Suppressed SUSY for the SU(5) Grand Unified Supergravity Theory

JOHN A. DIXON∗
CAP
EDMONTON, CANADA

Abstract

This paper starts with the most basic SU(5) Grand Unified Theory, coupled to Supergravity. Then it builds a new theory, incorporating the ideas of Suppressed SUSY. Suppressed SUSY is an alternative to the spontaneous breaking of SUSY. It does not need an invisible sector or explicit soft breaking of SUSY. It varies the content of the supermultiplets while keeping the restrictive nature of SUSY. For the simple model and sector constructed here, Suppressed SUSY has only three dimensionless parameters, plus the Planck mass. At tree level, this predicts a set of 8 different new masses, along with a cosmological constant that is naturally zero. The X and Y vector bosons get Planck scale masses $2\sqrt{10} g_5 M_P$. The five scalar multiplets that accompany the Higgs, and the Gravitino, all get colossally huge ‘SuperPlanck’ scale masses of order $M_{SP} \approx 10^{17} M_P$ from a see-saw mechanism that arises from the theory. This new mass spectrum, the well-known SU(5) weak angle problem, and the cosmological constant value, should serve as guides for further modifications for the new Action.

1. SUSY GUTs tend to be unpredictable: Grand Unified Models coupled to Supergravity tend to be unpredictable, because there are so many possibilities [1]. Added to this are the problems from spontaneous SUSY breaking, with its sum rules and fine-tuning [2,3,4,5,6,7,8,9,10]. These have given rise to the ideas of the invisible sector and explicit soft breaking, which are incorporated into the ‘Minimal Supersymmetric Standard Model’ (‘MSSM’), with over a hundred new parameters [9,10].

2. The MSSM has not yet been confirmed experimentally: Even using many parameters, attempts to understand the elementary particles or astrophysics using the MSSM have, so far, not met with success. Many recent experiments testing the MSSM at the LHC have been fruitless [10]. There is a mounting belief that Squarks and Gluinos, for example, will never be found experimentally [11].

3. Two ‘Obvious Features’ of SUSY: It seems obvious that if Supersymmetry (‘SUSY’) is relevant to physics, then superpartners should exist and be detectable. It also seems obvious that there should be a mass splitting between particles and their superpartners by way of spontaneous breaking of SUSY. These obvious conclusions stem from the incorporation of the SUSY algebra into the transformations of the quantized fields of the theory.

4. Suppressed SUSY provides a way to avoid these ‘Obvious Features’: Like every other gauge theory, local SUSY gives rise to a Master Equation [12,13,14,15,16,17,18,19,20]. The Master Equation incorporates the intricate, complicated and highly restrictive nature of local SUSY. This paper shows, using an example, how we can keep the Master Equation, but change the quantum field theory, by using an ‘Exchange Transformation’ [21,22,23,24,25]. An Exchange Transformation has the form of a canonical transformation of the Master Equation, but it interchanges sources and fields. The result is a different theory with a new Master Equation, because fields are quantized and sources are not. The problems [26,27,28,8] arising from the need for the spontaneous breaking of SUSY are avoided. But the intricate, complicated and highly restrictive nature of local SUSY survives, and it governs the physics in detail.

∗jadixg@gmail.com
5. Example of an Exchange Transformation: The Exchange Transformation used here is set down in Paragraphs 35 and 36 below. The Exchange Transformations are always linear, homogeneous, and invertible. So the new Action and Master Equation are ‘formally identical’ to the old ones, except that the symbols and properties for some of the fields and sources get changed. As a result, the new Action is divided into a quantized Action and a source Action in a different way from the original Action.

6. The Scalar Potential for Supergravity: For the simplest case\(^2\), the scalar potential function \(V\) for supergravity has the form [1]:

\[
V = e^{(\kappa^2 \sum z j^2)} \left\{ \sum_j \left| \frac{\partial W}{\partial z_j} + \kappa^2 z_j W \right|^2 - 3\kappa^2 |W|^2 \right\} + \frac{1}{2} \sum_\alpha |D_\alpha|^2 \tag{1}
\]

Here \(W(z)\) is the superpotential, \(D_\alpha(z, \bar{z})\) are the auxiliary \(D\) terms for the Yang-Mills theory, and \(z^i\) are the chiral scalar fields in the action, with complex conjugates \(\bar{z}_i\). The constant \(\frac{1}{\kappa} = \frac{M_{\text{Planck}}}{\sqrt{8\pi}} = 2.4 \times 10^{18}\) GeV = \(M_P\) is the ‘reduced’ Planck mass\(^3\).

