NNLO versus NLO multi-jet merging for top-pair production including electroweak corrections

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In this proceedings we compare phenomenological predictions for differential distributions in top-quark pair production at the LHC. In particular we consider NNLO QCD fixed-order predictions and parton-level predictions based on NLO QCD multi-jet merging following the MEPS@NLO scheme. In both predictions NLO electroweak (EW) corrections are incorporated in different approximations. We focus on several transverse-momentum distributions and on the top-quark invariant mass distribution, both highly relevant for the ongoing physics program at the LHC. We discuss comparisons between the different considered approximations and their advantages and disadvantages for different distributions.

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1 Introduction

In this proceedings we compare phenomenological predictions for differential distributions in top-quark pair production at the LHC with 13 TeV, using two different approximations: fixed-order NNLO QCD combined with electroweak (EW) corrections at NLO, and multi-jet merging at NLO also incorporating EW corrections. The latter is applicable for predictions at the particle level.

The NNLO QCD predictions for top-pair production are based on the results of Refs. [1, 2], which have been combined with EW corrections in Refs. [3, 4]. Besides the well known NLO EW corrections of relative \(O(\alpha^2)\) [5, 6, 7, 8, 9, 10] these combinations further included all subleading Born and one-loop contributions, first considered in [10] and based on the MadGraph5_aMC@NLO framework [11, 12].

The NLO multi-jet merged predictions are based on the MEPS@NLO framework in Sherpa [13, 14, 15] and EW corrections are incorporated in an approximation integrating out QED radiation effects at the level of the NLO calculation [16]. Predictions for top-pair production merged up to one extra jet at NLO QCD+EW and 2 additional jets at LO in this approximation have been presented in [17], which serves as basis for the MEPS@NLO predictions considered in this proceedings. These predictions will be denoted as MEPS@NLO, while multi-jet merged predictions without EW corrections will be denoted as MEPS@NLO QCD. In Ref. [17] also full fixed-order one-loop predictions have been presented for top-pair and top-pair production in association with an extra jet. It was found that the relative EW corrections behave universally between the two processes, indicating a factorization of QCD and EW higher-order corrections.

We focus on different transverse-momentum \((p_T)\) distributions and on the top-quark invariant mass. In particular, we consider the \(p_T\) of the top quark and anti-quark, \(p_T(t)\) and \(p_T(\bar{t})\), of the leading and trailing top, \(p_T(t_1)\) and \(p_T(t_2)\), and their average value at the histogram level, \(p_T,avt \equiv (p_T(t) + p_T(\bar{t}))/2 = (p_T(t_1) + p_T(t_2))/2\).

2 Input parameters and scales

In order to exclude other possible sources of discrepancies in the comparison, we have used for both the approximations the same input parameters, which we conventionally set equal to those of Ref. [3]. Also the same PDF set NNPDF3.1luxQED [18] has been used, which includes a photon density à la LUXqed [19, 20]. Although the calculation of Ref. [17] does not include photon-induced contributions, it has been shown in Ref. [3] that with this set of PDFs they are at the permille level and therefore negligible for the comparison in this work.

For what concerns the choice of the central value of the factorisation and renormalisation scales, different choices, both physically motivated, are employed in the two different calculations. In the case of NNLO predictions their central values \(\mu\) depend on the distribution
that is considered,

\[ \mu = \frac{m_T(t)}{2} \text{ for the } p_T(t) \text{ distribution}, \quad (1) \]

\[ \mu = \frac{m_T(\bar{t})}{2} \text{ for the } p_T(\bar{t}) \text{ distribution}, \quad (2) \]

\[ \mu = \frac{H_T}{4} = \frac{1}{4} \left( m_T(t) + m_T(\bar{t}) \right) \text{ for all other distributions}, \quad (3) \]

and have been identified in Ref. [21] via the “Principle of Fastest Convergence”. Instead, in the multi-jet merging approximation, a CKKW scale [22, 23] has been used, with a scale \( \mu_{\text{core}} \) for the \( pp \rightarrow t\bar{t} \) hard process set equal to

\[ \mu_{\text{core}} = \frac{1}{2} \left( \frac{1}{s} + \frac{1}{m_T^2 - t} + \frac{1}{m_T^2 - \bar{u}} \right)^{-\frac{1}{2}}, \quad (4) \]

which in practice weights the contributions of the different colour flows involved in the process [24, 25].

It is very interesting to notice that

\[ \text{for } p_T(t) \rightarrow \infty \quad \mu_{\text{core}} \Rightarrow \frac{1}{2} \sqrt[4]{\frac{5}{4}} p_T \sim \frac{p_T}{2}, \quad (5) \]

\[ \text{for } E(t) \rightarrow \infty \text{ and } p_T(t)/E(t) \rightarrow 0 \quad \mu_{\text{core}} \Rightarrow \frac{m_T}{2} \sim \frac{H_T}{4}, \quad (6) \]

\[ \text{for } E(t) \rightarrow 0 \quad \mu_{\text{core}} \Rightarrow \frac{1}{2} \sqrt[4]{\frac{5}{4}} m_t \sim \frac{H_T}{4}, \quad (7) \]

where \( E(t) \) is the energy of the top quark. In other words, even though the scale choice in the two different calculations have been identified on the basis of very different principles, they are very close in the full phase-space: in the boosted regime (5), at high \( m(t\bar{t}) \) (6), and at the threshold (7). Moreover, (5)-(7) suggest that the observable-dependent scale choices in (1)-(3) can actually be derived from a single definition at the fully differential level.

