A Search for Millilensing Gamma-Ray Bursts in the Observations of Fermi GBM

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Abstract

Millilensing of gamma-ray bursts (GRBs) is expected to manifest as multiple emission episodes in a single triggered GRB with similar light-curve patterns and similar spectrum properties. Identifying such lensed GRBs could help improve constraints on the abundance of compact dark matter. Here we present a systemic search for millilensing among 3000 GRBs observed by the Fermi GBM up to 2021 April. Eventually we find four interesting candidates by performing an autocorrelation test, hardness test, and time-integrated/resolved spectrum test. GRB 081126A and GRB 090717A are ranked as the first-class candidates based on their excellent performance in both temporal and spectrum analysis. GRB 081122A and GRB 110517B are ranked as the second-class candidates (suspected candidates), mainly because their two emission episodes show clear deviations in part of the time-resolved spectrum or in the time-integrated spectrum. Considering a point-mass model for the gravitational lens, our results suggest that the density parameter of lens objects with mass $M_L \sim 10^6 M_\odot$ is larger than $1.5 \times 10^{-3}$.

Unified Astronomy Thesaurus concepts: Gamma-ray bursts (629)

1. Introduction

In general relativity, light from very distant sources would be deflected by intervening masses, which is called the gravitational lensing effect. Depending on the positions of the source, lens, and observer and the mass and shape of the lens, gravitational lensing can manifest itself by making one source produce multiple images, or altering the source image through shearing and convergence (Schneider et al. 1992). Due to the rapid progress in time domain surveys, many searches have been carried out for gravitational lensing of explosive transients because such systems have been widely proposed as promising cosmological and astrophysical probes (see Oguri 2019 for a review). For instance, transients strongly lensed by intervening galaxies could be used to improve constraints on cosmological parameters such as the Hubble constant $H_0$ (Liao et al. 2017; Li et al. 2018); differences of time delays between multiple images among different particles, energy, or messengers could be used for testing fundamental physics from the propagation speed (Biesiada & Piórkowska 2009; Collett & Bacon 2017; Fan et al. 2017); and statistical search results of lensed transients with different timescales could be applied to derive constraints on the abundance of compact dark matter in different mass ranges (Munoz et al. 2016; Ji et al. 2018; Liao et al. 2020; Zhou et al. 2022a).

As one of the most violent explosions in the universe, gamma-ray bursts (GRBs) are bright enough to be detected in the high-redshift range up to at least $z \sim 10$ (Salvaterra et al. 2009; Tanvir et al. 2009), so they have long been proposed as the most promising transients for searching for the gravitational lensing effect (Paczynski 1987; Mao 1992; Li & Li 2014). Thanks to the successful operation of several dedicated detectors, e.g., the Burst And Transient Source Experiment (BATSE) on Compton Gamma Ray Observatory (CGRO; Meegan et al. 1992), the Burst Alert Telescope (BAT) on the Neil Gehrels Swift Observatory (Gehrels et al. 2004; Barthelmy et al. 2005), and the Gamma-Ray Burst Monitor (GBM) on the Fermi Observatory (Meegan et al. 2009), $\sim 10^4$ GRBs have been detected. Two kinds of searches for lensed GRBs have been widely carried out:

1. Searching for independently triggered GRB pairs with similar light curves and spectra, but with different fluxes and a small positional offset (Nemiroff et al. 1994; Veres et al. 2009; Davidson et al. 2011; Li & Li 2014; Hurley et al. 2019; Ahlgren & Larsen 2020). This is usually called a macrolensing event, for which the lens candidates are galaxies or clusters of galaxies (with mass $> 10^{10} M_\odot$).

2. Searching for two emission episodes in a single triggered GRB with similar light-curve patterns and similar spectrum properties. This is usually called a millilensing event, for which the lens candidates could be an intermediate-mass black hole (Paynter et al. 2021), compact dark matter (Nemiroff et al. 1993; Ji et al. 2018), star cluster, or Population III star (Hirose et al. 2006), with mass $10^4 - 10^5 M_\odot$.

