Revising the Evolutionary Stage of HD 163899: The Effects of Convective Overshooting and Rotation

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Abstract

We revise the evolutionary status of the B-type supergiant HD 163899 based on the new determinations of the mass–luminosity ratio, effective temperature, and rotational velocity, as well as on the interpretation of the oscillation spectrum of the star. The observed value of the nitrogen-to-carbon abundance fixes the value of the rotation rate of the star. Now, more massive models are strongly preferred than those previously considered, and it is very likely that the star is still in the main-sequence stage. The rotationally induced mixing manifests as the nitrogen overabundance in the atmosphere, which agrees with our analysis of the HARPS spectra. Thus, HD 163899 probably belongs to a group of evolved nitrogen-rich main-sequence stars.

Key words: stars: early-type – stars: individual (HD 163899) – stars: oscillations – supergiants

1. Introduction

HD 163899, a B2 Ib/II supergiant, was proposed by Saio et al. (2006) as a prototype candidate of a new class of pulsating stars termed slowly pulsating B-type supergiants (SPBsg). The star was analyzed by many groups (Saio et al. 2006; Godart et al. 2008; Daszyńska-Daszkiewicz et al. 2013a; Ostrowski & Daszyńska-Daszkiewicz 2015), but all of these studies share the same drawback, which is the lack of proper determination of the basic stellar parameters. The spectral classification of HD 163899 was done on the basis of photometric indices by Klare & Neckel (1977) and Schmidt & Carruthers (1996) and the very crudely estimated basic parameters. Therefore, the main problem was establishing the evolutionary stage of the star. Saio et al. (2006) and Godart et al. (2009) claimed that it was a supergiant during the first crossing toward the red giant branch (RGB). Godart et al. (2008) suggested that HD 163899 could be a main-sequence (MS) star, but lower masses derived from the old estimations of stellar parameters did not support this statement. In such a case the enormously large value of overshooting from the convective core was required. In our previous paper (Ostrowski & Daszyńska-Daszkiewicz 2015) we studied the possibility that HD 163899 is a supergiant on the blue loop in the core helium burning phase. We found that for certain values of the convective overshooting parameter and metallicity, blue loops can reach early B spectral types for stars with masses in the range of 14–20 $M_\odot$, but such models could not reproduce the observed frequency range and the number of detected modes. Therefore, we concluded that the star would rather be in the phase of shell hydrogen burning. In this paper, we determine for the first time the basic parameters from high-resolution spectra. We also derive the value of the projected rotational velocity, $V_{\text{rot}} \sin i$, which was not available in the previous studies. Moreover, we discuss the measured abundances of CNO elements and use them to apply constraints on parameters of the models, especially the rotation rates. Stellar rotation is an important source of element mixing and, together with other mixing mechanisms (convection, semi-convection, convective overshooting), can greatly affect the evolution and internal structure of the star, as well as observed surface abundances. Here, we focus on these issues and study their effects on stellar models using the MESA code.

The values of convective overshooting studied here, $f \lesssim 0.03$, are fairly typical among the values considered in modern stellar modeling or calibrations with observational data (e.g., Stancliffe et al. 2015). The grids of Ekström et al. (2012) and Brott et al. (2011) use a step overshooting instead of the exponential formula adopted here (Equation (1); Herwig 2000). Ekström et al. (2012) use $\alpha_{\text{ov}} = 0.1$, which roughly corresponds to $f = 0.01$ ($\alpha_{\text{ov}} \approx 10^3$; Moravveji 2015). The value of $f = 0.03$, which gives good results in our calculations, is rather high but comparable to the value adopted by Brott et al. (2011): $\alpha_{\text{ov}} = 0.335$. They also adopted much more efficient semi-convective mixing with $\alpha_{\text{SC}} = 1.0$, using the same prescription of Langer et al. (1983) as we do. On the other hand, Ekström et al. (2012) utilize the Schwarzschild criterion for convective instability and hence do not take semiconvection into account. Recently, Moravveji et al. (2015) tried to build a seismic model to reproduce 19 frequencies of the SPB star KIC 10526294 using MESA models. They obtained the best fit for the models with overshooting of $f = 0.017$–0.018; however, the mass of the star was in the range of [3.15, 3.25] $M_\odot$, which is significantly lower than for the stars we study. More interesting insight about the value of overshooting can be found in the seismic modeling of two hybrid β Cephei/SPB pulsators: 12 Lacertae (Daszyńska-Daszkiewicz et al. 2013b) and γ Pegasi (Walczak et al. 2013). In both cases the models that fit the observed frequencies can be found within a quite wide range of overshooting parameter ($\alpha_{\text{ov}} \sim 0.2–0.45$) because obviously other factors such as metallicity or opacities affect seismic models. In this paper, we choose the value of the core overshooting parameter of $f = 0.02$ and 0.03, which reasonably lies within the accepted range.

Stellar pulsations are potentially powerful probes of the stellar interior. They can yield constraints on parameters of stellar models and theory. For self-excited pulsation certain conditions have to be fulfilled, and hence the changes in the internal structure at some stages of evolution can prevent a propagation of pulsational modes. For a long time it was thought that g-mode pulsations cannot be excited in massive

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### 2. Stellar Parameters of HD 163899

HD 163899 aroused more interest when Saio et al. (2006) detected 48 frequencies in the MOST light curve. These light variations were explained by the existence of $g$- and $p$-mode pulsations. However, an obstacle in the interpretation of the HD 163899 oscillations was the problem with determination of the evolutionary stage of the star because of the uncertain values of the basic parameters such as effective temperature or surface gravity.

