Triply charmed baryons mass decomposition from lattice QCD

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We present the first lattice QCD calculation about the mass decomposition of triply charmed baryons with $J^P$ as $\frac{3}{2}^-$ and $\frac{5}{2}^-$. The quark mass term $\langle H_M \rangle$ contributes about 66% to the mass of $\frac{3}{2}^-$ state, which is slightly lower than that of the meson system with the same valence charm quark. Furthermore, based on our results, the total contribution of sea quarks, the gluons and the QCD anomaly accounts for about a quarter of the mass of these two triply charmed baryons. The mass difference of $\frac{3}{2}^-$ and $\frac{5}{2}^-$ states is mainly from the quark energy $\langle H_K \rangle$ of the QCD energy-momentum tensor. For comparison, the mass splitting is also calculated under the framework of the constituent quark model.

PACS numbers:
Keywords:

I. INTRODUCTION

Ever since the discovery of the $J/\psi$ particle in 1974 [1, 2], charm physics has become an important research field of hadron physics. At the same time, more and more charmed hadrons were discovered in experiments. Despite the fact that the first charmed baryon was identified by Mark-II in 1979 [3], the first doubly charmed baryon $\Xi^{++}$ was observed by LHCb [4] until recently. However, there is no signal of triply charmed baryons from experiments. In recent years, the Large Hadron Collider (LHC) shows its greater potential for the discovery of the charmed baryons, such as the excited singly charmed baryon states $\Lambda'^*(X)$ [5], the doubly charmed baryon $\Xi^++$ [4, 6, 7], the charmed strange baryon $\Xi_c(X)$ states [8] and $\Omega_c(X)$ states [9]. Besides the singly and doubly charmed baryons, there is still a good chance for LHC to discover the missing triply charmed baryon [10–12].

On the other hand, theoretical studies of triply charmed baryons can not only provide clues for experiments, but also deepen our understanding of heavy flavor baryon. At present, a lot of works have focused on the study of the mass spectra of the triply charmed baryons, such as quenched [13] and unquenched lattice QCD [14–24], relativistic [25] and non-relativistic quark model [26–30], QCD sum rule [31–33], Faddeev equation [34–36], diquark model [37, 38], variational method [39, 40], bag model [41, 42], hypercentral constituent quark model (HCQM) [43, 44], Regge theory [45] and Bethe-Salpeter equation [46]. The mass of the ground state given in most literature is 4800±50 MeV. However, in addition to spectroscopy, decomposing the mass of a hadron under the QCD framework gives us more information about the internal structure of a hadron. The mass decomposition of a hadron state under the QCD framework originated from the consideration of the dynamical mechanism between the quark and gluon of the nucleon. It was first proposed in Ref. [47], which opened up a new area for the study of the hadron structure. Recently, some researches of hadron mass decomposition from lattice QCD have been presented, such as the mass decomposition for proton [48], light and heavy meson [49] and charmonium [50]. The mass decomposition of hadron states reveals the complexity of hadron structure under the QCD framework and deepens our understanding of the hadron masses and their intrinsic properties.

Stimulated by the work of the mass decomposition of the charmonia and charmonium-like states, we explored the internal structure of the triply charmed baryons in this work. The calculations are performed on two different configurations with 2+1 flavor domain wall fermion and Iwasaki gauge action for two triply charmed baryons with $J^P$ as $\frac{3}{2}^-$ and $\frac{5}{2}^-$. In this work, we calculated the hadron mass $M$, the valence charm quark mass contribution $\langle H_M \rangle$ and the valence charm quark momentum fraction $\langle x \rangle$. Then the other component of the hadron mass were obtained through the decomposition formula of QCD energy momentum tensor (EMT) and the trace sum rule.

In order to compare the hadron mass decomposition from the QCD framework with the phenomenological model, we also calculated the mass decomposition of these two triply charmed baryons under the constituent quark model. The conventional charmonium spectrum is well described in the constituent quark model. From the framework of the quark model, the masses of charmonium come from three parts, the mass and kinetic energy of the constituent quarks and the potential energy between them. From the picture of the constituent quark model, the charmonium hyper-fine splitting
is mainly from the spin-spin interaction of the one gluon-exchange potential. The previous results of heavy meson mass decomposition from lattice QCD appear to be consistent with the picture of the constituent quark model \[49\]. In this study we also explore the connections between the phenomenological models and QCD theory through the mass decomposition of the triply charmed baryons.

The remainder of our article is organized as follows. In Sec. II, we introduce the formula for the hadron mass decomposition under the QCD framework in detail, as well as the relationship between the hadron mass and the two-point function, the hadron matrix element and the three-point function. In Sec. III, we introduce the details of the numerical simulation, including the configuration information, the effective mass, the effective matrix element, and our fitting results. In Sec. IV, we discuss our results and compare them with the constituent quark model. At last, a brief summary is given in Sec. V.