Up to now, all searches for macrolensing events have yielded null results. On the other hand, however, several candidates of millilensing events have been proposed. Paynter et al. (2021) claim the first convincing evidence for millilensing events in the light curve of BATSE GRB 950830 and thus claim the existence of intermediate-mass black holes. Later on, an...
increase of millilensing GRB candidates from the data of Fermi GBM was claimed (Kalantari et al. 2021; Veres et al. 2021; Wang et al. 2021; Yang et al. 2021). For instance, Wang et al. (2021) and Yang et al. (2021) made independent analyses of GRB 200716C and argued that it showed millilensing signatures. Veres et al. (2021) made an exhaustive temporal and spectral analysis to claim that GRB 210812A shows strong evidence in favor of the millilensing effects.

Inspired by the claims of these individual cases, it is essential to operate a systematic search for millilensing events in the entire Fermi sample because (1) after a decade of operation, thousands of GRBs were detected by the Fermi satellite, and new candidates may be found; and (2) Fermi GBM is monitoring the whole sky almost full-time, and the millilensing event rate for Fermi GRBs could provide powerful probes of the small-scale structure of the universe.

Most recently, Kalantari et al. (2021) applied the autocorrelation method to the Fermi sample and found one more millilensing candidate (GRB 090717A). But Mukherjee & Nemiroff (2021b) later argued that the light curves of two pulses in GRB 090717 differ at about the 5σ confidence level; therefore, GRB 090717 does not present a compelling example of gravitational lensing. Nevertheless, in the analysis of Kalantari et al. (2021), spectral properties for each source have not been taken into account (not even the hardness test). In existing case studies, it can be seen that spectral analysis plays a vital role in identifying the authenticity of candidates. For instance, Mukherjee & Nemiroff (2021a) found cumulative hardness discrepancies between the two pulses in GRB 950830 and thus argued that the case for GRB 950830 involving a gravitational lens may well be considered intriguing but should not be considered proven.

In their study of GRB 210812A, Veres et al. (2021) show that a time-resolved spectrum is a more powerful tool than hardness to help justify the lensing effect. Taking advantage of its two main instruments (GBM and LAT), it is possible to study the γ-ray spectra of Fermi GRBs in unprecedented detail. In this work, we intend to conduct a comprehensive analysis of the most complete Fermi samples at present to systematically search for millilensing events. We will first apply the autocorrelation method to analyze light curves of each source in the sample and make the preliminary screening of candidates in combination with the hardness test (see Section 2.1). Then, we will analyze the time-resolved spectrum for each candidate for further justification (see Section 3). Conclusion and discussion are presented at the end (see Section 4).

2. Preliminary Candidate Selection

2.1. Data

In this work, we use the data observed by the Fermi GBM, which consists of 14 detector modules: 12 sodium iodide (NaI) detectors, covering the energies 8 keV–1 MeV, and 2 bismuth germanate (BGO) detectors, covering 200 keV to 40 MeV (Meegan et al. 2009). The data were downloaded from Fermi Science Support Center’s (FSSC) FTP site, which contains 3000 GRBs up to 2021 April. For each burst, we use the time-tagged event (TTE) data for both spectral and temporal analysis, which records each photon’s arrival time with 2 μs temporal resolution, as well as information regarding in which of the 128 energy channels the photon registered. The prebinned, eight-energy-channel (CTIME) data were used to carry out the background subtraction. In the temporal analysis (including the autocorrelation test and hardness test), we only use the data from the NaI with the highest signal-to-noise ratio (S/N). In the spectral analysis, data from BGO detectors are involved.

2.2. Autocorrelation Test

For each Fermi GRB, we produce its light curve from the TTE data with 0.01 s resolution for short GRBs and 0.1 s resolution for long GRBs. The background was determined by fitting first-order polynomials without the signal part. In order to search for two emission episodes with similar light-curve patterns, we apply the autocorrelation test to each GRB, with the following function:

$$ a(\Delta t) = \sum f(t) \ast f(t + \Delta t) / N\sigma^2, $$

(1)

where $\sigma$ is the standard deviation for the light curve $f(t)$, $\Delta t$ is relative displacement for autocorrelation, and $N$ is the bin number of the light curve.

