Multi-class Label Noise Learning via Loss Decomposition and Centroid Estimation

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20 December 2021

Abstract

In real-world scenarios, many large-scale datasets often contain inaccurate labels, i.e., noisy labels, which may confuse model training and lead to performance degradation. To overcome this issue, Label Noise Learning (LNL) has recently attracted much attention, and various methods have been proposed to design an unbiased risk estimator to the noise-free dataset to combat such label noise. Among them, a trend of works based on Loss Decomposition and Centroid Estimation (LDCE) has shown very promising performance. However, existing LNL methods based on LDCE are only designed for binary classification, and they are not directly extendable to multi-class situations. In this paper, we propose a novel multi-class robust learning method for LDCE, which is termed “MC-LDCE”. Specifically, we decompose the commonly adopted loss (e.g., mean squared loss) function into a label-dependent part and a label-independent part, in which only the former is influenced by label noise. Further, by defining a new form of data centroid, we transform the recovery problem of a label-dependent part to a centroid estimation problem. Finally, by critically examining the mathematical expectation of clean data centroid given the observed noisy set, the centroid can be estimated which helps to build an unbiased risk estimator for multi-class learning. The proposed MC-LDCE method is general and applicable to different types (i.e., linear and nonlinear) of classification models. The experimental results on five public datasets demonstrate the superiority of the proposed MC-LDCE against other representative LNL methods in tackling multi-class label noise problem. Keywords: Multi-class Classification; Label Noise; Loss Decomposition; Centroid Estimation.

1 Introduction

With the availability of large quantities of well-annotated datasets, recent learning methods such as deep learning have been proven successful in various practical tasks. However, it is very expensive to collect large-scale well-annotated data in some fields, such as medical image analysis, speech translation, natural language processing, and so on. Currently, several strategies with low cost have been developed to collect labeled data for many tasks, such as automatic web crawlers and crowdsourcing. However, these strategies inevitably introduce many incorrect labels, due to the limitation of technologies and human expertise, leading to dramatic performance degradation of learning models.
Zhang et al (2021b); Wang et al (2021). Therefore, developing effective Label Noise Learning (LNL) algorithms is highly demanded in various real-world applications.

Up to now, different LNL methods have been proposed to deal with the label noise problem Han et al (2020b) over the last few years, and they can be roughly divided into three categories. The first category is sample selection Kumar et al (2010); Jiang et al (2015); Han et al (2018) which relies on the memorization effect of neural networks to select probable correctly-labeled examples. To be specific, neural networks tend to overfit the small-loss examples which are considered as clean data in the early learning stage, then gradually overfit the large-loss examples which are likely to be contaminated by label noise. Therefore, many methods based on choosing small-loss examples are proposed to improve LNL performance. The second category is label correction Brodley and Friedl (1999); Reed et al (2014); Tanaka et al (2018) which attempts to identify and correct the potentially incorrect labels through joint optimization of label purification and network weights. The third category is loss correction Natarajan et al (2013); Ghosh et al (2015); Van Rooyen et al (2015) which modifies the loss functions to be further minimized and makes them robust to noisy labels.

Among the above three categories of methods, loss correction has shown very promising performance due to a solid mathematical foundation, and a popular way to convert the conventional losses to the robust ones is based on loss decomposition and centroid estimation (LDCE), such as Labeled Instance Centroid Smoothing (LICS) Gao et al (2016), µSGD Patrini et al (2016), and Centroid Estimation with Guaranteed Efficiency (CEGE) Gong et al (2020). These methods aim to estimate the real data centroid under a clean set using the observed noisy data so that a noise-robust loss function can be obtained. However, the above works can only deal with binary classification tasks and can hardly be applied to multi-class cases. The reasons are two-fold. First, they only focus on decomposing the binary classification loss such as hinge loss and perceptron loss. As a result, it is difficult to apply these models to multi-class classification tasks. Second, they need to use the facts that \(y^2 = 1\) for the label \(y = +1\) or \(-1\) and the positive label \(+1\) and negative label \(-1\) only differ in the sign. Unfortunately, these facts do not hold under multi-class cases anymore. Therefore, developing LNL methods based on LDCE for the multi-class classification task is highly demanded.

To this end, we propose a new Multi-Class LNL method via loss decomposition and Centroid Estimation (termed MC-LDCE) to deal with LNL problems. Specifically, we propose to decompose the multi-class classification loss (e.g., mean squared loss) into a label-independent part and a label-dependent part, so that the multi-class label noise only affects the second part. Then by defining a new form of data centroid, we observe that the label-dependent part is strongly related to such centroid which critically governs the model robustness. Further, by investigating the mathematical expectation of centroid under the noisy dataset as well as introducing an elementary row transformation matrix, such data centroid can be estimated which leads to an unbiased risk estimator to the noise-free case for multi-class learning. Besides, our MC-LDCE is quite general and independent of the classification models, which does not need auxiliary clean data unlike some existing methods such as Ren et al (2018) and Veit et al (2017). The experimental results on typical benchmarks and real-world noisy datasets show that MC-LDCE outperforms the representative LNL methods under different types of multi-class label noise.

