Attenuation Zones of Combined Periodic Foundations with Different Combination Patterns

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Abstract. One-dimensional periodic foundations with single-size unit cells are difficult to possess attenuation zones with both low frequencies and wide bandwidths. Combined one-dimensional periodic foundations are proposed by combing unit cells with different sizes in tandem in this paper. Attenuation zones of combined one-dimensional periodic foundations with different combination patterns are compared. The present study is useful for the design of one-dimensional periodic foundations to isolate low frequency seismic waves.

Keywords. Combined periodic foundation, attenuation zones, combination pattern, frequency response function.

1. Introduction

Developing effective seismic isolation methods has been the research target in the field of seismic isolation for the past three decades. According to the studies of solid state physics, periodic structures have unique dynamic attribute of frequency attenuation zones (AZs) [1-3]. Waves with frequencies in the attenuation zones cannot propagate in periodic structures. Bao and Shi [4] studied one-dimensional periodic foundation which constituted of alternating concrete and rubber layers, and found that the thickness ratio of concrete and rubber has an effect on AZs of the one-dimensional periodic foundation. Xiang et al [5] studied one-dimensional periodic foundation proposed in Bao and Shi [4] by shaking table test. The experimental results in [5] show that the periodic foundation can isolate the seismic wave frequency in the attenuation zone. However, above mentioned two studies [4, 5] used continues rubber as the matrix layer, which is difficult to obtain AZs with low frequencies and wide frequency band widths. Shi et al [6] used rubber block instead of rubber layer, and the one-dimensional periodic foundation of the attenuation zone of 2.15-15Hz is obtained. Witarto [7] extended periodic foundation to pipeline isolation, and the research results show that the periodic foundation can also effectively reduce the seismic response of pipeline systems.

Most of seismic wave frequencies are concentrated in low frequencies. Hence, to achieve the AZs with low frequencies and wide frequency band widths has become a research goal. Jain et al [8] proposed a combined periodic foundation for seismic isolation of bridges by combining two types of unit cells with different sizes. The numerical analysis results show that attenuation zones of the combined periodic foundation become wider and lower significantly than those of periodic foundations made of single-size unit cells, which is a good evidence that combined periodic foundations can be designed to obtain AZs with low frequencies and wide frequency band widths. However, for one-dimensional combined periodic foundation, there are two combination patterns, which are “ABCDABCD” and “ABABCDCD”, respectively, where A, B, C and D represent materials of the combined periodic foundation. However, the effect of two combination patterns on AZs of combined periodic foundations has not been investigated in previous studies.
In the present paper, computational method of attenuation zones for shear waves in one-dimensional periodic foundation with single-size unit cell is given in section 2. In section 3, two combined periodic foundations are proposed by using two different combination patterns, and frequency response functions of these two combined periodic foundations are calculated to investigate the isolation mechanism difference of them. Finally, section 4 gives some conclusions.

2. Computational Method

Figure 1 shows the configuration of a one-dimensional periodic foundation and its superstructure, where the self-weight of superstructure on the periodic foundation can be reduced to uniformly distributed homogeneous initial stress. Consider a harmonic SH wave propagating in the z-direction with angular frequency $\omega$ and wave vector $k = k_x \mathbf{e}_x$, where $k_x$ and $\mathbf{e}_x$ are the phase constants and unit wave vectors in the $x$-direction, respectively. Here, $u$ is defined as displacement in $x$-direction. The field equation of SH wave in one-dimensional periodic foundation can be written as

$$\frac{\partial^2 u^{(e)}(x, z)}{\partial z^2} + \frac{\partial^2 u^{(e)}(x, z)}{\partial x^2} \sigma^{(e)}_x = \rho^{(e)} u^{(e)}$$

where the superscript $e$ denotes the material of each layer; $\tau^{(e)}_x$, $\sigma^{(e)}_x$ and $\rho$ are shear stress, initial stress and material density, respectively.

![Figure 1. Configuration of a one-dimensional periodic foundation and its superstructure.](image)

The strain and constitutive equations of one-dimensional periodic foundation can be written as

$$\gamma^{(e)} = \frac{\partial u^{(e)}}{\partial x}, \quad \tau^{(e)}_x = \frac{1}{G^{(e)}} \gamma^{(e)}$$

where $\gamma^{(e)}$ and $G$ represent the shear strain and shear modulus, respectively.

By applying the weak form quadrature element method [9], the governing equations for SH waves in the one-dimensional periodic foundation including the effect of initial stress can be expressed as:

$$\left[ K(k) + K^0(k) - \omega^2(k)M \right] d = 0$$

where $K(k)$, $K^0(k)$, and $M$ are the stiffness matrix, geometric stiffness matrix, and mass matrix of
the typical unit cell, respectively; \( \mathbf{d} \) is the nodal displacement vector. When the wave vector \( \mathbf{k} \) is given, the eigenfrequencies \( \omega \) can be determined by equation (3). Dispersion curve can be obtained by changing the wave vector \( \mathbf{k} \) in the first Brillouin zone. Then, attenuation zones can be identified with the frequency gaps between adjacent dispersion branch.

