Performance considerations of ultrasonic distance measurement with well defined properties

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Abstract. Conventional ultrasonic distance measurement systems based on narrow bandwidth ultrasonic bursts and amplitude detection are often used because of their low costs and easy implementation. However, the achievable results strongly depend on the actual environments where the system is implemented: in case of well defined objects that are always located near the measurement direction of the system, in general good results are obtained. If arbitrary objects are expected that are moreover located in arbitrary positions in front of the sensor, strongly object dependent areas where objects are detected with decreasing accuracy towards their borders must be taken into account. In previous works we developed an ultrasonic measurement system that provides accurate distance measurement values within a well defined detection area that is independent of the reflection properties of the objects. This measurement system is based on the One Bit Correlation method that is described in the following. To minimise its implementation efforts, it is necessary to examine the influence of the system parameters as e.g. the correlation length to the results that are expected in case of different signal to noise ratios of the received signal. In the following, these examinations are shown and the obtained results are discussed that allow getting a well conditioned system that makes best use of given system resources.

1. Introduction
The use of ultrasonic signals is a well known principle for distance measurement, object detection and scene recognition, which is popular due to its low costs and robustness. In contrast to optical sensors that require clean and well illuminated environments, ultrasonic sensors are mainly insensitive to dust, atomized spray and other contaminations. However, conventional ultrasonic sensors based on time of flight evaluation of narrowband bursts suffer from several problems. Due to the amplitude detection algorithm, achievable results are connected with received signal amplitudes, which depend on various conditions: as the emitted signal amplitude depends on the angle of radiation, the signal amplitude at the reflecting object depends on its position towards the ultrasonic transmitter. Secondly, the reflected signal amplitude strongly depends on the shape of the reflecting object. This leads to varying detection areas: weakly reflecting objects located near the rotational axis of the transmitter cannot be distinguished from well reflecting objects located at greater angles. In other words, well reflecting objects are detected in a much wider area than weakly reflecting ones.

In previous works [1] we developed an ultrasonic distance measurement method which provides an accurate time-of-flight measurement within a well defined and object independent detection area. This amplitude independent characteristic is achieved by the One Bit Correlation method that only
evaluates the sign but not the amplitude of the incoming signal. In the following, this signal processing algorithm is described and analyzed.

2. One Bit Correlation method
At first, the One Bit Correlation algorithm digitizes the sampled incoming signal (compare Figure 1). The received bit stream is stored in a shift register that is shifted at each sampling time. The content of the shift register is compared bitwise with a previously stored reference pattern. The number of coincident bits represents the correlation result $k$ at this point of time. The maximum achievable result is equal the length of the shift register $N$ (in case of full correlation). In case of $k = 0$ full anti correlation is found (full correlation with inverse signal). In case of arbitrary signals a correlation result around $k \approx N/2$ is expected. It is convenient to normalize the correlation result $k$ in

$$\kappa = \frac{2k - N}{N} = \left[-1\ldots + 1\right].$$

Therefore, arbitrary signals lead to a relative correlation result of $\kappa = 0$, while full correlation leads to $\kappa = 1 (=100\%)$.

The advantages of the One Bit Correlation with respect to the conventional analog pulse compression are the following:

- The use of simple comparator instead of multi bit analog to digital converters (ADCs) significantly simplifies the receiver circuit especially in cases where wide amplitude ranges are expected as it is common in case of ultrasonic applications.
- As the method evaluates only the sign but not the amplitude of a signal, varying signal amplitudes lead to constant correlation results independent of the signal amplitudes as only the signal phase is evaluated.
- The One Bit Correlation Method is well implementable in field programmable gate arrays (FPGA), which allows real time evaluation of the algorithm.

3. Robustness
In the following chapter, the robustness of the one bit correlation method is evaluated with statistical means. At first, the effect of the correlation length $N$ is discussed and secondly the influence of white Gaussian noise added to the received signal is evaluated. Finally, the combination of both aspects allows an estimation of the required correlation length depending of the assumed signal to noise ratio of the received signal.

