Underwater Acoustic OFDM Channel Estimation with Unknown Sparsity

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Abstract. Underwater acoustic channels estimation accuracy seriously affects the demodulation performance of the receiver in underwater acoustic (UWA) orthogonal frequency division multiplexing (OFDM) communications. Many channel estimation algorithms based on compressed sensing (CS) have been proposed. However, these algorithms usually needs the information of sparsity, while the information of sparsity is unavailable in many UWA OFDM communications. In this paper, a novel channel estimation algorithm based on fractional Fourier transform (FRFT) and orthogonal matching pursuit (OMP) is proposed, FRFT is introduced to process the synchronization signal, so sparsity is estimated. OMP is adopted to reconstruct channel impulse response (CIR). The most innovative feature of the proposed algorithm is the estimation of sparsity, which makes OMP algorithm more practical. Simulation results show that the proposed algorithm outperforms the least square (LS) and in UWA OFDM communication.

1. Introduction

Underwater acoustic (UWA) channel is a complex time-varying, frequency-varying and spatially variable multipath channel, which brings huge challenges for high quality and robust communication. Compared with the traditional single-carrier communication system, OFDM has higher spectrum utilization, better anti-multipath capability, and a simpler equalizer, thus becoming a research hotspot [1]-[4]. In order to overcome the multipath interference caused by UWA channel, accurate channel estimation and channel equalization algorithms are required.

Many channel estimation methods have been proposed for UWA OFDM communication, such as traditional LS and minimum mean square error (MMSE) algorithm. This kind of method is sensitive to noise and has a poor performance in accuracy. UWA channel is considered to be sparse both in time and frequency domain, which provides possibility to apply CS theory to UWA channel estimation. One of the most widely used UWA channel estimation methods based on CS is greedy algorithms, such as matching pursuits (MP) [5], OMP [6], stagewise orthogonal matching pursuit (StOMP) [7], regularized orthogonal matching pursuit (ROMP) [8], compressive sampling matching pursuit (CoSaMP) [9], and subpulse pursuit (SP) [10]. These algorithms show a good performance with a prior information of sparsity. However, the number of non-zero coefficients (sparsity) of a signal is not available in many practical UWA communications, in order to solve the problem, literature [11] propose SAMP algorithm which approach the original signal gradually by adjusting the step size adaptively, but the algorithm is ineffective at low signal-to-noise ratio (SNR).

This paper is devoted to solving the problem of OFDM channel estimation in UWA channels. We propose a channel estimation algorithm especially for the information of sparsity is unavailable. Firstly,
sparsity is estimated by processing synchronous signal. Secondly, OMP is adopted to reconstruct CIR, LS algorithm is also simulated for comparison. Simulation results show that the proposed algorithm has a better reconstruction precision performance for UWA channel compared with LS algorithm.

2. SIGNAL MODEL

Fig. 1. OFDM packet structure

Fig. 1 is the transmitted OFDM packet structure [12], the former segment is synchronous data, a Chirp signal is used for frame synchronization, the latter segment is OFDM data. Assume \( T_g \) denote the CP interval and \( T \) the symbol duration, thus the total OFDM block duration is \( T' = T + T_g \). The subcarrier spacing is \( \Delta f = 1/T \) and the frequency of the \( k \)-th subcarrier can be expressed as:

\[
\nu_k = f_c + k\Delta f, \quad k = -K/2,\cdots,K/2-1
\]

where \( f_c \) is the subcarrier frequency \( K \) is the number of subcarrier, hence the bandwidth is \( B = K\Delta f \).

Assume \( d[k] \) is the information symbol transmitted on the \( k \)-th subcarrier, so the transmitted signal in passband can be written as :

\[
x(t) = \text{Re}\{\sum_{k=-K/2}^{K/2-1} d[k] e^{j2\pi \nu_k t}\}, \quad t \in [0,T + T_g]
\]

where \( g(t)=1, t \in [0,T] \), otherwise \( g(t)=0 \). The CIR of UWA channel after Doppler estimation and compensation can be expressed as:

\[
h(t) = \sum_p A_p \delta(t - \tau_p)
\]

where \( A_p \) and \( \tau_p \) is the amplitude and time-varying delay of \( p \)-th path, \( p \) is the multipath number. After passing through UWA channel, the received signal can be expressed as:

\[
y(t) = x(t) * h(t) + n(t)
\]

where \( n(t) \) is additive Gaussian noise, after Fourier transform on the both sides of equation (4), we can get:

\[
Y = XH + N
\]

where \( Y \) and \( X \) is the Fourier transform of \( y \) and \( x \) respectively. \( H \) and \( N \) is the Fourier transform of CIR and additive Gaussian noise, where \( H \) can be expressed as:

\[
H[i] = \sum_{j=0}^{K-1} h(j) e^{-j2\pi j/N}, i = 0,1,\cdots,K-1
\]

