Cost-Optimal System Performance Maps for Laser-Accelerated Sailcraft

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Abstract

Breakthrough Starshot is an initiative to explore the Centauri system using laser-accelerated sailcraft. Earlier work produced a point design for a 0.2 c mission carrying 1 g of payload. The present work widens the design space to missions having 0.1 mg to 100 kt payload, 0.0001 c to 0.99 c cruise velocity. To make comparisons easier, assumed technology figures of merit are those of the earlier work. Also, the beam director may now draw up to 5 GW of power directly from the grid to augment the power drawn from its energy storage system.

Augmenting stored energy with grid power shrinks beam director capital cost by up to 5 orders of magnitude for smaller missions, enabling new possibilities: A 0.1 mg microbiome accelerated to 0.01 c in only 4 min by a 17 cm sail and $21M beam director, expending only $12k worth of energy to do so. A 10 kg Solar system cubesat accelerated to 0.001 c (63 au/yr) by a 77 m sail and $610M beam director, expending $58M worth of energy per mission.

A progression from cost-optimized point designs to whole performance maps has been made possible by replacing numerical trajectory integration with closed-form equations. Consequently, the system model computes point designs 1-2 orders of magnitude faster than before. Resulting maps reveal different solution regimes characterized by their performance-limiting constraints.

The performance maps also reveal a family of missions that accelerate at Earth gravity. The heaviest such mission is a 7.4 km diameter 100 kt vessel (equivalent to 225 International Space Stations) that is accelerated for 21 d to achieve 0.07 c, reaching the Centauri system within a human lifetime. While unthinkable at this time, the required 380 PW peak radiated power (twice terrestrial insolation) might be generated by space solar power or fusion within a few centuries. Regardless, it is now possible to contemplate such a mission as a laser-accelerated sailcraft.

Keywords: Breakthrough Starshot, beam-driven sail, beamed energy propulsion, model-based systems engineering

1. Motivation

In work that was published in 2018 [1], I formulated a system model for laser-accelerated sailcraft missions and used it to compute a cost-optimal point design for a 0.2 c mission to the Centauri system (carrying 1 g of payload), and for a 0.01 c precursor mission (carrying only 7 mg of payload). Beyond point designs, this earlier work began to explore what is possible using laser-driven sails by plotting how cost-optimal missions vary over a wide range of cruise velocities. These plots were difficult to produce because computational complexity made the model run slowly, sometimes taking an hour or more to converge upon a single design. Also, for a design to converge, the model’s numerical solvers had to start with valid search ranges over all iterated variables, and these ranges had to be manually adjusted as cruise velocity (or other mission requirements) changed. Both these reasons made the plotting procedure far from automatic, and so I did not attempt to plot how cost-optimal missions vary with payload mass.

Two years later, Witten [2] suggested a phalanx of low-cost laser-driven sailcraft to probe for the gravitational field of a primordial black hole in the outer Solar system; an idea that was also examined by Christian, Hoang, and Loeb [3, 4]. The sailcraft would be like the ones envisioned in the Breakthrough Starshot project, but cruise slowly at 0.001 c and carry orders of magnitude heavier payloads. Hoang & Loeb [4] estimated that at this cruise velocity, a spacecraft should have greater than 10 g mass for the gravitational signal to be distinguishable from drag noise and magnetic noise. In response, I used the system model to generate point designs for 0.001 c (63 au/yr) precursor missions carrying 1 kg, 10 kg, and 100 kg payloads. For the same reasons as before, this work did not attempt to plot how cost-optimal designs vary with payload mass. Ultimately, we combined these mission concepts and point designs together with other ideas into a white paper [5], which was sent to the National Academy of Sciences for consideration as part of the Planetary Science and Astrobiology Decadal Survey.

Around this time, NASA was asking for similar information: Starting in FY2017, the United States Congress directed NASA to undertake an interstellar mission technology assessment [6] toward the goal of launching a probe to Alpha Centauri by the year 2069 in commemoration of the one-hundredth anniversary of the Apollo-11 moon landing. Consequently, NASA held three extramural workshops to identify and evaluate technology concepts. Litchford & Sheehy [7] summarized these activities and suggested phased science goals and missions to incrementally develop the interstellar propulsion, spacecraft sub-
systems, and operational experience, needed to mount a fully-integrated scientific probe to nearby stars. As far as laser-driven sailcraft are concerned, NASA’s need was to understand how the approach scales to heavier and slower missions spanning the inner Solar system through to the Oort cloud.

Ever since the Breakthrough Starshot initiative launched in 2016, interstellar precursor mission concepts have been collected and viewed as potential stepping stones toward the Centauri system mission. To be a stepping stone, a precursor mission would need to retire risks associated with the Centauri system mission, turning leaps of faith into inevitable outcomes, one step at a time. For example, an early idea was to send sub-scale sailcraft to probe the nearby interstellar magnetic field and dust environment. This idea addressed key uncertainties that affect how Centauri sailcraft will mitigate dust impacts. The precursor mission’s slower cruise velocity would allow earlier launch via a smaller and less powerful beamer, and without first having to master precision trajectories, develop imaging sensors, or to meet nearly such difficult data return requirements as the Centauri mission. Another precursor mission candidate has been Turyshev’s solar gravitational lens mission [8], whose 550 au starting distance can be reached by laser-driven sailcraft within a decade. This mission’s need for precision maneuvering and imaging sensors is shared by a class of missions that catch up with interstellar objects passing through the Solar system [9]. These and other precursor mission concepts have involved slower cruise velocities and very different payload masses than the Centauri system mission, making them difficult to characterize using the 2018 Starshot system model.

In summary, what has been needed is a faster system model with improved convergence and autonomy over a wider range of mission parameters, and a system-level performance map that varies payload mass as well as cruise velocity. This is what has motivated the present work.

2. Changes Since the 2018 System Model

At its core, the system model describes a laser beam’s propagation from a ground-level beamer (beam director) to a spaceborne sailcraft (sail and craft, which may be discrete or integrated) [1], and the sailcraft’s resulting motion. It relates key design parameters that determine the system capex (capital expense) and opex (operational expense). Optimizers then vary the inputs to find values that minimize opex and capex. Compared to the 2018 system model, aspects are changed as follows.

2.1. Nomenclature

Powers and efficiencies have been renamed with subscripts 1 to 4 corresponding to power’s passage from an electrical grid through to a sailcraft. This sequence is shown in fig. 1, a Sankey diagram that is extended to include frame transformations.

Following the figure from grid through to sailcraft, a grid supplies power $P_1$ to an energy storage facility co-located with a beamer, corresponding to the ‘wallplug’ power at the organizational interface between the power provider and Starshot as its customer. After minor losses due energy conversion, storage, and local transmission, the energy storage subsystem provides power $P_2$ to the beamer. Within the beamer, the lasers’ electrical efficiency is relatively low, and there are various smaller loss mechanisms as well. After these losses, the beamer radiates power in the direction of a sailcraft. Radiated power $P_3$ relates to the earlier powers via,

$$ P_3 = \eta_{23} P_2 = \eta_{12} \eta_{23} P_1. \tag{1} $$

Power transfer efficiencies $\eta_{12}$ and $\eta_{23}$ also relate energies (in addition to powers) from the grid to the beamer because they are invariant over the course of a pulse. Through space and thereafter, the efficiencies vary with time.

Between the beamer and the sailcraft there is a delay due to the finite speed of light, given by

$$ \frac{P_3(t)}{\eta_{34}(t)} = P_3 \left(1 - \frac{z(t)}{c}\right). \tag{2} $$

The delay $z(t)/c$ lengthens as the sailcraft accelerates away. If $t=0$ when the first photons reach the sailcraft, then the photons’ path losses $(1-\eta_{34})$ depend on the sailcraft’s initial separation from the beamer $z_0 \equiv z(t=0)$. This means that the first photons depart the beamer at time $z_0/c$.