Recently, HD 163899 was a target object in a SPACEINN spectroscopic project, which supported the MOST satellite mission (Ranier et al. 2016). The spectral type of B2 II, effective temperature $T_{\text{eff}} = 23961$ K, surface gravity log $g = 3.1$, projected rotational velocity $V \sin i = 48–49$ km s$^{-1}$, and metallicity [Fe/H] = 0.05 dex were assigned for HD 163899 in the SPACEINN archive data (http://www.brera.inaf.it/spaceinn/data/). This is in good agreement with earlier classifications of this star as a B2 I–II object based on photometric measurements (Klare & Neckel 1977; Schmidt & Carruthers 1996).

### Table 1

| No. | Spectrum | MJD (days) | Exp. time (s) | S/N | $\lambda$ (Å) |
|-----|----------|------------|---------------|-----|---------------|
| 1   | HD 163899-2010625-2320 | 2,455,738.48708 | 600 | 141 | 3785–6910 |
| 2   | HD 163899-2010625-20053 | 2,456,103.54619 | 600 | 136 | 3785–6910 |
| 3   | HD 163899-2010625-0103  | 2,456,103.55345 | 1600 | 212 | 3785–6910 |

### Table 2

| No. | Model | $T_{\text{eff}}$ (K) | $\log g$ (cgs) | $\zeta_t$ (km s$^{-1}$) | He/H | C/H | Abundances N/H | O/H |
|-----|-------|----------------------|----------------|------------------------|------|-----|---------------|-----|
| 1   | BG    | 15,000–30,000        | 2.50–4.75      | 2, 10                  | 0.10 | 3.31 $\times$ 10$^{-4}$ | 8.32 $\times$ 10$^{-5}$ | 6.76 $\times$ 10$^{-4}$ | GS98 |
| 2   | CN    | 15,000–30,000        | 2.50–3.00      | 10                     | 0.20 | 1.65 $\times$ 10$^{-4}$ | 4.16 $\times$ 10$^{-4}$ | 6.76 $\times$ 10$^{-4}$ | GS98 |
| 3   | T1    | 15,000–24,000        | 3.00–3.75      | 2–20                   | 0.085 | 3.28 $\times$ 10$^{-4}$ | 8.24 $\times$ 10$^{-5}$ | 6.76 $\times$ 10$^{-4}$ | GS98 |
| 4   | T2    | 20,000–25,000        | 3.50           | 2–15                   | 0.085 | 2.69 $\times$ 10$^{-4}$ | 6.76 $\times$ 10$^{-5}$ | 4.90 $\times$ 10$^{-4}$ | AGSS09 |
| 5   | T3    | 23,000               | 3.00           | 15                     | 0.085 | 3.28 $\times$ 10$^{-4}$ | (1.65–2.23)$\times$10$^{-4}$ | 6.70 $\times$ 10$^{-4}$ | GS98 |
| 6   | T4    | 23,000               | 3.00           | 15                     | 0.15, 0.20 | 3.28 $\times$ 10$^{-4}$ | 1.83 $\times$ 10$^{-4}$ | 6.70 $\times$ 10$^{-4}$ | GS98 |

### Note

The BG and CN models were calculated by Lanz & Hubeny (2007), while the T1–T4 models were calculated with the updated atom model of carbon (see text) in this paper.

B-type stars after the terminal-age main sequence (TAMS; Pamyatnykh 1999; Pamyatnykh & Ziomek 2007). The reason was strong radiative damping expected in a dense radiative helium core where the Brunt–Väisälä frequency reaches huge values. The situation changed when Saio et al. (2006) identified 48 frequency peaks below 3 day$^{-1}$ and with the maximum amplitude of a few millimagnitudes from the data obtained by the MOST satellite for HD 163899 (B2 Ib/II supergiant). The authors attributed these frequencies to some eigenmodes at an intermediate convective zone (ICZ) related to the hydrogen-burning shell. The instability analysis performed by other groups (Godart et al. 2008; Daszyńska-Daszkiewicz et al. 2013a; Ostrowski & Daszyńska-Daszkiewicz 2015) confirmed that both $p$- and $g$-modes can be excited in the post-MS massive stellar models by the $\kappa$-mechanism acting in the metal opacity bump. Thus, HD 163899 has become a prototype of SPB’s stars. Other candidates have been also proposed (e.g., Lefever et al. 2007). In this paper, for the first time, pulsational properties of HD 163899 are confronted with rotating models calculated for a well-established set of stellar parameters and rotational velocity derived from high-resolution spectroscopy.

The structure of this paper is as follows. In Section 2, we present the new determinations of basic stellar parameters for HD 163899. The MESA evolutionary models are presented in Section 3, where we discuss the influence of convective overshooting and stellar rotation on the internal structure and surface composition of the star. In Section 4.2, we present pulsational calculations and interpret the observed and theoretical frequency distributions obtained for HD 163899. The last section contains conclusions.
The depths of these features are less than 2.5%, the radial velocities are about $-45$ km s$^{-1}$, and they can be related to variable stellar wind, pulsations, or binarity. Given the limited spectroscopic data at our disposal, we cannot exclude the possibility that HD 163899 has a pulsating lower-mass companion.

