Re-analysis of the $D^*D\pi$ coupling in the light-cone QCD sum rule

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Abstract

The recent measurement from the CLEO experiment presents the $DD^*\pi$ coupling, $17.9\pm0.3\pm1.9$. This value is much larger than any of QCD sum rule predictions available in literature. We report that, with a relevant treatment of the continuum subtraction as well as with the asymptotic form of the twist-2 pion wave function, the light-cone QCD sum rule can provide the coupling comparable to the experimental value. The stability of the resulting sum rule becomes much better with these corrections.

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I. INTRODUCTION

Recently, the CLEO collaboration [1] presents a new measurement of the $D^*D\pi$ coupling, $g_{D^*D\pi} = 17.9 \pm 0.3 \pm 1.9$. Though this value is somewhat consistent with quark model predictions [2] and lattice calculations [3], it is much larger than any of QCD sum rule predictions, which gives some skepticism in accepting the various QCD sum rule results. In particular, the conventional QCD sum rules relying on the short-distance expansions [4, 5, 6, 7] consistently restrict the coupling $g_{D^*D\pi} \leq 10$. In these sum rules, QCD inputs are well-constrained by the low-energy theorem. Thus, the current disagreement may due to the fact that various pion matrix elements appearing in the conventional sum rules may not converge fast enough in the short-distance expansions. One may need to do a partial summation to all order of those matrix elements.

The other method is to construct QCD sum rules from three-point function [8], which leads to the coupling around 6, again much smaller than the experimental value. Its recent extension [9] using exponential type parametrization for the form factor seems to give a much larger value around 15. However, it should be remembered that the operator product expansion (OPE) of a three-point function leads to an unphysical behavior at high momentum transfers [10], which in fact was one of the motivation for constructing light-cone QCD sum rules (LCQSR) relying on the expansions along the light-cone. Also, LCQSR may improve the conventional QCD sum rules by having the pion wave functions that encode the partial summation to all order of the pion matrix elements. However, LCQSR [11, 12] yields the $D^*D\pi$ coupling only about 12, still smaller than the experimental value. Thus, as far as the $D^*D\pi$ coupling is concerned, all the sum rules do not provide the experimental value. Furthermore as quark model and lattice calculations [2, 3] agree with the experiment, it is important to re-investigate the existing sum rules and look for a way to reach an agreement with them.

One crucial input in the prediction of LCQSR is the twist-2 pion wave function at the middle point $\varphi_\pi(0.5)$. Refs.[11, 12] in their sum rules use $\varphi_\pi(0.5) \sim 1.2$ obtained by comparing two different Lorentz structures of the light-cone sum rule for the pion-nucleon coupling [13]. However, as the pion-nucleon sum rule has a strong dependence on the Lorentz structure [14] considered, $\varphi_\pi(0.5) \sim 1.2$ is questionable. Thus, one may attribute the current discrepancy in the $D^*D\pi$ coupling to the uncertainty of $\varphi_\pi(0.5)$. Indeed, there are some suggestions that
the twist-2 pion wave function should be of the asymptotic form whose value at the middle point is \( \varphi_\pi(0.5) \sim 1.5 \) \[15, 16\]. As we will discuss below, this new input only increases the \( D^*D\pi \) coupling by 9 \%, still not enough to reproduce the experimental value. Certainly, the existing sum rules need further improvements.

In this letter, we suggest one possibility to improve the existing LCQSR. Specifically, we show that the continuum subtraction in the existing LCQSR is mathematically ill-defined. A similar suggestion was reported in Refs. \[17, 18, 19\]. In particular, we demonstrate that the OPE of the form \( \int_0^1 du/[m_c^2 - up_1^2 - (1-u)p_2^2] \) should not entail the continuum subtraction if one strictly follows the QCD duality assumption in constructing the continuum contributions. This correction combined with the asymptotic twist-2 pion wave function leads to the \( D^*D\pi \) coupling comparable to its experimental value. The resulting sum rule is stable against the variation of the Borel mass.

