SU(3) systematization of baryons

V. Guzey

Institut für Theoretische Physik II, Ruhr-Universität Bochum, D-44780 Bochum, Germany

M.V. Polyakov

Institut für Theoretische Physik II, Ruhr-Universität Bochum, D-44780 Bochum, Germany, and
Petersburg Nuclear Physics Institute, Gatchina, St. Petersburg 188300, Russia

Abstract

We review the spectrum of all baryons with the mass less than approximately 2000-2200 MeV using methods based on the approximate flavor SU(3) symmetry of the strong interaction. The application of the Gell-Mann–Okubo mass formulas and SU(3)-symmetric predictions for two-body hadronic decays allows us to successfully catalogue almost all known baryons in twenty-one SU(3) multiplets. In order to have complete multiplets, we predict the existence of several strange particles, most notably the Λ hyperon with \( J^P = 3/2^- \), the mass around 1850 MeV, the total width approximately 130 MeV, significant branching into the \( \Sigma \pi \) and \( \Sigma(1385)\pi \) states and a very small coupling to the \( N\overline{K} \) state. Assuming that the antidecuplet exists, we show how a simple scenario, in which the antidecuplet mixes with an octet, allows to understand the pattern of the antidecuplet decays and make predictions for the unmeasured decays.
Contents

1 Introduction 4

2 SU(3) classification of octets 20

2.1 Accuracy of the Gell-Mann–Okubo formula 21

2.2 Universal SU(3) coupling constants and the barrier and phase space factors 23

2.3 Multiplet 1: Ground-state octet 26

2.4 Multiplet 15: (8, 5/2^+)=(1680, 1820, 1915, 2030) 26

2.5 Multiplet 12: (8, 5/2^-)=(1675, 1830, 1775, 1950) 28

2.6 Multiplets 8 and 7: (8, 3/2^-)=(1520, 1690, 1670, 1820) and (1, 3/2^-)=Λ(1520) 31

2.7 Multiplets 9 and 6: (8, 1/2^-)=(1535, 1670, 1560, 1620-1725) and (1, 1/2^-)=Λ(1405) 34

2.8 Multiplet 3: (8, 1/2^+)=(1440, 1600, 1660, 1690) 37

2.9 Multiplet 4: (8, 1/2^+)=(1710, 1810, 1880, 1950) 39

2.10 Multiplet 17: (8, 3/2^+)=(1720, 1890, 1840, 2035) 41

2.11 Multiplet 14: (8, 1/2^-)=(1650, 1800, 1620, 1860-1915) 42

2.12 Multiplet 11: (8, 3/2^-)=(1700, 1850, 2000, 2150) 43

3 SU(3) classification of decuplets 48

3.1 Multiplet 2: (10, 3/2^+)=(1232, 1385, 1530, 1672) 48

3.2 Multiplet 20: (10, 7/2^+)=(1950, 2030, 2120, 2250) 51

3.3 Multiplet 18: (10, 5/2^+)=(1905, 2070, 2250, 2380) 51

3.4 Multiplet 16: (10, 3/2^+)=(1920, 2080, 2240, 2470) 54

3.5 Multiplet 10: (10, 1/2^-)=(1620, 1750, 1900, 2050) 55

3.6 Multiplet 13: (10, 3/2^-)=(1700, 1850, 2000, 2150) 57

3.7 Multiplet 19: (10, 1/2^+)=(1910, 2060, 2210, 2360) 57

3.8 Multiplet 5: (10, 3/2^+)=(1600, 1690, 1900, 2050) 58
1 Introduction

After some thirty years of recess, hadron spectroscopy becomes again a lively and exciting field of research in hadronic physics. The recent wave of interest in light baryon spectroscopy was initiated by the report of the discovery of the explicitly exotic baryon with strangeness $+1$, now called the $\Theta^+$ [1]. The properties of the $\Theta^+$ and the other members of the antidecuplet were predicted in the chiral quark soliton model [2]. While at the time of writing of this report, the fate of the $\Theta^+$ and $\Xi^{--}$ [3], yet another explicitly exotic member of the antidecuplet, is uncertain and is a subject of controversy, the $\Theta^+$ has nevertheless already contributed to hadron physics by popularizing it and forcing critical assessments and advances of the used experimental methods and theoretical models.

The situation with the $\Theta^+$ has revealed that many more surprises in the spectrum of light baryons could be hiding. In particular, the existence of new light and very narrow nucleon resonances was recently suggested [4]. In addition, the problem of missing resonances predicted in the constituent quark model as well as the explanation of the light mass of the Roper resonance $N^*(1440)$ and the $\Lambda(1405)$ still await their solutions, see [5] for a recent review.

In light of the expected advances in the baryon spectroscopy, it is topical to systematize the spectrum of presently known baryons according to the flavor SU(3) group. Last time this was done in 1974 [6] when many baryons were not yet known. In this work, we review the spectrum of all baryons with the mass less than approximately 2000-2200 MeV using general and almost model-independent methods based on the approximate flavor SU(3) symmetry of strong interactions. We successfully place almost all known baryons in twenty-one SU(3) multiplets and, thus, confirm the prediction [6] that the approximate SU(3) symmetry works remarkably well. In order to complete the multiplets, we predict the existence of several strange particles. They appear underlined in this review, see e.g. the Table of Contents. Among them, the most remarkable is the $\Lambda$ hyperon with $J^P = 3/2^-$, the mass around 1850 MeV, the total width $\approx 130$ MeV, significant branching into the $\Sigma\pi$ and $\Sigma(1385)\pi$ states and a very small coupling to the $N\bar{K}$ state. Our analysis gives a model-independent confirmation of the constituent quark model prediction that there should exist a new $\Lambda$ baryon with the mass between 1775 MeV and 1880 MeV, which almost decouples from the $N\bar{K}$ state [7–9].

The hypothesis of the approximate SU(3) symmetry of strong interactions put forward by Gell-Mann and Ne’eman in the early 60’s was probably the most successful and fruitful idea for the systematization of elementary particles, see a classic compilation of original papers [11]. A natural assumption that the part of the strong interaction that violates SU(3) symmetry is proportional to the mass of the strange quark\footnote{The original formulation used fictitious leptons to build the fundamental SU(3) representation. The quarks would be invented shortly after.} resulted in very suc-
cessful Gell-Mann–Okubo (GMO) mass formulas describing the mass splitting inside a given SU(3) multiplet. The approximate SU(3) symmetry has not only enabled to bring order to the spectroscopy of hadrons but has also allowed to predict new particles, which were later confirmed experimentally. The most famous prediction in the baryon sector is the last member of the ground-state decuplet, Ω(1672), see [11], whose spin and parity are still (!) not measured but rather predicted using SU(3).

Because of their simplicity and almost model-independence, the approximate SU(3) symmetry of strong interactions and the resulting systematics of strongly interacting particles have become a classic textbook subject, see [12–14]. Theoretical methods based on the approximate flavor SU(3) symmetry work very efficiently as a bookkeeping tool. For instance, a few still missing baryon resonances can be identified and their masses and partial decay widths can be predicted, as we shall demonstrate. Since the approach does not involve internal degrees of freedom of QCD and makes only very general assumptions, its applicability is mostly limited to the description of the baryon spectrum, certain gross features of strong decays and static properties of baryons. As soon as the microscopic structure of hadrons is of interest, one needs to use other (dynamical) approaches such as quark models, effective theories or lattice gauge calculations.

**Basics of flavor SU(3)**

The exact flavor SU(3) symmetry of strong interactions predicts the existence of definite representations or multiplets: singlets (1), octets (8), decuplets (10), antidecuplets (T), twenty-seven-plets (27), thirty-five-plets (35), etc., where the numbers in the bold face denote the dimension of the representation (the number of particles in the multiplet). A common feature of these multiplets is that they all have zero triality [13]. Thus, from the pure SU(3) point of view, there is nothing mysterious in the existence of the antidecuplet. Actually, exotic baryons with positive strangeness (then called Z-resonances) have been mentioned in the literature dealing with SU(3) multiplets since the late 60’s [12].

Except for the antidecuplet, all known hadrons belong to singlet, octet and decuplet representations, which naturally follows from the Clebsh-Gordan series for mesons and baryons, respectively,

\[
3 \otimes 3 = 1 \oplus 8 ,
3 \otimes 3 \otimes 3 = 1 \oplus 8 \oplus 8 \oplus 10 .
\]  

(1)

All states, which do not belong to singlets, octets or decuplets and have the baryon number \( |B| \leq 1 \), are called exotic [12,13]. For instance, the exotic antidecuplet representation can only be constructed out of at least four quarks and one antiquark

\[
3 \otimes 3 \otimes 3 \otimes \bar{3} = (3)1 \oplus (8)8 \oplus (4)10 \oplus (2)\bar{10} \oplus (3)27 \oplus 35 .
\]  

(2)
Every particle in a given SU(3) multiplet is uniquely characterized by its isospin $I$, the $z$-component of the isospin $I_3$ and hypercharge $Y$. Therefore, it is customary to represent the particle content of SU(3) multiplets in the $I_3 - Y$ axes. The octet, decuplet and antidecuplet are presented in Figs. 1, 2 and 3.

\[ \begin{array}{c}
\Sigma^- & \Sigma^0 & \Sigma^+ \\
-1/2 & 0 & 1/2 \\
\Xi^- & \Xi^0 & \Xi^+ \\
-3/2 & 0 & -1/2
\end{array} \]

Fig. 1. SU(3) octet.

Today one groups all experimentally known baryons into singlets, octets, decuplets and the antidecuplet (if it is experimentally confirmed). There is no experimental evidence for the need in other (higher) SU(3) representations such as $27$ and $35$. However, these representations are discussed in the literature in relation to the antidecuplet [15–17].

In Nature and in QCD, the flavor SU(3) symmetry is broken by non-equal masses of the up and down quarks and the strange quark. This justifies the hypothesis that the SU(3)-violating part of the Hamiltonian transforms like the eighths component of an octet SU(3) representation with zero isospin and hypercharge. This can be understood by noticing that the mass term in the Hamiltonian, $m_s ss$, where $m_s$ is the mass of the strange quark, which is responsible for the flavor SU(3) breaking, has no net strangeness and isospin and transforms under SU(3) like the direct sum of the octet and singlet representations, $3 \otimes \bar{3} = 1 \oplus 8$. Then the symmetry breaking is induced by the octet component.

For practical applications, it is useful to work in the tensor notation of deSwart [18]. In this notation, each operator has the form $T_{YI}^{\mu}$, where $\mu$ labels the SU(3) representation, $Y$ denotes the hypercharge and $I$ denotes the isospin. Therefore, the SU(3) breaking part of the Hamiltonian is proportional to $T_{00}^8$. 
This observation allows one to write down a general formula for the mass of the baryon, which belongs to the SU(3) multiplet of the dimension $\mu$ and has the hypercharge $Y$ and the isospin $I$, as a perturbative expansion in terms of powers of the SU(3)-violating.
Hamiltonian

\[ M_B^\mu = M_0^\mu + \sum_\gamma A_\gamma^\mu \left( \begin{array}{cc} \mu & 8 \\ YI & 00 \end{array} \right) \left( \begin{array}{c} \mu_\gamma \\ YI \end{array} \right). \] (3)

In this equation, we retained only the SU(3)-symmetric term \( M_0^\mu \) and the term linear in the SU(3) symmetry breaking operator – the phenomenological constants \( A_\gamma^\mu \) are proportional to the difference between the strange quark mass \( m_s \) and the mass of the \( u \) and \( d \) quarks. Since the resulting representation \( \mu \) can appear several times in the tensor product of the initial \( \mu \) and \( 8 \), the final representation contains the degeneracy label \( \gamma \). The factors in the brackets are the so-called SU(3) isoscalar factors, which are known for all SU(3) multiplets and transitions [18]. Equation (3) is the most general form of the Gell-Mann–Okubo mass formula for an arbitrary baryon multiplet \( \mu \).

The tensor product \( 8 \otimes 8 \) contains two octet representations,

\[ 8 \otimes 8 = 1 \oplus 8 \oplus 8 \oplus 10 \oplus 10 \oplus 10 \oplus 27, \] (4)

which means that, according to Eq. (3), the masses of the \( N, \Lambda, \Sigma \) and \( \Xi \) members of any octet can be expressed in terms of three constants: the overall mass of the octet \( M_0^8 \) and two constants \( A_1^8 \) and \( A_2^8 \). Therefore, evaluating the corresponding isoscalar factors and eliminating the unknown constants, one obtains the Gell-Mann–Okubo mass formula for the octet

\[ \frac{1}{2} (m_N + m_\Xi) = \frac{1}{4} (3 m_\Lambda + m_\Sigma). \] (5)

In the case of decuplets, the tensor product \( 8 \otimes 10 \) contains only one decuplet representation,

\[ 8 \otimes 10 = 8 \oplus 10 \oplus 27 \oplus 35, \] (6)

which means that the masses of the \( \Delta, \Sigma, \Xi \) and \( \Omega \) states of any decuplet can be expressed in terms of two free constants: \( M_0^{10} \) and \( A_1^{10} \). Consequently, there are two independent relations among the masses – the so-called equal spacing rule

\[ m_\Sigma - m_\Delta = m_\Xi - m_\Sigma = m_\Omega - m_\Xi. \] (7)

Along the similar lines, the equal spacing rule can be derived for the antidecuplet

\[ m_{N_{10}} - m_{\Omega^+} = m_{S_{10}} - m_{N_{10}} = m_{\Xi_{10}} - m_{S_{10}}. \] (8)
We now briefly discuss a useful concept of the $U$-spin, which makes very transparent the derivation of SU(3) predictions for the relations among the magnetic moments of a given multiplet \cite{19} as well as other SU(3) electromagnetic predictions. The operator of the electromagnetic current,

$$ J_\mu = \frac{2}{3} \bar{u} \gamma_\mu u - \frac{1}{3} \bar{d} \gamma_\mu d - \frac{1}{3} \bar{s} \gamma_\mu s , \quad (9) $$

contains the $I = 0$ and $I = 1$ components and, thus, transforms in a complicated way under isospin rotations. However, if instead of the $(I_3, Y)$-basis, one characterizes a given SU(3) multiplet by the third component of the so-called $U$-spin and the corresponding hypercharge $Y_U$, see e.g. \cite{13,20},

$$ U_3 = -\frac{1}{2} I_3 + \frac{3}{4} Y , \quad Y_U = -Q , \quad (10) $$

the operator of the electromagnetic current $J_\mu$ becomes proportional to $T_{00}^8$ since $\bar{u} \gamma_\mu u$ is the $U$-spin singlet and $\bar{d} \gamma_\mu d + \bar{s} \gamma_\mu s$ is invariant under $U$-spin rotations. Therefore, in the $(U_3, Y_U)$-basis the derivation of SU(3) relations among electromagnetic transition amplitudes proceeds as in the case of the Gell-Mann–Okubo mass formula. The decomposition of an SU(3) octet and the antidecuplet in the $(U_3, Y_U)$-basis is presented in Fig. 4. In the figure, $\Sigma^0 = -1/2 \Sigma^0 + (\sqrt{3}/2) \Lambda$ and $\bar{\Lambda} = -(\sqrt{3}/2) \Sigma^0 - (1/2) \Lambda$ \cite{20}.

Below we consider two applications of the $U$-spin: SU(3) predictions for the magnetic moments of the octet and the transition magnetic moments of the antidecuplet.

It follows from the assumption of the $U$-spin conservation that the magnetic moments (and electric charges) of all members of the same $U$-spin multiplet are equal. From the left panel of Fig. 4, one then immediately obtains that

$$ \mu_\Sigma^- = \mu_\Xi^- , \quad \mu_{\Sigma^0} = \mu_n , \quad \mu_p = \mu_{\Sigma^+} . \quad (11) $$

Additional relations are obtained from the observation that the operator of the electromagnetic current is proportional to $T_{00}^8$ in the $(U_3, Y_U)$-basis. This allows one to write the magnetic moments of an SU(3) octet in the form, which closely resembles Eq. (3) for the

\[9\]
Fig. 4. An octet and the antidecuplet in the \((U_3, Y_U)\)-basis.

octet mass splitting,

\[
\mu_B = \sum C_\gamma \begin{pmatrix} 8 & 8 \\ Y_U U & 00 \end{pmatrix} \begin{pmatrix} 8 \gamma \\ Y_U U \end{pmatrix},
\]

(12)

where \(C_\gamma = 1, 2\) are two free coefficients. A direct evaluation of the isoscalar factors in Eq. (12) and the use of the auxiliary isospin relation \(\Sigma^0 = (\Sigma^+ + \Sigma^-)/2\) allows one to derive the following relations among the eight magnetic moments of the octet and the \(\mu_{\Sigma^0\Lambda}\) matrix element for the \(\Sigma^0 \rightarrow \Lambda \gamma\) transition (we also list the relations of Eq. (11) for completeness)

\[
\mu_{\Sigma^-} = \mu_{\Xi^-} = -\mu_p - \mu_n, \\
\mu_{\Xi^0} = \mu_n, \\
\mu_\Lambda = -\mu_{\Sigma^0} = \frac{1}{2} \mu_n, \\
\mu_p = \mu_{\Sigma^+}, \\
\mu_{\Sigma^0\Lambda} = \frac{\sqrt{3}}{2} \mu_n.
\]

(13)

Turning to the antidecuplet-octet transition magnetic moments, we immediately see from Fig. 4 that, in the SU(3) limit, the transition magnetic moments involving \(\Xi^{-}, \Theta^+, N_{10}^+\)
and \( \Xi \) are exactly zero since the transition is prohibited by the \( U \)-spin conservation. The remaining non-zero transition magnetic moments can be parameterized in the following form

\[
\mu_{B_1B_2} = C_{\Pi \to \Omega} \left( \begin{array}{cc} 8 & 8 \\ Y_U & 00 \\ Y_U \end{array} \right),
\]

(14)

where \( C_{\Pi \to \Omega} \) is a free constant. Since the two relevant (for the transitions between the states with \( Y_U = 0,1 \) of the antidecuplet and the octet) isoscalar factors in Eq. (14) are equal, we have the following simple relations between the transition magnetic moments

\[
\mu_{\Sigma^{-} \Sigma^{-}} = \mu_{\Xi^{-} \Xi^{-}} = \mu_{N_{10}^{0}N_{10}^{0}} = \mu_{N_{10}^{0}N_{10}^{0}},
\]

\[
\mu_{\Sigma^{-} \Lambda} = \frac{\sqrt{3}}{2} \mu_{N_{10}^{0}N_{10}^{0}},
\]

where \( Y_U \) is a free constant. Since the two relevant (for the transitions between the states with \( Y_U = 0,1 \) of the antidecuplet and the octet) isoscalar factors in Eq. (14) are equal, we have the following simple relations between the transition magnetic moments

Effects of the SU(3) breaking in the transition magnetic moments were considered in [21].

It is important to note that the simple consequence of the \( U \)-spin conservation that \( \mu_{p_{10}} = 0 \) and \( \mu_{n_{10}} \neq 0 \) leads to the dramatic prediction that the photoexcitation of the nucleon-like member of the antidecuplet, \( N_{10} \), is suppressed on the proton target and, hence, predominantly takes place on the neutron target [22], see also the discussion in Sect. 4.

**SU(3) systematization: main objectives, methods and results**

The goal of the SU(3) systematization of baryons is to group known baryons into SU(3) singlets, octets and decuplets. Besides the requirement that all particles in the same multiplet must have the same spin and parity, one subjects the baryons to two tests. The first and the basic test is the GMO mass formulas, see Eqs. (5) and (7), which are indeed very useful for the systematization of baryons.

For the fine tuning of the multiplets, however, the GMO mass relations may be useless. First, it might happen that there are several candidates (there are several baryons with the correct or unmeasured spin-parity within a given mass range) for a member of a given multiplet. The mass formulas will work equally well for all candidates and, thus, will not be able to pick out the right one. Second, states from different multiplets with the same spin-parity can mix. The mass formulas are not particularly sensitive to the typically small mixing since mixing parameters will enter the GMO mass relation as a second order in \( m_s \) correction. Therefore, the multiplet decomposition of a given physical state cannot be determined from the GMO mass formulas. Third, if a multiplet is incomplete, i.e. some of
its members are yet unknown, the mass formulas may have no predictive power. Fourth, there is no objective criterion of the accuracy of the GMO formulas for octets.

The next test, which the group of baryons has to pass in order to be placed in the same SU(3) multiplet, is based on the SU(3) analysis of two-hadron partial decay widths [6]. The requirement is that the SU(3) predictions for the partial decay widths of the multiplet in question should describe (e.g. in the sense of the $\chi^2$ fit) the corresponding experimental values. The SU(3) prediction for the partial decay widths has the form

$$\Gamma (B_1 \to B_2 + P) = |g_{B_1 B_2 P}|^2 \cdot \text{barrier factor} \cdot \text{phase space factor},$$  \hspace{1cm} (16)

where $B_1$ is the initial baryon; $B_2$ and $P$ are the final baryon and pseudoscalar meson, which are normally stable and belong to the ground-state multiplets; $g_{B_1 B_2 P}$ are the SU(3) symmetric coupling constants; “barrier factor” takes into account spins and parity of the involved hadrons and the relative orbital moment of the final $B_2 + P$ system; ”phase space factor” is the usual kinematic phase space factor. In Eq. (16), one explicitly assumes that the only source of the SU(3) symmetry breaking is the different physical masses entering the barrier and phase space factors.

Another possibility would be to assume that the part of the strong interaction Hamiltonian responsible for the decays contains an explicit SU(3) symmetry breaking term [23]. However, in this case, the beauty, simplicity and predictive power of the whole approach will be largely lost since one will have to introduce a number of unknown parameters.

The main attraction of the assumption of the SU(3)-symmetric coupling constants $g_{B_1 B_2 P}$ is the possibility to relate all decays of a given multiplet using only a few phenomenological constants. This assumption can be tested by performing the $\chi^2$ analysis of the measured decays. Below we give two important examples, which we shall use in our numerical analysis. In this work, we adopt the notation of [6].

For the $8 \to 8 + 8$ decays ($B_1$, $B_2$ and $P$ belong to the octet representations), the $g_{B_1 B_2 P}$ coupling constants in the SU(3) symmetric limit are parameterized in terms of two constants $A_s$ and $A_a$ (the tensor product $8 \otimes 8$ contains two octets labelled $8_S$ and $8_A$)

$$g_{B_1 B_2 P} = A_s \begin{pmatrix} 8 & 8 \\ Y_2I_2 & Y_PI_P \\ Y_1I_1 \end{pmatrix} + A_a \begin{pmatrix} 8 & 8 \\ Y_2I_2 & Y_PI_P \\ Y_1I_1 \end{pmatrix}.$$  \hspace{1cm} (17)

In this equation, $Y_1,2$ and $I_1,2$ are hypercharges and isospins of the baryons $B_{1,2}$; the corresponding symbols with the subscript $P$ refer to the pseudoscalar meson. In practice, it is more convenient to perform the $\chi^2$ fit using an alternative pair of parameters $A_8$ and
\[
A_8 = \frac{\sqrt{15}}{10} A_s + \frac{\sqrt{3}}{6} A_a, \quad \alpha = \frac{\sqrt{3}}{6} A_a.
\]  
(18)

With this parameterization, \(A_8\) is directly determined by the generally well-measured \(N \to N \pi\) partial decay width and \(\alpha\) is determined by the often measured \(\Sigma \to \Sigma \pi\) partial decay width.

For the \(8 \to 10 + 8\) decays (\(B_2\) belongs to the ground-state decuplet), the corresponding coupling constants can be expressed in terms of a single universal coupling constant \(A'_8\)

\[
g_{B_1B_2P} = A'_8 \left( \begin{array}{ccc}
10 & 8 \\
Y_2I_2 & Y_P I_P & 8
\end{array} \right). 
\]  
(19)

Similarly to Eq. (19), the coupling constants for the \(10 \to 8 + 8\) and \(10 \to 10 + 8\) decays are expressed in terms of just one coupling constant, see Sect. 3.

It is well-known that states with the same spin and parity from different unitary multiplets can mix. The GMO mass relations and the SU(3) relations among the coupling constants are modified in the presence of the mixing. Let us illustrate this with the physically important example of mixing between an octet and a singlet \(\Lambda\) baryon. Introducing the mixing angle \(\theta\), the physical mostly octet state \(|\Lambda_8\rangle\) and the mostly singlet state \(|\Lambda_1\rangle\) can be written as linear superpositions of the bare \(|\Lambda_0^8\rangle\) and \(|\Lambda_0^1\rangle\) states

\[
|\Lambda_8\rangle = \cos \theta |\Lambda_0^8\rangle + \sin \theta |\Lambda_0^1\rangle, \\
|\Lambda_1\rangle = -\sin \theta |\Lambda_0^8\rangle + \cos \theta |\Lambda_0^1\rangle.
\]  
(20)

While the bare states are eigenstates of the (idealized) SU(3)-symmetric Hamiltonian, the physical states are eigenstates of the Hamiltonian with SU(3) symmetry breaking terms (mass eigenstates). The GMO mass relation for the octet becomes

\[
\frac{1}{2} (m_N + m_\Xi) = \frac{1}{4} \left( 3 m_8^0 + m_\Sigma \right) = \frac{1}{4} \left( 3 \left( m_8 \cos^2 \theta + m_1 \sin^2 \theta \right) + m_\Sigma \right),
\]  
(21)

where the mass of \(|\Lambda_0^0\rangle\) denoted as \(m_8^0\) is expressed in terms of the physical masses of the octet (\(m_8\)) and singlet (\(m_1\)) \(\Lambda\) baryons.

The predictive power of the mass relation has reduced since a new free parameter, the mixing angle \(\theta\), has been introduced. Equation (21) also illustrates that, if the mixing angle is small, it is inconsistent and also impractical to determine it from the modified
GMO mass formula because the mixing angle enters Eq. (21) as a second order correction in the mass of the strange quark, $\theta \propto O(m_s)$, which was neglected in the derivation of the GMO mass relations.

