Comparative study of cracked rotor responses for simply supported end conditions and fluid film bearings

Sumit Srivastava and Skylab P Bhore
Rotor Dynamics and Vibration Diagnostics Lab, Department of Mechanical Engineering, Motilal Nehru National Institute of Technology, Allahabad 211004, Uttar Pradesh, India
sumitsrivastava49@gmail.com, skylabpbhore@mnnit.ac.in

Abstract. In this paper, a comparative study of cracked rotor responses for simply supported end conditions and fluid film bearings is presented. A 2 degree of freedom (dof) Jeffcott rotor model is used. A cracked rotor is represented by switching crack model. Fluid film bearings is modelled by short bearing approach given by Ocvirk theory. Stiffness damping coefficients of the bearings are incorporated in the rotor equation of motion. The equation of motion is solved by using Runge-Kutta numerical integration method. Cracked rotor responses are analysed by using Fast Fourier Transform (FFT) and orbit plots. It is found that for simply supported end conditions, cracked rotor shows 1X, 2X, 3X, 4X and 5X frequency components to be significant in horizontal direction. For fluid film bearings, cracked rotor shows 1X, 2X, 3X, 5X and 7X frequency components to be significant in horizontal direction.

1. Introduction
Fault diagnosis in turbomachines is becoming very popular. It prevents a catastrophic failure and gives a continuous system health information. Most of the turbomachinery works in harsh operating conditions such as high temperature, high speed, and cyclic loading etc. It leads to development of fatigue crack. Identification of cracks is very important. If, it is identified in time it prevents catastrophic failure. A lot of work on fatigue failure cracked rotors has been done in the past. There are basically two types of crack models. Switching crack and breathing crack models are pretty common in use. Jun et al [1] have analyzed the vibrational response of cracked rotor with breathing and switching crack model. Gasch [2] have studied dynamic behavior Laval rotor with a transverse crack. They have used simple crack model and derived a non-linear equation. Stability analysis is carried out with floquets method. Shinou [3] have presented the detection of cracks in rotors based on the 2X and 3X super harmonic frequency components and crack unbalance interaction. He found that the detection of the crack using one half or one third subcritical resonances is possible and acceptable to the industrial community. Darpe and Patel [4] have studied the influence of the crack breathing models on the non-linear vibrational characteristics of the cracked rotors. The effect of variation of different parameters such as rotational speed, damping etc. are studied for different levels of unbalance, crack depth etc. Guo et al [5] have investigated experimentally the transverse crack in a Jeffcott rotor. They shown that the resonances at one third and one half of critical speed are prevalent in cracked rotor. Wang et al [6] have investigated on the instability due to transverse crack in the anisotropic rotor bearing system. Hu et al [7] have experimentally studied the characteristics of rotor with rub and crack using intrawave frequency modulation. Vashisht and Peng [8] used switching control strategy and short time Fourier transform for detection of a crack in the rotor supported on ball bearing. Ren et al [9] have presented cracked rotor...
response analysis with uncertain parameters with probabilistic based rotor parameters. Wei et al [10] investigated the non-uniform shaft under transverse crack using transfer matrix method. Peng and He [11] have studied the effect of crack location on the whirl motion of cracked rotor along with stability analysis.

From the literature it is found that most of the research on cracked rotor is considered for the simply supported shaft end conditions. However, rotors supported on fluid film bearings is not considered. Fluid film bearings is considered as a most common bearing in turbomachinery due to its salient features. Therefore, cracked rotor response analysis with fluid film bearings is essential. In this paper, a comparative study of cracked rotor responses for simply supported end conditions and fluid film bearings is presented. A 2-dof, Jeffcott rotor model is used. Cracked Shaft is represented by switching crack model. A fluid film bearing in the rotor is modelled using short bearing approach (Ocvirk theory). Rotor responses are obtained for both the conditions and comparison is presented.

2. Mathematical Model

A 2 dof Jeffcott rotor is considered. In order to represent the transverse crack in the shaft a switching crack model given by Gash [2] is used. Two types of boundary conditions for the rotor as shown in Figure 1 (a) and (b) are presented.

