More on direct CP violation in $b \to dJ/\psi$ decays

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Abstract

Direct CP violation can occur in $B$-meson decays of the type $b \to d\bar{c}c$, where the charm-anticharm pair forms a $J/\psi$. The CP asymmetry requires the contribution to the amplitude from decays into other states, which rescat ter into $dJ/\psi$ via final state interactions. In particular, states with the same quark content as $dJ/\psi$ can contribute. A perturbative calculation, based on a quark level description of the rescattering process, gives an asymmetry of about 1%, due to this effect. This makes it the dominant contribution to the asymmetry, as suggested by an earlier estimate, based on an hadronic picture.
PACS: 13.25.+m, 11.30.Er, 12.15.Ff.
1 Introduction

Direct CP violation in B-meson decays may generate an asymmetry

\[ a_{CP} = \frac{\Gamma(B \rightarrow f) - \Gamma(\bar{B} \rightarrow \bar{f})}{\Gamma(B \rightarrow f) + \Gamma(\bar{B} \rightarrow \bar{f})}, \]  

between the rates for the CP conjugated processes \( B \rightarrow f \) and \( \bar{B} \rightarrow \bar{f} \), even in the absence of \( B - \bar{B} \) mixing. In the standard model, it is predicted that the most significant asymmetries of this type, which tend to be fairly small \([1, 2, 3]\), occur for Cabibbo suppressed decays. Hence, a large number of \( B \)-mesons is necessary, and the observation of the asymmetries in eq. (1) may be best achieved at hadronic accelerators. The decays \( b \rightarrow q c \bar{c} (q = s, d) \), where the charm-anticharm pair forms a \( J/\psi \), are particularly suitable for a hadronic machine, given the clean signature from \( J/\psi \rightarrow l^+ l^- \). It was pointed out recently by Dunietz \([4]\), and later investigated in more detail in ref. \([5]\), that a CP asymmetry of type (1) appears in these decays. A rough estimate suggested that it could be of order 1%, in decays of the type \( b \rightarrow d J/\psi \), which is the expected reach of an experiment with a sample of \( 10^{10} \) \( B \)-mesons (the analogous asymmetry in \( b \rightarrow s J/\psi \) is suppressed by a factor of \( \sin^2 \theta_C \)).

The CP asymmetry in \( b \rightarrow d J/\psi \) stems from the interference between the dominant tree amplitude and a small absorptive amplitude that contributes coherently. That is due to the process \( b \rightarrow i \rightarrow d J/\psi \), where \( i \) denotes on-mass-shell intermediate states with quark content \( du \bar{u} \) or \( dc \bar{c} \). For the former, the process is OZI suppressed, and the rescattering to \( d J/\psi \) occurs at the order \( \alpha_s^3 \) or via the electromagnetic interaction. It generates an asymmetry of the order of a few \( \times 10^{-3} \) \([4]\). As for the intermediate states with the same quark content, \( dc \bar{c} \), as the final state, it was first pointed out by Wolfenstein \([6]\) that they can contribute to the CP asymmetry in exclusive decays, or in semi-inclusive decays such as \( b \rightarrow d J/\psi \) (although, their contribution is absent in the case of the inclusive decay \([2]\)). In ref. \([5]\), the nature of this effect was discussed in terms of the on-mass-shell hadronic states that form the intermediate state. This approach is reviewed in section 2; it only allows for a rough estimate which suggests that a contribution to the CP asymmetry of about 1% is possible. In section 3, I look at this effect from a different perspective, and try to obtain a better estimate of its contribution to the asymmetry. In the spirit of quark-hadron duality, the collection of hadronic intermediate states is replaced by the corresponding \( dc \bar{c} \) quark configuration.
The final state scattering is then treated perturbatively in $\alpha_s$; the absorptive part of the amplitude and the ensuing CP asymmetry are evaluated at the lowest order. The results obtained are discussed in section 4.