II. THE CONTINUUM CONSTRUCTION

In this section, we demonstrate a possible modification in obtaining the continuum subtraction in the existing LCQSR \[11, 12\]. The leading OPE in the existing LCQSR takes the form

\[
\int_0^1 \frac{du}{m_c^2 - up_1^2 - (1-u)p_2^2}
\]

where \( m_c \) the charm quark mass and \( p_1, p_2 \) are the two momenta associated with the coupling vertex. The leading OPE in Refs. \[11, 12\] also contains the pion wave function in the numerator of the integral but here we suppress that for a mathematical simplicity. The main issue regarding the continuum subtraction is not affected by this simplicity.

It was claimed in Ref. \[11, 12\] that, after the continuum subtraction and the double Borel transformation, the OPE, Eq.(1), appears in the final sum rule as the factor

\[
M^2(e^{-m_c^2/M^2} - e^{-S_0/M^2})
\]

where \( M^2 \) is the reduced mass of the two Borel masses associated with the double Borel transformations. We will show that the continuum subtraction factor, \( M^2e^{-S_0/M^2} \), comes from a mathematically spurious term and it should not be a part of the final sum rule.
To determine the continuum subtraction for the given OPE, one needs to determine the spectral density $\rho(s_1, s_2)$ firstly from the double dispersion relation,

$$
\int_0^1 \frac{du}{m_c^2 - u p_1^2 - (1 - u) p_2^2} = \int_0^\infty ds_1 \int_0^\infty ds_2 \frac{\rho(s_1, s_2)}{(s_1 - p_1^2)(s_2 - p_2^2)}.
$$

(3)

Note that the lower boundaries of the integrals in the right-hand side should be chosen such a way that the entire region of $\rho(s_1, s_2) \neq 0$ is included. According to the prescription given in Ref.[11], the spectral density is given by

$$
\rho(s_1, s_2) = \delta(s_1 - s_2) \theta(s_1 - m_c^2).
$$

(4)

An alternative way to obtain this spectral density is to take the imaginary part of Eq.(1) with respect to the two momenta, $p_1^2$ and $p_2^2$.

A common practice in LCQSR is to (1) put this spectral density back to the double dispersion integral, (2) restrict the integral below the continuum threshold $S_0$ (according to QCD duality), $\int_0^\infty ds_1 \int_0^\infty ds_2 \rightarrow \int_0^{S_0} ds_1 \int_0^{S_0} ds_2$, and (3) take the double Borel transformations. This procedure as advocated by Belyaev et.al. [11] precisely yields the simple prescription given in Eq. (2). We want to point out that the second step is dangerous as it yields a spurious contribution. To show this, let us see how the spectral density Eq.(4) reproduces the OPE Eq.(1) within the double dispersion relation Eq.(3). We put the spectral density Eq.(4) in the double dispersion relation and obtain

$$
\int_0^\infty ds_1 \int_0^\infty ds_2 \frac{\rho(s_1, s_2)}{(s_1 - p_1^2)(s_2 - p_2^2)} = \int_0^1 du \int_0^\infty ds_1 \frac{\theta(s_1 - m_c^2)}{(s_1 - u p_1^2 - (1 - u) p_2^2)^2}.
$$

(5)

Note, we have used the Feynman parameterization to get an integral over the Feynman parameter $u$. We then perform the integration by part to separate into the two terms

$$
\int_0^1 du \int_0^\infty ds_1 \frac{\delta(s_1 - m_c^2)}{s_1 - u p_1^2 - (1 - u) p_2^2} - \int_0^1 du \frac{\theta(s_1 - m_c^2)}{s_1 - u p_1^2 - (1 - u) p_2^2} \bigg|_{s_1=\infty}^{s_1=0}.
$$

One can see that the first term precisely yields the anticipated OPE while the second term, though vanishes under the present boundaries, is an unnecessary spurious term. When QCD duality is applied (i.e. restricting the integral below the continuum threshold $S_0$), it is mathematically sensible to apply to the first term only. Any contribution from the second term is ill-defined as the second term is mathematically spurious. However one can show that the continuum subtraction factor $M^2 e^{-s_0/M^2}$ comes from the spurious second term. When the integral interval in Eq. (3) is switched to $0 \sim S_0$, the second term becomes $\int_0^1 du / [S_0 -$
up^2 - (1 - u)p_2^2$, which no longer vanishes. Then under the double Borel transformations the second term precisely yields the continuum subtraction factor $M^2 e^{-S_0/M^2}$. Therefore, the continuum subtraction factor $M^2 e^{-S_0/M^2}$ should be dropped in the final expression of the sum rule as it is from the mathematically spurious term.

