η_c production at LHC and indications on the understanding of J/ψ production

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We present a complete evaluation for the prompt η_c production at the LHC at next-to-leading order in \(\alpha_s\) in nonrelativistic QCD. By assuming heavy quark spin symmetry, the study of η_c production results in a very strong constraint on the upper bound of the color-octet long distance matrix element \(\langle O^{J/\psi}(S_0^{[6]}) \rangle\) of J/ψ. We find this upper bound is consistent with our previous study of the J/ψ yield and polarization and can give good descriptions for the measurements, but in conflict with most other theoretical studies. This may provide important information for understanding the mechanism of charmonium production.

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\begin{itemize}
\item Introduction.—The production of heavy quarkonium at hadron colliders is an ideal process to study QCD in both perturbative and nonperturbative domains. In recent years, the study on this subject has been significantly improved. On the experiment side, abundant LHC data of the yields and polarizations of prompt heavy quarkonium production are accumulated. On the theory side, one of the main improvements are the higher order calculations in the framework of nonrelativistic QCD (NRQCD) factorization \([1]\), where the inclusive production cross section of a quarkonium state Q in pp collisions can be expressed as
\end{itemize}

\[ \frac{d\sigma_{pp\to Q+X}}{d\tau} = \sum_n \frac{d\sigma_{pp\to Q\bar{Q}[n]+X}}{d\tau} \langle O^Q(n) \rangle. \] (1)

Here, \(d\sigma_{pp\to Q\bar{Q}[n]+X}\) are the short-distance coefficients (SDCs) for producing a heavy quark pair Q\bar{Q} with quantum number n, and \(\langle O^Q(n) \rangle\) are the long-distance matrix elements (LDMEs) for Q. The SDCs can be computed in perturbative QCD as partonic cross sections convoluted with parton distributions, and for the large \(p_T\) (the transverse momentum of Q) spectrum, they behave roughly as some powers of \(1/p_T\). The LDMEs, although nonperturbative, can be arranged as a series in powers of \(p_T\) (the relative velocity of the Q and the \(\bar{Q}\) in the rest frame of Q) \([1]\). For example, for the production of J/ψ, the sum over n is usually truncated at order \(v^4\) and only four LDMEs are essential, which are

\begin{itemize}
\item \(\langle O^{J/\psi}(S_1^{[1]}) \rangle \sim O(1)\), \(\langle O^{J/\psi}(S_0^{[6]}) \rangle \sim O(v^3)\),
\item \(\langle O^{J/\psi}(S_1^{[8]}) \rangle \sim O(v^4)\), \(\langle O^{J/\psi}(T_0^{[8]}) \rangle \sim O(v^4)\).
\end{itemize}

(2)

In the last few years, complete next-to-leading order (NLO) QCD corrections for the J/ψ hadroproduction have been accomplished by three groups independently \([2,4]\). Although the three groups obtained consistent SDCs, they had different philosophies on how to fit the color-octet (CO) LDMEs. As a result, the three groups got significantly different CO LDMEs, and gave different predictions/descriptions for the polarization of J/ψ. Specifically, our group found that the J/ψ polarization can be explained by NLO NRQCD \([3]\), whereas the other two groups conclude that they can not explain the polarization data \([4,5]\). More recently, two other groups performed independent fit for the CO LDMEs \([6,7]\), and both concluded that the J/ψ polarization can be understood by a \(S_0^{[6]}\) channel dominance mechanism, which was first proposed as one possibility to explain the J/ψ polarization in Ref. \([2]\) and reemphasized in Ref. \([4]\).

