Electromagnetic interaction between a rising spherical particle in a conducting liquid and a localized magnetic field

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Abstract. Lorentz force velocimetry (LFV) is a non-contact electromagnetic flow measurement technique for electrically conductive liquids. It is based on measuring the flow-induced force acting on an external permanent magnet. Motivated by extending LFV to liquid metal two-phase flow measurement, in a first test we consider the free rising of a non-conductive spherical particle in a thin tube of liquid metal (GaInSn) initially at rest. Here the measured force is due to the displacement flow induced by the rising particle. In this paper, numerical results are presented for three different analytical solutions of flows around a moving sphere under a localized magnetic field. This simplification is made since the hydrodynamic flow is difficult to measure or to compute. The Lorentz forces are compared to experiments. The aim of the present work is to check if our simple numerical model can provide Lorentz forces comparable to the experiments. The results show that the peak values of the Lorentz force from the analytical velocity fields provide us an upper limit to the measurement results. In the case of viscous flow around a moving sphere we recover the typical time-scale of Lorentz force signals.

1. Introduction
Two-phase flows in an electrically conductive liquid occur in a number of metallurgical processes. For example in continuous casting of steel, argon bubbles are injected in order to prevent clogging of the submerged entry nozzle, to mix the melt in the mold, and to remove slag particles via the free surface. Hence, liquid metal two-phase flows are not only of fundamental interest but also of practical importance. Among the various methods to measure them [1], one promising candidate is Lorentz force velocimetry (LFV) [2, 3].

LFV is based on the electromagnetic induction when a static localized magnetic field is applied to a conductive liquid. The current density \( j \) is induced according to Ohm’s law for moving conductors, i.e.

\[
    j = \sigma (E + u \times B),
\]

where \( \sigma \) is the electrical conductivity of the liquid, \( E \) is the electrical field, and \( u \) is the liquid velocity, respectively. In turn, according to Ampère’s law, this current density \( j \) is connected with the so-called secondary magnetic field \( b \), i.e.
\[ j = \frac{1}{\mu_0} \nabla \times b, \]  
(2)

where \( \mu_0 \) is vacuum magnetic permeability. In general, the magnetic field \( B \) is the sum of the applied magnetic field \( B_0 \) and the secondary magnetic field \( b \). The ratio between \(|b|\) and \(|B_0|\) is roughly proportional to the magnetic Reynolds number \( Re_m \), defined as

\[ Re_m = \mu_0 \sigma u L, \]  
(3)

where \( u \) is the liquid velocity, and \( L \) is the characteristic length of the flow. In most industrial applications (including our experiment), \( Re_m \ll 1 \), so that \( B \approx B_0 \) [4]. Therefore, the Lorentz force density can be defined by the simplified expression

\[ f_L = j \times B_0. \]  
(4)

Upon integrating this Lorentz force density over the volume, we obtain the braking force induced in the moving conductor according to the relation,

\[ F_L = \int_V j \times B_0 dV. \]  
(5)

The magnitude of this force can be determined by simple scaling arguments, whereby the force is proportional to the flow velocity \( u \),

\[ F_L \propto \sigma u B_0^2 L^3. \]  
(6)

In LFV we measure the counter force to this flow-braking Lorentz force, which acts on the source of the magnetic field, i.e. the external magnet system. When the small permanent magnet is used, the measured force can be used to evaluate the local velocity in the vicinity of the magnet. Earlier work has demonstrated the capability of LFV to detect a particle rising in liquid metal at rest [5]. The aim of the present work is to check if our simple numerical model can provide Lorentz forces comparable to the experimental results for a particle rising.

This paper is structured as follows: in Section 2 the experimental setup is explained. In Section 3 we present our numerical model. In Section 4, we present the experimental and numerical results and discussions. The concluding remarks are given in Section 5.

