Phase transition between the isovector to isoscalar pairing correlations in deformed $N = Z$ nuclei

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Abstract. We investigate the pairing correlations on $N = Z$ nuclei. In particular, the isoscalar (IS) pairing mode in neutron-proton ($np$) pairing is discussed in detail because the IS $np$ pairing is closely related to the tensor force in the nucleon-nucleon interaction. Recent experiments demonstrated that the IS pairing might be enhanced from the M1 spin strength distribution data and deuteron, which is a consequence of the IS pairing, can be detected in $^{16}O(\text{p}, \text{p}^\prime \text{d})^{14}N$ reaction. The present study illustrates that the IS $np$ pairing can be condensed not only in $S=0, L = \text{odd states}$ but also in $S=1, L = \text{even states}$. The former is usually treated as $T=0$ $np$ pairing. The latter is associated with deuteron and has not been fully treated in the BCS theory for the nuclear structure. In particular, the latter $np$ pairing can be ejected as deuteron from the energetic probes in the experiments. Detailed analyses of the $np$ pairing are performed mainly on $^{16}O$ and extended to other $N = Z$ nuclei.

1. Introduction

Pairing interactions inside nuclei originate from the residual interaction remained in a mean field [1]. One expects two types of the pairing interactions, like-pairing from the neutron-neutron ($nn$) and the proton-proton ($pp$), and unlike-pairing from the neutron-proton ($np$) interaction. The like-pairing has only isovector ($T=1$) mode, while the unlike-pairing has both isovector (IV) and isoscalar (IS) modes. In the coupling scheme of its time reversed state the unlike IV pairing has $J = \text{even couplings}$ and the unlike IS coupling has $J = \text{odd couplings}$ with their allowed momenta as summarized in Table I. Main contribution to the unlike-pairing is believed to come from the IV spin-singlet ($T=1$, $J = 0$) and IS spin-triplet ($T= 0$, $J = 1$) coupling, but the contributions beyond $J \geq 1$ cannot be negligible [2, 3]. In deformed nuclei, this angular momentum is not a good quantum number due to the rotational symmetry breaking. This is prescribed by the angular momentum projection $\Omega$ which can be a good quantum number with the parity in the Nilsson basis.

The IS spin-triplet $np$ mode ($T=0$, $S=1$) in Table 1 is closely related to tensor force. The tensor force in the nucleon-nucleon ($N$-$N$) interaction is one of important ingredients...
for understanding the nuclear force. Deuteron ground state could \((J^\pi = 1^+, T = 0)\) not be understood without the tensor force. Also numbers of the phase shift analysis of N-N scattering data demonstrate the attractive tensor force for \(3S_1\) and \(3D_1\) state. Effects of the tensor force are being extensively studied by many papers. Recent study regarding the roles of the tensor force inside nuclei demonstrated that the tensor interaction is attractive between \(j_\prec\) and \(j_\succ\) states, but it is repulsive for \(j_\prec\) and \(j_\succ\) states \([4, 5]\). The tensor force is thought to be mostly repulsive and plays important roles in the competition of the residual interaction against the mean field \([9, 10, 11]\). But, the shell evolution by the deformation of nuclei may affect the tensor force property.

Along these lines, recent experiments searching directly the tensor force effect are quite important. For example, Ref. \([6]\) exhibited that the tensor force can manifest itself by the deuteron detection through \((p,pd)\) reactions on \(^{16}\)O target. They found a strong population of the \(J=1, T=0\) state and a very weak population of \(J=0, T=1\) state by observing large cross sections due to the excitations (corresponding to \(L=0\) and \(2\) transitions) in the residual nucleus \(^{14}\)N. It implies that the unlike-pairing correlations coupled through a higher (relative) angular momentum state are inevitable to search for the deuteron breakup reactions and also for studying the pairing correlations inside nuclei. Not only proton beams, but also electron beams are also available for looking for the deuteron breakup related to the tensor force inside nuclei as suggested in Ref. \([7]\).

