Dephasing in semiconductor–superconductor structures by coupling to a voltage probe

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Abstract

We study dephasing in semiconductor–superconductor structures caused by coupling to a voltage probe. We consider structures where the semiconductor consists of two scattering regions between which partial dephasing is possible. As a particular example we consider a situation with a double-barrier junction in the normal region. For a single-mode system we study the conductance both as a function of the position of the Fermi level and as a function of the barrier transparency. At resonance, where the double-barrier is fully transparent, we study the suppression of the ideal factor-of-two enhancement of the conductance when a finite coupling to the voltage probe is taken into account.

KEYWORDS: Andreev scattering, quantum interference, dephasing

1. Introduction

Charge transport through a normal conductor–superconductor (NS) interface is accompanied by a conversion of quasiparticle current to a supercurrent. In the Andreev reflection, by which the conversion occurs, an electron-like quasiparticle in the normal conductor (with an excitation energy lower than the energy gap of the superconductor) incident on the NS interface is retroreflected into a hole-like quasiparticle (with reversal of its momentum and its energy relative to the Fermi level) and a Cooper pair is added to the condensate of the superconductor [1]. For an ideal NS interface, a signature of Cooper pair transport and the Andreev scattering is a doubling of the conductance compared to the normal state conductance.
A theoretical framework for studying scattering at NS interfaces is provided by the Bogoliubov–de Gennes (BdG) formalism [2] where the scattering states are eigenfunctions of the BdG equation which is a Schrödinger-like equation in electron-hole space (Nambu space).

For a phase-coherent normal region, the conducting properties can be found using a scattering matrix approach based on the BdG equation. In the linear-response regime in zero magnetic field, Beenakker [3] found that the sub-gap conductance \( G \equiv \partial I/\partial V \) is given by

\[
G_{NS} = 2G_0 \text{Tr} \left( tt^\dagger \left[ 2 - tt^\dagger \right]^{-1} \right)^2 = 2G_0 \sum_{n=1}^{N} \frac{T_n^2}{(2 - T_n)^2},
\]

which, in contrast to the Landauer formula [4, 5, 6, 7] for the normal state conductance,

\[
G_N = G_0 \text{Tr} tt^\dagger = G_0 \sum_{n=1}^{N} T_n,
\]

is a non-linear function of the transmission eigenvalues \( T_n (n = 1, 2, \ldots, N) \) of \( tt^\dagger \). Here \( G_0 = 2e^2/h \) is the quantum unit of conductance for a spin-degenerate system and \( t \) is the \( N \times N \) transmission matrix of the normal region, \( N \) being the number of propagating modes. The conductance formula holds for an arbitrary disorder potential and is a multi-channel generalization of a conductance formula first obtained by Blonder, Tinkham and Klapwijk [8] who considered a delta function potential as a model for the interface barrier potential. Eq. (1) is computationally convenient since standard methods developed for quantum transport in normal conducting semiconductor structures can be applied to calculate the transmission matrix \( t \) or the corresponding transmission eigenvalues \( T_n \).

Equations (1) and (2) apply to two-probe structures where the length \( L \) of the disordered normal region is much shorter than the phase-correlation length over which quasiparticle propagation remains phase-coherent.

The effect of coupling a mesoscopic region to a voltage probe was first used by Büttiker in the context of the role of quantum phase-coherence in normal conducting series resistors [7] and subsequently in the study of the cross-over from coherent to sequential tunneling in series of barriers [10]. In the work of Chang and Bagwell [11] the model of Büttiker [10] was used as a normal-metal probe controlling the Andreev-level occupation in a Josephson junction. In this paper we apply the model of Büttiker to a two-probe semiconductor-superconductor junction with a phase-coherent normal region. However, here we consider the case where the dephasing reservoir is constituted by a real voltage probe. As a general model we consider a normal region with two arbitrary scattering regions (characterized by scattering matrices \( S_1 \) and \( S_2 \)) separated by a coherent ballistic conductor of length \( L \) to which the voltage probe couples, see (a) in Fig. 1.
Figure 1: Scattering scheme for a two probe structure with two scattering regions (characterized by the S–matrices $S_1$ and $S_2$) between which dephasing is caused by coupling to a voltage probe. The left probe (1) is in the normal state and the right probe (2) is a metal which can be either in the normal or superconducting state. The voltage probe could e.g. be an STM tip. Panel (a) shows the generic structure, (b) corresponds to the present model for the voltage probe, (c) to the phase-coherent regime with no coupling to the voltage probe, and (d) to the sequential tunneling regime with full coupling to the voltage probe. The arrows indicate possible directions of current flow and the arrows with dotted midsections indicate equilibration of quasiparticles.

