CMB Constraints on Brane Inflation

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(Dated: November 17, 2020)

We investigate the primordial phase of the Universe in the context of brane inflation modeled by Bogomol’nyi-Prasad-Sommerfield (BPS) domain walls solutions of a bosonic sector of a 5D supergravity inspired theory. The solutions are embedded into five dimensions and it is assumed that they interact with each other due to elastic particle collisions in the bulk. A four-dimensional arctan-type inflaton potential drives the accelerated expansion phase and predicts observational quantities in good agreement with the currently available Cosmic Microwave Background data.

I. INTRODUCTION

Measurements of the temperature fluctuations of the Cosmic Microwave Background (CMB) have provided strong observational support for the inflationary scenario [1–3] (see also [4–7] for different points of view of the current observational status of inflation). Although compatible with the simplest slow-roll scenarios of inflation, and showing a preference for plateau over monomial potentials, these observations have also severely constrained specific class of models that emerged as an attempt to explain early accelerated phase of the Universe [5, 10–12].

On the other hand, the aforementioned models have also been constrained from theoretical arguments in the realm of fundamental theories such as supergravity/string theories. Since these theories live in ten or eleven dimensions, an important mechanism such as compactification of extra dimensions is necessary. However, there is an additional difficulty in producing four-dimensional effective theories able to describe inflation [13]. Alternatively, it is hard to produce four-dimensional theories that can develop a de Sitter vacuum, which is crucial for the existence of an
inflationary phase of our Universe. This is because the obtained inflaton potentials are normally very steep, which do not meet the essential criteria for developing sufficiently inflation. More recently this problem has been reconsidered in a similar concept now well-known as swampland conjectures (see e.g. [17,21]).

In a previous communication [22], some of us considered that Bogomol’nyi-Prasad-Sommerfield (BPS) solutions of truncated (or at least inspired) supergravity theories in five-dimensions can induce four-dimensional models that are not constrained by compactification because they are induced due to force along inter-brane distance. So even if inflaton potentials coming from the dimensional reduction cannot in general produce sufficiently inflation, one can still expect that such inter-brane force can produce the desired inflaton potential to describe inflation. In the setups [22] and more recently in [24] the authors considered elastic collision of bulk particles with parallel domain walls (thick branes) embedded in five-dimensions. The resonant tunneling effect that affects transmission rate through the barriers related to the parallel domain walls induces an attractive force that associated with the reflection rate allows us to find an attractive arctan-type potential for the inflaton field. We have indeed followed previous attempt by Dvali and Tye in the context of producing brane inflation with extra dimensions and several sources of force. In our setup, however, we mainly consider the dominance of particle collisions and the electric force of possibly charged domain walls [24].

In this paper, we test the observational viability of a four-dimensional arctan-type inflaton potential in light of the currently available CMB data from Planck collaboration. We organize this work as following: Section (II) present the theoretical motivation for the brane scenario considered here, that comes from a supergravity inspired model. Section (III) shows the slow-roll analysis for the potential induced on the brane, where we put theoretical constraints on the inflationary parameters. We expose in Section (IV) the method used to estimate the cosmological parameters, as well as the observational dataset used to do this estimation, and the main results obtained in the analysis. Finally, we summarized the main conclusions in Section (V).

II. THE ARCTAN MODEL

To the best of our knowledge the arctan model described in this section is the simplest model that can be found with minimal assumptions in the context of brane inflation in the realm of a string/supergravity inspired theories. These fundamental theories have a lot of constraints on the inflaton potentials, with some that run from those that invoke time-varying compact hyperbolic
manifold [15] to others that take advantage of flux compactification [16].

The potential analysed in this work was inspired by the brane inflation scenario of Ref. [23] and constructed in Ref. [24]. In this scenario, the universe is described as a (3+1) dimensional thick domain wall (thick brane) embedded into a five-dimensional bulk. The interaction with another parallel brane due to elastic collisions of bulk particles induces acceleration of the universe.

