Self-consistent solution of the Schwinger-Dyson equations
for the nucleon and meson propagators

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Abstract

The Schwinger-Dyson equations for the nucleon and meson propagators are solved self-consistently in an approximation that goes beyond the Hartree-Fock approximation. The traditional approach consists in solving the nucleon Schwinger-Dyson equation with bare meson propagators and bare meson-nucleon vertices; the corrections to the meson propagators are calculated using the bare nucleon propagator and bare nucleon-meson vertices. It is known that such an approximation scheme produces the appearance of ghost poles in the propagators. In this paper the coupled system of Schwinger-Dyson equations for the nucleon and the meson propagators are solved self-consistently including vertex corrections. The interplay of self-consistency and vertex corrections on the ghosts problem is investigated. It is found that the self-consistency does not affect significantly the spectral properties of the propagators. In particular, it does not affect the appearance of the ghost poles in the propagators.
I. INTRODUCTION

The development of relativistic many-body theories for the nucleus is one of the most important goals of contemporary nuclear theory. Models based on the methods of relativistic quantum field theory have been developed for more than two decades.

The starting point for understanding the many-nucleon problem is a description of the elementary processes in vacuum: the nucleon propagator, meson-nucleon scattering, and the \( N - N \) interaction. Successes and difficulties with relativistic meson-nucleon field theory have been the subject of papers for more than half a century. We will certainly not detail the history here, but note that a nagging inconsistency in (almost) all calculations has been the appearance of ghost poles.

Brown, Puff and Wilets [1], for example, calculated the nucleon propagator by summing all planar meson diagrams with one nucleon line using \( \pi^- \), \( \rho^- \), and \( \omega \)-mesons. No cutoffs were introduced. The renormalized nucleon propagator was well-defined and self-consistent, but contained a pair of conjugate complex poles located approximately one GeV off the real and imaginary axes. The full propagator, including these unphysical poles, was used with some success to describe the isovector nucleon magnetic moment, \( \pi \)-nucleon scattering [2], and nucleon-nucleon scattering [3]. (The last did require cut-offs in the \( N - N \) interaction, but yielded better chi-squared fits to scattering data with fewer parameters than the then current Paris potential). The inclusion of the complex poles was essential. Nevertheless, the occurrence of the complex poles remained an enigma.

Several interpretations of the appearance of the poles have been proffered, including the statement that it is a signal of the inconsistency of any local, relativistic field theory, and that a field theory with asymptotic freedom (e.g. QCD) is required.

The program of the previous section was driven by the interpretation that the appearance of the ghosts is an artifact of the approximations, and that progressively better calculations should lead to the receding or elimination of the ghosts, but that for consistency one must keep the ghosts as they emerge from the calculations at each stage.

Another interpretation is that it is an effective theory, and that one should be prepared to introduce further parameters to ensure physical quantities.

In a recent paper [4], the problem of ghosts poles in the nucleon propagator was inves-
tigated. The appearance of the ghost poles is related to the short distance behavior of the model interactions [1]; asymptotically free theories appear to be free of ghost poles [3]. An interesting possibility to eliminate the complex poles is the regularization of the theory by means of vector meson dressing of nucleon-meson vertices. It is known that in a theory with neutral vector mesons there are vertex corrections that generate a strongly damped vertex function in the ultraviolet [6]. In quantum electrodynamics, such corrections give rise to the Sudakov form factor [7]. When the Sudakov form factor, generated by massive vector mesons, is included in the HF approximation to the Schwinger-Dyson equation (SDE) for the nucleon propagator, the ghost poles disappear. A similar result was obtained by Allendes and Serot [8] earlier in the study of the ghost pole in the meson propagator. Those authors concluded that the Sudakov corrected propagator is free of ghost poles.

It is the purpose of the present paper to solve self-consistently the coupled system of Schwinger-Dyson equations (SDE) for the nucleon and meson propagators and investigate the role of self-consistentency on the appearance of ghost poles in the propagators. Vertex corrections are introduced by means of form factors.

