Analytic Resummation and Power Corrections for DIS and Drell–Yan

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Dimensional continuation is applied to resummed expressions for the DIS and Drell-Yan partonic cross sections, to regularize the Landau pole. Simple analytic expression are obtained, encoding information about nonperturbative power-suppressed effects.

1 Introduction

Resummations of perturbation theory are a valuable tool in perturbative QCD, for theoretical as well as phenomenological applications. From a phenomenological point of view, resummations extend the applicability of perturbation theory to regions of phase space which are characterized by the presence of logarithms of large ratios of kinematical scales. From a theoretical point of view, resummations highlight the inherent limitations of the perturbative expansion, and provide a useful tool to estimate the size and shape of power-suppressed, nonperturbative corrections.

Nonperturbative effects must be present in order to compensate for the fact that resummed expressions for QCD cross sections are typically ill-defined, due to the presence of the Landau pole in the running coupling on the integration contour for the relevant scale variable. Schematically, for a single scale process, a resummation will yield expressions of the form

\[ f_a(Q^2) = \int_0^{Q^2} \frac{dk^2}{k^2} (k^2)^a \alpha_s(k^2). \]  

A common way to interpret such expressions is to expand the integrand in powers of \( \alpha_s(Q^2) \) and evaluate the integral term by term. One reconstructs then a perturbative expansion, and the singularity of the integral reappears as a factorial behavior of the large order perturbative coefficients. Borel transformation shows that this factorial behavior corresponds to a power-suppressed...
ambiguity in the resummed expression,

\[ \delta f_a(Q^2) \propto \left( \frac{\Lambda^2}{Q^2} \right)^a , \tag{2} \]

as one could easily have guessed directly from Eq. (1).

It is apparent that it would be useful to have a gauge invariant regularization scheme for such singularities, arising from the Landau pole. It was pointed out in Ref. 2 that dimensional regularization is just such a scheme. It is well known that, in \( d = 4 - 2\epsilon \), the \( \beta \) function acquires \( \epsilon \) dependence, so that

\[ \beta(\epsilon, \alpha_s) \equiv \mu \frac{\partial \alpha_s}{\partial \mu} = -2\epsilon \alpha_s + \hat{\beta}(\alpha_s) , \tag{3} \]

where \( \hat{\beta}(\alpha_s) = -b_0 \alpha_s^2/(2\pi) + O(\alpha_s^3) \). As a consequence, the running coupling also becomes dimension dependent. At one loop,

\[ \alpha \left( \frac{\mu^2}{\mu_0^2}, \alpha_s(\mu_0^2), \epsilon \right) = \alpha_s(\mu_0^2) \left[ \left( \frac{\mu^2}{\mu_0^2} \right)^\epsilon - \frac{1}{\epsilon} \left( 1 - \left( \frac{\mu^2}{\mu_0^2} \right)^\epsilon \right) \frac{b_0}{4\pi} \alpha_s(\mu_0^2) \right]^{-1} . \tag{4} \]

It is easy to see that the running coupling in Eq. (3) has a qualitatively different behavior with respect to its four dimensional counterpart. First of all, it vanishes as \( \mu^2 \to 0 \) for \( \epsilon < 0 \), as appropriate for infrared regularization. This is a consequence of the fact that the one loop \( \beta \) function, for \( \epsilon < 0 \), has two distinct fixed points: the one at the origin in coupling space is now a Wilson–Fisher fixed point, whereas the asymptotically free fixed point is located at \( \alpha_s = -4\pi\epsilon/b_0 \). Furthermore, the location of the Landau pole becomes \( \epsilon \) dependent, and it is given by

\[ \mu^2 = \Lambda^2 \equiv Q^2 \left( 1 + \frac{4\pi\epsilon}{b_0 \alpha_s(Q^2)} \right)^{-1/\epsilon} . \tag{5} \]

The pole is not on the real axis in the \( \mu^2 \) plane, i.e. not on the integration contour of resummed formulas, provided \( \epsilon < -b_0 \alpha_s(Q^2)/(4\pi) \). We then expect resummed expressions such as Eq. (1) to be integrable for general \( \epsilon \): scale integrals will yield RG invariant analytic functions of \( \epsilon \) and \( \alpha_s \), with the singularity corresponding to the Landau pole replaced by a cut. This will be verified below for a few relatively simple QCD amplitudes and cross sections.

