Origin of craters on Phoebe: comparison with Cassini’s data

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ABSTRACT

Context. Phoebe is one of the irregular satellites of Saturn. The images taken by the Cassini-Huygens spacecraft have allowed us to analyze its surface and the craters on it.

Aims. We study the craters on Phoebe produced by both Centaur objects from the scattered disk (SD) and plutinos that have escaped from the 3:2 mean motion resonance with Neptune and compare our results with the observations by Cassini.

Methods. We use previously developed simulations of trans-Neptunian objects and a method that allows us to derive the number of craters and the cratering rate on Phoebe.

Results. We determine the number of craters and the largest crater on Phoebe produced by Centaurs in the present configuration of the solar system. We present a new normalized rate of encounters of Centaurs with Saturn of $\dot{F} = 7.1 \times 10^{-11}$ per year, from which we can infer the current cratering rate on Phoebe for each crater diameter.

Conclusions. Our study and comparison with observations suggest that the main crater features on Phoebe are unlikely to have been produced in the present configuration of the solar system but that they must have been created instead when the SD were depleted in the early solar system. If this is indeed what happened and the craters were produced when Phoebe was a satellite of Saturn, then it must have been captured, very early on, in the evolution of the solar system.

Key words. methods: numerical – Kuiper belt: general

1. Introduction

Phoebe is one of a number of irregular satellites of Saturn. It has a retrograde orbit, which suggests that it was captured by Saturn instead of being formed “in situ” (e.g. Pollack et al. 1979). Moreover, Phoebe’s composition is close to that derived from bodies such as Triton and Pluto, and differs from that of the regular satellites of Saturn; implying that Phoebe could have been a captured body of the outer solar system (Johnson & Lunine 2005).

On 11 June 2004, the Cassini-Huygens spacecraft encountered Phoebe in a fly-by that passed within 2000 km of Phoebe’s surface. This encounter allowed Cassini to analyze Phoebe’s surface in detail and improve our understanding derived from the previous Voyager data. Buratti et al. (2004) analyzed and characterized the physical properties of the surface using photometric data from Cassini VIMS (Visual and Infrared Spectrometer), concluding that it is rough and dusty, perhaps owing to a history of out-gassing or a violent collisional history as suggested by Nesvorny et al. (2003). Hendrix & Hansen (2008) analyzed the first UV spectra of Phoebe with the Cassini Ultraviolet Imaging Spectrograph (UVIS) during the Cassini spacecraft fly-by and detected water ice. Using VIMS data, Cruikshank et al. (2004) concluded that Phoebe’s surface is rich in organics, which is compatible with the low albedo of Phoebe. Porco et al. (2005) calculated a mean density of Phoebe of $1.63 \text{ g cm}^{-3}$ based on the volume and the mass determined by measuring the perturbation of the spacecraft’s trajectory during the Phoebe fly-by. If Phoebe’s surface were a mixture of rock and ice, the previous density would be compatible with a porosity lower than $\sim 40\%$ (Porco et al. 2005). Johnson & Lunine (2005) analyzed the relation between the composition and the probable porosity of Phoebe and found that if Phoebe were derived from the same compositional reservoir as Pluto and Triton, Phoebe’s measured density would be consistent with a porosity of $\sim 15\%$.

Giese et al. (2006) presented the results of a photogrammetric analysis of the high-resolution stereo images of Phoebe. In particular, they obtained a mean figure radius of 107.2 km and derived a digital terrain model of the surface that describes significant morphological detail. The images revealed that on Phoebe there are mainly simple crater shapes, the only exception being the largest impact crater Jason with a diameter of $\sim 100$ km. Several of the smaller craters have pronounced conical shapes that might be indicative of porous, low compacting material on the surface of Phoebe.

Kirchoff & Schenk (2010) reexamined the impact crater distribution for the mid-sized Saturnian satellites. For Phoebe, they found that the crater-size frequency distribution has relatively constant values for crater diameters $D \leq 1$ km, but then has a sudden and confined dip around $D \sim 1.5$ km. Beyond this dip, the crater-size frequency distribution undergoes a slow increase. This behavior is unique to the Saturnian satellite system and it is probably connected to Phoebe’s origin.

Zhanle et al. (2003) calculated cratering rates for the satellites of the outer planets. They used impact rates on the giant planets obtained by Levison & Duncan (1997) and independent constraints on the number of ecliptic comets. Their results are later compared with ours.