2 Related Work

LNL is an important branch of weakly-supervised learning Gong et al (2021); Zhang et al (2019); Gong et al (2019); Zhang et al (2021a) which has attracted intensive research over the past decades. We review three major types of existing LNL methods, including sample selection, label correction, and loss correction.

Sample selection. The methods of this type try to select the correctly-labeled examples according to different criteria. For example, Jiang et al. Jiang et al (2018) proposed MentorNet to teach another student network to select the examples with probably correct labels during training. However, such a selection method cannot overcome the inferiority of accumulated error caused by sample-selection bias. To overcome such drawbacks, Han et al. Han et al (2018) proposed Co-teaching to train two networks simultaneously and update itself with the data selected by its peer network. As for Co-teaching+ Yu et al (2019), it improves Co-teaching by only selecting the small-loss instances with different predictions from two networks. To further explore the information inherited from data, Wei et al. Wei et al (2020) proposed to use a joint loss to select small-loss examples, so that more data with the consensus of two networks can be selected.

Label correction. Label correction is a quite intuitive solution that identifies the possible incorrectly-labeled data and then corrects their labels for reliable training Brodley and Friedl (1999). However, such clean data identification and correction can be imprecise. Therefore, Samel et al. Samel and Miao (2018) presented a new active deep denoising
We define \( l : \mathbb{R} \times \mathcal{Y} \rightarrow \mathbb{R} \) as the loss function that penalizes the difference between the model output \( h(x) \) and the ground-truth label \( y \) under traditional supervised learning. Thus, the empirical risk on a clean set \( S \) can be formulated as

\[
\hat{R}(h, S) = \frac{1}{n} \sum_{i=1}^{n} \ell(h(x_i), y_i),
\]

(4.1)
where \( h \) is the shorthand for \( h(x) \) in this paper if no confusion is incurred. Similar to Eq. (4.1), we can define the empirical risk on corrupted data set \( \tilde{S} \) as Eq. (4.2).

\[
\tilde{R}(h, \tilde{S}) = \frac{1}{n} \sum_{i=1}^{n} \ell(h(x_i), \tilde{y}_i). 
\]  

(4.2)

Because of the unavailability of the ground-truth \( \{y_i\}_{i=1}^{n} \), the \( \tilde{R}(h, \tilde{S}) \) can be deviated from the real \( \tilde{R}(h, S) \). It is expected to find an unbiased estimator \( \tilde{R}(h, \tilde{S}) \) for \( \tilde{R}(h, S) \) on the noisy set \( \tilde{S} \), so that the negative impact caused by the noisy label can be eliminated.

As mentioned earlier, our method is based on LDCE to deal with multi-class LNL, while previous works Patrini et al (2016); Gong et al (2020) based on LDCE are designed to solve the binary LNL. By decomposing the mean squared loss and expressing the decision function as \( h(x; W) = \langle W, x \rangle \). Eq. (4.1) can be reformulated as:

\[
\tilde{R}(h, S) 
= \frac{1}{n} \sum_{i=1}^{n} \|y_i - W^\top x_i\|_2^2 
= \frac{1}{n} \sum_{i=1}^{n} (y_i^\top y_i - 2y_i^\top W^\top x_i + x_i^\top WW^\top x_i) 
= \frac{1}{n} \sum_{i=1}^{n} (y_i^\top y_i + x_i^\top WW^\top x_i) - 2 \frac{1}{n} \sum_{i=1}^{n} y_i^\top W^\top x_i.
\]  

(4.3)

It should be noted that when the label vector \( y_i \) follows the form of one-hot encoding, \( y_i^\top y_i = 1 \) always holds. Besides, from the knowledge of linear algebra, the following equation holds, which is

\[
y_i^\top W^\top x_i = \text{trace}(y_i^\top W^\top x_i) = \text{trace}(W^\top x_i y_i^\top).
\]  

(4.4)

Therefore, according to Eq. (4.4), Eq. (4.3) can be derived as

\[
\tilde{R}(h, S) 
= \frac{1}{n} \sum_{i=1}^{n} (1 + x_i^\top WW^\top x_i) - 2 \frac{1}{n} \sum_{i=1}^{n} \text{trace}(W^\top x_i y_i^\top) 
= (1 + \frac{1}{n} \sum_{i=1}^{n} x_i^\top WW^\top x_i) - 2 \frac{1}{n} \text{trace}(W^\top \sum_{i=1}^{n} x_i y_i^\top) 
= (1 + \frac{1}{n} \sum_{i=1}^{n} x_i^\top WW^\top x_i) - 2 \text{trace}(W^\top \tilde{\mu}(S)),
\]  

where the empirical centroid \( \tilde{\mu}(S) \) of the clean data set \( S \) is defined as

\[
\tilde{\mu}(S) = \frac{1}{n} \sum_{i=1}^{n} x_i y_i^\top.
\]  

