Influence of Loading Rate on Mechanical Behavior of Soil Specimen Considering Inertia Term

Binbin Xu\textsuperscript{1,2,3,*}

\textsuperscript{1}Tianjin Port Engineering Institute Co. Ltd. of CCCC First Harbor Engineering Co. Ltd., Tianjin, 300222, China
\textsuperscript{2}CCCC First Harbor Engineering Co. Ltd., Tianjin, 300461, China
\textsuperscript{3}Key Lab. of Geotechnical Engineering of Tianjin, Tianjin, 300222, China
\textsuperscript{4}Key Lab. of Geotechnical Engineering, Ministry of Communication, Tianjin, 300222, China
\textsuperscript{*}Corresponding author’s e-mail: xubinbin@tpei.com.cn

Abstract. The bifurcation or the strain localization of the soil specimen has been researched from the aspect of mechanical point. In the paper, the importance of the inertia term on the realization of a uniform deformation field is explained numerically. The effect of the uniform deformation is similar to the loading condition with infinitely slow loading, where there is no migration of the pore water. Also there is significant influence of inertia term on the apparent behavior of the soil specimen.

1. Research background

It is generally regarded that the strain localization or the material bifurcation on the soil specimen is derived from the uniform deformation field based on theoretical analyses [1-8]. A dynamic strain localization analysis is carried out by Zhang et al. [9] and the emphasis is put on the performance of soil constitutive model. However, in the above literature the influences of several factors including the soil constitutive model and the interaction between the pore water and the soil skeleton are discussed in detail. Another key factor of the influence of the acceleration on the formation of the shear band is ignored. In previous calculation, the deformation distribution is found to be related with the distribution of the pore water pressure. Meanwhile recently the degree of the pore water pressure is resulted from the rapid loading rate and varied accompanying different deformation modes. The main purpose of the paper is to analyze the realization of a completely uniform deformation field numerically based on the finite deformation theory. In the calculation, the inertial term will be taken into consideration as a dynamic problem which is different from the traditional quasi-static assumption.

2. Realization of uniform deformation field

For a calculation model of the soil specimen, usually the rectangular specimen under the plane strain condition is used and there is no material imperfection or geometrical bifurcation. The constraint condition is symmetrical with the top and bottom ends free in horizontal direction. The bottom surface is fixed in vertical direction and the top surface is applied a constant strain rate vertically. If we use this model to realize the uniform deformation the uniformity would soon lose when the inertia term is...
involved. The reason may be resulted in the dynamic strain propagation inside the specimen and the movement of the node is not at the same pace.

The updated Lagrangian method in the finite element calculation is used and the incremental constitutive equation of the soil skeleton instead of the total constitutive equation is employed. When the material time derivative of the acceleration is used, a new term which is called jerk term would be derived and there is a third order simultaneous ordinary differential equations towards time.

2.1 Boundary conditions

The boundary conditions and the partition of the finite element mesh for the specimen used in the calculation are shown in Figure 1. Two-dimension place strain condition is assumed in the calculation and a rectangular of 3.5cm width and 8.0cm height is used to represent the soil specimen. The specimen is completely saturated and isotropically consolidated in the initial state. The soil parameters and initial values used in the calculation is demonstrated in Table 1. For the mechanical stage of the specimen, it is assumed that the soil is remoulded and isotropically consolidated into 1471.5kPa and then isotropically unloaded to 294.3kPa. There are totally 70 elements in the horizontal direction and 160 elements in the vertical direction.

![Figure 1. Element mesh partition and boundary conditions](image)

Table 1 Elasto-plastic parameters and initial values

| Elasto-plastic parameters | Initial conditions |
|---------------------------|--------------------|
| Critical state index M    | 1.55               |
| NCL intercept N           | 2.0                |
| Compression index λ̂       | 0.108              |
| Swelling index Ł           | 0.025              |
| Poisson's ratio ν          | 0.3                |
| Degradation index of OC m | 0.2                |
| Specific volume v₀        | 1.747              |
| Stress ratio η₀           | 0.0                |
| Degree of structure 1/R₀* | 1.0                |
| Degree of overconsolidation 1/R₀ | 5.0 |
| Degree of anisotropy 𝜈₀    | 0.0                |
| Soil particle density ρₛ (g/cm³) | 2.65 |
| Coefficient of permeability k (cm/s) | 3.7 × 10⁻⁸ |

2.2 Calculation results
Two vertical loading velocities are adopted to evaluate the influence of the inertia term: (a) $10^{-5}$ cm/sec and (b) $10^3$ cm/sec. Due to the numerical error, there is some oscillation in the initial stage if the given values are applied on the nodes with velocity and acceleration in the horizontal direction and it can be ignored. The variation of the distribution of the pore water pressure is shown in Figure 2 compared with the deformation of the specimen. The apparent behavior of the specimen is shown in Figure 3 when the whole specimen is viewed as one mass. Usually there are four relationships in the apparent behavior: the apparent deviator stress $q$-axial strain $\varepsilon_a$ relationship, the deviator stress $q$-mean effective stress $p'$ relationship (in other words, the effective stress path), the pore water pressure $u$-axial strain $\varepsilon_a$ relationship, and the specific volume $v$-mean effective stress $p'$ relationship. For both case (a) and case (b) there is a uniform shear strain inside the specimen. However, for case (a) the distribution of the pore water pressure is actually also uniform inside the specimen. For case (b), the distribution of the pore water pressure is no longer uniform, which is related to the initial values applied to the nodes. It can be seen that the closer the place is to the centre the larger the pore pressure is. Even though, the apparent behavior between the deviator stress and the shear strain is quite similar to the perfect path calculated by the soil constitutive model for the same soil based on the uniform field.

