COSMOLOGICAL REDSHIFT-SPACE DISTORTION ON CLUSTERING OF HIGH-REDSHIFT OBJECTS: CORRECTION FOR NONLINEAR EFFECTS IN THE POWER SPECTRUM AND TESTS WITH N-BODY SIMULATIONS

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ABSTRACT

We examine the cosmological redshift-space distortion effect on the power spectrum of objects at high redshifts, which is an unavoidable observational contamination in general relativistic cosmology. In particular, we consider the nonlinear effects of density and velocity evolution using high-resolution N-body simulations in cold dark matter models. We find that theoretical modeling on the basis of the fitting formulae of nonlinear density and velocity fields accurately describes the numerical results, especially in quasi-nonlinear regimes. These corrections for nonlinear effects are essential in order to use the cosmological redshift-space distortion as a cosmological test. We perform a feasibility test to derive constraints from the future catalogs of high-redshift quasars using our theoretical modeling and results of N-body simulations. Applying the present methodology to the future data from ongoing surveys of high-redshift galaxies and quasars will provide a useful tool to constrain a geometry of the universe.

Subject headings: cosmology: theory — dark matter — large-scale structure of universe — galaxies: distances and redshifts — methods: n-body simulations — quasars: general

1. INTRODUCTION

Observational cosmology is now entering a new and exciting phase where the high-redshift universe can be directly and systematically mapped by observation. Lyman break galaxies at \( z \approx 3 \) (Steidel et al. 1996, 1998) already placed a strong constraint on the nature and evolution of the hosting halos at high redshifts (Jing & Suto 1998). The Sloan Digital Sky Survey (SDSS) will complete a homogeneous catalog of Sloan Digital Sky Survey (SDSS) will complete a homogeneous catalog of \( \sim 10^5 \) quasars (QSOs) even extending to \( z \geq 5 \); in fact, the highest \( z \) QSO as of this writing (SDSSp J033829.31 +002156.3 at \( z = 5.00 \)) was discovered from the SDSS commissioning data (Fan et al. 1999). Haiman & Loeb (1999) predict that the Chandra X-Ray Observatory will detect \( \sim 100 \) QSOs at \( z \geq 5 \) per its \( 17' \times 17' \) field of view.

Those data sets will necessarily and significantly change the way of research of the clustering of objects at high redshifts. Because of the statistical limitations of existing catalogs of QSOs, it has been common to infer their clustering amplitude and evolution in a model-dependent and indirect manner (but see, e.g., La Franca, Andreani, & Cristiani 1998); an interesting example of this methodology is the use of the fluctuation in the X-ray background (e.g., Lahav, Piran, & Teyerer 1997; Teyerer et al. 1998). In the near future, however, one will be able to analyze the QSO redshift catalogs in order to detect clustering at \( z \sim 5 \) as is routinely repeated for the existing galaxy redshift samples at \( z \lesssim 0.1 \).

In extracting the cosmological information from such catalogs of objects at high redshifts, there are several observational “contaminations” that one has to keep in mind, including

Linear redshift-space (velocity) distortion. — In the linear theory of gravitational evolution of fluctuations, any density fluctuations induce a corresponding peculiar velocity field, which results in the systematic distortion of the pattern of distribution of objects in redshift space. The analytical expression for the linear distortion was first given by Kaiser (1987) and then later elaborated by a number of people (see Hamilton 1998 for an excellent review).

Nonlinear redshift-space (velocity) distortion. — Virialized nonlinear objects have isotropic and large velocity dispersions. This “finger-of-God” effect significantly suppresses the observed amplitude of correlation on small scales (Davis & Peebles 1983; Suto & Suginoaha 1991). An empirical expression for this effect in the power spectrum was discussed by Peacock & Dodds (1994, 1996, hereafter PD) and Cole, Fisher, & Weinberg (1994, 1995).

Cosmological redshift-space (geometry) distortion. — While the geometry of the local universe is well approximated as Euclidean, the global structure of the universe should be properly described by a general relativistic model. In particular, the comoving separation of a pair of objects at \( z \gg 0.1 \) is not determined only by their observable angular and redshift separations (\( \delta \theta \) and \( \delta z \)) without specifying the geometry, or equivalently the matter content, of the universe (i.e., the cosmological density parameter \( \Omega_0 \) and the dimensionless cosmological constant \( \lambda_0 \)). This generates a nontrivial anisotropy in the clustering pattern of objects, particularly at \( z \gtrsim 1 \) (Alcock & Paczyński 1979; Matsubara & Suto 1996; Ballinger, Peacock, & Heavens 1996; Popowski et al. 1998).

Cosmological light cone effect. — All cosmological observations are carried out on a light cone, the null hypersurface of an observer at \( z = 0 \), and not on any constant-time hypersurface. Thus the clustering amplitudes and the shapes of objects should naturally evolve even within the survey volume of a given observational catalog. Unless we restrict the objects to a narrow bin of \( z \) at the expense of statistical significance, a proper understanding of the data requires a theoretical model to take account of the average over the light cone, which has been extensively discussed by our group (Nakamura, Matsubara, & Suto 1998; Matsubara, Suto, & Szapudi 1997; Yamamoto & Suto 1999; Nishioka & Yamamoto 1999; Suto et al. 1999; Yamamoto, Nishioka,
surveys of objects at et al. 2000).

possible effects (e.g., Bardeen et al. 1986; Mo & White 1996; Fry 1996; Jing 1998; Taruya, Koyama, & Soda 1999; Suto 1999) and others (e.g., Mataresse et al. 1997; Sugiyama 1995). In order to separate the resulting effect due to the shape of density fluctuations from those due to

The first two effects are already important in any shallow surveys of objects at \( z \approx 0 \) (see, e.g., Hatton & Cole 1998), and the last three effects become progressively important as the survey becomes deeper.

