Angular distribution analysis of $B \rightarrow J/\psi K^*$ and resolving discrete ambiguities in the determination of $\phi_1$

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We discuss the angular distribution analysis of $B \rightarrow J/\psi K^*$ decays and a way to resolve discrete ambiguities in the determination of the unitary triangle $\phi_1(=\beta)$. We study the status of factorization hypothesis in the color-suppressed $B$ meson decays: $B \rightarrow J/\psi K^*(*)$ within the general factorization approach and QCD-factorization method.

1. Introduction

Among a hundred nonleptonic two body decays of $B$ mesons, the process $B \rightarrow J/\psi K^*$ has lots of interests in many aspects: First of all, it was firstly observed color-suppressed process with a large branching ratio in $B$ meson decays.\(^{1-5}\) The vector-vector decays $B^0 \rightarrow J/\psi K^{*0}(K^{*0} \rightarrow K_s^0 \pi^0)$ is a mixture of CP-even and CP-odd eigenstates since it can proceed via an S,P,D wave decays. By using both angular and time distribution analysis, we can separate the CP-even one from CP-odd eigenstate and determine the angle $\phi_1(=\beta)$ of the unitarity triangle without any dilution effects in a manner similar to which the CP-odd eigenstate $B^0 \rightarrow J/\psi K^0$ is used. In addition, the angular distribution analyses on both $B^0 \rightarrow J/\psi K^*$ and $B_s^0 \rightarrow J/\psi \phi$ can be used to resolve 4-fold ambiguities in the measurement of $\sin 2\phi_1$.

The recent measurement by BaBar\(^4\) has confirmed the earlier CDF\(^2\) observation that there is a nontrivial strong phase difference between polarized amplitudes indicating final-state interactions. However no such evidence has been seen yet by CLEO\(^3\) and Belle\(^4\). It is interesting to check if the current QCD-approaches for $B$ hadronic decays predicts a departure from factorization. Therefore, the measurements of various helicity amplitudes in $B \rightarrow J/\psi K^*$ decays will provide a powerful tool for testing factorization and differentiating various theoretical models\(^6-9\) in which the calculated nonfactorizable term have real and imaginary parts.

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On the other hand, precise measurement of the CP asymmetry in the \( B \to J/\psi K^{(*)} \) decays is important for new physics search by comparing with one of \( B \to \phi K \) with high degree of accuracy.\(^{10,11}\) This measurement is experimentally accessible at the early stage of the asymmetric \( B \) factories. The \( B \to \phi K \) decays arise from penguin (loop) effects, while the \( B \to J/\psi K^{(*)} \) decays involve dominant tree amplitudes. The search for different CP asymmetries in the \( B \to J/\psi K^{(*)} \) and \( \phi K \) decays, with the common source from \( B^0 - \bar{B}^0 \) mixing, provides a promising way to discover new physics\(^ {12,13} \): a difference of \( |A_{CP}(J/\psi K^{(*)}) - A_{CP}(\phi K)| > 5\% \) would be an indication of new physics.

2. Angular Analysis and Resolving Discrete Ambiguities

The measurement of \( \sin 2\beta \) has a four-fold ambiguity: \( \phi_1, \pi/2 - \phi_1, \pi + \phi_1, 3\pi/2 - \phi_1 \) with \( 0 < \beta < 2\pi \). In order to resolve this ambiguity, one need to determine the signs of \( \cos 2\phi_1 \) and \( \sin \phi_1 \) in addition to the value of \( \sin 2\phi_1 \).

