Regression analysis as an alternative method of determining the Economic Order Quantity and Reorder Point

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ABSTRACT

The goal of the paper is to show how the problem of determining the Economic Order Quantity and Reorder Point could be solved with the use of a regression model. The method proposed by the authors of the paper is an alternative to the methods in which a mathematical EOQ formula is used and also to the simulation method. The intention of the authors was to find a solution which would allow, on the one hand, to obtain results as close as possible to real parameters and on the other, could be useful for decision makers in companies.

With the use of a specially elaborated model, simulations have been conducted for different parameters (value, weight, demand of a commodity, logistics costs) which first proved the importance of the optimization of deliveries to a warehouse. Second, based on the results of these simulation, model regression models have been worked out and used for finding delivery quantities and reorder point. Both regression models are statistically significant, a slightly better value is obtained by the model created for the reorder point.

1. Introduction

Optimization of the size of a delivery to a warehouse and a level of inventories has been the subject of scientific research for many years. Since the elaboration of the basic EOQ model in 1913 (Erlenkotter, 1989), many models, which are a modification of this classic formula, have been developed. The problem is considered in the literature as an important one and EOQ models are recognized as useful tools to optimize the size of the delivery and inventory levels (Agarwal, 2014). The authors of the paper, on the basis of their own research, also think that decisions in the field of supply and inventory management can significantly affect the economic efficiency of a company, which means that research aimed on solving this problem is important also from the practical point of view. However, these decisions can have much wider effects than it would appear from many mathematically developed models. That could mean that they may not be useful for optimizing deliveries, especially in the case when transport costs have to be taken into consideration and demand for goods is variable and difficult-to-predict.

To solve complex problems in which many different factors should be considered, one should probably look for other methods. Such a method proposed by the authors of this paper is a simulation because it is very difficult to capture the complex and often non-linear relationships between process parameters and their economic efficiency in the form of a mathematical formula.

This paper is a continuation of the research conducted by the authors Milewski (2019) concerning the problem of optimization of deliveries and determining the so-called Economic Order Quantity. With the use of a specially elaborated model, simulations were conducted. The results of the simulations proved that the delivery size and the level of safety stocks significantly affect the economic performance of enterprises. The impact, however, depends on different factors – value of goods, stability of demand, costs of logistics processes.

The simulations also proved that not only the level of safety stocks but also frequencies and sizes of deliveries affect the level of logistics customer service measured by the accessibility of stocks. The probability of a shortage of stock of a given item is reduced if the delivery volume is larger. That means that a given level of customer service can be achieved with an even lower level of safety stocks than would result from the Reorder Point model. The conclusion that follows is that both these problems should not be treated separately because the level of delivery size affects not only the costs of a delivery (placing an order and transportation costs) and inventory and warehousing costs but also the costs of the so-called lost sales.

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Taking into consideration such complex relations, there is doubt whether the simplified models that use aggregated data can be used for solving such complex problems. However, to perform the simulations, especially to solve the problem presented here, a model must be built. In addition, the user of such a model (e.g. an employee of the procurement department in a given company) would have to carry out simulations, which requires more effort compared to a method using e.g. the EOQ model. So, there is a need for a method, which would allow simplifying the decision-making process.

The method proposed by the author for finding the optimal batch size is to use a regression model. Such a model would be a simple decision-making tool that could be used by practitioners in companies responsible for inventory management and delivery of goods from suppliers. Such a method is quite an innovative approach to the problem of “Economic Order Quantity” and, in the author’s opinion, can make a significant contribution to theory of inventory management. The method presented in this paper stands as the alternative for the existing operational research methods used to solve this type of optimization problems.

2. Literature review and problem background

The main problem associated with the implementation of not only the Economic Order Quantity formula (EOQ) but, in general, the optimization methods is a large number of factors, which should be considered, what make it difficult to use a mathematical formula. Because of this, for years enormous amount of research has been carried out to transform the classic EOQ formula to incorporate these complex relations.

As some authors state the vast majority of inventory models are excessively complicated, conceptual or distorted and consequently difficult in comprehension and applying them in practice. They should also take into account specificity of branch of economy or a given company (Kevin and Nwosu, 2022). The literature also recognizes the problem of determining the correct criterion for optimizing the size of a batch of deliveries to the warehouse and the level of inventory, which may not be the minimum cost but the maximum profits (Swain et al. 2018; Samal et al., 2022). Conducting research in this field is of a great importance also from the practical point of view, because inventory management has a significant influence on financial performance of companies (Anantadjava et al., 2021). As an alternative to this criterion, there may be total costs including lost sales costs or opportunity costs (Juhi et al., 2022). This approach is used by the authors of this article.

