QCD color superconductivity in compact stars: color-flavor locked quark star candidate for the gravitational-wave signal GW190814

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At sufficiently high densities and low temperatures matter is expected to behave as a degenerate Fermi gas of quarks forming Cooper pairs, namely a color superconductor, as was originally suggested by Alford, Rajagopal and Wilczek. The ground state is a superfluid, an electromagnetic insulator that breaks chiral symmetry, called the color-flavor locked phase. If such a phase occurs in the cores of compact stars, the maximum mass may exceed that of hadronic matter. The gravitational-wave signal GW190814 involves a compact object with mass $2.6M_\odot$, within the so-called low mass gap. Since it is too heavy to be a neutron star and too light to be a black hole, its nature has not been identified with certainty yet. Here, we show not only that a color-flavor locked quark star with this mass is viable, but also we calculate the range of the model-parameters, namely the color superconducting gap $\Delta$ and the bag constant $B$, that satisfies the strict LIGO constraints on the equation of state. We find that a color-flavor locked quark star with mass $2.6M_\odot$ satisfies the observational constraints on the equation of state if $\Delta \geq 200$MeV and $B \geq 83$MeV/fm$^3$ for a strange quark mass $m_s = 95$ MeV/$c^2$, and attains a radius $(12.7 - 13.6)$km and central density $(7.5 - 9.8)$10$^{14}$g/cm$^3$.

I. INTRODUCTION

The LIGO/Virgo Collaboration announcement of the signal GW190814 [1] initiated a discussion on the nature of the merger’s secondary component with mass $2.59^{+0.08}_{-0.09}M_\odot$. There are theoretical and observational uncertainties regarding the maximum mass of neutron stars [1–8] and the lower mass of black holes [9–11], which do not allow concluding with certainty on the nature of this secondary GW190814 component. The heaviest, observed, neutron star is $2.01 \pm 0.04M_\odot$ [12] and PSR J0740+6620 may host a $2.14^{+0.10}_{-0.09}M_\odot$ neutron star [13], while stellar evolution seems to not allow stellar black holes to be formed with mass less than $5M_\odot$ [3, 11]. Consequently, the existence and nature of compact objects in the mass range $\sim [2.5, 5]M_\odot$, the ‘low mass gap’, are highly uncertain [3, 11].

A primary candidate as the secondary GW190814 member is a stellar black hole [3, 11], but still multitude other proposals have been put forward. These include a primordial black hole [18–21], a heavy neutron star with stiff equation of state [22–24], a fast pulsar [25–29], a compact star grown via accretion [30–31], an anisotropic star [32–33], a spinning compact star with deconfinement phase transition [34], a strange star [35], an up-down quark star [36], a compact star with a population III star progenitor [37], and modified gravity scenarios [38, 39].

In the present work we propose to consider yet another possibility; namely the case of QCD color superconductivity. It has been shown by Alford, Rajagopal and Wilczek [40] (see also [41, 42]) that at sufficiently high density, quarks of different color and flavour form Cooper pairs with the same Fermi momentum. The quarks are electrically neutral and electrons cannot be present [41]. This superfluid ground state was called a color-flavor locked (CFL) phase. Later the CFL phase for up, down and strange quarks was argued to exist inside compact stars [43–50], called CFL quark stars. Studies of the structure of these objects have revealed that color superconductivity allows for large maximum masses [48, 51, 52]. Using the constraints on the equation of state suggested by LIGO from the combined study of GW170817 and GW190814 [1] we will narrow down the CFL parameters, namely the color superconducting gap $\Delta$ and the bag constant $B$, that allow for an equation of state of a $2.6M_\odot$ CFL quark star that satisfies these constraints.

The plan of our work is the following. In the next section we briefly review the structure equations describing hydrostatic equilibrium of a CFL quark star interior solutions. In section III we calculate the region in the parameter space of a $2.6M_\odot$ CFL quark stars for which the LIGO constraints are satisfied. Stability conditions are also discussed. Finally, in the last section we close with some concluding remarks.