Once the factor $M^2 e^{-S_0/M^2}$ is dropped, then the resulting sum rule does not depend on the continuum threshold, indicating that there is no continuum contribution. Intuitively, absence of the continuum contribution may seem strange because the current in the correlation function can couple to higher resonances as well as the lowest resonance. But one can show that $\alpha_s$ corrections to the perturbative part can give a continuum contribution within our prescription as the corrections are logarithmic functions of $s_1$ and $s_2$ [20]. Therefore one can pick up small continuum contribution [at an order $O(\alpha_s)$] and, in the present calculation without $\alpha_s$ correction, having no continuum contribution is not against the intuitive picture.

### III. RE-CALCULATION OF THE $D^*D\pi$ COUPLING

Having suggested a modification in the previous LCQSR, we now re-analyse the sum rule for the $D^*D\pi$ coupling. We take the sum rule formula for the $D^*D\pi$ coupling from Ref.[11] and obtain the solid curve in Fig. 1. This is the same curve that was presented in Ref.[11]. Based on this curve, Ref.[11] obtained $f_D f_{D^*} g_{D^*D\pi} = 0.51$ GeV$^2$, which yields $g_{D^*D\pi} = 12.5$ if we use $f_D = 170$ MeV and $f_{D^*} = 240$ MeV obtained from two-point vacuum sum rules. The dashed curve is obtained when we take the asymptotic twist-2 pion wave function, $\varphi_\pi(0.5) = 1.5$, while keeping all other parameters fixed as in Ref.[11]. This correction increases the coupling only by 9%. Thus, with this correction only, LCQSR cannot still reproduce the experimental value of the coupling, 17.9.

However the correction coming from the continuum factor is substantial. The dot-dashed curve is obtained when the continuum subtraction factor $M^2 e^{-S_0/M^2}$ is taken out in the sum rule according to our suggestion in Sec.II. One clearly sees that this curve can provide a much larger coupling. Furthermore, the curve has a minimum around which the variation with respect to the Borel mass is minimized. This indicates that, around the minimum, the sum rule is well saturated by the OPE terms included in the calculation. For a qualitative estimate, instead of doing a detail analysis, we take the minimum of the curve to calculate the coupling. The minimum gives $f_D f_{D^*} g_{D^*D\pi} = 0.66$ GeV$^2$, which in turn yields the coupling
$g_{D^*D\pi} = 16.2$. This value is only 10 % smaller than the experimental one. The other source of error is the uncertainty in the decay constants $f_D$ and $f_{D^*}$. Indeed, from Ref.[11], the decay constants are known to be $f_D = 170 \pm 10$ MeV and $f_{D^*} = 240 \pm 20$ MeV. If we take these uncertainties into account we obtain the range

$$14.1 \leq g_{D^*D\pi} \leq 18.75$$

which certainly overlaps with the experimental value.

Of course, there are other possibilities to improve the existing LCQSR. For example, the current LCQSR is up to twist-4. There might be some contributions from higher twists. Also the twist-4 element denoted by $\delta^2$ is currently known to be 0.2 GeV$^2$ based on QCD sum rule calculation of Novikov et al. [21]. This value however crucially depends on the factorization assumption for the four-quark condensate $\langle (\bar{q}q)^2 \rangle$. Based on the rho meson sum rule [22], the four-quark condensate can be much larger, which may shift $\delta^2$ to a larger value. Nevertheless, what we want to emphasize is that all these improvements are expected to be less than 10 % because, otherwise, the twist expansion can not be valid. To be comparable with the CLEO experiment, one needs a correction of order 40 % in the existing LCQSR calculation. Therefore, the continuum correction that we are addressing in this work provides the most important modification to the existing LCQSR and it helps to reach an agreement with the CLEO experiment. More interesting is that, with this improvement, LCQSR provides the coupling comparable to the quark model [2] and lattice calculations [3].

Acknowledgments

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FIG. 1: $f_D^g D^* D_\pi$ versus the Borel mass in the light-cone QCD sum rule. The solid line is from the sum rule of Ref. [1]. The dash curve is obtained when $\varphi_\pi(0.5) = 1.5$ is used. The dot-dashed curve is obtained with the continuum correction.