Brane Supersymmetry Breaking and the Cosmological Constant: Open Problems

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\textbf{Abstract}

It has recently been argued that non–BPS brane world scenarios can reproduce the small value of the cosmological constant that seems to have been measured. Objections against this proposal are discussed and necessary (but not sufficient) conditions are stated under which it may work. At least \( n = 2 \) extra dimensions are needed. Also, the mass matrix in the supergravity sector must satisfy \( \text{Str} \ M^2 = 0 \). Moreover, the proposal can be ruled out experimentally if Newton’s constant remains unchanged down to scales of \( 10 \ \mu m \). If, on the other hand, such a “running Newton constant” is observed, it could provide crucial experimental input for superstring phenomenology.

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1. Introduction

The energy density $\rho$ of the vacuum enters Einstein’s equations in the form of an effective cosmological constant $\lambda$:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \lambda g_{\mu\nu} \quad \text{with} \quad \lambda = 8\pi G \rho , \quad (1.1)$$

where $G$ is Newton’s constant. Such a cosmological constant $\lambda$ curves four-dimensional space–time, giving it a curvature radius $R_{\text{curv}}$ of order $\lambda^{-\frac{1}{2}}$. Only part of the curvature of the universe is due to the cosmological constant; other contributions come from visible and dark matter. This is accounted for by a factor $3\Omega_\Lambda$:

$$3\Omega_\Lambda R_{\text{curv}}^{-2} \sim \lambda .$$

$\Omega_\Lambda$ seems to have been measured to be $\Omega_\Lambda \sim \frac{2}{3}$ [1]. For a spatially flat universe, which is the case that seems to be realized in nature, this curvature radius implies an expanding universe with metric

$$ds^2 \sim -dt^2 + e^{Ht} d\vec{x}^2 \quad \text{with} \quad 3\Omega_\Lambda H^2 \sim \lambda . \quad (1.2)$$

The Hubble constant $H$ in our universe is of order

$$H \sim 10^{-33} \text{eV} \sim (10^{26} \text{m})^{-1}.$$ 

The cosmological constant problem is the question why this is so small, given that we expect much larger contributions to the energy density of the vacuum from Standard Model physics. Let us briefly recall why we expect such contributions [2].

First of all, there are classical effects. E.g., in the course of electroweak symmetry breaking the Higgs field is supposed to roll down its potential, thereby changing the vacuum energy density by an amount of order $(\text{TeV})^4$. This would give a curvature radius of the universe in the millimeter range, which is not what we observe. Of course, the Higgs potential could be shifted so that its Minimum is at zero. Equivalently, a bare cosmological constant could be added in (1.1) that exactly cancels the cosmological constant induced by $\rho$. But this is just the fine–tuning problem: why should the bare cosmological constant be fine–tuned to exactly cancel the contributions from later phase transitions?
Even if we do find a reason why the minimum of the Higgs potential should be zero, the problem does not go away. There are other condensates in the Standard Model, such as the chiral and gluon condensates in QCD after chiral symmetry breaking. They should contribute an amount of order \( (\Lambda_{QCD})^4 \) to the cosmological constant, which would translate into a curvature radius of the universe in the kilometer range.

And even if one did not believe in chiral and gluon condensates, there would still remain what is perhaps the most mysterious aspect of the problem: the cosmological constant does not seem to receive contributions from the zero–point energy of the Standard Model fields. A harmonic oscillator has a ground state energy \( \frac{1}{2} \hbar \omega \). In a field theory we have one oscillator for each momentum \( k \) with frequency

\[
\omega = \sqrt{k^2 + m^2}.
\]

In the case of the photon (with \( m = 0 \)), integrating over \( k \) gives, for each of the two polarizations, a divergent contribution

\[
\lambda = 8\pi G \int_{|k| \leq \Lambda} \frac{d^3k}{(2\pi)^3} \frac{1}{2} \hbar \omega \sim \frac{1}{2\pi} \frac{\Lambda^4}{m_{Pl}^2}
\]

