Quantitative evaluation of erosive cavitation pressure field from pits in material: fact or myth?

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Abstract. Material pitting in a cavitating flow has been used for a long time as an indicator of the vague ‘cavitation intensity’ concept. Periodically, some researchers suggest pitting tests as a “simple” means to provide quantitative measurements of the amplitude of the impulsive pressures in the cavitation field, especially when combined with Tabor’s formula or with simple finite element computations with static loads. This paper examines the viability of such a method using fully coupled bubble dynamics and material response, and strongly concludes that the commonly accepted idea is a myth, as different loading scenarios with the same amplitude of the cavitation impulsive pressure result in different pit aspect ratios.

1. Introduction
In order to characterize cavitation erosive fields for different operating conditions (e.g. different flow speeds, cavitation sources, or small scale laboratory accelerated erosion tests versus full scale real operation), a reliable method to evaluate the cavitation field ‘intensity’ is needed. As expanded upon in [1] the cavitation field intensity is defined by a distribution function, \( N(P_{\text{peak}}) \), where \( N \) is the number of impulsive pressure peaks per unit exposed area and unit time of given amplitude, \( P_{\text{peak}} \). To this, we should add the distribution of time widths of the pressure peaks [2]. Acoustic pressure measurements are probably the best that can be done presently to determine this intensity despite limitations in the high frequency response and in the size of the sensors [1,2]. However, with advances in instrumentation and miniaturization, these limitations will be reduced over time. Another method, which has been used for at least a whole century [3,4] is to “use the material as the pressure sensor”. Short duration cavitation pitting tests within the incubation period, where non-overlapping pits are produced, are conducted and the peak pressures are deduced from the pit geometric characteristics using the Tabor relationship [1,5]. Recently, finite element method analysis was also conducted to obtain a substitute to Tabor equations [6] using however the same basis for the load, i.e. time independent idealized constant pressure loads. Since in such approaches the load is also characterized by over-simplified parameters, almost a one-to-one relationship between the loads and the pit geometry characteristics is also found.

In order to investigate more rigorously whether cavitation pit geometric characteristics can be used along with Tabor relationship to deduce the pressure amplitude responsible for the pit, we conducted fully coupled computations of bubble dynamics/material response and compared the Tabor-predicted pressure to the measured one.

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peak pressure from the resulting pit to the actually peak pressure generated by the bubble collapse and applied to the material. The numerical approach is documented with details in [7–9].

2. Analytic relation between strain and pit geometry

In Tabor’s approach [5], a pit is assumed to be a spherical indentation characterized by its diameter $D$ and depth $h$ (figure 1) and the mean strain, $\varepsilon_m$, in the material around the pit can be estimated from:

$$\varepsilon_m = 0.1 \frac{D}{R} = 0.8 \frac{h}{D} \left(\frac{h}{D}\right)^{-1.4}, \quad (1)$$

where $R$ is the sphere radius that fits in the pit. The maximum strain is related to the mean strain after assuming an exponential strain distribution in the depth direction [10] by:

$$\varepsilon_{\text{max}} = \varepsilon_m \left(1 + \theta\right), \quad (2)$$

where $\theta$ is the exponent of the exponential strain distribution, which can be obtained experimentally. The values reported in [7,10] are used in this study. From the maximum strain, the maximum stress can be deduced using the stress-strain curve of the material. The maximum stress occurs at the surface of the material, and in this approach is assumed to be equal to the peak of the impulsive pressure at the material surface.

