The linear regression method of the RC circuit for electrical impedance characterization

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Abstract. An impedance bode plot modelling simulation has been conducted with the regression method to determine the RC model. The aim of the research is to determine the RC model and determine the resistance and capacitance of the equivalent circuit. The study uses nine RC models which are modelled into a linear regression equation and the equivalent model is determined based on the smallest error. Gradient and regression constants are used to determine the resistance and capacitance of the equivalent model. The results in the RC series show that the equivalent model is the first model with a resistance 998.580 Ω and a capacitance 9.253 pF with an error 2.4%. While the program implementation of the RC parallel produces a resistance 999.809 Ω and a capacitance 9.995 pF with an error 0.042%.

1. Introduction

Animals and plants develop from cells and tissues. For example, the human body is a biological subject which is a very complex structure constructed from several tissues [1]. Biological cells, which contain intracellular fluid (ICF), a cell membrane with or without a cell wall, are suspended in extracellular fluid (ECF) and exhibit frequency-dependent behavior to electrical signals. Biological cells and tissues produce complex bioelectric impedances [2, 3, 4] which depend on the tissue composition and the frequency of the electrical signal [2, 3, 4, 5]. Therefore the electrical impedance frequency response of biological tissues is strongly influenced by physiological and physiochemical status and varies of subject.

The past several decades, a number of impedance-based non-invasive tissue characterization techniques such as bioelectric impedance analysis (BIA), electrical impedance spectroscopy (EIS), electrical impedance plethysmography (IPG), impedance cardiography (ICG), and electrical impedance tomography (EIT) have been developed, proposed and much research work has been done on this method for noninvasive tissue characterization and disease diagnosis [6].

The impedance spectrum can be used to study the overall anatomy and physiology of organic tissue. The plots of impedance parameters include Nyquist plots (imaginary Z versus real Z parts), bode plots (impedance versus frequency, conductivity versus frequency plots, and permittivity versus frequency plots), which are popular tools for visualizing various aspects of bio-impedance [7, 8].

The device used to obtain bioelectric impedance spectrum data is EIS [9]. The spectrum is then analyzed so that information on the group or type of object can be obtained. In order to obtain this information, this research was conducted, namely by building an alternative model of the object but it was still limited to a model containing R and C with a linear approach.
2. Method
In this study, the impedance parameter used is the impedance bode plot with the regression modelling method. The impedance as frequency response is modelled in such a way that it becomes a regression equation. The first step is to select an impedance plot bode model consisting of a resistor and/or capacitor components, with series, parallel, or mixed circuit models. The second step is to model the impedance equation from the selected impedance plot bode models into a regression equation with impedance as the dependent variable and frequency as the independent variable. The third step is to create a regression program and input the frequency and impedance values obtained from the measurement. The RC circuit model with the smallest error is considered to be the model closest to the sample material. From the regression equation, the gradient and regression constants are obtained. Gradient and regression constants are used to determine the resistance and capacitance impedance of the material.

2.1 Selection of RC models
Determine the RC model and derive its impedance equation. The impedance equation of the circuit model is then modeled into a regression equation. The linear regression equation is obtained from the impedance equation, the bode plot of impedance and modulus. This study uses nine RC circuit models which are shown in Table 1.

| No | Circuit models | Equation |
|----|----------------|----------|
| 1  | ![Circuit 1](image1) | \[ Z = R + \frac{1}{j\omega C} \]
\[ F(Z) = R + \frac{1}{j\omega C} \]
\[ |F|^2 = R^2 + \frac{1}{\omega^2 C^2} \] (1) |
| 2  | ![Circuit 2](image2) | \[ \frac{1}{Z} = \frac{1}{R} + j\omega C \]
\[ F(Z) = \frac{1}{R} + j\omega C \]
\[ |F|^2 = \frac{1}{R^2} + \omega^2 C^2 \] (2) |
| 3  | ![Circuit 3](image3) | \[ \frac{1}{Z} = \frac{1 + jR\omega C}{2R} \]
\[ F(Z) = \frac{1 + jR\omega C}{2R} \]
\[ |F|^2 = \frac{1}{4R^2} + \frac{C^2}{4} \omega^2 \] (3) |
| 4  | ![Circuit 4](image4) | \[ \frac{1}{Z} = \frac{3 + jR\omega C}{2R} \]
\[ F(Z) = \frac{3 + jR\omega C}{2R} \]
\[ |F|^2 = \frac{9}{4R^2} + \frac{C^2}{4} \omega^2 \] (4) |
| 5  | ![Circuit 5](image5) | \[ Z = \frac{R}{2} + \frac{1}{2j\omega C} \]
\[ F(Z) = \frac{R}{2} + \frac{1}{2j\omega C} \]
\[ |F|^2 = \frac{R^2}{4} + \frac{1}{4C^2 \omega^2} \] (5) |
2.2. The solving of the regression equation program code
The algorithm for solving the regression equation follows:

a. Input frequency and impedance values.
b. Calculate $x_i$ as a function of frequency and the $y_i$ as a function of impedance.
c. Calculate $\sum x_i$, $\sum y_i$, $\sum x_i^2$ and $\sum x_i y_i$
d. Create a coefficient matrix.
e. Calculate variable matrix.
f. Calculate resistance and capacitance.
g. Calculate error regression approach of each model.