On the other hand, the study of the unlike-pairing correlations inside nuclei have been continued to find some evidences in \(N = Z\) nuclei because one expects a strong overlap of neutron and proton wave functions in \(N \sim Z\) nuclei. For example, Ref. \([12]\) has argued that some of heavy \(N = Z\) nuclei may have IS dominance or coexistence with IV mode. Our previous papers discussed that even \(sd-\) and \(pf-\) shell nuclei may have such IS dominance for some deformed nuclei, which depends on the IS pair strength \([14, 15]\). Recent data of the M1 spin transition \([8]\) reported that the IV contribution may be strongly quenched, and consequently the IS pair strength can be enhanced in \(sd-\) shell \(N = Z\) nuclei. It implies that the IS pair can be condensed inside nuclei. We demonstrated that the enhanced IS pairing brings about the IS condensation with the deformation in the \(N = Z\) nuclei. In fact, the relationship between the IS pairing in the tensor force and the deformation is not clear because of the shell evolution along the deformation. In particular, the competition of the pairing interaction in the residual interaction and the deformation in the mean field remains to be discussed. In this work, we focus on the unlike-pairing correlations keeping higher angular momentum by taking Goswami formalism for the pairing interaction in a deformed BCS approach (DBCS) based on the coupling of a time reversal state.

| Pairing Types | T   | S     | L             | J            | Allowed |
|---------------|-----|-------|----------------|--------------|---------|
| Like          | \(T = 1\) | \((\alpha \bar{\alpha}) S = 0\) | \(L = 0.2,4,\ldots\) (E) | \(J = 0.2,4,\ldots\) (E) | Y       |
|               | \((\alpha\alpha)(\bar{\alpha}\bar{\alpha}) S = 1\) | \(L = 1.3,5,\ldots\) (O) | \(J = 0,1,2,3,\ldots\) (E and O) | No       |
| Unlike        | \(T = 1\) | \((\alpha \bar{\alpha}) S = 0\) | \(L = 0.2,4,\ldots\) (E) | \(J = 0.2,4,\ldots\) (E) | Y       |
|               | \((\alpha\alpha)(\bar{\alpha}\bar{\alpha}) S = 1\) | \(L = 1.3,5,\ldots\) (O) | \(J = 0,1,2,3,\ldots\) (E and O) | No Data |
|               | \(T = 0\) | \((\alpha \bar{\alpha}) S = 0\) | \(L = 1.3,5,\ldots\) (O) | \(J = 1.3,5,\ldots\) (O) | Y       |
|               | \((\alpha\alpha)(\bar{\alpha}\bar{\alpha}) S = 1\) | \(L = 0.2,4,\ldots\) (E) | \(J = 1,1,2,3),(3,4,5,\ldots\) (E and twice O) | Y       |

Table 1. IV and IS pairing scheme for like- and unlike-pairing interactions. Deuteron ground state \((J^\pi = 1^+, T = 0)\) is composed of \((LS,T) = (0,1,0) = 3S_1\) and \((2,1,0) = 3D_{1,2,3}\) in the last low.
2. Formalism

Since the theoretical framework for the DBCS had already been detailed in our previous papers [14, 15], we briefly summarize the basic formula. We start from the following nuclear Hamiltonian

\[
H = H_0 + H_{\text{int}},
\]

\[
H_0 = \sum_{\rho_\alpha\alpha'} c_{\rho_\alpha\alpha'} c_{\rho_\alpha\alpha'}^\dagger,
\]

\[
H_{\text{int}} = \sum_{\rho_\alpha\beta_\alpha'\rho_\beta\beta'} V_{\rho_\alpha\beta_\alpha'\rho_\beta\beta'} \rho_\alpha\beta_\alpha' |\gamma\gamma'\rangle \langle \gamma\gamma'| c_{\rho_\alpha\beta_\alpha'} c_{\rho_\beta\beta'} c_{\rho_\alpha\alpha'}^\dagger c_{\rho_\beta\beta'}^\dagger,
\]

where Greek letters denote proton or neutron single particle states (SPSs) with a projection \( \Omega \) of a total angular momentum on a nuclear symmetry axis. \( \rho_\alpha (\rho_\alpha = \pm 1) \) is a sign of the total angular momentum projection of a state. Isospins of the particles are denoted by Greek letters with prime.

