Study on Control Survey of Super-long Tunnels for Water Diversion Project in Central Yunnan

Xin Zhang, Zhijuan Shen and Zhe Yang
Changjiang Institute of Survey, Planning, Design and Research, Changjiang Spatial Information Technology Engineering Co. Ltd., Wuhan, Hubei, 430010, China
*Corresponding author’s e-mail: zsimba@qq.com

Abstract. The paper carries out an analysis and study on through error of typical tunnels for water diversion project in central Yunnan. Firstly, the paper confirms that lateral through error has the largest impact on through error of tunnels and makes an estimation and error analysis from two aspects of GNSS network error outside tunnel and control network error inside tunnel, then determines to use weight functions and equilateral traverse to calculate the lateral through error. Based on calculation of error inside and outside tunnel, calculation equations of lateral through error are determined and a stimulation is calculated to propose an allowable value of lateral through error of tunnels 20~65km long. The study can provide an important theoretical basis and numerical reference for project construction.

1. Introduction
Water diversion project in central Yunnan is the most important national key water conservancy projects in construction in China, and the largest inter-basin water diversion project in southwest China. There are numerous water delivery structures along the water delivery route as long as 661 km, especially up to 63 typical tunnels. Among those, there are 2 tunnels with single length more than 50 km, and the longest Xiang LuShan tunnel is about 63km, besides 7 tunnels with single length about 20~50km, 12 tunnels of 10~20km long and 11 tunnels of 5~10km long. The total length of all tunnels is up to 610 km. It has reached more than 90% of the total route length of water diversion project in central Yunnan. Normally, a tunnel with single length more than 10km is called super-long tunnel. While the number of super-long tunnels is up to 21 in this project. Consecutive super-long tunnels increase difficulties for design of construction control network [1,2]. Additionally, the existing survey specifications [3~5] only provide numerical reference of through error for tunnels of excavation length within 20km in opposite directions, meanwhile, they require special technical design for tunnels longer than 20km. For water diversion project in central Yunnan, there are 9 tunnels longer than 20km; therefore, there are possibilities for excavation length longer than 20km in opposite directions.

The precision of control network for super-long tunnel is mainly restricted by its through error, and through error consists of lateral, longitudinal and vertical components. From current survey technologies and project requirements, lateral through error is the most difficult of control, while longitudinal and vertical through errors are easier to solve [6]. Therefore, to solve lateral through error is crucial in precision analysis of super-long tunnels. Tunnel lateral through error comes from control survey error outside tunnel, traverse survey error inside tunnel, connection survey error and construction error. Among those, construction error is usually can be ignored, and no connection survey error exists when there is only one through plane. As a result, the paper focuses on control survey error outside tunnel and traverse survey error inside tunnel, and then deduces tunnel lateral
through error.

2. **GNSS network error analysis outside tunnel**

GNSS control network is established outside tunnel for water diversion project in central Yunnan. In general, it is hard to stimulate the raw or generated observations of GNSS network. But if a GNSS base line vector is projected on the surface of a certain reference ellipsoid and further on a Gauss plane, it is a line with known length and direction as a matter of face. Therefore, we can consider GNSS network as a plane network with observed side length and directions. Based on the plane network, we can analyse the impact imposed on tunnel through error. Specific error analysis methods include the weakest point error method, weight function method, zero point error ellipsoid method, etc [7].

![Figure 1 Sketch of through error analysis](image)

2.1 **The weakest point error method**

The method is constrained adjustment for GNSS network with one known point and one known direction. As shown in figure 1, tunnel axis perpendicular to through plane is taken as x-axis, and axis parallel to through plane as y-axis in coordinate system of this method. Generally, point at tunnel entrance and azimuth from the point at one tunnel opening to one at another tunnel opening are known. After two dimensional constrained adjustments, mean square error of Y coordinate of the weakest point can be considered as the impact value of GNSS network survey on lateral through error.

