MSGUT : THE NEXT AVTAR

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We report the viability of the Type I seesaw mechanism in 3 generation renormalizable SO(10) GUTs based on the $210 - 10 - 126 - 120$ Higgs system. The $120$ -plet and $10$-plet Higgs fit charged fermion masses while small $126$-plet couplings enhance Type I seesaw neutrino masses to viable values and make the fit to light fermions accurate. For the 3 generation CP conserving case we display accurate charged fermion fits ($\chi^2_{\text{tot}} < .2$) which imply Type I neutrino masses $10^2$ to $10^3$ times larger than the $10 - 126$ scenario. The correct ratio of neutrino mass squared splitting, large ($\theta^1_{\text{PMNS}}$) and small ($\theta^2_{\text{PMNS}}$) mixing angle are achievable. $\theta^2_{\text{PMNS}}$ is however small and indicates that -as in the Type I $10 - 126$ case - a fully realistic fit to the lepton mixing data also requires CP violation.

1. Introduction

The discovery of neutrino oscillations has driven an intense wave of research into supersymmetric seesaw mechanisms, Left-right symmetric models and particularly the minimal Supersymmetric Grand Unified Theory (MSGUT) with Higgs multiplets $210 - 10 - 126 - 126$. GUT scale Spectra, threshold effects and fermion spectrum fits have all received much attention. Until last year this ‘Babu-Mohapatra’ (BM) program seemed successful and accurate generic fits to all known fermion data using Type I, Type II and mixed seesaw mechanisms were obtained. However it was just assumed that the required overall scale and relative strength of the Type I and Type II seesaw masses could be realized in appropriate Susy GUTs. The first survey of this question in the MSGUT revealed serious difficulties in obtaining Type II over Type I dominance and also in obtaining large enough Type I neutrino masses. Using a convenient parametrization of the MSGUT spectra and couplings in terms of the single “fast” parameter ($x$) which controls MSGUT symmetry breaking, as emphasized by the authors of the first reference in [8], a complete proof was then given of the failure of the Seesaw mechanism in the context of the MSGUT and confirmed by another group. We also suggested a fix for the problem of too small seesaw masses using an additional $120$plet. In our fitting ansatz the small $126$-plet couplings give appreciable contributions only to light charged fermion masses and enhance the Type I seesaw masses to viable values since the Type I seesaw masses are inversely proportional to the $126$ coupling. The 2-3 generation case was first analyzed as a toy model of the dominant core of the complete hierarchical fermion mass system. Consistency required $m_b - m_s = m_\tau - m_\mu$ at the GUT scale $M_X$ and predicted near maximal PMNS mixing for wide parameter ranges. In the current contribution we report on the extension of our 2 generation toy model to the 3 generation CP conserving case using a procedure based on an expansion in a Wolfenstein type parameter around the dominant 23 generation core of the fermion hierarchy.

1.1. Seesaw Failure in the MSGUT

The Type I and Type II seesaw Majorana masses of the light neutrinos in the MSGUT are $\langle h, f \rangle$ are proportional to $10, 126$
Yukawas):

\[ M^I_u = (1.70 \times 10^{-3} \text{eV}) \, F_I \, \hat{n} \, \sin \beta \]
\[ M^I_{\nu} = (1.70 \times 10^{-3} \text{eV}) \, F_{II} \, \hat{f} \, \sin \beta \]
\[ \hat{n} = (\hat{h} - 3f)\hat{f}^{-1}(\hat{h} - 3f) \]

Typical BM-Type II fits require \( R = F_I / F_{II} \leq 10^{-3} \) so it not be overwhelmed by Type I values it implies. Such \( R \) values are un-achievable in the MSGUT. In Type I values it implies. Such \( R \) values are un-achievable in the MSGUT. In Type I fits values of \( F_I \sim 100 \) are needed but are not achievable anywhere over the complex \( x \) plane. 

1.2. The new \( 10-120-1 \bar{2}6 \) scenario

To resolve the difficulty we proposed that \( \bar{2}6 \) couplings be reduced much below the level where they are important for 2-3 generation masses and introduced \( 120 \) plet for charged fermion mass fitting (previously accomplished by \( \bar{2}6 \) couplings comparable to those of the \( 10 \)-plet). Fermion masses in such GUTs are (\( m \): Dirac, \( M \): Majorana):

\[ \hat{m}^u = v(\hat{h} + \hat{f} + \hat{g}) \]
\[ \hat{m}_{\nu} = v(\hat{h} - 3\hat{f} + r_3\hat{g}) \]
\[ \hat{m}^d = v(r_1\hat{h} + r_2\hat{f} + r_6\hat{g}) \]
\[ \hat{n} = (\hat{h} - 3\hat{f} - r_5\hat{g})\hat{f}^{-1}(\hat{h} - 3\hat{f} + r_5\hat{g}) \]

where \( r_i \) are coefficients fixed by the SUSY GUT. We assume CP conservation i.e. fermion Yukawa couplings and coefficients \( r_i \) are both real. Note that with only \( r_i \) complex and the Yukawa couplings real (i.e. spontaneous CP violation) the resulting NMSGUT has only 12 Yukawa coupling parameters i.e. 3 less than in the MSGUT with complex Yukawas!

