Insight into $f_0(980)$ through the $B_{(s)}$ charmed decays

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Abstract

Through analyzing the $B_{(s)}$ charmed decays $B^0 \rightarrow \bar{D}_0 f_0(980)$ and $B_s \rightarrow \bar{D}_0 f_0(980)$ within the framework of the PQCD factorization approach and comparing with the current data, we find that there are two possible regions for the $f_0(980) - f_0(500)$ mixing angle $\theta$: one is centered at $34^\circ \sim 38^\circ$ and the other is falls into $142^\circ \sim 154^\circ$. The former can overlap mostly with one of allowed angle regions extracted from the decay $B^0 \rightarrow \bar{D}_0 f_0(500)$. The branching fractions of $B_s$ decay modes are less sensitive to the mixing angle compared with those of B decay modes. Especially, for the decay $B_s \rightarrow D^0 f_0(980)$, its branching fraction changes only slightly between $(1.2 \sim 1.8) \times 10^{-7}$ when the mixing angle $\theta$ runs from $0^\circ$ to $180^\circ$. All of our results support the picture that the $f_0(980)$ is dominated by two quark component in the B decay dynamic mechanism. Furthermore, the $ss$ component is more important than the $q\bar{q} = (u\bar{u} + d\bar{d})/\sqrt{2}$ component. This point is different from $f_0(500)/\sigma$. Last but not least, our picture is not in conflict with the popular four-quark explanation.

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I. INTRODUCTION

Up to now the quark-level substructure of scalar mesons is still not well understood. Especially, the slight scalars mesons, including $f_0(500)$, $f_0(980)$, $K_0^*(800)$, and $a_0(980)$, which form an SU(3) flavor nonet and are considered as either two quark states or tetraquark states (di-quark and anti-di-quark structure) as originally advocated by Jaffe [1]. Certainly, there are other different SU(3) scenarios about scalar mesons [2]. If one considers these light scalar mesons as two quark states, $qar{q}$ structure, there are experiments indicate that the heaviest one $f_0(980)$ and the lightest one $f_0(500)$ in this SU(3) nonet must have a mixing

$$|f_0(980)⟩ = |s\bar{s}⟩ \cos \theta + |n\bar{n}⟩ \sin \theta, \quad |f_0(500)⟩ = -|s\bar{s}⟩ \sin \theta + |n\bar{n}⟩ \cos \theta,$$

where $|n\bar{n}⟩ \equiv (u\bar{u} + d\bar{d})/\sqrt{2}$. For the mixing angle $\theta$, there are several different values from experimental measurements. A mixing angle $\theta = 34° \pm 6°$ was determined from the decays $J/\Psi \to f_0\phi, f_0\omega$, and $31° \pm 5°$ or $42° \pm 7°$ from the decays $D_{(s)} \to f_0(980)\pi, f_0(980)K$, while a range $35° < |\theta| < 55°$ was given from the analysis of three body decay $D_s^+ \to \pi^+\pi^+\pi^-$. An analysis of $f_0(980) - f_0(500)$ mixing by using the light cone QCD sum rules [3], yielded $\theta = 27° \pm 13°$ and $\theta = 41° \pm 11°$. The value of $\theta \sim 34°$ or $\sim 146°$ was obtained in the decays $B_s \to J/\psi f_0(980), J/\psi\sigma$ [4]. Ochs [5] found $\theta = 30° \pm 3°$ by averaging over several decay processes. The authors of Ref. [6] provided a limit on the mixing angle $\theta < 29°$ at 90% confidence. As we know, the mixing between $f_0(980) - \sigma$ is something like that in $\eta - \eta'$, but with much more uncertainties. In order to explain the $K - \eta'$ puzzle, some complex mixing mechanisms including gluon even $\eta_c$ meson in $\eta - \eta'$ were also considered [7, 8]. This led people to conjecture that $f_0(980)$ and $f_0(500)$ may not be simple quark-antiquark states, perhaps there exist more complicated structure except the $f_0(980) - \sigma$ mixing.

Recently, the decays $B_{(s)} \to \bar{D}f_0(500), \bar{D}f_0(980)$ were measured by LHCb collaboration [10, 11]:

$$B(B^0 \to \bar{D}^0 f_0(500)) = (11.2 \pm 0.8 \pm 0.5 \pm 2.1 \pm 0.5) \times 10^{-5}, \quad (2)$$
$$B(B^0 \to \bar{D}^0 f_0(980)) = (1.34 \pm 0.25 \pm 0.10 \pm 0.46 \pm 0.06) \times 10^{-5}, \quad (3)$$
$$B(B_s^0 \to \bar{D}^0 f_0(980)) = (1.7 \pm 1.0 \pm 0.5 \pm 0.1) \times 10^{-6}. \quad (4)$$

where the first and the second uncertainties are statistical and experimental systematic errors, respectively, the third one is from the model-dependent error. We see there exist larger statistical error in the $B_s^0$ decay and the model-dependent error in the fist two $B^0$ decays. By using these new data, we will try to constrain the mixing angle between $f_0(980)$ and $\sigma$ through these $B_{(s)}$ decays in the perturbative QCD (pQCD) approach. There was a work about constraining the mixing angle through $B_s^0 \to J/\Psi f_0(980), J/\Psi\sigma$ decays [4], but two different approaches were used in the same decay channel: the factorizable contribution and vertex corrections are calculated in the QCD Factorization (QCDF) approach, while the hard spectator scattering corrections are calculated in the pQCD approach. So one may suspect its rationality and reliability in determining the mixing angle between $f_0(980) - \sigma$. The B meson decays with a D meson involved in the final states have been studied in pQCD approach, such as $B \to DP, DV, DA$ [12, 13], here $P, V, A$ represent a pseudoscalar, vector and axial-vector meson, respectively. Most of the
predictions can well explain the experimental data. While an explicit calculation for the branching ratio of the decay \( B^0_s \rightarrow \bar{D}^0 f_0(980) \) gives \((3.5^{+1.26+0.50}_{-1.15-0.77}) \times 10^{-5}\) [16], which is quite different from the present experimental result. So we would like to systematically study the decays \( B_{(s)} \rightarrow D f(980) \) in the pQCD approach, including the CKM suppressed decays \( B_{(s)} \rightarrow D f_0(980) \). At last, the decays \( B_{(s)} \rightarrow \bar{D}^* f(980) , D^* f_0(980) \) are also considered.

