Fluidization of collisionless plasma turbulence

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In a collisionless, magnetized plasma, particles may stream freely along magnetic field lines, leading to “phase mixing” of their distribution function and consequently, to smoothing out of any “compressive” fluctuations (of density, pressure, etc.). This rapid mixing underlies Landau damping of these fluctuations in a quiescent plasma—one of the most fundamental physical phenomena that makes plasma different from a conventional fluid. Nevertheless, broad power law spectra of compressive fluctuations are observed in turbulent astrophysical plasmas (most vividly, in the solar wind) under conditions conducive to strong Landau damping. Elsewhere in nature, such spectra are normally associated with fluid turbulence, where energy cannot be dissipated in the inertial-scale range and is, therefore, cascaded from large scales to small. By direct numerical simulations and theoretical arguments, it is shown here that turbulence of compressive fluctuations in collisionless plasmas strongly resembles one in a collisional fluid and does have broad power law spectra. This “fluidization” of collisionless plasmas occurs, because phase mixing is strongly suppressed on average by “stochastic echoes,” arising due to nonlinear advection of the particle distribution by turbulent motions. Other than resolving the long-standing puzzle of observed compressive fluctuations in the solar wind, our results suggest a conceptual shift for understanding kinetic plasma turbulence generally: rather than being a system where Landau damping plays the role of dissipation, a collisionless plasma is effectively dissipationless, except at very small scales. The universality of “fluid” turbulence physics is thus reaffirmed even for a kinetic, collisionless system.

Significance

Two textbook physical processes compete to thermalize turbulent fluctuations in collisionless plasmas: Kolmogorov’s “cascade” to small spatial scales, where dissipation occurs, and Landau’s damping, which transfers energy to small scales in velocity space via “phase mixing,” also leading to dissipation. We show that, in a magnetized plasma, another textbook process, plasma echo, brings energy back from phase space and on average, cancels the effect of phase mixing. Energy cascades effectively as it would in a fluid system, and thus, Kolmogorov wins the competition with Landau for the free energy in a collisionless turbulent plasma. This reaffirms the universality of Kolmogorov’s picture of turbulence and explains, for example, the broad Kolmogorov-like spectra of density fluctuations observed in the solar wind.
that Landau fluid models, carefully constructed, might capture the relevant physics.

In the context of solar wind turbulence (and more generally, of collisionless plasma turbulence, of which the solar wind is a particularly well-diagnosed instance), the question of how the turbulent fluctuations’ energy (free energy) is thermalized must be tackled if one is to explain the measured spectra of “compressive” (density and magnetic field strength) perturbations. In the fluid (short mean free path) MHD theory, these perturbations correspond to the slow-wave and entropy modes. Compressions are supported in MHD by collisions, which prevent particles from freely streaming through and away, along the magnetic field. Because momentum and energy are conserved in each collision, the inertial-range fluctuations are undamped (viscosity and resistivity do dissipate them but at much smaller spatial scales). Instead, they are passively advected by the Alfvénic perturbations, including in situations when the latter are turbulent (2, 35). As passive tracers, compressive MHD perturbations should have a spectrum that follows the spectrum of the Alfvénic turbulence—and indeed, solar wind measurements show that they do (36–42). However, the solar wind plasma at 1 astronomical unit (AU), where these measurements are done, is essentially collisionless: its mean free path is approximately 1 AU. In such a plasma, compressive perturbations, while still passive with respect to the Alfvénic ones (2), are in fact subject to Landau damping [known in this context as Barnes (43) damping] at rates characterized by free streaming along field lines. Thus, the variances of density and field strength fluctuations are not conserved—they form part of the total compressive free energy, which also includes the variance of the perturbed ion distribution function and could be rapidly redistributed through all scales in velocity space—equivalently, to higher-velocity space moments—until it is ultimately dissipated by collisions. By this conventional argument, the wavenumber spectra of the compressive perturbations should decay more steeply in a collisionless plasma than in a collisional one, because at each scale, energy is removed into phase space at a rate at least similar to the rate at which it is passed to the next smaller scale. However, this is not observed. Not only do the observed compressive fluctuations have spectra that follow the Alfvénic fluctuations’ spectra, as if the flow of free energy to higher moments was substantially blocked, but they also display surprisingly fluid dynamics (46).

A range of possible explanations for this fluid behavior of a collisionless plasma has been mooted [e.g., that Landau damping might be quantitatively weak (35) or that compressive fluctuations, unlike the Alfvénic ones, do not develop small scales along the (perturbed) magnetic field lines, rendering the damping ineffective (2)]. What in fact happens is subtler: while the compressive fluctuations do have a parallel cascade and thus, do phase mix vigorously, much of their energy flux into phase space due to this phase mixing is on average canceled by a return flux from phase space due to the stochastic version (23) of the plasma echo phenomenon (47, 48). The result is effective suppression of Landau damping of the compressive fluctuations. With access to this loss channel inhibited, the density and magnetic field strength perturbations instead develop increasingly sharp, small-scale spatial features characterized by broad power law wavenumber spectra. Their free energy is ultimately thermalized by processes that occur at small spatial scales [at and below the ion Larmor radius (2)] outside the scope of this study.

