Class-specific residual constraint non-negative representation for pattern classification

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Abstract. Representation based classification method (RBCM) remains one of the hottest topics in the community of pattern recognition, and the recently proposed non-negative representation based classification (NRC) achieved impressive recognition results in pattern classification. However, there is no regularization term other than the reconstruction error term in the formulation of NRC, which may result in unstable solution leading to misclassification. To overcome this drawback of NRC, in this paper, we propose a class-specific residual constraint non-negative representation (CRNR) for pattern classification. CRNR introduces a class-specific residual constraint into the formulation of NRC, which encourages more homogeneous training samples to participate in the representation of the test sample. Based on the proposed CRNR, we develop a CRNR based classifier (CRNRC) for pattern classification. Experimental results on several benchmark datasets demonstrate the superiority of CRNRC over conventional RBCM as well as the recently proposed NRC. Moreover, CRNRC works better or comparable to some state-of-the-art deep approaches on diverse challenging pattern classification tasks. The source code of our proposed CRNRC is accessible at https://github.com/yinhefeng/CRNRC.

Keywords: pattern classification, representation based classification, non-negative representation, class-specific residual constraint.

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1 Introduction

During the past few years, representation based classification method (RBCM) has received considerable attention in various classification tasks, such as face classification\textsuperscript{1} and hyperspectral image classification.\textsuperscript{2} In the domain face recognition, the popular work is the sparse representation based classification (SRC).\textsuperscript{3} SRC treats all the training samples as a dictionary, and a test sample is sparsely coded over the dictionary, then the classification is accomplished by checking which class yields the least reconstruction error. SRC can achieve excellent recognition results even when the test samples are occluded or corrupted. To further increase the robustness of SRC, Lai \textit{et al.} \textsuperscript{4} developed a discriminative and compact coding (DCC) algorithm which introduces multiple error measurements into regression model. Wen \textit{et al.} \textsuperscript{5} proposed a structured occlusion coding (SOC)
framework which simultaneously separates the occlusion and classifies the test image by coding over the occlusion dictionary with a structured sparsity constraint. Cao et al. presented a robust face recognition method by using a low-rank dictionary decomposition technique and a low-rank projection matrix learning method. Zheng et al. designed an iterative re-constrained group sparse classification (IRGSC) method in which weighted features and groups are collaboratively adopted to encode structure information and discriminative information.

Another prevailing approach of RBCM is collaborative representation based classification (CRC). Zhang et al. argued that it is the collaborative representation (CR) mechanism rather than the $\ell_1$-norm sparsity that makes SRC powerful for classification. Similarly, Xu et al. introduced a discriminative sparse representation (DSR) method for robust face recognition via $\ell_2$ regularization. Inspired by DSR, Liu et al. proposed a sparsity augmented discriminative sparse representation based classification method which augments an $\ell_2$-norm regularization discriminative sparse representation with a sparse representation. Li et al. presented a sparsity augmented weighted collaborative representation based classification (SA-WCRC) for image classification. Although Zhang et al. offered a geometric interpretation of the classification mechanism of CRC, it is still obscure to understand its intrinsic principle. Afterwards, Cai et al. analyzed the classification mechanism of CRC from a probabilistic viewpoint and proposed a probabilistic collaborative representation based classifier (ProCRC). Based on ProCRC, Gou et al. presented two-phase probabilistic collaborative representation based-classification (TPCRC) which adopts the coarse to fine strategy.

Though SRC and CRC achieve impressive recognition results in various classification tasks, they cannot avoid negative entries in their coding vectors for test samples. The negative coefficients indicate negative data correlations between the test sample and the training samples. Inspired by
the principle of non-negative matrix factorization (NMF), Xu et al. proposed a non-negative representation based classifier (NRC) which introduces a non-negative constraint on the coding vector. Extensive experiments on diverse classification tasks demonstrate the superiority of NRC over many existing RBCM, including SRC, CRC and ProCRC. Nevertheless, besides the reconstruction error term, there is no other regularization terms in the formulation of NRC, which may produce unstable solution and lead to misclassification. To tackle this problem, we incorporate a class-specific residual constraint into the formulation of NRC and propose a class-specific residual constraint non-negative representation (CRNRC) for pattern classification. Through this constraint, more homogeneous training samples are promoted to participate in the representation of the test sample.