The layout of this paper is as follows. In Sec. III decay constants and light-cone distribution amplitudes of the relevant mesons are introduced. In Sec. IV we then analyze these decay channels using the PQCD approach. The numerical results and the discussions are given in Sec. IV. Conclusions are presented in the final part.

II. DECAY CONSTANTS AND DISTRIBUTION AMPLITUDES

For the wave function of the heavy \( B_{(s)} \) meson, we take

\[
\Phi_{B_{(s)}}(x, b) = \frac{1}{\sqrt{2N_c}}(P_{B_{(s)}} + m_{B_{(s)}})\gamma_5\phi_{B_{(s)}}(x, b). \tag{5}
\]

Here only the contribution of the first Lorentz structure \( \phi_{B_{(s)}}(x, b) \) is taken into account, since the contribution of the second Lorentz structure \( \bar{\phi}_{B_{(s)}} \) is numerically small [19] and can be neglected. For the distribution amplitude \( \bar{\phi}_{B_{(s)}}(x, b) \) in Eq. (5), we adopt the following model:

\[
\phi_{B_{(s)}}(x, b) = N_{B_{(s)}}x^2(1 - x)^2 \exp \left[ -\frac{M^2_{B_{(s)}}x^2}{2\omega_b^2} - \frac{1}{2}(\omega_b b)^2 \right], \tag{6}
\]

where \( \omega_b \) is a free parameter and taken to be \( \omega_b = 0.4 \pm 0.04(0.5 \pm 0.05) \) GeV for \( B(B_s) \) in numerical calculations, and \( N_B = 101.445 \) (\( N_{B_s} = 63.671 \)) is the normalization factor for \( \omega_b = 0.4 \) (0.5). For \( B_s \) meson, the SU(3) breaking effects are taken into consideration.

As for the wave functions of the \( D \) meson, we use the form derived in Ref. [20]

\[
\int \frac{d^4\omega}{(2\pi)^4} e^{ik \cdot \omega}\langle 0|\bar{c}(0)u(\omega)|\bar{D}^0 \rangle = -\frac{i}{\sqrt{2N_c}}[(\not{P}_D + m_D)\gamma_5 \beta \phi_D(x, b)], \tag{7}
\]

\[
\int \frac{d^4\omega}{(2\pi)^4} e^{ik \cdot \omega}\langle 0|\bar{c}(0)u(\omega)|\bar{D}^{*0} \rangle = -\frac{i}{\sqrt{2N_c}}[(\not{P}_{D^*} + m_{D^*})\not{\epsilon}_L \gamma_\beta \phi^{D^*}_{D}(x, b)], \tag{8}
\]

where \( \not{\epsilon}_L \) is the longitudinal polarization vector. In this work only the longitudinal polarization component is used. Here we take the best-fitted form \( \phi^{D}_{D}(x, b) \) from B to charmed meson decays derived in [22] as

\[
\phi_D(x, b) = \frac{f_D}{2\sqrt{2N_c}}6x(1-x)[1+C_D(1-2x)]\exp[-\frac{\omega^2 b^2}{2}]. \tag{9}
\]

For the wave function \( \phi_{D_s}(x, b) \), it has the similar expression as \( \phi_D(x, b) \) except with different parameters, and given as follows: \( f_D = 204.6\ MeV \), \( f_{D_s} = 257.5\ MeV \), and \( C_{D(s)} = 0.5 \) (0.4), \( \omega_{D(s)} = 0.1 \) (0.2) [21]. For the wave function \( \phi^{D^*}_{D}(x, b) \), we take the same distribution amplitude with that of the pseudoscalar meson \( \bar{D}(s) \) because of their
small mass difference, except with different decay constants \(f_D = 270\,\text{MeV}\) and \(f_{D^*} = 310\,\text{MeV}\) [22].

Since the neutral scalar meson \(f_0(980)\) cannot be produced via the vector current, we have \(\langle f_0(p) | \bar{q} \gamma_\mu q | 0 \rangle = 0\) (the abbreviation \(f_0\) denotes the \(f_0(980)\) for simplicity). Taking the \(f_0(980) - \sigma\) mixing into account, the scalar current \(\langle f_0(p) | \bar{q} \gamma_1 q | 0 \rangle = \langle f_s \rangle \bar{s} s\) can be written as:

\[
\langle f_0^n | \bar{d} d | 0 \rangle = \langle f_0^n | u \bar{u} | 0 \rangle = \frac{1}{\sqrt{2}} m_{f_0} \bar{f}_0^n, \quad \langle f_0^s | s \bar{s} | 0 \rangle = m_{f_0} \bar{f}_0^s,\tag{10}
\]

where \(f_0^{(n,s)}\) represent for the quark flavor states for \(n\bar{n}\) and \(s\bar{s}\) components of \(f_0\) meson, respectively. As the scalar decay constants \(\bar{f}_0^n\) and \(\bar{f}_0^s\) are very close [17], we can assume \(\bar{f}_0^n = \bar{f}_0^s\) and denote them as \(\bar{f}_0\) in the following.

The twist-2 and twist-3 LCDAs for the different components of \(f_0(980)\) are defined by:

\[
\langle f_0(p) | \bar{q}(z) \gamma_\mu q(0) \rangle = \frac{1}{\sqrt{2N_c}} \int_0^1 dx e^{ixp-z} \left\{ \bar{q} \Phi_{f_0}(x) + m_{f_0} \Phi_{f_0}^S(x) + m_{f_0} (\bar{q} q - 1) \Phi_{f_0}^T(x) \right\},\tag{11}
\]

where we assume \(f_0^n(p)\) and \(f_0^n(p)\) are the same and denote them as \(f_0(p), n_+\) and \(n_-\) are light-like vectors: \(n_+ = (1, 0, 0_T), n_- = (0, 1, 0_T)\). The normalization of the distribution amplitudes are related to the decay constants:

\[
\int_0^1 dx \Phi_{f_0}(x) = \int_0^1 dx \Phi_{f_0}^T(x) = 0, \quad \int_0^1 dx \Phi_{f_0}^S(x) = \frac{\bar{f}_0}{2\sqrt{2N_c}}.\tag{12}
\]