Matching mass terms above to renormalized MSSM mass matrices at \( M_X \) introduces unitary matrices (specifying the MSSM⊂MSGUT embedding) which are of vital relevance for calculation of exotic signatures:\n
\[ \hat{m}^u = V_u^T D_u Q \]
\[ \hat{m}^d = V_d^T D_d R \]
\[ \hat{m}^l = V_l^T D_l L \]

\( D_{u,d,l} \) are masses at \( M_X \) and \( V_u,Q,V_d,R = C^T Q,L,V_l \) unitary matrices (\( C \) is the CKM matrix). Rewriting \( V_{u,d,l} : V_d = \Phi_d R ; V_u = \Phi_u Q ; V_l = \Phi_l L \) and separating symmetric and antisymmetric parts:

\[ Z = \Phi^T D + D \Phi ; A = \Phi^T D - D \Phi \]

allows the problem to be reduced to that of determining the matrices \( \Phi_{u,d,l} ; D = RL^T \) and coefficient \( r_2 \) such that:

\[ \hat{S}_3 = \hat{S}_X + \hat{r}_2 \frac{TrZ_d}{TrZ_u}(\hat{S}_2 - \frac{(TrZ_d - TrZ_l)}{TrZ_d}) \hat{X} \]
\[ \hat{A}_l = C \hat{A}_d C^T - \hat{A}_u = 0 \]
\[ \hat{A}_l^2 = \hat{A}_d \mp D \hat{A}_l D^T = 0 \]
\[ \hat{X} = \frac{3Z_d + DZ_l D^T}{3TrZ_d + TrZ_l} \]
\[ \hat{S}_X = \hat{X} - C^T \frac{Z_u}{TrZ_u} C \]
\[ \hat{A}_{u,d,l} = A_{u,d,l}/\sqrt{A^2_{u,d,l}} \]

the ambiguous sign above corresponds to the branch of the 2 generation model that one is expanding about i.e. \( \hat{A}_2^\pm = 0 \) for \( \chi_l = \pm \chi_l \). The definitions are such that \( \hat{S}_2 = 0 \Rightarrow \hat{S}_3 = 0 \) reduces to \( \hat{S}_1 = 0 \).

These equations are too complicated to solve analytically in the 3 generation case. Our approach is to consider the 23 sector as the dominant “core” of the fermion mass hierarchy and to expand around it. With our reality assumption all unitary matrices can be given the parametrization (\( O_{ij} \) are orthogonal rotations): \( O = O_{23}(\chi)O_{13}(\phi)O_{12}(\theta) \).

In view of the encouraging results of our 2 generation model for the 23 dominated fermion hierarchy, we looked for solutions as an expansion (in powers of \( \epsilon = \sqrt{\theta_{23}} \sim \theta_{12} \sim .2 \)) around the 2 generation results. The 2 generation case gave \( \hat{S}_2 \sim O(\epsilon^3) \) and
\( \hat{f} = \frac{1}{2\epsilon} \hat{r}_2 Tr Z_d \hat{S}_2 \). The \( O(10^{-2}) \) suppression provided by the ratio \( d_3/v \) implies that \( \hat{f} \sim .01 \epsilon^{3+4} \). This ensures the enhancement of Type I neutrino masses that is the rationale for this fitting scenario.

\[ \chi^0_i = \chi^0_u = \chi^0_l = \chi^0_D \]

The angles \( \theta_{i} \) are crucial: the value of \( \theta_2 \) and the constraint \( d_3 = l_3 - l_2 + d_2 \) are still fixed by the expansion up to \( O(\epsilon^2) \) just as in the toy model. A simple criterion is to define \( \chi^2 \) functions measuring the deviation from central data and terminating the perturbation expansion when these are small. For the central quark masses and angles we used Das-Parida\(^9\) central values (for \( \tan^2(M_S) = 55 \) and at \( M_X = 2 \times 10^{16} \text{ GeV} \)) except for \( d_3 \) as explained above.

The Type I seesaw mass formula is

\[
\hat{M}_{\nu} = \frac{\nu^2}{2\epsilon} (\hat{h} \cdot \hat{r}_5 \hat{g} - 3 \hat{f})^T \hat{f}^{-1} (\hat{h} \cdot \hat{r}_5 \hat{g} - 3 \hat{f}) \equiv (1.70 \times 10^{-3} \text{eV}) R^T \hat{n} R F_1 \sin^2 \beta = L^T P D V P^T L
\]

\[ \mathcal{P} = D^1 N ; \quad m_{\text{sol}}^2 / m_{\text{atm}}^2 = \left| \frac{n^2_{\nu} - \hat{n}^2_{\nu}}{n^2_{\tau} - \hat{n}^2_{\tau}} \right| \]

\( \mathcal{P} \) is the PMNS matrix and \( D_v \) is the light neutrino masses at \( M_X \). \( N \) diagonalizes \( \hat{n} : \hat{n} = N \hat{n}_{\text{diag}} N^T \).

