Abstract—A hybrid encryption scheme is a public key encryption system that consists of a public-key part called the key encapsulation mechanism (KEM), and a (symmetric) secret-key part called data encapsulation mechanism (DEM); the public key part is used to generate a shared secret key between the two parties, and the symmetric key part is used to encrypt the message. Hybrid encryption schemes are widely used for secure communication over the Internet. In this paper we initiate the study of hybrid encryption in preprocessing model which assumes access to initial correlated variables by all the parties (including the eavesdropper). We define information theoretic KEM (iKEM) that together with a (computationally) secure DEM results in a hybrid encryption scheme in preprocessing model. We define security of each building block and prove a composition theorem that guarantees security of the final encryption system. We show that iKEM can be realized by a one-message SKA (OW-SKA) protocol with an extended security definition. Using a OW-SKA that satisfies this extended definition of security effectively allows the secret key that is generated by the OW-SKA to be used with symmetric key encryption system such as AES in counter mode. We discuss our results and future work including providing stronger security for the final encryption, and using information theoretic DEM to construct information theoretic encryption systems.

I. INTRODUCTION

Public key encryption (PKE) schemes are usually defined for restricted message spaces and so the ciphertext can hide a limited number of plaintext bits. A Hybrid encryption scheme consists of a public-key part and a (symmetric) secret-key part. The public key part is called key encapsulation mechanism (KEM), and generates a pair of, (i) a random symmetric key $K$, and (ii) a ciphertext $c$. The symmetric key part uses the generated key $K$ to encrypt the actual data to obtain the corresponding ciphertext $c'$ using an efficient data encapsulation mechanism (DEM) (e.g. such as counter mode of AES [1]). The pair $(c, c')$ allows the decryptor to first recover $K$ from $c$, and then use it to decrypt $c'$ and obtain the data. KEM/DEM paradigm was formalized by Cramer and Shoup [2] and has been widely in Internet protocols to implement public key cryptography in protocols such as TLS [3] and SSH protocols [4] and is incorporated in standards such as [5].

Today’s main constructions of KEM rely on the hardness of two computational problems, Discrete Logarithm (DL) and integer factorization problems, for both of which efficient quantum algorithms have been proposed by Shor [6]. KEMs that remain secure in presence of quantum computers (also called post-quantum secure), have constructed using hard problems in lattice theory [7], [8] or coding theory [9], [10] for which efficient quantum algorithms is not known. These constructions have significantly higher computation and communication cost, and will need updates with the new developments in computing and security technologies (e.g. updating parameters when new algorithms and attacks are found). We note that post-quantum security of hybrid encryption systems is primarily determined by the KEM part as security of symmetric key encryption schemes will not be significantly affected by quantum computers (one need to use longer keys).

In this paper we initiate the study of “KEM/DEM paradigm in preprocessing model”, where the specification of the protocol includes a joint distribution $D$ over $R_1\times\ldots\times R_n$, where $R_i$ is a finite domain. Before the start of the protocol, a (trusted) sampler samples correlated random values $(r_1,\ldots,r_n)$, and delivers $r_i$ to the party $P_i$ before the protocol starts (thus making it independent of the input). The model has been widely studied in cryptography with both positive and negative results on unconditionally secure computation with correlated randomness [11]. In information theoretic security a similar initial setup is considered for key agreement protocols and is referred to as source model [12], [13]: In a two party key agreement in source model, before protocol starts, a trusted sampler samples a public distribution $P(x,y,z)$ and gives the samples of $x$, $y$, and $z$, to Alice, Bob and Eve, respectively. An example of this setting was proposed by Maurer [13] and is known as satellite setting where a satellite broadcasts a random beacon that is received by protocol parties through their (independent) channels. Our main observation is that a one-message two-party key agreement (also called one-way SKA (OW-SKA) [14], [15]) can serve as a KEM in a hybrid encryption scheme and together with DEM, can provide post-quantum security for the encryption system. Our intuition is that using an information theoretically secure KEM will establish a key that will not depend on the computation power of the adversary, and since a DEM component that is implemented using an algorithm such as AES-256 will be safe against quantum computers [16], the combination of the two will provide post-quantum security.

The KEM in this model will use private randomness samples (of correlated random variables) that Alice and Bob hold, to establish a shared key and so the resulting hybrid encryption scheme in preprocessing model is not a public key encryption. In fact, the hybrid encryption system in preprocessing model will be neither public key, nor is symmetric key system that requires a shared secret key before the scheme is used. Rather, it will start with the initial correlated samples of the two parties for secure encryption and decryption of data. The hybrid encryption system will be computationally secure (although it can be extended to information theoretic case (see Discussion...
in Section V). A traditional OW-SKA cannot be directly used as a KEM because of the difference in the security definition of the two.

Overview of our main contributions. We formalize KEM/KEM paradigm in preprocessing model and prove a composition theorem that shows security of the resulting encryption system when appropriate definitions and security notions for KEM and DEM are used. We define information theoretic KEM (iKEM) and its security notions (Definitions 9 and 10) inline with computational KEM and using game-based security definition. In this approach security is defined as the success probability of an adversary in a game against a challenger, and is described by a probabilistic experiment with a well-defined success event. Lemma 2 relates this success probability to the statistical distance based security definition of OW-SKA. Security definition of DEM will be the same as in traditional hybrid encryption schemes and is recalled in Definition 7. We define a hybrid encryption in preprocessing model (Definition II) and its security notions (Definition II) against a computationally bounded adversary, and prove a composition theorem (Theorem II) that shows combining an iKEM and a DEM with appropriate security definitions results in a hybrid encryption system in preprocessing model with provable security with respect to the defined security notion.

As a concrete example of our results, in Section IV we extend the OW-SKA in [17] to provide iKEM security in the sense of Definition 10. By choosing parameters of the iKEM to output a secure 256 bit key, we can use AES-256 in counter model as the required DEM.

Discussion. Hybrid encryption system in preprocessing model and employing OW-SKA as an iKEM to establish the required key for DEM, are novel and have a number of important implications. Firstly, it will allow secure integration of information theoretic secure OW-SKA in encryption systems in practice. Maurer [13] noted the security challenges of using a securely established key in practice. Ben-or et al. [19] had noted that a direct application of an information theoretic secure key that is obtained through a quantum key distribution protocol for encryption may not result in a secure encryption system, and one needs to use stronger definition of security that takes into account composability of the key with other crypto systems. Similar results were also shown by Renner and König [20]. The security notions of iKEM allow composability of the key with computationally secure symmetric key encryption, such that the resulting encryption system satisfies CPA (Chosen Plaintext Attack) security notions of symmetric key encryption (with fixed number of plaintext queries).

Our work can be extended to define stronger security notions for iKEM (e.g. security against chosen ciphertext attack (CCA)), and stronger security notion from the final hybrid encryption in preprocessing model.

Secondly, we noted that hybrid encryption in preprocessing model provides post-quantum security. Our proposed construction of iKEM in source model can be used in wireless settings where a beacon broadcasts randomness. It is well known [12] that secret key agreement with information theoretic security requires initial correlated variables. An interesting direction of future work will be to extend our results to iKEMs using other physical layer assumptions for establishing a shared key using a single message.

Thirdly, hybrid encryption in preprocessing model leads to efficient encryption scheme with post-quantum security and without requiring a secure key agreement protocol. Alternatives to such an encryption would be to use a traditional KEM with post-quantum security, which because of high computation cost will become inaccessible to resource constrained IoT devices such as a smart lock that have long life. Using direct application of a key agreement protocol with post-quantum security will have a similar inefficiency drawback.