The self-consistent and practical way to establish whether the mixing takes place and to determine the value of the mixing angle(s) is to consider decays. In the context of the considered example, the physical octet and singlet coupling constants are

$$g_{\Lambda_8 B_2 P} = \cos \theta \, g_{\Lambda_8^0 B_2 P} + \sin \theta \, g_{\Lambda_8^1 B_2 P},$$

$$g_{\Lambda_1 B_2 P} = -\sin \theta \, g_{\Lambda_8^0 B_2 P} + \cos \theta \, g_{\Lambda_8^1 B_2 P},$$

where the SU(3) universal coupling constants $g_{\Lambda_8^0 B_2 P}$ are given by Eq. (17); the $1 \rightarrow 8 + 8$ coupling constants $g_{\Lambda_8^1 B_2 P}$ will be discussed later. If the introduction of the mixing improves the description of the data on decays of the $\Lambda_8$ and $\Lambda_1$ hyperons, then this serves as an unambiguous confirmation of the mixing hypothesis. This happens in the case of the mixing of the octet $\Lambda(1670)$ with the singlet $\Lambda(1405)$. Another example, when the mixing with a singlet $\Lambda$ baryon is established, is the mostly singlet $\Lambda(1520)$, which decays into the $\Sigma(1385)\pi$ final state. Since the $1 \rightarrow 10 + 8$ decay is forbidden by SU(3), the decay can take place only due to the mixing, presumably with the octet $\Lambda(1690)$.

It is important to note that due to the interference between the terms proportional to $g_{\Lambda_8^0 B_2 P}$ and $g_{\Lambda_8^1 B_2 P}$, the mixing angle enters the partial decay width of Eq. (16) in the first power and, hence, it can be determined consistently and more reliably than via the modified GMO mass relation.

To the best of our knowledge, the most recent and thorough SU(3) systematization of hadrons using the Gell-Mann–Okubo mass relations and the $\chi^2$ fit to the experimentally measured decays was performed by Samios, Goldberg and Meadows in 1974 [6]. We quote the main conclusion of that work: *The detailed study of mass relationships, decay rates, and interference phenomena shows remarkable agreement with that expected from the most simple unbroken SU(3) symmetry scheme.*

In addition to the well-known GMO mass formulas, there exist almost unknown similar relations for the total widths of particles in a given SU(3) multiplet derived by Weldon [24]. Using the perturbation theory for non-stationary states, it was shown that, except for the ground-state decuplet, the pattern of the total width splitting inside a given multiplet is the same as for the mass splitting. This means that the total widths of octet members obey the relationship

$$\frac{1}{2} (\Gamma_N + \Gamma_\Xi) = \frac{1}{4} (3 \Gamma_\Lambda + \Gamma_\Sigma).$$
For decuplets, the total widths should obey the equal spacing rule

$$\Gamma_\Sigma - \Gamma_\Delta = \Gamma_\Xi - \Gamma_\Omega = \Gamma_\Omega - \Gamma_\Xi.$$  \hspace{1cm} (24)

A similar equal-space relation holds for the antidecuplet. For the ground-state decuplet, the relation among the total widths is different [24]. For unstable particles, the total width is as intrinsic and fundamental as the mass and, therefore, it is not surprising that there is a relation among the total widths of baryons from the same multiplet.

Below we sketch the derivation of Weldon’s relations. Using Eq. (16) for the partial decay widths, one can sum over all possible decay modes of the initial baryon [25]

$$\sum_{Y_2,I_2,Y_P,I_P} \Gamma (B_1 \rightarrow B_2 + P) = \Gamma_0 + \sum_{i=1,2,P,\gamma} C_i \left[ \sum_{Y_2,I_2,Y_P,I_P,\delta} \left( \sum_{\delta} A_{\delta} \left( \begin{array}{ccc} \mu_2 & \mu_P & \mu_{1\delta} \\ Y_2 I_2 & Y_\delta I_P & Y_1 I_1 \end{array} \right) \right)^2 \left( \begin{array}{c} \mu_i \\ 8 \\ \mu_{i\gamma} \end{array} \right) \right].$$  \hspace{1cm} (25)

In this equation, \(\Gamma_0\) is the baryon width in the limit of unbroken SU(3) symmetry. The second term is proportional to the SU(3)-symmetric \(|g_{B_1,B_2P}|^2\) multiplied by the SU(3)-violating term coming from the Taylor expansion of the masses entering the barrier and phase volume factors about the central mass \(M_{0i}^2\) of the corresponding multiplets. A direct evaluation shows that, provided that all decay channels are open, Eq. (26) can be written in the following form

$$\sum_{Y_2,I_2,Y_P,I_P} \Gamma (B_1 \rightarrow B_2 + P) = \Gamma_0 + \sum_{\gamma} D_{\gamma} \left( \begin{array}{c} \mu_1 \\ 8 \\ \mu_{1\gamma} \end{array} \right) \left( \begin{array}{ccc} Y_1 I_1 & 00 \\ Y_1 I_1 \end{array} \right).$$  \hspace{1cm} (26)

A comparison to Eq. (3) demonstrates that the sums of all two-body hadronic partial decay widths satisfy the same relations as the baryon masses. Note that we have derived a more specific form of Weldon’s relations, which are valid not only for the total widths, but also for the sum of the partial decay widths into the fixed final representations. Summing over all possible final state SU(3) representations, one obtains Weldon’s relations for the total widths, Eqs. (23) and (24).

Our derivation is based on the assumption that all possible decay channels for a given set of multiplets \(\mu_1 \rightarrow \mu_2 + \mu_P\) are kinematically open. Therefore, the Weldon relations are especially useful for multiplets with heavy baryons for which enough decay modes are open.

In practice, however, Weldon’s formulas are of little use in the SU(3) systematization of baryons. The derivation of Eqs. (23) and (24) implies that all two-body decay channels of
a given baryon are open. For light baryons, some decays are kinematically prohibited and Weldon’s formulas are not expected to hold. In the opposite case when all decay channels are open, Weldon’s relations is a mere consequence of the approximate SU(3) symmetry and they do not supply any extra information, which would not be already present in the used formalism.

The list of SU(3) multiplets of baryons with the mass less than approximately 2000-2200 MeV, which one could find in the literature [12,6,26–30], is summarized in Table 1. The first column indicates SU(6)×O(3) supermultiplets; the second and third columns enumerate and indicate the type of the SU(3) representation, its spin and parity. The masses in the parenthesis are for \((N, \Lambda, \Sigma, \Xi)\) members of the octets and for \((\Delta, \Sigma, \Xi, \Omega)\) members of the decuplets. We give the modern values of the masses [10].

| \((56, L = 0)\) | 1 | \((8, \frac{1}{2}^+)\) | \((939, 1115, 1189, 1314)\) |
|---|---|---|---|
| 2 | \((10, \frac{3}{2}^+)\) | \((1232, 1385, 1530, 1672)\) |
| 3 | \((8, \frac{1}{2}^+)\) | \((1440, \ldots, \ldots)\) |
| 4 | \((8, \frac{1}{2}^+)\) | \((1710, \ldots, \ldots)\) |
| 5 | \((10, \frac{3}{2}^+)\) | \((1600, \ldots, \ldots)\) |

| \((70, L = 1)\) | 6 | \((1, \frac{1}{2}^-)\) | \(\Lambda(1405)\) |
|---|---|---|---|
| 7 | \((1, \frac{3}{2}^-)\) | \(\Lambda(1520)\) |
| 8 | \((8, \frac{3}{2}^-)\) | \((1520, 1690, 1670, 1820)\) |
| 9 | \((8, \frac{1}{2}^-)\) | \((1535, 1670, 1750, 1835)\) |
| 10 | \((10, \frac{1}{2}^-)\) | \((1620, \ldots, \ldots)\) |
| 11 | \((8, \frac{3}{2}^-)\) | \((1700, \ldots, \ldots)\) |
| 12 | \((8, \frac{5}{2}^-)\) | \((1675, 1830, 1775, 1950)\) |
| 13 | \((10, \frac{3}{2}^-)\) | \((1700, \ldots, \ldots)\) |
| 14 | \((8, \frac{5}{2}^-)\) | \((1650, \ldots, \ldots)\) |

| \((56, L = 2)\) | 15 | \((8, \frac{5}{2}^+)\) | \((1680, 1820, 1915, 2030)\) |
|---|---|---|---|
| 16 | \((10, \frac{3}{2}^+)\) | \((1920, \ldots, \ldots)\) |
| 17 | \((8, \frac{3}{2}^+)\) | \((1720, \ldots, \ldots)\) |
| 18 | \((10, \frac{5}{2}^+)\) | \((1905, \ldots, \ldots)\) |
| 19 | \((10, \frac{1}{2}^+)\) | \((1910, \ldots, \ldots)\) |
| 20 | \((10, \frac{7}{2}^+)\) | \((1950, 2030, 2120, 2250)\) |

Table 1
SU(3) multiplets known by 1974.
In order to determine the minimal number of possible SU(3) multiplets, it is useful to use as guide the SU(6) classification scheme, which combines the flavor SU(3) group with the spin SU(2) group \([12,13]\). SU(6) implies that quarks are non-relativistic and, hence, there is no reason that SU(6) is a symmetry of QCD. However, phenomenologically SU(6) works surprisingly well. Starting with three constituent quarks in the fundamental SU(6) representation, one obtains the following allowed multiplets

\[
6 \otimes 6 \otimes 6 = 20 \oplus 56 \oplus 70 \oplus 70,
\]  

(27)

where the 20 representation is totally antisymmetric, 56 is totally symmetric and 70 has mixed symmetry. It is a phenomenological observation that most likely only the 56 and 70 SU(6) representations are realized in Nature \([6]\). They have the following decomposition in terms of SU(3) representations \([12]\)

\[
\begin{align*}
56 &= (8, \frac{1}{2}) \oplus (10, \frac{3}{2}) , \\
70 &= (1, \frac{1}{2}) \oplus (8, \frac{1}{2}) \oplus (8, \frac{3}{2}) \oplus (10, \frac{1}{2}) ,
\end{align*}
\]

(28)

where the second number in the parenthesis denotes the total spin \(S\) of the multiplet.

Since parity is the quantum number beyond SU(6), in order to obtain multiplets of different parities, one couples the total spin \(S\) to the total orbital moment of the three quarks \(L\) (it corresponds to the group of spacial rotations O(3)]. The resulting baryon wave function must be symmetric [we always keep in mind the final antisymmetrization under color SU(3)]. Therefore, assuming that the radial part of the wave function is symmetric, one sees that the 56 representation admits only even \(L\), while the 70 representation accepts all \(L\). Then, coupling \(L = 0, 2\) to the total spin \(S\) of the SU(3) multiplets that belong 56 and coupling \(L = 1\) to \(S\) of the SU(3) multiplets that belong 70, see Eq. (28), one obtains the following decomposition in terms of SU(3) multiplets with different \(J^P\)

\[
\begin{align*}
(56, L = 0) &= (8, \frac{1}{2}^+) \oplus (10, \frac{3}{2}^+) , \\
(70, L = 1) &= (1, \frac{1}{2}^-) \oplus (1, \frac{3}{2}^-) \oplus 2 (8, \frac{1}{2}^-) \oplus 2 (8, \frac{3}{2}^-) \oplus (8, \frac{5}{2}^-) \\
&\quad \oplus (10, \frac{1}{2}^-) \oplus (10, \frac{3}{2}^-) , \\
(56, L = 2) &= (8, \frac{3}{2}^+) \oplus (8, \frac{5}{2}^+) \oplus (10, \frac{1}{2}^+) \oplus (10, \frac{3}{2}^+) \oplus (10, \frac{5}{2}^+) \\
&\quad \oplus (10, \frac{7}{2}^+) .
\end{align*}
\]

(29)
Therefore, the assumption that only the \((56, L = 0), (70, L = 1)\) and \((56, L = 2)\) supermultiplets of SU(6)×O(3) are possible indicates the existence of the seventeen SU(3) multiplets listed in Eq. (29). The actual number of multiplets is larger because new multiplets can be formed by radial excitations. For instance, Table 1 contains twenty SU(3) multiplets: seventeen listed in Eq. (29) and additional multiplets 3, 4 and 5, which can be thought of as radial excitations of the corresponding ground-state octet and decuplet multiplets.

We stress that without attempting to model the dynamics of the quark interaction, one can only accept as an empirical fact that Nature seems to use only the \((56, L = 0, 2)\) and \((70, L = 1)\) supermultiplets for the baryons with the mass less than approximately 2000-2200 MeV, as indicated in Table 1. Note that for heavier baryons, other, higher supermultiplets are needed. For instance, an octet and singlet with \(J^P = 7/2^-\), which do not fit to Eq. (29) and which supposedly belong to the \((70, L = 3)\) supermultiplet, were considered in [6].

The problem of missing resonances in constituent quark models partially is a consequence of the fact that in an attempt to derive the spectrum of baryons, the quark models have no reasons to prefer some SU(6)×O(3) supermultiplets the others. For example, the prediction for positive-parity baryons [31] include five low-lying SU(6)×O(3) supermultiplets: \((56, L = 0, 2), (70, L = 0, 2)\) and \((20, L = 1)\), which naturally makes the number of the predicted baryons larger than found (so far) in Nature. In contrast to the positive-parity baryons, the quark models predict the correct number of low-lying negative-parity baryons (the same as in Table 1) because they essentially employ only the \((70, L = 1)\) supermultiplet [7–9].

Now we discuss Table 1 in some detail. Multiplets 1, 2, 6-9, 12, 15 and 20 were considered in [6]. While all the multiplets are mentioned in [12], no SU(3) analysis of the decays was performed. In Table 1, we underlined genuine predictions of new particles: Kokkedee [12] predicted \(\Delta(1920)\) with \(J^P = 3/2^+\); Samios, Goldberg and Meadows [6] predicted \(\Xi(2120)\) and \(\Omega(2250)\) with \(J^P = 7/2^+\) and \(\Xi(1835)\) with \(J^P = 1/2^-\). All these particles, except for \(\Xi(1835)\), were later discovered. In addition, it was supposed in [12] that \(N(1440)\) and \(N(1710)\) with \(J^P = 1/2^+\) and \(\Delta(1600)\) with \(J^P = 3/2^+\) represent radial excitations of the corresponding ground-state multiplets and thus, belong to the \((56, L = 0)\) supermultiplet. Our analysis confirms this assumption.

The Review of Particle Physics 2004 (RPP) [10], in Sect. Quark model also gives the list of SU(3) multiplets, which we present in Table 2. Tables 1 and 2 are very similar except for the following differences. Multiplets 5, 16 and 19 are not mentioned in the RPP; octet 9 contains \(\Sigma(1620)\) instead of \(\Sigma(1750)\), which is assumed to belong to octet 14; octet 4 is assumed to belong to the \((70, L = 0)\) supermultiplet, which is not present in Table 1. Throughout this work, we shall refer to the multiplets as they are numbered in Table 1.

The aim of this review is to update the picture of SU(3) multiplets presented in Tables 1
Table 2
SU(3) multiplets from the Review of Particle Physics 2004.

| (56, $L = 0$) | 1 | $(8, \frac{1}{2}^+)$ | (939, 1116, 1193, 1318) |
| (56, $L = 0$) | 2 | $(10, \frac{3}{2}^+)$ | (1232, 1385, 1530, 1672) |
| (56, $L = 0$) | 3 | $(8, \frac{1}{2}^+)$ | (1440, 1600, 1660, ...) |
| (70, $L = 0$) | 4 | $(8, \frac{1}{2}^+)$ | (1710, 1810, 1880, ...) |
| (70, $L = 1$) | 6 | $(1, \frac{1}{2}^-)$ | $\Lambda(1405)$ |
| (70, $L = 1$) | 7 | $(1, \frac{3}{2}^-)$ | $\Lambda(1520)$ |
| (70, $L = 1$) | 8 | $(8, \frac{3}{2}^-)$ | (1520, 1690, 1670, 1820) |
| (70, $L = 1$) | 9 | $(8, \frac{1}{2}^-)$ | (1535, 1670, 1620, ...) |
| (70, $L = 1$) | 10 | $(10, \frac{1}{2}^-)$ | (1620, ..., ...) |
| (70, $L = 1$) | 11 | $(8, \frac{3}{2}^-)$ | (1700, ..., ...) |
| (70, $L = 1$) | 12 | $(8, \frac{5}{2}^-)$ | (1675, 1830, 1775, ...) |
| (70, $L = 1$) | 13 | $(10, \frac{3}{2}^-)$ | (1700, ..., ...) |
| (70, $L = 1$) | 14 | $(8, \frac{1}{2}^-)$ | (1650, 1800, 1750, ...) |
| (56, $L = 2$) | 15 | $(8, \frac{5}{2}^+)$ | (1680, 1820, 1915, 2030) |
| (56, $L = 2$) | 17 | $(8, \frac{3}{2}^+)$ | (1720, 1890, ..., ...) |
| (56, $L = 2$) | 18 | $(10, \frac{5}{2}^+)$ | (1905, ..., ...) |
| (56, $L = 2$) | 20 | $(10, \frac{7}{2}^+)$ | (1950, 2030, ..., ...) |

and 2 following the approach of [6]. First, Samios et al. chose not to use a number of baryons already present in the 1972 edition of the Review of Particle Physics [32] partially because of relatively weak evidence of their existence and partially because they corresponded to rather incomplete multiplets. Those of them, which are not listed in Tables 1, include (we give the values of the masses as they were in 1972)

\[
N(1860)\frac{3^+}{2}, \Lambda(1750)\frac{1^+}{2}, \Lambda(1860)\frac{3^+}{2}, \Lambda(1870)\frac{1^-}{2}, \Sigma(1480)?, \Sigma(1620)\frac{1^-}{2}, \\
\Sigma(1620)\frac{1^+}{2}, \Sigma(1690)?, \Sigma(1880)\frac{1^-}{2}, \Sigma(1940)\frac{3^-}{2}, \Sigma(2070)\frac{5^+}{2}, \Sigma(2080)\frac{3^+}{2}, \\
\Xi(1630)?,
\]

where the number next to the particle’s mass denotes its $J^P$.

Second, a number of new particles since 1972 were reported. These include
In addition, the values of many measured partial decay widths have changed their values and, in general, have become more precise. In our analysis, whenever possible, we try to use uniform sources of information on the partial decays widths. The partial decay widths of $N$ and $\Delta$ baryons are predominantly taken from [33], which appears to be the analysis preferred by the authors of the Review of Particle Physics [10]; the partial decay widths of $\Lambda$ and $\Sigma$ hyperons are taken from [34–36], which is by far the most recent and comprehensive analysis of strange particles.

For reader’s convenience, the final results of our SU(3) systematization of all baryons with the mass less than approximately 2000-2200 MeV are presented in Table 3. The underlined entries in the table are predictions of new particles, which are absent in the Review of Particle Physics.

This review is organized as follows. The SU(3) systematization of octets is presented in Sect. 2. The analysis is then repeated for decuplets in Sect. 3. In Sect. 4, we discuss the SU(3) analysis of the antidecuplet and consider its mixing with octet 3, which allows us to obtain a self-consistent picture of the antidecuplet decays. We discuss our results in Sect. 5.

In summary, we have succeeded in placing nearly all known baryons with the mass less than approximately 2000-2200 MeV into twenty-one SU(3) multiplets. In order to have complete multiplets, we predict the existence of a number of strange particles. The most remarkable among them is the $\Lambda$ hyperon with $J^P = 3/2^-$, the mass around 1850 MeV, the total width $\approx 130$ MeV, significant branching into the $\Sigma\pi$ and $\Sigma(1385)\pi$ states and a very small coupling to the $N\bar{K}$ state. This is remarkable because all other eleven $\Lambda$ hyperons, which are required for the consistency of our SU(3) picture, are known and have very high (three and four stars) status in the RPP [10]. Our prediction model-independently confirms the constituent quark model prediction that there should exist a new $\Lambda$ baryon with $J^P = 3/2^-$ in the 1775 – 1880 MeV mass range [7–9]. In addition, we show how SU(3) can be effectively applied for the systematization and predictions of the decays of the antidecuplet.

2 SU(3) classification of octets

In this section, we review the picture of SU(3) octets of baryons with the mass less than approximately 2000-2200 MeV using the the Gell-Mann–Okubo mass formulas and the $\chi^2$ fit to the measured partial decay widths. We scrutinize one octet at a time, starting
Table 3
The final list of SU(3) multiplets.

with the established octets considered in [6].

2.1 Accuracy of the Gell-Mann–Okubo formula

We mentioned in the Introduction that there is no objective criterion of the accuracy of the Gell-Mann–Okubo mass formula for octets. Below we define an estimator for the GMO relations for decuplets and discuss how the accuracy of the GMO relation for octets can be estimated. The same reasoning applies to Weldon’s formulas for total widths.
The accuracy of the GMO formula for decuplets can be estimated as follows. Introducing the average mass splitting, \( \langle z \rangle \), and the standard deviation, \( \Delta z \),

\[
\langle z \rangle = \frac{1}{3} (m_\Sigma - m_\Delta + m_\Xi - m_\Sigma + m_\Omega - m_\Xi) = \frac{1}{3} (m_\Omega - m_\Delta),
\]

\[
\Delta z = \frac{1}{\sqrt{2}} \left( (m_\Sigma - m_\Delta - z)^2 + (m_\Xi - m_\Sigma - z)^2 + (m_\Omega - m_\Xi - z)^2 \right)^{1/2},
\]

one can estimate the accuracy of the equal spacing rule by comparing \( \Delta z \) to \( \langle z \rangle \), e.g.

\[
\text{accuracy} = \frac{\Delta z}{\langle z \rangle}.
\]  

(33)

Note that the numerator has the order \( O(m_\Sigma^2) \) and the denominator has the order \( O(m_\Sigma) \). Thus defined accuracy does not depend on the overall position of the decuplet (the mass \( M_{10}^0 \)).

Turning to the accuracy of the GMO formula for octets, one observes that it is impossible to suggest an estimator of the accuracy which would have the form similar to Eqs. (32) and (33), since there is just one equation relating four masses. Therefore, one can estimate the accuracy of the octet GMO mass formula only qualitatively, for example, by comparing the mismatch between the right and left hand sides of Eq. (5),

\[
\Delta M \equiv \frac{1}{2} (m_N + m_\Xi) - \frac{1}{4} (3 m_\Lambda + m_\Sigma),
\]

(34)

to the typical hadronic scale of approximately 1 GeV or to some average mass of the octet in question.

Of course, one might argue that using three free parameters of the GMO mass formula for the octet (3), one can perform a \( \chi^2 \) fit to the four experimental masses and, thus, one can objectively judge how well the GMO mass formula works. However, the SU(3)-symmetric mass \( M_{0}^\mu \) has nothing to do with the mass splitting and the original idea of Gell-Mann and Okubo and, hence, should not be used in assessing the accuracy of the GMO formula.

Replacing the masses by the total widths in Eqs. (32) and (33), one obtains an estimate for the accuracy of Weldon’s formula for decuplets, see Eq. (24). Similarly to the GMO formula, there is no prescription for the estimation of the accuracy of Weldon’s formula for octets, see Eq. (23). It is not even clear what hadronic scale the mismatch between the left and right hand sides of Eq. (23) should be compared to since 1 GeV cannot be used. Hence, in this work we will not actively use Weldon’s relations.
2.2 Universal SU(3) coupling constants and the barrier and phase space factors

The assumption that the approximate flavor SU(3) symmetry is violated only by the different masses of the baryons and is exact for their decays, gives the possibility to relate all decays of a given multiplet using only a few phenomenological constants. For the decays of octets, these are \( A_8 \) and \( \alpha \) of Eq. (17) and \( A'_8 \) of Eq. (19).

In addition to octets, we will consider decays of two SU(3) singlets, which are represented by the \( \Lambda(1405) \) and \( \Lambda(1520) \) hyperons. The \( 1 \rightarrow 8 + 8 \) coupling constants have the form

\[
g_{B_1B_2P} = A_1 \begin{pmatrix} 8 & 8 & 1 \\ Y_2T_2 & Y_P T_P & 0 \\ 0 & 0 & 0 \end{pmatrix},
\]

where \( A_1 \) is a free parameter to be determined from the \( \chi^2 \) fit to the decay rates. The coupling constants for all decay modes of octets and singlets in terms of \( A_8, \alpha, A'_8 \) and \( A_1 \) are summarized in Tables 4 and 5.