(1.3)

to the cosmological constant, where \( \hbar G = \frac{m_{Pl}^2}{m} \) and \( \Lambda \) is a large–momentum cutoff. How large does this cutoff \( \Lambda \) have to be? \([1.3,1.2]\) tell us that \( \Lambda \) is, on a logarithmic scale, half–way between the Planck mass and the Hubble constant. So \( \Lambda \) is several milli-eV, corresponding to a Compton wave length in the micrometer range. This is even larger than the wave–length of visible light: we can see with our bare eyes that there is no such cutoff in nature!

There is one idea that can explain a zero cosmological constant: supersymmetry. In supersymmetric theories, the contributions from bosons and fermions to \( \lambda \) have opposite signs and cancel. But in order to explain the smallness of the Hubble constant, supersymmetry would have to remain unbroken at least up to scales of the order of the above cutoff \( \Lambda \), i.e. up to milli-eV scales (see figure 1), which is of course ruled out for the Standard Model.

It has been emphasized that the nearly vanishing cosmological constant should be the main clue as to how nature breaks supersymmetry [3]. Based on [1, 2, 3], it has recently been argued [4] that supersymmetry breaking may generate only a tiny cosmological constant, if we assume that we live on a non–BPS 3–brane soliton that is embedded in a \((4 + n)\)–dimensional supergravity background. This situation can be obtained by compactifying
some string theory on a \((6 - n)\)-dimensional manifold. In such “brane world models,” there is a fundamental difference between the Standard Model fields, which originate from and live only on the brane, and the supergravity fields, which are allowed to propagate everywhere in the bulk.

What has been argued is the following (see [7]):

1. Supersymmetry remains indeed unbroken up to micrometer scales, but only in the bulk supergravity sector.

2. On the brane, supersymmetry is broken at much larger scales of order \(TeV\). As in [8], the brane is in fact assumed to be the origin of supersymmetry breaking: we start with a supersymmetric string compactification that is perturbed by one or more stable 3-brane solitons, parallel to the 4 uncompactified coordinates; those solitons are assumed to be non-BPS, so they break all supersymmetries (and not just half of them).

3. In this situation, where supersymmetry breaking originates in the matter sector (on the brane) and is transmitted to the gravity sector (the bulk), one naturally expects the corresponding scales \(m_{\text{brane}}\) and \(m_{\text{bulk}}\) of supersymmetry breaking to be related by \(m_{\text{bulk}} \sim (m_{\text{brane}}^2/m_{\text{Pl}})\) [8]. Thus, \(m_{\text{brane}}\) is half-way between \(m_{\text{bulk}}\) and \(m_{\text{Pl}}\) on a logarithmic scale (see figure 1). If \(m_{\text{brane}} \sim TeV\), then supersymmetry in the bulk supergravity sector remains unbroken up to scales of order \(m_{\text{bulk}} \sim meV\) (milli-eV).

4. Supersymmetry breaking on the brane creates a huge brane vacuum energy of order \((TeV)^4\). But as in the mechanism of Rubakov and Shaposhnikov [4], it has been argued that this brane vacuum energy does not curve the four-dimensional spacetime parallel to the brane and is instead absorbed by the curvature transverse to the brane.

5. The only fields whose vacuum energy contributes to the four-dimensional curvature (and therefore to the cosmological constant) are then the bulk supergravity fields. Under two important assumptions that will be stated, with bulk supersymmetry being broken at \(meV\) (milli-eV) scales this indeed gives a Hubble constant of roughly the observed order of magnitude.
6. Given the values of the Hubble constant and the Planck mass, we can reverse the argument and try to predict the precise masses of superpartners not only on the brane, but also in the bulk. This mass spectrum of superpartners of the graviton in the milli–eV range should be observable in measurements of Newton’s constant in the micrometer range. In our scenario, these predictions depend on the detailed non-BPS soliton solution. Therefore the spectrum of supergravity masses should offer a window through which we can probe what kinds of branes our string compactification contains.

\[
\begin{array}{cccc}
10^{19}\text{GeV} & \text{TeV} & \text{meV} & 10^{-33}\text{eV} \\
\hline
\text{Planck scale} & \text{brane susy} & \text{bulk susy} & \text{Hubble constant breaking breaking} \\
\end{array}
\]

Figure 1: Evenly spaced hierarchies on a logarithmic scale.

