Research on Combination of Multiple Methods for Spacecraft Propellant Consumption Prediction

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Abstract. In order to further improve the accuracy of propellant gauging during the geosynchronous satellite transfer orbit, combined with the existing propellant gauging method, a time series analysis method for satellite propellant consumption fusion algorithm is proposed. The error characteristics of the prediction data are analyzed according to the principle of different gauging methods. On the basis of this, the variance of each gauging method is estimated by time series analysis and the optimal weighted fusion estimation based on least squares method is given. The practical application shows that the fusion algorithm can make full use of the advantages of different gauging methods and provide a more reasonable estimation of the real-time propellant consumption.

1. Introduction

For geosynchronous satellites, the propellant carrying capacity is the main factor limiting the life of the satellite. The propellant prediction will also affect the satellite's de-orbiting operation. Therefore, the high-precision propellant remaining prediction and the propellant depletion time are important tasks of satellite on-orbit management.

For the orbital transfer phase, the commonly used propellant gauging methods include the BK (book-keeping) method, the PVT (pressure, volume, temperature) method, and the ultrasonic flowmeter method. The measuring equipment of the BK method and the PVT method is simple, and there is no special requirement for the hardware of the satellite propulsion system. Compared with the PVT method, the BK method and the ultrasonic flowmeter method have higher accuracy, but rely on the integration method, so there is a cumulative error. In practical applications, the method with the highest accuracy is generally used as the main prediction method, and the other methods are used for recheck, and the accuracy of the propellant remaining gauging cannot be further improved. This article is aimed at the characteristics of different gauging methods. Considering the accumulative error effect of propellant gauging methods, a fusion method of propellant consumption prediction based on least squares method and time series analysis is proposed. Practical application shows that this method can effectively utilize multiple gauging methods, provide a new way to reduce the uncertainty of satellite propellant consumption estimation.

2. Propellant consumption gauging method

The main real-time gauging methods during orbital transfer phase are as follows [1~4]:

2.1. Bookkeeping method

The BK method first calculates the flow rate of the propellant based on the telemetry parameters and
the ground calibration data, and then accumulates flow rate by working time to obtain the propellant consumption. This method is simple but practical and its uncertainty directly from the uncertainty of the flow rate during LAE and thruster firings. So the BK method has high accuracy especially when the apogee engine is working. The BK method is currently only available for liquid propellant gauging. It requires fully reliable test data of propulsion system performance, but there is accumulative error.

2.2. PVT method
The PVT method is based on the Ideal gas law. The method calculates the pressurizing gas (Helium) volume in the propellant tank according to the telemetry data including pressure and temperature data of pressurizing gas, and then obtains the liquid volume and liquid mass in the propellant tank according to the total volume of the propellant tank and propellant density. The structure of the PVT method is particularly simple, and there is no accumulative error. The PVT accuracy greatly depends on the pressure transducer accuracy and decreases when the amount of propellant in the tank decreases.

2.3. Ultrasonic flowmeter method
The basic principle of the ultrasonic flowmeter method is to calculate the flow velocity of the liquid by measuring the propagation velocity of the ultrasonic pulse in the liquid. The ultrasonic flowmeter is installed at the outlet of the propellant tank, and the real-time flow of the propellant in each tank (including the parallel tanks) can be measured, and the total propellant expelled from the propellant tank can be obtained by integration, thereby calculating the propellant remaining in the tank. Ultrasonic flowmeter has a large uncertainty for small flow, and it relies on the integral method to obtain the propellant remaining amount, so the accuracy of the ultrasonic flowmeter method increases with time.

In general, the ultrasonic flowmeter method has the highest single-measurement accuracy; the BK method accuracy is lower, PVT method has the lowest accuracy compared with the other two gauging methods[5~6]. However, if the accumulative effect is considered, the accuracy of these methods will change. Therefore, the selection of a suitable propellant prediction fusion method can maximize the gauging accuracy.

3. Calculation model

3.1. Error characteristic analysis
For different measurement methods, the measurement data $X_i$ can be regarded as the sum of the true value plus the noise signal, ie

$$X_i = X + e_i \quad i = 1,2,3$$  \hspace{1cm}(1)$$

Where, $X$ denotes the true value, $e_i$ denotes the error, the subscript “$i$ ”denotes the gauging method. It is assumed that each measurement $X_i$ is independent of each other and is an unbiased estimate of the true value, and the measurement variance is $\sigma_i^2$.

