A Study of Power Suppressed Contributions in $J/\psi \to p\bar{p}$ Decay

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Abstract

The power suppressed amplitude which describes the Pauli ($\sigma^{\mu\nu}$) coupling in the $J/\psi \to p\bar{p}$ decay is calculated within the effective field theory framework. It is shown that at the leading-order approximation this contribution is factorisable and the overlap with the hadronic final state can be described by collinear matrix elements. The obtained contribution depends on the nucleon light-cone distribution amplitudes of twist-3 and twist-4. This result is used for a qualitative phenomenological analysis of existing data for $J/\psi \to p\bar{p}$ decay: branching ratio and the angular distribution in the cross section $e^+e^- \to J/\psi \to p\bar{p}$. It is found that the power corrections provide a large numerical effect.
1 Introduction

Hadronic exclusive decays of heavy charmonia remain one of the most challenging subject for theoretical investigations, see for instance reviews in Refs. [1,2]. The effective field theory framework provides the most effective approach for calculations of the corresponding decay amplitudes. Such a framework involves the nonrelativistic expansion with respect to a small heavy quark velocity $v$ [3,4] and the expansion with respect to a small ratio $\lambda^2 \sim \Lambda/m_Q$ where $\Lambda$ is the typical hadronic scale. A decay amplitude can be represented as a superposition of the hard and soft contributions associated with the hard and soft scales, respectively. The nonperturbative long distance dynamics is described by the well defined, process independent matrix elements. The nice feature of the effective field theory framework is the systematic power of the various counting contributions with respect to small parameters $v$ and $\lambda$. This allows one to perform a systematic calculations and better understand complicated underlying partonic dynamics.

The power counting is closely associated with the hard partonic subprocess and helicities of the initial and final hadrons. This leads to the well known helicity selection rule, see e.g. Refs. [5,6]. A typical feature of hard processes is that their amplitudes are dominated by contributions with the helicity conserving partonic amplitudes. The partonic configurations, which require a helicity flip or involve the angular momentum, are suppressed by the powers of the small $\lambda$. Nevertheless, there are many indications that corresponding subleading amplitudes might be relevant for an understanding of many exclusive decays. In particular such power suppressed corrections are required for a description of charmonium decays into baryon-antibaryon pair [1].

The first calculation of such charmonia decay was done in Ref. [5]. Later the various decays of $S$- and $P$-wave charmonia were studied in many publications, see Refs. [1,2,6] and references therein. In all of these calculations the subleading amplitudes as a rule have been neglected. However, in some cases such a naive estimate does not agree with experimental data. For instance, the ratio $\Gamma[\chi_{c0} \rightarrow pp]/\Gamma[\chi_{c1} \rightarrow pp] \simeq 36$ [7] is very large despite the $\chi_{c0}$ amplitude is suppressed by small $\lambda^2$. An understanding of such effects definitely requires a careful study of subleading power corrections.

In this paper we consider the effect of power suppressed corrections in the description of $J/\psi \rightarrow pp$ decay. Such a reaction is simpler than the mentioned $P$-wave decays because $1S$-charmonium state is a ground state. As in the case of the electromagnetic source the corresponding decay amplitude is described by the Dirac ($\gamma^\mu$) and by the Pauli ($\sigma^\mu\nu$) vertices and the second one is suppressed according to pQCD helicity selection rule. Therefore in many theoretical considerations this amplitude is often discarded.

On the other hand, this amplitude can provide a substantial effect in description of the polar angular distribution of baryon-antibaryon pairs produced in the exclusive decay of the $J/\psi$. The corresponding angular distribution can be written as a function of the angle $\theta$ between the nucleon or antinucleon direction and the beam as follows:

$$\frac{dN}{d\cos \theta} = N(1 + \alpha \cos \theta),$$

(1)

where $N$ is an overall normalisation. The coefficient $\alpha = 1$ in the limit of infinite mass of heavy quark [5]. The simple kinematical effect from the nucleon mass $m_N/M_\psi \neq 0$ yields [8]

$$\alpha = \frac{1 - 4m_N^2/M_\psi^2}{1 + 4m_N^2/M_\psi^2} \simeq 0.455.$$  

(2)

The angular distribution was measured in many experiments [9–16]. The most accurate measurements [16] give the value $\alpha \simeq 0.59 \pm 0.01$. The difference with the simple prediction in
Eq. (2) can only be explained by the amplitude associated with the Pauli coupling ($\sigma^{\mu\nu}$). The more sophisticated phenomenological models with massive constituent quarks were considered in Refs. [17,19]. Despite various assumptions about the QCD underlying dynamics, these calculations give a reliable description of the angular distribution. A calculation of the angular coefficient $\alpha$ within the systematic framework does not depend on a model of hadron dynamics and might help better understand the role of the power suppressed contributions.

In the present work the subleading amplitude is computed within the effective field theory framework. Such calculation involves the twist-4 light-cone distributions amplitudes of the nucleon which can be associated with the three-quark component of the nucleon wave function. The required nucleon matrix elements have been already studied using QCD sum rules in Refs. [20–22] and in the lattice calculations [23, 24]. The interesting observation which can be done from these results is that the twist-4 matrix elements associated with the three quark in $P$-wave configuration are quite large comparing to the leading twist-3 matrix element. This can lead to a large power corrections because charmonium mass is not large enough.

The paper is organised as follows. In Sec.2 we introduce notations, kinematics and provide the known leading-twist results for the decay amplitude. In Sec.3 we briefly describe the calculation of the subleading amplitude $A_2$ and provide the corresponding analytical results. We show that for the $S$-wave charmonia such amplitude is also factorisable and is described by the well defined convolution integral of the hard partonic amplitude with the light-cone distribution amplitudes (LCDAs) of twist-3 and twist-4. The obtained result is used with various models of the LCDAs for the qualitative numerical estimates in Sec.4. In Sec.5 we discuss the obtained results. In Appendices A and B we provide the information about the nonperturbative matrix elements and discuss useful technical details.

2 Definitions, kinematics and the leading-twist amplitude

It is convenient to describe the decay $J/\psi(P) \to p(k)\bar{p}(k')$ in the charmonia rest frame

$$P = M_\psi \omega, \quad \omega = (1, \bar{0}).$$

The outgoing momenta $k$ and $k'$ are directed along the $z$-axis and read

$$k = (M_\psi/2, 0, 0, M_\psi \beta/2), \quad k' = (M_\psi/2, 0, 0, -M_\psi \beta/2), \quad \beta = \sqrt{1 - 4m_N^2/M_\psi^2},$$

where $m_N$ is the nucleon mass. We use the auxiliary light-cone vectors

$$n = (1, 0, 0, -1), \quad \bar{n} = (1, 0, 0, 1).$$

Any four-vector $V$ can be expanded as

$$V = V_+ \frac{n}{2} + V_- \frac{\bar{n}}{2} + V_\perp,$$

where $V_+ = (Vn) = V_0 + V_3$, $V_- = (V\bar{n}) = V_0 - V_3$. The light cone expansions of particle momenta are given by

$$P = M_\psi \frac{1}{2}(n + \bar{n}), \quad k \simeq M_\psi \frac{\bar{n}}{2}, \quad k' \simeq M_\psi \frac{n}{2}.$$  

The decay amplitude $J/\psi \to p\bar{p}$ is defined as

$$\langle k, k' | i\hat{T} \rangle | P \rangle = (2\pi)^4 \delta(P - k - k') iM,$$
Therefore this amplitude is usually neglected. The leading-order in \( \alpha \) as the small ratio \( \lambda \) amplitudes can be computed expanding over the small relative heavy quark velocity \( v \) where the mass of the heavy quark is much large than the typical hadronic scale \( m \). The collinear convolution integral is given by

\[
\sum_{\lambda} \epsilon_{\psi}^{\mu}(P, \lambda) \epsilon_{\psi}^{\nu}(P, \lambda) = -g^{\mu\nu} + \frac{P^\mu P^\nu}{M_\psi^2}.
\]

The scalar amplitudes \( A_1 \) and \( A_2 \) describe the decay process. Within the effective field theory, where the mass of the heavy quark is much large then the typical hadronic scale \( m_Q \gg \Lambda \), these amplitudes can be computed expanding over the small relative heavy quark velocity \( v \) and over the small ratio \( \lambda^2 \sim \Lambda/m_Q \). The power counting predicts that the amplitude \( A_2 \) is suppressed as

\[
A_2/A_1 \sim \lambda^2.
\]

Therefore this amplitude is usually neglected. The leading-order in \( \alpha_s \) expression for the amplitude \( A_1 \) was calculated a long time ago in Refs. [5,25,26] and can be written as [26]

\[
A_1^{(0)} = \frac{f_\psi}{m_Q^2} \frac{f_N^2}{m_Q^4} (\pi \alpha_s)^3 \frac{10}{81} I_0,
\]

where the collinear convolution integral is given by

\[
I_0 = \frac{1}{4} \int D\gamma_i \frac{1}{y_1 y_2 y_3} \int Dx_i x_1 x_2 x_3 \left\{ \frac{y_1 x_3}{D_1 D_3} \varphi_3(y_i) \varphi_3(x_i) + \frac{2y_1 x_2}{D_1 D_2} T_1(y_i) T_1(x_i) \right\}.
\]

with \( D_i = x_i + y_i - 2x_i y_i \) and \( \varphi_3(x_i) \equiv \varphi_3(x_1, x_2, x_3) \). The couplings \( f_\psi \) and \( f_N \) in Eq.\((12)\) are related with the long distance matrix elements of charmonia and nucleon, respectively. The explicit definitions are given in Appendix A. The nucleon light-cone distribution amplitudes \( \varphi_3(y_i) \) and \( T_1(x_i) \) are related with the three quark component of the nucleon wave function and describe the distribution of the quark momenta in the nucleon wave function at zero transverse separation. They depend on the quark light-cone fractions \( 0 < x_i < 1 \) which satisfy momentum conservation condition \( x_1 + x_2 + x_3 = 1 \). Therefore the convolution integrals in Eq.\((12)\) have a \( \delta \)-function in the measure

\[
Dx_i = dx_1 dx_2 dx_3 \delta(1 - x_1 - x_2 - x_3).
\]

The LCDA \( T_1 \) in Eq.\((13)\) is not independent and related with \( \varphi_3 \) by Eq.\((A.13)\). Hence the amplitude \( A_1 \) depends on the one twist-3 LCDA \( \varphi_3 \). The different models for this nonperturbative function will be discussed later.

From Eq.\((12)\) it follows that \( A_1 \sim v^3 \lambda^3 \alpha_s^3 \) which is obtained from the fact that \( f_N \sim \Lambda^2 \) and in NRQCD \( f_\psi \sim m_Q^2 v^3 \). The scaling behaviour of order \( v^3 \) is the minimal possible power behaviour according to NRQCD power scaling. The helicity flip amplitude \( A_2 \) is suppressed by additional power \( \lambda^2 \) due to the twist-4 nucleon LCDA which can be associated with the three quarks in \( P \)-wave state or with the 3-quark-gluon component of the wave function. Only such configurations can provide a description of the long distance collinear overlap with the nucleon state in this case. This is a direct consequence of the helicity conservation in the hard subprocess. In many hard processes such subleading amplitudes often do not possess collinear factorisation because of overlap between collinear and soft regions. Formally such an overlap
leads to the endpoint singularities in the collinear integrals. The well known example is the nucleon electromagnetic form factor $F_2$\cite{27}. However the short distance annihilation of $Q\bar{Q}$ pair into three gluons in of $J/\psi$ decay has different properties. None of the three virtual gluons can have ultrasoft momentum because the coupling of the ultrasoft gluon with the heavy quark is suppressed at least by one additional power of the small velocity $v$. Therefore the annihilation to hard and ultrasoft gluons are suppressed. This observation allows one to conclude that the helicity suppressed amplitude $A_2$ can also be factorised in the hard and soft contributions like the amplitude $A_1$. Therefore the amplitude $A_2$ can also be calculated within the standard collinear factorisation framework.