Power reaching the sailcraft can be viewed in different ways, each of which has particular utility: $P_4$, the power received in the quasistatic frame, is used with power transfer efficiency calculations that assume a static target relative to the source, such

![Figure 1: Power and efficiency relationships](image-url)
as the Goubau equations in the next section. $P_{4b}$, the power received in the sailcraft co-moving frame, is integrated with respect to time to find the radiated energy $Q$ (see section 2.3.3). $P'_4$, the power received in the sailcraft relativistic rest frame, can be constant when the sailcraft is accelerating at its thermal limit (see section 2.3.1). These sailcraft powers are related by

$$P_{4b} = (1 - \beta) P_4,$$

(3)

$$P'_4 = \left(\frac{1}{1 + \beta}\right) P_{4b},$$

(4)

where factor $1 - \beta$ is the fractionally-reduced rate at which photons hit the receding sailcraft, and factor $1/(1 + \beta)$ is the Doppler frequency downshift of beamer-radiated photons as perceived by the receding sailcraft. This latter factor is also the fractional photon energy loss, because photon frequency $\nu$ is linearly proportional to photon energy ($E = h\nu$). Multiplying the photon energy factor by the rate factor yields the power factor derived by Einstein [10] relating power between frames,

$$P'_4 = \left(\frac{1 - \beta}{1 + \beta}\right) P_4.$$

(5)

2.2. Goubau Beam

A Goubau beam [11] describes near-optimal power transfer between finite diameter optics [12]. It is appropriate when the intensity distribution across a transmitting optic can be tailored. This is the case for a phased array, which consists of very many elements whose intensities are individually controlled and varied for optimal performance throughout the sailcraft’s acceleration. A Gaussian beam is a poor fit for this situation because it is accurate only in the limit of energy transfer between infinite diameter optics, resulting in oversized optics costing billions of dollars extra. A top-hat beam describes energy transfer between a finite diameter optic and an infinite diameter optic and is also non-optimal when optics’ capex dominates, as it can do for laser-driven sailcraft.

Goubau beam transfer efficiency $\eta_g$ is approximated by

$$\eta_g(a) = \begin{cases} \eta_{g+}(a) & \text{if } a > \sqrt{2 \frac{1}{1 + \beta}} \\ \eta_{g-}(a) & \text{otherwise} \end{cases}$$

(6)

$$\eta_{g+}(a) = \frac{1}{4b^2} \left( a^4 + 2a^2b + 4a^2b - 8b^4 + 4b^2 \right)^{-\frac{1}{2}},$$

(7)

$$\eta_{g-}(a) = \left( \frac{a^2}{2} - \frac{a^6}{32} + \frac{7a^{10}}{4096} \right)^{\frac{1}{2}},$$

(8)

where $a(\tau_g) = \sqrt{\frac{\tau_d}{\tau_g}}$ and $b = \frac{a^2}{\tau_g}$. In this expression, transfer efficiency depends on just a single dimensionless parameter $\tau_g$, which in turn depends on the product of (circular) optic radii $R_s$ and $R_g$, their separation $\tau$, and beam wavelength $\lambda$. The expression for $\tau_g$ used in earlier work [11][13] contains an erroneous factor of two that traces back to the caption of Figure 2 in Goubau’s book chapter [11]. The correct expression for $\tau_g$ is

$$\tau_g = \frac{\pi \lambda}{\sqrt{\lambda^2 - \lambda^2}},$$

(9)

Earlier numerical results are unaffected because the code did not directly calculate $\tau_g$, instead calculating $a$ using an alternate (correct) expression.

Subsequent to Goubau [11], works by Brown [14]; Hansen, McSpadden, and Benford [12]; and Goubau himself [15] use a different dimensionless grouping, usually called $d$, to parameterize transfer efficiency,

$$\tau_d \equiv \frac{\sqrt{\lambda^2 - \lambda^2}}{\lambda} = \frac{\pi R_s R_g}{\lambda^2} \equiv \frac{z_{GR}}{2}. \quad (10)$$

Here, I have reduced the expression to a simple ratio by introducing a geometric mean of Rayleigh ranges $z_{GR} \equiv \pi R_s R_g / \lambda$: If the beamer were to emit a Gaussian beam, its Rayleigh range would be $\pi R_s^2 / \lambda$, and if the sailcraft were to emit a Gaussian beam its Rayleigh range would be $\pi R_g^2 / \lambda$, so $z_{GR}$ is the geometric mean of these. I have also added subscripts to the two
different versions of \( \tau \) to clarify which is which and how they relate to each other.

For completeness, a dimensionless grouping

\[
N_T \equiv R_T \rho_s / \lambda z = \tau_d / \pi
\]  

(11)
is occasionally called the target (or sail) Fresnel number and used instead of \( \tau_d \) or \( \tau_s \). It is distinct from the conventional Fresnel number, which does not incorporate target size.

The system model now adopts the use of \( \tau_d \) (instead of \( \tau_s \) or \( N_T \)) because \( \tau_d \) lends itself to a greatly simplified expression for transfer efficiency:

\[
\eta_d = 1 - e^{-\tau_d},
\]  

(12)
The lineage of this approximation is a little unclear [16,17]. It is stated without proof by Benford starting in 2007 [18,19], but Dickinson and a colleague at NASA JPL were likely the first to deduce it (in 1998 or prior) by fitting it to the curve in Figure 2 of the 1970 article by Goubau [15]. Regardless, Figure 2 of the present paper shows that eq. (12) differs from eq. (6) by less than 3%. Equation (12) is now used instead of eq. (6) because it is simpler and quicker to calculate.

Finally, the power transfer efficiency from the beamer to the sailcraft is given by,

\[
\eta_{3d}(t) = \eta_d \eta_d(z(t)),
\]  

(13)
where \( \eta_d \) accounts for scattering and absorption losses as the beam transits the atmosphere, and \( \eta_d \) is calculated from eqs. (10) and (12) above.

2.3. Closed-Form Trajectory Equations

The quasi-1D motion of a sailcraft in thermal equilibrium in its rest frame is described by [11],

\[
F = (A + 2R) \frac{P'_4}{c},
\]  

(14)
where \( F \) is the force acting on the sailcraft due to the incident beam power \( P'_4 \), \( A \) is sailcraft absorptance, \( R \) is sailcraft reflectance, and \( c \) is the speed of light in free space.

To relate \( P'_4 \) to beamer and beam propagation characteristics, the equation of motion is transformed from the sailcraft rest frame to the quasistatic frame: Force remains unchanged because a pure force (that does not add net heat to the sailcraft) is equal in both frames. In the quasistatic frame, force is related to acceleration by

\[
F = \frac{dp}{dt} = \frac{d}{dt}(\gamma m_0 \beta c) = m_0 \gamma \frac{d}{dt} \left( \frac{E_0}{c} \beta \frac{d\beta}{dt} \right),
\]  

(15)
where \( p \) is relativistic momentum, \( t \) is time measured in the quasistatic frame, \( \beta \) is sailcraft speed as a fraction of light speed, \( \gamma \equiv 1/\sqrt{1-\beta^2} \) is the Lorentz factor, \( m_0 \) is the sailcraft rest mass, and \( E_0 \) is the sailcraft rest energy famously equal to \( m_0 c^2 \).

Equating eq. (14) to eq. (15) yields the equation of motion in a more convenient form,

\[
\frac{E_0}{A + 2R} \frac{d\beta}{dt} = \frac{P'_4}{c \gamma^3} = \frac{P_4}{D^2 \gamma^3},
\]  

(16)
where the square of the Doppler factor \( D^2 = (1 + \beta)/(1 - \beta) \) converts \( P'_4 \) to \( P_4 \) as per eq. (5).