We consider the scalar bosonic sector of a supergravity theory in 5D with Lagrangian given by [22, 25–28]:

\[ e^{-1}L_{sugra} = -\frac{1}{4}M_3^3R(5) + G_{AB}\partial_\mu\Phi^A\partial^\mu\Phi^B - \frac{1}{4}G^{AB}\frac{\partial W(\Phi)}{\partial \Phi^A}\frac{\partial W(\Phi)}{\partial \Phi^B} + \frac{1}{3}M_3^3W(\Phi)^2, \]

(1)

where \( G_{AB} \) is the metric on the real scalar field space and \( e = |\det g_{\mu\nu}|^{1/2} \). \( R(5) \) is the Ricci scalar and \( 1/M_s \) represents the five-dimensional Planck length. The superpotential \( W(\Phi) \) is normally constrained and so is the five-dimensional scalar potential. As such, an effective four-dimensional theory with the induced inflaton potential with sufficient flatness to produce enough inflation is hard to find. However, it is possible to find such inflaton potential by assuming a few conditions if one takes the advantage of the following mechanism. In our model we just need the BPS domain wall solutions embedded in the 5D bulk can suffer interactions, with mainly interaction due to an attractive potential induced by bulk particle collisions with the transmission coefficient

\[ T = \frac{4}{(4\theta^2 + \frac{1}{4\theta^2})\cos^2 L + 4\sin^2 L}, \]

(2)

through two parallel domain walls. Here \( L \equiv L(r) \) is a function of distance \( r \) that separates the barriers related to the equation of small perturbations around the domain walls and \( \theta \) encodes information about the height and thickness of each barrier in terms of the colliding particles energy. Since the reflection coefficient changes because of the resonant tunneling effect that increases transmission rate as the barriers are brought close together, few bulk particles are reflected by the domain walls and then a very small force acts to them.

On the other hand, if one brings the domain walls far from each other the transmission rate decreases and so the reflection rate is increased and a stronger attractive force is experienced by the domain walls. By associating such force with the reflection rate it is possible to find an attractive potential as a function of the inflaton field

\[ V(\phi) = K\beta \arctan \left( \frac{\phi}{\beta} \right), \]

(3)

where the inter domain walls distance \( r \) was associated with inflaton \( \phi \sim \sqrt{T_{wall}r} \), being \( T_{wall} \) the domain wall tension — for further details see [24].
We will study some theoretical and observational predictions of the potential, hereafter named arctan model, with its first and second derivatives with respect to the field $\phi$ given by:

$$V'(\phi) = \frac{K \beta^2}{\beta^2 + \phi^2} \quad \text{and} \quad V''(\phi) = -\frac{2K}{\beta} \left( \frac{\beta^2}{\phi^2 + \beta^2} \right)^2. \quad (4)$$

The potential behavior $V(\phi)$ as a function of the field $\phi$ and its derivatives are displayed in Fig. (1). Note that the higher the value of $\beta$ the higher is the slope of the potential. However, this increase is large enough to allow the slow-roll regime to continue to happen and a plateau-type inflationary potential is obtained.

### III. SLOW-ROLL ANALYSIS

The slow-roll regime is characterized by the parameters $\epsilon$ and $\eta$, such that the conditions $\epsilon, \eta \ll 1$ are satisfied $^1$. For the arctan potential considered here, these two parameters are written in function of the potential and its first and second derivatives with respect to $\phi$ as:

$$\epsilon = \frac{1}{2 \beta^2 \arctan^2 \left( \frac{\phi}{\beta} \right) \left( 1 + \frac{\phi^2}{\beta^2} \right)^2}, \quad (5)$$

and

$$\eta = -\frac{2}{\beta^2 \arctan \left( \frac{\phi}{\beta} \right) \left( 1 + \frac{\phi^2}{\beta^2} \right)^2}. \quad (6)$$

When the condition $\epsilon = 1$ is satisfied we can define the value of the field at the end of inflation, $\phi_{end}$. However, we could not invert Eq. (5) directly, instead, we did it numerically considering

