Proton Structure Function Measurements from HERA

Jörg Gayler
DESY, Notkestrasse 85, 22603 Hamburg, Germany
E-mail: gayler@mail.desy.de

Abstract. Measurements of proton structure functions made in neutral and charged current interactions at HERA are discussed, covering four-momentum transfers $Q^2$ from about 0.5 GeV$^2$ to 30,000 GeV$^2$. The results include the rise of the structure function $F_2$ towards small $x$ and electro-weak effects at high $Q^2$. QCD fits made by the H1 and ZEUS collaborations provide both, parton densities with uncertainties, and precise $\alpha_s$ determinations.

1. Introduction

The proton is probably the most studied hadron. Whereas general parameters like the mass are measured to an accuracy of about $10^{-7}$, the internal properties are known at best at the few percent level. The internal structure, as probed in hard interactions, is described in terms of parton density functions (pdfs). These are determined, in particular, in lepton nucleon scattering experiments. Such measurements are important for two reasons, they provide an important testing ground for QCD, but also because the pdfs are needed to make predictions for other reactions, e.g. $\bar{p}p$ collisions.

In inclusive $e^+p$ ($e^-p$) scattering the proton structure can be probed by $\gamma$ or $Z^0$ exchange, i.e. by neutral current (NC) interactions ($ep \rightarrow eX$), or by $W^+$ ($W^-$) exchange, i.e. by charged current (CC) interactions ($ep \rightarrow \nu X$). The NC differential cross section can be expressed in terms of three structure functions, $\tilde{F}_2$, $\tilde{F}_3$ and $\tilde{F}_L$:

$$d^2\sigma_{NC}^\pm/dxdQ^2 = \frac{2\pi\alpha^2}{xQ^4}[Y_+ \cdot \tilde{F}_2 \mp Y_- \cdot x\tilde{F}_3 - y^2 \cdot \tilde{F}_L] \equiv \frac{2\pi\alpha^2}{xQ^4}\tilde{\sigma}_{NC}^\pm,$$

where $Y_{\pm} = 1 \pm (1 - y)^2$. Here, $Q^2 = -q^2$ with $q$ being the four-momentum of the exchanged gauge boson, $x = Q^2/2(P \cdot q)$, the momentum fraction of the proton carried by the parton participating in the interaction, and $y = (P \cdot q)/(P \cdot k)$, the inelasticity, where $k(P)$ is the four-momentum of the incident electron (proton). The structure function $\tilde{F}_2$ is the dominant contribution in most of the phase space and in leading order (LO) QCD can be written in terms of the quark densities $\sim x \sum_q e_q^2(q(x) + \bar{q}(x))$.

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The term \( x \tilde{F}_3 \) contributes significantly at \( Q^2 \gtrsim M_Z^2 \) and is to LO \( \sim x \sum_q (q(x) - \bar{q}(x)) \), that is, it is given by the valence quarks. The longitudinal contribution \( \tilde{F}_L \) is important in Eq. (1) only at large \( y \). At small \( x \), to order \( \alpha_s \), \( \tilde{F}_L \sim \alpha_s g \), where \( g \) is the gluon density.

Similarly, the CC cross section can be written

\[
d^2\sigma_{\text{CC}}^\pm/dx dQ^2 = \frac{G_F^2}{2\pi} \left( \frac{M_W^2}{Q^2 + M_W^2} \right)^2 \cdot \tilde{\sigma}_{\text{CC}}^\pm ,
\]

where \( G_F \) is the Fermi coupling constant.

In LO \( \tilde{\sigma}_{\text{CC}}^\pm = x[(\bar{u}(x) + \bar{c}(x)) + (1 - y)^2(d(x) + s(x))] \)
and \( \tilde{\sigma}_{\text{CC}}^\pm = x[(u(x) + c(x)) + (1 - y)^2(\bar{d}(x) + \bar{s}(x))] \).

The \( d \)-quark density is therefore directly accessible in \( e^+p \to \bar{\nu}_eX \) scattering avoiding the nuclear corrections necessary in electron deuteron scattering.

