GPDs of the nucleons and elastic scattering at high energies

O. V. Selyugin

BLTP, Joint Institute for Nuclear Research, 141980 Dubna, Moscow region, Russia

Abstract. Taking into account the electromagnetic and gravitational form factors, calculated from a new set of $t$-dependent GPDs, a new model is built. The real part of the hadronic amplitude is determined only through complex $s$. In the framework of this model the quantitative description of all existing experimental data at $52.8 \leq \sqrt{s} \leq 1960$ GeV, including the Coulomb range and large momentum transfers $(0.0008 \leq |t| \leq 9.75$ GeV$^2$), is obtained with only 3 fitting high energy parameters. The comparison with the preliminary data of the TOTEM Collaboration at an energy of 7 TeV is made.

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1 Introduction

The dynamics of strong interactions finds its most complete representation in elastic scattering at small angles. Only in this region of interactions we can measure the basic properties of the non-perturbative strong interaction which defines the hadron structure: the total cross section, the slope of the diffraction peak and the parameter $\rho(s,t)$. Their values are connected, on the one hand, with the large-scale structure of hadrons and, on the other hand, with the first principles which lead to the theorems on the behavior of the scattering amplitudes at asymptotic energies [1-2].

There are indeed many different models for the description of hadron elastic scattering at small angles [3-4]. They lead to the different predictions for the structure of the scattering amplitude at asymptotic energies, where the diffraction processes can display complicated features [5]. This concerns especially the asymptotic unitarity bound connected with the Black Disk Limit (BDL) [6-7]. It was assumed in the Chow-Yang model [7] that the hadron interaction is proportional to the overlapping of the matter distribution of the hadrons and in the Wu and Yang [7] that the matter distribution is proportional to the charge distribution of the hadron. Then many models used the electromagnetic form factors of hadron, but, in most part they change its form to describe the experimental data, as was made in the famous Bourrely-Soffer-Wu model [9]. The parameters of the obtained form factor are determined by the fit of the differential cross sections.

Now we present a model, based on the assumption that the hadron interaction is sensitive to the generalized parton distributions (GPDs) whose moments can be represented over momentum transfer in the form of two different distributions: charge and matter, separately. Hence, this model uses the exact electromagnetic and gravitational form factors determined by one function - generalized parton distributions (GPDs). So both form factors are independent of the fitting procedure of the differential cross sections. Note that the form of the GPDs is determined, on the one hand, by the deep-inelastic processes and, on the other hand, by the measure of the electromagnetic form factor from the electron-nucleon elastic scattering. Hence, the form of the electromagnetic form factor (first momentum of GPDs) determines the form of the second form factor (second momentum of GPDs). This picture is supported by the good description of the experimental data in the Coulomb-hadron interference region and large momentum transfer at high energies by one amplitude with a few free parameters. The impact of the hard pomeron contribution to the elastic differential cross sections is very important for understanding of the properties of the QCD in the non-perturbative regime [10]. Note that the real part of the hard pomeron is essentially larger than the real part of the soft pomeron. Now in [11] it is suggested that such a contribution can be explained by the preliminary result of the TOTEM Collaboration on the elastic proton-proton differential cross sections. In our model, the real part of the hadronic amplitude is determined only through complex $s$ satisfying the cross symmetric relation. In the framework of this model, the quantitative description of all existing experimental data at $52.8 \leq \sqrt{s} \leq 1960$ GeV, including the Coulomb range and large momentum transfers $0.0008 \leq |t| \leq 9.75$ GeV$^2$, is obtained with only 3 fitting high energy parameters. The comparison of the predictions of the model at 7 TeV and preliminary data of the TOTEM collaboration are shown to coincide well. There is some small place, especially in
the region of the diffraction dip, for the small correction contributions which are determined by the odderon, and possibly the spin-dependent part of the scattering amplitude which gives a small contribution at large momentum transfer. In the framework of the model, only the Born term of the scattering amplitude is introduced. Then the whole scattering amplitude is obtained as a result of the unitarization procedure of the hadron Born term that is then summed with the Coulomb term. The Coulomb-hadron interference phase is also taken into account. The essential moment of the model is that both parts of the Born term of the scattering amplitude have the positive sign, and the diffraction structure is determined by the unitarization procedure.

