Abstract  We show that dark energy and dark matter can be described simultaneously by ordinary Einstein gravity interacting with a single scalar field provided the scalar field Lagrangian couples in a symmetric fashion to two different spacetime volume-forms (covariant integration measure densities) on the spacetime manifold – one standard Riemannian given by $\sqrt{-g}$ (square-root of the determinant of the pertinent Riemannian metric) and another non-Riemannian volume-form independent of the Riemannian metric, defined in terms of an auxiliary antisymmetric tensor gauge field of maximal rank. Integration of the equations of motion of the latter auxiliary gauge field produce an a priori arbitrary integration constant that plays the role of a dynamically generated cosmological constant or dark energy. Moreover, the above modified scalar field action turns out to possess a hidden Noether symmetry whose associated conserved current describes a pressureless “dust” fluid which we can identify with the dark matter completely decoupled from the dark energy. The form of both the dark energy and dark matter that results from above class of models is insensitive to the specific form of the scalar field Lagrangian. By adding appropriate perturbation, which breaks the above hidden symmetry and along with this it couples dark matter and dark energy, we also suggest a way to obtain growing dark energy in the present universe’s epoch without evolution pathologies.

Keywords  modified gravity theories, non-Riemannian volume forms, $\Lambda$-CDM, Noether symmetries

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1 Introduction

The late time Universe appears to be dominated by two components, both of them “non-luminous” or “dark”. The dominant contribution, about 70% of the energy density of the universe is well described by a cosmological constant term, as introduced originally by Einstein and has been also given the name “dark energy”. This contribution leads to gravitational repulsion. The cosmological constant or dark energy is not diluted by the expansion of the universe. The other subdominant contribution, about 25% of the energy density of the universe is well described by a pressureless fluid, which is called “dark matter”. As opposed to the dark energy it is gravitational attractive and it gets diluted by the universe expansion, it can form structures, etc.

Dark energy was observationally discovered rather recently through the observation of type Ia supernova [1].

Dark matter was first postulated in the 1930s, separately by J. Oort and F. Zwicky, due to the anomaly of the orbital velocity of some stars in the Milky Way galaxy and the orbital velocity of galaxies in clusters. A recent review of dark matter is given in Ref.[2], reviews of dark energy can be found in [3] and a review of both dark matter and dark energy in [4].

In this paper we study a class of models providing a unified description of dark energy and dark matter starting from a well-defined gravity-scalar-field Lagrangian action constructed by means of both standard Riemannian as well as an alternative non-Riemannian (i.e., independent of the pertinent Riemannian metric) volume forms (covariant integration measure densities) on the spacetime manifold. The introduction of such modified “two-measure” gravity-matter theory (the general class of “two-measure” gravity models was origi-
nally proposed in Refs. [5] opens the possibility to obtain both dark energy and dark matter from a single scalar field, as it was already observed in [6]. This was further generalized in [7] by the inclusion of another field with phantom-like kinetic energy so as to produce growing dark energy. In the present paper we will achieve growing dark energy in a different way that does not invoke phantom kinetic terms and without introducing additional fields.

In a recent paper [8] a model providing unifying description of dark energy and dark matter was proposed by studying thermodynamics of cosmological systems where a constraint on the pressure being a constant was introduced from the very beginning. In the present case we start from a well-defined Lagrangian action principle for a modified gravity-scalar-field system which produces systematically the constant pressure constraint in a self-consistent dynamical way as part of the pertinent equations of motion.

Here we will proceed to discover the fundamental reasons how modified gravity-matter models, generalizing those studied in Ref. [6], succeed to describe simultaneously both dark matter and dark energy. We find that this is realized due to:

(i) The existence of a hidden (strongly nonlinear) Noether symmetry of the underlying single scalar field Lagrangian, that implies a conservation law from which it follows that there is conserved current giving rise to the dark matter component.

(ii) An a priori arbitrary integration constant appears in a dynamical constraint on the scalar field Lagrangian, which plays the role of a dynamically generated cosmological constant and provides the dark energy component. The fact that the latter arises from an integration constant makes the observed vacuum energy density totally decoupled from the parameters of the matter Lagrangian.

Both fundamental features (i)-(ii) arise in a way completely independent of the specific form of the scalar field Lagrangian and the details of the scalar field dynamics.

Other treatments that unify dark energy and dark matter have appeared before, for example, the Chaplygin gas models [9,10].

