Consideration of nonlinear properties of new structural materials

I S Gelver¹, A V Kolunin² and A S Shudykin²

¹Omsk State Transport University, 35, prospect Marksa, Omsk, 644046, Russia
²Branch of Federal State-Owned «Khrulev Military Educational Institution of Logistics» of the Ministry of Defense of the Russian Federation in Omsk (OABII), 14th military town, Omsk, 644098, Russia

Abstract. In recent years, new structural materials (composite and polymer materials, non-ferrous alloys) are widely used in mechanical engineering. For example, in the production of high-pressure vessels, in order to lighten the design, a thin-walled aluminum alloy liner is installed in a strong carbon fiber shell. However, such materials are among the physically nonlinear ones, when, even with minor deformations, there are significant deviations from the Hooke's law. Therefore, the development of methods to account for this effect and assessment of its impact on the stress-strain state of the design is an actual problem. In this paper, the authors justify the possibility of using the finite element method for nonlinear relationship between stresses and strains. They propose to approximate a real deformation diagram by a cubic parabola. As a result, the calculation data are obtained in the form similar to the linear theory of the finite element method.

1. Introduction

Currently, aluminum alloys [1], as well as polymeric materials manufactured in terms of modern technologies of processing of hydrocarbon raw materials are widely used in the production of structures for various purposes both in Russia and abroad. As you know, polymer materials are increasingly used in industry and substitute steel products. For such materials, the resistance to external loads is nonlinear. Figure 1 presents a real deformation diagram for the aluminum alloy AMg6, where there is a deviation from the Hooke's law.
2. Problem statement
Modern computer software systems (Ansys, Nastran, Lira, etc.) provide for the possibility of nonlinear analysis, which does not exclude the need to use the basic relations of construction mechanics, taking into account the deviation from the Hooke's law at the stage of development of the preliminary design. In this paper, we propose to approximate a real deformation diagram by a cubic parabola. The next step is to clarify the well-known procedure of the finite element method, widely known in the linear theory of material resistance, taking into account the physical nonlinearity.

3. Theory
At the stage of development of the preliminary design of a new construction from physically nonlinear materials under some design conditions, the allowable stresses are assumed to be equal to the values, under which there is a significant deviation from the Hooke’s law. The need to take into account this effect is justified in [2].

As shown by the authors of [3], the most acceptable generalization of the Hooke's law, resulting from the decomposition of the potential energy of deformation of the lattice points of the material in the Taylor series, taking into account the second term, is the approximation of the real deformation diagram by a cubic parabola of the form:

\[ \sigma = E \cdot \varepsilon - E_1 \cdot \varepsilon^3 \]  

(1)

where E is Young's modulus, E1 is a constant determined by the elastic behaviour of the material. In [4], it is shown that the elastic behaviour E1 is determined from the expression:

\[ E_1 = \frac{0.148 \cdot E^3}{\sigma_f^2} \]  

(2)

The problem of approximation of a real deformation diagram by a cubic parabola (1) can be solved, as shown by the authors of [5], using the program Origin.
To do this, select the "Non-linear Curve Fit..." option in the "Analysis" tab. In the opened window, use the "New" button to create a user function "User-Defined" with the number of parameters 2 ("Number of Param.") and in the field "Definition" enter the command consisting of the following key combination: y=P1*x-P2*x^3.

Figure 1. Stress-strain diagram of the alloy AMg6
Figure 2. Approximation of the experimental deformation diagram of the alloy AMg6 by the program Origin

Figure 2 shows the results of this program for the aluminum alloy AMg6. Here, the straight line reflects Hooke's law, the middle line – the dependence (2), and the lower parabola is constructed by the formula (1), and the elastic characteristics for the alloy AMg6 have the following values: $E=72$ hPa, $E_1=2.8 \times 10^{15}$Pa, $P_1=(75\pm2)$hPa, $P_2=(2.98\pm0.38)\times10^{15}$Pa.

Approximation by a cubic parabola gives an opportunity to present the potential energy of strain in the form of the linear part $U_0$ and nonlinear supplement $U_1$. The expression for $U_0$ is common in the linear theory of strain, and for the nonlinear supplement, it can be represented in the form:

$$U_1 = \sum_k \beta_k \int_0^l \{\lambda_k(z)\}^4 dz$$  \hspace{1cm} (3)

where $\beta_k$ is resistance characteristic, determined by the inertia moment of the higher order, $\lambda_k(Z)$ is generalized deflection of the rod finite element. The accumulation of deflections with respect to $k$ means strain accounting in one of the planes under consideration. The integration is carried out along the entire length of the rod.

