Prediction of clear-air turbulence induced by short gravity waves

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Abstract. A new auxiliary clear-air turbulence (CAT) predictor based on a triggering mechanism for clear-air turbulence, namely short (500m-20000m) gravity waves excited by shallow convection is considered. Case studies are presented in which the predictor is compared with other widely used CAT indices. One case is based on NWP (numerical weather prediction) model and other on on real sounding. Also comparison of real life sounding results with model generated ones is performed.

1. Introduction

Clear-air turbulence (CAT) i.e. turbulence encountered mostly at higher altitudes, from about 3000m up to the tropopause, in cloudless air or inside thin cirrus clouds, creates considerable problems for air traffic. It has rather complex physics and may have various causes. Forecasting this phenomenon is of great practical importance to the aircraft crews but poses considerable challenge to meteorologists. Most of the presently used practical methods are based on empirical, statistical relations between the observed cases of CAT and some particular features of the atmospheric soundings or synoptic maps, mainly related to the presence of the vertical or horizontal shear which is often generated by the jet streams and by frontogenetical processes. The probability of CAT encounter is usually expressed by various so called CAT indices based mostly on the Richardson number and on certain parameters of the frontogentic function (e.g. (Colson & Panofsky, 1965; Dutton, 1980; Elrod & Knapp, 1992; Sharman & Wolff, 2006)). Performance of the forecasting techniques based on such indices is rather moderate. Since they do not take into account the details of the CAT triggering mechanisms, they have a tendency to overpredict CAT occurrence in typical situations and underpredict it in atypical ones. Both the available observational data and outputs of the NWP models are used to calculate indices, but most available data have insufficient resolution for the effective forecasting of CAT as a phenomenon on a relatively small scale. Looking for more detailed mechanisms of CAT triggering than turbulent energy balance underlying the concept of the Richardson number, Haman (Haman, 1962) proposed gravity waves excited by the low level cumulus convection. Such waves propagate upwards and in certain vertical temperature and wind profiles their amplitude increases with height until they begin to break and overturn. In the present paper this concept is exploited in the analysis of the atmospheric profiles obtained from direct measurements and
from the NWP model. We develop a systematic procedure of identifying conditions favourable to the development of CAT caused by such mechanism of wave amplification.

The main idea (Haman, 1962) is that the low level cumulus convection can be seen as a process periodic in space and time with the period roughly equal to the typical life time of a single cumulus cell, i.e. several minutes, and the wave-length corresponding to the typical distance between neighbouring clouds. In the stable layer above the cloud tops this process excites a spectrum of standing, harmonic waves with wave vectors in various directions. The dominating component in this spectrum can be expected to have similar period and wavelength as the underlying convective process and can be decomposed into a pair of travelling waves. The spatial structure, i.e. the dependence of the vertical displacement amplitude on height, is governed by a variant of the linear Taylor-Goldstein or Scorer equation. This is valid provided that background stratification of temperature and wind can be assumed horizontally homogeneous on the scale of the wave-length and that scale is small enough to neglect the Coriolis force. In practice this is usually the case. In order to solve the equation the amplitude of the vertical displacement at level zero (taken at the top of the convective layer) must be estimated as well as its first derivative at this level. In order to do this harmonic spatial and temporal distribution of vertical and horizontal velocities components at the ceiling of convection is assumed, for more detail see (Haman, 1962). We thus have a Cauchy problem for the Haman’s variant of the Scorer equation (Haman, 1962). We solve this numerically looking for solutions with the amplitude increasing with altitude until the non-linear terms become significant, the linearisation is no longer valid and overturning followed by turbulence can be expected.

Since the Scorer equation depends on the wind component in the direction of the wave-vector, the basic idea of the numerical experiment is to look for such directions in which the wind and temperature profiles permit solutions with the amplitude increasing indefinitely with altitude. Such solutions of the linearised equations would indicate a possible occurrence of CAT. By studying this for various realistic wind and temperature profiles taken from the NWP model we retrieve the general statistical properties of the phenomenon.

