Design of A Novel Feed-Forward Control Strategy for A Non-Minimum Phase System

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Abstract: This paper proposes a new feed-forward control strategy for a plant with non-minimum phase dynamics. A Feed-Forward controller is very essential for controlling plants with time-varying reference signals. However, designing this type of controller is a non-trivial problem in case the plant dynamics is non-minimum-phase. This is because, this controller design involves the concept of inversion of the plant model and inverse of a non-minimum-phase plant model is unstable or non-causal. For this problem, usually an approximate inversion of a plant model is applied. An approach with corrected-approximate-inverse (CAI) method is available for feed forward controller design. After analyzing the results and discussion of this CAI technique, it is seen that although it seems to be working perfectly for small duration of time but its performance is unsatisfactory in case of large span of time. Therefore, in this paper, a new feed-forward control strategy has been designed to erase the above said problem. This method has adopted a simple version of an internal plant-model structure in its time domain. The fixed-structure feed forward controller that is constructed using this method is usually a linear amalgamation of a reference trajectory and its time-derivatives with suitable weighting factors. This control strategy has been verified with appropriate simulation results applied to a studied plant and results are compared with that of the CAI technique.

Keywords: Non minimum phase system, feed forward design, system inverse, CAI method

1. INTRODUCTION

In close-loop control system applications, feedback control and feed-forward control are two essential concepts. The main difference between feedback control and feed-forward control is that feedback control is less sensitive to modelling error. Feed-forward control is used in cascade with feedback control. The advantage of feed-forward control is that it compensates the effect of disturbance before it affects plant output and stability of control system is not affected. Recently, utilization of feed-forward controllers is rapidly increasing in desirable trajectory tracking problems due to its almost null tracking errors. A number of feedforward controllers are available in literature [1-2]. After detail investigation on these controllers, it is found that all of them are classical controllers. Performances of these controllers are highly dependable on plant dynamics. Hence, designs of these controllers are very complex as linearization of hysteresis nonlinearity is for controller design.
In [3], an exact unstable inverse method or a higher order non-causal-series expansion method is introduced. Here, the zeroth-order series or Non-minimum-phase (NMP) zeros ignore (NPZ-Ignore), ZPETC, and ZMETC based approximation techniques are used for designing plant model-inversion. Although this technique makes the design problem simple but it suffers from more complexity when the controllers are implemented. Another approach to output tracking of non-minimum-phase systems is proposed in [4]. The method is a preview-based stable-inversion type method. It has used the desired future output trajectory to set the feed-forward controller input. The limitation of this method is that the optimal boundary condition of the unstable internal dynamics needs to be found at the end of the preview time period to obtain the optimal preview-based inverse of plant for controller design. To overcome this problem, a controller with inversion-based methodology is proposed in [5]. In this paper, the controller is an inversion-based feed forward control. This controller design has used inverse of the plant to be controlled using a stable inversion method. However, performance of this inverse feed-forward controller model is not robust for parameter variation of plant and disturbances. Feed-forward controller with CAI concept is proposed in [6]. This controller works under some finite range of frequency or for short duration of time. Error tracking error increases as we increase the duration. So for long duration, we design a feed-forward controller by using an integrator. Tracking of reference trajectory is improved by using proposed controller. Hence error is decreased and mismatch transfer function is also reduced.

Therefore, a novel controller with robust-system-inversion approach is introduced our paper. In this paper, a novel control design methodology is proposed to optimally integrate the inversion-based feed forward control with the robust feedback control. The design consists of an inversion-based 2-DOF control system. The main contributions of this paper are as follows.
1. to achieve guaranteed tracking performance of the system using the proposed the feed-forward controller with uncertainties in plant dynamics of given bound,
2. to improve the bandwidth of the feedback control path without affecting stability of the feedback control and
3. comparison the effectiveness of proposed controller with that CAI controller applied to the studied plant that has non-minimum phase dynamics.

2. PROBLEM FORMULATION

Let us consider a 2-DOF control system with a linear-time-invariant SISO plant as shown in Fig.1. In this system, \( G_{ff}(s) \) and \( G_{fb}(s) \) are feed-forward and feedback controllers respectively. Parameters \( y_d \) and \( y \) are reference and output of the system respectively. Similarly, \( u_{ff}(s) \) and \( u_{fb}(s) \) are feed-forward and feedback input to the plant respectively. The transfer function of the plant \( G_{p}(s) \) is defined as follows.

\[
G_{p}(s) = \frac{y(s)}{u(s)} = \frac{b_0 s^n + \ldots + b_1 s + b_0}{a_n s^n + \ldots + a_1 s + a_0} = \frac{B(s)}{A(s)}
\]

Where, \( b_0 \neq 0 \). Here, the plant dynamics can be both stable/unstable and minimum phase/non-minimum phase dynamics. The feed-forward control design problem of this system is 2-DOF in nature. Normally, 2-DOF structure includes either plant inversion feed-forward or closed loop inversion feed-forward techniques. In this paper, the plant inversion feed-forward technique has been used. In this case, \( y_d \) indicates a smooth reference trajectory, \( G_{fb}(s) \) is a
proportional stabilizer controller given by $k_p$. The tracking error of the system is given by as follows.

$$e(t) = y_d(t) - y(t)$$

$$\text{(2)}$$

![Diagram](image)

Fig.1. 2-DOF system with the plant inversion feed-forward controller

To design Feed-forward control, various methods have been proposed in [2, 6-8]. Basically, the following relationship is applied in all of the cases.

$$G_{ff}(s) = G_0^{-1}(s)$$

$$\text{(3)}$$

The methods based on eq (3) improve the output tracking performance of the system as compared to that of the ideal feedback control in case of small model uncertainty and plant dynamic is minimum phase type. To overcome this limitation, an approximate-inverse method is proposed in [9]. In another method, the unstable exact inverse of plant is approximated by some stable transfer function [10]. This method is simple and includes only current value of reference signal and its time derivatives.