(4.6)

Note that the dataset centroid \( \tilde{\mu}(S) \) defined here is different from that defined in previous works Patrini et al (2016); Gong et al (2020) which target binary classification, and this is essential for our method to handling multi-class classification. Corresponding to Eq (4.6), the mathematical expectation of the centroid on the entire distribution \( \mathcal{D} \) is defined as

\[
\mu(D) = \mathbb{E}_{(X,Y) \sim \mathcal{D}}[XY^\top],
\]  

(4.7)
Algorithm 1 The overall algorithm of MC-LDCE.

1: Input: Noisy training dataset \( \tilde{S} = \{(x_i, \tilde{y}_i)\}_{i=1}^n \);
2: Estimate the transition matrix \( T \) via VolMinNet Li et al (2021);
3: Compute all class priors \( \pi_1, \cdots, \pi_c \) via Eq. (4.14);
4: Compute \( M \) via Eq. (4.11);
5: Compute the estimated centroid of \( S \) via Eq. (4.12);
6: Compute the unbiased risk estimator \( \tilde{R}(h, \tilde{S}) \) via Eq. (4.13);
7: Use any off-the-shelf solver to optimize the model (e.g., linear model or CNN) by employing the \( \tilde{R}(h, \tilde{S}) \) as the loss function;
8: Output: Optimal parameters \( W \).

4.2 Centroid Estimation

We estimate \( \hat{\mu}(S) \) of the clean set \( S \) via the centroid \( \hat{\mu}(\tilde{S}) \) of the noisy set \( \tilde{S} \). To this end, we investigate the mathematical expectation of the centroid on the noisy set, which can be formulated by

\[
E_{\tilde{Y}}[X\tilde{Y}^\top|(X,Y)] = \sum_{i=1}^c \pi_i E_{\tilde{Y}}[X\tilde{Y}^\top|(X,Y = e_i)],
\]

where \( e_i \) represents a one-hot vector of which only the \( i \)-th element is 1, so \( Y = e_i \) denotes that the example belongs to the \( i \)-th class. Besides, \( \pi_i = P(Y = e_i) \) stands for the prior probability of the \( i \)-th class. To compute the value of Eq. (4.8), we need the following definition:

Definition 1 Suppose there are two one-hot label vectors, where \( y_i \) has the \( i \)-th element being 1, and \( y_j \) has the \( j \)-th element being 1 where \( i \neq j \). Then the two vectors can be converted using an imputation matrix \( K_{i\rightarrow j} \), which is

\[
y_j = K_{i\rightarrow j}y_i,
\]

where \( K_{i\rightarrow j} \) can be obtained by swapping the \( i \)-th row and the \( j \)-th row of an identity matrix \( I \).

Therefore, for the \( j \)-th class, we introduce the imputation matrix \( K_{i\rightarrow j} \) defined in Definition 1, and then have

\[
E_{\tilde{Y}}[X\tilde{Y}^\top|(X = e_i)] = \sum_{j=1}^c T_{ij}XY^\top K_{i\rightarrow j}^\top,
\]

where \( T \) is the noise transition matrix defined in Section 3. The estimation for this matrix can be completed by employing some off-the-shelf tools such as the method in Liu and Tao (2015); Xia et al (2019). In this work, we use the state-of-art method VolMinNet Li et al (2021) to estimate \( T \).

Based on Eq. (4.10), Eq. (4.8) can be further derived as

\[
E_{\tilde{Y}}[X\tilde{Y}^\top|(X,Y)] = \sum_{i=1}^c \pi_i \sum_{j=1}^c T_{ij}XY^\top K_{i\rightarrow j}^\top
\]

\[
= XY^\top \left[ \sum_{i=1}^c \pi_i \sum_{j=1}^c T_{ij} K_{i\rightarrow j}^\top M \right].
\]

Here we denote \( M = \sum_{i=1}^c \pi_i \sum_{j=1}^c T_{ij} K_{i\rightarrow j}^\top \). Thus, the unbiased estimator \( \tilde{\mu}(S) \) can be formulated as

\[
\tilde{\mu}(S) = \hat{\mu}(\tilde{S})M^\dagger,
\]

where the \( M^\dagger \) stands for the pseudo inverse matrix of \( M \).
Table 1: The characteristic of CIFAR-10, MNIST, FASHION-MNIST, SVHN and Animal-10N.

| Dataset       | # train  | # test  | size       |
|---------------|----------|---------|------------|
| MNIST         | 60,000   | 10,000  | 28x28      |
| FASHION-MNIST | 60,000   | 10,000  | 28x28      |
| CIFAR-10      | 50,000   | 10,000  | 32x32x3    |
| SVHN          | 73,257   | 26,032  | 32x32x3    |
| Animal-10N    | 50,000   | 5,000   | 64x64x3    |