7. \(V\) and the cosmological constant: This \(V\) in (1) is the scalar non-derivative term in \(-e^{-1}L\) where \(L\) is the Lagrangian and \(e\) is the vierbein determinant\(^4\). Clearly, if the Vacuum Expectation Value (‘VEV’), \(\langle V \rangle \neq 0\), this term yields a ‘cosmological constant term’ \(\int d^4 x e \langle -V \rangle\) in the action.

8. \(V\) is not positive definite: The \(V\) in equation (1) is manifestly not positive definite because of the term \(-3\kappa^2 |W|^2\). The discovery that the scalar potential (1) is not positive definite quickly led to a number of efforts to find a spontaneously broken theory of SUSY with zero cosmological constant. The negative term \(-3e^{(\kappa^2 \sum z j^2)}\kappa^2 |W|^2\) in (1) could be fine tuned against the positive term from the non-zero auxiliary VEV contributions in (1), so that the net contribution to \(\langle V \rangle\) was zero. Since it is known that the cosmological constant is very tiny, although perhaps non-zero [29,30], this was deemed necessary to make contact with cosmology.

9. Development of the New ‘Counting Form’ Theory with a naturally zero cosmological constant: This paper constructs a theory with Suppressed SUSY, starting from the SU(5) Grand Unified Supergravity Theory. The crucial step is the rewriting of \(V\) in the ‘Counting Form’ (2) below. This suggests the special superpotential in Paragraph 12 and the Exchange Transformation used in Paragraph 14, and explained more fully in Paragraphs 35 and 36. Detailed Mathematica calculations of the mass spectrum are described, starting in Paragraph 19. Some general remarks about the Master Equation start in Paragraph 31, and then the detailed results are summarized in Paragraph 37.

10. Alternate Form of the Scalar Potential \(V\) The expression (1) can also be written:

\[
V = e^{(\kappa^2 \sum z_j \bar{z}_j)} \left\{ \sum_j \left| \frac{\partial W}{\partial z_j} \right|^2 + \kappa^2 \left( N - 3 + \kappa^2 \sum_j z_j \bar{z}_j \right) |W|^2 \right\} + \frac{1}{2} \sum_\alpha (D_\alpha)^2 \tag{2}
\]

where we define the Field Counting Operator:

\[
N = \sum_j \left( z_j \frac{\partial}{\partial z_j} + \bar{z}_j \frac{\partial}{\partial \bar{z}_j} \right) \tag{3}
\]

\(^2\)Coupling Supergravity to chiral matter is a surprisingly complicated subject, as the authors of [1] candidly acknowledge. A little confidence, and a check on normalizations, can be gained by applying this formula (1), and also its variation in (2) and (3), to get formula (19.27) of [1], as discussed on page 398 of [1] for the Polonyi model.

\(^3\)Quoting p 574 of [1], we note that \(\kappa^2 = \sqrt{8\pi G}\) and \(M_{\text{Planck}} = G^{-\frac{1}{2}} = 1.2 \times 10^{19}\) GeV.

\(^4\)The universal but confusing practice is that the letter \(e\) stands for \(e = 2.71828\) as well as the determinant of the vierbein, depending on context.
11. Just the Higgs Sector: For the Higgs sector of the model of interest here the foregoing can be written [32,33]:

\[ V = e^{\alpha^2}(\sum_{i=1}^{5} (H_L^i H_{Ri}^i) + \sum_{i=1}^{4} S^a S^a) \left\{ \sum_{i=1}^{5} \left( \frac{\partial W}{\partial H_L^i} \right)^2 + \left( \frac{\partial W}{\partial H_{Ri}^i} \right)^2 \right\} + \sum_{i=1}^{24} \left| \frac{\partial W}{\partial S^a} \right|^2 \]

\[ + \kappa^2 \left[ \sum_{i=1}^{5} \left( H_L^i H_{Lj}^i + H_{Ri}^i H_{Rj}^i \right) + \sum_{a=1}^{24} S^a S^a \right] \left| W \right|^2 + \frac{1}{2} \sum_{a=1}^{24} (D^a)^2 \]

where \( D^a \) can be found below in Equation (21), and

\[ N = \sum_{i=1}^{5} \left( H_L^i \frac{\partial}{\partial H_L^i} + H_{Ri} \frac{\partial}{\partial H_{Ri}} \right) + \sum_{a=1}^{24} \left( S^a \frac{\partial}{\partial S^a} \right) + * \]