2 The hadronic description

The effect of the final state interactions in a given decay amplitude, $A_i \equiv A(B \to i)$, can be described by the S-matrix, $S = 1 + iT$, for the scattering among different final states. When the amplitudes in $T$ can be treated perturbatively [6, 7],

$$A_i \simeq A_i^{(0)} + i \frac{1}{2} \sum_j T_{ij} A_j^{(0)},$$  \hspace{1cm} (2)

where $A_j^{(0)}$ are the weak decay amplitudes in the absence of the final state interactions. It is the interference between the different terms on the RHS that generates the CP violating quantity

$$\Delta_i \equiv |A_i|^2 - |\bar{A_i}|^2 = \sum_j \Delta_j^i$$  \hspace{1cm} (3)

with

$$\Delta_j^i = 2T_{ij} \text{Im}\{A_j^{(0)*} A_i^{(0)}\}.$$  \hspace{1cm} (4)

Clearly, $\Delta_i^i = 0$: the rescattering of the final state does not contribute to the asymmetry, since it does not generate a term in eq. 2 with a different CP-odd phase than that of $A_i^{(0)}$. For the case of the inclusive decay $b \to d\bar{c}c$, this means that there is no contribution to the asymmetry from the intermediate state $d\bar{c}c$. However, the situation is different for the exclusive or semi-inclusive cases [8]. In ref. [5] the decay $B^- \to J/\psi\pi^-$ was examined, as an example. Including the absorptive part that is due to the intermediate states $X = D^0D^-, D^{*-}D^0, J/\psi\rho^-, \ldots$, that have the same quark content as the final state $J/\psi\pi^-$, the decay amplitude is

$$A(B^- \to J/\psi\pi^-) = V_{cb} V_{cd}^* T_{\psi\pi^-} + V_{tb} V_{td}^* P_{\psi\pi^-}$$

$$\phantom{A(B^- \to J/\psi\pi^-)} + i \frac{1}{2} \sum_X (V_{cb} V_{cd}^* T_X + V_{tb} V_{td}^* P_X) A(X \to J/\psi\pi^-).$$  \hspace{1cm} (5)
The weak amplitudes include both tree and penguin contributions, proportional to $V_{cb}V_{cd}^*$ and $V_{tb}V_{td}^*$, respectively. The penguin/tree ratios $P_{\psi\pi^-}/T_{\psi\pi^-}$ and $P_X/T_X$ will in general be different, and so the dispersive and absorptive parts of the amplitude in eq. 5 will have different CP-odd phases. Then, the states $X$ will contribute to the CP asymmetry with

$$a_{CP} \simeq Im\left\{ \frac{V_{tb}V_{td}^*}{V_{cb}V_{cd}^*} \sum_X T_{\psi\pi^-}^* T_X A(X \rightarrow J/\psi\pi^-) \right\} \left( \frac{P_X}{T_X} - \frac{P_{\psi\pi^-}}{T_{\psi\pi^-}} \right).$$

There is no reliable way of calculating the scattering amplitudes $A(X \rightarrow J/\psi\pi^-)$ at the hadronic level (moreover, a large number of hadronic states, including those with larger multiplicity, should be included). In ref. [5], the asymmetry due to some of the intermediate states $X$ was estimated, leaving the ratio

$$\xi_X \equiv \frac{T_{\psi\pi^-}^* T_X A(X \rightarrow J/\psi\pi^-)}{|T_{\psi\pi^-}|^2}$$

as an undetermined parameter. Contributions to the asymmetry of about

$$a_{CP} = \xi_X \times 1\% \times \frac{\eta}{0.4}$$

were found ($\eta = -Im\{(V_{cb}V_{td})/(V_{cb}V_{cd})\}$, in the Wolfenstein parametrization of the CKM matrix).