For the BK method and the ultrasonic flow meter method, the measurement result is obtained by accumulating the result at the last sampling time, thus, the propellant consumption at the current time can be expressed as

$$X_i(k) = X_i(k-1) + q(k)T + e_i(k) \quad i = 1,2$$  \hspace{1cm}(2)$$

Where, $q$ is the instantaneous propellant consumption within one sampling period; $T$ is the sampling period, equation (2) can be expressed as
\[ X_i(k) = \sum_{j=1}^{k} q(j)i + \sum_{j=1}^{k} e(j) \quad i = 1, 2 \] (3)

Obviously, the total error characteristic of the BK method and the ultrasonic flow meter method has an accumulative effect. However, the propellant consumption of PVT method is obtained according to the change of the state of the pressurizing gas at the current time and the initial time. The current propellant consumption is independent of the propellant consumption at the previous moment, so the propellant consumption can be expressed as

\[ X_j(k) = X(k) + e_j(k) \] (4)

3.2. fusion algorithm

The propellant consumption gauging methods involve many uncertain factors, especially the PVT method and the BK method. Some uncertain factors cannot be accurately defined, such as engine performance degradation, helium leak, and mixture ratio deviation. In order to obtain real-time propellant consumption fusion results, a large amount of real-time measurement data can be utilized to estimate the variance.

For different measurement methods \( p, q \), the measured values are \( X_p \), \( X_q \) respectively, and the corresponding measurement error is \( e_p \), \( e_q \), and if the measurement methods are not related to each other, then the autocovariance function \( R_{pp} \) of \( X_p \) satisfies[7–9]

\[ R_{pp} = E(X_p - X)(X_p - X) = E(X_p^2) - E(X^2) \] (5)

The cross-covariance \( X_p, X_q \) variance function \( R_{pq} \) satisfies

\[ R_{pq} = E(X_p - X)(X_q - X) = E(X_pX_q) - E(X^2) \] (6)

The variances \( X_p \) are available as follows

\[ R_{pp} - R_{pq} = E(X_p^2) - E(X_pX_q) = E(e_p^2) = \sigma_p^2 \quad p \neq q \] (7)

Assuming the number of propellant consumption calculation data is \( k \), the time domain estimate of \( R_{pp} \) is \( R_{pp}(k) \), the time domain estimate of \( R_{pq} \) is \( R_{pq}(k) \), suppose

\[ \hat{D}_p(k) = X_p(k)X_p(k) - X_p(k)X_q(k) \] (8)

The estimated values of \( \hat{D}_p(k) \) are as follows:

\[ D_p(k) = R_{pp}(k) - R_{pq}(k) = \frac{1}{k} \left( \sum_{i=1}^{k} X_p(k)X_p(k) - \sum_{i=4}^{k} X_p(k)X_q(k) \right) \] (9)

Further available:

\[ D_p(k) = \frac{k-1}{k} D_p(k-1) + \frac{1}{k} (X_p(k)X_p(k) - X_p(k)X_q(k)) \] (10)

Where, \( X_p(k)X_q(k) = \frac{1}{n-1} \sum_{q=1}^{n} X_p(k)X_q(k), n \) denotes the number of gauging methods.

For the BK method,
\[
\tilde{D}_i(k) = \left[ \sum_{j=1}^{k} q(j) \psi + \sum_{j=1}^{k} e_1(j) \right]^2 - \left[ \sum_{j=1}^{k} q(j) \psi + \sum_{j=1}^{k} e_1(j) \right] \left[ \sum_{j=1}^{k} q(j) \psi + \sum_{j=1}^{k} e_2(j) \right]
\]
\[
= \left[ \sum_{j=1}^{k} e_1(j) \right]^2 + \sum_{j=1}^{k} q(j) \psi \cdot \left[ e_1(j) - e_2(j) \right] - \sum_{j=1}^{k} e_1(j) \cdot \sum_{j=1}^{k} e_2(j)
\]  

Assume \( e_i \) is independent of the instantaneous flow \( q \), combined with the nature of the time domain estimate, then

\[
E(\tilde{D}_i(k)) = E\left( \sum_{j=1}^{k} e_1^2(j) \right) = k\sigma_i^2
\]  

It can be noted that for the BK method and the ultrasonic flow meter method, the statistical characteristics of the error signal change with time, and the mathematical expectation of the real-time variance of the propellant consumption increases with time. Therefore, \( \tilde{D}_i(k) \) satisfies,

\[
\frac{\tilde{D}_i(k)}{k} = o\left(\sigma_i^2\right)
\]

Assume \( E_p(k) = \frac{D_p(k)}{k} \), the iterative relationship can be written as:

\[
E_p(k) = \frac{k-1}{k} E_p(k-1) + \frac{1}{k} \tilde{D} p(k)
\]  

