Link Scheduling Using Graph Neural Networks

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Abstract—Efficient scheduling of transmissions is a key problem in wireless networks. The main challenge stems from the fact that optimal link scheduling involves solving a maximum weighted independent set (MWIS) problem, which is known to be NP-hard. In practical schedulers, centralized and distributed greedy heuristics are commonly used to approximately solve the MWIS problem. However, most of these greedy heuristics ignore important topological information of the wireless network. To overcome this limitation, we propose fast heuristics based on graph convolutional networks (GCNs) that can be implemented in centralized and distributed manners. Our centralized heuristic is based on tree search guided by a GCN and 1-step rollout. In an distributed MWIS solver, a GCN generates topology-aware node embeddings that are combined with per-link utilities before invoking a distributed greedy solver. Moreover, a novel reinforcement learning scheme is developed to train the GCN in a non-differentiable pipeline. Test results on medium-sized wireless networks show that our centralized heuristic can reach a near-optimal solution quickly, and our distributed heuristic based on a shallow GCN can reduce by nearly half the suboptimality gap of the distributed greedy solver with minimal increase in complexity. The proposed schedulers also exhibit good generalizability across graph and weight distributions.

Index Terms—MWIS, graph convolutional networks, wireless networks, scheduling, reinforcement learning.

I. INTRODUCTION

A FUNDAMENTAL problem in managing wireless networks is to efficiently schedule transmissions. In general, the scheduling problem involves determining which links should transmit and when they should transmit, along with other relevant parameters such as transmit power, modulation, and coding schemes [2], [3], [4]. In this paper, we focus on link scheduling in wireless networks with time-slotted orthogonal multiple access, in which a time slot comprises a scheduling phase followed by a transmission phase [5], [6]. This problem is associated with many real-world applications of both ad-hoc and infrastructure-based wireless networks, e.g., battlefield communications, vehicular/flying ad-hoc networks, wireless sensor networks [7], [8], device-to-device (D2D) communications [9], cloud-radio access networks (CRAN) [10], wireless backhaul networks [5], [11], multihop relay networks for mmWave and THz bands communications [12], [13], and internet of things [14], [15]. Compared to contention-based multiple access, e.g., CSMA-CA, scheduled multiple access is often preferred due to its increased spectrum utilization efficiency [16]. This benefit, however, comes with the associated challenge that optimal scheduling involves solving a maximum weighted independent set (MWIS) problem [1], [2], [3], [4], [5], [6], [10], [17], [18], [19], [20], [21], [22]. An MWIS is the independent set (IS) that achieves maximal total weight on a node-weighted graph, where an IS is a set of vertices not connected by any edges. The MWIS problem is NP-hard – the running time of exactly solving it grows as an exponential function of the size of the graph [22], [23]. The idea of scheduling links by selecting the MWIS of the conflict graph of a network was first established in [2], and has been developed since then [3], [4], [5], [6], [10], [17], [18], [19], [20], [21], and [22]. Such link scheduling schemes involve two major tasks: 1) A per-link utility function to compute the weights associated with activating each link, and 2) An efficient (possibly distributed) MaxWeight [2], [18] scheduler as an approximate solver for the associated MWIS problem.

In terms of the per-link utility design, queue length [2], [3], [17], link rate [10], their product [18], and ratio [6], and the age of information [24] have been used in the past, along with some more theoretically-grounded variations [4]. Moreover, routing decisions can be involved in the per-link utility. In backpressure routing [2], [25], the per-link utility for scheduling is the queue length of a flow decided by a routing algorithm for throughput-optimal routing [2] or a virtual queue length for delay-optimal routing [25]. Utility functions can promote not only traffic metrics, but also fairness across the network [26]. Besides analytical functions, the per-link utilities can also be generated by a reinforcement learning (RL) agent with a continuous action space [11], where the transition of network state is modeled as a Markov decision process.

For MaxWeight scheduling, a good MWIS solver is defined by high quality and low complexity, and should be agnostic to the choice of the per-link utility function. The quality of a solution is measured by its approximation ratio (AR), its ratio to the optimal solution. Further, low communication and computational complexity can reduce scheduling overhead. Many heuristics have been developed for lower complexity and/or better performance [3], [6], [10], [17], [18], [19], [20], [21], [27], [28]. Centralized MWIS solvers, with limited scalability but high AR provided by the global information of the network, can be used as schedulers for infrastructure-based...
networks [5], [9], [10], [11], [27], and for theoretical exploration of network capacity [4]. On the other hand, distributed MWIS solvers are preferred for practical scheduling in wireless ad-hoc networks since they can reduce the computational overhead through parallelism, and eliminate the need for relaying packets over the network by limiting communications to local neighborhoods. Moreover, distributed scheduling increases network robustness against a single point of failure at the fusion center. Without global information, distributed MWIS solvers seek to achieve reasonable performance efficiently, e.g., with linear [6], [10], [19], [20], logarithmic [3], [17], [18], or constant [17], [21], [28] local communication complexity (referred to as local complexity), which is defined as the required rounds of information exchange in the neighborhood (referred to as local exchange). As the common choice of practical scheduling schemes, distributed greedy heuristics [3], [17], [18] seeks to mimic an iterative process of adding the link with the largest utility to the partial solution and removing other links that might interfere with it.

Motivated by the success of machine learning in other fields, data-driven learning-based solutions for resource allocation problems in wireless networks have been proposed over the last few years [29], [30], [31], [32], [33], [34], [35]. A common practice in most of these approaches is to parameterize a function of interest using multi-layer perceptrons (MLP) or convolutional neural networks (CNNs), which are not well-suited for problems in wireless communications since they do not exploit the underlying topology. This led to several approaches that tried to adjust CNNs to the wireless setting [29], [36], [37], [38]. Here, we adopt an alternative direction [34], [39], in which graph neural networks (GNNs) [40], [41], [42], [43] are used to incorporate the topology of the wireless network into the learning algorithm.

Our approach is also in line with the recent trend of using deep learning to find approximate solutions to combinatorial problems on graphs [44], [45]. More precisely, we integrate graph convolutional networks (GCNs) [46], [47], [48] into fast and/or distributed algorithmic frameworks to find solution of high quality in a timely and/or distributed manner. Our centralized MWIS solver adopts 1-step lookahead rollout [49, Ch. 2.3] to further guide the tree search in [45], to reach a good solution by traversing the search tree only once, rather than hundreds of thousands of times as in [45], thus reducing the complexity by several orders of magnitude. Meanwhile, distributed schedulers are preferred in large networks for improved time complexity, scalability, and robustness against single-point-of-failure, since the parallelization of solution construction not only lowers the time complexity, but also eliminates the need for full knowledge of the graph. Our distributed MWIS solver has a novel modular structure where a GCN-based node embedding module is followed by a distributed greedy heuristic, thus exploiting the efficiency of the latter while raising graph-awareness through the use of the GCN. Moreover, the GCN can be trained in a computationally efficient manner – i.e., without the need to exactly solve any MWIS problem – and, although the training must be centralized, the execution is fully distributed.

### A. Contributions

We develop efficient approximate MWIS solvers suitable for link scheduling in wireless networks. Specifically: 1) We propose the first GCN-based distributed MWIS solver for link scheduling by combining the topology-awareness of GCNs and the efficiency of distributed greedy solvers; 2) We propose fast centralized MWIS solvers based on a GCN-guided tree search that can achieve near optimal performance on medium-sized graphs with hundreds of nodes; 3) We develop a customized scheme of graph-based deterministic policy gradient to train the GCNs embedded in a non-differentiable downstream pipeline with the help of efficient heuristics. Our reinforcement learning scheme has better performance and computational efficiency than alternative supervised learning schemes, and complements the existing approaches of zeroth-order optimization [50] and surrogate gradient [51] for training pipelines with blackbox/combinatorial module; and 4) Through numerical experiments, we demonstrate the superior performance of the proposed method in single and multi-channel scheduling as well as its generalizability over different graph types and weight distributions.

### B. Paper Outline

The rest of this paper is organized as follows. Related work is reviewed in Section II. The system model and the formulation of the scheduling problem are introduced in Section III. The proposed GCN-based centralized MWIS solver is described in Section IV, followed by our GCN-based distributed MWIS solver in Section V. Our reinforcement learning scheme is introduced in Section VI. In Section VII, numerical experiments illustrate the performance of our proposed solutions in comparison with current state-of-the-art methods. Section VIII wraps up the paper with a short conclusion and a discussion on future directions.

### C. Notation

The notational convention and descriptions of major notations are listed in Table I.

