The Accelerating Universe: A Gravitational Explanation

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Abstract

The problem of explaining the acceleration of the expansion of the universe and the observational and theoretical difficulties associated with dark matter and dark energy are discussed. The possibility that GR does not correctly describe the large-scale structure of the universe is considered and an alternative gravity theory is proposed as a possible resolution to the problems.

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1 Introduction

The recent surprising observational discovery that the expansion of the universe is accelerating [1] has led to an increasing theoretical effort to understand this phenomenon. The attempt to interpret the data by postulating a non-zero positive cosmological constant is not satisfactory, because it is confronted by the two serious issues of why the estimates from the standard model and quantum field theory lead to preposterously large values of the cosmological constant, and the coincidence of matter and dark energy dominance today [2].

If we simply postulate a repulsive force in the universe associated with a charge density, then we might expect that this force could be responsible for generating the acceleration of the universe. However, for a homogeneous and isotropic universe the net charge density would be zero, although for a finite range force with a small mass there will exist a non-zero charge density [3]. The effect of a Maxwell-type force would be to lower or raise the total energy, leaving the form of the Friedmann equation unchanged. Thus, we would still have to invoke exotic forms of energy with an equation of state, $p = w \rho$, where $w$ is negative and violates the positive energy theorems. For a non-zero cosmological constant $w = -1$.

In addition to the dark energy problem, we are still confronted with the puzzle of dark matter. Any observational detection of a dark matter candidate has eluded us and the fits to galaxy halos using dark matter models are based on several parameters depending on the size of the galaxy being fitted. The dark matter model predictions disagree with observable properties of galaxies [4].

Another problem with the dark energy hypothesis is the serious challenge to present day particle physics and string theory from the existence of cosmological horizons, which arise in an eternally accelerating universe [5]. Several resolutions
of this problem have been proposed \[6\] but a cosmological horizon does produce a potentially serious crisis for modern particle physics and string theory.

Challenging experimental results are often the precursors of a shifting of scientific paradigms. We must now entertain the prospect that the discovery of the mechanism driving the acceleration of the universe can profoundly change our description of the universe.

Given the uneasy tension existing between observational evidence for the acceleration of the universe and the mystery of what constitutes dark matter and dark energy, we are tempted to reconsider the question of whether Einstein’s gravity theory (GR) is correct for the large scale structure of the universe. It agrees well with local solar system experimental tests and for the data obtained for observations of the binary pulsar PSR 1913+16. However, this does not preclude the possibility of a breakdown of the conventional Einstein equations for the large-scale structure of the universe. The standard GR cosmological model agrees well with the abundances of light elements from big bang nucleosynthesis (BBN), and the evolution of the spectrum of primordial density fluctuations, yielding the observed spectrum of temperature anisotropies in the cosmic microwave background (CMB). Also, the age of the universe and the power spectrum of large-scale structure agree reasonably well with the standard cosmological model. However, it could be that additional repulsive gravitational effects from an alternative gravity theory could agree with all of the results in the early universe and yet lead to significant effects in the present universe accounting for its acceleration \[7\].

When contemplating alternative gravity theories, one is impressed with the mathematical and physical robustness of GR. It is not easy to change the structure of GR without running into consistency problems. A fundamental change in the predictions of the observational data will presumably only come about from a non-trivial alteration of the mathematical and geometrical formalism that constitutes GR. From the cosmological standpoint, such theories as Jordan-Brans-Dicke \[8\] theories of gravity will not radically change the Friedmann equation in the present universe. Recent developments in brane-bulk cosmological models \[9\] have led to alterations of the Friedmann equation but only for the very early universe corresponding to high energies.

This tempts us to return to a physically non-trivial extension of GR called the nonsymmetric gravity theory (NGT). This theory was extensively studied over a period of years, and a version of the theory was discovered that was free of several possible inconsistencies such as ghost poles, tachyons, exotic asymptotic behaviour and other instabilities \[10, 11, 12, 13\]. Further research is needed to fully understand such problems as its Cauchy development and the deeper meaning of the basic gauge symmetries underlying the theory.

