Estimation of aircraft distances using transponder signal strength information

John H. Mott

Abstract: The Federal Aviation Administration has recently mandated the installation of transponders that provide position reporting (Extended Mode S) in aircraft operating in most types of domestic controlled airspace by January 1, 2020. The resulting proliferation of aircraft transponder data has accelerated the potential for the use of such data in measuring operations counts at nontowered airports, as it may be easily collected with an inexpensive receiver and analyzed with appropriate algorithms. While many of the data (Basic Mode S and Mode C) do not include aircraft position information, the portion of Extended Mode S data that do may be used to directly compute the distance of the corresponding aircraft from a receiver located at an airport of interest. This article describes a method by which these computed distances may be utilized to calibrate an adaptive digital filter that can subsequently estimate distances for the remainder of the transponder records that do not provide position information. The digital filter is a combined first-order Butterworth low-pass filter and a Rayleigh maximum likelihood estimator for the signal variance. The resulting distance estimates from two different antenna installations exhibited median absolute deviations of 0.92 and 1.12 nm per transponder record, respectively, within 5.0 nm of the receiver. These accuracies are sufficient for the estimation of aircraft operations counts at nontowered airports.

Subjects: Intelligent Systems; Technology; Aerospace & Aviation Engineering

Keywords: adaptive estimation; digital filtering; Rayleigh channels; transponders

ABOUT THE AUTHORS

John H. Mott received the BS and MS degrees in electrical engineering from the University of Alabama, and the PhD degree in civil engineering from Purdue University. He is currently an Associate Professor in the School of Aviation & Transportation Technology at Purdue University in West Lafayette, Indiana. Professor Mott serves as Director of the Advanced Aviation Analytics Institute for Research, a Purdue University Center of Research Excellence, and as Executive Editor of the Journal of Aviation Technology & Engineering. In addition, he is a commercially-rated pilot and flight instructor. His research interests are in the areas of signal processing, stochastic modeling of transportation operations, and aerospace applications of Bayesian inference.

PUBLIC INTEREST STATEMENT

Current efforts to modernize the US air traffic control system have resulted in a recent FAA mandate for the installation of Extended Mode S transponders in all aircraft operating in most types of domestic controlled airspace by January 1, 2020. This has accelerated the potential for transponder data to be used in measuring parameters associated with airport operations. This article describes a method by which computed distances of aircraft transmitting position information may be used to calibrate an adaptive digital filter that can subsequently estimate distances for the remainder of the aircraft that do not transmit position information. The resulting distance estimates are of sufficient accuracy to estimate aircraft operations counts at airports.
1. Introduction
Accurate counts of aircraft operations at unmonitored or partially-monitored general aviation airports are important due to their role in the allocation of funds for airport development and improvement. While approximately $1 billion in federally allocated funds are provided annually to domestic small commercial and general aviation airports for improvement projects, fewer than 270 of these 2,950 airports have either full- or part-time air traffic personnel available to register operations counts. The unmet need for accurate registration of aircraft operations at these airports led the author to develop a means of employing a low-cost ground-based transponder data collection unit to facilitate the estimation of the proximity of transponder-equipped general aviation aircraft utilizing signal strength information provided by the unit (Mott & Bullock, 2017). Aircraft proximity can be used in conjunction with other parameters to determine when operations occur.

The use of Extended Mode S transponder data with its accompanying GPS-derived position information for determining aircraft distances is reasonably straightforward. Empirical observations made in the US and Europe suggest that the average percentage of aircraft transmitting Extended Mode S data is only 25–30% (McNamara, Mott, & Bullock, 2016); data published by the Aerospace Electronics Association, however, indicates that the actual percentage of Extended Mode S-equipped aircraft in the United States is substantially less. As of March, 2015, only 10,949 domestic aircraft were equipped with Extended Mode S equipment out of approximately 204,000 aircraft on the federal registry, or about 5.5%; the FAA (2015) places this figure at 7% at the end of CY 2014. Regardless of the precise penetration percentage, the salient point is that the more prevalent Basic Mode S and Mode C signals contain aircraft position information limited to barometric altitude only; hence, the estimation of the proximity of the associated aircraft to the receiver is a problem of considerable difficulty.

