The Very Idea of Dynamic Semantics: An Overview from the Underground

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1 Introduction

Imagine someone complained as follows: “Our standard accounts of the semantics of Natural Languages are sadly inadequate, and a crucial source of their inadequacy is that they do not take account of or reflect the dynamics of language; they are simply too static.” One might well agree, yet wonder more exactly what the complainant was talking about. Thinking of the work of Wittgenstein, Austin, Searle and others, one might respond, “Oh, you mean that inadequate attention has been paid to uses of language other than the statement-making use; that standard semantic accounts have ignored all the kinds of things we do with language.” But, no, that’s not the complaint they have in mind; the plaintiff is perfectly happy accepting the centrality of the statement-making use of language and the correlative centrality of indicative sentences. One might then reflect on the fact that the roots of our standard accounts are to be found in the work of Frege, Russell, and others on the logic of mathematics and that there are no dynamics within mathematical structures. The domain of mathematics is a domain of unchanging entities and unchangeable relations, but we most often use natural languages to describe and report on things that change and act on one another. But, no, you are told, this is not the source of the dissatisfaction, either.

Finally you decide to ask for some examples, some phenomena that motivate the complaint—phenomena no adequate account of which can be had without moving to a more dynamic, less static framework. Wouldn’t you be surprised when given the following list of examples?

1. If Hans owns a donkey, he pets it.
2. Every semanticist who owns a donkey pets it.
3. A donkey came to my office this morning. It had a theory about people anaphora.
Perhaps the third example does suggest something, something about the importance of sequences of sentences, but otherwise, you would be at a loss—unless, of course, you knew a lot about work in natural language semantics over the last decade or so. So, what is the problem with these sentences and for whom is it a problem? Whose oxen do these sentences gore?

1.1 The Phenomena

First let us make clear the intended, but troublesome readings of our sentences

1. If Hans owns a donkey, he pets it.
2. Every semanticist who owns a donkey pets it.
3. A donkey came to my office this morning. It had a theory about people anaphora.

On the readings intended, the occurrences of ‘it’ in these sentences are [meant to be] semantically dependent on the corresponding (co-indexed) indefinites. The problem is that it’s not clear that, or how, this dependence is mediated by the syntactic structures. Consider, that is, the following diagnosis:

The phenomena all involve anaphoric pronouns outside the scope of their [indefinite] NP antecedents.

One might wonder whether all cases of semantic dependence of pronouns on NP’s of various kinds are to be classified as anaphoric; in any event, the theory-ladenness of this description is carried by its use of a notion of scope. Where is the operator whose scope is in question, and what is that scope?

Part of the intended formal model is clear enough: anaphoric pronouns are to be modeled by variables and their quantified antecedents are to be modeled by some form of variable-binding quantificational operator. There are various possibilities. We won’t go into any in any detail; we simply want to note that it is not at all obvious that there are any variables in English. There are, of course, pronouns, though not many: nobody seems to treat English as an n-variable fragment (for relatively small n) of some standard quantificational language. If there are no variables, then there are no variable-binding operators, either. (Or they would all have only vacuous occurrences.) We can grant that pronouns have uses on which they are semantically dependent on other noun phrases in ways that are similar to, and for certain purposes can be adequately modeled by the relation between a variable $x_i$ and a variable-binding quantificational operator $Qx_i$ that binds it. But this means no more than what it says. In particular, one should be careful about identifying elements/aspects of a model, even a very good one, with elements/aspects of the phenomena modeled. So one should be careful about claiming that there are variables and variable-binding operators in English.
In the foregoing, we have been ignoring the existence of real honest-to-goodness variables in informal mathematics, physics, etc. Consider, “Let \( A \) and \( B \) be sets; then \( A = B \) if and only if, for each \( x \), \( x \) is a member of \( A \) if and only if \( x \) is a member of \( B \).” By my lights, this is English all right, and \( A, B, x \) are variables. But it is well to remember that the use of variables in informal mathematics is ambiguous and that the interpretation of variables, context dependent. If in the course of a discussion of integers, the formula \( x + 7 = -1 \) appears, we are almost surely meant to understand the variable as existentially quantified. If we see instead \( x + 7 = 7 + x \), we will take the quantification to be universal. Finally, if we see \( x + y = z \) in that same discussion, we won’t know how to take it—except that we won’t take all three as universally quantified—without further hints from the context.

Even assuming that there are variables and variable-binding operators, it is not clear that there is a unique, well-defined notion of the scope of these operators. One way of understanding the phenomenon of donkey anaphora is precisely this: One can opt for a precise notion of scope, say as determined by something like c-command, but then also note that a quantifier phrase can have pronouns or anaphoric definite descriptions coreferential with or dependent on it that are arbitrarily far from it, not even within the same sentence, and hence that being within the scope of such a phrase is not a necessary condition for being anaphorically dependent on it. Donkey anaphors or discourse anaphors are not bound by those phrases, though semantically dependent on them. This motivates accounts that conceive the semantics of anaphoric but unbound pronouns and anaphoric definite descriptions as on the model of demonstratives, but demonstratives with the special feature that the entity to which they are directed must be made salient by linguistic means. Or one can do what \[1\], \[2\], \[3\], and \[4\] do and alter the syntax (and semantics) of certain variable-binding quantifiers, by altering the notion of their scope or their selectivity, degree, etc. Or one can provide two notions of scope: an analogue of the standard syntactic one, the smallest wff. following the operator—this is akin to the notion of c-command—and a semantic one, semantic scope, capturing the observed facts about dependency, but quite undetermined by syntactic structure.