**Theoretical Framework**

Motivated by observational evidence that turbulence in the solar wind consists of low-frequency fluctuations (compared with the ion Larmor frequency), with negligible energy in the fast magnetosonic modes, we tackle the problem in the framework of “kinetic reduced magnetohydrodynamics” (KRMHD) (2), which is the long-wavelength limit of gyrokinetics (34, 49, 50) and the anisotropic reduction of kinetic MHD (3, 51). The electrons are isothermal in this approximation, and therefore, only the ions require a kinetic treatment. The collisional limit of KRMHD is the well-known “reduced MHD” (52, 53).

The applicability of KRMHD is limited by a few constraints. Most importantly, these include magnetized ions (λ ≫ ρi, where ρi is the thermal ion Larmor radius and λ is any target fluctuation’s scale length measured across the magnetic field), spatial anisotropy (λ ≪ ℓi, where ℓi is a fluctuation’s scale length along the local magnetic field), and low frequency (ω ≪ Ωi, where ω represents any dynamical frequency of interest and Ωi is the ion Larmor frequency). The fluctuating fields must be small in comparison with their background values (ordered ~ ω/Ωi ∼ λ/ℓi), and the background magnetic field is assumed to be (locally) straight, with constant magnitude. The ion mean free path λonly can be long or short compared with ℓi without violating KRMHD ordinances. Similarly, the ratio of the sound speed to the Alfvén speed (vth/√β, where β is the ratio of the ion thermal to magnetic energies) can be large or small. KRMHD is well suited to the study of the inertial-range turbulence in the solar wind and elsewhere.

Under this approximation, Alfvénic fluctuations are completely unaffected by the compressive fluctuations, and the compressive fluctuations are affected only by the Alfvénic fluctuations and not by one another (2).

**Alfvén Waves.** In KRMHD, Alfvénic fluctuations are purely transverse, incompressible, nonlinearly interacting Alfvén-wave packets, which propagate in both directions along a straight, static background magnetic field B0 = B0z in a plasma with mean mass density ρ0. Two scalar stream (flux) functions Φ = Φ(r, t) and Ψ = Ψ(r, t) describe the fluctuating field-perpendicular flow velocity u = z × ∇Φ and magnetic field b = −z × ∇Ψ, respectively; the latter is in units of the Alfvén speed vA = B0/√4πμ0ρ0. Only the z components of the vorticity ∇ × u and current ∇ × b are nonzero, equal to Vz Φ and VzΨ, respectively.

The Elsässer representation Z± = u ± b (54) brings to the fore key features of Alfvénic fluctuations in full MHD. In our reduced theory, the Elsässer potentials ζ± = Φ ± Ψ and “vorticities” ω± = Vzζ± are useful. In terms of these functions, the KRMHD equations for Alfvénic fluctuations are

\[
\left( \frac{\partial}{\partial t} + v_A \frac{\partial}{\partial x} \right) \omega^± = -\{\zeta^±, \omega^±\} - \{\partial_x \zeta^±, \partial_x \zeta^±\},
\]

where \{f, g\} = ∂x f ∂x g − ∂x f ∂x g. In the final term in Eq. 1, summation over j = x, y is implied.

Important Alfvénic phenomena are easily read from Eq. 1. The linear terms (on the left-hand side) describe perturbations propagating up and down the background magnetic field with speed vA. According to the right-hand side, only counterpropagating perturbations interact. The compressive fluctuations do not appear. Note that the local direction of the magnetic field is determined by the Alfvén waves, because B0/B ≈ z × b/vA. KRMHD separately tracks fluctuations of the magnetic field strength dB, as described below.

**Ion Kinetics.** Compressive fluctuations, namely those of density (dn) and pressure, are calculated via moments of the ion distribution function perturbed from a Maxwellian equilibrium; the perturbed magnetic field strength dB is obtained from these via

Theorem 1

\[\text{Theorem 1:} \quad \text{If } f(x, t) \text{ is a solution of the KRMHD equations, then }\]

\[\text{the time evolution of the density } n(x, t) \text{ is given by } n(x, t) = \int f(x, v, t) \, d^3v,\]

where \[\int f(x, v, t) \, d^3v = \int f(x, v, t) \, dv_x \, dv_y \, dv_z \text{ is the Boltzmann integral over speed space.}\]
pressure balance (across the mean field $B_0$). In KRMHD, as a result of the straight $B_0$ equilibrium geometry, magnetic momentum conservation, and the restriction to long wavelengths ($\lambda \gg \rho_i$), one can integrate the perturbed distribution function over perpendicular velocities $v_{\perp}^i$, retaining only the $v_{\parallel}^i$ and $v_{\perp}^i$ moments, which are required to obtain $\delta n$ and $\delta B$. The ion kinetics are thus encoded by two scalar field equations, $g = g^{(1)}(t, v_{\parallel}^i, \ell)$, $i = 1, 2$, which are particular linear combinations of the $v_{\parallel}^i$ and $v_{\perp}^i$ moments, chosen to produce two decoupled kinetic equations (2):

$$
\frac{dg^{(i)}}{dt} + v_{\parallel}^i \nabla g^{(i)} + v_{\perp} F_0 \nabla \phi^{(i)} = 0,
$$

