The Mu and Tau Number of Supernovae

C. J. Horowitz\textsuperscript{*} and Gang Li\textsuperscript{†}

Nuclear Theory Center and Department of Physics,
Indiana University, Bloomington, IN 47405

(March 27, 2022)

The neutrino-nucleon cross section is slightly larger than that for $\bar{\nu} - N$. Therefore, $\bar{\nu}$ will escape more quickly from core collapse supernovae leaving the star $\nu$ rich. A diffusion formalism is used to calculate the time evolution of the mu and tau lepton number densities. These quickly reach steady state equilibria. We estimate that a protoneutron star with a maximum temperature near 50 MeV will contain over 50\% more $\nu_\mu$ and $\nu_\tau$ than $\bar{\nu}_\mu$ and $\bar{\nu}_\tau$. Supernovae may be the only known systems with large mu and or tau lepton numbers.

$97.60.Bw, 11.30.Fs, 95.30.Cq$

Mu and tau neutrinos and antineutrinos are produced copiously in (core collapse) supernovae. These neutrinos with tens of MeV energies can not undergo charged current reactions. Therefore, it is assumed $\bar{\nu}$ have identical distributions.

However, the cross section for $\nu$-nucleon elastic scattering is somewhat larger then that for $\bar{\nu} - N$ scattering. This allows $\bar{\nu}$ to escape the star more easily leaving the supernova $\nu$ rich. In this paper, we examine the effects of recoil and or weak magnetism corrections to the $\nu-N$ cross sections. These corrections are of order $E/M$, where $E$ is the neutrino energy and $M$ the nucleon mass. They lift the degeneracy between neutrinos and antineutrinos. To our knowledge, all previous supernovae works (see for example [1]) assume equal $\nu$ and $\bar{\nu}$ cross sections.

We find a large excess of $\nu$ over $\bar{\nu}$ in the star and a large mu and tau lepton number for supernovae. Possible implications of this for the neutrino signal, sensitivity to new physics, nucleosynthesis and other phenomena are discussed at the end of this paper. A diffusion formalism is used for the neutrino transport. This is adequate for our purposes since most of the effect comes from the core of the protoneutron star well inside the neutrino sphere.

We focus on mu and tau neutrinos. For these, the physics is very simple and clear. We explicitly discuss mu neutrinos. All of our results apply unchanged to $\tau$. The effect is also present for electron neutrinos. Indeed, it will change the ratio of $\nu_e$ to $\bar{\nu}_e$ and may produce significant consequences by changing the proton fraction. However, for $\nu_e$ there are also charged current reactions so the situation is more complicated. Therefore, we postpone a discussion of $\nu_e$ until the end of this paper.

It is a simple matter to expand the $\nu-N$ elastic cross section $d\sigma/d\Omega$ to first order in $E/M$, see ref. [2] for example,

\[
\frac{d\sigma}{d\Omega} = \frac{G_F^2 E^2}{4\pi^2} \left\{ [2c_\nu^2(1+x) + c_\nu^2(3-x)] \left[ 1 - 3 \frac{E}{M} (1 - x) \right] + 4c_\nu(c_e + F_2) \frac{E}{M} (1 - x) \right\},
\]

(1a)

with $x = \cos \theta$ for scattering angle $\theta$. Here the plus sign is for $\nu$ and the minus sign for $\bar{\nu}$. The vector ($c_e$, $F_2$) and axial ($c_\nu$) couplings are given in Table I. The transport cross section $\sigma = \int d\Omega d\sigma/d\Omega(1 - x)$ is,

\[
\sigma = \frac{2G_F^2 E^2}{3\pi} \left[ c_\nu^2(1 - 3 \frac{E}{M}) + c_\nu^2(5 - 21 \frac{E}{M}) + 8c_\nu(c_e + F_2) \frac{E}{M} \right].
\]

(1b)

The weak anomalous moment $F_2$ describes the $\sigma_{\mu\nu}q^\nu$ coupling of the $Z$ to the nucleon. It could have significant strange quark contributions. Furthermore, the large value of $F_2$ is important for this paper. A measurement of $F_2$ is underway using parity violating electron scattering [3]. For simplicity we neglect strange quark contributions to $F_2$ and $c_\nu$ in this paper.

