1. Introduction

The realization that a precise test of CKM unitarity can be obtained from the first-row constraint $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$ (with $|V_{ub}|^2$ negligible) has sparked a new interest in good measurements of quantities related to $|V_{us}|$. As we discuss in the following sections, $|V_{us}|$ can be determined using semileptonic kaon decays; the experimental inputs are the BRs, lifetimes, and form-factor slopes. One problem that frequently plagues the interpretation of older BR measurements is lack of clarity in the treatment of radiative effects. All new measurements of kaon decays with charged particles are fully inclusive of radiation.

1.0.1. $K_{e3}$ Decays The semileptonic kaon decay rates still provide the best means for the measurement of $|V_{us}|$ because only the vector part of the weak current contributes to the matrix element $\langle \pi | J_\alpha | K \rangle$. In general, $\langle \pi | J_\alpha | K \rangle = f_+(t)(P+p)_{\alpha} + f_-(t)(P-p)_{\alpha}$, where $P$ and $p$ are the kaon and pion four-momenta, respectively, and $t = (P-p)^2$. The form factors $f_+$ and $f_-$ appear because pions and kaons are not point-like particles, and also reflect both SU(2) and SU(3) breaking. For vector transitions, the Ademollo-Gatto theorem ensures that SU(3) breaking appears only to second order in $m_s - m_{ud}$. In particular, $f_+(0)$ differs from unity by only 2–4%. When the squared matrix element is evaluated, a factor of $m_\pi^2/m_K^2$ multiplies all terms containing $f_-(t)$. This form factor can be neglected for $K_{e3}$ decays. For the description of $K_{\mu 3}$ decays, it is customary to use $f_+(t)$ and the scalar form factor $f_0(t) = f_+(t) + [t/(m_K^2 - m_\pi^2)] f_-(t)$.

The semileptonic decay rates, fully inclusive of radiation, are given by

$$\Gamma^i(K_{e3,\mu 3}) = \frac{C^2_i G^2 M^5}{768\pi^3} S_{\text{EW}} |f_+^{K^0}(0)|^2$$

$$\times I_{e3,\mu 3}(1 + 2\delta_{i,\text{em}} + 2\delta_i, SU(2)).$$

In the above expression, $i$ indexes $K^0 \rightarrow \pi^\pm$ and $K^\pm \rightarrow \pi^0$ transitions, for which $C^2_i = 1$ and 1/2, respectively. $G$ is the Fermi constant, $M$ is the appropriate kaon mass, and $S_{\text{EW}}$ is the universal short-distance electroweak correction. The $\delta_{i,\text{em}}$ terms are the long-distance radiative corrections, which depend on the meson charges and lepton masses. The form factors are written as $f_+(0) = f_+(0)\hat{f}_+(0)$, with $\hat{f}_+(0) = 1$. $f_+(0)$ reflects SU(3)- and SU(2)-breaking corrections. $f_+(0)$ is calculated for $K^0$ transitions, $\delta_{K^\pm, SU(2)} = (2.3 \pm 0.2)%^3$ accounts for the SU(2)-breaking corrections. $I_{e3,\mu 3}$ is the integral of the Dalitz-plot density over the physical region and includes $|\hat{f}_+(0)|^2$.

The vector form factor $f_+$ is dominated by the vector $K\pi$ resonances, the closest being the $K^*(892)$. The natural form for $\hat{f}_+(t)$ is then:

$$\hat{f}_+(t) = \frac{M^2_{\pi^0}}{M^2_{\pi^0} - t}.$$
It is also customary to expand the form factor in powers of \( t \) as
\[
\hat{f}_+(t) = 1 + \lambda_+ \frac{t}{m_{\pi^+}} + \frac{\lambda_+}{2} \left( \frac{t}{m_{\pi^+}} \right)^2.
\]

1.0.2. \( K \to \mu\nu \) decays High-precision lattice quantum chromodynamics (QCD) results have recently become available and are rapidly improving. The availability of precise values for the pion- and kaon-decay constants \( f_\pi \) and \( f_K \) allows use of a relation between \( \Gamma(K_{\mu2})/\Gamma(\pi_{\mu2}) \) and \( |V_{us}|^2/|V_{ud}|^2 \), with the advantage that lattice-scale uncertainties and radiative corrections largely cancel out in the ratio:
\[
\frac{\Gamma(K_{\mu2})}{\Gamma(\pi_{\mu2})} = \frac{|V_{us}|^2}{|V_{ud}|^2} \frac{m_K}{m_{\pi^+}} \left(1 - \frac{m_{\pi^+}^2}{m_K^2} + \frac{m_{\pi^+}^2}{m_K^2} \right)^2 \times (0.9930 \pm 0.0035),
\]
where the precision of the numerical factor due to structure-dependent corrections can be improved.

