In-plane magnetic field effect on hole cyclotron mass and $g_z$ factor in high-mobility SiGe/Ge/SiGe structures

I.L. Drichko,¹ V.A. Malyshev,¹ I.Yu. Smirnov,¹ L.E. Golub,¹ S.A. Tarasenko,¹ A.V. Suslov,² O.A. Mironov,3,4 M. Kummer,5 and H. von Känel5

¹A. F. Ioffe Physico-Technical Institute of Russian Academy of Sciences, 194021 St.Petersburg, Russia
²National High Magnetic Field Laboratory, Tallahassee, FL 32310, USA
³Department of Physics, University of Warwick, Coventry, CV4 7AL, United Kingdom
⁴International Laboratory of High Magnetic Fields and Low Temperatures, 53-421 Wroclaw, Poland
⁵Laboratorium für Festkörperphysik ETH Zürich, CH-8093 Zürich Switzerland

The high-frequency (ac) conductivity of a high quality modulation doped GeSi/Ge/GeSi single quantum well structure with hole density $p=6\times10^{11}\text{cm}^{-2}$ was measured by the surface acoustic wave (SAW) technique at frequencies of 30 and 85 MHz and magnetic fields $B$ of up to 18 T in the temperature range of 0.3 – 5.8 K. The acoustic effects were also measured as a function of the tilt angle of the magnetic field with respect to the normal of the two-dimensional channel at $T=0.3$ K.

It is shown, that at the minima of the conductivity oscillations, holes are localized on the Fermi level, and that there is a temperature domain in which the high-frequency conductivity in the bulk of the quantum well is of the activation nature. The analysis of the temperature dependence of the conductivity at odd filling factors enables us to determine the effective $g_z$ factor. It is shown that the in-plane component of the magnetic field leads to an increase of the cyclotron mass and to a reduction of the $g_z$ factor.

We developed a microscopic theory of these effects for the heavy-hole states of the complex valence band in quantum wells which describes well the experimental findings.

PACS numbers: 73.63.Hs, 73.50.Rb

I. INTRODUCTION

Modulation doped SiGe/Ge/SiGe structures with a two-dimensional (2D) hole gas are attractive systems for both fundamental and applied studies since they are compatible with silicon-based technology and, at the same time, have a record high hole mobility among all group IV semiconductors. As compared to silicon-based metal-oxide-semiconductor field-effect-transistor structures, they are also characterized by a strong spin-orbit coupling and a large and strongly anisotropic $g$-factor, which is of interest for the study of spin-related phenomena. However, the details of the band structure and the quantum transport in p-SiGe/Ge/SiGe systems have not yet been sufficiently explored. It is known, that due to the lattice constant mismatch the 2D hole channel is located in strained Ge so that, the ground subband is formed by heavy-hole (hh) states while the light-hole (lh) subband is split off by a hundred meV. The splitting should suppress the lh-lh mixing and lead to a strong anisotropy of the lh $g$-factor tensor with vanishingly small in-plane component $g_{zz}$. One can therefore expect that the transport properties of the hole channel in SiGe/Ge/SiGe are determined by the normal component of the magnetic field only, as they are in a strained p-type channel of AlAs/GaV semiconductors. Such a behavior has been observed in p-Ge/SiGe multilayer structures by studying the resistivity oscillations in tilted magnetic fields up to a tilt angle of $60^{\circ}$. What concerns us about the absolute value of the lh out-of-plane $g$-factor, $|g_z|$, is that the data available in literature vary from 20.4 for bulk Ge to 16.5 for strained Ge to 14.2 and 5.8 for p-Ge/GeSi multilayers, and to 1 for a single quantum well. These drastically different values may be the result of many-body effects and spectrum nonparabolicity, which are also responsible for the observed dependence of the in-plane effective mass on the hole density.

In this paper, we report a comprehensive study of the high-frequency conductivity of the high-mobility 2D hole gas embedded in a SiSiGe/Ge/SiGe structure in tilted magnetic fields. The measurements are carried out by means of the contactless surface acoustic wave technique. It probes the “bulk” electric properties of the two-dimensional system and provides information on the 2D hole energy spectrum unaffected by chiral edges which play a key role in conventional four-probe measurements of the quantum Hall effect. The experimental data allow us to determine the hole cyclotron mass from the temperature dependence of Shubnikov-de Haas (SdH) oscillations and the effective $g_z$ factor for our samples. We find that both the cyclotron mass and the $g_z$ factor can be tuned by applying an in-plane magnetic field. Raising the in-plane field component $B_\parallel$ leads to an increase of the cyclotron mass and decrease of the $g_z$ factor. We have developed a microscopic theory of these effects for the complex valence band of germanium and have shown that the theory describes well the experimental data.