The origin of craters on Phoebe is therefore unclear. However, the main population of objects that can produce craters on Phoebe are Centaurs, since they are the small body objects...
that cross the orbits of the giant planets, in particular the orbits of Saturn, and then its satellites.

Centaurs are transient bodies between their source in the trans-Neptunian population and the Jupiter family comets. They come mainly from a subpopulation in the trans-Neptunian zone, the scattered disk objects (SDOs). The SDOs are bodies with perihelion distances \( q \) greater than 30 AU and smaller than ~39 AU that can cross the orbit of Neptune and eventually evolve into the giant planetary zone, where they cross both the orbits of these planets and of their satellites (Di Sisto & Brunini 2007; Levison & Duncan 1997). The secondary source of Centaurs are plutinos and the low-eccentricity trans-Neptunian objects (Di Sisto et al. 2010; Levison & Duncan 1997). Plutinos are trans-Neptunian objects located in the 3:2 mean motion resonance with Neptune at \( a \sim 39.5 \) AU. They are “protected” by the 3:2 mean motion resonance with Neptune but are long-term escapers that are presently escaping from the resonance (Morbidelli 1997). In this paper, we study the production of term escapers that are presently escaping from the resonance (Morbidelli 1997). In this paper, we study the production of term escapers that are presently escaping from the resonance (Morbidelli 1997). In this paper, we study the production of term escapers that are presently escaping from the resonance (Morbidelli 1997).

2. The number of SDOs

Cratering rates depend on the number and size of the impactor population. Thus we must know the real initial number of SDOs to calculate the total number of collisions on Phoebe. We may then, estimate the total number of present SDOs.

Parker & Kavelaars (2010a) re-characterized the orbital sensitivity of several published pencil-beam surveys. They found that these surveys were sensitive to distant populations such as SDOs and Sedna-like objects. Using this result, Parker & Kavelaars (2010b) derived new upper limits for these distant populations, which they used to determine the number of SDOs. To do this, they performed a model that considered two laws for the radial distance distribution of SDOs. On the one hand, they took a power-law distribution \( SDOs \propto r^{-1.5} \) and obtained a maximum population of \( N(d > 100 \text{ km}) = 3.5 \times 10^5 \). On the other hand, they took a uniform radial distance distribution obtaining in this case a maximum population of \( N(d > 100 \text{ km}) = 25 \times 10^5 \). In this paper, we assume that the number of SDOs greater than \( d = 100 \text{ km} \) is equal to \( N(d > 100 \text{ km}) = 3.5 \times 10^5 \) since the considered radial distance distribution is consistent with the one obtained by Di Sisto & Brunini (2007). The total population of SDOs of diameter greater than \( d_0 \) is then given by \( N(d > \text{d}_0) = 3.5 \times 10^5 (d_0/100)^{-1.5} \), where \( d_0 \) must be in km and \( s \) is the index of the differential size distribution. Some authors have found a single power-law size distribution for TNOs (Petit et al. 2000; Fraser et al. 2008). However, other papers suggest that the size distribution function (SDF) of TNOs might have a break at a diameter of ~60 km (Bernstein et al. 2004; Gil Hutton et al. 2009; Fraser & Kavelaars 2009; Fuentes & Holman 2008; Fuentes et al. 2009). The differential power-law indices of smaller TNOs (of size \( d < 60 \text{ km} \)) found by the aforementioned five papers are \( s_2 = 2.8, 2.4, 1.9, 2.5 \), and 2, respectively. Hence, there seems to be evidence of a break in the size distribution of the TNO population. In particular, we assume here that this break is also valid for all the dynamical classes of TNOs.

Fig. 1. Cumulative number of SDOs and plutinos according to the size distribution laws described in the text.

Elliot et al. (2005) accounts for the SDF of each dynamical class in the TN region. Specifically for SDOs, they found that the differential size distribution index of the brightest objects is \( s_1 = 4.7 \). Taking into account the assumed break in the SDF of SDOs, we assume here that the power-law SDF of SDOs breaks at \( d > 60 \text{ km} \) to an index of between 3.5 and 2.5. We analyze those indices as limiting cases that define the upper and lower limits of a range for the population of SDOs and then the production of craters on Phoebe. The higher value of \( s_2 = 3.5 \) corresponds to a population in a steady-state (Dohnanyi 1969), which could be the case for the smallest SDOs (Gil Hutton et al. 2009).

