The Gauge Technique in QED$_{2+1}$

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Abstract

The Gauge Technique has been applied to QED$_{2+1}$ in the quenched case with infrared subtraction. The behaviour of the fermion propagator near the threshold is then found to be

$$S(p) \rightarrow \frac{(\gamma \cdot p + m)}{(p^2 - m^2)^\varsigma} \exp\left(\frac{-\eta \varsigma}{2}\right),$$

where $\varsigma = e^2/(4\pi m)$ and this is gauge invariant except the exponential factor. We also find a spectral function in the Landau and Yennie like gauge. The propagators $S(p)$ are expressed in terms of $\Phi(z, 1, \varsigma)$ explicitly. The vacuum expectation value $\langle \bar{\psi} \psi \rangle$ is gauge independent but divergent. Thus dynamical mass generation does not occur.
1 Introduction

The non-linear integral Dyson-Schwinger equation has been extensively analysed with a particular vertex ansatz or in the quenched approximation to deal with the dynamical symmetry breaking in Quantum Field Theory. Sometimes the approximations made do not satisfy the Ward-Takahashi (W-T) Identity. In any case one may use the divergence of the axial-vector current to show dynamical chiral symmetry breaking via the axial Ward identity,

\[ \partial_{\mu} \langle T(J_5^\mu(x)\bar{\psi}(y)\gamma_5\psi(y)) \rangle = -2\delta(x-y)\langle \bar{\psi}\psi \rangle \quad \text{and} \quad \langle \bar{\psi}\psi \rangle = -\text{tr}(S_F(x)) \]

in QED$_{4,3}$ and QCD$_4$, where the right hand side of the above equation depends on the dynamics. Thus an effective mass, induced by gauge interaction of the fermions, with a non vanishing order parameter $\langle \bar{\psi}\psi \rangle \neq 0$ has been found which is similar to the gap equation in superconductivity [4]. This is the familiar scenario of dynamical symmetry breaking. But there remain ambiguities as they consider only continuum contributions in Euclidian space of the fermion self energy and the structure of the propagator is not clear. Thus we have an interest to see what type of solutions exist in Minkowski space which satisfy the gauge identities. We shall discuss the structure of the fermion propagator in Minkowski space using the gauge technique, which obeys the vertex W-T identity. The Gauge Technique, which is based on dispersion relations, leads to a linear Dyson-Schwinger equation that admits an analytic solution [3]. On the other hand in the ladder approximation and in the Landau gauge, W-T is valid only at one loop level of perturbation theory in non-linear Dyson-Schwinger equation approach. Atkinson and Blatt[2] have studied the singularity of the propagator after an analytic continuation of the Dyson-Schwinger equation to Minkowski space; the only physically meaningful answer for the propagator is a branch point(cut) on the real axis associated with zero mass photon, but the analytic continuation from Euclidean to Minkowski space is not unique and the result depends on the way the vertex is treated; sometimes it leads to a complex singularity [2]. Hereafter we confine ourselves in QED$_{2+1}$. Because of infrared divergences in QED$_{2+1}$, the infrared behaviour was modified [1,2] by introducing massless fermions into the photon vacuum polarization since it affects the low energy behaviour of the photon. In this paper, we analyse a linearized form of the Dyson-Schwinger equation for the fermion propagator in quenched approximation using the gauge technique, where we treat the 2-spinor representation of fermions in (2+1) dimensions and we do not introduce massless sources [1,2]. In our analysis, the structure of the propagator has an essential singularity at $p^2 = m^2$ in arbitrary gauges. We also examine the problem of dynamical mass generation and find that the vacuum expectation value $\langle \bar{\psi}\psi \rangle$ is gauge independent but divergent. We conclude that dynamical mass generation does not occur.
2 Zeroth Gauge approximation

W-T identities between Green function in gauge theory are well known. Thus, with photon legs amputated, the first few identities read

\[ k^\mu S(p) \Gamma_\mu(p, p - k) S(p - k) = S(p - k) - S(p), \]

\[ k^\mu S(p') \Gamma_{\nu\rho}(p'k'; pk) S(p) = S(p') \Gamma_\nu(p', p' + k') S(p' + k') - S(p - k') \Gamma_\nu(p - k', p) S(p) \]  

