Detecting TeV Gamma Rays from Gamma Ray Bursts by Ground Based Muon Detectors

Nayantara Gupta
Department of Theoretical Physics,
Indian Association for the Cultivation of Science,
Jadavpur, Calcutta 700 032, INDIA.

Pijushpani Bhattacharjee
Indian Institute of Astrophysics, Bangalore 560 034, INDIA.

Abstract

The possibility of detecting Gamma-Ray Bursts (GRBs) in TeV energy range using large area muon detectors like AMANDA and Lake Baikal detector is examined. These detectors can detect TeV energy photons by detecting the secondary muons created by the TeV photons in the Earth’s atmosphere. We calculate the expected number of muons and the signal to square root of noise ratios in these detectors due to TeV gamma-rays from individual GRBs for various assumption on their luminosity, distance from the observer (redshift), gamma-ray integral spectral index, maximum energy cutoff of the photon spectrum, and duration, including the effect of the absorption of TeV photons in the intergalactic infrared radiation background. We also calculate the expected rate of detectable TeV-GRB events in these detectors using a recent determination of the luminosity and redshift distributions of the GRBs in the Universe. For reasonable ranges of values of various parameters, we find about 1 event in 20 years in AMANDA (and a similar number for BAIKAL), while the event rate can be significantly larger (by factors of 10 or more depending on the area of the detector) in the proposed next generation detectors such as ICECUBE. Although, for the specific forms of the luminosity- and redshift distributions assumed, the average rate of expected detection is low, occasional nearby ($z \lesssim 0.1$), high luminosity ($\gtrsim 10^{54}$ erg/ sec), long duration ($\gtrsim 10$’s of seconds), and sufficiently hard spectrum (gamma-ray integral spectral index < 1) GRBs can, however, be detected even by AMANDA. Detection (or even non-detection) of TeV photons from GRBs in coincidence with satellite-borne detectors (which detect mainly sub-MeV photons) will be able to provide important new insights into the characteristics of GRBs and their emission mechanisms and, in addition, provide limits on the strength and spectrum of the intergalactic infrared background which affects the propagation of TeV photons from cosmological sources.

1tpng@mahendra.iacs.res.in
2pijush@iiap.ernet.in
1 Introduction

Gamma Ray Bursts (GRBs) are among the most powerful astrophysical phenomena in the Universe; see e.g., Ref. [1] for a recent review. Because of the limited sizes of the satellite-borne GRB detectors (BATSE, BeppoSAX, for example), GRBs have been detected mostly in the sub-MeV energy region where the photon number flux is sufficiently large for typical power-law photon spectra of GRBs. However, GRB emission extending to $\sim$10 GeV is known as in the case of the famous long-duration burst GRB 940217 [2] detected by the EGRET instrument on board the Compton Gamma Ray Observatory (CGRO), suggesting the possibility that GRBs may also emit much higher energy photons, perhaps even extending to TeV energies as in some highly energetic Active Galactic Nuclei (AGN) of the “blazar” class. For power-law spectra falling with energy, the photon number flux at TeV energies may be too low for TeV photons from GRBs to be detected by the satellite-borne detectors. However, ground-based detectors can in principle detect TeV photons from GRBs by detecting the secondary particles (electrons, muons) of the atmospheric showers generated by these photons in the Earth’s atmosphere. A variety of techniques (see, e.g., [3] for a review) allow determination of the energy and direction of the shower-initiating primary photon with good accuracy.

Indeed, three major ground-based gamma ray detectors, the Tibet air shower array [4], the HEGRA-AIROBICC Cherenkov array [5] and the Milagro water-Cherenkov detector [6] have independently claimed evidence for possible TeV $\gamma$-ray emission from sources in directional and temporal coincidence with some GRBs detected by BATSE. The estimated energy in TeV photons in each case has been found to be about 1 to 2 orders of magnitude larger than the corresponding sub-MeV energies measured by BATSE. Since TeV photons are efficiently absorbed in the intergalactic infrared (IR) background due to pair production [7], only relatively close by (i.e., low redshift) GRBs, for which the absorption due to IR background is insignificant, can be observed at TeV energies, which would explain the fact that only a few, not all, of the BATSE-detected GRBs in the fields of view of the individual ground detectors have been claimed to be detected at TeV energies.

While more GRBs would have to be detected in TeV energies before any firm conclusions can be drawn, the above discussions already point to the possibility that not only may the spectra of some, if not all, GRBs extend to TeV energies, but that the energy emitted in TeV photons may even constitute the dominant part of the total energy emitted by individual GRBs. This would happen, for example, if the photon spectrum, extending to TeV region, is hard with a differential power-law index $\gamma < 2$. In such situations, while the total number of photons in the entire spectrum is dominated by those at the lower cutoff energy of the spectrum, the total energy is
dominated by the upper cutoff energy. In other words, it is possible that the total burst energies of individual GRBs calculated from the measured fluences in the keV – MeV energies by the satellite detectors may only be a small fraction of the actual total energies of the individual bursts, most of which could be carried away by the relatively few TeV photons rather than by the abundant sub-MeV photons.

Indeed, photon differential spectrum with power-law index $\gamma < 2$ would be expected in most situations if the radiation arises from synchrotron radiation of power-law distributed particles (electrons or protons): Recall that the power-law index of the synchrotron photon differential spectrum is $(p + 1)/2$ where $p$ is the power-law index of differential number spectrum of the particles (electrons or protons). Thus, one can expect $\gamma < 2$ as long as $p < 3$, which is typically the case for particles accelerated in shocks.

