Phenomenological Constraints on Supersymmetric Models with an Anomalous $U(1)$ Flavor Symmetry

Galit Eyal

Department of Particle Physics, Weizmann Institute of Science, Rehovot 76100, Israel

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We investigate supersymmetric models in which an anomalous $U(1)_X$ symmetry explains the Yukawa hierarchy, and the related $D_X$-term plays a role in supersymmetry breaking. We use a bottom-up approach to model building. Phenomenological viability leads to a scenario with degenerate squark and slepton spectra and with heavy gauginos. Features of a Kähler potential that allows for such a scenario are described.

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1 Introduction

Two open questions in the framework of Supersymmetry (SUSY) are the mechanism of SUSY breaking and the structure of flavor parameters. In a particularly interesting class of models, a single, anomalous $U(1)_X$ flavor symmetry plays an important role in answering these two questions.

SUSY breaking with an anomalous $U(1)_X$ has been introduced in refs. [1, 2], and further analyzed in ref. [3]. This mechanism includes a $U(1)_X$ gauge symmetry which is anomalous due to a non-vanishing value of

$$\delta_{GS} = \frac{1}{192\pi^2} \sum_i q_X^i,$$

where $q_X^i$ is the $U(1)_X$ charge of the $i$ chiral superfield. Without loss of generality $\delta_{GS}$ is taken to be positive. The anomaly cancels by a non-trivial transformation of the dilaton ($S$) superfield according to the Green-Schwarz mechanism [4]. This results in the generation of a Fayet-Iliopoulos term:

$$\xi^2 = -\frac{1}{2} \delta_{GS} M_p^2 K',$$

where $K$ is the dimensionless Kähler potential and $K' \equiv \frac{\partial K}{\partial S}$. All the chiral superfields charged under the Standard Model (SM) gauge group are assigned positive (or zero) charges under $U(1)_X$, in order not to break any of these symmetries at a high-energy scale. There is, however, a single SM-singlet field with a negative $U(1)_X$ charge ($\phi_-$). Minimization of the scalar potential of the complete model results in SUSY breaking with the following non-zero vacuum expectation values (vevs):

$$\langle \phi_- \rangle, \quad \langle F_S \rangle, \quad \langle D_X \rangle.$$  

Depending on the details of the model, there might be additional fields that receive non-zero vevs.

The $U(1)_X$ symmetry breaking produces a naturally small parameter:

$$\epsilon = \frac{\langle \phi_- \rangle}{M_p} \sim \frac{\xi}{M_p}.$$  

This small parameter motivates the use of the $U(1)_X$ as a horizontal flavor symmetry [1]-[4]. We take $\epsilon$ to be of order of the Cabbibo angle, $\lambda \sim 0.2$. (In our framework $\delta_{GS} = O(\lambda^2 - \lambda)$, which implies $K' = O(1 - \lambda)$.) The anomalous $U(1)_X$ is an interesting flavor symmetry since the $D_X$-term contributes to the soft masses of light scalar fields which carry a $U(1)_X$ charge and induces non-degeneracy among them.

In this work we use a bottom-up approach to analyze models with a single anomalous $U(1)_X$ that acts as a horizontal symmetry. (We assume that there are no additional non-anomalous horizontal $U(1)$s.) The requirement that the $U(1)_X$ symmetry explains the mass parameters in the fermion sector leads to an almost unique charge assignment for the Supersymmetric-SM matter fields. This charge assignment leads to only mild alignment.
between the fermions and sfermions \[8\]. We assume that all sfermion masses are below, or just at, the TeV scale. (Models with heavy first two sfermion generations, the motivation for them and their potential problems, have been discussed in the literature \[7\], \[1\]-\[2\]. We do not consider them here.) Consequently, in order to satisfy phenomenological constraints, the squark spectrum as well as the slepton spectrum have to be roughly degenerate at low-energy, so the horizontal symmetry is not manifest in the squark and slepton spectra. We describe the possible high-energy scenarios that produce such a spectrum. Assuming that the high-energy parameters are determined by the vevs in eq. (3), the required relations between the $F_S$-term, $D_X$-term and the derivatives of the Kähler potential that need to be satisfied, within the different scenarios, are analyzed.