Finally, the unbiased risk estimator $\tilde{R}(h, \tilde{S})$ to $\hat{R}(h, S)$ under noisy set $\tilde{S}$ can be obtained by substituting Eq. (4.12) to Eq. (4.5), which can be represented as

$$\tilde{R}(h, \tilde{S}) = 1 + \frac{1}{n} \sum_{i=1}^{n} x_i^\top W W^\top x_i - 2\text{trace}(W \hat{\mu}(\tilde{S}) M^\top).$$

(4.13)

### 4.3 Class Prior Estimation

Note that in Eq. (4.11), it needs to obtain the class prior $\pi_1, \ldots, \pi_c$, thus we present how to estimate them based on the noise transition matrix $T$ in this subsection, and we will describe the corruption process from clean labels to noisy labels. The element $T_{ij} = P(\tilde{Y} = e_j | Y = e_i)$ in the matrix represents the label flip rate from the $i$-th class to the $j$-th class as defined before. Obviously, $\sum_{j=1}^{c} T_{ij} = 1$. The class priors are defined as $\pi_1 = P(Y = e_1), \pi_2 = P(Y = e_2), \ldots, \pi_c = P(Y = e_c)$ which can be easily obtained by solving the following equation

$$
\begin{align*}
P(\tilde{Y} = e_1) &= T_{11}\pi_1 + T_{21}\pi_2 + \cdots + T_{c1}\pi_c \\
&\vdots \\
P(\tilde{Y} = e_i) &= T_{1i}\pi_1 + T_{2i}\pi_2 + \cdots + T_{ci}\pi_c , \\
&\vdots \\
P(\tilde{Y} = e_c) &= T_{1c}\pi_1 + T_{2c}\pi_2 + \cdots + T_{cc}\pi_c ,
\end{align*}
$$

(4.14)

where the left-hand side of every equation can be directly estimated based on the noisy $\tilde{S}$.

### 4.4 Summary of the Proposed Method

From Subsection 4.1 to Subsection 4.3, we see that the proposed MC-LDCE approach decomposes the multi-class classification loss (e.g., mean squared loss) into a label-independent part and a label-dependent part, and then directly estimates the label-dependent part via centroid estimation, which makes it can solve the multi-class LNL problem. It is worth noting that MC-LDCE is a simple yet effective LNL algorithm, which is flexible and compatible with different types of classification models $h(x)$ (e.g., deep nonlinear models and linear models). The detailed steps of our method are summarized in Algorithm 1.

### 5 Experiments

In this section, we first provide the experimental settings, including the characteristics of datasets, selected backbone, and some implementation details. Then, we present the experimental results on both simulated and real-world noisy datasets with deep classification models. Further, we validate the robustness of our MC-LDCE when a linear classification model is deployed.
Table 2: Network architectures of the adopted six-layer CNN and two-layer MLP.

|                      | six-layers CNN | two-layers MLP |
|----------------------|---------------|---------------|
| 3 × 3 conv, 128 LReLU| 2 × 2 max-pool, stride 2, dropout, p = 0.25 |
| 3 × 3 conv, 128 LReLU| dense 784 → 256 |
| 3 × 3 conv, 128 LReLU| 256 LReLU |

Table 3: Average test accuracy and the corresponding standard deviation on CIFAR-10, MNIST, FASHION-MNIST, and SVHN over the last ten epochs. The best results are marked in bold.

