Neutrinoless double beta decay can constrain neutrino dark matter

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Abstract

We examine how constraints can be placed on the neutrino component of dark matter by an accurate measurement of neutrinoless double beta (0νββ) decay and the solar oscillation amplitude. We comment on the alleged evidence for 0νββ decay.
The detection of neutrinoless double beta decay would imply the violation of lepton number conservation. The process could be induced by Majorana neutrino mass terms, or by less trivial modifications of the standard model. Here we consider the former possibility wherein there are exactly three left-handed neutrino states with Majorana masses \(^1\). The measurement of \(0\nu \beta \beta\) decay, together with what has been learned from studies of solar and atmospheric neutrinos, has direct consequences on the spectrum of neutrino masses and therefore on the effects of neutrinos on structure formation. We define what will be necessary to determine the neutrino component of dark matter from terrestrial experiments.

The charged-current eigenstates are related to the mass eigenstates by a unitary transformation

\[
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix} = U V \begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3
\end{pmatrix},
\]

where \(s_i\) and \(c_i\) are the sines and cosines of \(\theta_i\), and \(V\) is the diagonal matrix \((1, e^{i\phi_2}, e^{i(\phi_3+\delta)})\).

In Eq. (1), \(\phi_2\) and \(\phi_3\) are additional phases for Majorana neutrinos that are not measurable in neutrino oscillations; if \(CP\) is conserved, the phases in \(UV\) are either 0 or \(\pi\).

The solar neutrino data favor the Large Mixing Angle solution with \(0.6 \leq \sin^2 2\theta_3 \leq 0.98\), and \(2 \times 10^{-5} \text{ eV}^2 \leq \Delta_s \leq 4 \times 10^{-4} \text{ eV}^2\) at the 3\(\sigma\) C.L. \(2\). Atmospheric neutrino data imply \(\sin^2 2\theta_1 \geq 0.85\) and \(1.1 \times 10^{-3} \text{ eV}^2 \leq \Delta_a \leq 5 \times 10^{-3} \text{ eV}^2\) at the 99\% C.L. \(3\). The CHOOZ reactor experiment imposes the constraint \(\sin^2 2\theta_2 \leq 0.1\) at the 95\% C.L. \(4\). \(\Delta_s\) and \(\Delta_a\) are the mass-squared differences relevant to solar and atmospheric neutrino oscillations, respectively.

We choose the mass ordering \(m_1 < m_2 < m_3\) with \(m_i\) non-negative. There are two possible neutrino mass spectra:

\[
\Delta_s = m_2^2 - m_1^2, \quad \Delta_a = m_3^2 - m_2^2, \quad \text{ (normal hierarchy),} \quad (2)
\]
\[
\Delta_s = m_3^2 - m_2^2, \quad \Delta_a = m_2^2 - m_1^2, \quad \text{ (inverted hierarchy),} \quad (3)
\]

where in either case \(\Delta_a \gg \Delta_s\) in accord with the previously described experimental data. For the \textit{normal hierarchy} (Case I), mixing is given by Eq. (1). The limit on \(\theta_2\) implies that there is very little mixing of \(\nu_e\) with the heaviest state. In Case I solar neutrinos oscillate primarily between the two lighter mass eigenstates. For the \textit{inverted hierarchy} (Case II),
solar neutrinos oscillate primarily between the two nearly degenerate heavier states. In this case the mixing is described by interchanging the roles of \( m_1 \) and \( m_3 \). With a mixing matrix obtained from Eq. (1) by interchange of the first and third columns of \( UV \), the parameters governing neutrino oscillations (\( \theta_i \) and \( \delta \)) retain the same import as those in Case I. The limit on \( \theta_2 \) again implies that for Case II there is very little mixing of \( \nu_e \) with the lightest state.

The rate of \( 0\nu\beta\beta \) decay depends on the magnitude of the \( \nu_e-\nu_e \) element of the neutrino mass matrix \( [5] \), which is

\[
M_{ee} = |c_2^2 c_3^2 m_1 + c_2^2 s_3^2 m_2 e^{i\phi_2} + s_2^2 m_3 e^{i\phi_3}|, 
\]  
(Case I),

\[
M_{ee} = |c_2^2 s_3^2 m_3 + c_2^2 s_3^2 m_2 e^{i\phi_2} + s_2^2 m_1 e^{i\phi_3}|, 
\]  
(Case II).

The masses \( m_i \) may be determined from the lightest mass \( m_1 \) and the mass–squared differences. Since the solar mass–squared difference is very small it can be ignored; then setting \( m_1 = m \) and \( \Delta_a = \Delta \),

\[
m_2 = m, 
\]

\[
m_3 = \sqrt{m^2 + \Delta}, 
\]  
(Case I),

\[
m_2 = m_3 = \sqrt{m^2 + \Delta}, 
\]  
(Case II).

