Sound speed resonance of the stochastic gravitational waves background

Yi-Fu Cai, Chunshan Lin, Bo Wang, and Sheng-Feng Yan

Introduction. The current understanding of the nature is based on the theory of general relativity (GR) and the standard model (SM) of particle physics. While these theories remain extremely successful, it is crucial to question how one may probe new physics beyond them. One approach is to search for observable effects of possible deviations from some constants on which the theories depend. In particular, the propagation speed of gravitational waves (GWs) $c_g$ is a fundamental constant in the context of GR, which characterizes how rapidly a change in the distribution of energy and momentum of matter or gravitational field itself results in subsequent alteration of the gravitational field over a distance in spacetime. In GR GWs propagate at the same speed as light $c$, which was confirmed with high precision $-3 \times 10^{-15} \leq c_g/c - 1 \leq 7 \times 10^{-16}$ at low redshift regime by the multi-messenger observation GW170817 [1]. However, the observational limits are considerably weak at large scales, and hence, in principle $c_g$ could deviate from $c$ that indicates a possibility of new physics beyond GR [2].

Recently, the North American Nanohertz Observatory for Gravitational Waves (NANOGrav) PTA experiment has reported the results of search for the stochastic background of GWs based on its 12.5-year data set [3], which indicates it is possible to probe new physics beyond GR with GW experiments in near future.

According to terrestrial experiments, new physics beyond SM of particle physics is expected to exist at energy scales above $\sim 13$ TeV [4]. It is easy to exceed this scale in the very early universe and leave us clues of new physics beyond both SM and GR, for instance, quantum gravity theories [5]. If we concern gravitation only, modified gravity (MG) is one of the explicit and classical way for probing physics beyond GR. Many theories of MG have been studied and given fruitful cosmological applications including dark energy, GWs and early universe [2, 6], such as torsion gravity [7–10], scalar-tensor theories [11–13], massive gravity [14, 15], etc. The Lovelock’s theorem [16, 17] implies that GR is the unique theory, if it is from a local gravitational action which contains only second derivatives of the 4-dimensional spacetime metric. Any modification of GR should violate this theorem and lead to new phenomena. One remarkable phenomenon, which inspires the current study, is the time-dependent propagation of GWs. This phenomenon exists in many theories of MG, for instance, the Horndeski theory [18, 19], and theories beyond Horndeski [11, 20–22] (see also [23] for a review). It also exists in the recently proposed 4-dimensional Einstein-Gauss-Bonnet (EGB) gravity [24], where the coupling constant of the Gauss-Bonnet term is rescaled by a singular factor $1/(D - 4)$ and the 4-dimensional theory is defined in the limit $D \to 4$. Moreover, in the dictionary of cosmological effective field theory (EFT) [25], the operator $M_4(t)$ can lead to a time-dependent sound speed of GWs as well.

In this article, we focus on one particular type of time dependence, i.e. oscillation. In physical systems, the phenomenon of parametric resonance would occur if some parameters or degrees of freedom oscillate. The parametric resonance is widely applied in condensed matter physics [34, 35], and also appears in cosmological processes, namely, cosmic preheating [26–29], sound speed resonance (SSR) during inflation [30–33], and so on. As primordial fluctuations can be resonantly amplified via the SSR, the corresponding density perturbations can lead to the formation of primordial black holes (PBHs) in post-inflationary phases [30], and a stochastic background of GWs can be significantly induced through non-linear process [31]. Thus, it follows an interesting ques-
tion whether the GW production becomes more efficient when the resonance could take place in the gravity sector directly. For instance, a broad resonance for primordial GWs could occur if graviton obtains an oscillating mass [36, 37]. In this article, we introduce the oscillatory sound speed of GWs in the early universe and examine if any detectable effects could be achieved via parametric resonance. As shall be shown in the present study, we found that GWs can be considerably enhanced at small scales in the very early universe and hence the underlying new physics can be examined in the GW astronomy.