II. SAMPLE AND METHOD

The experiments were carried out on a p-type SiGe/Ge/SiGe heterostructure with a single hole channel (sample K6016). The layer structure of the sample is illustrated in Fig. 1. The two-dimensional hole gas has a density of $p \approx 6\times10^{11}\text{cm}^{-2}$ and mobility...
\[ \mu \approx 6 \times 10^4 \text{ cm}^2/(\text{V-s}) \text{ at } 4.2 \text{ K.} \] The structure was grown by low-energy plasma-enhanced chemical vapor deposition (LEPECVD) on a Si(001) substrate by making use of the large dynamic range of growth rates. The buffer, graded at the rate of about 10%/\mu m to the final Ge content of 70%, and the 4-\mu m-thick Ge0.7Si0.3 layer were grown at the high rate of 5 \div 10 \text{ nm/s} gradually lowering the substrate temperature \( T_s \) from 720°C to 450°C. The active part, consisting of a 20-mm-thick pure Ge layer sandwiched between cladding layers with a Ge content of about 60%, and the Si cap were grown at the low rate of about 0.3 \text{ nm/s} at \( T_s = 450°C \). Modulation doping was achieved by introducing dilute diborane pulses into the cladding layer at a distance of about 30 nm above the channel. In the structure grown on a relaxed Si0.3Ge0.7 buffer layer, the Ge channel is compressively strained in the interface plane due to the lattice mismatch of about 1.2%, which leads to a splitting between the hh and lh subbands. Following Ref. 17, we estimate the hh-lh splitting \( \Delta \approx 100 \text{ meV} \) for our sample.

The properties of the 2D hole gas are studied by a contactless acoustoelectric method. The technique was first employed in Ref. 18 for GaAs/Al\(_{x}\)Ga\(_{1-x}\)As heterostructures. The experimental setup is illustrated in Fig. 1b. A surface acoustic wave (SAW) is excited on the surface of a piezoelectric LiNbO\(_3\) platelet by an inter-digital transducer. The SAW propagating along the lithium niobate surface induces a high-frequency electric field which penetrates into the hole channel located in the SiGe/Ge/SiGe structure slightly pressed to the piezoelectric platelet by means of springs. The field produces an ac electrical current in the channel. As the result of the interaction of the SAW electric field with holes, the wave attenuates and its velocity is modified, governed by the high-frequency conductivity \( \sigma_{ac} \). This “sandwich-like” experimental configuration enables contactless acoustoelectric experiments on non-piezoelectric 2D systems, such as SiGe/Ge/SiGe. The measurements were done at SAW frequencies of 30 and 85 MHz, in external magnetic fields \( B \) of up to 18 T, and in the temperature range of 0.3 – 5.8 K. The samples were mounted on a one-axis rotator, which enabled us to change the angle \( \Theta \) between the quantum well (QW) normal and the magnetic field.

**III. EXPERIMENTAL RESULTS**

Figure 2 shows the dependencies of the SAW attenuation change \( \Delta \Gamma \equiv \Gamma(B) - \Gamma(0) \) and normalized change of the SAW velocity \( \Delta v/v(0) \equiv [v(B) - v(0)]/v(0) \) on the magnetic field \( B \) applied along the QW normal \( z \). The dependencies are presented for different temperatures. All curves contain pronounced oscillations, which are caused by the formation of Landau levels in the two-dimensional hole gas and microscopically are similar to the oscillations of the magnetoconductivity in the Shubnikov-de Haas and quantum Hall effects. We note that \( \Gamma(0) \) is negligibly small compared to \( \Gamma(B) \) due to very high electric conductivity of the hole gas at zero magnetic field, \( \sigma(0) \approx 6 \times 10^{-3} \text{ Ω}^{-1} \).

From the experimentally measured values of the SAW absorption and the relative change of the SAW velocity, one can calculate the real \( \sigma_1 \) and imaginary \( \sigma_2 \) components of the high-frequency conductivity of the hole channel by using Eqs. (1) and (2) of Ref. 13. Below, we focus on the real part of the ac conductivity only since it enables us to determine the parameters of the energy spectrum. The corresponding dependence of \( \sigma_1 \) on the magnetic field for different temperatures is presented in Fig. 3. The magnetic field dependence of the conductivity contains the Shubnikov-de Haas oscillations evolving into the integer quantum Hall effect at strong fields. The positions of the even and odd filling factors \( \nu \) corresponding to the orbital and spin splitting of the Landau levels, respectively, are shown by vertical arrows.