Considering all this, the number of SDOs of a diameter larger than \( d_0 \) is given by

\[
N(d > d_0) = C_0 \left( \frac{1 \text{ km}}{d} \right)^{s_2-1} \quad \text{for} \quad d \leq 60 \text{ km},
\]

\[
N(d > d_0) = 3.5 \times 10^5 \left( \frac{100 \text{ km}}{d} \right)^{-1} \quad \text{for} \quad d > 60 \text{ km},
\]

where \( C_0 = 3.5 \times 10^5 100^{s_2-1}(60)^{s_2-1} \) by continuity for \( d = 60 \text{ km} \), \( s_1 = 4.7 \), and \( s_2 = 2.5 \) and 3.5. This law is plotted in Fig. 1 with the two breaks that we considered.

3. SDO collisions on Phoebe

To study the collisions of SDOs on Phoebe and the contribution of that population to the cratering history of the satellite, we use some of the outputs of the numerical simulation performed in Di Sisto & Brunini (2007). In that paper, we numerically integrated 1000 objects from the SD (95 real + 905 fictitious) and studied their evolution in the Centaur zone under the gravitational attraction of the Sun and the four giant planets. The computations were followed for 4.5 Gyr, or until the test body either collided with a planet, was ejected, or entered the region inside Jupiter’s orbit (\( r < 5.2 \text{ AU} \)). In that paper, we also stored in a file the encounters of the fictitious SDOs with the planets and registered the time of the encounter, the minimum distance to the planet (\( q \)), and the relative velocity at this distance (\( v(q) \)). From these data, we can calculate the number of encounters with Saturn within the Hill’s sphere of the planet. Using the particle-in-a-box approximation and assuming that the geometry of the encounters is isotropic it is possible to calculate the number of collisions on Phoebe (\( N_c \)) using the relation

\[
\frac{N_c}{N_c^o} = \frac{\nu_1 R_P^2}{v(R) R^2}.
\]

where \( N_c \) is the number of encounters with Saturn inside its Hill’s sphere of radius \( R \), \( R_P \) is the radius of Phoebe, \( v(R) \) is
the mean relative encounter velocity of SDOs when entering the Hill’s sphere of the planet, and \( v_i \) is the collision velocity of SDOs on Phoebe. The \( v(R) \) can be calculated from the values of \( v(q) \) registered in the outputs of our simulations using the relation

\[
v(R) = v^2(q) + 2Gm \left( \frac{1}{R} - \frac{1}{q} \right),
\]

where \( G \) is the constant of gravitation and \( m \) is the mass of Saturn.

The collision velocity on Phoebe is computed assuming that the geometry of collisions is isotropic, hence that

\[
v_i = \sqrt{v^2_0 + v^2_p},
\]

where \( v_0 \) is Phoebe’s orbital velocity and \( v_p \) is the mean relative velocity of SDOs when they cross the orbit of Phoebe. This velocity was computed in the same way as \( v(R) \) appropriately using Eq. (3) from the values of \( v(q) \) registered in our outputs. All the afore mentioned velocities and the radius and orbital velocity of Phoebe are shown in Table 1.

Equation (2) provides the number of collisions on the surface of Phoebe in relation to the number of encounters with Saturn that we had registered in our previous run.

Many papers based on theoretical and observational work argue that the initial mass of the trans-Neptunian region was ~100 times higher than its present mass, and decayed to nearly its present value in at most 1 Gyr (see e.g. Morbidelli et al. 2008). The simulation of Di Sisto & Brunini (2007) studies the evolution of SDOs in the present configuration of the solar system, that is when the SD is expected to have roughly reached its present mass and dynamic state, ~3.5 Gyr ago. We can estimate the total number of collisions on Phoebe in the past ~3.5 Gyr by rescaling Eq. (2) to account for the total SDOs population.

