Parameter design optimization of the crank-rocker engine using the FMINCON function in MATLAB

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Abstract. This study presents an optimization approach in the design selection of parameters of the Crank-Rocker engine. This engine works on the principle of a four-bar crank-rocker mechanism, and the purpose of this study is to find the basic geometry of the mechanism that satisfies three required objectives which are to determine the optimized rocker stroke, to achieve maximum output torque, and to attain specific time ratio to enhance engine performance. A MATLAB model was used to solve this synthesis problem and a comparison between different algorithms was introduced to determine the best algorithm for the purpose. The results show that the FMINCON function is reliable and the suggested optimization method can be used to obtain the most suitable parameters of the crank-rocker engine.

1. Introduction

In 2018, a new single-piston engine called Crank-Rocker (CR) engine has been developed by UTP research teams at Universiti Teknologi Petronas (UTP). The main concept of this engine is based on the oscillating movement of a curved piston inside a curved cylinder, and this new configuration is expected to improve engine thermodynamic performance [1]. The author recommended further optimization of the engine configuration for better performance by optimizing the CR engine basic geometry. To achieve this objective, starting with the kinematic study of the engine mechanism is the key to build a successful working mechanism [1]–[4].

Normally, there are multiple factors to be considered in mechanism synthesis, such as linkages’ length, corresponding motion angles, and other variables depending on the area of applications. With such many aspects of design variables, conventional mechanism synthesis methods are not feasible to achieve all desired variables easily [5]. As part of the system modelling and optimization, it is crucial to know what type of function to be used in the optimization process. R.V. Rao et al. [6], have introduced new optimization function, called ‘Teaching–Learning-Based Optimization (TLBO)’. The concept of this function is population-based optimization and can simply be explained through the function name. It suggests that there is an influence when learning between learners and teacher. The novel function goes through two-stages, the first one is the teacher phase where information is passed to the learners, and the second phase is the interaction among the learners themselves. The authors found that the new function is more efficient in optimization process and can be implemented on different engineering applications. On the other hand, S. Sleesongsom et al. [7], have improved new variant in the TLBO...
method and called it self-adaptive population size teaching-learning based optimization (SAP-TLBO). This method was used to avoid timing constraints and for the application of path synthesis problems. The suggested method showed superior performance when compared to the original function.

Suwin Sleesongsom et al. [8] proposed new constraints handling technique for four-bar mechanism path generation, and the new algorithm was called evolutionary algorithms (EAs). Comparing the proposed algorithm to six other algorithms in solving three different optimization problems, it is found that this new algorithm is best when handling constraints, while TLBO is best for mechanism synthesis. R. Sancibrian [9] developed a mathematical optimization method called the Generalized Reduced Gradient (GRG) method. This method has been used by mathematicians but not for the dimensional synthesis purposes. The mathematical reformulation of the suggested function for mechanism synthesis optimization was found to be accurate, robust and efficient.

In a different perspective, design variables and optimization objective functions are important elements when performing optimization. Some studies consider the quality and number of design variables to be crucial during optimization which also include the determination of other sub-parameters such as defining constraints, bounds and initial values. The coherence and ability to determine each value depends on designer’s vision and abilities that would finally lead to the optimal results out of the optimization process. Galal A. et al. [10], introduced optimization study of four-bar planar mechanism in which two positions of coupler motions were considered. The mathematical model involved six nonlinear equations where the transmission angle was kept as a variable. A case study was introduced to verify the validity of the method. Also in another study, Galal A. et al. [11], used the former method to perform a mechanism synthesis for a specific objective, where in this case the objectives were to compute the predetermined stroke and time ratio.

Sun Jian [12] used ADAMS as an analytical and optimization software. In a model set up, the author decided the minimum value of transmission angle to be the design objective. And he suggested that this factor was important to determine effective mechanism motion. Ankur J. et al. [13], implemented the Freudenstein-Chebyshev method to perform five precision of coupler points mechanism synthesis. The structural error resulted from the objective function was optimized using least square function, thus the proposed method succeeded to reduce the order of error compared to the previous studies.

