Optomechanical Metamaterials: Dirac polaritons, Gauge fields, and Instabilities

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Freestanding photonic crystals can be used to trap both light and mechanical vibrations. These "optomechanical crystal" structures have already been experimentally demonstrated to yield strong coupling between a photon mode and a phonon mode, co-localized at a single defect site. Future devices may feature a regular superlattice of such defects, turning them into "optomechanical arrays". In this letter we predict that tailoring the optomechanical band structure of such arrays can be used to implement Dirac physics of photons and phonons, to create a photonic gauge field via mechanical vibrations, and to observe a novel optomechanical instability.

Studies of light interacting with nanomechanical motion have progressed rapidly in recent years. Current successes in the field of cavity optomechanics include the radiative cooling of a nanomechanical mode to the quantum ground state [1, 2], strong coupling physics [3], state transfer [4, 5], radiation-mechanics entanglement [6], and many more (for a review see [7]). Recently, a new frontier is opening up: first steps have been taken towards exploring setups with more optical and vibrational modes, e.g. by coupling two mechanical or optical modes to investigate issues such as synchronization [8, 9], Brillouin cooling [10], phonon lasing [11], or wavelength conversion [12, 13]. A setup particularly well suited for this avenue consists in so-called optomechanical crystals [1, 14–18]. A superarray of such defects (an "optomechanical array") is the next logical step forward in this development. First theoretical studies have indicated that these arrays could exhibit functionalities such as slow light [19], quantum information processing [20], synchronization [21,23] and quantum many-body physics [24,26]. In the present letter, we predict novel features that can be obtained by engineering the optomechanical band structure of these arrays, creating, in effect, optomechanical metamaterials with tailored properties.

We consider a 1D or 2D lattice of identical optomechanical cells, comprising a localized photon mode \( \hat{a}_j \) and a phonon mode \( \hat{b}_j \):

\[
\hat{H}_J = \frac{\Omega}{\hbar} \hat{b}_j^\dagger \hat{b}_j - \Delta \hat{a}_j^\dagger \hat{a}_j + \alpha L (\hat{a}_j^\dagger \hat{a}_j - \hat{a}_j \hat{a}_j^\dagger) - g_0 (\hat{b}_j^\dagger \hat{b}_j + \hat{b}_j \hat{b}_j^\dagger) \hat{a}_j^\dagger \hat{a}_j. \tag{1}
\]

Here, \( \Omega \) is the phonon mode eigenfrequency, and a laser of amplitude \( \alpha L \) and detuning \( \Delta \equiv \omega_L - \omega_{\text{phot}} \) drives uniformly all the cells. The overlap between the evanescent tails of the localized modes leads to tunneling of photons and phonons, which we describe in a tight-binding model (see also [21,22,23,28]). This is similar in spirit to the Hubbard model (or, even more closely, the Holstein model), but for photons and phonons. Our formulas will be general, but in concrete examples we will assume nearest-neighbor hopping \( \hat{H}_\text{hop} = -\hbar \sum_{\langle ij \rangle} J \hat{a}_i^\dagger \hat{a}_j + K \hat{b}_i^\dagger \hat{b}_j + h.c. \). Then the full Hamiltonian reads \( \hat{H} = \hat{H}_S + \hat{H}_\text{hop} + \hat{H}_\text{diss} \), where \( \hat{H}_S = \sum_j \hat{H}_j + \hat{H}_\text{hop} \) and \( \hat{H}_\text{diss} \) describes the coupling to the environment, including photon (phonon) decay at rate \( \kappa \) (\( \Gamma \)).

For laser driving below a finite threshold (see below), the light amplitudes \( \alpha \) and mechanical displacements \( \beta = (\hat{b}_j) \) at all cells reach a uniform steady state, which leads to a shifted detuning \( \tilde{\Delta} \) (see Supplemental Material [29]). We linearize the dynamics around the steady solution, which is an excellent approximation [7] for current values of \( g_0 \). In a plane-wave basis one finds

\[
\hat{H}_S \approx \hbar \sum_k \Omega_k \hat{b}_k^\dagger \hat{b}_k - \Delta_k \hat{a}_k^\dagger \hat{a}_k - g (\hat{a}_k^\dagger + \hat{a}_{-k}) (\hat{b}_k^\dagger + \hat{b}_{-k}). \tag{2}
\]

Here, \( \hat{a}_k \) and \( \hat{b}_k \) are the photonic and phononic Bloch

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Figure 1: Two-dimensional Optomechanical Array: (a) A patterned optomechanical crystal slab supports localized photonic and phononic defect modes with on-site optomechanical interaction. The defects form a 2D superlattice, in which neighboring sites couple via photon and phonon tunneling, and the optical modes are driven by a single laser beam with frequency \( \omega_L \). (b) Light will be back-reflected from the structure into various momenta whose projections onto the plane differ by reciprocal lattice vectors. The arrows indicate elastic scattering of the pump light. In addition, around each elastic scattering peak, there will be inelastically scattered signals due to the optomechanical interaction (schematically indicated by the blue and red dots). These reveal the optomechanical band structure, as discussed in the main text.
modes, where $\hat{a}_k = N^{-1/2} \sum_j e^{-ikr_j} (a_j - \alpha)$ for a Bravais lattice of $N$ sites (likewise for $\hat{b}_k$). We have introduced the band structure of the free phonons $\Omega_k = \Omega - Kf(k)$ (with $f(k) = 2\cos k\alpha$ for a 1D chain of lattice constant $a$) and the photonic dispersion $\Delta_k = \Delta + Jf(k)$. The coupling constant $g = g_0\alpha$ of the linearized interaction is enhanced by the light amplitude $\alpha$. It comprises beam-splitter terms $\hat{a}_k\hat{b}_k^{-\dagger}$ involving Bloch modes with equal quasimomentum and two-mode squeezing terms $\hat{a}_k\hat{b}_k^{-\dagger}$ involving modes with opposite quasimomentum.

Both fluctuations and dissipation can be included via the input-output formalism, which yields the following quantum Langevin equations:

$$\dot{\hat{a}}_k = (i\Delta_k - \kappa/2)\hat{a}_k + ig(\hat{b}_k + \hat{b}_k^{-\dagger}) + \xi_k$$
$$\dot{\hat{b}}_k = (-i\Omega_k - \Gamma/2)\hat{b}_k + ig(\hat{a}_k + \hat{a}_k^{-\dagger}) + \eta_k,$$  

where $\xi_k$ and $\eta_k$ are the noise operators, see Supplemental Material [29].

