Twisted Dust Acoustic Waves in Dusty Plasmas

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Abstract

We examine linear dust acoustic waves (DAWs) in a dusty plasma with strongly correlated dust grains, and discuss possibility of a twisted DA vortex beam carrying orbital angular momentum (OAM). For our purposes, we use the Boltzmann distributed electron and ion density perturbations, the dust continuity and generalized viscoelastic dust momentum equations, and Poisson’s equation to obtain a dispersion relation for the modified DAWs. The effects of the polarization force, strong dust couplings, and dust charge fluctuations on the DAW spectrum are examined. Furthermore, we demonstrate that the DAW can propagate as a twisted vortex beam carrying OAM. A twisted DA vortex structure can trap and transport dust particles in dusty plasmas.

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I. INTRODUCTION

Charged dust grains \([1]\) and dusty plasmas \([2]\) are ubiquitous in cosmic and astrophysical environments \([3–5]\), such as interstellar media, molecular dusty clouds, star forming dust clouds, Eagle nebula, and supernovae remnants, etc. They are also found in our solar system, e.g. in planetary rings systems \([6–8]\), in interplanetary media \([9]\) due to the presence of cometary dust particles, on the Martian surface as dust devils, on the surface of Sun and on moon, as well as in the Earth’s mesosphere \([10,12]\), in space as charged dust debris \([13]\) produced when satellites are destructed, and near space propulsion vehicles \([14]\) [for future spacecrafts that go nearer to the Sun (such as Solar Orbital and Solar Probe Plus) and Lunar Atmosphere and Dust Environment Explorer mission to the moon] for exploring composition of dust grains and their role in collective dust-plasma interactions) due to rocket exhausts. Charged dust particles, which could be of different sizes (ranging from micron-sized to nanometer sized), are naturally formed in industrial processing \([15]\) for nanotechnology and in magnetic fusion reactors \([16]\). Furthermore, low-temperature dusty plasmas are also produced in laboratory devices \([17]\) for fundamental studies in a new environment (e.g. on board International Space Station for examining the behaviour of dusty plasmas under microgravity conditions \([18]\)) that does not exist in the usual electron-ion plasma without dust. It emerges that dusty plasmas are of broad interdisciplinary interest in physical sciences, and share ideas with other fields, e.g. condensed matter physics and astrophysics. Furthermore, dusty plasmas are also finding applications in medical and biological sciences \([19]\).

A neutral dust particle in plasmas is charged both negative and positive due to a variety of physical processes \([20–25]\), including absorption of electrons from the background plasma \([20]\), photo emissions \([12]\), triboelectric effects \([25]\), etc. A dusty plasma is usually composed of electrons, positive ions, negative or positive dust grains, and neutral atoms. When the interaction potential energy \((= Z_d^2 e^2 / d\), where \(Z_d\) is the dust charge state, \(e\) the magnitude of the electron charge, and \(d\) the inter-grain spacing or the Wigner-Seitz radius) between two neighboring dust particles is much larger (smaller) than the dust kinetic energy \(k_B T_d\), where \(k_B\) is the Boltzmann constant and \(T_d\) the dust particle temperature, the dusty plasma is in a strongly (weakly) coupled state.

More than half a century ago, Wuerker et al. \([26]\) showed that electrically charged iron and
aluminium particles having diameters of a few micrometers can be contained in a confined space by alternating and dc electric fields. Under the three-dimensional focusing alternating gradient focusing force and the Coulomb repulsive force, charged iron and aluminium particles form a crystalline array, which can be melted and reformed. This seems to be the first indication of ordered dust particle structures in an external confined potential. However, following Ichimaru’s idea [27] of one-component strongly coupled electron system, Ikezi [28] postulated the solidification of charged dust particles in a dusty plasma [2, 38–41], when the dusty plasma $\Gamma = Z_d^2 e^2 / d k_B T_d$ is close to 172. Such values of $\Gamma$ can be achieved in low-temperature laboratory dusty plasma discharges at room temperatures owing to the large $Z_d$ a micron-size dust grain would acquire by absorbing electrons from the background plasma. The formation of dust Coulomb crystals and ordered dust particle structures have since been observed in the sheath region of many laboratory experiments [29–34]. The ordered dust particle structures are attributed to attractive forces [35, 36, 41] between negative dust grains due to ion focusing and ion wakefields in a dusty plasma sheath with streaming ions, as well as due to overlapping Debye spheres [37] and dipole-dipole interactions [2].

