Module wise absorption study of pyramidal composite gamma detector

Pintu Bhattacharya¹, Ritesh Kshetri²,³*, Ankhi Samui ² and Anneswa Chakraborty¹

Abstract

We report the investigation of the module-wise absorptions of a sixteen element composite detector arranged in the shape of a four level pyramid. Using the simplifying assumptions of the isotropic scattering of gamma-rays, equal absorption probabilities of successive gamma scatterings inside the detector, and up to fourth order interactions of gamma-rays, the gamma-ray absorptions in each module of the composite detector are studied. This basic study gives us a quantitative estimate of the contribution of each module to the full energy peak efficiency during the addback mode. Predictions have been compared with that of the four element stacked detector.

Keywords

Mathematical modeling, phenomenology, gamma-ray, detector, spectroscopy.

AMS Subject Classification

60G99, 97M10, 00A71, 97K50.

1 Department of Physics, Sidho-Kanho-Birsha University, Purulia-723104, West Bengal, India.
2 Department of Physics, University of Burdwan, Bardhaman-713104, West Bengal, India.
*Corresponding author: ritesh.kshetri@gmail.com

Article History: Received 10 February 2020; Accepted 18 May 2020

©2020 MJM.

Contents

1 Introduction ........................................ 693
2 Our model ......................................... 694
3 Discussion ........................................ 697
4 Summary and Conclusion .................... 697
References ......................................... 698

1. Introduction

Modern gamma arrays play a pivotal role in nuclear spectroscopic experiments and comprise of detectors having various shapes and sizes [1–5], including the composite detectors [1, 2]. As the name suggests, the composite detector consists of several closely packed detector modules in close geometry. When a gamma-ray is incident on and interacts with one of the modules of the composite detector, it can be either absorbed or scattered outside the module. Scattered out gamma-ray (having lower energy) can interact with another module or escape the composite detector and is lost. If we could associate probability with each of these processes, then we can understand the scattering and absorption processes inside the composite detector. Based on this probabilistic understanding of gamma interaction process, we have investigated the operation of various composite gamma-ray detectors and estimated performance parameters like the addback factor, the peak-to-total ratio and the peak-to-background ratio in terms of a few probabilities [6–12]. The addback mode of operation of these composite detectors [1, 2, 6] is responsible for their increased full energy peak efficiency and lower background. Recently, we have investigated the operation of the pyramidal detectors [10], where we investigated the overall addback factor and the fold distribution of its three configurations. It is important to note that our previous modeling papers [6–9], used a generalised approach of composite gamma detector modeling, and has shown reasonable agreement with available experimental or simulation data (see section 3.3 of ref. [6], and section 4.3 of ref. [7]).

The present approach is simpler mainly relying on the isotropic scattering approximation using a single probability - the full energy peak detection efficiency in a single module [10–12]. The experimental validation of the present modeling approach has been extensively discussed in our very recent work on Clover germanium detector [11]. In the present work, we have made a detailed investigation to find the modules which have major contribution to the addback process of the four level pyramidal detector which is a sixteen element composite detector, schematically shown in figure 1(a). We have also compared our present case with that of the four element stacked detector [12] (shown in figure 1(b)), which
resembles the pyramidal detectors without the side modules.

2. Our model

The incident gamma-ray is assumed to be of pencil beam type and will be incident on the central detectors of the four level pyramidal detector as shown by red arrow in the three dimensional schematic diagram of figure 1(a). We have also considered the schematic two dimensional diagram (for the sake of modeling) in figure 1(c). Although there are a total of sixteen modules, owing to the symmetry of the problem based on the direction of incident gamma-rays, we have named the modules — A, B, C, D, E, F, G, H, I and J, such that there are two modules each naming — E, F, G, H, I and J.

As shown in figure 1(c), the incident gamma-ray could first interact with any of the modules — A, B, C, or D. We have named these four scenarios: configuration 1 — 4, respectively.

Let us denote \( A_X \) as the number of gamma-rays absorbed in module X, \( S_X \) as the number of gamma-rays scattered from module X, and \( S_X' \) as the number of gamma-rays scattered to module X, where X could be any of the modules. In our calculations, we have assumed equal absorption probabilities of successive gamma scatterings inside the detector. We have considered up to the fourth order interactions of gamma-rays for these modules having cubical shape. Let the probability of full energy peak absorption in a single detector module be \( x \), then the scattering out probability is \( (1-x) \).  

For first interaction:
\[
\begin{align*}
A_A &= Nx, \\
S_A &= N(1-x), \\
S_B &= N(1-x) (\text{this is entire flux falling on module B}),
\end{align*}
\]

For second interaction:
\[
\begin{align*}
A_B &= S_B \times x = N(1-x)x, \text{ (assuming entire flux falling on module B interacts with it)} \\
S_A &= N(1-x)(1-x), \\
S_C &= N(1-x)(1-x), \\
S_G &= N(1-x)(1-x).
\end{align*}
\]

The module-wise gamma-rays absorptions for configuration 1 is shown in table 1, where \( \eta = Nx \) and \( \delta = \frac{1}{6}(1-x) \). We have summarised the details of our calculations for configurations 2 - 4 in tables 2 - 4.

