Realistic Type IIB Supersymmetric Minkowski Flux Vacua

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We show that there exist supersymmetric Minkowski vacua on Type IIB toroidal orientifolds with general flux compactifications where the RR tadpole cancellation conditions can be relaxed elegantly. Then we present a realistic Pati-Salam like model. At the string scale, the gauge symmetry can be broken down to the Standard Model (SM) gauge symmetry, the gauge coupling unification can be achieved naturally, and all the extra chiral exotic particles can be decoupled so that we have the supersymmetric SMs with/without SM singlet(s) below the string scale. The observed SM fermion masses and mixings can also be obtained. In addition, the unified gauge coupling, the dilaton, the complex structure moduli, the real parts of the Kähler moduli and the sum of the imaginary parts of the Kähler moduli can be determined as functions of the four-dimensional dilaton and fluxes, and can be estimated as well.

PACS numbers: 11.10.Kk, 11.25.Mj, 11.25.-w, 12.60.Jv

Introduction – One of the great challenging and essential problems in string phenomenology is the construction of the realistic string vacua, which can give us the low energy supersymmetric Standard Models (SMs) without exotic particles, and can stabilize the moduli fields. With renormalization group equation running, we can connect such constructions to the low energy realistic particle physics which will be tested at the upcoming Large Hadron Collider (LHC). During the last a few years, the intersecting D-brane models on Type II orientifolds \([1]\), where the chiral fermions arise from the intersections of D-branes in the internal space \([2]\) and the T-dual description in terms of magnetized D-branes \([3]\), have been particularly interesting \([4]\).

On Type IIA orientifolds with intersecting D6-branes, many non-supersymmetric three-family Standard-like models and Grand Unified Theories (GUTs) were constructed in the beginning \([5]\). However, there generically existed uncancelled Neveu-Schwarz-Neveu-Schwarz (NSNS) tadpoles and the gauge hierarchy problem. To solve these problems, semi-realistic supersymmetric Standard-like and GUT models have been constructed in Type IIA theory on the \(T^6/(Z_2 \times Z_2)\) orientifold \([6, 7]\) and other backgrounds \([8]\). Interestingly, only the Pati-Salam like models can give all the Yukawa couplings. Without the flux background, Pati-Salam like models have been constructed systematically in Type IIA theory on the \(T^6/(Z_2 \times Z_2)\) orientifold \([9]\). Although we may explain the SM fermion masses and mixings in one model \([9]\), the moduli fields have not been stabilized, and it is very difficult to decouple the chiral exotic particles. To stabilize the moduli via supergravity fluxes, the flux models on Type II orientifolds have also been constructed \([10, 11]\). Especially, some flux models \([11]\) can explain the SM fermion masses and mixings. However, those models are in the AdS vacua and have quite a few chiral exotic particles that are difficult to decoupled.

In this paper, we consider the Type IIB toroidal orientifold with the Ramond-Ramond (RR), NSNS, non-geometric and S-dual flux compactifications \([12]\). We find that the RR tadpole cancellation conditions can be relaxed elegantly in the supersymmetric Minkowski vacua, and then we may construct the realistic Pati-Salam like models \([13]\). In this paper, we present a concrete simple model which is very interesting from the phenomenological point of view and might describe Nature. We emphasize that we do not fix the four-dimensional dilaton via flux potential. The point is that the fixed values for dilaton and Kähler moduli can be estimated as well. This is a blessing in disguise from a cosmological point of view \([14]\).

Type IIB Flux Compactifications – We consider the Type IIB string theory compactified on a \(T^6\) orientifold where \(T^6 = T^2 \times T^2 \times T^2\) whose complex coordinates are \(z_i, i = 1, 2, 3\) for the \(i^{th}\) two-torus, respectively. The orientifold projection is implemented by gauging the symmetry \(\Omega R\), where \(\Omega\) is world-sheet parity, and \(R\) is given by

\[
R: (z_1, z_2, z_3) \rightarrow (-z_1, -z_2, -z_3)
\]