While one of the primary scientific goals for cosmological surveys is a proper understanding of the clustering evolution of luminous objects at high redshifts, one can probe the geometry of the universe combining the standard theories of structure formation. This is what we will explore in the present paper. Our previous work (Suto et al. 1999) examined the two-point correlation function in cosmological redshift space and found that the nonlinear peculiar velocity field, or finger-of-God, cannot be neglected even on fairly large scales. Thus here we focus on the power spectrum, the nonlinear effects of which are easier to describe theoretically (PD; Ballinger et al. 1996). We focus on the mass power spectrum and do not consider the biasing explicitly below, while the same methodology is applicable if the bias is linear and scale independent. In reality, however, the bias, especially for galaxies and quasars, may be too complicated to be analytically tractable (i.e., Dekel & Lahav 1999; Taruya, Koyama, & Soda 1999). This issue will be studied in detail and described elsewhere (H. Magira, Y. P. Jing, A. Taruya, & Y. Suto 2000, in preparation).

The rest of the paper is organized as follows. First we describe a theoretical modeling of the power spectrum of dark matter including the cosmological redshift-space distortion in addition to the linear and nonlinear velocity distortions (§ 2). The theoretical model prediction is checked and calibrated against the N-body simulations. Next we examine the feasibility of estimating cosmological parameters by analyzing the anisotropy in the monopole and quadrupole moments of power spectra of objects at high redshifts with the SDSS QSO sample specifically in mind (§ 3). Finally § 4 is devoted to discussion and conclusions.

2. MODELING NONLINEAR REDSHIFT-SPACE DISTORTION ON THE POWER SPECTRUM

2.1. Distortion Due to the Peculiar Velocity

The power spectrum distorted by the peculiar velocity field (neglecting the cosmological distortion for the moment), \( P^S(k; z) \), is known to be well approximated by the following expression (Peacock & Dodds 1994; Cole et al. 1995):

\[
P^S(k_\perp, k_\parallel; z) = P^R(k; z) \left[ 1 + \beta(z) \left( \frac{k_\perp}{k} \right)^2 \right] D[k_\parallel \sigma_p(z)],
\]

where \( k_\perp \) and \( k_\parallel \) are the comoving wavenumber perpendicular and parallel to the line of sight of an observer, and \( P^R(k; z) \) is the power spectrum in real space. The second factor in the right-hand side of equation (1) represents the linear redshift-space distortion derived by Kaiser (1987) adopting the distant-observer approximation and the scale-independent linear bias \( b(z) \). Then \( \beta(z) \) is defined by

\[
\beta(z) = \frac{1}{b(z)} \frac{d \ln D(z)}{d \ln a} \approx \frac{1}{b(z)} \left\{ \Omega^{0.6}(z) + \frac{\dot{\lambda}(z)}{70} \left[ 1 + \frac{\Omega(z)}{2} \right] \right\},
\]

\[
\Omega(z) = \left[ \frac{H_0}{H(z)} \right]^2 (1 + z)^3 \Omega_0,
\]

and

\[
\dot{\lambda}(z) = \left[ \frac{H_0}{H(z)} \right] \lambda_0,
\]

where \( H(z) \) is the Hubble parameter at redshift \( z \): \( H(z) = H_0 \sqrt{\Omega_0 (1 + z)^3 + (1 - \Omega_0 - \lambda_0)(1 + z)^3 + \lambda_0} \).

The finger-of-God effect is modeled by the damping function \( D[k_\parallel \sigma_p] \), which is the Fourier transform of the distribution function \( f_\parallel(v_{12}) \) of pairwise peculiar velocities in real space. It has often been assumed that \( f_\parallel(v_{12}) \) is exponential with a scale-independent pairwise velocity dispersion, \( \sigma_p \). While this exponential function has been adopted in the literature (Cole et al. 1994, 1995; PD; Ballinger et al. 1996), its predictability was not fully checked against N-body simulations; the previous studies used a value of \( \sigma_p \) determined a priori from the simulation data themselves. In reality, however, the formula is useful for the present purpose only if \( \sigma_p(z) \) can be computed from a given \( P^R(k; z) \) and a set of cosmological parameters (\( \Omega_0, \lambda_0, \sigma_8, \) and \( H_0 \)). In the next subsections, we will show that \( P^S(k_\perp, k_\parallel; z) \) combined with the existing fitting formulae for nonlinear density and velocity fields is in excellent agreement with the results of our high-resolution N-body simulations.

2.2. Pairwise Velocity Distribution Function from N-Body Simulations

In order to quantify various effects of redshift-space distortion and to examine the validity of theoretical predictions, we use a series of high-resolution N-body simulations by Jing & Suto (1998). The simulations assume representative cosmological models in cold dark matter (CDM) cosmogonies (Table 1). Each model has three different realizations and employs \( N = 256^3 \) dark matter particles in a periodic comoving cube of box size \( L_{box} = 300 \, h^{-1} \) Mpc. We use the transfer function of Bardeen et al. (1986; BBKS) characterized by the shape parameter \( \Gamma \), and the fluctuation amplitude is normalized by using \( \sigma_8 \) from the cluster abundance (Kitayama & Suto 1997). In the conventional CDM models, the shape parameter \( \Gamma \) is written in terms of \( \Omega_0, h, \) and \( \Omega_0 \), the baryon density parameter:

\[
\Gamma = \Omega_0 h \exp \left( -\Omega_b - \sqrt{2m\Omega_b}/\Omega_0 \right),
\]

(Sugiyama 1995). In order to separate the resulting effect due to the shape of density fluctuations from those due to
the geometry and evolution of the universe, we consider cases where $\Gamma$ is treated as an independent parameter or given by equation (6) in § 3. In the latter case, we assume that each simulation adopts $h \equiv \Gamma/\Omega_0$ neglecting $\Omega_b$ for definiteness.

First we compute the distribution function $f(v_{12})$ of pairwise velocity $v_{12}$ for all the simulation particles in order to determine the functional form of the damping function $D[k || \sigma_P]$. Figure 1 plots the pairwise peculiar velocity distribution function at $z = 0$ and $z = 2.2$ in three different

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**TABLE 1**

**Simulation Model Parameters**

| Model                | $\Omega_0$ | $\lambda_0$ | $\Gamma^*$ | $\sigma_8$ | $N$     | $h^b$  | $A^c$ | Realizations |
|----------------------|-------------|--------------|------------|------------|---------|--------|--------|-------------|
| SCDM (Standard CDM)  | 1.0         | 0.0          | 0.5        | 0.6        | $256^3$ | 0.50   | 0.60   | 3           |
| LCDM ($\lambda$ CDM) | 0.3         | 0.7          | 0.21       | 1.0        | $256^3$ | 0.70   | 0.52   | 3           |
| OCDM (Open CDM)      | 0.3         | 0.0          | 0.25       | 1.0        | $256^3$ | 0.83   | 0.55   | 3           |

* The shape parameter of the power spectrum.