As we shall see in the following, NGT can describe the current data on the accelerating universe and the dark matter halos of galaxies, gravitational lensing and cluster behaviour, as well as the standard results such as BBN, the solar system tests and the binary pulsar PSR 1913+16, without invoking the need for dominant,
exotic dark matter and dark energy.

2 NGT Action and Field Equations

The nonsymmetric \( g_{\mu\nu} \) and \( \Gamma^\lambda_{\mu\nu} \) are defined by \([10, 11, 12, 13, 14]\):

\[
g_{\langle\mu\nu\rangle} = \frac{1}{2}(g_{\mu\nu} + g_{\nu\mu}), \quad g_{[\mu\nu]} = \frac{1}{2}(g_{\mu\nu} - g_{\nu\mu}),
\]

and

\[
\Gamma^\lambda_{\mu\nu} = \Gamma^\lambda_{\langle\mu\nu\rangle} + \Gamma^\lambda_{[\mu\nu]},
\]

(2)

The contravariant tensor \( g^{\mu\nu} \) is defined in terms of the equation

\[
g^{\mu\nu}g_{\sigma\nu} = g_{\nu\mu}g^{\nu\sigma} = \delta^\mu_\sigma.
\]

(3)

The Lagrangian density is given by

\[
\mathcal{L}_{\text{NGT}} = \mathcal{L} + \mathcal{L}_M,
\]

(4)

where

\[
\mathcal{L} = g^{\mu\nu}R_{\mu\nu}(W) - 2\Lambda \sqrt{-g} - \frac{1}{4} \mu^2 g^{\mu\nu} g_{[\nu\mu]}
\]

\[
- \frac{1}{6} g^{\mu\nu} W_{\mu} W_{\nu} + g^{\mu\nu} J_{[\mu} \phi_{\nu]},
\]

(5)

and \( \mathcal{L}_M \) is the matter Lagrangian density \((G = c = 1)\):

\[
\mathcal{L}_M = -8\pi g^{\mu\nu} T_{\mu\nu}.
\]

(6)

Here, \( g^{\mu\nu} = \sqrt{-g} g^{\mu\nu} \), \( g = \text{Det}(g_{\mu\nu}) \), \( \Lambda \) is the cosmological constant and \( R_{\mu\nu}(W) \) is the NGT contracted curvature tensor:

\[
R_{\mu\nu}(W) = W^\beta_{\mu\nu,\beta} - \frac{1}{2}(W^\beta_{\mu\beta,\nu} + W^\beta_{\nu\beta,\mu}) - W^\alpha_{\alpha\nu} W^\beta_{\mu\beta} + W^\beta_{\alpha\beta} W^\alpha_{\mu\nu},
\]

(7)

defined in terms of the unconstrained nonsymmetric connection

\[
W^\lambda_{\mu\nu} = \Gamma^\lambda_{\mu\nu} - \frac{2}{3} \delta^\lambda_\mu W_{\nu},
\]

(8)

where

\[
W_{\nu} = \frac{1}{2}(W^\lambda_{\mu\nu} - W^\lambda_{\nu\mu}).
\]

Eq.(8) leads to the result

\[
\Gamma_{\mu} = \Gamma^\lambda_{[\mu\lambda]} = 0.
\]
The contracted tensor \( R_{\mu\nu}(W) \) can be written as
\[
R_{\mu\nu}(W) = R_{\mu\nu}(\Gamma) + \frac{2}{3} W_{[\mu,\nu]},
\]
where
\[
R_{\mu\nu}(\Gamma) = \Gamma_{\mu
u,\beta}^{\gamma} - \frac{1}{2} \left( \Gamma_{(\mu\beta),\nu}^{\gamma} + \Gamma_{(\nu\beta),\mu}^{\gamma} \right) - \Gamma_{\alpha\nu}^{\gamma} \Gamma_{\mu\beta}^{\alpha} + \Gamma_{(\alpha\beta)}^{\gamma} \Gamma_{\mu\nu}^{\alpha}.
\]

The term in Eq.(9):
\[
g^{\mu\nu} J_{[\mu \phi_{\nu}]} ,
\]
contains the Lagrange multiplier fields \( \phi_{\mu} \) and a source vector \( J_{\mu} \).