(1)

where \( S \) is a complete electron propagator and \( \Gamma \) stands for the fully amputated connected Green function; coupling constants have been factorized out of eq(1). In QED the propagators \( S \) and \( D \) of fermion and photon, and the vertex part \( \Gamma \mu \) play a central role via Dyson-Schwinger equations

\[ 1 = Z_2(\gamma \cdot p - m + \delta m) S(p) - i e^2 Z_2 \int \frac{d^n k}{(2\pi)^n} S(p) \Gamma_\mu(p, p - k) S(p - k) \gamma_\nu D^{\nu\mu}(k) \]  

(2)

\[ D^{-1}_{\mu\nu}(k) = Z_3[k^2 g_{\mu\nu} - k_\mu k_\nu(1 - \eta^{-1})] + i e^2 Z_2 T \int \frac{d^n p}{(2\pi)^n} \gamma_\nu S(p) \Gamma_\mu(p, p - k) S(p - k) \]  

(3)

\[ \Gamma_\mu(p, p - k) = Z_2 \gamma_\mu - i e^2 Z_2 \int \frac{d^n p'}{(2\pi)^n} \gamma_\lambda S(p') \Gamma_{\nu\mu}(p'k'; pk) D^{\lambda\nu}(k'). \]  

(4)

where \( \eta \) is a covariant gauge parameter and we are working in \( n \)-dimensions at this stage.

In the gauge technique one seeks solutions to equations (2)-(4) in the form given above. To this end, begin with the Lehmann-Kallen spectral representation for the fermion propagator in the form

\[ S(p) = (\int_{-\infty}^{\infty} + \int_{m}^{\infty} (dw \rho(w)) \frac{1}{\gamma \cdot p - w + i\epsilon(w)} \]  

(5)

with

\[ \rho(w) = \epsilon(w) \rho(w) \quad \text{where} \quad \epsilon(w) = \theta(w) - \theta(-w). \]

Since

\[ S(p - k) - S(p) = \int dw \rho(w) \gamma \cdot p - w \gamma \cdot k \frac{1}{\gamma \cdot (p - k) - w}, \]  

(6)

the simplest possible (but by no means unique) solution of (1) is to take

\[ S(p') \Gamma_\mu^{(0)}(p', p) S(p) = \int dw \rho(w) \gamma \cdot p' - w \gamma_\mu \frac{1}{\gamma \cdot p - w}. \]  

(7)

The above formula represents a bare vertex weighted by a spectral function \( \rho(w) \) for an electron of mass \( w \). If the ansatz for the vertex in equation (7) is used in quenched approximation, equation (2) is written in three dimensions as

\[ Z_2^{-1} = (\gamma \cdot p - m_0) S(p) - i e^2 \int \frac{d^3 k}{(2\pi)^3} S(p) \Gamma_\mu^{(0)}(p, p - k) S(p - k) D^{\nu\mu}(k) \]  

(8)
\[
\begin{align*}
&= (\gamma \cdot p - m_0) \int \frac{\rho(w)dw}{\gamma \cdot p - w - i\epsilon} - \frac{ie^2}{(2\pi)^3} \int \frac{d^3k}{\gamma \cdot p - w} \frac{1}{\gamma \cdot p - w} \gamma\mu \\
&\quad \times \frac{1}{\gamma \cdot (p - k) - w} \gamma\nu \cdot (g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}(1 - \eta)) \frac{1}{k^2} \\
&= \int \frac{\rho(w)dw}{\gamma \cdot p - w} (\gamma \cdot p - m_0 + \Sigma(p, w)), \tag{8}
\end{align*}
\]

where \(\Sigma(p, w)\) is obtained from lowest-order self energy of the fermion with mass \(w\). Recalling \(Z_{-1/2} = \int \rho(w)dw\), equation (8) can be written in the renormalized form

\[
0 = \int \frac{\rho(w)dw}{\gamma \cdot p - w + i\epsilon(w) - m} + \Sigma(p, w) - \Sigma(w, w) = \int \rho(w)dw \frac{w - m + \Sigma(p, w) - \Sigma(w, w)}{\gamma \cdot p - w + i\epsilon(w)} . \tag{9}
\]

Taking the imaginary part of equation (9) yields the integral equation for the spectral function if we replace \(\gamma \cdot p = w\),

\[
\epsilon(w)(w - m)\rho(w) = \frac{1}{\pi} \int \rho(w')dw' \frac{\Im \Sigma(w, w')}{w - w'} . \tag{10}
\]

QED\(_{2+1}\) is a super renormalizable theory but there are infrared singularities. The threshold cut opens at the \(p = m_0\). We may carry out a subtraction at some point, rewriting \((w - m_0 + \Sigma(p, w)) \rightarrow (w - m + \Sigma(p, w) - \Sigma(w, w))\), where \(m_0 = m + \Sigma(w, w)\) represents a mass renormalization within the integral.