II. FORMALISM

A. Mass decomposition from the QCD EMT

In this paper, we adopt the decomposition of QCD energy-momentum tensor proposed by Ji \[47\], which is also used in Ref. \[48–50\]. The energy-momentum tensor of QCD is written as

\[
T^{\mu\nu} = \frac{1}{\bar{\psi}D^{\mu}\gamma^\nu\psi} + \frac{1}{2}F^{\mu\nu}F_{\mu\nu},
\]

(1)

where \(\bar{D}^{\mu} = \bar{D}^{\nu}D^{\mu}\bar{D}\) is the gauge-covariant derivative, \(F^{\mu\nu}\) is the color field strength tensor. The QCD Hamiltonian and the hadron mass could be written in terms of the energy-momentum tensor

\[
H_{\text{QCD}} = \int d^3x T^{00}(0, x),
\]

\[
M = \frac{\langle H|H_{\text{QCD}}|H \rangle}{\langle H|H \rangle} = \langle T^{00} \rangle.
\]

The QCD Hamiltonian could be decomposed as

\[
H_{\text{QCD}} = H_E^{(0)} + H_M + H_E^{(\rho)} + \frac{1}{4} H_a,
\]

where

\[
H_E^{(0)} = \sum_f \int d^3x \bar{\psi}(f)i(\bar{D} \cdot \gamma)\psi(f),
\]

\[
H_M = \sum_f \int d^3x \bar{\psi}(f)m_f\psi(f),
\]

\[
H_E^{(\rho)} = \int d^3x \frac{1}{2} (E^2 + B^2),
\]

\[
H_a = \int d^3x \gamma_m \sum_f \bar{\psi}(f)m_f\psi(f) + \frac{\beta(g)}{2g} F^{\mu\nu}F_{\mu\nu}.
\]

Here \(\gamma_m\) is the quark mass anomalous dimension, \(\sum_f\) denotes the summation of quark flavors, \(\beta(g)\) is the \(\beta\) function. Hence, the hadron mass can be decomposed as

\[
M = \langle T^{00} \rangle = \langle H_M \rangle + \langle H_E^{(\rho)} \rangle + \langle H_E^{(\rho)} \rangle + \frac{1}{4} \langle H_a \rangle.
\]

(3)

Here \(\langle H_M \rangle\) and \(\langle H_a \rangle\) are scale and renormalization scheme independent, while the quark energy \(\langle H_E^{(\rho)} \rangle\) and glue field energy \(\langle H_E^{(\rho)} \rangle\) have the dependency of the energy scale and renormalization scheme respectively. With the help of the equation of motion, the renormalized quark and gluon energy can be re-defined as

\[
\langle H_E^R \rangle = \frac{3}{4} \langle x_q^R M \rangle - \frac{3}{4} \langle H_M \rangle,
\]

\[
\langle H_E^R \rangle = \frac{3}{4} \langle x_g^R M \rangle.
\]

(4)

Here \(\langle x_q^R \rangle\) and \(\langle x_g^R \rangle\) are the renormalized quark and gluon momentum fraction respectively. They are related as \[51\]

\[
\langle x_q^R \rangle = 1 - \langle x_{\gamma}^R \rangle.
\]

Following Ref. \[50\], the total valence charm quark contribution can be further defined as:

\[
\langle H_E^R \rangle = \langle H_M \rangle + \langle H_a \rangle,
\]

(5)

Considering the trace sum rule \[52\]

\[
M = \langle T^{00} \rangle = \langle H_M \rangle + \langle H_a \rangle,
\]

(6)

the QCD anomaly term \(\langle H_a \rangle\) could be derived, provided with the hadron mass \(M\) and the quark mass term \(\langle H_M \rangle\). Then we can get the mass decomposition of a hadron state, provided with the hadron mass \(M\), quark momentum fraction \(\langle x_q^R \rangle\) and the quark mass term \(\langle H_M \rangle\), according to Eq. (3) and Eq. (6).

B. Two-point and three-point function

From the expression in Section. II A, we can calculate the hadron mass \(M\), the quark mass \(\langle H_M \rangle\) and the renormalized quark momentum fraction \(\langle x_q^R \rangle\) (the following denoted as \(\langle x_q \rangle\) for convenience) of a hadron state.

Refer to the basic operator of the \(\Omega\) baryon \[53\], the basic structure of the triply charmed baryon operator is as follows:

\[
O^a(\vec{x}, t) = e^{abc}e_q^R e_q^R \bar{q}^c(\vec{x}, t) T C \gamma^\mu e_q^R(\vec{x}, t)c^a_q(\vec{x}, t).
\]

(7)

\(C = \gamma_2 \gamma_4\) is the \(C\)-parity operator; \(a, \beta, \gamma\) represent the Dirac indices; \(a, b, c\) are the color indices; and \(T\) is the transpose operator. In order to get the baryon operator with definite \(P\) parity, we introduce the \(P\) parity projector:

\[
P_\pm = \frac{1}{2}(1 \pm \gamma_4).
\]

(8)
On the other hand, we refer to the specific form of the spin projector from [54] to get the definite spin for the baryon operator,

\[
\begin{align*}
P^{\alpha\nu} &= \delta^{\alpha\nu} - \frac{1}{4} \gamma^\mu \gamma^\nu, \\
P^{\alpha\nu} &= \frac{i}{2} \gamma^\mu \gamma^\nu.
\end{align*}
\]

In our study, only the spatial component of the triply charmed baryon operator is considered. Therefore, the baryon operator with a definite \( J^P \) quantum number can be expressed as:

\[
O'(\vec{x}, t) = (P_{s})_{00'} \times \sum_j \langle P_j \rangle^{ij}_{\mu\gamma} \varepsilon^{abc} \left[ c^a_q(\vec{x}, t)^T (C\gamma^j)_{ab} e^b_c(\vec{x}, t) \right] c^c_q(\vec{x}, t).
\]

