Strategies for energy pricing to modify energy consumption using reward process functions

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ABSTRACT
The consumption pattern in the energy sector could be modified by adopting innovative strategies on both sides of energy supply and energy demand. The energy consumption could be given a great cut by relying on research-based strategies, target-oriented investments, and modern technologies. Also an effective way to reduce the energy consumption is efficient management of energy demand, which has many implications including fair pricing. By fair pricing or fairer pricing we mean each consumer to pay according to their amount of consumption on a progressive trend—the more energy the unit uses, the more it has to pay on average. A way to get closer to fair pricing is applying the nonlinear reward functions. The present article seeks to calculate $E[Z(t)]$, the mean of consumption cost till the time $t$, and $E[T_t]$, the mean time of passing from a certain price level to the next price level of $z$, with $Z(t)$ as a reward process and $\rho$ as the nonlinear reward function.

1. Introduction
In recent decades, there has been a dramatic rise in energy consumption across the world. From among all types of energy carriers, electricity, though rather expensive, is the cleanest and, one may say, the most important one. The fact is that modern life is not imaginable without electricity and electrical devices as an even short-time blackout would simply paralyze urban life at any corner of the globe. But as greater energy consumption means greater energy production, and greater production means more energy resources and more costs, the world finds itself in dire need of some major energy consumption modifications.

In Iran, a number of constructive steps have been made to modify energy consumption; some good examples are: preparing energy consumption tags, establishing the national laboratory of energy saving, optimizing energy consumption in industries, monitoring electricity consumption in residential places, designing a consumption optimization software package, and preparing programs to raise public statistical awareness. Meanwhile, there is still much potential for optimizing energy consumption in the country. Such measures will result in less energy demand, less need for construction of more power stations, improvement in production and consumption patterns, more efficient use of the current system, and more chance of attending the international energy markets by more national surpluses. Electricity consumption modification may happen on both sides of supply and demand. On the supply side, introducing modifications to the electricity pricing pattern could be of great effect as many economists believe that modification of the charging mechanism in the energy sector would lead to improving
social and economic behavior. The following studies provide some evidence.

Saffarinia\(^1\) introduced the theory of the formation and changing of customers’ attitude and behavior in his book *Psychology of the Change in Attitude and Behaviour of Energy Consumers*. Mousaee\(^2\) insisted on the role of culture and its impact on consumption pattern. He studied the relations between culture and components and consumption patterns using a descriptive analysis method and assuming the society and its subsets as a system and its subsystems.

Bakhtiar and Yazdani\(^3\) studied the influence of price, income, and the number of natural gas subscribers’ factors in addition to keeping an eye on the latest reserves status and the amount of production and consumption in Iran and the whole world. They came up with a model by using time series data from 1360 to 1387, which proved that natural gas is an essential commodity in the consumer basket, as gas consumption income elasticity is approximately 23% and price elasticity stands at 5.7%. According to the results, they mentioned that in the near future the country will face serious problems unless natural gas consumers receive efficient management and control. As a result, they see the optimal gas consumption as a dire need while suggesting pricing modification, improving public awareness, and reward strategies as effective policies for a breakthrough.

Hamidizadeh\(^4\) worked on awareness, attitude, and behavior of electricity subscribers of the Tehran Power Company. In his article, he studied consumption behavior of residential electricity subscribers in a framework of a systematic approach to the management of electricity consumption. As he suggests, the first step to recognize the external environment is to define the electricity consumption strategy in the country.

Higher income elasticity and lower price elasticity in the short run in rapidly growing countries may impose pressure on energy demand in the domestic and international markets.\(^5\) In one study, Whitford\(^6\) investigated estimation of several political action effects of energy prices. The obtained results of this study show that the structure and performance of oil and natural gas markets will be one important effect of price shocks in the United States. Whitford\(^6\) illustrated how price fluctuations helped lead to the emergence of a political agenda accompanied by several interferences, as revealed through Granger causality tests on change in the legislative agenda.

