Research on CNC machine tools reliability modeling based on weighted least squares method

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Abstract. Reliability test for a certain numerical control machine tool is carried out. Failure data is collected. On this basis, a reliability model is established. Inference is a Weibull distribution. The reliability model is also analyzed. The weighted least squares method is used to estimate the scale parameter alpha and the shape parameter beta of Weibull distribution. Research shows, this method can effectively establish the reliability model.

1. Introduction
The reliability modeling of CNC machine tools can be established by the probability distribution of its failure time [1]. When studying the failure time of a certain type of CNC machine tool, it will be found that there is a certain law in its distribution. Therefore, the reliability model of the CNC machine tool will be established, and the life performance of the machine tool will be mathematically expressed, and then some interesting performance parameters can be solved mathematically. Finally, the reliability level positioning and evaluation of this type of CNC machine tool can be performed. It has important guiding significance for the engineering management and application of CNC machine tools [2].

2. Establish the distribution model of CNC machine tool failure time
We checked the breakdown maintenance record of a certain type of CNC machining center which operated in the company. After screening, the man-made fault data was removed, and 52 experimental failure data were obtained. The experimental data were processed by the fixed time truncation method. The experimental data, i.e. interfault interval time, are arranged in order from small to large, and the interfault interval time is divided into 14 equal intervals according to the empirical formula [3]. The data after sorting is shown in Table 1---failure data frequency table.
### Table 1. CNC Fault data frequency table

| No. | \( t_i / h \) | \( t_r / h \) | \( t_m / h \) | \( m_r(i) / \text{time} \) | \( n_i / \text{time} \) | freq | grand total |
|-----|----------------|----------------|----------------|----------------|----------------|-----|-------------|
| 1   | 64.6401        | 337.6538       | 201.1469       | 14             | 14             | 0.2692 | 0.2692      |
| 2   | 337.6538       | 610.6675       | 474.1606       | 24             | 10             | 0.1923 | 0.4615      |
| 3   | 610.6675       | 883.6812       | 747.1743       | 34             | 10             | 0.1923 | 0.6538      |
| 4   | 883.6812       | 1156.6949      | 1020.188       | 39             | 5              | 0.0962 | 0.75        |
| 5   | 1156.6949      | 1429.7086      | 1293.202       | 41             | 2              | 0.0385 | 0.7885      |
| 6   | 1429.7086      | 1702.7223      | 1566.215       | 43             | 2              | 0.0385 | 0.8269      |
| 7   | 1702.7223      | 1975.7360      | 1839.229       | 46             | 3              | 0.0577 | 0.8846      |
| 8   | 1975.7360      | 2248.7497      | 2112.243       | 46             | 0              | 0      | 0.8846      |
| 9   | 2248.7497      | 2521.7634      | 2385.257       | 49             | 3              | 0.0577 | 0.9423      |
| 10  | 2521.7634      | 2794.7771      | 2658.27        | 50             | 1              | 0.0192 | 0.9615      |
| 11  | 2794.7771      | 3067.7908      | 2931.284       | 50             | 0              | 0      | 0.9615      |
| 12  | 3067.7908      | 3340.8045      | 3204.298       | 50             | 0              | 0      | 0.9615      |
| 13  | 3340.8045      | 3613.8182      | 3477.311       | 51             | 1              | 0.0192 | 0.9808      |
| 14  | 3613.8182      | 3886.8321      | 3750.325       | 52             | 1              | 0.0192 | 1           |

The plane coordinate system is established. The abscissa of the coordinate system is the time median value of each interval, and the ordinate of the coordinate system is the probability density observation value \( \hat{f}(t) \) of each interval. The calculation process of \( \hat{f}(t) \) is as follows:

\[
\hat{f}(t) = \frac{n_i}{n \Delta t_{i}}
\]  

Where \( n_i \): the frequency of failures in each group of fault intervals; 
\( n \): total frequency of early failures, this test is 52 times; 
\( \Delta t_i \): group distance, 273h.

From Table 1, the curve of the probability density function that can be fitted is shown in Figure 1.

It can be seen from Figure 1 that the probability density curve of the time interval of CNC machine tool does not have a single peak shape, and the probability density function \( f(t) \) of the fault interval time monotonously decreases, \( f'(t) < 0 \). Therefore, the distribution function \( F(t) \) should be convex, which may be an exponential distribution or a Weibull distribution [4],[5]. The empirical distribution function \( F(n)(t) = i/n \) is fitted, and the failure time distribution function curve is shown in Figure 2.

![Figure 1. Probability intensity function f (t) curve.](image1)

![Figure 2. Experience distribution function F (t) curve.](image2)

It can be seen from Fig. 2 that the curve trend graph is convex and has no inflection point. It can be seen that the distribution of the fault interval of the CNC machine tool may be an exponential distribution or a Weibull distribution [6]. The exponential distribution is a special case when the shape parameter \( \beta \) is equal to 1 in the Weibull distribution. Therefore, the fault time obeys the Weibull distribution.
For the convenience of research, the original time data is shown in Table 2.