To find the sailcraft’s distance from the beamer \( z(t) \), the earlier system model [1] uses the definition of \( \beta \),

\[
\beta \equiv \frac{1}{c} \frac{dz}{dt},
\]  

(17)
to transform eq. (16) into a second order ODE,

\[
\frac{d^2 z}{dt^2} = \frac{c(A + 2R) P_4}{E_0 D^2 \gamma^3},
\]  

(18)
numerically integrating it to obtain \( z(t) \) and auxiliary quantities.

In contrast, the present work uses the identity,

\[
\frac{d\beta}{dt} = \frac{dz}{dt} \frac{d\beta}{dz} = c \frac{\beta}{\gamma} \frac{d\beta}{dz},
\]  

(19)
to transform eq. (16) into a first order ODE,

\[
\frac{d\beta}{dz} = \frac{A + 2R P_4}{c E_0 D^2 \gamma^3 \beta},
\]  

(20)
Expressions for \( P_4 \) turn out to lead to closed-form solutions for \( \beta(z) \) (and auxiliary quantities) in the sailcraft temperature-limited regime and the beamer power-limited regime.

2.3.1. Sailcraft Temperature-Limited Regime

In the sailcraft temperature-limited regime, assuming constant \( A \), \( P'_4 \) remains constant at its upper limit of \( P'^+_4 \). Also assuming constant \( R \) and \( E_0 \), dimensionless range \( \tilde{z} \) can be defined as

\[
\tilde{z} \equiv \frac{(A + 2R) P'^+_4 \gamma}{c E_0},
\]  

(21)
Equations (5) and (21) simplify eq. (20) to a temperature-limited equation of motion,

\[
\int \frac{\beta}{(1 - \beta^2)^{3/2}} d\beta = \int d\tilde{z},
\]  

(22)
which directly integrates to

\[
\Delta \tilde{z} = \Delta \gamma.
\]  

(23)
This simple closed-form relation between speed and distance inverts to

\[
\beta = \frac{1}{(\Delta \gamma + y_0)^2},
\]  

(24)
where \( y_0 \) is \( \gamma \) at \( \Delta \tilde{z} = 0 \).

Simple equations also relate sailcraft acceleration duration \( t_s \) to speed and distance: Defining dimensionless duration \( \tilde{t} \) as

\[
\tilde{t} \equiv (A + 2R) \frac{P'^+_4}{E_0} t_s,
\]  

(25)
eq. (17) becomes
\[
\beta = \frac{d\tilde{z}}{dt}. \tag{26}
\]
Equating this with eq. (24) yields
\[
\int dt = \int \frac{d\tilde{z}}{\sqrt{1 - \frac{1}{(\Delta z^2 + \eta_0)^2}}}, \tag{27}
\]
which integrates to
\[
\tilde{t}_{c} = \Delta(\gamma\beta) \tag{28}
\]
given the initial condition \(\Delta \tilde{z} = 0\) at sailcraft acceleration duration \(\tilde{t}_{c} = 0\). As can be illustrated on a spacetime diagram, sailcraft acceleration duration relates to the radiated beam duration \(t_{b}\) via
\[
t_{b} = t_{c} - \frac{\Delta \tilde{z}}{c}, \tag{29}
\]
where both durations are measured by a clock at the beamer. In dimensionless form, and using eq. (23), this reduces to
\[
\tilde{t}_{b} = \frac{1}{D_{0}} - \frac{1}{D}. \tag{30}
\]

The acceleration duration \(t'_{c}\) measured by a clock on the sailcraft is deduced by starting with a well-known relation for relativistic proper time \(t'\):
\[
dt' = \frac{dt}{\gamma(t)}. \tag{31}
\]
Time \(t\) is substituted for \(\beta\) using the equation of motion eq. (16). After integrating,
\[
\tilde{t}'_{c} = \ln \left( \frac{D}{D_{0}} \right). \tag{32}
\]

The ratio between the time experienced by the sailcraft’s payload and the beam duration experienced at the beamer is therefore,
\[
\frac{\tilde{t}'_{c}}{\tilde{t}_{b}} = \frac{\tau_{c}}{\tau_{d}} = \ln \left( \frac{D}{D_{0}} \right). \tag{33}
\]
This ratio tends to \(D_{0}\) in the limit \(D \to D_{0}\), meaning that the payload experiences at least the duration of the accelerating beam, even if cruise velocity approaches nearly light speed.

2.3.2. Beamer Power-Limited Regime

In the beamer power-limited regime, \(P_{3}\) remains constant at its upper limit of \(P_{3}^{*}\), so that eq. (2) becomes
\[
P_{4} = \eta_{34}(\tilde{z})P_{3}^{*}. \tag{34}
\]

The same dimensionless quantities used in the temperature-limited case can be kept by defining an efficiency-like factor,
\[
\eta_{T} = \frac{\eta_{P}}{P_{3}^{*}}\tilde{z}_{GR}. \tag{35}
\]
Assuming constant \(A, R, \eta_{34}\), and \(E_{0}\), and substituting eqs. (10), (12), (21), (34) and (35) into eq. (20) yields,
\[
\int \frac{\beta}{(1 + \beta)^{1/2}(1 - \beta)^{5/2}} d\beta = -\eta_{T} \int \frac{1 - r_{d}^{2}}{r_{d}^{2}} d\tau_{d}. \tag{36}
\]
The result takes the form,
\[
\eta_{T}I_{T} = I_{\beta} + K, \tag{37}
\]
where \(K\) is a constant,
\[
I_{\beta} \equiv \frac{1}{3} \left[ 1 - \frac{(1 - 2\beta)(1 + \beta)^{1/2}}{(1 - \beta)^{1/2}} \right], \tag{38}
\]
and, similar to the result obtained by Rather at al. (31).
\[
I_{T} \equiv \sqrt{\pi} \text{erfc}(\tau_{d}) + \frac{1 - e^{-\tau_{d}^{2}}}{\tau_{d}}. \tag{39}
\]

GIVEN initial sailcraft position \(\tilde{z}_{0}\) and speed \(\beta_{0}\), \(K\) is calculated using the three equations above. Given \(K\) and a subsequent position \(\tilde{z}_{1}\), speed \(\beta_{1}\) is found by numerically solving eq. (38). As \(\tilde{z} \to \infty\), \(\tau_{d} \to 0\) and so \(I_{T} \to \sqrt{\pi}\). This implies that \(\beta\) tends to a finite limit \(\beta_{\infty}\). Given \(K\), \(\beta_{\infty}\) is found by numerically solving eq. (38). This especially useful procedure shows whether or not a specified cruise velocity can be reached before attempting more involved calculations.

Beam duration cannot be found in the same way as before (by using \(\beta = d\tilde{z}/dt\) and directly integrating \(t\) on one side of the equation and \(\tilde{z}\) on the other) because eq. (38) is an implicit function of \(\beta\). Instead, beam duration is found by realizing that radiated power is constant (\(P_{3}^{*}\)) in this regime:
\[
t_{b} = \frac{Q_{3}}{P_{3}^{*}}, \tag{40}
\]
where \(Q_{3}\) is the energy radiated by the beamer while in the power-limited regime. The equation by which \(Q_{3}\) is calculated turns out to be the same for both the temperature-limited and power-limited regimes; it is derived in the next section.