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$^1$ These conditions guarantee that the field slowly rolls down its potential until its minimum and that the expansion rate, $H$, is almost a constant.
the values of $\phi_{\text{end}}$ and $\beta$ which satisfy $\epsilon(\phi_{\text{end}}) = 1$. The results are two solutions: one is valid for $\beta, \phi_{\text{end}} > 0$ and the second is valid for $\beta, \phi_{\text{end}} < 0$, hereafter solution 1 and 2, respectively. Thus, we fit two polynomial functions of 15th order for both solutions of $\phi_{\text{end}}$ in terms of $\beta$.

In order to obtain the amplitude $K$ of the potential \([3]\), we consider the primordial power spectrum of curvature perturbations, determined at the scale when $\phi_{\ast}$ as

\[ P_R = \frac{V(\phi)}{24\pi^2 \epsilon} \bigg|_{k=k_{\ast}} \]  \hspace{1cm} (7)

where $\ast$ refers to the scale CMB that crosses the Hubble horizon during inflation. The value of $P_R(k_{\ast})$ is determined by Planck normalization to $2.0933 \times 10^{-9}$ for the pivot choice $k_{\ast} = 0.05\text{Mpc}^{-1} \,[29]$. Hence, using the potential $V(\phi)$ given by Eq. (3) and $\epsilon$ given by (5), and inverting for $K$, we obtain:

\[ K = \frac{12\pi^2 P_R(k_{\ast})}{\beta^3 \arctan^3 \left( \frac{\phi_{\ast}}{\beta} \right) \left( 1 + \frac{\phi_{\ast}^2}{\beta^2} \right)^2}. \]  \hspace{1cm} (8)

We can find the value of the field $\phi_{\ast}$ using the expression for the number of e-folds, since the horizon crossing moment up to the end of inflation

\[ N = \int_{\phi_{\text{end}}}^{\phi_{\ast}} \frac{d\phi}{\sqrt{2\epsilon}} \]

\[ = \left( \frac{\phi_{\ast}^3}{3\beta} + \phi_{\ast} \beta \right) \arctan \left( \frac{\phi}{\beta} \right) - \frac{\phi_{\ast}^2}{6} - \frac{\beta^2}{3} \ln \left( 1 + \frac{\phi_{\ast}^2}{\beta^2} \right) \Bigg|_{\phi_{\text{end}}}. \]  \hspace{1cm} (9)

We then consider the pivot scale for which the CMB mode crosses the Hubble horizon during inflation to be $N = 55$. Since the previous equation cannot be solved analytically, we must use numerical methods to solve it, in order to find the values of $\phi_{\ast}$ and $\beta$ that satisfy $N = 55$. Thus, it is possible to interpolate a polynomial fit and obtain a function for $\phi_{\ast}$. Similarly, in order to find the value of the field at the beginning of inflation we solved numerically the equation for the number of e-folds, considering $N = 70$, and then interpolate a polynomial fit for $\phi_{\ast}$. For both cases, $\phi_{\ast}$ and $\phi_{\ast i}$, the polynomial functions obtained were of 15th order. Through these three quantities we are able to find the values of $\beta$ where the slow-roll conditions are fully met: $0.01 < \beta < 10$, which can be used as the prior in our analysis.

Lastly, we calculate the inflationary parameters for the arctan potential, i.e. the spectral index $n_s$ and the tensor-to-scalar ratio $r$, which can be written as

\[ n_s = 1 - \frac{1}{\beta^2 \arctan \left( \frac{\phi_{\ast}}{\beta} \right) \left( 1 + \frac{\phi_{\ast}^2}{\beta^2} \right)^2} \left[ 4 - \frac{3}{2 \arctan \left( \frac{\phi_{\ast}}{\beta} \right)} \right], \]  \hspace{1cm} (10)
Figure 2: The cosmological observables $n_s$ and $r$ for the Arctan potential [3], considering different values of the parameter $\beta$ and two values for the number of e-folds, $N = 50$ and $N = 60$, respectively. The contours are the 68% and 95% confidence level regions obtained from Planck (2018) CMB data using the pivot scale $k_* = 0.05\text{Mpc}^{-1}$.