1.1. \( K_L \) decays

1.1.1. \( K_L \) lifetime KLOE has performed a fit to the proper-time distribution for \( K_L \to 3\pi^0 \) decays, which can be isolated with high purity and high and uniform efficiency. The KLOE result is \( \tau_{K_L} = 50.92 \pm 0.30 \) ns.

1.1.2. \( K_L \) BRs Recent measurements of dominant \( K_L \) branching ratios from KTeV, KLOE, and NA48 are in disagreement with averages of previous measurements.

KLOE has performed a measurement of the absolute \( K_L \) BRs \(^8\) of the four dominant (99.5% of all decays) \( K_L \) decay modes. The presence of a \( K_L \) is tagged by observation of a \( K_S \to \pi^+\pi^- \) decay. The value of these absolute BRs depend on the \( K_L \) lifetime value, which enters into the calculation of the geometrical efficiency. This allows KLOE, by applying the constraint \( \Sigma B = 1 \), to obtain another independent value of \( K_L \) lifetime: \( \tau_{K_L} = 50.72 \pm 0.36 \) ns.

A precise determination of the six largest \( K_L \) branching ratios has been obtained by KTeV \(^9\) by measuring the following ratios: \( B_{K_{L,03}}/B_{K_{L,e3}}, \ B_{+0}/B_{K_{L,e3}}, \ B_{00}/B_{K_{L,e3}}, \ B_{+}/B_{K_{L,e3}}, \) and \( B_{00}/B_{000} \).

A measurement of the ratio \( B_{K_{L,03}}/B_{2T} \) \(^10\) has been performed by NA48, where the 2T “decay mode” include all modes with 2 charged particles in the final state \( B_{2T}=1.0048-B_{000} \). A preliminary measurement of \( B_{000} \) from NA48 is reported in Ref. \(^11\).

We perform an average of KLOE, KTeV, NA48 results, and the \( K_L \) lifetime measurement from Ref. \(^12\) by applying the constraint that the \( K_L \) BRs must sum to unity. Correlation among channels are taken into account. The results are given in Tab. 1.

| \( B_{K_{L,03}} \) | \( B_{K_{L,e3}} \) | \( B_{000} \) | \( B_{+0} \) | \( \tau_{K_L} \) (ns) |
|---|---|---|---|---|
| m | 40.46 | 26.97 | 19.69 | 12.52 | 51.10 |
| \( \delta \) | 0.08 | 0.07 | 0.12 | 0.06 | 0.19 |

1.2. \( K_S \to \pi e \nu \) Using the \( K_L \) tag, obtained by observing the interaction of a \( K_L \) in the calorimeter, KLOE has isolated a very pure sample of \( \sim 13,000 \) semileptonic \( K_S \) decays and accurately measured the BR for \( K_S \to \pi e \nu \). KLOE obtains \( B_{K_S e3} = (7.046 \pm 0.092) \times 10^{-4} \). \(^13\)

1.3. \( K^\pm \) decays

1.3.1. \( K^\pm \) lifetime The measurements of the \( K^\pm \) lifetime listed in the PDG compilation \(^14\) exhibit poor consistency. The KLOE preliminary result \( \tau_{K^\pm} = 12.367 \pm 0.078 \) ns, obtained from a fit to the proper time distribution, is in agreement with the PDG average.

1.3.2. \( K^\pm \) BRs A campaign for new measurements of the dominant charged kaon
branching ratios is ongoing at KLOE, NA48, and ISTRA+. In the following, we only consider measurements in which the effects due to photon radiation have been explicitly taken into account.

The ratio $B_{K^{\pm}e^3}/B_{K^{\pm}0}$ has been measured by E865 \cite{15}, NA48 \cite{11}, and ISTRA+ \cite{16}.

Measurements of $B_{K^{\pm}\mu^3}/B_{K^{\pm}e^3}$ have been performed by NA48 \cite{11}, and by E246 \cite{17}.

KLOE has measured the absolute BRs of $K^+ \rightarrow \mu^+\nu$, $K^\pm \rightarrow \pi^0\mu^\pm\nu$, $K^\pm \rightarrow \pi^0\mu^\pm\nu$, and $K^\pm \rightarrow \pi^0\pi^0\pi^0$ \cite{20}.

