Cluster Structure of Disoriented Chiral Condensates in Rapidity Distribution

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Abstract

We study the creation of disoriented chiral condensates with some initial boundary conditions that may be expected in the relativistic heavy ion collisions. The equations of motion in the linear $\sigma$-model are solved numerically with and without a Lorentz-boost invariance. We suggest that a distinct cluster structure of coherent pion production in the rapidity distribution may emerge due to a quench and may be observed in experiments.

PACS numbers: 11.30.Rd, 12.38.Mh, 14.40.Aq

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*This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Divisions of High Energy Physics and Nuclear Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098, and by the Natural Sciences and Engineering Research Council of Canada.

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Recently much attention has been paid to an interesting proposal to observe a disoriented chiral condensate (DCC) in high energy collisions [1, 2, 3, 4]. Such events can be signaled by a coherent pion emission along some particular isospin direction in the collision domain. A probability distribution of neutral pions is characterized by a non-binomial function if all isospin directions are equally probable,

\[ P(r) = \frac{1}{2} \frac{1}{\sqrt{r}}, \]  

where \( r = n_{\pi^0}/(n_{\pi^0} + n_{\pi^\pm}) \). This behavior may have already been observed in the so-called Centauro events in cosmic ray collisions [5]. The central theoretical work is to understand the mechanism for creating such a DCC domain in relativistic heavy ion collisions. Rajagopal and Wilczek [6] have suggested that the expansion of highly relativistic debris from collisions “quenches” the high temperature field configurations such that the long wavelength modes will grow with time, leading to a correlated domain. It is recently reported, however, that the correlation size may be very small [7].

In this paper, we shall study the creation of the disoriented chiral condensate with a more realistic initial condition that may be expected in the high energy collisions. We shall re-examine the idea of a quench based on a linear \( \sigma \)-model and suggest that indeed, following a quench, correlated pion fields will populate near the light cone where the leading collision particles are expanding at speed of light. The production of the unusually rich (or poor) \( \pi^0 \)'s will cluster into groups in the rapidity distribution. We are inspired by the consistency of our results with some recent events of JACEE experiments [8] and confirm the general picture of a “Baked Alaska” sketched by Bjorken, Kowalski and Taylor [9].

We assume that following a quench the possible DCC is to be described by classical low energy effective interactions of pions at zero temperature

\[ \mathcal{L} = \int d^4x \left\{ \frac{1}{2} \partial_\mu \phi_i \partial^\mu \phi_i - \frac{\lambda}{4} (\phi_i \phi_i - v^2)^2 + H\sigma \right\} \]  

where \( \phi_i \equiv (\sigma, \pi) \) stands for a vector in internal space. \( H\sigma \) is an explicit chiral symmetry-breaking term which is responsible for the mass of the pseudo-goldstone
bosons, the pions. We look for a solution to the classical equations of motion of \( \phi \) with a given initial condition. Clearly the initial condition is essential to determine whether or not there will be DCC in the system. For example, if the \( \phi \) and \( \dot{\phi} \) initially align in the \( \sigma \) direction throughout the space, then the system does not have any pion field at any time. What might be the typical initial configurations? In a quench, the system begins at a temperature well above the chiral phase transition point \( T_c \). There are thermal fluctuations in all internal directions. Therefore, it is appropriate to assume that initially the space averages \( \langle \phi \rangle \sim 0 \) and \( \langle \dot{\phi} \rangle \sim 0 \) but \( \langle \phi^2 \rangle \neq 0 \) and \( \langle \dot{\phi}^2 \rangle \neq 0 \) both for \( \sigma \) and \( \pi \) fields. In addition, we are modelling a situation applied to highly relativistic collisions. The system should satisfy the space-time geometry of such a collision where the incident nuclei are, shortly after the collision, highly Lorentz-contracted “pancakes” receding in opposite longitudinal direction from the collision point \([10]\). An approximate \( 1+1 \) Lorentz invariance of the system is indeed inspired by the existence of a central-plateau structure in the rapidity distribution of produced particles in high energy cosmic-ray events \([11]\) and \( pp \) or \( p\bar{p} \) collisions \([12]\). Of course, the above picture should be modified at large transverse distance, comparable to the nuclei radii. However, in most of the transverse area, the field changes rather slowly in the transverse directions compared with the rapid longitudinal expansion. Bearing this approximation in mind, we shall assume a uniform field in the transverse direction, i.e. \( \phi \) is only a function of \( t \) and \( z \). In terms of more convenient coordinates, the proper time \( \tau = \sqrt{t^2 - z^2} \) and the (spatial)rapidity variable \( \eta = \frac{1}{2} \ln \frac{t+z}{t-z} \), the equations of motion read,

\[
\frac{1}{\tau} \frac{\partial}{\partial \tau} \left( \tau \frac{\partial \sigma}{\partial \tau} \right) - \frac{1}{\tau^2} \frac{\partial^2 \sigma}{\partial \eta^2} = -\lambda \sigma \left( \sigma^2 + \pi^2 - v^2 \right) + H; \quad (3)
\]

\[
\frac{1}{\tau} \frac{\partial}{\partial \tau} \left( \tau \frac{\partial \pi}{\partial \tau} \right) - \frac{1}{\tau^2} \frac{\partial^2 \pi}{\partial \eta^2} = -\lambda \pi \left( \sigma^2 + \pi^2 - v^2 \right). \quad (4)
\]

