Inflation in String Theory and how you can get out of it.

E. Papantonopoulos
National Technical University of Athens, Physics Department, Zografou Campus, GR 157 80, Athens, Greece.

Abstract
Inflation is now a basic ingredient of the modern cosmology. After reviewing the basic characteristics of inflation, we briefly discuss inflation in string and brane theories, focusing on the problem of the exit from inflation in these theories. We present a model, based on type-0 string theory, in which our universe after the inflationary phase, passes smoothly to a flat Robertson-Walker universe.

Talk given at the 2nd Hellenic Cosmology Workshop, National Observatory of Athens (Penteli), 19-20 April 2001.
Introduction

Inflation was introduced in cosmology [1] in order to solve three basic problems of standard cosmology. The first problem is the Particle Horizon. The particle horizon is the ‘instantaneous’ distance at time $t$ traveled by the light since the beginning of time and is given by

$$d_{H}(t) = a(t) \int_{0}^{t} \frac{dt'}{a(t')}.$$  \hspace{1cm} (1)

The particle horizon gives the distance at which causal contact has been established at time $t$. The present particle horizon is $d_{H} = 2H_{0}^{-1} = 6000h^{-1} \text{Mpc}$ where $H_{0}$ is the Hubble parameter at the present time and $h$ a parameter with values $(0.4 \leq h \leq 0.8)$. Recent results from COBE indicate that the Cosmic Background Radiation (CBR) is uniform in temperature at large scales. The CBR was emitted at the time of decoupling of matter and radiation. If one uses the standard cosmology, the particle horizon at the decoupling time can be calculated and found $\approx 0.168h^{-1} \text{Mpc}$. Then if we allow this distance to expand till the present time, then we will find $\approx 184h^{-1} \text{Mpc}$. This means that in the past there was not enough time for the photons to communicate their temperature to all direction of the now visible universe.

The particle horizon during inflation (exponential expansion) is

$$d(t) = e^{Ht} \int_{t_{i}}^{t} \frac{dt'}{e^{Ht'}}$$  \hspace{1cm} (2)

and grows as fast as $a(t)$. Therefore the successful inflationary model in which the particle horizon exceeds $2H_{0}^{-1}$ solves the Particle Horizon Problem, because the light rays emitted at the decoupling time have enough time to communicate the temperature everywhere and produce a uniform spectrum of CBR, as it is observed today [2].

The second problem of the standard cosmology is known as the Flatness Problem. The present energy density, $\rho$, of the universe has been observed to lie in the relative narrow range $0.1\rho_{c} \leq \rho \leq 2\rho_{c}$, where $\rho_{c}$ is the critical energy density corresponding to flat universe. Then using the Einstein equations one finds $\frac{\rho - \rho_{c}}{\rho_{c}} = 3(8\pi G \rho_{c})^{-1} \frac{k}{a^{2}}$ which is equal to $a$ for a matter dominated universe. If we use the fact that $a = t^{\frac{2}{3}}$ in a matter dominated universe, we can calculate that at the time of decoupling $\frac{\rho - \rho_{c}}{\rho_{c}} \approx 10^{-16}$. The standard big-bag cosmology cannot offer a plausible explanation why the early energy
density of the universe is so finely closed to its critical value. Inflation offers a very convincing argument why $\rho = \rho_c$ at the very early times. Because of the exponential growth of the scale factor $a$, the term $\frac{k}{a^2}$ gets suppressed driving $\frac{\rho - \rho_c}{\rho_c} \ll 1$ in a natural way. In fact inflation implies that the present universe is flat to a great accuracy.

The third problem is the Monopole Problem [3]. At early times in the expansion, the physics of the universe is described by particle theory. These theories predict that at the expansion proceeds as the universe cools, phase transitions occur which are natural consequences of symmetry breaking. During these phase transitions various particles are produced like magnetic monopoles. If one calculates the number of monopoles produced during, for example, the electroweak phase transition, finds that the dominated matter of our universe should consists out of monopoles, contradicting with all experimental results. Inflation solves the Monopole Problem by diluted the primordial monopole density.