2.1. Determination of \( \text{sign}(\cos 2\phi_1) \)

Using interference between opposite CP amplitudes in \( J/\psi K^{(*)} \) and \( J/\psi \phi \) can help to determine the sign of \( \cos 2\phi_1 \). The interference term between CP-even and CP-odd amplitude which can be obtained from the transversity analysis contains a term in \( \cos 2\phi_1 \). For instance,

\[
\text{Im}[A_\perp(t)A_{||}^*(t)] \sim \text{Im}[A_\perp(0)A_{||}^*(0)] \cos \Delta m t - \text{Re}[A_\perp(0)A_{||}^*(0)] \eta \cos 2\phi_1 \sin \Delta m t. \tag{1}
\]

Observeables in transversity frame for \( J/\psi(K^{(*)})_{CP} \) is given by in Table I.

| Time-dependent Obs. | Time-dependence | Time-independence |
|---------------------|----------------|-------------------|
| \( |A_{||}|^2 \) (CP=+) | \( \sin \Delta m t \) | \( \sin 2\phi_1 \) |
| \( |A_{\perp}|^2 \) (CP=-) | \( \sin \Delta m t \) | \( \cos \phi(A_{||}) - \phi(A_0) \) |
| \( |A_0|^2 \) (CP=+) | \( \sin \Delta m t \) | \( \sin \phi(A_{||}) - \phi(A_0) \) \( \sin 2\phi_1 \) |
| \( \text{Re}[A_{||}A_0^*] \) | constant | \( \cos \phi(A_{||}) - \phi(A_0) \) |
| \( \text{Im}[A_{||}A_0^*] \) | \( \sin \Delta m t \) | \( \cos \phi(A_{||}) - \phi(A_0) \) \( \cos 2\phi_1 \) |
| \( \text{Im}[A_{\perp}A_0^*] \) | \( \cos \Delta m t \) | \( \sin \phi(A_{||}) - \phi(A_0) \) \( \cos 2\phi_1 \) |
| \( \text{Im}[A_{\perp}A_0^*] \) | \( \cos \Delta m t \) | \( \sin \phi(A_{||}) - \phi(A_0) \) |

Since experiments measured interference terms in the angular distribution with \( \text{Re}(A_{||}A_0^*), \text{Im}(A_{\perp}A_0^*) \) and \( \text{Im}(A_{\perp}A_{||}^*) \), there exists a phase ambiguity with \( \phi(A_0) = 0 \):

\[
\phi_{||} \rightarrow -\phi_{||}, \quad \phi_{\perp} \rightarrow \pm \pi - \phi_{\perp}, \quad \phi_{\perp} - \phi_{||} \rightarrow \pm \pi - (\phi_{\perp} - \phi_{||}). \tag{2}
\]
Angular distribution analysis of $B \to J/\psi K^*$ and resolving discrete ambiguities on the determination $\phi_1$.  

It is easy to check that there is a sign ambiguity on $\cos[\phi(A_{\perp}) - \phi(A_{||})]$ and on $\cos\phi(A_{\perp})$. Therefore a sign ambiguity on $\cos 2\phi_1$ remains. 

There are two solutions for the relative phases according to BaBar measurement\textsuperscript{4} as an example:

$$\phi_{\perp} = -0.17 \pm 0.17, \quad \phi_{||} = 2.50 \pm 0.22, \quad \Rightarrow \quad |H_+| < |H_-|, \quad (3)$$

where the phases are measured in radians. The other allowed solution is

$$\phi_{\perp} = -2.97 \pm 0.17, \quad \phi_{||} = -2.50 \pm 0.22, \quad \Rightarrow \quad |H_+| > |H_-|. \quad (4)$$

As pointed out by Suzuki\textsuperscript{14}, the solution (3) indicates that $A_{||}$ has a sign opposite to that of $A_{\perp}$ and hence $|H_+| < |H_-|$, in contradiction to what expected from factorization. Therefore, we prefer to solution (4) to compare with the factorization approach. Obviously there is a $3\sigma$ effect that $\phi_{||}$ is different from $\pi$ and this agrees with the CDF measurement\textsuperscript{3}. However, such an effect is not observed by Belle\textsuperscript{5} and CLEO\textsuperscript{2} (see Table 4).

In fact, We can determine unambiguously the strong phase of $\cos 2\phi_1$ term by studying the angular distribution analysis of $B_s \to J/\Psi\phi$ with SU(3) flavour symmetry\textsuperscript{15}.