| Dataset      | Noise Type and Rate | GCE Zhang and Sabuncu (2018) | Co-teaching+Yu et al (2019) | JoCoR Wei et al (2020) | SIGU Han et al (2020) |
|--------------|---------------------|-------------------------------|-----------------------------|------------------------|------------------------|
| CIFAR-10     | Symmetry-20%        | 80.73 ± 0.04                 | 78.75 ± 0.04                | 84.85 ± 0.03           | 84.55 ± 0.03           |
|              | Symmetry-60%        | 57.40 ± 0.08                 | 48.78 ± 0.37                | 69.07 ± 0.07           | 69.10 ± 0.07           |
|              | Pairflip-20%        | 79.03 ± 0.03                 | 74.99 ± 0.08                | 83.63 ± 0.06           | 83.46 ± 0.06           |
|              | Pairflip-40%        | 60.01 ± 0.05                 | 51.73 ± 0.07                | 62.95 ± 0.09           | 62.80 ± 0.09           |
| MNIST        | Symmetry-20%        | 95.88 ± 0.01                 | 96.79 ± 0.03                | 96.32 ± 0.03           | 96.00 ± 0.03           |
|              | Symmetry-60%        | 93.95 ± 0.01                 | 93.83 ± 0.07                | 94.10 ± 0.06           | 93.75 ± 0.06           |
|              | Pairflip-20%        | 95.93 ± 0.01                 | 97.08 ± 0.04                | 95.27 ± 0.02           | 95.10 ± 0.02           |
|              | Pairflip-40%        | 95.16 ± 0.01                 | 91.57 ± 0.09                | 95.52 ± 0.07           | 95.20 ± 0.07           |
| FASHION-MNIST | Symmetry-20%        | 86.22 ± 0.01                 | 87.48 ± 0.05                | 87.42 ± 0.04           | 87.00 ± 0.04           |
|              | Symmetry-60%        | 84.27 ± 0.01                 | 76.64 ± 0.03                | 83.92 ± 0.08           | 83.60 ± 0.08           |
|              | Pairflip-20%        | 86.38 ± 0.02                 | 86.89 ± 0.09                | 87.62 ± 0.03           | 87.20 ± 0.03           |
|              | Pairflip-40%        | 85.18 ± 0.02                 | 69.10 ± 0.06                | 82.90 ± 0.06           | 82.60 ± 0.06           |
| SVHN         | Symmetry-20%        | 81.29 ± 0.01                 | 93.02 ± 0.05                | 78.40 ± 0.02           | 78.10 ± 0.02           |
|              | Symmetry-60%        | 56.24 ± 0.01                 | 72.13 ± 0.27                | 36.49 ± 0.05           | 36.10 ± 0.05           |
|              | Pairflip-20%        | 92.90 ± 0.01                 | 92.55 ± 0.03                | 77.15 ± 0.02           | 77.00 ± 0.02           |
|              | Pairflip-40%        | 83.95 ± 0.01                 | 72.49 ± 0.09                | 54.96 ± 0.03           | 54.60 ± 0.03           |

5.1 Experiments with Deep Classification Models

In this part, we equip our MC-LDCE with deep classification models and compare it with several representative deep robust methods.

5.1.1 Basic Setup

Simulated NoisyDatasets. We verify the effectiveness of our approach on four manually corrupted datasets (i.e., CIFAR-10, MNIST, FASHION-MNIST, and SVHN). The statistics of the used datasets are summarized in Table 1. Specifically, FASHION-MNIST and MNIST consist of 60,000 images for training and 10,000 images for testing, with the number of classes and the scale of each image being 10 and 28 × 28, respectively. While CIFAR-10 and SVHN are also considered 10-class datasets, with the the scale of each image being 32 × 32 × 3. CIFAR-10 contains 50,000 training images and 10,000 test images. SVHN contains 73,257 training images and 26,032 test images. Note that all the original datasets are clean. Following the common setting in Han et al (2018); Yu et al (2019); Wei et al (2020), we corrupted the training sets manually by using Sym-flipping and Pair-flipping noise transition matrices Han et al (2020b), with the noise rate being \{20\%, 60\%\} and \{20\%, 40\%\}, respectively. To be specific, the Sym-flipping structure models the scenario where the class of clean label can uniformly flip into other classes, and the Pair-flipping structure models the situation where the class of a clean label can flip into its adjunct class instead of a far-away class.

Real-world Noisy Dataset. Animal-10N is introduced by Song et al (2019) recently, which contains five pairs of confusing animals. The images are crawled from several online search engines including Bing and Google using the predefined labels as the search keyword. All label noise on Animal-10N is introduced by human mistakes, and the
overall noise rate on the training dataset is around 8% while the test dataset is clean. This dataset contains 50,000 RGB images used for training and 5,000 RGB images for testing, and the resolution of each image is $64 \times 64$.

**Compared Baselines.** We compare our MC-LDCE with several popular robust learning methods, including:

- Co-teaching+ Yu et al (2019) trains two deep neural networks simultaneously and lets them teach each other given every mini-batch.
- JoCoR Wei et al (2020) trains two networks and utilizes co-regularization to reduce the diversity of the two networks and combat the noisy labels.
- SIGUA Han et al (2020a) adopts gradient descent on “good” data while using a learning-rate-reduced gradient ascent on “bad” data.
• Generalized Cross-Entropy (GCE) Zhang and Sabuncu (2018) changes the loss function to make the trained neural network more robust in noisy label situations.

It is worth noting that we do not compare with other unbiased loss correction methods, such as Natarajan et al (2013); Gao et al (2016); Patrini et al (2016), since they are not applicable to multi-class cases.

Network Architectures. We adopt a six-layer Convolutional Neural Network (CNN) as the backbone for CIFAR-10, SVHN and Animal-10N, and two-layer Fully Connected Neural Network (MLP) for FASHION-MNIST and MNIST, which are both widely used in the related literature Han et al (2018); Wei et al (2020); Han et al (2020a); Malach and Shalev-Shwartz (2017). The detailed network architectures are summarized in Table 2.