2. Experimental setup

The diagram of the experimental setup for a sphere rising in liquid GaInSn is shown in figure 1. As described in [5], a cylindrical glass tube with an inner diameter of \( D_\text{in} = 20 \text{ mm} \) is used. The tube is filled with alloy GaInSn as working fluid. A fishing line is pulled straight through the tube by springs. The line goes through the central hole of a sphere of 6mm diameter and thus, the particle rising path is restricted by the fixed fishing wire. The particle is made of plastic and not conducting, and the effects of magnetic permeability is neglected. The particle rises freely due to buoyancy. A cubic permanent magnet (NdFeB 48 material) of 12 mm side-length is attached to the LFV sensor. The magnetization points directly into the tube. The sensor detects the vertical component of the flow-induced Lorentz forces. Its resolution is 1 \( \mu N \) with cut-off frequency 6.3 Hz. Upward forces are set to be positive in this study. Without the sphere motion, LFV detects only the gravity force of the permanent magnet. When the sphere rises through the near-magnet region, some local liquid flow and eddy currents occur due to the displacement effect of the sphere, and Lorentz forces are generated. Hence, using this setup, a difference measurement of the Lorentz forces can be made.
In the experiment, shortly after we start the LFV measurement, the spoon (see figure 1) is removed and the particle starts to rise, following the straight fishing line. When the displacement flow by the particle rising affects the region permeated by the magnetic field, the Lorentz force is generated. In LFV we measure the counter force to this flow-braking Lorentz force. After seeing that the particle reaches the top free surface, we stop LFV measurement. And then the particle is pulled down to the bottom of the tube and we wait till the liquid comes to rest again. Then we repeat this process. It is difficult to observe the particle itself or to measure the rising velocity $u_0$ of the particle. We assume that the averaged particle velocity is similar to that in [6, 7], which lead us to $u_0 = 0.22 \text{m/s}$. This velocity is used in the following experimental and numerical results. We assume the liquid velocity is in the similar range as particle velocity. Therefore, the particle Reynolds number $Re_d$ can be estimated as follow

$$Re_d = \frac{u_0 d_0 \rho}{\mu} = 3570.6,$$

where $u_0 = 0.22 \text{ m/s}$ is the mean velocity of the particle, $d_0 = 6 \text{ mm}$ is the diameter of the particle, $\rho = 6.492 \text{ g/cm}^3$ is the density of GaInSn, and $\mu = 0.0024 \text{ Pa\cdot s}$ is the dynamic viscosity of GaInSn, respectively. For the Hartmann number $Ha$ we obtain

$$Ha = B_0 d_0 \sqrt{\frac{\sigma}{\mu}} = 10.7,$$

where $B_0 = 47 \text{ mT}$ is the magnetic flux density at the center of tube, and $\sigma = 3.46 \times 10^6 \text{ S/m}$ is the conductivity of GaInSn at 20 °C. This value demonstrates that in the present experiment the Lorentz force dominates the friction effects. As expected, the magnetic Reynolds number $Re_m$ is small according to equation (3),

$$Re_m = \mu_0 \sigma u_0 d_0 = 0.0057,$$

which suggests that the secondary magnetic field can be neglected in the numerical simulation due to $Re_m \ll 1$. Finally, to describe the ratio of the Lorentz forces to the inertial forces, we
have the interaction parameter (Stuart number) $N$,

$$N = \frac{\sigma B_0^2 d_0}{\rho u_0} = 0.0321,$$

which is small in our experiments. We use the so-called "kinematic approach" [8] for our numerical simulation, although $Ha$ is not small, which may modify the flow near the boundaries. Here the velocity is prescribed by the analytical solutions of the flow around a moving sphere.

3. Numerical model

The geometry of numerical model is shown in figure 2, which is equivalent to the experiment. Here $L_1 = 180\text{mm}$ and $d_1 = D_1 = 20\text{mm}$ are the length and diameter of the cylindrical domain of the conducting liquid, respectively. The diameter of the rigid sphere is $d_2 = d_0 = 6\text{mm}$. The side-length of the cubic permanent magnet is $w = 12\text{mm}$. The axial distance between the magnet and the sphere is $L_2 \in [-90, 90]\text{mm}$, and $L_3 = 4\text{mm}$ represents the distance between the magnet surface and the liquid boundary.

![Figure 2. Schematic of the geometry in simulation (view on the $x-z$ plane).](image)

![Figure 3. Spherical coordinate.](image)

In the present case, we want to compute the electromagnetic induction of a pre-defined velocity field and a localized magnetic field. Thus the governing equations read

$$\nabla^2 \Phi = \nabla \cdot (u \times B_0),$$

$$j = \sigma [-\nabla \Phi + (u \times B_0)],$$

$$f_L = j \times B_0,$$

where $u = (u_x, u_y, u_z)$ is the velocity field in the reference system, and $\Phi$ is the electrical potential. The imposed magnetic field of the permanent magnet is calculated from the analytical solution presented by Furlani [9], which is determined upon integration of

$$B_0(x, y, z) = \frac{\mu_0 M_s}{4\pi} \sum_{k=1}^{2} (-1)^k \int_{y_1}^{y_2} \int_{x_1}^{x_2} \frac{[(x-x')x + (y-y')y + (z-z_k)z]}{[(x-x')^2 + (y-y')^2 + (z-z_k)^2]^{5/2}} dxdyd' .$$

(14)
In equation (14), $M_s$ is the surface magnetization of the magnet, $(x_1, x_2)$, $(y_1, y_2)$, $(z_1, z_2)$ are the coordinates of the magnet's corners, $x$, $y$, $z$ are the unit vector in the $x,y,z$ directions, respectively. To describe the velocity field using the analytical solutions, a spherical coordinate system is introduced and fixed on the center of the sphere (see figure 3):

$$x = r \cos \theta, \quad y = r \sin \theta \cos \phi, \quad z = r \sin \theta \sin \phi. \quad (15)$$

