Small-x Structure Functions and QCD Pomeron

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The recent progress in color BFKL-Regge phenomenology of small-\(x\) DIS on nucleons, pions and photons is reviewed.

As noticed by Fadin, Kuraev and Lipatov\textsuperscript{1} and discussed in detail by Lipatov\textsuperscript{2}, the incorporation of asymptotic freedom (AF), i.e. the running QCD coupling, into the BFKL equation makes the QCD pomeron a series of moving Regge poles. The trajectories of the running BFKL poles have been calculated by us and B.G. Zakharov in the color dipole (CD) approach to running BFKL equation\textsuperscript{3,4}. The contribution of each pole to scattering amplitudes satisfies the seasoned Regge factorization which is a basis of our BFKL-Regge expansion for small-\(x\) DIS in the CD basis\textsuperscript{3,4}. We review here recent applications of the BFKL-Regge factorization to DIS off pions and photons and the charm structure function of the proton. More details and references to early works on the subject are found in\textsuperscript{5,6,7}.

In the CD basis the beam-target interaction is viewed as a scattering of color dipoles \(r\) and \(r'\) in both the beam \((b)\) and target \((t)\) particles. Once the beam and target independent dipole-dipole cross section \(\sigma(x, r, r')\) is known one can calculate \(\sigma^{bt}(x)\) making use of the CD factorization

\[
\sigma^{bt}(x) = \int dz d^2rd^2r' |\Psi_b(z, r)|^2 |\Psi_t(z', r')|^2 \sigma(x, r, r'),
\]

where \(|\Psi_b(z, r)|^2\) and \(|\Psi_t(z', r')|^2\) are probabilities to find a CD, \(r\) and \(r'\) in the beam and target, respectively. Here we emphasize that all the beam and target dependence is contained in the CD distributions \(|\Psi_b(z, r)|^2\) and \(|\Psi_t(z, r)|^2\). The CD BFKL-Regge factorization uniquely prescribes an expansion of the dipole-dipole total cross section \(\sigma(x, r, r')\) in eigen-functions \(\sigma_m(r)\) of the CD BFKL equation

\[
\sigma(x, r, r') = \sum_m C_m \sigma_m(r) \sigma_m(r') \left(\frac{x_0}{x}\right)^{\Delta_m}.
\]

The AF exacerbates the well known infrared sensitivity of the CD BFKL equation and infrared regularization is called upon: infrared freezing of \(\alpha_S\) and...
finite propagation radius $R_c$ of perturbative gluons were consistently used in our CD approach to BFKL equation since 1994. The past years the both concepts have become widely accepted.

The leading eigen-function $\sigma_0(r)$ for the ground state with intercept $\Delta_0 = \Delta_{\text{IP}}$ is node free, subleading $\sigma_m(r)$ have $m$ radial nodes. With our infrared regulator the intercept of the leading pole trajectory is found to be $\Delta_{\text{IP}} = 0.4$ and $\Delta_m$ are found to follow closely the Lipatov’s law $\Delta_m = \Delta_0/(m + 1)$. For our infrared regulator the first node of $\sigma_1(r)$ is located at $r = r_1 \simeq 0.05 - 0.1 \text{ fm}$, for solutions with $m \geq 4$ the higher nodes are located at a very small $r$ way beyond the resolution scale $1/\sqrt{Q^2}$. In practical evaluation of $\sigma^{bl}$ we can truncate expansion (2) at $m = 3$ lumping in the term with $m = 3$ contributions of all singularities with $m \geq 3$.

The expansion coefficients $C_m$ in eq.(2) are fully determined by the boundary condition $\sigma(x_0, r, r')$. The very ambitious program of description of $F_2(x, Q^2)$ starting from the, perhaps excessively restrictive, but appealingly natural, two-gluon exchange boundary condition at $x_0 = 0.03$ has been launched by us in and met with remarkable phenomenological success.

Figure 1: Predictions from the CD BFKL-Regge factorization for the pion structure function $F_2^\pi(x_{Bj}, Q^2)$ (solid lines) vs. the experimental data from the H1 are shown. The different components of $F_2^\pi(x_{Bj}, Q^2)$ are shown: the valence contribution (dashed-dotted), the non-perturbative soft contribution (dashed), the Leading Hard + Soft + Valence Approximation $F_2^{\text{LHSVA}}(x_{Bj}, Q^2)$ (dotted) and the sum of the soft and leading hard pole contributions without valence component (long dashed).

The exchange by perturbative gluons is a dominant mechanism for small dipoles $r \lesssim R_c$, interaction of large dipoles is modeled by the non-perturbative, soft mechanism which we approximate here by a factorizable soft pomeron with intercept $\alpha_{\text{soft}}(0) - 1 = \Delta_{\text{soft}} = 0$, i.e., flat vs. $x$ at small $x$. Then the extra term $C_{\text{soft}} \sigma_{\text{soft}}(r)\sigma_{\text{soft}}(r')$ must be added in the r.h.s. of expansion (3). At moderately small $x$ we include a contribution from DIS on valence quarks.
It is convenient to introduce the eigen structure functions \((m = \text{soft}, 0, 1, 2, \ldots)\)

\[
f_m(Q^2) = \frac{Q^2}{4\pi^2\alpha_{em}} \sigma_m^*(Q^2),
\]

where \(\sigma_m^*(Q^2) = \langle \gamma^* | \sigma_m(r) | \gamma^* \rangle\). Then \((m = \text{soft}, 0, 1, 2, \ldots)\)

\[
\sigma^{\gamma^*}(x, Q^2) = \sum_m C_m \sigma_m^*(Q^2) \sigma^t_m \left( \frac{x_0}{x} \right)^{\Delta_m} + \sigma_{\text{val}}^{\gamma^*}(x, Q^2),
\]

\[
F_{2t}(x, Q^2) = \sum_m A_m f_m(Q^2) \left( \frac{x_0}{x} \right)^{\Delta_m} + F_{\text{val}}^{2t}(x, Q^2),
\]

where the target \((t = p, \pi, \gamma, \gamma^* \ldots)\) dependence comes exclusively from \(\sigma_m^* = \langle t | \sigma_m(r) | t \rangle = \int dz d^2r |\Psi_t(z, r)|^2 \sigma_m(r)\).

