A REMARK ON TORSORS FOR AFFINE GROUP SCHEMES

MICHAEL WIBMER

ABSTRACT. We present an elementary proof of the fact that every torsor for an affine group scheme over an algebraically closed field is trivial. This is related to the uniqueness of fibre functors on neutral tannakian categories.

1. INTRODUCTION

Clearly, every torsor for an affine group scheme of finite type over an algebraically closed field is trivial. However, it is not clear if this also holds without the finite type assumption. This question was raised in [Den18], where some partial positive results were obtained: When the affine group scheme $G$ is written as a projective limit of affine group schemes of finite type over an index set $I$, then all $G$-torsors are trivial in the following two cases: $I$ is countable or the cardinality of the algebraically closed base field is strictly larger than the cardinality of $I$. The main result discussed in this short note is the following:

**Theorem 1.1.** Let $G$ be an affine group scheme over an algebraically closed field $k$ and let $X$ be a $G$-torsor. Then $X$ is trivial, i.e., $X(k) \neq \emptyset$.

As an application one obtains:

**Corollary 1.2.** Any two neutral fibre functors on a neutral tannakian category over an algebraically closed field are isomorphic.

We note that a different proof of Corollary 1.2 is outlined in [Del]. In fact, Corollary 1.2 implies Theorem 1.1 because by [DMS12 Theorem 3.2 (b)], for $\omega$ the forgetful functor on the category $\text{Rep}(G)$ of finite dimensional $k$-linear representations of $G$, the functor $\eta \mapsto \text{Isom}^G(\omega, \eta)$ is an equivalence of categories between the category of neutral fibre functors on $\text{Rep}(G)$ and the category of $G$-torsors. Cf. Remark 6.4.4 in [Con20].

In this note we present an elementary proof of Theorem 1.1 that relies on a general principle guaranteeing the non-emptiness of a projective limit. We also show that (over an arbitrary base field) any $G$-torsor can be written as a projective limit of affine $G$-spaces of finite type that are torsors for quotient groups of $G$ (Proposition 2.3).

2. PROOFS

We begin by fixing our notation. Throughout $k$ is a field; our base field. All schemes (including group schemes), products, tensor products and morphisms are assumed to be over $k$ unless the contrary is indicated.

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We will often identify a scheme \( X \) with its functor of points \( R \leadsto X(R) \) from the category of \( k \)-algebras to the category of sets. For an affine scheme \( X \) we denote with \( k[X] \) its \( k \)-algebra of global sections.

Let \( G \) be an affine group scheme. By a closed subgroup of \( G \) we mean a closed subgroup scheme of \( G \). A \( G \)-space is a scheme \( X \) together with a \( G \)-action (from the right) \( X \times G \to X \), \( (x,g) \mapsto x.g \). A morphism of \( G \)-spaces is a \( G \)-equivariant morphism of schemes. A \( G \)-torsor is a \( G \)-space \( X \) such that \( X \times G \to X \times X \), \( (x,g) \mapsto (x,x.g) \) is an isomorphism. For an affine \( G \)-space \( X \) the centralizer \( C_G(X) \) is defined by

\[
C_G(X)(R) = \{ g \in G(R) \mid x.g = x \ \forall \ x \in X(R') \ \text{and all} \ R \text{-algebras} \ R' \}.
\]

Then \( C_G(X) \) is a normal closed subgroup of \( G \) ([DG70 Chapter II, Theorem 3.6 c]) and \( X \) is a \( G/C_G(X) \)-space. Following [Mil72 Def. 5.5.] we call a morphism \( G \to H \) of affine group schemes a quotient map if it is faithfully flat (equivalently, the dual map \( k[H] \to k[G] \) is injective [Wat79 Section 14]).

