A Uniform Fixpoint Approach to the Implementation of 
Inference Methods for Deductive Databases

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Abstract. Within the research area of deductive databases three different database tasks have been deeply investigated: query evaluation, update propagation and view updating. Over the last thirty years various inference mechanisms have been proposed for realizing these main functionalities of a rule-based system. However, these inference mechanisms have been rarely used in commercial DB systems until now. One important reason for this is the lack of a uniform approach well-suited for implementation in an SQL-based system. In this paper, we present such a uniform approach in form of a new version of the soft consequence operator. Additionally, we present improved transformation-based approaches to query optimization and update propagation and view updating which are all using this operator as underlying evaluation mechanism.

1 Introduction

The notion deductive database refers to systems capable of inferring new knowledge using rules. Within this research area, three main database tasks have been intensively studied: (recursive) query evaluation, update propagation and view updating. Despite of many proposals for efficiently performing these tasks, however, the corresponding methods have been implemented in commercial products (such as, e.g., Oracle or DB2) in a very limited way, so far. One important reason is that many proposals employ inference methods which are not directly suited for being transferred into the SQL world. For example, proof-based methods or instance-oriented model generation techniques (e.g., based on SLDNF) have been proposed as inference methods for view updating which are hardly compatible with the set-oriented bottom-up evaluation strategy of SQL.

In this paper, we present transformation-based methods to query optimization, update propagation and view updating which are well-suited for being transferred to SQL. Transformation-based approaches like Magic Sets [1] automatically transform a given database schema into a new one such that the evaluation of rules over the rewritten schema performs a certain database task more efficiently than with respect to the original schema. These approaches are well-suited for extending database systems, as new algorithmic ideas are solely incorporated into the transformation process, leaving the actual database engine with its own optimization techniques unchanged. In fact, rewriting techniques allow for implementing various database functionalities on the basis of one common inference engine. However, the application of transformation-based approaches with respect to stratifiable views [17] may lead to unstratifiable recursion within the rewritten schemata. Consequently, an elaborate and very expensive inference mechanism is generally required for their evaluation such as the alternating fixpoint computation or the residual program approach proposed by van Gelder [20] resp. Bry [10]. This is also the case for the kind of recursive views proposed by the SQL:1999 standard, as they cover the class of stratifiable views.

As an alternative, the soft consequence operator together with the soft stratification concept has been proposed by the author in [2] which allows for the efficient evaluation of Magic Sets transformed rules. This efficient inference method is applicable to query-driven as well as update-driven derivations. Query-driven inference is typically a top-down process whereas update-driven approaches are usually designed bottom-up. During the last 6 years, the idea of combining the advantages of top-down and bottom-up oriented inference has been consequently employed to
enhance existing methods to query optimization \cite{3} as well as update propagation \cite{6} and to develop a new approach to view updating. In order to handle alternative derivations that may occur in view updating methods, an extended version of the original soft consequence operator has to be developed. In this paper, this new version is presented, which is well-suited for efficiently determining the semantics of definite and indefinite databases but remains compatible with the set-oriented, bottom-up evaluation of SQL.

2 Basic concepts

A Datalog rule is a function-free clause of the form \( H_1 \leftarrow L_1 \land \cdots \land L_m \) with \( m \geq 1 \) where \( H_1 \) is an atom denoting the rule’s head, and \( L_1, \ldots, L_m \) are literals, i.e., positive or negative atoms, representing its body. We assume all deductive rules to be safe, i.e., all variables occurring in the head or in any negated literal of a rule must be also present in a positive literal in its body. If \( A \equiv p(t_1, \ldots, t_n) \) with \( n \geq 0 \) is a literal, we use \( \text{vars}(A) \) to denote the set of variables occurring in \( A \) and \( \text{pred}(A) \) to refer to the predicate symbol \( p \) of \( A \). If \( A \) is the head of a given rule \( R \), we use \( \text{pred}(R) \) to refer to the predicate symbol of \( A \). For a set of rules \( R \), \( \text{pred}(R) \) is defined as \( \cup_{r \in R} \{ \text{pred}(r) \} \). A fact is a ground atom in which every \( t_i \) is a constant.

A deductive database \( D \) is a triple \( \langle F, R, I \rangle \) where \( F \) is a finite set of facts (called base facts), \( I \) is a finite set of integrity constraints (i.e., positive ground atoms) and \( R \) a finite set of rules such that \( \text{pred}(F) \cap \text{pred}(R) = \emptyset \) and \( \text{pred}(I) \subseteq \text{pred}(F \cup R) \). Within a deductive database \( D \), a predicate symbol \( p \) is called derived (view predicate), if \( p \in \text{pred}(R) \). The predicate \( p \) is called extensional (or base predicate), if \( p \in \text{pred}(F) \). Let \( H_D \) be the Herbrand base of \( D = \langle F, R, I \rangle \). The set of all derivable literals from \( D \) is defined as the well-founded model \cite{21} for \( \langle F \cup R \rangle \): \( M_D := I^+ \cup \neg \cdot I^- \) where \( I^+, I^- \subseteq H_D \) are sets of ground atoms and \( \neg \cdot I^- \) includes all negations of atoms in \( I^- \). The set \( I^+ \) represents the positive portion of the well-founded model while \( \neg \cdot I^- \) comprises all negative conclusions. The semantics of a database \( D = \langle F, R, I \rangle \) is defined as the well-founded model \( M_D := I^+ \cup \neg \cdot I^- \) for \( F \cup R \) if all integrity constraints are satisfied in \( M_D \), i.e., \( I \subseteq I^+ \). Otherwise, the semantics of \( D \) is undefined. For the sake of simplicity of exposition, and without loss of generality, we assume that a predicate is either base or derived, but not both, which can be easily achieved by rewriting a given database.