More recently, a “mimetic” dark matter model was proposed [11] based on a special covariant isolation of the conformal degree of freedom in Einstein gravity, whose dynamics mimics cold dark matter as a pressureless “dust”. Also, the cosmological implications of the “mimetic” matter were studied in some detail (second Ref. [11]). For further generalizations and extensions of “mimetic” gravity, see Refs. [12].

Models of explicitly coupled dark matter and dark energy described in terms of two different scalar fields were proposed in Ref. [13].

As a final introductory remark let us briefly describe the usefulness of employing the formalism based on alternative non-Riemannian spacetime volume-forms, i.e., alternative covariant integration measure densities in gravity-matter Lagrangian actions independent of the pertinent Riemannian metric. The latter have profound impact in any field theory models with general coordinate reparametrization invariance – general relativity and its extensions, strings and (higher-dimensional) membranes as already studied in a series of previous papers on this subject [5,14,15].

Although formally appearing as (almost) “pure-gauge” dynamical degrees of freedom [1] the non-Riemannian volume-form fields trigger a number of remarkable physically important phenomena:

– Non-Riemannian volume-form formalism in gravity-matter theories naturally generates a dynamical cosmological constant as an arbitrary dimensionful integration constant. At this point it resembles the earlier proposed unimodular gravity formulated as a fully generally covariant theory within the framework of Dirac’s constraint Hamiltonian method [13].

3 Unimodular gravity became further an object of active studies – for the latest developments, especially path integral quantization, equivalence with the fully diffeomorphism invariant formulation, and further references, see [21]. On the other hand, the non-Riemannian volume-form approach goes well beyond the dynamical cosmological constant generation and has significantly broader scope. Namely, unimodular gravity in its generally covariant form (Eq.(18) in [13], which appears as a particular case of a gravity theory with a non-Riemannian volume-form) is equivalent to standard general relativity (on classical level, except that the cosmological constant is an integration constant). On the other hand, generic non-Riemannian-volume-form-modified gravity theories are non-trivial extensions to general relativity; see also the next points here below.

– Employing two different non-Riemannian volume-forms generates several independent arbitrary integration constants leading to the construction of a new class of gravity-matter models, which produce

\footnote{For a detailed canonical Hamiltonian analysis a’la Dirac of gravity-matter theories with several independent non-Riemannian spacetime volume-forms, see [15] and Appendix A in [17]; see also Section 2 below for the simple case of one non-Riemannian volume form.}

\footnote{The original idea of unimodular gravity is in Einstein’s works [19]; in more modern context it appeared in [20].}
an effective scalar potential with two infinitely large flat regions [22,16]. This allows for a unified description of both early universe inflation as well as of present dark energy epoch.

- A remarkable feature is the existence of a stable initial phase of non-singular universe creation preceding the inflationary phase – stable "emergent universe" without "Big-Bang" [22].

- Within non-Riemannian-modified-measure minimal $N = 1$ supergravity the dynamically generated cosmological constant triggers spontaneous supersymmetry breaking and mass generation for the gravitino (supersymmetric Brout-Englert-Higgs effect) [23]. Applying the same non-Riemannian volume-formalism to anti-de Sitter supergravity allows to produce simultaneously a very large physical gravitino mass and a very small positive observable cosmological constant [23] in accordance with modern cosmological scenarios for slowly expanding universe of the present epoch [1].

- Adding interaction with a special nonlinear ("square-root" Maxwell) gauge field (known to describe charge confinement in flat spacetime) produces various phases with different strength of confinement and/or with deconfinement, as well as gravitational electrovacuum “bags” partially mimicking the properties of MIT bags and solitonic constituent quark models (for details, see [24]).

In Section 2 we briefly describe the basics of the non-Riemannian volume-form (modified measure) approach, including elucidating the meaning of the dynamically generated cosmological constant (i.e., dark energy appearing as an arbitrary integration constant in a dynamical constraint on the scalar field Lagrangian) from the point of view of the canonical Hamiltonian formalism. In Section 3 we derive the hidden symmetry and the associated Noether conserved current of the present modified-measure gravity-scalar-field model leading to the "dust-fluid" interpretation of a part of the scalar field energy density, i.e., dark matter. In Section 4 few implications for cosmology are considered. We briefly discuss perturbing our modified-measure gravity-scalar-field model which breaks the above crucial hidden symmetry and triggers (upon appropriate choice of the perturbation) a growing dark energy in the present day universe' epoch without invoking any pathologies of "cosmic doomsday" or future singularities kind [24].