The prospect trend of energy demand in Iran has been analyzed using a long-range energy alternative planning system (LEAP) model by Khorasani et al.\(^7\) They explored the energy consumption structure in Iran and its historical trend and introduced the key assumptions of a LEAP model as comprising different economic growth, population growth, and urban settlement perspectives in combination with several assumptions on energy-saving policy rules.

Khoabi et al.\(^8\) studied the relationship between electricity prices and economic growth in South Africa. They used a multivariate analysis on the time period 1985–2014. The autoregressive distributed lag (ARDL) bounds test is applied to detect a long-run relationship among the variables. The obtained results show that there is a long-run relationship between electricity supply, economic growth, electricity prices, trade openness, employment, and capital.

In the research of an empirical analysis of electricity demand in Pakistan, Alter and Syed\(^9\) employed cointegration and vector error correction analysis in order to detect long- and short-run dynamics between electricity demand and its determinants. They used time series data for Pakistan from 1970 to 2010. The results of the Johansen cointegration test show that variables integrate in the long run and error correction term reflects the convergence of variables toward equilibrium. Electricity acts as a necessity in the short run and luxury in the long run. Finally, the results show that effective price and income policies, group pricing policy, and peak-load pricing policy should be exercised in electricity demand management.

In Iran, Ojand and Nazari\(^10\) calculated the pricing electricity for the electricity market household segment with the help of the peak load pricing method for each season in their peak load and off peak times.

The article progresses as following: *Section 2* discusses the method of electricity pricing in Iran; *Section 3* introduces the reward function approach; *Section 4* discusses the nonlinear reward function for calculating the energy consumption price; and *Section 5* is dedicated to the results of the study. An appendix containing the required definitions comes at the end.

### 2. Electricity pricing in Iran

In Iran, the government can easily and significantly reduce the energy consumption as it is in control of almost the whole energy supply chain. The government can considerably reduce the energy consumption through scientific research, target-oriented investment, and use of modern technologies in the energy sector. But the most effective factor in reducing energy consumption is an efficient energy demand management, which contains such topics as pricing, education, culture, reward, norm, lifestyle, renewable energies, material optimization, and retrieve.

In Iran, the increasing block tariffs (IBT), also called progressive tariffs, tiered rates, or inverted block rates, is a model used for pricing the residential electric customers. In this model the subscriber is charged higher as the consumption surges. The IBT defines a base- or zero-tariff as the lowest charging for consumption up to a certain limit. The zero-tariff, which is appropriate to the household essential needs, charges below the real production cost as an incentive for clients not to cross the limit line. Following the base block, each successive block is tagged with a higher price. It means the more subscribers consume, the higher the average price they have to pay. The model pushes the clients toward energy saving while reducing the pressure on low-income subscribers.

The model, however, is not faultless, especially when it contains only a few blocks as in our country. In Iran the model defines a 3-tariff structure, including the zero-tariff. As there is a sudden price increase at the beginning of the second or third block and the counter readings falling at the beginning of a range and those at the end of it are attached to
the same tariff, many clients would feel treated unfairly. In addition, if we consider that the national power company (Tavanir) is greatly in debt for the huge subsidy it pays, we would feel the need for model correction—an improved version of IBT.

To make the model fairer, there are two choices: (a) adding to the blocks—a 10-block structure, for example, and (b) adopting a nonlinear function instead of a linear function for each block, which means having a continuous surfing of price for each kWh over the non-zero blocks. The present study discusses the latter choice based on the reward processes functions.

3. Reward processes

3.1 Reward process definition

A stochastic or random process, which can be written as \( X(t) : t \in \tau \), can be defined as a collection of random variables indexed based on an index set. A stochastic process can be classified based on its state space and its index set. Some examples of stochastic processes are the Bernoulli process, random walk, Wiener process, and Poisson process.