A variation of the action
\[
S = \int d^4x \mathcal{L}_{\text{NGT}}
\]
yields the field equations in the presence of matter sources:
\[
G_{\mu\nu}(W) + \Lambda g_{\mu\nu} + S_{\mu\nu} = 8\pi(T_{\mu\nu} + K_{\mu\nu}),
\]
where
\[
G_{\mu\nu}(\Gamma) = \Gamma_{\mu\nu,\beta}^{\gamma} - \frac{1}{2} \left( \Gamma_{(\mu\beta),\nu}^{\gamma} + \Gamma_{(\nu\beta),\mu}^{\gamma} \right) - \Gamma_{\alpha\nu}^{\gamma} \Gamma_{\mu\beta}^{\alpha} + \Gamma_{(\alpha\beta)}^{\gamma} \Gamma_{\mu\nu}^{\alpha}.
\]

The contribution from the variation of \( (5) \) with respect to \( g_{\mu\nu} \) is given by
\[
K_{\mu\nu} = -\frac{1}{8\pi} \left( J_{[\mu \phi_{\nu}]} - \frac{1}{2} g_{\mu\nu} (g^{[\alpha\beta]} J_{[\alpha \phi_{\beta}]} \right).
\]

Moreover, the contribution from the variation of \( (5) \) with respect to \( \sqrt{-g} \) is given by
\[
K_{\mu\nu} = -\frac{1}{8\pi} \left[ J_{[\mu \phi_{\nu}]} - \frac{1}{2} g_{\mu\nu} (g^{[\alpha\beta]} J_{[\alpha \phi_{\beta}]} \right].
\]

The variation of \( \phi_{\mu} \) yields the constraint equations
\[
g^{[\mu\nu]} J_{\nu} = 0.
\]

We have not varied the source vector \( J_{\mu} \).

If we use (7), then (8) becomes
\[
K_{[\mu\nu]} = -\frac{1}{8\pi} J_{[\mu \phi_{\nu}]}.
\]

If we specify \( J_{\mu} \) to be \( J_{\mu} = (0,0,0,J_0) \), then (8) corresponds to the three constraint equations
\[
g^{[0]} = 0.
\]
After eliminating the Lagrange multiplier field $\phi_{\mu}$ from the field equations (10), we get
\begin{equation}
G_{\mu\nu}(W) + \Lambda g_{(\mu\nu)} + S_{(\mu\nu)} = 8\pi T_{(\mu\nu)},
\end{equation}
\begin{equation}
\epsilon^{\mu\nu\alpha\beta} J_\alpha (G_{[\mu\nu]}(W) + \Lambda g_{[\mu\nu]} + S_{[\mu\nu]}) = 8\pi \epsilon^{\mu\nu\alpha\beta} T_{[\mu\nu]},
\end{equation}
where $\epsilon^{\mu\nu\alpha\beta}$ is the Levi-Civita symbol.

The generalized Bianchi identities
\begin{equation}
[g^{\alpha\nu}G_{\rho\nu}(\Gamma) + g^{\nu\alpha}G_{\nu\rho}(\Gamma)]_\alpha + g^{\mu\nu,\rho} G_{\mu\nu} = 0,
\end{equation}
give rise to the matter response equations
\begin{equation}
g_{\mu\rho} T^{\mu\nu,\rho} + g_{\mu\nu} T^{\nu\mu,\rho} + (g_{\mu\rho,\nu} + g_{\rho\nu,\mu} - g_{\mu\nu,\rho}) T^{\mu\nu} = 0.
\end{equation}

A study of the linear approximation has proved that the present version of NGT described above does not possess any ghost poles or tachyons in the linear approximation [12]. This cures the inconsistencies discovered by Damour, Deser and McCarthy in an earlier version of NGT [15]. Moreover, the instability discovered by Clayton [16] for both massless and massive NGT in a Hamiltonian formalism, associated with three of the six possible propagating degrees of freedom in the skew symmetric sector, is eliminated from the theory. This is implemented in the NGT action by the covariant constraint equations (15).

