A Novel Pseudoerror Monitor

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Received 10 April 2003; Revised 14 September 2003; Recommended for Publication by Tomohiko Taniguchi

The error rate (ER) is a crucial criterion in evaluating the performance of a digital communication system. Many ER estimation methods have been described in the literature. Among them, the pseudoerror monitoring solution has attracted special attention due to its consistent performance in different environments and distinctive blind estimation capability, that is, estimating the ER without needing any prior knowledge of the transmitted information. In this paper, a novel pseudoerror monitor (PEM) design, the kernel PEM, is developed. Incorporating the strength of the probability density function (pdf) approximation technique, the proposed design has remarkable advantage of being able to produce statistically consistent ER estimate within a much shorter observation time. Simulation results are given in support of this claim.

Keywords and phrases: error rate estimation, pseudoerror monitor, density function approximation.

1. INTRODUCTION

One of the primary goals of a digital communication system is to provide users with reliable data transmission service. Being the most straightforward measure of the reliability of data transmission, not surprisingly, the error rate (ER) has been widely recognized as a crucial criterion in evaluating the performance of a digital communication system. Many ER estimation methods have been described in the literature, for example, the error counting solution [1], the parameter estimation solution [1, 2, 3, 4, 5], the probability density function (pdf) approximation-based solution [6, 7, 8], the pseudoerror monitoring solution [1, 9, 10, 11, 12, 13, 14, 15], and so forth. Among them, the pseudoerror monitoring scheme has attracted special attention due to its distinctive blind estimation capability and consistent performance in various environments. The conventional pseudoerror monitor (PEM) designs, however, require a relatively long observation time to produce statistically reliable estimates at low ERs. In this study, a novel PEM design, termed kernel PEM, has been developed. By exploiting the pdf approximation technique, the proposed design successfully reduces the observation time without degrading the overall quality of the ER estimate.

This paper is organized as follows. In Section 2, the principle of the pseudoerror monitoring approach is introduced. In Section 3, the kernel density-approximation technique is reviewed. Section 4 describes the kernel PEM design, summarizes its advantages, and proposes an iterative method to attain the optimum estimation. Simulation results are given in Section 5 to demonstrate the superiority of the proposed design over its conventional counterparts. Section 6 concludes this paper.

2. PSEUDEOERROR MONITORING

In pseudoerror monitoring, the observed events that are relatively more likely to be erroneous are treated. These events are not necessarily the real transmission errors. The most direct benefit of this strategy is to relieve the error counting monitor from the high dependence on the prior knowledge of the transmitted information. Furthermore, the observation time needed for generating statistically consistent ER estimate can be reduced significantly too.

In conventional pseudoerror monitoring, several secondary transmission channels are constructed, and controlled amounts of signal degradations are introduced (or the error criteria are released), to make the error events occur more frequently. Such errors are often referred to as pseudo errors. As a consequence, the ER is amplified and a sufficiently large number of pseudo errors can be recorded within a much shorter observation time. The estimates of the pseudoerror rates (PERs), resulted from counting the numbers of pseudo errors, are then extrapolated to estimate the ER.
The accuracy of the ER estimate calculated as above is dependent on the extrapolation method used. A simple and generally acceptable extrapolation can be performed by treating the logarithmic ER as a linear function of a suitably defined degradation parameter, such as the signal degradation factor \( [9] \). For secondary channels with signal degradation factors of \( d_1 \) and \( d_2 \), we can extrapolate the PER estimates \( \hat{P}_{d1} \) and \( \hat{P}_{d2} \), respectively, to have the desired ER estimate \( \hat{P}_0 \) as follows:

\[
\log \hat{P}_0 = \frac{d_1 \log \hat{P}_{d2} - d_2 \log \hat{P}_{d1}}{d_1 - d_2}.
\]

Many PEM designs have been described in the literature. These schemes face the same challenge when they are applied to fast-varying channels, that is, the long observation time. This problem can be relieved by adding in more signal degradations or further relaxing the error criteria. However, since the discrepancy between the extrapolation and the actual error pattern can be too big sometimes, this solution may suffer a serious drop in the estimation accuracy. In some cases, the resultant ER estimate may be too biased to be useful to serve as a performance indicator.

