MOReL : Model-Based Offline Reinforcement Learning

Rahul Kidambi*\(^1\), Aravind Rajeswaran*\(^2\), Praneeth Netrapalli\(^3\), Thorsten Joachims\(^1\)

Abstract

In offline reinforcement learning (RL), the goal is to learn a successful policy using only a dataset of historical interactions with the environment, without any additional online interactions. This serves as an extreme test for an agent’s ability to effectively use historical data, which is critical for efficient RL. Prior work in offline RL has been confined almost exclusively to model-free RL approaches. In this work, we present MOReL, an algorithmic framework for model-based RL in the offline setting. This framework consists of two steps: (a) learning a pessimistic MDP model using the offline dataset; (b) learning a near-optimal policy in the learned pessimistic MDP. The construction of the pessimistic MDP is such that for any policy, the performance in the real environment is lower bounded by the performance in the pessimistic MDP. This enables the pessimistic MDP to serve as a good surrogate for the purposes of policy evaluation and learning. Overall, our framework MOReL is amenable to detailed theoretical analysis, enables easy and transparent design of practical algorithms, and leads to state-of-the-art results on widely used offline benchmark tasks.

1 Introduction

The availability and use of large datasets have enabled tremendous advances in computer vision [1], speech recognition [2], and natural language processing [3, 4]. In these fields, it is customary to first collect large datasets [5, 6, 7], then train deep learning models on these datasets, and finally deploy these models on various platforms (e.g. smartphones). In contrast, reinforcement learning (RL) [8] is typically viewed as an online learning process. An RL agent repeatedly uses interactions with the environment to improve the policy; thus data collection and policy learning happen simultaneously. However, a direct embodiment of this trial and error learning process often leads to slow and sample inefficient learning that is often feasible only in the presence of a simulator [9, 10]. Similar to progress in other fields of AI, the ability to effectively learn from large offline datasets may hold the key to unlocking the sample efficiency of RL agents.

Offline RL (also known as batch RL) is the setting where an agent must learn a highly rewarding policy using only a static offline dataset that has already been collected by one or more logging (also known as behavioral) policies [11]. In this setting, no additional online interactions with the environment are allowed. Since the data has already been collected, the offline RL paradigm abstracts away aspects of data collection (exploration), and allows prime focus on data-driven learning of policies using large offline datasets. This abstraction is particularly relevant for safety sensitive applications like healthcare and industrial automation where careful oversight by a domain expert is necessary for taking exploratory actions or deploying new policies [12, 13]. Additionally, large volumes of historical data are already available in many applications like autonomous driving, recommendation systems, and industrial automation. These fields can directly utilize the offline RL framework to improve currently deployed policies. Finally, algorithms developed for offline

\*Equal contributions. 1 Cornell University, 2 University of Washington, 3 Microsoft Research, India. Correspond to: rkidambi@cornell.edu, aravraj@cs.uw.edu, praneeth@microsoft.com, tj@cs.cornell.edu
RL can also serve as useful sub-routines for data-driven policy learning in the standard online RL paradigm, with the view of solving online RL as a sequence of offline RL problems. Since offline RL uses a static dataset for sequential decision making, it offers unique challenges. Over the course of learning, the offline RL agent has to use the static dataset to evaluate and reason about candidate policy updates that deviate from the logging policy. This offline policy evaluation can be difficult since the new policy’s state-action visitation distribution may differ significantly from that of the logging policy. Furthermore, this offline policy evaluation can become progressively harder over the course of learning as the policy deviates more and more from the logging policy. This change in distribution as a result of policy updates is often referred to as \textit{distribution shift} and constitutes a major challenge in offline RL. This distribution shift is particularly exacerbated in MDPs with continuous state-action spaces owing to challenges in function approximation and generalization [14, 15].

In principle, it is possible to use any off-policy RL algorithm (e.g. Q-learning, actor-critic) with an offline dataset. However, recent work suggests that direct use of such algorithms yield poor results due to the aforementioned distribution shift [16, 17, 18]. To overcome this challenge, prior work has proposed modifications such as the use of Q-network ensembles [18] and regularizing the learned policy towards the data logging policy [16, 19, 17, 20]. In this endeavor, prior work has been confined almost exclusively to model-free RL methods [21, 16, 17, 19, 18, 20, 22] (see Section 2 for more details).

In contrast, model-based RL (MBRL) presents an alternate set of approaches for using offline datasets. In particular, a dynamics model of the environment can be learned using the offline data, which can subsequently be used to guide policy search. MBRL enables the use of generic priors (e.g. smoothness, Physics etc.) when constructing models, as well as the ability to use a wide collection of planning algorithms once the model has been learned, including MPC [23, 24], MCTS [25], dynamic programming [26], and policy optimization [27]. These advantages have enabled MBRL to be highly sample efficient in the online RL setting, often substantially outperforming model-free counterparts [28, 29, 30, 31]. In this backdrop, we study the pertinent question of how to effectively use model-based RL techniques in the offline RL setting.

Our Contributions Our main contribution in this work is the development of \textbf{MOReL} (Model-based \textbf{O}ffline \textbf{R}einforcement \textbf{L}earning), a novel MBRL framework for the offline RL setting. MOReL is amenable to theoretical analysis, enables easy and transparent design of practical algorithms, and leads to state-of-the-art results on widely used offline benchmark tasks. We summarize the key ideas and contributions below:

- \textbf{Algorithmic framework description :} MOReL consists of two steps:
  - Construction of a pessimistic MDP model using the offline dataset.
  - Planning or policy optimization on the above pessimistic MDP.

- \textbf{Constructing the pessimistic MDP :} The key idea in our algorithmic framework is the construction of a pessimistic MDP which partitions the state space into “known” and “unknown” regions. To construct such an MDP, we first learn a model of the MDP’s transition dynamics using the offline dataset. Subsequently, we construct an \textit{Unknown State-Action Detector} (USAD) that takes as input a query state-action pair and returns “known” if we can guarantee that the learned model is approximately accurate for the query (and unknown otherwise). The pessimistic MDP uses the learned transition models in known regions, and forces a transition to a low-reward absorbing state from unknown regions. As we will see, this construction has the property that for any policy, the performance in the pessimistic MDP is a lower bound for the performance in the true MDP (i.e. environment). This enables the pessimistic MDP to serve as a good surrogate for the true MDP, for the purposes of policy search.
Why model-based for offline RL? As suggested earlier, distribution shift and over-estimation bias are the major challenges in offline RL. A general template to mitigate these challenges is to perform regularization towards the data logging policy. This is intended to restrict policy search to only those policies whose visitation distribution is similar to that of the data logging policy. Since model-free algorithms do not explicitly construct a model or directly deal with visitation distributions, they are often forced to directly regularize in the policy space [16, 17, 20]. In other words, the policy search is restricted to those policies that satisfy $D(\pi(\cdot|s), \pi_b(\cdot|s)) \leq \epsilon$, where $\pi_b$ is the data logging policy and $D$ is a probabilistic distance (e.g. KL divergence). However, this can be overly restrictive. We make the key observation that policies which differ substantially in the conditional distribution can still produce very similar state visitation distributions.

The pessimistic MDP enables us to directly regularize in the space of state visitation distributions, enabling the search over a potentially richer set of policies, when compared with model-free alternatives. The pessimistic MDP provides regularization by aggressively punishing policies that drift and visit unknown parts of the state space, thereby pruning them during policy search.

- **Theoretical contributions**: We show that MOREL achieves the following guarantees:
  - Every policy’s value on MOREL’s pessimistic MDP construction approximately lower bounds its value on the true MDP (i.e. the environment). This motivates the use of pessimistic MDP as a good surrogate for policy evaluation and learning.
  - We introduce a new notion, called unknown state hit time, that captures instance specific complexity of the offline RL problem. We provide performance guarantees for the MOREL framework in terms of this quantity and further relate this quantity to the mismatch in the support of state action visitation distribution of the optimal policy with the static offline dataset.
  - Our bound shows that if the static dataset spans the full support of state-action visitation distribution of an optimal policy, MOREL outputs a near-optimal policy whose sub-optimality goes to zero as the size of dataset increases. Further, our bound captures the graceful degradation of the performance of MOREL when the static dataset does not cover the full support of any optimal policy. In contrast, guarantees from prior work become degenerate and pathologically loose when the offline dataset has insufficient support.
  - We also prove the tightness of our bound by showing that the suboptimality of any offline RL algorithm needs to scale similarly (up to log factors) in the worst case.

