We begin by reviewing the noncommutative supersymmetric tubular configurations in the matrix theory. We identify the worldvolume gauge fields, the charges and the moment of R-R charges carried by the tube. We also study the fluctuations around many tubes and tube-D0 systems. Based on the supersymmetric tubes, we have constructed more general configurations that approach supersymmetric tubes asymptotically. These include a bend with angle and a junction that connects two tubes to one. The junction may be interpreted as a finite-energy domain wall that interpolates U(1) and U(2) worldvolume gauge theories. We also construct a tube along which the noncommutativity scale changes. Relying upon these basic units of operations, one may build physical configurations corresponding to any shape of Riemann surfaces of arbitrary topology. Variations of the noncommutativity scale are allowed over the Riemann surfaces. Particularly simple such configurations are Y-shaped junctions.
1 Introduction

When external R-R field strengths are turned on, the lower dimensional D-branes may respond to the external fields like the dielectric material placed in an external electric field\[1\]. A spherical D2 branes may be formed by D0’s placed in an external R-R four form field strength and the world-volume gauge theory becomes noncommutative. Recently, it is found that the tubular D2-brane formed by D0’s and strings may also be self supported from the collapse by its own worldvolume gauge fields\[2, 3\]. This was originally realized in the Born-Infeld theory description of D2-branes\[2\]. The corresponding description from the matrix model was found thereafter including many supersymmetric tubes\[3\]. This implies that there are no static forces between parallel tubes of various sizes. The worldvolume gauge field theory is more accessible in this setting and the tube-D0 systems are described in terms of the solitonic excitations of the worldvolume gauge theory. The super D-helix is shown to be related to the supersymmetric tube by the T-duality transformation\[4\]. The supergravity solutions describing the tubular branes are identified recently and the dipole moment for the R-R four form field strength has been computed from the solutions\[5\].

In this note, we shall first review the previous construction of tubes from the matrix model. We shall identify the background magnetic field and electric field on the tube. The strength of electric field turns out to be critical while the strength of the background magnetic component is arbitrary. We identify also the number of fundamental strings stretched along the tube. The tube does not carry net D2-brane charges. The moments carried by the tubes for the R-R four form field strength will be shown to agree to that of the supergravity solutions. We then study some fluctuations around the tube-D0 system and tubes with different radii.

We move on the main subject of this note. Namely, we shall construct more general physical configurations of tubes. For certain set of initial configuration to be physical, they ought to satisfy the Gauss law constraint. We will construct such configurations that approach supersymmetric tubes in the asymptotic region. The regions in which the BPS equations are violated will be kept finite with the excitation energy bounded. Within these conditions, we ask most general configurations allowed by the system.

We shall first show that the basic construct for building up general tubes involves a junction of two tubes to one as well as a bending of tube. We shall also construct a tube along which the noncommutativity scale varies. In a certain sense, these basic operations are local excitations out of many asymptotically supersymmetric tubes. In particular, the junction is a sort of local finite size domain wall interpolating U(2) noncommutative worldvolume gauge theory to U(1) theory; a similar type of domain walls between different gauge groups was considered in Ref. \[6\]. The domain wall carries a finite energy. We shall not address the dynamics of such domain walls.

To construct general tubular configurations, we combine the basic operations along the axis of the tube. This way one may construct, for example, a junction which connects a tube to \( p \) tubes or one tube splitting to two tubes and recombing to one along the \( z \) direction. If one repeats such operations, one can build generic Riemann surfaces of an arbitrary topology with varying noncommutativity scale.

The plan of this paper is as follows. In section 2, we shall review the previous construction and identify the background electromagnetic field, the charges and the moment of the tube. In
section 3, the fluctuations around multiple tubes will be discussed. A particular interest will be the pattern of the symmetry breaking of tube configurations with different radii. We shall further analyze the fluctuation spectra of tube-D0 systems. This will correspond to the spectrum of 0-2 strings connecting D0’s to the tube. In section 4, we shall study the basic construct for the generic configurations. These will include the joint of two tube to one as well as the bending. We shall show that the noncommutativity scale may vary along the tube. Utilizing these basic operations, one may construct arbitrary Riemann surfaces with varying noncommutativity scale. Last section comprises concluding remarks.

2 Supersymmetric Tube Solutions

To discuss the tube configurations, we begin with the matrix model Lagrangian \[ L = \frac{1}{2R} \text{tr} \left( \sum_I (D_0 X_I)^2 + \frac{1}{(2\pi\alpha')^2} \sum_{I<J} [X_I, X_J]^2 + \text{fermionic part} \right) \] (1)

where \( I, J = 1, 2, \cdots, 9, R = g_s l_s \) is the radius of tenth spatial direction and \( \alpha' \equiv l_s^2 \) is related to the eleven dimensional Planck length \( l_{11} \) by \( l_{11} = (Ra')^{\frac{3}{4}} \). The scales \( R \) and \( 2\pi\alpha' \) will be omitted below by setting them unity and we shall recover them whenever necessary. As is well known, this Lagrangian can be thought of describing \( N \) D-particles if one takes all the dynamical variables as \( N \times N \) matrices.

Let us first describe relevant BPS equations we like to solve. For this, we shall turn on only first three components of the matrices \( X_I \). Then the Gauss law reads
\[
[X, D_0 X] + [Y, D_0 Y] + [Z, D_0 Z] = 0.
\] (2)

Using the Gauss law constraint, the bosonic part of the Hamiltonian can be rewritten as
\[
H = \frac{1}{2} \text{tr} \left( (D_0 X \pm i[Z, X])^2 + (D_0 Y \pm i[Z, Y])^2 + (D_0 Z)^2 - [X, Y]^2 + 2C_J \right) \geq \text{tr} C_J
\] (3)

where \( \text{tr} C_J \) is the central charge defined by
\[
\text{tr} C_J = \pm \frac{i}{2} \text{tr} \sum_{i=1}^3 [X_i, Z(D_0 X_i) + (D_0 X_i)Z].
\] (4)

The saturation of the BPS bound occurs if the BPS equations
\[
[X, Y] = 0, \quad D_0 Z = 0, \quad D_0 X \pm i[Z, X] = 0, \quad D_0 Y \pm i[Z, Y] = 0
\] (5)

hold together with the Gauss law constraint. On the choice of gauge \( A_0 = \frac{1}{2\pi\alpha'} Z \), the BPS equations of the upper sign imply that all the fields are static. Hence the system of equations reduce to
\[
[X, Y] = 0, \quad [X, [X, Z]] + [Y, [Y, Z]] = 0,
\] (6)