The collective behavior of dusty plasmas involving an ensembles of charged particles was recognized through the prediction of the dust acoustic wave (DAW) by Shukla [42] at the First Capri Workshop on Dusty Plasmas in May of 1989, where he suggested the existence of the nonlinear DAW in the presence of Boltzmann distributed electrons and ions, and massive charged dust particles. Shukla’s idea was then worked out in the first paper [43] on linear and nonlinear DAWs. It must be stressed that there does not exist a counterpart of the DAW in an electron-ion plasma without charged dust grains, since the DAW is supported by the dust particle inertia, and the restoring force comes from the pressures of the inertialess Boltzmann distributed electrons and ions. Thus, similar to the Alfvén wave in an electron-ion magnetoplasma without dust, the DAW is of fundamental importance in laboratory and space plasmas. The DAW is usually excited by an ion streaming instability [45, 46], and has a frequency much smaller than the dust plasma frequency, extending into the infra-sonic frequency range when the dust particles are anomalously heated. The low-frequency (of the order of 10 Hz) DA fluctuations were first indirectly observed in the experiment of Chu et al. [29] prior to dust particle crystallization, and have since been spectacularly observed in many laboratory experiments world-wide [47, 49], and also in the Earth’s ionosphere [50]. Thus, the existence of the DAW in dusty plasmas have been demonstrated at kinetic levels,
and the visual images of the dust acoustic wavefronts by naked eyes are possible.

In this paper, we revisit the linear DAW in a dusty plasma by incorporating the effects of dust particle correlations, the polarization force due to interactions between thermal ions and highly charged dust grains, and dust charge fluctuations (DCFs). A linear dispersion relation is derived and analyzed. The underlying physics of the DAW has been put on the firm footing. Furthermore, we also discuss the possibility of a twisted dust acoustic wave (TDAW) carrying OAM [we also refer it as a dust acoustic vortex (DAV) beam]. A TDAW or a DAV beam can, in turn, be used for trapping and transporting charged dust grains from one region to another in laboratory and space dusty plasmas.

II. THEORETICAL CONSIDERATION

Let us consider an unmagnetized dusty plasma composed of inertialess electrons and ions, as well as strongly correlated negative dust particles of uniform sizes. In the presence of ultra-low frequency DAWs, with \( \omega \ll \nu_{en}, \nu_{in} \ll k^2V_{Te,Ti}^2/|\omega| \), where \( \omega \) is the wave frequency, \( \nu_{en} (\nu_{in}) \) the electron (ion)-neutral collision frequency, \( k \) the wave number, and \( V_{Te} (V_{Ti}) \) the electron (ion) thermal speed, both electrons and ions obey the Boltzmann law (deduced from the balance of the electric force and pressure gradients of the electrons and ions), since they can be considered inertialess on the timescale of the DAW period, and rapidly thermalize due to collisions with neutrals. Thus, the electron and ion number density perturbations \( (n_{e1,i1} \ll n_{e0,i0}) \) are, respectively, given by

\[
n_{e1} \approx n_{e0} \frac{e^2}{k_B T_e}, \tag{1}
\]

and

\[
n_{i1} \approx -n_{i0} \frac{e^2}{k_B T_i}, \tag{2}
\]

where \( n_{e0} \) and \( n_{i0} \) are the unperturbed electron and ion number densities, respectively, \( \phi \) the electrostatic potential of the DAW, and \( T_e (T_i) \) the electron (ion) temperature. At equilibrium, we have the quasi-neutrality condition, viz. \( n_{i0} = n_{e0} + Z dn_{d0} \), where \( Z_{d0} \) is the average number of electrons residing on a dust grain, and \( n_{d0} \) the unperturbed dust number density.