3. Results and Discussion
Concrete and rubber are selected as the periodic foundation materials, which \( \rho=2300 \text{ kg/m}^3 \), \( E=25 \text{ GPa} \), \( \nu=0.330 \) are the Mass density, Young modulus, Poisson ratio of the concrete, which \( \rho=1300 \text{ kg/m}^3 \), \( E=1.37 \times 10^4 \text{ GPa} \), \( \nu=0.463 \) are the Mass density, Young modulus, Poisson ratio of the rubber, respectively. In the case discussed later, the initial stress is taken as \( \sigma_{0z}^e = 0.4G_t \), where \( G_t \) is the shear modulus of the rubber layer.

The dimension of unit cell 1 is \( a=0.5 \text{ m} \), in which the thickness of concrete is \( a_1=0.4 \text{ m} \) and the thickness of the rubber layer is \( a_2=0.1 \text{ m} \), as show in figure 2(a); the dimension of unit cell 2 is \( b=1 \text{ m} \), in which the concrete thickness is \( b_1=0.6 \text{ m} \), and the rubber layer thickness is \( b_2=0.4 \text{ m} \), as showed in figure 2(b). Unit cell 3 is the combination of the unit cell 1 and unit cell 2, as shown in figure 2(c).

![Figure 2](image2.png) **Figure 2.** Periodic foundation with (a) unit cell 1, (b) unit cell 2 and (c) unit cell 3.

Figures 3, 4 and 5 show that the dispersion curves for shear waves in periodic foundation with unit cells 1, 2 and 3, respectively. The shadow areas of the figures 3, 4 and 5 are the AZs. It can be seen from figures 3 and 4 that the first attenuation zone of the smaller size unit cell is wider and higher. It can be found from figure 5 that the first AZ of the periodic foundation with unit cell 3 is lower and narrower than both those of periodic foundations with unit cell 1 and 2. Moreover, the combination of unit cell 1 and unit cell 2 introduces several flat passbands.

![Figure 3](image3.png) **Figure 3.** Dispersion curves for shear waves in periodic foundation with unit cell 1. **Figure 4.** Dispersion curves for shear waves in periodic foundation with unit cell 2. **Figure 5.** Dispersion curves for shear waves in periodic foundation with unit cell 3.

Consider two periodic foundations with different combination patterns. The first one is composed of two unit cell 1 and two unit cell 2 in tandem, as shown in figure 6 (a). The second one is composed of two unit cell 3, as shown in figure 6 (b).
The frequency response function (FRF) of the top of the foundation is defined as $20\log_{10}\left(\frac{u_t}{U_s}\right)$, where $U_s$ is the displacement excitation input from bottom, and $u_t$ is the displacement amplitude output of the top of the foundation. Negative FRF means which the displacement response of the top is much less than bottom input displacement excitation. The unit displacement along the $x$-direction is loaded to the foundation bottom of periodic foundation 1 and periodic foundation 2 in figure 6, and the FRFs are plotted in figure 7.

**Figure 6.** Configuration of (a) Periodic foundation 1 and (b) Periodic foundation 2.

The blue and red areas in figure 7 correspond to the AZs of periodic foundations 1 and 2, respectively. The AZs of periodic foundation 1 in figure 7, i.e., the blue areas, are obtained by the union of AZs of figures 3 and 4. The AZs of periodic foundation 2 in figure 7, i.e., the red areas, are the AZs in figure 5. It can be found from figure 7 that the first AZ of periodic foundation 2 is lower and narrower than that of periodic foundation 1. Except for the passbands over 5 Hz of dispersion curves of periodic foundation 2, the AZs of periodic foundation 1 and periodic foundation 2 are almost the same.

**Figure 7.** FRFs of periodic foundation 1 and periodic foundation 2.

4. Conclusions
In this paper, frequency response functions of two periodic foundations with different combination patterns are calculated to investigate the isolation mechanism difference of them. The following conclusions can be drawn:

(1) The width of the first AZ of periodic foundation 2 of the “ABCDABCD” combination pattern is narrower than that of periodic foundation 1 of the “ABABCDABCD” combination pattern.

(2) AZs of periodic foundation 1 of the “ABABCDABCD” combination pattern can be obtained by the
union of AZs of periodic foundations with unit cell 1 and unit cell 2.

(3) In sufficiently high frequency, the AZs of periodic foundation 1 and periodic foundation 2 are almost the same except for the frequency passbands.

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