The effect of “pre-formed” hadron potentials on the dynamics of heavy ion collisions and the HBT puzzle

Qingfeng Li, 1,* Marcus Bleicher, 2 and Horst Stöcker 1,2,3
1) Frankfurt Institute for Advanced Studies (FIAS), Johann Wolfgang Goethe-Universität, Max-von-Laue-Str. 1, D-60438 Frankfurt am Main, Germany
2) Institut für Theoretische Physik, Johann Wolfgang Goethe-Universität, Max-von-Laue-Str. 1, D-60438 Frankfurt am Main, Germany
3) Gesellschaft für Schwerionenforschung, Darmstadt (GSI), Germany

The nuclear stopping, the elliptic flow, and the HBT interferometry are calculated by the UrQMD transport model, in which potentials for “pre-formed” particles (string fragments) from color flux-tube fragmentation as well as for confined particles are considered. This description provides stronger pressure at the early stage and describes these observables better than the default cascade mode (where the “pre-formed” particles from string fragmentation are treated to be free-streaming). It should be stressed that the inclusion of potential interactions pushes down the calculated HBT radius $R_0$ and pulls up the $R_S$ so that the HBT time-related puzzle disappears throughout the energies from AGS, SPS, to RHIC.

PACS numbers: 25.75.Gz,25.75.Dw,24.10.Lx

One of the most important discoveries from SPS and RHIC experiments in the beginning of this century is that a new form of matter is produced in high energy ion collisions that seems to behave as a strongly interacting quark gluon plasma (sQGP) [1, 2, 3, 4]. Furthermore, at RHIC energies, the sQGP seems to be a nearly “perfect liquid” with small viscosity [5, 6]. Although hydrodynamical studies are a useful tool for the theoretical investigations with small viscosity [5, 6]. Although hydrodynamical studies are a useful tool for the theoretical investigations at RHIC energies, hydrodynamics fails at lower energies. Therefore, a relativistic dynamic transport approach, in which an entire equilibrium is not pre-assumed, is necessary to explore the excitation functions of various observables. In addition, the transport model has the advantage of observing the whole phase-space evolution of all particles involved in a microscopic fashion.

In order to detect this new matter - quark gluon plasma (QGP) - many theoretical suggested observables have been discussed and argued frequently. Among these, the Hanbury-Brown-Twiss interferometry (HBT) or Femtoscopy technique has been used widely to extract the spatio-temporal information of the particle freeze-out source. It would be a vital discovery if there is a nontrivial transition in the spatio-temporal characteristics of the source, when going from low to high beam energies [7]. Experimentally, the HBT parameters have been scanned thoroughly over the energies from SIS, AGS, SPS, up to RHIC, unfortunately, no obvious discontinuities do appear [8].

However, this “null” result does not mean that the HBT technique comes to an end. Non-Gaussian effects might shadow the possible energy dependence of the HBT parameters (“E-puzzle”), and this topic has been quickly improving in the recent years [9, 10, 11]. Secondly, even with a Gaussian parametrization, we found that the HBT time-related puzzle (“t-puzzle”) is present at almost all energies from AGS to RHIC [12]. In order to understand the origins of these “puzzles”, it is necessary to dig deeper into the dynamics of the heavy ion collisions.

The hydrodynamic model, in which various kinds of equations of state (EoS) with latent heats are considered, successfully explained the elliptic flow $v_2$ at transverse momentum $p_t < 2\text{GeV}/c$ at RHIC energies [13, 14, 15]. However, hydrodynamics failed to explain the experimental $R_0/R_S$ data [8] and the references therein), i.e. the ratio between the HBT radii in outward and sideward directions, where the “t-puzzle” initially occurred. But recently, it has been realized that a direct (fully “apple-to-apple”) comparison between hydro calculations and data is required [16].

In the transport models, the effective EoS is softened by the existence of resonances, but the re-scattering between particles will reinforce to some extent the stiffness of the dense matter. At SPS and RHIC energies, the color-string excitation and fragmentation is further considered for particle production and subsequent evolution in the ultra-relativistic/relativistic quantum molecular dynamics (UrQMD/RQMD) hadron-string transport models. The formation time of the hadrons from string fragmentation is determined by a “yo-yo” mode [17, 18]. During the formation time, the “pre-formed” particles (string fragments that will be projected onto hadron states later on) are usually treated to be free-streaming, while reduced cross sections are only included for leading hadrons. The idea of “pre-formed” hadrons is a well-

*E-mail address: liqf@fias.uni-frankfurt.de
known concept in the description of deep inelastic scattering data (DIS) [19, 20]. It has also been found that the “pre-formed” hadrons from the color fluxtube fragmentation behave already very hadronlike, before the hadron formation is fully completed [21]. In previous calculations [17, 18, 22], the interaction of the newly created “pre-formed” particles from the string fragmentation is usually not taken into account for simplicity. The facts have shown that the effective EoS at the early stage in the past model simulations is too soft and the early pressure is inadequate to match the calculated elliptic flow to the experimentally observed flows at RHIC energies [23, 24]. In order to make up for this discrepancy in the early-stage EoS, it is essential to modify the dynamics of the early stage with/without new degrees of freedom, such as done in a multi-phase transport (AMPT) [11], the hadronic rescattering (HRM) [25], the quark molecular dynamics (qMD) [26], and the parton hadron string dynamics (PHSD) [27] model.