Comparison of the temperature dependence of the conductivity at odd and even filling factors allows us to de-
The absolute value of the $g_z$ factor. The procedure is the following. For odd filling factors, we found a temperature range (0.5-5.8 K) where the conductivity in the oscillation minima is of activation nature and described by the Arrhenius law

$$\sigma_{1}^{\text{odd}} \propto \exp \left( \frac{-\Delta_{\text{odd}}}{2k_BT} \right).$$

Here $\Delta_{\text{odd}}$ is the activation energy and $T$ is the temperature. Thus, the slope of the linear dependence of $\ln \sigma_1$ on $1/T$ yields the activation energy. The corresponding Arrhenius plots for the filling factors $\nu=3$, 5, and 7 together with linear fits are presented in Fig. 3. The activation energy is given by $\Delta_{\text{odd}} = \Delta_Z - \Gamma_B$, where $\Delta_Z = |g_z| \mu_0 B_z$ is the Zeeman splitting, $\mu_0$ is the Bohr magneton, and $\Gamma_B$ is the Landau level broadening. The latter also depends on the magnetic field, the calculation in the self-consistent Bohr approximation yields $\Gamma_B = C \sqrt{\mu_0 B_z}$ with $C$ being the field-independent parameter $\frac{1}{19.20}$.

For even filling factors, the activation conductivity is also described by the Arrhenius law

$$\sigma_{1}^{\text{even}} \propto \exp \left( \frac{-\Delta_{\text{even}}}{2k_BT} \right),$$

where $\Delta_{\text{even}} = \hbar \omega_c - \Delta_Z - \Gamma_B$, $\hbar \omega_c = \hbar e B_z/(m_e c)$ is the energy spacing between the orbital Landau levels and $m_e$ is the cyclotron mass. The cyclotron mass for our Ge/SiGe structure $m_e \approx 0.1m_0$ is known with high accuracy from analysis of the Shubnikov-de Haas oscillations $\frac{15}{15}$. The Arrhenius plots for the even filling factors $\nu=10$, 12, and 14 together with linear fits are presented in Fig. 4.

The best fit of the activation conductivity for odd and even filling factors by Eqs. (1) and (2) yields $|g_z| = 6.7 \pm 0.3$ and $C = 0.64 \pm 0.06 \text{meV T}^{-1/2}$. The extracted dependence of the Zeeman splitting $\Delta_Z$ on the magnetic field is shown in the inset in Fig. 3. Note, that the theoretical estimation $C = \sqrt{2\hbar^2/(\pi m_e c \tau_q)}$ (see Ref. 20) yields the very close value $C \approx 0.69 \text{meV T}^{-1/2}$ for the quantum relaxation time $\tau_q \approx 1 \text{ps}$ calculated for our sample from the Shubnikov-de Haas oscillations at the temperature 1.7 K. The obtained value of the out-of-plane $g$-factor differs from that of the heavy holes in bulk Ge, $|g_{bulk}| = |6K| \approx 20.4$. We attribute the difference to renormalization of the energy spectrum in Ge quantum wells due to size quantization, strain, and interaction effects. The dispersion of heavy holes in quantum wells is typically non-parabolic so that the in-plane effective mass and $g$-factor depend on the Fermi energy. We also note that in the above analysis we neglected possible oscillations of the hole $g$-factor in the magnetic field due to exchange interaction. While the exchange contribution to the $g$-factor may be important for 2D electron systems, experimental results and theoretical analysis reveal that it is suppressed in GaAs-based hole systems $\frac{19}{19}, \frac{20}{20}$. The problem of the exchange interaction in strained Ge-based systems requires further study and is out of the scope of this paper.

![FIG. 3: (Color online) Dependence of $\sigma_1$ on the magnetic field at different temperatures for $B \parallel z$ and for a SAW frequency $f = 30 \text{MHz}$. The positions of integer filling factors are marked by arrows.](image)

![FIG. 4: (Color online) Dependence of $\ln \sigma_1$ on $1/T$ for (a) odd and (b) even filling factors. Lines are the result of linear fitting. Inset shows the obtained Zeeman splitting vs perpendicular magnetic field for odd filling factors.](image)
structures resulting in the splitting between the hh and lh subbands of about 100 meV which exceeds the Fermi energy of 14 meV. Therefore, the in-plane component of the hh \( g \)-factor vanishes and the Zeeman splitting is determined by \( B_\parallel \). Similar dependencies are also presented in Fig. 3, where the conductivity oscillations corresponding to small filling factors \( \nu = 2, 3 \), and 4 are shown as a function of the total magnetic field. With increasing the tilt angle, the positions of the oscillation minima are shifted towards higher magnetic fields while the oscillation amplitudes remain almost the same. Note, however, that study of conductivity at small filling factors requires high \( B_\parallel \) and therefore for \( \nu = 4 \) we were limited by 60° tilt angle.

