Sensitivity to Dark Sector Scales from Gravitational Wave Signatures

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ABSTRACT: We consider gravitational sound wave signals produced by a first-order phase transition in a theory with a generic renormalizable thermal effective potential of power law form. We find the frequency and amplitude of the gravitational wave signal can be related in a straightforward manner to the parameters of the thermal effective potential. This leads to a general conclusion; if the mass of the dark Higgs is less than 1% of the dark Higgs vacuum expectation value, then the gravitational wave signal will be unobservable at all upcoming and planned gravitational wave observatories. Although the understanding of gravitational wave production at cosmological phase transitions is still evolving, we expect this result to be robust.

KEYWORDS: Gravitational Wave, Cosmological phase transition, Dark Higgs

ArXiv ePrint: 2203.11736
1 Introduction

With the advent of gravitational wave (GW) detection [1], and the development of new instruments (see Table 1, and references therein), gravitational wave signals have become among the most promising signatures for new physics beyond the Standard Model (SM) in the early Universe. In particular, cosmological first order phase transitions in the early Universe can generate gravitational wave signatures [2–6] which can be observed at current and upcoming experiments (for reviews, see [7–13]). Considerable work has focused on determining if particular models can produce GW signals which are observable at upcoming experiments. There has been a recent flurry of works examining extensions of the Standard Model which could provide gravitational wave signatures [14–63], including models with a focus on dark sectors and dark matter [64–100].

In this work, we approach this problem from a different angle. We scan the parameter space of a simple and well-motivated thermal effective potential with a single dark Higgs field, which can arise from a wide class of UV-complete models. We correlate the properties of the GW signal arising from a first-order phase transition with the parameters of the effective potential (for other work examining general models, as well as model and signal features and classes, see for example, [101–110], including cosmological contraints [111], non-gaussianities [112], and circular polarization [113]).

The result of this analysis is a characterization of the sound wave GW signal which can arise from any UV-complete model whose thermal effective potential is polynomial and renormalizable. Interestingly, we find that dimensional analysis and scaling relations control much of the analysis, allowing us to relate specific features of the GW signal to specific parameters of the effective potential.

As a specific example, we find that amplitude of the gravitational wave signal is strongly related to the ratio of the dark Higgs mass to the dark Higgs vacuum expectation value (vev) at zero temperature. If the dark Higgs mass is less than $O(1\%)$ of the dark Higgs vev, then the GW signal will be too small to be observed at any upcoming experiments. Because the
amplitude of the signal scales as several powers of ratio of dark Higgs mass to vev, our result is robust. Indeed, theoretical work connecting the amplitude of a gravitational wave signal to the underlying parameters of the phase transition is constantly evolving (for example, more refined calculations for determining the kinetic energy fraction [114–116], the wall velocity [117–119], and adoption of more general parameterized forms for the frequency spectra [120] which can account for the variations from different studies, such as [121–129]), leading one to wonder how well one can trust any even state-of-the-art result. But in order to change our bound by an order-of-magnitude, one would need a correction to the gravitational wave signal of many orders of magnitude, giving our result some robustness to future developments.

We also find a general relationship between the symmetry-breaking vev and peak frequency of the GW signal. We then characterize the optimal instruments for probing generic first-order phase transitions, in terms of the symmetry-breaking scale at zero temperature. But it is also important to point out what we do not do – we do not provide a complete analysis of any particular UV-complete model. We assume a thermal effective potential for a single scalar field which is renormalizable and of the polynomial form. Although well-motivated, for any particular model, this may or may not be a good approximation to the thermal effective potential. Our results apply directly to models for which this thermal effective potential is a good approximation, while for other models, our results are at best indicative.

The plan of this paper is as follows. In Section 2, we parameterize the thermal effective potential, and characterize the allowed phase transitions and resulting gravitational wave signals. In Section 3, we describe the results of our scans over the parameter space of the thermal effective potential. We conclude in Section 4 with a discussion of our results.

2 First-Order Phase Transitions with a Renormalizable Scalar Potential

We consider a thermal effective potential for a single real scalar field \( \phi \). We consider a renormalizable polynomial potential [65] of the form

\[
V(T, \phi) = \Lambda^4 \left[ \left( -\frac{1}{2} + c \left( \frac{T}{v} \right)^2 \right) \left( \frac{\phi}{v} \right)^2 + b \frac{T}{v} \left( \frac{\phi}{v} \right)^3 + \frac{1}{4} \left( \frac{\phi}{v} \right)^4 \right],
\]

where \( V(T = 0, \phi) \) is minimized at \( \phi = \pm v \), and where the mass of the dark Higgs (the excitation of \( \phi \) about this minimum) is given by \( m^2/v^2 = 2(\Lambda/v)^4 \). This form of the potential arises, for example, from thermal corrections in the high temperature limit, where the dimensionless coefficients \( b \) and \( c \) depend on the details of the UV model. We assume \( \Lambda/v \ll 1 \), ensuring that effective potential has a perturbative quartic coupling. We also take \( b < 0 \), as this coefficient arises in the high temperature limit from fermion loops which contribute with a negative sign to the effective potential.