2.2 **Weight function method**

Weight function method is also called unknown coordinate weight function method. In detailed application, figure 1 can be still seen as reference. During control survey, J, C are regarded as points at entrance and exit, A, B as orientation points; in entrance section coordinates is propagated to through point EJ from coordinates of point J and coordinate azimuth α of JA via traverse points A1/A2/A3……., while in exit section coordinates is propagated to through point EC from coordinates of point C and coordinate azimuth β of CB via traverse points B1/B2/B3……. When studying effect control survey outside tunnel made on through error, since the impact value of traverse inside tunnel on through error is not calculated, traverse inside tunnel can be replaced by a virtual traverse. Angle α/β to virtual through points EJ and EC inside tunnel and length SJE/SCE are not observations outside tunnel, thus their errors can be left out. Consequently, impact of GNSS control survey outside tunnel on through error can be concluded as point position error of J and C, and coordinate azimuth error of JA and CB orientations. Detailed equation analysis is shown as follows:

$$
\Delta X = (1 - \Delta Y_e \frac{\Delta Y_{Ja}}{S_{Ja}}) \delta X_J + \Delta Y_e \frac{\Delta X_{Ja}}{S_{Ja}} \delta Y_J + \Delta Y_{Ja} \frac{\Delta Y_{Ja}}{S_{Ja}^2} \delta X_A - \Delta Y_{Ja} \frac{\Delta X_{Ja}}{S_{Ja}^2} \delta Y_A \\
- (1 - \Delta Y_{ce} \frac{\Delta Y_{ce}}{S_{ce}}) \delta X_C - \Delta Y_{ce} \frac{\Delta X_{ce}}{S_{ce}} \delta Y_C - \Delta Y_{ce} \frac{\Delta Y_{ce}}{S_{ce}^2} \delta X_B + \Delta Y_{ce} \frac{\Delta X_{ce}}{S_{ce}^2} \delta Y_B
$$

(1)
\[ d \Delta_y = (1 - \Delta X_{AE} \frac{\Delta Y_{AE}}{S_{AE}^2}) \delta Y_y + \Delta X_{BE} \frac{\Delta Y_{BE}}{S_{BE}^2} \delta Y_B - \Delta X_{AE} \frac{\Delta Y_{AE}}{S_{AE}^2} \delta X_A + \Delta X_{BE} \frac{\Delta Y_{BE}}{S_{BE}^2} \delta Y_B \]

\[-(1 - \Delta X_{EB} \frac{\Delta Y_{EB}}{S_{EB}^2}) \delta Y_y - \Delta X_{BE} \frac{\Delta Y_{BE}}{S_{BE}^2} \delta X_C - \Delta X_{EB} \frac{\Delta Y_{EB}}{S_{EB}^2} \delta X_B + \Delta X_{EB} \frac{\Delta Y_{EB}}{S_{EB}^2} \delta Y_B \]

The longitudinal through coordinate error is derived from Equation (1), and lateral error from Equation (2). Analysis on Equation (2) suggests that given an independent tunnel coordinate system, the impact value of GNSS plane control network outside tunnel is connected with the positions and precisions of entrance, exit points and their corresponding orientation points, and the position of through point. According to covariance propagation law, its mean square error shown in Equation (3):

\[ M_y = \pm \sigma \left( f^T Q_{XX} f \right)^{1/2} \]

In Equation (3), \( M_y \) is mean square error of lateral tunnel through, \( \sigma \) is posterior variance of unit weight for control network, \( Q_{XX} \) is correlation coefficient matrix of unknown coordinates, \( f \) is coefficient matrix of Equation (2). Therefore, \( Q_{XX} \) can be obtained, and multiplies posterior variance of unit weight of control network, variance and covariance of coordinate difference of through point is then obtained, in the meantime, zero point relative error ellipsoid of through point can be calculated. The lateral projection of error ellipsoid on through plane is lateral through error of control network outside tunnel namely.

2.3 Zero point error ellipsoid method

This method is to regard \( \alpha, \beta, S_{AE} \) and \( S_{CE} \) in Figure 1 as virtual measurements without error, and take \( E_I, E_C \) as control points into calculation. On the basis of parameter adjustment, relative error ellipsoid of through point can be determined, and the projection of its major semi axis on through plane denotes mean square error of lateral through:

\[ M_y = \pm \left( E^2 \cos^2 \varphi + F^2 \sin^2 \varphi \right)^{1/2} \]

In the Equation (4), \( \varphi \) denotes azimuth of Y axis when the major semi axis of the ellipsoid is initial direction.

