well with the magnitude gap distribution from our simulations.

For low-mass halos, the distribution of magnitude gaps is peaked at zero. However, this peak shifts away from zero as we move to larger halo masses. This figure establishes that if galaxies populate halos according to the CLF (which is
Figure 1. Distribution of the difference in magnitudes between the brightest and the second brightest galaxy predicted by simulations in which we populate galaxies in halos of different mass according to the conditional luminosity function. The solid histograms show the distribution of magnitude gaps in halos of different mass (shown using different colors) for our fiducial model in which we assume that the central galaxy (defined to be drawn from the central CLF) is the brightest in the halo. The solid curves show the analytical prediction based on Equation (6). The dashed histograms show the corresponding result for the case when we allow satellites to be brighter than the central galaxy.

(A color version of this figure is available in the online journal.)

Figure 2. Distribution of magnitude gaps (similar to Figure 1) for the case when the number of satellites is equal to 5 and 64 is shown in the left and right-hand panels, respectively, and assuming that the central galaxy is the brightest in the halo.

(A color version of this figure is available in the online journal.)

supported by several observations such as galaxy group catalog, and the observations of abundance clustering and galaxy lensing from SDSS, the magnitude gap should have more information about the halo mass, in addition to that conveyed by richness alone. At fixed richness, higher mass halos tend to have larger magnitude gaps, in agreement with Hearin et al. (2012). Our result that the magnitude gap distribution for low-mass halos is peaked at zero, and shifts to larger magnitude gaps for larger mass halos, may appear to be exactly the opposite of the result presented in van den Bosch et al. (2007). However, note that the magnitude gap distributions we present are at fixed richness and halo mass ($P(f_\ell | N, M)$), while the magnitude gap distributions shown in the different panels in Figure 5 of van den Bosch et al. (2007) correspond to groups with varying richness (thus corresponding to $P(f_\ell | M)$, see Equation (13)), due to the underlying mass–richness relation. We will shortly consider the effect of changing richness on the magnitude gap distribution.

First, we consider the effect of relaxing the assumption that the centrals are the brightest in their halos. Note that in this case, the magnitude gap could be either between the central and the brightest satellite, or between the two brightest satellites, if the halo has two or more satellites brighter than the putative central galaxy (see the Appendix). The resultant magnitude gap is shown with a dashed histogram. For low-mass halos, it can be hardly distinguished from the case when we demand the central to be the brightest. It can be also seen that for all halo masses the distribution of magnitude gaps for $\Delta m_{12} > 0.5$ is consistent with the case when the central galaxy is assumed to be the brightest. This is also expected since the satellite CLF in our model dies exponentially at the bright end. Therefore if there is a satellite galaxy brighter than the central galaxy, the magnitude gap is not expected to be extremely large. Therefore, the few cases when the satellite galaxy is brighter cause a small but noticeable increase in the probability distribution at the small magnitude gap end.

We show the results of varying the number of satellites in Figure 2. The left-hand panel shows the magnitude gap distribution in halos of different mass, when the number of satellites equals 5, while the right-hand panel shows the same for number of satellites equal to 64. As the number of satellites increases (decreases) the magnitude gap tends to be smaller (larger), as expected (and qualitatively consistent with the results presented in Figure 5 of van den Bosch et al. 2007). The analytical expectation (from Equation (6)) is shown as a solid curve; it describes the simulation results accurately, and is shown as a sanity check.

The parameter $p$ governs the exponential cutoff at the bright end of the satellite CLF (see Equation (4)). Based upon the analysis of offsets of the line-of-sight velocities and projected position of the brightest galaxy relative to the mean of the other group members, Skibba et al. (2011) concluded that the value of
$\rho$ ought to be closer to unity instead of the fiducial value of two that we assume (see also Reddick et al. 2012). Therefore, we also show the effect of varying the parameter $\rho$ on the magnitude gap distribution in Figure 3. We have verified that the predictions based upon Equation (6) that we show in the figure also agree with detailed simulations. As expected, decreasing the value of $\rho$ causes the satellite CLF to fall less rapidly at the bright end which results in smaller magnitude gaps.

Regardless of these details, it is clear that the results from this section establish that if galaxies populate halos according to the CLF, then at fixed richness the magnitude gap distribution should depend upon the halo mass, in a manner which is qualitatively consistent with Hearin et al. (2012).

4. LUMINOSITY DISTRIBUTION OF THE BRIGHTEST AND SECOND BRIGHTEST GALAXY

We would like to now investigate the result presented in Paranjape & Sheth (2012). They demonstrate that the luminosity distribution of the brightest and the second brightest galaxies in the group catalog of Berlind et al. (2006) is consistent with their expected distribution if the luminosity of galaxies in each of the groups were randomly sampled from the global luminosity function of galaxies. To verify their result, we construct Monte Carlo galaxy catalogs in which galaxy luminosities are drawn from the global luminosity function, $\Phi(L)$,

$$\Phi(L) = \int \Phi(L|M)n(M)dM,$$  

(14)

where $n(M)$ is the halo mass function. In practice, we randomly sample (with replacement) from the luminosities of galaxies in the entire previous catalog, while maintaining the richness of the group they belong to, thus effectively sampling the galaxy luminosities in every group from the global luminosity function of galaxies.