To illustrate the mechanism of CRNRC, we carry out an experiment on the MNIST dataset. This dataset contains images for digits 0-9, and 50 images per class are selected to form the training set. The 500 images are arranged in an order of [0, 1, 2, . . . , 9], thus the training data matrix is denoted by $X = [X_0, X_1, \ldots, X_9]$, and we choose a test sample from the tenth class (i.e., digit 9). The coding vector and residual obtained by NRC are shown in Figs. 1 (a) and (b), respectively, while the coding vector and residual obtained by CRNRC are shown in Figs. 1 (c) and (d), respectively. From Fig. 1 (b), one can see that the fifth class has the least residual, i.e., the test sample is recognized as digit 4 even though the tenth class has dominant coefficients in Fig. 1 (a). From Fig. 1 (d), we can observe that the tenth class results in the minimum residual, i.e., the test sample is correctly recognized as digit 9. When we have a closer look at the coding vector of NRC and CRNRC, for the fifth class (indices 201-250), the number of nonzero entries of NRC and CRNRC are 16 and 14, respectively. While for the tenth class (indices 451-500), the number of nonzero entries of NRC and CRNRC are 15 and 19, respectively. By introducing the class-specific resid-
ual constraint into NRC, more coefficients are concentrated on the correct class, thus improved performance of our proposed CRNRC can be expected.

Our main contributions are summarized as follows,

- A class-specific residual constraint non-negative representation (CRNR) scheme is proposed.

Moreover, a CRNR based classifier (CRNRC) is designed for pattern classification.

- The resulting optimization problem of CRNR is elegantly solved under the framework of alternating direction method of multipliers (ADMM).\textsuperscript{16}

- Our proposed CRNRC outperforms conventional RBCM and works better or comparable to
some state-of-the-art deep approaches on various standard datasets.

The rest of this paper is structured as follows: Section 2 reviews the related work. Section 3 presents our CRNRC approach. Experimental evaluation on several benchmark databases is presented in Section 4. Finally, Section 5 concludes this paper.

2 Related Work

In this section, we will briefly review some related work, including SRC, CRC and NRC. To begin with, we present some notations used throughout this paper. Suppose we have \( n \) training samples from \( K \) classes, and the training data matrix is denoted by \( X = [X_1, X_2, \ldots, X_K] = [x_1, x_2, \ldots, x_n] \in \mathbb{R}^{d \times n} \), where \( X_i \) is the data matrix of the \( i \)-th class. The \( i \)-th class has \( n_i \) training samples and \( \sum_{i=1}^{K} n_i = n \) (\( i = 1, 2, \ldots, K \)), \( d \) is the dimensionality of vectorized samples.

2.1 Sparse Representation based Classification

In SRC, a test sample \( y \in \mathbb{R}^d \) is firstly expressed as a sparse linear superposition of all the training data, then the classification is done by checking which class yields the minimum reconstruction error, the objective function of SRC can be formulated as,

\[
\min_c \| y - Xc \|_2^2 + \lambda \| c \|_1
\]  (1)

where \( \lambda > 0 \) is a balancing parameter. Once we obtain the coefficient vector \( c \) of \( y \), the test sample \( y \) is classified according to the following formulation,

\[
\text{identity} (y) = \arg \min_i \| y - X_i c_i \|_2
\]  (2)
where \( c_i \) is the coefficient vector that corresponds to the \( i \)-th class.

### 2.2 Collaborative Representation based Classification

Zhang et al. \(^8\) presented collaborative representation based classification (CRC) algorithm, which replaces the \( \ell_1 \)-norm in SRC with the \( \ell_2 \)-norm constraint, the objective function of CRC is formulated as follows,

\[
\min_c \| y - Xc \|_2^2 + \lambda \| c \|_2^2 \tag{3}
\]

CRC has the following closed-form solution,

\[
c = (X^TX + \lambda I)^{-1}X^Ty \tag{4}
\]

where \( I \) is an identity matrix. Let \( P = (X^TX + \lambda I)^{-1}X^T \), we can see that \( P \) is only determined by the training data matrix \( X \). Therefore, when given all the training data, \( P \) can be pre-computed, which makes CRC very efficient. CRC employs the following regularized residual for classification,

\[
\text{identity} (y) = \arg \min_i \frac{\| y - X_i c_i \|_2}{\| c_i \|_2} \tag{5}
\]

### 2.3 Nonnegative Representation based Classification

SRC and CRC have become two representative approaches in RBCM. However, the coding vector of conventional RBCM contains negative entries. The test sample should be better expressed by homogeneous samples with non-negative representation coefficients. Moreover, Lee and Seung\(^{14}\) pointed out that it is unsuitable to approximate the test sample by allowing the training samples to cancel each other out with complex additions and subtractions. Therefore, Xu et al. \(^{15}\) proposed
the following non-negative representation model by imposing the non-negative constraint on the coding vector,

$$
\min_c \| y - Xc \|_2^2, \text{ s.t. } c \geq 0
$$

(6)

Similar to SRC, NRC employs the class specific residual to classify the test sample, i.e., identity $y = \arg \min_i \| y - X_i c_i \|_2$.

