Enhanced $B \to \mu \bar{\nu}$ Decay at Tree Level as Probe of Extra Yukawa Couplings

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With no New Physics seen at the LHC, a second Higgs doublet remains attractive and plausible. The ratio $R_{B}^{\mu\tau} = B(B \to \mu \bar{\nu})/B(B \to \tau \bar{\nu})$ is predicted at 0.0045 in both the Standard Model and the type II two Higgs doublet model, but it can differ if extra Yukawa couplings exist in Nature, which we claim in 1977 [1], removes flavor changing neutral Higgs (FCNH) or extra Yukawa couplings and propose the ratio of $B_{\mu\tau}$ data [5]. In this Letter, we explore the issue of extra Yukawa couplings in general, as it was stressed [2] that the existence of FCNH, or extra Yukawa couplings, if any new symmetry exists, it seems to be at rather high scale. In this light, we ought to reexamine the validity of any presumed symmetry. In this Letter we reflect on the usual discrete $Z_2$ symmetry imposed on two Higgs doublet models (2HDM) to conform with the Natural Flavor Conservation (NFC) condition [1].

Although extra Higgs bosons remain elusive, given the existence of one doublet that contains the element of a second doublet is quite plausible. The type II 2HDM, where up- and down-type quarks receive mass from different doublets, arises automatically with SUSY, making it the most popular 2HDM. The NFC condition, proclaimed in 1977 [1], removes flavor changing neutral Higgs (FCNH) couplings by demanding each mass matrix receives only one contributing Yukawa matrix. But this removes all extra Yukawa couplings that naturally should exist in a 2HDM! With discovery of $m_h < m_t$, however, it was stressed [2] that the existence of FCNH, or extra Yukawa couplings in general, is an experimental issue, as attested by the ATLAS and CMS pursuit [3] of $t \to ch$, $uh$, and further augmented by the hint [4] of $h \to \tau \mu$ in Run 1 data of CMS, although it disappeared with Run 2 data [3]. In this Letter, we explore the issue of extra Yukawa couplings and propose the ratio of $B \to \mu \bar{\nu}$ over $B \to \tau \bar{\nu}$ decay rates, $R_{B}^{\mu\tau}$, as a potential leading probe at the upcoming Belle II experiment.

In the LHC era, a handful of so-called “flavor anomalies” [6] do exist, but the trend at the recent Winter conferences is “softening”. Belle announced a new measurement [7] of the $R_D$ and $R_{D^*}$ ratios (rate of $B \to D^{(*)}\tau\nu$ over $B \to D^{(*)}\mu\nu$) using semileptonic tag, with results consistent with SM, and the world average tension with SM expectation decreases from $3.8\sigma$ to $3.1\sigma$. For the clean $R_K$ ratio (rate of $B \to K^{\mp}\mu^\pm \bar{\nu}$ over $B \to Ke^+\gamma$), using Run 2 data taken in 2016, LHCb finds [8] consistency with SM expectation, which differs from the Run 1 result. In any case, these “anomalies” need confirmation with full Run 2 data, as well as scrutiny by Belle II. Our proposed $R_{B}^{\mu\tau}$ ratio adds to the list, and could be an early NP probe at Belle II.

The Belle experiment recently remeasured [9] $B(B \to \mu \bar{\nu}) = (5.3 \pm 2.0 \pm 0.9) \times 10^{-7}$, (Belle 2019) (1) which supersedes the recently published $(6.46 \pm 2.22 \pm 1.60) \times 10^{-7}$ [10]. Although the central value dropped slightly, the improved systematic error moves the significance up from $2.4\sigma$ to $2.8\sigma$. The $B \to \ell \bar{\nu}$ decay branching fraction in SM is well known,

$$B(B \to \ell \bar{\nu})^{SM} = |V_{ub}|^2 f_B^2 G_F^2 m_B^2 |m_{\ell}| (1 - m_B^2/m_H^2)^2,$$

where the helicity suppression factor, $m_B^2/m_H^2$, makes it quite rare, but also more susceptible to NP effects. Using $f_B = 190$ MeV from FLAG [11], and the exclusive value $|V_{ub}|^{excl.} = 3.70 \times 10^{-3}$ from PDG [12], we find $B(B \to \ell \bar{\nu})^{SM} \sim 3.92 \times 10^{-7}$. Belle data allow a mild enhancement, echoing a decade-old result from BaBar [13] based on a smaller data set.