For each burst, we can plot its $a(\Delta t)$–$\Delta t$ correlation curve. In principle, if there are no similar emission episodes in the light curve, the $a(\Delta t)$–$\Delta t$ curve should show a smoothly downward trend with the highest peak at $\Delta t = 0$. Otherwise, if similar emission episodes indeed exist, multiple peaks would be superposed on the background decay component. Note that the residual noise may also lead to small wiggles in the $a(\Delta t)$–$\Delta t$ curve for dim GRBs; here we regard it as an effective peak only when the peak height is $\sigma$ higher than the background. A total of 170 candidates, whose $a(\Delta t)$–$\Delta t$ curve contains at least one effective peak (peak at $\Delta t = 0$ does not count), are selected as the preliminary candidates. $\Delta t$ of the highest effective peak could be taken as the possible millilensing time delay, whose uncertainty could be estimated by adding Poisson noise to the original light curve (Ukwatta et al. 2010; Hakilla et al. 2018; Ji et al. 2018; Veres et al. 2021, for a similar approach).

2.3. Hardness Test

The hardness test has been widely used to justify the lensing effect (Kalantari et al. 2021; Paynter et al. 2021; Veres et al. 2021; Wang et al. 2021), based on the hypothesis that the flux ratio between gravitationally lensed pulses should not depend on energy (Paczynski 1987). In this work, we adopt three energy channels to carry out the hardness test: low-energy channel (8–50 keV), medium-energy channel (50–110 keV), and high-energy channel (110–323 keV).

For each preliminary candidate, we first plot their light curves in different energy channels. Based on the autocorrelation results, we can divide each light curve into two similar episodes (we first selected the interval containing the first episode by visually identifying contiguous temporal bins with significant signal and then selected the second interval with the same length as the first one, but with a certain time delay). Then, we define hardness ratios $HR_{HM}$ and $HR_{SM}$ for each episode as

$$ HR_{ij} = \frac{N_i - B_i}{N_j - B_j}, $$

(2)
\[ N \] are the total photon counts of the episode, \( B \) are the corresponding background photon counts, and \( i, j = \{ L, M, H \} \) indicate the channel index. The uncertainty of the HR\(_{ij}\) could be estimated as

\[
\Delta \text{HR}_{ij} = \left[ \frac{N_i}{(N_i - B_i)^2} + \frac{B_i}{(N_i - B_i)^2} \right]^{\frac{1}{2}} + \left[ \frac{N_j(N_i - B_i)^2}{(N_j - B_j)^3} + \frac{B_j(N_i - B_i)^2}{(N_j - B_j)^3} \right]^{\frac{1}{2}},
\]

where Poisson noise is considered. Here we require the preliminary candidate to pass the hardness test only when their HR\(_{HM}\) and HR\(_{ML}\) for different episodes are consistent with the mean value within the 1\(\sigma\) region. Eventually we have four candidates passing the hardness test, i.e., GRB 081122A, GRB 081126A, GRB 090717A, and GRB 110517B. The information of these four bursts are listed in Table 1.

### 3. Final Candidate Selection

In order to make a better justification on the final candidates, for each candidate we have analyzed and compared the time integral spectrum and time-resolved spectrum of the two emission episodes in detail. The spectral fitting is performed by using the Markov Chain Monte Carlo (MCMC) method with an automatic code “McSpecfit” (Zhang et al. 2018a), and in the analysis of

| GRB Name     | \( \Delta t \) (s) | HR\(_{HM}\) Episode 1 | HR\(_{HM}\) Episode 2 | HR\(_{ML}\) Episode 1 | HR\(_{ML}\) Episode 2 | \( f \) | \( M_\text{[1+L(z)]} \) (\( M_\odot \)) |
|--------------|---------------------|-----------------------|-----------------------|-----------------------|-----------------------|------|------------------|
| GRB 081126A  | 30.6 ± 0.3          | 0.98 ± 0.10           | 0.84 ± 0.11           | 0.59 ± 0.06           | 0.57 ± 0.07           | 1.35 ± 0.02 | 5.1^{+1.1}_{-0.4} \times 10^6 |
| GRB 090717A  | 42.1 ± 0.2          | 0.63 ± 0.04           | 0.61 ± 0.07           | 0.44 ± 0.02           | 0.44 ± 0.04           | 1.69 ± 0.01 | 4.02^{+0.05}_{-0.05} \times 10^6 |
| GRB 081122A  | 13.7 ± 0.3          | 0.77 ± 0.06           | 0.63 ± 0.11           | 0.67 ± 0.05           | 0.56 ± 0.07           | 2.22 ± 0.07 | 8.6^{+0.4}_{-0.4} \times 10^5 |
| GRB 110517B  | 17.2 ± 0.1          | 0.51 ± 0.06           | 0.54 ± 0.07           | 0.52 ± 0.05           | 0.48 ± 0.04           | 1.04 ± 0.02 | 2.2^{+1.1}_{-1.1} \times 10^7 |

