Magnetised winds in dwarf galaxies

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ABSTRACT

Context. The generation and the amplification of magnetic fields in the current cosmological paradigm are still open questions. The standard theory is based on an early field generation by Biermann battery effects, possibly at the Epoch of Reionisation, followed by a long period of field amplification by galactic dynamos. The origin and the magnitude of the inter-galactic magnetic field is of primordial importance in this global picture, as it is considered to be the missing link between galactic magnetic fields and cluster magnetic fields on much larger scales.

Aims. We are testing whether dwarf galaxies are good candidates to explain the enrichment of the Inter-Galactic Medium (IGM): after their disc formation and trigger galactic dynamos, supernova feedback will launch strong winds, expelling magnetic field lines in the IGM.

Methods. We have performed Magneto-Hydrodynamics simulations of an isolated dwarf galaxy, forming self-consistently inside a cooling halo. Using the RAMSES code, we have for the first time simulated the formation of a magnetised supernova-driven galactic outflow. This simulation is an important step towards a more realistic modelling using fully cosmological simulations.

Results. Our simulations reproduce well the observed properties of magnetic fields in spiral galaxies. The formation and the evolution of our simulated disc lead to a strong magnetic field amplification: the magnetic field in the final wind bubble is one order of magnitude larger than the initial value. The magnetic field in the disc, essentially toroidal, is growing linearly with time as a consequence of differential rotation.

Conclusions. We discuss the consequence of this simple mechanism on the cosmic evolution of the magnetic field: we propose a new scenario for the evolution of the magnetic field, with dwarf galaxies playing a key role in amplifying and ejecting magnetic energy in the IGM, resulting in what we call a “Cosmic Dynamo” that could contribute to the rather high field strengths observed in galaxies and clusters today.

Key words. galaxies: formation – galaxies: evolution – galaxies: magnetic fields – methods: numerical

1. Introduction

The origin of magnetic fields in the Universe is a long-standing puzzle in cosmology. Although measuring magnetic field in cosmic structures is very challenging, we have strong evidence that magnetic fields exist in many galaxies, with magnitudes between 1 and 10 $\mu$G at a level close to equipartition (see the review of Beck et al. 1999). Magnetic fields of the order of several tens of $\mu$G are also detected in large galaxy clusters (see Carilli & Taylor 2002 or Govoni & Feretti 2004 and references therein). However, in the very low density Intergalactic Medium (IGM), magnetic fields still remain undetected on large scale (Blasi et al. 1999).

From a theoretical point of view, small-scale fluctuating magnetic fields could be generated from quantum effects during the Big–Bang (Turner & Widrow 1988 for example). The amplitude and the correlation length of such primordial fields should both be quite small (Grasso & Rubinstein 2001). Magnetic fields greater than a few nG would also have been detected as a specific source of anisotropy in the Cosmic Microwave Background. More extreme scenarios can also be ruled out: for example, a uniform magnetic field greater than $10^{-1}$ $\mu$G at our epoch would have led to unexpected anisotropies in the expansion of the Universe (Cheng et al. 1994). Note also that the relic of any magnetic field generated in the early universe, if it is greater than $10^{-7}$ $\mu$G today, would have broken the symmetry between left and right neutrinos (spin transition) during the period of neutrino production (Enqvist et al. 1993; Sciama 1994).

When the first structures collapse ($z \approx 10$), the Biermann Battery (see Kulsrud et al. 1997) is believed to have generated magnetic fields from pure collisional microscopic processes. These magnetic fields appear mainly at shocks and ionization fronts, where the motions of electrons and ions are decoupled, creating small microscopic currents. It was shown that this battery effect can generate magnetic fields of the order of $10^{-13}$ $\mu$G field in the IGM (Gnedin et al. 2001). More recently, shocks around primordial mini-halos and the associated first genera-
tion of massive stars were also considered as important sources of magnetic field at high redshift (Xu et al. 2008).

In order to reconcile the very small values quoted above with observations, we need strong amplification processes up to the observed level of magnetic field strength in galaxies and clusters. Gravitational contraction of the frozen-in magnetic field will not be sufficient, since we expect in this case that $B$ scales as $r^{2/3}$.

Using cosmological simulations of galaxy cluster formation, it was shown that one could reproduce the observed field magnitude by considering both gravitational contraction and turbulent stirring of the magnetised gas (Roettiger et al. 1999; Dolag et al. 1995, 2002, 2005; Sijl et al. 2004; Brüggen et al. 2005; Subramanian et al. 2006; Asai et al. 2007; Dubois & Teyssier 2008b). In cooling flow clusters, radiative losses are probably boosting this amplification mechanism even further up (Dubois & Teyssier 2008a) as confirmed by recent observations (Carilli & Taylor 2002). When an active galactic nucleus is releasing energy into the intra-cluster medium, a non-cool core develops and the magnetic field that one needs an initial magnetic field of at least $10^{-5}$ G in the IGM to reach $\mu$G values in cluster cores. Note that the finite resolution of these numerical experiments leads probably to an underestimation of the turbulent amplification of the magnetic field. Nevertheless, this justifies indirectly rather low values for the magnetic field in the IGM, but significantly larger than Biermann Battery generated fields.

In order to explain the origin of this diffuse cosmic magnetic field, Bertone et al. (2006) proposed galactic winds as a possible solution to explain the enrichment of the IGM by metals and magnetic fields. In this scenario, magnetic fields are amplified inside galactic discs by the so-called Galactic Dynamo (Kulsrud 1999), and in a second phase, supernova-driven winds are expelling field lines into the surrounding IGM.

The basic idea of the Galactic Dynamo (Parker 1971, Ferriere 1992a, 1992b, Brandenburg et al. 1995, Kulsrud 1999, Shukurov 2004) is that small-scale turbulence (precisely cyclonic motions that could result from supernova explosions, Balsara et al. 2004, Gissinger et al. 2009) in the Interstellar Medium (ISM) modify sufficiently the small-scale magnetic field to make the large-scale component grow significantly. An additional term $\nabla \times (\alpha B)$ is added to the induction equation that represents this dynamo amplification of the field, where $\alpha \propto < u, \nabla \times u >$ tensor stands for the cyclonic motions of the gas. If one can correctly approximate the $\alpha$ parameter (Ferriere 1992a,b), it is possible to show that the magnetic field can grow in a few Gyr up to its equipartition value (Ferriere & Schmitt 2000). Note that the dynamo theory works only if the galaxy can loose magnetic helicity (Brandenburg & Subramanian 2005), even though it is still debated that the small scale saturation of the $\alpha$ effect really occurs (Field 1995). Numerical simulations of a very small patch (kpc size) of a galaxy from Gressel et al. (2008) demonstrated that such a fast dynamo occurs if supernovae explosions can carry some magnetic helicity from the disc to the hot wind. However one has to reach a tremendous resolution power (parsec scale) in cosmological situations to provide such a long-term dynamo. Galactic winds have been considered as a nice explanation for suppressing some magnetic helicity from the disc, but no clear demonstration of the effect has been performed in a cosmological context. Another issue with galactic dynamo is that the amplification mechanism is probably too slow, especially if one considers recent magnetic field observations in high-redshift galaxies beyond $z = 1$ (Bernet et al. 2008). These measurements suggest that magnetic amplification must occur very quickly, in less than 5 Gyr. It is also plausible that a fast growth of the field due to turbulent amplification in the first halos at redshift $z > 10$ could provide an equipartition field before the first galaxies form (Arshakian et al. 2009). Of course it has to be proven that small scale intense magnetic fields are able to generate this large-scale magnetic field observed in galaxies, for this purpose, alternative theories must be explored.

Parker (1992) first suggested that the production of cosmic rays in supernova remnants could generate stronger buoyancy in the ISM and amplify the magnetic field. Simulations of a cosmic ray pressurised medium have shown that it leads to a rapid growth of magnetic fields if they are strongly diffused in the ISM (Hanasz et al. 2004, 2009a,b). Rees (1987) considered Biermann battery effects occurring at the surface of massive stars. More generally, one can consider any stellar dynamo mechanism as an efficient amplification mechanism inside stars (Brun et al. 2004), followed by the release of this magnetic field into the ISM, thanks to stellar winds or supernova explosions. Although the efficiency of stellar dynamo is rather uncertain (Brandenburg & Subramanian 2005), and the magnetic energy released into supernova remnants poorly constrained (Kennel & Coroniti 1984 for the Crab nebula and Helfand et al. 2001 for the Vela nubula), this stellar origin represents a very appealing perspective to explain a fast magnetic field generation in high-redshift galaxies.

