Non-Standard Neutrino Self-Interactions Can Cause Neutrino Flavor Equipartition Inside the Supernova Core

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We show that non-standard neutrino self-interactions can lead to total flavor equipartition in a dense neutrino gas, such as those expected in core-collapse supernovae. In this first investigation of this phenomenon in the multi-angle scenario, we demonstrate that such a flavor equipartition can occur on very short scales, and therefore very deep inside the newly formed proto-neutron star, with a possible significant impact on the physics of core-collapse supernovae. Our findings imply that future galactic core-collapse supernovae can appreciably probe non-standard neutrino self-interactions, for certain cases even when they are many orders of magnitude smaller than the Standard Model terms.

I. INTRODUCTION

Core-collapse supernova (CCSN) explosions, caused by the death of massive stars, are among the most energetic astrophysical settings [1–4]. During the explosion, a huge amount of energy is reassessed of which almost 99% is in the form of neutrinos of all flavors. Given the short duration of the burst, the number density of the neutrinos in such environments is so huge that the neutrino-neutrino interactions play a crucial role in their flavor evolution [5–10].

In this paper, we study the impact of non-standard neutrino self-interactions (νNSSI) on their flavor evolution in dense neutrino media. Such νNSSI are allowed in some of the beyond Standard Model (BSM) theories of particle physics [11, 12]. While a BSM scalar mediator results in the presence of a trivial mass term of the form $\bar{\nu}_\alpha \nu_\alpha$ in the Lagrangian [13], a vector mediator leads to the effective Lagrangian $\mathcal{L}_{\text{eff}} \supset G_{\nu}^2 |\bar{\nu}_\alpha \gamma^\mu \nu_\alpha|^2$, which is very similar to the neutrino-neutrino interaction Lagrangian of the SM except that it now allows for new interaction terms coupling neutrinos with different flavors via $G_{\alpha\beta}$'s (see Ref. [14] for a discussion on how the effective single-particle Hamiltonian obtained from the Lagrangian can have a different structure). Needless to say, the SM can be recovered by setting $G_{\alpha\beta} = \delta_{\alpha\beta}$. The non-standard components of $G$ are related to the vector mediator mass $m_V$ and the coupling strength $g_V$, by $|G_{\alpha\beta}| \propto g_V^2 / m_V^2$.

The current constraints on νNSSI are rather weak and mostly model dependent, namely $|G_{\alpha\beta}|$ can be in general a couple of orders of magnitude larger than one (see, e.g., Fig. 1 of Ref. [15]). Though laboratory constraints are expectedly looser, stronger limits are feasible from the astrophysical neutrinos and the early Universe.

As for the constraints from laboratory processes, despite the fact that the number density of neutrinos is very low to allow for any direct detection of νNSSI, it can be indirectly probed by the measurements of other particles decay/interactions. Of particular interest is the measurement of the invisible width of the Z-boson [16, 17] (see also a nice improvement of this in Ref. [18]).

νNSSI can also impact the physics of the early universe, in a number of ways. For example, one can derive constraints on νNSSI noting that if the mediator is light enough, then it could be in thermal equilibrium in the early universe, which changes the number of relativistic degrees of freedom and can be probed through Big Bang Nucleosynthesis [19]. νNSSI can also affect Cosmic Neutrino Background properties, meaning that it can be constrained via the Cosmic Microwave Background through its temperature angular power spectra [20, 21].

νNSSI can be constrained by the observation of high-energy cosmic neutrinos as well. This comes from the fact that high-energy neutrinos can be attenuated by their propagation in the Cosmic Neutrino Background if νNSSI is strong enough [22–28]. Besides, CCSNe have also been considered as a probe of νNSSI [29–31].

In order to study the flavor evolution of neutrinos in the presence of νNSSI, we solve the Liouville-von Neumann equation ($c = \hbar = 1$) [32]

$$id_t \varrho_p = \left[ \frac{UM^2 \varrho_p^+}{2E_p} + H_m + H_{\nu\nu, p} \cdot \varrho_p \right],$$

where $p$ is the neutrino momentum, $E_p = |p|$, $\nu = p/E_p$, and $M^2$ are the energy, velocity, and mass-square matrix of the neutrino, respectively, and $U$ is the Pontecorvo–Maki–Nakagawa– Sakata matrix. Moreover, $H_m$ is the contribution from the matter term which is proportional to matter (electron) density [33, 34], and $H_{\nu\nu}$ is the neutrino potential stemming from the neutrino-neutrino forward scattering in the presence of νNSSI [35–39],

$$H_{\nu\nu, p} = \sqrt{2} G_F \int \frac{d^3 p'}{(2\pi)^3} \{G(\varrho_{p'} - \bar{\varrho}_{p'}) \mathcal{G} + G \text{ Tr}[(\varrho_{p'} - \bar{\varrho}_{p'}) \mathcal{G}] \},$$

where $G = 1$ in the SM. The diagonal components of $\mathcal{G}$ constitute the flavor-preserving νNSSI, whereas the off-diagonal components show flavor-violating νNSSI.