The CD BFKL approach predicts uniquely that for light flavours subleading eigen structure functions \(f_{m \geq 1}(Q^2)\) have their first node at \(Q^2 \sim 20 - 60\) GeV\(^2\). In this range of \(Q^2\) the proton, pion and real photon structure functions are well reproduced by the Leading Hard+ Soft+Valence Approximation (LHSVA) which gives a unique handle on the intercept \(\alpha_{IP}(0) = 1 + \Delta_0\) of the leading hard BFKL pole. This point is illustrated in fig. 1 where the dotted curve represents the LHSVA for the pion structure function and is hardly distinguishable from the solid curve for the full fledged BFKL-Regge expansion.

The numerical results for pion can be well approximated by \(F_{2\pi}(x, Q^2) \simeq \frac{2}{3} F_{2p} \left( \frac{x_0}{x}, Q^2 \right)\). This additive quark rule derives for the hard component from \(R_c^2 \ll \langle r_0^2 \rangle, \langle r_\gamma^2 \rangle\), whereas for the soft component it derives from approximate
equality of the quark-quark separation proton and quark-antiquark separation in the pion. The agreement with the recent H1 data \cite{9} is very good.

In CD approach open charm is excited from $c\bar{c}$ color dipoles of small size,

$$\frac{4}{Q^2 + 4m_q^2} \lesssim r^2 \lesssim \frac{1}{m_q^2}$$

which for a broad range of $Q^2 \lesssim 100 \text{ GeV}^2$ is close to the position of the first node of subleading $\sigma_m(r)$. Because the soft contribution to charm production is negligible small, this entails the dominance by rightmost hard BFKL pole, see fig. 2. Here the l.h.s box shows the ratio of subleading-to-rightmost BFKL pole. Recall that this point about charm excitation being a clean probe of the hard most BFKL pole has been made by us and B.G.Zakharov already in 1994 \cite{8}. The found nice agreement with the experimental data from ZEUS Collaboration \cite{10} on the charm structure function of the proton (fig. 3) and open charm photoproduction \cite{11} (fig. 2) strongly corroborates our 1994 prediction $\Delta_{\mathbf{IP}} = \alpha_{\mathbf{IP}}(0) - 1 \approx 0.4$ for the intercept of the rightmost hard BFKL pole.

The virtuality $P^2$ of the ‘target’ photon in $\gamma^*(Q^2)\gamma^*(P^2)$ scattering offers still further tests of BFKL-Regge factorization, which gives the parameter free prediction ($m = \text{soft, 0, 1, 2, ...}$)

$$\sigma_{\gamma^*\gamma^*}(x, Q^2, P^2) =$$

$$\frac{(4\pi^2 \alpha_{em})^2}{Q^2 P^2} \sum_m C_m f_m(Q^2) f_m(P^2) \left( \frac{3x_0}{2x} \right)^{\Delta_m} + \sigma_{\gamma^*\gamma^*_{\text{val}}}(x, Q^2, P^2).$$

The quasi-valence (reggeon) correction is important at not so small $x$.  

Figure 3: Prediction from CD BFKL-Regge factorization for the charm structure function of the proton $F_{cc}^2(x_BJ, Q^2)$ vs. the experimental data from ZEUS \cite{10}. The legend of curves is the same as in fig. 1.
The CD BFKL approach predicts uniquely that that because of the node effect at $Q^2 \sim 20$ GeV$^2$ in which region of $Q^2$ the rightmost hard BFKL pole contribution will dominate. Recently the L3 collaboration reported the first experimental evaluation of the vacuum exchange in equal virtuality $\gamma^*\gamma^*$ scattering. Their procedure of subtraction of the non-vacuum reggeon and/or the Quark Parton Model contribution is described in References 12, arguably the subtraction uncertainties are marginal within the present error bars. In fig. 4 we compare our predictions to the L3 data shown vs. the variable $Y = \log(W^2/\sqrt{Q^2P^2})$. The agreement of our estimates with the experiment is good, the contribution of subleading hard BFKL exchange is negligible within the experimental error bars.

Our predictions for the photon structure function are parameter-free and are presented in fig. 5 in comparison with the recent L3 and OPAL data.
A comparison of the solid and dotted curves shows clearly that subleading hard BFKL exchanges are numerically small in the experimentally interesting region of $Q^2$, the rightmost hard BFKL pole exhausts the hard vacuum contribution for $2 \lesssim Q^2 \lesssim 100$ GeV$^2$.

**Conclusions:** The BFKL-Regge factorization has lead to a remarkable progress in relating parameter-free to each other structure functions of various targets. The understanding of nodal properties of eigenfunctions of CD BFKL equation has shed a light on when and why the hard contribution to structure functions is dominated by the contribution from the rightmost hard BFKL pole. There is a mounting evidence for the intercept of the rightmost hard BFKL pole $\Delta P = 0.4$ as predicted by us in 1994.

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