The difference in transport cross sections, to order $E/M$, is

\[
D = \frac{\sigma_{\nu} - \sigma_{\bar{\nu}}}{\sigma_{\nu} + \sigma_{\bar{\nu}}} = \frac{8c_\nu(c_e + F_2) E}{c_\nu^2 + 5c_\nu^2} \frac{E}{M} = \delta \frac{E}{M}.
\]

(2)

The coefficient $\delta$ is 3.32 for neutrons and 2.71 for protons (using Table I). We assume that nucleon elastic scattering dominates the opacity, so $D$ will also give the difference of mean free paths, $D = (\lambda_{\bar{\nu}} - \lambda_{\nu})/(\lambda_{\bar{\nu}} + \lambda_{\nu})$.

\*email: charlie@iucf.indiana.edu

\†email: ganli@indiana.edu
A simple diffusion equation for neutrinos is,

$$\frac{\partial}{\partial t} n(E) - \frac{1}{3} \nabla \cdot \lambda \nabla n(E) = 0.$$  (3)

We subtract a similar equation for $\bar{\nu}$ to get,

$$\frac{\partial}{\partial t} [n(E) - \bar{n}(E)] - \frac{1}{3} \nabla \cdot \lambda \nabla [n(E) - \bar{n}(E)] = -\frac{1}{3} \nabla \cdot (\Delta \lambda) \nabla [n(E) + \bar{n}(E)],$$  (4)

with $\lambda = (\lambda_\bar{\nu} + \lambda_\nu)/2 = \lambda_0 E_0^2/E^2$ and

$$\Delta \lambda = (\lambda_\bar{\nu} - \lambda_\nu)/2 = \lambda D.$$  (5)

Here the reference mean free path $\lambda_0$ is,

$$\lambda_0^{-1} = \frac{2G_\star^2 E_0^2}{3\pi} (v_\nu^2 + 5c_a^2) \rho_n,$$  (6)

with $E_0$ an arbitrary reference energy and $\rho_n$ the density of neutrons. For simplicity, $E/M$ terms are dropped in Eq. (6). Note that we assume pure neutron matter. A nonzero proton fraction should not change our results very much since $\delta$ in Eq. (2) is similar for protons and neutrons.

We integrate Eq. (4) over energy $\int d^3E/(2\pi)^3 n(E) = \rho$, assume local thermodynamic equilibrium and work to lowest order in the $\nu$ chemical potential $\mu$ over the temperature $T$,

$$\rho - \bar{\rho} = \frac{\mu T^2}{6}, \quad \rho + \bar{\rho} = \frac{3\zeta(3)}{2\pi^2} T^3,$$  (7a)

to get,

$$-\frac{\pi^2}{E_0^2} \frac{\partial}{\partial t} (T^2 \mu) + \nabla \cdot \lambda_0 \nabla \mu = \frac{\pi^2}{6} \frac{\delta}{M} \nabla \cdot \lambda_0 \nabla T^2.$$  (7b)

This equation describes the time evolution of the muon number density in the star. We also calculate the lepton number current,

$$J_\nu - J_\bar{\nu} = -\frac{\lambda_0 E_0^2}{6\pi^2} \nabla (\mu - \frac{\pi^2}{6} \frac{\delta}{M} T^2).$$  (8)

The first term in Eq. (8) describes the conventional diffusion of lepton number while the second term comes from the diffusion of neutrino pairs which produce a lepton number current because of the difference in mean free paths $\Delta \lambda$. In steady state equilibrium $J_\nu - J_\bar{\nu} = 0$ giving for the chemical potential

$$\mu = \frac{\pi^2}{6} \frac{\delta}{M} T^2,$$  (9)

a remarkably simple result.

We now discuss numerical solutions of Eq. (7b) to bound the diffusion time for muon number in a supernova. To our knowledge there have been no previous estimates of this time. We find that muon number diffuses faster than the thermal energy. This may be because the heat capacity is proportional to the large baryon number. Furthermore, a low energy $\nu$ with a long mean free path can effectively transport muon number [4]. Since diffusion is fast, we will assume steady state equilibrium and use Eq. (9) for some later results.

