Fuzzy Modeling Control and Simulation of a Class of Complex Dynamical Systems

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Abstract. In this paper, an equivalent fuzzy financial dynamic system model is established by using T-S fuzzy modeling method. According to Lyapunov stability theory, a fuzzy feedback controller is designed to realize chaos control. The results of theoretical proof and numerical simulation show that the fuzzy control scheme is effective. The fuzzy controller can simulate human's experience knowledge and is simple and easy to implement. It can also be used to control general nonlinear chaotic systems.

Keywords: Complex system modeling; Stability; Chaos control.

1. Introduction
Chaos control has become the core problem in the field of nonlinear science. It has a profound impact on classical mechanics, physics, communication, control and ecology. Due to the sensitive dependence of chaotic motion on disturbance and the unpredictability of long-term movement trend, the emergence of chaos in the system means that the system itself may have inherent instability. Therefore, it is very important to control chaos effectively, and the key is to find an efficient and simple control strategy.

There are feedback control[6], gain control[7] and pulse control[8] in the research of economic chaotic system control, but there are few intelligent control methods, and intelligent control plays an important role in the control process of complex objects. Therefore, this paper establishes a T-S fuzzy equivalent dynamic system model for a class of financial dynamic systems, and then according to Lyapunov stability theory, the paper establishes a T-S fuzzy equivalent dynamic system model. A simple fuzzy feedback controller is designed to realize chaos control. The fuzzy controller can simulate human's experience knowledge and is easy to implement. It can also be used to control general nonlinear chaotic systems.

2. T-S Fuzzy Financial Dynamic System Model

2.1. Original Financial System Model[1]
The financial dynamic system model given in reference [1] is as follows:
This is a chaotic financial system consisting of the production of sub blocks, money, securities sub block and labor sub block. In the formula: $\bar{x}$ expressed interest rate, $\bar{y}$ express investment demand, $\bar{z}$ express price index, $a$ represents the amount of savings, $b$ represents the investment cost, and $c$ represents the commodity demand elasticity. 

Make $x = \bar{x}, y = \bar{y} - 1/b, z = \bar{z}$. Then (1) the type is changed into:

$$
\begin{align*}
\dot{x} &= \bar{x} + (\bar{y} - a)x \\
\dot{y} &= 1 - by - x^2 \\
\dot{z} &= -x - cz \\
\end{align*}
$$

After transformation, $P_0 (0,0,0)$ is the equilibrium point of (2).

2.2. Fuzzy Financial Dynamic System Modeling

Introduction of T-S fuzzy model[3]:

Fuzzy control is based on fuzzy set theory, fuzzy language and fuzzy logic, it is the application of fuzzy mathematics in control system, it is a nonlinear intelligent control. Fuzzy control is a method to control the object by using the knowledge of human, it is usually expressed in the form of "if condition, then" , so popularly called language control, the control object model, which is generally used to be unable to be expressed in strict mathematics, experience and knowledge of the use of human (expert) is well controlled. Using human intelligence, fuzzy control method is fuzzy control.

T-S fuzzy model contains conditional statements as "if $x$ is $A_i$ then $y = f(x)$" Where $y = f(x)$ is a linear function of $x$, it is a nonlinear model in nature, for the dynamic characteristics of a complex system. its main idea is to use complex surfaces in multi dimensional space with many planes. By establishing a set of linear equations to represent the local rule of each local region, then using the membership function to connect the group of linear equations to form a global function.

Advantages of T-S fuzzy model:

- Integrated expert control experience, in the form of if-then rules, the characteristics of knowledge expression;
- The local linear model can be used to analyze and design the modern control theory (pole assignment, state feedback, predictive control, etc.);
- According to the system's input and output data can be identified, with quantitative and qualitative characteristics of the integration of knowledge;
- To represent highly nonlinear complex system with fewer fuzzy rules, can be easily extended to multi input and multi output fuzzy system, conveniently tuning parameters.

Therefore, in view of the complex nonlinear system modeling and control, in the fuzzy model, T-S fuzzy model can be said is the most potential, one of the most advantages, is also one of the most studied.