It is important to note that while the coupling constants \( A_8, A'_8 \) and \( A_1 \) are free parameters, SU(6) makes unique predictions for \( \alpha \), which is related to the so-called \( F/D \) ratio. This can be seen as follows. Since the tensor product of two \( 8 \) representations contains two \( 8 \)'s, see Eq. (4), the effective Lagrangian describing \( 8 \rightarrow 8 + 8 \) is parameterized in terms of two free constants \( g_0 \) and \( \alpha \) [11]

\[
\mathcal{L}_{\text{int}} = 2i g_0 B^j_1 \left[ \alpha F^{jk} + (1 - \alpha) D^{jk} \right] B^k_2 P^j,
\]

where \( F^{jk}_i = -if_{ijk} \) and \( D^{jk}_i = d_{ijk} \) with \( f_{ijk} \) and \( d_{ijk} \) the antisymmetric and symmetric SU(3) structure constants; \( B^j_1 \) and \( P^j \) denote the baryon and meson octet fields. The ratio of the coupling constants in front of \( F^{jk}_i \) and \( D^{jk}_i \) is called the \( F/D \) ratio

\[
F/D = \frac{\alpha}{1 - \alpha}.
\]

From this, one immediately obtains the relation between \( \alpha \) and \( F/D \)

\[
\alpha = \frac{F/D}{1 + F/D}.
\]

While in SU(3) the ratio \( F/D \) is unconstrained, SU(6) makes unique predictions for \( F/D \) [37]. The SU(6) predictions for \( F/D \) and \( \alpha \) are summarized in Table 6. In the table, in the second column, \( S \) denotes the spin, which is coupled to the orbital moment \( L \) to give
| Decay mode | $g_{B_{1}B_{2}P}$ | $g_{B_{1}B_{2}P}$ | $g_{B_{1}B_{2}P}$ |
|------------|-------------------|-------------------|-------------------|
| $N \rightarrow N\pi$ | $\sqrt{3}A_{8}$ | $\sqrt{3}A_{8}$ | $\sqrt{3}A_{8}$ |
| $\rightarrow N\eta$ | $[(4\alpha - 1)/\sqrt{3}]A_{8}$ | $[(4\alpha - 1)/\sqrt{3}]A_{8}$ | $[(4\alpha - 1)/\sqrt{3}]A_{8}$ |
| $\rightarrow \Sigma K$ | $\sqrt{3}(2\alpha - 1)A_{8}$ | $\sqrt{3}(2\alpha - 1)A_{8}$ | $\sqrt{3}(2\alpha - 1)A_{8}$ |
| $\rightarrow \Lambda K$ | $-(2\alpha + 1)/\sqrt{3}A_{8}$ | $-(2\alpha + 1)/\sqrt{3}A_{8}$ | $-(2\alpha + 1)/\sqrt{3}A_{8}$ |
| $\rightarrow \Delta\pi$ | $-2/\sqrt{5}A'_{8}$ | $-2/\sqrt{5}A'_{8}$ | $-2/\sqrt{5}A'_{8}$ |
| $\rightarrow \Sigma^* K$ | $1/\sqrt{5}A'_{8}$ | $1/\sqrt{5}A'_{8}$ | $1/\sqrt{5}A'_{8}$ |
| $\Lambda \rightarrow N\bar{K}$ | $\sqrt{2/3}(2\alpha + 1)A_{8}$ | $1/2A_{1}$ | $1/2A_{1}$ |
| $\rightarrow \Sigma\pi$ | $2(\alpha - 1)A_{8}$ | $\sqrt{6/4}A_{1}$ | $\sqrt{6/4}A_{1}$ |
| $\rightarrow \Lambda\eta$ | $2/\sqrt{3}(\alpha - 1)A_{8}$ | $-(\sqrt{2/4})A_{1}$ | $-(\sqrt{2/4})A_{1}$ |
| $\rightarrow \Xi K$ | $\sqrt{2/3}(4\alpha - 1)A_{8}$ | $-1/2A_{1}$ | $-1/2A_{1}$ |
| $\rightarrow \Sigma^*\pi$ | $-\sqrt{15}/5A'_{8}$ | $-\sqrt{15}/5A'_{8}$ | $-\sqrt{15}/5A'_{8}$ |
| $\rightarrow \Xi^* K$ | $\sqrt{10}/5A'_{8}$ | $\sqrt{10}/5A'_{8}$ | $\sqrt{10}/5A'_{8}$ |
| $\Sigma \rightarrow \Sigma\pi$ | $2\sqrt{2}\alpha A_{8}$ | $2\sqrt{2}\alpha A_{8}$ | $2\sqrt{2}\alpha A_{8}$ |
| $\rightarrow \Lambda\pi$ | $-2/\sqrt{3}(\alpha - 1)A_{8}$ | $-2/\sqrt{3}(\alpha - 1)A_{8}$ | $-2/\sqrt{3}(\alpha - 1)A_{8}$ |
| $\rightarrow N\bar{K}$ | $\sqrt{2}(2\alpha - 1)A_{8}$ | $\sqrt{2}(2\alpha - 1)A_{8}$ | $\sqrt{2}(2\alpha - 1)A_{8}$ |
| $\rightarrow \Sigma\eta$ | $-2/\sqrt{3}(\alpha - 1)A_{8}$ | $-2/\sqrt{3}(\alpha - 1)A_{8}$ | $-2/\sqrt{3}(\alpha - 1)A_{8}$ |
| $\rightarrow \Xi K$ | $-\sqrt{2}A_{8}$ | $-\sqrt{2}A_{8}$ | $-\sqrt{2}A_{8}$ |
| $\rightarrow \Delta\bar{K}$ | $2\sqrt{30}/15A'_{8}$ | $2\sqrt{30}/15A'_{8}$ | $2\sqrt{30}/15A'_{8}$ |
| $\rightarrow \Sigma^*\pi$ | $-\sqrt{30}/15A'_{8}$ | $-\sqrt{30}/15A'_{8}$ | $-\sqrt{30}/15A'_{8}$ |
| $\rightarrow \Sigma^*\eta$ | $-\sqrt{5}/5A'_{8}$ | $-\sqrt{5}/5A'_{8}$ | $-\sqrt{5}/5A'_{8}$ |
| $\rightarrow \Xi^* K$ | $\sqrt{30}/15A'_{8}$ | $\sqrt{30}/15A'_{8}$ | $\sqrt{30}/15A'_{8}$ |

Table 4
The SU(3) universal coupling constants for $8 \rightarrow 8 + 8$, $8 \rightarrow 10 + 8$ and $1 \rightarrow 8 + 8$ decays.

In the calculation of partial decay widths using Eq. (16) the total angular moment of a given SU(3) multiplet. Therefore, SU(6) predictions for $\alpha$ for the octets with $J = 1/2$ and $J = 3/2$, which belong to the $(70, L = 1)$ representation (octets 8, 9, 11 and 14), are ambiguous since $\alpha$ can be either 0.625 or $-0.5$. In our analysis, we shall use the SU(6) predictions for $\alpha$ as a rough guide.

The second ingredient in the calculation of partial decay widths using Eq. (16) is the barrier and phase space factors. Since we assumed that the only source of SU(3) symmetry breaking is non-equal masses of baryons in multiplets, the phase space factor, which is
| Decay mode | $g_{B_1B_2P}$ | $g_{B_1B_2P}$ | $g_{B_1B_2P}$ |
|------------|--------------|--------------|--------------|
| $\Xi \rightarrow \Xi\pi$ | $\sqrt{3} (2\alpha - 1) A_8$ | | |
| $\rightarrow \Lambda K$ | | $(4\alpha - 1)/\sqrt{3} A_8$ | |
| $\rightarrow \Sigma K$ | | $\sqrt{3} A_8$ | |
| $\rightarrow \Xi\eta$ | | $-(2\alpha + 1)/\sqrt{3} A_8$ | |
| $\rightarrow \Xi^∗\pi$, $\Xi^∗\eta$ | | $-\sqrt{5}/5 A'_8$ | |
| $\rightarrow \Sigma^∗K$ | | $\sqrt{5}/5 A'_8$ | |
| $\rightarrow \Omega K$ | | $\sqrt{10}/5 A'_8$ | |

Table 5
Continuation of Table 4.

| SU(6) | (SU(3), S) | $F/D$ [37] | $\alpha$ |
|-------|------------|------------|----------|
| 56    | (8, 1/2)   | $\frac{2}{3}$ | $\frac{2}{3}$ |
| 70    | (8, 1/2)   | $\frac{5}{3}$ | $\frac{5}{3}$ |
|       | (8, 3/2)   | $-\frac{1}{3}$ | $-\frac{1}{2}$ |

Table 6
SU(6) predictions for $F/D$ [37] and $\alpha$. In the second column, $S$ denotes the spin of SU(3) multiplets.

well-defined in the SU(3)-symmetric limit, can be multiplied by any function of the ratio of the baryon masses. This introduces ambiguity in the choice of the phase space factor. In our analysis, we use the convention of Samios et al. [6], which captures the main features of relativistic kinematics and provides the dimensionless coupling constants

$$
\text{barrier factor} = \left(\frac{k}{M}\right)^{2l},
$$

$$
\text{phase space factor} = \left(\frac{k}{M_1}\right) M,
$$

(39)

where $k$ is the center-of-mass momentum of the final particles; $M_1$ is the mass of $B_1$; $M = 1000$ MeV is the dimensional parameter; $l$ is the relative orbital moment of the outgoing $B_2 P$ system. The orbital moment $l$ is found by requiring the conservation of parity and the total angular moment in the decay.
2.3 Multiplet 1: Ground-state octet

The Gell-Mann–Okubo mass formula for the ground-state octet works with very high precision,

\[ \Delta M = \frac{1}{2} (m_N + m_\Xi) - \frac{1}{4} (3 m_A + m_\Sigma) \approx -8 \text{ MeV}. \]  

The mismatch between the left and the right hand sides of the GMO mass formula, \( \Delta M \), is less than one per cent of the individual baryon masses. The ground-state octet is of course stable against two-body hadronic decays so that the \( \chi^2 \) analysis of its decays cannot be performed. However, as discussed in [6], one can attempt to examine how well SU(3) describes the relation between the phenomenological \( NN\pi \), \( \Lambda pK^- \) and \( \Sigma pK^- \) coupling constants. Unfortunately, the two latter coupling constants are poorly known and rather ambiguous, which does not allow one to really test SU(3).

A better test of SU(3) for the ground-state octet can be performed by considering SU(3) predictions (the so-called Cabbibo theory) for semi-leptonic \( B_1 \to B_2 l \nu_l \) decays. The conclusion of the analysis [38] is that the Cabbibo theory works with limited accuracy: the 3-parameter \( \chi^2 \) fit to 21 semi-leptonic decays gives \( \chi^2/d.o.f = 44.3/18 \). The quality of the fit improves when SU(3)-breaking and other effects are taken into account [38].

Therefore, judging by Eq. (40) and keeping in mind the limited success of the Cabbibo theory of the hadronic semi-leptonic decays, one can conclude that the approximate SU(3) works well for the ground-state octet.

The discussion of the accuracy of the SU(3) predictions for the magnetic moments of the ground-state octet is a separate subject and is beyond the scope of this paper. We refer the interested reader to [6].

2.4 Multiplet 15: \((8, 5/2^+) = (1680, 1820, 1915, 2030)\)

This is a well-established octet present both in Tables 1 and 2: the \( N, \Lambda \) and \( \Sigma \) members of the octet have a four-star rating in the RPP; the \( \Xi \) state has a three-star rating and \( J^P \geq 5/2^+ \).

The masses of all the states are known with high accuracy. Taking the RPP face values for the masses, the mismatch between the left and right hand sides of the GMO mass formula is smaller than 0.5% of the lowest mass involved, \( \Delta M = 11.3 \text{ MeV} \). Also, the Weldon’s relation among the total widths is satisfied with a fair accuracy (we use the values from
\[ \frac{1}{2} (\Gamma_N + \Gamma_\Xi) = 80 \text{ MeV} \quad \text{vs.} \quad \frac{1}{4} (3\Gamma_\Lambda + \Gamma_\Sigma) = 90 \text{ MeV}. \] (41)

The SU(3) predictions for the partial decay widths of the considered octet are obtained using Eq. (16) with the universal coupling constants summarized in Tables 4 and 5 and with the barrier factor of Eq. (39) with \( l = 3 \) for the \( 8 \to 8 + 8 \) decays and with \( l = 1 \) for the \( 8 \to 10 + 8 \) decays. We perform the \( \chi^2 \) fit to selected experimentally measured partial decay widths using the MINUIT program [39]. We separately fit the \( 8 \to 8 + 8 \) and \( 8 \to 10 + 8 \) decays because, as practice shows, SU(3) works significantly worse for the decays involving decuplets. The coupling constants \( A_8, \alpha \) and \( A'_8 \) are free parameters of the fit. The fit is considered successful, if the resulting value of the \( \chi^2 \) function per degree of freedom is few units. A detailed discussion of the \( \chi^2 \) fit and its interpretation can be found in Sect. Probability of the RPP [10].

As fitted experimental observables we take partial decay widths and square roots of the product of two partial decay widths. The latter quantities can be both positive and negative, depending on the relative phase between the two involved decay amplitudes. This sign is an important test for SU(3) which predicts definite relative signs of the coupling constants, see Tables 4 and 5. Note that the use of the interference of two amplitudes as a fitted observable is an improvement over the analysis of Samios, Goldberg and Meadows [6], who used only decay rates.

Also, if the two partial decay widths that interfere correspond to the \( 8 \to 8 + 8 \) and \( 8 \to 10 + 8 \) decays, we ignore the sign of the interference and convert it to the \( 8 \to 10 + 8 \) decays width (provided that the corresponding \( 8 \to 8 + 8 \) partial width is known), which is then used in the fit.

Throughout this review, we try to use similar sources of experimental information on decays rates, which are all summarized in the Review of Particle Physics [10]. Whenever possible, we do not use the average values or estimates, but we rather prefer to use the original references. For \( N \) and \( \Delta \) baryons, we predominantly use the analysis of Manley and Saleski [33], which appears to be preferred by the RPP. Also, some of the decays are taken from the analysis of Vrana, Dytman and Lee [40]. Most of the decay rates of strange baryons are taken from the analysis of Gopal et al. [34–36], which provides the most recent and comprehensive analysis of strange baryons. In several cases, we also use [41].

Table 7 summarizes the results of our \( \chi^2 \) fit to the nine observables of the considered octet. The observables used in the fit are underlined. The right column presents the SU(3) predictions for the fitted observables as well as for other observables not used in the fit.

As one can see from Table 7, the absolute values and the relative signs of the measured
decay rates are reproduced well. An examination shows that the value of $\chi^2$ is dominated by the $\sqrt{\Gamma_{N\pi}\Gamma_{\Sigma\pi}}$ of $\Lambda(1820)$ and $\Sigma(1915)$. In order to lower the $\chi^2$ to the acceptable level, we increased the experimental error on these two observables by the factor 1.5.

This error manipulation requires an explanation. It is commonly believed that the accuracy of SU(3) predictions is approximately 30%. In the case of two-body hadronic decays, this means that we expect that SU(3) predictions can correspond to $\chi^2/d.o.f. \approx 1$ only when the experimental errors on the fitted observables are about 30%. For the particular example of octet 15, this means that some errors can be increased by the factor of 1.5 and still do not exceed 30%. Also, an analysis of Bukhvostov [42] shows that the results of physical measurements do not follow the conventional Gaussian distribution – the tail of the actual probability distribution is much larger than expected on the basis of the Gaussian distribution. This effect can be roughly simulated by increasing the dispersion of the Gaussian distribution (experimental errors) by the factor $2^{-3}$.

It is a phenomenological observation that approximate SU(3) works worse for decays involving decuplets and, in particular, for the $8 \rightarrow 10 + 8$ decays [6]. In the considered case, we increase the error on the fitted $\Gamma_{\Delta\pi}$ by the factor 1.5. According to our logic (see the discussion above), this is legitimate procedure since the resulting error is still $\approx 30%$.

The $\chi^2$ fits to the seven $8 \rightarrow 8 + 8$ and two $8 \rightarrow 10 + 8$ decays presented in Table 7 give

$$A_8 = 52.0 \pm 1.3, \quad \alpha = 0.39 \pm 0.02, \quad \chi^2/d.o.f. = 7.85/5,$$
$$A'_8 = 19.2 \pm 3.4, \quad \chi^2/d.o.f. = 3.44/2.$$  \hfill (42)

The obtained values of the coupling constants are close to those obtained in [6]. The $\chi^2$ values are larger because the experimental errors, which we use in our analysis, are smaller. The value of $\alpha$ is in excellent agreement with the SU(6) prediction $\alpha = 0.4$, see Table 6.

Note that the predicted large $\Gamma_{\Sigma\pi}$ of $\Xi(2030)$ is in agreement with the experiments [10], which indicate that this decay rate is the largest. While the predicted value of $\Gamma_{\Sigma\pi}$ appears to be larger than $\Gamma_{\text{tot}}$, the total width of $\Xi(2030)$ is poorly known and varies from $\Gamma_{\text{tot}} = 16 \pm 5 \text{ MeV}$ to $\Gamma_{\text{tot}} = 60 \pm 24 \text{ MeV}$ [10].

Based on our results presented in Table 7 and Eq. (42) we conclude that SU(3) works well for the considered octet.

2.5 Multiplet 12: $(8, 5/2^-) = (1675, 1830, 1775, 1950)$

This is an established octet present both in Tables 1 and 2: the $N$, $\Lambda$ and $\Sigma$ members of the octet have a four-star rating in the RPP; the $\Xi$ state has a three-star rating, but its
| Mass and width (MeV) | Observables | Experiment (MeV) | SU(3) pred. (MeV) |
|---------------------|-------------|-----------------|-------------------|
| N(1684)             | $\Gamma_{N\pi}$ | 97.3 $\pm$ 7.0  | 93.2              |
| $\Gamma = 139 \pm 8$ | $\Gamma_{\Delta\pi}$ | 19.5 $\pm$ 4.3 $\times$ 1.5 | 9.0               |
|                     | $\Gamma_{N\eta}$ | 0.00 $\pm$ 0.01 | 0.2               |
| $\Lambda(1823)$    | $\Gamma_{N\overline{K}}$ | 44.7 $\pm$ 3.3  | 44.09             |
| $\Gamma = 77 \pm 5$ | $\sqrt{\Gamma_{N\overline{K}}\Gamma_{\Sigma\pi}}$ | $-21.6 \pm 2.7 \times 1.5$ | $-29.6$           |
|                     | $\sqrt{\Gamma_{N\overline{K}}\Gamma_{\Lambda\pi}}$ | $-6.9 \pm 3.1$ | $-4.8$            |
|                     | $\Gamma_{\Sigma(1385)\pi}$ | 3.7 $\pm$ 2.4  | 5.9               |
|                     | $\Gamma_{\Sigma\pi}$ | 19.8            |                   |
| $\Sigma(1920)$     | $\Gamma_{N\overline{K}}$ | 3.9 $\pm$ 2.6  | 4.9               |
| $\Gamma = 130 \pm 10$ | $\sqrt{\Gamma_{N\overline{K}}\Gamma_{\Sigma\pi}}$ | $-24.7 \pm 4.3 \times 1.5$ | $-13.6$           |
|                     | $\sqrt{\Gamma_{N\overline{K}}\Gamma_{\Lambda\pi}}$ | $-11.7 \pm 4.0$ | $-11.4$           |
|                     | $\Gamma_{\Sigma\pi}$ | 37.4            |                   |
|                     | $\Gamma_{\Lambda\pi}$ | 26.3            |                   |
| $\Xi(2025)$        | $\Gamma_{\Sigma\overline{K}}$ | 46.9            |                   |
| $\Gamma = 21 \pm 6$ | $\Gamma_{\Xi\pi}$ | 4.1             |                   |

Table 7
SU(3) analysis of $(8, 5/2^+)$=$(1680, 1820, 1915, 2030)$.

spin and parity are unknown.

The masses of all the states are known with high accuracy. Taking the RPP estimates for the masses, the mismatch between the left and right hand sides of the GMO mass formula is tiny, $\Delta M = -3.8$ MeV. The Weldon’s relation among the total widths is satisfied exactly

$$\frac{1}{2} (\Gamma_N + \Gamma_\Xi) = 110 \text{ MeV} \quad \text{vs.} \quad \frac{1}{4} (3 \Gamma_\Lambda + \Gamma_\Sigma) = 109 \text{ MeV}. \quad (43)$$

For this octet, the barrier factor is calculated with $l = 2$ for the $8 \rightarrow 8 + 8$ and $8 \rightarrow 10 + 8$ decays, see Eq. (39).

Table 8 summarizes the results of our $\chi^2$ fit to nine observables of the considered octet. As
| Mass and width (MeV) | Observables | Experiment (MeV) | SU(3) pred. (MeV) |
|----------------------|-------------|------------------|-------------------|
| \( N(1676) \)       | \( \Gamma_{N\pi} \) | 74.7 ± 4.6       | 73.3              |
| \( \Gamma = 159 \pm 7 \) | \( \Gamma_{\Delta\pi} \) | 83.2 ± 5.2       | 78.3              |
|                      | \( \Gamma_{N\eta} \) | 0.00 ± 0.01      | 3.9               |
|                      | \( \Gamma_{\Lambda K} \) | 0.02             |                   |
| \( \Lambda(1831) \) | \( \Gamma_{N\bar{K}} \) | 8.0 ± 3.1        | 4.0               |
| \( \Gamma = 100 \pm 10 \) | \( \sqrt{\Gamma_{N\bar{K}}\Gamma_{\Sigma\pi}} \) | -17.0 ± 3.5      | -18.5             |
|                      | \( \Gamma_{\Sigma(1385)\pi} \) | 24.9 ± 10.8     | 58.6              |
|                      | \( \Gamma_{\Sigma\pi} \) |                   | 86.0              |
|                      | \( \Gamma_{\Lambda\eta} \) |                   | 5.0               |
| \( \Sigma(1775) \)  | \( \Gamma_{N\bar{K}} \) | 54.8 ± 4.9       | 55.9              |
| \( \Gamma = 137 \pm 10 \) | \( \sqrt{\Gamma_{N\bar{K}}\Gamma_{\Sigma\pi}} \) | 17.8 ± 3.0       | 14.9              |
|                      | \( \sqrt{\Gamma_{N\bar{K}}\Gamma_{\Lambda\pi}} \) | -38.4 ± 5.0      | -42.3             |
|                      | \( \Gamma_{\Sigma(1385)\pi} \) | 11.7 ± 1.7 \times 2 | 6.7               |
|                      | \( \Gamma_{N\bar{K}} \) |                   | 55.9              |
|                      | \( \Gamma_{\Lambda\pi} \) |                   | 32.0              |
| \( \Xi(1950) \)    | \( \Gamma_{\Sigma\bar{K}} \) | 21.9             |                   |
| \( \Gamma = 60 \pm 20 \) | \( \Gamma_{\Xi\pi} \) | 84.3             |                   |
|                      | \( \Gamma_{\Xi(1530)\pi} \) | 14.2             |                   |

Table 8
SU(3) analysis of \( (8, 5/2^-) = (1675, 1830, 1775, 1950) \).

one can see from Table 7, the absolute values and the relative signs of the measured \( 8 \rightarrow 8 + 8 \) decay rates are reproduced well. At the same time, SU(3) significantly overestimates the \( \Gamma_{\Lambda(1830) \rightarrow \Sigma(1385)\pi} \), which results in a very large \( \chi^2 \), even after we have increased the error on another fitted partial decay width, \( \Gamma_{\Sigma(1775) \rightarrow \Sigma(1385)\pi} \), by the factor of two.

Let us examine this in some more detail. The value of \( \Gamma_{\Lambda(1830) \rightarrow \Sigma(1385)\pi} \) in Table 8 was obtained by combining the results of two different analyses: \( \sqrt{\Gamma_{\Lambda(1830) \rightarrow \Sigma(1385)\pi}\Gamma_{\Lambda(1830) \rightarrow N\bar{K}}} \) from [41] with \( Br(\Lambda(1830) \rightarrow N\bar{K}) \) and \( \Gamma_{\Lambda(1830)} \) from [36]. Since these are two completely different analyses, one can imagine that the central value of \( \Gamma_{\Lambda(1830) \rightarrow \Sigma(1385)\pi} \) is much
more uncertain than indicated by our estimate of its experimental error. Note also that the same situation was encountered in the analysis of Samios et al. [6]: the SU(3) prediction $\Gamma_{\Lambda(1830) \rightarrow \Sigma(1385)\pi} = 55.1 \text{ MeV}$ was significantly larger than the experimental value $\Gamma_{\Lambda(1830) \rightarrow \Sigma(1385)\pi} = 27 \pm 26 \text{ MeV}$. However, the 100% experimental error resulted in an acceptably low $\chi^2$.

The $\chi^2$ fits to the six $8 \rightarrow 8 + 8$ and three $8 \rightarrow 10 + 8$ decays presented in Table 8 give

$$A_8 = 26.8 \pm 0.7, \quad \alpha = -0.23 \pm 0.02, \quad \chi^2/d.o.f. = 3.59/4,$$

$$A'_8 = 158.7 \pm 4.9, \quad \chi^2/d.o.f. = 12.66/2.$$  (44)

The obtained values of the coupling constants are close to those obtained in [6]. The value of $\alpha$ is lower than the SU(6) prediction $\alpha = -0.5$, see Table 6, which was also observed in the analysis of [6].

Note that the sum of the predicted partial decay widths of $\Xi(1950)$ is larger than the RPP estimate for the total width of this hyperon. However, $\Gamma_{\text{tot}}$ of $\Xi(1950)$ is not known well and could be much larger than the RPP estimate [10]. Therefore, with the present level of accuracy, SU(3) predictions for $\Xi(1950)$ cannot be ruled out.

In conclusion, based on our results presented in Table 8 and Eq. (44) we conclude that SU(3) works rather well for the $8 \rightarrow 8 + 8$ decays of the considered octet. At the same time, SU(3) fails to describe the $\Gamma_{\Lambda(1830) \rightarrow \Sigma(1385)\pi}$ partial decay width. However, since the experimental value of $\Gamma_{\Lambda(1830) \rightarrow \Sigma(1385)\pi}$ is extracted by combining two different partial wave analyses, it is intrinsically very uncertain, which might be the cause of the inconsistency.

2.6 Multiplets 8 and 7: $(8, 3/2^-) = (1520, 1690, 1670, 1820)$ and $(1, 3/2^-) = \Lambda(1520)$

It has been known since the early 70’s that the $\Lambda(1520)$ and $\Lambda(1405)$ hyperons, which were thought to be SU(3) singlets [43], in reality are not pure singlets but are mixed with $\Lambda$ hyperons from octets with the corresponding quantum numbers. The direct evidence for the mixing exists only for $\Lambda(1520)$, which decays into the $\Sigma(1385)\pi$ final state. Since a singlet cannot decay into the $10 + 8$ final states, the decay can take place only through the mixing, presumably with $\Lambda(1690)$ from octet 8.

The considered octet is well-established and is present both in Tables 1 and 2: the $N$, $\Lambda$ and $\Sigma$ members of the octet have a four-star rating in the RPP; the $\Xi$ state has a three-star rating.

The masses are known with high accuracy. Taking the RPP estimates for the masses, the mismatch between the left and right hand sides of the GMO mass formula is $\Delta M = -15 \text{ MeV}$, i.e. it less than 1% of the involved masses. This illustrates that while the GMO
mass relation is satisfied with very high accuracy, the mixing might still place. In other words, the GMO mass formulas are not sensitive to small mixing, which is the case in the considered example.

