Research on Visual Control System of Inverted Pendulum Based on Pixel Displacement

Bing Liu*, Bin Zhan, Changda Zhang, Lisi Yang

School of Mechatronic Engineering and Automation, Shanghai University, 200072

*Corresponding author e-mail: liu3371472@shu.edu.cn.

Abstract: In inverted pendulum visual servo control system (IPVSCSs), how to achieve the fast and effective measurement of the state information of the inverted pendulum is an important task. In this paper, the characteristics of the inverted pendulum are considered and a series of image processing and positioning algorithm of the pendulum are proposed. Furthermore, the inverted pendulum control system model based on the pixel displacement is established and the LQR controller is designed. Finally, the feasibility and superiority of the proposed method are verified by real-time control experiments.

1. Introduction

The inverted pendulum system (IPS) is a typical unstable and nonlinear system, which is usually used as an ideal experimental platform to carry out control theory research and teaching [1-2]. In the traditional IPS, encoder is usually used to measure the state information of the cart and the pendulum [3-4]. With the rapid development of visual sensing technology [5], visual detection has been gradually applied to complex industrial control field, e.g., industrial production and medical, but the processing and calculation of visual image will bring adverse factors such as image processing and calculation time, calculation error, etc., which will inevitably affect the performance of control system. In order to study and analyze the impact of visual sensor on the performance of control system, scholars use visual sensor instead of traditional encoder to build the inverted pendulum visual servo control system (IPVSCS) [6].

At present, many methods, such as the template matching and inverse trigonometric function [7-8], are used to obtain the state information of the cart and the pendulum in the IPS. However, those methods can not directly reflect the relationship between the image information and the performance of the control system. Therefore, how to establish the quantitative relationship between the image pixels of the inverted pendulum and the state information of the inverted pendulum to realize the visual servo stability control of the IPS is an open problem.

In this paper, the quantitative relationship between the pixel displacement of inverted pendulum images and the actual physical displacement is described based on the principle of camera imaging, and the control model of the IPVSCS based on pixel displacement is established to realize the stable control of IPS.

2. Experimental platform of the IPVSCS

This section describes the framework of the IPVSCS and a series of image processing methods for obtaining the state information of inverted pendulum, i.e., the cart position and the pendulum angle.
2.1. The framework of the IPVSCS
The framework of the IPVSCS is shown in figure 1. It mainly composed of inverted pendulum (including a cart, a swing rod and a slide), visual sensing device (including an industrial camera and a light source), image processing unit (including a PC and motion control card), electric control box (including controller and servo driver) and servo motor.

In the IPVSCS, the inverted pendulum image which is captured by the industrial camera is first sent to the image processing unit. Then, the state information of the pendulum and the cart are obtained through a series of image processing algorithms (containing select image area, edge detection, feature points locating and calculate state information). Furthermore, the motion control card transforms the status information into the corresponding pulse control quantity to drive the servo motor to achieve stable control of the IPVSCS.

![Diagram of IPVSCS framework](image)

**Figure 1. Framework of the IPVSCS.**

Due to the encoder is replaced by visual sensor, the state information of the cart and pendulum is not able to acquired directly through the inverted pendulum image, so it is necessary to process and locate the image of inverted pendulum, and then calculate the state information of the cart and the pendulum. However, due to the inverted pendulum image has a large amount of data, it will take a long time to process the whole image, which not only affects the selection of control cycle but also affects the system control performance.

In order to solve the above problems, the characteristics of the inverted pendulum are considered and the image processing and positioning algorithm for the inverted pendulum is proposed. Then the quantitative relationship between the image pixel displacement and the physical displacement of the inverted pendulum is given, and the calculation time is statistically analyzed.

2.2. Visual image processing and positioning algorithm
In order to effectively calculate the state information of the inverted pendulum, i.e., the cart position and the pendulum angle, a series of image processing methods and positioning algorithm for the inverted pendulum images are proposed. Firstly, the regional image containing cart motion areas and pendulum motion areas (figure 2a) is selected from the inverted pendulum to reduce the delay of image processing. Then, the edge of the cart motion area and the pendulum motion area is extracted through Canny detector. The flow chart of image processing and positioning algorithm for inverted pendulum is shown in figure 2.
For the inverted pendulum image after edge detection, we need to use the feature point positioning algorithm to acquire the state information of the cart and the pendulum rod. Taking the first pixel in the upper left corner of the image as the origin \(O\), the image row index value as the \(y\)-axis, and the column index value as the \(x\)-axis, the feature points \(P_1\) and \(P_2\) are set respectively in cart motion areas and pendulum motion areas of the inverted pendulum image, and then the abscissa and ordinate values of the feature points \(P_1\) and \(P_2\) are obtained respectively by the row scanning method.

\[
x_{\text{pf}} = \frac{d}{p}
\]

Among (1), \(f\) and \(d\) are respectively the distance between the focal length of the camera and the imaging plane of the camera to the inverted pendulum plane. \(p\) and \(x\) are respectively pixel displacement and physical displacement.
In the process of stability control, the driving force exerted by the motor on the cart is to keep the pendulum stick in the stable state of vertical upward, so we pay more attention to the dynamic changes of the pendulum stick near the vertical direction. In figure 5, the linearization error of the deflection angle of the inverted pendulum rod is shown.

