The Structure Function $F_2^\gamma(x, Q^2)$ at LEP2

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Abstract

The unique nature of the photon can be investigated in hitherto unexplored kinematic regions at LEP2. We discuss the theoretical significance of deep inelastic measurements and present a prescription that allows a theoretically and experimentally sensible separation of the so-called 'anomalous' and 'hadronic' components of the target photon. We perform preliminary studies regarding the ability to reconstruct the $\gamma^*\gamma$ CM energy (and hence $x$) and the usefulness of the easier to measure electron structure function.

1. The unique nature of the photon

Deep inelastic scattering of electrons off hadronic targets teaches us a great deal regarding the dynamical substructure of hadrons. If the target is a photon then we are provided with a unique opportunity to examine the interplay between the non-perturbative phenomena associated with hadronic bound states and purely perturbative QCD. This is a result of the dual nature of the photon, which can be seen to interact as a fundamental gauge boson or as a hadron. In fig.(1) we illustrate the so-called 'anomalous' and 'hadronic' contributions to the deep inelastic process (we show only the lowest order QED contribution to the 'anomalous' process). The need for a

hadronic component arises, for massless quarks, even at the QED level (i.e. $\gamma^*\gamma \rightarrow q\bar{q}$) where one encounters a collinear divergence when integrating over the transverse momentum of the

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exchanged quark. Of course the divergence can be regulated by introducing a quark mass, but this is perhaps unphysical since we anticipate that the region of low transverse momentum will be subject to large non-perturbative corrections. As a result, it is more sensible to introduce some factorisation scale and an associated non-perturbative component (often modelled by vector dominance ideas). However, we expect that there will be some deep inelastic events that are completely perturbative in nature and some that are inherently non-perturbative. Clearly it would be interesting to isolate such components and to study their relative contributions to the inclusive cross section, $F_2^\gamma(x, Q^2)$. To quantify this separation, we would like to propose the following definition of hadronic and anomalous events (in fact, it is very similar to that being used already by the HERA physicists to isolate so-called ‘direct’ enriched events in photoproduction [1]).

It is appropriate to work in a frame in which the $\gamma^*$ and target $\gamma$ are collinear. The ideal frame would be their Breit frame, in which the virtual photon is purely longitudinal ($q_\mu = (0, 0, Q)$), and collides head-on with the target photon. However, since the target photon’s direction is not known exactly, we propose the Breit frame of the virtual photon and the target electron beam. Further studies need to be performed in order to investigate the feasibility of performing this Lorentz transformation (which only requires sufficiently good resolution of the tagged electron), and the extent to which using the beam direction instead of the photon direction smears the results. Fig.1 illustrates how a typical ‘hadronic’ event and a typical ‘anomalous’ event would look in the Breit frame. The ‘anomalous’ events are characterised by the fact that all of the radiation is at high $p_T$ and it is this property we exploit. Like the HERA experimentalists, we define the variable, $x_\gamma$:

$$x_\gamma = \frac{\sum (E - p_z)_{\text{jets}}}{\sum (E - p_z)_{\text{all}}}.$$  \hspace{1cm} (1)

Where the numerator sum runs over all particles in jets and the denominator sum runs over all particles (both computed in the collinear frame where the +ve $z$-direction is defined by the direction of the virtual photon). Importantly, this quantity is invariant under longitudinal boosts. For ‘hadronic’ events, there are no high-$p_T$ particles produced (to leading order, the current jet and photon remnant are collinear and have zero $p_T$) and so the sum over jets is zero. Providing the experiments are able to identify at least some of the target remnant, this means that $x_\gamma = 0$ for ‘hadronic’ events. For the ‘anomalous’ events, all particles are emitted at high $p_T$, and as such should be assigned into jets, i.e. $x_\gamma = 1$. This is a remarkably clean separation of the two components and so a considerable smearing of the distributions can be tolerated. A quantitative definition of ‘hadronic’ and ‘anomalous’ is now established by making a cut on $x_\gamma$.