We use the analytical solutions of the flow generated by a sphere moving in the $x$-direction. One option to compute the velocity field is to use the analytical solution of the potential flow around a moving sphere [10]. The velocity potential $\phi'$ is

$$\phi' = -\frac{u_0 R^3 \cos \theta}{2r^2}, \quad (16)$$

where $u_0$ is the velocity of the sphere, $R$ is the sphere radius, $r$ is the radial coordinate, and $\theta$ is the polar angle between radial direction and flow direction, respectively. Then the velocity field reads

$$u_x = \frac{\partial \phi'}{\partial x}, \quad u_y = \frac{\partial \phi'}{\partial y}, \quad u_z = \frac{\partial \phi'}{\partial z} \quad (17)$$

The second and third options are to use the Stokes solution of viscous flow due to a moving sphere at low Reynolds-number [11]. In "free-slip" condition the radial and angular components of velocity are written as

$$u_r = u_0 R \cos \theta \frac{3R^3 - R^3}{2r^3}, \quad u_\theta = u_0 \sin \theta \left(-\frac{3R}{4r} - \frac{R^3}{4r^3}\right), \quad u_\phi = 0. \quad (18)$$

Under "no-slip" condition the radial and angular components of velocity are

$$u_r = u_0 \cos \theta \left(\frac{3R^3}{2r} - \frac{R^3}{2r^3}\right), \quad u_\theta = u_0 \sin \theta \left(-\frac{3R}{4r} - \frac{R^3}{4r^3}\right), \quad u_\phi = 0. \quad (19)$$

Then we implement equation (18) or (19) into our geometry as

$$u_x = u_r \cos \theta - u_\theta \sin \theta, \\
 u_y = u_r \sin \theta \cos \phi + u_\theta \cos \theta \cos \phi, \\
 u_z = u_r \sin \theta \sin \phi + u_\theta \cos \theta \sin \phi. \quad (20)$$

Finally, we may solve equations (11-13) by implementing equations (14-20), respectively, and receive the re-action force $F_L'$ on the permanent magnet as

$$F_L' = -\int_V f_L dV. \quad (21)$$

The simulations in this paper were done by the commercial code COMSOL Multiphysics. The finite Element Method (FEM) is used in order to determine the forces acting in the fluid. The solution domain is a cylinder with a sphere cut out. For this model we use the hybrid tetrahedral mesh. In the computational domain, the elements are uniformly distributed over the $x$-axis. Smaller elements are applied in the region where the gradient of velocity field is large. The influence of discretization was examined by using meshes with different numbers of elements. The mesh used in subsequent computations in section 4 is shown in figure 4, where the surface mesh of the sphere can be seen as well. The effect of mesh quality on the Lorentz forces is checked and shown in figure 5. Here we define the deviation of force amplitude $D_e$ by

$$D_e = \frac{F_x - F_{x0}}{F_{x0}}, \quad (22)$$

where $F_{x0}$ is the peak value of the force in the case of maximum number of mesh elements. The influence of mesh quality on Lorentz forces is below $10^{-4}$. 

4. Results and discussion

The LFV measurement results of three cases are shown in figure 6 by setting the force peaks to the same time (1.667s). The Lorentz force signals vary significantly when the particle moves through the test region. The LFV sensor detects the displacement flow in the near-magnet region. The time-scales of these signals agree with each other. However the peak values of them are different, varying from 54 $\mu$N to 85 $\mu$N. The reproducibility of the force measurement is not good. This may be due to the lateral displacement of the particle during rising, since the hole through the particle is of 1mm diameter and the fishing line is of 0.1mm diameter. Another reason can be unsteady turbulent wake structures downstream of the particle [7, 12]. Additionally, we observe an asymmetry of the experimental signals, which is likely to be caused by the asymmetry between the flow upstream and downstream of the sphere. The long tail of signals represents the flow in the wake.

As for the numerical simulation, the analytical solution of the magnetic flux density is validated by comparing it to the measurement results. We observe a good agreement between them, which are both shown in figure 7. At the point of $z = 0$mm, the magnetic flux $B_z = 47mT$, which leads to the interaction parameter $N = 0.0321$ in equation (10). However, $B_z = 284mT$
at the boundary of the liquid, which leads to a much higher interaction parameter near the wall. However, the velocity is smaller there.

The three different analytical solutions of the flow around a sphere, i.e. potential flow around sphere (equation 16,17), free-slip viscous flow around sphere (equation 18,20), no-slip viscous flow around sphere (equation 19,20), are implemented and shown in figure 8 on the line \((x = 0, y = 0)\).