The acceleration duration \(t'_{c}\) measured by a clock on the sailcraft is (again) deduced by starting with the well-known relation for relativistic proper time \(t'\):
\[
dt' = \frac{dt}{\gamma(t)}. \tag{41}
\]
In this case, \(t\) cannot be substituted for \(\beta\) using the equation of motion because \(P_{3}\) is constant and not \(P_{3}^{*}\). Letting \(dQ_{3} = P_{3}^{*}dt\),
\[
dt' = \frac{dQ_{3}}{P_{3}^{*}}. \tag{42}
\]
An expression for \(dQ_{3}\), eq. (45), is derived in the next section, and the solution procedure to find \(t_{b}\) and \(t_{c}\) is similar to the procedure to find \(Q_{3}\). The results show that a power-limited payload can experience less than the duration of the accelerating beam, which is qualitatively different from the temperature-limited result that the payload experiences at least the duration of the accelerating beam. The power-limited \(t_{c}\) must behave this way: As acceleration tends to zero while the beam is still on, time runs a factor of \(\gamma\) slower for the sailcraft payload than the beamer.
2.3.3. Radiated Energy

Energy $Q_3$ is radiated by the beamer toward the sailcraft. It determines the energy cost to accelerate each sailcraft toward its destination, and it drives the beamer’s energy storage capacity. $Q_3$ is therefore essential to calculating system cost. The cost minimization procedure, given shortly, calculates $Q_3$ within its innermost iteration loop alongside the other trajectory equations. Therefore, we seek a faster and more accurate way to find $Q_3$ than numerical integration.

The essence of finding $Q_3$ is to integrate beamer-radiated power $P_3(t)$ with respect to time:

$$Q_3 = \int_{z_0/c}^{z_f/c} P_3(t) dt = \int_{z_0/c}^{z_f/c} \frac{P_4\left(\frac{t+\frac{3\beta}{c}}{3}\right)}{\eta_{34}\left(\frac{t+\frac{3\beta}{c}}{3}\right)} dt.$$ \hfill (43)

Acceleration is defined to commence when $t=0$, which means that the pulse begins and ends $z_0/c$ before it reaches the initial position of the sailcraft. Section 2.1.1 has already described the relationship between various powers, with eq. \ref{eq:2} relating radiated power $P_3$ (at time $t$) to received power $P_4$ after a delay of $z(t)/c$. To allow the equation of motion, eq. \ref{eq:16}, to eliminate the unknown power $P_4$, the origin of $t$ is first shifted by $z(t)/c$:

$$Q_3 = \int_{0}^{\frac{3\beta}{c}} P_4(t) \frac{(1-\beta)P_4(t)}{\eta_{34}(t)} dt.$$ \hfill (44)

This origin shift has the effect of widening the pulse length (consistent with sailcraft’s acceleration duration being longer than the radiated pulse duration, as can be shown on a space-time diagram) and reduces the power by a factor of $1-\beta$ (as described by eq. \ref{eq:2}). Finally, the equation of motion eq. \ref{eq:16} eliminates the unknown power $P_4$ and transforms the integral from the time domain to the velocity domain:

$$Q_3 = \frac{E_0}{\eta_4 (A + 2R)} \int \gamma \frac{1}{1-\beta} \eta_d \frac{d\beta}{d\eta_d}. \hfill (45)$$

Defining dimensionless energy $\tilde{Q}_3$ as

$$\tilde{Q}_3 \equiv \frac{\eta_4 (A + 2R)}{E_0}, \hfill (46)$$

using eq. \ref{eq:12}, and writing $\gamma$ in terms of $\beta$, yields a solvable energy integral:

$$\tilde{Q}_3 = \int \frac{1}{(1-\beta) \sqrt{1-\beta^2}} \frac{1}{1-e^{-2\beta\eta_d}} d\beta.$$ \hfill (47)

This result does not constrain any particular power to be constant and is therefore general; applicable to both temperature-limited and power-limited regimes. Choosing a regime affects the solution through $\tau_d(\beta)$ because $\tau_d$ relates to $\tilde{Q}_3$ via eq. \ref{eq:10}, $z$ to $\tilde{z}$ via eq. \ref{eq:21}, and $\tilde{z}$ to $\beta$ via explicit eq. \ref{eq:25} in the temperature-limited regime, and via implicit eq. \ref{eq:37} in the power-limited regime. Unfortunately, the resulting expressions do not integrate to simple closed-form equations for either regime.

Minimum $\tilde{Q}_3$ occurs when $\eta_d$ is unity. In this special case, the indefinite integral reduces to the Doppler factor, providing a useful closed-form lower bound:

$$\tilde{Q}_3 = \int \frac{1}{(1-\beta) \sqrt{1-\beta^2}} \frac{1}{1-e^{-2\beta\eta_d}} d\beta = \frac{1 + \beta}{\sqrt{1-\beta^2}} = D.$$ \hfill (48)

In the general case, eq. \ref{eq:37}’s integrand is expanded into a Taylor series to find $\tilde{Q}_3$, without numerical integration. This series does not converge in the vicinity of non-analytic points $\beta=1$ and $\tau_d=0$ (corresponding to $z=\infty$, and to $\beta=\beta_\infty$ in the power-limited regime), but trajectories do not pass through these values anyway; they only approach them. For reasons given shortly, the series is expanded about the target speed $\beta_1$ and not the initial speed.

$$\Delta \tilde{Q}_3 = \int_0^{\tau_0} \sum_{n=0}^{N} a_n(\beta_1) B^n d\beta = \sum_{n=0}^{N} \frac{a_n(\beta_1) B^{n+1}}{(n+1)!}.$$ \hfill (49)

where $B \equiv \beta - \beta_1$ is a negative quantity. The coefficient $a_n(\beta_1)$ is given by

$$a_n(\beta_1) = \lim_{\beta \rightarrow \beta_1} \frac{d^n}{dB^n} \left[ \frac{1}{B} - \frac{1}{1-e^{-2\eta_d}(1-B)\sqrt{1-B^2}} \right].$$ \hfill (50)

The system model uses algorithmic differentiation \ref{eq:22} to compute eq. \ref{eq:50}. For this purpose, $\tau_d$ relates to $\tilde{z}$ via eq. \ref{eq:10}. Then, in the temperature-limited regime, $\tilde{z}$ relates to $\beta$ via eq. \ref{eq:25}. Or in the power-limited regime, eq. \ref{eq:37} is numerically solved to provide $\tau_d(\beta_1)$, and the higher derivatives are computed from eq. \ref{eq:50}’s differential form combined with eq. \ref{eq:10}’s differential form,

$$\frac{d\tau_d}{d\beta} = \frac{1}{\eta_\tau (1-\beta)^2 \sqrt{1-\beta^2}} \frac{\tau_d^2}{1-e^{-2\eta_d}},$$ \hfill (51)

where

$$\eta_\tau \equiv \frac{\eta_0 (A + 2R) P_4^2 z_{GR}}{c E_0}.$$ \hfill (52)

The system model computes eq. \ref{eq:50} using double-precision floating-point numbers. The radius of convergence $B_r$ is estimated by a fairly conservative form of ratio test,

$$B_r = \max \left\{ \frac{a_0}{a_1}, \ldots, \frac{a_{n-1}}{a_n} \right\}. \hfill (53)$$

Finite precision numbers clash with the exponential term $e^{-2\eta_d(\beta)}$ to make $B_r$ estimates inaccurate when the Taylor series is expanded about $\beta=0$: $B_r$ sometimes extends beyond $\beta_\infty$, missing the endpoint entirely and corrupting the $\Delta \tilde{Q}_3$ estimate. For this reason, the series is expanded about $\beta_1$, the trajectory’s target speed, which has a smaller $B_r$.

The system model employs successive Taylor expansions (via algorithmic differentiation to 12th order) to ‘hop’ backwards from the target speed $\beta_1$ to the earlier speed $\beta$, with each hop contributing energy to $\Delta \tilde{Q}_3$. Hop distance $B$ is initially set to $B_r$. The system model considers that a hop has converged when the series’ final term contributes 1 part in $10^6$ or less of the energy in that hop. If the hop does not converge, then $B$ is halved and the hop is retried. Retries continue until the hop converges or the model reaches a minimum hopping interval, equal to $(\beta_1 - \beta)/10^6$. 

6
2.3.4. Solution Procedure

For each trajectory, the initial sailcraft temperature (sailcraft’s equilibrium temperature just as acceleration commences) is calculated via a simple energy balance.