Let us first sketch the proof of Theorem 1.1. We are given a \( G \)-torsor \( X \) and we would like to show that \( X(k) \neq \emptyset \). We can write \( X \) as a projective limit \( X = \varinjlim X_i \) of \( G \)-spaces \( X_i \) of finite type. So \( X(k) = \varinjlim X_i(k) \). Since we assume \( k \) to be algebraically closed, the \( X_i(k) \)'s are non-empty. However, a projective limit of non-empty sets may well be empty. A standard condition to guarantee the non-emptiness of a projective limit of sets is that the sets are compact Hausdorff topological spaces with continuous transition maps ([RZ10 Prop. 1.1.4].) Unfortunately, the \( X_i(k) \)'s equipped with the Zariski topology are not Hausdorff and so another approach is needed. The following lemma (see [HM57 Prop. 2.7] or [Ste75 Theorem 2.1]) provides a more refined criterion to show that a projective limit is non-empty.

**Lemma 2.1.** Let \( I \) be a directed set and let \(( (X_i)_{i \in I}, (\varphi_{i,j})_{i \leq j} \) be a projective system of topological spaces. If the \( X_i \)'s are non-empty compact T1 spaces and the \( \varphi_{i,j} \)'s are closed maps, then \( \varinjlim X_i \) is non-empty.

Returning to the above discussion, the \( X_i(k) \)'s are compact T1 spaces with respect to the Zariski topology. However, the transition maps need not be closed and so Lemma 2.1 cannot be applied directly. A different topology is needed. We first show that the \( X_i \)'s can be chosen in such a way that \( X_i \) is a \( G/C_G(X_i) \)-torsor. Using this property, we show that the subsets of \( X_i(k) \) that are finite unions of orbits of the form \( x.H(k) \) with \( x \in X_i(k) \) and \( H \) a closed subgroup of \( G \), are the closed subsets of a topology on \( X_i(k) \); the orbit topology. With respect to the orbit topology \( X_i(k) \) is a compact T1 space and the transition maps are continuous and closed. Thus Lemma 2.1 applied to the projective system of the \( X_i(k) \)'s equipped with the orbit topology yields Theorem 1.1.

To make the above sketch precise, we will use the action of \( G \) on \( k[X] \). Let \( G \) be an affine group scheme and \( X \) an affine \( G \)-space. The \( G \)-action \( X \times G \to X \) induces a functorial (left) action of \( G \) on \( k[X] \). For any \( k \)-algebra \( R \), the group \( G(R) \) acts on \( k[G] \otimes R \) by \( R \)-algebra automorphisms. Identifying \( k[X] \otimes R \) with the set of morphisms from \( X_R \) to \( k^1_R \), the action of \( g \in G(R) \) on \( f \in k[X] \otimes R \) is given by \( g(f)(x) = f(x.g) \) for \( x \in X(R') \) and any \( R \)-algebra \( R' \). The invariant ring under this action is

\[
k[X]^G = \{ f \in k[X] \mid g(f \otimes 1) = f \otimes 1 \ \forall \ g \in G(R) \ \text{and any} \ k \text{-algebra} \ R \}.
\]

It is a \( k \)-subalgebra of \( k[X] \). Note that for a normal closed subgroup \( N \) of an affine group scheme \( G \) acting via right-multiplication on \( G \), we have \( k[G]^N = k[G/N] \). See e.g. [Wat79 Section 16.3].
Lemma 2.2. Let $X \times G \to X$ and $Y \times H \to Y$ be actions of affine group schemes on affine schemes. With respect to the diagonal action of $G \times H$ on $X \times Y$ we have

$$k[X \times Y]^{G \times H} = k[X]^G \otimes k[Y]^H.$$  

Proof. Note that for a $k$-algebra $R$ and $g \in G(R)$, $h \in H(R)$, the action of $(g, h)$ is given by

$$(g, h) \colon k[X \times Y] \otimes R \to (k[X] \otimes R) \otimes_R (k[Y] \otimes R) \xrightarrow{\rho \otimes \rho} (k[X] \otimes H) \otimes_R (k[Y] \otimes H) = k[X \times Y] \otimes R.$$  

So the inclusion $k[X]^G \otimes k[Y]^H \subseteq k[X \times Y]^{G \times H}$ is clear.