On the theoretical side, it has been suggested [8, 9] that if protons are accelerated to energies above $\sim 10^{20}$ eV in GRBs as envisaged in the scenario [10] in which GRBs are the sources of the ultrahigh energy cosmic rays extending to energies above $10^{20}$ eV, then the synchrotron radiation of those ultrahigh energy protons in the magnetic field within the GRB would produce TeV photons. Thus, while the synchrotron radiation of electrons produce the keV–MeV flux, the TeV flux would be produced by synchrotron radiation of the protons with energy above $10^{20}$ eV. Whether or not a GRB would be bright in TeV would, in this scenario, be determined by the efficiency of energy transfer from protons to electrons within the GRB [9]. It has also been suggested [11, 12] that while the TeV photons emitted by GRBs at large redshifts would be absorbed through $e^+ e^-$ pair production on the intergalactic infrared background, the resulting electromagnetic cascades initiated by the produced pairs could produce the observed extragalactic diffuse gamma ray background in the GeV energy region. It is thus clear that confirmation of TeV gamma ray emission from GRBs would have major implications not only for the physics and astrophysics of GRBs, but also for the strength and redshift evolution of the intergalactic infra-red background.

High energy primary gamma rays above a few hundred GeV create “air-showers” of secondary particles by interacting with Earth’s atmosphere. Among the secondary particles are muons which can be detected, and the energy and direction of the shower initiating parent photon reconstructed (to a directional accuracy of typically a degree), by using the relatively shallow underground muon detectors such as AMANDA [13], Lake Baikal detector [14] and the proposed ICECUBE [15], and surface muon detector such as MILAGRO [1]. The first three are facilities primarily for detecting high energy neutrinos from cosmic sources — the neutrinos are detected by detecting the muons produced by the neutrinos in ice (AMANDA and ICECUBE) or water (Lake Baikal) — while MILAGRO is a water-Cherenkov detector of “all” particles (including muons)
primarily for detecting gamma rays above a few hundred GeV from astronomical sources.

The advantages of using these muon detectors over the conventional air-Cherenkov telescopes for doing TeV gamma ray astronomy have been expounded in Refs. [16, 17]. Compared to air-Cherenkov telescopes, the muon detectors cover much larger fraction of the sky with large duty cycles. For example, the AMANDA detector at the South Pole covers more than a quarter of the sky with essentially 100% efficiency. Being at relatively shallow depths — of order 1 km — the “neutrino” detectors are sensitive to muons with energies of a few hundred GeV and thereby to parent photons of energy of order a TeV and above. MILAGRO detector’s muon detection threshold is smaller — about 1.5 GeV — because of its location on the surface, which makes it sensitive to even lower energy gamma rays.

As pointed out in Ref. [17], these muon detectors are particularly suited for detecting TeV gamma rays from transient sources like GRBs — the otherwise large background of down-going cosmic-ray induced atmospheric muons can be significantly reduced because, with the information on the time and duration of a burst provided by satellite observation, the background is integrated only over the relatively short duration of a typical GRB which is of order few seconds.

In this paper we calculate, following Ref. [17], the expected number of muons and the signal to square root of noise ratios in AMANDA and Lake Baikal detectors due to TeV gamma-rays from individual GRBs for various assumption on their luminosity, distance from the observer (redshift), gamma-ray integral spectral index, maximum energy cutoff of the photon spectrum, and duration, including the effect of the absorption of TeV photons in the intergalactic infrared radiation background. We carefully take into account all relevant cosmological effects (in a cosmological constant dominated present Universe, see below for details). Our results for number of muons for individual GRBs differ significantly (especially in the case when absorption in the intergalactic IR background is taken into account) from those obtained in Ref. [17] for the same set of GRB parameters. We also calculate the expected rate of detectable TeV-GRB events in AMANDA-like detectors using a recent determination of the luminosity and redshift distributions of the GRBs in the Universe.

This paper is organized as follows: In Sec. 2 we discuss the parametrization of the TeV emission from the GRBs and the formalism we have used to calculate the number of secondary muons produced in the atmosphere from these TeV gamma rays of the GRBs. In Sec. 3 the results of our calculation of the number of muons expected and the signal to square root of noise ratios due to TeV photons from individual GRBs as functions of various GRB parameters are discussed. The expected rate of detection of
2 Parametrizations of the Photon Spectrum of GRBs and the Secondary Muons in Gamma-Ray Induced Showers in Earth’s Atmosphere

2.1 Photon Spectrum of GRBs

We shall assume the emission spectrum of the GRB to be constant during the emission time $\delta t_e$, and represented by $dN_\gamma/dE_\gamma = KE_\gamma^{-\alpha-1}$, giving the total number of photons emitted during the entire burst per unit energy at energy $E_\gamma$ between a minimum and a maximum energy, $E_{\gamma \min}$ and $E_{\gamma \max}$, respectively. Here $\alpha > 0$ is the integral spectral index and $K$ is a normalization constant, which are related to the total energy emitted from the burst, $E_{GRB}$, by

$$E_{GRB} = K \int_{E_{\gamma \min}}^{E_{\gamma \max}} E_\gamma^{-\alpha} dE_\gamma,$$

from which we get

$$K = E_{GRB} \times \begin{cases} (-\alpha + 1) \left( E_{\gamma \max}^{-\alpha+1} - E_{\gamma \min}^{-\alpha+1} \right)^{-1} & \text{for } \alpha \neq 1 \\ [\ln (E_{\gamma \max}/E_{\gamma \min})]^{-1} & \text{for } \alpha = 1. \end{cases}$$

Following Ref. [17], we shall use $E_{\gamma \min} = 1 \text{ MeV}$ since negligible amount of energy is emitted below that energy, and assume that the spectrum continues to TeV energies with the same integral spectral index $\alpha$. Further, as discussed in the Introduction, we shall assume the photon spectrum to be sufficiently hard with differential spectral index $\gamma < 2$, i.e., integral spectral index $\alpha < 1$.

The luminosity, $L$, and the total energy emitted, $E_{GRB}$, are related as

$$E_{GRB} = L \times \delta t_e.$$

For a GRB source located at a cosmological distance corresponding to a redshift $z$, the observed burst duration is $\delta t = (1 + z)\delta t_e$. For our purpose, a GRB is completely specified by the parameters $L$, $z$, $\delta t_e$, $\alpha$ and $E_{\gamma \max}$.