Disjunctive Datalog extends Datalog by disjunctions of literals in facts as well as rule heads. A disjunctive Datalog rule is a function-free clause of the form \( \bigvee_{1 \leq k \leq n} A_k \leftarrow B_1 \land \cdots \land B_m \) with \( m, n \geq 1 \) where the rule’s head \( A \) is a disjunction of positive atoms, and the rule’s body \( B_1, \ldots, B_m \) consists of literals, i.e., positive or negative atoms. A disjunctive fact \( f \equiv f_1 \land \cdots \land f_k \) is a disjunction of ground atoms \( f_i \) with \( i \geq 1 \). \( f \) is called definite if \( i = 1 \). We solely consider stratifiable disjunctive rules only, that is, recursion through negative predicate occurrences is not permitted \cite{17}. A stratification partitions a given rule set such that all positive derivations of relations can be determined before a negative literal with respect to one of those relations is evaluated. The semantics of a stratifiable disjunctive databases \( D \) is defined as the perfect model state \( P \cdot M_D \) of \( D \) iff \( D \) is consistent \cite{11}.

3 Transformation-Based Approaches

The need for a uniform inference mechanism in deductive databases is motivated by the fact that transformation-based approaches to query optimization, update propagation and view updating are still based on very different model generators. In this section, we briefly recall the state-of-the-art with respect to these transformation-based techniques by means of Magic Sets, Magic Updates and Magic View Updates. The last two approaches have been already proposed by the author in \cite{6} and \cite{7}. Note that we solely consider stratifiable rules for the given (external) schema. The transformed internal schema, however, may not always be stratifiable such that more general inference engines are required.
3.1 Query Optimization

Various methods for efficient bottom-up evaluation of queries against the intensional part of a database have been proposed, e.g. Magic Sets [1], Counting [9], Alexander method [19]. All these approaches are rewriting techniques for deductive rules with respect to a given query such that bottom-up materialization is performed in a goal-directed manner cutting down the number of irrelevant facts generated. In the following we will focus on Magic Sets as this approach has been accepted as a kind of standard in the field.

Magic Sets rewriting is a two-step transformation in which the first phase consists of constructing an adorned rule set, while the second phase consists of the actual Magic Sets rewriting. Within an adorned rule set, the predicate symbol of a literal is associated with an adornment which is a string consisting of letters b and f. While b represents a bound argument at the time when the literal is to be evaluated, f denotes a free argument. The adorned version of the deductive rules is constructed with respect to an adorned query and a selected sip strategy [18] which basically determines for each rule the order in which the body literals are to be evaluated and which bindings are passed on to the next literal. During the second phase of Magic Sets the adorned rules are rewritten such that bottom-up materialization of the resulting database simulates a top-down evaluation of the original query on the original database. For this purpose, each adorned rule is extended with a magic literal restricting the evaluation of the rule to the given binding in the adornment of the rule’s head. The magic predicates themselves are defined by rules which define the set of relevant selection constants. The initial values corresponding to the query are given by the so-called magic seed. As an example, consider the following stratifiable rules $\mathcal{R}$

$$o(X, Y) \leftarrow \neg p(Y, X) \land p(X, Y)$$
$$p(X, Y) \leftarrow e(X, Y)$$
$$p(X, Y) \leftarrow e(X, Z) \land p(Z, Y)$$

and the query $?\neg o(1, 2)$ asking whether a path from node 1 to 2 exists but not vice versa. Assuming a full left-to-right sip strategy, Magic Sets yields the following deductive rules $\mathcal{R}_{ms}$

$$o_{bb}(X, Y) \leftarrow m_{o_{bb}}(X, Y) \land \neg p_{bb}(Y, X) \land p_{bb}(X, Y)$$
$$p_{bb}(X, Y) \leftarrow m_{p_{bb}}(X, Y) \land e(X, Z) \land p_{bb}(Z, Y)$$
$$m_{p_{bb}}(X, Y) \leftarrow m_{o_{bb}}(X, Y) \land \neg p_{bb}(Y, X)$$
$$m_{p_{bb}}(Z, Y) \leftarrow m_{p_{bb}}(X, Y) \land e(X, Z)$$

as well as the magic seed fact $m_{o_{bb}}(1, 2)$. The Magic Sets transformation is sound for stratifiable databases. However, the resulting rule set may be no more stratifiable (as is the case in the above example) and more general approaches than iterated fixpoint computation are needed. For determining the well-founded model of general logic programs, the alternating fixpoint computation by Van Gelder [20] or the conditional fixpoint by Bry [10] could be used. The application of these methods, however, is not really efficient because the specific reason for the unstratifiability of the transformed rule sets is not taken into account. As an efficient alternative, the soft stratification concept together with the soft consequence operator [2] could be used for determining the positive part of the well-founded model (cf. Section 4).