Our concluding remarks are contained in the last Section 5.

2 Gravity-Matter Formalism With a Non-Riemannian Volume-Form

Our starting point is the following non-conventional gravity-scalar-field action (for simplicity we use units with the Newton constant $G_N = 1/16\pi$):

$$S = \int d^4 x \sqrt{-g} R + \int d^4 x (\sqrt{-g} + \Phi(B)) L(\varphi, X) \ ,$$

(1)

which in fact is a simple particular case of the general class of the so called "two-measure" gravity-matter theories proposed more than a decade ago [5]. The notations we are using are as follows:

- The first term in (1) is the standard Hilbert-Einstein action; $\sqrt{-g} \equiv \sqrt{-\det g_{\mu\nu}}$ is the standard Riemannian integration measure density with $g_{\mu\nu}$ being the standard Riemannian spacetime metric.

- $\Phi(B)$ denotes an alternative non-Riemannian generally covariant integration measure density defining an alternative non-Riemannian volume form on the pertinent spacetime manifold:

$$\Phi(B) = \frac{1}{3!} \epsilon^{\mu\nu\kappa\lambda} \partial_\mu B_{\nu\kappa\lambda} \ ,$$

(2)

where $B_{\mu\nu\lambda}$ is an auxiliary maximal rank antisymmetric tensor gauge independent of the Riemannian metric.

$B_{\mu\nu\lambda}$ [2] will also be called "measure gauge field" [1].

- $L(\varphi, X)$ is general-coordinate invariant Lagrangian of a single scalar field $\varphi(x)$ of a generic "k-essence" form [20]:

$$L(\varphi, X) = \sum_{n=1}^{N} A_n(\varphi) X^n - V(\varphi) \ ,$$

(3)

$$X \equiv -\frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi \ ,$$

i.e., a nonlinear (in general) function of the scalar kinetic term $X$.

Varying (1) w.r.t. $g^{\mu\nu}$, $\varphi$ and $B_{\mu\nu\lambda}$ yield the following equations of motion, respectively:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{1}{2} T_{\mu\nu} \ ,$$

(4)

$$T_{\mu\nu} = g_{\mu\nu} L(\varphi, X) + \left(1 + \frac{\Phi(B)}{\sqrt{-g}}\right) \frac{\partial L}{\partial \varphi} \partial_\mu \varphi \partial_\nu \varphi \ ;$$

(5)

In the original papers [5] an alternative parametrization of $B_{\mu\nu\lambda}$ through 4 auxiliary scalar fields \( \phi^i \) \( i = 1,...,4 \) was used: $B_{\mu\nu\lambda} = \frac{1}{2} \epsilon_{IJKL} \phi^i \partial_\mu \phi^i \partial_\nu \phi^j \partial_\kappa \phi^j \partial_\lambda \phi^j $, so that $\Phi(B) = \frac{1}{7!} \epsilon^{\mu\nu\kappa\lambda \epsilon \iota \mu \kappa \lambda} \partial_\phi \phi^i \partial_\nu \phi^j \partial_\iota \phi^\iota \partial_\kappa \phi^\iota \partial_\lambda \phi^\iota \partial_\nu \phi^j = \det [\frac{2\phi}{\lambda}]$. In a recent study [25] of general relativity as an extended canonical gauge theory a similar Jacobian representation of the covariant integration measure has appeared in terms of additional scalar fields. However, unlike the present case in the construction of Ref. [25] the additional scalar fields enter also in the proper Lagrangian.
\begin{align}
\frac{\partial L}{\partial \dot{\varphi}} + (\Phi(B)+\sqrt{-g})^{-1} \partial_{\mu} \left[ (\Phi(B)+\sqrt{-g})g^{\mu \nu} \partial_{\nu} \frac{\partial L}{\partial \varphi} \right] &= 0 ; \tag{6}
\end{align}

\partial_{\mu} L(\varphi, X) = 0 \quad \rightarrow \quad L(\varphi, X) = -2M = \text{const} , \tag{7}

where \( M \) is arbitrary integration constant\(^4\) (the factor 2 is for later convenience).

