Magnon driven domain wall motion with Dzyaloshinskii-Moriya interaction

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We study domain wall (DW) motion induced by spin waves (magnons) in the presence of Dzyaloshinskii-Moriya interaction (DMI). The DMI exerts a torque on the DW when spin waves pass through the DW, and this torque represents a linear momentum exchange between the spin wave and the DW. Unlike angular momentum exchange between the DW and spin waves, linear momentum exchange leads to a rotation of the DW plane rather than a linear motion. In the presence of an effective easy plane anisotropy, this new mechanism is significantly more efficient than angular momentum transfer in moving the DW.

The manipulation of domain wall (DW) motion has been extensively studied in the past few years due to potential applications in logic devices and data storage technology [1-5]. A DW can be driven by an applied field [6], microwaves [7], spin transfer torque [8] and spin waves (magnons) [9-11]. Spin waves can drive the DW effectively since they carry magnonic spin current. In general, when the spin waves travel through the DW, the DW acquires a negative velocity – relative to the propagation direction of the spin waves – due to conservation of angular momentum [11], although positive velocities have been observed in micromagnetic simulations at special frequencies [10,12,13].

Angular momentum conservation plays a crucial role in spin wave induced DW motion: when the spin wave passes through the DW, the magnonic spin current changes its sign, which generates a torque and the DW moves in order to absorb this torque. Magnons can be considered as particles with angular momentum $\pm \hbar$ and linear momentum $\hbar k$ [11]. When the spin wave is reflected, linear momentum is transferred to the DW which results in DW motion [12,13]. The difference between these two mechanisms is that the DW moves in opposite directions [13,14]. In this article we demonstrate, by using micromagnetic simulation and a one-dimensional (1d) analytical DW model, that spin waves passing through a domain wall in the presence of Dzyaloshinskii-Moriya interaction (DMI) and an easy-plane anisotropy drive the domain wall very effectively. We attribute this to linear momentum transfer and show that this effect can be an order of magnitude more efficient than the better known angular momentum transfer.

The DMI is an antisymmetric interaction induced by spin-orbit coupling due to broken inversion symmetry in lattices or at the interface of magnetic films [17]. The DMI can lead to chiral magnetic orders such as skyrmions and spin spirals [17-20]. In addition, the DMI has brought new phenomena for DW dynamics driven by fields [21] or charge currents [22]. The DMI has been found both for bulk materials such as MnSi [23] and magnetic interfaces [20]. In this work we focus on bulk DMI with micromagnetic energy density $\varepsilon_{\text{dmi}} = Dm \cdot (\nabla \times m)$ where $D$ is the DMI constant and $m$ is the normalized magnetization.

We consider a quasi-1d nanowire with exchange interaction, DMI and two effective anisotropies which can be interpreted as representing an approximation of the demagnetization field of a thin film strip. One anisotropy $K$ is the uniaxial anisotropy along the $x$-axis, and the other effective $K_\perp$ is an easy $xy$-plane anisotropy. The total free energy for the wire along the $x$ axis is

$$E = S \int \left[ A(\nabla m)^2 - K m_x^2 + K_\perp m_z^2 + \varepsilon_{\text{dmi}} \right] dz,$$

where $S$ is the cross-sectional area of the wire and $A$ is the exchange constant.

The dynamics of the magnetization $m$ is governed by the Landau-Lifshitz-Gilbert (LLG) equation

$$\frac{\partial m}{\partial t} = -\gamma m \times H_{\text{eff}} + \alpha m \times \frac{\partial m}{\partial t},$$

where $\gamma$ ($>0$) is the gyromagnetic ratio and $\alpha$ is the Gilbert damping. The effective field $H_{\text{eff}}$ is calculated as the functional derivative $H_{\text{eff}} = -1/(\mu_0 M_s) \delta E/\delta m = 2/(\mu_0 M_s) [A \nabla^2 m - D \nabla \times m + K m_x e_x - K_\perp m_z e_z]$ with $M_s$ the saturation magnetization and $\mu_0$ the vacuum permeability.