* The dimensionless Hubble constant defined through $h \equiv \Gamma/\Omega_0$ neglecting the baryon density parameter $\Omega_b = 0$.

* The normalization factor of $\sigma_8$, $\Omega_0$ relation determined from the X-ray cluster abundance (Kitayama & Suto 1997).

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**Fig. 1.—Pairwise peculiar velocity distribution for different cosmological models at $z = 0$ and $z = 2.2$.** We compute the number of particles within a velocity bin of $\Delta v_{12} = 200$ km s$^{-1}$. Results from $N$-body simulations at different pair separations, $r = 46$ (filled circles), 23 (stars), 11 (open circles), and 4.1 (crosses) $h^{-1}$ Mpc are shown. Solid lines in each panel represent the best fit to the exponential distribution function (eq. [7]) treating $\sigma_P$ as a free parameter. Dashed and dotted lines display the exponential and Gaussian functions, respectively, with $\sigma_P = \sigma_P^{\text{sim}}$. 

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models. Different symbols correspond to the results for different comoving separations. As simple analytical models for \(f_i(v_{12})\), we consider an exponential,

\[ f_i(v_{12}) = \frac{1}{\sqrt{2 \pi \sigma_p}} \exp \left( -\frac{|v_{12}|^2}{2 \sigma_p^2} \right), \tag{7} \]

and a Gaussian,

\[ f_i(v_{12}) = \frac{1}{\sqrt{2 \pi \sigma_p}} \exp \left( -\frac{v_{12}^2}{2 \sigma_p^2} \right). \tag{8} \]

The former is suggested to be a good approximation on nonlinear scales both observationally (Davis & Peebles 1983) and theoretically (Efstathiou et al. 1988; Ueda, Itoh, & Suto 1993; Suto 1993; Seto & Yokoyama 1998; Juszkiewicz et al. 1998), and the latter should be the case if the underlying density field is purely random Gaussian.

Solid lines in Figure 1 represent the best-fit exponential distribution for results at \(r = 45.5 \, h^{-1} \, \text{Mpc}\), treating \(\sigma_p\) as a free parameter. The corresponding best-fit values are listed in Table 2.

Figure 1 indicates that neither an exponential nor a Gaussian function fits the numerical results completely. Thus it is reasonable that the best-fit values in Table 2 are somewhat different from the pairwise dispersions, \(\sigma_{p,\text{sim}}\), directly evaluated from the \(N\)-body data. Dashed and dotted lines represent the exponential and Gaussian curves, respectively, both of which adopt \(\sigma_{p,\text{sim}}\) as the velocity dispersion \(\sigma_p\).

With this result in mind, however, we would still like to adopt the above distribution functions mainly for simplicity and definiteness. Thus it seems that the above procedure is accurate enough for our present purpose. Thus the remaining task is to predict \(\sigma_p\) given a set of cosmological model parameters. In a previous paper (Suto et al. 1999), we find that a fitting formula by Mo, Jing, & Börner (1997; hereafter MJB) is in excellent agreement with our numerical results. Figure 2 plots \(\sigma_{p,\text{sim}}\) as a function of the pair separation \(r\) for three models at \(z = 0\) and 2.2. It is clear that \(\sigma_{p,\text{sim}}\) asymptotically approaches the MJB formula:

\[ \sigma_{p,\text{MJB}}^2 \equiv \Omega(z) H_0^2 \left[ 1 + \frac{1 + z}{D(z)} \right] \int_0^\infty \frac{D^2(z')}{(1 + z')^2} \, dz' \times \int_0^\infty \frac{dk}{k^3} \frac{\Delta^2_{\text{NL}}(k, z)}{k^2}. \tag{9} \]

Note that the original expression in MJB corresponds to the proper velocity, and the above equation (9) is converted in the comoving redshift space by multiplying a factor \([(1 + z) c(z)]^2 = [(1 + z) H_0/H(z)]^2\); see § 2.3 and Suto et al. (1999).

In order to examine whether the above prescription for the theoretical predictions is really consistent with the numerical simulations, we compute the multipole of the power spectrum in redshift space:

\[ P_0^{\delta}(k; z) \equiv \frac{2l + 1}{2} \int_{-1}^1 d \mu P_{\delta}(k, \mu; z) L_2(\mu), \tag{10} \]

where \(\mu \equiv k_{\parallel}/k\) is the direction cosine and \(L_2\) are the Legendre polynomials. Figure 3 shows the comparison of theoretical predictions and our numerical simulations of the angle-averaged power spectrum, \(P_0^{\delta}(k; z)\) at \(z = 0\) and 2.2.

Our theoretical predictions are based on the combination of the PD formula for \(\Delta^2_{\text{NL}}(k, z)\) in real space and the MJB formula for \(\sigma_p\). For definiteness we adopt an exponential distribution function (7) for the pairwise velocity, and the corresponding damping function in \(k\) space is a Lorentzian:

\[ D[k, \mu\sigma_p] = \frac{1}{1 + (k^2 \mu^2 \sigma_p^2)/2}. \tag{11} \]

Then equations (1) and equation (11) are analytically integrated; specifically the monopole and quadrupole moments are expressed as

\[
P_0^{\delta_0}(k, z) = \left[ A(\kappa) + \frac{5}{2} B(\kappa) + 3^2 C(\kappa) \right] P^{\text{R}}(k, z), \tag{12}
\]

\[
P_2^{\delta_0}(k, z) = \left( \frac{5}{2} B(\kappa) - A(\kappa) \right) \beta(z)
\times \left( \frac{4}{3} B(\kappa) + 3 C(\kappa) - B(\kappa) \right) + \beta^2(z) \left( \frac{3}{2\kappa^2} \left[ 1 - C(\kappa) \right] - \frac{3}{2} C(\kappa) \right) \right) P^{\text{R}}(k, z), \tag{13}
\]

\[
A(\kappa) = \frac{1}{\kappa} \arctan (\kappa), \tag{14}
\]

\[
B(\kappa) = \frac{3}{\kappa^2} \left[ 1 - A(\kappa) \right], \tag{15}
\]

and

\[
C(\kappa) = \frac{5}{3\kappa^2} \left[ 1 - B(\kappa) \right], \tag{16}
\]

with \(\kappa(z) = k \sigma_p(z)/\sqrt{2H_0}\).