After substituting equation (6) into equation (5), we can get:

\[
Y = \hat{X}FH + N
\]

where \( \hat{X} \) is a diagonal matrix which is constructed by \( X \), \( F \) is the Fourier transform matrix and can be expressed as:

\[
F = \begin{bmatrix}
1 & 1 & \cdots & 1 \\
1 & \exp(-j2\pi 1/N) & \cdots & \exp(-j2\pi L/N) \\
\vdots & \vdots & \ddots & \vdots \\
1 & \exp(-j2\pi \frac{N-1}{N}) & \cdots & \exp(-j2\pi \frac{(N-1)(L-1)}{N})
\end{bmatrix}
\]

3. THE CHANNEL ESTIMATION ALGORITHM BASED ON FRFT AND OMP

OMP is a classic greedy algorithm in CS theory and there are many improvement algorithms based on OMP, such as StOMP, ROMP, CoSaMP, and SP. When applying these algorithms in UWA channel estimation, the sparsity of channel is a critical parameter for estimation accuracy, so we propose a
channel estimation algorithm based on FRFT and OMP. FRFT is introduced to process the synchronization signal so that sparsity is estimated and OMP is adopted to reconstruct CIR.

3.1. The estimation of sparsity
The commonly used synchronous signal in UWA communication is Chirp, which has a good anti-multipath and anti-noise performance. Chirp can be expressed as:

\[ s(t) = \begin{cases} A \exp(j(2\pi f_0 t + k t^2)), & t \in [-T/2, T/2] \\ 0, & \text{others} \end{cases} \]  

(9)

after passing the channel in equation (3), the received signal can be expressed as:

\[ r(t) = h(t) * s(t) + n(t) \]

= \sum_p A_p s(t - \tau_p) + n(t)  

(10)

The FRFT of a continuous signal can be expressed as:

\[ S_p(u) = \int_{-\infty}^{\infty} \exp(-j\pi \alpha \cot \alpha) \exp(j\pi (\alpha - 2u \csc \alpha + \tau^2 \cot \alpha)) s(t) \, dt \]

\[ = \begin{cases} \frac{1}{2\pi} \int_{\pi/2}^{\pi/2} \exp(j\pi (\alpha - 2u \csc \alpha + \tau^2 \cot \alpha)) s(t) \, d(\alpha \neq n\pi) \\ \frac{1}{2\pi} \int_{\pi/2}^{\pi/2} \exp(j\pi (\alpha - 2u \csc \alpha + \tau^2 \cot \alpha)) s(t) \, d(\alpha = 2n\pi) \\ \frac{1}{2\pi} \int_{\pi/2}^{\pi/2} \exp(j\pi (\alpha - 2u \csc \alpha + \tau^2 \cot \alpha)) s(t) \, d(\alpha = (2n+1)\pi) \end{cases} \]

(11)

The FRFT of the Chirp signal in equation (9) is given as:

\[ S_p(u) = A \sqrt{1 - j \cot \alpha} \int_{-\tau}^{\tau} [\pi (k + \cot \alpha) \tau^2 + 2f(u - u \csc \alpha) + u^2 \cot \alpha] \, dt \]

\[ = \frac{1}{\sqrt{2\pi}} \int_{\pi/2}^{\pi/2} \exp(j\pi (\alpha - 2u \csc \alpha + \tau^2 \cot \alpha)) s(t) \, d(\alpha \neq n\pi) \]

(12)

After substituting equation (10) into equation (11), we can get:

\[ R_p(u) = \sum_p AA_p S_p(u - \tau_p \cos \alpha) \]

\[ \times \exp(j\pi \gamma^2 \sin \alpha \cos \alpha - j2\pi \alpha \gamma \sin \alpha) + N_p(u) \]

(13)

where \( S_p \) is shown in equation (12) and \( N_p(u) \) is Gaussian white noise components after FRFT, the rotation factor \( \alpha \) is adjusted to get the optimal estimation of \( \hat{\alpha} \) in which Chirp signal is represented as an impulse function, different time delays generate a series of impulses in FRFT domain.

The sparsity of UWA channel, as well as the number of multipath time domain. By calculating the peak number in equation (13) which is higher than the threshold, we can obtain the sparsity of UWA channel.