---

1During the absorption process of an incident gamma-ray in a medium, the photoelectric effect plays a crucial role [2]. The probability of absorption by photoelectric effect \( \propto E^{-3.5} \) (\( E \) being energy of gamma-ray), so a high value of \( x \) could correspond to low energy of a gamma-ray.
| Interaction | A | B | C | D | E | F | G | H | I | J | Total absorption |
|------------|---|---|---|---|---|---|---|---|---|---|------------------|
| 1st        | $\eta$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $\eta$           |
| 2nd        | 0 | $\eta \delta$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $\eta \delta$    |
| 3rd        | $\eta \delta^2$ | 0 | $\eta \delta^2$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $4 \eta \delta^2$ |
| 4th        | 0 | $4 \eta \delta^3$ | 0 | $\eta \delta^3$ | 0 | 0 | 0 | 0 | 0 | 0 | $9 \eta \delta^3$ |

| Interaction | A | B | C | D | E | F | G | H | I | J | Total absorption |
|------------|---|---|---|---|---|---|---|---|---|---|------------------|
| 1st        | 0 | $\eta$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $\eta$           |
| 2nd        | $\eta \delta$ | 0 | $\eta \delta$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $4 \eta \delta$  |
| 3rd        | 0 | $4 \eta \delta^2$ | 0 | $\eta \delta^2$ | 0 | 0 | 0 | 0 | 0 | 0 | $9 \eta \delta^2$ |
| 4th        | $4 \eta \delta^3$ | 0 | $9 \eta \delta^3$ | 0 | $6 \eta \delta^3$ | 0 | $12 \eta \delta^3$ | 0 | $4 \eta \delta^3$ | 0 | $35 \eta \delta^3$ |
### Table 3. Gamma-ray absorptions for configuration 3

| Interaction | A | B | C | D | E | F | G | H | I | J | Total absorption |
|-------------|---|---|---|---|---|---|---|---|---|---|------------------|
| 1st         | 0 | 0 | η | 0 | 0 | 0 | 0 | 0 | 0 | 0 | η                |
| 2nd         | 0 | ηδ | 0 | ηδ | 0 | 2ηδ | 0 | 0 | 0 | 0 | 4ηδ              |
| 3rd         | ηδ² | 0 | 4ηδ² | 0 | 4ηδ² | 0 | 2ηδ² | 0 | 15ηδ²           |
| 4th         | 0 | 9ηδ³ | 0 | 8ηδ³ | 0 | 18ηδ³ | 0 | 6ηδ³ | 0 | 41ηδ³           |

### Table 4. Gamma-ray absorptions for configuration 4

| Interaction | A | B | C | D | E | F | G | H | I | J | Total absorption |
|-------------|---|---|---|---|---|---|---|---|---|---|------------------|
| 1st         | 0 | 0 | 0 | η | 0 | 0 | 0 | 0 | 0 | 0 | η                |
| 2nd         | 0 | 0 | ηδ | 0 | 2ηδ | 0 | 0 | 0 | 0 | 0 | 3ηδ              |
| 3rd         | 0 | ηδ² | 0 | 3ηδ² | 0 | 4ηδ² | 0 | 2ηδ² | 0 | 10ηδ²           |
| 4th         | ηδ³ | 0 | 8ηδ³ | 0 | 12ηδ³ | 0 | 6ηδ³ | 0 | 6ηδ³ | 2ηδ³ | 35ηδ³           |
3. Discussion

Comparing the module-wise contributions from tables 1–4, we find that the modules which have maximum contribution during the various interactions are:

- Second interaction - two modules B, and C (contribution being $2 \eta \delta$)
- Third interaction - two modules B, and C (contribution being $5 \eta \delta^2$)
- Fourth interaction - module C (contribution being $17 \eta \delta^3$)

From all the tabular results, we can now estimate the total absorbed count which is the sum of absorbed counts from all interactions in all modules.

Total absorbed counts $= 4N(x + \alpha)$

where $\alpha = x \delta [3 + \frac{19}{2} \delta + 30 \delta^2]$, and $\delta = \frac{1}{6}(1 - x)$

Table 5 shows the details of total module wise absorptions in decreasing order. We observe that the modules receiving the incoming flux of gamma-rays (i.e., A, B, C, and D) contribute the most. In particular, modules C and B have almost equal contribution, which is highest among others. The lowest contribution is by module J, which is the farthest from central modules, followed by equal contributions from modules H and I. The ratio of the contribution of single module F relative to the overall contributions is given by

$$F_{abs} = \frac{\delta [1 + 4 \delta + 11 \delta^2]}{4[1 + 3 \delta + \frac{19}{2} \delta^2 + 30 \delta^3]}$$

The contributions of both modules C and F are plotted in figure 3(i) as a function of probability x. We observe the contributions from the module F increases with decreasing value of x (or increasing gamma energy), as a result of increased addback at higher energies. In case of module C, there is an approx. 5% decrease in contributions at higher energies.