Incidentally, Cole et al. (1995) derived analogous expressions for the exponential distribution function of one-dimensional (i.e., not pairwise) velocity. This corresponds to a damping function:

\[ D[k, \mu\sigma_p] = \frac{1}{1 + (k^2 \mu^2 \sigma_p^2)/2}, \tag{17} \]

with \(\sigma_p\) being the one-dimensional velocity dispersion. Then they obtained

\[
P_0^{\delta_0}(k, z) = \left[ \bar{A}(\kappa) + \frac{5}{2} B(\kappa) + 3^2 C(\kappa) \right] P^{\text{R}}(k, z), \tag{18}
\]

\[
P_2^{\delta_0}(k, z) = \left( \frac{5}{2} B(\kappa) - A(\kappa) \right) \beta(z)
\times \left( \frac{4}{3} B(\kappa) + 3 C(\kappa) - B(\kappa) \right) + \beta^2(z) \left( \frac{5}{2\kappa^2} \left[ 1 - C(\kappa) \right] - \frac{3}{2} C(\kappa) \right) \right) P^{\text{R}}(k, z), \tag{19}
\]

\[
\bar{A}(\kappa) = \frac{1}{\sqrt{2\kappa^2}} \arctan (\kappa) + \frac{1}{2 + \kappa^2}, \tag{20}
\]

\[
\bar{B}(\kappa) = \frac{6}{\kappa^2} \left[ A(\kappa) - \frac{2}{2 + \kappa^2} \right], \tag{21}
\]

and

\[
\bar{C}(\kappa) = -\frac{10}{\kappa^2} \left[ B(\kappa) - \frac{2}{2 + \kappa^2} \right]. \tag{22}
\]
TABLE 2
PAIRWISE PECULIAR VELOCITY DISPERSIONS AT z = 0 AND 2.2

| Model   | z   | \(\sigma_{P,\text{MJB}}\) (km s\(^{-1}\)) | \(\sigma_{P,\text{sim}}\) (km s\(^{-1}\)) | \(\sigma_{P,\text{fit}}\) (km s\(^{-1}\)) | Fitting Range |
|---------|-----|--------------------------------------|--------------------------------------|--------------------------------------|--------------|
| SCDM    | 0   | 580                                  | 592                                  | 493                                  | 900 – 2900   |
| LCDM    | 0   | 582                                  | 606                                  | 563                                  | 900 – 2900   |
| OCDM    | 0   | 599                                  | 603                                  | 602                                  | 900 – 2900   |
| SCDM    | 2.2 | 164                                  | 166                                  | 113                                  | 100 – 1100   |
| LCDM    | 2.2 | 381                                  | 379                                  | 273                                  | 900 – 2900   |
| OCDM    | 2.2 | 368                                  | 365                                  | 287                                  | 900 – 2900   |

\(a\) A relative pairwise peculiar velocity dispersion directly evaluated from the N-body data at \(r = 42\ h^{-1}\) Mpc.

\(b\) A relative pairwise peculiar velocity dispersion evaluated through a fit to the exponential distribution function.

\(c\) A range of \(v_{12}\) for the fit.

with \(\tilde{\kappa}(z) = k\sigma_z(z)/H_0\). If the velocity correlation is neglected, then \(\sigma_z\) is equal to \(\sigma_p/\sqrt{2}\), and the above expressions agree with our results up to \(O(\tilde{\kappa}^2)\) for \(\tilde{\kappa} \ll 1\). While we use our expression in what follows, the result is insensitive to this choice because we mainly use the range \(\tilde{\kappa} \ll 1\) for the statistical analysis.

As Figure 3 indicates, our theoretical predictions in redshift space (solid lines) are in good agreement with the simulation results (filled circles) especially for \(k \lesssim 1\ h\ Mpc^{-1}\). For larger \(k\), they start to deviate from each other; while this might be ascribed to the approximation of the velocity

\[ P(k,z) \]
Fig. 4.—Two-dimensional power spectra in cosmological redshift space; (a) \( z = 0 \), (b) \( z = 2.2 \). In each figure, the top panels display the predictions with the PD mass power spectrum and linear velocity distortion. The middle panels display the predictions with the PD mass power spectrum and nonlinear velocity distortion. The Lorentzian (solid lines) and Gaussian (dotted lines) are adopted for the damping function describing the nonlinear velocity distortion (with \( \sigma_p = \sigma_{p,\text{MB}} \)). The bottom panels display the power spectrum calculated from \( N \)-body simulations with all the particles \((N = 256^3)\).
distribution function, a similar discrepancy is recognized even in real space (dotted lines and open circles). As a matter of fact, we found that this is mainly ascribed to the smoothing effect resulting from the cloud-in-cell interpolation in computing the Fourier transform; we used 512^3 grids in estimating the power spectrum of simulation data. When we apply the correction for the smoothing effect in real space using the method described in Jing (1992), the results (crosses) agree better with the PD formula. Thus the discrepancy in the strongly nonlinear regime in Figure 3 is not real. In any case this does not affect our conclusions in this paper because we mainly use the range of k ≤ 0.2 h Mpc^{-1} for the later analysis.

2.3. Theoretical Predictions of the Power Spectrum in Cosmological Redshift Space

In previous subsections, we have shown using our high-resolution N-body simulations that the power spectrum distorted by the peculiar velocity field (without the cosmological distortion) can be predicted fairly accurately by a combination of the existing fitting formulae. The expression for the power spectrum in cosmological redshift space has been derived by Ballinger et al. (1996). In this subsection, we simply summarize their result using our own notation adopted in the paper for definiteness. Then we examine the extent to which the overall predictions are consistent with the N-body results in later subsections.