Most of the successful models of inflation incorporate a scalar field the inflaton. The scalar field appears in the action with a kinetic energy term and a potential term. The interplay between potential and kinetic energy of the inflaton field gives all the dynamics in an inflationary model. During inflation, the inflaton field slowly rolls down the nearly flat potential until it reaches its minimum. Fluctuations of the inflaton field around the minimum of the potential reheats the universe, and then involves to its present epoch.

We all now believe, and recent observational data make this believe stronger, that the universe in its early evolution had passed from an inflationary epoch. The crucial question however is, how long the inflation lasted and how it ended. All the inflationary models have to solve the three basic problems we discussed. The solution of these problems impose bounds on the duration of the inflation. These bounds depend on the parameters of the theory, but all these bounds give a very short period of inflation. The end of inflation stars when the inflaton field rolls down to the minimum of the potential and the reheating starts. The reheating of the universe is a complex physical problem [4] which depends on the physical parameters of the theory and in some models a fine tuning of parameters is needed.

As we already discussed, the inflaton field plays a central rôle in inflation. This scalar field, usually is added by hand in the action, and the resulting effective theory is studied. Nevertheless, there are models in the literature, mostly based on Grand Unification Schemes, where the inflaton and its potential arise naturally from the scalar-field content of the theory.
2 Inflation in String and High Dimensional Theories

String theory is a high dimensional theory, which attempts to unify the electroweak, strong and gravity theories. Cosmological string models [5] can be obtained if one dimensionally reduces the string theory to four dimensions and looks for time dependent solutions in a Robertson-Walker background. The massless fields that survive this procedure, in the simplest possible string theory, are the dilaton field, the graviton field and the action field. The theory that describes such a theory is

$$\int d^4x \sqrt{-g} \left[ R - \frac{1}{2} (D\phi)^2 - \frac{1}{2} e^{2\phi} (Db)^2 - \frac{1}{3} e^\phi \delta c \right]$$

(3)

where $\phi$ is the dilaton field, $b$ is the action field and $\delta c$ is the central charge deficit defined by $c = 22 + \delta c$ with $c$ the central charge. Using this action one can write down the equations of motion (the $\beta$-functions of the theory) for the graviton, dilaton and action fields. If we allow only time dependence and choose a Robertson-Walker background, these equations can be solved to give a linear expanding universe [6].

The action given in (3) is the most general action for any bosonic string theory dimensionally reduced to four-dimensions. So it is interesting to find that the cosmology of such a theory is of a linear expanding universe. To get an inflationary solution we must add a potential for the dilaton field. Such a term does not arise classically and has to be generated by string loop effects or by non-perturbative phenomena.

Another attempt to string cosmology was to interpret the dilaton field of the string theory as the inflaton [7], the scalar field which drives the inflation. The advantage of such the approach is that the usual four-dimensional Einstein gravity is naturally part of the theory and you can study the very early cosmological evolution, evading even the initial singularity. The disadvantage of such theories is that they invoke on a dilaton potential for which we know very few things and that it is very difficult to end the inflationary phase and to go in a smooth way to the usual Robertson-Walker expanding universe.

We can find an inflationary solution and eventual exit from it in a string theory, if we consider non-critical strings. In non-critical strings the $\beta$-functions of the theory (the classical equations of motion) are modified in
such a way as to restore the conformal invariance [8]. Then the resulting equations of motion can be solved analytically or numerically and study cosmological evolution. In a toy two dimensional model we had studied such a theory [9]. The action of this model is

\[ \int d^2x \sqrt{-g} \left[ R - 2(\nabla\phi)^2 + 2e^{2\phi}(\nabla T)^2 + V(T) - e^{2\phi}Q^2 \right] \quad (4) \]

where \( T \) is the tachyon field with a potential \( V(T) \) and \( Q \) is a time dependent quantity similar to the central charge of (3). Solving the modified equations of motion, supplemented by the Curci-Paffuti equation we get an inflationary phase followed in a smooth way by an expanding Robertson-Walker universe [9]. In the next section will discuss a four-dimensional realistic theory, based on non-critical strings in which the inflationary phase is terminated and then followed by the usual expanding phase.