The impact of νNSSI on the coherent scatterings of neutrinos in a dense neutrino gas was first addressed in Ref. [36], in a single-angle two-flavor scenario. It was then investigated more in Refs. [40, 41] where a similar single-angle model was employed and it was shown that νNSSI can lead to flavor instabilities in both normal and
inverted mass orderings. It was also particularly demonstrated that the flavor-violating $\nu_{\text{NSSI}}$ lead to spectral splits during the SN neutronization burst.

In this paper, we provide the first multi-angle investigation of $\nu_{\text{NSSI}}$ in a dense neutrino gas. Our results suggest that some of the most important insights obtained in the single-angle scenarios (Sec. II A) should be artefacts of the limitations of such models. To be more specific, we show that the flavor-violating $\nu_{\text{NSSI}}$ can lead to flavor equipartition in a multi-angle neutrino gas on scales determined by the neutrino number density (Sec. II B).

II. TWO-FLAVOR SCENARIO

To study the flavor evolution of neutrinos in the presence of $\nu_{\text{NSSI}}$, we consider a neutrino gas in the two-flavor scenarios ($\nu_x$ and $\nu_y$ with $x = \mu, \tau$, and their antiparticles) in which neutrinos are emitted uniformly with emission angles in the range $[-\theta_{\text{max}}, \theta_{\text{max}}]$. We then consider a number of examples in a single-energy single-angle one-dimensional (1D) gas, a single-energy multi-angle 1D gas, a single-energy multi-angle 2D gas, and a multi-energy multi-angle 1D gas, respectively. This sort of models have been extensively used in the literature (e.g., Ref. [42]). Also as it is usual in studying collective neutrino oscillations, we ignore the matter potential assuming that it is constant and can be rotated away by transforming into a co-rotating frame. The dropping of the matter term is specifically motivated once one considers neutrino flavor evolution deep inside the PNS where on short scales, the neutrino gas and the ambient medium can be considered to be isotropic and homogenous to a very good degree. In addition, except for our single-energy single-angle model, we here assume that the strength of the week interaction defined as, $\mu = \sqrt{2}G_F n_{\nu_x}$, is constant, and the vacuum frequency is set to be $|\omega| = 1$ km$^{-1}$ for the single energy cases.

In the two-flavor scenario, the $\nu_{\text{NSSI}}$ matrix can be written as,

$$G = \begin{pmatrix} 1 + \gamma_{ee} & \gamma_{ex} \\ \gamma_{xe}^* & 1 + \gamma_{xx} \end{pmatrix}.$$

Following Refs. [36], one can write the coupling matrix as,

$$G = \frac{1}{2}(g_01 + g \cdot \sigma),$$

where $g_0 = 2 + \gamma_{ee} + \gamma_{xx}$, $g_1 = 2\text{Re}(\gamma_{ex})$, $g_2 = 2\text{Im}(\gamma_{ex}^*)$, and $g_3 = \gamma_{ee} - \gamma_{xx}$, of which $g_2$ can be absorbed by a redefinition of the neutrino phases. Note that $g_0$ provides a measure of the overall strength of the (nonstandard) weak interactions. From now on, we set $g_0 = 2$ so that all the other quantities are normalised by $g_0$. The coupling matrix can now be written as,

$$G = \begin{pmatrix} 1 + g_3 & g_1 \\ g_1 & 1 - g_3 \end{pmatrix}.$$  

We here assume that $|g_3| < 1$, so that the diagonal components of $G$ are positive. This implies that one can always set $g_0$ in such a way that the contributions from $\nu_{\text{NSSI}}$ to the diagonal components have the same sign as those of the SM, as might be naively expected because the diagonal $\nu_{\text{NSSI}}$ contribution should be $\propto |g_{V}|^2$ and the sign should not depend on the phase of $g_{V}$ (though we can not exclude the possibility of the opposite). This is unlike Ref. [40], where such a limitation was not considered and a number of noticeable phenomena were reported for the cases with $|g_3| > 1$.