2 Historical Background

Formal languages were first devised precisely to overcome what were felt to be the inadequacies, for certain delimited purposes, of informal, natural languages.\[1\] In particular, the original purpose was to investigate the logical foundations of mathematics and more generally, to analyze systematically logically valid

\[1\]The notion of a formal language as used here is not a precise technical notion. We have in mind formalisms devised by particular humans at particular moments of history, for some practical or theoretical purpose, and amenable—and intended to be amenable—to precise mathematical characterization, at least with respect to certain syntactic properties.
inferences and to delimit the logically valid from the invalid. Thus, Peano, in the Preface to “The Principles of arithmetic, presented by a new method,” says:

Questions that pertain to the foundations of mathematics still lack a satisfactory solution. The difficulty has its main source in the ambiguity of language...I have denoted by signs all ideas that occur in the principles of arithmetic, so that every proposition is stated only by means of these signs...With these notations, every proposition assumes the form and the precision that equations have in algebra; from the propositions thus written other propositions are deduced, and in fact by procedures that are similar to those used in solving equations. This is the main point of the whole paper. ([8], p. 85)

Frege, in the Preface to his “Begriffschrift,” notes that he had set out to determine how much of arithmetic could be established by logic alone:

To prevent anything intuitive from penetrating here unnoticed, I had to bend every effort to keep the chain of inferences free of gaps. In attempting to comply with this requirement in the strictest possible way, I found the inadequacy of language to be an obstacle...This deficiency led me to the idea of the present ideography. Its first purpose, therefore, is to provide us with the most reliable test of the validity of a chain of inferences and to point out every presupposition that tries to sneak in unnoticed...(p. 5f)

It is quite clear that one central purpose of these new formal languages was to enable one to state effective tests of the property of being a proof and the relation of a sentence being correctly inferred from others. Correlatively, a central inadequacy of ordinary language was that one could not do this for inferences stated in such language—the language of life (“Sprache des Lebens”). Frege goes on to explain the relation between his formalism and ordinary language.

I believe that I can best make the relation of my ideography to ordinary language clear if I compare it to that which the microscope has to the eye. Because of the range of its possible uses and the versatility with which it can adapt to the most diverse circumstances, the eye is far superior to the microscope. Considered as an optical instrument, to be sure, it exhibits many imperfections, which ordinarily remain unnoticed only on account of its intimate connection to our mental life. But, as soon as scientific goals demand great sharpness of resolution, the eye proves to be insufficient. The microscope, on the other hand, is perfectly suited to precisely such goals, but that is just why it is useless for all others.

This ideography, likewise, is a device invented for certain scientific purposes, and one must not condemn it because it is not suited to others. ([8], p. 6)
In the first sections of work, Frege returns to this theme again, and in a way that is especially relevant to the concerns of this essay:

In ordinary language, the place of the subject in the sequence of words has the significance of a distinguished place, where we put that to which we wish especially to direct the attention of the listener. This, may, for example, have the purpose of pointing out a certain relation of the given judgment to others and thereby making it easier for the listener to grasp the entire context. Now all those peculiarities of ordinary language that result only from the interaction of speaker and listener—as when, for example the speaker takes the expectations of the listener into account and seeks to put them on the right track even before the complete sentence is enunciated—have nothing that answers to them in my formula language, since in a judgment I consider only that which influences its possible consequences. Everything necessary for a correct inference is expressed in full, but what is not necessary is generally not indicated; nothing in left to guesswork. ([8], p. 12. All emphases in the original.)

Logical formalisms, on this view, are analytical tools (“scientific devices”) created to help scientists study a certain range of phenomena. They are more than this, though: they are analytical tools by way of being models of the phenomena to be analyzed. In the case at hand the phenomena included aspects of the expression of thoughts about mathematical entities and, most centrally, certain features of proofs in mathematics. Of course, neither Frege nor Peano thought that proofs in their formal systems were adequate models of any other feature of proofs in mathematics beyond that of exhibiting in a systematic and effectively determinable way the relation of logical consequence between axioms/postulates and theorems.

3 Programming Languages as Analogues, not Metaphors

We noted in §3, that formal languages were objects to be studied, and to be used in the study of natural phenomena, but that they were not devised to be used directly to communicate. There are, though, formal languages that have been devised precisely for the purposes of communication: programming languages. Of course, these are used to communicate with computers primarily and with other humans only secondarily, and for many purposes the communication is

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2It is clear enough that the languages of informal mathematics in which mathematics is actually done and in which results are presented and explained are annexes to ordinary language.
thought of as only-one way. Despite this, and despite their view of the computational metaphor, Groenendijk and Stokhof point to work on the logic of programs, in particular to Dynamic Logic, as a source of insight and technical detail for their own contributions.

Dynamic Logic ([12, 13]) is a formal system for reasoning about and proving properties of programs from languages designed to be executed by state machines. Such programs can, in general, be thought of as associated with transformations of states; starting from an initial (input) state, the execution of a program will cause the machine to go through a sequence of states, and perhaps to halt in a final (output) state. Dynamic logic abstracts from the intermediate states and associates with a program a binary relation between input and output states. As for the states themselves, in Propositional Dynamic Logic these are completely abstract atoms, as they are in the most abstract version of Kripke-style semantics for modal logics. Indeed, PDL is a multi-modal logic, with each atomic program determining an associated normal accessibility relation. In First-Order Dynamic Logic, states are modeled by first-order assignments or valuations and programs determine binary relations between such valuations. This idea reappears in Dynamic Predicate Logic.