The observed spectrum is determined by two factors: (a) the photons energies are redshifted by a factor $(1 + z)$ due to the expansion of the universe, and (b) GeV
TeV photons are absorbed in the intergalactic space due to $e^+ e^-$ pair production on the intergalactic starlight and infrared background photons. The probability for a photon emitted with energy $E_\gamma$ by a source at redshift $z$ to reach the earth is $e^{-\tau(E_\gamma,z)}$, where $\tau$ is the optical depth for absorption due to the pair production process. We have used the optical depths given in [7], where a parametrization has been given for $\tau(E_\gamma,z)$ for small $z$ which, as we shall see, is appropriate for our purpose.

The observed photon energy $E_{\gamma 0}$ and its energy at the source, $E_\gamma$, are related as $E_{\gamma 0} = E_\gamma / (1 + z)$. Thus, at the top of the atmosphere, the observed cutoff energy of the source is $E_{\gamma 0,max} = E_{\gamma,max} / (1 + z)$. Assuming isotropic emission from the source, the total number of photons from a single GRB at redshift $z$ per unit energy at energy $E_{\gamma 0}$ that strike per unit area of the Earth at the top of the atmosphere is

$$\frac{d\phi_0}{dE_{\gamma 0}} = \frac{K}{4\pi r^2(z)} E_{\gamma 0}^{-\alpha - 1} (1 + z) e^{-\tau(E_\gamma,z)}, \quad (4)$$

where $r(z)$ is the comoving radial coordinate distance of the source. For a spatially flat Universe (which we shall assume to be the case) with $\Omega_\Lambda + \Omega_m = 1$, where $\Omega_\Lambda$ and $\Omega_m$ are, respectively, the contribution of the cosmological constant and matter to the energy density of the Universe in units of the critical energy density, $3H_0^2/8\pi G$, the radial coordinate distance $r(z)$ is given by [18]

$$r(z) = \int_0^z \left( \frac{c}{H_0} \right) \frac{dz'}{\sqrt{\Omega_\Lambda + \Omega_m (1 + z')^3}}. \quad (5)$$

Here $c$ is the speed of light, $H_0$ is the Hubble constant in the present epoch. In our calculations, we shall use $\Omega_\Lambda = 0.7$, $\Omega_m = 0.3$, and $H_0 = 65$ km sec$^{-1}$ Mpc$^{-1}$.

### 2.2 Number of Muons in a Photon-Induced Air Shower

Photons of sufficiently high energy striking the earth’s atmosphere interact with the air nuclei to produce pions; these photo-pions are the major source of muon production by photons in atmospheric air-showers, among other possible physical processes like direct pair production of muons by photons or photo-produced charm decays. The number of muons with energy above $E_\mu$ in an air-shower produced by a single photon of energy $E_{\gamma 0}$ striking the atmosphere can be parametrized [19, 17], for $E_\mu$ in the range 0.1 TeV to 1 TeV, by the formula

$$N_\mu(E_{\gamma 0}, > E_\mu) \approx \frac{2.14 \times 10^{-5}}{\cos \theta} \frac{1}{(E_\mu / \cos \theta)} \frac{E_{\gamma 0}}{(E_\mu / \cos \theta)}, \quad (6)$$

where $\theta$ is the zenith angle of the photon source, and all energies are in TeV units.
The above parametrization is valid for $E_{\gamma 0}/E_{\mu} \geq 10^{17}$, which is adequate for the purpose of calculating the muon rates in the AMANDA and Baikal detectors both of which have muon thresholds of the order of a few hundred GeV [17].

The total number of muons with energy in excess of $E_{\mu}$ in a detector of effective area $A$ due to a single GRB at redshift $z$ can then be written as

$$N_\mu(> E_{\mu}) = A \int_{E_{\gamma \text{th}}}^{E_{\gamma \text{max}0}(z)} dE_{\gamma 0} \frac{d \phi_0}{dE_{\gamma 0}}(z) N_\mu(E_{\gamma 0}, > E_{\mu}), \quad (7)$$

where $E_{\gamma \text{th}} \simeq 10 \times E_{\mu}/\cos \theta$ is the minimum photon energy needed to produce muons of energy $E_{\mu}$ in the atmosphere [17], $\theta$ being the zenith angle of the photon source. Using Eq. (4) in (7) we finally have

$$N_\mu(> E_{\mu}) = AK \frac{K(1 + z)^{-\alpha}}{4\pi r^2(z)} \int_{E_{\gamma \text{th}}}^{E_{\gamma \text{max}0}(z)} dE_{\gamma 0} E_{\gamma 0}^{-(\alpha+1)} e^{-\tau(E_{\gamma 0}(1+z),z)} N_\mu(E_{\gamma 0}, > E_{\mu}), \quad (8)$$

where $K$ is a function of the GRB parameters $L$, $z$, $\delta t_e$, $\alpha$ and $E_{\gamma \text{max}}$, and is given by Eqns. 2 and 3.

### 3 TeV Photon Signal of GRBs in AMANDA and Lake Baikal Muon Detectors

The threshold muon energy for detection of vertical muons in the AMANDA detector [13] of effective area $A \simeq 10^4$ m$^2$ is $\sim$ 350 GeV. For the Lake Baikal detector [14] the vertical muon threshold energy is $\sim$ 150 GeV and the effective area is $\sim$ 10$^3$ m$^2$. The signal in the detector, $S = N_\mu(E_{\mu})$, must be compared with the square root of noise, $\sqrt{N}$, given by

$$\sqrt{N} = \sqrt{A \times \delta t \times I_\mu(\theta) \times \delta \theta^2}, \quad (9)$$

where $\delta t$ is the observed duration of the burst, $I_\mu(\theta)$ is the background muon intensity (i.e., number of muons per unit area, time and solid angle) due to cosmic ray induced atmospheric showers [13, 17], and $\delta \theta$ is the angular resolution of the detector which is typically a few degrees. The smallest value of $n \equiv S/\sqrt{N}$ necessary to consider an excess signal as a positive detection depends on the search strategy of the particular detector.
TABLE I
Expected number of muons in AMANDA detector for a GRB at \( z = 0.1 \)

