Monte Carlo Inter-Cycle Correlation Reduction with the Application of the Modified Power Method

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Abstract. The modified power method has been applied to reduce the inter-cycle correlation of the Monte Carlo simulation in this study. Previously it has been developed and implemented in the Monte Carlo simulation to accelerate the source convergence. The convergence of the original Monte Carlo simulation determined by the decay rate of the 1st mode in the source, which will finally converge to the 0th mode. The 1st mode is also the most important origin of the inter-cycle correlation of the sources of different cycles. The modified power method can give the first several eigenmode solutions at the same time, so it is efficient for both source convergence acceleration and inter-cycle correlation reduction by subtracting the higher modes from the source. The 1D slab problem is modeled in this study to demonstrate the performance of the modified power method.

Introduction

The Monte Carlo (MC) method has been widely used in neutron transport simulation of the criticality analysis of the nuclear power systems. To solve the criticality eigenvalue and the corresponding eigenvector (the fission source), the power method is exclusively adopted, for with the cycle/generation concepts are introduced. The convergence rate, i.e., the number of iterations/cycles required for the eigenvector/source convergence, is determined by the dominance ratio \( k_1/k_0 \), where \( k_0 \) and \( k_1 \) are the 0th and 1st eigenvalues. This means the convergence may be very slow if the dominance ratio of the system is close to 1.0. Many source convergence acceleration techniques have been developed to solve the high dominance ratio problems, such as the coarse mesh finite difference method, which adopts the solution of a lower order operator (diffusion) to accelerate the transport solution. In our previous works, the modified power method (MPM) has been adopted to accelerate the source convergence, which can be thought as a kind of coarse mesh rebalance technique. It can also generate multiple eigen-solutions, and the convergence acceleration of the fission source, i.e., the fundamental/0th eigenmode, can be achieved by the subtraction of the other modes from the fundamental/0th mode.

On the other hand, the inter-cycle correlation is also related to the higher modes. For the high dominance ratio problems, there is strong inter-cycle correlation, as the biases introduced in previous cycle will decay very slow, while the decay rate is also determined by the eigenvalue ratios, especially the dominance ratio \( k_1/k_0 \). Therefore, the modified power method should be also efficient for reducing the inter-cycle correlation.

The theory of the modified power method will be briefly reviewed in the next section, followed by the numerical tests about the 1D slab problems and the conclusion.

The Modified Power Method

In the original MC simulation, one particle only carries one weight to represent the source that will finally converge to the fundamental mode. For the MPM, we introduced a weight vector for one particle, each element of the vector corresponding to one independent source, which will finally
converge to one eigenmode. Suppose there are $N$ modes to be solved, and the whole space is divided into $N$ sub regions. The basic idea of the MPM is to force the local eigenvalues to be the same with the linear combination of the $N$ modes:

$$WX = VXK,$$

(1)

where $V, W \in \mathbb{R}_{N \times N}$ are the weight integrals over the sub regions at the beginning and end of one cycle, $X \in \mathbb{R}_{N \times N}$ contains the linear combination coefficients that each column can combine the $N$ sources to get one mode, and $K \in \mathbb{R}_{N \times N}$ is a diagonal matrix that contains the first $N$ eigenvalues. With tallied $V$ and $W$, the $X$ and $K$ can be solved, and $X$ is then used to update the sources.

**Numerical Tests**

The 400-cm 1D slab is modeled, while the multi-group cross sections are from the C5G7 benchmark for the 8.7% MOX fuel-clad macroscopic cross sections. 400 inactive / 400 active cycles / 1,000,000 histories per cycle are used for the Original MC simulation, while 200 inactive / 400 active cycles / 1,000,000 histories per cycle are used for the MPM with 8 modes solved (MPM8). Two initial sources are tested with the Original MC: one is a flat source over the whole length, the other is a flat source on the half of the slab. It is expected that the results are similar with different initial sources.

The eigenvalues results are listed in Table 1. The dominance ratio of this problem is 0.97, so it is expected that the Original MC simulation will have slow source convergence and big inter-cycle correlation.

The Shannon Entropy results are shown in Figure 1. It can be confirmed that 400 and 200 inactive cycles are enough for the Original MC and MPM8 simulations to be well converged. As expected, the MPM8 can dramatically reduce the number of inactive cycles required for the fission source to be converged.

![Figure 1. The Shannon Entropy results.](image-url)
The first 8 fission source eigenmodes are shown in Figure 2.

![Figure 2. The first 8 fission source eigenmodes by the MPM8.](image)

The autocorrelation coefficient (ACC) results with different mesh tallies are shown in Figures 3-4. The ACC results of the Original MC show several peaks in the inner regions, which are caused by the higher modes, especially the 1st and 2nd eigenmodes. While for the MPM, as the higher modes can be subtracted from the 0th mode, the ACC results are much lower than the Original MC and close to 0.
Figure 3. The ACC results of the 400-cm slab with 100 mesh tallies.

Figure 4. The ACC results of the 400-cm slab with 10 mesh tallies.
As a contrast, the 10-cm slab is modeled as an example of the low dominance ratio problem. The results are shown in Table 1 and Figures 5-6. In this case the ACC results of different simulations cannot be distinguished due to the low dominance ratio nature of the problem.

Table 1. The eigenvalue results.

|                  | 400-cm slab | 10-cm slab |
|------------------|-------------|------------|
| Original-1, k0   | 1.13776 ± 0.00002 | 0.20207 ± 0.00001 |
| Original-2, k0   | 1.13779 ± 0.00002 | 0.20205 ± 0.00002 |
| MPM8, ki (ki/k0) |             |            |
| i=0.7            | 1.13773 ± 0.00002 (1.00) | 0.20207 ± 0.00001 (1.00) |
|                  | 1.10923 ± 0.00006 (0.97) | 0.07299 ± 0.00001 (0.36) |
|                  | 1.06459 ± 0.00007 (0.94) | 0.04113 ± 0.00001 (0.20) |
|                  | 1.00850 ± 0.00006 (0.89) | 0.02802 ± 0.00001 (0.14) |
|                  | 0.94514 ± 0.00007 (0.83) | 0.02115 ± 0.00001 (0.10) |
|                  | 0.87848 ± 0.00007 (0.77) | 0.01694 ± 0.00001 (0.08) |
|                  | 0.81189 ± 0.00007 (0.71) | 0.01411 ± 0.00001 (0.07) |
|                  | 0.74747 ± 0.00007 (0.66) | 0.01209 ± 0.00001 (0.06) |

Figure 5. The ACC results of the 10-cm slab with 100 mesh tallies.
Figure 6. The ACC results of the 10-cm slab with 10 mesh tallies.

**Conclusion**

The modified power method has been applied to reduce the inter-cycle correlation of the Monte Carlo simulation. As the higher modes are the main cause of the inter-cycle correlation, it is expected that the inter-cycle correlation can be reduced if the higher modes can be subtracted from the source. The performance of the MPM is well demonstrated with the 1D slab problems. Further tests will be conducted based on more practical problems like the 3D whole core problem.

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