In section 2 we describe the general framework in which we work. In section 3 the choice of charge assignments is explained. Two specific examples and a general analysis of the phenomenologically viable scenarios are given in section 4. Section 5 contains some concluding remarks.

2 The Framework

We assume that the leading contributions to the relevant dimensionful parameters come from the vevs in eq. (3). Any contributions by additional vevs for scalar fields or $F$-terms are sub-dominant. Then we can make the following statements concerning the flavor and Higgs parameters at high-energy \[13\]-\[15\]:

(i) The Yukawa parameters depend on the $U(1)_X$ charges of the matter fields \[16\]. For example, consider the following down quark mass matrix element:

$$Y_{d_{11}}^d \phi_d q_{11} \bar{d}_{11} = \frac{a_{d_{11}}^d}{M_p^d} \phi_d^n q_{11} \bar{d}_{11} \rightarrow M_{d_{11}}^d = a_{d_{11}}^d e^n \langle \phi_d \rangle .$$

(ii) Diagonal elements of the sfermion mass-squared matrices, $\tilde{m}_i^2$, receive $\tilde{m}_3/2$ (flavor independent, universal) and $D_X$ (flavor dependent) contributions as follows:

$$\tilde{m}_i^2 = m_{3/2}^2 - q_{X_i} \langle D_X \rangle .$$

When the vanishing of the cosmological constant is imposed, the gravitino mass $\tilde{m}_{3/2}$ is given by:

$$\tilde{m}_{3/2}^2 = \frac{1}{3} K'' | \langle F_S \rangle |^2 .$$

(iii) Gaugino masses, $\tilde{m}_1^2$, receive universal contributions:

$$\tilde{m}_1^2 = \frac{\langle F_S \rangle}{\langle S + S^* \rangle} .$$
Below we take $\langle S \rangle \simeq 2$.

(iv) Off-diagonal elements in the mass-squared matrices are suppressed compared to the diagonal terms for two reasons: First, they receive contributions proportional to $F$-terms smaller than $F_S$, and, second, they are suppressed by the horizontal symmetry.

(v) A-terms are proportional to the Yukawa couplings $[13]$:

$$A_{ij}^f = \langle F_S \rangle K' Y_{ij}^f.$$  \hfill (9)

Using eq. (8) we get:

$$A_{ij}^f = \langle S + S^* \rangle \tilde{m}_i^{3/2} K' Y_{ij}^f.$$  \hfill (10)

Later we take $Y_{33}^u = O(1)$. In order to avoid large contributions by the corresponding A-term to the renormalization group equations (RGE) that might lead to a negative stop mass, we need to impose (especially when the gauginos are heavy) $K' \leq O(\lambda)$. Once this is imposed, the A-term contributions are negligible.

(vi) Higgs sector mass parameters:

We assume that the $\mu$ term is not generated in the superpotential. It can be generated by the Giudice-Masiero mechanism $[14, 17]$:

$$\mu = a_\mu \epsilon^{q_{Xh_d}} \tilde{m}_{3/2},$$  \hfill (11)

where $a_\mu$ is an $O(1)$ coefficient and the suppression by powers of $\epsilon$ comes from the need to make the term $U(1)_X$ symmetric. We always set $q_{Xq_3} = q_{Xu_3} = q_{Xh_u} = 0$ in order to have $Y_{33}^u = O(1)$, and $q_{Xh_d} \geq 0$ in order to avoid breaking the SM symmetry at high-energy. The other relevant Higgs parameters are given by $[13]$

$$B = 2 \mu \tilde{m}_{3/2},$$  \hfill (12)

$$m_{h_u}^{o2} = \tilde{m}_{3/2}^2,$$  \hfill (13)

$$m_{h_d}^{o2} = \tilde{m}_{3/2}^2 - q_{Xh_d} \langle D_X \rangle,$$  \hfill (14)

$$\tan \beta \sim \frac{m_{h_u}^{o2} + m_{h_d}^{o2}}{B}.$$  \hfill (15)

If the Higgs masses are not dominated by the $D_X$-term then $\tan \beta = O(\epsilon^{-q_{Xh_d}})$.