### 2 A simple example: the quark form factor

The electromagnetic quark form factor is perhaps the simplest QCD amplitude to which the present ideas may be applied. In the massless theory, with dimensional regularization of infrared and collinear divergences, it is expressed
in terms of a single scalar RG invariant form factor, $\Gamma(Q^2/\mu^2, \alpha_s(\mu^2), \epsilon)$. Because it depends on a single scale $Q^2$, the resummed form factor can be expressed explicitly in terms of standard analytic functions to all orders in (exponentiated) perturbation theory, full results being available up to two loops. Resummation of the form factor can be achieved by deriving an evolution equation which, in dimensional regularization, takes the form

$$Q^2 \frac{\partial}{\partial Q^2} \log \left[ \frac{\Gamma(Q^2/\mu^2, \alpha_s(\mu^2), \epsilon)}{\Gamma(Q^2/\mu^2, \alpha_s(\mu^2), \epsilon)} \right] = \frac{1}{2} \left[ K(\epsilon, \alpha_s(\mu^2)) + G\left(\frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \epsilon\right) \right].$$

(6)

The functions $K$ and $G$, whose perturbative expansions are known up to two loops, are characterized by the fact that they are additively renormalizable, with the same anomalous dimension function $\gamma_K(\alpha_s)$, to preserve the RG invariance of the form factor. Further, $K$ is a pure counterterm. Dimensional regularization implies the simple boundary condition $\Gamma(0, \alpha_s(\mu^2), \epsilon) = 0$, so that the evolution equation can be explicitly solved yielding

$$\Gamma\left(\frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \epsilon\right) = \exp \left\{ \frac{1}{2} \int_{0}^{-Q^2} \frac{d\xi^2}{\xi^2} \left[ K(\epsilon, \alpha_s) + G\left(\frac{-1}{\tau}, \pi(\xi^2), \epsilon\right) \right] \right\}.$$

(7)

Remarkably, using Eq. (3) and changing variables from the scale to the coupling itself, $d\mu/\mu = d\alpha/\beta(\epsilon, \alpha)$, all integrals in Eq. (7) can be explicitly performed to the desired order in the perturbative expansion of the functions $K$ and $G$. The resulting analytic functions are RG invariant to the relevant perturbative order, and display the expected (cut) singularity associated with the Landau pole. At the one-loop level, for example, one finds

$$\log \Gamma\left(\frac{-Q^2}{\mu^2}, \alpha_s(\mu^2), \epsilon\right) = \log \Gamma(-1, \alpha_s(Q^2), \epsilon)$$

$$= -\frac{2C_F}{b_0} \left\{ \frac{1}{\epsilon} \text{Li}_2\left[ \frac{a(Q^2)}{a(Q^2) + \epsilon} \right] + C(\epsilon) \log \left[ 1 + \frac{a(Q^2)}{\epsilon} \right] \right\},$$

(8)

where $a(Q^2) = b_0 \alpha_s(Q^2)/(4\pi)$ and $C(\epsilon) = 3/2 + O(\epsilon)$. Eq. (8) resums the two leading towers of IR-collinear poles of the form factor, and may also be used to study the behavior of $\Gamma$ in the vicinity of the singular, physical limit $\epsilon \to 0$. One finds

$$\log \Gamma(-1, \alpha_s(Q^2), \epsilon) = \frac{2C_F}{b_0} \left[ -\frac{\zeta(2)}{\epsilon} + \frac{1}{a(Q^2)} + O(\epsilon, \log \epsilon) \right].$$

(9)
Notice the universal, exponentiated single pole, which does not depend upon the energy, nor upon the coupling. Its residue is not affected by two-loop corrections. Notice also the presence of a term behaving like a (fractional) power-suppressed correction. Although in this case such a term is of no direct physical interest, its presence emphasizes that the present formalism may be suited to study power corrections for more realistic QCD cross sections. This will be discussed in the following, using as examples DIS and the Drell–Yan cross section.

3 Analytic resummation and power corrections for factorized cross sections

Resummation of threshold \((x \to 1)\) logarithms, both for DIS and Drell-Yan, was performed at NNLO level in \([5]\). A formulation closer to the present approach was later given in \([6]\). Applying the latter formalism, consider the following expression for the Mellin transform of \(F_2(x, Q^2/\mu^2, \alpha_s(\mu^2), \epsilon)\), where one resums leading logarithms of \(N\),

\[
F_2 \left( N, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \epsilon \right) = F_2 \left( 1, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \epsilon \right) \exp \left[ \frac{C_F}{\pi} \int_0^1 dz \frac{z^{N-1} - 1}{1 - z} \right] \\
\times \int_0^{1-z} \frac{d\xi^2}{\xi^2} \bar{\alpha} \left( \frac{\xi^2}{\mu^2}, \alpha_s(\mu^2), \epsilon \right). 
\]

Integration of the running coupling around \(\xi^2 = 0\) generates the leading collinear divergences, which can be factorized by subtracting the resummed \((\overline{MS})\) parton distribution

\[
\psi \left( N, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \epsilon \right) = \exp \left[ \frac{C_F}{\pi} \int_0^1 dz \frac{z^{N-1} - 1}{1 - z} \right] \\
\times \int_0^{Q^2} \frac{d\xi^2}{\xi^2} \bar{\alpha} \left( \frac{\xi^2}{\mu^2}, \alpha_s(\mu^2), \epsilon \right). 
\]

The IR and collinear finite resummed partonic DIS cross section is then defined by taking the ratio of Eqs. \((10)\) and \((11)\), as \(\tilde{F}_2 = F_2/\psi\).