Specifically we study the case where the total current is conserved, but, due to the coupling to the voltage probe the non-equilibrium distribution of electrons and holes equilibrates to a Fermi–Dirac distribution. In Nambu space this corresponds to a decay of electron-hole states in the normal region. As a particular example we consider a semiconductor-superconductor structure with a normal region containing a double-barrier junction of length $L$ and model the cross-over from vanishing dephasing to full dephasing in the conducting properties.

We note that in our approach the voltage probe is real as it is also the case in the recent work of Gramespacher and Büttiker [12] where a tunneling tip is coupled weakly to the normal side of a two-probe NS structure. As in the original work of Büttiker [9, 10] the voltage probe could also be considered as a fictitious probe used in modeling a finite dephasing length of the normal region. However, we note that the model only accounts for dephasing caused by inelastic scattering and that thermal dephasing, which is the dominant dephasing mechanism in most semiconductor–superconductor structures, can not be accounted for by this model [13]. Dephasing due to a finite temperature can in principal be incorporated by calculating the full energy dependence of the S–matrices, see e.g. [14, 15, 16, 17], or through Green functions methods, see e.g. [17] and references therein.

The paper is organized as follows: In Section II the S–matrix formalism is introduced, in Section III we formulate our general model and present the corresponding scattering scheme. In Section IV we present results of an application
of our scattering scheme to the problem of double-barrier tunneling. Finally, in Section V discussion and conclusions are given.

2. Scattering matrix formalism

Using the Büttiker voltage probe \[10\] we need to consider four-probe structures (two real and two probes accounting for the voltage probe) and we therefore need multi-probe generalizations of Eqs. (1) and (2). The scattering approach to dc transport in superconducting hybrids follows closely the scattering theory developed for non-superconducting mesoscopic structures, see e.g. the monograph of Datta [18]. For a multi-probe system, the normal state current is given by the multi-probe conductance formula of Büttiker \[7\] which in the linear-response limit gives the following current in lead \(p\):

\[ I_p = \sum_{q=1}^{M} G_{pq} [V_p - V_q], \]

\[ G_{pq} = G_0 \text{Tr} S_{pq} S_{pq}^\dagger, \]  \hspace{1cm} (3)

where \(q = 1, 2, 3, \ldots, M\) labels the \(M\) probes and \(V_q\) is the potential of probe \(q\). The S–matrix \(S\) is an \(M \times M\) block-matrix with matrices \(S_{pq}\) describing scattering of an electron state in lead \(q\) into an electron state in lead \(p\). These matrices are \(N_p \times N_q\) matrices where \(N_p\) and \(N_q\) are the number of modes in lead \(p\) and \(q\), respectively. In the linear-response limit, the S–matrices are evaluated at the Fermi level.

For an NS interface with multiple normal probes, Lambert, Hui and Robinson [14] considered the sub-gap current which in the linear-response regime is given by the formula

\[ I_p = \sum_{q=1}^{M} G_{pq} [V_q - V], \]

\[ G_{pq} = G_0 \text{Tr} \left( \hat{1} \delta_{pq} + S_{eh} \{ S_{pq} \}^\dagger - S_{ee} \{ S_{pq} \}^\dagger \right), \]  \hspace{1cm} (4)

where \(\hat{1}\) is an \(N_p \times N_p\) identity matrix, \(\delta_{pq}\) is the Kronecker delta symbol and \(V\) is the potential of the superconductor. The S–matrix in electron-hole space is given by

\[ S = \begin{pmatrix} S_{ee} & S_{eh} \\ S_{he} & S_{hh} \end{pmatrix}, \]  \hspace{1cm} (5)

where the sub-matrix \(S_{ee}\) (\(S_{hh}\)) is for normal scattering of an electron (hole) state in lead \(q\) to an electron (hole) state in lead \(p\), and \(S_{he}\) (\(S_{eh}\)) is for Andreev scattering of an electron (hole) state in lead \(q\) to a hole (electron) state in lead \(p\).