3 Cosmological Solutions

For the case of a spherically symmetric field, the canonical form of $g_{\mu\nu}$ in NGT is given by
\begin{equation}
g_{\mu\nu} = \begin{pmatrix}
-\alpha & 0 & 0 & w \\
0 & -\beta & f\sin\theta & 0 \\
0 & -f\sin\theta & -\beta\sin^2\theta & 0 \\
-w & 0 & 0 & \gamma
\end{pmatrix},
\end{equation}
where $\alpha, \beta, \gamma$ and $w$ are functions of $r$ and $t$. We have
\[\sqrt{-g} = \sin\theta[(\alpha\gamma - w^2)(\beta^2 + f^2)]^{1/2}.\]

For a comoving coordinate system, we obtain for the velocity vector $u^{\mu}$ which satisfies the normalization condition $g_{(\mu\nu)}u^{\mu}u^{\nu} = 1$:
\[u^0 = \frac{1}{\sqrt{\gamma}}, \quad u^r = u^\theta = u^\phi = 0.\]

From Eq.(17), we get $w = 0$ and only the $g_{[23]}$ component of $g_{\mu\nu}$ is different from zero. The vector $W_\mu$ can be determined from
\begin{equation}
W_\mu = -\frac{2}{\sqrt{-g}} g_{\mu\sigma} S^{[\rho\sigma]},
\end{equation}
where \( s_{\mu\alpha}g^{(\alpha\nu)} = \delta^\nu_\mu \). For the spherically symmetric field with \( w = 0 \), it follows from (17) and (24) that \( W_\mu = 0 \).

The energy-momentum tensor for a fluid is

\[
T^{\mu\nu} = (\rho + p)u^\mu u^\nu - pg^{\mu\nu} + B^{\mu\nu},
\]

where \( B^{\mu\nu} \) is a skew symmetric source tensor.

We can prove from a Killing vector analysis that for a homogeneous and isotropic universe massless NGT requires that \( f(r, t) = 0 \) \cite{14}. It follows that for massless NGT all strictly homogeneous and isotropic solutions of NGT cosmology reduce to the Friedmann-Robertson-Walker (FRW) solutions of GR. For the case of massive NGT, it will be possible to obtain strictly homogeneous and isotropic solutions. In the following, we shall solve the field equations for a spherically symmetric inhomogeneous universe and then approximate the solution by assuming that the inhomogeneities are small. We expand the metric \( g_{(\mu\nu)} \) as

\[
g_{(\mu\nu)} = g^{HI}_{(\mu\nu)} + \delta g_{(\mu\nu)},
\]

where \( g^{HI}_{(\mu\nu)} \) denotes the homogeneous and isotropic solution of \( g_{(\mu\nu)} \) and \( \delta g_{(\mu\nu)} \) are small quantities which break the maximally symmetric solution with constant Riemannian curvature. We shall simplify our calculations by assuming that the density \( \rho \) and the pressure \( p \) only depend on the time \( t \). Moreover, we assume that the mass parameter \( \mu \approx 0 \) and we neglect any effects due to the antisymmetric source tensor \( B^{(\mu\nu)} \).

It is assumed that a solution can be found by a separation of variables

\[
\alpha(r, t) = h(r)R^2(t), \quad \beta(r, t) = r^2S^2(t).
\]

From the field equations, we get

\[
\frac{\dot{R}}{R} - \frac{\dot{S}}{S} = \frac{1}{2}Zr,
\]

where \( \dot{R} = \partial R/\partial t \) and \( Z \) is given by

\[
Z = \frac{\dot{\beta}'f^2}{\beta^3} - \frac{5\beta'\beta f^2}{2\beta^4} - \frac{\dot{\alpha}\beta'f^2}{2\alpha\beta^3} + \frac{2\dot{\beta}ff'}{\beta^2} - \frac{3f'\dot{f}}{2\beta^2} + \frac{\dot{\alpha}ff'}{2\alpha\beta^2} + \frac{2\beta'f\dot{f}}{\beta^3}.
\]

Let us assume that \( Z \approx 0 \), then from (28) we find that \( R \approx S \) and the metric line-element takes the form

\[
ds^2 = dt^2 - R^2(t)\left[h(r)dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2)\right].
\]
From the conservation laws, we get
\[ \dot{p} = \frac{1}{R^3} \frac{\partial}{\partial t} [R^3(\rho + p)]. \tag{31} \]