The hadron mass can be obtained from the two-point correlation function

\[
C_2(\vec{p}, t) = \sum_{\vec{x}} e^{-i\vec{p} \cdot \vec{x}} \langle O(\vec{x}, t) O'(\vec{0}, 0) \rangle = \sum_n A_n e^{-M_n t} t \rightarrow \infty \rightarrow A_0 e^{-M_0 t}, \tag{9}
\]

while the quark mass term \( \langle H_M \rangle \) and the quark momentum fraction \( \langle \chi \rangle_q \) can be extracted from the three-point function

\[
C_3(\vec{p}, \vec{q}, t', \vec{O}) = \sum_{\vec{x}} e^{-i\vec{p} \cdot \vec{x}} e^{-i\vec{q} \cdot \vec{x}} \langle O(\vec{x}, t) \hat{O}(\vec{y}, t') O'(\vec{0}, 0) \rangle, \tag{10}
\]

where \( \hat{O}(\vec{y}, t') \) refers to the current operator in the corresponding process. Here, we only considered the contribution of the valence charm quark. According to the Hamiltonian of the valence charm quark mass term \( \langle H_M \rangle \), the current operator is written as

\[
\hat{H}_M(\vec{y}, t') = m_c \bar{c}(\vec{y}, t') c(\vec{y}, t'), \tag{11}
\]

where \( m_c \) refers to the valence charm quark mass. For the valence charm quark momentum fraction \( \langle \chi \rangle_q \), the current operator is

\[
\hat{\chi}_q(\vec{y}, t') = \frac{1}{2} \bar{c}(\vec{y}, t')(\gamma_4 \overleftrightarrow{D}_4 - \frac{1}{3} \gamma_i \overleftrightarrow{D}_i) c(\vec{y}, t'). \tag{12}
\]

### III. NUMERICAL DETAILS

The 2+1 flavor domain wall fermion and Iwasaki gauge action configurations provided by the RBC/UKQCD cooperation group are used in this calculation [55, 56]. Table I shows the parameters of these gauge ensembles. For the valence charm quark, we use the overlap fermion with exact chiral symmetry on the lattice, in which the Hamiltonian of the valence charm quark mass \( \langle H_M \rangle \) is renormalization scale and scheme independent [57].

For the hadron mass, we can extract it by fitting the two-point correlation function directly. Considering the unphysical oscillatory behavior caused by the Domain Wall fermion [58], we take the fitting function of the two-point function as:

\[
C_2(t) = A_0 e^{-M_0 t}(1 + A_1 e^{-\delta m t}) + W(-1) e^{-M_1 t}. \tag{13}
\]

where \( e^{-\delta m t} \) represents the contribution from the excited state and \( e^{-M_1 t} \) is the oscillation term. \( A_0 \) and \( A_1 \) are free parameters.

As usual, we take the definition of effective mass as following:

\[
m_{\text{eff}} = \ln \left( \frac{C_2(t + 1)}{C_2(t)} \right). \tag{14}
\]

The effective mass of two triply charmed baryons with \( J^P \) as \( \frac{3}{2}^- \) and \( \frac{3}{2}^+ \) obtained from two different configurations are shown in Fig. 1, where the dark color band is our fitting range and the light color band shows the extrapolation results. Table II shows the masses of two different triply charmed baryons.

![Image](324x244 to 561x325)

**TABLE I:** The parameters for the configurations [50]

| ensemble | \( L \times T \) | a(\text{fm}) | \( m_c(\text{MeV}) \) | \( m_o(\text{MeV}) \) | \( N_{\text{cfg}} \) |
|----------|----------------|-------------|----------------|----------------|-------------|
| 32I      | \( 32^3 \times 64 \) | 0.0828(3)   | 300            | 0.493          | 305         |
| 48I      | \( 48^3 \times 96 \) | 0.0711(3)   | 278            | 0.410          | 205         |

**TABLE II:** The hadron mass for two states on \( 32^3 \times 64 \) (top) and \( 48^3 \times 96 \) (bottom) configurations, where the fitting range and \( \chi^2/d.o.f \) are shown in the table.

| ensemble | \( J^P \) | \( M(\text{GeV}) \) | \( \chi^2/d.o.f \) |
|----------|--------|----------------|----------------|
| 32I      | \( \frac{3}{2}^+ \) | 4.804(20) | 10-27 \( 0.76 \) |
|          | \( \frac{3}{2}^- \) | 5.064(51) | 10-25 \( 1.4 \) |
| 48I      | \( \frac{3}{2}^+ \) | 4.793(21) | 13-38 \( 0.42 \) |
|          | \( \frac{3}{2}^- \) | 5.071(27) | 10-25 \( 0.85 \) |

![Image](54x116 to 550x756)

**FIG. 1:** Effective mass for two states on \( 32^3 \times 64 \) (top) and \( 48^3 \times 96 \) (bottom) configurations.

We can extract the corresponding hadron matrix elements from the corresponding three-point function. Similar to the effective mass, the corresponding effective matrix elements are
defined as:

\[ \langle H_M \rangle(t) = R(t, \hat{H}_M) - R(t-1, \hat{H}_M), \]
\[ M(x)_q(t) = R(t, \hat{x}) - R(t-1, \hat{x}), \]  \hspace{1cm} (15)

where \( R(t, \hat{O}) = C_3(t, \hat{O})/C_2(t) \) is a ratio function. \( C_3(t, \hat{O}) \) is a three-point function about the operator \( \hat{O} \) from Eq. (10), which had summed all the possible intermediate time from source to sink because of the sequential source method we took [59]. When \( t \) is large enough, \( R(t, \hat{O}) - R(t-1, \hat{O}) \) approach the matrix elements we need.