| \( L \) (ergs/sec) | \( E_{\gamma\text{max}} \) (TeV) | \( \alpha \) | No. of muons \( \theta = 0^\circ \) | \( S/\sqrt{N} \) \( \theta = 0^\circ \) | No. of muons \( \theta = 60^\circ \) | \( S/\sqrt{N} \) \( \theta = 60^\circ \) |
|------------------|------------------|-------|-----------------|-----------------|-----------------|-----------------|
| \( 10^{52} \)    | 10.              | 1.0   | 0.06            | 0.214           | 0.0034          | 0.03            |
|                  | 10.              | 0.8   | 0.19            | 0.635           | 0.0101          | 0.1             |
|                  | 10.              | 0.5   | 0.38            | 1.276           | 0.0264          | 0.23            |
| \( 10^{52} \)    | 5.               | 1.0   | 0.03            | 0.103           | 0.0000          | 0.0             |
|                  | 5.               | 0.8   | 0.09            | 0.32            | 0.0000          | 0.0             |
|                  | 5.               | 0.5   | 0.22            | 0.744           | 0.0000          | 0.0             |
| \( 10^{54} \)    | 10.              | 1.0   | 6.404           | 21.404          | 0.3480          | 3.16            |
|                  | 10.              | 0.8   | 19              | 63.55           | 1.1300          | 10.31           |
|                  | 10.              | 0.5   | 38.18           | 127.68          | 2.6000          | 23.70           |
| \( 10^{54} \)    | 5.               | 1.0   | 3.09            | 10.33           | 0.0000          | 0.0             |
|                  | 5.               | 0.8   | 9.727           | 32.52           | 0.0000          | 0.0             |
|                  | 5.               | 0.5   | 22.28           | 74.46           | 0.0000          | 0.0             |

In Table I we present for illustration the result of our calculations of the number of muons expected in AMANDA and the corresponding signal to square root of noise ratios \( (S/\sqrt{N}) \) due to a single GRB at redshift \( z = 0.1 \), for various possible values of its source luminosity \( (L) \), maximum energy cutoff at source \( (E_{\gamma\text{max}}) \), and integral spectral index \( (\alpha) \). The duration of the burst is taken to be \( \delta t = 11 \) sec, somewhat intermediate between short- and long duration bursts. The angular resolution of the AMANDA detector has been taken to be \( \delta \theta = 1^\circ \). The muon numbers are given for two different values of the zenith angle \( \theta \) of the source. The absorption of the TeV photons in the intergalactic infrared and optical backgrounds has been taken into account in calculating the numbers shown in Table I for which we have taken the optical depths corresponding to the higher level of the intergalactic infrared radiation field \( (\text{IIRF}) \) given in Ref. [7] in order to have conservative estimates of the number of muons produced. Throughout this paper we use optical depths corresponding to this higher level of IIRF whenever we include the effect of absorption of the TeV photons in the intergalactic space.

Clearly, the number of muons ("signal") and the signal-to-noise ratios \( (S/\sqrt{N}) \) depend crucially on the various GRB parameters. Some of these dependencies are illustrated in more details in Figures below. As pointed out in [17], for a given set of other
parameters, the smaller number of muons at the larger zenith angle is a consequence of the increase of the energy threshold with zenith angle because the muons have to traverse a greater amount of matter to reach the detector. Indeed, for too low value of the source gamma-ray energy cutoff $E_{\gamma max}$, for sufficiently large zenith angle, the threshold gamma-ray energy $E_{\gamma th}$ required to produce muons above the threshold energy of the detector can be larger than $E_{\gamma max}$, thus yielding no muons. Also, as expected, for a given luminosity $L$ and spectral index $\alpha \leq 1$, higher source cutoff energy $E_{\gamma max}$ yields higher number of muons as well as higher $S/\sqrt{N}$. However, the effect of decrement of the signal due to smaller $E_{\gamma max}$ can, in some cases, be over-compensated for by a harder source spectrum (i.e., smaller value of $\alpha$) yielding a relatively larger signal.

Fig. 1 and Fig. 2 show the number of muons expected in AMANDA as a function of gamma-ray integral spectral index of the GRB source assuming $L = 3 \times 10^{52}$ ergs/ sec, $\delta t_e = 1$ sec (so that $E_{GRB} = 3 \times 10^{52}$ ergs), $E_{\gamma max} = 10$ TeV, and $z = 0.1$, for zenith angles $\theta = 0^\circ$ (Fig. 1) and $\theta = 60^\circ$ (Fig. 2). The results for the BAIKAL detector are shown in Fig. 3. Curves are shown for both without and with absorption of the TeV photons in the IIRF. In Fig. 1 and Fig. 2, the results of Ref. [17] are also shown for comparison.

From Fig. 1 and Fig. 2, we see that there are significant differences between our results and those of Ref. [17] for the same set of GRB parameters, with our results for the number of muons being always consistently lower than those of Ref. [17]. In the case of absence of absorption, we guess the difference between our results and those of [17] could be due to our using a specific cosmological model — as already mentioned, we use the “standard” cosmological model with a spatially flat Universe with $\Omega_A = 0.7$, $\Omega_m = 0.3$ and $H_0 = 65$ km sec$^{-1}$ Mpc$^{-1}$ — whereas Ref. [17] does not explicitly mention the values of various cosmological parameters used.