However, it can be noted that when the value \( k \) is increased to a certain value, the correction effect is minimal. In order to improve the adaptability of the fusion algorithm, optimize the algorithm by adding data window. It means that only the latest batch of continuous data is processed and calculated, and the previous historical data is discarded. When the data window width is \( N \), the recursion formula is as follows:

\[
D_i(k) = \begin{cases} 
0 & k = 0 \\
\frac{k-1}{k} D_p(k-1) + \frac{1}{k} \tilde{D}_p(k) & 1 \leq k \leq N \\
\frac{N-1}{N} D_p(k-1) + \frac{1}{N} \tilde{D}_p(k) & k > N
\end{cases}
\]

\[
E_i(k) = \begin{cases} 
0 & k = 0 \\
\frac{k-1}{k} E_p(k-1) + \frac{1}{k} \tilde{D}_p(k) & 1 \leq k \leq N \\
\frac{N-1}{N} E_p(k-1) + \frac{1}{N} \tilde{D}_p(k) & k > N
\end{cases}
\]

Where, equation (16) is suitable for the BK method and the ultrasonic flow meter method, equation (15) is suitable for the PVT method.

According to the constraint that the variance of the fusion result is the smallest, it is easy to find the optimal real-time propellant consumption dynamic weighted fusion result by least square method.
\[
\hat{X}(k) = \omega_1(k)X_1(k) + \omega_2(k)X_2(k) + \omega_3(k)X_3(k)
\]  
(17)

Where the weighting factor is
\[
\omega_i(k) = \frac{1}{D_i(k)}
\]
(18)

4. Results analysis

In order to test the actual effect of the fusion algorithm, taking the fusion of the BK method and the PVT method as an example, the sampling period is 1 second and the data window width is 1000. The propulsion system uses a 490N engine, and the orbital firing time is 4280s, 4921s, and 3976s, respectively.

The calculation results of the propellant consumption in the three ignition cycles are showed in figure1–6. Figure 1, 3, 5 are the comparison of the oxidant consumption predictions, Figure 2, 4, 6 are the comparison of the fuel consumption predictions.

It can be noted that in the initial stage, the uncertainty of the BK method is significantly smaller than that of the PVT method. Therefore, the weighting coefficient of the BK method is larger, and the fusion results are closer to the BK method. As time increases, the error of the BK method accumulates. The proportion of the weighting coefficient gradually becomes smaller. These results are consistent with the theoretical calculation results of the uncertainty of BK method and PVT method.

The comparison of the calculation results of propellant consumption before and after fusion is shown in Table 1. “O” denotes the oxidant consumption; “F” denotes the fuel consumption, the subscript denotes the gauging method.

It can be seen from Table 1 that the accuracy of the BK method is significantly better than the PVT method in the initial stage of measurement. With the accumulation of time, the calculation result of the PVT method is more reliable. The fusion results of the propellant consumption are consistent with the trend of uncertainty, and the data is more reasonable than only using the BK method or the PVT method.

![Figure 1. Comparison of oxidant consumption predictions during 1\textsuperscript{st} orbit transfer.](image1)

![Figure 2. Comparison of fuel consumption predictions during 1\textsuperscript{st} orbit transfer.](image2)
Figure 3. Comparison of oxidant consumption predictions during 2nd orbit transfer.

Figure 4. Comparison of fuel consumption predictions during 2nd orbit transfer.

Figure 5. Comparison of oxidant consumption predictions during 3rd orbit transfer.

Figure 6. Comparison of fuel consumption predictions during 3rd orbit transfer.

Table 1. Comparison of fusion results with theoretical values (units/kg)

|       | O_{Theoretical} | O_{BK}   | O_{PVT}   | O_{Fuzzy} | F_{Theoretical} | F_{BK}   | F_{PVT}   | F_{Fuzzy} |
|-------|-----------------|----------|-----------|-----------|-----------------|----------|-----------|-----------|
| 1     | 439.723         | 434.458  | 456.191   | 443.712   | 262.783         | 260.251  | 262.783   | 265.563   |
| 2     | 951.097         | 940.439  | 960.569   | 949.567   | 567.105         | 562.209  | 572.309   | 566.554   |
| 3     | 1369.558        | 1354.170 | 1374.849  | 1363.961  | 815.362         | 807.462  | 815.745   | 811.311   |

5. Conclusion

In this paper, the error characteristics of the prediction data are analyzed according to the principle of different gauging methods. Based on this, the variance of each measurement method is estimated by time domain estimation method. The optimal weighted fusion results based on the least squares method of variance are given. The practical application shows that the fusion algorithm can make full use of the advantages of different gauging methods and provide a more reasonable real-time estimation of the propellant consumption, which provides a new way to reduce the uncertainty of satellite propellant remaining prediction.

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