Double droplet splashing on a thin liquid film with a pseudopotential lattice Boltzmann method

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ABSTRACT
This paper studies the interaction of two droplets splashing on a stationary film. A source term is included in the large-density-ratio pseudopotential lattice Boltzmann method to achieve tuneable surface tension. This model offers excellent numerical accuracy and stability for droplet impacts on liquid films. The influence of the Reynolds number, Weber number, film thickness, and horizontal/vertical distance between the droplets on the crown geometry evolution is investigated. The energy loss during the impact process and the velocity discontinuity in the liquid film are the two key factors affecting the stability and evolution process of the crown. A smaller Reynolds number or thicker liquid film enhances the energy loss and decreases the velocity discontinuity, leading to more stable side and central jets. An increase in the horizontal distance between the droplets reduces the velocity discontinuity, causing the central jet height to decrease. An increase in the Weber number does not affect the energy loss or velocity discontinuity, but the lower surface tension leads to a dramatic deformation in both the central and side jets. A vertical distance between the two droplets causes an asymmetrical evolution of the crown geometry, and postpones the breakup time of the central jet.

1. Introduction
After droplets impact a solid wall, they gather and form a liquid film on the surface; later droplets then impact the liquid film. This phenomenon is widespread in nature and industrial applications, and the whole process includes a series of complex multiphase flows. Understanding how droplets interact with the liquid film and each other upon impact is important when studying spray coating, inkjet printing, pesticide spraying, and soil erosion by raindrops.

Earlier studies have found that the evolution of a droplet impacting a liquid film is dominated by several non-dimensional parameters, such as the film thickness, geometry, and other properties of the wall. A large number of studies have examined the underlying complex dynamics of a single droplet splashing a stationary thin film (Alghoul et al., 2011; Burzynski & Bansmer, 2018; Cossali et al., 2004; Hung et al., 2011; Josserand & Zaleski, 2003; Rioboo et al., 2003; Vander Wal et al., 2006; Wang, Dandekar, et al., 2018). According to the comprehensive experimental studies of Huang and Zhang (2008), the impact mode of droplet splashes on a thin film can be classified as one of five types: fusion, rebound, partial rebound, crown, and splash. An important study was carried out by Yarin and Weiss (1995), in which a quasi-one-dimensional model revealed the discontinuity of the kinematic velocity in the liquid film. The discontinuity of the kinematic velocity corresponds to the formation of the crown jet and exerts a great influence on the evolution process of the crown. Yarin and Weiss (1995) also indicated that the non-dimensional crown radius is a function of the non-dimensional time. However, the relationship between the two parameters was not clearly defined. Gao and Li (2015) built an empirical model through theoretical analysis and experimental studies, and predicted the variation of the crown root radius and jet stretch rate with respect to time.

Compared with experimental studies, numerical simulations can provide deep insights into the flow field during the evolution process. Macroscopic multiphase...
models have typically used the Navier–Stokes equations and an interface tracking model (Guo et al., 2014; Mosavi et al., 2019; Motzkus et al., 2011; Shamshirband et al., 2020; Shetabivash et al., 2014; Wang et al., 2020). Several studies have analyzed the evolution process of the flow field and found that the instability of the crown rim causes the jet to break up (Allen, 1975; Nikolopoulos et al., 2007; Roisman et al., 2006). However, some discrepancies still exist, with Roisman et al. (2006) and Nikolopoulos et al. (2007) stating that the breakup is caused by the Plateau–Rayleigh instability, while Allen (1975) suggests that the breakup is caused by the Rayleigh–Taylor instability.

The lattice Boltzmann method (LBM) has been applied to simulations of multiphase phenomena in which there is a dramatic change in the interface. The most widely used LBM multiphase models fall into four categories, namely the color model (Grunau et al., 1993), pseudopotential model (Shan & Chen, 1993, 1994), free-energy model (Swift et al., 1995; Swift et al., 1996), and phase-field model (He et al., 1999). Compared with the traditional macroscopic numerical method, pseudopotential LBM approaches have several attractive characteristics (He, Zhang, Xu, et al., 2020; He, Zhang, Yang, et al., 2020; Li, Luo, et al., 2016): (i) the convection terms are composed of linear equations, which avoids the complex treatment of the nonlinear convection term; (ii) the pressure is solved from the ideal or non-ideal equation of state (EOS), removing the need to solve the Poisson equation; (iii) the gas–liquid interface is automatically formed by the interaction force between particles, without using the interface tracking model; (iv) the boundary treatment is relatively simple, especially for walls with different contact angles; (v) the pseudopotential model gives better numerical results than traditional methods (Ezzatneshan, 2017); and (vi) the governing equation can be divided into stream and collision components, which is conducive to parallel computing.

Lee and Lin (2005) first proposed the LBM phase-field model to simulate a droplet impact on a film with a high density ratio. This model must determine the directional gradient, which requires considerable computing resources. Based on this model, the multiple-relaxation-time (MRT) collision model was introduced by Mukherjee and Abraham (2007) and the effects of the density and viscosity ratios were studied. Recently, improved pseudopotential models have been used by Fallah Kharmiani et al. (2016) and Yuan et al. (2020) to study the impact process, allowing the breakup of the crown rim to be clarified. In most natural phenomena, large numbers of droplets splashing on a liquid film mean that the droplets not only interact with the liquid film, but also with each other. However, most studies based on LBM have focused on a single droplet impacting the liquid film. To the best of our knowledge, few studies have used LBM to examine the underlying evolution mechanics of multiple droplets impacting a liquid film (Li, Jia, et al., 2016; Raman et al., 2015; Wang, Wang, et al., 2018). Raman et al. (2015) first investigated the evolution of the crown following the impact of two droplets on a liquid film using a phase-field model, and found that a critical non-dimensional film thickness produces the highest side and central jets. The authors also indicated that the growth rate of the crown height and radius decreases with increasing gas density. In their study, the crown did not break up, although the governing Reynolds (Re) and Weber (We) numbers were set as 500 and 8000, respectively. Additionally, the side jets rolled inward, which was inconsistent with experimental results (Wang, Dandekar, et al., 2018). Furthermore, Li, Jia, et al. (2016) simplified the model of Lee and Lin (2005) and studied the same phenomenon with different initial impact parameters. However, Re and We in their study were limited to a narrow range, and no satellite droplets were formed.

Note that the aforementioned studies (Li, Jia, et al., 2016; Raman et al., 2015; Wang, Wang, et al., 2018) related to two droplets splashing were based on the LBM phase-field model – the present paper describes the first systematic study of double-droplet impacts on liquid films using the LBM pseudopotential model. In this model, a source term for tuneable surface tension is applied, thus enlarging the range of We. This strategy has been successfully applied in previous research (Fallah Kharmiani et al., 2016; Yuan et al., 2020). We analyze the effects of We, Re, and the non-dimensional film thickness on the crown structure. The kinetic energy of the liquid phase and the velocity discontinuity in the film are introduced to investigate the instability of the side and central jets. The effects of the non-dimensional horizontal and vertical distance between two droplets on the evolution process of the side and center jet height and crown radius are also studied. The remainder of this paper is organized as follows. The introduction of a source term to achieve tuneable surface tension in the pseudopotential LBM is described in section 2. Mesh convergence is analyzed and the model is validated by a droplet splashing on a film in section 3, with the results quantitatively compared with those from a semi-empirical model (Gao & Li, 2015). We then study the influence of different parameters on the evolution processes in section 4. Finally, the conclusions to this study are presented in section 5.
2. Model description

The original pseudopotential model (Shan & Chen, 1993, 1994) has several evident drawbacks, including: (1) poor numerical stability for high-density-ratio multiphase phenomena; (2) high spurious currents at the gas–liquid interface; (3) a non-tunable surface tension; and (4) thermodynamic inconsistency. Significant efforts have been made to overcome these drawbacks. In this section, a large-density-ratio pseudopotential model with tuneable surface tension and thermodynamic consistency is proposed.