The typical DW structures described by the energy [1] for the case $D = 0$ are head-to-head and tail-to-tail DWs, and the former is shown in Fig. 1(a). By using spherical coordinates $\theta = \theta(x)$ and $\phi = \phi(x)$, the magnetization unit vector $m$ is expressed as $m = (\cos \theta, \sin \theta \cos \phi, \sin \theta \sin \phi)$, and the total micromagnetic energy [1] reads

$$E = S \int \left[ A(\theta'^2 + \sin^2 \phi \phi'^2) - D \phi' \sin^2 \theta 
+ K \sin^2 \theta (1 + \kappa \sin^2 \phi) \right] dx,$$

where $\kappa = K_\perp / K$ and $'$ represents the derivative with respect to $x$. Using standard variational calculus, we
arrive at two coupled differential equations for $\theta$ and $\phi$,

$$2A\theta'' = \sin \theta (2A\phi'' + K (1 + \kappa \sin^2 \phi) - D \phi'),$$
$$\sin \theta (2A\phi'' - K_\perp \sin 2\phi) = 2 \cos \theta (D - 2A\phi')\theta,'$$

(4)

with the boundary conditions $\theta' = 0$ and $\phi' = 1/\xi$ for $x = \pm \infty$ where $\xi = 2A/D$ is the characteristic length \[^{20}\]. We are searching for the head-to-head DW solution, therefore the ansatz $\cos \theta = -\tanh(x/\Delta)$ is used, where $\Delta$ is the domain wall width. Initially, we consider the case of $\kappa = 0$ (i.e. $K_\perp = 0$) which preserves the rotational symmetry. We assume that $\phi$ is a linear function of space $x$, i.e., $\phi(x) = x/\xi$. Inserting it back to Eq. (4), we obtain $\Delta = \sqrt{A/(K - A/\xi^2)}$. In the absence of DMI, the DW width reduces to $\Delta = \sqrt{A/K}$ which is the well known Bloch wall width. Therefore the static one-dimensional head-to-head DW profile can be expressed as \[^{22}\]

$$m_x = -\tanh(x/\Delta),$$
$$m_y = \text{sech}(x/\Delta) \cos(x/\xi),$$
$$m_z = \text{sech}(x/\Delta) \sin(x/\xi).$$

(5)

Fig. (b) shows the DW profile using Eq. (5) for $K_\perp = 0$ with lines, and the red dashed line depicts the micromagnetic simulation result of $m_z$ for $K_\perp^2 = 6 \times 10^5 J/m^3$. The rotational symmetry breaks for $K_\perp > 0$ and the $z$-component of the magnetization $m_z$ is suppressed by the easy plane anisotropy. For $K_\perp \gg K_\perp$, the DW profile is close to the profile for $D = 0$. The DW configuration \[^{5}\] is not stable if the DMI constant is larger than the critical value $D_c = 2\sqrt{AK}$ \[^{22}\], and the presence of $K_\perp > 0$ increases this threshold.

We assume that the spin wave can be described by a small fluctuation $u = u(x)$ and $v = v(x)$ around $\mathbf{m}_0$, where $\mathbf{m}_0 = (\cos \theta_0, \sin \theta_0 \cos \phi_0, \sin \theta_0 \sin \phi_0)$ is the static domain wall profile Eq. (5),

$$\mathbf{m} = \mathbf{m}_0 + [u(x)e_\phi + v(x)e_\theta]e^{-i\omega t},$$

(6)

where $\sqrt{u^2 + v^2} \ll 1$, $e_\phi = (0, -\sin \phi_0, \cos \phi_0)$, $e_\theta = (-\sin \theta_0, \cos \theta_0 \cos \phi_0, \cos \theta_0 \sin \phi_0)$, and $\omega$ is the spin wave frequency. By following the treatment in Ref. \[^{11}\], we obtain for the $K_\perp = 0$ case,

$$Av'' - K \cos(2\theta_0) = -i\omega/\gamma_0,$$
$$Av'' - K \sin(2\theta_0) = i\omega/\gamma_0,$$

(7)

where we define $K = K - D^2/(4A)$ and $\gamma_0 = 2\gamma/(\mu_0 M_s)$. By introducing the complex variable $\psi = u - iv$, Eq. (7) can be written as a time-independent Schrödinger-type equation with reflectionless potential \[^{24}\],

$$\hat{H}\psi(x) = (1 + q^2)\psi(x),$$

(8)

where $\xi = x/\Delta$ and the operator is $\hat{H} = -d^2/dx^2 + 1 - 2\text{sech}^2(\xi)$. The eigenvalues $1 + q^2 = \omega/(\gamma_0 K)$ define the spin wave dispersion relation inside the DW, which is plotted in Fig. (c) (magenta line) with wavevector $k = q/\Delta$. The above discussion is only valid for wavelengths smaller than the domain wall size, which corresponds to wave vectors greater than $k_m \sim 1/(2\Delta)$. The propagating wave excitations can be expressed as $\psi(x, t) = \rho_k e^{i\Omega t} (\text{tanh}(\xi - iq))$ where $\Omega = q - \omega t$ represents the sine or cosine type waves and $\rho_k$ the wavevector dependent spin wave amplitude \[^{29}\]. The reflectionless property for spin waves holds even in the presence of the easy plane anisotropy \[^{27}\]. Interestingly, the dispersion relation inside the DW is symmetric in the reduced wavevector $q$ even though the wall is twisted by the DMI. However, due to the exponential decay of the DW profile when moving away from the DW centre, the magnetization is uniform in the domains and the dispersion relations become asymmetric outside the DW \[^{19}\]\[^{28}\],

$$\omega = \gamma_0 (K + Ak^2 \pm Dk).$$

(9)