3.2 Update Propagation

Determining the consequences of base relation changes is essential for maintaining materialized views as well as for efficiently checking integrity. Update propagation (UP) methods have been proposed aiming at the efficient computation of implicit changes of derived relations resulting from explicitly performed updates of extensional facts [13][14][16][17]. We present a specific method for update propagation which fits well with the semantics of deductive databases and is based on the soft consequence operator again. We will use the notion update to denote the 'true' changes caused by a transaction only; that is, we solely consider sets of updates where compensation effects (i.e., given by an insertion and deletion of the same fact or the insertion of facts which already existed, for example) have already been taken into account.
The task of update propagation is to systematically compute the set of all induced modifications starting from the physical changes of base data. Technically, this is a set of delta facts for any affected relation which may be stored in corresponding delta relations. For each predicate symbol $p \in \text{pred}(D)$, we will use a pair of delta relations $(\Delta_p^+, \Delta_p^-)$ representing the insertions and deletions induced on $p$ by an update on $D$. The initial set of delta facts directly results from the given update and represents the so-called UP seeds. They form the starting point from which induced updates, represented by derived delta relations, are computed. In our transformation-based approach, so-called propagation rules are employed for computing delta relations. A propagation rule refers to at least one delta relation in its body in order to provide a focus on the underlying changes when computing induced updates. For showing the effectiveness of an induced update, however, references to the state of a relation before and after the base update has been performed are necessary. As an example of this propagation approach, consider again the rules for relation $\tau$ from Subsection 3.1. The UP rules $R^\Delta$ with respect to insertions into $e$ are as follows:

\[
\Delta_p^+(X, Y) \leftarrow \Delta_p^+(X, Y) \land \neg p^{\text{old}}(X, Y) \\
\Delta_p^+(X, Y) \leftarrow \Delta_p^+(X, Z) \land p^{\text{new}}(Z, Y) \land \neg p^{\text{old}}(X, Y) \\
\Delta_p^+(X, Y) \leftarrow \Delta_p^+(Z, Y) \land e^{\text{new}}(X, Z) \land \neg p^{\text{old}}(X, Y)
\]

For each relation $p$ we use $p^{\text{old}}$ to refer to its old state before the changes given in the delta relations have been applied whereas $p^{\text{new}}$ is used to refer to the new state of $p$. These state relations are never completely computed but are queried with bindings from the delta relation in the propagation rule body and thus act as a test of effectiveness. In the following, we assume the old database state to be present such that the adornment $\text{old}$ can be omitted. For simulating the new database state from a given update so called transition rules [16] are used. The transition rules $R^\tau$ for simulating the required new states of $e$ and $p$ are:

\[
e^{\text{new}}(X, Y) \leftarrow e(X, Y) \land \neg \Delta_e^+(X, Y) \\
e^{\text{new}}(X, Y) \leftarrow \Delta_e^+(X, Y) \\
p^{\text{new}}(X, Y) \leftarrow e^{\text{new}}(X, Y) \\
p^{\text{new}}(X, Y) \leftarrow e^{\text{new}}(X, Z) \land p^{\text{new}}(Z, Y)
\]

Note that the new state definition of intensional predicates only indirectly refers to the given update in contrast to extensional predicates. If $R$ is stratifiable, the rule set $R \cup R^\Delta \cup R^\tau$ will be stratifiable, too (cf. [6]). As $R \cup R^\Delta \cup R^\tau$ remains to be stratifiable, iterated fixpoint computation could be employed for determining the semantics of these rules and the induced updates defined by them. However, all state relations are completely determined which leads to a very inefficient propagation process. The reason is that the supposed evaluation over the two consecutive database states is performed using deductive rules which are not specialized with respect to the particular updates that are propagated. This weakness of propagation rules in view of a bottom-up materialization will be cured by incorporating Magic Sets.

**Magic Updates**

The aim is to develop an UP approach which is automatically limited to the affected delta relations. The evaluation of side literals and effectiveness tests is restricted to the updates currently propagated. We use the Magic Sets approach for incorporating a top-down evaluation strategy by considering the currently propagated updates in the dynamic body literals as abstract queries on the remainder of the respective propagation rule bodies. Evaluating these propagation queries has the advantage that the respective state relations will only be partially materialized. As an example, let us consider the specific deductive database $D = (F, R, I)$ with $R$ consisting of the well-known rules for the transitive closure $p$ of relation $e$:

\[
R: \\
p(X, Y) \leftarrow e(X, Y) \\
p(X, Y) \leftarrow e(X, Z), p(Z, Y)
\]

\[
F: \\
\text{edge}(1,2), \text{edge}(1,4), \text{edge}(3,4) \\
\text{edge}(10,11), \text{edge}(11,12), \ldots, \text{edge}(98,99), \text{edge}(99,100)
\]
Note that the derived relation $p$ consists of 4098 tuples. Suppose a given update contains the new tuple $e(2, 3)$ to be inserted into $D$ and we are interested in finding the resulting consequences for $p$. Computing the induced update by evaluating the stratifiable propagation and transition rules would lead to the generation of 94 new state facts for relation $e$, 4098 old state facts for $p$ and 4098 + 3 new state facts for $p$. The entire number of generated facts is 8296 for computing the three induced insertions $\Delta^+_p(1, 3), \Delta^+_p(2, 3), \Delta^+_p(2, 4)$ with respect to $p$.