If the initial sailcraft temperature is less than the prescribed limit, then the power-limited equations of section 2.3.2 are used to solve for the sailcraft state at the point where the target velocity is reached.

Else if the initial sailcraft temperature is greater than or equal to the prescribed limit, then the temperature-limited equations of section 2.3.1 are used instead. In this case, the model solves for the power radiated by the beamer such that the sailcraft equilibrium temperature equals its limit. As the sailcraft accelerates away, radiated power increases to maintain the sailcraft temperature at its limit despite increasing beam spillage. There comes a point where the beamer reaches its maximum rated radiated power. Thereafter, the sailcraft’s temperature begins to fall. This transition point between the temperature-limited and power-limited equations is calculated by solving the temperature-limited equations for the distance at which the beamer radiates at its maximum rated power. If the target velocity is less than the velocity at the transition point, then the temperature-limited equations are used to solve for the sailcraft state as acceleration ends. Else the power-limited equations are used (with the transition point as the initial condition). In this latter case, the expended energy (needed by the cost model, for example) is then the temperature-limited energy from the start to the transition point plus the power-limited energy from the transition point to the point where the target velocity is reached.

2.3.5. Comparison with the 2018 System Model Results

The system model is now used to reproduce the 0.2 c trajectory that was originally produced by the 2018 system model [1]. To compare algorithms, all available input digits are given:

- 4.065 203 538 694 140 3 m sailcraft diameter
- 2.657 83 km beamer diameter
- 203.698 926 469 033 67 MW beamer peak radiated power

Other constants, if needed, are those used in 2018 and repeated shortly in table 1 [1] with two exceptions: First, the Stefan-Boltzmann constant has been updated in accordance with the 2019 redefinition of the SI base units and is greater than the earlier value by 1 part in 10^23. Second, the 2018 system model code mishandles the transatmospheric propagation efficiency \( \eta_o \), ignoring it in scaling the sailcraft incident power, but including it in sizing the beamer energy storage. To bypass the effect of this error, \( \eta_o = 1 \) is used for computations presented within this subsection.

To produce plots with time along the x-axis, the earlier system model outputs the time and other quantities from its numerical integration steps, whereas the new system model has time as an input (not an output) and recalculates outputs that are then collected in a table. For the new model in the temperature-limited regime, time enters through eq. (28). In the power-limited regime, time is calculated from eq. (40), and the system model numerically solves for \( \beta \), from which all else is calculated.

The resulting trajectory, shown in fig. 3, is difficult to distinguish from the corresponding plot produced by the 2018 system model [1]. The sailcraft transits from temperature-limited to power-limited regimes just slightly past the 5 min mark in both plots. The qualitative behavior is the same in each regime, validating the closed-form equations’ correct behavior against the numerically-integrated result.

Comparing numbers, outputs from the new code are close to the old code: 0.136 99 au vs. 0.136 03 au (0.7% increase) in acceleration distance, 80 236 GJ vs. 80 105 GJ (0.2% increase) in radiated energy, and 557.94 s vs. 555.33 s (0.5% increase) in acceleration duration. The new expression for beam transmission efficiency differs from the old one by up to 3% as discussed in section 2.2; however, it makes little difference to the result.

Acceleration distance in the new model is a function of cruise velocity and constants only, so the computed values are exact. Yet the difference between distance estimates in the two codes tends to 0.7% as the numerical integrator’s step size vanishes in the old code. The RK45 numerical integration scheme used in the old code is likely the source of the distance discrepancy and a contributing factor in the others.

2.4. Cost Model

The earlier cost model has been extended to include the case where power is accepted from an external grid. Pulse energy supplied in this way does not need storage, thus lowering the beamer capex \( C_c \), now given by

\[
C_c = k_o A_1 + k_1 P_{13}^+ + k_2 Q_2. \tag{54}
\]

Beamer area \( A_1 \) and peak radiated power \( P_{13}^+ \) are dependent variables of the system model. Factors \( k_o, k_1, k_2 \) are independent user-supplied values for cost per unit area, cost per unit power, and cost per unit energy stored; they are technology figures of merit. Stored pulse energy \( Q_2 \) is modified to account for maximum available electric transmission line power \( P_{13}^+ \),

\[
Q_2 = \begin{cases} 
0 & \text{if } P_{13}^+ \leq \eta_{13}P_{13}^+ t_p \\
\frac{\eta_{13}P_{13}^+ t_p \left( 1 - \frac{1}{\eta_{13}} \right)}{\eta_{13}P_{13}^+ t_p - \eta_{12}P_{13}^+ \Delta t_g} & \text{if } \frac{P_{13}^+}{P_{13}^+} \leq \eta_{13} \leq \eta_{13}, \frac{P_{13}^+}{P_{13}^+} > \eta_{14}, \frac{P_{13}^+}{P_{13}^+} < \eta_{12} \eta_{23} \\
\frac{\eta_{13}P_{13}^+ t_p \left( 1 - \frac{1}{\eta_{13}} \right)}{\eta_{13}P_{13}^+ t_p - \eta_{12}P_{13}^+ \Delta t_g} & \text{otherwise,}
\end{cases}
\tag{55}
\]

where \( \eta_{13} \equiv \eta_{12} \eta_{23} \) and \( \eta_{14} \equiv \eta_{13} \eta_{35} \).

If there is sufficient power from the grid \( (P_{13}^+ < \eta_{13} P_{13}^+) \), then no energy storage is needed and the sailcraft is accelerated by grid power alone. Else if grid power initially heats the sail to its maximum temperature \( (P_{13}^+ > \eta_{13} P_{13}^+) \), then the power supplied by the grid is constant at its maximum \( P_{13}^+ \) and can be multiplied by the pulse duration \( t_p \) to obtain the pulse energy supplied by the grid, \( \eta_{12} P_{13}^+ t_p \). Storage is therefore required for the radiated energy \( Q_3/\eta_{23} \) minus that supplied by the grid. Otherwise, the sailcraft is initially power-limited. If this case, no stored energy is needed until time \( t_{13} \), after which any energy in excess of that supplied by the grid is instead supplied from storage. This excess energy is defined as \( \Delta Q_g \equiv Q_1 - Q_{3g} \).
### Section 2.5 Cost Minimization Procedure

As before, the system model uses nested optimizations to ensure that the sailcraft reaches its specified cruise velocity and that all system elements are sized to minimize cost. This new solution procedure, shown in fig. 4, no longer explicitly calculates the minimum beamer diameter for which cruise velocity is reachable (thus deleting former iterations 3a and a4). Instead, the traditional golden-section search logic has been modified to accept an additional boolean value (cruise velocity reached) that signals whether the function being minimized (cost) is valid for this input value. This means that if cost is evaluated for a given point design, but that point design does not turn out to reach the specified cruise velocity, then the search region shrinks accordingly, and optimization continues thereafter.

Thus far, the deliberately vague word ‘cost’ has been used.

---

The energy radiated up to time \( t_p \) minus the energy radiated up to \( t_g \). From time \( t_g \) to the pulse’s cutoff at time \( t_p \), the grid supplies its maximum \( P_1^* \), so \( P_1^*(t_p - t_g) \) is subtracted from the required energy storage. \( Q_3 \) and \( t_g \) are obtained by solving the temperature-limited equations of section 2.3.1 such that \( P_1 = P_1^* \).

Opex \( C_o \) is approximated to pulse energy cost, and pulse energy originates from the grid whether or not it is stored in an intermediate step:

\[
C_o = k_g Q_1. \tag{56}
\]

These equations imply that efficiencies \( \eta_{12} \) and \( \eta_{23} \) do not depend on whether radiated energy \( Q_3 \) is supplied directly from the grid or via intermediate storage. This approximation can be revisited in future.