In the case when absorption of TeV photons in the IIRF is included, the difference between our results and those of [17] is even more than that in the case of no absorption. The flux of the TeV photons, and hence the number of muons in the detector, depend quite sensitively on the optical depth of the TeV photons, which in turn is governed by the strength of the IIRF. A relatively higher (lower) level of IIRF gives relatively larger (smaller) optical depth, resulting in relatively smaller (larger) number of muons in the detector. Since the level of IIRF is not known with certainty, we have used optical depths corresponding to the higher level of IIRF from Ref. [17] in order to obtain conservative estimates of the number of muons in the detector. However, Ref. [17] does not explicitly mention whether the optical depths corresponding to the higher or the lower level of the IIRF given in Ref. [17] was used in their actual calculation, and so it is difficult to find the exact reasons for the differences between
our results and those of Ref. [17]. For redshift \( z = 0.1 \) our results differ from the results of Ref. [17] by almost a factor of ten in the case when the effect of absorption of the TeV photons in the IIRF is included.

The above discussion demonstrates the fact that the detectability of possible TeV gamma-rays from GRBs in ground based muon detectors depends sensitively on the various cosmological parameters and the strength and spectrum of the IIRF in addition to the intrinsic GRB parameters. Conversely, within the context of the “standard cosmological model”, the detection or non-detection of TeV photons from GRBs can significantly constrain the strength and spectrum of the IIRF, provided the intrinsic GRB parameters such as their total luminosity, redshift and duration are known with reasonable accuracy from independent observations.

In Fig. 4 the number of muons expected in AMANDA from a GRB is shown as a function of the redshift of the GRB for \( E_{\gamma_{\text{max}}} = 10 \text{ TeV} \) and for two different cases, namely, \( L = 3 \times 10^{52} \text{ ergs/sec} \), \( \alpha = 1.0 \), and \( L = 10^{54} \text{ ergs/sec} \), \( \alpha = 0.5 \). The absorption of TeV photons is included with optical depth corresponding to the higher level of IIRF from [7]. As expected, the number of muons from a GRB falls steeply with increasing redshift due to absorption of TeV photons, and so detection is possible only for relatively nearby (low redshift) bursts.

Fig. 5. shows the expected number of muons (for \( \theta = 0^\circ \)) in AMANDA as a function of the total luminosity of the GRB for a fixed duration \( \delta t = 1 \text{ sec} \) of the burst. In Figs. 6–8 we show the effect of varying the physical parameters of a GRB on the signal to square root of noise ratios in the AMANDA detector. It is clear that high signal to square root of noise detection of TeV photons from GRBs in AMANDA (or other ground based muon detectors of similar area and energy threshold) will be possible mostly for relatively nearby (\( z \lesssim 0.1 \)), long duration (\( \gtrsim \) several tens of seconds), high luminosity (\( \gtrsim \) few \( \times 10^{52} \text{ ergs/sec} \)) and hard spectrum (\( \alpha < 1 \)) GRBs.

| BATSE Number | 50–300 keV Luminosity \((L_L)\) from [20] (ergs/sec) | Redshift \( z \) | Burst duration \((\delta t)\) (sec) | 1 MeV–10 TeV Luminosity \((L_H)\) (ergs/sec) |
|--------------|---------------------------------|--------|-----------------|------------------|
| 6707         | \(2.63 \times 10^{45}\)         | 0.0085 | 20.6            | \(2.48 \times 10^{48}\) |
| 2123         | \(0.18 \times 10^{50}\)         | 0.1    | 22.0            | \(10.06 \times 10^{51}\) |
| 2316         | \(0.64 \times 10^{49}\)         | 0.1    | 29.2            | \(7.58 \times 10^{51}\) |
| 3055         | \(0.61 \times 10^{50}\)         | 0.2    | 40.6            | \(2.12 \times 10^{54}\) |
| 3352         | \(0.62 \times 10^{50}\)         | 0.2    | 46.3            | \(1.86 \times 10^{54}\) |
| 4368         | \(6.04 \times 10^{51}\)         | 0.4    | 36.5            | \(0.88 \times 10^{60}\) |
In Table II we list some of the nearby GRBs from the BATSE catalogue whose luminosities in the 50 to 300 keV photon energy band are known and whose redshifts have been estimated in Ref. [20] by using a conjectured variability – luminosity relationship for GRBs which is calibrated using seven GRBs whose redshifts are known independently from afterglow studies. We have calculated the luminosities for photon energies in the range 1 MeV to 10 TeV required for these GRBs to produce $S/\sqrt{N} = 1.5$ at zenith angle $\theta = 0^\circ$ in AMANDA. We assume $\alpha = 1.0$ for these GRBs. The luminosities so calculated, denoted by $L_H$, are displayed in the last column of Table II. The intergalactic absorption of TeV photons corresponding to the higher IIRF level from [7] has been taken into account in this calculation.

From Table II we see that the high energy (1 MeV to 10 TeV) luminosities ($L_H$) required to detect TeV photons from these GRBs in AMANDA are much higher than their luminosities $L_L$ in the lower energy (50 keV to 300 keV) band. No TeV photons from these GRBs have been reported to be detected by AMANDA. The most likely explanation is that these GRBs did not emit sufficient number of TeV photons that would create sufficient number of muons above the threshold of AMANDA and/or the intergalactic absorption of TeV photons is even stronger than what we assumed.

### 4 Average Rate of Detectable TeV GRBs in Muon Detectors

In the previous sections we have studied the detectability of TeV photons from individual GRBs in muon detectors by calculating the number of muons produced and the signal to noise ratios as functions of various intrinsic parameters of the bursts such as their luminosities, integral spectral index, redshift, and burst duration.

In this section we estimate the expected rate of positive detection of TeV photons from all GRBs in the Universe in muon detectors such as AMANDA. This requires the knowledge of the forms of the luminosity function (LF) and the redshift distribution (RD) of the GRBs in the Universe. Unfortunately, direct measurements of redshifts exist for only a few GRBs (from optical afterglow measurements), and so the RD and the LF of the GRB population are not known with certainty. Nevertheless, recently there have been attempts to indirectly derive the redshifts (and hence luminosity) distributions of GRBs from the BATSE data exploiting various features of the light curves of individual GRBs in the BATSE data.
4.1 Luminosity Function and Redshift Distribution of GRBs

Recently, authors of Ref. [20] have found that the luminosities of seven GRBs in the BATSE catalogue with known redshifts (measured from their optical afterglows) are correlated (as a power law) with the variability of the individual bursts, the variability being defined as the normalized variance of the observed 50–300 keV light curve about a fitted smooth light curve. Based on this observation, the authors of Ref. [20] have conjectured a variability \( (V) \)-luminosity \( (L) \) relationship which is hypothesized to be true for all GRBs in the BATSE catalogue. Use of this conjectured \( V-L \) relationship, which is calibrated using the seven GRBs with known redshifts, then allowed estimation of the redshifts of about 220 GRBs in the BATSE catalogue, which in turn allowed construction of a LF and a RD for GRBs in general.