Figure 1. Temporal and spectral analysis for GRB 081126A. The left panels show light curves in different energy bands and the corresponding autocorrelation curve for each band. The right panels show comparison of the temporal structure and spectrum parameters of the two emission episodes.
time-resolved spectrum, the time bin for each slice is manually selected to ensure sufficient S/N, so as to give a reliable fitting. For each fitting, we adopt four different spectral models, including signal power law, cutoff power law, Band function, and blackbody function. We use the Bayesian information criteria (BIC; Schwarz 1978) to test the goodness of each model, which shows that in most slices of all candidates the Band function and cutoff power-law models are clearly better than the signal power-law and blackbody models, while the cutoff power law performs slightly better than the Band function. Therefore, we decided to use the best-fitting parameters of the cutoff power-law model to compare the similarity of the two emission episodes. We discuss each candidate in detail below.

3.1. GRB 081126A

GRB 081126A was triggered on 2008 November 26 21:34:09.065 UT ($T_0$) by Fermi GBM (Holl & Margutti 2008; trigger number 081126899) with a duration of 54.145 ± 0.923 s ($T_{90}$, the time interval between 5% and 95% of the cumulative flux). The analysis results are shown in Figure 1. Autocorrelation analysis suggests that the time delay between two emission episodes is 30.6 ± 0.3. We select data from $T_0 - 1.7$ s to $T_0 + 8.3$ s as the first episode and shift it to the second episode with $\Delta t = 30.6$ s. The hardness ratios of two episodes are consistent both for the first episode ($HR_{HM} = 0.98 \pm 0.10$, $HR_{ML} = 0.59 \pm 0.06$) and for the second episode ($HR_{HM} = 0.84 \pm 0.11$, $HR_{ML} = 0.57 \pm 0.07$). Here we performed a so-called “$\chi^2$ test” for testing the light-curve similarity of two episodes, which considers the binned light curves of the two pulses as representing two distributions and asks whether they are consistent with coming from the same parent distribution (Mukherjee & Nemiroff 2021a). Following the same algorithm as Mukherjee & Nemiroff (2021a), we calculate the minimum $\chi^2$ value for GRB 081126A, which is 100.94 for 100 degrees of freedom with the 0.1 s time bin, corresponding to a $p$-value of 0.46.

The time-integrated spectrum parameters of two episodes are $E_{\text{peak}1} = 336^{+40}_{-26}$, $\alpha_{\text{peak}1} = -0.69^{+0.06}_{-0.10}$ and $E_{\text{peak}2} = 238^{+28}_{-21}$, $\alpha_{\text{peak}2} = -0.61^{+0.12}_{-0.10}$. As shown in Figure 1, we divide each episode into six slides to compare the time-resolved spectrum. Considering that the detection of GRB 081126A has a relatively high S/N, and based on its excellent performance in light-curve and spectrum analysis, we rank it as the first-class candidate for millilensing event. Moreover, based on the early detection on the afterglow of GRB 081126A in the UV filters, Figure 2. Temporal and spectral analysis for GRB 090717A.
it is suggested that GRB 081126A should have a redshift of approximately $2.8 < z < 3.8$ (Bhat & van der Horst 2008; Holl et al. 2008), which further increases the possibility for GRB 081126A being a millilensing event.

For a lensed GRB, one can estimate the redshift mass of the lens source with

$$M_L(1+z_L) = \frac{c^3 \Delta t}{2G \left( f^{-1} \frac{d}{df} + \ln f \right)},$$

where $f$ is the amplification ratio between two images, $z_L$ is the redshift of the lens source, $G$ is the gravitational constant, and $c$ is the speed of light (Mao 1992). Here $f$ could be calculated with the total counts and background counts of two episodes as $f = (N_1 - B_1)/(N_2 - B_2)$. For GRB 081126A, we have $f = 1.35 \pm 0.02$. In this case, the redshift mass of the lens source can be estimated as $M_L(1+z_L) = 5.1_{-0.3}^{+0.5} \times 10^6 M_\odot$.

