PARTON-HADRON DUALITY: 
RESONANCES AND HIGHER TWISTS *

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Abstract

We explore the physics of the parton-hadron duality in the nucleon structure functions appearing in lepton-nucleon scattering. We stress that the duality allows one to extract the higher-twist matrix elements from data in the resonance region, and learn about the properties of resonances if these matrix elements are known. As an example, we construct the moments of $F_2(x, Q^2)$ for the low and medium $Q^2$ region, and from which we study the interplay between higher twists and the resonance contributions.

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In electron-nucleon scattering, one probes the substructure of the nucleon with virtual photons of mass $Q^2$ and energy $\nu$. Before the advent of Quantum Chromodynamics (QCD), Bloom and Gilman \[1\] discovered an interesting phenomenon about the nucleon structure function $W_2(\nu, Q^2)$, measured at SLAC. Simply speaking, when expressed in terms of the improved scaling variable $\omega' = 1 + W^2/Q^2$, where $W$ is the final-state hadron mass, the scaling function $F_2(Q^2, \omega') = \nu W_2/m_N$ in the resonance region ($W < 2$ GeV) roughly averages to (or duals) that in the deep-inelastic region ($W > 2$ GeV). Referring to a similar phenomenon observed in hadron-hadron scattering, they called it parton-hadron duality. Moreover, the occurrence of the duality appears to be local, in the sense that it exists for each interval of $\omega'$ corresponding to the prominent nucleon resonances. In fact, the assumption of an exact local duality allows an approximate extraction of the nucleon’s elastic form factor from the deep-inelastic scaling function!

An explanation of the Bloom-Gilman duality in QCD was offered by de Rujula, Georgi, and Politzer in 1977 \[2\]. Following the operator product expansion, they studied the moments of the scaling function in the Nachtmann scaling variable $\xi = 2x/(1 + \sqrt{1 + 4x^2 m_N^2/Q^2})$, where $x = Q^2/2m_N\nu$. They argued that the $n$-th moment $M_n(Q^2)$ of $F_2$ has the following twist expansion,

$$M_n(Q^2) = \sum_{k=1}^{\infty} \left( \frac{nM_0^2}{Q^2} \right)^{k-1} B_{n,k}(Q^2),$$

(1)

where $M_0^2$ is a mass scale $\sim (400 \sim 500$ MeV$)^2$ and $B_{n,k}(Q^2)$ depends logarithmically on $Q^2$ and is roughly on the order of $B_{n,0}$. According to Eq. (1), there exists a region of $n$ and $Q^2$ ($n \leq Q^2/M_0^2$), where the higher twist contribution is neither large nor negligible, and where the dominant contribution to the moments comes from the low-lying resonances. The appearance of local duality reflects the very existence of this region. A more recent study on duality can be found in Ref. \[3\].

While these original studies of the parton-hadron duality were largely qualitative, enormous progress has been made in understanding QCD in the past twenty years. The radiative corrections have been evaluated to the next-to-leading order for the twist-two part of the scaling function \[4\]; the structure of the higher twist expansion has been clarified to the order of $1/Q^2$ and some at the order of $1/Q^4$ \[5\]. The physics of the parton-hadron duality has been exploited ingeniously in the vacuum correlation functions, from which a powerful technique for calculating hadron properties from QCD—the QCD sum rule method—has emerged \[6\]. Experimentally, a large body of lepton-nucleon scattering data has been collected in the past 25 years \[7\]. With the CEBAF facility becoming available for making systematic, high precision measurements in the resonance region, it is timely to re-examine duality in its original context, and further explore the physics content of this important concept.

In this Letter we seek to sharpen the explanation of the duality offered by authors in Ref. \[2\], with a few crucial differences. First, we choose to work with the moments of Cornwell-Norton, instead of those of Nachtmann, thereby avoiding the unphysical region of $\xi > \xi(x = 1)$. Second, we look for a way to describe more clearly the contribution of the resonances to the moments. Finally, we emphasize a thorough exploitation of the consequences of duality. We furnish our discussions with the example of $F_2$, for which the abundant data allow an accurate construction of its moments in the low and medium $Q^2$
region. These moments offer a unique opportunity for studying the effects of higher twists and the resonance contributions.