The Weldon’s relation among the total widths should be modified in the presence of mixing. The final result is analogous to Eq. (21). The accuracy of the modified Weldon’s relation is fair (we use the value of the mixing angle $\theta$ from Eq. (46) that follows)

$$\frac{1}{2} (\Gamma_N + \Gamma_\Xi) = 74 \text{ MeV} \quad \text{vs.} \quad \frac{1}{4} [3 (\Gamma_{\Lambda(1690)} \cos^2 \theta + \Gamma_{\Lambda(1520)} \sin^2 \theta) + \Gamma_\Sigma] = 59 \text{ MeV} \quad (45)$$

For the considered case of $J = 3/2^-$, the barrier factor is calculated with $l = 2$ for the $8 \to 8 + 8$ decays and with $l = 0$ for the $8 \to 10 + 8$ decays. Because of the mixing, we simultaneously fit the octet and singlet decay rates. Therefore, we have five free parameters: $A_8$, $\alpha$, $A_1$ and the mixing angle $\theta$ are determined from the $\chi^2$ fit to the $8 \to 8 + 8$ and $1 \to 8 + 8$ decay rates; $A'_8$ is determined from the fit to the $8 \to 10 + 8$ decay rates. Note that the $\Lambda(1520) \to \Sigma(1385)\pi$ decay is not allowed kinematically and, hence, cannot be used in our fit.

Table 9 summarizes the results of our $\chi^2$ fit to eleven observables of the considered octet and singlet. As one can see from Table 9, the absolute values and the relative signs of the measured decay rates are reproduced well.

An examination shows that the value of the $\chi^2$ function is dominated by the $\sqrt{\Gamma_{NKK}\Gamma_{\Sigma\pi}}$ of $\Lambda(1690)$ and by the $\sqrt{\Gamma_{NKK}\Gamma_{\Lambda\pi}}$ of $\Sigma(1670)$. In this respect we note that the modern value of the strength of the $\Sigma(1670) \to \Lambda\pi$ decay is smaller than the experimental value used in the analysis of Samios et al. [6]. Also, the experimental value of $\sqrt{\Gamma_{NKK}\Gamma_{\Sigma\pi}}$ of $\Lambda(1690)$, which is reported by our standard source [34] and which we used in our analysis, is the smallest compared to all other measurements [10]. In order to take the theoretical uncertainty in the measurement of $\sqrt{\Gamma_{NKK}\Gamma_{\Sigma\pi}}$ for $\Lambda(1690)$ into account, we increase the experimental error on this observable by 1.5. Note also that there is large ambiguity in the value of $\Gamma_{N\to \Delta\pi}$. In our analysis, we use the $S$-wave value ($l = 0$) for $\Gamma_{N\to \Delta\pi}$ of [40].

The $\chi^2$ fits to the observables underlined in Table 9 give

$$A_8 = 42.7 \pm 1.5, \quad \alpha = 0.74 \pm 0.03, \quad A_1 = 175.4 \pm 7.0, \quad \theta = (26 \pm 2)^0, \quad \chi^2/\text{d.o.f.} = 8.28/4, \quad A'_8 = 12.7 \pm 0.9, \quad \chi^2/\text{d.o.f.} = 2.03/2 \quad (46)$$

The obtained values of the coupling constants and the mixing angle are very close to those obtained in [6]. The value of $\alpha$ is in fair agreement with the SU(6) prediction $\alpha = 0.625$, see Table 6.

32
Table 9
SU(3) analysis of \((8, 3/2^-) = (1520, 1690, 1670, 1820)\) and \((1, 3/2^-) = \Lambda(1520)\).

One should note that, in principle, there is another candidate for the \(\Sigma\) member of the considered octet – \(\Sigma(1580)\). We studied this possibility and found that the inclusion of \(\Sigma(1580)\) into octet 8 leads to a larger value of \(\chi^2\) than in Eq. (46): \(\chi^2/d.o.f. = 12.0/4\). Therefore, we take \(\Sigma(1670)\) as the \(\Sigma\) member of octet 8.

Taking the mixing between \(\Lambda(1690)\) and \(\Lambda(1520)\) into account, the accuracy of the GMO mass formula improves. The mismatch between the left and right hand sides of Eq. (21) reduces to \(\Delta M = 8.3 \text{ MeV}\).

Based on the results presented in Table 9 and the acceptably low value of \(\chi^2\), see Eq. (46),
we conclude that SU(3) works rather well for octet 8, whose Λ(1690) hyperon is mixed with Λ(1520).

2.7 Multiplets 9 and 6: \((8, 1/2^-) = (1535, 1670, 1560, 1620-1725)\) and \((1, 1/2^-) = \Lambda(1405)\)

The particle content of octet 9 is not established. In the analysis of Samios et al. [6], it was assumed that the Σ member of the considered octet is Σ(1750). However, the decay rates of Σ(1750) were not used in the \(\chi^2\) analysis. As we shall argue, the inclusion of Σ(1750) in octet 9 leads to very large \(\chi^2\). Therefore, one cannot assign Σ(1750) to octet 9 without mistrusting the data on the Σ(1750) decays. In Table 2, the Σ member of octet 9 is assumed to be Σ(1620). This choice also gives unacceptably large \(\chi^2\), \(\chi^2/d.o.f. > 5\). The good description of the decays of the considered octet is achieved only by assigning Σ(1560) to octet 9.

The Ξ member of the octet is unknown. Using the modified GMO mass formula, we predict its mass to lie in the interval 1620 < \(m_\Xi\) < 1725 MeV. The decay rates of Ξ are predicted using SU(3), see Table 10.

The \(\Lambda(1405)\) has a 100\% branching ratio into the Σπ final state. Therefore, the mixing between \(\Lambda(1405)\) and \(\Lambda(1670)\) from octet 9 can be established only indirectly, by noticing that the mixing provides a better simultaneous \(\chi^2\) fit to the decays of \(\Lambda(1405)\) and octet 9.

Since the \(N\) and Λ states of the considered octet are established, we first perform a \(\chi^2\) fit to the decays of \(N(1535)\), \(\Lambda(1670)\) and \(\Lambda(1405)\). An examination shows that if for the \(N(1535)\) decay rates one takes the results of [33], the \(\chi^2\) fit fails because the large partial widths of \(N(1535)\) are incompatible with the rather narrow Λ(1670). Therefore, in order to obtain a sensible result, we use the RPP estimates for the total width and \(Br(N\pi)\) of \(N(1535)\). Note that the value of \(\Gamma_{N\to N\pi}\) used in the analysis of Samios al. [6] is two times smaller than its modern value [33].

Next we begin trying different candidates for the Σ state of the considered octet. The Σ(1750) does not fit this octet because SU(3) predicts the \(\Gamma_{\Sigma(1750)\to N\overline{K}}\), which is several times larger than the experimental value. The Σ(1620) cannot be assigned to the octet because of the same reason. Finally, Σ(1690) cannot be a member of the considered octet because SU(3) predicts \(\Gamma_{\Sigma(1690)\to N\overline{K}}/\Gamma_{\Sigma(1690)\to \Lambda\pi} \approx 2.5\), which seriously contradicts the experimental value \(\Gamma_{\Sigma(1690)\to N\overline{K}}/\Gamma_{\Sigma(1690)\to \Lambda\pi} \approx 0.4 \pm 0.25\) or even smaller [10].

In the appropriate mass range, the last remaining candidate for the Σ member of octet 9 is Σ(1560) with the two-start rating and unknown spin and parity. The only measured observable of Σ(1560), \(\Gamma_{\Sigma\pi}/(\Gamma_{\Sigma\pi} + \Gamma_{\Lambda\pi})\), fits nicely the decays of octet 9 so that we assign Σ(1560) to octet 9.
Table 10 summarizes the results of our $\chi^2$ fit to the eight observables of the considered octet and singlet. The absolute values and the relative signs of the measured decay rates are reproduced well. The barrier factor is calculated with $l = 0$ for the $8 \to 8 + 8$ decays and with $l = 2$ for the $8 \to 10 + 8$ decays.

| Mass and width (MeV) | Observables | Experiment (MeV) | SU(3) pred. (MeV) |
|----------------------|-------------|-----------------|-------------------|
| $N(1535)$ $\Gamma_{N\pi}$ | 67.5 $\pm$ 27.0 | 46.2 |
| $\Gamma = 150 \pm 50$ $\Gamma_{\Delta\pi}$ | 0 $\pm$ 6 | 3.9 |
| $\Gamma_{N\eta}$ | 63.8 $\pm$ 28.3 | 20.2 |
| $\Lambda(1670)$ $\Gamma_{N\overline{K}}$ | 5.5 $\pm$ 1.3 | 5.2 |
| $\Gamma = 29 \pm 5$ $\sqrt{\Gamma_{N\overline{K}}\Gamma_{\Sigma\pi}}$ | $-9.0 \pm 1.8$ | $-9.7$ |
| $\Gamma_{\Lambda\eta}$ | 8.7 $\pm$ 2.8 | 6.1 |
| $\Gamma_{\Sigma(1385)\pi}$ | 4.6 $\pm$ 3.6 | 1.9 |
| $\Gamma_{\Sigma\pi}$ | 18.1 |
| $\Lambda(1405)$ $\Gamma_{\Sigma\pi}$ | 50 $\pm$ 2 | 50.1 |
| $\Sigma(1560)$ $\Gamma_{\Sigma\pi}/(\Gamma_{\Sigma\pi} + \Gamma_{\Lambda\pi})$ | 0.35 $\pm$ 0.12 | 0.38 |
| $\Gamma = 15 - 79$ $\Gamma_{\Sigma\pi}$ | | 22.1 |
| $\Gamma_{\Lambda\pi}$ | 36.4 |
| $\Gamma_{N\overline{K}}$ | 78.9 |
| $\Gamma_{\Sigma(1385)\pi}$ | 0.6 |
| $\Xi(1620-1725)$ $\Gamma_{\Xi\pi}$ | | $\approx 100$ |
| $\Gamma_{\Lambda\overline{K}}$ | | $\approx 15$ |

Table 10
SU(3) analysis of $(8, 1/2^-) = (1535, 1670, 1560, 1620-1725)$ and $(1, 1/2^-) = \Lambda(1405)$

The $\chi^2$ fits to the observables underlined in Table 10 give

$$A_8 = 7.1 \pm 1.1, \quad \alpha = -0.53 \pm 0.15,$$

$$A_1 = 13.1 \pm 4.9, \quad \theta = (-48 \pm 9)^0, \quad \chi^2/d.o.f. = 1.67/2,$$

$$A'_8 = 93.9 \pm 56.0, \quad \chi^2/d.o.f. = 0.99/1. \quad (47)$$
The obtained values of the coupling constants and the mixing angle are very different from those obtained in [6]

\[
A_8 = 5.2 \pm 0.5, \quad \alpha = -0.28 \pm 0.06, \\
A_1 = 26.2 \pm 1.8, \quad |\theta| = (16 \pm 5)^0, \quad \chi^2/\text{d.o.f.} = 3.2/1, \\
A'_8 = 262 \pm 58, \quad \chi^2 = 0. \tag{48}
\]

The large difference between our analysis and that of [6] is the result of very significant changes in the measured decay rates of the considered octet since 1974. Our value of \( \alpha \) is in good agreement with the SU(6) prediction \( \alpha = -0.5 \), see Table 6.

The obtained mixing angle is very large. The reason for this is as follows. The small \( \Gamma_{\Lambda(1670) \rightarrow N K} \) forces \( \Gamma_{N(1535) \rightarrow N\pi} \) to be also small, which is in conflict with the experimental measurement. The mixing of \( \Lambda(1670) \) with the fairly wide \( \Lambda(1405) \) enables one to simultaneously have sufficiently large \( A_8 \) and small \( g_{\Lambda(1670) \rightarrow N K} \), but the resulting mixing angle has to be large in order to provide the sufficient compensation of the octet and singlet contributions to \( g_{\Lambda(1670) \rightarrow N K} \), see Eq. (22). Note also that quark model calculations support our finding (which contradicts the analysis of [6]) that the mixing angle associated with \( \Lambda(1405) \) is larger than the mixing angle associated with \( \Lambda(1520) \) [7,8]. One should keep in mind, however, that none of the existing quark model calculations is able to reproduce the small mass of \( \Lambda(1405) \). Finally, as we explained above, the value of \( \theta \) is indirectly affected by the \( \Gamma_{N(1535) \rightarrow N\pi} \), which is known with almost 50% uncertainty. Therefore, the error on the obtained value of the mixing angle \( \theta \) is most likely much larger than we quote in Eq. (47).

Since the \( \Xi \) member of octet 9 is not known, one can predict its mass using the modified Gell-Mann–Okubo mass formula, Eq. (21). Using \( \theta = -48^0 \) in Eq. (21) leads to \( m_\Xi = 1534 \) MeV, which is probably too small. This is another reason to believe that the true mixing angle is smaller than given by our \( \chi^2 \) fit. Also, from the theoretical point of view, the large value of the mixing angle is not welcome because this means that our basic assumption of small SU(3) violations proportional to the mass of the strange quark is not legitimate. If one uses smaller values of \( \theta \) in Eq. (21), for instance \( 15^0 < |\theta| < 35^0 \), then one obtains larger values for the mass of the \( \Xi \), \( 1620 < m_\Xi < 1725 \) MeV. We assume that this is the true range, where the mass of the missing \( \Xi \) baryon lies. The decay rates of \( \Xi \) in Table 10 were predicted assuming \( m_\Xi = 1650 \) MeV.

Based on the results presented in Table 10 and the low value of \( \chi^2 \), see Eq. (47), we conclude that SU(3) works rather well for octet 9, whose \( \Lambda(1670) \) baryon is mixed with \( \Lambda(1405) \). Based on the modified Gell-Mann–Okubo mass formula and taking into account significant ambiguities in the extraction of the mixing angle from the \( \chi^2 \) fit to the decay rates of octet 9, we predict the mass of the missing \( \Xi \) state of octet 9 in the range \( 1620 < m_\Xi < 1725 \) MeV. This \( \Xi \) hyperon should have \( J^P = 1/2^- \) and a very large partial decay width in the \( \Xi \pi \) final state, see Table 10.
Note also that the Weldon’s relation among the total widths of the considered octet is badly violated: the $\Lambda(1670)$ mixed with $\Lambda(1405)$ and the $\Sigma(1560)$ (even if we use $\Gamma_{\Sigma(1560)} = 80$ MeV) are too narrow for the $\Gamma_{N(1535)} = 150$ MeV. A possible explanation is that the octet is too light for the Weldon’s relation to work (see the relevant discussion in Sect. 1).

2.8 Multiplet 3: $(8, 1/2^+) = (1440, 1600, 1660, 1690)$

The $N$, $\Lambda$ and $\Sigma$ members of the considered octet are well-established: $N(1440)$ has a four-star rating in the RPP and $\Lambda(1600)$ and $\Sigma(1660)$ have a three-star rating. Ignoring the one-star $\Xi(1620)$, the RPP contains only one candidate for the $\Xi$ member of octet 3, the $\Xi(1690)$ with a three-star rating and unknown spin and parity. Therefore, we assume that $\Xi(1690)$ belongs to the considered octet, see also [44].

The masses of the baryons from octet 3 are known with a significant uncertainty since different analyses reporting these baryons give rather different predictions for the masses. Taking the average RPP values, the mismatch between the left and right hand sides of the GMO mass formula is $\Delta M = -50$ MeV, which is $\approx 3\%$ of the mass of $N(1440)$. If one takes the actual measured values for the masses, $m_N = 1462 \pm 10$ MeV [33], $m_\Lambda = 1568 \pm 20$ MeV and $m_\Sigma = 1670 \pm 10$ MeV [36], and $m_\Xi = 1690$ (the RPP average), the mismatch becomes only $\Delta M = -17.5$ MeV. Therefore, while the precision of the GMO mass formula for octet 3 is worse than for the previously considered cases of octets 1, 12 and 15, it is still at a few percent level, i.e. rather high.

The Weldon’s relation among the total widths is at most qualitative

$$\frac{1}{2} (\Gamma_N + \Gamma_\Xi) = 211 \text{ MeV } \text{ vs. } \frac{1}{4} (3 \Gamma_\Lambda + \Gamma_\Sigma) = 125 \text{ MeV}. \quad (49)$$

Again, like in the case of octet 9, light masses of the considered octet might explain the failure of the Weldon’s relation.

Based on the observation that the GMO mass formula works only with a $\approx 3\%$ accuracy for octet 3, it was argued by Diakonov and Petrov [44] that this serves as an indication of the mixing between octet 3 and the antidecuplet (the mixing takes place for $N$ and $\Sigma$ states of the octet and the antidecuplet). While, indeed, mixing with the antidecuplet, whose $N$ and $\Sigma$ states are heavier than $N(1440)$ and $\Sigma(1660)$, respectively, might improve the accuracy of the generalized GMO mass formula, the $\chi^2$ analysis of the decays of octet 3 is very weakly affected by the possible mixing.

As we shall show in Sect. 4, the current data on the decays of the antidecuplet strongly favor mixing of the antidecuplet with some non-exotic or exotic multiplet. Assuming that the antidecuplet has $J^P = 1/2^+$ as predicted in the chiral quark soliton model [2], the
antidecuplet can mix with octets 1, 3 and 4 (decuplet 19 is probably too heavy to be mixed with the antidecuplet). In Sect. 4, we consider the scenario that the antidecuplet mixes with octet 3. It is important to emphasize that since the resulting mixing angle is small and the intrinsic SU(3) coupling constant for the antidecuple is expected to be small, mixing with the antidecuplet does not affect the decay rates of octet 3. Therefore, in the following analysis, we consider decays of octet 3 as if it were unmixed.

| Mass and width (MeV) | Observables | Experiment (MeV) | SU(3) pred. (MeV) |
|---------------------|-------------|-----------------|-------------------|
| N(1462)            | \( \Gamma_{N\pi} \) | 269.8 \( \pm \) 26.2 \times 2.5 | 152.1 |
| \( \Gamma = 391 \pm 34 \) | \( \Gamma_{\Delta\pi} \) | 86.0 \( \pm \) 12.3 \times 1.5 | 86.0 |
| \( \Lambda(1568) \) | \( \Gamma_{\Lambda\pi} \) | 26.7 \( \pm \) 6.5 | 28.7 |
| \( \Gamma = 116 \pm 20 \) | \( \sqrt{\Gamma_{N\pi} \Gamma_{\Sigma\pi}} \) | \(-18.6 \pm 5.6 \times 2.5 \) | \(-34.8 \) |
| & \( \Gamma_{\Sigma\pi} \) | | 42.3 |
| & \( \Gamma_{\Sigma(1385)\pi} \) | | 70.4 |
| \( \Sigma(1670) \) | \( \Gamma_{\Lambda\pi} \) | | |
| \( \Gamma = 152 \pm 20 \) | \( \Gamma_{\Sigma\pi} \) | 18.2 \( \pm \) 5.1 | 18.8 |
| & \( \sqrt{\Gamma_{N\pi} \Gamma_{\Sigma\pi}} \) | \(-24.3 \pm 5.6 \) | \(-20.3 \) |
| & \( |\sqrt{\Gamma_{N\pi} \Gamma_{\Lambda\pi}}| \) | < 6.1 | \(-27.5 \) |
| & \( \Gamma_{\Sigma\pi} \) | 21.8 |
| & \( \Gamma_{\Lambda\pi} \) | 40.1 |
| & \( \Gamma_{\Sigma(1385)\pi} \) | 43.4 |
| \( \Xi(1690) \) | \( \Gamma_{\Xi\pi} \) | | |
| \( \Gamma < 30 \) | \( \Gamma_{\Xi(1530)\pi} \) | 11.5 | 2.7 |

Table 11
SU(3) analysis of \((8, 1/2^+)=(1440, 1600, 1660, 1690)\).

Using the experimental values and errors for the decay rates of \(N(1440)\) [33] and \(\Lambda(1600)\) [34], we find that the \(\chi^2\) fit fails because it is impossible to simultaneously accommodate a very wide \(N(1440)\) with a large branching into the \(N\pi\) final state with moderate partial decay widths of \(\Lambda(1600)\). In order to obtain sensible results of the \(\chi^2\) fit, we increase the experimental errors on \(\Gamma_{N(1440)\rightarrow N\pi}\) and \(\sqrt{\Gamma_{\Lambda(1600)\rightarrow N\pi} \Gamma_{\Lambda(1600)\rightarrow \Sigma\pi}}\) by the factor 2.5.

While the exact value of the multiplication factor is somewhat arbitrary, it is clear that the experimental information on the decays of \(N(1440)\) and \(\Lambda(1600)\) is ambiguous.
ous [10]. In particular, different analyses of \( N(1440) \) reporting its total width and the branching ratio into the \( N\pi \) final state give conflicting results; the experimental value of \( \sqrt{\Gamma_{\Lambda(1660)} \to N\pi} \Gamma_{\Lambda(1660)} \to \Sigma\pi \), which comes from our standard source [34] and which we used in our analysis, is much smaller than all other measurements. One way to take the mentioned experimental inconsistency into account is to introduce the multiplication factor as we did, see also [42]. Another possibility would be to replace the \( \chi^2 \) criterion of the SU(3) testing by a new parameter-testing criterion, which would also help to eliminate ”bugs” in experiments and their analysis [45].

Table 11 summarizes the results of our \( \chi^2 \) fit to six observables of the considered octet. The barrier factor is calculated with \( l = 1 \) for the \( 8 \to 8 + 8 \) and \( 8 \to 10 + 8 \) decays. As one can see from Table 11, SU(3) describes the decay rates of octet 3 fairly well, except for the \( \Gamma_{N(1440)} \to N\pi \) and \( |\sqrt{\Gamma_{N\pi}}\Gamma_{\Lambda\pi}| \) of \( \Sigma(1660) \). However, one has to admit that the latter observable is poorly known [10].

The \( \chi^2 \) fits to the underlined observables in Table 11 give

\[
A_8 = 32.4 \pm 2.5, \quad \alpha = 0.27 \pm 0.03, \quad \chi^2/d.o.f. = 4.75/3, \\
A'_8 = 229. \pm 16.4, \quad \chi^2 = 0. 
\]

The value of \( \alpha \) qualitatively agrees with the SU(6) prediction \( \alpha = 0.4 \), see Table 6.

We conclude that SU(3) works sufficiently well for octet 3.

2.9 Multiplet 4: (8, 1/2+) = (1710, 1810, 1880, 1950)

The \( N, \Lambda \) and \( \Sigma \) members of the considered octet are established, see Table 2. The \( N(1710) \) and \( \Lambda(1810) \) have a three-star rating and \( \Sigma(1880) \) has a two-star rating and the known spin and parity. The \( \Xi \) member of octet 4 is missing. We shall estimate its mass using the GMO mass formula.

The total width of \( N(1710) \) is known with a large ambiguity. In order to use the same experimental sources for all multiplets, we use for \( \Gamma_{tot} = 480 \pm 230 \) MeV [33], which is much larger than the values of \( \Gamma_{tot} \) from other experiments and the RPP average.

Table 12 summarizes the results of our \( \chi^2 \) fit to six observables of octet 4. The barrier factor is calculated with \( l = 1 \) for the \( 8 \to 8 + 8 \) and \( 8 \to 10 + 8 \) decays.

The \( \chi^2 \) fits to the underlined observables in Table 12 give

\[
A_8 = 14.9 \pm 1.1, \quad \alpha = 0.32 \pm 0.03, \quad \chi^2/d.o.f. = 1.73/2, 
\]
| Mass and width (MeV) | Observables     | Experiment (MeV) | SU(3) pred. (MeV) |
|----------------------|-----------------|------------------|-------------------|
| $N(1717)$            | $\Gamma_{N\pi}$ | $43.2 \pm 28.2$  | 80.0              |
| $\Gamma = 480 \pm 230$ | $\Gamma_{\Delta\pi}$ | $187.2 \pm 97.6$ | 56.4              |
|                     | $\Gamma_{N\eta}$ | $28.8 \pm 14.6$  | 0.3               |
| $\Lambda(1841)$     | $\Gamma_{N\overline{K}}$ | $39.4 \pm 8.1$  | 37.9              |
| $\Gamma = 164 \pm 20$ | $\sqrt{\Gamma_{N\overline{K}}\Gamma_{\Sigma\pi}}$ | $-39.4 \pm 8.1$ | $-34.6$            |
|                     | $\Gamma_{\Sigma(1385)\pi}$ | $22.1 \pm 25.1$ | 36.0              |
|                     | $\Gamma_{\Sigma\pi}$ |                   | 31.6              |
|                     | $\Gamma_{\Lambda\pi}$ |                   | 3.9               |
| $\Sigma(1826)$      | $\Gamma_{N\overline{K}}$ | $5.1 \pm 1.9$   | 5.0               |
| $\Gamma = 85 \pm 15$ | $\Gamma_{\Sigma\pi}$ |                   | 13.6              |
|                     | $\Gamma_{\Lambda\pi}$ |                   | 13.0              |
|                     | $\Gamma_{\Sigma(1385)\pi}$ |                   | 7.3               |
| $\Xi(1950)$         | $\Gamma_{\Xi\pi}$ |                   | 5.9               |
|                     | $\Gamma_{\Sigma\overline{K}}$ |                   | 32.4              |
|                     | $\Gamma_{\Xi(1530)\pi}$ |                   | 9.1               |

|Table 12|

SU(3) analysis of $(8, 1/2^+) = (1710, 1810, 1880, 1950)$.  

$A'_8 = 44.9 \pm 14.4$,  \quad \chi^2/d.o.f. = 2.02/1. \quad (51)$

The value of $\alpha$ compares well to the SU(6) prediction $\alpha = 0.4$, see Table 6.