Therefore we fitted the effective hadron matrix elements we constructed directly to extract the quark mass contribution \( \langle H_m \rangle \) and the quark momentum fraction \( \langle x \rangle_q \). When we only consider the contribution of the first order energy of the excited state, the fitting function of the effective matrix elements can be written as:

\[ \langle H_M \rangle(t) = \langle H_M \rangle + A_1'e^{-\delta m t} + tA_2'e^{-\delta m t}, \]
\[ M(x)_q(t) = M(x)_q + A_3'e^{-\delta m t} + tA_4'e^{-\delta m t}. \]  \hspace{1cm} (16)

where \( A_1', A_2', A_3' \) and \( A_4' \) are all free parameters.

The effective matrix elements of valence charmed quark mass \( \langle H_M \rangle \) and valence charmed quark momentum fraction \( \langle x \rangle_q \) are shown in Fig. 2 and Fig. 3, where the dark color band is our fitting range and the light color band shows the extrapolation results. Table III and Table IV are the fitting results of the corresponding hadron matrix elements. Our results reveal that, on two different configurations, the valence charmed quark mass of the orbital excited state \( \frac{1}{2}^- \) is slightly smaller than that of the ground state \( \frac{1}{2}^+ \).

![FIG. 2: Effective matrix elements of valence charmed quark mass \( \langle H_M \rangle \) for two states on 32\(^3\) \times 64 (top) and 48\(^3\) \times 96 (bottom) configurations.](Image)

![FIG. 3: Effective matrix elements of valence charmed quark momentum fraction \( \langle x \rangle_q \) for two states on 32\(^3\) \times 64 (top) and 48\(^3\) \times 96 (bottom) configurations.](Image)

### TABLE III: The charmed quark mass \( \langle H_M \rangle \) for two states on 32\(^3\) \times 64 (top) and 48\(^3\) \times 96 (bottom) configurations, where the fitting range and \( \chi^2/d.o.f \) are shown in the table.

| ensemble | \( \langle H_M \rangle \) (GeV) | \( [t_{min} - t_{max}] \) | \( \chi^2/d.o.f \) |
|----------|-------------------------------|--------------------------|------------------|
| 32I \( \frac{1}{2}^+ \) | 3.192(13) | 10-27 | 0.062[18] |
| 48I \( \frac{3}{2}^- \) | 3.114(60) | 11-16 | 0.38[6] |

### IV. DISCUSSION

#### A. Mass spectrum

A comparison of our results of the mass of two triply charmed baryons with other lattice QCD collaborations are listed in Table V. For the ground state \( \frac{1}{2}^+ \), the central value of the mass calculated by other collaborations are higher than 4.7 GeV, except for Ref.[13]. In particular, the central value of the ground state triply charmed baryon mass from Ref.[20] is slightly higher than 4.8 GeV. Our results are reasonably consistent with the previous studies. As for the orbital excited states \( \frac{3}{2}^- \), there are only a few literature calculated its mass. Combined with the error we obtained, our results are in a good agreement with the previous literature.

#### TABLE IV: The charmed quark momentum fraction \( \langle x \rangle_q \) for two states on 32\(^3\) \times 64 (top) and 48\(^3\) \times 96 (bottom) configurations, where the fitting range and \( \chi^2/d.o.f \) are shown in the table.

| ensemble | \( \langle x \rangle_q \) | \( [t_{min} - t_{max}] \) | \( \chi^2/d.o.f \) |
|----------|-------------------|--------------------------|------------------|
| 32I \( \frac{1}{2}^+ \) | 0.7960(18) | 10-27 | 0.096[18] |
| 48I \( \frac{3}{2}^- \) | 0.7653(33) | 13-38 | 0.089[26] |
| 48I \( \frac{3}{2}^- \) | 0.7623(42) | 7-24 | 1.1[18] |
TABLE V: The mass of the ground state \( \Omega_{cc}(1^+S_{3/2}) \) and the orbital excited state \( \Omega_{cc}(1^-P_{3/2}) \) calculated by us and other lattice QCD collaborations. We show the number of the flavor \( (N_f) \), lattice spacings \( (a) \), the pion mass \( (m_\pi) \), the actions of the relevant sea \( (S^\text{sea}) \) and valence charm \( (S^\text{val}) \) quarks from these literatures. Where HISQ and RHQA are abbreviations for highly-improved staggered quark and relativistic heavy-quark action respectively.

| Collaboration          | \( N_f \) | \( a (fm) \) | \( m_\pi (GeV) \) | \( S^\text{sea} \) | \( S^\text{val} \) | \( \Omega_{cc}(1^+S_{3/2})(GeV) \) | \( \Omega_{cc}(1^-P_{3/2})(GeV) \) |
|------------------------|-----------|-------------|-----------------|----------------|----------------|---------------------|---------------------|
| [Ours]32[1]            | 2+1       | 0.0828(3)   | 0.3             | Domain-wall   | Overlap        | 4.804(20)           | 5.064(51)           |
| [Ours]48If             | 2+1       | 0.0711(3)   | 0.278           | Domain-wall   | Overlap        | 4.793(21)           | 5.071(27)           |
| T.W.Chiu et al. [13]   | quench    | 0.0882(30)  | -               | Wilson        | Domain-wall    | 4.681(28)           | 5.066(48)           |
| R.A.Briceno et al. [14] | 2+1+1     | 0.06-0.12   | 0.220-0.310     | HISQ          | RHQA           | 4.761(52)(21)(6)    | -                   |
| PACS-CS [15]           | 2+1       | 0.0899      | 0.135(6)        | Clover        | RHQA           | 4.789(22)           | -                   |
| HSC [16]               | 2+1       | 0.0351(2)   | 0.390           | Clover        | RHQA           | 4.763(6)            | 5.124(13)           |
| Z.S.Brown [17]         | 2+1       | 0.085-0.11  | 0.227-0.419     | Domain-wall   | RHQA           | 4.796(8)(18)        | -                   |
| K.U.Can [18]           | 2+1       | 0.0907(13)  | 0.156(7)(2)     | Wilson        | Clover         | 4.769(6)            | -                   |
| ETM [19]               | 2+1+1     | 0.065-0.094 | 0.210-0.430     | Twisted Mass  | Twisted Mass   | 4.734(12)(11)(9)    | -                   |
| TRJQCD [20]            | 2+1       | 0.0907(13)  | 0.156(9)        | Clover        | Clover         | 4.817(12)           | 5.083(67)           |
| C.Alexandrou et al. [21]| 2         | 0.0938(3)(2)| 0.130           | Twisted Mass  | clover-improved| 4.746(4)(32)        | -                   |
| C.Alexandrou et al. [22]| 2         | 0.0561(1)-0.0891(0) | 0.260-0.450 | Twisted Mass  | Osterwalder-Seiler | 4.6769(46)(30) | - |
| S.Durr et al. [23]     | 2         | 0.0728(5)(19)| 0.280           | Wilson        | Brillouin      | 4.774(24)           | -                   |
| TWQCD [24]             | 2+1+1     | 0.063       | 0.280           | Domain-wall   | Domain-wall    | 4.766(5)(11)        | 5.168(37)(51)       |