2.1. Pseudopotential lattice Boltzmann method

Yu and Fan (2010) indicated that the LBM with an MRT model exhibits higher numerical stability and smaller spurious currents than with the Bhatnagar–Gross–Krook collision operator. The corresponding particle distribution function with source terms is expressed as (Yu & Fan, 2010):

\[ f_\alpha(x + e_\alpha \Delta t, t + \Delta t) = f_\alpha(x, t) - \sum_\alpha \tilde{A}_{\alpha\beta}(f_\beta - f_\beta^{eq}) \times |(x,t) - \sum_\beta (L_{\alpha\beta} - \tilde{L}_{\alpha\beta})S_{\beta} \Delta t_{(x,t)} + C_{\alpha}, \]

where \( x \) is the node position, \( e_\alpha \) is the discrete velocity in the \( \alpha \)th direction, \( f_\alpha \) and \( f_\alpha^{eq} \) are the particle distribution functions and equilibrium distribution functions, \( \Delta t \) is the time step, \( I \) is the unit matrix, \( \tilde{A} \) is the relaxation matrix, \( S_{\alpha} \) is the forcing term, and \( C_{\alpha} \) is the source term that is used to achieve different values of the surface tension. For incompressible flows, the equilibrium distribution \( f_\alpha^{eq} \) is given as (Li et al., 2013):

\[ f_\alpha^{eq} = \omega_\alpha \rho \left[ 1 + \frac{e_\alpha \cdot v}{c_s^2} + \frac{(e_\alpha \cdot v)^2}{2c_s^4} - \frac{v \cdot v}{2c_s^2} \right], \]

where \( \omega_\alpha \) denotes the weights: for a two-dimensional model with nine discrete velocities (D2Q9), \( \omega_0 = 4/9, \omega_{1\sim 4} = 1/9, \omega_{5\sim 8} = 1/36. c_s = c/\sqrt{3} \) is the lattice sound speed, \( c = \Delta x / \Delta t \) is the lattice constant. \( \rho \) is the macroscopic density and \( v \) is the equilibrium velocity, set to be equal to the macroscopic velocity in the present study. In the D2Q9 lattice model, \( e_\alpha \) is given by (Yu & Fan, 2010):

\[ e_\alpha = \begin{cases} (0,0) & \alpha = 0 \\ \sqrt{2}c \cos((\alpha - 1)\pi/2), \sin((\alpha - 1)\pi/2) & \alpha = 1 \sim 4 \\ \sqrt{2}c \cos((2\alpha - 9)\pi/4), \sin((2\alpha - 9)\pi/4) & \alpha = 5 \sim 8 \end{cases}. \]

The diagonal relaxation matrix \( \tilde{A} \) with different relaxation coefficients is defined as (Li et al., 2013):

\[ \tilde{A} = M^{-1}AM = \text{diag}(\tau_\rho^{-1}, \tau_\rho^{-1}, \tau_\xi^{-1}, \tau_\xi^{-1}, \tau_q^{-1}, \tau_q^{-1}, \tau_v^{-1}, \tau_v^{-1}), \]

where \( \rho \) is the density, \( e \) is the energy, \( \xi \) is the energy square, \( j \) is the momentum, and \( v \) is the kinematic viscosity. Note that \( \tau_\rho \) is related to the kinematic viscosity by \( v = 1/[c_s^2(\tau_\rho - 0.5)\Delta t] \). The relaxation parameters in Equation (4) are designated as \( \tau_\rho^{-1} = \tau_j^{-1} = 1.0, \tau_\xi^{-1} = \tau_\xi^{-1} = 1.1, \) and \( \tau_q^{-1} = 1.1 \) in the present study. The transformation matrix \( M \) can be written as (Yu & Fan, 2010):

\[ M = \begin{bmatrix} -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 4 & -1 & -1 & -1 & -1 & 2 & 2 & 2 \\ 4 & -2 & -2 & -2 & -2 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & -1 & -1 \\ 0 & 0 & -2 & 0 & 2 & 1 & -1 & -1 \\ 0 & 0 & -1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 \end{bmatrix}, \]

where \( M^{-1} \) is the inverse of \( M \). The equilibrium moment \( \mathbf{m}^{eq} \) can be obtained by transforming \( \mathbf{f}^{eq} \) into the momentum space with \( \mathbf{m}^{eq} = \mathbf{M}^{-1} \mathbf{f}^{eq} \) (Yu & Fan, 2010):

\[ \mathbf{m}^{eq} = \rho (1, -2 + 3(u_x^2 + u_y^2), 1 - 3(u_x^2 + u_y^2), u_x, -u_x, u_y, -u_y, u_x, u_y)^T. \]

2.2. Source terms

The external force term \( \mathbf{S} \) proposed by Li et al. (2013) is introduced here, as this provides better numerical stability and enables larger liquid/gas density ratios to be achieved:

\[ \mathbf{S} = \rho \begin{bmatrix} 0 \\ 6(u_x F_x + u_y F_y) + \frac{12e|F_x|^2}{\psi^2 \Delta t (\tau_\rho - 0.5)} \\ -6(u_x F_x + u_y F_y) - \frac{12e|F_x|^2}{\psi^2 \Delta t (\tau_\rho - 0.5)} \\ F_x \\ F_y \\ 2(u_x F_x - u_y F_y) \\ u_x F_x + u_y F_y \end{bmatrix}, \]

where \( \varepsilon \) is a tuneable parameter for thermodynamic consistency. \( \mathbf{F}_{int} \) is the force between particles, which is given
where $G = -1$ is the interaction strength and $\psi$ is the pseudopotential. $w_\alpha$ denotes the weights for the interaction force, and $w_0 = 0$, $w_1 - 4 = 1/3$ and $w_5 - 8 = 1/12$ (Shan, 2008). The non-ideal EOS is applied here, where $\psi$ can be calculated as (Yuan & Schaefer, 2006):

$$\psi(\rho) = \sqrt{\frac{2(p_{\text{EOS}} - \rho c^2)}{Gc^2}}. \quad (9)$$

Yuan and Schaefer (2006) found that the application of the Carnahan–Starling (C–S) EOS can achieve higher numerical stability in the pseudopotential model. Thus, $p_{\text{EOS}}$ is calculated as:

$$p_{\text{EOS}} = \rho RT \left[ 1 + b \rho/4 + (b \rho/4)^2 - (b \rho/4)^3 \right] \left[ 1 - b \rho/4 \right]^3 - a \rho^2, \quad (10)$$

where $a = 0.4963 R_g^2 T_c^2 / p_c$, $b = 0.1873 R_g T_c / p_c$, $T_c$ is the critical temperature, and $p_c$ is the critical pressure. The parameters in Equation (10) are $a = 0.25$, $b = 4$, and $R_g = 1$, as used by Yuan and Schaefer (2006).