The introduction of branes into cosmology had offered another exiting possibility. According to the brane scenario we are living on a three dimensional brane, where also the gauge fields are confined. The brane is embedded in a higher dimensional space in which the gravitational field lives. There are a lot of cosmological models in the literature that exhibit brane-inflation. We can separate these models into two categories. In the first one [10], the brane is a static solution of the underling theory and the cosmological evolution is due to the time development of the fields, while in the second category [11] the cosmological evolution is due to the movement of the brane in the bulk.

We had studied [12, 13] a brane moving in a particular background of a type-0 string background. What we found is that an energy density (time dependent cosmological constant) was induced on the brane, because of its movement in the bulk. This energy density gives an inflationary phase in the cosmological evolution of the brane-universe which is terminated when the brane is far way and does not feel the gravitational field of the other branes.

Motivated by brane cosmology we had studied [14] the four-dimentional Einstein equation with a time dependent cosmological constant of the form \( \Lambda = \frac{1}{\log t} \). If \( a \) is the scale factor of the universe, the Einstein equation which gives the cosmological evolution of the \( a(t) \) in the presence of a cosmological constant of the form \( \Lambda = \frac{1}{\log t} \) is

\[ \frac{\ddot{a}}{a} = \left( 1 - \frac{3\gamma}{2} \right) \left( \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} \right) + \gamma \frac{1}{2 \log t} \quad (5) \]
where $\gamma$ appears in the equation of state $p = (\gamma - 1)\rho$. For a flat universe $k = 0$, the above equation has two distinct solutions one of rapid growth (inflation) and the other of logarithmic growth (slow expansion). These solutions are connected in a smooth way, without any singularity.

3 Cosmological Evolution of a Four-Dimensional String Model Based on Type-0 String Theory

We consider a ten-dimensional string theory of type-0 \cite{15}. We dimensionally reduce the ten-dimensional action to the four-dimensional space-time on a brane, assuming that all our fields depend only upon the time. We choose two different fields to parametrize the internal space. The first field sets the scale of the fifth dimension, while the other parametrize a conformally flat five-dimensional space. In the same way, we dimensionally reduce the modified
(because of the off-criticality) $\beta$-functions of the ten-dimensional theory, to the effective four-dimensional $\beta$-functions, assuming a Robertson-Walker form of the four-dimensional metric. The resulting equations of motion, supplemented by the equation Curci-Paffuti, are too complicated to be solved analytically. We follow a systematic method, of the quasi-linear systems \[16\], to solve them numerically. The action of the theory in ten dimensions is

$$S = \int d^{10}x \sqrt{-G} \left[ e^{-2\Phi} \left( R + 4(\partial_M \Phi)^2 - \frac{1}{4}(\partial_M T)^2 \right) - \frac{1}{4}m^2 T^2 - Q^2 \right] - \frac{1}{4}(1 + T + \frac{T^2}{2}) |\mathcal{F}_{MNPST}|^2 \] (6)

where capital Greek letters denote ten-dimensional indices, $\Phi$ is the dilaton, $T$ is the tachyon field of mass $m^2 < 0$. In our analysis we have ignored higher-order terms in the tachyon potential. The quantity $\mathcal{F}_{MNPST}$ denotes the appropriate five-form of type-0 string theory, which couples to the tachyon field in the Ramond-Ramond (RR) sector via the function $f(T) = 1 + T + \frac{T^2}{2}$, and $Q^2$ plays the role of the central charge.

We need first to consider the dimensional reduction of the ten-dimensional action to the four-dimensional space-time on the brane. This procedure is achieved by assuming the following ansatz for the ten-dimensional metric:

$$G_{MN} = \left( \begin{array}{ccc} g^{(4)}_{\mu\nu} & 0 & 0 \\ 0 & e^{2\sigma_1} & 0 \\ 0 & 0 & e^{2\sigma_2} I_{5 \times 5} \end{array} \right) \] (7)

where lower-case Greek indices are four-dimensional space time indices, and $I_{5 \times 5}$ denotes the $5 \times 5$ unit matrix. We have chosen two different scales for internal space. The field $\sigma_1$ sets the scale of the fifth dimension, while $\sigma_2$ parametrize a flat five dimensional space. In the context of cosmological models, we are dealing with here, the fields $g^{(4)}_{\mu\nu}, \sigma_i, i = 1, 2$ are assumed to depend on the time $t$ only.

