Nuclear stopping and flow in heavy ion collisions and the in-medium NN cross section

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Abstract

We present transport calculations for heavy ion reactions in which the mean field and the in-medium nucleon-nucleon cross section are consistently based on the same effective interaction, i.e. the in-medium T-matrix from microscopic Dirac-Brueckner calculations. Doing so, the stopping in central reactions in terms of the recently proposed \( \vartheta_{\text{tt}} \) observable and the correlation to the behavior of the directed flow is investigated. The relation to the nuclear shear viscosity is discussed.

Key words: Heavy ion collisions at intermediate energies, Dirac-Brueckner-Hartree-Fock, nuclear stopping, collective flow, shear viscosity

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One of the primary goals in studying heavy ion reactions at intermediate energies is the investigation of the nuclear equation-of-state (EOS) at supra-normal densities and/or high temperatures [1]. In a hydrodynamical picture the time evolution of such a reaction can be understood in terms of pressure gradients which build up in the compressed zone and drive the dynamics. Originally it was therefore expected that a direct determination of the nuclear EOS via collective flow observables should be possible [2]. However, the situation turned out to be more complex: The system does not behave like an ideal fluid but binary collisions of the nucleons lead to a viscous behavior. Moreover, over most of the reaction time, in particular during the compressional phase, the system is even out of \textit{local} equilibrium [3,4]. E.g. experimental evidence for incomplete stopping even in central reactions has been reported in [5]. Thus the hydrodynamical limit is not reached in relativistic heavy ion reactions except of the extreme ultra-relativistic regime [6].

On the other hand, microscopic transport models turned out to be an ade-
quate tool for the description of the reactions dynamics at intermediate energies. The physical input of such Boltzmann type semi-classical models are the nuclear mean field $U$ and the nucleon-nucleon (NN) cross section $\sigma$. Both are determined by the effective two-body interaction in the medium, i.e. the inmedium T-matrix; $U \sim \Re T \rho, \quad \sigma \sim \Im T$, respectively $d\sigma/d\Omega \sim |T|^2$. However, in most practical applications phenomenological mean fields and cross sections are used. Adjusting the known bulk properties of nuclear matter around the saturation point one tries to constrain the models for supra-normal densities with the help of heavy ion reactions [7,8]. Medium modifications of the NN cross section are often disregarded which works, from a practical point of view, astonishingly well. However, in particular kinematics regimes a sensitivity of dynamical observables such as in-plane ($v_1$) and elliptic ($v_2$) flow [9] or transverse energy transfer [10] to the NN cross section has been observed.

In this letter we present calculations were both, the mean field and the inmedium cross sections are consistently based on the relativistic Dirac-Brueckner (DB) T-matrix. The standard Dirac-Brueckner ladder approximation [11] accounts for two body-correlations to lowest order in the Brueckner hole-line expansion. Microscopic DB mean fields have already been successfully applied to heavy ion reactions [4] but a consistent treatment of mean field and cross section has not been performed so far. In a consistent truncation scheme two-body correlations are maintained at the same level in the drift term and the collision integral of the kinetic equation [12].

At finite densities the NN cross section is suppressed for two reasons: (1) Kinematics, i.e. the cross section scales with the in-medium nucleon mass as $\sigma_{\text{eff}} \sim (M^*/M)^2 \sigma$ [13,14]. (2) At low energies an additional suppression due to Pauli-blocking of the intermediate states in the Bethe-Salpeter (BS) equation appears. The Pauli-Operator $Q = (1 - f)(1 - f)$ in (1) projects intermediate states onto unoccupied phase space regions. The mean field, respectively the Hartree-Fock self-energy, is obtained by a summation over the Fermi-sea $\Sigma = -itr[TG_F]$. This leads to dressed quasi-particles carrying effective masses and kinetic momenta

$$M^* = M + \Re \Sigma_s(k_F), \quad k^{*\mu} = k^\mu + \Re \Sigma^{\mu}(k_F)$$

given by the real part of the, in general complex, self-energy. For on-shell scattering the NN cross section is given by

$$d\sigma = \frac{(M^*)^4}{s^{*4}\pi^2}|T|^2d\Omega.$$


Fig. 1. Elastic in-medium neutron-proton cross sections at various Fermi momenta \( k_F \) as function of the laboratory energy \( E_{\text{lab}} \). The free cross section (\( k_F = 0 \)) is compared to the experimental total \( np \) cross section [15].

