A Resonance Model for $\pi N \rightarrow Y K$ and $\pi \Delta \rightarrow Y K$

Reactions for Kaon Production in Heavy Ion Collisions

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Abstract

In a resonance model the reactions $\pi N \rightarrow Y K$ and $\pi \Delta \rightarrow Y K$ are studied. For the reactions $\pi N \rightarrow \Lambda K$ and $\pi \Delta \rightarrow \Lambda K$, the resonances $N(1650)(J^P = \frac{1}{2}^-)$, $N(1710)(\frac{1}{2}^+)$ and $N(1720)(\frac{3}{2}^+)$ are included as intermediate states. For the reactions $\pi N \rightarrow \Sigma K$, the resonances $N(1710)(\frac{1}{2}^+)$, $N(1720)(\frac{3}{2}^+)$ and $\Delta(1920)(\frac{3}{2}^+)$ are considered, while for the $\pi \Delta \rightarrow \Sigma K$ reactions the intermediate resonances are $N(1710)(\frac{1}{2}^+)$ and $N(1720)(\frac{3}{2}^+)$. Besides these resonances in the $s$-channel, the $t$-channel $K^*(892)$-exchanges are also taken into account as a smooth background.

The relevant coupling constants for the meson-baryon vertices are obtained (except for $\Delta(1920)$) from the experimental decay branching ratios of the relevant resonances. All isospin channels of the $\pi N \rightarrow Y K$ and $\pi \Delta \rightarrow Y K$ cross sections are calculated. By comparing the calculated results with the available experimental data, we find that the total cross sections of the $\pi N \rightarrow Y K$ reactions can be explained by the resonance model. The $\pi \Delta \rightarrow Y K$ cross sections, for which no experimental data are available, are predicted theoretically.

Parametrizations of the calculated total cross sections for all different isospin channels are given for the use of kaon productions in heavy ion collisions. The differential cross sections are also studied.

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1 Introduction

The purpose of this paper is to study the \( \pi N (\text{and} \Delta) \rightarrow Y K \) (\( \Delta \) stands for \( \Delta(1232) \), \( Y \) stands for either the \( \Sigma \) or the \( \Lambda \)) reactions for kaon productions in heavy ion collisions.

One of the main goals of intermediate energy heavy ion physics is to determine the equation of state (EOS) of nuclear matter. Theoretical studies show that the kaons produced in heavy ion collisions are sensitive to the EOS [1][2]. To calculate the kaon production in heavy ion collisions one needs two ingredients [3][4]:

1. A transport theory to describe the evolution of the heavy ion colliding systems.
2. The elementary cross sections of kaon production in the BB and \( \pi B \) collisions (here \( B \) stands for either a nucleon \( N \) or a \( \Delta(1232) \)).

Therefore, in order to obtain a definite conclusion about the EOS one needs the reliable elementary cross sections for the kaon production as well as the dynamical transport models.

One usually uses transport models such as BUU/VUU [3] and QMD [3] to simulate the dynamical evolution of projectile and target systems. In these models collision terms and the nuclear mean field are included. By including the nuclear mean field one hopes to determine the EOS. In the collision terms, the following reactions are included as main processes:

\[ B + B \rightarrow B + B \] (baryon elastic collision)
\[ N + N \rightarrow N + \Delta \] (\( \Delta \) production)
\[ N + \Delta \rightarrow N + N \] (\( \Delta \) absorption)
\[ N + N \rightarrow N + N + \pi \] (pion production)
\[ \Delta \rightarrow N + \pi \] (\( \Delta \) decay)
\[ \pi + N \rightarrow \Delta \] (pion absorption)

then, kaons can be produced mainly through the following reactions:

\[ B + B \rightarrow B + Y + K \]
and

\[ \pi + B \rightarrow Y + K. \]

The Lorentz-invariant differential multiplicity for kaons for a given impact parameter \( b \) in heavy ion collisions is connected with the elementary cross section by

\[
E \frac{d^3 N(b)}{d^3 p} = \sum_{BB_{\text{coll}}} \int \left( E \frac{d^3 \sigma_{BB \rightarrow BY K}(\sqrt{s_{BB}})}{d^3 p}/\sigma_{BB}^{\text{tot}}(\sqrt{s_{BB}}) \right) \left[ 1 - f(r, p, t) \right] \frac{d\Omega}{4\pi} \\
+ \sum_{\pi B_{\text{coll}}} E'' \frac{d^3 \sigma_{\pi B \rightarrow Y K}(\sqrt{s_{\pi B}})}{d^3 p''}/\sigma_{\pi B}^{\text{tot}}(\sqrt{s_{\pi B}}),
\]

where the primed and the double-primed quantities are in the center-of-momentum (c.m.) frames of the two colliding baryons (\( BB \)) and pion-baryon (\( \pi B \)), respectively. The unprimed quantities are in the c.m. frame of the two nuclei. \( \sigma_{BB}^{\text{tot}}(\sqrt{s_{BB}}) \) and \( \sigma_{\pi B}^{\text{tot}}(\sqrt{s_{\pi B}}) \)
are the total cross sections for two baryons \((BB)\) and the pion-nucleon system \((\pi N)\) as functions of c.m. energy \(\sqrt{s_{BB}}\) and \(\sqrt{s_{\pi B}}\), respectively. The factor \([1 - f(r, p, t)]\) stands for the Pauli blocking correction for the final nucleon \(N\) in the \(BB \rightarrow NYK\) reaction due to the surrounding nuclear medium.

The Lorentz-invariant differential kaon-production cross section in heavy ion collisions is then given by:

\[
E \frac{d^3 \sigma}{d^3 p} = 2\pi \int db \frac{bE d^3 N(b)}{d^3 p}. \tag{2}
\]

From eq.(1) we see that the elementary cross sections are directly related to the kaon yields in heavy ion collisions. Thus, it is important to have good elementary kaon-production cross sections.

However, most elementary cross sections needed in studying heavy ion collisions are not well known experimentally. Therefore one has to use the available experimental data as far as possible. In the case when no experimental data are available one has to rely on symmetry considerations and theoretical models to extrapolate the available experimental data.

The elementary kaon-production cross section in \(BB\) collisions used in eq.\((1)\) are usually taken from Randrup and Ko\([7]\). In their works, the \(N\Delta \rightarrow NYK\) and \(\Delta\Delta \rightarrow NYK\) total cross sections are obtained from \(NN \rightarrow NYK\) cross section by symmetry consideration in isospin space, and the \(\Delta\) is treated as spin-\(\frac{1}{2}\) (not \(\frac{3}{2}\)) and isospin-\(\frac{3}{2}\) particle. Moreover, the coupling constants relevant to the \(\Delta\) are assumed to be the same as for the nucleon \(N\).

The elementary \(K^+\)-production cross section in \(\pi B\) collisions needed in eq.\((1)\) are usually taken from Cugnon and Lombard’s parametrization\([8]\). In this parametrization the isospin averaged cross sections are obtained from only three available experimental data assuming proton and neutron \((N = Z)\) symmetry. This symmetry is, in principle, valid for the light nuclei, and not appropriate for collisions between heavy nuclei with a large neutron excess, i.e. \(N > Z\). Furthermore the parametrization for the reaction \(\pi\Delta \rightarrow YK\) is not given, hence the contribution of this channel to kaon production in the heavy ion collision is unclear.

By using the elementary cross section of Randrup and Ko, one finds that \(N\Delta \rightarrow NYK\) gives the main contribution to the total kaon yield in heavy ion collisions\([2]\). However, the elementary cross section for this channel is experimentally unknown. Thus a more sophisticated study of this channel is still necessary. In order to determine the cross section for the reaction \(N\Delta \rightarrow NYK\), one needs at first to study the reaction \(\pi\Delta \rightarrow YK\) according to Randrup and Ko’s model\([7][9]\).

Since in heavy ion collisions many pions are produced, and the threshold of \(\pi N(\text{or}\Delta) \rightarrow YK\) is lower than in \(BB \rightarrow NYK\), the reaction \(\pi N(\text{or}\Delta) \rightarrow YK\) seems to be important as a secondary process besides the \(BB \rightarrow NYK\) channel for kaon production. According to a study of the Giessen group, the \(\pi N \rightarrow \Lambda K\) process is dominant for kaon production in proton-nucleus reactions\([10]\).

Recently, in the simulation codes of BUU and QMD the isospin dependence has been considered. One distinguishes the members of the nucleon isospin-doublet (\(p\) and \(n\)), of the pion isospin-triplet (\(\pi^+, \pi^0\) and \(\pi^-\)) and of the delta isospin-quartet (\(\Delta^{++}, \Delta^+, \Delta^0, \) and \(\Delta^-\))\([11][12]\). This enables us to calculate the kaon production in an isospin dependent
way, hence one needs information on all the possible isospin channels for kaon production. This means that the isospin-averaged elementary cross sections of $BB \to BYK$ and $\pi N(\text{or} \Delta) \to YK$ can not be used for this purpose. For an isospin-dependent description of kaon production, one needs all the isospin channels. However, the cross sections for some isospin channels are not available in experiment. Therefore one needs a theoretical model to evaluate all the unknown isospin channels.

On the other hand, in order to study medium effects for kaon production, a dynamical model for the elementary cross sections is needed, since one can not simply obtain the in-medium cross sections from a parametrization of experimental data of the corresponding cross sections in free space.

As a first step, we concentrate on the $\pi B \to YK$ reactions. Once we know this cross sections we can calculate the second term in eq.(1). This channel is also the basis to study $BB \to BYK$, since the parameters of the $\pi B \to YK$ reactions are ingredients for calculating the reaction $BB \to BYK$ in the present model.

In this paper, we present results for all $\pi N(\text{and} \Delta) \to YK$ reactions using a resonance model. Part of this work has been briefly reported in [13]. By a resonance model we mean that the $\pi N(\text{and} \Delta) \to YK$ reactions happen via intermediate resonance states, which decay both into $\pi N(\text{or} \Delta)$ and $YK$. The coupling constants in the meson-baryon-resonances can be determined from the relevant branching ratios of the resonances. The cross sections of $\pi N(\text{and} \Delta) \to YK$ are obtained coherently from the square of the sum of all resonance amplitudes. Besides these $s$-channel resonances which give peaks in the total cross sections, the $t$-channel $K^*(892)$-exchange is also included. It provides a smooth background.

2 Experimental data for the resonance model

We study the reactions $\pi N(\text{and} \Delta) \to YK$ for intermediate energies, i.e. for the range of the $\pi N(\text{and} \Delta)$ invariant mass from the threshold for $YK$ production to about 3 GeV. According to the “Review of Particle Properties”[14][15], the relevant resonances to be included as intermediate states are $N(1650)(J^P = \frac{1}{2}^-)$, $N(1710)(\frac{1}{2}^+)$, $N(1720)(\frac{3}{2}^+)$ and $\Delta(1920)(\frac{3}{2}^+)$. In this section we list the properties of the resonances and of $K^*(892)$ relevant to our calculations.