- **Experimental Results**: We develop a practical instantiation of the proposed algorithmic framework, the details of which are presented in Section 4.3. The practical instantiation implements the USAD using an ensemble of randomly initialized dynamics models and plans in the MDP with familiar policy gradient methods like NPG [32, 33, 28]. We evaluate our algorithm in the standard continuous control benchmarks in OpenAI gym modified for the batch setting as done in a number of recent works [16, 17, 20], and find that our algorithm obtains state of the art (SOTA) results in a majority of the tasks. In particular, out of 20 environment-dataset configurations, our algorithm obtains SOTA results in 12, and performs competitively in the rest. In contrast, the best prior algorithm obtains SOTA results in only 4 configurations.

2 Related Work

Our work takes a model-based approach to offline RL. We review related work pertaining to both of these domains in this section.
2.1 The offline RL setting

Offline RL, as a problem setting, dates at least to the work of Lange et al. [11]. In this setting, an RL agent is provided access to a typically large offline dataset, using which it has to produce a highly rewarding policy. This has direct applications in fields like healthcare [34, 35, 36], recommendation systems [37, 38, 39, 40], dialogue systems [41, 19, 42], and autonomous driving [43]. We refer the readers to the review paper of Levine et al. [44] for an overview of potential applications. On the algorithmic front, prior work in offline RL can be broadly categorized into three groups as described below.

Importance sampling  The first approach to offline RL is through importance sampling. In this approach, trajectories from the offline dataset are directly used to estimate the policy gradient, which is subsequently corrected using importance weights. This approach is particularly common in contextual bandits literature [45, 37, 38] where the importance weights are relatively easier to estimate due to the non-sequential nature of the problem. For MDPs, Liu et al. [46] present an importance sampling based off-policy policy gradient method by estimating state distribution weights [47, 48, 49]. The work of Liu et al. [46] also utilizes the notion of pessimism by optimizing only over a subset of states visited by the behavioral policy. They utilize importance weighted policy gradient (with estimated importance weights) to optimize this MDP. However, their work does not naturally capture a notion of generalization over the state space. Moreover, their results require strong assumptions on the data collecting policy in the sense of ensuring support on states visited by the optimal policy. Our framework, MOReL, provides the same guarantees under identical assumptions, but we also show that the performance of MOReL degrades gracefully when these assumptions aren’t satisfied.

Dynamic programming  The overwhelming majority of recent algorithmic work in offline RL is through the paradigm of approximate dynamic programming. In principle, any off-policy algorithm based on Q-learning [50, 51] or actor-critic architectures [52, 14, 53] can be used with a static offline dataset. However, recent empirical studies confirm that such a direct extension leads to poor results due to the challenges of overestimation bias in generalization and distribution shift. To address overestimation bias, prior work has proposed approaches like ensembles of Q-networks [18, 16, 19]. As for distribution shift, the principle approach used is to regularize the learned policy towards the data logging policy [16, 17, 20]. Different regularization schemes, such as those based on KL-divergence and maximum mean discrepancy (MMD), have been considered in the past. Wu et al. [20] perform a comparative study of such regularization schemes and find that they all perform comparably. Finally, a recent line of work [22, 54] focuses on obtaining provably convergent methods for minimizing the (one-step) Bellman error using Duality theory. While they show promising results in continuous control tasks in the online RL setting, their performance in the offline RL setting is yet to be studied.

Model-based RL  The interplay between model-based methods and offline RL has only been sparsely explored. The work of Ross & Bagnell [55] theoretically studied the performance of MBRL in the batch setting. In particular, the algorithm they analyzed involves learning a dynamics model using the offline dataset, and subsequently planning in the learned model without any additional safeguards. Their theoretical results are largely negative for this algorithm, suggesting that in the worst case, this algorithm could have arbitrarily large sub-optimality. In addition, their sub-optimality bounds become pathologically loose when the data logging distribution does not share support with the distribution of the optimal policy. In contrast, we present a novel algorithmic framework that constructs and pessimistic MDP, and show that this is crucial for better empirical results and sharper theoretical analysis.
2.2 Advances in model-based RL

Since our work utilizes model-based RL, we review the most directly related work in the online RL setting. Classical works in MBRL have focused extensively on tabular MDPs and linear quadratic regulator (LQR). For tabular MDPs (in the online RL setting), the first known polynomial time algorithms were the model-based algorithms of $E^3$ [56] and R-MAX [57]. More recent work suggests that model-based methods are minimax optimal for tabular MDPs when equipped with a wide restart state distribution [58]. However, these works critically rely on the tabular nature of the problem. Since each table entry is typically considered to be independent, and updates to any entry do not affect other entries, tabular MDPs do not afford any notion of generalization. The metric-$E^3$ [59] algorithm aims to overcome this challenge by considering an underlying metric space for state-actions that enables generalization. While this work provides a strong theoretical basis, it does not directly provide a practical algorithm that can be used with function approximation. Our work is perhaps conceptually closest to $E^3$ and metric-$E^3$ which partitions the state space into known and unknown regions. However, we note that our work is in the offline RL setting, where we show that our pessimistic MDP construction plays a crucial role. Furthermore, direct practical instantiations of $E^3$ and metric-$E^3$ with rich function approximators have remained elusive.

In recent years, along with an explosion of interest in deep RL, MBRL has emerged as a powerful class of approaches for sample efficient learning. Modern MBRL methods (typically in the online RL setting) can support the use of flexible function approximators like neural networks, as well as generic priors like smoothness and approximate knowledge of physics [60], enabling the learning of accurate models. Furthermore, MBRL can draw upon the rich literature on model-based planning including model predictive control (MPC) [23, 24, 61, 62], search based planning [25, 63], dynamic programming [26, 64], and policy optimization [32, 65, 33, 27, 53]. These advances in MBRL have enabled highly sample efficient learning in widely studied benchmark tasks [30, 29, 66, 67, 28], as well as in a number of challenging robotic control tasks like aggressive driving [61], dexterous hand manipulation [68, 28], and quadrupedal locomotion [69]. Among these works, the recent work of Rajeswaran et al. [28] demonstrated state of the art results with MBRL in a range of benchmark tasks, and forms the basis for our practical implementation. In particular, our model learning and policy optimization subroutines are extended from the MAL framework in Rajeswaran et al. [28]. However, our work crucially differs from it due to the pessimistic MDP construction, which we show is important for success in the offline RL setting.

3 Problem formulation

Markov Decision Processes (MDP) We define an MDP using the tuple $\mathcal{M} = \{S, A, r, P, \rho_0, \gamma\}$, where, $S$ is the state-space, $A$ is the action-space, $r : S \times A \rightarrow [-R_{\text{max}}, R_{\text{max}}]$ is the reward function, $P : S \times A \rightarrow S$ is the transition kernel, $\rho_0$ is the initial state distribution, and $\gamma$ the discount factor. A policy defines a mapping from states to a probability distribution over actions, $\pi : S \times A \rightarrow \mathbb{R}_+$. The value of a policy $\pi$ at an arbitrary state $s$ is defined as:

$$V^\pi(s, \mathcal{M}) = \mathbb{E}_{a_t \sim \pi(\cdot|s_t), s_{t+1} \sim P(\cdot|s_t, a_t)} \left[ \sum_{t=0}^{\infty} \gamma^t r(s_t, a_t) | s_0 = s \right].$$

(1)

The value depends on three quantities: the state of interest $s$, the policy $\pi$, and the transition dynamics according to the MDP $\mathcal{M}$. The expectation involves taking actions according to the policy, i.e. $a_t \sim \pi(\cdot|s_t)$, and transition dynamics according to the MDP, i.e. $s_{t+1} \sim P(\cdot|s_t, a_t)$. The performance of a policy and some start state distribution $\beta$ is given by $J_\beta(\pi, \mathcal{M}) := \mathbb{E}_{s \sim \beta} [V^\pi(s, \mathcal{M})]$. Generally, we are interested in
optimizing the performance according to the MDP start state distribution, resulting in the optimization

\[
\max_\pi J_{\rho_0}(\pi, \mathcal{M}) := \mathbb{E}_{s \sim \rho_0} [V^\pi(s, \mathcal{M})].
\] (2)

To avoid notation clutter, we also suppress the dependence on \(\rho_0\) when understood from context, i.e. \(J(\pi, \mathcal{M}) \equiv J_{\rho_0}(\pi, \mathcal{M})\). We denote the optimal policy using \(\pi^*_\mathcal{M} := \arg \max_\pi J_{\rho_0}(\pi, \mathcal{M})\). Typically, a class of parameterized policies \(\pi_\theta \in \Pi(\Theta)\) are considered, and the parameters are optimized.