The fields here are the scalar terms for the usual chiral Higgs type superfields. The scalar field \( S^a \) is in the adjoint of SU(5), \( H_L^i \) is a 5 of SU(5), and \( H_{Ri} \) is a \( 5' \) of SU(5). To be scalars in a chiral superfield, all of these need to be complex, including the \( S^a \) (even though the adjoint is a real representation). Note:

\[ S_j^i = S^a T_j^{ai} \quad T_j^{ai} T_j^{bi} = 2 \delta^{ab} \quad [T^a, T^b] = i f^{abc} T^c \quad \text{Tr}[S^2] = 2 S^a S^a \quad T^a = (T^a)^\dagger \]

12. A Special Form for the Superpotential: Here is the superpotential \( W \) that we will use here for the Higgs sector. This is designed so that the cosmological constant will be zero after the Exchange Transformations and gauge symmetry breaking:

\[ W = e^{\alpha^2}(2 H_L^i H_{Ri} + \frac{1}{2} \text{Tr} S^2) M_P^2 \sqrt{g_1 \text{Tr}(SSS) - g_2 H_L^i S_j^i H_{Rj}} \]

The factor \( M_P^2 \) is needed to get the dimensions right. The parameters \( g_1 \) and \( g_2 \) are dimensionless. This \( W \) is clearly quite strange, but it satisfies some strange requirements, explained in the rest of the paper. There are really just two masses here which we could write under the square root as \( M_1^2 = g_1 M_P^2 \) and \( M_2^2 = g_2 M_P^2 \).

13. Action of the Counting Operator \( N \): The reason for choosing the square root in equation (8) is that the Counting Operator (6) works simply with the resulting dimensions in \( W \):

\[ (N - 3) \sqrt{g_1 \text{Tr}(SSS) - g_2 H_L^i S_j^i H_{Rj}} = 0 \]

This follows because \( N \) counts fields, and the number of fields in the expression is \( 2 \times \frac{1}{2} = 3 \). Next we note that for any constant \( a \):

\[ \left[ N, e^{\alpha^2}(2 H_L^i H_{Ri} + \frac{1}{2} \text{Tr} S^2) \right] = 2 a \kappa^2 \left( 2 H_L^i H_{Ri} + \frac{1}{2} \text{Tr} S^2 \right) e^{\alpha^2}(2 H_L^i H_{Ri} + \frac{1}{2} \text{Tr} S^2) \]

We would add

\[ W_{\text{Matter}} = g_3 H_L^1 T_{ij} F^j + g_4 H_{Ri} T_{jk} T_{lm} \epsilon^{ijklm} + \kappa g_6 H_L^i S_j^i T_{lj} F^j + \kappa g_7 H_{Ri} S_j^i T_{jk} T_{lm} \epsilon^{ijklm} + O(\kappa^3) \]

to include the matter here. As usual in SU(5) theories, the Squarks and Sleptons are in F and T, where \( F^i \) is another 5, and \( T_{ij} \) is the antisymmetric \( 10 \). The dimensionless constants \( g_3, g_4, g_6, g_7 \) are really matrices in the flavour space of the quarks and leptons, and we will discuss them very little here. We would need to add suitable terms in the exponential in (2) also. Anyway, the Exchange Transformations will remove the scalars \( F^i \) and \( T_{ij} \) from the theory, but the related fermion terms do remain of course. The matter scalars get transformed to Zinn sources, since they are Squarks and Sleptons. Similar techniques were used in [21,22]. But those papers were not based on Supergravity, and there is a big difference, as we see here.