![Figure 1. (a) Simply supported end conditions (b) Fluid film bearings end support](image)

Equation of motion of cracked rotor system with simply supported end conditions using switching crack model can be written as-

\[
m\ddot{Y} + c\dot{Y} + [k_0 - \frac{1}{2}\phi(2k_0 - \dot{k}_\xi - \dot{k}_\eta + (\dot{k}_\eta - \dot{k}_\xi)\cos 2\theta)]Y - \frac{1}{2}\phi([\dot{k}_\eta - \dot{k}_\xi]\sin 2\theta) Z = mu_0\omega^2\cos(\theta) - mg
\]

\[
m\ddot{Z} + c\dot{Z} + [k_0 - \frac{1}{2}\phi(2k_0 - \dot{k}_\xi - \dot{k}_\eta - (\dot{k}_\eta - \dot{k}_\xi)\cos 2\theta)]Z - \frac{1}{2}\phi([\dot{k}_\eta - \dot{k}_\xi]\sin 2\theta) Y = mu_0\omega^2\sin(\theta)
\]  

(1)

In the above equation m is the rotor mass, c is shaft damping coefficient, ε is unbalance eccentricity, ω is angular speed of rotor, Y and Z are vertical and horizontal degree of freedom of rotor, k_0 is uncracked rotor stiffness, φ is switching function, \( \dot{k}_\xi \) and \( \dot{k}_\eta \) are stiffnesses in rotational coordinate system when crack is fully open, \( \theta \) is angular displacement of rotor.

Similarly, equation of motion of cracked rotor system with switching crack model supported on fluid film bearing can be written as
\[ mY'' + cY + [k_0 - \frac{1}{2}\phi\{2k_0 - \hat{k}_\xi - \hat{k}_\eta + (\hat{k}_\eta - \hat{k}_\xi)\cos 2\theta\}]Y - \frac{1}{2}\phi\{(\hat{k}_\eta - \hat{k}_\xi)\sin 2\theta\}Z = F_{by} + mu_m\omega^2\cos(\theta) - mg \]

\[ mZ'' + cZ + [k_0 - \frac{1}{2}\phi\{2k_0 - \hat{k}_\xi - \hat{k}_\eta + (\hat{k}_\eta - \hat{k}_\xi)\cos 2\theta\}]Z - \frac{1}{2}\phi\{(\hat{k}_\eta - \hat{k}_\xi)\sin 2\theta\}Y = F_{bz} + mu_m\omega^2\sin(\theta) \]  

where, \( F_{by} \) and \( F_{bz} \) are bearing reaction forces and given as follows

\[
\begin{pmatrix}
F_{by} \\
F_{bz}
\end{pmatrix}
= - \begin{bmatrix}
K_{YY} & K_{YZ} \\
K_{ZY} & K_{ZZ}
\end{bmatrix}
\begin{pmatrix}
Y \\
Z
\end{pmatrix}
- \begin{bmatrix}
C_{YY} & C_{YZ} \\
C_{ZY} & C_{ZZ}
\end{bmatrix}
\begin{pmatrix}
\dot{Y} \\
\dot{Z}
\end{pmatrix}
\]  

(3)

In the equation (3) bearing reaction forces are function of stiffness and damping coefficients. The stiffness and damping coefficients are obtained by using short bearing Ocvirk theory [12].

The Ocvirk number can be written as,

\[
S_S = \frac{D\omega^2L^3}{8fc_0^2}
\]  

(4)

where, \( D \) is bearing diameter, \( \omega \) is angular speed, \( L \) is bearing length, \( f \) is the load on journal, \( c_0 \) is the bearing radial clearance, \( \nu \) is oil viscosity. In order to find the eccentricity ratio for given load and bearing characteristics Ocvirk have given following polynomial,

\[
e^8 - 4e^6 + [6 - S_S^2(16 - \pi^2)]e^4 - 4 + \pi^2S_S^2 + 1 = 0
\]  

(5)