While some form of ADS-B equipment (either Extended Mode S or UAT) is required on all general aviation aircraft operating within much of the controlled airspace within the domestic airspace structure by January 1, 2020, it appears that the requirement may not be met by this deadline, based on the reluctance of operators to equip aircraft due in part to concerns related to cost. The US Air Force has publicly stated that it will likely miss this deadline, as well (AOPA, 2016).

It is apparent, then, that Basic Mode S and Mode C equipment will be in use for some time to come. Because of this, the need to employ the use of Basic Mode S and Mode C signals in the aircraft operations counting process is evident. The research described in this article focuses on a means of extracting distance information from Basic Mode S and Mode C aircraft transponder signals without incorporating multilateration; that is, sufficient information for determination of the proximity of the aircraft should ideally be provided by a single field-based receiver. The software utilized to examine the received transponder data and perform the distance estimation process is coded in the statistical language R (R Core Team, 2014), due to that language’s open-source nature and provision of applicable statistical routines.

2. Determining signal strength and estimating distance
Estimates of the distances from aircraft to a fixed receiver that are sufficiently accurate to be utilized in the registration of airport operations may be derived using a stochastic channel model in conjunction with an adaptive digital filtering process. There are numerous challenges, however, involved in creating such estimates. The transponders themselves, among which exists individual variations in transmitted power, are installed in aircraft that are in motion at velocities of up to 250 knots with respect to the receiving antenna. Various fading phenomena in the transmission channel result from fluctuations in atmospheric properties and the presence of structures that interfere with line-of-sight reception, and include small-scale fading due to multipath interference and large-scale fading due to shadowing (Al-Raie, 2010; Charalambous & Menemenlis, 2001; Sklar, 1997). The different fading modes result in the propriety of different channel models in different situations, depending on the geometry between transmitter and receiver (Bernado, Zemen, Tufvesson, Molisch, & Mecklenbrauker, 2015; Matolak & Frolik, 2011). Vertical pattern lobing (Shaw & Simolunas, 1970),
which affects both the transmitting and receiving antennas (in terms of signal power transmitted by
the former and receiving sensitivity of the latter), may occur when the dipole antenna is located
greater than a half-wavelength above the ground plane. This phenomenon leads to null areas in
antenna radiation and reception patterns, resulting in missed reports, and is therefore also a con-
cern. It is evident, then, that careful analysis of the problem and judicious use of the appropriate
signal processing tools are essential in the solution of the estimation problem.

While the estimation of distances between a transmitter and receiver using transponder signal
strength information is not a new concept, previous efforts to employ signal strength data to deter-
mine distances of transmitting aircraft from a moving receiver, such as those described in (Brodegard,
Ryan, & Ryan, 1991), have not resulted in measurements of sufficient accuracy to be utilized in
counting airport operations.

Secondary surveillance radar installations interrogate aircraft transponders on a frequency of
1030 MHz (L-band) in a pulse amplitude/pulse position-modulated format; these interrogations are
acknowledged on a 1090 MHz carrier frequency (Petrochilos, 2002; Vidmar, n.d.). There are several
different modes, or modulation patterns, that are used in the interrogation process to elicit different
information from the various types of transponders. Mode A and Mode C transmit a four-digit octal
identification code to the interrogating station; Mode C also includes barometric Gillham-encoded
altitude information. Mode S (in standard DF4 format; see Table 1) includes a 24-bit data stream that
contains an ICAO-issued identifier for the transmitting aircraft. Extended Mode S in DF17 format,
periodically transmitted without interrogation, includes altitude and position information in an ap-
pended 56-bit data field.

The algorithm described here processes Basic Mode S, Extended Mode S, and Mode C data output
by a 1090 MHz tuner/demodulator combination hosted by a Raspberry Pi single-board computer
running a modified version of the open-source software dump1090 (Sanfilippo & Robb, 2014). These
data consist of eight closely chronologically-spaced values of relative signal strength for each tran-
sponder transmission that is received. Generally, these transmissions are received every few sec-
onds; the interval between transmissions is, however, irregular and dependent on the frequency of
interrogations from ground-based secondary surveillance radar stations and of uninterrogated
Mode S DF17 squitter transmissions. These eight values constitute a signal strength vector that may
be combined into a single scalar quantity that can, in turn, be used to represent the signal strength
in the manner described below. A detailed flowchart of the distance estimation process is depicted
in Figure 1.