### 3 Quark level description

In view of the difficulties of the approach outlined in the previous section, an approximate prescription may provide a more complete calculation of the absorptive amplitude. It amounts to replacing the collection of the hadronic intermediate states by the corresponding quark configuration $d\bar{c}\bar{c}$ (in the spirit of the quark-hadron duality). Then, the scattering to $dJ/\psi$ is treated perturbatively in the strong coupling constant, which is chosen at the $m_b$ scale (for $m_b \approx 5.0 GeV$ and $\Lambda_{MS}^{(d)} \approx 200 MeV$, $\alpha_s(m_b) \approx 0.23$). Notice that at zeroth order in $\alpha_s$, there are no intermediate states (other than $dJ/\psi$) that contribute to the absorptive amplitude, as the $J/\psi$ resonance is below the $D - \bar{D}$ threshold, and its overlap with $\psi'$ can be neglected. At the order $\alpha_s$, the presence of the gluon removes the kinematical constraint, and the
intermediate states where $c\bar{c}$ appears in a color octet (i.e., states in the $c-\bar{c}$ continuum) will contribute. The intermediate states with $c\bar{c}$ in a color singlet will be neglected: they can only contribute at higher orders in $\alpha_s$, due to the color selection rule, and the weak decay amplitude into that configuration is color suppressed.

The amplitude for the decay $b \to dJ/\psi$, in the absence of final state scattering, is

$$A^{(0)}_{d\psi} = V_{cb}V_{cd}^*T_{d\psi} + V_{tb}V_{td}^*P_{d\psi}. \quad (9)$$

The tree and penguin terms are calculated from the effective Hamiltonian

$$H_{eff} = -\frac{G_F}{\sqrt{2}} [V_{ub}V_{ud}^* (Q_1 + Q_2) + V_{cb}V_{cd}^* (Q_3 + Q_4)] + V_{tb}V_{td}^* \sum_{k=3}^{6} C_k Q_k + h.c.], \quad (10)$$

where

\begin{align*}
Q_1 &= \bar{d}\gamma^\mu(1-\gamma_5)b\bar{l}\gamma_\mu(1-\gamma_5)l \\
Q_2 &= \bar{l}\gamma^\mu(1-\gamma_5)b\bar{d}\gamma_\mu(1-\gamma_5)l \\
Q_3 &= \sum_{l=u,d,s,c,b} \bar{d}\gamma^\mu(1-\gamma_5)b\bar{l}\gamma_\mu(1-\gamma_5)l \\
Q_4 &= \sum_{l=u,d,s,c,b} \bar{l}\gamma^\mu(1-\gamma_5)b\bar{d}\gamma_\mu(1-\gamma_5)l \\
Q_5 &= \sum_{l=u,d,s,c,b} \bar{d}\gamma^\mu(1-\gamma_5)b\bar{l}\gamma_\mu(1+\gamma_5)l \\
Q_6 &= -2 \sum_{l=u,d,s,c,b} \bar{l}(1-\gamma_5)b\bar{d}(1+\gamma_5)l. \quad (11)
\end{align*}

In the leading-logarithm approximation, the Wilson coefficients at the scale $m_b$ (and for $\Lambda_{\overline{MS}}^{(4)}$ as above) are

\begin{align*}
C_1 &= 0.25 \\
C_2 &= -1.11 \\
C_3 &= 0.011 \\
C_4 &= -0.026 \\
C_5 &= 0.008 \\
C_6 &= -0.032. \quad (12)
\end{align*}
Then,
\[ T_{d\psi} = \frac{G_F}{\sqrt{2}}(C_1 + \frac{1}{N_c}C_2)m_{\psi}f_{\psi}\epsilon_\mu^*\bar{u}_d\gamma^\mu(1 - \gamma_5)u_b, \] (13)
and
\[ P_{d\psi} = \frac{G_F}{\sqrt{2}}[C_3 + C_5 + \frac{1}{N_c}(C_4 + C_6)] \times m_{\psi}f_{\psi}\epsilon_\mu^*\bar{u}_d\gamma^\mu(1 - \gamma_5)u_b. \] (14)