We also consider the measurement of $B_{K^{\pm}\mu^3}/B_{K^{\pm}0}$ inclusive of radiation performed by PS183 \cite{21}, and the PDG average for $K^\pm \rightarrow \pi^\pm\pi^0\pi^0$ \cite{14}.

The average of the dominant $K^\pm$ BRs is given in Table \ref{tab:brs}. We use the constraint that the $K^\pm$ BRs must sum to unity taking into account correlations.

Table 2. Average for $K^\pm$ BRs(%) and lifetime

| $B_{K^{\pm}e^3}$ | $B_{K^{\pm}\mu^3}$ | $B_{K^{\pm}\mu^2}$ | $B_{K^{\pm}0}$ | $\tau_{K^\pm}(ns)$ |
|-----------------|-----------------|-----------------|----------------|----------------|
| 5.048           | 3.383           | 63.56           | 20.77          | 12.384         |
| m               |                  |                 |                |                |
| 0.031           | 0.027           | 0.13            | 0.12           | 0.022          |

1.4. $K_{f3}$ form factor

This measurement is particularly delicate. Mainly because the effect of the form factor is maximal near the end of the $t$ spectrum where the phase space goes to zero.

Figure 1 compares KLOE results for $\lambda^I_+$ and $\lambda^\prime I_+$ \cite{22} with those from other experiments \cite{23,24,25}. Measurements of the scalar form factor slope have been performed by ISTRA+ using $K_{u^3}$ and KTeV using $K_{L\mu3}$. Since the experimental accuracies are larger than the expected SU(2) breaking correction effects we average measurements from $K^\pm$ and $K_L$. The result, with correlations taken into account, is given in Tab.3.

Table 3. Averages for form factor slopes

| $\lambda^I_+$ | $\lambda^\prime I_+$ | $\lambda_0$ |
|---------------|----------------------|-------------|
| 0.0250 ± 0.0008 | 0.0016 ± 0.0003 | 0.0158 ± 0.0010 |

1.5. $V_{us}$ and CKM unitarity

The available data set on $K_{f3}$ decays allows multiple determinations of $|V_{us}|$. Following the derivation in Section 1.0.1, we compute the value of $f^{K^0}_+(0)|V_{us}|$ from the decay rates for the five semileptonic decay processes measured. The results are shown in Fig.2. We use the $K_S$ lifetime from the PDG \cite{13} while all other BR measurements and lifetime values are discussed in Sections 1.1, 1.2, and 1.3.

Figure 2. $f^{K^0}_+(0)|V_{us}|$ for the five semileptonic decay modes.
To extract the value of $|V_{us}|$, one needs an estimate of $f_+^{K^0}(0)$. The original calculation of Leutwyler & Roos $^{26}$, $f_+^{K^0}(0) = 0.961 \pm 0.008$, has been confirmed recently by lattice QCD calculations $^{27}$. A considerable improvement, $f_+^{K^0}(0) = 0.9680 \pm 0.0016$, has been achieved very recently $^{28}$. We obtain $|V_{us}| = 0.2249 \pm 0.0019$ using $f_+^{K^0}(0)$ from Ref.$^{26}$, or $|V_{us}| = 0.2232 \pm 0.0006$ using $f_+^{K^0}(0)$ from Ref.$^{28}$. To test CKM unitarity we use $\delta = |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1$, with $|V_{ud}| = 0.97377 \pm 0.00027$ $^{29}$. We obtain a good agreement with unitarity, $\delta = -0.0012 \pm 0.0010$, using $f_+^{K^0}(0)$ from Leutwyler & Roos $^{26}$. A $3 \sigma$ deviation is found, $\delta = -0.0020 \pm 0.0006$, using the recent calculation for $f_+^{K^0}(0)$ of Ref.$^{28}$.

Following the derivation in Section 1.0.2, we evaluate the ratio $|V_{us}|/|V_{ud}|$. We use the KLOE measurement of $K^0_{\mu2}$, all other experimental inputs from PDG, and $f_K/f_x = 1.1983^{+0.017}_{-0.018}$ $^{30}$. We obtain $|V_{us}|/|V_{ud}| = 0.2294 \pm 0.0026$ in agreement with both values obtained from $K_{\ell3}$ decays. The interplay of all measurements is shown in Fig.3.

Conclusions

New precise experimental results and lattice calculations allow to test of CKM unitarity at $10^{-3} – 10^{-4}$ level. A deviation of about $3 \sigma$, found using a new determination of $f_+^{K^0}(0)$, has to be confirmed.

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