Using again \(d\mu/\mu = d\alpha/\beta(\epsilon, \alpha)\), one easily performs the scale integrals, obtaining the compact RG invariant expression

\[
\tilde{F}_2 \left( N, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \epsilon \right) = F_2 \left( 1, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \epsilon \right) \exp \left[ -\frac{4\pi C_F}{b_0} \int_0^1 \frac{z^{N-1} - 1}{1 - z} \right] \\
\times \log \left( \frac{\epsilon + a((1 - z)Q^2)}{\epsilon + a(Q^2)} \right),
\]

4
manifestly finite, though ambiguous due to the cut, as $\epsilon \to 0$.

As was done for the form factor, the expected power correction can be evaluated by taking the limit $\epsilon \to 0$ with $\alpha_s(Q^2)$ fixed. One is lead to

$$\log \left[ \frac{\hat{F}_2(N, 1, \alpha_s(Q^2), 0)}{\hat{F}_2(1)} \right] = -\frac{4\pi C_F}{b_0} \sum_{k=0}^{N-2} I_k (\alpha_s(Q^2)) \ , \quad (13)$$

where

$$I_k (\alpha_s(Q^2)) = \int_0^1 dz z^k \log [1 + a(Q^2) \log(1 - z)] \ . \quad (14)$$

Each of these integrals carries an ambiguity due to the cut, which is easily seen to be proportional to integer powers of $\exp(-1/\epsilon(Q^2))$, as expected. Collecting the leading power corrections thus identified one finds

$$\delta \hat{F}_2 (N, \alpha_s(Q^2)) \propto N \Lambda^2 \frac{Q^2}{\mu^2} \left( 1 + O \left( \frac{1}{N} \right) + O \left( \frac{\Lambda^2}{Q^2} \right) \right) \ , \quad (15)$$

as expected in DIS.

The resummed expression for the Drell-Yan partonic cross section, at the leading log $N$ level, is very similar. One finds

$$\hat{\sigma}_{DY} (N, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \epsilon) = \frac{\sigma_{DY} (N, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \epsilon)}{\psi^2 (N, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \epsilon)} \ , \quad (16)$$

where $\sigma_{DY}$ differs from $\hat{F}_2$ because of a factor of two in the exponent (due to the presence of two radiating quarks in the initial state for the DY process), and because phase space dictates that the upper limit of the scale integration should be $(1 - z)^2 Q^2$ instead of $(1 - z)Q^2$. Thus one finds

$$\log \left[ \frac{\hat{\sigma}_{DY} (N, 1, \alpha_s(Q^2), 0)}{\hat{\sigma}_{DY} (1)} \right] = -\frac{8\pi C_F}{b_0} \sum_{k=0}^{N-2} I_k (2 \alpha_s(Q^2)) \ , \quad (17)$$

which is twice the DIS result with $a(Q^2) \to 2 a(Q^2)$. Then

$$\delta \hat{\sigma}_{DY} (N, \alpha_s(Q^2)) \propto N \frac{\Lambda}{Q} \left( 1 + O \left( \frac{1}{N} \right) + O \left( \frac{\Lambda}{Q} \right) \right) \ . \quad (18)$$

This $\Lambda/Q$ correction is known to cancel in the full Drell-Yan cross section, provided a suitable subset of non–logarithmic terms are included in the resummation \textsuperscript{7}. Eq. (18), however, is the result that must be expected from a LL resummation, in agreement with \textsuperscript{8}.
4 Outlook

Dimensional regularization is useful to regulate in a gauge invariant way the Landau singularity which characterizes resummed expressions for QCD amplitudes and cross sections. The resulting formulas are simple and transparent, and they encode information on the all–order structure of infrared and collinear divergences, as well as on the parametric size of nonperturbative, power–suppressed corrections to factorized cross sections. Applying the formalism to DIS and to the Drell–Yan process reproduces known results at the LL level. Possible interesting generalizations include applications to existing resummations for event shapes in $e^+e^-$ annihilation and for the production of coloured final states in hadronic collisions.

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