Numerical Simulation of Supercavitating Flows using a Viscous-Potential Method

Ji-Hye Kim and Byoung-Kwon Ahn
Dept. of Naval Architecture and Ocean Engineering, College of Engineering, Chungnam National University, 99 Daehakro(St), Yuseong-gu, Daejon, Korea
E-mail: bkahn@cnu.ac.kr

Abstract. A numerical method was developed to predict the supercavity around axi-symmetric bodies. Employing potential flow, the proposed method computes the cavity shape and drag force, which are the important features of practical concern for supercavitating objects. A method to calculate the frictional drag acting on the wetted body surface was implemented, which is called the viscous-potential method. The results revealed details of the drag curve appearing in the course of an increase in speed and cavity growth. In addition, the supercavity and drag features of the actual shape of the supercavitating torpedo were investigated according to the different depth conditions.

1. Introduction
When the speed of a submerged body increases and the cavity is sufficient to cover the entire body, it is called a supercavity. The cavity reduces the drag forces acting on the body and can help an underwater vehicle moves faster. This study focused on the cavitator located in front of the body, which generates a cavity and develops it until the natural supercavity covers the entire body. In the field of marine hydrodynamic applications, traditional methods to predict the extent and behavior of a cavity on the surface of a propeller blade are based mostly on the linearized lifting surface theory [1]. The surface panel method was developed to improve the accuracy near the high curvature section [3], [2]. Following that studies, present method distributed the normal dipoles and sources on the cavitator and the cavity surface to solve the supercavitating flow problem generated by various types of cavitators. The normal dipoles and sources on the cavity surface are moved to the newly computed cavity surface, where the boundary conditions are satisfied again. This iteration process is repeated until the results converge. All the results were validated by a comparison with existing analytic and empirical solutions through the previous research [4]. In addition, a viscous-potential method was introduced to compensate for the effects of viscosity.

2. Numerical Formulation
Geometric definition of cavitator and generated supercavity are shown in Figure 1 and positive x-axis direction is the direction in the wake. $S_B$ is the cavitator surface, $S_C$ is the supercavity surface and $\hat{n}$ is the unit normal vector from the surface to the outer flow. The induced velocity potential, $\phi$, which is satisfied with the governing equation in the infinite fluid domain surrounded by boundary surface is also satisfied with the Green theorem. We divided the whole body into lots of panels. And we considered the piecewise constant distribution of the normal dipoles on the whole surface and the
sources only on the cavity surface. The pressure and strength of the source distributions of the axisymmetric cavity along the cavity surface were assumed to be the same as that of the two dimensional ones.

![Geometric definition of body and supercavity](image)

**Figure 1.** Geometric definition of body and supercavity

Then the total potential, $\Phi_i$, on the $i^{th}$ panel can be expressed by

$$
\Phi_i = U_\infty \cdot \mathbf{S} + \sum_{j=0}^{N^D - 1} \frac{\mu_j}{2\pi} \int_{S_{Dj}} \frac{\partial \ln r}{\partial n} dS + \sum_{k=0}^{N^S - 1} \frac{q_k}{2\pi} \int_{S_{Ck}} \ln r dS
$$

(1)

where $N^D$, $N^S$ and $N^F$ is the number of normal dipoles, sources, and dipoles only on the cavitator. And $i, j$ are the index numbers from zero to $N^D - 1$ and $k$ is the index number from zero to $N^S - 1$. $U_\infty$ is the oncoming velocity, $\mathbf{S}$ is the position vector and $\mu_j, q_k$ is the strength of the normal dipoles and sources. To know the effect of the existence of the supercavity on the flow around it, first we assume the length of the cavity and then find the tangential velocity on the cavity surface. Cavitation number and tangential velocity are constant on the cavity surface. And with the thickness correction factor, we make new cavity shapes. On this stage, we check that the new cavity surface is satisfied with the cavity closure condition and if it is not, code return to the first stage and give the new correction cavity as an initial geometry on the first stage. After iteration process, if the cavity closure condition is satisfied, then code computes the force and pressure distribution around the cavity surface. In this paper, we did not consider the influence of the wake behind the supercavity, because the lift force can be ignored in this case as compared with the drag force acting on the cavitators. And we considered a dummy body behind the cavitator in order to investigate more practical cases. Before the cavity covers the entire body, some parts of body surface are wet with water. In the case of turbulent flow, the coefficient of the friction force is strongly affected by the surface roughness [5],

$$
C_f = \left[ 1.89 - 1.62 \log_{10} \left( \frac{e}{L} \right) \right]^{-2.5}
$$