Given an initial condition, Eqs. (3) and (4) completely determine the space-time evolution of the system.

The simplest case to solve Eqs. (3) and (4) is when the system has an initial Lorentz-boost invariance, i.e.,

\[
\phi(\tau_0, \eta) = \phi_0, \quad (5)
\]
where $\phi_0$ is independent of $\eta$. The lack of $\frac{\partial}{\partial \tau} \frac{\partial}{\partial \eta}$ in the Klein-Gordon operator guarantees that Eq. (3) is sufficient to maintain the boost invariance at any time (there is no need to impose $\frac{\partial}{\partial \tau} \phi(\tau_0, \eta) = \text{constant}$). To model a quench, we assume that the system is initially in a symmetric phase and lies on the top of the “Mexican Hat” of the potential,

$$\pi(\tau_0) = 0, \quad \sigma(\tau_0) \simeq 0.$$  

(6)

To obtain a non-trivial solution for $\pi$ field, we allow a small initial “kick” of the system, i.e. a non-zero velocity $\frac{\partial \phi}{\partial \tau}(\tau_0) \neq 0$. In this case, since the system starts from one point $z \sim 0$ at $t \sim 0$ (the rapidity dependence of $\phi$ decouples and the motion is zero-dimensional), the correlation in isospin direction is automatically 100\%. That is, if the initial kick is in $\pi^0$-direction, then $\pi^\pm = 0$ at all time such that there will be only $\pi^0$ production. A numerical solution is plotted in Fig. 1 where we have used the following standard parameter input: $v = 87.4 \text{ MeV}$, $H = (119 \text{ MeV})^3$ and $\lambda = 19.97$ so that $f_\pi = 92.5 \text{ MeV}$, $m_\pi = 135 \text{ MeV}$ and $m_\sigma = 600 \text{ MeV}$. The initial condition we used in Fig. 1 is $\phi(\tau_0) = 0$, $\frac{\partial \phi(\tau_0) / \partial \tau = (1, 5, 0, 0)}{\text{MeV/fm}}$, at $\tau_0 = 1 \text{ fm}/c$. The general feature of the solution is following. The $\sigma$ field grows from zero and takes about 1 fm/c proper time to reach the true vacuum expectation value $\langle \sigma \rangle \sim f_\pi$, while the $\pi$ field oscillates around zero rather slowly and eventually tends to zero when the proper time gets large $\tau = \tau_{\text{max}} \sim 30 \text{ fm}/c$. In terms of the real space-time variables $t$ and $z$, the disoriented chiral condensate develops in the interior region of the receding pancakes, surrounded by the normal vacuum in the exterior; as time ($t$) evolves, the DCC will be squeezed toward the light cone, leaving behind a normal vacuum where $\langle \pi \rangle = 0$ in the central region inside the light cone. For $t \gg \tau_{\text{max}}$, the $\pi$ field is found only in a small region $\Delta z \simeq \frac{\tau_{\text{max}}^2}{2t}$ near the light front while the $\sigma$ field occupies most of the space inside the light cone, surrounded by a thin shell of DCC. This picture supports a space-time geometry of DCC sketched in [9].

While the essential feature of the above solution may be generic, the real situation is more complicated. One complication is that the system may not start from one point shortly after the collision and there may not be a Lorentz-boost invariance in the system. In this case, the initial condition like Eq. (3) is not appropriate. If the system
initially has a distribution in rapidities and is not correlated in isospin directions, the
the crucial question would be whether a quench can yield a coherent pion field at a
later time.