A. Single-angle scenario

For the sake of comparison and in order to build some useful insights, we here first provide our results of the single-angle calculations where we assume that in our model, all of the neutrino beams experience exactly the same flavor evolution. To be consistent with the previous works [36, 40], we assume that the neutrino number density is decreasing as a function of the propagated distance such that $\mu = 3 \times 10^4 (10/r)^4$ km$^{-1}$, motivated by the supernova physics. As indicated in the left and middle panels of Fig. 1, while for the normal mass ordering (NO) the final content of $\nu_e$ remains mostly unaffected by the flavor-violating $\nu_{\text{NSSI}}$, it can be almost completely depleted in the inverted mass ordering (IO). These results show a fantastic agreement with the results presented in Ref. [36], in spite of the difference between the geometries of the employed models.

The impact of nonzero flavor-preserving $\nu_{\text{NSSI}}$ is indicated in the right panel of Fig. 1. As can be clearly seen, the presence of nonzero $g_3$ tends to suppress the flavor conversions once $g_3 \gtrsim g_1$. As discussed in the following, this insight which was pointed out also in Ref. [40], survives in the multi-angle scenario.

B. Multi-angle scenario

Having discussed the single-angle scenario, we now turn our focus to the multi-angle simulations. As indicated in the left panel of Fig. 2, even very tiny flavor-violating $\nu_{\text{NSSI}}$ can lead to an almost perfect flavor equipartition on scales $\sim (g_1 \mu)^{-1}$, as long as $g_1 \mu \gtrsim 100 \omega_{\text{atm}}$. On the other hand, we have confirmed that 10% flavor conversions can be induced once $g_1 \mu \sim 10 \omega_{\text{atm}}$. Though the equipartition occurs in the exact manner for $g_1 \gtrsim 0.5$, it holds on average for smaller $g_1$’s where the average is taken over a few oscillations. As a matter of fact, what matters here is the ratio $H_{\nu e - \text{diag}} / H_{\nu e}$. This can be better understood from the right panel of Fig. 2, which presents the angular distributions of $\nu_e$ survival probabilities for $g_1 = 0.1$ and $0.5$, at $r \times g_1 \mu = 30$. While the angular distribution of $F_{\nu_e}$ shows large amplitude modulations for $g_1 = 0.5$, the different angle beams
are oscillating in phase for $g_1 = 0.1$. This explains the immediate flavor equipartition for $g_1 = 0.5$. 

In addition, nonzero flavor-preserving $\nu$NSSI can significantly suppress the flavor conversions provided that $g_3 \gtrsim g_1$, as demonstrated in the middle panel of Fig. 2. Note that this is consistent with the insight obtained in the single-angle scenario.

Although for smaller values of the flavor-violating $\nu$NSSI ($g_1 \lesssim 0.5$) the flavor equipartition occurs only on average in our 1D neutrino gas, the story could be different in MD models. This is specifically plausible considering the fact that in a MD neutrino gas, each neutrino beam arriving at an arbitrary point (coming from different points on the source) should have already experienced a couple of large amplitude oscillations. Assuming that these beams have evolved somewhat independently, one should naively expect an exact equipartition in a multi-angle MD neutrino gas even for smaller $g_1$’s. This is illustrated clearly in Fig. 3, for a calculation in which $g_1 = 0.1$ and $g_3 = 0$. Here we have considered a 2D ($x$ and $z$) multi-angle neutrino gas in which a periodic boundary condition is imposed along $x$, and the neutrino flavor evolution is followed along $z$. This model is similar to the one used in Refs. [43–45]. Though the neutrino gas experiences a few large amplitude oscillations initially, the coherence is lost afterwards and an almost perfect flavor equipartition is reached.

It is illuminating to note that the flavor equipartition discussed above (which is observed for $g_1 \gtrsim 0.5$ even in 1D models, but also observable for smaller $g_1$’s in MD
models) is purely a multi-angle effect. Indeed, it occurs due to the occurrence of large amplitude modulations in the angular distributions of neutrinos.

Though the behaviour observed in Fig. 3 is consistent with our insight that there should be an exact flavor equipartition in a MD gas, we alert the reader that we do not consider this topic as being settled since we could only run our 2D model simulations for very few points in the parameter space of $\mu-\alpha-g_1$ ($\alpha = n_{\nu_e}/n_{\nu_{\mu}}$). This turns out to be indeed a very unstable problem from the numerical point of view and we postpone a thorough investigation of this effect to a future work.