We show the behavior of the spectral index and the tensor-to-scalar ratio for different values of $\beta$ in Fig. (2), considering the number of e-folds given in Eq. (9) ranging from $N = 50$ to $N = 60$. We used the number of e-folds in this range in order to compare the model predictions to the Planck data, which are displayed as the confidence regions corresponding to 68% and 95%, obtained from the latest release of Planck CMB data [29]. Notice that the value of $n_s$ decreases ($r$ increases) as $\beta$ increases. The results for the $n_s - r$ plan indicates that just part of the range of $\beta$ considered ($5 < \beta < 10$ for $N = 50$) are in agreement with Planck data (within 95% C.L.). Therefore, we consider the range of $\beta$ which totally match the slow-roll regime and is in accordance with Planck data to be $5 < \beta < 10$.

It is also worth mentioning that this result differs from a previous analysis of this model, where the authors found that values of $\beta$ of order $10^{-2}$ are in good agreement with Planck data [24], which is out of the allowed range we found. However, one should mention that they performed their analysis examining an approximation for the slow-roll regime where $\phi \gg \beta$, which is not the case considered in this work.
IV. ANALYSIS AND RESULTS

In order to produce the theoretical predictions for the arctan model, we adopt a version of the latest version of the Code for Anisotropies in the Microwave Background CAMB [30], to include the $\beta$ parameter. The version of the Boltzmann solver we used, MODECODE [31], is proper to deal with inflationary potentials, by calculating numerically the dynamics, i.e. the Friedmann and Klein-Gordon equations, and the perturbations along with the Fourier components associated with curvature perturbations produced by the fluctuations of the scalar field $\phi$. Then we construct the Primordial Power Spectrum (PPS) which can be translated to the temperature power spectrum of the CMB.

In general, the method to implement MODECODE consists in choosing the potential $V(\phi)$, write its first and second derivatives with respect to the field $\phi$, consider the initial condition to the field, $\phi_{ini}$, and then the code solves the dynamics equations to obtain $H$ and $\phi$. At last, the PPS is obtained through the solutions of the Mukhanov-Sasaki equations [32]. The PPS can be used to obtain the predictions for the temperature power spectrum of CMB, shown in Fig. (3), considering different values of $\beta$. We observe that the main effect of the $\beta$ parameter is to change the amplitude of the temperature power spectrum, which reinforces as an appropriate range for our analysis the flat prior of $5 < \beta < 10$.

In order to constrain the cosmological parameters associated with the arctan model, we perform a Markov Chain Monte Carlo (MCMC) analysis using the latest version of CosmoMC code [33].
We vary the usual cosmological variables: the baryon and cold dark matter density, the ratio between the sound horizon and the angular diameter distance at decoupling, and the optical depth: \( \{ \Omega_b h^2, \Omega_c h^2, \theta, \tau \} \), respectively. Also, the value of the parameter \( \beta \) is chosen according to the considerations made above. We consider purely adiabatic initial conditions, fix the sum of neutrino masses to 0.06 eV and the universe curvature to zero, and also vary the nuisance foregrounds parameters \( [1] \). In Table (I), we show the flat priors used in the analysis to all the cosmological parameters.

The data set considered in this analysis comes from the latest Planck (2018) Collaboration release \[29\] and considers the high multipoles Planck temperature data from the 100-,143-, and 217-GHz half-mission T maps, and the low multipoles data by the joint TT, EE, BB and TE likelihood, where EE and BB are the E- and B-mode CMB polarization power spectrum and TE is the cross-correlation temperature-polarization (hereafter “PLA18”). We also consider an extended data set, combining the CMB data along with i) Baryon Acoustic Oscillations (BAO) from the 6dF Galaxy Survey (6dFGS) \[34\], Sloan Digital Sky Survey (SDSS) DR7 Main Galaxy Sample galaxies \[35\], BOSSgalaxy samples, LOWZ and CMASS \[36\] and ii) the tensor amplitude of B-mode polarization, used to constrain the parameters associated with the tensor spectrum coming from 95, 150, and 220 GHz maps, coming from the Keck Array and BICEP2 Collaborations \[37, 38\] analysis of the BICEP2/Keck field, in combination with Planck high-frequency maps to remove polarized Galactic dust emission (hereafter “BKP15”).