We assume that shortly after the collision, the typical configuration is that of a
thermal random fluctuation at high temperature. The fluctuation is governed by the
temperature. Similarly to Ref. [6], we choose \( \phi(\tau_0) \) and \( \partial \phi(\tau_0)/\partial \tau \) randomly according
to gaussian distributions so that
\[
\langle \phi(\tau_0, \eta) \rangle_\eta = \langle \frac{\partial}{\partial \tau} \phi(\tau_0, \eta) \rangle_\eta = 0
\]
but \( \langle \phi^2(\tau_0, \eta) \rangle_\eta = v^2/4 \) and \( \langle (\frac{\partial}{\partial \tau} \phi(\tau_0, \eta))^2 \rangle_\eta = v^2 \). By solving equations (3) and (4),
we find that as the proper time evolves, the average value \( \langle \sigma \rangle_\eta \) grows from zero quickly
and takes its final value (unlike the case with boost-invariant initial c ondition, the
oscillation of \( \langle \sigma \rangle_\eta \) around \( f_\pi \) is quickly damped at large \( \tau \); \( \langle \pi \rangle_\eta \) grows from zero and
oscillates around zero slowly in the same way as in Fig. 1. Again, \( \langle \pi \rangle_\eta \) decreases to
zero at about \( \tau \simeq 20 \sim 30 \) fm/c. This clearly indicates a strong correlation among
the isospin directions in rapidities. Indeed, we can define a correlation function at a
given \( \tau \),
\[
C(\tau, \eta - \eta') = \frac{\pi(\tau, \eta) \cdot \pi(\tau, \eta')}{\pi^2(\tau, \eta) + \pi^2(\tau, \eta')} .
\]
We find that after about 1 fm/c evolution in proper time (which is the time scale
that it takes for the average value of \( \sigma \) field to reach its final value \( \langle \sigma \rangle_\eta \sim f_\pi \)),
\( C(\tau, \eta - \eta') \) changes from a zero value corresponding to a typical random distribution
to an exponential distribution with non-vanishing width as shown in Fig. 2. For small
\( \Delta \eta = \eta - \eta' \), \( C(\tau, \Delta \eta) \) can be fit by an exponential function
\[
C(\tau, \Delta \eta) \propto \exp(-c_0 m_\pi \tau |\Delta \eta|) \quad (\tau - \tau_0 \geq 1 \text{ fm/c})
\]
where \( c_0 \sim 0.4 \). At \( \tau > 20 \sim 30 \) fm/c, the correlation disappears as indicated by
the average value \( \langle \pi \rangle_\eta \). A similar result is derived in [13] based on a non-linear \( \sigma \)-
model in 1+1 dimensions where the pion fields are the phases of the order parameter.
However, in our model, we find that if initially \( \langle \phi \rangle^2_\eta \simeq f_\pi^2 \), a correlated distribution
does not occur. This clearly shows that the emergence of a correlation in isospin orientation is mainly due to the unstable long wavelength modes of the pion fields when $\langle \phi \rangle^2 < f_\pi^2$. Our result unambiguously points to the success of the “quench” mechanism for creating DCC (i.e. a high temperature initial condition plus zero temperature equations of motion).

Eq. (9) indicates that a rapidity interval in which the pion field is correlated in isospin direction could be as large as $2 \sim 3$. Our conclusion coincides with the recent cosmic ray experiment [8] where the dominant neutral pions are found in an interval $\Delta \eta \sim 3$. Eq. (9) also suggests that the correlation occurs mainly in the region near the light cone where $\tau$ is small. At a given large $t$, a region of ordinary vacuum where $\langle \pi \rangle_\eta$ vanishes effectively is found inside a shell of coherent pion field. However, the cluster structure of pions radiated from the coherent pion field may occur anywhere in the whole rapidity region. We should also emphasize that the width of the cluster size in our study depends on the initial time $\tau_0$ when the quenching happens. For sufficiently large initial time, the width can become very small.

Although the size of correlated DCC domains may be small in space, especially at a large $t$ because of the contracting effect due to the relativistic expansion, we conclude that the DCC production can be distinctly observed as a cluster structure in rapidities. What is less clear is whether there is indeed a quench after the collision. Strictly speaking, the quench condition is an idealization of the more complex situation where the temperature relaxes to zero only gradually. For example, the hydrodynamics suggests a temperature drop according to $T = T_0 (\tau_0 / \tau)^{1/3}$ [10]. In this case, equations (3) and (4) may not be appropriate. Clearly, more theoretical work is needed to examine if DCC can be created in the process of cooling.

**Acknowledgements**

We wish to thank M. Asakawa, J. Bjorken, M. Suzuki and Dandi Wu for very useful discussions. This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Divisions of High Energy Physics and
Nuclear Physics of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098, and by the Natural Sciences and Engineering Research Council of Canada.
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**Figure Captions**

FIG. 1. Proper time evolution of $\sigma$ and $\pi_1$ fields following a Lorentz-boost invariant initial condition at $\tau_0 = 1 \text{ fm}/c$.

FIG. 2. The correlation function $C(\tau, \Delta \eta)$ of DCC. The initial configuration is that of a thermal fluctuation at $\tau_0 = 1 \text{ fm}/c$. 

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Fig. 1
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Fig. 2