The \( \Lambda(1600) \): A Strange Hybrid Baryon

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We use the method of QCD sum rules to investigate a possible hybrid baryon with the quantum numbers of the \( \Lambda \). Using a current composed of uds quarks in a color octet and a gluon, a strange hybrid, the \( \Lambda_H \), is found about 500 MeV above the \( \Lambda \), and we identify it as the \( \Lambda(1600) \) Using our sigma/glueball model we predict a large branching fraction for the \( \Lambda_H \rightarrow \Lambda + \sigma(\pi\pi \text{ resonance}) \), and the experimental search for this decay mode could provide a test of the hybrid nature of the \( \Lambda(1600) \).

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I. INTRODUCTION

One of the most important aspects of hadron spectroscopy for providing new information about Quantum Chromodynamics (QCD) is the identification of hybrid hadrons, mesons and baryons that have gluonic valence components. These are typical exotics, hadrons that cannot be explained in standard quark models. Since some exotic mesons can be identified by their quantum numbers, great deal of work has been done on hybrid mesons. Hybrid baryons are more difficult to identify experimentally, although there are clear theoretical definitions. For example, a nonstrange baryon or a baryon with strangeness = -1 with three-quark components in a configuration with color \( \neq 0 \), while the three quarks plus gluon have zero color is certainly a hybrid. Strangeness = +1 baryons are exotic, but cannot be characterized as hybrids in the sense that they must involve an anti-strange quark rather than valence glue.

There have been theoretical studies of hybrid baryons for two decades, including bag model of nonstrange hybrids calculations \[ [1,2,3] \] and of strange hybrids \[ [4] \], models of the \( P_{11}(1440) \) (Roper) resonance as a hybrid \[ [5,6] \], QCD Sum Rule studies \[ [7,8,9] \] and flux tube models \[ [10,11] \]. A major problem is to identify the hybrids. Electroproduction \[ [6] \] and large decay branching ratio into the sigma \( \pi - \pi \) resonance \[ [12] \] have been suggested for tests of the Roper resonance as a hybrid; and possible tests of hybrids related to the flux tube model have been suggested \[ [13] \]. See the recent review by Barnes \[ [14] \] for a discussion of hybrid baryons and possible ways to identify them.

In the early bag model studies of nonstrange hybrid baryons \[ [1,2,3] \] there was a range of solutions of about 1.5-1.9 GeV for the lightest hybrid. Using a bag model for the study of \( \Lambda \)-like strange hybrid baryons, i.e., using (udsg) configurations with the uds quarks in a color octet, in Ref. \[ [4] \] a narrower range of solutions was found with the choice of parameters used, and the authors concluded the the lightest \( \Lambda \)-type hybrid might be the three-star \( \Lambda(1600) \).

In the present work we use the QCD sum rule method as in Ref. \[ [8] \] to estimate the mass of the lightest strange baryon with the quantum numbers of the \( \Lambda \) : \( J^P = \frac{1}{2}^+ \), \( S = -1 \), \( I = 0 \). One advantage of the sum rule method is that the composite field operator, called the current, for the particle, for which we use the symbol \( \Lambda_H \), is a clearly defined hybrid, with the three quarks having nonzero color. Our conclusion in the present work is that the \( \lambda(1600) \) is a hybrid. The theoretical calculations is similar to that of our previous work in which we concluded that the Roper is a good hybrid baryon candidate. Note that the excitation energy of the \( P_{11}(1440) \) resonance above the nucleon is similar to that of the \( \Lambda(1600) \) above the \( \Lambda \). We also use the arguments of Ref. \[ [2] \] to suggest a test of the \( \Lambda(1600) \) as a hybrid.

II. QCD SUM RULE ESTIMATE OF THE MASS OF THE \( \Lambda_H \)

The current operator for the \( \Lambda_H \) which we use is the form used in Ref. \[ [8] \]
\[ \eta_{\Lambda H} = \epsilon^{a b c} [u^a(x) C \gamma^\mu d^b(x) \gamma^\alpha G^c_{\mu \alpha}(x) [\frac{\lambda^e}{2} s(x)]^c, \]  

(1)

where (a,b,c,e) are color indices, \( \lambda^e \) is the generator of the SU(3) color group, (u,d,s) refer to up, down and strange quarks, and \( G_{\mu \nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu] \) is the gluonic tensor, with \( A_\mu = A_\mu^a \lambda^a/2 \) the color field. The corresponding correlator is

\[ \Pi_{\Lambda H}(x) = \langle 0 | T[\eta_{\Lambda H}(x) \bar{\eta}_{\Lambda H}(0)] | 0 \rangle > \]

\[ = \epsilon^{a b c} \epsilon^{a' b' c'} \lambda^a \gamma_\alpha S^\alpha_{\alpha'}(x) \gamma_\beta \gamma_5 < 0 | G^{m \alpha}_{\mu\alpha}(x) G^{n \alpha}_{\nu\beta}(0) | 0 > \frac{\lambda^m_{\nu} \lambda^n_{\nu}}{2}, \]

(2)

where \( S_f \) is the flavor \( f \) quark propagator and \( C \) is the charge conjugation operator. The correlator has two independent terms, which in momentum space can be written as

\[ \Pi(q)_{\Lambda H} = i \int e^{i q x} < 0 | T[\eta(x) \bar{\eta}(0)] | 0 > = \Pi_1(q^2) q + \Pi_2(q^2), \]

(3)

with \( q = \gamma_\mu q^\mu \).