The luminosity distributions of the brightest and the second brightest galaxies in each halo for both the catalogs are shown in the upper left and right hand panels of Figure 4, respectively. The peak of the magnitude distribution of the brightest galaxies in Catalog A is at a slightly higher value of luminosity compared to Catalog B. On the other hand, the magnitude distribution of the second brightest galaxies shows a tail towards larger luminosities in Catalog B compared to that in Catalog A. However, the plot also shows that the differences are not that huge, and detecting such differences in the magnitude distributions will require a large sample of groups.

From our large sample of Monte Carlo groups, we now restrict ourselves to selecting sample sizes (~350) which are similar to those used by Paranjape & Sheth (2012). We show the results of one of the random realizations in the bottom panel of Figure 4. We also obtain the corresponding cumulative distributions and use the Kolmogorov–Smirnov (K-S) statistic to compare the distributions from the two catalogs. The $p$-values from the K-S test are indicated in the corresponding panels and these values imply that the luminosity distributions from the two catalogs, when downsamled to the size of the catalog that Paranjape & Sheth (2012) use, are consistent with each other. To show that this particular random realization is not a statistical fluke, in Figure 5, we show the distribution of $p$-values from K-S tests carried out on 1000 random samples similar in size to the catalog used by Paranjape & Sheth (2012). The distribution of $p$-values from the K-S test peaks at values larger than 0.1, which highlights the difficulty in distinguishing between the magnitude distributions from the two catalogs with a small sample size. This suggests that the group catalog used by Paranjape & Sheth (2012) does not have enough number statistics to detect the difference between the luminosity distributions of the brightest (or the second brightest) galaxies in the cases corresponding to the two catalogs.

It is well known that the luminosity of central galaxies depends upon the halo mass in which they reside. However, it is also known that at the massive end, the luminosity of central galaxies is a weak function of halo mass, e.g., based on two-point statistics such as the projected galaxy–galaxy correlation function, its dependence upon luminosity of galaxies
Figure 4. Comparison between the luminosity distributions of the brightest and the second brightest galaxy in the halos present in the two mock galaxy catalogs are shown in the left- and right-hand panels, respectively. Upper panels: the solid line shows the luminosity distribution when galaxies are populated in halos according to random draws from the CLF (Catalog A), while the dashed histogram shows the distribution when galaxies are populated according to random draws from the global luminosity function (Catalog B), maintaining the richness of halos. Bottom panels: same as the upper panels but for a catalog with sample size comparable to the one used by Paranjape & Sheth (2012). The differences in the distribution from the two catalogs, as quantified by the p-values from the K-S test, are indicated in each panel.

(A color version of this figure is available in the online journal.)

(Zehavi et al. 2005, 2011; Zheng et al. 2007), and the projected galaxy–matter correlation function probed by the galaxy–galaxy lensing measurements (Mandelbaum et al. 2006; Cacciato et al. 2009, 2012), or other probes such as satellite kinematics (More et al. 2009b, 2011) and subhalo abundance matching (Moster et al. 2010; Behroozi et al. 2010; Yang et al. 2012). This, coupled with the fact that the satellite fraction is very low at the bright end, could be a reason why the differences in the magnitude distribution of the brightest galaxy between the two different catalogs are not that large.

We note that this insensitivity of the magnitude distributions to the underlying halo occupation distribution was also pointed out by Paranjape & Sheth (2012). Instead of considering the magnitude distributions, the mean of the magnitude distributions may be better suited to detecting the differences between the two scenarios (S. Shen et al., in preparation). Alternatively, Paranjape & Sheth (2012) suggest the use of two-point statistics such as the luminosity-marked correlation function, in order to distinguish between the two scenarios. Based on the radial dependence of the marked correlation function, they concluded that the galaxy luminosities in groups cannot be drawn from a global luminosity function. However, their analysis does not directly address whether this is due to a CLF which varies with mass or a result of environmental dependences of the luminosity function.

5. SUMMARY

Recently, Hearin et al. (2012) suggested that the magnitude gap between the two brightest galaxies in a given halo at fixed richness contains additional information about the halo mass. Their claim was based upon an analysis of the galaxy group catalog constructed from the SDSS by Berlind et al. (2006). If correct, the magnitude gap information can be used to reduce the scatter in the mass–richness relation in galaxy clusters, which is important for the use of optically identified galaxy clusters as cosmological probes. However, they claimed that their result is at odds with the results presented in Paranjape & Sheth (2012) who investigated the distribution of magnitudes of the brightest and second brightest galaxies, from the same
of the brightest and the second brightest galaxies will also be present if the sample size of groups grows, even the luminosity distribution does contain extra information about the mass of a halo. As the group catalog. Paranjape & Sheth (2012) showed that these differences cannot be meaningfully detected given the small sample size that Paranjape & Sheth (2012) use in their study. This shows that the luminosities sampled from the overall galaxy luminosity function independent of halo mass. This would imply that the magnitude gap just depends upon richness and does not contain extra information about the halo mass.