Galactic dynamos, or other field generation mechanisms in the ISM, together with galactic winds, should therefore play an important role in the generation of magnetic fields in the IGM. In this respect, this is probably inside dwarf galaxies that this double mechanism occurs: dwarf galaxies are very numerous in the early universe, and they host most of the cosmic material at early times (Bertone et al. 2006; Donnert et al. 2009). Dwarf galaxies are also easily disrupted by galactic winds, in contrast to Milky Way like galaxies, from which metal and magnetic fields probably never escaped (Dubois & Teyssier 2008b and references therein). Kronberg et al. (1993) proposed a scenario where primeval galaxies launch strong starbursts that generate a $10^{-5}$ G magnetic field. They assume that galactic winds are able to reach Mpc scales and that they substantially amplify the field during the outburst.

Understanding the evolution of magnetic fields in dwarf galaxies is therefore an important goal. Performing Magnetohydrodynamics (MHD) numerical simulations with SPH, Kotarba et al. (2009) studied the amplification of the magnetic field by differential rotation and by the spiral pattern of the gaseous disc. Another step was undertaken by Wang & Abel (2009), who performed MHD numerical simulations with AMR of a dwarf galaxy formed by the collapse of a cooling halo. They studied in details the gas fragmenta-
tion and the magnetic field energy amplification due to turbulent, vortical motions induced by the gravitational instability in the disc. Starting with an initially uniform field of $10^{-7} \mu G$ in the halo, they reached equipartition after only a few rotations. They haven’t simulated, however, the effect of supernova feedback in driving strong outflows out of the galaxy. Bertone et al. (2006), on the other hand, focused their study on the effect of galactic winds and how they might enrich the Universe with metals and magnetic fields. They used for that purpose a semi-analytical model, assuming equipartition magnetic fields inside their galactic discs and following the winds evolution through cosmic times.

In this paper, our goal is to model the evolution of magnetic fields in a dwarf galaxy together with the formation of a galactic wind, in order to compute the amount of magnetic energy that escapes from the disc. This outgoing magnetic energy flux is the key quantity that we need in order to estimate the efficiency of IGM enrichment. For that purpose, we simulate an isolated, cooling halo, in the same spirit of Dubois & Teyssier (2008b) and Wang & Abel (2009), and form a small galactic disc in the halo centre. We solve numerically the ideal MHD equations, in the presence of self-gravity, with standard galaxy formation physics (cooling, heating, star formation, supernova feedback). The main parameter in our study is the starting magnetic field that permeates the initial halo. We therefore vary its strength, assuming a simple but realistic initial field geometry. We will also consider the case of zero initial magnetic field in the halo, in a model where seed fields are introduced within supernova bubbles, implementing the stellar feedback scenario suggested by Rees (1987) in a way similar to Hanasz et al. (2009). In Sect. 4, we present the numerical set-up of our simulations, as well as some details in our implementation of galaxy formation physics, and in Sect. 5, we discuss more specifically our initial conditions. In Sect. 4 and 5, we present our results on the magnetic amplification in galactic discs, and in Sect. 5, we show how the IGM can be enriched by magnetised galactic winds. We finally comment the implications of this work in Sect. 7.

2. Numerical methods

2.1. Gas dynamics with magnetic fields

We use the Adaptive Mesh Refinement code RAMSES (Teyssier 2002) to solve the full set of ideal MHD equations

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0,$$

$$\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u} - \mathbf{B} \otimes \mathbf{B} + P_0 \mathbf{I}) = 0,$$

$$\frac{\partial E}{\partial t} + \nabla \cdot ((E + P_0) \mathbf{u} - \mathbf{B} \otimes \mathbf{u}) = 0,$$

$$\frac{\partial \mathbf{B}}{\partial t} \times (\mathbf{u} \times \mathbf{B}) = 0,$$

using a Godunov scheme with Constrained Transport that preserves the divergence of the magnetic field (Teyssier et al. 2006, Fromang et al. 2006). In the previous system, $\rho$ is the plasma density, $\rho \mathbf{u}$ is the plasma momentum, $\mathbf{B}$ is the magnetic field, $E = 0.5\rho u^2 + E_{th} + B^2/8\pi$ is the total energy and $E_{th}$ is the thermal energy. This system of conservation laws is closed using the Equation of State (EoS) for an ideal mono-atomic gas, where the total pressure is given by $P_{tot} = (\gamma - 1)E_{th} + B^2/8\pi$, with $\gamma = 5/3$. The MHD solver in the RAMSES code has been tested using idealized cases, as well as more realistic astrophysical situations in Fromang et al. (2006). It has been used for the first time in the cosmological context by Dubois & Teyssier (2008a) to study the evolution of a cooling flow cluster, for which it produced results in good agreement with previous studies (Dolag et al. 2005, Brüggen et al. 2005). Because it is based on Constrained Transport, the magnetic divergence exactly vanishes in an integral sense; the total magnetic flux across each AMR cell boundary is zero. The electric field is computed at cell edges, using a 2D Riemann solver, in order to upwind the edge-centered electric field with respect to the 4 neighboring cell states. The 2D Riemann solver is based on a generalisation of the 1D HLLD Riemann solver of Miyoshi & Kusano (2005), assuming a 5-wave piecewise constant Riemann solution. We also account for self-gravity in the gas and stellar distribution, assuming a static potential for the dark matter component. The Poisson equation is solved with isolated boundary conditions, using Dirichlet boundary conditions given by a multipole approximation of the mass distribution in the box. The gravitational force is added as a source term in both the momentum and the energy conservation equations.

2.2. Cooling, star formation and supernova feedback

In order to describe the formation and evolution of a galactic disc, we model gas cooling in the halo as well as in the star forming disc using a look-up table for the metallicity–dependent radiative cooling function of Sutherland & Dopita (1993) for a mono–atomic gas of H and He with a standard metal mixture. This cooling function is added as a sink term in the energy equation (3). The metal mass fraction is advected as a passive scalar, with a source term coming from supernova explosions in the simulated ISM.

Gas is not allowed to cool down to arbitrarily small temperature. Following the general ideas presented in Yepes et al. (1997) and in Springel & Hernquist (2003), we use a density–dependent temperature floor to account for the unresolved turbulence in the ISM, modelling this complex multiphase medium with an effective EoS. In this paper, we use the following temperature floor $T_{\text{min}} = T_0(n_{\text{th}}/n_0)^{0.5}$, where $n_0$ is the star formation density threshold (see below) and $T_0 = 10^4$ K. The effect of this temperature floor is to prevent the gas from fragmenting down to parsec scales, well below our resolution limit. This is in contrast with the study presented in Wang & Abel (2005), where the gas temperature was allowed to cool down to 300 K, triggering strong disc fragmentation and massive clumps formation.

Star formation is implemented using a standard approach based on the Schmidt law, spawning star particles as a random Poisson process. The reader can refer to Rasera & Teyssier (2006) and Dubois & Teyssier (2008a) for more details. We choose here the following star formation parameters: an effi-
ciency of 5% per free–fall–time and a star formation density threshold of n0 = 0.1 H.cm^{-3}. Each stellar particle represents a single stellar population of total mass m∗ = n0 m0 Δx^3 ≈ 10^4 M_⊙. This stellar particle should therefore be considered as a Stellar Cluster (SC) rather than an individual star.

After several Myr, the first massive stars explode in supernova, giving rise to large expanding bubbles and ultimately to the formation of a large scale galactic wind. A mass fraction of f_{SN} = 10% in massive stars M > 8 M_⊙ (taken from any standard Initial Mass Function, e.g. Kroupa 2001) is considered for our stellar clusters. Modelling supernova feedback is a condition of the computational domain.

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### 3.1. Initial hydrostatic halo structure

We use the same set of initial conditions as the one presented in Dubois & Teyssier (2008b), namely an isolated dark matter halo hosting a cooling gaseous component. This idealized situation has been used many times to study various aspects of galaxy formation (Springel & Hernquist 2003; Dubois & Teyssier 2008b; Kaufmann et al. 2009). Let us recall here only the most important aspects. Dark matter is modelled with a static density distribution following a NFW (Navarro et al. 1996) profile such that

\[
\rho = \frac{\rho_s}{r/r_s(1 + r/r_s)^2},
\]

where \(\rho_s\) and \(r_s\) are the characteristic density and radius. This profile can be also be parametrized in terms of concentration \(c = r_{200}/r_s\) and Virial mass as

\[
M_{200} = 4\pi\rho_s^3 r_{200}^3 \left(\log(1 + c) + \frac{1}{1 + c}\right),
\]

within the virial radius \(r_{200}\). The virial mass is defined here at an overdensity equal to 200 times the critical density \(\rho_c\). We also assume \(H_0 = 70\) km/s/Mpc.

The gas is initially in hydrostatic equilibrium with the same profile than dark matter, with a total gas fraction of \(f_h = 15\%\). We consider that the halo is slowly rotating, with an angular momentum distribution consistent with the average specific angular momentum profile measured in cosmological simulations (Bullock et al. 2001).