It can be noticed from the previous literatures that many approaches have been introduced with different optimization methods and algorithms. Even though mathematical models were developed in all cases, handling the models with various conditions such as system constraints, mechanism synthesis objective or even solution methodology, the final answers were still different from each case. However, these artificial optimization methods are considered more convenient and efficient for solving complicated problems than the old style methods which can handle only simple and limited design parameters [5].

In this paper, we aim to implement an optimized algorithm to solve a real-life problem which is to design the main dimensional configuration of the Crank-Rocker engine. The solution of ten geometrical design variables is considered, and all related constraints were predefined as per desired working conditions. Moreover, three main objectives were used for design criteria namely engine stroke, maximum torque delivered and specific mechanism time ratio.

To this point, an introductory literature about mechanism synthesis, optimization process methods and applications were introduced. Next, a literature on crank-rocker mechanism position analysis has been illustrated. Then, design optimization method and implementation has been constructed. Finally, results and discussions were introduced.

2. Position analysis of the crank-rocker mechanism

Figure 1 represents the basic geometry of the crank rocker mechanism, consisting of four linkages with related angles. The distances between the fixtures are identified as Ground link L₁, the Crank L₂, the Coupler L₃ and the Rocker L₄. The crank angle, θ₂ rotates from zero⁰ to 360⁰. The coupler angle with reference to the ground is θ₃.
The angle between the coupler and rocker is called transmission angle $\gamma$, while the last one is the rocker angle $\theta_4$. These angles may vary as per governing equations (1-6), listed below[14]:

$$
\gamma = \cos^{-1} \left[ \frac{L_3^2 + L_4^2 - AO_2^2}{2(L_3)(L_4)} \right] 
$$

Where:

$$
AO_2 = \sqrt{L_1^2 + L_2^2 - 2(L_1)(L_2)\cos(\theta_2)} \quad (2)
$$

$$
\theta_3 = 2\tan^{-1} \left[ \frac{-L_2 \sin\theta_2 - L_4 \sin(\gamma)}{L_1 + L_3 - L_2 \cos\theta_2 - L_4 \cos(\gamma)} \right] \quad (3)
$$

$$
\theta_4 = 2\tan^{-1} \left[ \frac{L_2 \sin\theta_2 + L_4 \sin(\gamma)}{L_2 \cos\theta_2 + L_1 + L_4 - L_3 \cos(\gamma)} \right] \quad (4)
$$

The throw angle $\phi$, which is the angle between the maximum (P1) to the minimum (P2) rocker position, can be calculated by

$$
\phi = \theta_4(p1) - \theta_4(p2) \quad (5)
$$

Also, the imbalance angle $\beta$, which is the angle between the coupler configuration at the two limiting positions can be calculated by

$$
\beta = \theta_2(p1) - \theta_2(p2) \quad (6)
$$

3. Design optimization approach

Even though four-bar mechanism is considered a simple mechanism, the need to develop optimization method is crucial to perform efficient synthesis, especially with large number of design variables and various constraints. In this paper, MATLAB software is used to perform the optimization process. Once the mathematical model was derived, a program was constructed based on the required algorithm. Shown below are the steps taken for the optimization process.
3.1 Problem formulation
It is desired to design a crank-rocker mechanism that satisfies Grashof’s law, where the sum of the shortest link length $s$ and the longest link length $l$ is less than the sum of the length of other two links $q$ and $p$:

$$s + l < q + p$$  \hspace{1cm} (7)

Also, a specific value of rocker stroke is set to the desired engine stroke $S_d$, where $S_d = 33.1$ mm. Another parameter, the desired time ratio $Q_d$ of piston-travel is assumed to have a specific value for the purpose of engine performance enhancement, where $Q_d = 1.15$. The formulation for this optimization problem is for the system to deliver maximum output torque. This can be accomplished by minimizing the deviated value of the transmission angle $\gamma$ to be as close as possible to a value of $\gamma_d = 90^\circ$ [15].