The equation under consideration is:

\[ s'' + \left( \frac{u'}{u - \epsilon} - \frac{g}{c^2} - \beta \right) s' + \left( \frac{g \beta}{(u - \epsilon)^2} - k^2 \right) s = 0 \]  

Here \( z \) denotes altitude, \( ' \) denotes \( d/dz \), \( s(z) \) is the wave amplitude (vertical profile of vertical displacement from the mean flow), \( k = |k| \) is the length of the wave vector \( \mathbf{k} \), \( u = k^{-1} \mathbf{u} \cdot \mathbf{k} \) is the wind speed component in the direction \( \mathbf{k} \), \( \omega \) is the angular frequency, \( \epsilon = \frac{\omega}{k} \) is the phase velocity of the wave characterized by \( \mathbf{k} \) and \( \omega \), \( T \) is temperature, \( g \) is gravity acceleration, \( \Gamma = (g/c_p) \) is adiabatic lapse rate, \( \beta = \Gamma + T' \) is the stability of the air, and \( c \) is the speed of sound.

2. Single real-life sounding

We present a case study in which we solve Eqn. 1 for the atmospheric sounding from the Valentia Observatory, Ireland at 1200UTC on 17 July 2010. The data were taken from the University of Wyoming public data repository. The sounding was chosen so that it indicates high probability of shallow cumulus convection. The equilibrium level for the profile was located at approximately 1700m amsl and this is the assumed ceiling of cumulus convection. The wave is expected to overturn and break when linearisation ceases to be valid. Let \( K = \left| \frac{u - \epsilon}{u} \right| (2\pi)^{-1} k |s| + |s'| \) be the ratio of nonlinear terms magnitude to the magnitude of linear terms in momentum equation. We assume that linearization is no longer valid when \( K \geq 1 \).

Since the detailed spectrum of the convection-excited waves is difficult to determine, the experiment was started under the assumption of uniform amplitude distribution among all wave-vector directions, wavelengths in the range 500-20000m and periods of 600-1800s, numerically...
represented by 1542 uniformly spaced values. For each wave 19 values of initial \( s'(z_0) \) have been tried. For each case \( z_b \) such that \( K(z_b) = 1 \) was calculated and assumed to be the ‘wave breaking level’, provided that the critical level \( z_c \) such that \( u(z_c) = \epsilon \) was not encountered below. In the latter case \( z_b = z_c \) has been assumed. For each \( z \in [1800\,\text{m}, 16500\,\text{m}] \) average density of breaking waves in 100m-thick bins (that is number of \( z_b \) values divided by 100m) was calculated. This quantity was called wave breaking density (WBD). Prominent local maxima of WBD are supposed to be the most likely levels of CAT formation. In Fig. 1 we show the comparison of WBD with vertical distributions of some other CAT prediction criteria. Inverse richardson number indicates favourable conditions when sufficiently large (in (Sharman & Wolff, 2006) it is \( 1/Ri > 2 \)) or negative; vertical wind shear indicates turbulence generation levels when large (Kelvin-Helmhotz instability) yet what large means depends on background conditions that is why it is rarely used as standalone index; Colson-Panofsky index indicates turbulence when positive and for Endlich index larger absolute values indicate higher probability.

![Figure 1](image)

**Figure 1.** (a) Wave breaking density (b) Inverse Richardson number (c) Vertical wind shear (d) Colson-Panofsky index (e) Endlich empirical index.

In the analysed case WBD shows that atmosphere can be divided in two layers. The lower layer (up to approximately 6000m) where excited waves are breaking quite densely (45% of waves achieve \( K = 1 \)) and upper where this density is smaller (only 16% of waves reach \( K = 1 \)). This distinction is not reflected by any other index. WBD also shows another significant peak at 7750m. This peak is above a thin layer of negative Richardson number \( Ri \). No other index indicates the existence of that peak. Above that height WBD is roughly in agreement with \( Ri \) but consecutive peaks show falling intensity.