2. Electro-weak Effects

The \( e^+p \) and \( e^-p \) data on NC and CC interactions at high \( Q^2 \) are summarised in Fig. 1a. In NC, \( d\sigma/dQ^2 \sim 1/Q^4 \) due to photon exchange. At \( Q^2 \approx 100 \text{ GeV}^2 \) the cross section is about a factor 1000 larger than the CC cross section which varies as \( \sim 1/(Q^2 + M_W^2) \).
However, we observe that at \( Q^2 \gtrsim M_Z^2, M_W^2 \), \( \sigma_{\text{CC}} \approx \sigma_{\text{NC}} \) illustrating electro-weak unification in deep inelastic scattering (DIS).

A closer look at Fig. 1b shows that the \( e^-p \) cross sections are above those of \( e^+p \). In the CC case, this follows from the valence contribution which is \( \sim u_v(x) \) for \( e^-p \) scattering and \( \sim (1 - y^2) \cdot d_v(x) \) for \( e^+p \). In the NC case, this difference is seen in \( \tilde{F}_i \) contain also \( M_Z \) terms originating from \( Z \) exchange.
more detail in Fig. 3 b) which shows $x\tilde{F}_3$ which is dominated by the $\gamma Z^0$ interference term. Taking the electro-weak couplings into account, $x\tilde{F}_3 \gamma Z \sim 2u_v + d_v$. Future, more precise HERA measurements of $x\tilde{F}_3$ will provide an interesting consistency check for the valence quark densities based on NC $ep$ scattering only.

3. Recent QCD Analyses of DIS data

In the standard DIS QCD analyses a parameterisation of the pdfs at a starting scale $Q^2_0$ is assumed, which are evolved to higher $Q^2$ using the NLO DGLAP equations [3]. The parameters at $Q^2_0$ are determined by a fit of the calculated cross sections or $F_2$ values to the data. The analyses differ mainly in the amount of data used, the handling of systematic errors, the parameterisations at $Q^2_0$, and the treatment of heavy quarks. Results of such analyses were recently presented by H1 and ZEUS, leading to pdfs with associated uncertainties.

![Figure 2. $F_2^{em}$, i.e., $F_2$ due to $\gamma$ exchange, from HERA and fixed target experiments compared with the the ZEUS NLO fit [2].](image)

The H1 2000 QCD fit [4] used the H1 $ep$ NC data and BCDMS $\mu p$ data. The primary purpose was a determination of the gluon density $g(x)$ and the strong coupling constant $\alpha_s$. For this reason, besides $g(x)$ only two functions were parametrised at $Q^2_0$, one for the valence and one for the sea quark contribution, with small corrections.

The preliminary H1 2002 pdf fit [5], which includes in addition the H1 CC and the BCDMS $\mu d$ data, determines $g(x)$ and also the four up and down combinations...
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$U = u + c, \bar{U} = \bar{u} + \bar{c}, D = d + s, \text{ and } \bar{D} = \bar{d} + \bar{s}$ from which the valence densities $u_v = U - \bar{U}$ and $d_v = D - \bar{D}$ are derived. Fitting the H1 data alone gives essentially the same pdfs, but with increased uncertainties at large $x$. In this case the sensitivity to $d(x)$ is mainly due to the $e^+p$ CC data.

The recent ZEUS analysis \[6\] uses ZEUS NC data, $\mu p$ and $\mu d$ data from BCDMS, NMC and E665, and CCFR $\nu Fe$ data. Results on $g(x)$, $u_v(x)$, $d_v(x)$, the total sea and $\bar{d} - \bar{u}$ are given.

The ZEUS and H1 NLO fits describe the data very well (Fig. 2).

The fits follow the steep rise of $F_2$ at small $x$ which is driven by $g(x)$. The question remains whether the DGLAP approach is good enough at small $x$ where $\alpha_s \ln 1/x$ terms are neglected. The parameterisations for the $x$ dependence at $Q_0^2$ are indeed flexible, but the $Q^2$ dependence of the data is well described by DGLAP evolution without further parameters.

The resulting pdfs of the fits are compared in Fig. 3.

![HERA: PDF determination](image1.png)

![ZEUS](image2.png)

**Figure 3.** a) Comparison of pdfs of the prel. H1 2002 pdf fit [4] with the ZEUS NLO fit [2] b) comparison of the ZEUS fit with the global analyses CTEQ6M [7] and MRST2001 [6].