The lightest mass is related to the sum of neutrino masses (\( \Sigma = \Sigma m_i \)) via

\[
\Sigma = 2m + \sqrt{m^2 + \Delta}, 
\]  
(Case I),

\[
\Sigma = m + 2\sqrt{m^2 + \Delta}, 
\]  
(Case II).

For a given value of \( M_{ee} \), the minimum possible value of \( m \) is obtained if the three contributions to \( M_{ee} \) are in phase, \( i.e., \phi_2 = \phi_3 = 0 \). Thus

\[
m_{\text{min}} = \frac{M_{ee} c_2^2 - s_2^2 \sqrt{M_{ee}^2 + \Delta \cos 2\theta_2}}{\cos 2\theta_2}, 
\]  
(Case I),

\[
m_{\text{min}} = \frac{c_2^2 \sqrt{M_{ee}^2 - \Delta \cos 2\theta_2} - s_2^2 M_{ee}}{\cos 2\theta_2}, 
\]  
(Case II).

The maximum possible value of \( m \) is obtained if the the two smaller contributions to \( M_{ee} \) are out of phase with the largest contribution (\( i.e., \phi_2 = \phi_3 = \pi \) when \( c_3 > s_3 \)). Then

\[
m_{\text{max}} = \frac{M_{ee} c_2^2 |\cos 2\theta_3| + s_2^2 \sqrt{M_{ee}^2 + \Delta (c_2^4 \cos^2 2\theta_3 - s_2^4)}}{c_2^4 \cos^2 2\theta_3 - s_2^4}, 
\]  
(Case I),

\[
m_{\text{max}} = \frac{M_{ee} s_2^2 + c_2^2 |\cos 2\theta_3| \sqrt{M_{ee}^2 - \Delta (c_2^4 \cos^2 2\theta_3 - s_2^4)}}{c_2^4 \cos^2 2\theta_3 - s_2^4}, 
\]  
(Case II).
The allowed ranges for Σ are determined from Eqs. (8) and (9). Because θ_2 is small (sin^2 2θ_2 ≤ 0.1 or s^2_2 ≤ 0.026), its value does not significantly affect the result. (We have confirmed this result numerically). The limits on Σ (for θ_2 = 0) are

\[
2M_{ee} + \sqrt{M_{ee}^2 + \Delta} \leq \Sigma \leq \frac{2M_{ee} + \sqrt{M_{ee}^2 + \Delta \cos^2 2\theta_3}}{|\cos 2\theta_3|},
\]

where the plus sign applies to the normal hierarchy and the minus sign to the inverted hierarchy. The bounds depend on only two oscillation parameters: the scale of atmospheric neutrino oscillations (Δ) and the amplitude of solar neutrino oscillations (sin^2 2θ_3).

Figure 1 shows the allowed bands for Σ and M_{ee} with several possible values of θ_3 within its 3σ allowed range. We have fixed Δ = 3 × 10^{-3} eV^2 and θ_2 = 0. The solid line is the θ_3-independent lower bound on Σ from Eq. (14). The several dashed lines (labelled by sin^2 2θ_3) are upper bounds on Σ. The bands between solid and dashed lines are the (θ_3-dependent) allowed domains of Σ and M_{ee}. The bands in Fig. 1b terminate at M_{ee} = 0.055 eV because M_{ee} ≥ √Δ for Case II. It is possible that CP violation in the neutrino sector is absent or negligible. In this case, the point defined by Σ and M_{ee} must lie on one of the bounding lines of the allowed region.

If the recent evidence that 0.05 eV ≤ M_{ee} ≤ 0.84 eV at the 95% C.L. [8] is borne out, this would imply that 0.1 eV ≲ Σ ≲ 20 eV, which using Ω_ν h^2 = Σ/(93.8 eV) translates to

\[
0.001 ≲ Ω_ν h^2 ≲ 0.2,
\]

where Ω_ν is the fraction of the critical density contributed by neutrinos and h is the dimensionless Hubble constant (H_0 = 100h km s^{-1} Mpc^{-1}). CMB measurements and galaxy cluster surveys already constrain Σ to be smaller than 4.4 eV (Ω_ν h^2 ≲ 0.05) at the 95% C.L. [7]. Data from the MAP satellite should either determine Σ or tighten this constraint to about 0.5 eV in the near future [9]. A more stringent upper bound on Σ from terrestrial experiments must await the precise determination of θ_3 (such as is anticipated from KamLAND [10]), and a firmer measurement (or constraint) on M_{ee} [11].

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Figure 1: $\Sigma$ versus $M_{ee}$ for the a) normal hierarchy and b) inverted hierarchy. The solid line is the $\theta_3$-independent lower bound on $\Sigma$ and the dashed lines are the upper bounds for different values of $\theta_3$. For the inverted hierarchy, $M_{ee} \geq \sqrt{\Delta}$. The 95% C.L. bounds from tritium beta decay [8] and cosmology [7] are shown.