However, the application of the Magic Updates rewriting with respect to the propagation queries \{\(\Delta^+_p(Z, Y), \Delta^+_p(X, Y), \Delta^+_p(X, Z)\)\} provides a much better focus on the changes to $e$. Within its application, the following subquery rules

\[
\begin{align*}
\text{m}_{\Delta^+_{\text{fb}}}^\text{ev}(Z) & \leftarrow \Delta^+_e(X, Z) \\
\text{m}_{\Delta^+_{\text{fb}}}^\text{ev}(Z) & \leftarrow \Delta^+_p(Z, Y)
\end{align*}
\]

are generated. The respective queries $Q = \{m_{\Delta^+_{fb}}^\text{new}, m_{\Delta^+_{fb}}^{\text{new}}, \ldots\}$ allow to specialize the employed transition rules, e.g.,

\[
\begin{align*}
e_{\text{fb}}^\text{ev}(X, Y) & \leftarrow \text{m}_{\Delta^+_{fb}}^\text{ev}(Y) \land e(X, Y) \land \neg \Delta^+_e(X, Y) \\
e_{\text{fb}}^\text{ev}(X, Y) & \leftarrow \text{m}_{\Delta^+_{fb}}^\text{ev}(Y) \land \Delta^+_e(X, Y)
\end{align*}
\]

such that only relevant state tuples are generated. We denote the Magic Updates transformed rules $\mathcal{R} \cup \mathcal{R}^+ \cup \mathcal{R}^+_\mathcal{R}$ by $\mathcal{R}^+_{\text{mu}}$. Despite of the large number of rules in $\mathcal{R}^+_{\text{mu}}$, the number of derived results remains relatively small. Quite similar to the Magic sets approach, the Magic Updates rewriting may result in an unstratifiable rule set. This is also the case for our example where the following negative cycle occurs in the respective dependency graph:

\[
\Delta^+_p \xrightarrow{\text{pos}} \text{m}_{\Delta^+_{bb}} \xrightarrow{\text{pos}} \text{P}_{bb} \xrightarrow{\text{new}} \Delta^+_p
\]

In [6] it has been shown, however, that the resulting rules must be at least softly stratifiable such that the soft consequence operator could be used for efficiently computing their well-founded model. Computing the induced update by evaluating the Magic Updates transformed rules leads to the generation of two new state facts for $e$, one old state fact and one new state fact for $p$.

The entire number of generated facts is 19 in contrast to 8296 for computing the three induced insertions with respect to $p$.

### 3.3 View Updates

Bearing in mind the numerous benefits of the afore mentioned methods to query optimization and update propagation, it seemed worthwhile to develop a similar, i.e., incremental and transformation-based, approach to the dual problem of view updating. In contrast to update propagation, view updating aims at determining one or more base relation updates such that all given update requests with respect to derived relations are satisfied after the base updates have been successfully applied. In the following, we recall a transformation-based approach to incrementally compute such base updates for stratifiable databases proposed by the author in [7]. The approach extends and integrates standard techniques for efficient query answering, integrity checking and update propagation. The analysis of view updating requests usually leads to alternative view update realizations which are represented in disjunctive form.

**Magic View Updates**

In our transformation-based approach, true view updates (VU) are considered only, i.e., ground atoms which are presently not derivable for atoms to be inserted, or are derivable for atoms to be deleted, respectively. A method for view updating determines sets of alternative updates (called VU realization) satisfying a given request. There may be infinitely many realizations and even realizations of infinite size which satisfy a given VU request. In our approach, a breadth-first search
is employed for determining a set of minimal realizations. A realization is minimal in the sense that none of its updates can be removed without losing the property of being a realization. As each level of the search tree is completely explored, the result usually consists of more than one realization. If only VU realizations of infinite size exist, our method will not terminate.

Given a VU request, view updating methods usually determine subsequent VU requests in order to find relevant base updates. Similar to delta relations for UP we will use the notion VU relation to access individual view updates with respect to the relations of our system. For each relation \( p \in \text{pred}(R \cup F) \) we use the VU relation \( \nabla_p^+ (\bar{x}) \) for tuples to be inserted into \( D \) and \( \nabla_p^- (\bar{x}) \) for tuples to be deleted from \( D \). The initial set of delta facts resulting from a given VU request is again represented by so-called VU seeds. Starting from the seeds, so-called VU rules are employed for finding subsequent VU requests systematically. These rules perform a top-down analysis in a similar way as the bottom-up analysis implemented by the UP rules. As an example, consider the following database \( D = (F, R, I) \) with \( F = \{r_2(2), s(2)\} \), \( I = \{ic(2)\} \) and the rules \( R \):

\[
\begin{align*}
p(X) &\leftarrow q_1(X) & q_1(X) &\leftarrow r_1(X) \land s(X) \\
p(X) &\leftarrow q_2(X) & q_2(X) &\leftarrow r_2(X) \land \neg s(X) \\
ic(2) &\leftarrow \neg au(2) & au(X) &\leftarrow q_2(X) \land \neg q_1(X)
\end{align*}
\]

The corresponding set of VU rules \( \mathcal{R}^\nabla \) with respect to \( \nabla_p^+ (2) \) is given by:

\[
\begin{align*}
\nabla_{q_1}^+(X) \lor \nabla_{q_2}^+(X) &\leftarrow \nabla_p^+(X) \\
abla_{r_1}^-(X) &\leftarrow \neg \nabla_{q_1}^+(X) \land \neg r_1(X) \\
abla_{s}^-(X) &\leftarrow \nabla_{q_1}^+(X) \land \neg s(X) \\
\end{align*}
\]

In contrast to the UP rules from Section 3.2, no explicit references to the new database state are included in the above VU rules. The reason is that these rules are applied iteratively over several intermediate database states before the minimal set of realizations has been found. Hence, the apparent references to the old state really refer to the current state which is continuously modified while computing VU realizations. These predicates solely act as tests again queried with respect to bindings from VU relations and thus will never be completely evaluated.