3 Calculation of the amplitude $A_2$

The hard coefficient function is given by the hard $Q\bar{Q}$ annihilation into three gluons which further creates light quark-antiquark pairs forming the final $p\bar{p}$ state, the typical diagram is shown in Fig.1. The nucleon matrix elements are described by the operators of twist-3 and twist-4. In present calculation we take into account only the three-quark operators of twist-4 and neglect the quark-gluon operators. Such approximation is based on the assumption that the quark-gluon matrix elements are relatively small, see e.g. Ref.\cite{20}. The properties of the twist-4 LCDAs were studied in Refs.\cite{20,22,25,29}. For the convenience of the reader we provide a brief description of these matrix elements and corresponding LCDAs in Appendix A.

In general, the calculation of amplitude $A_2$ is quite standard. We compute the diagrams in the momentum space writing the appropriate projections for the collinear matrix elements. For that purpose we need the projection for the twist-4 matrix element. As a rule, the projections of higher twist matrix elements include the derivatives with respect to quark transverse momenta. Corresponding formulae can be derived easily from the parametrisation of such a matrix element for the corresponding off light-cone correlator. The technical problem is that for a nucleon matrix element one has to consider many various Dirac structures which makes the calculation quite complicated. In this work we define the twist-4 projection related with the three-quark operators which are built from the large components of the collinear quark fields and transverse derivatives. Such matrix elements allow one to simplify the calculations and the final expression for the amplitude. Below we briefly discuss the main steps of our calculation. In order to make simpler the connection with the nucleon matrix elements in Appendix A we compute the amplitude for the time reversal process $p + \bar{p} \rightarrow J/\psi$ with $(k + k')^2 = M_{\psi}^2$. 

Figure 1: One of the six hard diagrams which contribute to the hard coefficient function.
The contribution of diagrams in Fig. can be written as

$$\int d^4z_1 dz_2 dz_3 \int d^4z_1 dz_2 dz_3 \; D(z_1', z_2) \; \langle 0| Q_\alpha(z_1') Q_\beta(z_2') |0 \rangle \times \langle 0| u_1(z_1) u_2(z_2) d_3(z_3) \; \bar{u}_{1'}(z_1) \bar{u}_{2'}(z_2) \bar{d}_{3'}(z_3) | k, k' \rangle.$$  \hspace{1cm} (15)

Here, the function $D_{\beta\alpha}(z_1', z_2)$ represents the sum of the hard diagrams in position space. This function depends from spinor and colour indices, which are not shown for simplicity. We also imply the factorisation of heavy quark sector and therefore we write in Eq. the product of the charmonia and proton-antiproton matrix elements. The light quark operator is constructed from the $u$- and $d$- quark fields and we simplify the writing of the spinor indices

$$\bar{u}_{\sigma_i} \equiv u_i, \quad \bar{u}_{\sigma_i'} \equiv \bar{u}_{i'}.$$  \hspace{1cm} (16)

The colour indices are also not shown for simplicity.

The calculation of the heavy quark matrix element in NRQCD is a well known technique, therefore, we skip the details and only write the final expression

$$\langle 0| Q_\alpha(z_1') Q_\beta(z_2') |0 \rangle \sim e^{im0} \; \sigma \; \eta^\prime \; f_\psi,$$  \hspace{1cm} (17)

where the constant $f_\psi$ is defined in Eq. (A.1).

Consider the proton-antiproton matrix element

$$M_h = \langle 0| u_1(z_1) u_2(z_2) d_3(z_3) \; \bar{u}_{1'}(z_1) \bar{u}_{2'}(z_2) \bar{d}_{3'}(z_3) | k, k' \rangle.$$  \hspace{1cm} (18)

We rewrite the given quark operator as the product of the collinear operators of twist-3 and twist-4. For that purpose, we expand the quark fields into large and small collinear components

$$\psi(x) = \frac{\bar{p}_i \bar{p}_j}{4} \psi(x) + \frac{\bar{p}_i \bar{p}_j}{4} \psi(x) = \xi_h(x) + \eta_h(x), \quad \bar{\psi}(x) = \bar{\psi}(x) \frac{\bar{p}_i \bar{p}_j}{4} + \bar{\psi}(x) \frac{\bar{p}_i \bar{p}_j}{4} = \bar{\xi}_n(x) + \bar{\eta}_n(x).$$  \hspace{1cm} (19)

The effective field theory counting rules imply

$$\bar{\xi}_n(x_+) \sim \xi_h(x_-) \sim \lambda^2, \quad \partial_\perp \xi_h(x_-) \sim \partial_\perp \bar{\xi}_n(x_-) \sim \lambda^4, \quad \eta_h(x) \sim \bar{\eta}_n(x) \sim \lambda^4.$$  \hspace{1cm} (20)

At the next step one has to perform the multipole expansion of the fields. To our accuracy we need

$$\psi(x) \simeq \xi_h(x_-) + (x_+ \partial_\perp) \xi_h(x_-) + \eta_h(x_-), \quad \bar{\psi}(x) \simeq \bar{\xi}_n(x_+) + (x_+ \partial_\perp) \bar{\xi}_n(x_+) + \bar{\eta}_n(x_+),$$  \hspace{1cm} (21)

where we introduced short notations for the light-cone arguments of the fields

$$x_- \equiv (x \bar{n}) \frac{n}{2}, \quad x_+ \equiv (x n) \frac{n}{2}.$$  \hspace{1cm} (22)

We assume that the collinear and hard fields can be completely decoupled (factorised) in the effective Lagrangian, which can be written as the sum of three contributions corresponding to the hard, $n$- and $\bar{n}$-collinear sectors. Such factorisation implies that the leading-order collinear gluon fields

$$\bar{n} \cdot A^{(n)}(x_+) \sim n \cdot A^{(n)}(x_-) \sim O(\lambda^0),$$  \hspace{1cm} (23)

are factorised into the collinear Wilson lines, which ensure the colour gauge invariance. The detailed discussion of this factorisation is quite complicated and requires a separate consideration. Therefore, we accept this fact as a plausible assumption. The factorisation of the hard
and collinear sectors implies that the operator in (18) will be modified by the redefinition of the collinear fields

$$\xi_n(x_-) \rightarrow W^\dagger_n(x_-)\xi_n(x_-), \quad \xi_n(x_+) \rightarrow \xi_n(x_+) W_n(x_+),$$

$$\eta_n(x_-) \rightarrow W^\dagger_n(x_-)\eta_n(x_-), \quad \eta_n(x_+) \rightarrow \eta_n(x_+) W_n(x_+),$$

where the collinear Wilson lines $W_{n,\bar{n}}$ are defined as

$$W_n(x_+) = P \exp i g \int_{-\infty}^{0} ds \ n \cdot A^{(n)}(x_+ n/2 + s n),$$

$$W_{\bar{n}}(x_-) = P \exp i g \int_{-\infty}^{0} ds \ n \cdot A^{(\bar{n})}(x_- n/2 + s n).$$

The terms with the transverse derivatives of the quark fields must be redefined as

$$\partial_\perp \xi_n(x_-) \rightarrow W^\dagger_n(x_-)\partial_\perp \xi_n(x_-) = \left[ W^\dagger_n(x_-)\partial_\perp W_n(x_-) \right] W^\dagger_n\xi_n(x_-) + \partial_\perp W^\dagger_n\xi_n(x_-),$$

where we assume that the derivative must be applied only inside the square brackets. The contribution with $\left[ W^\dagger_n\partial_\perp W_n \right]$ is not gauge invariant, but it can be associated with the quark-gluon operators. This term must be combined with the diagrams with emissions of collinear transverse gluons from the quark lines. The sum of such contributions gives the gauge invariant quark-gluon operators of twist-4. However, we neglect such operators and therefore we can skip this contribution. Hence, we can write

$$\partial_\perp \xi_n(x_-) \rightarrow \partial_\perp W^\dagger_n(x_-)\xi_n(x_-).$$

Notice that this term is already gauge invariant.

After the factorisation of hard and collinear sectors, the relevant contributions to the hadronic matrix element (18) can be written as

$$M_h \simeq \langle 0 | O(z_{i+})|^\text{tw3}_3 | k' \rangle \langle 0 | O(z_{i-})|^\text{tw4}_4 | k \rangle + \langle 0 | O(z_{i+})|^\text{tw4}_4 | k' \rangle \langle 0 | O(z_{i-})|^\text{tw3}_3 | k \rangle,$$

where the leading twist-3 operators read

$$[O(z_{i+})]^\text{tw3}_3 = \bar{\chi}_1(z_{i+}) \bar{\chi}_2(z_{i+}) \bar{\chi}_3(z_{i+}), \quad [O(z_{i-})]^\text{tw3}_3 = \chi_1(z_{i-}) \chi_2(z_{i-}) \chi_3(z_{i-}).$$

In these formulas we used the standard notation for the gauge invariant blocks

$$\bar{\chi}(z_+) = \bar{\xi}_n(z_+) W_n(z_+), \quad \chi(z_-) = W^\dagger_n(x_-)\xi_n(x_-).$$

We do not show explicitly the quark flavour assuming that Dirac indices 1, 2 correspond to the $u$-quarks. We also do not indicate explicitly the collinearity ($n$ and $\bar{n}$) of the fields explicitly, assuming that the field arguments allows one to conclude about the collinear sector.

The obtained twist-4 operators include the fields with the transverse derivatives $\partial_\perp \chi$. For the nucleon case one finds

$$[O(z_{i-})]^\text{tw4}_3 = -i (z_1 - z_3)^\alpha_1 \left[ i \partial_\perp \alpha \chi_1(z_{i-}) \right] \chi_2(z_{i-}) \chi_3(z_{i-})$$

$$- i (z_2 - z_3)^\alpha_2 \chi_1(z_{i-}) \left[ i \partial_\perp \alpha \chi_2(z_{i-}) \right] \chi_3(z_{i-}) - \frac{1}{2} \left[ (i \partial_\perp)^\gamma \gamma \partial_\perp \chi(z_{i-}) \right]_1 \chi_2(z_{i-}) \chi_3(z_{i-})$$

$$- \frac{1}{2} \chi_1(z_{i-}) \left[ (i \partial_\perp)^\gamma \gamma \partial_\perp \chi(z_{i-}) \right]_2 \chi_3(z_{i-}) + \frac{1}{2} [i \partial_\perp \alpha \chi_1(z_{i-}) \chi_2(z_{i-}) \left[ (i \partial_\perp)^\gamma \gamma \partial_\perp \chi(z_{i-}) \right]_3$$

$$+ \frac{1}{2} \chi_1(z_{i-}) \left[ i \partial_\perp \alpha \chi_2(z_{i-}) \right] \left[ (i \partial_\perp)^\gamma \gamma \partial_\perp \chi(z_{i-}) \right]_3.$$
The first two terms in rhs of Eq.(33) appear from to the multipole expansion of the fields (this was explained above). The remnant terms in rhs of Eq.(33) appear from the small collinear components $\eta(z_i)$. The latter can be rewritten using QCD EOM

$$W_n^\dagger(x_-)\eta_n(x_-) = -W_n^\dagger(x_-)\eta_n(x_-) = -\frac{n}{2}(in\partial)^{-1}W_n^\dagger(x_-)\eta_n(x_-)$$

$$= -\frac{n}{2}(in\partial)^{-1}\left[W_n^\dagger(x_-)i\partial_n W_n(x_-)\right]\chi_n(x_-) = \frac{n}{2}(in\partial)^{-1}i\partial_n\chi_n(x_-)$$

where we again neglected the contribution with $[W_n^\dagger(x_-)i\partial_n W_n(x_-)]$, which gives the quark-gluon operator. We also used that the matrix element of any operator with the total transverse derivative vanishes $\langle 0|\partial_\perp O|k\rangle = 0$ and therefore can be neglected. This gives

$$\chi_1(z_{1-})\chi_2(z_{2-})\partial_\perp\alpha\chi_3(z_{3-}) \approx -[\partial_\perp\alpha\chi_1(z_{1-})]\chi_2(z_{2-})\chi_3(z_{3-})$$

$$-\chi_1(z_{1-})[\partial_\perp\alpha\chi_2(z_{2-})]\chi_3(z_{3-}).$$

(35)