In the next step, the observed spectra were compared with non-LTE atmosphere models listed in Table 2. First, the synthetic spectra calculated by Lanz & Hubeny (2007) were broadened for the projected rotational velocities of $V_{\text{rot}} \sin i = 20, 40, 50, 60, 70, \text{and } 80$ km s$^{-1}$. The spectra were also convolved with an instrumental profile and then interpolated to a denser grid of $T_{\text{eff}}$, $\log g$, and $V_{\text{rot}} \sin i$. Rather than separately determining the effective temperature, $T_{\text{eff}}$, and gravity, $\log g$, as in a traditional spectroscopic analysis, we made use of the parameter $\mathcal{L}' = 4 \log T_{\text{eff}} - \log g - 10.609$, which can be uniquely assigned to each spectrum. The values of $\mathcal{L}'$ were measured from the four hydrogen lines H$_{\delta}$, H$_{\gamma}$, H$_{\beta}$, and H$_{\alpha}$. The best fits correspond to the minimum value of the rms deviation (RMSD), and its logarithmic values as a function of $\mathcal{L}'$ are plotted in Figure 1. As one can see, there is good agreement for all studied hydrogen lines. However, H$_{\delta}$ is most suitable for our purposes, because of the smallest blending effect by metallic lines. The quality of the solution can be judged from Figure 2, where we show an example of the fit of the theoretical spectra for $\mathcal{L}' = 3.85$ to the two hydrogen lines, H$_{\beta}$ and H$_{\alpha}$ (top panels), the helium line He I 4388 (bottom right panel), and the region with strong nitrogen lines (bottom left panel). To reproduce the He I and N II line profiles, not only did we have to increase the microturbulent velocity from 2 to 10 km s$^{-1}$, but also we had to use atmosphere models enriched in nitrogen and helium. These are the CN models, in the notation of Lanz & Hubeny (2007), which have the helium abundance increased to He/H = 0.2 by number, the nitrogen abundance increased by a factor of 5, and the carbon abundance halved. These spectral features are signatures of a surplus of helium and, in particular, of an overabundance of nitrogen in HD 163899.

The overabundance of nitrogen for HD 163899 is also seen in the diagram $W(N)/W(C)$ versus $W(N)/W(O)$, shown in Figure 3, where W(C), W(N), and W(O) mean the equivalent widths of carbon lines at 5132.947, 5133.282, 5139.174, 5143.495, 5145.165, and 5151.085 Å, nitrogen lines at 5666.630, 5676.020, 5679.560, and 5686.210 Å, and oxygen at 4661.632 Å, respectively. These lines were selected because of the smallest contributions of blends (see Figure 2, where the N II lines were plotted). The LH models were calculated with the C II atom model, which includes 17 individual levels and

![Figure 1](image-url) Logarithmic values of the RMSD as a function of the parameter $\mathcal{L}'$ for the four hydrogen lines. The best-fit solutions correspond to the minimum value of RMSD obtained for the solar chemical abundance (GS98) and the microturbulent velocity $\zeta = 2$ km s$^{-1}$.
five superlevels. This atom model does not have a sufficient number of individual levels to obtain correct values of equivalent widths of C II lines located in the visual spectral range. Therefore, we calculated the new non-LTE atmosphere models (T1–T4 in Table 2) with the updated C II atom model, which now includes 34 levels individually and five superlevels.

The predicted ratios of the equivalent widths, shown as triangles in Figure 3, were calculated for selected stellar parameters assuming GS98 (Grevesse & Sauval 1998) and AGSS09 (Asplund et al. 2009) chemical compositions (see Table 2). The models shown as red diamonds and green circles in Figure 3 were calculated for increased abundances of nitrogen (red diamonds) and increased abundances of both helium and nitrogen (green circles). The observations are shown with plus signs for three spectra of HD 163899. As one can see, only models with an increased abundance of nitrogen are able to fit the observations. This conclusion does not depend on the other parameters of the models.

The value of $\mathcal{L} = 3.85 \pm 0.1$ dex results from the best-fit procedure mentioned above. This solution also has well-established values of $T_{\text{eff}} = 23000 \pm 1000 \, K$, $\log g = 3.0 \pm 0.15$, and $V_{\text{rot}} \sin i = 65 \pm 5 \, \text{km} \, \text{s}^{-1}$. The accuracy of these parameters was estimated by considering grids of the LH and T atmosphere models for different chemical compositions and microturbulent velocities assuming that there is no contribution by an additional source of radiation in the cores of hydrogen lines. Otherwise, $\log g = 3.0$ should be regarded as a minimum value of the surface gravity. For instance, if an additional light contributes by an amount of about 30% in the cores of hydrogen lines, then $\log g$ increases to about 3.5 dex. This is the main source of uncertainty in our analysis.

In the case of slowly and moderately rotating stars, the parameter $\mathcal{L}$ can be directly compared with $\mathcal{L} = \log (L/M) - \log (L/\odot) M_{\odot}$ constrained from evolutionary models and spectroscopic Hertzsprung–Russell (H-R) diagrams introduced by Langer & Kudritzki (2014). These diagrams with $\mathcal{L}$ versus $T_{\text{eff}}$ or versus $\log g$ can be used to derive stellar parameters such as mass, luminosity, age, etc. An additional advantage is that spectroscopic H-R diagrams are not influenced by interstellar extinction and distance to the star.