3.1. Correlation length
If no signal is received, the probability of each bit to be set is assumed to be $p = 0.5$. Therefore, the probability to get a full correlation result of $\kappa = 100\%$ is $p_{100\%} = p^N$. The general case of $k$ coincident bits is calculated using the binomial coefficient $c_k$, which allows finding the probability $p_k$ to reach a specific amount of matching bits by
\[ p_k = c_k \cdot p_k^k \cdot q^{N-k} = \frac{N!}{k!(N-k)!} \cdot \frac{N!}{k!(N-k)!} \cdot \left( \frac{1}{2} \right)^N. \] (2)

For large numbers of \( N \) the binomial distribution can be approximated by the normal distribution. A detailed evaluation is found in [2] and the final result is shown in the following: assuming a given probability \( P_{\kappa_{\text{min}}} \) to reach at least a correlation result of \( \kappa_{\text{min}} \), this value \( \kappa_{\text{min}} \) is calculated by the following formula, using the error integral \( Q(x) \) defined in (5):

\[ \kappa_{\text{min}} = \frac{Q^{-1}(P_{\kappa_{\text{min}}})}{\sqrt{N}} + \frac{1}{N}. \] (3)

In Figure 2, some examples of evaluated results are shown: the minimum expected relative correlation result \( \kappa_{\text{min}} \) which is exceeded with a probability of at least \( P_{\kappa_{\text{min}}} \) is plotted over the correlation length \( N \). As expected, with increasing correlation length noise suppression increases leading to lower correlation maxima occurring with the same probability.

![Figure 2. Expected minimum relative correlation result \( \kappa_{\text{min}} \) depending on probability of occurrence \( P_{\kappa_{\text{min}}} \) over the correlation length \( N \).](image)

3.2. Influence of added white Gaussian noise

In case of undisturbed received signals, correlation maxima in the range of \( \kappa = 100\% \) are expected. However, due to several reasons (e.g. electrical noise inside the receiver and surrounding acoustic noise) disturbances occur at the received signal. In the following, the influence of white Gaussian noise, characterized by the signal to noise ratio, is examined.

In case of the One Bit Correlation, additional noise does not effect the result as long as it does not cause a sign change of the received signal. In general, the probability that white Gaussian noise exceeds an amplitude \( A_{\text{defined}} \) can be calculated by the error integral \( Q(x) \) as shown in equation (5).

\[ P_{\text{error}} = Q(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{x^2}{2}} \, dx \quad \text{with} \quad x = \frac{A_{\text{defined}}}{A_{\text{noise}}}. \] (5)

As the actual signal amplitude \( A(t) \) changes during time, also the error probability varies and, therefore, the final result depends on the amplitude distribution of the used signal. In case of a chirp as used in previous works, the amplitude distribution equals the distribution function obtained by a sinusoidal signal. In [2] a detailed evaluation of the mean error probability \( P_{\text{error,mean}} \) considering the
effective error probability of each sample and the amplitude distribution function of the used signal, is shown. The final result is

\[
P_{error,mean}(SNR_{sig}) = \frac{1}{\pi} \frac{2}{\sqrt{1 - A^2}} Q\left(10^{-20} \sqrt{20} A\right) dA
\]

and the expected correlation maximum is given by \(\kappa_{max} = 1 - P_{error,mean}(SNR_{sig})\). To verify this theoretical result, it is compared with simulation results: therefore, 50 measurements are taken at various values of signal to noise ratio values in the range of \(-10dB \leq SNR \leq 30dB\) and the evaluated values of correlation maxima are compared with the theoretical results of equation (6). As shown in Figure 3, good coincidence between simulation and theoretical evaluation is obtained.

4. Conclusion
To prevent the ultrasonic measurement system from producing erroneous results, on the one hand the probability of exceeding a given threshold of the correlation result should be as low as possible (see section “Correlation length”). On the other hand, measurement signals must be detected even if they are disturbed until a certain extent (see section “Influence of added white Gaussian noise”). In Figure 4 the results of these two considerations are plotted.

The solid line shows the decrease of the correlation maxima due to additional white Gaussian noise and the dashed lines show the borders of correlation results that are exceeded with a given probability for different correlation lengths \(N\). This figure also allows an estimation of the necessary correlation length for a given signal to noise ratio: the assumed signal to noise ratio leads to a maximum threshold value that is required to detection signals disturbed by this SNR (e.g. a SNR of 0dB requires a threshold \(\kappa_{thres} < 60\%\) to be able to detect these disturbed signals). If a maximum error rate probability of \(10^{-9}\) is tolerated, the correlation length \(N\) must be set to at least \(N_{min} \geq 100\). This allows getting a well conditioned measurement system that makes best use of expensive system resources.

References
[1] Elmer H and Schweinzer H 2004 Ultrasonic Distance Measurement system with a Well Defined and Adjustable Detection Area: Proc. IEEE Sensors (Vienna, AT, 24-27 Oct 2004) 437-440
[2] Elmer H 2005 Improved Ultrasonic Distance Measurement in Air: Accepted PhD thesis at Vienna University of Technology