In the present work we use both, mean fields and in-medium cross sections obtained in DB calculations of the Tübingen group [11,14] utilizing the Bonn A one-boson-exchange potential as the bare NN interaction \( V \) which yields very reasonable saturation properties for infinite nuclear matter. Fig. 1 shows the energy dependence of the in-medium neutron-proton (\( np \)) cross section [14] at Fermi momenta \( k_F = 0.0, 1.1, 1.34, 1.7 \text{ fm}^{-1} \), corresponding to \( \rho \sim 0, 0.5, 1, 2 \rho_0 \) (\( \rho_0 = 0.16 \text{ fm}^{-3} \) is the nuclear matter saturation density). The presence of the medium leads to a substantial suppression of the cross section which is most pronounced at low laboratory energy \( E_{\text{lab}} \) and high densities where, in addition to the \( (M^*/M)^2 \) scaling of (3) the Pauli-blocking of intermediate states is most efficient. At larger \( E_{\text{lab}} \) asymptotic values of 15-20 mb are reached. However, not only the total cross section but also the angular distributions are affected by the presence of the medium. The initially highly forward-backward peaked \( np \) cross sections becomes much more isotropic at finite densities [14] which is mainly do to the Pauli suppression of soft modes (\( \pi \)-exchange) and corresponding higher partial waves in the T-matrix.

Heavy ion collisions are described by the relativistic Boltzmann-Uehling-Uhlenbeck (RBUU) transport equation, which has the form [12]

\[
\left[ k^\mu \partial_\mu + (k^\nu F_{\mu\nu} + M^* \partial_\nu M^*) \partial_\nu k^\nu \right] f(x, k^*) = \frac{1}{2(2\pi)^9} \int \frac{d^3 k_2}{E_{k_2}^*} \frac{d^3 k_3}{E_{k_3}^*} \frac{d^3 k_4}{E_{k_4}^*} W(kk_2|k_3k_4) \left[ f_3 f_4 \tilde{f}_2 \tilde{f}_4 - f_2 f_3 \tilde{f}_4 \right] \tag{4}
\]

where the single particle distribution function is given by \( f_i = f(x, t; k_i^*) \) and
the hole-distribution by \( \tilde{f}_i = (1 - f(x, t; k_i^*)) \). The collision integral accounts explicitly for final state Pauli-blocking while the in-medium scattering amplitude includes the Pauli-blocking of intermediate states.

The dynamics of the lhs of eq. (4), the drift term, is determined by the mean field. Here the attractive scalar field \( \Sigma_S \) enters via the effective mass \( M^* \) and the repulsive vector field \( \Sigma_\mu \) via kinetic momenta \( k_\mu^* \) and via the field tensor \( F^{\mu\nu} = \partial^\mu \Sigma^\nu - \partial^\nu \Sigma^\mu \). The in-medium cross sections enter into the collision integral via the transition amplitude

\[
W = (2\pi)^4 \delta^4 (k + k_2 - k_3 - k_4) (M^*)^4 |T|^2
\]

with relation (3).

It was found that in relativistic heavy ion collisions the phase space distribution is not equilibrated during much of the reaction [4]. This modifies the in-medium T-matrix and thus the mean field and the cross section via the intermediate state Pauli operator. To account for this approximation, the mean field is described in the Local (phase space) Configuration Approximation (LCA) where the anisotropic phase space is taken into account locally in a 2-Fermi-sphere configuration [4]. In a fully consistent treatment one should treat the in-medium NN scattering in the same way, i.e. for a scattering of a nucleon to a 2-Fermi sphere configuration which is not available yet. To make the discussion more transparent here, we will discuss results of transport calculations where the mean field is always treated in the LCA approximation.