**Offline RL.** In the offline RL setting, we are provided with a static dataset of interactions with the environment. For this data collection, one or more data logging (or behavioral) policies can be used. We denote a data logging policy using \(\pi_b\). The data logging policy is typically known, although we do not require it in our formulation. The collected dataset consists of tuples of the format: \(D = \{(s_i, a_i, r_i, s'_i)\}_{i=1}^N\). In many applications, the dataset is also time indexed, thereby providing trajectory level information.

The goal in offline RL is to design an algorithm that takes as input the offline dataset \(D\) to output a policy \(\pi_{\text{out}}\) with minimal sub-optimality, i.e. \(J(\pi_{\text{out}}, \mathcal{M}) - J(\pi_{\text{out}}, \mathcal{M})\). In general, it may not be possible to find the optimal policy (i.e. \(\pi^*_\mathcal{M}\)) in offline RL, even when provided with an infinite sized dataset. Indeed, our theoretical results in Section 4.2 formalizes this claim for any offline RL algorithm. Thus, we aim to design algorithms that would result in as low a sub-optimality as possible.

**Model-Based RL (MBRL)** In MBRL, we construct an explicit model of the world to aid policy search. Specifically, we construct another MDP \(\hat{\mathcal{M}} = \{S, A, r, \hat{P}, \hat{\rho}_0, \gamma\}\), which has the same state and action space as the original MDP/environment \(\mathcal{M}\). Data collected by interactions with the environment are used to learn an approximate transition dynamics \(\hat{P}\). For simplicity, we assume that the reward function is known and same across the true MDP \(\mathcal{M}\) and the model \(\hat{\mathcal{M}}\). If required, the reward function can be learned using the dataset as well, which is generally simpler compared to the transition dynamics. Finally, \(\hat{\rho}_0\) is the initial state distribution, which can be identical to \(\rho_0\) if it is known, or can be learned using the dataset. Analogous to the definition for \(\mathcal{M}\), we use \(J_{\hat{\rho}_0}(\pi, \hat{\mathcal{M}})\) or simply \(J(\pi, \hat{\mathcal{M}})\) to denote performance of the policy \(\pi\) in \(\hat{\mathcal{M}}\). If \(J(\pi, \hat{\mathcal{M}})\) closely tracks \(J(\pi, \mathcal{M})\), we can use the model as a surrogate for policy search. However, learning such an accurate model using a static offline dataset can be difficult. Some policies may visit parts of the state space where we do not have sufficient data/support in \(D\). In subsequent sections, we outline approaches to construct effective models for the offline RL setting.

## 4 Algorithmic framework

In this section, we first describe an idealized version of our algorithmic framework, MOREL. We then present the main theoretical results based on this idealized algorithmic framework. Finally, we describe a practical instantiation of this algorithmic framework that we use in our experimental results.

### 4.1 Idealized Framework

We describe MOREL in Algorithm 1. The primary intuition for our algorithmic framework involves constructing a special pessimistic MDP, which has the property that it can lower bound the performance of any policy in the environment (original MDP). This property enables the pessimistic MDP to serve as a good surrogate for optimization. It avoids the model-bias and over-estimation issues typically associated with model based RL, as elaborated below. We now describe the steps in our algorithmic framework.
Algorithm 1 MOReL: Model Based Reinforcement Learning in the offline RL setting

1: **Require** Dataset $\mathcal{D}$
2: Learn approximate dynamics model $\hat{P} : S \times A \rightarrow S$ using $\mathcal{D}$.
3: Construct $\alpha$-USAD, $U^\alpha : S \times A \rightarrow \{\text{TRUE}, \text{FALSE}\}$ using $\mathcal{D}$ (see Definition 1).
4: Construct the pessimistic MDP $\hat{M}_p = \{S \cup \text{HALT}, A, r_p, \hat{P}, \hat{\rho}_0, \gamma\}$ (see Definition 2).
5: (OPTIONAL) Use a behavior cloning approach to estimate the behavior policy $\hat{\pi}_b$.
6: $\pi_{\text{out}} \leftarrow \text{PLANNER}(\hat{M}_p, \pi_{\text{init}} = \hat{\pi}_b)$
7: **Return** $\pi_{\text{out}}$.

**Learning the dynamics model:** The first step involves using the offline dataset to learn an approximate dynamics model $\hat{P}(\cdot|s,a)$. A number of approaches can be used to achieve this, including maximum likelihood estimation, or other techniques from generative modeling [70, 71, 72]. Since the offline dataset may not have sufficient support in all parts of the state-action space, we cannot expect the model to be globally accurate. A naive MBRL approach would entail directly planning using the learned approximate model. However, such an approach without any safeguards may result in the RL agent erroneously over-predicting the rewards in parts of the state space with minimal data support, ultimately resulting in a highly sub-optimal policy [55]. We overcome this challenge of model-bias by modifying the MDP as described below.

**Unknown state-action detector (USAD):** To ensure that policies do not visit states where the model is inaccurate, we will partition the state space into known and unknown regions based on the accuracy of learned model. Subsequently, we will aim to restrict policy search to only those policies that remain within the known states. To perform this partitioning, we will use an $\alpha$–USAD as defined below.

**Definition 1.** ($\alpha$-USAD) Given a state-action pair $(s, a)$, the unknown state action detector is defined as:

$$U^\alpha(s, a) = \begin{cases} \text{FALSE (i.e. Known)} & \text{if } D_{TV}(\hat{P}(\cdot|s,a), P(\cdot|s,a)) \leq \alpha \text{ can be guaranteed} \\ \text{TRUE (i.e. Unknown)} & \text{otherwise} \end{cases}. \quad (3)$$

Intuitively, USAD provides confidence about where the learned model is accurate. It flags those state-actions for which the model is guaranteed to be accurate (in total variation distance) as “known”, while flagging the other state-actions where such a guarantee cannot be ascertained as “unknown”. Note that USAD is based on our ability to guarantee the accuracy of the model, and is not an inherent property of the model. In other words, there could be states where the model is actually accurate, but we may flag them as unknown due to our inability to guarantee accuracy. This is important to ensure that the USAD can be implemented with only the dataset, and does not require any additional information about the true dynamics. Two factors affect the ability to guarantee (generalization) accuracy: (a) data availability – are there sufficient data points “close” to the query in the dataset; (b) the features or representations used – certain features and representations, such as those based on knowledge of physics, might be more amenable to generalization guarantees. This broadly suggests that larger datasets and research in representation learning can be beneficial for offline RL.

**Pessimistic MDP construction:** Next, we construct a pessimistic MDP using the learned dynamics model and the aforementioned USAD. This is based on the following definition:
Definition 2. The $(\alpha, \kappa)$-pessimistic MDP is described by tuple $\hat{M}_p := \{ S \cup \text{HALT}, A, r_p, \hat{P}_p, \hat{\rho}_0, \gamma \}$. Here, $S$ and $A$ are the set of states and actions in the original MDP (environment). HALT is an additional absorbing state we introduce into the state space of $\hat{M}_p$. $\hat{\rho}_0$ is the initial state distribution learned from the offline dataset. $\gamma$ is the discount factor, again from the original MDP. We use the modified reward and transition dynamics given by:

$$
\hat{P}_p(s'|s, a) = \begin{cases} 
\delta(s' = \text{HALT}) & \text{if } U^\alpha(s, a) = \text{TRUE} \text{ or } s = \text{HALT} \\
\hat{P}(s'|s, a) & \text{otherwise}
\end{cases}
$$

$$
r_p(s, a) = \begin{cases} 
-\kappa & \text{if } s = \text{HALT} \\
r(s, a) & \text{otherwise}
\end{cases}
$$

We use $\delta(s' = \text{HALT})$ to indicate the Dirac delta function, which forces the MDP to transition to the special state HALT from every unknown state-action pair. For this special state, we use the reward of $-K$, while other state-actions receive the same reward as in the environment. In this work, we assume the reward function of the environment is known for simplicity. This is a reasonable assumption for many applications like robotics and operations research. If required, the reward function can also be learned from the offline dataset, and it is often much easier to learn compared to the transition dynamics.