The effects of running from high-energy to low-energy are implemented following ref. $[18]$. We define an average high-energy squark mass-squared $\tilde{m}_q^{o2}$ and an average high-energy slepton mass-squared $\tilde{m}_\ell^{o2}$. We denote:

$$X_q^o = \frac{\tilde{m}_{1/2}^o}{\tilde{m}_q^{o2}}, \quad X_\ell^o = \frac{\tilde{m}_{1/2}^o}{\tilde{m}_\ell^{o2}}.$$  \hfill (16)

At the low-energy scale we get the following average values (neglecting contributions from A-terms and $O(m_Z^2)$ corrections):

$$\tilde{m}_q^2 \simeq \tilde{m}_q^{o2} + 7 \tilde{m}_{1/2}^{o2},$$  \hfill (17)

$$\tilde{m}_\ell^2 \simeq \tilde{m}_\ell^{o2} + 0.3 \tilde{m}_{1/2}^{o2}.$$  \hfill (18)
\[ X_q = \frac{\tilde{m}_3^2}{\tilde{m}_q^2} \simeq \frac{9X_q^0}{1 + 7X_q^0} \rightarrow X_q = [0, \frac{9}{7}), \quad (19) \]
\[ X_\ell = \frac{\tilde{m}_1^2}{\tilde{m}_\ell^2} \simeq \frac{0.16X_\ell^0}{1 + 0.3X_\ell^0} \rightarrow X_\ell = [0, 0.53). \quad (20) \]

The quark and lepton \( U(1)_X \) charges are assigned in such a way that they reproduce the flavor parameters:

\[ (m_u : m_c : m_t) \sim (\lambda^7 : \lambda^4 : 1), \quad (21) \]
\[ (m_d : m_s : m_b) \sim (\lambda^7 : \lambda^5 : \lambda^3), \quad (22) \]
\[ (m_\mu : m_\tau) \sim (\lambda^8 - \lambda^9 : \lambda^5 : \lambda^3), \quad (23) \]
\[ |V_{us}| \sim \lambda, \quad |V_{cb}| \sim \lambda^2, \quad |V_{ub}| \sim \lambda^3. \quad (24) \]

We do not impose additional constraints on the charges, as required by the Green-Schwarz mechanism for example, since we assume that there could be additional superheavy matter fields in the model that are vector-like under the SM gauge group but chiral under \( U(1)_X \). \( \tan \beta \) is in the range \( 1 - \lambda^{-2} \). We found no reason to prefer one choice over the other.

### 3 The Fermion Sector

In our examples we choose \( q_{Xh_d} = 0 \) and consequently \( \tan \beta \sim 1 \). With our requirements the charges of the quarks are fixed uniquely, but there is still freedom left in the lepton sector. Different lepton charges lead to different contributions to Flavor Changing Neutral Currents (FCNC). The charges we choose to use are given in table \( 3 \). The choice \( q_{X\ell_1} = 4, q_{X\ell_2} = q_{X\ell_3} = 3 \) and \( q_{X\ell_1} = 5 \) would have led, for example, to larger contributions to \( Br(\mu \rightarrow e\gamma) \). A different charge assignment was given in ref. [7], but there a deviation of factors up to \( O(10) \) in the mass ratios was allowed, and a spectrum of heavy squarks was produced.