The connection of the microscopic BUU approach to a hydrodynamical picture is established via macroscopic transport coefficients such as the shear viscosity \( \eta \) and the heat capacity [16]. Though defined close to the equilibrium limit, transport coefficients give insight in the bulk properties of excited nuclear matter. In particular the shear viscosity is closely connected with the in-medium cross section. Quantum effects enter via the final state Pauli blocking for final states and for intermediate states in the BS-equation (1). The cross section is not only reduced at finite density but also becomes more isotropic which increases the average momentum transfer and reduces the viscosity of the system.

It is of great interest where such effects are experimentally accessible in heavy ion reactions. While the close connection of collective flow observables to the mean field is well established [1,7,8,4], the experimental access to in-medium modifications of the cross section is not so straightforward. Stopping observables are in principle natural tools to study the effectiveness of binary collisions. However, the comparison to experimental rapidity distributions up to now did not deliver conclusive information on in-medium modifications of the cross section.
Very recently, the FOPI collaboration at GSI performed a systematic analysis of nuclear stopping in the most central reactions over a wide energy range from $0.1 \div 1.5$ AGeV [17]. The complete phase space coverage allowed not only the measurement of longitudinal rapidity distributions but also those transverse to the beam direction. The data showed that even in most central reactions the system is not completely equilibrated which is reflected in a broader longitudinal distribution. A quantitative measure for the equilibration was introduced by Reisdorf et al. [17] by the ratio

$$\text{var}_{tl} = \frac{\sigma^2(y)}{\sigma^2(z)}$$

of the longitudinal ($z$) and transverse ($y$) variances of the rapidity distributions. Hence, a value of $\text{var}_{tl} \leq 1$ indicates incomplete stopping. It was further observed that stopping (in central reactions) and transverse flow (in semi-peripheral reactions) show a strong correlation in that the excitation function for both observables has a broad maximum around 0.4-0.6 AGeV [17].

Fig. 2 shows the incident energy dependence of the observable $\text{var}_{tl}$ and the scaled directed flow $P^{(0)}_{xdir} = (\sum_i \text{sign}(y_i)p^i_x)/P_{proj}$, where the sum runs over all charged particles. A comparison to experimental data [17] requires an accurate determination of the centrality class in the same way as in the experiment. In particular, this is important for the observable $\text{var}_{tl}$ due to its high sensitivity to the centrality selection. We have performed a systematic study on the impact parameter dependence of the flow and stopping observables. $P^{(0)}_{xdir}$ was theoretically determined both for the centrality interval PM4 (top-right panel in Fig. 2), corresponding to $b=2-6$ fm and for the fixed impact parameter $b=4$ fm (top-left panel in Fig. 2), corresponding to the maximum magnitude of directed flow. The last condition was used in determining $P^{(0)}_{xdir}$ experimentally [17]. A consistent determination of the stopping observable $\text{var}_{tl}$ and the highest centrality class is more difficult due to its high sensitivity on the centrality selection. In Ref. [17] the observable ERAT was used for the experimental determination of central collisions. Comparisons with IQMD simulations [18] provide an impact parameter range $b \sim 2 - 2.5$ fm, consistent with previous studies [4]. However, because of numerical fluctuations of the ERAT distribution a determination of the impact parameter to better than $\delta b = 0.25 - 0.5$ fm is not possible, which essentially affects the final result of $\text{var}_{tl}$. The fluctuations of $\text{var}_{tl}$ according to these $\delta b$ are indicated in Fig. 2 (bottom) as error bars. Furthermore, as for the experimental data [17], $\text{var}_{tl}$ is determined for central reaction ($b \leq 2.5$ fm) by Gaussian fits to the final rapidity distributions. The theoretical calculations are performed for two cases, i.e. using free and in-medium total and differential NN cross sections (solid and dashed lines in Fig. 2, respectively).