HD 163899 is now more luminous and slightly hotter than assumed in the previous papers (e.g., Figure 4 in Saio et al. 2006). As a result, it is now more massive. The older spectral classification of HD 163899 was B2 Ib/II, but the value of $\log g = 3.0$ derived in this paper corresponds better to B2 II.

### 3. Evolutionary Models

The evolutionary models were computed by means of the MESA code (Paxton et al. 2011, 2013, 2015). We assumed the
where $D_{\text{conv}}$ is the diffusion coefficient derived from mixing length theory (MLT) at a user-defined location in the convective zone ($f_{\text{MLT}}H_p$ off the boundary of a convective zone), $H_p$ is the pressure scale height at that location, $z$ is the distance in the radiative layer away from that location, and $f$ is an adjustable parameter. In regions stable according to the Ledoux criterion for convection but unstable according to the Schwarzschild criterion, we use a semiconvective mixing with the scheme of Langer et al. (1983, 1985), and the standard value of the efficiency parameter is $\alpha_{\text{MLT}} = 1.82$ (calibration of Choi et al. 2016). The exponential formula of Herwig (2000) for overshooting from the convective regions has been used, i.e.,

$$D_{\text{OV}} = D_{\text{conv}} \exp \left( - \frac{2z}{fH_p} \right),$$

where $D_{\text{conv}}$ is the diffusion coefficient derived from mixing length theory (MLT) at a user-defined location in the convective zone ($f_{\text{MLT}}H_p$ off the boundary of a convective zone), $H_p$ is the pressure scale height at that location, $z$ is the distance in the radiative layer away from that location, and $f$ is an adjustable parameter. In regions stable according to the Ledoux criterion for convection but unstable according to the Schwarzschild criterion, we use a semiconvective mixing with the scheme of Langer et al. (1983, 1985), and the standard value of the efficiency parameter is $\alpha_{\text{MLT}} = 0.01$.

The rotation in MESA is implemented in the framework of the shellular approximation, which means that the angular velocity, $\Omega$, is constant over isobars (e.g., Meynet & Maeder 1997). Mixing of chemical elements and transport of angular momentum are treated in the diffusion approximation (Endal & Sofia 1978; Pinsonneault et al. 1989; Heger et al. 2000). This approach is consistent with implementation of all other mixing processes in MESA such as convection, semiconvection, overshooting, etc. Different rotational mixing mechanisms were taken into account: dynamical shear instability, Solberg–Høiland instability, secular shear instability, Eddington–Sweet circulation, and Goldreich–Schubert–Fricke instability. We neglected all effects of magnetic field; hence, the effects of the Spruit–Tayler dynamo were not taken into account (Spruit 2002). The adopted efficiency of rotational mixing is based on the calibration of Brott et al. (2011).

We calculated models with initial values of rotation in a range of 20%–50% of the critical velocity on the zero-age main sequence (ZAMS). The critical rotational rate is given by $\Omega_{\text{crit}}^2 = GM/R_{\text{eq}}^2$ within the Roche model, where $R_{\text{eq}}$ is the equatorial radius. These angular velocities at ZAMS correspond to rotational velocities of about $V_{\text{rot}} = 140–350$ km s$^{-1}$ for all models in the considered range of masses. The rotation rate evolves with time owing to changes in the internal structure and angular momentum transport. Mass loss is parameterized using the prescription of Vink et al. (2001), which is often used for massive stellar models.

### 3.1. The Evolutionary Stage of HD 163899

New determinations of the basic parameters of HD 163899 and the projected value of the rotational velocity, $V_{\text{rot}} \sin i$, once again opened up the possibility that the star can be an evolved MS star. In Figure 4, we showed the luminosity-to-mass ratio, $L'/M'$, versus effective temperature, $\log T_{\text{eff}}$, for models with initial masses of 18, 20, 22, 24, and 26 $M_\odot$. The chosen masses are suitable to reproduce the observed parameters of the star. The presented evolutionary tracks cover the phase from ZAMS up to the point where $\log T_{\text{eff}} = 4.2$. In the left panel, we show the effect of the convective overshooting parameter, $f$, for the fixed value of the rotation rate at ZAMS. We set 0.2 $f_{\text{MLT}}$, which corresponds to the rotational velocity $V_{\text{rot}} \approx 140$ km s$^{-1}$ for all masses presented here. In the right panel, the effect of rotation is shown for $f = 0.03$. The width of the MS is mostly determined by the overshooting from the hydrogen core (left panel of Figure 4), whereas the faster rotation shifts the evolutionary track up to the higher values of luminosity (right panel of Figure 4). As we will see in the next subsection, the moderate rotation has a significant impact on the internal structure. In turn, the internal structure determines the pulsational properties of a star, which we will explore hereafter.

With the new parameters, the star is more luminous than it was believed to be before, and slightly higher values of the effective temperature are preferred as well. These new values have two major consequences. First, the required initial mass for models of HD 163899 should now be at least as high as about 20 $M_\odot$. For the previous estimates of stellar parameters, we found that models with masses of around 15–16 $M_\odot$ are the most appropriate (Ostrowski & Daszyńska-Daszkiewicz 2015). Second, MS models are now contained in the error box if sufficiently high values of the convective overshooting parameter are used ($f \geq 0.02$; see the left panel of Figure 4).