Independently, authors of Ref. [21] have found a correlation between luminosity and the so-called “lag” \( (\tau_{\text{lag}}) \), which is the time delay between the peaks in the light curves of individual GRBs. It was then pointed out in Ref. [22] that the above two relationships, namely the \( V-L \) and the \( \tau_{\text{lag}}-L \) relationships, if true in general, would together imply a \( V-\tau_{\text{lag}} \) relationship which could be tested directly on the measured data for BATSE observed GRBs without reference to their redshifts. Indeed, Ref. [22] claims strong concordance of the above mentioned \( V-\tau_{\text{lag}} \) relationship with the observed BATSE data for 112 GRBs, based on which a LF has been derived for GRBs in general, which is a broken power-law with a break at \( L \approx 2 \times 10^{52} \text{ erg/sec} \), with LF going as \( L^{-1.7 \pm 0.1} \) below the break and \( L^{-2.8 \pm 0.2} \) above the break. The RD giving the number density of GRBs as a function of redshift is also derived [22], which goes as \((1 + z)^{2.5 \pm 0.3}\).

We must remember that the LF mentioned above refers to the luminosity in the BATSE band (50–300 keV), which we have denoted above by \( L_L \). We are, however, interested in the LF expressed as a function of the luminosity in the 1 MeV — 10 TeV interval, which we have denoted above by \( L_H \). It is, however, easy to see that, for a single power-law photon spectrum across all energies, which we assume for simplicity in this paper, the LF expressed as a function of \( L_H \) has the same functional form as that in terms of \( L_L \), except that the break of the LF occurs at a different value of luminosity which depends on the integral spectral index \( \alpha \). For a given GRB with a definite \( L_L \) and \( L_H \), one can see that the two are related as

\[
\frac{L_H}{L_L} = \frac{(10 \text{ TeV})^{-\alpha+1} - (1 \text{ MeV})^{-\alpha+1}}{(300 \text{ keV})^{-\alpha+1} - (50 \text{ keV})^{-\alpha+1}} \equiv C(\alpha),
\]

which is valid for all GRBs assumed to have the same integral spectral index \( \alpha \).
Based on the above considerations, we shall assume the following LF for GRBs:

\[
\frac{dN}{dL_H} \propto \begin{cases} 
L_H^{-1.7} & \text{for } L_H < L_{H*}, \\
L_H^{-1.1} L_H^{-2.8} & \text{for } L_H \geq L_{H*},
\end{cases}
\]

(11)

where the break luminosity \( L_{H*} \) is given by \( L_{H*} = C(\alpha) L_{L*} \), with \( L_{L*} \simeq 2 \times 10^{52} \text{ erg/sec} \) being the break of the LF derived in Ref. [22].

Below, we shall often drop the subscript \( H \) in \( L_H \), and so, unless otherwise specified, \( L \) will mean \( L_H \).

For the rate-density of GRBs (number of bursts per unit volume and unit time) in the Universe as a function of redshift, we will assume it to be proportional to \( (1 + z)^{2.5} \) [22] for the redshift range of our interest (up to \( z \sim 4 \)). This redshift distribution follows star formation rate (SFR) [23] for \( z < 2 \), but unlike the observed SFR based on optical observations, which generally yield an SFR that either levels off or falls with redshift for \( z > 2 \), the above GRB rate increase monotonically with \( z \) at least up to \( z \sim 5 \) [22] and possibly even up to \( z \sim 10 \) [20]. The suggestion [20, 22] is that the SFR based on optical observations may have underestimated the true SFR at high redshifts because of cosmological reddening effects, and since gamma rays are not subject to the reddening effect, the GRB rate may indeed reflect the true SFR at high redshifts provided, of course, the physical connection between the GRB phenomenon and the star formation process is clearly understood by future studies.

With the LF and rate-density (i.e., redshift distribution) of GRBs specified above, we now proceed to calculate the expected average rate of detection of TeV GRBs in the muon detectors. In doing this we follow the formalism described in Ref. [24].

### 4.2 Calculation of the Rate of TeV GRBs

The first step is to calculate the redshift up to which a GRB of a given \( L, E_{\gamma\text{max}}, \alpha \) and \( \delta t \) will be detectable at a zenith angle \( \theta \) in a given detector. By detectable we mean the statistical significance of the signal, i.e., the signal to square root of noise ratio, \( S/\sqrt{N} \), is larger than some preassigned value.
Table III illustrates the values of maximum redshifts up to which GRBs of various $L$ are detectable by AMANDA at zenith angle $\theta = 0^\circ$ with $S/\sqrt{N} \geq 1.5$, assuming $E_{\gamma, \text{max}} = 10\,\text{TeV}$, $\alpha = 0.8$ and $\delta t = 20\,\text{sec}$, with absorption of TeV photons taken into account. Fig. 9 shows this in more details for three different values of the integral spectral index $\alpha$. The regions below the curves in this figure are the allowed regions for positive detections with $S/\sqrt{N} \geq 1.5$.