The twist-2 LCDA \(\Phi_{f_0}(x)\) can be expanded in terms of Gegenbauer polynomials as:

\[
\Phi_{f_0}(x) = \frac{1}{2\sqrt{2N_c}} \bar{f}_0 6x(1-x) \left[ B_0 + \sum_{m=1} B_m C_m^{3/2}(2x - 1) \right],\tag{13}
\]

with the decay constant \(\bar{f}_0 = 0.18 \pm 0.015\,\text{GeV}\) [23]. It is noticed that all the even Gegenbauer momentums vanish due to the charge conjugation invariance. As for the odd Gegenbauer momentums, only the first term is kept and the value of the coefficient is taken as \(B_1 = -0.78 \pm 0.08\) [17]. For the twist-3 LCDA, we also take the first term of the Gegenbauer expansion, i.e. the asymptotic form,

\[
\Phi_{f_0}^T(x) = \frac{1}{2\sqrt{2N_c}} \bar{f}_0 (1-2x), \quad \Phi_{f_0}^S(x) = \frac{1}{2\sqrt{2N_c}} \bar{f}_0 (1-x)\tag{14}.
\]

### III. THE PERTURBATIVE QCD CALCULATION

The weak effective Hamiltonian \(H_{\text{eff}}\) for the charmed \(B_{(s)}\) decays \(B_{(s)} \to \bar{D} f_0(980), \bar{D}^* f_0(980)\), is composed only by the tree operators and given by:

\[
H_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cb}^* V_{ub} (\mu_1 O_1(\mu) + \mu_2 O_2(\mu)),\tag{15}
\]
where the tree operators are written as:

\[ O_1 = (\bar{c}_a b_\beta)_{V-A} (\bar{D}_\beta u_\alpha)_{V-A} \]
\[ O_2 = (\bar{c}_a b_\alpha)_{V-A} (\bar{D}_\beta u_\alpha)_{V-A} \]

with \( D \) represents \( d(s) \). These decays with larger CKM matrix elements, say the \( \bar{b} \to \bar{d} \) transition, \( |V_{ub}V_{ud}| = 0.04 \) are called CKM allowed decays. Another kind of decays \( B(s) \to D^0 f_0, D^{*0} f_0, D^+ f_0, D^{*+} f_0 \) with smaller CKM matrix elements (in case of \( b \to d \) transition, \( |V_{ub}V_{cd}| = 0.00093 \)) are called CKM suppressed decays and the corresponding weak effective Hamiltonian is given as:

\[ H_{eff} = \frac{G_F}{\sqrt{2}} V^*_{ub} V_{cd} [C_1(\mu)O_1(\mu) + C_2(\mu)O_2(\mu)]. \]

Here we take the decay \( B^0 \to D^0 f_0 \) as an example, whose leading-order Feynman diagrams are shown in Figure 1. The Feynman diagrams on the first row are for the emission types, where Figs.(a) and (b) are the factorizable diagrams, Figs.(c) and (d) are the nonfactorizable ones, their amplitudes are written as:

\[ \mathcal{F}^D_{B \to f_0} = 8 \pi C_F M_B^4 f_D \int_0^1 dx_1 dx_2 \int_0^\infty b_1 b_2 b_3 b_4 \phi_B(x_1, b_1)(1 + x_2)\phi_f(x_2) + r_f(1 - 2x_2) \]
\[ \times (\phi^*_{f_0}(x_2) + \phi^*_{f_0}(x_2)) E_{e}(t_a) h_e(x_1, x_2(1 - r_D^2), b_1, b_2) S_i(x_1) \]
\[ + 2r_f \phi_{f_0}(x_2) E_{e}(t_b) h_e(x_2, x_1(1 - r_D^2), b_2, b_1) S_i(x_1), \]

\[ \mathcal{M}^D_{B \to f_0} = 32 \pi C_F m_B^4 / \sqrt{2} N_C \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 b_2 b_3 b_4 \phi_B(x_1, b_1)\phi_D(x_3, b_3) \]
\[ \times \left\{ \left[ (x_3 - 1)\phi_{f_0}(x_2) + r_f x_2(\phi^*_{f_0}(x_2) - \phi^*_{f_0}(x_2)) - 4r_f r_c r_D \phi^*_{f_0}(x_2) \right] \right\} \]
\[ \times E_{en}(t_c) h_{en}(x_1, x_2(1 - r_D^2), x_3, b_1, b_3) + E_{en}(t_d) h_{en}(x_1, x_2(1 - r_D^2), x_3, b_1, b_3) \]
\[ \times \left[ (x_2 + x_3)\phi_{f_0}(x_2) - r_f x_2(\phi^*_{f_0}(x_2) + \phi^*_{f_0}(x_2)) \right] \],

with the mass ratios \( r_{f_0} = m_{f_0}/M_B, r_D = m_D/M_B, \) and \( r_c = m_c/M_B \). The evolution factors evolving the scale \( t \) and the hard functions of the hard part of factorization amplitudes
are listed as:

\[ E_e(t) = \alpha_s(t) \exp[-S_B(t) - S_f(t)], \]
\[ E_{en}(t) = \alpha_s(t) \exp[-S_B(t) - S_f(t) - S_D(t)|b_1=b_2], \]
\[ h_e(x_1, x_2, b_1, b_2) = K_0(\sqrt{x_1 x_2 m_B b_1}) \left[ \theta(b_1 - b_2)K_0(\sqrt{x_2 m_B b_1})I_0(\sqrt{x_2 m_B b_2}) \right. \\
\left. + \theta(b_2 - b_1)K_0(\sqrt{x_2 m_B b_2})I_0(\sqrt{x_2 m_B b_1}) \right], \]
\[ h_{en}^j(x_1, x_2, x_3, b_1, b_3) = \left[ \theta(b_1 - b_3)K_0(\sqrt{x_1 x_2 (1 - r_D^2)} m_B b_1)I_0(\sqrt{x_1 x_2 (1 - r_D^2)} m_B b_3) \right. \\
\left. + (b_1 \leftrightarrow b_3) \left( \frac{K_0(A_j m_B b_3)}{2} \right)_{A_j^2 \geq 0} \right. \\
\left. \left( \frac{\pi}{2} H_0^{(1)}(\sqrt{|A_j^2|m_B b_3}) \right)_{A_j^2 \leq 0} \right], \]

with the variables \( A_j^2(j = c, d) \) listed as:

\[ A_c^2 = r_c^2 - (1 - x_1 - x_3)(x_2(1 - r_D^2) + r_D^2), \]
\[ A_d^2 = (x_1 - x_3)x_2(1 - r_D^2). \]

The hard scale \( t \) and the expression of Sudakov factor in each amplitude can be found in the Appendix. As we know, the double logarithms \( \alpha_s ln^2 x \) produced by the radiative corrections are not small expansion parameters when the end-point region is important. In order to improve the perturbative expansion, threshold resummation of these logarithms to all order is needed, which leads to a quark jet function:

\[ S_t(x) = \frac{21+2c \Gamma(3/2 + c)}{\sqrt{\pi} \Gamma(1 + c)} [x(1 - x)]^c, \]

with \( c = 0.32 \). It is effective to smear the end point singularity with a momentum fraction \( x \rightarrow 0 \). This factor will also appear in the factorizable annihilation type amplitudes.