3.2 Design variables
This study considers ten design variables to reach optimal performance of this engine, and the variables are $L_1, L_2, L_3, L_4, \theta_2, \gamma, \varphi, \beta, S,$ and $Q$. Each of these parameters are used in the mathematical modeling of this system, then introduced to the program. The inputs to the program are the crank angle $\theta_2$ which rotates $360^\circ$ and the imbalance angle $\beta$ that represents the mechanism time ratio, while the rests are to be optimized using the MATLAB code. All the sampling variables are changed into a single form of variable for ease of programming, and the variable is referred as $X$ and tagged from 1 to 10, i.e. $[X_1, X_2, \ldots, X_{10}]$, in correspondence to the previous variables.

3.3 Objective function
The objective function ($f$) consists of engine stroke, $S$, time ratio $Q$ and transmission angle $\gamma$. The objective function is based on square error function, and the purpose is to minimize the error between the optimized and design values. The mathematical expression is [7]:

$$f = \min[(S - S_d)^2 + (Q - Q_d)^2 + (\gamma - \gamma_d)^2]$$  \hspace{1cm} (8)

Where:

$$S = L_4 * \varphi$$  \hspace{1cm} (9)

$$Q = \frac{180 + \beta}{180 - \beta}$$  \hspace{1cm} (10)

$$\gamma = \gamma_{\text{max}} - \gamma_{\text{min}}$$  \hspace{1cm} (11)

The position of the maximum and minimum values of the transmission angles occurs when the crank position becomes coincident with the ground link (figure 2). Moreover, at the same time the solution domain also has to satisfy all constraints given in the next step.

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**Figure 2.** Minimum and maximum positions of the Transmission angle
The analytical value of the maximum and minimum values of the transmission angles can be calculated using the following formula[14]:

\[\gamma_{\text{max}} = \cos^{-1} \left[ \frac{L_3^2 + L_4^2 - (L_1 + L_2)^2}{2L_3 L_4} \right] \]  
\[\gamma_{\text{min}} = \cos^{-1} \left[ \frac{L_3^2 + L_4^2 - (L_1 - L_2)^2}{2L_3 L_4} \right] \]  

3.4 Constraints Identification

Besides equations 1 to 13 given in section 2 and 3, which govern the geometry of the crank rocker mechanism, the complete rotation of the crank \(L_2\) should also be considered. Other constraints for the system are Grashof’s law, which was mathematically introduced as:

\[s = \min(L_1, L_2, L_3, L_4)\] Shortest link length  
\[l = \max(L_1, L_2, L_3, L_4)\] Longest link length  
\[2 \times (s + l) < (L_1 + L_2 + L_3 + L_4)\] Grashof’s condition

The maximum and minimum values of the transmission angle \(\gamma\), should be between \(45^0\) and \(135^0\) as a rule of thumb [14]. For the purpose of limiting the solution space domain that reduces the function effort to find possible solutions, predetermined upper and lower bound of the geometry configurations are set as in table 1.

Table 1. Design variables lower and upper bounds.

| \(L_1/\) (mm) | \(L_2/\) (mm) | \(L_3/\) (mm) | \(L_4/\) (mm) | \(\theta_2/\) (Radian) | \(\gamma/\) (Radian) | \(\Phi/\) (Radian) | \(B/\) (Radian) | \(S/\) (mm) | \(Q/\) (mm) |
|---|---|---|---|---|---|---|---|---|---|
| **Lower** | 30 | 10 | 35 | 20 | 0 | \(\pi/4\) | \(\pi/6\) | \(\pi/10\) | 10 | 1 |
| **Bound** | | | | | | | | | | |
| **Upper** | 200 | 200 | 200 | 200 | 2\(\pi\) | \(3/4\pi\) | \(\pi/2\) | \(\pi/4\) | 80 | 1.5 |
| **Bound** | | | | | | | | | | |

3.5 Optimization algorithm

Since the derived mathematical model in this optimization study is multi-variable with non-linear constraints, a MATLAB function called FMINCON is used to optimize the constructed model. This function is a gradient based function and can be used to search and find all possible local minima that satisfy the given objectives. The iteration process starts with an initial guess by the algorithm, and stops when all setup criteria are met. If the first-order optimization is fulfilled by the last iteration, the result is considered as a local minimum that satisfies system needs[16].