It is convenient to express the potential in terms of the scale-free quantities \( \tilde{\phi} = \phi/v \) and \( \tilde{T} = T/v \). Essentially, the symmetry-breaking scale \( v \) is used as the scale against which
all other energies are measured. Defining $\tilde{V}(\tilde{T}, \tilde{\phi}) = \Lambda^{-4} V(T, \phi)$, we have

$$\tilde{V}(\tilde{T}, \tilde{\phi}) = \left( -\frac{1}{2} + c\tilde{T}^2 \right) \tilde{\phi}^2 + b\tilde{T} \tilde{\phi}^3 + \frac{1}{4} \tilde{\phi}^4. \quad (2.2)$$

We can now consider the generic behavior of this thermal effective potential. At finite temperature, the potential can have up to three extrema, at $\phi = 0$ and at factor of 2 under the radical is fixed

$$\tilde{\phi} = -\frac{3b\tilde{T} \pm \sqrt{9b^2\tilde{T}^2 + 4(1 - 2c\tilde{T}^2)}}{2}. \quad (2.3)$$

If $c\tilde{T}^2 < (1/2) + (9/8)b^2\tilde{T}^2$, then the discriminant is positive, and three distinct extrema exist. In that case, one extremum is a local maximum, while the other two are local minima, and at least one is a symmetry-breaking minimum.

$V(T, \phi = 0) = 0$, where this extremum at $\phi = 0$ is a local minimum for $c\tilde{T}^2 > 1/2$ and a local maximum for $c\tilde{T}^2 < 1/2$. The condition for $V(T, \phi) = 0$ for $\phi > 0$ is

$$c\tilde{T}^2 \leq \frac{1}{2} + b^2\tilde{T}^2. \quad (2.4)$$

We can thus classify the behavior of the potential as the Universe cools from high temperature. At sufficiently high temperature, the symmetry-preserving extremum is always a local minimum at $V = 0$. But for $c/b^2 < 1$, the potential has two other zeros, with a global minimum between them, even at high temperature. In this case, there is no phase transition, since the Universe is always in the symmetry-breaking phase. We thus restrict ourselves to the case $c/b^2 > 1$.

For $c/b^2 > 1$, then at high temperature there is a global minimum at $\phi = 0$ and a local minimum at $\phi_+ > 0$, with a local maximum at $\phi_-$ between them. But at low enough temperature ($\tilde{T}_C = 1/\sqrt{2(c - b^2)}$), we find $V(\phi_+) = 0$.

For $T < T_C$, $\phi_+$ is now the global minimum, and a first-order phase transition is possible. The transition will not occur until the nucleation temperature, $T_N$, at which time the bubble nucleation rate is larger than the Hubble expansion rate. But at sufficiently low temperature ($T < 1/\sqrt{2c}$), the potential barrier goes to zero; the symmetry-preserving extremum is now a local maximum, and if the phase transition has not yet completed, then $\phi$ can simply roll to the global minimum, yielding a second-order phase transition.

Note that our analysis of the minima of the potential depended only on the parameters $b$ and $c$, as $v$ appears only as a scale parameter, and $\Lambda$ has factored out. But the dependence on $\Lambda$ returns when we determine the nucleation temperature. The bubble nucleation rate per unit volume ($p(T)$) is given by $p(T) = T^4 \exp[-S_E/T]$, where $S_E$ is the Euclidean action for the radial bounce solution between the symmetry-preserving and symmetry-breaking vacua. The nucleation temperature $T_N$ is the temperature at the nucleation time $t_N$, at which a fraction $e^{-1}$ of the Universe has tunneled to the global minimum of the potential. The nucleation time is given approximately by the relation $p(t_N) t_N^4 = 1$. Assuming a radiation-dominated universe with $g_* = O(100)$ relativistic degrees of freedom, we then
\[
\frac{S_E}{T_N} = 140 + \mathcal{O}(\log T_N/\text{TeV}). \tag{2.5}
\]

\(S_E\) is the Euclidean action evaluated on the bounce solution to the radial equation \[131, 133\]
\[
\frac{d^2 \phi}{dr^2} + \frac{2}{r} \frac{\partial \phi}{\partial r} = \frac{\partial V_E(T, \phi)}{\partial \phi}. \tag{2.6}
\]

where \(V_E(T, \phi)\) is the temperature and field dependent potential in the Euclidean action. This solution interpolates between \(\phi = 0\) and \(\phi = \phi_+\). We will utilize an analytic approximation to the bounce solution for this form of the effective potential \[134\], given by
\[
S_E = 4.85M^3 \left[ 1 + \frac{\alpha}{4} \left( 1 + \frac{2.4}{1 - \alpha} + \frac{0.26}{(1 - \alpha)^2} \right) \right], \tag{2.7}
\]
where
\[
M^2 \equiv 2\Lambda^4 \left( \frac{cT^2}{v^2} - \frac{1}{2} \right), \quad E \equiv -\frac{b\Lambda^4}{v^4}, \quad \alpha \equiv \frac{M^2\Lambda^4}{2E^2T^2v^4}. \tag{2.8}
\]

For certain models, this analytic solution was compared to the Euclidean action computed using \texttt{CosmoTransitions} \[131\], a Python package that numerically computes the properties of phase transitions in scalar field theories, and good agreement was found. A more detailed comparison was performed in \[135\], similarly finding good agreement.