The QCD sum rule calculation is very similar to that in Refs. [8,9]. The operator product expansion converges rapidly at high \( Q^2 = -q^2 \), which is achieved via a Borel transform. The diagrams included for \( \Pi_1(q^2) \) are shown in Fig. 1.

FIG. 1. Diagrams for \( \Pi_1(q^2) \) for the OPE including dimension 6 condensates
One finds from the diagrams of Fig. 1 that

\[ \Pi_1(Q^2) = \frac{1}{45\pi^6} Q^8 \ln(Q^2) - \frac{17}{36\pi^5} <0|\alpha_s G^2|0 > Q^4 \ln(Q^2) \]
\[ + \left( \frac{512}{9\pi^2} <0|\bar{q}q|0 >^2 - \frac{2}{3\alpha_s\pi^5} <0|g^3 f G^3|0 > \right) Q^2 \ln(Q^2), \]

and from the diagrams of Fig. 2 that

\[ \Pi_2(Q^2) = \frac{16}{9\pi^4} <0|\bar{s}s|0 > Q^6 \ln(Q^2) - \frac{20}{\pi^4} <0|\bar{q}\sigma \cdot G q|0 > Q^4 \ln(Q^2), \]

where \( Q^2 = -q^2 \), and for the condensates the values used are: quark condensate \( <0|\bar{q}q|0 > = -(0.25)^3 \) GeV\(^3\), mixed condensate \( <0|\bar{q}\sigma \cdot G q|0 > = 0.8(0.25^3) \) GeV\(^5\), the gluon condensate \( <0|\alpha_s G^2|0 > = 0.038 \) GeV\(^4\), and the triple gluon condensate \( <0|g^3 f G^3|0 > = (0.6)^8 \) GeV\(^6\). The convention of Ref. [7] has been used. In deriving Eqs. (4,5) the quark masses have been neglected. Also the strange quark condensate \( <0|\bar{s}s|0 > \) is expected to be about 20% smaller than the u,d quark condensates, but this difference is neglected as it does not change our results significantly.

The sum rule method involves equating the dispersion relation for the correlator to the form obtained from the OPE expansion, using a Borel transform to ensure that the OPE converges rapidly and that the continuum contribution to the dispersion relation is not too large to find the mass given by the pole term. For our problem we must take into account that the \( \Lambda \) as well as the \( \Lambda_H \) can contribute to the correlator given by Eqs. (1,2), since the \( \Lambda \) can have a (udsg) component. Thus the form of the dispersion relation for the correlator, called the phenomenological side of the sum rule, is

\[ \Pi_{1\text{phen}}(Q^2) = \frac{c_1}{Q^2 + M_\Lambda^2} + \frac{c_2}{Q^2 + M_{\Lambda H}^2} + \text{continuum} \]
\[ \Pi_{2\text{phen}}(Q^2) = \frac{c_1 M_\Lambda}{Q^2 + M_\Lambda^2} + \frac{c_2 M_{\Lambda H}}{Q^2 + M_{\Lambda H}^2} + \text{continuum}. \]

After the Borel transform, in which the \( Q^2 \) variable is replaced by Borel mass \( M^2 \), the phenomenological side becomes

\[ \Pi_{1\text{phen}}(M^2) = c_1 e^{-M_\Lambda^2/M^2} + c_2 e^{-M_{\Lambda H}^2/M^2} + \text{continuum} \]
\[ \Pi_{2\text{phen}}(M^2) = c_1 M_\Lambda e^{-M_\Lambda^2/M^2} + c_2 M_{\Lambda H} e^{-M_{\Lambda H}^2/M^2} + \text{continuum}. \]

The constants \( c_1, c_2 \) in Eq. (6) contain important information about the wave function, but can be eliminated in the analysis by using the technique of Ref. [8]. The continuum is approximated by replacing the Borel transform.
\[ B_{Q^2 \to M^2}(Q^2 \ln Q^2) = -\ln M^{2(l+1)} \]  

by

\[ B_{Q^2 \to M^2}^c(Q^2 \ln Q^2) = -M^{2(l+1)}(1 - \sum_{n=1}^{l} \frac{(s_l/M^2)^n}{n!} e^{-s_l/M^2}), \]

where \( s_l \) is the threshold \( E_{c.m.}^2 \), which is chosen by matching the phenomenological to the theoretical expressions for the correlator.