*There is a change from \[ l_{11} \] in the definition of the eleven dimensional Planck length \( l_{11} \). The difference is a numerical factor by \( (2\pi)^{\frac{3}{2}} l_{11}^{\text{old}} = l_{11}^{\text{new}}. \)
where the latter comes from the Gauss law constraint. Before providing the representations of the algebra, let us count the remaining supersymmetries of the states specified by the nontrivial representation of the algebra. The supersymmetric variation of the fermionic coordinates $\psi$ is

$$\delta \psi = \left( D_0 X^I \gamma_I + \frac{i}{2} \{ X^I, X^J \} \gamma_{IJ} \right) \epsilon + \tilde{\epsilon},$$

where $\epsilon$ and $\tilde{\epsilon}$ are respectively real spinors of 16 components parameterizing total 32 supersymmetries. Using the BPS equations, the invariance condition becomes

$$2(D_0 X \gamma_1 + D_0 Y \gamma_2) \Omega_+ \epsilon + \tilde{\epsilon} = 0,$$

where the projection operators $\Omega_\pm$ are $(1 \pm \gamma_3)/2$. This is solved by $\epsilon = \Omega_- \eta$ and $\tilde{\epsilon} = 0$ with $\eta$ arbitrary. The kinematical supersymmetries parametrized by $\tilde{\epsilon}$ are completely broken while half of remaining sixteen supersymmetries are left unbroken. Thus the configuration preserves a quarter of 32 supersymmetries of the matrix model, which is in agreement with the Born-Infeld or the supergravity description of the tubes $[2, 5]$.

Among the solutions of the BPS equations, a tube or multiple tubes are described by the algebra,

$$[z, x] = il y, \quad [y, z] = il x, \quad [x, y] = 0,$$

with $X_i = x_i$. The length scale $l$ is the noncommutativity parameter of the worldvolume gauge theory.

The algebra in (9) is realized as follows. Let us introduce variables $x_\pm$ by

$$x_\pm = x \pm iy.$$

It is clear that $x_-, x_+ = x^2 + y^2 \equiv \rho^2$ is a Casimir operator.

We are interested in the following irreducible representation of the algebra $[9]$,

$$x_+|n\rangle = \rho |n + 1\rangle, \quad z|n\rangle = l(n + \epsilon)|n\rangle \quad \epsilon \in [0, 1),$$

where $|n\rangle (n \in \mathbb{Z})$ is the basis for the original matrix variables. Because the system is only invariant under the finite translations to the $\hat{z}$ direction by $ml$ ($m \in \mathbb{Z}$), there is this nontrivial parameter $le$ characterizing the continuous translation modulo the finite translations. Below we shall set $\epsilon$ to zero for simplicity.

As $\rho^2$ is Casimir operator and can be regarded as a number, we can represent the $x_\pm$ with the angular variable as follows

$$x_\pm = \rho e^{\pm i\theta}$$

with periodic Hermitian operator $\theta$. Then $e^{\pm i\theta}|n\rangle = |n \pm 1\rangle$ and $[z, e^{\pm i\theta}] = \pm le^{\pm i\theta}$. It is obvious that our BPS configuration describes a noncommutative tube of radius $\rho$ in three dimensions. The coordinates $(\theta, z)$ on the tube would be noncommutative.

Any well-defined operator can be presented as

$$f(z, \theta) = \sum_{n=-\infty}^{\infty} \int_{-\pi}^{\pi} \frac{dk}{2\pi} \hat{f}_n(k) e^{in\theta + ikz}.$$
The range of $k$ is determined by the fact that the $z$ operator has discrete eigenvalues. Also any operator can be represented as $f = \sum_{n,m} f_{nm} |n\rangle \langle m|$ in the matrix theory. Two representations are related by

$$f_{nm} = \int_{-\pi}^{\pi} \frac{dk}{2\pi} f_{n-m}(k) e^{i(n+m)k}.$$  

(14)

The multiplication of operators on the noncommutative tube is well defined. One may instead introduce the $*$-product of ordinary functions on the corresponding commutative tube. In the Fourier representation of ordinary functions, their $*$-product should lead to the Fourier representation which we would get as the product of operators. Thus, the $*$ product of two ordinary functions $g$ and $h$ would be

$$g * h = \left[ e^{i\phi(\partial_{\theta} \partial_{\theta'}, - \partial_{\theta} \partial_{\theta'})} g(\theta, z) h(\theta', z') \right] \theta = \theta', z = z'.$$

(15)

In addition, the spatial integration $\int d\theta dz$ on the tube corresponds to $2\pi \rho \ell$. Since above $*$-product implies that $\theta * z - z * \theta = il$, the minimal area is in a rough sense given by $2\pi \rho \ell$. Since the circumference of the tube is $2\pi \rho$, one may regard the noncommutativity scale $\ell$ as a minimal distance in the $z$ direction. Indeed the discreteness of the spectrum of $z$ is consistent with this observation. Moreover, $1/(2\pi \rho \ell)$ corresponds to the area density of the the constituent D0-branes. The total number of D0-branes is $N = \text{tr} I = \frac{1}{2\pi \rho \ell} \int dzd\theta \rho = L_z / \ell$, where $L_z$ is the length of the tube in the $z$ direction. Hence the D0brane density per unit length in the $z$-direction is $1/\ell$.

At this point, let us work out the relation of the supersymmetric tube to the string theory configuration. For this, we shall work out the charges involved, the worldvolume electromagnetic field and the dipole moment of the tube for the R-R four form field strength. For the comparison to string theory, we shall restore $R$ and $2\pi \alpha'$ in this part.

The energy for the supersymmetric tube is given by the central charge,

$$E = \text{tr} C_J = \frac{l^2}{g_s l_s (2\pi l_s^3)^2} \text{tr} x_+ - x_+ = \frac{l^2 \rho^2}{(2\pi)^2 g_s l_s^3} \text{tr} I = \frac{1}{(2\pi)^2 g_s l_s^3} \frac{l \rho}{2 \pi l_s^3} \int dz d\theta \rho.$$  

(16)

We compare this with the total energy of D2-brane in the M-theory point of view. Namely, we expand the membrane Hamiltonian $\sqrt{p_{11}^2 + E_{M2}^2}$ to the leading order by $p_{11} + \frac{E_{M2}^2}{2p_{11}}$ where $p_{11} = N/R$ is the momentum along the tenth spatial direction and $E_{M2}^2$ is the total energy squared carried by the membrane except $p_{11}^2$. Thus the energy of the matrix model will be compared to

$$E = \frac{E_{M2}^2}{2p_{11}}.$$  

(17)

Hence, the membrane energy $E_{M2}$ is evaluated as

$$E_{M2} = \frac{1}{(2\pi)^2 g_s l_s^3} (2\pi \theta_{nc} \text{tr} I) \sqrt{2} = \frac{1}{(2\pi)^2 g_s l_s^3} V_T \sqrt{2},$$

(18)

where $V_T$ is the spatial volume of the tube and $\theta_{nc} = l \rho$. The factor $\sqrt{2}$ is due to the kinetic contribution that balances the one from the quartic potential.
For the worldvolume electromagnetic fields, let us first note that
\[ [\theta, z] \sim il . \] (19)
As in the case of the planar noncommutative D2-brane, we define the worldvolume gauge fields by
\[ \Theta = \theta + lA_z, \quad Z = z - lA_\theta . \] (20)
Using \([\theta, \cdot] = il\partial_z\) and \([z, \cdot] = -il\partial_\theta\), we are led to
\[ [\Theta, Z] = il^2 \left( \frac{1}{l} + F_{\theta z} \right) . \] (21)
Thus it is clear that the worldvolume background magnetic field is
\[ B = \frac{1}{l} . \] (22)
Further noting \(D_0\Theta = -lE_z\), the electric field on the tube may be evaluated as
\[ E_z = -\frac{1}{l}D_0\Theta \sim \frac{1}{l}(\partial_X \Theta D_0 X + \partial_Y \Theta D_0 Y) = -\frac{1}{lp^2}(XD_0 Y - YD_0 X) , \] (23)
where we have ignored the operator ordering problem. Using the explicit tube solution, one finds that
\[ E_z = \frac{1}{2\pi \alpha'} , \] (24)
which is in agreement with that of the Born-Infeld description of the supersymmetric tube in [3].