We have investigated both these studies within the framework of the CLF, which describes the halo occupation statistics of galaxies as a function of halo mass. The CLF and its variation with halo mass has been calibrated using observations of the abundance and clustering of galaxies, and the galaxy–galaxy lensing signal in the SDSS, and is consistent with results based upon the kinematics of satellite galaxies and abundance matching. We have shown that if galaxies populate halos according to the CLF and if the luminosities of central and satellite galaxies are drawn from their corresponding CLF in an uncorrelated manner, then the magnitude gap is expected to contain information about halo mass at fixed richness. We have presented analytical expressions for predicting the magnitude gap distribution at fixed richness as a function of halo mass and verified these expressions using Monte Carlo simulation of galaxy catalogs populated according to the CLF.

We have shown that the magnitude distributions of the brightest and the second brightest galaxies show significant differences between mock galaxy catalogs constructed by drawing galaxy luminosities according to the CLF and those constructed according to the luminosity function of galaxies. However, we have also shown that these differences cannot be meaningfully detected given the small sample size that Paranjape & Sheth (2012) use in their study. This shows that the magnitude distribution of the brightest and the second brightest galaxies is not the appropriate statistic to address the issue of how galaxies populate dark matter halos, at least given the current sample sizes.

These results suggest that the apparent tension between the two studies is due to small sample size used by Paranjape & Sheth (2012). The magnitude gap at fixed richness can and does contain extra information about the mass of a halo. As the sample size of groups grows, even the luminosity distribution of the brightest and the second brightest galaxies will also be able to distinguish between the two scenarios. In this paper, we have provided an analytical model based on the CLF to predict the magnitude gap distribution. We have also presented how the magnitude gap distribution can vary as some of our fiducial assumptions are changed.

It is also important to note that the CLF of galaxies in clusters can also be directly observed, albeit as a function of optical properties such as richness (see, e.g., Hansen et al. 2009). Such observations, when combined with halo mass indicators such as weak lensing, can in turn be used to better constrain CLF at the high-mass end, which will help to constrain our model for the magnitude gap, at fixed richness.

Finally, we would like to remark that we have assumed that the luminosities of the galaxies in every group are drawn from the CLF in an uncorrelated fashion. This assumption, however, needs to be thoroughly tested. For example, if bright satellite galaxies merge with the central galaxy due to dynamical friction, the central galaxy will become brighter at fixed halo mass, and the magnitude gap will correspondingly be larger. However, simultaneously the richness of the group will decrease (which will also cause the magnitude gap to be larger just due to the statistics of random draws from the CLF), thus making it difficult to disentangle the physical correlation from the effect due to changing richness.

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APPENDIX

MAGNITUDE GAP WHEN SATELLITES ARE ALLOWED TO BE BRIGHTER THAN THEIR CENTRAL GALAXIES

Based upon the analysis of offsets of the line-of-sight velocities and projected position of the brightest galaxy relative to the mean of the other group members, Skibba et al. (2011) concluded that there is a significant chance that in a halo of given mass, a satellite galaxy is brighter than the central galaxy. In Figure 1 presented in Section 3, we showed the magnitude gap distribution for the case when we allow satellite galaxies to be brighter than centrals. In this Appendix, we provide analytical expressions for the magnitude gap (between the two brightest galaxies) distribution in this case.

Note that since we allow for the possibility that the satellite galaxies can be brighter than the central galaxy, in a given halo, one of the following three mutually exclusive and collectively exhaustive cases may occur: (1) central galaxy still turns out to be the brightest, (2) central galaxy turns out to be second brightest, and (3) central galaxy is not one of the two brightest galaxies. The probabilities corresponding to these three cases are given by

$$P_i(M, N) = \int P_c(L|M) [P_s(< L|M)]^{N-1} dL,$$

(A1)
\[ P_2(M, N) = (N - 1) \int P_c(L|M)[1 - P_s(< L|M)] \]
\[ \times [P_s(< L|M)]^{N-1} dL, \quad (A2) \]

\[ P_3(M, N) = 1 - P_1(M, N) - P_2(M, N), \quad (A3) \]

respectively. The magnitude gap distribution can then be expressed as

\[ P(f_L, M, N) = P_1(M, N)(N - 1) \int_{L_{\text{min}}}^{\infty} dL' P_s(L'|M) \]
\[ \times P_c(L'/f_L|M) [P_s(< L'|M)]^{(N-2)} \]
\[ + P_2(M, N)(N - 1) \int_{L_{\text{min}}}^{\infty} dL' P_s(L'|M) \]
\[ \times P_c(L'/f_L|M) [P_s(< L'|f_L|M)]^{(N-2)} \]
\[ + P_3(M, N)(N - 1)(N - 2) \int_{f_{\text{min}}}^{\infty} dL' P_s(L'|M) \]
\[ \times P_c(L'/f_L|M) [P_s(< L'|f_L|M)]^{(N-3)}. \quad (A4) \]

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