Of the 1000 initial particles of our previous simulation (Di Sisto & Brunini 2007), 368 underwent 10,257 encounters within Saturn’s Hill sphere. Therefore, the total number of encounters with Saturn of the whole SDO population in the present configuration of the solar system is estimated as

\[
N_{a_{\xi}} = \left( \frac{368}{1000} \right) \left( \frac{10,257}{368} \right) N,
\]

where \( N \) is the number of different SDOs that have existed in the past 3.5 Gyr and can be inferred from Eq. (1). Here we assume that the present number of SDOs is roughly the same as it was 3.5 Gyr ago. Consequently, the total number of encounters between Saturn and the whole SDO population throughout the past 3.5 Gyr depending on the SDO diameter is given by

\[
N_{a_{\xi}} (d > d_0) = \left( \frac{10,257}{1000} \right) N (d > d_0).
\]

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\[
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\[
N_{a_{\xi}} (d > d_0) = \left( \frac{10,257}{1000} \right) N (d > d_0).
\]

From this equation and Eq. (2), the total number of collisions of SDOs with Phoebe over the past 3.5 Gyr again depending on the SDO diameter, is given by

\[
N_{c} (d > d_0) = \frac{v_i R^2_0}{v(R) R^2} N_{a_{\xi}} (d > d_0).
\]

Table 2 shows some values of \( N_{c} \) for certain values of the diameters of the impactors.

Depending on the values of \( s_2 \), the diameter of the largest SDO impactor onto Phoebe in the past 3.5 Gyr was calculated to range from 110 mts to 1.36 km.

### 4. Craters on Phoebe by SDOs

The estimation of the size of a crater produced by a particular impact has been extensively studied. Schmidt & Housen (1987) presented a set of power-law scaling relations for the crater volume based on laboratory experiments that simulate crater formation and point-source solutions. Holsapple (1993) also described the scaling law for impact processes in a review work. These derived scaling laws allow us to relate the effects of different sizes, velocities, and superfluid gravity and then obtain the size of a crater produced by a collision with a Solar System body. Holsapple & Housen (2007) present the updated scaling laws for cratering in a recent work dedicated to interpreting the observations of the Deep Impact event. These cratering laws are used here to calculate the size of the craters on Phoebe. Thus, the diameter \( D \) of a crater produced by an impactor of diameter \( d \) can be obtained from the general equation (Holsapple & Housen 2007)

\[
D = K_1 \left[ \frac{g d^3}{2 \mu \rho_s} \left( \frac{\rho_i}{\rho_s} \right)^{\frac{2}{3}} + \left( \frac{Y}{\mu \sqrt{\nu}} \right)^{\frac{1}{3}} \left( \frac{\rho_i}{\rho_s} \right)^{\frac{10+4 \nu}{3 \nu}} \right]^{\frac{1}{2}} d,
\]

\( \rho_i \) being the target density, \( g \) its superfluid gravity, \( Y \) its strength, \( \rho_s \) the density of the impactor, and \( v_i \) the collision velocity. This impact cratering scaling-law depends on two exponents, \( \mu \) and \( \nu \), and a constant, \( K_1 \), that characterize the different materials. The first term in the square brackets is a measure of the importance of gravity in the cratering event and the second is a measure of the importance of the target strength. Thus, if the first term is larger in value than the second term, the crater is under the gravity regime, and if the second term is instead larger we have the strength regime. The partition between the two size scales of impacts depends on the size of the event (Holsapple 1993).

Equation (8) is a convenient empirical smoothing function to span the transition between the gravity regime and the strength regime (Holsapple 1993). Since Phoebe is a small satellite with a relatively low gravity, the strength regime can be important for the smaller craters.

As Phoebe’s density (1.63 g cm\(^{-3} \)) is similar to sand and lower compacting material is found on its surface, we adopt

### Table 1. Physical parameters of Phoebe and velocities involved in the model (see Sect. 3).

| Parameter | Value |
|-----------|-------|
| \( R_p \) [km] | 107.2 |
| \( \rho_p \) [g cm\(^{-3} \)] | 1.634 |
| \( v_0 \) [km s\(^{-1} \)] | 1.71 |
| \( g_{\text{m}} \) [km s\(^{-2} \)] | 0.5532 |
| \( v(R) \) [km s\(^{-1} \)] | 4.06 |
| \( v_0 \) [km s\(^{-1} \)] | 4.96 |

### Table 2. Number of collisions of SDO impactors on Phoebe with diameters \( d > d_0 \) that produce craters with diameter \( D > D_0 \) \( (N_c (D > D_0)) \).