II. CFL QUARK STARS

Lugones and Horvath [53] have found that the CFL strange matter phase can be the true ground state of hadronic matter for a wide range of the parameters of the model (see also [48]), which are the QCD gap of Cooper pairs $\Delta$ and the bag energy density $B$ within an MIT bag model. They further derive a full equation of state and
an analytic approximation, that is

\[ P = \frac{3\mu^4}{4\pi^2\hbar^2c^5} + \frac{9\alpha\mu^2}{2\pi^2\hbar^3c^3} - B, \quad (1) \]

\[ \rho = \frac{9\mu^4}{4\pi^2\hbar^3c^5} + \frac{9\alpha\mu^2}{2\pi^2\hbar^3c^3} + \frac{B}{c^2}, \quad (2) \]

where

\[ \alpha = -\frac{1}{6}m_s^2c^4 + \frac{2}{3}\Delta^2 \quad (3) \]

and \( m_s \) is the strange quark mass. From the above we get

\[ \rho(P) = \frac{3P}{c^2} + \frac{4 B}{c^2} - \frac{9\alpha\mu(P)^2}{\pi^2\hbar^3c^5} \quad (4) \]

\[ \mu(P)^2 = -3\alpha + \left( \frac{9\alpha^2}{3} + \frac{4}{\pi^2}(P + B)\hbar^3c^3 \right)^{1/2} \quad (5) \]

that can be used to solve the Tolman-Oppenheimer-Volkoff (TOV) problem \[54,55]\n
\[ \frac{dP}{dr} = -\left(\rho(r) + \frac{P(r)}{c^2}\right) \frac{\frac{GM(r)}{r^2} + 4\pi G P(r)r}{1 - \frac{2GM(r)}{rc^2}}, \quad (6) \]

\[ \frac{dM}{dr} = 4\pi\rho(r)r^2. \quad (7) \]

We denote \( M = M(r) \) the mass of a compact star included inside a radius \( r \), and \( P = P(r) \), \( \rho = \rho(P(r)) \) are the pressure and mass density, respectively. We shall denote \( R \) the radius of the CFL-quark core and \( M_{QS} = M(r = R) \) the total mass. We integrate the TOV problem up to zero boundary pressure at the radius of the core and match with the exterior Schwarzschild metric \[56]\n
\[ ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \quad (8) \]

where the lapse function is given by

\[ f(r) = 1 - \frac{2GM_{QS}}{rc^2}, \quad r \geq R \quad (9) \]

The bag constant \( B \) induces a mass density-scale, characteristic for the system, which in turn induces a radius-scale and a mass-scale

\[ \rho_* \equiv \frac{B}{c^2}, \quad r_* \equiv \left( \frac{4\pi G}{c^2\rho_*} \right)^{-1/2}, \quad M_* \equiv \frac{r_*c^2}{G}. \quad (10) \]

We further introduce the dimensionless variables

\[ x = \frac{r}{r_*}, \quad \tilde{\rho} = \frac{\rho}{\rho_*}, \quad \tilde{P} = \frac{P}{\rho_*c^2}, \quad \tilde{M} = \frac{M}{M_*} \quad (11) \]

as well as the dimensionless quantities

\[ \lambda = \frac{9\alpha^2}{\pi^2 B\hbar^3c^3}, \quad \kappa(\tilde{P}) = \frac{\mu(\tilde{P})^2}{\alpha}. \quad (12) \]