### 3 Zeroth Green function

Now the self energy of the fermion can be written as \(\Sigma(p, m) = \gamma \cdot p \Sigma_1(p, m) + m \Sigma_2(p, m)\). (\(\Sigma_1\) and \(\Sigma_2\) are often referred to as the vector and scalar parts of the self-energy.) In the dispersion integral for the fermion self-energy

\[
\Sigma(p, m) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{dw}{\gamma \cdot p - w + i\epsilon} \Im \Sigma(w, m) ,
\]

\(\Sigma_1\) is given from the even function of \(w\) and \(\Sigma_2\) is given from odd function of \(w\) in \(\Im \Sigma(w, m)\). It is conventional to replace \(\gamma \cdot p = w\) in \(\Im \Sigma(p, m)\). In \(2+1\) dimensions to \(O(e^2)\) and in the photon gauge specified by \(\eta\), the self energy of the fermion is written as follows,

\[
\begin{align*}
\Im \Sigma(w, m) &= \frac{e^2}{16} \theta(w^2 - m^2) \left[ \frac{\eta w(w^2 + m^2)}{\sqrt{w^2} w^2} - \frac{2(2 + \eta)m}{\sqrt{w^2}} \right], \\
\Im \Sigma_1(w, m) &= \frac{e^2}{16} \theta(w^2 - m^2) \frac{\eta (w^2 + m^2)}{\sqrt{w^2} w^2}, \\
\Im \Sigma_2(w, m) &= -\frac{e^2}{16} \theta(w^2 - m^2) \frac{2(2 + \eta)}{\sqrt{w^2}} . \tag{11}
\end{align*}
\]
If we substitute the above expression into the equation (10) it can be expressed to this order,

$$\epsilon(w)(w-m)\rho(w) = \frac{e^2}{16\pi} \int \frac{\eta w (w^2 + w'^2) - 2(2 + \eta) w w' \rho(w') dw'}{\sqrt{w^2 w'^2 (w - w')}}. \quad (12)$$

We find that the integral diverges at the w = w'; however instead of introducing massless source to modify the photon propagator we can remedy the infrared divergence by replacing the selfenergy \( \sum(p, m) \to \sum(p, m) - \sum(m, m) \). Thus we subtract \( 3\Sigma(w, w') - \epsilon(w) 3\sum(w', w') \) to avoid the infrared divergence (i.e. removing points at \( \sqrt{p^2} = m \)). The integrand is modified in the following way

$$\frac{1}{w - w'} \eta((1 + \frac{w'^2}{w^2}) - 2) - 2(2 + \eta)(\frac{w'}{w} - 1) = \frac{(4 + \eta)}{w} - \frac{\eta}{w^2}. \quad$$

In this way the spectral function obeys the more sensible integral equation

$$\epsilon(w)(w-m)\rho(w) = \frac{\xi(\eta + 4)}{w} \left( \int_{-\infty}^{w} \rho(w')dw' - \int_{-\infty}^{-\epsilon(w)} \rho(w')dw' \right)$$

$$- \frac{\xi \eta}{w^2} \left( \int_{-\infty}^{w} \rho(w')dw' - \int_{-\infty}^{-\epsilon(w)} \rho(w')dw' \right), \quad (13)$$

where \( \xi = \frac{e^2}{16\pi} \). (The 3-D analogue of the 4-D Yennie type gauge is \( \eta = -4 \).)

The infrared behaviour of the solution is given by multiplying \( w^2 \) and set \( w = m \) in front of the integral in the right hand side of equation (13) thereby converting it into the approximate differential equation

$$m^2 \frac{d}{dw}[(w - m)\rho(w)] \approx m\xi(\eta + 4)(\rho(w) - \rho(-w)) - \xi\eta\rho(w + \rho(-w))$$

$$m^2 \frac{d}{dw}[(w + m)\rho(-w)] \approx m\xi(\eta + 4)(\rho(-w) - \rho(w)) - \xi\eta\rho(w + \rho(-w)),$$

from which we obtain near \( w = m \)

$$\rho\left(\frac{w}{m}\right) - \rho\left(-\frac{w}{m}\right) \approx \exp\left(-\frac{\zeta}{2}(\frac{w}{m} - 1)^{-1 + \xi}\right)\left(\frac{w}{m} + 1\right)^{-1 - \zeta}, \quad (14)$$