TABLE VI: Each part of the triply charmed baryon mass from two kinds of configurations, the upper are for 32I configurations, and the lower are for 48I configurations.

| \( J^p \) \( M (GeV) \) | \( \langle H_M \rangle(GeV) \) | \( \langle H_{\pi} \rangle(GeV) \) | \( \langle H_{p} \rangle(GeV) \) | \( \frac{1}{2} \langle H_{\gamma} \rangle(GeV) \) |
|--------------------------|-----------------|-----------------|-----------------|-----------------|
| \( \frac{1}{2} \)        | 4.804(20)       | 3.192(13)       | 0.474(17)       | 0.735(30)       | 0.403(6)         |
| \( \frac{3}{2} \)        | 5.064(51)       | 3.098(38)       | 0.669(56)       | 0.806(86)       | 0.492(16)        |
| \( \frac{5}{2} \)        | 4.793(21)       | 3.185(19)       | 0.362(22)       | 0.844(37)       | 0.4020(71)       |
| \( \frac{7}{2} \)        | 5.071(27)       | 3.114(60)       | 0.564(50)       | 0.904(84)       | 0.489(16)        |

B. Mass decomposition

Based on the Eq. (3) and Eq. (6), the other components can be calculated with the hadron mass \( M \), the valence charm quark mass term \( \langle H_M \rangle \) and the valence charm quark momentum fraction \( \langle x \rangle_q \). Fig. 4 and Table VI show the results of the hadron mass \( M \) and every part of the mass decomposition for two different states on two different gauge ensembles. On two gauge ensembles, the valence charm quark mass for ground state \( \frac{1}{2}^+ \) are 3.192(13) GeV and 3.185(19) GeV, are around 66% of the hadron mass. For conventional charmonium states [50], the valence charmed quark mass for ground state \( \eta_c(0^{++}) \) and \( J/\psi(1^{--}) \) are around 74% and 69% of the hadron mass. Thus, the ratio of the valence charmed quark masses for ground state triply charmed baryon to the whole baryon mass is slightly lower than that of the meson system with the same valence charm quark.

Based on the heavy quark expansion [52]

\[ \langle H_M^{\text{sea}} \rangle = \frac{2}{27} \left( \frac{1}{1 + \gamma_M(\mu)} - \langle H_M^{\text{val}} \rangle \right) + O(\alpha_s) \]

(17)

where \( \gamma_M(\mu) = 2 \alpha(\mu)/\pi \) is the quark anomalous dimension. If we take the same \( \alpha(\mu = m_c) \approx 0.37 \) in our study [60], we can estimate the sea charm quark mass \( \langle H_M^{\text{sea}} \rangle \) for ground state \( \frac{1}{2}^+ \) and orbital excited state \( \frac{3}{2}^- \). From our results, we get the sea charmed quark mass \( \langle H_M^{\text{sea}} \rangle \) are less than 100 MeV for two states on two different configurations. Thus we ignore the sea charm quark mass \( \langle H_M^{\text{sea}} \rangle \) in this study. As for the quark mass contribution of the light and strange quarks, we take into account their contribution in the charmonium is not more than 40 MeV in total [50]. Therefore, we tentatively ignore their contribution in this work. However, a more rigorous calculation is left for us in the future.

Based on Eq. (5), the total contribution of valence charmed quarks \( \langle H_M \rangle \) can be calculated with the hadron mass \( M \), the valence charmed quark mass \( \langle H_M \rangle \) and the valence charmed quark momentum fraction \( \langle x \rangle_q \). The total contribution of valence charmed quarks are 3.666(14) GeV and 3.547(18) GeV for \( \frac{1}{2}^+ \) state on two different configurations, are 76.31(43)% and 74.00(50)% of the baryon mass. For orbital excited state