The H1 and ZEUS results are consistent at the 5 to 10% level and also agree with the results of global analyses [4, 7]. This is remarkable in view of the different methods and the different data sets used.

The strong rise of the gluon density towards small $x$ leads to the prediction of a substantial $F_L$ contribution to the cross section which is consistent with the data [4].

In the central H1 and ZEUS fits, $\alpha_s(M_Z^2)$ is kept fixed. If treated as a free parameter, the results $\alpha_s(M_Z^2) = 0.1150 \pm 0.0017 ($exp$) \pm 0.0009 ($model$) (H1 2000 QCD fit [4])$ and $\alpha_s(M_Z^2) = 0.1166 \pm 0.0008 ($uncorr.$) \pm 0.0032 ($corr.$) \pm 0.0036 ($norm.$) \pm 0.0018 ($model$) (ZEUS analysis [3]) are obtained, which are competitive with other $\alpha_s$ determinations. However, theoretical uncertainties due to missing higher orders are estimated to be $\approx \pm 0.005 [4]$. This uncertainty are expected to be considerably reduced by full next to
4. The rise of $F_2$ towards low $x$

The rise of the proton structure function $F_2$ towards small $x$ has been discussed already in the early days of QCD. In the double asymptotic limit (large energies, i.e. small $x$, and large photon virtualities $Q^2$) the DGLAP evolution equations can be solved \[^9\] and $F_2$ is expected to rise approximately like a power of $x$ towards low $x$. Power like behaviour is also expected in the BFKL approach \[^10\]. However, it was soon realised \[^11\] that this rise must eventually be limited to satisfy unitarity constraints, perhaps as a result of gluon fusion in the nucleon. Experimentally, the rise towards small $x$ was first observed in 1993 in the HERA data \[^12\].

Now the improved precision of the data allows detailed study of the rise through the determination of \(\lambda \equiv -\left(\partial \ln F_2 / \partial \ln x\right)_{Q^2}\) as a function of $x$ and $Q^2$. The derivative $\lambda$ was shown \[^13\] to be constant within experimental uncertainties at $x < 0.01$ for fixed $Q^2$ in the range $0.5 \lesssim Q^2 \lesssim 150$ GeV$^2$, implying that the data are consistent with the behaviour $F_2 = c(Q^2) \cdot x^{-\lambda(Q^2)}$. Fitting this form to the HERA and fixed target data at $x < 0.01$, results in $\lambda$ values (Fig. 4) which rise logarithmically for $Q^2 \gtrsim 3.5$ GeV$^2$, that is in the region where perturbative QCD fits are thought to be valid.

![Figure 4](image.png)

**Figure 4.** $\lambda(Q^2)$ from fits of the form $F_2 = c(Q^2) \cdot x^{-\lambda(Q^2)}$ (results from refs. \[^13\],\[^14\]).

At small $Q^2$ the structure function $F_2$ can be related to the total virtual photon absorption cross section by $\sigma_{tot}^{\gamma^*p} = 4\pi\alpha^2 F_2/Q^2 \sim x^{-\lambda}/Q^2$, where the total $\gamma^*p$ energy squared is given by $s = Q^2/x$. For $Q^2 \to 0$ we can expect $\lambda(Q^2) \to 0.08$. This corresponds to the energy dependence observed in soft hadronic interactions $\sigma_{tot} \sim s^{\alpha_{FP}(0) - 1}$ with $\alpha_{FP}(0) - 1 \approx 0.08$ \[^13\] which is approximately reached at $Q^2 \approx 0.5$ GeV$^2$. 

NLO calculations which are expected to be completed soon \[^8\].
5. Conclusion

New improved data on inclusive $e^\pm p$ scattering have become available in recent years. At high $Q^2$, NC and CC interactions are consistent with the expectations of electro-weak theory and QCD.

H1 and ZEUS have performed DGLAP based pQCD analyses which describe their data very well and provide pdfs including uncertainties. The strong coupling $\alpha_s$ was determined with good experimental accuracy.

At low $x$, no significant deviation from a power behaviour $F_2 \sim x^{-\lambda}$ at fixed $Q^2$ is visible at present energies and $Q^2 \gtrsim 0.85 \text{ GeV}^2$. At $Q^2 \lessgtr 1 \text{ GeV}^2$, the rise with energy is similar to that observed in soft hadronic interactions.

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