To put eqn. 2.6 in scale-free form, we define \(\tilde{r} \equiv r(\Lambda^2/v)\), yielding
\[
\frac{d^2 \tilde{\phi}}{d\tilde{r}^2} + \frac{2}{\tilde{r}} \frac{\partial \tilde{\phi}}{\partial \tilde{r}} = \frac{\partial \tilde{V}_E(\tilde{T}, \tilde{\phi})}{\partial \tilde{\phi}}. \tag{2.9}
\]

The scale-free Euclidean action, \(\tilde{S}_E\), is obtained by integrating the scale-free Lagrangian, evaluated on the bounce solution of the above equation, with respect to \(d^3\tilde{r} \, d(1/\tilde{T})\), and is related to \(S_E\) by
\[
\frac{S_E}{T} = \frac{v^2 \tilde{S}_E}{\Lambda^2 \tilde{T}}. \tag{2.10}
\]

We thus find
\[
\frac{\tilde{S}_E}{T_N} \sim 140 \left( \frac{\Lambda}{v} \right)^2, \tag{2.11}
\]
where the left-hand side of eqn. 2.11 depends only on \(b, c\) and \(\tilde{T}_N\).

\section{2.1 Thermal Parameters of the Phase Transition}

We are now able to determine determine \(S_E(\tilde{T})\) and \(\tilde{T}_N\) for any parameters \(b, c\) and \(\Lambda/v\). From these, we can determine the thermal parameters of the phase transition, which in turn feed into the gravitational wave signature. These are speed parameter of the phase transition \((\beta/H)\), and the latent heat parameter \((\xi)\).
We can write the thermal parameters as

$$\left( \frac{\beta}{H} \right) = \left( \frac{\tilde{\beta}}{H} (b, c, \Lambda/v) \right) \times \left( \frac{\Lambda}{v} \right)^{-2},$$

$$\xi = \tilde{\xi} (b, c, \Lambda/v) \times \left( \frac{\Lambda}{v} \right)^4 \left( \frac{g_*}{100} \right)^{-1},$$

(2.12)

where

$$\tilde{\beta} (b, c, \Lambda/v) = \left( T \frac{d(\tilde{S}_E/\tilde{T})}{d\tilde{T}} \right)_{\tilde{T} = \tilde{T}_N},$$

$$\tilde{\xi} (b, c, \Lambda/v) = \left[ \left( \frac{3}{10 \pi^2 \tilde{T}_N^4} \right) \left( \Delta \tilde{V} - \tilde{T} \Delta \frac{d\tilde{V}}{d\tilde{T}} \right) \right]_{\tilde{T} = \tilde{T}_N},$$

(2.13)

and \( g_* \) is the number of relativistic degrees of freedom. Essentially, the parameter \( \Lambda/v \) controls an overall rescaling of the potential, and the power-law dependence of the thermal parameters on \( \Lambda/v \) in eq. 2.12 encodes the dependence of these parameters on that scaling.

Note that the scale-free thermal parameters \( (\tilde{\beta}/H) \) and \( \tilde{\xi} \) depend on \( \Lambda/v \) only through the determination of the scale-free nucleation temperature, \( \tilde{T}_N \). Thus, the actual thermal parameters \( (\beta/H) \) and \( \xi \) have a dependence on \( \Lambda/v \) which is nearly power-law. This dependence would be exactly power law if the nucleation temperature were exactly the same as the critical temperature. In fact, the nucleation temperature will be less than the critical temperature, leading to a deviation from power-law behavior. But we will see that this power-law behavior largely determines the maximum allowed gravitational wave signal. Moreover, this form of the thermal effective potential is a good approximation in the high temperature limit; if the nucleation temperature is significantly smaller than the critical temperature, this may not be a good approximation. A typical scale where the high-temperature approximation remains valid is given by \( T \gtrsim m \) (for example, in [65] the criteria \( T^2 > 2m^2 \) was employed). This can be translated to the constraint \( \tilde{T} \gtrsim \sqrt{2}(\Lambda/v)^2 \).

However, the size of the effects of non-power law terms that would be generated if this criteria is not met would be model dependent, thus we do not pursue that line of inquiry further in this work.

### 2.2 The Gravitational Wave Signal

First order phase transitions can produce gravitational waves via three different mechanisms: i) bubble collisions, ii) sound waves from bubble expansion in the fluid, and iii) turbulence in the fluid. In a runaway bubble expansion, where bubble wall velocities are not sufficiently slowed by friction from the fluid, bubble collisions can provide the dominant gravitational wave signal. This situation can arise, for example, for extremely supercooled systems [125, 136, 137]. The study of turbulence is ongoing, with the efficiency of converting kinetic energy into turbulent energy being one source of some uncertainty [109, 122, 129]. Thus, as we are not examining extremely supercooled situations, we will assume that the dominant gravitational wave signature arises from sound waves [138]. We can express the
amplitude ($h^2 \Omega_{sw}$) and frequency ($f_{sw}$) of this gravitational wave signal as

$$h^2 \Omega_{sw}(f) = h^2 \Omega^\text{max}_{sw}(f) \times \left(\frac{f}{f_{sw}}\right)^3 \left(\frac{7}{4 + 3(f/f_{sw})^2}\right)^{7/2},$$