![FIG. 5: (Color online) Dependence of \( \sigma_1 \) on the normal component \( B_z \) of the magnetic field for different tilt angles \( \Theta = (0 \pm 82)°; f = 30 \text{ MHz}, T = 0.3 \text{ K.} \)](image)

![FIG. 6: (Color online) (a) Dependence of \( \sigma_1 \) on the total magnetic field \( B_{\text{TOT}} \) for different tilt angles; (b) \( \sigma_1 \) vs \( B_z \) for the tilt angles \( \Theta = 0° \sim 80°; f = 30 \text{ MHz}, T = 0.3 \text{ K.} \) Arrows denote the positions of integer filling factors.](image)

The oscillation amplitude decreases. We emphasize that such a behavior is observed for both even and odd filling factors, see Fig. 4b. Therefore, it cannot be explained by changes in relative positions of the Landau levels since, in that case, the amplitudes of conductivity oscillations corresponding to even and odd \( \nu \) would change in antiphase. We attribute the observed features to the effect of the in-plane magnetic field on the hole cyclotron mass, \( g_\parallel \)-factor, and Landau level broadening in the complex valence band of germanium.

According to the Arrhenius law Eq. (1), the decrease of the oscillation amplitudes at odd filling factors indicates a decrease of the activation energy \( \Delta_{\text{odd}} \). We suggest that the decrease is caused by the reduction of the absolute value of the hole \( g \)-factor \( |g_z| \) and increase of the Landau level broadening in the in-plane magnetic field. To second order in \( B_\parallel \), the dependence of \( g_z \) and \( C \) on the magnetic field is given by

\[
g_z(B_\parallel) = g_z(0) + \alpha_s B_\parallel^2, \tag{3}
\]

\[
C(B_\parallel) = C(0) + \beta B_\parallel^2,
\]

where \( g_z(0) \) and \( C(0) \) are the \( g \)-factor and the parameter of the Landau level broadening at zero in-plane field, which are calculated above, \( \alpha_s \) and \( \beta \) are parameters. Below, see part IV, we present the microscopic theory of the Zeeman splitting of heavy-hole states in QWs and show that exactly such a dependence of the \( g_\parallel \)-factor on the in-plane field follows from the theory. Equations (1) and (3) yield

\[
\ln \sigma_i^{\text{odd}}(B_\parallel) = \ln \sigma_i^{\text{odd}}(0) + \alpha_s \mu_0 B_z + \beta \sqrt{B_z^2 - B_\parallel^2}, \tag{4}
\]

where we take into account that \( g_z < 0 \) for heavy holes in Ge, and \( \sigma_i^{\text{odd}}(0) \) is the conductivity in perpendicular magnetic field for given odd filling factor (an in-plane field independent term). To determine the field corrections to both the \( g_z \) factor and Landau level broadening we plot in Fig. 4 the dependence of \( \ln \sigma_i(B_\parallel) \) on \( B_\parallel \) for \( \nu = 5 \) and 7 in the range of activation behavior of conductivity. The dependencies are linear, as expected, and yield \( \alpha_s^{(\exp)} \approx 1.4 \times 10^{-3} \text{ T}^{-2}, \beta^{(\exp)} \approx 8 \times 10^{-5} \text{ meV T}^{-5/2} \). For the magnetic field \( B_z = 5.04 \text{ T} \) corresponding to \( \nu = 5 \) the effect of \( B_\parallel \) on the Zeeman splitting is more than twice as large as the effect of \( B_\parallel \) on the level broadening.