There is a difference between an Order Quantity and Quantity or a Size of a Delivery because a supplier can for example send fewer goods than ordered. The size of a delivery depends also on the availability of the warehouse space (Dordevic et al., 2017). So, it is the size of a delivery which affects, in fact, the performance of deliveries and this term will be used in the paper, especially that the model elaborated by authors include the costs of transport.

In optimizing the level of inventories, one should also take into account the specificity of the industry and stored goods that have different susceptibility to storage - such as agro products (Samal et al., 2022; Swain et al. 2018). This specificity is very clearly visible in the case of spare parts inventory management in e.g. car repair shops or in aviation industry (Al-Momani et al., 2020). Special models are used to manage such stocks like IMS model (Roslin et al., 2015). The specificity of the problem of optimizing inventory management is also expressed in a relatively larger number of factors that should be taken into account. For example, according to the results of research carried out by Al-Dulaimi and Emir (2019) revealed that the factors influencing inventory management of laptop spare parts are setup cost, holding cost, carrying cost, selling prices of laptop spare parts and reorder point besides transport cost incurred during the maintenance or delivery duration.

One of the important problems is non-linearity between the size of a delivery and the costs linked with it (Maitiy et al., 2021). For example, Rana and Eyob (2006) incorporated learning curves in setup costs in the EOQ formula. Because the learning effect reduces the average setup cost, the optimal size of a delivery becomes lower than those in the classical models.

Nonlinearity concerns also other parameters, what may require new methods (. For example, Taleizadeh et al. (2013) took into consideration the quantity discount and prepayment and proposed the Bees Colony Optimization (BCO) algorithm to solve this problem. The problem of quantity discount and also imperfect quality has been incorporated in the model proposed by Lin (2010) (see also: Chang (2011), Wahab and Jaber (2010), Salameh and Jaber (2000), Maddah and Jaber (2008), Jaber et al. (2008) and Cárdenas-Barrón (2012)).

Inventory costs themselves are not always constant and this problem (taking into account the uncertainty of the competitive market) has been taken into account in a two-warehouse EOQ model with interval-valued inventory cost (Shaikh et al., 2019). This problem, combined with the problem of volatility and difficulties in forecasting demand, is also the subject of research, an example of which may be the model developed by Cárdenas-Barrón et al. (2020) for the case of retail sales. Non-linearity refers also to the transport costs. Of course, transport costs can be used in a model in the same way as ordering costs, that is as a fixed amount of costs, what means that when the size of a delivery changes, a vehicle can be underutilized (Less Than A Vehicle Load). In practice, although such a situation can take place, a more common practice is to use a vehicle the capacity of which is adjusted to the size of a delivery as much as possible. 100 % is not always possible, so for some ranges of delivery sizes, transport costs will be in fact fixed. This, however, is another argument, why difficult it is to find an optimal size of a delivery.

Models that include transport costs, although not too numerous, have been described in some scientific publications (Birbil et al., 2015; Carter and Ferrin, 1996; Lee, 1986; Wasiaj, 2016). Moreover, Mendoza and Ventura (2008) tried even to incorporate to the models with transportation costs mentioned above problem of quantity discounts.

This is a particularly difficult factor to include in optimization models due to the above-mentioned changes in transport costs. These costs may change not only due to a change in the load capacity of a given means of transport in a given mode of transport, but also a change in the mode of transport – for example, the transition from road to rail transport, if larger batches of cargo are shipped. This is a particularly difficult factor to include in optimization models due to the above-mentioned changes in transport costs. These costs may change not only due to a change in the load capacity of a given means of transport in a given mode of transport, but also a change in the mode of transport - for example, the transition from road to rail transport, if larger batches of cargo are shipped. Such a change will have consequences not only for transport costs but also for the quality of services (delivery time, punctuality), which is an additional factor of the stock level.

Therefore, another factor appears here, namely the impact of the size of the delivery batch on the level of logistic customer service. So next problem, which according to the authors' opinion should be incorporated in models are the costs of lost sales (opportunity cost). This problem also has been addressed in the scientific literature (Singh et al., 2020). For example, San-Jos et al. (2009, 2015) have been working with this problem for years. They proposed solutions in which shortages are allowed and, during the stock out period, only a fraction of demand is partially backordered. What is also interesting and important, the authors take into consideration the structure of costs and the division on the fixed and variable costs, what refers to the backordering cost but also to the holding cost. Authors underline that "in some real-world situations, the unit cost of a backorder may not be linear".

"Taking into account the number of factors that should be taken into account in inventory optimization, the use of simulation models is proposed as an alternative to optimization (Febriani and Komarudin, 2022; Saeed Osman, 2022; Kopp et al., 2022)."