The momentum conjugated to the \(\theta\) coordinate carried by the tube will be
\[ \Pi_\theta = \frac{1}{g_s l_s} \rho^2 D_0 \Theta = \frac{1}{g_s l_s}(XD_0 Y - YD_0 X) = -\frac{\rho^2 l}{2\pi g_s l_s^3} I . \] (25)
Using the semiclassical Bohr-Sommerfeld quantization rule,
\[ \oint \Pi_\theta d\theta = 2\pi N_s \quad (N_s \in \mathbb{Z}) , \] (26)
we conclude that
\[ \frac{l \rho^2}{2\pi g_s l_s^3} = N_s , \] (27)
which implies that the radius of the tube is quantized semiclassically. The momentum \(\Pi_\theta\) in fact corresponds to the displacement current density in the \(z\)-direction and the integer \(N_s\) counts the number of fundamental strings stretched along the \(z\) direction.

The total angular momentum along the \(z\)-axis
\[ J_{\text{tot}} = \frac{1}{g_s l_s} \text{tr} (XD_0 Y - YD_0 X) = -\frac{l \rho^2}{2\pi g_s l_s^3} N = -N_s N , \] (28)
\[ ^1\text{Due to the compactness in the } \theta\text{-direction, only } x + iy = \rho e^{i\theta} \text{ or } X + iY \equiv \rho e^{i\theta} \text{ with } X^2 + Y^2 = \rho^2 \text{ are well defined. However, for the clarity of argument, we shall ignore this issue.} \]
is an integer and saturates the bound

$$|J_{\text{tot}}| \leq N s N.$$  \hfill (29)

(As will be shown below, tube-D0 systems are the examples where the bound is not saturated. See also the case of many tubes.) In fact this may be compared to the bound on the angular momentum per unit length in the z-direction in \[2\],

$$|j| \equiv |J_{\text{tot}}/L_z| \leq |\Pi B|.$$  \hfill (30)

Let us now turn to the problem of computing the dipole moment which generates the R-R four form field strength. For this purpose, we will use the nonabelian Chern-Simons couplings of D-particles to the R-R gauge fields,

$$S_{\text{CS}} = \mu_0 \int dt \text{tr} \left(C_t^{(1)} + C_t^{(1)} D_t \phi^I + \frac{i\lambda}{2} C_t^{(3)} [\phi^I, \phi^J] + \frac{i\lambda^2}{3} \phi^I \phi^J \phi^K F_t^{(4)}_{IJJK} + \text{h.o.t.} \right),$$  \hfill (31)

where $\mu_0^{-1} = (2\pi)^p g_s r_s^{p+1}$, $\lambda = 2\pi \alpha'$, $X^I = 2\pi \alpha' \phi^I$ and $F^{(p+1)}$ is the field strength corresponding to the R-R p-form potential, $C^{(p)}$. (See Ref. \[\text{[2]}\] for the details.) The second and the third terms do not contribute to the interaction since $X^I$ and $[X^I, X^J]$ are traceless on the tube solution. In particular, the vanishing of the third term implies there is no D2-brane charge carried by the tube configurations. This is as expected since such Dp-branes involving a compact direction of trivial cycle cannot carry net Dp-brane charges.

The first term is nonvanishing and tells us that the number of D-particles is given by $\text{tr} I$. The forth term is related to the dipole moment for the four form field strength. Noting

$$S_{\text{CS}}^{\text{dipole}} = \frac{i\mu_0 \lambda^2}{3} \int dt \text{tr} \phi^I \phi^J \phi^K F_t^{(4)}_{IJJK} = -\frac{1}{3} \left(\frac{2\pi}{2\pi}ight)^2 g_s r_s^3 \int dt dz \rho \int dt dz \rho F_t^{(4)}_{xyz},$$  \hfill (32)

we conclude that the dipole moment density of the tube is

$$d_2 = \frac{1}{3} \frac{\rho}{(2\pi)^2 g_s r_s^3} = \frac{1}{3} \mu \rho.$$  \hfill (33)

\section{3 Fluctuation Spectra of Multiple Tubes and Tube-D0 Systems}

In Ref. \[\text{[3]}\], the solutions for the multiple tubes and the tube-D0 are also constructed. We shall analyze the fluctuation spectra around these solutions. Due to the complexity of geometries, the fluctuation spectra are also quite involved. Hence we shall restrict our attention to two kinds of simple configurations. First is the symmetry breaking pattern of the concentric two tubes with different radii. The other is the spectra of tube-D0 strings when D0’s are located at the axis of the tube.

First let us recall that the BPS solution describing many parallel tubes are given by

$$X_+ = \sum_{a=0}^{p-1} \rho_a \sum_{n=-\infty}^{\infty} |(n+1)p + a\rangle \langle np + a| + \sum_{a=0}^{p-1} \xi_a \sum_{n=-\infty}^{\infty} |np + a\rangle \langle np + a|,$$

$$Z = \sum_{a=0}^{p-1} l_a \sum_{n=-\infty}^{\infty} n |np + a\rangle \langle np + a|,$$  \hfill (34)
where $X_+ = X + iY$ and $p$ is the number of tubes. Here $\rho_a$ is for the radius of each tube, $l_a$ is for the noncommutative parameter of each tube, and $\xi_a$ is for the position of the center of each tube in $(x,y)$ space. Of course one may add the position along the other dimensions. When all the noncommutativity parameters agree i.e. $l_a = l$, this background makes the worldvolume theory being a $U(p)$ noncommutative gauge theory. The $U(p)$ basis can be constructed by writing $|np + a\rangle\langle mp + b| = |n\rangle'\langle m'| T_{ab}$. Here $|n\rangle'$ is interpreted as a new basis for the space while $T_{ab}$ generates $U(p)$ algebra\[10\]. The total angular momentum for the many tubes is evaluated as $J_{\text{tot}} = -\sum_a l_a\rho_a^2 \text{tr}' I'$ where $I' = \sum_n |n\rangle'\langle n'|$ and $\text{tr}'$ is over the basis spanned by $|m\rangle'\langle n'|$. Since $\sum_a l_a\rho_a^2$ is the total number of strings through the tubes and $\text{tr}' = N_{\text{tot}}/p$, the angular momentum is related to the numbers of strings and D0’s by\[5\]

$$|J_{\text{tot}}| = \frac{1}{p} N_{\text{tot}} s N_{\text{tot}}. \quad (35)$$

To study the symmetric breaking involved with the multiple tubes, we shall consider the two concentric tubes with the same noncommutativity parameter but different radii. The two tubes are described by

$$x_+ = \sum_{a=0}^1 \rho_a \sum_{n=-\infty}^{\infty} |2(n+1) + a\rangle\langle 2n + a|,$$

$$z = l \sum_{a=0}^1 \sum_{n=-\infty}^{\infty} n |2n + a\rangle\langle 2n + a|. \quad (36)$$