Comprehensive comparison of the above 3 methods shows that the theoretical model of the weakest point error method is too simple to apply in through error calculations of super-long tunnels; models of zero point error ellipsoid is more precise, but known rough coordinates, priori mean square error of measurements and other values are needed, so the methods can be applied with more constraints; weight function method is the most precise and flexible, which is the method used for follow-up stimulated calculations in this study.

Due to the existing survey specifications, it provides numerical reference of through error only for tunnels with through length in opposite directions within 20km, and no value for those more than 20km. The part following proposes a stimulated calculation on tunnels with through length in opposite directions 20km~65km long. The study adopts a stimulation plan of triangulation network, supposing the precision of direction measurement is 0.7″, and that of length measurement is 5mm+1ppm, then carries out the plane network adjustment and stimulates through. With regard to GNSS plane control network, the results include lateral through mean square error and longitudinal through error. The specific values are displayed in Table 1, and show that lateral and longitudinal through errors increase evidently with tunnel through length in opposite directions increasing, and lateral through error is greater than longitudinal through error clearly, the impact on tunnel through of the latter can be neglected basically.

\[ \Delta \Delta_y = \cdots \]

\[ \Delta \Delta_A = \cdots \]

\[ \Delta \Delta_B = \cdots \]

\[ \Delta \Delta_C = \cdots \]
Table 1 Through mean square error calculations of GNSS network for tunnels 20~65km long

| Through mean square error (mm) | Through length in opposite directions of tunnels (km) |
|-------------------------------|-----------------------------------------------|
| Lateral                       | 20 | 25 | 30 | 35 | 40 | 45 | 50 | 55 | 60 | 65 |
| Lateral average               | 40 | 50 | 59 | 69 | 78 | 89 | 98 | 108 | 118 | 128 |
| Longitudinal                  | 6  | 7  | 9  | 10 | 12 | 13 | 15 | 16 | 17 | 19 |

3. Control Network error analysis inside tunnel

Due to the limited size of tunnel section (the diameter is not more than 10 meters generally), only long and narrow traverse network can be established inside tunnel. With excavation of tunnel, the errors of traverse length and angle measurements will accumulate to affect total through error. This chapter will analyses the calculation methods of lateral through error for traverse network inside tunnel.

3.1 Triangulation traverse method

According to error propagation law, angle measurement and length measurement of traverse are independent from each other. Therefore, traverse equations can be used to calculate lateral through error caused by angle and length measurement respectively, and total through error can be obtained with equal effect principle.

\[
M_y = \pm \frac{m_\beta}{\rho} \left( \sum R_x^2 \right)^{1/2} = \pm \frac{m_L}{L} \left( \sum d_y^2 \right)^{1/2} = \pm \left( \frac{m_\beta^2 + m_L^2}{N} \right)^{1/2}
\]

In Equation (5), \(m_\beta\) denotes lateral error that mean square error of angle measurement projected on through plane, (m); \(m_L\) denotes lateral error that mean square error of length measurement projected on through plane, (m); \(M_y\) denotes lateral through mean square error, (m); \(m_\beta\) denotes mean square error of traverse angle measurement, (″); \(m_L/L\) denotes relative mean square error of traverse sides; \(R_x\) denotes vertical distance from traverse point to through plane, (m); \(d_y\) denotes line length that traverse side projected on through plane, (m); \(N\) denotes number of measurement groups; \(\rho\) denotes constant 206 265″; contents in brackets means unit.

When using the method, distance \(R_x\) from each traverse point to through plane and length \(d_y\) of each traverse side projected on through plane, angle measurement mean square error \(m_\beta\) and length measurement precision \(m_L/L\), which can be determined by measuring equipment precision, are needed for calculations of Equation (5). If \(M_y\) is less than the allowance of tunnel lateral through mean square error, the corresponding plan can be implemented; otherwise equipment will be selected again or traverse route and survey plan will be adjusted to satisfy the demands of through precision.