The optomechanical band structure can now be studied by finding the (complex) eigenfrequencies $\omega(k)$ for the homogeneous part of equations (3). These are the roots of the quartic polynomial

$$[\omega + i\kappa/2 - \Delta_k] [\omega + i\Gamma/2 - \Omega_k] + 4\Delta_k\Omega_k g^2 = 0.$$  

They are complex numbers, where $-2\text{Im}\omega$ represents the intensity decay rate of the corresponding excitation. In the noninteracting case $g = 0$, the four bands describe photon (phonon) particles and holes with eigenfrequencies $\mp\Delta_k - \kappa/2 (\pm\Omega_k - i\Gamma/2)$, respectively. The phonon bands are flat compared to the photon bands, as for typical parameters $K \ll \Omega, J$.

For $|\Delta_k| \ll \Omega$, the mechanical oscillations are faster than the fluctuations of the field amplitude and can be adiabatically eliminated, leading to a two-mode squeezing Hamiltonian

$$\hat{H}_k/\hbar = - (\Delta_k + \hat{g})(\hat{a}_k^{-\dagger}\hat{a}_k + \hat{a}_k^{-\dagger}\hat{a}_k^{-\dagger}) - \hat{g}(\hat{a}_k\hat{a}_k^{-\dagger} + \text{c.c.}),$$

with $\hat{g} = 2g^2/\Omega$. Physically, two drive photons (arriving perpendicularly to the structure, i.e. at in-plane quasimomentum $k = 0$) are converted, via mechanically-mediated four-wave mixing, into a pair of counter-propagating Bloch photons in the array, see sketch in Fig. 2(a). Thus, in a 2D array, there will be entanglement in the momenta. This process then gives rise to an instability (for $4g^2/\kappa\Omega > 1$) towards an optomechanically-induced optical parametric oscillator. Already below threshold, it modifies the optical band in a distinct way, as can be seen in the highlighted region of Fig. 2(b) (gray box), shown as a close-up in Figs. 2(c-d). Above threshold, the array produces beams with opposite quasimomentum, entangled in their quadratures. Their intensity increases smoothly with the laser power while the intensity of the laser-driven pump mode at $k = 0$ saturates. This optical instability does not have any analogue in single-mode optomechanics.

We note that an array version of optomechanical self-oscillations, generated for blue-detuned drive, also exists (Fig. 3(i)), while strong-coupling physics on the red-detuned side leads to the formation of optomechanical polaritons (Fig. 3(i)).

Hitherto, we have described the behavior in simple lattices. We now turn to an optomechanical honeycomb lattice, see the sketch in Fig. 4(a). The tight-binding model for this non-Bravais lattice is well-known for graphene [30], but is now also studied for photonic crystals [31, 32]. The band structure includes special points where the upper and lower bands touch, forming so-called Dirac cones, which are robust, topologically protected structures. There, both the optical and mechanical band are described by a relativistic massless 2D Dirac equation, but with different velocities: fast photons (with velocity $v_O = 3aJ/2$) and slow phonons (velocity $v_M = 3aK/2$). For concreteness, we focus on excitations around the symmetry point $K = 2\pi (3^{-1/2}, 1)/3a$ and consider the quasimomentum $\delta k$ relative to that point. As usual, one can assign a binary degree of freedom $\sigma_\pm = \pm 1$ to excitations on the A/B sublattices and write the Hamiltonian (for either of the bands) as $\hbar v \hat{\sigma}_\pm \delta k = \hbar v (\hat{\sigma}_x \delta k_x + \hat{\sigma}_y \delta k_y)$. To investigate the effects of optomechanical interaction, we fo-
Figure 3: Band structure in the blue- and red-detuned regime, for an optomechanical array: (a-f) Close to the blue detuned sideband, $\Delta_k \approx \Omega$, a drive photon is parametrically converted into a photon-phonon pair with opposite quasimomenta; see Feynman diagram (a). Panels (b) and (c) show the associated frequencies and damping rates in the weak coupling regime $g \ll \kappa$, near the instability. The unstable region, where the damping rates become negative, is marked in yellow. Slightly above the instability threshold, a pair of optical and mechanical cones come close, in the strong sideband, for an optomechanical array: (a-f) Close to the blue detuned momentum space, see the cut panel (h) and the 3D plot panel (i).

Figure 4: Optomechanical Dirac Physics: (a) Sketch of the optomechanical honeycomb lattice. The unit cell contains the optomechanical cells A and B. The corresponding sublattices are generated by discrete translations by the lattice basis vectors $\vec{a}_1$ and $\vec{a}_2$. (b) 3D view of the optomechanical band structure at the red detuned sideband, $\Omega = -\Delta$. (c-d) Close-up close to a symmetry point for the interacting and unperturbed case, respectively. (e) Cut of the band structure for the same parameters ($v_M = 0.1\omega_0$) and (f) for large detuning $|\Delta| - \Omega = 3g$ where an avoided crossing appears. The bands with opposite helicity (plotted in different colors) display exact crossings.

cones shifted in frequency by $\sqrt{\delta \omega^2 + g^2}$. By varying $\delta \omega$ from a large positive value to a large negative one, the velocity of the upper pair of cones smoothly goes from $v_O$ to $v_M$ as the corresponding excitations turn from optical to mechanical. The frequency shift has its minimum value $g$ at the red detuned sideband $\delta \omega = 0$ where both cones describe polaritons with velocity $\bar{v}$, see Fig. 4(e). For large detunings $\delta \omega \gg g$, the cones with same helicity $\vec{\sigma} \cdot \delta \vec{k} / |\delta \vec{k}|$ hybridize away from the symmetry point displaying an avoided crossing for $v_O |\delta \vec{k}| \sim \delta \omega$ (f). Notice that the optomechanical interaction mixes the optical and the mechanical bands but, as in the standard Dirac Hamiltonian, the helicity remains conserved. This symmetry is responsible for the band crossings in Fig. 4.