Upon considering the fields to be time dependent only, restricting ourselves to the compactification (7), and assuming a Robertson-Walker form of the four-dimensional metric, with scale factor $a(t)$, the modified equations of motion are

$$-\frac{3\ddot{a}}{a} + \ddot{\sigma}_1 + 5\ddot{\sigma}_2 - 2\ddot{\Phi} + \dot{\sigma}_1^2 + 5\dot{\sigma}_2^2 + \frac{1}{4}\dot{T}^2 + e^{-2\sigma_1 + 2\Phi} f_5 f(T) = 0,$$

$$\ddot{a}a + a\ddot{a} \left( 2Q + \dot{\sigma}_1 + 5\dot{\sigma}_2 - 2\ddot{\Phi} \right) + e^{-2\sigma_1 + 2\Phi} f_5^2 f(T) a^2 = 0 ,$$
\[ \ddot{\sigma}_1 + 5\dot{\sigma}_1^2 + \frac{3}{a} \dot{\sigma}_1 + 2Q \dot{\sigma}_1 + 5\dot{\sigma}_1 \dot{\sigma}_2 - 2\dot{\sigma}_1 \dot{\Phi} + e^{-2\sigma_1 + 2\Phi} f^2(T) = 0 , \]
\[ 3\ddot{\sigma}_2 + 9\dot{\sigma}_2^2 + \frac{3}{a} \dot{\sigma}_2 + 2Q \dot{\sigma}_2 + \dot{\sigma}_1 \dot{\sigma}_2 - 2\dot{\sigma}_2 \dot{\Phi} - e^{-2\sigma_1 + 2\Phi} f^2(T) = 0 , \]
\[ 2\ddot{T} + 3\dot{a} \dot{T} + Q \dot{T} + \dot{\sigma}_1 \dot{T} + 5\dot{\sigma}_2 \dot{T} - 2\dot{T} \dot{\Phi} + \]
\[ m^2 T - 4e^{-2\sigma_1 + 2\Phi} f^2(T) f'(T) = 0 , \]
\[ \ddot{\Phi} + Q \dot{\Phi} + 6\frac{\dot{a}}{a} + 6\frac{\dot{a}^2}{a^2} + 2 \left[ -\ddot{\sigma}_1 - \frac{3a}{\sigma_1 - 5\dot{\sigma}_2 - 15\dot{\sigma}_2 - \sigma_1^2 - 15\sigma_2^2 - 5\dot{\sigma}_1 \dot{\sigma}_2 - \right. \]
\[ 2 \ddot{\Phi}^2 + 2\dot{\Phi} + 6\frac{\dot{a}}{a} \dot{\Phi} + 2\dot{\sigma}_1 \dot{\Phi} + 10\dot{\sigma}_2 \dot{\Phi} \right] - \frac{1}{4} \dot{T}^2 + \frac{1}{4} m^2 T^2 + Q^2 = 0 , \]
\[ C_5 = e^{-\sigma_1 + 5\sigma_2} f(T) f_5 , \]
\[ \Phi^{(3)} + Q \ddot{\Phi} + \ddot{Q} \dot{\Phi} + 12\frac{\dot{a}}{a^3} \left[ a \ddot{a} + \dot{a}^2 + Q a \dot{a} \right] - \ddot{T} \ddot{\Phi} - \dot{T} \dot{\Phi} + \]
\[ 4\dot{\sigma}_1 \dot{\sigma}_1^2 + 2\dot{\sigma}_1 \dot{\sigma}_2 + 2\dot{\sigma}_2^2 + Q \dot{\sigma}_2 = 0 \]

where \( f'(T) \) denotes functional differentiation with respect to the field \( T \), the dots denote time derivatives, \( \Phi^{(3)} \) denotes triple time derivative.

To solve the above system numerically, we separate the fields in their asymptotic values plus fields which tend asymptotically to zero. Substituting these fields back to the equations we let the system evolve in time backwards [17].