Already at this point it is important to stress that the scalar field dynamics is determined entirely by the first-order differential equation - the dynamical constraint Eq.\((7)\) \((X - V(\varphi)) = -2M \) in the simplest case of \((3)\). The standard second order differential equation \((6)\) is in fact a consequence of \((7)\) together with the energy-momentum conservation \( \nabla^\mu T_{\mu \nu} = 0 \).

The physical meaning of the “measure” gauge field \( B_{\mu \nu \lambda} \) as well as the meaning of the integration constant \( M \) are most straightforwardly seen within the canonical Hamiltonian treatment of \((\text{the scalar field part of})\)\(^4\) \((4)\). Namely, upon introducing the short-hand notations:

\[ \Phi(B) = \partial_{\mu} B^\mu = B + \partial_i B^i , \]

\[ B \equiv B^0 = \frac{1}{3!} \epsilon^{mkl} B_{mkl} , \quad B^i \equiv -\frac{1}{2} i^{kl} B_{0kl} , \tag{8} \]

we have for the canonically conjugated momenta \( \pi_B, \pi_{B^i} \) and \( p_\varphi \) w.r.t. \( B, B^i \) and \( \varphi \):

\[ \pi_{B^i} = 0 \quad , \quad \pi_B = L(\varphi, X) , \]

\[ p_\varphi = (B + \partial_i B^i + \sqrt{-g}) \frac{\partial L}{\partial \dot{\varphi}} . \tag{9} \]

The first relations in \((9)\) represent primary Dirac first-class constraints and, therefore, their canonically conjugate coordinates \( B^i \) (“electric”) component of the auxiliary “measure” gauge field \( B_{\mu \nu \lambda} \), cf. \((8)\)) are pure-gauge degrees of freedom – in fact they are Lagrange multipliers for secondary Dirac first-class constraints (see Eq.\((14)\) below). From the second relation in \((9)\) we obtain the velocity \( \dot{\varphi} = \varphi'(\varphi, \pi_B) \) as function of the canonical variables (in the simplest case of \((3)\) \( L(\varphi, X) = X - V(\varphi) \)):

\[ \dot{\varphi} = N^i \partial_i \varphi + N \sqrt{h^{ij} \partial_i \varphi \partial_j \varphi + 2(V(\varphi) + \pi_B)} , \tag{10} \]

where we have used the standard ADM parametrization for the Riemannian metric:

\[ ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt) . \tag{11} \]

Finally, from the last relation in \((9)\) we obtain the velocity \( B \) as a function of the canonical variables. Thus, inserting \((10)\) and the second relation \((9)\) in the expression for the canonical scalar field Hamiltonian:

\[ \mathcal{H}_m = p_\varphi \dot{\varphi} + \pi_B B - (B + \partial_i B^i + \sqrt{-g}) L(\varphi, X) \tag{12} \]

we arrive at the result:

\[ \mathcal{H}_m = N^i (\partial_i \varphi p_\varphi) + N \left[ p_\varphi \sqrt{h^{ij} \partial_i \varphi \partial_j \varphi + 2(V(\varphi) + \pi_B)} - \sqrt{h} \pi_B \right] - \partial_i B^i \pi_B . \tag{13} \]

i.e., scalar field canonical Hamiltonian being linear combination of first-class constraints only.

The last term in \((13)\) shows that \( B^i \) are canonical Lagrange multipliers for the secondary Dirac first-class constraints:

\[ \partial_i \pi_B = 0 \quad \rightarrow \quad \pi_B = \text{const} \equiv -2M . \tag{14} \]

The latter implies that also \( B \) (the “magnetic” component of the auxiliary “measure” gauge field \( B_{\mu \nu \lambda} \), cf. \((8)\)) is a pure-gauge degree of freedom. Clearly, Eqs. \((14)\) are the canonical Hamiltonian analog of Eq.\((7)\) within the Lagrangian formalism. Therefore, the meaning of the arbitrary integration constant \( 2M \) is the minus value of conserved Dirac-constrained canonical momentum conjugated to the “pure-gauge” magnetic component of the “measure” gauge field \( B_{\mu \nu \lambda} \). Moreover, the second term in \((13)\) shows that \( M \) plays the role of a dynamically generated cosmological constant.