The effect in 2HDM II is also well known [14],

$$B(B \to \ell \bar{\nu})^{2HDM II} = r_H B(B \to \ell \bar{\nu})^{SM},$$

where

$$r_H = \left(1 - t_3^2 \frac{m_b m_{H_2}^2}{(m_b + m_u) m_{H_1}^2}\right)^2 \equiv \left(1 - t_3^2 \frac{m_{H_2}^2}{m_{H_1}^2}\right)^2,$$

$t_3 \equiv \tan \beta$ is the ratio of vacuum expectation values (v.e.v.) of the two scalar doublets, and $m_{H_1}$ is the mass of $H^+$. As $r_H$ is $m_{t,\tau}$-independent, one has

$$R_{B}^{\mu\tau} = \frac{B(B \to \mu \bar{\nu})}{B(B \to \tau \bar{\nu})} = \frac{m_{H_2}^2 (m_{H_2}^2 - m_{H_1}^2)^2}{m_{H_1}^2 (m_{H_2}^2 - m_{H_1}^2)^2} \approx 0.0045,$$

for both SM and 2HDM II, as stressed in a recent review [15]. It is also independent of $|V_{ub}|$. Together with $B \to \tau \bar{\nu}$ being consistent [12] with SM, one usually does not expect $B(B \to \mu \bar{\nu})$ to deviate from SM, as reflected in the Belle II Physics Book [16] discussion.
Belle II can check to some precision whether $R_{\mu/e}^{\mu/e} \simeq 0.0045$ holds; a deviation would not only be beyond SM, it would rule out 2HDM II convincingly. We shall demonstrate that, in 2HDM without $Z_2$, i.e. if there exist extra Yukawa couplings, the $B \rightarrow \mu\bar{\nu}$ rate can become enhanced, or suppressed, compared with SM expectation, while the $B \rightarrow \tau\nu$ rate would be consistent with SM. Thus, a Belle II measurement of $B(B \rightarrow \mu\bar{\nu})$, plus a refined measurement of $B(B \rightarrow \tau\nu)$, would open up a probe of extra Yukawa couplings that have been of interest lately at the LHC, namely $t \rightarrow c(hb)$ and $h \rightarrow \tau\mu$ search, but involving the $H^+$ boson.

Formalism.— In Eq. (5), we dropped the $\ell$ index to the $\bar{\nu}$ (see Eqs. (4) and (3)), as the $\bar{\nu}_\ell$ flavor is not measured by experiment, which is relevant for 2HDM without $Z_2$. Called 2HDM III earlier [1] to distinguish from the usual 2HDM II or I, 2HDM without $Z_2$ followed the Cheng-Sher ansatz [2], that a trickle-down hierarchical mass-mixing pattern of $\sqrt{m_t m_{tb}}$ may loosen the need for NFC [1] to forbid FCNH couplings. With the discovery of the $h$ boson, a discrete symmetry started to appear ad hoc [2], that the existence of FCNH, such as $tch$ or $h\tau\mu$ couplings, should be up to experiment to decide.

We assume for each type of charged fermion $F = u, d, \ell$, there is a second set of Yukawa matrices $\rho^F_\ell$ from a second scalar doublet, where some trickle-down flavor pattern helps hide the effects, in particular from FNCH couplings. It was revealed recently [11] that “NFC protection against FCNH can be replaced by approximate alignment, together with a flavor organizing principle reflected in SM itself”, and that the Cheng-Sher $\sqrt{m_t m_{tb}}$ pattern may be too strong an assumption.