2. Examples of 3 generation fits

We have obtained a number of examples of acceptable fits -modulo the assumption that CP violation is neglected- by going to high orders in \( \epsilon \) (sometimes as high as \( O(\epsilon^2) \)). For reasons of space we confine ourselves to quoting the values found for one such fit:

\[ \hat{r}_1 = \frac{15.27}{\hat{r}_2 = 0.255 ; \hat{r}_6 = 0.0187 ; \hat{r}_7 = 0.023305}. \]

\[ \hat{h} = \begin{pmatrix} -0.000299 & 0.000039 & -0.02227 \\ 0.000039 & -0.13482 \\ -0.02227 & -0.13482 & 0.481333 \end{pmatrix} \]

\[ \hat{f} = \begin{pmatrix} 0.000132 & 0.000277 & -0.00011 \\ 0.000277 & -0.0000146 & 0.000057 \\ -0.0001 & 0.000057 & 0.000040 \end{pmatrix} \]

\[ \hat{g} = \begin{pmatrix} 0 & 0.001644 & -0.02478 \\ -0.001644 & 0. & -0.119710 \\ 0.02478 & 0.119710 & 0 \end{pmatrix} \]

The eigenvalues of \( \hat{h}, \hat{f}, \hat{g} \) are

\[ \hat{h} : 0.52 ; 0.028 ; 1.92 \times 10^{-5} \]

\[ \hat{f} : 1.33 \times 10^{-4} ; 1.11 \times 10^{-4} ; 0.17 \times 10^{-4} \]

\[ \hat{g} : \pm 0.122 ; 0 \]

The premises of our\(^5,6,7\) scenario are indeed respected. Then the reconstructed values of the charged fermion masses are (in GeV):

\[ M_U = \{95.148, 0.211, 0.00077\}; M_D = \{1.584, 0.0298, 0.0015\}; M_l = \{1.629, 0.0753, 0.00036\} \]

and the CKM angle magnitudes:

\[ \theta_{12} = 0.227 ; \theta_{13} = 0.00216; \theta_{23} = 0.038 \]

The charged fermion fit \( \chi^2 \) values are \( \chi^2_{\text{m}} = 0.126 ; \chi^2_{\text{CKM}} = 0.16 ; \chi^2_{\text{tot}} = 0.142 \).

Fixing \( r'_5 \) by imposing \( R_{ijk} = \left| \frac{n^2_{\nu} - \hat{n}^2_{\nu}}{n^2_{\tau} - \hat{n}^2_{\tau}} \right| \).

.32 gives 6 solutions which are however 3 closely related pairs. One finds that in only one case is \( \theta_{23} \) large and \( \theta_{13} \) reasonably small. However the value of the 12 sector mixing is very low. On the other hand the large (about 200 times larger than the Type I fits in the \( 10^{1-126} \) scenario: see below) value of the largest \( \hat{n} \) eigenvalue together with the satisfactory value of the mass squared splitting ratio means that the problem with too small neutrino masses is unlikely to appear even for generic values of the GUT scale breaking, leave alone regions where the coefficient function \( F_1 \) is itself large.
In the almost viable case $r'_0 = -0.333$; $\phi_D = 0.053$; $\theta_D = 0.435$; $\chi_D = 0.0257$ and $n_1 = 113.8$; $n_2 = 20.045$; $n_3 = 1.97 \times 10^{-5}$, while $\sin^2 \theta_{12}^{PMNS} = 0.073$; $\sin^2 \theta_{23}^{PMNS} = 0.77$; $\theta_{13}^{PMNS} = 0.176$.

3. Discussion, Conclusions and Outlook

We have reported progress towards a completely realistic fit of all known charged fermion and neutrino mass data using the mass relations and RG evolution common to any SO(10) SUSY GUT with a $10 - 120 - \overline{126}$ Higgs system. Specifically, we have shown that in the quasi realistic 3 generation but CP preserving real case we are able to obtain accurate charged fermion fits, neutrino mass parameters and a PMNS mixing pattern that can be large in the 23 sector and small in the 13 sector. The remaining deficiencies, namely simultaneous large mixing in the 12 sector and the fit of the MSSM CKM CP phase in the first quadrant can presumably be remedied in the complex 3 generation case, in close analogy with the $10 - \overline{126}$ case where a successful Type I fit could be found only when CP violation was introduced. Very recently another group has implemented our scenario in the $120$ extended MSGUT with spontaneous CP violation and an ad-hoc $Z_2$ symmetry imposed to improve tractability. Using the “downhill simplex” method of non linear fitting they obtain a very accurate and realistic 3 generation fit\(^{10}\). With the completion of this program we will be in possession of a well defined SUSY GUT compatible with all low energy data as well as information on the embedding of the MSSM in the MSGUT (coded in the Unitary matrices $\Phi_{a,d,l,D}$) which emerges as the most valuable corollary product of the fitting procedure.\(^4\) It is only then that we will be able to enter the third phase of the GUT program in which the exotic process ($\Delta B \neq 0$, LFV etc.) predictions will finally be linked sufficiently tightly to low energy data as to make the search for exotic processes a falsifiability test rather than a hopeful check on a lottery bet.

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