Our main results are as follows. The cosmological evolution of our universe passes through the following phases. At very early times, the universe starts from the initial singularity. Then enters a phase where the physical dimensions are formed. At this stage \( \sigma_1, \sigma_2 \) and \( a \) are comparable in magnitude. The phase of inflation follows, during which the scale factor grows exponentially (for a short time though), while the internal space contracts with very negative values of \( \ddot{\sigma}_1 \) and \( \ddot{\sigma}_2 \). Finally the universe enters a phase, where it expands slowing until it reaches the asymptotically flat space. The internal space continuous to contract but with very slow rate (with \( \ddot{\sigma}_1 \) and \( \ddot{\sigma}_2 \) positive) until it reaches a constant value. The fields \( \sigma_1 \) and \( \sigma_2 \) scale differently. The field \( \sigma_2 \) very soon freezes to a value much lower than the value of \( \sigma_1 \), indicating the the fifth dimension can be much larger than the other five dimensions. In Fig.2 the evolution of the scale factor is shown.

The dilaton field at the singularity is infinite, indicating that the gravity
is very strong. Then at the second phase of evolution, the strength of gravity is weakened because the dilaton field drops linearly. Then during inflation the gravity becomes more weaker, and finally at the exit, the dilaton field continuous to drop linearly. The tachyon field during the evolution, falls continuously until it becomes zero. The RR-field, from zero value at the singularity, grows until it reaches a constant value at the exit of inflation.

4 Summary

We have presented a cosmological model based on a type-0 string theory. The type-0 string theory is rich in its content. Except the graviton and the dilaton field, it includes also a tachyon field which couples to an RR five-form field. To avoid tachyon instabilities the tachyon field has to take values at the minimum of its potential. Considering the ten-dimensional action of the type-0 field theory one can get at the conformal point the $\beta$-functions of the theory. Reducing them to four-dimensions and assuming that all the fields are time dependent we get an effective four-dimensional theory. We found that this theory in a Robertson-Walker background has an inflationary phase but we cannot exit from this phase in a smooth way [17].

We believe that the central issue of inflationary cosmology is not how we can get an inflationary phase, but how we can exit from it. In view of the recent discussion [18] of the exit from inflation in string theory, and in general on the consistent quantization of de-Sitter Universes [19], we think that it
is useful to propose new mechanisms to exit from inflation. In our previous work [4], we had proposed a mechanism based on non-critical strings, to exit from inflation. We had consider a two dimensional model with a tachyon field. The presence of the tachyon field was crucial for an inflationary phase to go smoothly to a Robertson-Walker phase.

In this work we applied this mechanism to a realistic four-dimensional model. We modified the $\beta$-functions of the ten-dimensional theory, assuming that the string theory is non-critical. This hypothesis, introduces new terms in the $\beta$-functions of the theory. Physically these terms express the fact that our non-critical string theory is performing small oscillations around the conformal point.

The modified $\beta$-functions supplemented with the Curci-Paffuti equation, are then reduced to four-dimensions. Assuming an homogeneous and spherically symmetric background, we solve numerically the resulting equations. The numerical analysis of the system of the coupled differential equations uses the quasi-linear method. Because we want the solution asymptotically to approach the Minkowski space with a linear dilaton, we separate the fields in their asymptotic values plus fields which tend asymptotically to zero. Substituting these fields back to the equations we leave the system to evolve in time backwards. What we find is that the scale factor after the initial singularity, enters a short inflationary phase and then in a smooth way goes to flat Minkowski space. The fields $\sigma_1$ and $\sigma_2$ which parametrise the internal space have an interesting behaviour. The field $\sigma_1$ which sets the scale of the fifth dimension, during inflation contracts until it reaches a constant value. After the inflation maintains this value until the universe involves to an asymptotically flat space. The field $\sigma_2$ which parametrise the flat five-dimensional space freezes to a constant value which is much smaller than the value of the constant of the fifth dimension. Thus we see that cosmological evolution may lead to different scales if we decompose the extra dimensions. It is important finally to notice, that this difference in scales of the extra dimensions is due to the fact that in our theory the gravity is very weak asymptotically.

