The nimbus of away-side jets
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Abstract

The conical structure around the away-side jets is discussed. The equations of in-medium gluodynamics are proposed. Their classical lowest order solution is explicitly shown for a color charge moving with constant speed. For nuclear permittivity larger than 1 it describes the shock wave induced by emission of Cherenkov gluons. The values of real and imaginary parts of nuclear permittivity are estimated from fits of RHIC data. Specific effects at LHC energies are described.

The conical structure around away-side jets has been observed in high-energy central nucleus-nucleus collisions at RHIC [1, 2, 3]. It can be explained as the emission of Cherenkov gluons by a parton passing through a quark-gluon medium. The properties and evolution of the medium are widely debated. At the simplest level it is assumed to consist of a set of current quarks and gluons. The collective excitation modes of the medium may, however, play a crucial role. Phenomenologically their impact would be described by the nuclear permittivity of the matter corresponding to its response to passing partons. Namely this approach is most successful for electrodynamical processes in matter. Therefore, it is reasonable to modify the QCD equations by taking into account collective properties of the quark-gluon medium [4]. Strangely enough, this was not done earlier. For the sake of simplicity we consider here the gluodynamics only.

The classical lowest order solution of these equations coincides with Abelian electrodynamical results up to a trivial color factor. One of the most spectacular of them is Cherenkov radiation and its properties. Now, Cherenkov gluons take the place of Cherenkov photons [5, 6]. Their emission in high-energy hadronic collisions is described by the same formulae but with the nuclear permittivity in place of the usual one. Actually, one considers them as quasiparticles, i.e. quanta of the medium excitations leading to shock waves with properties determined by the permittivity.

Another problem of this approach is related to the notion of the rest system of the medium. It results in some specific features of this effect at LHC energies.

To begin, let us recall the classical in-vacuum Yang-Mills equations

\[ D_\mu F^{\mu\nu} = J^\nu, \quad F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu - ig[A^\mu, A^\nu], \]

where \( A^\mu = A^\mu_a T_a; \) \( A_a(A^0_a = \Phi_a, A_a) \) are the gauge field (scalar and vector) potentials, the color matrices \( T_a \) satisfy the relation \( [T_a, T_b] = if_{abc} T_c, \) \( D_\mu = \partial_\mu - ig[A_\mu, \cdot], \) \( J^\nu(\rho, j) \) a classical source current, the metric \( g^{\mu\nu}=\text{diag}(+,−,−,−). \)

The chromoelectric and chromomagnetic fields are \( E^\mu = F^{\mu0}, \) \( B^\mu = -\frac{1}{2}e^{\mu\nu} F^{\nu j} \) or, as functions of the gauge potentials in vector notation,

\[ E_a = -\text{grad}\Phi_a - \frac{\partial A_a}{\partial t} + gf_{abc} A_b \Phi_c, \quad B_a = \text{curl}A_a - \frac{1}{2}gf_{abc}[A_b A_c]. \]
Herefrom, one easily rewrites the in-vacuum equations of motion (1) in vector form. We do not show them explicitly here (see [4]) and write down the equations of the in-medium gluodynamics using the same method as in electrodynamics. We introduce the nuclear permittivity and denote it also by $\epsilon$, since this will not lead to any confusion. After that, one should replace $E_a$ by $\epsilon E_a$ and get

$$\epsilon(\text{div} E_a - gf_{abc} A_b E_c) = \rho_a, \quad \text{curl} B_a - \epsilon \frac{\partial E_a}{\partial t} - gf_{abc}(\epsilon \Phi_b E_c + [A_b B_c]) = j_a.$$ (3)

The space-time dispersion of $\epsilon$ is neglected here.