The effect of the rotationally induced mixing can be seen in stellar atmospheres. In Section 2, we found an overabundance of nitrogen from the spectral analysis of HD 163899. To compare this result with theoretical predictions, in Figure 5 we present the ratio of mass abundances of $^{14}$N and $^{12}$C, $X(\text{N})/X(\text{C})$, versus the ratio of mass abundances of $^{14}$N and $^{16}$O, $X(\text{N})/X(\text{O})$, for values derived from atmosphere models with nitrogen enrichment (black diamonds; Section 2) and for evolutionary models in the error box of HD 163899 on the MS (red circles) and supergiant phase (blue open circles).
can be compared to the area in Figure 3 where T3 and T4 models, as well as observational values of equivalent width ratios for HD 163899, are located. We did not show atmosphere models without nitrogen enrichment in Figure 5 because they are not suitable for the studied star. In the rotating models the nitrogen abundance is increasing during most of the MS evolution and during the phase of second contraction, whereas for the nonrotating models (not shown) the abundance of $^{14}$N is constant during the studied phases of the evolution. Thus, it seems that the nitrogen overabundance of HD 163899 can be reproduced only by rotating models.

The parameters of studied stellar models are chosen to reproduce the abundance ratios of X(N)/X(C) and X(N)/X(O) suitable for the atmosphere models of HD 163899 (X(N)/X(C) [0.58, 0.80], X(N)/X(O) [0.21, 0.30]) and the determined parameters of the star within their error boxes (Section 2). For comparison, X(N)/X(C) = 0.29 and X(N)/X(O) = 0.12 for the AGSS09 mixture. The full selection of models that fulfill these criteria is presented in Figure 5. We used the observed values to constrain the rotational velocity for models with different combinations of initial mass and overshooting. Generally, with higher initial mass, the rotation rate required to obtain the observed nitrogen overabundance is lower.

The overshooting parameter also influences the surface abundances because it prolongs the MS phase, during which the rotational mixing is efficient. For all selected models the initial rotation rate is in the range of 0.26–0.30 $\Omega_{\text{crit}}$. The linear increase of the surface nitrogen abundance is clearly visible in Figure 5: the MS models are divided into two separate groups, one with lower values of X(N)/X(C) and X(N)/X(O) and one with higher values. The first group corresponds to the models with $\Omega = 0.28 \Omega_{\text{crit}}$ and the second to $\Omega = 0.30 \Omega_{\text{crit}}$. This bifurcation is not visible for the supergiant models because of the additional effect of overshooting. On the MS almost all models in the error box have $f = 0.03$, whereas supergiants have $f = 0.01$ or 0.02. That leads to continuous visible distribution of points despite the same step in $\Omega$ in the studied grid ($\Delta \Omega = 0.02$).

### 3.2. Internal Structure

Modeling massive stars is a difficult task, due to a number of uncertainties regarding mixing mechanisms and the sensitivity of global stellar parameters such as log $T_{\text{eff}}$ or log $L$ to the internal structure. The mixing parameters control the size and efficiency of convective and semiconvective zones and determine ranges of convective overshooting and the occurrence of zones with rotationally induced instabilities. These different mixing layers are crucial for the transport of energy, chemical elements, and angular momentum and, through their influence on the available amounts of nuclear fuel, the rates of energy generation from thermonuclear reactions, total flux of the star, and other important components of the model. The recent examples of massive star modeling can be found, e.g., in Brott et al. (2011), Ekström et al. (2012), Choi et al. (2016), and Georgy et al. (2013) for very low metallicity.

Changes of the internal structure of a star during its evolution can be easily followed using the Kippenhahn diagrams. In Figure 6, we show such diagrams for two models with initial masses of 22 $M_\odot$ and convective overshooting parameter $f = 0.02$ (top panel) and $f = 0.03$ (bottom panel). In both cases the rotation rate at ZAMS is 30% of $\Omega_{\text{crit}}$. The structure of a star is shown as a function of a calculated model number, and the plots cover the evolution from ZAMS to the point where log $T_{\text{eff}}$ = 4.0 after the overall contraction. Convective zones, convective overshooting, and semiconvective zones are shown with orange, blue, and green, respectively. The radiative zones are depicted as white areas. The total luminosity (red line), as well as luminosities from hydrogen and helium burning (blue and violet lines, respectively), is also shown. The end of the MS is around model number 1400 (log $T_{\text{eff}}$ ≈ 4.39) for the case shown in the top panel ($M = 22 M_\odot$, $f = 0.02$) and around model number 1660 (log $T_{\text{eff}}$ ≈ 4.35) for the bottom panel ($M = 22 M_\odot$, $f = 0.03$).

The effects of overshooting and rotation on stellar structure are very complex. When the rotation is slow or neglected and no or small overshooting is assumed, thin embedded
semiconvective layers may occur in convective zones at some evolutionary stages. It is common in MESA models with masses higher than 18 $M_\odot$, but it may also be visible in less massive models. The properties of semiconvective layers have been discussed by, for example, Langer (1991), Mowlavi et al. (1993), Chiosi (2009), and Moore & Garaud (2016). Such a nonsmooth structure has a rather small effect on the shape of evolutionary tracks or the energy output of the models in the considered range of effective temperatures ($\log T_{\text{eff}} > 4.0$), but strongly modifies the chemical composition gradient, $\nabla_N$. As a consequence, such features greatly affect the Brunt–Väisälä frequency. High-order gravity modes, which are common in evolved massive stellar models (see Section 4.1), are very sensitive to the fine structure of a star, especially around the intermediate convective zone (ICZ; e.g., Gautschy 2009).