In this note, these steps will be reviewed in more detail and the following hidden assumptions and objections against this scenario will be discussed:

1. **The issue of boundary conditions:** due to obstructions to finding a global solution for the background geometry, it has been argued [9] that the Rubakov-Shaposhnikov mechanism cannot be extended to models with a single extra dimension [10]. It will be argued that in order to avoid similar problems, at least two extra dimensions are needed.

2. **The issue of nearby curved solutions:** it remains an open question whether supersymmetry in the bulk is good enough to ensure that the solution where all of the brane vacuum energy is absorbed in the curvature transverse (rather than parallel) to the brane is the stable one.
3. It must be demonstrated in concrete examples that $m_{\text{bulk}}$ is really in the milli–eV range. Even when this is the case, the bulk fields produce a curvature radius of the universe that is too big (of solar system scales), unless $\text{Str } M_{\text{bulk}}^2 = 0$.

4. It must be assumed that Kaluza Klein modes do not contribute to the 4–dimensional cosmological constant, e.g. because their mass splittings might be very small.

5. Our scenario can be ruled out experimentally if Newton’s constant is observed to remain unchanged down to scales of $10 \mu m$. This is more an advantage than a problem.

Points 1 and 2 refer to the Rubakov–Shaposhnikov mechanism in classical supergravity and are discussed in section 2. Points 3–5 refer to one–loop supergravity corrections and are discussed in section 3. Section 4 contains an (optimistic) conclusion.

That the Rubakov–Shaposhnikov mechanism should be combined with supersymmetry away from the brane to adress the “first cosmological constant problem” (why $\lambda$ is zero [11]) was – as far as the author is informed – first suggested in [6]. What was argued in [7] is that if this classical mechanism works, then one–loop supergravity corrections can also solve the “second cosmological constant problem” (why $\lambda$ is tiny but nonzero), and that this could be tested by looking for superpartners of the graviton with Compton wavelengths in the micrometer range.

2. Classical Objections

In this section we discuss the classical approximation to the bulk supergravity theory. The world–brane theory, though, is assumed to be treated to all orders in and beyond perturbation theory.

Let us first make the setup more precise. We assume that the $n$ extra dimensions are compactified on some compact manifold $\mathcal{M}$. The brane itself is a soliton solution of the supergravity equations of motion; its characteristic size should be of order $\sqrt{\alpha'}$, the fundamental scale in string theory. In this setup it is natural to identify $\sqrt{\alpha'}$ with the (inverse) scale $m_{\text{brane}}$ of supersymmetry breaking in the Standard Model (on the brane), since the brane is
the origin of supersymmetry breaking. There is one free parameter in our discussion,

$$\tau^2 = \frac{Vol(B)}{Vol(M)} \sim \frac{m_{brane}^2}{m_{Pl}^2},$$

the ratio of the volumes of $M$ and of $B$, the ball in which the brane intersects $M$. $\tau$ is related to the hierarchy between the Planck scale and the supersymmetry breaking scale in the Standard Model similarly as in [12, 13]. Our scenario will predict $\tau \sim 10^{-15}$. We do not attempt to explain the origin of this hierarchy; at best, we hope to reduce the cosmological constant problem to the hierarchy problem.