FIG. 4: The mass decomposition for two states on 32I \( \times 64 \) (top) and 48I \( \times 96 \) (bottom) configurations.
From the perspective of the constituent quark model \[30\], the Hamiltonian of a triply charmed baryon includes the confinement potential and the one gluon exchange potential. The specific form of the Hamiltonian is shown in the literature \[30\]. The parameters of the constituent quark model, the sum of the quark kinetic energy, the confinement potential and the one gluon exchange potential, is actually the kinetic and potential energy of three valence charm quark. Under the framework of QCD, the mass splitting between the ground state $\frac{3}{2}^+$ and the orbital excited state $\frac{3}{2}^-$ of the triply charmed baryons almost comes from the quark energy. It seems that the mass splitting between the ground state $\frac{3}{2}^+$ and the orbital excited state $\frac{1}{2}^-$ of the triply charmed baryons from the picture of the constituent quark model is consistent with our results from lattice QCD. However, we hope there will be more accurate calculations to confirm this in the future. Furthermore, we got the difference between the confinement potential of the ground state $\frac{3}{2}^+$ and the orbital excited state $\frac{1}{2}^-$ is 160 MeV. The one gluon exchange potential are negative for two different states and their difference is 170 MeV, because of the repulsive Coulomb force between three different quarks. Our results reveal that the mass splitting between the ground state $\frac{3}{2}^+$ and the orbital excited state $\frac{1}{2}^-$ of two different triply charmed baryon mainly comes from the joint action of the Hamiltonian of the confinement potential and the Coulomb potential under the framework of the constituent quark model.

### TABLE VII: Each part of the triply charmed baryon mass calculated from the quark model, $M$ is the hadron mass of triply charmed baryons

| $J^P$ | $M(GeV)$ | $M_q(GeV)$ | $T(GeV)$ | $V_C(GeV)$ | $V_G(GeV)$ |
|---|---|---|---|---|---|
| $\frac{3}{2}^+$ | 4.83 | 4.45 | 0.53 | 0.47 | -0.63 |
| $\frac{3}{2}^-$ | 5.16 | 4.45 | 0.54 | 0.63 | -0.46 |
| $\frac{1}{2}^-$ | 5.16 | 4.45 | 0.54 | 0.63 | -0.46 |

The previous results of the meson mass decomposition indicate that the charmonium hyper-fine splitting mainly comes from the quark energy $\langle H_E \rangle$ [49]. It is shown that the mass splitting of the orbital excited state and the ground state also comes from the quark energy of the energy-momentum tensor of QCD even if it is extended to the baryon system containing the same valence charm quark.

As we can see in Fig. 4 and Table. VI, the mass differences between the $\frac{3}{2}^+$ state and the $\frac{3}{2}^-$ state are 0.260(55) GeV and 0.278(34) GeV on two different configurations. From the results of every part of our mass decomposition from Table VI, the valence charmed quark masses of ground state $\frac{3}{2}^+$ are about 100 MeV heavier than the orbital excited state $\frac{1}{2}^-$. But they are largely canceled by the QCD anomaly term $\frac{1}{2}\langle H_E \rangle$. Within the errors of our results, the differences of the gluon energy ($H_G$) between two different states are very small. Thus the mass differences between the ground state $\frac{3}{2}^+$ and the orbital excited state $\frac{1}{2}^-$ are mainly from the quark energy $\langle H_E \rangle$.

In addition, we also studied the mass decomposition of the ground state $\frac{3}{2}^+$ and orbital excited state $\frac{1}{2}^-$ triply charmed baryons under the framework of the constituent quark model. From the perspective of the constituent quark model \[30\], the Hamiltonian of a triply charmed baryon includes the constituent quark mass, the kinetic energy term, confinement potential and one gluon exchange potential,

$$H = M_q + T + V_C + V_G,$$

(18)

where $M_q$ is the constituent quark mass, and $T$ is the kinetic energy, $V_C$ is the confinement potential, and $V_G$ is the one gluon exchange potential. The specific form of the Hamiltonian is shown in the literature \[30\]. The parameters of the non-relativistic quark model we adopted are the same as \[30\]. The values of each part of the triply charmed baryon mass calculated from the quark model are shown in the Table VII.

From the constituent quark model, we got the masses of the ground state $\frac{3}{2}^+$ and the orbital excited state $\frac{1}{2}^-$ are 4.83 GeV and 5.16 GeV respectively. Their mass splitting comes from three parts: the quark kinetic energy, the confinement potential and the one gluon exchange potential. Their constituent quark mass are almost the same. From the picture of the constituent quark model, the sum of the quark kinetic energy, the confinement potential and the one gluon exchange potential is actually the kinetic and potential energy of three valence charm quark.

In this work, we make a research for the mass decomposition of two triply charmed baryons with $J^P$ as $\frac{3}{2}^+$ and $\frac{1}{2}^-$ from lattice QCD, which is helpful to reveal the internal structure of the triply-charmed baryons.

We found that, the valence charmed quark masses ($H_M$) are about 66% of the hadron mass for ground state $\frac{3}{2}^+$. Compare with the results of the charmonium-like states, the valence charmed quark masses for ground state triply charmed baryon is slightly lower than that of the meson system with the same valence charm quark.

On two different configurations, the total contribution of valence charmed quarks of $\frac{3}{2}^+$ state are 76.31(43)% and 74.00(50)% of the baryon mass; the total contribution of the valence charmed quarks of $\frac{3}{2}^-$ state are about 74.4(1.2)% and 72.53(66)% of the baryon mass. This shows that the total contribution of the sea quark, the gluon energy and the QCD anomaly accounts for about a quarter of the mass composition of the triply-charmed baryon.