There is an extensive literature on calculations of nuclear matter and finite nuclei properties based on the Walecka scalar-vector model [9]. In general, the applications have been performed using Hartree-Fock (HF) type of approximations. In a relativistic HF approximation, the single-nucleon propagator is calculated by solving self-consistently the (SDE) using bare meson propagators and bare meson-nucleon vertices. An additional approximation has been the neglect of the quantum vacuum of the nucleon propagator. The corrections to the meson propagators are usually calculated considering the vacuum polarization correction using nucleon propagators with an effective mass. Although the nucleon propagator is solved self-consistently by means of the SDE, the self-consistency is only partial, since the meson propagators used are the bare ones. The meson propagators satisfy their own SDE, which require for their solution the nucleon propagator. A self-consistent solution requires the consideration of the coupled system of nucleon and meson SDE’s.

Besides the lack of self-consistency, the neglect of the quantum vacuum in the nucleon sector is a major limitation. It is exactly the nontrivial nature of the vacuum of a relativistic quantum field theory that motivates the introduction of models which go beyond the usual
nonrelativistic approach. However, severe difficulties arise in including the vacuum effects beyond the one-loop Hartree approximation. The inclusion of these vacuum corrections leads to catastrophic results due to the presence of the ghost poles in the propagators. Among other things, the ghosts lead to a large imaginary part to the nuclear matter energy.

The paper is organized as follows. In section II we present the model for the interacting nucleon-meson system. We briefly review the spectral representation of the propagators and their inverses and discuss the renormalization procedure. In section III we discuss the coupled system of Schwinger-Dyson equations for the nucleon and meson propagators in terms of their spectral representations. Section IV presents the method of solution of the equations and presents our numerical results. Conclusions are presented in section V.

II. THE MODEL

In this paper we consider a model field theory with nucleons ($\psi$), pions ($\vec{\pi}$), and vector isoscalar mesons ($\omega$). The model Lagrangian density is

$$
\mathcal{L} = \bar{\psi}(i\gamma_\mu \partial^\mu - ig_0\gamma_5 \vec{\pi} \cdot \vec{\tau} - g_0\gamma_\mu \omega^\mu)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m_\omega^2 \omega_\mu \omega^\mu + \frac{1}{2}\partial_\mu \vec{\pi} \cdot \partial^\mu \vec{\pi} - \frac{1}{2}m_\pi^2 \vec{\pi} \cdot \vec{\pi},
$$

where $F^{\mu\nu} = \partial^\mu \omega^\nu - \partial^\nu \omega^\mu$.

As usual, the nucleon propagator is defined by

$$
G_{\alpha\beta}(x' - x) = -i < 0 | T[\psi_\alpha(x')\bar{\psi}_\beta(x)] | 0 > ,
$$

where $|0>$ represents the physical vacuum state. The $\pi$- and $\omega$-meson propagators are defined respectively by

$$
D^{ij}_\pi(x' - x) = -i < 0 | T[\pi^i(x')\pi^j(x)] | 0 > ,
$$

and

$$
D^{\mu\nu}_\omega(x' - x) = -i < 0 | T[\omega^\mu(x')\omega^\nu(x)] | 0 > .
$$

The Schwinger-Dyson equations for the nucleon and meson propagators in momentum space are given by the following expressions, Fig. 1,
(a) nucleon:

\[ G(p) = G^{(0)}(p) + G^{(0)}(p) \Sigma(p) G(p), \]  

\[ \Sigma(p) = -3i g_{0\pi}^2 \int \frac{d^4 q}{(2\pi)^4} \gamma_5 D_{\pi}(q^2) G(p-q) \Gamma_5(p-q,p;q) \]
\[ + i g_{0\omega}^2 \int \frac{d^4 q}{(2\pi)^4} \gamma_\mu D_{\omega}^{\mu\nu}(q^2) G(p-q) \Gamma_\nu(p-q,p;q), \]  

(b) pion:

\[ D_{\pi}^{ij}(q^2) = D_{\pi}^{(0)ij}(q^2) + D_{\pi}^{(0)ik}(q^2) \Pi_{\pi}^{kl}(q^2) D_{\pi}^{lj}(q^2), \]
\[ \Pi_{\pi}^{ij}(q^2) = i g_{0\pi}^2 \int \frac{d^4 p}{(2\pi)^4} Tr[\gamma_5 \tau_i G(p) \Gamma_5 \tau_j(p,p+q;q) G(p+q)]. \]  