In terms of potentials these equations are cast in the form

$$\Delta A_a - \epsilon \frac{\partial^2 A_a}{\partial t^2} = -j_a - gf_{abc}(\frac{1}{2} \text{curl}[A_b, A_c] + \frac{\partial}{\partial t}(A_b \Phi_c) + [A_b \text{curl} A_c] - \epsilon \Phi_b \frac{\partial A_c}{\partial t} - \epsilon \Phi_b \text{grad} \Phi_c - \frac{1}{2} g f_{cmn} [A_b[A_m A_n]] + g \epsilon f_{cmn} \Phi_b A_m \Phi_n),$$ (4)

$$\Delta \Phi_a - \epsilon \frac{\partial^2 \Phi_a}{\partial t^2} = -\rho_a \epsilon + gf_{abc}(2A_b \text{grad} \Phi_c + A_b \frac{\partial A_c}{\partial t} + \epsilon \frac{\partial \Phi_b \Phi_c}{\partial t} + g^2 f_{amn} f_{nbl} A_m A_l \Phi_b.$$ (5)

If the terms with coupling constant $g$ are omitted, one gets the set of Abelian equations, that differ from electrodynamical equations by the color index $a$ only.

The crucial distinction between the in-vacuum and in-medium equations is that there is no radiation (the field strength is zero in the forward light-cone and no gluons are produced) in the lowest order solution in vacuum, and it is admitted in medium, because $\epsilon$ takes into account the collective response (color polarization) of the nuclear matter.

Cherenkov effects are especially suited for treating them by classical approach to (4), (5). Their unique feature is independence of the coherence of subsequent emissions on the time interval between these processes. The lack of balance of the phase $\Delta \phi$ between emissions with frequency $\omega = k/\sqrt{\epsilon}$ separated by the time interval $\Delta t$ (or the length $\Delta z = v \Delta t$) is given by

$$\Delta \phi = \omega \Delta t - k \Delta z \cos \theta = k \Delta z (\frac{1}{v \sqrt{\epsilon}} - \cos \theta) \tag{6}$$

up to terms that vanish for large distances. For Cherenkov effects the angle $\theta$ is

$$\cos \theta = \frac{1}{v \sqrt{\epsilon}}. \tag{7}$$

The coherence condition $\Delta \phi = 0$ is strictly valid independent of $\Delta z$. This is a crucial property specific for Cherenkov radiation only. The fields ($\Phi_a, A_a$) and the
classical current for in-medium gluodynamics can be represented by the product of the electrodynamical expressions ($\Phi, A$) and the color matrix $T_a$.

Let us recall the Abelian solution for the current with velocity $v$ along $z$-axis:

$$j(r, t) = v \rho(r, t) = 4\pi g v \delta(r - vt).$$ (8)

In the lowest order the solutions for the scalar and vector potentials are related $A^{(1)}(r, t) = \epsilon v \Phi^{(1)}(r, t)$ and

$$\Phi^{(1)}(r, t) = \frac{2g \theta(vt - z - r_\perp \sqrt{\epsilon v^2 - 1})}{\epsilon \sqrt{(vt - z)^2 - r_\perp^2 (\epsilon v^2 - 1)}}$$ (9)

Here $r_\perp = \sqrt{x^2 + y^2}$ is the cylindrical coordinate; $z$ symmetry axis. The cone

$$z = vt - r_\perp \sqrt{\epsilon v^2 - 1}$$ (10)

determines the position of the shock wave due to the $\theta$-function in (9). The field is localized within this cone and decreases with time as $1/t$ at any fixed point. The gluons emission is perpendicular to the cone (10) at the Cherenkov angle (7).

Due to the antisymmetry of $f_{abc}$, the higher order terms ($g^3, ...$) are equal to zero for any solution multiplicative in space-time and color as seen from (4), (5).

The expression for the intensity of the radiation is given by the Tamm-Frank formula (up to Casimir operators) that leads to infinity for constant $\epsilon$. The $\omega$-dependence of $\epsilon$ (dispersion), its imaginary part (absorption) $\epsilon_2$ and chromomagnetic permeability can be taken into account [4].