Adding the well-known canonical Hamiltonian of the Hilbert-Einstein action (upto a total derivative term \((28)\)) the total canonical Hamiltonian of the gravity-scalar-field model \((1)\) is the following linear combination of the first-class constraints:

\[ \mathcal{H}_{\text{total}} = N^i \mathcal{H}_i + N \mathcal{H}_0 - \partial_i B^i \pi_B \tag{15} \]

\[ \mathcal{H}_i \equiv -2D_j \pi^j + \partial_i \varphi p_\varphi , \tag{16} \]

\[ \mathcal{H}_0 \equiv \frac{1}{\sqrt{h}} \left( \pi_{ij} \pi^{ij} - \frac{1}{2} (\pi_i^2)^2 - \sqrt{h} R^{(3)}(h) \right) + p_\varphi \sqrt{h^{ij} \partial_i \varphi \partial_j \varphi + 2(V(\varphi) + \pi_B)} - \pi_B \sqrt{h} . \tag{17} \]
The only requirement is that the kinetic term on the specific form of the scalar field Lagrangian (3) be positive.

For more details about the canonical Hamiltonian treatment of gravity-matter theories with non-Riemannian volume-forms we refer to [16,17].

3 Hidden Symmetry, Conservation Laws and “Dust” Fluid Interpretation

We go back to the Lagrangian formalism and consider Eq. (7). Multiplying its differential form \( \partial_\mu L(\varphi, X) \equiv \partial_\mu \frac{\partial}{\partial \varphi} \frac{\partial \mathcal{L}}{\partial X} + \partial_\mu X \frac{\partial X}{\partial \varphi} = 0 \) by the factor \(-\frac{1}{2} g^{\mu \nu} \partial_\nu \varphi \) we get the following equivalent form of the dynamical Lagrangian constraint (7):

\[
\partial L = \frac{\partial L}{\partial \varphi} \frac{\partial}{\partial \varphi} \frac{\partial L}{\partial X} = 0 .
\]

(18)

Inserting \( \frac{\partial L}{\partial \varphi} \) from (18) into \( \varphi \)-equations of motion (10) we immediately rewrite the latter in the following current-conservation law form (for later convenience we multiplied both sides by the numerical factor \( \sqrt{2} \)):

\[
\partial_\mu \left[ \left( \Phi(B) + \frac{\sqrt{-g}}{\sqrt{X}} \right) 2 \sqrt{X} g^{\mu \nu} \partial_\nu \varphi \frac{\partial L}{\partial X} \right] = 0.
\]

(19)

or, equivalently, in a covariant form:

\[
\nabla_\mu J^\mu = 0 , \quad J^\mu \equiv \left( 1 + \frac{\Phi(B)}{\sqrt{-g}} \right) 2 \sqrt{X} g^{\mu \nu} \partial_\nu \varphi \frac{\partial L}{\partial X} .
\]

(20)

In fact we find a hidden (strongly nonlinear) Noether symmetry of the original action (1) which produces \( J^\mu \) (20) as a genuine Noether conserved current. Indeed, the action (1) is invariant (modulo total derivative) under the following nonlinear symmetry transformations:

\[
\delta \varphi = \epsilon \sqrt{X} , \quad \delta_2 g_{\mu \nu} = 0 , \quad \delta \mathcal{B}^\mu = -\epsilon \frac{1}{2 \sqrt{X}} g^{\mu \nu} \partial_\nu \varphi \left( \Phi(B) + \frac{\sqrt{-g}}{\sqrt{X}} \right) ,
\]

(21)

where the short-hand notations (5) are used. Under (21) the action (1) transforms as \( \delta S = \int d^4 x \delta \varphi \left( L(\varphi, X) \delta \mathcal{B} + \frac{\partial L}{\partial X} \delta X \right) \). Then, the standard Noether procedure yields precisely \( J^\mu \) (20) as the pertinent conserved current.

Let us particularly stress, that the existence of the hidden symmetry (21) of the action (1) does not depend on the specific form of the scalar field Lagrangian (4). The only requirement is that the kinetic term \( X \) must be positive.