3.2. Channel estimation with unknown sparsity
The estimated sparsity in section 3.1 can be regarded as a input parameter for OMP channel estimation, OMP channel estimation algorithm can be demonstrated as follows:

**Input:** sensing matrix \( A \), sampled vector \( y \), sparsity \( K \);

**Output:** reconstructed \( \hat{x} \) of the input signal;

**Step1:** Initialization trivial \( \hat{x} = 0 \), initial residue \( r_0 = y \), empty finalist \( \Lambda_0 = \emptyset \), let \( t = 1 \), \( j = 1 \);

**Step2:** Calculate \( u = \text{abs}[A^T r_{t-1}] \), select the maximum value of \( u \), thus we get \( \lambda_j = \arg \max_{j \neq \lambda_i} |(A^T r_{t-1})| \);

**Step3:** Update set \( \Lambda_t = \Lambda_{t-1} \cup \{\hat{\lambda}_j\} \), record the rebuild atoms set of sensing matrix \( A_j = \{A_{t-1}, r_{\lambda}_j\} \);

**Step4:** Find the LS solution of \( y = A \hat{\theta}_j \), thus \( \hat{\theta}_j = \arg \min_\theta \|y - A \theta\| = (A^T A)^{-1} A^T y \);

**Step 5:** Update the residual \( r_{\text{new}} = y - A \hat{\theta}_j, t = t+1 \);

**Step 6:** If \( t > K \), then go to step8, otherwise go to step2;
Step 7: Calculate $\hat{x} = \psi \hat{\theta}$.

4. SIMULATION
In this section, the performance of the proposed algorithm was investigated and compared with the traditional LS channel estimation algorithm.

4.1. Experiment 1
In order to verify the performance of the proposed algorithm, we adopt Bellhop ray model [13] to simulate the underwater acoustic channel. The horizontal distance between the receiver and the transmitter is 1 km, the average depth of the water body is 100 m, the depth of the transmitter and receiver is both 10 m and the frequency of the sound wave is 50 kHz. The number of sound beam is 9. Synchronous signal is Chirp, the Chirp rate is 20000 Hz/s and the initial frequency is 2000 Hz. The signal-to-noise (SNR) is 5 dB, simulation results are shown below.

Fig. 2. The sound speed profile

Fig. 3. The normalized impulse response

Fig. 2 is the input sound speed profile for Bellhop channel model and Fig. 3 is the normalized impulse response of Bellhop channel. It is clear that there are nine sound lines, which means the sparsity of the channel is nine. Fig. 4 is the FRFT of Chirp signal, we can find that there are nine peaks which is corresponding to Fig. 3. With the excellent anti-noise performance of Chirp signal and FRFT, the peaks is clear and the other interference is much smaller than the peaks, so we can estimate the sparsity of UWA channel accurately.
4.2. EXPERIMENT 2

In order to explore the channel estimation performance of the proposed algorithm, we adopt the Bellhop channel in experiment 1, the FFT points of OFDM system is 1024, the carrier frequency is 24 KHz, the bandwidth is 12 KHz, the CP length is 25.6 ms, the symbol duration is 102.4 ms. We adopt convolutional coding with 0.5 code rate and random interleaver. The modulation type is QPSK and comb pilot is adopted. We define the mean square error (MSE) as:

$$\text{MSE} = \mathbb{E} \left[ \| H - \hat{H} \|_2^2 \right]$$  \hspace{1cm} (14)

where $H$ is the frequency response of Bellhop channel, $\hat{H}$ is the frequency response of estimated channel.

Fig. 5 is the MSE performance curve of LS and the proposed algorithm with SNR, as we can see from the figure, it is clear that all the MSE of the algorithm decrease with the increase of SNR. The proposed algorithm enhance about 2dB MSE compared with LS, which means LS has a poor performance for noise while the proposed performs better. On the other hand, the MSE performance of the proposed algorithm confirms the creditability of the estimated sparsity.
Fig. 6. Uncoded BER performance

Fig. 7. Coded BER performance

Fig. 6 is the uncoded BER performance of the two algorithms while Fig. 7 is the coded BER performance. It is clear that the BER decrease with the increase of SNR, and the proposed algorithm can enhance about 1 dB BER performance compared with LS algorithm. On the other hand, the BER performance can improved about 2~3 dB with convolutional coding.

The results show the proposed algorithm is superior to LS channel estimation algorithm in UWA OFDM communication. The simulation results are consistent with the theoretical analysis.

5. CONCLUSION

Aiming at the problem that traditional channel estimation algorithm needs the information of sparsity in UWA channel estimation, this paper proposed a practical channel estimation algorithm based on FRFT and OMP. FRFT is used to estimate the sparsity which is the innovation point of this paper. Simulation results show that the proposed algorithm can improve MSE of channel estimation and BER performance of UWA OFDM communication. It can also be used with other greedy channel estimation algorithms, such as: StOMP, SP, ROMP and CoSaMP channel estimation algorithms, which has great application prospects.

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