Recently, the experimental data of STAR and PHENIX [1, 2] were fitted [8] with account of the imaginary part of $\epsilon$ and emission of pions and $\rho$-mesons within the Cherenkov cone. The results are presented in Table 1 (for more details see [8]).

| Experiment | $\theta_{\text{max}}$ | $\epsilon_1$ | $\epsilon_2$ |
|------------|----------------------|--------------|--------------|
| STAR       | 1.04 rad             | 3.95         | 0.8          |
| PHENIX     | 1.27 rad             | 9.5          | 1.8          |

The real parts $\epsilon_1$ are quite large while the imaginary parts are small so that $(\epsilon_2/\epsilon_1)^2 \approx 0.04 \ll 1$. Different values of $\epsilon_1$ for STAR and PHENIX are related to different positions of hump maxima in these experiments.

The theoretical attempts to estimate the nuclear permittivity from first principles are not very convincing [9, 10, 11, 6, 12]. Therefore, I prefer to use the general formulae of the scattering theory for the nuclear permittivity. It is related to the refractive index $n$ of the medium $\epsilon = n^2$ and the latter one is expressed [13] through...
the real part of the forward scattering amplitude of the refracted quanta $\text{Re}F(0^\circ, E)$ by

$$\text{Ren}(E) = 1 + \Delta n_R = 1 + \frac{6m^3\nu}{E^2}\text{Re}F(E) = 1 + \frac{3m^3\nu}{4\pi E}\sigma(E)\rho(E).$$  \hspace{1cm} (11)

Here $E$ denotes the energy, $\nu$ the number of scatterers within a single nucleon, $m_\pi$ the pion mass, $\sigma(E)$ the cross section and $\rho(E)$ the ratio of real to imaginary parts of the forward scattering amplitude $F(E)$.

Thus the emission of Cherenkov gluons is possible only for processes with positive $\text{Re}F(E)$ or $\rho(E)$. Unfortunately, we are unable to calculate directly in QCD these characteristics of gluons and have to rely on analogies and on our knowledge of the properties of hadrons. The only experimental facts we get for this medium are brought about by particles registered at the final stage. They have some features in common, which (one may hope!) are also relevant for gluons as the carriers of the strong forces. Those, first, are the resonant behavior of amplitudes at rather low energies and, second, the positive real part of the forward scattering amplitudes at very high energies for hadron-hadron and photon-hadron processes as measured from the interference of the Coulomb and hadronic parts of the amplitudes. $\text{Re}F(0^\circ, E)$ is always positive (i.e., $n > 1$) within the low-mass wings of the Breit-Wigner resonances. This shows that the necessary condition for Cherenkov effects $n > 1$ is satisfied at least within these two energy intervals. This fact was used to describe experimental observations at SPS, RHIC and cosmic ray energies. The asymmetry of the $\rho$-meson shape at SPS \[14\] and azimuthal correlations of in-medium jets at RHIC \[2, 1\] were explained by emission of comparatively low-energy Cherenkov gluons \[15, 16\]. The parton density and intensity of the radiation were estimated. In its turn, cosmic ray data \[17\] at energies corresponding to LHC require very high-energy gluons to be emitted by the ultrarelativistic partons moving along the collision axis \[5\]. The specific predictions at LHC stemming from this observation were discussed elsewhere \[18\]. Let us note the important difference from electrodynamics, where $n < 1$ at high frequencies. The energy of the forward moving partons at LHC would exceed the thresholds above which $n > 1$. Then both types of experiments can be done, i.e. the $90^\circ$-trigger and non-trigger forward-backward partons experiments. The predicted results for $90^\circ$-trigger geometry are similar to those at RHIC. The non-trigger Cherenkov gluons should be emitted within the rings at polar angles of tens degrees in c.m.s. at LHC by the forward moving partons (and symmetrically by the backward ones) according to some events observed in cosmic rays \[17, 16\]. This is the new prediction for LHC.

To conclude, the in-medium gluodynamics leads quite naturally to the prediction of Cherenkov gluons emitted within the nuclear medium if $\epsilon > 1$. The experimental data about the nimbus of away-side jets obtained at RHIC have been well fitted by these formulae with complex nuclear permittivity. Quite large values of its real part are estimated from fits to experimental data. Therefrom one concludes that the density of scatterers $\nu$ is rather high (about 10-20 per a hadron). The imaginary
part is comparatively small. The specific predictions at LHC energies are waiting for their verification.

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