The effect of layered semiconvection on properties and period spacing of high-order $g$-modes has been studied by Miglio et al. (2008) and more recently by Belyaev et al. (2015). Both papers relied on the adiabatic approximation. For high-order $g$-modes, discontinuities in the ICZ may lead to the emergence of additional acoustic cavities. Therefore, when one computes pulsational instability, the resulting oscillation spectra might contain fictitious unstable modes.

Models with moderate rotation rate, $\Omega \simeq 0.3 \Omega_{\text{crit}}$, which we found suitable for the analyzed star, are less prone to the occurrence of layered semiconvection. This is due to more efficient rotationally induced mixing. Examination of Figure 6 confirms this fact: the ICZ is free of embedded semiconvection in both presented cases. The values of the convective overshooting parameter used in the considered models are in the range $f = 0.01$–0.03. The convective overshooting has two important effects on the studied models. First, higher values of $f$ lead to a wider MS (Section 3.1, Figure 4). Second, with the more efficient overshooting from the hydrogen core, the ICZ during the MS evolution either is thinner or does not appear at all. This effect may be evaluated by comparing the top ($f = 0.02$) and bottom ($f = 0.03$) panels of Figure 6. These conclusions are true for all studied masses.

4. Interpretation of the Observed Frequencies of HD 163899

The analysis of the MOST light curve of HD 163899 by Saio et al. (2006) revealed 48 frequency peaks in the range [0.02, 2.85] day$^{-1}$. The observations spanned 37 days; thus, the Rayleigh resolution limit, $1/T$, is about 0.03 day$^{-1}$. Therefore, the frequency $\nu = 0.021$ should not be considered to be intrinsic to the star. Moreover, some frequencies listed in Saio et al. (2006) do not fulfill the $S/N$ requirement. The commonly adopted criterion for the frequency detection is that $S/N$ should be around 4 (Breger et al. 1993). Besides, the photometric amplitudes increase toward the low frequencies that can be interpreted as the “red noise” excess (e.g., Blomme et al. 2011). The fit of the power-law shape, $\nu^{\alpha}$, gives the value $\alpha = 0.23$ with the power $\alpha = -0.34(4)$. Thus, some of these frequencies can have a stochastic origin. Another possibility is that HD 163899 is a binary and the pulsations come from a companion. If the latter has a mass less than about 7 $M_\odot$ and is a fast rotator, then the oscillation spectrum can be easily explained by the pulsational models (e.g., Townsend 2005; Dziembowski et al. 2007).

4.1. Pulsational Models

In this section, we allow the hypothesis that the observed frequencies are associated with the heat-driven pulsations that come from HD 163899 itself. For this purpose, we accepted frequencies that have $S/N \geq 3.9$ and excluded the lowest frequency around 0.02 day$^{-1}$. Thus, we considered 44 frequencies out of 48 in total (see the top panel of Figure 8 below).

We computed pulsations for massive stellar models satisfying the $N/C$ criterion discussed in the previous sections. In Figure 7, we put the representative models on the $\mathcal{L}$ versus $\log T_{\text{eff}}$ diagram, together with the evolutionary tracks. The presented models have initial masses $M_0 = 22, 24$, and 26 $M_\odot$, overshooting parameter $f = 0.02$ and 0.03, and initial rotation rates at ZAMS of 0.28 and 0.30 $\Omega_{\text{crit}}$. The corresponding values of the rotational frequency, $\nu_{\text{rot}}$, of the models are of the order of 0.06 day$^{-1}$; thus, the effects of rotation on stellar pulsation can be safely neglected because the spin parameter, $s = 2\nu_{\text{rot}}/\nu_{\text{puls}}$, is less than 1 for all but three frequencies.

Interestingly, the three lowest frequencies, 0.0431, 0.0726, and 0.1035 day$^{-1}$, show spacing of about 0.03 day$^{-1}$, which is half of the rotational frequencies of the models. The value of 0.03 day$^{-1}$ is close to the Rayleigh resolution limit, but there is also a possibility that these three peaks are a rotational split triplet with $\ell = 1$ because the Ledoux constant for high-order $g$-modes is about 0.5.

The pulsations were computed with the nonadiabatic code of Dziembowski (1977), and the results are presented for the two
representative models in Figure 8. The middle and bottom panels show the instability parameter, \( \eta \), as a function of the mode frequency, \( \nu \), for an evolved MS model with \( \epsilon = M_{\odot}^{22.88} \) (\( \epsilon = M_{\odot}^{240} \)), \( f = 0.03 \), \( \log L = \log 4.397 \), \( \log T_{\text{eff}} = \log 5.190 \) and a supergiant model with \( \epsilon = M_{\odot}^{24.47} \) (\( \epsilon = M_{\odot}^{260} \)), \( f = 0.02 \), \( \log L = \log 4.401 \), \( \log T_{\text{eff}} = \log 5.310 \), respectively.

The instability parameter, \( \eta \), measures the net energy gained by a mode during one pulsational cycle and is defined as (Stellingwerf 1978)

\[
\eta = \frac{W}{\int_0^R \left[ \frac{dW}{dr} \right] dr},
\]

where \( W \) is the global work integral. This definition gives normalization \( \eta \in [-1, 1] \), and the mode is excited if \( \eta > 0 \).