(c) omega:

\[ D_{\omega}^{\mu\nu}(q^2) = D_{\omega}^{\mu\nu(0)}(q^2) + D_{\omega}^{\mu\rho(0)}(q^2) \Pi_{\omega}^{\rho\sigma}(q^2) D_{\omega}^{\sigma\nu}(q^2), \]
\[ \Pi_{\omega}^{\mu\nu}(q^2) = -i g_{0\omega}^2 \int \frac{d^4 p}{(2\pi)^4} Tr[\gamma_\mu G(p) \Gamma_\nu(p,p+q;q) G(p+q)]. \]  

In the above equations, \( \Gamma_5(p,p+q;q) \) and \( \Gamma_\mu(p,p+q;q) \) are the three-point \( \pi \)-nucleon and \( \omega \)-nucleon vertex functions, respectively. They satisfy their own Schwinger-Dyson equations. These relate the three-point functions to four-point vertices and so on \emph{ad infinitum}. In practice one has to truncate this infinite set. In this paper we truncate the SDE's by postulating a specific form for the three-point functions (see below).

Next, we discuss the spectral representations of the propagators and their inverses. We do not intend to review the subject of spectral representations, we simply make use of the relevant equations for the purposes of the present paper. We refer the reader to Refs. [11] - [12] for an extensive discussion on the subject. Let us start with the nucleon propagator. The spectral representation of the nucleon propagator (in momentum space) can be written as

\[ G(p) = \int_{-\infty}^{+\infty} d\kappa \frac{A(\kappa)}{\not{q} - \kappa + i\epsilon}. \]
$A(\kappa)$ is the spectral function. It represents the probability that a state of mass $|\kappa|$ is created by $\psi$ or $\bar{\psi}$, and as such it must be non-negative. Negative $\kappa$ corresponds to states with opposite parity to the nucleon.

Defining the projection operators

$$P_{\pm}(p) = \frac{1}{2} \left( 1 \pm \frac{\not{p}}{w_p} \right),$$

where

$$w_p = \sqrt{p^2} = \begin{cases} \sqrt{p^2}, & \text{if } p^2 > 0 \\ i\sqrt{-p^2}, & \text{if } p^2 < 0. \end{cases}$$

$G(p)$ can be rewritten conveniently as

$$G(p) = P_+(p)\tilde{G}(w_p + i\epsilon) + P_-(p)\tilde{G}(-w_p - i\epsilon),$$

where $\tilde{G}(z) = \pm(w_p + i\epsilon)$, is given by the dispersion integral

$$\tilde{G}(z) = \int_{-\infty}^{+\infty} d\kappa \frac{A(\kappa)}{z - \kappa}.$$ 

The inverse of the propagator can also be written in terms of the projection operators $P_{\pm}(p)$ as

$$G^{-1}(p) = P_+(p)\tilde{G}^{-1}(w_p + i\epsilon) + P_-(p)\tilde{G}^{-1}(-w_p - i\epsilon).$$

Since $A(\kappa)$ is supposed to be non-negative, it is simple to show that $\tilde{G}(z)$ can have no poles or zeros off the real axis. This is known as the Herglotz property. Now, if $\tilde{G}(z)$ possesses the Herglotz property, then so does $\tilde{G}^{-1}(z)$. This permits us to write a spectral representation for $\tilde{G}^{-1}(z)$,

$$\tilde{G}^{-1}(z) = z - M_0 - \tilde{\Sigma}(z) = z - M_0 - \int_{-\infty}^{+\infty} d\kappa \frac{T(\kappa)}{z - \kappa}.$$ 

The function $\tilde{\Sigma}(z)$ is related to the $\Sigma(q)$ of Eq. (5) by the projection operators $P_{\pm}(q)$ as in Eq. (16). If $T(\kappa)$ is non-negative, the Herglotz property is also satisfied for $\tilde{G}^{-1}(z)$.