Consider a pair of objects located at redshifts z_1 and z_2 whose redshift difference δz ≡ z_1 − z_2 is much less than the mean redshift $z ≡ (z_1 + z_2)/2$. Then the observable separations of the pair perpendicular and parallel to the line-of-sight direction, $x_\perp$ and $x_\parallel$, are given as $z \delta \theta / H_0$ and $\delta z / H_0$, respectively ($\delta \theta$ is the angular separation of the pair in the sky). Then the mapping of the comoving separation in real space ($x_\perp$, $x_\parallel$) to that in cosmological redshift distortion (CRD) space is expressed as

$$x_\perp(z) = x_\perp / c_\perp(z), \quad x_\parallel(z) = x_\parallel / c_\parallel(z), \quad (23)$$

where $c_\perp(z) = H_0 / (H(z))$, $c_\parallel(z) = H_0 (1 + z) d_\perp(z) / z$, and $d_\perp(z)$ is the angular diameter distance (Matsubara & Suto 1996; Ballinger et al. 1996; Suto et al. 1999).

Then the power spectrum in the CRD space is

$$p^{(CRD)}(k_\perp, k_\parallel; z) = \frac{1}{c_\perp(z)^2 c_\parallel(z)} p^{(R)} \left( k_\perp / c_\perp(z), k_\parallel / c_\parallel(z); z \right), \quad (24)$$

where $k_\perp$ and $k_\parallel$ are the CRD wavenumber perpendicular and parallel to the line-of-sight direction.

Substituting equation (1), equation (24) reduces to

$$p^{(CRD)}(k_\perp, k_\parallel; z) = \frac{1}{c_\perp(z)^2 c_\parallel(z)} \times p^{(R)} \left[ \frac{k_\perp^2}{c_\perp(z)^2} + \frac{k_\parallel^2}{c_\parallel(z)^2}; z \right] \times \left[ \frac{k_\perp^2}{c_\perp(z)^2} + \frac{k_\parallel^2}{c_\parallel(z)^2} \right]^{-2} \times \left[ \frac{k_\perp^2}{c_\perp(z)^2} + \frac{\beta(z) + 1}{c_\parallel(z)^2} k_\parallel^2 \right]^2 \times D \left[ \frac{k_\parallel \sigma_\parallel(z)}{c_\parallel(z)} \right]. \quad (25)$$

Introducing

$$k_s ≡ \sqrt{k_\perp^2 + k_\parallel^2}, \quad \mu_s ≡ k_\parallel / k_s, \quad \text{and} \quad \eta ≡ c_\parallel / c_\perp, \quad (26)$$

the above result is rewritten as

$$p^{(CRD)}(k_s, \mu_s; z) = \frac{1}{c_\perp(z)^2 c_\parallel(z)} \times p^{(R)} \left( k_s / c_\perp(z), \sqrt{1 + \frac{1}{\eta(z)^2} - \frac{\mu_s^2}{2}}; z \right) \times \left\{ 1 + \left[ \frac{1}{\eta(z)^2} - 1 \right] \mu_s^2 \right\}^{-2} \times \left\{ 1 + \frac{1 + \beta(z)}{\eta(z)^2} k_\parallel^2 \right\}^2 \times D \left[ k_\parallel \sigma_\parallel(z) \right] / c_\parallel(z). \quad (27)$$

where we adopt equation (11) for the damping function.

2.4. Comparison of the Predicted Anisotropy of the Power Spectrum with N-Body Simulations

In order to compute the power spectrum in CRD space, we employ the distant-observer approximation, calculate the power spectrum in comoving space, and finally transform it to that in the observed frame according to equation (24). We adopted this indirect procedure since we are mainly interested in the anisotropies in the power spectrum. As we will show in § 3, however, even the angle-averaged power spectrum alone is useful as a cosmological test. Therefore in practice one would not have to take this route but could estimate the power spectrum directly in CRD space because the distribution of objects obtained by observations has already been in the observed frame. We made sure that the estimate of the angle-averaged power spectrum from simulation data is almost identical even if we first transform the simulation volume into the CRD space and then compute the power spectrum.

In Figures 4a and 4b, we show the contours of the power spectrum in CRD space at $z = 0$ and 2.2, respectively. In each figure, the top panels correspond to theoretical predictions taking account of the linear velocity distortion alone but with the PD formulae for the power spectrum. The middle panels include the correction for the nonlinear finger-of-God using the MJB formulae in equation (27), and the bottom panels plot the results from N-body simulations. As Cole et al. (1994) emphasized, the finger-of-God effect is appreciable even at $k_s ∼ 0.2 h$ Mpc^{-1} where the nonlinearity in the density field is rather small.

In the middle panels, solid and dotted contours refer to the prediction using a Lorentzian and a Gaussian for the damping function, respectively. Despite the fact that the exponential does not perfectly match the distribution function of pairwise relative peculiar velocity derived from simulations, the agreement of theory and simulation in Figure 4 is rather good in this quasi-nonlinear regime. Nevertheless the direct analysis using the contour curves is still sensitive to the statistical noise in the data, and we instead are forced to use the multipole expansion as in our previous paper (Suto et al. 1999):

$$P^{(CRD)}(k_s; z) = \frac{2l + 1}{2} \int_{-1}^{1} d\mu_s P^{(CRD)}(k_s, \mu_s; z) L_l(\mu_s). \quad (28)$$
Fig. 5.—Monopole of power spectra for different cosmologies at (upper panels) \( z = 0 \) and (lower panels) \( z = 2.2 \). Filled circles and crosses correspond to the results from \( N \)-body simulations, which are evaluated from all particles (\( 1.7 \times 10^7 \)) and selected particles (\( 5 \times 10^4 \)), respectively. Heavy lines correspond to the predictions including cosmological redshift distortion effect with an exponential (solid lines) and Gaussian (dotted lines) damping function, respectively. For reference, the predictions neglecting the geometrical effect at \( z = 2.2 \) (light lines) are shown (eq. [12]).

