NUCLEAR INTERACTIONS: THE CHIRAL PICTURE

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Received (Day Month Year)
Revised (Day Month Year)

Chiral expansions of the two-pion exchange components of both two- and three-nucleon forces are reviewed and a discussion is made of the predicted pattern of hierarchies. The strength of the scalar-isoscalar central potential is found to be too large and to defy expectations from the symmetry. The causes of this effect can be understood by studying the nucleon scalar form factor.

Keywords: chiral symmetry; nucleons; pions.

PACS Nos.: 13.75.Gx, 21.30.Fe

1. CHIRAL SYMMETRY

The outer components of nuclear forces are dominated by pion-exchanges and involve just a few basic subamplitudes, describing pion interactions with either nucleons or other pions. The simplest process \( N \rightarrow \pi N \), corresponding to the emission or absorption of a single pion by a nucleon, is rather well understood and gives rise to the one-pion exchange \( NN \) potential (\( OPEP \)). The scattering reaction \( \pi N \rightarrow \pi N \) comes next and determines both the very important two-pion exchange term in the \( NN \) force and the leading three-body interaction, as shown in Fig. 1.

\begin{figure}[h]
\centering
(a) \hspace{1cm} (b) \hspace{1cm} (c)
\end{figure}

Fig. 1. Free \( \pi N \) amplitude (a) and two-pion exchange two-body (b) and three-body (c) potentials.

The theoretical understanding of the \( \pi N \) amplitude proved to be very challenging and a suitable description was only produced by means of chiral symmetry. This framework provides a natural explanation for the observed smallness of \( \pi N \) scattering lengths and plays a fundamental role in Nuclear Physics. Nowadays, the use of chiral symmetry in low-energy pion-interactions is justified by \( QCD \).
The small masses of the quarks $u$ and $d$, treated as perturbations in a chiral symmetric lagrangian, give rise to a well defined chiral perturbation theory (ChPT). Hadronic amplitudes are then expanded in terms of a typical scale $q$, set by either pion four-momenta or nucleon three-momenta, such that $q \ll 1$ GeV. This procedure is rigorous and many results have the status of theorems. In general, these theorems are written as power series in the scale $q$ and involve both leading order terms and chiral corrections. The former can usually be derived from tree diagrams, whereas the latter require the inclusion of pion loops and are the main object of ChPT. At each order, predictions for a given process must be unique and the inclusion of corrections cannot change already existing leading terms.

The relationship between chiral expansions of the $\pi N$ amplitude and of two-pion exchange (TPE) nuclear forces is discussed in the sequence. For the $\pi N$ amplitude, tree diagrams yield $O(q, q^2)$ terms and corrections up to $O(q^4)$ have already been evaluated, by means of both covariant (CF) and heavy baryon (HBF) formalisms. In the case of the $NN$ potential, the leading term is $O(q^0)$ and given by the OPEP. The tree-level $\pi N$ amplitude yields TPE contributions at $O(q^2, q^3)$ and corrections at $O(q^4)$ are available, based on both HBF and CF. Tree-level $\pi N$ results also determine the leading $O(q^3)$ three-body force and partial corrections at $O(q^4)$ begin to be derived. As this discussion suggests, $O(q^4)$ corrections to both two- and three-nucleon forces require just the $O(q^3)$ $\pi N$ amplitude.

The full empirical content of the $\pi N$ amplitude cannot be predicted by chiral symmetry alone. Experimental information at low energies is usually encoded into the subthreshold coefficients introduced by Höhler and collaborators which can, if needed, be translated into the low-energy contants (LECs) of chiral lagrangians. Therefore, in order to construct a $O(q^3)$ $\pi N$ amplitude, one uses chiral symmetry supplemented by subthreshold information, as indicated in Fig. 2. The first two diagrams correspond to the nucleon pole, whereas the other ones represent a smooth background. The third graph reproduces the Weinberg-Tomozawa contact interaction, the fourth one summarizes LEC contributions and the last two describe medium range pion-cloud effects.

![Fig. 2. Representation of the $\pi N$ amplitude at $O(q^3)$.](image-url)
2. TWO-BODY POTENTIAL

With the purpose of discussing the problem of predicted $\times$ observed chiral hierarchies, in this section we review briefly results obtained by our group\textsuperscript{4,5} for the TPE-NN potential at $O(q^4)$. This component is determined by the three families of diagrams shown in Fig. 3. Family $I$ begins at $O(q^2)$ and implements the minimal realization of chiral symmetry\textsuperscript{9}, whereas family $II$ depends on $\pi\pi$ correlations and is $O(q^4)$. They involve only the constants $g_A$ and $f_\pi$ and all dependence on the LECs is concentrated in family $III$, which begins at $O(q^3)$.