In finite honeycomb arrays, photon/phonon edge states would appear, depending on the type of edge, as in graphene. A potential landscape can be introduced by modifying locally the properties of the optomechanical cells, which would give rise to effects such as Klein tunneling.

We now show that by adopting a slightly different lattice topology and time-dependent laser driving, one can engineer an artificial magnetic field for the photons inside an optomechanical array. Recently, it has been predicted that an effective gauge field $A_{\text{eff}}$ can be introduced in a photonic square lattice comprising two sublattices A and B by electro-optically modulating the nearest-neighbor coupling constants: $J_{ij} = J \cos(\omega_{\text{ext}} t + \phi_{ij})$ [34]. For resonant modulation, $\omega_{\text{ext}} = \omega_A - \omega_B$, a photon hopping from site $i$ on A to a neighboring site $j$ on B picks up the phase of the modulation $\phi_{ij}$ like a charged particle subjected to the gauge potential $\int_{r_i}^{r_j} A_{\text{eff}} \cdot dr = \phi_{ij}$.
amplitude is front engineering is applied. Suppose the 'carrier' beam injected along wave guides). However, the same effect would make use of laser beams whose intensity is independent on different links. A naive approach for very few cells rally at each link cell, with different modulation phases requires modulating the incoming laser intensity temporally.

In our proposal, the nearest neighbor coupling between the optical sublattices $A$ and $B$ would be mediated by an intermediate lattice $C$; see Fig. 5(a). Driving the optomechanical cells on this lattice $C$ with a sinusoidal time-dependent power gives rise to an oscillating radiation pressure force and thus classical (large-amplitude) oscillations of the mechanical mode at the link: $\beta(t) = \beta \exp(-i\omega_{\text{ext}}t)$. These oscillations weakly modulate the eigenfrequency $\omega_C(t)$ of an optical mode $\hat{a}_C$ at the link: $\omega_C(t) = \omega_C + 2g_0|\beta|\cos(\omega_{\text{ext}}t + \phi)$, $g_0|\beta| \ll \omega_{\text{ext}}$. The effective coupling $J_{\text{eff}}\exp[i\phi]$ between $\hat{a}_A$ and $\hat{a}_B$ is mediated by virtual tunneling through the modulated mode $\hat{a}_C$ (with coupling constants $J_A$ and $J_B$, respectively) accompanied by the exchange of a phonon. Here, $\phi$ is set by the phase of the laser power modulation whereas $J_{\text{eff}} = g_0|\beta|J_AJ_B/[2(\omega_A - \omega_C)(\omega_B - \omega_C)]$.

Creating the artificial magnetic field for the photons requires modulating the incoming laser intensity temporally at each link cell, with different modulation phases on different links. A naive approach for very few cells would make use of laser beams whose intensity is individually controlled (either with tightly focused beams or injected along wave guides). However, the same effect can be obtained with no more than two laser beams for an entire large optomechanical array, provided that wavefront engineering is applied. Suppose the 'carrier' beam amplitude is $E_1 = E_{10}e^{-i\omega_1t}$ and the 'modulation' beam amplitude is $E_2 = E_{20}e^{-i(\omega_2 + \omega_{\text{ext}})t + i\phi(x,y)}$, where $\phi$ is an imprinted phase shift depending on the position within the array plane. Then the intensity, and thus the local radiation pressure force driving the mechanical oscillations, is $|E_{10}|^2 + |E_{20}|^2 + 2\Re[E_{10}E_{20}e^{-(\omega_{\text{ext}}t + \phi(x,y))}]$, which is what was needed. For the resulting mechanical oscillations $\beta(t)$, there will be an extra, but constant and thus irrelevant phase shift depending on the relation between $\omega_{\text{ext}}$ and the mechanical frequency $\Omega$ on the links. For resonant drive, the amplitude $|\beta|$ on the links will be enhanced by the mechanical quality factor, and thus much larger than any spurious amplitude on other cells (which can be chosen off-resonant with $\omega_{\text{ext}}$), which could be further suppressed by engineering the intensity pattern $|E_{20}(x,y)|^2$ as well.

Our proposal demonstrates how optomechanical systems can contribute to the recent efforts in creating gauge fields for photons [32–36]. The resulting optomechanical Hofstadter butterfly fractal level scheme is displayed in Fig. 5 for a square lattice. The photonic magnetic field can be implemented in arbitrary lattices (like the honeycomb lattice) and it could be combined with magnetic fields for phonons, based on [37–38].

The realizability of optomechanical arrays based on optomechanical crystals has been shown by theoretical studies including ab-initio simulations [15,23]. Moreover, nanocavity arrays in photonic crystals have been demonstrated already in experiments [39]. The parameters for the phenomena analyzed here are within experimental reach and we estimate the effects of disorder to be minor for realistic array sizes, see Supplemental Material [29].

In summary, we have shown how to tailor the flow of photons and phonons in an optomechanical metamaterial, whose band structure can be tuned via the driving laser. Future studies could deal with the rich nonlinear dynamics of competing unstable modes, the additional effects brought about by nonlinearities on the single-photon level, the influence of more complex spatial patterns imprinted by the driving laser, or photon/transport phenomena.

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See Supplemental Material at ... for the array stationary light amplitudes and mechanical displacements, the derivation of the optomechanical band structure on a honeycomb lattice, and of the Langevin equations including the resulting noise spectra. Also discussed there is the detailed derivation of the phonon-induced gauge field for photons and the discussion of the experimental observability of the investigated phenomena, including an analysis of the effects of disorder.
Stationary states and Hamiltonian in momentum space

The steady state light amplitude $\alpha = \langle a_i \rangle$ and mechanical displacements $\beta = \langle b_j \rangle$ can be immediately computed by plugging the mean field ansatz in the full nonlinear equations of motion. The resulting equations have the same form as the standard equations for a single optomechanical cell \[1\]

$$\alpha = \alpha_L/|\Delta + 2g_0\alpha - \nu_O + i\kappa/2|, \quad \beta = g_0|\alpha|^2/(\Omega + \nu_M).$$ \hfill (S.8)

