Topics on Nuclear Structure with Electroweak Probes

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Abstract. We study some relevant aspects of complex nuclei structure using electroweak probes within the framework of self-consistent mean field theories with Skyrme density-dependent two-body interactions, including pairing and spin-isospin RPA correlations where necessary. We apply the formalism to the study of single and double beta decays as normal modes of the system, as well as to the analysis of parity-violating electron scattering by nuclei. Finally, we profit from the studied processes to draw some conclusions on the neutrino nature (eigenstates mixing).

1. Introduction

We present in this work some of our recent theoretical results on nuclear structure using electroweak probes, including double beta decay ($^{76}$Ge - $^{76}$Se), parity-violating elastic electron scattering by nuclei ($^{208}$Pb, $^{12}$C) and beta particle spectra in single beta decays ($^{187}$Re, $^{3}$H). We examine various nuclear structure properties such as active-shell spectroscopic factors, nuclear deformation, neutron distributions in nuclei or Coulomb and weak interactions between nuclei and scattered or emitted beta particles, among others. The processes under study set stringent tests on the capability of theory to make reliable predictions. The theoretical scheme that we follow in describing the nuclear structure properties lies on the adiabatic time-dependent Hartree-Fock approximation which provides a unified framework for a large variety of nuclear properties: sizes and shapes, density profiles, fission barriers, excitation energies, transition strengths or even energy transfers between single-particle and collective modes \cite{1}. This theory has not yet been fully implemented to explore, for instance, the coupling between different nuclear modes or the dilution of multiphonon states, but many applications have been made to the study of electric excitations (mainly dipole, quadrupole and octupole) around equilibrium, as well as of spin-isospin excitations. Many applications of the latter have been put forward by our group in the last years.

In the following section we focus on the spectroscopic factors of the double-beta decay partners $^{76}$Ge and $^{76}$Se that have been recently measured. We show that a modification of the spin-orbit terms of the Skyrme energy density functional and the Hartree-Fock potential are needed to reproduce the measured spectroscopic factors. We also show that fair agreement with experiment is obtained for the single beta Gamow-Teller branches and for the double-beta decay
matrix elements. In the next section we discuss parity-violating elastic electron scattering by nuclei and its potential application to the precise determination of neutron densities in nuclei. We study nuclear isovector and isoscalar densities for $N > Z$ and for $N = Z$ stable nuclei obtained within the self-consistent mean field approximation. We compare the values of the parity-violation asymmetry (PVA) at low and intermediate $q$-values for different $N/Z$ and $A$ values. Distorted wave calculations of PVA are shown and are compared to plane wave impulse approximation. In the last section we show our results on the energy spectrum of the electron emitted in the beta decay of $^{187}$Re, including the distortion of its wave function due to the Coulomb interaction with the daughter nucleus. We will exploit this information to analyze the effect on the spectrum of a hypothetical keV-mass contribution to the emitted neutrino.

2. Active-shell spectroscopic factors in beta decaying nuclei: $^{76}$Ge→$^{76}$Se case and the single and double beta decay nuclear matrix elements

The search for the neutrino mass is pushing strongly experimental and theoretical work on nuclear double-beta decay because of the crucial role of a possible detection of the zero-neutrino mode. However to settle the absolute scale of the neutrino mass, the precise theoretical calculation of the nuclear matrix element is equally important as the detection of the zero-neutrino mode. An important ingredient of those calculations are the spectroscopic factors.

The best studied case of two-neutrino double-beta decay is that of $^{76}$Ge going to $^{76}$Se. The relevant protons and neutrons involved in the double-beta process are those in the valence shells $1p_{3/2}$, $1p_{1/2}$, $0f_{5/2}$, $0g_{9/2}$, whose occupation probabilities have been recently measured by Schiffer et al. (for neutrons) and by Kay et al. (for protons) [2]. These data are not reproduced by previous mean-field calculations using Woods-Saxon potential, and although shell-model calculations have also been carried out with occupation probabilities in good agreement with experimental ones, the corresponding single particle energies are not available in the literature. In the deformed Hartree-Fock calculation of this work we allow the Skyrme force parameters to vary and search for a parametrization that provides better agreement with the experimental occupations in the valence shells of $^{76}$Ge and $^{76}$Se. We focus on the effect of the two-body spin-orbit strength, $W_0$, that appears to be of great relevance to describe properly the proton and neutron valence shells occupations. In addition, the use of the experimental ground-state deformations contributes to the agreement between the calculated occupations and the experimental ones [3].

