Pion and Kaon Masses and Pion Form Factors from Dynamical Chiral-Symmetry Breaking with Light Constituent Quarks

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Abstract. Light constituent quark masses and the corresponding dynamical quark masses are determined by data, the quark-level linear $\sigma$ model, and infrared QCD. This allows to define effective nonstrange and strange current quark masses, which reproduce the experimental pion and kaon masses very accurately, by simple additivity. In contrast, the usual nonstrange and strange current quarks employed by the Particle Data Group and Chiral Perturbation Theory do not allow a straightforward quantitative explanation of the pion and kaon masses.

Keywords: Light constituent quarks, dynamical quark mass, effective current quark mass, pion and kaon masses, pion form factors, dynamical chiral-symmetry breaking, quark-level linear $\sigma$ model

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INTRODUCTION

The pion is commonly accepted to be massless in the chiral limit (CL). Not only is its physical mass a good measure of chiral-symmetry breaking (ChSB), but also the related nonstrange constituent quark mass $\hat{m}$. In the present short note, we shall show that $\hat{m}$ can be additively decomposed into a bulk part called dynamical quark mass ($m_{\text{dyn}}$), associated with chiral-symmetric strong interactions, and a smaller part called current quark mass ($m_{\text{cur}}$), which arises from ChSB in the electroweak sector. This effective $m_{\text{cur}}$ turns out to be precisely half the pion mass. Moreover, the strange constituent quark mass allows a similar decompostition as well, with the kaon mass being the simple sum of the effective nonstrange and strange current quark masses.

QUARK-MASS DIFFERENCE $m_d - m_u \approx 4$ MeV

A simple estimate of the mass difference between a down and an up quark gives

\[
\begin{align*}
\left\{ m_{K^0} - m_{K^+} \quad m_{\Sigma^-} - m_{\Sigma^+} \right\} & \implies m_d - m_u \approx 4 \text{ MeV}.
\end{align*}
\]

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Note that this holds for both current and constituent quarks. On the other hand, from the proton magnetic moment we can derive \[1\] an average constituent quark mass as

\[
\hat{m} = (m_u + m_d)/2 = 337.5 \text{ MeV}.
\]

Using Eq. (1), this yields the constituent masses

\[
m_u \approx 335.5 \text{ MeV}, \quad m_d \approx 339.5 \text{ MeV}.
\]

We can also obtain $\hat{m}$ in the context of the quark-level linear $\sigma$ model (QLL$\sigma$M), via the Goldberger-Treiman relation \[1, 2\]

\[
\hat{m} \approx f_{\pi}g = 93 \text{ MeV} \times \frac{2\pi}{\sqrt{3}} \approx 337.4 \text{ MeV}.
\]

The agreement with the value in Eq. (2) is remarkable.

**DYNAMICAL QUARK MASS** $m_{\text{dyn}}$

A bulk dynamical CL nonstrange quark mass can be estimated as

\[
m_{\text{dyn}} \approx \frac{m_N}{3} = 313 \text{ MeV}.
\]

A check via the CL pion charge radius gives

\[
m_{\text{dyn}} \approx \frac{hc}{r_{\pi}^{\text{CL}}} = \frac{197.3 \text{ MeV} \cdot \text{fm}}{0.63 \text{ fm}} = 313 \text{ MeV},
\]

employing vector-meson dominance or the QLL$\sigma$M to predict

\[r_{\pi}^{\text{CL}} = 0.63 \text{ fm}.
\]

Alternatively, using infrared QCD, with $\alpha_s \approx 0.5$ at a 1 GeV cutoff, we get

\[
m_{\text{dyn}} = \left[ \frac{4\pi}{3} \alpha_s \langle -\bar{q}q \rangle \right]^{\frac{1}{2}} \approx 313 \text{ MeV},
\]

for the commonly accepted value of the quark condensate

\[
\langle -\bar{q}q \rangle \approx (245 \text{ MeV})^3.
\]