### 3. NWP model based aerial forecast

Since the numerical solution of the Eqn. 1. is quite fast, the procedure like this can be repeated in a relatively short time for range of profiles generated by NWP model. The case chosen covers Germany on 13 April 2011 1200UTC. This choice is a result of accidental light to moderate CAT observation over shallow convection cumulus clouds by one of the authors during flight Warsaw - Paris over north Western Germany at approximately 1230UTC unfortunately precise location of the observation is unknown. Such choice is dictated by the fact that comparative studies with a set of CAT observations is not finished before the deadline of this paper. Background NWP model is NLR COAMPS v. 3.2 run at University of Warsaw. Data used are located on 30 vertical levels with horizontal resolution of 39km. For the analysis 6h forecast was used. The procedure described in section 2 was performed for each point in horizontal grid. Equation was solved with use of modified LSODA solver (Petzold, 1983). This experiment was also run under assumption of uniform distribution among all wave-vector directions, wavelengths in the range
500-20000m and periods of 600-1800s, numerically represented by 945 uniformly spaced values. For each wave 2 values of initial $s'(z_0)$ have been tried resulting in total of 1890 waves per grid point. Results are presented at the altitude 10500m amsl being one of typical cruising levels for commercial airliners.

![Figure 2](image1.png)

**Figure 2.** Background conditions at 10500m amsl on 13.04.2011 1200UTC according to COAMPS model (a) Wind speed in m/s (colors) and direction (arrows); (b) temperature (K)

![Figure 3](image2.png)

**Figure 3.** Satellite photograph taken on 13.04.2011 1200UTC (source: sat24.com)

Fig. 2 presents weather conditions overview (output from the model). Conditions for turbulence generation were rather favourable in south east of considered territory due to jet stream but classical methods indicate no other possible region. However there are shallow convection clouds present in the north of Germany with obviously good conditions for gravity wave propagation above.

It is clear from Fig. 4 that no turbulence is forecasted over interesting cumulus area by means of classical Ellrod indices which are essentially based on wind field deformation. Also forecasters
would rather concentrate on jet stream that is present relatively near. On the contrary WBD (Fig. 5) detects increased turbulent activity over northern parts of the cumulus fields. One can also see that though WBD agrees in many regions with increased TI (especially TI2) differences in predicted intensity (if larger WBD indeed corresponds to greater intensity) are quite large. WBD is also less susceptible to model anomaly present over northern Italy.

4. Profiles influence comparison

NWP models are never perfect in their results and this influences forecast quality based on their output (see e.g. (Behne, 2008)). That is why at this early stage in our research we must ask question whether forecast such as presented in previous section has large errors generated by model profile imperfection. One must note that the vertical profiles produced by mesoscale NWP model (especially one with horizontal resolution of 39km) represent rather averaged profile on quite large area than really pointwise accurate forecast.

Another feature making simple comparison not so obvious procedure is the fact that balloon performing sounding is carried by the wind sometimes as far as 200km from where they had
been launched. Therefore either one must control location of the sounding or pick a sounding that should remain more or less in the range of grid resolution from the start location (here 39km).

We will present comparison performed in the latter manner - that is we have chosen a sounding corresponding to the area and time of the study presented in the previous section that is performed rather in weak wind conditions. Assuming that typical ascent rate of the balloon is about 5m/s easy calculation shows that 1kt wind will carry the balloon by approximately 0,1m per each meter of ascent. This results in a ‘weak wind limitation’ (maximum wind speed that is sufficient to carry the balloon away by more than 39km in a 15km ascent) equal to $V_{\text{max}} = 26\text{kt}$. On this basis we have chosen sounding from Lindenberg made on 14 April 2011 1200UTC acquired from University of Wyoming archive. NWP data is again from COAMPS model 39km grid 6h forecast.

![Figure 6. Lindenberg sounding (solid line) comparison with COAMPS results (dashed line) (a) temperature profile and (b) WBD](image)

Although model output approximates real sounding fairly however it is considerably smoother for each feature (e.g. temperature on Fig. 6a)). This smoothing cancels out small inversions in the lower troposphere which are in turn very important for WBD immediately above. This makes WBD probably useless at altitudes up to about 9km when applied to NWP model output. However results on the important from air traffic point of view altitudes ranging from 9km to 14km are quite well reproduced thus indicating that it is worthwhile to develop WBD for NWP models. This aspect is yet under research in order to precisely determine applicability conditions.

5. Further development

Though results obtained in above work seem promising the procedure of obtaining WBD is still numerically much more expensive than any of the classical indices. Modifications have to be imposed on initial conditions as well as considered wave set. Solution procedure must be simplified and better handling of critical levels needs to be implemented. Tests of the presented method against CAT observations are under way as well as determination of accuracy of WBD.
when applied to NWP model output. Unfortunately both tasks have not been completed before the deadline of submission of this paper, and their results will be published later.

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