Next, we calculate the fraction, $f(L, \cos \theta)$, of all bursts of given luminosity $L$ that are detectable at a zenith angle $\theta$ in the sense defined above. To do this, we first define, for a burst of luminosity $L$ at redshift $z$ at a zenith angle $\theta$ with respect to a given detector, a quantity $J(L, z, \cos \theta)$ such that $J(L, z, \cos \theta) = 1$ if the burst is detectable and $J(L, z, \cos \theta) = 0$ if it is not. With the redshift distribution of GRBs as specified above, we can write

$$f(L, \cos \theta) = \frac{\int_0^{z_{\text{max}}} J(L, z, \cos \theta)(1 + z)^{2.5} \frac{dV}{dz} dz}{\int_0^{z_{\text{max}}}(1 + z)^{2.5} \frac{dV}{dz} dz},$$

where $z_{\text{max}}$ is the maximum redshift of the GRBs in the Universe, and $dV(z)$ is the volume element of the Universe between redshifts $z$ and $z + dz$, which is given by

$$\frac{dV}{dz} = \frac{4\pi}{(1 + z)^3} \frac{d}{dz} \left[r^3(z)/3\right].$$

In the above expression $r(z)$ is the comoving distance as defined in Eq. (5).

In our numerical calculations we shall take $z_{\text{max}} = 4$.

Finally, folding the fraction $f$ with the LF given in Eq. (11) and the zenith angle distribution of the bursts, $dS(\cos \theta)/d \cos \theta$, we write the expected TeV GRB rate in muon detector, $R_{\text{TeV}}$, as
\[ R_{\text{TeV}} = R_{\text{total}} \int_{L_{\text{min}}}^{L_{\text{max}}} \int_{0.5}^{1} \frac{dN(L)}{dL} f(L, \cos \theta) \frac{dS(\cos \theta)}{d \cos \theta} dL d(\cos \theta), \]  

where \( \frac{dN(L)}{dL} \) is now the normalized LF and \( R_{\text{total}} \) is the total observed GRB rate, and we assume that the detector can view GRBs with good efficiency between zenith angles \( \theta = 0^\circ \) and \( \theta = 60^\circ \). We shall assume a uniform zenith angle distribution for the bursts and set \( \frac{dS(\cos \theta)}{d \cos \theta} = 1/2 \). Also we take \( L_{\text{min}} = 10^{51} \text{erg/sec} \) and \( L_{\text{max}} = 10^{56} \text{erg/sec} \).

BATSE observes about 1 burst per day with a detection efficiency of 0.3. Thus, the GRB rate deduced from BATSE observations is \( R_B \sim 1000 \text{yr}^{-1} \). Assuming that BATSE can observe GRBs up to a maximum redshift \( z_{\text{max}} = 4 \) we set \( R_{\text{total}} = R_B \).

We have calculated \( R_{\text{TeV}} \) for AMANDA for a wide range of values of the intrinsic parameters specifying the bursts. These rates turn out to be rather low. For example, with \( E_{\gamma_{\text{max}}} = 10 \text{TeV}, \alpha = 0.8 \) and \( \delta t = 20 \text{sec} \), and requiring \( S/\sqrt{N} \geq 1.5 \) (the same parameter set as used in obtaining the numbers in Table III), we get \( \sim 1 \) detectable TeV GRB in 25 years in AMANDA. The rate depends somewhat on \( \alpha \); for example, for \( \alpha = 0.5 \) with other parameters kept same as above, the rate improves to \( \sim 1 \) burst in 20 years, while for \( \alpha = 1 \), the rate is \( \sim 1 \) in 40 years. In ICECUBE, assuming its effective area will be about 10 times larger, the above expected rates will be correspondingly larger by the same factor, assuming same threshold energy for muon detection as in AMANDA.

Note also that the above rates were calculated for only a 1.5\( \sigma \) detectability; rates for higher-\( \sigma \) detectability will be even lower.

### 4.3 Discussion

The reasons for the rather low detectability rate of TeV photons from GRBs in existing muon detectors are clear: The absorption of TeV photons in the intergalactic infrared background allows detection of TeV photons from only relatively nearby \( (z < 0.1) \) bursts as clear from Fig. 9, unless the luminosity of the burst is relatively large \( (\gtrsim 10^{53} \text{erg/sec}) \). However, for a LF falling with \( L \), and a redshift distribution scaling like the SFR (which increases with \( z \)), low-redshift and high-luminosity GRBs are relatively rare. Nevertheless, as we have demonstrated in Figs. 6–9, occasional nearby GRBs of sufficiently long duration, large luminosity and hard spectrum are likely to be detectable in existing detectors like AMANDA, and will certainly be detectable in the next generation detectors such as ICECUBE.
5 Summary and Conclusions

It is possible that GRBs emit not only sub-MeV photons as detected in satellite-borne detectors, but also higher energy photons extending to TeV energies. For a GRB photon spectrum falling with energy, as is usually the case, the non-detection of TeV photons in satellite-borne detectors could be due to their limited size. On the other hand, for sufficiently hard photon spectrum with integral spectral index $\alpha < 1$, although the total number of photons is dominated by those at the low (keV – MeV) energy end, the total energy of the burst would be carried away mostly by the few high (say, TeV)- energy photons. Thus, the actual total energy emitted in a burst may be much larger than the total energy estimated from the observed fluence of sub-MeV photons in the satellite detectors, which, if true, would have tremendous implications for the theories of origin of GRBs. It is, therefore, very important to study possible ways of detecting possible TeV photons from GRBs.

Refs. [16, 17] suggested that TeV photons from GRBs might be detectable in the existing muon detectors such as AMANDA, Lake Baikal, and Milagro and future detectors such as the proposed ICECUBE, by detecting the muons in the atmospheric showers created by TeV photons. Following the calculations of Ref. [17], we have made detailed calculations of the detectability of possible TeV photons from individual GRBs by AMANDA-type detectors, as functions of various GRB parameters such as their luminosity, duration, integral spectral index, and redshift. While our calculations generally yield lower number of expected muons per burst than that calculated in Ref. [17] for the same set of GRB parameters, we conclude that sufficiently high luminosity, long duration, and hard spectrum nearby GRBs would still be detectable in AMANDA-class detectors and will certainly be detectable in next generation detectors such as ICECUBE. We have also estimated the expected rate of TeV GRB events in these detectors using recent information on the luminosity function of GRBs and their redshift distribution in the Universe. Our conservative estimates, including the effect of absorption of TeV photons in the intergalactic space, show that while the expected average rate of TeV GRB events in AMANDA is rather low — about one GRB in 20 years for sufficiently hard (integral spectral index $\alpha = 0.5$) spectrum, the rate in up-coming bigger detectors such as ICECUBE could be about 10 times larger, i.e., about 1 in every couple of years.