3.2 Equilateral traverse method

Actually, most tunnel traverse can be considered as underground traverse of equal and straight lines; in this condition, traverse length measurement error will not affect lateral through error but only angle measurement error will impose an impact on lateral through error [6]. Then Equation (5) can be simplified as:

\[
M_y = \pm \left( \frac{m_\beta^2}{N} \right)^{1/2} = \pm \frac{m_\beta}{\rho} \left( \frac{\sum R_x^2}{N} \right)^{1/2}
\]

For traverse of equal and straight lines, the square sum of \(R_x\) can be extended as follows:

\[
\sum R_x^2 = n^2 S^2 + (n-1)^2 S^2 + \ldots + S^2 = \frac{n(n+1)(2n+1)}{6} S^2 = \frac{n^2 S^2(n+1.5)}{3}
\]
In the equation, \( n \) denotes traverse point number, \( S \) denotes average length of traverse lines, the equation of two values and the total length \( L \) of traverse is:

\[
L = n \cdot s
\]  

Substitute Equation (7) and (8) into Equation (18), and consider there is only one group of measurements, which means \( N=1 \), the equation is:

\[
M_y = \pm \frac{Lm_\beta}{\rho} \left( \frac{L}{3S} + \frac{1}{2} \right)^{1/2}
\]  

(9)

A stimulated calculation of traverse through error inside tunnel can be conducted based on Equation (9). Suppose the mean square error \( m_\beta \) of traverse angle measurement is 0.7″, the average length of traverse lines is 300m, 400m, 500m, 600m, 700m, the through mean square errors of traverse network for tunnels with through length in opposite directions of 20~65km are calculated respectively. Of which, the equation between tunnel through length in opposite directions \( L_0 \) and total traverse length \( L \) is:

\[
L_0 = 2L
\]  

(10)

For example, when tunnel through length in opposite directions is 20km, the traverse is 10km long at both sides respectively.

This study adopts equilateral traverse method to calculate control network error inside tunnel, detailed results are shown is Table 2. When average length of traverse lines remains unchanged, the longer the tunnel through length in opposite directions, the greater the lateral through error, which is consistent with error change of GNSS control network outside tunnel. However, when tunnel through length in opposite directions remains unchanged, lateral through error will be decreased effectively if the average length of traverse lines increases.

| Traverse average length (m) | Tunnel through length in opposite directions (km) |
|-----------------------------|---------------------------------------------------|
| 20                          | 60 65                                             |
| 25                          | 30 35 40 45 50 55 60 65                          |
| 30                          |                                                   |
| 35                          |                                                   |
| 40                          |                                                   |
| 45                          |                                                   |
| 50                          |                                                   |
| 55                          |                                                   |
| 60                          |                                                   |
| 65                          |                                                   |

4. The determination of lateral through error
Through error analysis is carried out for GNSS network outside tunnel and traverse network inside tunnel in the above-mentioned chapters, and valid numerical values are acquired. Based on the calculations, this chapter will further determine the total values of lateral through error, and propose the distribution suggestion of error allowance.

4.1 The condition of single through plane
In the condition of single through plan in tunnel construction, the total calculation equation of through mean square error can be derived from calculation methods of impact values outside and inside tunnel and error propagation law as follows:

\[
M_y = \left( M_{GPS}^2 + M_{JD}^2 + M_{CD}^2 \right)^{1/2}
\]  

(11)

Additionally, under the circumstance of single through plane, the plane is designed in the centre of the line between tunnel entrance and exit, consequently, the errors of open traverse network at
entrance and exit are identical, which means:

\[ M_{ID} = M_{CD} = M_D \]  

Hence Equation (11) can be simplified as:

\[ M_y = \left( M_{GPS}^2 + 2M_D^2 \right)^{1/2} \]  

Besides, the allowance \( \Delta y \) of lateral through error is:

\[ \Delta y = 2M_y \]  

Which means the allowance of lateral through error is 2 times of lateral through mean square error. According to the analysis above, lateral through mean square error and its allowance of tunnels 20–65km long can be calculated, shown in Table 3.