Related Works. Cramer and Shoup [2] formalized KEM/DEM paradigm and proved that CCA security of KEM and DEM as a public key and a symmetric key encryption, respectively, leads to CCA security of the final (public key) hybrid encryption system. This is the strongest commonly used security notion for encryption systems. Kurosawa and Desmedt [21] presented a hybrid encryption scheme that shows that CCA-secure hybrid encryption can be achieved using a weaker security notion for KEM (the KEM in their construction was later shown to be not CCA secure [22]). The existence of strongly secure hybrid encryption schemes (CCA secure) from weaker primitives was also studied by Abe et al. [23] and Shacham [24]. The necessary and sufficient conditions on the security of DEM and KEM parts to achieve a desired level of security by the hybrid encryption is studied in [25].

Study of secret key agreement in source model was initiated by Maurer [13] and Ahlswede and Csiszár [12], with many followup work for different physical layer setups. One-way secret key (OW-SK) capacity was introduced by Ahlswede and Csiszár [12], who derived OW-SK capacity. Holenstein and Renner [14] considered one-way SKA (OW-SKA) protocols and gave constructions that achieve OW-SK capacity. There are a number of capacity achieving OW-SKA constructions [26], [27], [28], [17], in some cases [17] with explicit lower bound on finite key length.

Cryptographic protocols that have been studied in pre-processing model include oblivious transfer [29] and multi-party computation (MPC) protocols [30], [11]. The source model in information theoretic key agreements uses a similar initialization phase [31].

Organization. Preliminaries are reviewed in I. Our main contribution is in Section III where we propose the hybrid encryption in preprocessing model to discuss its security. A practical construction of our scheme is given in IV concluding remarks are discussed in V.

II. PRELIMINARIES

A. Notations

We denote random variables (RVs) with upper-case letters, (e.g., X), and their realizations with lower-case letters, (e.g., x). Calligraphic letters denote sets and size of the set ‘X’ is denoted by |X|. \( U_X \) denotes a random variable with uniform distribution over ‘X’ and \( U_y \) denotes a random variable with uniform distribution over \( \{0,1\}^y \).

Functions are denoted with sanserif fonts e.g., f(·). We use the symbol ‘−’ to assign a constant value (on the right-hand side) to a variable (on the left-hand side). Similarly, we use,
\textsuperscript{38}, to assign to a variable either a uniformly sampled value from a set or the output of a randomized algorithm.

The probability mass function (p.m.f) of an RV $X$ is denoted by $P_X$ and $P_X(x) = \Pr(X = x)$. Col$(X)$ is the collision probability of random variable $X \in \mathcal{X}$ is defined as the probability that two independent samples of $X$ are equal. That is $Col(X) = \sum_x \{Pr[X = x]\}^2$.

For two random variables $X$ and $Y$, $P_{XY}$ denotes their joint distribution, and $P_{X|Y}$ denotes their conditional distribution.

The statistical distance between two corresponding RVs $X$ and $Y$ defined over a common alphabet $\mathcal{T}$, is given by

\[
\text{SD}(X; Y) = \max_{W \in \mathcal{T}} (\Pr(X \in W) - \Pr(Y \in W))
\]

where $\Pr(X \in W) = \sum_{x \in X} \Pr(X = t)$.

The min-entropy $H_\infty(X)$ of random variable $X \in \mathcal{X}$ with distribution $P_X$ where $P_X(x) \in [0,1], x \in \mathcal{X}$, is defined by $H_\infty(X) = -\log(\max_x P_X(x))$. The average conditional min-entropy is commonly defined as $\tilde{H}_\infty(X|Y) = -\log \mathbb{E}_{y \in Y} \max_x P_{X|Y}(x|y)$.

Randomness extractors map a random variable with a guaranteed entropy, to a random variable from a smaller set that is statistically close (in terms of the statistical distance) to a uniform random variable. See [33] and references therein for more details. One of the well known constructions for randomness extractors is by using (Strong) Universal Hash Families (UHF) via the so called Leftover Hash Lemma (LHL) [34]. We will use a variation of the LHL, called the generalized LHL [35] later in this paper.

**Definition 1** (Strong Universal Hash Family [36]). A family of functions $\{h_s : \mathcal{X} \rightarrow \mathcal{Y}\}_{s \in \mathcal{S}}$ is a Strong Universal Hash family if for any $x \neq x'$ and any $a, b \in \mathcal{Y}$, $\Pr[h_S(x) = a \wedge h_S(x') = b] = \frac{1}{|\mathcal{Y}|^2}$, where the probability is over the uniform choices over $\mathcal{S}$.

**Lemma 1** (Generalized LHL). For two possibly dependant random variables $A \in \mathcal{X}$ and $B \in \mathcal{Y}$, applying a universal hash function (UHF) $\{h_S : \mathcal{X} \rightarrow \{0,1\}\}_{s \in \mathcal{S}}$ on $A$ can extract a uniformly random variable whose length $\ell$ will be bounded by the average min-entropy of $A$, given $B$, and the required closeness to the uniform distribution. That is

\[
\text{SD}(B, S, (h_S(A)); (B, S, U_\ell)) \leq \frac{1}{2} \sqrt{2\ell - H_\infty(A|B)},
\]

where $S$ is the randomly chosen seed of the hash function family, and the average conditional min-entropy is defined above.

**B. One-way Secret Key Agreement (OW-SKA)**

One-way secret key agreement was first considered by Ahlswede [12]. Ahlswede considered source model where Alice and Bob have samples of correlated RVs $X$ and $Y$, and Eve has their side-information $Z$, and variables are obtained through a joint public distribution $P_{XYZ}$. “Forward key capacity” in this setting is defined for key establishment protocols in which there is a single message from Alice to Bob. These protocols are later called “one-way secret key agreement” (OW-SKA) [14].

**Definition 2** (OW-SKA [14]). Let $\lambda$ denote the security parameter and $\ell$ denote the length of the shared key $(\lambda, \ell) \in \mathbb{N}$ and $P_{XYZ} = \{P^x_{XYZ}[n' \in \mathbb{N}]\}$ be a family of distributions over $\mathcal{X} \times \mathcal{Y} \times \mathcal{Z}$. A one-way secret-key agreement (OW-SKA) protocol consists of the the function $m(\lambda, \ell) : \mathbb{N} \times \mathbb{N} \rightarrow \mathbb{N}$ that specifies $n' = m(\lambda, \ell)$, a (probabilistic) function family with parameters $\lambda$ and $\ell$, $\{t_{\text{Alice}} : \mathcal{X} \rightarrow \mathcal{K}_{\lambda, \ell}\}$, mapping $x \in \mathcal{X}$ to a bit string $k_A \in \{0,1\}^\ell$ (the secret key) and $c \in \mathcal{C}$ (the communication); and a function family, $\{t_{\text{Bob}} : \mathcal{Y} \times \mathcal{C} \rightarrow \mathcal{K}_{\lambda, \ell}\}$ mapping $c \in \mathcal{C}$ and $y \in \mathcal{Y}$ to a bit string $k_B \in \{0,1\}^\ell$.

The goal of secret-key agreement is to establish a key $k = k_A = k_B$ that appears uniformly random to Eve.