Steps a untill g are repeated for another models, with the $x_i$ and $y_i$ for each model given by equation (1), (2), (3), (4), (5), (6), (7), (8), (9) respectively.

2.3 Test of program code
The data is obtained from the simulation of each model over a certain frequency range, while the resistance and capacitance are set at 1 kΩ and 10 pF, respectively. Program performance testing is carried out nine times. The first program test input is the frequency and impedance of the calculation of the first model equation. If the output of the first model program produces the smallest error compared to other models, it means that the program is correct. The second to nine program test methods are the same, except that the input impedance is the impedance calculated from the second, third, and ninth models. The program code is considered correct if the program output is close to the initial set is the model whose impedance is used as the program input.
2.4 Data analysis
The compiled program is then tested with experimental data. The data are impedance data measured from RC series and RC parallel. The program inputs are frequency and impedance, and the output is resistance, capacitance, and error of each model. The model which has the smallest error is considered as the equivalent model.

3. Result and Discussion
The simulation data of each model is then matched to nine model equations. The impedance at a frequency 10 kHz to 100 kHz obtained from the simulation is used as input. The output are the resistance, capacitance and error of nine models used. The results for models 1 to 9 are shown in Figure 1 and Table 2.
Figure 1. Suitability between data and linear regression model from 9 models
Tables 2 show that the model with the smallest error in a row is that the first model produces an error of 0.129 with a resistance of 1.000 kΩ and a capacitance of 9.995 pF. The second model produces an error of 0.129 with a resistance of 999.999 Ω and capacitance 9.995 pF. It shows that the results of the program test correspond to the input. The input impedance is based on the impedance modulus.

Implementation of experimental data is carried out using impedance measurement data in the RC circuit. The first is the RC series, while the second is the RC parallel. They use 1 kΩ of resistor and 10 pF of capacitor. Measurements are conducted at a frequency of 10 kHz to 100 kHz with intervals of 10 kHz. The bode plot of the series RC is shown in Figure 2 and the parallel RC is shown in Figure 3.
The ‘o’ in Figure 2(a) is an experimental data, while the green line is a linear approach to the model. According to the impedance modulus equation in the series RC circuit from equation (1), the x-axis of the series RC plot bode represents the value of \( \frac{1}{\omega^2} \) and the y-axis represents \(|Z|^2\). The program input is the measured impedance along with the frequency and the output being the resistance, capacitance, and error of nine models. The results of the whole RC model are shown in Figure 2(b). Based on Figure 2 (b), it shows that the first model has the smallest regression error of nine models. So it can be concluded that among nine models, the first model is the closest to the data. The program implementation of the experimental data indicate that the regression method is quite suitable for determining the series RC model.

The implementation of the second experimental data was carried out using the impedance measurement data on the parallel RC. Just like in the series RC, the parallel RC impedance measurements are also carried out at a frequency of 10 kHz to 100 kHz with 10 kHz intervals. The parallel RC bode plot is shown in Figure 3.

![Figure 3](image)

Figure 3. (a) Bode plot of RC parallel, and (b) analysis result

The graph of the experimental data is shown by ‘o’, while the green line is a linear approximation of the experimental data in figure 3(a). The impedance modulus equation for the parallel RC given by equation (2), the x-axis of the parallel RC bode plot represents of \( \omega^2 \), and the y-axis represents \(|\frac{1}{Z}|^2\).

As in the program implementation for the RC series, the program input is the measured impedance along with the frequency with the output being the resistance, capacitance, and error of the nine models. Figure 3 (b) shows that the program output in the second model has the smallest error compared to the other models. It indicates that the second model is equivalent to the RC parallel.

4. Conclusion

Development of the bode plot model to determine the RC model can be done using the linear regression method. The implementation of the RC series program produces 998.580 \( \Omega \) of resistance and 9.253 pF of capacitance with an approximate error 2.4\%. While the implementation of the RC parallel program resulted 999.809 \( \Omega \) of of resistance and 9.995 pF of capacitance with a regression error 0.042\%.
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