With our charge assignment the following mass matrices are produced:

\[ M^u \sim \langle \phi_u \rangle \begin{pmatrix} e^7 & e^5 & e^3 \\ e^6 & e^4 & e^2 \\ e^4 & e^2 & 1 \end{pmatrix}, \quad M^d \sim \langle \phi_d \rangle \begin{pmatrix} e^7 & e^6 & e^5 \\ e^6 & e^5 & e^4 \\ e^5 & e^4 & e^3 \end{pmatrix}, \quad (25) \]
\[ M^\ell \sim \langle \phi_d \rangle \begin{pmatrix} e^9 & e^7 & e^6 \\ e^7 & e^5 & e^4 \\ e^6 & e^4 & e^3 \end{pmatrix}. \quad (26) \]
This is the form of the matrices at high energy. The diagonalizing matrices for the fermions are:

\[
V_{L}^{u} \sim \begin{pmatrix} 1 & \epsilon & \epsilon^3 \\ \epsilon & 1 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & 1 \end{pmatrix}, \quad V_{R}^{u} \sim \begin{pmatrix} 1 & \epsilon^2 & \epsilon^4 \\ \epsilon^2 & 1 & \epsilon^2 \\ \epsilon^4 & \epsilon^2 & 1 \end{pmatrix}, \quad (27)
\]

\[
V_{L}^{d} \sim \begin{pmatrix} 1 & \epsilon & \epsilon^3 \\ \epsilon & 1 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & 1 \end{pmatrix}, \quad V_{R}^{d} \sim \begin{pmatrix} 1 & \epsilon \\ \epsilon & 1 \\ 1 & 1 \end{pmatrix}, \quad (28)
\]

\[
V_{L}^{\ell} \sim V_{R}^{\ell} \sim \begin{pmatrix} 1 & \epsilon^2 & \epsilon^3 \\ \epsilon^2 & 1 & \epsilon \\ \epsilon^3 & \epsilon & 1 \end{pmatrix}. \quad (29)
\]

The size of the gaugino-fermion-sfermion flavor changing couplings is determined by the above matrices and by the diagonalizing matrices for the sfermions. However, the off-diagonal entries in the diagonalizing matrices for the fermions given above are dominant and they determine the couplings. Only a mild alignment is found [8].

### 4 The Sfermion Sector

The charge assignments above lead to non-universal contributions to the masses of the sfermions at high-energy (eq. (6)). Since in our scenario there is only mild alignment, and we do not consider the heavy squark scenario, only a degeneracy among the squarks and among the sleptons at low energies can avoid too large FCNC. The limits on FCNC parameters are taken from ref. [19], updated with the new bound on \(Br(\mu \rightarrow e\gamma)\) [20], where the \(\delta\)'s are defined as, for example,

\[
(\delta^{d}_{LR})_{12} \sim (K_{R}^{d})_{11}(K_{R}^{d})_{12}^{*} \frac{\tilde{m}_{d_{R1}}^{2} - \tilde{m}_{d_{R2}}^{2}}{\tilde{m}_{q}^{2}} \sim (V_{R}^{d})_{12}^{*} \frac{\tilde{m}_{d_{R1}}^{2} - \tilde{m}_{d_{R2}}^{2}}{\tilde{m}_{q}^{2}}. \quad (30)
\]

Here \(K_{R}^{d}\) denotes gluino couplings to right-handed down quarks and ‘right-handed’ down squarks.

The assumptions made in section 2 regarding the size of the different soft terms, and particularly the A-terms, imply that the limits on \((\delta^{f}_{LR})_{ij}\) do not pose additional constraints.

There are two ways in which to achieve the necessary degeneracy at low energies:

(i) Heavy gauginos induce degeneracy through RGE (see eqs. (17)-(18)).

(ii) Large universal contributions to sfermion masses (see eq. (6)).

Below we first examine two limiting cases, heavy and light gauginos, and then give a general analysis of the possible scenarios.