In general, the integral in Eq. (17) needs renormalization. The usual mass and wave-function renormalizations are performed by imposing the condition that the renormalized
propagator has a pole at the physical nucleon mass $M$, with unit residue. This implies that
the renormalized propagator $\tilde{G}_R(z)$, defined as

$$\tilde{G}_R(z) \equiv \tilde{G}(z)/Z_2,$$

is given by the following expression:

$$\tilde{G}_R(z) = \int_{-\infty}^{+\infty} d\kappa \frac{A_R(\kappa)}{z - \kappa}.$$  \hspace{1cm} (19)

The renormalized inverse is given by

$$\tilde{G}_R^{-1}(z) = (z - M) \left[ 1 - (z - M) \int_{-\infty}^{+\infty} d\kappa \frac{T_R(\kappa)}{(\kappa - M)^2(z - \kappa)} \right].$$  \hspace{1cm} (20)

In the above expressions, $A_R(\kappa) = A(\kappa)/Z_2$ and $T_R(\kappa) = Z_2 T(\kappa)$. In terms of renormalized quantities, $Z_2$ can be written as

$$Z_2 = 1 - \int_{-\infty}^{+\infty} d\kappa \frac{T_R(\kappa)}{(\kappa - M)^2} = \left[ \int_{-\infty}^{+\infty} d\kappa A_R(\kappa) \right]^{-1}.$$  \hspace{1cm} (21)

The spectral functions $A_R(\kappa)$ and $T_R(\kappa)$ are related by

$$A_R(\kappa) = \delta(\kappa - M) + |\tilde{G}_R^{-1}(\kappa(1 + i\epsilon))|^{-2} T_R(\kappa)$$
$$\hspace{2cm} \equiv \delta(\kappa - M) + \tilde{A}_R(\kappa).$$  \hspace{1cm} (23)

Let us now consider the spectral representations of the meson propagators. The isospin structure of the $\pi$-meson propagator is such that $D_{ij}^{\pi}(q^2) = \delta^{ij} D_\pi(q^2)$. For $D_\pi(q^2)$ one can write the spectral representation

$$D_\pi(z) = \int_0^{\infty} d\sigma^2 \frac{\rho_\pi(\sigma^2)}{z - \sigma^2},$$

where $\rho_\pi(\sigma^2)$ is the pion spectral function. It represents the probability that a state of mass $\sqrt{\sigma^2}$ is created by the pion field and as such it must be non-negative. The meaning of the complex variable $z$ is that the physical propagator $D_\pi(q^2)$ is the limit of $D_\pi(z)$ when $z \to q^2 + i\epsilon$.

Using the SDE for the pion, Eq. (7), the inverse of $D_\pi(z)$ can be written in terms of the pion self-energy $\Pi(z)$ as
Similarly to the case of the nucleon, one can write a spectral representation for \( D^{-1}_\pi(z) \)

\[
D^{-1}_\pi(z) = z - m^0_\pi - \Pi_\pi(z). \tag{26}
\]

The renormalized propagator is again obtained by fixing the pole position at the physical mass, and the residue at the pole equal to one. The renormalized propagator \( D_{\pi R}(z) \), defined as

\[
D_{\pi R}(z) \equiv D_\pi(z)/Z_{3\pi}, \tag{28}
\]

is then

\[
D_{\pi R}(z) = \int_0^\infty d\sigma^2 \frac{\rho_{\pi R}(\sigma^2)}{z - \sigma^2}. \tag{29}
\]

Its inverse is given by

\[
D^{-1}_{\pi R}(z) = (z - m^2_\pi) \left[ 1 - (z - m^2_\pi) \int_0^\infty d\sigma^2 \frac{S_{\pi R}(\sigma^2)}{(\sigma^2 - m^2_\pi)^2(z - \sigma^2)} \right]. \tag{30}
\]

The spectral functions are defined as \( \rho_{\pi R}(\sigma^2) = \rho_{\pi}(\sigma^2)/Z_{3\pi} \) and \( S_{\pi R}(\sigma^2) = Z_{3\pi} S_\pi(\sigma^2) \).