Planning in the pessimistic MDP The final step in MOREL is to plan in the constructed pessimistic MDP. A variety of techniques can be used for this, including MPC, search based planning, dynamic programming (e.g. value iteration), or policy optimization, as appropriate based on context. For the idealized case, we assume that we have a planning oracle which can return an $\epsilon$-suboptimal policy in the pessimistic MDP.

4.2 Theoretical results

In this section, we present our main theoretical result showing a performance guarantee for Algorithm 1. The bounds will depend on the following quantity, called hitting time.

Definition 3. (Hitting time) Given an MDP $M$, starting state distribution $\rho_0$, state-action pair $(s, a)$ and a policy $\pi$, the hitting time $T^\pi_{(s,a)}$ is defined as the random variable denoting the first time action $a$ is taken at state $s$ by $\pi$ on $M$, and is equal to $\infty$ if $a$ is never taken by $\pi$ from state $s$. For a set of state-action pairs $S \subseteq S \times A$, we define $T^\pi_S \triangleq \min_{(s,a) \in S} T^\pi_{(s,a)}$.

We are now ready to present our main result.

Theorem 1. (Policy value with pessimism) The value of any policy $\pi$ on the original MDP $M$ and its $(\alpha, R_{\text{max}})$-pessimistic MDP $\hat{M}_p$ satisfies:

$$
J_{\hat{\rho}_0}(\pi, \hat{M}_p) \geq J_{\rho_0}(\pi, M) - \frac{2R_{\text{max}}}{1-\gamma} \cdot D_{TV}(\rho_0, \hat{\rho}_0) - \frac{2\gamma R_{\text{max}}}{(1-\gamma)^2} \cdot \alpha - \frac{2R_{\text{max}}}{1-\gamma} \cdot \mathbb{E} \left[ T^\pi_{U} \right],
$$

$$
J_{\hat{\rho}_0}(\pi, \hat{M}_p) \leq J_{\rho_0}(\pi, M) + \frac{2R_{\text{max}}}{1-\gamma} \cdot D_{TV}(\rho_0, \hat{\rho}_0) + \frac{2\gamma R_{\text{max}}}{(1-\gamma)^2} \cdot \alpha,
$$

where $T^\pi_{U}$ denotes the hitting time of unknown states $U \triangleq \{ (s, a) : U^\alpha(s, a) = \text{TRUE} \}$ by $\pi$ on the true MDP $M$. 

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We first note that Theorem 1 implies that the output policy $\pi_{\text{out}}$ of Algorithm 1 satisfies

$$ J_{\rho_0}(\pi_{\text{out}}, \mathcal{M}) \geq J_{\rho_0}(\pi^*, \mathcal{M}) - \frac{4R_{\text{max}}}{1 - \gamma} \cdot D_{TV}(\rho_0, \hat{\rho}_0) - \frac{4\gamma R_{\text{max}}}{(1 - \gamma)^2} \cdot \alpha - \frac{4R_{\text{max}}}{1 - \gamma} \cdot \mathbb{E} \left[ \gamma^{T_{U}^*} \right]. $$

Theorem 1 says that the value of any policy on the $(\alpha, R_{\text{max}})$ pessimistic MDP $\mathcal{M}_p$ is close to its value on the original MDP $\mathcal{M}$, where the closeness depends on three quantities: i) the total variation distance between the true starting state distribution and that learned from the dataset, $D_{TV}(\rho_0, \hat{\rho}_0)$, ii) the maximum total variation distance $\alpha$ between the learned dynamics model $\hat{P}(\cdot|s, a)$ and the true model $P(\cdot|s, a)$ over all known states i.e., $\{(s, a)|U^\alpha(s, a) = False\}$ and, iii) the hitting time $T_{U}^*$ of unknown states $U$ on the original MDP $\mathcal{M}$ under the original policy $\pi^*$. As the size of the dataset increases, $D_{TV}(\rho_0, \hat{\rho}_0)$ and $\alpha$ approach zero. So the key quantity determining the bound is $\mathbb{E} \left[ \gamma^{T_{U}^*} \right]$. In order to get better intuition into this term and be able to compare our result with prior results, we simplify this quantity using discounted state-action visitation distribution, which for a policy $\pi$ and MDP $\mathcal{M}$ is defined as follows:

$$ d^{\pi,\mathcal{M}}(s', a') = (1 - \gamma) \sum_{t=0}^{\infty} \gamma^t P(s_t = s', a_t = a'|s_0 \sim \rho_0, \pi, \mathcal{M}). $$

**Lemma 2.** (Hitting time and visitation distribution) For any set $U \subseteq S \times A$, and any policy $\pi$, we have $\mathbb{E} \left[ \gamma^{T_{U}^*} \right] \leq \frac{1}{1 - \gamma} \cdot d^{\pi,\mathcal{M}}(U)$.

**Proof.** The proof is rather straightforward. We have

$$ \mathbb{E} \left[ \gamma^{T_{U}^*} \right] \leq \sum_{(s', a') \in U} \mathbb{E} \left[ \gamma^{T_{(s',a')}^*} \right] \leq \sum_{(s', a') \in U} \sum_{t=0}^{\infty} \gamma^t P(s_t = s', a_t = a'|s_0 \sim \rho_0, \pi, \mathcal{M}) $$

$$ = \frac{1}{1 - \gamma} \sum_{(s', a') \in U} d^{\pi,\mathcal{M}}(s', a') = \frac{1}{1 - \gamma} \cdot d^{\pi,\mathcal{M}}(U). $$

Lemma 2 and Theorem 1 give us the following corollary bounding the suboptimality of Algorithm 1 in terms of the probability mass of unknown state actions $U$ under the discounted state visitation distribution $d^{\pi^*,\mathcal{M}}$ of optimal policy $\pi^*$.

**Corollary 3.** (Upper bound) Suppose the offline dataset is large enough so that $\alpha = D_{TV}(\rho_0, \hat{\rho}_0) = 0$. Then, the output $\pi_{\text{out}}$ of Algorithm 1 satisfies:

$$ J_{\rho_0}(\pi_{\text{out}}, \mathcal{M}) \geq J_{\rho_0}(\pi^*, \mathcal{M}) - \frac{4R_{\text{max}}}{1 - \gamma} \cdot \mathbb{E} \left[ \gamma^{T_{U}^*} \right] \geq J_{\rho_0}(\pi^*, \mathcal{M}) - \frac{4R_{\text{max}}}{(1 - \gamma)^2} \cdot d^{\pi^*,\mathcal{M}}(U). $$

To put this in context, note that most previous results such as [16, 46], assume that $d^{\pi^*,\mathcal{M}}(U_D) = 0$, where $U_D \equiv \{(s, a)|s, a, r, s' \notin \mathcal{D}\} \supseteq U$ is the set of state action pairs that do not occur in the offline dataset, and guarantee finding optimal policies for the offline RL problem under this assumption. Our result improves upon this in three ways:

1. The set $U_D$ is replaced by the smaller set $U$, leveraging the generalization ability of learned dynamics model $\hat{P}(\cdot|s, a)$ to states $(s, a)$ that have not been seen in the dataset.
Figure 1: This example shows that the suboptimality of any offline RL algorithm is at least \( R_{\text{max}} \frac{d^{\pi^*,\mathcal{M}}(\mathcal{U}_D)}{(1-\gamma)^2 \cdot \log \frac{1}{1-\gamma}} \) in the worst case and hence Corollary 3 is tight. The states 1, 2, \( \cdots \), \( k + 1 \) in the MDP are depicted under the circles. The actions \( a_1, a_2, a_3 \), rewards and transitions are depicted on the arrows connecting the states. The actions taken by the behavior (i.e. the data collection) policy are depicted in blue. See Proposition 4 and its proof for more details.