The amplitudes for the Feynman diagrams on the second row can be obtained by the Feynman rules and are given as:

\[ \mathcal{M}^{\tilde{D}}_{\text{ann}} = 32\pi C_f m_B^4 / \sqrt{2N} \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_3 \phi_B(x_1, b_1) \phi_D(x_3, b_3) \times \{ E_{an}(t_e) h^e_{an}(x_1, x_2, x_3, b_1, b_3) [x_3 \phi_f_0(x_2) \right. \\
\left. + r_D r_f_0 ((x_2 - x_3 - 3) \phi_f_0^*(x_2) + (x_2 + x_3 - 1) \phi_f_0(x_2)) \right] \\
\left. + E_{an}(t_f) h^f_{an}(x_1, x_2, x_3, b_1, b_3) [(x_2 - 1) \phi_f_0(x_2) \right. \\
\left. + r_D r_f_0 ((1 + x_3 - x_2) \phi_f_0^*(x_2) + (x_2 + x_3 - 1) \phi_f_0(x_2)) \right] \}, \]

\[ \mathcal{F}^{D}_{\text{ann}} = -8\pi C_f f_B m_B^4 \int_0^1 dx_2 dx_3 \int_0^\infty b_2 db_2 b_3 db_3 \phi_D(x_3, b_3) \{ [(1 - x_2) \phi_f_0(x_2) - 2r_f_0 r_D \right. \\
\left. \times x_2 \phi_f_0^*(x_2) + 2r_D r_f_0 (x_2 - 2) \phi_f_0(x_2)] E_{af}(t_g) h_{af}(x_3, (1 - x_2)(1 - r_D^2), b_3, b_3) \right. \\
\left. + E_{af}(t_h) h_{af}(x_2, x_3 (1 - r_D^2), b_2, b_3) [-x_3 \phi_f_0(x_2) + 2r_D r_f_0 (x_3 + 1) \phi_f_0^*(x_2)] \}.  \]
Similarly, \( F_{\text{ann}}^D(M_{\text{ann}}^D) \) are the (non)factorizable annihilation type amplitudes, where the evolution factors \( E \) evolving the scale \( t \) and the hard functions of the hard part of factorization amplitudes are listed as:

\[
E_{\text{an}}(t) = \alpha_s(t) \exp[-S_B(t) - S_D(t) - S_{f_0}(t)]_{b_3 = b_3},
\]

\[
E_{af}(t) = \alpha_s(t) \exp[-S_D(t) - S_{f_0}(t)],
\]

\[
h_{\text{an}}^j(x_1, x_2, x_3, b_1, b_3) = \frac{i\pi}{2} \left[ \theta(b_1 - b_3)H_0^{(1)}(\sqrt{x_2x_3(1 - r_f^2)}m_Bb_1)J_0(\sqrt{x_2x_3(1 - r_f^2)}m_Bb_3)
\right.
\]

\[
+ (b_1 \leftrightarrow b_3)] \left( \frac{K_0(L_j m_B b_1)}{\sqrt{|L_j|^2 m_B b_1}} \text{ for } L_j^2 \geq 0 \right)
\]

\[
\left. \frac{1}{2} H_0^{(1)}(\sqrt{|L_j|^2 m_B b_1}) \text{ for } L_j^2 \leq 0 \right),
\]

\[
h_{af}(x_2, x_3, b_2, b_3) = \left( \frac{i\pi}{2} \right)^2 H_0^{(1)}(\sqrt{x_2x_3m_Bb_2})
\]

\[
\times \left[ \theta(b_2 - b_3)H_0^{(1)}(\sqrt{x_2x_3m_Bb_2})J_0(\sqrt{x_2x_3m_Bb_3}) + (b_2 \leftrightarrow b_3) \right],
\]

where the definitions of \( L^2_j(j = e, f) \) are written as:

\[
L_e^2 = r_b^2 - (1 - x_3)(1 - (1 - x_2)(1 - r_f^2)) - x_1),
\]

\[
L_f^2 = x_3(x_1 - (1 - x_2)(1 - r_f^2)).
\]

The functions \( H_0^{(1)}, J_0, K_0, I_0, \) which appear in the upper hard kernel \( h_e, h_{e, \text{an}}, h_{\text{an}}, h_{af} \) are the (modified) Bessel functions, which are obtained from the Fourier transformations of the quark and gluon propagators. Combining above amplitudes, one can easily to write down the total decay amplitudes of each considered channel

\[
\mathcal{A}(B^0 \to D^0 f_0(980)) = \frac{G_F}{\sqrt{2}} V_{cb} V_{ud} (F^D_{B \to f_0} a_2 + M^D_{B \to f_0} C_2 + M^D_{\text{ann}} C_2 + F^D_{\text{ann}} a_2),
\]

\[
\mathcal{A}(B^0 \to D^0 f_0(980)) = \frac{G_F}{\sqrt{2}} V_{ub} V_{cd} (F^D_{B \to f_0} a_2 + M^D_{B \to f_0} C_2 + M^D_{\text{ann}} C_2 + F^D_{\text{ann}} a_2),
\]

\[
\mathcal{A}(B^*_s \to D^0 f_0(980)) = \frac{G_F}{\sqrt{2}} V_{cb}^* V_{us} (F^D_{B \to f_0} a_2 + M^D_{B \to f_0} C_2 + M^D_{\text{ann}} C_2 + F^D_{\text{ann}} a_2),
\]

\[
\mathcal{A}(B^0 \to D^0 f_0(980)) = \frac{G_F}{\sqrt{2}} V_{ub}^* V_{cs} (F^D_{B \to f_0} a_2 + M^D_{B \to f_0} C_2 + M^D_{\text{ann}} C_2 + F^D_{\text{ann}} a_2),
\]

\[
\mathcal{A}(B^+ \to D^0 f_0(980)) = \frac{G_F}{\sqrt{2}} V_{ub}^* V_{cd} (F^D_{B \to f_0} a_1 + M^D_{B \to f_0} C_2/3 + M^D_{\text{ann}} C_2/3 + F^D_{\text{ann}} a_1),
\]

\[
\mathcal{A}(B^+ \to D^+ f_0(980)) = \frac{G_F}{\sqrt{2}} V_{ub}^* V_{cs} (F^D_{B \to f_0} a_1 + M^D_{B \to f_0} C_2/3 + M^D_{\text{ann}} C_2/3 + F^D_{\text{ann}} a_1)
\]

and likewise for the corresponding decays with the pseudoscalar meson \( D \) replaced by the vector meson \( D^* \).

