NONLOCAL CONDENSATES AND CURRENT-CURRENT CORRELATORS WITHIN THE INSTANTON LIQUID MODEL

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The quark and gluon nonlocal condensates and current-current correlators are discussed within the instanton liquid model.

1 Gluon field strength correlator.

The non-perturbative vacuum of QCD is densely populated by long-wave fluctuations of gluon and quark fields. The order parameters of this complicated state are characterized by the vacuum matrix elements of various singlet combinations of quark and gluon fields, condensates: \[ \langle \bar{q}q \rangle, \langle F_{\mu\nu}F^{\mu\nu} \rangle, \langle q(\sigma_{\mu\nu}F_{\mu\nu})q \rangle, \text{etc.} \] The nonzero quark condensate \[ \langle \bar{q}q \rangle \] is responsible for the spontaneous breakdown of chiral symmetry, and its value was estimated a long time ago within the current algebra approach. The nonzero gluon condensate \[ \langle F_{\mu\nu}F^{\mu\nu} \rangle \] through trace anomaly provides the mass scale for hadrons. Its value was estimated within the QCD sum rule (SR) approach and first evidence on its existence has been obtained in. The values of low-dimensional condensates were obtained phenomenologically from the QCD SR analysis of the current-current correlators in various hadron channels. In it was proposed to study the effects of the gluon condensate on a quarkonium state in order to estimate its absolute value.

The nonlocal vacuum condensates or vacuum correlators describe the distribution of quarks and gluons in the non-perturbative vacuum. Physically, it means that vacuum quarks and gluons can flow through the vacuum with nonzero momentum. From this point of view the standard vacuum expectation values (VEVs) like \[ \langle \bar{q}q \rangle, \langle \bar{q}D_{\mu}q \rangle, \langle F_{\mu\nu} \rangle, \ldots \] appear as expansion coefficients of the quark \[ M(x) = \langle \bar{q}(0)\hat{E}(0,x)q(x) \rangle \] and gluon \[ D_{\mu\nu,\rho\sigma}(x) \] correlators in a Taylor series in the variable \[ x^2/4. \] The correlator \[ D_{\mu\nu,\rho\sigma}(x) \] of gluonic field strengths

\[
D_{\mu\nu,\rho\sigma}(x-y) \equiv \left\langle : \text{Tr} F_{\mu\nu}(x) \hat{E}(x,y) F_{\rho\sigma}(y) \hat{E}(y,x) : \right\rangle,
\]

may be parameterized in the form consistent with general requirements of the gauge and Lorentz symmetries as

\[
D_{\mu\nu,\rho\sigma}(x) \equiv \frac{1}{24} \left\langle F^2 : \right\rangle \left\{ (\delta_{\mu\rho}\delta_{\nu\sigma} - \delta_{\mu\sigma}\delta_{\nu\rho})[D(x^2) + D_1(x^2)] + \right\} \]

\[
+ \left( x_{\mu}x_{\rho}\delta_{\nu\sigma} - x_{\mu}x_{\sigma}\delta_{\nu\rho} + x_{\nu}x_{\sigma}\delta_{\mu\rho} - x_{\nu}x_{\rho}\delta_{\mu\sigma} \right) \frac{\partial D_1(x^2)}{\partial x^2},
\]

where \[ \hat{E}(x,y) = P \exp (i \int_{x_0}^{y} A_{\mu}(z) dz^\mu) \] is the path-ordered Schwinger phase factor (the integration is performed along the straight line) required for gauge invariance and \[ A_{\mu}(z) = A^a_{\mu}(z) \frac{\lambda^a}{2}, F_{\mu\nu}(x) = F^a_{\mu\nu}(x) \frac{\lambda^a}{2}, F^a_{\mu\nu}(x) = \partial_{\mu}A^a_{\nu}(x) - \partial_{\nu}A^a_{\mu}(x) + f^{abc}A^b_{\mu}(x)A^c_{\nu}(x). \]
The $P-$exponential ensures the parallel transport of color from one point to other. In [2] $\langle F_{\mu\nu}^2 \rangle = \langle F_{\mu\nu}^{a}(0)F_{\mu\nu}^{a}(0) \rangle$ is a gluon condensate, and $D(x^2)$ and $D_1(x^2)$ are invariant functions which characterize nonlocal properties of the condensate in different projections. The form factors are normalized at zero by the conditions $D(0) = \kappa$, $D_1(0) = 1 - \kappa$, that depend on the dynamics considered. For example, for the self-dual fields $\kappa = 1$, while in the Abelian theory without monopoles the Bianchi identity provides $\kappa = 0$.