The matrix element of twist-3 light-cone operator is given in Eq.(A.7). The expression for the twist-4 matrix element is more complicated. Using the Fierz identities

$$\chi_1(z_{1-})\chi_2(z_{2-}) = -\frac{1}{8}[\hat{g}C]_{21}\chi(z_{1-})C\gamma_5\chi(z_{2-}) - \frac{1}{8}[\hat{g}\gamma_5 C]_{21}\chi(z_{1-})C\gamma_5\chi(z_{2-})$$

$$+ \frac{1}{8}[\hat{g}\gamma_\perp C]_{21}\chi(z_{1-})C\gamma_\perp\alpha\chi(z_{2-}).$$

(36)

we rewrite this operator as a sum

$$O(z_{i-}) = O_V(z_{i-}) + O_A(z_{i-}) + O_T(z_{i-}),$$

(37)

where the operators $O_V$, $O_A$ and $O_T$ correspond to the three projections in Eq.(36), respectively. Consider the operator $O_V$. It can be written as a sum of two terms, which only include $\partial_\perp^0\chi_1(z_{1-})$ or $\partial_\perp^0\chi_2(z_{2-})$, respectively

$$O_V(z_{i-}) = O_{V1}(z_{i-}) + O_{V2}(z_{i-}).$$

(38)

From Eq.(33) one finds

$$O_{V1}(z_{i-}) = \frac{1}{8}[\hat{g}C]_{21}\left(i\partial_\perp\alpha\chi(z_{1-})\right)C\gamma_5\chi(z_{2-})\chi_3(z_{3-})$$

$$+ \frac{1}{8}[\hat{g}\gamma_\perp C]_{21}\left((in\partial)^{-1}i\partial_\perp\alpha\chi(z_{1-})\right)C\gamma_5\chi(z_{2-})\chi_3(z_{3-})$$

$$- \frac{1}{8}[\hat{g}C]_{21}\left(i\partial_\perp\alpha\chi(z_{1-})\right)C\gamma_\perp\alpha\chi(z_{2-})\left((i\partial_\perp)^{-1}\gamma_\perp\alpha\chi(z_{3-})\right)_3.$$

(39)

The matrix element of this operator can be easily computed with the help of Eq.(A.25). This gives

$$\langle 0|(i\partial_\perp\alpha\chi(z_{1-})) C\gamma_5\chi(z_{2-})\chi_3(z_{3-})|k\rangle = m_N [\gamma_\perp\alpha\gamma_5 N]_3 FT \left[\frac{1}{x_1}V_1(x_i)\right],$$

(40)

$$\langle 0|\partial_\perp\alpha\chi(z_{1-})|C\gamma_5\chi(z_{2-})\left((i\partial_\perp)^{-1}\gamma_\perp\alpha\chi(z_{3-})\right)_3|k\rangle$$

$$= m_N [\hat{g}\gamma_\perp\alpha\gamma_5 N]_3 \left[\hat{g}C\right]_{12} FT \left[\frac{1}{x_3}V_1(x_i)\right],$$

(41)
where symbol FT denotes the Fourier transformation. With the help of these equations we obtain

\[
\langle 0 | O_{V1}(z_{i-}) | k \rangle = k_+ m_N \frac{1}{8} [\not{\nu} C]_{12} [\gamma_5 \gamma_5 N]_{3} \langle z_{1} - z_{3} \rangle_{\perp} \text{FT} | V_1(x_i) \rangle
\]

\[
+ \frac{1}{16} m_N [\gamma_{\perp} \gamma_5 N]_{3} [\not{\nu} \gamma_{\perp\alpha} \not{\nu} C]_{12} \text{FT} \left[ \frac{1}{x_1} V_1(x_i) \right]
\]

\[
- \frac{1}{16} m_N \langle \gamma_{\perp} \gamma_{\perp\alpha} \gamma_5 N \rangle_{3} [\not{\nu} C]_{12} \text{FT} \left[ \frac{1}{x_3} V_1(x_i) \right].
\]

(42)

Now we have the all required matrix elements \[17\], \[A.7\] and \[42\], which we need in order to calculate the appropriate contribution to the amplitude \(A_2\)

\[
A_2[[O(z_{i+})]_{tw3}, O_{V1}(z_{i-})] = \int dz'_1 dz'_2 dz'_3 \int dz_1 dz_2 dz_3 D(z'_i, z_j)
\]

\[
\times \langle P | Q_{\alpha}(z'_1)Q_{\beta}(z'_2) | 0 \rangle \langle 0 | [O(z_{i+})]_{tw3} | k^\prime \rangle \langle 0 | O_{V1}(z_{i-}) | k \rangle.
\]

(43)

One can use the fact that all our diagrams have the same structure with respect to spinor indices

\[
[D(z'_i, z_j)]_{123,1'2'3'} = D_{\mu_1 \mu_2 \mu_3}(z'_i, z_j) \left[ \gamma_{\mu_1} \right]_{1'1} [\gamma_{\mu_2}]_{2'2} [\gamma_{\mu_3}]_{3'3},
\]

(44)

where the \(\gamma\)-matrices \(\gamma_{\mu_i}\) originate from the light quark-gluon vertices in the diagrams as in Fig.2. Substitution of the matrix elements from \[17\], \[A.7\] and \[42\] and contractions of the spinor indices yields

\[
A_2[V_1, V_1] = \int dz'_1 dz'_2 dz'_3 \int dz_1 dz_2 dz_3 e^{i m_Q(z'_1 + z'_2)} \int Dy_i V_1(y_i) e^{-i(k'_1 z_1) - i(k'_2 z_2) - i(k'_3 z_3)}
\]

\[
\times T_{3g \rightarrow pp}[V_1] f_{\psi} \frac{1}{4} \text{Tr} \left[ (1 - Q_{\psi}) D_{\mu_1 \mu_2 \mu_3} (z'_i, z_j) \right],
\]

(45)

with

\[
T_{3g \rightarrow pp}[V_1] = - \frac{1}{4} m_N \tilde{V}_5 \gamma_5 \gamma_{\mu_3} \gamma_{\perp} \gamma_5 N \frac{1}{4} \text{Tr} \left[ \gamma_{\mu_1} \not{\nu} C \left( C \not{\nu} \gamma_{\mu_2} \right)^\dagger \right] \frac{\partial}{\partial k_{1\perp}^\alpha} \text{FT} | V_1(x_i) \rangle
\]

\[
+ \frac{m_N}{16} \tilde{V}_5 \gamma_5 \gamma_{\mu_3} \gamma_{\perp} \gamma_5 N \frac{1}{4} \text{Tr} \left[ \gamma_{\mu_1} \not{\nu} \gamma_{\perp\alpha} \not{\nu} C \left( C \not{\nu} \gamma_{\mu_2} \right)^\dagger \right] \text{FT} \left[ \frac{1}{x_1} V_1(x_i) \right]
\]

\[
- \frac{m_N}{16} \tilde{V}_5 \gamma_5 \gamma_{\mu_3} \not{\nu} \gamma_{\perp} \gamma_5 \gamma_{\perp\alpha} \gamma_5 N \frac{1}{4} \text{Tr} \left[ \gamma_{\mu_1} \not{\nu} C \left( C \not{\nu} \gamma_{\mu_2} \right)^\dagger \right] \text{FT} \left[ \frac{1}{x_3} V_1(x_i) \right].
\]

(46)

For simplicity we show in this expression only the contribution with the twist-3 DA \(V_1(y_i)\).

The term with the transverse derivative \(\partial/\partial k_{1\perp}^\alpha\) in \[46\] arises from the contribution with \(i(z_1 - z_3)^\alpha\) in Eq.\[42\]. In order to describe the transition \(i(z_1 - z_2)_{\perp} \rightarrow \partial/\partial k_{1\perp}\), we used that the corresponding contribution can be written as

\[
i(z_1 - z_3)_{\perp} \text{FT} | V_1(x_i) \rangle = - \frac{\partial}{\partial k_{1\perp}^\alpha} \int Dx_i e^{-i(k_1 z_1) - i(k_2 z_2) - i(k_3 z_3)} V_1(x_i) \bigg|_{k_{1\perp} = 0},
\]

(47)

where partonic momenta have the transverse components

\[
k_1 = x_1 k + k_{1\perp}, \quad k_2 = x_2 k + k_{2\perp}, \quad k_3 = x_3 k - k_{1\perp} - k_{2\perp}, \quad k \simeq k + \frac{\bar{n}}{2},
\]

(48)
Now we can perform the integrations over $dz'_i$ and $dz_j$ in [45]. This corresponds to the Fourier transformation of the diagrams $D^{μ_1 ν_2 μ_3}(z'_i, z_j)$ to the momentum space

$$D^{μ_1 ν_2 μ_3}(z'_i, z_j) \rightarrow D^{μ_1 ν_2 μ_3}(k'_i, k_j).$$

This gives

$$A_2[V_1 V_1] = \int D_{y_i} V_i (y_i) \int D_{x_i} V_1 (x_i) \ T_{3g→p̅p} \ \frac{1}{4} \ Tr \left[ (1 - ϕ) \bar{Q}_ν D^{μ_1 ν_2 μ_3}(k'_i, k_j) \right]_{k_{1} = 0}, \quad (50)$$

where

$$T_{3g→p̅p} = \frac{1}{4}m_N \bar{V}_5 γ_μ γ_5 γ_μ γ_5 N \frac{1}{4} \ Tr \left[ γ_μ γ_3 \bar{ψ} C \left(C γ_5 \gamma_μ \right)^{\top} \right] \ \frac{∂}{∂ k_{1\perp}}$$

$$+ \frac{1}{x_1} \frac{m_N}{16} \bar{V}_5 γ_μ γ_5 γ_5 N \frac{1}{4} \ Tr \left[ γ_μ γ_3 \bar{ψ} C \left(C γ_5 \gamma_μ \right)^{\top} \right]$$

$$- \frac{1}{x_3} \frac{m_N}{16} \bar{V}_5 γ_μ γ_5 γ_5 \bar{ψ} C \left(C γ_5 \gamma_μ \right)^{\top} \frac{1}{4} \ Tr \left[ γ_μ \bar{ψ} C \left(C γ_5 \gamma_μ \right)^{\top} \right]. \quad (51)$$

The analytical expression in $D^{μ_1 ν_2 μ_3}(k'_i, k_j)$ describes the contributions of the heavy quark line with the gluon vertices (with the indices $μ$) and the three gluon propagators as in the diagrams in Fig. [4]. The light quark momenta defined as $k'_i = y_i k_n / 2$ and as in Eq.[48]. The similar contributions must be also obtained for the other operators in Eq.[37].