The two-probe case \((M = 2)\) was studied by Takane and Ebisawa [19] and
subsequently by Beenakker [3] who derived Eq. (1) within the Andreev approximation [1] and the rigid boundary condition for the pairing potential of the superconductor. Within these approximations, the interface acts as a phase-conjugating mirror and the interface scattering can be described by an S–matrix which at the Fermi level is given by [1, 3]

\[
S_A = \begin{pmatrix}
\hat{0} & e^{i(\varphi - \frac{\pi}{2})}\hat{1} \\
e^{-i(\varphi + \frac{\pi}{2})}\hat{1} & \hat{0}
\end{pmatrix},
\]

(6)

where \(\varphi\) is the phase of the pair potential of the superconductor.

The form of the BdG equation [2] in the normal region where the pairing potential \(\Delta\) is zero means that the elastic scattering in the normal region due to an arbitrary disorder potential is characterized by the block-diagonal S–matrix

\[
S_N = \begin{pmatrix}
S & \hat{0} \\
\hat{0} & S^*
\end{pmatrix},
\]

(7)

where \(S\) is the normal state S–matrix for electrons (the S–matrix entering e.g. Eq. (3)) and \(S^*\) is the corresponding S–matrix for scattering of holes. The S–matrix in Eq. (6) entering Eq. (4) is given by the composite result \(S = S_A \otimes S_N\) where we use the notation of Datta [18] with the meaning of \(\otimes\) found by elimination of internal current amplitudes.

3. Scattering scheme

We consider an NS structure with a normal region containing two scattering regions

\[
\mathcal{S}_1 = \begin{pmatrix}
r_1 & t_1' \\
t_1 & r_1'
\end{pmatrix}, \quad \mathcal{S}_2 = \begin{pmatrix}
r_2 & t_2' \\
t_2 & r_2'
\end{pmatrix},
\]

(8)

connected by a ballistic conductor of length \(L\). The free propagation of electron-like quasiparticles is described by

\[
U = \begin{pmatrix}
\hat{0} & X \\
X & \hat{0}
\end{pmatrix},
\]

(9)

with elements given by \(X_{nn'} = \delta_{nn'} \exp(ik_nL)\). For a wire of width \(W\) with a hard-wall confining potential the longitudinal wave vector \(k_n\) at the Fermi level is given by

\[
k_n = k_F \sqrt{1 - (n\pi/k_FW)^2}.
\]

We connect voltage probe to the center of the conductor, see panel (b) in Fig. 1. Using the model of Büttiker [10] the conductor is now characterized by the unitary matrix
which is a generalization of the S–matrix in Ref. \[10\] with the matrix $X$ taking the finite length of the conductor into account. Here $0 \leq \zeta \leq 1$ is a parameter determining the coupling of the conductor to the voltage probe. The scattering from lead $q$ to lead $p$ is given by the sub-matrices $\{S_{\phi}\}_{pq}$ which are zero along the diagonal since there is no back-scattering from the conductor and the other zeros reflect that lead 1 couples only to leads 2 and 3, lead 2 only to leads 1 and 4 etc. The basic idea is that current can flow either directly through the phase-coherent conductor, as shown in panel (c) of Fig. \[4\], or in the sequential way, as in panel (d). Using the scheme in panel (b), the cross-over from coherent to sequential tunneling can be modeled by changing the coupling from $\zeta = 0$ to $\zeta = 1$. In this approach, probes 3 and 4 have a common reservoir (that of the voltage probe) and quasiparticles entering probe 3 are re-injected through probe 4, and vice versa, so that the common reservoir of probes 3 and 4 does not supply or draw any net current ($I_3 + I_4 = 0$).

The composite S–matrix of the normal region $S = S_2 \otimes S_{\phi} \otimes S_1$ can now be found. Eliminating the internal current amplitudes and introducing the matrices 

\[
\gamma_{12} \equiv \left[ \hat{i} - (1 - \zeta) Xr_2 Xr_1' \right]^{-1} \quad \text{and} \quad \gamma_{21} \equiv \left[ \hat{i} - (1 - \zeta) Xr_1' Xr_2 \right]^{-1}
\]