Considering that the surface tension is related to the gas/liquid density ratio in the original LBM, and cannot be adjusted independently, the parameters are limited to a very small range. Therefore, the source term proposed by Li and Luo (2013) is introduced to achieve tuneable surface tension:

$$C = \left( 0, 1.5 \tau_e^{-1} (Q_{xx} + Q_{yy}), -1.5 \tau_e^{-1} (Q_{xx} + Q_{yy}), 
0, 0, 0, 0, -\tau_v^{-1} (Q_{xx} - Q_{yy}), -\tau_v^{-1} (Q_{xy}) \right), \quad (11)$$

and $Q$ is solved from (Li & Luo, 2013):

$$Q = \kappa \frac{G}{2} \psi(x) \sum_{\alpha} w_\alpha (\psi(x + e_\alpha \Delta t) - \psi(x)) e_\alpha e_\alpha, \quad (12)$$

where the parameter $\kappa$ is used to obtain tuneable surface tension. In the present study, the corresponding surface tension is obtained from Laplace's law; however, the process of determining the surface tension is tedious (He, Zhang, Xu, et al., 2020), and is not presented here.

The macroscopic density $\rho$ and velocity $u$ are calculated through (Yuan & Schaefer, 2006):

$$\rho = \sum_{\alpha} f_\alpha, \quad \rho u = \sum_{\alpha} f_\alpha e_\alpha + \frac{1}{2} F \Delta t, \quad (13)$$

where $F = F_m + G + \ldots$ is the resultant force and $G$ is the gravitational force, which is calculated as

$$G = (\rho(x) - \rho_g) g, \quad (14)$$

where $g$ is the gravitational acceleration and $\rho_g$ is the initial density of the gas.

### 3. Mesh analysis and model validation

The pseudopotential model proposed in this section was validated in our earlier study through Laplace’s law (He, Zhang, Xu, et al., 2020; Yuan et al., 2020), and the corresponding surface tension values were obtained. This section presents simulation results for a single droplet splashing on a film to validate our model. The results are quantitatively compared with the analytical theory in an earlier study (Gao & Li, 2015).

Figure 1 shows a schematic diagram of a single droplet impacting on a liquid film. The left, right, and top boundaries are set as non-equilibrium boundaries, and the bottom is set as a no-slip boundary, which is expressed as (Li et al., 2014):

$$f_2 = f_4,$$
$$f_5 = f_7 - 0.5(f_1 + f_3) - 0.25 \Delta t (F_x + F_y),$$
$$f_6 = f_8 + 0.5(f_1 + f_3) + 0.25 \Delta t (F_x - F_y).$$

$r_0$ is the droplet radius, $U_0$ is the initial impact velocity, and $H$ is the liquid film height. $r_e$ is the crown root radius, defined as the horizontal distance between the inner crown root and the droplet center, as shown in Figure 1. $h$ is the crown height, defined as the distance between the highest point of the crown and the gas–liquid interface of the liquid film.

Combined with the liquid density $\rho_l$, viscosity of the liquid phase $\nu_l$, and surface tension $\sigma$, all of the aforementioned parameters determine several important non-dimensional parameters related to the crown evolution process, including

$$\text{Re} = \frac{2r_0 U_0}{\nu_l}, \quad \text{We} = \frac{2 \rho_l r_0 U_0^2}{\sigma}, \quad (16)$$

$$h^* = \frac{H}{2r_0}, \quad t^* = \frac{U_0 t}{2r_0},$$

where $h^*$ is the non-dimensional film thickness and $t^*$ is the non-dimensional time. The non-dimensional radius
of the crown $r^*$ and the non-dimensional crown height $h^*$ are also introduced, and are normalized as

$$ r^* = \frac{r_c}{2r_0}, h^* = \frac{h}{2r_0}, \quad (17) $$

In our study, the density field of the droplet is initialized as:

$$ \rho(x, y) = \frac{\rho_l + \rho_g}{2} + \frac{\rho_l - \rho_g}{2} \tanh \left\{ \frac{2}{w} \left[ \sqrt{(x-x_0)^2 + (y-y_0)^2 - R_0^2} \right] \right\}, \quad (18) $$

where $(x_0, y_0)$ is the droplet center. The density field of the liquid film is initialized as:

$$ \rho(x, y) = \frac{\rho_l + \rho_g}{2} + \frac{\rho_l - \rho_g}{2} \tanh \left\{ \frac{2(y_1 - H)}{w} \right\}, \quad (19) $$

where $y_1$ is the interface location. Considering the numerical stability and physical reality (Li et al., 2013), the interface width $w$ in Equations (18) and (19) is chosen as 5 lattice units ($lu$).

The temperature is chosen as $T = 0.5 T_c$ and the liquid/gas density ratio is 720, where $T_c = 0.0249$ in the present study. The viscosity relaxation time $\tau_\nu$ is chosen as 0.575, with the corresponding Re and We set to 500 and 87.8, respectively. Yarin and Weiss (1995) indicated that the main parameters affecting the crown evolution are Re and We, and gravity has little influence on the crown evolution process when the droplet is small. Thus, the gravitational acceleration in the present study is set to $g = 0$.

Four grid densities are considered, which are $500lu \times 150lu$, $800lu \times 240lu$, $1000lu \times 300lu$, and $2000lu \times 600lu$. The corresponding droplet radii are $25lu$, $40lu$, $50lu$, and $100lu$. To compare the simulation results quantitatively, the initial Reynolds number is held at 1000 and the Weber number is fixed to 87.8. The variation in the root radius versus non-dimensional time is then examined.

The jet shape of different mesh densities at non-dimensional time $t^*$ under different grid densities is shown in Figure 2. However, the shape of the crown does not vary with the grid density. The main difference is in the size of the satellite droplets, because the LBM pseudopotential model is a diffuse method and there exists a gas–liquid interface thickness with several grids, which is related to the parameter $a$ in the C–S EOS. According to the earlier research of Li et al. (2013), the thickness is $5lu$ with $a = 0.25$. When the initial droplet radius is reduced to $25lu$, the interface thickness is $1/5$ of the radius, which leads to the simulation results becoming unphysical and unable to fully capture the shape of the satellite droplets.

Gao and Li (2015) developed a model for predicting the evolution process of the crown root radius and jet stretch rate:

$$ r^* = \beta \sqrt{t^*} + \frac{1}{\sqrt{6h^*}} - \left( \frac{1}{3h^*} - \frac{1}{\sqrt{6h^*}} \right)^{1/2}, \quad (20) $$

where $\beta$ is used to determine the energy loss coefficient, and is solved from (Gao & Li, 2015):

$$ \beta = \left( \frac{2\lambda^2}{3h^*} \right)^{1/4}. \quad (21) $$
The energy loss factor $\lambda$ is obtained from a large number of experimental results (Gao & Li, 2015):

$$
\lambda = \frac{0.26}{Re^{-0.05}We^{0.07}h^{0.34}},
$$

(22)

In our study, the energy loss factor is $\lambda = 0.416$, and the corresponding $\beta$ is 0.824. The simulation results are compared with the theoretical analysis in Figure 3(a). The numerical results for all four grid densities are consistent with the semi-empirical theoretical analysis at $t^* = 0.4 - 1.4$. There are small discrepancies between the numerical results and the theoretical analysis for $t^* = 1.4 - 2.0$. This may be caused by the breakup of the crown, which is not considered in the theoretical analysis. For the grid density $500lu \times 150lu$, oscillations occur for $t^* = 1.4 - 2.0$, whereas the other grid densities retain smooth curves. Furthermore, the curves for $1000lu \times 300lu$ and $2000lu \times 600lu$ almost overlap with each other. Taking computational efficiency, precision, and stability into consideration, a grid density of $1000lu \times 300lu$ is suitable.

