Neutron Transverse-Momentum Distributions and Polarized $^3$He within Light-Front Hamiltonian Dynamics

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Abstract The possibility to extract the quark transverse-momentum distributions in the neutron from semi-inclusive deep inelastic electron scattering off polarized $^3$He is illustrated through an impulse approximation analysis in the Bjorken limit. The generalization of the analysis at finite momentum transfers in a Poincaré covariant framework is outlined. The definition of the light-front spin-dependent spectral function of a $J=1/2$ system allows us to show that within the light-front dynamics only three of the six leading twist T-even transverse-momentum distributions are independent.

Keywords Transverse momentum distributions · Light-front dynamics · $^3$He target

1 Introduction

As is well known, most of the proton spin is carried by the quark orbital angular momentum, $L_q$, and by the gluons. Information on the quark transverse momentum distributions (TMDs) [1] and then on $L_q$ can be accessed through non forward processes, as semi-inclusive deep inelastic electron scattering (SIDIS). In Ref. [2] the possibility to extract information on the neutron TMDs from experimental measurements of the single spin asymmetries (SSAs) on $^3$He was proposed. In particular SSAs allow one to...
experimentally distinguish the Sivers and the Collins asymmetries, expressed in terms of different TMDs and fragmentation functions (ff) [1,3]. A large Sivers asymmetry was measured in $^3\text{He}(e,e'\pi)x$ [4] and a small one in $^3\text{He}(e,e'\pi)x$ [5]. This puzzle has attracted a great interest in obtaining new information on the neutron TMDs.

2 Neutron properties and a polarized $^3\text{He}$ target

A polarized $^3\text{He}$ is an ideal target to study the neutron, since at a 90% level a polarized $^3\text{He}$ is equivalent to a polarized neutron. Dynamical nuclear effects in inclusive deep inelastic electron scattering $^3\text{He}(e,e'\pi)x$ (DIS) were evaluated with a realistic spin-dependent spectral function for $^3\text{He}$, $P_{\sigma,\sigma'}(p,E)$ [6]. It was found that the formula

$$A_n \approx \frac{1}{p_n f_n} \left( A_{exp}^{\text{pp}} - 2p_p f_p A_{exp}^{\text{nn}} \right),$$

($f_p, f_n$ dilution factors)

can be safely adopted to extract the neutron information from $^3\text{He}$ and proton data and it is actually used by experimental collaborations. The nuclear effects are hidden in the proton and neutron "effective polarizations" $p_p = -0.023, p_n = 0.878$ [3].

To investigate if an analogous formula can be used to extract the SSAs in $^3\text{He}$ the process $^3\text{He}(e,e'\pi)x$ was evaluated in the Bjorken limit and in impulse approximation (IA), i.e. the final state interaction (FSI) was considered only between the two-nucleon which recoil. In IA, SSAs for $^3\text{He}$ involve convolutions of $P_{\sigma,\sigma'}(p,E)$, with TMDs and ff. Ingredients of the calculations were: i) a realistic $P_{\sigma,\sigma'}(p,E)$ for $^3\text{He}$ [7], obtained using the AV18 interaction ii) parametrizations of data for TMDs and ff, whenever available; iii) models for the unknown TMDs and ff. As shown in Fig. 1, in the Bjorken limit the extraction procedure through the formula successful in DIS works nicely for the Sivers SSA and the same was shown to occur for the Collins SSA [8].

In [3] the calculation was performed in the Bjorken limit. To study relativistic effects in the actual experimental kinematics we adopted [4] the light-front (LF) form
of Relativistic Hamiltonian Dynamics (RHD) introduced by Dirac. Indeed the RHD of an interacting system with a fixed number of on-mass-shell constituents, plus the Bakamijan-Thomas construction of the Poincaré generators allow one to generate a description of SIDIS off $^3$He which is fully Poincaré covariant.