"The problem of the impact of decisions made in the area of logistics on the level of logistic customer service and transport costs is also often presented in the publications of one of the authors of this article (Milewski, 2014, 2019), who proposes that this factor should be taken into"
account in the models of logistics processes. However, this inclusion increases the complexity of the problem and makes it even more difficult to use mathematical formulas to find optimal parameters of logistic processes.

However, even if individual (partial) problems have been included in the literature, there is still a need to include them all in one model.

3. Simulation as a tool for optimization

The literature on the problem of inventory optimization is abundant and many variants of the classic EOQ model have already been developed. At the same time, both the model itself and the possibility of using the optimization method with the use of a mathematical formula are criticized, because as some authors state they are based on unrealistic assumptions. A simulation method is proposed as an alternative (Davoli and Melloni, 2012). Obtained, thanks to the simulation methods may differ from the ones obtained from the classic EOQ (Dellino et al., 2010; Shadkam and Bijari, 2017). The simulation method allows to take into account the variability of demand and lead time in calculating the optimal order quantity (Mulya, 2020).

Over the recent years, in literature, there have been a lot of papers approaching the optimization problem using simulation. It is used wherever constructing mathematical or physical models is very difficult or wherever it is impossible to accurately analyze complex processes. The use of analytical methods is generally impractical because mathematical models for realistic cases are usually too complex to be solved. Obviously, the physical experimentation suffers from technical- and cost-related limitations. In fact, a modelling and simulation approach is the only practical recourse for exploring performance of the large-scale situations that exist in reality (Petrovic et al., 1998, Terzi and Cavalieri, 2004).

Unfortunately, unexplained randomness is a common and unavoidable characteristic among real-world systems, e.g. uncertainty of demand. The simulation modelling as an evaluative tool for stochastic systems has facilitated the ability to obtain performance measure estimates under any given system configuration (Scott and Harmonosky, 2005).

Modeling and simulation can better reflect real processes and their efficiency and allow finding better solutions than optimization methods. However, their use may be difficult for decision-makers regarding these processes in enterprises. Therefore, there is a need to develop a tool that, on the one hand, will be easy to use and, on the other hand, will allow to obtain results as close to the real ones as possible. Such a tool can be a regression model that will be developed on the basis of the parameters of real processes.

Regression models are also used in the inventory optimization process, but rather for forecasting and demand, which also has a significant impact on being able to establish the correct inventory level (Dash et al., 2021).

Authors of the paper conducted simulations with the use of the Author’s simulation model which remodels the real processes on a daily basis.

In order to show how complex the problems of optimizing decisions made in the sphere of logistics, the authors conducted simulations using two methods that differ in the level of data aggregation (see Figure 1).

The first method is based on the use of a simulation model called by the authors “detailed model”. This model reflects the logistic processes (their parameters, costs) on a daily basis and allows to simulate various delivery strategies (size, frequency, type of means of transport, delivery time, level of logistic customer service).

The construction of the model and its functioning is based on the following principles:

- demand is variable and the distribution of demand is normal (standard deviation of demand) – these are commercial commodities, which are purchased and re-sold by a trading company
- A receiver of goods is responsible for transport Goods are transported on EURO pellets
- Transport is organized by an external transportation company, so transportation costs depend on freight rates, which decrease when sizes of deliveries increase
- Inventory costs include costs of capital costs and warehousing costs
- Goods are stored in a rented warehouse and costs of warehousing depend on the sizes of deliveries and rates for warehousing

Total yearly costs (Eq. (1)) are a sum of the costs of logistics processes, which can be calculated with the use of the following formula:

\[ T_{C_{log}} = \sum_{i=0}^{n} (C_{I} + C_{W} + C_{T} + C_{LS}) \]  

where:
- \( T_{C_{log}} \) – Total yearly costs
- \( n \) – number of working days during a year
- \( C_{I} \) – Inventory costs (value of goods x inventory costs) per day \( i \);
- \( C_{W} \) – Warehousing costs (number of units x costs of storing per unit) per day \( i \);
- \( C_{T} \) – Transportation costs (costs of transportation by a vehicle of a given capacity) per day \( i \);
- \( C_{LS} \) – Costs of lost sales (quantity of goods which were not sold x value of goods) per day \( i \);

Costs of lost sales are the results of the level of the logistics customer service, which is measured as accessibility of goods in a warehouse for customers.

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**Figure 1.** The process of finding optimal process parameters using methods of optimization (“aggregate model”) and simulation (“detailed model”) and comparison of results.
The results of these simulations were the basis for the development of a regression model that can be a useful decision-making tool. The model was described later in the article.