2. The suboptimality bound on output policy \( \pi_{\text{out}} \) is extended to the setting where the full support coverage assumption is violated i.e., \( d^{\pi^*,\mathcal{M}}(\mathcal{U}) > 0 \).

3. The suboptimality bound on \( \pi_{\text{out}} \) is stated in terms of unknown state hitting time \( T_{\pi^*,\mathcal{U}} \), which can be significantly better than a bound that depends only on \( d^{\pi^*,\mathcal{M}}(\mathcal{U}) \). More concretely, one can construct simple MDPs where \( \mathbb{E}[\gamma^{T_{\pi^*,\mathcal{U}}}] \) is much smaller than \( \frac{1}{1-\gamma} \cdot d^{\pi^*,\mathcal{M}}(\mathcal{U}) \), so the result in Theorem 1 is much stronger than that of Corollary 3. On the other hand, the following proposition shows that Corollary 3 is tight up to logarithmic factors.

**Proposition 4.** (Lower bound) For any discount factor \( \gamma \in [0.95, 1) \), support mismatch \( \epsilon \in \left( 0, \frac{1-\gamma}{\log \frac{1}{1-\gamma}} \right] \) and reward range \([-R_{\text{max}}, R_{\text{max}}]\), there is an MDP \( \mathcal{M} \), starting state distribution \( \rho_0 \), optimal policy \( \pi^* \) and a dataset collection policy \( \pi_b \) such that:

- \( d^{\pi^*,\mathcal{M}}(\mathcal{U}_D) \leq \epsilon \), and
- any policy \( \hat{\pi} \) that is learned solely using the dataset collected with \( \pi_b \) satisfies:
  \[
  J_{\rho_0}(\hat{\pi}, \mathcal{M}) \leq J_{\rho_0}(\pi^*, \mathcal{M}) - \frac{R_{\text{max}}}{4(1-\gamma)^2} \cdot \frac{\epsilon}{\log \frac{1}{1-\gamma}},
  \]

where we recall that \( \mathcal{U}_D \overset{\text{def}}{=} \{(s, a) : (s, a, r, s') \notin \mathcal{D} \text{ for any } r, s'\} \) denotes state action pairs not contained in the static dataset \( \mathcal{D} \).

We see that for \( \epsilon < \frac{1-\gamma}{\log \frac{1}{1-\gamma}} \), the lower bound obtained by Proposition 4 on the suboptimality of any offline RL algorithm matches the upper bound of Corollary 3 up to an additional log factor. For \( \epsilon > \frac{1-\gamma}{\log \frac{1}{1-\gamma}} \), Proposition 4 also implies (by choosing \( \epsilon' = \frac{1-\gamma}{\log \frac{1}{1-\gamma}} < \epsilon \)) that any offline algorithm must suffer at least constant factor suboptimality in the worst case.

### 4.3 Practical Implementation of MOReL

We present a practical implementation of MOReL (see algorithm 1) through model-based natural policy gradient [28]. This entails learning an ensemble of dynamics models using the offline dataset, constructing a pessimistic MDP using the ensemble, and finally optimizing a policy in the pessimistic MDP using natural policy gradient [32, 33, 28]. We present the details of each step below.
Dynamics model learning: Our model representation and learning closely follows the approach from Rajeswaran et al. [28]. We consider a class of Gaussian dynamics models so that \( \hat{P}(\cdot|s, a) \) has the form:

\[
\hat{P}(\cdot|s, a) \equiv \mathcal{N}(f_\phi(s, a), \Sigma).
\] (4)

The mean is parameterized using a flexible function approximator like neural network. The specific parameterization we use is:

\[
f_\phi(s, a) = s + \sigma_\Delta \text{MLP}_\phi \left( \frac{s - \mu_s}{\sigma_s}, \frac{a - \mu_a}{\sigma_a} \right)
\] (5)

where \( \mu_s, \sigma_s, \mu_a, \sigma_a \) are the mean and standard deviations of the states and actions computed from \( D \). \( \sigma_\Delta \) is the standard deviation of the differences in the dataset, i.e. of \( \Delta = s' - s \). Such a parameterization ensures some degree of local continuity since the neural network (MLP) has to only learn the state differences. We learn the parameters by maximizing the log likelihood, or equivalently, for the neural network parameters as

\[
\min_{\phi} \mathbb{E}_{(s,a,r,s') \sim D} \left[ \left\| (s' - s) - \sigma_\Delta \text{MLP}_\phi \left( \frac{s - \mu_s}{\sigma_s}, \frac{a - \mu_a}{\sigma_a} \right) \right\|_2^2 \right]
\] (6)

Unknown state-action detector: Our approach to partitioning the state space into known and unknown regions is inspired by \( E^3 \) and metric-\( E^3 \). The classical metric-\( E^3 \) algorithm [59] for online RL views a state-action pair as known if there exists \( k \)–neighbors within a specified distance threshold, where the distance metric is appropriately chosen. This approach, whilst incorporating the powerful notion of generalization in online RL, is difficult in practice due to computation and storage constraints. Furthermore, the correct choice of distance metric may be difficult and highly problem specific.

In this work, we instead develop a practical heuristic for choosing known vs unknown states based on approximate uncertainty quantification. The notion of uncertainty helps in separating out known regions where confidence is high from unknown regions. A number of approaches have been studied to perform uncertainty quantification with rich function approximators [73, 74, 75, 62]. We follow the simplistic approach of estimating uncertainty using ensembles of models as suggested in Osband et al. [73] and Lowrey et al. [62]. We train an ensembles of models as outlined in eq. 6, and compute the discrepancy across the ensembles for a query \((s, a)\) pair as

\[
disc(s, a) = \max_{i, j} \left\| f_{\phi_i}(s, a) - f_{\phi_j}(s, a) \right\|_2,
\] (7)

where \( f_{\phi_i} \) and \( f_{\phi_j} \) are different members of the model ensemble. With this, we implement USAD as

\[
U_{\text{practical}}(s, a) = \begin{cases} 
\text{FALSE (i.e. Known)} & \text{if } disc(s, a) \leq \text{threshold} \\
\text{TRUE (i.e. Unknown)} & \text{if } disc(s, a) > \text{threshold}
\end{cases}
\] (8)

where we treat threshold as a tunable hyperparameter for the algorithm.

Policy optimization In order to optimize the policy in the pessimistic MDP, we use the normalized natural policy gradient algorithm [32, 33, 76, 28]. Specifically, we use the MJRL code base\(^1\). This has close parallels to related policy gradient algorithms like TRPO [65] and PPO [27]. We refer readers to the MJRL repository and the appendix of Rajeswaran et al. [28] for further details about the algorithm implementation.

\(^1\)The MJRL repository can be found at https://github.com/aravindr93/mjrl
5 Experiments

Environments and partially trained policies: Following recent works in offline RL [16, 17, 20], we consider four continuous control tasks: Hopper-v2, HalfCheetah-v2, Ant-v2, Walker2d-v2 from OpenAI gym [77] simulated with MuJoCo [78]. In several applications, we typically have access to data collected using a partially trained sub-optimal policy interacting with the environment. To simulate this setting, following guidelines from prior work [16, 17, 20], we obtain a partially trained policy $\pi_p$ by running TRPO [65] in these environments until the policy reaches a value of 1000, 4000, 1000, 1000 respectively for the four environments. This partially trained policy in conjunction with different additional exploration strategies are used to collect the datasets, as outlined below (see appendix B for more details).

Datasets and exploration strategies: For each environment, we use a combination of a behavior policy $\pi_b$, a noisy variant of the behavior policy $\tilde{\pi}_b$, and a “purely” random stochastic process $\pi_r$ to collect several datasets. This protocol is based on prior work of Wu et al. [20]. Each dataset contains the equivalent of 1 million timesteps of interactions with the environment. The noisy variant of the behavior policy, $\tilde{\pi}_b$, refers to different noise processes on top of the behavioral policy $\pi_b$. These datasets are referred to as eps-1, eps-3, Gauss-1, Gauss-3 in order to represent various levels of epsilon-greedy noise or Gaussian noise added to the behavior policy. We use epsilon-greedy to denote the process where with certain probability the behavioral policy is chosen, and with the remaining probability, a uniformly random action is chosen. The Gaussian case involves an additive Gaussian noise on top of $\pi_b$ at every timestep. In addition, we also consider the basic setting where our entire dataset is collected only using the behavior policy $\pi_b$, without any additional noise process (we call this Pure). Refer to the appendix for more details.