The operator \( c_{\rho_\alpha\alpha'}^\dagger (c_{\rho_\alpha\alpha'}) \) in Eq. (1) stands for a usual creation (destruction) operator of a real particle in the state of \( \alpha \rho_\alpha \). The Hamiltonian, represented by real particles in Eq. (1), is then transformed to a quasiparticle representation by the following deformed BCS(DBCS) transformation for the \( \alpha \) state

\[
\begin{pmatrix}
    a_1^\dagger \\
    a_2^\dagger \\
    a_3 \\
    a_4
\end{pmatrix}_\alpha = \begin{pmatrix}
    u_{1p} & u_{1n} & v_{1p} & v_{1n} \\
    u_{2p} & u_{2n} & v_{2p} & v_{2n} \\
    -v_{1p} & -v_{1n} & u_{1p} & u_{1n} \\
    -v_{2p} & -v_{2n} & u_{2p} & u_{2n}
\end{pmatrix}_\alpha \begin{pmatrix}
    c_1^\dagger \\
    c_2^\dagger \\
    c_3 \\
    c_4
\end{pmatrix}_\alpha,
\]

where \( u \) and \( v \) coefficients are calculated by the following 4 \times 4 DBCS equation

\[
\begin{pmatrix}
    \epsilon_p - \lambda_p & 0 & \Delta_{\rho\rho} & \Delta_{\rho\overline{n}} \\
    0 & \epsilon_n - \lambda_n & \Delta_{\rho\overline{n}} & \Delta_{\overline{n}\rho} \\
    \Delta_{\rho\rho} & \Delta_{\rho\overline{n}} & -\epsilon_p + \lambda_p & 0 \\
    \Delta_{\rho\overline{n}} & \Delta_{\overline{n}\rho} & 0 & -\epsilon_n + \lambda_n
\end{pmatrix}_\alpha \begin{pmatrix}
    u_{\alpha'n'} \\
    u_{\alpha'n} \\
    v_{\alpha'n'} \\
    v_{\alpha'n}
\end{pmatrix}_\alpha = E_{\alpha''} \begin{pmatrix}
    u_{\alpha''n'} \\
    u_{\alpha''n} \\
    v_{\alpha''n'} \\
    v_{\alpha''n}
\end{pmatrix}_\alpha.
\]

Here \( E_{\alpha''} \) is an energy of a quasiparticle 1 and 2 denoted as \( \alpha'' \) in the \( \alpha \) state. We include \( np \) and \( \overline{np} \) pairings in addition to the like-pairing (\( pp \) and \( \overline{nn} \)) correlations. The pairing potentials in Eq. (3) are permitted between the nucleons in a time-reversed state \( (\overline{\alpha\alpha}) \) [17]. But the unlike-pairing may have \( (\alpha\alpha) \) pairing as well as \((\overline{\alpha\overline{\alpha}}) \) pairing associated with the tensor force [18], which pairings are effectively included in the present framework discussed later on.

In the DBCS, the conventional quasiparticle is mixed with a particle state and its hole state. But, if we include the \( np \) pairing the quasiparticle is also mixed with additional mixing of proton and neutron. Furthermore the quasiparticle state in the present approach is mixed with different particle states in a spherical basis because each deformed state (basis) is represented by a linear combination of the spherical state (basis) (see Fig. 1 at Ref. [13]) up to higher \( J \) states keeping the angular momentum projection \( \Omega = 0 \) in the deformed pairing matrix elements. This feature is one of additional merits due to the inclusion of deformation in the DBCS approach.