Let us define the angular momentum profile

\[
j(r) = j_{\text{max}} \frac{M(< r)}{M_{200}},
\]

with a spin parameter

\[
\lambda = \frac{J/|E|^{1/2}}{G M_{200}^{3/2}}.
\]

Using this particular setting, Springel & Hernquist (2003) and Dubois & Teyssier (2008b) showed that large-scale outflows are produced only in low-mass halos with \(M_{200} \leq 10^{11} M_⊙\). As we are interested in the role of galactic winds in the enrichment of the IGM, we consider in this paper only the case of a small mass halo with \(M_{200} = 10^{10} M_⊙\). The halo spin parameter is chosen to be \(\lambda = 0.1\).

The box size is set equal to 3 times the Virial radius or \(L = 150.7\) kpc, and is covered by a coarse grid with 64^3 cells. We consider 4 additional levels of refinement, so that our effective resolution reaches 1024^3 or \(\Delta x = 147\) pc. Our refinement strategy is based on two criteria. First, we refine a cell if its mass exceeds 8 times our mass resolution indicator \(m_{\text{res}} = 2 \times 10^{-6} M_{200}\). Second, we refine a cell if the local Jeans length falls below 4 cells, so that we prevent the disc from artificially fragmenting (Truelove et al. 1997; Bate & Burkert 1997; Machacek et al. 2001). The initial AMR grid was built from the coarse mesh with this refinement strategy. We start initially with a total \(\sim 2 \times 10^6\) cells which number remains roughly constant with time. The number of cells in the maximum level of refinement (mainly located in the gaseous disc) is \(\sim 4.10^4\). After 3 Gyr the total number of stars is \(\sim 10^8\).
3.2. Initial magnetic field

Although the choice of the initial magnetic field is quite crucial, the magnetic field topology is usually chosen quite arbitrarily in the literature. Using an initial set up very similar to the one presented here, Wang & Abel (2009) considered a constant initial magnetic field \( B_z = 10^{-9} \) G. If the initial halo field is the product of the collapse of some large scale cosmological structure, the field should rather scale as \( \rho_{\text{gas}}^{2/3} \). This scaling has been observed in cluster–size halo in cosmological MHD simulations (Sigl et al. 2004; Dolag et al. 2005; Brüggen et al. 2005; Dubois & Teyssier 2008a). In the sphericaly symmetric set-up we are considering, each collapsing gas shell will compress and amplify significantly the frozen-in field, before any disc-like dynamo comes into play. The initial magnetic field amplitude profile is therefore quite important. In a different context, Kotarba et al. (2009) considered the case of a pre-existing galactic disc. The initial magnetic field was also constant, but this time \( B_z = \text{constant} \), the field lines lying exactly in the disc plane. As argued by these authors, differential rotation quickly takes over and create a strong toroidal field in the star–forming disc, so that the initial topology is quickly forgotten.

We nevertheless propose here to consider a more realistic initial magnetic field profile, with a dipole–like topology. In order to preserve the \( \nabla \cdot \mathbf{B} = 0 \) constraint we compute the magnetic field from its potential vector \( \mathbf{A} \)

\[
\mathbf{B} = \nabla \times \mathbf{A}.
\]

\( A \) is computed assuming that each cell in the computational domain contain a small magnetic dipole with strength proportional to \( \rho_{\text{gas}}^{2/3} \). Every dipole have the same direction along the \( z \) axis, so that they all add up to a global large scale dipole–like topology on the halo scale. The potential vector is written as

\[
A(x, y, z) = B_{\text{IGM}} \left( \frac{\rho_{\text{gas}}(r)}{\rho_{\text{IGM}}} \right)^{2/3} \begin{pmatrix} -y \\ +x \\ 0 \end{pmatrix},
\]

where \( \rho_{\text{IGM}} = \Omega_b \rho_\text{b} \), with the baryon density \( \Omega_b = 0.04 \) and \( r \) is the spherical radius. Each component of the potential vector is averaged over the corresponding cell edge, and the magnetic field is computed using the integral form of the curl operator on each cell face. One can see on Figure1 that the initial magnetic field topology is roughly dipolar, preferentially aligned with the \( z \) axis, and that the field strength decreases strongly with radius.

Figure2 shows the magnetic field profile along the \( z \) axis for different values of the magnetic field in the IGM \( B_{\text{IGM}} \). The magnetic field strength decreases down to 2 orders of magnitude from the halo centre to the outer parts at \( 2r_{\text{200}} \). Results from cluster scale cosmological simulations seem to favor values of \( B_{\text{IGM}} \approx 10^{-6} - 10^{-4} \) \( \mu \)G in order to explain the observed magnetic field strength inside the core of large X–ray clusters (Carilli & Taylor 2002, Govoni & Feretti 2004 and references therein). This value is however to be considered as an upper limit since these cosmological simulation have limited resolution and are believed to underestimate the magnetic field amplification from the initial IGM value to the final cluster’s core strength. In this paper, \( B_{\text{IGM}} \) is a free parameter that we allow to vary between \( 10^{-7} \) \( \mu \)G, deep into the pure induction regime for which the Lorentz force remains negligible and \( 10^{-4} \) \( \mu \)G, our maximum value. Wang & Abel (2009) initial magnetic field of \( B_z = 10^{-3} \) \( \mu \)G in the halo corresponds roughly to our case with \( B_{\text{IGM}} \approx 3 \times 10^{-5} \) \( \mu \)G, using the \( \rho^{2/3} \) scaling and a mean overdensity of 200 in the halo.
3.3. Zero–gradient boundary conditions

A complex numerical issue for MHD flows is to design proper boundary conditions. We are dealing with an isolated halo, for which we consider that the environment is a very low density, homogeneous medium. It is convenient in this case to use outflow boundary conditions: gas is allowed to flow freely out of the computational domain. Outflow boundary conditions can be implemented using a zero-gradient condition at the domain boundary for all hydro variable (density, pressure and the 3 components of the velocity). For the magnetic field, however, we need to be more careful in order to enforce the divergence free condition. We use the zero-gradient condition only for the transverse magnetic field components which we consider that the environment is a very low density, homogeneous medium. It is convenient in this case to use a linear extrapolation for the normal component of the velocity field. For the magnetic field, however, we need to be more careful in order to enforce the divergence free condition. We use the zero-gradient condition only for the transverse magnetic field components $B_L$ (perpendicular to the unit vector normal to the surface), and we use a linear extrapolation for the normal component $B_B$ (parallel to the unit vector). We make sure that the divergence of the magnetic field takes the same value on each side of the domain boundary, namely zero.

4. Disk formation with magnetic field

When radiative cooling is turned on, our hydrostatic halo cools down from the inside out. Each shell looses pressure support and sinks towards the centre of the halo. The magnetic field lines are frozen in the free–falling plasma. Although initially the field lines are mainly vertical (perpendicular to the future galaxy), they are entrained and bent by the collapsing gas, so that the radial component finally dominates. Figure 1 compares the topology of the initial magnetic field and the magnetic field after 3 Gyr of evolution. One clearly sees that the field lines close to the disc are almost horizontal (with a strong radial component). The radial component changes sign above and below the plane, a natural consequence of the pinching of the field lines by the collapsing gas. Because of angular momentum conservation, each collapsing shell will speed up, flatten and form a centrifugally supported disc. As we will show below, the galactic velocity field will quickly generate a toroidal field component. This toroidal field is a consequence of the field lines being wrapped up and amplified by the rotating disc.

One should also note that the halo magnetic field keep its initial magnetic dipole-like structure, even after 3 Gyr evolution. This a direct consequence of the choice of our initial conditions. We will see that when a galactic wind develops, it strongly suppress the memory of the initial configuration of the field.

We show in Figure 2 the initial magnetic field amplitude along the z axis of the halo. We consider in this study 3 different cases: $B_{IGM} = 10^{-6} \mu G$, $B_{IGM} = 10^{-5} \mu G$ and $B_{IGM} = 10^{-4} \mu G$. We also show in Figure 2 for comparison the magnetic field inside the disc. As we will show below, the galactic velocity field will quickly generate a toroidal field component. This toroidal field is a consequence of the field lines being wrapped up and amplified by the rotating disc.

After the disc forms, the magnetic field is amplified by rotation. If the resulting field is strong enough, the Lorentz force can significantly affect the dynamics of the flow, and alter the formation of the disc itself. The simulation with $B_{IGM} = 10^{-4} \mu G$ is quite extreme in this respect: the magnetic field is so strong that it prevents the formation of the thin, centrifugally supported disc, leaving instead a low density, highly magnetised torus, in which star formation is very weak. The magnetic field lines become harder to deform by gas motions, except along the z axis, where field lines remains vertical and do not prevent gas from collapsing. This is clearly shown in Figure 3 where we can compare the disc morphology in our 3 scenarios. The 3 maps of the $\beta$ parameter are useful to discriminate between the high magnetic field case $B_{IGM} = 10^{-4} \mu G$, for which $\beta$ is very small in most of the torus region, demonstrating that the torus is magnetically supported against gravity, and the low magnetic field case $B_{IGM} = 10^{-6} \mu G$, for which the disc we obtain is indistinguishable from the pure hydro case, and the magnetic field inside the disc is one order of magnitude below the equipartition value. Only in the intermediate case $B_{IGM} = 10^{-5} \mu G$ do we get a thermally supported disc, with a magnetic field roughly in equipartition only inside the disc ($\beta \approx 1$).