\section*{IV. NUMERICAL RESULTS AND DISCUSSIONS FOR B_{(s)} DECAYS}

We use the following input parameters for numerical calculations \cite{21}:

\[
f_B = 190MeV, f_{B_s} = 230MeV, M_B = 5.28GeV, M_{B_s} = 5.37GeV,
\]

\[
\tau_B^\pm = 1.638 \times 10^{-12}s, \tau_{B^0} = 1.519 \times 10^{-12}s, \tau_{B_s} = 1.512 \times 10^{-12}s,
\]

\[
M_{B^0} = 1.869GeV, M_{D^*_s} = 1.968GeV, M_{D^*_s} = 2.007GeV, M_{D^*_s} = 2.112GeV.
\]
For the CKM matrix elements, we adopt the Wolfenstein parametrization and the updated values $A = 0.814, \lambda = 0.22537, \rho = 0.117 \pm 0.021$ and $\eta = 0.353 \pm 0.013$ [21].

In the $B(s)$-rest frame, the decay rates of $B(s) \to D^{(*)} f_0(980)$ can be written as:

$$BR(B(s) \to D^{(*)} f_0(980)) = \frac{\tau_{B(s)}}{16\pi M_B} (1 - r_f^2) A,$$

where $A$ is the total decay amplitude of each considered decay, which has been given in last section.

Using the input parameters and the wave functions as specified in this section and Sec.II, we give the dependencies of the branching ratios $BR(B^0 \to \bar{D}^0 f_0(980))$ and $BR(B_s \to \bar{D}^0 f_0(980))$ on the mixing angle $\theta$ shown in Fig.2. Combining these two panels,

![Figure 2](image)

**FIG. 2:** Dependencies of the branching ratios $BR(B^0 \to \bar{D}^0 f_0(980))$ (left) and $BR(B_s \to \bar{D}^0 f_0(980))$ (right) on the mixing angle $\theta$. In each panel, the solid (blue) curve represents the central value of the theoretical prediction, and the two dashed (red) curves correspond to the upper and lower limits. On the left panel, the shaded band shows the allowed region and the horizontal bisector the central value of $BR(B^0 \to \bar{D}^0 f_0(980)) = (1.34 \pm 0.54) \times 10^{-5}$ for data. On the right panel, for the large uncertainties with the branching ratio, only the half width band is given, that is to say, the upper edge line represents the center value of data $BR(B_s \to \bar{D}^0 f_0(980)) = (1.7 \pm 1.1) \times 10^{-6}$, and the lower edge line represents the experimental lower limit.

one can find that the allowed mixing angle lies in the range $135^\circ < \theta < 158^\circ$ at the large angle region. It is not strange that, as mentioned before, the large mixing angle $\theta \sim 146^\circ$ is also obtained in the analysis of $B_s \to J/\psi f_0(980), J/\psi \sigma$ decays [4]. In the following we mainly discuss the region with the mixing angle less than $90^\circ$. For the branching ratio of the decay $B^0 \to \bar{D}^0 f_0(980)$, the experimental value $(1.34\pm0.54)\times10^{-5}$ with $2.5\sigma$ can give a stronger constrain on the mixing angle, and in the range of $29^\circ < \theta < 46^\circ$, the central theoretical values agree well with the data. But if the theoretical uncertainties are included, the range will become wider. Although the branching ratio $Br(B_s \to \bar{D}^0 f_0(980))$ with large uncertainty can not give stringent constrain on the value of the mixing angle,
we can get some hints from the data: If we take the mixing angle $\theta = 0^\circ$, that is, we consider that the scalar meson $f_0(980)$ is composed entirely of two quark component $s\bar{s}$, the corresponding branching ratio is about $1.4 \times 10^{-6}$, which is a little lower than the experimental value. If we consider the small mixing with $q\bar{q} = (u\bar{u} + q\bar{q})/\sqrt{2}$, the branching ratio will get an enhancement for the interference between the two different kinds amplitudes from the different quark components, the maximal value for the branching ratio can be obtained at the mixing angle $\theta = 19^\circ$, and arrives at $1.56 \times 10^{-6}$ (shown in the right panel of Fig.2). But if we take such small mixing angle, say about $20^\circ$, it will make the branching ratio of the decay $B^0 \rightarrow D^0 f_0(980)$ undershoot the shaded band in the left panel of Fig.2, which represents the experimental allowed region. While the mixing angle $\theta$ between $f_0(980)$ and $f_0(500)$ should not be too large, say larger than $70^\circ$. If so, the predicted branching ratios of both the decays $B_\pm \rightarrow D^0 f_0(980)$ and $B^0 \rightarrow D^0 f_0(980)$ will deviate from the data even with the large errors taken into account. So we get the conclusion that the two quark component should be dominant for B meson decays in dynamic mechanism. Furthermore, the $s\bar{s}$ component is more important than the $q\bar{q}$ component. But it is not in conflict with the dominant four-quark structure in explaining the mass degeneracy of $f_0(980)$ and $a_0(980)$, and the narrower decay width of $f_0(980)$ than that of $f_0(500)$. In the following, we will discuss the mixing angle by considering the ratio of branching fractions. There are some advantages in considering the ratio, because one can eliminate the systematic errors on the experimental side, and avoid the hadronic uncertainties, such as the decay constants and the Gegenbauer moments of the final states on the theoretical side. From the data, one can find that the ratio of these two branching fractions $BR(B^0 \rightarrow D^0 f_0(980))/BR(B_\pm \rightarrow D^0 f_0(980)) = 7.88 \pm 5.60$. Unfortunately, here the uncertainty is mainly from the statistical error in the decay $B_\pm \rightarrow D^0 f_0(980)$, so the errors of the ratio are not much improved compared to those of the branching ratio of each decay mode. Certainly, here we consider a simple method, maybe there is a much better approach for the experimentalists to greatly reduce the errors from this ratio. So we advice to accurately measure this ratio in experiment, because it is important to further restrict the mixing angle $\theta$ between $f_0(980)$ and $f_0(500)(\sigma)$. The ratio can change in a very large range with the mixing angle taking different values, especially for $\theta = 90^\circ$, the branching ratio of $B_\pm \rightarrow D^0 f_0(980)$ is very small and will be exactly equal to zero if the contribution from $q\bar{q} = (u\bar{u} + d\bar{d})/\sqrt{2}$ is turned off, while $BR(B^0 \rightarrow D^0 f_0(980))$ arrives its maximal value. Then it will be meaningless for the ratio, not mentioning the errors. For the sake of comparison, we give two regions for the mixing angle shown in Fig.3. If combining these four panels in Fig.2 and Fig.3 together, one will get two further shrunken mixing angle ranges $22^\circ < \theta < 58^\circ$ and $141^\circ < \theta < 158^\circ$. In view of present large uncertainties from data and theory, it will be difficult to get an unitary value for the mixing angle. But even if more precise data are available, we still can not get the unitary value. This argument might be reasonable that there must be some influence from other components in $f_0(980)$, such as gluon, four quark component, and $K\bar{K}$ threshold effect, which we can not handle at present. Nevertheless, one can not deny that the two quark component in $f_0(980)$ is dominant in B decay dynamic mechanism, and the $s\bar{s}$ component is more important than the $q\bar{q}$ component.

