GAMOW-TELLER STRENGTH DISTRIBUTIONS
for $\beta\beta$-DECAYING NUCLEI WITHIN
CONTINUUM-QRPA

S.Yu. Igashov$^1$, V.A. Rodin$^2$, M.H. Urin$^{1,3}$, A. Faessler$^2$

$^1$ Moscow Engineering Physics Institute (State University), Russia
$^2$ Institute for Theoretical Physics, University of Tuebingen, Germany
$^3$ Kernfysisch Versneller Instituut, University of Groningen, The Netherlands

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Abstract

A version of the pn-continuum-QRPA is outlined and applied to describe the Gamow-Teller strength distributions for $\beta\beta$-decaying open-shell nuclei. The calculation results obtained for the pairs of nuclei $^{116}$Cd-Sn and $^{130}$Te-Xe are compared with available experimental data.

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1 Introduction

Description of weak interaction in nuclei is often a challenge for models of nuclear structure. Numerous calculations of the nuclear $\beta\beta$-decay amplitudes well illustrate this statement (see, e.g. Refs. [1] and references therein). Uncertainties in theoretical calculations of the Gamow-Teller (GT) $2\nu\beta\beta$-decay amplitude $M^{2\nu}_{\beta\beta}$ have stimulated experimental studies of the GT($^\pm$)-strengths of the $1^+$ states virtually excited in the decay process (see, e.g. Refs. [2, 3]).

As a double charge-exchange process, $2\nu(0\nu)\beta\beta$-decay is enhanced by nucleon pairing which is due to the singlet part of the particle-particle (p-p) interaction. The discrete quasiboson version of the quasiparticle RPA (pn-dQRP) which accounts for the nucleon pairing is usually applied to calculate the $\beta\beta$-decay amplitudes in open-shell nuclei [1]. In spite of differences in model parameterizations of the nuclear mean field and residual interaction in the particle-hole (p-h) and p-p channels, all pn-dQRP calculations reveal marked sensitivity of the amplitude $M^{2\nu}_{\beta\beta}$ to the ratio $g_{pp}$ of the triplet to singlet strength of the p-p interaction. Physical reasons for such a general feature of all calculations were analyzed in Ref. [4], where they were attributed to violation of the spin-isospin SU(4) symmetry in nuclei. An identity transformation of the amplitude into sum of two terms was used in Ref. [4]. One term, which is due to the p-p interaction only, depends linearly on $g_{pp}$ and vanishes at $g_{pp} = 1$ when the SU(4)-symmetry is restored in the p-p sector of a model Hamiltonian. The second term is a smoother function of $g_{pp}$ at $g_{pp} \sim 1$, but
exhibits a quadratic dependence on the strength of the mean-field spin-orbit term, which is the main source of violation of the spin-isospin SU(4)-symmetry in nuclei.

Understanding of general properties of the amplitude $M^{2\nu}_{GT}$ helps to improve reliability of evaluation of $\beta\beta$-decay amplitudes. For a quantitative analysis, we use here an isospin-selfconsistent pn-continuum-QRPA (pn-cQRPA) approach of Ref. [5], where this approach was applied to evaluate the GT($^\mp$) strength distributions in single-open-shell nuclei. In the reference the full basis of the single-particle (s-p) states was used in the p-h channel along with the Landau-Migdal forces, while the nucleon pairing was described within the simplest version of the BCS-model based on discrete basis of s-p states. A rather old version of the phenomenological isoscalar nuclear mean field (including the spin-orbit term) was used in Refs. [5, 4], as well.

The first application of the pn-cQRPA approach of Ref. [5] to description of the $\beta\beta$-decay observables in several nuclei has been given recently in Ref. [7]. Realistic (zero-range) forces have been used in the p-p channel to describe the nucleon pairing within the BCS model realized on a rather large discrete+quasidiscrete s-p basis.

The pn-cQRPA approach of Refs. [5, 7] is further extended here by using a modern version of the phenomenological isoscalar mean field (including the spin-orbit term) deduced in Ref. [6] from the isospin-selfconsistent analysis of experimental single-quasiparticle spectra in double-closed-shell nuclei. In the present contribution we give a brief overview of the approach and its applications to description of different GT strength functions for the pairs of nuclei $^{116}$Cd-Sn and $^{130}$Te-Xe.