Besides the new matter which is dominant at the early stage, the potentials between confined hadrons which play a role during almost all time of the collision process deserve attention as well. In the previously low energy UrQMD calculations, the Hamiltonian $H$ served for the EoS consists of the kinetic energy $T$ and the effective two-body interaction potential energy $U$. The potential energy $U$ is indispensable, which includes the two-body and three-body (which can be approximately written in the form of two-body interaction) Skyrme- (also called as the density dependent terms), Yukawa-, and Coulomb-terms [14, 18]. Recently, in order to be more successfully applied in the intermediate energy region ($E_\text{c} \lesssim 2A$ GeV), more terms are considered [28, 29], those are, the density-dependent symmetry potential (essential for isospin-asymmetric reactions at intermediate and low energies) and the momentum-dependent term. At higher energies, i.e., AGS/FAIR and SPS, the Yukawa- and symmetry- potentials of baryons becomes negligible, while the Skyrme- and the momentum-dependent part of potentials still influence the whole dynamic process of HICs though weakly. In Ref. [30], the effects of the mean field with momentum dependence on collective flows from HICs at 2-158A GeV energies were studied by a Jet AA Microscopic Transportation Model (JAM) and it has been found that the momentum dependence in the nuclear mean field is important for the understanding of the proton collective flows at AGS and even at SPS energies. Further, by adopting the same soft EoS with momentum dependence (SM-EoS) on confined baryons, we also found that the calculated $R_0/R_S$ ratio from HICs at AGS energies can be vastly improved when compared to the measured data [12, 31].

In this work, we will follow this idea and investigate the HBT radii at higher beam energies, i.e., SPS and RHIC. Furthermore, the other observables, such as the rapidity distributions of net protons ($p-\bar{p}$) and pions, and the elliptic flows of charged hadrons, are checked at the same time. As an initial attempt, we consider potentials for both formed and “pre-formed” particles: 1), for formed baryons, the SM-EoS [30] is chosen. The Coulomb potential is also considered, as in the low-energy calculations [28, 31]. 2), for formed mesons, only the Coulomb potential is optional in calculations, while other potentials are not considered as in previous calculations at low energies [28, 31]. 3), for “pre-formed” particles from string fragmentation, the similar density dependent (Skyrme-like) terms [49] as the formed baryons are used, but without the Yukawa, the Coulomb, and the momentum dependent terms. 4), the “pre-formed” mesons act like “pre-formed” baryons but with a reduction factor ($2/3$) due to the quark-number difference. Furthermore, 5), there is no potential interaction between “pre-formed” baryons and formed baryons so far. Correspondingly, the hadronic density is calculated by $\rho_h = \sum_{i,j} c_{i,j} \rho_{ij}$, where $c_{i,j} = 1$ for Baryons, $2/3$ for “pre-formed” mesons, and 0 for formed mesons. $\rho_{ij}$ is a Gaussian in coordinate space. As stated in [30, 31], the relativistic effect (Lorentz transformation) on the relative distance and relative momentum between two particles $i$ and $j$ has been considered.

To set the stage, Fig. 1 shows the rapidity dependence of the multiplicities of net protons (top) and negatively charged pions (bottom) for central ($< 5\%$ of total cross section $\sigma_T$) Pb+Pb collisions at $E_\text{c} = 158A$ GeV. The experimental data (solid stars) are taken from [32, 33], and the open stars represent the data reflected at mid-rapidity. The dash-dot-dotted lines are the cascade mode calculations, the dashed lines represent the calculations with SM-EoS of confined baryons, while the solid lines illustrate the results with the additional density-dependent potentials for “pre-formed” particles. While the cascade calculations give a reasonable rapidity distribution for the pions, the rapidity distribution of net protons is Gaussian - as seen in previous UrQMD calculations [22], and not fully consistent with the data (which has a two-bump structure at $y \sim \pm 1.2$). After considering the baryonic potentials, the distribution only changes slightly at projectile and target rapidities. However, when the density-dependent EoS for “pre-formed” particles is additionally taken into account, one observes that two bumps in the rapidity distribution of net protons appear. This implies that, due to the interactions of “pre-formed” particles at early stage, the additional pressure increases the effective stopping power as well as the central density at the compression stage. The total pion multiplicity remains basically unchanged on a level below 5% at this beam energy. Thus, the current pion distribution with potentials is acceptable for further investigations.

