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Gas mixing enhancement in minichannels using a rotationally oscillatory circular cylinder

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Abstract. Oscillating structures and actuators can induce flow kinematics that enhances mixing. This approach is specifically effective for mixing enhancement in meso-scale channels, where the flow kinematics can be actively controlled using micro-electro-mechanical-systems (MEMS). In this paper, numerical results for mixing of two incompressible ideal gas (Schmidt number of 1.0) streams through a 2D minichannel via a rotationally oscillating circular cylinder are presented and discussed. Simulations are performed for blockage ratio of D/H=1/3 and Reynolds number of 100 and oscillation amplitudes of \(\frac{\pi}{3}\), \(2\frac{\pi}{3}\), \(\frac{\pi}{2}\) and \(\pi\) for subharmonic (\(F < 1\)), harmonic (\(F = 1\)) and superharmonic (\(F > 1\)) regimes. Numerical results indicate that mixing performance is improved by about 70% compared to the plane channel at oscillation amplitude of \(\pi\) and excitation frequency of 25% higher than the natural frequency of vortex shedding of a stationary cylinder. It is shown that the mixing efficiency is increased by increasing of amplitude in all the cases except at very low excitation frequencies. This study also shows that when the excitation frequency is equal to the vortex shedding frequency the maximum power is required for mixing of two gases.

1. Introduction

Effective mixing is a major problem in many MEMS devices where mixing of two or more fluids is the main goal. Because of small dimensions and low velocities, flow regime in MEMS devices is often laminar and thus effective mixing may require long length and time which is not of practical interest. Therefore, innovative methods for fluid mixing are required. Pulsed-flow mixing, electrokinetic instability, hydrodynamic instability effects due to vortex shedding and boundary modification are methods that may be used for mixing enhancement in MEMS devices [1-7]. In the present investigation, attention is focused on the fluid mixing in a straight 2D channel via vortex shedding from a rotationally oscillating circular obstacle.

A number of researchers have investigated fluid flow structure around a circular cylinder in external flow because of its numerous engineering applications. However, only a few studies in the literature have studied fluid flow past a stationary or oscillating confined cylinder with the objective of either heat transfer or fluid mixing augmentation. One of the earliest studies on this subject was reported by Fu and Tong [8]. They performed numerical simulations to examine the effects of transversely oscillating cylinder on the cooling of an array of heated blocks. Their results showed that heat transfer markedly increases when the excitation frequency is in the lock-in regime. Recently, Celik et al. [9] have studied laminar flow past a transversely oscillating circular cylinder in a 2D channel. They found that the shedding frequency of laminar flow past a confined stationary cylinder is two times higher than the vortex shedding frequency of external flow past a cylinder. More recently, Celik et al. [10] have examined the effects of transversely oscillating cylinder on the wall heat transfer. According to their results, transversely oscillating cylinder significantly enhances the wall heat transfer. Maximum heat transfer enhancement was observed for the frequency ratio of \(F = 0.75\) and faster oscillations was found to be not beneficial. In another study, Celik et al. [11] have investigated mixing enhancement in
the channel via a transversely oscillating cylinder. They found that the most favorable mixing performance occurs when the oscillation frequency is in lock-in regime.

In practice, the transverse oscillation occupies the channel height and demands more complicated movement mechanism, control and set-up than a rotationally oscillating cylinder. In the present paper, we investigated the utilization of a rotationally oscillating cylinder with various amplitudes and frequencies for mixing of two gases. For each frequency and amplitude, the mixing efficiency and the required power are calculated.

2. Problem Definition and Governing Equations
In this section, the geometry, the governing equations and the numerical method are described.