(1) $N(1650)$ $L_{2I_{2J}} = S_{11}$; $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$:

Mass $m \approx 1650$ MeV
Full width $\Gamma_{\text{full}} = 145$ to 190 MeV $\approx 150$ MeV
Partial decay modes and branching fractions:

$N(1650) \to \begin{cases} 
N\pi & 60 - 80\%, \text{ we use } 70\% \\
\Lambda K & \approx 7\% \\
\Delta\pi & < 10\%, \text{ we use } 5\%
\end{cases}$
\( \text{(2) } N(1710) \ L_{2I2J} = P_{11}; I(J^P) = \frac{1}{2}(1^+) \): \\
Mass \( m \approx 1710 \text{ MeV} \) \\
Full width \( \Gamma_{\text{full}}=50 \text{ to } 250 \text{ MeV} \approx 100 \text{ MeV} \) \\
Partial decay modes and branching fractions:
\[
N(1710) \rightarrow \begin{cases} 
N\pi & 10 - 20\%, \text{we use } 15\% \\
\Lambda K & 5 - 25\%, \text{we use } 15\% \\
\Delta\pi & 10 - 25\%, \text{we use } 17.5\% \\
\Sigma K & 2 - 10\%, \text{we use } 6\% \\
\end{cases}
\]

\( \text{(3) } N(1720) \ L_{2I2J} = P_{13}; I(J^P) = \frac{1}{2}(3^+) \): \\
Mass \( m \approx 1720 \text{ MeV} \) \\
Full width \( \Gamma_{\text{full}}=100 \text{ to } 200 \text{ MeV} \approx 150 \text{ MeV} \) \\
Partial decay modes and branching fractions:
\[
N(1720) \rightarrow \begin{cases} 
N\pi & 10 - 20\%, \text{we use } 15\% \\
\Lambda K & 3 - 10\%, \text{we use } 6.5\% \\
\Delta\pi & 5 - 15\%, \text{we use } 10\% \\
\Sigma K & 2 - 5\%, \text{we use } 3.5\% \\
\end{cases}
\]

\( \text{(4) } \Delta(1920) \ L_{2I2J} = P_{33}; I(J^P) = \frac{3}{2}(3^+) \): \\
Mass \( m \approx 1920 \text{ MeV} \) \\
Full width \( \Gamma_{\text{full}}=150 \text{ to } 300 \text{ MeV} \approx 200 \text{ MeV} \) \\
Partial decay modes and branching fractions:
\[
\Delta(1920) \rightarrow \begin{cases} 
N\pi & 5 - 20\%, \text{we use } 12.5\% \\
\Sigma K & 1 - 3\%, \text{we use } 2\% \\
\end{cases}
\]

\( \text{(5) } K^*(892) I(J^P) = \frac{1}{2}(1^-) \): \\
Mass \( m \approx 892 \text{ MeV} \) \\
Full width \( \Gamma_{\text{full}}=49.8 \text{ MeV} \) \\
Partial decay modes and branching fractions:
\[
K^*(892) \rightarrow K\pi \ 100\%
\]

3 Formalism

In the present study, the mesons, baryons and baryon-resonances are treated as fundamental fields. Due to the fact that the relevant hadrons have finite sizes, form factors are included at every vertex.

In section 3.1 to 3.4 we give the effective interaction Lagrangians, and the formulas for the transition amplitudes and cross sections. Throughout this work SU(2) symmetry in isospin space is assumed. In detail we treat \( \pi N \rightarrow \Lambda K, \pi N \rightarrow \Sigma K, \pi\Delta \rightarrow \Lambda K, \pi\Delta \rightarrow \Sigma K \) in the following subsections §3.1, §3.2, §3.3 and §3.4 separately.
3.1 $\pi + N \rightarrow \Lambda + K$ reactions

From the experimental data listed in §2, three resonances $N(1650)(\frac{1}{2}^-)$, $N(1710)(\frac{1}{2}^+)$ and $N(1720)(\frac{3}{2}^+)$ contribute to the $\pi N \rightarrow \Lambda K$ reactions. In addition $K^*(892)$-exchange can also be responsible for the same reaction. The relevant Feynman diagrams are shown in fig. 1:

The effective interaction Lagrangians needed in describing the vertices are given by:

$$L_{\pi NN(1650)} = -g_{\pi NN(1650)} \left( \bar{N}(1650) \tau N \cdot \phi + \bar{N} \tau \bar{N}(1650) \cdot \bar{\phi} \right),$$  \(3\)

$$L_{\pi NN(1710)} = -ig_{\pi NN(1710)} \left( \bar{N}(1710) \gamma_5 \tau N \cdot \phi + \bar{N} \gamma_5 \bar{N}(1710) \cdot \bar{\phi} \right),$$  \(4\)

$$L_{\pi NN(1720)} = \frac{g_{\pi NN(1720)}}{m_{\pi}} \left( \bar{N}(1720) \tau N \cdot \partial_{\mu} \bar{\phi} + \bar{N} \tau \bar{N}(1720) \cdot \partial_{\mu} \bar{\phi} \right),$$  \(5\)

$$L_{K\Lambda N(1650)} = -g_{K\Lambda N(1650)} \left( \bar{N}(1650) \Lambda K + \bar{K} \Lambda \bar{N}(1650) \right),$$  \(6\)

$$L_{K\Lambda N(1710)} = -ig_{K\Lambda N(1710)} \left( \bar{N}(1710) \Lambda \gamma_5 K + \bar{K} \gamma_5 \Lambda \bar{N}(1710) \right),$$  \(7\)

$$L_{K\Lambda N(1720)} = \frac{g_{K\Lambda N(1720)}}{m_K} \left( \bar{N}(1720) \Lambda \partial_{\mu} K + \partial_{\mu} \bar{K} \Lambda \bar{N}(1720) \right),$$  \(8\)

$$L_{K^*(892)\Lambda N} = -g_{K^*(892)\Lambda N} \left( \bar{N} \gamma^\mu \Lambda K^*_\mu(892) + \frac{\kappa}{m_N + m_\Lambda} \bar{N} \sigma^{\mu\nu} \Lambda \partial_{\mu} K^*_\nu(892) + \text{h.c.} \right),$$  \(9\)

$$L_{K^*(892)K\pi} = i f_{K^*(892)K\pi} \left( \bar{K} \gamma^\mu K^*_\mu(892) \cdot \partial_{\mu} \bar{\phi} - (\partial_{\mu} \bar{K}) \gamma^\mu K^*_\mu(892) \cdot \bar{\phi} \right) + \text{h.c.},$$  \(10\)

In the above Lagrangians, $N$, $N(1650)$, $N(1710)$, $\Lambda$ stand for spin-$\frac{1}{2}$ Dirac spinors to describe the corresponding particles. Spin-$\frac{3}{2}$ Rarita-Schwinger fields $\psi^\mu = N^\mu(1720)$ with mass $m$ satisfy the equations[16]:

$$(i\gamma \cdot \partial - m)\psi^\mu = 0,$$  \(11\)

$$\gamma_\mu \psi^\mu = 0,$$  \(12\)

$$\partial_\mu \psi^\mu = 0,$$  \(13\)

and $\bar{\phi}$ stands for the pion field. In isospin space nucleon fields are expressed by $N = (p, n)^T$, where the superscript "$^T$" means the transposition operation, similar expressions can also be written for the nucleon resonances. The meson field operators appearing in eqs. (3)-(11) are related to the physical representations as follows, $K = (K^+, K^0)^T$, $\bar{K} = \ldots$
\(K^-, K^0\), \(K^*(892) = (K^*_\mu(892)^+, K^*_\mu(892)^0)^T\), \(\bar{K}^*(892) = (K^*_\mu(892)^-, K^*_\mu(892)^0)\), 
\[\pi^\pm = \frac{1}{\sqrt{2}}(\phi_1 \mp i\phi_2), \pi^0 = \phi_3.\]

For the propagators \(i\Sigma_F(p)\) of the spin-\(\frac{1}{2}\) and the propagators \(iG^{\mu\nu}(p)\) of the spin-\(\frac{3}{2}\) resonances\(^{17}\) we use

\[i\Sigma_F(p) = i \frac{\gamma \cdot p + m}{p^2 - m^2 + i\Gamma_{\text{full}}} \quad \text{and} \]

\[iG^{\mu\nu}(p) = i \frac{P^{\mu\nu}(p)}{p^2 - m^2 + i\Gamma_{\text{full}}},\]

with

\[P^{\mu\nu}(p) = -(\gamma \cdot p + m) \left[ g^{\mu\nu} - \frac{1}{3} \gamma^\mu \gamma^\nu - \frac{1}{3m} (\gamma^\mu p^\nu - \gamma^\nu p^\mu) - \frac{2}{3m^2} p^\mu p^\nu \right],\]

respectively, where \(m\) and \(\Gamma_{\text{full}}\) stand for the mass and the full decay width of the corresponding baryon resonance.

According to the Lagrangians given by eqs. (3)-(10), the amplitudes for the \(\pi N \rightarrow \Lambda K\) reactions are given by

\[\mathcal{M}_{\pi^0 p \rightarrow \Lambda K^+} = -\mathcal{M}_{\pi^0 n \rightarrow \Lambda K^0} = \mathcal{M}_{a1} + \mathcal{M}_{b1} + \mathcal{M}_{c1} + \mathcal{M}_{d1}\]  
\[\mathcal{M}_{\pi^+ n \rightarrow \Lambda K^+} = \mathcal{M}_{\pi^- p \rightarrow \Lambda K^0} = \sqrt{2} (\mathcal{M}_{a1} + \mathcal{M}_{b1} + \mathcal{M}_{c1} + \mathcal{M}_{d1})\]

where the resonance amplitudes \(\mathcal{M}_{a1}\) (\(N(1650)\) intermediate state), \(\mathcal{M}_{b1}\) (\(N(1710)\) intermediate state), \(\mathcal{M}_{c1}\) (\(N(1720)\) intermediate state) and the \(K^*(892)\)-exchange amplitude \(\mathcal{M}_{d1}\) are given by

\[\mathcal{M}_{a1} = \frac{g_{\pi NN}(1650) g_{K\Lambda N}(1650) \bar{u}_\Lambda(p_\Lambda, s_\Lambda)(\gamma \cdot p + m_{N(1650)}) u_N(p_N, s_N)}{p^2 - m_{N(1650)}^2 + i m_{N(1650)} \Gamma_{\text{full}}^{N(1650)}} \]

\[\mathcal{M}_{b1} = \frac{-g_{\pi NN}(1710) g_{K\Lambda N}(1710) \bar{u}_\Lambda(p_\Lambda, s_\Lambda) \gamma_5 (\gamma \cdot p + m_{N(1710)}) \gamma_5 u_N(p_N, s_N)}{p^2 - m_{N(1710)}^2 + i m_{N(1710)} \Gamma_{\text{full}}^{N(1710)}} \]

\[\mathcal{M}_{c1} = \frac{-g_{\pi NN}(1720) g_{K\Lambda N}(1720) p_K \mu p_{\pi \nu} \bar{u}_\Lambda(p_\Lambda, s_\Lambda) P_{\nu N(1720)}^{\mu}(p) u_N(p_N, s_N)}{m_{\pi} m_K (p^2 - m_{N(1720)}^2 + i m_{N(1720)} \Gamma_{\text{full}}^{N(1720)})} \]

\[\mathcal{M}_{d1} = \frac{f_{K^*(892)K^-}}{(p_\Lambda - p_N)^2 - m_{K^*(892)}^2} \bar{u}_\Lambda(p_\Lambda, s_\Lambda) \left[ \gamma_\mu - i \frac{\kappa}{m_N + m_\Lambda} \sigma_{\alpha\mu}(p_\Lambda - p_N) \right] \cdot (p_\pi + p_K)_\nu \left[ g^{\mu\nu} - \frac{(p_\Lambda - p_N)^\mu (p_\Lambda - p_N)^\nu}{m_{K^*(892)}^2} \right] u_N(p_N, s_N).\]

Here \(u_N(p_N, s_N)\) and \(u_\Lambda(p_\Lambda, s_\Lambda)\) in eqs. (13)-(22) are the spinors of the nucleon \(N\) and the lambda \(\Lambda\), with four-momentum \(p_N\), spin \(s_N\) and four-momentum \(p_\Lambda, s_\Lambda\), respectively. \(p\) is the four-momentum of the resonance. \(p_\pi, p_K\) are the four-momenta of the pion and the kaon, respectively.
We obtain the absolute values of the effective coupling constants by equating the experimental branching ratios given in § 2 with the theoretical calculations given in appendix § 5.1.