Before discussing the fluctuation spectra of the above tubes, let us consider the equations satisfied by the transverse scalars in the background of a tube. Turning on only one component, $X_4$, the equation of motion becomes

$$[\partial_t - iz, [\partial_t - iz, X_4]] + [x_i, [x_i, X_4]] = 0. \quad (37)$$

First, let us rewrite the equation in the continuum limit. To this end, we note

$$[z, \cdot ] = -il\partial_\theta,$$

$$[x, \cdot ] = -il y\partial_z + O(l^2), \quad [y, \cdot ] = il x\partial_z + O(l^2), \quad (38)$$

where the first line is exact and, in the second line, we have ignored order $l^2$ terms appearing due to the discreteness of $z$-direction. Using these relations, the above scalar equation can be written as

$$\left(\partial_t - \frac{l}{2\pi\alpha'} \partial_\theta\right)^2 X_4 - \left(\frac{l}{2\pi\alpha'}\right)^2 \left(\frac{1}{\rho^2} \partial_\theta^2 + \partial_z^2\right) X_4 + O(l^3) = 0, \quad (39)$$

where we have restored our units. Introducing $\theta' = \theta + \frac{l}{2\pi\alpha'} t$, the above equation becomes a standard wave equation on a cylinder,

$$\partial_{\theta'}^2 X_4 - \left(\frac{l}{2\pi\alpha'}\right)^2 \left(\frac{1}{\rho^2} \partial_\theta^2 + \partial_z^2\right) X_4 + O(l^3) = 0. \quad (40)$$

The extra terms in the wave equation (39) are produced by the presence of the background electric field. As shown in [3], the continuum limit of the worldvolume gauge theory on the tube produced
by the fluctuation of $X,Y,Z$ is the noncommutative U(1) Yang-Mills-Higgs system with a peculiar Chern-Simons term. Here the coordinate $\theta'$ were used and the geometry is flat and noncommutative. The origin of the Chern-Simons term in the worldvolume theory is not clear.

Let us now turn to the case of the two concentric tubes with different radii. Here we expect that an analog of the spontaneous symmetry breaking occurs. Namely, the breaking of the $U(2)$ symmetry to $U(1) \times U(1)$ from the view point of the worldvolume gauge theory. However, the symmetry breaking pattern is more involved. For example, the wave equations for each $U(1)$ component of the transverse scalar are dependent upon the scale $\rho_a$ explicitly as we can see in (39). The effective metric and the Yang-Mills couplings are also dependent upon the radii. This behavior is quite different from that of the Yang-Mills theory on the parallel flat Dp-branes where the effective Yang-Mills coupling or the metric do not depend on the transverse separations.

The W-boson corresponds to the strings connecting one tube to the other. The mass produced by the radial separation may be found by considering the quadratic fluctuations around background two-tube configuration. Indeed, one may explicitly verify that the extra mass squared term produced by the separation is proportional to the radial separation squared,

$$\left(\Delta \rho\right)^2 \equiv |\rho_0 - \rho_1|^2.$$  \hfill (41)

Let us now discuss the probe of the supersymmetric tube using D0 solitons. In Ref.[3], multi soliton solutions describing D0’s are constructed. For this we introduce a shift operator defined by

$$S = \sum_{n=0}^{\infty} |n + m\rangle \langle n| + \sum_{n=-\infty}^{-1} |n\rangle \langle n|.$$  \hfill (42)

It satisfies the relations

$$SS^\dagger = I - P, \quad S^\dagger S = I,$$  \hfill (43)

where the projection operator $P$ is defined by $P = \sum_{a=0}^{m-1} |a\rangle \langle a|$. The general soliton solutions including the moduli parameters are given by

$$\bar{X}_i = S x_i S^\dagger + \sum_{a=0}^{m-1} \lambda^a_i |a\rangle \langle a|, \quad \bar{X}_s = \sum_{a=0}^{m-1} \varphi^a_s |a\rangle \langle a|,$$  \hfill (44)

with the index $s$ referring to the transverse scalar $X_4$ to $X_9$. Unlike the solitons of the noncommutative Yang-Mills theory describing a planar D2-brane[1, 2, 3, 4, 5], the solutions we constructed here are BPS saturated states of eight supersymmetries. Namely, the solution satisfies the original BPS equations and no further supersymmetries are broken by the presence of the solitons. The moduli are describing the positions of D0 branes in the 9-dimensional space. The appearance of moduli further support the view point that the configurations are describing not holes in tubes but extra D0-branes that may even fly off the tube. Since there are extra D0’s, the bound in (23) is no longer saturated. Namely the total angular momentum is smaller than the number of strings multiplied by the total number of D0’s.

For simplicity of the analysis in studying the fluctuation around this background, we shall set all the moduli parameters to zero. This corresponds to D0’s located at the origin of the nine spatial
dimensions, which is at the axis of the supersymmetric tube. (For the gauge invariant positions of noncommutative solitons, see Ref. [12].)