As it is well known from the standard case of single-mode optomechanics, the eigenfrequency of the optical mode is shifted by the radiation pressure (in the main text we have incorporated this shift in the effective detuning $\Delta = \Delta + 2g_0\beta$). Moreover, the optical and mechanical eigenfrequencies are shifted by the coupling to the modes in the neighboring cells. The dependence of the corresponding eigenfrequency shifts $\nu_O$ and $\nu_M$ on the hopping term in the Hamiltonian is discussed below. In the main text, we have considered a real positive amplitude $\alpha$. This does not imply any loss of generality as the phase of $\alpha$ can be fixed by a global gauge transformation when all the cells are driven with the same phase. For $|\alpha_L| > [\kappa^2(\Omega + \nu_M)/(6\sqrt{3}g_0^2)]^{1/2}$, there exists a $\Delta$-interval where Eq. (S.8) has three solutions. As it is well known from the standard case of single-mode optomechanics, one of the solutions is always unstable. On the other hand, the stability analysis of the remaining two solutions is specific of the optomechanical metamaterial and is carried out in the main text. Since for appropriate values of the damping rates there are two stable solutions, it is possible to observe a hysteresis of the band structure as a function of the driving parameters ($\Delta$ or $\alpha_L$).

In a Bravais lattice, the stationary solutions depend on the lattice geometry only via the eigenfrequency shifts $\nu_O$ and $\nu_M$. For the nearest neighbor hopping considered in the main text we have $\nu_O = -zJ$ and $\nu_M = -zK$, where $z$ is the coordination number. On the other hand, for a general short range hopping $\hat{H}_{\text{hop}} = -\hbar \sum_{ij} J(i-j)\hat{a}_j^\dagger \hat{a}_i + K(i-j)\hat{b}_j^\dagger \hat{b}_i + \text{c.c.}$, we find $\nu_O = -\sum_r J(r)$ and $\nu_M = -\sum_r K(r)$.

The Hamiltonian in momentum space Eq. (2) of the main text follows directly from the plane-wave ansatz for the Bloch modes and the linearization of the optomechanical interaction $\hat{a}_j^\dagger \hat{a}_j (\hat{b}_j^\dagger + \hat{b}_j) \approx \text{const} + 2\beta \hat{\delta} \hat{a}_j^\dagger \hat{a}_j + \alpha (\hat{d}_j + \hat{d}_j^\dagger)((\hat{\delta} \hat{b}_j + \hat{\delta} \hat{b}_j^\dagger))$ where $\hat{\delta} = \hat{a}_j - \alpha$ and $\hat{\delta} = \hat{b}_j - \beta$.

Optomechanical band structure of an array on a honeycomb lattice

The two sublattices of a honeycomb lattice are mapped onto each other by a rotation by a $\pi$-angle around the center of the unit cell, a transformation belonging to the crystallographic point group, see Fig. 4(a) of the main text. Due to this symmetry, for a driving below a finite threshold, the stationary light amplitudes and mechanical displacements are independent not only of the cell but also of the sublattice, $\alpha = \langle a_{is} \rangle$ and $\beta = \langle b_{is} \rangle$ (index $s$ indicates the sublattice $s = 1, 2$ correspond to sublattice A and B, respectively). In the tight-binding approximation, they are given by Eq. (S.8) with $\nu_O = -3J$ and $\nu_M = -3K$.

Since we describe both photons and phonons in the tight-binding approximation, their Bloch modes have the same wavefunctions $|\psi_s(\vec{k})\rangle e^{i\vec{k}\cdot\vec{r}_s}$ (the eigenfunctions of the tight-binding model of electrons in graphene \[2\]). They are defined by $\delta\hat{a}_{js} = N^{-1/2} \sum_{\vec{k} s'} e^{i\vec{k}\cdot\vec{r}_j} \langle s'|\psi_s(\vec{k})\rangle \hat{a}_{\vec{k}s'}$, $\hat{H}_{\text{hop}} = -\hbar \sum_{\vec{k}s} f_s(\vec{k}) J\hat{a}_{\vec{k}s}^\dagger \hat{a}_{\vec{k}s} + K\hat{b}_{\vec{k}s}^\dagger \hat{b}_{\vec{k}s} + \text{const.}$ and likewise for $\hat{b}_{\vec{k}s}$ ($|s = 1, 2\rangle$ indicate an excitation on sublattice A and B, respectively). From the above definitions and the definition of $\hat{H}_{\text{hop}}$ (in the main text), one immediately derives the eigenvalue problem $f_s(\vec{k})|\psi_s(\vec{k})\rangle = [d'(\vec{k})\hat{\sigma}_x - d''(\vec{k})\hat{\sigma}_y]|\psi_s(\vec{k})\rangle$, where $d'(\vec{k})$ and $d''(\vec{k})$ are the real and imaginary part of $d(\vec{k}) = 1 + e^{i\vec{k}\cdot\vec{a}_1} + e^{i\vec{k}\cdot\vec{a}_2}$ and $\hat{\sigma}$ is a set of Pauli matrices, $\hat{\sigma}_z|s\rangle = (-)^{s+1}|s\rangle$. Notice that the linearized optomechanical interaction is then diagonal on the common Bloch basis of photons and phonons $\sum_{js} (\delta\hat{a}_{js} + \delta\hat{a}_{js}^\dagger)((\hat{\delta} \hat{b}_{js} + \hat{\delta} \hat{b}_{js}^\dagger)) = \sum_{\vec{k}s} (\hat{a}_{\vec{k}s}^\dagger + \hat{a}_{\vec{k}s})((\hat{\delta} \hat{b}_{\vec{k}s} + \hat{\delta} \hat{b}_{\vec{k}s}^\dagger)$.
Hence, equation (4) of the main text for the optomechanical band structure of a Bravais lattice applies also to each band of the honeycomb lattice separately. In this case, \( \Delta_{ks} = \Delta + J f_s(\bar{k}) \) and \( \Omega_{ks} = \Omega - K f_s(\bar{k}) \) with \( \Delta = \Delta + 2g^2/(\Omega - 3K) \) and \( f_s = (-)^s |d(\bar{k})| \).