A reward process is a specific type of stochastic process based on a semi-Markov process that has applications in industry sectors for calculating the cumulative reward value (e.g., Electricity consumption cost until time \( t \)).

Assume that \( \{J(t) : t \geq 0\} \) is a semi-Markov process (SMP) with \( \{\{v_n, \tau_n\}, n \in \mathbb{Z}_+\} \) as the Markov renewal process (MRP), where \( \{v_n\} \) is the state space and \( \{\tau_n\} \) is the index set. For more information in this field and required definitions such as Markov chain/process, Markov renewal process, and semi-Markov process, please see Appendix and Refs.11-12. When the reward function is linear \( (\rho : N \rightarrow R \text{ where } \rho(k, x) = time \text{ in consumption state}) \), then the accumulated reward until time \( t \) would be:

\[
Z(t) = \int_0^t \rho(v(\tau))d\tau.
\]

This means in the reward process (1), the accumulated reward in a given state is the linear function of time in that state. In the other words, linear reward functions behave as IBT. This sort of reward function has been studied by many researchers including Refs.13-15. But in order to calculate the consumption price more fairly, the reward function must be nonlinear to be used in cost calculation policies such as consumed electricity. Accordingly, Soltani15 defined reward function in nonlinear state \( \rho(k, x) \) as \( \rho : N \times R \rightarrow R \) a special case of which is the polynomial \( \rho(k, x) = kx^n; n \geq 1 \). He also introduced Z reward process by a nonlinear function:

\[
Z(t) = \sum_{n=1}^{t} \rho(J_n, \tau_{n+1} - \tau_n) + \rho(J(t), X(t)),
\]

where \( X(t) \) is known as age process (Poisson) and denotes the last state of consumption or the last transition time.

\[
X(t) = t - \tau_n, \quad n = \sup\{m : \tau_m \leq t\}.
\]

Fair calculation of electricity price for any consuming sector could help to improve consumption patterns. In fact, charging electricity subscribers based on nonlinear reward functions would modify sudden unfair increases in electricity bills. It would make the charging pattern much fairer compared to linear reward function and IBT approaches.

Two important values of \( E[Z_{\rho}(t)] \) and \( E[T_z] \)—the former stands for expected cumulative reward until time \( t \) (cost of consumed electricity until time \( t \)) and the latter shows expected first passage time from a certain level (the first expected time when the consumed electricity cost reaches to \( z \))—were studied by Ref.16 and Ref.17 methods, respectively:

\[
T_z = \inf\{t > 0 : Z(t) = z|Z(0) = 0\}.
\]

3.2 Calculation of \( E[Z_{\rho}(t)] \)

As said before, an important value in calculating consumed energy is the amount of the expected price till the time \( t \), which will be discussed later.

Assume that

\[
B^i_{kj} = \int_0^{+\infty} x^i \rho(k, u) A_j(\sqrt{dx})
\]

and

\[
\theta^i_j = \int_0^{+\infty} \int_0^{+\infty} x^i \rho(u, d\tau) A_j(\sqrt{dx})
\]

where \( B^i = [B^i_{kj}] \) and \( \phi^i_j = [\delta_{kj} \theta^i_j] \) calculates \( E[Z_{\rho}(t)] \) asymptotically when \( t \rightarrow +\infty \).

Theorem 1: Assume that \( Z_{\rho}(t) \) is the reward process based on (2) with reward function of \( \rho(k, x) \) and also assume that \( B^i \) and \( \phi^i_j \) are available for \( i, j = 0, 1, 2 \), then when \( t \rightarrow +\infty \):

\[
E[Z_{\rho}(t)] = p'(\cdot)(H_0B^0 - H_1B^1 + H_1\phi^0_j + H_1B^t) \in o(1)
\]

which if,

\[
A_i = \int_0^{+\infty} x^i A(\sqrt{dx}),
\]

\[
m_1 = \pi' A_1 \in
\]

and

\[
H_1 = \frac{1}{m_1} \pi' \pi
\]

and \( \pi' \) is a unique stationary distribution, so that \( \pi' = A(\infty)\pi' \), then:

\[
H_0 = \frac{1}{m_1} \pi' \{ -A_1 + \frac{1}{2m_1} A_2 \pi' \}
\]

\[
+ \{ Z_0 - \frac{1}{m_1} \pi' A_1 \} \{ P - \frac{1}{m_1} A_1 \pi' \} + I
\]

where \( Z_0 \) is defined as \( Z_0 = \{I - P + \pi' \pi'\}^{-1} \).
3.3 Calculation of $E [T_2]$