Performing the calculations of the traces one obtains the final expression for the helicity flip amplitude

$$A_2 = \frac{2m_N^2}{M_N^2} \ \varphi \ (πα_s)^{3/4} \ \frac{10}{81} \ J, \quad (52)$$

where the dimensionless convolution integral $J$ reads

$$J = \frac{λ_1}{f_N} \ (J_1[V_1, V_i] + J_2[A_1, V_i] + J_3[V_1, A_i] + J_4[A_1, A_i] + J_5[T_1, T_{ij}]). \quad (53)$$

In square brackets we show the LCDAs, which enter in the integrands. Each integral in Eq.[53] is given by

$$J_n[X_1, Y_i] = \int D_{y_i} X_i (y_i) \int D_{x_i} x_i x_3 \ \left\{ \frac{K_n(Y_i; x_i, y_i)}{D_1 D_3} + \frac{L_n(Y_i; x_i, y_i)}{D_1 D_2} + \frac{N_n(Y_i; x_i, y_i)}{D_2 D_3} \right\}, \quad (54)$$

where

$$D_i = x_i (1 - y_i) + (1 - x_i) y_i. \quad (55)$$

The factors $D_i$ appear from the heavy quark propagators. It is easy to understand that the three groups in Eq.[54] are related with the three groups of the diagrams which have appropriate configurations of the heavy quark momenta. The analytical expressions for the coefficients $\{K_n, L_n, N_n\}$ read

$$K_1(V_i; x_i, y_i) = V_1(x_i) \left( \frac{y_3 - x_3}{x_1} + 2 \frac{x_1 - y_1}{x_3} + 2x_1 + 2y_1 - 2y_3 \right) + V_2(x_i) 2 \left( \frac{x_1 - y_1}{x_3} + \frac{x_1 x_3}{x_2} + x_1 + y_1 \right), \quad (56)$$
\[ L_1(V_i; x_i, y_i) = V_1(x_i) \left( \frac{y_2 - x_2}{x_1} - \frac{4x_1x_2}{x_3} - 2x_2 \right) + V_2(x_i) \left( \frac{y_1 - x_1}{x_2} - \frac{4x_1x_2}{x_3} - 2x_1 \right), \] (57)

\[ N_1(V_i; x_i, y_i) = V_1(x_i) 2 \left( \frac{x_2 - y_2}{x_3} + \frac{x_2x_3}{x_1} + x_2 + y_2 \right) + V_2(x_i) \left( \frac{y_3 - x_3}{x_2} + 2\frac{x_2 - y_2}{x_3} + 2x_2 + 2y_2 - 2y_3 \right), \] (58)

\[ K_2(V_i; x_i, y_i) = V_1(x_i) \left( \frac{y_3 - x_3}{x_1} - 2x_1 + 2y_1 - 2y_3 \right) + V_2(x_i) 2 \left( y_1 - x_1 - \frac{x_1x_3}{x_2} \right), \] (59)

\[ L_2(V_i; x_i, y_i) = V_1(x_i) \left( \frac{y_2 - x_2}{x_1} - 2x_2 \right) + V_2(x_i) \left( \frac{x_1 - y_1}{x_2} + 2x_1 \right), \] (60)

\[ N_2(V_i; x_i, y_i) = V_1(x_i) 2 \left( x_2 - y_2 + \frac{x_2x_3}{x_1} \right) + V_2(x_i) \left( \frac{x_3 - y_3}{x_2} + 2x_2 - 2y_2 + 2y_3 \right), \] (61)

\[ K_3(A_i; x_i, y_i) = A_1(x_i) \left( \frac{y_3 - x_3}{x_1} + 2x_1 - 2y_1 - 2y_3 \right) + A_2(x_i) 2 \left( x_1 - y_1 - \frac{x_1x_3}{x_2} \right), \] (62)

\[ L_3(A_i; x_i, y_i) = A_1(x_i) \left( \frac{y_2 - x_2}{x_1} + 2x_2 - 4y_2 \right) + A_2(x_i) \left( \frac{x_1 - y_1}{x_2} + 4y_1 - 2x_1 \right), \] (63)

\[ N_3(A_i; x_i, y_i) = A_1(x_i) 2 \left( y_2 - x_2 + \frac{x_2x_3}{x_1} \right) + A_2(x_i) \left( \frac{x_3 - y_3}{x_2} - 2x_2 + 2y_2 + 2y_3 \right), \] (64)

\[ K_4(A_i; x_i, y_i) = A_1(x_i) \left( \frac{y_3 - x_3}{x_1} - 2\frac{y_1 - x_1}{x_3} - 2y_3 + 2y_1 + 2x_1 \right) + A_2(x_i) 2 \left( \frac{x_1 - y_1}{x_3} + \frac{x_1x_3}{x_2} + x_1 + y_1 \right), \] (65)

\[ L_4(A_i; x_i, y_i) = A_1(x_i) \left( \frac{y_2 - x_2}{x_1} - 4 \frac{x_1x_2}{x_3} - 2x_2 \right) + A_2(x_i) \left( \frac{y_1 - x_1}{x_2} - 4 \frac{x_1x_2}{x_3} - 2x_1 \right), \] (66)

\[ N_4(A_i; x_i, y_i) = A_1(x_i) 2 \left( \frac{x_2 - y_2}{x_3} + \frac{x_2x_3}{x_1} + y_2 + x_2 \right) + A_2(x_i) \left( \frac{y_3 - x_3}{x_2} - 2\frac{x_2 - y_2}{x_3} - 2y_3 + 2y_2 + 2x_2 \right), \] (67)

\[ K_5(Tij; x_i, y_i) = \left( T_{21} - T_{41} \right) (x_i) 2 \left( \frac{x_3 - y_3}{x_1} + 2x_3 \right) + \left( T_{22} - T_{42} \right) (x_i) \left( \frac{4x_1x_3}{x_2} \right), \] (68)

\[ L_5(Tij; x_i, y_i) = \left( T_{21} - T_{41} \right) (x_i) 2 \left( \frac{x_2 - y_2}{x_1} + 2y_2 \right) + \left( T_{22} - T_{42} \right) (x_i) 2 \left( \frac{x_1 - y_1}{x_2} + 2y_1 \right), \] (69)
\[ N_5(T_{ij}; x_i, y_i) = (T_{21} - T_{41})(x_i) \left( -\frac{4x_2x_3}{x_1} \right) + (T_{22} - T_{42})(x_i)2 \left( \frac{x_3 - y_3}{x_2} + 2x_3 \right). \] (70)

The various LCDAs which enter in the expressions in Eqs. (56)-(70) are discussed in Appendix A. Notice that the contributions with DAs \( \mathcal{V}_2, \mathcal{A}_2 \) and \( T_{22} - T_{42} \) can be reduced to the contributions with \( \mathcal{V}_1, \mathcal{A}_1 \) and \( T_{21} - T_{41} \) with the help of the symmetry relations, see Eqs. (A.37), (A.38) and (A.44). The formulas (52)-(70) represent the main result of this work.

Let us shortly discuss the properties of the obtained convolution integrals. These integrals are well defined, i.e. they do not have singularities from the endpoint regions as it was expected. In order to see this, consider, for instance, the integral with \( \mathcal{V}_1(y_i) \mathcal{V}_1(x_i) \) LCDAs. The properties of these functions allow one to rewrite them as

\[ \mathcal{V}_1(y_i) = 120y_1y_2y_3 \tilde{V}(y_i), \quad \mathcal{V}_1(x_i) = 3x_1x_2x_3 \tilde{V}(x_i), \] (71)

where the functions \( \tilde{V}(y_i) \) and \( \tilde{V}(x_i) \) are some nonsingular functions when momentum fractions are small \( y_i, x_i \sim 0 \). For the twist-4 DA \( \mathcal{V}_1(x_i) \) this follows from Eq. (A.34). Substituting (71) into the convolution integral, we obtain

\[ J_1[\mathcal{V}_1, \mathcal{V}_1] = 360 \int D y_i \tilde{V}(y_i) \int D x_i \left( \frac{\tilde{K}_i(x_i, y_i)}{D_1 D_3} + \frac{\tilde{L}_i(x_i, y_i)}{D_1 D_2} + \frac{\tilde{N}_i(x_i, y_i)}{D_2 D_3} \right), \] (72)

where the coefficients \( \tilde{K}_i, \tilde{L}_i, \tilde{N}_i \) depend on \( \tilde{V}(x_i) \). The endpoint singularities can be produced by the most singular contributions in these coefficients with the factors \( 1/x_i \). Consider for instance the terms with \( 1/x_1 \) which are dangerous in the limit \( x_1 \to 0 \). The most singular terms with \( 1/x_1 \) give

\[ \frac{\tilde{K}_i(x_i, y_i)}{D_1 D_3} + \frac{\tilde{L}_i(x_i, y_i)}{D_1 D_2} + \frac{\tilde{N}_i(x_i, y_i)}{D_2 D_3} = \tilde{V}_1(x_i) \left( \frac{y_3 - x_3}{D_1 D_3} + \frac{y_2 - x_2}{D_1 D_2} + \frac{2x_2x_3}{D_2 D_3} \right) + \ldots \] (73)

\[ = 2\tilde{V}_1(x_i) \frac{x_2x_3(1 - 2y_1) + y_2y_3}{D_1 D_2 D_3} + \ldots = F(x_1, x_2, y_i), \] (74)

where for simplicity dots denote the contributions without \( 1/x_1 \). Hence, we see that the dangerous singularity \( 1/x_1 \) cancel. The resulting expression is power suppressed in the endpoint region \( x_1 \sim 0 \) \( (\eta \ll 1) \)

\[ J_1[\mathcal{V}_1, \mathcal{V}_1]_{\text{ms}} \sim \int D y_i \tilde{V}(y_i) \int_0^\eta dx_1 \int_0^1 dx_2 \tilde{V}_1(0, x_2, \bar{x}_2) F(0, x_2, y_i) \]

\[ \sim \eta \int D y_i \tilde{V}(y_i) \int_0^1 dx_2 \tilde{V}_1(0, x_2, \bar{x}_2) F(0, x_2, y_i), \] (75)

The cut-off parameter \( \eta \) is small and can be associated with the power suppressed scale \( \eta \sim \lambda^2 \). We see that the integral over small fraction \( x_1 \) in the endpoint limit \( x_1 \sim \eta \) is power suppressed. The function \( F(0, x_2, y_i) \) does not have any power singularities in the other endpoint regions and therefore the total integral is of order \( \eta \). This means that the contributions from the endpoint regions are power suppressed and the integrals over the momentum fractions are well defined. The same conclusions are also true for the other contributions with \( 1/x_2 \) and \( 1/x_3 \) and for the integrals with other combinations of LCDAs.

The absence of the endpoint divergencies is closely related with the suppression of the ultrasoft gluons with momentum \( k_\mu \sim m_Q \lambda^2 \) in the heavy quark annihilation. One can show without the explicit calculation that the singular terms, arising due to the ultrasoft gluon in the individual diagrams, will cancel in the sum of all diagrams. This consideration is briefly discussed in Appendix B. The cancellation of the endpoint singularities provides a good check of the obtained expressions for the hard coefficient functions.
4 Phenomenology

In this section the obtained amplitude $A_2$ is used for a qualitative analysis of the branching ratio and angular behaviour of the cross section $e^+e^- \to J/\psi \to p\bar{p}$. Except of the corrections, associated with the higher Fock components of hadronic wave functions, there are also relativistic corrections associated with the charmonium wave function. These corrections are formally suppressed by the power of $v^2$, at the same time the hard annihilation mechanism is strongly suppressed by power(s) of the small $\alpha_s$. In the Coulomb limit $m_Q v^2 \gg \Lambda$ the colour-octet contribution can be described by the annihilation with one hard and two ultrasoft gluons. Such contribution is associated with the colour-octet operator and referred as the octet contribution. This contribution is obviously of order $\alpha_s(m_c^2)$. We assume that $\alpha_s((m_Q v^2)^2) \gg \alpha_s(m_c^2)$ and that it can be estimated to be of order one for the real world. Then a simple estimate for the decay amplitudes gives

$$\frac{A_{\text{oct}}}{A_{\text{sing}}} \sim v^2 \alpha_s / \alpha_s^3 \sim v^2 / \alpha_s^2,$$

where all $\alpha_s$ are defined at the hard scale $\sim m_c^2$. For realistic charmonium $v_c^2 \simeq 0.3$, which is comparable with the value of $\alpha_s(2m_c^2) \simeq 0.3$. This indicates that the colour-octet mechanism can potentially provide a sufficiently large correction, which can be associated with the soft-overlap mechanism (the ultrasoft gluons in the Coulomb limit). Such corrections are sensitive to a long distance behaviour of the charmonium and hadronic wave functions. This could lead to a strong violation of the ratio $Q$ which is expected from the hard annihilation which depends only from the wave function at the origin and therefore one expects that

$$Q = \frac{\text{Br}[J/\psi \to p\bar{p}]}{\text{Br}[\psi(2S) \to p\bar{p}]} \approx \frac{\text{Br}[J/\psi \to e^+e^-]}{\text{Br}[\psi(2S) \to e^+e^-]}.$$ (77)

However this relation is satisfied to a very good accuracy $Q_{p\bar{p}} = 0.139$, $Q_{e^+e^-} = 0.133$. This observation allows one to assume that the dominant effects in $p\bar{p}$ decay is provided by colour-singlet mechanism, which is proportional to the charmonium wave function at the origin.