2.1. Characteristics of received transponder signals
It is important at this stage to gain a deeper understanding of the characteristics of the received
Mode S and Mode C signals with respect to timing and amplitude. The single-board host computer

| Downlink Format | Length (Bytes) | Squittered | Content                                      |
|-----------------|---------------|------------|----------------------------------------------|
| DF0             | 56            | No         | Air-to-Air (Traffic Collision Avoidance)    |
| DF4             | 56            | No         | Surveillance, Altitude                      |
| DF5             | 56            | No         | Surveillance, Identity                      |
| DF11            | 56            | Yes        | Mode S All-Call Reply (Identification Squitter) |
| DF16            | 112           | No         | Air-to-Air (Traffic Collision Avoidance)    |
| DF17            | 112           | Yes        | 1090 Extended Squitter                     |
| DF20            | 112           | No         | Address, Flight Status, Altitude           |
| DF21            | 112           | No         | Address, Flight Status, Identity           |
| DF24            | Up to 224     | No         | Address, Flight Status, Identity, Extended Message |
provides an output representing the 8-bit signal strength of the last eight values of each received transponder message; hence, these values range from 0–255. The computer samples the output of the demodulator at a frequency of 2 MHz; therefore, the interval between samples is 0.5 μsec. The computer buffer is by design last-in, first-out (LIFO), so the last eight signal strength values of each received message (data vector) are made available for analysis.

Mode C interrogations are broadcast from both approach control and Air Route Traffic Control Center (ARTCC) radar installations. These interrogations, as the principal component of Secondary Surveillance Radar (SSR), are broadcast from antennas that are collocated with primary radar, which is used by air traffic control computers to determine the azimuth and range of aircraft within the facility’s designated airspace. The most common type of radar installation is ASR-9; 135 of these systems are currently deployed across the United States. The ASR-9 antenna has a rotational speed of 12.5 rpm and therefore requires 4.8 s for a 360° scan of its associated airspace. Because an aircraft may be interrogated by multiple SSR interrogators, dependent on position, the timing of Mode C replies generated by the aircraft is stochastic in nature. However, if one assumes that an aircraft is within range of a single SSR site, the expected frequency of Mode C replies is 0.208 Hz.

Mode S, in civilian surveillance installations, currently employs nine different message formats consisting of either 56 or 112 frames, with each frame occurring at an interval of 1 μsec. These message formats are summarized in Table 1.

The format of primary interest in this research is the DF17 format, also known as extended squitter (ADS-B). This is a 112-frame format that requires no external interrogation and contains aircraft identification and position information, in addition to other parameters such as altitude, heading, and air/ground status. While the other formats require SSR interrogation in order to generate replies from aircraft, the DF17 format is squittered (transmitted without interrogation) by the aircraft. The DF17 squitter with both aircraft identification and position occurs every 5 s when the aircraft is airborne and every 10 s when the aircraft is on the ground, while a basic identification squitter without position information (DF11 format) occurs at a rate of 2 Hz when the aircraft is airborne, or 0.2 Hz when the aircraft is on the ground.

As the volume of Mode S traffic increases in a particular volume of airspace, pseudorandom squitter transmissions may interfere with each other and cause the resulting transmissions to be indecipherable. However, as Orlando demonstrated (Orlando, 1995), approximately 400 aircraft can be handled in a given volume of airspace with an average 5-s reception reliability of at least 99.5%. This
Figure 2 describes the timing values of the Mode S data vector and the intervals between received vectors. Note that in both the Mode C and Mode S cases, the interval between each of the signal level values retrieved from the software-defined radio is much shorter (by some six orders of magnitude) than the interval between received transponder messages.

A case study should prove effective at illustrating the relationships between aircraft position and received signal level. Figure 3 depicts a Cessna 172 aircraft transmitting DF17 Mode S signals, N170TH, departing from Runway 23 at KLAF on April 1, 2016, at approximately 10:30 am. The recorded latitudes/longitudes are overlaid on a Google Earth map in the figure, and the times are recorded next to each plot of the aircraft position. The total linear distance depicted between the first and last timestamps is 15.75 nm.

A scatterplot of received signal levels vs. time is shown in Figure 4. In addition, a second-order polynomial regression line is shown. The coefficient of determination of the regression line ($R^2$) is 0.8689, with corresponding correlation coefficient 0.9321, indicating an excellent fit to the data. Note the gap of approximately one minute during which no data was received; this may be due to interference from another aircraft or some other signal anomaly. Regardless, the regression line shows a clear decrease in signal strength as distance increases, which is expected.

