Killing gauge for the 0-brane on $AdS_2 \times S^2$ coset superspace

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Abstract

How to gauge fix $\kappa$-symmetry for the super 0-brane action on $AdS_2 \times S^2$ in Killing gauge properly is discussed in order to find the superconformal mechanics which describes super 0-brane probes moving on $AdS_2 \times S^2$. The dependence on the coordinate frame for the proper Killing gauge is considered and the subtleties of gauge-fixing $\kappa$-symmetry in Killing gauge are analysed explicitly. It is found that the Killing gauge works indeed without the incompatibility if the magnetic charge of the super 0-brane is nonzero.

Keywords: 0-brane; supersymmetry; curved space; AdS/CFT correspondence

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Recently, there has been much interest in the AdS/CFT correspondence [1], which relates string theory on $\text{AdS}_{p+2} \times S^{D-P-2}$ to extended superconformal theories in $p+1$ dimensions. In view of the AdS/CFT conjecture, it is important to understand the formulation of superstrings and super p-branes on these curved spaces. In [2], the type IIB Green-Schwarz (GS) superstring action was constructed in $\text{AdS}_5 \times S^5$ background in terms of supercoset formalism. This action possesses global $SU(2,2 \mid 4)$ super-invariance, has $\kappa$-symmetry and 2D reparametrization invariance as its local symmetries, and reduces to the conventional type IIB GS superstring action in the flat background limit. The other related construction for GS superstring, super D3-brane, D1-brane on $\text{AdS}_5 \times S^5$, and super M-branes on $\text{AdS}_4 \times S^7$ and $\text{AdS}_7 \times S^4$ have been discussed in [3]-[8]. The GS superstring and super p-brane actions on $\text{AdS}_3 \times S^3$ and $\text{AdS}_2 \times S^2$ have been constructed in [3] and [10]. The gauge-fixing of $\kappa$-symmetry was carried out in Killing gauge [13] or supersolvable algebra approach [14, 15]. However, the $\kappa$-symmetry gauge-fixing and quantization seem still to pose some difficulties [16, 17, 18]. In [16], it was argued that there is an incompatibility between Killing gauge and the static vacuum solution for super p-brane actions on $\text{AdS}_{p+2} \times S^{D-P-2}$ superbackgrounds. Since the D3-brane action on $\text{AdS}_5 \times S^5$ is very complicated [4], while the super 0-brane action on $\text{AdS}_2 \times S^2$ was constructed in supercoset formalism [10] only recently, it is quite interesting to see how the incompatibility mentioned in [16] appears explicitly and whether it is possible to simplify the super 0-brane action on $\text{AdS}_2 \times S^2$ in Killing gauge.

On the other hand, the radial motion of a superparticle with zero angular momentum near the horizon of an extreme Reissner-Nordström black hole ($\text{AdS}_2 \times S^2$) is found to be described by an $Osp(1 \mid 2)$-invariant superconformal mechanics [19], and it was argued in [19] that the full superparticle dynamics should be invariant under the larger $SU(1,1 \mid 2)$ superconformal group because this is the superisometry group of $\text{AdS}_2 \times S^2$. This full dynamics describes not only the radial motion of the superparticle, but also its motion on $S^2$. In [20], the authors tried to construct a $SU(1,1 \mid 2)$-invariant action from the worldline superfield formalism, but, due to technical difficulties, the explicit $SU(1,1 \mid 2)$-invariant

1 A slightly different construction for superstring on $\text{AdS}_2 \times S^2$ and $\text{AdS}_3 \times S^3$ was discussed in [11] and [12].

2 As the superstring $\kappa$-symmetry projector is different from the reduced D3-brane projector in the static vacuum solution, such an incompatibility does not exist for GS superstring.
action has not been obtained. Then it is interesting to see how the explicit action, which
describes the super 0-brane dynamics on $\text{AdS}_2 \times S^2$ with underlying $\text{SU}(1, 1 \mid 2)$ invariance is constructed in the bosonic and fermionic coordinates of the super 0-brane.