The Cornwell-Norton moments of a scaling function $F(x, Q^2)$ are defined as,

$$M_n(Q^2) = \int_0^1 dx x^{n-2} F(x, Q^2),$$  \hspace{1cm} (2)

where the upper limit includes the elastic contribution. According to the operator production expansion, the moments can be expanded in powers of $1/Q^2$,

$$M_n(Q^2) = \sum_{k=0}^{\infty} E_{nk}(Q^2/\mu^2) M_{nk}(\mu^2) \left( \frac{1}{Q^2} \right)^k,$$  \hspace{1cm} (3)

where $E_{nk}$ are the dimensionless coefficient functions which can be calculated perturbatively as a power series in the strong coupling constant $\alpha_s(Q^2)$,

$$E_{nk}(Q^2/\mu^2) = \sum_{i=0}^{\infty} \alpha_s^i(Q^2) e_{nk}^i,$$  \hspace{1cm} (4)

and $M_{nk}(\mu^2)$ are the nucleon matrix elements of local operators composed of quark and gluon fields. The renormalization scale ($\mu^2$) dependence cancels in the product of the two quantities; however, when we talk about them separately, $\mu^2$ is chosen to be the hadron mass scale. The terms beyond the first in Eq. (3) are called the higher-twist corrections, which include both the target mass corrections and the true higher-twist effects.

The double expansions in Eq. (3) are asymptotic at best. Non-perturbative effects can invalidate both expansions at higher orders, and can mix the two, rendering the separation of radiative and power corrections ambiguous [8]. In the following discussion, however, we assume that in the $Q^2$ region of our interest, the size of the twist-four term $(1/Q^2)$ is significantly larger than the smallest term in the asymptotic expansion for $E_{n0}$, beyond which the evaluation of $E_{n0}$ cannot be improved by including higher-order terms, and so the ambiguity in defining the higher-twist corrections can be neglected [8]. We shall henceforth focus only on the structure of the twist expansion.

Following Ref. [9], we assume the ratio of the twist-four term to the leading twist in each moment is approximately $nM_0^2$, where $M_0$ is a scale characterizing the matrix elements of the twist-four operators. We further assume that the twist expansion is an asymptotic series in the parameter $nM_0^2/Q^2$. According to the above assumptions, we can classify the higher-twist contributions to the moments. Consider the $n - Q^2$ plane as shown in Fig. (a), which is separated into three regions by two solid lines. Region A is defined by $nM_0^2 \ll Q^2$, where the higher-twist effect are negligible. Region B is where the higher-twist corrections become important but stay perturbative, and thus only the first few terms in the twist-expansion are of practical importance. Region C is where the higher-twist effects become non-perturbative, and the power-expansion loses meaning. It is in this third region that the resonance physics dominates the behavior of the moments and the quantum coherence, inherent to resonance production, defies a description of the scattering in terms of a finite number of quarks and gluons. In the later part of the paper, we will show that the first assumption is consistent with the behavior of the lower moments for $F_2$.

Now we consider the resonance contribution to the Cornwell-Norton moments by examining the $x - Q^2$ plane shown in Fig. (b), in which the resonance region is approximately
above the curve $W = 2$ GeV. For a large, fixed $Q^2$ (say 15 GeV$^2$), the resonance contribution to the lowest few moments is very small, and can be neglected. When $n$ increases, the resonance contribution is weighted more and becomes significant. We can use a dashed line, as $Q^2$ varies, in the $n - Q^2$ plane to indicate the separation of the two cases. The dashed line certainly cannot be in region A, because the non-resonance experimental data have already detected the higher-twist effects [11]. If the dashed line is in the region C, then the perturbative higher-twist effects have nothing to do with resonance physics. The most exciting possibility is when the dashed line lies in the region B, and this is what happened in reality.

When the dashed line is located in region B, then in the left portion of it, the following statements are true: 1). the higher-twist corrections are perturbative, so the moments are not too different from those at larger $Q^2$, and 2). the resonance contribution to the moments are significant. Thus in this region, the resonances must organize themselves to follow the deep-inelastic contribution apart from a perturbative higher-twist correction, or conversely, the structure of the higher-twist expansion constrains the behavior of the resonance contribution. The degree of duality is determined by the size of this region: the larger the region, the more the moments are constrained, and the more local the duality will be.

Why should duality occur at all in QCD? On one hand, the quark transverse momentum in the nucleon, which governs the magnitude of the higher-twists, is about 400 MeV. This makes the higher-twist corrections perturbative down to very small $Q^2$. On the other hand, the resonance contribution to the moments are already significant at $Q^2 \sim 5$ GeV$^2$ for low $n$. Thus the occurrence of the duality seems unavoidable, unless QCD had two widely different scales.

The consequences of duality, like duality itself, are two-fold. If one knows data in the resonance region, one can extract the matrix elements of the higher-twist operators. The extraction, of course, is limited by our ability to calculate higher-order radiative corrections, about which we have nothing to say. On the other hand, if one knows the higher-twist matrix elements from other sources, such as lattice QCD calculations, one can utilize them to extract the properties of the resonances. This second use of duality has been pursued extensively in the QCD sum rule calculations, from which a large number of interesting results has been obtained [4]. In the present case, however, the number of higher-twist matrix elements is large, and they are difficult to estimate in general. This severely limits our ability to check, for example, the internal consistency of the duality predictions.