The Rayleigh scattering effect is evident in Figure 4. The received signal levels are well spread as the aircraft departs the airport and the terrain, buildings, and other obstructions interfere with signal reception. The levels become more tightly clustered as the aircraft moves further away from the receiver to a distance of approximately 16 nautical miles.

2.2. Determining received power

The signal strength vector $U$, received for each transponder transmission and consisting of eight 8-bit values of relative signal strength, $u_k$, that are output from the host computer, represents the sampled amplitude of the envelope of signal received by the software-defined radio (SDR). Once a scaled version $X$ is formed from $U$, it may be filtered and the Rayleigh maximum likelihood estimator for the variance applied to yield $V_{REC}$, the scalar estimate of the received RMS voltage over the eight
observations, which is subsequently converted to average received power and used to determine \( \hat{d} \), an estimate of the distance of the aircraft from the receiver.

The receiver is a combination of a low-IF tuner and quadrature amplitude demodulator. The signal strength information output by the demodulator is not a true power level, but is a single-byte representation obtained from the magnitudes of the in-phase and quadrature components of the received signal.

The derivation of the expression for the distance from the transmitting aircraft to a receiver is given by (1), and may be written as

\[
d = 10^{\left(\frac{P_{\text{TRANS}} - P_{\text{REC}} - 32.44 \times 20 \log f - \text{other gains}}{20}\right)}
\]

where \( P_{\text{TRANS}} \) is transmitted power in watts, \( P_{\text{REC}} \) is received power, \( d \) is given in km and \( f \) in MHz. Antenna gains and the insertion loss are represented in the other gains term. The transmitter power is assumed to be fixed at 250 W, a reasonable assumption supported by FAA TSO C74c (1973).
Distances between two points of known latitude and longitude were calculated for the purpose of this research using the haversine formula.

Aircraft transponders use a form of Manchester-encoded pulse amplitude modulation (with additional pulse position modulation) (Petrochilos, 2002), which can be demodulated easily. The modulation scheme is more properly described as ITU Class A1D; i.e. double-sideband amplitude modulation with a single channel containing digital information with no modulating subcarrier and utilized for data transmission. In general, for quadrature amplitude-modulation,

\[
P_{\text{AVG}} = \frac{1}{2R} \left( I^2 + Q^2 \right) = P_{\text{REC}}
\]

where \( P_{\text{AVG}} \) is the average transmitted power and \( R \) is the real part of \( Z_0 \), the characteristic impedance of free space, or 376.73 \( \Omega \).

The transmitted signal passes through the signal channel and is received by the SDR. The ADC portion of the SDR unit acquires 8-bit in-phase \( (I_{\text{SDR}}) \) and quadrature \( (Q_{\text{SDR}}) \) samples at a frequency of 28.8 MHz; these samples are buffered and retrieved by the software running on the host device at a frequency of 2.0 MHz, as noted earlier. They are then converted to a magnitude that is mapped to an 8-bit value \( x_k \), as

\[
x_k = \frac{u_k}{f_u} = \sqrt{I_{\text{SDR}}^2 + Q_{\text{SDR}}^2}.
\]

Because the SDR in-phase and quadrature components in (3) differ from the transmitted components as a result of passage through the channel, the magnitude of the received signal must be estimated.
Consider first the deterministic case in which no scattering is assumed. Figure 5 depicts this ideal channel model, in which only noiseless losses occur as described by the Friis equation.

The received signal level is

\[ V_{SDR} = \frac{x_k}{\sqrt{2}}. \]  

(4)

At maximum gain, the relationship between the indicated power and received power (Schrödle, 2015) is given by

\[ P_{SDR_{\text{dm}}} = 0.988 P_{REC_{\text{dm}}} + 88.52 \]  

(5)

where \( P_{SDR_{\text{dm}}} \) is the power in dBm indicated by the output of the demodulator as measured by calibrated spectrum analyzer software and \( P_{REC_{\text{dm}}} \) is the actual received power in dBm.

Then

\[ P_{REC_{\text{dm}}} = 1.012146 P_{SDR_{\text{dm}}} - 89.59514 \]  

(6)

and

\[ V_{REC} = \sqrt{P_{REC_{\text{dm}}} R} = \sqrt{376.73 \times 10^{2.02492 \log x_k - 11.835081}}. \]  

(7)

Again, equation (7) is deterministic in the sense that it does not include the effects of the channel noise.