The pairing potentials in Eq. (3) are calculated in the deformed basis by using \( G \)-matrix calculated from the realistic Bonn CD potential for the N-N interaction as follows

\[
\Delta_{\rho\rho} = \Delta_{\rho_{\overline{p}\rho}} = -\sum_{J,a,c} \left[ \sum_{\gamma} g_{\rho\rho} F_{\alpha\alpha\gamma}^{J0} F_{\gamma\gamma'}^{J0} G(\alpha\gamma, J, T = 1) (u_{1p}^*, v_{1p} + u_{2p}^*, v_{2p}) \right],
\]

\[
\Delta_{\rho_{\overline{n}\rho}} = \Delta_{\rho_{\overline{n}\overline{n}}} = -\sum_{J,a,c} \left[ \sum_{\gamma} g_{\rho_{\overline{n}\rho}} F_{\alpha\alpha\gamma}^{J0} F_{\gamma\gamma'}^{J0} G(\alpha\gamma, J, T = 1) \right] Re (u_{1n}^*, v_{1p} + u_{2n}^*, v_{2p}),
\]

\[
+ \left[ \sum_{J,a,c} g_{\rho_{\overline{n}\rho}}^{J0} F_{\alpha\alpha\gamma}^{J0} F_{\gamma\gamma'}^{J0} i G(\alpha\gamma, J, T = 0) \right] Im (u_{1n}^*, v_{1p} + u_{2n}^*, v_{2p}) ,
\]
where $F_{\alpha K}^{JK}$, $B_\alpha^a$, $B_\alpha^a$, $(-1)^J a_{\text{pp}}$, $C_{\text{pp}, \text{pp}, \text{pp}, \text{pp}}$, $C_{\text{pp}, \text{pp}, \text{pp}, \text{pp}}$ ($K = \Omega_\alpha - \Omega_\alpha$) was introduced with an expansion coefficient $B_\alpha$ [13]

$$B_\alpha^a = \sum_{N_{n_z} \Sigma} C_{\text{pp}, \text{pp}, \text{pp}, \text{pp}} N_{n_z} \Sigma \ b_{N_{n_z} \Sigma} \ A_{N_{n_z} \Sigma}^{N_{n_z} \Sigma} = <N_0 | \Lambda | N \bar{z} \Lambda > .$$

Detailed formula used for the coefficient $B_\alpha^a$ and the overlap integral $A_{N_{n_z} \Sigma}^{N_{n_z} \Sigma}$ are presented in Ref. [16]. The $T = 0$ pairing contribution is included as an imaginary term in the $np$ pairing potential in Eq.(5). $K$ is a projection number of a total angular momentum $J$ onto the $z$ axis and selected as $K = 0$. The Brueckner $G(aacc JT)$ matrix represents the state-dependent pairing matrix element (PME) calculated in the spherical basis. We sum up all possible $J$ values of the coupling of two-particle state assigned by $(aa)$ or $(cc)$ in the spherical basis, which has the $K = 0$ projection. This sum of $J$ values is due to the expansion of the deformed state by the spherical states $(a)$ or $(c)$. $\Lambda_{\text{odd}}$ is obtained from Eq. (4) by replacing $p$ by $n$.

As for the IS $np$ pairing we have two modes, spin-singlet ($S=0$) and spin-triplet ($S=1$), for which we take into account $J = \text{odd}$ (even and odd) cases for the $S=0$ ($S=1$) states, respectively. In fact we need an extended pairing scheme by $(\alpha \alpha)$ and $(\bar{\alpha} \bar{\alpha})$ components, which requires $8 \times 8$ transformation matrix instead of Eq.(3) [19]. Within the present $4 \times 4$ scheme, in which we include only $\bar{p}p$ and $\bar{p}p$ pairing correlations, we effectively take into account these $T = 0$ channel in the $\alpha$ and $\bar{\alpha}$ coupling by multiplying a factor 2 to the $T = 0$ pairing matrices for the $\alpha \bar{\alpha}$ configurations [19].