The mass of the hybrid is obtained from the expression

\[ M_{\Lambda H} = \frac{\frac{d}{dM^2}[\Pi_2(M^2)e^{M^2_{\Lambda}/M^2}]}{\frac{d}{dM^2}[\Pi_1(M^2)e^{M^2_{\Lambda}/M^2}]} . \]

The sum rules for \( \Pi_1 \) and \( \Pi_2 \) are quite stable, and the solution give a hybrid mass about 500 MeV above the \( \Lambda \) mass. An error of about 15% is expected with this method. There we conclude that

\[ M_{\Lambda H} \simeq 1600 \pm 200 \text{ MeV}, \]

and that the \( \Lambda(1600) \) is our candidate for the lightest strange hybrid, with the quantum numbers of the \( \Lambda(1115) \).

### III. THE \( \sigma \) DECAY OF THE \( \Lambda_H \)

A test of the hybrid nature of the \( \Lambda(1600) \) follows from our glueball/sigma model [15,16]. In studies of the scalar hadrons, the mixing of scalar mesons and scalar glueballs was found to be important for mesons and glueballs with masses above 1 GeV. Moreover, a light glueball solution was obtained with a mass 300-600 MeV, which is the mass of the broad \( I=0, L=0 \) \( \pi - \pi \) resonance, which was found in an analysis of \( \pi - \pi \) scattering [17], which we call the sigma. This leads to our glueball/sigma model, a model based on a coupled-channel picture with the glueball pole driving the \( \pi - \pi \) resonance, motivated in part by the large branching ratios for decays of glueball candidates into channels with sigmas [18]. This model successfully explains [19] the observed [20] \( D^+ \) charm meson decay into a \( \sigma(\pi\pi) \) resonance, while this decay channel is not found for the \( D_s^+ \) decay.

Using the current operator for the \( \Lambda \)

\[ \eta_\Lambda = \epsilon^{abc}[u^a(x)C\gamma^\mu d^b(x)]\gamma_5 \gamma_\mu s(x)^c , \]

the decay of the \( \Lambda_H \) into a \( \Lambda \) and a sigma can be estimated by the external field method [21] from the \( \Lambda_H, \Lambda \) two-point function in an external Sigma field

\[ \Pi(q)_{\Lambda\Lambda H} = i \int e^{iqx} < 0|T[\eta_\Lambda(x)\bar{\eta}_{\Lambda H}(0)]J_\sigma|0 >, \]

where \( J_\sigma \) is the sigma current. E.g., if one were investigating the decay into the pion channel, which is a forbidden \( \Lambda_H \to \Lambda \) decay, the current would be \( J^\pi = i\bar{q}\gamma_5 q \). The calculation is illustrated in Fig. 3 for the lowest dimensional diagram.
The glueball sigma model leads to the evaluation of the sigma-gluonic coupling

\[< G_{\mu\nu}^a J_\sigma G_{\mu\nu}^a > = g_\sigma < G_{\mu\nu}^a G_{\mu\nu}^a > . \] (14)

The sigma-glue coupling constant, \( g_\sigma \), was estimated from the 356 Mev width of the \( \sigma(\pi\pi) \) resonance \cite{17} to be \( g_\sigma \simeq 700 \) MeV. Because of the unknown constants in the sum rule calculation one can only predict ratios of decay widths. For the study of the Roper as a hybrid we were able to predict the ratio of the sigma decay width to the pion decay width. Since the only decay channels that have been measured for the \( \Lambda(1600) \) are \( \Lambda(1600) \rightarrow N\bar{K} \) and \( \Lambda(1600) \rightarrow \Sigma\pi \), and the single-pion + \( \Lambda \) channel is not allowed, we are not able to make a specific prediction here. Noting that the decay \( P_{11}(1440) \rightarrow N + (\pi\pi)_{I=0,L=0} \) has a 5-10 \% branching fraction, we predict a sizable \( \Lambda_H \rightarrow \Lambda + \sigma \) branching fraction. The main background will be the nonresonant \( \pi - \pi \) channels, so that for the sigma channel the \( (I=0,L=0) \) resonance must be extracted. Also, it would be difficult to measure the ratio \( \Lambda_H \rightarrow \Sigma\sigma/\Lambda_H \rightarrow \Sigma\pi \), for which our model could give a prediction, since there is very little of phase space to map out to extract the \( \sigma \) resonance due to the mass of the \( \Sigma \).

In conclusion, I believe that a careful experimental study of the \( \Lambda(1600) \rightarrow \Lambda\pi\pi \), with an analysis to extract the sigma resonance, could be a valuable test of the hybrid nature of this resonance. There have been no experiments on \( \Lambda(1600) \) decays for past twenty years, and this resonance is an excellent candidate for a strange hybrid baryon.

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