**Theoretical background.** The oscillatory sound speed of GWs can arise in a wide class of MG theories. For instance, in the Horndeski theory, the sound speed of GWs takes \( c_g^2 = \mathcal{F}_T/\mathcal{G}_T \) [23], where \( \mathcal{F}_T \) and \( \mathcal{G}_T \) are functions of scalar field and its temporal derivative to be determined by the underlying theory. If we assume that this scalar field dominates over the universe during reheating, its coherent oscillation can then give rise to the oscillatory behaviour of \( c_g^2 \). For the 4-dimensional EGB gravity [24], \( c_g^2 \) receives a correction proportional to \( \dot{H}/M^2_{\text{Pl}} \) with \( H \) being the Hubble parameter and the dot representing for the time derivative, and it can also be oscillatory around the reheating epoch. Similar oscillatory corrections can be realized in many scalar-tensor theories beyond Horndeski [11, 20, 21, 23]. Without loss of generality, we in this article perform the model-independent analysis by parameterizing the sound speed of GWs as,

\[
c_g^2 = 1 - \frac{\alpha}{(1 + \tau/\tau_0)^2} \cos^2(k_\star \tau) ,
\]

where \( \alpha \) is conformal time, \( \tau_0 \) and \( k_\star \) are the characteristic time scale and wave number, respectively. We demand \( \alpha > 0 \) to avoid the superluminal propagation. The specific time dependence form in the Eq. (1) is well motivated, as we would generally expect a scalar field oscillates about its minimum during reheating, which gives rise to the cosine factor, while the amplitude of the oscillation decreases due to the Hubble friction as the universe expands, which gives rise to the time dependent factor in the denominator of the Eq. (1). For simplicity, we assume that the correction to the cosmological background is negligible and thus the background evolution is the same as the one depicted by the standard big bang cosmology. From the EFT point of view [25], this type of model could be achieved by assuming a sizeable and oscillatory \( \mathcal{M}_2(t) \) operator, while keeping other higher dimensional operators small. In passing, we comment that the sound speed of GWs reduces to unity in the late time limit \( \tau \to \infty \) and thus satisfies the stringent constraint from the multi-messenger observation GW170817.

The oscillatory correction term in Eq. (1) leads to the narrow parametric resonance at the sub-Hubble scale, given the proper values of the model parameters. It is of observable interest if this scale corresponds to the frequencies of current or near future GW detectors. We are thus more interested in the sub-Hubble modes, where \( k \gg a'/a \) with \( a' \) being the scale factor of the universe and the prime being the derivative with respect to conformal time. At this scale, we can safely neglect the Hubble friction term, as well as the possible mass term, which stems from the canonical normalisation of the GWs quadratic action. Hence, by introducing \( x = k/\tau \), the equation of motion for GWs can be approximately expressed in form of the Mathieu equation,

\[
\frac{\partial^2 h_k}{\partial x^2} + [A - 2q \cos(2x)] h_k = 0 ,
\]

where \( A = k^2/k_*^2 - 2q \) and \( q = \alpha k^2/[4k_*^2(1 + x/\alpha)]^2 \).

The primary character of the solutions is an exponential instability \( h_k \propto \exp(\mu_k^{(n)} x) \) within certain resonance bands, where \( \mu_k^{(n)} \) is called the Floquet Exponent. The first resonance band is the most efficient, which appears between \( A \in (1 - q, 1 + q) \) with \( \mu_k^{(1)} \approx q/2 \) for \( q \ll 1 \) due to the cosmic expansion, a mode enters the resonance band, undergoes the exponential growth for a short period, and then automatically exits this band. Thus, the amplification of GWs can become controllable.