In Dynamic Logic, these binary relations model actual state transformations; in particular, those transformations associated with the central programming construct of the class of programming languages under study. That construct is the simple assignment. Where $x$ is a program variable or identifier and $e$ is e.g., a numerical expression, an assignment takes the following form:

$$ x := e $$

Identifiers refer to abstract locations (cells, registers); these are the components of the store or memory of a state machine. Assignments change state (store, memory). After executing the assignment command above, the state of the machine will be just as it was before except possibly with respect to the value stored in the cell named or referred to by $x$; the contents of $x$ will be the value of the term $e$.\footnote{It should be obvious how this understanding of assignments relates to the treatment of quantification in first-order logic; the most direct connection, however, is in terms of so-called random assignments, a nondeterministic assignment in which an arbitrary member of a certain data type is assigned to a location. See below.}

Notice that variables can also occur on the right hand side:

$$ x := x + 1 $$

The occurrence of $x$ on the right is unlike that of $x$ on the left. The term $x+1$ does not refer to the successor, in some fixed ordering, of the location referred to by $x$. Rather the term on the right is a numerical expression, referring to
the number which is the successor of the number that is the content (value) of location $x$ prior to the execution of the assignment.\footnote{In this respect, the occurrence on the right is akin to that of a free variable being used to model some referential relation between an utterance of a sentence with a context-dependent element, like a demonstrative, and the object referred to by that element in that utterance.}

Most real state-based or imperative programming languages provide constructs, called _declarations_, for allocating and initializing locations, often restricted by sorts or data types, as follows:

```plaintext
begin integer x;
  x := 7;
  x := x^2;
  print(x)
end
```

Here we also have an instance of a _block_, in this case a simple instance in which a new variable is declared with an initial value and then a sequence of simple commands follows which have that variable as a (so-called) formal parameter. These commands constitute the _scope_ of the variable.\footnote{One may compare explicit block structure with Groenendijk and Stokhof’s $\lozenge$ operator.} Here the scope is easily determinable from the program text. Things are not always so simple. Some programming languages have dynamic scoping rules governing parameters in procedures, so that the arguments to a procedure are only determined at run-time, when the procedure is invoked. More to the point, a given syntactic characterization of a language, e.g., via an inductive definition of the set of well-formed programs, does not determine the nature of the scope mechanisms. As we shall see below, this determination can be provided in various ways; roughly such a determination is provided by specifying how machines to execute the programs are supposed to operate.\footnote{The possibility of dynamic vs. lexical or static scoping also arises in purely functional languages as well. See \cite{4} for a use of this difference, in an imperative programming language paradigm, in modeling the availability of alternative interpretations of ellipsed verb phrases.} Still, here let us assume that the scope of a variable is always determinable _statically_, from the text of the program. The same is not true of the _extent or lifetime_ of a variable or location.

We have said that declarations allocate and initialize a location; the extent of a variable (or of a variable declaration) is its computational duration—the length of (notional) time during which the allocation associated with the declaration is in force. In some programming languages, the extent of a variable ends when the control exits the block in which the variable was declared; it lasts through temporary jumps out of the block, however. In other languages, explicit constructs ending extents (releasing or deallocating the location) are provided. In some though, extents are indefinite and cannot in any way be determined statically from the program text. In such cases, the implementation of the language will provide _garbage collection_ facilities for retrieving those locations that have become inaccessible for reuse.
Indeed what we just said about extents, or what we might have said about dynamically scoped variables, can also be said about *semantic scope*. Consider the scope of the existential quantifier, according to Groenendijk and Stokhof. It is not determinable statically, from the text or discourse—at least not until we have reached the end of the text or discourse, and this need not be determinable from any point in the text short of the end.

Though Groenendijk and Stokhof reject the computational metaphor, van Eijck and De Vries seem to accept it and accept, in particular, an interpretation of PDL according to which assignments, in the sense of valuations, model aspects of the cognitive states of listeners and natural language sentences are, like programs, associated with transformations of cognitive states. Natural language sentences are akin to programs, executed, that is uttered, so as to bring about changes in the cognitive states of one’s audience.

Perhaps a closer analogy to program variables or locations is to be found in Kamp’s *discourse markers* or Heim’s *indices*. One can think of the discourse representation construction algorithm as a program that, typically at least, begins by declaring a number of (sorted) variables, with (perhaps) determinate scopes but indeterminate extents. The assignments involved in the initializations, however, are sometimes, e.g., when associated with an indefinites *an N*, not simple, but rather random assignments, of the form:

\[
\begin{align*}
\text{begin } x_N; \\
x := ?;
\end{align*}
\]

whose meaning command is to assign a random value of sort $N$ to $x$.