In [8], one has shown that the instanton model of the QCD vacuum provides a way to construct nonlocal vacuum condensates. Within the effective single instanton (SI) approximation one has obtained the expressions for the nonlocal gluon $D^{\mu,\nu,\rho,\sigma}_{I}(x)$ and quark $M_{I}(x)$ condensates and derived the average virtualities of quarks $\lambda^2_q$ and gluons $\lambda^2_g$ in the QCD vacuum. The behavior of the correlation functions demonstrates that in the SI approximation the model of nonlocal condensates can well reproduce the behavior of the quark and gluon correlators at short distances.

In [10], it was suggested that the instanton $A_{\mu}^{CI}(x)$ is developed in the physical vacuum field $b_{\mu}(x)$ interpolating large-scale vacuum fluctuations. One has found that at small distances the instanton field dominates, and at large distances it decreases exponentially $\sim \exp \left[-\frac{2}{3}(\eta_g|x|)^{3/2}\right]$ unlike the power-like decreasing SI. It is important to note that the form of this asymptotics is independent of the model for the background field and the driven parameter $\eta_g \sim \left(\frac{N_c}{9(N_c^2 - 1)}R \langle F^2_{b(b)} \rangle\right)^{\frac{1}{3}}$, where $R$ is the correlation length and $\langle F^2_{b(b)} \rangle$ is the background field contribution to the gluon condensate, only weakly depends on it. This solution is called the constrained instanton (CI) [11].

The knowledge of the constraint-independent parts of CI allowed us to construct the solution in the ansatz form

$$A_{\mu}^{CI,\alpha}(x) = \pi^0_{\psi \mu} \frac{x_{\mu}}{x^2} \varphi_{g}(x^2), \quad \varphi_{g}(x^2) = \frac{\bar{\rho}^2(x^2)}{x^2 + \bar{\rho}^2(x^2)}$$

where the notation $\bar{\rho}^2(x^2) = a_{4/3} \eta_g^2 x^2 K_{4/3} \left[\frac{2}{7}(\eta_g x)^{3/2}\right]$, $\rho^2(0) = \rho^2$ is introduced. By translational invariance the center of CI can be shifted in [3] from the origin to an arbitrary position $x_0$: $x \rightarrow x - x_0$. The constrained instanton model introduces two characteristic scales (correlation lengths). The short distance behavior of the correlation functions is proportional to the instanton size and dominated by the single instanton contribution. The large correlation length is physically related to the confinement size $R$. One sees that the $D(x^2)$ structure is close to the SI induced function with the exponential asymptotics being developed at large distances.

The gluon field strength correlation functions $D(x^2)$ and $D_1(x^2)$ have been found in [10] and the results are shown in Fig. [1]. We see that the results of the constrained instanton model calculations are in a remarkable agreement with the results of lattice QCD simulations: 1) The $D_1(x^2)$ structure is much smaller than the $D(x^2)$ function [12][10]; 2) the correlation length of the gluon field correlator is much smaller than for the quark field correlators [13][14]; 3) the path dependence of the gluon field strength correlator has similar features in both approaches [14][15].
Within the constrained instanton model there is a firm prediction\textsuperscript{10}, that if the localized gluon classical field (constrained instanton) of the general form

\[ A^{CI,a}_\mu(x) = \eta^{a}_\nu \frac{x_\nu}{x^2} \varphi_g(x^2), \]  

(4)

with properties \( \varphi_g(x^2 \to 0) = 1 \) and \( \varphi_g(x^2 \to \infty) \to 0 \) faster than any power of \( 1/x \), then the correlation functions must have zeros in preasymptotic region on the background of their exponential decay. Thus the predicted asymptotics is of kind

\[ D(x) \sim \exp(-\Lambda x) \cos(bx + \delta(x)) \]  

(5)

rather than pure exponential one. The appearance of oscillations is possible exclusively due to the Schwinger string phase factor (link) and so is very interesting\textsuperscript{9}. For interpretation of these oscillations as specific “confining” behaviour of the propagator-like functions see\textsuperscript{16}.