| \( d_0 \) [km] | \( D_0 \) [km] | \( N_c (D > D_0) \) |
|--------------|--------------|------------------|
| 0.081        | 1            | 2–1180           |
| 0.445        | 5            | 0–16             |
| 0.969        | 10           | 0–2              |
| 3.48         | 30           | 0                |

From this equation and Eq. (2), the total number of collisions of SDOs with Phoebe over the past 3.5 Gyr again depending on the SDO diameter, is given by

\[
N_{c} (d > d_0) = \frac{v_i R^2_0}{v(R) R^2} N_{a_{\xi}} (d > d_0).
\]
Fig. 2. Cumulative number of craters with diameters greater than $D_h$ in the past 3.5 Gyr, produced by Centaurs from SDOs on Phoebe. Filled line corresponds to the differential power-law index $s_2 = 3.5$ and the dotted line to $s_2 = 2.5$.

\begin{equation}
K_1 = 1.03, \mu = 0.41, \text{ and } \nu = 0.4, \text{ which correspond to either sand or cohesive soil in Holsapple \& Housen (2007). This value of } \mu \text{ corresponds to materials with a porosity of } \sim 30-35\% \text{ (Holsapple \& Schmidt 1987), which is compatible with the ranges of Phoebe's predicted porosity. The value for dry soils from Holsapple (1993), i.e. } Y = 0.18 \text{ mpa, is used for the strength.}
\end{equation}

The calculated densities of TNOs vary considerably from \sim 0.5-3 g cm$^{-3}$. Although a dimension-density trend has been suggested (Sheppard et al. 2008; Perna et al. 2009), more data would be required to confirm this. In addition, as crater experiments do not account for variations in the impactor material, there is no data to precisely determine the dependence of the crater size on the impactor density (Schmidt \& Housen 2007; Housen \& Holsapple 2003). Therefore, we assume that $\rho_i = \rho_s$, which is also between the lowest and highest calculated densities in the trans-Neptunian region. By taking all this into account, the diameter of a crater on Phoebe for a given impactor diameter can be calculated from Eq. (8) using

\begin{equation}
D = 1.03 \left[ \frac{gd}{2l_i^2} \right] + \left( \frac{Y}{\rho_i \sigma_i^2} \right)^{1.205} d^{-0.17}.
\end{equation}

This equation describes simple bowl-shape craters but, as previously mentioned in the introduction, Cassini images of Phoebe have detected simple crater shapes with the only exception being Jason with a diameter of \sim 100 km (Giese et al. 2006). Hence, we use Eq. (9) to calculate the diameters of all craters on Phoebe without any additional correction for transient-to-final size. By combining Eqs. (7) and (9), it is possible to calculate the number of craters on Phoebe according to the diameter of the crater. Figure 2 shows the cumulative number of craters, with diameters greater than a given value for the two size-distribution power laws for smaller SDOs on Phoebe. We note that the different slopes in the number of craters for each curve is due to the difference in both indices $s_2$ considered. As we previously noted, there is a limit to the impactor diameter that corresponds to the transition between the gravity regime and the strength regime. This diameter can be obtained by equating the first and second terms of Eq. (9). This limit to the impactor diameter is $d_h = 367$ mts, which produces a limit crater of $D_h = 4.2$ km. Thus, for crater diameters $D < D_h$, the production of craters on Phoebe is under the strength regime and for $D > D_h$, the production of craters is under the gravity regime. We note that $D_h$ depends strongly on the assumed value of strength, which is actually unknown. Then considering other values of the strength for less cohesive soils such as terrestrial dry desert alluvium of $Y = 65$ kpa (Holsapple \& Housen 2007) and surface lunar regolith $Y = 10$ kpa (Holsapple 2011), $D_h$ could take the values 1.5 km and 233 mts, respectively. In the following, we will assume that $D_h = 4.2$ km but it must be taken into account that $D_h$ can be as small as 233 mts.