![Figure 2. Variation of pore pressure (kPa) distribution for $10^{-5}$ cm/s and $10^3$ cm/s](image-url)
3. Influence of inertia term

In order to realize the uniform deformation, the initial values including the velocity, the acceleration and the pore pressure are designated to the nodes and the elements in advance inside the specimen to eliminate the influence of the inertia term. Obviously, it is quite different from the practical indoor experiment. In this section, the initial values mentioned above are removed from the nodes and the elements. Also two velocities (a) $10^{-5}$ cm/sec and (b) $10^3$ cm/sec are applied on the top surface and the results are shown as follows.

The progress of the shear strain and the deformation mode are shown in Figure 4 for case (a) and the corresponding apparent behavior of this case is shown in Figure 5. As can be seen, the deformation of the specimen is still uniform until the shear strain is over 10% when the loading rate is extremely slow. As the loading proceeds the symmetry in the horizontal direction is maintained while the symmetry in the vertical direction becomes asymmetrical. It can be ascribed that the deformation is transmitting from the top to the bottom and the inertia term cannot be ignored. Figure 5 shows the apparent behavior. As can be seen, the apparent behavior becomes different from the perfect path after the shear strain is over 16%. The distribution of the shear strain inside the specimen is shown in Figure 6 for case (b). At this case, the load rate is very large. Due to the influence of the inertia term, the symmetry in the vertical direction loses soon after the loading. The shear strain firstly occurs at the top surface and then propagates towards the bottom. There is almost no migration of pore water inside the specimen when the apparent behavior is different from the perfect path.
Figure 5. Apparent behavior for case (a)

Figure 6. Distribution of shear strain inside the specimen for case (b)

4. Conclusions
Using the finite element method, a series numerical calculation is carried out by taking the inertia term into consideration under the plane strain condition. The undrained boundary and constant cell pressure are set during the calculation and the conclusion are as follows:

1. When the initial values including the velocity and the acceleration are set to the node the uniform deformation can be realized no matter how large the loading rate is.

2. When the initial values are set to be zero inside the specimen, the symmetry in the vertical direction can not be maintained, which is similar to the practical experiment.

3. When the loading rate is very fast both the shear strain and the pore water inside the specimen propagates from the top surface to the bottom surface. The apparent behavior is different from the perfect path due to the influence of the inertia term.

References
[1] R. Hill and J. W. Hutchinson. Bifurcation phenomena in plane tension test, Mechanical Physical Solids, 1975: Vol. 23, 239-264.
[2] J. W. Rudnicki, J. R. Rice. Conditions for the localization of deformation in pressure-sensitive dilatant materials, Journal of Mechanical Physical Solids, 1975: Vol. 23, 371–394.
[3] L. Szabó. Shear band formulations in finite strain elasto-plasticity, International Journal of Solid Structure, 1994: Vol. 31 No. 9, 1291-1308.

[4] J. R. Rice. The localization of plastic deformation, Proc. 14th IUTAM Congress; Theoretical and applied mechanics, Amsterdam, Netherlands, 1976: pp. 207-220.

[5] C. Yatomi, A. Yashima, A. Iizuka and I. Sano. General theory of shear bands formation by a non-coaxial Cam-clay model, Soils and Foundations, 1989: Vol. 29, No. 3, 41-53.

[6] M. Khojastehpour and K. Hashiguchi. Axisymmetric bifurcation analysis in soils by the tangential-subloading surface model, Journal of Mechanical Physical Solids, 2004: Vol. 52, No. 10, 2235-2262.

[7] S. Kimoto, F. Oka and Y. Higo: Strain localization analysis of elasto-visco-plastic soil considering structural degradation, Computer Methods Apply Mechanical Energy, 2004: Vol. 193, Issue 27-29, 2845-2866.

[8] K. Ikeda, Y. Yamakawa and S. Tsutumi. Simulation and interpretation of diffuse mode bifurcation of elastoplastic solids, Journal of Mechanical Physical Solids, 2003: Vol. 51, Issue 9, 1649–1673.

[9] H. W. Zhang, L. Sanavia and B. A. Schrefler. Numerical analysis of dynamic strain localization in initially water saturated dense sand with a modified generalized plasticity model, Computers and Structures, 2001: Vol. 79, No. 4, 441-459.