Form factors (denoted by $F$ and $F_{K^*}(892)K\pi$ below) are introduced. They reflect the finite size of the hadrons. These form factors must be multiplied to each vertex of the interactions $\Pi$. Thus, the coupling constants are obtained from the branching ratios in the rest frame of the resonances.

For the form factors in baryon-baryon-meson vertices we use

$$F(|\vec{q}|) = \frac{\Lambda_c^2}{\Lambda_c^2 + \vec{q}^2},$$

with $|\vec{q}|$ being the magnitude of three-momentum $\vec{q}$ of the mesons, and $\Lambda_c$ being the cut off parameter.

For the $K^*(892)$-$K\pi$ vertex we adopt the form factor of ref. [18].

$$F_{K^*}(892)K\pi(|\frac{1}{2}(|\vec{p}_K - \vec{p}_\pi|)|) = C |\frac{1}{2}(|\vec{p}_K - \vec{p}_\pi|)| \exp \left(-\beta \frac{1}{2}(|\vec{p}_K - \vec{p}_\pi|)^2\right).$$

In evaluating the cross sections in the c.m. frame of the $\pi N$ system, each coupling constant $g_{PBB*}$, $g_{K^*(892)AN}$ and $f_{K^*(892)K\pi}$ appearing in eqs. [19]-[22] must be replaced by $g_{PBB*} \rightarrow g_{PBB*} F(|\vec{q}_P|)$, $g_{K^*(892)AN} \rightarrow g_{K^*(892)AN} F(|\vec{q}_\Lambda - \vec{q}_N|)$ and $f_{K^*(892)K\pi} \rightarrow f_{K^*(892)K\pi} F_{K^*}(892)K\pi(|\frac{1}{2}(|\vec{p}_K - \vec{p}_\pi|)|)$. Here $B^*$ stands for the nucleon resonances. $B$ indicates a baryon ($N(938)$ or $\Lambda$) and $P$ stands for pseudoscalar-meson ($\pi$ or $K$), respectively.

The (spin unpolarized) differential cross section for $\pi^0 p \rightarrow \Lambda K^+$ in the c.m. frame of $\pi^0 p$ is given by

$$d\sigma(\pi^0 p \rightarrow \Lambda K^+) = \frac{1}{(8\pi\sqrt{s})^2} \sum_{all\ spins} |M_{\pi^0 p \rightarrow \Lambda K^+}|^2,$$
\[ \sigma(p^0 p \rightarrow \Lambda K^+) = \int d\Omega \frac{d\sigma(p^0 p \rightarrow \Lambda K^+)}{d\Omega}. \]  

(29)

Similar expressions for other isospin channels for \( K^+ \) and \( K^0 \)-production can be obtained in the same way as in eqs. (17) and (18).

### 3.2 \( \pi + N \rightarrow \Sigma + K \) reactions

Besides \( K^* \)-exchange, there are three resonances contributing to the \( \pi N \rightarrow \Sigma K \) reactions (see §2). They are \( N(1710)(\frac{1}{2}^+), N(1720)(\frac{3}{2}^+) \) and \( \Delta(1920)(\frac{3}{2}^+) \). The Feynman diagrams are shown in fig. 2.

In addition to eqs. (4)-(5) and eq. (10), the interaction Lagrangians needed in describing the vertices relevant for the reactions \( \pi N \rightarrow \Sigma K \) are given by:

\[ \mathcal{L}_{\pi N \Delta(1920)} = \frac{g_{\pi N \Delta(1920)}}{m_{\pi}} \left( \Delta^\mu(1920) \overleftrightarrow{T}_\mu N \cdot \partial_\mu \phi + \overrightarrow{N} \overrightarrow{T}_\mu \Delta^\mu(1920) \cdot \partial_\mu \phi \right), \]  

(30)

\[ \mathcal{L}_{K\Sigma N(1710)} = -ig_{K\Sigma N(1710)} \left( \overrightarrow{N}(1710) \gamma_5 \tau \cdot \overrightarrow{\Sigma} K + \overrightarrow{K} \overrightarrow{\Sigma} \cdot \tau \gamma_5 N(1710) \right), \]  

(31)

\[ \mathcal{L}_{K\Sigma N(1720)} = \frac{g_{K\Sigma N(1720)}}{m_K} \left( \overrightarrow{N}(1720) \tau \cdot \overrightarrow{\Sigma} \partial_\mu K + (\partial_\mu K) \overrightarrow{\Sigma} \cdot \tau \partial^\mu N(1720) \right), \]  

(32)

\[ \mathcal{L}_{K\Sigma \Delta(1920)} = \frac{g_{K\Sigma \Delta(1920)}}{m_K} \left( \overrightarrow{\Delta}(1920) \overrightarrow{T}_\mu \cdot \overrightarrow{\Sigma} \partial_\mu K + (\partial_\mu K) \overrightarrow{\Sigma} \cdot \tau \partial^\mu \Delta(1920) \right), \]  

(33)

\[ \mathcal{L}_{K^*(892)\Sigma N} = -g_{K^*(892)\Sigma N} \left( \overrightarrow{N} \gamma^\mu \tau \cdot \overrightarrow{\Sigma} K_\mu^*(892) + \frac{\kappa}{m_N + m_\Sigma} \overrightarrow{N} \sigma^{\mu\nu} \tau \cdot \overrightarrow{\Sigma} \partial_\mu K_\nu^*(892) + \text{h.c.} \right), \]  

(34)

In the eqs. above \( \Sigma \) stands for the spin-\( \frac{1}{2} \) Dirac spinor of the \( \Sigma \) particle. \( \Delta^\mu(1920) \) stands for the spin-\( \frac{3}{2} \) Rarita-Schwinger field of \( \Delta(1920) \), which satisfies the equations (11)-(13). \( \kappa \) is the ratio of the tensor coupling constant to the vector coupling constant. \( \overrightarrow{T} \) is the isospin transition operator defined by

\[ \overrightarrow{T}_{Mm} = \sum_{\ell=\pm 1,0} (1 - \frac{1}{2} m |\frac{3}{2} M) \hat{e}_\ell, \]

with

\[ \begin{align*}
\hat{e}_1 &= -\frac{1}{\sqrt{2}}(1, i, 0) \\
\hat{e}_0 &= (0, 0, 1) \\
\hat{e}_{-1} &= \frac{1}{\sqrt{2}}(1, -i, 0)
\end{align*} \]

and \( \tau \) are the Pauli matrices. \( \Delta^\mu(1920) = (\Delta^\mu(1920)^{++}, \Delta^\mu(1920)^{+}, \Delta^\mu(1920)^0, \Delta^\mu(1920)^-) \). The \( \Sigma \) field operators appearing in eqs. (31)-(34) are related to the physical representations by \( \Sigma^\pm = \frac{1}{\sqrt{2}}(\Sigma_1 \mp i \Sigma_2), \Sigma^0 = \Sigma_3 \).
The required amplitudes for the cross section are given by

\[ M_{π^+p → Σ^+K^+} = M_{π^−n → Σ^−K^0} = (M_{c2} + 2M_{d2}), \]

\[ M_{π^−p → Σ^−K^+} = M_{π^+n → Σ^+K^0} = 2 \left( M_{a2} + M_{b2} + \frac{1}{6}M_{c2} \right), \]

\[ M_{π^+n → Σ^0K^+} = M_{π^−p → Σ^+K^0} = \sqrt{2} \left( M_{a2} + M_{b2} - \frac{1}{3}M_{c2} - M_{d2} \right), \]

\[ M_{π^0p → Σ^0K^+} = M_{π^0n → Σ^0K^0} = M_{a2} + M_{b2} + \frac{2}{3}M_{c2} + M_{d2}, \]

where the single resonance amplitudes \( M_{a2} \) (with the \( N(1710) \) resonance), \( M_{b2} \) \( (N(1720)) \), \( M_{c2} \) \( (Δ(1920)) \), and the \( K^*(892)\)-exchange amplitude \( M_{d2} \) in the above equations are given by

\[ M_{a2} = \frac{-g_{πNN(1710)}g_{KΣN(1710)}}{p^2 - m_N^2(1710) + im_N(1710)\Gamma_N(1710)} \] \( |u_N(p_N, s_N)| \),

\[ M_{b2} = \frac{g_{πNN(1720)}g_{KΣN(1720)}PKμPπν\bar{u}Σ(p_Σ, s_Σ)P_{N(1720)}(p)u_N(p_N, s_N)}{m_πm_K(p^2 - m_N^2(1720) + im_N(1720)\Gamma_N(1720))}, \]

\[ M_{c2} = \frac{g_{πΔ(1920)}g_{KΣΔ(1920)}PKμPπν\bar{u}Σ(p_Σ, s_Σ)P_{Δ(1920)}(p)u_N(p_N, s_N)}{m_πm_K(p^2 - m_Σ^2(1920) + im_Δ(1920)\Gamma_Δ(1920))}. \]

\[ M_{d2} = \frac{f_{K^*(892)Kπ}g_{K^*(892)ΣN}}{(p_Σ - p_N)^2 - m_{K^*(892)}^2} \bar{u}Σ(p_Σ, s_Σ) \left[ \gamma_μ - i\frac{κ}{m_N + m_Σ}σ_{νμ}(p_Σ - p_N)^α \right] \cdot (p_π + p_K)_ν \left( g_μν - \frac{(p_Σ - p_N)μ(p_Σ - p_N)ν}{m_{K^*(892)}^2} \right) u_N(p_N, s_N). \]

Form factors are inserted at each vertex in eqs. (35)-(43) in the same way as in §3.1.