Notice that we have not supplied an *end* for this block, precisely because in general this can not be determined unless we are given the text or discourse as a whole. Of course, this means that not only is the extent of $x$ indeterminate, so too is its scope. In our analytic practice, in which we are presenting theories or models of the use of natural language, we can stipulate that even though our object of study is, e.g., the understanding of discourses or texts as we (as subjects) encounter them, we (as scientists) have access to the completed discourse or text. Still, we must realize that one discourse is or can followed by another, after interruptions no doubt, where the later is about and is intended to be recognized to be about the very same objects as the earlier. Does what we mark as the end of a discourse or text mark the end of the extent (or scope) of a variable? Or do we rather rely on something akin to a garbage collection mechanism for retrieving free cognitive cells? This question is connected, of course, to the question of the accessibility of sub-DRS’s within the single DRS that results from processing a given text or discourse. But in the present context, these odd questions are really about the human cognitive architecture, in particular about interactions between short-term, on-line processing of linguistic

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7 This view can be seen as a high-tech version of Grice’s perspective on language and meaning.
information and long-term memory structures. Even more particularly, they are questions about the management of relatively persistent notions of individuals about whom we gain information in many different ways and over extended and interrupted periods of time.

3.1 Extending the Analogy

Of course it can not be surprising that any theory of natural language use that involves reference to the processes of understanding will involve reference to our cognitive architecture. The same is true for all fully adequate accounts of programming languages.

Programming languages can be thought of, at least partially, on the model of more familiar formal languages for mathematics. This is especially true for so-called functional languages; one can think of the meaning of a program (or term) in such a language as denoting some entity in a mathematical structure—as opposed to an algorithm or set of processes or sequence of actions—and of the denotations of complex terms in general as some function of the denotations of their constituents. This is the project of denotational semantics of programming languages. Even for functional languages, however, the question of the computational content of such a semantic account arises. For, as we noted above, programming languages are languages meant to be used, by us, to control the behavior of other entities—in this case, of computers. Thus, the equations of a denotational semantic account are often themselves conceived of as programs in a functional programming language whose modes of execution must then be specified in some more operational way. Or these equations may be given a direction in a term rewriting system and this latter operationalized.

Quite generally, then the design of a programming language is at the same time the design of an abstract machine or family of machines to execute programs in that language. Of course, the level of abstraction necessary and useful for an implementor of the language (on a real, commercially available) machine is quite different from that useful to a programmer. More directly, perhaps, the design of a programming language involves the specification of the behavior we intend to induce by the execution of programs from that language. This specification, whether by way of an abstract machine, by way of defining the transitions between abstractly characterized machine configurations, or by way of a definition of the relation between configurations and results (values) constitutes an operational semantics for the programming language.

We have just urged that a fully adequate semantics for a programming language will involve both a denotational and an operational semantics; but, of course, this is not enough. Surely we want to relate the two accounts in some systematic and illuminating way. One way is as follows. We can assume that the denotational account will provide a notion of denotational equivalence for programs; so where \( \tau_1, \tau_2 \) are programs, and \( [\cdot] \) is the semantic function specified by the account, we will have cases in which \( [\tau_1] = [\tau_2] \). As hinted above,
things are a bit more complicated on the operational side. To simplify, though, we can say that two programs are operationally equivalent when they can be substituted, one for the other, in a larger context, a larger encompassing program, without altering the behavior of that larger program. So we assume we can delimit a set of program contexts $C[]$ and propose the following (schematic) definition:

- **Operational Equivalence** Two expressions $\tau_1, \tau_2$, are operationally equivalent iff they induce the same behavior in all program contexts—iff for any program context, $C[]$,

$$C[\tau_1] =_{\text{oper}} C[\tau_2]$$

Then the desired fits between operational and denotational semantics are called *correctness* and *full abstractness*:

- **Correctness** $[\tau_1] = [\tau_2] \Rightarrow C[\tau_1] =_{\text{oper}} C[\tau_2]$
- **Full abstractness** $C[\tau_1] =_{\text{oper}} C[\tau_2] \Rightarrow [\tau_1] = [\tau_2]$

The requirement, in particular, of full abstractness is rather like that of compositionality: it is a methodological ideal—a good thing if you can get it, but it can be quite hard to attain and the price can be too high. There are relatively easy cases, though. Indeed, as we move away from functional programming languages, we may find that the gap between natural denotational accounts and illuminating operational accounts narrows; in particular, it may be that the underlying mathematical space in which the denotations of programs is to be found is best conceived of as consisting of fairly abstractly conceived configurations.

One such example is precisely that of the standard state-transformation, ‘dynamic’ semantics for sequential while-programs, which is provably fully abstract with respect to the partial correctness behavior of such programs, as axiomatized for instance, by Propositional Dynamic Logic or by way of Hoare triples. A partial correctness assertion about a program states that if the program is executed in a state that meets a certain condition (involving the values assigned to locations/variables), then a certain other condition will be true if and when the program terminates.

Van Eijck and de Vries use Hoare logic to reason about the ‘programs’ expressible in Dynamic Predicate Logic, which they, unlike Groenendijk and Stokhof, understand in a straightforwardly computational way. In a sense, their soundness and completeness results establish correctness and full abstraction for PDL with its the state-transformation semantics.

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8For which, see below Section
9The standard approach to proving full abstractness for imperative languages within the denotational paradigm consists in defining a typed λ-calculus (with fixed points) as a metalanguage, e.g., with explicit functions over stores or states and providing semantics for that language in terms of a particular family of mathematical structures called *domains*. 10for both partial and total correctness assertions
3.1.1 An Abstract Machine for DRT?

In our discussion of correctness and full abstractness, we noted that one must specify some notion of behavior and of behavioral equivalence and that one could do this in many different ways and at many different levels of abstraction. In the case mentioned above, of partial correctness behavior, we are interested only in the input-output behavior of programs, with respect only to highly abstract properties of undecomposed states. To get a feel for what might be involved in a more ful-bodied conception of behavior, let us turn briefly to a consideration of Kamp’s conception, which, though certainly not straightforwardly computational, is ‘mentalistic’ and oriented toward issues of cognitive processes.