Conversely, assume that $\sum a_i \otimes b_i \in (k[X] \otimes k[Y])^{G \times H}$. We may assume that the $a_i$’s are $k$-linearly independent. For any $k$-algebra $R$ and $h \in H(R)$ we have

$$(1, h)(\sum a_i \otimes b_i \otimes 1) = \sum a_i \otimes h(b_i \otimes 1) = \sum a_i \otimes b_i \otimes 1 \in k[X] \otimes k[Y] \otimes R$$

As the $a_i$’s are $k$-linearly independent we can conclude that $h(b_i \otimes 1) = b_i \otimes 1$, i.e., $b_i \in k[Y]^H$.

Now, assuming that the $b_i$’s are $k$-linearly independent, a similar argument shows that the $a_i$’s must lie in $k[X]^G$. Thus $\sum a_i \otimes b_i \in k[X]^G \otimes k[Y]^H$. □

It is well known (see e.g. [Wat79, Section 3.3]) that every affine group scheme is a projective limit of affine algebraic groups. The following proposition shows that a similar statement is true for torsors.

Proposition 2.3. Let $G$ be an affine group scheme and let $X$ be an affine $G$-torsor. Then $X$ can be written as a projective limit $X = \lim_{\leftarrow i \in I} X_i$ of affine $G$-spaces $X_i$ of finite type such that every $X_i$ is a $G/C_G(X_i)$-torsor.

Proof. If an abstract group $G$ acts (from the right) on a set $X$ such that $X$ is a $G$-torsor, then for any normal subgroup $N$ of $G$ the set $X/N$ of $N$-orbits in $X$ is a $G/N$-torsor under the action $X/N \times G/N \to X/N$, $(xN, gN) \mapsto x.gN$. This is the idea for the construction of the $X_i$’s. However, to avoid a discussion of the existence of $X/N$ (as an affine scheme) in our context, we will mainly work with the invariant rings.

Let $N$ be a normal closed subgroup of $G$ such that $G/N$ is algebraic (i.e., of finite type). Then $N$ acts (form the right) on $X$ and on $G$. Let $\rho \colon k[X] \to k[X] \otimes k[G]$ be the dual of the action $X \times G \to X$. We claim that $\rho$ restricts to a map $k[X]^N \to k[X]^N \otimes k[G]^N$.

We have a (right) action of $N \times N$ on $X \times G$ given by $(x, g), (n_1, n_2) = (x.n_1, gn_2)$ for $x \in X(R)$, $g \in G(R)$, $n_1, n_2 \in N(R)$ and $R$ a $k$-algebra. According to Lemma 2.2 the invariants $k[X \times G]^N \otimes k[G]^N$ with respect to this action are equal to $k[X]^N \otimes k[G]^N$. It thus suffices to show that $\rho$ maps an $N$-invariant $f \in k[X]$ to an $(N \times N)$-invariant, i.e., we have to show that $f(x.g) = f(x.n_1 gn_2)$ for $n_1, n_2 \in N(R)$, $x \in X(R')$, $g \in G(R')$, where $R$ is a $k$-algebra and $R'$ an $R$-algebra. But since $g^{-1}n_1 gn_2 \in N(R')$, we have $f(x.g) = f(x.n_1 g^{-1}n_2) = f(x.n_1 gn_2)$ by the $N$-invariance of $f$.

Thus $\rho$ restricts to a well-defined map $\rho_N \colon k[X]^N \to k[X]^N \otimes k[G]^N$. Setting $X_N = \text{Spec}(k[X]^N)$ we thus have an action $X_N \times G/N \to X_N$ of $G/N$ on $X_N$. We claim that $X_N$ is a $G/N$-torsor.

The dual $\psi \colon k[X] \otimes k[X] \to k[X] \otimes k[G]$ of the isomorphism $X \times G \to X \times X$, $(x, g) \mapsto (x, x.g)$ is an isomorphism. Therefore, the dual $\psi_N \colon k[X]^N \otimes k[X]^N \to k[X]^N \otimes k[G]^N$ of $X_N \times G/N \to X_N \times X_N$, $(x, g) \mapsto (x, x.g)$ is at least injective.