From the QCD energy-momentum tensor, the mass differences between the ground state $\frac{3}{2}^+$ and the orbital excited state $\frac{3}{2}^-$ are from the quark energy $\langle H_E \rangle$. On the other hand, the differences almost comes from the kinetic and potential energy of three valence charm quarks in the constituent quark model. It seems that the mass splitting between the ground state $\frac{3}{2}^+$ and the orbital excited state $\frac{1}{2}^-$ of the triply charmed baryons from the picture of the constituent quark model is consistent with our results from lattice QCD.
Acknowledgement

The authors L. C. Gui, W. Sun and J. Liang, as members of the χQCD collaboration, thank the RBC collaboration for providing us their DWF gauge configurations. This work is supported by the Natural Science Foundation of China under grant No.12175036, No.11935017, No.12175073, No.12205311, No.12222503. J. Liang is also supported by Guangdong Major Project of Basic and Applied Basic Research, China (No.2020B0301030008). We thank Hui-hua Zhong for useful discussion. The computations were performed on the Xiangjiang-1 cluster at Hunan Normal University (Changsha) and the Southern Nuclear Science Computing Center (SNSC) and the HPC clusters at Institute of High Energy Physics (Beijing) and China Spallation Neutron Source (Dongguan) and the ORISE Supercomputer.

[1] J. J. Aubert et al. [ES98], Experimental Observation of a Heavy Particle J′ [J], Phys. Rev. Lett. 33, 1404-1406 (1974)
[2] J. E. Augustin et al. [SLAC-SP-017], Discovery of a Narrow Resonance in e+e− Annihilation [J], Phys. Rev. Lett. 33, 1406-1408 (1974)
[3] G. S. Abrams, M. S. Alam, C. A. Blocker, A. Boyarski, M. Breidenbach, D. L. Burke, W. C. Carithers, W. Chinowsky, M. W. Coles and S. Cooper, et al. Observation of Charmed Baryon Production in e+e− Annihilation [J], Phys. Rev. Lett. 44, 10 (1980)
[4] R. Aaij et al. [LHCb], Observation of the doubly charmed baryon Ξ+c [J], Phys. Rev. Lett. 119, no.11, 112001 (2017)
[5] R. Aaij et al. [LHCb], Study of the Δ(3) p amplitude in Λc0 → Δ(3) pπ− decays [J], JHEP 05, 030 (2017)
[6] R. Aaij et al. [LHCb], First Observation of the Doubly Charmed Baryon Decay Ξ+c+ → Ξ+cπ− [J], Phys. Rev. Lett. 121, no.16, 162002 (2018)
[7] R. Aaij et al. [LHCb], Observation of the doubly charmed baryon decay Ξ+c+ → Ξ+cπ− [J], JHEP 05, 038 (2022)
[8] R. Aaij et al. [LHCb], Observation of New Ξ+c Baryons Decaying to Λc+K− [J], Phys. Rev. Lett. 124, no.22, 222001 (2020)
[9] R. Aaij et al. [LHCb], Observation of five new narrow Ωb states decaying to Ξ−cK− [J], Phys. Rev. Lett. 118, no.18, 182001 (2017)
[10] V. A. Saleev, Omega(ccc) production via fragmentation at LHC [J], Mod. Phys. Lett. A 14, 2615-2620 (1999)
[11] Y. Q. Chen and S. Z. Wu, Production of Triply Heavy Baryons at LHC [J], JHEP 08, 144 (2011) [erratum: JHEP 09, 089 (2011)]
[12] H. He, Y. Liu and P. Zhuang, Ωb production in high energy nuclear collisions [J], Phys. Lett. B 746, 59-63 (2015)
[13] T. W. Chiu and T. H. Hsieh, Baryons in lattice QCD with exact chiral symmetry [J], Nucl. Phys. A 755, 471-474 (2005)
[14] R. A. Briceno, H. W. Lin and D. R. Bolton, Charmed-Baryon Spectroscopy from Lattice QCD with Nf = 2 + 1 + 1 Flavors [J], Phys. Rev. D 86, 094504 (2012)
[15] Y. Namekawa et al. [PACS-CS], Charmed baryons at the physical point in 2+1 flavor lattice QCD [J], Phys. Rev. D 87, no.9, 094512 (2013)
[16] M. Padmanath, R. G. Edwards, N. Mathur and M. Peardon, Spectroscopy of triply-charmed baryons from lattice QCD [J], Phys. Rev. D 90, no.7, 074504 (2014)
[17] Z. S. Brown, W. Detmold, S. Meinel and K. Orginos, Charmed bottom baryon spectroscopy from lattice QCD [J], Phys. Rev. D 90, no.9, 094507 (2014)
[18] K. U. Can, G. Erkol, M. Oka and T. T. Takahashi, Look inside charmed-strange baryons from lattice QCD [J], Phys. Rev. D 92, no.11, 114515 (2015)
[19] C. Alexandrou, V. Drach, K. Jansen, C. Kallidonis and G. Koutsou, Baryon spectrum with Nf = 2 + 1 + 1 twisted mass fermions [J], Phys. Rev. D 90, no.7, 074501 (2014)
[20] H. Bahiyar, K. U. Can, G. Erkol, P. Gubler, M. Oka and T. T. Takahashi, Charmed baryon spectrum from lattice QCD near the physical point, Phys. Rev. D 102, no.5, 054513 (2020)
[21] C. Alexandrou and C. Kallidonis, Low-lying baryon masses using Nf = 2 twisted mass clover-improved fermions directly at the physical point [J], Phys. Rev. D 96, no.3, 034511 (2017)
[22] C. Alexandrou, J. Carbonell, D. Christaras, V. Drach, M. Gravin and M. Papinutto, Strange and charm baryon masses with two flavors of dynamical twisted mass fermions [J], Phys. Rev. D 86, 114501 (2012)
[23] S. Durr, G. Koutsou and T. Lippert, Meson and Baryon dispersion relations with Brillouin fermions [J], Phys. Rev. D 86, 114514 (2012)
[24] Y. C. Chen et al. [TWQCD], Lattice QCD with Nf = 2 + 1 + 1 domain-wall quarks [J], Phys. Lett. B 767, 193-198 (2017)
[25] A. P. Martynenko, Ground-state triply and doubly heavy baryons in a relativistic three-quark model [J], Phys. Lett. B 663, 317-321 (2008)
[26] B. Silvestre-Brac, Spectrum and static properties of heavy baryons [J], Few Body Syst. 20, 1-25 (1996)
[27] W. Roberts and M. Pervin, Heavy baryons in a quark model [J], Int. J. Mod. Phys. A 23, 2877-2890 (2008)
[28] J. Vijande, A. Valcarce and H. Garcilazo, Constituent-quark model description of triply heavy baryon nonperturbative lattice QCD data, Phys. Rev. D 91, no.5, 054011 (2015)
[29] G. Yang, J. Ping, P. G. Ortega and J. Segovia, Triply heavy baryons in the constituent quark model [J], Chin. Phys. C 44, no.2, 023102 (2020)
[30] M. S. Liu, Q. F. Liu and X. H. Zhong, Triply charmed and bottom baryons in a constituent quark model [J], Phys. Rev. D 101, no.7, 074031 (2020)
[31] J. R. Zhang and M. Q. Huang, Deciphering triply heavy baryons in terms of QCD sum rules [J], Phys. Lett. B 674, 28-35 (2009)
[32] Z. G. Wang, Analysis of the Triply Heavy Baryon States with QCD Sum Rules [J], Commun. Theor. Phys. 58, 723-731 (2012)
[33] T. M. Aliev, K. Azizi and M. Savec, Properties of triply heavy spin-3/2 baryons [J], J. Phys. G 41, 065003 (2014)
[34] H. Sanchis-Alepuz, R. Alkofer, G. Eichmann and R. Williams, Model Comparison of Delta and Omega Masses in a Covariant Faddeev Approach [J], PoS QCD-TNT-II, 041 (2011)
[35] M. Radin, S. Babaghadrat and M. Monemzadeh, Estimation of heavy baryon masses Ωccc± and Ωbhb by solving the Faddeev equation in a three-dimensional approach [J], Phys. Rev. D 90, no.4, 047701 (2014)
[36] S. X. Qin, C. D. Roberts and S. M. Schmidt, Spectrum of light- and heavy-baryons [J], Few Body Syst. 60, no.2, 26 (2019)
[37] K. Thakkar, A. Majethiya and P. C. Vinodkumar, Magnetic moments of baryons containing all heavy quarks in the
quark-diquark model [J], Eur. Phys. J. Plus 131, no.9, 339 (2016)