Here, we considered modes with the harmonic degree up to 4. Although the disk averaging factor, \( b_\ell \), drops significantly between \( \ell = 2 \) and \( \ell = 3 \), the geometrical effect in the light variation, \((\ell - 1)(\ell + 2)\), increases with \( \ell \) (Daszyńska-Daszkiewicz et al. 2006). Thus, it is justified to take into account modes higher than \( \ell = 2 \) for interpretation of space data.

The models presented in Figure 8 are confined in the error box of HD 163899 and have the N/C abundance as determined from observations. As one can see, the two oscillation spectra are dissimilar. In the case of the MS model, there are two distinctive global maxima of \( \eta(\nu) \): one related to the high-order g-modes and
The oscillation spectrum of the supergiant model looks quite different. In the low-frequency range, below about 1.1 day$^{-1}$, individual pulsational modes become unstable. The higher-frequency modes are all stable, in particular, the radial modes located at the low frequencies and the second one, at higher frequencies, related to the $p$- and mixed modes. The fundamental radial mode is unstable in this model and has a frequency of 1.44 day$^{-1}$. The total range of unstable modes is $[0.07, 1.44]$ day$^{-1}$.

**Figure 8.** Comparison of the observed frequency spectrum of HD 163899 (top panel; see Table 1 in Saio et al. 2006) and theoretical frequencies calculated for an MS model with a mass of $22.88 \, M_\odot$ (middle panel) and a supergiant model with a mass of $24.47 \, M_\odot$ (bottom panel). Modes with harmonic spherical degree up to $\ell = 4$ are shown.
are not excited. In our previous studies (Daszyńska-Daszkiewicz et al. 2013a), we obtained for lower-mass supergiant models the consecutive pattern of the local minima and maxima of \( \eta \) at low frequencies. This pattern was related to the partial trapping of pulsation modes. For more massive and rotating models, it seems to be not so clearly visible.

The main difference between the MS and supergiant models is the substantially higher number of unstable modes in the MS models. However, as one can see, none of the models can account for the whole range of the observed frequencies. The noteworthy feature of the observed spectrum is a rather well coverage of the frequency range \([0.04, 1.75]\) day\(^{-1}\) and a few single peaks above 1.9 day\(^{-1}\). The main problem with the models is the lack of unstable modes with the lowest and highest detected frequencies. In our next step, we will check whether the disagreement can be solved if the rotational splitting of pulsational frequencies is included.

### 4.2. Distribution of Pulsational Frequencies

For a more reliable comparison of the observed frequencies of HD 163899 with the theoretical ones, we calculated the frequency distributions predicted by the pulsational models and confronted them with the observed one. To this end, we took into account rotational splitting for all the unstable frequency modes \((\eta > 0)\) according to the formula

\[
\nu = \nu_0 + m(1 - C_{nf})\nu_{rot},
\]

where \( m \) is the azimuthal order, \( C_{nf} \) is the Ledoux constant resulting from pulsational models, and \( \nu_{rot} \) is the rotational frequency. The value of \( \nu_{rot} \) was determined for the rotational velocity, \( V_{rot} \), and the radius, \( R \), of a given model. In all cases \( \nu_{rot} \) was of the order of 0.06 day\(^{-1}\).

From all unstable modes, we selected those that had theoretical photometric amplitudes in the Johnson \( V \) passband higher than the minimum observed amplitudes in the \textit{MOST} data. The value of this minimum amplitude is \( A_{\min} = 0.39 \) mmag. We computed the photometric amplitudes according to the linear formula of Daszyńska-Daszkiewicz et al. (2002). This formula contains one pulsational parameter that is not provided in the framework of the linear theory. This is the intrinsic mode amplitude, \( \varepsilon \), defined by the local radial displacement of the photosphere

\[
\delta r(R, \theta, \phi) = R \text{Re}\{\varepsilon Y^m_l e^{-ik_{rot} t}\},
\]

where \( Y^m_l \) is spherical harmonic and \( \text{Re} \) means the real part. Other symbols have their usual meanings.

We estimated the maximum value of \( \varepsilon \) taking the highest observed amplitude from the \textit{MOST} data, which is \( A \approx 4 \) mmag, at the lowest frequency \( \nu = 0.0431 \) day\(^{-1}\) and assuming that it corresponds to the dipole axisymmetric mode. The value of the inclination angle, \( i \), was obtained from the equation \( i = \arcsin(65/V_{rot}) \). We got \( \varepsilon_{\max} \) of about 0.01 independently of the model. For each unstable mode, we randomly drew the value of \( \varepsilon \) from the range \([0, 0.01]\). Then, for the obtained inclination angles, we calculated the photometric amplitudes in the Johnson \( V \) passband. We chose this passband because for the considered range of \((T_{eff}, \log g)\) the pulsational amplitude in this band is close to the \textit{MOST} amplitude.

In Figure 9, we show the theoretical values of the photometric amplitudes as a function of mode frequency for the MS model (left panel) and for the supergiant model (right panel). These are the same models used in Figure 8. As one can see, the criterion \( A_V > 0.39 \) mmag cuts many frequencies, but still a large number of them remain. This can be better estimated if the results are depicted as histograms for the frequencies with the photometric amplitudes \( A_V > 0.39 \) mmag. In Figure 10, we show the histograms for the observed frequencies (top left panel) and for the theoretical ones of the two MS models differing in mass (top right and bottom left panels) and a supergiant model (bottom right panel). The models in the right panels are the same as considered in Figure 8.