The oscillations corresponding to even filling factors are also damped out with increasing the tilt angle \( \Theta \), see Fig. 4a. This indicates a decrease of the activation energy \( \Delta_{\text{even}} \) which can be attributed to the increase of the cyclotron mass \( m_\parallel \) and the Landau level broadening by the in-plane component of the magnetic field. Similar behavior was observed for \( n \)-type 2D systems in a number of papers and ascribed to the increase of \( m_\parallel \). To second order in \( B_\parallel \), the effect is phenomenologically described by

\[
\frac{m_\parallel}{m_\parallel(B_\parallel)} = \frac{m_\parallel}{m_\parallel(0)} - \alpha_c B_\parallel^2, \tag{5}
\]
where \( m_0 \) is the free electron mass and \( \alpha_e \) is a parameter. From the Arrhenius law at even filling factors, Eq. 2, we obtain

\[
\ln \sigma_1^{\text{even}}(B) = \ln \sigma_1^{\text{even}}(0) + \frac{2(\alpha_e - \alpha_s)B_2 + \beta B_2^2}{2k_BT},
\]

where \( \sigma_1^{\text{even}}(0) \) is the conductivity in perpendicular magnetic field for given even filling factor (the term independent of \( B_2^2 \)).

Figure 7 shows the dependence of \( \ln \sigma_1 \) on \( B_2^2 \) measured at different \( B_2 \) corresponding to the filling factors \( \nu = 10, 12, \) and 14. In accordance with Eq. 6, the dependencies are linear. Fitting the experimental data by Eq. 6, we obtain \( \alpha_e^{(\text{exp})} \approx 6 \times 10^{-3} \ T^{-2} \). Comparing \( 2\alpha_e^{(\text{exp})} \) with \( \alpha_s^{(\text{exp})} \), we conclude that the dominant contribution to the variation of \( \Delta_\text{even} \) is due to change of the cyclotron mass.

Below we calculate the parameters \( \alpha_s \) and \( \alpha_e \) determining the corrections to the spin splitting and cyclotron mass, respectively, for the heavy-hole subband in QWs and compare them with the values obtained from our experiment.

### IV. THEORY

We describe the effect of the in-plane magnetic field on the cyclotron mass and \( g_z \) factor in the framework of the Luttinger model. In the axial approximation, the effective Hamiltonian of holes in Ge quantum wells in an external magnetic field has the form

\[
\mathcal{H} = H_0 + U(z) + H_Z + V,
\]

where \( H_0 \) is the Luttinger Hamiltonian for zero in-plane momentum,

\[
H_0 = \frac{1}{2m_0} \left( \gamma_1 + \frac{5}{2} \gamma \right) I - 2\gamma J_z^2 p_z^2,
\]

\( \gamma_1 \) and \( \gamma \) are the Luttinger parameters, \( J_i \) \((i = x, y, z)\) are the 4x4 matrices of the angular momentum \( 3/2 \), \( I \) is the identity matrix, \( p_z = -i\hbar \partial / \partial z \), \( U(z) \) is the diagonal matrix of confinement potentials which are different for the heavy-hole and light-hole subbands due to strain, \( H_Z \) is the Zeeman Hamiltonian,

\[
H_Z = -2\mathcal{K}\mu_0 \mathbf{J} \cdot \mathbf{B},
\]

\( \mathcal{K} \) is the parameter of the Zeeman splitting of hole states at the \( \Gamma \) point of the Brillouin zone in bulk material, \( V \) is the contribution to the Luttinger Hamiltonian accounting for the in-plane momentum and magnetic field,

\[
V = \frac{1}{2m_0} \left( \gamma_1 + \frac{5}{2} \gamma \right) I \left( P_x^2 + P_y^2 \right) - \frac{\gamma}{m_0} \left( J_x^2 P_x^2 + J_y^2 P_y^2 + 2\{J_z J_y\} \{P_x P_y\} \right),
\]

\( \mathbf{P} = -i\hbar \nabla - (e/c)\mathbf{A} \), \( e > 0 \) is the hole charge, \( \mathbf{A} \) is the vector potential of the magnetic field \( \mathbf{B} \), and the braces denote the symmetrized product \( \{CD\} = (CD + DC)/2 \). Other possible contributions to the effective Hamiltonian beyond the Luttinger model, e.g., \( \propto B^3 \) or \( \propto B^4 \), do not seem to give a substantial contribution to the cyclotron mass and \( g_z \) factor in moderate in-plane magnetic fields.