The expression for the decay width reads

$$\Gamma[J/\psi \to p\bar{p}] = \frac{M_{\psi}\beta}{12\pi} \left(|G_M|^2 + \frac{2m_N^2}{M_{\psi}^2} |G_E|^2 \right),$$

where we introduced the helicity amplitudes

$$G_M = A_1 + A_2, \quad G_E = A_1 + \frac{M_{\psi}^2}{4m_N^2} A_2.$$ (79)

The amplitude $A_1$ can be written as

$$A_1 \simeq A_1^{(0)} \left(1 + \frac{m_N^2}{M_{\psi}^2} I_1 \right),$$ (80)

where $A_1^{(0)}$ describes the leading-twist contribution. The second term with the dimensionless constant $I_1$ describes the possible power correction. The expression for $A_1^{(0)}$ is given in Eq. (12) and can be written as a product

$$A_1^{(0)} = A_0 I_0,$$ (81)
where we introduced the convenient normalisation factor

$$A_0 = \frac{f_\psi}{m_Q^2} \frac{f_{N}^2}{m_Q^4} (\pi \alpha_s)^{\frac{10}{81}}. \tag{82}$$

The subleading term in Eq. (80) is unknown but together with $A_2$ it provides the next-to-leading power contribution to $G_M$. The constant $I_1$ can be understood as a dimensionless ratio of the collinear integrals. The amplitude $A_2$ in Eq. (52) can also be written as

$$A_2 = A_0 \frac{2m_N^2}{M^2_\psi} J. \tag{83}$$

Using (81)-(83) one finds

$$G_E \simeq A_0 I_0 \left(1 + \frac{1}{2} J/I_0 \right), \quad G_M \simeq A_0 I_0 \left(1 + \frac{m_N^2}{M^2_\psi} I_1 + \frac{2m_N^2}{M^2_\psi} J/I_0 \right). \tag{84}$$

Obviously, the power corrections in $G_M$ in Eq. (84) are the part of the total power suppressed contribution.

The exact expression for the coefficient $\alpha$ describing the angular distribution in Eq. (1) reads

$$\alpha = \frac{|G_M|^2 - \frac{4m_N^2}{M^2_\psi} |G_E|^2}{|G_M|^2 + \frac{4m_N^2}{M^2_\psi} |G_E|^2}. \tag{85}$$

If $A_1 \gg A_2$, then $A_2$ can be neglected in $G_M$ and $G_E$ and one receives the expression from Eq. (2). The effect from the total power correction can be obtained by the substitution of expressions in Eq. (84). Obviously, this observable does not depend on the normalisation $A_0$ which has large uncertainty from the value of $\alpha_s$.

The accurate measurements carried out by BESIII provide \[16\]

$$\alpha = 0.595 \pm 0.012. \tag{86}$$

From the known $\alpha$ one easily gets the ratio $|G_E| / |G_M| \[16\]

$$|G_E| / |G_M| = 0.832 \pm 0.015. \tag{87}$$

This result allows one to conclude that the effect from the amplitude $A_2$ is not negligible if one wants to accurately get the value of $\alpha$.

Below we consider the qualitative analysis of $\alpha$ and the the branching ratio. In this analysis we consider the power correction to $A_1$ provided by the constant $I_1$ as a free real parameter. The integrals $I_0$ and $J$ are given in Eqs. (13) and (53). Their values depend on the models of LCDAs.

For the nucleon twist-3 LCDA we will use the model with the truncated conformal expansion from Ref. \[21\]

$$\varphi_3(x_i) \simeq 120x_1x_2x_3 (1 + \varphi_{10} P_{10}(x_i) + \varphi_{11} P_{11}(x_i) + \varphi_{20} P_{20}(x_i) + \varphi_{21} P_{21}(x_i) + \varphi_{22} P_{22}(x_i)) \tag{88}$$

where

$$P_{10}(x_i) = 21(x_1 - x_3), \quad P_{11}(x_i) = 7(x_1 - 2x_2 + x_3), \tag{89}$$
The coupling \( f_N \) and the coefficients \( \varphi_{ij} \) are multiplicatively renormalizable and corresponding anomalous dimensions can be found in Refs. \[22,28\].

The required twist-4 LCDAs reads

\[
\Phi_4(x_i) = f_N \Phi_4^{WW}(x_i) + \lambda_1 \Phi_4(x_i),
\]

\[
\Psi_4(x_i) = f_N \Psi_4^{WW}(x_i) - \lambda_1 \Psi_4(x_i).
\]

The functions with the index \( WW \) correspond to the Wandzura-Wilczek contributions, which are defined by the \( \varphi_3(x_i) \). The explicit expressions for these functions were obtained in Refs. \[28,29\].

To our accuracy they read

\[
\Phi_4^{WW}(x_i) \simeq -40 \left( 2 - \frac{\partial}{\partial x_3} \right) x_1 x_2 x_3 - 20 \sum_{k=0}^1 \varphi_{1k} \left( 3 - \frac{\partial}{\partial x_3} \right) x_1 x_2 x_3 P_{1k}(x_i) - 12 \sum_{k=0}^2 \varphi_{2k} \left( 4 - \frac{\partial}{\partial x_3} \right) x_1 x_2 x_3 P_{2k}(x_i).
\]

The formula for \( \Psi_4^{WW}(x_i) \) can be obtained from (95) by following substitutions in the \( rhs \):

\( \partial/\partial x_3 \to \partial/\partial x_2 \) and \( P_{nk}(1,2,3) \to P_{nk}(2,1,3) \). Notice that the differentiations must be computed with the unmodified expressions of the polynomials \( P_{nk}(x_i) \) in Eqs. (89) and (92) and only after that one can apply the condition \( x_1 + x_2 + x_3 = 1 \).

For the genuine twist-4 functions in Eqs. (93) and (94) we also use the truncated conformal expansions

\[
\Phi_4(x_1, x_2, x_3) = 24 x_1 x_2 \left( 1 + \eta_{10} \mathcal{R}_{10}(x_3, x_1, x_2) - \eta_{11} \mathcal{R}_{11}(x_3, x_1, x_2) \right),
\]

\[
\Psi_4(x_1, x_2, x_3) = 24 x_1 x_3 \left( 1 + \eta_{10} \mathcal{R}_{10}(x_2, x_3, x_1) + \eta_{11} \mathcal{R}_{11}(x_2, x_3, x_1) \right),
\]

where

\[
\mathcal{R}_{10}(x_1, x_2, x_3) = 4 (x_1 + x_2 - 3/2 x_3), \quad \mathcal{R}_{11}(x_1, x_2, x_3) = \frac{20}{3} (x_1 - x_2 + x_3/2).
\]

The twist-4 moments \( \lambda_1, \eta_{10} \) and \( \eta_{11} \) are multiplicatively renormalizable, see the details in Refs. \[22,28\].

The four-dimensional convolution integrals can be easily computed numerically. For the models with \( \varphi_{20} = \varphi_{21} = \varphi_{22} = 0 \) we obtained the following expressions

\[
I_0 = 120^2 \left( 0.1054 + 0.0659 \varphi_{10} + 0.0304 \varphi_{11} + 0.4882 \varphi_{10} \varphi_{11} + 0.0013 \varphi_{10}^2 + 0.0889 \varphi_{11}^2 \right),
\]

\[
\frac{\lambda_1}{f_N} (J_1[V_1, V_i] + J_3[V_1, A_i]) = 720 \frac{\lambda_1}{f_N} (1.2069 + 5.0168 \eta_{10} - 0.5996 \eta_{11}
+ (3 \varphi_{10} - \varphi_{11}) (19.2724 + 18.1453 \eta_{10} - 11.453 \eta_{11})
+ 720 (12.1675 - 5.3808 \varphi_{10} + 6.6111 \varphi_{11} - 399.8760 \varphi_{10} \varphi_{11}
+ 1203.0400 \varphi_{10}^2 - 0.3793 \varphi_{11}^2) ).
\]
\[ \frac{\lambda_1}{f_N} (J_2[A_1, V_I] + J_4[A_1, A_i]) = 720 (\varphi_{10} + \varphi_{11}) \]
\[ \times \left( \frac{\lambda_1}{f_N} (18.9843 + 2.6497 \eta_{10} + 33.2066 \eta_{11}) - 11.7872 + 43.1562 \varphi_{10} + 83.4753 \varphi_{11} \right). \]  

(101)

\[ \frac{\lambda_1}{f_N} J_5[T_1, T_{ij}] = 2880 \frac{\lambda_1}{f_N} \eta_{11} (-0.1918 + 10.5689 \varphi_{11}) \]
\[ + 2880 (10.2441 - 43.9569 \varphi_{11} + 104.9574 \varphi_{11}^2). \]  

(102)

The contributions with the higher coefficients \( \varphi_{2i} \) have only been used for the COZ model (see detail below) and the corresponding integrals have been computed numerically.

For our estimates we consider the parameters obtained from the QCD sum rules and from the analysis of the light-cone sum rule for the nucleon electromagnetic form factors (ABO model). We also consider the models with the parameters which were recently obtained by lattice calculations in Ref. [24].

As a kind of different scenario, we consider for twist-3 DA \( \varphi_3 \) the model \( \varphi^{IJ} \) from Ref. [26] (COZ-model). The twist-4 DAs in this case include the appropriate WW-terms and we also add the simplest genuine twist-4 contribution with the moment \( \lambda_1 \). Therefore, we denote this model with the sign plus. The values of the corresponding parameters are given in the Table 1.

| model   | \( f_N, \text{GeV}^2 \) | \( \varphi_{10} \) | \( \varphi_{11} \) | \( \varphi_{20} \) | \( \varphi_{21} \) | \( \varphi_{22} \) | \( \lambda_1/f_N \) | \( \eta_{10} \) | \( \eta_{11} \) |
|---------|---------------------|----------------|----------------|---------------|---------------|---------------|----------------|----------------|----------------|
| ABO     | \( 4.8 \times 10^{-3} \) | 0.047          | 0.047          | 0             | 0             | 0             | -6.27          | -0.038         | 0.13           |
| Lattice | \( 3.5 \times 10^{-3} \) | 0.18           | 0.12           | 0             | 0             | 0             | -12.68         | 0              | 0              |
| COZ+    | \( 4.9 \times 10^{-3} \) | 0.154          | 0.182          | 0.38          | 0.054         | -0.146        | -6.27          | 0              | 0              |

Table 1: The LCDA parameters for the different models at \( \mu^2 = 4 \text{ GeV}^2 \).