In terms of the renormalized quantities, \( Z_{3\pi} \) is given by

\[
Z_{3\pi} = 1 - \int_0^\infty d\sigma^2 \frac{S_{\pi R}(\sigma^2)}{(\sigma^2 - m^2_\pi)^2} \tag{31}
\]

\[
= \left[ \int_0^\infty d\sigma^2 \rho_{\pi R}(\sigma^2) \right]^{-1}. \tag{32}
\]

The spectral functions \( \rho_{\pi R} \) and \( S_{\pi R} \) are related by

\[
\rho_{\pi R}(q^2) = \delta(q^2 - m^2_\pi) + |D^{-1}_{\pi R}|^2 S_{\pi R}(q^2) \tag{33}
\]

\[
\equiv \delta(q^2 - m^2_\pi) + \bar{\rho}_{\pi R}(q^2). \tag{34}
\]

Let us now consider the \( \omega \)-meson propagator. Since the baryon current is conserved the \( \omega \)-meson self-energy \( \Pi^{\mu\nu}_\omega(q^2) \), must satisfy

\[
q_\mu \Pi^{\mu\nu}_\omega(q^2) = q_\nu \Pi^{\mu\nu}_\omega(q^2) = 0. \tag{35}
\]
Therefore, the Lorentz structure of $\Pi^{\mu\nu}_{\omega}$ must be

$$\Pi^{\mu\nu}_{\omega}(q^2) = \left(g^{\mu\nu} - q^\mu q^\nu / q^2\right) \Pi_{\omega}(q^2). \quad (36)$$

Substituting this in the SDE for $\omega$-meson propagator, Eq. (9), $D^{\mu\nu}_{\omega}(q^2)$ can be written as

$$D^{\mu\nu}_{\omega}(q^2) = -g^{\mu\nu} D_{\omega}(q^2), \quad (37)$$

where

$$D_{\omega}(q^2) = \frac{1}{q^2 - m_{\omega}^2 - \Pi_{\omega}(q^2) + i\epsilon}. \quad (38)$$

Terms proportional to $q^\mu q^\nu$ in Eq. (37) can be neglected when using $D^{\mu\nu}_{\omega}$ in Eq. (6), because of current conservation.

The spectral representation of $D_{\omega}$ is

$$D_{\omega}(z) = \int_0^\infty d\sigma^2 \frac{\rho_{\omega}(\sigma^2)}{z - \sigma^2}. \quad (39)$$

As in the case of the pion, one can write the Cauchy representation for the inverse of the $\omega$-meson propagator as

$$D_{\omega}^{-1}(z) = z - m_{\omega}^2 - \int_0^\infty d\sigma^2 \frac{S_{\omega}(\sigma^2)}{z - \sigma^2}. \quad (40)$$

Renormalization proceeds as for the pion. The renormalized quantities are given by expressions similar to the ones for the pion, Eqs. (29, 30, 32, 34), with the $\pi$ indices replaced by $\omega$ indices.

### III. SCHWINGER-DYSON EQUATIONS

We start with the nucleon SDE, Eq. (5). It can be written as

$$G^{-1}(p) = G^{-1}(p) - \Sigma(p), \quad (41)$$

where $\Sigma(p)$ is given by Eq. (3). To proceed, we need to specify the form of the vertex functions $\Gamma_5(p, p + q; q)$ and $\Gamma^{\mu}(p, p + q; q)$. In the usual HF approximation, $\Gamma_5(p, p + q; q) = \gamma^5$, and $\Gamma^{\mu}(p, p + q; q) = \gamma^{\mu}$. In this paper we consider vertex functions written as
\[ \Gamma^i_5(p_1, p_2; q) = \tau^i \gamma_5 F_5(p_1, p_2; q) \]  
\[ \Gamma^\mu(p_1, p_2; q) = \gamma^\mu F_V(p_1, p_2; q), \]  

where \( F_5(p_1, p_2; q) \) and \( F_V(p_1, p_2; q) \) are scalar functions.

Substituting Eqs. (42,43) and the spectral representations for \( G(q), D_\pi \) and \( D_\omega \) in the integral for \( \Sigma(q) \), Eq. (3), and applying the projection operators \( P_\pm(p) \) to Eq. (31), one obtains:

\[ T_R(\kappa) = \int_{-\infty}^{+\infty} d\kappa' K(\kappa, \kappa') A_R(\kappa') \]  

where \( K(\kappa, \kappa') \) is given by

\[ K(\kappa, \kappa') = \frac{1}{2|\kappa|^3} \left[ (\kappa - \kappa')^2 - 2 \kappa' m^2 \right] \theta(\kappa^2 - (|\kappa'| + m^2)^2) F_5(\kappa, \kappa'; m), \]  

and

\[ K_\omega(\kappa, \kappa'; m^2) = \frac{1}{2|\kappa|^3} \left[ (\kappa - \kappa')^2 - 2 \kappa' m^2 \right] \theta(\kappa^2 - (|\kappa'| + m^2)^2) F_V(\kappa, \kappa'; m). \]

\( \bar{\rho}_{\pi R}(\sigma^2) \) is related to \( S_{\pi R} \) as shown in Eqs. (33-34) (similarly for \( \bar{\rho}_{\omega R}(\sigma^2) \)).