In IA the LF hadronic tensor for the $^3$He nucleus is:

$$ W^{\mu\nu}(\xi^2, z, \tau, \mathbf{h}, S_{\text{He}}) \propto \sum_{\sigma, \sigma'} \sum_{\tau} \sum_{\epsilon_0}^{\epsilon_{\text{max}}} \int_{M_{\text{p}}^2}^{(M_{\text{p}}-M_{\text{p}})^2} dM_f^2 d\xi \int_{P_M^2}^{P_{\text{max}}} \frac{dP_{\perp}}{sin \theta} (\tau, \mathbf{q}, \mathbf{h}, \mathbf{P}) P^{\mu\nu}_{\sigma\sigma'}(\tau, \mathbf{h}, \mathbf{P}) \mathcal{P}_{\sigma\sigma'}(\mathbf{k}, \epsilon_0, S_{\text{He}}) $$

where $\mathbf{v} = \{v^0, v^1, v^2, v^3\}$, $u_{\sigma\sigma'}^{\mu\nu}(\tau, \mathbf{q}, \mathbf{h}, \mathbf{P})$ is the nucleon hadronic tensor and $\mathcal{P}_{\sigma\sigma'}(\mathbf{k}, \epsilon_0, S_{\text{He}})$ is the LF nuclear spectral function given in terms of the unitary Melosh Rotations, $D^\gamma_{\sigma}(R_M(\mathbf{k}))$, and the instant-form spectral function $S^\gamma_{\sigma_1\sigma_1}(\mathbf{k}, \epsilon_0, S_{\text{He}})$:

$$ \mathcal{P}_{\sigma\sigma'}(\mathbf{k}, \epsilon_0, S_{\text{He}}) \propto \sum_{\sigma_1, \sigma_1'} D^\gamma_{\sigma_1}(R_M(\mathbf{k})) S^\gamma_{\sigma_1\sigma_1}(\mathbf{k}, \epsilon_0, S_{\text{He}}) D^\gamma_{\sigma_1}(R_M(\mathbf{k})) S^\gamma_{\sigma_1\sigma_1}(\mathbf{k}, \epsilon_0, S_{\text{He}}) $$

Notice that $S^\gamma_{\sigma_1\sigma_1}$ is given in terms of three independent functions, $B_0, B_1, B_2$ [7].

We are now evaluating the SSAs using the LF hadronic tensor, at finite values of $Q^2$. The preliminary results are quite encouraging, since, as shown in Table 1, LF longitudinal and transverse polarizations weakly differ and we find that in the Bjorken limit the extraction procedure works well within the LF approach as it does in the non-relativistic case. Furthermore the effect of the finite integration limits in the actual JLAB kinematics [9], instead of the ones in the Bjorken limit, is small and will be even smaller in the JLAB planned experiments at 12 GeV [2].

Concerning the FSI, we plan to include the FSI between the jet produced from the hadronizing quark and the two nucleon system through a Glauber approach [10].

| $\int dE dp \frac{1}{4} Tr(\mathcal{P}_{\sigma\sigma}) S_{\lambda_1=\xi}$ | $\int dE dp \frac{1}{4} Tr(\mathcal{P}_{\sigma\sigma}) S_{\lambda_1=\bar{\xi}}$ |
|-----------------|-----------------|
| proton NR       | -0.02263        |
| proton LF       | -0.02231        | 0.87805        | 0.87248        |
| neutron NR      | 0.87805         | 0.87248        |
| neutron LF      | 0.87494         |

### 3 The $J = 1/2$ LF spectral function and the nucleon LF TMDs

The TMDs for a $J = 1/2$ system are introduced through the q-q correlator

$$ \Phi(k, P, S)_{\alpha\beta} = \int d^4z e^{ik\cdot z} \langle PS | \tilde{\psi}_q(z) \psi_{q0}(0) | PS \rangle = \frac{1}{2} \left\{ A_1 P + A_2 S_L \gamma_5 P + A_3 P \gamma_5 \mathbf{S} \right\} $$

so that the six twist-2 T-even TMDs, $A_i$, $\tilde{A}_i$ ($i = 1, 3$), can be obtained by proper traces of $\Phi(k, P, S)$. Let us consider the contribution to the correlation function from on-mass-shell fermions

$$ \phi_p(k, P, S) = \frac{(k_{\perp n} + m)}{2m} \Phi(k, P, S) \frac{(k_{\perp n} + m)}{2m} = \sum_{\sigma} \sum_{\sigma'} u_{\sigma}(\mathbf{k}, \sigma') u_{\sigma}(\mathbf{k}, \sigma') \Phi(k, P, S) u_{\sigma}(\mathbf{k}, \sigma') u_{\sigma}(\mathbf{k}, \sigma') $$

$$ \frac{(k_{\perp n} + m)}{2m} $$

$$ \phi_p(k, P, S) = \frac{(k_{\perp n} + m)}{2m} \Phi(k, P, S) \frac{(k_{\perp n} + m)}{2m} = \sum_{\sigma} \sum_{\sigma'} u_{\sigma}(\mathbf{k}, \sigma') u_{\sigma}(\mathbf{k}, \sigma') \Phi(k, P, S) u_{\sigma}(\mathbf{k}, \sigma') u_{\sigma}(\mathbf{k}, \sigma') $$