3.2. GRB 090717A

GRB 090717A was triggered on 2009 July 17 00:49:32.108 UT ($T_0$) by Fermi GBM (Kara et al. 2009; trigger number 090717034) with a duration of $65.537 \pm 1.577$ s. The analysis results are shown in Figure 2. Autocorrelation analysis suggests that the time delay between two emission episodes is $42.1 \pm 0.2$ s. We select data from $T_0$ to $T_0 + 17$ s as the first episode and shift it to the second episode with $\Delta t = 42.1$ s. The hardness ratios of two episodes are consistent both for the first episode ($HR_{HM} = 0.63 \pm 0.04$, $HR_{ML} = 0.44 \pm 0.02$) and for the second episode ($HR_{HM} = 0.61 \pm 0.07$, $HR_{ML} = 0.44 \pm 0.04$). The $\chi^2$ test obtains $\chi^2_{min} = 196.7$ for 230 degrees of freedom with the 0.1 s time bin, which corresponds to a $p$-value of 0.95.9

The time-integrated spectrum parameters of two episodes are $E_{peak1} = 161_{-53}^{+80}$, $\alpha_{peak1} = -1.11_{-0.04}^{+0.03}$, and $E_{peak2} = 158_{-12}^{+13}$, $\alpha_{peak2} = -1.03_{-0.06}^{+0.05}$. As shown in Figure 2, for the time-integrated spectrum, the $\alpha$ and $E_{peak}$ of two episodes are remarkably consistent within the 1$\sigma$ confidence level. We then divide each episode into 10 slides to compare the time-resolved

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9 It is worth noticing that the result of the “$\chi^2$ test” is very sensitive to the time bin sizes of the light curve. For instance, Mukherjee & Nemiroff (2021a) found a relatively low $p$-value when the time bin for GRB 090717A is adopted as 1.024 s. Comprehensive analysis, especially the time-resolved spectral analysis, is thus essential for justifying millilensing effects.
spectrum, and the spectrum parameters of two episodes are consistent within the 1σ confidence level in every slide except the α of slide 5, 6 and E_{peak} of slide 2, 8, which are consistent within the 2σ confidence level. Considering its temporal and spectral analysis results, we rank GRB 090717A as the first-class candidate that may have experienced the millilensing effect, with the amplification ratio as $f = 1.69 \pm 0.01$ and the redshift mass of the lens source as $M_L(1 + z_L) = 4.03^{+0.05}_{-0.03} \times 10^6 M_\odot$.

3.3. GRB 081122A

GRB 081122A was triggered on 2008 November 22 12:28:12.211 UT ($T_0$) by Fermi GBM (McBreen 2008; trigger number 081122520) with a duration of 23.30 ± 2.11 s. The analysis results are shown in Figure 3. Autocorrelation analysis suggests that the time delay between two emission episodes is 13.7 ± 0.3 s. We select data from $T_0 - 0.4$ s to $T_0 + 4.0$ s as the first episode and shift it to the second episode with $\Delta t = 13.7$ s. The hardness ratios of two episodes are consistent both for the first episode (HR_{HM} = 0.77 ± 0.06, HR_{ML} = 0.67 ± 0.05) and for the second episode (HR_{HM} = 0.63 ± 0.11, HR_{ML} = 0.56 ± 0.07). The $\chi^2$ test obtains $\chi^2_{\text{min}} = 97.98$ for 88 degrees of freedom with the 0.05 s time bin, which corresponds to a $p$-value of 0.21. Here a 0.05 s time bin is adopted because the total duration of GRB 081122A is relatively short.

The time-integrated spectrum parameters of two episodes are $E_{\text{peak}1} = 220^{+116}_{-111}$, $\alpha_{\text{peak}1} = -0.61^{+0.05}_{-0.03}$ and $E_{\text{peak}2} = 189^{+37}_{-20}$, $\alpha_{\text{peak}2} = -0.80^{+0.10}_{-0.12}$. As shown in Figure 3, we divide each episode into six slides to compare the time-resolved spectrum. The spectrum parameters of two episodes are consistent within the 1σ confidence level for each slide. The overall spectral analysis results provide positive evidence to support GRB 081122A being a millilensing candidate. However, we notice that there are two peaks in the first emission episode but that the light curve of the second episode seems not to have such a feature, which may be due to the low S/N. We thus rank it as a second-class candidate that may have experienced a millilensing effect. With the amplification ratio $f = 2.22 \pm 0.07$, the redshift mass of the lensing source can be estimated as $M_L(1 + z_L) = 8.6^{+0.4}_{-0.3} \times 10^6 M_\odot$.