**Acknowledgments**

This Talk is based on work done in collaboration with G. Diamandis, B. Georgalas, N. Mavromatos and I. Pappa. We thank the organizers of the 2nd Hellenic Cosmology Workshop, National Observatory of Athens (Penteli), 19-
20 April 2001, for their kind invitation.

References

[1] A. H. Guth, Phys. Rev. D 43, (1981) 347; A. D. Linde “Particle Physics and Inflationary cosmology, Harwood Academic Publishers, London, 1990.

[2] G. F. Smoot et. al., Astrophys. J. Lett. 396 (1992) L1; C. L. Bennett et al., Astrophys. J. Lett. 464 (1996) 1.

[3] G. ’t Hooft, Nucl. Phys. B 79 (1974) 276; A. Polyakov, JETP lett. 20 (1974) 194.

[4] A. D. Linde, JETP Lett. 38 (1983) 149; Phys. Lett. B 129 (1983) 177.

[5] M. S. Turner, Phys. Rev. D 28 (1983) 1243.

[6] For a review on string cosmological models see J. E. Lidsey, D. Wands, E. J. Copeland, Phys.Rept. 337 (2000) 343, [hep-th/9909061].

[7] I. Antoniadis, C. Bachas, John Ellis and D.V. Nanopoulos, Phys. Lett. B211 383 (1988); Nucl. Phys. B 328 117 (1989).

[8] M. Gasperini and G. Veneziano, Astropart. Phys. 1 (1993) 317.

[9] J. Ellis, N. E. Mavromatos and D. V. Nanopoulos, J. Chaos, Solitons and Fractals 10, 345 (eds. C. Castro and M.S. El Naschie, Elsevier Science, Pergamon 1999) [hep-th/9805120].

[10] G. A. Diamandis, B. C. Georgalas, N. E. Mavromatos and E. Papantonopoulos, Phys. Lett. B 461, 57 (1999) [hep-th/9903043].

[11] N.Kaloper and A.Linde, Phys. Rev. D 59 (1999) 101303 [hep-th/9811141]; N.Arkani-Hamed, S.Dimopoulos, N.Kaloper and J.March-Russell, [hep-ph/9903224]; P.Kanti, I.I.Kogan, K.A.Olive and M.Pospelov, Phys. Lett. B 468 (1999) 31 [hep-ph/9909481]; P.Binetruy, C.Deffayet and D.Langlois, [hep-th/9905012]; C.Csaki, M.Graesser, C.Kolda and J.Terning, Phys. Lett. B 462 (1999) 34 [hep-ph/9906513].
[11] H.A.Chamblin and H.S.Reall, hep-th/9903225; A.Chamblin, M.J. Perry and H.S.Reall, J.High Energy Phys. 09 (1999) 014 [hep-th/9908047]; P.Kraus, JHEP 9912:011 (1999) [hep-th/9910149]; A.Kehagias and E.Kiritsis, JHEP 9911:022 (1999) [hep-th/9910174].

[12] E. Papantonopoulos, Proceedings of the 9th Marcel Grossmann Meeting, 2000, [hep-th/0011051].

[13] E. Papantonopoulos and I. Pappa, Mod. Phys. Lett. A15, 2145 (2000), [hep-th/0001183]; Phys. Rev. D63, 103506, (2001), [hep-th/0103101].

[14] E. Papantonopoulos and I. Pappa, [gr-qc/0103101].

[15] I. Klebanov and A.A. Tseytlin, Nucl. Phys. B546, 155 (1999); Nucl. Phys. B547, 143 (1999).

[16] S. Nemytskii and B. Stepanov, ”Qualitative Theory of Differential Equations”,1960 Princeton University Press.

[17] G.A. Diamandis, B.C. Georgalas, N.E. Mavromatos, E. Papantonopoulos and I. Pappa, [hep-th/0107124].

[18] S. Hellerman, N. Kaloper and L. Susskind, [hep-th/0101180]; W. Fischer, A. Kashani-Poor, R. McNees, S. Paban, [hep-th/0104181]; A.P. Billyard, A.A. Coley and J.E. Lidsey, J. Math. Phys. 41 (2000) 6277.

[19] E. Witten, [hep-th/0106109]