**Table 2.** CNC data preparation of the time between failure

| No. | t/h  | F(t)  | x    | y    | w    |
|-----|------|-------|------|------|------|
| 1   | 64.6401 | 0.0134 | 4.1689 | -4.3089 | 0.0002 |
| 2   | 65.9736 | 0.0324 | 4.1883 | -3.4118 | 0.0005 |
| 3   | 83.9963 | 0.0515 | 4.4308 | -2.9393 | 0.0008 |
| 4   | 97.2660 | 0.0706 | 4.5774 | -2.6142 | 0.0011 |
| 5   | 112.0853 | 0.0897 | 4.7193 | -2.3647 | 0.0015 |
| 6   | 127.7783 | 0.1088 | 4.8503 | -2.1614 | 0.0019 |
| 7   | 172.8452 | 0.1279 | 5.1524 | -1.9892 | 0.0025 |
| 8   | 183.2456 | 0.1469 | 5.2108 | -1.8393 | 0.0032 |
| 9   | 201.0289 | 0.1660 | 5.3034 | -1.7062 | 0.0039 |
| 10  | 223.8819 | 0.1851 | 5.4111 | -1.5862 | 0.0047 |
| 11  | 238.6965 | 0.2042 | 5.4752 | -1.4766 | 0.0055 |
| 12  | 277.1630 | 0.2233 | 5.6246 | -1.3756 | 0.0065 |
| 13  | 286.9000 | 0.2424 | 5.6591 | -1.2817 | 0.0075 |
| 14  | 377.3666 | 0.2615 | 5.9332 | -1.1938 | 0.0088 |
| 15  | 409.8423 | 0.2805 | 6.0158 | -1.1109 | 0.0102 |
| 16  | 429.0539 | 0.2996 | 6.0616 | -1.0325 | 0.0117 |
| 17  | 466.1987 | 0.3187 | 6.1446 | -0.9577 | 0.0134 |
| 18  | 468.3603 | 0.3378 | 6.1492 | -0.8863 | 0.0150 |
| 19  | 494.5240 | 0.3569 | 6.2036 | -0.8178 | 0.0167 |
| 20  | 499.0165 | 0.3760 | 6.2126 | -0.7518 | 0.0185 |
| 21  | 541.4939 | 0.3950 | 6.2943 | -0.6880 | 0.0204 |
| 22  | 554.1994 | 0.4141 | 6.3175 | -0.6262 | 0.0223 |
| 23  | 567.1557 | 0.4332 | 6.3406 | -0.5661 | 0.0243 |
| 24  | 608.1875 | 0.4523 | 6.4105 | -0.5075 | 0.0264 |
| 25  | 641.4046 | 0.4714 | 6.4637 | -0.4502 | 0.0287 |
| 26  | 647.8827 | 0.4905 | 6.4737 | -0.3942 | 0.0310 |
| 27  | 650.1414 | 0.5095 | 6.4772 | -0.3391 | 0.0332 |
| 28  | 714.2768 | 0.5286 | 6.5713 | -0.2849 | 0.0357 |
| 29  | 748.0857 | 0.5477 | 6.6175 | -0.2314 | 0.0384 |
| 30  | 753.4073 | 0.5668 | 6.6246 | -0.1785 | 0.0410 |
| 31  | 776.7845 | 0.5859 | 6.6552 | -0.1260 | 0.0437 |
| 32  | 790.5871 | 0.6050 | 6.6728 | -0.0739 | 0.0465 |
| 33  | 855.1085 | 0.6240 | 6.7512 | -0.0220 | 0.0495 |
| 34  | 857.7482 | 0.6431 | 6.7543 | 0.0299 | 0.0525 |
| 35  | 904.5030 | 0.6622 | 6.8074 | 0.0819 | 0.0556 |
| 36  | 1020.2738 | 0.6813 | 6.9278 | 0.1341 | 0.0592 |
| 37  | 1024.1225 | 0.7004 | 6.9316 | 0.1867 | 0.0628 |
| 38  | 1056.3151 | 0.7195 | 6.9625 | 0.2399 | 0.0665 |
| 39  | 1056.4215 | 0.7385 | 6.9626 | 0.2938 | 0.0702 |
| 40  | 1312.0918 | 0.7576 | 7.1794 | 0.3488 | 0.0748 |
| 41  | 1376.8026 | 0.7767 | 7.2275 | 0.4050 | 0.0796 |
| 42  | 1480.8325 | 0.7958 | 7.3004 | 0.4629 | 0.0848 |
| 43  | 1690.2152 | 0.8149 | 7.4326 | 0.5228 | 0.0907 |
| 44  | 1837.1358 | 0.8340 | 7.5160 | 0.5853 | 0.0971 |
| 45  | 1882.0524 | 0.8531 | 7.5401 | 0.6511 | 0.1037 |
| 46  | 1933.0479 | 0.8721 | 7.5669 | 0.7212 | 0.1105 |
| 47  | 2307.5872 | 0.8912 | 7.7440 | 0.7968 | 0.1186 |
| 48  | 2313.9118 | 0.9103 | 7.7467 | 0.8802 | 0.1267 |
| 49  | 2440.6515 | 0.9294 | 7.8000 | 0.9748 | 0.1352 |
| 50  | 2756.3471 | 0.9485 | 7.9217 | 1.0871 | 0.1449 |
| 51  | 3525.1286 | 0.9676 | 8.1677 | 1.2321 | 0.1572 |
| 52  | 3886.8321 | 0.9866 | 8.2653 | 1.4622 | 0.1708 |
For the two parameter Weibull distribution, the probability distribution function is [6],[7]:

\[ F(t) = 1 - e^{-\left(\frac{t}{\alpha}\right)^\beta} \]  \hspace{1cm} (2)