The internal momentum of the \( c\bar{c} \) pair that forms the \( J/\psi \) is neglected, and the decay constant \( f_{\psi} \) is defined by
\[ <J/\psi|\bar{c}\gamma_\mu c|0> = m_{\psi}f_{\psi}\epsilon_\mu^*. \] (15)

The same Hamiltonian gives the amplitude for the decay \( b \to dJ/\psi \), where the charm-anticharm pair forms a color octet,
\[ A^{(0)}_8 = V_{cb}V_{cd}^*(T^V_8 + T^A_8) + V_{tb}V_{td}^*(P^V_8 + P^A_8). \] (16)

The superscripts \( V \) and \( A \) designate the terms that correspond to the \( c\bar{c} \) pair in a vector and in an axial-vector state, respectively. The latter are given by
\[ T^A_8 = -\frac{G_F}{\sqrt{2}}C_2\frac{1}{2}\bar{u}_d\gamma^\mu(1 - \gamma_5)\lambda^a u_b\bar{u}_c\gamma_\mu\gamma_5\lambda^a v_c \] (17)
and
\[ P^A_8 = -\frac{G_F}{\sqrt{2}}(C_4 - C_6)\frac{1}{2}\bar{u}_d\gamma^\mu(1 - \gamma_5)\lambda^a u_b\bar{u}_c\gamma_\mu\gamma_5\lambda^a v_c. \] (18)

As for the former, it is shown below that they do not contribute to the absorptive part of the \( b \to dJ/\psi \) amplitude, if the final state scattering is treated at the order \( \alpha_s \).

The \( c\bar{c} \) color octet can scatter to the \( J/\psi \) by exchanging a gluon with the \( d \)-quark. As pointed out earlier, this gluon exchange is required not only by the color constraint, but also for kinematical reasons, since the \( J/\psi \) is below the \( c\bar{c} \) continuum (a similar effect was discussed in ref. [3] in relation with the CP asymmetry in the radiative \( b \) decays). The convolution of the scattering amplitude \( A(d(c\bar{c})_8 \to dJ/\psi) \) with \( A^{(0)}_8 \) gives the contribution to the absorptive part of the \( b \to dJ/\psi \) amplitude:
\[ A^{\text{absorptive}}_{d\psi} = i\frac{1}{2} \sum \int d\Phi A(d(c\bar{c})_8 \to dJ/\psi)A^{(0)}_8. \] (19)
The summation is over the spin and color, and the integral is over the phase space of the intermediate state quarks. The integration was done analytically, and the rather cumbersome result is given in the Appendix.

The scattering amplitude \( A(d(c\bar{c})_8 \rightarrow dJ/\psi) \) is the sum of two terms, that correspond to the diagrams with a gluon exchanged between the \( d \)-quark and either the \( c \)- or the \( \bar{c} \)-quarks. As long as the internal momentum of the \( J/\psi \) is neglected, the corresponding terms in the expression for \( A_{\text{absorptive}} \) are related by charge conjugation, and they are equal in magnitude. Because, the \( J/\psi \) is a vector state (\( C = -1 \)), the two terms have the same sign when the \( A_8^{(0)} \) amplitude produces \( c\bar{c} \) as an axial vector. Whereas they have the opposite sign, and they cancel each other, when the \( c\bar{c} \) octet forms a vector (this is nothing else than a manifestation of Furry’s theorem). The amplitude for the decay \( b \rightarrow dJ/\psi \), with the final state scattering included to order \( \alpha_s \), is then

\[
A_{d\psi} = V_{cb}V_{cd}^*T_{d\psi} + V_{tb}V_{td}^*P_{d\psi}
\]

\[
+ \frac{1}{2} \sum \int d\Phi A(d(c\bar{c})_8 \rightarrow dJ/\psi)(V_{cb}V_{cd}^*T_8^A + V_{tb}V_{td}^*P_8^A),
\]

where the dispersive terms are given in eqs. \([13]\) and \([14]\), and the absorptive part is given in the Appendix. The interference between the terms with relative CP-odd and CP-even phases in the RHS gives

