ON GRIBOV’S IDEAS ON CONFINEMENT

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Abstract

I comment on possible relations of Gribov’s ideas on mechanism of confinement with some phenomena in QCD and in supersymmetric gauge theories.

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1 Gribov’s mechanism of confinement of color

V.N. Gribov developed his ideas on confinement in QCD over the course of more than ten years. While I had a privilege of multiple discussions on the subject with him, it was difficult for me to follow. I tried to relate some of his points to subjects I knew from QCD and from supersymmetric gauge theories. These discussions were origin of the comments presented here.

First, I will try to review briefly Gribov’s ideas following his last two papers.1,2 His starting point is that confinement is due to light quarks — the mechanism which is similar to supercritical phenomena for large Z in QED. These phenomena are related with fermion bound states in a strong field. There is no such state for bosons. For this reason Gribov believed there were no glueballs in pure gluodynamics, and the theory presents scaling behavior.

To realize his picture of confinement in QCD Gribov introduced a number of new points:

- Formulation of QCD with no divergences and no need for renormalization. Both perturbative and nonperturbative phenomena are described by Green functions.
- In QCD ultraviolet and infrared regimes are strongly interrelated to give a specific confining solution.
- Chiral symmetry and corresponding Goldstone bosons — pions, play a special role in confinement. In particular, there is a short distance component in the pion wave function — a core which is pointlike. Equations are changed because of this.

What is the supercritical phenomenon in QED? A heavy nucleus with $Z > Z_{cr} \sim 1/\alpha$ makes the vacuum of light charged fermions unstable. It forms a bound state with a positron at short distances (an electron component of pair goes away). As a result an effective shift, $Z \rightarrow Z_1 = Z - 1$, occurs for distances larger than the bound state size.

A similar picture is proposed for QCD based on the growth of effective charge at large distances. Only colorless states are stable. The phenomenon is described by Gribov’s equation for the fermion propagator $G(q)$

$$
\left( \frac{\partial}{\partial q_{\mu}} \right)^2 G^{-1} = g(q) \frac{\partial G^{-1}}{\partial q_{\mu}} G \frac{\partial G^{-1}}{\partial q_{\mu}} - \frac{3}{16\pi^2 f_{\pi}^2} \{i\gamma_5, G^{-1}\} G \{i\gamma_5, G^{-1}\} \quad (1)
$$

where the running coupling $g(q)$

$$
g(q) = \frac{4}{3} \frac{\alpha(q)}{\pi}
$$
comes from the solution for the gluon propagator, and the second term is due to the pointlike structure of the pion.

The quantity $\partial G^{-1}/\partial q_\mu$ in the r.h.s. of the Gribov’s equation is the effective vertex in the theory. The gluon propagator (in Feynman gauge) is of the form

$$-\frac{g_{\mu\nu}}{q^2} 4\pi \alpha(q).$$

A qualitative picture for running of the effective charge, $\alpha(q)$, emerging from the solution for the gluon propagator is given in Fig. 1 where the scale $\lambda \sim \Lambda_{QCD}$.

**Figure 1:** Running of the effective charge $\alpha(q)$

Gribov’s derivation of Eq. (1) for the fermion propagator is illustrated by Fig. 2. Acting by $(\partial/\partial q_\mu)^2$ on the gluon propagator, $1/(p-q)^2$, one gets

$$\left(\frac{\partial}{\partial q}\right)^2 \Gamma q = \Gamma q \Gamma q \Gamma q$$

**Figure 2:** Diagrammatic form of Gribov’s equation

$\delta^4(p-q)$. It gives the first term in r.h.s. of Eq. (1) (without the second term which is due to the pointlike component of the pion). For a detail analysis of Gribov’s equation I refer readers to Ewerz’s work.