2.1. Geometry
Figure 1 demonstrates the schematic of the computational domain of the present investigation. Two fully-developed laminar ideal gas flows with a parabolic velocity profile and mean velocity of \( U_m \) enter the channel from the upper and the lower parts of the channel. Two streams are initially separated by a plate of infinitesimal thickness. The separation plate and channel lengths are 2D and 31D, respectively. The circular obstacle is positioned at 4D away from the channel inlet. The circular cylinder is subjected to forced rotational oscillations that result in time periodic tangential velocity \( V_\theta \) on the cylinder according to following formula:

\[
V_\theta = \theta_{\text{max}} \pi f_e D \sin(2\pi f_e t)
\]

where \( f_e \) and \( \theta_{\text{max}} \) are the excitation frequency and the maximum rotation angle, respectively.

In this study, computations are carried out for a blockage ratio of 1/3, Reynolds number of 100 and Schmidt number of 1. The maximum rotation angle varies between 0 and \( \pi \), whilst frequency ratio (F) is ranged between 0.25 and 2.0. The frequency ratio (F) is defined as the ratio of the excitation frequency \( f_e \) to the vortex shedding frequency of flow around a stationary confined cylinder (f).

![Figure 1. Schematic of the geometry studied.](image)

2.2. Governing Equations and Numerical Method
The governing equations of flow and scalar for an unsteady incompressible flow are written as:

Continuity:

\[
\frac{\partial U_j}{\partial x_j} = 0
\]
Momentum:

$$\frac{\partial U_i}{\partial t} + \frac{\partial (U_i U_j)}{\partial x_j} = -\frac{1}{\rho} \frac{\partial P}{\partial x_i} + \nu \frac{\partial^2 U_i}{\partial x_i \partial x_j}$$  \hspace{1cm} (3)

Scalar:

$$\frac{\partial c}{\partial t} + \frac{\partial (U_i c)}{\partial x_j} = d \frac{\partial^2 c}{\partial x_i \partial x_j}$$  \hspace{1cm} (4)

where $\rho$, $\nu$, $c$ and $d$ are the density, the kinematic viscosity, concentration and the mass diffusivity of the fluid, respectively.

Computations of the present work are obtained using OpenFOAM code. The pressure field is linked to the velocity field using SIMPLE algorithm. Second order upwind scheme is employed for approximation of nonlinear convective terms in momentum and scalar equations, while temporal derivatives in equations (3) and (4) are discretized using the second order Crank-Nicolson scheme. In the simulations, uniform inlet concentrations are assumed for the upper and lower streams. No-slip boundary condition is considered for all solid boundaries. Zero gradient concentration boundary condition is applied on the cylinder surface and on the channel walls. At the channel outlet, zero Neumann boundary condition is applied for both velocity and concentration. The computational domain is meshed into 21600 quadrilateral structured cells. A series of grid-independency tests have been carried out to assess the accuracy of the results of the present investigation. The outcome of these studies has shown that the grid used here is sufficiently fine to produce grid-independent results.

3. Results and Discussion

In this section, the numerical results of present investigation are presented and discussed to study the effects of excitation frequency and amplitude of oscillations on the mixing performance.

The criterion used here to evaluate the mixing performance is mixing efficiency defined by Erickson and Li \[12\] as:

$$\eta(x) = \frac{1 - \int_0^y [c(x,y) - c_m] dy}{\int_0^y [c_0(x,y) - c_m] dy} \times 100$$  \hspace{1cm} (5)

where $c_m$ is the concentration in perfect mixing condition ($c_m = 0.50$), $c_0$ is the concentration for an unmixed condition ($c_0 = 1.0$ and 0 for the lower and upper portion of the channel, respectively).