\[
\Delta = |A_{d\psi}|^2 - |\bar{A}_{d\psi}|^2 = 2Im\{V_{cb}V_{cd}^*V_{tb}V_{td}\} \sum \int d\Phi A(d(c\bar{c})_8 \rightarrow dJ/\psi)
\times \left( P_8^{A\dagger}T_{d\psi}^\dagger - T_8^{A\dagger}P_{d\psi}^\dagger \right),
\]

where the summation includes the spin and color of the \( b \)- and \( d \)-quarks, and the \( J/\psi \) polarization. It follows that

\[
\Delta = -Im\{V_{cb}V_{cd}^*V_{tb}V_{td}\}[(C_4 - C_6)(C_1 + \frac{1}{N_c}C_2)
- C_2(C_3 + C_5 + \frac{1}{N_c}(C_4 + C_6))]
\times G_F^2\alpha_s m_b^3 m_\psi f_\psi^2 \frac{32}{3} \frac{(1 - z)^2(z^2 - 3)}{\sqrt{z}(2 - z)^2},
\]

\[
(22)
\]
with \( z = (m_\psi/m_b)^2 \). This gives the CP asymmetry, in the semi-inclusive decay \( b \to dJ/\Psi \),

\[
a_{CP} \simeq \frac{|A_{d\psi}|^2 - |\bar{A}_{d\psi}|^2}{2|V_{cb}V_{cd}^*T_{d\psi}|^2} \nonumber
\]

\[
= \text{Im}\left\{ \frac{V_{tb}V_{td}^*}{V_{cb}V_{cd}^*} \left( C_4 - C_6 \right) (C_1 + \frac{1}{N_c}C_2) - C_2(C_3 + C_5 + \frac{1}{N_c}(C_4 + C_6)) \right\} \nonumber
\]

\[
\times \alpha_s \frac{(1-z)(z^2-3)}{9(1+2z)(2-z)^2}.
\]

(23)

Following the usual prescription of setting \( N_c = \infty \) (so that the strength of the color suppression in \( \Gamma(b \to dJ/\psi) \) is in good agreement with what is measured in the analogous decays of the type \( b \to sJ/\psi \) \[9\],

\[
a_{CP} = 1.1\% \times \frac{\eta}{0.4}.
\]

(24)

4 Conclusion

The value of the CP asymmetry in eq. (24) confirms the earlier suspicion \[5\] that the contribution from the \( dc\bar{c} \) intermediate states may be important. Indeed, if the assumptions in which this calculation is based are correct (namely, the use of a quark configuration for the intermediate state, and an expansion in \( \alpha_s \) for the final state scattering), the absorptive part of the \( b \to dJ/\psi \) amplitude is dominated by the rescattering from states that contain a \( c\bar{c} \) pair in a color octet, \( i.e. \) states in the continuum above the \( D - \bar{D} \) threshold. Their contribution to the CP asymmetry is somewhat lowered by the fact that it must be proportional to a ratio of penguin to tree amplitudes (both are necessary in order to generate a relative CP-odd phase). Still, it dominates over the contribution from the OZI suppressed process \( b \to du\bar{u} \to dJ/\psi \) \[5, 10\].

An important source of uncertainty is the very bothersome fact that, at present, the strength of the color suppression in decays such as \( b \to qJ/\psi \) (\( q = s \) or \( d \)) is not well understood. The prescription of dropping all non-leading terms in \( 1/N_c \), that I adopted in here, allows to reproduce the branching ratios that have been measured, but it is not well founded theoretically, nor confirmed by data from other types of \( B \) decays \[3\]. For the moment, it
provides a systematic framework to derive quantitative predictions. However, it is quite possible that some new mechanism is at work that would dominate the decay rate, and most likely affect the value of the CP asymmetry. Hence the interest in pursuing an experimental search, given the potential of present and future facilities for probing the asymmetry close to the level predicted in here.