Using the analyzing program “correlation afterburner” (CRAB v3.0β), which is contributed by S. Pratt [34, 35], we calculate the HBT correlator and further fit it as a three-dimensional Gaussian form under the Pratt convention, i.e., the longitudinally comoving system (LCMS), which is expressed as
seen that the two-body mesonic Coulomb potential be-
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we show a comparison of calculations between with and
calculations with various potential treatments are shown
mode are shown by lines with down-triangles while the
HBT-radii $R_{OL}$ (at midrapidity) for central
S
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2
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and the lines with triangles at $\sqrt{s_{NN}} = 200$ GeV. So far
the question of a steeper $k_T$-dependence of HBT radii
than data at high SPS and RHIC energies, similar to the
AMPT transport model calculations with parton scatter-
ings [11], is still open.

The most exciting results show up in the transverse
space. Roughly speaking, the nuclear potentials lead to
a smaller $R_O$ (especially at large $k_T$) but a larger $R_S$
(specially at small $k_T$) generally allowing for a better
description of the $k_T$ dependent data. Certainly, our
present crude approach is only semi-realistic as a descrip-
tion of the dynamic processes at the early stage of the
reaction. However, one can still come to the conclusion
that the potentials help to obtain a better description of
the pion freeze-out in the transverse space.

In Fig. 3 we depict the excitation function of the
$R_O/R_S$ ratio at small $k_T$: at AGS and SPS energies, the
results at $k_T$ 150 ± 50 MeV/c are shown, while at RHIC
energies, the $k_T$ is set to 200 ± 50 MeV/c. The exper-
imental data within the transverse momentum regions
are compared with the calculations with and without
potentials. As seen before [12, 44, 45], in the cascade mode
of the pair relative momen-
tum $i$ direction, Fig. 2 shows the transverse mo-
mmentum $k_T$ dependence ($k_T = (p_{1T} + p_{2T})/2$) of the
HBT-radii $R_L$, $R_O$, and $R_S$ (at midrapidity) for central
HICs at AGS ($E_b = 2A$, 4A, 6A, and 8A GeV), SPS
($E_b = 20A$, 30A, 40A, 80A, and 158A GeV), and RHIC
($\sqrt{s_{NN}} = 130$ and 200 GeV) energies. The physical cuts
used are same as those listed in [12]. The data are from
the E895, NA49, STAR, and PHENIX collaborations
[36, 37, 38, 39, 40, 41]. The calculations with cascade
mode are shown by lines with down-triangles while the
calculations with various potential treatments are shown
with other lines with different symbols. At $E_b = 2A$ GeV,
we show a comparison of calculations between with and
without Coulomb potentials for confined mesons. It is
seen that the two-body mesonic Coulomb potential be-
fore freeze-out affects the HBT radii only very weakly at
about $k_T < 100$ MeV/c, and the ratio between $R_O$ and
$R_S$ values is not altered consequently. At AGS energies,
besides the nuclear potentials, the Coulomb potential of
confined mesons is also considered (solid lines with cir-
cles), while at SPS and RHIC energies, the Coulomb po-
tential for confined mesons is switched off in order to
save computing times and to avoid problems with non-
local interactions. It is known that, below AGS energies
hadrons are dominantly produced from the decay of reso-
nances so that the considered potentials for “pre-formed”
particles from string fragmentations have no effect on the
HBT radii [42, 43]. At higher energies “pre-formed” par-
ticles play a dominant role for the early stage dynamics,
which can be seen, e.g., by the comparison of solid lines
and the lines with triangles at $\sqrt{s_{NN}} = 200$ GeV. So far
the question of a steeper $k_T$-dependence of HBT radii
than data at high SPS and RHIC energies, similar to the
AMPT transport model calculations with parton scatter-
ings [11], is still open.

In Eq. (1) the $\lambda$ is the incoherence factor, $R_L$, $R_O$, and
$R_S$ are the Pratt radii in longitudinal, outward, and
sideward directions, while the cross-term $R_{OL}$ plays a
role at large rapidities. $q_i$ is the pair relative moment-
tum in the $i$ direction. Fig. 2 shows the transverse mo-
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\[
C(q_O, q_S, q_L) = 1 + \lambda \exp(-R_L^2 q_L^2 - R_O^2 q_O^2 - R_S^2 q_S^2 - 2R_{OL} q_O q_L)
\]