We can now rewrite \( T_{\mu \nu} \) (5) and \( J^\mu \) (20) in the following relativistic hydrodynamical form (taking into account (7)):

\[
T_{\mu \nu} = \rho_0 u_\mu u_\nu - 2M g_{\mu \nu} , \quad J^\mu = \rho_0 u^\mu ,
\]

(22)

where the integration constant \( M \) appears as dynamically generated cosmological constant and:

\[
\rho_0 \equiv \left( 1 + \frac{\Phi(B)}{\sqrt{-g}} \right) 2 \sqrt{X} \frac{\partial L}{\partial X} , \quad u_\mu \equiv \frac{\partial \varphi}{\sqrt{2X}} (\text{note } u^\mu u_\mu = -1) .
\]

Comparing (22) with the standard expression for a perfect fluid stress-energy tensor \( T_{\mu \nu} = (\rho + p) u_\mu u_\nu + pg_{\mu \nu} \), we see that:

\[
p = -2M , \quad \rho = \rho_0 + 2M \quad \text{with } \rho_0 \text{ as in (23)} ,
\]

(23)

i.e. the fluid tension is constant and negative, whereas \( \rho_0 \) (23) and \( 2M \) are the rest-mass and internal fluid energy densities, respectively (for general definitions, see i.e. (30)).

The energy-momentum tensor (22) consists of two parts with the following interpretation according to the standard A-CDM model [21,22,33] (using notations \( p = p_{DM} + p_{DE} \) and \( \rho = \rho_{DM} + \rho_{DE} \) in (24)):

- Dark energy part given by the second cosmological constant term in \( T_{\mu \nu} \) (22), which arises due to the dynamical constraint on the scalar field Lagrangian (7), or equivalently, by (24) with \( p_{DE} = -2M , \rho_{DE} = 2M \);
- Dark matter part given by the first term in (22), or equivalently, by (24) with \( p_{DM} = 0 , \rho_{DM} = \rho_0 \) (\( \rho_0 \) as in (23)), which in fact describes a dust.

Indeed, the covariant conservation laws for the energy-momentum tensor (22) \( \nabla_\mu T^\mu_{\nu} = 0 \) and the \( J \)-current (20) acquire the form:

\[
\nabla_\mu \left( \rho_0 u_\mu u_\nu \right) = 0 , \quad \nabla_\mu \left( \rho_0 u_\mu \right) = 0 ,
\]

(25)

both of which implying the geodesic equation for the “dust fluid” 4-velocity \( u_\mu \):

\[
u u_\mu u_\nu = 0 .
\]

(26)

To conclude this section let us point out that the hidden symmetry transformation of the scalar field (first Eq. (21)) can be equivalently represented as a specific field-dependent coordinate shift of the \( \varphi \)-field (taking into account the definition of \( X \) in (4)):

\[
\delta \varphi(x) = \epsilon \sqrt{X} = \varphi(x + \epsilon \zeta(x)) - \varphi(x) = \epsilon \zeta^\mu(x) \partial_\mu \varphi(x) , \quad \zeta^\mu = -\frac{1}{\sqrt{2}} u^\mu .
\]

(27)

Accordingly, the dust 4-velocity transforms under the hidden symmetry (21) or (24) as:

\[
\delta \varphi = \epsilon (g^{\mu \nu} + u^\mu u^\nu) \frac{\partial \varphi}{\sqrt{2X}} .
\]

(28)
4 Implications for Cosmology

Let us now consider the modified gravity-scalar-field model (1) with the hidden symmetry (24) describing simultaneously dark matter and dark energy in the context of cosmology. To this end let us take the Friedmann-Lemaître-Robertson-Walker (FLRW) metric (see e.g. (29)):

$$ds^2 = -dt^2 + a^2(t) \left[ \frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right],$$

(29)

and consider the associated Friedmann equations:

$$\frac{\dot{a}}{a} = -\frac{1}{12}(\rho+3p), \quad H^2 + \frac{K}{a^2} = \frac{1}{6} \rho, \quad H = \frac{\dot{a}}{a},$$

(30)

describing the universe’s evolution. In the present case we have for pressure $p$ and the full energy density $\rho$ the explicit expressions (24). Also now $\phi = \phi(t)$, so that $X = \frac{1}{2} \dot{\phi}^2$ and $u_\mu = (1,0,0,0)$.