One can think of the discourse representation construction algorithm as specifying the reader or parser for an interpreter for an abstract machine whose input is a stream of sentences and whose output (when it is initialized with a starting DRS) is a stream of DRS’s or a single over-all DRS for a given discourse or text. In what follows, we shall presume that for a given text or discourse, however this is delimited, a single DRS is produced. So the reader produces a new DRS, $DRS'$ given a sentence and the DRS $DRS$ produced so far. The additions to $DRS$ are determined by the parsing of the sentence into constituents, the commands that each type of constituent triggers, together with the input $DRS$.

We might characterize the set of configurations $\Gamma$ for a DRT machine as follows:

$$\Gamma = \langle Cst^*, Com^* \rangle \times DRS \times Val^* \times DRS'$$

where $Cst$ is the set of types of constituents, $Com$ is the set of commands, $DRS$, the set of discourse structures $DRS = \langle DM, Con \rangle$ and $Val = DM + Con$. If there are no more constituent-command pairs on the stack, the system terminates, with $DRS'$ as the final result. The stack of values is for holding intermediate results of the interpretation of subentential constituents. The questions raised before about the extent of discourse markers would be answered by a detailed specification of an interpreter for this abstract machine.

Thus imagine that one thought of giving an operational semantics for English by way of specifying the discourse representation construction algorithm as an interpreter for a state machine (our minds) whose configurations were as sketched above. One would want to specify some appropriate notion of behavior and of behavioral equivalence. Here’s a first very simple idea. Where texts are finite sequences of sentences:

- **Operational Equivalence** Two sentences $s_1, s_2$, are operationally equivalent iff they induce the same final $DRS$ in all texts—iff for any text, $C[]$, 
\[ C[s_1] = \text{oper} C[s_2] \]

where \( C[\,] \) is the DRS that results from executing the DRS construction algorithm, initialized with the empty DRS, on the text \( C \).

Of course, one may have to specify much else to yield any determinate analysis in this regard: in particular the perceptual capacities of the agent and the nonlinguistic knowledge base and knowledge base routines. e.g., access and inference routines, exploited by the language understanding algorithm. Or perhaps not: perhaps there is enough of an autonomous language-understanding module to justify talk of the purely linguistically accessible content of a text—what John Perry and I call the purely reflexive semantic content of the text. Waving all that aside, do we have any reason for thinking that there are many operationally equivalent sentence pairs? Perhaps the sentence is the wrong ‘unit’; perhaps only larger (or smaller) stretches of text will do. What category or categories of linguistic expression correspond to programs—to items that are directly executable and whose execution leads to observable behavior, observable transformations of configurations?

## 4 Proofs as Another Source of Analogies

Let us return to a simple instance of the most straightforward example of so-called discourse anaphora:

A man walked in. He sat down.

Heim, Kamp, Van Eijck and de Vries all suggest something like the following processing story:

1. The processor is to focus on an arbitrary man, call him \( a \),
2. such that \( a \), the man chosen, walked in;
3. then the processor is to add to the information about that man that he sat down.

The process in question is akin to that of opening up a file for a man, labeling that file \( a \) and incrementally entering new information about whomever it is that is called \( a \). We have urged above that that process is analogous in certain ways to declaring, allocating and initializing a location named by a variable and then invoking procedures involving that variable. It is also analogous to certain steps in the construction of proofs—but to what steps, in what kinds of proof? A first suggestion is to look to the rule of Existential Elimination (\( \exists E \)) in a Gentzen-Prawitz style natural deduction system(13):

In this style of proof, one does not prove things directly from existentially quantified sentences (\( \exists x \)\( \Phi \)); rather one must assume \( \Phi[x/a] \) for some otherwise
indeterminate entity labeled \( a \) and then use this assumption, together with whatever other sentences are available in the proof as so far constructed, to derive \( \Psi \), whereupon one can discharge the assumption.

How is one to think of the labels, typically called parameters, introduced by \( (\exists E) \), and eliminated by Universal Introduction \( (\forall I) \)? Syntactically they are atomic (individual) terms; but they are not variables, for they are not bindable, nor are they individual constants, for they are not to be thought of as in the domain of interpretation functions.\(^{11}\) Moreover, though this is often not made explicit, derivations are presumed to involve only sentences, formulae with no parameters (and no free variables), as undischarged assumptions—basic premises—and only sentences as conclusions. In a sense, parameters are not expressions in the language over which the provability predicate or derivation relation is being defined. They play a role only in the construction of proofs or derivations; only \textit{in media res}. Parameters are proof-dependent. Indeed, they can actually be shown to be inference-step dependent, that is, associated with exactly one application of either \( \exists E \) or \( \forall I \).\(^{12}\)

Some of these features of parameters are analogous to certain characteristics of discourse markers (especially those associated with indefinites) in Kamp’s theory or indices in Heim’s. In these latter cases, of course, the dependency involved is to texts or discourses, not proofs. Indeed, there is a sense in which these devices lead a double life, akin to parameters, or to program variables, as we urged above, \textit{during} the processing of a text, but then akin to quantified variables when the processing of the text has produced the final output DRS for interpretation. In these dynamic accounts, then, attention is drawn to the fact that there are intermediate stages or steps involved in the production of the final semantically evaluable/interpretable item (DRS or wff or file). We shall now turn to a brief examination of an analogous phenomenon in alternative derivation systems.

