Bremsstrahlung Dileptons in Ultra-relativistic Heavy Ion Collisions

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Abstract

We consider production of dilepton pairs through coherent electro-magnetic radiation during nuclear collisions. We show that the number of pairs produced through bremsstrahlung is about two orders of magnitude smaller than the yield measured by the CERES collaboration. Therefore, coherent bremsstrahlung can be ruled out as an explanation for the observed enhancement of low mass dileptons in CERES and HELIOS.
Production of low mass dileptons in ultra-relativistic heavy ion collisions is consid-
ered a useful probe of possible in-medium changes of hadrons due to the onset of chiral
restoration (see [1] for a recent review). Dilepton are penetrating probes, i.e. once
produced they do not re-interact with the hadronic environment and thus, provide in-
fomation about the early stages of the collision where high temperatures and densities
are reached. Since vector mesons such as $\rho$, $\omega$ and $\Phi$ have direct decay channels into
dileptons, possible in-medium changes of the vector mesons masses can be observed in
the dilepton invariant mass spectrum.

There has been a major surge of interest in dilepton mass spectrum due to a recent
experimental observation of an enhanced dilepton yield at invariant masses of about
400MeV in S + Au and Pb + Au collisions in SPS compared to proton induced collisions
as reported by CERES collaboration [2]. There is also a similar enhancement reported
by HELIOS collaboration at more forward rapidities [3]. Although the decay of the final
state hadrons explain the data well in proton induced collisions, they can not explain
the current SPS data for S + Au and Pb + Au in the mass region around $\sim$ 400 MeV.

A major source of this enhancement is simply pion annihilation which is not present
in the proton-nucleus system. Including pion annihilation and contributions from other
hadronic reactions the calculations reach the lower end of the sum of statistical and
systematic errors of the CERES data (see [4] for a compilation of different calculations).
An additional enhancement can be achieved [3, 6] if one assumes that the mass of the
$\rho$ is lowered according to the conjectures of [7]. Another possible enhancement arises
from an in-medium modified pion dispersion relation. While this effect is small if one
considers the modification of the pion dispersion in a pion gas [8], it is considerably larger
if, in addition, one takes the effect of baryons into account [4]. Consistency of the latter
scenario with the observed pion to baryon ratio is presently debated.

Another source of dilepton production is simply the bremsstrahlung due to the de-
celeration of the incoming nuclei during the collision. Motivated by a recent result of
Mishustin et.al. [10], we investigate whether this source could account for part of the
observed enhancement of dilepton numbers as reported by CERES collaboration. In
reference [10] only the production of $\omega$ mesons due to the deceleration and their subse-
quent decay into dileptons has been taken into account. This corresponds to considering
the iso-scalar part of the electro-magnetic current. However, one also has to take into
account the iso-vector part, i.e. production of $\rho$-mesons and their subsequent decay into
dileptons. These two amplitudes, iso-scalar and iso-vector, interfere destructively and as
a result the dilepton production cross section from the coherent deceleration scales like
the square of the charge and not the square of the baryon number, as assumed in [10].
We should also note that the point like limit of this process addressed here has been
considered in [11].

As already mentioned, one expects that the coherent radiation will be enhanced by a factor of $Z_1 Z_2$ in nuclear as compared to proton induced collisions. Here $Z_1$ and $Z_2$ are the atomic numbers of the colliding nuclei. The enhancement factor $Z_1 Z_2$ follows from assuming coherent radiation off of charged nuclei. This assumption is an approximation which should be valid when one considers photon virtualities (invariant dilepton masses) which are much smaller than inverse size of individual nucleons. Therefore, as one goes to higher and higher invariant dilepton masses, this approximation will cease to be valid. Also, the ratio of photon virtuality to center of mass energy of the collision should be small so that there is also longitudinal coherence. Even though incoherent radiation will become as important and eventually dominate over coherent radiation as one goes to higher and higher masses, the coherent radiation will always be there as a background and so therefore, it should be understood. Here we try to provide an upper limit on this background.

1 Radiation from Decelerating Nuclei

Let us consider a typical ultra-relativistic nuclear collision where nuclei A and B move towards each other with very high but constant velocities. In order to simplify the extremely complicated process of nuclear collisions, we will assume that the main effect of the collision is deceleration of each nucleus while they are passing through each other. Since electric charges in the nuclei are decelerated, they emit photons. All time like photons with virtuality $q^2 > 0$ which subsequently decay into dileptons are emitted during this time. After a passing time $t \sim \frac{R_A + R_B}{\gamma}$, they move on with reduced but constant velocity. We will ignore all subtleties associated with expansion of nuclei in the transverse direction during the collision and, for simplicity, assume a Gaussian form for the charge distribution.