Fig. (c) shows the asymmetric dispersion relations outside the DW. A spin wave that has its $k$-vector pointing in the same direction as the local magnetization $\mathbf{m}$ follows the $\omega_+$ branch of the dispersion relation, and if $k$ and $m$ point in opposite directions, the $\omega_-$ branch is the relevant one. The dispersion relation \[^{9}\] also suggests that the wavevector changes by $D/A$ when the spin wave passes through the DW if the frequency of the spin wave remains the same. The spin wave becomes
elliptical rather than circular if $K_{\perp} > 0$ and the corresponding dispersion relation outside the DW becomes $\omega_{\perp} = \gamma_{0}\sqrt{(K + Ak^2)(K + K_{\perp} + Ak^2) \pm Di}$. [19].

To study the DW dynamics, micromagnetic simulations have been performed using a 1d mesh with length 2000 nm and cellsize 2 nm. The parameters used are typical of Permalloy: the exchange constant $A = 1.3 \times 10^{-11} J/m$, the saturation magnetization $M_s = 8.0 \times 10^5 A/m$ and the damping coefficient $\alpha = 0.01$. The spin waves are excited locally in the region $-1000 \leq x \leq -998 nm$ by a linearly polarized microwave $h(t) = h_0 \sin(\omega t)e_\perp$ with $h_0 = 1 \times 10^5 A/m$. The initial domain wall is located at $x_0 = -50 nm$, and to prevent spin wave reflection the damping coefficient is increased linearly [10] from 0.01 to 0.5 in the region $800 \leq x \leq 100 nm$.

The spin wave traveling in the $+x$ direction induces DW motion. Fig. 2 shows the DW velocity as a function of frequency for $K_{\perp} = 0$. The DW velocity is $v_t = -\frac{\rho^2}{T}V_g$, where $V_g = \frac{\partial u_{\perp}}{\partial k}$ is the spin wave group velocity and $\rho$ is the spin-wave amplitude [11]. For a circular spin wave, i.e. for $K_{\perp} = 0$, by inserting the dispersion relation inside the DW we have $V_g = 2\gamma_0 Ak$ which is the same as for the case $D = 0$. The DW magnitude of the velocity decreases as the frequency of the spin wave increases. The reason for this is that the spin wave amplitude decays exponentially as the spin wave propagates. To quantify this, we assume the spin wave has the form $\rho(x)e^{i(kx-\omega t)}e^{-x/T}$, and obtain $\Gamma = 2\gamma_0 Ak(1 + \alpha^2)/(\alpha\omega)$ [20], which is plotted in the inset of Fig. 2. From Fig. 2 we can see that the DMI has only a small influence on the DW velocity, in this case where $K_{\perp} = 0$ and the spin waves are circular.

We now repeat the study for Fig. 2 above with $K_{\perp} > 0$ and where the spin waves are elliptical. Fig. 3 shows the DW velocity as a function of spin wave frequency for $K_{\perp} = 6 \times 10^5 J/m^2$. As in the $K_{\perp} = 0$ case, the DW velocity is negative if the DMI constant $D$ is $>0$, and is enhanced by the DMI. The DW velocity is zero if the frequency is less than the cut-off frequency $f_{cut} \sim 16.3 GHz$. The DW velocity first increases, and then decreases with the frequency due to the spin wave amplitude decaying. However, for the $D < 0$ case the DW velocity is positive in a large frequency range, which is different from the reports in previous studies [10, 12, 14].

To understand this novel phenomenon, we recall the dispersion relation [9] outside the DW and assume the wavevector of a magnon before and after passing through the DW to be $k_1$ and $k_2$, respectively. The change in wavevector $\delta k = k_2 - k_1$ leads to a momentum change $\delta p = h\delta k$ for each magnon. The excited magnon density is $n = \frac{M_s}{\gamma} \frac{1}{2} \rho^2$ [12] and for elliptical spin waves we choose $\rho^2 = u_0v_0$ where $u_0$, $v_0$ are fluctuation amplitudes in $e_\perp$ and $e_\parallel$. The linear momentum of a DW is $P_{DW} = M_s/\gamma \int \phi \sin \theta (\partial \phi / \partial x) dx = 2\phi M_s/\gamma$ [20] and conservation of linear momentum [15] gives $dP_{DW}/dt = -dP_{magnons}/dt = -nV_g \rho \phi$, i.e., $\phi = -\frac{1}{T} k^2 V_g\delta k$. We can introduce an effective field along the $x$ direction by using the spherical form of the LLG equation,

$$H_x = \dot{\phi}/\gamma = -\frac{1}{4} \rho^2 \delta k V_g/\gamma. \quad (10)$$