To study dynamics of the system, we choose the gauge \( A_0 = Z \) and consider small fluctuations of the matrix variable around the solution (44)

\[
Z = \bar{Z} + P \delta Z P + \bar{P} \delta Z \bar{P},
X_+ = \bar{X}_+ + P \delta X_+ P + \bar{P} \delta X_+ \bar{P},
X_- = P \delta X_- P + \bar{P} \delta X_- \bar{P}
\]

(45)

where \( \bar{P} = I - P \). We shall insert this expression to the original matrix model Lagrangian and find a new effective Lagrangian to the quadratic order of the fluctuations. As in the case of the Abelian-Higgs system [12] or D2-D0 system [13], the linear fluctuation of \( \bar{P} \delta X \) are described by the original worldvolume dynamics of the tube without \( D0 \)'s. This would corresponds to the dynamics of tube-tube strings. On the other hand, the linear fluctuation of \( P \delta X \) are described by the original matrix model with \( m \times m \) matrix variables. This implies that the fluctuations describe the 0-0 strings connecting \( m \) \( D0 \)'s. These two kinds of fluctuations decouple from the rest at the order of linear fluctuation. The remaining fluctuations would correspond the tube-0 strings. In order to find the spectra of these strings, we shall find the corresponding equations of motion to the linear order. For this purpose, we will use the following matrix variables,

\[
T_1 = \frac{1}{\rho} P \delta X_+ \bar{P} \bar{X}_- S,
T_2 = \frac{1}{\rho} P \delta X_- \bar{P} \bar{X}_+ S,
K = P \delta Z \bar{P} S
\]

(46)

where all of these are complex \( m \times \infty \) matrices, e.g. \( M_{an} (a = 0, \cdots, m-1 \text{ and } n \in \mathbf{Z}) \). With straightforward computations, one finds the following set of linearized equations,

\[
\frac{d}{dt} \left( 2 \ddot{K} - i \rho (\dot{T}_1 + \dot{T}_2) \right) = 0,
\dot{T}_1 + \frac{\rho^2}{2} (T_1 - T_2) - i \rho \dot{K} + 2i \dot{T}_1 (z - l) = 0
\dot{T}_2 + \frac{\rho^2}{2} (T_2 - T_1) - i \rho \dot{K} + 2i \dot{T}_2 (z + l) = 0.
\]

(47)

There is also one equation from the Gauss law but it is not independent. The equations above show that there is no mixing between matrix components with different indices. The first equation can be easily integrated and one obtains \( 2 \dot{K} = i \rho (T_1 - T_2) \). (The integration constant is set to zero.) Using this relation, we eliminate \( K \) from above equations and get

\[
(\ddot{T}_1)_{an} + 2i l (n-1)(\dot{T}_1)_{an} + \rho^2 (T_1)_{an} = 0
(\ddot{T}_2)_{an} + 2i l (n+1)(\dot{T}_2)_{an} + \rho^2 (T_2)_{an} = 0
\]

(48)

We now introduce new matrix variables by

\[
U = P \delta X_+ \bar{P} S,
V = P \delta X_- \bar{P} S
\]

(49)
and the equations of motion for these now become

\begin{align*}
\ddot{U}_{an} + 2iln \dot{U}_{an} + \rho^2 U &= 0 \\
\ddot{V}_{an} + 2iln \dot{V}_{an} + \rho^2 V &= 0. 
\end{align*}

(50)

For the transverse coordinates $X^s$, their fluctuations are governed by the same equations, i.e.

\begin{equation}
(\partial_t^2 + 2iln \partial_t + \rho^2)T_{an}^s = 0, 
\end{equation}

(51)

where we define $T^s = P\delta X^s \bar{P}$. The corresponding angular frequency for all these modes is

\begin{equation}
\omega_n^\pm = \frac{1}{2\pi\alpha'} \left( \ln \pm \sqrt{\rho^2 + (ln)^2} \right),
\end{equation}

(52)

where we have restored the units. The first term in the parenthesis is again coming from the fact that there is the nonvanishing electric field or angular momentum carried by the tube configuration. The spectra are independent of the D0-brane index $a$ as it should be. The second contribution, $M_n \equiv \frac{1}{2\pi\alpha'} \sqrt{\rho^2 + (ln)^2}$, in the frequency is also the one expected. It corresponds to the mass of the string stretched between D0’s at the origin to the point on the tube at $z = ln$. Namely, the mass for such string is given by the string tension multiplied by the stretched length, which agrees precisely with $M_n$. Thus we conclude that the geometry seen by D0-probe is a tube extended in the z-direction with radius $\rho$. Finally, we like to compute the contributions of above modes to the Hamiltonian of the system. But evaluating the contribution to $H$ requires the terms in the second order in the fluctuation due to the couple to the background energy. Instead, we will evaluate $\tilde{E} \equiv H - \text{tr} C_J$. Since $C_J$ is conserved, the combination is also conserved. The straightforward evaluation is given by

\begin{equation}
\tilde{E} = \frac{1}{2} \sum \left( \ddot{U}_{an}^2 + \ddot{V}_{an}^2 + \rho^2 \dot{U}_{an}^2 + \dot{V}_{an}^2 + \sum_s \left[ (\dot{T}_{an}^s)^2 + M_n^2 (T_{an}^s)^2 \right] \right). 
\end{equation}

(53)

Here $K$ is eliminated again and the above form explicitly shows that there are indeed only two independent oscillators; one may regard $K$ as a gauge degree of freedom. The disparity between the transverse modes and $U, V$ is mainly due to subtraction of the central charge from the Hamiltonian.

### 4 Junctions and Bends of Tubes

So far we have described the characteristics of the supersymmetric tubes by looking at the charges or worldvolume fluctuations. In this section we like to describe the junctions of three tubes that approach asymptotically the supersymmetric tubes extended along the same direction. Then such junctions may be considered as local excitations in a certain sense because the remaining supersymmetries are broken locally at the junctions. If one splits, for example, a tube to two concentric tubes of equal radii, the junction will interpolate the U(1) noncommutative worldvolume theory on one tube to the U(2) noncommutative gauge theory on the concentric tubes. The interpolation is local because it occurs along the $z$ directions and the area of the excited region is finite. The excitation of energy of this domain wall is also finite. In short, it is a domain wall interpolating between U(1) and U(2) noncommutative gauge theories.
The second element of deformation will be a bend of tube that approaches asymptotically supersymmetric configurations. In particular, we shall construct a bend of tube that makes an asymptotic angle. The region where the bending occurs will be local. We call this type of configuration as bend with angle \((\kappa, \nu)\) where the angles are defined as follows. If the lower part of the tube is extended in the \(z\)-direction, \(\kappa\) is the angle of the upper part of the tube with \(z\)-axis and \(\nu\) the azimuthal angle. By performing double bending operations, one may construct a bend of tube with displacement, which connects a tube of the \(z\) direction centered at \((\xi_x, \xi_y)\) in the \((x, y)\) space to the other tube at \((\xi'_x, \xi'_y)\). This of course can be done with the bending of angle \((\kappa, 0)\) at some point of tube and the other bending of angle \((\kappa, \pi)\) at another point. We also construct such configuration directly and study more details of parameters involved. The remaining basic construct is a tubular configuration along which the noncommutativity scale changes.

Combining these configurations, one may construct arbitrary Riemann surfaces. For example, a tube with one hole can be easily constructed; for two tubes that are appropriately bended, one joins the lower ends as well as the upper ends by the junctions.