We start by presenting the sfermion mass-squared matrices at high-energy, allowing for a universal contribution. We give only the diagonal elements because the off-diagonal ones are, as mentioned above, suppressed:

\[
\tilde{M}_{LL}^{q^2} = \tilde{m}^{2} \left( \frac{3 + z}{z} \right), \quad (31)
\]
\[ \tilde{M}_{RR}^2 = \tilde{m}^2 \begin{pmatrix} 4 + z \\ 2 + z \\ z \end{pmatrix}, \quad \tilde{M}_{RR}^d = \tilde{m}^2 \begin{pmatrix} 4 + z \\ 3 + z \\ 3 + z \end{pmatrix}, \quad (32) \]

\[ \tilde{M}_{LL}^2 = \tilde{m}^2 \begin{pmatrix} 5 + z \\ 3 + z \\ 2 + z \end{pmatrix}, \quad \tilde{M}_{RR}^e = \tilde{m}^2 \begin{pmatrix} 4 + z \\ 2 + z \\ 1 + z \end{pmatrix}, \quad (33) \]

where

\[ \tilde{m}^2 = - \langle D_X \rangle, \quad z = - \frac{\tilde{m}_{3/2}^2}{\langle D_X \rangle}. \quad (34) \]

Here \( \tilde{m}_q^2 \simeq (2.2 + z)\tilde{m}^2 \) and \( \tilde{m}_l^2 \simeq (3 + z)\tilde{m}^2 \).

### 4.1 Heavy Gauginos: \( \tilde{m}_{1/2}^q \geq \tilde{m}_{q,l}^q \)

The gauginos are heavy when the following relations are fulfilled (eqs. (35)-(38)):

\[ \frac{\langle F_S \rangle^2}{\langle S + S^* \rangle^2} \gtrsim \tilde{m}_{3/2} \quad \rightarrow \quad \frac{1}{\langle S + S^* \rangle^2} \gtrsim \frac{1}{3} K'' \quad \rightarrow \quad K'' \leq O(\lambda), \quad (35) \]

and

\[ \frac{\langle F_S \rangle^2}{\langle S + S^* \rangle^2} \gtrsim - \langle D_X \rangle \quad \rightarrow \quad - \frac{\langle D_X \rangle}{\langle F_S \rangle^2} < O(\lambda). \quad (36) \]

In this scenario the degeneracy of the sfermion masses at low-energy is enhanced compared to the degeneracy at high-energy as given by the mass-squared matrices in eqs. (37)-(41).

Let us take for example \( X_q^o = 4 \). The gaugino masses at high-energy are estimated to be \( \tilde{m}_{1/2}^q = X_q^o \tilde{m}_q^2 \). The sfermion mass-squared matrices have the following form at low-energy:

\[ \text{diag}\{\tilde{M}_{LL}^q\} \simeq \tilde{m}^2 \{64 + 29z, \ 63 + 29z, \ 61 + 29z\}, \quad (37) \]
\[ \text{diag}\{\tilde{M}_{RR}^u\} \simeq \tilde{m}^2 \{65 + 29z, \ 63 + 29z, \ 61 + 29z\}, \quad (38) \]
\[ \text{diag}\{\tilde{M}_{RR}^d\} \simeq \tilde{m}^2 \{65 + 29z, \ 64 + 29z, \ 64 + 29z\}, \quad (39) \]
\[ \text{diag}\{\tilde{M}_{LL}^e\} \simeq \tilde{m}^2 \{8 + 2z, \ 6 + 2z, \ 5 + 2z\}, \quad (40) \]
\[ \text{diag}\{\tilde{M}_{RR}^e\} \simeq \tilde{m}^2 \{7 + 2z, \ 5 + 2z, \ 4 + 2z\}, \quad (41) \]

and

\[ X_q \simeq 1.2, \quad X_l \simeq 0.3. \quad (42) \]

In eqs. (37)-(41) we give the same correction to all squarks (eq. (17)) and to all sleptons (eq. (18)). This is an approximation. 'Left-handed', 'right-handed', up and down sfermions run differently from high-energy to low-energy. However, this does not effect our final qualitative results.
For small $z$ the high-energy scalar masses are not-universal. At low-energy the spectrum is:

$$\tilde{m}_q \sim \tilde{m}_3 > \tilde{m}_\ell > \tilde{m}_\gamma.$$  \hfill (43)

The ratio between squark and slepton masses is about 3.5. We can easily see that the squark masses are quite degenerate. The slepton masses are degenerate but to a lesser extent. This degeneracy is enough to avoid too large FCNC even for vanishing $z$: $\tilde{m}_{3/2} \rightarrow 0$ ($K'' \rightarrow 0$) (here we compare to limits given in ref. [19, 20] for $\tilde{m}_q \simeq 1 TeV$). However, the Higgs sector parameters require that $\tilde{m}_{3/2}$ is of order of the electro-weak (EW) scale. In the above case this requires $z \sim 1$ (eqs. (34)).