we get

\[
S = \begin{pmatrix}
\frac{r_1 + (1 - \zeta) t_1 t_2' \gamma_{12} Xr_2 Xr_1}{\sqrt{1 - \zeta} t_2 \gamma_{21} Xr_1 T_{t_1}} & \frac{\sqrt{1 - \zeta} t_1' \gamma_{12} X r_2'}{\sqrt{1 - \zeta} t_2 \gamma_{21} X r_1 T_{t_1}} \\
\frac{\sqrt{t_1' \gamma_{12} X^{1/2}}}{\sqrt{1 - \zeta} (1 - \zeta) X^{1/2} r_2 \gamma_{21} X T_{t_1}} & \frac{\sqrt{t_1' \gamma_{12} X r_2'}}{\sqrt{1 - \zeta} (1 - \zeta) X^{1/2} r_2 \gamma_{21} X T_{t_1}} \\
\frac{\sqrt{r_2' + (1 - \zeta) t_2 \gamma_{21} X r_1^2}}{\sqrt{1 - \zeta} X^{1/2} r_2' \gamma_{12} X T_{t_1}} & \frac{\sqrt{t_2' \gamma_{21} X r_2^2}}{\sqrt{1 - \zeta} X^{1/2} r_2' \gamma_{12} X T_{t_1}} \\
\frac{\sqrt{t_2' \gamma_{21} X}}{\sqrt{1 - \zeta} X^{1/2} r_2' \gamma_{12} X T_{t_1}} & \frac{\sqrt{t_2' \gamma_{21} X}}{\sqrt{1 - \zeta} X^{1/2} r_2' \gamma_{12} X T_{t_1}}
\end{pmatrix},
\]

which together with Eq. (3) gives the normal state conducting properties.

In order to apply Eq. (11) for the conducting properties of the superconducting state we need to calculate the two block-matrices $S_{\text{ee}}$ and $S_{\text{he}}$ of the composite S–matrix $S = S_2 \otimes S_{\phi} \otimes S_3$. Since the phase $\varphi$ of the pairing potential for a 2DEG-S system (in contrast to S-2DEG-S systems) can be arbitrary and the conducting properties are independent of $\varphi$, we may for simplicity let $\varphi = 0$. Eliminating internal current amplitudes we get
\[
S^{ne} = \begin{pmatrix}
S_{11} - S_{12} S_{22}^{*} \Gamma_{22} S_{21} & 0 & S_{13} - S_{12} S_{22}^{*} \Gamma_{22} S_{23} & S_{14} - S_{12} S_{22}^{*} \Gamma_{22} S_{24} \\
0 & 0 & 0 & 0 \\
S_{31} - S_{32} S_{22}^{*} \Gamma_{22} S_{21} & 0 & S_{33} - S_{32} S_{22}^{*} \Gamma_{22} S_{23} & S_{34} - S_{32} S_{22}^{*} \Gamma_{22} S_{24} \\
S_{41} - S_{42} S_{22}^{*} \Gamma_{22} S_{21} & 0 & S_{43} - S_{42} S_{22}^{*} \Gamma_{22} S_{23} & S_{44} - S_{42} S_{22}^{*} \Gamma_{22} S_{24}
\end{pmatrix}
\] (12)

\[
S^{he} = \begin{pmatrix}
-i S_{12} \Gamma_{22} S_{21} & 0 & -i S_{12} \Gamma_{22} S_{23} & -i S_{12} \Gamma_{22} S_{24} \\
0 & 0 & 0 & 0 \\
-i S_{32} \Gamma_{22} S_{21} & 0 & -i S_{32} \Gamma_{22} S_{23} & -i S_{32} \Gamma_{22} S_{24} \\
-i S_{42} \Gamma_{22} S_{21} & 0 & -i S_{42} \Gamma_{22} S_{23} & -i S_{42} \Gamma_{22} S_{24}
\end{pmatrix}
\] (13)

where the matrix \( \Gamma_{22} \equiv [1 + S_{22} S_{22}^{*}]^{-1} \) has been introduced. The zero matrices in row 2 and column 2 reflect that the superconductor (probe 2) does not carry any quasiparticle current due to the gap in the density of states. At the interface, the quasiparticle current is transformed to a supercurrent carried by the Cooper pairs of the condensate so that current is conserved. For a finite bias exceeding the energy gap of the superconductor, the current in the superconducting probe will be carried by both quasiparticles and Cooper pairs.