We calculate the hole energy spectrum in the symmetric heterostructure subjected to the magnetic field in two steps. First, we solve the Schrödinger equation for the case of \( H_Z = V = 0 \) and find the envelope functions \( \varphi_{mn}(z) \) and \( \varphi_{lm}(z) \) and energies \( E_{hn} \) and \( E_{lm} \) of the heavy-hole and light-hole states, respectively, where \( n, m = 1, 2, \ldots \) are the subband indices. Each state is two-fold degenerate. Then we use perturbation theory and calculate the in-plane effective mass for the \( h1 \) subband, which becomes anisotropic in the presence of the in-plane magnetic field, as well as the Zeeman splitting at the subband bottom. We assume that the magnetic
field $\mathbf{B}$ is oriented in the $(yz)$ plane and choose the vector potential in the form $\mathbf{A} = (zB_y, xB_z, 0)$. The corrections to the in-plane mass and the Zeeman splitting are proportional to $B_y^2$ and emerge in the second, third, and fourth orders of the perturbation theory. The knowledge of the in-plane effective masses and the Zeeman splitting enables us to obtain the quasi-classical structure of the Landau levels.

The calculation shows that the energy spectrum in the subband $h_1$ has the form

$$E_{h_{1,\pm}}^{(N)} = \frac{\hbar e}{c m_c(B_y)} (N + 1/2) \pm \frac{\mu_0 g_z(B_y)B_y}{2}$$.

where $m_c(B_y) = \sqrt{m_x(B_y)m_y(B_y)}$ is the cyclotron mass, $m_x(B_y)$ and $m_y(B_y)$ are the in-plane effective masses in the directions perpendicular and parallel to the in-plane component of the magnetic field, respectively, and $N$ is an integer number. The in-plane component of the $g$-factor tensor vanishes at $B = 0$ in the uniaxial approximation$^{28}$ and therefore gives only higher order corrections ($\propto B^5$) to the Zeeman splitting for tilted magnetic fields.

The $g_z$ factor determining the spin splitting of Landau levels is given by Eq. (3) where the value in the perpendicular field $g_z(0)$ has the form$^{28}$

$$g_z(0) = -6\gamma + \frac{12\gamma^2}{m_0^2} \sum_n \frac{|p_{h_{1,ln}}|^2}{E_{ln} - E_{h1}}$$

$\alpha_s = \frac{12\gamma^2 e^2}{m_0 c^2} \sum_{n,m,k} \left[ \frac{z_{h_{1,ln}}^2}{E_{ln} - E_{h1}} + \frac{\gamma_1 + \gamma(z^2)_{h_{1,ln}}|p_{h_{1,ln}}|^2}{2 m_0 (E_{ln} - E_{h1})^2} - \frac{\gamma_1 - \gamma}{m_0} \left< \frac{p_{h_{1,ln}}}{E_{h1}(E_{ln} - E_{h1})} \right> \left< \frac{p_{2h_{1,ln}}}{E_{ln} - E_{h1}} \right> \left< \frac{p_{h_{1,ln}}}{E_{ln} - E_{h1}} \right> \left< \frac{p_{2h_{1,ln}}}{E_{ln} - E_{h1}} \right> \left< \frac{p_{h_{1,ln}}}{E_{ln} - E_{h1}} \right> \left< \frac{p_{2h_{1,ln}}}{E_{ln} - E_{h1}} \right> \right]$$.

Here we take into account that the matrix elements $z_{\mu n,\mu' n'} = \left< \mu n | z | \mu' n' \right>$ and $z_{\mu n,\mu' n'} = \left< \mu n | z^2 | \mu' n' \right>$ are real while the matrix elements $p_{\mu n,\mu' m}$ are purely imaginary ($\mu, \mu' = h, l$). The major contribution to $\alpha_s$ comes typically from the terms containing only one light-hole energy $E_{h0}$ in the denominator.

Figure 6 shows the dependence of $\alpha_s$ responsible for the $g_z$ factor renormalization on the Luttinger parameters calculated after Eq. (13) for strain-free rectangular QWs with infinitely high barriers. In this model, the dependence of $\alpha_s$ on the QW width $a$ is simplified to \( \alpha_s \propto a^4 \). The curves in Fig. 6 are plotted for $a = 200$ Å. It follows from the calculation that the correction to the $g_z$ factor caused by the in-plane magnetic field depends on the material parameters.

The in-plane masses are given by

$$\frac{m_0}{m_{x,y}(B_y)} = \frac{m_0}{m_\parallel} - (\alpha_s \pm \delta)B_y^2$$.

where $m_\parallel$ is the in-plane mass at zero magnetic field,

$$\frac{1}{m_\parallel} = \frac{\gamma_1 + \gamma}{m_0} - \frac{6\gamma e^2}{m_0^2} \sum_n \frac{|p_{h_{1,ln}}|^2}{E_{ln} - E_{h1}}$$.