The dynamics of dust particles in a dusty plasma is governed by the hydrodynamic...
equations composed of the dust continuity equation
\[
\frac{\partial n_{d1}}{\partial t} + n_{d0} \nabla \cdot \mathbf{v}_d = 0,
\] (3)
and the generalized dust fluid momentum equation
\[
\left(1 + \tau_m \frac{\partial}{\partial t}\right) \left[\frac{\partial \mathbf{v}_d}{\partial t} + \nu_d \mathbf{v}_d - \frac{Z_{d0} e}{m_d} (1 - R) \nabla \phi + \frac{\mu_d k_B T_d}{\rho_d} \nabla n_{d1}\right] = \frac{\eta}{\rho_d} \nabla^2 \mathbf{v}_d + \frac{\xi}{\rho_d} \nabla (\nabla \cdot \mathbf{v}_d),
\] (4)
where \(n_{d1} \ll n_{d0}\) and \(\mathbf{v}_d\) are the dust number density and dust fluid velocity perturbations, respectively, \(m_d\) the dust mass, \(\rho_d = n_{d0} m_d\) the dust mass density, \(R = Z_{d0} e^2 / 4 k_B T_i \lambda_{Di}\) is a parameter determining the effect of the polarization force \(\overline{52}\) that arises due to the interaction between thermal ions and negative dust grains, \(\mu_d n_{d0} k_B T_d \equiv P_d\) the effective dust thermal pressure, \(\mu_d = 1 + (1/3) u(\Gamma) + (\Gamma/9) \partial u(\Gamma) / \partial \Gamma\) the compressibility, \(\Gamma = Z_{d0}^2 e^2 / d k_B T_d\) the ratio between the dust Coulomb and dust thermal energies, \(d = (3/4 \pi n_{d0})^{1/3}\) the Wigner-Seitz radius, \(u(\Gamma)\) is a measure of the excess internal energy of the system, which reads \(\overline{58, 59}\) \(u(\Gamma) \simeq -(\sqrt{3}/2) \Gamma^{3/2}\) for \(\Gamma \leq 1\) (viz. a liquid-like state), and \(u(\Gamma) = -0.80 \Gamma + 0.95 \Gamma^{1/4} + 0.19 \Gamma^{-1/4} - 0.81\) in a range \(1 < \Gamma < 200\). The coupling parameter in dusty plasmas including the shielding of a negative dust grain by electrons and ions reads \(\Gamma_g = \Gamma \exp(-\kappa)\), where \(\kappa = a_d / \lambda_D\), \(a_d\) is the inter-dust grain spacing, and \(\lambda_D = \lambda_{De} \lambda_{Di} / (\lambda_{De}^2 + \lambda_{Di}^2)^{1/2}\) the effective Debye radius of dusty plasmas, with \(\lambda_{De} = (k_B T_e / 4 \pi n_{e0} e^2)^{1/2}\) and \(\lambda_{Di} = (k_B T_i / 4 \pi n_{i0} e^2)^{1/2}\) being the ion and electron Debye radii, respectively. The dust-neutral collision frequency is \(\overline{57}\) \(\nu_{dn} = (8/3) \sqrt{2 \pi m_n n_r^2 V_{Tn} / m_d}\), where \(m_n\) is the neutral mass, \(n_n\) the neutral number density, \(r_d\) the dust grain radius, \(V_{Tn} = (k_B T_n / m_n)^{1/2}\) the neutral thermal speed, and \(T_n\) the neutral gas temperature. The visco-elastic properties of the dust fluids are characterized by the relaxation time \(\overline{55, 56}\) \(\tau_m = [(\xi + 4 \eta/3) / n_{d0} T_d] / [1 - \mu_d + 4 u(\Gamma)/15]\), involving the shear and bulk viscosities \(\eta\) and \(\xi\). There are various approaches for calculating \(\eta\) and \(\xi\), which are widely discussed in the literature \(\overline{59}\). We note that the generalized viscoelastic dust momentum Eq. (4) is an extension of Kaw and Sen \(\overline{51}\), by including the effects of the polarization force \(\overline{52}\). The viscoelastic momentum equation, similar to Eq. (4), has also been used in the study of collective phenomena in fluids \(\overline{54}\) and in one-component plasmas with strongly correlated electrons \(\overline{55, 56}\).