| Models | \( M_{QS} = 2.6M_\odot \) |
|-----------------|---------------------|
| \( \Delta[\text{MeV]} \) | \( B[\text{MeV}\cdot\text{fm}^3] \) | \( R[\text{km]} \) | \( \rho_0[10^{14}\text{g/cm}^3] \) |
| 200 | 83 | 13.51 | 8.97 |
| 210 | 85-87 | 13.62-13.41 | 8.33-9.10 |
| 220 | 89-91 | 15.30-13.31 | 8.49-9.24 |
| 230 | 91-94 | 13.56-13.30 | 8.07-9.02 |
| 240 | 95-98 | 13.44-13.18 | 8.29-9.22 |
| 250 | 97-102 | 13.47-13.07 | 8.01-9.44 |
| 260 | 99-105 | 13.48-13.03 | 7.81-9.36 |
| 270 | 102-108 | 13.41-12.98 | 7.87-9.34 |
| 280 | 104-112 | 13.40-12.86 | 7.76-9.65 |
| 290 | 105-115 | 13.45-12.80 | 7.51-9.69 |
| 300 | 108-118 | 13.36-12.74 | 7.65-9.78 |

TABLE I: Bag constant, \( B \), and quark semiconductor gap, \( \Delta \), for \( m_s = 95\text{MeV}/c^2 \) that allow for a CFL quark star, as a GW190814-candidate, with \( M_{QS} = 2.6M_\odot \) which meets the LIGO constraints of Figure 1(b). On the third and fourth columns are depicted the radius and central density values of the quark star for each model. These radius values correspond to compactness \( 0.28 - 0.30 \).

The problem is reformulated in the dimensionless format

\[ \frac{d\tilde{P}}{dx} = -\left( \tilde{\rho}(x) + \tilde{P}(x) \right) \frac{\tilde{M}(x)}{1 - \frac{2M(x)}{x}}, \quad (13) \]

\[ \frac{d\tilde{M}}{dx} = \tilde{\rho}(x)x^2. \quad (14) \]

with

\[ \tilde{\rho}(\tilde{P}) = 3\tilde{P} + 4 - \lambda \cdot \kappa(\tilde{P}) \quad (15) \]

\[ \kappa(\tilde{P}) = -3 + 3\left(1 + \frac{4}{3\lambda}(\tilde{P} + 1)^{1/2} \right) \quad (16) \]

where

\[ \lambda = 0.118682 \left( \frac{\alpha}{[100\text{MeV}]^2} \right)^2 \left( \frac{B}{100\text{MeV}/\text{fm}^3} \right)^{-1} \quad (17) \]

and \( \alpha \) is given in \[56\]. We may also write

\[ \rho_* = 1.7827 \cdot 10^{14}\text{g/cm}^3 \left( \frac{B}{100\text{MeV}/\text{fm}^3} \right)^{-1/2} \quad (18) \]

\[ r_* = 24.518\text{km} \left( \frac{B}{100\text{MeV}/\text{fm}^3} \right)^{-1/2} \quad (19) \]

\[ M_* = 16.5989M_\odot \left( \frac{B}{100\text{MeV}/\text{fm}^3} \right)^{-1/2}. \quad (20) \]

### III. ANALYSIS

The green shaded region of Figure 1(b) designates the LIGO constraints (see Figure 8 of \[15\]) on the equation of state (EoS) imposed by the combined analysis of
GW170817 and GW190814. Setting $m_s = 95 \text{ MeV}/c^2$, we calculate the range of parameter values for $\Delta, B$ for which these constraints are met, for a CFL quark star with $M_{QS} = 2.6 \, M_\odot$, as in Table I. The radius of the corresponding CFL quark star is found to lie within the range (12.7 – 13.6) km, that corresponds to compactness values $C_{QS} \equiv GM_{QS}/R = 0.28 – 0.30$. The CFL quark star is almost twice as compact than a canonical neutron star satisfying the same LIGO constraints on the EoS, which attains compactness $C_{NS} = 0.16$ \cite{1607.03999}.

In Figure 1(a) we demonstrate for three concrete models that the maximum mass exceeds $2.6 \, M_\odot$, and therefore the CFL EoS considered here allows for a quark star with such high mass. In Figure 1(b) we show for the same three models that the LIGO constraints are satisfied for a CFL quark star with mass $M_{QS} = 2.6 \, M_\odot$.