The main results of our analysis are displayed in Table (II) and Fig. (4), where we present the main constraints on the cosmological parameters for the arctan model. Note that all the primary cosmological parameters are in good agreement with the \( \Lambda \)CDM standard model, at least in 2\( \sigma \) \[29\]. We obtain a good constraint on the tensor-to-scalar ratio of \( r_{0.002} = 0.0296 \pm 0.0006 \), while for the \( \Lambda \)CDM model we find \( r_{0.002} < 0.109 \) \[29\]. Also, this is the first analysis that put a tight restriction on the \( \beta \) parameter using observational data, i.e., \( \beta = 7.1 \pm 0.2 \) (within 68\% confidence level). It is

| Parameter | Prior Ranges |
|-----------|--------------|
| \( \Omega_b h^2 \) | [0.005 : 0.1] |
| \( \Omega_c h^2 \) | [0.001 : 0.99] |
| \( \theta \) | [0.5 : 10.0] |
| \( \tau \) | [0.01 : 0.8] |
| \( \beta \) | [5 : 10] |
Table II: 68% confidence limits for the cosmological parameters for Arctan model using PLA18+BAO+BK15 data.

| Primary parameters | Derived parameters |
|--------------------|--------------------|
| $\Omega_b h^2$     | $H_0$              |
| $0.02223 \pm 0.019$ | $67.84 \pm 0.39$   |
| $\Omega_c h^2$     | $\Omega_m$         |
| $0.1185 \pm 0.0009$ | $0.307 \pm 0.005$  |
| $\theta$           | $\Omega_\Lambda$  |
| $1.04106 \pm 0.00040$ | $0.693 \pm 0.005$  |
| $\tau$             | $n_s$              |
| $0.057 \pm 0.007$   | $0.9744 \pm 0.0002$|
| $\beta$            | $r_{0.002}$        |
| $7.1 \pm 0.2$       | $0.0296 \pm 0.0006$|

Important to note that we analyzed the arctan potential without making any approximation that can limit the power of constraining the $\beta$ parameter from the slow-roll analysis. In fact, this kind of analysis was performed in [24] that values for $\beta$ of order $10^{-2}$, where the authors considered an approximation of type $\phi \gg \beta$.

Lastly, the confidence regions at 68% and 95% and the posterior probability distribution for some primary parameters of the model are shown in Fig. (4). Notice that the values of the cosmological parameters are well inside the bounds obtained previously for the $\Lambda$CDM model [29]. Particularly, our analysis obtains tighter constraints for all the parameters, being more prominent the result obtained for the tensor-to-scalar ratio parameter. We also notice a strong correlation between $\beta$ and $r$. Finally, we show the best-fit values of the temperature power spectrum for the model, which accommodate the CMB data as good as $\Lambda$CDM cosmology.

V. CONCLUSIONS

In this paper, we analyzed an inflationary model retrieved in a brane cosmology scenario, considering that the period of inflation occurred in a 3D domain wall immersed in a five-dimensional Minkowski space in the presence of a stack of $N$ parallel domain walls. We then studied the theoretical and observational predictions for arctan-type inflaton model and obtained constraints on its inflationary parameters.

Regarding the slow-roll analysis, it indicates that for the range of values $0.01 < \beta < 5$ ($N = 60$) we should rule the arctan-type model out since the model predictions are outside the $2\sigma$ contour region allowed by the CMB data. We demonstrate that this result contrasts with the analysis performed in [24], that obtained values of $\beta$ of order $10^{-2}$ in agreement with the CMB data.