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**Figure 1:** The amplitudes \( D \) (top lines) and \( D_1 \) (bottom lines) (all normalized by \( D(0) \)) versus physical distance \( x \), for the instanton size \( \rho = 0.3 \) fm and parameters \( (\rho \eta_0)^2 = 0 \) (solid lines) and \( (\rho \eta_0)^2 = 1 \) (dashed lines).

**Figure 2:** The Adler function from the ILM contributions: dynamical quark loop (short dashed), quark + chiral loops + vector mesons (full line) versus the ALEPH data (dashed). The dash-dotted line is the prediction of the constituent quark model (extended NJL) and the dotted line is the asymptotic freedom prediction, \( 1/4\pi^2 \).

The instanton model predicts the behaviour of nonperturbative part of gluon correlation functions in the short and intermediate region assuming that it is dominated by instanton vacuum component, while the large-scale asymptotics is dominated by the background field.

\*\textsuperscript{9}In many works devoted to theoretical analysis of the gluon field strength correlators the Schwinger link is neglected and so the important effect of oscillations is missed.
The gluon correlators are the base elements of the stochastic model of vacuum\[7\]. They are also often used in the description of high-energy hadron diffractive scattering amplitudes\[17,18\]. However, as it was shown in\[19\] the reduction of these amplitudes to the gluon correlators is only valid for the Abelian model like that considered in\[17\]. As explicit calculations made within the instanton model show in general case these amplitudes are process dependent\[19,20\].

2 Hadronic current-current correlators.

The transition from perturbative regime of QCD to nonperturbative one has yet remained under discussion. At high momenta the fundamental degrees of freedom are almost massless quarks. At low momenta the nonperturbative regime is adequately described in terms of constituent quarks with masses dynamically generated by spontaneous breaking of chiral symmetry. The instanton model of QCD vacuum provides the mechanism of dynamical quark dressing in the background of instanton vacuum and leads to generation of the momentum dependent quark mass that interpolates these two extremes. Still it is not clear how an intuitive picture of this transition may be tested at the level of observables. Below we demonstrate that the Adler function and the amplitudes related to the anomalous triangle diagram depending on spacelike momenta may serve as the appropriate quantity.

The Adler function defined as the logarithmic derivative of the current-current correlator can be extracted from the experimental data of ALEPH and OPAL collaborations on inclusive hadronic $\tau$ decays. From theoretical point of view it is well known that in high-energy asymptotically free limit the Adler function calculated for massless quarks is a nonzero constant. From the other side in the constituent quark model this function is zero at zero virtuality. Thus the transition of the Adler function from its constant asymptotic behaviour to zero is very indicative concerning the nontrivial QCD dynamics at intermediate momenta. Below we intend to show that the instanton liquid model which is a nonlocal chiral quark model (ILM) describes this transition correctly\[21\]. The use in the calculations of a covariant nonlocal low-energy quark model based on the self-consistent approach to the dynamics of quarks has many attractive features as it preserves the gauge invariance, is consistent with the low-energy theorems, as well as takes into account the large-distance dynamics controlled by the hadronic bound states.

In ILM in the chiral limit the vector currents correlator has a transverse character\[22\]:

$$\Pi_{\mu\nu}^v(Q^2) = \left( g_{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right) \Pi_{v}^{ILM}(Q^2),$$

(6)

where the polarization function is given by the sum of the dynamical quark loop, the intermediate vector mesons and the higher order mesonic loop contributions. The lowest order spectral representation of the polarization function consists of zero width vector resonances and two-meson states. The dynamical quark loop under condition of analytical confinement has no singularities in physical space of momenta.

