Higher order corrections to inclusive semileptonic B decays

Andrea Alberti

Università di Torino, Dip. di Fisica & INFN Torino, I–10125, Italy

Abstract

We describe the computation of the $O(\alpha_S)$ corrections to the Wilson coefficients of the kinetic and chromomagnetic operators in inclusive semileptonic $B$ decays. We can thus evaluate the complete correction at order $\alpha_S^2 \Lambda_{\text{QCD}}^2 / m_b^2$ to the semileptonic width and the first two leptonic moments.

Keywords:
OPE, HQET, B meson, semileptonic decay

1. Introduction

The study of semileptonic $B$ decays (first initiated in [1, 2]) has its foundation on an Operator Product Expansion together with an heavy quark expansion. The OPE expresses the differential decay rate (and consequently the total width, leptonic and hadronic moments) as a double series in $\alpha_S$ and $\Lambda_{\text{QCD}} / m_b$. The leading term reproduces the free quark decay, while subsequent corrections take into account interactions inside the meson and further reduce the theoretical uncertainty.

Currently several corrections are known: up to $O(\alpha_S^2)$ in the free decay [3, 4, 5], while the nonperturbative terms already calculated include $O(\alpha_S^2 \Lambda_{\text{QCD}}^2 / m_b^2)$ [2], $O(\Lambda_{\text{QCD}}^3 / m_b^3)$ [6] and a first investigation of $O((\Lambda_{\text{QCD}} / m_b)^4)$ [7]. The terms in the heavy mass expansion are expressed as B-meson matrix elements of local operators, growing in dimension: they are customarily parametrized as $\mu_{\mu}^2$ and $\mu_G^2$ at order $O(\Lambda_{\text{QCD}}^2 / m_b^2)$ and $\rho_D$, $\rho_{LS}$ at $O(\Lambda_{\text{QCD}}^3 / m_b^3)$.

Regarding the perturbative corrections to these parameters, there has been a first investigation, about $\mu_{\mu}^2$ alone, in reference [8] and then the complete calculation for both $\mu_{\mu}^2$ and $\mu_G^2$ has been carried out in the two papers relevant to this proceeding, [9] and [10].

Quite recently the coefficient of $\mu_G^2$ for the total width has been calculated again in the limit $m_c \rightarrow 0$ [11], yielding results compatible with our own.

The method we used to perform the calculation can be traced back to [12], where it was employed for the simpler process $B \rightarrow X \gamma$, then it has been further developed in [9] for the coefficient of $\mu_G^2$ and finally extended to the full calculation of $\mu_G^2$ in [10].

2. The inclusive semileptonic decay

The triple differential decay distribution for an inclusive semileptonic decay is

$$\frac{d\Gamma}{dq^2 dE_x dE_e} = \frac{1}{2} \sum_{\text{spin}} \delta(p_t) \left| <X,\nu|H_w|\bar{B}> \right|^2$$

(1)

where $p_t = p_B - p_e - p_\nu - p_X$ and labels $e, \nu$ and $X$ represent the final lepton, neutrino and hadronic state $X_e$: $q$ is the transferred momentum to the leptonic couple and $H_w$ the weak Hamiltonian

$$H_w = \frac{4GF}{\sqrt{2}} V_{cb} J_L^\mu \bar{e} \gamma^\mu P_L \nu_c; \quad J_L^\mu = \bar{e} \gamma^\mu P_L b$$

(2)

The first step to simplify eq. (1) is to express it as the product of an hadronic tensor $W^{a\bar{b}}$ and a leptonic tensor $L^{a\bar{b}}$, which is possible since leptons do not interact strongly (and we work at leading order in the ew interactions):