To further test the mechanism of quarkonium production and to clarify the confusion mentioned above, it is crucial to have more measurements. Very recently, the LHCb Collaboration measured the differential cross section of prompt η_c production for the first time \([3]\). This measurement will not only be significant for studying η_c production, but also provide important information for J/ψ production via the heavy quark spin symmetry (HQSS) \([1]\). In this letter, we will study the η_c prompt production at NLO in \(\alpha_s\) within the framework of NRQCD factorization. We find that, by combining the η_c production data and HQSS, the above mentioned confusion for the J/ψ production will tend to become clear.

\begin{itemize}
\item Relationship between J/ψ and η_c production.—Before concentrating on the η_c production, let us first explain in some details why it will provide important clues to the J/ψ production.
\end{itemize}

For the J/ψ production, with a relative large \(p_T\) cutoff (\(p_T > 7\) GeV), our group found that \([2]\) only two linear combinations of the three CO LDMEs in Eq. (2) can be well constrained by fitting the CDF data \([10]\) of the yields
of $J/\psi$ production, which gives
\[ M_0 = \langle O^{J/\psi}(S^0[1]) \rangle + \frac{r_0}{m_c^2} \langle O^{J/\psi}(P^0[1]) \rangle, \]
\[ M_1 = \langle O^{J/\psi}(S^1[1]) \rangle + \frac{r_1}{m_c^2} \langle O^{J/\psi}(P^0[1]) \rangle, \]
where $r_0 = 3.9$ and $r_1 = -0.56$ for the CDF window, and the corresponding values are $M_0 = (7.4 \pm 1.9) \times 10^{-2}$ GeV$^2$ and $M_1 = (0.05 \pm 0.02) \times 10^{-2}$ GeV$^3$. Roughly speaking, the SDCs for the LDMEs $M_0$ and $M_1$ defined in (3) have mainly $p_T^6$ and $p_T^4$ behaviors [2], respectively. These two $p_T$ behaviors dominate the $J/\psi$ production in the region $p_T > 7$ GeV. The coefficient $r_0$ and $r_1$ change slightly with the rapidity interval but almost do not change with the center-of-mass energy $\sqrt{s}$ (see Table I in Ref. [1]). Thus, the CMS yield data [12] for $J/\psi$ production can be also well described by the same LDMEs in Eq. (3). Importantly, we further found that the transversely polarized cross section for direct $J/\psi$ production at NLO is almost proportional to the combined LDME
\[ M'_1 = \langle O^{J/\psi}(S^1[1]) \rangle - 0.52 \langle O^{J/\psi}(P^0[1]) \rangle / m_c^2 \]
for the CDF and CMS window, which is very close to the $M_1$ in (3). Since the value of $M_1$ is much smaller than that of $M_0$ in Eq. (4), one can expect that the polarizations will be dominated by $M_0$ at least in the intermediate $p_T$ region, which tends to give unpolarized results [3]. We emphasize here that the above expectation is independent of the exact values of the three CO LDMEs in Eq. (4), as long as $M_0$ and $M_1$ are fixed by Eq. (3). This can be seen from the fact that, by varying the value of $\langle O^{J/\psi}(S^1[1]) \rangle$ in Table I of Ref. [3], the resulted polarizations are similar and basically unpolarized [3]. Based on Eq. (3), assuming all CO LDMEs to be positive, we updated our results for the polarization of direct $J/\psi$ production together with that of the feeddown contributions from $\chi_c$ and $\psi(2S)$ in Ref. [13], which are roughly consistent with the LHC data. Needless to say, further constraints on the CO LDMEs from data are the main way to test the above solution for $J/\psi$ polarization.

The cross section for $\eta_c$ production can also be expressed as Eq. (1). Similar to the case for $J/\psi$, four LDMEs are needed up to relative order $v^4$ for the direct $\eta_c$ production, which are $\langle O^{\eta_c}(S^0[1]) \rangle$, $\langle O^{\eta_c}(S^1[1]) \rangle$, $\langle O^{\eta_c}(P^0[1]) \rangle$ and $\langle O^{\eta_c}(P^1[1]) \rangle$. The dominant feeddown contribution through $h_c \rightarrow \eta_c$ introduces two other LDMEs at relative order $v^2$: $\langle O^{h_c}(P^1[1]) \rangle$ and $\langle O^{h_c}(S^1[1]) \rangle$. Superficially, it appears that six channels would be involved in the fit to data, but in fact, some of them are not important. The relative importance of these channels should depend on the power counting both in $v$ and in $\delta = m_\varphi / p_T$, where $m_\varphi$ is the mass of the charmonium. The powers of $v$ can be estimated by the velocity scaling rules [1]. The powers of $\delta$ can be determined by QCD factorization for quarkonium production [14], which shows that all channels have a leading power (LP), $p_T^4$, component at the current order in $\alpha_s$. However, because of the relative importance of next-to-leading power (NLP), $p_T^6$, contributions for some channels [13], will behave almost as $p_T^6$ within a large range of $p_T$.