where $F_0 = \exp(-v_{\perp}^2/m_0)$ is the background Maxwellian distribution. Ions are accelerated along the moving field lines by the parallel electric field and the mirror force. For each $g^{(1)}$, the appropriate linear combination of these forces is accounted for by its potential $\phi^{(1)} = \alpha^{(1)} \int dv_{\parallel}^i g^{(1)}(v_{\parallel}^i)$, where the constant prefactors $\alpha^{(1)}$ depend on plasma parameters—the ratios of ion thermal to magnetic energies (“plasma beta,” $\beta_i = 8\pi n_i T_i/B_0^2$), of the ion to electron temperatures ($\tau \equiv T_i/T_e$), and of the ion to electron charge ($Z \equiv q_i/e$). The explicit expressions are $\alpha^{(1)} = (\tau Z - 1/\beta_i \pm \kappa)^{-1}$, with $+ \tau Z - 1/\beta_i$, and $- \tau Z - 1/\beta_i$, and $\kappa = [(1 + \tau Z)^2 + \beta_i^2]^1/2$. At any time location, and $\delta n$ and $\delta B$ can be determined as linear combinations of $\phi^{(1)}$ and $\phi^{(2)}$, with coefficients that also depend on the plasma parameters:

$$
\frac{\delta n}{n_0} = \frac{1}{2\kappa} \left[ \frac{\phi^{(1)}}{\alpha^{(1)}} + \frac{2\tau}{Z\beta_i} \frac{\phi^{(2)}}{\alpha^{(2)}} \right],
$$

$$
\frac{\delta B}{B_0} = \frac{1}{2\kappa} \left[ \frac{\phi^{(2)}}{\alpha^{(2)}} - \left( 1 + \frac{Z}{\tau} \right) \frac{\phi^{(1)}}{\alpha^{(1)}} \right],
$$

where $\sigma = 1 + \tau Z - 1/\beta_i + \kappa$. The particular forms of these coefficients or of $\alpha^{(i)}$ are not important for the forthcoming discussion, which hinges just on the mathematical structure of Eq. 2. We shall drop the superscripts of $g$ and $\phi$ wherever this causes no ambiguity.

The apparent simplicity of Eq. 2 is an intentional feature of the KRMHD model, but a few subtleties deserve mention. The equations for $g^{(1)}$ and $g^{(2)}$ are totally decoupled from one another but not from the background Alfvenic perturbations, which move the plasma and the field lines around. This effect enters via the “convective” derivatives $d/dt \equiv \partial/\partial t + \mathbf{u} \cdot \nabla_{\perp}$ and $\nabla_{\parallel} \equiv \partial/\partial \ell + (b/\alpha) \cdot \nabla_{\perp}$.

In the presence of Alfvenic fluctuations, since the magnetic field lines are “frozen” into the plasma, the Alfvenic flows $\mathbf{u}$ move the field lines $b$ and the plasma together. Thus, Eq. 2 might seem to be a linear equation that has been reexpressed in a well-defined Lagrangian frame (2). However, after about one eddy turnover time, structure in $b$ develops at arbitrarily small spatial scales (55–57). Thus, in the presence of any resistivity (not shown in Eq. 1 but included in our simulations and in reality), the field lines’ identities and thus, the Lagrangian frame are lost. Consequently, a density fluctuation (for example) that is momentarily aligned with the local magnetic field develops finer parallel wavelengths in about one eddy turnover time.\(^1\) This process of field line dissipation and replacement manifests as a forward parallel cascade of compressive fluctuations by background Alfvenic fluctuations, such as we observe in our simulations (see Fig. 3).

The derivative operators in Eq. 2 are independent of $v_{\perp}$, and the $v_{\parallel}^i$ integrals extend from $-\infty$ to $+\infty$. These properties and the appearance of the Maxwellian function $F_0(v_{\parallel})$ make Hermite polynomials a natural orthogonal basis for $g(v_{\parallel})$. That is, one can decompose

$$
g(v_{\parallel}) = \sum_{\ell=0}^{\infty} H_{\ell}(v_{\parallel}/\sqrt{m_0}) F_0(v_{\parallel}) \frac{g_{\ell}}{\sqrt{2^{\ell}/\ell!}} \frac{1}{v_{\parallel}},
$$

where $H_{\ell}(x) = \left(-1\right)^{\ell} e^{2x} (d/dx)^{\ell} e^{-2x}$ is the Hermite polynomial of degree $\ell$. It is convenient to describe the velocity space structure of $g(v_{\parallel})$ in terms of the spectral coefficients $g_{\ell}$. Just as a fluid cascade is described in terms of a flux of energy from low to high wavenumbers, one may describe collisionless damping as a flux of free energy from low $m$ to high $m$.

In this context, it is appropriate to state explicitly what we mean by free energy (2). It follows immediately from Eq. 2 that it has a quadratic invariant:

$$
W = \int d^3\mathbf{r} \left( \int d v_{\parallel} \frac{g^{(2)}}{2F_0} + \frac{\phi^{(1)}}{2\alpha} \right) = \frac{1}{2} \sum_{\ell,k} \left( \int g_{\ell,k} |g_{\ell,k}|^2 + \alpha |g_{0,k}|^2 \right),
$$

where $V$ is the volume of the domain, and the second expression follows from the Parseval theorem for the Fourier and Hermite functions and provides an explicit motivation for discussing free-energy flows in the $(m, k)$ phase space. It is not hard to show that $W$ is minus the entropy of the perturbed distribution plus the energy of the fluctuating magnetic field (up to a factor of $T_i$), which is what motivates the term “free energy.”