Using the GMO mass formula for the considered octet, we find that the mass of the missing $\Xi$ state of the octet should be around 1950 MeV (this value is obtained using either the average RPP values for the masses or the values used in Table 12). Note that the $\Xi(1950)$, which exists in the RPP and which we assigned to octet 12, does not fit octet 4 because, for instance, SU(3) predicts a very large $\Gamma_{\Sigma\overline{K}}/\Gamma_{\Lambda\overline{K}}$, which contradicts the data on the $\Xi(1950)$ [10].

Our attempt to estimate the total width of the predicted $\Xi(1950)$ using the Weldon’s relation failed: the central value of the $N(1710)$ total width, $\Gamma_{N(1710)} = 480$ MeV, is way too large compared to the total widths of $\Lambda(1810)$ and $\Sigma(1880)$. However, since the total width of $N(1710)$ is very uncertain, the failure of the Weldon’s relation does not
mean that the particle assignment for the octet is wrong – it rather means that the large uncertainties in the measured total widths preclude the use of the Weldon’s formula.

2.10 Multiplet 17: \((8, 3/2^+) = (1720, 1890, 1840, 2035)\)

The \(N\) and \(\Lambda\) members of the considered octet are established, see Table 2. They have a four-star rating in the RPP. An examination of the list of available \(\Sigma\) baryons shows that the only candidate for the \(\Sigma\) member of the considered octet is \(\Sigma(1840)\) with a one-star rating and with the proper \(J^P = 3/2^+\). The \(\Xi\) member of octet 17 is missing. We shall estimate its mass using the GMO mass formula.

The masses and total widths of the baryons in this octet are known rather poorly. In our \(\chi^2\) analysis, for \(N(1720)\) we use the results of [33] and for \(\Lambda(1890)\) we use the results of [36].

Table 13 summarizes the results of our \(\chi^2\) fit to three observables of octet 4. The barrier factor is calculated with \(l = 1\) for the \(8 \to 8 + 8\) decays. Since the \(8 \to 10 + 8\) decays of \(N(1720)\) and \(\Lambda(1890)\) are known very poorly, we do not attempt to fit them. Therefore, we do not determine the \(A_8'\) coupling constant.

| Mass and width (MeV) | Observables | Experiment (MeV) | SU(3) pred. (MeV) |
|----------------------|-------------|-----------------|-------------------|
| \(N(1717)\)         | \(\Gamma_{N\pi}\) | 49.4 ± 30.1      | 16.2              |
| \(\Gamma = 380 \pm 180\) | \(\Gamma_{N\eta}\) | 1.0              |                   |
| \(\Lambda(1897)\)   | \(\Gamma_{N\bar{K}}\) | 14.8 ± 2.5       | 14.9              |
| \(\Gamma = 74 \pm 10\) | \(\sqrt{\Gamma_{N\bar{K}}\Gamma_{\Sigma\pi}}\) | -6.7 ± 2.4       | -7.0              |
|                      | \(\Gamma_{\Sigma\pi}\) |                 | 3.2               |
| \(\Sigma(1840)\)    | \(\Gamma_{N\bar{K}}\) | 0.1              |                   |
| \(\Gamma = 65 - 120\) | \(\Gamma_{\Sigma\pi}\) | 8.5              |                   |
|                      | \(\Gamma_{\Lambda\pi}\) | 1.2              |                   |
| \(\Xi(2035)\)       | \(\Gamma_{\Lambda\bar{K}}\) | 2.2              |                   |
|                      | \(\Gamma_{\Sigma\bar{K}}\) | 9.8              |                   |

Table 13

SU(3) analysis of \((8, 3/2^+) = (1720, 1890, 1840, 2035)\).
The $\chi^2$ fits to three observables of octet 17 gives

$$A_8 = 6.7 \pm 0.7, \quad \alpha = 0.56 \pm 0.12, \quad \chi^2/d.o.f. = 1.23/1. \quad (52)$$

The value of $\alpha$ compares well to the SU(6) prediction $\alpha = 0.4$, see Table 6.

It is important to note that since the resulting value of $\alpha$ is so close to 0.5, where the coupling constant $g_{\Sigma \to N}\pi$ vanishes, small variations in $\alpha$ lead to large variations in the predicted $\Gamma_{\Sigma \to N}\pi$, see the relevant discussion in [6]. Therefore, the SU(3) predictions for $\Gamma_{\Sigma \to N}\pi$ in Table 13 should not be taken too literally.

Substituting $m_N = 1720$ MeV, $m_\Lambda = 1890$ MeV and $m_\Sigma = 1840$ MeV in the GMO mass formula, we determine the mass of the missing $\Xi$ member of the considered octet, $m_\Xi = 2035$ MeV.

As a consequence of small fitted partial decay widths of the considered octet, SU(3) predicts that all partial decay widths in Table 13 are rather small. As a result, the total width of the predicted $\Xi(2035)$ appears to be of the order of 10 MeV. However, if $8 \to 10 + 8$ decays contribute significantly to the large total width of $N(1720)$ (the present experimental information is uncertain), then this would automatically imply that the $8 \to 10 + 8$ partial decay widths of all baryons in the considered octet are large. Naturally, this would significantly increase our estimate of the total width of the predicted $\Xi(2035)$. Until the situation with the $8 \to 10 + 8$ decays is settled, we hypothesize that $\Xi(2035)$ is rather narrow. One should also mention that the Weldon’s relation among the total widths is violated: $N(1720)$ appears to be too wide for the octet. Note, however, that $\Gamma_{N(1720)}$ is very uncertain [10] and, hence, because of this, the Weldon’s formula cannot be used.

2.11 Multiplet 14: $(8, 1/2^-)=(1650, 1800, 1620, 1860-1915)$

The $N$ and $\Lambda$ members of the considered octet are well-established and well-studied experimentally: $N(1650)$ and $\Lambda(1800)$ have a four-star and three-star rating in the RPP, respectively. For the $\Sigma$ member of the considered octet, the RPP contains several candidates in the appropriate mass range. Table 2 assigns the $\Sigma(1750)$ to octet 14. However, our $\chi^2$ analysis demonstrates that $\Sigma(1750)$ cannot belong to octet 14 because SU(3) predicts the positive $\sqrt{\Gamma_{N}\Gamma_{\Sigma}\Gamma_{\pi}}$, while the experiment gives a negative value for this observable. We show that a good $\chi^2$ value is obtained only if the $\Sigma$ member of the considered octet is identified with the $\Sigma(1620)$ with a two-star rating and the known spin and parity. Since the $\Xi$ state of octet 14 is missing, we predict its mass using the GMO mass formula.

We begin our $\chi^2$ analysis by fitting the four underlined in Table 12 decay rates of $N(1650)$ and $\Lambda(1800)$. Since the important for the fit branching ratio of the $N \to N\pi$ decay is
known with large experimental uncertainty, we use the RPP estimates for $\Gamma_{\text{tot}}$ and $Br(N\pi)$ of $N(1650)$. After the initial $\chi^2$ fit is successful, we test the candidates for the $\Sigma$ member of octet 14 by adding their decay rates to the $\chi^2$ fit. The best result is obtained with the $\Sigma(1620)$ when its decay properties are taken from [46].

As to the other two candidates, SU(3) significantly overestimates the $\Gamma_{N\pi}/\Gamma_{\Lambda\pi}$ and $\Gamma_{\Sigma\pi}/\Gamma_{\Lambda\pi}$ of $\Sigma(1690)$ and predicts the positive $\sqrt{\Gamma_{N\pi}\Gamma_{\Sigma\pi}}$ for $\Sigma(1750)$, which results in an unacceptably large value of $\chi^2$ because the experiment gives a negative $\sqrt{\Gamma_{N\pi}\Gamma_{\Sigma\pi}}$ with a sufficiently small error.

Table 14 summarizes the results of our $\chi^2$ fit to eight observables of the considered octet, when $\Sigma(1620)$ is used. The barrier factor is calculated with $l = 0$ for the $8 \rightarrow 8 + 8$ decays and with $l = 2$ for the $8 \rightarrow 10 + 8$ decays.

The $\chi^2$ fits to the underlined observables in Table 14 give

$$A_8 = 8.3 \pm 0.5, \quad \alpha = 0.79 \pm 0.05, \quad \chi^2/\text{d.o.f.} = 3.99/4,$$
$$A'_8 = 30.4 \pm 8.8, \quad \chi^2/\text{d.o.f.} = 0.08/1. \quad (53)$$

The value of $\alpha$ qualitatively agrees with the SU(6) prediction $\alpha = 0.625$, see Table 6.

The mass of the missing $\Xi$ member of the octet is estimated using the GMO mass formula. Using the RPP estimates for the masses of $N$, $\Lambda$ and $\Sigma$ states, we obtain $m_{\Xi} = 1860$ MeV. However, the mass of $\Lambda(1800)$ is known with a large uncertainty. Using for the masses the values used in Table 14, we obtain $m_{\Xi} = 1913$ MeV. Therefore, based on our SU(3) analysis of octet 14, we predict the existence of a new $\Xi$ resonance with $J^P = 1/2^-$, the mass in the 1860-1915 MeV range and large branching ratios to all allowed decay channels, see Table 14. In addition, using the Weldon’s relation for the total widths, one can estimate the total width of the predicted $\Xi$. Using the central values for the total widths listed in Table 14, we derive from Eq. (23) that $\Gamma_{\Xi(1860-1915)} \approx 220$ MeV.

### 2.12 Multiplet 11: $(8, 3/2^-) = (1700, 1850, 1940, 2045)$

Octet 11 is remarkable because this is the only octet in our scheme, which is missing the $\Lambda$ member. All other eleven $\Lambda$ resonances, which are required to complete the picture of light SU(3) multiplets in Table 1, are known very well and have three and four-star ratings in the RPP.

The octet opens with the well-established three-star $N(1700)$. In addition, in the required mass range, one can offer a candidate for the $\Sigma$ member of the considered octet – the three-star $\Sigma(1940)$. Since octet 11 lacks two states, we cannot use the GMO mass formula.
| Mass and width (MeV) | Observables | Experiment (MeV) | SU(3) pred. (MeV) |
|----------------------|-------------|-----------------|-------------------|
| $N(1659)$            | $\Gamma_{N\pi}$ | $108.8 \pm 26.8$ | 68.2              |
| $\Gamma = 150 \pm 10$ | $\sqrt{\Gamma_{N\pi} \Gamma_{\Lambda K}}$ | $-33.0 \pm 4.4$ | $-31.7$          |
|                     | $\Gamma_{\Delta \pi}$ | $2.8 \pm 1.9$ | 2.4               |
|                     | $\Gamma_{N\eta}$ | $12.1 \pm 3.1$ | 22.7              |
| $\Lambda(1841)$     | $\Gamma_{N\Xi}$ | $82.1 \pm 11.6$ | 92.8              |
| $\Gamma = 228 \pm 20$ | $\sqrt{\Gamma_{N\Xi} \Gamma_{\Sigma \pi}}$ | $-18.2 \pm 11.5$ | $-17.7$          |
|                     | $\Gamma_{\Sigma(1385)\pi}$ | $2.0 \pm 2.0$ | 2.4               |
|                     | $\Gamma_{\Sigma \pi}$ |                     | 3.4               |
|                     | $\Gamma_{\Xi \pi}$ |                     | 15.2              |
| $\Sigma(1620)$      | $\Gamma_{N\Xi}$ | $14.3 \pm 4.59$ | 10.4              |
| $\Gamma = 65 \pm 20$ | $\sqrt{\Gamma_{N\Xi} \Gamma_{\Sigma \pi}}$ | $26.0 \pm 8.9$ | 27.9              |
|                     | $\Gamma_{\Sigma \pi}$ |                     | 75.0              |
|                     | $\Gamma_{N\Xi}$ |                     | 10.4              |
| $\Xi(1860 - 1915)$  | $\Gamma_{\Xi \pi}$ |                     | 19.9              |
|                     | $\Gamma_{\Lambda \pi}$ |                     | 31.1              |
|                     | $\Gamma_{\Sigma \pi}$ |                     | 53.8              |
|                     | $\Gamma_{\Xi \eta}$ |                     | 27.7              |

Table 14
SU(3) analysis of $(8, 1/2^-)=(1650, 1800, 1620, 1860-1915)$.

to estimate the mass of the missing $\Lambda$. Instead, we notice that for the seven unmixed octets considered so far, the mass difference between the $N$ and $\Lambda$ baryons is on average 150 MeV. Therefore, we assume that the mass of the missing $\Lambda$ hyperon of octet 11 is 1850 MeV. Note that the mass of $N(1700)$, which we use as a reference point, is itself known with a large uncertainty: $m_{N(1700)} = 1650 - 1750$ MeV according to the RPP estimate [10]. Therefore, the uncertainty in the predicted mass of $\Lambda(1850)$ is approximately 50 MeV.

The mass of the missing $\Xi$ member of the octet is then estimated using the GMO mass formula. Using $m_N = 1700$ MeV, $m_\Sigma = 1940$ MeV and $m_\Lambda = 1850$ MeV, we obtain $m_\Xi = 2045$ MeV.
Next we turn to two-body hadronic decays. Since for \( N(1700) \) the total width and the important branching into the \( N\pi \) final state are known with large ambiguity, we use the RPP estimates in our \( \chi^2 \) analysis. Note that according to the analysis of [34], both \( \sqrt{\Gamma_{NK}\Gamma_{\Lambda\pi}} \) and \( \sqrt{\Gamma_{NK}\Gamma_{\Sigma\pi}} \) of \( \Sigma(1940) \) are negative. This contradicts SU(3): expecting that \( \alpha \) will be close to its SU(6) prediction \( \alpha = -1/2 \), we notice that SU(3) requires that the signs of \( \sqrt{\Gamma_{NK}\Gamma_{\Lambda\pi}} \) and \( \sqrt{\Gamma_{NK}\Gamma_{\Sigma\pi}} \) should be opposite, see Table 4. SU(3) also requires the opposite signs, if \( \Sigma(1940) \) belongs to a decuplet. Therefore, we reverse the sign of \( \sqrt{\Gamma_{NK}\Gamma_{\Sigma\pi}} \). This is consistent with the analysis [47], which reports the positive value for \( \sqrt{\Gamma_{NK}\Gamma_{\Sigma\pi}} \), which is somewhat larger (no errors are given) than the value from the analysis [34].

Table 15 summarizes the results of our \( \chi^2 \) fit to three observables of the considered octet. The barrier factor is calculated with \( l = 2 \).

The \( \chi^2 \) fit to the five underlined observables in Table 15 gives

\[
A_8 = 8.3 \pm 3.5, \quad \alpha = -0.70 \pm 0.54, \quad \chi^2/\text{d.o.f.} = 0.42/1, \\
A'_8 = 67.2 \pm 31.0, \quad \chi^2/\text{d.o.f.} = 0.8/1. \quad (54)
\]

The central value of \( \alpha \) compares well to its SU(6) prediction \( \alpha = -1/2 \). However, since we used only two fitted observables, which depend on \( \alpha \), the error on the obtained value of \( \alpha \) is large.

Note that in order to convert the experimentally measured \( \sqrt{\text{Br}(N\overline{K})\text{Br}(\Delta\overline{K})} \) of \( \Sigma(1940) \) into the corresponding \( \Gamma_{\Delta\overline{K}} \) used in the fit, we used the SU(3) prediction \( \Gamma_{\Sigma(1940)\rightarrow N\overline{K}} = 37.4 \text{ MeV} \). Also, we chose to fit only the D-wave \( (l = 2) \) \( 8 \rightarrow 10 + 8 \) decays because the S-wave \( N(1700) \rightarrow \Delta\pi \) branching ratio is rather uncertain and is smaller than the corresponding D-wave branching [10].

An examination of SU(3) predictions in Table 15 shows that the sum of predicted two-body hadronic decay widths significantly underestimates the known total widths of \( N(1700) \) and \( \Sigma(1940) \): \( \Gamma_{N(1700)}^{\text{SU(3)}} = 28 \text{ MeV} \) vs. \( \Gamma_{N(1700)} = 100 \pm 50 \text{ MeV} \) and \( \Gamma_{\Sigma(1940)}^{\text{SU(3)}} = 77 \text{ MeV} \) vs. \( \Gamma_{\Sigma(1940)} = 300 \pm 80 \text{ MeV} \). In both cases, the central value of the total width is underestimated by the factor of \( 3.5 - 4 \). Therefore, in order to obtain a realistic estimate for the total width of the predicted \( \Lambda(1850) \), we simply multiply the sum of the SU(3) predictions for the two-body hadronic decays by the factor four

\[
\Gamma_{\Lambda(1850)} = 4 \times 32 \text{ MeV} \approx 128 \text{ MeV}. \quad (55)
\]

We can estimate the total width of the predicted \( \Xi(2045) \) in a similar way

\[
\Gamma_{\Xi(2045)} = 4 \times 66 \text{ MeV} = 264 \text{ MeV}. \quad (56)
\]
Now that all total widths are in place, one can check how accurately Weldon’s relations for the total widths are satisfied. Substituting the central values of the total widths into Eq. (23), we obtain

\[ \frac{1}{2} (\Gamma_N + \Gamma_\Xi) = 182 \text{ MeV} \quad \text{vs.} \quad \frac{1}{4} (3 \Gamma_\Lambda + \Gamma_\Sigma) = 171 \text{ MeV}. \] (57)

While the nice agreement seen in Eq. (57) should not be taken literally because of large

| Mass and width (MeV) | Observables         | Experiment (MeV) | SU(3) pred. (MeV) |
|----------------------|---------------------|------------------|-------------------|
| \( N(1700) \)       | \( \Gamma_{N\pi} \) | 10.0 ± 7.1       | 7.9               |
| \( \Gamma = 100 \pm 50 \) | \( \Gamma_{N\eta} \) | 0 ± 1            | 2.0               |
| \( \Delta_\pi, D\text{-wave} \) |                      | 14.4 ± 17.0      | 17.9              |
| \( \Lambda(1850) \)  | \( \Gamma_{N\overline{K}} \) |               | 0.2               |
|                      | \( \Gamma_{\Sigma\pi} \) |               | 17.8              |
|                      | \( \Gamma_{\Lambda\eta} \) |               | 1.2               |
|                      | \( \Gamma_{\Sigma(1385)\pi} \) |           | 12.8              |
| \( \Sigma(1940) \)   | \( \sqrt{\Gamma_{N\overline{K}}\Gamma_{\Sigma\pi}} \) | 24.0 ± 13.6     | 18.4              |
| \( \Gamma = 300 \pm 80 \) | \( \sqrt{\Gamma_{N\overline{K}}\Gamma_{\Lambda\pi}} \) | −18.0 ± 10.2    | −22.2             |
|                      | \( \Gamma_{\Delta_\pi\overline{K}}, D\text{-wave} \) | 47.2 ± 42.0     | 10.6              |
|                      | \( \Gamma_{\Sigma\pi} \) |               | 9.1               |
|                      | \( \Gamma_{\Lambda\pi} \) |               | 13.1              |
|                      | \( \Gamma_{N\overline{K}} \) |               | 37.4              |
|                      | \( \Gamma_{\Sigma(1385)\pi} \) |           | 5.4               |
| \( \Xi(2045) \)      | \( \Gamma_{\Xi\pi} \) |               | 39.5              |
|                      | \( \Gamma_{\Lambda\overline{K}} \) |               | 12.3              |
|                      | \( \Gamma_{\Sigma\overline{K}} \) |            | 4.8               |
|                      | \( \Gamma_{\Xi(1530)\pi} \) |           | 6.7               |
|                      | \( \Gamma_{\Sigma(1385)\overline{K}} \) |         | 2.6               |

Table 15

SU(3) analysis of \( (8, \frac{3}{2}^-) \)=(1700, 1850, 1940, 2045).
uncertainties in the measured total widths of \( N(1700) \) and \( \Sigma(1940) \), it still illustrates that our method of estimating the total widths of \( \Lambda(1850) \) and \( \Xi(2045) \) has a certain merit.

A remarkable property of \( \Lambda(1850) \) is its vanishingly small coupling to the \( NK \) state, see Table 15. This is a consequence of the fact that \( \alpha = -0.70 \pm 0.54 \), which strongly suppresses the \( g_{\Lambda(1850) \to NK} \) coupling constant, \( g_{\Lambda \to NK} = \sqrt{2/3(2\alpha + 1)}A_8 \), see Table 4. In the SU(6) limit, \( \alpha = -1/2 \), see Table 6, which leads to \( g_{\Lambda \to NK} = 0 \). Therefore, our prediction that \( \Lambda(1850) \) very weakly couples to the \( NK \) final state is rather model-independent.

As follows from Table 15, SU(3) predicts that the \( \Lambda(1850) \) has significant branching ratios into the \( \Sigma \pi \) and \( \Sigma(1385)\pi \) final states. This suggests that one should experimentally search for the \( \Lambda(1850) \) in production reactions using the \( \Sigma \pi \) and \( \Sigma(1850)\pi \) invariant mass spectrum.

The existence of a new \( \Lambda \) hyperon with \( J^P = 3/2^- \) was predicted in different constituent quark models. In 1978, Isgur and Karl predicted that the new \( \Lambda \) has the mass 1880 MeV and a very small coupling to the \( NK \) state. The latter fact is a consequence of SU(6) selection rules and explains why this state was not observed in the \( NK \) partial wave analyses [34–36]. In a subsequent analysis, Isgur and Koniuk explicitly calculated the partial decay widths of the \( \Lambda(1880) \) and found that \( \Gamma_{NK} \) is small, while \( \Gamma_{\Sigma\pi} \) and \( \Gamma_{\Sigma(1385)\pi} \) are dominant [48].

More recent calculations within the constituent quark model framework also predict the existence of a new \( \Lambda \) with \( J^P = 3/2^- \), but with somewhat different masses: the analysis of L"oring, Metsch and Petry [8] (model A) gives 1775 MeV; the analysis of Glozman, Plessas, Varga and Wagenbrunn [9] gives \( \approx 1780 \) MeV. Note also that the analysis [8] predicts that the \( \Lambda \) very weakly couples to the \( NK \) state.

We would like to emphasize that while many results concerning the new \( \Lambda \) were previously derived in specific constituent quark models with various assumptions about the quark dynamics, we demonstrate that they are actually model-independent and follow directly from flavor SU(3) symmetry.

In conclusion, the existence of a new \( \Lambda \) hyperon with \( J^P = 3/2^- \) is required by the general principle of the flavor SU(3) symmetry of strong interactions. Our SU(3) analysis predicts that its mass is \( \approx 1850 \) MeV and the total width is \( \approx 130 \) MeV. We predict that \( \Lambda(1850) \) has a very small coupling to the \( NK \) state and large branching ratios into the \( \Sigma\pi \) and \( \Sigma(1385)\pi \) final states. Therefore, \( \Lambda(1850) \) can be searched for in production reactions by studying the \( \Sigma\pi \) and \( \Sigma(1385)\pi \) invariant mass spectra. The fact that the total width of \( \Lambda(1850) \) is not larger than \( \approx 130 \) MeV makes the experimental search feasible. In addition, in order to have the complete octet, we predict the existence of a new \( \Xi \) baryon with \( J^P = 3/2^- \), the mass \( \approx 2045 \) MeV and the total width \( \approx 265 \) MeV.
3 SU(3) classification of decuplets

In this section, we perform the SU(3) classification of eight decuplets of Table 1. Since the \( \Delta \) baryons that open those decuplets are well-established, SU(3) requires the existence of the corresponding \( \Sigma, \Xi \) and \( \Omega \) members of the considered decuplets. In many cases those states are missing – SU(3) then makes predictions for their spin and parity and estimates of their masses and decay widths. Similarly to the analysis of the octets presented above, the main tools of our analysis are the equal spacing rule for the mass splitting in a given decuplet, Eq. (7), and SU(3) predictions for the partial decay widths.

The bulk of the experimental information on two-body hadronic decays of decuplets comes from \( 10 \to 8 + 8 \) decays. In the SU(3) limit, the \( B_1 \to B_2 P \) coupling constants are parameterized in terms of a single universal constant \( A_{10} \)

\[
g_{B_1 B_2 P} = A_{10} \left( \begin{array}{c|c} 8 & 8 \\ \hline Y_2 Y_2 & 10 \\ \hline Y_2 T_2 & Y_1 T_1 \end{array} \right) . \tag{58} \]

In addition, there is less precise and complete information on \( 10 \to 10 + 8 \) decays, whose coupling constants are parameterized in terms of a constant \( A'_{10} \)

\[
g_{B_1 B_2 P} = A'_{10} \left( \begin{array}{c|c} 10 & 8 \\ \hline Y_2 T_2 & 10 \\ \hline Y_2 T_2 & Y_1 T_1 \end{array} \right) . \tag{59} \]

The coupling constants for all possible decay channels of decuplets are summarized in Table 16.

The SU(3) classification of decuplets is somewhat more ambiguous than for the octets [6]. Since there are fewer free parameters for the \( \chi^2 \) fit to the decays rates into the \( 8 + 8 \) final state (\( A_{10} \) for decuplets vs. \( A_8 \) and \( \alpha \) for octets), we expect that the \( \chi^2 \) fit should be somewhat less successful. We begin our analysis by first considering the decuplets already analyzed in [6].