Implementation Details. For a fair comparison, all experiments are conducted on an RTX2080-Ti GPU. The backbone network architectures are the same for all the methods, and we implement the compared baselines using their default parameters suggested by the original papers. For our MC-LDCE, the model is trained over 200 epochs, and we adopt the Adam algorithm to optimize our model with a momentum of 0.9. The initial learning rate is set to 0.001, and will be linearly decreased after 80 epochs. The batchsize is set to 128.

5.1.2 Experimental Results

Results on CIFAR-10. Figure 1 plots the test accuracy vs. number of epochs on CIFAR-10. In the easiest Symmetry-20% case, the test accuracy of all compared methods increases steadily over the increase of epochs, which demonstrates their robustness. However, when meeting with a harder case, i.e., 60% symmetric noise, Co-teaching+ and GCE first reach a very high level and then decrease gradually, which is because of the memorization effect of neural networks. To be specific, when the training proceeds, the neural network will tend to overfit the noisy examples which will lead to a decline in test accuracy. While the accuracy of our method increases steadily and finally exceeds all the others, verifying the robustness of our MC-LDCE for extremely corrupted datasets (more than 50% data are corrupted). As for pairflip noise, we can see that our MC-LDCE outperforms the competitors with a large margin. For example, MC-LDCE exceeds the second best method with 1.83% and 6.85% in Pairflip-20% case and Pairflip-40% case, respectively. Thus, the proposed MC-LDCE is effective in dealing with both symmetric and pairflip label noise.

Results on MNIST. For MNIST, we evaluate the proposed method with synthetic label noise, i.e., symmetric label noise with the noise rate in {20%, 60%} and pairflip label noise with the noise rate in {20%, 40%}. We run five individual trials for all the compared methods under each noise level. Figure 2 (a) and (b) plot the test accuracy curves on MNIST with 20% and 60% noise rates under symmetric label noise. Figure 2 (c) shows the test accuracy curves with the noise rate of 20% under the pairflip label noise. Table 3 provides us the test accuracies and the corresponding standard deviations of all compared methods. From the results, we can see that the accuracy of our MC-LDCE increases steadily over the increase of epochs, and our method outperforms other compared baselines finally, which indicates the effectiveness of our MC-LDCE in dealing with noisy labels.

Results on FASHION-MNIST. Figure 1 shows the test accuracy vs. number of epochs on FASHION-MNIST. Similarly, the accuracy of our MC-LDCE grows steadily over the increase of epochs and outperforms the other compared baseline gradually. As shown in Table 3, our method on FASHION-MNIST consistently outperforms all the compared methods on all label noise cases, which demonstrate the superiority of the proposed method.

Results on SVHN. The comparison results on SVHN with different types of noise and different noise rates are shown in Figure 4 and Table 3. From the comparison results, it can be seen that our MC-LDCE grows stably with the epoch increasing and gradually outperforms the compared methods with a large margin, especially for JoCoR, GCE, and Co-teaching+. In addition, as shown in Table 3, our MC-LDCE can consistently achieve the best or the second best performance among all the compared methods. It is noted that the additional experimental results on four simulated noisy datasets can be found in the supplementary material.
Table 4: Average test accuracy on Animal-10N. The best results are marked in **bold**.

| Method                     | Accuracy (%) |
|----------------------------|--------------|
| GCE Zhang and Sabuncu (2018)  | 68.7 ± 0.04  |
| Co-teaching+ Yu et al (2019)   | 69.7 ± 0.11  |
| JoCoR Wei et al (2020)        | 75.7 ± 0.12  |
| SIGUA Han et al (2020a)       | 74.0 ± 0.21  |
| MC-LDCE                      | 76.6 ± 0.23  |

Table 5: Average test accuracy on CIFAR-10 using a linear classification model. The best results are marked in **bold**.

| Method                      | 20%       | 40%       | 60%       |
|-----------------------------|-----------|-----------|-----------|
| ULE Natarajan et al (2013)  | 74.8±0.050| 61.7±0.075| 41.5±0.068|
| µSGD Patrini et al (2016)   | 74.1±0.009| 72.6±0.012| 71.6±0.001|
| RP Northcutt et al (2017)   | 77.9±0.015| 64.6±0.009| 47.4±0.006|
| LNSI Wei et al (2019)       | 84.7±0.006| 83.8±0.006| 77.4±0.006|
| SCD Luo et al (2021)        | 86.5±0.007| 84.5±0.006| 77.6±0.020|
| MC-LDCE                     | 87.1±0.359| 85.1±0.441| 79.7±0.884|

Results on Animal-10N. Similar to the experimental settings on CIFAR-10\(^1\), we run five individual trials for all compared methods on Animal-10N. Note that we do not apply any data augmentation or pre-processing procedures. Table 4 shows the average test accuracies and corresponding standard deviations of all compared methods on Animal-10N, where we can see that our MC-LDCE achieves the highest classification accuracy among all comparators. Therefore, the proposed MC-LDCE is effective in handling real-world label noise.