**Parametric resonance.** In our case, inflation can be applied to set initial conditions for primordial GWs. Namely, quantum fluctuations of spacetime can appear at sub-Hubble scales, so that they behave as plane waves with \( h_k(\tau) \cdot aM_{\text{Pl}}/2\sqrt{2} = e^{-ic_s h_k \sqrt{\gamma^2/2k^2}} \), where \( c_s \) denotes the sound speed of GWs during inflation, and hence is not necessarily the same as the parameterization given in Eq. (1). Note that, for a broad class of MG theories, \( c_s \) generally deviates from unity due to the presence of possible high-order derivative terms. During inflation, the exponential expansion stretches the modes out of the Hubble horizon, which eventually yields the power spectrum to be: \( \Delta^2(k, \tau) = \frac{k^n}{a^4} |h_k(\tau)|^2 \approx 2 \left( H/c_s M_{\text{Pl}} \right)^2 k^{n_s}/\pi^2 \), where \( n_s \) is the spectral index of GWs.

The parametric resonance, which is triggered by the coherent oscillation of a scalar in certain MG theories, may occur during or after reheating epoch. As we have explained, the modes could enter and exit the resonance bands due to the cosmic expansion. Thus, the Mathieu equation Eq. (2) is oversimplified for the physics that we are interested. A full picture requires us to bring back the redshift term in the equation of motion of GWs,

\[
h_k''(\tau) + 2H h_k'(\tau) + c_g^2 k^2 h_k(\tau) = 0 ,
\]

where \( \mathcal{H} \) is the conformal Hubble constant \( \mathcal{H} \equiv a'(\tau)/a(\tau) \). For an accelerating background, \( a(\tau) = H(\tau - \tau_0)^{-\gamma} \), with the normalization \( |\mathcal{H} - \tau_0| = 1 \) and \( \mathcal{H} \equiv \gamma/H_0 \) [47, 48], where \( \tau_0 = 1.01 \) is the present conformal time, and \( \gamma = 2.265 \), \( \tau_0 = 2.101 \), \( H_0 = 67.4 \text{km s}^{-1}\text{Mpc}^{-1} \) according to the CMB observation [49]. The conformal wavenumber \( k \) is related to
the physical frequency at \( \tau \) via \( f(\tau) = k/(2\pi\alpha(\tau)) \). This generalized Mathieu equation can be resolved numerically and the results are reported in Fig. 1. We mention that, at the very beginning of radiation domination, the initial amplitudes of sub-Hubble modes were originally set by those produced during inflation and thus our analysis is consistent with the initial condition discussed previously. In Fig. 1, we present the evolution of stochastic GWs under parametric resonance by choosing two sets of parameter values, and accordingly, there are two characteristic frequency bands with one being sensitive to space-based detectors (left panel) and the other terrestrial detectors (right panel).

We show that, resonance takes place and manifestly enhances primordial GWs, which reaches its maximum when \( \tau \leq \tau_0 \). For space-based detectors, like LISA, we concern the mode with \( k = 3.9 \times 10^{16} \), which corresponds to the frequency \( f = 0.0066 \) for present just within the sensitive range of LISA. While the mode with \( k = 6.8 \times 10^{20} \) corresponding to the frequency \( f = 115 \) Hz, are sensitive to terrestrial experiments, namely LIGO, ET, CE, etc. In general, the amplitude of primordial GWs decays dramatically with time because of Hubble expansion. However, for enhanced modes, resonance sustains about \( \Delta \tau \sim 10^{-15} \) around \( \tau_0 = 7.5 \times 10^{-15} \) for space-based experiments and \( \Delta \tau \sim 10^{-19} \) around \( \tau_0 = 3.7 \times 10^{-19} \) for terrestrial experiments. Each of them contributes to \( 10^4 \) and \( 10^5 \) times enhancement due to exponential factor \( \exp(\alpha k_\ast \Delta \tau/16) \), respectively.

Other than the mode evolution, we use the characteristic spectrum \( h_c(f) \) to measure the energy of GWs as \( h_c(f) \equiv h(k, \tau_H)/(2\sqrt{T}) \) in observations, where a notation of spectrum \( h(f, \tau_H) \equiv \sqrt{\Delta^2(k, \tau_H)} \) is also used [46]. Thus, we illustrate the characteristic spectrum of each mode evolving to the end of radiation domination in Fig. 2. We show the first resonance band in this figure. The resonance band spans a very narrow interval \( \Delta k \sim q k_\ast \) on the spectrum due to the narrow resonance for both cases. In the meanwhile, the amplification magnitude around \( k = 3.9 \times 10^{16} \) and \( k = 6.8 \times 10^{20} \) consistent with their evolutions along with \( \tau \). When GWs continue to evolve to the present accelerating era, it shall be within the detection frequency band of space-based and terrestrial experiments, respectively.