Generally the microscopic calculations reproduce the qualitative behavior of
the data. The stopping power as well as the scaled directed flow have a broad maximum as a function of beam energy. The use of in-medium cross sections has only little impact on the collective flow as expected. The flow is slightly reduced but at high energies the data are still somewhat overestimated. This is understood from the momentum dependence of the nucleon-nucleus optical potential. Within the microscopic Dirac-Brueckner approach [11] the optical potential is too repulsive at energies above 0.6 AGeV and does not reproduce the empirically observed saturation of the optical potential [4]. In heavy ion collisions this leads to an overestimation of the collective flow, which has been cured adopting ad-hoc phenomenological parameterizations of the optical potential [8]. The main reason for the decrease of the flow at high energies is the opening of inelastic channels (pion, kaon production etc.). Such inelasticities are included in the collision part of the transport calculation but not on the level of the mean field. The theoretical understanding of the bare NN interaction above the inelastic threshold is still an open issue [19].
The energy dependence of $\var$, can be understood more easily in a qualitative way: at low energies Pauli blocking is most efficient, suppressing 2-body collisions which leads to a higher transparency. With increasing energy the blocking becomes ineffective whereas the elementary cross section drops. Hence the maximum reflects the combination of final state Pauli blocking and a decreasing cross section. For a quantitative description of the data the additional reduction of the in-medium cross section by Pauli blocking of intermediate states and $M^*$ scaling is seen to be essential.

The interplay between Pauli blocking and the NN cross section can be seen from Fig. 3. This figure shows the isospin averaged free cross section multiplied by average Pauli blocking factors for the initial local momentum configurations in the central cell of a heavy ion reaction. These are given by two nuclear matter currents of density $\rho_0$ separated by the beam velocity which in momentum space correspond to Lorentz boosted Fermi ellipsoids. The blocking factors are evaluated for two representative nucleons in the centers of the ellipsoids. One clearly sees how the interplay between Pauli blocking and decreasing cross section leads to a broad maximum in the effective cross sections at about 0.5 GeV which appears in the same region as that of $\var$.

It is also important to obtain an understanding of the stopping behavior in terms of nuclear matter bulk properties. However, ideal one-fluid hydrodynamics which is most directly related to nuclear matter bulk properties completely fails at SIS energies [17]. In contrast, quantum effects such as Pauli blocking of two-body collisions are essential for the degree of stopping. In Ref. [16] nuclear transport coefficients have been derived from the microscopic BUU equation. In the Boltzmann statistical limit these correspond to the first order Chapman-Enskog coefficients. At large density the microscopic transport coefficients differ, however, essentially from the Chapman-Enskog results [16]. According to [16] the shear viscosity of a nuclear liquid can be parameterized
as a function of $\rho$ and $T$

$$\eta \sim \frac{1700}{T^2} \left( \frac{\rho}{\rho_0} \right)^2 \left[ 1 + \frac{22}{1 + T \cdot 10^{-3}} \left( \frac{\rho}{\rho_0} \right)^{0.7} + \frac{5.8\sqrt{T}}{1 + 160T^{-2}} \right]$$  \hspace{1cm} (7)

Fig. 4 shows $\eta$ according to eq. (7) as a function of $T$ for various $\rho$ together with the values which have been extracted from central Au+Au reactions. The $T - \rho$ for which $\eta$ is evaluated have been determined in the central cell at the stage of maximal compression. The temperature was thus extracted by fits of Fermi distribution to the local phase space as delivered from the transport calculations [3]. From Fig. 4 one sees that $\eta$ exhibits a minimum as function of beam energy at $E_{\text{Lab}} = 0.25$ AGeV which is close to the maximum of $\text{var}_{\text{t1}}$.

In summary, we performed for the first time RBUU calculations where the mean field and the in-medium cross section are consistently based on the same in-medium Dirac-Brueckner T-matrix. We investigated the recently proposed observable $\text{var}_{\text{t1}}$ as a measure for the degree of stopping and equilibration in central reactions. In-medium effects on the NN cross section are essential in order to reproduce these data. Qualitatively, the behavior of the $\text{var}_{\text{t1}}$ excitation function can be understood by the interplay between the decreasing NN cross section and final state Pauli blocking. The interpretation of nuclear bulk properties in terms of transport coefficients indicates that the shear viscosity is minimal in the regime where maximal stopping occurs.
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