Acknowledgements

I wish to thank Lincoln Wolfenstein, Isi Dunietz and Per Ernstrom for enlightening discussions and helpful criticism. This work was partly supported by the Natural Science and Engineering Research Council of Canada.

Appendix

The scattering amplitude \( A(d(\bar{c}c)_{s} \rightarrow dJ/\psi) \) is the sum of a term

\[
\bar{t} = \alpha_{s} \pi \bar{u}_{d} \gamma^{\mu} \lambda^{a} u'_{d} \bar{v}'_{c} \gamma^{\mu} \lambda^{a} v_{c} \frac{1}{(p'_{d} - p_{d})^{2}}
\]  

(25)

(the quantities \( u'_{d}, \bar{v}'_{c}, p'_{d}, \) and later \( p'_{c} \), correspond to the intermediate state quarks), that corresponds to the gluon exchange between the \( d \)- and the \( \bar{c} \)-quarks, and an analogous term \( t \) due to the gluon exchange between the \( d \)- and the \( c \)-quarks. In the expression for the absorptive amplitude in eq. 19, the contributions from \( t \) and \( \bar{t} \) are related by charge conjugation, and it follows that

\[
A_{d\psi}^{\text{absorptive}} = i \sum \int d\Phi \bar{t}(V_{cb}V_{cd}^{*}T_{A}^{A} + V_{tb}V_{td}^{*}P_{S}^{A}).
\]  

(26)

The tree and penguin terms, \( T_{A}^{A} \) and \( P_{S}^{A} \), are given in eqs. 17 and 18, and

\[
d\Phi \equiv (2\pi)^{4} \delta^{4}(p_{c} + p_{d} - p'_{c} - p'_{d}) \frac{d^{3}p'_{c}}{(2\pi)^{3} 2p'_{c}^{0}} \frac{d^{3}p'_{d}}{(2\pi)^{3} 2p'_{d}^{0}}.
\]  

(27)

The hadronization of the \( c\bar{c} \) pair, that forms the \( J/\psi \) in the final state, is described by a single parameter: the magnitude of the \( J/\psi \) wavefunction at
the origin or, equivalently, the decay constant defined in eq. [15]. The relations

\[< J/\psi | \bar{c} \sigma_{\mu \nu}^c | 0 > = if_\psi (p_{\psi \mu} \epsilon_\nu^* - p_{\psi \nu} \epsilon_\mu^*)\]

\[< J/\psi | \bar{c} \sigma_{\mu \nu}^c \gamma_5 | 0 > = \frac{1}{2} i \epsilon_{\mu \nu \alpha \beta} < J/\psi | \bar{c} \sigma_{\alpha \beta}^c | 0 >\]

are also useful. Summing over the spin and color of the intermediate state quarks, and integrating over their phase space, I obtain the following result

\[A_{absorptive}^{d\psi} = -i \frac{G_F}{2\sqrt{2}} \alpha_s f_\psi [V_{cb} V_{cd}^* C_2 + V_{tb} V_{td}^* (C_4 - C_6)] \frac{8}{9} \Omega\]  

(29)

where

\[\Omega = m_\psi \epsilon^{\sigma s} \{ \bar{u}_d \gamma_\sigma (1 - \gamma_5) u_b \} (C_1 + C_2) \frac{p_{b \sigma}}{2m_\psi} + D + 3G\]

\[-\bar{u}_d (1 + \gamma_5) u_b (C_1 + C_2) \frac{m_b p_{b \sigma}}{2m_\psi}\]

\[-\bar{u}_d \gamma_\nu \gamma_\sigma \gamma_\mu (1 - \gamma_5) u_b (C_1 - C_2) i \epsilon^{\alpha \mu \beta \nu} \frac{p_{b \alpha} p_{b \beta}}{4m_\psi}\]

\[+\frac{1}{4} (p_\nu^s \epsilon^{\delta s} - p_\delta^s \epsilon^{\nu s}) \{ -\bar{u}_d \gamma_\delta (1 + \gamma_5) u_b \frac{m_b}{m_\psi} (C_1 + C_2) p_{d \delta} + 2D p_{\psi \delta} \]