2 Coordinate representation of the pn-cQRPA equations and GT strength functions

In formulation of a version of the pn-cQRPA we follow Ref. [5], where the pn-dQRPA equations originally written in terms of the forward $X_s^{(-)}$ and backward amplitudes $Y_s^{(-)}$ are transformed in equivalent equations for the 4-component radial transition density $\rho_i^{(-)}(s, r)$. The latter is defined in terms of the $X, Y$ amplitudes by Eqs. (39), (40) of Ref. [5] and related to the GT($^\mp$) excitations having the wave functions $|1^+\mu, s\rangle$ and energies $\omega_s$. The expression for the transition density $\rho_i^{(\mp)}(s, r)$ related to GT($^\mp$) excitations follows from that for $\rho_i^{(-)}(s, r)$ by the substitution $p \leftrightarrow n$. Hereafter, we often use the notations of Ref. [5] (except for the energies) and refer to some equations from this reference. Limiting ourselves in this contribution to description of the GT transitions only, we omit the quantum numbers indices $J = S = 1, L = 0$ for spin-monopole excitations. The spin-angular variables in all expressions are separated out as well.

The pn-dQRPA solutions $\omega_s$ are related to the excitation energies $E_{x,s}^{(\mp)}$ measured from the ground-state energy $E_0(Z \pm 1, N \mp 1)$ of the corresponding daughter nuclei as:

$$\omega_s \pm (\mu_p - \mu_n) = \omega_s^{(\mp)} = E_{x,s}^{(\mp)} + Q_b^{(\mp)}.$$  

Here, $\mu_{p(n)}$ is the chemical potential for the proton (neutron) subsystem found from the known BCS equations, $Q_b^{(\mp)} = E_b(Z, N) - E_b(Z \pm 1, N \mp 1)$ are the total binding-energy differences, $\omega_s^{(\mp)}$ are the excitation energies measured from $E_0(Z, N) - \sum_a m_a = -E_b(Z, N)$ ($m_a$ is the nucleon mass). The energies $\omega_s^{(\mp)}$ are usually described by a model Hamiltonian.
The system of equations for $\rho_i^{(s,s)}(s,r)$ (Eq.(41) of Ref. [5]) contains the explicit expression for the $4 \times 4$ matrix of the free two-quasiparticle propagator $A_{ik}^{(+)}(r,r',\omega_s)$ (Eq.(43) of Ref. [5]: $A_{ik}^{(+)} = A_{ik}^{(-)}(p \leftrightarrow n)$). These propagators are the main quantities in description of charge-exchange excitations within the pn-QRPA. In particular, in terms of $A_{ik}$ one can formulate a Bethe-Salpeter-type equation for the effective propagator $\tilde{A}_{ik}(r,r',\omega_s)$ [7]. The spectral expansion of $\tilde{A}_{ik}$ in terms of $\rho_i(s,r)$ allows one to express the pn-QRPA strength functions in terms of the effective propagator, or, equivalently, in terms of the 4-component effective fields [5]. Some relevant formulas are shown below.

The GT$^{(+)}$ strength functions, corresponding to the external fields (probing operators) $\tilde{V}_\mu^{(+)}(r) = \sum_a V_\mu^{(+)}(a)$, $V_\mu^{(+)} = \tau^{(+)} \sigma_\mu$, are defined as follows:

$$S^{(+)}(\omega) = \sum_s |\langle 1^+, s || \tilde{V}^{(+)} || 0 \rangle|^2 \delta(\omega - \omega_s)$$

(2)

with GT strengths $B^{(+)}(GT) = |\langle 1^+, s || \tilde{V}^{(+)} || 0 \rangle|^2$. The strength function $S^{(-)}(\omega)$ can be expressed in terms of the corresponding effective field $\tilde{V}_i^{(-)}(r)$, which is different from the external one $V_i^{(-)}(r) = \delta_{i1}$ due to the residual interaction [5]:

$$S^{(-)}(\omega) = -\frac{3}{\pi} Im \sum_i \int A_{i1}^{(-)}(r,r',\omega) \tilde{V}_i^{(-)}(r',\omega) dr dr'$$

(3)

$$\tilde{V}_i^{(-)}(r,\omega) = \delta_{i1} + \frac{F_i(1)}{4\pi r^2} \sum_k \int A_{ik}^{(-)}(r,r',\omega) \tilde{V}_k^{(-)}(r',\omega) dr'. $$

(4)

The residual interaction here is supposed to be of zero-range type with intensities $F_i^{(1)}$:

$F_1^{(1)} = F_2^{(1)} = 2G'$, $F_3^{(1)} = F_4^{(1)} = G_1$. For the $0^+$ p-h and p-p channels the corresponding strengths are: $F_1^{(0)} = F_2^{(0)} = 2F'$ and $F_3^{(0)} = F_4^{(0)} = G_0$, respectively. Dimensionless values $g'/C$, $f'/F'C$, $(C = 300 MeV \cdot fm^3)$ are the well-known Landau-Migdal p-h strength parameters. The same parameterization we use for the p-p interaction strengths: $g_1 = G_1/C$, $g_0 = G_0/C$. For calculation of $S^{(+)}(\omega)$ one can use Eqs. (3), (4) with substitution $p \leftrightarrow n$ [5]. An alternative way is based on the symmetry properties of $A_{ik}$: $A_{11}^{(+)} = A_{22}^{(-)}$. As a result, we get the expression for $S^{(+)}(\omega)$ in terms of $A_{ik}^{(-)}$ and $\tilde{V}_i^{(-)}$. This expression is obtained from Eqs. (3), (4) with the substitution $1 \rightarrow 2$.

The nuclear GT$^{(-)}$ amplitude for $2\nu\beta\beta$-decay into the ground state $|0'\rangle$ of the product nucleus $(N - 2, Z + 2)$ is given by the expression:

$$M^{2\nu}_{GT} = \sum_s \langle 0' || \tilde{V}^{(-)} || 1^+, s \rangle \langle 1^+, s || \tilde{V}^{(-)} || 0 \rangle$$

(5)

where $\omega_s = E_s - \frac{1}{2}(E_0 + E_{0'}) = E_{x,s} + \frac{1}{2}(Q_b^{(-)} + Q_b^{(+)}').$ To calculate $M^{2\nu}_{GT}$ within the pn-QRPA, the vacua $|0\rangle$ and $|0'\rangle$ should be identified. As a result of such identification, one has $\omega_s = \frac{1}{2}(\omega_s^{(-)} + \omega_s^{(+)}') \approx \omega_{s'},$ in accordance with Eq. (1).

The amplitude (5) can be expressed in terms of a “non-diagonal” GT$^{(-)}$ strength function $S^{(-)}(\omega)$:

$$M^{2\nu}_{GT} = \int \omega^{-1} S^{(-)}(\omega) d\omega,$$

(6)

where $S^{(-)}(\omega)$ is defined as follows:

$$S^{(-)}(\omega) = \sum_s \langle 0' || \tilde{V}^{(-)} || 1^+, s \rangle \langle 1^+, s || \tilde{V}^{(-)} || 0 \rangle \delta(\omega - \omega_s).$$

(7)
The corresponding pn-QRPA expression for $S^{(-)}$ is:

$$S^{(-)}(\omega) = -\frac{3}{\pi} Im \sum_i \int A_{2i}^{(-)}(r, r', \omega) \tilde{V}_{i[1]}^{(-)}(r', \omega) dr dr'.$$

(8)

An alternative expression for $M_{GT}^{2\nu}$ is obtained in terms of the “non-diagonal” static polarizibility [7]:

$$M_{GT}^{2\nu} = -\frac{3}{2} \sum_i \int A_{2i}^{(-)}(r, r', \omega = 0) \tilde{V}_{i[1]}^{(-)}(r', \omega = 0) dr dr'.$$

(9)

Decomposition of the amplitude (5) into two terms [4]

$$M_{GT}^{2\nu} = (M_{GT}^{2\nu})' + \bar{\omega}_{GT R} EWSR^{(-)},$$

(10)

$$EWSR^{(-)} = \sum_s \bar{\omega}_s \langle 0' || \hat{V}^{(-)} || 1^+ + s \rangle \langle 1^+ + s || \hat{V}^{(-)} || 0 \rangle,$$

(11)

where $\bar{\omega}_{GT R}$ is the energy of GT$^{(-)}$ giant resonance (GTR), allows us to clarify the sensitive dependence of $2\nu\beta\beta$-decay amplitude as a function of $g_{pp}$ (for details, see Ref. [4]). The “non-diagonal” energy-weighted sum rule $EWSR^{(-)}$ is straightforwardly expressed in terms of the strength function $S^{(-)}$ of Eq. (6):

$$EWSR^{(-)} = \int \omega S^{(-)}(\omega) d\omega,$$

(12)

again supposing the QRP A vacuum $|0'\rangle$ is identified with that of $|0\rangle$.