\[ n = \frac{\langle O^{\eta_c}(S^0[1]) \rangle}{\langle O^{\eta_c}(S^1[1]) \rangle} \approx \frac{\langle O^{J/\psi}(S^1[1]) \rangle}{\langle O^{J/\psi}(S^1[1]) \rangle} = 3.9 GeV^3. \] (5)

The theoretical uncertainties by varying $m_c$, $\mu_f$ and $\mu_r$ have been studied thoroughly in our earlier publications [2, 3, 11, 13], where one found that the uncertainties can be estimated by a systematical error of about 30%.

As mentioned above, only the channels $S^0[1]$ and $S^1[1]$ are essential for the $\eta_c$ production at the LHCb window. However, with the fixed value of $\langle O^{\eta_c}(S^0[1]) \rangle$ in Eq. (6), we find that the LHCb data are almost saturated by the contribution from the CS channel, which is denoted by the solid lines in Fig. 1. Similar results have been found in Ref. [20] with only the LO SDCs and a relative smaller CS LDME. The similarity is mainly caused by that the

\[ n = \frac{\langle O^{\eta_c}(S^0[1]) \rangle}{\langle O^{\eta_c}(S^1[1]) \rangle} = \frac{\langle O^{J/\psi}(S^0[1]) \rangle}{\langle O^{J/\psi}(S^0[1]) \rangle} = 3.9 GeV^3. \] (6)
NLO calculation gives only a modest correction factor for $S_0^{[1]}$ channel. Therefore, the saturation can hardly be avoided if one choose the CS LDME as large as that in Eq. (6).

However, the above result does not mean that there is no contribution from the $S_1^{[3]}$ channel. On the one hand, although there are large uncertainties of the data, one can roughly find in Fig. 1 that the slope of data is different from the contribution of $S_0^{[1]}$ channel itself. On the other hand, the value in Eq. (6) is not exact, but with at least an uncertainty of order $v^2 \sim 0.3$ because of modeling of potential, relativistic corrections, HQSS broken, and so on. These uncertainties may generate some room for $S_1^{[3]}$ channel to contribute.

Unfortunately, it is very hard at present to determine the exact value of $\langle O^{\eta_c S_1^{[3]}} \rangle$ due to the large uncertainties from both data and theory. But we are able to give a very safe upper bound for $\langle O^{\eta_c S_1^{[3]}} \rangle$. To this end, we let the data be saturated by the $S_1^{[3]}$ channel only, which gives $\langle O^{\eta_c S_1^{[3]}} \rangle = (1.46 \pm 0.20) \times 10^{-2} \text{ GeV}^3$. Since the value of $\langle O^{\eta_c S_1^{[3]}} \rangle$ has been sufficient amplified, we choose the central value as the upper bound for the LDME. To give a lower bound, we assume the $\langle O^{\eta_c S_1^{[3]}} \rangle$ to be positive $\eta_c$, which should be acceptable because of the following reason. Since the renormalization dependence of LDME $\langle O^{\eta_c S_1^{[3]}} \rangle$ is at higher order in $v^2$, $\langle O^{\eta_c S_1^{[3]}} \rangle$ can be approximated as the probability for the $c\bar{c}$ pair in $S_1^{[3]}$ configuration to evolve into $\eta_c$, which should be positive in general sense. We thus get the range of $\langle O^{\eta_c S_1^{[3]}} \rangle$,

$$0 < \langle O^{\eta_c S_1^{[3]}} \rangle < 1.46 \times 10^{-2} \text{ GeV}^3.$$  \hspace{1cm} (7)

By using the HQSS relation, the result in Eq. (7) can be viewed as another constraint on the three CO LDMEs for $J/\psi$ other than Eq. (6). We thus constrain all three CO LDMEs of $J/\psi$ into a finite range.