The purpose of this paper is that we explore the possibility of how to simplify the super 0-brane action on $\text{AdS}_2 \times S^2$ in Killing gauge and avoid the incompatibility in $[16]$ in order to find the super 0-brane action on $\text{AdS}_2 \times S^2$ with underlying $\text{SU}(1, 1 \mid 2)$ invariance in terms of the bosonic and fermionic coordinates of the super 0-brane. To achieve this goal, we exploit the super 0-brane action on $\text{AdS}_2 \times S^2$ built out of the Cartan 1-forms $L^a, L^{a'}$ and $L^I$ $[10]$, which has global $\text{SU}(1, 1 \mid 2)$-invariance, and is invariant under local $\kappa$-symmetry and one-dimensional reparametrization symmetry. The crucial feature of the super 0-brane action on $\text{AdS}_2 \times S^2$ is that it contains two free parameters $A$ and $B$, which can be interpreted as the electric and magnetic charges of the super 0-brane. To gauge fix $\kappa$-symmetry in Killing gauge $[13]$ and to avoid the incompatibility $[16]$, we choose the magnetic charge ($B$) of the super 0-brane to be nonzero. The 0-brane projector on $\text{AdS}_2 \times S^2$ is given by $\mathcal{P}_\pm = \frac{1}{2}(\delta^{ij} \pm \gamma^0 \epsilon^{ij})$, where the signs depend on the choice of the coordinate frames. First we consider the Killing gauge in the coordinate frame (8) whose Killing horizon is at $r = \infty$ $[19]$. We find that the proper Killing gauge is $\mathcal{P}_- \Theta = \Theta_- = 0$, which makes $(\mathcal{M}^2_{\text{fix}})D \Theta_+ = 0$, and $(D \Theta_+)^I = (\mathcal{A} d \theta_+)^I$, thus we can simplify the Cartan 1-forms $L^a, L^{a'}$ and $L^I$. If we work in AdS coordinates, the situation is reversed, instead of $\Theta_-$ we have to put $\Theta_+ = 0$ as Killing gauge to simplify the Cartan 1-forms. The rule is that for a metric $g_{00} \sim r^l$, we pick $\Theta_- = 0$ if $l < 0$, but we have to choose $\Theta_+ = 0$ if $l > 0$. With the simplified expression for the Cartan 1-forms, the $\kappa$-symmetry gauge fixed super 0-brane action on $\text{AdS}_2 \times S^2$ is obtained, which can be considered as the supersymmetric generalization of the action in $[19]$ with underlying $\text{SU}(1, 1 \mid 2)$ invariance. To get $\kappa$-symmetry gauge fixed super 0-brane action, we have taken the magnetic charge of the super 0-brane to be nonzero. To see the incompatibility explicitly, we choose the parameter $B = 0$ and $A > 0$, then the classical static vacuum solution exists. For this static solution the gauge fixed super 0-brane action vanishes, and the $\kappa$-symmetry transformation is reduced to $\delta_\kappa \Theta_+^I = \kappa_+^I$, which means that the gauge fixing should be $\Theta_+^I = 0$ instead of $\Theta_-^I = 0$ in (9), that is, the usual Killing gauge