Evaluating these rules using model generation with disjunctive facts leads to two alternative updates, insertion \( \{r_1(2)\} \) and deletion \( \{s(2)\} \), induced by the derived disjunction \( \nabla_{q_1}^+(2) \lor \nabla_{s}^-(2) \). Obviously, the second update represented by \( \nabla_{s}^-(2) \) would lead to an undesired side effect by means of an integrity violation. In order to provide a complete method, however, such erroneous/incomplete paths must be also explored and side effects repaired if possible. Determining whether a computed update will lead to a consistent database state or not can be done by applying a bottom-up UP process at the end of the top-down phase leading to an irreparable constraint violation with respect to \( \nabla_{s}^-(2) \):

\[
\nabla_{s}^-(2) \Rightarrow \Delta_{q_1}^+(2) \Rightarrow \Delta_{q_2}^+(2) \Rightarrow \Delta_{ic}^+(2) \Rightarrow \text{false}
\]

In order to see whether the violated constraint can be repaired, the subsequent view update request \( \nabla_{ic}^+ \) with respect to \( D \) ought to be answered. The application of \( \mathcal{R}^\nabla \) yields

\[
\begin{align*}
\nabla_{ic}^+(2) &\Rightarrow \nabla_{aux}^+(2) \Rightarrow \nabla_{q_2}^+(2), \nabla_{q_2}^-(2) \Rightarrow \text{false} \\
\nabla_{ic}^+(2) &\Rightarrow \nabla_{q_1}^+(2) \Rightarrow \nabla_{q_2}^+(2), \nabla_{q_2}^-(2) \Rightarrow \text{false}
\end{align*}
\]

showing that this request cannot be satisfied as inconsistent subsequent view update requests are generated on this path. Such erroneous derivation paths will be indicated by the keyword false. The reduced set of updates - each of them leading to a consistent database state only - represents the set of realizations \( \Delta_{ic}^+(2) \).

An induced deletion of an integrity constraint predicate can be seen as a side effect of an ‘erroneous’ VU. Similar side effects, however, can be also found when induced changes to the database caused by a VU request may include derived facts which had been actually used for deriving this view update. This effect is shown in the following example for a deductive database \( D = (R, F, I) \) with \( R = \{h(X) \leftarrow p(X) \land q(X) \land 1, i \leftarrow p(X) \land \neg q(X)\} \), \( F = \{p(1)\} \), and \( I = \emptyset \).
Given the VU request $\nabla_h^+(1)$, the overall evaluation scheme for determining the only realization $\{\Delta^+_q(1), \Delta^-_p(c^{new})\}$ would be as follows:

\[
\nabla_h^+(1) \Rightarrow \nabla_q^+(1) \Rightarrow \Delta^+_q(1) \Rightarrow \Delta^-_i \Rightarrow \nabla_i^+ \downarrow \\
\Rightarrow \nabla_q^-(1), \nabla_q^+(1) \Rightarrow false
\]

The example shows the necessity of compensating side effects, i.e., the compensation of the 'deletion' $\Delta^-_i$ (that prevents the 'insertion' $\Delta^+_q(1)$) caused by the tuple $\nabla_i^+(1)$. In general the compensation of side effects, however, may in turn cause additional side effects which have to be 'repaired'. Thus, the view updating method must alternate between top-down and bottom-up phases until all possibilities for compensating side effects (including integrity constraint violations) have been considered, or a solution has been found. To this end, so-called VU transition rules $R^\Sigma$ are used for restarting the VU analysis. For example, the compensation of violated integrity constraints can be realized by using the following kind of transition rule $\Delta^-_{ic}(\vec{c}) \rightarrow \nabla^+_{ic}(\vec{c})$ for each ground literal $ic(\vec{c}) \in I$. VU transition rules make sure that erroneous solutions are evaluated to false and side effects are repaired.

Having the rules for the direct and indirect consequences of a given VU request, a general application scheme for systematically determining VU realizations can be defined (see[7] for details). Instead of using simple propagation rules $R \cup \Delta \cup R^\Delta$, however, it is much more efficient to employ the corresponding Magic Update rules. The top-down analysis rules $R \cup \Delta \cup \Sigma$ and the bottom-up consequence analysis rules $R^\Delta \cup R^\Sigma$ are alternating applied. Note that the disjunctive rules $R \cup \Sigma$ are stratifiable while $R^\Delta \cup R^\Sigma$ is softly stratifiable such that a perfect model state [H11] and a well-founded model generation must alternately be applied. The iteration stops as soon as a realization for the given VU request has been found. The correctness of this approach has been already shown in [7].