$$\frac{d\Gamma}{dq^2 dE_x dE_e} = 2G_F^2 |V_{cb}|^2 W_{a\bar{b}} L^{a\bar{b}}$$

(3)
the leptonic tensor is fairly simple
\[ L^{\alpha \beta} = 2 \left( p^\mu p^\nu \epsilon^{\mu \nu \alpha \beta} + p^\nu p^\mu \epsilon^{\mu \nu \alpha \beta} - p_\alpha p_\beta - \epsilon^{\alpha \beta \mu \nu} p_\mu p_\nu \right) \] (4)
while the hadronic tensor still contains unknown matrix elements
\[ W^{\alpha \beta} = \left( \frac{2 \pi^3}{2m_B} \sum_{x_i} \delta^3(p_i) \langle \bar{B} J^\alpha_{L}(x_i) \rangle^\beta_{B} \right) \] (5)
with \( J^\beta_{L} \) the usual left-handed weak current (described in eq. (2)). To further manipulate \( W_{\alpha \beta} \) we can relate it to the discontinuity of a time-ordered product across a cut. So by defining
\[ T^{\alpha \beta} = -i \int d^4x e^{-iqx} \left( \frac{1}{2m_B} \right) \langle \bar{B} T \left[ J^\alpha_{L}(x) J^\beta_{L}(0) \right] B \rangle \] (6)
we obtain the relation
\[ W_{\alpha \beta} = \frac{1}{\pi} \text{Im} T_{\alpha \beta} \] (7)
Up to this point we are still handling tensors with two indices, in order to simplify things a bit in this regard we can express the whole tensor \( W^{\alpha \beta} \) in terms of structure functions \( W_i \):
\[ W^{\alpha \beta} = -g^{\alpha \beta} W_1 + \gamma^\alpha \gamma^\beta W_2 - i \epsilon^{\alpha \beta \mu \nu} v_\mu q_\nu W_3 \] (8)
where \( v \) is the four-velocity of the \( B \) meson: \( p_B = m_B v \).
There would be two additional functions \( W_4 \) and \( W_5 \) factoring structures containing \( q \), but since we are dealing with massless leptons, the leptonic tensor yields zero when contracted with \( q \):
\[ q^\alpha L_{\alpha \beta} = q^\beta L_{\alpha \beta} = 0 \] (9)
and so only \( W_1, W_2 \) and \( W_3 \) contribute in the end. Eq. (7) thus becomes
\[ W_i = -\frac{1}{\pi} \text{Im} T_i, \quad i = 1, 2, 3 \] (10)

3. The operator product expansion

Having related \( W_i \) to a product of currents (eq. (6) and eq. (10)), we can now employ an operator product expansion and express the latter as the sum of local operators:
\[ T \left[ J^\alpha_{L}(x) J^\beta_{L}(0) \right] = \sum_i C_i(x) O_i(0) \] (11)
in order to do so we consider an initial \( b \) quark which is slightly off-shell (being inside a \( B \) meson), \( p_B = m_B v + k \) with \( k \) of the order of \( \Lambda_{\text{QCD}} \), and expand in \( k \) up to \( O(k^2) \). If we then use QCD perturbation theory to calculate the coefficients of the \( O_i \) operators, we obtain a double series in \( \alpha_s \) and \( \Lambda_{\text{QCD}}/m_B \):
\[ T^{ij} = \sum_{n \geq 3} \sum_{j \geq 0} \left( \frac{\Lambda_{\text{QCD}}}{m_B} \right)^{n-3} \left( \frac{\alpha_s}{\pi} \right)^{j(i)} \langle B O^{\alpha \beta}_{j}(x) B \rangle \] (12)
where \( n \) is the dimension of the operator.
It has to be noted that we could have followed the same steps with the addition of an external soft gluon and we would have expected a result similar to eq. (12), but possibly with different coefficients and operators. This consideration will play an important role in the determination of the coefficient of \( \mu_{\alpha}^2 \).