As expected, the rotationally split multiplets populate the gap between the two maxima of \( \eta(\nu) \) in Figure 8 of the MS pulsational models and fill the lowest frequency range in both models. First of all, we have a much higher number of theoretical modes than observed. This is not a surprise, as the result recalls the old problem in the linear theory of stellar pulsations, namely, we observe significantly fewer modes than we can predict. An example from the \textit{MOST} observations is HD 163830, the SPB star with 20 frequencies detected comparing to hundreds of unstable theoretical modes (Aerts et al. 2006).

Moreover, a sudden increase of frequencies toward the lower frequencies occurs for both MS and supergiant models. Thus, even reducing the number of frequencies, we cannot reproduce the shape of their distribution. The highest observed frequencies also pose a challenge; there are seven peaks above \( \nu = 1.8 \) day\(^{-1}\).

There can be a few reasons for this. First, we do not know the mode selection mechanism, and our simple random selection of the values of \( \varepsilon \) can be very likely incorrect. Besides, our crude approximation about the value of the intrinsic mode amplitude, \( \varepsilon \), can be incorrect; a frequency peak with the highest photometric amplitude does not necessarily have the highest intrinsic amplitude. Second, maybe there are still some inaccuracies in computations of stellar structures resulting from uncertainties in calibrations of the free parameters describing, e.g., convection or efficiency of rotational mixing. The other source of uncertainties arises from the opacity data. It is well known that models with the standard opacities cannot account for pulsations in some early B-type MS stars (Daszyńska-Daszkiewicz et al. 2016). It is also possible that some other mechanism, like stochastic processes, is responsible for the light variations of HD 163899, e.g., subsurface convection. Finally, the frequencies may originate from a lower-mass companion if the star is a binary.

### 5. Conclusions

The goal of this paper was to revise the evolutionary status of HD 163899 using the new determinations of its basic stellar parameters. For the first time, it was possible to determine \( \log T_{eff}, \log L/M, \) and rotational velocity of the star with high precision. The new values of these parameters suggest that the star is more luminous, hotter, and more massive than was previously thought. In all former papers treating HD 163899 it was assumed that the star was beyond the MS, either during shell hydrogen burning or during core helium burning on the blue loop. That was a valid premise because of the available determination of the spectral type and also because the expected mass of the stars was lower. The most obvious result...
Figure 9. Theoretical photometric amplitudes as a function of the mode frequency for MS (left panel) and supergiant (right panel) models with initial masses of 24 and 26 $M_{\odot}$, respectively (the same models as in Figure 8). The horizontal line, located at $A_V = 0.39$ mmag, depicts the lowest amplitude detected in the MOST data for HD 163899 with $S/N = 3.9$.

Figure 10. Histogram for the observed frequencies of HD 163899 (top left panel) and the theoretical histograms calculated for MS models with initial masses of 24 and 22 $M_{\odot}$ (top right and bottom left panels) and for a supergiant model (bottom right panel) with initial mass of 26 $M_{\odot}$. Unstable pulsational modes with degrees $\ell = 0–4$ and $A_V > 0.39$ mmag were considered. The modes were rotationally split according to the value of $\nu_{\text{rot}}$ given in each panel.
related directly to the current higher mass estimations is the presence of MS models in the error box of HD 163899. The MS models in the error box are rather evolved and close to TAMS. They also require a mixing mechanism that leads to broadening of the MS. We found that a higher value of convective overshooting parameter ($f > 0.02$) is needed to obtain MS models in the error box. However, rotation is a key ingredient in our models. Not only is it necessary to reproduce the observed rate, but the rotational mixing also greatly helps to deal with various problems related to modeling of massive stars.

The analysis of the HARPS spectra led to the conclusion that HD 163899 is a nitrogen-rich star. The measured nitrogen excess is a clear manifestation of efficient mixing in this star, which we have successfully reproduced by invoking rotational mixing. The observed value of the equivalent ratios, $W(N)/W(C)$ and $W(N)/W(O)$, allowed us to estimate the rotation to be about 25%–30% of the critical value of the angular velocity.

We were not able to reproduce the distribution of the observed frequencies with standard pulsational models. Possibly our simple approach to modeling the frequency distribution is not adequate and does not compensate for the lack of a mode selection mechanism. Moreover, there are still many uncertainties in evolutionary models concerning mixing processes and opacity data. Consequently, pulsational models can be incorrect. Other scenarios involve stochastic processes that can cause the light variations of HD 163899. Finally, some features in the spectra may be evidence of a binarity. Given a high percentage of binaries among B- and O-type stars (e.g., Sana et al. 2012; de Mink et al. 2014), it is very likely that this is the case. This fact could explain also the observed frequencies if they would come from a lower-mass companion.

To decide which explanation is true or more likely, further observations and analysis of HD 163899 are needed. In particular, time-series spectroscopy could bring a breakthrough in the interpretation of the variability of the star. Given the fact that MS models can also represent HD 163899, this star may not be the prototype of the SPBsg class, which encourages additional searches in the ongoing or upcoming space missions such as K2, TESS, and PLATO 2.0.

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