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3By “underlying $\text{SU}(1, 1 \mid 2)$ invariance” we mean that upon gauge fixing $\kappa$-symmetry, its superconformal transformations are non-linearly realized on the remaining fields $[21, 23]$. 
is incompatible with the classical static vacuum solution. Since in AdS coordinates, the proper gauge fixing for \( \kappa \)-symmetry is \( \Theta_+ = 0 \), naively it seems the incompatibility could be resolved. However, we find that in AdS coordinates, to make the action (34) vanish in the static solution (31), we have to choose \( A = -m \). Then the \( \kappa \)-symmetry transformation is changed into \( \delta_\kappa \Theta^I_+ = \kappa^I_+ \), which indicates that the gauge fixing should be \( \Theta_- = 0 \) instead of \( \Theta^I_+ = 0 \). Thus the incompatibility between Killing gauge and static solution cannot be smoothed out by a change of the coordinate frame. But when the magnetic charge \( B \) is nonzero, this incompatibility can be avoided and the simplification of the super 0-brane action on \( \text{AdS}_2 \times S^2 \) in Killing gauge works, since the static vacuum solution does not exist for \( B \neq 0 \). Finally the invertibility of the fermionic kinetic operator is discussed, and we find that if we choose gauge fixing properly for the coordinate \( e \), the Killing gauge fixing is acceptable.

Now let us consider the super 0-brane action on \( \text{AdS}_2 \times S^2 \) background described in terms of the supercoset formalism\(^4\). The interesting property of \( \kappa \)-symmetry of GS actions in coset superspaces was discussed in \(^5\).
\[ m = \sqrt{A^2 + B^2} \]  

where the expression for \( \kappa \)-symmetry includes the parameters A and B, and \( m = \sqrt{A^2 + B^2} \) occurs as a consequence of \( \kappa \)-symmetry of the super 0-brane action.

The invariant 1-forms \( L^I = L^I_{s=1}, \hat{L}^\alpha = \hat{L}^\alpha_{s=1} \) are given by

\[
L^I_s = \left( \sinh \left( s \frac{M}{2} \right) \right) D\Theta^I \\
\hat{L}^\alpha_s = c^\alpha_{\hat{m}}(x) dx^{\hat{m}} + 4\hat{\Theta}^I \Gamma^\alpha \left( \sinh \left( s \frac{M}{2} \right) \right) D\Theta^I
\]

where \( x^{\hat{m}} \) and \( \Theta^I \) are the bosonic and fermionic super 0-brane coordinates and for the \( SU(1,1 \mid 2) \) superalgebra we have

\[
(M^2)^{IL} = -\epsilon^{IJ} \gamma(\gamma_a \Theta^I \Theta^L \gamma^a + \gamma \otimes \gamma_a \gamma_a^I \Theta^L \gamma \otimes \gamma^a) \\
+ \frac{1}{2} \epsilon^{KL} (-\gamma_{ab} \Theta^I \Theta^K \gamma_{ab} + \gamma_{a'b'} \Theta^I \Theta^K \gamma_{a'b'}) \gamma,
\]

\[
(D\Theta)^I = \left[ d + \frac{1}{4} (\omega^{ab} \gamma_{ab} + \omega^{a'b'} \gamma_{a'b'}) \right] \Theta^I \\
+ \frac{1}{2} \epsilon^{IJ} (e^a \gamma_a \gamma - e^{a'} \gamma_{a'}) \Theta^I
\]

where the Dirac matrices are split in a ‘2+2’ way.

Before gauge fixing the \( \kappa \)-symmetry for the super 0-brane action on \( AdS_2 \times S^2 \), we choose the coordinates

\[
ds^2 = -\left( \frac{2M}{r} \right)^4 dr^2 + \left( \frac{2M}{r} \right)^2 dr^2 + M^2 \left( d\chi^2 + \sin^2 \chi d\phi^2 \right)
\]

where the Killing horizon in these coordinates is at \( r = \infty \). In the following discussion, we put \( M = 1 \) for simplicity, and finally we recover \( M \) by dimension analysis.