To find the normal state conductance \( G_N \) we apply Eq. (3). From Kirchhoff’s law \( \sum \mathcal{I}_p = 0 \) and the condition \( \mathcal{I}_3 + \mathcal{I}_4 = 0 \) for the voltage probe get

\[
\begin{pmatrix}
\mathcal{I}_1 \\
0
\end{pmatrix} = \begin{pmatrix}
\mathcal{G}_{12} + \mathcal{G}_{13} + \mathcal{G}_{14} & -\mathcal{G}_{13} - \mathcal{G}_{14} \\
-\mathcal{G}_{31} - \mathcal{G}_{41} & \mathcal{G}_{31} + \mathcal{G}_{32} + \mathcal{G}_{41} + \mathcal{G}_{42}
\end{pmatrix} \begin{pmatrix}
V_1 - V_2 \\
\bar{V} - V_2
\end{pmatrix},
\] (14)

where \( \bar{V} = V_3 = V_4 \) is the common potential of the voltage probe reservoir. The conductance \( G_N = \partial \mathcal{I}_1 / \partial (V_1 - V_2) \) is therefore given by

\[
G_N = \mathcal{G}_{12} + \mathcal{G}_{13} + \mathcal{G}_{14} - \frac{(\mathcal{G}_{13} + \mathcal{G}_{14})(\mathcal{G}_{31} + \mathcal{G}_{41})}{\mathcal{G}_{31} + \mathcal{G}_{32} + \mathcal{G}_{41} + \mathcal{G}_{42}}.
\] (15)

In the superconducting state we apply Eq. (3) and in the same way we find that

\[
\begin{pmatrix}
\mathcal{I}_1 \\
0
\end{pmatrix} = \begin{pmatrix}
\mathcal{G}_{11} & \mathcal{G}_{13} + \mathcal{G}_{14} \\
\mathcal{G}_{31} + \mathcal{G}_{41} & \mathcal{G}_{33} + \mathcal{G}_{34} + \mathcal{G}_{43} + \mathcal{G}_{44}
\end{pmatrix} \begin{pmatrix}
V_1 - V_2 \\
\bar{V} - V_2
\end{pmatrix},
\] (16)

so that the conductance \( G_{NS} = \partial \mathcal{I}_1 / \partial (V_1 - V_2) \) is given by

\[
G_{NS} = \mathcal{G}_{11} - \frac{(\mathcal{G}_{13} + \mathcal{G}_{14})(\mathcal{G}_{31} + \mathcal{G}_{41})}{\mathcal{G}_{33} + \mathcal{G}_{34} + \mathcal{G}_{43} + \mathcal{G}_{44}}.
\] (17)

Despite the minus signs in Eqs. (13) and (17), both \( G_N \) and \( G_{NS} \) are positive definite which follows from the unitarity, time-reversal, and electron hole symmetries of the S–matrices. Eqs. (15) and (17) together with Eqs. (11), (12), and (13) form the basis for our calculations of the conducting properties of two-probe semiconductor–superconductor junctions with a normal region containing
two scattering regions separated by a phase-coherent region which couples to a voltage probe.

The sequential tunneling limit ($\zeta = 1$) corresponds to series connected resistors, see panel (d) in Fig. [1]. Thus we can in general combine Eqs. (11) and (12) to obtain

$$G_N \stackrel{\zeta \to 1}{\longrightarrow} G_0 \left( \frac{1}{\sum_n T_{1,n}} + \frac{1}{\sum_n T_{2,n}} \right)^{-1}, \quad G_{NS} \stackrel{\zeta \to 1}{\longrightarrow} G_0 \left( \frac{1}{\sum_n T_{1,n}} + \frac{1}{2 \sum_n (2-T_{2,n})^2} \right)^{-1},$$

(18)

where $T_{n,1}$ and $T_{n,2}$ are the transmission eigenvalues of $t_1 t_1^\dagger$ and $t_2 t_2^\dagger$, respectively.

The conductance in the zero-coupling limit ($\zeta = 0$) can of course be calculated directly from Eqs. (11) and (12) with the transmission eigenvalues $T_{n,12}$ of $t_{12} t_{12}^\dagger$, where $t_{12}$ is the transmission matrix of the composite $S$–matrix $S_{12} = S_2 \otimes U \otimes S_1$, see e.g. the structure of Eq. (8).