Approximate alignment emerged at the LHC [20] with Run 1 data, reflecting the fact that the $h$ boson appears rather close [21] to the SM Higgs. This means the two CP-even scalars, $h^0$ and $H^0$, do not mix much, or in 2HDM II notation, the mixing angle $\cos(\alpha - \beta)$ is rather small, which is now affirmed by Run 2 data [22,23]. But in 2HDM without $Z_2$ (which we now call g2HDM), $\tan \beta$ is unphysical, hence we use the cos $\gamma$ notation [19].

The $tch$ coupling is actually $\rho^u_c \cos \gamma$ ($\rho^u_d$ is already constrained by flavor physics to be small [24,25]), while the $h\tau\mu$ coupling is $\rho^\tau \cos \gamma$, and approximate alignment can account for suppressed $t \rightarrow ch$ or $h \rightarrow \tau\mu$ transitions, without invoking tiny $\rho^u_c$ or $\rho^\tau$ couplings for $H^0, A^0$.

The fundamental $H^+$ Yukawa couplings in g2HDM,

$$\bar{u}(V^\dagger R - \rho^u V L)d H^+ - \bar{\nu}(\rho^\tau R)\ell H^+ + \text{H.c.,}$$

are independent of $\cos \gamma$, where $V$ is the CKM matrix, $L, R \equiv (1 \pm \gamma_3)/2$, and $u, d, \ell$ are in matrix notation. They give rise to the branching fraction

$$B(B \rightarrow \ell\bar{\nu}) = B(B \rightarrow \ell\bar{\nu})^{\text{SM}} \times \sum_{\ell' = e, \mu, \tau} \left| \delta_{\ell\ell'} - \frac{m^2_{\ell'}}{m^2_{\ell}} - \frac{m^2_{\ell'}}{m^2_{\ell}} S_{L\ell}' \right|^2,$$

where we sum over $\bar{\nu}_\ell$ flavor explicitly, and $i$ is also summed over. Expanding $\sum_i \rho_i V_{ui} = \rho_{0b} V_{ub} + \rho_{tb} V_{tb} + \rho_{tb} V_{td} \equiv \rho_{0b} V_{ub}$, we drop the $\rho_{0b}$ and $\rho_{tb}$ terms as they are constrained severely at tree level by $B_s$ and $B_d$ meson mixings. Expanding $\sum_i \rho^*_{ti} V_{ib} = \rho^*_{0b} V_{ib} + \rho^*_{tb} V_{ib} + \rho^*_{tb} V_{td} \equiv \rho^*_{0b} V_{ib}$, we drop $\rho_{0b}$ term as it is constrained by $D^0$ mixing, and $\rho_{tb}$ as it is suppressed by mass-mixing hierarchy, while both terms are CKM suppressed. Note that our result is not affected by the PMNS matrix in the neutrino sector, so long that it is unitary.

After some rearrangement, the factor becomes

$$\sum_{\ell' = e, \mu, \tau} \left| \delta_{\ell\ell'} - \frac{m^2_{\ell'}}{m^2_{\ell}} \frac{m^2_{\ell'}}{m^2_{\ell}} S_{L\ell}' \right|^2,$$

where $\lambda_\ell = \sqrt{2} m_{\ell}/v$ is the Yukawa coupling of lepton $\ell$, with $v \simeq 246$ GeV. Although $\bar{m}_b = \sqrt{\bar{m}_b}/v$ is defined similarly, but since it arises as “mass” from hadronic matrix element, we need to run $\bar{m}_b$ to $m_H$ scale. We note that, taking $\rho_{tb} = -\lambda_\tau \tan \beta$ and $\rho_{\ell \ell'} = -\lambda_\tau \tan \beta \delta_{\ell\ell'}$, and setting the $\rho_{tb}$ term to zero, one recovers the usual $\tau\mu$ factor of 2HDM II. But given the $V_{tb}/V_{ub}$ enhancement factor, one can ignore $\rho_{tb}/\bar{m}_b = O(1)$ in g2HDM, so long that $\rho_{tb}/\bar{m}_b$ is not as small as $|V_{tb}/V_{ub}| \simeq 0.004$.