3. KERNEL DENSITY FUNCTION APPROXIMATION

The subject of density function approximation has long been a hot research topic in statistics and it has been studied extensively in the literature (see \([16, 17]\) and the references therein). Among the existing solutions, the kernel approximation method is the most widely studied and perhaps the most successful method in practice. A kernel pdf estimator can be constructed as follows:

\[
\hat{f}(x) = \frac{1}{nh} \sum_{i=1}^{n} K \left( \frac{x - X_i}{h} \right),
\]

where \( x \) is the random variable of interest, \( X_i \) is the \( i \)th sample of \( x \), \( n \) is the number of the samples used for the approximation, \( h \) is a positive smoothing parameter, \( \hat{f} \) is the approximate of the actual pdf \( f \), and \( K \) is a kernel function that satisfies

\[
\int_{-\infty}^{+\infty} K(x) dx = 1.
\]

Function \( K \) is usually, but not always, selected to be a density function, such as the standard Gaussian function. It follows from (2) that the density approximate \( \hat{f} \) is also a density function. The value of \( h \) determines the amount of details of the samples that will be masked in the approximation process. If \( h \) is set too small, the spurious fine structure will become visible, and if \( h \) is set too large, some important features of the distribution will be obscured. The optimum value of \( h \) is affected by many factors, for example, the choice of the kernel, the actual density, the criterion used to evaluate the pdf approximate, and so forth. If the concerned statistics is a Gaussian distribution with a variance of \( \sigma^2 \), the optimum smoothing parameter for the standard Gaussian kernel can be found to be \([16]\)

\[
h_o = 1.06 \sigma n^{-1/5},
\]

where \( h_o \) is optimum in the sense of minimizing the mean integrated square error (MISE), that is,

\[
\text{MISE}(\hat{f}) = E \left\{ \int \left[ \hat{f}(x) - f(x) \right]^2 dx \right\}.
\]

Obviously, the MISE criterion measures the global accuracy of the resultant pdf approximate.

4. KERNEL PSEUDOERROR MONITORING

4.1. Principle

The pdf approximation technique can be readily applied in ER estimation as follows:

\[
\hat{P}_0 = \sum_m \left( P_{sm} \cdot \int_{ER_m} \hat{f}_m(x_m) dx_m \right),
\]

where \( P_{sm} \) is the probability that the \( m \)th \((m = 0, 1, \ldots, M - 1)\) symbol is transmitted, \( x_m \) is the corresponding decision statistics, \( \hat{f}_m \) is the pdf approximate of \( x_m \), and \( ER_m \) denotes the error region of \( x_m \). Assume that all the \( M \) symbols are equiprobable, that is, \( P_{sm} = 1/M \), and they suffer the same degree of corruption during the transmission, that is, \( f_m \) can only be identified by its mean value. The ER estimator in (6) can be accordingly simplified to

\[
\hat{P}_0 = \int_{ER} \hat{f}(x) dx,
\]

where \( x \) is an arbitrary decision statistics. The ER can now be estimated in two successive steps: approximate the pdf of a decision statistics, and then calculate its integration over the relevant error region. Rather than using some specific types of events as the error counting method and the conventional pseudoerror monitoring method do, the density approximation-based scheme exploits the information carried by all the observations. Consequently, it cuts down the cost on the observation time significantly.