### 2.2. Determination of sign($\sin\phi_1$)

The determination of sign($\sin\phi_1$) leaves the ambiguity of $\phi_1 \to \pi + \phi_1$. However, it needs some model-dependent input. By comparing the coefficients of $\sin \triangle m t$ of $J/\Psi K_0$ vs $D^+D^-$, we obtain :

$$S_{J/\Psi K_0} = - \sin 2\phi_1,$$

$$S_{D^+D^-} = \frac{\sin 2\phi_1 - 2|R_{DD}| \sin \phi_1 \cos \delta_{DD}}{1 + |R_{DD}|^2 - 2|R_{DD}| \cos \phi_1 \cos \delta_{DD}} \quad (5)$$

which can give the sign($\sin\phi_1$) if sign($\cos 2\phi_1$) and sign($\cos\delta_{DD}$) are known. The sign($\cos 2\phi_1$) could be determined by method(2-1).

The determination of sign($\cos\delta_{DD}$) need model-dependent input.

$$S_{J/\Psi K_0} + S_{D^+D^-} = 2 |R_{DD}| \cos \delta_{DD} \cos 2\phi_1 \sin \phi_1. \quad (6)$$

If $\cos\delta_{DD} > 0$, we obtain the relation :

$$\text{sign}[S_{J/\Psi K_0} + S_{D^+D^-}] = \text{sign}[\cos 2\phi_1 \sin \phi_1] \quad (7)$$

In Standard Model sign($\cos 2\phi_1$) is positive.

### 3. Test of Factorization in $B \to J/\psi K^*$ decay

By using the angular distribution analysis in the transversity basis, we can measure precisely both their magnitudes and phases of the three different helicity amplitudes, denoted by $H_0, H_-, \text{ and } H_+$. These observations can provide a crucial way to test
not only the naive factorization method but also recent improved QCD-approaches in which non-factorizable term can be calculable. Also it would answer the question of the existence of the final state interactions, which is strong enough to flip the quark spin in color-suppressed B decays.

3.1. General Factorization Approach for $J/\psi K^{\ast}$

It has been well known that the factorization approach (naive or generalized) fails to explain simultaneously the production ratio $R = \mathcal{B}(B \rightarrow J/\psi K^{\ast})/\mathcal{B}(B \rightarrow J/\psi K)$ and the fraction of longitudinal polarization $\Gamma_{L}/\Gamma$ in $B \rightarrow J/\psi K^{\ast}$ decay

| Experiments | CDF | CLEO | BaBar | Belle |
|-------------|-----|------|-------|-------|
| $R$         | 3.40 | 3.11 | 1.53 $\pm$ 0.32 | 1.45 $\pm$ 0.26 | 1.38 $\pm$ 0.11 | 1.43 $\pm$ 0.13 |
| $\Gamma_{L}/\Gamma$ | 0.47 | 0.46 | 0.61 $\pm$ 0.14 | 0.52 $\pm$ 0.08 | 0.60 $\pm$ 0.04 | 0.60 $\pm$ 0.05 |

In the general factorization approach, non-factorizable term is directly proportional to the factorizable piece and have the same phase as factorized one. So the theoretical difficulty can be understandable because we assumed the parameter $a_{2}$ to be universal according to the factorization hypothesis, namely $a_{2}^{h}(J/\psi K^{\ast}) = a_{2}(J/\psi K)$ where $h = 0, +, -$ refer to the helicity states $00, ++$ and $--$ respectively. In this case, the amplitudes are relatively real and there is no significant signature of the final state interaction, which is agreed with experimental results of CLEO and Belle, but contradicted to BaBar and CDF results.