Although the side detector modules (E – J) do not get the incident flux, but their presence increases the efficiency due to their overall contributions in the addback mode. This could be observed if we compare the present pyramidal detector with the four element stacked detector [12], the latter resembling the pyramidal detector where only the central detectors are present. As worked out in our recent work [12], for the four element stacked detector:

Total absorbed counts $= 4N(x + \alpha')$

where $\alpha' = x \delta [3 + \frac{5}{2} \delta + 4 \delta^2]$, and $\delta = \frac{1}{6}(1 - x)$

The ratio (R) of the absorbed counts of the four level pyramidal detector and the four element stacked detector is shown in figure 3(ii). It is observed that at higher energies (lower value of x), the addback contribution from the twelve modules of the pyramidal detector cause in increase in full energy peak efficiency by $\approx 1.4$ times compared to that of the four level stacked detector.

4. Summary and Conclusion

We present a novel approach of understanding the module

Table 5. Details of the total gamma-ray absorptions in a module (in descending order)

| Module | Total absorbed gamma-rays ($T_{abs}$) |
|--------|--------------------------------------|
| C      | $\eta [1 + 2 \delta + 5 \delta^2 + 17 \delta^3]$ |
| B      | $\eta [1 + 2 \delta + 5 \delta^2 + 13 \delta^3]$ |
| D      | $\eta [1 + \delta + 4 \delta^2 + 9 \delta^3]$ |
| A      | $\eta [1 + \delta + 2 \delta^2 + 5 \delta^3]$ |
| F      | $2 \eta \delta [1 + 4 \delta + 11 \delta^2]$ |
| G      | $2 \eta \delta [1 + 3 \delta + 9 \delta^2]$ |
| H      | $2 \eta \delta^2 (1 + 3 \delta)$ |
| I      | $2 \eta \delta^2 (1 + 5 \delta)$ |
| J      | $2 \eta \delta^3$ |
Module wise absorption study of pyramidal composite gamma detector — 698/698

Absorptions in a composite gamma detector in addback mode. Considering isotropic scattering of gamma-rays and their partial as well as complete absorption, we have obtained a quantitative estimate of the contribution of each module to the full energy peak efficiency during the addback mode. Results are obtained in terms of only one parameter — the absorption probability in a single detector module. We have compared our present case with that of the four element stacked detector.

References

[1] J. Eberth and P. Von Brentano and W. Teichert and H. G. Thomas and A.V.D. Werth and R.M. Lieder and H. Jager and H. Kammerling and D. Kutchin and K.H. Maier and M. Berst and D. Gutknecht and R. Henck, Development of a composite Ge detector for EUROBALL, Progress in Particle and Nuclear Physics, 28 (1992), 495–504.

[2] J. Eberth and J. Simpson, From Ge(Li) detectors to gamma-ray tracking arrays – 50 years of gamma spectroscopy with germanium detectors, Progress in Particle and Nuclear Physics, 60 (2008), 283–337.

[3] D. Weisshaar and A.Gade and T.Glasmacher and G.F.Grinyer and D.Bazin and P.Adrich and T.Baugher and J.M. Cook and C.Aa.Diget and S.McDaniel and A.Ratkiewicz and K.P.Siwek and K.A.Walsh, CAESAR-A high-efficiency CsI(Na) scintillator array for in-beam gamma ray spectroscopy with fast rare-isotope beams, Nuclear Instruments and Methods in Physics Research A, 624 (2010), 615–623.

[4] S. Takeuchi and T.Motobayashi and Y.Togano and M.Matsushita and N.Aoi and K.Demichi and H. Hasegawa and H.Murakami, DALI2: A NaI(Tl) detector array for measurements of gamma rays from fast nuclei, Nuclear Instruments and Methods in Physics Research A, 763(2014), 596–603.

[5] M. Doncel and B. Cederwall and S. Mart and B. Quintana and A. Gadea and E. Farnea and A. Algors, Conceptual design of a high resolution Ge array with tracking and imaging capabilities for the DESPEC (FAIR) experiment, Journal of Instrumentation, 10 (2010), P06010, 1–14.

[6] R. Kshetri, Modeling an array of encapsulated germanium detectors, Journal of Instrumentation, 7(2012) P04008, 1–20.

[7] R. Kshetri, A first principle approach for encapsulated type composite detectors, Journal of Instrumentation, 7(2012) P07006, 1–21.

[8] R. Kshetri, Modeling of clover detector in addback mode, Journal of Instrumentation, 7 (2012) P07008, 1–17.

[9] R. Kshetri and P. Bhattacharya, A novel approach for modeling the cluster detector and the SPI spectrometer, Pramana journal of physics, 83 (2014), 817–827.

[10] A. Samui and R. Kshetri, Understanding the operation of gamma-ray detectors arranged in the shape of a pyramid, Journal of Instrumentation, 14 (2019) T09008, 1–17.

[11] P. Sarkar and R. Kshetri, Single parameter modeling approach for the Clover detector, AIP Conference Proceedings (Accepted).