The differential cross section for \( π^+p → Σ^+K^+ \) in the c.m. frame of \( π^+p \) is given by

\[ \frac{dσ(π^+p → Σ^+K^+)}{dΩ} = \frac{1}{(8π\sqrt{s})^2} \frac{|\tilde{q}_f|}{|\tilde{q}_i|} \frac{1}{2} \sum_{all\; spins} |M_{π^+p → Σ^+K^+}|^2, \]

where \( \tilde{q}_f \) stands for the outgoing three-momenta of the \( Σ^+ \) and the \( K^+ \) in the c.m. system, and \( \tilde{q}_i \) stands for the incoming three-momenta of the \( π^+ \) and the \( p \). We have, in the c.m. frame of \( πN \) system,

\[ |\tilde{q}_f| = |\tilde{p}_Σ| = |\tilde{p}_K| = \frac{λ^2(s, m_Σ^2, m_K^2)}{2\sqrt{s}} \]

(45)
and

$$|\vec{q}_i| = |\vec{p}_N| = |\vec{p}_\pi| = \frac{\sqrt{s} \bar{m}_N^2}{2\sqrt{s}}. \tag{46}$$

The total cross section for $\pi^+p \to \Sigma^+K^+$ reads

$$\sigma(\pi^+p \to \Sigma^+K^+) = \int d\Omega \frac{da(\pi^+p \to \Sigma^+K^+)}{d\Omega}. \tag{47}$$

The same expressions as eqs. (44) and (47) for other isospin dependent reactions can be obtained by eqs. (36)-(39). From eqs. (37) and eq. (38), we derive

$$\frac{d\sigma(\pi^0n \to \Sigma^-K^+)}{d\Omega} = \frac{d\sigma(\pi^+n \to \Sigma^0K^+)}{d\Omega}. \tag{48}$$

The $K^0$ and the $K^+$ production have the same amplitudes given by eqs. (35)-(39), for example, from eq. (35), one finds:

$$\frac{d\sigma(\pi^-n \to \Sigma^-K^0)}{d\Omega} = \frac{d\sigma(\pi^+p \to \Sigma^+K^+)}{d\Omega}. \tag{49}$$

### 3.3 $\pi + \Delta(1232) \to \Lambda + K$ reactions

From the experimental data listed in § 2, three resonances $N(1650)(\frac{1}{2}^-)$, $N(1710)(\frac{1}{2}^+)$, $N(1720)(\frac{3}{2}^+)$ contribute to the $\pi\Delta(1232) \to \Lambda K$ reactions. Due to isospin conservation $K^*(892)$-exchange cannot contribute to the $\pi\Delta(1232) \to \Lambda K$ reactions. The relevant Feynman diagrams are shown in fig. 3:

Besides eqs. (3)-(8), the following three interaction Lagrangian are needed in describing the vertices of the $\pi\Delta \to \Lambda K$ reactions:

$$L_{\pi\Delta N(1650)} = i \frac{g_{\pi\Delta N(1650)}}{m_\pi} \left( \bar{N}(1650) \gamma_5 \overleftrightarrow{\partial} \Delta^\mu \cdot \partial_\mu \phi + \Delta^\mu \overleftrightarrow{\partial} \gamma_5 N(1650) \cdot \partial_\mu \phi \right), \tag{50}$$

$$L_{\pi\Delta N(1710)} = \frac{g_{\pi\Delta N(1710)}}{m_\pi} \left( \bar{N}(1710) \overleftrightarrow{\partial} \Delta^\mu \cdot \partial_\mu \phi + \Delta^\mu \overleftrightarrow{\partial} \gamma_5 N(1710) \cdot \partial_\mu \phi \right), \tag{51}$$

$$L_{\pi\Delta N(1720)} = -ig_{\pi\Delta N(1720)} \left( \bar{N}(1720) \gamma_5 \overleftrightarrow{\partial} \Delta^\mu \cdot \phi + \Delta^\mu \overleftrightarrow{\partial} \gamma_5 N^\mu(1720) \cdot \phi \right). \tag{52}$$

The amplitudes for the reactions are given by

$$M_{\pi^-\Delta^+ \to \Lambda K^0} = -M_{\pi^+\Delta^- \to \Lambda K^0} = - (M_{a3} + M_{b3} + M_{c3}), \tag{53}$$

$$M_{\pi^0\Delta^+ \to \Lambda K^+} = M_{\pi^0\Delta^- \to \Lambda K^-} = \frac{2}{3} (M_{a3} + M_{b3} + M_{c3}), \tag{54}$$

$$M_{\pi^+\Delta^0 \to \Lambda K^+} = -M_{\pi^-\Delta^+ \to \Lambda K^0} = \frac{1}{\sqrt{3}} (M_{a3} + M_{b3} + M_{c3}). \tag{55}$$
Here the amplitudes \( M_{a3} \) (with \( N(1650) \) as the intermediate state), \( M_{b3}(N(1710)) \) and \( M_{c3}(N(1720)) \) corresponding to each of the diagrams (a), (b) and (c) given in fig. 3 are given by (the coefficients in eqs. (53)-(55) are from isospin):

\[
M_{a3} = \frac{-g_{\pi\Delta N(1650)}g_{K\Lambda N(1650)}}{m_\pi}\frac{p_\mu u_\Lambda(p_\Lambda, s_\Lambda) (\gamma \cdot p + m_\Lambda N(1710)) \gamma_5 u_\Delta(p_\Delta, s_\Delta)}{p^2 - m_\Lambda^2 N(1650) + i m_N(1650) \Gamma_{N(1650)}^{full}},
\]

\[
M_{b3} = \frac{-g_{\pi\Delta N(1710)}g_{K\Lambda N(1710)}}{m_\pi}\frac{p_\mu u_\Lambda(p_\Lambda, s_\Lambda) \gamma_5 (\gamma \cdot p + m_\Lambda N(1710)) u_\Delta(p_\Delta, s_\Delta)}{p^2 - m_\Lambda^2 N(1710) + i m_N(1710) \Gamma_{N(1710)}^{full}},
\]

\[
M_{c3} = \frac{g_{\pi\Delta N(1720)}g_{K\Lambda N(1720)}}{m_\kappa}\frac{p_\kappa u_\Lambda(p_\Lambda, s_\Lambda) F_{\mu\nu}^\Delta(p_\Delta, s_\Delta)}{p^2 - m_\kappa^2 N(1720) + i m_N(1720) \Gamma_{N(1720)}^{full}}.
\]

The Rarita-Schwinger spinor-vector \( u_\Delta(p_\Delta, s_\Delta) \) of the delta’s with momentum \( p_\Delta \) and spin \( s_\Delta \), \( u_\Delta(p_\Delta, s_\Delta) \) satisfies in momentum space the eqs. (11)-(13).

\[
(\gamma \cdot p_\Delta - m_\Delta) u_\Delta(p_\Delta, s_\Delta) = 0,
\]

\[
\gamma_\mu u_\Delta(p_\Delta, s_\Delta) = 0,
\]

\[
p_\Delta u_\Delta(p_\Delta, s_\Delta) = 0.
\]

Form factors are inserted at each vertex in eqs. (56)-(58) in the same way as in § 3.1 and § 3.2.

The differential cross section for \( \pi^-\Delta^{++} \rightarrow \Lambda K^+ \) in the c.m. frame of \( \pi^-\Delta^{++} \) is given by

\[
\frac{d\sigma(\pi^-\Delta^{++} \rightarrow \Lambda K^+)}{d\Omega} = \frac{1}{(8\pi\sqrt{s})^2} \frac{|\vec{q}|}{|\vec{q}|} \frac{1}{4} \sum_{\text{all spins}} |M_{\pi^-\Delta^{++} \rightarrow \Lambda K^+}|^2,
\]

where

\[
|\vec{q}| = |\vec{p}_\Lambda| = |\vec{p}_K| = \frac{\lambda_2(s, m_\Lambda^2, m_\kappa^2)}{2\sqrt{s}}
\]

and

\[
|\vec{q}_\Delta| = |\vec{p}_\Delta| = |\vec{p}_\pi| = \frac{\lambda_2(s, m_\Delta^2, m_\pi^2)}{2\sqrt{s}}
\]

with

\[
s = (p_\Delta + p_\pi^2) = p^2.
\]

The total cross section for \( \pi^-\Delta^{++} \rightarrow \Lambda K^+ \) reads

\[
\sigma(\pi^-\Delta^{++} \rightarrow \Lambda K^+) = \int d\Omega \frac{d\sigma(\pi^-\Delta^{++} \rightarrow \Lambda K^+)}{d\Omega}.
\]

The relationships between other isospin channels and the above channel can be easily obtained from eqs. (53)-(55).
3.4 \( \pi + \Delta(1232) \rightarrow \Sigma + K \) reactions

According to the data listed in § 2, two resonances contribute to the \( \pi\Delta \rightarrow \Sigma K \) reactions: \( N(1710)(\frac{1}{2}^+) \) and \( N(1720)(\frac{3}{2}^+) \). Further the t-channel \( K^*(892) \)-exchange is also relevant for these reactions. The Feynman diagrams are shown in fig. 4.

In addition to Lagrangians given by eqs. (31)-(32), (51)-(52) and eq. (10), one more interaction Lagrangian needed in describing the \( \pi\Delta \rightarrow \Sigma K \) reactions is given by

\[ \mathcal{L}_{K^*(892)\Sigma \Delta} = -i g_{K^*(892)\Sigma \Delta} \left( \bar{K}^*_\mu(892) \gamma^\mu \gamma_5 \Delta + \bar{\Delta} \gamma_5 \gamma^\mu \bar{K}^*_\mu(892) \right). \]  

The amplitudes for the \( \pi\Delta \rightarrow \Sigma K \) reactions are given by:

\[
\mathcal{M}_{\pi^-\Delta^+\rightarrow \Sigma^0 K^+} = \mathcal{M}_{\pi^+\Delta^-\rightarrow \Sigma^0 K^0} = -(M_{a4} + M_{b4}), \tag{68}
\]

\[
\mathcal{M}_{\pi^-\Delta^+\rightarrow \Sigma^- K^+} = -\mathcal{M}_{\pi^+\Delta^-\rightarrow \Sigma^+ K^0} = -\sqrt{\frac{2}{3}}(M_{a4} + M_{b4}), \tag{69}
\]

\[
\mathcal{M}_{\pi^0\Delta^+\rightarrow \Sigma^+ K^+} = \mathcal{M}_{\pi^0\Delta^-\rightarrow \Sigma^- K^0} = -\mathcal{M}_{c4}, \tag{70}
\]

\[
\mathcal{M}_{\pi^0\Delta^+\rightarrow \Sigma^0 K^+} = -\mathcal{M}_{\pi^0\Delta^-\rightarrow \Sigma^0 K^0} = -\sqrt{\frac{2}{3}}M_{c4}, \tag{71}
\]

\[
\mathcal{M}_{\pi^0\Delta^0\rightarrow \Sigma^+ K^+} = \mathcal{M}_{\pi^0\Delta^0\rightarrow \Sigma^- K^0} = \frac{1}{\sqrt{3}}(2M_{a4} + 2M_{b4} + M_{c4}), \tag{72}
\]

\[
\mathcal{M}_{\pi^+\Delta^+\rightarrow \Sigma^+ K^+} = -\mathcal{M}_{\pi^-\Delta^0\rightarrow \Sigma^- K^0} = -\sqrt{\frac{2}{3}}M_{c4}, \tag{73}
\]

\[
\mathcal{M}_{\pi^+\Delta^0\rightarrow \Sigma^+ K^+} = \mathcal{M}_{\pi^-\Delta^-\rightarrow \Sigma^0 K^0} = \frac{1}{\sqrt{3}}(M_{a4} + M_{b4} + 2M_{c4}), \tag{74}
\]

\[
\mathcal{M}_{\pi^+\Delta^-\rightarrow \Sigma^- K^+} = -\mathcal{M}_{\pi^+\Delta^+\rightarrow \Sigma^+ K^0} = \sqrt{2}(M_{a4} + M_{b4} + M_{c4}). \tag{75}
\]