For circular spin waves $\delta k = D/A$, and thus the corresponding effective field is $H_x = -\frac{C}{4} D k_{\perp}/\gamma$. In the $\kappa > 0$ case (i.e. for $K_{\perp} > 0$), the spin wave is elliptical and $\delta k$ is a function of the frequency, as shown in Fig. 3(a). The presence of a non-zero $K_{\perp}$ suppresses the wavevector change, especially for low frequency spin.
waves. The DW velocity $v_d$ induced by this effective field $H_x$ in the presence of damping can be obtained using the rigid DW model [31],

$$v_d = \frac{\gamma \Delta H_x}{\alpha} \rho \sqrt{1 + \frac{\kappa^2}{2} \left(1 - \sqrt{1 - h^2}\right)},$$

where $h = H_x/\alpha H_{K \perp}$ and $H_{K \perp} = 2K_{\perp}/(\mu_0 M_s)$. For $\kappa \gg 1$ the DW width $\Delta$ approaches $\Delta_0$. The total velocity is the sum of the established $v_d$ and $v_e$, which correspond to the linear and angular momentum conservations, respectively.

The total velocity is plotted in Fig. 3 with lines, which shows a good agreement with the simulation results. The DW can rotate freely if $K_{\perp} = 0$ and the DW velocity induced by the field $H_x$ is $v_0 = \alpha \Delta \gamma_0 H_x/(1 + \alpha^2)$. We can establish that $v_0 \sim 10^{-4}$ m/s, which could explain why the linear momentum exchange is not significant for the DW motion shown in Fig. 2.

So far the effective field is introduced by linear momentum conservation. In the following section we recheck this using the LLG equation. The LLG equation (2) with zero damping is rewritten to describe the spin conservation law [32],

$$\frac{\partial \mathbf{m}}{\partial t} + \frac{\partial \mathbf{j}_e}{\partial x} = \tau_n + \tau_d$$

where $\mathbf{j}_e = \gamma_0 \mathbf{A} \mathbf{m} \times \nabla \mathbf{m}$ is the exchange spin current associated with localized spin. The spin source or sink $\tau_n = -\gamma_0 \mathbf{m} \times \left[K_m \mathbf{e}_x - K_{\perp} m_x \mathbf{e}_z\right]$ and $\tau_d = \gamma_0 D \mathbf{m} \times (\nabla \times \mathbf{m})$ come from the anisotropy and DMI, respectively. The average DW velocity can be computed as the overall change, and is more efficient in moving the DW. This linear momentum exchange between spin waves and DW exists in addition to the angular momentum exchange between the spin wave and the DW. The effect of the linear momentum is equivalent to an effective field and the direction of the field depends on the sign of DMI constant and DW profile. This linear momentum exchange between spin waves and DW exists in addition to the angular momentum exchange, and is more efficient in moving the DW.

By using the DW profile [3] it is found that the overall contributions of the $x$-component torques $\tau_n$ and $\tau_d$ are zero, i.e., $\int (\tau_n^x) dx = \int (\tau_d^x) dx = 0$. However, the contribution of the $z$-component of the DMI is nonzero, i.e., $\int (\tau_d^z) dx = -\int \frac{2}{\gamma_0} D k m_y dx$, represents an additional torque rotating the DW plane. By introducing an effective field $H_z^0$ in the $x$ direction such that the total torque on the DW equals the torque $\tau_d^z$, we obtain $H_z^0 = \int (\tau_d^z/\gamma) dx/\int m_y dx = -\frac{2}{\gamma_0} D k m_0$, which is in exact agreement with the analysis above.

Fig. 4(b) shows a contour plot of the DW velocity as a function of $K_{\perp}$ and DMI constant $D$. The figure is approximately symmetric in the DMI constant, with a biased velocity originating from the angular momentum exchange between the spin wave and the DW. The DW velocity is always negative if $D > 0$. There exist some optimal areas in which the DW has the highest velocity, and this area depends on the frequency of the spin wave.

In conclusion, we have studied DW motion induced by spin waves in the presence of DMI. We found that the DMI exerts an extra torque which rotates the DW plane when the spin wave passes through the DW. This torque represents a linear momentum exchange between the spin wave and the DW. The effect of the linear momentum is equivalent to an effective field and the direction of the field depends on the sign of DMI constant and DW profile. This linear momentum exchange between spin waves and DW exists in addition to the angular momentum exchange, and is more efficient in moving the DW.

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