We can relate the number of dileptons produced in this process to the Fourier transform of the correlator of the electro-magnetic currents of the colliding nuclei. It is given by [12]

$$\frac{dN_{\pi+}}{d^4p} = \frac{\alpha^2}{6\pi^2} \frac{1}{p^4} (p^\mu p^\nu - p^2 g^{\mu\nu}) W_{\mu\nu}(p)$$  \hspace{1cm} (1)

where $p^\mu$ is the virtual photon momentum, $\alpha = 1/137$ is the electro-magnetic coupling
constant and $W_{\mu\nu}$ is the Fourier transform of the product of the electro-magnetic currents:

$$W_{\mu\nu}(p) = \int d^4x d^4y e^{-ip(x-y)}J_\mu(x)J_\nu(y). \quad (2)$$

Our task is now simple, we just need to write down an electro-magnetic current corresponding to an extended charge density with (proper) time dependent velocity. It is \[13\]:

$$J^R_\mu(x) = \int d\tau v_\mu(\tau)\delta[v_\nu(x') - z^\nu(\tau)]\left[1 + a_\nu(x') - z^\nu(\tau)\right]f[(x - z)^2]. \quad (3)$$

with

$$z_0(\tau) = z_0(T) + \int_T^\tau d\tau'\gamma(\tau')$$

and

$$z_3(\tau) = z_3(T) + \int_T^\tau d\tau'\gamma(\tau')\beta(\tau').$$

We choose the initial time $T = -\infty$ for convenience. $f[(x - z)^2]$ is a properly normalized but otherwise arbitrary charge profile at this point.

Here, $\tau$ is the proper time and $a_\mu = \frac{dv_\mu}{d\tau}$ is the corresponding acceleration (deceleration) with

$$v_\mu(\tau) = \gamma(\tau) \begin{pmatrix} 1 \\ 0 \\ 0 \\ \beta(\tau) \end{pmatrix} \quad (4)$$

being the four velocity and $\gamma(\tau) = [1 - \beta^2(\tau)]^{-1/2}$. It is easy to verify that this current is conserved. The acceleration term $a_\nu$ looks peculiar and arises out of a consistent application of the concept of simultaneity in special relativity to an extended (but still rigid) charged object and is essential for current conservation in the case of a spatially extended charge distribution (see \[13\] for a nice illustration of this). It will drop out when one takes the point charge limit of this expression.

Expression (3) is the current for a charged nucleus moving from left ($z = -\infty$) to right ($z = +\infty$) with velocity $\beta$. In order to get the current of a charged nucleus moving
from right to left, we simply take \( \beta \to -\beta \) in the current of the right moving nucleus. The total current to be used in (2) is the sum of the currents of the right moving and left moving nuclei. The last step is to determine the velocity \( \beta \). We will assume that both nuclei have a constant initial velocity \( \beta_i \) until they collide at \( \tau = \tau_i \). During the collision, from \( \tau_i \) to \( \tau_f \), velocity changes in a non-trivial way. After \( \tau = \tau_f \), both nuclei have again a constant but reduced velocity \( \beta_f \).

To proceed further, we need to take a specific form for the nuclear charge distribution \( f[(x - z)^2] \). For simplicity we will use a Gaussian profile

\[
f[(x - z)^2] = \rho_0 \exp \left[ \frac{(t - z_0(\tau))^2 - (\vec{x} - \vec{z}(\tau))^2}{2\sigma^2} \right]
\]

(5)

Here \( \rho_0 \) and \( \sigma \) are related to atomic number \( Z \) and radius of nucleus \( R \) by

\[
\rho_0 = Z (2\pi R^2 / 3)^{-\frac{3}{2}}, \quad \sigma^2 = \frac{1}{3} R^2
\]

A more realistic profile would be a Woods-Saxon shape. However, since we want to provide an upper limit of the bremsstrahlung contribution, Gaussian distribution is a reasonable approximation. For a given charge, it is more narrow and will lead to a larger dilepton yield at finite momentum than the corresponding Woods-Saxon profile.

In order to model the deceleration phase, one could take the velocity during the collision to be a linearly changing function of time but it is perhaps more realistic to take the rapidity, rather than the velocity, to be a linearly changing function of time. We therefore take the rapidity during the collision to be

\[
y(\tau) = y_i + \frac{\Delta y}{\Delta \tau} \tau
\]

(6)

where \( \beta(\tau) = \tanh[y(\tau)] \), \( \Delta y = y_f - y_i \), \( \Delta \tau = \tau_f - \tau_i \) and the initial and final rapidities and times are to be determined by experimental considerations.