(2.14)

where

$$f_{sw} = \tilde{f}_{sw}(b, c, \Lambda/v) \times \left(\frac{\Lambda}{v}\right)^{-2} \left(\frac{T_N}{100 \text{ GeV}}\right) \left(\frac{g_*}{100}\right)^{1/6},$$

$$h^2 \Omega^\text{max}_{sw} = h^2 \Omega^\text{max}_{sw}(b, c, \Lambda/v) \times \left(\frac{\Lambda}{v}\right)^{10+8n} \left(\frac{g_*}{100}\right)^{-5/3-2n} \times \left[1 - \frac{1}{\sqrt{1 + 2\tau_{sh}H_s}}\right],$$

(2.15)

and [114]

$$\tilde{f}_{sw}(b, c, \Lambda/v) = 8.9 \times 10^{-3} \text{mHz} \left(\frac{1}{v_w}\right) \left(\frac{\tilde{\beta}}{H}\right) \left[\times \left(\frac{z_p}{10}\right) \frac{1}{\sqrt{1 + 2\tau_{sh}H_s}}\right],$$

$$h^2 \Omega^\text{max}_{sw}(b, c, \Lambda/v) = 8.5 \times 10^{-6} \left(\frac{\Gamma}{4/3}\right)^2 \left(\frac{\tilde{\beta}}{H}\right)^{-1} v_w,$$

$$\tilde{\kappa}_f \sim \tilde{\xi}^n.$$

(2.16)

Here, $v_w$ is the sound speed, which determines the exponent $n$; for $v_w \sim 1$, we have $n = 1$, while for $v_w \sim 0.5$, we have $n \sim 2/5$ [114]. $\Gamma$ is the adiabatic index, which we will take to be $\Gamma = 4/3$ [65], and $z_p \sim 10$ [123, 135].

There are additional factors listed in brackets [135], which depend on the details of the model, encoded in quantities such as the time scale for turbulence ($\tau_{sh}$) and the Hubble scale at which the source becomes active ($H_s$). Indeed, as pointed out in [139], there are a variety of suppression factors which can arise from a more diligent treatment of gravitational wave production, which can suppress the signal amplitude by more than an order of magnitude. As we begin with an effective potential for the scalar, but no detailed microphysics, it is not possible to attain this level of precision. But for our purpose, it is not necessary. Since the gravitational wave amplitude scales as more than 10 powers of $(\Lambda/v)$, this analysis will be sufficient to determine classes of models which will be easily inaccessible to all upcoming experiments. Moreover, although future theoretical developments may uncover additional correction factors, unless their collective size is several orders of magnitude, they will not change our results appreciably. For similar reasons, it is sufficient for us define the thermal parameters at the nucleation temperature, rather than the percolation temperature.

In a similar vein, there are a variety of theoretical uncertainties with both the first-order phase transition and the associated gravitational wave signature, and there has been much recent theoretical progress, including studies of gauge dependence, renormalization group equations, and the one-loop effective potential [140, 141], discussions within an effective field theory framework [142–144], issues related to the perturbative expansion including its scale dependence [145–148], and non-perturbative models and resummation and use of
numerical simulations (for example, see [115, 121–123, 125, 126, 149–155]). It would be interesting to incorporate these effects in a more detailed analysis, but it is not necessary for our purpose here.

The main quantities we are interested in are the peak amplitude of the GW signal \( h^2\Omega_{\text{sw}}^{\text{max}} \), and the frequency at which that peak is obtained \( (f_{\text{sw}}) \). If the nucleation temperature were equal to the critical temperature, then \( h^2\Omega_{\text{sw}}^{\text{max}} \) and \( f_{\text{sw}} \) would be independent of \( \Lambda/v \), the dependence of the GW signal parameters on \( \Lambda/v \) would be entirely determined by the scaling relations in eq. 2.12. We would then find

\[
(h^2\Omega_{\text{sw}}^{\text{max}})_{T_N=T_C} \propto \xi^{2(n+2)} (\frac{\beta}{H})^{-1} \propto (\frac{\Lambda}{v})^{10+8n},
\]

\[
(f_{\text{sw}})_{T_N=T_C} \propto \frac{\beta}{H} v \propto (\frac{\Lambda}{v})^{-2} v,
\]

(2.17)

where \( (\Lambda/v)^4 = m^2/2v^2 \), and the power-law dependence on \( \Lambda/v \) is induced by the dependence of the thermal parameters on the overall scale of the potential.

Reducing \( m/v \) by an order of magnitude would reduce the amplitude of the GW signal by a factor of at least \( 10^5 \) (taking \( n = 0 \)). We thus expect to find a sharp minimum for \( m/v \); for dark Higgs masses which are sufficiently smaller than the symmetry-breaking scale, the amplitude of the GW signal will be unobservable.

Note, however that although the amplitude of the GW signal depends on \( m/v \), it has no actual dependence on \( v \) itself, a result which could be anticipated from dimensional analysis. But the symmetry-breaking scale does enter into \( f_{\text{sw}} \), to which it is directly proportional. Thus far, our analysis has been completely scale-free. We now see that the energy scale of the phase transition only enters in the last step, setting the frequency scale of the GW signal.

Some of these considerations will be modified once we include the dependence of the nucleation temperature on \( (\Lambda/v) \). But we expect that this modification should not change the result substantially; if \( T_N \ll T_C \), then the phase transition is very supercooled, and this form of the thermal effective potential may in any case not be a good approximation to the thermal potential of a UV complete model. We will see from more detailed numerical calculation that this intuition is correct.