4. Heavy quark effective theory

Even after having found all operators in eq. (12) with the respective coefficients, we must still calculate their matrix elements between \( B \) meson states. This could be done in QCD, like the rest of the calculation so far, but it is convenient to switch to HQET, an effective theory specifically tailored around the idea of heavy quarks being very close to their mass shell.
In practice what we do is substituting the old quark field in terms of the new one
\[ b(x) = e^{-im_B v x} \left( 1 + \frac{iD}{2m_B} \right) b_\nu(v) \] (13)
and then take advantage of all the properties that hold true inside HQET in order to simplify the result and minimize the number of operators we have to deal with.
The equation of motion is of particular importance
\[ iD \cdot b_\nu = -(iD^2)^\nu b_\nu \] (14)
it implies that no operator appears at \( O(\Lambda_{\text{QCD}}/m_B) \) which cannot be written in terms of higher dimension operators. So in the end we have one operator at dimension three, describing the decay of a free quark, which is better to express in QCD:
\[ O^\nu_b = \tilde{b} \gamma^\nu b \] (15)
and then two operators at dimension five, the kinetic operator and the chromomagnetic operator:
\[ O_2^{\alpha \nu} = \frac{1}{2} b_\nu [iD^\nu, iD^\alpha] b_\nu \] (16)
\[ O_3^{\alpha \nu} = \frac{1}{2} \bar{b}_\nu g_5 G_5^{\alpha \nu} b_\nu \] (17)
As becomes apparent from eq. (17), the chromomagnetic operator contains a gluon field, so its coefficient can be calculated only from the current product emitting an
The operators are parametrized as follows:

\[ \frac{1}{M_B} < \bar{B} O_i^{\mu} \bar{B} > = 2 \nu^\mu \]  

\[ \frac{1}{M_B} < \bar{B} O_i^{\mu} \bar{B} > = -\frac{2 \mu_2^2}{\delta - 1} (g^{\mu\nu} - \nu^\mu \nu^\nu) \]  

\[ \frac{1}{M_B} < \bar{B} O_i^{\mu} \bar{B} > = \frac{2 \mu_2^2}{\delta - 1} (g^{\mu\nu} - \nu^\mu \nu^\nu) \]  

having neglected higher order power corrections and introducing dimension \( \delta = 4 - 2\epsilon \).

When expressing the final amplitude what really matters, rather than the operators themselves, are their matrix elements evaluated between \( B \) states, so we can now rewrite eq. (12) as

\[ W = W^{(0)} + \frac{\mu_2^2}{2m_b^2} W^{(1)} + \frac{\mu_G^2}{2m_b^2} W^{(G,0)} + \frac{\alpha_S}{\pi} \left[ W^{(1)} + \frac{\mu_2^2}{2m_b^2} W^{(1)} + \frac{\mu_G^2}{2m_b^2} W^{(G,1)} \right] (21) \]

5. One loop calculation

At tree level the use of HQET is sufficient to solve most of the problems, but carrying out the calculation at one loop (contributing diagrams shown in figure 1) presents many more challenges: namely lengthier expressions, loop integrals, and divergencies to be cancelled through renormalization. The latter deserve some deeper discussion, in order to get rid of all the poles we need to be precise when matching HQET with QCD. We are equating

\[ T = \left( c_{\mu}^{i,j} + \frac{\alpha_s}{4\pi} c_{\mu}^{i,j,l} + O(\alpha_s^2) \right) < O^{(i)} > \]  

where \( < O > = < \bar{B} O \bar{B} > \) and \( \{\alpha\} \) is a set of indices; the lhs contains the poles coming from the diagrams in figure 1 and the counterterms from mass and wave function renormalization, while on the rhs we have \( O(\alpha) \) contributions from the one-loop matrix elements (zero in the case of \( O_2 \) and \( O_3 \)) as well as the renormalization of each operator.

After having taken into account all the counterterms, both the lhs and the rhs become ultraviolet free, but still retain infrared divergencies: since QCD and HQET reproduce the same infrared behaviour, these will cancel out and so the one-loop Wilson coefficient \( c_{\mu}^{i,j,l} \) (what we wanted to calculate in the first place) will be finite.

The relevant counterterms are:

\[ \delta Z^{\mu\nu}_{\text{kin}} = -C_F \frac{3}{\epsilon} (g^{\mu\nu} - \nu^\mu \nu^\nu) \frac{\alpha_S}{4\pi} + ... \]  

\[ \delta Z^{\mu\nu}_{\text{chromo}} = \frac{C_A}{\epsilon} (g^{\mu\nu} - \nu^\mu \nu^\nu) \frac{\alpha_S}{4\pi} + ... \]  

where \( \xi \) is the Feynman gauge and bare quantities are defined as follows:

\[ \left[ c_{\mu}^{i,j,l} \right]_{\text{bare}} = Z^{\mu\nu}_{b_{\alpha}} Z^{\mu\nu}_{\alpha} c_{\mu}^{i,j,l} \]  

6. Results

The calculation we have briefly sketched has been carried out in ref. [9] and [10], respectively for the one-loop coefficient of \( \mu_2^2 \) and \( \mu_G^2 \). The full analytic result for the structure functions \( W_i \) can be found in the Appendix of each paper.