Non–BPS stable branes have been presented, e.g., in [14, 15], and in the present context in [16]. Although these branes are stable in the sense that they have no world–brane tachyons, they are not stable under variations of the moduli of the compactification geometry. The models in [17, 18] might be viable, provided the tachyons appearing there can indeed be interpreted as Higgs fields. Of course, if no truly stable non–BPS solitons can be found, one can always make the ansatz that one of the BPS branes in a supersymmetric string compactification that involves space–time filling 3–branes is perturbed out of the BPS ground state, then relaxes back slowly to the supersymmetric equilibrium.

Let us now discuss the mechanism that has been held responsible for absorbing the brane vacuum energy in the bulk. For the metric near the brane, we make the ansatz

$$ds^2 = dr^2 + e^{2\alpha(r)} \hat{g}_{\mu\nu} dx^\mu dx^\nu + \tilde{g}_{ab} dx^a dx^b,$$

where $\mu, \nu \in \{0, 1, 2, 3\}$, $a, b \in \{4, ..., 3 + n\}$, $r$ is the distance from the brane, $\hat{g}_{\mu\nu}$ is proportional to the metric of the four–dimensional low–energy effective theory and $\tilde{g}$ is the metric on the compactification manifold $M$.

The $4 + n$–dimensional spacetime can be split into two regions: the region near the brane, where supersymmetry is broken; and the bulk region, where supersymmetry in the four–dimensional sense (on slices parallel to but far away from the brane) is assumed to be unbroken. The classical supergravity equations of motion can be solved separately in both regions and can then be matched at their boundary.

In the bulk region, unbroken four–dimensional supersymmetry implies

$$\hat{R}_{\mu\nu} = 0,$$
at least in the absence of four–form field strengths or expectation values of other supergravity fields.

As for the brane region, there should be a horizon near $r = 0$, where the classical supergravity approximation breaks down. We therefore restrict the discussion to a region $r > \epsilon$ where derivatives with respect to $r$ are small, with some cutoff $\epsilon$. The issue of boundary conditions at $r = \epsilon$ will be discussed shortly.

To lowest order in derivatives with respect to $r$, the Einstein equations are

\[ R_{mn} - \frac{1}{2} g_{mn} R = \lambda(r) g_{mn}. \]

Here $m, n \in \{0, \ldots, 3 + n\}$, and $\lambda(r)$ is the cosmological constant in the vicinity of the brane that is created by brane supersymmetry breaking and by the flux that emanates from the brane in the case where the brane carries some charge. We can split

\[ R_{\mu\nu} = \hat{R}_{\mu\nu} - e^{2\alpha} \hat{g}_{\mu\nu} \{\Box^{(n)} + 4(\nabla^{(n)} \alpha)^2\} \]
\[ R_{ab} = \tilde{R}_{ab} + 4\{\nabla_a \nabla_b \alpha + \nabla_a \alpha \nabla_b \alpha\}, \]

where $\Box^{(n)}, \nabla^{(n)}$ act on the closed transverse space. Plugging this into the Einstein equations yields:

\[ \hat{R}_{\mu\nu} = k^2 \hat{g}_{\mu\nu} \]
\[ k^2 = e^{2\alpha} \left\{-\frac{2}{2 + n} \lambda(r) + 4(\nabla^{(n)} \alpha)^2 + \Box^{(n)} \alpha\right\} \]
\[ \tilde{R}_{ab} = -\frac{2}{2 + n} \lambda \tilde{g}_{ab} - 4\{\nabla_a \nabla_b \alpha + \nabla_a \alpha \nabla_b \alpha\}. \]

$k^2$ is an integration constant which, by definition, is the four–dimensional cosmological constant. If solutions to these equations can be found for all $k$, then there is a one–parameter family of possibilities, labelled by $k$, of what one can do with the brane cosmological constant $\lambda(r)$: at one end of this one–parameter family, the transverse curvature described by $\hat{\alpha}$ and $\tilde{R}$ is zero and $\lambda(r)$ goes into the four–dimensional cosmological constant $k^2$. At the other end, $k^2$ is zero and the brane cosmological constant $\lambda(r)$ is completely absorbed by the transverse curvature.