4. Double-barrier tunneling

As a specific example we consider double-barrier tunneling with two identical barriers which we model by delta function potentials $V(\mathbf{r}) = H [\delta(z) + \delta(z + L)]$. Here $L$ is the separation of the two barriers and $H$ is the strength of the potentials. The $S$–matrices are given by

$$S_1 = S_2 = \begin{pmatrix} r_\delta & t_\delta \\ t_\delta & r_\delta \end{pmatrix}, \quad (t_\delta)_{nn'} = \delta_{nn'} \frac{1}{1 + i Z/\cos \theta_n}, \quad (r_\delta)_{nn'} = \delta_{nn'} \frac{-i Z/\cos \theta_n}{1 + i Z/\cos \theta_n},$$

(19)

where $Z \equiv H/hv_F$ is a normalized barrier strength and $\cos \theta_n \equiv k_n/k_F$ takes the transverse momentum of mode $n$ into account [21, 22].

In the phase-coherent limit we calculate the transmission matrix $t_{12}$ corresponding to the composite $S$–matrix $S_{12} = S_2 \otimes U \otimes S_1$. Since the transmission is diagonal, $t_{12} t_{12}^\dagger$ has transmission eigenvalues given by

$$T_{12,n} = \left( 1 + 2 Z_c^2(\theta_n) \left[ 1 + \cos \vartheta_n + 2 Z_c(\theta_n) \sin \vartheta_n + Z_c^2(\theta_n) \left( 1 - \cos \vartheta_n \right) \right] \right)^{-1},$$

(20)

where $Z_c(\theta_n) \equiv Z/\cos \theta_n$ and $\vartheta_n \equiv 2k_F L \cos \theta_n$. Comparing to the transmission eigenvalues $T_{1,n} = T_{2,n} = \left[ 1 + Z_c^2(\theta_n) \right]^{-1}$ of the individual barriers it is seen that for the double-barrier system, $T_{12,n}$ can be equal to unity even for a finite value of $Z$ which is in contrast to the result for $T_{1,n} = T_{2,n}$. This is one of the important differences between phase-coherent quantum transmission and classical transmission probabilities. At the values of $Z$ where mode $n$ becomes fully transparent a peak in the conductance is to be expected. Apart from the
Figure 2: Double-barrier junction with a normalized barrier strength $Z = 0.05$ and an aspect ratio $L/W = 3$. Panel (c) shows the normal state conductance $G_N$, (b) the conductance $G_{NS}$ of the superconducting state, and (a) the normalized conductance $G_{NS}/G_N$ for different values of the dephasing parameter $\zeta$. The plots are shown as a function of $k_F W / \pi < 2$ corresponding to only a single propagating mode. Notice the different scales on the conductance axes.
trivial solution $Z = 0$ we find that $Z = -\cos \theta_n \cot(\theta_n/2)$ which means that the mode can be fully transparent for $\pi \leq \theta_n \leq 2\pi$ (modulo $2\pi$) only.

As a numerical example we consider a double-barrier junction with an aspect ratio $L/W = 3$. Fig. 2 shows the conductance as a function of $k_F W/\pi$ for $Z = 0.05$. For $k_F W/\pi < 1$ there are no propagating modes and for $1 < k_F W/\pi < 2$ a single mode is propagating. In panel (c), the normal state conductance $G_N$ increases from zero to approximately $G_0$ at the onset of the first propagating mode. As the coupling parameter is increased from 0 towards 1, $G_N$ approaches $G_0/2$ and it actually becomes a little lower than $G_0/2$ due to the finite barrier strength (at $k_F W/\pi = 2$, Eq. (18) gives $G_N/G_0 = 150/301$).

In the coherent regime, oscillations caused by size-resonances are seen and as the sequential tunneling regime is approached these resonances vanish. In panel (b), the conductance $G_{NS}$ in the superconducting state shows the same overall behavior as seen for $G_N$, even though the size-resonances are much more pronounced. These size-resonances are even more pronounced in the normalized conductance $G_{NS}/G_N$ shown in panel (a). As the coupling parameter is increased the resonances vanish and the normalized conductance is suppressed below the ideal factor-of-two for the enhancement of $G_{NS}$ compared to $G_N$. For these parameters, ideal transmission and maxima in the conductance are expected for $k_F W/\pi \simeq 1.018, 1.113, 1.296, 1.531, 1.808$ which is in full agreement with the plots in Fig. 2 where these numbers are indicated by arrows.

