Dijet correlations in $pp$ collisions at RHIC

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We compare results of $k_t$-factorization approach and next-to-leading order collinear-factorization approach for dijet correlations in proton-proton collisions at RHIC energies. We discuss correlations in azimuthal angle as well as correlations in two-dimensional space of transverse momenta of two jets. Some $k_t$-factorization subprocesses are included for the first time in the literature. Different unintegrated gluon/parton distributions are used in the $k_t$-factorization approach. The results depend on UGDF/UPDF used. Limitations due to leading jet condition are discussed.
1. Introduction

The jet correlations are interesting in the context of recent detailed studies of hadron-hadron correlations in nucleus-nucleus [1] and proton-proton [2] collisions. Those studies provide interesting information on the dynamics of nuclear and elementary collisions. Effects of geometrical jet structure were discussed recently in Ref. [3]. No QCD calculation of parton radiation was performed up to now in this context. Before going into hadron-hadron correlations it seems indispensable to better understand correlations between jets due to the QCD radiation. In this paper we address the case of elementary hadronic collisions in order to avoid complicated and not yet well understood nuclear effects. Our analysis should be considered as a first step in order to understand the nuclear case in the future. In leading-order collinear-factorization approach jets are produced back-to-back. These leading-order jets are therefore not included into correlation function, although they contribute a big ($\frac{1}{2}$) fraction to the inclusive cross section.

The truly internal momentum distribution of partons in hadrons due to Fermi motion (usually neglected in the literature) and/or any soft emission would lead to a decorrelation from the simple kinematical back-to-back configuration. In the fixed-order collinear approach only next-to-leading order terms lead to nonvanishing cross sections at $\phi \notin \pi$ and/or $p_{1T} \notin p_{2T}$ (moduli of transverse momenta of outgoing partons). In the $k_t$-factorization approach, where transverse momenta of gluons entering the hard process are included explicitly, the decorrelations come naturally in a relatively easy to calculate way. In Fig.1 we show $k_t$-factorization processes discussed up to now in the literature [5, 6, 7]. The soft emissions, not explicit in our calculation, are hidden in model unintegrated gluon distribution functions (UGDF). In our calculation the last objects are assumed to be given and are taken from the literature [8, 9, 10].

In addition we include two new processes (see Fig.2), not discussed up to now in the context of $k_t$-factorization approach. We shall discuss their role at the RHIC energy $W = 200$ GeV.

Furthermore we compare results obtained within the $k_t$-factorization approach and results obtained in the NLO collinear-factorization. Here we wish to address the problem of the relation between both approaches. We shall identify the regions of the phase space where the hard 2 ! 3

![Figure 1: Diagrams for $k_t$-factorization approach included in the literature. We shall call them $A_1$ and $A_2$ for brevity.](image-url)
2. Formalism

The cross section for the production of a pair of partons (k,l) can be written as

\[
\frac{d\sigma}{d^2p_1d^2p_2} = \sum_{i,j,k,l} \frac{z}{16\pi^2} \frac{d^2k_1}{\pi} \frac{d^2k_2}{\pi} \frac{1}{(x_1x_2s)^2} (ij) \delta(k_1 + k_2 - p_1 - p_2) \mathcal{F}_i(x_1k_1^2) \mathcal{F}_j(x_2k_2^2); \tag{2.1}
\]

where

\[
x_1 = \frac{m_1^2}{s} e^{y_1} + \frac{m_2^2}{s} e^{y_2}; \tag{2.2}
\]

\[
x_2 = \frac{m_1^2}{s} e^{y_1} + \frac{m_2^2}{s} e^{y_2}; \tag{2.3}
\]

and \(m_1^2\) and \(m_2^2\) are so-called transverse masses defined as \(m_i^2 = \frac{q_i^2}{s} + m^2\), where \(m\) is the mass of a parton. In the following we shall assume that all partons are massless. The objects denoted by \(\mathcal{F}_i(x_1k_1^2)\) and \(\mathcal{F}_j(x_2k_2^2)\) in the equation (2.1) above are the unintegrated parton distributions in hadron \(h_1\) and \(h_2\), respectively. They are functions of longitudinal momentum fraction and transverse momentum of the incoming (virtual) parton.