The electromagnetic and hadronic parts of the elastic scattering amplitudes used in the model are presented in the second and third sections. In the fourth section, we introduce the hadron form factors obtained from the first and second momenta of GPDs. Our fitting procedure and the description of the high energy differential cross sections of the proton-proton and proton-antiproton scattering are presented in the fifth and sixth sections. Also, we stretch our model on 7 TeV and compare the model calculations with the preliminary data of the TOTEM Collaboration. Then, the model calculations are compared with the experimental data at low energies $\sqrt{s} = 30$ GeV. In the seventh section and in the conclusion, we show and discuss the model calculations for the total cross sections and the value of $\rho(s,t)$ obtained in the framework of the model.

2 Electromagnetic part of the hadron scattering amplitude

The differential cross sections of nucleon-nucleon elastic scattering can be written as the sum of different helicity amplitudes:

$$\frac{d\sigma}{dt} = \frac{2\pi}{s^2}[|\Phi_1|^2 + |\Phi_2|^2 + |\Phi_3|^2 + |\Phi_4|^2 + 4|\Phi_5|^2].$$

The total helicity amplitudes can be written as $\Phi_1(s,t) = F_1^h(s,t) + F_1^{em}(s,t)e^{i\psi(s,t)}$, where $F_1^h(s,t)$ comes from the strong interactions, $F_1^{em}(s,t)$ from the electromagnetic interactions and $\varphi(s,t)$ is the interference phase factor between the electromagnetic and strong interactions. The hadron part of the amplitude with spin-flip is neglected in this approximation, as usual at high energy.

The electromagnetic amplitude can be calculated in the framework of QED. In the high energy approximation, it can be obtained for the spin-non-flip amplitudes:

$$F_1^{em}(t) = \alpha f_1^2(t) \frac{s - 2m^2}{t}; \quad F_3^{em}(t) = F_1^{em};$$

and for spin-flip amplitudes:

$$F_2^{em}(t) = \alpha f_2^2(t) \frac{4m^2}{4m^2}; \quad F_4^{em}(t) = -F_2^{em}(t), \quad F_5^{em}(t) = \alpha \frac{s}{2m\sqrt{|t|}} f_1(t) f_2(t),$$

where the form factors are:

$$f_1(t) = \frac{4m^2 - (1 + k) t}{4m^2 - t} Gd(t); \quad f_2(t) = \frac{4m^2 k}{4m^2 - t} Gd(t);$$

with $k$ relative to the anomalous magnetic moment, and $Gd(t)$ has the conventional dipole form

$$Gd(t) = \frac{1}{(1 - t/0.71)^2}.$$ 

3 Main hadronic amplitude

The model is based on the representation that at high energies a hadron interaction in the non-perturbative regime is determined by the reggenized-gluon exchange. The cross-even part of this amplitude can have 2 non-perturbative parts, possible standard pomeron - $P_{2np}$ and cross-even part of the 3 non-perturbative gluons ($P_{3np}$). The interaction of these two objects is proportional to two different form factors of the hadron. This is the main assumption of the model. Of course, we can not insist on the origin of the second term of the scattering amplitude. It can be a different nature. However, in any case, it has the cross-even properties and positive sign. The second important assumption is that we chose the slope of the second term 4 time smaller a slope of the first term, by analogy with the two pomeron cut. Both terms have the same intercept.

The form factors are determined by the General parton distributions of the hadron (GPDs). The first form factor, corresponding to the first momentum of GPDs is the standard electromagnetic form factor - $G(t)$. The second form factor is determined by the second momentum of GPDs - $A(t)$. The parameters and $t$-dependence of the GPDs are determined by the standard parton distribution functions, so by the experimental data on the deep inelastic scattering, and by the experimental data for the electromagnetic form factors (see [13]).