Although it seems possible to estimate the ER directly by integrating the pdf approximate obtained over the real-error region, this solution, termed kernel real-error monitoring, is not feasible in practice. The ER estimate obtained in this way is very sensitive to the authenticity of the error decisions. It follows that in order to produce a good ER estimate, this solution may suffer a serious drop in the estimation accuracy. The conventional pseudoerror monitoring solution described previously works successfully in blind ER estimation, but fails to provide sufficient reduction in the observation time. The kernel real-error monitoring solution, on the other side, may reduce the observation time, but it is incapable of giving satisfactory performance in blind state. The idea of the
The superiority of the proposed kernel PEM design is also evident by its flexibility in adjusting the operation of the monitor. Since the objective of estimating the ER is to provide a reliable indicator of the system performance, the consistency of the ER estimate is usually more important than the absolute value of the ER itself [1]. In conventional PEM designs, other than increasing the observation time, the only method of improving the consistency is to define wider...
pseudoerror regions, or equivalently, add in larger amount of signal degradation. As has been mentioned earlier, this approach may introduce unbearable bias, and in some cases, it may even lead to misjudgement of the system performance. In the kernel PEM scheme, better consistency is the immediate outcome of using a larger smoothing parameter. Although it also suffers certain loss of accuracy, this approach is advantageous in not needing to change the orders of the ER estimates, that is, lower ERs are mapped to smaller values and vice versa. Consequently, in the proposed scheme, the increment of the estimation bias will not show distinctive destructive effect on the final evaluation of the system performance. Moreover, the adoption of a narrower pseudoerror region reduces the error introduced by linear extrapolation, and this may be helpful in counteracting the loss of accuracy caused by oversmoothing the samples.

4.3. Optimum smoothing parameter

For a given operational environment and an observation time, the performance of a kernel PEM is determined mainly by the value of the smoothing parameter and the size of the pseudoerror regions. The former factor dominates the statistical properties of the pdf approximate, while the latter determines the amount of error introduced by the integration in PER estimation and the extrapolation in ER calculation. Since controlling the smoothing effect is more flexible, effective, and reliable, it is highly recommended to be used as the main means of adjusting the behavior of the monitor. Modifying the thresholds, on the other side, should be kept out of consideration unless the previous scheme alone cannot fulfill the requirement. In this study, we discuss the optimum smoothing effect for fixed modified thresholds, that is, fixed setting of the pseudoerror regions.

The smoothing parameter given in (4) works quite well in the simulations conducted. However, it requires the variance of the noise to be known a priori, otherwise, a relatively costly noise variance estimator has to be implemented. Furthermore, inaccurate knowledge or estimate of the variance may seriously degrade the performance of the monitor. Modifying the thresholds, on the other side, should be kept out of consideration unless the previous scheme alone cannot fulfill the requirement. In this study, we discuss the optimum smoothing effect for fixed modified thresholds, that is, fixed setting of the pseudoerror regions.

The smoothing parameter given in (4) works quite well in the simulations conducted. However, it requires the variance of the noise to be known a priori, otherwise, a relatively costly noise variance estimator has to be implemented. Furthermore, inaccurate knowledge or estimate of the variance may seriously degrade the performance of the monitor. However, the situation becomes more complicated when the order of the process is smaller, and the optimal smoothing parameter cannot be obtained using the iterative method described previously. Figure 2b shows the effect of using a larger smoothing parameter, where $h$ is redefined to be 0.1 while $r_1$ and $r_2$ take the same value. Figure 2e illustrates the effect of using wider pseudoerror regions, where $r_1$ and $r_2$ are set to 0.2 and 0.4, respectively, and $h$ takes the corresponding optimum value 0.035. For ease of comparison, the theoretical ERs are displayed in the figures with dashed lines. As is clearly illustrated, the consistency of the ER estimate can be enhanced by increasing the value of the smoothing parameter or by extending the coverage of the pseudoerror regions.

The result obtained with a threshold modification monitor is shown in Figure 3, where the operation conditions remain unchanged, and $n$, $r_1$, and $r_2$ are set to 10000, 0.2, and 0.4, respectively. It can be seen that although the observation time is much longer and the pseudoerror regions are much wider, the conventional monitor is still unable to
6. CONCLUSION

By combining the strengths of the conventional PEM and the kernel real-error monitor, the proposed kernel PEM has been shown to perform better than both. Compared with the
conventional PEM, the proposed monitor is superior in that it significantly reduces the observation time. Compared with the kernel real-error monitor, the proposed method has a better performance in blind state. Overall, the kernel PEM design has great potential to be applied in practice to offer fast and statistically consistent blind ER estimate.

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