3.2. QCD-improved factorization approach for $J/\psi K^{\ast}$

The QCD-improved factorization approach allows us to compute the nonfactorizable corrections in the heavy quark limit since only hard interactions between the $(BV_{1})$ system and $V_{2}$ survive in the $m_{b} \rightarrow \infty$ limit. In this approach, the light-cone distribution amplitudes (LCDAs) play an essential role. It is shown that non-factorizable terms contribute differently to each helicity amplitude and to different decay modes so that $a_{2}^{0}(J/\psi K^{\ast}) > a_{2}^{+}(J/\psi K^{\ast}) \neq a_{2}^{-}(J/\psi K^{\ast})$ and $a_{2}(J/\psi K) > a_{2}^{h}(J/\psi K^{\ast})$. With non-relativistic and asymptotic type $J/\psi$ LACDs, and upto twist-3 LCDAs for $K^{\ast}$, we find that (i) for $B \rightarrow J/\psi K$, twist-3 hard spectator interaction are equally important as twist-2 contributions, (ii) however, for $B \rightarrow J/\psi K^{\ast}$, the spectator and final state interactions from leading twist contributions play an important role in the dominant longitudinal component, which is safe from the infrared divergence and induce $|a^{0}(J/\psi K^{\ast})| \sim 0.14$ different from $|a^{0}(J/\psi K)| \sim 0.2$. 
Angular distribution analysis of $B \rightarrow J/\psi K^*$ and resolving discrete ambiguities on the determination $\phi_1$. 5

Table 4.
Normalized spin amplitudes and their phases (in radians) in $B \rightarrow J/\psi K^*$ decays calculated in various form-factor models using QCD factorization. The branching ratios given in the Table are for $B^+ \rightarrow J/\psi K^{*+}$. For comparison, experimental results form CDF, CLEO, BaBar and Belle are also exhibited.

| Model       | $|A_h|^2$ | $|A_\perp|^2$ | $|A_\parallel|^2$ | $\phi_\perp$ | $\phi_\parallel$ | $B(10^{-3})$ |
|-------------|----------|---------------|-------------------|-------------|------------------|--------------|
| BSWI^{29}   | 0.43     | 0.33          | 0.24              | -3.05       | -2.89            | 0.76         |
| BSWI^{25}   | 0.38     | 0.36          | 0.26              | 3.13        | -3.12            | 0.73         |
| LF^{26}     | 0.41     | 0.34          | 0.25              | -3.09       | -2.95            | 0.69         |
| NS^{27}     | 0.40     | 0.34          | 0.25              | -3.10       | -2.99            | 0.70         |
| Yang^{28}   | 0.38     | 0.36          | 0.25              | -3.12       | -3.11            | 0.64         |
| BB^{20}     | 0.41     | 0.34          | 0.25              | -3.04       | -3.05            | 0.77         |
| MS^{31}     | 0.40     | 0.35          | 0.25              | -3.08       | -3.05            | 0.75         |
| YYK^{37}    | 0.44     | 0.32          | 0.23              | -2.99       | -2.95            | 0.84         |
| CLEO        | 0.52 ± 0.08 | 0.16 ± 0.09  | -3.03 ± 0.46      | -3.00 ± 0.37 | 1.41 ± 0.31    |
| CDF         | 0.59 ± 0.06 | 0.13^{+0.13}_{-0.11} | 0.28 ± 0.12 | -2.58 ± 0.54 | -2.29 ± 0.47 |
| BaBar       | 0.60 ± 0.04 | 0.16 ± 0.03  | 0.24 ± 0.04       | -2.97 ± 0.17 | -2.50 ± 0.22 | 1.37 ± 0.14 |
| Belle       | 0.60 ± 0.05 | 0.19 ± 0.06  | -3.15 ± 0.21      | -2.86 ± 0.25 | 1.29 ± 0.14 |

As shown in Table 4, we obtained small $a_h^b$ which can explain only half of the data for the branching ratio. We also got relatively small fraction of the longitudinal polarizaion component, but large fraction for $|A_\parallel|^2$.

From somehow negative results, we conclude that it is needed to understand more correctly the LCDAs of heavy ($c\bar{c}$)-state and the power $\Lambda/m_b$ corrections within QCD-factorization method.

To get more understanding on factorization in color-suppressed decays including charmonium states, we suggest that the study on $B \rightarrow \eta_c K^{(*)}$ will provide a good test of the factorization hypothesis.\textsuperscript{24} We expect $Br(B \rightarrow K^+ \eta_c) = (1.14 \pm 0.31) \times 10^{-3}$, which can be observed in near future.

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