[38] P. L. Yin, C. Chen, G. Krein, C. D. Roberts, J. Segovia and S. S. Xu, Masses of ground-state mesons and baryons, including those with heavy quarks [J], Phys. Rev. D 100, no.3, 034008 (2019)

[39] Y. Jia, Variational study of weakly coupled triply heavy baryons [J], JHEP 10, 073 (2006)

[40] J. M. Flynn, E. Hernandez and J. Nieves, Triply Heavy Baryons and Heavy Quark Spin Symmetry [J], Phys. Rev. D 85, 014012 (2012)

[41] A. Bernotas and V. Simonis, Heavy hadron spectroscopy and the bag model [J], Lith. J. Phys. 49, 19-28 (2009)

[42] P Hasenfratz, R. R. Horgan, J. Kuti and J. M. Richard, Heavy Baryon Spectroscopy in the QCD Bag Model [J], Phys. Lett. B 94, 401-404 (1980)

[43] Z. Shah and A. K. Rai, Masses and Regge trajectories of triply heavy Ωccc and Ωbbb baryons [J], Eur. Phys. J. A 53, no.10, 195 (2017)

[44] B. Patel, A. Majethiya and P. C. Vinodkumar, Masses and Magnetic moments of Triply Heavy Flavour Baryons in Hypercentral Model [J], Pramana 72, 679-688 (2009)

[45] X. D. Ji, A QCD analysis of the mass structure of the nucleon,” Phys. Rev. Lett. 74, 1071-1074 (1995)

[46] Y. B. Yang, J. Liang, Y. J. Bi, Y. Chen, T. Draper, K. F. Liu and Z. Liu, Proton Mass Decomposition from the QCD Energy Momentum Tensor [J], Phys. Rev. Lett. 121, no.21, 212001 (2018)

[47] Y. B. Yang, Y. Chen, T. Draper, M. Gong, K. F. Liu, Z. Liu and J. P. Ma, Meson Mass Decomposition from Lattice QCD [J], Phys. Rev. D 91, no.7, 074516 (2015)

[48] R. Horsley et al. [QCDSF and UKQCD], A Lattice Study of the Glue in the Nucleon [J], Phys. Lett. B 714, 312-316 (2012)

[49] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Remarks on Higgs Boson Interactions with Nucleons [J], Phys. Lett. B 78, 443-446 (1978)

[50] J. Liang et al. [CLQCD], Spectrum and Bethe-Salpeter amplitudes of Ω baryons from lattice QCD [J], Chin. Phys. C 40, no.4, 041001 (2016)

[51] R. D. Mawhinney [RBC and UKQCD], 2+1 Flavor Domain Wall Fermion QCD Lattices: Ensemble Production and (some) Properties [J], [arXiv:1912.13150 [hep-lat]].