Let us first consider the construction of the simple junction. Here we shall use the gauge \(A_0 = Z\). The tube for \(z \to -\infty\) approaches the supersymmetric tube with radius \(\rho\) whereas, for \(z \to \infty\), it approaches two tubes with radii \(\alpha\) and \(\beta\). The ansatz for the solution is then

\[
Z = Z + l \left\{ \sum_{n=-\infty}^{-(b+1)} n |n\rangle \langle n| + l \sum_{n=0}^{\infty} n \left( |2n+1\rangle \langle 2n+1| + |2n+2\rangle \langle 2n+2| \right) \right\},
\]

\[
X_+ = X_+ + \rho \sum_{n=-\infty}^{-(b+1)} |n+1\rangle \langle n| + \sum_{n=a}^{\infty} \left( \alpha |2n+1\rangle \langle 2n-1| + \beta |2n+2\rangle \langle 2n| \right),
\]

where \(Z\) and \(X_+\) are \((2a + b + 1) \times (2a + b + 1)\) matrices. The basis of these finite matrices is spanned by \(|-b\rangle, |-b+1\rangle, \cdots, |2a-1\rangle, |2a\rangle\). We insert this ansatz to the Gauss law and find an equation

\[
[X_+, [X_-, Z]] + [X_-, [X_+, Z]] = 2l\alpha^2 |2a-1\rangle \langle 2a-1| + 2l\beta^2 |2a\rangle \langle 2a| - 2l\rho^2 \langle -b| \langle -b| - b\rangle.
\]  (55)

By the Gauss law, the boundary conditions for \(Z\) and \(X_+\) are given as

\[
\langle -b | Z | -b \rangle = -lb, \quad \langle 2a-1 | Z | 2a-1 \rangle = \langle 2a | Z | 2a \rangle = la,
\]  (56)

and

\[
\left( Z + l(b-1) \right) X_+ | -b \rangle = 0, \\
\left( Z - l(a-1) \right) X_- | 2a-1 \rangle = \left( Z - l(a-1) \right) X_- | 2a \rangle = 0.
\]  (57)

Taking trace of Eq. (55), one obtains the relation

\[
l\rho^2 = l\alpha^2 + l\beta^2.
\]  (58)

Since \(l (\text{radius})^2 \sim \# \) of fundamental strings, the above relation implies that the number of fundamental strings is preserved through the junction. Finding most general solutions of the above is quite involved. Let us work out the simplest case, i.e. \(a = 1\) and \(b = 0\). In this case, the \(3 \times 3\) matrix \(Z\) is fixed completely as

\[
Z = l |1\rangle \langle 1| + l |2\rangle \langle 2|.
\]  (59)
Figure 1: A skeleton graph for the supersymmetric tube is depicted first. Here the box with number \( i \) denotes the basis \( |i\rangle \) and its position corresponds to the eigenvalue of the diagonal matrix \( Z \). The arrow from the box \( i \) to the box \( j \) indicates that there is a nonvanishing component of \( |j\rangle \langle i| \) in \( X^+ \). The second figure is for the simple junction in (59) and (60) with \( \varphi = 0 \).

by the condition (56). Imposing the condition in (57) on \( X^+ \) and solving (55), one finds that

\[ X^+ = |0\rangle \langle \psi| + |\phi\rangle \langle 0| \quad \text{(60)} \]

with

\[ |\psi\rangle = \alpha \cos \varphi |1\rangle + \beta \sin \varphi |2\rangle, \quad |\phi\rangle = -\alpha \sin \varphi |1\rangle + \beta \cos \varphi |2\rangle. \quad \text{(61)} \]

This will be the generic configuration (within the ansatz) which is consistent with the Gauss law. (Here we do not include some trivial phase factors that amount to the gauge parameters.) It does not satisfy the equations of motion but serves as a valid initial configuration. Staring from this configuration, the system will eventually evolve toward lower energy configurations. We call such initial configuration as physical one. Let us compute the energy of the junction which is, of course, conserved during the later time evolution. For this purpose, we shall compute the following energy,

\[ \tilde{E} = H - \text{tr} C_J \equiv \frac{1}{2} \text{tr} \left( \partial_0 X^i \partial_0 X^i - [X,Y]^2 \right). \quad \text{(62)} \]

For the above initially static configuration, the energy is evaluated as

\[ \tilde{E} = (\alpha^2 \sin^2 \varphi + \beta^2 \cos^2 \varphi)^2 + \frac{1}{4}(\alpha^2 + \beta^2)(\alpha^2 \sin^2 \varphi + \beta^2 \cos^2 \varphi) + \frac{1}{4}(\alpha^4 \sin^2 \varphi + \beta^4 \cos^2 \varphi). \quad \text{(63)} \]

When \( \alpha \geq \beta \), the energy is minimized if \( \sin \varphi = 0 \); the energy at the minimum becomes

\[ \tilde{E}_{\text{min}} = \frac{1}{16\pi^2 g_s l_s^2} \left( 5\beta^4 + \rho^2 \beta^2 \right) = \frac{g_s l_s}{4 l^2} N_\beta (5N_\beta + N_\rho), \quad \text{(64)} \]

where \( N_\beta \) and \( N_\rho \) are respectively the numbers of fundamental strings through tubes of radii \( \beta \) and \( \rho \). Note that the expression is valid only when \( \sqrt{2}\beta \leq \rho \) due to the condition \( \beta \leq \alpha \). The
corresponding configuration is depicted in Figure 1. We shall comment more on the energetics of the configuration.

Let us now consider the bend without asymptotic angles. The corresponding ansatz will be

\[
Z = Z + l \sum_{n=-\infty}^{-(b+1)} n|n\rangle\langle n| + l \sum_{n=0}^{\infty} n|n\rangle\langle n|,
\]

\[
X_+ = X_+ + \rho \sum_{n=-\infty}^{-(b+1)} |n+1\rangle\langle n| + \sum_{n=0}^{\infty} (\alpha|n+1\rangle\langle n| + \Delta|n\rangle\langle n|),
\]

(65)

where we take \(\Delta\) real using the rotational symmetry in the \((x,y)\) space. Here \(Z\) and \(X_+\) are \((a+b+1)\times(a+b+1)\) matrices with basis spanned by \(|-b\rangle, |b+1\rangle, \cdots, |a-1\rangle, |a\rangle\). We again insert this ansatz to the Gauss law and find

\[
[X_+, [X_-, Z]] + [X_-, [X_+, Z]] = 2l \alpha^2 |a\rangle\langle a| - 2l \rho^2 (-|b\rangle\langle b| - \Delta|Q\rangle\langle Q|).
\]

(66)

where

\[
\Delta Q = \Delta(Z - la)X_+|a\rangle\langle a| + \Delta|a\rangle\langle a|X_+(Z - la) + (h.c.).
\]

(67)

By the Gauss law, the boundary conditions for \(Z\) and \(X_+\) are set by

\[
(-b|Z|-b) = -lb, \quad (a|Z|a) = la,
\]

(68)

and

\[
\left(Z + l(b-1)\right)X_+|b\rangle = 0, \quad \left(Z - l(a-1)\right)X_-|a\rangle = 0.
\]

(69)

The left side of Eq. (66) and \(\Delta Q\) are traceless. This implies that

\[
l \rho^2 = l \alpha^2,
\]

(70)

which states that the number of fundamental strings is preserved through the bend. Let us consider the case \(a = 1\) and \(b = 0\). The resulting most general configuration is

\[
Z = l|1\rangle\langle 1|, \quad X_+ = \frac{\rho}{\sqrt{2}}(|0\rangle\langle 1| - |1\rangle\langle 0|)
\]

(71)

where again we set some gauge trivial phase factors to zero. This bend with displacement is depicted in Figure 2. There is essentially no free parameter in this case. The shifted energy \(\tilde{E}\) is evaluated as

\[
\tilde{E} = \frac{1}{2} \rho^2 (\rho^2 + \Delta^2).
\]

(72)

Contrary to the naive expectation, the energy does not approach to zero when \(\Delta = 0\). This is because there is a discontinuity at \(\Delta = 0\). Namely, when \(\Delta = 0\), there appears a new parameter.