At large $q$ the solution of Eq. (1) is

$$G^{-1}(q) \rightarrow Z^{-1} \left[ m - \frac{\nu_1}{q^2} + \frac{\nu_2}{q^4} \right]$$

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where in the limit of vanishing coupling, i.e. at $\alpha = 0$, parameters $Z$, $m$, $\nu_1$, $\nu_2$ are arbitrary constants. For perturbative $\alpha(q)$ these parameters become running as

$$Z(q) = Z_0 \left( \frac{\alpha(q)}{\alpha_0} \right)^{\gamma_Z}, \quad m(q) = m_0 \left( \frac{\alpha(q)}{\alpha_0} \right)^{\gamma_m}, \quad \nu_{1,2}(q) = \nu_{1,2}^0 \left( \frac{\alpha(q)}{\alpha_0} \right)^{\gamma_{1,2}} \tag{4}$$

At $m_0 = 0$, $\nu_0^0 \neq 0$ the solution exhibits a spontaneous chiral symmetry breaking, the coefficient $\nu_3^1$ is related to the quark condensate,

$$\nu_3^1 \propto \langle 0 | \bar{q}q | 0 \rangle \tag{5}$$

Matching this large $q$ behavior with an absence of singularity at $q \to 0$ gives what Gribov calls a confined solution. The solution features a complicated analytical structure, it leads to an unusual filling of positive and negative energy levels.

The next step for Gribov was to calculate the meson spectrum — he was in the process.

## 2 SUSY gauge theories

Supersymmetric gauge theories present an easier object than standard QCD. They are thoroughly studied now, and one particular lesson we have learned is that the matter content of a theory is important for its phase. This lesson contradicts the “quenched” approach usually taken in QCD when the gluon dynamic is presumed to play a dominant role.

The crucial role of light matter (where light fermions are accompanied by light bosons (pions) automatically) is explicitly seen. Theories with $SU(N_c)$ gauge group and different number of flavors, $N_f$, produce different phases depending on $N_f$. In particular, supersymmetric gluodynamics ($N_f = 0$) is a confining theory, at $N_f = N_c - 1$ we get the Higgs phase, while in the so called conformal window,

$$\frac{3}{2} N_c \leq N_f \leq 3 N_c,$$

we deal with scaling theories.

Thus, changing the number of light flavors, we are dramatically changing the phase of the theory. Although, the SUSY example does not support Gribov’s point about scaling in the case of pure glue, it demonstrates that his general idea about the crucial role of light matter finds its confirmation in the SUSY world.
3 Duality between pion and quark propagators in OPE

The comment below is relevant to the problem of the pointlike structure in the pion. I am addressing here the relation between the pion and the certain term in the OPE which was derived in [5].

Let us consider polarization of a vacuum by the axial current $a_\mu = \bar{u} \gamma_\mu \gamma_5 d$,

$$\Pi_{\mu\nu}(q) = \int dx e^{ixq} \langle 0| T\{a_\mu(x), a_\nu(0)\}|0 \rangle.$$  \hspace{1cm} (6)

At large $q$ we can use perturbation theory (see Fig. 3) to fix the Operator Product Expansion coefficients. OPE allows to write $\Pi_{\mu\nu}$ as

$$\Pi_{\mu\nu} = \frac{m_u + m_d}{q^2} q_\mu q_\nu \langle 0|\bar{u}u + \bar{d}d|0 \rangle + \text{transversal terms} \hspace{1cm} (7)$$

(terms containing quadratic and higher powers of $m_q$ are neglected). It is clear from Fig. 3 that the longitudinal term in the polarization operator at large $q$ is produced by linear in $m_q$ term in the quark propagator.

On the other hand we can compare this with the pion contribution to $\Pi_{\mu\nu}$, (see Fig. 4), which is equal to

$$\Pi_{\mu\nu}^{(\pi)} = -f_\pi^2 q_\mu q_\nu \frac{1}{q^2 - m_\pi^2}.$$ \hspace{1cm} (8)

For the longitudinal part it gives

$$q^\mu \Pi_{\mu\nu}^{(\pi)} = -f_\pi^2 q_\nu \frac{q^2}{q^2 - m_\pi^2} = -f_\pi^2 q_\nu \left[ \frac{m_\pi^2}{q^2} + \mathcal{O}(m_\pi^4) \right]. \hspace{1cm} (9)$$
The leading term in this expansion in powers of $m_\pi^2$ is polynomial in $q$, and for this reason it is not relevant. The non-polynomial pion contribution is linear in $m_\pi^2$. It is simple to verify that higher states contribute to $q^\mu \Pi_{\mu\nu}$ only starting from $m_\pi^4$ terms. Comparing Eq. (9) with the longitudinal part of $\Pi_{\mu\nu}$ given by Eq. (7), we see that

$$f_\pi^2 m_\pi^2 = -(m_u + m_d) \langle 0 | \bar{u}u + \bar{d}d | 0 \rangle.$$  

(10)

This is a well-known relation between the quark and pion masses.