In figure 5, it can be seen when the pendulum angle is small, e.g., $\theta \in [-10^\circ, 10^\circ]$, tan$\theta$ can be approximately linearized into $\theta$, and the following formula can be obtained:

$$\theta = \arctan \frac{p_2 - p_1}{\Delta} \approx \frac{p_2 - p_1}{\Delta}$$

(2)

3. The inverted pendulum control system model based on pixel displacement

Traditional models of the IPS, e.g., acceleration input model and force input model [9], all use Newton mechanics analysis method to model the IPS. The state-space equation is as follows:

$$
\begin{bmatrix}
\dot{x} \\
\dot{\theta} \\
\ddot{\theta}
\end{bmatrix}
=
\begin{bmatrix}
0 & 0 & 1 & 0 & x \\
0 & 0 & 0 & 1 & \theta \\
0 & 0 & 0 & 0 & \dot{\theta}
\end{bmatrix}
+\begin{bmatrix}
0 \\
0 \\
\frac{ml}{J}
\end{bmatrix}
\begin{bmatrix}
u
\end{bmatrix}
$$

(3)

Among (3), $u$ is the input signal of acceleration and $x$ is the cart position and $\theta$ is the pendulum angle, and the specific values of other parameters are shown in table 1.

| Parameter | Value |
|-----------|-------|
| $l$       | 0.025m|
| $m$       | 0.109kg|
| $J$       | 0.009kg.m$^2$|
| $g$       | 9.81m/s$^2$|

Furthermore, (1) and (2) are substituted into (3) to obtain the state space equation of the inverted pendulum control system based on pixel displacement:
\[
\begin{bmatrix}
\dot{p}_1 \\
\dot{p}_2 \\
\dot{p}_1 \\
\dot{p}_2 \\
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 \\
-\frac{ml}{J} & \frac{ml}{J} & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
p_1 \\
p_2 \\
p_1 \\
p_2 \\
\end{bmatrix} +
\begin{bmatrix}
0 \\
0 \\
1 \\
\frac{ml\Delta d}{J} + 1 \\
\end{bmatrix} u
\]

(4)

Among (4), \( u \) is the input signal of acceleration, \( p_1 \) and \( p_2 \) are pixel displacements of the feature points \( P1 \) and \( P2 \), respectively.

4. Image processing delay analysis and controller design

In order to analyze the advantages of the proposed method in this paper, firstly, real-time statistics are made on the system delay and the results are shown in figure 6. According to [12], when the inverted pendulum can achieve stable control, the upper limit of system delay should be less than or equal to 0.038s. Therefore, the proposed method in this paper can meet the real-time requirements of the IPVSCS.

![Image processing and computational delay](image)

Figure 6. Image processing and computational delay.

Furthermore, \( LQR \) theory [10] is used to calculate the control rate \( K \) required by closed-loop system (4) simulation and real-time control experiment. The weight matrix of \( LQR \) is selected as

\[
Q =
\begin{bmatrix}
50 & 0 & 0 & 0 \\
0 & 230 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}, R = 10
\]

Finally, the control gain matrix \( K \) can be obtained by solving \( Riccati \) [10] equation:

\[
K =
\begin{bmatrix}
-361.89 & 312.39 & -88.38 & 57.17 \\
\end{bmatrix}
\]

5. Real time control experiment analysis

In [11], the state of the cart and pendulum is obtained by using the encoder. In [12], the state information of the inverted pendulum, i.e., the cart position and the pendulum angle, is obtained by using the visual mark. In order to prove the actual effectiveness and superiority of the proposed method, under the premise of LQR theory is adopted in controller design, respectively to different physical models with different status information acquisition in real-time control experiment. The results are shown in figure 7.

In figure 7, it can be seen that the stability of our method is worse than [11], but better than [12]. This is because using the vision sensor to measure the state information will produce the image processing calculation error, which will lead to the system stability performance degradation and produce a certain amplitude of oscillation.
6. Conclusion
In this paper, the visual servo stability control of inverted pendulum based on pixel displacement is studied. The quantitative relationship between the cart position and the pendulum angle and the image pixel is calculated directly to save calculation time. Finally, the feasibility and superiority of the proposed method are compared and analyzed by experiments.

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