3.1 Multiplet 2: \( (10, 3/2^+)=(1232, 1385, 1530, 1672) \)

Naturally, the ground-state decuplet (decuplet 2) is very well established (all its states have a four-star rating in the RPP) and its decay rates are known with very high precision. Because of the small experimental errors on the measured two-body hadronic partial decay widths, an attempt of the \( \chi^2 \) fit to the decay rates returns an unacceptably large \( \chi^2 \). At this
| Decay mode             | $g_{B_1B_2P}$       | $g_{B_1B_2P}$       |
|-----------------------|----------------------|----------------------|
| $\Delta \rightarrow N\pi$ | $-(\sqrt{2}/2)A_{10}$ |                      |
| $\rightarrow \Sigma K$   | $(\sqrt{2}/2)A_{10}$  |                      |
| $\rightarrow \Delta\pi$   |                      | $(\sqrt{10}/4)A'_{10}$ |
| $\rightarrow \Delta\eta$   |                      | $-(\sqrt{2}/4)A'_{10}$ |
| $\rightarrow \Sigma^*K$   |                      | $1/2A'_{10}$         |
| $\Sigma \rightarrow \Lambda\pi$ |                     | $-1/2A_{10}$         |
| $\rightarrow \Sigma\pi, \Xi K$ |                     | $(\sqrt{6}/6)A_{10}$  |
| $\rightarrow N\overline{K}$   |                     | $-(\sqrt{6}/6)A_{10}$  |
| $\rightarrow \Sigma\eta$   |                     | $1/2A_{10}$          |
| $\rightarrow \Sigma^*\pi, \Xi^*K, \Delta\overline{K}$ |                     | $(\sqrt{3}/6)A'_{10}$  |
| $\rightarrow \Sigma^*\eta$   |                     | $0$                 |
| $\Xi \rightarrow \Xi\pi, \Xi\eta, \Sigma\overline{K}$ |                     | $1/2A_{10}$         |
| $\rightarrow \Lambda\overline{K}$   |                     | $-1/2A_{10}$         |
| $\rightarrow \Xi^*\pi, \Xi^*\eta$ |                     | $(\sqrt{2}/4)A'_{10}$  |
| $\rightarrow \Sigma^*\overline{K}$   |                     | $(\sqrt{2}/2)A'_{10}$  |
| $\rightarrow \Omega K$   |                     | $1/2A'_{10}$         |
| $\Omega \rightarrow \Xi\overline{K}$   |                     | $A_{10}$             |
| $\rightarrow \Xi^*\overline{K}, \Omega\eta$ |                     | $(\sqrt{2}/2)A'_{10}$  |

Table 16
The SU(3) universal coupling constants for $10 \rightarrow 8 + 8$ and $10 \rightarrow 10 + 8$ decays.

point, we have two options. First, one can increase the experimental errors in several times because one should not expect that SU(3) works with a few percent accuracy. Second, one can try other models for the kinematic phase space factor. As shown by Samios et al. [6], there are models for the phase space factor, which give a much smaller $\chi^2$ with the same experimental input. In our analysis, we used the former approach and increased the experimental errors on all four used decay rates of the ground-state decuplet by factor two.

The Gell-Mann–Okubo mass formula (the equal splitting rule) works very well for decuplet 2. Using Eq. (32), one finds that the standard deviation from the average spacing, $\Delta z$, is much smaller than the average spacing $\langle z \rangle$, $\Delta z ≈ 5.7$ MeV and $\langle z \rangle = 146.7$ MeV.
The Weldon’s relation for the total widths of the ground-state decuplet reads [24]

\[
\frac{1}{4} \Gamma_\Delta + \frac{3}{4} \Gamma_\Xi = \frac{1}{4} \Gamma_\Omega + \frac{3}{4} \Gamma_\Sigma. \tag{60}
\]

The use of the RPP values for the total widths gives

\[
\frac{1}{4} \Gamma_\Delta + \frac{3}{4} \Gamma_\Xi = 36.3 \text{ MeV} \quad \text{vs.} \quad \frac{1}{4} \Gamma_\Omega + \frac{3}{4} \Gamma_\Sigma = 26.8 \text{ MeV}. \tag{61}
\]

Note the values of the total widths have changed since 1977, when the agreement of Eq. (60) was much better [24].

| Mass and width (MeV) | Observables | Experiment (MeV) | SU(3) pred. (MeV) |
|----------------------|-------------|------------------|-------------------|
| \(\Delta(1232)\)    | \(\Gamma_{N\pi}\) | \(118 \pm 4 \times 2\) | 100.0             |
| \(\Gamma = 118 \pm 4\) |
| \(\Sigma(1385)\)    | \(\Gamma_{\Lambda\pi}\) | \(31.5 \pm 1.0 \times 2\) | 33.1              |
| \(\Gamma = 35.8 \pm 0.8\) | \(\Gamma_{\Sigma\pi}\) | \(4.30 \pm 0.72 \times 2\) | 5.1               |
| \(\Xi(1530)\)       | \(\Gamma_{\Xi\pi}\) | \(9.1 \pm 0.5 \times 2\) | 10.4              |
| \(\Gamma = 9.1 \pm 0.5\) |
| \(\Omega(1672)\)    |                        |                   |                   |
| \(\Gamma \approx 0\) |

Table 17
SU(3) analysis of \((10, 3/2^+) = (1232, 1385, 1530, 1672)\).

Table 17 summarizes the results of our \(\chi^2\) fit. The barrier factor is calculated with \(l = 1\). The \(\chi^2\) fit gives

\[
A_{10} = 142.1 \pm 3.0, \quad \chi^2/\text{d.o.f.} = 7.59/3. \tag{62}
\]

As discussed in the beginning of this subsection, the goodness of the \(\chi^2\) fit can be improved by using a different phase space factor. For instance, the \(\chi^2\) fit with the phase space factor of Eq. (39) multiplied by the \(M/M_1\) factor, where \(M_1\) is the mass of the decaying baryon and \(M = 1000\) MeV, gives a much lower \(\chi^2\)

\[
A_{10} = 166.2 \pm 3.5, \quad \chi^2/\text{d.o.f.} = 1.46/3. \tag{63}
\]
This conclusion was first obtained in [6]. Note also that the improvement of the $\chi^2$ value by modifying the phase space factor is so dramatic only for decuplet 2. For other decuplets, whose spin and parity are different and decay rates are not known with such good precision, the change of the phase space factor does not systematically lead to the improvement of the fit. Therefore, throughout our analysis, we shall use the phase space factor as given by Eq. (39).

In summary, SU(3) works very well for decuplet 2.

### 3.2 Multiplet 20: $(10, 7/2^+)= (1950, 2030, 2120, 2250)$

Multiplet 20 was considered by Samios et al. [6] in 1974, when only the $\Delta(1950)$ and $\Sigma(2030)$ members of the decuplet were known. Using the equal spacing rule, the new $\Xi(2120)$ and $\Omega(2250)$ resonances were predicted (we use the modern RPP average masses). Later those baryons were discovered: $\Xi(2120)$ has a one-star rating in the RPP and $\Omega(2250)$ has a three-star rating. The spin and parity of the both baryons are not known.

Using the RPP estimates for the masses, we find that the equal spacing rule works with mediocre accuracy for decuplet 20: $\Delta z = 26.5$ MeV and $\langle z \rangle = 100$ MeV. Note that the equal spacing rule for the total widths predicted by the Weldon’s formula does not hold.

Table 18 summarizes the results of our $\chi^2$ fit to the six underlined observables. The barrier factor is calculated with $l = 3$.

The $\chi^2$ fit to the underlined observables in Table 18 gives

$$
A_{10} = 60.7 \pm 1.0, \quad \chi^2/{\text{d.o.f.}} = 4.02/3,
A'_{10} = 94.1 \pm 7.1, \quad \chi^2/{\text{d.o.f.}} = 4.63/1.
$$

Since the $\Xi(2120)$ has only a one-star rating in the RPP, we do not consider the fact that the sum of the predicted partial decays widths is greater than the total widths of $\Xi(2120)$ as a contradiction. If the decay rates of $\Xi(2120)$ are measured in the future, they can be included in the $\chi^2$ fit, which will adjust the resulting $A_{10}$ and $A'_{10}$ to the experiment.

### 3.3 Multiplet 18: $(10, 5/2^+)= (1905, 2070, 2250, 2380)$

The content of multiplet 18 is not established: only the obvious four-star $\Delta(1905)$ member is listed in Tables 1 and 2. However, an examination of the RPP shows that one can offer candidates within a suitable mass range for all members of the considered decuplet. These
| Mass and width (MeV) | Observables | Experiment (MeV) | SU(3) pred. (MeV) |
|----------------------|-------------|----------------|-----------------|
| ∆(1950)              | Γ_{N\pi}    | 114 ± 4        | 115.8           |
| Γ = 300 ± 7          | Γ_{Δ\pi}    | 57.6 ± 8.8     | 58.4            |
| √Γ_{N\pi}Γ_{ΣK}      | -15.9 ± 1.6 | -21.2          |                 |
| Σ(2030)              | Γ_{N\pi}    | 32.7 ± 5.5     | 24.9            |
| Γ = 172 ± 10         | √Γ_{N\pi}Γ_{Λ\pi} | 31.0 ± 3.9 | 30.4 |
|                       | √Γ_{N\pi}Γ_{Σπ} | -25.8 ± 5.4 | -19.9 |
|                       | Γ_{Δ\pi}    | 23.2 ± 9.6     | 2.7             |
|                       | Γ_{Λ\pi}    |                | 31.1            |
| Ξ(2120)              | Γ_{Ξπ}      | 18.8           |                 |
| Γ = 25 ± 12          | Γ_{Λ\pi}    | 23.1           |                 |
|                       | Γ_{Σ\pi}    | 13.1           |                 |
| Ω(2250)              | Γ_{Ξ\pi}    | 55.7           |                 |

Table 18
SU(3) analysis of (10, 7/2+)=(1950, 2030, 2120, 2250).

are the one-star Σ(2070) with \( J^P = 5/2^+ \), the two-star Ξ(2250) with unknown spin and parity and the two-star Ω(2380) with unknown spin and parity. Assuming that multiplet 18 consists of ∆(1905), Σ(2070), Ξ(2250) and Ω(2380), we find that the equal spacing rule for the mass splitting works with fair accuracy: \( Δz = 25.7 \) MeV and \( ⟨z⟩ = 158 \) MeV. For the total widths, the equal spacing rule is qualitatively fulfilled, see the values in Table 19.

Table 19 summarizes the results of our \( χ^2 \) fit to the underlined observables. The barrier factor is calculated with \( l = 3 \) for the \( 10 \rightarrow 8 + 8 \) decays and with \( l = 1 \) (\( P \)-wave) for the \( 10 \rightarrow 10 + 8 \) decays. Note that in order to have a successful fit, we increased the experimental error on the \( Γ_{Σ(2070)→N\pi} \) by the factor two (the resulting experimental error is still smaller than 30%, see the relevant discussion in Sect. 2). Note also that SU(3) predicts the negative sign for \( √Γ_{N\pi}Γ_{Σπ} \) of any Σ member of decuplets, while \( √Γ_{N\pi}Γ_{Σπ} > 0 \) for Σ(2070) experimentally [49]. Since the analysis of [49] is not our standard source of information on hyperons [34–36], we ignore this inconsistency.
| Mass and width (MeV) | Observables | Experiment (MeV) | SU(3) pred. (MeV) |
|----------------------|-------------|----------------|------------------|
| Δ(1881 ± 18)         | $\Gamma_{N\pi}$ | 39.2 ± 11.6 | 45.1 |
| $\Gamma = 327 \pm 51$ | $\sqrt{\Gamma_{N\pi}\Gamma_{\Sigma K}}$ | $-4.9 \pm 1.2$ | $-5.7$ |
|                     | $\Gamma_{\Delta\pi}$ | 75.2 ± 12.2 | 75.2 |
| Σ(2051 ± 25)         | $\Gamma_{N\overline{K}}$ | $24.0 \pm 2.6 \times 2$ | 16.3 |
| $\Gamma = 300 \pm 30$ | $\sqrt{\Gamma_{N\overline{K}}\Gamma_{\Sigma\pi}}$ | $31.2 \pm 6.8$ | $-13.1$ |
|                     | $\Gamma_{\Lambda\pi}$ | 24.1 |  |
|                     | $\Gamma_{\Sigma\pi}$ | 10.5 |  |
|                     | $\Gamma_{\Sigma(1385)\pi}$ | 10.4 |  |
| Ξ(2250)              | $\Gamma_{\Xi\pi}$ | 25.6 |  |
| $\Gamma = 60 - 150$  | $\Gamma_{\Lambda\overline{K}}$ | 32.4 |  |
|                     | $\Gamma_{\Sigma\overline{K}}$ | 20.9 |  |
|                     | $\Gamma_{\Sigma(1385)\overline{K}}$ | 63.3 |  |
| Ω(2380)              | $\Gamma_{\Xi\overline{K}}$ | 89.6 |  |
| $\Gamma = 26 \pm 23$ | $\Gamma_{\Xi(1385)\overline{K}}$ | 58.4 |  |

Table 19
SU(3) analysis of (10, 5/2+)=(1905, 2070, 2250, 2380).

As can be seen from Table 19, SU(3) predicts large partial decay rates of Ω(2380), whose sum exceeds the estimate for the total width of Ω(2380). However, until the decay rates of Ω(2380) are measured and used in the $\chi^2$ fit, one cannot conclude that our predictions for Ω(2380) are ruled out (see also the appropriate discussion in the previous subsection).

The $\chi^2$ fit to the underlined observables in Table 19 gives

\[
A_{10} = 45.9 \pm 3.4, \quad \chi^2/d.o.f. = 2.89/2, \\
A'_{10} = 39.7 \pm 3.2, \quad \chi^2 = 0.
\]  

Based on the sufficient accuracy of the Gell-Mann–Okubo mass formula and the results presented in Table 19 and Eq. (65), we conclude that SU(3) works well for decuplet 18. However, one should keep in mind the problem with the sign of $\sqrt{\Gamma_{N\overline{K}}\Gamma_{\Sigma\pi}}$ for Σ(2070).
3.4 Multiplet 16: \((10, 3/2^+) = (1920, 2080, 2240, 2470)\)

The content of multiplet 16 is known even worse than that of decuplet 18: decuplet 16 opens with a three-star \(\Delta(1920)\), see Table 1, and the other members are not established. Assuming that the mass splitting for the decuplet is around 150 MeV (recall \(\langle z \rangle \approx 150\) MeV for decuplets 2 and 18) and using the RPP estimate for the mass of the \(\Delta(1920)\) (its mass is rather uncertain), we find in the RPP a candidate for the \(\Sigma\) member of the decuplet – the two-star \(\Sigma(2080)\) with the appropriate \(J^P = 3/2^+\). Using the 160 MeV splitting between the \(\Delta(1920)\) and \(\Sigma(2080)\), we estimate that the mass of the \(\Xi\) member of the decuplet could be around 2240 MeV and the mass of the \(\Omega\) member of the decuplet could be around 2400 MeV. While the RPP contains no \(\Xi\) baryons around 2240 MeV, there is a two-star \(\Omega\) with the mass 2470 MeV. In order to use up as many baryons from the RPP as possible, we assume that \(\Omega(2470)\) belongs to decuplet 16.

The total widths for this decuplet are known with very large uncertainty. Therefore, the Weldon’s equal spacing rule can give only a very rough estimate for the total width of the predicted \(\Xi(2240)\): we estimate that \(\Gamma_{\Xi(2240)} \approx 100 - 150\) MeV.

The \(\chi^2\) analysis of the decay rates of decuplet 16 can be hardly performed: the decay rates of \(\Delta(1920)\) are known with large ambiguity and the data on the only measured observable of \(\Sigma(2080)\) [50] does not come from our standard source of information on hyperons [34–36]. Moreover, SU(3) predicts the positive value for \(\sqrt{\Gamma_{NK}\Gamma_{\Lambda\pi}}\) for \(\Sigma(2080)\), which conflicts with the data. Therefore, we perform the \(\chi^2\) fit using the RPP estimates for the mass, total width and branching ratios of \(\Delta(1920)\) and ignore the \(\sqrt{\Gamma_{NK}\Gamma_{\Lambda\pi}}\) of \(\Sigma(2080)\). Consequently, the resulting SU(3) predictions should not be taken too seriously: we use only two observables for the fit, which have very large experimental errors.

Table 20 summarizes the results of our \(\chi^2\) fit. The barrier factor is calculated with \(l = 1\) for the \(10 \to 8 + 8\) decays. We chose not to give predictions for the \(10 \to 10 + 8\) decays because they would have been based on the poorly measured \(\Gamma_{\Delta(1920)\to\Delta\pi}\) partial decay width.

The \(\chi^2\) fit to the two underlined observables in Table 20 gives

\[
A_{10} = 15.4 \pm 2.0, \quad \chi^2/d.o.f. = 0.02/1. \quad (66)
\]

In summary, we propose that decuplet 16 contains the known \(\Delta(1920), \Sigma(2080)\) and \(\Omega(2470)\) baryons and the unknown \(\Xi\) resonance with the mass around 2240 MeV, the sum of two-body partial decay widths \(\approx 30\) MeV and the total width \(\approx 100 - 150\) MeV. It is important to emphasize that unlike for all previously considered SU(3) multiplets, the SU(3) analysis of the partial decay widths of decuplet 16 fails because SU(3) predicts the positive \(\sqrt{\Gamma_{NK}\Gamma_{\Lambda\pi}}\) for \(\Sigma(2080)\), which contradicts all the available data on this observ-
| Mass and width (MeV) | Observables | Experiment (MeV) | SU(3) pred. (MeV) |
|----------------------|-------------|-----------------|------------------|
| $\Delta(1920)$       | $\Gamma_{N\pi}$ | 25 ± 15         | 23.2             |
| $\Gamma = 200$       | $\sqrt{\Gamma_{N\pi}\Gamma_{\Sigma K}}$ | −10.4 ± 3.0 | −10.6 |
| $\Sigma(2080)$       | $\sqrt{\Gamma_{\Lambda K}\Gamma_{\Lambda\pi}}$ | −18.6 ± 7.4 | 9.2 |
| $\Gamma = 186 ± 48$  | $\Gamma_{\Lambda K}$ | 7.5 |
|                     | $\Gamma_{\Lambda\pi}$ | 11.2 |
| $\Xi(2240)$         | $\Gamma_{\Xi\pi}$ | 10.0 |
|                     | $\Gamma_{\Lambda K}$ | 11.0 |
|                     | $\Gamma_{\Sigma K}$ | 9.1 |
| $\Omega(2470)$       | $\Gamma_{\Xi K}$ | 47.0 |
| $\Gamma = 72 ± 33$   |             |                |                  |

Table 20
SU(3) analysis of ($^{10}$, $3/2^+$)=(1920, 2080, 2240, 2470).

able [10]. Therefore, the particle assignment in decuplet 16 is essentially done using only the GMO mass formula and it driven by the desire to use up as many baryons with the appropriate spin, parity and mass as possible.

3.5 Multiplet 10: ($^{10}$, $1/2^-$)=(1620, 1750, 1900, 2050)

Decuplet 10 opens with a four-star $\Delta(1620)$, see Tables 1 and 2. The RPP contains two potential candidates for the $\Sigma$ member of the decuplet: $\Sigma(1690)$ and $\Sigma(1750)$. We assume that decuplet 10 contains $\Sigma(1750)$ because it gives the larger mass splitting (we always keep in mind the approximately 150 MeV mass splitting observed in the best studied case of decuplet 2) and because decuplet 10 is the only place where $\Sigma(1750)$ can be fitted in. The $\Xi$ and $\Omega$ members of the decuplet are missing. We predict their masses assuming the 150 MeV mass splitting.

Table 21 summarizes the results of our $\chi^2$ fit to the available decay rates of decuplet 10. Since the decay rates of $\Delta(1620)$ and $\Sigma(1750)$ are rather ambiguous, we use the RPP estimates for their total widths and branching ratios. The barrier factor is calculated with $l = 0$ for the $^{10} \rightarrow 8 + 8$ decays and with $l = 2$ ($D$-wave) for the $^{10} \rightarrow ^{10} + 8$ decays.
| Mass and width (MeV) | Observables | Experiment (MeV) | SU(3) pred. (MeV) |
|----------------------|-------------|-----------------|------------------|
| Δ(1620)             | Γ_{N\pi}   | 37.5 ± 7.5      | 25.1             |
| Γ = 150             | Γ_{Δ\pi}   | 61.5 ± 19.5     | 61.9             |
| Σ(1750)             | Γ_{Λπ}     | 22.5 ± 13.5     | 7.1              |
| Γ = 90              | √|Γ_{Λπ}Γ_{Λ\pi}| 3.6 ± 2.7       | 8.9              |
|                     | √|Γ_{Λπ}Γ_{Σπ}| -8.1 ± 4.5      | -6.9             |
|                     | Γ_{Σ(1385)π} | 20.7 ± 35.1    | 5.9              |
| Ξ(1900)             | Γ_{Ξπ}     | 9.7             |                  |
|                     | Γ_{Λπ}     | 9.7             |                  |
|                     | Γ_{Σπ}     | 8.3             |                  |
|                     | Γ_{Ξ(1530)π} | 9.1             |                  |
| Ω(2050)             | Γ_{ΞK}     | 32.8            |                  |
|                     | Γ_{Ξ(1530)K} | 0.4             |                  |

Table 21
SU(3) analysis of (10, 1/2−)=(1620, 1750, 1900, 2050).

The χ^2 fit to the underlined observables in Table 21 gives

\[ A_{10} = 12.4 ± 1.2, \quad \chi^2/d.o.f. = 7.99/3, \]
\[ A_{10}' = 221.4 ± 34.8, \quad \chi^2/d.o.f. = 0.18/1. \]  \hspace{1cm} (67)

Note that an even better value of χ^2 can be obtained by using Σ(1690) as the Σ member of decuplet 10. However, as we explained above, the assignment of Σ(1690) to decuplet 10 would spoil the overall picture of SU(3) multiplets by leaving out Σ(1750).

In order to have a complete multiplet, we predict the existence of two new strange resonances with \( J^P = 1/2^- \): a Ξ baryon with the mass around 1900 MeV and an Ω baryon with the mass around 2050 MeV. In the estimate of their masses, we assumed that \( m_Ξ - m_Σ(1750) = m_Ω - m_Ξ = 150 \) MeV. As follows from Table 21, the sum of two-body partial decay widths of Ξ(1900) and Ω(2050) is at the level of 40 MeV. Also, judging by the central values of the Δ(1620) and Σ(1750) total widths, the total widths of the predicted Ξ and Ω should be \( 50 - 60 \) MeV.
3.6 Multiplet 13: (10, 3/2−) = (1700, 1850, 2000, 2150)

Decuplet 13 is the least established SU(3) multiplet we have considered so far: it opens with a four-star ∆(1700), but other members are missing and the RPP contains no candidates. By analogy with our previous analysis, we assume that the mass difference between the members of decuplet 13 is 150 MeV. This allows us to estimate the masses of the missing Σ, Ξ and Ω states: \( m_\Sigma \approx 1850 \text{ MeV}, \ m_\Xi \approx 2000 \text{ MeV} \) and \( m_\Omega \approx 2150 \text{ MeV} \). The decay rates of the missing resonances are predicted by fitting the SU(3) predictions to the measured decay rates of ∆(1700), see Table 22.

Table 22 presents the results of our \( \chi^2 \) fit to the decay rates of ∆(1700). Since our standard source of information on non-strange particles [33] reports the total width of ∆(1700), which is much larger than the values obtained in many other analyses, we use the RPP estimates for the total width and branching ratios of ∆(1700). The barrier factor is calculated with \( l = 2 \) for the \( 10 \rightarrow 8 + 8 \) decays and with \( l = 0 \) for the \( 10 \rightarrow 10 + 8 \) decays.

The coupling constants \( A_{10} \) and \( A'_{10} \), which are used to make the predictions summarized in Table 22, are determined from the \( \chi^2 \) fit to the two underlined observables in Table 22

\[
A_{10} = 48.3 \pm 8.0, \quad A'_{10} = 29.8 \pm 6.7.
\]

(68)

Note that since the partial decay width \( \Gamma_{\Delta(1700)\rightarrow\Delta \pi} \) is large, SU(3) predicts large \( 10 \rightarrow 10 + 8 \) decays widths for other members of decuplet 13. Based on the results in Table 22, we can estimate the sum of two-body partial decay widths of the the predicted baryons:
\[
\Gamma_{\Sigma(1850)}^{2\text{-body}} \approx 70 \text{ MeV}, \quad \Gamma_{\Xi(2000)}^{2\text{-body}} \approx 130 \text{ MeV} \quad \text{and} \quad \Gamma_{\Omega(2150)}^{2\text{-body}} \approx 110 \text{ MeV}.
\]

3.7 Multiplet 19: (10, 1/2+) = (1910, 2060, 2210, 2360)

Decuplet 19 is very similar to previously considered decuplet 13: only the four-star ∆(1910) is known, while the other members are missing in the RPP. Their masses are estimated using the equal spacing rule with the 150 MeV mass difference: \( m_\Sigma \approx 2060 \text{ MeV}, \ m_\Xi \approx 2210 \text{ MeV} \) and \( m_\Omega \approx 2360 \text{ MeV} \). The decay rates of the missing resonances are predicted by fitting the SU(3) predictions to the RPP estimates for the decay rates of ∆(1910), see Table 23.

Table 23 summarizes our results for decuplet 19. The barrier factor is calculated with \( l = 1 \). The coupling constants \( A_{10} \) and \( A'_{10} \), which are used to make the predictions summarized in Table 23, are

\[
A_{10} = 24.2 \pm 4.0, \quad A'_{10} = 8.7 \pm 1.4.
\]

(69)
### Table 22

SU(3) analysis of \((10, 3/2^-) = (1700, 1850, 2000, 2150)\).

| Mass and width (MeV) | Observables | Experiment (MeV) | SU(3) pred. (MeV) |
|----------------------|-------------|-----------------|-------------------|
| \(\Delta(1700)\)    | \(\Gamma_{N\pi}\) | 45.0 ± 15.0     | 45.0              |
| \(\Gamma = 300\)    | \(\Gamma_{\Delta\pi}\) | 126.0 ± 57.0    | 126.0             |
| \(\Sigma(1850)\)    | \(\Gamma_{N\Sigma}\) | 12.2            |                   |
|                      | \(\Gamma_{\Lambda\pi}\) | 20.2            |                   |
|                      | \(\Gamma_{\Sigma\pi}\) | 8.8             |                   |
|                      | \(\Gamma_{\Sigma(1385)\pi}\) | 15.6           |                   |
|                      | \(\Gamma_{\Delta\Sigma}\) | 12.1           |                   |
| \(\Xi(2000)\)       | \(\Gamma_{\Xi\pi}\) | 14.9            |                   |
|                      | \(\Gamma_{\Lambda\Xi}\) | 16.3            |                   |
|                      | \(\Gamma_{\Sigma\Xi}\) | 9.4             |                   |
|                      | \(\Gamma_{\Xi(1530)\pi}\) | 22.0          |                   |
|                      | \(\Gamma_{\Sigma(1385)\Xi}\) | 67.2           |                   |
| \(\Omega(2150)\)    | \(\Gamma_{\Xi\Omega}\) | 45.5            |                   |
|                      | \(\Gamma_{\Xi(1530)\Omega}\) | 64.8          |                   |

Summing the SU(3) predictions for the partial decays widths, we obtain the following estimates: \(\Gamma_{\Sigma(2060)}^{2\text{-body}} \approx 75\ MeV\), \(\Gamma_{\Xi(2210)}^{2\text{-body}} \approx 85\ MeV\) and \(\Gamma_{\Omega(2360)}^{2\text{-body}} \approx 90\ MeV\). Note that the total widths could be significantly larger, for instance, by a factor 1.5.