More exotic solutions exist. For example \( (N - 3) \sqrt{\frac{\text{Tr}(S^2)}{H_L^i H_{Rj}}} = 0 \). But it seems that this will have undesirable singularities in field space when \( H_L \cdot H_R \rightarrow 0 \). However \( (N - 3) \sqrt{\frac{\text{Tr}(S^2)}{\text{Tr}(S^2)}} = 0 \) is less pathological, and combinations of solutions like this should probably be considered. Here we look at the simplest case for a start.
14. An example of an Exchange Transformation: To get the Suppressed SUSY field theory from the above SU(5) Grand Unified Supergravity Theory, we will implement the field dependent part of a simple but important Exchange Transformation, as follows. A more complete explanation of this can be found below in paragraph 35:

\[ H^i_L \rightarrow \frac{1}{\sqrt{2}} H^i, \quad \overline{H}^i_L \rightarrow \frac{1}{\sqrt{2}} \overline{H}^i, \quad H^i_R \rightarrow \frac{1}{\sqrt{2}} H^i, \quad \overline{H}^i_R \rightarrow \frac{1}{\sqrt{2}} H^i, \quad S^a \rightarrow \frac{1}{\sqrt{2}} K^a, \quad S^i \rightarrow \frac{1}{\sqrt{2}} K^a \]  
(12)

We also eliminate all of the Sleptons and Squarks and Higgsinos and Gauginos:

\[ F^i \rightarrow 0, \quad T_{ij} \rightarrow 0; \quad \chi^i_L \rightarrow 0, \chi^j_R \rightarrow 0, \quad \chi^{i} S_{ij} \rightarrow 0, \quad \lambda^a \rightarrow 0 \]  
(13)

So the result is that the theory retains, as quantized fields, only half of the original Higgs Bosons, plus the Leptons, Quarks, and Gauge Bosons, plus the Gravitino and the Graviton. All the fields we set to zero are taken into Zinn sources, so they are really still there, but performing a different role. The total action, including all the Zinn source terms, is really the same, but with some changes of names and roles.

15. New Form of W: The above transformations on (8) yield the simpler, and real, expression \( W_R \):

\[ W \rightarrow W_R = e^{-\frac{\iota}{2} \kappa^2 (\frac{1}{4} \text{Tr}(K K) + H \cdot \overline{H})} M_p^\Delta \left( 2^{-\frac{4}{3}} g_1 \text{Tr}(K K) - g_2 H \cdot K \cdot \overline{H} \right) \]  
(14)

The rest of the original expression is still present, but we can ignore it for now because it contains Zinn sources which do not get quantized.

16. The Kahler Metric: Note that the Kahler metric term in (4) becomes:

\[ \kappa^2 \left( \frac{1}{2} \text{Tr}(S \overline{S}^T) + H_L \cdot \overline{H}_L + H_R \cdot \overline{H}_R \right) \rightarrow \kappa^2 \left( H^i \overline{H}_i + \frac{1}{4} \text{Tr} K^2 \right) \]  
(15)

We get the same expression from the term in the exponential in the superpotential (8):

\[ \kappa^2 \left( 2H^i R^i + \frac{1}{2} \text{Tr} S^2 \right) \rightarrow \kappa^2 \left( H^i \overline{H}_i + \frac{1}{4} \text{Tr} K^2 \right) \]  
(16)

17. The Scalar Potential Simplifies: The field dependent part of the scalar potential (5) after this Exchange Transformation is of the form:

\[ V_R = e^{\kappa^2 (H^i \overline{H}_i + \frac{1}{4} \text{Tr} K^2)} \left\{ \left( \sum_{i=1}^{5} \left| \frac{\partial W_R}{\partial H^i} \right|^2 + \frac{1}{2} \sum_{a=1}^{24} \left( \frac{\partial W_R}{\partial K^a} \right)^2 \right) + \kappa^2 W_R \left[ 2N_{\text{Real}} - 3 + \kappa^2 \left( H^i \overline{H}_i + \frac{1}{4} \text{Tr} K^2 \right) \right] \right\} + \frac{g_2}{2} \sum_{a=1}^{24} \left( H^i T^{ai}_j \overline{H}_i - \overline{H}_i T^{ai}_j H^j + i f^{abc} K^b K^c \right)^2 \]  
(17)

In the above, as far as the fields are concerned, using the definitions in (35):

\[ N \rightarrow N_{\text{Real}} = H^i \frac{\partial}{\partial H^i} + \overline{H}_i \frac{\partial}{\partial \overline{H}_i} + K^a \frac{\partial}{\partial K^a} \]  
(19)