3 Class-specific Residual constraint Nonnegative Representation

In this section, first we present the model of our proposed CRNR, then we design a CRNR based classifier (CRNRC) for pattern classification.

3.1 Proposed Model

From Equation (6), we can see that apart from the reconstruction error term, there is no other regularized terms in the objective function of NRC. As demonstrated by the illustration in Section 1, due to lack of regularization, NRC would result in misclassification. To alleviate this problem, we incorporate a class-specific residual constraint into the formulation of NRC, and the objective function of our CRNR is formulated as follows,

$$
\min_c \| y - Xc \|_2^2 + \lambda \sum_{i=1}^{K} \| y - X_i c_i \|_2^2, \text{ s.t. } c \geq 0
$$

(7)

where $\lambda > 0$ is a balancing parameter. The first term in Equation (7) is the collaborative representation term and the second term is the class-specific residual constraint. One can see that when $\lambda = 0$, CRNR is degenerated to NRC. Therefore, NRC can be viewed as a special version of CRNRC.
3.2 Optimization

We adopt an alternative strategy to solve the NRCR model. By introducing an auxiliary variable $z$, Equation (7) can be rewritten as,

$$\min_{c, z} \|y - Xc\|_2^2 + \lambda \sum_{i=1}^{K} \|y - X_i c_i\|_2^2, \text{ s.t. } z = c, z \geq 0 \quad (8)$$

Equation (8) can be solved by the alternating direction method of multipliers (ADMM), and the Lagrangian function of Equation (8) is,

$$L(c, z, \delta, \mu) = \|y - Xc\|_2^2 + \lambda \sum_{i=1}^{K} \|y - X_i c_i\|_2^2 + \langle \delta, z - c \rangle + \frac{\mu}{2} \|z - c\|_2^2 \quad (9)$$

where $\delta$ is the Lagrange multiplier and $\mu > 0$ is a penalty parameter. The optimization of Equation (9) can be solved iteratively by updating $c$ and $z$ once at a time. The detailed updating procedures are presented as follows.

**Update $c$:** Fix the other variables and update $c$ by solving the following problem,

$$\min_{c} \|y - Xc\|_2^2 + \lambda \sum_{i=1}^{K} \|y - X_i c_i\|_2^2 + \frac{\mu}{2} \|z_t - c + \frac{\delta_i}{\mu}\|_2^2 \quad (10)$$

Suppose $X_i'$ is a matrix that has the same size as $X$, and $X_i'$ only consists of samples from the $i$-th class, i.e., $X_i' = [0, \ldots, X_i, \ldots, 0]$, Equation (10) can be reformulated as,

$$\min_{c} \|y - Xc\|_2^2 + \lambda \sum_{i=1}^{K} \|y - X_i' c_i\|_2^2 + \frac{\mu}{2} \|z_t - c + \frac{\delta_i}{\mu}\|_2^2 \quad (11)$$

Compute the partial derivative of Equation (11) with respect to $c$ and set it to zero, we can obtain
the following closed-form solution,

\[
c_{t+1} = (X^T X + \lambda \sum_{i=1}^{K} (X_i')^T (X_i') + \frac{\mu}{2} I)^{-1} (1 + \lambda) X^T y + \frac{\mu z_t + \delta_t}{2}
\]  

(12)

*Update z*: To update \(z\), we fix variables other than \(z\) and solve the following problem accordingly,

\[
\min_z \| z - \left( c_{t+1} - \frac{\delta_t}{\mu} \right) \|^2_2, \text{ s.t. } z \geq 0
\]  

(13)

The solution to \(z\) is given by,

\[
z_{t+1} = \max(0, c_{t+1} - \frac{\delta_t}{\mu})
\]  

(14)

where the max operator performs element by element.