Figures 5 and 6 plot \( P_0^{\text{CRD}}(k_s) \) and \( P_2^{\text{CRD}}(k_s) \), the monopole and quadrupole moments of the power spectra, at \( z = 0 \) and 2.2. Filled circles and crosses represent the simulation results with all particles (\( 1.7 \times 10^7 \)) and randomly selected particles (\( 5 \times 10^4 \)), respectively. The quoted error bars refer to the dispersion among three different realizations in the case of all particles and among 24 subsamples in total (i.e., eight randomly selected subsamples for each realization) in the case of \( 5 \times 10^4 \) particles. Note that the major difference in amplitude between the results with and without the geometrical effect in Figure 5 comes from the overall normalization factor in equation (24). This is, however, a matter of definition to some extent, and what is important here is the difference among the cosmological models, which is much smaller than the factor \( c_{M}(z) / c_{A}(z) \) (Ballinger et al. 1996; Matsubara & Suto 1996).

Numerical integration of equations (27) and (28) adopting the PD and MJB fitting formulae yields the corresponding theoretical predictions, which are plotted in heavy solid and dotted lines for the exponential and Gaussian velocity distribution functions. Analytical results neglecting the cosmological distortion (eq. [12]) are shown in light lines for reference. In any case the difference due to the modeling is negligible at \( z = 0 \), and the distortion at \( z = 2.2 \) is dominated by the geometrical effect.

Figures 5 and 6 indicate that the predictions based on the exponential velocity distribution function reproduce the simulation results fairly accurately, especially on large scales (small \( k_s \)). Also it is interesting and important to note that even the monopole component is significantly affected by the cosmological distortion. Figure 7 displays the predictions for the quadrupole to monopole ratio. In principle these statistics are superior to either the monopole or the quadrupole in the sense that they are independent of the overall normalization of the power spectrum (e.g., Cole et al. 1995); in fact the ratio becomes constant, \( (4 \beta/3 + 4 \beta^2/7)/(1 + 2\beta/3 + \beta^2/5) \), in a linear regime (i.e., without finger-of-god and geometrical effects), which is plotted in dashed lines in the figure. On the other hand, the ratio lies between \(-2 \) and 1, and the practical usefulness crucially depends both on the data quality and the accuracy of the theoretical predictions (see \( \S \mbox{ 3} \) below). Incidentally the simulation results seem to disagree with the predictions at \( z = 0 \), but this is mainly because the predictions adopt the exponential damping function, although the Gaussian is more appropriate for simulation results at \( z = 0 \) (see also Figs. 5 and 6).

In order to understand the model-dependence of the moments, we show the extent to which it is sensitive to the assumed velocity dispersions \( \sigma_p \) in Figure 8 for \( P_0^{\text{CRD}}(k_s) \) and to a set of parameters (\( \Omega_M, \Omega_0, \) and \( \Gamma \)) in Figures 9, 10, and 11 for \( P_0^{\text{CRD}}(k_s), P_2^{\text{CRD}}(k_s), \) and \( P_2^{\text{CRD}}(k_s)/P_0^{\text{CRD}}(k_s) \), respectively. Figure 8 implies that the predictions on scales \( k_s \lesssim 0.2 \) h Mpc\(^{-1} \) are very accurate and not sensitive to the adopted \( \sigma_p \). Thus the usefulness of our proposed cosmological redshift-space distortion.
Fig. 6.—Same as Fig. 5 but for the quadrupole moment of the power spectra.

Fig. 7.—Quadrupole to monopole ratio of power spectra for different cosmologies at (upper panels) $z = 0$ and (lower panels) $z = 2.2$. Filled circles and crosses correspond to the results from $N$-body simulations, which are evaluated from all particles ($1.7 \times 10^7$) and selected particles ($5 \times 10^4$), respectively. Heavy lines correspond to the predictions including cosmological redshift distortion effect with an exponential damping function. For reference, the predictions neglecting the geometrical effect at $z = 2.2$ (light lines) are shown. The dashed lines indicate the ratio expected in a linear redshift distortion model.
Fig. 8.—Dependence of the monopole of power spectra on the velocity dispersions at (upper panels) $z = 0$ and (lower panels) $z = 2.2$ in CRD space; $\sigma_p$ is set to be $0.8\sigma_{p,MB}$ (dotted lines), $\sigma_{p,MB}$ (solid lines), and $1.2\sigma_{p,MB}$ (dashed lines). Filled circles and crosses correspond to the results from $N$-body simulation, which are evaluated from all particles ($1.7 \times 10^7$) and selected particles ($5 \times 10^4$), respectively.

Fig. 9.—Dependence of the monopole of power spectra in cosmological redshift space at $z = 2.2$ on $\Omega_0$. We consider both (left panel) open ($\Omega_0 = 0$) and (right panel) spatially flat ($\Omega_0 = 1 - \Omega_m$) models. The shape parameter $\Gamma$ is fixed as (upper panels) $0.7\Omega_0$ and (lower panels) $0.2$. The fluctuation amplitude $\sigma_8$ is fixed as unity for simplicity.
Fig. 10.—Same as Fig. 9 but for the quadrupole moment of the power spectra

Fig. 11.—Same as Fig. 9 but for the quadrupole to monopole ratio
logical test is crucially dependent on the observational data quality on the quasi-linear scales, which are presumably achievable only with the upcoming SDSS QSO surveys. It should be noted that there is a small but clear systematic difference between $N = 1.7 \times 10^7$ and $5 \times 10^4$ in Figures 5–8, despite the fact that we subtracted the shot noise due to the finite number of sampled particles. Actually we were not able to understand the origin of this effect, but the effect is smaller than the other uncertainties involved in the present analysis.

Incidentally Figure 9 indicates that the behavior of $P_0^{(CRD)}(k_0)$ is degenerate for a certain set of the parameters since its shape and amplitude are sensitive both to $\Omega_0$ and $\lambda_0$ through the correction factors, $c_1(z)$ and $c_2(z)$, and to $\Gamma$ through the shape of the power spectrum in real space. Such a degeneracy on the density parameter is apparent for open models of $\Gamma = 0.7 \Omega_0$ and for flat models of $\Gamma = 0.2$. Fortunately, as shown by Figure 10, a measurement of the quadrupole $P_2^{(CRD)}(k_0)$ may help break the degeneracy especially for the density parameter as low as $\sim 0.2$. Combining other cosmological tests, such as the cosmic microwave background, clustering statistics, cluster abundances, etc., may further narrow the space of the cosmological parameters. In the next section we will investigate the feasibility of measuring the cosmological parameters by combining the first two moments of the redshift power spectrum from a survey like the Sloan QSO survey and the cluster abundance.