## II. RELATED WORK

The MWIS problem has been studied for decades. Centralized solvers based on exact, heuristic, and hybrid algorithms have been developed [27], [52], [53], [54], [55], [56], [57]. To find the exact solution, the MWIS problem can be formulated following integer programming, maximum satisfiability, or graph coloring approaches, and solved via mixed integer programming solvers [58] based on branch-and-bound schemes [52], [53], [54]. For general graphs, exact solvers only work on medium-sized graphs of up to hundreds of vertices, since the MWIS problem is NP-hard. For real-world graphs, however, their structural properties can be utilized to improve the efficiency of exact solvers [5], [57]. For networks with a tree topology, the MWIS problem can be solved efficiently [5]. Moreover, a full suite of rule-based graph reduction techniques has been developed to exploit the hierarchical structure of large real-world graphs [57], which can drastically reduce the effective size of graphs being processed in iterative...
frameworks and allow the exact solvers to work for some real-word graphs of up to millions of vertices. For medium to large-sized graphs that are unsolvable by the exact solvers, heuristics based on local search can often obtain approximate solutions with high quality [59], [60], [61], [62]. In addition, quantum approximate optimization combining quantum and classical computing has been recently proposed for heuristic solvers [28], [63]. Nonetheless, the aforementioned centralized MWIS solvers are not suitable for link scheduling due to complexity [5], since centralized link scheduling generally requires solving the MWIS problem on small graphs within milliseconds. Our centralized MWIS solver is specifically designed to work on general graphs at low complexity in terms of time and communication.

For practical scheduling in wireless ad-hoc networks, distributed MWIS solvers with low communication and computational complexity are usually preferred. Distributed MWIS solvers [6], [10], [19], [20], [21], [28], [64] construct a solution through an iterative procedure of a round of local exchanges between a vertex and its neighbors, followed by a phase of processing on each vertex. Thus, it is best to describe the complexity of distributed MWIS solvers with local (communication) complexity, defined as the number of rounds of local exchanges between each vertex and its neighbors. In [6], [19], and [10], a solution is obtained in 2 steps, each with a linear local complexity $O(V)$. In [20], a solution is constructed through $O(V)$ iterations of combining feasible local solutions at each vertex and exchanging the results with its neighbors. Compared with distributed solvers with linear local complexity $O(V)$ [6], [10], [19], [20], Ising-formulated MWIS solvers [21], [28], [64] require a fixed number of rounds (e.g. tens to hundreds) of local exchanges to emulate the cooling process of atoms with magnetic spin. The distributed greedy solvers [17], [18] have an average local complexity of $O(\log V)$, and the worst-case local complexity of $O(V)$ on certain graphs. In particular, the local greedy solver (LGS) [17] selects vertices with the largest

| Symbols | Descriptions |
|---------|--------------|
| $(\cdot)^{-1}$ | transpose operator, $\odot$: element-wise product operator, $|\cdot|$ | cardinality of a set. |
| $\alpha$ | learning rate |
| $\gamma, \gamma(\cdot)$ | ratio of total utilities of solutions found by GCN-based solver and greedy solver, $\gamma = \mu(\psi_{\text{GCN}})/\mu(\psi_{\text{GR}})$. |
| $\Theta_{l}^{g}, \Theta_{l}^{s}$ | the sets of trainable parameters of the layer $l$ of GCN in (3). |
| $\lambda$ | $\lambda$: arrival rate |
| $\mu, \mu(s)$ | $\mu = E(\tau)/\lambda$: traffic load, $\mu(s)$: saturation traffic load |
| $\sigma_{l}(\cdot)$ | activation function of layer $l$ of GCN |
| $\phi, \phi$ | $\phi$: an empty set (or queue), $\phi_{l}$: an empty graph |
| $\Psi_{g}(\cdot, \Xi)$ | parameterized function of GCN defined on graph $G$, with a set of trainable parameters $\Xi$. |
| $\Omega_{s}, \Omega_{g}$ | $\Omega_s$: distribution of network state $(G, S, \omega)$, $\Omega_g$: distribution of state $S = (G, S)$. |
| $\Omega_{s}^{\text{CG}}$ | conditional distribution of $\omega$ given state $S = (G, S)$. |
| $\tilde{B}, b$ | $\tilde{B}$: branching factor of search tree, $b \in \{1, \ldots, B\}$: index of a child state (branch) of current state |
| $c, c_{u}, c_{v}(\cdot)$ | $c$: vector of control messages from all links, $c_{u} = c_{v}(\cdot)$: control message from link $v$ to its neighbors |
| $d(v, d)$, $d(v)$ | $d(v)$: degree of vertex $v$, $d(v)$: average vertex degree of graph $G$. |
| $\mathbb{E}(\cdot), \mathbb{I}(\cdot)$ | $\mathbb{E}(\cdot)$: expectation, $\mathbb{I}(\cdot)$: indicator function |
| $g_{l}(\cdot)$ | $g_{l}(\cdot)$: function of centralized greedy heuristic, $g_{l}(\cdot)$: function of distributed greedy heuristic |
| $g_{d}(\cdot)$ | Conflict graph $G_{d}$ composed of a set of vertices $V$ and a set of edges $E$. |
| $g_{d}(V, E)$ | the objective function of trainable parameters $\Xi$ in the formulation in Section VI. |
| $\nabla, \nabla^{(E)}$ | $\nabla$: gradient, $\nabla^{(E)}$: gradient of objective function $J(\Xi)$. |
| $k, K$ | $k$: sub-channel index, $K$: number of sub-channels. |
| $L, L'$ | $L$: the total number of layers of GCN, $L \in \{1, \ldots, L\}$: the index of a layer. |
| $m$ | number of edges formed by a new vertex in the preferential attachment process in Barabási–Albert model. |
| $N$ | maximum number of iterations in the truncated LGS-N. |

| Symbols | Descriptions |
|---------|--------------|
| $N_{G}(V)$ | the set of immediate neighbors of vertex $v$ on graph $G$. |
| $N(v)$ | normal distribution with mean $a$ & standard deviation $b$. |
| $O(\cdot)$ | big $O$ notation provides an upper bound on the growth rate of the function, for complexity |
| $p$ | probability of edge appearance in Erdős–Rényi model |
| $q(v), q_{w}$ | $q(v) = q_{w}$, the queue length of link $v$, $q$: the vector of queue lengths on all links $V$. |
| $Q(S, \omega)$ | $Q$ vector that captures the contribution of each dimension of action $\omega$ to the Q-value $Q(S, \omega)$. |
| $r(\cdot), r_{w}$ | $r(\cdot) = r_{w}$: link rate of link $v$, $r$: the vector of link rates for all links $V$. |
| $S, \omega$ | $S = (G, S)$: state in the training formulation in Section VI. |
| $S_{\text{CG}}$, $S_{G}$ | $S_{\text{CG}}$: solution from greedy MWIS solver, $S_{G}$: solution from GCN-based MWIS solver. |
| $\omega_{v}(\cdot)$ | $\omega_{v}(\cdot)$: utility value on link $v$, $\omega_{v}(\cdot)$: total utility on independent set $v$. |
| $u_{v}$ | $u_{v}$: the vector of utility values on all links $V$, $u_{v}$: utility value on link $v$. |
| $U(a, b)$ | uniform distribution between $a$ and $b$. |
| $v, \nu_{v}$ | $v \in \mathbb{R}^{V}$: an arbitrary vertex (link) on the conflict graph $G$ (network). |
| $\nu_{v}$, $\nu_{v}^{*}$ | $\nu_{v}^{*}$: the optimal solution of MWIS problem. |
| $\psi_{G_{d}}$, $\psi_{G_{CG}}$ | $\psi_{G_{d}}$: solution from greedy MWIS solver, $\psi_{G_{CG}}$: solution from GCN-based MWIS solver. |
| $V, \nu_{v} \in \{0,1\}^{V}$: the indicator vector of set $v$ w.r.t. $V$, $\phi_{v}':$ the indicator vector of $\phi_{G_{CG}}$ w.r.t. $V$. |
| $V$ | graph size, the number of vertices on graph $G$, $V = |V|$. |
| $w, w_{v}$ | $w_{v}$: vector of topology-weighted utilities, $w = x \odot u$. |
| $x, x_{i}$ | $x_{i}$: Upright bold lower-case symbol denotes a column vector, $x_{i}$: the $i$th element of vector $x$. |
| $X_{i}$ | $X_{i}$: Upright bold upper-case symbol denotes a matrix, $X_{i}$: element at row $i$ and column $j$ of matrix $X$. |
| $X_{i}$, $X_{i}$ | $X_{i}$: the entire row $i$ of matrix $X$, $X_{i}$: the entire column $j$ of matrix $X$. |
| $z, Z, Z(G)$ | node embeddings generated by GCN (based on $G$), either as a vector $z \in \mathbb{R}^{V}$ or a matrix $Z \in \mathbb{R}^{V \times B}$. |
weights among their neighbors with a built-in tie resolution mechanism, and then excludes the neighbors of the selected vertices. Randomization is introduced in [18] to improve the complexity of LGS, which is deterministic; a vertex is selected if its weight exceeds a prescribed fraction of the maximal weight of its neighbors. Our distributed MWIS solver departs from existing work by incorporating topological information in the solution through a trainable node embedding procedure.