### 3 Calculation of strength function within the pn-cQRP A

Starting from the coordinate representation of the pn-dQRPA equations outlined above, we are able to take exactly into account the s-p continuum in the p-h channel and, therefore, to formulate a version of the pn-cQRP A. The pairing problem is solved on a rather large basis of bound+quasibound proton and neutron s-p states within the present version of the model. To take the s-p continuum into account, the following transformations of the expression for $A_{ik}(r, r', \omega)$ [5] are done: (i) the Bogolyubov coefficients $v_\lambda$, $u_\lambda$ and the quasi-particle energies $E_\lambda$ are approximated by their non-pairing values $v_\lambda = 0$, $u_\lambda = 1$, and $E_\lambda = \epsilon_\lambda - \mu$ for those s-p states ($\lambda$), which lie far above the chemical potential (i.e. $\epsilon_\lambda - \mu \gg \Delta_\lambda$), (ii) the Green function of the s-p radial Schrödinger equation $g_\lambda(r, r', \epsilon) = \sum_\epsilon \chi_\lambda(\epsilon - \epsilon_\lambda + i0)^{-1} \chi_\lambda(r) \chi_\lambda(r')$, which is calculated via the regular and irregular solutions of this equation, is used to perform explicitly the sum over the s-p states in the continuum. As a result, the properly transformed free two-quasiparticle propagator $A$ is obtained, upon which a corresponding version of the pn-cQRP A is based.

The solution of the pairing problem is simplified by using the “diagonal” approximation for the p-p interaction for the $0^+$ neutral channel. In this approximation the nucleon-pair operators are assumed to be formed only from the pair of nucleons occupying the same s-p level $\lambda$. The nucleon pairing is described with the use of the Bogolyubov transformation with the gap parameter $\Delta_\lambda$ dependent on $\lambda$. The same number $N_{b+qb}$ of bound+quasibound states forming the basis of the BCS problem is used for both the neutron and proton subsystems. These numbers are shown in Table 1 for nuclei in question.
In evaluation of total binding energies within the model (that is necessary to evaluate the pairing energies $E_{pair}$) the blocking effect for odd nuclei is taken into account. In description of the nucleon pairing, different values of the p-p interaction strength parameters $g_{0,n}$ and $g_{0,p}$ for the neutron and proton subsystems are used. These values are found from comparison of the calculated and experimental pairing energies for nuclei under consideration (Table 1).

Table 1: The phenomenological mean field parameters ($U_0$, $U_{SO}$ and $a$), singlet and triplet p-h and p-p interaction strengths ($f'$, $g'$, $g_{pp}$) used in calculations.

| Pair of nuclei | $U_0$, MeV | $U_{SO}$, MeV-fm$^2$ | $a$, fm | $f'$ | $g_{0,n}$ | $g_{0,p}$ | $N_{b+gb}$ | $g'$ | $g_{pp}$ |
|----------------|------------|------------------------|--------|-----|----------|----------|-----------|-----|--------|
| $^{116}$Cd-$^{116}$Sn | 51.62      | 34.08                  | 0.618  | 1.06| 0.388    | 0.333    | 22        | 0.77| 1.0    |
| $^{130}$Te-$^{130}$Xe | 51.74      | 34.025                 | 0.628  | 1.09| 0.356    | 0.364    | 22        | 0.88| 0.99   |

The mean field consists of the phenomenological isoscalar part (including the spin-orbit term) along with the isovector and Coulomb part (Eq. (1) of Ref. [5]). The parameterization of the Woods-Saxon-type isoscalar part contains two strength ($U_0$, $U_{SO}$) and two geometrical ($r_0$, $a$) parameters [6]. The mean field isovector part (the symmetry potential) is calculated in an isospin-selfconsistent way (Eqs. (7), (35) of Ref. [5]) via the neutron-excess density and Landau-Migdal strength parameter $f'$. The mean Coulomb field is also calculated selfconsistently via the proton density. All densities are calculated with taking into account the nucleon pairing. Five above-listed model parameters found in Ref. [6] for a number of double-closed-shell nuclei are properly interpolated for nuclei under consideration (see Table 1; $r_0 = 1.27$ fm is taken for all nuclei).

The values of the Landau-Migdal strength $g'$ listed in Table 1 are obtained by fitting the experimental GTR energy in calculations of the GT$^(-)$ strength function. The p-p interaction strength $g_1$ (or its relative value $g_{pp} = 2g_1/(g_{0,n} + g_{0,p})$) is considered as a free parameter. It can be adjusted to reproduce the experimental $M^{2\nu}_{GT}$ value (the corresponding values are listed in the last column of Table 1).