If we assume that \( \beta \gg f \), then the equations of motion become \(^{13, 14}\)
\[ 2b(r) + \ddot{R}(t)R(t) + 2\dot{R}^2(t) - R^2(t)W(r, t) = 4\pi R^2(t)[\rho(t) - p(t)], \tag{32} \]
\[ - \ddot{R}(t)R(t) + \frac{1}{3} R^2(t)Y(t) = \frac{4\pi}{3} R^2(t)[\rho(t) + 3p(t)], \tag{33} \]
where
\[ 2b(r) = \frac{h'(r)}{rh^2(r)}. \]

The functions \( W \) and \( Y \) are given by
\[ W = \frac{\alpha' \beta' f^2}{2 \alpha^2 \beta^3} - \frac{\beta'' f^2}{\alpha \beta^3} + \frac{\dot{\alpha} \beta f^2}{2 \alpha \beta^4} + \frac{5 \beta^2 f^2}{2 \alpha \beta^3} - \frac{\alpha f \dot{f}}{2 \alpha \beta^2} - \frac{\alpha' f' f''}{2 \alpha^2 \beta^2} - \frac{f f''}{\alpha \beta^2} - \frac{4 f f' \beta'}{\alpha \beta^3} + \frac{3 f^2}{2 \alpha \beta^2}; \tag{34} \]
\[ Y = \frac{\ddot{\beta} f^2}{\beta^3} - \frac{5 \beta^2 f^2}{2 \beta^4} - \frac{3 f^2}{2 \beta^2} + \frac{4 \beta f \ddot{f}}{\beta^3} - \frac{f \dddot{f}}{\beta^2}. \tag{35} \]

Within our approximation scheme, \( W \) and \( Y \) can be expressed in the form
\[ W = \frac{h' f^2}{h^2 r^5 R^6} - \frac{2 f^2}{h r^6 R^6} + \frac{2 \ddot{R}^2 f^2}{r^4 R^6} + \frac{10 f^2}{h r^6 R^6} - \frac{\dot{R} f \dot{f}}{r^4 R^5} - \frac{h' f f'}{2 h^2 r^4 R^6} - \frac{f f''}{h^4 R^6} - \frac{8 f f'}{h^5 R^6} + \frac{3 f^2}{2 h r^4 R^6}; \tag{36} \]
\[ Y = \frac{2(\ddot{R}^2 + R \dddot{R}) f^2}{r^4 R^6} - \frac{10 \dddot{R}^2 f^2}{r^4 R^6} - \frac{3 f^2}{2 r^4 R^4} + \frac{8 \dddot{R} f \dot{f}}{r^4 R^5} - \frac{f \dddot{f}}{r^4 R^4}. \tag{37} \]

Eliminating \( \dddot{R} \) by adding \(^{(32)} \) and \(^{(33)} \), we get
\[ \ddot{R}^2 + b = \frac{8 \pi}{3} \rho R^2 + QR^2, \tag{38} \]
where
\[ Q = \frac{1}{2} W - \frac{1}{6} Y. \tag{39} \]

From \(^{(33)} \) we obtain
\[ \ddot{R} = -\frac{4 \pi}{3} \rho(R + 3p) + \frac{1}{3} R Y. \tag{40} \]

We can write Eq.\(^{(38)} \) as
\[ H^2 + \frac{b}{R^2} = \Omega H^2, \tag{41} \]
where \( H = \dot{R}/R \),

\[ \Omega = \Omega_M + \Omega_Q, \]  

(42)

and

\[ \Omega_M = \frac{8\pi\rho}{3H^2}, \quad \Omega_Q = \frac{Q}{H^2}. \]  

(43)

If \( b = 0 \), then we get \( \Omega = 1 \) and

\[ H^2 = \frac{8\pi}{3\rho + Q}. \]  

(44)

The line element now takes the approximate form of a flat, homogeneous and isotropic FRW universe

\[ ds^2 = dt^2 - R^2(t)[dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)]. \]  

(45)