then

$$\rho\left(\frac{p}{m}\right) \equiv \rho\left(\frac{p}{m}\right) - \rho\left(-\frac{p}{m}\right) \approx (\frac{p^2 - m^2}{2m^2})^{-1 + \xi}\exp\left(-\frac{\zeta \eta}{2}\right), \quad (15)$$

$$S(p)p^2 \approx \frac{(\gamma \cdot p + m)}{(p^2 - m^2)} \left(\frac{m^2}{p^2 - m^2}\right)^2 \exp\left(-\frac{\zeta \eta}{2}\right), \quad \zeta = \frac{e^2}{4\pi m}. \quad (16)$$

This is a new feature of QED_{2+1}. It shows the gauge independence near the threshold. There is no special gauge in which the fermion has a free pole near \( p^2 = m^2 \). In QED_{4+1}, the Yennie gauge \( \eta = -3 \) produces a free particle pole (Abrikosov 1956) in the lowest approximation [3]:

$$S(p)p^2 \approx \frac{(\gamma \cdot p + m)}{(p^2 - m^2)} \left(\frac{m^2}{p^2 - m^2}\right)^{\alpha(\eta-3)/2\pi}. \quad (17)$$
It is often helpful to split the spectral function in odd and even parts by [1,3]
\[ \rho(w) = \epsilon(w)[w\rho_1(w) + m\rho_2(w)]. \]
But in our case this is unnecessary, as shown below. Introducing dimensionless variables, \( \frac{w}{m} = \omega, \varsigma = \frac{2}{m^2} \), we rewrite the equation (13) as
\[ \epsilon(\omega)(1 - \varsigma) = \varsigma \omega \left( \int_1^\omega \epsilon(\eta + 1) \, d\eta' - \int_{\omega \epsilon(\omega)}^{\omega - 1} \omega' \rho(\omega') \, d\omega' \right) + \frac{\varsigma\eta}{\omega^2} \left( \int_1^{\omega \epsilon(\omega)} \omega' \rho(\omega') \, d\omega' \right). \] (18)

It is easy to see that the eqn (18) can be converted into a first order differential equation in the Landau gauge \( \eta = 0 \), as well as in the Yennie like gauge \( \eta = -4 \). We separate the equation in the different region of \( \omega \) by \( \epsilon(\omega) = \theta(\omega) - \theta(-\omega) \).

In fact for those gauges the differential equations read
\[
\begin{align*}
\frac{d}{d\omega}(\omega(\omega - 1)\rho(\omega)) &= \varsigma(\rho(\omega) - \rho(-\omega)), \omega > 0 \\
\frac{d}{d\omega}(\omega(\omega + 1)\rho(-\omega)) &= \varsigma(\rho(-\omega) - \rho(\omega)), \omega < 0 (\omega \rightarrow -\omega), \eta = 0, \\
\frac{d}{d\omega}(\omega^2(\omega - 1)\rho(\omega)) &= \varsigma\omega(\rho(\omega) + \rho(-\omega)), \omega > 0 \\
\frac{d}{d\omega}(\omega^2(\omega + 1)\rho(-\omega)) &= \varsigma\omega(\rho(-\omega) + \rho(\omega)), \omega < 0, \eta = -4
\end{align*}
\] (19)

and the spectral function solutions are given by
\[
\begin{align*}
\rho(\omega) &= -\frac{C_1}{\varsigma \omega(\omega - 1)} - \frac{C_2}{\omega(\omega - 1)} \frac{(\omega - 1)^\varsigma}{\omega + 1} - \frac{C_1}{\omega(\omega - 1)} \Phi(1 + \omega \frac{1 + \omega}{1 - \omega}, 1, -\varsigma) \\
\rho(-\omega) &= \frac{C_1}{\omega(\omega + 1)} \Phi(1 + \omega \frac{1 + \omega}{1 - \omega}, 1, -\varsigma) + \frac{C_2}{\omega(\omega + 1)} \frac{(\omega - 1)^\varsigma}{\omega + 1}, \eta = 0
\end{align*}
\] (21)