After some simple algebra one obtains a handy formula:

\[
\frac{d\sigma}{dp_1dp_2} = \frac{1}{2} \frac{1}{4\pi} \frac{z}{16\pi^2} \frac{d^2k_1}{\pi} \frac{d^2k_2}{\pi} \sum_{i,j,k,l} \frac{1}{16\pi^2(x_1x_2s)^2} (ij) \delta(k_1 + k_2 - p_1 - p_2) \mathcal{F}_i(x_1k_1^2) \mathcal{F}_j(x_2k_2^2); \tag{2.4}
\]
This 5-dimensional integral is now calculated for each point on the map \( p_1, p_2 \).

Up to now we have considered only processes with two explicit hard partons. Now we shall discuss also processes with three explicit hard partons. In Fig. 3 we show a typical \( 2!3 \) process and kinematical variables needed in the description of the process. We select the particle 1 and 2 as those which correlations are studied. This is only formal as all possible combinations are considered in real calculations.

\[ x_2 = \frac{P_{1}^z}{s} \exp(y_1) + \frac{P_{2}^z}{s} \exp(y_2) + \frac{P_{3}^z}{s} \exp(y_3); \]

After a simple algebra [4] we get finally:

\[ d\sigma = \frac{1}{64\pi^4 s^2} x_1 g_1(x_1 \mu^2) x_2 g_2(x_2 \mu^2) \frac{1}{s^2} x_1 p_1 dp_1 x_2 p_2 dp_2 d\phi dy_1 dy_2 dy_3; \]

where \( \phi \) is restricted to the interval \((0,\pi]\). The last formula is very useful in calculating the cross section for particle 1 and particle 2 correlations.

3. Results

Let us concentrate first on \( 2!2 \) processes calculated within \( k_t \)-factorization approach.

In Fig. 4 we show two-dimensional maps in \((p_1, p_2)\) for all \( k_t \)-factorization processes shown in Fig. 1 and Fig. 2. Only very few approaches in the literature include both gluons and quarks and antiquarks. In the calculation above we have used Kwieciński UPDFs with exponential nonperturbative form factor \((b_0 = 1 \text{ GeV}^{-1})\) and the factorization scale \( \mu^2 = (p_{1, \text{min}} + p_{1, \text{max}})^2 = 4 = 100 \text{ GeV}^2 \).

In Fig. 5 we show fractional contributions (individual component to the sum of all four components) of the above four processes on the two-dimensional map \((y_1, y_2)\). One point here requires a...
Figure 4: Two-dimensional distributions in $p_{1T}$ and $p_{2T}$ for different subprocesses $gg \rightarrow gg$ (left upper), $gg \rightarrow q\bar{q}$ (right upper), $gq \rightarrow gq$ (left lower) and $qg \rightarrow qg$ (right lower). In this calculation $W = 200$ GeV and Kwieciński UPDFs with exponential nonperturbative form factor ($b_0 = 1$ GeV$^{-1}$) and $\mu^2 = 100$ GeV$^2$ were used. Here integration over full range of parton rapidities was made.

(better clarification. Experimentally it is not possible to distinguish gluon and quark/antiquark jets. Therefore in our calculation of the $(y_1, y_2)$ dependence one has to symmetrize the cross section (not the amplitude) with respect to gluon – quark/antiquark exchange ($y_1 \rightarrow y_2, y_2 \rightarrow y_1$). While at midrapidities the contribution of diagram $B_1 + B_2$ is comparable to the diagram $A_1$, at larger rapidities the contributions of diagrams of the type B dominate. The contribution of diagram $A_2$ is relatively small in the whole phase space. When calculating the contributions of the diagram $A_1$ and $A_2$ one has to be careful about collinear singularity which leads to a significant enhancement of the cross section at $\phi = 0$ and $y_1 = y_2$, i.e. in the one jet case, when both partons are emitted in the same direction. This is particularly important for the matrix elements obtained by the naive analytic continuation from the formula for on-shell initial partons. The effect can be, however, easily eliminated with the jet-cone separation algorithm [4].