### 3 Results

In Fig. 1 we compare NNLO and MEPS@NLO predictions for \( p_T(t) \) and \( p_T, \text{av}_t \) distributions. In the first inset we show the scale uncertainties† for the two different approximations, both normalised over the central value of the NNLO one. As can be seen, besides the first bin, for both distributions the uncertainty bands from the two different predictions overlap. 

As expected, the scale uncertainty at NNLO is much smaller than in the MEPS@NLO predictions. The information about the EW corrections is instead displayed in the second inset. We show, for both NNLO and MEPS@NLO, the ratio of their predictions with and

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*At high \( m(t\bar{t}) \), top (anti)quarks are mainly produced in the peripheral region, due to \( t- \) and \( u- \)channel diagrams, therefore \( p_T(t)/E(t) \rightarrow 0 \).

†At NNLO we have used 7-point variation, while at MEPS@NLO the 9-point variation. However, the off-diagonal values are within the uncertainty band and thus this difference has no impact.
without including EW contributions, at their central scales. Moreover, for the NNLO, we consider also the case in which we have combined QCD and EW approach in the additive approach, see Ref. [3] for details. These plots further support the multiplicative approach, since the MEPS@NLO value is much closer to the NNLO in the multiplicative approach than in the additive one.

Using the same layout, in Fig. 2 we compare NNLO and MEPS@NLO predictions for $p_T(t_1)$ and $p_T(t_2)$ distributions. As can be seen, there are large discrepancies between the two approximations. The reason is that these two distributions are pathological at fixed order. Indeed, since $p_T(t_1) > p_T(t_2)$, with fixed order calculations a jet-veto on additional radiation is indirectly applied at small values of $p_T(t_1)$ and leads to large uncontrolled terms for $p_T(t_1) \lesssim m_t$. On the contrary, adding shower effects and thus multiple radiations, this effect is automatically cured in MEPS@NLO. The same argument applies for $p_T(t_2) \gtrsim m_t$. Comparing the impact of the electroweak corrections is therefore not meaningful in these phase-space regions. Outside these two regions ($p_T(t_1) \gtrsim m_t$ or $p_T(t_2) \lesssim m_t$), where fixed-order calculations are reliable, the two calculations are again compatible and NNLO scale uncertainty is much smaller than at MEPS@NLO. Moreover, as can be seen in Fig. 2, the impact of electroweak corrections in MEPS@NLO and NNLO (in the multiplicative approach) is again very similar. We remark that averaging the distributions in the plots on the left and the right in Fig. 2 the $p_T,av_t$ distributions of Fig. 1 is obtained, where all these issues are not present.

In the left plot in Fig. 3 we show the same kind of plot for the $m(t\bar{t})$ distribution. As can be seen, in this case the agreement is far to be perfect, especially in the tail of the distribution. We have checked that this is not due to the particular procedure that is used in the NLO merging in MEPS@NLO by comparing the purely QCD predictions with the one obtained with FxFx [26], which also provides NLO-merged predictions with a different procedure. The plot on the right of Fig. 3 shows this comparison. We have also verified that
Figure 2: Comparison between MEPS@NLO and NNLO predictions for the $p_T(t_1)$ and $p_T(t_2)$.

setting the scales exactly to the same values in the NNLO and MEPS@NLO approaches reduces but does not completely eliminate this tension. Further work is therefore necessary in order to fully understand the origin of this discrepancy between the two approaches. Both approaches correctly take into account the contributions from one hard real emission at one-loop and two hard real emissions at tree level; a possible origin of the discrepancy may be due to the missing two-loop terms in the MEPS@NLO and/or the missing shower effects in the NNLO.

4 Conclusions and Outlook

In this proceeding we have compared two different approximations for phenomenological predictions of top-quark pair production at the LHC: NNLO QCD combined with EW corrections at NLO, and MEPS@NLO multi-jet merging at NLO also including EW corrections. We have considered different distributions at 13 TeV: $p_T(t)$, $p_T(t_1)$, $p_T(t_2)$, and $m(t\bar{t})$. In the case of $p_T(t)$ and $p_T(t_{av})$, the two approximations are compatible and, as expected, NNLO predictions have much smaller scale uncertainties. Thus, this approximation is more suitable for precision studies on parton-level distributions. On the other hand, fixed-order calculations can be pathological and indeed this is the case for $p_T(t_1)$ and $p_T(t_2)$ distributions in the regions $p_T(t_1) \lesssim m_t$ and $p_T(t_2) \gtrsim m_t$, respectively, where the MEPS@NLO is superior to fixed-order calculations.

We have also shown that the relative effects induced by EW corrections in MEPS@NLO is much closer to those observed at NNLO with EW corrections combined in the multiplicative approach than in the case when they are simply added (additive approach).

For the specific case of $m(t\bar{t})$ a tension between the two approaches is present, especially
in the far tail, therefore further work is necessary in order to fully understand its origin. Additional plots regarding this aspect can be found at the repository:

http://www.precision.hep.phy.cam.ac.uk/results/ttbar-nnloqcd-nloew/

We conclude that an NNLO+PS calculation would be desirable for precise predictions in the full phase-space and especially for future studies. For this purpose, we remark the relevance of EW corrections and we have further supported the superiority of the multiplicative approach for their combination with those from QCD origin, especially in the boosted regime.

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