Hence, the Born term of the elastic hadron amplitude can be written as

$$F_k^{Born}(s,t) = h_1 G^2(t) F_a(s,t) \left(1 + \frac{r_1}{s^{0.5}}\right) + h_2 A^2(t) F_b(s,t) \left(1 + \frac{r_2}{s^{0.5}}\right),$$

where $F_a(s,t)$ and $F_b(s,t)$ has the standard Regge form

$$F_a(s,t) = \hat{s}^{\xi_1} e^{B(s)/t}; \quad F_b(s,t) = \hat{s}^{\xi_1} e^{B(s)/t},$$

with $G(t) = G_E(t)$ is the Sachs electric form factor relative to the first moment of GPDs and $A(t)$ relative to the second moment of GPDs.

$$G(t) = \frac{L_1^4}{(L_1^2 - t)^2} \frac{4m^2 - (1 + k) t}{4m^2 - t} \quad \text{(8)}$$

$$A(t) = \frac{L_2^4}{(L_2^2 - t)^2}. \quad \text{(9)}$$
with the parameters: \( L_1^2 = 0.71 \text{ GeV}^2; \ L_2^2 = 2 \text{ GeV}^2 \).

\[
\hat{s} = s \ e^{-i \pi/2}/s_0; \quad s_0 = 1 \text{ GeV}^2. \tag{10}
\]

The slope of the scattering amplitude has the standard logarithmic dependence on the energy.

\[
B(s) = \alpha' \ln(\hat{s}). \tag{11}
\]

with \( \alpha' = 0.24 \text{GeV}^{-2} \).

The final elastic hadron scattering amplitude is obtained after unitarization of the Born term. So in first, we have to calculate the eikonal phase

\[
\chi(s, b) = \frac{1}{2\pi} \int d^2q \ e^{ib \cdot q} \ F_h^{\text{Born}}(s, q^2), \tag{12}
\]

and then obtain the final hadron scattering amplitude

\[
F_h(s, t) = is \int b \ J_0(bq) \ G(s, b) \ db. \tag{13}
\]

\[
G(s, b) = 1 - \exp[-\chi(s, b)]. \tag{14}
\]

All these calculations are carried out by the FORTRAN program.

### 4 Hadron form factors

As was mentioned above, all the form factors are obtained from the GPDs of the nucleon [16]. The electromagnetic form factors can be represented as first moments of GPDs

\[
F_h^q(t) = \int_0^1 \ dx \ \mathcal{H}^q(x, t); \quad F_L^q(t) = \int_0^1 \ dx \ \mathcal{E}^q(x, t), \tag{15}
\]

following from the sum rules [18][19].

Recently, there were many different proposals for the \( t \)-dependence of GPDs. We introduced a simple form for this \( t \)-dependence [16] based on the original Gaussian form corresponding to that of the wave function of the hadron. It satisfies the conditions of non-factorization, introduced by Radyushkin, and the Burkhardt condition on the power of \( (1 - x)^n \) in the exponential form of the \( t \)-dependence. With this simple form we obtained a good description of the proton electromagnetic Sachs form factors. Using the isotopic invariance we obtained good descriptions of the neutron Sachs form factors without changing any parameters [16].

We shall use this model of GPDs to obtain the second momentum form factor of the nucleon. Taking instead of the electromagnetic current \( J^\mu \) the energy-momentum tensor \( T_{\mu\nu} \) together with a model of quark GPDs, one can obtain the gravitational form factor of fermions

\[
\int_{-1}^{1} dx \ \left[ H(x, \Delta^2, \xi) \pm E(x, \Delta^2, \xi) \right] = A_q(\Delta^2) \pm B_q(\Delta^2). \tag{16}
\]

For \( \xi = 0 \) one has

\[
\int_{0}^{1} dx \ [ \mathcal{H}(x, t) \pm E(x, t)] = A_q(t) \pm B_q(t). \tag{17}
\]

The integration of the second momentum of GPDs over \( x \) gave the momentum-transfer representation of the form factor (see Fig.1). We approximate this by the dipole form

\[
A(t) = L_2^2/(L_2^2 - t)^2. \tag{18}
\]

with the parameter \( L_2^2 = 2.0 \text{ GeV}^2 \).