GNNs have been recently proposed to approximate the solution to combinatorial problems [44], [65] including the maximal (unweighted) independent set (MIS) problem [45], the Boolean satisfiability problem [66], the traveling salesman problem [67], and the maximum constraint satisfaction problems [68]. These learning-based solvers often prioritize sub-optimality gap over time complexity, while ours do the opposite. The GCN-guided tree search in [45] randomly traverses a search tree predicted by a GNN, for many times, in order to find as many candidate solutions as possible, and outputs the best one at timeout. Similar to [45], our centralized solver also builds its solution by traversing a search tree predicted by a GNN. However, we adopt a rollout strategy [49] to further guide the tree traversal so that we can find a good solution fast by traversing the tree only once. Moreover, the aforementioned works [44], [45], [65], [66], [67] propose centralized solvers whereas, to the best of our knowledge, we provide the first fully distributed GCN-based solver to the MWIS problem.

In addition, our results demonstrate meaningful contributions of GNN to the enhanced quality of our centralized and distributed solvers, whereas the contribution of GNN to the tree-search in [45] is questionable [69]. Finally, we develop a customized RL scheme to train the GCN embedded in a non-differentiable pipeline, which complements the existing approaches of zeroth-order optimization [50] and surrogate gradient [51].

Fig. 1. Wireless multihop network with orthogonal access. (a) Connectivity graph of the network. (b) Example of orthogonal access in an FDMA system, where the spectrum is divided into sub-channels, time is divided into time slots, and each spectral-temporal slot can be accessed by at most one link in a set of potentially interfering links.

III. SYSTEM MODEL AND PROBLEM STATEMENT

A. System Model

Consider a wireless multihop network as illustrated in Fig. 1(a), where the existence of link \( (i, j) \) implies that user \( i \) and user \( j \) can communicate with each other. Since we will ultimately focus on a conflict graph whose vertices represent links in the wireless network, we denote an arbitrary link \( (i, j) \) as \( v \). A flow \( f \) describes the stream of packets from a source user to a destination user. A flow may pass through multiple links determined by a routing scheme. For each link, there is a queuing system \( q \) for packets of all the flows.

The wireless network adopts a multiple access scheme that divides the spectrum resource into a set of orthogonal sub-channels, \( \mathcal{K} = \{1, \ldots, K\} \), and time slots. The wireless channel is assumed to be stationary and ergodic network wide, and invariant within a time slot, i.e., the coherence time of the channel is assumed to exceed the duration of a time slot. In Fig. 1(b), an example of a frequency division multiple access (FDMA) system is illustrated.\(^1\) The channel state information of link \( v \) on sub-channel \( k \) at time slot \( t \) is denoted by \( h^{(t)}_{k}(v) \). We assume an orthogonal access scheme where each slot of the spectral-temporal grid in Fig. 1(b) can only be accessed by one link out of a set of potentially interfering links.

There are two formulations of interference constraints in the literature: physical distance model and hop distance model [23]. In this work, the interference in the system is considered to follow a physical distance model. For example, two links interfere with each other if their incident users are within a certain distance such that their simultaneous transmission will cause the outage probability to exceed a certain level. Depending on the air-interface technology and antenna systems, the interference zone of a link can be different from its connectivity zone. A link is assumed to be able to learn its interfering neighbors by monitoring the channel and/or beacon signals. Moreover, mutually interfering links are assumed to be able to exchange control messages, e.g., with low-rate modulation and coding schemes. Notice that, in principle, the interference zone of a link would depend on the transmit power of the corresponding user and hence possibly vary with time. To simplify the analysis and avoid this dependence, we consider a scenario in which all the users transmit at power levels that do not vary with time. In general, our approaches work on any conflict graph, no matter how it is constructed.

B. Single-Radio Single-Channel Scheduling

We first consider a wireless network with only one sub-channel, in which each user is equipped with one half-duplex radio interface. The interference relationship between links of the wireless multihop network is described by a conflict graph \( \mathcal{G} \), where a vertex in the conflict graph represents a link in the wireless network and the presence of an edge in \( \mathcal{G} \) encodes the fact that the corresponding links interfere with each other. In the rest of this paper, we focus on the conflict graph \( \mathcal{G} \) which we assume to be known; see, e.g., [70] for its estimation, and the supplemental materials [71, Sec. V] for the complexity of its construction. Recall that an independent (vertex) set (IS) in a graph is a set of vertices such that there are no edges between any two vertices in the set. From the definition of \( \mathcal{G} \), only wireless links that form an IS in \( \mathcal{G} \) can communicate simultaneously in time and frequency under the constraint of orthogonal access.

\(^1\)Alternatively, in code division multiple access (CDMA) systems, channels are implemented by orthogonal codes.
The state of the system at time slot \( t \) can be described by the tuple \((\mathcal{G}(t), \mathbf{q}(t), \mathbf{f}(t), \mathbf{h}(t))\) consisting of the conflict graph \( \mathcal{G}(t) \), queue lengths \( \mathbf{q}(t) \), flows \( \mathbf{f}(t) \), and channel states \( \mathbf{h}(t) \). Since the scheduling is conducted at each time slot \( t \), for notational simplicity we omit the superscript \( t \), denoting the system state as \((\mathcal{G}, \mathbf{q}, \mathbf{f}, \mathbf{h}))\). In this setting, the task of optimal link scheduling can be described as selecting a set of non-interfering links, \( \mathbf{v} \subseteq \mathcal{V} \), on which to transmit in order to maximize some utility \( u(\mathbf{v}) = f(\mathbf{v}; \mathcal{G}, \mathbf{q}, \mathbf{f}, \mathbf{h}) \) that is parameterized by the current state of the system. As is customary [4], we model here the utility of the set of links \( \mathbf{v} \) (or the set of vertices in the conflict graph) as the sum of utilities associated with each link, i.e., \( u(\mathbf{v}) = \sum_{v \in \mathbf{v}} u(v) \), leading to the following formal problem statement.

**Problem 1:** Consider a conflict graph \( \mathcal{G}(\mathcal{V}, \mathcal{E}) \), where \( \mathcal{V} \) and \( \mathcal{E} \) describe all the links and their conflict relationships in the wireless network, respectively, and a utility function \( u : \mathcal{V} \rightarrow \mathbb{R}_{+} \). The optimal scheduling is given by selecting a subset of vertices \( \mathbf{v}^* \subseteq \mathcal{V} \) such that

\[
\mathbf{v}^* = \underset{\mathbf{v} \subseteq \mathcal{V}}{\operatorname{arg\ max}} \sum_{v \in \mathbf{v}} u(v) \quad (1a)
\]

subject to

\[
(v_i, v_j) \notin \mathcal{E}, \forall v_i, v_j \in \mathbf{v}. \quad (1b)
\]

With the statement of Problem 1, the optimal scheduling at each temporal slot is transformed to an MWIS problem in the corresponding graph. Indeed, we want to choose non-neighboring vertices in the conflict graph (i.e., non-interfering links in the wireless network) such that the total utility is maximized. As discussed in Section I and formally introduced here, this utility \( u \) is a function of the current state of the network with many existing variants [3], [4], [6], [10], [17], [18]. The MaxWeight scheduler described by Problem 1 is not tied to a specific utility function, but rather can work with any utility function.

**C. Multi-Channel Scheduling**

There are two ways to extend the MaxWeight scheduler defined in Problem 1 to networks with a set of orthogonal sub-channels \( \mathcal{K} \): 1) Sequentially solve a set of single-channel scheduling tasks with state \((\mathcal{G}^k, \mathbf{q}^k, \mathbf{f}, \mathbf{h}^k)\) for each sub-channel \( k \in \mathcal{K} \), where the queue lengths \( \mathbf{q}^k \) for sub-channel \( k \) depend on the schedules on sub-channels \( \{1, \ldots, k-1\} \). The complexity and scheduling overhead of this approach grows linearly with the number of channels \( |\mathcal{K}| \). 2) Solve the MWIS problem on a single multi-channel conflict graph, for which detailed construction methods can be found in [72] and [73]. The multi-channel conflict graph is \( |\mathcal{K}| \) times larger than the single-channel conflict graph. With a heuristic solver of logarithmic complexity, the second approach can reduce the complexity from \( O(|\mathcal{K}| \log |\mathcal{V}|) \) to \( O(\log |\mathcal{K}| \log |\mathcal{V}|) \) at the cost of poorer relative performance on larger graphs as illustrated in Section VII-B. In this paper, we focus on single-channel scheduling while numerically evaluating the second approach for multi-channel scheduling.

**D. Greedy Heuristics**

A centralized greedy solver (CGS) [27], denoted as \( \hat{\mathbf{v}}_{\text{Gr}} = g_c(\mathcal{G}, \mathbf{u}) \), estimates \( \hat{\mathbf{v}}_{\text{Gr}} \) that approximates the solution to (1) in an iterative fashion by first adding to \( \hat{\mathbf{v}}_{\text{Gr}} \) the vertex with the largest utility, deleting its neighbors as potential candidates, and repeating this procedure until all vertices are either added to \( \hat{\mathbf{v}}_{\text{Gr}} \) or deleted, as detailed in Algorithm 1 in [71].