\(^1\) Note that, in the absence of resistivity, the parallel electric field associated with Alfvenic fluctuations is zero, and one might, therefore, mistakenly believe that the potentials $\phi$ describe all parallel electric fields also in resistive KRMHD. In fact, the moving field lines associated with the Alfvenic fluctuations can change their topology in the presence of resistivity and generate parallel electric fields in the process. In KRMHD, the compressive fluctuations cannot see these resistive parallel electric fields; they are not included in $\phi$. Thus, when field lines resistively reconnect, KRMHD compressive fluctuations are undisturbed but get relabeled as the field line identities change. It is this relabeling that causes the compressive fluctuations to inherit the parallel structure of the Alfvenic fluctuations.

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phase (6, 7). A more complete analysis is required to recover Landau’s full story, but the essential connection between phase mixing and the damping of fields that is exposed here is authentic. Velocity space integrals decay even as $g_{m\parallel}(v_f)$ itself only becomes more oscillatory in $v_f$ without decaying. If time were reversed, the accumulated phase would unmix, and the original $\phi$ perturbations would grow in time until the unmixing was complete.

It is not easy to reverse time, but the scattering of a compressive wave by an Alfvén wave that is propagating in the opposite direction along the field line has the same mathematical effect: such an interaction causes the compressive wave’s $k_{\perp}$ to change sign, the phase subsequently unmixes, and the field perturbation regenerates. The most familiar example of this phase unmixing is the textbook phenomenon of plasma echo (47, 48). The idea of a stream of stochastic echoes produced by a sequence of nonlinear interactions has been the object of several recent studies of electrostatic ($b = 0$) plasma turbulence (23, 24, 60, 61). Here, we explore the possibility of stochastic echoes in a plasma with moving field lines ($b \neq 0$). As described above, the compressive fluctuations couple to the Alfvénic fluctuations in a nontrivial way, as the field lines along which the particles stream are being advected by the same flows as the particles’ distribution function $g$. Although there is as yet no complete theory for this problem, our numerical results suggest that elements of the electrostatic theory (23, 61) are germane. This generalization opens the way to understanding kinetic turbulence in heliospheric (solar wind) and similarly collisionless astrophysical plasmas, which are usually well into the electromagnetic regime.

Below, we show evidence that nonlinear stochastic echoes are present and that they can have profound effects on observable quantities. Our key finding, best illustrated by Fig. 1, is that, in the inertial range, the average net flux of free energy to smaller velocity-space scales (higher Hermite moments) is very small; Landau damping is largely suppressed by stochastic echoes. The values of $k_{\perp}$ and $m$ of the ion distribution function $g_m$ are effectively energetically insulated from each other.

![Image](image_url)

**Fig. 1.** The flux (II, I) (Eqs. 9 and 11) of free energy $W$ (Eq. 6) of the kinetic field $g^{(1)}$ normalized to its injection rate as a function of the Hermite number $m$ (vertical) and perpendicular wavenumber $k_{\perp}$ (horizontal). Colors represent the magnitude of the flux, and arrows represent its streamlines. Away from the stirring (at small $m$ and small $k_{\perp}$) and damping (at large $m$ and/or large $k_{\perp}$), the time-averaged net turbulent free-energy flux to high $m$ is very small; Landau damping is largely suppressed by stochastic echoes (return flux from high to low $m$), and therefore, different Hermite moments $g_m$ of the ion distribution function are effectively energetically insulated from each other.

**Numerical Experiment**

**Numerical Setup.** Our simulations track the evolution of driven and damped compressive fluctuations in a sea of driven and damped Alfvénic turbulence for a number of the latter’s eddy turnover times—well into the statistically stationary state, and therefore, statistical averages can be reliably calculated.

We solve Eqs. 1 and 2 spectrally in Fourier–Hermite space. Namely, we solve Eq. 2 in the form of a number of coupled fluid equations for the Hermite moments $g_m$, defined by Eq. 5:

$$\frac{d g_m}{dt} + v_B \nabla \left( \frac{m + 1}{2} g_{m+1} + (1 + \alpha d_m) \right) \sqrt{\frac{m}{2}} g_{m-1} = 0$$

and $\phi = \alpha g_m$. We advance these equations in time with a third-order modified Williamson algorithm (a four-step, low-storage Runge–Kutta method). The spatial dependence of $\xi_k$ and $g_m$ is expressed by discrete, triple periodic Fourier series, and we use the standard pseudospectral approach to solve the linear and nonlinear terms containing spatial derivatives. The simulation domain is a cube of size $2\pi$ in each direction. The code units are set by this and by $v_B = 1$. The nonlinear terms are partially dealtised using a phase shift method (62). The total number of spectral modes is, therefore, smaller than the total number of mesh points used to evaluate the nonlinear terms; we report the spatial size of a given simulation in terms of the latter. The simulations presented here have spatial resolution from $256^3$ (see Fig. 4) to $512^3$ (see Figs. 1–6).