### 5 Fitting procedure

The model has only 3 high energy fitting parameters and 2 low energy parameters, which reflect some small contribution coming from the different low energy terms. (see Table 1). We take all existing experimental data in the energy range \( 52 \leq \sqrt{s} \leq 1960 \text{ GeV} \) and the region of the momentum transfer \( 0.0008 \leq -t \leq 9.75 \text{ GeV}^2 \) of the elastic differential cross sections of proton-proton and proton-antiproton data [20][21]. So we include the whole Coulomb-hadron interference region where the experimental errors are remarkably small. We do not include the data on total cross sections \( \sigma_{\text{tot}}(s) \) and \( \rho(s) \), as their values were obtained from the differential cross sections especially in the Coulomb-hadron interference region. Including such data decreases \( \chi^2 \). We also do not include the interpolated and extrapolated data of Amaldi [22].

In the fitting procedure we calculate the minimum in \( \sum_{i=1}^{N} \chi_i^2 \) related with the statistical errors \( \sigma_i \). The systematical errors are taken into account by the additional normalization coefficient \( n_k \) for the \( k \) series (the experiment) of the experimental data

\[
\chi^2 = \sum_{i=1}^{N} \frac{n_k \ E_i(s, t) - T_i(s, t)}{\sigma_i^2(s, t)}. \tag{19}
\]
where $T_i(s, t)$ are the theory predictions and $n_k E_i(s, t)$ are the data points allowed to shift by the systematical error of the $k$-experiment (see, for example [23,24].

In the region of the small momentum transfer the systematic errors are of an order of 2%÷5%. For most part the additional normalization are in the region 0.95÷1.05. At large momentum transfer the order of the systematical errors is 10%÷20%. In this case, the additional normalization is situated in the region 0.8÷1.2.

For the non-normalized experimental data of the UA4/2 Collaboration [25], which have very small statistical errors, we take the normalization determined in [26]. Our correction normalization is obtained from the fitting procedure in this case $n_{UA42} = 0.95$.

As a result, one obtains $\sum \chi^2/N \simeq 1.8$ where $N = 975$ is the number of experimental points. Of course, if one sums the systematic and statistical errors, the $\sum \chi^2/N$ decreases, to 1.4. Note that the parameters of the model are energy-independent. The energy dependence of the scattering amplitude is determined only by the single intercept and the logarithmic dependence on $s$ of the slope.

Note that there are some separate points ( $n = 17$) at the different energies and momentum transfer which give $\sum_{n=1}^{17} \chi^2_n = 260$. However, we do not remove such points.

### 6 Description of the differential cross sections

The differential cross sections for proton-proton elastic scattering at $\sqrt{s} = 52.8$ GeV are presented in Fig. 2(left panel) and 3(top panel). At this energy there are experimental data at small (beginning at $-t = 0.001$ GeV$^2$) and large (up to $-t = 10$ GeV$^2$) momentum transfers. The model reproduces both regions and provides a qualitative description of the dip region at $-t \approx 1.4$ GeV$^2$, for $\sqrt{s} = 53$ GeV$^2$ and for $\sqrt{s} = 62.1$ GeV$^2$ (Fig.3(top) and Fig.4(left panels)).

Now let us examine the proton-antiproton differential cross sections (see Fig.2(right panel)). In this case at small momentum transfer the Coulomb-hadron inter-

![Fig. 2. $d\sigma/dt$ at $\sqrt{s} = 52.8$ GeV and at small $t$ for $pp$ (left) and for $p\bar{p}$ (right).](image1)

![Fig. 3. $d\sigma/dt$ at $\sqrt{s} = 52.8$ GeV at large $|t|$ for $pp$ (top) and for $p\bar{p}$(bottom panel)](image2)
The interference term plays an important role and has the opposite sign. The model describes the experimental data well. Slightly worse than $pp$ is the description of $p\bar{p}$ of differential cross sections at $\sqrt{s} = 53$ GeV, especially in the diffraction minimum (Fig. 3(bottom panel)).

Maybe, this shows an additional odderon contribution. Note that at $\sqrt{s} = 62.2$ GeV for $p\bar{p}$ scattering the description of the differential cross sections is essentially better (Fig. 4).

In Fig. 5, the description of the proton-antiproton scattering at $\sqrt{s} = 541$ GeV and at $\sqrt{s} = 1800$ GeV is shown. In this cases, the Coulomb-hadron interference term is large, especially at $\sqrt{s} = 541$ GeV as $t$ is very small. The good description of the experimental data shows that the energy dependence of the real part of scattering amplitude obtained in the model corresponds to the real physical situation.