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1If there was a four–form field strength $F^{(4)}$, one could have a supersymmetric solution with $\hat{R}_{\mu\nu} \sim F^{(4)}_{\mu\alpha\beta\gamma} F^{(4)*}_{\nu\alpha\beta\gamma}$. 

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This is the mechanism of Rubakov and Shaposhnikov [4]. They remarked that they had merely geometrically reformulated the fine-tuning problem: now the question is, why should \( k \) be fine-tuned to be exactly zero? But in our case, it has been argued that matching to the solution in the region away from the brane picks the solution with \( k = 0 \) [3].

In the following we mention two potential problems with this step of the argument. First, the issue of boundary conditions, and second, the question whether supersymmetry in the bulk is good enough to select the solution with \( k = 0 \).

As for the first problem, if we want to have a closed compactification manifold, we must actually impose boundary conditions on \( \alpha(r) \) and \( \tilde{g}(r) \) both at \( r = \epsilon \) and at large \( r \). But there may be global obstructions to our ability to impose such boundary conditions.

As an illustration of the type of obstructions that may occur, consider the case of two extra dimensions, i.e., branes of co-dimension 2. Those are analogous to cosmic strings, and they typically produce a deficit angle \( \gamma \) in the transverse two-dimensional metric \( \tilde{g} \). The boundary condition at \( r = \epsilon \) in this case says that the deficit angle is

\[
\gamma \propto T ,
\]

where \( T \) is the brane tension. To get a closed geometry we must have several branes with deficit angles \( \gamma_i \). The deficit angles must add up to \( 2\pi \) (or a multiple \( n \) thereof), so there is a sum rule, analogous to that in [4]:

\[
2n\pi = \sum_i \gamma_i \propto \sum_i T_i .
\]

It can be satisfied for BPS branes, where the deficit angle is fixed by a BPS bound and often a rational fraction of \( 2\pi \). But if we break supersymmetry on one of the branes, then the brane cosmological constant, and thereby the brane tension and the deficit angle are out of control. Since only a discrete set of brane cosmological constants can be absorbed in the transverse curvature, there is no continuous “Rubakov–Shaposhnikov modulus” that can be adjusted to absorb this arbitrary brane tension.

However, there does not seem to be any such global obstruction to absorbing an arbitrary tension for branes of codimension 3 or more. In those cases, the transverse geometry can simply react to an increase in brane tensions by changing its shape and size. The above
discussion is over–simplified, because there is also the warp factor \( \alpha(r) \) that enters the equations (2.3–2.5). As demonstrated in [4] (there in the case of an \( r \)–independent \( \lambda \)), if \( \alpha \) is included, then there may be ranges of \( \lambda \) where solutions with compact transverse manifolds exist even for \( n = 2 \).

As discussed in [9], however, the situation is worse in the case of a single extra dimension. In this case, either the fine–tuning problem is hidden in the boundary conditions near a singularity, or the extra dimension is noncompact. We assume a compact manifold \( \mathcal{M} \), so let us assume in the following that the number of extra dimensions is

\[
n \geq 2
\]

and that \( k \) is indeed a modulus of the classical solution that can be adjusted to zero.

But in this case we still have the second problem mentioned above. There are nearby curved solutions with \( k \neq 0 \). What picks the solution with \( k = 0 \)? The argument that has been given above for selecting \( k = 0 \) is supersymmetry in the bulk. But won’t supersymmetry breaking on the brane backreact on the bulk already at the classical supergravity level, thus invalidating this argument?