For the above equation, there will be only two real roots and only one is between 0 & 1. The stiffness and damping coefficients can be obtained using following equations

\[
K_{ZZ} = 4h_o\left[\pi^2(2\epsilon^2 + 16\epsilon^2)\right]
\]

\[
K_{ZY} = h_o\frac{\pi\left[\pi^2(1-\epsilon^2)^2 - 16\epsilon^4\right]}{\epsilon\sqrt{1-\epsilon^2}}
\]

\[
K_{YZ} = -h_o\frac{\pi\left[\pi^2(1-\epsilon^2)(1+2\epsilon^2) + 32\epsilon^2(1+\epsilon^2)\right]}{\epsilon\sqrt{1-\epsilon^2}}
\]

\[
K_{YY} = 4h_o\left[\pi^2(1+2\epsilon^2) + 32\epsilon^2(1+\epsilon^2)\right]
\]

\[
K_{ZZ} = h_o\frac{2\pi\sqrt{1-\epsilon^2}(\pi^2(1+2\epsilon^2) - 16\epsilon^2)}{\epsilon}
\]

\[
C_{ZZ} = C_{ZY} = -8h_o\left[\pi^2(1+2\epsilon^2) - 16\epsilon^2\right]
\]

\[
C_{YY} = h_o\frac{2\pi\left[\pi^2(1-\epsilon^2)^2 + 48\epsilon^2\right]}{\epsilon\sqrt{1-\epsilon^2}}
\]

\[
C_{ZZ} = C_{ZY} = -8h_o\left[\pi^2(1+2\epsilon^2) - 16\epsilon^2\right]
\]

(6)

Where, \( h_o = \frac{1}{\left[\pi^2(1-\epsilon^2)^2 + 16\epsilon^2\right]^{1/2}} \)

The dimensional stiffness and damping coefficients can be obtained as follows,

\[
K_e = \frac{f}{c_0}\begin{bmatrix}
K_{ZZ} & K_{ZY} \\
K_{YZ} & K_{YY}
\end{bmatrix};
C_e = \frac{f}{c_0\omega}\begin{bmatrix}
C_{ZZ} & C_{ZY} \\
C_{ZY} & C_{YY}
\end{bmatrix}
\]  

(7)
Table 1. Physical Parameters of rotor bearing system

| Parameters                      | Values               |
|------|----------------------|
| Young’s Modulus, E              | $2 \times 10^{11}$ N/m² |
| Mass of Disc, m                 | 6 kg                 |
| Diameter of shaft, d            | 0.025 m              |
| Length of shaft, l              | 0.7 m                |
| Unbalance Eccentricity, $u_{un}$ | $1.0597 \times 10^{-5}$ m |
| Damping coefficient, c          | 182.56 Ns/m          |
| Bearing length, L               | 0.01 m               |
| Oil viscosity, $\nu$            | 0.1 Ns/m²            |
| Bearing load, $f$               | 58.8 N               |
| Bearing radial clearance, $c_0$ | $173.5 \times 10^{-6}$ m |

3. Validation of Mathematical Model

Equation (1) and (2) obtained are solved using Runge-Kutta fourth order numerical integration method. For this a MATLAB code is written. The specification of rotor bearing system is given in Table 1. Cracked rotor response for simply supported end conditions is obtained as shown in Figure (2) and validated with Tejas et al [13]. It is found that results are in good agreement.

![Figure 2. (a) Orbit plot, Present study (b) Orbit plot, Tejas et al. [13] for crack depth ratio 0.15 operating at 1/2nd of natural frequency](image)

4. Results and Discussion

In this section cracked rotor response analysis with simply supported shaft end conditions and supported with fluid film bearings is presented. Rotor horizontal, vertical responses, FFT’s and orbit plots are used for the analysis.

4.1. For simply supported end conditions

As discussed in section 2, Equation (1) is solved and responses are obtained. Rotor responses for crack depth ratio 0.15 operating at 1/5th of $\omega_n$ is shown in Figure 3. The bending natural frequency of rotor is 48.07 Hz. The vibration amplitude in vertical direction is greater than that of horizontal direction. This may be due to the gravity load prevalent in the vertical direction. FFT plot shows that 1X frequency component is greater than 2X, 3X and 5X frequency components. 2X frequency components in both horizontal and vertical response has significant vibration amplitude. In addition, super synchronous
frequency components such as 3X and 5X are also present in horizontal direction response. Results shows that vibration response in horizontal direction is more sensitive than that of vertical direction response. This may be due to the inertia force in vertical direction. Orbit plot shows a complex orbit with multiple knots.