**Definition 3** (Secure OW-SKA Protocol). Let $C$ denote the RV corresponding to the message that is communicated from Alice to Bob over the public channel. Eve sees $C$ and has the side information $Z$, a random variable distributed over $\mathcal{Z}$. A OW-SKA protocol on $\mathcal{X} \times \mathcal{Y}$ is secure on a probability distribution family $P_{XYZ}$ if for $\lambda, \ell \in \mathbb{N}$, the OW-SKA protocol outputs a $(\epsilon, \sigma)$-Secret Key (in short $(\epsilon, \sigma)$-SK) $K$, an RV over $\mathcal{K}$ that satisfies the following reliability and security properties:

\[
\text{(reliability)} \quad \Pr[K_A = K_B = K] \geq 1 - \epsilon,
\]

\[
\text{(security)} \quad \text{SD} ((K, F, Z); (U_{K_{\lambda, \ell}}, F, Z)) \leq \sigma,
\]

where $K_A$ and $K_B$ are random variables corresponding to $t_{\text{Alice}}()$ and $t_{\text{Bob}}()$ functions respectively, and $\epsilon$ and $\sigma$ are small non-negative numbers.

**C. Hybrid Encryption**

A hybrid encryption scheme is a public-key encryption (PKE) scheme that uses (i) a special PKE, known as KEM, that is used to encrypt a symmetric key that is decrytable by Bob to establish a shared key between Alice and Bob, and (ii) a symmetric key encryption schemes, known as DEM, to encrypt an arbitrarily long message.

In the following, we use $\lambda$ to denote a parameter that determines security level of the system, and use the unary representation $1^\lambda$ that is commonly used in cryptography. In the rest of this section the attacker is assumed to be computationally bounded.

**Definition 4** (Key Encapsulation Mechanism (KEM) [37]). A KEM KEM $= \{\text{Kem.Gen}, \text{KEM.Enc}, \text{KEM.Dec}\}$ for an associated key space $\text{KeySP}(\lambda) = \mathcal{K}$, is a triple of algorithms defined as follows:

1. $\text{Kem.Gen}(1^\lambda)$ is a randomized key generation algorithm that takes the security parameter $\lambda \in \mathbb{N}$ returns a public and secret-key pair $(pk, sk)$.
2. $\text{KEM.Enc}(pk)$ takes a public key $pk$ and outputs a ciphertext $c$, and a key $k \in \mathcal{K}$.
3. $\text{KEM.Dec}(sk, c)$ is a deterministic decapsulation algorithm that takes a secret key $sk$ and a ciphertext $c$, and returns a key $k \in \mathcal{K}$, or $\perp$ that denotes failure, where private and public key spaces are $\mathcal{SK}$ and $\mathcal{PK}$, respectively, and a ciphertext space is $\mathcal{C}$. That is $sk \in \mathcal{SK}$, $pk \in \mathcal{PK}$, and $c \in \mathcal{C}$.
A KEM KEM is $\epsilon$-correct if for all $(sk, pk) \leftarrow \text{Kem.Gen}(\lambda)$ and $(c, k) \leftarrow \text{KEM.Enc}(pk)$, it holds that $\Pr[\text{KEM.Dec}(sk, c) \neq k] \leq \epsilon$, where probability is over the choices of $(sk, pk)$ and the randomness of KEM.Enc().

Security of KEM is defined as indistinguishability of the generated key from a random string against an attacker that may have access to the decryption algorithm (attacker can always access the encryption algorithm using the public key). Access to the decryption oracle is by sending a ciphertext to a decryption oracle and receiving the corresponding plaintext, or $\perp$ that denotes invalid ciphertext. This is known as Chosen Ciphertext Attack (CCA) security. CCA1 and CCA2 refer to disallowing or allowing ciphertext queries before or after the challenge ciphertext is seen. An attack without any access to the decryption algorithm is called Chosen Plaintext Attack (CPA). We use notations and formalization of [25]. Let the KEM be a KEM scheme and let $\text{Kem}$ and $\text{DEM}$ algorithms with

\begin{align*}
\text{KeySP} &\equiv (\text{Gen}, \text{Enc}, \text{Dec}) \\
\text{IND-CPA} &\equiv \text{IND-CPA}(\lambda) \\
\text{IND-CCA1} &\equiv \text{IND-CCA1}(\lambda) \\
\text{IND-CCA2} &\equiv \text{IND-CCA2}(\lambda)
\end{align*}

\begin{align*}
\text{Security of KEM: IND-CPA, IND-CCA1, IND-CCA2} &\equiv \text{Security of KEM: IND-CPA, IND-CCA1, IND-CCA2} \\
\text{KEM} &\equiv \text{Kem} \\
\text{DEM} &\equiv \text{DEM}
\end{align*}

\begin{align*}
\text{Pr}[k \leftarrow \text{Kem.Gen}(\lambda); (c, k) \leftarrow \text{KEM.Enc}(pk); k_0 \leftarrow k^*; k_1 \leftarrow K_{\lambda}; b \leftarrow \{0, 1\}; \\
\text{KEM-Dec}(c, k_{b}) = b - 1,
\end{align*}

where

\begin{align*}
\text{atk} &\equiv \text{atk} \\
\text{O}_\text{enc}(\cdot) &\equiv \text{O}_\text{dec}(\cdot) \\
\text{O}_\text{dec}(\cdot) &\equiv \text{O}_\text{dec}(\cdot)
\end{align*}

Let $\text{atk} \in \{\text{cpa, cca1, cca2}\}$ and $\lambda \in \mathbb{N}$, let $\text{Adv}^{\text{kind}}(\text{atk}) \equiv \text{atk}$.

\begin{align*}
\text{Pr}[k \leftarrow \text{Kem.Gen}(\lambda); (c, k) \leftarrow \text{KEM.Enc}(pk); k_0 \leftarrow k^*; k_1 \leftarrow K_{\lambda}; b \leftarrow \{0, 1\}; \\
\text{KEM-Dec}(c, k_{b}) = b - 1,
\end{align*}

where

\begin{align*}
\text{atk} &\equiv \text{atk} \\
\text{O}_\text{enc}(\cdot) &\equiv \text{O}_\text{dec}(\cdot) \\
\text{O}_\text{dec}(\cdot) &\equiv \text{O}_\text{dec}(\cdot)
\end{align*}

A KEM is $\sigma$-IND-ATK secure, if for all computationally bounded adversaries $A$, $\text{Adv}^{\text{kind}}(\text{atk}) \leq \sigma$.

Definition 6 (Data Encapsulation Mechanism (DEM) [37]). A DEM DEM = (DEM.Enc, DEM.Dec) associated to a key space $\text{KeySP}(\lambda) = K_{\lambda}$ consists of two algorithms:

1. DEM.Enc$(k, m)$ encrypts message $m$ under the uniformly chosen key $k \in K_{\lambda}$ and outputs a ciphertext $c$.
2. DEM.Enc$(c, k)$ decrypts the ciphertext $c$ using the key $k$ to get back a message $m$ or the special rejection symbol $\perp$.

Similar to [37], we assume encryption and decryption algorithms are deterministic, and that the scheme is (perfectly) correct (i.e. "sound" in the terminology of [37]), and for all $k \in K_{\lambda}$, and all message $m$, we have $\Pr[\text{DEM.Dec}(k, \text{DEM.Enc}(k, m)) = m] = 1$.