The $J^\mu$-current conservation (25) now reads:

$$\nabla^\mu (\rho_0 u_\mu) = 0 \rightarrow \frac{d}{dt} (a^3 \rho_0) = 0 \rightarrow \rho_0 = \frac{c_0}{a^3},$$

(31)

where the last relation is the typical cosmological dust solution (see e.g. (32)) with $c_0 = \text{const}$. Inserting in (31) the explicit expression (26) for $\rho_0$ we obtain a solution for the non-Riemannian integration measure density $\Phi(B) = c_0 (2X \frac{dL}{d\phi})^{-1} - a^3$, or in the simplest case for the scalar Lagrangian ($L = \frac{1}{2} \dot{\phi}^2 - V(\phi)$):

$$\Phi(B) = \frac{c_0}{\dot{\phi}^2} - a^3.$$

(32)

Let us particularly stress, that the solution (31) for the dust (dark matter) energy density $\rho_0$ (last relation in (24)) does not depend on the specific form of the scalar Lagrangian (cf. (3)) and the details of the dynamics of $\phi(t)$:

$$L(\phi, X) = \frac{A_1(\phi)}{2} \dot{\phi}^2 + \frac{A_2(\phi)}{4} \dot{\phi}^4 + \cdots - V(\phi).$$

(33)

Taking into account (24) and (31), the Friedmann equations (30) acquire the form:

$$\frac{\ddot{a}}{a} = -\frac{1}{12} \left( \frac{c_0}{a^3} - 4M \right), \quad \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = \frac{1}{6} \left( \frac{c_0}{a^3} + 2M \right).$$

(34)

and, thus, the solution for $a = a(t)$ does not depend either on the specific form of the scalar Lagrangian (33) and the details of the dynamics of $\phi(t)$. Exact solution for $a(t)$ of the second Eq. (34) when $k = 0$ was given in Ref.[31]. In the general case including radiation, exact solutions for $a(t)$ in terms of elliptic functions can be found in Ref.[34].

In fact, concerning the cosmological solutions of (31) and (34), the only requirement for $L(\phi, X)$ (33) comes from the dynamical constraint Eq. (7) on (33):

$$L(\phi, X) = -2M \rightarrow V(\phi) > 2M.$$

(35)

In general the inequality $V(\phi) > 2M$ might define classical forbidden regions for $\phi(t)$ (where $V(\phi) < 2M$), including turning points $\phi_0$ (where $V(\phi_0) = 2M$). In view of later applications (see discussion about obtaining growing dark energy below) we will demand:

$$V(\phi) > 2M \quad \text{for all } \phi,$$

(36)

so that we will have a purely monotonic behaviour for $\phi(t)$ (cf. (38) below).

The dynamics of the scalar field $\phi(t)$ itself is given by the first-order differential equation (35). Although it does not affect the cosmological solutions, nevertheless, it is worth mentioning the following property. Taking time derivative on both sides of (35) we obtain second order evolution equation for $\phi(t)$:

$$\ddot{\phi}(\phi, X) = \frac{A_1(\phi)}{2} \dot{\phi}^2 + \frac{A_2(\phi)}{4} \dot{\phi}^4 + \cdots - \frac{\partial V}{\partial \phi} = 0.$$

(37)

In particular, for the standard scalar Lagrangian $L = \frac{1}{2} \dot{\phi}^2 - V(\phi)$ Eqs. (35) and (37) read, accordingly:

$$\dot{\phi}^2 = 2V(\phi) \rightarrow \int_{\phi(0)}^{\phi(t)} \frac{d\phi}{2V(\phi) - 2M} = \pm t,$$

(38)

$$\phi - \frac{\partial V}{\partial \phi} = 0,$$

(39)

where we specifically stress on the opposite sign in the force term in the second order $\phi$-equation of motion (39). Due to the dynamical constraint on $V(\phi)$ in (38) and choosing the + sign the integral in (38) yields $\phi(t)$ monotonically growing with $t$.

Let us now consider a perturbation of the initial modified-measure gravity-scalar-field action (1) by some additional scalar potential $U(\phi)$ independent of the initial potential $V(\phi)$:

$$\tilde{S} = \int d^4x \sqrt{-g} R + \int d^4x (\sqrt{-g} + \Phi(B)) L(\phi, X) - \int d^4x \sqrt{-g} U(\phi).$$

(40)

An important property of the perturbed action (40) is that once again the scalar field $\phi$-dynamics is given by
the unperturbed dynamical constraint Eq. (7), in particular, by Eq. (37) or (38) in the case of FLRW metric (29). Let us strongly emphasize that the latter are completely independent of the perturbing scalar potential $U(\varphi)$.

The associated scalar field energy-momentum tensor now reads (cf. Eqs. (22) and (23)):

$$\tilde{T}_{\mu\nu} = \rho_0 u_\mu u_\nu + g_{\mu\nu} \left(-2M - U(\varphi)\right)$$

$$\equiv (\tilde{\rho} + \tilde{p}) u_\mu u_\nu + \tilde{p} g_{\mu\nu} ,$$

$$\tilde{\rho} = \rho_0 + 2M + U , \quad \tilde{p} = -2M - U ,$$

where again notations (23) are used.