If there was no warp factor, this would certainly be the case. The brane would want to have \( k \) of order \( TeV \), while the bulk – being supersymmetric – would want to have \( k = 0 \). The bulk – being much larger – dominates (equivalently, the 4d Newton constant is small), but the “compromise” between bulk and brane would still be \( k \sim (mm)^{-1} \), which would be way too big. Due to the warp factor, though, at least at the classical supergravity level the brane doesn’t care, while the bulk still wants to be supersymmetric. So it seems the compromise could be \( k = 0 \).

But even if so, what about supergravity loop corrections? The modulus \( k \), like any modulus in a string compactification, will become an effective 4d field, which may receive a complicated potential from loops of the bulk fields. Even if this potential is small due to the smallness of Newton’s constant, it is not clear that the potential is such that its minimum is at \( k = 0 \). Unfortunately this remains an open problem of the scenario outlined in section 1.

\[2\] Thanks to Tom Banks for pointing this out; see also [3].
3. One–loop objections

Having mentioned this problem, let us now assume for the sake of argument that it can be resolved, and that the Rubakov–Shaposhnikov mechanism works in the sense that all the brane vacuum energy gets absorbed in the bulk. Let us then consider the one–loop vacuum energy in the bulk sector and ask whether it really gives us the right order of magnitude for the cosmological constant. In other words, assuming that the “first cosmological constant problem” is solved by the Rubakov–Shaposhnikov mechanism, do one–loop supergravity corrections really solve the “second cosmological constant problem” as suggested in [7]?

The one–loop vacuum energy in the bulk sector of the low–energy effective theory vanishes as long as supersymmetry is unbroken in the bulk. But supersymmetry breaking is transmitted from the brane to the bulk. Intuitively, because of the small overlap of the supergravity wave functions with the brane, the supersymmetry breaking scale in the bulk should be suppressed by the same volume factor as before:

\[
\frac{m_{2,\text{bulk}}^2}{m_{2,\text{brane}}^2} \sim \frac{\text{Vol}(B)}{\text{Vol}(M)} \sim \frac{m_{2,\text{brane}}^2}{m_{\text{Planck}}^2} .
\]  

(2.6)

To actually compute the bulk masses precisely, one should find the eigenvalues of the various wave operators in the gravitational background of the non–BPS soliton:

\[
0 = \Box \phi = \Box^{(4)} \phi + \tilde{g}^{ab} \nabla_a \nabla_b \phi
\]  

(2.7)

\[
0 = \mathcal{D} \psi = \mathcal{D}^{(4)} \psi + \mathcal{D}^{(n)} \psi
\]  

(2.8)

and the same for the Rarita–Schwinger operator, etc. The lowest eigenvalues of the wave operators on \(M\) are then the masses \(m_0, m_{1/2}, m_{3/2}, \ldots\) of the superpartners of the graviton.

At least in simple toy models, such masses are indeed typically suppressed by the volume factor. E.g., consider the case of a scalar field in one extra dimension (without gravity), with a Schrödinger–type equation of motion

\[
\phi'' + V(x)\phi + m^2 \phi = 0 .
\]

If we make the toy model ansatz

\[
x \in [0,L] , \ V(x) = \Theta\left(\frac{\epsilon}{2} - |x - \frac{L}{2}|\right) ,
\]

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then the zero mode will be of the form

\[
\phi(x) \sim \cos mx \quad \text{for} \quad x < \frac{L}{2} - \frac{\epsilon}{2} \quad \text{with} \quad m^2 = \frac{\epsilon}{L} \quad \text{(2.9)}
\]

\[
\sim \cos m(L - x) \quad \text{for} \quad x > \frac{L}{2} + \frac{\epsilon}{2}. \quad \text{(2.10)}
\]

The term \((\epsilon/L)\) is indeed the volume factor. Similar toy models in higher dimensions yield similar results. Generalizing the discussion to the three–brane geometry is left for the future; let us just note that it seems important to “get out of the throat”, i.e. to not only restrict the discussion to the near–horizon limit \(AdS_5 \times S^5\).