The amplitudes \( M_{a4} \) (with \( N(1710) \) as an intermediate resonance), \( M_{b4}(N(1720)) \) and \( M_{c4}(K^*(892)-exchange) \) corresponding to diagrams (a), (b) and (c) in fig. 4 are given as follows (isospin factors are included by the coefficients in eqs. (68)-(75)):

\[
M_{a4} = \frac{-g_{\pi\Delta N(1710)g_{K\Sigma N(1710)}}}{m_\pi} \left( \frac{p_{\pi\mu} \bar{u}_\Sigma(p_{\Sigma}, s_{\Sigma}) \gamma_5 \left( \frac{\gamma_\mu \gamma_5}{p + m_{N(1710)} - i m_{N(1710)}} \right) u_\Delta(p_\Delta, s_\Delta)}{p^2 - m_{N(1710)}^2 + i m_{N(1710)}} \right), \tag{76}
\]

\[
M_{b4} = \frac{g_{\pi\Delta N(1720)g_{K\Sigma N(1720)}}}{m_K} \left( \frac{p_{K\mu} \bar{u}_\Sigma(p_{\Sigma}, s_{\Sigma}) P_{N(1720)}^{\mu\nu}(p) \gamma_5 u_\Delta(p_\Delta, s_\Delta)}{p^2 - m_{N(1720)}^2 + i m_{N(1710)}} \right), \tag{77}
\]

\[
M_{c4} = \frac{if_{K^*(892)K^*g_{K^*(892)\Delta}}}{(p_{\Sigma} - p_\Delta)^2 - m_{K^*(892)}^2} \bar{u}_\Sigma(p_{\Sigma}, s_{\Sigma}) \gamma_5 u_\Delta(p_\Delta, s_\Delta) \left( g_{\mu\nu} - \frac{(p_{\Sigma} - p_\Delta)_{\mu}(p_{\Sigma} - p_\Delta)_{\nu}}{m_{K^*(892)}^2} \right). \tag{78}
\]

Note that the \( N(1650) \) resonance gives contributions to the \( \pi\Delta \rightarrow \Lambda K \) reactions but does not give contributions to the \( \pi\Delta \rightarrow \Sigma K \) reactions.
Form factors are inserted at each vertex in eqs. (76)-(78) in the same way as in § 3.1, § 3.2 and § 3.3.

The differential cross section for the reaction $\pi^− Δ^{++} → Σ^0 K^+$ in the c.m. frame of $πΔ$ reads

$$\frac{dσ(π^− Δ^{++} → Σ^0 K^+)}{dΩ} = \frac{1}{(8π\sqrt{s})^2} \frac{|\vec{q}_f|}{4} \sum_{all \, spins} |M_{π^− Δ^{++} → Σ^0 K^+}|^2,$$

(79)

where

$$|\vec{q}_f| = |\vec{p}_Σ| = |\vec{p}_K| = \frac{λ_{12}^2(s, m_Σ^2, m_K^2)}{2\sqrt{s}}$$

(80)

and

$$|\vec{q}_i| = |\vec{p}_Δ| = |\vec{p}_π| = \frac{λ_{12}^2(s, m_Δ^2, m_π^2)}{2\sqrt{s}}.$$

(81)

Other isospin channels and $K^0$ productions can be obtained by eqs. (68)-(75).

4 Results and Discussions

In this section we give the results and discuss the cross sections of the reactions given in section 3. We treat in detail the reactions $πN → ΛK$, $πN → ΣK$, $πΔ → ΛK$, $πΔ → ΣK$ in the subsections § 4.1, § 4.2, § 4.3 and § 4.4, respectively. In each subsection we first discuss the total and then the differential cross sections.

4.1 $π + N → Λ + K$ reactions

The total cross section for $π^0 + p → Λ + K^+$ is given by eq. (29). Since all the resonances contributing to the $π + N → Λ + K$ reactions have isospin $1/2$, different isospin channels of the $π + N → Λ + K$ reactions differ only by an overall factor. The cross section for $π^- + p → Λ + K^0$ is two times the one of $π^0 + p → Λ + K^+$ according to eqs. (17) and (18). The amplitudes appearing in eqs. (17) and (18) are given by eqs. (19)-(22).

In evaluating these amplitudes we need eight coupling constants (i.e. $g_{πNN(1650)}$, $g_{πNN(1710)}$, $g_{KΛN(1650)}$, $g_{KΛN(1710)}$, $g_{KAN(1720)}$, $g_{K(892)ΛN}$, $f_{K(892)Kπ}$) and cutoff parameters. From the eight coupling constants, the absolute values of seven can be determined by equating eqs. (92)-(98) given in appendix 5.1 with the experimental branching ratios listed in § 2. One coupling constant, i.e. $g_{K(892)ΛN}$, is treated as a free parameter in the present work. As mentioned in § 3, we include form factors in the decay branching ratios as well as in the scattering amplitudes throughout this work (see also eqs. (23) and (24)). By choosing the cutoff values as : $Λ_c = 0.8$ GeV for $N(1650)$, $N(1710)$ and $N(1720)$, we obtain the coupling constants in table 1. For the $K(892)ΛN$ vertex we use $g_{K(892)ΛN} = 0.45$, $Λ_c = 1.2$ GeV. The cut off parameters $C$ and $β$ in $F_{K(892)Kπ}$ are $C = 2.72$ fm and $β = 8.88 \times 10^{-3}$ fm$^2$ used in ref. [13], and the resulting coupling constant $f_{K(892)Kπ}$ is also shown in table 1.
According to eqs. (17), (18) and (25) we sum the amplitudes of the different resonances, and obtain the cross sections by squaring the sum of the amplitudes. Thus we include all interference terms. (The problem with the signs of the coupling constants is discussed below.)

First, we show the total cross section for the reaction $\pi^- + p \rightarrow \Lambda + K^0$ without interference terms in fig. 5(a). The experimental data are from ref. [21]. Since the experimental branching ratios can determine the square of the coupling constants only, the relative signs of the interference terms are not determined (in the present case $2^{4-1} = 8$ possibilities). We have tested all these possibilities. Fig. 5(b) shows the result with the interference terms with one choice of the relative signs. In selecting this sign combination we choose the one which gives the best results compared with the experimental data for both the total and the differential cross section, we then choose this sign combination in all further investigations. We also found that other relative sign combinations give a better total cross section, however do not give a good differential cross section.

A single resonance or $K^*(892)$-exchange can not explain the experimental data. The contribution from $N(1650)$ is larger than from the other resonances. But one must include all the resonances from the particle booklet which decay into $\pi N$ and $K\Lambda$ for the description of kaon production.

The $s$-channel resonances gives a Breit-Wigner form as expected, whereas the $t$-channel $K^*$-exchange is necessary to explain the long tail in the total cross section.

Figs. 6(a), 6(b) and 6(c) show the results for differential cross sections of the reaction $\pi^- + p \rightarrow \Lambda + K^0$ at the pion beam momentum 0.980 GeV/c, 1.13 GeV/c and 1.455 GeV/c respectively. The experimental data of Knasel et al. [21] and Saxon et al. [22] are shown in the figures with error bars. One sees that the theoretical results are in roughly good agreement with the experimental data.

We mention here that in the work of Brown et al. [23], the isospin-averaged total cross section of $\pi^+ + N \rightarrow \Sigma + K$ is parametrized in the Breit-Wigner form by including the $N(1650)$, the $N(1710)$, the $N(1720)$. The interference terms between the resonances are not taken into account. Since the amplitudes are not given, they can not study the differential cross sections.

Our parametrization for the $\pi^- p \rightarrow \Lambda K^0$ cross section is given by

$$\sigma(\pi^- p \rightarrow \Lambda K^0) = \frac{0.007665(\sqrt{s} - 1.613)^{0.1341}}{(\sqrt{s} - 1.720)^2 + 0.007826} \text{ mb},$$

for $\sqrt{s}$ smaller than the $\Lambda K$ production threshold of 1.613 GeV, the above cross section is zero. Other isospin channels can be related to the reaction of $\pi^- p \rightarrow \Lambda K^0$ by eqs. (17) and (18).

### 4.2 $\pi + N \rightarrow \Sigma + K$ reactions

The total and differential cross sections of the $\pi^+ + p \rightarrow \Sigma^+ + K^+$ reaction are given by eqs. (47) and (44), respectively. The expressions for other isospin dependent reactions can be written in the same way. The amplitudes for each channel are shown in eqs. (40)-(43).
In evaluation of the amplitudes we need eight coupling constants (i.e. $g_{\pi NN(1710)}$, $g_{\pi NN(1720)}$, $g_{\pi N\Delta(1920)}$, $g_{K\Sigma N(1710)}$, $g_{K\Sigma N(1720)}$, $g_{K^*\Sigma N}$ and $f_{K^*\Sigma K\pi}$) and cutoffs. Among these eight coupling constants, three (i.e. $g_{\pi NN(1710)}$, $g_{\pi NN(1720)}$ and $f_{K^*\Sigma K\pi}$) are already given in table 1. The coupling constant and the cut off parameter of the $\Sigma - N - K^*(892)$ vertex is assumed to be the same as $\Lambda - N - K^*(892)$ vertex in order to introduce a minimum of free parameters, i.e. $g_{K^*\Sigma N}$=$g_{K^*\Sigma K\pi}$=0.45. The other four coupling constants (i.e. $g_{\pi N\Delta(1920)}$, $g_{K\Sigma N(1710)}$, $g_{K\Sigma N(1720)}$ and $g_{K\Sigma \Delta(1920)}$) are determined by comparing eqs. (33)-(39) given in appendix 5.2 with the experimental branching ratios listed in § 2. (The values of $g_{\pi N\Delta(1920)}$ and $g_{K\Sigma \Delta(1920)}$ obtained in this way will be called “set 2”, the reason will get clear below.) By using the cutoff value $\Lambda_C = 0.8$ GeV for the $N(1710)$, the $N(1720)$ as in § 3.1, and $\Lambda_C = 0.5$ GeV for the $\Delta(1920)$, these four coupling constants are shown in table 2.