Then, using identities like (11) and (10), it is easy to show that the following is true:

\[ \left[ 2N_{\text{Real}} - 3 + \kappa^2 \left( H^i \overline{H}_i + \frac{1}{4} \text{Tr} K^2 \right) \right] W_R = 0 \]  
(20)

Recall (5) and (12). We note that:

\[ D^a = g_5 \left( H^i T^{ai}_j \overline{H}_i - H_R T^{ai}_j \overline{H}_R + i f^{abc} S^c S^c \right) \rightarrow g_5 \left( H^i T^{ai}_j \overline{H}_i - H_R T^{ai}_j \overline{H}_R + i f^{abc} K^b K^c \right) = 0 \]  
(21)
because $K^a$ is real and $f^{abc}$ is antisymmetric. For $H^i$ the two contributions from $H_L$ and $H_R$ in (12) cancel each other in (21).

18. **Simplification of V and Generation of VEVs:** Given the form (14), (20) and (21) become identically zero for the fields. So now (17) and (18) reduce to just the first line (17). This form suggests that we can look for nontrivial solutions where this potential $V_R$ is evaluated at the VEVs of the Scalar fields, after shifts like (26) below, as in (27).

19. **Mathematica Calculations:** Most of the calculations in this paper were done using Mathematica. A Mathematica Notebook which contains, and explains, the various calculations, is supplied in [34]. There are 34 real scalar fields, and 24 real vector fields. These mix together in complicated ways, and that is a lot to keep track of. The full expressions for the various invariants are large. The calculation time is quite short (5 minutes). A fundamental technique, which is crucial here, is the expansion of series in the tiny dimensionless parameter $f$ in Equation (34).

20. **Detailed Notation used in Mathematica:** Now we convert the scalars to real component fields as follows ($K_i^j = \sum_{a=1}^{24} K_a T_{1i}^a$):

$$
K_i^j = \begin{pmatrix}
K_3 + K_a \sqrt{\frac{2}{3}} & K_1 + iK_2 & K_4 + iK_5 & K_6 + iK_7 & K_8 + iK_9 & K_{10} + iK_{11} & K_{13} + iK_{14} & K_{15} + iK_{16} & K_{17} + iK_{18} & K_{19} + iK_{20} \\
K_1 - iK_2 & K_3 + K_a \sqrt{\frac{2}{3}} & K_6 + iK_7 & K_8 + iK_9 & K_{10} + iK_{11} & K_{13} + iK_{14} & K_{15} + iK_{16} & K_{17} + iK_{18} & K_{19} + iK_{20} \\
K_4 - iK_5 & K_6 - iK_7 & K_3 + K_a \sqrt{\frac{2}{3}} & K_8 + iK_9 & K_{10} + iK_{11} & K_{13} + iK_{14} & K_{15} + iK_{16} & K_{17} + iK_{18} & K_{19} + iK_{20} \\
K_{13} - iK_{14} & K_{15} - iK_{16} & K_{17} - iK_{18} & K_{11} + \frac{i}{2}K_{12} & K_9 + iK_{10} & K_{13} + iK_{14} & K_{15} + iK_{16} & K_{17} + iK_{18} & K_{19} + iK_{20} \\
K_{19} - iK_{20} & K_{21} - iK_{22} & K_{23} - iK_{24} & K_9 - iK_{10} & \frac{i}{2}K_{12} - K_{11} & K_{13} + iK_{14} & K_{15} + iK_{16} & K_{17} + iK_{18} & K_{19} + iK_{20}
\end{pmatrix}
$$

$$H^i = \frac{1}{\sqrt{2}} \{K_{25} + iK_{30}, K_{26} + iK_{31}, K_{27} + iK_{32}, K_{28} + iK_{33}, K_{29} + iK_{34}\}$$

(23)

To get the Realized form $W_R$, we just substitute (22) and (23) into (14) and (17). So our scalar potential is simply

$$V_{\text{Real After ET}} = e^{\frac{1}{2} \kappa^2 \left( \sum_{i,j=1}^{34} K_i^j K_i^j \right)} \left\{ \frac{1}{2} \sum_{j=1}^{34} \left( \frac{\partial W_R}{\partial K_i^j} \right)^2 \right\}$$