We make the above discussion more concrete and quantitative by using the example of the $F_2$ scaling function, for which rich data exist in an extended kinematic region. Most of the low $Q^2$ data were taken in late 60’s and early 70’s at SLAC and DESY, and they nearly cover the whole resonance region at large $x$. The data were fitted by Brasse et al. [9] to a function with three parameters for each fixed $W$. In Ref. [10], Bodek et al. have made a more extensive but different fit, covering higher $Q^2$ resonance data. The deep-inelastic data were systematically taken by SLAC, BCDMS, EMC, and other collaborations during the 70's and 80's, and they have recently been shown to be consistent with each other [11]. New measurements from NMC at CERN has extended these data to lower $Q^2$ and $x$ [12]. In Fig. 2, we have shown the $F_2$ data as a function of Bjorken $x$ at $Q^2 = 0.5$, 1.0, 2.0, 4.0, 8.0 and 16.0 GeV$^2$ from the two fits [10,12] made in different kinematic regions.
The salient features of the data can be summarized as follows. At high-$Q^2$, the data is almost entirely deep-inelastic except for a small resonance contribution at large $x$. The scaling function near $x = 0$ shows a rise due to perturbative QCD effects. As $Q^2$ decreases, small bumps become visible and slide toward low $x$. These prominent excitations are believed to be the $\Delta(1232)$, $S_{11}(1535)$ or $D_{13}(1520)$, and $F_{15}(1680)$ resonances. The resonance excitations become very strong near $Q^2 = 2$ GeV$^2$ and clearly dominates $F_2$ below $Q^2 = 1$ GeV$^2$. As $Q^2 \to 0$ the data is compressed toward $x = 0$ due to simple kinematics. At $Q^2 = 0$, the whole photo-production physics is shrunk to $x = 0$. Of course, one should not forget about the elastic contribution, which contributes a delta-function at $x = 1$.

To understand the role of the resonances in the Cornwell-Norton moments, we plot in Fig. 3 the ratio of the resonance part to the total, where the resonance contribution is defined by a cut on $W < 2$ GeV. If one uses ten percent as a measure of the importance of the resonance contribution, then this threshold is reached for the lowest moment ($n = 2$) at $Q^2 \sim 4$ GeV$^2$. For higher moments, the transition occurs approximately at $2n$ GeV$^2$. This is quite surprising because the non-perturbative physics becomes potentially important at $Q^2 = 16$ GeV$^2$ for the 8th moment! At $Q^2 = 8$ GeV$^2$, the same moment receives fifty percent of the contribution from the resonance region. The dashed line in Fig. 3(a) roughly corresponds to the ten percent line shown in Fig. 3.

The data on the $F_2$ moments can be used to extract the matrix elements of higher-twist operators. To effect this, we first subtract the twist-two part of the contribution. We use a parton distribution (CTEQ2, [13]) fitted to a large number of data on hard processes, and calculate the moments for each quark flavor and gluon distribution at some large $Q^2$ (=20 GeV$^2$ in our case). Then we evolve these moments to lower $Q^2$ using the perturbative QCD formula accurate to next-to-leading order. Theoretical errors in evolution are mainly generated from uncertainty in $\Lambda_{QCD}$ and unknown higher-order terms in the coefficient functions. In our work, we take $\Lambda_{QCD} = 260 \pm 50$ GeV [14], and the resulting error is added to the experimental error which is taken to be 3% uniformly, yielding the total error on the residue. The target mass corrections are further subtracted from the moments according to the formula in Ref. [15]. In Fig. 4(a), we show the moments as a function of $Q^2$ and the twist-two part plus the target mass corrections (solid lines). The residual moments, which are entirely higher twist effects, are shown in Fig. 4(b) as functions of $1/Q^2$.

We choose to fit the $Q^2$ evolution of the moments with a pure twist-four contribution,

$$\Delta M_n(Q^2) = a_n \left( \frac{\alpha(Q^2)}{\alpha(1)} \right)^{\gamma_n} \frac{1}{Q^2}$$

where we have included phenomenologically the leading-log effects with an adjustable exponent. The fitted $\gamma_n$ represents an average of the anomalous dimensions of the spin-$n$, twist-four operators, weighted by the size of individual matrix element. The coefficient $a_n$ is a simple sum of the twist-four matrix elements at the scale $\mu^2 = 1$ GeV$^2$. Inclusion of a twist-six term creates strong correlations among the parameters and renders the fits indeterminate. Thus we have neglected such a term by restricting the fit to the region with $Q^2 > n$, where the twist-six contribution is presumably small.