Furthermore, the stretch rate of the jet obtained by the LBM with a grid density of $1000lu \times 300lu$ is compared with the theoretical results in Figure 3(b). The vertical jet stretch rate is described as (Gao & Li, 2015):

$$
S_t = \left[ t^* + t_i + 2\tau_0 \right]^{-1},
$$

(23)

where $t_i$ is the end time of the deformation phase, given by (Gao & Li, 2015):

$$
t_i = \left( r_i - 0.5 \right) / \lambda,
$$

(24)

and $r_i$ is calculated from the non-dimensional height $h^*$ as (Gao & Li, 2015):

$$
r_i = \left( \frac{1}{6h^*} \right)^{1/2}.
$$

(25)

$\tau_0$ is the time shift, obtained as (Roisman & Tropea, 2002):

$$
\tau_0 = \frac{1}{2\lambda} \left( 1 - \frac{1}{\sqrt{6h^*}} \right).
$$

(26)

The stretch rate obtained by the LBM is consistent with the theoretical analysis during the no-breakup period for $t^* = 0.5 - 1.5$, although some discrepancies appear when $t^* > 1.5$ after the satellites have formed. This may be caused by the semi-empirical model not considering the influence of crown breakup. Some oscillations can also be observed in the stretch rate curve, and these exist even when the grid density is increased to $2000lu \times 600lu$. Further investigations are needed to understand these phenomena.
4. Numerical results and discussion

The computational and boundary conditions are shown in Figure 5(a). The size of the computational domain is set as $1201\,lu \times 651\,lu$. The boundaries are as indicated in Figure 1. Two droplets of radius $r_0 = 50\,lu$ are located in the computational domain. The impact velocity of the droplets is set as $U_0 = 0.125\,lu \cdot tu^{-1}$. The horizontal distance between the two droplets is $S_x$, and the vertical distance between them is $S_y$. $H$ is the liquid film height. The parameters used to quantify the crown structure are presented in Figure 5(b). $r$ is the crown rim radius, $h$ is the crown height, and $S_l$ and $S_r$ are the left and right spread lengths, respectively. The subscripts $l$ and $r$ indicate the left crown and the right crown, respectively. $h_c$ is the central jet height. The density ratio is set as $\rho_l / \rho_g = 720$.

To provide a deeper insight into the crown dynamics, we introduce the total kinetic energy of the liquid phase and investigate the crown deformation and breakup from an energy perspective.

$$Eu = \sum_{(x,y)} \rho f(x,y)u^2,$$

where $\rho f$ is the density of the liquid grid nodes, distinguished by the discriminant $\rho f \geq 0.5(\rho_l + \rho_g)$. The discontinuity of the velocity in the liquid film is also discussed.

The influence of several non-dimensional parameters on the impact dynamics is investigated systematically in the following subsections:

1. **Reynolds number (Re).**
2. **Weber number (We).**
3. **Non-dimensional liquid film thickness ($h^*$).**
4. **Non-dimensional horizontal distance between two droplets ($S_x^e = S_x / 2r_0$).**
5. **Non-dimensional vertical distance between two droplets ($S_y^e = S_y / 2r_0$).**

4.1. Reynolds number

The influence of Re on the crown evolution is studied with three different Reynolds values. Raman et al. (2015) and Li, Jia, et al. (2016) studied the influence of Re by adjusting the impact velocity and liquid viscosity. Their results show that the crown height increases as Re rises, but the evolution of the non-dimensional extension length overlaps with different Re numbers (the non-dimensional extension length is defined as $S_a^* = S_a^e + S_r^e$, where $S_l^e = S_l / r$ and $S_r^e = S_r / r$ are the left and right non-dimensional spread lengths). The impact velocity of the droplets is kept constant, and different Re numbers are obtained by changing the liquid viscosity $\nu_l$. The Re numbers considered here are $Re =$

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**Figure 4.** Relationship between the non-dimensional time and crown root radius, and the linear fit between them. The hollow points correspond to the evolution process in the no-breakup period, and the solid points correspond to the evolution process after the crown has broken up.
200, 500, and 1000. The other parameters are as follows: 
We = 87.8, non-dimensional liquid film height $h^* = 0.25$, non-dimensional horizontal distance $S_x^* = 2.0$, and non-dimensional vertical distance $S_y^* = 0$.

The crown geometry at $t^* = 2.5$ is shown in Figure 6. The central jet is formed by the collision of two crown jets between two droplets. The central jet heights are the same for Re = 500 and 1000, and are greater than for Re = 200, which is consistent with the study of Li, Jia, et al. (2016). At $t^* = 2.5$, both the left and right crowns break up, and satellite droplets are formed with Re = 500 and 1000. In a numerical study by Raman et al. (2015), inward bending of the left and right crowns was observed at high density ratios, a phenomenon that did not concur with earlier studies (Wang, Dandekar, et al., 2018). In the present study, the left and right crown jets bend outwards during the evolution process. Furthermore, a gas–liquid interface was observed in the central jet of the simulation results reported by Raman et al. (2015), which is not consistent with the actual physical phenomenon. The colliding crowns merge into a central jet in our study, which is qualitatively consistent with earlier simulation results (Li, Jia, et al., 2016), and no gas–liquid interface appeared in the central jet.

Due to the symmetrical geometry of the crown, only the crown rim radius and height of the left crown are analyzed. The relationship between the left crown rim radius and non-dimensional time is shown in Figure 7(a) for

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**Figure 5.** (a) Schematic diagram of two droplets splashing on a film. (b) Schematic diagram of the crown geometry.

**Figure 6.** Crown geometry of the density contour line $\rho = 0.2(\rho_g + \rho_l)$ at non-dimensional time $t^* = 2.5$ with We = 87.8, $h^* = 0.25$, $S_x^* = 2.0$, and $S_y^* = 0$. 
the different Reynolds numbers. Before the breakup of the left crown, the radius increases as Re rises, because the energy lost in the collision process decreases as Re increases, as shown in Figure 7(d). At the point at which the left crown breaks, the rim radius is greatest in the case of Re = 200. However, the rim radius with Re = 1000 is still greater than for Re = 500. The left crown height varies with time under all Re values, as shown in Figure 7(b). Before the crown breaks, the crown height increases with increasing Re. This is because the deformation of the crown is dominated by inertia, and the attenuation of the inertial force is slowed by the decrease in liquid viscosity. As the left crown breaks, the height is greatest with Re = 1000. The left crown height with Re = 500 almost overlaps that with Re = 200. Figure 7(c) depicts the evolution of the central jet height with respect to non-dimensional time. The central jet breakup occurs from $t^* = 0.75 - 1.0$ for Re = 1000. Due to some defects in the pseudopotential model, the droplets are ‘eaten’, and no satellite droplets are observed on the central jet in Figure 6.