As far as chiral orders of magnitude are concerned, on finds that the various components of the force begin as follows\textsuperscript{5}: $O(q^2) \rightarrow V_{SS}^+, V_T^+, V_C^-$ and $O(q^3) \rightarrow V_C^+, V_L^+, V_L^-, V_{SS}^+$. As $V_{SS}^+$, where the superscripts (+) and (−) refer to terms proportional to either the identity or $\tau^{(1)} \cdot \tau^{(2)}$ in isospin space. An interesting feature of these results is that the role played by family $II$ is completely irrelevant. On the other hand, family $I$ dominates almost completely the components $V_L^+, V_T^+$ and $V_{SS}^+$, whereas family $III$ does the same for $V_C^+, V_T^-$ and $V_{SS}^-$.

The relationship between the $OPEP[= O(q^0)]$ and $TPEP[= O(q^3)]$ contributions to the $V_{SS}^-$ and $V_T^-$ profile functions is shown in Fig. 4, where it is possible to see that the chiral hierarchy is respected.
In Fig. 5, the two central components $V_C^- = \mathcal{O}(q^2)$ and $V_C^+ = \mathcal{O}(q^3)$ are displayed side by side and two features are to be noted. The first one concerns the favorable comparison with the empirical Argonne potentials in both cases. The second one is that $|V_C^+| \sim 10|V_C^-|$ in regions of physical interest, defying strongly the predicted chiral hierarchy. This problem will be further discussed in the sequence.

Violations of the chiral hierarchy are also present in the drift potential, which corresponds to kinematical corrections due to the fact that the two-body center of mass is allowed to drift inside a larger system. In terms of Jacobi coordinates, it is represented by the operator

$$V(r)^\pm = V(r)^\pm, \, \text{cm} + V_D^\pm \, \tilde{\Omega}_D$$

$$\tilde{\Omega}_D = \frac{1}{4\sqrt{3}} (\sigma^{(1)} - \sigma^{(2)}) \cdot r \times (\vec{\nabla}_\rho).$$

The profile function $V_D^+$ together with $V_{LS}^+$, are displayed in Fig. 6. Drift corrections begin at $\mathcal{O}(q^4)$ and, in principle, should be smaller than the spin-orbit terms, which begin at $\mathcal{O}(q^3)$. However, in this channel, the hierarchy is again not respected.

Fig. 6. Isospin even drift (full and dotted lines) and spin-orbit (dashed line) potentials.
3. THREE-BODY POTENTIAL

The leading term in the three-nucleon potential, known as $TPE-3NP$, has long range and corresponds to the process shown in fig.1c, in which a pion is emitted by one of the nucleons, scattered by a second one, and absorbed by the last nucleon. In this case, the intermediate $\pi N$ amplitude, which is $O(q)$ for free pions, becomes $O(q^2)$ and the three-body force begins at $O(q^3)$. The first modern version of this component of the force was produced by Fujita and Miyazawa, its chiral structure has bee much debated since the seventies and, nowadays, a sort of consensus has been reached about its form. The leading $TPE-3NP$ has a generic structure given by

$$V_{L}(123) = -\frac{\mu}{(4 \pi)^2} \left\{ \delta_{ab} \left[ a \mu - b \mu^3 \nabla_{12} \cdot \nabla_{23} \right] + d \mu^3 i \epsilon_{bac} \tau_c^{(2)} \cdot \nabla_{12} \times \nabla_{23} \right\} \times \left[ (g_A \mu/2 f_\pi) \tau_a^{(1)} \cdot \sigma^{(1)} \cdot \nabla_{12} \right] \left[ (g_A \mu/2 f_\pi) \tau_b^{(3)} \cdot \sigma^{(3)} \cdot \nabla_{23} \right] Y(x_{12}) Y(x_{23}),$$

where $\mu$ is the pion mass and $a$, $b$ and $d$ are strength parameters, determined by either LECs or subthreshold coefficients.

The evaluation of $O(q^4)$ corrections requires the inclusion of single loop effects and is associated with a large number of diagrams, which are being calculated by Epelbaum and collaborators. In order to produce a feeling for the structure of these corrections, we discuss a particular set of processes belonging to the $TPE-3NP$ class, considered recently. Full results involve expressions which are too long and cumbersome to be displayed here. However, their main qualitative features can be summarized in the structure $V(123) = V_L(123) + [V_{\delta L}(123) + \delta V(123)]$, where $V_L$ is the leading term shown above and the factors within square brackets are ChPT corrections. The function $V_{\delta L}$ can be obtained directly from $V_L$, by replacing $(a, b, c) \rightarrow (\delta a, \delta b, \delta c)$, where the $\delta$s indicate changes smaller than 10%. This part of the ChPT correction corresponds just to shifts in the parameters of the leading component. The term $\delta V(123)$, on the other hand, represents effects associated with new mathematical functions involving both non-local operators and complicated propagators containing loop integrals, in place of the Yukawa functions. The strengths of these new functions are determined by a new set of parameters $e_i$, which are also typically about 10% of the leading ones.