In evaluating $\bar{m}_b$ at $m_H$ scale, we follow PDG [22] by first calculating the $\bar{M}\bar{S}$ running mass $m_{\bar{m}}(m_b)$ at the pole mass, then evolve to scale $\mu = m_H$ by $m_{\bar{m}}(\mu) = \mathcal{O}((\alpha_s(\mu))/\mathcal{O}(\alpha_s(m_b))) m_{\bar{m}}(m_b)$, where the function $\mathcal{O}(x)$ is taken with four-loop accuracy and we use $\bar{M}\bar{S}$ three-loop scale at $\mu$ in five-flavor scheme.

Results.— To make contact with experiment, we recast in the notation of BaBar’s $R_{\text{B}}(\mu)$ paper [25], the form of which the new Belle analysis [9] has followed,

$$B(B \rightarrow \ell\bar{\nu}) = B(B \rightarrow \ell\bar{\nu})^{\text{SM}} \times \sum_{\ell' = e, \mu, \tau} \left| \delta_{\ell\ell'} - \frac{m^2_{\ell'}}{m^2_{\ell}} S_{L\ell}' \right|^2,$$

where $S_{L\ell}'$, at $m_b$ scale, are the ratios of NP Wilson coefficients (of 4-Fermi operators) with SM ones. We note that a $+S_{L\ell}'$ contribution, proportional to $\rho_{tb}$ as seen in Eq. (5), is negligible in g2HDM because of the $|V_{tb}/V_{ub}|$ enhancement of $S_L$. Keeping both $S_{L\ell}$ and $S_{L\ell}'$, Belle had to assume reality to make a 2D plot [9]. But, sourced in Yukawa couplings, they are clearly complex.

Including the subtlety of scale dependence for $\bar{m}_b$, we have the correspondence,

$$S_{L\ell}' = \frac{m_b(\mu_0)}{m_b(\mu)} v^2 \rho_{\ell \ell'} \rho^*_{tb} V_{tb}/V_{ub},$$

where $\mu_0$ is at $m_H$ scale. We note that Yukawa couplings are dimension-4 terms in the Lagrangian. For leptonic $B^+$ decay, QCD correction is easy to match with 4-Fermi operators. But it was the insight on $|\rho_{tb}|$ vs $|\rho_{tb}^* V_{tb}/V_{ub}|$ that allowed us to drop the $S_{L\ell}$ term.
Ignoring $\ell^\prime = e$, i.e. taking $\rho_{\ell\ell^\prime}$ as negligible, we get,

$$B(B \to \mu \bar{\nu}) = B(B \to \mu \bar{\nu})^{SM} \times \left( 1 - \frac{m_B^2}{m_b m_\mu} S_L^{\mu \mu} + \frac{m_B^2}{m_b m_\mu} S_L^{\mu \mu} \right),$$  \hspace{1cm} (11)$$

where the $S_L^{\mu \mu}$ effect from diagonal $\rho_{\mu \mu}$ coupling interferes with SM (the “1”), while the $S_L^{\mu \mu}$ effect from off-diagonal $\rho_{\tau \mu}$ adds in quadrature. Setting $\rho_{\mu \mu} = 0$, we illustrate in Fig. 1 (left) projection of Belle $B \to \mu \bar{\nu}$ result in $|S_L^{\mu \mu}| - \phi_\mu$ plane; and (right) $B(B \to \mu \bar{\nu})$ vs $|S_L^{\mu \mu}|$ (blue dashed line). The $\pm 1\sigma$ and $\pm 2\sigma$ allowed regions are in dark and light pink shades, and black dotted (red solid) line denotes Belle central (SM) value. The gray shaded region below SM value in right panel is theoretically inaccessible. The MS mass $m_\mu(m_b)$ is used.

The Belle result on $|S_L^{\mu \mu}|$, Eq. (11), is consistent with SM, and would be measured in due time by Belle II [16]. But if one has $+1\sigma$ or even $+2\sigma$ enhancement over the central value, then earlier discovery is possible. Belle data constrain $|S_L^{\mu \mu}| \lesssim 0.019$, the same as imaginary $S_L^{\mu \mu}$ in Fig. 1 (left).