The description of compressive fluctuations provided by Eq. 2 is appropriate for the inertial range, but the production of fluctuations at large scales and their removal at small scales in a simulation require one to add forcing and dissipation, respectively. The forcing occurs at $m = 1$ and low wavenumbers ($\nu_m |k_{\perp}|, |k_z|, |k_\parallel| = 1, 2$). The strength of the forcing can be arbitrary, because Eq. 2 is linear in $g$, and therefore, the amplitude of the compressive fluctuations is arbitrary. Our forcing is designed so as to keep the rate of injection of the compressive free energy exactly constant.

We add two forms of dissipation to Eq. 2 to absorb energy at small scales in $r$ (“hyperviscosity”) and $v_f$ (“hypercollisionsality”) without changing the dynamics at large scales. We tested multiple specific forms of dissipation in the course of this study. Our results require the presence of the dissipation at small scales but do not depend on its specific form. In the simulations reported here, we add $-\kappa k_{\perp}^2 g_m - r m^2 g_m$ to the right-hand side of Eq. 7 with $m \geq 2$. The values of $\kappa$ and $r$ are chosen so that this dissipation is significant only for modes with the largest values of $k_{\perp}$ and $m$ but removes energy without creating bottleneck effects or reflections.

In practice, the Hermite series of $g(v_f)$ (Eq. 5) is truncated at some $m = M$. In the absence of collisions, the Hermite Eq. 7 fail to close, because the evolution of each $g_m$ depends on $g_{m+1}$ (and $g_{m-1}$). However, with the hypercollisional cutoff described above, one can take $g_{m+1} = 0$ to a very good approximation as long as $M$ is chosen large enough. This is our approach. The simulations reported here have resolution from $M = 32$ (see Figs. 1–6) to $M = 128$ (see Fig. 4).

The Alfvén waves are also stirred and damped. We stir them by forcing at low wavenumbers (the same as for the kinetic field.
and in the velocity field only. The forcing is designed to keep
the injected power $\epsilon$ exactly constant while maintaining the rate
of cross-helical injection exactly zero (63). The magnitude of $\epsilon$
allows us to control the character of the resulting Alfvénic tur-
bulence: weaker injection produces weakly interacting Alfvén waves, but increasing it pushes the system toward strong, criti-
cally balanced turbulence (44, 45). This is the relevant regime
that we study here. It is achieved when $\epsilon = 1$ in code units. Like
the kinetic field, the Alfvénic fields are also dissipated by eighth-
order hyperviscosity and “hyperresistivity,” artificially set to be
numerically equal.

The plasma parameters $\beta_i$, $\tau$, and $Z$ are all set to unity. They
enter via the constants $\alpha^{(i)}$ in the definitions of the potentials
$\phi^{(i)}$ in terms of $g^{(i)}$ and affect their Landau damping rates. They
are also needed to compute the relative amplitudes of $\delta n$ and $\delta B$ via Eqs. 3 and 4. The results presented here do not depend
significantly on these as long as they are all approximately one.

**Perpendicular Spectra.** Let us first establish that we are dealing
with a system that exhibits some familiar features of turbulence.

A snapshot of the density fluctuation field in the plane per-
pendicular to the background magnetic field is shown in Fig. 2
together with time-averaged perpendicular wavenumber spec-
tra of the density $\delta n$ ($E^n$), magnetic field strength $\delta B$ ($E^B$),
and the Alfvénic fluctuations of velocity $u$ ($E^u$) and magnetic
field $b$ ($E^b$). The spectral slope of the latter follows quite well
the $k_{\perp}^{-3/2}$ scaling expected for RMHD turbulence (64–68). To a
good approximation, the compressive fluctuations’ spectra track
the Alfvénic velocity spectrum (Fig. 2, Inset), as one might expect
an undamped passive scalar to do (69, 70). We do not observe a
significant steepening of the compressive spectra compared with
the Alfvénic ones, which would have had to happen had there
been Landau damping of compressive fluctuations depleting
their energy cascade at each scale (23). This is circumstantially
consistent with an inhibition of Landau damping. It is also essen-
tially consistent with what is observed in the solar wind (36–42).

**Existence of Parallel Cascade.** Critically balanced Alfvénic fluctu-
ations become progressively more anisotropic at finer scales (44,
45), with parallel and perpendicular correlation scales related by
$\ell_\parallel \propto \lambda^{1/2}$ (64, 65, 71). Consequently, their parallel spectrum is
expected to scale as $k_{\parallel}^{-3/2}$ (45, 65, 72) [and indeed, does in the
solar wind (9, 67, 73–75)]. The question that we address here is
whether the kinetic field $g$ advected by them “inherits” this parallel
cascade.

Because $\ell_\parallel \gg \lambda$, parallel correlations can only be measured
correctly along the fluctuating field (66, 67, 76). We identify the field
lines by tracing a set of them from 100,000 randomly chosen
points in the simulation domain at a given instant in time,
record the values of fluctuating quantities as functions of the dis-
tance along each field line, and then, calculate parallel spectra
we use a fourth-order Runge–Kutta method to integrate the
field lines and cubic spline interpolation to determine values of
the fluctuating fields on them).