*Update \(\delta\)*: The Lagrange multiplier \(\delta\) is updated according to the following formulation,

\[
\delta_{t+1} = \delta_t + \mu (z_{t+1} - c_{t+1})
\]  

(15)

The detailed procedures of solving the Equation (7) are described in Algorithm 1.

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**Algorithm 1** Solve Equation (7) via ADMM

**Input:** Test sample \(y\), training data matrix \(X\), balancing parameter \(\lambda\), \(tol > 0\), \(\mu > 0\) and the maximum iteration number \(T\).

1. Initialize \(z_0 = c_0 = \delta_0 = 0\);
2. while not converged do
3. Update \(c\) by Eq. (12);
4. Update \(z\) by Eq. (14);
5. Update \(\delta\) by Eq. (15);
6. end while

**Output:** Coding vectors \(z\) and \(c\).
Algorithm 2 Our proposed CRNRC algorithm

**Input:** Training data matrix $X = [X_1, X_2, \ldots, X_K] \in \mathbb{R}^{d \times n}$, test data $y \in \mathbb{R}^d$ and balancing parameter $\lambda$.
1: Normalize the columns of $X$ and $y$ to have unit $\ell_2$ norm;
2: Obtain the coding vector $c$ of $y$ on $X$ by solving the CRNR model in (7);
3: Compute the class-specific residuals $r_i = \|y - X_i c_i\|_2$;

**Output:** $\text{label}(y) = \arg\min_i (r_i)$

3.3 Classification

For the test sample $y \in \mathbb{R}^d$, first we obtain its coding vector $c$ over the entire training data $X$, then the test sample is designated into the class that yields the least residual, i.e., $\text{identity}(y) = \arg\min_i \|y - X_i c_i\|_2$, where $c_i$ is the coding vector that belongs to the $i$-th class. The complete process of our proposed CRNRC is summarized in Algorithm 2.

4 Experiments

In this section, we evaluate the classification performance of CRNRC on diverse benchmark datasets: two face databases including AR\textsuperscript{17} and Extended Yale B\textsuperscript{18} databases, two handwritten digit datasets including USPS\textsuperscript{19} and MNIST\textsuperscript{20} datasets, and four large-scale datasets including the Stanford 40 dataset,\textsuperscript{21} the Oxford 102 Flowers dataset,\textsuperscript{22} the Aircraft dataset\textsuperscript{23} and the Cars dataset.\textsuperscript{24} We compare the classification accuracy of CRNRC with NSC,\textsuperscript{25} linear SVM, SRC,\textsuperscript{3} CRC,\textsuperscript{8} CROC,\textsuperscript{26} ProCRC\textsuperscript{12} and NRC.\textsuperscript{15} In addition, we also compare CRNRC with the state-of-the-art (SOTA) methods on the four large-scale datasets.

4.1 Face Recognition

4.1.1 Experiments on the AR Database

The AR database\textsuperscript{17} consists of more than 4000 color images of 126 subjects (70 men and 56 women), these images have variations in facial expressions, illumination conditions and occlu-
sions, example images from this database are shown in Fig. 2. Following the experimental settings in Ref. 15, in our experiments, we use a subset with only illumination and expression changes that contains 50 male subjects and 50 female subjects from the AR database. For each individual, 7 images from Session 1 are used as training samples, and the other 7 images from Session 2 as test samples. All the images are firstly cropped to $60 \times 43$ pixels and projected to a subspace of dimensions 54, 120, and 300 by PCA. Experimental results are summarized in Table 1, the balancing parameter $\lambda$ of CRNRC under dimensions 54, 120 and 300 is set to be 0.001, 0.01 and 0.001, respectively. One can see that our proposed CRNRC achieves the highest recognition accuracy under all the three reduced dimensions.

![Fig 2 Example images from the AR database.](image)

4.1.2 Experiments on the Extended Yale B Database

The Extended Yale B database\textsuperscript{18} contains 2414 face images from 38 individuals, each having 59-64 images. These images have illumination variations, example images from this database are shown in Fig. 3. The original images are of $192 \times 168$ pixels. In our experiments, all the images are resized to $54 \times 48$ pixels. 32 images per subject are randomly selected for training and the remaining for
Table 1 Recognition accuracy (%) of competing approaches on the AR database.