The result of our fit is shown in Table I. The correction to the $n = 2$ moment (the famous momentum sum rule) is best determined, yielding a characteristic higher-twist scale of 500 MeV. From this, we determine that the twist-four contribution to the momentum sum rule
at $Q^2 = 2 \text{ GeV}^2$ is 0.015, about ten percent of the total. The exponent of the leading-log contribution increases gradually with $n$, in accord with general expectations. The near constancy of the twist-four contribution is in sharp contrast with the fast decrease of the leading-twist contribution with increasing $n$. It confirms, though, the speculation that the higher-twist contribution become more important for higher moments, and is a precursor for the onset of the resonance region. In QCD, this can be explained by an increasing number of twist-four operators compensated by a decrease in strength of individual matrix elements. The pattern of the moments indicates a twist-four distribution negative at small $x$, positive at large $x$ and peaked near $x = 1$, qualitatively consistent with the fits in Ref. [11], where the resonance data were entirely ignored.

Finally, we test the assumption about the higher-twist matrix elements in Eq. (1). We show in the fourth column of Table I the ratio of the higher-twist matrix elements and the twist-two part. From this, we extract an effective $M_0$ by dividing by $n$ and taking the square root. The result is shown in the fifth column and is approximately $n$-independent, although there is a slight hint of $M_0$ getting larger for larger $n$. However, this should not be taken too seriously because of the errors and limited number of moments. If fifty percent of the higher-twist contribution is taken as an indication that the twist-expansion is getting non-perturbative, we find a $Q^2$ for each moment where the transition takes place. For $n = 2$, this is about 0.3 GeV$^2$. For higher moments, this happens at about $n - 1$ GeV$^2$. The line which separates regions B and C in Fig. 1 roughly corresponds to this. Thus the existence of the duality zone is clearly established beyond any doubt.

To illustrate the other use of duality, one could, for instance, use the higher-twist contribution extracted from the pure deep-inelastic region (as done in [11]), or from some theoretical calculations, to determine the nucleon’s elastic form factor. However, we feel that the higher-twist matrix elements have not been determined in other methods to a sufficient accuracy to allow a quantitative extraction of the resonance properties. Qualitatively, however, knowing the higher-twist contribution will surely improve the nucleon form factor extracted in Ref. [2], which shows a systematic deviation from the directly measured $G_M$, a clear indication of higher-twist effects.

To sum up, we explored in this work the physics of the parton-hadron duality. We emphasized that the existence of duality allows one to determine the higher twist matrix elements from data in the resonance region, or alternatively, knowing the matrix elements enables one to determine the properties of the resonances. We studied the duality picture offered by the $F_2$ scaling function, and extracted the matrix elements of the lowest few spin, twist-four operators. Clearly, this study can be applied straightforwardly to the spin-dependent structure function $G_1$ once more data becomes available.

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FIGURES

FIG. 1. a). Three regions of differing importance to higher twists: Region A, negligible higher
 twists; Region B, perturbative higher twists; and region C, the twist-expansion breaks down.
b). Kinematic regions corresponding to the resonance and deep-inelastic scattering.

FIG. 2. Scaling function obtained from the fits to experimental data in Refs. [10,12].

FIG. 3. Ratio of the moments from the resonance region, including the elastic contribution, to
 that of the total.

FIG. 4. a). Moments as functions of $Q^2$, extracted from the scaling function in Fig. 2. The
 solid lines refer to the contribution from the leading twist and target-mass corrections. b). Residue
 moments from the higher-twist contribution. The solid lines are the fits described in the text.
TABLES

TABLE I. Extracted twist-four matrix elements $a_n$, effective anomalous dimension $\gamma_n$, ratio to the leading twist contribution, and the effective mass scale $M_0$.

| $n$ | $a_n$ (GeV$^2$) | $\gamma_n$ | $a_n/(E_{n0}M_{n0})$ | $M_0$ |
|-----|----------------|------------|----------------------|-------|
| 2   | 0.030 ± 0.003  | 1.0 ± 0.5  | 0.14                 | 0.26  |
| 4   | 0.042 ± 0.013  | 1.5 ± 0.5  | 1.00                 | 0.50  |
| 6   | 0.047 ± 0.021  | 2.5 ± 0.5  | 2.47                 | 0.64  |
| 8   | 0.038 ± 0.018  | 2.5 ± 0.5  | 3.45                 | 0.66  |
| 10  | 0.052 ± 0.025  | 3.5 ± 0.5  | 4.73                 | 0.69  |
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$F_2(x, Q^2)$

Bjorken $x$
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