(2)

where $e$ is the surface roughness and $L$ is the length of the plate. Eq. (2) was used to calculate the water friction force on the surface and the value was fixed to 0.0032 in this paper. Until now, the term is introduced only in the pure liquid part and the value of the coefficient is dependent on each practical cases. We validated our method by comparisons with the analytic solutions, empirical values, and the results of the simulations by using ANSYS-FLUENT.

### 3. Results and Discussion

The model is close to the real-size practical model, which is similar to the supercavitating torpedo, called Shkval, which was developed in Russia. The body length is 8.2 m and the maximum body diameter is 533 mm. The maximum speed is known to be 200 knots or greater [6]. The diameter of the cavitator, $D_C$, of the proposed analysis model is 250 mm, the maximum diameter of the body, $D_B$ is 533 mm, and the length of a dummy body is 8 m. The model is assumed to move 10 m below sea level. The water density is 998.2 kg/m$^3$ and the vapor pressure is 2,300 Pa according to the water temperature, 20° C.
Table 1. Predicted supercavity and hydrodynamic force

| V (m/s) | σ   | Potential Drag(kN) | Viscous-Potential Drag(kN) | L_s [m] | D_s [m] |
|---------|-----|---------------------|-----------------------------|---------|---------|
| 27.80   | 0.511 | 14.40               | 27.55                       | 0.4     | 0.33    |
| 39.67   | 0.251 | 24.09               | 50.11                       | 1.0     | 0.43    |
| 52.58   | 0.143 | 38.56               | 80.78                       | 2.0     | 0.54    |
| 63.11   | 0.099 | 53.39               | 107.89                      | 3.0     | 0.62    |
| 71.11   | 0.078 | 66.44               | 126.63                      | 4.0     | 0.69    |
| 78.06   | 0.065 | 79.06               | 135.24                      | 5.0     | 0.75    |
| 85.89   | 0.053 | 94.67               | 142.06                      | 6.0     | 0.80    |
| 91.68   | 0.047 | 107.19              | 138.43                      | 7.0     | 0.85    |
| 97.04   | 0.042 | 119.50              | 129.06                      | 8.0     | 0.89    |
| 106.78  | 0.035 | 143.66              | 143.66                      | 10.0    | 0.97    |
| 115.53  | 0.030 | 167.32              | 167.32                      | 12.0    | 1.05    |

The characteristic features of the supercavity developed by the cavitator according to the different speed or cavitation number can then be predicted, and more detail information is shown in Table 1. $L_s$ and $D_s$ is the predicted cavity length and maximum diameter. Figure 3 shows the drag forces according to each velocity conditions. The speed and acting forces can be determined according to different depth conditions.

Figure 3. Predicted drag force; comparison of pressure and friction drag
Figure 4 shows the speed and drag forces below each sea level conditions when the predicted cavity length, which are non-dimensionalized with the diameter of the cavitator, are 30, 40, 50, and 60.

4. Conclusions

In recent years, the practicality of supercavitation has been growing with the increased engineering applicability of the high speed underwater objects. The important features of a supercavity are the cavity shape and drag force acting on the body according to the operating conditions. In this study, a potential based numerical method was developed to predict the supercavity around various shapes of the cavitator. Based on the reliability of the two-dimensional analysis, a three-dimensional analysis was developed. The numerical results were compared with the existing analytic and empirical solutions, and showed reasonable agreement. By employing the coefficient of the friction force, a viscous-potential method was suggested and the drag forces for a certain model of a specific shape were calculated. The characteristics of drag forces appearing in the course of the cavity growth with increasing speed were determined. The supercavity of a practical model considering the influence of the depth conditions was also investigated.

Acknowledgement

This work was carried out with the support of National Research Foundation of Korea(NRF-2014M3C1A9060785).

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