It is quite interesting to see thanks to these simulations that dwarf galaxies formation can be almost suppressed (or at least star formation within them) by a strong enough magnetic field in the IGM. It is even more interesting to notice that the value of the magnetic field that we require from cosmological simulation of galaxy cluster ($B_{IGM} \approx 10^{-5} - 10^{-4} \mu G$) is precisely the same critical value above which dwarf galaxy formation can be regulated, if not suppressed, by magnetic fields. This point has important consequences that we will address in the conclusion.
Fig. 3. Slice through the (Oxz) plane of the gas density in log H/cm$^{-3}$ (first row), of the temperature in log K (second row), of the magnetic amplitude in log $\mu$G (third row) and the $\beta$ parameter in log $\beta$ (fourth row) for an initial magnetic field of $B_{\text{IGM}} = 10^{-4}$ $\mu$G (left column), $B_{\text{IGM}} = 10^{-5}$ $\mu$G (middle column) and $B_{\text{IGM}} = 10^{-6}$ $\mu$G (right column) at $t \approx 3$ Gyr. Picture size is 40 kpc.
5. A Galactic Dynamo?

We now discuss more quantitatively the amplification mechanism of magnetic energy in the galactic disc. We use for that purpose the so-called “Galactic Dynamo” theory [Parker 1971, Wielebinski & Krause 1993, Shukurov 2004], for which the mean large scale magnetic field evolution is governed by a modified induction equation

\[
\frac{\partial B}{\partial t} = \nabla \times (\alpha \times B) + \nabla \times (\alpha B) + \nabla \times (\eta \nabla \times B),
\]

(12)

where parameter \(\alpha\) represents the field amplification due to small scale turbulent motions driven by various small scale phenomena (supernova explosion, cloud-cloud collision or collapsing vortex modes as explained in Ferriere 1992a, Efstathiou 2000 and Kulcsár & Zweibel 2008) and parameter \(\eta\) represents magnetic diffusivity induced by turbulent transport and reconnection. The standard \(\alpha\)-\(\Omega\) dynamo theory relies on the \(\alpha\) term to amplify the radial magnetic field \(B_r\) and on the large-scale induction term to amplify the toroidal field \(B_\theta\). In the present simulation, however, we do not include any explicit \(\alpha\) and \(\eta\) terms in our induction equation. The original goal was to rely on our implementation of supernova feedback to induce helical motions self-consistently. Probably because of our limited resolution (around 150 pc) and the fact that we use a subgrid model based on an effective EoS, we didn’t get any \(\alpha\) effect: field amplification in our case is due only to differential rotation. Gressel et al. (2008), simulating a local patch a galaxy with much more resolution elements, obtained a galactic dynamo driven by supernovae explosion and differential rotation. They outlined that the rotation is the critical driver for the dynamo to operate an exponential growth of the field.

An important issue is that we attempt to model a quiescent star-forming galaxy with an EoS mimicking the multiphase nature of the ISM gas, i.e. the gas pressure is artificially boosted to take into account the effect of supernovae heating of the ISM. The direct consequence is that the gas distribution is very smooth in the disc (no small scale clumping can develop), and as a consequence, supernovae explosions have a very low impact on the ISM. A next step would be to simulate the evolution of starburst galaxy, within which large clumps \((-100\ \text{pc}\ \text{size})\) would be resolved and within which we could hope that supernovae are able to produce a small-scale dynamo effect. In order to get a global dynamo effect, additional physics would also be needed: non-ideal effects like Ohmic or turbulent dissipation and cosmic rays have been already identified as key ingredients to get a strong dynamo (see Hanasz et al. 2009a for instance).

An investigation of the magnetic energy growth within the disc reveals that there is no substantial increase of the magnetic field when SN explosions are allowed in the galaxy (figure 5). There is 3 effects: first, due to supernovae heating (both turbulent and thermal), the early magnetic field amplification appears weaker than in the no-SN case. As a result, the magnetic energy is slightly lower than without SNII. Second, the longer term magnetic field amplification appears slightly more efficient in presence of SN. Finally, the large-scale galactic wind carry some of the magnetic energy available in the disc, producing a slight decrease in comparison of the simulation without feedback. As a result, the final magnetic energy value is almost equal to the final energy without SN explosions. This is a very weak and subtle effect. We notice that we reproduce roughly the same energy evolutions (kinetic, thermal and magnetic) than Wang & Abel (2009). Interestingly, both kinetic and internal energies are lower when feedback proceeds: this is partly
due to the gas consumption by the star forming process, and to the ejection of some material outside from the galaxy.

If we assume that in the disc, radial and vertical motions are much smaller than circular motions, we can assume \( v_r = 0 \) and \( v_z = 0 \). After collapse, the magnetic field in the disc appears mainly horizontal, so we also assume \( B_z = 0 \) in the disc. In these conditions, the induction equation writes in cylindrical coordinates \((r, \theta)\) and \((z)\)

\[
\partial_t B_r = 0 \tag{13}
\]

and

\[
\partial_t B_\theta = \frac{v_\phi}{r} \partial_\phi B_\theta = \Omega \partial_\phi (r B_r) \tag{14}
\]

where we introduce the angular velocity \( \Omega(r) \), and with \( \nabla \cdot B = 0 \). In the later equation, the last term, which is the dominant in the induction equation, is kept in order to predict the evolution of the tangential component. This result closely matches the measurements of Kotarba et al. (2009). In order to simplify the problem, we assume that the radial component of the field follows a power law \( B_r \propto r^\alpha \). Such an approximation is relevant here: the initial magnetic field is proportional to \( r^{3/2} \), which could be approximated by \( r^{-3/3} \). Thus, it comes

\[
\partial_t B_\theta = (\alpha + 1) \Omega r B_r \tag{15}
\]

For a flat rotation curve, we finally obtain the following well-known result that for \( v_\phi \approx V_{200} \), \( B_r \) remains constant, and \( B_\theta \) grows linearly with time as

\[
B_\theta = (\alpha + 1) B_r \frac{V_{200}}{r} \tag{16}
\]

where \( B_r = B_{\text{IGM}} \). Thus, it comes

\[
E_B(t)/E_B(0) = 1 + (\Omega_G t)^2 , \tag{17}
\]

where \( \Omega_G^2 = \langle (\alpha + 1)^2 \Omega^2 \rangle > \) is the average (magnetic energy–weighted) angular velocity squared of the galaxy. This simple analytical formula compares very well with the actual magnetic energy amplification we have measured in our simulations, as can be seen in Figure 2. We have used for the pure \( \Omega \) amplification model \( \Omega_G = 19 \text{ Gyr}^{-1} \), which corresponds to a disc magnetic scale length \( r_d \approx 1.9 \text{ kpc} \) for a circular velocity of \( V_{200} = 35 \text{ km/s} \) (with \( \Omega_G \approx V_{200}/r_d \)). Only in our high magnetic field case, \( B_{\text{IGM}} = 10^{-4} \mu \text{G} \), do we see a significant deviation from this simple model. This is due to magnetic braking: angular momentum is removed from the disc by Alfvén waves propagating along the more rigid field lines [Hennebelle & Fromang 2008]. The angular velocity of the disc is therefore slowly decreasing. For all the other cases, magnetic braking is negligible and the magnetic amplification is very similar to the pure \( \Omega \) amplification model.

We have estimated the effect of mass resolution on the magnetic amplification. Figure 4 shows the measured amplification in 4 different cases: our fiducial with \( m_{\text{res}} = m_0 \), and 3 lower mass resolution with \( m_{\text{res}} = 5, 25 \) and \( 100 \times m_0 \). The collapsing halo is therefore described with fewer and fewer AMR cells (3.10^6, 8.10^5, 4.10^5 and 3.10^5 total cells in decreasing order of \( m_{\text{res}} \)). The disc is however always refined at the maximum level of refinement, with a spatial resolution of 150 pc. As a consequence, only our lowest resolution simulation shows a significant deviation from the fiducial case, which can be considered as fully converged, within the framework of our sub-grid effective EoS.

Our simulation results compare very well with the work presented in Kotarba et al. (2009), where these authors report a similar magnetic amplification using 2 different Smooth Particle Hydrodynamics (SPH) codes. They used various force softening length, down to 100 pc, quite similar to our grid spacing. They also used a similar physical model, since they didn’t allow the gas to cool below 10^4 K. Although their SPH simulations were all initialized as an initial Milky–Way disc configuration, their results are quite consistent with a pure \( \Omega \) amplification. In a few cases only did they obtain a faster, exponential growth of magnetic energy. These cases all corresponds to numerical schemes where the magnetic divergence was allowed to deviate significantly from zero. They are probably associated with numerical instabilities. In the majority of the SPH results, for which the magnetic divergence was well–behaved, the magnetic energy was growing like \( r^2 \).