Similar analysis is done for crack depth ratio of 0.44 and results are shown in Figure (4). It is observed that the rotor responses are highly nonlinear in both horizontal and vertical direction. This may be due to higher crack depth ratio which leads to more variation in stiffness of the shaft. The FFT plot shows 1X, 2X, 3X, 4X, 5X, 6X, 7X frequency components in both X and Y direction. It is found that 1X, 2X and 5X frequency components are prevalent. As compared to the FFT plot obtained for crack depth ratio 0.15, additional frequency component at 4X for crack depth ratio of 0.44 is observed. In both the cases, frequency component 1X, 2X and 5X shows significant vibration amplitude and can be used to detect the crack in the rotor system. The orbit plot shows complex shape with multiple petals and knots.

Figure 3. (a) Vertical Response (b) Horizontal Response (c) Vertical response FFT (d) Horizontal response FFT (e) Orbit plot for crack depth ratio of 0.15 operating at 1/5th of natural frequency
Figure 4. (a) Vertical Response (b) Horizontal Response (c) Vertical response FFT (d) Horizontal response FFT (e) Orbit plot for crack depth ratio of 0.44 operating at 1/5th of natural frequency

4.2. For fluid film bearings
As discussed in section 2, Equation (2) is solved with fluid film bearing stiffness and damping coefficients and responses are obtained. Cracked rotor response with fluid film bearing for crack depth ratio 0.15 operating at 1/5th of $\omega_n$ is shown in Figure (5). The response in both vertical and horizontal direction is influenced by cracked rotor stiffness variation and fluid film reaction forces. The FFT plot shows 1X, 2X, 3X and 5X frequency. However, as crack depth increases it is found that the nonlinear
behavior of cracked rotor becomes significant. Figure (6) shows cracked rotor responses at crack depth ratio of 0.44. As compared to the responses obtained at crack depth ratio of 0.15, responses show strong nonlinear behavior of cracked shaft with fluid film bearings. FFT plot shows 1X, 2X, 3X, 4X, 5X, 6X, 7X, 8X, 9X frequency components. It is observed that 1X, 2X, 3X, 5X and 7X frequency component are dominant. It is found that as compared to simply supported end conditions 8X, 9X, 10X frequency component are present in case of rotor supported with fluid film bearings. The orbit plot shows a simple shape with one knot and two petals.

Figure 5. (a) Vertical Response (b) Horizontal Response (c) Vertical response FFT (d) Horizontal response FFT (e) Orbit plot for crack depth ratio of 0.15 operating at 1/5th of natural frequency
Figure 6. (a) Vertical Response (b) Horizontal Response (c) Vertical response FFT (d) Horizontal response FFT (e) Orbit plot for crack depth ratio of 0.44 operating at $1/5^{th}$ of natural frequency
In this paper cracked rotor responses for two shaft end conditions viz simply supported end conditions and fluid film bearings are considered. The simply supported end condition is purely theoretical boundary condition. However fluid film bearing shaft end conditions is more realistic. Therefore, the cracked rotor responses with fluid film bearing end condition are more correct. In the literature, the simply supported shaft end condition is approximated as anti-frictional bearing shaft end condition.

5. Conclusion
A comparative study of cracked rotor response for simply supported end conditions and fluid film bearings is presented.
- For simply supported end conditions, cracked rotor shows 1X,2X,3X,4X and 5X frequency components to be significant in horizontal direction. For lower crack depth ratios 4X frequency components is absent. The orbit plot shows a complex shape with multiple petals and knots.
- For fluid film bearings, cracked rotor shows 1X,2X,3X,5X and 7X frequency components to be significant in horizontal direction. For lower crack depth ratios higher order frequency components such as 4X,6X,8X,9X are absent. The orbit plot shows simple shape with one knot and two petals.
- The results can be useful for detecting crack in the rotor bearing system.

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