Unlike the $\pi + N \rightarrow \Lambda + K$ reactions described in § 3.1 and § 4.1, the resonances $N(1710)$, $N(1720)$, $\Delta(1920)$ contribute to these reactions, due to the mixture of the isospin-$\frac{1}{2}$ of the nucleon resonances ($N(1710)$ and $N(1720)$), and isospin-$\frac{3}{2}$ of the $\Delta(1920)$ resonance, the different isospin channels of the $\pi + N \rightarrow \Sigma + K$ reactions differ not only by a overall factor but also in shape. This point can be seen from eqs. (33)-(39). In these equations the amplitudes $M_{a2}$, $M_{b2}$, $M_{c2}$ and $M_{d2}$ contribute to different isospin channels of the $\pi + N \rightarrow \Sigma + K$ reactions with different weights. Namely the $\Delta(1920)$ and $K^*(892)$-exchange distinguish different isospin channels of $\pi + N \rightarrow \Sigma + K$ reactions. The role of the $\Delta(1920)$ in the $\pi + N \rightarrow \Sigma + K$ reactions was studied in our previous work [13].

For the $\pi + N \rightarrow \Sigma + K$ reactions three experimental data sets exist in the literature:

1. $\pi^+ + p \rightarrow \Sigma^+ + K^+$,
2. $\pi^+ + p \rightarrow \Sigma^- + K^+$,
3. $\pi^- + p \rightarrow \Sigma^0 + K^0$.

All these three channels have indeed different shapes, i.e. a different energy dependence of the total cross section. First, we discuss the reaction $\pi^+ + p \rightarrow \Sigma^+ + K^+$. In this channel the initial and the final states have charge two. Therefore only the double charged resonance $\Delta(1920)^{++}$ and the $K^*(892)$-exchange can contribute (see eq. (33)). The result is shown in the dashed line of fig. 7(a). The cross section is considerably underestimated. This means that the contributions from other $\Delta$ resonances with masses near 1.9 GeV can not be neglected[2]. Thus we consider the $\Delta(1920)$ as an effective resonance that simulates also other $\Delta$ resonances in this energy region that contribute to this reaction.

In this work we scale the two coupling constants $g_{K\Sigma \Delta(1920)}$ and $g_{\pi N\Delta(1920)}$ of the $\Delta(1920)$ to account for other $\Delta$ resonance contributions by fitting the $\pi^+ + p \rightarrow \Sigma^+ + K^+$ experimental data. Their values are also shown in table 2. These two scaled coupling constants $g_{K\Sigma \Delta(1920)}$ and $g_{\pi N\Delta(1920)}$ are also used in all other channels of our calculations. All other coupling constants remain unchanged. The calculations with these two scaled coupling constants are called “set 1”. The results with these two coupling constants unscaled as determined from the branching ratios are also shown in the fig. 7 for reference ( “set 2”). We believe that the “set 1” parameters produce reliable results and should be

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[2] In the compilation of “Review of Particle Properties” (Phys. Rev. D50 1173 (1994)) there are $\Delta(1900)$, $\Delta(1905)$, $\Delta(1910)$, $\Delta(1930)$, $\Delta(1940)$, $\Delta(1950)$ contribute to this reaction, however the value of branching ratios are still not yet determined.
used for kaon production in heavy ion collisions.

Fig. 7(b) shows the result of the \( \pi^- + p \rightarrow \Sigma^- + K^+ \) reaction. We find that “set 1” results are in good agreement with the data, especially for the second peak, while “set 2” can not reproduce the second peak. This means that the two scaled coupling constants give a reliable description also of these data, although fitted to another reaction.

According to eq. (77) and eq. (78) the cross sections for \( \pi^0 + n \rightarrow \Sigma^- + K^+ \) and \( \pi^+ + n \rightarrow \Sigma^0 + K^+ \) reactions are the same as for the \( \pi^- + p \rightarrow \Sigma^0 + K^0 \) reaction. We thus compare the calculations of the reactions \( \pi^0 + n \rightarrow \Sigma^- + K^+ \) and \( \pi^+ + n \rightarrow \Sigma^0 + K^+ \) with the experimental data for \( \pi^- + p \rightarrow \Sigma^0 + K^0 \) in fig. 7(c). From fig. 7(c) we see that these reactions are also in good agreement with the experimental data. From figs. 7(a)-7(c) we see that the experimental data for different isospin channels have indeed a different shape. The theoretical calculations reproduce these different shapes. For the reaction \( \pi^0 + p \rightarrow \Sigma^0 + K^+ \) no data are available. Since this cross section is important for heavy ion collisions, we give the results for “set 1” parameters in eq(86).

Since in the present approach we scaled the two coupling constants to take into account other \( \Delta \) resonances around 1.9 GeV, we can not demand that the differential cross section can also be reproduced. The differential cross section strongly depends on the quantum numbers of the resonance. The differential cross section can be studied in the future if the branching ratios of the relevant resonances are well determined.

In order to use these cross sections in heavy ion collisions, we parameterize all the channels based on “set 1” parameters as follows:

They are:

\[
\begin{align*}
\sigma(\pi^+ p \rightarrow \Sigma^+ K^+) &= \frac{0.03591(\sqrt{s} - 1.688)^{0.9541}}{(\sqrt{s} - 1.890)^2 + 0.01548} + \frac{0.1594(\sqrt{s} - 1.688)^{0.01056}}{(\sqrt{s} - 3.000)^2 + 0.9412} \text{ mb}, \quad (83) \\
\sigma(\pi^- p \rightarrow \Sigma^- K^+) &= \frac{0.009803(\sqrt{s} - 1.688)^{0.6021}}{(\sqrt{s} - 1.742)^2 + 0.006538} + \frac{0.006521(\sqrt{s} - 1.688)^{1.4728}}{(\sqrt{s} - 1.940)^2 + 0.006248} \text{ mb}, \quad (84) \\
\sigma(\pi^+ n \rightarrow \Sigma^0 K^+) &= \sigma(\pi^0 n \rightarrow \Sigma^- K^+) = \frac{0.05014(\sqrt{s} - 1.688)^{1.2878}}{(\sqrt{s} - 1.730)^2 + 0.006455} \text{ mb}, \quad (85) \\
\sigma(\pi^0 p \rightarrow \Sigma^0 K^+) &= \frac{0.003978(\sqrt{s} - 1.688)^{0.5848}}{(\sqrt{s} - 1.740)^2 + 0.006670} + \frac{0.04709(\sqrt{s} - 1.688)^{2.1650}}{(\sqrt{s} - 1.905)^2 + 0.006358} \text{ mb}, \quad (86)
\end{align*}
\]

where, all parametrizations given above should be understood to be zero below the threshold \( \sqrt{s} \leq 1.688 \) GeV. These parametrizations are especially useful for kaon production simulation codes for those channels where no experimental data are available.

4.3 \( \pi + \Delta(1232) \rightarrow \Lambda + K \) reactions

The total and differential cross sections of the reaction \( \pi^- \Delta^{++} \rightarrow \Lambda K^+ \) are given by eqs. (60) and (62), respectively. The same expressions for other isospin dependent reactions can be written in the same way. The amplitudes for each channels are given by eqs. (56)-(58).
We need six coupling constants (i.e. \( g_{\pi N(1650)} \), \( g_{\pi N(1710)} \), \( g_{\pi N(1720)} \), \( g_{K\Sigma N(1650)} \), \( g_{K\Delta N(1710)} \) and \( g_{K\Sigma N(1720)} \)), three of them (i.e. \( g_{K\Sigma N(1650)} \), \( g_{K\Sigma N(1710)} \) and \( g_{K\Sigma N(1720)} \)) are given in \( \S \) 4.1 and listed in table 1. Three other coupling constants are obtained from the experimental data listed in \( \S \) 2 and eqs. (103)-(105) given in appendix 5.3. By choosing the cutoff values as: \( \Lambda_c = 0.8 \) GeV for \( N(1650) \), \( N(1710) \) and \( N(1720) \), i.e. in the same way as in \( \S \) 4.1 and \( \S \) 4.2, we obtain these three coupling constants as listed in table 3.

Since all the resonances that contribute to the reactions \( \pi + \Delta \rightarrow \Lambda + K \) have isospin-\( \frac{1}{2} \), the different isospin dependent reaction channels have all the same shape (see eqs. (53)-(55). This is the same as for the \( \pi + N \rightarrow \Lambda + K \) reactions.

We find that the contribution from the \( N(1710) \) resonance is larger (about 60\% of the total cross section at peak position) than from other resonances.

For this reaction one has four possible relative sign combinations. Eq.(87) shows the total cross section of the reaction \( \pi^- \Delta^{++} \rightarrow \Lambda K^+ \) without interference terms. A careful study did show that the interference terms do not affect the final results appreciably for this reaction.

\[
\sigma(\pi^- \Delta^{++} \rightarrow \Lambda K^+) = \frac{0.006545(\sqrt{s} - 1.613)^{0.7866}}{(\sqrt{s} - 1.720)^2 + 0.004852} \text{ mb},
\]

This parametrization should be understood to be zero below the threshold (\( \sqrt{s} \leq 1.613 \) GeV). This parametrization is useful for codes which simulate kaon production since no experimental data are available.

We find that the angular distribution \( \pi^- \Delta^{++} \rightarrow \Lambda K^+ \) is almost constant for all bombarding energies.

4.4 \( \pi + \Delta(1232) \rightarrow \Sigma + K \) reactions

We need six coupling constants (i.e. \( g_{\pi N(1710)} \), \( g_{\pi N(1720)} \), \( g_{K\Sigma N(1710)} \), \( g_{K\Sigma N(1720)} \), \( g_{K^*(892)\Sigma \Delta} \), \( f_{K^*(892)K\pi} \)), three of them (\( g_{K\Sigma N(1710)} \), \( g_{K\Sigma N(1720)} \), \( f_{K^*(892)K\pi} \)) are given in \( \S \) 4.2 and table 2. Two coupling constant \( (g_{\pi N(1710)} \), \( g_{\pi N(1720)} \)) are given in \( \S \) 4.3 and table 3. One remaining coupling constant \( g_{K^*(892)\Sigma \Delta} \) is assumed to be \( \sqrt{3}g_{K^*(892)\Sigma N} \) where \( \sqrt{3} \) comes from the different normalization of the operator \( \vec{r} \) for isospin-\( \frac{1}{2} \) and \( \vec{T} \) for isospin-\( \frac{3}{2} \). The cut off parameter \( \Lambda_c \) for vertex \( K^*(892)\Sigma \Delta \) is the same as \( K^*(892)YN \) used in \( \S \) 4.1 and \( \S \) 4.2.

According to the different reaction amplitudes given by eqs. (58)-(75), one sees that the \( K^*(892)\)-exchange distinguishes channels with different isospin projections. Hence, among eight possible reaction channels there are five different independent reactions which differ not only by a overall factors, they are

1. \( \pi^- \Delta^{++} \rightarrow \Sigma^0 + K^+ \),
2. \( \pi^0 \Delta^0 \rightarrow \Sigma^- + K^+ \),
3. \( \pi^+ \Delta^0 \rightarrow \Sigma^0 + K^+ \),
4. \( \pi^+ \Delta^- \rightarrow \Sigma^- + K^+ \) and
5. \( \pi^0 \Delta^{++} \rightarrow \Sigma^+ + K^+ \).