The total kinetic energy of the liquid phase varies with non-dimensional time during the evolution process, as shown in Figure 7(d). For the three different Reynolds numbers, $E_u$ decreases over time. During the evolution of $E_u$, three steep drops are observed, corresponding to the adjustment of the initial flow field, the droplets impacting the liquid film, and the crown colliding to form a
central jet. During the second steep drop, the decrease in $E_u$ is smaller at higher values of $Re$, which indicates that more kinetic energy is consumed in the collision process at lower $Re$. With more residual $E_u$, the side and central jets become more unstable, leading to a significant deformation and a greater likelihood of breaking up. Figure 8 shows the flow field of the central jet and the liquid film for $Re = 200$ and $1000$, indicating that the velocity in the liquid film below the central jet is moving in the opposite direction. With the increase in $Re$, the thickness of the liquid film under the central jet decreases, and the velocity discontinuity inside the liquid film increases. This leads to an increase in the velocity of the central jet and greater deformation.

The horizontal velocity discontinuity in the liquid film is now analyzed. According to the width of the central jet, the velocity $20lu$ from the $x$-axis central line is selected, and the horizontal velocity difference at $h^*_y = 0.2h^*$, $0.4h^*$, and $0.8h^*$ is presented in Figure 9. The horizontal velocity discontinuity at $t^* = 1.25$ for the different $Re$ numbers is presented in Figure 9(a). The horizontal velocity discontinuity increases by about 19.3% at $h^*_y = 0.8h^*$ as the $Re$ number increases from 200 to 1000, whereas the equivalent change is approximately $-2.7$% at $h^*_y = 0.2h^*$. However, at $t^* = 2.5$, the differences in the horizontal velocity discontinuity decrease to 5.1% and 1.3% for $h^*_y = 0.8h^*$ and $0.2h^*$, respectively. The reason for the enhanced central jet as $Re$ increases can therefore be largely attributed to the discontinuity in the horizontal velocity on the surface.

4.2. Weber number

In this section, the effect of $We$ on the crown evolution is studied. Fallah Kharmiani et al. (2016) showed that, as $We$ increases, the kinetic energy consumed by surface tension decreases and the crown becomes more unstable. The crown height increases, but its thickness decreases as $We$ rises. The crown root radius is independent of $We$. In the present study, different $We$ numbers are obtained by changing the surface tension $\sigma$. The Weber numbers are set as $We = 69.0$, $87.8$, $139.4$, and $1165.5$. Figure 10 shows the crown geometry at $t^* = 2.5$. The central jet height increases as $We$ increases from 69.0 to 139.4, and no satellite droplets form until $We$ reaches 1165.5. As the Weber number increases, the rim of the crown becomes thinner and more unstable on both sides, and the satellite droplets decrease in size. Although the crown rims

![Figure 8. Velocity field of the central jet and the liquid film with (a) $Re = 200$ and (b) $Re = 1000$ at $t^* = 2.5$.](image)

![Figure 9. Horizontal velocity discontinuity with different Re numbers at (a) $t^* = 1.25$ and (b) $t^* = 2.5$.](image)
exhibit different geometries, the crown roots remain consistent, and the radius of the jet root spreading in the radial direction is not affected by We. The results agree well with those from earlier studies.

Figure 11(a) shows that the left crown rim radius varies with non-dimensional time. It can be observed that, for \( \text{We} = 1165.5 \), the left crown breaks four times, compared with only one breakup for the other three Weber numbers. As We increases, the first breakup of the crown occurs earlier, and in the early stages, the crown rim radius evolves in almost the same way under the different We numbers. When \( t^* = 2.5 \), the radius of the crown rim is the largest with \( \text{We} = 139.4 \), while the radii of the other three conditions are very similar. The variation in the left crown height with non-dimensional time is shown in Figure 11(b). The interface stability decreases with increasing surface tension, and more kinetic energy is consumed. Although the crown jet breaks many times at \( \text{We} = 1165.5 \), the left crown height is still higher than for the other three We numbers. Both the left crown height and the central jet increase with increasing We. The central jet does not break until \( \text{We} = 1165.5 \), for which the central jet breaks at \( t^* = 2.5 \). Before breakup occurs, the central jet height increases as We rises. Figure 11(d) shows the evolution of the kinetic energy of the liquid phase under different We numbers. The \( E_{\text{kin}} \) value of the impact loss is almost the same under all four We numbers, indicating that We has little effect on the energy loss during the impact process. The side jets break up many times with \( \text{We} = 1165.5 \) (due to some unphysical phenomena that occur within the pseudopotential model), and the satellite droplets are ‘eaten’, which leads to an increase in the kinetic energy loss during the crown evolution process. Figure 12 shows the flow field of the central jet and the liquid film below when \( \text{We} = 69 \) and 1165.5. The change in We makes little difference to the velocity discontinuity inside the liquid film. However, the surface tension decreases as We increases, leading to the central jet becoming less stable. The flow velocity at the end of the central jet is larger with higher We values, and the central jet is more prone to deforming and breaking up.

Furthermore, the horizontal velocity discontinuity with different We numbers is analyzed. The horizontal velocity discontinuity at \( t^* = 1.25 \) with different We numbers is presented in Figure 13(a). For all We numbers, the horizontal velocity discontinuity is almost the same at different depths in the liquid film. However, for \( t^* = 2.5 \), the horizontal velocity discontinuity becomes increasingly different at each depth of the liquid film, and the difference reaches about 40% at the surface. A stronger surface tension will consume more kinetic energy, leading to a stable central and lower central jet.

4.3. Liquid film thickness

Earlier studies indicated that the liquid film thickness exerts a great influence on the crown evolution process. Gao and Li (2015) obtained a semi-empirical formula from many experimental data, and noted that the film thickness has a greater effect on the crown radius and vertical stretch rate evolution process than We and Re. A previous numerical study (Yuan et al., 2020) indicated that an increase in film thickness will lead to an increase in energy consumption during the collision process and an increase in the splash angle, because more kinetic energy is used to squeeze the liquid film for the crown. Snapshots of two droplets splashing on different non-dimensional film thicknesses are presented in Figure 14. As the liquid film thickness decreases, the left liquid crown becomes more prone to break up with a smaller splash angle. During the impact process, the energy loss decreases as the film becomes thinner. The vertical interaction area between the droplet and the liquid film decreases as the non-dimensional liquid film thickness decreases, which weakens the restriction of the liquid film to the crown at both sides and reduces the splash angle. Note that there are different liquid densities in the film because the LBM is a diffuse model, and so the entrapped gas in the liquid film cannot maintain a bubble form and gradually dissolves in the liquid film, before being discharged to the gas phase.

Figure 15 shows the variation of the left crown radius, left crown height, and central jet height with time. Before the liquid crown breaks, the splash angle increases with increasing liquid film thickness, leading to a decrease in the left crown radius. The left liquid crown also breaks earlier with a thinner liquid film, as shown in Figure 15(a). Before the breakup of the left crown, its height reaches a maximum at \( h^* = 0.25 \), which is consistent with previous results. Figure 15(c) presents the relationship between the central jet height and non-dimensional
Figure 11. Influence of We on time evolution of (a) left crown radius, (b) left crown height, (c) central jet height, and (d) total kinetic energy of the liquid phase with $Re = 500$, $h^* = 0.25$, $S_x^* = 2.0$, and $S_y^* = 0$.