**Figure 3:** 0.2c trajectory and related quantities calculated by the closed-form system model
because the minimized quantity has changed from capex to,

\[ C \equiv C_c + n_o C_o \]  

(57)

instead, where \( n_o \) is the number of sailcraft accelerations to minimize the cost for, \( C_c \) is the single-acceleration capex, and \( C_c \) is the capex. This change is made because minimizing capex alone gives too little regard to opex, which can be very high for smaller grid-dominant missions (see section 3.2). Specifically, the optimizer shrinks the beamer, reducing capex but potentially increasing opex by decreasing energy transfer efficiency. Minimizing \( C \) instead of \( C_c \) ensures that the optimizer tries to balance capex and opex.

3. System Performance Maps

System performance maps show how system-level figures of merit (and other engineering quantities) vary with cruise velocity and payload mass. Each plotted point corresponds to a point design computed by the system model. It is the intent of these maps to provide a sense of the cost, power, and physical scales involved at the earliest stage of conceptualizing new missions. With this in mind, the lightest payload in the map is 0.1 mg, enough to carry \( 10^5 \) human cells. The heaviest is 100 kt, enough to carry 225 International Space Stations. The slowest cruise velocity in the map is 0.0001 c, which is less than the record 0.0005 c reached by the Parker Solar Probe (via conventional propulsion with gravity assists). The fastest is 0.99 c, at which speed the 4.4 ly journey to Alpha Centauri, for example, is experienced as 225 d in the sailcraft’s rest frame.

Other constants used in computing the maps are summarized in table 1. These values are largely unchanged from before, so the descriptions and justifications are not repeated here. Only the former “wallplug to laser efficiency” is changed; it is decomposed into \( \eta_1 \) and \( \eta_2 \) because the cost model now requires this distinction. \( \eta_1 \) is chosen such that the “wallplug to laser efficiency” is unchanged, allowing comparisons with earlier results.

Table 1: System model constants

| 1.06 \( \mu \)m wavelength |
|---------------------------|
| 60 000 km initial sail displacement from laser source |
| 0.2 g m\(^{-2}\) areal density |
| \( 10^{-8} \) spectral normal absorptance at 1.06 \( \mu \)m |
| 70% spectral normal reflectance at 1.06 \( \mu \)m |
| 625 K maximum temperature |
| 0.01 total hemispherical emittance (2-sided, 625 K) |
| \$0.01 W\(^{-1}\) laser cost (\( k_l \)) |
| \$500 m\(^{-2}\) optics cost (\( k_o \)) |
| \$50 kWh\(^{-1}\) storage cost (\( k_s \)) |
| \$0.1 kWh\(^{-1}\) grid energy cost (\( k_g \)) |
| 100% grid to storage efficiency (\( \eta_{12} \)) |
| 50% storage to laser efficiency (\( \eta_{23} \)) |
| 70% transatmospheric propagation efficiency (\( \eta_o \)) |
| 100 operations included in cost minimization (\( n_o \)) |

3.1. Stored Energy Only

Assuming zero grid input (in addition to the inputs given in table 1), yields the map shown in fig. 4. As would be expected, the beamer’s capex (plot [1,3]) where [r,c] denotes the row and column number, with [1,1] corresponding to the top left) grows as the payload gets heavier, cruise speeds up, or any combination thereof. Maximum capex is set at \$10T because Flyvbjerg[24] estimates worldwide annual megaproject spending to be \$6T to \$9T circa 2014. Greater capex than that is colored in magenta, whereas less than \$1M minimum is colored in cyan.

In general, magenta signals greater than maximum and cyan signals less than minimum throughout the maps. Undesirable limits tend toward the color red in the plots, and desirable limits tend toward blue. Intermediate values are colored in gray and white because the two neutral tones show more detail.

Plots for which the limits are neither good nor bad use black as one bound and white as the other. In this case, intermediate values are plotted in blue because this highlights details that would otherwise be lost in gray.

The map’s dynamics can be reasoned by starting with the primary quantities obtained from iterations 1, 2, and 3b in fig. 4. Sailcraft diameter, beamer power, and beamer diameter respectively. The corresponding maps are plots [1,1], [2,1], and [3,1] in fig. 6. Sail diameter and peak radiated beam power are unconstrained, whereas beamer diameter is maximally 10 km. When the optimizer (iterations 1, 2, and 3b) reaches this limit, the limit becomes an immovable obstacle to lowering cost, and therefore a driving constraint.

When beamer diameter is maximal, the optimizer varies sailcraft diameter and peak radiated beam power instead, and in this way the solution’s dynamics qualitatively change within...
Figure 5: System performance map for a sailcraft that is accelerated by a beamer whose pulse energy is entirely stored locally to the beamer. All x-axes are \( \beta \) at cutoff (equal to cruise velocity) and are plotted on a linear scale. All y-axes are payload mass and are plotted on a logarithmic scale.
Figure 6: System performance map for a sailcraft that is accelerated by a beamer whose pulse energy is entirely stored locally to the beamer. All x-axes are $\beta$ at cutoff (equal to cruise velocity) and are plotted on a logarithmic scale. All y-axes are payload mass and are plotted on a logarithmic scale.
Figure 7: System performance map for a sailcraft that is accelerated by a beamer whose pulse energy is augmented by transmission line to a regional or national grid. All x-axes are $\beta$ at cutoff (equal to cruise velocity) and are plotted on a logarithmic scale. All y-axes are payload mass and are plotted on a logarithmic scale.
the uniform red region of plot [3,1]. For example, the beamer’s mean radiant exitance (plot [5,2]) tends to be between 1 kW m\(^{-2}\) to 1000 kW m\(^{-2}\) until maximum beamer diameter is reached, then rapidly intensifies a thousand-fold (because area is constrained). This more intense region reddens in the radiant exitance plot, signaling undesirability. Ultimately, physics limits its radiant exitance to \(10^9\) kW m\(^{-2}\) for ground-based beamers because the surrounding air breaks down into a plasma \([24]\), and to \(10^{22}\) kW m\(^{-2}\) for space-based beamers because free space transparency is limited by electron-positron pair production \([25]\).

Another example of how constraints affect map dynamics is the sailcraft temperature, which is limited to 625 K. Figure 6's bottom-left plot and the plot above it show the sailcraft temperature at the start and finish of its acceleration. For missions spanning a wide range of cruise velocities and payload masses, sailcraft temperature is never a constraint. Whereas in other regions, temperature is a constraint at acceleration’s start (plot [5,1]). When sailcraft temperature becomes a constraint, it is not directly optimized as beam diameter is, but instead determines whether the sailcraft’s trajectory is modeled by the temperature-limited equations in section 2.3.1 or the power-limited equations in section 2.3.2.

The impossibility of better than perfect energy transfer acts as a boundary condition imposed by physics itself (instead of the user). Figure 6 shows beam transmission efficiency as acceleration begins (plot [1,2]) and ends (plot [2,2]). For heavy enough payloads, efficiency is unity throughout. In regions where beam transmission efficiency has reached unity, one might wonder why the optimizer still chooses the maximum beam diameter in some cases? A smaller beamer would reduce capex without affecting the efficiency. However, capex (essentially all energy storage cost in this regime) is not the optimized quantity. The optimized quantity is given by eq. (57), so pulse energy cost rules in this regime. Doubling the beamer diameter halves the sailcraft diameter, in turn reducing sailcraft mass, in turn reducing sailcraft energy. Therefore, the optimizer has a different reason to maximize beam diameter, even in cases where the payload fraction (plot [6,3]) approaches unity.

A summary of regimes (plot [6,4]) combines the key constraints of maximum beam diameter, maximum sail temperature, and perfect beam transmission efficiency. Each regime is driven slightly differently, so the solution behaves slightly differently within it. Piecing together these various behaviors is a way to understand and verify the maps.