In the strong coupling limit \( g \gg \kappa \) where dissipative effects are less important and for red detuned driving \(-\Delta \gg g \) where the two-mode squeezing part of the interaction is negligible, the system is described by a particle-conserving second-quantized Hamiltonian. In this regime, we can switch to a single-particle picture introducing the polariton eigenfunctions \( |\psi_\ell(\bar{k})\rangle e^{i\bar{k} \cdot \mathbf{r}_j} \) and eigenenergies \( \hbar \omega_\ell(\bar{k}) \). They are defined by

\[
\delta \hat{\alpha}_{js}(t) \approx N^{-1/2} \sum_{\bar{k} \ell = 1, \ldots, 4} e^{i[\bar{k} \cdot \mathbf{r}_j - \omega_\ell(\bar{k})t]} \langle 1, s |\psi_\ell(\bar{k}) \rangle \hat{c}_{\bar{k} \ell} \\
\delta \hat{b}_{js}(t) \approx N^{-1/2} \sum_{\bar{k} \ell = 1, \ldots, 4} e^{i[\bar{k} \cdot \mathbf{r}_j - \omega_\ell(\bar{k})t]} \langle 2, s |\psi_\ell(\bar{k}) \rangle \hat{c}_{\bar{k} \ell}.
\]

On the left-hand side, \( \delta \hat{\alpha}_{js}(t) \) and \( \delta \hat{b}_{js}(t) \) are solutions of the corresponding Heisenberg equations (the approximation consists in neglecting the two-mode squeezing part of the optomechanical interaction). On the right-hand side, the quantum numbers \(|s', s\rangle\) denotes optical/mechanical excitations \((s' = 1/2)\) on sublattice A/B \((s = 1/2)\). From the Heisenberg equations for \( \delta \hat{\alpha}_{js}(t) \) and \( \delta \hat{b}_{js}(t) \), we derive the time-independent single particle Schrödinger equation \( \hbar \omega_\ell(\bar{k})|\psi_\ell(\bar{k})\rangle = \hat{H}_H|\psi_\ell(\bar{k})\rangle \) where

\[
\hat{H}_H / \hbar = \omega + \delta \hat{\omega}_x/2 - (\bar{v} + \delta \hat{\omega}_x/2)|d^\dagger(\bar{k})\delta_x + d(\bar{k})\delta_y| - g \hat{\tau}_x.
\]

Here, we have introduced the parameters \( \omega = (\Omega + |\Delta|)/2 \), \( \delta \omega = |\Delta| - \Omega \), \( \bar{v} = (v_0 + v_M)/2 \), and \( \delta \bar{v} = v_0 - v_M \), and the set of Pauli matrices \( \hat{\tau}_x, \hat{\tau}_y \) on \(|s', s\rangle = (-)^{s + s'} |s', s\rangle \). The spectrum \( \omega_\ell \) is shown in Fig. 4 of the main text. By expanding \( d(\bar{k}) \) around the symmetry point \( \bar{K} = 2\pi(3^{-1/2}, 1)/3a \) we arrive to the optomechanical Dirac equation \( \hat{H}_D \).

**Generalization to multiband lattices**

The above analysis applies also to any symmetric multiband lattice, that is a lattice formed by sublattices any of which can be mapped to another by a transformation belonging to the crystallographic point group, e.g., an optomechanical array on a Kagome lattice (which has three bands). Due to the symmetry, the stationary light amplitudes and displacements are independent of the sublattice and the cell, \( \alpha = \langle a_{\bar{k}s} \rangle \) and \( \beta = \langle b_{\bar{k}s} \rangle \) \((s = 1, \ldots, M)\). They are given by Eq. (3.8) with \( v_0 = -\sum_{r_s} J_{ss}(r) \) and \( v_M = -\sum_{r_s} K_{ss}(r) \). For nearest neighbor hopping, photons and phonons have the same Bloch wavefunctions and the band index \( s \) is a conserved quantity. Therefore, formula (4) of the main text for the band structure applies to each band separately. The coupling to non-nearest neighbor sites can be regarded as a perturbation introducing a small interaction between bands with different band index \( s \) which leads to additional resonances where the optical-vibrational mixing is enhanced. For a general multiband lattice, the stationary solutions depend on the sublattice and the coupling between different bands is not necessarily small.

**Derivation of the Langevin equations**

The Langevin equations (3) of the main text follow by assuming the weak linear coupling to a Markovian heat bath \( \mathbb{B} \). The latter describes other mechanical and electronic degrees of freedom in the sample (for the phononic Bloch modes \( \hat{b}_s \)) and the electromagnetic field in free space (for the photonic Bloch modes \( \hat{A}_s \)). If we assume an infinitely extended system with discrete translational symmetry in the xy-plane, then the in-plane symmetry under discrete translation by the lattice unit vectors is preserved in the full Hamiltonian \( \hat{H} \) which incorporates also these degrees of freedom. Therefore we will assume that Bloch modes with different quasimomentum are coupled to independent baths. With this assumption, we find the noise correlators \( \langle \hat{\xi}_{k}(t)\hat{\xi}_{k}^\dagger(0) \rangle = \kappa \delta_{k,k'} \delta(t) \) and \( \langle \hat{\eta}_{k}(t)\hat{\eta}_{k}^\dagger(0) \rangle = \Gamma(\bar{n} + 1) \delta_{k,k'} \delta(t) \) and \( \langle \hat{\eta}_{k}(t)\hat{\eta}_{k}(0) \rangle = \Gamma \bar{n} \delta_{k,k'} \delta(t) \). Here \( \bar{n} \) is the bosonic occupation number, \( \bar{n} = (\exp[h\Omega/k_B T] - 1)^{-1} \). In the generic case, the relaxation rates \( \Gamma \) and \( \kappa \) depend on the quasimomentum. Quasimomentum independent relaxation rates correspond to independent fluctuations on different lattice sites.