Thus, we see, that the specific term in OPE is valid all the way from large to small $q$. At small $q$, i.e. at large distances, it is given by the pion, but it keeps the same form at large $q$, i.e. at short distances, as if the pion were pointlike.

The derivation of the duality between short and long distances demonstrated above does not imply any core in the pion. In this sense it looks as an argument against the Gribov’s idea of the pointlike structure in the pion.

4 Different scales in QCD

A support for a pointlike structure in the pion comes, in my view, from the observation of few scales in QCD, which are numerically quite different.

To remind you of the old story, let me start with the low-energy theorem for the trace of energy-momentum tensor

$$\int dx e^{iqx} \left\langle 0 | \mathcal{T} \left\{ \frac{\alpha_s}{\pi} G_{\mu\nu}(x), \frac{\alpha_s}{\pi} G_{\gamma\delta}^2(0) \right\} | 0 \right\rangle = \frac{32\pi}{b} \left\langle 0 | \frac{\alpha_s}{\pi} G_{\mu\nu}^2(0) | 0 \right\rangle,$$

(11)

where $b = 11/3 N_c - 2/3 N_f$. Numerically, the r.h.s. is quite large and leads to the large momentum scale $\lambda_G^2$ in the correlator of two $G^2$,

$$\lambda_G^2 = 20 \text{ GeV}^2,$$

(12)

i.e. in the glueball channel with quantum numbers $0^+$. This scale should be compared with the scale $m_\rho^2 = 0.6 \text{ GeV}^2$ in the isovector quark channel with quantum numbers $1^-$. The order of magnitude difference is quite significant.

The large scale the gluonium $0^+$ channel does not imply that the mass of the lowest state in this channel should be large, rather it tells us that the onset of parton-hadron duality is at higher energies in this channel as compared, say, with the isovector quark $1^-$ channel. I try to illustrate this by Fig. 5.

In the quark channel with the $\rho$ meson quantum numbers the duality appears very early, it is enough to integrate over the $\rho$ meson peak to have it. In the gluonium $0^+$ channel one needs to include some number of higher excitations
to see the duality. Numerically, 20 GeV$^2$ is the duality interval in $s$ for the $0^+$ gluonium channel versus 2 GeV$^2$ for the $\rho$ channel.

Gribov’s view that there is no confinement in pure glue could be related to the regime at $E^2 < 20$ GeV$^2$.

The existence of a much smaller spatial scale in gluon channels was confirmed by studies in frameworks of the instanton liquid model and by lattice calculations. A particularly interesting signal is given by the small size of the lowest $0^+$ gluonium. It is about 0.2 fm versus 1 fm for a “normal” hadronic size.

Another channel with small spatial size is the pseudoscalar $\bar{u}\gamma_5d$ channel $0^-$,

$$\langle 0 | \bar{u} \gamma_5 d | \pi \rangle \sim \frac{m_\pi^2}{m_q} f_\pi.$$  \hspace{1cm} (13)

Numerically, $m_\pi/m_q$ is large. As a result, the scale $f_\pi \sim 130$ MeV in the matrix element $\langle 0 | \bar{u} \gamma^\mu \gamma_5 d | \pi \rangle$ of the axial current becomes

$$\frac{m_\pi^2}{m_u + m_d} \approx 2$ GeV \hspace{1cm} (14)$$

in the $\bar{q}\gamma_5 q$ channel. This large scale is the key ingredient of the penguin mechanism for $\Delta I = 1/2$ enhancement in weak decays of strange particles (it also makes theory consistent with the recent measurement of $\epsilon'/\epsilon$).

I suggest that Gribov’s pointlike core in the pion could be a reflection of this large momentum scale.
5 Concluding remarks and acknowledgments

Gribov constructed in his mind a very physical picture of the hadronic world which is only partially reflected in his publications. While some of his ideas look very unconventional I believe that they could be the source of a much deeper understanding of QCD than we currently have. In my comments I limited myself to a very few examples where the depth of Gribov’s ideas can be seen.

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