Other details We use 2 hidden layer MLPs with 512 ReLU activated nodes each for representing the dynamics model, use an ensemble of four such models for building the USAD, and our policy is represented with a 2 hidden layer MLP with 32 tanh activated nodes in each layer. The dynamics model is learnt using Adam [79] and the policy parameters are learnt using model-based natural policy gradient steps [28]. We track the performance of the policy learning algorithm by performing rollouts in the real environment, but these rollouts are used only for the purpose of tracking the learning progress, and are never made available to the learning algorithm in any way. Similar protocols are used in prior work as well[16, 17, 20].

Questions: The experiment section aims to primarily answer the following questions:

1. **Results for different environment-exploration configurations:** How does the performance of the practical variant of MOReL compare to that of recent offline RL algorithms for the different environment-exploration configurations outlined above?

2. **Impact of the behavior policy’s quality:** How does the quality (i.e. performance) of the data logging policy, and by extension the dataset, impact the performance of the policy outputted by MOReL?

3. **Ablation study - impact of the pessimistic MDP construction:** Does MOReL’s pessimistic MDP construction help alleviate the challenge of distribution shift and enable effective model utilization? How does MOReL’s performance compare against the naïve model-based RL approach of directly planning using the learned model without any safeguards (like the pessimistic MDP)?
5.1 MOReL’s performance with different exploration strategies

This section compares the results obtained by MOReL with recent state of the art algorithms for the offline RL setting. In particular, our comparisons are with BCQ [16], BEAR [17], and different variants of BRAC [20]. We present results for the four environments and the five exploration settings, totalling 20 different variants. The results are summarized in Table 1. We reproduce results of prior algorithms (from Wu et al. [20]), and include our results as the final row in each table. We observe that our algorithm obtains the state of the art (SOTA) results for 12 out of the 20 possible environment-exploration configurations, and presents competitive results in the remaining configurations. In contrast, the next best approach, MMD_yp [20], obtains SOTA results for only 4 out of 20 configurations.

5.2 Quality of the logging policy

This section aims to shed light on the significance of the quality of behavior policy used for logging the dataset used by an offline RL algorithm. In particular, our theoretical bounds indicate that if the behavior policy doesn’t visit parts of the state space spanned by an optimal policy, it is impossible for any offline RL approach to return a high performing policy even if the size of the dataset grows large. To study this empirically, we consider two datasets. The first dataset is collected using a partially trained policy without any additional exploratory process (called Pure-partial) and the second is collected with just a purely random process (Gaussian), i.e. a policy that hasn’t been trained at all (called Pure-random). Table 2 presents results of MOReL for the two datasets, as well as the value of the logging policy. We find that MOReL leads to significant improvement over the value of the logging policy in both the cases. However, it is clear that the value of the output policy in the Pure-partial case far exceeds the value in the Pure-random case. This confirms that offline RL cannot find a near-optimal policy if the behavioral policy does not visit all states visited by the optimal policy.

Table 2: Value of the policy outputted by MOReL when working with a dataset collected with a random policy (Pure-random) and a partially trained policy (Pure-partial). The value of the behavior policy is indicated within the parenthesis. All results are averaged over 5 random seeds.

| Environment    | Pure-random | Pure-partial |
|----------------|-------------|--------------|
| Hopper-v2      | 2354 ± 443  | 3642 ± 54    |
| HalfCheetah-v2 | 2698 ± 230  | 6028 ± 192   |
| Walker2d-v2    | 1290 ± 325  | 3709 ± 159   |
| Ant-v2         | 1001 ± 3    | 3663 ± 247   |
Table 1: Results in the four environments and five exploration configurations. 0 represents overflow/divergence for Q-learning based algorithms. Table and results for baselines are reproduced from Wu et al. [20]. Underline indicates best baseline and bold indicates the SOTA result.

| Environment: Ant-v2 | Partially trained policy: 1241 |
|---------------------|---------------------------------|
| Algorithm           | Pure (E1) | Eps-1 (E2) | Eps-3 (E3) | Gauss-1 (E4) | Gauss-3 (E5) |
| SAC [53]            | 0         | -1109      | -911       | -1071        | -1498        |
| BC                  | 1235      | 1300       | 1278       | 1203         | 1240         |
| BCQ [16]            | 1921      | 1864       | 1504       | 1731         | 1887         |
| BEAR [17]           | 2100      | 1897       | 2008       | 2054         | 2018         |
| MMD_vp [20]         | 2839      | 2672       | 2602       | 2667         | 2640         |
| KL_vp [20]          | 2514      | 2530       | 2484       | 2615         | 2661         |
| KL_dual_vp [20]     | 2626      | 2334       | 2256       | 2404         | 2433         |
| W_vp [20]           | 2646      | 2417       | 2409       | 2474         | 2487         |
| MMD_pr [20]         | 2583      | 2280       | 2285       | 2477         | 2435         |
| KL_pr [20]          | 2241      | 2247       | 2181       | 2263         | 2233         |
| KL_dual_pr [20]     | 2218      | 1984       | 2144       | 2215         | 2201         |
| W_pr [20]           | 2241      | 2186       | 2284       | 2365         | 2344         |
| Best Baseline       | 2839      | 2672       | 2602       | 2667         | 2661         |
| MOReL (Ours)        | 3663      | 3732       | 3468       | 3880         | 3796         |

| Environment: Hopper-v2 | Partially trained policy: 1202 |
|------------------------|---------------------------------|
| Algorithm              | Pure (E1) | Eps-1 (E2) | Eps-3 (E3) | Gauss-1 (E4) | Gauss-3 (E5) |
| SAC [53]               | 0.2655    | 661.7      | 701        | 311.2        | 592.6        |
| BC                     | 1330      | 129.4      | 828.3      | 221.1        | 284.6        |
| BCQ [16]               | 1543      | 1652       | 1632       | 1599         | 1590         |
| BEAR [17]              | 0         | 1620       | 2213       | 1825         | 1720         |
| MMD_vp [20]            | 2291      | 2282       | 1892       | 2255         | 1458         |
| KL_vp [20]             | 2774      | 2360       | 2892       | 1851         | 2066         |
| KL_dual_vp [20]        | 1735      | 2121       | 2043       | 1770         | 1872         |
| W_vp [20]              | 2292      | 2187       | 2178       | 1390         | 1739         |
| MMD_pr [20]            | 2334      | 1688       | 1725       | 1666         | 2097         |
| KL_pr [20]             | 2574      | 1925       | 2064       | 1688         | 1947         |
| KL_dual_pr [20]        | 2053      | 1985       | 1719       | 1641         | 1551         |
| W_pr [20]              | 2080      | 2089       | 2015       | 1635         | 2097         |
| Best Baseline          | 2774      | 2360       | 2892       | 2255         | 2097         |
| MOReL (Ours)           | 3642      | 3801       | 3750       | 3682         | 3642         |
## Environment: HalfCheetah-v2
### Partially trained policy: 4206

| Algorithm               | Pure (E₁) | Eps-1 (E₂) | Eps-3 (E₃) | Gauss-1 (E₄) | Gauss-3 (E₅) |
|-------------------------|-----------|------------|------------|--------------|--------------|
| SAC [53]                | 5093      | 6174       | 5978       | 6082         | 6090         |
| BC                      | 4465      | 3206       | 3751       | 4084         | 4033         |
| BCQ [16]                | 5064      | 5693       | 5588       | 5614         | 5837         |
| BEAR [17]               | 5325      | 5435       | 5149       | 5394         | 5329         |
| MMD_vp [20]             | 6207      | 6307       | 6263       | **6323**     | **6400**     |
| KL_vp [20]              | 6104      | 6212       | 6104       | 6219         | 6206         |
| KL_dual_vp [20]         | **6209**  | 6087       | **6359**   | 5972         | 6340         |
| W_vp [20]               | 5957      | 6014       | 6001       | 5939         | 6025         |
| MMD_pr [20]             | 5936      | 6242       | 6166       | 6200         | 6294         |
| KL_pr [20]              | 6032      | 6116       | 6035       | 5969         | 6219         |
| KL_dual_pr [20]         | 5944      | 6183       | 6207       | 5789         | 6050         |
| W_pr [20]               | 5897      | 5923       | 5970       | 5894         | 6031         |
| Best Baseline           | 6207      | 6307       | 6263       | 6323         | 6400         |
| MOReL (Ours)            | 6028      | **6350**   | 6109       | 6171         | 6101         |