Beam pulse duration (measured at the beamer in plot [4,2]) varies over 1 min to \(10^7\) min (2 \(\times\) 10\(^{-6}\) yr to 20 yr). Any trajectory lasting more than a few minutes will have to contend with orbital obstacles and the Earth’s rotation relative to a sailcraft’s intended destination. Beam interruption or handoff to other beamers will require the beam to re-enter the sailcraft.

Surprisingly, system energy efficiency (plot [6,2]) climbs with sailcraft cruise velocity, tending toward 22% as missions aim for relativistic speeds. This high-efficiency region somewhat corresponds to the region in which the sailcraft is most temperature-limited (the red region in plot [6,1]).

Opex/capex ratio (plot [2,3]) is \(n_e C_e / C_c\) (not \(C_e / C_c\)) and might therefore be expected to be \(O(1)\) after cost optimization. However, the ratio is \(\sim 1/5\) for missions carrying more than 10 g of payload. For lighter missions, the ratio decreases further, approaching \(1/100\) minimum at around 0.02 c cruise velocity. Therefore, it is incorrect to assume that capex and opex (total for 100 missions) are roughly equal for cost-optimal point designs.

Energy storage (plot [3,3]) is the dominant capex for missions with heavier payloads than 10 g; laser (plot [4,3]) and optics (plot [5,3]) expenses prove negligible. For missions with lighter payloads than 10 g, storage, lasers, and optics expenses all matter, with optics being roughly half the capex except for relativistic cruise velocities. Lasers trade off against storage for the other half of the capex. Laser cost is least important for missions having around 0.5 c cruise velocity, and also for slower missions having 0.001 c cruise velocity or less.

Sailcraft carrying heavier payloads than 10 g are mostly payload mass (plot [6,3] shows sailcraft payload fraction) until their cruise velocity approaches light speed. When payload mass dominates, the sail may yet have greater extent than the payload, or it may be regarded as an optical layer that coats the payload. In the latter case, the payload would have at least the diameter required of the sail. For example, if the sailcraft were a gas-filled balloon containing a garden, then the sail might be regarded as a special outer coating on the balloon, and all inside would be regarded as the payload. This example neglects the problematic dynamics of balloons \([26]\), and it neglects provisions to deflect the transmitted fraction of the beam from the absorbing parts of the payload.

System-level performance in the relativistic limit is more easily discerned by plotting the cruise velocity on a linear scale, as done in fig. 5. The sailcraft payload fraction (plot [6,3] as before) is now seen to decline as cruise velocity increases. Payload energy cost (plot [2,4]) starts to climb into the $B/\text{kg}$ range despite (or thanks to) the region of high system energy efficiency, which on a linear scale is broad and occupies a great deal of the mission design space. Though it is hard to tell from plot [4,4], the ratio of beam duration measured on board the sailcraft relative to that measured at the beamer is a weak function of payload mass.

Previously, it had been found that it is cost optimal to oversize the beamer’s maximum radiated power, ramping up from low initial power to compensate for increasing losses as sailcraft accelerate away (eventually saturating at a maximum value). The initial to maximum radiated power ratio (plot [3,4]) shows that this is not universally the way to save money: For slower cruise velocities than 0.02 c, it is always cheapest for the beamer to radiate maximum power throughout acceleration. For cruise velocities faster than 0.2 c, it is always cheapest to oversize beamer power. And for cruise velocities between, it depends on the payload mass. The regimes summarized in plot [6,4] help to see that cost favors overpowered beamers when the sailcraft is initially temperature-limited.

### 3.2. Stored Energy Augmented by Grid

The prior section has predicted that energy storage capex dominates beamer capex for missions carrying heavier payloads than 10 g (as shown in fig. 5, plot [4,3]). To lower the energy
storage cost, power drawn from beamer storage can be augmented by a regional or national grid thus reducing the energy storage needed at the beamer site.

Grid power is limited by the transmission line’s capacity and the generating capacity available via that grid. At this time, grids in many countries routinely transport multi-gigawatt power levels, so this performance map assumes that up to 5 GW is available to the beamer for an indefinite period. Transmission line capex is not included in the system cost, nor is the electrical substation’s cost; these costs are regarded as being amortized into the $0.1 kWh−1 grid energy rate.

Grid-connected maps are shown in fig. 7 (the page after non-grid maps in fig. 6 to make comparisons easier). Grid augmentation creates a new solution branch that capitalizes on the grid connection, changing other system specifications to maximize this advantage. When the payload and speed are low enough, this grid-dominant branch supplies the lowest capex, and when they are high enough, the grid connection fades into insignificance, leaving the solution almost as it was without grid input. The maps show a discrete dividing line, on the left side of which the optimizer chooses the grid-dominant solution branch, and on the right side of which the optimizer chooses solutions that are essentially unchanged from the previous map.

The full dividing line between solution branches is most easily discernible in the opex to capex ratio (plot [2,3]). The left side of this plot shows opex exceeding capex, meaning the beamer becomes cheaper to build than to operate 100 times.

As the dividing line is approached from the grid-dominant (lower left) side, the optimizer wrings all it can from the power-limited transmission line, rapidly increasing the beamer diameter (plot [1,1]) leading to the thin gray outline in plot [6,3] as the sailcraft’s payload fraction drops. There is a wobbliness to the dividing line in some sections. Nevertheless, all plotted points are valid point designs. Also, the capex (plot [1,5]) and opex (plot [2,5]) of the grid-dominant region compared to those in fig. 7 and fig. 8 show that the optimizer is not picking noticeably more expensive solutions than the storage-only map. If the map is in error, it errs on the side of higher-cost missions.

Grid input reduces mission capex by 1-5 orders of magnitude, with 1-3 orders of magnitude being more typical. The biggest savings are for slow and heavy missions. For example, a 10 kg payload 0.001 c mission falls from $26B to $610M, as shown in table 2. This 40-fold drop occurs while the pulse energy cost and system energy efficiency remain comparable in both cases. The optimizer chooses 2.5 GW peak radiated power from the beamer (as it does over a sizable region of plot [2,1] of fig. 6) because that corresponds to the maximum 5 GW input power while shrinking the required energy storage to none. Laser cost is therefore fixed at $25M for this solution family. The sailcraft’s mass and speed fix the kinetic energy, which in turn drive the pulse duration for an effectively-fixed beam power. Because the pulse takes 15 times longer to accelerate the sailcraft, it gets farther away before cruise velocity is attained. To avoid greater beam spillage, which is expensive, the optimizer quadruples the sailcraft diameter and doubles the beamer diameter.

Grid input changes which beamer subsystems drive capex too. Referring to fig. 8 once again, storage cost (plot [3,3]) drops to zero within the grid-dominant region, leaving optics and lasers as the two major expenses. Only within the grid-

Table 2: 10 kg payload 0.001 c (6.3 au/yr) mission point designs

| No grid input | Grid input |
|---------------|------------|
| Capex         | $26B       | $610M |
| - of which lasers | $320M | $25M |
| - of which optics | $160M | $590M |
| - of which storage | $26B | $0 |
| Opex per mission | $51M | $58M |
| Sys. energy efficiency | 0.025% | 0.024% |
| Sail diameter | 18 m | 77 m |
| Beamer diameter | 0.64 km | 1.2 km |
| Peak radiated power | 32 GW | 2.5 GW |
| Pulse duration | 8.1 h | 120 h |
| Initial acceleration | 1.0 g’s | 0.08 g’s |

Table 3: 10 kg payload 0.0001 c (6.3 au/yr) mission point designs

| No grid input | Grid input |
|---------------|------------|
| Capex         | $25M       | $21M |
| - of which lasers | $9.8M | $9.9M |
| - of which optics | $13M | $11M |
| - of which storage | $2.8M | $0 |
| Opex per mission | $5.5k | $12k |
| Capex/payload | $2250/kg | $210/kg |
| Opex/payload | $55B/kg | $120B/kg |
| Sys. energy efficiency | 0.0089% | 0.0046% |
| Sail diameter | 16 cm | 17 cm |
| Beamer diameter | 180 m | 170 m |
| Peak radiated power | 980 MW | 990 MW |
| Pulse duration | 1.7 min | 3.7 min |
| Initial acceleration | 9500 g’s | 8300 g’s |

Table 4: 0.1 mg payload 0.01 c mission point designs

| No grid input | Grid input |
|---------------|------------|
| Capex         | $25M       | $21M |
| - of which lasers | $9.8M | $9.9M |
| - of which optics | $13M | $11M |
| - of which storage | $2.8M | $0 |
| Opex per mission | $5.5k | $12k |
| Capex/payload | $2250/kg | $210/kg |
| Opex/payload | $55B/kg | $120B/kg |
| Sys. energy efficiency | 0.0089% | 0.0046% |
| Sail diameter | 16 cm | 17 cm |
| Beamer diameter | 180 m | 170 m |
| Peak radiated power | 980 MW | 990 MW |
| Pulse duration | 1.7 min | 3.7 min |
| Initial acceleration | 9500 g’s | 8300 g’s |

This obvious improvement was not originally considered because it would have made negligible difference to the Centauri system mission cost.