The distributed implementation of CGS is denominated as local greedy solver (LGS) [17], denoted as \( \hat{\mathbf{v}}_{\text{Gr}} = g_d(\mathcal{G}, \mathbf{u}) \). As detailed in Algorithm 1, if a vertex \( v \) has the largest weight in the neighborhood (line 5), it is marked as \( +1 \) (line 6) and added to the solution set \( \hat{\mathbf{v}}_{\text{Gr}} \) (line 10), then \( v \) broadcasts a control message to its neighbors, who then mark themselves as \( -1 \) (line 7). Next, the unmarked vertices form the residual graph \( \mathcal{G}' \) (line 12). In practice, the LGS has a built-in tie-breaking mechanism based on an initial assignment of identification numbers to each vertex in the conflict graph that does not require additional information exchanges in the case of a tie (in line 5). Notice that these local exchanges are between vertices in the conflict graph. By construction, \( \hat{\mathbf{v}}_{\text{Gr}} \) is guaranteed to follow the IS constraint in (1b) but the suboptimality gap \( u(\mathbf{v}^*) - u(\hat{\mathbf{v}}_{\text{Gr}}) \) might be large since CGS and LGS do not fully consider the topology of \( \mathcal{G} \).

Both CGS and LGS have linear computational complexity \( O(|\mathcal{V}|) \). However, as distributed MWIS solvers are parallel by nature, we focus on their local complexity. The LGS has a logarithmic average local complexity \( O(\log |\mathcal{V}|) \) on random graphs and a linear worst-case complexity \( O(|\mathcal{V}|) \) on path graphs with increasing vertex weights along the path as illustrated in Fig. 2 [17].

**IV. GRAPH CONVOLUTIONAL NETWORK-GUIDED TREE SEARCH**

An enhanced centralized scheduler could improve the performance of infrastructure-based wireless multihop networks of small to medium sizes, such as D2D communications [9], CRAN [10], wireless backhaul networks [5], [11], and

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**Algorithm 1** Local greedy solver \( \hat{\mathbf{v}}_{\text{Gr}} = g_d(\mathcal{G}, \mathbf{u}) \) [17]

**Input:** \( \mathcal{G}, \mathbf{u} \)

**Output:** \( \hat{\mathbf{v}}_{\text{Gr}} \)

1: \( \hat{\mathbf{v}}_{\text{Gr}} \leftarrow \emptyset; \mathcal{G}'(\mathcal{V}', \mathcal{E}') \leftarrow \mathcal{G}(\mathcal{V}, \mathcal{E}); c = 0 \)
2: while \( \mathcal{G}' \neq \phi \) do
3:   for all \( v \in \mathcal{V}' \) do
4:     \( v \) exchanges \( u(v) \) with its neighbors \( \forall v_i \in \mathcal{N}_{G'}(v) \)
5:     if \( u(v) > \max_{v_i \in \mathcal{N}_{G'}(v)} u(v_i) \) then
6:       \( c(v) \leftarrow +1; v \) broadcasts a control message
7:       \( c(v_i) \leftarrow -1, \forall v_i \in \mathcal{N}_{G'}(v) \)
8:   end if
9: end for
10: \( \hat{\mathbf{v}}_{\text{Gr}} \leftarrow \{v|v \in \mathcal{V}', c(v) = 1\} \)
11: \( \mathcal{V}' \leftarrow \{v|v \in \mathcal{V}', c(v) = 0\} \), update \( \mathcal{G}' \) with new \( \mathcal{V}' \)
12: end while

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Fig. 2. Example path graph with 5 vertices of increasing weights on which LGS requires 5 iterations to complete as the worst case.
A. Algorithmic Framework

Our methodology consists of defining a search tree of candidate solutions and then traversing it iteratively with multiple strategies. The search is formulated as a Markov decision process, of which the possible state transitions of each iteration form a search tree, as illustrated in Fig. 3, which we explain in more detail in this section. The solver finds an approximate solution by traversing from the root of the search tree to a terminal node. A node in the search tree represents an intermediate search state $x_n = < G', u', v', N_G(v') >$, where $G'$ is the residual graph, $u'$ is the corresponding residual utility vector collecting the utilities $u(v)$ for all $v \in G'$, $v'$ is the partial solution for $G$, and $N_G(v')$ is the set of all the vertices adjacent to some vertex in the partial solution. A state transition would be triggered by the action of adding a vertex in $G'$ to the partial solution $v'$. The root node of the search tree, $x_0 = < G, u, \emptyset, \emptyset >$, is the initial state generated from the input conflict graph $G$ and corresponding utility vector $u$. At a terminal node, the residual graph is empty $G' = \emptyset$ and $v'$ is a feasible approximate solution to Problem 1. Each non-terminal node has $B$ children nodes, where $B$ is the branching factor of the search tree as further discussed in Sections IV-B and IV-D. Next, we explain the iterative traversal of the search tree through the exemplary rollout strategy in Section IV-B.

B. GCN-Guided Centralized Rollout Search

In order to reach a good solution, the branching factor $B$ should be configured to create a large search tree with a large search space. In the approach outlined so far, it may take a long time for the solver to find a good terminal node in a large search space. To reach a good terminal node quickly, we introduce GCN-guided centralized rollout search (GCN-CRS), which employs 1-step rollout search [49] to further guide the tree search, as illustrated in Fig. 3 and Algorithm 2.

Algorithm 2 is explained as follows. On initialization (line 1), the root node $x_0 = < G, u, \emptyset, \emptyset >$ is pushed into the search queue $Q$. In each iteration, the solver predicts $B$ vertices based on the current state (steps 1-5), estimates the Q-value of adding each predicted vertex to the partial solution (step 6), and then proceeds with the action of the highest Q-value (step 7). In step 1, a non-terminal node $x_n = < G', u', v', N_G(v') >$ is randomly popped from the search queue as the current state. In GCN-CRS, $x_n$ is the only item in the search queue, but other algorithm variations discussed in Section IV-D can have more items in the queue. In step 2, the residual graph $G'$ and its corresponding utility vector $u' = [u(v)]_{v \in G'}$ are mapped by an $L$-layered GCN to a node embedding vector, as $z' = \Psi_G(u'; \Xi)$, where $z' = [z'(v)]_{v \in V'} \in \mathbb{R}^{L'}$ contains the topology-aware scaling factors of all vertices in $G'$, $\Psi_G$ is the GCN defined on the graph $G'$ (as detailed in Section IV-C), and $\Xi$ represents the collection of trainable parameters of the GCN. In step 3 (line 5 of Algorithm 2), a vector of topology-aware utilities of all vertices in $G'$ is created through the element-wise product $w' = z' \odot u'$. In step 4 (line 6), vector $w'$ is sorted in descending order into $\hat{w}'$, and vector $\hat{v}'$ collects

\begin{itemize}
  \item the input conflict graph $G$,
  \item the set of vertices $u$,
  \item the set of vertices $v$,
  \item the set of vertices $w$,
  \item the set of vertices $\emptyset$,
  \item the set of vertices $G$,
  \item the set of vertices $u$,
  \item the set of vertices $v$,
  \item the set of vertices $w$,
  \item the set of vertices $\emptyset$,
  \item the set of vertices $G$,
  \item the set of vertices $u$,
  \item the set of vertices $v$,
  \item the set of vertices $w$,
  \item the set of vertices $\emptyset$,
  \item the set of vertices $G$,
  \item the set of vertices $u$,
  \item the set of vertices $v$,
  \item the set of vertices $w$,
  \item the set of vertices $\emptyset$,
  \item the set of vertices $G$,
  \item the set of vertices $u$,
  \item the set of vertices $v$,
  \item the set of vertices $w$,
  \item the set of vertices $\emptyset$,
  \item the set of vertices $G$,
  \item the set of vertices $u$,
  \item the set of vertices $v$,
  \item the set of vertices $w$,
  \item the set of vertices $\emptyset$,
  \item the set of vertices $G$,
  \item the set of vertices $u$,
  \item the set of vertices $v$,
  \item the set of vertices $w$,
  \item the set of vertices $\emptyset$.
\end{itemize}
the vertices corresponding to the sorted vector \( \tilde{\mathbf{w}}' \). In step 5 (lines 7-11), the first \( B \) vertices in \( \tilde{\mathbf{v}}' \) (with the largest topological-aware utilities) are used to predict \( B \) respective children nodes (next states) of the node \( x_n \) (current state). A child node \( < G'_b, \tilde{\mathbf{u}}'_b, \mathbf{v}'_b, \mathcal{N}_G(\mathbf{v}'_b) > \) is predicted by vertex \( v'^{(n)}_b = \tilde{\mathbf{v}}'_b, \forall b \in \{1, \ldots, B\} \) as:

\[
G'_b = G' \setminus (\{v'^{(n)}_b\} \cup \mathcal{N}_G(v'^{(n)}_b)), \quad \mathbf{v}'_b = \mathbf{v}' \cup \{v'^{(n)}_b\}.
\]

In step 6 (line 12), the Q-value of the \( b \)th child node \( x'_{n+1} \) (the action of adding \( v'^{(n)}_b \) to the partial solution) is estimated by 1-step lookahead rollout as \( Q(x'_{n+1}) = \mathbf{v}(x_{n+1}^b) + f_h(x_{n+1}^b) \). Here, \( \mathbf{v}(x_{n+1}^b) \), the utility of vertex \( v_{n+1}^b \), is the immediate reward of transitioning from the node \( x_n \) to node \( x_{n+1}^b \). The score of the \( b \)th child node \( f_h(x_{n+1}^b) \) is the total utility of a solution to the MWIS problem defined on the residual graph \( (G'_b, \tilde{\mathbf{u}}'_b) \), obtained by an efficient guiding heuristic, which serves as the estimated reward of traversing from state \( x_{n+1}^b \) to a terminal state. Lastly, in step 7 (line 14-19), the GCN-CRS proceeds to the child node with the largest Q-value, and ties are broken randomly. If the selected child node (next state) is a non-terminal node, it is pushed into the search queue. Otherwise, the solver outputs \( \mathbf{v}' \) as the full solution.