Since in the strength regime the crater diameter depends linearly on the impactor diameter, the relation between the cumulative number of craters on Phoebe and the crater diameter follows the same power-law relation as that followed by the number of SDOs. For $D < 4.2$ km, the cumulative number of craters on Phoebe follows a power law with a cumulative index of 1.5 and 2.5, according to the value of $s_2 = 2.5$ or $s_2 = 3.5$ respectively. For $D > 4.2$ km, this is in the gravity regime, the crater diameter does not depend linearly on the impactor diameter. Therefore, we fit a power law for the cumulative number of craters on Phoebe depending on the crater diameter of index 2.8. Kirchoff \& Schenk (2010) found that the crater size frequency distribution for Phoebe has a cumulative index of 2.348 for $D = 0.15-1$ km and 1 for $D = 1-4$ km. We can see that, for very small craters, this index is very similar to our value of $s_2 = 3.5$ or cumulative index of 2.5. This is consistent with the size distribution of very small objects being expected to approach a Donnanyi size distribution ($s_2 = 3.5$) and the craters then produced by those small projectiles (that are in the strength regime) following the same power-law size distribution. In addition Kirchoff \& Schenk (2010) found that for $D = 1-4$ km, Phoebe's crater distribution has a shallow slope and this implies that Phoebe has a deficiency of craters with $D \sim 1.5$ km. This change in slope cannot be explained by our method and is inconsistent with our proposed contribution of Centaurs from the SD to the craters on Phoebe, unless the SDF of SDOs that we consider is different. In all cases, additional study of another source of craters on Phoebe, such as planetocentric objects, is needed and also of the connection of this source to the origin of the irregular satellite itself.

According to the differential size distribution index $s_2$, the largest crater on Phoebe produced by a Centaur from the SD has a diameter of between 1.4 km and 13.5 km. Table 2 shows the cumulative number of craters on Phoebe greater than certain diameters produced by Centaurs from the SD in the current configuration of the solar system in the past 3.5 Gyr. Since the largest crater on Phoebe has a diameter of \sim 100 km, it is unlikely to have been produced by a recent collision with an SDO, as we discuss in a following section.

5. Rate of SDO collisions onto Phoebe

From our outputs, we can calculate the number of encounters within the Hill’s sphere of Saturn as a function of time. In Fig. 3, we plot the normalized cumulative number of encounters as a function of time. The whole plot can be fitted by a log-function given by $f(t) = a + b \log t$, where $a = -3.24$ and $b = 0.19$. The total cumulative number of encounters with Saturn for all diameters at each time can be obtained from the plot and Eq. (6). We calculated the number of collisions onto Phoebe from the number of encounters with Saturn using our output data (Eq. (7)), and for each time in the integration. The cumulative number of

\footnote{Web page http://keith.aa.washington.edu/craterdata/scaling/index.htm Accessed August 9, 2011}
This linear approximation allows us to calculate a present rate of collisions onto Phoebe for all diameters as a function of time can be obtained by multiplying the fraction of encounters obtained from Fig. 3 by the number of collisions $N_c(d > d_0)$.

As we can see from Fig. 3 the rate of encounters and the rate of collisions onto Phoebe was high at the beginning but it has decreased until the present. During the first few Myrs, the shape of the curve is purely arbitrary because of initial conditions, but then begins to stabilize and to be significant. During the past ~3.5 Gyr, the rate has been almost constant. It is indeed possible to fit a linear relation to the past 3.5 Gyr of Fig. 3 given by $g(t) = F_1 + c$, where $F = 7.1 \times 10^{-11}$ and $c = 0.69$. This linear approximation allows us to calculate a present rate of collisions for a given diameter. The slope of this linear function $F = 7.1 \times 10^{-11}$ is the present normalized rate of encounters of SDOs with Saturn per year. To obtain the present rate of encounters with Saturn for each diameter, we must multiply $F$ by $N_c(d > d_0)$ (obtained from Eq. (6)). We can evaluate the present rate of collisions onto Phoebe for each diameter by multiplying $F$ by $N_c(d > d_0)$ (obtained from Eq. (7)). Similarly, the current rate of cratering on Phoebe for craters larger than a given diameter can be obtained from the current rate of collisions and the relation in Eq. (9) between the diameter of the impactor and that of the crater. Thus, for example the current cratering rate on Phoebe of Centaurs from SDOs that produce craters with $D > 1$ km is between $1.4 \times 10^{-10}$ and $8.3 \times 10^{-8}$ craters per year (depending on the $s_2$ value), which is at least ~80 craters with $D > 1$ km in the past Gyr. The current cratering rate of craters with $D > 5$ km is at least $1.14 \times 10^{-9}$ craters per year, which is ~1 crater with $D > 5$ km in the past Gyr.