3.1. Effects of frequency and amplitude

The effect of oscillating cylinder on the mixing of two gases is a function of the excitation frequency and the amplitude. Figure 2 demonstrates the variation of mixing efficiency along the channel for various non-dimensional excitation frequencies ($F$) and amplitudes ($\theta_{\text{max}}$). It is seen that a considerable mixing is achieved compare to the plane channel (P. Ch.) by the insertion of the cylinder in the channel. For the stationary cylinder, as expected the maximum mixing efficiency ($\approx 80\%$) occurs at the channel outlet. A relatively long length ($\approx 20D$) is however required to reach this maximum mixing performance. Depending on the excitation frequency, oscillation of the cylinder with the amplitude of $\theta_{\text{max}} = \pi/3$ can either enhance or diminish the mixing performance. Slow oscillation of the cylinder with frequency ratio of 0.25 is seen to have insignificant influence on the efficiency compare to the stationary cylinder. By increasing the frequency ratio to 0.50 the mixing
efficiency becomes lower than those obtained for $F = 0.0$ and 0.25. As the frequency ratio, approaches to the lock-in regime ($F = 0.75$), the beneficiary of the rotation starts to appear. For the frequency ratio of 0.75, the mixing efficiency substantially increases to more than 90% at the channel outlet. For higher frequency ratios of 1.00 and 1.25, the efficiency is not changed much at the channel outlet, but fully-mixed condition is reached at the shorter distances from the channel inlet. More specifically, for $F = 1.25$, the mixing efficiency reaches its maximum value at $x/D \approx 11$, whilst for $F = 0.75$ this condition occurs at $x/D \approx 22$. For super-harmonic oscillation ($F > 1$), the mixing efficiency drops with increase in excitation frequency. Note that for maximum excitation frequency ($F = 2$), the mixing efficiency distribution approaches to that of the plane channel. For amplitude of $\pi/2$, the stationary cylinder appears to be more effective for fluid mixing than the rotating cylinder in subharmonic regime under ($F = 0.25$ and 0.5). Similar to the previous case, the frequency ratio of 0.75 substantially enhances the mixing efficiency, giving the peak mixing efficiency at the outlet. Further increase of frequency ratio to $F = 1.0$ and 1.25 gives lower efficiencies but shorter mixing lengths. At this amplitude, the frequency ratio of 1.5 provides mixing levels similar to those obtained for $F = 0.75$. Once again, further increase of $F$ to 1.75 and 2 reduces the mixing efficiency in a way that least overall mixing efficiency is reached for $F = 2.0$.

Figure 2. Effect of frequency and amplitude on the mixing efficiency $Re = 100$ and $Sc = 1.0$.

The overall trend of efficiency curves for higher amplitudes of $2\pi/3$ and $\pi$ are similar to those obtained for the lower amplitudes of $\pi/3$ and $\pi/2$. It is clearly seen that by increasing the oscillation amplitude the mixing efficiency increases. For $\theta_{\text{max}} = \pi$, the maximum mixing efficiency is obtained
in lock-in regime \((0.75 < F < 1.25)\). In this regime, the mixing efficiency is insensitive to the excitation frequency. The above results can be interpreted by examining the effects of vortex shedding on the mixing process. To this end, figure 3 demonstrates the vorticity and the concentration contours in the channel for amplitude of \(\pi\) and frequency ratios of 0.50, 1.25 and 2.00. These contours correspond to the instant that cylinder starts to rotate anticlockwise. At the lowest excitation frequency, the generated vortices are too few to effectively mix two streams. As a result, a little mixing occurs in the bulk fluid which is consistent with the efficiency distribution shown in figure 2. At the highest excitation frequency \((F = 2.0)\), while more vortices are generated, but too small in size to vigorously mix the two fluids. Consequently, unmixed fluids are clearly visible in the end of the channel. At frequency ratio of \(F = 1.25\), the optimum condition for mixing seems to be established. This condition creates more strong vortices and thus well mixing condition is observed for \(x/D > 11\).

![Figure 3](image)

**Figure 3.** Vorticity and concentration contours along the channel for \(\theta_{\text{max}} = \pi\), \(\text{Re} = 100\) and \(\text{Sc} = 1.0\).