Now let us consider the following configurations. We make the bend of total displacement \(\Delta\) by repeating \(k\) times of the unit bend with displacement \(\Delta/k\). The unit bend with displacement \(\Delta/k\) will cost the energy \(\tilde{E}\) by \((1/2)\rho^2 (\rho^2 + \Delta^2/k^2)\). The total cost of energy will then be

\[
\tilde{E}_k = \frac{1}{2} \rho^2 (k \rho^2 + \Delta^2/k).
\]

(73)
If \( k \) were an arbitrary real number, the value
\[
\tilde{E}_{\text{min}} = \rho^3 \Delta 
\] (74)
would be the minimum at \( k = \Delta/\rho \). Thus it appears that the bend with displacement might be stabilized with its excited region kept finite. We shall, however, prove below that this is not the case.

To verify (73) for \( k = 2 \), let us consider the case of \( a = 2 \) and \( b = 0 \) for a given nonvanishing real positive \( \Delta \). They are given by
\[
Z = 2l |2\rangle \langle 2| + l |1\rangle \langle 1|, \quad X_+ = \delta |2\rangle \langle 2| + \frac{\rho}{\sqrt{2}} \left( e^{i\theta_1} |0\rangle \langle 1| - e^{-i\theta_1} |0\rangle \langle 1| \right) + \frac{\rho}{\sqrt{2}} \left( e^{i\theta_2} |0\rangle \langle 2| - e^{-i\theta_2} |2\rangle \langle 0| \right),
\] (75)
where \( \theta_1 \) and \( \theta_2 \) are respectively the arguments of \( \delta \) and \( \Delta - \delta \). Here again we set some gauge trivial phase factors to zero. The shifted energy \( \tilde{E} \) is given by
\[
\tilde{E} = \frac{1}{2} \rho^2 \left( 2\rho^2 + |\delta|^2 + |\Delta - \delta|^2 + \rho^2 \sin^2(\theta_1 - \theta_2) \right).
\] (76)
The minimum is achieved when \( \theta_1 = \theta_2 = 0 \) and \( \delta = \Delta/2 \) with its value
\[
\tilde{E} = \frac{1}{2} \rho^2 \left( 2\rho^2 + \frac{|\Delta|^2}{2} \right),
\] (77)
which agrees with (74) for \( k = 2 \).

Another basic operation is the change of the noncommutativity scale within a tube. The configuration starts with the noncommutativity scale \( l_1 \) for \( z < 0 \) and becomes a tube with the
noncommutativity scale $l_2$ for $z > 0$. One simple example is described by

$$Z = l_1 \sum_{n=-\infty}^{-1} n|n\rangle\langle n| + l_2 \sum_{n=0}^{\infty} n|n\rangle\langle n|,$$

$$X_+ = \rho_1 \sum_{n=-\infty}^{-1} |n+1\rangle\langle n| + \rho_2 \sum_{n=0}^{\infty} |n+1\rangle\langle n|,$$  \hspace{1cm} (78)

which we illustrate in Figure 2 and Figure 3. One may easily verify that this satisfies the Gauss law constraint when $l_1 \rho_1^2 = l_2 \rho_2^2$. This implies that the number of fundamental strings is preserved through the change of noncommutative scale. The excitation energy for this configuration reads

$$\tilde{E} = \frac{1}{8}(\rho_1^2 - \rho_2^2)^2.$$  \hspace{1cm} (79)

We now discuss the bend with angle. First let us note that there are rotational symmetries in the target space $(X, Y, Z)$. For example, a rotation by an angle $\gamma$ in the $(X, Z)$ plane may produce another supersymmetric tube if one starts from a tube with its axis along the $z$-direction. Choosing a gauge $A_0 = \cos \gamma Z + \sin \gamma X$, the rotated supersymmetric tube solution is given by

$$Z = \cos \gamma \sum_{n=-\infty}^{\infty} \ln|n\rangle\langle n| - \sin \gamma \sum_{n=-\infty}^{\infty} \frac{\rho}{2}(|n+1\rangle\langle n| + |n\rangle\langle n+1|),$$

$$X = \cos \gamma \sum_{n=-\infty}^{\infty} \frac{\rho}{2}(|n+1\rangle\langle n| + |n\rangle\langle n+1|) + \sin \gamma \sum_{n=-\infty}^{\infty} \ln|n\rangle\langle n|,$$

$$Y = \sum_{n=-\infty}^{\infty} \frac{\rho}{2i}(|n+1\rangle\langle n| - |n\rangle\langle n+1|).$$  \hspace{1cm} (80)

To obtain the bended configuration with angle, we shall combine the above solution with the supersymmetric tube with $\gamma = 0$. For this purpose, we shall work in a gauge $A_0 = \sum_{n=-\infty}^{\infty} \ln|n\rangle\langle n|$. The Gauss law for a static configuration will then be

$$[X_i, [X_i, A_0]] = 0.$$  \hspace{1cm} (81)

\hspace{1cm} 

Figure 3: The shape of tube, along which the noncommutativity scale changes, is illustrated in the first figure. The second represents a Y-shaped junction. The last picture is for a tube with two holes.
Figure 4: A junction constructed by $k$ steps repeatedly using splitting and joining. In the splitting or joining, the smaller (radius)$^2$ is given by $N_\beta/(kl)$. Here we illustrate the case of $k = 4$ for simplicity.