Consider next the practical case of the noisy signal channel. This channel is depicted in Figure 6.

The random variable representing the power received over the length-8 sequence of signal values, \( P_{REC} \), must now be estimated as \( \hat{P}_{REC} \). It is most convenient to obtain an estimate, \( \hat{P}_{SDR} \), of \( P_{SDR} \), which is now a random variable. The estimate \( \hat{P}_{SDR} \) is formed by filtering the received random voltage vector \( X \) (a scaled version of \( U \)) with a discrete first-order Butterworth low pass filter to eliminate noise at higher frequencies and estimating the variance of the resulting filter output sequence \( Y_k \) with the Rayleigh maximum likelihood estimator for the variance of the signal. This estimate serves to account for the shift in the Doppler power spectral density, discussed below, of the signal as a result of the aircraft velocity relative to the receiver, for uncertainties in the transmission channel due primarily to multipath interference, and for noise generated by the software-defined radio and processing electronics.
2.3. Creating a channel model

The relative velocity of the aircraft being tracked results in a spreading effect of the channel which is captured in its Doppler power spectral density (DPSD). The deterministic DPSD is based on a stochastic short-term fading model proposed by Aulin (1979). Aulin’s model geometry is shown in Figure 7.

The signal broadcast from the aircraft transponder is of the form

\[ y(t) = I_r(t) \cos \omega t - I_r(t) \sin \omega t \]  \hspace{1cm} (8)

The model employed here assumes that at each point between the transmitter and receiver, the total received wave consists of the superposition of \( N \) plane waves, each having traveled via a different path as a result of multipath scattering. The \( n \)th wave is characterized by its field vector \( E_n(t) \) given by

\[ E_n(t) = I_n(t) \cos \omega_c t - Q_n(t) \sin \omega_c t \]  \hspace{1cm} (9)

where \( I_n(t) \) and \( Q_n(t) \) are the in-phase and quadrature components of the wave, respectively. Superposition yields

\[ E(t) = \sum_{n=1}^{N} E_n(t) = I(t) \cos \omega_c t - Q(t) \sin \omega_c t \]  \hspace{1cm} (10)

\[ I(t) = \sum_{n=1}^{N} c_n \cos(\omega_n t + \theta_n) \]  \hspace{1cm} (11)

\[ Q(t) = \sum_{n=1}^{N} c_n \sin(\omega_n t + \theta_n) \]  \hspace{1cm} (12)

For large \( N \), an application of the central limit theorem shows that the resulting sum of the in-phase and quadrature components \( I(t) \) and \( Q(t) \) is a Gaussian random process. The in-phase and quadrature components have mean \( \mu = E[I(t)] = E[Q(t)] \) and variance \( \sigma^2 = \text{Var}[I(t)] = \text{Var}[Q(t)] \), respectively. The received signal amplitude, the quantity in which we are interested, is the square root of the sum of the squares of these components. Because the in-phase and quadrature components are both Gaussian, the square of their sum follows a chi-square distribution, and the square root follows a chi distribution. The chi distribution with two degrees of freedom is the Rayleigh distribution. In cases where the line-of-sight component is very weak, as one might expect from signals broadcast from low altitudes and received at low reception angles, the mean \( \mu \approx 0 \), and the received signal amplitude is approximately Rayleigh distributed. As the line-of-sight component strengthens,
the signal amplitude more closely approximates a Rician distribution. However, the scattering resulting in the multipath interference assumed to be present in this particular case is expected to be predominately Rayleigh-distributed. Rayleigh scattering is associated with atmospheric phenomena such as precipitation, haze, and clouds, and with multipath interference; both result in a relatively weak line-of-sight signal component (Sang-Hoon, Jeong-Hun, & Young-Mok, 2016; Van Der Pryt & Vincent, 2015).

The autocorrelation function is

$$ R_s(\tau) = \frac{E_0}{2} E[\cos(\omega \tau)] \cos(\omega_0 \tau) $$

(13)

If we assume that \( \beta \) in Figure 7 is fixed and that \( \theta \) in (8) and (9) is uniformly distributed on \([0, \pi]\),

$$ E[\cos(\omega \tau)] \approx s_0(\tau) = \frac{E_0}{2} J_0 \left( \frac{2\pi \nu \tau}{\lambda} \right) $$

(14)

where \( J_0 \) is a zero-order Bessel function of the first kind.