First, consider a uniform star of density $\rho_0 = 5 \times 10^{14}$ g/cm$^3$ and (baryon) mass $M_{tot} = 1.5$ Solar masses. This has a radius $R$ of 11.25 km. A lower density surface will only speed up diffusion. Likewise we neglect nucleon Pauli blocking corrections to $\lambda_0$ in Eq. (6). These will also increase the mean free path. Perhaps the most extreme case is for the center of the star to be hot. Muon number must diffuse all the way to $r=0$. Therefore we consider a simple temperature distribution characteristic of the later stages of the protoneutron star cooling (perhaps 10 sec. after collapse [1]),

$$T(r) = T_0(1 - M(r)),$$  (10)
with $M(r)$ the enclosed mass divided by $M_{\text{tot}}$. For a uniform density $M = (r/R)^3$. We neglect the small time dependence of the temperature during the short simulation.

We start from the initial condition $\mu = 0$ inside the star and adopt a somewhat arbitrary surface boundary condition that $\mu$ is in equilibrium, given by Eq. (9), at the surface for all times. Our results are not very sensitive to this choice. Figure 1 shows $\mu$ as a function of time for a central temperature $T_0$ of 35 MeV. Muon number diffuses into the center of the star so that eventually $\mu(r = 0)$ rises to its equilibrium value given by Eq. (9). This rise happens quickly with $\mu$ reaching half of its equilibrium value by $t \approx 0.25$ seconds. In comparison it takes several seconds for the temperature distribution to change from surface peaked to one with a maximum at $r = 0$ [5]. We conclude that lepton number diffusion is fast and expect $\mu$ to be near its equilibrium value, Eq. (9).

Equation (9) implies a density asymmetry,

$$\frac{\rho - \bar{\rho}}{\rho + \bar{\rho}} = \frac{\pi^2}{9\zeta(3)} T = \frac{\pi^4}{54\zeta(3)} \frac{\delta}{M} T = 4.98 \frac{T}{M}.$$  

(11)

This is large, 0.186 at $T = 35$ MeV. Equations (9) and (11) can be understood as follows. The density of $\nu$ rises above that for $\bar{\nu}$ until the larger $\nu$ density times a shorter mean free path balances $\bar{\rho} \lambda_{\nu}$ for antineutrinos.

We integrate $\rho - \bar{\rho}$ over the star assuming Eq. (9) and the temperature profile in Eq. (10). This gives,

$$N - \bar{N} = \int d^3r (\rho - \bar{\rho}) = \frac{\pi^3}{135} \frac{\delta T_0}{M} \left( \frac{RT_0}{\hbar c} \right)^3 f_1.$$  

(12)

Here $T_0$ is a characteristic maximum temperature and the profile factor $f_1 = 15 \int_0^R r^2 dr/R^3[T(r)/T_0]^4$ is one for Eq. (10) and is expected to be of order unity for other temperature profiles. We also calculate the total number of muon neutrinos,

$$N + \bar{N} = \int d^3r (\rho + \bar{\rho}) = \frac{\zeta(3)}{2\pi} \left( \frac{RT_0}{\hbar c} \right)^3 f_2,$$  

(13)

with the profile factor $f_2 = 12 \int_0^R r^2 dr/R^3[T(r)/T_0]^3$. For the temperature distribution in Eq. (10), $f_2 = 1$.

For example at $T_0 = 50$ MeV we have,

$$N - \bar{N} = 9.40 \times 10^{53},$$  

(14)

$$N + \bar{N} = 4.43 \times 10^{54},$$  

(15)

or $N = 2.69 \times 10^{54}$ and $\bar{N} = 1.74 \times 10^{54}$. This is a remarkably large asymmetry. The star contains 54% more $\nu_\mu$ than $\nu_\tau$. One might expect $E/M$ correction terms to be small. However, the coefficient $\delta$ is large and the temperature, 50 MeV, is high. We note that the asymmetry depends only linearly on $T_0$, see Eq. (11). Thus a decrease in $T_0$ will not decrease the asymmetry greatly.