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FIG. 2: Transverse momentum $k_T$ dependence of the HBT radii $R_L$, $R_O$, and $R_S$ (at midrapidity) for central HICs at AGS ($E_b = 2A, 4A, 6A, \text{and} 8A \text{ GeV}$), SPS ($E_b = 20A, 30A, 40A, 80A, \text{and} 158A \text{ GeV}$), and RHIC ($\sqrt{s_{NN}} = 130 \text{ and} 200 \text{ GeV}$) energies. The data are indicated by solid stars, which are from E895, NA49, STAR, and PHENIX collaborations [36, 37, 38, 39, 40, 41]. The calculations with cascade mode are shown by lines with down-triangles. At AGS energies, the calculations with potentials for confined baryons (SM-EoS) as well as a Coulomb potential for confined mesons are shown by the lines with circles. At AGS $E_b = 2A \text{ GeV}$, SPS, and RHIC energies, the results with potentials for both “pre-formed” particles and confined baryons but without Coulomb potential for mesons are shown by lines, while the lines with up-triangles correspond to the calculations without any potentials for “pre-formed” particles but with potentials for confined baryons.

Therefore, our explanation of the HBT time related puzzle is different from the investigations in Ref. [46] where Cramer, et al., considered a single-pion optical potential which simulates its interactions with the dense medium. While in this work, the microscopic “nuclear” potentials are considered for “pre-formed” hadrons from the string fragmentation. The Hamilton’s equations of motion of the particles represented by Gaussian wave packets are solved microscopically. After that, the freeze-out phase space is analyzed numerically by the CRAB program, in which the plain and distorted (by Coulomb and nuclear potentials after freeze-out) wave functions are optional.

Let us finally check the modification of the early transverse expansion by a look into the elliptic flow. In the upper plots of Fig. 4 we present the transverse momentum $p_t$ dependence of the elliptic flow $v_2$ ($v_2 = \langle (p_{t,x}^2 + p_{t,y}^2)/p_t^2 \rangle$), where $p_t^2 = p_x^2 + p_y^2$ of charged pions from Au+Au collisions at RHIC $\sqrt{s_{NN}} = 200 \text{ GeV}$. Two sets of centralities are chosen: $5\% < \sigma/\sigma_T < 10\%$ (in (a)) and $20\% < \sigma/\sigma_T < 30\%$ (b). To avoid non-flow effects [47], the four-particle integral cumulant $v_2$ data ($v_2 \{4\}$) are shown with stars. Similar to those in Fig. 3, the UrQMD calculations without and with potentials are illustrated.
for comparison with data. It is clear that due to the absence of early pressure, the elliptic flow in the cascade mode (dash-dot-dotted lines with open rectangles) can not grow as fast as the data with the increase of transverse momentum in both centralities. After switching on the SM-EoS for confined baryons (dashed lines with open up-triangles), the flow is seen to increase slightly especially at high transverse momenta. With the density dependent potentials for “pre-formed” particles (solid lines with open down-triangles), the flow is driven down and deviates from data especially at large centralities. This phenomenon calls for a more complete description of the dynamics of HICs at high energies in which the (in)elastic scatterings of particles (confined hadrons and deconfined partons) should be considered in more detail. To make a whole comparison, we show in Fig. 4 (c) the centrality dependence of the ratio between different theoretical $v_2$ values of charged hadrons and the four-particle cumulant data ($v_2^{th}/v_2^{exp}$), here the flows are integrated over $p_t$ and $\eta$. As stated before, the flow obtained from the cascade calculation is only about 55% of the experimental one. The contribution of potentials of confined baryons to flow is only at the level of about 4%. While the potentials of “pre-formed” hadrons increase the flow in more central collisions. At large centrality, the potential effects are relatively weak.

In summary, besides a soft equation of state with momentum dependence, which is required especially at low energies, a density dependent (Skyrme-like) potential has been tested for the “pre-formed” particles from string fragmentation in the UrQMD transport model. Although the form of the potential for the new phase is simple and rough (ideally, the EoS of the new phase should be based on the first-principle lattice QCD calculations [48]), it provides new insights into the dynamics of HICs. It reproduces the proper stopping power for net protons. It also improves the elliptic flow at large transverse momenta. Most importantly however, it decreases the HBT radius $R_0$ but increases the $R_S$ so that the experimental duration time related quantity — here the $R_0/R_S$ ratio — can be reasonably reproduced throughout the energies from AGS, SPS, up to RHIC. Although some open problems remain due to the simple treatment of the interaction of the particles at the early stage.

**Acknowledgements**

We would like to thank S. Pratt for providing the CRAB program and acknowledge support by the Frankfurt Center for Scientific Computing (CSC). We also...
thank M. Gyulassy, H. J. Drescher, and S. Haussler for helpful discussions. Q. Li thanks the Frankfurt Institute for Advanced Studies (FIAS) for financial support. This work is partly supported by GSI, BMBF, and Volkswagenstiftung.

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