Security of DEM against CPA, CCA1, and CCA2 is defined in [37] and is the same as the corresponding definitions for symmetric encryption schemes as defined in [38]. DEM is a symmetric key primitive and so unlike KEM in which access to encryption oracle is free, access to encryption oracle is a resource. CPA security of DEM allows the attacker to have access to encryption oracle. Herranz et al. [25] considered two more one-time attacks for DEM, known as one-time (OT) attack and one-time (adaptive) chosen-ciphertext attacks (OTCCA) that correspond to passive attack and chosen-ciphertext attack after observing the challenge, respectively. OT attack is a CPA attack where the adversary does not have any access to the encryption oracle. Security a DEM is formalized by bounding the indistinguishability advantage of an adversary A denoted by $\text{Adv}^{\text{ind}}(\text{atk})(\lambda)$ and defined in the following:

Definition 7 (Security of DEM: IND-OT, IND-OTCCA, IND-CPA, IND-CCA1, IND-CCA2 [25]). Let $\text{DEM} = \{\text{DEM.Enc}, \text{DEM.Dec}\}$ be a DEM scheme with KeySP$(\lambda) = K_{\lambda}$ and let $A = (A_1, A_2)$ be an adversary. For $atk \in \{\text{ot, otcca, cpa, cca1, cca2}\}$ and $\lambda \in \mathbb{N}$, let $\text{Adv}^{\text{ind}}(\text{atk})(\lambda) \equiv \text{atk}$.

\begin{align*}
\text{Pr}[k \leftarrow \text{Kem.Gen}(\lambda); (st, m_0, m_1) \leftarrow A_1(\text{Kem.Gen}(\lambda)); b \leftarrow \{0, 1\};
\text{DEM.Enc}(k, m_b); A_2(\text{Kem.Gen}(\lambda)); \text{DEM.Enc}(k, m_{\overline{b}}); b \leftarrow \{0, 1\};
\text{DEM.Dec}(c, b) = b - 1,
\end{align*}

where

\begin{align*}
\text{atk} &\equiv \text{atk} \\
\text{O}_\text{enc}(\cdot) &\equiv \text{O}_\text{dec}(\cdot) \\
\text{O}_\text{dec}(\cdot) &\equiv \text{O}_\text{dec}(\cdot)
\end{align*}

A DEM is $\sigma$-IND-ATK for $\text{atk} \in \{\text{OT, OTCCA, CPA, CCA1, CCA2}\}$ if for all adversaries $A$, $\text{Adv}^{\text{ind}}(\text{atk})(\lambda) \leq \sigma$.

Definition 8 (Hybrid PKE (HPKE) [37]). An HPKE $\text{HPKE}_{\text{KEM,DEM}} = (\text{HPKE.Gen, HPKE.Enc, HPKE.Dec})$ uses a pair of KEM $\text{KEM} = (\text{Kem.Gen, Kem.Enc, Kem.Dec})$ and DEM $\text{DEM} = (\text{DEM.Enc, DEM.Dec})$ algorithms with a common key space $\text{KeySP}(\lambda) = K_{\lambda}$, and consists of three algorithms for key generation, encryption and decryption defined below.

\begin{align*}
\text{Algo HPKE.Gen}(\lambda) &\equiv \text{Algo HPKE.Gen}(\lambda) \\
(p, k) &\leftarrow \text{Kem.Gen}(\lambda, D) \\
\text{Algo HPKE.Enc}(pk, m) &\equiv \text{Algo HPKE.Enc}(pk, m) \\
c_e &\leftarrow \text{DEM.Enc}(k, m) \\
\text{Return (c_e, c_2)} &\equiv \text{Return (c_e, c_2)}
\end{align*}

\begin{align*}
\text{Algo HPKE.Enc}(sk, c_1, c_2) &\equiv \text{Algo HPKE.Enc}(sk, c_1, c_2) \\
k &\leftarrow \text{KEM.Dec}(y, c_1) \\
\text{if } \perp \leftarrow \text{iKEM.Dec}(y, c_1) \\
\text{Return } k &\equiv \text{DEM.Dec}(c_2, k) \\
\text{Return m} &\equiv \text{Return m}
\end{align*}

Fig. 1: Hybrid public-key encryption

The following composition theorem gives security of hybrid encryption [25] (Theorem 5.1).

Theorem 1 (IND-ATK KEM + IND-ATK’ DEM ⇒ IND-ATK PKE). [25] Theorem 5.1] For $\text{atk} \in \{\text{CPA, CCA1, CCA2}\}$
and \( \text{ATK}' \in \{\text{OT}, \text{OTCCA}\} \), if \( \text{iKEM} \) is a secure KEM under \( \text{IND-ATK} \) attacks and \( \text{DEM} \) is a secure DEM under \( \text{IND-ATK}' \) attacks, then the hybrid public key encryption scheme \( \text{HPE}_{\text{iKEM,DEM}} \) is a secure public key encryption scheme under \( \text{IND-ATK} \) attacks, where for \( \text{ATK} \in \{\text{CPA, CCA}\} \), \( \text{ATK}' = \text{OT} \) and for \( \text{ATK} = \text{CCA2} \), \( \text{ATK}' = \text{OTCCA} \).

In Section III we prove a similar composition theorem for \( \text{iKEM} \) and \( \text{DEM} \) with specific security notions.

### III. HYBRID ENCRYPTION IN PREPROCESSING MODEL

In the preprocessing model Alice, Bob and the attacker have access to their corresponding samples of a joint distribution. The distribution is public but the samples are private inputs of the parties. A hybrid encryption in preprocessing model, denoted by \( \text{HE}_{\text{iKEM,DEM}} \), uses a pair of algorithms \( \text{iKEM} \) with information theoretic security, and \( \text{DEM} \) with computational security to construct a hybrid encryption.

We first define information theoretic KEM (\( \text{iKEM} \)) and give its security notions, and then describe the \( \text{HE}_{\text{iKEM,DEM}} \) system that uses a \( \text{DEM} \) as defined in Definition 6.

#### A. KEM in Preprocessing Model (\( \text{iKEM} \))

An \( \text{iKEM} \) allows Alice and Bob to use their samples of correlated randomness and a single message from Alice to Bob, to obtain a shared key that is secure against an eavesdropper (a wiretapper) with side information that is represented by their initial random samples.

**Definition 9 (\( \text{iKEM} \)).** An \( \text{iKEM} \) is defined by a triple of algorithms \( \text{iKEM}.\text{Gen}, \text{iKEM}.\text{Enc} \) and \( \text{iKEM}.\text{Dec} \), as follows:

1. \( \text{iKEM}.\text{Gen}(1^\lambda, \mathcal{D}) \) the generation algorithm takes the security parameter \( \lambda \in \mathbb{N} \) and a publicly known family of distributions \( \mathcal{D} \), and provides private inputs to Alice and Bob, and possibly Eve, denoted by \( x, y \), and \( z \), respectively.
2. \( \text{iKEM}.\text{Enc}(x) \), the encapsulation algorithm, is a probabilistic algorithm that takes as input Alice’s random string \( x \) and outputs a ciphertext/key pair \( (c, k) \).
3. \( \text{iKEM}.\text{Dec}(y, c) \), the decapsulation algorithm, is a deterministic algorithm that takes as input the receiver’s random string \( y \) and ciphertext \( c \), and outputs key \( k \) or special symbol \( \bot \) (\( \bot \) implies that the ciphertext was invalid).