3 Results

In this section, we describe our strategy for scanning over the parameter space of the thermal effective potential and then present the results of the scan. First, we briefly specify the conditions for a parameter space point to be of interest.

- \( \Lambda/v < 1 \): This condition ensures that the zero-temperature quartic coupling is perturbative.
- \( c/b^2 > 1 \): This condition is required for a phase transition to occur at all. If it is not satisfied, then the symmetry-breaking minimum is the global minimum even at high temperature.
• \( c\tilde{T}_N^2 > 1/2 \): This is necessary in order for the transition to be first-order. If this is not satisfied, then the barrier disappears before the nucleation temperature is reached, leading to a smooth phase transition.

• \( \beta/H > 1 \): This condition is needed for the bubble growth rate to exceed the Hubble expansion, so that bubbles grow. This is equivalent to \( \beta/H > (\Lambda/v)^2 \).

• \( \xi > 0 \): a first order transition will only proceed if the latent heat parameter is positive at the nucleation temperature. This is equivalent to \( \tilde{\xi} > 0 \).

It may seem unusual that the latent heat parameter can be negative. It is easy to demonstrate that, if the phase transition occurs at the critical temperature, then the latent heat parameter must be positive. But it may become negative if the phase transition is sufficiently supercooled. As we have already mentioned, such a transition would potentially stretch the effective potential beyond its regime of validity. But in any case, if the latent heat parameter is negative, no GW signal will be produced.

The scale-free Euclidean action, \( \tilde{S}_E \), is determined by \( \tilde{T} \) and the parameters \( b \) and \( c \). For any choice of parameters \( b \), \( c \) and \( \Lambda/v \), the nucleation temperature \( \tilde{T}_N \) is approximately determined by the solution of the equation

\[
\frac{\tilde{S}_E(b,c,\tilde{T}_N)}{140\tilde{T}_N} = \left(\frac{\Lambda}{v}\right)^2.
\]  

In Figures 1 and 2, we plot contours of \( \tilde{S}_E(b,c,\tilde{T})/140\tilde{T} \) in the \((b,c)\)-plane, for eight different choices of \( \tilde{T} \). We can then solve eqn. 3.1 by setting \( (\Lambda/v)^2 \) equal to the \( \tilde{S}_E/140\tilde{T} \), for a value of \( \tilde{T} \) which is taken as the nucleation temperature. These figures thus encode all choices of the parameters \((b,c,\Lambda/v)\) for which the scale-free nucleation temperature is given by a particular choice \( \tilde{T}_N \). Note, we only plot parameters points which are of interest, as defined by the criteria described at the beginning of this section.

Note that the range of parameter space which is of interest is very small for \( \tilde{T}_N \sim 1 \), near \((b,c) \sim (-\sqrt{2},2)\). More parameter space is available for \( \tilde{T}_N < 1 \), and for relatively large \( \tilde{T}_N \), the available parameter space is focused near the curve \( c = b^2 \). But as \( \tilde{T}_N \) becomes small, the high temperature approximation becomes less valid; in that case, it is not clear if this effective potential can be realized from a UV complete model. In particular, note that, although we do not consider the range \( c > 25 \) for reasons of computational ease, that range in any case corresponds to small \( \tilde{T}_N \).

In Figures 3 and 4, we plot contours of \( \tilde{\beta}/H \) in the \((b,c)\)-plane, for different choices of \( \tilde{T}_N \). Only parameter points satisfying the criteria described in the beginning of this section are considered. In particular, for any choice of \( \tilde{T}_N \), only points in the \((b,c)\)-plane are shown for which there exists some choice of \( \Lambda/v \) consistent with the given \( \tilde{T}_N \).

Similarly, in Figures 5 and 6, we plot contours of \( \tilde{\xi} \) in the \((b,c)\)-plane for different choices of \( \tilde{T}_N \). Note that unless \( \tilde{T}_N \ll 1 \), we find \( \tilde{\xi} \lesssim \mathcal{O}(10) \) throughout the entire parameter space. This implies that, unless the phase transition occurs well below the symmetry breaking
Figure 1: Contours of $\tilde{S}_{E}/140\tilde{T}$ are shown in the $(b,c)$ plane for smaller values of $\tilde{T}$: $\tilde{T} = 0.15$ (upper left), $0.20$ (upper right), $0.25$ (lower left), and $0.30$ (lower right). The contours also satisfy the conditions: $c \geq b^2$, $c\tilde{T} \geq 1/2$, $0 \leq \tilde{S}_{E}/140\tilde{T} \leq 1$, $\tilde{\beta}/H \geq \tilde{S}_{E}/140\tilde{T}$ and $\tilde{\xi} \geq 0$.

scale, we will find

$$h^2\Omega_{sw}^{max} < O(0.1),$$
$$h^2\Omega_{sw}^{max} < O(0.1) \left(\frac{\Lambda}{v}\right)^{10+8n}$$

(3.2)