As a feedback, the other two CO LDMEs for direct $\eta_c$ production can be estimated by the HQSS relations $\eta_c$,

$$\langle O^{\eta_c S_1^{[3]}} \rangle = \langle O^{J/\psi S_1^{[3]}} \rangle / 3,$$

$$\langle O^{\eta_c S_1^{[1]}} \rangle = 3 \langle O^{J/\psi S_1^{[1]}} \rangle.$$  \hspace{1cm} (8)

As for the feeddown contribution through $h_c \to \eta_c \gamma$, the two relevant LDMEs can be estimated again by the HQSS relations

$$\langle O^{h_c S_1^{[3]}} \rangle = 3 \langle O^{\chi_c S_1^{[3]}} \rangle,$$

$$\langle O^{h_c S_1^{[1]}} \rangle = 3 \langle O^{\chi_c S_1^{[1]}} \rangle,$$  \hspace{1cm} (9)

where the LDMEs for $\chi_c$ have been determined in Ref. [19, 21]. Combining the LDMEs estimated by the relations in Eqs. (8) and (9) and the SDCs calculated up to NLO in $\alpha_s$, we show the sizes of the contributions from these channels in Fig. 1 all of which are smaller than that for the CS channel by about one or two orders of magnitude as expected. Thus, the upper bound of the value of $\langle O^{\eta_c S_1^{[3]}} \rangle$ given in Eq. (7) will not be changed even these new contributions are taken into account. In addition, to provide an order of magnitude estimation of the contributions from the $S_1^{[3]}$ channel, we use a half of the upper bound of $\langle O^{\eta_c S_1^{[3]}} \rangle$ as its input, and the results are shown as the middle-width dashed lines in Fig. 1. The theoretical errors, which are indicated by the blue band in Fig. 1, are mainly from the uncertainties of the LDME $\langle O^{\eta_c S_1^{[3]}} \rangle$ in Eq. (7).

**Indications on the $J/\psi$ production.** Let us go back to the problem of the $J/\psi$ production. Since the three CO LDMEs for $J/\psi$ can be constrained better by Eqs. (6) and (7) using the HQSS relation Eq. (6), we update our predictions for both yields and polarizations of $J/\psi$ prompt production, which are shown in Fig. 2. The details for these calculations have been explained in Ref. [13]. Compared with the old results given in Ref. [13], the new predictions for the CMS window are...
almost unchanged. This is because, for the CMS window, the prediction for yield is only sensitive to the LDMEs \( M_0 \) and \( M_1 \) defined in Eq. (3), and that for polarizations is only sensitive to \( M'_1 \), which is given in (4) and very close to \( M_1 \) as mentioned above. Thus, these predictions can hardly be influenced by the extra constraint in Eqs. (5) and (7). On the other hand, since \( r_1 \) in the forward rapidity interval is smaller than that in the central rapidity interval \( (9)_1 \), the relative large and positive \( \langle O^{J/\psi}(\bar{P}^8_0) \rangle \), which is indicated by Eqs. (9), (10) and (11), will imply that the transversely polarized component of the cross section should be further reduced in the forward rapidity interval comparing with that in the central one. As a result, our new prediction of the polarization for the LHCb window with the new constraint Eqs. (5) and (7) tends to be constrained strongly to the longitudinal polarization side. This slightly improves the consistency between the theoretical predictions and the experimental measurements compared with that in Ref. [12].

The above calculations and analyses indicate that the new constraint Eq. (7) can hardly change our previous conclusions on the \( J/\psi \) production \( 2, 3, 4, 5, 7, 8 \). One should note that the HQSS in Eq. (4) could be violated up to relative order \( v^2 \). But the violation at this level will not change our conclusion qualitatively.