**Correctness of \( \text{iKEM} \):** Security of \( \text{iKEM} \) is against an attacker with unlimited computational power that in addition to its side information, can query the encryption and decapsulation algorithms. We thus, consider two types of oracles, \( \text{O}_{\text{enc}}(\cdot) \) and \( \text{O}_{\text{dec}}(\cdot) \), and their corresponding attacks, Encryption Oracle Attack (\( \text{EnO} \)) and Chosen Ciphertext Attack (\( \text{CCA} \)), respectively. A query to \( \text{O}_{\text{enc}}(\cdot) \) does not have any input, and outputs a pair \( (c, k) \) where \( k \) and \( c \) are a key and the corresponding ciphertext that is obtained by using the secret input of Alice and other system’s public information. A query to \( \text{O}_{\text{dec}}(\cdot) \) is a ciphertext \( c \) that is chosen by the attacker, and will result in \( \text{O}_{\text{dec}}(\cdot) \) to output either a key \( k \), or \( \bot \), indicating that \( \text{iKEM}.\text{Dec} \) can/cannot generate a valid key for the presented \( c \).

We consider three types of attackers: an attacker with no access to encapsulation or decapsulation oracles (\( \text{OT} \) attack), an attacker with access to \( q_e \) encapsulation queries (\( q_e-\text{EnO} \) attack), and an attacker that has access to a total of \( q_c \) queries to the decapsulation and/or encryption oracles (\( q_c-\text{CCA} \) attack). The corresponding security notions are denoted by \( \text{IND-OT}, \text{IND-}q_e-\text{EnO}, \text{IND-}q_c-\text{CCA} \), respectively. For a given security level \( \lambda \), the number of queries affect parameters of the \( \text{iKEM} \). We use \( A^U = (A^U_1, A^U_2) \) to denote an adversary with “\( U \)”-unbounded computation that uses algorithm \( A^U_1 \) before seeing the challenge, and passes the learnt information (its state) to algorithm \( A^U_2 \) that is executed after seeing the challenge. Security a \( \text{iKEM} \) is formalized by bounding the information theoretic key indistinguishability (\( \text{ikind} \)) advantage of an adversary \( A^U \) denoted by \( \text{Adv}_{\text{iKEM,A}^U}^{\text{ikind}}(\lambda, q) \) and defined in the following:

**Definition 10 (\( \text{Security of } \text{iKEM}: \text{IND-OT, IND-}q_e-\text{EnO, IND-}q_c-\text{CCA} \)).** Let \( \text{iKEM} = (\text{iKEM}.\text{Gen}, \text{iKEM}.\text{Enc}, \text{iKEM}.\text{Dec}) \) be an \( \text{iKEM} \) scheme and let \( A^U = (A^U_1, A^U_2) \) be an unbounded adversary. For \( \text{atk} \in \{\text{ot}, q_e-\text{enO}, q_c-\text{cca}\}, \ q \in \{q_e, q_c\} \) and \( \lambda \in \mathbb{N} \), let \( \text{Adv}_{\text{iKEM,A}^U}^{\text{ikind},\text{atk}}(\lambda, q) \).\text{e} \text{e}

\[ \text{Pr}[\text{iKEM}.\text{Gen}(1^\lambda); s_1 \leftarrow A^U_1(\cdot); k^* \leftarrow \text{K}_\lambda; b \leftarrow \{0,1\} A^U_2(\cdot); c^* \leftarrow \text{dec}(b); c^* = c] = 1, \]

where

| \( \text{atk} \)   | \( \text{O}_{\text{enc}}(\cdot) \) | \( \text{O}_{\text{dec}}(\cdot) \) |
|---------------|----------------|----------------|
| ot            | \( \varepsilon \) | \( \varepsilon \) |
| \( q_e-\text{enO} \) | \( \text{iKEM}.\text{Enc}_.(\cdot) \) | \( \varepsilon \) |
| \( q_c-\text{cca} \) | \( \text{iKEM}.\text{Dec}_.(\cdot) \) | \( \text{iKEM}.\text{Dec}_.(\cdot) \) |

An \( \text{iKEM} \) is \( \sigma \)-\( \text{IND-ATK} \) secure for \( \sigma \in \{q_e, q_c\} \), and \( \text{ATK} \in \{\text{ot}, q_e-\text{enO}, q_c-\text{CCA}\} \), if for all adversaries \( A^U \), \( \text{Adv}_{\text{iKEM,A}^U}^{\text{ikind},\text{atk}}(\lambda, q) \leq \sigma \).

The following lemma shows that the distinguishing advantage of the adversary \( A^U \) in Definition 10 is bounded by the the statistical distance of the generated key with uniform distribution, given adversary’s view of the game. This lemma can be seen as a special case of [39, Lemma 4], where the random system is an \( \text{iKEM} \).

**Lemma 2.** Let \( \nu_{A^U}^{q_e-\text{enO}} = (v_1^{q_e-\text{enO}}, \ldots, v_{q_e}^{q_e-\text{enO}}) \) for \( v_i^{q_e-\text{enO}} \in K_\lambda \times C \) denote the encryption oracle’s responses to adversary \( A^U \)’s queries in the \( q_e \)-bounded \( \text{EnO} \) attack, and \( \nu_{A^U}^{q_c-\text{cca}} \) denote the corresponding random variable (that is probabilistic according to the randomness of \( \text{iKEM}.\text{Enc} \) algorithm and the distribution of Bob’s private and public inputs). The \( \text{iKEM} \) is
σ-indistinguishable against $q_e$-bounded $EnO$, if and only if for all adversaries $A^U$, we have

$$SD((Z, C^*, K^*, V^q_{\text{EnO}}); (Z, C^*, U_{K^*}, V^q_{\text{EnO}})) \leq \sigma,$$

where random variables $Z$, $C^*$ and $K^*$ correspond to $z$, the initial correlated random string received by the adversary, and the challenge ciphertext and key pair $(c^*, k^*)$, respectively.

**Proof of Lemma 2** The proof has two parts: (a) the iKEM indistinguishable if the statistical distance is bounded, and (b) if the iKEM is indistinguishable then the statistical distance is bounded. We show each part separately:

(a) Suppose a given iKEM is σ-indistinguishable. Then holds. Because if it doesn’t, there exist a set $W \subset Z \times K \times C$ for which

$$\left| Pr[(Z, K^*, C^*) \in W], V^q_{\text{EnO}} \right| - \left| Pr[(Z, U_{K^*}, C^*) \in W], V^q_{\text{EnO}} \right| > \sigma$$

We use $W$ and define an adversary algorithm $A^U$ that for any $(z, c^*, k^*) \in W$ outputs zero. This allows $A^U$ to gain an advantage $Adv_{iKEM,A^U}(\lambda) > \sigma$, and this contradicts the assumption (that the iKEM is σ-indistinguishable). Therefore the statistical distance is less than $\sigma$.

(b) Suppose holds, then let $F_{A^U} : Z \times K \times C \rightarrow \{0, 1\}$ be an arbitrary function that takes $A^U$’s inputs $(z, c^*)$, $k^*$ and $V^q_{\text{EnO}}$ and output 0 or 1. Then we have

$$Adv_{iKEM,A^U}(\lambda, q_e) \leq \max_{F_{A^U}} \left| Pr[F_{A^U}(Z, C^*, K^*, V^q_{\text{EnO}}) = 1] \right| - \left| Pr[F_{A^U}(Z, C^*, U_{K^*}, V^q_{\text{EnO}}) = 1] \right|.$$

Let $W \subset Z \times K \times C$ be the set for which $Pr[(Z, C^*, K^*) \in W], V^q_{\text{EnO}}] - Pr[(Z, C^*, U_{K^*}) \in W], V^q_{\text{EnO}}]$ is maximized, then define $F_{A^U}(\cdot)$ to be 1 only if its input is in $W$. From the definition of the **statistical distance** [1], it is easy to see that

$$Adv_{iKEM,A^U}(\lambda, q_e) \leq \max_{F_{A^U}} \left| Pr[F_{A^U}(Z, C^*, K^*, V^q_{\text{EnO}}) = 1] \right| - \left| Pr[F_{A^U}(Z, C^*, U_{K^*}, V^q_{\text{EnO}}) = 1] \right| = SD((Z, C^*, K^*, V^q_{\text{EnO}}); (Z, C^*, U_{K^*}, V^q_{\text{EnO}})) \leq \sigma \blacksquare$$

**Corollary 1.** The iKEM in **Definition 9** is IND-OT secure if and only if:

$$SD((Z, C^*, K^*); (Z, C^*, U_{K^*})) \leq \sigma,$$

where random variables $Z$, and $(C^*, K^*)$ correspond to $z$, and the pair of challenge ciphertext and key $(c^*, k^*)$, respectively.