In GCN-CRS, the search tree is traversed along a single path and the search terminates when it reaches the first terminal node. The performance of the GCN-CRS is guaranteed to be no worse than (and largely dictated by) the guiding heuristic [49]. In our case, the CGS is selected as the guiding heuristic due to its linear complexity and determinism. Specifically, the score of the \( b \)th child node is obtained as \( f_h(x_{n+1}^b) = \mathbf{v}(x_{n+1}^b) \), where \( \mathbf{v}(x_{n+1}^b) \) is obtained by either the vanilla CGS as \( \mathbf{v}_{Gr} = g_c(G'_b, \tilde{\mathbf{u}}'_b) \), or an enhanced CGS as \( \mathbf{v}'_{Gr} = g_c(G'_b, \mathbf{w}'_b) \), where \( \mathbf{w}'_b = [\mathbf{w}(v) | v \in G'_b] \). The operations on the \( B \) branches in lines 5 and 9, respectively, Moreover, the operations on the \( B \) branches in lines 5-13 can be parallelized. The GCN-CRS solvers using vanilla CGS and enhanced CGS as guiding heuristics are denoted as GCN-CRS-v and GCN-CRS-e, respectively. The training method of the GCN is detailed in Section VI.

C. Graph Convolutional Network Design

Our GCN has an \( L \)-layer structure as follows: Given the input feature as \( \mathbf{X}^0 = \mathbf{u}' \), then \( \mathbf{z}' = \mathcal{P}_G(u'; \Xi) = \mathbf{X}^L \), where an intermediate \( l \)-th layer of the GCN is given by

\[
\mathbf{X}^l = \sigma_l \left( \mathbf{X}^{l-1} \Theta^l_0 + \mathbf{L} \mathbf{X}^{l-1} \Theta^l_1 \right).
\]

In (3), \( \mathbf{L} \) is the normalized Laplacian of \( G \), \( \Theta^l_0, \Theta^l_1 \in \mathbb{R}^{m_l \times m_l} \) are the trainable parameters of the \( l \)th layer in the collection of \( \Xi \), \( g_{l-1} \) and \( g_l \) are the dimensions of the output features of layers \( l-1 \) and \( l \), respectively, and \( \sigma_l(\cdot) \) is the activation function. The dimension of the input feature is \( g_0 = 1 \). The activation functions of the input and hidden layers are selected as leaky ReLUs. By setting the dimension of the output layer as \( g_L = 1 \) with linear activation, the GCN generates a node embedding vector, \( \mathbf{z}' \in \mathbb{R}^{V'} \), as a topological scaling factor. As explained earlier, the prediction vector is then computed as \( \mathbf{w}' = \mathbf{z}' \odot \mathbf{u}' \). The GCN is trained by RL with the help of an efficient CGS. Since both our proposed centralized and distributed solutions share the same training mechanism, we defer its explanation to Section VI. Notice that implementation of GCN in (3) is just one viable option, other implementations of GNNs, e.g., in [48], could also be used in our framework.

D. Reference GCN-Guided Centralized Solvers

Next, we propose two reference solvers with which to compare GCN-CRS: GCN-guided centralized random tree search (GCN-CRTS) and GCN-guided centralized greedy search (GCN-CGS). They are obtained by modifying the state-of-the-art solvers for the unweighted MIS problem in [44] and [45], respectively, making them compatible with our MWIS setting. GCN-CRTS and GCN-CGS follow an iterative procedure similar to that of GCN-CRS, while employing different traversal strategies.

The idea of GCN-CRTS is to reach as many random terminal nodes as possible in a given time interval to increase its chance of finding a good solution from them. Randomized search ensures the equal chance of reaching each terminal node, which is implemented by unfolding a non-terminal node randomly popped from the search queue and randomly pushing non-terminal nodes along the path to the search queue. The GCN-CRTS is almost identical to the MIS solver in [45], in which a node embedding matrix \( \mathbf{Z}' \in [0, 1]^{V' \times B} \) is generated by a GCN with a slightly different output structure, with only two modifications: 1) Using \( \mathbf{W}' = \mathbf{Z}' \odot \mathbf{u}' \mathbf{1}^\top \) instead of \( \mathbf{Z}' \) as the prediction matrix. 2) For computational efficiency, synthetic random graphs are used as the training data for supervised learning, where the label vectors are generated by heuristics instead of exactly solving the NP-hard problem. Specifically, the labels are generated by selecting the best solution from two guiding heuristics: linear programming [74] and centralized greedy solver. The implementation of the GCN-CRTS is detailed in Section II and Algorithm 2 of [71].

The GCN-CGS is modified from the deep Q network in [44] by replacing the Node2Vec module with a GCN. For a non-terminal node \( < G'_b, \mathbf{u}' > \), the GCN takes \( (\mathbf{G}'_b, \mathbf{u}') \) as input and generates node embedding \( \mathbf{z}' \in \mathbb{R}^{V'} \) as the Q-values of the action space \( A' = V' \), whereas the branching factor \( B = |V'| \) is no longer a hyperparameter. The action is selected by an \( \epsilon \)-greedy method. The next state \( (\mathbf{G}''_b, \mathbf{u}'') \) is generated according to (2). The search terminates upon reaching a terminal node. More details of the implementation of GCN-CGS can be found in Section III and Algorithm 3 of [71].

Both GCN-CRTS and GCN-CGS benefit from the flexible input dimensions of GCN and the iterative algorithmic framework, so that they can generalize to large graphs and outperform the training heuristics [45]. The GCN-CGS can reach a good terminal node quickly. However, the GCN-CRTS will take a relatively long time to find a good solution (e.g., several minutes), making it unsuitable for link scheduling.

E. Computational Complexity

The computational complexity of the \( l \)th layer of the GCN is \( \mathcal{O}(d \mathbf{g}_l \mathbf{g}_{l-1}) \), where \( d \) is the average degree of a vertex
in the input graph. By assuming $g_l = g, \forall l \in \{1, \ldots, L\}$, an $L$-layered GCN has a computational complexity of $O(LV^g \hat{d})$. In tree search, reaching a terminal node requires an average of $V/d$ steps, therefore the computational complexity of finding a solution is $O(LV^2 g^2)$. Without timeout, the GCN-CRTS requires an exponential complexity of $O(LV^2 g^2 B^V)$ to reach all $B^V$ terminal nodes. To find a good solution on graphs of hundreds of vertices, a timeout of several minutes is usually required for GCN-CRTS. The computational complexity of GCN-CRS is $O(LV^2 g^2 + BV^2 / \hat{d})$, since the guiding heuristic of CGS will be executed $B$ times on each node of the search tree, adding a complexity of $O(BV^2 / \hat{d})$ with an average of $V/d$ passes. The computational complexities of the presented centralized MWIS solvers are summarized in Table II.