However, the upper bound of \( \langle O^{J/\psi}(S_0^{[n]}(N_0) \rangle \) obtained in Eq. (7) along with HQSS disagree with all other NLO NRQCD fits in the literature \( 2, 4, 7, 8 \). In Refs. \( 2, 4, 7, 8 \), the \( \langle O^{J/\psi}(S_0^{[n]}(N_0) \rangle \) is found to be well constrained, with the value \( 0.0304 \pm 0.0035 \) GeV\(^3\), \( 0.097 \pm 0.009 \) GeV\(^3\) and \( 0.099 \pm 0.022 \) GeV\(^3\), respectively. While in Ref. \( 8 \), the authors argued that \( S_0^{[n]}(N_0) \) will dominate the \( J/\psi \) production, and thus their \( \langle O^{J/\psi}(S_0^{[n]}(N_0) \rangle \) should be at least larger than 0.07 GeV\(^3\). As we discussed above, Eq. (7) gives a very safe upper bound for \( \langle O^{J/\psi}(S_0^{[n]}(N_0) \rangle \), so the contradiction with these NLO NRQCD fits means that either HQSS is essentially broken or there is some flaw in those theoretical works, if the LHCb data \( 6 \) are reliable.

It is very interesting to compare the work of Ref. \( 7 \) with ours. In addition to our complete NLO NRQCD results \( 2 \), the crucial element that Ref. \( 7 \) includes is a very large LP contribution at next-to-next-to-leading order (NNLO) in \( \alpha_s \). We believe that it is mainly this extra LP contribution that changes the theoretical curve of \( \bar{P}^8_0 \) channel, and then results in a \( S_0^{[n]}(N_0) \) dominant conclusion in Ref. \( 7 \). If HQSS is good, the most natural way to solve the contradiction is that the NNLO correction for NLP contribution may be also significant, and that it is needed to calculate the LP contribution and the NLP contribution to the same order in \( \alpha_s \). Based on the QCD factorization up to NLP \( 14 \) and the method to calculate the partonic hard part at NLP \( 26 \), the NNLO correction for NLP contribution should be achieved very soon. Then the validation of HQSS for charmonium production will be tested on a more rigorous base.

Summary.—We compute the cross section of prompt \( \eta_c \) production at LHC at the NLO in \( \alpha_s \) within the framework of NRQCD factorization. We demonstrate that only \( S_0^{[1]} \) and \( S_0^{[8]} \) channels are essential for the \( \eta_c \) production. We find that the data measured by the LHCb Collaboration \( 9 \) tend to be saturated by the contributions from the CS channel. This strongly constrains
the CO LDME $\langle O^{\eta_c} (S_0^{[8]}) \rangle$ for $\eta_c$, which can be related to $\langle O^{J/\psi} (S_0^{[8]}) \rangle$ for $J/\psi$ by the HQSS relation given in Eq. (2). With the help of this new information, all three CO LDMEs of $J/\psi$ can be constrained well into a finite range. Then, we update our predictions of yields and polarization in $J/\psi$ prompt production, which are a little bit better than, but roughly the same as, the old ones [2, 3, 12]. Therefore, the prompt production of $\eta_c$ and $J/\psi$ can be understood in the same theoretical framework. Moreover, we find that other previous works [3, 4, 7, 8], where $\langle O^{J/\psi} (S_0^{[8]}) \rangle$ is well constrained by fitting the data of $J/\psi$ production, overestimate the value of $\langle O^{J/\psi} (S_0^{[8]}) \rangle$ unless HQSS is broken dramatically.

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Note added.—When our calculation was finished and the manuscript was being prepared for publication, an independent study of $\eta_c$ production in NLO NRQCD framework was reported in Ref. [27]. These authors conclude that with HQSS the $\eta_c$ data is in conflict with all NLO NRQCD fits to $J/\psi$ production data, which is significantly different from ours. The reason is that these authors misunderstood our fit in Ref. [5]. In fact, as we emphasized clearly in Ref. [5], our fit implies that the $S_0^{[8]}$ can not be constrained at all, and it can be even as small as zero.

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