The parameter $\alpha_c$, which determines renormalization of the cyclotron mass in the in-plane field, and the parameter $\delta$ describing the mass anisotropy have the form

$$\alpha_c = \alpha_s/2 + \xi + \xi_1$$, \hspace{1cm} \delta = \xi + \xi_2$$,

where $\alpha_s$ is given by Eq. (13).
The coupling of the cyclotron to the cyclotron mass renormalization comes from the Luttinger parameters in strain-free QW of width and α. We note that the dominant contribution of the magnetic field given by the first term in ξ, Eq. (15), and is similar to that in electron systems. However, in contrast to the conduction band where the effective mass is modified only for the direction perpendicular to the magnetic field, in hole systems both components of the effective mass tensor are renormalized.

Equations (13) and (16) enable one to calculate the renormalizations of the cyclotron mass and Zeeman splitting in hole systems caused by the in-plane magnetic field.

V. DISCUSSION AND SUMMARY

The experimental results discussed above demonstrate that, in SiGe/Ge/SiGe quantum wells, the in-plane component of the magnetic field leads to a decrease of the effective g\textsubscript{c} factor and to an increase in the hole cyclotron mass. The effects are described by Eqs. (8) and (10), respectively, which also follow from the microscopic theory, with the fitting parameters α\textsubscript{s}\textsuperscript{(exp)} ≈ 1.4 × 10\textsuperscript{-3} T\textsuperscript{-2} and α\textsubscript{c}\textsuperscript{(exp)} ≈ 6 × 10\textsuperscript{-3} T\textsuperscript{-2}.

Figure 10 shows the theoretical dependence of the coefficients α\textsubscript{c} and α\textsubscript{s} describing the effective mass and g\textsubscript{c} factor renormalization, respectively, on the QW width. The curves are calculated for a rectangular Ge quantum well with the infinitely high barriers, taking into account the strain-induced splitting of the hh and lh subbands (solid curves) and neglecting the strain (dashed curves). Both α\textsubscript{c} and α\textsubscript{s} increase with the QW width. The coefficient α\textsubscript{c} is almost independent of strain since its domi-
nent part is determined by the structure of the hh subbands only. In contrast, the Zeeman splitting renormalization occurs due to the mixing of the hh and lh subbands and $\alpha_s$ is therefore sensitive to the hh-lh splitting induced by strain. The dependence of $\alpha_s$ on strain is more pronounced for wide QWs where the hh-lh splitting due to size quantization is comparable to or smaller than the strain-induced splitting. The absolute values of $\alpha_c$ and $\alpha_s$ extracted from the experiment correspond to the calculated values for the QW width of about 100 Å according to Fig. 10. The nominal QW width in the studied sample is 200 Å, but the effective length of the hole confinement might be considerably smaller due to the built-in electric field produced by ionized dopants incorporated in the barrier above the QW (see Sec. II). This electric field pushes the holes to the upper interface thereby reducing the effective QW width.

To summarize, we have performed contactless acoustoelectric measurements of the high-frequency conductivity of the two-dimensional hole gas in a p-SiGe/Ge/SiGe structure subjected to a strong magnetic field. It has been shown that in certain temperature domains and integer filling factors the conductivity is of the activation nature. The analysis of the activated conductivity at odd and even filling factors in perpendicular magnetic field enabled us to determine $|g_z| \approx 6.7$. By applying a tilted magnetic field we observed that at fixed normal component of the field, the conductivity in oscillation minima increases with increasing $\Theta$ ($\Theta > 60^\circ$) at both even and odd filling factors for $\nu \geq 5$. Such a behavior is attributed to the decrease of the cyclotron frequency and Zeeman splitting of holes and the broadening of the Landau levels by the in-plane component of the magnetic field. We have developed a microscopic theory of the heavy-hole cyclotron mass and $g_z$ factor renormalization in the framework of the Luttinger Hamiltonian. This theory describes the experimental data and predicts the renormalization to be more pronounced in wide quantum wells.

The authors would like to thank E. Palm, T. Murphy, J.-H. Park, and G. Jones for technical assistance and V.A. Volkov and V.S. Khrapai for useful discussions. This work was supported by Russian Foundation for Basic Research, the Program “Spintronika” of Branch of Physical Sciences of RAS, and the U.M.N.I.K grant 16906. National High Magnetic Field Laboratory is supported by NSF Cooperative Agreement No. DMR-1157490, the State of Florida, and the U.S. Department of Energy.