The Doppler frequency shifts on the multipath rays caused by the relative motion between the aircraft and the ground-based receiver result in a Doppler power spectral density (DPSD), which is the Fourier transform of the autocorrelation function of the received signal. In Figure 7, \( \alpha_n \) and \( \beta_n \) represent the angles of the incident wave on the receiver relative to the aircraft with respect to the \( x \) and \( z \) axes. Given the assumptions noted previously, the deterministic DPSD is given by

$$ S(f) = \frac{E_0}{4\pi f_{\max}} \frac{1}{\sqrt{1 - \left( \frac{f}{f_{\max}} \right)^2}} $$

(15)

where \( |f| < f_{\max} \), the maximum Doppler frequency. This frequency is given by

$$ f_{\max} = f_c \frac{v}{c} $$

(16)

where \( f_c \) is the 1090 MHz carrier frequency, \( v = v_{\text{SOURCE}} \) is the velocity of the aircraft relative to the receiver, and \( c \) is the speed of light. From Jakes (1974), the peak amplitude of the received electric field vector, \( E_{0p} \) is

$$ E_{0p} = 2 \sqrt{\text{Var} [I(t)]} = 2 \sqrt{\text{Var} [Q(t)]}. $$

(17)

2.4. The filtering and estimation process

The Doppler power density spectrum that represents the received signal is characterized by a rather sharp cutoff at a particular (variable) maximum Doppler frequency. This type of frequency spectrum lends itself well to low-pass filtering to remove unwanted high-frequency components related to noise. Consequently, the use of a digital adaptive first-order low-pass Butterworth filter to filter the received voltage vector, \( V_{\text{REC}} \) is proposed. This filter is relatively easy to implement from a programming perspective and requires a single coefficient that may be optimized using an appropriate optimization algorithm. The optimization algorithm adjusts the coefficient to minimize the mean square distance estimation error over a set of signal strength values from aircraft of known distances. The particular optimization algorithm used is the bounded, limited-memory Broyden-Fletcher-Goldfarb-Shanno algorithm (L-BFGS-B) (Byrd, Lu, Nocedal, & Zhu, 1995). This algorithm is available as a function in R, offers a reasonably rapid rate of convergence, and provides the ability to set boundaries on the search. Because the condition \( \alpha > 0 \) must be true, this is imposed as a constraint on the optimization routine to reduce computation time.
A first-order Butterworth filter has a normalized s-domain transfer function of

\[ H_N(s) = \frac{1}{s + 1} \]  

(18)

The conversion of an analog filter design to a digital design may be accomplished by an impulse-invariant design procedure suggested by Oppenheim and Schafer (Oppenheim & Schafer, 2013). The resulting difference equation is

\[ Y_k = \frac{K}{1 + K} X_k + \frac{K}{1 + K} X_{k-1} + \frac{1 - K}{1 + K} Y_{k-1}. \]  

(19)

where

\[ K = \tan^{-1}(\pi c). \]  

(20)

The parameter \( K \) incorporates both the sampling frequency and the cutoff frequency. The sampling frequency, as described earlier, is 2.0 MHz. The appropriate design cutoff frequency can be estimated from Figure 8 (for example, an aircraft velocity of 120 knots results in a DPSD frequency of at least 220 Hz, requiring a considerably higher cutoff frequency, since the filter has a single-pole rolloff). For a nominal \( K \) value of 0.01, for example, the cutoff frequency of the digital filter can be calculated from (20), and is 6.366 kHz. However, it is unnecessary to specify this frequency. Because both of these parameters are contained in (20), the parameter \( K \) can simply be calculated using the L-BFGS-B algorithm with boundary constraints over a data subset of known distances. Thus, the optimal value of \( K \) can be determined and used over the remainder of the data-set.