Since the superstring \( \kappa \)-symmetry projector differs from the D3-brane projector in \( AdS_5 \times S^5 \), the Killing spinor gauge works for the GS superstring action on \( AdS_5 \times S^5 \) and a similar conclusion holds for the GS superstring action on \( AdS_2 \times S^2 \). In [16], it is argued that there is incompatibility between Killing spinor gauge for the D3-brane action on \( AdS_5 \times S^5 \) and the static vacuum solution of the D3-brane equation of motion. Similarly, this incompatibility also exists for the 1-brane action on \( AdS_3 \times S^3 \) and for the 0-brane action on \( AdS_2 \times S^2 \). To make use of the Killing spinor gauge and avoid the incompatibility, we
have to choose the magnetic charge (B) of super 0-brane to be nonzero (the reason will become clear below). If we define $P_\pm = \frac{1}{2}(\delta^{IJ} \pm \gamma^0 \epsilon^{IJ})$, $P_\pm \Theta = \Theta_\pm$, one may wonder which component, $\Theta_+$ or $\Theta_-$, is put to zero for gauge fixing $\kappa$-symmetry in the coordinate frame (8). Even though the projector is indicated by the full 0-brane Killing spinor [13, 5], as we will see below, it is coordinate frame dependent.

In the coordinate frame (8), the proper gauge is

$$P_\pm \Theta = \Theta_\pm = 0, \quad \epsilon^{IJ} \Theta_+^I = -\gamma^0 \Theta_+^I. \quad (9)$$

We will show that in this gauge we have $(\mathcal{M}^2_{\text{fix}}) D \Theta_+ = 0$, which means that all terms of the $(\mathcal{M}^2_{\text{fix}}) D \Theta_+$ for $n > 0$ vanish. Since the coordinate frame (8) possesses the property $\omega^0 e^0 = 0$, one has

$$\epsilon^{IJ}(D \Theta_+)^J = -\gamma^0 (D \Theta_+)^I. \quad (10)$$

Here should emphasize that, to get (10), $\omega^0 e^0 = 0$ plays a crucial role. With (9) and (10), one can easily show that if $\{\gamma^0, U\} = 0$, one has

$$\bar{\Theta}_+^I U D \Theta_+^I = 0. \quad (11)$$

By exploiting (9), (10) and (11), one gets

$$(\mathcal{M}^2_{\text{fix}})^L D \Theta_+^L = -\epsilon^{IJ} \gamma \left( \gamma_a \Theta_+^J \bar{\Theta}_+^L \gamma^a + \gamma \otimes \gamma_a \Theta_+^I \bar{\Theta}_+^L \gamma \otimes \gamma^a \right) D \Theta_+^L + \frac{1}{2} \epsilon^{KL} \left( -\gamma_{ab} \Theta_+^I \bar{\Theta}_+^L \gamma^a + \gamma_a \gamma^a \Theta_+^I \bar{\Theta}_+^L \right) \gamma D \Theta_+^L = 0. \quad (12)$$

Then the 1-forms are simplified as

$$(L_+^I)_+ = s D \Theta_+^I, \quad (L_+^I)_- = 0$$

$$L_0^s = \left( \frac{2}{r} \right)^2 d\tau + s^2 \bar{\Theta}_+^I \gamma^0 D \Theta_+^I$$

$$L_r^s = \frac{2}{r} dr, \quad L_\chi^s = d\chi, \quad L_\phi^s = \sin \chi d\phi. \quad (13)$$

Since we are interested in a ‘2 + 2’ splitting, we need work out $D \Theta_+$ explicitly in the coordinate frame (8), which can be expressed as

$$D \Theta_+^I = \left[ d \left( \frac{1}{4}(\omega_{ab} \gamma_{ab} + \omega_a^{a'b'} \gamma^a \gamma^b) \right) \right] \Theta_+^I$$

6If we choose $\Theta_+$ to be zero instead of $\Theta_-$, we cannot obtain (11) in the coordinate frame (8).