The maximum sensitivity of LISA is roughly $h^2\Omega_{sw} \sim O(10^{-13})$, implying that LISA will be insensitive to any model for which $\Lambda/v \lesssim O(10^{-1})$. Note that we can directly relate the parameters of the thermal effective potential to the mass and vev of the dark Higgs at zero temperature through the relation $(\Lambda/v)^2 = m/\sqrt{2}v$. We thus see that these simple scaling arguments imply a powerful connection between gravitational wave observations of
Figure 2: Contours of $\tilde{S}_E/140\tilde{T}$ are shown in the $(b,c)$ plane for larger values of $\tilde{T}$: $\tilde{T} = 0.40$ (upper left), $0.50$ (upper right), $0.60$ (lower left), and $0.80$ (lower right). The contours also satisfy the conditions: $c \geq b^2$, $c\tilde{T} \geq 1/2$, $\alpha \leq 1$, $0 \leq \tilde{S}_E/140\tilde{T} \leq 1$, $\tilde{\beta}/H \geq \tilde{S}_E/140\tilde{T}$ and $\tilde{\xi} \geq 0$.

a cosmological phase transition and laboratory probes of the hidden sector at zero temperature. In particular, if the mass of the dark Higgs particle excitation is less than $O(1\%)$ of the dark Higgs vev, then the gravitational wave signal arising from condensation of the dark Higgs in the early Universe would have an amplitude too small to be observed at LISA. If the hidden sector is coupled to the Standard Model, then the dark Higgs mass and vev can be probed at fixed target or beam dump experiments, thus determining if a gravitational wave signal can be seen. Note that, although the maximum sensitivity of BBO will be 4-5 orders of magnitude better than LISA this leads to much less than an order of magnitude improvement in minimum value of $m/v$ for models which can be probed with gravitational waves.
Figure 3: Contours of $\tilde{\beta}/H$ are shown in the $(b,c)$ plane for the allowed parameter space for lower values of $\tilde{T}_N$: $\tilde{T}_N = 0.15$ (upper left), 0.20 (upper right), 0.25 (lower left), and 0.30 (lower right). The contours also satisfy the conditions: $c \geq b^2$, $c\tilde{T}_N \geq 1/2$, $\alpha \leq 1$, $0 \leq \tilde{S}_E/140\tilde{T}_N \leq 1$, $\tilde{\beta}/H \geq \tilde{S}_E/140\tilde{T}_N$ and $\xi \geq 0$.

We can provide an even tighter bound on the amplitude of the gravitational wave signal by accounting for the dependence of $h^2\Omega_{s_{\text{sw}}}^{\text{max}}$ on $\xi$ and $\tilde{\beta}/H$. In Figure 7a, we scan over parameters $(b, c, \Lambda/\nu)$, plotting each point on the $(h^2\Omega_{s_{\text{sw}}}^{\text{max}}, \Lambda/\nu)$-plane. We have assumed $\nu_w = 1$. If we could ignore the effects of supercooling, then we would expect from Eq. 2.17 that the set of points with the largest values of $h^2\Omega_{s_{\text{sw}}}^{\text{max}}$ correspond to the choice of $(b, c)$ for which $h^2\Omega_{s_{\text{sw}}}^{\text{max}}$ is largest, and lie along the line $h^2\Omega_{s_{\text{sw}}}^{\text{max}} \propto (\Lambda/\nu)^{-18}$. In fact, the slope of this line (plotted as the solid blue line in Fig. 7a) is $\sim 17.2$, which is only slightly different from that given above, indicating that the dependence of $h^2\Omega_{s_{\text{sw}}}^{\text{max}}$ on $\Lambda/\nu$ induced by the effects of supercooling is relatively minor. Similarly, in Figure 7b, we scan over the parameters $(b, c, \Lambda/\nu)$, plotting each point in the $(f_{s_{\text{sw}}}, \Lambda/\nu)$-plane. Again, we see that most of the points roughly follow the $f_{s_{\text{sw}}} / \nu \propto (\Lambda/\nu)^{-2}$ relation which we found earlier (ignoring the...
effects of supercooling), with a spread of roughly an order of magnitude in $f_{sw}/v$ for any $\Lambda/v$.

In Figure 8, we plot the set of points in the $(b,c)$-plane for which $h^2\Omega_{sw}^{\text{max}}$ is within 2% of the maximum for any given value of $\Lambda/v$. These points correspond to the $(b,c)$ values of the points near the solid blue line in Figure 7a, discussed above. We see that scatter in these points is small, another indication that the effects of supercooling are relatively minor (if these effects were negligible, then all parameter points on this line would have the same values of $b$ and $c$, differing only in the parameter $\Lambda/v$). We also plot shaded regions indicating values of $(b,c)$ for which a given nucleation temperature $T_N$ can be realized. We see that these models with maximal signal amplitude can only be realized with $T_N < 0.4$. 

Figure 4: Contours of $\tilde{\beta}/H$ are shown in the $(b,c)$ plane for the allowed parameter space for higher values of $T_N$: $T_N = 0.40$ (upper left), 0.50 (upper right), 0.60 (lower left), and 0.80 (lower right). The contours also satisfy the conditions: $c \geq b^2$, $cT_N \geq 1/2$, $\alpha \leq 1$, $0 \leq \tilde{S}_E/140\tilde{T}_N \leq 1$, $\tilde{\beta}/H \geq \tilde{S}_E/140\tilde{T}_N$ and $\tilde{\xi} \geq 0$. 