We find that the cross section of the reaction \( \pi^0 \Delta^{++} \rightarrow \Sigma^+ K^+ \), to which only the \( K^*(892)\)-exchange contribute (see eq.(77)), is negligible (smaller than 0.01 mb at its
maximum value). We plot the reactions (1)-(3) in fig. 8. The reaction $\pi^+\Delta^- \rightarrow \Sigma^-K^+$ has a maximum cross section of 0.15 mb with the peak position located at $\sqrt{s} = 1.75$ GeV. Due to the small contributions from the $K^*(892)$-exchange, the shape from the different reactions of (1) - (4) do not differ much especially at the peak positions.

We also find that the differential cross sections of the reactions $\pi^+\Delta(1232) \rightarrow \Sigma^+K^+$ are almost constant as a function of $\cos\theta_{cm}$ for all beam energies.

Now, we are in a position to give parametrizations of total cross sections $\pi^-\Delta^{++} \rightarrow \Sigma^0K^+$, $\pi^0\Delta^0 \rightarrow \Sigma^-K^+$, $\pi^+\Delta^0 \rightarrow \Sigma^0K^+$, and $\pi^+\Delta^- \rightarrow \Sigma^-K^+$ which are sufficient to describe all channels given in eqs. (68)-(75). Channel (5) is omitted, since only $K^*(892)$-exchange contributes, which is negligible as mentioned above. Since the signs of interference terms cannot be fixed by experimental data, we parameterize the results obtained without interference terms.

$$
\sigma(\pi^-\Delta^{++} \rightarrow \Sigma^0K^+) = 0.004959(\sqrt{s} - 1.688)^{0.7785} \frac{1}{(\sqrt{s} - 1.725)^2 + 0.008147} \text{ mb}, \quad (88)
$$

$$
\sigma(\pi^0\Delta^0 \rightarrow \Sigma^-K^+) = 0.006964(\sqrt{s} - 1.688)^{0.8140} \frac{1}{(\sqrt{s} - 1.725)^2 + 0.007713} \text{ mb}, \quad (89)
$$

$$
\sigma(\pi^+\Delta^0 \rightarrow \Sigma^0K^+) = 0.002053(\sqrt{s} - 1.688)^{0.9853} + 0.3179(\sqrt{s} - 1.688)^{0.9025} \frac{1}{(\sqrt{s} - 2.675)^2 + 44.88} \text{ mb}, \quad (90)
$$

$$
\sigma(\pi^+\Delta^- \rightarrow \Sigma^-K^+) = 0.01741(\sqrt{s} - 1.688)^{1.2078} \frac{1}{(\sqrt{s} - 1.725)^2 + 0.003777} \text{ mb}, \quad (91)
$$

The parametrizations for the cross sections $\sigma(\pi\Delta \rightarrow \Sigma K)$ in eqs. (88) to (91) should be understood to be zero below the threshold ($\sqrt{s} \leq 1.688$ GeV). These parametrizations are useful for codes which simulate kaon production since no experimental data are available.

4.5 Summary

In this paper, we have calculated the $\pi N \rightarrow YK$ and $\pi\Delta \rightarrow YK$ cross sections using a resonance model on the hadronic level. All resonances with known experimental branching ratios to $\pi N$ and $YK$ are included.

We constructed the interaction Lagrangians needed in the description of the vertices. For the reactions $\pi N \rightarrow \Lambda K$ our theoretical calculations are in reasonable good agreement with the experimental data both for differential and total cross sections. For the $\pi N \rightarrow \Sigma K$ reactions, we scaled the coupling constant related to the $\Delta(1920)$ resonance, to take into account also other $\Delta$ resonances in this energy region. To get a complete understanding of these reactions a better determination of the branching ratios also of other $\Delta$ resonances is necessary. For the $\pi\Delta \rightarrow YK$ reactions, we predict the total cross sections and find that the differential cross sections are almost isotopic.

For all different reaction channels with different charges of the participants the total cross sections are parameterized for the application in heavy ion collisions.
Acknowledgement: The authors express their thanks to Prof. R. Vinh Mau, Prof. H. Müther and Prof. E. Oset for useful discussions and are indebted to Prof. K. W. Schmid for providing the code used for fitting the theoretically calculated cross sections to the simple parametrisations given in this work.
5 Appendix

In this appendix, we list all the theoretical branching ratios of the resonances used in this work.

5.1 Branching ratios for $\pi + N \rightarrow \Lambda + K$ reactions

The branching ratios averaged over initial and summed over final spin and isospin states of the baryon resonances and $K^*(892)$ are given: (form factors have to be inserted at each vertex see $\S$ 3.1)

$$\Gamma(N(1650) \rightarrow N\pi) = 3 \frac{g_{\pi NN(1650)}^2}{4\pi} \left( \frac{m_N^2 + \vec{p}_N^2 + m_N}{m_{N(1650)}} \right) |\vec{p}_N|,$$  \hspace{1cm} (92)

with $|\vec{p}_N| = \frac{\lambda^4(m_{N(1650)}^2, m_N^2, m_{\pi}^2)}{2m_N(1650)}$

$$\Gamma(N(1710) \rightarrow N\pi) = 3 \frac{g_{\pi NN(1710)}^2}{12\pi m_N(1710)} \frac{m_{\pi}^2}{m_{N(1710)}} \left( \sqrt{m_N^2 + \vec{p}_N^2} - m_N \right) |\vec{p}_N|,$$ \hspace{1cm} (93)

with $|\vec{p}_N| = \frac{\lambda^4(m_{N(1710)}^2, m_{\pi}^2, m_N^2)}{2m_N(1710)}$

$$\Gamma(N(1720) \rightarrow N\pi) = 3 \frac{g_{\pi NN(1720)}^2}{12\pi m_N(1720)} \left( \sqrt{m_N^2 + \vec{p}_N^2} + m_N \right) |\vec{p}_N|,$$ \hspace{1cm} (94)

with $|\vec{p}_N| = \frac{\lambda^4(m_{N(1720)}^2, m_{\pi}^2, m_N^2)}{2m_N(1720)}$

$$\Gamma(N(1650) \rightarrow \Lambda K) = \frac{g_{\Lambda K N(1650)}^2}{4\pi} \frac{m_{\Lambda}^2 + \vec{p}_\Lambda^2 + m_{\Lambda}}{m_{N(1650)}} |\vec{p}_\Lambda|,$$ \hspace{1cm} (95)

with $|\vec{p}_\Lambda| = \frac{\lambda^2(m_{N(1650)}^2, m_{\Lambda}^2, m_K^2)}{2m_N(1650)}$

$$\Gamma(N(1710) \rightarrow \Lambda K) = \frac{g_{\Lambda K N(1710)}^2}{4\pi} \frac{m_{\Lambda}^2 + \vec{p}_\Lambda^2 - m_{\Lambda}}{m_{N(1710)}} |\vec{p}_\Lambda|,$$ \hspace{1cm} (96)

with $|\vec{p}_\Lambda| = \frac{\lambda^2(m_{N(1710)}^2, m_{\Lambda}^2, m_K^2)}{2m_N(1710)}$

$$\Gamma(N(1720) \rightarrow \Lambda K) = \frac{g_{\Lambda K N(1720)}^2}{12\pi m_N(1720)} \frac{m_{\Lambda}^2 + \vec{p}_\Lambda^2 + m_{\Lambda}}{m_{N(1720)}} |\vec{p}_\Lambda|,$$ \hspace{1cm} (97)

with $|\vec{p}_\Lambda| = \frac{\lambda^2(m_{N(1720)}^2, m_{\Lambda}^2, m_K^2)}{2m_N(1720)}$

$$\Gamma(K^*(892) \rightarrow K\pi) = 3 \frac{f_{K^*(892)K\pi}^2}{4\pi} \frac{2}{3m_{K^*(892)}^2} |\vec{p}_K|^3,$$ \hspace{1cm} (98)

with $|\vec{p}_K| = \frac{\lambda^2(m_{K^*(892)}^2, m_K^2)}{2m_{K^*(892)}}$. 


\[ |\vec{p}_K| = \frac{\lambda^2_{\pi K}(m_K^2, m_K^2, m_K^2)}{2m_K^2} \]

5.2 Branching ratios for \( \pi + N \rightarrow \Sigma + K \) reactions

Besides eqs. (23)-(24) and eq. (28), the decay width needed in determining the effective coupling constants are given by:

\[ \Gamma(\Delta(1920) \rightarrow N\pi) = g_{\pi N\Delta(1920)}^2 \frac{\sqrt{m_N^2 + p_N^2 + m_N}}{m_{\Delta(1920)} m_{\pi}^2} |\vec{p}_N|^3, \]  
with \[ |\vec{p}_N| = \frac{\lambda^2_{\pi N}(m_{\Delta(1920)}, m_{\Delta(1920)}, m_{\pi}^2)}{2m_{\Delta(1920)}} \]

\[ \Gamma(N(1710) \rightarrow \Sigma K) = 3 g_{K\Sigma N(1710)}^2 \frac{\sqrt{m_{\Sigma}^2 + p_{\Sigma}^2 - m_{\Sigma}}}{m_{N(1710)} m_{\pi}^2} |\vec{p}_{\Sigma}|^3, \]  
with \[ |\vec{p}_{\Sigma}| = \frac{\lambda^2_{\pi \Sigma}(m_{N(1710)}, m_{\Sigma}, m_{\pi}^2)}{2m_{N(1710)}} \]

\[ \Gamma(N(1720) \rightarrow \Sigma K) = 3 g_{K\Sigma N(1720)}^2 \frac{\sqrt{m_{\Sigma}^2 + p_{\Sigma}^2 + m_{\Sigma}}}{m_{N(1720)} m_{K}^2} |\vec{p}_{\Sigma}|^3, \]  
with \[ |\vec{p}_{\Sigma}| = \frac{\lambda^2_{\pi \Sigma}(m_{N(1720)}, m_{\Sigma}, m_{\pi}^2)}{2m_{N(1720)}} \]