In the introduction, the contribution to the cosmological constant from a single massless bosonic degree of freedom has been given, assuming a short–distance cutoff \(\Lambda\). In the presence of masses \(m_i\), this formula generalizes (expanding in powers of \(\frac{m_i}{\Lambda}\)) to

\[
\lambda = 3\Omega_\Lambda H^2 = \frac{1}{2\pi m_{Pl}^2} \sum_i (-1)^{F_i} \left\{ \Lambda^4 + m_i^2 \Lambda^2 - m_i^4 \log \frac{\Lambda^2}{m_i^2} + \ldots \right\}, \quad \text{(2.11)}
\]

where \(F_i\) is the fermion number. In our case, \(m_i\) are the masses of the supergravity degrees of freedom labelled by \(i\) (i.e., \(i\) runs from 1 to 256 in the case of maximal supergravity).

The \(\Lambda^4\) term vanishes, since in a theory with spontaneously broken supersymmetry such as this one (where the classical soliton solution does not respect the supersymmetry of the underlying Lagrangean) there is an equal number of bulk bosons and bulk fermions. What about the \(\Lambda^2\) term? It is proportional to the supertrace of the square of the mass matrix in the bulk supergravity sector. The question is then, is

\[
Str \ M^2_{\text{bulk}} = 0 ?
\]

We will come back to this shortly. For the moment, let us assume that we can find a situation where this is the case. Then the relevant term for the cosmological constant is the last one in (2.11). Up the the logarithmic factor, which is of order 100 if we identify the cutoff \(\Lambda\) with the fundamental scale \(m_{\text{brane}}\) of the theory, this last term indeed reproduces the four equal steps on the logarithmic scale of hierarchies drawn in figure 1, using (2.10).

In the absence of concrete classical solutions for non–BPS stable branes, we can not yet work out precise supergravity masses from (2.7, 2.8) etc. However, working out the numbers
for some randomly picked patterns of supergravity masses, the majority of predictions are in the range

$$m_{\text{brane}} \sim 2 - 6 \, \text{TeV}, \quad m_{\text{bulk}} \sim 0.5 - 5 \, \text{meV}.$$ 

Both of these ranges are the subject of planned measurements. The latter prediction is not yet ruled out by experiment \[19\]. Since the contributions to $\lambda$ go like the 4th power of the mass, it is quite possible that one superpartner of the graviton (whichever is the heaviest one) produces the lion’s share of the cosmological constant. If so, then this heaviest superpartner should not be heavier than $5 \, \text{meV} \sim (40 \, \mu \text{m})^{-1}$, otherwise it would produce a Hubble expansion that is too fast.

If $\text{Str } \mathcal{M}_{\text{bulk}}^2$ was nonzero, the curvature radius of the universe would be at the third mark in the diagram in figure 1. This roughly corresponds to the distance between the planets in the solar system, which is obviously too small. In theories where global supersymmetry is spontaneously broken, $\text{Str } \mathcal{M}_{\text{bulk}}^2$ is known to be zero at tree level. But here we have local supersymmetry and there is no guarantee that the supertrace vanishes. So $\text{Str } \mathcal{M}_{\text{bulk}}^2 = 0$ is an important consistency criterion that must be checked for each non–BPS soliton background individually. However, since finding backgrounds with $\text{Str } \mathcal{M}_{\text{bulk}}^2 = 0$ involves no fine–tuning, this may in fact be a blessing: it might allow one to throw out most of the candidate non–BPS solitons. Situations with $\text{Str } \mathcal{M}_{\text{bulk}}^2 = 0$ have been considered in \[20\].

Even when $\text{Str } \mathcal{M}_{\text{bulk}}^2 = 0$ at tree level, it will generally not be zero after loop corrections; however, since we are concerned only about supergravity loops, these loop corrections are suppressed by additional powers of Newton’s constant and should therefore be small.