For completeness in Fig. 5 we show azimuthal angle dependence of the cross section for all four components. There is no sizeable difference in the shape of azimuthal distribution for different components.

The Kwieciński approach allows to separate the unknown perturbative effects incorporated via nonperturbative form factors and the genuine effects of QCD evolution. The Kwieciński distribu-
Figure 5: Two-dimensional distributions of fractional contributions of different subprocesses as a function of $y_1$ and $y_2$ for $gg \rightarrow gg$ (left upper), $gg \rightarrow q\bar{q}$ (right upper), $gq \rightarrow gq$ (left lower) and $qg \rightarrow qg$ (right lower). In this calculation $W = 200$ GeV and Kwieciński UPDFs with exponential nonperturbative form factor and $b_0 = 1$ GeV$^{-1}$ were used. The integration is made for jets from the transverse momentum interval: $5$ GeV $< p_{1t}, p_{2t} < 20$ GeV.

The functions have two external parameters:

- the parameter $b_0$ responsible for nonperturbative effect (for details see [4]),
- the evolution scale $\mu^2$ (for details see [4]).

While the latter can be identified physically with characteristic kinematical quantities in the process $\mu^2 = p_{1t}^2, p_{2t}^2$, the first one is of nonperturbative origin and cannot be calculated from first principles. The shapes of distributions depends, however, strongly on the value of the parameter $b_0$ in which the initial momentum distribution is encoded. This is demonstrated in Fig. 7 where we show angular correlations in azimuth for the $gg \rightarrow gg$ subprocess. The smaller $b_0$ the bigger decorrelation in azimuthal angle can be observed. In Fig. 8 we show also the role of the evolution scale in the Kwieciński distributions. The QCD evolution embedded in the Kwieciński evolution equations populate larger transverse momenta of partons entering the hard process. This significantly increases the initial (nonperturbative) decorrelation in azimuth. For transverse momenta of the order of 10 GeV the effect of evolution is of the same order of magnitude as the effect due to the nonperturbative physics of hadron confinement. For larger scales of the order of $\mu^2$. 
Figure 6: The angular correlations for all four components: $gg \to gg$ (solid), $gg \to q\bar{q}$ (dashed) and $gq \to gg$ (dash-dotted). The calculation is performed with the Kwieciński UPDFs and $b_0 = 1$ GeV$^{-1}$. The integration is made for jets from the transverse momentum interval: $5 \text{ GeV} < p_{t1}, p_{t2} < 15 \text{ GeV}$ and from the rapidity interval: $-4 < y_1, y_2 < 4$.

100 GeV$^2$, more adequate for jet production, the initial condition is of minor importance and the effect of decorrelation is dominated by the evolution. Asymptotically (infinite scales) there is no dependence on the initial condition provided reasonable initial conditions are taken.

Figure 7: The azimuthal correlations for the $gg \to gg$ component obtained with the Kwieciński UGDFs for different values of the nonperturbative parameter $b_0$ and for different evolution scales $\mu^2 = 10$ (on line blue), 100 (on line red) GeV$^2$. The initial distributions (without evolution) are shown for reference by black lines.

In Fig.8 we show the maps for different UGDFs and for $gg \to ggg$ processes in the broad range
of transverse momenta $5 \text{ GeV} < p_{1T}, p_{2T} < 20 \text{ GeV}$ for the RHIC energy $W = 200 \text{ GeV}$. In this calculation we have not imposed any particular cuts on rapidities. We have not imposed also any cut on the transverse momentum of the unobserved third jet in the case of $2 \rightarrow 3$ calculation. The small transverse momenta of the third jet contribute to the sharp ridge along the diagonal $p_{1T} = p_{2T}$. Naturally this is therefore very difficult to distinguish these three-parton states from standard two jet events. In principle, the ridge can be eliminated by imposing a cut on the transverse momentum of the third (unobserved) parton [4]. There are also other methods to eliminate the ridge and underlying soft processes which is discussed in Ref. [4].