$$ \frac{(k_{\perp n} + m)}{2m} $$
and let us identify $\bar{u}_{LF} \Phi(k, P, S) u_{LF}(k, \sigma)$ with the LF nucleon spectral function, $\mathcal{P}_{\sigma}(k, \sigma; S)$. In a reference frame where $P_\perp = 0$, the following relation holds between the off-mass-shell minus component $k^-$ of the struck quark and the spectator diquark energy $\epsilon_S$:

$$k^- = \frac{M^2}{P^+} - \frac{(\epsilon_S + m) 4m + |k_\perp|^2}{P^+ - k^+}$$  \hspace{1cm} (5)

The traces of $[\gamma^+ \Phi_p(k, P, S)]$, $[\gamma^+ \gamma_5 \Phi_p(k, P, S)]$, and $[k_\perp \gamma^+ \gamma_5 \Phi_p(k, P, S)]$ can be obtained in terms of the TMD’s, $A_i$, $\tilde{A}_i$ ($i = 1, 3$), through Eq. (4). However these same traces can be also expressed through the LF spectral function, since

$$\frac{1}{2P^+} \text{Tr} \left[ \gamma^+ \Phi_p(k, P, S) \right] = \frac{k^+}{2mP^+} \text{Tr} \left[ \mathcal{P}(k, \epsilon_S, S) \right]$$ \hspace{1cm} (6)

$$\frac{1}{2P^+} \text{Tr} \left[ \gamma^+ \gamma_5 \Phi_p(k, P, S) \right] = \frac{k^+}{2mP^+} \text{Tr} \left[ \sigma_z \mathcal{P}(k, \epsilon_S, S) \right]$$ \hspace{1cm} (7)

$$\frac{1}{2P^+} \text{Tr} \left[ k_\perp \gamma^+ \gamma_5 \Phi_p(k, P, S) \right] = \frac{k^+}{2mP^+} \text{Tr} \left[ k_\perp \cdot \sigma \mathcal{P}(k, \epsilon_S, S) \right]$$ \hspace{1cm} (8)

In turn the traces $\frac{1}{2} \text{Tr}(\mathcal{P} I)$, $\frac{1}{2} \text{Tr}(\mathcal{P} \sigma_z)$, $\frac{1}{2} \text{Tr}(\mathcal{P} \sigma_i)$ ($i = x, y$) can be expressed in terms of three scalar functions, as in the $^3$He case, and known kinematical factors. Then in the LF approach with a fixed number of particles the six leading twist TMDs, $A_i$, $\tilde{A}_i$ ($i = 1, 3$), can be expressed in terms of the previous three independent scalar functions.

4 Conclusion

A realistic study of $^3\text{He}(e, e'\pi)X$ in the Bjorken limit was performed in IA. Nuclear effects in the extraction of the neutron information were found to be under control. An analysis at finite $Q^2$ with a LF spectral function is in progress in order to test the extraction procedure of the neutron information from $^3\text{He}(e, e'\pi)X$ experiments. The relations found among the six leading twist T-even TMDs from general properties within LF dynamics and a fixed number of degrees of freedom show that only three of the six T-even TMDs are independent. These relations are precisely predicted within LF dynamics, and could be experimentally checked to test the LF description of SIDIS in the valence region.

References

1. V. Barone, A. Drago, P. G. Ratcliffe, Physics Reports 359, 1 (2002)
2. G. Cates et al., E12-09-018
3. S. Scopetta, Physical Review D 75, 054005 (2007)
4. A. Airapetian et al. The HERMES Collaboration, Physical Review Letters 94, 012002 (2005)
5. V. Yu. Alexakhin et al. COMPASS Collaboration, Physical Review Letters 94, 202002 (2005)
6. C. Ciofi degli Atti, E. Pace, G. Salmè, S. Scopetta, Physical Review C 48, R968 (1993)
7. C. Ciofi degli Atti et al., Physical Review C 46, R 159 (1992); A. Kievsky et al., Physical Review C 56, 64 (1997)
8. A. Del Dotto, E. Pace, G. Salmè, S. Scopetta, Il Nuovo Cimento C 35, 101 (2012)
9. X. Qian, Physical Review Letters 107, 072003 (2011)
10. C. Ciofi degli Atti, L. Kaptari, Physical Review C 83, 044602 (2011)