| Dim. | 54  | 120 | 300 |
|------|-----|-----|-----|
| NSC  | 70.7| 75.5| 76.1|
| SVM  | 81.6| 89.3| 91.6|
| SRC  | 82.1| 88.3| 90.3|
| CRC  | 80.3| 90.1| 93.8|
| CROC | 82.0| 90.8| 93.7|
| ProCRC | 81.4| 90.7| 93.7|
| NRC  | 85.2| 91.3| 93.3|
| CRNRC| 85.7| 91.3| 94.0|

Table 2 Recognition accuracy (%) of competing approaches on the Extended Yale B database.

| Dim. | 84  | 150 | 300 |
|------|-----|-----|-----|
| NSC  | 91.2| 95.3| 96.6|
| SVM  | 93.4| 95.8| 96.9|
| SRC  | 95.5| 96.9| 97.7|
| CRC  | 95.0| 96.3| 97.8|
| CROC | 95.5| 97.1| 98.2|
| ProCRC | 93.4| 95.3| 96.2|
| NRC  | 96.7| 97.2| 98.4|
| CRNRC| 96.8| 97.3| 98.4|

testing. The resized images are projected to a subspace of dimensions 84, 150, and 300 by PCA.

Table 2 lists the classification accuracy of the competing approaches, the balancing parameter $\lambda$ of CRNRC under dimensions 84, 150 and 300 is set to be 0.001. It can be seen that CRNRC consistently outperforms its competing approaches in all cases.
Table 3 Recognition accuracy (%) of competing approaches on the USPS database.

|     | 50   | 100  | 200  | 300  |
|-----|------|------|------|------|
| NSC | 91.2 | 92.2 | 92.8 | 92.8 |
| SVM | 91.6 | 92.5 | 93.1 | 93.2 |
| SRC | 89.1 | 91.2 | 92.9 | 93.8 |
| CRC | 89.8 | 90.8 | 91.5 | 91.5 |
| CROC | 91.9 | 91.3 | 91.7 | 91.8 |
| ProCRC | 90.9 | 91.9 | 92.2 | 92.2 |
| NRC | 90.3 | 91.6 | 92.7 | 93.0 |
| CRNRC | 92.3 | 93.6 | 94.6 | 94.6 |

4.2 Handwritten Digit Classification

4.2.1 Experiments on the USPS Dataset

The USPS dataset is composed of 9298 images for digit numbers of ten classes, i.e. from 0 to 9. The training set and test set contain 7291 and 2007 images, respectively. All the images are resized into $16 \times 16$ pixels. $N$ ($N=50, 100, 200, 300$) images per class from the training set are randomly selected for training and all the images in the test set for testing. Experiments are repeated for 10 times and the average results are recorded. Experimental results are shown in Table 3, the balancing parameter $\lambda$ of CRNRC is set to be 0.1 in all cases. We can observe that CRNRC achieves the best recognition results under all scenarios. With the increasing of the number of training images, recognition accuracy of all competing approaches improves steadily.

4.2.2 Experiments on the MNIST Dataset

The MNIST dataset has 60,000 training samples and 10,000 testing samples for digit numbers from 0 to 9. The original images are of size $28 \times 28$. We randomly selected $N$ ($N=50, 100, 300, 500$) samples per class from the training set for training, and utilize all the samples in the test set for testing. Experiments are repeated for 10 times and the average results are reported. Recognition accuracy of competing approaches is presented in Table 4, the balancing parameter $\lambda$ of CRNRC...
Table 4 Recognition accuracy (%) of competing approaches on the MNIST database.

| N   | 50    | 100   | 300   | 500   |
|-----|-------|-------|-------|-------|
| NSC | 91.6  | 92.7  | 84.8  | 71.3  |
| SVM | 86.6  | 88.5  | 90.8  | 91.1  |
| SRC | 80.1  | 85.6  | 89.3  | 92.7  |
| CRC | 86.3  | 88.2  | 89.2  | 89.4  |
| CROC | 91.0 | 92.3  | 89.9  | 89.3  |
| ProCRC | 87.6 | 89.6  | 92.8  | 93.8  |
| NRC | 86.1  | 88.5  | 90.7  | 91.7  |
| CRNRC | 89.2 | 92.1  | **94.4** | **95.1** |

is set to be 0.1 in all cases. One can see that when the number of training images per class is 50 and 100, CRNRC underperforms NSC. Unfortunately, with the increase of the number of training images, the classification performance of NSC dramatically decreases. The reason behind this phenomenon lies in that NSC employs the class-specific training samples to represent the test sample, and the test sample can be well expressed by the training data when the number of training images per class is 50 and 100. Nevertheless, the training data matrix will be singular when the number of training images per class is 300 and 500, which will deteriorate the performance of NSC.