The allowed region in the parameter space is then depicted graphically in Figure 2. Our results show that for a given energy gap $\Delta$, the bag constant $B$ takes values in a certain range, which becomes wider as $\Delta$ increases.

We further study stability and physical requirements for the solutions of Table I. Firstly, we stress that CFL quark matter is absolutely stable if the energy per baryon is smaller than the neutron mass, $m_n = 939 \text{ MeV}/c^2$. This condition is satisfied if

$$S \equiv B h^3 c^3 + \frac{m_s^2 m_n^2 c^8}{12 \pi^2} - \Delta^2 m_n^4 c^4 - \frac{m_n^4 c^8}{108 \pi^2} < 0. \quad (21)$$

In Figure 3(a) we demonstrate that this stability condition is satisfied for all parameter values of Table I.

In addition, the $2.6 \, M_\odot$ solution satisfies the stability condition of the adiabatic index \cite{57335815}

$$\Gamma \equiv \frac{1}{c^2} \frac{dP}{d\rho} \left[1 + \rho \frac{v^2}{c^2}\right] > \frac{4}{3}, \quad (22)$$

for all models of Table I. We demonstrate this in Figure 3(b) for the three models of Figure 1. For the same three models we demonstrate in Figure 3(c) that the causality condition

$$0 < v_s^2 \equiv \frac{dP}{d\rho} < c^2 \quad (23)$$

is also satisfied, where $v_s$ denotes the speed of sound.

Finally, we require \cite{55461054, 61146255} that the strong energy condition

$$\rho c^2 + P \geq 0, \quad \rho c^2 + 3P \geq 0 \quad (24)$$

is satisfied. This is true for all models of Table I as it is evident in Figure 1(b) for our three indicative models. Note that since the strong energy condition is satisfied it follows that all other energy conditions are satisfied, namely the weak energy condition ($\rho \geq 0$, $\rho c^2 + P \geq 0$), the null energy condition ($\rho c^2 + P \geq 0$) and the dominant energy condition ($\rho c^2 \geq |P|$).
FIG. 3: Stability and physical requirements. (a) The quantity $S$ defined in (21) with respect to the superconducting gap $\Delta$ for the marginal values of $B$ for all models of Table I. Stability of the CFL phase requires $S < 0$. (b) The adiabatic index $\Gamma$ with respect to the radius in the interior of a 2.6M⊙ CFL quark star for the three models of Figure 1. Stability requires $\Gamma > \frac{4}{3}$. (c) The velocity of sound squared $v_s^2$ with respect to the radius in the interior of a 2.6M⊙ CFL quark star for the three models of Figure 4. Causality requires $v_s < c$.

IV. CONCLUSIONS

In the present work we have investigated the possibility that strange quark stars in the CFL phase comprise the secondary GW190814 component, with an observed mass at $M_{QS} = 2.6$ M⊙. QCD superconductivity effects lead to a non-linear, but still analytical, EoS characterized by three parameters, namely the bag constant $B$, the superconducting energy gap $\Delta$ as well as the mass of the strange quark $m_s$. Assuming for the latter a numerical value compatible with the Particle Data Group review at $m_s = 95$ MeV/c², we obtain the region in the $(B - \Delta)$ plane for which the stringent LIGO constraints on the EoS are met. In particular, we have obtained the $M - R$ relationships, and our main numerical results show that the EoS adopted here can support a CFL quark star with a mass $M_{QS} = 2.6$ M⊙ provided that the quark semiconductor gap $\Delta \geq 200$ MeV, and that the bag constant $B \geq 83$ MeV/fm³ with a range of values depicted in Figure 2. For these values, we further verify that stability, causality and energy conditions are met, suggesting that our solutions are physical within the context of General Relativity. The radius of the corresponding CFL quark star is found to lie within the range $(12.7 - 13.6)$km, while the central density $\rho_0$ is found to take values in the range $(7.5 - 9.8) \times 10^{14}$ g/cm³.

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