and
\[
\begin{align*}
\rho(\omega) &= \frac{C_1}{\omega^2(\omega - 1)} \frac{(\omega - 1)^\varsigma}{\omega + 1} - \frac{C_2}{\varsigma \omega^2(\omega - 1)} \frac{(\omega - 1)^\varsigma}{\omega + 1} + \frac{C_2}{\omega^2(\omega - 1)} \Phi(1 + \omega \frac{1 + \omega}{1 - \omega}, 1, -\varsigma) \\
\rho(-\omega) &= \frac{C_1}{\omega^2(\omega + 1)} \frac{(\omega - 1)^\varsigma}{\omega + 1} - \frac{C_2}{\varsigma \omega^2(\omega + 1)} \frac{(\omega - 1)^\varsigma}{\omega + 1} + \frac{C_2}{\omega^2(\omega + 1)} \Phi(1 + \omega \frac{1 + \omega}{1 - \omega}, 1, -\varsigma), \\
\eta &= -4,
\end{align*}
\] (22)

where
\[
\int \frac{d\omega}{\omega(\omega - 1)} \frac{(\omega + 1)^\varsigma}{\omega - 1} = (\frac{w + 1}{w - 1})^\varsigma \Phi(1 + \frac{w}{1 - w}, 1, -\varsigma),
\]

6
\[ \int_0^1 \frac{t^{\nu-1} dt}{1 - zt} = \Phi(z, 1, \nu), \]

\[ \Phi(z, a, \nu) = \sum_{n=0}^{\infty} \frac{z^n}{(\nu + n)^a} = \frac{1}{\Gamma(a)} \int_0^{\infty} \frac{t^{a-1} e^{-vt}}{1 - ze^{-t}} dt. \]  \tag{23}

In order to agree with the free field limit \( \rho(\omega) \to \delta(\omega - 1) \), where the condition \( \int \rho(\omega) d\omega = 1 \) holds, the spectral function \( \rho(\omega) \) is normalized in the Landau gauge to

\[ \rho(\omega) = \frac{\varsigma}{m} \theta(\omega - 1) \frac{1}{\omega(\omega - 1)} \left( \frac{\omega - 1}{\omega + 1} \right)^{\varsigma}, \]

\[ \rho(-\omega) = \frac{\varsigma}{m} \theta(\omega - 1) \frac{1}{\omega(\omega + 1)} \left( \frac{\omega - 1}{\omega + 1} \right)^{\varsigma}. \]  \tag{24}

This leads to quantities \( Z_2, m_0 \) for small \( \varsigma \),

\[ Z_2^{-1} = \int_{-1}^{1} \rho(\omega) d\omega = \int_{-1}^{1} (\rho(\omega) - \rho(-\omega)) d\omega = 1 \]

\[ m_0 Z_2^{-1} = m \int_{-1}^{1} \omega(\rho(\omega) + \rho(-\omega)) d\omega = m. \]  \tag{25}

More generally

\[ \int \rho(\omega) f(\omega) d\omega = \int_{-1}^{1} (\rho(\omega) f(\omega) - \rho(-\omega) f(-\omega)) d\omega. \]

Therefore

\[ S(p) = 2\varsigma \int_{-1}^{1} d\omega \frac{\gamma \cdot p + m}{p^2 - m^2} \frac{1}{\omega^2 - 1} \left( \frac{\omega - 1}{\omega + 1} \right)^{\varsigma}, \]  \tag{26}

it can be expressed in terms of the higher transcendental function

\[ \Phi\left( \frac{p - m}{p + m}, 1, \varsigma \right), \Phi\left( \frac{p + m}{p - m}, 1, \varsigma \right), \]

and

\[ S(p) = \frac{\gamma \cdot p + m}{p^2 - m^2} (1 + \varsigma \epsilon(p) \Phi\left( \frac{p + m}{p - m}, 1, \varsigma \right)). \]  \tag{27}

Notice that the point \( p = \infty \) corresponds to a branch point at \( z = 1 \) and the point \( p = m \) to \( z = \infty \) or 0. In the gauge \( \eta = -4 \), the spectral function with correct normalization is instead given by

\[ \rho(\omega) = \frac{\varsigma}{m} \theta(\omega - 1) \frac{1}{\omega^2(\omega - 1)} \left( \frac{\omega - 1}{\omega + 1} \right)^{\varsigma}, \]

\[ \rho(-\omega) = \frac{\varsigma}{m} \theta(\omega - 1) \frac{1}{\omega^2(\omega + 1)} \left( \frac{\omega - 1}{\omega + 1} \right)^{\varsigma}. \]  \tag{28}
leading to

\[ Z_{2}^{-1} = 1 + 2\varsigma - 4\varsigma^{2}\Phi(-1, 1, \varsigma) \rightarrow 1 \]

\[ m_{0}Z_{2}^{-1} = m \]  

and

\[ S(p) = 2\varsigma \int_{1}^{\infty} \frac{d\omega}{p^{2} - m^{2}\omega^2} \left( \frac{\gamma \cdot p}{\omega^2 - 1} + \frac{m}{\omega^2 - 1} \right) \]  