To see that $\psi_N$ is surjective, we consider the $(N \times N)$-invariants on both sides of the isomorphism $\psi$ (Lemma 2.2). Note however, that $\psi$ is not $(N \times N)$-equivariant. Anyhow, to show that $\psi_N$ is surjective, it suffices to show that $\psi(f) \in (k[X] \otimes k[G])^{N \times N}$ for $f \in k[X] \otimes k[X]$ implies $f \in (k[X] \otimes k[X])^{N \times N}$. But $\psi(f) \in (k[X] \otimes k[G])^{N \times N}$ means that

$$f(x, x.g) = \psi(f)(x, g) = \psi(f)(x.n_1, g.n_2) = f(x.n_1, x.n_1 gn_2)$$

(1)
for \( n_1, n_2 \in N(R), x \in X(R'), g \in G(R'), R \) a \( k \)-algebra and \( R' \) an \( R \)-algebra.

Given a \( k \)-algebra \( R, \tilde{n}_1, \tilde{n}_2 \in N(\tilde{R}), \) an \( \tilde{R} \)-algebra \( \tilde{R}' \) and \( x_1, x_2 \in X(\tilde{R}') \), we can write \( x_2 = x_1, g \) for a unique \( \tilde{g} \in G(\tilde{R}') \). Then, using \( (1) \) with \( R = \tilde{R}' = \tilde{R}, \) \( x = x_1, \) \( g = \tilde{g}, \) \( n_1 = \tilde{n}_1, \) \( n_2 = \tilde{g}^{-1}\tilde{n}_1^{-1}g\tilde{n}_2 \), we have

\[
 f(x_1, x_2) = f(x_1, x_1, g) = f(x_1, \tilde{n}_1, x_1, \tilde{n}_1, g^{-1}g^{-1}g\tilde{n}_2) = f(x_1, \tilde{n}_2, x_1, g\tilde{n}_2) = f(x_1, \tilde{n}_1, x_2, \tilde{n}_2).
\]

Thus \( f \in [k[X] \otimes k[X]]^{N \times N} = k[X]^N \otimes k[X]^N \) as desired and we can conclude that \( X_N \) is a \( G/N \)-torsor. Thus \( R \) is a \( k \)-algebra and \( G \) is a \( k \)-algebra.

For \( F \) space comodules. So \( \mathcal{R} \) of \( \mathcal{R} \) and \( \mathcal{A} \) of \( \mathcal{A} \). Note that \( \mathcal{R} \) and \( \mathcal{A} \) are finitely generated \( \mathcal{A} \)-algebras. Thus \( \mathcal{R} \) and \( \mathcal{A} \) are finitely generated \( \mathcal{A} \)-algebras.

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We next show that \( k[X] \) is the directed union of the \( k[X]^N \)'s. Because each \( k[X]^N \) is a finitely generated \( k \)-algebra, it suffices to show that every finite subset \( F \) of \( k[X] \) is contained in some \( k[X]^N \).

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Note that \( k[X] \) is the directed union of the \( k[X]^N \)'s. Since \( k[X] \) is the directed union of the \( k[X]^N \)'s, we see that \( X = \varprojlim X_N \), where the projective limit is taken over the set of all closed normal subgroups \( N \) of \( G \) such that \( G/N \) is algebraic. This index set is a directed set with respect to the partial order defined by \( N \subseteq N' \) if \( N' \subseteq N \).

To finish the proof it remains to verify that \( C_G(X_N) = N \). Since the action of \( G \) on \( X_N \) factors through \( G/N \), surely \( N \subseteq C_G(X_N) \). Conversely, if \( R \) is a \( k \)-algebra and \( g \in C_G(X_N)(R) \), then the image \( \overline{g} \) of \( g \) in \( (G/N)(R) \) acts trivially on \( X_N(R') \) for every \( R \)-algebra \( R' \). Since \( X_N \) is an \( G/N \)-torsor we must have \( \overline{g} = 1 \), i.e., \( g \in N(R) \). Thus \( C_G(X_N) \subseteq N \) and consequently \( C_G(X_N) = N \).

The following lemma introduces the orbit topology on \( X(k) \), where \( X \) is a \( G \)-space such that \( X \) is a \( G/C_G(X) \)-torsor.