4. Result and Discussion

The optimization process was successfully ended into an optimal result, where all optimality and constraints tolerances were met. Figure 3 shows that the iteration count stopped after 33 iterations and the best objective function value was 0.0118.
For the purpose of studying the behavior of the algorithm, another code was formulated to find the historical information during the optimization process. The selected values for the optimized variables versus the iteration process are listed in table 2.

**Table 2. Design variable values for selected points in optimization process.**

| Iteration count no. | 1       | 6       | 13      | 22      | 28      | 32      |
|---------------------|---------|---------|---------|---------|---------|---------|
| L1                  | 30      | 30.2072 | 30.3323 | 36.6478 | 36.6467 | 35.3995 |
| L2                  | 10      | 12.6452 | 12.7014 | 14.8846 | 14.9308 | 14.7035 |
| L3                  | 35      | 35.0056 | 35.0055 | 35      | 35      | 35.0269 |
| L4                  | 35      | 34.7249 | 34.6185 | 29.7369 | 29.8044 | 27.2553 |
| γ                   | 0.7854  | 0.8145  | 0.8206  | 1.1654  | 1.171   | 1.2264  |
| φ                   | 0.5236  | 0.9537  | 0.9568  | 1.1081  | 1.112   | 1.2115  |
| β                   | 0.3142  | 0.6036  | 0.5992  | 0.3142  | 0.3142  | 0.3142  |
| S                   | 10      | 33.1173 | 33.1238 | 32.9685 | 33.1424 | 33.0267 |
| Q                   | 1       | 1.2125  | 1.2108  | 1.105   | 1.1053  | 1.1053  |
| Fval                | 218.8894| 0.5762  | 0.5669  | 0.1888  | 0.1635  | 0.0118  |

For the purpose of algorithm effectiveness, a comparison with another optimization algorithm is introduced in table 3. Three other methods are used for these illustrations, namely Multi objective goal attainment (FGOALATTAIN); unconstrained nonlinear constraints function (FMINSEARCH) and Simulated Annealing Algorithm (SIMULANNEALBND).
Table 3. Optimization results comparison for different algorithms.

| variable | FMINCON | FGOALATTAIN | FMINSEARCH | SIMULANNEALBND |
|----------|---------|-------------|------------|----------------|
| L1       | 33.2290 | 33.2290     | 40.6407    | 44.5693        |
| L2       | 14.0313 | 14.0313     | 11.4312    | 63.7502        |
| L3       | 35.0    | 35.0        | 85.7671    | 68.0031        |
| L4       | 21.0730 | 21.0730     | 38.2168    | 63.2938        |
| θ₂       | 0.0     | 0.0         | -0.0125    | 2.7992         |
| γ        | 1.4720  | 1.4720      | 1.2435     | 1.5549         |
| φ        | 1.5708  | 1.5708      | 0.8660     | 0.5240         |
| β        | 0.3142  | 0.3142      | -0.0070    | 0.3265         |
| S        | 33.1015 | 33.1015     | 10.3856    | 17.7884        |
| Q        | 1.1053  | 1.1053      | 1.2150     | 1.4423         |
| Fval     | 0.0118  | 0.0118      | 0.0591     | 3.9776         |
| Iteration no. | 33      | 35         | 250       | 6331          |
| Fun. Count | 379     | 459        | 547       | 6572          |

The result shows that FMINCON and FGOALATTAIN algorithms have obtained the same and best results with better performance count for FMINCON with regard to optimization timing and number of utilized functions. On the other hand, SIMULANNEALBND function has shown more deviation in the objective function value (Fval) from the minimum value, and this means that it is the least desired function to be used in such conditions and design application.

5. Conclusion
In this paper, an optimization process was introduced to conduct the best design approach for the Crank-Rocker engine. A MATLAB code was constructed using FMINCON function to perform the optimization process with ten design variables, three objectives, and system linear and non-linear constraints were implemented. The optimization process led into a result that met all desired objectives with less computational efforts when comparing to other methods. This method can be implemented effectively in the design optimization of the Crank-Rocker engine for better mechanism synthesis. However, more analytical study on the optimized design is needed to validate the results.

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