6 Acknowledgment

One of us (NG) wishes to thank IIA, Bangalore for hospitality where a major part of this work was done. PB acknowledges partial support under the NSF US-India cooperative research grant # INT-9714627.
References

[1] T. Piran, Phys. Rep. 314, 575 (1999).
[2] K. Hurley et al, Nature 372, 652 (1994).
[3] R.A. Ong, Phys. Rep. 305, 95 (1998).
[4] M. Amenomori et al, Astron. Astrophys. 311, 919 (1996).
[5] L. Padilla et al, Astron. Astrophys. 337, 43 (1998).
[6] R. Atkins et al, Astrophys. J. 533, L119 (2000); also see http://www.lanl.gov/milagro/themilagrocollaboration.html.
[7] F. W. Stecker and O. C. de Jager, Astron. Astrophys. 334, L85 (1998).
[8] T. Totani, Astrophys. J 509, L81 (1998).
[9] T. Totani, Astrophys. J. 536, L23 (2000).
[10] E. Waxman, Phys. Rev. Lett. 75, 386 (1995); M. Milgrom and V. Usov, Astrophys. J. 449, L37 (1995); M. Vietri, Astrophys. J. 453, 883 (1995).
[11] R. A. Vazquez, astro-ph/9810231.
[12] T. Totani, Astropart. Phys. 11, 451 (1999).
[13] F. Halzen (for AMANDA Collaboration) Nucl. Phys. Proc. Suppl. 77, 474 (1999); E. Andres et al (AMANDA collaboration), Astropart. Phys. 13, 1 (2000).
[14] Ch. Spiering (for BAIKAL Collaboration), Prog. Part. Nucl. Phys. 40, 391 (1998); V. A. Balkanov et al (BAIKAL Collaboration), Astropart. Phys. 14, 61 (2000).
[15] The ICECUBE proposal: see http://pheno.physics.wisc.edu/icecube/.
[16] F. Halzen, T. Stanev, and G. Yodh, Phys. Rev. D 55, 4475 (1997).
[17] J. Alvarez-Muñiz and F. Halzen, Astrophys. J. 521, 928 (1999).
[18] C.W. Misner, K.S. Thorne and J.A. Wheeler, *Gravitation* ( W. H. Freeman, San Francisco, 1973).
[19] F. Halzen, K. Hikasa and T. Stanev, Phys. Rev. D 34, 2061 (1986).
[20] E. E. Fenimore and E. Ramirez-Ruiz, astro-ph/0004176.

[21] J.P. Norris, G. Marani and J. Bonnell, Astrophys. J. 534, 248 (2000).

[22] B. E. Schaefer, M. Deng and D.L. Band, astro-ph/0101461.

[23] C.C. Steidel et al, Astrophys. J. 519, 1 (1999).

[24] S. Vernetto, Astropart. Phys. 13, 75 (2000).

[25] J.C. Charlton and M.S. Turner, Astrophys. J. 313, 495 (1987).
Figure 1: Number of muons per burst at zenith angle $\theta = 0^\circ$ in AMANDA as a function of the integral spectral index of the GRB photon spectrum, for a GRB at redshift $z = 0.1$, total energy $E_{\text{GRB}} = 3 \times 10^{52}$ erg, and maximum cutoff energy $E_{\gamma \text{max}} = 10$ TeV. Curves are shown for with and without absorption of the photons in the intergalactic infrared field (IIRF). Corresponding curves from the calculation of Ref. [17], indicated above by A H (their Fig. 1) for the same set of parameter values as above, are also shown for comparison. In the case of absorption, our results are almost a factor of 10 below those in A H (see text for discussion).
Figure 2: Same as Fig. 1, but for zenith angle $\theta = 60^\circ$
Figure 3: Number of muons per burst in the Lake Baikal detector as a function of the integral spectral index of the GRB photon spectrum, for the same set of GRB parameters as in Fig. 1. Curves are shown for with and without absorption of the photons in the IIRF and for two different zenith angles $\theta = 0^\circ$ and $\theta = 60^\circ$. 
Figure 4: Number of muons per burst in the AMANDA detector at zenith angle \( \theta = 0^\circ \) as a function of the redshift of the burst for various GRB parameters as indicated, taking into account the absorption of photons in the IIRF corresponding to the higher level of IIRF (see text).
Figure 5: Number of muons per burst in the AMANDA detector at zenith angle $\theta = 0^\circ$ as a function of the luminosity of the burst for various GRB parameters as indicated. Curves are shown for both with absorption (dashed curves) and without absorption (solid curves) in the IIRF. The upper solid or dashed curves are for maximum spectral cutoff of the source at $E_{\gamma\text{max}} = 10$ TeV and the lower ones for $E_{\gamma\text{max}} = 5$ TeV.
Figure 6: The signal to square root of noise ratio in the AMANDA detector as a function of the integral spectral index of the GRB photon spectrum, corresponding to the muon numbers shown in Figs. 1 and 2.
Figure 7: The signal to square root of noise ratio in the AMANDA detector as a function of redshift of the GRB, corresponding to the muon numbers shown in Fig. 4.
Figure 8: The signal to square root of noise ratio in the AMANDA detector as a function of the duration of the GRB.
Figure 9: The maximum redshift of a GRB that can be detected with signal to square root of noise ratio $S/\sqrt{N} \geq 1.5$ in AMANDA as a function of the luminosity of the burst for several choices of integral spectral index $\alpha$ and for the indicated values of $E_{\gamma \text{max}}$ and burst duration. The regions below the respective curves are the allowed regions for positive detection.