The dominant contribution to the vector current correlator at space-like momentum transfer is given by the loop of light quarks with dynamical momentum dependent mass.
\[ M(k)^{22} \text{ with the result}^b \]

\[ \Pi_{V}^{Q_{\text{Loop}}} (Q^2) = \frac{4N_c}{Q^2} \int \frac{d^4k}{(2\pi)^4} \frac{1}{D_+ D_-} \left\{ M_+ M_- + \left[ k_+ k_- - \frac{2}{3} k_\perp^2 \right]_{\text{ren}} \right\} \]

\[ + \frac{4}{3} k_\perp \left[ \left( M^{(1)} (k_+, k_-) \right)^2 (k_+ k_- - M_+ M_-) - \left( M^{(2)} (k_+, k_-) \right)^{(1)} \right] \] +

\[ + \frac{8N_c}{Q^2} \int \frac{d^4k}{(2\pi)^4} \frac{M (k)}{D (k)} \left[ M' (k) - \frac{4}{3} k_\perp M^{(2)} (k, k + Q, k) \right], \]

where the notations \( k_\pm = k \pm Q/2 \), \( k_\perp^2 = k_+ k_- - \frac{(k+q)(k-q)}{4} \), \( M_\pm = M(k_\pm) \), \( D(k) = k^2 + M^2(k) \), \( D_\pm = D(k_\pm) \) are used. We also introduce the finite-difference derivatives defined for an arbitrary function \( F(k) \) as

\[ F^{(1)}(k, k') = \frac{F(k') - F(k)}{k'^2 - k^2}, \quad F^{(2)}(k, k', k'') = \frac{F^{(1)}(k, k'') - F^{(1)}(k, k')}{k''^2 - k'^2}. \] (8)

The expression for \( \Pi_{V}^{Q_{\text{Loop}}} (Q^2) \) is formally divergent and needs proper regularization and renormalization procedures which are symbolically noted by \([\ldots]_{\text{ren}} \) for the divergent term. At the same time the corresponding Adler function is well defined and finite.

Also we have checked that there is no pole in the vector correlator as \( Q^2 \rightarrow 0 \), which simply means that photon remains massless with inclusion of strong interaction. In the limiting cases the corresponding Adler function satisfies general requirements of QCD

\[ A_{V}^{\text{ILM}} (Q^2 \rightarrow 0) = O \left( Q^2 \right), \quad A_{V}^{\text{ILM}} (Q^2 \rightarrow \infty) = \frac{N_c}{12\pi^2} + \frac{O_Y^V}{Q^2} + O \left( Q^{-4} \right). \] (9)

The leading high \( Q^2 \) asymptotics comes from the \( \left[ k_+ k_- - \frac{2}{3} k_\perp^2 \right]_{\text{ren}} \) term in (7), while the subleading asymptotics is driven by "tachionic" or \( < A^2 > \) term with coefficient\(^{22}\)

\[ O^Y_2 = -\frac{N_c}{2\pi^2} \int_0^\infty du u M(u) M'(u) D(u). \] (10)

It is possible to integrate Eq. (10) in the dilute liquid approximation, \( u >> M^2(u) \),

\[ O^Y_2 \approx \frac{N_c}{4\pi^2} M^2_q \approx 4.7 \times 10^{-3} \text{ GeV}^2, \] (11)

which is close to exact result\(^{22}\) and phenomenological estimate from\(^{23}\).

By using set of parameters found in ILM\(^{21}\) the Adler function in the vector channel is presented in Fig. 2.

\(^b\)Within the context of ILM, the integrals over the momentum are calculated by transforming the integration variables into the Euclidean space, \((k^0 \rightarrow ik_+, k^2 \rightarrow -k^2)\).
Since discovery of anomalous properties\cite{24,25} of the triangle diagram with incoming two vector and one axial-vector currents\cite{26}, many new interesting results have been gained. Recently, the interest to triangle diagram has been renewed due to the problem of accurate calculation of higher-order hadronic contributions to muon anomalous magnetic moment via the light-by-light scattering process, that cannot be expressed as a convolution of experimentally accessible observables and need to be estimated from theory.