**EFFECTIVE CURRENT QUARK MASS VIA QCD**

Away from the CL, we define the effective current quark mass as

\[
\hat{m}_{\text{cur}} = \hat{m} - m_{\text{dyn}},
\]

where $\hat{m}$ is the constituent quark mass, and the dynamical mass $m_{\text{dyn}}$ runs as

\[
m_{\text{dyn}}(P^2) \sim P^{-2}.
\]
according to QCD. On the \( \hat{m} = 337.5 \) MeV mass shell, selfconsistency then requires

\[
m_{\text{dyn}}(p^2 = \hat{m}^2) = \frac{m_{\text{dyn}}^3}{\hat{m}^2} = \frac{(313)^3}{(337.5)^2} \text{MeV} = 269.2 \text{MeV}.
\]

This yields

\[
\hat{m}_{\text{cur}} = (337.5 - 269.2) \text{MeV} = 68.3 \text{MeV},
\]

near the pion-nucleon sigma term

\[
\sigma_{\pi N} = (55 \pm 13) \text{MeV} \quad [3], \quad \sigma_{\pi N} = (66 \pm 9) \text{MeV} \quad [4], \quad \sigma_{\pi N} = (64 \pm 8) \text{MeV} \quad [5].
\]

Note that both \( \sigma_{\pi N} \) and \( \hat{m}_{\text{cur}} \) vanish in the CL.

**PION \bar{q}q MASS**

With the effective current quark mass derived in Eq. (13), we get a \( \bar{q}q \) pion mass

\[
m_{\pi} = 2\hat{m}_{\text{cur}} = 136.6 \text{MeV},
\]

almost midway between the observed \( m_{\pi^0} = 134.98 \) MeV and \( m_{\pi^+} = 139.57 \) MeV!

**PION FORM-FACTOR RATIO**

In the QLLσM, the conserved-vector-current pion form-factor ratio is predicted as

\[
\frac{F_\pi^A(0)}{F_\pi^V(0)} = 1 - \frac{1}{3} = \frac{2}{3},
\]

for \( q^2 = 0 \). Here, the terms 1 and 1/3 are due to quark and meson loops, respectively, and the relative minus sign is a Feynman rule. The result is very near data \([6]\) at

\[
\left(0.0116 \pm 0.0016\right) / \left(0.017 \pm 0.008\right) = 0.68 \pm 0.33.
\]

**KAON \bar{q}q MASSES**

Chiral-symmetry-breaking experimental kaon masses (given \( m_d - m_u \approx 4 \) MeV as above) are computed as (neglecting small experimental errors)

\[
m_{K^+(\bar{s}u)} = m_{\text{s,cur}} + m_{\text{u,cur}} = \hat{m}_{\text{cur}} \left[1 + \left(\frac{m_{\text{s}}}{\hat{m}}\right)_{\text{cur}}\right] - 2 \text{MeV} = 493.677 \text{MeV},
\]

\[
m_{K^0(\bar{s}d)} = m_{\text{s,cur}} + m_{\text{d,cur}} = \hat{m}_{\text{cur}} \left[1 + \left(\frac{m_{\text{s}}}{\hat{m}}\right)_{\text{cur}}\right] + 2 \text{MeV} = 497.648 \text{MeV}.
\]
For $\hat{m}_\text{cur} \approx 68.3$ MeV (see Eq. (13)), this gives in both cases

$$\left( \frac{m_s}{\hat{m}} \right)_\text{cur} \approx 6.257,$$

which compares well to the light-plane result [7]

$$\left( \frac{m_s}{\hat{m}} \right)_\text{cur} \approx 6-7,$$

and to other theory work [8], but not to the chiral-perturbation-theory (ChPT) predictions

$$\left( \frac{m_s}{\hat{m}} \right)_\text{cur} \approx 25-30 \text{ and } \hat{m}_\text{cur} \sim 5 \text{ MeV}.$$ (22)

**ChPT ALTERNATIVE**

The value for $(m_s/\hat{m})_\text{cur}$ presently adopted by the Particle Data Group [6] and the ChPT [9] research community is as large as 25–30, with the small nonstrange current-mass value $\hat{m}_\text{cur} = 2.5-5.5$ MeV being strongly biased by the ChPT estimate

$$\hat{m}_\text{cur} = \frac{(f_\pi m_\pi)^2}{2\langle -\bar{q}q \rangle} \approx 5.5 \text{ MeV},$$

for $f_\pi \approx 93$ MeV. On the basis of these scales, the quantitative explanation of $m_\pi$ and $m_K$ appears, however, rather cumbersome and “unnatural”, contrary to what we observe in our aforementioned scheme resulting from the QLL$\sigma$M and dynamically broken QCD.

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