The second (and relatively simpler) method is based on the use of an aggregated model that uses the average annual parameters of the processes and their costs as an alternative to the EOQ formula. A modified Economic Order Quantity formula could be used, which takes into account, in addition to the cost of order fulfillment and inventory maintenance costs, also storage and transport costs. There is however important problem. If the processes of storage and transport are carried out as part of external services provided by companies providing logistics services. In such situation the problem of changes in the prices of these services appears, when the size of the delivery batch changes. So, for example, if the model has entered data corresponding to the size of a batch of 8 pallets, and the model has calculated that the optimal batch of delivery is 10 pallets, then further calculations should be made. The optimization process actually becomes more complex, which is in contradiction to the idea of optimization, which is supposed to simplify the decision-making process. In fact even the use of the EOQ formula several calculations would have to be performed. Because of that the authors decided to use a model of total costs, perform simulations with use of it and compare results to find an optimal Order Quantity.

The model consists of two parts: model of total costs of deliveries and model of total costs of safety stocks. The first module consists of:

- Sizes of deliveries
- Costs corresponding to these sizes (costs of transportation and ordering and costs of inventories (capital and warehousing costs)

An algorithm finds an optimal size of a delivery for which total costs of deliveries are minimal.

In the second module, an optimal Reorder Point is being found (and a level of safety stocks corresponding to it) for which total costs of inventories and costs of lost sales are minimal.

Total costs of deliveries (Eq. (2)) and keeping a safety stocks (Eq. (3)) have been calculated with the use of the following formulas:

\[ T_{CD} = T_{IO} + T_{IQ} = n^* (FRQ_i * di + OC) + \frac{1}{2} * Qi * PPx * i_k + Q_i * WR_l * n_{I_P} \]  

\[ T_{SS} = T_{LS} = SS_m + (PPx * i_k + WR_l * n_{I_P}) + (100% - CSL)^* (SPx - PPx) \]  

where:

- \( T_{CD} \) – Total costs of deliveries
- \( T_{SS} \) – Total costs of safety stocks
- \( T_{IO} \) – Transportation costs and ordering costs
- \( T_{IQ} \) – Costs of inventories (Capital and warehousing) of the size of a delivery \( Q \)
- \( n \) – number of deliveries in a year
- \( FRQ \) – freight rate for a quantity of delivery \( i \)
- \( di \) – distance of transport \( j \)
- \( OC \) – costs of one order sent to a supplier
- \( PPx \) – Purchase price of a commodity \( x \)
- \( i_k \) – inventory costs \( k \)
- \( WR_l \) – Warehousing rate \( l \)
- \( n_{I_P} \) – number of days in a year
- \( T_{LS} \) – Total Costs of lost sales
- \( CSL \) – Customer Service Level measured by accessibility of inventories on stock
- \( SPx \) – sales price of a commodity \( x \)

Simulations carried out using these two methods were aimed at checking whether “aggregated models” could be a useful decision-making tool compared to “detailed models” (Figure 1).

3.1. Simulation results

The assumptions for optimization and simulations are presented in Tables 1 and 2. Results of the simulations obtained with use of the first method are presented in Tables 3 and 4.

There are visible relations between the value of goods, amount of sales and a distance which affects transport costs. For the cheapest goods, optimal sizes are the biggest, what is understandable as costs of holding inventories are relatively lower for this group of commodities. When transportation costs increase due to longer distances (or higher freight rates), the sizes of deliveries also increase.

The strength of the influence of these factors is, however, difficult to explain. The results of some variants seem surprising – especially for medium value goods. It is difficult to find an explanation for such a large change in the delivery lot (from 8 pallets to 16) when the sales volume changes. This may be further evidence of the existence of complex relationships between processes and their efficiency.

On the other hand, however, the results confirm that for cheaper goods the sizes of deliveries should be bigger and for more expensive – smaller. For the most expensive ones, the delivery sizes almost do not change or change slightly, even if transport costs increase significantly.

It is also worth to pay attention to a few interesting results of the simulations.

First – in all cases, the optimal level of availability of stocks in a warehouse is on a very high level; although, the highest level is in the case of the most expensive goods. That is understandable and, on the one hand, is a proof that the construction of the model is proper and the simulations have been conducted properly. On the other hand, it confirms the opinions stressed in the scientific literature about the importance of the logistics customer service. That could lead to another conclusion that if a company does not offer a high level of this service, it probably does not understand what the consequences of such a strategy are, because they do not perform total costs calculations.

Second, the size of a delivery depends among others on the value of goods, what is also understandable. Yet, although for cheaper goods, the optimal quantities are bigger, still smaller quantities are optimal, which can be transported with the use of road transport. That means that although transport costs are important, using e.g. rail transport means increasing considerably the level of inventories and costs associated with them. That also could be the explanation for the low share of this mode of transport in freight transport at least in Poland.