Equation (14) is the muon lepton number of the supernova. Furthermore, it also gives the $\tau$ number. We predict that supernovae are the only known systems with large $\mu$ and or $\tau$ number. This could impact new physics. For example, matter enhanced $\nu$ oscillations could depend on the $\nu$ density [6]. A weak long range force that couples only to $\mu$ or $\tau$ number is difficult to observe. One may be able to use supernovae to set limits on such forces.

We have found a large asymmetry for neutrinos recoiling against heavy nucleons. It is possible that the protoneutron star undergoes a transition from hadronic to a pure or mixed quark matter phase which could also contain strange quarks [7]. In a quark phase, one expects recoil corrections not of order $E/M$ but $E/m_q$ where $m_q \approx M/3$ is a constituent quark mass. This might produce even larger separations of $\nu$ from $\bar{\nu}$ which could lead to an observable signature of quark matter.

Perhaps the simplest model of quark matter is to assume the neutrinos scatter from nearly free quarks and that the structure of the quarks are simple without anomalous moments. Evaluating Eq. (2) for a down or strange quark using the couplings in table I gives, $D \approx 1.01E/(M/3) = 3.03E/M$. This is comparable to the neutron value $D = 3.32E/M$. The absence of the factor $F_2$ reduces the quark value and approximately cancels the enhancement from the small

---

1Steady state equilibrium is reached even faster for a surface peaked temperature distribution.
mass. Therefore, we do not expect a strong sensitivity to a transition to quark matter. However, this discussion does emphasize that the separation between neutrinos and antineutrinos is occurring deep in the protoneutron star and may be sensitive to the properties of dense matter.

We now discuss many-body corrections to Eq. (2) assuming a hadronic phase. The effects on the nucleons of relativistic kinematics, Fermi motion, Pauli blocking and nuclear mean fields[8,9] have been studied in ref. [10]. One might think that $M$ in Eq. (2) will be replaced by the Dirac mass $M^* < M$ increasing the asymmetry. However, the anomalous moment term $F_2$ is defined with the free nucleon mass, see Eq. (28) of ref. [11]. Therefore its contributions do not increase. We find that $D$ in Eq. (2) is almost unchanged in the medium.

Burrows and Sawyer [12] argue that RPA correlations will greatly reduce cross sections in dense matter. However, fully relativistic RPA calculations using an interaction that is consistent with the equation of state give smaller corrections [10,13]. Furthermore, RPA corrections should effect $\nu$ and $\bar{\nu}$ in about the same way. Therefore, we do not expect significant RPA changes to the cross section difference of Eq. (2).

We now discuss the neutrino signal from a supernova. We divide the time into three periods. For a short initial time interval, the star is not in steady state equilibrium. During this period, which we estimate may only last for tens to hundreds of nsec, the star radiates significantly more $\bar{\nu}$ than $\nu$. Next, the star will radiate equal numbers of $\nu$ and $\bar{\nu}$ in steady state equilibrium for a relatively long period. Finally, as the star cools it must radiate away its lepton number. Therefore, there will be an ending period where more $\nu$ are radiated than $\bar{\nu}$. However, the fractional difference between $\nu$ and $\bar{\nu}$ during this time is expected to be small because the star cools slowly.

The number of neutrinos in the star at any one time, Eq. (15) is much less than the total number of muon neutrinos radiated (of order $10^{57}$). Therefore, the first phase, with its excess $\bar{\nu}$, can only last for a fraction of the total time. One should consider the possibility of observing the excess $\bar{\nu}$ during the initial phase. However, it may be difficult. Future work should calculate the neutrino signal in more detail.

If the star undergoes a prompt collapse to a black hole, it will take the large lepton number of Eq. (14) with it. Assuming lepton number does not couple to a long range force, it will simply be lost down the black hole. However, the brief neutrino signal which precedes a prompt collapse could be significantly antineutrino rich. Depending on the time of the collapse, neutrinos from the more symmetric second and third periods may never escape the star.