The triangle amplitude involving the axial current $A$ and two electromagnetic currents (one soft $\tilde{V}$ and one virtual $V$), which can be viewed as a mixing between the axial and vector currents in the external electromagnetic field, were considered recently in\cite{27,28,29}. This amplitude can be written as a correlator of the axial current $j^5_\lambda$ and two vector currents $j_\nu$ and $\tilde{j}_\mu$:

$$\tilde{T}_{\mu\nu\lambda} = - \int d^4x d^4y e^{iqx-iky} \langle 0 | T \{ j_\nu (x) \tilde{j}_\mu (y) j^5_\lambda (0) \} | 0 \rangle, \quad (12)$$

with the tilted current being for the soft momentum photon vertex. In the specific kinematics when one photon ($q_2 \equiv q$) is virtual and another one ($q_1$) represents the external electromagnetic field and can be regarded as a real photon with the vanishingly small momentum $q_1$ depends only on two invariant functions, longitudinal $w_L$ and transversal $w_T$ with respect to axial current index,

$$\tilde{T}_{\mu\nu\lambda}(q_1, q) = \frac{1}{4\pi^2} \left[ -w_L (q^2) q^\lambda q^\rho q^\sigma \varepsilon_{\rho\mu\nu\lambda} + w_T (q^2) \left( q^2 q_1^\rho q_2^\sigma \varepsilon_{\rho\mu\nu\lambda} - q^\rho q_1^\lambda q_2^\sigma \varepsilon_{\rho\mu\nu\lambda} + q^\lambda q_1^\rho q_2^\sigma \varepsilon_{\rho\mu\sigma\nu} \right) \right]. \quad (13)$$

Both structures are transversal with respect to vector current, $q^\rho \tilde{T}_{\mu\nu\lambda} = 0$. As for the axial current, the first structure is transversal with respect to $q^\lambda$ while the second is longitudinal and thus anomalous.

In the local theory the one-loop result for the invariant functions $w_T$ and $w_L$ is\cite{30}

$$w_L^{1-\text{loop}} = 2 w_T^{1-\text{loop}} = \frac{2N_c}{3} \int_0^1 \frac{d\alpha}{\alpha(1-\alpha)} \frac{1}{q^2 + m_f^2}, \quad (14)$$

where the factor $N_c/3$ is due to color number and electric charge. In the chiral limit, $m_f = 0$, one gets the result for space-like momenta $q$ ($q^2 \geq 0$) $w_L (q^2) = 2w_T (q^2) = 2/q^2$.

The appearance of the longitudinal structure is the consequence of the axial Adler-Bell-Jackiw anomaly\cite{24,25}. For the nonsinglet axial current $A^{(3)}$ there are no perturbative\cite{40} and nonperturbative\cite{31} corrections to the axial anomaly and, as consequence, the invariant function $w_L^{(3)}$ remains intact when interaction with gluons is taken into account. Recently, it was shown that the relation

$$w_{LT} (q^2) \equiv w_L (q^2) - 2w_T (q^2) = 0, \quad (15)$$

\footnote{Here and below the small effects of isospin violation is neglected, considering $m_f \equiv m_u = m_d$.}
which holds in the chiral limit at the one-loop level \(3\), gets no perturbative corrections from gluon exchanges in the iso-triplet case. Nonperturbative nonrenormalization of the nonsinglet longitudinal part follows from the ’t Hooft consistency condition, i.e. the exact quark-hadron duality realized as a correspondence between the infrared singularity of the quark triangle and the massless pion pole in terms of hadrons. OPE analysis indicates that at large \(q\) the leading nonperturbative power corrections to \(w_T\) can only appear starting with terms \(\sim 1/q^6\) containing the matrix elements of the operators of dimension six.\(^{33}\) Thus, the transversal part of the triangle with a soft momentum in one of the vector currents has no perturbative corrections nevertheless it is modified nonperturbatively. However, for the singlet axial current \(A^{(0)}\) due to the gluonic \(U_A(1)\) anomaly there is no massless state even in the chiral limit. Instead, the massive \(\eta'\) meson appears.