Another important value is $E [T_2]$ which refers to the expected time for the cost of consumed energy reaches the predetermined value of $z$.

**Theorem 2**: for the reward function introduced at (2), this value is calculated as follows:

$$ E[T_2] = \frac{m_1}{m_1^*} z + p\left(\cdot\right)|H_0^*\rho A_{D,1} - H_0^*\Phi| \epsilon, \quad +o(1), \ z \to \infty \quad (12) $$

where

$$ m_1 = \pi A_1 \epsilon, \quad (13) $$

$$ m_1^* = \pi B_1 \epsilon, \quad (14) $$

$$ H_0^* = \frac{1}{m_1^*} \epsilon \pi^* \quad (15) $$

$$ H_0^* = \frac{1}{m_1^*} \epsilon \pi^* \{ -B_1 + \frac{1}{2m_1^*} B_1 \epsilon \pi\} + \{Z_0 - \frac{1}{m_1^*} \epsilon \pi^* B_1 Z_0\} \{ P - \frac{1}{m_1^*} B_1 \epsilon \pi\} + I, \quad (16) $$

where

$$ Z_0 = \{I - P + \epsilon \pi^*\}^{-1}. \quad (17) $$

$
\rho_D(x)$ is a diagonal matrix with $\rho(., x)$ elements; and we have for $k=1, 2, \ldots$

$$ y_{D,k}(x) = \int_0^x \rho_{D,k}^0(t)dt, \quad (18) $$

and

$$ B_k = \int_0^{+\infty} \rho_{D,k}^0(t)A(dx) \quad (19) $$

then

$$ \Phi_k = \int_0^{+\infty} y_{D,k}(x)dA_D(x). \quad (20) $$

### Tables

**Table 1. Calculations with linear reward function.**

| Household | Consumption value [kW] | Consumption time [kWh] | $\rho(k, x) = kx$ | Price paid | Difference between costs |
|-----------|------------------------|------------------------|-------------------|------------|------------------------|
| Household 1 | 3                      | 8                      | 24                | 24x10^2=240 | 560                   |
| Household 2 | 10                     | 8                      | 80                | 80x10^2=800 |                       |

*Each [kW] is equivalent to 10 monetary units.

**Table 2. Calculations with nonlinear reward function.**

| Household | Consumption value [kW] | Consumption time [kWh] | $\rho(k, x) = kx^2$ | Price paid | Difference between costs |
|-----------|------------------------|------------------------|-------------------|------------|------------------------|
| Household 1 | 3                      | 8                      | 192               | 192x2**=384 | 896                   |
| Household 2 | 10                     | 8                      | 640               | 640x2**=1280 |                       |

**Each [kW] is equivalent to 2 monetary units.**

See Ref.\textsuperscript{11} for the proof.

### 4. Results and Discussion

In the present section the application of nonlinear functions and their merits for fair pricing is discussed. First, two examples of nonlinear reward processes functions are provided and are followed by a review of the current status and the status in which a reward function is used.

**Example 1.** With some simple data, we will show that why the nonlinear reward function is a more equitable means of charging. Take two households, one of which consumes 3 kWh and the other one 10 kWh in a given period (8 hours). With linear function the consumption of the two households amounts to 24 and 80 kWh, respectively, while with nonlinear reward function, the numbers will be 192 and 640 kWh. Now, if we compute the cost by considering 10 monetary units in the first and 2 monetary units in the second case, the difference between their bills would be 560 and 896 monetary units, respectively. These charges seem fairer for their habits of consumption and help to modify the consumption pattern. In Tables 1 and 2 there have been carried out calculations with linear and nonlinear reward functions.