**Observational constraints.** As shown in Fig. 3, it is straightforward to evaluate the GWs to present and make comparison to the noise curves of experiments. From the left panel, by adopting the same parameter set in modes evolution, the peak on characteristic spectrum with almost \( 10^4 \) times amplification is detectable to LISA. We find that the most intense resonance takes place at redshift \( z \sim 6 \times 10^{15} \), at which \( \alpha'/\alpha = 1.7 \times 10^{14} \ll k \sim 10^{15} \), which is self-consistent with condition of Mathieu equation approximation given above Eq. (2). Moreover, the energy scale of the universe at that redshift is around \( \sim \text{TeV} \). As for terrestrial experiments in right panel of Fig. 3, the spectrum is enhanced about 5 orders of magnitude with a peaky feature around characteristic frequency. In this case, the most intense resonance takes place at \( z \approx 1.2 \times 10^{20} \), at which \( \alpha'/\alpha = 3 \times 10^{18} \ll k \sim 6 \times 10^{20} \) is consistent to the assumptions for Mathieu equation Eq. (2), and the corresponding energy scale is \( \sim 10^6 \) TeV. These features indicate that at early universe with energy scales \( \gtrsim \text{TeV} \), a broad class of scalar-tensor theories could leave us observable footprint of new physics on primordial GWs spectrum via the sound speed resonance of the GWs.

It is shown that the peak of such spectrum is an extremely narrow band in the detection frequency range of the detectors, it satisfies most astrophysical bounds. However, the signal-to-noise ratio (SNR) provide the lower bound for the model parameters, and defined as

\[
\text{SNR} = \frac{h_c(f)}{h_{\text{detector}}(f)},
\]

where \( h_{\text{detector}}(f) \) is the noise curve for GWs detectors. To illustrates the detectable parameter space, the SNR distributions with respect to \( \alpha \) and characteristic wave number \( k_\ast \) are shown in Fig. 4 under the detection of LISA, ET and CE, respectively. We usually choose \( \text{SNR} > 1 \) as a threshold for detectable primordial GWs signals. For LISA detections, we set \( \text{SNR} = h_c(f_0)/h_{\text{LISA}}(f_0) \), and \( f = f_0 \) maximizes the value of \( h_c(f)/h_{\text{LISA}}(f) \). At the meanwhile, we define \( \text{SNR} = h_c(f_0)/h_{\text{ET}}(f_0) \) for ET and \( \text{SNR} = h_c(f_0)/h_{\text{CE}}(f_0) \) for CE, where \( f = f_0 \) maximizes the value of \( h_c(f)/h_{\text{ET}}(f) \). As a result, LISA and LIGO will be able to detect such signals via viable speed of GWs with the parameter space of the upper-right corner in these contour maps.