5.3 Branching ratios for \( \pi + \Delta(1232) \rightarrow \Lambda + K \) reactions

\[ \Gamma(N(1650) \rightarrow \Delta\pi) = 2 g_{\pi \Delta N(1650)}^2 \frac{m_{N(1650)} (E_{\Delta} - m_{\Delta})}{6\pi m_{\pi}^2 m_{\Delta}^2} |\vec{p}_{\Delta}|^3, \]  
with \[ |\vec{p}_{\Delta}| = \frac{\lambda^2_{\pi \Delta}(m_{N(1650)}, m_{\Delta}^2, m_{\pi}^2)}{2m_{N(1650)}} \]

\[ \Gamma(N(1710) \rightarrow \Delta\pi) = 2 g_{\pi \Delta N(1710)}^2 \frac{m_{N(1710)} (E_{\Delta} + m_{\Delta})}{6\pi m_{\pi}^2 m_{\Delta}^2} |\vec{p}_{\Delta}|^3, \]  
with \[ |\vec{p}_{\Delta}| = \frac{\lambda^2_{\pi \Delta}(m_{N(1710)}, m_{\Delta}^2, m_{\pi}^2)}{2m_{N(1710)}} \]

\[ \Gamma(N(1720) \rightarrow \Delta\pi) = 2 g_{\pi \Delta N(1720)}^2 \frac{m_{N(1720)} (E_{\Delta} + m_{\Delta})}{36\pi} |\vec{p}_{\Delta}|^3, \]
\[
\cdot \left( \frac{m_\Delta}{m_{N(1720)}} \right) \left[ \left( \frac{E_\Delta}{m_\Delta} \right) - 1 \right] \left[ 2 \left( \frac{E_\Delta}{m_\Delta} \right)^2 - 2 \left( \frac{E_\Delta}{m_\Delta} \right) + 5 \right],
\]

with \(|\vec{p}_\Delta| = \frac{\sqrt{\frac{m_{N(1720)}^2}{2m_{N(1720)}}}}{\lambda \frac{m_\Delta^2}{m_{N(1720)}^2} m_\pi^2 m_\Delta}\).
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Table 1: Calculated coupling constants

| $B^*$ (resonance) | $\Gamma^{full}$ (MeV) | $\Gamma_N$ (%) | $g_{\pi NB^*}^2$ | $\Gamma_{\Lambda K}$ (%) | $g_{\Lambda K B^*}^2$ |
|-------------------|----------------------|----------------|------------------|------------------------|------------------|
| $N(1650)$         | 150                  | 70.0           | 1.41             | 7.0                    | $6.40 \times 10^{-1}$ |
| $N(1710)$         | 100                  | 15.0           | 2.57             | 15.0                   | $4.74 \times 10^{+1}$ |
| $N(1720)$         | 150                  | 15.0           | $5.27 \times 10^{-2}$ | 6.5                    | 3.91             |

$K^*_{(892)}$ $K\pi$ $g_{K^*_{(892)}\Lambda N}^2$

$6.89 \times 10^{-1}$

$(\Gamma=50$ MeV, $\Gamma_{K\pi} = 100\%)$

Table 1
The calculated coupling constants and the experimental branching ratios used for the calculations of the coupling constants.
Table 2: Calculated and fitted coupling constants

| $B^*$ (resonance) | $\Gamma^{full}(MeV)$ | $\Gamma_{N\pi}(\%)$ | $g_{NB}^2$ | $\Gamma_{\Sigma K}(\%)$ | $g_{K\Sigma B}^2$ |
|-------------------|----------------------|---------------------|-----------|------------------|-----------------|
| $N(1710)$         | 100                  | 15.0                | (see table 1) | 6.0              | $4.5 \times 10^{+1}$ |
| $N(1720)$         | 150                  | 15.0                | (see table 1) | 3.5              | 3.15            |
| $\Delta(1920)$ (set 1) | $-$            | $-$                | (1.44)      | $-$              | (3.83)         |
| $\Delta(1920)$ (set 2) | 200            | 12.5                | $4.17 \times 10^{-1}$ | 2.0              | 1.11            |

\[
\frac{f_{K^*(892)K\pi}^2}{g_{K^*(892)\Sigma N}^2} = 2.03 \times 10^{-1} (=g_{K^*(892)\Lambda N}^2)
\]

Table 2
The calculated or fitted coupling constants and the data used for the calculations. The values in brackets stand for the coupling constants obtained by fitting to the total cross section for the $\pi^+ p \rightarrow \Sigma^+ K^+$ reaction (set 1). The value of $\frac{g_{K^*(892)\Sigma N}^2}{4\pi}$ is the fitted value to the $\pi^+ p \rightarrow \Sigma^+ K^+$ channel when zero tensor coupling for $K^*(892)\Sigma N$ interaction ($\kappa = 0$) is applied and the rescaled coupling constants for $\Delta(1920)$ are used.
Table 3: Calculated coupling constants

| $B^\ast$(resonance) | $\Gamma_{\text{full}}(MeV)$ | $\Gamma_\Delta(\%)$ | $g_{\pi B^*}^2$ | $\Gamma_{\Lambda K}(\%)$ | $g_{K\Lambda B^*}^2$ |
|---------------------|-----------------|------------------|----------------|-----------------|----------------|
| $N(1650)$           | 150             | 5.0              | $6.56 \times 10^{-1}$ | 7.0             | (see table 1) |
| $N(1710)$           | 100             | 17.5             | $1.85 \times 10^{-2}$ | 15.0            | (see table 1) |
| $N(1720)$           | 150             | 10.0             | $1.12 \times 10^{+1}$ | 6.5             | (see table 1) |

Table 3
The calculated coupling constants and the experimental branching ratios used for the calculations of the coupling constants.
Figure captions

Fig. 1
The processes contributing to the $\pi N \rightarrow \Lambda K$ reactions. The diagrams are corresponding to the different intermediate resonance states: (a) : $N(1650) I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$ (b) : $N(1710) \frac{1}{2}(\frac{1}{2}^+)$, (c) : $N(1720) \frac{1}{2}(\frac{3}{2}^+)$ and (d) : t-channel $K^*(892)$-exchange, respectively.

Fig. 2
The processes contributing to the $\pi N \rightarrow \Sigma K$ reactions. The diagrams are corresponding to the different intermediate resonance states: (a) : $N(1710) I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$, (b) : $N(1720) \frac{1}{2}(\frac{3}{2}^+)$, (c) : $\Delta(1920) \frac{3}{2}(\frac{3}{2}^-)$ and (d) : t-channel $K^*(892)$ exchange, respectively.

Fig. 3
The processes contributing to the $\pi \Delta \rightarrow \Lambda K$ reactions. The diagrams are corresponding to the different intermediate resonance states: (a) : $N(1650) I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$ (b) : $N(1710) \frac{1}{2}(\frac{1}{2}^+)$ and (c) : $N(1720) \frac{1}{2}(\frac{3}{2}^+)$, respectively.

Fig. 4
The processes contributing to the $\pi \Delta \rightarrow \Sigma K$ reactions. The diagrams are corresponding to the different intermediate resonance states: (a) : $N(1710) I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$, (b) : $N(1720) \frac{1}{2}(\frac{3}{2}^+)$ and (d) : t-channel $K^*(892)$ exchange, respectively.

Fig. 5
The total cross section for the reaction $\pi^- p \rightarrow \Lambda K^0$. The experimental data with error bars are taken from [20]. The theoretical calculation are shown with the solid lines for the results without interference terms (fig. (a)) and with interference terms (fig. (b)).

Fig. 6
The differential cross section for the reaction $\pi^- p \rightarrow \Lambda K^0$ as a function of $\cos \theta$ in the c.m. frame at different bombarding energies. (a), (b) and (c) show the reactions at pion beam momenta 0.980 GeV/c (corresponding to $\sqrt{s} = 1.66$ GeV), 1.13 GeV/c ($\sqrt{s} = 1.742$ GeV) and 1.455 GeV/c ($\sqrt{s} = 1.908$ GeV), respectively. The experimental data with error bars for (a) and (b) are taken from [21]. The experimental data with error bars for (c) are from [22]. The theoretical calculations (solid lines) include interference terms.
Fig. 7
(a). The calculated and experimental [20] total cross sections for the $\pi^+p \rightarrow \Sigma^+K^+$ ($\pi^-n \rightarrow \Sigma^-K^0$) reaction. The solid line and the dashed line stand for the set 1 and set 2 parameters, respectively. Set 1 are the relevant coupling constants for $\Delta(1920)$ and $g_{K^*(892)K\pi}$ adjusted to the cross section $\pi^+p \rightarrow \Sigma^+K^+$ with the $\Delta(1920)$ as an s-channel intermediate state resonance and other coupling constants are determined from the branching ratios. Set 2 uses the same value of $g_{K^*(892)K\pi}$ as that of set 1 and all other coupling constants determined from the branching ratios.
(b). The calculated total cross sections for the $\pi^-p \rightarrow \Sigma^-K^+$ ($\pi^+n \rightarrow \Sigma^+K^0$) reaction.
(c). The calculated total cross sections for the $\pi^+n \rightarrow \Sigma^0K^+$ and $\pi^0n \rightarrow \Sigma^-K^+$ ($\pi^0p \rightarrow \Sigma^+K^0$ and $\pi^-p \rightarrow \Sigma^0K^0$) reactions.

Fig. 8
(a). The calculated total cross sections for the $\pi^-\Delta^{++} \rightarrow \Sigma^0K^+$ ($\pi^+\Delta^- \rightarrow \Sigma^0K^0$) reactions. The solid line and the dashed lines stand for the results without and with the inclusion the interference terms, respectively. Note that the largest and the smallest results are displayed for the four possibilities arising from the possible signs of the coupling constants and thus the interference terms.
(b). The calculated total cross sections for the $\pi^0\Delta^0 \rightarrow \Sigma^-K^+$ ($\pi^0\Delta^+ \rightarrow \Sigma^+K^0$) reactions.
(c). The calculated total cross sections for the $\pi^+\Delta^0 \rightarrow \Sigma^0K^+$ ($\pi^-\Delta^+ \rightarrow \Sigma^0K^0$) reactions.
Fig. 1

\( \Lambda \quad K \)

\( \text{N(1650) } \frac{1}{2}(\frac{1}{2}^-) \)

\( N \quad \pi \)

(a)

\( \Lambda \quad K \)

\( \text{N(1710) } \frac{1}{2}(\frac{1}{2}^+) \)

\( N \quad \pi \)

(b)

\( \Lambda \quad K \)

\( \text{N(1720) } \frac{1}{2}(\frac{3}{2}^+) \)

\( N \quad \pi \)

(c)

\( \Lambda \quad K \)

\( \text{K^*(892) } \frac{1}{2}(1^-) \)

\( N \quad \pi \)

(d)
Fig. 2

\[
\begin{array}{llll}
\Sigma & \quad K & \Sigma & \quad K & \Sigma & \quad K \\
N(1710)_{\frac{1}{2}(1^+)} & \quad \pi & \quad N(1720)_{\frac{1}{2}(3^+)} & \quad \pi & \quad \Delta(1920)_{\frac{3}{2}(3^+)} & \quad \pi \\
N & \quad \pi & \quad N & \quad \pi & \quad N & \quad \pi \\
\end{array}
\]

\[
\begin{array}{ll}
\Sigma & \quad K \\
N & \quad \pi \\
K^*(892)_{\frac{1}{2}(1^-)} & \quad \pi \\
\end{array}
\]
Fig. 3

(a) \( \Lambda K N(1650) \frac{1}{2}(\frac{1}{2}^-) \)

(b) \( \Lambda K N(1710) \frac{1}{2}(\frac{1}{2}^+) \)

(c) \( \Lambda K N(1720) \frac{1}{2}(\frac{3}{2}^+) \)
Fig. 4

(a) $\Sigma \rightarrow K \Delta \pi$, $N(1710)_{1/2}^{1/2}(1^+)$

(b) $\Sigma \rightarrow K \Delta \pi$, $N(1720)_{1/2}^{3/2}(3^+)$

(c) $\Sigma \rightarrow K \Delta \pi$, $K^*(892)_{1/2}^{1/2}(1^-)$
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