The results from this analysis are shown in Fig. 3 alongside an
instantaneous snapshot of field lines, in which they are “painted”
with the values of the density fluctuation field. One can see that
the field lines wander widely across the domain. The resulting
parallel wavenumber spectra are steeper than the perpendicular
wavenumber spectra as expected. The $k_{\parallel}^{-1}$ scaling is well sat-
sified for $E^n$, whereas the Alfvénic velocity spectrum $E^u$ is
a little steeper. The compressive spectra closely track $E^n$ (Fig. 3,
Middle Inset), confirming the existence of a “parallel cascade” of
these passive fields (contrary to what was believed in ref. 2 on the
basis of linearity of Eq. 2 in a suitably well-behaved Lagrangian
frame). Fig. 3, Bottom reinforces this conclusion: the parallel
coherence lengths of the compressive fluctuations scale as $\lambda^{1/2}$,
as do those of the Alfvénic fluctuations.

The existence of a parallel cascade is an important finding of
this study. As we have argued above, it occurs because mag-
netic fields reconnect at every scale over a time comparable
with the correlation time associated with this scale (55–57) and
thus, cannot preserve their identity over more than one parallel
correlation scale of the Alfvénic turbulence—thus, any density
perturbation that extends along a field line will be broken up and
decorrelated on the same parallel scale (hence, the tracking of
the Alfvén spectra by the compressive ones).

It is the presence of the parallel cascade that makes the ques-
tion of the efficiency of Landau damping relevant: its rate
$\sim |k_{\parallel}|^{1/2}$ will be of the same order as the Alfvén frequency

**Fig. 2.** Perpendicular cascade. (Upper) Snapshot of the density fluctuation field $\delta n$ in the plane ($x, y$) perpendicular to the mean magnetic field. (Lower) Perpendicular spectra of the Alfvénic (velocity $E^u$ and magnetic $E^b$), density ($E^n$), and magnetic field strength ($E^B$) fluctuations. (Inset) Ratio of the den-
sity to Alfvénic velocity spectra, $E^n/E^u$. The Alfvénic spectra are normalized
to the total mean Alfvénic energy, and compressive spectra are normalized
to the total mean free energy $W$ (Eq. 6).
Finally, having collected all of this circumstantial evidence, these are of interest in the context of the rapid recent advances in both instrumentation and computing, meaning that they can now be measured directly both in space (10) and in fully kinetic simulations (14, 17, 77).

In the absence of background Alfvénic turbulence, the time- and space-averaged Hermite spectrum of a forced and Landau damped kinetic field is \( \langle |g_m|^2 \rangle \propto m^{-1/2} \) (8, 59). This corresponds to a constant finite free energy flux from small to large Hermite numbers. As a consequence, there is a dissipative anomaly associated with the collision operator: in the limit of vanishing collisionality, the collisional dissipation stays finite, enabling the removal of free energy from the system at a collisionality-independent phase mixing rate. With turbulence, we find the spectrum to be a little steeper than \( m^{-1} \), as shown in Fig. 4. The collisional dissipation rate associated with such a spectrum would vanish in the limit of small collisionality (61), indicating that refinement of scales in velocity space is not sufficient to process into heat the finite free energy injected by forcing and thus, that the principal cascade route must be via smaller spatial scales. Indeed, Fig. 5, where we show the combined collisional and hyperviscous dissipation in the \((k_\perp, m)\) phase space, confirms that the majority of the free energy is thermalized at small spatial scales, not at high \( m \).

**Free-Energy Fluxes.** Finally, having collected all of this circumstantial (but observationally testable) evidence for the “fluidization” of kinetic turbulence, let us return to Fig. 1, where the fluxes of free energy in the \((k_\perp, m)\) space are plotted. Comparison of the fluxes toward smaller scales in velocity space (higher Hermite numbers) with the fluxes toward smaller scales in position space (higher wavenumbers) provides the most direct numerical evidence supporting the claim that, in the inertial range, Landau damping is suppressed rather than enhanced.

The compressive free energy, \( W \), defined by Eq. 6 is conserved in the inertial range of our simulations. Indeed, forcing is implemented spectrally (in \( m \) and \( k_\perp \)) and is, therefore, exactly zero outside of a few lowest modes. The dissipation rate (shown in Fig. 5) is negligibly small away from the highest resolved \( m \) and \( k_\perp \). For most \( m \) and \( k_\perp \) modes, there are, therefore, neither sinks nor sources of free energy other than coupling to other modes. Fluxes from mode to mode can then

\[ k_\parallel v_A \] (taking \( \beta_i = 1 \)) and therefore, in a critically balanced turbulence, of the same order as the nonlinear cascade rate at every scale. It, therefore, a priori matters as an energy-removal channel, and the discovery that if in fact does not is a non-trivial one.

**Phase Space Spectra and Dissipation.** Let us now complete the characterization of the free-energy distribution in phase space by studying the structure of the fluctuations of \( g \) in velocity space in terms of its Hermite spectra. Other than helping us build our case for fluidization of kinetic turbulence, these are of
be defined in such a way that the free energy in a wavenumber shell \([k'_⊥ = k_⊥ \text{ and at Hermite number } m, W_m(k_⊥)] = \sum_{k'_⊥} \sum_{|k'_⊥| = k_⊥} (1 + \alpha \delta_{m,0})|g_{m',k'_⊥}|^2/2\), satisfies, away from the forcing and dissipation,

\[
\frac{\partial W_m(k_⊥)}{\partial t} = -[\Gamma_m(k_⊥) - \Gamma_{m-1}(k_⊥)] - \frac{\partial \Pi_m(k_⊥)}{\partial k_⊥},
\]