A heavy gaugino scenario with $X_o^q = 4$ and $z \sim 1$ can be realized for example with $\tilde{m} \sim \tilde{m}_{3/2} \sim 100 GeV$, $K'' \sim \lambda^2 - \lambda^3$ and $-\frac{(D_X)}{|(F_s)|^2} \sim \lambda^3$.

4.2 Light Gauginos: $\tilde{m}_{1/2}^o \ll \tilde{m}_{q,\ell}^o$

Light gaugino scenarios are limited by the following conditions:

Naturalness : $\tilde{m}_t \lesssim 1 TeV,$  \hfill (44)

Experiment : $\tilde{m}_q \gtrsim 0.2 TeV.$  \hfill (45)

This imposes:

$$X_q > 0.04 \quad \rightarrow \quad X_q^o > 0.0045.$$  \hfill (46)

(The scenario in ref. [21] violates the above bound.)

The gauginos are light when the following relation is fulfilled (eq. (3)):

$$\frac{(F_S)^2}{\langle S + S^* \rangle^2} \ll \tilde{m}_{3/2}^2 - q_i \langle D_X \rangle.$$  \hfill (47)

The mass-squared matrices of the sfermions are similar to the ones given in eqs. (31)-(33), because the corrections coming from the gaugino masses are very small. We see that the squark and slepton masses-squared are of the same size.

We take for example the case $X_q^o = 0.04$, with $X_q = 0.3$ at low-energy. In order to avoid too large FCNC we need $z > 30$ (here again we compare to limits given in ref. [19] for $\tilde{m}_q \simeq 1 TeV$; lighter squarks would have required larger $z$ in order to avoid too large contributions to FCNC). The sleptons are heavy enough and $z$ is large enough so that, with our choice of charge assignments, there is no stronger limit on $z$ coming from the lepton sector. This is a scenario of universality:

$$\tilde{m}_{3/2}^2 \gg -\langle D_X \rangle \quad \text{and} \quad K'' \gg O(\lambda).$$  \hfill (48)

The typical spectrum is:

$$\tilde{m}_q \sim \tilde{m}_3 > \tilde{m}_\ell > \tilde{m}_\gamma.$$  \hfill (49)

Taking $X_q^o = 0.04$ and $z \sim 31$ we can have for example $\tilde{m} \sim 170 GeV$, $\tilde{m}_{3/2} \sim 900 GeV$, $K'' \sim \lambda^{-1}$ and $-\frac{(D_X)}{|(F_s)|^2} \sim \lambda^2$. 

7
4.3 General Analysis

Once \( X_q^o \) is chosen (\( X_q^o > 0.0045 \)), we can find a minimal value of \( z \), \( z_* \), such that for \( z > z_* \) there are models in which \( U(1)_X \) is the only horizontal symmetry, and there is no problem with too large FCNC. Requiring that the degeneracy between the low-energy squarks solves the supersymmetric \( \epsilon_K \) problem \([22]\) (allowing CP violating phases of \( O(1) \)), we find a stronger lower bound on \( z \) which we denote by \( z_{**} \). \( z_* \) and \( z_{**} \) are shown in fig. 1 as a function of \( X_q^o \) (the calculation is for \( \tilde{m}_q = 1 \) TeV; if the sfermions are lighter, the lower limit on \( z \) is higher). As can be seen in the graph, the heavier the gauginos are (the larger \( X_q^o \) is), the smaller the universal contribution to scalar masses (\( z \)) is allowed to be.