Using $|S_L^{\mu \mu}| = 0$, we illustrate in Fig. 2 (left) the Belle average $B(B \to \tau \bar{\nu}) \simeq (9.1 \pm 2.2) \times 10^{-5}$ from PDG in the $|S_L^{\tau \tau}| - \phi_{\tau \tau}$ plane, with notation analogous to Fig. 1. Likewise, Fig. 2 (right) plots $B(B \to \tau \bar{\nu})$ vs $|S_L^{\tau \tau}|$, with $S_L^{\tau \tau} = 0$. Note that the bands in Fig. 2 appear narrower not so much as due to plotting, but reflects $B(B \to \tau \bar{\nu})$ being better measured than $B(B \to \mu \bar{\nu})$.

Interpretation in g2HDM.— Interpreting in the fundamental g2HDM, that plausibly holds extra Yukawa couplings sheds light on the underlying physics.

The Belle result on $B(B \to \mu \bar{\nu})$, Eq. (11), is consistent with SM, and would be measured in due time by Belle II [16]. But if one has $+1\sigma$ or even $+2\sigma$ enhancement over the central value, then earlier discovery is possible. From Fig. 1 we find $+1\sigma$ to $+2\sigma$ enhancement correspond to $|S_L^{\mu \mu}| \in (0.006, 0.009)$ for the constructive (negative) case, $0.015, 0.019$ for the imaginary case, and $(0.038, 0.041)$ for the destructive (positive) case. The incoherent, second effect of Eq. (11) has the same parameter range as imaginary $S_L^{\mu \mu}$, but for $|S_L^{\mu \mu}| \in (0.015, 0.019)$, i.e. with $\rho_\tau$ index.

From Eq. (10), we find

$$|S_L^{\mu \mu}| \simeq 150|\rho_{\ell \ell^\prime} \rho_{\mu \mu}|(300 \text{ GeV}/m_H)^2.$$  \hspace{1cm} (13)$$

We take $m_H = 300$ GeV as benchmark for sake of the largest effect, but also because the usual $H^+$ mass bound from 2HDM II does not apply (some discussion can be found in Refs. [24] and [28]), given the many new flavor parameters. The most promising constructive case of $|S_L^{\mu \mu}| \in (-0.009, -0.006)$ needs $|\rho_{\mu \mu} \rho_{\mu \mu}| \simeq (4-6) \times 10^{-5}$, and would grow as $(m_H/300 \text{ GeV})^2$. So, what do we know, or can infer, about $|\rho_{\mu \mu}|$ and $|\rho_{\mu \mu}|$? Given that $H^+$ effect is normalized to SM, Eq. (8) offers a clue: $\rho_{\mu \mu}$ is “normalized” against $\lambda_{\mu} \simeq 0.0006$, the charged lepton...
Yukawa coupling, while $\rho_{\mu}$ is normalized to $\hat{m}_b \sim 0.015$, the “effective” Yukawa coupling from $m_b$ evaluated at $m_H$ scale. The combined $\lambda_b \hat{m}_b \sim 1 \times 10^{-5}$ suggests the $S^\mu_L$ mechanism falls short of enhancing $B(B \to \mu \bar{\nu})$ even for the most optimistic case, let alone the larger $|S^\mu_L|$ needed for imaginary or destructive cases.

Before turning to more detailed scrutiny of plausible $\rho_{\mu}$ and $\rho_{ttu}$ strengths, it is instructive to take a look at the second mechanism, i.e. via $\bar{\nu}_\tau$ flavor. $|S^\tau_L|^2 \in (0.015, 0.019)$ is the same as the incoherent case of imaginary $S^\mu_L$, hence $|\rho_{\tau \mu} \rho_{tt}| \gtrsim 10^{-4}$ may not appear promising at all. However, recall that up until early 2017, values of $|\rho_{\tau \mu}|$ as large as 0.26 had been entertained, which was due to the hint for $h \to \tau \mu$ in CMS Run 1 data. Although the hint disappeared with Run 2 data, we caution that it could reflect approximate alignment, i.e. a small cosine value. As we discuss below, if we allow $\rho_{\tau \mu} \sim \lambda_{\tau}$, then the combined $\lambda_{\tau} \hat{m}_b \sim 1.5 \times 10^{-4}$ does seem to allow the $\rho_{\tau \mu}$ mechanism to enhance $B(B \to \mu \bar{\nu})$.