Thus, the model contains 64 O3-planes. In order to cancel the negative RR charges from these O3-planes, we introduce the magnetized D(3+2n)-branes which are filling up the four-dimensional Minkowski space-time and wrapping \(2n\)-cycles on the compact manifold. Concretely, for one stack of \(N_a\) D-branes wrapped \(m_a^{th}\) times on the \(i^{th}\) two-torus \(T^2_i\), we turn on \(n_a^{th}\) units of magnetic fluxes \(F^a_i\) for the center of mass \(U(1)_a\) gauge factor on \(T^2_i\), such...
TABLE I: General spectrum for magnetized D-branes on the Type IIB \( T^6 \) orientifold.

| Sector | Representation |
|--------|----------------|
| \( aa \) | \( U(N_a) \) vector multiplet |
|        | 3 adjoint multiplets |
| \( ab + ba \) | \( I_{ab}(N_a, N_b) \) multiplets |
| \( ab' + b'a \) | \( I_{a'b'}(N_a, N_b) \) multiplets |
| \( aa' + a'a' \) | \( \frac{1}{2}(I_{aa'} - I_{a'O}) \) symmetric multiplets |
|        | \( \frac{1}{2}(I_{aa'} + I_{a'O}) \) anti-symmetric multiplets |

that

\[
m_a \frac{1}{2\pi} \int_{T^2} F_a^i = n_a^i ,
\]

where \( m_a \) can be half integer for tilted two-torus. Then, the D9-, D7-, D5- and D3-branes contain 0, 1, 2 and 3 vanishing \( m_a \)'s, respectively. Introducing for the \( i^{\text{th}} \) two-torus the even homology classes \( [O_i] \) and \( [T^2_i] \) for the point and two-torus, respectively, the vectors of the RR charges of the \( a^{\text{th}} \) stack of D-branes and its image are

\[
[I_a] = \prod_{i=1}^{3} (n_a^i [O_i] + m_a^i [T^2_i]) ,
\]

\[
[I_a'] = \prod_{i=1}^{3} (n_a^i [O_i] - m_a^i [T^2_i]) ,
\]

respectively. The “intersection numbers” in Type IIA language, which determine the chiral massless spectrum, are

\[
I_{ab} = [I_a] \cdot [I_b] = \prod_{i=1}^{3} (n_a^i m_b^i - n_b^i m_a^i) .
\]

Moreover, for a stack of \( N \) D(2n+3)-branes whose homology classes on \( T^6 \) is (not) invariant under \( \Omega R \), we obtain a \( USp(2N) \) (\( U(N) \)) gauge symmetry with three anti-symmetric (adjoint) chiral superfields due to the orbifold projection. The physical spectrum is presented in Table I.

The flux models on Type IIB orientifolds with four-dimensional \( N \) = 1 supersymmetry are primarily constrained by the RR tadpole cancellation conditions that will be given later, the four-dimensional \( N \) = 1 supersymmetric D-brane configurations, and the K-theory anomaly free conditions. For the D-branes with worldvolume magnetic field \( F_a^i = n_a^i / (m_a^i \chi_i) \) where \( \chi_i \) is the area of \( T^2_i \) in string units, the condition for the four-dimensional \( N \) = 1 supersymmetric D-brane configurations is

\[
\sum_i (\tan^{-1}(F_a^i)^{-1} + \theta(n_a^i)\pi) = 0 \mod 2\pi ,
\]

where \( \theta(n_a^i) = 1 \) for \( n_a^i < 0 \) and \( \theta(n_a^i) = 0 \) for \( n_a^i \geq 0 \). The K-theory anomaly free conditions are

\[
\sum_a N_a n_a^1 m_a^2 m_a^3 = \sum_a N_a n_a^1 m_a^2 n_a^3 = \sum_a N_a n_a^1 m_a^3 n_a^3
\]

\[
= \sum_a N_a n_a^1 n_a^2 m_a^3 = 0 \mod 2 .
\]

And the holomorphic gauge kinetic function for a generic stack of D(2n+3)-branes is given by \(^{13, 15}\)

\[
f_a = \frac{1}{\kappa_a} (n_a^1 n_a^2 n_a^3 s - n_a^1 m_a^2 m_a^3 t_1 - n_a^2 m_a^2 m_a^3 t_2 - n_a^3 m_a^2 m_a^3 t_3) ,
\]

where \( \kappa_a \) is equal to 1 and 2 for \( U(n) \) and \( USp(2n) \), respectively.