In summary, ChPT gives rise both to small changes in already existing coefficients and to the appearance of many new mathematical structures. The latter are the most interesting ones, since they may be instrumental in explaining effects such as the $A_y$ puzzle.

4. THE CHIRAL PICTURE

Chiral symmetry has already been applied to about 20 components of nuclear forces, allowing a comprehensive picture to be assessed. According to ChPT, the various effects begin to appear at different orders and the predicted hierarchy is displayed in the table below.
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beginning

TWO-BODY TWO-BODY THREE-BODY

OPEP TPEP TPEP

\[ O(q^0) \quad V_T^{-}, V_{SS}^{-} \]
\[ O(q^2) \quad V_D^{-} \]
\[ O(q^4) \quad V_{LS}^{-}, V_T^{-}, V_{SS}^{-}, V_C^{-}, V_{LS}^{+} \]
\[ d; a, b \]
\[ V_D^{-}; V_Q^{-}, V_D^{+} \]
\[ e_i \]

In Ref.5, the relative importance of \( O(q^2) \), \( O(q^3) \) and \( O(q^4) \) terms in each component of the TPEP-NNP has been studied. In general, convergence at distances of physical interest is satisfactory, except for \( V_{CS}^{+} \), where the ratio between \( O(q^4) \) and \( O(q^3) \) contributions is larger than 0.5 for distances smaller than 2.5 fm.

As far as the relative sizes of the various dynamical effects are concerned, one finds strong violations of the predicted hierarchy when one compares \( V_{CS}^{+} \) with \( V_{CS}^{-} \) and \( V_{D}^{-} \) with \( V_{LS}^{+} \), as discussed above. It is interesting to note that, in both cases, the unexpected enhancements occur in the isoscalar sector. The numerical explanation for this behavior is that some of the LECs used in the calculation are large and generated dynamically by delta intermediate states. However, it is also possible that perturbation theory may not apply to isoscalar interactions at intermediate distances. This aspect of the problem is explored in the next section.

5. SCALAR FORM FACTOR

The structure of \( V_{CS}^{+} \) was scrutinized in Ref.5 and found to be heavily dominated by a term of the form

\[ V_{CS}^{+}(r) \sim -\left(4/f_\pi^2\right) \left[(c_3 - 2c_1) - c_3 \nabla^2 / 2\right] \hat{\sigma}_{NN}(r), \]

where the \( c_i \) are LECs and \( \hat{\sigma}_{NN} \) is the leading contribution from the pion cloud to the nucleon scalar form factor. This close relationship between \( \hat{\sigma}_{NN} \) and \( V_{CS}^{+} \) indicates that the study of the former can shed light into the properties of the latter.

The nucleon scalar form factor is defined as

\[ \langle N(p') | -L_{sb} | N(p) \rangle = \sigma_N(t) \bar{u}(p') u(p), \]

where \( L_{sb} \) is the symmetry breaking lagrangian. It has already been expanded up to \( O(q^4) \) and receives its leading \( O(q^2) \) contribution from a tree diagram associated with the LEC \( c_1 \). Corrections at \( O(q^3) \) and \( O(q^4) \) are produced by two triangle diagrams, involving nucleon and delta intermediate states. In configuration space, the scalar form factor is denoted by \( \hat{\sigma} \) and one writes

\[ \hat{\sigma}_N(r) = -4c_1 \mu^2 \delta^3(r) + \hat{\sigma}_{NN}(r) + \hat{\sigma}_{N\Delta}(r), \]

where \( \hat{\sigma}_{NN} \) and \( \hat{\sigma}_{N\Delta} \) are the finite-range triangle contributions.

The symmetry breaking lagrangian can be expressed in terms of the chiral angle \( \theta \) as \( L_{sb} = f_\pi^2 \mu^2 (\cos \theta - 1) \). The ratio \( \hat{\sigma}_N(r)/(\mu^2 f_\pi^2) = (1 - \cos \theta) \) describes the density
of the $q\bar{q}$ condensate around the nucleon and is displayed in Fig. 7. One notes that it vanishes at large distances and increases monotonically as one approaches the center. This means that the function $\tilde{\sigma}_N(r)$ becomes meaningless beyond a critical radius $R$, corresponding to $\theta = \pi/2$, since the physical interpretation of the quark condensate requires the condition $q\bar{q} > 0$. In Ref. 15, the condensate was assumed to no longer exist in the region $r < R$ and the $\pi N$ sigma-term was evaluated using the expression

$$\sigma_N = \frac{4}{3}\pi R^3 f_\pi^2 \mu^2 + 4\pi \int_R^\infty dr \ r^2 \tilde{\sigma}_N(r).$$

This procedure yields $43 \text{ MeV} < \sigma_N < 49 \text{ MeV}$, depending on the value adopted for the $\pi N\Delta$ coupling constant, in agreement with the empirical value $45\pm8 \text{ MeV}$. This picture of the nucleon scalar form factor is sound and can be used to gain insight about $V_C^\pm$.