where \(\Gamma_m(k_⊥)\) is the Hermite flux and \(\Pi_m(k_⊥)\) is the Fourier flux. The Hermite coupling is essentially local (it involves only neighboring Hermite numbers \(m - 1, m, m + 1\)), and therefore, it is easy to read off Hermite fluxes from Eq. 7:

\[
\Gamma_m(k_⊥) = \pi \sum_{m'=0}^{m} \int d^3r \frac{1}{V} (1 + \alpha \delta_{m',0})|g_{m',k'_⊥}|^2 /
\]

\[
\sum_{m' \leq m} \left[ \frac{m'^2 + 1}{2} g_{m' + 1} + (1 + \alpha \delta_{m',1}) \frac{m'^2}{2} g_{m' - 1} \right],
\]

where the Fourier ring filter applied to a function is defined by

\[
[g_{m}]_{k'_⊥}(r) = \sum_{k'_⊥} \sum_{|k'_⊥| = k_⊥} e^{i k'_⊥ \cdot r} g_{m,k'_⊥}.
\]

Fluxes in Fourier space can be nonlocal but are commonly defined and studied nonetheless (78):

\[
\Pi_m(k_⊥) = (1 + \alpha \delta_{m,0}) \frac{2\pi}{L_x L_y} \int d^3r \frac{1}{V} [g_{m}]_{k'_⊥} \cdot \nabla_{k'_⊥} [g_{m}]_{k'_⊥},
\]

where \(L_x L_y = (2\pi)^2\) is the cross-section of the box, and the high- and low-pass Fourier filters are defined by

\[
[g_{m}]_{k'_⊥}(r) = \sum_{k'_⊥} \sum_{|k'_⊥| > k_⊥} e^{i k'_⊥ \cdot r} g_{m,k'_⊥},
\]

\[
[g_{m}]_{k'_⊥}(r) = \sum_{k'_⊥} \sum_{|k'_⊥| \leq k_⊥} e^{i k'_⊥ \cdot r} g_{m,k'_⊥}.
\]

While the 2D phase space flux \((\Pi, \Gamma)\) is not a unique quantity, defined up to arbitrary circulations in the \((k_⊥, m)\) space, it is a useful one, and absent any obviously spurious such circulations, it gives a good representation of the paths that free energy takes to cross the gap between the forcing and dissipation scales.

It is \((\Pi, \Gamma)\) vs. \((k_⊥, m)\) as defined by Eqs. 9 and 11, which is plotted in Fig. 1. On average, at forcing scales (low \(k_⊥\)), a significant amount of free energy flows directly to high \(m\). This flux is restricted to the forcing band of \(k_⊥\) modes, because in this range, the nonlinear interactions have not yet managed to set up a return echo flux. This is in sharp contrast with the situation in the inertial range, where the energy flows mostly to small spatial scales. Recall that, in the inertial range, the parallel cascade of compressive fluctuations should make Landau damping stronger in the presence of turbulence than otherwise, because free streaming particles have less distance to travel to smooth out spatial fluctuations. Recall also that this parallel cascade is critically balanced, which means that Landau damping should be able to “keep up” with nonlinear spatial mixing at every scale in the inertial range. In reality (as represented by the simulations), Landau damping is manifestly rapid enough to be important at forcing scales but (on average) far weaker than perpendicular spatial mixing in the inertial range; therefore, it is reasonable to conclude that there is another process that is canceling on average the flux to high \(m\). This is the most explicit signature of stochastic echoes that can be diagnosed in these simulations.6

The conclusion from Fig. 1 is that, in the inertial range, individual Hermite moments are, in effect, energetically insulated from each other on average. The compressive turbulence is “fluidized.”

Discussion

Summary. We have presented a case for effective fluidization of compressive fluctuations in collisionless (or weakly collisional) plasma turbulence. Compressive fluctuations are advected passively by the ambient Alfvénic turbulence and turn out to inherit its parallel structure (Fig. 3). This means that their phase mixing (Landau damping) rate \(\sim |k|/\nu_B\) is in principle comparable with the frequency \(\nu_B\) of the Alfvén motions and therefore, in a critically balanced turbulence (44, 45) to the rate at which these motions push the compressive free energy to smaller perpendicular scales. However, what might have been thought of as an effective dissipation at every scale fails to remove energy efficiently from the low-order moments of the ion distribution function (Fig. 1) and thus, steepens the inertial-range spectra of, for example, density and magnetic field strength fluctuations. Instead, it follows faithfully the spectrum of the advecting Alfvénic field (Fig. 2) as a well-behaved fluid passive scalar would do (69, 70). The reason for this turns out to be that nonlinear advection effectively nullifies phase mixing (except at the forcing scales), with free-energy fluxes in the inertial range largely confined within each moment of the distribution function, taking its energy from large to small spatial scales where it dissipates (Fig. 5). We interpret this behavior as resulting from statistical cancellation of phase mixing by plasma echoes. These are excited, because the nonlinear advection causes phase mixing modes propagating toward smaller-velocity space scales to couple to antiphase mixing ones, propagating back to larger scales—the result is a return flux from phase space (23).