Figure 12. Velocity field of the central jet and the liquid film with (a) $We = 69$ and (b) $We = 1165.5$ at $t^* = 2.5$. 
Figure 13. Horizontal velocity discontinuity with different We numbers at (a) $t^* = 1.25$ and (b) $t^* = 2.5$.

Figure 14. Snapshots of two droplets splashing on liquid films of different thicknesses: (a) $h^* = 0.1$, (b) $h^* = 0.25$, (c) $h^* = 0.5$, (d) $h^* = 0.75$ at non-dimensional time $t^* = 2.5$ with $Re = 500$, $We = 87.8$, $S^*_x = 2.0$, and $S^*_y = 0$.

time, and the evolution processes of $h^* = 0.1$ and $h^* = 0.25$ are consistent with each other. The maximum central height occurs when the non-dimensional liquid film thickness $h^* = 0.5$. As the non-dimensional liquid film thickness increases from 0.5–0.75, the central jet height decreases. Figure 15(d) shows the evolution of the total kinetic energy of the liquid phase with different film thicknesses $h^*$. The $E_{k1}$ loss increases as the liquid film thickness increases, which is consistent with experimental results (Gao & Li, 2015). When $h^* = 0.5$, although more kinetic energy is consumed in the impact process than for $h^* = 0.1$ and 0.25, the splash angle is also greater. The combined effects of energy loss and splash angle cause the central jet to reach its maximum height with $h^* = 0.5$. Comparing the velocity field in the central jet and the lower liquid film with $h^* = 0.25$ and 0.75, the greater liquid film thickness produces a smaller difference in velocity discontinuity, which makes little difference to the central jet height, as shown in Figure 16.

4.4. Horizontal distance between two droplets

The relationship between the evolution of the liquid crown and the horizontal distance between the two droplets is now investigated. Note that the left liquid crown radius and height do not vary with the horizontal
distance between the two droplets, so we only study the evolution of the central jet. Some discrepancies exist in earlier studies. Both Li, Jia, et al. (2016) and Raman et al. (2015) studied the central jet evolution process for non-dimensional horizontal distances $S^*_x$ from 1.5–2.1. Raman et al. (2015) found that the central jet height increases with increasing horizontal distance $S^*_x$, whereas Li, Jia, et al. (2016) reported that the central jet height reaches a maximum when $S^*_x = 1.8$.

In the present study, horizontal distances of $S^*_x = 1.5$, 1.8, 2.0, and 2.5 are considered. The other parameters are fixed as follows: $Re = 500$, $We = 87.8$, $h^* = 0.25$, and $S^*_y = 0$. Snapshots of two droplets splashing with $S^*_x = 1.5$ and 2.0 are presented in Figure 17. An earlier study noted that the crown root thickness increases with time during the early stage of the evolution process when a single droplet impacts the liquid film. An increase in the distance between the two droplets leads to an increase in the evolution time before impact, which means that the central jet root thickness also increases. Yarin and Weiss (1995) noted that the jet stability is related to the kinematic discontinuity in the liquid film. The kinematic discontinuity decreases with increasing horizontal distance. The simulation results show that when $S^*_x = 1.5$, the central jet breaks. However, for $S^*_x = 2.0$, the central jet remains stable.

**Figure 15.** Influence of liquid film thickness on time evolution of (a) left crown radius, (b) left crown height, (c) central jet height, and (d) total kinetic energy of the liquid phase with $Re = 500$, $We = 87.8$, $h^* = 0.25$, and $S^*_y = 0$. 
Figure 16. Velocity field of the central jet and the liquid film with (a) \( h^* = 0.25 \) and (b) \( h^* = 0.75 \) at \( t^* = 2.5 \).

Figure 17. Snapshots of impact with different horizontal distances between the two droplets: (a) \( S_{x}^* = 1.5 \), (b) \( S_{x}^* = 2.0 \) at non-dimensional time \( t^* = 2.5 \) with \( Re = 500 \), \( We = 87.8 \), \( h^* = 0.25 \), and \( S_{y}^* = 0 \).

The time evolution of the non-dimensional height of the central jet with different horizontal distances is presented in Figure 18(a). Before the central liquid jet breaks, the central jet height decreases as the horizontal distance increases, as does the central jet vertical stretch rate. However, the only case in which two breaks are observed is when \( S_{x}^* = 1.5 \). After that, the central jet reaches its maximum height at \( S_{x}^* = 1.8 \). The kinetic energy evolution process is depicted in Figure 18(b). The curves showing the evolution of the energy almost overlap for \( t^* = 0 - 0.4 \). However, the time of jet impact is delayed as the horizontal distance rises, and the kinetic energy loss becomes faster as \( S_{x}^* \) decreases. Comparing the flow field inside the central jet and the liquid film, the velocity discontinuity is greater when \( S_{x}^* = 1.5 \) than when \( S_{x}^* = 3.0 \), which leads to a higher central jet height that is more prone to breaking up (Figure 19).

Furthermore, the horizontal velocity discontinuity with different horizontal distances is presented in Figure 20. At both \( t^* = 1.5 \) and \( 2.5 \), a decrease in \( S_{x}^* \) enhances the horizontal velocity discontinuity of the liquid film surface, although the horizontal velocity discontinuity remains almost the same in the middle of the liquid film and at the bottom. The horizontal velocity discontinuity with \( S_{x}^* = 1.5 \) is 1.59 times that with \( S_{x}^* = 2.5 \) at \( t^* = 1.5 \), which destabilizes the central jet and makes satellite droplets more prone to form. The main reason can be ascribed to the larger horizontal distance, which leads to a longer propagation time before the crown collapse, consumes more kinetic energy, and leads to a smaller vertical stretch rate, as shown in Figure 18(a). The horizontal velocity discontinuity with \( S_{x}^* = 1.5 \) is 0.75 times that with \( S_{x}^* = 3.0 \) when \( t^* = 2.5 \), which means that not only the height of the central jet, but also its width and stability, are affected.

4.5. Vertical distance between two droplets

In the previous subsections, we considered the case in which two droplets hit the liquid surface at the same time. Roisman and Tropea (2002) noted that the time interval between the droplets impacting the liquid film plays an important role in the crown evolution process. Therefore,
Figure 18. Influence of horizontal distance between two droplets on time evolution of (a) central jet height and (b) total kinetic energy of the liquid phase with $Re = 500$, $We = 87.8$, $h^* = 0.25$, and $S_{y}^* = 0$.

Figure 19. Velocity field of the central jet and the liquid film with (a) $S_{x}^* = 1.5$ and (b) $S_{x}^* = 3.0$ at $t^* = 2.5$.

Figure 20. Horizontal velocity discontinuity with different horizontal distances at (a) $t^* = 1.25$ and (b) $t^* = 2.5$. 
the influence of the vertical distance between droplets on the central jet evolution process is now studied. The vertical distances are set as $S_y^* = 0.13, 0.25,$ and $0.38$, corresponding to non-dimensional time intervals of $0.13, 0.25,$ and $0.38$ between the two droplets impacting the surface.

Due to the time interval between the two droplets impacting the liquid surface, the evolution of the right crown is slower than that of the left crown. The vertical impact point of the two crowns decreases with increasing vertical distance between the two droplets. After the collision occurs, the higher crown bends outward and a satellite droplet is formed. The droplet size increases with increasing vertical distance, as shown in Figure 21. After impact, the central jet exhibits asymmetric evolution.