3.2. Comparison of results

With the use of the above formulas, the simulation experiments has been conducted. Model verification and validation is an important step in simulation modelling. Several preliminary verification techniques were used, like: internal consistency checks, checking the correctness of the model (check errors). To validate the model a certain number of experiments was run with different set of inputs and the calculated outputs (delivery quantity and reorder point) were compared with output that give reliable results.

The results of the simulations with the use “detailed” model presented above were compared with the results of the calculations with the use of “aggregated” model. The results of this comparison are presented in Tables 5, 6, 7, and 8. The percentage differences mean how much the

| Commodity | Weight of a commodity [kg/pallet] | Purchase price [PLN/kg] | Sales price [PLN/kg] |
|-----------|----------------------------------|-------------------------|---------------------|
| 1         | 500                              | 2                       | 3                   |
| 2         | 200                              | 80                      | 180                 |
| 3         | 300                              | 800                     | 1000                |
results obtained with the use of aggregated model differ from the results obtained by the “detailed” model.

Although some regularity can be seen; yet, it is very doubtful if such an approach could be helpful in finding optimal solutions.

For cheapest products and relatively lower levels of sales and both lower and higher standard deviations results are the same (0% differences – see Table 5). However for longer times of deliveries the differences are 100%. The differences become bigger for higher sales. What is a big surprise the size of the standard deviation is irrelevant here. So it seems that aggregated models can be useful for optimization of sizes of deliveries of goods sold in small quantities and delivered rather quickly from a supplier.

As for the safety stocks and costs associated with them the situation is quite different – for all cases the differences are big, which raises doubts as to whether these methods are useful when sales are volatile and quite different from a supplier.

deliveries of goods sold in small quantities and delivered rather quickly from a supplier.

As for the safety stocks and costs associated with them the situation is quite different – for all cases the differences are big, which raises doubts as to whether these methods are useful when sales are volatile and difficult to predict (Table 6). Moreover (and which is difficult to explain at this stage), the impact of the length of the order fulfillment time on the level of the safety margin is exactly the opposite to that in the case of EOQ.

However, it is more difficult to identify the regularities here if the results for more expensive goods are analyzed (Tables 7 and 8). In the case of 2 goods, there are more different ones, and in the case of 3, they decrease as the delivery time increases. The simulations assumed that these goods are not only more expensive, but also have different loading parameters and different transport and storage susceptibility, which has an impact on the costs of these processes. Therefore, the profitability of a specific delivery strategy is the result of various factors, which makes it difficult to determine the correctness of the impact of a given strategy on costs. The conclusion is therefore that most likely the method that most accurately reflects the relationship between delivery parameters.

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**Table 2. Typical freight rates of road carriers in Poland in 2018.**

| Loading capacity [pallets/vehicle] | 8  | 16 | 24 | 34 | 68 | 136 | 170 |
|-----------------------------------|----|----|----|----|----|-----|-----|
| Freight rates [PLN/km]           | 2.2| 3.1| 3.7| 4.0| 7.6| 13.6| 16  |

Source: Data obtained from the Polish market of road transport.

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**Table 3. Results of simulations with the “detailed” model – lower average sales.**

| Average sales [kg/day]| Standard deviations [kg/day]| Time of delivery [days]| Warehousing costs [PLN/pallet/day]| Inventory costs |
|-----------------------|-----------------------------|------------------------|----------------------------------|----------------|
| 5000                  | 50                          | 1                      | 0.7                               | 20%            |
| Distance [km]         | 200                         | 400                    | 800                               | 1200           |
| Commodity             | Optimal sizes of deliveries | [pallets]              |                                   |                |
| 1                     | 24                          | 34                     | 34                                | 34             |
| 2                     | 16                          | 34                     | 34                                | 34             |
| 3                     | 4                           | 6                      | 6                                 | 8              |

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**Table 4. Results of simulations with the “detailed” model – higher average sales.**

| Average sales [kg/day]| Standard deviations [kg/day]| Time of delivery [days]| Warehousing costs [PLN/pallet/day]| Inventory costs |
|-----------------------|-----------------------------|------------------------|----------------------------------|----------------|
| 10000                 | 50                          | 1                      | 0.7                               | 20%            |
| Distance [km]         | 200                         | 400                    | 800                               | 1200           |
| Commodity             | Size of a delivery [pallets]|                       |                                   |                |
| 1                     | 34                          | 34                     | 170                               | 442            |
| 2                     | 34                          | 34                     | 34                                | 34             |
| 3                     | 18                          | 18                     | 20                                | 24             |