Wang & Abel (2009), on the other hand, have used an AMR code with a much better spatial resolution (down to 25 pc) and have included cooling processes down to 300 K. They were able to resolve, in a dwarf galaxy similar to the one studied here, the fragmentation of gas clumps and the associated vortex modes. They however didn’t consider star formation and supernova feedback. They concluded that the main amplification mechanism in their simulations was differential rotation:
even at the 25 pc resolution they have reached, they have not captured any turbulent dynamo and their associated $\alpha$ effect: magnetic energy was also growing like $t^2$ in their case.

To support even further the simple $\Omega$ amplification model to explain our simulation results, we now analyze in more details the magnetic field morphology in our disc galaxy. We consider for that purpose 3 different cases: 1- formation of the disc with only cooling, 2- formation of the disc with cooling and star formation and finally 3- formation of the disc with cooling, star formation and supernova feedback. The magnetic field morphology is shown in Figure 6. We also represent in the same figure the magnetic pitch angle defined as

$$\tan \theta_p = \frac{B_r}{B_\theta}.$$  

(18)

$\theta_p$ gives the angle between the magnetic field within the horizontal plane and its azimuthal component. $\theta_p = 0^\circ$ corresponds to pure azimuthal field lines and $\theta_p = 90^\circ$ to pure radial field lines. We must underline that pitch angles inferred from observations are obtained with synthetic polarisation maps from radio emission. In the following, we extract a slice of the magnetic field through the galactic plane to get the pitch angle as defined by equation (18), assuming that it provides a good comparison with observational synthetic polarisation if the pitch is independent of the vertical height (it is roughly the case in these simulations).

Without any star formation process, the magnetic field in the galactic disc is almost purely azimuthal: we can see from the left plot in Figure 6 that in this case the pitch angle is reaching a maximum value of $\theta_p = -2^\circ$. The gas disc is stabilized by the effective EoS, so that no density perturbation can break the almost perfect azimuthal symmetry (only modes with $m = 0$ emerge). After collapse, the radial component of the magnetic field points away from the centre of symmetry above the galactic plane and towards the centre below the plane. Using Equation (14), we see that the toroidal magnetic field also changes sign above and below the plane: the field is said to be anti-symmetric. This kind of particular symmetry is called the A0 mode (anti-symmetric and $m = 0$) and emerges naturally in halo without strong disc–halo interactions (like galactic winds or galactic fountains) (Beck et al. 1996; Beck 2009). The Milky-Way, for example, is believed to be a good example of such a A0 magnetic field in the halo. (Han et al. 1997; Sun et al. 2008).

The situation appears to be quite different when star formation in taken into account. Because mass is removed from the gas disc to form stars, our effective EoS reaches lower density and the gas sound speed is smaller. The disc Toomre parameter get smaller, closer to 1, and a strong spiral mode can develop. We see from the central plot of Figure 6 that the magnetic field is not axisymmetric anymore. The strong den-

\[\text{Fig. 6. Magnetic amplitude in log } \mu \text{G with magnetic vectors overplotted (upper panels) and the pitch angle } \theta_p \text{ in degrees (bottom panels) for the galaxy without star formation at } t \approx 3 \text{ Gyr (left panels), for the galaxy with star formation and without feedback at } t = 4 \text{ Gyr (middle panels), and for the galaxy with star formation and supernova feedback at } t = 3 \text{ Gyr (right panels) in the upper part of the equatorial plan. Initial magnetic field is } B_{\text{IGM}} = 10^{-5} \mu \text{G. Picture size is } 40 \text{ kpc.}\]
sity perturbation triggers fluctuation in the angular velocity, so that $\Omega(r)$ is not monotone anymore, but fluctuates around $V_{200}/r$. We can see again from Equation (14) that, when $B_r$ changes sign, the corresponding toroidal component can decrease, or even change sign. We see indeed in Figure 6 that the toroidal field almost reverts its main direction between the spiral arm and the inter-arm regions. As a consequence, the pitch angle amplitude $|\theta_p|$ is rather high in the low density, inter-arm regions ($|\theta_p| \approx 40^\circ$ (with both negative and positive signs) and is lower within the spiral arms with $|\theta_p| \approx 20^\circ$ (always with negative values). These values are in good agreement with the observed pitch angles in nearby galaxies (Beck et al. 2005, Rohde et al. 1999, Patrikeev et al. 2006). The same relation between the pitch angle and the spiral arms is also observed within spiral galaxies: magnetic field is aligned with the optical spiral arms (Krause et al. 1989, Neininger et al. 1991, Berkhuijsen et al. 1997, Beck 2007), and it also appears sometimes that stronger ordered magnetic fields are seen within the inter-arm regions (Ehle et al. 1994, Frick et al. 2000).

When supernova explosions are finally included in the model, the previous magnetic field topology is conserved. The spiral arm is somewhat weaker, because the increased velocity dispersion due to supernova blast waves stabilizes the disc. The disc appears less coherent and more perturbed by small scales perturbations. Note that this is precisely the small scale turbulence we need for the $\alpha$ effect, but despite this small scale perturbations, the radial component did not grow significantly in our simulation. The magnetic energy evolution conserves the same properties than in the run without star formation: the differential rotation of the disc is the main driver of the galactic magnetic field in these simulations. We probably need sub–parsec scale resolution in order to resolve supernova induced cyclonic motions that can drive a turbulent dynamo, like in Balsara et al. (2004). Besides the lack of small scale amplification of the radial component of the field, our simulation recovers most of the qualitative features of Galactic dynamo models, that predict similar pitch angle distribution and magnetic field topologies (Krasheninnikova et al. 1989, Donner & Brandenburg 1991, Elstner et al. 1992). The existence of this small scale $\alpha$ effect in galaxies was recently shown by direct simulations of Gressel et al. (2008), but the problem of its saturation is still under debate (Brandenburg & Subramanian 2005 for example). The value of the parameter $\alpha$ needs to be rather high in order to obtain a fast enough galactic dynamo. The conservation of the total magnetic helicity in the galaxy makes large value for $\alpha$ rather difficult to obtain if there is no exchange of magnetic helicity with the halo. In this paper, we rather focus on the other consequence of supernova feedback, namely the formation of a strong galactic wind, and how this wind transports magnetic energy.

6. Magnetic field in the galactic wind

We now study in more details the properties of the supernova–driven galactic wind that forms during our simulations. We restrict ourselves to the analysis of the case $B_{\text{GM}} = 10^{-5} \mu G$: we obtained in this case a normal star–forming disc, and in the same time a magnetic field close to equipartition in the gaseous disc. A larger value of $B_{\text{GM}}$ would result in the formation of a non–star–forming torus, while a smaller value would give rise to a weakly magnetised galactic disc, significantly below equipartition.

In agreement with the simulations shown in Dubois & Teyssier (2008b), the galactic wind appears at 1.5 Gyr and fully develops during the next Gyr. Figure 2 shows 4 different snapshots (from 1 to 3 Gyr) of the magnetic field in a plane perpendicular to the disc. These snapshots illustrate the 3 different phases of the galactic wind, described in more details in Dubois & Teyssier (2008b): 1- the bubbling phase, when the supernova luminosity is too weak to break the ram-pressure of the infalling gas ($t = 1$ Gyr in Fig. 7), 2- the snow-plow phase, when the supernova luminosity is greater than the ram-pressure of the infalling gas, and the wind starts to propagate through the halo ($t = 1.5$ Gyr and $t = 2$ Gyr in Fig 7) and 3- the blow away phase, when the wind has reached its final, stationary shape with a characteristic nozzle-like structure ($t = 3$ Gyr in Fig. 7).

We can see from Figure 7 that the magnetic field in the wind is highly turbulent, with an average amplitude around $10^{-3} \mu G$, but showing strong variations within the wind, with highly magnetised filament (as high as $10^{-1} \mu G$) entrained by low density bubbles (as low as $10^{-4} \mu G$). The turbulent structure of the magnetic field is typical of convective flows, with buoyantly rising vortices and their associated shearing motions. The magnetic field lines appear however mainly colinear to the flow, a magnetic configuration that is actually observed in
galaxies with outflows (Brandenburg et al. 1993; Chyży et al. 2006; Heesen et al. 2009). Whether the turbulent flow we observe in the galactic wind amplifies the magnetic field is unclear in our simulation. As we discuss below, the wind magnetic energy is clearly decaying because of the overall expanding flow. Moreover, in the wind region, our spatial resolution is slightly degraded (around 300 pc), because the grid has been derefinied in this low density region. This suggests that we are not resolving the small scale magnetic energy amplification due to the turbulent convective flow, leading to a potential underestimation of the magnetic energy in the wind.