Fourier transforming the current to momentum space, dividing the proper time interval into three different regions and performing the proper time integration in the initial and final stages where the velocity is constant, we get:

\[
J^R_3(p) = Z \exp \left[ -\frac{1}{6} R^2 p_\tau^2 \right] \left\{ \frac{\exp \left[ -\frac{1}{6} R^2 \gamma_i^2 (\beta_i p^0 - p_3^2)^2 \right]}{i(p^0 - \beta_i p_3 - i\epsilon)} \right\}
\]
\[- \frac{\exp \left[ - \frac{1}{6} R^2 \gamma_f^2 (\beta_f p^0 - p^3)^2 + i \frac{\Delta \tau}{\Delta y} [p^0 (\gamma_f \beta_f - \gamma_i \beta_i) - p^3 (\gamma_f - \gamma_i)] \right]}{\beta_f} \int_{p^0 - \beta_f p_3 + i \epsilon} \gamma(\tau) \beta(\tau) \left[ \frac{1}{3} R^2 \frac{\Delta y}{\Delta \tau} \gamma(\tau) [\beta(\tau) p^0 - p^3] \right] \times \exp \left[ - \frac{1}{6} R^2 \gamma^2(\tau) [\beta(\tau) p^0 - p^3]^2 + i \frac{\Delta \tau}{\Delta y} [p^0 (\gamma(\tau) \beta(\tau) - \gamma_i \beta_i) - p^3 (\gamma(\tau) - \gamma_i)] \right] \right]. \tag{7}

where we have set \( \tau_i = 0 \) without loss of generality. The first two terms (proportional to \( \beta_i \) and \( \beta_f \)) correspond to the contribution of the initial and final stages of the collision respectively while the last term is the contribution of the time interval when the nuclei are passing through each other and has to be evaluated numerically. Also, in the last term corresponding to the contribution of the deceleration region, extended structure of the source is responsible for the second term (proportional to area of the nucleus, \( R^2 \)) in the bracket :

\[
\left[ 1 - \frac{1}{3} R^2 \frac{\Delta y}{\Delta \tau} \gamma(\tau) [\beta(\tau) p^0 - p^3] \right] \tag{8}
\]

and will be absent in collision of point charges.

Using current conservation \( p_0 J_0(p) = p_3 J_3(p) \) and also \( J_\mu = J_\mu^R + J_\mu^L \), we then have

\[
\frac{d^4 N_{\ell+\ell^-}}{dM dy dp_t} = \frac{\alpha^2}{6\pi^3} \frac{1}{M} \left[ 1 - \frac{p_3^2}{p_0^2} \right] J_3(p) J_3^\dagger(p). \tag{9}
\]

As a consistency check, it is easy to show that one does not get either physical (light like) or time like photons without acceleration.

Below, we plot our results for the number of dileptons produced for two different values of collision times with and without the CERES acceptance cuts for \( \Delta y = 2.4 \) \[14\]. From here, it is clear that bremsstrahlung is irrelevant for the observed enhancement of dilepton spectrum even with our Gaussian charge distribution which would clearly over estimate the number of produced dileptons and that a different mechanism is needed. Variation of our parameters like rapidity change \( \Delta y \) and collision time \( \Delta \tau \) does not change our results appreciably even in the extreme case of \( \Delta y = y_i \) i.e. full stoppage of nuclei after collision.

We also plot the transverse momentum spectrum of the produced dileptons, integrated over rapidity and dilepton invariant mass range [200 – 600 MeV] and compare it with CERES data. Both graphs have the peculiarity that as one increases the time it
takes the nuclei to pass through each other from 1 fm to 4 fm, the number of dileptons increases in contrast to what one expects intuitively (at least for high invariant masses). This is again due to the extended structure of the source. Unlike the case of a point charge, we have an inherent scale in the problem, namely $R$, the nuclear radius.

![Graph showing number of dileptons produced for 1 fm and 4 fm collision times with and without CERES cuts.](image)

**Figure 1:** Number of dileptons produced for 1 fm and 4 fm collision times with and without CERES cuts.

Physically, there are two competing effects which are responsible for this initial increase in the number of emitted dileptons as one increases the collision time. The first effect is just what one expects; as charges decelerate over a longer time interval, they emit fewer dileptons. The second effect is due to the finite time required to build up coherence among different parts of the extended source. In other words, if the nucleus decelerates too quickly, there is no time for the different points in the extended source to communicate and coherent emission takes place only from a fracture of the source which has had time to react. As we increase the collision (deceleration) time, we increase the fraction of the nucleus which can be considered a coherent source and as a result, we have more dileptons emitted. After some time, the whole nucleus is emitting dileptons coherently after which the first effect takes over and the number of dileptons starts decreasing as we increase the collision time any further.
Figure 2: $P_t$ spectrum of dileptons produced for 1 fm and 4 fm with and without CERES cuts integrated over rapidity and mass in the mass range between 200 and 600 MeV.

It is interesting to note that dilepton mass spectrum due to deceleration of a point charge (not shown here) shows a (modulated) periodicity which depends on the time it takes the nuclei to pass through each other and in principle could be used to determine this time. However, this structure is totally wiped out by the Gaussian charge distribution and it is unlikely that it could be experimentally useful in determining the collision time for a realistic charge distribution.

In summary, we consider production of dilepton pairs due to coherent bremsstrahlung in ultra-relativistic heavy ion collisions. We provide an estimate of the upper limit for this contribution using a Gaussian charge distribution. We find coherent bremsstrahlung to be a negligible source for dileptons.

This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics Division of the Department of Energy, under contract No. DE-AC03-76SF00098 and DE-FG02-87ER40328.

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