The perturbed energy-momentum (41) conservation $\nabla^\mu T_{\mu\nu} = 0$ now implies (cf. Eqs. (24) and (26)):

$$\nabla^\mu \left(\rho_0 u_\mu\right) - \sqrt{2\lambda} \frac{\partial U}{\partial \varphi} = 0 , \quad u_\mu \nabla^\mu u_\nu = 0 .$$

While we again obtain the geodesic equation for the dark matter “fluid” 4-velocity, in the perturbed case the action (40) does not any more possess the hidden symmetry (24) and, therefore, the conservation of the Noether current $J^\mu = \rho_0 u^\mu$ (22) is now replaced by the first Eq. (43). In the case of FLRW metric (29) the latter acquires the known form:

$$\frac{d}{dt} \left(a^3 \tilde{\rho}\right) + \tilde{\rho} \frac{d}{dt} a^3 = 0 ,$$

where the notations (42) for the total perturbed energy density and pressure are used.

As already stressed above, the dynamics of the scalar field does not depend at all on the presence of the perturbing scalar potential $U(\varphi)$. Therefore, if we choose the perturbation $U(\varphi)$ in (10) to be a growing function at large $\varphi$ (e.g., $U(\varphi) \sim e^{\alpha \varphi}$, $\alpha$ small positive) then, when $\varphi(t)$ evolves through (38) to large positive values, it (slowly) “climbs” the perturbing potential $U(\varphi)$ and according to the expression $2M + U(\varphi)$ for the dark energy density (cf. (11)), the latter will (slowly) grow up! Let us emphasize that in this way we obtain growing dark energy of the “late” universe without any pathologies in the universe’ evolution like “cosmic doomsday” or future singularities [24].

Taking another example of perturbation in (10) of the type $U(\varphi) \sim \tanh(\alpha \varphi)$ for large $\varphi$, then after (slowly) growing up the dark energy density $2M + U(\varphi)$ will asymptotically (for $t \to +\infty$) approach a finite constant value.

5 Conclusions

Let us recapitulate the main points above:

- Employing a non-Riemannian volume-form (alternative covariant integration measure density independent of the Riemannian metric) in the modified-measure gravity-scalar-field action (1) produces naturally a dynamically generated cosmological constant (identified as dark energy) in the form of an arbitrary integration constant in solving the equations of motion (7) corresponding to the auxiliary “measure” gauge fields.
- The modified-measure gravity-scalar-field action (1) possesses a hidden Noether symmetry (21) acting on the scalar field and the “measure” gauge fields (but leaving the Riemannian metric untouched), whose associated Noether conserved current (20) provides a relativistic hydrodynamical interpretation of the energy-momentum tensor (22) describing two decoupled matter components – a “dust” (dark matter) and a constant negative pressure (dark energy) ones.
- The above unified description of dark energy and dark matter is insensitive w.r.t. the specific form of the scalar field Lagrangian (which might be of higher order “k-essence” type) and the details of the underlying dynamics of the scalar field.
- Upon appropriate perturbing the modified-measure gravity-scalar-field action (1), which breaks the above hidden symmetry, we find a way to obtain growing dark energy in the present universe’s epoch without evolution pathologies.

Straightforward quantization (e.g., via functional integral) of the scalar field action in (1), which is required to study possible quantum radiative instabilities within the cosmological constant problem, does not allow the use the standard quantum field theoretic methods (standard perturbative expansion, Feynman diagrams and their renormalization). This is due to the essential nonlinearity (square root) in the expression for the corresponding scalar field canonical Hamiltonian (13) (even in flat spacetime $N = 1$, $h_{ij} = \delta_{ij}$) and, especially, because it is linear (instead of the usual quadratic) function of the conjugated canonical momentum $p_\varphi$.

Canonical Hamiltonian quantization of the full gravity-scalar-field action (1) were studied in (35) in the reduced case of FLRW cosmological metric (29) and purely time-dependent scalar field $\varphi$. Upon appropriate change of variables the corresponding quantum Wheeler-DeWitt equation was reduced (in the case of zero FLRW spatial curvature) to the Schrödinger equation for inverted harmonic oscillator.

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