7Unlike GS superstring on $\text{AdS}_5 \times S^5$, there only if $[\Gamma^{0123}, U] = 0$, (11) holds.
\[ + \frac{1}{2} \epsilon^{IJ} (\epsilon^a \gamma_a - e^{\alpha'} \gamma_{a'}) \Theta_+^I \]
\[ = \frac{\Lambda}{r} d(r \Lambda^{-1} \Theta_+)^I, \]  
(14)

where we have used the fact that

\[ \frac{1}{4} \delta^{IJ} \omega^{a'b'} \gamma_{a'b'} - \frac{1}{2} \epsilon^{IJ} e^{\alpha'} \gamma_{a'} = (\Lambda d \Lambda^{-1})^{IJ} \]  
(15)

with

\[ \Lambda = e^{-\chi E \gamma_{32}/2} e^{-\phi \gamma_{23}/2}, \]  
(16)

where \( E \) is defined in (4), and in deriving (14), we have exploited the property \( \omega^{01} + e^0 = 0 \). If we define new variables

\[ \theta_+^I = r (\Lambda^{-1} \Theta_+)^I, \quad \Theta_+^I = \frac{1}{r} (\Lambda \theta_+)^I \]  
(17)

then \( D \Theta_+^I \) turns into

\[ (D \Theta_+)^I = \left( \frac{\Lambda}{r} d \theta_+ \right)^I. \]  
(18)

With (18), the 1-forms can be further reduced to

\[ \left( L_s^I \right)_+ = \frac{s}{r} (\Lambda d \theta_+)^I, \quad \left( L_s^I \right)_- = 0, \]
\[ L_s^0 = \left( \frac{2M}{r} \right)^2 \left( d \tau + \frac{s^2}{4} \theta_+^0 \gamma_0 d \theta_+^I \right), \]
\[ L_s^r = \frac{2M}{r} dr, \quad L_s^\chi = M d \chi, \quad L_s^\phi = M \sin \chi d \phi, \]  
(19)

where we have exploited the explicit expression for \( \Lambda \), the relation \( \gamma_{a'} = c^{-1} c^{a'T} c' \) \( [10] \), and the dependence on \( M \) has been recovered. We notice that the dependence of the 1-forms on \( \Lambda \) has been removed.

To get (19), we have heavily exploited the equation \( \omega^{01} + e^0 = 0 \), and the gauge fixing for \( \kappa \)-symmetry has to be taken as \( \Theta_- = 0 \) in order to use (10) and (14) to get \( M_{fix}^2 D \Theta_+ = 0 \). If we instead choose \( \Theta_+ = 0 \) to gauge fix the \( \kappa \)-symmetry, the simplification cannot be carried out since (10) and (14) fail in the coordinate frame (8). However, in AdS (spherical) coordinates\(^8\)

\[ ds^2 = - \left( \frac{r}{M} \right)^2 d \tau^2 + \left( \frac{M}{r} \right)^2 dr^2 + M^2 (d \chi^2 + \sin^2 \chi d \phi^2) \]  
(20)

\(^8\)To get (20), we need do the transformation \( r \to 2 Mr^{-\frac{1}{2}} \) from (8).
the Killing horizon is at \( r = 0 \) and we have \( \omega^{01} - e^0 = 0 \). From the above discussion we know that the proper gauge fixing for \( \kappa \)-symmetry is
\[
\mathcal{P}_+ \Theta = \Theta_+ = 0, \quad e^{IJ} \Theta_+^I = \gamma^0 \Theta_+^I,
\]
which reverses the role of \( \Theta_+ \) and \( \Theta_- \). Moreover, we have
\[
e^{IJ} (D \Theta_-)^J = \gamma^0 (D \Theta_-)^I,
\]
\[
(M^2_{fix})^{IL} (D \Theta_-)^L = 0,
\]
\[
(D \Theta_-)^I = r^\frac{l}{2} (\Lambda d\theta_-)^I,
\]
\[
\theta_-^I = r^{-\frac{l}{2}} (\Lambda^{-1} \Theta_-)^I
\]
and the corresponding 1-forms in AdS coordinates are simplified as
\[
(L^I_\delta)_- = s r^\frac{l}{2} (\Lambda d\theta_-)^I, \quad (L^I_\delta)_+ = 0
\]
\[
L^0_\delta = \frac{r}{M} \left( d\tau + s^2 \bar{\Theta}_-^I \gamma^0 \theta_-^I \right),
\]
\[
L^r_\delta = \frac{M}{r} dr, \quad L^\chi_\delta = M d\chi, \quad L^\phi_\delta = M \sin \chi d\phi.
\]
What we learned is that, for a given coordinate frame, the Killing gauge fixing for \( \kappa \)-symmetry is unique: For \( g_{00} \sim r^l \) we have to put \( \Theta_- = 0 \) when \( l < 0 \), and we should choose \( \Theta_+ = 0 \) when \( l > 0 \).