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**Table 5. Comparison of results of simulations for commodity 1 with the use of “detailed” and “aggregated” simulation models for EOQ.**

| Average sales [kg/day]| Time of delivery [days]| Standard deviations [kg/day]| Inventory costs |
|-----------------------|------------------------|-----------------------------|----------------|
| 5000                  | 50                     | 100                         | 50             |
| 10000                 | 10000                  | 50                          | 100            |
| [days]                | [days]                 | Standard deviations [kg/day]|           |
| 1                     | 0%                      | 0%                          | 89%           |
| 2                     | 0%                      | 0%                          | 78%           |
| 3                     | 0%                      | 0%                          | 78%           |
| 4                     | 100%                    | 100%                        | 67%           |

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**Table 6. Comparison of results of simulations for commodity 1 with the use of “detailed” and “aggregated” simulation models for ROP.**

| Average sales [kg/day]| Time of delivery [days]| Standard deviations [kg/day]| Inventory costs |
|-----------------------|------------------------|-----------------------------|----------------|
| 5000                  | 100                    | 50                          | 100            |
| 10000                 | 10000                  | 50                          | 100            |
| [days]                | [days]                 | ROP differences             |               |
| 1                     | 120%                    | 113%                        | 124%           |
| 2                     | 71%                     | 67%                         | 73%            |
| 3                     | 32%                     | 30%                         | 34%            |
| 4                     | 33%                     | 30%                         | 34%            |

**Table 7. Comparison of results of simulations for commodity 2 with the use of “detailed” and “aggregated” simulation models for EOQ.**

| Average sales [kg/day]| Time of delivery [days]| Standard deviations [kg/day]| Inventory costs |
|-----------------------|------------------------|-----------------------------|----------------|
| 5000                  | 0                      | 0                          | 0              |
| 10000                 | 10000                  | 0                          | 0              |
| [days]                | [days]                 | EOQ differences             |               |
| 1                     | 0%                      | 0%                          | 25%            |
| 2                     | 0%                      | 0%                          | 25%            |
| 3                     | 75%                     | 75%                         | 25%            |
| 4                     | 75%                     | 100%                        | 25%            |

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**Table 8. Comparison of results of simulations for commodity 3 with the use of “detailed” and “aggregated” simulation models for EOQ.**

| Average sales [kg/day]| Time of delivery [days]| Standard deviations [kg/day]| Inventory costs |
|-----------------------|------------------------|-----------------------------|----------------|
| 5000                  | 0                      | 0                          | 0              |
| 10000                 | 10000                  | 0                          | 0              |
| [days]                | [days]                 | EOQ differences             |               |
| 1                     | −50%                    | −50%                        | −50%           |
| 2                     | −50%                    | −50%                        | −50%           |
| 3                     | −25%                    | −25%                        | −25%           |
| 4                     | 0%                      | 0%                          | 0%             |
(delivery volume and safety stock) is a simulation method in which a model is used that reflects the real processes as accurately as possible.

4. Regression analysis as a tool to simplify optimisation

As a novelty, in this paper, the authors present a new methodology using calculations from a “detailed” simulation model to create a simplified regression model. Such a model (formula for finding optimal sizes of deliveries) would allow managers to easily implement it in real inventory management practice. The framework of this methodology is shown in Figure 2.

From the results calculated with the use of complex computer simulation models, the authors wanted to present a simpler way to obtain comparable results for managers. Therefore, it was decided to use the regression method to obtain a simplified formula for calculating the desired values of searched variables: delivery quantity and reorder point. During the analysis of real phenomena and processes, even in relatively simple situations, we are not able to explain them completely. Therefore, when describing interdependencies between them, we usually use some simplified models of real interdependencies. Thus, under the concept of a model, we can understand a useful form of presenting empirical data. When approaching the model building process, we must take a certain compromise between excessive simplification of reality and the desire to include too detailed data.

The sample size in the regression model contains more than 700 elements, obtained as results from the simulation model. In this paper, the regression analysis is used to explain the impact of changes in independent variables (from x1 to x9) on the dependent variables (y1 and y2).

Dependent variables are:
- y1 – delivery quantity
- y2 – reorder point

Independent variables are:
- x1 – weight of a commodity
- x2 – average sales
- x3 – standard deviation of average sales
- x4 – time of delivery
- x5 – purchase price
- x6 – sales price
- x7 – distance
- x8 – warehousing costs
- x9 – inventory costs

Authors started with a comprehensive model containing all potential, subject to testing, factors affecting the phenomenon we are considering. Then the components of the initial were tested, extensive model to identify less extensive sub-models that explain the phenomenon in question in an adequate manner. Finally, among these potential sub-models, the simplest one was selected, which, on the principle of saving, we treat as “the best” describing the studied phenomenon.