Special attention should be paid to the asymmetry of the central jet. We, therefore, study the central jet height and the relative horizontal position of the highest point in the evolution process. The relative horizontal position $x_c^*$ of the highest point is introduced to determine the horizontal deviation of the central jet. This is defined as:

$$x_c^* = \frac{(x_{\text{max}} - 0.5lx)}{2r_0},$$

where $x_{\text{max}}$ is the horizontal coordinate of the highest point and $lx$ is the width of the computational domain. As shown in Figure 22(a), the formation time of the central jet increases as the vertical distance between the droplets increases; however, the crown heights in the later stage almost overlap. Figure 22(b) shows that the relative horizontal coordinate of the highest position varies with non-dimensional time, the central jet is inclined to the left with all three vertical distances, and the offset distance decreases with increasing vertical distance. Figure 22(c) shows the evolution process of the total kinetic energy of the liquid phase with the different vertical distances. As we are using the same Re, We, and film thickness, the energy loss during the impact is very similar, resulting in the same central jet height. However, due to the different impact times and positions, $x_c^*$ is different. The flow field in the center jet and the lower liquid film with $S_y^* = 0.13$ and $0.38$ at $t^* = 2.5$ are shown in Figure 23. The difference in the velocity field in the liquid film is small. However, the flow field is quite different within the central jet. Due to the later collision time in the case of $S_y^* = 0.38$, the velocity of the central jet cannot adjust to become normal to the wall, which causes the difference in $x_c^*$.

5. Conclusions

A high-density-ratio pseudopotential LBM with tunable surface tension has been developed to investigate the dynamics of two droplets impacting a liquid film. The model was validated by the benchmark problem of a single droplet splashing on the film. Quantitative verification of the model was performed. The crown and central jet evolution processes were investigated with different Reynolds numbers, Weber numbers, liquid film thicknesses, horizontal distances, and vertical distances. The following conclusions can be drawn:
Figure 22. (a) Central jet height, (b) relative horizontal position of the highest point, and (c) total kinetic energy of the liquid phase with respect to non-dimensional time $t^*$ at different vertical distances.

Figure 23. Velocity field of the central jet and the liquid film with (a) $S_y^* = 0.13$ and (b) $S_y^* = 0.38$ at $t^* = 2.5$. 
(1) The breakup of the liquid crown occurs earlier and the central jet height increases as the Reynolds number increases. Once the central jet height reaches a threshold, it no longer varies with the Reynolds number.

(2) As the Weber number increases, the crown breaks up more easily and the central jet height increases.

(3) The increase in liquid film thickness not only consumes more energy in the impact process, but also affects the splash angle of the jet and changes the normal velocity of the central jet. Initially, the central jet height increases with increasing liquid film thickness. Once the thickness reaches a threshold, the central jet begins to decrease with further increases in liquid film thickness.

(4) The kinematic discontinuity decreases with delayed collision time, and so the central jet height decreases with increasing horizontal distance between the droplets.

(5) A vertical distance between the two droplets leads to an asymmetric evolution of the crown geometry. The offset distance between the highest point and the satellite droplet size of the central jet increases with increasing vertical distance between droplets.

Note that the basic features of two droplets impacting on the liquid film of a central axis have been captured in the present study. However, Tanaka et al. (2011) reported that the impact point and breakup mode of the central jet should be examined in three dimensions. Further investigations using a three-dimensional pseudopotential LBM are currently underway to study the asymmetric evolution of the crown geometry. Additionally, the splashing of multiple droplets on stationary and moving liquid films will be the focus of future work.

Acknowledgements
This work was supported by the China National Key R&D Plan (grant number 2016YFC0402006) and the National Natural Science Foundation of China (grant number 51979183).

Disclosure statement
No potential conflict of interest was reported by the author(s).

Funding
This work was supported by the National Key Research and Development Program of China [grant number 2016YFC0402006] and the National Natural Science Foundation of China [grant number 51979183].

References
Alghoul, S. S., Eastwick, C. N., & Hann, D. B. (2011). Normal droplet impact on horizontal moving films: An investigation of impact behaviour and regimes. *Experiments in Fluids*, 50(5), 1305–1316. https://doi.org/10.1007/s00348-010-0991-0

Allen, R. F. (1975). The role of surface tension in splashing. *Journal of Colloid and Interface Science*, 51(2), 350–351. https://doi.org/10.1016/0021-9797(75)90126-5

Burzynski, D. A., & Bansmer, S. E. (2018). Droplet splashing on thin moving films at high Weber numbers. *International Journal of Multiphase Flow*, 101, 202–211. https://doi.org/10.1016/j.ijmultiphaseflow.2018.01.015

Cossali, G. E., Marengo, M., Coghe, A., & Zhdanov, S. (2004). The role of time in single drop splash on thin film. *Experiments in Fluids*, 36(6), 888–900. https://doi.org/10.1007/s00348-003-0772-0

Ezzatneshan, E. (2017). Study of surface wettability effect on cavitation inception by implementation of the lattice Boltzmann method. *Physics of Fluids*, 29(11), 113304. https://doi.org/10.1063/1.4990876

Fallah Kharmiani, S., Passandideh-Fard, M., & Niazmand, H. (2016). Simulation of a single droplet impact onto a thin liquid film using the lattice Boltzmann method. *Journal of Molecular Liquids*, 222, 1172–1182. https://doi.org/10.1016/j.molliq.2016.07.092

Gao, X., & Li, R. (2015). Impact of a single drop on a flowing liquid film. *Physical Review E*, 92(5), 053005. https://doi.org/10.1103/PhysRevE.92.053005

Grunau, D., Chen, S., & Eggert, K. (1993). A lattice Boltzmann model for multiphase fluid flows. *Physics of Fluids A: Fluid Dynamics*, 5(10), 2557–2562. https://doi.org/10.1063/1.858769

Guo, Y., Wei, L., Liang, G., & Shen, S. (2014). Simulation of droplet impact on liquid film with CLSVOF. *International Communications in Heat and Mass Transfer*, 35, 26–33. https://doi.org/10.1016/j.icheatmasstransfer.2014.02.006

He, X., Chen, S., & Zhang, R. (1999). A lattice Boltzmann scheme for incompressible multiphase flow and its application in simulation of Rayleigh–Taylor instability. *Journal of Computational Physics*, 152(2), 642–663. https://doi.org/10.1006/jcph.1999.6257

He, X., Zhang, J., & Xu, W. (2020). Study of cavitation bubble collapse near a rigid boundary with a multi-relaxation-time pseudo-potential lattice Boltzmann method. *AIP Advances*, 10(3), 035315. https://doi.org/10.1063/1.5142243

He, X., Zhang, J., Yang, Q., Peng, H., & Xu, W. (2020). Dissolution process of a single bubble under pressure with a large-density-ratio multicomponent multiphase lattice Boltzmann model. *Physical Review E*, 102(6), Article 063306. https://doi.org/10.1103/PhysRevE.102.063306

Huang, Q., & Zhang, H. (2008). A study of different fluid droplets impacting on a liquid film. *Petroleum Science*, 5(1), 62–66. https://doi.org/10.1007/s12182-008-0010-8

Hung, Y.-L., Wang, M.-J., Liao, Y.-C., & Lin, S.-Y. (2011). Initial wetting velocity of droplet impact and spreading: Water on glass and parafilm. *Colloids and Surfaces A: Physicochemical and Engineering Aspects*, 384(1), 172–179. https://doi.org/10.1016/j.colsurfa.2011.03.061