The flavor equipartition discussed in this section is most relevant for the neutrino gas inside the PNS. This can be understood considering the following observations. First, the PNS is the most sensitive zone to the $\nu$NSSI due to the highest neutrino number densities existing there. Hence as long as $g_1 \mu \gg \max(\omega, l_{\text{col}}^{-1})$, the $\nu$NSSI-induced conversions can dominate the vacuum frequency oscillations and the collisional processes ($l_{\text{col}}$ represents the collisional scales). In addition, if the neutrino gas is already in flavor equipartition at the neutrinosphere, then it should not change at larger radii (though we emphasise that this equipartition exists only in an average sense). One should also note that for such a problem which shows variations on very short scales and in which MD effects are important, 1D large scales simulations, as the one discussed in Fig. 1, should not be expected to provide very useful insights.

The flavor equipartition in the presence of $\nu$NSSI can even occur in the absence of vacuum mixing, i.e., once $\omega, \theta_{\nu} = 0$, where $\theta_{\nu}$ is the vacuum mixing angle. This is because the instability here is solely arising from the flavor-violating $\nu$NSSI and has nothing to do with the vacuum mixing. To be more specific, the occurrence of flavor equipartition is not sensitive to $\omega$ (or the neutrino mass ordering) as long as $g_1 \mu \gg |\omega|$, as expected from our discussions. Here we have considered a multi-angle multi-energy 1D model which initially consists of a pure $\nu_e$ gas. Needless to say, such an equipartition should be easier to measure given the fact that it should exist for all the energy bins.

In Appendix A, we provide a simple intuitive understanding of the roles of $g_1$, $g_3$, and $\omega$ in flavor instabilities in the linear regime, which was previously missed in the literature.

### III. Conclusion

We have studied neutrino flavor evolution in dense neutrino media in the presence of $\nu$NSSI, for the first time in a multi-angle model. We demonstrate that although some of the insights obtained in the single-angle scenario
also apply to the multi-angle model, some others should be an artefact of the single-angle approximation. In particular, we show that the dense neutrino gas can reach flavor equipartition on very short scales ($\sim g_1^{-1} \mu^{-1}$) in the presence of $\nu$NSSI. We also illustrate that this effect can be suppressed by flavor-preserving component if $g_3 \gtrsim g_1$.

The $\nu$NSSI-induced flavor equipartition is of most relevance to the neutrino flavor evolution inside the PNS and the accretion disks of neutron star merger (NSM) remnants. This simply comes from the fact that the neutrino number density is very high in such environment. In addition, if neutrinos are already in flavor equipartition on the surface of the neutrino emitter, then they might be expected to maintain this equipartition also at larger radii, unless something destroys it.

The equipartition caused by the $\nu$NSSI inside the PNS can have important observational and theoretical consequences. On the one hand, considering specially the possible equipartition during the neutronization burst, one can probe $g_1$ down to the values $g_1 \gtrsim 10^{-6}$. On the other hand, considering the high neutrino number densities expected inside the PNS ($n_{\nu} \sim 10^{46}$ cm$^{-3}$), the CCSN physics should be sensitive to $g_1 \gtrsim 10^{-8}$.

Though our study points out the interesting possibility of total flavor equipartition in a dense neutrino gas in the presence of $\nu$NSSI, it has still several important limitations. First, though equipartition discussed in this work seems to be a robust phenomenon in the two-flavor scenario, the situation can be different in the three-flavor regime. This is specially expected considering the fact that in the three-flavor scenario, there are more than one flavor-violating $\nu$NSSI parameters. Then the expected equilibrium can in principle be different from equipartition. Second, the MD simulations of neutrino flavor evolution in the presence of $\nu$NSSI, as we also pointed out in the text, are highly prone to numerical instabilities. Although we could run a few simulations and we observed equipartition in MD models as presented in Fig. 3, a systematic exploration of MD effects remains to be addressed in a future work. In addition, neutrinos can also experience non-standard interactions with matter ($\nu$NSI). Though such interactions are well constrained in model-dependent scenarios [48], the model-independent bounds are relatively loose [49]. The impact of $\nu$NSI on neutrino flavor evolution has been extensively studied for the neutrino gas above the PNS and accretion disks of NSM remnants [14, 50–53]. Considering the equipartition induced by $\nu$NSI, one might then wonder if strong $\nu$NSI can also lead to equipartition inside the PSN.