**Proof.** The proof follows from **Lemma 2** and noting that for IND-OT security query is allowed for the adversary and $V^q_{\text{EnO}}$ is empty. ■

**B. DEM in Preprocessing Model**

Hybrid encryption in preprocessing model will use the DEM definition **Definition 6** with security notions as in **Definition 7** and defined against a computationally “B”ounded adversary that will be denoted by $A^B$.

**C. Hybrid Encryption using iKEM**

Hybrid encryption in preprocessing model uses private samples of correlated variables as the key material in an iKEM with information theoretic security against (unbounded attacker $A^U$), and a DEM with computational security (bounded attacker $A^B$) and provides a computationally secure encryption system.

**Definition 11.** [Hybrid Encryption (HE) in Preprocessing Model] Let $iKEM = (iKEM.Gen, iKEM.Enc; iKEM.Dec)$ and DEM $= (DEM.Enc, DEM.Dec)$ be a pair of iKEM and DEM defined with the same security parameter $\lambda$ and the same key space $KeySp(\lambda) = K_{\lambda}$, for each $\lambda$. We define a hybrid encryption in preprocessing model denoted by $HE_{iKEM,DEM} = (HE.Gen, HE.Enc, HE.Dec)$ using an iKEM and a DEM, as follows.

**Algorithm HE.Gen(1^\lambda)**

$x, y, z \leftarrow iKEM.Gen(1^\lambda, D)$

Return $(x, y, z)$

**Algorithm HE.Enc(x, m)**

$k \leftarrow iKEM.Enc(x, c_1)$

if $1 \leftarrow iKEM.Dec(y, c_1)$

Return $\bot$ and $m \leftarrow DEM.Dec(c_2, k)$

Return $m$

**Remark 1.** A hybrid encryption scheme in preprocessing model uses private samples and so access to the encryption oracle (CPA) corresponds to an attack where the attacker sees the output of the encryption system on messages of its choice. Each call to the HE.Enc counter oracle uses the same private samples $x$, but different encryption key. This is different from access to encryption oracle in DEM definition, where the secret key that is used for encryption of data is fixed.

We consider three security notions for hybrid encryption in preprocessing model, depending on the attacker’s access to the encryption and decryption oracles. Unlike traditional definition of security in computational setting where the number of queries can grow with the input size of the algorithm ($\lambda$), we will consider a fixed number of queries. This is because of security requirement of iKEM that is defined for the fixed number of queries. Security against adversaries with access to a fixed number of queries has also been considered in computational cryptography for reasons such as providing concrete constructions [40], [41]. The scheme parameters will depend on the number of oracle accesses.

We define one-time CPA attack, denoted by IND-OT, for an attacker with no oracle access (passive attacker), inline with OT attack in DEM. We also define IND-\$q_e\$-vCPA (indistinguishability against “variable key” chosen plaintext attack) where the attacker has access to a fixed number of encryption queries to HE.Enc counter oracle. We use “variable key” to emphasize the DEM key in each query will be freshly generated.

Decryption queries will be defined similar to that of HPKE, against the decryption oracle HE.Dec. Finally, we define
IND-\text{q}-\text{CCA} security of the hybrid encryption in preprocessing model where the attacker has access to a total of $q_e$ oracle queries, where the oracles can be \text{HE.En}_{e}(\cdot), \text{or \text{HE.Dec}_{e}(\cdot)} (encryption and decryption oracles).

**Definition 12.** [IND-OT, IND-q_p-\text{vCPA}, IND-q_e-\text{CCA} security of hybrid encryption in preprocessing model] Let \text{HE}_{\text{IKEM,DEM}} = (\text{HE.Gen,HE.Enc,HE.Dec}) be a hybrid encryption in preprocessing model using an iKEM \text{iKEM} = (\text{iKEM.Gen,\text{iKEM.Enc,\text{iKEM.Dec}}) and a DEM \text{DEM} = (\text{DEM.Enc,DEM.Dec}). Let $A^B = (A^B_1,A^B_2)$ be a computationally bounded adversary. For $q \in \{q_p, q_e\}$ and $atk \in \{\text{ot}, \text{q_p-vcpa}, \text{q_e-cca}\}$ and $\lambda \in \mathbb{N}$, let $Adv^{\text{ind-atk}}_{\text{HE,AB}}(\lambda, q) = \Pr[(x, y, z) \leftarrow \text{HE.Gen}(\lambda^\lambda)]$

\[ (st, m_0, m_1) \xleftarrow{\$} A^B_1(\cdot,\cdot,\cdot); b \xleftarrow{\$} \{0, 1\}; \]

\[ (c^*) \xleftarrow{\$} \text{DEM.Enc}(k, m_b); A^B_2(\cdot,\cdot,\cdot); \]

\[ (c^*), st = b) - 1, \]

where

| $atk$         | $O_{\text{enc}}(\cdot)$ | $O_{\text{dec}}(\cdot)$ |
|--------------|-----------------|-----------------|
| $\text{q}_p$-\text{vcpa}$ | $\text{HE.Enc}_e(\cdot)$ | $\text{HE.Enc}_e(\cdot)$ |
| $\text{q}_e$-\text{cca}$ | $\text{HE.Enc}_e(\cdot)$ | $\text{HE.Dec}_e(\cdot)$ |

A hybrid encryption scheme \text{HE}_{\text{IKEM,DEM}} in preprocessing model is $\sigma$-IND-ATK for $q \in \{q_p, q_e\}$ and $\text{ATK} \in \{\text{CA, q_p-vcpa, q_e-cca}\}$ if for all adversaries $A^B$, $Adv^{\text{ind-atk}}_{\text{HE,AB}}(\lambda, q) \leq \sigma$.

The following composition theorem for hybrid encryption shows that, an IND-OT secure iKEM and an IND-OT secure DEM implies an IND-OT secure HE, and a $q$-EnO secure iKEM and an IND-OT secure DEM implies a $q$-VC-CPA secure HE.

**Theorem 2 (IND-ATK iKEM + IND-OT DEM ⇒ IND-ATK HE).** For a security parameter $\lambda \in \mathbb{N}$, let DEM denote a computationally secure $\sigma'$-IND-OT secure DEM, and iKEM denote an information theoretically secure $\epsilon$-correct $\sigma$-IND-ATK secure iKEM, where $\text{ATK} \in \{\text{OT, q-EnO}\}$, and assume iKEM and DEM have compatible key spaces $\text{KeySP}(\lambda) = \mathcal{K}\lambda$. Then, the hybrid encryption scheme \text{HE}_{\text{IKEM,DEM}} is a computationally secure IND-ATK secure hybrid encryption in preprocessing model with security against a computationally bounded adversary $A^B = (A^B_1, A^B_2)$ and $Adv^{\text{ind-atk}}_{\text{HE,AB}}(\lambda, q) \leq \epsilon + \sigma + \sigma'$.

where for $\text{ATK} = \text{OT}$, $\text{ATK}' = \text{OT}$ and for $\text{ATK} = \text{q-EnO}$, $\text{ATK}' = \text{q-vcCPA}$.