The meson self-energies, \( S_{\pi R}(q^2) \) and \( S_{\omega R}(q^2) \), are obtained using the spectral representation of the nucleon propagator in Eqs. (35,36). \( S_{\pi R}(q^2) \) is given by:

\[ S_{\pi R}(q^2) = S_{\pi}(M, M; q^2) \]
\[ + 2 \int_{-\infty}^{+\infty} d\kappa \bar{A}_R(\kappa) S_{\pi}(M, \kappa; q^2) \]
\[ + \int_{-\infty}^{+\infty} dk d\kappa' \bar{A}_R(\kappa) \bar{A}_R(\kappa') S_{\pi}(\kappa, \kappa'; q^2), \]  

where \( \bar{A}(\kappa) \) is defined in Eq. (24), and
\[ S_\pi(\kappa, \kappa'; q^2) = \left( \frac{g_\pi^2}{4\pi^2} \right) \left[ 1 - \frac{(\kappa - \kappa')^2}{q^2} \right] \left[ q^4 - 2q^2(\kappa^2 + \kappa'^2) + (\kappa^2 - \kappa'^2)^2 \right]^{\frac{1}{2}} \times \Theta(q^2 - (|\kappa| + |\kappa'|)^2) F_5(\kappa, \kappa'; q) , \]  

(49)

with \( g_\pi^2 = Z_2 g_{0\pi} \) and \( g_\omega^2 = Z_2 g_{0\omega} \).

For the \( \omega \)-meson, we have the same expression as in Eq. (48), with the indice \( \pi \) replaced by \( \omega \) and

\[ S_\omega(\kappa, \kappa'; q^2) = \left( \frac{g_\omega^2}{8\pi^2} \right) \left[ 1 - \frac{(\kappa - \kappa')^2}{q^2} \right] + \frac{1}{3q^4} \left[ q^4 - 2q^2(\kappa^2 + \kappa'^2) + (\kappa^2 - \kappa'^2)^2 \right] \times [q^4 - 2q^2(\kappa^2 + \kappa'^2) + (\kappa^2 - \kappa'^2)^2]^{\frac{1}{2}} \Theta(q^2 - (|\kappa| + |\kappa'|)^2) F_V(\kappa, \kappa'; q) . \]  

(50)

**IV. NUMERICAL RESULTS**

The problem now consists in finding \( A_R(\kappa) \), \( \rho_{\pi R}(\sigma^2) \), and \( \rho_{\omega R}(\sigma^2) \). The strategy adopted is to solve the equations by iteration; it proceeds as follows:

1) start solving for \( A_R \) with the bare \( \pi \) and \( \omega \) propagators (\( \bar{\rho}_{\pi R} = \bar{\rho}_{\omega R} = 0 \)). This is the usual Hartree-Fock solution for the nucleon propagator including vertex corrections by means of the form factors of Eqs. (42, 43) [4];
2) with this \( A_R \), obtain \( S_R\pi(q^2) \) and \( S_R\omega(q^2) \) with Eq. (48);
3) then use Eq. (34) to obtain \( \bar{\rho}_{\pi R} \) and similarly \( \bar{\rho}_{\omega R} \);
4) use these \( \bar{\rho}_{\pi R} \) and \( \bar{\rho}_{\omega R} \) to obtain new \( K(\kappa, \kappa') \);
5) with this \( K(\kappa, \kappa') \), solve for \( A_R \); go back to 2), and cycle to convergence.