3.4. GRB 110517B

GRB 110517B was triggered on 2011 May 17 13:44:47.600 UT ($T_0$) by Fermi GBM (trigger number 110517573) with a duration of 23.04 ± 0.36 s. The analysis results are shown in Figure 4. Autocorrelation analysis suggests that the time delay between two emission episodes is 17.2 ± 0.1 s. We select data
from $T_0 - 4$ s to $T_0 + 9$ s as the first episode and shift it to the second episode with $\Delta t = 17.2$ s. The hardness ratios of two episodes are consistent both for the first episode ($HR_{\text{HI}} = 0.51 \pm 0.06$, $HR_{\text{ML}} = 0.52 \pm 0.05$) and for the second episode ($HR_{\text{HI}} = 0.54 \pm 0.07$, $HR_{\text{ML}} = 0.48 \pm 0.04$). The $\chi^2$ test obtains $\chi^2_{\text{min}} = 185.4$ for 130 degrees of freedom with the 0.1 s time bin, which corresponds to a p-value of 0.001.

The time-integrated spectrum parameters of two episodes are $E_{\text{peak1}} = 112^{+7}_{-6}$, $\alpha_{\text{peak1}} = -0.45^{+0.09}_{-0.09}$ and $E_{\text{peak2}} = 103^{+5}_{-5}$, $\alpha_{\text{peak2}} = -0.43^{+0.10}_{-0.10}$. As shown in Figure 4, for the time-integrated spectrum, the spectrum parameters of two episodes are consistent within the 1σ confidence level. After we divide each episode into 11 slides to compare the time-resolved spectrum, the $\alpha$ of two episodes are consistent within the 1σ region in every slide, but the $E_{\text{peak}}$ are only consistent within the 1σ region in three slides (27.3%), and the spectrum evolution of the $E_{\text{peak}}$ shows a different trend in two episodes, which cuts down the possibility for a millilensing event. We notice that there are multiple peaks in both emission episodes, where the width distribution of these peaks is similar, but there are some differences for the peak amplitude distribution between the two peaks (that is why we get a small p-value in the $\chi^2$ test). Considering its temporal and spectral analysis results, we rank GRB 110517B as a second-class candidate for a millilensing event. With the amplification ratio $f = 1.04 \pm 0.02$, the redshift mass of the lensing source can be estimated as $M_L(1 + z_L) = 2.2^{+1.1}_{-0.4} \times 10^7 M_\odot$.

### 4. Conclusion and Discussion

It has long been proposed that GRBs have the potential to be gravitationally lensed into multiple images, due to their high-redshift nature. When the lens source has mass $10^{4} - 10^{7} M_{\odot}$ (such as an intermediate-mass black hole, compact dark matter, star cluster, or Population III star), the delay time between different images would be on the order of $10^{-1}$ to $10^{2}$ s, which is comparable to the typical duration of GRBs. In this case, signals from two images would be collected within single trigger, manifesting as one GRB that contains two (or even more) emission episodes with similar properties in both the temporal and spectral domain. Such kinds of events are usually called millilensing events.

In previous works, a few GRBs have been proposed as the millilensing candidates, i.e., BATSE GRB 950830, Fermi GRB 200716C, and Fermi GRB 210812A. In this work, we conduct a systemic search for millilensing events from the Fermi GBM sample (3000 GRBs up to 2021 April). We select preliminary candidates by performing an autocorrelation test and a hardness test, and then we analyze and compare the time-integrated spectrum and time-resolved spectrum of the similar emission episodes. Eventually we find four interesting candidates that may have experienced a millilensing effect, e.g., GRB 081126A, GRB 090717A, GRB 081122A, and GRB 110517B. We rank GRB 081126A and GRB 090717A as first-class candidates based on their excellent performance in both temporal and spectral analysis. We rank GRB 081122A and GRB 110517B as second-class candidates (suspected candidates), mainly because their two emission episodes show clear deviations in part of the time-resolved spectrum or in the time-integrated spectrum. The ratio of the number of lensed candidates to the total number of GRBs in our sample is $2/3000 - 4/3000$. The redshift masses of the lensing source for our selected candidates are within the range of $8.6 \times 10^{5} M_{\odot} < M_L(1 + z_L) < 2.2 \times 10^{7} M_{\odot}$, which are very likely star clusters or supermassive black holes.

