Meson dominance implications for the bottomness preserving decays of the $B_{c}^{+}$ meson

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Encouraged by a recent observation of the $B_{c}^{+}$ → $B_{s}^{0} + \pi^{+}$ decay by the LHCb collaboration we present the meson dominance predictions for other weak decays of the $B_{c}^{+}$ into $B_{s}^{0}$ or $B_{s}^{0}$ in the form of branching ratios to the observed decay.

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The LHCb collaboration at the CERN LHC collider has recently announced [1] the observation of the bottomness–preserving (BP) decay $B_{c}^{+}$ → $B_{s}^{0} + \pi^{+}$ with significance in excess of five standard deviations independently in two decay channels of $B_{s}^{0}$. This sets a hope that similar BP decays of the $B_{c}^{+}$ meson may be found either in the already accumulated data or after the LHC reopen with higher energy and luminosity in 2015.

In this note we present the estimates of the branching ratios of other BP decays of the $B_{c}^{+}$ relative to the already observed one. We use the meson dominance (MD) model [2], which describes well [3] the meson decays that fall into the “external W-emission” category according to the quark-diagram nomenclature [4].

The decay of the $B_{c}^{+}$ (or, to be general, $P_{1}$) into the neutral meson $B_{s}^{0}$ ($P_{2}$) and the $\pi^{+}$ ($P_{3}$) is described by a diagram depicted in Fig. 1. The diagram contains first a strong interaction vertex entered by $P_{1}$, $P_{2}$, and a positive vector meson $V$ (here $D_{s}^{*+}$) with flavor quantum numbers required by the conservation laws. The vector meson then couples to the gauge boson $W^{+}$, which in turn couples to the outgoing pseudoscalar meson $P_{3}$ (here $\pi^{+}$). The corresponding partial decay width is given by

$$\Gamma(P_{1} \rightarrow P_{2} + P_{3}) = \frac{G_{F}^{2} \times X_{P_{1}P_{2}P_{3}} \times Z_{P_{1}} \times (x-y) \lambda^{1/2}(x, y, m_{3}^{2})}{16 \pi m_{1}^{2}},$$

(1)

where $x = m_{1}^{2}$, $y = m_{2}^{2}$, and function $\lambda$ is defined by

$$\lambda(x, y, z) = x^{2} + y^{2} + z^{2} - 2xy - 2xz - 2yz.$$

The definition of the dimensionless parameter $X_{P_{1}P_{2}P_{3}}$

$$X_{P_{1}P_{2}P_{3}} = \left| w_{V} V_{V} \frac{g_{V} P_{1} P_{2}}{g_{\rho}} \right|^{2},$$

includes the strong coupling constant $g_{V}$, the element $V_{V}$ of the CKM matrix pertinent to valence quark and antiquark of the meson $V$, and a dimensionless parameter $w_{V}$ ≈ 1 characterizing the deviation of the $V - W^{+}$ coupling from the $\rho^{+} - W^{+}$ one. At present, there is no chance of getting the value of $X_{B_{c}^{+} B_{s}^{0} D_{s}^{*+}}$ from its components. But as we are going to relate the branching fractions of the various $B_{c}^{+}$ → $B_{s}^{0}$ transitions, this unknown quantity will cancel.

The parameter $Z_{P}$, which is another important ingredient of Eq. (1), is defined in terms of the pseudoscalar decay constant $f_{P}$ and the CKM matrix element $V_{P}$ corresponding to the valence quark composition of a particular pseudoscalar meson $P$ by

$$Z_{P} = |f_{P} V_{P}|^{2}.$$

(2)

Its value is determined from the muonic decay width given by the formula [2]

$$\Gamma(P \rightarrow \mu^{+} + \nu_{\mu}) = \frac{G_{F}^{2}}{8 \pi} Z_{P} m_{\mu}^{2} m_{P} \left( 1 - \frac{m_{\mu}^{2}}{m_{P}^{2}} \right)^{2} \times [1 + \mathcal{O}(\alpha)].$$

We have included the radiative corrections $\mathcal{O}(\alpha)$ following [5]. The results obtained by using the experimental muonic decay widths [6] are shown in Table I.