**Proof.**

a) $\text{ATK} = \text{OT}$: For a given sample $\text{sam} = (x, y, z)$ generated by iKEM.Gen, we define a set of bad keys $\text{BK}_{\text{sam}}$ generated by iKEM.Enc, where

$\text{BK}_{\text{sam}} = \{k \leftarrow \text{iKEM.Enc}(x).\text{key} : \text{iKEM.Dec}(y, c) \neq k\}$.

According to the correctness of iKEM, for $k \xleftarrow{\$} \mathcal{K}\lambda$ we have $\Pr[k \in \text{BK}_{\text{sam}}] \leq \epsilon$.

We define two experiments as follows:

**EXP1** $= \{(x, y, z) \xleftarrow{\$} \text{HE.Gen}(\lambda^\lambda) ; (st, m_0, m_1) \xleftarrow{\$} A^B_1(\cdot,\cdot,\cdot); b \xleftarrow{\$} \{0, 1\}; (c^*) \xleftarrow{\$} \text{DEM.Enc}(k, m_b); A^B_2(\cdot,\cdot,\cdot); (c^*), st = b\} - 1,$

**EXP2** $= \{k \xleftarrow{\$} \mathcal{K}\lambda; (st, m_0, m_1) \xleftarrow{\$} A^B_1(\cdot,\cdot,\cdot); b \xleftarrow{\$} \{0, 1\}; (c^*) \xleftarrow{\$} \text{DEM.Enc}(k, m_b); A^B_2(\cdot,\cdot,\cdot); (c^*), st = b\} - 1$.

We have

\[ \Pr[\text{EXP2}] \leq \Pr[k \notin \text{BK}_{\text{sam}}] \cdot \Pr[k \xleftarrow{\$} \mathcal{K}\lambda; (st, m_0, m_1) \xleftarrow{\$} A^B_1(\cdot,\cdot,\cdot); b \xleftarrow{\$} \{0, 1\}; (c^*) \xleftarrow{\$} \text{DEM.Enc}(k, m_b); A^B_2(\cdot,\cdot,\cdot); (c^*), st = b + \Pr[k \in \text{BK}_{\text{sam}}] \]
Construction 1. The $iKEM_{iKEMOWSKA}$. The $iKEM_{iKEMOWSKA}$ will have the following algorithms.

Initialization: Let $\{h_s : \mathcal{X} \to \{0, 1\}^1\}_{s \in S}$ and $\{h'_s : \mathcal{X} \to \{0, 1\}^1\}_{s' \in S'}$ be two strongly universal hash families (SUHF). Also let $C = \{0, 1\}^* \times S \times S'$ and $K_\lambda = \{0, 1\}^\lambda$ denote the set of ciphertexts and keys. The relation between $t$, $t'$ and correctness and security parameters is given in [Theorem 2][17]. We recall these relations in Theorems 3 and 4. The three algorithms are as follows.

- **$iKEM_{iKEMOWSKA}.Gen(\lambda, \mathcal{P}_{XYZ})$:** The generation algorithm chooses an appropriate $\mathcal{P}_{XYZ}$ from $\mathcal{P}_{XYZ} = \{\mathcal{P}_{XYZ}'|n' \in \mathbb{N}\}$ according to $\lambda$ and samples the distribution to output the triplet $x, y, z$ of correlated samples, and privately gives them to Alice, Bob and Eve, respectively. That is
  $$(x, y, z) \xleftarrow{\$} iKEM_{iKEMOWSKA}.Gen(\lambda, \mathcal{P}_{XYZ}).$$

- **$iKEM_{iKEMOWSKA}.Enc(x)$:** The encapsulation mechanism $iKEM_{iKEMOWSKA}.Enc()$ samples $s', s \xleftarrow{\$} S'$ and $s \xleftarrow{\$} S$ for the seed of the strongly universal hash functions, and generates the key $k = h'_{s'}(x)$ and the ciphertext $c = (h_s(x), s', s)$. Thus
  $$(c, k) = ((h_s(x), s', s), h'_{s'}(x)) \xleftarrow{\$} iKEM_{iKEMOWSKA}.Enc(x).$$

- **$iKEM_{iKEMOWSKA}.Dec(y, c)$:** The decapsulation mechanism $iKEM_{iKEMOWSKA}.Dec(y, c)$ takes the private input of Bob, $y$, and the ciphertext $h_s(x), s', s$ as inputs, and outputs the key $h'_{s'}(x)$ or $\bot$. We have
  $$k = (h'_{s'}(x)) \xleftarrow{\$} iKEM_{iKEMOWSKA}.Dec(y, (h_s(x), s', s)).$$

The decapsulation algorithm works as follows:

1. Parses the received ciphertext to $(g, s', s), \text{where } g \text{ is a t-bit string}$.
2. Define the set,
   $$\mathcal{T}(X|y) \triangleq \{x : -\log \mathcal{P}_{XYZ}'(X|y) \leq \nu\},$$
   (8)

   For each vector $x \in \mathcal{T}(X|y)$, check $g \xleftarrow{\$} h_s(x)$.
3. Output $\hat{x}$ if it is the unique value of $x$ that satisfies $g = h_s(\hat{x})$; Else output $\bot$.

The value of $\nu$ depends on the correlation of $x$ and $y$: higher correlation corresponds to smaller $\nu$, and smaller set of candidates (see Theorem 1 for the precise relationship).

If successful, the decapsulation algorithm outputs a key $k = h'_{s'}(\hat{x})$; otherwise it outputs $\bot$.

For a given correlation between RVs $X, Y$ and $Z$ measured by the average conditional min-entropies $\tilde{H}_\infty(X|Y)$ and $\tilde{H}_\infty(X|Z)$, Theorem 3 gives the minimum length of the ciphertext to bound the error probability of the protocol by $\epsilon$, and for a given ciphertext length, gives the maximum number of key bits that can be established using $iKEM$ when the adversary is not allowed to make any queries (encapsulation or decapsulation oracles), in order to bound its advantage by $\sigma$ for any computationally unbounded adversary. Note that in contrast to [17], here we are not interested in achieving the secrecy capacity of the setting and for simpler representation, we can skip the last round of “entropy smoothing” [42] in deriving the relations between the scheme parameters and use min-entropy instead of the Shannon entropy as the measure of correlation between Alice and Bob’s random variables.

**Theorem 3.** (indistinguishable against an adversary with access to $q_e$ encryption queries (2\sigma_e-bounded $EnO$).