This article used the best subset method to simplify the model. Simple models are preferred for practical reasons, they are easier to retest, require a lower cost in the future, and also are easier to understand and appreciate.

Two analyses were carried out, separately for each of the dependent variables (y1 – delivery quantity and y2 – reorder point). For delivery quantity, the best subset method indicated a set of 8 variables: x1, x2, x3, x4, x5, x7, x8, x9. Table 9 presents the significance tests with effects sizes, p-value and t-test for delivery quantity.

From Table 9, one can observe that 7 from 8 left variables after the best subset regression method are statistically significant – the p-values are distinctly less than 0.05. Additionally, the assessment of the significance of structural parameters was carried out. To check the significance of structural parameters b0, b1, ..., bn, the t-test was used where t-statistic has t-test distribution with n-k-1 degrees of freedom.

Null and alternative hypotheses:
- H0: b_i = 0 (no linear relationship)
- H1: b_i ≠ 0 (linear relationship does exist)

The critical region is two-sided with the critical value that we read from the tables, the t-test distribution for a fixed level of significance α and n-k-1 degrees of freedom. If the value of t is in the critical region

![Figure 2. Conceptual framework scheme of simulation modelling with regression module for finding optimal sizes of deliveries.](image-url)
(calculated value of \( t > \) value of \( t \) from table), then we have to reject \( H_0 \) in favor of \( H_1 \). Otherwise, there is no basis to reject \( H_0 \). In the case of structural parameters from Table 9, all values (except \( x_3 \) – standard deviation of average sales) of \( t \) are in the critical region so a relationship does exist.

The regression model for \( y_1 \) – delivery quantity (Eq. (4)) is as follows:

\[
y_1 = 8.61 - 0.0109x_1 + 0.0118x_2 + 0.0055x_3 + 0.9804x_4 - 0.0165x_5 + 0.0202x_7 - 3.5423x_8 - 17.203x_9
\]

(4)

A similar situation can be seen in Table 10, which presents significance tests with effects sizes for the second dependent variable \( y_2 \) – reorder point. In this case, the best subset method also indicated a set of 8 variables: \( x_1, x_2, x_3, x_4, x_5, x_7, x_8, x_9 \).

From Table 10, one can observe that all variables are statistically significant – the \( p \)-values are distinctly less than 0.05. To check the significance of structural parameters in the table, there are also \( t \)-test values, in each case, they are in a critical region, so a relationship does exist.

The regression model for \( y_2 \) – reorder point (Eq. (5)) is as follows:

\[
y_2 = -530.058 -1.565x_1 + 2.468x_2 + 783.563x_4 + 0.043x_5 + 0.439x_7 - 54.869x_8 - 479.985x_9
\]

(5)

Furthermore, the model verification was carried out to check whether the econometric model is acceptable. The properties of standard deviations have been examined and it has also been checked, among other things, if the assessment of the model fits the empirical data. A number of indicators were calculated that verify the presented regression model. Coefficient of Determination, \( R^2 \) – is the portion of the total variation in the dependent variable that is explained by a variation in the independent variable, \( 0 \leq R^2 \leq 1 \). Standard Error of Estimate, \( s_x \) – is the standard deviation of the variation of observations around the regression line. Coefficient of variation \( V_x \) – is a standardized measure of dispersion of a probability distribution or frequency distribution. The model is better, the lower the value of this coefficient.

From Table 11, the \( R^2 \) parameter shows that 83.01% of changes in the delivery quantity are explained by the change in the set of independent variables \( (x_1, x_2, x_3, x_4, x_5, x_7, x_8, x_9) \). In the case of reorder point, it is even better – 90.42% changes in the delivery quantity are explained by the change in the set of independent variables \( (x_1, x_2, x_3, x_4, x_7, x_8, x_9) \). Looking at the parameter \( s_x \), one can say that estimated values of delivery quantity and reorder point differ from the real values by respectively 2.042 and 418.25; expressing this value in percent, we get part of the standard deviation of the random component in the average value of delivery quantity and reorder point 14.14% and 13.27%, respectively.

Summing up these results and the \( t \)-test value from Tables 8 and 9, it can be said that both regression models are statistically significant; a slightly better value is obtained by the model created for the reorder point, especially the parameter \( R^2 \) which explains as much as 90% of the variance of the model. Considering the large number of parameters, one may wonder if better results would not be achieved if the model were not simplified in such a way that fewer parameters would be entered by the user. For this purpose, the parameters have been aggregated and reduced to one unit – PLN/pallet. First, the value of purchase, sale and standard deviation were referred to 1 pallet. Secondly, transport costs were used in the model as the cost of transporting one pallet over a distance of 1 km. The use of such a synthetic unit makes it possible to take into consideration both the transport distance and possible changes in transport rates on the transport services market. Thirdly, inventory costs include total capital costs depending on the value and level of inventories, and the cost of warehouse services depending on the number of pallets. The results, however, are worse – the degree of matching with this approach was much worse (50%). The conclusion is that it is better to introduce more parameters to the model, which is more convenient for the user.