Now we are ready to calculate the branching ratio

$$\frac{B(B_{c}^{+} \rightarrow B_{s}^{0} + K^{+})}{B(B_{c}^{+} \rightarrow B_{s}^{0} + \pi^{+})} = \frac{Z_{K^{+}}}{Z_{\pi^{+}}} \sqrt{\frac{\lambda(m_{B_{c}^{+}}^{2}, m_{B_{s}^{0}}^{2}, m_{K^{+}}^{2})}{\lambda(m_{B_{c}^{+}}^{2}, m_{B_{s}^{0}}^{2}, m_{\pi^{+}}^{2})}} = (6.470 \pm 0.025) \times 10^{-2},$$

TABLE I. Parameters $Z_{P}$ characterizing the coupling of pseudoscalar mesons to the charged gauge boson and their sources.

| P  | $Z_{P}$ (MeV$^{2}$) | Source         |
|----|-------------------|----------------|
| $\pi^{+}$ | $(1.6158 \pm 0.0019) \times 10^{4}$ | $\pi^{+} \rightarrow \mu^{+} + \nu_{\mu}$ |
| $K^{+}$  | $(1.2307 \pm 0.0033) \times 10^{3}$ | $K^{+} \rightarrow \mu^{+} + \nu_{\mu}$ |

FIG. 1. Decay $B_{c}^{+}$ → $B_{s}^{0} + \pi^{+}$ in the MD model.
which shows that the decay $B_c^+ \to B_s^0 + K^+$ will not be probably observed soon.

Let us turn now to the semileptonic decays. The decay $B_c^+ (P_1) \to B_s^0 (P_2) + \ell^+ + \nu_{\ell}$ is in the MD model described by diagram in Fig. 2. The differential decay width in $t$, which is the square of the four-momentum transfer from $P_1$ to $P_2$, is given by

$$d\Gamma = \frac{G_F^2 X_{P1} X_{P2} \rho_{\mu \nu}}{3(4\pi m_1)^3} \left( \frac{t-z}{t^2} \right)^2 \lambda^{1/2}(x,y,t) \left( \frac{r}{r-t} \right)^2 \times \left[ (2t+z)\lambda(x,y,t) + \frac{3z}{r^2}(x-y)^2(r-t)^2 \right]. \quad (3)$$

Newly defined parameters are $r = m_1^2$ and $z = m_2^2$. Formula (3) is a simplified but equivalent version of Eq. (4.3) from Ref. [2]. After factorizing out the unknown multiplication constant we can perform the numerical integration of (3) and get the total semileptonic decay width (up to that multiplication constant). After dividing it by (1), the unknown $X$-factor cancels and we are getting

$$\frac{B(B_c^+ \to B_s^0 + \ell^+ + \nu_{\ell})}{B(B_c^+ \to B_s^0 + \pi^+)} = 0.392 \pm 0.006$$

and

$$\frac{B(B_c^+ \to B_s^0 + \mu^+ + \nu_{\mu})}{B(B_c^+ \to B_s^0 + \pi^+)} = 0.367 \pm 0.006.$$

We can also compare the branching fractions of the modes with different $B$-mesons in the final states, e.g.,

$$\frac{B(B_c^+ \to B_s^0 + \pi^+)}{B(B_c^+ \to B_s^0 + \pi^+)} = R_X \left( \frac{m^2_{B_s^0} - m^2_{B^0}}{m^2_{B_c^+} - m^2_{B^0}} \right) \times \frac{\lambda(m^2_{B_s^0}, m^2_{B^0}, m^2_{\pi^+})}{\lambda(m^2_{B_c^+}, m^2_{B^0}, m^2_{\pi^+})},$$

where

$$R_X = \frac{X_{B_c^+ B_s^0 D^*-}}{X_{B_c^+ B_s^0 D^*-}}$$

is an unknown quantity. We can get a crude estimate of its value if assuming that the light flavor SU(3) symmetry is not badly broken. Then we can write

$$R_X \approx \frac{|V_{cd}|^2}{|V_{cb}|^2} \approx 5.3 \times 10^{-2}$$

and

$$\frac{B(B_c^+ \to B_s^0 + \pi^+)}{B(B_c^+ \to B_s^0 + \pi^+)} \approx 7 \times 10^{-2}.$$

The semileptonic transitions of $B_c^+$ to $B^0$ will be suppressed relative to those to $B_s^0$ by the same factor.

To conclude: The semileptonic decays of the $B_c^+$ to $B_s^0$ meson are, based on the magnitude of their branching fractions, the best candidates for the experimental observation. The branching fraction of $B_c^+ \to B_s^0 + e^+ + \nu_e$ ($B_c^+ \to B_s^0 + \mu^+ + \nu_\mu$) is about thirty-nine (thirty-seven) per cent of that of the already observed decay $B_c^+ \to B_s^0 + \pi^+.$

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