In order to estimate the amount of magnetic energy that the wind can extract from the magnetised galactic disc, we have measured the magnetic energy flux in 2 thin shells of radius 4r_s ≈ 20 kpc and 9r_s ≈ 45 kpc. Results are shown in Figure 8. After the wind starts (around $t = 1.5$ Gyr), the magnetic energy flux quickly jumps to a value of $10^{52}$ erg/Gyr, and stays roughly constant during the next Gyr. The magnetic energy flux is decreasing when going to larger radii (roughly as $r^{-1}$), demonstrating that the effect of expansion is overcoming field amplification in the wind turbulence. It seems that the flux still grows after 2 Gyr. It is possible that when the wind reaches a stationary regime, this growth is correlated with the increase of magnetic energy within the disc (see figure 5).

It is enlightening to interpret our numerical results in the framework of the analytical theory of Bertone et al. (2006). This will also guide our discussion on magnetic field amplification in general. According to Bertone et al. (2006), the mag-

Fig. 7. Magnetic field in log µG units for the galaxy with star formation and supernova feedback with $B_{IGM} = 10^{-5}$ µG at different epochs $t = 1$ Gyr (upper left), $t = 1.5$ Gyr (upper right), $t = 2$ Gyr (bottom left) and $t = 3$ Gyr (bottom right). Picture size is 40 kpc.
netic energy flux which is injected at the base of the galactic wind can be written as

\[ E_{B,in} = \frac{B^2}{8\pi} 4\pi R_G^2 \rho_w, \]  

(19)

where \( B_{in} \) is the injected magnetic field, \( R_G \approx 7 \) kpc is the galactic disc radius and \( \rho_w \) is the wind velocity. In their analytical model, Bertone et al. (2006) assumed first that the magnetic energy could be considered as some isotropic pressure term. We see from the previous discussion that the convective turbulence in the wind justifies this assumption. They assume also that the injected magnetic field scales with the injected wind gas density as

\[ B_{in} = B_G \left( \frac{\rho_w}{\rho_G} \right)^{2/3}, \]  

(20)

where \( B_G \) and \( \rho_G \) are respectively the typical magnetic field and gas density inside the galactic disc. Finally, they related the wind gas density to the wind mass flux by

\[ M_w = \rho_w 4\pi R_G^2 u_w. \]  

(21)

Using the 3 previous equations, we can predict the magnetic energy flux in the wind to be

\[ E_{B,in} = E_{B,G} \frac{4u_w}{H_G} \left( \frac{M_w R_G}{3\mu_w M_{ISM}} \right)^{4/3}, \]  

(22)

where \( H_G \approx 300 \) pc is the disc thickness and \( E_{B,G} \) is the disc magnetic energy. We have plotted the predicted magnetic energy flux corresponding to Equation (22) in Figure 8 using the measured mass flux (\( M_w = 0.035 M_\odot/yr \)), disc magnetic energy (\( E_{B,G} = 5 \times 10^{53} \) erg) and disc mass (\( M_{ISM} \approx 10^9 M_\odot \)). We see that this formula is quite accurate, given the number of simplifying assumptions we have made. It slightly underestimates the measured flux, but less that a factor of 2. If we integrate this magnetic energy flux over \( t_w \approx 1 \) Gyr of wind activity, we find that roughly 6 % of the magnetic energy can be extracted by the galactic wind and funneled into the IGM. We note \( \epsilon_w \), this global wind efficiency. In the analytical model of Bertone et al. (2006), this efficiency writes

\[ \epsilon_w = \left( \frac{4u_w t_w}{H_G} \right) \left( \frac{M_w R_G}{3\mu_w M_{ISM}} \right)^{4/3}. \]  

(23)

A wind mass injection of 0.03 \( M_\odot/yr \) is a rather low value compared to starburst galaxies where winds are usually observed. Our simulation can be considered as a quiescent wind case. Typical starburst–driven winds show strong mass outflow as large as 10 \( M_\odot/yr \) (Martin 1998, 1999). On the other hand, the burst duration is probably much smaller, with \( t_w \approx 100 \) Myr (de Grijs et al. 2001). Overall, the efficiency of magnetic energy extraction probably lies above \( \epsilon_w \approx 10 \% \) for a typical star forming dwarf galaxy. The other extreme case one could also consider is when the starburst is so violent that the whole galaxy is destroyed. This scenario is likely to happen in the early universe inside small primordial galaxies, as shown by the recent cosmological simulations of Mashchenko et al. (2008). In this case, the efficiency obviously reaches \( \epsilon_w = 100 \% \).

We now want to estimate the magnetic field in the IGM that will result from the enrichment of the galactic wind. For this purpose, we use the analytical model developed by Bertone et al. (2006) and used recently in a cosmological simulation by Donnert et al. (2009). We write for the evolution of the magnetic energy in the wind blown bubble the following equation

\[ \frac{dE_B}{dr} = \dot{E}_{B,in} - \frac{R_w}{R_w} \dot{E}_B, \]  

(24)

\( \dot{E}_{B,in} \) is the magnetic energy injection rate, measured at the base of the expanding wind and \( R_w \) is the bubble radius (\( R_w \) its time derivative). The second term on the right-hand side of the previous equation stands for magnetic energy losses due to the expansion of the bubble. This equation also assumes that we can neglect the turbulent amplification of the magnetic field, which turned out to be true in the simulations we have pe-
formed here. Following Donnert et al. (2009), we consider the rather pessimistic case of a fast bubble expansion and writes \( R_w / R_w = 1 / t \). We also consider the solution of Equation (23) for which the initial bubble energy vanishes

\[
E_B \approx E_{B,0} t_w / 2 = \epsilon_w E_{B,G}.
\]  

(25)

We note that, in this scenario, half of the injected energy has been lost due to the expansion of the field lines. Using the cosmological wind model of Bertone et al. (2006), we consider a final wind radius of \( R_w = 800 \) kpc and a wind filling factor of \( f_w = 100 \% \) (both typical values for the universe at redshift zero), so that the final magnetic field in the IGM writes

\[
B_{IGM} = \sqrt{2 \pi \rho_e E_{B,G}} / R_w = 1.1 \times 10^{-4} \mu G.
\]  

(26)

In conclusion, the galactic wind has enriched the IGM with a magnetic field more than one order of magnitude larger than its initial value. Interestingly enough, the wind bubble magnetic field is proportional to the disc magnetic field. Indeed, the disc magnetic energy depends on the toroidal magnetic field in the disc as follows

\[
E_{B,G} = B^2_{G} / 8 \pi R^2_G H_G,
\]  

(27)

so that the new IGM value now writes

\[
B_{IGM} = B_{G} \sqrt{2 \pi \rho_e E_{B,G}} / 8R_w.
\]  

(28)

This last formula stands as a nice and concise summary of our work: we have shown with detailed MHD simulations that the final magnetic field in the expanding wind bubble inherits its value from the parent dwarf galaxy, where an \( \Omega \) effect has amplified the field by almost 2 orders of magnitude from its initial value.

7. Discussion

Using a rather idealized set up to model dwarf galaxy formation with MHD, we have followed the evolution of the magnetic field, from the collapse of an equilibrium halo, to the formation of a galactic disc and finally to the ejection of magnetic field lines by a supernova driven galactic wind. In order to illustrate further the mechanism powering up this cosmic cycle, for which dwarf galaxies play the central role, we have plotted in Figure 9 the magnetic phase space diagram, showing a mass weighted histogram as a function of magnetic field amplitude and gas overdensity. First, when the halo forms, the magnetic field is amplified by gravitational contraction from its initial IGM value to its final disc value

\[
B_r = B_{IGM} \left( \rho / \rho_{IGM} \right)^{2/3}.
\]  

(29)

We use here only the radial component \( B_r \), because just after collapse, the magnetic field is essentially radial in the disc (see discussion above and Fig. 1). We can see in the upper part of Figure 9 our initial configuration, where \( B = \rho \) are following this tight relation. Note here that we have neglected the additional amplification of the magnetic field due to turbulent motions in the halo. If we note \( R \) the initial Lagrangian radius of the collapsing halo and \( \lambda \) its spin parameter, we know from the standard cosmological disc formation model of Mo et al. (1993) that the final disc radius writes

\[
R_d = \lambda 200^{1/3} / R,
\]  

(30)

so that the initial disc radial component writes

\[
B_r = B_{IGM} \left( 200^{2/3} / \lambda \right).
\]  

(31)

In the lower panel of Figure 9, we see that the toroidal field in the disc has grown linearly with time due to differential rotation (the \( \Omega \) amplification), by a factor of 60, corresponding to the growth rate (Eq. 16) integrated over 3 Gyr. We finally get the growth rate of the toroidal magnetic field in the disc as a function of the initial magnetic field in the IGM as

\[
\partial_t B_r = B_r \Omega_G \approx B_{IGM} \left( 200^{2/3} / \lambda \right)\Omega_G.
\]  

(32)

In the lower panel of Figure 9 we see that this amplified magnetic field is funneled out of the galactic disc, following a \( \rho^{2/3} \) relation back to the IGM. This is the “galactic wind” branch that ends up the overall cycle. We have seen that the IGM magnetic field has been amplified by one order of magnitude during this cycle of roughly 3 Gyr.