Up to now we still do not analyze the decay $B^0 \rightarrow D^0 \sigma$, although the data of this channel is available. There are many uncertainties from the decay constant and the lightcone distribution amplitudes (LCDAs) of $\sigma$ meson. The authors of Ref. [17] assumed that $\sigma$ has the similar decay constant and LCDAs as those of $f_0(980)$, while the authors of
FIG. 3: Dependencies of the ratio between $\mathcal{BR}(B^0 \to \bar{D}^0 f_0(980))$ and $\mathcal{BR}(B_s \to \bar{D}^0 f_0(980))$ on the mixing angle $\theta$ at different regions. The shaded band shows the allowed region and the horizontal bisector the central value of $\mathcal{BR}(B^0 \to \bar{D}^0 f_0(980))/\mathcal{BR}(B_s \to \bar{D}^0 f_0(980)) = 7.88 \pm 5.60$ for data.

Ref.\cite{18} just took the same decay constant and LCDAs with those of $a_0(980)$. These two sets of parameters will generate very different results: If using the former, one will obtain small branching ratios which are far below the experimental lower limit in all the mixing angle region, but the predicted branching ratio will overlap with the data in some angle values by using the latter, which can be found in Fig.4. It shows that the decay constant and LCDAs of $\sigma$ is more close to those of $a_0(980)$, so they should have the similar quark components and structure. From Fig.4, we find that there also exist two allowed mixing angle regions $28^\circ \sim 64^\circ$ and $116^\circ \sim 152^\circ$, where the former region can overlap mostly with the allowed region $22^\circ \sim 58^\circ$ obtained from the analysis of $B^0 \to \bar{D}^0 f_0(980)$ and $B_s \to \bar{D}^0 f_0(980)$ decays. While the two large angle regions have less coincidence, it seems that the small angle region is more favored than the large one.

In order to predict other $B_{(s)}$ charmed decays, the mixing angle is taken as two values $34^\circ$ and $38^\circ$ (certainly, one can get similar branching ratios by taking $\theta = 142^\circ$ and $154^\circ$, if they can not be excluded by the future data), one of which is consistent with $\theta = 30^\circ \pm 3^\circ$ obtained by averaging over several processes \cite{5}. Then the branching ratios of these CKM suppressed decays $B^0 \to D^0 f_0(980), B_s \to D^0 f_0(980), B^+ \to D^+ f_0(980)$ and $B^+ \to D^{*+} f_0(980)$ are listed in Table I. When the pseudoscalar meson $D_{(s)}$ is replaced by the vector meson $D^{*}_{(s)}$ in our considering decays, and the branching ratios of the corresponding channels are listed in Table II. From our calculations, we find that the branching ratios of the $B_s$ decays are not very sensitive to the mixing angle $\theta$, especially for $\mathcal{BR}(B_s \to D^0 f_0)$, its value changes in the range of $(1.2 \sim 1.8) \times 10^{-7}$ when the mixing angle varies from $0^\circ$ to $180^\circ$. The reason is as follows: The amplitude from $s\bar{s}$ component has a large imaginary part and a small real part. It is contrary for the amplitude from $q\bar{q} = (u\bar{u} + d\bar{d})/\sqrt{2}$ component, where the real part is about one order larger than the
branching ratios of all the change trends make the total amplitude changes in a much milder cosine curve. The sine (cosine) law, but the later is stronger than the former, so the se two kinds of contrary amplitudes mixing through Eq.(1), respectively, the former (later) is dominated by the imaginary part. When the real and imaginary parts from the $s\bar{s}$ and $q\bar{q} = (u\bar{u} + d\bar{d})/\sqrt{2}$ amplitudes mixing through Eq.(1), respectively, the former (later) is dominated by the sine (cosine) law, but the later is stronger than the former, so these two kinds of contrary change trends make the total amplitude changes in a much milder cosine curve. The branching ratios of all the $B$ decay modes are dependent on the mixing angle via $\sin \theta$

FIG. 4: Dependence of the branching ratio $\mathcal{B}(B^0 \to \bar{D}^0 f_0(500))$ on the mixing angle $\theta$. The solid (blue) curve represents the central value of the theoretical prediction, and the two dashed (red) curves correspond to the upper and lower limits. The shaded band shows the allowed region and the horizontal bisector the central value of $\mathcal{B}(B^0 \to \bar{D}^0 f_0(500)) = (11.2 \pm 2.4) \times 10^{-5}$ for data.