2. Small-$x$ behaviour

There is already a great deal of data on the structure function, $F_2^\gamma(x, Q^2)$, in the region $x \gtrsim 0.01$ [2]. LEP2 will, for the first time, allow measurement in the lower $x$ region. In fig.2, we show the expected distribution of events in the $x$-$Q^2$ plane given 500 pb$^{-1}$ of $e^+e^-$ data and sensible cuts on the tagged electron (i.e. $E_e > E_{\text{beam}}/2$, $\theta_e > 1.7^\circ$, where $E_e$ and $\theta_e$ are the scattered electron’s energy and angle, and $E_{\text{beam}} = 87.5$ GeV). As can be seen from the figure, LEP2 can expect good statistics for $x \gtrsim 10^{-4}$. Also notice that LEP2 is able to measure a wide range in $x$ at a given $Q^2$. This is due to the variable target photon energy (since it is radiated off the incoming electron).
Opening up the small $x$ domain should reveal sensitivity to the dynamics that is responsible for the steep rise of the proton structure function, $F_2^p(x, Q^2) \sim x^{-0.3}$. So far this region has only been measured at HERA \[3\], so the universality of the rise could be tested at LEP2. This rise is much stronger than that predicted by the simple Regge pole contribution \[4\] ($F_2^p \sim x^{-0.08}$ at small $x$), confirming that perturbative physics plays an important role. Much effort has been dedicated to identifying the nature of the large perturbative contribution. Let us outline the basic ideas. Ultimately, the small $x$ rise is generated as a result of many soft gluon emissions which arise due to the singular nature of the gluon splitting function, i.e. $P_{gg} \sim 1/z$: soft gluons like to radiate even softer gluons. The dominant contribution therefore arises from graphs like the one in fig.3. In the conventional Dokshitzer, Gribov, Lipatov, Altarelli, Parisi (DGLAP) approach \[5\], the splitting functions (and appropriate coefficient functions) are expanded as power series in $\alpha_s$. At lowest order, the solution to the DGLAP evolution equations corresponds to those configurations where successive partons are emitted with much higher transverse momenta than any previous emissions, so that the parent parton of each emission can be considered collinear with the incoming hadron. This collinear approximation leads to the summation of all large logarithms in $Q^2$ (in leading order it is the sum of all terms $\sim (\alpha_s \ln Q^2)^n$). However, as $x$ falls, there is increased phase space for successively softer gluon emissions (the collinear approximation breaks down) and terms $\sim (\alpha_s \ln z)^n$ start to become more significant. These contributions (to the inclusive cross section) are summed up using the formalism of Balitsky, Fadin, Kuraev and Lipatov (BFKL) \[6\]. This latter formalism has created much interest since it provides a description of the elusive pomeron within QCD (i.e. it is that object that determines high energy scattering at short distances). It should however be appreciated that the DGLAP and BFKL formalisms are not completely disjoint. By summing an infinite subset of contributions in the expansion of the splitting (and coefficient functions) it is possible to incorporate the leading-twist BFKL contribution within the DGLAP approach \[7, 8\].

![Figure 2](image-url)
3. Feasibility

It is much harder to measure the \( x \)-dependence of the photon structure function than that of the proton. This is because one does not know the energy of the target photon and hence it is necessary to reconstruct the whole of the hadronic final state in order to extract the \( \gamma^*\gamma^* \) invariant mass (and hence \( x \)). In the left-hand plot of fig.4, we show the correlation of the observed invariant mass, \( W_{\text{vis}} \), and the generated invariant mass, \( W_{\text{true}} \). We define \( W_{\text{vis}} \) as simply the total invariant mass of hadrons in the region \( |\cos \theta_h| < 0.97 \), and show only events passing the electron cuts given earlier, with \( Q^2 < 10 \text{ GeV}^2 \) and \( W_{\text{vis}} > 2 \text{ GeV} \). We used the HERWIG Monte Carlo event generator, but similar results have been found using ARIADNE \[1\]. It is clear that the correlation is very poor and worsens as \( W_{\text{true}} \) rises (i.e. \( x \) falls). At large \( W_{\text{true}} \), the events tend to be increasingly boosted in the direction of the target photon, so more of the hadronic event is lost in the beam hole, and the number of events with \( W_{\text{vis}} \sim W_{\text{true}} \) decreases. At the larger \( x \) (lower \( W_{\text{true}} \)) values of the data so far collected, the correlation is
good enough that a reliable unfolding can be performed using relatively unsophisticated Monte Carlo programs. This will clearly not be the case at LEP2 and it is vital that effort is devoted to establishing a more sophisticated unfolding procedure. In the right-hand plot of fig.4, a very simple prescription (based along the lines of an idea by John Field [10]) has been used to define a reconstructed mass, $W_{\text{recon}}$, and a vastly improved correlation is found. The prescription utilises the information that transverse momentum is conserved, and that the lost mass is down the beam hole. A pseudo-particle is introduced that carries away the missing transverse momentum (ignoring the $p_T$ of the untagged electron) and has longitudinal momentum just sufficient to ensure that it remains unobserved. It is encouraging that such significant improvement is found using such a crude algorithm.

To conclude, let us say a few words about the electron structure function, $F_2(x_e, Q^2)$ (where $x_e = x z$ and $z$ is the photon energy fraction). This measurement can be made without unfolding. In fig.5, a variety of parametrisations for $F_2(x, Q^2)$ [11] are compared along with a similar comparison for $F_2(x_e, Q^2)$, both at $Q^2 = 10 \text{ GeV}^2$. One can see that those parametrisations that predict very different behaviours for the photon at small $x$ lead to very similar behaviours for the small $x_e$ electron structure function. This loss of sensitivity arises because the photon flux $f_{\gamma/e} \sim 1/z$ and so at small $x_e$ there is competition between large $z$-small $x$ and small $z$-large $x$. It can also be seen that not all the structure function predictions are reliable at the $x$ and $Q^2$ values at which LEP2 will provide data (nor were they ever intended to be so). It is clear that this must be improved before reliable predictions can be made for event rates or properties at LEP2 using the full range of structure function parametrisations.

Of course another very useful measurement that avoids the need to unfold is to measure both the electron and positron. This opens up the possibility of measuring the virtual photon structure function, which is of significant theoretical interest.
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