## Environment: Walker-v2
### Partially trained policy: 1439

| Algorithm               | Pure (E₁) | Eps-1 (E₂) | Eps-3 (E₃) | Gauss-1 (E₄) | Gauss-3 (E₅) |
|-------------------------|-----------|------------|------------|--------------|--------------|
| SAC [53]                | 131.7     | 213.5      | 127.1      | 119.3        | 109.3        |
| BC                      | 1334      | 1092       | 1263       | 1199         | 1137         |
| BCQ [16]                | 2095      | 1921       | 1953       | 2094         | 1734         |
| BEAR [17]               | 2646      | 2695       | 2608       | 2539         | 2194         |
| MMD_vp [20]             | 2694      | 3241       | **3255**   | 2893         | **3368**     |
| KL_vp [20]              | 2907      | 3175       | 2942       | **3193**     | 3261         |
| KL_dual_vp [20]         | 2575      | **3490**   | 3236       | 3103         | 3333         |
| W_vp [20]               | 2635      | 2863       | 2758       | 2856         | 2862         |
| MMD_pr [20]             | 2670      | 2957       | 2897       | 2759         | 3004         |
| KL_pr [20]              | 2744      | 2990       | 2747       | 2837         | 2981         |
| KL_dual_pr [20]         | 2682      | 3109       | 3080       | 2357         | 3155         |
| W_pr [20]               | 2667      | 3140       | 2928       | 1804         | 2079         |
| Best Baseline           | 2907      | 3490       | 3255       | 3193         | 3368         |
| MOReL (Ours)            | **3709**  | 3238       | 3231       | 3063         | 3129         |
5.3 Ablation study: The necessity of the Pessimistic MDP construction

In this experiment, we consider an ablation study of how a model-based offline RL algorithm performs with and without the pessimistic MDP construction. The pessimistic MDP construction allows the model to be utilized only in regions of the state space where it is deemed accurate. Our hypothesis is that by aggressively punishing and pruning policies that visit unknown parts of the state space, the pessimistic MDP provides a regularizing effect towards the state distribution in the offline dataset, thereby alleviating the difficulties of distribution shift. Furthermore, as highlighted in the theory section, the value of a policy in the pessimistic MDP construction approximately lowerbounds the value in the true MDP (i.e., the environment), which allows the value of the policy in the model to act as a surrogate of the true value. This allows for the model score to be utilized for purposes of policy evaluation/learning. To verify these aspects, we consider the Pure-partial and Pure-random datasets outlined in Section 5.2. We compare our MOReL approach with a naive MBRL algorithm that directly plans in the learned model without any safeguards offered by the pessimistic MDP construction. The results are summarized in Figures 2–3. We can draw the following observations regarding the importance of the pessimistic MDP construction:

- The pessimistic MDP construction appears to be particularly necessary in the harder locomotion tasks such as Hopper-v2 and Walker2d-v2 with termination conditions. In these environments, the policy can take an uncoordinated set of actions and fall over, exacerbating the distribution shift issue. Our results suggest that MOReL with the pessimistic MDP construction significantly outperforms naive MBRL.

- When policy improvement appears to be possible even without the use of pessimism, the learning becomes unstable and drops rather dramatically in performance after many iterations of policy optimization - this can be seen clearly for the Hopper-v2 and Half Cheetah-v2 environments in figure 2, whereas, the pessimism allows the learning to remain stable (with relatively minor variations) in these settings.

- Pessimism tends to play a larger role when the quality of data logging policy is poor, such as in the case of an untrained policy (Figure 3). In these situations, the data support is largely in unrewarding parts of the state space. When the policy optimization algorithm attempts to visit the rewarding parts of the state space, the model can become highly inaccurate, increasing the need for careful model safeguards when optimizing the policy.

Considering the Pure-partial case, one can observe that the green curve (i.e. the value of the policy on the learned model) tends to faithfully lowerbound the value of the policy on the true MDP (in blue) especially when deploying the planning algorithm in the proposed MOReL framework (figure 4). On

Figure 2: Plot of # of planning (NPG) iterations (x-axis) vs. value (y-axis) comparing MOReL (blue) with naive model-based planning (orange) for the Pure-partial case. Observe that the learning in the naive approach is highly unstable. MOReL utilizes pessimism to stabilize the learning process.
These results generally point towards the view that pessimism for a model-based offline RL approach allows compared to an approach that utilizes a vanilla model-based offline RL procedure (without pessimism).

This situation is precisely when the planning algorithm tends to begin over-exploiting the learned model.

On the other hand, when planning on the model-based framework without the safeguards of the pessimistic MDP construction, one can see that the score on the learned model tends to exceed the true score estimate. This situation is precisely when the planning algorithm tends to begin over-exploiting the learned model. These results generally point towards the view that pessimism for a model-based offline RL approach allows for stable and robust learning and, more generally, towards the view that this can never hurt performance compared to an approach that utilizes a vanilla model-based offline RL procedure (without pessimism).
6 Conclusions and discussion

In this paper, we introduced a new model based framework $\text{MOReL}$ for the offline RL problem. $\text{MOReL}$ incorporates both generalization and pessimism helping it perform policy search in known states that may not directly occur in the static offline dataset but can be predicted using the dataset, and at the same time do not drift into unknown states that cannot be predicted using the static offline data. Theoretically, we obtain bounds on the suboptimality of $\text{MOReL}$ which improve over those in prior work, and further show that this suboptimality bound cannot be improved upon by any offline RL algorithm in the worst case. Experimentally, we evaluate $\text{MOReL}$ in the standard continuous control benchmarks in OpenAI gym and show that it achieves state of the art performance in most tasks in the offline RL setting.

The modular structure of $\text{MOReL}$ comprising of model learning, uncertainty estimation and planning allows the use of a variety of approaches in each of these modules. While our instantiation of $\text{MOReL}$ in this paper uses simple and standard approaches, an interesting direction for future work is to explore the benefits of more sophisticated approaches such as multi-step prediction for model learning, prediction with abstention for uncertainty estimation and so on. Finally, $\text{MOReL}$’s modular structure allows it to automatically benefit from future progress in any of the modules.

Overall, our work demonstrates the benefits of model-based approaches over model-free approaches for the offline RL problem. In particular, model-based approaches offer the ability to constrain the policy to be supported on known state-action spaces without restricting it to be close to the behavior policy, which can lead to significant improvements in the final policy quality. More broadly, the fundamental question in offline RL is to understand the limits of achievable performance on a per-problem basis. While we obtain bounds that are tight in the worst case, a finer understanding of such limits will help make significant progress on the offline RL problem. From a practical point of view, an important question is that of policy evaluation. All current works (including this paper) perform policy evaluation using new trajectories from the true MDP and use these evaluations to decide when to stop training. A truly offline RL algorithm must only use the static offline dataset to perform this step as well.

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A Proofs for Section 4.2

In this section, we present the proofs of our main results Theorem 1 and Proposition 4.

Proof of Theorem 1.

\[ J_{\hat{\rho}_0}(\pi, \hat{M}_p) \geq J_{\rho_0}(\pi, M) - \frac{2R_{\max}}{1-\gamma} \cdot D_{TV}(\rho_0, \hat{\rho}_0) - \frac{2\gamma R_{\max}}{(1-\gamma)^2} \cdot \alpha - \frac{2R_{\max}}{1-\gamma} \cdot E[\gamma^{T_{\max}}], \quad \text{and} \]

\[ J_{\hat{\rho}_0}(\pi, \hat{M}_p) \leq J_{\rho_0}(\pi, M) + \frac{2R_{\max}}{1-\gamma} \cdot D_{TV}(\rho_0, \hat{\rho}_0) + \frac{2\gamma R_{\max}}{(1-\gamma)^2} \cdot \alpha, \]

The proof of this theorem is inspired by the simulation lemma of [56], with some additional modifications due to pessimism, and goes through the pessimistic MDP \( M_p \). We first show that

\[ J_{\hat{\rho}_0}(\pi, \hat{M}_p) \geq J_{\rho_0}(\pi, M) - \frac{2R_{\max}}{1-\gamma} \cdot D_{TV}(\rho_0, \hat{\rho}_0) - \frac{2\gamma R_{\max}}{(1-\gamma)^2} \cdot \alpha, \quad \text{and} \]

\[ J_{\hat{\rho}_0}(\pi, \hat{M}_p) \leq J_{\rho_0}(\pi, M) + \frac{2R_{\max}}{1-\gamma} \cdot D_{TV}(\rho_0, \hat{\rho}_0) + \frac{2\gamma R_{\max}}{(1-\gamma)^2} \cdot \alpha, \]

The main idea is to couple the evolutions of any given policy on the pessimistic MDP \( M_p \) and the model-based pessimistic MDP \( \hat{M}_p \) so that \((s_{t-1}, a_{t-1}) \eqdef (s_{t-1}^{M_p}, a_{t-1}^{M_p}) = (s_{t-1}^{\hat{M}_p}, a_{t-1}^{\hat{M}_p})\).