Considering the pair $^{116}$Cd-$^{116}$Sn, the GT$^(-)$ strength distribution calculated within the pn-dQRPA for the transition $^{116}$Sn$\rightarrow^{116}$Sb is shown in Fig. 1a (a small imaginary part is added to the s-p potential). To compare the calculation results with the $^{116}$Sn($^3$He, t) experimental data of Ref. [8], five centroids of the energy, $E_{x\cdot i}$, and their strength $x_i$ relative to the one of the GTR are evaluated (Fig. 1b). The value $g' = 0.77$ allows to reproduce the experimental GTR energy in the calculation. The GT$^(-)$ strength distribution is almost insensitive to the $g_{pp}$ value ($g_{pp} = 1.0$ is taken in the calculation). The GT$^{(+)}$ strength distribution for the transition $^{116}$Sn$\rightarrow^{116}$In is found more sensitive to $g_{pp}$. Only one $1^+$ state with $B^{(+)}(GT) = 0.47$ corresponding to the $1g_{9/2}^p \rightarrow 1g_{7/2}^n$ transition into the $^{116}$In ground state, is found in the calculation within the interval $E_x < 5$ MeV. This weak transition is allowed due to the neutron pairing in $^{116}$Sn. In the $^{116}$Sn(d, $^3$He) experiments four $1^+$ states in $^{116}$In were found within the interval $E_x \leq 3$ MeV with total strength $\sum_i B_i^{(+)}(GT) = 0.66$ [8]. Population of the $1^+$ states in $^{116}$In has also been studied in the $^{116}$Cd(p,n)-reaction [2]. The result $B^{(-)}(GT) = 0.26 \pm 0.02$ for excitation of the $^{116}$In ground state is only available now. Within the interval $E_x \leq 3$ MeV the calculated GT$^(-)$ strength distribution in $^{116}$In exhibits one $1^+$ state, corresponding to the back-spin-flip transition $1g_{7/2}^n \rightarrow 1g_{9/2}^p$ into the $^{116}$In ground state with the value 1.05 $B^{(-)}(GT)$ (for $^{116}$Sn-$^{116}$Sb this transition is Pauli blocked). The $2\nu\beta\beta$-decay amplitude
for the decay $^{116}\text{Cd} \to ^{116}\text{Sn}$ can barely be evaluated within the pn-QRPA because the proton shell is closed in $^{116}\text{Sn}$.

Coming to the pair $^{130}\text{Te}-^{130}\text{Xe}$, the value $g' = 0.88$ is found in the calculation by fitting the experimental GTR energy in $^{130}\text{I}$ [9]. Then the amplitude $M_{GT}^{2\nu}$ (6) (or (9)) and its decomposition (10) are calculated, as a function of $g_{pp}$ (Fig. 2). The corresponding experimental value $(M_{GT}^{2\nu})_{\text{exp}} = 0.03 \text{ MeV}^{-1}$ [10] can be reproduced in the calculation at $g_{pp} = 0.99$. The $2\nu\beta\beta$-decay strength function $\omega^{-1}S^{(--)}(\omega)$ is calculated for this value of $g_{pp}$ (Fig. 3a) along with the corresponding running sum $M_{GT}^{2\nu}(\omega) = \int_{\omega'}^{\omega} \omega'^{-1}S^{(--)}(\omega')d\omega'$ (Fig. 3b). Figs. 2 and 3 illustrate how the $M_{GT}^{2\nu}$ value for the decay $^{130}\text{Te} \to ^{130}\text{Xe}$ is formed. In particular, as one sees in Fig.3, the experimental studies of $B^{(+)}(GT)$ are not always sufficient for understanding partial contributions to $M_{GT}^{2\nu}$. The reason is that the intermediate states having a relatively large excitation energy and very small $B^{(+)}(GT)$ value (like the GTR) can nonetheless play essential role in formation of the $2\nu\beta\beta$-decay amplitude.

In conclusion, an isospin-selfconsequent version of the pn-cQRPA has been outlined and some its applications to description of charge-exchange excitations in open-shell spherical nuclei are presented. Although only general features of the low-energy strength distributions can be described within the approach, it seems applicable to analysis of $\beta\beta$-decay observables.

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Figure 1: The GT$^{(-)}$ strength function for $^{116}$Sn-$^{116}$Sb (a) and the relative (with respect to the GTR) strength of the low-energy $1^+$ peaks calculated within pn-cQRPA (b). The corresponding experimental data are taken from Ref. [8].

Figure 2: The calculated amplitude of $^{130}$Te $2\nu\beta\beta$-decay as a function of $g_{pp}$. Decomposition of Eq. (10) is also shown.
Figure 3: The GT $2\nu\beta\beta$-decay strength function (a) and the running sum (b) calculated for $^{130}\text{Te}$ at $g_{pp} = 0.99$