![Figure 1: The minimal value of \( z \) required, as a function of \( X_q^o \), in order to produce a viable low-energy spectrum, calculated from the contributions to the real (\( z_* \)) and imaginary (\( z_{**} \)) parts of (\( \delta^{d}_{12} \)) \( LL \), (\( \delta^{d}_{12} \)) \( RR \), for \( \tilde{m}_q = 1 \) TeV.](image)

Taking for each \( X_q^o \) the value of \( z \) such that \( z = z_* \), we get the mass spectrum displayed in fig. 2.

As mentioned before, the Higgs sector parameters require \( \tilde{m}_{3/2} \) to be of order of the EW scale. This sets an additional lower limit on viable values of \( z \). Taking \( \tilde{m}_q = 1 \) TeV we find for \( X_q^o \sim 2 \) the bound \( z \geq O(\lambda) \), rising for \( X_q^o \sim 4 \) to \( z \geq O(1) \).

An additional limit should be set, on the value of the ratio \( \tilde{m}_{1/2}^o/\tilde{m}_q^o \), in order to avoid problems with a potential that is not bounded from below \([23, 24]\). This is roughly \( X_q^o \lesssim 4 \). One should notice that this bound implies that scenarios in which the degeneracy between the low energy squarks solves the SUSY \( \epsilon_K \) problem require \( z > 12 \) (see \( z_{**} \) in fig. 1).
The ratio of the vevs is given by (see eqs. (8),(16),(34)):

$$\frac{-\langle D_X \rangle}{|\langle F_S \rangle|^2} = \frac{1}{\tilde{m}_o^2 X_q^o (S + S^*)^2}. \tag{50}$$

Choosing $X_q^o (X_q^o > O(\lambda^2))$ and $z (> z_*)$, the value of this ratio is found to be:

$$-\frac{\langle D_X \rangle}{|\langle F_S \rangle|^2} \leq O(\lambda^2) \tag{51}$$

(for smaller $X_q^o$ this ratio can rise up to $O(\lambda)$).

Even if additional vevs other than those appearing in eq. (8) contribute to the soft terms, in particular to $\tilde{m}_{3/2}$, fig. 4 gives an estimate of the amount of sfermion degeneracy required at the high-energy scale, relative to the gaugino mass, in order to arrive at a viable scenario in which the anomalous $U(1)_X$ flavor symmetry is involved in SUSY breaking, and fig. 2 gives the resulting spectra. Eqs. (50),(51) also hold for any composition of $\tilde{m}_{3/2}$.

Returning to the specific assumption of our model (dominance of the vevs in eq. (8)), we find that, for given $X_q^o$ and $z$, the Kähler potential obeys (see eqs. (7),(8),(16),(34)):

$$K'' = \frac{f(X_q^o, z)}{(S + S^*)^2} \tag{52}$$

with

$$f(X_q^o, z) = \frac{3z}{\tilde{m}_o^2 X_q^o} < \frac{3}{X_q^o}. \tag{53}$$

The bound in eq. (46) implies $K'' \leq O(\lambda^{-2})$. 

Figure 2: The low-energy sparticle spectrum (in GeV), $\{\tilde{m}_g, \tilde{m}_q, \tilde{m}_l, \tilde{m}_\gamma\} = f(X_q^o)$, calculated with $z = z_*$. (Here $\tilde{m}_q = 1 \text{ TeV}$ is imposed.)
4.4 A Viable Scenario

The above analysis is directed at building phenomenologically viable models without imposing any relations between the different vevs. In any case there is a large degeneracy, but the minimal required degeneracy to allow for phenomenologically viable models is given by parameters on the $z = z_*$ line. There are possible charge assignments for the leptons, including the one we chose, that predict (for particular $z$ and $X_q^o$) $Br(\mu \rightarrow e\gamma)$ at the experimental limit.