TABLE I: The CP-averaged branching ratios ($\times 10^{-6}$) of $B \to D f_0(980)$ obtained by taking the mixing angle $\theta = 34^\circ$ and $38^\circ$, respectively. The first uncertainty comes from the $\omega_b = 0.4 \pm 0.1(0.5 \pm 0.1)$ for $B(B_s)$ mesons, the second and the third uncertainties are from the decay constant $f_0 = 0.18 \pm 0.015$ GeV and the Gegenbauer moment $B_1 = -0.78 \pm 0.08$ of $f_0(980)$ meson, respectively, and the last one comes from $C_{D(s)} = 0.5(0.4) \pm 0.1$ for $D(s)$ meson.

| Branching Ratio | $34^\circ$ | $38^\circ$ |
|-----------------|------------|------------|
| $\mathcal{B}(B \to D^0 f_0) \times 10^{-9}$ | $4.45_{-1.42}^{+2.25} + 0.96 + 0.71 + 0.35$ | $5.39_{-1.71}^{+2.72} + 1.16 + 0.86 + 0.43$ |
| $\mathcal{B}(B_s \to D^0 f_0) \times 10^{-7}$ | $1.32_{-0.60}^{+1.02} + 0.30 + 0.21 + 0.19$ | $1.29_{-0.55}^{+0.99} + 0.29 + 0.21 + 0.18$ |
| $\mathcal{B}(B^+ \to D^0 f_0) \times 10^{-7}$ | $1.00_{-0.26}^{+0.37} + 0.16 + 0.06 + 0.01$ | $1.22_{-0.32}^{+0.45} + 0.19 + 0.08 + 0.01$ |
| $\mathcal{B}(B^+ \to D^+_s f_0) \times 10^{-6}$ | $2.30_{-0.67}^{+0.96} + 0.32 + 0.11 + 0.07$ | $2.97_{-0.83}^{+1.20} + 0.43 + 0.16 + 0.07$ |
TABLE II: Same as Table I except for the decays $B \rightarrow \bar{D}^*(D^*)f_0(980)$.

|                  | 34°                      | 38°                      |
|------------------|--------------------------|--------------------------|
| $BR(B \rightarrow \bar{D}^0 f_0)[\times 10^{-6}]$ | $7.40^{+2.33+1.32+2.32+0.75}_{-1.84-1.26-1.78-0.73}$ | $8.97^{+2.83+1.66+2.82+0.91}_{-2.23-1.52-2.16-0.89}$ |
| $BR(B_s \rightarrow \bar{D}^0 f_0)[\times 10^{-6}]$ | $1.63+0.72+0.31+0.48+0.20$ | $1.43+0.62+0.27+0.42+0.17$ |
| $BR(B \rightarrow \bar{D}^* f_0)[\times 10^{-9}]$ | $6.48^{+3.57+1.37+0.64+0.33}_{-2.24-1.23-0.56-0.31}$ | $7.86^{+4.33+1.66+0.78+0.40}_{-2.72-1.49-0.68-0.37}$ |
| $BR(B_s \rightarrow \bar{D}^* f_0)[\times 10^{-7}]$ | $2.06_{-0.98-0.41-0.18-0.17}^{+1.79+0.46+0.20+0.20}$ | $1.94_{-0.90-0.39-0.17-0.16}^{+1.63+0.44+0.19+0.19}$ |
| $BR(B^+ \rightarrow D^{*+} f_0)[\times 10^{-7}]$ | $2.07^{+0.69+0.38+0.16+0.02}_{-0.49-0.34-0.15-0.02}$ | $2.51^{+0.84+0.46+0.19+0.02}_{-0.60-0.42-0.19-0.02}$ |
| $BR(B^+ \rightarrow D^{*+} f_0)[\times 10^{-6}]$ | $5.00_{-1.21-0.88-0.39-0.06}^{+1.68+0.94+0.37+0.07}$ | $6.10_{-1.47-1.06-0.47-0.10}^{+2.04+1.14+0.45+0.08}$ |

(maybe with an initial phase), just like the left panel in Fig. 2, while those of $B_s$ decay modes are dependent on the mixing angle via $\cos \theta$ with a initial phase, just like the right panel in Fig. 2.

V. CONCLUSION

In this paper, first we analyze the decays $B \rightarrow \bar{D}^0 f_0(980)$ and $B_s \rightarrow \bar{D}^0 f_0(980)$ carefully in the PQCD factorization approach and find two possible regions for the mixing angle $\theta$, one is centered at $34^\circ \sim 38^\circ$ and the other is near $142^\circ \sim 154^\circ$. If the data of the decay $B^0 \rightarrow D^0 \sigma$ is also included, we find that the small angle region is more favored. Our analyses support that the two quark component in $f_0(980)$ is dominant in $B$ decay dynamic mechanism, and the $s\bar{s}$ component is more important than the $q\bar{q}$ component. Certainly other components, such as gluon, four quark component, and $K\bar{K}$ threshold effect may also give some more or less influences. It is noticed that our picture is not in conflict with the popular explanation of dominant four-quark component in $f_0(980)$. Then we predict the branching ratios of other $B(s) \rightarrow D(s) f_0(980), D^*(s) f_0(980)$ decay channels by fixing $\theta = 34^\circ$ and $38^\circ$, respectively and find that the branching ratios of $B_s$ decay modes are less sensitive to the mixing angle compared with those of $B$ decay modes. Especially, for the decay $B_s \rightarrow D^0 f_0$, its branching ratio changes in a small region between $(1.2 \sim 1.8) \times 10^{-7}$ with the mixing angle $\theta$ running from $0^\circ$ to $180^\circ$.

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Appendix A: Decay amplitudes

For the CKM suppressed decays, for example, $B \to D^0 f_0(980)$, their Feynman diagrams to leading order will be different from Fig.1, especially for the (non-)factorizable annihilation diagrams, where the positions of $D$ and $f_0(980)$ are exchanged compared with those of $B \to D^0 f_0(980)$ decay. But the factorizable emission diagrams are the same with each other, so $\mathcal{F}_{B\to f_0}^D = \mathcal{F}_{B\to f_0}^D$. Here we also list other amplitudes of these CKM suppressed decays:

\[
\mathcal{M}_{B\to f_0}^D = 32\pi C_f m_B^4 / \sqrt{2N_C} \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 b_1 b_3 b_3 b_3 \phi_B(x_1, b_1) \phi_D(x_3, b_3) \\
\times \{ [(x_3 - x_1) \phi_{f_0}(x_2) - r_{f_0} x_2 (\phi_{f_0}^* (x_2) - \phi_{f_0}^* (x_2)) ] \\
\times E_{en}(t_d) h_{en}^e(x_1, x_2, (1 - r_D^2), x_3, b_1, b_3) + E_{en}(t_c) h_{en}^e(x_1, x_2, (1 - r_D^2), x_2, b_1, b_3) \\
\times [(x_2 - x_2 + x_3 - 1) \phi_{f_0}(x_2) + r_{f_0} x_2 (\phi_{f_0}^* (x_2) - \phi_{f_0}^* (x_2))] \} ,
\]