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**Appendix A: Linear Stability Analysis**

In this section, we show that some of the insights developed in the nonlinear regime in the previous section, can be understood by linear stability analysis. For this purpose, we consider the simplest problem which can be solved analytically, i.e., a single-angle pure $\nu_\mu$ gas. Note that although such a single-angle model cannot provide us with any insight on flavor equipartition, it still can be very useful in understanding the roles of $g_1$, $g_3$, and $\omega$ in flavor instabilities.

In the linear regime, where the flavor conversion is still insignificant ($|\delta| \ll 1$), the neutrino density matrix can be written as,

$$\rho = \begin{pmatrix} 1 & \delta \\ \delta^* & 0 \end{pmatrix},$$  \hspace{1cm} (A1)

with the total Hamiltonian being

$$H = \frac{\omega}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \mu [G \rho G + G \text{Tr}(\rho G)],$$  \hspace{1cm} (A2)

where here $\mu$ captures all the information regarding the neutrino number density and the geometry.

Note that there is an important subtlety here: Unlike the conventional flavor stability analysis, one cannot remove the trace of $\rho$ in the presence of $\nu$NSSI. This is obvious given the fact that in the presence of $\nu$NSSI, trace of $\rho$ leads to a term $\propto G^2 + G \text{Tr}(G)$ in the Hamiltonian, which is not necessarily benign to the stability analysis. In the linear regime (ignoring terms nonlinear in $|\delta|$), the equation of motion for $\delta$ then becomes,

$$i \hbar \frac{d\delta}{dt} = (\omega + 4 \mu g_3^2 + 4 \mu g_1 - 2 \mu g_1^2)\delta - 2 \mu g_3^2 \delta^* - 2 \mu g_1 (1 + g_3).$$  \hspace{1cm} (A3)

This equation is very interesting in several aspects. Firstly, there is a constant term which can be nonzero for nonzero $g_1$. This implies that for large enough $g_1$, $\delta$ can experience a sudden enhancement on scales $\propto \mu^{-1} g_1^{-1} (1 + g_3)^{-1}$. However and though the right hand side of Eq. (A3) is initially controlled by this term, it can become finally subdominant once $|\delta|$ grows enough (which might be already in the nonlinear regime).

In addition, Eq. (A3) couples $\delta$ to $\delta^*$, which is different from the conventional linearised equation of motion for the flavor perturbations. This implies that the real and imaginary parts of $\delta$ can behave differently in the linear regime.

In order to get an impression of the stability of the neutrino gas in the presence of $\nu$NSSI, we assume that $\delta$ is already in a regime where the $2 \mu g_1 (1 + g_3)$ term can be ignored. Then one can find the following ordinary
eigenvalue equation in terms of the real and imaginary components of $\delta$:

$$\partial_t \begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix} = -\eta + 4\mu g_1^2 \begin{bmatrix} 0 & \eta \\ -\eta & 0 \end{bmatrix} \begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix},$$

(A4)

where $\eta = \omega + 4\mu g_3^2 + 4\mu g_3$. The eigenvalues of this equation can be easily found to be,

$$\lambda = \pm \sqrt{\eta(4\mu g_1^2 - \eta)}.$$

(A5)

Assuming $\eta > 0$, unstable solutions can only exist if $4\mu g_1^2 > \eta$, meaning that $g_1^2 > \omega/4\mu + g_3^2 + g_3$. This simple argument obviously shows the role of $\omega$ and $g_3$ in suppressing flavor instabilities discussed in the previous section, caused by the flavor-violating $\nu$SNI. Note, however, that this is the insight obtained ignoring the role of the $2\mu g_1(1 + g_3)$ term, and post our understanding regarding the roles of $g_1$, $g_3$, and $\omega$ in the flavor instabilities. Otherwise this simple argument has its own limitations, e.g., it fails when $\eta < 0$.

It should be kept in mind that the linear stability analysis presented here reaches very different results from the one in Ref. [41], given the following observations: (i) The trace of $\rho$ is not removed in this study, as it should not be (ii) The $\mu g_1^2 \delta^2$ term in the linearised equation of motion is the source of instability, meaning that one should not drop it in favor of the other terms. Then, for example, we here observe that the neutrino gas is always unstable as long as $g_1 \gtrsim g_3$, which is different from the results presented, e.g., in Fig. 1 of Ref. [41].