Because the right side of the map is virtually unchanged, the linear map of fig. 8 can be used in this case as well, with all visible changes being in the leftmost few columns of pixels.
4. Conclusions

The Starshot system model has been reformulated to use closed-form trajectory equations that do not need to be numerically integrated. As a result, the code computes point designs 1–2 orders of magnitude faster. Crucially, numerically-solved variable ranges no longer need to be manually adjusted, so point designs converge over the whole design space without user intervention. Speed and autonomy have combined to enable work to progress from cost-optimized point designs to whole performance maps. These maps have revealed differing solution regimes characterized by their performance-limiting constraints. Also, the design space has been expanded to include missions having 0.1 mg to 100 kt payload, 0.0001 c to 0.99 c (6.3 au/yr to 63 000 au/yr) cruise velocity, and 10 yr to 100 yr development time.

To make it easier to compare the new results with earlier ones, assumed technology figures of merit have been those used previously. Hereafter, estimates of energy efficiency and sailcraft optical properties are being improved by a growing body of research into beamers and sailcraft. Eventually, some constants will turn out to be whole models in their own right, so it remains for future work to update assumed figures of merit and incorporate new auxiliary models into the system model.

In the system model’s computations, the rate-determining step has become estimating the total energy radiated by the beamer. The energy integral has been replaced with a series approximation by transforming the integrand into a function of sailcraft speed, then Taylor expanding the resulting expression. Twelve terms of the series are computed via algorithmic differentiation. The series is recomputed many times to traverse from the sailcraft’s final velocity back to its initial velocity without exceeding the solution’s convergence radius. The speed and accuracy of the new scheme have surpassed the RK45 numerical integrator used before.

The system model has been compared to the 2018 version by recomputing the Centauri system mission trajectory. The resulting plot is difficult to distinguish from the original by eye, reproducing the same qualitative and quantitative behaviors. Upon closer inspection, the new acceleration distance is 0.7% longer than the old, the radiated energy is 0.2% greater, and the acceleration takes 0.5% longer. Acceleration distance is now a function of cruise velocity and constants only, so the computed values are exact. Therefore, the old code’s RK45 numerical integration scheme is likely responsible for most of the remaining difference, together with other factors like the simplified efficiency formula and the 2019 redefinition of SI base units, which noticeably alters the Stefan-Boltzmann constant. Also, the new code will have a finite but smaller error in its energy and time estimates resulting from the finite number of Taylor series terms and the double-precision floating-point representation.

Performance maps have been generated for the case in which the beamer draws all power from its on-site energy storage system, and for the case in which the beam director draws up to 5 GW directly from the grid (to augment the power drawn from its on-site energy storage system). Direct grid power has introduced a new solution branch that matches beamer power to transmission line capacity by increasing sail diameter and accelerating more slowly. The performance maps show a discrete dividing line (and not a continuous transition), to the left side of which the optimizer chooses the grid-dominant branch, and to the right side of which the optimizer ignores it. The wobbliness of the dividing line for lighter payloads than 100 kg is likely a numerical artifact caused by the optimizer, a nested golden-

| Capex/payload | No grid input | Grid input |
|---------------|---------------|------------|
| $190B/kg     | $190B/kg      |            |
| Opex/payload | $370M/kg      | $370M/kg   |
| Sys. energy efficiency | 1.7% | 1.7% |
| Sail diameter | 7.4 km | 7.4 km |
| Beamer diameter | 10 km | 10 km |
| Peak radiated power | 380 PW | 380 PW |
| Pulse duration | 20 d | 20 d |
| Acceleration duration | 21 d | 21 d |
| Initial acceleration | 1.3 g’s | 1.3 g’s |
| Final acceleration | 1.1 g’s | 1.1 g’s |

The Centauri system mission accelerates at 14 900 g’s, but the acceleration map (plot [3,2]) shows that other mission’s accelerations are rarely higher (20 600 g’s at most), and often far lower. Table [4] shows the 10 kg payload being accelerated very gently at 0.08 g’s. Interestingly, the white line extending from the left of plot [3,2] diagonally up to the top represents missions with 1 g initial acceleration. The heaviest such mission has 100 kt payload. As described in table [5], this accelerates to 0.07 c over the course of 20 d (plot [4,2]) using a 7.4 km diameter sail (plot [1,1]), arriving at the Centauri system 60 yr later; within a human lifetime. At today’s energy prices, the payload energy cost (plot [2,4]) is $370M/kg. Faster travel only ever increases this. Also, raising payload mass above 1 kg barely increases $/kg. Long before this, materials R&D would improve these figures of merit by increasing sail reflectance and decreasing sail absorbance relative to the values assumed here.
section search. This type of optimizer assumes a cost function with one minimum, yet may be seeing two (one corresponding to each solution branch). Despite this, all plotted points are valid point designs that reach the target cruise velocity, and the optimizer demonstrably picks close-to-‐optimal solutions near the dividing line. In future, the optimizer can be upgraded to choose the correct minimum in the vicinity of the dividing line, and this could expand the grid-‐dominant region into the storage-‐dominant region where peak radiated beam power is \(O(100 \text{GW})\) or less, lowering costs over a wider range of missions.

Solar system through to interstellar precursor missions are relatively slow and heavy, and the beamer needed to accelerate them was disappointingly expensive when optimized by the earlier system model. Introducing direct grid power has profoundly reduced these missions’ capex; they are 5 orders of magnitude cheaper in some cases, with 1-3 orders of magnitude being more typical. For example, direct grid power shrinks the capex for a 10 kg payload 0.001 c (63 au/yr) mission from $26B to $610M. Such a mission could be a Solar system cubesat that reaches Neptune (30 au) in 6 months, and/or the Solar Gravitational Lens threshold (550 au) in less than a decade. Accelerating such a cubesat would take $58M worth of pulse energy per mission assuming $0.1 \text{kWh}^{-1}$ grid energy cost. If the mission were to produce a gravitational map using 100 such cubesats, then $6B energy cost would be in the ballpark of the required 380 PW peak radiated power (twice the terrestrial insolation) might be generated by space solar power stations that are laser accelerated for 20 d to achieve 0.07 c, reaching the Centauri system within a human lifetime. At this scale, almost all the sailcraft’s mass resides in its payload and not the sail, so the sail mist be instead regarded as an optical coating (and a diameter requirement) that is applied to the payload. While unthinkable at this time, the required 380 PW peak radiated power (twice terrestrial insolation) might be generated by space solar power or fusion within a few centuries. The affordability of this mission’s energy, raw materials, and labor, undoubtedly scale with civilization’s capacity to generate energy and to automate away touch labor. Regardless of whether human civilization reaches this point or goes another way, it is now possible to contemplate such a mission as a laser-‐accelerated sailcraft.

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