(A1)

\[
\mathcal{M}_{ann}^{f_0} = 32\pi C_f m_B^4 / \sqrt{2N_C} \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 b_1 b_3 b_3 b_3 \phi_B(x_1, b_1) \phi_D(x_3, b_3) \\
\times \{ E_{en}(t_e) h_{en}^e(x_1, x_2, x_3, b_1, b_3) [(1 - r_b - x_2) \phi_{f_0}(x_2) \\
+ r_{f_0} r_{f_0} ((2 - 4r_b - x_2 - x_3) \phi_{f_0}^* (x_2) - (x_2 - x_3) \phi_{f_0}^* (x_2))] \\
+ E_{an}(t_f) h_{an}^e(x_1, x_2, x_3, b_1, b_2) [x_3 \phi_{f_0}(x_2) \\
+ r_{D} r_{f_0} ((x_2 + x_3) \phi_{f_0}^* (x_2) + (x_3 - x_2) \phi_{f_0}^* (x_2))] \} ,
\]

(A2)

\[
\mathcal{F}_{ann}^{f_0} = 8\pi C_f f_B m_B^4 \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_2 b_2 b_2 b_2 \phi_D(x_3, b_3) \{ [(r_D^2 - 1)x_3 \phi_{f_0}(x_2) \\
- 2r_{f_0} r_{D}(1 - r_D^2 + x_3) \phi_{f_0}^* (x_2)] E_{df}(t_d') h_{df}(x_1, (1 - x_2)(1 - r_D^2), b_3, b_2) \\
+ E_{af}(t_f') h_{af}(x_2, x_3 (1 - r_D^2), b_2, b_3) [(x_2 - 2r_D r_c) \phi_{f_0}(x_2) \\
+ 2r_D r_f (x_2 + 1) \phi_{f_0}^* (x_2) + (x_2 - 1) \phi_{f_0}^* (x_2)] \} .
\]

(A3)

Here we do not show the amplitudes of the decays $B_{(s)} \to \bar{D}*(D^*) f_0(980)$, because one can obtain them from those of the decays $B_{(s)} \to \bar{D}(D) f_0(980)$ by the substitutions $m_D \to m_{D^*}$, $f_D \to f_{D^*}$, $\phi_D \to \phi_{D^*}$, where the terms including $r_D^2$, $r_D r_f$ and $r_D r_c$ were neglected. It is similar for the decays involving $D_{s}^*$ meson.
Appendix B: Hard scales

\begin{align}
  t_a &= \max(\sqrt{x_2(1 - r_D^2)}m_B, 1/b_1, 1/b_2), \\
  t_b &= \max(\sqrt{x_1(1 - r_D^2)}m_B, 1/b_1, 1/b_2), \\
  t_{c,d} &= \max(\sqrt{x_1x_2(1 - r_D^2)}m_B, \sqrt{|A_{c,d}^2|}m_B, 1/b_1, 1/b_3), \\
  t_{e,f} &= \max(\sqrt{x_2x_3(1 - r_D^2)}m_B, \sqrt{|L_{e,f}^2|}, m_B, 1/b_1, 1/b_3), \\
  t_g &= \max((1 - x_2)(1 - r_D^2)m_B, 1/b_2, 1/b_3), \\
  t_h &= t_g' = \max(\sqrt{x_3(1 - r_D^2)}m_B, 1/b_2, 1/b_3), \\
  t_h' &= \max(\sqrt{x_2(1 - r_D^2)}m_B, 1/b_2, 1/b_3).
\end{align}

And the \(S_j(t)(j = B, D, f_0)\) functions in Sudakov form factors in Eq. (20), Eq. (21), Eq. (29) and Eq. (30) are listed as

\begin{align}
  S_B(t) &= s(x_1 \frac{m_B}{\sqrt{2}} b_1) + 2 \int_{1/b_1}^{t} \frac{d\hat{\mu}}{\hat{\mu}} \gamma_0(\alpha_s(\hat{\mu})), \\
  S_D(t) &= s(x_3 \frac{m_B}{\sqrt{2}} b_3) + 2 \int_{1/b_3}^{t} \frac{d\hat{\mu}}{\hat{\mu}} \gamma_0(\alpha_s(\hat{\mu})), \\
  S_{f_0}(t) &= s(x_2 \frac{m_B}{\sqrt{2}} b_2) + s((1 - x_2) \frac{m_B}{\sqrt{2}} b_2) + 2 \int_{1/b_2}^{t} \frac{d\hat{\mu}}{\hat{\mu}} \gamma_0(\alpha_s(\hat{\mu})),
\end{align}

where the quark anomalous dimension is \(\gamma_q = -\alpha_s / \pi\), and the expression of the \(s(Q,b)\) in one-loop running coupling constant is used

\begin{align}
  s(Q,b) &= \frac{A^{(1)}}{2\beta_1} \hat{q} \ln(\frac{\hat{q}}{\hat{b}}) - \frac{A^{(1)}}{2\beta_1} (\hat{q} - \hat{b}) + \frac{A^{(2)}}{4\beta_1^2} \left( \frac{\hat{q}}{\hat{b}} - 1 \right) \\
  &\quad - \left[ \frac{A^{(2)}}{4\beta_1^2} - \frac{A^{(1)}}{4\beta_1} \ln\left( \frac{e^{2\gamma_E-1}}{2} \right) \right] \ln(\frac{\hat{q}}{\hat{b}}),
\end{align}

with the variables are defined by \(\hat{q} = \ln[Q/(\sqrt{2}\Lambda)], \hat{b} = \ln[1/(b\Lambda)]\) and the coefficients \(A^{(1,2)}\) and \(\beta_1\) are

\begin{align}
  \beta_1 &= \frac{33 - 2n_f}{12}, A^{(1)} = \frac{4}{3}, \\
  A^{(2)} &= \frac{67}{9} - \frac{\pi^2}{3} - \frac{10}{27} n_f + \frac{8}{3} \beta_1 \ln(\frac{1}{2} e^{\gamma_E}),
\end{align}

where \(n_f\) is the number of the quark flavors and \(\gamma_E\) the Euler constant.

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