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Figure 5: Contours of $\tilde{\xi}$ are shown in the $(b, c)$ plane for the allowed parameter space for lower values of $\tilde{T}_N$: $\tilde{T}_N = 0.15$ (upper left), 0.20 (upper right), 0.25 (lower left), and 0.30 (lower right). The contours also satisfy the conditions: $c \geq b^2$, $c\tilde{T}_N \geq 1/2$, $\alpha \leq 1$, $0 \leq \tilde{S}_E/140\tilde{T}_N \leq 1$, $\tilde{\beta}/H \geq \tilde{S}_E/140\tilde{T}_N$ and $\tilde{\xi} \geq 0$.

Finally, in Figure 9, we scan over models, which are plotted in the $(f_{sw}, h^2\Omega_{sw}^{\text{max}})$-plane, taking $v_w = 1$. In particular, we scan over $b$, $c$, and $\Lambda/v$, with $v = 1$ MeV (brown), 100 GeV (green), 1 TeV (pink), and 1000 TeV (blue). We find $f_{sw} \propto v$, while for any choice of $v$, we roughly find $h^2\Omega_{sw}^{\text{max}} \propto f_{sw}^{-9}$, as one would expect from eqn. 2.17. We also plot the sensitivity of a variety of current and upcoming gravitational wave observatories in Fig. 9.

Note that the most sensitive upcoming experiments (BBO and ALIA) will probe models with symmetry-breaking scales in the range $\mathcal{O}(100 - 1000)$ GeV. For scenarios with new physics appearing at the MeV-scale, experiments such as THEIA would provide the leading sensitivity, while for scenarios with new physics appearing at the PeV-scale, experiments such as ET and CE would provide the leading sensitivity. But the absence of sensitivity for models with $m/v < \mathcal{O}(0.01)$ is largely independent of scale; even at the scales for which
Figure 6: Contours of $\tilde{\xi}$ are shown in the $(b, c)$ plane for the allowed parameter space for higher values of $\tilde{T}_N$: $\tilde{T}_N = 0.40$ (upper left), 0.50 (upper right), 0.60 (lower left), and 0.80 (lower right). The contours also satisfy the conditions: $c \geq b^2$, $c\tilde{T}_N \geq 1/2$, $\alpha \leq 1$, $0 \leq \tilde{S}_E/140\tilde{T}_N \leq 1$, $\beta/H \geq \tilde{S}_E/140\tilde{T}_N$ and $\tilde{\xi} \geq 0$.

future experiments will be most sensitive, this lower bound can be reduced by no more than a factor of a few. These experiments are described in Table 1.

Note from Figure 7a that, although the upper bound on $h^2\Omega_{sw}^{max}$ follows fairly closely the scaling relations we have derived, and in particular, grows steeply with $\Lambda/v$, the actual amplitude may be much smaller than this maximum. For example, the behavior of the SM electroweak phase transition is different, with the amplitude of a gravitational wave signal decreasing with $m_h/v$, and disappearing entirely for $m_h = 125$ GeV, at which point the phase transition is smooth. This is because the thermal effective potential for the Standard Model Higgs sector corresponds to a choice of parameters which lie well below the upper bound on $h^2\Omega_{sw}^{max}$. For sufficiently large $m_h/v$, the parameter $c$ becomes small enough that the potential barrier disappears before reaching the nucleation temperature. This example illustrates the point that, although this analysis provides an upper limit on
Figure 7: Plot of $h^2\Omega_{sw}^{\text{max}}$ (left) and $f_{sw}/v$ (right) against $\Lambda/v$ for a set of models obtained by scanning of $0 \leq \Lambda/v \leq 1$, $-5 \leq b \leq 0$ and $0 \leq c \leq 25$. The dashed lines in the left panel are the maximum sensitivities of LISA (green), BBO (beige), SKA (magenta) and CE (purple). The solid blue line in the left panel presents the maximum value of $h^2\Omega_{sw}^{\text{max}}$ obtained for a given $\Lambda/v$, for any $(b, c)$; this line has a slope of $\sim 17.2$.

the gravitational wave signal, it would be much more difficult to translate a measurement of $m/v$ into a prediction for the gravitational wave amplitude, or vice versa.

4 Conclusions

We have considered gravitational wave signals from sound waves in the context of an effective field theory with a renormalizable thermal potential exhibiting a first-order phase transition in the early Universe. We have considered a thermal effective potential of power law form, which would arise, for example, in the high temperature limit. This approach allows us to consider the gravitational wave signals produced by phase transitions in a wide class of models.

We find that for this class of models, there is a relatively clean relationship between the parameters of the gravitational wave signal and the parameters of the thermal effective potential. The effective potential can be expressed in terms of one dimensionful parameter, the vacuum expectation value of the dark Higgs ($v$), which sets the frequency scale of the gravitational wave signal. The amplitude of the gravitational wave signal is largely set by one dimensionless parameter, $\Lambda/v$, which sets the ratio of the potential energy scale to the dark Higgs vev. Since the amplitude of the gravitational wave signal scales as $\Lambda/v$ to a very high power, the dependence on the other dimensionless parameters is subdominant.

This approach leads to a striking conclusion. If the ratio of the dark Higgs mass to the dark Higgs vev is less than $\mathcal{O}(10^{-2})$, then the magnitude of the gravitational wave signal will
Figure 8: Plot in the $(b,c)$-plane of parameter points for which $h^2\Omega_{\text{sw}}^{\text{max}}$ is within 2% of solid blue line shown in Fig. 7a. The shaded regions indicate regions in the $(b,c)$-plane for which various nucleation temperatures $\tilde{T}_N$ can be realized.

be so small as to be undetectable by any upcoming observatories. This bound on sensitivity is nearly independent of the scale of new physics. This result is also robust against future developments in our understanding of the amplitude of gravitational sound waves produced by a first-order cosmological phase transition. Since the amplitude depends on $m_f/v$ raised to a high power, to modify our bound by an order of magnitude would require a new correction to the sound wave amplitude of several orders of magnitude.