So, one expects nonperturbative renormalization of the singlet anomalous amplitude \(w_L^{(0)}\) at momenta below \(q'\) mass. Below we demonstrate how the anomalous structure \(w_L^{(3)}\) is saturated within the instanton liquid model. We also calculate the transversal invariant function \(w_T\) at arbitrary space-like \(q\) and show that within the instanton model in the chiral limit at large \(q^2\) all allowed by OPE power corrections to \(w_T\) cancel each other and only exponentially suppressed corrections remain.\(^{28}\) The nonperturbative corrections to \(w_T\) at large \(q^2\) have exponentially decreasing behavior related to the short distance properties of the instanton nonlocality in the QCD vacuum.

Within the instanton liquid model the nondiagonal (\(VA\)) correlator of vector current and nonsinglet axial-vector current in the external electromagnetic field is given by

\[
\mathcal{T}_{\mu\nu\lambda}(q_1, q_2) = -2N_c \int \frac{d^4k}{(2\pi)^4} Tr \left[ \Gamma_\mu (k + q_1, k) S(k + q_1) \right] \\
\cdot \Gamma_\nu (k + q_1, k - q_2) S(k - q_2) \Gamma_\lambda (k, k - q_2) S(k),
\]

where the quark propagator \(S^{-1}(k) = \hat{k} - M(k)\). The effective vector and axial-vector vertices consist of the local and nonlocal parts and satisfy the Ward-Takahashi identities.\(^{22}\)\(^{14}\) The structure of the vector vertices guarantees that the amplitude is transversal with respect to vector indices \(\mathcal{T}_{\mu\nu\lambda}(q_1, q_2)q_1^\mu = \mathcal{T}_{\mu\nu\lambda}(q_1, q_2)q_2^\nu = 0\) and the Lorentz structure of the amplitude is given by \(\mu\nu\lambda\). The contribution of the diagram where all vertices are local to the invariant functions at space-like momentum transfer, \(q^2 = q_2^2\), are given by

\[
w_L^{(loc)}(q^2) = \frac{4N_c}{9q^2} \int \frac{d^4k}{\pi^2} \frac{1}{D_+ D_-} \left[ k^2 - 4\frac{(kq)^2}{q^2} + 3(kq) \right],
\]

\[
w_{LT}^{(loc)}(q^2) = 0,
\]

where we also consider the combination of invariant functions \(w_{LT}\), which show up nonperturbative dynamics. The notations used here and below are \(k_+ = k, k_- = k - q, k_L^2 = k_+ k_- - (k_+ q)(k_- q)\). At large \(q^2\) one gets \(w_L^{(loc)}(q^2 \to \infty) = \frac{2N_c}{3} - \frac{1}{q^2}\) showing saturation of the anomaly. The reason is that the leading large \(q^2\) asymptotics of \(\mu\nu\lambda\) is given by the configuration where the large momentum is passing through all quark
lines. Then the dynamical quark mass $M(k)$ reduces to zero and the asymptotic limit of triangle diagram with dynamical quarks and local vertices coincides with the standard triangle amplitude with massless quarks and, thus, it is independent of the model.

The contribution to the form factors when the nonlocal parts of the vector and axial-vector vertices are taken into account is given by

$$w_{L}^{(\text{nonloc})}(q^2) = \frac{4N_c}{3q^2} \int \frac{d^4k}{\pi^2} \frac{1}{D_+ D_-} \left\{ \frac{M_-}{M_+} \left[ M_+ - \frac{4}{3} M_+ k^2_{\perp} \right] - M^2(1)(k^+, k^-) \left( 2 \frac{(kq)^2}{q^2} - (kq) \right) \right\}.$$  \hspace{1cm} (19)

Summing analytically the local (17) and nonlocal (19) parts provides us with the result required by the axial anomaly\cite{28}

$$w_L(q^2) = \frac{2N_c}{3} \frac{1}{q^2}. \hspace{1cm} (20)$$

Fig. 3 illustrates saturation of the anomaly in the non-singlet and singlet cases. Note, that at zero virtuality the saturation of anomaly follows from anomalous diagram of pion decay in two photons. This part is due to the triangle diagram involving nonlocal part of the axial vertex and local parts of the photon vertices. The result (20) is in agreement with the statement about absence of nonperturbative corrections to longitudinal invariant function following from the 't Hooft duality arguments.