Initially, we considered bare vertices: \( F_5(p_1, p_2, q) = F_V(p_1, p_2, q) = 1 \), and investigated the role of the self-consistency on the spectral functions. We used the following values for the coupling constants and masses:

\[
\frac{g_\pi^2}{4\pi} = 14.6 \quad m_\pi = 0.144 \text{ } M \\
\frac{g_\omega^2}{4\pi} = 6.36 \quad m_\omega = 0.833 \text{ } M ,
\]

(51) (52)

where \( M \) is the nucleon mass.

The converged spectral functions \( A_R \), \( \rho_{\pi R} \), and \( \rho_{\omega R} \) are shown (without the delta functions) in Figs. 2-4. The solid (dashed) lines represent the self-consistent (not self-consistent)
solutions. The self-consistency does not affect the fermion spectral function perceptively, but it affects the meson functions, although not very importantly. It is interesting to note that the effect of the self-consistency is opposite in $\rho_\pi R$ and $\rho_\omega R$; it increases the former and decreases the last.

The role of the self-consistency on the appearance of ghost poles is shown in Table 1. The not self-consistent meson spectral functions are the ones obtained by calculating the nucleon loop in Figs. 1(b),1(c) using the bare nucleon propagators ($\bar{A}_R = 0$), i.e., these are the first order perturbative spectral functions. Clearly, the self-consistency does not change much the position of the poles and residues of the nucleon propagator, although it changes somewhat the ones of the meson propagators.

As discussed in Refs. [1], [4], the signal for the presence of ghosts in the nucleon propagator is revealed by the fact that the renormalization constant $Z_2$ calculated via the spectral function of the nucleon self-energy, $T_R(\kappa)$, gives $Z_2 = -\infty$. From this result one should have $Z_2^{-1} = 0$. However, in order to get the integral of $A_R$ equal to zero (see Eq. (22)) one has then to include the pair of complex conjugated poles in $\tilde{G}_R$ (the real parts of the residues are negative (see Table 1)). In the case of the renormalization constants of the $\pi$ and $\omega$ mesons, we obtain exactly the same result: the $Z_3$'s calculated via the spectral function of the self-energy, $S_R$, gives $Z_3 \to -\infty$, whereas to obtain zero for the integral over the spectral function of the propagator, $\rho_R$, the residue of the ghost pole has to be included.

In Ref. [4], the problem of ghosts poles in the nucleon propagator was investigated using form factors at the nucleon-meson vertices. Two types of form factors were used: (a) a Sudakov form factor, which is generated by vector meson dressing of the vertices and (b) a phenomenological form factor, of the monopole type. The conclusion there was that both types of form factors are able to kill the ghosts. However, as remarked in that reference, a proper extension of the Sudakov form factor to lower momenta is necessary for a better study of these issues. In this paper we use only the simple monopole form factor to investigate the interplay of self-consistency and vertex corrections on the spectral functions. As in Ref. [4], we use for $F_5(p_1, p_2, q)$ and $F_V(p_1, p_2, q)$ the following expressions:

$$F_5(p_1, p_2, q) = F_V(p_1, p_2, q) = \frac{1}{1 + |p_1^2/\Lambda^2|} \frac{1}{1 + |q^2/\Lambda^2|} \frac{1}{1 + |p_2^2/\Lambda^2|},$$  \hspace{1cm} (53)

where $\Lambda$ is an ultraviolet cutoff.
The modifications on the spectral functions due to the form factors can be seen in Figs. 5-7, where we plotted $A_R$, $\rho_{\pi R}$, and $\rho_{\omega R}$ (again without the delta functions) for two typical values of cutoffs, $\Lambda = M$ (solid), 1.25 $M$ (long-dashed). The (short-dashed) curves correspond to the case of $\Lambda = \infty$. The effect of the form factor is to increase $A_R(\kappa)$ for negative $\kappa$, a result already found in [4], and to increase (decrease) $\rho_{R\pi}$ ($\rho_{R\omega}$). In Ref. [4], it was found that for a $\Lambda < \Lambda_{\text{crit}} \approx 1.75$ $M$ the ghost poles in the nucleon propagator disappear. In the present case, we found that the self-consistency does not alter significantly this value; for $\Lambda \lesssim 1.60$ $M$, the ghosts of all propagators disappear.