If we adopt the enhanced IS pairing due to the IV quenching, we may multiply another weighting factor to the $T=0$ PME as discussed below. In the previous paper, we exploited a factor 1.5 for the enhanced IS pair, where we summed up to $J = 1, 2, 3, ... J_{\text{max}}$. But it turns out that we have to take into account twice $J = \text{odd}$ and once $J = \text{even}$ as shown in the last low of Table 1. It reduces the factor to 1.0. Thus there remained still a room for the IS enhancement. Therefore we allow some variation of the strength parameter $g_{np}^{IS}$, which has a value ranging from 1.0 to 1.7. On the while, other strength parameters ($g_{pp}, g_{nn}, g_{np}^{T=1}$), which are a kind of the renormalization constant due to the finite Hilbert particle model space, are fitted to reproduce empirical pairing gaps by the odd-even mass difference.

**3. Results**

In the following, we discuss the IV and IS $np$ pairings in $^{16}$O. Specifically, we investigate the contribution of high angular momentum couplings to the pairings. First, in Fig. 1, we illustrate the ratio of the IV (IS) to the total $np$ pairing gaps in terms of the deformation parameter $\beta_2$. For the enhanced IS pairing, we multiplied a weighting factor $g_{np}^{IS}$ from 1.0 to 1.7. With the increase of the deformation, the IS contribution to the $np$ pairing increases. This trend appears significantly with the increase of the weighting factor. In particular, beyond $\beta_2 > 0.2$ the IS contribution is comparable to that of the IV for $g_{np}^{IS} = 1.7$. Right panels show that the $J > 1$ components which correspond to the $L > 2$ components in Table I are increased compared to those in the left panel. Therefore the $np$ paring in high angular momentum state turns out to be non-negligible.

Figure 2 demonstrates the contribution from higher angular momenta in terms of the $g_{np}^{IS} = g_{np}^{IS}$. In general, the contribution to the $np$ pairing gap from $L \geq 2$, which is given as the difference of solid and dashed red lines, is smaller than that by $L = 0$ and 1, as expected. But, with increasing the weighting factor, the high angular momenta contribution to the IS pairing increases. If we fix the strength from some experimental data, such as the M1 spin strength data, how large the contribution to the $np$ pairing would be clear. This trend is explicitly shown in Fig. 2. The difference between the IS $J=1$ and IS $J = 1, 2, 3, ...$ becomes salient with the deformation and the $g_{np}^{IS}$. For the $\beta_2 = 0.5$ and $g_{np}^{IS} > 1.6$, the IS dominates the $np$ pairing

$$\Gamma_{\text{np}}^{\text{IS}} = \frac{g_{np}^{IS}}{\text{const}} \frac{1}{\Omega_\alpha - \Omega_\alpha} \ .$$

In the following, we discuss the IV and IS $np$ pairings in terms of the deformation parameter $\beta_2$. For the enhanced IS pairing, we multiplied a weighting factor $g_{np}^{IS}$ from 1.0 to 1.7. With the increase of the deformation, the IS contribution to the $np$ pairing increases. This trend appears significantly with the increase of the weighting factor. In particular, beyond $\beta_2 > 0.2$ the IS contribution is comparable to that of the IV for $g_{np}^{IS} = 1.7$. Right panels show that the $J > 1$ components which correspond to the $L > 2$ components in Table I are increased compared to those in the left panel. Therefore the $np$ paring in high angular momentum state turns out to be non-negligible.

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That is, the IS pairing can dominate the np gap contributions to the np illustrated. For 64 nuclei \( N = \text{medium-heavy and heavy} \) are obtained up to \( J \) \( = 1 \) \( (J = J_{\text{max}}) \) for the unlike-pairing in Table I. Pairing strength is fitted to empirical np gap.

Figure 1. (Color online) Ratio of IV and IS pairing interaction gap to the total np pairing gap by the enhanced \( T = 0 \) pairing (a)-(d)) for \( ^{16}\text{O} \). The np pairing gap is calculated by \( \delta_{\text{np}}^\text{emp} = H' - H \), where \( H' \) (H) does (not) include the np pairing [13]. Results in the left (right) hand side are obtained up to \( J = 1 \) \( (J = J_{\text{max}}) \) for the unlike-pairing in Table I. Pairing strength is fitted to empirical np gap.

Figure 2. (Color online) Same as Fig. 1, but in terms of \( g_{\text{np}}^{\text{eff}} \).

correlations in the ground state of \( ^{16}\text{O} \), where the contribution of \( L > 2 \) states amounts to 20%.