To get the superconformal mechanics for the super 0-brane on \( \text{AdS}_2 \times S^2 \), we consider the coordinate frame (8). To represent the WZ term in (1) as an integral we use the standard trick of rescaling \( \Theta \rightarrow \Theta_s \equiv s \Theta \),
\[
I_{WZ} = I_{WZ}(s = 1), \quad \partial_s I_{WZ}(s) = \int_{\partial \mathcal{M}_2} \partial_s H(s)
\]
and
\[
\partial_s H(s) = 2 \left( A e^{IJ} \bar{\Theta}_+^I L_s^J + B e^{IJ} \bar{\Theta}_+^I \Gamma_5 L_s^J \right),
\]
where we have used the following equation [10]:
\[
\delta \mathcal{L}_{WZ} = -2 \left( A e^{IJ} \bar{L}_s^I \delta \Theta^I + B e^{IJ} \bar{L}_s^I \Gamma_5 \delta \Theta^I \right)
\]
Then we have
\[ I_{WZ}(s = 1) = I_{WZ}(s = 0) + 2 \int_0^1 ds \int dt \left( A e^{IJ} \Theta^I L^J_s + B e^{IJ} \Theta^I \Gamma_5 L^J_s \right) \] (27)

In the coordinate frame (8), we get
\[ I_{WZ} = \int dt \left[ A \left( \frac{2M}{r} \right)^2 \dot{\tau} + B M \cos \chi \dot{\phi} + A e^{IJ} \Theta^I_+ D \Theta^J_+ \right], \] (28)

where we have used (10) and (11) to show that
\[ A^I_+ \Phi^I_+ = -\Theta^I_+ \gamma_0 (D \Theta^I_+) \] vanishes. We note that the first two terms in the brackets come from \( I_{WZ}(s = 0) \), which vanishes in the case of the GS superstring action. Then the \( \kappa \)-symmetry gauge fixed super 0-brane action on \( \text{AdS}_2 \times S^2 \) is
\[ I_{0\text{-brane}} = \int dt \left\{ -m \left( \frac{2M}{r} \right)^2 \left( \dot{\theta}_+ \right)^2 - \frac{r^2 \dot{\theta}_+^2}{4M^2} - M^2 \left( \frac{r}{2M} \right)^4 \left( \chi^2 + \sin^2 \phi \right) \right\}^{\frac{1}{2}} \]
\[ + \int dt \left\{ A \left( \frac{2M}{r} \right)^2 \left( \dot{\tau} - \frac{1}{2} \dot{\theta}_+ \right) + B M \cos \chi \dot{\phi} \right\}, \] (29)

where \( \theta_+ \) denotes \( \theta_+^1 \). Eq.(29) describes the dynamics of the super 0-brane in \( \text{AdS}_2 \times S^2 \) background, which generalizes the action given in [19]. By introducing the auxiliary coordinate \( e \), the above action is rewritten as
\[ I_{0\text{-brane}} = \int dt \left\{ \frac{1}{2} e^{-1} \left[ -\left( \frac{2M}{r} \right)^4 \left( \dot{\tau} + \frac{1}{2} \dot{\theta}_+ \right)^2 + \frac{4M^2 \dot{\theta}_+^2}{r^2} + M^2 \left( \chi^2 + \sin^2 \phi \right) \right] - \frac{1}{2} \epsilon m^2 \right\}^{\frac{1}{2}} \]
\[ + \int dt \left\{ A \left( \frac{2M}{r} \right)^2 \left( \dot{\tau} - \frac{1}{2} \dot{\theta}_+ \right) + B M \cos \chi \dot{\phi} \right\}. \] (30)