We turn on the NSNS flux \( h_0 \), RR flux \( e_i \), non-geometric fluxes \( b_{ij} \) and \( b_{ii} \), and the S-dual fluxes \( f_i, g_{ij} \) and \( g_{ii} \)^{12}. To avoid the subtleties, these fluxes should be even integers due to the Dirac quantization. For simplicity, we assume

\[
e_i = e , \quad b_{ij} = \beta , \quad b_{ii} = \bar{\beta} ,
\]

\[
f_i = f , \quad g_{ij} = -g_{ii} = g ,
\]

where \( i \neq j \). Then the constraint on fluxes from Bianchi identities is

\[
f \bar{\beta} = g \beta .
\]

The RR tadpole cancellation conditions are

\[
\sum_a N_a n_a^1 n_a^2 n_a^3 = 16 ,
\]

\[
\sum_a N_a n_a^1 m_a^2 m_a^3 = -\frac{1}{2} e \bar{\beta} ,
\]

\[
N_{NS7} = 0 , \quad N_{I7} = 0 ,
\]

where \( i \neq j \neq k \neq i \), and the \( N_{NS7} \) and \( N_{I7} \), denote the NS7 brane charge and the other 7-brane charge, respectively \(^{12}\). Thus, if \( e \bar{\beta} < 0 \), the RR tadpole cancellation conditions are relaxed elegantly because \(-e \bar{\beta}/2\) only needs to be even integer. Moreover, we have 7 moduli fields in the supergravity theory basis, the dilaton \( s \), three Kähler moduli \( t_i \), and three complex structure moduli \( u_i \). With the above fluxes, we can assume

\[
t \equiv t_1 + t_2 + t_3 , \quad u_1 = u_2 = u_3 \equiv u \.
\]

Then the superpotential becomes

\[
W = 3 i e u + i h_0 s - t (\beta u - i \bar{\beta} u^2) - st (f - i g u) .
\]

The Kähler potential for these moduli is

\[
K = -\ln(s + \bar{s}) - \sum_{i=1}^3 \ln(t_i + \bar{t}_i) - \sum_{i=1}^3 \ln(u_i + \bar{u}_i) .
\]
In addition, the supergravity scalar potential is
\[
V = e^K \left( K^{ij} D_i W D_j W - 3 |W|^2 \right),
\]
where $K^{ij}$ is the inverse metric of $K_{ij} = \partial_i \partial_j K$, $D_i W = \partial_i W + (\partial_i K) W$, and $\partial_i = \partial_{\phi_i}$, where $\phi_i$ can be $s$, $t$, and $u_i$. Thus, for the supersymmetric Minkowski vacua, we have
\[
W = \partial_s W = \partial_t W = \partial_u W = 0.
\] (15)
From $\partial_s W = \partial_t W = 0$, we obtain
\[
t = \frac{ih_0}{f - igu}, \quad s = -\frac{\beta}{f} u,
\] (16)
then the superpotential turns out
\[
W = \left( 3e - \frac{h_0 \beta}{f} \right) iu.
\] (17)
Therefore, to satisfy $W = \partial_u W = 0$, we obtain
\[
3ef = \beta h_0.
\] (18)
Because $\text{Re} s > 0$, $\text{Re} t > 0$ and $\text{Re} u > 0$, we require
\[
\frac{h_0}{g} < 0, \quad \frac{\beta}{f} < 0.
\] (19)