For a given GRB, the lensing optical depth can be written as (see Zhou et al. 2022a, 2022b for details)

$$\tau(f_L, z_S) = \int_0^{z_S} d\chi(z_L)(1 + z_L)^2 n_L f_L \sigma(M_{\text{len}}; z_L, z_S)$$

$$= \frac{3}{2} f_L \Omega_m \int_0^{z_S} \frac{H_0^2}{c^2 H(z_L)} \frac{D_L D_{\text{LS}}}{D_S} \times (1 + z_L) \chi^{f}_{\text{max}}(R_{f,\text{max}}),$$

where monochromatic mass distribution for the lens source is assumed, $n_L$ represents the fraction of the mass of the lens with mass $M_L$ to the total matter mass (dark matter mass + baryon mass), $z_S$ is the redshift of the GRB, $z_L$ is the redshift of the lens, $n_L$ is the comoving number density of the lens, $H(z_L)$ is the Hubble parameter at $z_L$, $H_0$ is the Hubble constant, and $\Omega_m$ is the present density parameter of matter. Here we define the maximum value of the normalized impact parameter as $\chi_{\text{max}}(R_{f,\text{max}}) = R_{f,\text{max}}^{1/2} - R_{f,\text{max}}^{-1/2}$, by requiring that the flux ratio of two lensed images is smaller than a critical value $R_{f,\text{max}}$ (in this work we take $R_{f,\text{max}} = 5$). If we accumulate a considerable number of GRBs with redshifts satisfying $N(z_S)$ distribution, the integrated optical depth of all these GRBs would be

$$\overline{\tau}(f_L) = \int d\chi \tau(f_L, z_S) N(z_S).$$

Consequently, the expected lensing fraction for GRBs would be

$$F = \frac{N_{\text{Lensed GRB}}}{N_{\text{GRB}}} = (1 - e^{-\overline{\tau}(f_L)}).$$

In this work, our results suggest that $F = 2/3000 - 4/3000$ (assuming $M_L \sim 10^6 M_\odot$), inferring $f_L = (0.5 - 1) \times 10^{-2}$. It is worth noting that although we have made a detailed analysis of each source in the sample, it cannot be ruled out that there are still some possible candidates that have not been screened out. The main reason is that when the time delay is shorter than the intrinsic timescale of the burst, the signals of different images will overlap. This situation is not easy to distinguish through autocorrelation analysis, especially when the signal itself has a complex structure. To be conservative, here we suggest to use $0.5 \times 10^{-2}$ as a lower bound estimate for the fraction of the mass of lens objects with mass $M_L \sim 10^6 M_\odot$ to the total matter mass, inferring that the density parameter of lens objects with mass $M_L \sim 10^6 M_\odot$ is larger than $1.5 \times 10^{-3}$.

Up to now, all candidates (including our findings and the findings of previous works) are proposed solely based on the gamma-ray data analysis. However, the physical origin of GRB prompt emission is still poorly understood (Zhang 2018). It cannot be excluded that a GRB might have two emission episodes with inherently similar temporal and spectral characteristics (Lan et al. 2018). Multiband observations would be essential to finally determine whether a GRB has really experienced the gravitational lensing effect (Chen et al. 2022). With the successful operation of many sky survey projects in

\[10\] Note that this burst is also included in the 3000 GRBs we searched. Since this source has been discussed in detail in two independent works, we do not list it as the candidate we found.
multiple bands, such as all-sky gamma-ray monitors (e.g., Gravitational wave high-energy Electromagnetic Counterpart All-sky Monitor; Zhang et al. 2019), sky survey detectors in the X-ray band (e.g., Einstein Probe; Yuan et al. 2018), and a wide-field-of-view monitoring system in optical band (e.g., Ground-based Wide Angle Camera system; Wei et al. 2016), more lensed GRBs are expected to be detected and accurately certified in the future.

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