In our estimates for the branching ratio we always assume that \( \mu^2 = 2m_c^2 \) and \( m_c = 1.5 \text{ GeV}^2 \), this gives \( \alpha_s \approx 0.3 \). The value of \( f_\psi \) is fixed by Eq. (A.3). Performing the required calculations we obtain the following results.

**ABO-model.** Setting \( I_1 = 0 \) as a starting point we obtain \( \alpha = -0.34 \). The negative value shows that in this case in Eq. (85) \( |G_M|^2 \leq \frac{4m_c^2}{f_N^2} |G_E|^2 \), i.e. the integral \( J \) in Eq. (84) provides a large contribution, which gives the large value of \( G_E \). Hence, we can conclude that the power correction described by \( I_1 \) must also be large in order to make \( \alpha \) close to the experimental value. The value \( I_1 \) can be estimated by fitting the value of \( \alpha \). In ABO model one needs a very large value \( I_1 \approx 90 \) which yields a very strong numerical effect in the branching ratio: \( 10^3 \text{Br}[J/\psi \to p\bar{p}] \approx 37.2 \). The experimental branching ratio is about an order of magnitude smaller [16]

\[ 10^3 \text{Br}[J/\psi \to p\bar{p}]_{\text{exp}} \approx 2.112 \pm 0.004. \]  

(103)

A possible solution of this discrepancy is that the value of \( A_2 \) is somewhat overestimated in the ABO model. The value of the branching ratio can only be substantially reduced taking a smaller coupling \( f_N \). The changing of other parameters can not provide a sufficiently large effect. In order to illustrate this observation we consider \( 10^3 f_N \approx 3.8 \text{ GeV}^2 \), \( \phi_{10} = 0.076 \), \( \phi_{11} = 0.028 \), \( \lambda_1/f_N \approx -10.5 \), \( \eta_{10} \approx -0.027 \) and \( \eta_{11} \approx 0.09 \) (remember that \( \mu^2 = 4 \text{ GeV}^2 \)). This yields

\[ I_1 = 60, \quad \alpha \approx 0.59, \quad 10^3 \text{Br}[J/\psi \to p\bar{p}] \approx 7.36, \]  

(104)
which is more close to the data. Nevertheless, the effect from power corrections in this case is large, see Eq.\(84\)

\[
\frac{m_N^2}{M_{\psi}^2} I_1 \simeq 5.62, \quad \frac{2m_N^2}{M_{\psi}^2} J/I_0 \simeq 2.46,
\]

which in the sum is about factor eight larger than the leading-twist contribution in \(G_M\). Such a large effect is not surprising because the leading-twist contribution in the ABO-model strongly underestimates the value of the branching ratio

\[
10^3 \text{Br}[J/\psi \to p\bar{p}]_{\text{LT}} \simeq 0.19.
\]

**Lattice model.** In this case the coupling \(f_N\) is smaller and \(\eta_{10} = 0\) and \(\eta_{11} = 0\). This leads to a smaller branching ratio and negative value of the angular distribution \((I_1 = 0)\)

\[
10^3 \text{Br}[J/\psi \to p\bar{p}] = 0.75, \quad \alpha = -0.18.
\]

Hence, in this case the large impact from the power corrections is also expected. The reliable description of the data can be obtained, for instance, using the value \(I_1 = 35\). This gives

\[
\alpha = 0.60, \quad 10^3 \text{Br}[J/\psi \to p\bar{p}] = 2.79.
\]

The numerical effect provided by the power corrections is still large

\[
\frac{m_N^2}{M_{\psi}^2} I_1 = 3.28, \quad \frac{2m_N^2}{M_{\psi}^2} J/I_0 = 1.35.
\]

**COZ+ model.** In this model it is very important to take into account the higher coefficients \(\varphi_{2i}\), which provide the dominant numerical effect. Corresponding model \(\varphi_3\) provides very large value for the leading-twist integral \(I_0\). Therefore contrary to the previous cases the leading-twist approximation gives quite reasonable value of the width\(^1\)

\[
10^3 \text{Br}[J/\psi \to p\bar{p}]_{\text{LT}} \simeq 5.17.
\]

At the same time the effects of the WW-contributions in the integral \(J\) are also sufficiently large providing the large amplitude \(A_2\). The next-to-leading twist correction with \(I_1 = 0\) gives

\[
10^3 \text{Br}[J/\psi \to p\bar{p}] \simeq 52.1, \quad \alpha = -0.19.
\]

The negative value for the \(\alpha\) again indicates the very large contribution of the integral \(J\) in \(G_E\), see Eq.\(84\). This effect is dominantly provided by the WW-part of the integral \(J_5\). In order to improve the value of \(\alpha\) we need larger value of \(G_M\), which can be achieved with the help of parameter \(I_1\), but this will further enhance the already large branching ratio. For instance

\[
I_1 = 35, \quad \alpha \simeq 0.58, \quad 10^3 \text{Br}[J/\psi \to p\bar{p}] \simeq 189.5.
\]

Therefore, we conclude that the large effect provided by the COZ twist-3 DA \(\varphi_3\) in \(A_2\) requires large power correction in the amplitude \(A_1\), which makes the consistent description of the data impossible.

The considered set of the LCDA models exclude many other models for \(\varphi_3\), which are known in the literature, see e.g. Refs.\(30,32\). We do not like to carry on a complicated phenomenological analysis having the incomplete power suppressed contribution. The main conclusion,

\(^1\)The numerical difference of this result with Ref.\(26\) is explained by the different values of \(f_\psi\)
which follows from these calculations, is a qualitative estimate of the possible effect of the power corrections in the amplitudes $A_i$. We find that they are expected to be quite large compared to the well known leading-twist contribution in $A_1$. The large power suppressed contribution in the amplitude $A_1$ is required in order to obtain a correct description of the angular distribution in the cross section $e^+e^- \rightarrow J/\psi \rightarrow p\bar{p}$. Such a scenario is compatible with the experimental data if the leading-twist approximation provides a relatively small value of the decay width. Qualitatively this agrees with the models motivated by the QCD sum rules [22] and by the lattice calculations [24]. Such a picture suggests that the twist-four LCDAs, describing the three quarks with orbital momentum $L = 1$, are very important and probably provide a dominant numerical effect in the description of the quarkonia decay amplitudes. This conclusion must be verified by the calculation of the subleading power correction to the amplitude $A_1$.

5 Discussion

We calculate of the power suppressed amplitude $A_2$, which describes the Pauli ($\sigma^{\mu\nu}$) vertex in the $J/\psi \rightarrow p\bar{p}$ decay amplitude. It is shown that this amplitude can be described within the standard QCD framework which is based on the factorisation of the hard and soft processes. The obtained result is used for a qualitative phenomenological analysis of the angular distribution of the cross section $e^+e^- \rightarrow J/\psi \rightarrow p\bar{p}$ and for the decay width. Using various models of nucleon LCDAs we obtain that the amplitude $A_2$ provides very large numerical contribution to the helicity amplitude $G_E = A_1 + M_2^2/(4m_N^2)A_2$. As a consequence, the power correction to the amplitude $A_1$ must also be large in order to obtain a reliable description of the angular distribution. We consider three different estimates of LCDAs which were obtained from QCD sum rules (ABO [22] and COZ [25] models) and from the lattice calculations [24]. We find that the COZ model does not allow one to consistently describe the data because of too large numerical effect from the power suppressed terms. The best agreement is observed for the parameters of the lattice calculations, which give a smaller value for the twist-3 normalisation coupling $f_N$ compared to the value obtained from the QCD sum rules. As a result, the corresponding leading twist contribution is numerically small and the dominant contribution is entirely associated with the power suppressed terms.

In the present analysis we do not calculate the power correction to the amplitude $A_1$. Such a calculation is more complicated and will be considered in a separate publication. Our phenomenological analysis indicates that the corresponding contribution must also be large and must numerically dominate over the leading-twist contribution. If this conclusion is correct, this means that the Fock component of the nucleon wave function, associated with the three quarks state with $L = 1$ ($P$-wave), provides a dominant effect in the description of the $J/\psi$ decay. In this case the charmonium and bottomonium decays into baryon-antibaryon can provide an interesting and important insight about the baryon wave functions.

Appendix

A Long distance matrix elements

Here we provide a brief summary of the required nonperturbative matrix elements and LCDAs. For the heavy quark sector we only need the NRQCD matrix element

$$\langle 0 | \chi^\dagger(0) \gamma^\mu \psi(0) | P \rangle = \epsilon^\mu_{\psi} f_\psi.$$  \hspace{1cm} (A.1)
The operator in \( (A.1) \) is constructed from the quark \( \psi_\omega \) and antiquark \( \chi_\omega^\dagger \) four-component spinor fields satisfying \( \varphi \psi_\omega = \psi_\omega, \varphi \chi_\omega = -\chi_\omega \). The coupling \( f_\psi \) is related with the radial wave function at the origin

\[
f_\psi = \sqrt{2M_{J/\psi}} \sqrt{\frac{3}{2\pi}} R_{10}(0).
\] (A.2)

The value \( R_{10}(0) \) is well known from various potential models, for instance for the Buchmuller-Tye potential \[33\]

\[
|R_{10}(0)|^2 \simeq 0.81 \text{GeV}^3.
\] (A.3)

One can also estimate this coupling from \( J/\psi \rightarrow e^+e^- \) decay using the well known formula for the leptonic width

\[
\Gamma[J/\psi \rightarrow e^+e^-] = \frac{16 \alpha_{em}^2}{9 M_{J/\psi}^2} |R_{10}(0)|^2 \left( 1 - \frac{16 \alpha_s}{3 \pi} \right).
\] (A.4)

This gives \( \text{Br}[J/\psi \rightarrow e^+e^-] = 5.97\%, \alpha_s = 0.3, \alpha_{em} = 1/130 \)

\[
|R_{10}(0)|^2 \simeq 0.76 \text{ GeV}^3,
\] (A.5)

which is quite close to the value \( (A.3) \).

The nucleon matrix elements are more complicated. In the definitions given below we use kinematics and notations introduced in the Section 2. For simplicity, in this Appendix we consider the matrix elements only for the nucleon state and we also imply the light-cone gauge

\[
\langle 0 | \gamma \cdotp A^{(i)}(x) | 0 \rangle = 0,
\] (A.6)

in order to simplify the formulas.