Taking a closer look at the possible strength of $\rho_{\mu \mu}$, we find $\rho_{\mu \mu} = \mathcal{O}(\lambda_{\mu})$ reasonable. This is because $\lambda_{\mu}$ arises from diagonalizing the mass matrix, but $\rho_{\mu \mu}$ comes from an orthogonal combination of the two unknown Yukawa matrices, going through the same diagonalization procedure. To avoid fine tuning, these two Yukawa matrices must each contain the “flavor organization” reflected in mass-mixing hierarchies, hence $\rho_{\mu \mu} = \mathcal{O}(\lambda_{\mu})$.

We treat $\rho_{\tau \mu}$ more liberally, as argued above for a rough yardstick of $|\rho_{\tau \mu}| \sim \lambda_{\tau} \sim 0.01$, since much larger $\rho_{\tau \mu}$ values have been considered only recently. The most relevant constraint actually comes from $\tau \to \mu \gamma$, where the two-loop mechanism constrains $|\rho_{\tau \mu}| \lesssim 0.01$ for $\rho_{tt} \sim 1$, but the bound weakens for weaker $\rho_{tt}$. Finally, having $|\rho_{\tau \mu}|$ up to $|\rho_{\tau \tau}| \sim \lambda_{\tau}$ is not unreasonable, just as $|\rho_{\mu \tau}|$ could be up to $|\rho_{\tau \tau}| \sim \lambda_{\tau}$, where $\rho_{tt}$ and $\rho_{tc}$ provide two possible CP violating sources for electroweak baryogenesis, which strongly motivates g2HDM. Thus, we suggest that $|\rho_{\tau \mu}| \lesssim 0.02$ is reasonable, and in any case, its value is an experimental issue.

For the common $\rho_{tu}$ factor, things are harder to discern. Taking $|\rho_{tu}| \sim \sqrt{2 m_\tau m_b} / v \sim 0.003$ would be a bit small, but it need not be that small, since the direct $t \to u \bar{u} h$ search bound \cite{[3]} is not so different from $t \to c h$, hence is still quite forgiving. In lack of a true yardstick, we take $|\rho_{tu}| \lesssim \hat{m}_b$ as a reasonable range.

Thus, even taking $|\rho_{\mu \mu}| \sim 3 \lambda_{\mu}$ and $|\rho_{tu}| \sim 2 \hat{m}_b$, $|\rho_{\mu \mu} \rho_{ttu}| \sim 5 \times 10^{-5}$ is only borderline in enhancing $B(B \to \mu \bar{\nu})$ for the most optimistic, constructive case, and in general would not quite suffice. However, even modest values of $|\rho_{\tau \mu}| \lesssim \lambda_{\tau}$ and $|\rho_{tu}| \lesssim \hat{m}_b$ give $|\rho_{\mu \mu} \rho_{ttu}| \lesssim 1.5 \times 10^{-4}$, allowing reasonable outlook for enhanced $B(B \to \mu \bar{\nu})$, even though the mechanism comes only in quadrature. For higher $m_H$, e.g. 500–600 GeV, enhancement can still be possible with $|\rho_{ttu}|$ and $|\rho_{tu}|$ at the upper reaches of our suggested range.