**Lemma 2.4.** Assume that \( k \) is algebraically closed and let \( G \) be an affine group scheme.

(i) Let \( X \) be an affine \( G \)-space of finite type such that \( X \) is a \( G/C_G(X) \)-torsor. Then the subsets of \( X(k) \) that are finite unions of orbits of the form \( x.H \) with \( x \in X(k) \) and \( H \) a closed subgroup of \( G \), are the closed subsets for a topology on \( X(k) \); the orbit topology. With respect to the orbit topology \( X(k) \) is a compact \( T_1 \) space.
(ii) Let \( \phi: X_2 \to X_1 \) be a morphism of affine \( G \)-spaces of finite type such that \( X_i \) is a \( G/C_G(X_i) \)-torsor \((i = 1,2)\). Then the map \( \phi_k: X_2(k) \to X_1(k) \) is continuous and closed with respect to the orbit topologies.

**Proof.** For (i), we first show that an orbit of the form \( x.H(k) \) with \( x \in X(k) \) and \( H \) a closed subgroup of \( G \) is a closed subset of \( X(k) \) with respect to the Zariski topology.

Set \( G' = G/C_G(X) \) and let \( H' \) denote the image of \( H \) in \( G' \). Then \( H \to H' \) is a quotient map and by [DG70, Chapter III, Cor. 7.6] the map \( H(k) \to H'(k) \) is surjective. Thus \( x.H(k) = x.H'(k) \). As \( X \) is a \( G' \)-torsor, the morphism \( G' \to X \), \( g' \mapsto x.g' \) is an isomorphism. In particular, \( G'(k) \to X(k) \) is a homeomorphism mapping the closed subset \( H'(k) \) to the closed subset \( x.H'(k) \). So \( x.H(k) \) is closed with respect to the Zariski topology and so is any finite union of such orbits.

Since \( X \) is of finite type, any descending chain of Zariski closed subsets of \( X \) is finite. Thus an arbitrary intersection of finite unions of orbits is in fact a finite intersection of finite unions of orbits. Therefore, to show that an arbitrary intersection of finite unions of orbits is itself a finite union of orbits, it suffices to show that the intersection of two orbits is again an orbit. So let \( H_1, H_2 \) be closed subgroups of \( G \) and \( x_1, x_2 \in X(k) \). If \( (x_1.H_1(k)) \cap (x_2.H_2(k)) \) is non-empty, then there exists an \( x \in X(k) \) such that \( x_1.H_1(k) = x.H_1(k) \) and \( x_2.H_2(k) = x.H_2(k) \). Moreover, as noted above, we have \( x.H_1(k) = x.H'_1(k) \) and \( x.H'_2(k) \) with \( H'_i \) the image of \( H_i \) in \( G' \). Then

\[
(x_1.H_1(k)) \cap (x_2.H_2(k)) = (x.H'_1(k)) \cap (x.H'_2(k)) = x.(H'_1(k) \cap H'_2(k)) = x.(H_1 \cap H_2)(k),
\]

where the second equality uses that \( G'(k) \) acts freely on \( X(k) \). Thus, if \( H \subseteq G \) denotes the inverse image of \( H_1 \cap H_2 \leq G' \) under the quotient map \( G \to G' \), then \( (x_1.H_1(k)) \cap (x_2.H_2(k)) = x.H(k) \).

Therefore the finite unions of orbits are indeed the closed sets of a topology on \( X(k) \). As noted above, a subset of \( X(k) \) that is closed with respect to the orbit topology is closed with respect to the Zariski topology. In particular, any descending chain of closed subsets with respect to the orbit topology is finite. Hence \( X(k) \) is compact with respect to the orbit topology. The points of \( X(k) \) are closed with respect to the orbit topology because they are the orbits of the trivial subgroup \( H = 1 \) of \( G \). This concludes the proof of (i).

For (ii), we first show that \( \phi_k: X_2(k) \to X_1(k) \) is surjective. Let \( x_1 \in X_1(k) \). The group \( G(k) \) acts transitively on \( X_1(k) \) because \( G(k) \to (G/C_G(X_1))(k) \) is surjective (again by [DG70, Chapter III, Cor. 7.6]) and \( X_1(k) \) is a \( (G/C_G(X_1))(k) \)-torsor. Thus, if \( x_2 \) is any element of \( X_2(k) \), there exists a \( g \in G(k) \) such that \( x_1 = \phi_k(x_2).g = \phi_k(x_2.g) \). Hence \( \phi_k \) is surjective.