In all of the above the anomalous $U(1)_X$ symmetry plays an important role in explaining the flavor parameters of the fermion sector. We are interested, however, in models in which this $U(1)_X$ plays an important role also in SUSY breaking. In particular, we assume that $\langle D_X \rangle$ is the dominant source of non-universality in the scalar masses and we are interested in the case that the contribution is not negligibly small compared to the universal one, say, $z < 10$. This requires, according to fig. 1, heavy gauginos ($X_o \sim 0.3$). This last statement is correct also when there are additional contributions to $\tilde{m}_{3/2}$. Under the specific assumption made here, that is to say dominance of the vevs in eq. (3), this implies $K'' \leq O(\lambda)$ (see eq. (52)) and consequently $K'' \lesssim (-K')$.

A viable scenario with $X_q^o = 2.5$ and $z = 0.4$ can be realized for example with $\tilde{m}_q \sim 1\, TeV$, $\tilde{m} \sim 145\, GeV$, $\tilde{m}_{3/2} \sim 90\, GeV$, $\tilde{m}_1 \sim 330\, GeV$, $K'' \sim 3$ and $-\langle D_X \rangle / \langle F_s \rangle \sim 3$.

The scenario requires the following relations to be obeyed:

- $-\langle D_X \rangle / \langle F_s \rangle = O(\lambda^2) - O(\lambda^3)$,
- $-\tilde{K}' = O(\lambda^2)$, with $|\tilde{K}'| = O(\lambda)$ and $\delta_{GS} = O(\lambda)$,
- $K'' = \frac{f(X_q^o, z)}{(S + S^*)^2}$ as given in eq. (52), leading to $K'' \leq O(\lambda)$.

Is there a Kähler potential that can fulfill these requirements? This question is beyond the scope of this work. However, it does seem to be a good sign that even the weak coupling form of the Kähler potential, $K = -\ln(S + S^*)$, comes close to fulfilling these requirements.

5 Conclusions

In this work a bottom-up approach is used in order to characterize the different possible models with an anomalous $U(1)_X$ flavor symmetry that is involved in SUSY breaking. Assuming that the leading contributions to the different soft-terms in the model come from the following vevs:

\[ \langle \phi_- \rangle, \quad \langle F_S \rangle, \quad \langle D_X \rangle, \quad \] (53)

the characteristics of the low-energy spectrum are given. The $U(1)_X$ horizontal symmetry explains naturally the smallness and hierarchy of the observed flavor parameters of fermions. Only mild alignment is produced. This leads, through the demand for suppression of FCNC, to scenarios with degenerate sfermions at low-energy. The horizontal symmetry is not manifest in the sfermion spectrum. The different possibilities for producing such a degenerate spectrum were described, ranging from universality scenarios to
scenarios with heavy gaugino. The different scenarios result in different relative sizes of squarks, sleptons and gaugino masses. This class of models allows for \( Br(\mu \to e\gamma) \) at the current experimental limit. For all the phenomenologically viable scenarios \( (X_q^o > O(\lambda^2)) \), the following is required:

\[
- \frac{\langle D_X \rangle}{|\langle F_S \rangle|^2} \leq O(\lambda^2).
\]

(54)

It seems impossible to build models in which the degeneracy is strong enough to solve the supersymmetric \( \epsilon_K \) problem while having the relevant CP violating phases of \( O(1) \). Approximate CP [25] is a possible solution to this problem.

Although there are different phenomenologically viable scenarios, the contributions of \( \langle D_X \rangle \) to scalar masses at high energy are significant only with heavy gauginos. In that case the dilaton Kähler potential needs to have the following features:

\[
|K'| = O(\lambda), \quad K'' \leq O(\lambda).
\]

(55)

We were led to the above scenarios without assuming any a-priori relationship between the different vevs. Whether it is possible to build a complete model in which the leading contributions come only from these three vevs, and with the required relations between them, is beyond the scope of this work.

While this work was near completion, a paper appeared [26] that deals with models similar to those described above. There, the starting point is an analysis of the Kähler potential, which is an approach different from ours. The implications of such models on \( Br(\mu \to e\gamma) \) were stressed.

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