Notice that the Langevin equations for a general multiband array are symmetric under the substitution \( k \rightarrow -k \). As a consequence, it is useful to consider the Bloch mode amplitudes \( \hat{X}^{(a)}_{k} = (\hat{a}_{k} + \hat{a}^\dagger_{-k})/\sqrt{2} \) and \( \hat{X}^{(b)}_{k} = (\hat{b}_{k} + \hat{b}^\dagger_{-k})/\sqrt{2} \). These amplitudes are actually the plane-wave coefficients of the fields \( \langle \delta \hat{a}_{\bar{k}j} + \delta \hat{a}^\dagger_{\bar{k}j} \rangle/\sqrt{2} \) and \( \langle \delta \hat{b}_{\bar{k}j} + \delta \hat{b}^\dagger_{\bar{k}j} \rangle/\sqrt{2} \). Here, we are interested ultimately in obtaining the eigenfrequencies defining the band structure. These can be obtained from the homogeneous part of the Langevin equations, i.e., the equations that result when taking the average and eliminating the noise terms. Thus, we now consider only the expectation values, \( X^{(a)}_{k} = \langle \hat{X}^{(a)}_{k} \rangle \), likewise for \( X^{(b)}_{k} \). Since the coefficients in the resulting set of coupled linear equations are real-valued, a monochromatic solution with complex eigenfrequency \( \omega_k \) must be accompanied by the complex conjugated solution with eigenfrequency \( -\omega_k^* \) (monochromatic solutions describing overdamped excitations can have purely imaginary eigenfrequencies). In the special case of a Bravais lattice (or a symmetric multi-band lattice with nearest neighbor hopping) the equations of mo-
tion for these quadratures are
\[ \hat{X}^{(a)} = -(\Delta_k^2 + k^2/4)X_k^{(a)} - \kappa \hat{X}^{(a)} - 2\Delta_k g X_k^{(b)}, \]
\[ \hat{X}^{(b)} = -(\Omega_k^2 + \Gamma^2/4)X_k^{(b)} - \Gamma \hat{X}^{(b)} + 2\Omega_k g X_k^{(a)}, \]

(S.9)

In this case, the eigenfrequency spectrum consists of the solutions of Eq. (4) of the main text. A graphical study of the polynomial equation shows that two scenarios are possible: i) either there are two pairs of complex conjugate eigenfrequencies or ii) one pair and two purely imaginary eigenfrequencies.

### Photon emission spectrum

From input output formalism, the intensity of the radiation emitted at frequency \( \omega_k - \omega \) and with in-plane quasimomentum \( k \) is proportional to the noise spectrum \( S(k, \omega) \equiv \int dt \exp[i\omega t]\langle \hat{a}^\dagger_k(t)\hat{a}_k \rangle \). By plugging the solution of the Langevin equations into the definition of \( S(k, \omega) \) and evaluating the correlators of the noise forces, we find

\[ S(k, \omega) = \frac{4\kappa g^4 \Omega^2 + \Gamma g \sigma_M(\omega)}{[N(\omega)]^2} \quad (S.10) \]

in terms of the analytical functions

\[ \sigma_M = g^2 |\chi_\omega(\omega)|^2 \left[ (n + 1)|\chi_M(\omega)|^2 + n|\chi_M(\omega)|^2 \right] \]
\[ N(\omega) = |\chi_\omega(\omega)\chi_M(\omega)\chi_\omega^*(\omega)|^2 + 4g^2 \Delta_k \Omega_k. \]

Here, we have introduced the free susceptibilities \( \chi_\omega(\omega) = \kappa^2 / (\omega + \Delta_k)^{-1} \) and \( \chi_M(\omega) = (\Gamma^2 / (\omega - \Omega_k)^{-1} \). Notice that \( N(\omega) \) coincides with the polynomial, in Eq. (4) of the main text, defining the band structure. When we extend \( S(k, \omega) \), which is defined for real-valued \( \omega \), to the complex \( \omega \)-plane, each eigenfrequency \( \omega_k \) belonging to the band structure is a second order pole of the photon emission spectrum. In the generic situation, the spectrum consists of four Lorentzian peaks whose position and FWHM correspond to the real and twice the imaginary part of the corresponding eigenfrequency \( \omega_k \).

Next, we derive a mapping between the noise correlators of the array and the noise correlators of linearized single-mode optomechanics (i.e. the standard system considered in the literature). We preliminary observe that the Langevin equations (3) of the main text have the same form as the standard Langevin equation for single mode optomechanics

\[ \dot{\hat{a}} = (i\Delta - \kappa/2)\hat{a} + ig(\hat{b} + \hat{b}^\dagger) + \xi \]
\[ \dot{\hat{b}} = (-i\Omega - \Gamma/2)\hat{b} + ig(\hat{a}^\dagger + \hat{a}) + \eta. \quad (S.11) \]

The key difference is that in the equations for the array [(3) of the main text] the pair of optical (mechanical) modes \( \hat{a}_k \) and \( \hat{a}_k^\dagger \ (\hat{b}_k \) and \( \hat{b}_k^\dagger \) that are coupled by the optomechanical interaction are not a hermitian conjugate pair (as the corresponding noise forces \( \xi_k \) and \( \xi_k^\dagger \) are not a hermitian conjugate pair). This difference is not relevant while computing the dynamical form factors as detailed below. Due to momentum conservation, the only non-zero noise correlators are

\[ S_{cd}(k, \omega) = \int dt e^{i\omega t} \langle \hat{c}_k(t)\hat{d}_{-k} \rangle \]

where \( \hat{c}_k, \hat{d}_k \) refer to either of \( \hat{a}_k, \hat{b}_k, \hat{a}_k^\dagger, \hat{b}_k^\dagger \). Since the array modes \( \hat{c}_k, \hat{d}_{-k} \) are governed by the same equations as the corresponding ladder operators \( \hat{c} \) and \( \hat{d} \) of single-mode optomechanics, \( \hat{c}_k(t) (\hat{d}_{-k}) \) is the same function of \( \Delta_k, \Omega_k, \xi_k, \eta_k, \xi_k^\dagger, \xi_{-k}, \eta_{-k}, \xi_k^\dagger, \eta_k^\dagger \) as \( \hat{c} \) \( \hat{d} \) of \( \Delta, \Omega, \xi, \eta, \xi^\dagger \) and \( \eta^\dagger \). For equal phonon bath temperature and decay rates \( \kappa \) and \( \Gamma \), also the correlators of the relevant noise forces coincide. From this, we immediately obtain the mapping between the noise spectra:

\[ S_{cd}(k, \omega) = \int dt e^{i\omega t} \langle \hat{c}(t)\hat{d} \rangle \bigg|_{\Delta=\Delta_k, \Omega=\Omega_k} \]

where the correlator on the right-hand-side is the one calculated for a single-mode optomechanical system. As Kubo formula expresses the susceptibilities as the difference between two noise correlators, a similar mapping obviously applies also to these functions. Hence, an optomechanical metamaterial will display many phenomena known from single-mode optomechanics, e.g. sideband cooling, photon-phonon entanglement, optomechanically induced transparency, etc. [1]. We emphasize though that in optomechanical arrays, phenomena that usually occur for distinct parameter sets can coexist, since \( \Delta_k \) varies in a finite range as a function of the quasimomentum. Note also that the mapping of course does not extend to the full nonlinear dynamics, where a wider range of momenta become mixed.