#### 3.8 Multiplet 5: \((10, 3/2^+) = (1600, 1690, 1900, 2050)\)

In the picture of SU(3) multiplets presented in Table 1, decuplet 5 can be interpreted as a radial excitation of the ground-state decuplet. Decuplet 5 opens with a well-established three-star \(\Delta(1600)\). The RPP contains one potential candidate for the \(\Sigma\) member of decuplet 5: the two-star \(\Sigma(1690)\) with unmeasured spin and parity. Note that the mass difference between \(\Sigma(1690)\) and \(\Delta(1600)\) is only 90 MeV (if one uses the RPP estimates for the masses), which is smaller than the typical 150 MeV mass difference in decuplets.
| Mass and width (MeV) | Observables | Experiment (MeV) | SU(3) pred. (MeV) |
|----------------------|-------------|-----------------|-------------------|
| $\Delta(1910)$      | $\Gamma_{N\pi}$ | $56.3 \pm 18.8$ | 56.3              |
| $\Gamma = 250$      | $\Gamma_{\Delta\pi}$ | $4.0 \pm 1.2$  | 4.0               |
|                     | $\Gamma_{\Sigma K}$ |                  | 11.1              |
| $\Sigma(2060)$      | $\Gamma_{N\overline{K}}$ | 17.8            |                   |
|                     | $\Gamma_{\Lambda\pi}$ | 26.4            |                   |
|                     | $\Gamma_{\Sigma\pi}$ | 14.7            |                   |
|                     | $\Gamma_{\Sigma(1385)\pi}$ | 0.5             |                   |
| $\Xi(2210)$         | $\Gamma_{\Xi\pi}$ | 22.9            |                   |
|                     | $\Gamma_{\Lambda\overline{K}}$ | 25.2            |                   |
|                     | $\Gamma_{\Sigma\overline{K}}$ | 20.6            |                   |
|                     | $\Gamma_{\Xi(1530)\pi}$ | 0.8             |                   |
|                     | $\Gamma_{\Sigma(1385)\overline{K}}$ | 2.5             |                   |
| $\Omega(2360)$      | $\Gamma_{\Xi\overline{K}}$ | 87.1            |                   |
|                     | $\Gamma_{\Xi(1530)\overline{K}}$ | 2.6             |                   |

Table 23
SU(3) analysis of (10, 1/2+)=(1910, 2060, 2210, 2360).

However, if decuplet 5 is mixed, for example with decuplet 2, this renders the equal spacing rule and the 150 MeV mass difference estimate inapplicable to decuplet 5. We shall not consider the mixing option here. We choose to assign $\Sigma(1690)$ to decuplet 5 because of the following two reasons. First, the decay rates of $\Sigma(1690)$ fit extremely well decuplet 5. In more detail, fixing the coupling constants $A_{10}$ and $A'_{10}$ from the decay rates of $\Delta(1600)$, SU(3) predictions for the measured ratios of the decay rates of $\Sigma(1690)$ are in a very good agreement with the experiment, see Table 24. Second, decuplet 5 is the only SU(3) multiplet, where the two-star $\Sigma(1690)$ can be placed, and we intend to systematize as many baryons existing in the RPP as possible.

The $\Xi$ and $\Omega$ members of decuplet 5 are missing. We assume that the missing $\Omega$ is 450 MeV heavier than the $\Delta(1600)$ (the equal spacing with the 150 MeV mass difference) and that $m_\Omega - m_\Xi = 150$ MeV. This gives $m_\Xi = 1900$ MeV and $m_\Omega = 2050$ MeV.
| Mass and width (MeV) | Observables | Experiment (MeV) | SU(3) pred. (MeV) |
|----------------------|-------------|-----------------|-------------------|
| \(\Delta(1600)\)    | \(\Gamma_{N\pi}\) | \(61.3 \pm 26.3\) | 61.3              |
| \(\Gamma = 350\)    | \(\Gamma_{\Delta\pi}\) | \(178.5 \pm 84.0\) | 178.5            |
| \(\Sigma(1690)\)    | \(\Gamma_{N\Xi}/\Gamma_{\Lambda\pi}\) | \(0.4 \pm 0.25\) or small | 0.54             |
| \(\Gamma = 100 - 240\) | \(\Gamma_{\Sigma\pi}/\Gamma_{\Lambda\pi}\) | \(0.3 \pm 0.3\) or small | 0.46             |
|                      | \(\Gamma_{\Sigma(1385)\pi}/\Gamma_{\Lambda\pi}\) | \(< 0.5\) | 0.58             |
|                      | \(\Gamma_{N\Xi}\) | | 11.4              |
|                      | \(\Gamma_{\Lambda\pi}\) | | 21.28             |
|                      | \(\Gamma_{\Sigma\pi}\) | | 9.8               |
|                      | \(\Gamma_{\Sigma(1385)\pi}\) | | 12.4              |
| \(\Xi(1900)\)       | \(\Gamma_{\Xi\pi}\) | | 20.8              |
|                      | \(\Gamma_{\Lambda\Xi}/\Gamma_{\Lambda\pi}\) | | 20.5              |
|                      | \(\Gamma_{\Sigma\Xi}/\Gamma_{\Lambda\pi}\) | | 12.9              |
|                      | \(\Gamma_{\Xi(1530)\pi}\) | | 32.5              |
|                      | \(\Gamma_{\Sigma(1385)\Xi}/\Gamma_{\Lambda\pi}\) | | 6.2               |
| \(\Omega(2050)\)    | \(\Gamma_{\Xi\Xi}\) | | 58.2              |
|                      | \(\Gamma_{\Xi(1530)\Xi}/\Gamma_{\Lambda\pi}\) | | 8.8               |

Table 24
SU(3) analysis of \((10, \frac{3}{2}^+)=(1600, 1690, 1900, 2050)\).

In the \(\chi^2\) fit, we used two decay rates of \(\Delta(1600)\). We used the average RPP values for the mass, total width and branching ratios of \(\Delta(1600)\). Note that when we use \(m_{\Delta(1600)} = 1600\) MeV, the measured \(\Delta(1600) \to \Sigma K\) decay is prohibited by kinematics and cannot be used in the fit. Since for \(\Sigma(1690)\) only ratios of the partial decay widths are known, it does not make sense to use the \(\Sigma(1690)\) measured observables in the fit. Table 24 summarizes our results for decuplet 5. The barrier factor is calculated with \(l = 1\).

The coupling constants \(A_{10}\) and \(A'_{10}\), which are used to make the predictions summarized in Table 24, are

\[
A_{10} = 38.2 \pm 8.2, \quad A'_{10} = 129.3 \pm 30.4
\]
Summing the SU(3) predictions for the partial decays widths, we estimate: \( \Gamma_{\Xi(1900)}^{2-\text{body}} \approx 95 \) MeV and \( \Gamma_{\Omega(2050)}^{2-\text{body}} \approx 70 \) MeV.

4 SU(3) analysis of the antidecuplet

As discussed in the Introduction, the recent wave of interest in the baryon spectroscopy was triggered by experimental indications of the existence of the exotic \( \Theta^+ \) baryon [1,51–60] and the exotic \( \Xi^- \) baryon [3]. These baryons are called exotic because, in the language of quark models, their quantum numbers cannot be obtained from three constituent quarks, i.e. the minimal Fock component of \( \Theta^+ \) and \( \Xi^- \) contains four quarks and one antiquark. Both \( \Theta^+ \) and \( \Xi^- \) are members of the multiplet called the antidecuplet (an \( \bar{10} \) SU(3) representation). The existence of the antidecuplet and the masses, total widths and decay rates of its members were predicted within the chiral quark soliton model by Diakonov, Petrov and Polyakov [2]. In addition to the \( \Theta^+ \) and \( \Xi^- \) reports, there was the observation of an exotic anti-charmed baryon state [61]. At the moment of writing of this report, the existence of \( \Theta^+ \) and \( \Xi^- \) is uncertain because there is a number of experiments and analyses, which do not see these states [62–75]. However, we expect that in the near future, there will appear thorough analyses trying to understand in detail why some some experiments see the \( \Theta^+ \) and some do not, see e.g. [76,77].

We assume that the antidecuplet exists and has spin and parity as predicted in the chiral quark soliton model, \( J^P = 1/2^+ \). The \( \Theta^+ \) is the lightest members of the antidecuplet with the mass approximately 1540 MeV and the \( \Xi_{\overline{10}} \) (\( \Xi^- \) is the member of the isoquadruplet referred to as \( \Xi_{\overline{10}} \)) is the heaviest member of the antidecuplet with the mass 1862 MeV, see Fig. 3. Only these two members of the antidecuplet can be considered established. Applying the equal spacing rule to the antidecuplet, see Eq. (8), we find that the mass spacing for the antidecuplet is 107 MeV. This means that the \( N_{\overline{10}} \) member of the antidecuplet should have the mass around 1650 MeV and the \( \Sigma_{\overline{10}} \) member of the antidecuplet should have the mass around 1760 MeV.

While no information is available about the \( \Sigma_{\overline{10}} \) member of the antidecuplet, candidates for the \( N_{\overline{10}} \) member were recently discussed in the literature. The partial wave analysis (PWA) of pion-nucleon scattering, which was modified for the search of narrow resonances, presented two candidate for the \( N_{\overline{10}} \) with masses 1680 MeV and 1730 MeV [78]. In both cases, \( \Gamma_{N_{\overline{10}} \rightarrow N\pi} < 0.5 \) MeV (the resonance is highly inelastic) and \( \Gamma_{\text{tot}} < 30 \) MeV.

Experimental evidence for a new nucleon resonance with the mass near 1670 MeV was recently obtained by the GRAAL collaboration [79]. The fact that the resonance peak is seen in the \( \gamma n \rightarrow n\eta \) process and is absent in the \( \gamma p \rightarrow p\eta \) process supplies a strong
piece of evidence that the resonance belongs to the antidecuplet because photoproduction of the antidecuplet is strongly suppressed on the proton target [22], see also the relevant discussion using the $U$-spin argument in Sect. 1. The position of the peak is very close to the 1680 MeV solution of [78].

There is another candidate for the $N_{10}$ member of the antidecuplet, which corresponds to the higher mass solution of the analysis of [78]. In gold-gold collisions at RHIC, the STAR collaboration observes a narrow peak at approximately 1734 MeV in the $\Lambda K^0_s$ invariant mass.

Therefore, the present experimental information on the properties of the antidecuplet can be summarized as follows. The $\Theta^+$ and $\Xi_{10}$ members are observed experimentally. These states have exotic quantum numbers and, hence, cannot mix with non-exotic baryons. The $\Theta^+$ and $\Xi_{10}$ are narrow. The $N_{10}$ member of $\Xi_{10}$ is not established, but there are at least two candidates for it. The present experimental data suggests that the pattern of the $N_{10}$ decays is the following: $\Gamma_{N_{10} \rightarrow N\pi} < 0.5$ MeV, $\Gamma_{N_{10} \rightarrow N\eta}$ is sizable (measurable) and $\Gamma_{N_{10} \rightarrow \Lambda K}$ is non-vanishing. There exists no information on the $\Sigma_{10}$ member of $\Xi_{10}$, except for an estimate based on the equal spacing rule, $m_{\Sigma_{10}} \approx 1760$ MeV.

This section is organized as follows. In Subsect. 4.1, we collect the SU(3) predictions for the antidecuplet decays and show that they are inconsistent with the antidecuplet decay pattern summarized above. This strongly implies that the non-exotic members of the antidecuplet, $N_{10}$ and $\Sigma_{10}$, are mixed with $N$ and $\Sigma$ from non-exotic multiplets. Another option, which we shall not consider here, is to assume that the antidecuplet is mixed with the 27-plet [15–17]. Among several possible scenarios of the mixing, in Subsect. 4.2 we examine the possibility that the $N_{10}$ and $\Sigma_{10}$ members of the antidecuplet mix with $N(1440)$ and $\Sigma(1660)$ of octet 3. We show that this can accommodate in a simple way all experimental information on the antidecuplet decays.

4.1 The antidecuplet decays: no mixing

Similarly to the $10 \rightarrow 8 + 8$ decays, the $\Xi_{10} \rightarrow 8 + 8$ decays of the antidecuplet are described in terms of one free parameter $A_{\Xi_{10}}$

$$g_{B_1B_2P} = -\frac{1}{\sqrt{5}} \left( \begin{array}{c|cc|cc|cc} 8 & 8 & \Xi_{10} & \hline Y_2T_2 & Y_P T_P & Y_1T_1 \end{array} \right).$$

(71)

The coupling constants for all possible decay channels of the antidecuplet are summarized in Table 25.
Using the SU(3) universal coupling constants of the $N_{10}$, it is easy to show that the emerging pattern of the $N_{10}$ decay rates is inconsistent with the trend discussed above. Indeed, treating the total width of the $\Theta^+$ as an input, one can readily determine $A_{10}$, which allows to unambiguously predict the $\Theta^+$ decay rates. Note that while the experimental determination of $\Gamma_{\Theta^+}$ is limited by the detector resolution and the experimental upper limit is $\Gamma_{\Theta^+} < 10 \text{ MeV}$, many theoretical analyses suggest that $\Gamma_{\Theta^+}$ is even smaller, of the order of several MeV or even less than 1 MeV [81–87].

In the following, we assume that $m_{N_{10}} = 1670 \text{ MeV}$. Figure 5 presents $\Gamma_{N_{10} \to N\pi}$ (the solid curve) and $\Gamma_{N_{10} \to N\eta}$ (the dashed curve) as functions of $\Gamma_{\Theta^+}$. As can be seen from the figure, SU(3) predicts that $\Gamma_{N_{10} \to N\pi} > \Gamma_{N_{10} \to N\eta}$. In addition, SU(3) predicts that $\Gamma_{N_{10} \to N\pi} < 0.5 \text{ MeV}$ only for $\Gamma_{\Theta^+} < 0.25 \text{ MeV}$. These two results contradict the analysis of [78] and the GRAAL result [79]. The only way to alter the pattern of the $N_{10}$ decays and to have consistency with the data is to introduce mixing. In the following subsection, we consider the scenario that the antidecuplet is mixed octet 3.

Table 25
The SU(3) universal coupling constants for $\Theta^+ \to N K$ are given in the following table.

| Decay mode | $g_{B1B2}P$ |
|------------|-------------|
| $\Theta^+ \to N K$ | $(1/\sqrt{5})A_{10}$ |
| $N_{10} \to N\pi$ | $1/(2\sqrt{5})A_{10}$ |
| $\to N\eta$ | $-1/(2\sqrt{5})A_{10}$ |
| $\to \Lambda K$ | $1/(2\sqrt{5})A_{10}$ |
| $\to \Sigma K$ | $1/(2\sqrt{5})A_{10}$ |
| $\Sigma_{10} \to N\bar{K}$ | $-(1/\sqrt{30})A_{10}$ |
| $\to \Sigma\pi$ | $(1/\sqrt{30})A_{10}$ |
| $\to \Sigma\eta$ | $-1/(2\sqrt{5})A_{10}$ |
| $\to \Lambda\pi$ | $1/(2\sqrt{5})A_{10}$ |
| $\to \Xi K$ | $1/(\sqrt{30})A_{10}$ |
| $\Xi_{10} \to \Xi\pi$ | $(1/\sqrt{10})A_{10}$ |
| $\to \Sigma\bar{K}$ | $-(1/\sqrt{10})A_{10}$ |
Fig. 5. The unmixed antidecuplet. The $\Gamma_{N^{10}_{10} \to N\pi}$ (the solid curve) and $\Gamma_{N^{10}_{10} \to N\eta}$ (the dashed curve) partial decay widths as functions of the total width of the $\Theta^+$, $\Gamma_{\Theta^+}$.

4.2 Mixing of the antidecuplet with octet 3

One possible way to suppress $\Gamma_{N^{10}_{10} \to N\pi}$ while enhancing $\Gamma_{N^{10}_{10} \to N\eta}$ is to mix the $N_{10}$ with $N(1440)$ of octet 3. This automatically implies that the $N_{10}$ is mixed with $\Sigma(1660)$ of octet 3 [44]. Naturally, because of their exotic quantum numbers, $\Theta^+$ and $\Xi_{10}$ do not mix with non-exotic baryons. Parameterizing the mixing in terms of the mixing angle $\theta$, the relevant $N_{10}$ coupling constants read (see Tables 4 and 25)

$$g_{N^{10}_{10} \to N\pi} = -\sin \theta \sqrt{3} A_8 + \cos \theta \frac{1}{2\sqrt{5}} A_{10},$$

$$g_{N^{10}_{10} \to N\eta} = -\sin \theta \left(4 \alpha - 1\right) A_8 - \cos \theta \frac{1}{2\sqrt{5}} A_{10},$$

where $A_8 = 32.4$ and $\alpha = 0.27$, see Eq. (50). Because of the relative minus sign between the terms proportional to $A_8$ and $A_{10}$ in the expression for $g_{N^{10}_{10} \to N\pi}$ and the positive relative sign between the two contributions to $g_{N^{10}_{10} \to N\eta}$, it is possible to simultaneously suppress $g_{N^{10}_{10} \to N\pi}$ and to keep $g_{N^{10}_{10} \to N\eta}$ sizable by a suitable choice of the mixing angle $\theta$.

This is illustrated in Fig. 6, where $\Gamma_{N^{10}_{10} \to N\pi}$ (solid curves) and $\Gamma_{N^{10}_{10} \to N\eta}$ (dashed curves)
are plotted as functions of the mixing angle $\theta$ for two values of the total width of the $\Theta^+$, $\Gamma_{\Theta^+} = 1$ MeV (left panel) and $\Gamma_{\Theta^+} = 3$ MeV (right panel). Note that $\Gamma_{N_{10} \rightarrow N \eta}$ very weakly depends on $\theta$ for small $\theta$ because the non-exotic contribution to $g_{N_{10} \rightarrow N \eta}$ is numerically very small.

Fig. 6. The $\Gamma_{N_{10} \rightarrow N \pi}$ (solid curves) and $\Gamma_{N_{10} \rightarrow N \eta}$ (dashed curves) partial decay widths as functions of the mixing angle $\theta$ for $\Gamma_{\Theta^+} = 1$ MeV (left panel) and $\Gamma_{\Theta^+} = 3$ MeV (right panel).

As can be seen from Fig. 6, the conditions $\Gamma_{N_{10} \rightarrow N \pi} < 0.5$ MeV [78] and $\Gamma_{N_{10} \rightarrow N \eta}$ is sizable [79] are provided, if $3^0 < \theta < 7^0$ for $\Gamma_{\Theta^+} = 1$ MeV and $6^0 < \theta < 10^0$ for $\Gamma_{\Theta^+} = 3$ MeV. It is important to emphasize that when the mixing angle $\theta$ is as small as we find, mixing with the antidecuplet does not affect the coupling constants of octet 3. This allows us to use the values of the parameters $A_8$, $\alpha$ and $A'_8$, which are determined for the unmixed octet 3.

The mixing angle $\theta_\Sigma$ for the $\Sigma_{10}$ and $\Sigma(1660)$ states is related to $\theta$ [44]

$$\theta_\Sigma(m_{\Sigma_{10}} - m_{\Sigma(1660)}) = \theta(m_{N_{10}} - m_{N(1440)}).$$  \hfill (73)
Using \( m_{N(1440)} = 1440 \text{ MeV}, \ m_{\Sigma(1660)} = 1660 \text{ MeV} \) and \( m_{\Theta^+} = 1540 \text{ MeV} \) and assuming that \( m_{\Sigma^{10}} \approx 1760 \text{ MeV} \) (the equal spacing rule estimate), we find that \( \theta_\Sigma \approx \theta \).

Table 26 summarizes SU(3) predictions for the decay rates of the antidecuplet mixed with octet 3, provided that the mixing angle \( \theta \) is chosen such that the \( \Gamma_{N^{10} \rightarrow N \pi} < 0.5 \text{ MeV} \).

| Partial decay widths | \( \Gamma_{\Theta^+} = 1 \text{ MeV} \) | \( \Gamma_{\Theta^+} = 3 \text{ MeV} \) |
|----------------------|----------------|----------------|
| \( \Gamma_{N_{10}^{\overline{10}} \rightarrow N \pi} \) | \(< 0.5\) | \(< 0.5\) |
| \( \Gamma_{N_{10}^{\overline{10}} \rightarrow N \eta} \) | \(0.65 - 0.67\) | \(1.94 - 1.95\) |
| \( \Gamma_{N_{10}^{\overline{10}} \rightarrow \Lambda K} \) | \(0.16 - 0.29\) | \(0.56 - 0.76\) |
| \( \Gamma_{N_{10}^{\overline{10}} \rightarrow \Delta \pi} \) | \(2.6 - 15.6\) | \(12.9 - 34.8\) |
| \( \Gamma_{\Sigma_{10}^{\overline{10}} \rightarrow N K} \) | \(0.11 - 0.50\) | \(0.49 - 1.18\) |
| \( \Gamma_{\Sigma_{10}^{\overline{10}} \rightarrow \Sigma \pi} \) | \(0.02 - 2.64\) | \(0.57 - 5.00\) |
| \( \Gamma_{\Sigma_{10}^{\overline{10}} \rightarrow \Sigma \eta} \) | \(0.04 - 0.08\) | \(0.15 - 0.20\) |
| \( \Gamma_{\Sigma_{10}^{\overline{10}} \rightarrow \Lambda \pi} \) | \(0.15 - 0.81\) | \(0.72 - 1.90\) |
| \( \Gamma_{\Sigma_{10}^{\overline{10}} \rightarrow \Sigma(1385)\pi} \) | \(0.33 - 1.96\) | \(1.6 - 4.3\) |
| \( \Gamma_{\Xi_{10}^{\overline{10}} \rightarrow \Xi \pi} \) | \(1.98\) | \(5.94\) |
| \( \Gamma_{\Xi_{10}^{\overline{10}} \rightarrow \Xi K} \) | \(1.08\) | \(3.23\) |

Table 26
SU(3) predictions for the decay rates of the antidecuplet mixed with octet 3.

The results presented in Table 26 deserve a discussion. The pattern of the \( N_{10}^{\overline{10}} \) decays complies with the following picture supported by experiments: the \( N_{10}^{\overline{10}} \rightarrow N \pi \) decay is suppressed [78], the \( N_{10}^{\overline{10}} \rightarrow N \eta \) decay is measurable [79] and \( N_{10}^{\overline{10}} \rightarrow \Lambda K \) decay is non-vanishing [80]. Note that while we assumed that \( m_{N_{10}^{\overline{10}}} = 1670 \text{ MeV} \), the qualitative picture of the \( N_{10}^{\overline{10}} \) decays does not change, if \( m_{N_{10}^{\overline{10}}} = 1734 \text{ MeV} \) [80] is used.

The \( N_{10}^{\overline{10}} \rightarrow \Delta \pi \) and \( \Sigma_{10}^{\overline{10}} \rightarrow \Sigma(1385)\pi \) decays are possible only due to the mixing because the \( \mathbf{10} \rightarrow 10 + 8 \) decays are not allowed. The corresponding coupling constants are

\[
g_{N_{10}^{\overline{10}} \rightarrow \Delta \pi} = - \sin \theta \frac{2}{\sqrt{5}} A'_8, \]
\[
g_{\Sigma_{10}^{\overline{10}} \rightarrow \Sigma(1385)\pi} = \sin \theta \frac{\sqrt{30}}{15} A'_8.\]
Despite the small mixing angle $\theta$, the large value of $A'_8$, see Eq. (50), provides large \( \Gamma_{N^{10} \rightarrow \Delta \pi} \).

It can be seen from Table 26 that the branching ratio of $\Sigma_{\pi\pi}$ into the $N\bar{K}$ final state is sizable, $Br(N\bar{K}) \approx 15 - 20\%$. This is important for the potential search for $\Sigma_{\pi\pi}$. Among the experiments reporting the $\Theta^+$ signal, there are four experiments, where the $\Theta^+$ is observed as a peak in the $pK_S$ invariant mass and strangeness is not tagged [54,56,57,59]. Since $\Sigma_{\pi\pi}$ has a sizable branching ratio into the same final state, the four experiments can give information on the $\Sigma_{\pi\pi} \rightarrow N\bar{K}$ decay. We shall consider this in detail.

The analysis of neutrino-nuclear (mostly neon) interaction data [54] clearly reveals the $\Theta^+$ peak as well as a number of other peaks in the $1650 < M_{pK_S} < 1850$ MeV mass region, which cannot be suppressed by the random-star elimination procedure, see Fig. 3 of [54]. Any of the peaks in the 1700-1800 MeV mass range is a good candidate for $\Sigma_{\pi\pi}$.

Similar conclusions apply to the SVD collaboration result [57]. Before the cuts aimed to enhance the $\Theta^+$ signal are imposed, the $pK_S$ invariant mass spectrum contains at least two prominent peaks in the 1700-1800 MeV mass range (see Fig. 5 of [57]), each of which can be interpreted as $\Sigma_{\pi\pi}$.