T-S financial system model:

Table 1 shows the bank for nearly ten years, the one-year deposit interest rate table
Table 1. Bank one-year deposit interest rate

| Date       | 2015.5.11 | 2015.3.1 | 2012.6.8 | 2011.4.6 | 2011.2.9 | 2010.12.26 |
|------------|-----------|----------|----------|----------|----------|------------|
| Interest   | 2.25      | 2.50     | 3.25     | 3.25     | 3.00     | 2.75       |
| Date       | 2010.10.20| 2008.10.30| 2008.10.9| 2007.9.15| 2007.3.18| 2006.8.19 |
| Interest   | 2.50      | 3.60     | 3.87     | 3.87     | 2.79     | 2.52       |

(Source: https://www.yinhang123.net/)

Experience with analog experts, using fuzzy mathematics, \( x \) can do fuzzy processing, let: \( z(t) = x(t) \), \( z(t) \) for fuzzy antecedent variables, according to table 1, may be the domain \( x \in [2,4] \), fuzzy processing for \( x \), by \( z(t) \), the maximum and minimum value can be set membership function as follows:

\[
M_1(z(t)) = \frac{z(t) - 2}{2}, \quad M_2(z(t)) = \frac{4 - z(t)}{2}
\]

As shown in Figure 1:

![Figure 1. Membership function diagram](image)

Fuzzy rules are as follows:

- \( R_1 \): If the \( z(t) \) is “big”, then \( \dot{X}(t) = A_1 X(t) \);
- \( R_2 \): If the \( z(t) \) is “small”, then \( \dot{X}(t) = A_2 X(t) \).

Among them: \( X(t) = (x,y,z)^T \)

\[
A_1 = \begin{bmatrix}
\frac{1}{b} & -a & 4 & 1 \\
-4 & -b & 0 & -c \\
-1 & 0 & -c & 0
\end{bmatrix}, \quad A_2 = \begin{bmatrix}
\frac{1}{b} & -a & 2 & 1 \\
-2 & -b & 0 & -c \\
-1 & 0 & -c & 0
\end{bmatrix}
\]

State variables of the original model, fuzzy and center weighted average solution by multiplicative reasoning and single point solution, T-S linear model is obtained:

\[
\dot{X}(t) = \frac{\sum_{i=1}^{2} w_i [A_i X(t)]}{\sum_{i=1}^{2} w_i}
\]
Among them: \( w_1 = M_1(z(t)) \), \( w_2 = M_2(z(t)) \), the theory proves that the T-S model (3) is equivalent to the model (2).

3. Fuzzy Control of Financial Dynamic System

3.1. System Dynamic Property Analysis

Stability analysis of equilibrium points:
When the system parameters are different, the stability of the equilibrium point will be affected, and the system (1) will bifurcate under certain conditions.
Numerical simulation:
The use of Simulink in the Matlab toolbox to simulate the chaotic financial system, such as Figure 2, take parameters \( a = 0.9, b = 0.2, c = 1.2 \), the initial conditions [3, 1, 5], to simulate by using Matlab software, when the simulation time was set to [0,10] and [0,1000], get the three-dimensional phase diagram of finance chaotic system (2) as shown in Figure 2, Figure 3, respectively. Figure 2, Figure 3 shows that as time extending, the chaotic behavior of the economic system become more and more prominent. Simulink module as shown in Figure 4.

![Figure 2](image-url) Simulation time for three-dimensional phase diagram at [0,10]

![Figure 3](image-url) Simulation time for three-dimensional phase diagram at [0,1000]

![Figure 4](image-url) Chaotic financial system Simulink module

The analysis shows that the different combinations of the structural parameters of the financial power system can affect the dynamic behavior, and even chaos.

3.2. System Chaos Control

State feedback control of T-S fuzzy financial system:
State feedback control, T-S fuzzy financial system model can be described as follows:
Fuzzy rules \( R_i \) If \( z(t) \) is \( L_i \), then \( \dot{X}(t) = A_i X(t) + B_i u(t) \), \( i = 1, 2 \)
is fuzzy set, \( A_i \) is system matrix, \( X(t) \) is state variable, \( B_i \) is input matrix, \( u(t) \) control input vector, \( z(t) \) is a fuzzy antecedent variable. For a given number of \((X(t),u(t))\), by product reasoning and single point fuzzy controller and center weighted average solution, the whole state equation of a continuous fuzzy system is as follows:

\[
\dot{X}(t) = \sum_{i=1}^{2} \frac{w_i(z(t))[A_iX(t) + B_iu(t)]}{\sum_{i=1}^{2} w_i(z(t))} = \sum_{i=1}^{2} \lambda_i(z(t))[A_iX(t) + B_iu(t)]
\]

(4)

In the formula: \( \lambda_i = \frac{w_i(z(t))}{\sum_{i=1}^{2} w_i(z(t))} \)

The feedback controller is designed using the parallel distributed compensation (PDC) \([5]\) algorithm. The distribution compensation algorithm is the same as the former one of the fuzzy rules for each control, according to the parallel distributed compensation algorithm, the local state feedback controller is designed.