The new single-particle energies and occupation probabilities obtained with this procedure have an effect on the matrix element of the two-neutrino double-beta decay $^{76}$Ge→$^{76}$Se and on the GT strength distributions of both nuclei going to the intermediate nucleus $^{76}$As. Theoretical results are shown in Fig. 1 together with experimental data (light lines). A first attempt of pnQRPA calculation using the Sk3 Skyrme force for spherical ground states is presented in the dashed black lines, whereas the solid black lines correspond to deformed ground states (experimental deformations) and using an increased value of the two-body spin-orbit strength for neutrons, $W_n = 200$ MeV fm$^3$. The deformed calculation agrees better with the experiment, as shown in the low energy regions of the GT (plus and minus) accumulated strength distributions (to the left) and in the running sum of the double-beta decay matrix element (to the right). The final value of the double-beta decay matrix element, reached by the running sum beyond 18 MeV, lies within the experimental range (shaded region) for the improved calculation, whereas the initial one (spherical, Sk3) is very far.
3. Parity-violation asymmetry in electron-nucleus scattering: $^{208}$Pb($\vec{e}, e$) and $^{12}$C($\vec{e}, e$) cases and the precise determination of neutron distributions

We show in Fig. 3 results on PV asymmetry in elastic scattering of polarized electrons by stable $N > Z$ and $N = Z$ nuclear targets, namely $^{208}$Pb and $^{12}$C respectively [4]. Their ground-state structures have been obtained from a Skyrme Hartree-Fock mean field with pairing interactions in BCS approximation. The curves in the figure correspond to the parity-violation asymmetry, which is proportional to the difference between the cross-sections of the scattered electrons polarized parallel and antiparallel to their momenta. Three curves (dark lines) are presented for $^{208}$Pb, and two for $^{12}$C (light lines). In the case of $^{208}$Pb, two calculations are in plane-wave (PW) Born approximation, one (solid line) corresponding to the actual proton and neutron form factors and the other one (dashed line) corresponding to the same function for both the proton and the neutron form factors, both normalized to 1. The latter shows a simple proportionality to $q^2$, whereas the former shows a complex pattern for which the actual nuclear structure is responsible. The divergences in this last curve are filled in or smoothed out when the distorted-wave (DW) calculation is carried out (dotted line). This calculation, which is the one to be compared to experimental data, takes into account the distortion of the incoming and outgoing electron’s wave function due to the presence of the nuclear Coulomb field, which is very important in this case (and not so much in the case of $^{12}$C).

The results for $^{12}$C in the same figure are for plane-wave calculations using the actual ground-state form factors (solid line), and the same proton and neutron form factors (dashed line). The latter shows again a simple $q^2$ dependence whereas the structure of the former is related to the isospin mixing in the nuclear ground state (nominally the isospin is $T = 0$ since it has $N = Z$).

4. Charged lepton spectrum in single-beta decay: $^{187}$Re case and the search for sterile neutrinos

The differential decay rate with respect to the energy of the electron emitted in a single beta minus decay depends on the nuclear transition matrix element and on several kinematic factors of the emitted leptons. The latter come from the differential density of asymptotic final states.
Figure 2. Parity-violation asymmetry of elastic electron scattering by $^{208}$Pb and by $^{12}$C. For each nucleus we show results of PW calculations (full lines) and also assuming equal proton and neutron form factors (dashed lines). For $^{208}$Pb we also show the result of a DW calculation using realistic proton and neutron densities (dotted line).