The IS condensation is revealed not only in \( ^{16}\text{O} \), but also it appears in other \( N = Z \) nuclei. That is, the IS pairing can dominate the np pairing gap with the larger deformation region even for medium-heavy and heavy \( N = Z \) nuclei. Similar calculations for each contribution for other nuclei \( ^{64}\text{Ge} \) and \( ^{108}\text{Xe} \) have been done and presented in Fig. 3. Likewise, the IV and IS pairing gap contributions to the np pairing gap along the deformation with the enhanced IS pairing are illustrated. For \( ^{64}\text{Ge} \), we find that the IS contribution becomes larger with the deformation even in the normal strength (left panel in Fig. 3). With the increase of the \( g_{\text{np}}^{\text{eff}} \), the IS contribution dominates the pairing. Similar behavior of the IS np pairing can also be conformed for \( ^{108}\text{Xe} \) in right panel in Fig. 3. Even for normal pairing strength, the IS dominates for \( \beta_2 \geq 0.1 \). Therefore, the IS dominance in the np pairing is apparent in this heavy nuclei.
4. Summary
In summary, we discussed that the IS pair condensation may occur in deformed $N=Z$ nuclei, such as $^{16}$O, $^{64}$Ge, and $^{108}$Xe through an abrupt phase transition from IV to IS interaction due to the enhanced $T=0$ np pairing correlations. For heavy nuclei such as $^{108}$Xe, the transition may happen more smoothly even with the normal $T=0$ pairing interaction. More detailed and systematic calculations regarding the IS pair condensation and the coexistence of the IV and IS phase for heavy $N\simeq Z$ nuclei are in progress.

5. References
[1] H. Sagawa, C. L. Bai and G. Colo 2016 Phys. Scr. 91 083011
[2] W. Satula, D. J. Dean, J. Gary, S. Mizutori, W. Nazarewicz 1997 Phys. Lett. B 407 103
[3] W. Satula, R. Wyss 2001 Phys. Rev. Lett. 86 4488; Phys. Rev. Lett. 87 052504
[4] Takaharu Otsuka, Toshio Suzuki, Rintaro Fujimoto, Hubert Grawe, and Yoshinori Akaishi 2005 Phys. Rev. Lett. 95 232502
[5] Isao Tanihata 2013 Phys. Scr. 152 014021
[6] S. Terashima et al. 2018 Phys. Rev. Lett. 121 242501
[7] R. Schiavilla, R. B. Wiringa, Steven C. Pieper, and J. Carlson 2007 Phys. Rev. Lett. 98 132501
[8] H. Matsubara et al. 2015 Phys. Rev. Lett. 115 102501
[9] Y. Urata, K. Hagino, and H. Sagawa 2017 Phys. Rev. C 96 064311
[10] Remi N. Bernard, Marta Anguiano 2016 Nucl. Phys. A 953 32
[11] Y. Suzuki, H. Nakada, and S. Miyahara 2016 Phys. Rev. C 94 024343
[12] Alexandros Gezerlis, G. F. Bertsch, and Y. L. Luo 2011 Phys. Rev. Lett. 106 252502
[13] Eunja Ha and Myung-Ki Cheoun 2015 Nucl. Phys. A 934 73
[14] Eunja Ha, Myung-Ki Cheoun, H. Sagawa 2018 Phys. Rev. C 97 024320
[15] Eunja Ha, Myung-Ki Cheoun, H. Sagawa, W. Y. So 2018 Phys. Rev. C 97 064322
[16] Eunja Ha, Myung-Ki Cheoun, H. Sagawa 2019 Phys. Rev. C 99 064304
[17] A. L. Goodman, G. L. Struble, J. Bar-Touv, A. Goswami 1970 Phys. Rev. C 2 380
[18] A. L. Goodman 1972 Nucl. Phys. A 186 475
[19] A. Goodman 1998 Phys. Rev. C 58 R3051

Figure 3. Same as Fig. 1, but for $^{64}$Ge and $^{108}$Xe.