In Fig. 6, $d\sigma/dt$ for $p\bar{p}$ elastic scattering at large $t$, at $\sqrt{s} = 546$ GeV (top) and $\sqrt{s} = 630$ GeV (bottom)
Figures 6 and 7 show the description of the experimental data at larger momentum transfers for $\sqrt{s} = 546$ GeV$^2$ and $\sqrt{s} = 630$ GeV$^2$ and for Tevatron energies $\sqrt{s} = 1800$ GeV and $\sqrt{s} = 1960$ GeV. It is clear that the model leads to a good description of these data in the region of the diffraction minimum without taking into account the odderon contribution. Hence, it is shown that very likely the intercept of the odderon is near 1.

On basis of this fit of the experimental data at $52.8 \leq \sqrt{s} \leq 1960$ GeV and $0.0008 \leq |t| \leq 9.75$ GeV$^2$ we obtained the fitting parameters (see Table 1). Taking into account these values of the parameters we extend the scope of the model and calculate the differential cross sections at 7 TeV for $pp$ elastic scattering. In Fig. 8, the comparison of the model calculations with the parameters, based at the fit of the existing experimental data at $52.8 \leq \sqrt{s} \leq 1960$ GeV, with the preliminary data of the TOTEM Collaboration are shown. Except the size of the diffraction minimum the coincidences are remarkable. Of course, if we include in the model some different correction terms, like odderon, the value of the fitting parameters of the model will be slightly change. However, we think that the basic properties of the model will not change in future.

Now let us see how it can be extend the scope of the model on a low energy. The calculation of the model of $d\sigma/dt$ for $\bar{p}p$ and $pp$ elastic scattering at $\sqrt{s} = 30.6$ GeV are compared with the experimental data on Fig.9 and Fig.10. There is a very good description at small momentum transfer for both the reactions - $pp$ and $\bar{p}p$. At large $t$ the model reproduces the differential cross sections only qualitatively. The position of the diffraction minimum corresponds to the experimental data. However, the form of the diffraction minimum obviously requires some additional small correction terms, possibly by the odderon con-

![Fig. 8. The comparison of the model calculations with the parameters, based on the fit of the experimental data at $52.8 \leq \sqrt{s} \leq 1960$ GeV, with the preliminary data of the TOTEM Collaboration at $\sqrt{s} = 7$ TeV.](image)

![Fig. 9. The model predictions of $d\sigma/dt$ for $\bar{p}p$ elastic scattering at $\sqrt{s} = 30.6$ GeV.](image)

![Fig. 7. $d\sigma/dt$ for $\bar{p}p$ elastic scattering at large $t$, at $\sqrt{s} = 1800$ GeV (top) and $\sqrt{s} = 1960$ GeV (bottom panel).](image)
tributions. This situation repeated the problem of describing the form of the diffraction minimum at $\sqrt{s} = 53$ GeV for $p\bar{p}$ elastic scattering.

7 Energy dependence of $\rho(s, t)$ and $\sigma_{\text{tot}}(s)$

The ratio of the real part to the imaginary part of the elastic scattering hadronic amplitude

$$\rho(s, t) = \frac{\text{Re}F_h(s, t)}{\text{Im}F_h(s, t)}$$

is very important as it reflects the $t$-dependence of both parts of the scattering amplitude, which are connected one to the other through the integral dispersion relations. The validity of this relation can be checked at LHC energies. The deviation can point out the existence of a fundamental length at TeV energies [27,28]. Usually, the value of $\rho(s, t = 0)$ is assumed to be small and to vary little with $t$: $\rho(s, t) \approx 0.14$. The differential cross sections at small momentum transfer $|t| \leq 0.05$ GeV$^2$, so-called Coulomb-hadron interference region, are determined by the interference of the Coulomb amplitude with the real part of the hadron amplitude. Hence, the $s$ and $t$ dependence of the real part of the hadron amplitude, which is reflected in $\rho(s, t)$, will determine the form of the differential cross sections.

In the model, the real part of $F_h(s, t)$ is determined only by the complex cross symmetric form of energy $\hat{s} = 3.6$ GeV.
The hadronic amplitude. A good description of the differential dependence of the differential cross sections is determined after fitting with the new data of the proton-proton scattering at LHC energies.

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