Acknowledgments

1  H. von Känel, M. Kummer, G. Isella, E. Müller, T. Hackbarth, Appl. Phys. Lett. 80, 2922 (2002).
2  E.L. Ivchenko and G.E. Pikus, Superlattices and Other Heterostructures: Symmetry and Optical Phenomena (Springer, Berlin, 1997).
3  S.A. Tarasenko, Fiz. Tverd. Tela 44, 1690 (2002) [Phys. Solid State 44, 1769 (2002)].
4  R.W. Martin, R.J. Nicholas, G.J. Rees, S.K. Haywood, N.J. Mason, and P.J. Walker, Phys. Rev. B 42, 9237 (1990); Proc. of the 20th Int. Conf. on the Physics of Semiconductors, Thessaloniki v.2, 909, Thessaloniki (1990). Tilting done at angles of 64°.
5  S.I. Dorozhkin, Sol. St. Comm. 72, 211 (1989), Phys. Rev. B 61, 7803 (2000).
6  S.Y. Lin, H.P. Wei, D.C. Tsui, J.F. Klem, S.J. Allen Jr., Phys. Rev. B 43, 12110 (1991). Tilting done at angles of 49.5°.
7  N.A. Gorodilov, O.A. Kuznetsov, L.K. Orlov, R.A. Rubtsova, A.L. Chernov, N.G. Shelushina, G.L. Shtrapenin, Pis’ma Zh. Eksp. Teor. Fiz. 56, 409 (1992) [JETP Letters 56, 394 (1992)].
8  Yu.G. Arapov, N.A. Gorodilov, O.A. Kuznetsov, V.N. Neverov, L.K. Orlov, R.A. Rubtsova, G.I. Kharus, A.L. Chernov, N.G. Shelushina, G.L. Shtrapenin, Fiz. Tekh. Poluprovodn. 27, 1165 (1993) [Semiconductors 27, 642 (1993)].
9  Yu.G. Arapov, V.N. Neverov, G.I. Harus, N.G. Sheleshina, M.V. Yakunin, O.A. Kuznetsova, L. Ponomarenko and A. de Visser, Low Temp. Phys. 30, 867 (2004). [Fiz. Nizk. Temp. 30, 1157 (2004)].
10  A.V. Nenashev, A.D. Vdvurechenkii, and A.F. Zinovieva, Phys. Rev. B 77, 205301 (2003).
11  A.V. Chernenko, N.G. Kalugin, O.A. Kusnetsov, JETP 87, 337 (1998) [Zh. Eksp. Teor. Fiz. 114, 619 (1998)].
12  R. Moriya, Y. Hoshi, Y. Inoue, S. Masubuchi, K. Sawano, Y. Shiraki, N. Usami, T. Machida, Bull. APS 58, 2922 (2003).
13  T. Irisawa, M. Myronov, O.A. Mironov, E.H.C. Parker, K. Nakagawa, M. Murata, S. Koh, Y. Shiraki, Appl. Phys. Lett. 82, 1425 (2003).
14  B. Rössner, B. Batlogg, H. von Känel, D. Christina, G. Isella, Mat. Sc. Semicond. Proc. 9, 777 (2006).
15  I.I. Drichko, A.M. Diakonov, E.V. Lebedeva, I.Yu. Smirnov, O.A. Mironov, M. Kummer, and H. von Känel, J. Appl. Phys. 106, 094305 (2009).
16  C. Rosenblad, H.R. Deller, A. Dommann, T. Meyer, P.
The vanishingly small value of the in-plane $g$-factor $g_\parallel$ is additionally supported by the fact that no change in the ac conductivity is observed in the pure in-plane magnetic field.

V. S. Khrapai, E. V. Deviatov, A. A. Shashkin, V. T. Dolgopolov, Proc. NGS 10 IPAP Conf. Series 2, 105 (2001).

V. E. Kozlov, S. I. Gubarev, I. V. Kukushkin, JETP Letters 94, 397 (2011) [Pis'ma v Zh. Eksp. Teor. Fiz. 94, 429 (2011)].

A. T. Hatke, M. A. Zudov, L. N. Pfeiffer, K. W. West, Phys. Rev. B 85, 241305(R) (2012).

X. Marie, T. Amand, P. Le Jeune, M. Paillard, P. Renucci, L. E. Golub, V. D. Dymnikov and E. L. Ivchenko, Phys. Rev. B 60, 5811 (1999).

Th. Wimbauer, K. Oettinger, A. L. Efros, B. K. Meyer, and H. Brugger, Phys. Rev. B 50, 8889 (1994).