4 Consequence Operators and Fixpoint Computations

In the following, we summarize the most important fixpoint-based approaches for definite as well as indefinite rules. All these methods employ so-called consequence operators which formalize the application of deductive rules for deriving new data. Based on their properties, a new uniform consequence operator is developed subsequently.

4.1 Definite Rules

First, we recall the iterated fixpoint method for constructing the well-founded model of a stratifiable database which coincides with its perfect model [17].

**Definition 1.** Let $D = (F, R)$ be a deductive database, $\lambda$ a stratification on $D$, $R_1 \cup \ldots \cup R_n$ the partition of $R$ induced by $\lambda$, $I \subseteq H_D$ a set of ground atoms, and $[[R]]_I$ the set of all ground instances of rules in $R$ with respect to the set $I$. Then we define

1. the immediate consequence operator $T_R(I)$ as
   \[
   T_R(I) := \{H \mid H \in I \lor \exists r \in [[R]]_I : r \equiv H \leftarrow L_1 \land \ldots \land L_n \text{ such that } L_i \in I \text{ for all positive literals } L_i \text{ and } L \notin I \text{ for all negative literals } L_j \equiv \neg L\},
   \]

2. the iterated fixpoint $M_n$ as the last Herbrand model of the sequence
   \[
   M_1 := 1fp (T_{R_1}, F), M_2 := 1fp (T_{R_2}, M_1), \ldots, M_n := 1fp (T_{R_n}, M_{n-1}),
   \]
   where $1fp (T_R, F)$ denotes the least fixpoint of operator $T_R$ containing $F$. 


3. and the iterated fixpoint model \( M^i_D \) as
\[
M^i_D := M_n \cup \neg \cdot M_n.
\]
This constructive definition of the iterated fixpoint model is based on the immediate consequence operator introduced by van Emden and Kowalski. In [17] it has been shown that the perfect model of a stratifiable database \( D \) is identical with the iterated fixpoint model \( M^i_D \) of \( D \).

Stratifiable rules represent the most important class of deductive rules as they cover the expressiveness of recursion in SQL:1999. Our transformation-based approaches, however, may internally lead to unstratifiable rules for which a more general inference method is necessary. In case that un-stratifiability is caused by the application of Magic Sets, the so-called soft stratification approach proposed by the author in [2] could be used.

**Definition 2.** Let \( D = \langle \mathcal{F}, \mathcal{R} \rangle \) be a deductive database, \( \lambda^s \) a soft stratification on \( D \), \( \mathcal{P} = P_1 \cup \ldots \cup P_n \) the partition of \( \mathcal{R} \) induced by \( \lambda^s \), and \( I \subseteq \mathcal{H}_D \) a set of ground atoms. Then we define:
1. the soft consequence operator \( T^p_\mathcal{P}(I) \) as
\[
T^p_\mathcal{P}(I) := \begin{cases} I & \text{if } T^p_\mathcal{P}(I) = I \text{ for all } j \in \{1, \ldots, n\} \\ T^p_\mathcal{P}(I) & \text{with } i = \min\{j \mid T^p_\mathcal{P}(I) \supseteq I\}, \text{ otherwise.} \end{cases}
\]
where \( T^p_\mathcal{P} \) denotes the immediate consequence operator.
2. and the soft fixpoint model \( M^p_D \) as
\[
M^p_D := \text{lfp}(T^p_\mathcal{P}, J) \cup \neg \cdot (\text{lfp}(T^p_\mathcal{P}, J)).
\]
Note that the soft consequence operator is based upon the immediate consequence operator and can even be used to determine the iterated fixpoint model of a stratifiable database [6]. As an even more general alternative, the alternating fixpoint model for arbitrary unstratifiable rules has been proposed in [12] on the basis of the eventual consequence operator.

**Definition 3.** Let \( D = \langle \mathcal{F}, \mathcal{R} \rangle \) be a deductive database, \( I^+, I^- \subseteq \mathcal{H}_D \) sets of ground atoms, and \( [[\mathcal{R}]]_{I^+} \) the set of all ground instances of rules in \( \mathcal{R} \) with respect to the set \( I^+ \). Then we define:
1. the eventual consequence operator \( \hat{T}_\mathcal{R}(I^-) \) as
\[
\hat{T}_\mathcal{R}(I^-)(I^+) := \{H \mid H \in I^+ \lor \exists r \in [[\mathcal{R}]]_{I^+} : r \equiv H \leftarrow L_1 \land \ldots \land L_n \text{ such that } L_i \in I^+ \text{ for all positive literals } L_i \text{ and } L \notin I^- \text{ for all negative literals } L_j \equiv \neg L\},
\]
2. the eventual consequence transformation \( \hat{S}_D \) as
\[
\hat{S}_D(I^-) := \text{lfp}(\hat{T}_\mathcal{R}(I^-), J),
\]
3. and the alternating fixpoint model \( M^a_D \) as
\[
M^a_D := \text{lfp}(\hat{S}_D, \emptyset) \cup \neg \cdot \text{lfp}(\hat{S}_D, \emptyset)),
\]
where \( \hat{S}_D \) denotes the nested application of the eventual consequence transformation, i.e.,
\[
\hat{S}_D(I^-) = \hat{S}_D(\hat{S}_D(I^-)).
\]
In [12] it has been shown that the alternating fixpoint model \( M^a_D \) coincides with the well-founded model of a given database \( D \). The induced fixpoint computation may indeed serve as a universal model generator for arbitrary classes of deductive rules. However, the eventual consequence operator is computationally expensive due to the intermediate determination of supersets of sets of true atoms. With respect to the discussed transformation-based approaches, the iterated fixpoint model could be used for determining the semantics of the stratifiable subset of rules in \( \mathcal{R}_{ms} \) for query optimization, \( \mathcal{R}^\Delta_{ms} \) for update propagation, and \( \mathcal{R}^\Delta_{ms} \cup \mathcal{R}^\nabla \) for view updating. If these rule sets contain unstratifiable rules, the soft or alternating fixpoint model generator ought be used while the first has proven to be more efficient than the latter [2]. None of the above mentioned consequence operators, however, can deal with indefinite rules necessary for evaluating the view updating rules \( \mathcal{R} \cup \mathcal{R}^\nabla \).
4.2 Indefinite Rules