Zhanle et al. (2003) calculate cratering rates in the satellites of the outer planets. They obtained a cratering rate on Phoebe for craters with $D > 10$ km of $8.6 \times 10^{-11}$ year$^{-1}$. In this study, we derive this rate to be between $2.7 \times 10^{-12}$ and $1.4 \times 10^{-10}$ craters per year, depending on $s_2$, our value for $s_2 = 3.5$ being very similar that of Zhanle et al. (2003).

6. The contribution of escaped plutinos to the craters on Phoebe

Plutinos might be another source of craters on Phoebe. Di Sisto et al. (2010) study the post-escape evolution of plutinos after escaping from the 3:2 mean motion resonance with Neptune, and in particular their contribution to the population of Centaurs. In that work, the present authors performed two sets of numerical simulations first to identify the plutinos that have recently escaped from the resonance and second to follow their evolution under the influence of the Sun and the four giant planets. This numerical simulation considered the evolution of plutinos in the present configuration of the solar system, this is as in our approach here for SDOs, when the trans-Neptunian region is expected to have reached roughly its present mass and dynamical state, ~3.5 Gyr ago. Following the same analysis that we made for SDOs described in Sects. 3 and 4, we calculated the number of craters on Phoebe produced by escaped plutinos in the past 3.5 Gyr, and also the largest impactor and crater. In the numerical simulations of Di Sisto et al. (2010), we started with 20000 initial massless particles in the 3:2 mean motion resonance, 671 of which underwent 20459 encounters within Saturn’s Hill sphere during the integration. We found that the mean relative encounter velocity of plutinos on entering the Hill’s sphere of Saturn is $v(R) = 4.57 \text{ km s}^{-1}$, that the mean relative velocity of plutinos when they intersect the orbit of Phoebe is $v_1 = 5.12 \text{ km s}^{-1}$, and that the collision velocity of plutinos on Phoebe is $v_1 = 5.4 \text{ km s}^{-1}$. In our present study, we take the present number of plutinos from de Elía et al. (2008), but consider that the size distribution breaks at $d \sim 60$ km with the upper and lower power-law indices of $s_2 = 3.5$ and 2.5, as we adopted for SDOs to be consistent (see Sect. 2). These indices infer the highest and lowest number of plutinos, hence the highest and lowest production of craters on Phoebe. The present cumulative number of plutinos is then given by

$$N(>D) = C \left( \frac{1 \text{ km}}{D} \right)^{p} \text{ for } D \leq 60 \text{ km},$$

$$N(>D) = 7.9 \times 10^{6} \left( \frac{1 \text{ km}}{D} \right)^{3} \text{ for } D > 60 \text{ km},$$

where $C = 7.9 \times 10^{6} (60)^{p-3}$ by continuity for $D = 60$ km and the cumulative power-law index $p$ has the values 2.5 and 1.5 ($p = s - 1$). This law is plotted in Fig. 1.

On the basis of all of this and our method described above, we found that the largest plutino that escaped and had an impact on Phoebe has a diameter between 1.5 mts and 102 mts, and produces a crater of between 19.3 mts and 1.3 km respectively, depending on the power index $p$ of the size distribution of plutinos. In addition, we inferred the number of craters on Phoebe produced by escaped plutinos. We reproduced at least two craters larger than 1 km on Phoebe that were created by plutinos. Comparing this with the values obtained for the contribution of SDOs, we found the number of craters produced by escaped plutinos on Phoebe is negligible with respect to those caused by SDOs. In addition, the largest craters are produced by SDOs.

7. Discussion

In the previous sections, we have calculated the production of craters on Phoebe considering the present population in the SD and plutinos. However – as we said – many papers based on theoretical and observational work argue that the initial mass of the TN region was ~100 times higher than the present mass (see e.g. Morbidelli et al. 2008). Observations predict a current mass of the Kuiper belt that is very small compared to that required for models to reproduce the objects that we see. The mass depletion due to a strong dynamical excitation of the Kuiper belt is thought to be the scenario for this “mass deficit problem”. Several models have been developed to describe the mass depletion; the last
model to describe this mechanism was the “Nice model” where the Kuiper belt had to have been significantly depleted before the time of the LHB (Levison et al. 2008). The “Nice Model” assumes that the giant planets were initially in a more compact region of between ~5.5 and ~14 AU in size and a planetesimal disk of a total mass of ~35 $M_\oplus$ that extends beyond the orbits of the giant planets up to ~34 AU. The interaction between the planets and planetesimals makes the giant planets migrate for a long time removing particles from the system. After a time ranging from 350 My to 1.1 Gy, Jupiter and Saturn cross their mutual 1:2 mean motion resonance. The eccentricities of Uranus and Neptune then increase causing them to penetrate into the planetesimal disk, destabilizing all of this disk and scattering the planetesimals all over the solar system.