Turning to $B \to \tau \bar{\nu}$, we take $\rho_{\tau \tau} = \mathcal{O}(\lambda_{\tau})$ and $\rho_{\mu \tau} \sim \lambda_{\tau}$. For constructive case, we see from Fig. [2] \cite{[1]} that $S^\tau_L \in (-0.065, -0.035)$ for +1σ to +2σ enhancement, which suggests $|\rho_{\tau \tau} \rho_{tt}| \sim (2.3-4.3) \times 10^{-4}$ for $m_H \approx 300$ GeV. But with $|\rho_{\tau \tau} \rho_{tt}| \sim \lambda_{\tau} \hat{m}_b \sim 1.5 \times 10^{-4}$, it falls short of enhancement. Enhancement from SM is possible in the constructive case only when $|\rho_{\tau \tau} \rho_{tt}|$ is at the upper reaches of $\sim 6 \lambda_{\tau} \hat{m}_b$, but gets easily damped by larger $m_H$. For the second effect, Fig. [2] \cite{[1]} suggests $|S^\tau_L|^2 \in (0.15, 0.2)$, much larger than the constructive case. We find from $\rho_{\mu \tau} \rho_{tt} \lesssim \lambda_{\tau} \hat{m}_b$ that this mechanism cannot enhance $B \to \tau \bar{\nu}$ rate.

Thus, one expects $B(B \to \tau \bar{\nu})$ in g2HDM to be consistent with SM. On the other hand, $B(B \to \mu \bar{\nu})$ could be better enhanced from SM expectation.

**Discussion.**— $K \to \mu \bar{\nu}$ decay is not constraining as both coherent and incoherent effects are suppressed by $|V_{us} V_{ub} / V_{ub} V_{us}| (m_K^2 \hat{m}_b / m_b^2 \hat{m}_s) \sim 0.0003$, while $K \to e \bar{\nu}$ is even more SM-like. The same argument goes with pion decays, and the effect in $D^+, D_s$ decays is also rather weak. For $B_c$, we do not see how $B_c \to \mu \bar{\nu}$, $\tau \bar{\nu}$ rates can be measured. Thus, $B \to \mu \bar{\nu}$ provides the unique probe of extra Yukawa couplings of g2HDM, whereas $B \to \tau \bar{\nu}$ is expected to be SM-like. Taking the $R_B^{\mu \tau}$ ratio eliminates the main uncertainties associated with $|V_{ub}|$. It is
interesting that the $S^\mu_B$ mechanism could also suppress $B(B \to \mu \bar{\nu})$ (see lower left region of Fig. 1[1]), but would take a longer time for Belle II to uncover.

What about $\mu \to e \nu \bar{\nu}$ and $\tau \to \ell \nu \bar{\nu}$ decays? As these are dominated by $V-A$ theory, the vector currents couple via $g \sim O(1)$, without helicity suppression. In contrast, since $|\rho_{\mu\mu}| \lesssim |\rho_{\tau\tau}| = O(\lambda_\tau) \ll g$ are the largest Yukawa couplings that enter, together with $M^2_W/M^2_H$ suppression, Nature has quite an effective mechanism in hiding the extra Yukawa coupling effects in the lepton sector. For example, given the extreme lightness and abundance of the electron, $\rho_{e\mu}$ and $\rho_{e\tau}$ must be very small, we expect $\mu \to e \nu \bar{\nu}$ and $\tau \to e \nu \bar{\nu}$ to be SM-like to high precision. Similar arguments hold for $B \to X_u \ell\bar{\nu}, \pi \ell \bar{\nu}$ decays, which are plagued further by hadronic uncertainties.

Nature does hide well the effect of extra Yukawa couplings in $H^+\to\ell\nu$ mediated low energy processes. For $B \to \mu \bar{\nu}$, it is more helicity suppressed than $B \to \tau \bar{\nu}$, with $\rho_{\tau\mu}$ giving $\mu_\tau$ final state, and $b \to u$ transition on quark side giving $V_{tb}/V_{ub}$ enhancement of $\rho_{\mu\nu}$, both of which can happen only in g2HDM. Our imprecise knowledge of these two FCNH couplings allow for enhancement $B(B \to \mu \bar{\nu})$ probes the extra Yukawa coupling combination $\rho_{\tau\mu}\rho_{\mu\nu}$.

But early measurements that found $B \to \tau \bar{\nu}$ 

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