As we noted above, in Gentzen-Prawitz style systems of natural deduction, the structure of the proof associates with a given parameter \( a \) occurring in the proof a unique inference, an application of either \( (\exists E) \) or \( (\forall I) \), in which that parameter is involved as the proper parameter of the particular application.\(^{13}\) In fact not all derivations that meet the required derivation construction requirements meet this last condition (of \textit{purity}), but they can all be transformed quite straightforwardly into derivations that do. This is not in general the case for systems of natural deduction that replace \( (\exists E) \) with a rule \( (\exists \text{Inst}) \) of existential instantiation, and \( (\forall I) \) with \( (\forall G) \).

These systems are strikingly different from Gentzen-Prawitz systems in ways that go beyond the difference in the two troublesome quantifier rules; we will

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\(^{11}\)Below we shall discuss proof systems in which there is not a separate set of parameters, but only individual variables. In such systems, too, variables introduced by the rule of Existential Instantiation \( (\exists \text{Inst}) \) have rather special, derivation-dependent properties.

\(^{12}\)See \[\text{15}\], especially the definition of \textit{pure} parameters.

\(^{13}\)The terminology is Prawitz’s.
not go into any detail here. But we do want to make two points about them: (i) derivations in these systems are, like derivations in Hilbert-style systems, sequences of formulas, not (proper) trees and (ii) it is much more natural to think of these ‘(∃Inst)’ systems as being constructed in one direction, down from the premises. Of course, derivations in such systems can be constructed by a combination of downward (forward) and upward (backward) directed steps; but one cannot define for them a middle, as one can for Gentzen-Prawitz derivations in normal form. Indeed, one cannot define an interesting normal form for them, either. They are not motivated by the kinds of symmetries at the heart of Gentzen-Prawitz style systems; what Prawitz calls the inversion principle and proof reduction (normalization) plays no part in their design or presentation, or metatheory.

As for the rules themselves, (∃Inst) is stronger than (∃E); in particular, one can infer conclusions directly from existentials, though only under certain restrictions. These restrictions tend to be of two sorts, local and global. The local restrictions have the same force as the restrictions on (∃E) and (∀I), involving occurrences of the instantial term (parameter or variable) above the inference in question. The global restrictions, on the other hand, apply only to a completed proof or derivation.

For instance, Quine imposes the following ordering condition on flagged variables, variables that play the role of instantial terms for (∃Inst) and (∀G) analogous to that of parameters in Prawitz-style proofs:

It must be possible to list the flagged variables of a deduction in some order $V_1, \ldots, V_n$ such that, for each number $i$ from 1 to $n - 1$, $V_i$ is free in no line in which $V_{i+1}, \ldots, V_n$ is flagged. (14)

This condition (and less troubling flagging condition) are required for the proof of soundness to go through:

Thus it is that we are unwarranted in supposing the last line of a deduction to be implied by its premises unless what we have is a finished deduction; one whose flagged variables are not free in the last line nor in premisses of the last line. (14. Emphasis in the original.)

There are various ways to interpret this lack of determinacy with respect to the property of being a sound derivation (lack of line-by-line soundness). In Fine’s generic interpretation it corresponds to the fact that one can not determine the meaning of an instantial term (flagged variable/parameter) until
one knows what application of generalization, especially what application of \( \forall G \), will eliminate the term and one can’t know this with certainty until all parameters introduced in the derivation have been eliminated by generalization. Moreover he notes that if we try to construct the ordering required by the ordering principle as we proceed with the construction of the derivation, we may have to revise this ordering.

Here we shall take a different perspective, largely because of connections with work done in the tradition of E-type approaches to discourse anaphora, especially [11].

4.1 Epsilon-terms

Hilbert introduced \( \& \)-terms and designed the \( \& \)-calculus as part of his attempt to justify classical infinitistic results by purely finitary methods, in particular by transforming nonfinitistic proofs into finitistic (combinatorial) proofs ([18, 19]). Since, on the natural conception, the existential and universal quantifiers are infinitary operators (‘abbreviations’ of infinitary disjunctions or conjunctions), Hilbert eliminated them in favor of complex singular terms that can be seen as embodiments of the axiom of choice, which he took to be central to classical reasoning. Thus, for each predicate \( A[x] \), he introduced a term \( \&x(A[x]) \) and an axiom scheme (the logical \( \& \)-axiom:

\[
A[x] \rightarrow A(\&x(A[x]))
\]

The scheme justifies the following definitions of the quantifiers:

1. \((\exists x A[x])^* := A^*[\&x A^*[x]]\)
2. \((\forall x A[x])^* := A^*[\&x \neg A^*[x]]\)

\( A \) may contain other free variables, \( x_1, \ldots \); thus, in general, \( \&x(A[x]) \) can be understood as the value of a choice function, with parameters corresponding to those free variables. In what follows, though we shall ignore Hilbert’s own motivation and shall imagine a formalism that includes both quantifiers and \( \& \)-terms, and in which the definitions above are turned into the corresponding inference rules (\( \exists \text{Inst} \)) and (\( \forall G \))—in the case of (\( \forall G \)) the definition must be read from right to left. But this version of these rules requires no restrictions, local or global, on occurrences of instantial terms elsewhere in a derivation. This system, an abuse of Hilbert’s, can be seen to be a conservative extension over first-order logic, essentially by way of Hilbert’s second \( \& \)-theorem.