Variating action (30) with respect to the variable \( \chi \), we have \( \dot{\chi} \sim B \sin \chi \dot{\phi} \), which shows that we can interpret \( B \) as the magnetic charge of the super 0-brane.

To get (29), we have assumed the magnetic charge of the super 0-brane to be nonzero. When we choose the parameter \( B = 0 \) and \( A > 0 \), we have \( A = m \) from (5) and there is a classical static vacuum solution of the super 0-brane equation of motion following from (29) [19],
\[ \tau = t, \ r = \text{constant}, \ \chi = \text{constant}, \ \phi = \text{constant}, \ \theta_+ = 0. \] (31)

For this static solution, the action (29) vanishes \( (A > 0, \ B = 0) \), which is called the no-force condition, since there is no potential which can push the super 0-brane probe to the
boundary of AdS\(_2\). When \(A > 0, B = 0\), the \(\kappa\)-symmetry projector is reduced to \(\Gamma = -\mathcal{E}\gamma^0\) and (2) can be written as

\[
\delta_\kappa \Theta^I = (\delta^I J + \epsilon^{IJ} \gamma^0) \kappa^J, \quad \delta_\kappa \Theta^I_+ = \kappa^I_+, \quad \delta_\kappa \Theta^I_- = 0,
\]

which shows that the gauge fixing for \(\kappa\)-symmetry should be chosen as \(\Theta^I_+ = 0\) instead of \(\Theta^I_- = 0\) in (3), that is, the usual Killing spinor gauge is incompatible with the classical static vacuum solution (31) which was first mentioned in [16].

However, from (21) we know that in AdS coordinates (20) the proper gauge fixing for \(\kappa\)-symmetry is \(\mathcal{P}_+ \Theta = \Theta_+ = 0\). Hence we would like to see whether choosing a different coordinate frame could avoid the above incompatibility. In the AdS coordinates (27) yields

\[
I_{WZ} = \int dt \left( -\frac{Ar}{M} \dot{\tau} + BM \cos \chi \dot{\phi} + \frac{2Ar \dot{\theta}_- \gamma^0 \theta_-}{M} \right),
\]

where \(\theta_-\) denotes \(\theta_1\), and the \(\kappa\)-symmetry gauge fixed super 0-brane action becomes

\[
I_{0\text{-brane}} = \int dt \left\{ -m \left[ \left( \frac{r}{M} \right)^2 \left( \frac{\dot{\tau} + 2\dot{\theta}_- \gamma^0 \dot{\theta}_-}{\dot{\tau}} \right)^2 - \frac{M^2 \dot{\phi}^2}{\dot{\tau}^2} - M^2 \left( \dot{\chi}^2 + \sin^2 \chi \dot{\phi}^2 \right) \right] \right\}^{\frac{1}{2}} - \int dt \left\{ \frac{Ar}{M} \left( \dot{\tau} - 2\dot{\theta}_- \gamma^0 \theta_- \right) - BM \cos \chi \dot{\phi} \right\}.
\]

When \(B = 0\), to make the action (34) vanish in the static solution (31), we should choose \(A = -m\). Then the \(\kappa\)-symmetry projector is reduced to \(\Gamma = \mathcal{E}\gamma^0\) and (2) turns into

\[
\delta_\kappa \Theta^I = (\delta^I J - \epsilon^{IJ} \gamma^0) \kappa^J, \quad \delta_\kappa \Theta^I_+ = \kappa^I_+, \quad \delta_\kappa \Theta^I_- = 0,
\]

which indicates that the gauge fixing for \(\kappa\)-symmetry should be \(\Theta^I_- = 0\) instead of \(\Theta^I_+ = 0\). Thus the incompatibility between the Killing gauge and the static solution cannot be smoothed out by a change of the coordinate frame. When the magnetic charge \(B\) is nonzero, however, this incompatibility can be avoided and the above simplification of the super 0-brane action on AdS\(_2\) \(\times\) \(S^2\) in the Killing gauge works, since in this case the static vacuum solution does not exist.