For fixed frequency ratio of \(F=1.25\), figure 4 illustrates the effect of the oscillation amplitude on the mixing. Similar to figure 3, these contours correspond to the instant that the cylinder starts to rotate anticlockwise. Zones represented by A, B and C are selected to aid the interpretation of the results. Zone A shows the region of lower flow immediately after the cylinder. This figure shows that the height of this region increases with increase of the amplitude. Zone B represents the portion of the lower fluid where is affected by the vortex motion. As shown, for \(\theta_{\text{max}} = \pi/3\), this zone is attached to the lower stream. By increasing the amplitude to \(2\pi/3\), this zone displaces to higher altitudes but still does not detach. It is seen that zone B is separated from the lower stream at \(\theta_{\text{max}} = \pi\). The region on
the lower wall, where there is continuous unmixed fluid, is denoted by C. While this zone starts at \( x/D \approx 6 \) for \( \theta_{\text{max}} = \pi/3 \), it approaches the cylinder as the oscillation amplitude increases. This shows that mixing enhancement increases with increase of the oscillation amplitude.

**Figure 4.** Vorticity and concentration contours along the channel for \( F = 1.25 \), \( Re = 100 \) and \( Sc = 1.0 \).

### 3.2. Power requirement

This section investigates the total power requirement for the proposed mixing mechanism. In order to calculate the mechanical power for oscillation, the average torque required for the oscillation is first computed by

\[
\bar{T} = \frac{1}{4} \int_0^{2\pi} \mu \frac{\partial U}{\partial n} D^2 d\theta
\]

where \( n \) is the direction normal to the surface.

Subsequently, the required mechanical power for one cycle of oscillation is obtained using

\[
\text{Mechanical power} = \frac{1}{\tau} \int_0^\tau \bar{T}(t) \dot{\theta}(t) dt
\]
Considering $\bar{P}$ as the time average pressure in one cycle of oscillation

$$\bar{P} = \frac{1}{\tau} \int_0^\tau P dt$$

(8)

pressure drop along the channel and pumping power are obtained as follows

$$\Delta P = \left( \frac{1}{H} \int_0^H \bar{P} dy \right)_{\text{inlet}} - \left( \frac{1}{H} \int_0^H \bar{P} dy \right)_{\text{outlet}}$$

(9)

Pumping power $= AU_m \Delta P$

(10)

where $A$ and $U_m$ are channel cross-sectional area and mean velocity, respectively.

Subsequently, we calculated the power ratio, $Po$, which is the ratio of the required power for oscillating cylinder in a channel (i.e., sum of the oscillation and pumping powers) to the required pumping power of a plane channel flow (obtained by multiplying the pressure drop in the channel with the sustained volumetric flow rate). As shown in figure 5, for each excitation frequency, the required power increases with increase of the oscillation amplitude. It is interesting to note that the total required power increases with the frequency ratio up to $F = 1.00$ and then decreases by increasing the frequency in the superharmonic region. It is worth mentioning that the same trend is observed for the pumping power. The mechanical power increases as the frequency increases up to $F = 2.00$. Another point about mechanical power is that its increase is mainly due to increase in amplitude and frequency as the average shear stress on the cylinder does not change considerably in the investigated range of amplitudes and frequencies.

**Figure 5.** Total required power of the system versus frequency, $Re = 100$ and $Sc = 1.0$. 

Conclusions
Utilization of a rotationally oscillating cylinder as a gas mixer in a 2D minichannel investigated numerically at a constant Reynolds number of 100 and a wide range of amplitude and frequencies. In many applications, considering the cost of instruments for rotating the cylinder, use of a stationary cylinder can provide a relatively good mixing at the end of the channel (up to 80% for Pe = 100). Oscillation of cylinder with subharmonic frequencies (F = 0.25 and F = 0.50) and superharmonic frequencies (F = 2.00) resulted in less mixing performance than dose a stationary cylinder. In general, for all conditions investigated, considering the mixing length and efficiency, frequency ratio of 1.25 can be considered as the optimum frequency ratio. Except at very low frequency (F = 0.25), mixing performance increases as the oscillation amplitude increases. The total power required for the system increases as the oscillation amplitude increases. At a fixed amplitude, in subharmonic frequencies (F < 1), the required power increases as the excitation frequency increases. In contrast, at superharmonic region (F > 1), less power is required. Further work is in progress to consider flow compressibility and rarefaction effects.

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