5.2 Experimental Results with Linear Model

In this part, we equip our MC-LDCE with a linear classification model and compare it with several statistical learning-based robust methods. The compared methods include: 1) Unbiased Logistic Estimator (ULE) Natarajan et al (2013), 2) µ Stochastic Gradient Descent (µSGD) Patrini et al (2016), 3) Spectral Cluster Discovery (SCD) Luo et al (2021), 4) Rank Pruning (RP) Northcutt et al (2017), and 5) Label Noise handling via Side Information (LNSI) Wei et al (2019). Note that the first two approaches are originally designed for binary classification tasks, so we use the one-vs-all strategy to apply them to multi-class cases. Details of all the compared methods can be found in the supplementary materials. In the experiments, we evaluate the proposed method on corrupted CIFAR-10. To be specific, we randomly pick up 30,000 images from CIFAR-10 across different classes and corrupt them with different levels of symmetric noise. The classification accuracies of all the compared approaches under different noise levels are shown in Table 5. It is worth noting that the proposed MC-LDCE consistently outperforms all the competitors under various noise levels, which again demonstrates the superiority of MC-LDCE in dealing with label noise.

6 Conclusion

In this paper, we propose a novel multi-class LNL method termed MC-LDCE to deal with the label noise problem. In the proposed MC-LDCE, we decompose the multi-class classification loss (e.g., mean squared loss) into label-independent and label-dependent parts, and directly estimate the label-dependent part via centroid estimation. Our MC-LDCE is the first method based on LDCE to deal with multi-class LNL problems. Furthermore, as our MC-LDCE is independent of the classification model, we conduct intensive experiments by using deep and linear models on both simulated and real-world noisy datasets. Experimental results demonstrate that our MC-LDCE outperforms other representative LNL methods.

Acknowledgement. This research is supported by NSF of China (Nos: 61973162, 62172228), NSF of Jiangsu Province (No: BZ2021013), and the Fundamental Research Funds for the Central Universities (Nos: 30920032202, 1024 for Animal-10N, while 512 is for CIFAR-10, as the sizes of their images are different.
References

Brodley CE, Friedl MA (1999) Identifying mislabeled training data. Journal of artificial intelligence research 11:131–167

Gao W, Wang L, Zhou ZH, et al (2016) Risk minimization in the presence of label noise. In: Proceedings of the AAAI Conference on Artificial Intelligence, vol 30

Ghosh A, Manwani N, Sastry P (2015) Making risk minimization tolerant to label noise. Neurocomputing 160:93–107

Gong C, Shi H, Liu T, Zhang C, Yang J, Tao D (2019) Loss decomposition and centroid estimation for positive and unlabeled learning. IEEE transactions on pattern analysis and machine intelligence 43(3):918–932

Gong C, Yang J, You JJ, Sugiyama M (2020) Centroid estimation with guaranteed efficiency: A general framework for weakly supervised learning. IEEE Transactions on Pattern Analysis and Machine Intelligence

Gong C, Wang Q, Liu T, Han B, You JJ, Yang J, Tao D (2021) Instance-dependent positive and unlabeled learning with labeling bias estimation. IEEE Transactions on Pattern Analysis and Machine Intelligence

Han B, Yao Q, Yu X, Niu G, Xu M, Hu W, Tsang I, Sugiyama M (2018) Co-teaching: Robust training of deep neural networks with extremely noisy labels. Advances in neural information processing systems 31

Han B, Niu G, Yu X, Yao Q, Xu M, Tsang I, Sugiyama M (2020a) Sigua: Forgetting may make learning with noisy labels more robust. In: International Conference on Machine Learning, PMLR, pp 4006–4016

Han B, Yao Q, Liu T, Niu G, Tsang JW, Kwok JT, Sugiyama M (2020b) A survey of label-noise representation learning: Past, present and future. arXiv preprint arXiv:201104406

Hu M, Han H, Shan S, Chen X (2019) Weakly supervised image classification through noise regularization. In: Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition, pp 11,517–11,525

Jiang L, Meng D, Zhao Q, Shan S, Hauptmann AG (2015) Self-paced curriculum learning. In: Twenty-Ninth AAAI Conference on Artificial Intelligence

Jing L, Zhou Z, Leung T, Li LJ, Fei-Fei L (2018) MentorNet: Learning data-driven curriculum for very deep neural networks on corrupted labels. In: International Conference on Machine Learning, PMLR, pp 2304–2313

Jin J, Li Y, Chen CP (2021) Pattern classification with corrupted labeling via robust broad learning system. IEEE Transactions on Knowledge and Data Engineering

Jindal I, Nokleby M, Chen X (2016) Learning deep networks from noisy labels with dropout regularization. In: 2016 IEEE 16th International Conference on Data Mining (ICDM), IEEE, pp 967–972