We can now follow a suggestion of [20] and consider abbreviating all \( \& \)-terms in sound derivations by single letters; the derivations will then look just like derivations in (\( \exists \text{Inst} \))-(\( \forall G \)) systems. The restrictions on parameters or

\[17\] The import of (\( \forall G \)) is as follows: if even the ideal or arbitrarily chosen non-\( A \) item is \( A \)-ish, then everything is \( A \)-ish.
flagged variables then take the form, collectively, of the requirement that all such instantial terms can be uniquely disabbreviated. And the indeterminacy *in media res* of the property of being a sound derivation takes the form of an ambiguity as to what *ε*-term such a term abbreviates, that is, what the matrix of the term is.

We have noted that the interpretation of a parameter or flagged variable introduced at a given stage in a (*∃Inst*)-(*∀G*) derivation depends on what happens subsequently in the derivation, e.g., on what applications of (*∀G*) it is involved in. This is, then, our last example in the list:

1. semantic scope of an NP (or of a quantifier)
2. scope of a program variable
3. extent of an identifier
4. interpretation of a parameter or flagged variable
5. matrix of an *ε*-term

In all these cases, we have indeterminacy of the semantic significance of an expression, relative to (a static analysis of) that part of the linguistic context {text/discourse/program/derivation} that has been encountered up to the point of the occurrence of the expression in question. In the case of natural languages and programming languages, we are dealing with communication tools that are intended to be used by and for agents of various sorts, with certain known processing characteristics. In the case of natural languages, the agents on both (all) sides of the communicative situation are humans—or machines programmed to behave, when encountering natural language utterances, as much like humans in certain respects as is possible. We often know how much we can leave indeterminate in a given communicative situation for we have some pretheoretical sense of the procedures by which we make the required determinations. In the case of programming languages, we can and should fully characterize the various dimensions of indeterminacy and the procedures by which these are resolved at run-time.

But what of derivations? Natural deduction systems, of both of the kinds we have mentioned, are defined over formal languages, but they are meant to model certain aspects of informal deductive reasoning. In particular, the (*∃Inst*)-(∀G) systems might be said to be more faithful models of an aspect of our informal reasoning that we often ignore or wish away: namely the fact that we often times don’t really know what we’re doing or how we should go about deriving a given sentence from a set of premises. This is an odd virtue, of course, and not one that purveyors of such systems are likely to boast about. Moreover, on the one hand, these systems, e.g., Quine’s in *Methods of Logic*, are provably sound and complete and, on the other, much the same possibilities of false-starts and somewhat blind proof-search are also available in Gentzen-Prawitz systems.
Still, we claim that the (∃Inst)-(∀G) systems, in use in actual proof construction, more faithfully reflect these features of informal reasoning than the more elegant and metamathematically satisfactory ND systems and once again, this is largely due to the feature sketched above: that the precise derivational import of a line is not in general determined by the structure of the derivation at the stage when the line is added.

5 Concluding Philosophical Considerations

In their discussion, in [1], of the Principle of Compositionality, Groenendijk and Stokhof note that it is primarily a methodological principle, to be adhered to when possible, to be eschewed if the empirical or computational or philosophical costs are too high. One philosophical consideration has to do with what might be called the autonomy of semantics, in particular its autonomy with respect to psychological or cognitive or mentalist issues involved in theorizing about processes involved in natural language understanding. “The best way to go about it is to carry on semantics as really a discipline of its own, not to consider it a priori a branch of cognitive science...” In particular, they deny the force of the metaphor that they feel underlies some, at least, of the attraction of positing a level of mental representation of the content or meaning of natural language utterances:

Our own opinion, for what it is worth, is that the calculating mind is a metaphor rather than a model. It is a powerful metaphor, no doubt, on which many branches of ‘cognitive’ science are based, and sometimes it can be helpful, even insightful. But it remains a way of speaking, rather than a true description of the way we are. ([1])

Quite independent of the computational metaphor for mind, there is something odd about the claim of the autonomy of the semantics of natural languages. Semantics, at least semantics in the model-theoretic vein of the school of Tarski, is an application of set theory. Indeed, the theorems of a Tarski-style definition of truth or satisfaction are theorems of applied (impure) set theory; they can even be made into theorems of pure set theory through a little coding. Set theory is a part of mathematics, and it is certainly autonomous from psychology (at least it is in my opinion, for what it is worth). But natural languages and their use in the expression of thought and in communication are natural, contingent, empirical phenomena, involving essentially, or so it would seem, things going on in people’s minds, as well things going on outside their skulls. This does not constitute an argument against the autonomy of semantics, though it might constitute the beginning of one; but it does suggest that it is not at all

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18 Much more can and should be said about the structure of derivations, about, for instance, the centrality of the subformula property in constraining derivations; but it shall have to wait for another occasion.
clear that one should not consider the semantics of natural languages a branch of cognitive science, however computationally this latter is itself conceived. Finally, Groenendijk and Stokhof return to the theme of the inadequacies of natural language that we have seen raised by Frege and Peano. Allow me to quote at length:

It may be the case, though, that for some the acceptance of level of logical representation springs forth from a positive philosophical conviction, viz., a belief in the deficiencies of natural language as a means to convey meaning. Now such there may be (or not) when we consider very specialized kinds of theoretical discourse, such as mathematics...In such cases, clearly there is room for extension and revision, for regimentation and confinement. But that is not what is at stake here. Here, it turns on the question whether natural language structures themselves, as we encounter them in spoken or written language, then and there are in need of further clarification in order to convey what they are meant to convey. In this matter, semantics, we feel, should start from the premiss that natural language is all right. If anything is a perfect means to express natural language meaning, natural language is. It can very well take care of itself and is in no need of (psycho)logical reconstruction and improvement in this respect. (1)