In Fig. 3 we consider the $Z$-dependence of the conductance for a Fermi wave vector corresponding to $k_F W/\pi = 1.9$ so that only one mode is propagating. Panels (b) and (c) show the conductance $G_{NS}$ in the superconducting state and the normal state conductance $G_N$, respectively, as a function of the normalized barrier strength $Z$ for different values of the coupling parameter $\zeta$. For these parameters, a peak in the conductance is expected for $Z \simeq 1.626$ which is indeed what is clearly seen in the coherent tunneling limit ($\zeta = 0$) where this number is indicated by an arrow on the $Z$-axis. As the sequential tunneling regime is approached ($\zeta \to 1$) the peak is suppressed and an overall lowering of the conductance is seen. However, dephasing may also suppress a destructive interference in the coherent transmission resulting in an increased conductance. In this particular case this is seen for $Z > 2.2$ in both $G_N$ and $G_{NS}$ and also in a range around $Z \sim 0.7$ in $G_{NS}$ where the conductance curves are crossing. Panel (a) shows the corresponding normalized conductance $G_{NS}/G_N$ which has the value of two for ideal Andreev scattering and decreases below the ideal factor-of-two enhancement when $Z$ is increased so that normal scattering becomes dominant.

For $\zeta = 1$ and $Z = 0$ Eq. (18) gives that $G_N/G_0 = 1/2$, $G_{NS}/G_0 = 2/3$, and $G_{NS}/G_N = 4/3$. These numbers are born out by the numerical calculations as indicated by arrows on the conductance axes in Fig. 3 and they can also be seen in Fig. 4 where there is a very low but finite barrier strength.

In experiments, the barrier strength will often not be a tunable parameter. However, for a given barrier strength the conductance can be lowered or enhanced due to phase-coherent quantum interferences in the double-barrier junc-
Figure 3: Double-barrier junction with an aspect ratio $L/W = 3$ and a Fermi wave vector corresponding to $k_F W/\pi = 1.9$. Panel (c) shows the normal state conductance $G_N$, (b) the conductance $G_{NS}$ in the superconducting state, and (c) the normalized conductance $G_{NS}/G_N$ as a function of the normalized barrier strength $Z$. The dephasing parameter $\zeta$ is varied from 0 to 1 in steps of 0.05. Notice the different scales on the conductance axes.
tion. Thus, increasing the coupling to the voltage probe, the dephasing can give rise to both an increasing or decreasing conductance depending on the specific parameters of the double-barrier junction.

5. Discussion and conclusion

In semiconductor structures and also semiconductor-superconductor junctions the conducting properties depend strongly on whether the length scale $L$ of the device is much smaller than the phase-correlation length $L_c$ or it is comparable to $L_c$. Another dephasing mechanism could be a coupling to a voltage probe. In this paper we have studied the effect of coupling a voltage probe to two-probe semiconductor-superconductor structures with two scattering potentials in the normal region.

As a particular example we have studied the cross-over from phase-coherent to sequential tunneling in a double-barrier semiconductor-superconductor junction. For a finite coupling to the voltage probe, we find that size-resonances of course become less pronounced. This may increase or decrease the conductance when increasing the temperature depending on the specific parameters of the double-barrier junction. However, we also find an overall lowering of the conductance and a suppression of the ideal factor-of-two enhancement of the conductance compared to the normal state conductance.

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20. Unitarity yields \( \sum_{q=1}^{M} S_{pq} \{ S_{pq}^{\text{ec}} \}^{\dagger} + S_{pq}^{\text{eh}} \{ S_{pq}^{\text{eh}} \}^{\dagger} = \hat{1} \) (\( \hat{1} \) being an \( N_p \times N_p \) unit matrix) and time-reversal symmetry together with the electron-hole symmetry yield \( \text{Tr} \{ S_{pq}^{\text{ec}} \}^{\dagger} = \text{Tr} \{ S_{qp}^{\text{ec}} \} \) and \( \text{Tr} \{ S_{pq}^{\text{eh}} \}^{\dagger} = \text{Tr} \{ S_{qp}^{\text{eh}} \} \) (at the Fermi level in zero magnetic field); We are grateful to S. Datta (private communication) for pointing out the usefulness of these relations in checking the numerics; See also Refs. [14, 15, 16, 17].
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