Linear transformation of equation (2) yields:

The linear transformation of formula (2) can be obtained:

\[ \ln \left[ \ln \left( \frac{1}{1-F(t)} \right) \right] = \beta \ln t - \beta \ln \alpha \]  \hspace{1cm} (3)

Assume that the equation of linear regression is \( y = A + Bx \).

Let \( y = \ln \left[ \ln \left( \frac{1}{1-F(t)} \right) \right] \), \( x = \ln t \).

\[ J(A, B) = \sum_{i=1}^{n} w_i \left( y_i - (A + Bx_i) \right)^2 \]  \hspace{1cm} (4)

Where: \( w_i \) is the weight coefficient, which can be obtained by the following formula:

\[ w_i = \frac{\sum_{j=1}^{n} x_j}{\sum_{j=1}^{n} (y_j - \bar{y})^2} \]  \hspace{1cm} (5)

The \( r \) in the formula (5) is the number of failures occurring during the truncation time \( T \), and \( n \) is the total number of machine tools. Where \( t \) is the recorded equipment failure time. When \( J(\alpha, \beta) \) is the smallest, the estimated quantities \( \hat{\alpha} \) and \( \hat{\beta} \) of the two parameters of the corresponding Weibull distribution can be obtained.

3. Estimate the Parameters

The parameters \( A \) and \( B \) are estimated by the weighted least squares method [8],[9], and the formula is derived from formula (4):

\[ \hat{A} = \frac{\sum_{i=1}^{n} (w_i x_i) \sum_{i=1}^{n} (w_i y_i) \sum_{i=1}^{n} (w_i x_i y_i) - (\sum_{i=1}^{n} (w_i x_i))^2 \sum_{i=1}^{n} (w_i y_i) - (\sum_{i=1}^{n} (w_i y_i))^2 \sum_{i=1}^{n} (w_i x_i)^2}{(\sum_{i=1}^{n} (w_i x_i)^2)^2} \]  \hspace{1cm} (6)

\[ \hat{B} = \frac{\sum_{i=1}^{n} (w_i x_i)^2 \sum_{i=1}^{n} (w_i y_i) - (\sum_{i=1}^{n} (w_i x_i))^2 \sum_{i=1}^{n} (w_i x_i y_i)}{(\sum_{i=1}^{n} (w_i x_i))^2 (\sum_{i=1}^{n} (w_i x_i)^2)} \]  \hspace{1cm} (7)

Calculated by Table 2, we can get \( \hat{A} = -6.92437055 \), \( \hat{B} = 1.010024793 \)

The linear regression equation is \( y = -6.92437055 + 1.010024793x \)

Substituting formula (4), \( J(\hat{A}, \hat{B}) = 0.018253037 \)

Therefore, \( \hat{\beta} = 1.010024793 \), \( \alpha = e^{\frac{\hat{A}}{\hat{B}}} = 949.2233608 \)

Thus, the distribution function \( F(t) \) and the failure rate function \( \lambda(t) \) of the reliability model of the numerically controlled machine tool are:

\[ F(t) = 1 - \exp \left[ \left( \frac{t}{949.2233} \right)^{1.01} \right] \]

\[ \lambda(t) = \frac{1.0100}{949.2233} \left( \frac{t}{949.2233} \right)^{0.01} \]

If the traditional least squares method and the weighted least squares method are used separately, the parameters are estimated. And the values \( A, B \) and \( J(\hat{A}, \hat{B}) \) can also be obtained. The two method compares the simulation results of MATLAB operation with figure 3. It can be seen from the diagram that the weighted least squares method is superior to the traditional least squares method.
Figure 3. Comparison of two algorithms.

The comparison of the specific values of the two algorithms is shown in Table 3.

| algorithms                      | A             | B             | J (A, B)         |
|---------------------------------|---------------|---------------|-----------------|
| the traditional least squares method | -8.09077047  | 1.176034385   | 2.104828858     |
| the weighted least squares method | -6.92437055  | 1.010024793   | 0.018253037     |

The data in Table 3 are the results of computer operation of the two algorithms. The constraint function \(J(A, B)\) is the best when it is the weighted least squares method.

4. Conclusion

In this paper, the fault time data of a certain CNC machine tool is studied. The reliability model is established. The fault interval is inferred to the Weibull distribution, and the weighted least squares method is used to optimize the target constraint function. The parameters of the Weibull distribution model are obtained. Thereby, the reliability law of the time-varying variation of this kind of CNC machine tool is obtained. This paper provides a new method for reliability modeling and complex nonlinear optimization of model parameters, laying the foundation for reliability growth and maintenance allocation and estimation.

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