Finally, we consider electron neutrinos. The neutral current effect that we have calculated for $\nu_e$ and $\nu_e$ will also apply to $\nu_e$. In addition, charged current reactions have similar weak magnetism corrections. These terms will increase the $\nu_e$ cross section with respect to $\bar{\nu}_e$. Thus, the number of $\nu_e$ compared to $\bar{\nu}_e$ will increase in the star. This could have a significant effect on the dynamics of the explosion and or nucleosynthesis by increasing the proton fraction. We will discuss this further in a later work [5].

The separation between neutrinos and antineutrinos involves parity violation. If the weak interactions conserved parity, the $\nu$ and $\nu$ cross sections would be equal (to lowest order in $G_F$). Thus, the large mu and tau lepton numbers of a supernova can be considered a macroscopic manifestation of parity violation.

In this paper we have calculated the effects of recoil or weak magnetism corrections to $\nu - N$ cross sections. These are of order the neutrino energy $E$ over the nucleon mass $M$, $E/M$ and make the $\nu - N$ cross section larger than that for $\bar{\nu} - N$. A diffusion formalism was used to follow the time evolution of lepton number. It quickly reaches a steady state equilibrium where a larger $\nu$ density compensates for the longer $\bar{\nu}$ mean free path. For a maximum temperature near 50 MeV, we estimate that protoneutron stars contain over 50 % more $\nu_\mu$ and $\nu_\tau$ than $\bar{\nu}_\mu$ and $\bar{\nu}_\tau$. Core collapse supernovae may be the only known systems with large mu and or tau lepton numbers.

\[1\] Adam Burrows and James M. Lattimer, Ap J. 307 (1986) 178. W. Keil and H.-Th. Janka, A & A. 296 (1995) 145. R. W. Mayle and J. R. Wilson in Supernovae: The Tenth Santa Cruz Summer Workshop in Astronomy and Astrophysics ed. S. E. Woosley (Springer, New York: 1991) p333. R. L. Bowers and J. R. Wilson, ApJ. Suppl. 50 (1982) 115. S. W. Bruenn and W. C. Haxton, ApJ. 376 (1991) 678. S. W. Bruenn, ApJ. Suppl. 58 (1985) 771.

\[2\] G. T. Garvey, W. C. Louis and D. H. White, Phys. Rev. C 48 (1993) 761.

\[3\] B. Mueller et al., Phys. Rev. Lett. 78 (1997) 3824.

\[4\] S. W. Bruenn and T. Dineva, ApJ. 458 (1996) L71.

\[5\] C. J. Horowitz and Gang Li, to be published.

\[6\] Alan Kostelecky and Stuart Samuel, Phys. Rev. D. 52 (1995) 3184.

\[7\] G. Baym and S. A. Chin, Phys. Lett. 62B (1976) 241.

\[8\] J. D. Walecka, Ann. Phys. (NY) 83 (1973) 491.
TABLE I. Vector and axial couplings for neutral current scattering from neutrons, protons and up and down quarks. We assume $\sin^2\theta=0.23$ and $g_a=1.26$.

| Coupling | $\nu-n$ | $\nu-p$ | $\nu-u$ | $\nu-d$ |
|----------|---------|---------|---------|---------|
| $c_v$    | $-\frac{1}{2}$ | $\frac{1}{2} - 2\sin^2\theta$ | $\frac{1}{2} - \frac{3}{2}\sin^2\theta$ | $-\frac{1}{2} + \frac{3}{2}\sin^2\theta$ |
| $c_a$    | $-\frac{g_a}{2}$ | $\frac{g_a}{2}$ | $\frac{g_a}{2}$ | $-\frac{g_a}{2}$ |
| $F_2$    | -0.972 | 1.029 | 0 | 0 |

FIG. 1. Muon neutrino chemical potential vs. radius. The temperature distribution is assumed to be given by Eq. (10) with a maximum central temperature of $T_0=35$ MeV. The curves are for times of 0.02 (dotted) to 1 second (dot-dashed). The solid curves are for intermediate times of (bottom to top) 0.1, 0.2, 0.3, 0.4, 0.5, and 0.6 seconds.