Inspecting Fig. 7 (right), one learns that the hierarchy predicted by ChPT is subverted for distances smaller than 1.5 fm, since the $O(q^3)$ delta becomes more important than the $O(q^2)$ nucleon. On the other hand, the good prediction obtained for the nucleon $\sigma$-term (and also for the $\Delta\sigma$-term\textsuperscript{15}) indicates that the functions $\tilde{\sigma}_{NN}(r)$ and $\tilde{\sigma}_{N\Delta}(r)$ can be trusted up to the critical radius $R \sim 0.6$ fm. Just outside this radius, the chiral angle is close to $\pi/2$, indicating that the pion cloud is non-perturbative in that region. This picture is supported by Fig. 5 (right) since, at least up to 1 fm, the prediction for $V_C^+$ agrees well with the Argonne phenomenological potentials. This leads to our main conclusion, namely that the range of validity of calculations based on nucleon and delta intermediate states is wider than that predicted by ChPT.

**Acknowledgments**

It was a great pleasure participating in the Chiral 2007 meeting and I would like to thank the organizers for the very nice conference, for the warm and friendly hospitality, and for supporting my stay in Osaka.
References

1. T. Becher and H. Leutwyler, Eur. Phys. Journal C 9, 643 (1999); JHEP 106, 17 (2001).
2. N. Fettes and U-G. Meissner, Nucl. Phys. A 693, 693 (2001); ibid. A 676, 311 (2000).
3. N. Kaiser, R. Brockman and W. Weise, Nucl. Phys. A 625, 758 (1997); N. Kaiser, Phys. Rev. C64, 057001 (2001); Phys. Rev. C65, 017001 (2001); E. Epelbaum, W. Glöckle and U-G. Meissner, Nucl. Phys. A 637, 107 (1998); ibid. A 671, 295 (2000); D.R. Entem and R. Machleidt, Phys. Rev. C 66, 014002 (2002).
4. R. Higa and M.R. Robilotta, Phys. Rev. C 68, 024004 (2003).
5. R. Higa, M.R. Robilotta and C. A. da Rocha, Phys. Rev. C69, 034009 (2004).
6. I. Ishikawa and M.R. Robilotta, Phys. Rev. C 76, 014006 (2007).
7. V. Bernard, E. Epelbaum, H. Krebs and Ulf-G. Meissner, preprint nucl-th/0712.1967.
8. G. Höhler, group I, vol.9, subvol.b, part 2 of Landoldt-Bornstein Numerical data and Functional Relationships in Science and Technology, ed. H.Schopper, 1983; G.Höhler, H.P.Jacob and R.Strauss, Nucl.Phys. B 39, 273 (1972).
9. C. A. da Rocha and M. R. Robilotta, Phys. Rev. C 49, 1818 (1994).
10. Wiringa, R.B., Smith, R.A., and Ainsworth, T.L.; Phys. Rev. C 29, 1207 (1984); Wiringa, R.B., Stocks, V.G.J., and Schiavilla, R.; Phys. Rev. C 51, 38 (1995).
11. M.R.Robilotta, Phys. Rev. C 74, 044002 (2006).
12. J.Fujita and H.Miyazawa, Prog.Theor.Phys. 17, 360 (1957).
13. S-N. Yang, Phys. rev. C 10, 2067 (1974); S.A. Coon, M.D. Scadron, P.C. McNamee, B.R. Barrett, D.W.E. Blatt and B.H.J. McKellar, Nucl. Phys. A 317, 242 (1979); S.A. Coon and W. Glöckle, Phys. Rev. C 23, 1790 (1981), H. T. Coelho, T. K. Das, and M. R. Robilotta, Phys. Rev. C 28, 1812 (1983); M.R. Robilotta and H.T. Coelho, Nucl. Phys. A 460, 645 (1986); S.A. Coon and H.K. Han, Few Body Syst. 30, 131 (2001).
14. M.R. Robilotta, proceedings of the Fujita-Miyazawa 3NF Symposium (FM50), Tokyo, October 2007.
15. Cavalcante, I.P., Robilotta, M.R., Sá Borges, J., Santos, D.O., and Zarna, R.G.S., Phys. Rev. C 72, 065207 (2005).