Therefore it is written as

\[ S(p) = \gamma \cdot p \left[ 1 + \frac{\varsigma m^{2}}{p^{2} - m^{2}} \epsilon(p) \frac{m + m}{p + m} \Phi(p - m, 1, \varsigma) \right] \]

\[ -2\varsigma \frac{\gamma \cdot p}{p^{2} - m^{2}} (1 - 2(\varsigma - 1)\Phi(-1, 1, \varsigma)) \]

\[ + \frac{m}{p^{2} - m^{2}} [1 + \varsigma \epsilon(p) \Phi(p - m, 1, \varsigma)] \]  

It is interesting to examine the possible occurrence of confinement and dynamical mass generation in our model. In general \( Z_{2} = 0 \) is a compositeness condition and \( m_{0}Z_{2}^{-1} = 0 \) is a signpost of dynamical mass generation. However in our case two conditions are not satisfied. But the vacuum expectation value \( \langle \bar{\psi}\psi \rangle \) is a gauge invariant quantity in general. We examine the order parameter in the above two gauges and find

\[ \langle \bar{\psi}\psi \rangle = - \text{itr} S(x) = 2 \int \frac{d^{3}p}{(2\pi)^{3}} \int dw \rho(w) \frac{w}{p^{2} + w^{2}} \]

\[ = -\infty(\eta = 0, -4) \]  

Here we used the dimensional regularization. In our case the order parameter is gauge independent but divergent. We deduce that in our approximation there is no dynamical mass generation.

### 4 Improved Vertex function

Direct substitution of our solution \( \rho(w) \) in two gauges (21) and (22) into equation (7), gives the zeroth the vertex function,

\[ S(p)\Gamma_{\mu}^{(0)}S(p') = \int \text{d}w \rho(w) \frac{1}{\gamma \cdot p - w} \gamma_{\mu} \frac{1}{\gamma \cdot p' - w}. \]  

We may improve the vertex by adding transverse combinations of terms such as

\[ [(p_{\mu} \gamma \cdot p' + p'_{\mu} \gamma \cdot p - p \cdot p' \gamma_{\mu}) + ((p + p')_{\mu} + i(p' - p)_{\alpha} \epsilon_{\alpha\mu\nu} \gamma_{\nu})w + w^{2} \gamma_{\mu}]] \]

in the numerator of the integral, allowing for a parity violating contribution. One may also include a form factor proportional to a linear combination of \( \gamma_{\mu} \) and \( (p + p')_{\mu} \) and try to ensure that on-shell quantities are gauge invariant.
5 Summary

Previously, to soften the infrared divergence, massless fermions were introduced into photon vacuum polarization [1]. In this work we have instead analyzed the quenched case and made an infrared subtraction to avoid the infrared singularity. We obtained the solution of the spectral function $\rho(\omega)$ and the momentum space propagator $S(p)$ in the Landau gauge and in the Yennie like gauge ($\eta = -4$); these are simple functions like in $QED_{3+1}$[3]. Here we summarize the differences between two approximations. The former shows the gauge invariance of the spectral function near the mass shell, the high energy behaviour of the spectral function is $1/p^2$ and the cut structure near the mass shell is given by massless fermion-loop $\rho(p) = (p - m)^{-1-e^2/\pi c^2}$ for $p - m \ll e^2$ and $c = e^2 N/8$ for $N$ massless fermions; the integral equation for $\rho(\omega)$ was not solved analytically because of its complexity, due to higher order corrections. In our approximation the high energy behaviour is $1/p^2$ in the Landau gauge and $1/p^4$ in the Yennie like gauge, and structure near the mass shell is determined by coupling constant mass ratio $\varsigma = e^2/(4\pi m)$. Most importantly we arrived at is the gauge independence of the cut near the mass shell $p = m$. The order parameter is gauge independent but divergent, so there is no dynamical mass generation. In contrast to (3+1) dimensions, the gauge dependence near the mass shell is very weak except for the exponential factor. Since $QED_{2+1}$ is super renormalizable, ordinary renormalization group is not relevant, but the gauge technique is non-perturbative, and we have succeeded in finding the infrared behaviour of the fermion propagator as well as its full structure in terms of $\Phi(z, 1, \varsigma)$.

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7 References

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