Kumar M, Packer B, Koller D (2010) Self-paced learning for latent variable models. Advances in neural information processing systems 23

Li X, Liu T, Han B, Niu G, Sugiyama M (2021) Provably end-to-end label-noise learning without anchor points. In: International Conference on Machine Learning, PMLR, pp 6403–6413

Liu T, Tao D (2015) Classification with noisy labels by importance reweighting. IEEE Transactions on pattern analysis and machine intelligence 38(3):447–461

Luo Y, Han B, Gong C (2021) A bi-level formulation for label noise learning with spectral cluster discovery. In: Proceedings of the Twenty-Ninth International Conference on International Joint Conferences on Artificial Intelligence, pp 2605–2611
Malach E, Shalev-Shwartz S (2017) Decoupling “when to update” from “how to update”. Advances in Neural Information Processing Systems 30

Masnadi-Shirazi H, Vasconcelos N (2008) On the design of loss functions for classification: theory, robustness to outliers, and savageboost. Advances in neural information processing systems 21

Natarajan N, Dhillon IS, Ravikumar PK, Tewari A (2013) Learning with noisy labels. Advances in neural information processing systems 26

Northcutt CG, Wu T, Chuang IL (2017) Learning with confident examples: Rank pruning for robust classification with noisy labels. arXiv preprint arXiv:170501936

Patrini G, Nielsen F, Nock R, Carioni M (2016) Loss factorization, weakly supervised learning and label noise robustness. In: International conference on machine learning, PMLR, pp 708–717

Reed S, Lee H, Anguelov D, Szegedy C, Erhan D, Rabinovich A (2014) Training deep neural networks on noisy labels with bootstrapping. arXiv preprint arXiv:14126596

Ren M, Zeng W, Yang B, Urtasun R (2018) Learning to reweight examples for robust deep learning. In: International conference on machine learning, PMLR, pp 4334–4343

Samel K, Miao X (2018) Active deep learning to tune down the noise in labels. In: Proceedings of the 24th ACM SIGKDD International Conference on Knowledge Discovery & Data Mining, pp 685–694

Sheng VS, Zhang J, Gu B, Wu X (2017) Majority voting and pairing with multiple noisy labeling. IEEE Transactions on Knowledge and Data Engineering 31(7):1355–1368

Song H, Kim M, Lee JG (2019) Selfie: Refurbishing unclean samples for robust deep learning. In: International Conference on Machine Learning, PMLR, pp 5907–5915

Tanaka D, Ikami D, Yasaske T, Aizawa K (2018) Joint optimization framework for learning with noisy labels. In: Proceedings of the IEEE conference on computer vision and pattern recognition, pp 5552–5560

Van Rooyen B, Menon A, Williamson RC (2015) Learning with symmetric label noise: The importance of being unhinged. Advances in neural information processing systems 28

Veit A, Alldrin N, Chechik G, Krasin I, Gupta A, Belongie S (2017) Learning from noisy large-scale datasets with minimal supervision. In: Proceedings of the IEEE conference on computer vision and pattern recognition, pp 839–847

Wang Q, Han B, Liu T, Niu G, Yang J, Gong C (2021) Tackling instance-dependent label noise via a universal probabilistic model. arXiv preprint arXiv:210105467

Wei H, Feng L, Chen X, An B (2020) Combating noisy labels by agreement: A joint training method with co-regularization. In: Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition, pp 13,726–13,735

Wei Y, Gong C, Chen S, Liu T, Yang J, Tao D (2019) Harnessing side information for classification under label noise. IEEE Transactions on Neural Networks and Learning Systems 31(9):3178–3192

Xia X, Liu T, Wang N, Han B, Gong C, Niu G, Sugiyama M (2019) Are anchor points really indispensable in label-noise learning? Advances in Neural Information Processing Systems 32

Yu X, Han B, Yao J, Niu G, Tsang I, Sugiyama M (2019) How does disagreement help generalization against label corruption? In: International Conference on Machine Learning, PMLR, pp 7164–7173

Zhang C, Ren D, Liu T, Yang J, Gong C (2019) Positive and unlabeled learning with label disambiguation. In: IJCAI, pp 4250–4256

Zhang C, Gong C, Liu T, Lu X, Wang W, Yang J (2021a) Online positive and unlabeled learning. In: Proceedings of the Twenty-Ninth International Conference on International Joint Conferences on Artificial Intelligence, pp 2248–2254
Zhang C, Wang Q, Liu T, Lu X, Hong J, Han B, Gong C (2021b) Fraud detection under multi-sourced extremely noisy annotations. In: Proceedings of the 30th ACM International Conference on Information & Knowledge Management, pp 2497–2506

Zhang Z, Sabuncu M (2018) Generalized cross entropy loss for training deep neural networks with noisy labels. Advances in neural information processing systems 31