We have also investigated the effect of the self-consistency on the ghosts-free spectral functions, i.e. for several $\Lambda$’s smaller than $\Lambda_{\text{crit}}$, we compared the self-consistent and not self-consistent spectral functions. Surprisingly, the effect of the self-consistency is negligible, and is almost invisible when one plots the spectral functions.

Although on physical grounds one expects that the cutoffs for the $\pi$ and $\omega$ vertices have different values, we used the same value for both, since in this work we are mostly interested in the qualitative effects. The consequences of the modifications induced by the form factors on physical observables deserves a separate study. Work in this direction is in progress.

V. CONCLUSIONS AND PERSPECTIVES

In this paper we have solved self-consistently the coupled set of Schwinger-Dyson equations for the nucleon and $\pi$ and $\omega$ mesons in the vacuum. The set of equations was truncated by postulating a three-point meson-nucleon vertex function. The understanding of the vacuum properties of the nucleon and meson propagators is a necessary first step towards the study of the properties of nucleon and meson in nuclear matter, as well as those of nuclear matter and finite nuclei. Although many of such properties have been studied using relativistic quantum field models, the vacuum polarization effects in medium have invariably been neglected.

The main conclusion of our investigation is the surprising result that the self-consistency does not modify significantly the spectral properties of the propagators. The appearance or disappearance of the ghost poles in the propagators is not affected by the self-consistency.

One important aspect regarding the vacuum of meson-nucleon effective theories that
was not yet satisfactorily investigated is the role of the three-point meson-nucleon vertex functions. In particular, the interplay of the infrared and ultraviolet sectors of the $\omega$-nucleon three-point vertex is extremely important to the problem of ghosts poles, as shown in the recent studies of Refs. (8, 4). Work in this direction is in progress.

The effects of the self-consistency on the nucleon and meson propagators in nuclear matter, in connections to the problem of ghost poles remains an open problem, although work in this direction has recently been communicated [13].

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FIGURES

FIG. 1. Diagrammatic representation of the Schwinger-Dyson equations for the full (a)nucleon, (b)pion, and (c)omega propagators. The solid, wavy and dashed lines represent respectively the nucleon, the \( \omega \), and the \( \pi \). The blobs represent full propagators and vertices.

FIG. 2. Self-consistent(solid) and not self-consistent (dashed ) nucleon spectral function \( A_R(\kappa) \). \( \kappa \) is in units of the nucleon mass \( M \) and \( A_R(\kappa) \) is in units of \( M^{-1} \). The curves are multiplied by 5 for negative \( \kappa \).

FIG. 3. Self-consistent(solid) and not self-consistent (dashed) \( \pi \) spectral function \( \rho_{\pi R}(\sigma^2) \). \( \sigma^2 \) is in units of \( M^2 \) and \( \rho_{\pi R}(\sigma^2) \) is in units of \( M^{-2} \).

FIG. 4. Self-consistent(solid) and not self-consistent (dashed) \( \omega \) spectral function \( \rho_{\omega R}(\sigma^2) \). The units are the same as in Fig. 3.

FIG. 5. \( A_R(\kappa) \) for different values of the cutoff: \( \Lambda = M \) (solid), 1.25 \( M \) (long-dashed), \( \infty \) (short-dashed). Units are the same as in Fig. 2. The short-dashed curve is multiplied by 5 for negative \( \kappa \).

FIG. 6. \( \rho_{\pi R}(\sigma^2) \) for same \( \Lambda \)'s as in Fig. 5. Units are the same as in Fig. 3.

FIG. 7. Same as in Fig. 6 for \( \rho_{\omega R}(\sigma^2) \).
TABLE I. Ghost poles. The first value is the pole position and the second is the residue at the pole. The nucleon poles are in units of $M$ and of the mesons are in units of $M^2$.

|      | Self-consistent | Not self-consistent |
|------|-----------------|---------------------|
| $N$  | $1.06 \pm 1.25i$| $-0.77 \pm 0.20i$   |
| $\pi$| $-1.04$         | $1.08$              |
| $\omega$ | $-3.50$       | $-1.30$            |
|      | $1.05 \pm 1.26i$| $-0.77 \pm 0.20i$   |
| $\pi$| $-1.44$         | $1.13$              |
| $\omega$ | $-5.68$       | $-1.49$            |