It should be noticed that the velocity is non-zero on walls, since the distance to the wall is not very large. The axial velocities are in opposite direction in these cases. The viscous forces exert shear on the surrounding liquid and thereby cause the upward velocities on the central plane \(x = 0\), which decays with \(r^{-1}\). For potential flow there is a downward motion on the symmetry plane \(x = 0\) since forces are transmitted by pressure only. The decay \(r^{-3}\) of the velocity with the distance is also far more rapid than that of a viscous flow. We do not expect the different analytical flows to correspond closely to the actual flows in the experiment. Nevertheless, they may approximate the behavior of the real flows in some respects. E.g., the potential flow should be a reasonable solution upstream of the particle when the Reynolds number is large, and the viscous no-slip flow may provide a reasonable bound on the velocity magnitude upstream of the sphere at smaller Reynolds numbers. Downstream of the sphere the actual flow should certainly differ significantly from all three analytical solutions due to flow separation and wake formation. It may be presumed that the slowly decaying viscous velocity distributions provide an indication of the maximal force signals.

![Figure 7. The magnetic flux density \(B_z\).](image7)

![Figure 8. The velocity profile \(u_z\).](image8)

We set the permanent magnet to different locations, and we observe the development of the eddy currents distributions and the Lorentz force distributions, which are shown on the \(x - z\) symmetry plane in figure 9 and 10, respectively. The intensities of the eddy current in figure 9 increase when the magnet approaches the sphere, and they penetrate much deeper into the liquid. The intensities of the eddy current reach the maximum value, when the magnet is at the same position as the sphere. Similarly in figure 10, the Lorentz force densities increase, and penetrate deeper into the liquid as the magnet approaches the sphere.

The peak values of the Lorentz forces of the different velocity fields are shown in table 1. The Lorentz force of viscous no-slip flow is the highest one, because in this case the velocity spreads further to the wall, and the flow in the near-magnet region contributes more to the total force. The peak value of the Lorentz force of the potential flow has opposite sign to that of the
Figure 9. The simulation results of the eddy current distributions on the $x - z$ plane (magnitude of $j$ is shown).

Figure 10. The simulation results of the Lorentz force density distributions on the $x - z$ plane (magnitude of $f_L$ is shown).

Table 1. The simulation result of the Lorentz forces.

| The velocity field       | The forces on magnet ($\mu$N) |
|--------------------------|-----------------------------|
| potential flow           | -49.1                       |
| free-slip viscous flow   | 471.0                       |
| no-slip viscous flow     | 725.0                       |

viscous flow, because of the different velocity fields in figure 8. The disagreements between the peak values of the Lorentz forces in the experiments and in the simulation may be due to the presence of the wake and insufficient decay of the viscous velocity fields towards the walls. In addition, the velocity field could be affected by the Lorentz forces, although the effects should
not be very significant since $N$ is small. Nevertheless, in the upstream region the computational results provide a reasonable range for the measured Lorentz forces.

From simulation we receive total Lorentz forces that depend on the axial distance $L_2$ between the magnet and the sphere as well. In order to compare the experimental and numerical results, the time $t$ in measurement is transformed to a distance $L'$ by

$$L' = u_0(t - t_0)$$

where $t_0 = 1.667s$ is the reference time point of force peaks in figure 6. The Lorentz forces of measurements and simulations are normalized by their peak values,

$$F_{\text{norm}} = \frac{F_L}{F_{L,\text{max}}}$$

where $F_{\text{norm}}$ is the normalized Lorentz force, and $F_{L,\text{max}}$ is the peak values of the re-action force on permanent magnet in experiment or simulation. Thus they can be compared in figure 11. We observe an agreement of the time-scale between the experiment and the Stokes solution of flow around a moving sphere, both about 1s. All the forces of simulation are symmetrical because of the symmetries of $B$ field and the velocity field of the analytical solutions. However, the measurements are asymmetrical due to the fact that the flows upstream and downstream of the sphere are not the same. The large force variations downstream of the sphere may be related to the unsteady wake structures.

![Figure 11. Comparison of Lorentz force between the experiments and simulations.](image)

5. Conclusion
In this paper, we present Lorentz force measurements of a spherical particle rising in liquid metal initially at rest. Three different types of analytical solutions of flow around a moving sphere are applied, respectively, and the Lorentz forces are compared to experiments. We observe an agreement of the typical time-scale of Lorentz forces using Stokes solution of viscous flow around sphere. In view of many simplifications, the peak values of Lorentz force from analytical velocity fields provide us a maximum range for the measurement results. Accurate numerical simulations of the flow will be required to reproduce the force signals quantitatively.
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