Numerical result are not as lengthy and can be discussed here. Considering on-shell quark masses of \( m_b = 4.6\text{GeV} \) and \( m_c = 1.15\text{GeV} \) we obtain a total width of

\[ \Gamma_{B \rightarrow X_{c\bar{c}}} = \Gamma_0 \left[ (1 - 1.78 \frac{\alpha_S}{\pi}) \left( 1 - \frac{\mu_2^2}{2m_b^2} \right) - (1.94 + 2.42 \frac{\alpha_S}{\pi} \frac{\mu_G^2(m_b)}{m_b^2} \right) \right] (26) \]

with \( \Gamma_0 = G_F^2 \frac{m_b^5}{8\pi^3} \left( 1 - 8\rho + 8\rho^3 - \rho^4 - 12\rho^3 \ln(\rho) \right) / 192\pi^3 \) and \( \rho = m_b^2 / m_c^2 \). The parameter \( \mu_G^2 \) is renormalized at the scale \( \mu = m_b \). Assuming \( \alpha_S = 0.25 \), the one-loop correction increases the \( \mu_G^2 \) coefficient by 7%.

Then there are the first and second central leptonic moments

\[ < E_{\ell} > = 1.41\text{GeV} \left[ (1 - 0.02 \frac{\alpha_S}{\pi}) \left( 1 + \frac{\mu_2^2}{2m_b^2} \right) - (1.19 + 4.20 \frac{\alpha_S}{\pi} \frac{\mu_G^2(m_b)}{m_b^2} \right) \right] (27) \]
\[
\ell_2 = 0.183 \text{GeV}^2 \left[ 1 - 0.16 \frac{\alpha_s}{\pi} + 4.98 \frac{\mu_b^2}{m_b^2} \right] - 0.37 \frac{\alpha_s}{\pi} \frac{\mu_b^2}{m_b^2} \left( 2.89 + 8.44 \frac{\alpha_s}{\pi} \frac{\mu_b^2(m_b)}{m_b^2} \right)
\]

where \( \ell_2 = \langle E_T^2 \rangle - \langle E_T \rangle^2 \), here the relative NLO correction to the \( \mu_b^2 \) coefficient are bigger than in the width, amounting to +28% and +23.

All the above can also be calculated inside the kinetic scheme (see [13, 14]) with cutoff \( \mu_{\text{kin}} = 1 \text{GeV} \) and they yield:

\[
\Gamma_{B - \pi, e^+ e^-} = \Gamma_0 \left[ 1 - 1.11 \frac{\alpha_s}{\pi} - 2m_b \frac{\mu_b^2}{m_b^2} \right] + 0.99 \frac{\alpha_s}{\pi} \frac{\mu_b^2}{m_b^2} \left( 1.94 + 3.46 \frac{\alpha_s}{\pi} \frac{\mu_b^2(m_b)}{m_b^2} \right)
\]

\[
\langle E_T \rangle = 1.41 \text{GeV} \left[ 1 - 0.05 \frac{\alpha_s}{\pi} - 2m_b \frac{\mu_b^2}{m_b^2} \right] - 0.44 \frac{\alpha_s}{\pi} \frac{\mu_b^2}{m_b^2} \left( 1.19 + 3.21 \frac{\alpha_s}{\pi} \frac{\mu_b^2(m_b)}{m_b^2} \right)
\]

\[
\ell_2 = 0.183 \text{GeV}^2 \left[ 1 - 0.24 \frac{\alpha_s}{\pi} + 4.98 \frac{\mu_b^2}{m_b^2} \right] - 3.89 \frac{\alpha_s}{\pi} \frac{\mu_b^2}{m_b^2} \left( 2.89 + 7.01 \frac{\alpha_s}{\pi} \frac{\mu_b^2(m_b)}{m_b^2} \right)
\]

Here \( \mu_b^2 \) coefficients always increase with the addition of the NLO corrections, respectively by +15%, +20% and +20%.