The HERMES [56] and ZEUS [59] $pK_S$ invariant mass spectra extend only up to 1.7 MeV and, therefore, do not allow to make any conclusions about the $\Sigma_{\pi\pi}$.

In addition to the $pK_S$ invariant mass spectrum, the HERMES collaboration also presents the $\Lambda\pi$ invariant mass spectrum in order to see if the observed peak in the $pK_S$ final state is indeed generated by the $\Theta^+$ and not by some yet unknown $\Sigma^*$ resonance [88]. The $\Lambda\pi$ invariant mass spectrum has no resonance structures except for the prominent $\Sigma(1385)$ peak. According to our analysis, the $\Gamma_{\Sigma_{\pi\pi} \rightarrow \Lambda\pi}$ partial decay width is small, when $\Gamma_{\Sigma_{\pi\pi} \rightarrow N\bar{K}}$ is large (this correlation is not explicitly indicated in Table 26). This correlation seems to be exactly what is needed to comply with the non-observation of $\Sigma_{10}^{10}$ in the HERMES $\Lambda\pi$ invariant mass spectrum.

In summary, the [54,57] data contain an indication for a narrow $\Sigma_{\pi\pi}$ member of the antidecuplet in the 1700-1800 MeV mass range and the [56,59,88] data do not rule out its existence. Obviously, a dedicated search for the $\Sigma_{10}^{10}$ signal in the $pK_S$ and $\Lambda\pi$ invariant mass spectra is needed in order to address several key issues surrounding this least known member of the antidecuplet.

It is curios that one can offer a candidate $\Sigma_{\pi\pi}$ state, the one-star $\Sigma(1770)$ with $J^P = 1/2^+$, which has been known for almost three decades [10,34]. We would like to emphasize that the $\Sigma(1770)$ was not used in any of the non-exotic multiplets we considered in Sects. 2 and 3. The $\Sigma(1770)$ has the $14 \pm 4\%$ branching ratio in the $N\bar{K}$ final state and poorly known but still probably rather small branching ratios into the $\Lambda\pi$ and $\Sigma\pi$ final states. The total width of the $\Sigma(1770)$ is $\Gamma_{\text{tot}} = 72 \pm 10$ MeV, which is much larger than the
sum of the partial decay widths in Table 26. However, if other decay channels contribute significantly, this will reduce the inconsistency between the SU(3) predictions for $\Gamma_{\text{tot}}$ and the experimental value. The possibility of large three-body partial decay widths of $\Sigma(1770)$ was studied in [89].

Turning to $\Xi_{10}$ we observe that, although in the considered mixing scenario $\Xi_{10}$ cannot mix, its partial decay rates are rather large for the antidecuplet because of the large phase space factor.

We considered a possible scenario of the antidecuplet mixing with octet 3, which allowed us to have a phenomenologically consistent picture of the $\mathbf{10}$ decays. Mixing of the antidecuplet with octet 3 was also considered in [44], where the decays were not considered and the mixing angles $\theta$ and $\theta_{\Sigma}$ were determined using the modified Gell-Mann–Okubo mass formulas for the octet and the antidecuplet. We demonstrated in a simple way that even very insignificant mixing dramatically affects the antidecuplet decays. Of course, since the antidecuplet decays are virtually unknown, many more mixing scenarios are possible and were considered in the literature. For instance, the antidecuplet can mix with octet 4 (the emerging picture is similar to the case of mixing with octet 3), with the ground-state octet [2,78], simultaneously with the ground-state octet and exotic $27$-plet and $35$-plet [15,16], simultaneously with the ground-state octet and octets 3 and 4 [90].

5 Conclusions and discussion

We have analyzed 20 multiplets of baryons with the mass less than approximately 2000-2200 MeV indicated in Table 1 and the antidecuplet using the Gell-Mann–Okubo mass formulas and SU(3) predictions for partial decay widths. We confirm the main conclusion of the 1974 analysis by Samios, Goldberg and Meadows [6] that the simple scheme based on flavor SU(3) symmetry of the strong interaction describes remarkably well the mass splitting and the decay rates of all considered multiplets. The main result of our analysis is the final list of SU(3) multiplets, which is presented in Table 27. The underlined entries in the table are predictions of new particles, which are absent in the Review of Particle Physics.

An examination of the RPP baryon listing shows that we have cataloged all four and three-star baryons with the mass less than approximately 2000-2200 MeV and almost all baryons with the weaker rating in the same mass region. In the following, we shall discuss the baryons, for which we could not find a place in Table 27. We will ignore baryons with the mass greater than 1900 MeV, which can be interpreted as radial excitations of the corresponding lighter baryons presented Table 27 or as states opening higher SU(6)$\times$O(3) supermultiplets [6]. One example is the $J^P = 1/2^-$ nonet consisting of the
The final list of SU(3) multiplets.

\[ N(2190), \Lambda(?), \Sigma(?), \Xi(?) \text{ octet} \] mixed with the \( \Lambda(2100) \) singlet, which can be thought to belong to the \( (70, L = 3) \) supermultiplet [6]. The remaining unused baryons are the one-star \( \Delta(1750) \), the one-star \( \Sigma(1480) \) and \( \Sigma(1770) \), the two-star \( \Sigma(1580) \) and the one-star \( \Xi(1620) \), which we shall discuss below.

It was suggested in [4] that \( \Sigma(1480) \) and \( \Xi(1620) \) might be members of a new light octet, whose \( N \) member (called \( N' \)) is predicted to have the mass around 1100 MeV and the vanishingly small total width. In addition, the \( g_{N'N\pi} \) coupling constant was predicted to be

Note that the \( \Lambda \), \( \Sigma \) and \( \Xi \) members of the considered octet have disappeared or have changed their masses since 1974 such that their RPP candidates cannot be easily established.
strongly suppressed compared to the usual $NN\pi$ coupling constant, $g_{NN\pi}/g_{NN\pi} \leq 0.01$. In our notation, this corresponds to the vanishingly small $A_8$ coupling constant. However, the experimental information on the decays of $\Sigma(1480)$ and $\Xi(1620)$ is too sketchy to perform a $\chi^2$ analysis of the decay rates. It is interesting to observe that the 1972 edition of the Review of Particle Physics [32] contained a possible candidate for the $\Lambda$ member of this superlight octet, $\Lambda(1330)$, which disappeared in the 2004 edition of the RPP.

As we discussed in Sect. 4, the $J^P = 1/2^+$ and the mass of $\Sigma(1770)$ make it a potential candidate for the $\Sigma_{\pi\pi}$ member of the antidecuplet.

Experimental evidence for the $\Delta(1750)$ is too weak to make any hypothesis concerning its place in our SU(3) scheme. The same applies to $\Sigma(1580)$. While it has a two-star status, this state is not seen in the analyses, which we use as our primary sources of information on hyperons [34–36].

In order to have a complete picture of unitary multiplets, we predict a number of new strange baryons, whose properties are summarized in Table 28. In the table, besides the predicted spin and parity, we also give estimates for the mass, the sum of two-body partial decay widths $\Gamma^{2\text{-body}}$ and the total width $\Gamma^{\text{tot}}$. The latter quantity is given only for the cases, where a meaningful estimate could be done. The last column lists final states with large branching ratios.

In Table 28, the most remarkable prediction is the existence of the $\Lambda$ hyperon with $J^P = 3/2^-$, the mass around 1850 MeV, $\Gamma^{2\text{-body}} \approx 32$ MeV, $\Gamma^{\text{tot}} \approx 130$ MeV and very small coupling to the $N\bar{K}$ state. This is the only missing $\Lambda$ resonance, which is needed to complete octet 11 – all other eleven $\Lambda$ hyperons, see Table 1, are known very well and have three and four-star ratings in the RPP. Note that the existence of a $\Lambda$ hyperon with $J^P = 3/2^-$ in the 1775-1880 mass range, which almost decouples from the $N\bar{K}$ state, is also predicted in the constituent quark model [7–9]. Our analysis suggests that the missing $\Lambda$ baryon can be searched for in production reactions by studying the $\Sigma\pi$ and $\Sigma(1385)\pi$ invariant mass spectra.

As can be seen from Table 28, we predict the existence of ten new $\Xi$ baryons. In this respect, one should mention the recent interest in double-strangeness baryon spectroscopy [91].

In addition to twenty multiplets of Table 1, we have an additional twenty-first multiplet in Table 27 – the antidecuplet. While the existence of the antidecuplet is under debate, we assumed that the antidecuplet does exist, has $J^P = 1/2^+$ and contains the $\Theta^+(1540)$ and $\Xi^{--}(1862)$ as its lightest and heaviest states. The nucleon-like member of $\Sigma_{10}$, $N_{10}$, is identified with the new nucleon resonance at 1670 MeV seen by GRAAL [79] and predicted by the PWA analysis of pion-proton scattering [78]. The mass of the $\Sigma$-like member of $\Sigma_{10}$, $\Sigma_{10}$, is predicted using the equal spacing rule, $m_{\Sigma_{10}} \approx 1760$ MeV.

Using the scarce information on the antidecuplet decays, we showed that $N_{10}$ must mix...
with the \( N \) member of another multiplet. We examine the scenario that the \( N_{10} \) and \( \Sigma_{10} \) members of the antidecuplet mix with \( N(1440) \) and \( \Sigma(1660) \) of octet 3. We showed that this can accommodate in a simple way all experimental information on the antidecuplet decays. Prediction for the unmeasured \( \Xi \) decays were made.

One should not have the impression that our SU(3) analysis of two-body baryon decays was flawless. In several cases, we had to increase experimental errors by hand in order to claim that the \( \chi^2 \) fit was successful. Also, we encountered three cases, when SU(3) predicted incorrectly the sign of the interference observables. These are \( \sqrt{\Gamma_N \Gamma_{\Sigma\pi}} \) for \( \Sigma(2070) \)

| Particle | \( J^P \) (multiplet) | Mass (MeV) | \( \Gamma_{2\text{-body}} \) (MeV) | \( \Gamma_{\text{tot}} \) (MeV) | Large branchings |
|----------|------------------------|------------|-------------------------------|-------------------------------|------------------|
| \( \Lambda \) | \( 3/2^- \) (11) | 1850 | 32 | 130 | \( \Sigma\pi, \Sigma^*\pi \) |
| \( \Sigma \) | \( 1/2^+ \) (21) | 1760 | 10 | \( \Sigma\pi, \Sigma^*\pi \) |
| \( \Sigma \) | \( 3/2^- \) (13) | 1850 | 70 | \( \Lambda\pi, N\overline{K}, \Sigma^*\pi \) |
| \( \Sigma \) | \( 1/2^+ \) (19) | 2060 | 75 | \( \Lambda\pi, N\overline{K}, \Sigma\pi \) |
| \( \Xi \) | \( 1/2^- \) (9) | 1620-1725 | 115 | \( \Xi\pi, \Lambda\overline{K} \) |
| \( \Xi \) | \( 1/2^- \) (14) | 1860-1915 | 135 | 220 | \( \Sigma\overline{K}, \Lambda\overline{K}, \Xi\eta \) |
| \( \Xi \) | \( 1/2^- \) (10) | 1900 | 33 | 50 – 60 | \( \Xi\pi, \Sigma\overline{K}, \Lambda\overline{K}, \Xi^*\pi \) |
| \( \Xi \) | \( 3/2^+ \) (5) | 1900 | 95 | \( \Xi\pi, \Lambda\overline{K}, \Xi^*\pi \) |
| \( \Xi \) | \( 1/2^+ \) (4) | 1950 | 50 | \( \Sigma\overline{K}, \Xi^*\pi \) |
| \( \Xi \) | \( 3/2^- \) (13) | 2000 | 130 | \( \Sigma^*\overline{K}, \Xi^*\pi, \Lambda\overline{K}, \Xi\pi \) |
| \( \Xi \) | \( 3/2^+ \) (17) | 2035 | 15 | \( \Sigma\overline{K} \) |
| \( \Xi \) | \( 3/2^- \) (11) | 2045 | 66 | 264 | \( \Xi\pi, \Lambda\overline{K} \) |
| \( \Xi \) | \( 1/2^+ \) (19) | 2210 | 85 | \( \Lambda\overline{K}, \Xi\pi, \Sigma\overline{K} \) |
| \( \Xi \) | \( 3/2^+ \) (16) | 2240 | 35 | 100 – 150 | \( \Lambda\overline{K}, \Xi\pi, \Sigma\overline{K} \) |
| \( \Omega \) | \( 1/2^- \) (10) | 2050 | 35 | \( \leq 50 – 60 \) | \( \Xi\overline{K} \) |
| \( \Omega \) | \( 3/2^+ \) (5) | 2050 | 70 | \( \Xi\overline{K}, \Xi^*\overline{K} \) |
| \( \Omega \) | \( 3/2^- \) (13) | 2150 | 110 | \( \Xi\overline{K}, \Xi^*\overline{K} \) |
| \( \Omega \) | \( 1/2^+ \) (19) | 2360 | 90 | \( \Xi\overline{K} \) |

Table 28
Predicted baryons.
(decuplet 18), $\sqrt{\Gamma_{NK}\Gamma_{\Sigma\pi}}$ for $\Sigma(1940)$ (octet 11) and $\sqrt{\Gamma_{NK}\Gamma_{\Lambda\pi}}$ for $\Sigma(2080)$ (decuplet 16).

Acknowledgments

We thank Ya.I. Azimov, P. Pobilitsa, Fl. Stancu and I. Strakovsky for useful discussions. This work is supported by the Sofia Kovalevskaya Program of the Alexander von Humboldt Foundation.
References

[1] T. Nakano et al. [LEPS Collaboration], Phys. Rev. Lett. 91 (2003) 012002 [arXiv:hep-ex/0301020].

[2] D. Diakonov, V. Petrov and M. V. Polyakov, Z. Phys. A 359 (1997) 305 [arXiv:hep-ph/9703373].

[3] C. Alt et al. [NA49 Collaboration], Phys. Rev. Lett. 92 (2004) 042003 [arXiv:hep-ex/0310014].

[4] Y. I. Azimov, R. A. Arndt, I. I. Strakovsky and R. L. Workman, Phys. Rev. C 68 (2003) 045204 [arXiv:nucl-th/0307088].

[5] S. Capstick and W. Roberts, Prog. Part. Nucl. Phys. 45 (2000) S241 [arXiv:nucl-th/0008028].

[6] N. P. Samios, M. Goldberg and B. T. Meadows, Rev. Mod. Phys. 46 (1974) 49.

[7] N. Isgur and G. Karl, Phys. Rev. D 18 (1978) 4187.

[8] U. Loring, B. C. Metsch and H. R. Petry, Eur. Phys. J. A 10 (2001) 447 [arXiv:hep-ph/0103290].

[9] L. Y. Glozman, W. Plessas, K. Varga and R. F. Wagenbrunn, Phys. Rev. D 58 (1998) 094030 [arXiv:hep-ph/9706507].

[10] S. Eidelman et al. [Particle Data Group], Phys. Lett. B 592 (2004) 1.

[11] M. Gell-Mann and Y. Ne’eman, The Eightfold Way, W.A. Benjamin, Inc., Amsterdam and New York, 1964.

[12] J.J.J. Kokkedee, The quark model, W.A. Benjamin, Inc., Amsterdam and New York, 1969.

[13] D.B. Lichtenberg, Unitary symmetry and elementary particles, Academic Press, New York and London, 1970.

[14] F.E. Close, An introduction to quarks and partons, Academic Press, London, 1979.

[15] J. R. Ellis, M. Karliner and M. Praszalowicz, JHEP 0405 (2004) 002 [arXiv:hep-ph/0401127].

[16] M. Praszalowicz, Acta Phys. Polon. B 35 (2004) 1625 [arXiv:hep-ph/0402038].

[17] R. Bijker, M. M. Giannini and E. Santopinto, Eur. Phys. J. A 22 (2004) 319 [arXiv:hep-ph/0310281].

[18] J. J. de Swart, Rev. Mod. Phys. 35 (1963) 916.

[19] S. R. Coleman and S. L. Glashow, Phys. Rev. Lett. 6 (1961) 423.

[20] Yu. V. Novozhilov, Introduction to elementary particle physics, Pergamon Press, 1975.
[21] H. C. Kim, M. Polyakov, M. Praszalowicz, G. S. Yang and K. Goeke, Phys. Rev. D 71 (2005) 094023 [arXiv:hep-ph/0503237].

[22] M. V. Polyakov and A. Rathke, Eur. Phys. J. A 18 (2003) 691 [arXiv:hep-ph/0303138].

[23] P. N. Dobson, S. Pakvasa and S. F. Tuan, Hadronic J. 1 (1978) 476.

[24] H. A. Weldon, Phys. Rev. D 17 (1978) 248.

[25] V. Guzey and M. V. Polyakov, Annalen Phys. 13 (2004) 673.

[26] A. W. Martin, Nuovo Cim. 32 (1964) 1645.

[27] R. D. Tripp, D. W. G. Leith, A. Minten and e. al., Nucl. Phys. B 3 (1967) 10.

[28] S. Pakvasa and S. F. Tuan, Nucl. Phys. B 8 (1968) 95.

[29] P. N. Dobson, Phys. Rev. 176 (1968) 1757.

[30] D. E. Plane et al., Nucl. Phys. B 22 (1970) 93.

[31] N. Isgur and G. Karl, Phys. Rev. D 19 (1979) 2653 [Erratum-ibid. D 23 (1981) 817].

[32] P. Söding et al. [Particle Data Group], Phys. Lett. B 39 (1972) 1.

[33] D. M. Manley and E. M. Saleski, Phys. Rev. D 45 (1992) 4002.

[34] G. P. Gopal, R. T. Ross, A. J. Van Horn, A. C. McPherson, E. F. Clayton, T. C. Bacon and I. Butterworth [Rutherford-London Collaboration], Nucl. Phys. B 119 (1977) 362.

[35] W. Cameron et al. [Rutherford-London Collaboration], Nucl. Phys. B 131 (1977) 399.

[36] G. P. Gopal, RL-80-045 Invited talk given at 4th Int. Conf. on Baryon Resonances, Toronto, Canada, Jul 14-16, 1980.

[37] F. J. Gilman, M. Kugler and S. Meshkov, Phys. Rev. D 9 (1974) 715.

[38] A. Garcia and P. Kielanowski, The Beta Decay Of Hyperons, Lecture Notes in Physics 222, Springer-Verlag, Berlin and Heidelberg, 1985.

[39] F. James, MINUIT minimization package reference manual. CERN program library Long Writeup D506.

[40] T. P. Vrana, S. A. Dytman and T. S. H. Lee, Phys. Rept. 328 (2000) 181 [arXiv:nucl-th/9910012].

[41] W. Cameron et al. [Rutherford-London Collaboration], Nucl. Phys. B 143 (1978) 189.

[42] A. P. Bukhvostov, arXiv:hep-ph/9705387.

[43] R. D. Tripp, R. O. Bangerter, A. Barbaro-Galtieri and T. S. Mast, Phys. Rev. Lett. 21 (1968) 1721.

[44] D. Diakonov and V. Petrov, Phys. Rev. D 69 (2004) 094011 [arXiv:hep-ph/0310212].
[45] J. C. Collins and J. Pumplin, arXiv:hep-ph/0105207.

[46] W. Langbein and F. Wagner, Nucl. Phys. B 47 (1972) 477.

[47] B. R. Martin and M. K. Pidcock, Nucl. Phys. B 126, 266 (1977); Nucl. Phys. B 126 (1977) 285; Nucl. Phys. B 127 (1977) 349.

[48] R. Koniuk and N. Isgur, Phys. Rev. D 21 (1980) 1868 [Erratum-ibid. D 23 (1981) 818].

[49] D. F. Kane, Phys. Rev. D 5 (1972) 1583.

[50] M. J. Corden, G. F. Cox, A. Dartnell, I. R. Kenyon, S. W. O’Neale, K. C. T. Sumorok and P. M. Watkins, Nucl. Phys. B 104 (1976) 382.

[51] V. V. Barmin et al. [DIANA Collaboration], Phys. Atom. Nucl. 66 (2003) 1715 [Yad. Fiz. 66 (2003) 1763] [arXiv:hep-ex/0304040].

[52] S. Stepanyan et al. [CLAS Collaboration], Phys. Rev. Lett. 91 (2003) 252001 [arXiv:hep-ex/0307018].

[53] V. Kubarovsky et al. [CLAS Collaboration], Phys. Rev. Lett. 92 (2004) 032001 [Erratum-ibid. 92 (2004) 049902] [arXiv:hep-ex/0311046].

[54] A. E. Asratyan, A. G. Dolgolenko and M. A. Kubantsev, Phys. Atom. Nucl. 67 (2004) 682 [Yad. Fiz. 67 (2004) 704] [arXiv:hep-ex/0309042].

[55] J. Barth et al. [SAPHIR Collaboration], Phys. Lett. B 572 (2003) 127 [arXiv:hep-ex/0307083].

[56] A. Airapetian et al. [HERMES Collaboration], Phys. Lett. B 585 (2004) 213 [arXiv:hep-ex/0312044].

[57] A. Aleev et al. [SVD Collaboration], arXiv:hep-ex/0401024.

[58] M. Abdel-Bary et al. [COSY-TOF Collaboration], Phys. Lett. B 595 (2004) 127 [arXiv:hep-ex/0403011].

[59] S. Chekanov et al. [ZEUS Collaboration], Phys. Lett. B 591 (2004) 7 [arXiv:hep-ex/0403051].

[60] A. Aleev et al. [SVD Collaboration], arXiv:hep-ex/0509033.

[61] A. Aktas et al. [H1 Collaboration], Phys. Lett. B 588 (2004) 17 [arXiv:hep-ex/0403017].

[62] J. Z. Bai et al. [BES Collaboration], Phys. Rev. D 70 (2004) 012004 [arXiv:hep-ex/0402012].

[63] S. Schael et al. [ALEPH Collaboration], Phys. Lett. B 599 (2004) 1.

[64] B. Aubert et al. [BABAR Collaboration], arXiv:hep-ex/0408064; arXiv:hep-ex/0408037.

[65] I. Abt et al. [HERA-B Collaboration], Phys. Rev. Lett. 93 (2004) 212003 [arXiv:hep-ex/0408048].

75
[66] D. O. Litvintsev [CDF Collaboration], Nucl. Phys. Proc. Suppl. 142 (2005) 374 [arXiv:hep-ex/0410024].

[67] K. Stenson [FOCUS Collaboration], arXiv:hep-ex/0412021.

[68] K. Abe et al. [BELLE Collaboration], arXiv:hep-ex/0409010.

[69] C. Pinkenburg [PHENIX Collaboration], J. Phys. G 30 (2004) S1201 [arXiv:nucl-ex/0404001].

[70] A. R. Dzierba, D. Krop, M. Swat, S. Teige and A. P. Szczepaniak, Phys. Rev. D 69 (2004) 051901 [arXiv:hep-ph/0311125].

[71] A. R. Dzierba, C. A. Meyer and A. P. Szczepaniak, J. Phys. Conf. Ser. 9 (2005) 192 [arXiv:hep-ex/0412077].

[72] H. G. Fischer and S. Wenig, Eur. Phys. J. C 37 (2004) 133 [arXiv:hep-ex/0401014].

[73] M. Zaveryaev, arXiv:hep-ex/0501028.

[74] M. Battaglieri, R. De Vita, V. Kubarovsky, L. Guo, G. S. Mutchler, P. Stoler and D. P. Weygand [the CLAS Collaboration], arXiv:hep-ex/0510061.

[75] K. H. Hicks [CLAS Collaboration], arXiv:hep-ex/0510067.

[76] A. I. Titov, B. Kampfer, S. Date and Y. Ohashi, Phys. Rev. C 72 (2005) 035206 [Erratum-ibid. C 72 (2005) 049901] [arXiv:nucl-th/0506072].

[77] S. Chekanov [H1 Collaboration], arXiv:hep-ex/0510057.

[78] R. A. Arndt, Y. I. Azimov, M. V. Polyakov, I. I. Strakovsky and R. L. Workman, Phys. Rev. C 69 (2004) 035208 [arXiv:nucl-th/0312126].

[79] V. Kuznetsov [GRAAL Collaboration], arXiv:hep-ex/0409032.

[80] S. Kabana [STAR Collaboration], arXiv:hep-ex/0406032.

[81] R. A. Arndt, I. I. Strakovsky and R. L. Workman, Phys. Rev. C 69 (2004) 035208 [arXiv:nucl-th/0308012].

[82] A. Casher and S. Nussinov, Phys. Lett. B 578 (2004) 124 [arXiv:hep-ph/0309208].

[83] J. Haidenbauer and G. Krein, Phys. Rev. C 68 (2003) 052201 [arXiv:hep-ph/0309243].

[84] R. N. Cahn and G. H. Trilling, Phys. Rev. D 69 (2004) 011501 [arXiv:hep-ph/0311245].

[85] W. R. Gibbs, Phys. Rev. C 70 (2004) 045208 [arXiv:nucl-th/0405024].

[86] A. Sibirtsev, J. Haidenbauer, S. Krewald and U. G. Meissner, Phys. Lett. B 599 (2004) 230 [arXiv:hep-ph/0405099].

[87] D. Diakonov and V. Petrov, arXiv:hep-ph/0505201.

[88] W. Lorenzon [HERMES Collaboration], arXiv:hep-ex/0411027.
[89] A. Hosaka, T. Hyodo, F. J. Llanes-Estrada, E. Oset, J. R. Pelaez and M. J. Vicente Vacas, Phys. Rev. C 71 (2005) 045205 [arXiv:hep-ph/0411311].

[90] V. Guzey and M. V. Polyakov, arXiv:hep-ph/0501010.

[91] Workshop Cascade Physics: A New Window on Baryon Spectroscopy, December 1-3, 2005, Jefferson Lab, Newport News, VA USA, http://conferences.jlab.org/cascade/.