3. PROPOSED FEED-FORWARD CONTROLLER

This paper provides simple rules of feed-forward control design for dynamics of the studied plant in the form of the linear combination of a reference trajectory and its successive time derivatives similar to [12] as follows.

$$u_{ff}(t) = p_0 y_d^{(0)} + \ldots + p_1 y_d^{(1)} + p_0 y_d$$

$$\text{(4)}$$

where, $\mu \in N$ and $(p_0, \ldots, p_0) \in R$ are the design parameter. In this design problem, the following assumptions have been considered.

i. The degree of numerator is greater than or equal to the degree of denominator i.e. system (1) is at least proper ($n \geq m$) and they do not have any common factors.

ii. Polynomial $B(s)$ can be factorized as follows.

$$B(s) = B^p(s)B^\alpha(s)$$

$$\text{(5)}$$

where

$$B^\alpha(s) = \beta_0 s^\theta + \ldots + \beta_\alpha s + 1$$

$$\text{(6)}$$
and
\[ B^\delta(s) = \alpha_0 s^\delta + \cdots + \alpha_s s + 1 \]  
(7)

where \( \delta + \theta = m \). Now,

\[ G_{\phi}(s) \downarrow \frac{U_{\phi}(s)}{E(s)} = \frac{L(s)}{M(s)} \]  
(8)

\[ G_{FF}(s) \downarrow \frac{U_{FF}(s)}{E_{a}(s)} = P(s) = p_0 s^\mu + \cdots + p_s s^\mu + p_0 \]  
(9)

\[ G_{E}(s) \downarrow \frac{E(s)}{Y_a(s)} = \frac{1-G_0(s)G_{FF}(s)}{1+G_0(s)G_{\phi}(s)} \]  
(10)

Eq (10) can be rewritten as follows.

\[ G_{E}(s) = S(s)[1-G_0(s)G_{FF}(s)] = S(s) \]  
(11)

where \( S(s) \) and \( \Gamma(s) \) are closed-loop sensitivity and feed forward mismatch transfer function respectively such that if

\[ (s) \equiv 0 \Leftrightarrow G_{FF}(s) \downarrow G_0^{-1}(s) \]  
(12)

Here, a nominal design of feed-forward controller is practically possible if \( G_0^{-1}(s) \) is stable and proper. But it is not applicable if plant \( G_0(s) \) is non-minimum phase [1, 11]. Therefore, the following approximation procedure is followed. From eq (11), we get

\[ G_E(s) = \frac{1}{B(s)} \frac{\frac{A(s)}{B(s)}p(s)}{1+\frac{A(s)}{B(s)}L(s)} = \frac{M(s)[A(s)-B(s)P(s)]}{M(s)A(s)+B(s)P(s)} = \frac{M(s)W(s)}{H(s)} \]  
(13)

where, \( G_{FF}(s) \downarrow P(s) \) and \( W(s) \) is user-defined weighting function to impose the requirement for bandwidth defined as follows.

\[ W(s) = \sum_{j=0}^{n} a_j s^j - \sum_{i=0}^{m} \sum_{j=0}^{i} b_j p_j s^{j+i} \]  
(14)

4. RESULTS AND DISCUSSION

To describe the designing of the proposed controller, an example with the following components has been used. Here, the plant is represented by a second order unstable transfer function with a single positive zero as \( \gamma_d = 0.5\sin(0.4t) + 1.0\sin(0.2t) + 1.5\sin(0.1t) \),

\[ G(s) = \frac{-0.4s+1}{0.3s^2 + 0.8s - 1.5} \]  
and \( G_{\phi}(s) = k_p = 1.6 \). The values of controller parameters computed by using eqs (18) and (19) are shown in table I.
TABLE I. Computed Coefficient of CAI based and proposed method based controllers

| Parameters | Values (CAI method) | Values (Proposed method) |
|------------|---------------------|--------------------------|
| $p_0$      | -1.5                | -1.5                     |
| $p_1$      | 0.200               | 0.0005                   |
| $p_2$      | 0.380               | 0.012                    |

Now by using these values, the sensitive function and mismatch transfer function are computed as follows. \( S(s) = \frac{0.3s^2 + 0.8s - 1.5}{0.3s^2 + 0.16s + 0.1} \) and \( \Gamma(s) = \frac{0.152s^3}{0.3s^2 + 0.8s - 1.5} \). The MATLAB/SIMULINK of plant with the proposed controller is shown in Fig.2. To verify efficacy of the proposed method, it is compared with that of the CAI method as shown in Fig.3 (a). From this figure, it can be seen that the response with CAI method is hugely oscillating in nature compared to that of proposed method. To check the results properly, the simulation results in case of CAI method is limited to 100s and it seems that output signal of plant 1 is exactly track by reference trajectory with nearly zero steady state error. However, for long span of time, this method fails. But, from the response of the proposed method is matching the reference for up to 2000s also (Fig.3 (c)). CAI exactly track the at (15 to 25),(50 to 70) and (80 to 90) sec. whereas proposed controller track at each period of time. Mismatch transfer is also reduced for these cases. Therefore, performance of the studied system in case of proposed method is better than that of the CAI method.

![Fig.2. MATLAB/ SIMULINK model for proposed controller](image-url)
A new feed-forward controller is proposed in this paper. The tested system is a non-minimum phase type plant. To justify the efficacy of the proposed controller, a MATLAB/SIMULINK model is constructed. Its tracking results are compared with that of the CAI based controller.
Although, both controllers are designed to yield small tracking error with guarantee zero steady state error, proposed controller can improve performance for long duration.

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