Taking into account the range of $\beta$ allowed by the data, we also performed the parameter estimation using the latest CMB temperature data combined with BAO and B-mode polarization...
data. The results of the MCMC analysis for the primary and derived cosmological parameters show an excellent match to the latest cosmological data and with the predictions for the ΛCDM model (as we can see in Fig. (5)). Particularly, we have obtained the following bound on the tensor-to-scalar ratio $r_{0.002} = 0.0296 \pm 0.0006$ at 68% C.L., which in principle can be be detected by the future CMB experiments that are planned to have sensitivities of order $\Delta r \sim 0.001$ [39, 40].

We also obtained tight constraints on the $\beta$ parameter that scales the primordial inflationary potential, to be $\beta = 7.1 \pm 0.2$. Especially, when considering the bestfit value of $\beta$, we set the prediction for the inflationary parameters $n_s$ and $r$ right within the 1σ region allowed by the CMB data and also yield an excellent fit to the temperature power spectrum, as good as the one predicted by the ΛCDM model, displayed in Fig. 5.
Therefore, this analysis corroborates the observational viability of the arctan-type model and establishes the general theoretical predictions of the model. A more robust statistical analysis considering a Bayesian model selection [10, 41–46] will appear in a forthcoming communication.

Acknowledgments

R.M.P. Neves is supported by Coordenação de Aperfeiçoamento de Pessoal de Nível Superior (CAPES). S. Santos da Costa thanks the financial support from the Programa de Capacitação Institucional (PCI) do Observatório Nacional/MCTI. F.A. Brito acknowledges CNPq (Grant nos. 312104/2018-9 and 439027/2018-7) and PRONEX/CNPq/FAPESQ-PB (Grant no. 165/2018), for partial financial support. J. Alcaniz is supported by CNPq (Grant nos. 310790/2014-0 and 400471/2014-0) and Fundação de Amparo à Pesquisa do Estado do Rio de Janeiro FAPERJ (Grant no. 233906). We also thank the authors of the ModeCode (M. Mortonson, H. Peiris and R. Easther) and CosmoMC (A. Lewis) codes. Finally, we acknowledge the computational support of the Observatório Nacional Data Center where this work was developed.