**Proof.** Each query to the encapsulation oracle gives a pair of matching key and ciphertext $(c, k)$ to the adversary. The vector $v_A^{\epsilon,\sigma_e} = (v_1^{\epsilon,\sigma_e}, \ldots, v_N^{\epsilon,\sigma_e})$ is the vector of adversary’s received responses to their $EnO$ queries, and reveals information about $X$ to them. The remaining uncertainty about $X$ that can be used for key extraction is $\tilde{H}_\infty(X|V_1^{\epsilon,\sigma_e} = v_1^{\epsilon,\sigma_e})$, where $v_i^{\epsilon,\sigma_e} = (c_1, k_1)$ and $c_i = (c_{0i}, s_i, s'_i)$. If we choose $S$ and $S'$ (in $h_S(X)$ and $h'_S(X)$) in the $i$th query’s response, $c_i = (c_{0i}, s_i, s'_i)$, be $s_i$ and $s'_i$. From [32] Lemma 2.2(b), for $C_0 \subseteq \{0, 1\}^t$ and $K_1 \subseteq \{0, 1\}^t$ we have $\tilde{H}_\infty(X|Z, C_0, K_1) \geq \tilde{H}_\infty(X|Z) - t - \ell$, and from [32] Lemma 2.2(a), $\tilde{H}_\infty(X|Z, C_0 = c_0, K_1 = k_1) \geq \tilde{H}_\infty(X|C_0, K_1) - \log(1/\delta)$ with probability at least $1 - \delta$ over the choice of $(c_0, k_1)$. Let $\delta = \frac{\epsilon}{q_e}$. Thus

$$\tilde{H}_\infty(X|Z, V_1^{\epsilon,\sigma_e} = v_1^{\epsilon,\sigma_e}) = \tilde{H}_\infty(X|Z, C_1 = c_0, K_1 = k_1) \geq \tilde{H}_\infty(X) - t - \ell - \log(q_e/\sigma_e),$$

with probability at least $1 - \frac{\epsilon}{q_e}$. This is adversary’s maximum uncertainty about $X$ after making a query to the encapsulation oracle, and so each query decreases the remaining min-entropy of $X$ by at most $t + \ell + \log(q_e/\sigma_e)$ with probability at least $1 - \frac{\epsilon}{q_e}$. Thus, after $q_e$ queries we have $\tilde{H}_\infty(X|Z, V_A^{\epsilon,\sigma_e}) = v_A^{\epsilon,\sigma_e} \geq \tilde{H}_\infty(X|Z) - q_e(t + \ell + \log(q_e/\sigma_e))$ with probability at least $(1 - \frac{\epsilon}{q_e})^{q_e}$, and since from Lemma 11

$$SD((Z, h_S(X), S, S', (h'_S(X))); (Z, h_S(X), S, S', U_d)) \leq \frac{1}{2} \sqrt{2t + \ell - \tilde{H}_\infty(X|Z)},$$

(10)
we have
\[
\text{SD}\left((Z, S^{q+1}, S'^{q+1}, h_S(X), h'_S(X), v^{q_{\text{ono}}})\right);
\]
\[
\left((Z, S^{q+1}, S'^{q+1}, h_S(X), U_t, v^{q_{\text{ono}}})\right)
\]
\[
\leq \frac{1}{2} \sqrt{2q_{(q+1)}(t+\log(q_\epsilon/\sigma_e))-\hat{R}_\infty(X|Z),}
\]
with probability \((1 - \frac{q_{\epsilon}}{q})^q\). Since \(t \leq \frac{2+2\log q_\epsilon + \hat{R}_\infty(X|Z) - \log(q_\epsilon/\sigma_e)}{q-1}\), the above statistical distance is bounded by \(\sigma_e\) with probability \((1 - \frac{q_{\epsilon}}{q})^q\) and by 1 otherwise. Thus we have,
\[
\text{SD}\left((Z, S, S', h_S(X), h'_S(X), v^{q_{\text{ono}}})\right);
\]
\[
\left((Z, S, S', h_S(X), U_t, v^{q_{\text{ono}}})\right) \leq \left(1 - \frac{q_{\epsilon}}{q} \right)^q \sigma_e + \left(1 - \frac{q_{\epsilon}}{q} \right)^q \sigma_e \leq 2\sigma_e,
\]
where (1) inequality is since \(1 - \frac{q_{\epsilon}}{q} \leq 1\) and \(1 - \frac{q_{\epsilon}}{q} \leq \sigma_e\) due to Bernoulli’s inequality stating for \(t \geq 1\) and \(0 \leq x \leq 1\), inequality \(xt \geq 1 - (1 - x)^t\) holds. Finally, for \(C^* = (h_S(X), S', S)\), the inequality (4) is satisfied . That is we have \(2\sigma_e\)-indistinguishability against \(q_e\) EnO.

V. CONCLUDING REMARKS
In this work, we formalized KEM/DEM paradigm in preprocessing model. This formalization is with the help of defining an iKEM that encapsulates information theoretic keys. We showed the key generated by an iKEM scheme can be safely used in computational DEMs (such as AES). The security of the combined scheme depends on the security of the iKEM key against well defined attacks (OT, EnO and CCA). These attacks model information leakages to the adversary through the encryption or decryption algorithms. We initiated the study of this new paradigm but our work can be studied under more general frameworks discussed bellow.

On the composability of iKEM: Informally, the correctness and security conditions of an iKEM can be rephrased in a composable framework such as the UC framework or the Constructive Cryptography framework as follows: The iKEM constructs a shared secret key (the ideal world) using resources (correlated randomness) in the preprocessing phase, and an authenticated communication channel (the real world), such that the two worlds are indistinguishable from computational point of view. We show this by combining the correctness and indistinguishability conditions under a singular bound on the statistical distance of the random variables in two words:

Lemma 3. For an \(\epsilon\)-correct, \(\sigma\)-IND-OT iKEM in Definition, let KeySP = \(K_\alpha\) and \(k_A\) and \(k_B\) be the keys obtained by Alice and Bob, and \(K_A\) and \(K_B\) be the corresponding random variables, respectively. That is, iKEM.Enc\((x)\).key = \(k_A\) and iKEM.Dec\((y, c)\) = \(k_B\). Then
\[
\text{SD}\left((Z, C^*, K_A, K_B); (Z, C^*, U_{K_A}, U_{K_B})\right) \leq \sigma + \epsilon,
\]
\[\text{(11)}\]
We show this for an IND-OT iKEM. The proof for IND-EnO security is the same.

Proof. A \(\sigma\)-IND-OT iKEM satisfies:
\[
\text{SD}\left((Z, C^*, K_A); (Z, C^*, U_{K_A})\right) \leq \sigma
\]
\[\text{(1)}\]
\[
\text{SD}\left((Z, C^*, K_A, K_B); (Z, C^*, U_{K_A}, U_{K_B})\right) \leq \sigma.
\]
\[\text{(12)}\]
On the other hand, according to the correctness condition
\[
\text{SD}\left((Z, C^*, K_A, K_B); (Z, C^*, U_{K_A}, U_{K_B})\right) \leq \epsilon
\]
\[\text{(13)}\]
and finally, from (12) and (13) and by the application of the triangle inequality, we have (11). ■

A computationally bounded HE: In Section, we studied the security of the hybrid encryption scheme in preprocessing model against a computationally bounded adversary. Security against an unbounded adversary can be achieved by encrypting messages under the key from an iKEM scheme using a symmetric encryption scheme that is secure against a computationally unbounded adversary. Although in this work we assumed the DEM used in the construction of the hybrid encryption scheme is deterministic, this condition can be relaxed to expand the application of iKEM key to other information theoretic symmetric encryption schemes. Probabilistic information theoretic symmetric key encryption schemes are proposed in [35, 46, 47].

Remark 2. The composability of these schemes is not studied (in particular the composability of [45] is an open problem) but in case of positive answer to the composability of these schemes, the iKEM key can be used in these schemes to construct a HE secure scheme against computationally unbounded adversary.

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