Fuzzy control rules \( R_i \): If \( z(t) \) is \( L_i \), then \( u(t) = -K_iX(t), i = 1,2\), get the whole state feedback control law:

\[
u(t) = -\sum_{i=1}^{2} \frac{w_i(z(t))K_iX(t)}{\sum_{i=1}^{2} w_i(z(t))} = -\sum_{i=1}^{2} \lambda_i(z(t))K_iX(t)\]

bring it into (4), get the closed-loop system of fuzzy T-S financial model:

\[
\dot{X}(t) = \sum_{i=1}^{2} \sum_{j=1}^{2} \lambda_i(z(t))\lambda_j(z(t))(A_i - B_iK_j)X(t)
\]

(5)

A little change can be:

\[
\dot{X}(t) = \sum_{i=1}^{2} \sum_{j=1}^{2} \lambda_i(z(t))\lambda_j(z(t))G_{ii}x(t) + 2\sum_{i<j} G_{ij}\lambda_i(z(t))\lambda_j(z(t))\frac{G_{ij} + G_{ji}}{2}
\]

Among them:\( G_{ij} = A_i - B_iK_j, \quad i, j = 1,2 \)

Stability of T-S fuzzy financial system based on feedback control:

According to the Lyapunov stability theory, the sufficient conditions for the global asymptotic stability of the system (5) can be given.

**Theorem 1**\([5]\) If there is a positive definite symmetric matrix, the linear matrix inequality is lower:

\[
G_{ii}^T P + PG_{ii} < 0, \quad i = 1,2
\]

(6)

\[
\left(\frac{G_{ij} + G_{ji}}{2}\right)^T P + P\left(\frac{G_{ij} + G_{ji}}{2}\right) < 0, \quad i \leq j
\]

(7)

set up, the system (5) is asymptotically stable, among them: \( G_{ij} = A_i - B_iK_j \).

The stability condition of the T-S fuzzy financial system

For the T-S financial system, finding the feedback coefficient matrix \( K_i, i = 1,2 \) of Theorem 1.
To simplify the problem, the $K_i, i = 1, 2$ is the same as the $k_i, i = 1, 2$

$$B_i = \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}, \quad i = 1, 2,$$
below to find $K_i, i = 1, 2$

$$A_1 = \begin{bmatrix} \frac{1}{b} - a & 4 & 1 \\ -4 & -b & 0 \\ -1 & 0 & -c \end{bmatrix}, \quad A_2 = \begin{bmatrix} \frac{1}{b} - a & 2 & 1 \\ -2 & -b & 0 \\ -1 & 0 & -c \end{bmatrix} = \begin{bmatrix} 4.1 & 2 & 1 \\ -2 & -0.1 & 0 \\ -1 & 0 & -1.2 \end{bmatrix},$$

$$G_{11} = \begin{bmatrix} 4.1 - 3k_1 & 4 & 1 \\ -4 & -0.2 - 3k_1 & 0 \\ -1 & 0 & -1.2 - 3k_1 \end{bmatrix}, \quad G_{21} = \begin{bmatrix} 4.1 - 3k_1 & 2 & 1 \\ -2 & -0.2 - 3k_1 & 0 \\ -1 & 0 & -1.2 - 3k_1 \end{bmatrix},$$

$$G_{12} = \begin{bmatrix} 4.1 - 3k_2 & 4 & 1 \\ -4 & -0.2 - 3k_2 & 0 \\ -1 & 0 & -1.2 - 3k_2 \end{bmatrix}, \quad G_{22} = \begin{bmatrix} 4.1 - 3k_2 & 2 & 1 \\ -2 & -0.2 - 3k_2 & 0 \\ -1 & 0 & -1.2 - 3k_2 \end{bmatrix},$$

take $P = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, you can get: $k_1 > 4.1/3, k_1 + k_2 > 8.2/3$, may wish to take

$$k_1 = 1.5, k_2 = 1.5$$

3.3. Numerical Simulation
When $a = 0.9, b = 0.2, c = 1.2$, the initial condition is $[3, 1, 5]$, T-S fuzzy financial model is simulated by Matlab software. Figure 5 shows, the unstable system is stable, and the control is effective.

![Figure 5. Simulation of the feedback control system (5)](image_url)

4. Conclusion and Discussion
According to the financial dynamic system model proposed by the author in reference [1], this paper uses the T-S fuzzy control method to realize the chaos control of a class of chaotic systems, and analyzes and verifies the effectiveness. It can be concluded that: first, the improper combination of the parameters in the chaotic system may cause the system to appear chaos, or may be the system in a stagnant and rigid state, The change of some variables affects the dynamic behavior of the system. Thirdly, the intelligent control strategy can make the chaotic system tend to be stable. Fuzzy controller
can simulate human's experience knowledge and is simple and easy to implement. It can also be used to control general nonlinear chaotic systems

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