V. DISTRIBUTED MWIS SOLVER USING GRAPH NEURAL NETWORKS

Our goal in the design of a distributed solver is to decrease the suboptimality gap of the baseline LGS described in Section III-D while keeping its two main advantages: 1) Low computational complexity, and 2) Can be implemented in a distributed manner with low communication cost. To achieve this goal, LGS with modified weights is proposed to solve Problem 1. More precisely, mimicking the development of our centralized solvers, instead of considering the vanilla utilities $u(v)$ we consider graph-aware utilities $w(v) = z(v)u(v)$, where the scalar node embedding $z(v)$ encodes a relevant topological feature of vertex $v$. Intuitively, if vertex $v$ is a high-degree link that may interfere with many other links in the wireless network, then $z(v)$ should downscale the utility $u(v)$ since scheduling $v$ would preclude many other links from the schedule. By contrast, if vertex $v$ has low-degree in the conflict graph $\mathcal{G}$ (e.g., an isolated link in the original wireless network) then $z(v)$ should amplify $u(v)$. In summary, $z(v)$ should be a topology-aware scaling that reduces the MWIS suboptimality gap and, to be consistent with our goal, should also be attainable in a distributed manner with low communication and computational cost. With these requirements in mind, we propose to obtain a vectorized node embedding $z(v) = \Psi_{\mathcal{G}}(\mathbf{S})$, where $\Psi_{\mathcal{G}}$ is an $L$-layered GCN defined on $\mathcal{G}$, which is the same distributable GCN used in our centralized solver as described in (3). Since the normalized Laplacian $\mathcal{L}$ is a local operator on $\mathcal{G}$, it should be noted that $z(v)$ can be computed locally at each vertex $v$ through $L$ rounds of local exchanges with its neighbors. Specifically, by avoiding the use of global operations, such as network-wide softmax activations or normalizations, the system level update of the $l$th layer of a GCN in (3) can be implemented in a fully distributed manner via the following local operation on link $v \in \mathcal{V}$,

$$\mathbf{X}_{v^*}^{l+1} = \sigma_l \left( \Theta_0 + \mathbf{X}_{v^*}^l \Theta_0^T + \sum_{u \in \mathcal{N}(v)} \frac{\mathbf{X}_{u^*}^{l-1}}{d(v)d(u)} \mathbf{X}_{v^*}^l \right)$$

(4)

where $\mathbf{X}_{v^*}^{l+1} \in \mathbb{R}^{1 \times g_l}$ is the $v^*$th row of matrix $\mathbf{X}^l$ in (3), capturing the features on vertex $v$, $\mathcal{N}(v)$ denotes the neighbor set of vertex $v$, and $d(\cdot)$ is the degree of a vertex. In practice, a link $v$ can track its degree by counting its interfering neighbors during local exchanges. The expression in (4) shows that the local computational complexity of link $v$ scales
linearly with its degree, which is small in many practical scenarios. The major complexity of GCN-LGS comes from local exchanges. Next, the approximate solution is estimated as \( \hat{z}(G, w) \) through the LGS [17] detailed in Algorithm 1, where the graph-aware utilities \( w = z \odot u \).

\[ \text{Algorithm 1, where the graph-aware utilities} \ w = z \odot u. \]

**B. Architectural Variations**

Apart from the aforementioned baseline architecture, the GCN-LGS admits several useful variations. The first set of variations is associated with the iterative structure, as illustrated in Fig. 4. First, the baseline LGS can be truncated to \( N \) iterations, denoted as \( \text{GCN-LGS-N} \), in order to achieve a constant local complexity at the cost of the quality of solution, which is a key aspect to promote scalability since the constant local complexity is independent of the graph size \( V \). In the second variation, denoted as \( \text{GCN-LGS-it} \), the GCN can be placed before each inner iteration of LGS (i.e., before line 3 in Algorithm 1), so that the input of the LGS iteration \( w^{t'} = z' \odot u' \), where \( z' = \Psi_{G'}(u'; \Xi) \), is based on the residual graph \( (G', S') \) from the previous iteration, rather than the input graph \( (G, S) \) as in the baseline GCN-LGS, as detailed in Algorithm 4 in [71]. The GCN-LGS-it can further improve the performance over the baseline GCN-LGS at the cost of higher local complexity.

The second variation refers to the choice of input features as \( S = 1 \). We can think of the node embedding \( z = \Psi_G(1; \Xi) \) generated by a featureless GCN as a *topological embedding*, denoted as \( z(G) \), which can be reused until the network topology changes. In practice, \( z(G) \) can be reused for a coherent window of \( T \) time slots that matches the pace of topological change to further reduce the computational and communication complexities of a distributed scheduler.

Moreover, by reusing the topological embedding generated \( L \) time slots earlier, i.e., use \( w^{(t)} = z(G)^{(t-L)} \odot u^{(t)} \) in time slot \( t \), the additional local complexity of GCN for GCN-LGS can be reduced to zero, since \( z(G)^{(t-L)} \) and the intermediate features \( X' \) can be piggybacked to the local exchange of \( d^{(t)} \) and \( u^{(t)} \) at the beginning of time slot \( t \). An exemplary GCN-LGS with 1-layer GCN and \( w^{(t)} = z(G)^{(t-1)} \odot u^{(t)} \) is illustrated in Fig. 5. Compared to LGS, this GCN-LGS only incurs larger control messages for the first round of local exchange, additional local computational complexity, and slight topological mismatch between consecutive time slots. The robustness of GCN-LGS to topological mismatch is evaluated in Section VII-A.

Note that a scheduled link \( v \) in LGS can start to transmit right after broadcasting a control message \( c_v^{(t)} \) to mute its interfering neighbors, without the need to wait until every link in the network has been determined or until a maximum number of local exchanges has been reached. Under this scheme, the impact of the scheduling overhead of LGS (and GCN-LGS) on spectrum utilization efficiency is further reduced without truncation.

**C. Local Communication Complexity**

The worst and average local complexities of the baseline and proposed distributed MWIS solvers are listed in Table III. Without reusing the topological embedding, the average local complexity of the GCN-LGS solver is \( O(L + \log V) \), where \( O(L) \) is the local complexity of the GCN and \( O(\log V) \) is the average local complexity of LGS (as discussed in Section III-D). For LGS truncated to \( N \) iterations, the local complexity of GCN-LGS-N is \( O(L + N) \). In this way, the local computational and communication costs of our distributed schedulers can be controlled by modifying the number of layers \( L \) in the GCN. Moreover, by reusing the topological embedding, the average local complexity of GCN-LGS is further reduced to \( O(\log V) \).

**VI. CENTRALIZED TRAINING**

Having discussed the rationale and the mechanics of the downstream architectures, we are left to discuss how to train the parameters \( \Xi \) in the GCN. Compared to the typical supervised or semi-supervised settings in which GCNs are employed, our proposed downstream pipelines face two challenges. First, it is generally infeasible to obtain the optimal solution \( v^* \) (the labels for supervised learning) of a simulated training instance \( (G', S, u) \), since this would require solving an NP-hard problem. Second, the output \( z \) of the GCN is related to the objective to be maximized \( u(\hat{v}_{GCN}) \) through a non-differentiable discrete function, e.g., CGS and LGS, which prevents the gradients w.r.t. the objective being back-propagated to the GCN. To overcome these two challenges, we develop an RL scheme of graph-based deterministic policy gradient that trains the GCN based on the performance of our algorithm relative to an efficient greedy algorithm.
We formulate the scheduling in each time slot as a single step episode, of which the state is $S = (G, S)$, the multi-dimensional continuous action is $z = \Psi_G(S, \Xi) \in \mathbb{R}^V$, and the return equals the reward $\gamma = u(\hat{v}_{\text{GCN}})/u(\hat{v}_{\text{Gr}})$. Our objective is to find the optimal set of parameters $\Xi$ that maximizes the expected return for network state $(G, S, u)$ drawn from a target distribution $\Omega$

$$\Xi^* = \arg\max_{\Xi \in \mathbb{R}^V} J(\Xi),$$  

subject to

$$J(\Xi) = \mathbb{E}_{(G, S, u) \sim \Omega} [\gamma(G, u, z)],$$

$$\gamma(G, u, z) = u(\hat{v}_{\text{GCN}})/u(\hat{v}_{\text{Gr}}),$$

$$z = \Psi_G(S, \Xi),$$

$$\hat{v}_{\text{GCN}} = g_d(G, u \circ z), \hat{v}_{\text{Gr}} = g_r(G, u).$$  

Notice that functions $g_d(\cdot)$ and $g_d(\cdot)$ are respectively the efficient CGS and LGS as detailed in Section III-D, thus circumventing the need to exactly solve the MWIS problem. According to the deterministic policy gradient theorem [75, e.q. (9)], the gradient of $J(\Xi)$ can be found by

$$\nabla J(\Xi) = \mathbb{E}_{(G, S, u) \sim \Omega} [\nabla \Psi_G(S, \Xi) \nabla z Q(S, z)]$$

$$\approx \mathbb{E}_{(G, S, u) \sim \Omega} [\nabla \Psi_G(S, \Xi) Q(S, z)].$$  

In (6), vector $Q(S, z) \in \mathbb{R}^V$ approximates the contribution of each dimension of action $z$ to the Q-value $Q(S, z) = \mathbb{E}_{u \sim \Omega_u} [\gamma(G, u, z)]$, $\Omega_u$ is the distribution of state $S = (G, S)$, and $\Omega_u$ is the conditional distribution of $u$ under state $S$. Indeed, if the Q-value $Q(S, z)$ were to be a differentiable function of the action $z$, we would have $Q(S, z) = \nabla z Q(S, z)$. However, since $Q(S, z)$ is non-differentiable due to the non-differentiable function $g_d(\cdot)$ in (5e), we propose a proxy for the true credit assignment vector of action $z$

$$Q(S, z) = \mathbb{E}_{u \sim \Omega_u} [\gamma(G, u, z)] \hat{v}, \quad \hat{v} = [\mathbb{I}_{u_{\text{GCN}}}(v)|v \in V],$$  

where $\hat{v} \in \{0, 1\}^V$ is the indicator vector of $u_{\text{GCN}}$. The intuition behind (7) is that the more likely a link $v \in V$ is scheduled under state-action pair $(S, u)$, the more it contributes to $Q(S, z)$. Based on (6) and (7) and a learning rate $\alpha \in (0, 1)$, we can update the parameters $\Xi$ through the following stochastic gradient ascent

$$\Xi \leftarrow \Xi + \alpha \nabla J(\Xi), \quad \nabla J(\Xi) = \gamma(G, u, z) \nabla \Psi_G(S, \Xi) \hat{v}. $$