4.3 Large Scale Pattern Classification

To fully evaluate the performance of CRNRC, in this subsection we compare it with conventional RBCM and state-of-the-art approaches on four large scale datasets, i.e., Stanford 40 Actions dataset, the Oxford 102 Flowers dataset, the Aircraft dataset and the Cars dataset. First we will give a description of these datasets, then we will evaluate our proposed CRNRC and its competing approaches on these datasets.

The Stanford 40 Actions dataset is composed of 9532 images of humans performing 40 actions, such as reading book, throwing a frisbee and brushing teeth, example images from this dataset are shown in Fig. 4 (a). Each action class has 180-300 images. Following the common
training-testing split settings presented in Ref. 15, 100 images per class are randomly chosen for training and the remaining for testing.

The Oxford 102 Flowers dataset\textsuperscript{22} contains 8189 images from 102 flower classes, example images from this dataset are shown in Fig. 4 (b). The flowers chosen to be flower commonly occurring in the United Kingdom. Each class has 40-258 images, in which the images have large scale, pose and light variations.

The Aircraft dataset\textsuperscript{23} includes 10,000 images of aircraft spanning 100 aircraft models. The models appear at different scales, design structures, and appearances, making this dataset challenging for visual classification task, example images from this dataset are shown in Fig. 4 (c). We use the same experimental settings as in Ref. 15 to conduct our experiments.

The Cars dataset\textsuperscript{24} has 16,185 images of 196 classes of cars, example images from this dataset are shown in Fig. 4 (d). According to the standard split scheme, 8144 images are used as the training samples and the other 8041 images as the testing samples.

### 4.3.1 Evaluation with Deep Features

Following the experimental setting in Ref. 15, on the Stanford 40 Actions dataset and the Oxford 102 Flowers dataset, VGG-19\textsuperscript{27} is employed to extract CNN features (referred to as VGG19 features), and the final feature dimension of each image is 4096 for these two datasets. For the Aircraft and Cars datasets, a VGG-16 network\textsuperscript{27} is used to extract features and then these deep features are fed into CRNRC and its competing approaches. Recognition accuracy of competing approaches on the above four datasets are recorded in Table 5, the balancing parameter $\lambda$ of CRNRC on the Stanford 14 dataset, Oxford 102 dataset, Aircraft dataset and Cars dataset is set to be $1e^{-3}$, $1e^{-5}$, 0.1 and 0.1, respectively. As we can see, CRNRC achieves the best results on the four datasets.
Fig 4  Example images from four large scale datasets. (a) Example images from the Stanford 40 dataset. (b) Example images from the Flower 102 dataset. (c) Example images from the Aircraft dataset. (d) Example images from the Cars dataset.
Table 5 Recognition accuracy (%) of competing approaches on the four fine-grained datasets.

| Methods | Stanford 40 | Flower 102 | Aircraft | Cars  |
|---------|------------|------------|----------|------|
| Softmax | 77.2       | 87.3       | 85.6     | 88.7 |
| NSC 25  | 74.7       | 90.1       | 85.5     | 88.3 |
| SRC 3   | 78.7       | 93.2       | 86.1     | 89.2 |
| CRC 8   | 78.2       | 93.0       | 86.7     | 90.0 |
| CROC 26 | 79.2       | 93.1       | 86.9     | 90.3 |
| ProCRC 12 | 80.9     | 94.8       | 86.8     | 90.1 |
| NRC 15  | 81.1       | 95.3       | 87.3     | 90.7 |
| CRNRC   | **81.3**   | **95.3**   | **87.7** | **91.1** |

Specifically, CRNRC makes an improvement of 0.2%, 0.4% and 0.6% over NRC on the Stanford 40, Aircraft and Cars datasets, respectively.

4.3.2 Comparison with SOTA Methods

In this section, we evaluate CRNRC with SOTA methods. As mentioned earlier, on the Stanford 40 Actions and Oxford 102 Flowers datasets, VGG19 features are used in NRC and CRNRC, while on the Aircraft and Cars datasets, VGG16 features are employed by NRC and CRNRC. It should be noted that the compared CNN based approaches exploit more sophisticated network architectures or features than our utilized VGG19 features.