### 3. FEASIBILITY OF DETERMINING THE COSMOLOGICAL PARAMETERS FROM THE POWER SPECTRUM IN COSMOLOGICAL REDSHIFT SPACE

In the previous section we have demonstrated that the theoretical predictions for $P_0^{(CRD)}(k_0; z)$ and $P_2^{(CRD)}(k_0; z)$ are quite accurate for $k_0 \lesssim 0.2$ h Mpc$^{-1}$. Thus we finally examine whether the cosmological test using these moments leads to any useful constraints on the cosmological parameters. For this purpose, we use the results from the $N$-body simulation models (Table 1) at $z = 2.2$ and perform the statistical analyses as follows.

Our theoretical model is specified by a set of $\Omega_0$, $\lambda_0$, $\sigma_8$, and $h$. Since the results of our $N$-body simulations are scalable with respect to $h$, we assume that $h$ is related to the shape of the spectrum through the relation $h \equiv \Omega_0/\Gamma$, neglecting the baryon contribution $\Omega_B$ (Sugiyama 1995) for simplicity. In order to take account of the statistical limitation, we do not use all the simulation particles but randomly select particles. For a given number of selected particles, we generate 24 mock samples (eight randomly selected subsamples for each realization) for each cosmological model. Then we assign the errors to the simulation data from the ensemble average over the 24 samples. In order to avoid a strongly nonlinear effect, we use the spectrum in the range $k_0 \lesssim 0.2$ h Mpc$^{-1}$ and perform the $\chi^2$ analysis.

Figures 12 and 13 display the confidence level contours from the $\chi^2$ analysis of $P_0^{(CRD)}(k_0; z)$ or $P_2^{(CRD)}(k_0; z)$ on the $\Omega_0$-$\sigma_8$ and $\Omega_0$-$\lambda_0$ planes, respectively. To be specific, we compute the $\chi^2$ defined by

$$
\chi^2 = \sum_{i=1}^{10} \left[ \frac{P_0^{R, S}(k_{0,i}) - P_{\text{model}}(k_{0,i})}{\Delta P_{\text{sim}}(k_{0,i})} \right]^2,
$$

where the wavenumber is sampled between 0.02 and 0.2 h Mpc$^{-1}$ as

$$
k_{0,i} = \frac{2\pi}{L_{\text{box}} i} \quad (i = 1, \ldots, 10).
$$

The indices $R$ and $S$ denote the three different realizations and eight randomly selected subsamples for each cosmological model, and in evaluating equation (29) we randomly choose one mock sample from 24 ($= 3 \times 8$) simulated mock samples in total. For $\Delta P_{\text{sim}}(k_{0,i})$ we use the dispersion among 24 mock samples assuming that this corresponds to a cosmic variance:

$$
P_{\text{sim}}(k_{0,i}) = \frac{1}{24} \sum_{R=1}^{3} \sum_{S=1}^{8} P_{\text{sim}}^{R,S}(k_{0,i})
$$

The resulting confidence level is computed from the reduced $\chi^2$ for 8 degrees of freedom, i.e., 10 data points minus two free parameters, either $(\Omega_0, \sigma_8)$ or $(\Omega_0, \lambda_0)$. We perform the above analysis for the monopole and the quadrupole separately and also combine the monopole and quadrupole analyses. We have adopted the $\chi^2$ technique to quantify the errors for the measured parameters because it is simple, but perhaps other statistical methods, e.g., the maximum likelihood, may better quantify the measurement errors in the actual application to future observational data.

Figures 12a, 12c, 13a, and 13c assume that $\Gamma = \Omega_0 h$, while the other panels in these two figures treat $\Gamma$ as an independent parameter and fix the value as specified by the simulation (Table 1). We display the results for $N = 5 \times 10^5$ (bottom panels), $5 \times 10^4$ (middle panels), and $5 \times 10^3$ (top panels) from the entire sample of 256$^3$ particles at $z = 2.2$. For reference, the quasar luminosity function of Boyle, Shanks, & Peterson (1988) predicts that the number of quasars per $\pi$ steradian brighter than 19 mag in B is about 4500 either for $0.9 < z < 1.1$ or $1.9 < z < 2.1$ (for the $\Omega_0 = 1$ and $\lambda_0 = 0$ model). If we use the extrapolation of the luminosity function to $z = 5$ by Wallington & Narayan (1993), a total number of $\sim 10^3$ QSOs in $0 \lesssim z \lesssim 5$ is expected to be cataloged in the upcoming SDSS QSO sample. It should be noted that the best-fit parameters in these figures are sometimes a bit different from the true values that we use in the simulations. This is simply because the figures represent results for one particular mock sample. We made sure that the best-fit values are in good agreement with the true values if we replace $P_{\text{sim}}^{R,S}(k_{0,i})$ in equation (29) by $P_{\text{sim}}(k_{0,i})$.

We repeat the analysis in Figure 12 by fixing either $\lambda_0 = 0$ or $\lambda_0 = 1 - \Omega_0$. As anticipated, the present analyses with either $P_0^{(CRD)}(k_0)$ or $P_2^{(CRD)}(k_0)$ alone do not determine the cosmological parameters completely. However, the error contours of the monopole do not align with those of the quadrupole completely; in some cases the contours of the monopole and the quadrupole are somewhat orthogonal, though the error contours of the quadrupole are generally larger than those of the monopole. These results indicate that a combination of the first two moments can, as expected, constrain the cosmological parameters more stringently. Figure 14 shows the confidence levels of such a joint constraint on $(\sigma_8, \Omega_0)$. An encouraging feature noted from this figure is that the flat and open universes with
Fig. 12.—The confidence contours on the $\Omega_0$-$\sigma_8$ plane from the $\chi^2$ analysis of the monopole and quadrupole moments of the power spectrum in cosmological redshift space at $z = 2.2$: (a) $\Gamma = \Omega_0 h$ for $P_{10}^{\text{CRD}}(k)$, (b) $\Gamma = \Gamma_{\text{sim}}$ for $P_{10}^{\text{CRD}}(k)$, (c) $\Gamma = \Omega_0 h$ for $P_{20}^{\text{CRD}}(k)$, and (d) $\Gamma = \Gamma_{\text{sim}}$ for $P_{20}^{\text{CRD}}(k)$. The long-dashed, heavy thick, light thick, and thin lines correspond to 0.5, 1, 2, and 3 $\sigma$ confidence levels. We randomly selected (top panels) $N = 5 \times 10^3$, (middle panels) $N = 5 \times 10^4$, and (bottom panels) $N = 5 \times 10^5$ particles from $N$-body simulations. Solid and dotted lines represent results for open and spatially flat models, respectively. The crosses indicate the true values of our simulations. We also show the constraint on $\Omega_0$ and $\sigma_8$ from the cluster abundance by thin lines in the middle panels.
Fig. 12c