(A) Linear reward function ($\rho(k, x) = kx$)

(B) Nonlinear reward function ($\rho(k, x) = kx^2$)

**Example 2.** As mentioned above, another advantage of using this method is being able to assign separate prices to different consumption conditions. Suppose that the community’s consumption status contains three different states as follows.

The first state is when the total consumption of the whole area (M) is less than 5 (M<5). The second state is when 5 ≤ M<10, and the third state is when M≥ 10. Then the consumption costs will be different in these three states. For instance, the consumption cost would not be the same in the morning and at evening. This will prove a preventive policy in line with modification of consumption pattern. If we assume that a client consumes based on the following pattern:

$$ a(x) = \frac{d}{d(x)} A(x) = \begin{bmatrix} 0 & 0.8(2e^{-2x}) & 0.2(2e^{-2x}) \\ 0.8(e^{-2x}) & 0 & 0.2(e^{-2x}) \\ 0.8(e^{-2x}) & 0.2(e^{-2x}) & 0 \end{bmatrix} $$

Then the linear and nonlinear functions would be $\rho(k, x) = \rho_0 x$, where $\rho_0 = 1$, $\rho_1 = 2$, $\rho_3 = 3$ and $\rho(k, x) = \rho_0 x^2$, where $\rho_0 = 1$, $\rho_1 = 2$, $\rho_3 = 3$, respectively, and if the initial probability vector is $\pi^* = (0.3, 0.2, 0.5)$, then for linear form...
Now, if we look for the first passage time of 1,000 monetary units, we will find that this time is 535 hours in linear state and 293 hours in nonlinear state.

The following two models of IBT and reward processes functions are compared to find out the differences. Given the discussions in the previous sections, both models are initially kind of tiered rates or progressive tariffs models. But the fact is that the nonlinear reward function shows a fairer treatment compared to the IBT.

The national power grid sustains a great loss annually for the huge subsidy it pays to the subscribers, which means a great financial pressure on the government. In the current model, the more one consumes electricity, the more they receive subsidy. Consider Figure 1; it shows two households, both of which consume more than 4 kWh and less than 7 kWh; both are allocated the same tariff, but the first one has consumed a bit more than 4 kWh and the second one a bit less than 7 kWh. So some injustice is felt here. Now if we apply the reward processes functions, though both households are attached to the same tariff, the second household will get a fatter bill compared to the linear IBT model.

Table 3 is the base for calculation of the households’ electricity consumption cost in Iran. According to the table, a household with 150 kWh consumption would be charged based on the first and second rows of the table: 100 kWh at price of 270 rials and 50 kWh at 320 rials, which hands in a total of 43,000 rials.

Now let’s have a more tangible comparison between two households: a household with electricity consumption of 201 kWh and another one with 300 kWh. Both are treated the same for the first 200 kWh. But for the rest, which falls in the range of 720 rials tariff, they would be treated unfairly, as the first household uses the third tariff subsidy for only 1 kWh, while the second household uses the subsidy for 100 kWh.

5. Conclusion

The reward process is a specific type of stochastic process with numerous applications especially in industry. The process could be used for modifying the consumption patterns to improve them. There are two crucial values in reward processes: the first one is cumulative reward until a particular time, and the second one is the first time we expect to reach a predetermined amount of reward, known as the first passage time.

The E[Z0(T)] and E[T2] values are the urgent values in the consumed energy topic, calculated in the present article by two different methods. However, their distribution, which is very important in practice, is not yet obtained. By computing the two distributions, we would be able to modify and improve the consumption patterns.