Although our simulations, in conjunction with the work presented in Bertone et al. (2006) and Donnert et al. (2009), provide a nice explanation for the origin of an IGM magnetic field, at a level as high as \( 10^{-3} \mu G \), assuming equipartition magnetic fields in dwarf galaxies, we still need to explain the origin of these intense galactic fields, given that we start our cosmic evolution with a very small initial value \( B_{IGM} \approx 10^{-14} \mu G \) just after reionization. We can’t rely on the \( \Omega \) amplification alone, since in this case, the magnetic field grows only linearly with time. In the context of isolated galaxies, the only source of additional amplification comes from small scales turbulent and cyclonic motions (the \( \alpha \) effect). This turbulent dynamo is believed to amplify the radial component of the galactic field, the toroidal component being generated and amplified by the \( \Omega \) effect.

We know however that galaxies are far from being isolated. They form by accretion of diffuse IGM gas, usually in the form of dense, cold filaments known as cold streams (Dekel et al. 2009), but also by accretion of satellite galaxies, orbiting around the central galaxy before being incorporated more or less violently into the final disc. These 2 important processes define the hierarchical scenario of galaxy formation (Silk & White 1978, White & Rees 1978), and probably explain most of the properties of present day galaxies (Governato et al. 2007, Mayer et al. 2008).

We propose here to look at the galactic dynamo problem from a different angle. Instead on relying only on the small scale dynamo to amplify the radial component of the field, we should also consider as a source of radial field: 1-the toroidal
magnetic field coming from accreted satellite galaxies and 2-the accreted IGM magnetic field previously enriched by galactic winds or 3- the accreted IGM magnetic field previously enriched by entirely destroyed progenitor galaxies. On one hand, we know from Equation (28) that the enriched magnetic field is proportional to the toroidal field of the parent galaxy, and on the other hand, we know from Equation (32) that the growth rate of the toroidal field of the main galaxy is proportional to the initial IGM magnetic field.

Although a correct description of satellites and cold streams accretion requires detailed cosmological simulations of galaxy formation with MHD, we can speculate here that our proposed cycle of accretion and ejection of magnetic field lines (see Fig. 9) probably provides a dynamo loop, for which the radial field component is amplified by accretion of satellite galaxies and of the magnetically enriched IGM. A very rough and speculative analytical description of this effect can be obtained by injecting Equation (32) into Equation (28),

\[ \dot{B}_{\text{IGM}} \approx B_{\text{IGM}} \sqrt{\frac{3\epsilon_0 R_w^2 H_G}{8R_w^2}} \frac{R^2}{R_G} \Omega_G \approx B_{\text{IGM}} \sqrt{0.1 \epsilon_0 \Omega_G}, \quad (33) \]

where we have exploited the result of Bertone et al. (2005) that \( R_w \approx 3R \) (a reasonably good approximation according to their Figure 2) and we have assumed a typical disc aspect ratio \( H_G/R_G \approx 1/20 \) and a typical spin parameter \( \lambda \approx 0.05 \). We see from the previous equation that now the magnetic field amplification is exponential, and not linear anymore, because we have connected the radial component to the progenitor galaxies’ toroidal component. Although we have clearly not demonstrated that this cosmic dynamo actually works, we want to stress that the cosmological environment might play an important role, namely by accretion of amplified field lines from diffuse gas or satellite galaxies. The central engines of this cosmic dynamo are the dwarf galaxies, because they are easily disrupted by galactic winds, and therefore provide a important source of diffuse radial field, and also because they are the progenitors of our Milky Way Galaxy.

Some caveats to this Cosmic Dynamo scenario must be pointed out. We assumed that \( B_{\text{IGM}} \) is regenerated in a pure radial mode to get that exponential growth. Even though magnetic field lines in large-scale bipolar outflows are mainly perpendicular to the disc (see Heesen et al. 2009 for example), they carry turbulence that produces non-radial component of the field. Propagation of the outflow can be affected by the particular geometry of the galactic environment and leads to irregular distributions of the magnetic field components. Moreover magnetic fields can contribute constructively or destructively to the \( B_{\text{IGM}} \) when large-scale bubbles percolate and can substantially alter the final configuration of the field. A very rough way of taking into account these effects on the radial component is to consider that each component of the field is equally distributed, then equation (33) is simply affected by a factor \( 1/\sqrt{3} \).

While the \( \alpha-\Omega \) dynamo comes from the connection between small scale turbulence and galactic differential rotation, the proposed Cosmic Dynamo originates from the coupling of a large scale gas cycle (cosmological accretion and wind ejection) and differential rotation. We see from Equation (33) that this effect is probably very fast, especially at high redshift, for which the angular velocity of dwarf galaxies is much higher than today \( \Omega_G \approx (1+z)^{3/2} \). The actual growth rate will however depend crucially on many complicated aspects of galaxy formation that are beyond the scope of this paper, such as the wind filling factor as a function of redshift (Bertone et al. 2005), the field geometry during satellite mergers, the exact 3-dimensional propagation of wind bubbles within the cosmic web, etc. Here it is a priori assumed, that the galactic wind expansion in conjunction with some mergers happen within the 3 Gyr. Thus it has to be shown, that a similar growth could be achieved with an initial field configuration, which is similar to the evolved wind field.

We believe that this Cosmic Dynamo, in conjunction with a strong galactic dynamo, should be able to account for the fast magnetic field amplification we see in our universe, especially in light of the recent measurements performed on high redshift galaxies up to redshift 1 (Bernet et al. 2008). Because the existence of an efficient \( \alpha-\Omega \) Dynamo is questionable in dwarf galaxies due to low rotation of the disc and the lack of relativistic particles (Klein, 1991), it is important to explore alternative scenarios. It is possible to rely on the ideas discussed by Rees (1987), for which stars are able to generate their own magnetic field by stellar dynamos. We have tested this scenario in the Appendix, showing that for reasonable assumptions, we can reach magnetic field close to equipartition in a few Gyr.

We have also demonstrated with our cooling halo simulations that if the IGM magnetic field is high enough (namely \( B_{\text{IGM}} \approx 10^{-4} \mu G \)), the Lorentz force can prevent the formation of a new generation of dwarf galaxies. The formation of a magnetically supported torus prevent any subsequent star formation, and therefore breaks the central engine of our Cosmic Dynamo. This has the interesting consequence of providing a saturation mechanism, explaining naturally why our Cosmic Dynamo leads to a final value of \( B_{\text{IGM}} \approx 10^{-5} \) to \( 10^{-4} \mu G \). While the \( \alpha-\Omega \) dynamo saturates when the small scale turbulent magnetic field reaches equipartition in the disc and magnetic tension becomes able to suppress the small scale dynamo (see Subramanian, 1999, Brandenburg & Subramanian, 2005), the Cosmic Dynamo saturates when the large scale, cosmological magnetic field prevents dwarf galaxies’ disc formation, or severely affects star formation within them.

The evolution of magnetic fields in galaxies can now be addressed in a cosmological context. In a more or less near future, we propose to extend the present work using much higher resolution simulations, the ultimate goal being to resolve the small scale \( \alpha \) effect in our isolated, cooling halo simulations. Another ambitious objective would be to address the same problem in a fully cosmological environment, capturing the \( \alpha \) effect with some subgrid model, but accounting properly for satellite accretion and wind bubble propagation.

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Appendix A: Stellar magnetic feedback scenario

Rees (1987) was the first to suggest that Biermann battery effects at the surface of stars could create magnetic fields from a zero field. We also know that convective stars can amplify the magnetic energy extremely fast (Brun et al. 2004) close to its equipartition value by dynamo effects like in the Sun. In this scenario, massive stars $> 10 M_\odot$ which ones will release their magnetic energy in the ISM in powerful explosions, are the key ingredient for generating galactic magnetic fields. In the most massive stars, it seems that there is no convective envelope allowing for a very fast dynamo, but differential rotation could be extremely strong (Maeder et al. 2008) and can substantially increase any seed field. In some massive stars the convective zone can be in the central part (Zahn et al. 2007) and the dynamo can be effective (Charbonneau & MacGregor 2001). These stars have also a very short life-time (a few Myr), any amplification process must have an $e$-folding time extremely small. Some models predict that instabilities in the external radiative envelope are able to sustain a sufficiently fast growth of the field (MacDonald & Mullan 2004, Mullan & MacDonald 2005). Though it remains difficult to conclude whether magnetic field are appreciably amplified, there are increasing observational evidence for strong magnetic fields in massive stars (Aleian et al. 2008; Hubrig et al. 2008).