To show that \( \phi_k \) is continuous with respect to the orbit topologies, it suffices to show that the inverse image of an orbit is an orbit. So let \( H \) be a closed subgroup of \( G \) and \( x_1 \in X_1(k) \). We would like to show that \( \phi_k^{-1}(x_1.H(k)) \) is an orbit. As noted in the proof of (i), we have \( x_1.H(k) = x_1.H'(k) \), where \( H' \) denotes the image of \( H \) in \( G/C_G(X_1) \). In other words, we may assume that \( C_G(X_1) \leq H \). Since \( \phi_k \) is surjective, there exists an \( x_2 \in X_2(k) \) such that \( \phi_k(x_2) = x_1 \). We claim that \( \phi_k^{-1}(x_1.H(k)) = x_2.H(k) \). Clearly, \( x_2.H(k) \subseteq \phi_k^{-1}(x_1.H(k)) \). For the reverse inclusion, let \( x'_2 \in \phi_k^{-1}(x_1.H(k)) \). Since \( G(k) \) acts transitively on \( X_2(k) \), there exists a \( g \in G(k) \) such that \( x'_2 = x_2.g \). Then

\[
x_1.g = \phi_k(x_2).g = \phi_k(x_2.g) = \phi_k(x'_2) \in x_1.H(k).
\]

Hence there exists an \( h \in H(k) \) such that \( x_1.g = x_1.h \). Since \( X_1(k) \) is a \( (G/C_G(X_1))(k) \)-torsor, we have \( gh^{-1} \in C_G(X_1)(k) \). But \( C_G(X_1) \leq H \) and so \( g \in H(k) \). Thus \( x'_2 = x_2.g \in x_2.H(k) \). Therefore \( \phi_k^{-1}(x_1.H(k)) = x_2.H(k) \) and \( \phi_k \) is continuous with respect to the orbit topologies.

To see that \( \phi_k \) is closed with respect to the orbit topologies, it suffices to see that \( \phi_k \) preserves orbits. But this follows immediately from the \( G(k) \)-equivariance of \( \phi_k \). \[\square\]

We are now prepared to prove our main results.
Proof of Theorem 1.1. We first note that $X$ is an affine scheme. Indeed, $X$ and $G$ become isomorphic over some field extension $K$ of $k$. So $X_K$ is an affine scheme. By faithfully flat descent, $X$ is an affine scheme ([Gro71, Exposé VIII, Cor. 5.6]).

By Proposition 2.3 we may write $X$ as a projective limit $X = \varprojlim_{i \in I} X_i$ of affine $G$-spaces $X_i$ of finite type such that each $X_i$ is a $G/C(G)$-torsor. In particular, $X(k) = \varprojlim_{i \in I} X_i(k)$.

By Lemma 2.4 each $X_i(k)$ is a compact T1 space with respect to the orbit topology. Moreover, the transition maps $X_j(k) \to X_i(k)$ ($j \geq i$) are continuous and closed with respect to the orbit topologies. Thus Lemma 2.1 applied to the projective system of the $X_i(k)$’s equipped with the orbit topology shows that $X(k)$ is non-empty. □

Proof of Corollary 1.2. Let $\omega_1, \omega_2$ be two neutral fibre functors on a neutral tannakian category over an algebraically closed field $k$. Then $G = \text{Aut} \otimes (\omega_1)$ is an affine group scheme and $\text{Isom}(\omega_1, \omega_2)$ is a $G$-torsor ([DM82, Theorem 3.2]). By Theorem 1.1 the $G$-torsor $\text{Isom}(\omega_1, \omega_2)$ has a $k$-point, i.e., $\omega_1$ and $\omega_2$ are isomorphic. □

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MICHAEL WIBMER, INSTITUTE OF ANALYSIS AND NUMBER THEORY, GRAZ UNIVERSITY OF TECHNOLOGY, KOPERNIKUS-GASSE 24, 8010 GRAZ, AUSTRIA. https://sites.google.com/view/wibmer
Email address: wibmer@math.tugraz.at