The invertibility of the fermionic kinetic operator in action (30) can be seen from the quadratic term in the fermionic part

\[
\mathcal{L}_2 \sim \bar{\theta}_+ \Omega \gamma^0 \theta_+, \quad \Omega = \left( \frac{2M}{r} \right)^2 \left[ e^{-1} \left( \frac{2M}{r} \right)^2 \dot{\tau} + A \right].
\]
Since A is positive in the coordinate frame with the static gauge \( \tau = t \), \( \Omega^2 \) is nonzero, provided that we choose the gauge fixing for \( e \) properly. This shows that the Killing gauge fixing is acceptable in the above derivation.

Up to now, we have obtained the \( \kappa \)-symmetry gauge fixed super 0-brane action on \( \text{AdS}_2 \times S^2 \) with underlying \( SU(1,1|2) \) invariance, which is the supersymmetric generalization of the action in [19]. If we put \( \theta_+ = 0 \) and \( B = 0 \), our bosonic Lagrangian is reduced to that in [19] with \( A = q \). Due to the restriction from \( \kappa \)-symmetry (9), we have \( m = q \), but the other two cases for \( m > q \) and \( m < q \) cannot appear from our construction. It seems that we can add the following term to the WZ term in order that we could introduce more free parameters:

\[
\mathcal{L}_{WZ} \rightarrow \mathcal{L}_{WZ} + \tilde{\mathcal{H}}
\]

with

\[
\tilde{\mathcal{H}} = \tilde{A} \delta^{IJ} L^{\alpha\alpha'}(C \gamma)_{\alpha\beta} C_{\alpha' \beta'} \wedge L^{\beta\beta'} + \tilde{B} s^{IJ} L^{\alpha\alpha'} C_{\alpha \beta'}(C \gamma')_{\alpha' \beta} \wedge L^{\beta\beta'} ,
\]

where \( s^{IJ} = (1, -1) \). One can easily check

\[
d\tilde{\mathcal{H}} = 0, \quad \delta \tilde{\mathcal{H}} = -2 d[\tilde{A} \delta^{IJ} \bar{L}_0(\gamma \otimes 1) \delta \theta^J + \tilde{B} s^{IJ} \bar{L}_0(1 \otimes \gamma') \delta \theta^J],
\]

so we can define a new \( \kappa \)-symmetry projector. However, when we demand \( \Gamma^2 = 1 \), we have \( \tilde{A} = \tilde{B} = 0 \), that is, we cannot introduce more free parameters to the action in this way. Then it is interesting to see whether it is possible to find a more general super 0-brane action on \( \text{AdS}_2 \times S^2 \).

In [24] it was observed that the quantum gravity on \( \text{AdS}_2 \) is a conformal theory on a strip which exhibits the symmetries of the Virasoro algebra. Motivated by [24], the model in [19] was studied to see if one can find the generators of the full Virasoro algebra [25]. It was shown that for the model in [19] with one dynamical variable, one can find generators of the full Virasoro algebra, but the central charge always vanishes. It would be interesting to know whether there is a way to determine the central charge and normal-ordering conventions for the present model, and to see whether a non-vanishing central charge could be found for the gauge-fixed super 0-brane model on \( \text{AdS}_2 \times S^2 \), which is related to the \( \text{AdS}_2/\text{CFT}_1 \) correspondence [24].
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