**Model** – Choosing $e \beta = -12$, we present the D-brane configurations and intersection numbers in Table II and the resulting spectrum in Table III. The anomalies from the global $\text{U}(1)$s of $U(4)_C$, $U(2)_L$ and $U(2)_R$ are cancelled by the Green-Schwarz mechanism, and the gauge fields of these $U(1)$s obtain masses via the linear $B \wedge F$ couplings. So, the effective gauge symmetry is $SU(4)_C \times SU(2)_L \times SU(2)_R$. In order to break the gauge symmetry, on the first two-torus, we split the $a$ stack of D-branes into $a_1$ and $a_2$ stacks with 3 and 1 D-branes, respectively, and split the stack of D-branes into $c_1$ and $c_2$ stacks with 1 D-brane for each one. Then, the gauge symmetry is further broken down to $SU(3)_C \times SU(2)_L \times U(1)_{t_{3R}} \times U(1)_{B-L}$. We can break the $U(1)_{t_{3R}} \times U(1)_{B-L}$ gauge symmetry down to the $U(1)_Y$ gauge symmetry by giving vacuum expectation values (VEVs) to the vector-like particles with quantum numbers $(1, 1, 1, 2, -1)$ and $(1, 1, -1, 2, 1)$ under $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ from $a_2 c_1'$ D-brane intersections. Similar to the discussions in Ref. 3, we can explain the SM fermion masses and mixings via the Higgs fields $H_u$, $H_u'$, $H_d$ and $H_d'$ because all the SM fermions and Higgs fields arise from the intersections on the first torus. To decouple the chiral exotic particles, we assume that the $T_R^i$ and $S^i_R$ obtained VEVs at about the string scale, and their VEVs satisfy the D-flatness $U(1)_R$. The chiral exotic particles mass $M_{\text{str}}$ is the string scale, and we neglect the $O(1)$ coefficients in this paper. In addition, the vector-like particles $S^i_L$ and $\overline{S}^i_R$ in the anti-symmetric representation of $SU(2)_L$ can obtain the VEVs close to the string scale while keeping the D-flatness $U(1)_L$. Thus, we can decouple all the Higgs bidoublets close to the string scale except one pair of the linear combinations of the Higgs doublets for the electroweak symmetry breaking at the low energy by fine-tuning the following superpotential
\[
W \supset \Phi_i (S^i_L \Phi' + \overline{S}^i_R \overline{\Phi'}) + \Phi_i (T^i_R \Phi' + \overline{S}^i_R \overline{T}^i_R)
\]
\[
+ \frac{1}{M_{\text{str}}} (S^i_L S^j_R \phi_i + S^i_L T^j_R \phi_i + \overline{S}^i_R \overline{S}^j_R \Phi' + \overline{S}^i_R \overline{T}^j_R)
\]. (21)

In short, below the string scale, we have the supersymmetric SMs which may have zero, one or a few SM singlets from $S_1^L$, $\overline{S}_1^R$, and/or $S_2^R$. And then the low bound on the lightest CP-even Higgs boson mass in the minimal supersymmetric SM can be relaxed if we have the SM singlet(s) at low energy [10].

**Table II**: D-brane configurations and intersection numbers.

| $\Phi$ | $(a', m')$ | $n_a$ | $n_{a'}$ | $b'$ | $c'$ | $O3$ |
|-------|-------------|------|---------|------|------|-----|
| $a$   | $(1, 0) \times (1, 1)$ | 1    | 0       | 0    | 0    | 0   |
| $b$   | $(1, -1) \times (1, 1)$ | 0    | 0       | 0    | 0    | 0   |
| $c$   | $(1, 1) \times (1)$ | -6   | -1      | -1   | -1   | -1  |
| $O3$  | $(1, 0) \times (1, 0)$ | -6   | -1      | -1   | -1   | -1  |

**Table III**: The chiral and vector-like superfields, and their quantum numbers under the gauge symmetry $SU(4)_C \times SU(2)_L \times SU(2)_R \times U(1)$.

| Quantum Number | $Q_1$ | $Q_2$ | $Q_3$ | Field |
|----------------|-------|-------|-------|-------|
| $ab$           | 3     | 4     | 1     | $F_{1(L), L(R)}$ |
| $ac$           | 3     | 4     | 1     | $F_{1(R), L(R)}$ |
| $cs$           | 6     | 1     | 0     | $T^i_R$ |
| $ca$           | 6     | 1     | 0     | $S^i_R$ |
| $c'c''$        | 3     | 4     | 1     | $X^i$ |
| $bc$           | 6     | 1     | 0     | $\Phi_i$ |
| $bc'$          | 6     | 1     | 0     | $\Phi_i$ |
| $bb'$          | 6     | 1     | 0     | $S^i_L$ |