In training, we draw a batch of $G^{(i)}$ from the training dataset and $u^{(i)} \in U(0, 1)$, collect the tuple $u^{(i)}, z^{(i)}, \hat{v}_{\text{GCN}}, \gamma^{(i)}$ of each forward pass $i$, and then update the GCN by (8). Intuitively, (8) encourages the GCN to generate the solution $\hat{v}_{\text{GCN}}$ for input $(G^{(i)}, u^{(i)})$ by an amount proportional to its quality $\gamma^{(i)}$. Therefore, the aggregate effect of (8) on a batch of $(G^{(i)}, u^{(i)})$ is moving $\Xi$ towards generating solutions of larger $\gamma$. To prevent overfitting, after each update of $\Xi$, the GCN-LGS is tested on a small independent validation dataset drawn from the target distribution $\Omega$, and the $\Xi$ that yields the best $\gamma$ on the validation dataset is kept as $\Xi^*$. 

**VII. NUMERICAL EXPERIMENTS** 

The performance of the GCN-based MWIS solvers is evaluated on synthetic random graphs and as schedulers in wireless networks. The comparative baselines are CGS, LGS [17], and message passing (MP) [6]. To evaluate the contribution of the GCNs in our solvers, we also replace the GCNs in our solvers by random values [69] $(z_r \in \{0, 1\})$, denoted as Random-LGS and Random-CRS) and a local 5-layer perceptron taking node degree as input and trained in the same way as GCN (denoted as MLP(S)-LGS and MLP(S)-CRS). Threshold local greedy [18] and Ising [21] are not included since they perform worse than LGS/CPS. The quality of an approximate solution $\hat{v}$ is evaluated by its approximation ratio $(AR) u(\hat{v})/u(\nu^*)$, where the optimal solution $\nu^*$ is obtained by solving the computationally expensive integer programming formulation of MWIS [6], [19] using the Gurobi solver [58].

The synthetic conflict graphs for training and testing are generated from the Erdő-Rényi (ER) [76] and Barabási-Albert (BA) [77] models. The ER model is completely determined by two parameters: the number of vertices $V$, and the probability of edge-appearance $p$. The BA model is also determined by two parameters: the number of vertices $V$ and the number of edges, $m$, that each new vertex forms during the preferential attachment process. In the experiments, we set $m = Vp = \bar{d}$ so that graphs from the ER and BA models have the same expected average degree. By default, the vertex utilities are drawn following a uniform distribution $u(v) \sim \mathbb{U}(0, 1)$.

The hyperparameters of the evaluated GCNs are as follows: the numbers of layers $L \in \{1, 5, 20\}$, the size of every hidden layer is $g_l = 32$ for $L > 1$. The $L$-layered GCN is denoted as GCN($L$). To balance exploration and computational efficiency, the branching factor is selected as $B = 32$, the same as in [45]. Each GCN is trained on a set of 5000 random graphs drawn from an ER model unless otherwise specified. The training set comprises 5000 graphs of size $V \in \{100, 150, 200, 250, 300\}$ and expected average degree $\bar{d} \in \{2, 5, 7.5, 10, 12.5\}$ (200 graphs per $(V, \bar{d})$), and 900 graphs of size $V \in \{30, 100\}$ and edge probability $p \in \{0.1, 0.2, \ldots, 0.9\}$ (50 graphs per $(V, p)$). The timeout of GCN-CRTS is set to 5 minutes. The GCN in GCN-CRTS is trained by supervised learning with a maximum of 200 epochs as explained in Section II-D of [71]. The GCNs in GCN-CRS and GCN-LGS are trained as described in Section VI, for which the settings include a batch size of 200 for experience replay, 25 epochs, and periodic gradient reset. This configuration is used for the training of GCN-CGS as described in Section III of [71].

**A. Performance on Synthetic Random Graphs**

1) Results on ER Graphs: First, the GCN-based MWIS solvers, baseline heuristics, and optimal solver are tested on a set of 500 ER graphs of size $V \in \{100, 150, 200, 250, 300\}$ and average degree $\bar{d} \in \{2, 5, 10, 15, 20\}$, with 20 instances for each pair of $(V, \bar{d})$. The average ARs of the tested solvers are listed in Table IV (under the ‘ER set’ column) for the entire test set, and illustrated in Figs. 6(a) and 6(b) as a function of the average degree and size of the tested graphs, respectively.

Training typically takes 30 minutes on a workstation with a specification of 16GB memory, 8 cores and GeForce GTX 1070 GPU. The source code is published at https://github.com/zhongyuanzhao/distgcn
Table IV

| Solver       | Train | ER set | BA set | Tapt. |
|--------------|-------|--------|--------|-------|
| CGS, LGS [10] | -     | 0.897  | 0.858  | 0.921 |
| MP [Paschalisidis15] | - | 0.907 | 0.892 | - |
| GCN(20)-CRTS | ER    | 0.989  | 0.993  | -     |
| GCN(5)-CGS   | ER    | 0.976  | 0.972  | 0.960 |
| GCN(1)-CRS-v | ER    | 0.978  | 0.979  | 0.995 |
| GCN(1)-CRS-e | ER    | 0.985  | 0.986  | 0.996 |
| Random-CRS   | -     | 0.952  | 0.943  | -     |
| MLP(5)-CRS   | ER    | 0.970  | 0.969  | -     |
| GCN(1)-LGS   | ER    | 0.932  | 0.937  | 0.954 |
| GCN(1)-LGS-it | ER   | 0.936  | 0.942  | 0.956 |
| Random-LGS   | -     | 0.873  | 0.837  | -     |
| MLP(5)-LGS   | ER    | 0.917  | 0.902  | -     |

Note that the same trained GCN model of GCN(1) is used by solvers of GCN(1)-CRS-v, GCN(1)-CRS-e, GCN(1)-LGS, GCN(1)-LGS-it, and GCN(1)-LGS-N for N = 3 and N = 4. The relative performance of all the tested solvers decreases on larger and denser graphs. Intuitively, the size of the MWIS decreases as the graph becomes denser (the average degree increases), hence a wrong selection of a vertex incurs a higher cost in the AR. Also, the MWIS problem is harder on larger graphs due to the larger search space.

Among the centralized solvers, the GCN-based centralized solvers outperform the vanilla CGS on average AR by a gap of 7.7% to 9.2%. The GCN(20)-CRTS achieves the top average AR of 0.989, but is also most sensitive to the graph size (see Fig. 6(b)), since the larger search space lowers its chance of finding a good solution before timeout. Compared to GCN(20)-CRTS, our proposed GCN(1)-CRS-v reduces its runtime by at least two orders of magnitude (i.e., from 5 minutes to up to a few seconds), at the cost of 1.2% average AR, and is less sensitive to graph size. It demonstrates that rollout search can reach a good solution quickly with the same trained model. Note that the performance of rollout search is largely determined by its guiding heuristic. With the guiding heuristic of enhanced CGS described in Section IV-B, the AR of GCN(1)-CRS-e can be boosted by an average of 0.8% from that of GCN(1)-CRS-v. On the ER test set, the GCN(5)-CGS performance is similar to that of the GCN(1)-CRS-v with slight advantage on denser and larger graphs. It shows that GCN can match the vanilla CGS in estimating the Q-values when the test set matches the training set. Moreover, GCN(5)-CGS further reduces the runtime by an order of magnitude on those worst-case instances (i.e., from seconds to sub-seconds) and has the lowest sensitivity to graph size. In comparison, the runtime of the highly optimized exact Gurobi solver on the worst-case instances is 4 hours.

Our proposed GCN-guided distributed solvers, GCN(1)-LGS and GCN(1)-LGS-it, also outperform the baselines of LGS and MP. Prepending a 1-layer GCN improves LGS by 3.5% on average AR, and this difference is more conspicuous (close to 5%) in the more challenging case of denser graphs. Although MP can find the optimal solution on some sparsely connected graphs (i.e., small average degree), it becomes virtually the same as LGS when the average degree increases to the range of typical conflict graphs in practice (i.e., average degree of 10 to 20). The GCN(1)-LGS-it outperforms GCN(1)-LGS by 0.4% on average, and its advantage is mostly on larger and denser graphs, as feeding the residual graph to GCN(1) improves the consistency. On the other hand, the LGS-3, LGS-4, GCN(1)-LGS-3, and GCN(1)-LGS-4 achieve average ARs of 0.883, 0.896, 0.923, and 0.931, respectively. From the curves of these truncated solvers in Fig. 6, we can find that improperly truncating the LGS can severely degrade the performance on larger and denser graphs, on which LGS requires more iterations to complete. As shown in [1], increasing the number of layers L from 1 to 3 does not yield better performance. Our GCN-based solvers performs notably better than their counterparts based on MLP(5) and random values, where MLP(5)-LGS outperforms vanilla LGS while Random-LGS does not. It shows that our training method allows both GCN and MLP to learn meaningful representations, and GCN is more representative than MLP.