Assuming that such a coupling can be performed in the first step, since \( \|P(s, a) - \hat{P}(s, a)\|_1 \leq \alpha \), this coupling can be performed at each subsequent step with probability \( 1 - \alpha \). The probability that the coupling is not valid at time \( t \) is at most \( 1 - (1 - \alpha)^t \). So the total difference in the values of the policy \( \sigma \) on the two MDPs can be upper bounded as:

\[ |J_{\hat{\rho}_0}(\pi, \hat{M}_p) - J_{\rho_0}(\pi, M_p)| \leq \frac{2R_{\max}}{1-\gamma} \cdot D_{TV}(\rho_0, \hat{\rho}_0) + \sum_{t} \gamma^t (1 - (1 - \alpha)^t) \cdot 2 \cdot R_{\max} \]

\[ \leq \frac{2R_{\max}}{1-\gamma} \cdot D_{TV}(\rho_0, \hat{\rho}_0) + \frac{2\gamma R_{\max}}{(1-\gamma)^2} \cdot \alpha. \]

We now argue that

\[ J_{\rho_0}(\pi, M) \geq J_{\rho_0}(\pi, M) - \frac{2R_{\max}}{1-\gamma} \cdot E[\gamma^{T_{\max}}], \quad \text{and} \]

\[ J_{\rho_0}(\pi, M) \leq J_{\rho_0}(\pi, M). \]

For the first part, we see that the evolution of any policy \( \pi \) on the pessimistic MDP \( M_p \), can be coupled with the evolution of \( \pi \) on the actual MDP \( M \) until \( \pi \) encounters an unknown state. From this point, the total rewards obtained on the pessimistic MDP \( M_p \) will be \(-R_{\max}\), while the maximum total reward obtained by \( \pi \) on \( M \) from that point on is \( \frac{R_{\max}}{1-\gamma} \). Multiplying by the discount factor \( E[\gamma^{T_{\max}}] \) proves the first part.

For the second part, consider any policy \( \pi \) and let it evolve on the MDP \( M \) as \((s, a, s')\). Simulate an evolution of the same policy \( \pi \) on \( M_p \), \((s, a, s')\), as follows: if \((s, a) \in S_{Ak}\), then \( s'_{M_p} = s'_{M} \) and if \((s, a) \in U\), then \( s'_{M_p} = \text{HALT} \). We see that the rewards obtained by \( \pi \) on each transition in \( M_p \) is less than or equal to that obtained by \( \pi \) on the same transition in \( M \). This proves the second part of the lemma. \(\Box\)
Proof of Proposition 4. We consider the MDP in Figure 1, where we set $k = 10 \log \frac{1}{\gamma}$. The MDP has $k+1$ states, with three actions $a_1, a_2$ and $a_3$ at each state. The rewards (shown on the transition arrows) are all 0 except for the action $a_1$ taken in state $k+1$, in which case it is 1. Note that the rewards can be scaled by $R_{\text{max}}$ but for simplicity, we consider the setting with $R_{\text{max}} = 1$. It is clear that the optimal policy $\pi^*$ is to take the action $a_1$ in all the states. The starting state distribution $\rho_0$ is state 1 with probability $\rho_0 = \frac{1}{(1-\gamma)\log \frac{1}{1-\gamma}}$ and state $k+1$ with probability $1 - \rho_0$. The actions taken by the data collection policy are shown in blue. Since the dataset consists only of (state, action, reward, next state) pairs $(1, a_1, 0, 2), (2, a_2, 0, 1)$ and $(k+1, a_1, 1, k+1)$ we see that $U_D = (S \times A) \setminus \{(1, a_1), (2, a_2), (k+1, a_1)\}$ and $d^{\pi^*}(U_D) = (1-\gamma) \cdot (k-1) \cdot \rho_0 \leq \epsilon$ proving the first claim. Since none of the remaining states and actions are seen in the dataset, after permuting the actions if necessary, the time taken by any policy learned from the dataset, to reach state $k+1$ starting from state 1 is at least $\exp (k/5) \geq (1-\gamma)^{-2}$. So, the value of any policy $\tilde{\pi}$ learned from the dataset is at most $\frac{1-\rho_0}{1-\gamma} + \rho_0 \cdot \frac{1-\gamma^{(1-\gamma)^{-2}}}{1-\gamma} = \frac{1}{1-\gamma} - p_0 \cdot \frac{1-\gamma^{(1-\gamma)^{-2}}}{1-\gamma} \leq \frac{1}{1-\gamma} - \frac{3}{4(1-\gamma)}$, where we used $\gamma \in [0.95, 1)$ in the last step. On the other hand, the value of $\pi^*$ is at least $\frac{1-\rho_0}{1-\gamma} + \rho_0 \cdot \left(\frac{1}{1-\gamma} - k\right)$. So the suboptimality of any learned policy is at least $p_0 \cdot \left(\frac{3}{4(1-\gamma)} - k\right) = p_0 \cdot \left(\frac{3}{4(1-\gamma)} - 10 \log \frac{1}{1-\gamma}\right) \geq \frac{p_0}{4(1-\gamma)}$, where we again used $\gamma \in [0.95, 1)$ in the last step. Substituting the value of $p_0$ proves the proposition.

B Additional Experimental Details

Details of environments: As normally done in MBRL literature with OpenAI gym tasks [80, 81, 82, 28], we reduce the planning horizon for the environments to 400 or 500. Similar to [82, 28], we append our state parameterization with center of mass velocity to compute the reward from observations.

Types of policies: We build off the experimental setup of [20]. Towards this, we first go over some notation. Firstly, let $\pi_b$ represent the behavior policy, $\pi_r$ is a random policy that picks actions according to a certain probability distribution (for e.g., Gaussian $\pi^u_b$/Uniform $\pi^u_r$ etc.), $\pi_g$ a partially-trained policy, which one can assume is better than a random policy in value. Let $\pi^u_b(q)$ be a policy that plays random actions with probability $q$, and sampled actions from $\pi_r$ with probability $1-q$. Let $\pi^u_b(\beta)$ be a policy that adds zero-mean Gaussian noise with standard deviation $\beta$ to actions sampled from $\pi_r$.

Following prior work [20], we consider a partially trained data logging policy $\pi_b$. We consider five different cases, each corresponding to adding different kinds of exploratory noise to $\pi_b$, as described below.

(\text{E1}) \textbf{Pure}: The entire dataset is collected with the data logging (behavioral) policy $\pi_b$.

(\text{E2}) \textbf{Eps-1}: 40% of the dataset is collected with $\pi_b$, another 40% collected with $\pi^u_b(0.1)$, and the final 20% is collected with a random policy $\pi_r$.

(\text{E3}) \textbf{Eps-3}: 40% of the dataset is collected with $\pi_b$, another 40% collected with $\pi^u_b(0.3)$, and the final 20% is collected with a random policy $\pi_r$.

(\text{E4}) \textbf{Gauss-1}: 40% of the dataset is collected with $\pi_b$, another 40% collected with $\pi^u_b(0.1)$, and the final 20% is collected with a random policy $\pi_r$.

(\text{E5}) \textbf{Gauss-3}: 40% of the dataset is collected with $\pi_b$, another 40% collected with $\pi^u_b(0.3)$, and the final 20% is collected with a random policy $\pi_r$. 25