Next, we consider the gauge coupling unification and moduli stabilization. The real parts of the dilaton and
Kähler moduli in our model are [13]

\[
\begin{align*}
\text{Res} &= \frac{\sqrt{6}e^{-\phi_4}}{4\pi}, \quad \text{Ret}_1 = \frac{\sqrt{6}e^{-\phi_4}}{2\pi}, \\
\text{Ret}_2 &= \frac{\sqrt{6}e^{-\phi_4}}{12\pi}, \quad \text{Ret}_3 = \frac{\sqrt{6}e^{-\phi_4}}{6\pi},
\end{align*}
\]

where \( \phi_4 \) is the four-dimensional dilaton. From Eq. (17), we obtain that the SM gauge couplings are unified at the string scale as follows

\[
g_{SU(3)_C}^{-2} = g_{SU(2)_L}^{-2} = 3 g_{U(1)_Y}^{-2} = \frac{\sqrt{6}e^{-\phi_4}}{2\pi}.
\]

Using the unified gauge coupling \( g^2 \simeq 0.513 \) in supersymmetric SMs, we get

\[
\phi_4 \simeq -1.61.
\]

For moduli stabilization, we first obtain \( t \) from Eqs. [10] and (22)

\[
\text{Ret} = \frac{3\sqrt{6}e^{-\phi_4}}{4\pi}, \quad \text{Im}t = \pm \sqrt{\frac{3\beta h_0 - 27 e^{-2\phi_4}}{8\pi^2}}.
\]

Thus, we have

\[
\begin{align*}
\text{Im} s &= \frac{1}{3} \text{Im} t + \frac{\beta}{g}, \\
\text{Re} u &= -\frac{\sqrt{6}e^{-\phi_4}}{4\pi \beta}, \quad \text{Im} u = \frac{f}{3\beta} \text{Im} t - \frac{f}{g}.
\end{align*}
\]

Let us present a set of possible solutions to the fluxes

\[
\begin{align*}
h_0 &= -18\eta, \quad e = 6\eta, \quad \beta = 2\eta', \\
\beta &= -2\eta, \quad f = -2\eta', \quad g = 2\eta,
\end{align*}
\]

where \( \eta = \pm 1 \) and \( \eta' = \pm 1 \). Choosing \( \phi_4 = -1.61, \eta = \eta' = 1 \), we obtain the numerical values for the moduli fields

\[
\begin{align*}
\text{Res} &= \text{Re} u = 0.975, \quad \text{Ret}_1 = 1.95, \\
\text{Ret}_2 &= 0.325, \quad \text{Ret}_3 = 0.650, \\
\sum_{i=1}^{3} \text{Im} t_i &= \pm 4.30, \quad \text{Im} s = \text{Im} u = \mp 1.43 + 1.
\end{align*}
\]

**Conclusions** — We showed that the RR tadpole cancellation conditions can be relaxed elegantly in the supersymmetric Minkowski vacua on the Type IIB toroidal orientifold with general flux compactifications. And we presented a realistic Pati-Salam like model in details. In this model, we can break the gauge symmetry down to the SM gauge symmetry, realize the gauge coupling unification, and decouple all the extra chiral exotic particles around the string scale. We can also generate the observed SM fermion masses and mixings. Furthermore, the unified gauge coupling, the dilaton, the complex structure moduli, the real parts of the Kähler moduli and the sum of the imaginary parts of the Kähler moduli can be determined as functions of the four-dimensional dilaton and fluxes, and can also be estimated.

**Acknowledgments** — This research was supported in part by the Mitchell-Heep Chair in High Energy Physics (CMC), by the Cambridge-Mitchell Collaboration in Theoretical Cosmology (TL), and by the DOE grant DE-FG03-95-Er-40917 (DVN).