The estimated received power at the SDR, \( \hat{P}_{\text{SDR}} \), to be used to compute received distance, may be determined using the Rayleigh maximum-likelihood estimate for the signal variance,

\[ \hat{\sigma}^2 = \frac{1}{2N} \sum_{k=1}^{N} Y_k^2 \]  

(21)

Figure 8. Doppler power density spectrum with overlay of first-order Butterworth filter \( (f_c = 220 \text{ Hz}) \).
which yields

\[ P_{SDW} = \frac{\overline{Y^2_k}}{2} \]  

where \( \overline{Y^2_k} \) is the average value of the squares of the individual values \( Y_k \in \{ Y_1, \ldots, Y_N \} \) of the filtered signal strength vector. This value serves as input to a deterministic channel model represented by the Friis transmission equation, which represents various power losses in the transmission channel, and may be used in conjunction with (6) and (7) to obtain the distance estimate, \( \hat{d} \).

A depiction of a typical Doppler power density spectrum with an overlaid squared magnitude plot of a first-order Butterworth filter is provided in Figure 8. Note that the cutoff frequency, \( f_c \), is 220 Hz, and that the squared magnitude of the filter response curve intersects the DPSD at \( S(f_{\max})^2 \).

The filtering process described here is an adaptive process that can be optimized over large data sets to yield the desired levels of accuracy of \( \hat{d} \). It is important to realize that the tuning of the filter in an optimal manner based on existing environmental and positioning conditions can occur in a dynamic manner such that the roughly 7% of aircraft broadcasting Extended Mode S data with position information can be used to calibrate the filter parameters for the 89% of aircraft broadcasting Mode A, Mode C or Elementary Mode S signals.

3. Results and discussion

Figure 9 illustrates the filtering process using the previous example of the departure of N170TH from Runway 23 at KLAF to show received signal strength as a function of time. The scaled output of the adaptive Butterworth filter is overlaid on the graph, as is the output of the Rayleigh estimator. The effect of the filtering and estimating process is easy to discern from the figure. The resulting distance estimate is extremely accurate within 2.5 nm of the receiver, with somewhat reduced accuracy outside of that distance. Note that the distance errors appear to correlate strongly with excursions of the Rayleigh estimate (and of the filtered signal), suggesting that the use of a higher-order filter with sharper cutoff properties may further improve the distance estimate.

It is also interesting to note the variation in signal clustering with distance in Figure 9. Again, the dispersion in the signal level values is larger when the aircraft is departing the airport at low altitudes due to multipath interference from buildings, terrain, and other obstructions. This is clearly indicative of Rayleigh fading. As the aircraft climbs and proceeds away from the airport, the dispersion decreases as the line-of-sight signal components increase relative to the non-line-of-sight components, a situation that is more indicative of Rician fading. While a difference maximum likelihood variance estimator may be more appropriate in the case of Rician fading, the distance estimation application of interest is the counting of aircraft operations at airports. This particular application is concerned primarily with aircraft that are close to the receiver (at the airport); hence, the Rayleigh estimator is preferred.

The distance estimation error was defined as \( \overline{e_{total}} = \sum_{i=1}^{N} (d_i - \hat{d}_i) \). The estimation error was calculated for two data sets; the first data-set (Li, Mott, & Bullock, 2017b, 2017c) was collected over a period of 30 days in 2016 from a permanent antenna-receiver installation atop the Terminal building (TERM) at the Purdue University Airport in Lafayette, Indiana, while the second was collected from a temporary antenna-receiver combination in Niswonger Hall (NISW) at the same airport over a period of 30 days in 2016.

The estimation error using data collected from the TERM installation over a 30-day period fit a Pearson Type VII distribution (a leptokurtic distribution to which the Student’s t-distribution is related) with mean –0.192 nm, variance 1.265, skewness –2.0, and kurtosis 16.95. The standard deviation was 1.12 nm. The median absolute deviation was 0.92 nm. Median absolute deviation (the median of the absolute value of the difference of the residuals from the median) is preferred as a
measure of dispersion because of its relatively greater resilience to outliers than that of the standard deviation (Leys, Ley, Klein, Bernard, & Licata, 2013).

A total of 102,079 distinct observations were recorded over this period. A histogram of the distance estimation error for the TERM installation, with an overlay of the corresponding Pearson Type VII distribution, is shown in Figure 10.
The distance estimation error using data collected over a different 30-day period from the NISW installation (Li, Mott, & Bullock, 2017a) was also approximately Pearson Type VII-distributed. The mean error over a 30-day period of the NISW data was 1.75 nm, while the variance, skewness, and kurtosis were 20.2, 5.5, and 48.6, respectively. The median absolute deviation was 1.12 nm. There were 57,066 distinct observations over this period.