When calculating dijet correlations in the standard NLO ($2 \rightarrow 3$) approach we have taken all possible dijet combinations. This is different from what is usually taken in experiments [11], where correlation between leading jets are studied. In our notation this means $p_{3T} < p_{1T}$ and $p_{3T} < p_{2T}$. When imposing such extra condition on our NLO calculation we get the dash-dotted curve in Fig. 9. In this case $d\sigma = d\phi = 0$ for $\phi < 2\pi$. This vanishing of the cross section is of purely kinematical origin. Since in the $k_T$-factorization calculation only two jets are explicit, there is no such an effect in this case. This means that the region of $\phi < 2\pi$ should be useful to test models of UGDFs. For completeness in Fig. 10 we show a two-dimensional plot $(p_{1T}, p_{2T})$ with imposing the leading-jet condition. Surprisingly the leading-jet condition removes a big part of the two-dimensional space. In particular, regions with $p_{2T} > 2p_{1T}$ and $p_{1T} > 2p_{2T}$ cannot be populated.
Figure 9: Dijet azimuthal correlations $d\sigma/d\phi$ for the $gg \rightarrow gg$ component and different UGDFs as a function of azimuthal angle between the gluonic jets. In this calculation $W = 200$ GeV and $-1 < y_1, y_2 < 1$, $5 \text{ GeV} < p_{1T}, p_{2T} < 20 \text{ GeV}$. Here the thick-solid line corresponds to the Kwieciński UGDF, the dashed line to the Kharzeev-Levin type of distribution and the dotted line to the BFKL distribution. The two thin solid (on line black) lines are for NLO collinear approach without (upper line) and with (lower line) leading jet restriction.

Figure 10: Cross section for the $gg \rightarrow ggg$ component on the $(p_{1T}, p_{2T})$ plane with the condition of leading jets (partons).

via 2 ! 3 subprocess \(^1\). There are no such limitations for 2 ! 4, 2 ! 5 and even higher-order processes. Therefore measurements in “NLO-forbidden” regions of the $(p_{1T}, p_{2T})$ plane would test higher-order terms of the standard collinear pQCD. These are also regions where UGDFs can be tested, provided that not too big transverse momenta of jets are taken into the correlation in order to assure the dominance of gluon-initiated processes. For larger transverse momenta and/or

\(^1\)In LO collinear approach the whole plane, except of the diagonal $p_{1T} = p_{2T}$, is forbidden.
forward/backward rapidities one has to include also quark/antiquark initiated processes via unintegrated quark/antiquark distributions.

4. Summary

Motivated by the recent experimental results of hadron-hadron correlations at RHIC we have discussed dijet correlations in proton-proton collisions. We have considered and compared results obtained with collinear next-to-leading order approach and leading-order $k_t$-factorization approach.

In comparison to recent works in the framework of $k_t$-factorization approach, we have included two new mechanisms based on $gq!\,gg$ and $qg!\,qg$ hard subprocesses. This was done based on the Kwieciński unintegrated parton distributions. We find that the new terms give significant contribution at RHIC energies. In general, the results of the $k_t$-factorization approach depend on UGDFs/UPDFs used, i.e. on approximation and assumptions made in their derivation.

The results obtained in the standard NLO approach depend significantly whether we consider correlations of any jets or correlations of only leading jets. In the NLO approach one obtains $\frac{d\sigma}{d\phi} = 0$ if $\phi < 2=3\pi$ for leading jets as a result of a kinematical constraint. Similarly $\frac{d\sigma}{dp_1dp_2} = 0$ if $p_{1T} > 2p_{2T}$ or $p_{2T} > 2p_{1T}$.

There is no such a constraint in the $k_t$-factorization approach which gives a nonvanishing cross section at small relative azimuthal angles between leading jets and transverse-momentum asymmetric configurations. We conclude that in these regions the $k_t$-factorization approach is a good and efficient tool for the description of leading-jet correlations. Rather different results are obtained with different UGDFs which opens a possibility to verify them experimentally. Alternatively, the NLO-forbidden configurations can be described only by higher-order (NNLO and higher-order) terms. We do not need to mention that this is a rather difficult and technically involved computation.

What are consequences for particle-particle correlations measured recently at RHIC requires a separate dedicated analysis. Here the so-called leading particles may come both from leading and non-leading jets. This requires taking into account the jet fragmentation process.

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