Optimization-based method for interplanetary spacecrafts passive navigation

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Abstract. During deep space missions, planning and realization of dynamic operations (orbit and attitude correction manoeuvres, landing in target areas, etc.) require fast determination of spacecraft position. Due to the objective difficulties of interplanetary communication, it is interesting to solve this problem using the data from onboard sensors (in particular, the optical ones). What is more, most "transportable" of these devices give out "angular" (or quaternion) representation of reference objects’ positions instead of rectangular coordinates or at least distances. Thus, one faces the problem of determination of spacecraft state vector (in a rectangular coordinate system) from the data in "angular" coordinates in the coordinate system of measurement devices. We suggest to determine the spacecraft state vector at the required time as a solution to the problem of minimizing the quadratic cost that determines the difference between the "simulated" measurements (i.e., those that would have been made on the "simulated" trajectory generated by a current state vector approximation) and the real ones (i.e., those that were obtained from on-board sensors). To find a solution, we use a software implementation of the Levenberg-Marquardt method and a prototype of a software package that allows one to configure spacecraft sensors set and the measurement series parameters.

1. Introduction

During deep space missions, planning and realization of dynamic operations (orbit and attitude correction manoeuvres, landing in target areas, etc.) require fast determination of the spacecraft position. Due to the well-known facts (firstly, time delays and data transmission speed limitations in interplanetary communication and, secondly, inability to use global navigation systems) related to deep-space communication, it is necessary to solve this problem using the data from on-board sensors (in particular, the optical ones).

What is more, the most “transportable” of these devices give out “angular” (or quaternion) representation of reference objects’ positions instead of rectangular coordinates or at least distances. Thus, one faces with the problem of determination of spacecraft state vector (in rectangular coordinate system) from the data in “angular” coordinates of the coordinate systems of the measurement devices.
2. Problem Statement
We consider a model spacecraft quipped with a number of sensors. Each sensor is characterized by transformation between the spacecraft frame (SF) and the device frame (DF).

Each sensor i returns its measurement $\mu_i$ (typically, either a 3D target vector in DF or a pair of angular coordinates in DF).

To solve our problem, we use Levenberg–Marquardt algorithm LMA (see, e.g., [1], [2]) for quadratic cost minimization where the cost is square of the vector length.

To detect the state vector determination “quality”, we use the following quadratic cost:

$$\phi = \|\hat{r}(\hat{r}, \hat{v})\|^2 = \|\hat{\mu}(\hat{r}, \hat{v}) - \mu_{\text{meas}}\|^2 \rightarrow \min_{\hat{r}, \hat{v}}$$

where $\mu_{\text{mod}}$ is modeled measurements (measurement times from real cyclogram, spacecraft orientation from star sensors measurements, spacecraft position from movement model and current parameters collection, target positions from celestial mechanics model); $\mu_{\text{meas}}$ is real measurements.

Two technical definitions will be needed. The trajectory along which the real measurements were obtained will be called a real or target trajectory. It propagates from the real state vector. A determining or guidance one is the trajectory propagating from the real state vector approximation.

At each step of the method, we set optimization parameters (6D state vector) and propagate the “guidance” trajectory to check how close it is to the “target” one w.r.t. the measurement values. In other words, we get real measurements as an “image” of the real (target) trajectory and try to find the state vector that gives us the best guidance trajectory.

Variants with bad and good determinations are shown in Fig. 1 and Fig. 2, respectively.

![Fig. 1: “Bad” determination of real trajectory. The initial state vectors are far from one another, thus the cost value is big.](image1)

![Fig. 2: “Good” determination of real trajectory. The initial state vectors are close to each other, thus the cost value is small.](image2)

General scheme of the suggested method is shown in Fig. 4.
3. **Example with the initial data being *.csv data with reference trajectories**

To demonstrate our method capabilities, we use the following model situation:

- spacecraft equipment consists of two sensors: one to measure 3D direction to the Moon in the DF, and one to measure 3D direction to the Earth in the DF;
- the Earth sensor optical axis is pointed to “-Z” direction of the SF;
- the Moon sensor optical axis is pointed to “+X” direction of the SF;
- during the flight, the spacecraft changes its orientation w.r.t. J2000 axes such that the Moon sensor optical axis points to the Moon center;
- the spacecraft’s initial state vector in J2000 is obtained from the following Keplerian orbit:
  - orbit epoch is 22 Nov 2020 09:00:00.000 UTC;
  - semi-major axis is 300000 km;
  - eccentricity is 6.82424e-15;
  - inclination is 18 deg;
  - RAAN is 3.82044e-05;
  - argument of the perigee is 0.0 deg;
  - true anomaly is 360 deg;
- the program input data are as follows:
  - .csv file containing two state vectors: a “determined” one to get the initial approximation of the real one, and the real one at the time related to the last measurement;
  - .csv file containing the Moon measurements – the Moon trajectory in spacecraft-centered J2000 system;
  - .csv file containing the Earth measurements – the Earth trajectory in spacecraft-centered J2000 system.
- The Moon trajectory is generated using JPL DE430 ephemeris.

To compute a satellite trajectory, the program uses the following perturbation model:
- EGM96 gravity potential for the Earth field;
3.1. Test 1: The initial approximation is close to the real one.

![Method main window](image)

Start diff. in ECI = -234.3740852541; 4737.4550755021; 1537.7058205270;
dist = 4986.2762653830
Iterations: 5
Func evals: 42
Initial cost: 0.0882094
Final cost: 2.60246e-15
Pos diff: -1.37914; -0.518712; 0.658274, m.
Pos diff length: 1.61382 m.
Vel diff: -8.63498e-08; -3.29599e-06; -2.44258e-06, m/s.
Vel diff length: 4.10332e-06 m/s.

![Method main window](image)

3.2. Test 2

Pos diff: -1.37912; -0.51871; 0.658275, m.
Pos diff length: 1.6138 m.
Vel diff: -8.63498e-08; -3.29599e-06; -2.44258e-06, m/s.
Vel diff length: 4.10332e-06 m/s.
Start diff. in ECI = -385.3380155443; 9457.4616772392; 3070.2876504347; 
dist = 9950.8156961670
Iterations: 6
Func evals: 49
Initial cost: 0.358257
Final cost: 2.60246e-15
Pos diff: -1.37912; -0.51871; 0.658275, m.
Pos diff length: 1.6138 m.
Vel diff: -8.6155e-08; -3.29588e-06; -2.44255e-06, m/s.
Vel diff length: 4.1032e-06 m/s.

3.3. Test 3

![Figure 6: Method main window.](image)

Start diff. in ECI = -454.1743611150; 14191.5896626575; 4608.0149290888; 
dist = 14927.8696768912
Successful convergence.
Iterations: 6
Func evals: 49
Initial cost: 0.823395
Final cost: 2.60246e-15
Pos diff: -1.37913; -0.518711; 0.658275, m.
Pos diff length: 1.61381 m.
Vel diff: -8.62072e-08; -3.29593e-06; -2.44257e-06, m/s.
Vel diff length: 4.10326e-06 m/s.

4. Conclusions
We present an optimization-based method to determine a spacecraft state vector, that needs only the 
data obtained from on-board sensors. Our model sensors are optical ones, so theoretically one can 
create an on-board program system for passive navigation that only requires modern technologies. 
Certainly, it is necessary to provide some quality and quantity analysis of the method, with the main 
focus on the devices errors model and on hardware demands.

References

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