2) Generalizability: Next, we examine how well the GCN-based MWIS solvers generalize across graph models. 

For deployment, these presented runtimes can be further optimized.
For a given set of hyperparameters, two versions of GCN-based solvers are trained on two different training sets generated from ER and BA models, respectively, with other configurations identical to the default setting. Each version of this solver is then tested on the ER test set of the previous experiment and a BA test set with identical configuration. The ARs of the proposed and baseline solvers on these two test sets are illustrated as mean values in Table IV, and as box plots in Fig. 7(a), where means are marked by green triangles.

Given the heavy-tailed degree distribution of BA graphs [77], the topology-agnostic solvers of LGS and MP experience drops of 3.9% and 1.5% in AR, respectively. In contrast, GCN-based MWIS solvers, regardless of training set, achieve relatively consistent performance on both ER and BA test sets, e.g. an increase of 0.1% to 0.7% in AR on BA graphs, and outperform LGS and MP by a greater margin on BA graphs, as shown in Table IV and Fig. 7(a), which underscores the value of taking topology into account.

In general, GCN-based MWIS solvers tend to perform slightly better if they are trained on the same graph model of the test set, showing good transferability across graph models. To understand the impact of $L$, GCNs of 1 layer and 20 layers are evaluated for GCN-LGS. Deeper GCNs do not significantly improve the mean ARs, but tend to present smaller variance compared to shallower GCNs. Moreover, deep GCNs are more tuned to the training graph model. For example, GCN(20)-LGS attains the best performance when trained and tested in BA, but underperforms compared to GCN(1)-LGS when trained on one graph model and tested on the other. A similar pattern can be found in GCN-based centralized solvers, the solver trained and tested on the same graph model has a performance 0.5% greater than when trained and tested on different graph models. Results in Section VII-B further shows good transferability across per-link utility distributions.

3) Topology Mismatch: Lastly, we measure the robustness of the reusable topological node embedding $z(G)$ (introduced in Section V-B) against topology changes in wireless networks, which can be attributed to mobility and shadowing. The reusing strategy, denoted as GCN(1:reuse)-LGS, applies the topological embedding $z^{(0)}$ generated from a baseline graph $G^{(0)}$ to 100 instances of similar graphs $G^{(i)}$, $i \in \{1, \ldots, 100\}$ generated by replacing each edge in $G^{(0)}$ with a random new edge with a given probability denoted as the level of edge perturbation (e.g., the x-axis of Fig. 7(b)), and each $G^{(i)}$ is associated with a new realization of random utility $u^{(i)} \sim \mathbb{U}(0, 1)$. The tested baseline graphs are 500 random graphs generated by ER or BA models with $V \in \{100, 150\}$ and $m = \bar{d} \in \{2, 5, 10, 15, 20\}$. The ARs of GCN(1:reuse)-LGS versus the normalized edit distance are illustrated in Fig. 7(b), along with the LGS and the GCN(1)-LGS using $z^{(1)}$ for $G^{(i)}$, as the control group. On average, the reusable topological embedding $z^{(0)}$ can stand up to 30% of edge perturbation on ER graphs and 50% on BA graphs before its gain over LGS diminishes. Note that the control group is also slightly influenced by the topological change, since the underlying degree distribution of the similar graphs generated by our method will shift towards the ER model. Since the topology change of a wireless network is usually several orders of magnitude slower than the channel fading that defines the coherence time slot, this result shows that the reusing strategy detailed in Section V-B allows GCN-LGS to be implemented at the same local complexity of vanilla LGS.

B. GCN-Based Throughput-Optimal Scheduling

To understand how the performance of the proposed solvers on synthetic random graphs could be transferred to scheduling, throughput-optimal scheduling in wireless ad-hoc networks is simulated. These networks consist of 100 users randomly located in a square of area 250. A link is established if the distance between two users is smaller than 1, and two links interfere with each other if they have incident users within distance of 4. A 1-hop flow with random direction is created on each link. The exogenous arriving packets at each source node follow a Poisson arrival with a prescribed arrival rate $\lambda$. The link rate $r(v)$, defined as the number of packets that can be transmitted through link $v$ in a time slot, is drawn

![Boxplot of approximation ratios of GCN-based MWIS solvers trained on ER and BA models and tested on both settings.](image-url)

![Approximation ratios of reusing topological embeddings under topology mismatches.](image-url)
in which the traffic load varies from a single-radio wireless network with three sub-channels, throughout is identical to the average utility a scheduler $GCN(1)$-CRS-e and $GCN(1)$-CRS-v are near optimal.

of LGS by $42\%$ improvements over LGS by GCN-based schedulers. In over-scheduler are listed in Table IV (Thpt.), and their corresponding achieved by the tested schedulers w.r.t. the optimal scheduler $u(v) = q(v)r(v)$. 

in single channel scheduling, the size and density of the 3-channel conflict graph are both tripled, which would substantially increase the suboptimality gap of the heuristics, as shown in Fig. 6. Note that higher traffic load reduces the coefficient of variation (relative standard deviation) of per-link utility across the network, further enlarging the suboptimality gap. The normalized latencies of the tested heuristic schedulers w.r.t. the optimal scheduler, measured by normalized median backlog (smaller is better) by traffic load, are presented in Fig. 8(b). The overall ranking of the 6 tested heuristics on median backlog in 3-channel scheduling is the same as that on throughput in single channel scheduling. In the stable region, the median backlog of LGS decreases from 4.9 times of the optimal scheduler under lightweight traffic, to 1.95 times under near-saturation traffic. Our distributed schedulers, $GCN(1)$-LGS and $GCN(1)$-LGS-it, can respectively close the suboptimality gap of LGS by $22\%$ to $10\%$ and $54\%$ to $32\%$ between light-weight and near-saturation traffics. Our centralized schedulers, $GCN(1)$-CRS-v and $GCN(1)$-CRS-e, only increase the median backlog of the optimal scheduler by a maximum of $50\%$ and $60\%$, respectively. In addition, multi-channel schedulers based on solving a sequence of MWIS problems on sub-channels, denoted by dashed curves and postfix ‘(seq)’ in the legends in Fig. 8(b), outperform their counterparts on normalized latency only under light traffic loads, e.g., $\mu = 0.1, 0.2$.

In both single-channel and 3-channel scheduling, centralized scheduler $GCN(5)$-CGS performs only slightly better than $GCN(1)$-LGS-it, in contrast with its good performance on synthetic random graphs. This result shows that our GCN-based rollout search can generalize well to unseen graph size, graph density, and weight distribution, which is crucial for scalable scheduling in wireless multihop networks, whereas a naive integration of GCN and deep Q learning could not.

C. Concluding Remarks on Numerical Experiments

The numerical results show that a minimal GCN ($GCN(1)$) with only 2 trainable parameters [e.g., $\Theta_0^1, \Theta_1^0 \in \mathbb{R}$ in (4)], when integrated into various algorithmic frameworks, can
substantially close the suboptimality gap of the baseline greedy heuristics for the MWIS problem. The enhancement in an efficient heuristic can be transferred to improved efficiency or performance of more sophisticated exact and approximate solvers, which usually employ efficient heuristics as intermediate steps or to obtain warm-start solutions.

In practice, our centralized solvers require efficient implementations that leverage the sparsity of the graph to cope with the computational complexity listed in Table II. The distributed solver GCN(1)-LGS can be implemented by increasing the size of control message without additional local exchanges. Lastly, the advantage of GCN-based heuristics over the greedy baseline is more conspicuous for wireless networks that are denser, larger, and with more sub-channels.

VIII. Conclusion and Future Work

We proposed several graph-aware efficient MWIS solvers for link scheduling in wireless networks. The centralized solver uses a lightweight GCN to guide centralized rollout tree search, and can achieve near optimal performance on small- to middle-sized networks with a complexity of $O(V^2)$. By leveraging the topology-awareness and distributed nature of GCNs, the distributed solvers enhance the distributed greedy heuristic and retain its efficiency, and can achieve superior performance in larger and denser networks with a local communication complexity of $O(\log V)$. The GCN can be trained on simulated networks without exactly solving the NP-hard MWIS problem, and generalizes well across different types of graphs. Moreover, our approach is agnostic to the specific per-link utility, thus, it can be used in conjunction with many existing distributed scheduling protocols. In practice, our approach could perform poorly due to corrupted parameters of the GCN, or large estimation errors in per-link utility, e.g., link rate. Future research efforts include: 1) Incorporating state-awareness into the per-link utility by taking into account the causal relationship between network state transition and scheduling decision, 2) Considering the scheduling problem for wireless networks operating on non-orthogonal channels, and 3) Developing distributed online learning schemes to enable training in real world scenarios.

Acknowledgment

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