The twist-3 DAs are defined as \( (i, j, k) \) are the colour indices

\[
\left\langle 0 \left| \varepsilon^{ijk} u_\alpha^i(z_1-) u_\beta^j(z_2-) d_\sigma^k(z_2-) \right| k \right\rangle_{\text{tw3}} = \frac{1}{4} \left\langle 0 \left| \gamma \gamma^5 C \gamma_\alpha \gamma_\beta \right| N_\hbar \right\rangle \text{FT}[V_1(y_i)]
\]

\[
+ \frac{1}{4} \left\langle 0 \left| \gamma \gamma^5 C \right| N_\hbar \right\rangle \text{FT}[A_1(y_i)] + \frac{1}{4} \left\langle 0 \left| i \gamma_\sigma \gamma_\hbar k C \right| \gamma_\alpha \gamma_\beta \right\rangle \text{FT}[T_1(y_i)],
\] (A.7)

where

\[
\text{FT}[F(y_i)] = \int Dy_i \ e^{-iy_ik_\sigma/z_1+2 - iy_2k_\sigma'z_2+2 - iy_3k_\sigma'z_3+2 /2} F(y_1, y_2, y_3),
\] (A.8)

with

\[
Dy_i = dy_1 dy_2 dy_3 \delta(1 - y_1 - y_2 - y_3).
\] (A.9)

We also explicitly write the large component of the nucleon spinor

\[
N_\hbar = \frac{\not{\hbar} \not{\hbar}}{4} N(k).
\] (A.10)

Three DAs \( V_1, A_1 \) and \( T_1 \) can be combined into the one twist-3 DA \( \varphi_3 \) as

\[
V_1(x_1, x_2, x_3) = f_N \frac{1}{2} \left[ \varphi_3(x_1, x_2, x_3) + \varphi_3(x_2, x_1, x_3) \right]
\] (A.11)

\[
A_1(x_1, x_2, x_3) = f_N \frac{1}{2} \left[ \varphi_3(x_2, x_1, x_3) - \varphi_3(x_2, x_1, x_3) \right]
\] (A.12)

\[
T_1(x_1, x_2, x_3) = f_N \frac{1}{2} \left[ \varphi_3(x_1, x_3, x_2) + \varphi_3(x_2, x_3, x_1) \right].
\] (A.13)
where large collinear components \( \chi \) finds (in this section we denote \( z \) the coordinates expansion of the operator in the lhs

By calligraphic letters we denote the auxiliary LCDAs, which can be rewritten in terms of defined projection. The corresponding correlator reads

Even correlators have already been considered in Ref. \[22\]. Consider, for simplicity, the vector expressions for these matrix elements we need to consider off light-cone correlators. The chiral twist-4 LCDAs are defined as

In our calculation we use the matrix elements of twist-4 operators constructed from the large collinear components \( \chi_\beta \) \[32\] and their derivative \( \partial_\perp \chi_\beta \), see Eq. \[39\]. In order to find expressions for these matrix elements we need to consider off light-cone correlators. The chiral even correlators have already been considered in Ref. \[22\]. Consider, for simplicity, the vector projection. The corresponding correlator reads

By calligraphic letters we denote the auxiliary LCDAs, which can be rewritten in terms of defined above in Eq.\[A.14\] twist-4 LCDAs. The explicit expressions will be given below. Performing expansion of the operator in the lhs \[A.24\] according to formulas \[21\], expanding on the rhs the coordinates \( z_1 \sim (z_1 n)/2 + z_{1\perp} \) in \( z_{1\perp} \) and comparing the linear in \( z_{1\perp} \) contributions one finds (in this section we denote \( \xi_\beta(x) \equiv \xi(x) \) in order to simplify notations)

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\begin{equation}
\langle 0 | \varepsilon^{ijk} \xi^i(z_{1-}) C \eta \left[ i \partial_{\perp \alpha} \xi^j(z_{2-}) \right] \xi^k_\sigma(z_{3-}) | k \rangle = k_+ m_N \left[ \gamma_{\perp \alpha} \gamma_5 N \right]_\sigma \text{FT} [V_2]. \tag{A.26}
\end{equation}

For the axial projection one has
\begin{equation}
\begin{aligned}
- \left\langle 0 \left| \varepsilon^{ijk} u^i(z_{1}) C \gamma^\alpha \gamma_5 u^j(z_{2}) d^k_\sigma(z_{3}) \right| k \right\rangle &= k^\alpha \left[ N \right]_\sigma \text{FT} [A_1] + m_N \left[ \gamma^\alpha N \right]_\sigma \text{FT} [A_3] \\
&+ m_N i k^\alpha (z_{1\beta} \text{FT} [A_1] + z_{2\beta} \text{FT} [A_2] + z_{3\beta} \text{FT} [A_3]) \left[ \gamma^\beta N \right]_\sigma, \tag{A.27}
\end{aligned}
\end{equation}

The expansion around the light-cone direction gives
\begin{equation}
\begin{aligned}
\langle 0 | \varepsilon^{ijk} [i \partial_{\perp \alpha} \xi^i(z_{1-})] C \gamma^\alpha \gamma_5 \xi^j(z_{2-}) \xi^k_\sigma(z_{3-}) | k \rangle &= k_+ m_N \left[ \gamma_{\perp \alpha} N \right]_\sigma \text{FT} [A_1], \tag{A.28} \\
\langle 0 | \varepsilon^{ijk} \xi^i(z_{1-}) C \gamma^\alpha \gamma_5 [i \partial_{\perp \alpha} \xi^j(z_{2-})] \xi^k_\sigma(z_{3-}) | k \rangle &= k_+ m_N \left[ \gamma_{\perp \alpha} N \right]_\sigma \text{FT} [A_2]. \tag{A.29}
\end{aligned}
\end{equation}

We also need to consider the chiral-odd correlator
\begin{equation}
\begin{aligned}
- \left\langle 0 \left| \varepsilon^{ijk} u^i(z_{1}) C \sigma^{\mu \nu} u^j_\beta(z_{2}) d^k_\beta(z_{3}) \right| k \right\rangle &= i p^\mu \left[ \gamma^\mu \gamma_5 N \right] \text{FT} [T_1] + \frac{1}{2} m_N \left[ \sigma^{\mu \nu} \gamma_5 N \right] \text{FT} [T_7] \\
&+ (z_{1} - z_{3})^\mu p^\nu m_N \left[ \gamma_5 N \right] \text{FT} [T_{21}] + (z_{2} - z_{3})^\mu p^\nu m_N \left[ \gamma_5 N \right] \text{FT} [T_{22}] \\
&+ m_N i p^\nu (z_{1} - z_{3})_\beta \left[ \sigma^{\mu \beta} \gamma_5 N \right] \text{FT} [T_{41}] + m_N i p^\nu (z_{2} - z_{3})_\beta \left[ \sigma^{\mu \beta} \gamma_5 N \right] \text{FT} [T_{42}] \\
&- (\mu \leftrightarrow \nu). \tag{A.30}
\end{aligned}
\end{equation}

This equation yields
\begin{equation}
\begin{aligned}
\langle 0 | \varepsilon^{ijk} [i \partial_{\perp \alpha} \xi^i(z_{1-})] C \gamma^\beta \gamma_5 \xi^j(z_{2-}) \xi^k_\sigma(z_{3-}) | k \rangle &= g^\beta_1 m_N k_+ \left[ \gamma_5 N \right]_\sigma \text{FT} [T_{21}] \\
&- m_N k_+ \left[ i \sigma^{\alpha \beta} \gamma_5 N \right]_\sigma \text{FT} [T_{41}], \tag{A.31}
\end{aligned}
\end{equation}

\begin{equation}
\begin{aligned}
\langle 0 | \varepsilon^{ijk} \xi^i(z_{1-}) C \gamma^\beta \gamma_5 [i \partial_{\perp \alpha} \xi^j(z_{2-})] \xi^k_\sigma(z_{3-}) | k \rangle &= g^\beta_1 m_N k_+ \left[ \gamma_5 N \right]_\sigma \text{FT} [T_{22}] \\
&- m_N k_+ \left[ i \sigma^{\alpha \beta} \gamma_5 N \right]_\sigma \text{FT} [T_{42}]. \tag{A.32}
\end{aligned}
\end{equation}

The LCDAs which are denoted by calligraphic letters can be rewritten in terms of the light-cone LCDAs which are defined by the light-cone matrix element \( \langle A.14 \rangle \). For the LCDAs \( V_{1,2} \) and \( A_{1,2} \) such expressions are already derived in Ref. \cite{22}. We also recalculated these relations and find the same expressions. They read
\begin{equation}
V_1(x_i) + V_3(x_i) + V_3(x_i) = 0, \tag{A.33}
\end{equation}
\begin{equation}
4V_k(x_i) = x_3 V_2(x_i) + (-1)^k \left\{ (x_1 - x_2) V_3(x_i) - x_3 A_2(x_i) + x_3 A_3(x_i) \right\}, \tag{A.34}
\end{equation}
\begin{equation}
A_1(x_i) + A_3(x_i) + A_3(x_i) = 0, \tag{A.35}
\end{equation}
\begin{equation}
4A_k(x_i) = -x_3 A_2(x_i) + (-1)^k \left\{ (x_1 - x_2) A_3(x_i) + x_3 V_2(x_i) + \bar{x}_3 V_3(x_i) \right\}. \tag{A.36}
\end{equation}

Notice that
\begin{equation}
V_2(x_1, x_2, x_3) = V_1(x_2, x_1, x_3), \quad A_2(x_1, x_2, x_3) = - A_1(x_2, x_1, x_3), \tag{A.37}
\end{equation}
which follows from
\begin{equation}
V_2(2, 1, 3) = V_1(1, 2, 3), \quad A_2(2, 1, 3) = - A_1(1, 2, 3). \tag{A.38}
\end{equation}
The similar relations for the chiral-odd LCDAs $T_{ij}$ have not yet been considered. Our calculations yield (see the details below)

\begin{align}
T_{21}(x_i) - T_{41}(x_i) &= \frac{x_1}{2} (T_3 + T_7 + S_1 - P_1)(x_i) \tag{A.39} \\
&= \frac{x_1}{2} \left[ (V_3 - A_3)(3, 1, 2) + (V_2 - A_2)(2, 3, 1) \right], \tag{A.40} \\
T_{22}(x_i) - T_{42}(x_i) &= \frac{x_2}{2} (T_3 + T_7 - S_1 + P_1)(x_i) \tag{A.41} \\
&= \frac{x_2}{2} \left[ (V_3 - A_3)(3, 2, 1) + (V_2 - A_2)(1, 3, 2) \right], \tag{A.42} \\
T_{41} + T_{21} &= \frac{x_1}{2} (T_3 - T_7 + P_1 + S_1) = \frac{x_1}{2} \Xi_4(1, 2, 3), \tag{A.43} \\
T_{42} + T_{22} &= \frac{x_2}{2} (T_3 - T_7 - S_1 - P_1) = \frac{x_2}{2} \Xi_4(2, 1, 3), \tag{A.44}
\end{align}

where it was used that

\begin{equation}
T_{i}(2, 1, 3) = T_{i}(1, 2, 3), \quad S_{1}(2, 1, 3) = -S_{1}(1, 2, 3), \quad P_{1}(2, 1, 3) = -P_{1}(1, 2, 3). \tag{A.45}
\end{equation}

Consider the derivation of Eqs. \(\text{A.40}\)-\(\text{A.44}\). Let us introduce two twist-4 light-cone operators defined as

\begin{align}
O_1 &= \left[ \frac{\bar{\psi}\psi}{4} u(x_-) \right] C\sigma^{\mu\nu} \frac{\bar{\psi}\psi}{4} u(y_-) \left[ \frac{\bar{\psi}\psi}{4} d(z_-) \right]_{\sigma}, \tag{A.46} \\
O_2 &= \left[ \frac{\bar{\psi}\psi}{4} u(x_-) \right] C\sigma^{\mu\nu} \frac{\bar{\psi}\psi}{4} u(y_-) \left[ \frac{\bar{\psi}\psi}{4} d(z_-) \right]_{\sigma}, \tag{A.47}
\end{align}

where the projectors $\frac{\bar{\psi}\psi}{4}$ and $\frac{\bar{\psi}\psi}{4}$ are used in order to decompose collinear fields into large and small components, respectively

\begin{equation}
\frac{\bar{\psi}\psi}{4} u = \xi, \quad \frac{\bar{\psi}\psi}{4} u(x_-) = \eta. \tag{A.48}
\end{equation}

Using Eq. \(\text{A.34}\) we rewrite the first operator as

\begin{equation}
O_1 = -T_{i}^{\mu\nu\lambda} \left[ (in\theta)^{-1} \partial_{\perp\alpha} \xi(x_-) \right] C\sigma^{+\lambda} \xi(y_-) \left[ \xi(z_-) \right]_{\sigma}. \tag{A.49}
\end{equation}

where

\begin{equation}
T_{i}^{\mu\nu\lambda} = \left\{ g^{\alpha\nu} g^{\lambda\mu} + \frac{1}{2} \bar{n}^{\nu} n^{\nu} g^{\lambda\mu} - (\mu \leftrightarrow \nu) \right\}. \tag{A.50}
\end{equation}