For $w_{LT}(q^2)$ a number of cancellations takes place and the final result is quite simple\cite{28}

$$w_{LT}(q^2) = \frac{4N_c}{3q^2} \int \frac{d^4k}{\pi^2} \sqrt{\frac{M_-}{D_+ D_-}} \left\{ \sqrt{M_+} \left[ M_+ - \frac{2}{3} M_+ \left( k^2 + 2 \frac{(kq)^2}{q^2} \right) \right] - \frac{4}{3} k^2_{\perp} \left[ \sqrt{M_- M(1)(k^+, k^-)} - 2(kq) M'_+ \sqrt{M(1)(k^+, k^-)} \right] \right\}. \hspace{1cm} (21)$$

The behavior of $w_{LT}(q^2)$ is presented in Fig. 4. In the above expression the integrand is proportional to the product of nonlocal form factors $f(k^2_{\perp}) f(k^2_{\perp})$ depending on quark momenta passing through different quark lines. Then, it becomes evident that the large $q^2$ asymptotics of the integral is governed by the asymptotics of the nonlocal form factor $f(q^2)$ which is exponentially suppressed\cite{28}. Thus, within the instanton model the distinction between longitudinal and transversal parts is exponentially suppressed at large $q^2$ and all allowed by OPE power corrections are canceled each other. The instanton liquid model indicates that it may be possible that due to the anomaly the relation (15) is violated at large $q^2$ only exponentially.

The calculations of the singlet $V AV$ correlator results in the following modification of the nonsinglet amplitudes\cite{29}

$$w_L^{(0)}(q^2) = \frac{5}{3} w_L^{(3)}(q^2) + \Delta w_L^{(0)}(q^2), \hspace{1cm} (22)$$

$$w_{LT}^{(0)}(q^2) = \frac{5}{3} w_{LT}^{(3)}(q^2) + \Delta w_{LT}^{(0)}(q^2), \hspace{1cm} (23)$$
where

\[
\Delta w^{(0)}(q^2) = -\frac{5N_c}{9q^2} \left(1 - \frac{G'/G}{1 - G'\Delta_{PP}(q^2)}\right) \int \frac{d^4k}{\pi^4} \frac{\sqrt{M_+M_-}}{D_+^2D_-^2} \left[ M_+ - \frac{4}{3} M'_+ k_+^2 - M^{(1)}(k_+, k_-) \left( \frac{4}{3} \frac{(kq)^2}{q^2} + \frac{2}{3} k^2 - (kq) \right) \right].
\] (24)

Fig. 3 illustrates how the singlet longitudinal amplitude \(w^{(0)}_L\) is renormalized at low momenta by the presence of the \(U_A(1)\) anomaly. The behavior of \(w^{(0)}_{LT}(q^2)\) is presented in Fig. 4. Precise form and even sign of \(w^{(0)}_{LT}(q^2)\) strongly depend on the ratio of singlet and nonsinglet couplings \(G'/G\) and has to be defined in the calculations with more realistic choice of model parameters.

4 Conclusions

We have analyzed the gluon field strength correlator (nonlocal gluon condensate), the vector Adler function and the nondiagonal vector - axial-vector correlator in external field (anomalous triangle amplitude) for Euclidean (spacelike) momenta within an effective nonlocal chiral quark model motivated by the instanton model of QCD vacuum. The results on the gluon correlator are in good qualitative agreement with the lattice QCD simulations. The dominant contributions to the Adler function and the triangle amplitude come from the loop contributions of the light quark with dynamical momentum dependent mass. It is this contribution that provides the matching between low energy hadronized phase and the high energy QCD which is clearly seen in the behaviour of the Adler function (Fig. 2) and the \(w^{(0)}_{LT}\) combination of the triangle amplitudes (Fig. 4). The results obtained are close to estimates of the vector Adler function extracted directly from the ALEPH data on hadronic inclusive \(\tau\) decays and transformed by dispersion relations to the spacelike region.
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