3. FSI simulations of pit formation and pit geometry

In order to examine the validity of inferring the magnitude of the impulsive pressure generated by the collapse of a bubble, which resulted in the analyzed pit, we conducted an extensive series of finite element simulations of pit formation in a material exposed to two types of loads: a) fully coupled fluid structure interaction (FSI) simulations of bubble dynamics and material response; b) material deformation due to synthetic loading configurations where we varied systematically the amplitude, space extent and duration of the load [11]. The FSI simulations were conducted using well-validated 3D bubble dynamics codes and procedures [7–9], which modelled bubble shape deformation, reentrant jet development, compressible impact of the jet on the material with generation of a water hammer impact and emission of a shock wave. Bubble size, standoff from wall, and collapse driving pressure functions were varied to cover a range of FSI conditions. Figure 2 illustrates this for a particular condition taken from [8]. Figure 2a shows bubble outlines at different instants during the bubble collapse. Figure 2b and figure 2c show bubble shapes and pressure contours with very high pressures generated respectively by liquid-liquid impact at reentrant jet touchdown and by jet-wall impact. The liquid-liquid impact event generates a localized high pressure region which expands quasi spherically and propagate to the wetted material surface (first pressure peak in figure 3). The bubble ring left after the jet touchdown shrinks, then collapses and generates another high pressure wave (second peak in figure 3).

The dynamics of the material is modeled by a non-linear finite element method. Figure 3 illustrates the FSI computations with the pressures generated at the material surface center by the collapsing bubble and the corresponding time dependent response of the material illustrated by the development of the indentation depth with time. Due to the high pressure generated...
by the collapse, the material deforms elastically first, then exceeds the elastic limit to result in unrecoverable plastic deformation and a pit.

Four metals (Al 1100, Al 7075, NAB, and SS A2205) were modeled using elastic-plastic models. The stress-strain curves of these metals are given in [7]. An example of permanent deformation predicted from an FSI simulation is shown in figure 4. The profile of the permanent deformation generated on the surface resembles the experimentally observed pit shapes [1]. The figure also shows the circle, which corresponds to equations (1) and (2) spherical indentation assumption. In this case of a bubble-generated pit, the shape is closer to a cone rather than a sphere sector. Figure 5 also shows a pit shape resulting from a synthetic time and space Gaussian distribution pressure loading. The shape is rounder but it is still significantly different from a spherical shape.

A synthesis of the results from the set of cases studied is shown in figures 6 through 8. Figure 6 collects pit depth to diameter ratios, \( h/D \), which according to Tabors’ formula determines the load input. The figure clearly shows that the same impulsive pressure obtained by different bubble collapse mechanisms can produce very different \( h/D \) (see for example the data points along the dashed line in the figure). For the same impulsive pressure peak level, the shape of the pressure function can vary wildly, both in space and time, for different bubble sizes, standoffs, and collapse driving pressure functions. In addition, for large strains, strain and stress distributions in the material under the pit deviate from the simple relationship (2).

The \( h/D \) ratios are converted to maximum strain using (2) then using the stress-strain curve of each material, the maximum stress is deduced from the maximum strain. For each \( h/D \), this provides the supposed peak impulsive pressure applied to the material. Figure 7 summarizes the results and compares the actual input peak cavitation pressure to the one deduced from \( h/D \) and (2). This brings the data closer to a common curve because the procedure has accounted for differences in the various material strengths. However, this highlight that for all data, the maximum stress predicted from the pit is very far from the actual load applied (good correspondence would be the purple line in the plot).

The ratio of the pressure predicted from the pit geometry (\( h/D \)) and Tabor relation to the actually applied pressure peak is plotted in figure 8. The data by the Tabor relation forms roughly a curved band for all materials. Actual pressure over the pit-Tabor predicted pressure significantly increases with increasing loads exceeding 10 at the highest load values. The numerical computed maximum effective stress inside the material is also included for comparison. The stress from the Tabor relation agrees very well with this maximum effective stress (von Mises stress) recorded in the material (not at the surface). It is now evident that the stress from the Tabor relation is in agreement with the maximum effective stress in the material and not with the load itself. The maximum effective stress is only 10% - 60% of the peak load depending on the magnitude of the load.
4. Conclusions
Pits forming on a material from the impulsive pressure loading resulting from bubble collapse were simulated numerically. This enabled careful examination of the suitability of using material pits measurements and Tabor’s analytic relation to infer pressure peaks from the pit geometry. The study clearly showed that this approach is very inaccurate and actually provides the maximum effective (von Mises) stress inside the material and not the amplitude of the cavitation pressure peak. Additional information such as the time and space dependent pit characteristics would be needed to attempt a better correlation.

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