In [4], the author proposed a consequence operator for the efficient bottom-up state generation of stratifiable disjunctive deductive databases. To this end, a new version of the immediate consequence operator based on hyperresolution has been introduced which extends Minker’s operator for positive disjunctive Datalog rules [15]. In contrast to already existing model generation methods our approach for efficiently computing perfect models is based on state generation. Within this disjunctive consequence operator, the mapping red on indefinite facts is employed which returns non-redundant and subsumption-free representations of disjunctive facts. Additionally, the mapping min\_models\(F\) is used for determining the set of minimal Herbrand models from a given set of disjunctive facts \(F\). We identify a disjunctive fact with a set of atoms such that the occurrence of a ground atom \(A\) within a fact \(f\) can also be written as \(A \in f\). The set difference operator can then be used to remove certain atoms from a disjunction while the empty set as result is interpreted as false.

**Definition 4.** Let \(D = (F, R)\) be a stratifiable disjunctive database rules, \(\lambda\) a stratification on \(D\), \(R_1 \cup \ldots \cup R_n\) the partition of \(R\) induced by \(\lambda\), \(I\) an arbitrary subset of indefinite facts from the disjunctive Herbrand base \(\mathbb{I}\) of \(D\), and \([\{R\}]_I\) the set of all ground instances of rules in \(R\) with respect to the set \(I\). Then we define.

1. the disjunctive consequence operator \(T^\text{state}_R\) as

\[
T^\text{state}_R(I) := \text{red}(\{H \mid H \in I \lor \exists r \in [\{R\}]_I : r \equiv A_1 \lor \ldots \lor A_l \leftarrow L_1 \land \ldots \land L_n \\
\text{with } H = (A_1 \lor \ldots \lor A_l \lor f_1 \setminus L_1 \lor \ldots \lor f_n \setminus L_n \lor C) \\
\text{such that } f_i \in I \land L_i \in f_i \text{ for all positive literals } L_i \\
\text{and } L_j \not\in I \text{ for all negative literals } L_j \\
\text{and } (L_j \in M \Leftrightarrow \exists M \in \text{min\_models}(I) : \text{for at least one negative literal } L_j \\
\text{and } L_k \in M \text{ for all positive literals } L_k \\
\text{and } A_i \not\in M \text{ for all head literals of } r\})
\]

2. the iterated fixpoint state \(S_n\) as the last minimal model state of the sequence

\[S_1 := \text{lfp } (T^\text{state}_R, F), S_2 := \text{lfp } (T^\text{state}_R, S_1), \ldots, S_n := \text{lfp } (T^\text{state}_R, S_{n-1}),\]

3. and the iterated fixpoint state model \(\mathcal{MS}_D\) as

\[\mathcal{MS}_D := S_n \cup \neg \cdot \overline{S_n}.
\]

In [4] it has been shown that the iterated fixpoint state model \(\mathcal{MS}_D\) of a disjunctive database \(D\) coincides with the perfect model state of \(D\). It induces a constructive method for determining the semantics of stratifiable disjunctive databases. The only remaining question is how integrity constraints are handled in the context of disjunctive databases. We consider again definite facts as integrity constraints, only, which must be derivable in every model of the disjunctive database. Thus, only those models from the iterated fixpoint state are selected in which the respective definite facts are derivable. To this end, the already introduced keyword false can be used for indicating and removing inconsistent model states. The database is called consistent iff at least one consistent model state exists.

This proposed inference method is well-suited for determining the semantics of stratifiable disjunctive databases with integrity constraints. And thus, it seems to be suited as the basic inference mechanism for evaluating view updating rules. The problem is, however, that the respective rules contain unstratifiable definite rules which cannot be evaluated using the inference method proposed above. Hence, the evaluation techniques for definite (Section 4.1) and indefinite rules (Section 4.2) do not really fit together and a new uniform approach is needed.
5 A Uniform Fixpoint Approach

In this section, a new version of the soft consequence operator is proposed which is suited as efficient state generator for softly stratifiable definite as well as stratifiable indefinite databases. The original version of the soft consequence operator $T_s^P$ is based on the immediate consequence operator by van Emden and Kowalski and can be applied to an arbitrary partition $P$ of a given set of definite rules. Consequently, its application does not always lead to correct derivations. In fact, this operator has been designed for the application to softly stratified rules resulting from the application of Magic Sets. However, this operator is also suited for determining the perfect model of a stratifiable database.