The DA wave potential \(\phi\) is obtained from Poisson’s equation
\[
\nabla^2 \phi = 4 \pi e \left(n_{e1} - n_{i1} + Z_{d0} n_{d1} + Z_{d1} n_{d0}\right),
\] (5)
where the dust charge perturbation \(Z_{d1}(\ll Z_{d0})\) is determined from the charging equation

\[
\left( \frac{\partial}{\partial t} + \nu_1 \right) Z_{d1} = \frac{a_d \nu_2}{e} \phi. \tag{6}
\]

Here we have introduced the notations

\[
\nu_1 = a_d \left[ \frac{\omega_{pe}}{\lambda_{De}} \exp(-\eta_e) + \frac{\omega_{pi}}{\lambda_{Di}} \right], \tag{7}
\]

and

\[
\nu_2 = a_d \left[ \frac{\omega_{pe}}{\lambda_{De}} \exp(-\eta_e) + (1 + \eta_i) \frac{\omega_{pi}}{\lambda_{Di}} \right], \tag{8}
\]

with \(\eta_{e,i} = Z_{d0} e^2 / a_d k_B T_{e,i}\). Equation (6) reveals that the DCFs cause an adverse phase lag between \(Z_{d1}\) and \(\phi\), which lead to the DAW damping \([5,3]\). However, since the DAW frequency is much smaller that the dust charging frequency \(\nu_1\), one notices that \(Z_{d1}\) and \(\phi\) are in phase. Subsequently, there appears a decrease of the DAW phase speed, as shown in the subsection below.

From Eqs. (1), (2), (5) and (6) we readily obtain

\[
\left[ \left( \frac{\partial}{\partial t} + \nu_1 \right) (\nabla^2 - k_D^2) - k_q^2 \nu_2 \right] \phi = 4\pi e Z_{d0} (\frac{\partial}{\partial t} + \nu_1) n_{d1}, \tag{9}
\]

where \(k_D = (k_e^2 + k_i^2)^{1/2} \equiv 1/\lambda_D\), \(k_{e,i} = 1/\lambda_{De,Di}\), and \(k_q = (4\pi a_d n_{d0})^{1/2}\).

Furthermore, from Eqs. (3) and (4) we have

\[
\left( 1 + \tau_m \frac{\partial}{\partial t} \right) \left[ \frac{\partial^2}{\partial t^2} - V_{Td}^2 \nabla^2 \right] n_{d1} + \frac{Z_{d0} e (1 - R)}{m_d} \nabla^2 \phi = - \left[ \nu_d - \frac{(\xi + 4\eta_3/3)}{\rho_d} \right] \frac{\partial n_{d1}}{\partial t}. \tag{10}
\]

where \(V_{Td} = (\mu_d k_B T_d / m_d)^{1/2}\) is the effective dust thermal speed. Equations (9) and (10) are the desired equations governing the linear propagation of the modified DAWs.

### A. Modified DAWs

Within the framework of a plane-wave approximation, assuming that \(n_{d1}\) and \(\phi\) are proportional to \(\exp(-i\omega t + ik \cdot r)\), we Fourier analyze (9) and (10) and combine the resultant equations to obtain the dispersion relation for the modified DAWs

\[
1 + k^2 \lambda_D^2 + \frac{k^2 \lambda_D^2 \nu_2}{(\nu_1 - i\omega)} - \frac{\omega_d^2}{\omega(\omega + i\nu_d) - k^2 V_{Td}^2 + i\omega \nu_d} = 0, \tag{11}
\]
where \( \omega_d^2 = k^2 C_d^2 (1 - R) > 0 \), \( C_d = \omega_p d \lambda_D \), \( \omega_v = \eta_s k^2 / (1 - i \omega \tau_m) \), and \( \eta_s = (\xi + 4 \eta / 3) / m_d n_{d0} \).