7. Conclusions

We have calculated the \( O(\alpha_s) \) corrections to the Wilson coefficients of the kinetic and chromomagnetic operators in inclusive semileptonic B decays. The complete \( O(\alpha_s A_{QCD}/m_b^2) \) contribution to the width is just a few per mill, but the corrections to the first two leptonic moments are comparable to the experimental errors. In order to estimate their effect on \( V_{cb} \), these corrections have to be included in the global fit to the moments, which will be the subject of a future publication.

References

[1] I. I. Y. Bigi, N. G. Uraltsev and A. I. Vainshtein, Phys. Lett. B 293 (1992) 430 [Erratum-ibid. B 297 (1993) 477] [arXiv:hep-ph/9207214]; I. I. Y. Bigi, M. A. Shifman, N. G. Uraltsev and A. I. Vainshtein, Phys. Rev. Lett. 71 (1993) 496 [arXiv:hep-ph/9304225].

[2] B. Blok, L. Koryak, M. A. Shifman and A. I. Vainshtein, Phys. Rev. D 49 (1994) 3356 [Erratum-ibid. D 50 (1994) 3572] [arXiv:hep-ph/9307247]; A. V. Manohar and M. B. Wise, Phys. Rev. D 49 (1994) 1310 [arXiv:hep-ph/9308246].

[3] M. Jezebed and J. H. Kuhn, Nucl. Phys. B 314 (1989) 1; A. Czarnecki, M. Jezebed and J. H. Kuhn, Acta Phys. Polon. B 20 (1989) 961; M. Gremm and I. Stewart, Phys. Rev. D55 (1997) 1226; A. F. Falk, M. E. Luke, Phys. Rev. D57 (1998) 424 [hep-ph/9708327]; A. F. Falk, M. E. Luke, M. J. Savage, Phys. Rev. D53 (1996) 2491 [hep-ph/9507284]; M. Trott, Phys. Rev. D 70 (2004) 073003 [arXiv:hep-ph/0402120].

[4] V. Aquila, P. Gambino, G. Ridolfi and N. Uraltsev, Nucl. Phys. B 719 (2005) 77 [arXiv:hep-ph/0503083].

[5] K. Melnikov, Phys. Lett. B 666 (2008) 336 [arXiv:0803.0951 [hep-ph]]; S. Biswas and K. Melnikov, JHEP 1002 (2010) 089 [arXiv:0911.1442 [hep-ph]]; A. Pak and A. Czarnecka, Phys. Rev. Lett. 100 (2008) 241807 [arXiv:0803.0960 [hep-ph]]; Phys. Rev. D78 (2008) 114015. [arXiv:0808.3509 [hep-ph]]; P. Gambino, JHEP 1109 (2011) 055 [arXiv:1107.3100 [hep-ph]].

[6] M. Gremm and A. Kapustin, Phys. Rev. D 55 (1997) 6924 [hep-ph/9603448].

[7] T. Mannel, S. Turczyk and N. Uraltsev, JHEP 1011, 109 (2010) [arXiv:1009.4622 [hep-ph]].

[8] T. Becher, H. Boos and E. Lunghi, JHEP 0712 (2007) 062 [arXiv:0708.0855].

[9] A. Alberti, T. Ewerth, P. Gambino and S. Nandi, Nucl. Phys. B 870 (2013) 16 [arXiv:1212.5082].

[10] A. Alberti, P. Gambino and S. Nandi, JHEP (2014) 1 [arXiv:1311.7381 [hep-ph]].

[11] T. Mannel, A. A. Pivovarov and D. Rosenthal, arXiv:1405.5072 [hep-ph].

[12] T. Ewerth, P. Gambino and S. Nandi, Nucl. Phys. B 830 (2010) 278 [arXiv:0911.2175 [hep-ph]].

[13] Y. Amhis et al. [Heavy Flavor Averaging Group Collaboration], arXiv:1207.1158 [hep-ex], see also http://www.slac.stanford.edu/xorg/hfag/.

[14] P. Gambino and C. Schwanda, arXiv:1307.4551 [hep-ph].