\[-\bar{u}_d (1 - \gamma_5) u_b 2(C_1 + C_2 - 2D) \frac{p_{b \delta} p_{d \sigma}}{m_\psi} \]

\[+\bar{u}_d \gamma_\alpha (1 - \gamma_5) u_b \frac{2}{m_\psi} [(B - A) p_{d \delta} - A p_{b \delta}]\]

\[-\bar{u}_d \gamma_\delta (1 - \gamma_5) u_b ((C_1 + C_2) \frac{p_{b \delta}}{2m_\psi} + \frac{1}{2} D + 4G\]

\[+\bar{u}_d \gamma_\nu \gamma_\sigma \gamma_\delta (1 - \gamma_5) u_b (C_1 - C_2) \frac{m_b}{m_\psi} (C_1 + C_2) p_{d \delta} + 2D p_{\psi \delta} \]

\[+\bar{u}_d (1 - \gamma_5) u_b 2(C_1 + C_2 - 2D) \frac{p_{b \delta} p_{d \sigma}}{m_\psi} \]

\[+\bar{u}_d \gamma_\alpha (1 - \gamma_5) u_b \frac{2}{m_\psi} [(B - A) p_{d \delta} - A p_{b \delta}]\]

\[-\bar{u}_d \gamma_\delta (1 - \gamma_5) u_b [(C_1 + C_2) \frac{3p_{b \sigma}}{2m_\psi} + \frac{3}{2} D + 4G]\]
\[ +\bar{u}_d \gamma_\sigma \gamma_5 (1 + \gamma_5) u_b A \frac{m_b}{m_\psi} \]
\[ - \bar{u}_d \gamma_\sigma \gamma_5 \gamma_\delta (1 - \gamma_5) u_b (C_1 - C_2) i \epsilon^{\mu \nu \beta \gamma} \frac{p_{BA} p_{DB}}{4 m_\psi^2} \]. \quad (30)

The quantities \( A, B, C_1, C_2, D, F, \) and \( G \), are defined by
\[
4 \pi m_\psi^2 \int d\Phi \frac{1}{(p'_d - p_d)^2} p'_d \alpha = A p_\psi \alpha + B p_d \alpha \\
8 \pi m_\psi^2 \int d\Phi \frac{1}{(p'_d - p_d)^2} p'_d p'_e \beta = C_1 p_\psi \alpha p_d \beta + C_2 p_\psi \beta p_d \alpha + D p_\psi \beta p_\psi \alpha + \mathcal{F} p_d \alpha p_d \beta + \mathcal{G} m_\psi^2 g_\alpha \beta. \quad (31)
\]

Performing the integrals, it follows that
\[
A = -\frac{z}{2 - z} \\
B = I + \frac{2z}{(1 - z)(2 - z)} \\
C_1 = -\left( \frac{z}{2 - z} \right)^2 \\
C_2 = I + \frac{2z}{1 - z} - \left( \frac{z}{2 - z} \right)^2 \\
D = -\frac{z}{2 - z \right)^2 \\
G = -\frac{1}{2} \left( \frac{1 - z}{2 - z} \right). \quad (32)
\]

\((z = (m_\psi/m_b)^2)\). The divergent integral
\[
I = -\frac{z}{1 - z} \int_{-1}^{1} dx \frac{1}{1 - x} \quad (33)
\]
corresponds to a mass singularity (due to taking \( m_s = 0 \) or \( m_c = m_\psi/2 \)). However, the dependence on \( I \) cancels in the expression for \( \Delta \), and so the CP asymmetry is free of mass singularities (and of IR divergences).

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[9] See the discussion in ref. [5], and the references therein.

[10] The contribution from the $du\bar{u}$ intermediate state, with $u\bar{u}$ in a color octet, was not included in ref. [5]. Unlike the color singlet case, there is no 1-photon final state scattering and only the contribution of order $\alpha^3_s$ remains.