**Nonlinear enhancement of density perturbations.** Previously, we have shown that MG theories at the electroweak scale may give rise to a sharp peak on the stochastic GWs spectrum via the narrow parametric resonance. The frequency of the peak may lie within the sensitive range of LISA, given the proper parameter choice. Note that the LISA sensitive range is often relevant for the PBHs formation. Hence, it is very intriguing to further ask whether the sharp peak on the stochastic GWs spectrum at this scale may lead to extra contribution to density perturbations so that it might be related to the PBHs formation. In our case, it is possible for the peak of tensor modes to generate the overdense region at the nonlinear level without spoiling the effectiveness of the perturbative expansion, provided that (1) the peak is high enough, yet still less than unity, and (2) the tensor-tensor-scalar coupling is strong enough. At the sub-Hubble scale, GWs can source scalar modes at the nonlinear perturbation level, \( \zeta'' + 2H\zeta' - \partial^2 \zeta = \lambda_{\text{att}} \partial_i h_{jk} \partial_j \partial_k h_{ik} \), where \( \lambda_{\text{att}} \) is the scalar-tensor-tensor coupling coefficient.
In the Fourier space, the source term acquires the convolution over the whole momentum space. However, the most significant contribution comes from the resonance peak, and thus the scalar mode can be estimated as $\zeta_k \sim \lambda_{\text{att}} |h_k^2|$, where $|h_k^2|$ is the amplitude of the resonance peak of the GWs spectrum, which is bounded by the BBN constraint $\rho_{\text{GW}} \simeq \frac{1}{2(\pi^2 G)^2} \frac{k^2}{a} |h_k^2| < 0.05 \rho_{\gamma}$, where $\rho_{\gamma}$ is the radiation energy density. The Mathieu equation requires $k^2 > \mathcal{H}^2$ to have the narrow parametric resonance, and thus $|h_k^2| < 10^{-1}$ is bound from above. A peak with amplitude around $10^{-2}$ on scalar modes at the same scale, which might lead to the gravitational collapse of the matter and further lead to the PBHs formation, could be achieved if $\lambda_{\text{att}} > 1$, provided that the amplitude of the resonance peak saturates the BBN bound. To our best knowledge, this mechanism is proposed for the first time, and it requires more scrutiny. We will present much detailed analyses in the next project to examine such a hypothetical possibility.

**Conclusion and outlook.** In this article we reported a broad class of scalar-tensor theories as a origin of new physics which enhances the spectrum of primordial GWs to be observable. Such new physics induces a variable speed of GWs which could be parameterized as an oscillating $c_g$. We extend the idea of SSR to such GWs that are exponentially amplified by an instant parametric resonance at very early universe with $\gtrsim$ TeV energy scale. As for different observable frequency band, the corresponding GWs naturally start and stop resonating at characteristic moment during thermal history. With this mechanism, detectable signal on primordial GWs spec-
FIG. 3. GWs signal (purple curves) versus to noise curves of several experiments. The selection of parameters is consistent with that shown in Fig. 1 with $k$ being free. Left panel: Blue curve is the noise curve of LISA [50, 51]. Right panel: We show the noise curves of ET (red), CE (orange) [52, 53], LIGO A+ (green) [54], LIGO (black), Virgo (pink) and KAGRA (cyan) [55, 56], respectively.

FIG. 4. SNR distributions with respect to $\alpha$ and characteristic wave number $k$, for LISA, ET and CE, respectively.

trum opens an novel window for testing new physics beyond GR at TeV scales. As the temperature increasing to very close to the end of inflation, the observable characteristic frequency increasing which inspires the promotion on GWs observation frequency band. Note that, this effect is efficient at early universe owing to the variation of GWs speed, as the evolution of the universe, $c_g$ reduces to unity again, and all the physics governed by GR.

We note that the TeV energy scale, on which gravity is possibly modified, might be accessible for LHC as well as the next-generation colliders. We are prompted to ask whether the modification also leads to some new phenomenologies on collider physics. If one introduces the coupling $\phi^2 H^\dagger H$ (unless the shift symmetry of $\phi$ prohibits it) where $\phi$ arises from the MG sector and $H$ is the Higgs boson in the SM of particle physics, the particle collisions in colliders could lead to the excitation of the scalar boson $\phi$, which would subsequently decay to graviton as a tertiary product. However, the frequency of GWs generated in this process shall be at the TeV scale, which remain challenging to be probed by current and near future detectors.

We end by commenting that, in this article, we also put forward a novel mechanism of enhancing density perturbations by virtue of the nonlinear coupling between one scalar and two tensor modes. Although it is a byproduct of the present study, it deserves a detailed analysis on this mechanism in a follow-up project, in particular, on the possible realization of the PBHs formation. Additionally, we point out that, the nonlinear coupling among three tensor modes could also bring extra contribution to the background of stochastic GWs, which may yield the signal of the relatively flat spectrum within low frequency bands, namely the PTA and NANOGrav surveys.

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