4 Accelerating Expansion of the Universe

It follows from (40) that \( \ddot{R} > 0 \) when \( Y > 4\pi(\rho + 3p) \). If we assume that there is a solution for \( Q \) and \( Y \), such that they are small and constant in the early universe, then we will retain the good agreement of GR with the BBN era with \( \rho_{\text{rad}} \propto 1/R^4 \) and \( \rho_M \propto 1/R^3 \). As the universe expands beyond the BBN era at the temperatures, \( T \sim 1 \) MeV-60 kev, then \( Q \) begins to increase and reaches a slowly varying value with \( \Omega_Q^0 \sim 0.7 \) and \( \Omega_M^0 \sim 0.3 \), where \( \Omega_M^0 \) and \( \Omega_Q^0 \) denote the present values of \( \Omega_M \) and \( \Omega_Q \), respectively. These values can fit the combined supernovae, cluster and CMB data [1].

We observe from (36) and (37) that the dependence of \( Q \) and \( Y \) as the universe expands is a function of the behaviour of \( R \) and \( f \) and their derivatives. If \( f \) grows sufficiently with \( R \) as \( t \) increases, then \( Q \) and \( Y \) can dominate the matter contribution \( \rho_M \) as the universe evolves towards the current epoch. A detailed solution of the field equations is required to determine the dynamical behaviour of \( R \), \( f \), \( Q \) and \( Y \). However, we can obtain some knowledge of the qualitative behaviour of \( Y \) and \( Q \) by assuming that \( \dot{f} \sim f' \sim 0 \) and \( h = 1 \). Then, from (36) and (37) we obtain

\[ Y \approx \frac{2f^2}{r^4R^6}(R\ddot{R} - 4\dot{R}^2), \]  

(46)

and

\[ Q \approx \frac{f^2}{3r^4R^6}\left(\frac{12}{r^2} + 7\dot{R}^2 - R\ddot{R}\right). \]  

(47)

If, as the universe expands, the behaviour of \( f \) is \( f \sim R^a \) with \( a \geq 2 \), then we can satisfy \( Y > 0 \), \( Q > 0 \), \( \ddot{R} > 0 \) and \( \dddot{R} \) will increase as we approach the present epoch corresponding to an accelerating expansion.

We can explain the evolution of Hubble expansion acceleration within NGT, without violating the positive energy conditions. Both \( \rho \) and \( p \) remain positive
throughout the evolution of the universe. There is no need for a dark energy and a
cosmological constant. Thus, we avoid having to explain the unnatural and mysteri-
ous “coincidence” of matter and dark energy domination. The \( Q \) contribution to the
expansion of the universe increases at a slow rate up to a constant value today with
\( Y > 4\pi\rho \ (p \approx 0) \), and can then decrease to zero as the universe continues to expand,
avoiding an eternally accelerating universe. During this evolution, the cosmological
constant \( \Lambda = 0 \). It is then possible to avoid the existence of a cosmological horizon
and the problems it produces for quantum field theory and string theory \[5\].

5 NGT and Dark Matter

Galaxy dynamics observations continue to pose a problem for gravitational theories
and cosmology. The data for spiral galaxies are in sharp contradiction with Newto-
nian dynamics, for virtually all spiral galaxies have rotational velocity curves which
tend towards a constant value. The standard assumption is that dark matter exists
in massive, almost spherical halos surrounding galaxies. The standard hypothesis
is that about 90% of the mass is in the form of dark matter and dark energy and
this explains the flat rotational velocity curves of galaxies. This explanation is not
economical, for it requires three or more parameters to describe different kinds of
galactic systems and no satisfactory model of galactic halos exists \[3\].

A possible explanation of the galactic rotational velocity curves problem has been
obtained in NGT \[7\]. A derivation of the motion of test particles yields the total
radial acceleration experienced by a test particle in a static spherically symmetric
gravitational field for \( r \geq 0.2 \) kpc, due to a point source (we reinsert \( G \) and \( c \)) \[19\]:

\[
a(r) = -\frac{GM}{r^2} + \frac{\lambda C c^2 \exp(-r/r_0)}{r^2} \left( 1 + \frac{r}{r_0} \right) \tag{48}
\]

where \( \lambda, C \) and \( r_0 = 1/\mu \) are constants which remain to be fixed.