Taking the matrix element with the help of Eq. \(\text{A.25}\) one obtains

\begin{equation}
- \langle 0 | O_1 | k \rangle = i \bar{n}^{\nu} n^{\mu} \frac{m_N}{2} \left[ \gamma_5 N_{\bar{n}} \right] \sigma \text{ FT} \left[ \frac{1}{x_1} T_{21} \right] - m_N \frac{1}{2} \left[ \sigma_{\perp\perp} \gamma_5 N_{\bar{n}} \right] \sigma \text{ FT} \left[ \frac{1}{x_1} T_{41} \right] - (\mu \leftrightarrow \nu). \tag{A.51}
\end{equation}

On the other hand rewriting the operator \(\text{A.46}\) with the basic Dirac structures one finds

\begin{align}
O_1 &= \frac{i}{4} (\bar{n}^{\nu} n^{\mu} - \bar{n}^{\mu} n^{\nu}) \ u(x_-) Cu(y_-) \left[ \frac{\bar{\psi}\psi}{4} d(z_-) \right]_{\sigma} \\
&+ \frac{1}{2} \bar{\psi}_{\mu\nu} u(x_-) Cu(y_-) \left[ \frac{\bar{\psi}\psi}{4} d(z_-) \right]_{\sigma} + \frac{1}{2} u(x_-) C\sigma^{\mu\nu} u(y_-) \left[ \frac{\bar{\psi}\psi}{4} d(z_-) \right]_{\sigma} \\
&+ \frac{1}{8} (\bar{n}^{\nu} n^{\mu} - \bar{n}^{\mu} n^{\nu}) \ u(x_-) Cu(y_-) \left[ \frac{\bar{\psi}\psi}{4} d(z_-) \right]_{\sigma}. \tag{A.52}
\end{align}
Taking the matrix element in the rhs of this equation with the help of Eq. (A.14) and comparing with the Eq. (A.51) one obtains
\[ T_{21} - T_{41} = \frac{x_1}{2} (T_3 + T_7 + S_1 - P_1), \quad T_{21} + T_{41} = \frac{x_1}{2} (T_3 - T_7 + P_1 + S_1). \] (A.53)

The similar consideration for the operator \( O_2 \) in Eq. (A.47) gives
\[ T_{22} - T_{42} = \frac{x_2}{2} (T_3 + T_7 - S_1 + P_1), \quad T_{42} + T_{22} = \frac{x_2}{2} (T_3 - T_7 - S_1 - P_1). \] (A.54)

## B The cancellation of the ultrasoft gluon contributions

Here we briefly discuss the ultrasoft gluon limit. The contribution of the sum of diagrams as in Fig. 1 can be written as
\[ iM = \frac{f_\psi f_N \lambda_1 m_N}{m_Q^2 m_Q} J, \] (B.1)
where the dimensionless collinear convolution integral can be schematically written as
\[ J = m_Q^2 \int Dx_i \int Dy_i \; \Delta g_1 \Delta g_2 \Delta g_3 \; \hat{T}_{3g ightarrow pp}(x_i, y_i) \frac{1}{4} \text{Tr} \left[ \frac{1 - \gamma_5}{\epsilon} \psi D(k_i, k_j) \right]. \] (B.2)

For simplicity, we do not show the various indices. Notice also that the colour factors for all diagrams are the same. In Eq. (B.2) we have three gluon propagators
\[ \Delta g_i = \frac{(-i)}{(k_i + k'_i)^2} \simeq \frac{(-i)}{2(kk')} \frac{1}{x_i y_i}. \] (B.3)

The function \( \hat{T}_{3g ightarrow pp} \) describes the contribution from the light-quark vertices and from the projections of the nucleon matrix elements. The function \( D(k_i', k_j) \) describes the heavy quark lines with the quark-gluon vertices.

The scaling behaviour of the contribution in Eq. (B.1) is given by the following factors
\[ \frac{f_\psi}{m_Q^2} \sim v^3, \quad \frac{f_N \lambda_1 m_N}{m_Q^4 m_Q} \sim \left( \frac{\Lambda}{m_Q} \right)^5 \sim \lambda^{10}. \] (B.4)

The collinear integral is defined to be of order one: \( J \sim v^0 \). Therefore the ultrasoft region in \( J \) must give the contribution of order one.

Consider the ultrasoft gluon limit
\[ p_g = k_1 + k'_1 \sim m_Q v^2, \] (B.5)
that gives the counting for the small momentum fractions \( x_1 \sim y_1 \sim v^2 \). Such limit corresponds to the contribution from the endpoint region
\[ J_{us} \sim \int_0^\eta dx_i \int_0^\eta dy_i \frac{1}{x_i y_i} \int_0^1 dx_2 \int_0^1 dy_2 \; \Delta g_2 \Delta g_3 \]
\[ \times \hat{T}_{3g ightarrow pp}(x_i, y_i) \frac{1}{4} \text{Tr} \left[ (1 - \gamma_5) \psi D(k_i', k_j) \right], \] (B.6)
where the cut-off \( \eta \) can be understood as a factorisation scale separating the hard and ultrasoft domains. Our task is to estimate the scale behaviour of \( J_{us} \). For that we need to expand the integrand with respect to small fractions \( x_1 \sim y_1 \sim v^2 \).
The light-quark part $\hat{T}_{3g \rightarrow pp}(x_i, y_i)$ includes the twist-4 nucleon LCDAs $V_i(x_i)$, $A_i(x_i)$ and $T_{ij}(x_i)$ and twist-3 $\varphi_3(y_i)$. These functions must be also expanded with respect to the small fractions. We assume that the expansions of the LCDAs can provide only the positive powers of the small fractions. Therefore, it is quite reasonable here to consider only the asymptotic terms, which gives the contributions with the minimal powers of all fractions. Since one of our LCDAs is of twist-3, we immediately find that

$$
\hat{T}_{3g \rightarrow pp} \simeq \varphi_3(y_1) \hat{T}_{3g \rightarrow pp}^{tw4}(x_2) \sim y_1y_2y_3 \hat{T}_{3g \rightarrow pp}^{tw4}(x_2),
$$

where we assume that $x_3 \simeq 1 - x_2$. The asymptotic expressions for the twist-4 LCDAs can be easily obtained from the formulas in Appendix [A]. One finds

$$
V_i(x_i) \sim x_1x_2x_3, \quad A_i(x_i) \sim x_1x_2x_3, \quad T_{ij}(x_i) \sim x_1x_2x_3.
$$

(B.8)

The factor $\hat{T}_{3g \rightarrow pp}^{tw4}$ has the following schematic structure

$$
\hat{T}_{3g \rightarrow pp}^{tw4}(x_2) \simeq \sum_i \frac{1}{x_i}X_i(x_i) + X_i(x_i) \frac{\partial}{\partial k_{Li}},
$$

(B.9)

where $X_i$ denote one of twist-4 DAs in Eq. (B.3). The powers $1/x_i$ originate from the inverse derivatives $(i\eta \partial)^{-1}$ in the twist-4 operator in Eq. (33). From Eqs. (B.8) and (B.9) can be also seen that the terms with transverse derivatives are always suppressed by factor $v^2$ comparing to terms with $1/x_i$ and therefore can be neglected. Then one finds that $\hat{T}_{3g \rightarrow pp}^{tw4}(x_2) \sim O(v^0)$ which gives

$$
\hat{T}_{3g \rightarrow pp} \sim y_1y_2y_3 \hat{T}_{3g \rightarrow pp}^{tw4}(x_2) \sim O(v^2).
$$

(B.10)

Consider now the sum of the heavy quark subdiagrams $D(k_i', k_j)$. Performing expansions with respect to small fractions $x_1$ and $y_1$ one obtains that the most singular terms appear from the diagrams describing the attachments of the ultrasoft gluon to external vertices on the heavy quark line. It is convenient to divide such diagrams into two groups: the soft gluon vertex is associated with the external heavy quark or with the external heavy antiquark. Then the sum of all relevant diagrams reads

$$
\frac{1}{4} \text{Tr} \left[ (1 - \phi) \delta_{\psi} D^{\mu_1 \mu_2 \mu_3} \right] = \frac{1}{4} \text{Tr} \left[ (1 - \phi) \delta_{\psi} \gamma^{\mu_1} (mQ\phi + mQ - k_1 - k_1') D_{h}^{\mu_2 \mu_3} \right] \frac{-P(k_1 + k_1') + 2(k_1 k_1')}{-P(k_1 + k_1') + 2(k_1 k_1')},
$$

(B.11)

where $D_{h}^{\mu_2 \mu_3}$ describes the sum of the subdiagrams with the hard gluons. The expansion with respect to the small fractions yields

$$
\frac{1}{4} \text{Tr} \left[ (1 - \phi) \delta_{\psi} D^{\mu_1 \mu_2 \mu_3} \right]_{us} \simeq -\frac{1}{(kk')} \frac{1}{x_1 + y_1} \frac{1}{4} \text{Tr} \left[ (1 - \phi) \delta_{\psi} \gamma^{1} (mQ\phi + m) D_{h}^{\mu_2 \mu_3} \right] \frac{-P(k_1 + k_1') + 2(k_1 k_1')}{-P(k_1 + k_1') + 2(k_1 k_1')}
$$

$$
+ \frac{1}{4} \text{Tr} \left[ (1 - \phi) \delta_{\psi} D_{h}^{\mu_2 \mu_3} (-mQ\phi + mQ + k_1 + k_1') \gamma^{\mu_1} \right] \frac{-P(k_1 + k_1') + 2(k_1 k_1')}{-P(k_1 + k_1') + 2(k_1 k_1')},
$$

(B.12)

where $D_{h}^{\mu_2 \mu_3}$ describes the sum of the subdiagrams with the hard gluons. The expansion with respect to the small fractions yields

$$
\frac{1}{4} \text{Tr} \left[ (1 - \phi) \delta_{\psi} D^{\mu_1 \mu_2 \mu_3} \right]_{us} \simeq -\frac{1}{(kk')} \frac{1}{x_1 + y_1} \frac{1}{4} \text{Tr} \left[ (1 - \phi) \delta_{\psi} \gamma^{1} (mQ\phi + m) D_{h}^{\mu_2 \mu_3} \right] \frac{-P(k_1 + k_1') + 2(k_1 k_1')}{-P(k_1 + k_1') + 2(k_1 k_1')}
$$

$$
+ \frac{1}{4} \text{Tr} \left[ (1 - \phi) \delta_{\psi} D_{h}^{\mu_2 \mu_3} (-mQ\phi + mQ + k_1 + k_1') \gamma^{\mu_1} \right] + O(v^0)
$$

(B.12)

$$
\simeq -\frac{1}{(kk')} \frac{2m\omega}{(x_1 + y_1)} \text{Tr} \left[ (1 - \phi) \delta_{\psi} D_{h}^{\mu_2 \mu_3} \right] - \frac{1}{(kk')} \frac{-2m\omega}{(x_1 + y_1)} \text{Tr} \left[ (1 - \phi) \delta_{\psi} D_{h}^{\mu_2 \mu_3} \right] + O(v^0) = O(v^0).
$$

(B.13)
We see that each separate term in $D_{us}$ has the contribution of order $v^{-2}$ due to the factor $1/(x_1 + y_1)$ but these terms cancel in the sum. Substituting (B.10) and (B.13) in (B.6) one obtains

$$J_{us} \sim \int_0^\eta \frac{dx_1}{x_1} \int_0^\eta dy_1 \times O(v^0) \sim v^2,$$

(B.14)

Hence the contribution of the ultrasoft region is power suppressed. The same conclusion is also true for other regions where $x_i \sim y_i \sim v^2$. This result is in agreement with the Coulomb limit described by the potential NRQCD [34–39]. In this case the ultrasoft gluon vertices are suppressed by the small velocity $v$.

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