Josserand, C., & Zaleski, S. (2003). Droplet splashing on a thin liquid film. *Physics of Fluids*, 15(6), 1650–1657. https://doi.org/10.1063/1.1572815

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Lee, T., & Lin, C.-L. (2005). A stable discretization of the lattice Boltzmann equation for simulation of incompressible two-phase flows at high density ratio. *Journal of Computational Physics*, 206(1), 16–47. https://doi.org/10.1016/j.jcp.2004.12.001

Li, L., Jia, X., Liu, Y., & Su, M. (2016). Simulation of double droplets impact on liquid film by a simplified lattice Boltzmann model. *Applied Thermal Engineering*, 98, 656–669. https://doi.org/10.1016/j.applthermaleng.2015.12.050

Li, Q., & Luo, K. H. (2013). Achieving tunable surface tension in the pseudopotential lattice Boltzmann modelling of multiphase flows. *Physical Review E*, 88(5), Article 053307. https://doi.org/10.1103/PhysRevE.88.053307

Li, Q., Luo, K. H., Kang, Q. J., & Chen, Q. (2014). Contact angles in the pseudopotential lattice Boltzmann modelling of wetting. *Physical Review E*, 90(5), Article 053301. https://doi.org/10.1103/PhysRevE.90.053301

Li, Q., Luo, K. H., Kang, Q. J., He, Y. L., Chen, Q., & Liu, Q. (2016). Lattice Boltzmann methods for multiphase flow and phase-change heat transfer. *Progress in Energy and Combustion Science*, 52, 62–105. https://doi.org/10.1016/j.pecs.2015.10.001

Li, Q., Luo, K. H., & Li, X. J. (2013). Lattice Boltzmann modeling of multiphase flows at large density ratio with an improved pseudopotential model. *Physical Review E*, 87(5), Article 053301. https://doi.org/10.1103/PhysRevE.87.053301

Mosavi, A., Shamshirband, S., Salwana, E., Chau, K.-W., & Tab, J. H. M. (2019). Prediction of multi-inputs bubble column reactor using a novel hybrid model of computational fluid dynamics and machine learning. *Engineering Applications of Computational Fluid Mechanics*, 13(1), 482–492. https://doi.org/10.1080/19942060.2019.1613448

Motzkus, C., Gensdarmes, F., & Géhin, E. (2011). Study of the coalescence/splash threshold of droplet impact on liquid films and its relevance in assessing airborne particle release. *Journal of Colloid and Interface Science*, 362(2), 540–552. https://doi.org/10.1016/j.jcis.2011.06.031

Mukherjee, S., & Abraham, J. (2007). Crown behavior in drop impact on wet walls. *Physics of Fluids*, 19(5), 052103. https://doi.org/10.1063/1.2736085

Nikolopoulos, N., Theodorakakos, A., & Bergeles, G. (2007). Three-dimensional numerical investigation of a droplet impinging normally onto a wall film. *Journal of Computational Physics*, 225(1), 322–341. https://doi.org/10.1016/j.jcp.2006.12.002

Raman, K. A., Jaiman, R. K., Lee, T. S., & Low, H. T. (2015). Prediction of flow characteristics in the bubble column reactor by the artificial pheromone-based communication of biological ants. *Engineering Applications of Computational Fluid Mechanics*, 14(1), 367–378. https://doi.org/10.1080/19942060.2020.1715842

Shan, X. (2008). Pressure tensor calculation in a class of nonideal gas lattice Boltzmann models. *Physical Review E*, 77(6), Article 066702. https://doi.org/10.1103/PhysRevE.77.066702

Shan, X., & Chen, H. (1993). Lattice Boltzmann model for simulating flows with multiple phases and components. *Physical Review E*, 47(3), 1815–1819. https://doi.org/10.1103/PhysRevE.47.1815

Shan, X., & Chen, H. (1994). Simulation of nonideal gases and liquid-gas phase transitions by the lattice Boltzmann equation. *Physical Review E*, 49(4), 2941–2948. https://doi.org/10.1103/PhysRevE.49.2941

Shetabivash, H., Ommi, F., & Heidarinejad, G. (2014). Numerical analysis of droplet impact onto liquid film. *Physics of Fluids*, 26(1), 012102. https://doi.org/10.1063/1.4861761

Swift, M. R., Orlandini, E., Osborn, W. R., & Yeomans, J. M. (1996). Lattice Boltzmann simulations of liquid-gas and binary fluid systems. *Physical Review E*, 54(5), 5041–5052. https://doi.org/10.1103/PhysRevE.54.5041

Swift, M. R., Osborn, W. R., & Yeomans, J. M. (1995). Lattice Boltzmann simulation of nonideal fluids. *Physical Review Letters*, 75(5), 830–833. https://doi.org/10.1103/PhysRevLett.75.830

Tanaka, Y., Washio, Y., Yoshino, M., & Hirata, T. (2011). Numerical simulation of dynamic behavior of droplet on solid surface by the two-phase lattice Boltzmann method. *Computers & Fluids*, 40(1), 68–78. https://doi.org/10.1016/j.compfluid.2010.08.007

Vander Wal, R. L., Berger, G. M., & Mozes, S. D. (2006). The combined influence of a rough surface and thin fluid film upon the splashing threshold and splash dynamics of a droplet impacting onto them. *Experiments in Fluids*, 40(1), 23–32. https://doi.org/10.1007/s00348-005-0043-3

Wang, Y., Dandekar, R., Bustos, N., Pouliain, S., & Bourouiba, L. (2018). Universal rim thickness in unsteady sheet fragmentation. *Physical Review Letters*, 120(20), Article 204503. https://doi.org/10.1103/PhysRevLett.120.204503

Wang, F., Gong, L., Shen, S., & Guo, Y. (2020). Flow and heat transfer characteristics of droplet obliquely impact on a stationary liquid film. *Numerical Heat Transfer, Part B: Fundamentals*, 77(3), 228–241. https://doi.org/10.1080/10407790.2020.1713621

Wang, Y.-B., Wang, X.-D., Wang, T.-H., & Yan, W.-M. (2018). Asymmetric heat transfer characteristics of a double droplet impact on a moving liquid film. *International Journal of Heat and Mass Transfer*, 126, 649–659. https://doi.org/10.1016/j.ijheatmasstransfer.2018.05.161

Yarin, A. L., & Weiss, D. A. (1995). Impact of drops on solid surfaces: Self-similar capillary waves, and splashing as a new type of kinematic discontinuity. *Journal of Fluid Mechanics*, 283, 141–173. https://doi.org/10.1017/S0022112095002266

Yu, Z., & Fan, L.-S. (2010). Multirelaxation-time interaction-potential-based lattice Boltzmann model for two-phase flow.
Yuan, H., Li, J., He, X., Chen, L., Wang, Z., & Tan, J. (2020). Study of droplet splashing on a liquid film with a tunable surface tension pseudopotential lattice Boltzmann method. *Physical Review E, 82*(4), Article 046708. https://doi.org/10.1103/PhysRevE.82.046708

Yuan, P., & Schaefer, L. (2006). Equations of state in a lattice Boltzmann model. *Physics of Fluids, 18*(4), 042101. https://doi.org/10.1063/1.2187070