Several comments are in order. First, for \( \nu_d \ll |\omega| \ll 1 / \tau_m \), \( \nu_1 \), we have from (11)

\[
\omega^2 = k^2 U_d^2 + \frac{\omega_d^2}{1 + (k^2 + k^2 \nu_2 / \nu_1) \lambda_D^2},
\]

where \( U_d^2 = V_T^2 d + \eta^* k^2 / \tau_m \).

Second, in the long wavelength limit, viz. \( k^2 \lambda_D^2 \ll 1 \), Eq. (12) reduces to

\[
\omega^2 = k^2 U_d^2 + \frac{\omega_d^2}{(1 + k^2 \lambda_D^2 \nu_2 / \nu_1)}.
\]

Third, the well-known frequency of the DAW, in the absence of dust grain correlations, dust fluid viscosities, dust-neutral collisions, polarization and ion pressure effects, and DCFs, can be obtained from Eq. (11). We have the famous result [43]

\[
\omega = \frac{k C_d}{(1 + k^2 \lambda_D^2)^{1/2}}.
\]

Since \( C_d = Z_{d0} [n_{d0} k_B T_i / n_{i0} m_d (1 + \alpha_1)]^{1/2} \), where \( \alpha_1 = n_{e0} T_i / n_{i0} T_e \), we observe from Eq. (14) that the phase speed \( (\omega/k) \) of the long wavelength (in comparison with \( \lambda_D \)) DAWs is proportional to \( [k_B T_i / m_d (1 + \alpha_1)]^{1/2} \), dictating that the restoring force in the DAW comes from the pressures of the inertialess Boltzmann distributed electron and ion fluids, while the dust mass provides the inertia to sustain the wave. The wave dispersion [the \( k^2 \lambda_D^2 \)-term in Eq. (14)] arises owing to the departure from the quasi-neutrality condition in the perturbed density perturbations.

**B. Twisted DAWs**

We now discuss the possibility of a twisted DAV beam in the long-wavelength limit (viz. \( k^2 \lambda_D^2 \ll 1 \)), in which case the quasi-neutrality holds. Here we have

\[
n_{d1} = - \frac{n_{i0}}{Z_{d0} k_B T_i} (1 + \alpha_1 + \alpha_2) \phi,
\]

where \( \alpha_2 = a_d \nu_2 k_B T_i n_{d0} / \nu_1 e^2 n_{i0} \). Equation (15) depicts that the dust density compression is possible, since \( \phi < 0 \) (due to negative charge on dust grains) for the DAWs.

In order to study the property of a TDAW, we consider the limit \( \lambda_D^2 \nabla^2 \phi \ll \phi \) in Eq. (9) and combine the resultant equation with Eq. (10) to obtain the simple wave equation

\[
(\nabla^2 + K^2) n_{d1} = 0,
\]

7
where we have assumed that $n_{d1}$ is proportional to $\exp(-i\omega t)$, and have denoted $K^2 = \omega^2/U_*^2$ with $U_*^2 = V_{td}^2 + C_d^2(1 - R)/(1 + k_*^2\lambda_D^2\nu_2/\nu_1)$. We have assumed that $k^2\lambda_D^2 \ll 1$ and $\nu_d, \eta_*k^2 \ll |\omega| \ll 1/\tau_m, \nu_1$.

We now seek a solution of Eq. (16) in the form

$$n_{d1} = n_t(r) \exp(ikz), \quad (17)$$

where $n_t(r)$ is a slowly varying function of $z$. Here $r = (x^2 + y^2)^{1/2}$ and $k$ is the propagation wave number along the axial ($z-$) direction. By using Eq. (17) one can write Eq. (16) in a paraxial approximation as

$$\left( 2ik \frac{\partial}{\partial z} + \nabla^2_\perp \right) n_t = 0, \quad (18)$$

where $k = K \equiv \omega/U_*$ and $\nabla^2_\perp = (1/r)(\partial/\partial r)(r\partial/\partial r) + (1/r^2)\partial^2/\partial \theta^2$. We have used cylindrical coordinates with $r = (r, \theta, z)$.