On the Stanford 40 Actions dataset, AlexNet network, VGG19 network, EPM, and ASPD are used for comparison. EPM and ASPD are two leading approaches on action recognition for still images. The classification accuracy is shown in Table 6. One can see that CRNRC achieves the highest classification accuracy and outperforms NRC by 0.2%.

On the Flower 102 dataset, AlexNet network, VGG19 network, GMP, OverFeat, and NAC are employed for comparison. Classification results are presented in Table 7. We can observe that CRNRC has the same classification accuracy as NRC and NAC, and CRNRC achieves performance gains of 2.2% over VGG19.
Table 6 Recognition accuracy (%) of competing approaches on the Stanford 40 actions dataset.

| Methods   | AlexNet\textsuperscript{28} | EPM\textsuperscript{29} | ASPD\textsuperscript{30} | VGG19\textsuperscript{27} | NRC\textsuperscript{15} | CRNRC |
|-----------|-----------------------------|--------------------------|---------------------------|---------------------------|-------------------------|-------|
| Accuracy  | 68.6                        | 72.3                     | 75.4                      | 77.2                      | 81.1                    | 81.3  |

Table 7 Recognition accuracy (%) of competing approaches on the Flower 102 dataset.

| Methods   | GMP\textsuperscript{31} | OverFeat\textsuperscript{32} | AlexNet\textsuperscript{28} | VGG19\textsuperscript{27} | NAC\textsuperscript{33} | NRC\textsuperscript{15} | CRNRC |
|-----------|--------------------------|--------------------------------|-----------------------------|---------------------------|-------------------------|-------------------------|-------|
| Accuracy  | 84.6                     | 86.8                           | 90.4                        | 93.1                      | 95.3                    | 95.3                    | 95.3  |

For the Aircraft and Cars datasets, VGG16 network,\textsuperscript{27} Symbiotic,\textsuperscript{34} FV-FGC,\textsuperscript{35} and B-CNN method\textsuperscript{36} are used for comparison, classification accuracy of competing approaches is summarized in Table 8. Once again, CRNRC exhibits the best recognition performance, and it makes an improvement of 0.4% and 0.4% over NRC on the Aircraft dataset and Cars dataset, respectively.

4.4 Parameter Sensitiveness Analysis

To examine how the balancing parameter $\lambda$ influences the performance of CRNRC, we conduct experiments on the AR database. Experimental setting is the same as in Section 4.1.1 and the dimension of reduced samples is 300. Fig. 5 plots the recognition accuracy with varying $\lambda$. We can see that in quite a wide range of $\lambda$, CRNRC performs stable. Specifically, when $\lambda$ increases from 0 to 0.001, the classification accuracy of CRNRC also increases steadily. When $\lambda$ increases from 0.001 to 0.01, the performance of CRNRC drops a little. Larger value of $\lambda$ means that CRNRC will emphasize more on the class-specific residual constraint, which would undermine the collaborative mechanism of all training samples in representing the test sample. Therefore, we set a relatively small value for $\lambda$ in our experiments.

Table 8 Recognition accuracy (%) of competing approaches on the Aircraft and Cars datasets.

| Datasets | VGG16\textsuperscript{27} | Symbiotic\textsuperscript{34} | FV-FGC\textsuperscript{35} | B-CNN\textsuperscript{36} | NRC\textsuperscript{15} | CRNRC |
|----------|---------------------------|--------------------------------|---------------------------|-------------------------|-------------------------|-------|
| Aircraft | 85.6                      | 72.5                           | 80.7                      | 84.1                    | 87.3                    | 87.7  |
| Cars     | 88.7                      | 78.0                           | 82.7                      | 90.6                    | 90.7                    | 91.1  |
5 Conclusions

In this paper, we presented a class-specific residual constraint non-negative representation (CRNR) for pattern classification. Through this class-specific residual constraint, more homogeneous training samples are enforced to participate in the representation of the test sample. The proposed NRCR is solved by ADMM technique, and each subproblem has closed-form solution. Based on CRNR, we develop a CRNR based classifier (CRNRC). Experimental results on face databases, handwritten digit datasets and large-scale datasets validate that our proposed CRNRC outperforms NRC and several conventional RBCM, like SRC, CRC, CROC, and ProCRC. In this paper, we did not explicitly consider the situation that both the training and test samples are contaminated due to occlusion or corruption, thus in future, we will extend CRNRC to tackle the above scenarios.
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