5. Conclusions

The method proposed by the authors of the paper is an innovative solution for the problem of the Economic Order Quantity and an Optimal level of safety stocks.

The model used by the authors is assumed to be a model that reproduces the real processes implemented in supply chains in the most detailed way. The consequence, however, is that it is very extensive. It also requires appropriate software and efficient computer hardware, and, what is important from the user’s point of view, requires a certain level of framework to carry out the simulation. For this reason, the authors’ goal was to find a method that would simplify the decision-making process. Such a method was the use of a regression model, which was built on the basis of data obtained from simulations with the use of a simulation model.

The simulation model takes into consideration a relatively larger number of factors and relationships between them and between them and the economic efficiency. The optimization criterion is total costs, which, however, take into consideration specific cost categories not found in traditional accounting – capital costs of maintaining inventories and costs of lost sales. In addition to the capital costs of inventory calculated on quantity and value, storage costs are also included, which depend only on the quantity (or rather on the volume of the inventory).

Considering such a large number of factors and non-linear relationships between them, it is doubtful whether mathematical formulas can be effectively applied, especially if one considers the impact of the volume of deliveries not only on the costs of deliveries themselves but the level of service and, consequently, sales.

In order to demonstrate the usefulness of the method of using such a detailed and extensive simulation model, the results obtained with the use of this model were compared with the results obtained with the use of an “aggregated” model, i.e. a model in which average data were used. The
use of the “aggregated” model greatly simplifies the process of simulation and finding optimal delivery parameters. The results for relatively cheap goods, which daily demand is not high and significantly changes and delivery times are up to 3 days, are surprisingly similar. However, for the more expensive goods, the differences of results are very large. This means that the aggregate model would have a limited use.

The best results, i.e. the closest to those obtained using a detailed simulation model, can be obtained by using the regression model (90%). In the authors’ opinion, it constitutes an important contribution to theory and is a proposal to solve the problem of optimization of deliveries and stocks, which has been the subject of many studies for many years. It can also be a useful decision-making tool for practitioners without scientific background. With some however stipulations.

Calculations have been performed for a specific problem, when the demand is normally distributed and when one product is delivered in one delivery in one vehicle. Such situation is typical for example for deliveries of materials for production. Gauss distribution can be also characteristic for the sales of consumer goods sold in a retail shop. In case of distribution companies e.g. wholesalers the demand is dependent, what means, that it depends on the frequency of placing orders by customers and sizes of deliveries, which in turn can be result of decisions made by a supplier. Perhaps other distributions should be used e.g. Gamma, Poisson.

Costs of transport are based on the freight rates typical for the European transportation market, in which mainly road and rail transport are used. For optimization of deliveries in the global supply chains parameters of deliveries with the use of other modes of transport (maritime, air) should be taken into consideration.

In a situation when different products are purchased from one supplier or they are delivered from own distribution of a company to its shops or regional or local warehouses consolidated deliveries to a warehouse could be performed. Probably a vehicle of high loading capacity would be utilized. In such variant, a model would be simplified, and the results would be much different from the presented above. Probably optimal sizes and safety stocks would be smaller at lower costs of transport and better customer service.

The system on which the simulation model used in the authors’ simulations is based it is the continuous review system. This is one of the two classic stock renewal systems, which is based on the so-called information level – orders are placed when the available stock reaches a certain level. The alternative is the periodic review system in which orders are placed in a specific cycle, and their size depends on the level of available inventory at the time of the review. Perhaps parameters of deliveries (delivery sizes and safety stocks) would be different, but to prove this, the simulation model would have to be changed.

Probably the best solution would be not an optimization model but to find a compromise solution. Such a solution could be for example a model, which allows to calculate a level of stocks by a given level of customer service and then calculation the total costs of deliveries. To prove effectiveness of such solution both from theoretical and practical point of view further research is needed.

**Declarations**

**Author contribution statement**

Dariusz Milewski: Conceived and designed the experiments; Performed the experiments; Analyzed and interpreted the data; Contributed reagents, materials, analysis tools or data; Wrote the paper.

Tomasz Wiśniewski: Conceived and designed the experiments; Performed the experiments; Analyzed and interpreted the data; Wrote the paper.

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