### Table 3 Lateral through mean square error and its allowance of super-long tunnels

| Lateral through mean square error (m) | Tunnel length (km) |  
|--------------------------------------|-------------------|
|                                       | 20–25  | 25–30  | 30–35  | 35–40  | 40–45  | 45–50  | 50–55  | 55–60  | 60–65  |
| M_{GPS}                              | 50     | 59     | 69     | 78     | 89     | 98     | 108    | 118    | 128    |
| MD                                   | 126    | 165    | 207    | 252    | 301    | 352    | 405    | 461    | 519    |
| M_y                                  | 185    | 241    | 301    | 365    | 434    | 507    | 583    | 663    | 745    |
| \( \Delta y \)                        | 370    | 481    | 602    | 731    | 869    | 1013   | 1166   | 1325   | 1491   |

The suggested value in the last row are adjusted values of \( \Delta y \). And the adjustment principle is that the value of lateral through error allowance shall be greater than the stimulated calculation by a margin and also be an integer easy to remember. The suggested values in the last row are consistent with the study results of Professor Zhenglu Zhang[6] and others basically, but the suggested value of through error allowance for tunnels 45~50km long by the latter is 1000mm, which is 20mm less than the value proposed in this paper; and the results applies in tunnels only up to 50km long, which is slightly shorter than the tunnel length in this paper.

Regarding tunnels with length within 20km, multiple specifications such as in hydropower and water resources, high-speed railway provide allowance of lateral through error. This paper quotes the data from specification[4], shown in Table 4.

### Table 4 Lateral through error from survey specifications in hydropower and water resources construction

| Tunnel through length in opposite directions (km) | Lateral tolerance (mm) |
|-------------------------------------------------|------------------------|
| ≤5                                              | 100                    |
| 5~9                                             | 150                    |
| 9~14                                            | 300                    |
| 14~20                                           | 400                    |

4.2 The condition of multiple through planes

In underground tunnel construction, inclined shaft, vertical shaft and adit can be used in excavation from the middle part of tunnels, thereby multiple tunnel through planes are introduced [8]. An increase of each underground through plane means increased precision requirements distributed to each
through plane. Therefore, control network can be designed based on the calculations of every two adjacent through planes or through distance in opposite directions between through plane and entrance or exit, and with reference to lateral through error allowance in Table 3 and 4.

5. Conclusion
The paper mainly studies lateral through error which has the largest effect on tunnel through from the view that the precision of tunnel control network is primarily restricted by through error and based on the construction requirements of water diversion project in central Yunnan. The study is conducted in three phases, which are GNSS network error outside tunnel, control network error inside tunnel and the determination of lateral through error, and also does a stimulated calculation of lateral through error allowance for tunnels 20~65km long. The methods and numerical values proposed in this paper can be applied in water diversion project in Central Yunnan directly, and provide important theoretical basis and numerical reference for similar super-long tunnel construction.

References
[1] Yang A.M., Yan J.G., Yang C.H., et al. (2010) Study on construction surveying control network of South-to-North Water Diversion Middle Route Project. Yangtze River, 41(19):30-33.
[2] Zhang X., Yang A.M., Xu Q.F., et al. (2014) Key Technology on Establishment of an Independent Coordinate System for Water Diversion Project in Central Yunnan. Geomatics and Information Science of Wuhan University, 39(9): 1047-1051.
[3] Code for surveying of water resources and hydropower engineering. SL197-2013.
[4] Specification of construction survey in hydroelectric and hydraulic engineering. DL/T 5173-2012.
[5] Specification for construction survey of water and hydropower projects. SL52-2015.
[6] Zhang Z.L., Zhang S.L., Wu Z.G., Wang W.G., Zhou T., Zhang G. L. (2004) Research on the Allowable Value of Lateral Breakthrough Error for Super Long Tunnel from 20 to 50 km. Acta Geodaetica et Cartographica Sinica, 2004,33(1): 83-88.
[7] Du C.P. (2013) Analysis and program implementation of long tunnel through observation error. Southwest Jiaotong University Master Degree Thesis.
[8] Xu H. (2008) Summary of Control Survey Methods in Long Tunnel Construction. Tunnel construction, 2008, 28(5).