When a supernova is exploding, its magnetic field is diluted in a larger volume. We can approximate its final value by considering the flux conservation of the magnetic field. The uniform and averaged magnetic field in a spherical transformation must follow

$$B_2 \cong \left( \frac{R_1}{R_2} \right)^2 B_1 .$$

(A.1)

For example the Crab nebula with $B_1 \approx 100 \mu G$ within a $R_1 \approx 3.4 \text{ pc}$ radius (Kennel & Coroniti 1984) will have a $B_2 \approx 5.10^{-2} \mu G$ field within the radius $R_2 = 150 \text{ pc}$ of a superbubble. The case of the Crab nebula is a very strong particular case: a fast rotating pulsar inside the centre of the remnant could be responsible for this very high magnetic field. It is particularly difficult to explain how strong magnetic fields $B_1 \sim 10^6 \mu G$ in massive stars (Aleian et al. 2008, $R_1 \sim 5R_\odot \sim 3.10^6 \text{ km}$) can produce such high values in supernova remnants ($B_2 \sim 10^{-3} \mu G$ for $R_2 \sim 1 \text{ pc}$). This prediction does not take into account the possible amplifications stages during the rapid expansion of the blast wave, magnetic field can grow due to the cosmic ray pressure (Parker 1992), some strong Biermann battery effect (Hanayama et al. 2005), or turbulence (Bell 2004) that would allow stronger values in supernova remnants.

The basic idea to implement the release of magnetic field by superbubbles coming from supernova explosions is to put a fixed total magnetic energy within the bubble (ensuring that $\nabla \cdot B = 0$) with a random direction. We assume that within a $r_{\text{bubble}} \approx 150 \text{ pc}$ radius, each supernova contributes constructively for a certain amount of magnetic energy to the final release. We arbitrary choose that each superbubble (which corresponds to an assembly of several individual supernova explosions) releases a $10^{-3} \mu G$ field following a robust numerical implementation. This particular choice of $10^{-3} \mu G$ mag-
magnetic seeding is entirely heuristic and is set to match \( \mu G \) field after 1 Gyr of galactic evolution. To get a rough idea of the magnetic release by a single supernova inflating a larger bubble, we have to know the number of supernovae per star cluster particle \( N_{SN,\ast} = m_s \times \eta_{SN}/M_{SN} \approx 100 \), where we have assumed that \( M_{SN} = 10 M_\odot \) and for \( m_s = n_0 \times \Delta x^3 = 10^4 M_\odot \).

To simplify if we assume that each single supernova produces a magnetic field with the exact same intensity and direction, it comes that the associated magnetic field per supernova \((10 M_\odot)\) is \( B_{SN} = 10^{-5} \mu G \) (in a 150 pc radius bubble). We do not take into account the destructive contributions (magnetic field in opposite directions) but we do not also take into account the substantial increase coming from shocks between supernova explosions that generate instabilities, shear flows, cosmic rays, etc.

To implement a method of direct injection of the magnetic energy in the code, we must find a form of the magnetic field that ensures \( \nabla \cdot B = 0 \). For that purpose we will design a dipolar moment configuration such that the resulting magnetic field is embedded in the radius of the supernova explosion, and that the magnetic energy is not divergent in the centre in order to preserve the stability of the scheme even at different resolutions. Let us recall that the magnetic potential vector created by a unique magnetic dipole follows

\[
A(r) = \frac{4\pi}{\mu_0 r^3} (m \wedge r),
\]

where \( m \) is the magnetic dipolar moment. If we take \( m \) as a constant value, one can easily see that magnetic field will be infinite in the centre, then we must choose a certain distribution of dipolar moment elements such that the overall resulting moment respects the compact and convergent criterions. For that purpose we need \( m \) to grow as fast as \( r^4 \) and that sharply decreases down to zero at a certain radius \( r_{SN} \)

\[
m = r^4 \exp \left( -\frac{r}{r_{SN}} \right) m_0.
\]

With such a distribution of \( m \), we can compute the magnetic field as

\[
B = \nabla \wedge A.
\]

The resulting magnetic field computed on a 512\(^3\) grid within a 5\(r_{SN}\) box size (resolution element is 0.01 \(r_{SN}\)) is plotted in Figure A.1. We can renormalize the dipolar moment distribution such that the magnetic field gives a unit magnetic energy in regards to the measured magnetic energy \( E_m \), here

\[
m_0 = \frac{\sqrt{4\pi \mu_0 E_m}}{\mu_0 r_{SN}} = 0.85.
\]

This value is not very sensitive to the numerical resolution, for example, if \( \Delta x = 0.3 r_{SN} \), then \( m_0 = 1 \). That means that even if we poorly resolve the bubble (it is the case in our simulations \( r_{SN} \approx 2–3 \Delta x \)), the final magnetic energy is very close to the converged one.

We also add a saturation factor to avoid that the magnetic field excessively grows above the equipartition value, such that the resulting field is multiplied by

\[
f_{sat} = \frac{1}{1 + \left(0.1 \frac{r}{r_{SN}}\right)^4},
\]

where \( B \) is the value of the magnetic field inside the bubble and \( B_{eq} \) is the magnetic amplitude corresponding to its equipartition state with internal energy. For each bubble, the \( f_{sat} \) factor is the maximum of all individual \( f_{sat} \) factors computed on each cell within the supernova radius. Such saturation effects have been outlined in galactic dynamo simulations [Cattaneo & Hughes 1996].

![Figure A.1. Magnetic amplitude for the (A.3) dipolar distribution where \( m \) is aligned with the zaxis. Upper panel is a slice through the (Oyz) plane and bottom panel is through the (Oxy) plane. Magnetic vectors are overplotted. Picture size is 5\(r_{SN}\).](image-url)
generated in the disc but they tend to align with the global rotation within the disc as one can see in Figure A.2. After 3 Gyr, the amplitude has reached a value of a few \( \sim 10^{-2} \, \mu G \) in the central parts and its configuration is no more like a A0 type. As there is no more initial magnetic field, the A0 configuration does not appear. Moreover the randomisation of the magnetic field injection destroys any particular symmetry and reconnects the field through the equatorial plane. This effect explains the smaller scale of the magnetic field fluctuations but the global differential rotation of the galaxy maintains a preferred component of the field along the rotating vector. A direct consequence of the differential rotation is the amplification applied on magnetic seeds. A computation of the injected magnetic energy from supernovae with the star formation rate, gives \( E_{\text{inj}} \approx 10^{48} \text{erg}/\text{Gyr} \). Though the magnetic energy in the disc should reach \( 10^{48} \text{erg} \) after 1 Gyr, figure 5 shows that the energy measured in the galaxy for this simulation (blue line) is \( \sim 3 \times 10^{49} \text{erg} \) after 1 Gyr. This is an evidence that the differential rotation is still amplifying the seed field generated by supernovae.

Radial contribution is non-negligeable, the pitch angle \( |\theta_p| \) is everywhere greater than \( 10^\circ \) and goes up to \( 90^\circ \) in some regions of the galaxy. The \( 10^\circ \) minimum value is predictable: if the mean field amplitude is about \( \sim 10^{-2} \, \mu G \) preferentially toroidal, putting a \( 10^{-3} \, \mu G \) field in the supernova remnant along the radial component produces a \( |\theta_p| \approx 5^\circ \) pitch angle. Then the successive explosions and the differential rotation rise this value up to \( 90^\circ \).

As shown in figure 8 the magnetic energy flux is lower than when the simulation starts with an initial magnetic field by a factor \( \sim 40 \), but this value depends a lot on the particular choice of the magnetic seeding \( B_{\text{SN}} = 10^{-5} \, \mu G \). Thus, with a \( E_{\text{B,in}} \approx 10^{51} \text{erg Gyr}^{-1} \) at the bottom of the wind, we can predict that the final magnetic field within the hot tenuous bubble will reach \( \sim 10^{-5} \, \mu G \). It corresponds to the initial cosmological seeds required in large scale cosmological simulations (Dolag et al. 2005; Dubois & Teyssier 2008a). Relying on the assumption that some supernova remnants (Crab nebula, Vela nebula) are permeated with strong magnetic fields (Kennel & Coroniti 1984; Helfand et al. 2001) we verify that this mechanism can magnetise a dwarf galaxy and therefore produce a magnetised wind to enrich the IGM. Using a very low magnetic seed within supernova bubbles (5000 weaker than the field within the Crab nebulae), we showed that the galaxy rapidly reaches 1 \( \mu G \) in \( \sim 1 \) Gyr and is able to fill the IGM with a large-scale magnetic field of \( \sim 10^{-5} \, \mu G \). It is therefore sufficient to explain an early magnetisation of the Universe.

Fig. A.2. Magnetic amplitude in log \( \mu G \) units with magnetic vectors for the galaxy with star formation, supernova feedback, \( B_{\text{IGM}} = 0 \) and a magnetic seeding \( B_{\text{SN}} = 10^{-5} \, \mu G \) at \( t \approx 3 \) Gyr in the (Oyz) plane (upper panel), in the (Oxy) plane (middle panel) and the pitch angle (bottom panel). Picture size is 40 kpc.