We choose \( C \propto \sqrt{M} \) and fix \( \lambda \) to give

\[
a(r) = -\frac{G_\infty M}{r^2} + G_0 \sqrt{M} \frac{\exp(-r/r_0)}{r^2} \left( 1 + \frac{r}{r_0} \right) \tag{49}
\]

where \( G_\infty \) is defined to be the gravitational constant at infinity

\[
G_\infty = G_0 \left( 1 + \sqrt{\frac{M_0}{M}} \right) \tag{50}
\]

and \( G_0 \) is Newton’s gravitational constant.

These formulas were applied to explain the flatness of rotation curves of galaxies,
as well as the Tully-Fisher law \[20\], \( L \sim v^4 \), where \( v \) is the rotational velocity of a
galaxy and \( L \) is the luminosity. A derivation of \( v \) gives

\[
v^2 = \frac{3G_0 L}{r} \left\{ 1 + \sqrt{\frac{L_0}{L}} \left[ 1 - \exp(r/r_0) \left( 1 + \frac{r}{r_0} \right) \right] \right\} \tag{51}
\]
where $L_0 = 3M_0$. For distances less than $0.5 - 4$ kpc, the standard Newtonian law of gravity will apply. For $r_0 = 25$ kpc and $L_0 = 250 \times 10^{10} L_\odot$, an excellent fit to spiral galaxies was found \[17, 18\]. Moreover, a good fit to the Tully-Fisher law was also obtained.

Consider now the giant spiral galaxy M31 and our Galaxy in the local group. The center of M31 is approaching the center of the Galaxy at a velocity $\sim 119$ km/sec. The total mass of the local group should be very large, namely, the mass-to-light ratio should be $\sim 100 \left( M/L_\odot \right)$. This big ratio is normally explained using the dark matter hypothesis. The distance between M31 and the Galaxy is $\sim 700$ kpc, so the additional exponential force is vanishingly small, but what is left is the renormalized gravitational constant. Thus, the gravitational acceleration becomes

$$a(r) = -\frac{G_0 M^*}{r^2}, \quad (52)$$

where $M^* \sim 17M$ and from the observed mass-to-light ratio:

$$\frac{M^*}{L} \sim 100 \left( \frac{M}{L_\odot} \right), \quad (53)$$

we predict

$$\frac{M}{L} \sim 6 \left( \frac{M}{L_\odot} \right). \quad (54)$$

This agrees with the estimated ratio for luminous matter without using the dark matter assumption.

Gravitational lensing effects can also be accounted for in NGT. We find for the angle of deflection $\Delta \phi$, obtained in the post-Newtonian approximation

$$\Delta \phi = \frac{4G_0 \left( 1 + \sqrt{M_0/M} \right)}{c^2 R}, \quad (55)$$

where $M$ is the mass of the galaxy and $R$ is the distance between the galaxy center and the deflected ray. This prediction is close to the one obtained from the dark matter hypothesis.

If we calculate the acceleration expected in our solar system, we obtain

$$\delta a = \frac{a - a_{\text{NGT}}}{a_{\text{Newton}}} \approx \frac{1}{2} \sqrt{M_0} \left( \frac{r}{r_0} \right)^2. \quad (56)$$

For solar and terrestrial experiments, we find for $r_0 \sim 25$ kpc, $\delta a < 10^{-13}$, which is too small a deviation from Newton’s law to be detected with current experiments.

We must still account for the estimated value of $\Omega_M \sim 0.33 \pm 0.053 \ [21]$. Measurements of total baryon density give $\Omega_B h^2 = 0.020 \pm 0.001 \ [22]$. For $h \sim 0.7$, we get $\Omega_B \sim 0.04$. The usual hypothesis states that cold dark matter particles contribute $\Omega_{\text{CDM}} \sim 0.3$. We do expect that there is some dark matter in the universe in the
form of dark baryons and neutrinos with non-vanishing mass \((\leq 10\%)\). It remains to be seen whether an alternative gravity theory such as NGT can provide an explanation for the discrepancy between visible baryon matter and dark matter. At the era of structure formation, the contribution of \(\Omega_Q\) could be of order \(\Omega_Q \sim 0.3\), growing to its present day value of \(\Omega_Q^0 \sim 0.95\) so that \(\Omega^0 \sim 1\). NGT would then be required to explain the formation of galaxy structure without CDM. These are issues that require further investigation.

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