Beyond the model and the mechanism responsible for the mass depletion of the trans-Neptunian zone, we can assume that primitive SDOs (that were 100 times more numerous than the present ones) follow the same dynamical evolution as the present population when they enter the planetary zone as Centaurs. In the same way, we can then calculate, as we did in the previous sections and with the same model the cratering on Phoebe assuming an initial population of SDOs that is 100 times more numerous than the present population.

The above is only an estimate because we still need to ascertain the real initial scenario of formation of the solar system and in particular of SDOs. However, when a SDO enters the Centaur zone, within the orbit of Neptune, its dynamical evolution is governed by the giant planets and then the particular initial scenario can be considered secondary for the present study. In doing this, we find that the largest impactor onto Phoebe during the lifetime of the solar system has a diameter of between 2.4 to 8.6 km and produces a crater of between 21.6 and 64.2 km. The value corresponding to $s_2 = 3.5$ (64.2 km) agrees (within the expected errors and statistical fluctuations) with the observation of the largest crater “Jason” on Phoebe, which has a diameter of ~100 km. The number of craters larger than a given diameter can be obtained by increasing 100 times the values obtained in Sect. 4.

Cassini images of Phoebe have allowed us to study its surface and craters. Kirchoff & Schenk (2010) obtain, a cumulative crater density for $D \geq 5$ km of 2233±1117 based on crater counting from Cassini images. From our model, assuming an initial number of SDOs 100 times the present population, we obtain $N_c(D > 5 \text{ km}) = 12-1640$ (if $s_2 = 3.5$), again in good agreement with the values obtained by Kirchoff & Schenk (2010).

8. Conclusion

We have studied the production of craters on Phoebe from SDOs and escaped plutinos that have reached the Saturn zone in the present configuration of the solar system. We have found that the contribution of escaped plutinos is negligible with respect to the contribution of SDOs. We have found that both the number of craters and the largest crater on Phoebe produced by SDOs cannot reproduce the observations. However, when we assumed that the initial mass of the trans-Neptunian region was 100 times the present one, we were able to explain the craters produced by SDOs on Phoebe with the observed characteristics of the satellite if $s_2 = 3.5$.

Those considerations imply that the main cratering features of Phoebe must have been acquired when the SD had been depleted early in the evolution of the solar system. In addition, if the “Nice model” correctly describes the scenario of the origin of the solar system, the scattering inward of planetesimals by Neptune and Uranus in that model must be similar to the present scattering in our model, and the TNOs arrive at Saturn independently of these scattering events.

If this is indeed occurred and the main crater characteristics on Phoebe were produced when Phoebe was a satellite of Saturn, the close agreement of our model with the observations constrains the time at which Phoebe was captured, very early in the evolution of our solar system. This argument was also suggested by Levison et al. (2008).

We have found that the present normalized rate of encounters of SDOs with Saturn is $\dot{F} = 7.1 \times 10^{-11}$ per year. From this number, we have been able to predict the present cratering rate on Phoebe for each crater diameter.

We have compared the size crater distribution on Phoebe predicted by our model with the observations of craters by Kirchoff & Schenk (2010). Our crater size frequency distribution agree with that obtained by Kirchoff & Schenk for very small impactors that produce craters of $D = 0.15-1$ km. This distribution follows a power law with a cumulative index of 2.5 consistent with a Donahy size distribution. For craters of $D = 1-4$ km, Kirchoff & Schenk (2010) found a shallow slope and a deficiency of craters with $D \sim 1.5$ km. This change of slope cannot be explained by our method and the contribution of Centaurs from the SD. We should investigate the possibility of another source of craters on Phoebe, such as planetocentric objects, in addition to the connection of this source to the origin of the irregular satellite itself.

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