Lemma 1. Let $D = \langle F, R \rangle$ be a stratifiable database and $\lambda$ a stratification of $R$ inducing the partition $P$ of $R$. The perfect model $M_D$ of $\langle F, R \rangle$ is identical with the soft fixpoint model of $D$, i.e.,

\[ M_D = \text{lfp}(T_s^P, F) \cup \neg \text{lfp}(T_s^P, F). \]

Proof. This property follows from the fact that for every partition $P = P_1 \cup \ldots \cup P_n$ induced by a stratification, the condition $\text{pred}(P_i) \cap \text{pred}(P_j) = \emptyset$ with $i \neq j$ must necessarily hold. As soon as the application of the immediate consequence operator $T_{P_i}$ with respect to a certain layer $P_i$ generates no new facts anymore, the rules in $P_i$ can never fire again. The application of the incorporated $\min$ function then induces the same sequence of Herbrand models as in the case of the iterated fixpoint computation. □

Another property we need for extending the original soft consequence operator is about the application of $T^{\text{state}}$ to definite rules and facts.

Lemma 2. Let $r$ be an arbitrary definite rule and $f$ be a set of arbitrary definite facts. The single application of $r$ to $f$ using the immediate consequence operator or the disjunctive consequence operator, always yields the same result, i.e.,

\[ T^r(f) = T^{\text{state}}_P(f). \]

Proof. The proof follows from the fact that all non-minimal conclusions of $T^{\text{state}}$ are immediately eliminated by the subsumption operator $\text{red}$. □

The above proposition establishes the relationship between the definite and indefinite case showing that the disjunctive consequence operator represents a generalization of the immediate one. Thus, its application to definite rules and facts can be used to realize the same derivation process as the one performed by using the immediate consequence operator. Based on the two properties from above, we can now consistently extend the definition of the soft consequence operator which allows its application to indefinite rules and facts, too.

Definition 5. Let $D = \langle F, R \rangle$ be an arbitrary disjunctive database, $I$ an arbitrary subset of indefinite facts from the disjunctive Herbrand base of $D$, and $P = P_1 \cup \ldots \cup P_n$ a partition of $R$. The general soft consequence operator $T^{\text{g}}_P(I)$ is defined as

\[ T^{\text{g}}_P(I) := \begin{cases} I & \text{if } T_{P_i}(I) = I \text{ for all } j \in \{1, \ldots, n\} \\ T^{\text{state}}_{P_i}(I) & \text{with } i = \min\{j \mid T^{\text{state}}_{P_j}(I) \supseteq I\}, \text{ otherwise.} \end{cases} \]

where $T^{\text{state}}_{P_i}$ denotes the disjunctive consequence operator.

In contrast to the original definition, the general soft consequence operator is based on the disjunctive operator $T^{\text{state}}_{P_i}$ instead of the immediate consequence operator. The least fixpoint of $T^{\text{g}}_P$ can be used to determine the perfect model of definite as well as indefinite stratifiable databases and the well-founded model of softly stratifiable definite databases.
Theorem 1 Let $D = \langle F, \mathcal{R} \rangle$ be a stratifiable disjunctive database and $\lambda$ a stratification of $\mathcal{R}$ inducing the partition $\mathcal{P}$ of $\mathcal{R}$. The perfect model state $\mathcal{PS}_D$ of $\langle F, \mathcal{R} \rangle$ is identical with the least fixpoint model of $T^\mathcal{P}_F$, i.e.,

$$\mathcal{PS}_D = \text{lfp}(T^\mathcal{P}_F, F) \cup \neg \text{lfp}(T^\mathcal{P}_F, F).$$

Proof. The proof directly follows from the correctness of the fixpoint computations for each stratum as shown in [4] and the same structural argument already used in Lemma [1]. □

The definition of $\text{lfp}(T^\mathcal{P}_F, F)$ induces a constructive method for determining the perfect model state as well as the well-founded model of a given database. Thus, it forms a suitable basis for the evaluation of the rules $\mathcal{R}_{ms}$ for query optimization, $\mathcal{R}_{mu}$ for update propagation, and $\mathcal{R}_{mu} \cup \mathcal{R}_\tau$ as well as $\mathcal{R} \cup \mathcal{R}^\forall$ for view updating. This general approach to defining the semantics of different classes of deductive rules is surprisingly simple and induces a rather efficient inference mechanism in contrast to general well-founded model generators. The soft stratification concept, however, is not yet applicable to indefinite databases because ordinary Magic Sets can not be used for indefinite clauses. Nevertheless, the resulting extended version of the soft consequence operator can be used as a uniform basis for the evaluation of all transformation-based techniques mentioned in this paper.

6 Conclusion

In this paper, we have presented an extended version of the soft consequence operator for the efficient top-down and bottom-up reasoning in deductive databases. This operator allows for the efficient evaluation of softly stratifiable incremental expressions and stratifiable disjunctive rules. It solely represents a theoretical approach but provides insights into design decisions for extending the inference component of commercial database systems. The relevance and quality of the transformation-based approaches, however, has been already shown in various practical research projects (e.g. [58]) at the University of Bonn.

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