The solution of Eq. (18) can be written as a superposition of Laguerre-Gaussian (LG) modes [60], each of them representing a state of orbital angular momentum, characterized by the quantum number $l$, such that

$$n_t = \sum_{pl} n_{pl} F_{pl}(r, z) \exp(il\theta), \quad (19)$$

where the mode structure function is

$$F_{pl}(r, z) = F_{pl} X^{|l|} L_p^{|l|} (X) \exp(-X/2), \quad (20)$$

with $X = r^2/w^2(z)$, and $w(z)$ is the DA beam width. The normalization factor $F_{pl}$ and the associated Laguerre polynomial $L_p^{|l|}(x)$ are, respectively,

$$F_{pl} = \frac{1}{2\sqrt{\pi}} \left[ (l + p)! \right]^{1/2}, \quad (21)$$

and

$$L_p^{|l|}(X) = \frac{\exp(X)}{X^{|l|} p!} \frac{d^p}{dX^p} \left[ X^{l+p} \exp(-X) \right], \quad (22)$$

where $p$ and $l$ are the radial and angular mode numbers of the DAW orbital angular momentum state. In a special case with $l = 0$ and $p = 0$, we have a Gaussian beam.

The LG solutions, given by Eq. (19), describe the feature of a twisted DAV beam carrying OAM. In a twisted DAV beam, the wavefront would rotate around the beam’s propagation
direction in a spiral that looks like fusilli pasta (or a bit like a DNA double helix), creating a vortex and leading to the DAV beam with zero intensity at its center. A twisted DAV beam can be created with the help of two oppositely propagating three-dimensional DAWs that are colliding in a dusty plasma. Twisting of the DAWs would occur because different sections of the wavefront would bounce off different steps, introducing a delay between the reflection of neighboring sections and, therefore, causing the wavefront to be twisted due to an entanglement of the wavefronts, and take on the shape of the reflector. Thus, due to angular symmetry, Noether theorem guards OAM conservation even for a longitudinal DAV beam.

The energy flux of the DAV beam is given by \( W \mathbf{V}_g \), where \( \mathbf{V}_g = \frac{\partial \omega}{\partial \mathbf{k}} \) is the group velocity of the DAV beam and its energy density reads \[ W = \frac{\partial}{\partial \omega} \left[ \omega \epsilon(\omega, \mathbf{k}) \right]_{\omega=\omega_k} \frac{|\mathbf{E}|^2}{4\pi} \tag{23} \]

where \( \mathbf{E} = i\mathbf{k}\phi \), and for \( \nu_d, |\omega| \ll 1/\tau_m, \tau_1 \) and \( k^2 \lambda_D^2 \ll 1 \) we have \( \epsilon(\omega, \mathbf{k}) = (1/k^2 \lambda_D^2) + (k^2 \nu_2/k^2 \nu_1) - \omega_p^2(1 - R)/(\omega^2 - k^2 V^2_T d) \). Equation (15) exhibits a relationship between the electrostatic potential \( \phi \) and the dust density perturbation \( n_{d1} \). It turns out that the energy flux is independent of the mode number \( l \).

### III. SUMMARY AND CONCLUSIONS

To summarize, we have presented a new dispersion relation (11) for the DAWs in an unmagnetized dusty plasma that is composed of weakly correlated Boltzmann distributed electrons and ions in the wave potential, and strongly correlated highly-charged dust grains that follow the viscoelastic dust momentum equation. In our investigation, the effects of the polarization force and DCFs are incorporated. It is found that the contributions of both the polarization force and DCFs are to reduce the frequency of the DAWs. Furthermore, we have discussed the possibility of a twisted DAV beam. The latter can trap charged dust particles and transport them from one region to another. The present study of a twisted DAV beam can be useful for diagnostic purposes when the DAW frequencies are near the infra-sonic frequencies in laboratory, space and cosmic dusty plasmas. In closing, we mention that the importance of OAM of electromagnetic waves in the astrophysical context was recognized by Harwit [62], and is also the subject of current interest [63, 64] in connection with twisted
ultrasound pulses.

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