### Details of the derivation of the optomechanical gauge field for photons

We first consider the dynamics of the subsystem shown in the grey shaded region in Fig. 5 (a). It comprises a cell on each sublattice, A, B, C. This is sufficient to ultimately derive the effective coupling between A and B to leading order in perturbation theory. We denote the corresponding ladder operators by \( \hat{a}_A, \hat{a}_B, \) and \( \hat{a}_C, \) respectively. Since the second-quantized Hamiltonian is particle conserving, it is most convenient to switch to the single-particle picture. This means we solve the Schrödinger equation for the single-particle wave function \( \psi(t) \equiv \langle \alpha_A(t), \alpha_B(t), \alpha_C(t) \rangle \) of an exciton that
The quasi-degenerate quasienergy levels

Formally, we derive the effective Floquet Hamiltonian for the quasienergies \( \psi \) reads \( i\dot{\psi} = H_M \psi \) with

\[
H_M = \begin{pmatrix}
\omega_A & 0 & -J_A \\
0 & \omega_B & -J_B \\
-J_A & -J_B & \omega_C + 2g_0|\beta| \cos(\omega_{\text{ex}}t + \phi)
\end{pmatrix}
\]

The time periodicity ensures that there is a complete set of quasi-periodic solutions of the Schrödinger equation \( \psi_i(t + 2\pi/\omega_{\text{ex}}) = \exp[-i2\pi \omega_i/\omega_{\text{ex}}] \psi_i(t) \), \( i = 1, 2, 3 \). The quasienergies \( \omega_{i,m} \equiv \omega_i + m\Omega \) and the time periodic functions \( \phi_{i,m} \equiv \exp[i(\omega_i + m\Omega)t] \psi_i(t) \) \( m \in \mathbb{Z} \) are eigenvalues and eigenvectors of the Floquet Hamiltonian \( \mathcal{H} \equiv -i\partial_t + H_M \).

We want to compute an effective Schrödinger equation describing the resonant oscillations of couplings with frequency \( \omega_i \sim \omega_A \) on site A and \( \omega_i + \omega_{\text{ex}} \sim \omega_B \) on site B. Formally, we derive the effective Floquet Hamiltonian for the quasi-degenerate quasienergy levels \( \omega_A \) and \( \omega_B + \omega_{\text{ex}} \) (corresponding to the Floquet states \( \phi_{1,0}^{(0)} = (1, 0, 0) \) and \( \phi_{2,1}^{(0)} = (0, 1, 0) \) \( \exp[i\omega_{\text{ex}}t] \)) which incorporates their coupling to leading order in \( J_A/|\omega_A - \omega_C|, J_B/|\omega_B - \omega_C|, g/\omega_{\text{ex}} \). The leading order process consists in a virtual hopping transition to site C, the virtual absorption (emission) of a phonon and a virtual hopping transition from site C, see sketch of the Floquet level scheme in Fig. S6.

We consider the block of the Floquet Hamiltonian which includes only the unperturbed quasienergy levels involved in such a process

\[
\hat{H} = \begin{pmatrix}
\omega_A & 0 & -J_A & 0 \\
0 & \omega_B + \omega_{\text{ex}} & 0 & -J_B \\
-J_A & 0 & \omega_C & g_0|\beta| \\
0 & -J_B & g_0\beta^* & \omega_C + \omega_{\text{ex}}
\end{pmatrix}
\]

Applying third order perturbation theory for quasi-degenerate levels, we arrive at an effective block diagonal Floquet Hamiltonian. The block which describes the dynamics of the light on sites A and B reads

\[
\hat{H}_\text{eff} = \begin{pmatrix}
\tilde{\omega}_A & J_{\text{eff}} e^{-i\phi} \\
J_{\text{eff}} e^{i\phi} & \tilde{\omega}_B + \omega_{\text{ex}}
\end{pmatrix}
\]

with \( \tilde{\omega}_A = \omega_A + J_A^2/(\omega_A - \omega_C), \tilde{\omega}_B = \omega_B + J_B^2/(\omega_B - \omega_C) \) and \( J_{\text{eff}} = g_0|\beta| J_A J_B/(|\omega_A - \omega_C|(|\omega_B - \omega_C|) \). The Floquet Hamiltonian \( \hat{H}_\text{eff} \) is equivalent to the time-dependent Hamiltonian

\[
\hat{H}_\text{eff} = \left( J_{\text{eff}} e^{i(\omega_{\text{ex}}t + \phi)} \right) \tilde{\omega}_B
\]

or in a frame rotating with frequency \( \tilde{\omega}_A \) on site A and \( \tilde{\omega}_B \) on site B to the second-quantized Hamiltonian \( J_{\text{eff}} e^{-i\phi} \tilde{a}_A^\dagger \tilde{a}_B + \text{h.c.} \) (for resonant driving, \( \tilde{\omega}_A = \tilde{\omega}_B + \omega_{\text{ex}} \)). The same procedure can be applied to all nearest neighbors sites leading to an effective hopping term between each nearest neighbors. Hence, by appropriately tuning the driving frequency \( \omega_{\text{ex}} \) and phases \( \phi_{ij} \), we can create the desired gauge field for photons.