What is a potentially fatal problem with our scenario, though, is the following. So far, we have only considered the contribution of the Kaluza–Klein zero modes to both $\text{Str } \mathcal{M}_{\text{bulk}}^4$ and $\text{Str } \mathcal{M}_{\text{bulk}}^2$. But the higher Kaluza–Klein modes should also contribute. We must assume that $\text{Str } \mathcal{M}_{\text{bulk}}^2 = 0$ also for the Kaluza–Klein modes, and moreover, that they do not contribute to $\text{Str } \mathcal{M}_{\text{bulk}}^4$ – either because their mass splittings are tiny, or because of some other unexpected feature of the effective four–dimensional Kaluza–Klein gauge theory.\[^{3}\]

\[^{3}\text{E.g., one might speculate that the Kaluza–Klein modes are confined, the confined charge being Kaluza–Klein momentum; Confinement of Kaluza–Klein momentum has been discussed in \[21\], and in a three–dimensional context in \[22\].}\]
Boldly assuming that this is the case, here are the steps that would have to be followed in order to make the above predictions for future experiments more precise.

1. Make a list of string compactifications involving non–BPS stable 3–branes with co–dimension at least two, and find their supergravity solutions.

2. For each such compactification, compute the supergravity masses as a function of the volume factor $\tau^2$, by finding the eigenvalues of wave operators on $\mathcal{M}$ in these backgrounds. This will probably only be possible in an $\alpha'$ expansion.

3. Given these eigenvalues, compute $\text{Str} \, \mathcal{M}^2_{\text{bulk}}$ for each compactification. Compactifications with $\text{Str} \, \mathcal{M}^2_{\text{bulk}} \neq 0$ must be thrown out. Hopefully this leaves only a tiny set of candidate compactifications.

4. Compute $\text{Str} \, \mathcal{M}^4_{\text{bulk}}(\tau)$ and then use the observed ratio between the Planck and the Hubble constants to calibrate $\tau$.

5. From the supergravity masses, compute a “running Newton constant” $G_N(\mu)$: the effective Newton constant at micrometer scales should change due to the contribution from the exchange of particles such as the dilaton. Then compare the curve $G_N(\mu)$ with short–distance measurements of gravity in the micrometer range.

If no deviations from Newton’s constant are found at all down to scales of, say, 10$\mu$m, it is probably safe to say that our explanation for the small cosmological constant is wrong. On the other hand, if such deviations are found, they could be the beginning of an exciting period of experiment–driven superstring phenomenology. The curve $G_N(\mu)$ would encode information about the supergravity masses, and thereby about the type of non–BPS solitons that the “real–world” superstring compactification contains.
4. Conclusion

At least if the recent measurements of the cosmological constant can be trusted, then the size of the cosmological constant is such that it could be produced by a supermultiplet of particles with supersymmetry being broken in the milli–eV range. Whatever these particles are, they must be fundamentally different from the Standard Model particles, because they contribute to the cosmological constant, while the Standard Model particles obviously don’t.

The non–BPS brane world scenario provides precisely such a fundamental difference between the Standard Model particles, which live on the brane, and other particles (the supergravity particles), which live in the bulk. In addition, it provides a mechanism (the Rubakov–Shaposhnikov mechanism) which could at least in principle do the job of soaking up the Standard Model vacuum energy.

Most strikingly, the supersymmetry breaking scale of order milli–eV comes out naturally, if one assumes Standard Model supersymmetry breaking in the TeV range. This feels a bit like pieces of a puzzle falling into place. It may be part of the solution of the cosmological constant puzzle.

But obviously, what has been discussed here cannot be the whole story because of the problems mentioned – most notably, reasons are missing why the potential for the modulus $k$ should have its minimum at zero, and why higher Kaluza–Klein modes should be ignored.

Perhaps the best feature of our scenario is that it predicts contact with experiment. Clues about whether and how these problems can be solved will thus hopefully come from measurements of Newton’s constant in the range between 10 and 1000 $\mu m$, such as $[23]$.

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