The potential of external forces can be represented, as in the linear theory, in the following form:

$$A = -\sum_i R_i \omega_i$$ \hspace{1cm} (4)

As a result of the search for alternative solutions, it is possible to apply the Lagrange variational principle: if the body is at equilibrium, the functional of the potential energy $W$ on real displacements has stationary values, i.e. $\delta W = 0$.

To solve the problem of determining the nodal displacement, it is necessary to find the functions, on which $W$ has stationary values. It allows us to apply one of the approximate methods - the Ritz method, according to which:
The determination of partial derivatives from $U_0$ according to nodal displacements $\omega_i$ leads to the formation of the stiffness matrix $[K]$ known from the linear theory. Determination of partial derivatives from the potential of external forces $A$ forms the vector of nodal reactions:

$$\frac{\partial A}{\partial \omega_i} = -R_p$$  \hspace{1cm} (6)

New, in contrast to the linear theory of FEM, is the formation of the vector $\hat{G}$ in accordance with the procedure for determining the partial derivatives of a nonlinear addition to the potential energy $U_1$ according to displacements $\omega_i$, hereinafter referred to as a nonlinearity vector. It makes sense to dwell on this procedure.

The deflection of the rod taking into account the physical nonlinearity is approximated by the following polynomial:

$$\lambda_k = \alpha_{1k} + \alpha_{2k}z + \alpha_{3k}x^2 + \alpha_{4k}z^3$$  \hspace{1cm} (7)

The components of the nonlinearity vector will be determined from the following ratio:

$$\frac{\partial U_1}{\partial \omega_i} = 2\beta_k \int_{0}^{l} \left[\lambda_k''(z)\right]^3 \alpha_k''(z)dz$$  \hspace{1cm} (8)

where $\lambda_k''(z)$ is the second derivative of the generalized deflection;

$\alpha_k''(z)$ is the second derivative of the coefficients of decomposition.

Then the displacements of the finite element can be determined by solving a system of equations:

$$[M]\cdot \vec{W} - \vec{R}_p = \vec{G}$$  \hspace{1cm} (9)

where $[M]$ is a stiffness matrix;

$\vec{W}$ is a displacement vector;

$\vec{R}_p$ is a vector of nodal reactions:

The process of forming the resolving equations for the finite element system is carried out in the same way as in the linear theory of FEM.

4. Discussion of results

As shown by the analysis of the results of the experimental stress-strain dependence for physically nonlinear aluminum alloy AMg6 (Fig.2), the adopted nonlinear deformation law (1) enables to obtain a satisfactory approximation of the real deformation diagram.

The approximation parameters can be determined by numerical methods (Origin program) and analytically, using traditional characteristics of the mechanical properties of the material – Young's modulus, limit of yield strength. With this approach, it is possible to apply the relations of the linear theory of the finite element method.
5. Conclusions

The considered approach to the solution of the problem of accounting for the physical nonlinearity of the material allows us to obtain the basic relations in the form similar to the linear theory and convenient for implementation on a computer. The proposed method allows us to evaluate the effect of nonlinear properties of the structural material on the stress-strain state and to create the structures optimized for weight and strength characteristics.

References

[1] Zhuravleva L V 2004 Application of aluminum alloys in freight car building abroad Railway transport Issue 1-2 pp 21–36
[2] Gel’ver S A, Kolumin A V and Poyarkov S S 2018 Features of designing constructions of aluminum cases for light armored vehicles of the airborne forces Izvestiya of Tula State University 10 pp 534–539
[3] Matyash Ju I, Sosnovskiy Yu M and Krohin S N 2012 The use of modern physical-chemical approaches to the evaluation of thermal conductivity in terms of brake pads Izvestiya of the Transsib 3 (11) pp 40–46
[4] Gel'ver S A 2014 Non-linear static analysis in the development of high-performance land transport using the Ansys software package Improving the energy efficiency of land transport systems: Proceedings of the international scientific and practical conference. Ministry of Transport of the Russian Federation, Federal Agency of Railway Transport, JSC "Russian Railways", International Association of Transport Universities of Asia-Pacific Region (IATU APR) (Omsk State Transport University 2014) pp 160–165
[5] Gel'ver I S, Gel'ver S A and Drozdova I S 2018 Analysis of the current and finite stress-strain behavior of the train composition of railways under elastoplastic material response Izvestiya of the Transsib 2 (34) pp 13–20