The decrease in distance estimation accuracy of the NISW data relative to the TERM data may be attributed to a somewhat suboptimal antenna placement that was used in the collection of this particular data-set. The NISW receiving antenna was located indoors and was placed out of necessity in a horizontal position, which is not optimal for the reception of vertically-polarized transponder signals.

A total of 115,858 observations were recorded during this period. A histogram of the distance estimation error for the NISW data with an overlay of the Pearson Type VII distribution is depicted in Figure 11.

Table 2 provides the distance estimation error results in tabular format. Based on the comparison between the TERM results and NISW results, it is reasonable to assume that careful placement of the receiving antenna may improve the results.

The 30-day data sets from both the TERM and NISW installations were then used to compute aircraft operations counts estimates, using the methodology described by Mott and Bullock (2017). The 30-day estimated operations counts for the TERM installation, based on the distance estimates described in this section, differed from FAA-established counts over the same period by −10.2%, with a total of 12,177 actual operations. A Kolmogorov-Smirnov test comparing the empirical cumulative distributions of the estimated and FAA-established counts resulted in a p-value of 0.39, suggesting that the null hypothesis that the samples were obtained from the same distribution cannot be rejected. The 30-day estimated operations counts for the NISW installation likewise differed from the FAA counts by 7.65%, with a total of 7,877 actual operations. The p-value from the associated K-S test was 0.80, again suggesting the inability to reject the null hypothesis.
4. Conclusion and future research

The research presented here describes a means of estimating aircraft distances from a receiver at an airport using transponder signal strength information. The received signal strength data are processed using a digital adaptive first-order low-pass Butterworth filter in tandem with a Rayleigh maximum likelihood estimator of the signal variance. The single filter coefficient is calibrated by minimizing the mean squared error between estimated and computed distances of aircraft reporting GPS-based coordinates. Once the optimal calibration factor is calculated, the filter may be applied to accurately estimate the distances from the receiver of aircraft that are not reporting GPS-based positions.

Utilizing median absolute deviation as a metric, the resulting errors between distance estimates and calculated distances range from 0.92 nm at one installation to 1.12 nm at a second. Given the results from the comparisons of the estimated operations counts from these two installations over two separate data collection periods, it is reasonable to assume that the distance estimation method described herein produces estimates of sufficient accuracy to be useful in computing operations counts estimates at nontowered airports.

A limitation of the research that was conducted is related to the digital Butterworth filter. The order of the single-pole filter design that was implemented in this study could be increased to potentially improve the distance estimation results. A higher-order filter exhibits sharper frequency cutoff characteristics and could potentially result in more consistent values of the parameter $K$, thereby generating distance estimates of greater accuracy. In addition, this study did not employ the use of any potential correlation between successive received transponder records in the distance estimation process. While these records are substantially separate from one another in time relative to the spacing between the signal level values in a single record, it may be worthwhile to examine correlations between records to determine whether the use of that information could also improve the distance estimation process.

Also, it is possible to modify the maximum-likelihood variance estimator such that it transitions from a Rayleigh estimator to a Rician estimator as a function of distance to better match the scattering characteristics of the received signal to aircraft operating at a greater distance from the receiver.

It is apparent, then, that the use of the discrete adaptive Butterworth filter in conjunction with the Rayleigh maximum likelihood estimator provides a satisfactory means of estimating aircraft distances with sufficient accuracy to be useful in computing operations counts at nontowered airports. It is hoped that the suggestions for future research presented in this section will lead to improvements in the estimation algorithm and corresponding increases in aircraft distance estimation accuracy. It is also desired that additional applications for the estimation technique will be identified and implemented in the near future.

**Table 2. Distance estimation errors**

| Distance estimation method                  | Median absolute deviation (nm), KLAF-TERM | Median absolute deviation (nm), KLAF-NISW |
|--------------------------------------------|------------------------------------------|------------------------------------------|
| Adaptive Butterworth Filter + Rayleigh MLE | 0.92                                     | 1.12                                     |

**Funding**
The author received no direct funding for this research.

**Author details**

John H. Mott
E-mail: jhmott@purdue.edu
ORCID ID: http://orcid.org/0000-0002-2087-3971

**Citation information**
Cite this article as: Estimation of aircraft distances using transponder signal strength information, John H. Mott, Cogent Engineering (2018), 5: 1466619.