In Fig. 5(c) we show how the standard Hofstadter Butterfly is modified in presence of a red-detuned driving on each site (in addition to the driving on the links which creates the effective magnetic field). The resulting steady radiation enhances the optomechanical coupling of each localized optical mode to a mechanical mode on the same cell (with eigenfrequency \( \Omega \)). A equal detuning \( \Delta \) of the driving frequencies \( \omega_L^{(A)} \) and \( \omega_L^{(B)} \) on sublattice A and B, \( \Delta = \omega_L^{(A)} - \omega_A = \omega_L^{(B)} - \omega_B \) creates a monochromatic steady radiation on each cell \( \alpha_i = \langle \tilde{a}_i \rangle \) (in a frame rotating with the driving and within the RWA). Then, the onsite fluctuations \( \delta \tilde{a}_i = \tilde{a}_i - \alpha_i \) hops in the effective magnetic field and are coupled to the corresponding phononic modes by the standard linearized optomechanical interaction with coupling constant \( g_i \propto |\alpha_i| \). In the example in Fig. 5 we have chosen \( g_i = g \). This is a good approximation when the driving amplitude \( \alpha_L \) is the same on all sites and the detuning is larger than the effective coupling \( J_{\text{eff}} \) (then \( \alpha_i \approx \alpha_L/(\Delta + i\kappa/2) \) with a small magnetic-field induced site-dependent correction of order \( \sim \alpha_L J_{\text{eff}}/\Delta \)). Since the driving is red detuned we consider only the beam splitting terms and the spectrum in Fig. 5(c) is computed by diagonalizing the resulting single-particle optomechanical Hamiltonian. In particular, we plot a smeared density of states averaged over a small area \( d\phi \times d\omega \).

Discussion of the experimental realizability

Single-mode optomechanical systems based on a single vibrational and optical defect mode inside an optomechanical crystal have been experimentally realized [6,8]. The reported experimental parameters are very promising, for example \( g_0 = 2.2 \cdot 10^{-4} \Omega \) [9] corresponds to a linearized coupling strength \( g \approx 0.01\Omega \) for 2000 photons circulating in the cavity (a number reported in experiments [8]). The side band resolved regime is routinely reached, with ratios \( \kappa/\Omega \) in the range \( 0.01 - 0.1 \) [8,9] and mechanical quality factors up to almost \( Q \sim 10^6 \) were measured [9].

A number of theoretical investigations were carried out that indicate the feasibility of 2D optomechanical crystals
that support many optomechanically coupled modes with optical and mechanical tunnel coupling between neighboring sites as envisioned in this work. For defects at a distance of a few lattice constants of the underlying photonic crystal, one finds that the photon and phonon hopping amplitudes (J and K) can reasonably reach values up to 10% of the optical or vibrational mode frequencies, respectively. This leads to typical ratios $J/K$ of about $10^4$ corresponding to a much larger speed of photon propagation, unless special precautions are taken regarding the lattice design.

The observation of photon-phonon polaritons requires $4g > \kappa$ the so called strong coupling regime, which has been investigated experimentally \[12\]. The regime of self-induced mechanical oscillations requires $4g^2/\kappa \Gamma > 1$, which is also a prerequisite condition for side-band cooling and has been reached experimentally with high quality vibrational modes and cryogenic cooling \[8, 13\]. The observation of the twin-photon instability requires the more challenging condition $4g^2/\kappa \Omega > 1$. Coherent effects associated with the artificial gauge field will be observable in the strong coupling regime, $J_{\text{eff}} \gg \kappa$. Parameter $J_{\text{eff}}$ depends on the average phonon number $|\beta|^2$ in the classical mechanical oscillations, $J_{\text{eff}} = g_0 |\beta| \epsilon$ where $\epsilon$ is a small parameter which ensures the validity of the perturbative treatment. When the mechanics is driven close to its resonance we have $\beta \sim g_0 n/\Gamma$, where $n$ is the photon number in the pump mode. We arrive essentially at the condition $g^2 \epsilon / \Gamma \kappa > 1$.

We note that observation of the optomechanical band structure would not require ground state cooling (the effective temperature of the system would only determine the relative weight of spectral features observable in the emission spectrum discussed above or other quantities).

Other promising experimental platforms for optomechanical metamaterials include microdisks \[14, 16\] and microtoroids \[17–19\] on a microchip (which could be coupled via evanescent optical fields \[16\]) and superconducting circuits comprising microwave cavities parametrically coupled to vibrational modes. Recent experimental demonstrations of the latter systems employed micro- and nanomechanical beams \[20, 21\] and membranes \[13\] as mechanical elements. Prospects for both, mechanical and “optical” coupling in multi-mode systems are currently investigated \[22\].

Effects of disorder

We briefly discuss the effects of static disorder introduced during the fabrication of the optomechanical crystal structures.

Mathematically speaking, the wave functions of a single particle hopping on a disordered lattice are Anderson-localized both in 1D and 2D structures for any finite disorder strength. This would be the fate of the eigenstates of photons or phonons, respectively, and also of their optomechanically hybridized versions, in the absence of dissipation and non-linearities. However, due to the finite decay rates for photons and phonons, as well as the finite extent of the arrays, we have to be more quantitative and estimate the localization length.

A good first approximation to the actual situation is provided by the Anderson model \[23\] of disorder in a tight-binding lattice, with on-site potentials fluctuating randomly. In this model, the potential values (i.e. optical or mechanical mode frequencies for our case) are distributed evenly in the interval $[-W/2, +W/2]$, where $W$ denotes the disorder strength. Note that the qualitative results do not depend on the precise details of the assumed distribution. If we take the hopping matrix element to be $J$, which in our case stands for either photon or phonon hopping, the following results can be extracted from the extended literature on strong localization of electrons: In a 1D disordered chain, perturbation theory predicts the localization length (measured in sites) for a wave at frequency $\omega$ to be $24(4J^2 - \omega^2)/W^2$, i.e. maximal at the band center, here assumed to be at $\omega = 0$ \[24\]. In 2D, numerical calculations \[24\] indicate that the localization length reaches about 100 sites already for $W/J = 5$, and several hundred sites for $W/J = 4$. In conclusion, both for 1D and especially for 2D the localization length will be larger than the sample size for feasible optomechanical arrays, unless the fluctuations of the local optical (or mechanical) mode frequencies reach values on the order of the hopping matrix element itself. Therefore, strong localization effects should not show up in the experiments envisaged here. It might potentially be possible to observe the first precursors of strong localization (diffusion of waves and weak localization, or coherent backscattering for transport of photons through a slightly disordered optomechanical array), but we have to leave the analysis of these effects to future studies.

Current experiments indicate that the fluctuations of the on-site optical frequencies are at most about 10% of the hopping strength \[20\], i.e. well below the regime where any strong effects of disorder would be present.

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