Fig. 12d
Same as Fig. 12 but on the $\Omega_0 - \lambda_0$ plane. We adopt the value of $\sigma_8$ from eq. (33) with the quoted error bars.
Fig. 14a

Same as Fig. 12 but from the combined analysis of the monopole and quadrupole moments

Fig. 14b
Fig. 15a

Fig. 15b

Fig. 15.—Same as Fig. 13 but from the combined analysis of the monopole and quadrupole moments
Fig. 16a

Fig. 16b

Fig. 16.—Same as Fig. 13 but from the analysis of the quadrupole to monopole ratio.
\[ \sigma_8 = (A \pm 0.02) \times \begin{cases} \Omega_0^{0.35 - 0.82 \Omega_0 + 0.55 \Omega_0^2}, & (\lambda_0 = 1 - \Omega_0) \\
\Omega_0^{0.28 - 0.91 \Omega_0 + 0.68 \Omega_0^2}, & (\lambda_0 = 0) 
\end{cases} \]

are overlaid. The normalization factor \( A \) determined from the observed cluster abundance is 0.54.

Combining the cluster abundance with the test of this paper may determine the cosmological parameters \( \lambda_0 \) and \( \Omega_0 \). This is illustrated in Figure 13. The normalization \( \sigma_8 \) adopted in our simulations are slightly different from the value given by equation (33). Thus in the analysis presented in Figures 13 and 15, we accordingly change the normalization factor \( A \), keeping the same \( \Omega_0 \) dependence, and find the minimal \( \chi^2 \) by allowing a \( \pm 0.02 \) dispersion. Our adopted value of \( A \) for each model is summarized in Table 1. The monopole moment is sensitive to the density parameter \( \Omega_0 \) only, while the quadrupole measurement interestingly complements the monopole measurement in that it depends more strongly on the cosmological constant. Combining these two moments results in a joint constraint on \( (\Omega_0, \lambda_0) \), which is shown in Figure 15. Consistent with the results shown in Figure 14, the open and flat universes with \( \Omega_0 \leq 0.3 \) can be discriminated with the number of objects larger than \( 5 \times 10^4 \). It is also interesting to note that our constraints in Figure 15 are fairly orthogonal to those from the SN Ia (Perlmutter et al. 1999) and can even be combined to probe the cosmic equation of state in general (e.g., Garnavich et al. 1998).

Finally we repeat a similar analysis for the quadrupole to monopole ratio, the results of which are plotted in Figure 16. As anticipated, the resulting constraints are more sensitive to \( \Lambda_0 \) compared with those presented in Figure 15. On the other hand, we realize that our current modeling may not be sufficiently good to describe the ratio in practice; note that while the amplitudes of the monopole and the quadrupole span 2 orders of magnitude, their ratio is merely around between \(-2 \) and \(1 \), and thus accurate modeling and good data qualities are required for a robust estimation of the cosmological parameters. This is why some panels in Figure 16 do not have any contours with reasonable confidence levels.

4. CONCLUSIONS AND DISCUSSION

In this paper, we have examined the reliability and accuracy of theoretical modeling for the power spectrum in cosmological redshift space using high-resolution \(N\)-body simulations. Our main conclusion is that the cosmological test proposed by Ballinger et al. (1996) and Matsubara & Suto (1996) is in fact practically useful in constraining the cosmological parameters combined with the upcoming SDSS QSO sample. While an application of this methodology to the Lyman break galaxies is also interesting, the small number of total objects and cosmic variance seem to be the major difficulties in practice (Nair 1999). The results are admittedly complicated since many different and important contaminations are necessarily involved. In particular, our methodology is heavily dependent on the underlying mass power spectrum, which may be reconstructed from the redshift-space observation but with large uncertainties.

Thus we explicitly assume that the spectrum is completely specified by four free parameters, \( \Omega_0 \), \( \Lambda_0 \), \( \Gamma \), and \( \sigma_8 \), as in the case of CDM models. More importantly and realistically, any model for bias should add at least another free parameter (even in a time-independent and scale-invariant linear model, which is too idealistic). As a result, the estimated parameters are inevitably model dependent. Therefore in order to extract useful cosmological information, we have to combine the additional constraints on those parameters from independent cosmological consideration. Even so, the present model is a minimal requirement for understanding the clustering of high-redshift objects properly.

The remaining important problems that must be solved in order to improve this cosmological test include the higher order moment analysis, the biasing, and the light cone effects; first, we have shown that the anisotropy of the power spectrum in cosmological redshift space shows up already in the monopole and quadrupole moments, \( P^{(\text{CRD})}_0(k) \) and \( P^{(\text{CRD})}_2(k) \). On the other hand, this is why the resulting constraints are only weakly dependent on \( \Lambda_0 \), contrary to the original idea by Alcock & Paczynski (1979); note, however, that Ballinger et al. (1996) correctly recognized this difficulty. Utilizing the higher order moments will be another possibility, although the data will necessarily be noisier. Second, our analysis presented above is implicitly assumed to apply for data in a narrow redshift bin. In order to increase the number of available pairs and thus the statistical significance, it is crucial to take account of the cosmological light cone effect properly in theoretical predictions following Yamamoto & Suto (1999), Yamamoto, Nishioka, & Suto (1999), and Suto et al. (1999). Finally, we did not allow for possible biasing, which should further complicate the mapping of clustering statistics of luminous objects with that of the underlying mass distribution. These issues will be discussed elsewhere (Magira et al. 2000 in preparation).

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