How are we to reconcile this view with Frege’s? Assuming that one important component of the meaning of sentences lies in their logical interrelationships, and assuming that one can’t easily or systematically determine when one sentence is a logical consequence of another or of a finite set of others, it does seem that Frege’s view is that natural languages are not perfect means of expressing meaning. Also, how are to reconcile this view with what Groenendijk and Stokhof themselves say in [2]. Their view in that paper is that sentences of natural language convey partial information about the references of discourse markers (of which more later). But of course they realize that natural language sentences don’t themselves contain discourse markers, just as they don’t themselves contain indices, variables, or explicit scope indicators.

We need to choose a particular discourse marker in the translation [into the language of DMG], which is indicated by the index that occurs in the determiner itself. So the present approach assumes that we do not translate sentences as such, but indexed structures, i.e., sentences in which determiners, pronouns, and proper names are marked with indices. The function of the indexing mechanism

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18 Indeed, it should be noted that in “Dynamic Montague Grammar,” Groenendijk and Stokhof specify the content of a natural language sentences φ followed by ψ as “sets of propositions p such that after the processing of φ in state s, it holds that p holds after the processing of ψ.” Surely this is reference to processing by a mind of some sort.
is, of course, to stake out possible anaphoric relationships among constituents; if a pronoun is to be related anaphorically to an NP, a necessary (but not sufficient) condition is that both carry the same index.

In fact their earlier approach assumed something similar, though simpler. So natural language sentences, just as they are, as we encounter them in written and spoken language, seem not to be quite all right—at least not for purposes of systematic semantic analysis. Yet they are, perforce, all right for us, as users—speakers and hearers—of our natural languages. Frege hints at an explanation of this puzzle: as used, they are produced and meant to be encountered by intelligent agents—by us—who are either presumed or known to bring to bear on these encounters a vast store of knowledge, both quite general and context-specific, and skills of many sorts. This presumption of intelligence on the part of one’s audience, and the fact that it is so often shown to be well-grounded, allows speakers to leave much ‘to guesswork’; more exactly, the presumption is to the effect that something much more rational and systematic than mere guesswork on the part of one’s listeners can be depended on.

6 Conclusion

Let us return to the original diagnosis of the troublesome phenomena:

The phenomena all involve anaphoric pronouns outside the scope of their [indefinite] NP antecedents.

This has the advantage of being plausible to linguists, as it is grounded in widely accepted empirical generalizations; but it has the disadvantage of being purely negative. The more general diagnosis and corresponding positive program of research might be put as follows:

Donkey- or discourse-anaphora shows us that we cannot think of the semantic values of natural language expressions, in particular of sentences, as isolated entities, to be generated one per sentence, though in a systematic way, by a semantic account. Rather our semantic account must make room for possibilities of semantically significant interaction among sentences in sequence—for the ways in which the semantics of a sentence can be effected by sequences of sentences that are its ‘prefixes’ in a text and the ways in which it can, in turn, effect the semantics of sentences that come after it.

This has the ring of illuminating truth, but what truth? It certainly does not follow that the semantics of sentences cannot be adequately represented by sets of things, e.g., assignments, rather than sets of ordered pairs of things. Perhaps
the truth lies deeper: that contra Frege et al., the central unit of semantic analysis is not the sentence. Deeper still: the interactions among sentences of which we speak can only be mediated by, indeed grounded in, the cognitive activities and processes of agents who use language, by putting together sentences for certain purposes and inferring those purposes by understanding the sequences so constructed.

It is time to repeat the main claims of this essay.

Natural languages are used by intelligent agents to communicate with and influence one another. The same is true, though in a limited and unidirectional way, of programming languages. It is not true, in any direct way, of the formal languages developed as precise mathematical tools for analyzing mathematical proof. It was a crucial requirement that they wear their meanings on their sleeves—that no guesswork, inspired or not, be necessary to determine under what conditions a well-formed sentence belonging to such a language is true. In our view, the real import of the phenomenon of donkey anaphora is just that it is an especially simple and forceful reminder that natural languages are not like formal languages in that respect.

The revolution wrought by Montague consists in the fact that he was the first logician to apply sophisticated techniques of mathematical logic to a systematic analysis of the semantics of natural languages. This was especially striking coming from a student of a tradition according to which natural languages were not fit objects of such analysis; but in any event the revolution in attitude is more important than any detailed treatment of particular phenomena. The revolution is also more important than, and independent of, the philosophical attitude of its originator. Montague, like Tarski, took semantics to be a branch of applied set theory. We have argued that the semantics of natural languages cannot be so taken.

A complete theory of a programming language must include an account, at some level of abstraction, of the ‘psychology’ of the machines that execute the language. Just so, a complete theory of a natural language must include an account, at some level of abstraction, of the virtual machines in the heads of its users: us.

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\footnote{Given the sweeping nature of the conclusion, we consign to a footnote the reminder that in the treatments in \cite{3} and \cite{11}, the only aspect of our psychology involved is the fact that we process texts sequentially, left-to-right. And, of course, we are ignoring cataphora and conditionals in which the consequent has the bad manners to precede the antecedent. To deal with such cases adequately requires a story about revisions in the process of interpretation.}
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