[1] N. Aghanim et al. [Planck Collaboration], Astron. Astrophys. 594, A11 (2016)
[2] Planck Collaboration and P. A. R. Ade et al., Astron. Astrophys. 594, A20 (2016)
[3] Planck Collaboration and P. A. R. Ade et al., Astron. Astrophys. 594, A13 (2016)
[4] A. Ijjas, P. J. Steinhardt and A. Loeb, Phys. Lett. B 723, 261 (2013)
[5] A. Linde, [arXiv:1402.0520 [hep-th]]
[6] A. H. Guth, D. I. Kaiser and Y. Nomura, Phys. Lett. B 733, 112 (2014)
[7] R. H. Brandenberger, Class. Quant. Grav. 32, no. 23, 234002 (2015)
[8] A. H. Guth, Phys. Rev. D23, 347 (1981)
[9] A. D. Linde, Phys. Lett. B108, 389 (1982)
[10] S. Santos da Costa, M. Benetti and J. Alcaniz, JCAP 1803, 004 (2018)
[11] M. A. Santos, M. Benetti, J. S. Alcaniz, F. A. Brito and R. Silva, JCAP 1803, 023 (2018)
[12] L. Barosi, F. A. Brito and A. R. Queiroz, JCAP 0804, 005 (2008)
[13] G.W. Gibbons, Supersymmetry, Supergravity and Related Topics, eds. F. de Aguila, J.A. de Azcárraga and L.E. Ibañez (World Scientific, Singapore, 1985). J. Maldacena and C. Nuñez, Int. J. Mod. Phys. A16, 822 (2001)
[14] S. Kachru, R. Kallosh, A. Linde and S.P. Trivedi, Phys. Rev. D68, 046005 (2003)
[15] P.K. Townsend and M.N.R. Wohlfarth, Phys. Rev. Lett. 91, 061302 (2003)
[16] S. Kachru, R. Kallosh, A. Linde, J. Maldacena, L. McAllister and S.P. Trivedi, JCAP 0310, 013 (2003)
[17] S.K. Garg and C. Krishman, JHEP 11, 075 (2019).
[18] H. Ooguri, E. Palti, G. Shiu and C. Vafa, Phys. Lett. B788 (2019) 180-184
[19] W.H. Kinney, S. Vagnozzi and L. Vissinelli, Class. Quant. Grav. 36 (2019) 117001
[20] A. Mohammadi, T. Golanbari, S. Nasri and K. Saaidi, [arXiv:2006.09489 [gr-qc]]
[21] S. Das, Phys. Rev. D99, 083510 (2019)
[22] F. A. Brito, F. F. Cruz and J. F. N. Oliveira, Phys. Rev. D71, 083516 (2005)
[23] G. Dvali and S.-H. H. Tye, Phys. Lett. B450, 72 (1999)
[24] R. M. P. Neves, F. F. Santos and F. A. Brito, Phys. Lett. B 810, 135813 (2020)
[25] M. Cvetic, Int. J. Mod. Phys. A16, 819 (2001)
[26] F. A. Brito, M. Cvetic and S.-C. Yoon, Phys. Rev. D64, 064021 (2001)
[27] D. Bazeia, F. A. Brito and J. R. Nascimento. Phys. Rev. D68, 085007 (2003)
[28] D. Bazeia, F. A. Brito and F. G. Costa, Phys. Lett. B 661, 179 (2008)
[29] N. Aghanim et al. [Planck Collaboration], [arXiv:1807.06209 [astro-ph.CO]].
[30] A. Lewis, A. Challinor and A. Lasenby, Astrophys. J. 538, 473 (2000)
[31] M. J. Mortonson, H. V. Peiris and R. Easther, Phys. Rev. D 83, 043505 (2011)
[32] S. Weinberg, “Cosmology”, Oxford: OUP Oxford (2008).
[33] A. Lewis and S. Bridle, Phys. Rev. D 66, 103511 (2002)
[34] F. Beutler, C. Blake, M. Colless, D. Heath Jones, L. Staveley-Smith, L. Campbell, Q. Parker, W. Saunders, and F. Watson, Mon. Not. R. Astron. Soc.416, 3017 (2011)
[35] A. J. Ross, L. Samushia, C. Howlett, W. J. Percival, A. Burden, and M. Manera, Mon. Not. R. Astron. Soc.449, 835 (2015)
[36] L. Anderson et al.(BOSS Collaboration), Mon. Not. R. Astron. Soc.441, 24 (2014)
[37] P. A. R. Ade et al. (BICEP2 and Planck Collaborations), Phys. Rev. Lett. 114, 101301 (2015)
[38] P. A. R. Ade et al. (BICEP2 and Keck Array Collaborations), Phys. Rev. Lett. 116, 031302 (2016)
[39] P. Ade et al. [Simons Observatory Collaboration], JCAP 1902, 056 (2019)
[40] A. Suzuki et al., J. Low. Temp. Phys. 193, no. 5-6, 1048 (2018)
[41] M. Campista, M. Benetti and J. Alcaniz, JCAP 1709, 010 (2017)
[42] A. R. Liddle, Mon. Not. Roy. Astron. Soc. 377, L74 (2007)
[43] R. Trotta, Mon. Not. Roy. Astron. Soc. 378, 72 (2007)
[44] M. Benetti and J. S. Alcaniz, Phys. Rev. D 94, no. 2, 023526 (2016)
[45] L. L. Graef, M. Benetti and J. S. Alcaniz, JCAP 1707, 013 (2017)
[46] M. Benetti, S. J. Landau and J. S. Alcaniz, JCAP 1612, 035 (2016)