and from the shape factor:

$$
\frac{d\Gamma_l^\chi}{dE_e} = K \ B \ p_e \ p_\nu \ E_e \ (E_0 - E_e) \ F_l(Z', E_e) \ S_l(p_e, p_\nu) \ \theta(E_0 - E_e - m_\nu) \tag{1}
$$

where $K$ is a constant ($\sim 2 \cdot 10^{-36}$ keV$^{-4}$), $B$ is the squared nuclear reduced matrix element (dimensionless), $p_e$ and $p_\nu$ are the lepton momenta, and $E_e$ and $E_0$ are the electron total energy and maximum total energy respectively. The relativistic Fermi function $F_l(Z', E_e)$ accounts for the Coulomb interaction between the residual nucleus (charge $Z'$) and the emitted electron with an angular momentum $l$ with respect to the nucleus. The shape factor is a kinematic dependence coming from the total transition amplitude, which takes different forms depending on the nature of the decay (allowed or forbidden). For instance, for an allowed decay its value is simply 1, whereas for a unique first forbidden decay it depends on the orbital angular momentum quantum number $l$ of the emitted electron:

$$
S_0(p_e, p_\nu) = \frac{1}{3} p_\nu^2 \quad \text{and} \quad S_1(p_e, p_\nu) = \frac{1}{3} p_e^2 \tag{2}
$$

The shape and range (the latter given by the step function $\theta$) of the differential decay rate, Eq. 1, strongly depend on the mass $m_\nu$ of the considered neutrino $\chi$ [5]. In figure 3 left-panel we show the differential decay rate of the electron emitted in the beta decay of $^{187}$Re ($Q_\beta = 2.47$ keV) for different masses of the emitted neutrino. Only the leading contribution (electron $p$-wave) is shown. The massless case is a very good approximation to the usual light active neutrino (with mass $m_\nu \sim 1$ eV), whereas the keV-cases account for hypothetical heavy sterile neutrinos, which have been recently proposed as dark matter candidates [5]. The neutrino emitted in a beta decay should obviously be composed mainly of the light active neutrino, with only a very small mixing of the heavy sterile one. We will consider the following mixing of active-sterile neutrinos or antineutrinos:

$$
|\nu_e\rangle = \cos \theta \ |\nu_\alpha\rangle + \sin \theta \ |\nu_s\rangle \tag{3}
$$
Figure 3. Left-hand panel shows the differential beta decay rate (in arbitrary units of the order $10^{-37}$) of $^{187}$Re for different masses $m_\nu$ of the emitted neutrino. Right-hand panel shows the differential decay rate when the emitted neutrino is a mixing of a light neutrino ($m_{\nu_a} \sim 0$) with a heavy neutrino ($m_{\nu_s} = 1$ keV), the mixing angle being much too large in this plot for the sake of clarity.

In figure 3 right panel we show the effect on the $^{187}$Re differential decay rate when the neutrino emitted in association with the electron contains a mixing of a 1 keV sterile neutrino [5]. The mixing angle in this plot is unrealistically large in order to clearly show the kink produced in the electron spectrum when the neutrino has enough energy for the heavy neutrino to contribute ($E_\nu = Q_\beta - m_{\nu_s} = 1.47$ keV).

Acknowledgments
We acknowledge our collaborators Dr. Udías, Prof. Faessler, Prof. Donnelly for fruitful discussions. This work was supported by Ministerio de Ciencia e Innovación (Spain), Contracts FIS2008-01301 and CSPD-2007-00042@Ingenio2010, and UCM ‘Grupo de Física Nuclear’-910059.

References
[1] Villars F 1977 Nucl. Phys. A 285 269
Moya de Guerra E and Villars F 1977 Nucl. Phys. A 285 297
Moya de Guerra E and Villars F 1978 Nucl. Phys. A 298, 109
Moya de Guerra E 1982 Phys. Rev. Lett. 48 922
Moya de Guerra E 2001 The limits of the mean field, in An Advanced Course in Modern Nuclear Physics (Ed. Arias, J.M. and Lozano M.) Springer.
[2] Schiffer J P et al. 2008 Phys. Rev. Lett. 100 112501
Kay B P et al. 2009 Phys. Rev. C 79 021301(R)
[3] Moreno O, Moya de Guerra E, Arriguren P and Faessler A 2010 Phys. Rev. C 81 041303(R)
[4] Moreno O, Moya de Guerra E, Arriguren P and Udias J M 2010 J. Phys. G: Nucl. Part. Phys. 37 064019
Moreno O, Arriguren P, Moya de Guerra E, Udias J M, Donnelly T W and Sick I 2009 Nucl. Phys. A 828 306
[5] de Vega H J, Moreno O, Moya de Guerra E, Ramon Medrano M, Sanchez N G 2011 arXiv:1109.3452v2 [hep-ph].