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Appendix

Markov Chain/process:

1. The process $X(t)=X_0, X_1, X_2, \ldots$ is a discrete-time Markov chain with a state space of finite or countable, i.e. $i=\{1, 2, \ldots, N\}$, if it satisfies the Markov property:

$P(X_n=i|X_0=x_0, X_1=x_1, \ldots, X_{n-1}=x_{n-1}) = P(X_n=i|X_{n-1}=x_{n-1})$  \hspace{1cm} (1)

2. The Markov chain $X(t)$ is homogeneous if $P(X_{n+1}=j|X_n=i)=P(X_{n+1}=j|X_0=i)$, i.e. the transition probabilities $P_{ij}=P(X_{n+1}=j|X_n=i)$.  

Markov renewal processes:

Let $\tau_1, \tau_2, \ldots$ be IID random variables with $P[\tau_k=0]=1$. The partial sums is defined $S_n=\tau_1+\cdots+\tau_n, n \in N, S_0=0$ as the sequence $S_1, S_2, \ldots$ is increasing.

The process $(J, \tau) = (J_0, \tau_0, \tau_1, \ldots, \tau_n, n \in N)$ given by:

$P\{J_{n+1}=j, \tau_{n+1}+\tau_n \leq t|J_0, \ldots, J_n, \tau_0, \ldots, \tau_n\}$

$= P\{J_{n+1}=j, \tau_{n+1}+\tau_n \leq t|J_n\}$,  \hspace{1cm} (2)

where is called the Markov renewal process. On the other hand, the process $(J_n, S_n), n=0, 1, \ldots$ is called a Markov renewal process.

Semi-Markov process:

The process $f(t)=f_{D(t)}, \quad t \geq 0$ is called a semi-Markov process, where $D(t)=\max(n: \ S_n \leq t), \quad t \geq 0$ is a Markov renewal counting process ($D(t)$ counts a number of renewals in an interval $[0, t]$).

Semi-Markov matrix:

If the semi-Markov transition stochastic matrix ($\{f(t); t \geq 0\}$) is given as $A(x)=[a_{ij}(x)]$ where $a_{ij}(x)=P(J_n=j, T_{n+1}-T_n \leq x|J_n=i)$, then $A_{ij}(x)=\sum_{j \neq i} a_{ij}(x)$; $A_i(x)=1-A_{ij}(x)$. The joint distributions corresponding to the bivariate process ($\{J(t), X(t)\}, t \geq 0$) and the tri-variate process ($\{J(t), X(t), Z(t)\}, t \geq 0$) are, respectively, given by:

$G_{ij}(x, t)=P\{J(t)=j, X(t) \leq x|J(.)=i\}$,  \hspace{1cm} (3)

and

$F_{ij}(x, z, t)=P\{J(t)=j, X(t) \leq x, Z(t) \leq z|J(.)=i\}$,  \hspace{1cm} (4)

where the Laplace transitions of $A_{ij}(x), A_i(x)$ and their matrix form are, respectively,

\[ a_i(s) = \int_0^{+\infty} e^{-st}dA_i(x), \quad a(s) = [a(s)] \]  \hspace{1cm} (6)

and Laplace transform of $F_{ij}(x, z, t)$ is also denoted by:

\[ \phi_{ij}(v, w, s) = \int_0^{+\infty} \int_0^{+\infty} e^{-st-wz}F_{ij}(dx, dz, t)dt \]  \hspace{1cm} (7)

and from Ref.\[11]:

\[ \phi_{ij}(v, w, s) = [I-C(w, s)]^{-1}H_{ij}(w, v + s) \]  \hspace{1cm} (8)

where

\[ C_{ij}(w, s) = \int_0^{+\infty} e^{-wp(\theta, s)}A_{ij}(d\theta) \quad k, j = 1, 2, \ldots, N \]  \hspace{1cm} (9)

\[ H_{ij}(w, t) = \int_0^{+\infty} e^{-wp(j, t)}A_j(x)dx \quad j = 1, 2, \ldots, N. \]  \hspace{1cm} (10)