It is straightforward to verify that the following configuration

$$Z = \sum_{n=-\infty}^{-1} \ln|n\rangle\langle n| + \cos \gamma \sum_{n=0}^{\infty} \ln|n\rangle\langle n| - \sin \gamma \sum_{n=0}^{\infty} \frac{\rho}{2}(|n+1\rangle\langle n| + |n\rangle\langle n+1|),$$

$$X = \sum_{n=-\infty}^{-1} \frac{\rho}{2}(|n+1\rangle\langle n| + |n\rangle\langle n+1|) + \cos \gamma \sum_{n=0}^{\infty} \frac{\rho}{2}(|n+1\rangle\langle n| + |n\rangle\langle n+1|) + \sin \gamma \sum_{n=0}^{\infty} \ln|n\rangle\langle n|,$$

$$Y = \sum_{n=-\infty}^{\infty} \frac{\rho}{2i}(|n+1\rangle\langle n| - |n\rangle\langle n+1|)$$

(82)

does satisfy the Gauss law. In this configuration, the tube is along the z-direction for $z < 0$ while its axis is in the $(\sin \gamma, 0, \cos \gamma)$-direction for $z > 0$. The skeleton graph for this configuration is depicted in Figure 2. The bend occurs at $z = 0$. Its excitation energy may be computed using

$$\tilde{E} = \frac{1}{2} \text{tr} \left( \partial_0 X^i \partial_0 X^i - [X, Y]^2 - [X, Z]^2 - [Y, Z]^2 + [A_0, X^i][A_0, X^i] \right).$$

(83)

This energy density is also conserved in time and matches with $H - C_J$ asymptotically. Though complicated, the straightforward evaluation of the energy for the bend gives

$$\tilde{E} = \frac{\rho^4}{16} \left( 16 - (3 + \cos \gamma)^2 \right).$$

(84)

For $\gamma = 0$, the tube is straight and the corresponding excitation energy vanishes as it should be. The excitation energy increases as the angle grows and reaches its maximum for $\gamma = \pi$. For finite $\gamma$, we expect that the excitation energy may be lowered by making a bend gradually. However, we shall not explore such configurations. Finally, we like to comment on the bend with a displacement formed by combining double bends with angles. Namely it is achieved by one bending at $z = 0$ with angle $\gamma$ and the other bending at $z' = lk$ with angle $-\gamma$. The corresponding displacement will be $\Delta = lk \sin \gamma$. For a given $\Delta$, the excitation energy may be brought to arbitrarily small by sending $k$ to large with $\gamma \sim \Delta/(lk)$. Hence we see that the bend with displacement cannot be stabilized with a finite range of the excited region.
At this point let us discuss energetics involved with the junction constructed previously. As in Figure 3, one may construct the junction by \( k \) steps. In this case, the corresponding total excitation energy will be

\[
\tilde{E}_k = \frac{g s l_s}{4 l^2} N_\beta \left( N_\rho + \frac{N_\beta}{k^2} (11k - 6) \right).
\] (85)

Since \( \tilde{E}_k \) monotonically decreases as the integer \( k \) grows, it appears that the minimum occurs at \( k = \infty \). However, \( k \) cannot exceed \( N_\beta \) due to the quantization of \( l (\text{radius})^2 \). Hence the estimation of the minimum is

\[
\tilde{E}_\text{min} = \frac{g s l_s}{4 l^2} N_\beta \left( N_\rho + 11 - \frac{6}{N_\beta} \right).
\] (86)

The corresponding configuration is localized over a range \( \delta z \sim l N_\beta \). Such stabilization of the localized configuration occurs due to the semiclassical quantization of radius.

As we discussed in detail, the basic construct for the general Riemann surfaces will be the junction, the bend with angle and the connector with change of the noncommutativity scale. These basic local operations may be combined to produce arbitrarily complicated configurations with varying noncommutativity scale. These general configurations resemble string loop diagrams but everything occurs in spatial dimensions.

The number of fundamental strings stretched and the number of D0's play an important role in the understanding of such general configurations. The sum of the cross sectional area multiplied by the noncommutativity scale is preserved in any case. As we have shown above, this quantity is proportional to the total number of the stretched fundamental string along the tube. The D0 brane density along the tube is given by the inverse of the noncommutativity scale \( l \). For the given number of the stretched fundamental strings \( N_s \), the radius of the tube gets larger if the D0's are densely packed. Namely \( N_s \) and the density of D0-branes control the radius of tubes.

## 5 Conclusions

We have constructed the junction that separates one tube to two. One may also use this local operation to join two tubes to one. If one uses the junctions repeatedly, one may split one tube to arbitrary number of tubes or join arbitrary number of tubes to one. The operation of bend with angle makes a tube direct to an arbitrary spatial direction. If one combines the junction and the bend operation, one may construct a Y-shaped junction in Figure 3. The basic constructs are only locally excited, so are the combinations. For example, the configuration with two holes depicted in Figure 3 may be constructed by splitting a tube to two, adding appropriate bending, joining the two to one and repeating the whole operations again. In conclusion, one may have arbitrary Riemann surfaces by combining two basic operations. Furthermore if one adds noncommutativity-changing operations, one may have a varying noncommutativity over the Riemann surface.

From the view point of worldvolume gauge theories, the physical meaning of junctions is quite intriguing. When \( \alpha = \beta = \rho/\sqrt{2} \), the junction in [54] interpolates the U(1) noncommutative theory and the U(2) noncommutative gauge theory. Hence the junction works as a kind of domain
wall. Similarly by considering a tube splitting into many tubes of an equal radius, one finds that a domain wall interpolating the U(1) noncommutative theory to U(p) noncommutative gauge theory. Of course, the original matrix theory governs all these dynamics.

In this note, we first review the previous construction of supersymmetric tubes in the matrix model. We identified all the charges and the moment carried by the supersymmetric tubes. The symmetry breaking of many tubes with different radii is discussed. We then study the fluctuation spectra for the tube-D0 system. We construct more general physical configurations of tubes that are consistent with the Gauss law. The basic constructs are shown to be the junction, the bend with angle and the connector of tubes with different noncommutativity scales. Combining these basic operations one may construct even arbitrary Riemann surfaces with arbitrary topology. The noncommutativity scale may vary over the Riemann surfaces.

The tubes involve the background electric component that is critical. For the worldvolume theory, we have utilized only the spatial noncommutativity. The possible role of the electric field for the spacetime noncommutativity is not fully understood. In particular, the only invariant combination in the 2+1 dimensions is $(E^2 - B^2/\rho^2)(2\pi\alpha')^2 = 1 - (2\pi\alpha'/l')^2$. If one sends $\alpha'$ to zero while fixing $l\rho$, this corresponds to the NCOS limit\[16\]. Detailed investigation on the nature of the limit is necessary.

Finally the dynamical issues of the tubular configurations are not clearly understood. There is no intuitive understanding why there is no force between many parallel tubes. There is no force between the tube and D0 and again the intuitive understanding is lacking. One of approach in this direction would be the study of the worldsheet CFT of $p-p'$ strings as in Ref.\[17\]. Detailed dynamical investigation of all kind of the configurations will be quite interesting. However, due to complications, one needs better methods of organizing such dynamical processes. Further studies are required in this direction.

Acknowledgment We are especially benefitted from the extensive discussions with Kimyeong Lee at the early stage of this work. DB would like to thank J.-H. Cho, R. Gopakumar, M. Gutperle, A. Karch, Taejin Lee, S. Minwalla and A. Strominger for the enlightening discussions. DB also likes to acknowledge the Harvard Theory Group where part of this work was done. This work is supported in part by KOSEF 1998 Interdisciplinary Research Grant 98-07-02-07-01-5 and by UOS Academic Research Grant.

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