This result leads to several intriguing possibilities. For example, in scenarios of new MeV-scale physics, one may potentially be able to probe the symmetry-breaking scale with
Figure 9: Plot in the \((h^2 \Omega_{sw}, f_{sw})\)-plane of the gravitational wave signal obtained from a scan of parameter points \((b, c, \Lambda/v)\). The values of \(v\) used are 1 MeV, 100 GeV, 1 TeV and 1000 TeV (from left to right). Also plotted are the sensitivities of the following current and upcoming experiments: 1. EPTA [156], 2. NANOGrav [157, 158], 3. Gaia [159], 4. SKA [160], 5. THEIA [161], 6. LISA [12, 162, 163], 7. Taiji [164], 8. TianQin [165], 9. ALIA [166], 10. BBO [167, 168], 11. DECIGO [169, 170], 12. aLIGO [171, 172], 13. A+ [173], 14. ET [174], 15. CE [175].

forward detectors at high-luminosity beam experiments, which could potentially determine the mass and coupling of the dark photon. The detection of a gravitational wave signal from the corresponding first-order phase transition would then provide a lower bound on the Higgs mass. Similarly, it is possible that future high-energy beam experiments could produce a heavy (\(> \mathcal{O}(\text{TeV})\)) dark Higgs. In this case, the amplitude of a gravitational wave signal from the phase transition would correspond to an upper bound on the symmetry-breaking scale.
| Classifications based on frequency reach | Dark sector scale that can be probed | Type of experiments | Experiments |
|-----------------------------------------|-------------------------------------|---------------------|-------------|
| Very low frequency $10^{-9} - 10^{-8}$ Hz | $\leq 1 \text{ MeV}$ | Ground-based telescope using pulsar timing observation technique | EPTA (European Pulsar Timing Array) [156] |
|                                          |                                      | NANOGrav (North American Nanohertz Observatory for Gravitational Wave) [157, 158] | SKA (Square Kilometer Array) [160]: World’s largest radio telescope |
|                                          |                                      | Space-based telescope using astrometry | Gaia [159] |
| $\mu$-frequency $10^{-6} - 10^{-5}$ Hz | $\sim 100 \text{ MeV}$ | Space-based heliocentric constellation of satellites using Laser interferometry technique | $\mu$-Ares [176] |
| Low frequency $10^{-3} - 1$ Hz           | $100 \text{ GeV-1 TeV}$ | Space-based interferometer using Laser interferometry technique | LISA (Laser Interferometer Space Antenna) [12, 162, 163] |
|                                          |                                     | ALIA (Advanced Laser Interferometer Antenna) [166] | Taiji [164] |
|                                          |                                     | TianQin [165] | BBO (Big Bang Observer) [167, 168] |
|                                          |                                     | DECIGO (Deci-Hertz Interferometer Gravitational Wave Observatory) [169, 170] |
There are several avenues for further exploration. We have considered a power-law form for the thermal effective potential, which is expected to be a good approximation for renormalizable models, in the high-temperature limit. But there are a variety of models in which the phase transition is supercooled (see, for example [177–180]), and nucleation temperature is relatively low. In these cases, although the corrections to the form of the potential may be small, they need not be. There has been significant recent work regarding theoretical issues in finite temperature perturbation theory (see, for example, [145, 181]).

Similarly, we have assumed that the thermal effective potential is renormalizable. There are interesting models of new physics in which the phase transition can only be seen with the inclusion of non-renormalizable terms. It would be interesting to consider more general forms of the thermal effective potential, to better determine the extent to which the lessons found here generalize.

### Acknowledgments

We are grateful to Djuna Croon, Jeremy Sakstein, Tim Tait, and especially Graham White for useful discussions. The work of JK is supported in part by DOE grant DE-SC0010504. JR is supported by NSF grant AST-1934744. The work of BD and SG is supported in part by DOE grant DE-SC0010813. The work of SG is also supported in part by National Research Foundation of Korea (NRF) Grant No. NRF-2019R1A2C3005009 (SG). JBD

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**Table 1**: Description of upcoming gravitational wave observatories, including the frequency range to which they would be sensitive, and the corresponding symmetry-breaking scale.

| Classifications based on frequency reach | Dark sector scale that can be probed | Type of experiments | Experiments |
|------------------------------------------|--------------------------------------|---------------------|-------------|
| High frequency 10 – 100 Hz               | ~1000 TeV                            | Second generation   | aLIGO (Advanced LIGO) [171, 172] |
|                                          |                                      | Ground-based interferometer using Laser interferometry technique | A+ [173] (Quantum LIGO) |
|                                          |                                      | Third generation    | ET (Einstein Telescope) [174] |
|                                          |                                      | Ground-based interferometer using Laser interferometry technique | CE (Cosmic Explorer) [175] |
Acknowledges support from the National Science Foundation under grants no. PHY-1820801 and PHY-1748958.

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