Relation to Observations in Space. There is a long history of density and field strength spectra being measured in the solar wind and found to have the same slopes as the ambient Alfvénic turbulence (36–42). Why they do so in a collisionless plasma has

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6For electrostatic kinetic turbulence, where \(b = 0\), it is possible to decompose \(g_m\) explicitly into phase mixing and antiphase mixing components and thus, calculate directly the “forward” and “backward” free-energy fluxes in Hermite space (23, 61). However, this decomposition involves the sign of \(k_⊥\), which in our case, would have to be calculated with respect to the perturbed, turbulent magnetic field. We do not know how to make such a calculation mathematically rigorous and thus, do not use this decomposition.
remained a puzzle, if perhaps not always a fully appreciated one, in view of the community’s instinctive preference for fluid models. As solar wind has come to be regarded as a unique plasma physics laboratory, compressive fluctuations, under additional plasma physics-aware scrutiny, have stubbornly continued to manifest fluid behavior (46). Ours is a dedicated attempt to simulate these fluctuations drift kinetically using equations that are physically appropriate for them (2). Our results are manifestly in agreement with what observations have shown for many years.

In the solar wind, it has, relatively recently, become possible to probe beyond spectra and diagnose 3D structure of local correlations (79, 80). The physically motivated local basis includes the direction of the “local mean” magnetic field (the correlation length along it is denoted $\ell_\parallel$), the direction of the field’s perturbation $\mathbf{b}$ at the scale of interest (the corresponding correlation length is $\xi$), and the third direction, transverse to both (correlation length $\lambda$). The idea is that, if turbulence has a tendency toward local alignment between the two Alfvénic fields (64, 65, 81, 82), it must turn out that statistically not only $\lambda, \xi \ll \ell_\parallel$ (anisotropy) but also, $\lambda \ll \xi$ (alignment). In Fig. 6, we show representative contours of 3D correlation functions (“statistical eddy shapes”) calculated in this frame for both Alfvénic and compressive fluctuations. They are very similar to what is measured in the solar wind (79); Alfvénic “statistical eddies” are “pancakes” or “ribbons” with $\lambda \ll \xi \ll \ell_\parallel$, while the compressive structures have a more tabular aspect. Thus, it appears that simulated collisionless compressive fluctuations bear significant resemblance to the measured ones in a more detailed way than just having the same spectra—a reassuring fact.

With the extraordinary velocity space resolution afforded by the new magnetospheric multiscale mission satellite, phase space spectra have become measurable in space plasmas (10). Thus, the prediction of steep Hermite spectra (Fig. 4) is a falsifiable one. More generally, considerations of phase space turbulence can now be viewed as more than just theorizing about “under-the-hood” physics, because they deal with phenomena that are directly observable with available instruments.

Implications for Modeling Techniques. As we mentioned in the Introduction, there is a thriving industry of effective fluid models of collisionless plasmas, of which the most sophisticated strand is the Landau fluid closures (6, 7, 20, 21, 25–29). Their underlying idea is to assume that Landau damping removes free energy from low to high Hermite moments as effectively in a nonlinear system as in a linear one. This might seem to be the exact opposite of the main conclusion of this work. However, Landau fluid models have consistently been found to work better when more—but not necessarily many more—moments are kept compared with the standard fluid approximation. It might be argued that the art of crafting a good Landau fluid model is precisely to do it in such a manner as to capture the effect of the echoes within a minimal set of Hermite moments while setting the boundary (closure) condition at the maximum retained $m$ so as not to introduce or divert free-energy flows in a spurious way. With the echo effect and fluidization now explicitly part of one’s intellectual vocabulary, one might hope to revisit this task with renewed vigor, purpose, and insight.

An Implication for Astrophysical Theory. One cannot either adequately enumerate or indeed, anticipate all of the instances across the vast canvas of plasma astrophysics where the nature of collisionless plasma turbulence may prove to be of interest. We wish to highlight one problem that has a long history (83–85) and has recently seen a burst of activity (ref. 17 and references therein). It is a long-standing question in the theory of matter accretion onto black holes whether and to what degree plasma turbulence, which is excited in the accretion disk by instabilities driven by the Keplerian shear and helps enable accretion by transporting angular momentum, can be thermalized preferentially on ions rather than electrons or vice versa. This has implications for the relative amounts of energy radiated out by...
Implications for General (Plasma) Physics. The peregirgations and rearrangements of energy through a system’s phase space are a recurrent motif of theoretical physics. Turbulence theory is explicitly constructed to describe the energy’s thermalization routes, which bridge the usually vast separations between its injection and dissipation scales, producing rich, multiscale nonlinear structure in the process. In weakly collisional plasmas, these energy transfer routes are in a 6D phase space, with velocity space refinement (phase mixing) of the particles’ distribution functions in general as effective as spatial mixing in accessing dissipation mechanisms (2, 56). It is perhaps noteworthy that, in the case of inertial-range turbulence of a magnetized plasma, one of these forms of mixing turns out unambiguously to be the winner; spatial advection outperforms phase mixing and makes a collisionless plasma resemble a collisional fluid. Those who believe in the universality of nonlinear dynamics might be pleased by such an outcome. For plasma physicists, this is a sobering reminder that collisionless dissipation processes that make our subject so intellectually distinctive are not irreversible until they are consumed by collisional entropy production—and an intriguing demonstration that nonlinear effects can sometimes hinder them in favor of more “fluid-like” entropy-production mechanisms.

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