(24)

and our superpotential is simply:

$$W_R = e^{-\frac{1}{2} \kappa^2 \left( \sum_{i,j=1}^{34} K_i^j K_i^j \right)} M_P^2 \frac{1}{2} \sqrt{g_1 \text{Tr}(KKK)} - g_2 \sum_{K} H^i$$

(25)

21. **The Field Shift:** Now we can look for shifts of the scalar fields that leave the scalar potential (24) at zero energy. First we perform the following shift in $W_R$ in (25):

$$\{K_{11} \rightarrow K_{11} + M_{K1}, K_{12} \rightarrow K_{12} + M_{K2}, K_{29} \rightarrow K_{29} + M_{K9}\}$$

(26)

In the above, $M_{K1}, M_{K2}, M_{K9}$ are the three VEVs that we assume for these fields. The zero cosmological constant is maintained at this tree level by insisting that the VEV of the expression $V$ in (24) is still zero after the shifts. This is familiar from gauge symmetry breaking in rigid or global SUSY, uncoupled to supergravity. This requirement is trivial for all variables except $K_{11}, K_{12}$ and $K_{29} \equiv K_{H5}$. This results in three equations.

$$\left\langle \frac{\partial W_R}{\partial K_{11}} \right\rangle = \left\langle \frac{\partial W_R}{\partial K_{12}} \right\rangle = \left\langle \frac{\partial W_R}{\partial K_{29}} \right\rangle = 0$$

(27)

22. **Introducing a Scale $r$:** For the three equations in paragraph 21, we take

$$g_2 \rightarrow r g_1$$

(28)
This quickly reduces those equations to a dependency on \( r \) alone, provided that \( g_1 \neq 0 \). Then here are the equations from paragraph 21 after removing \( g_1, g_2 \) with the transformation (28):

\[
\begin{align*}
3rM_{K9}^2 \left( 2\sqrt{15}M_{K1}M_{K2} - 10M_{K1}^2 + 20M_{K2}^2 \right) - 8\sqrt{15}M_{K1}M_{K2} \left( 9M_{K1}^2 + M_{K2}^2 - 36M_{K3}^2 \right) &= 0, \\
4\sqrt{15} \left( 9M_{K1}^2 (4M_{K2}^2 - 2M_{K3}^2) + 12M_{K2}M_{K3}^2 - 2M_{K2}^4 \right) - 3rM_{K9} \left( 10M_{K1}M_{K2} + \sqrt{15} (4M_{K2}^2 - 2M_{K3}^2) \right) &= 0,
\end{align*}
\]

(29) \hspace{4cm} (30)

\[
3rM_{K9} \left( \sqrt{15}M_{K2} - 5M_{K1} \right) \left( M_{K3}^2 - 4M_{K2}^2 \right) - 4\sqrt{15}M_{K2}M_{K9} \left( 9M_{K1}^2 + M_{K2}^2 \right) = 0
\]

(31)

23. Introducing a tiny parameter \( f \): Then the following is a solution of the equations in Paragraph 22:

\[
\begin{align*}
M_{K1} &\to \frac{3\sqrt{r}(r+2)M_P}{2\sqrt{r^3 - 9r^2 + 72r + 108}}, \\
M_{K2} &\to \frac{\sqrt{15}(r-6)\sqrt{7}M_P}{2\sqrt{r^3 - 9r^2 + 72r + 108}}, \\
M_{K9} &\to -3\sqrt{2} \sqrt{\frac{(r-18)(r+2)}{r^3 - 9r^2 + 72r + 108}} M_P
\end{align*}
\]

(32) \hspace{4cm} (33)

Now we define \( f \), which is a very small ratio:

\[
f = \frac{M_{K9}}{M_P} = \frac{M_W}{g_5 M_P} = \frac{3.35 \times 10^{-17}}{g_5}
\]

(34)

Then the substitution (33) becomes

\[
-\frac{18(r-18)(r+2)}{r^3 - 9r^2 + 72r + 108} = f^2
\]

(35)

The equation (35) is a cubic equation for \( r \) in terms of \( f \). Mathematica can easily find the three solutions exactly in symbolic form, but those look complicated. They are easier to understand when we get Mathematica to expand them as power series in the tiny fraction \( f \):

\[
\left( r \to -\frac{18}{f^2} - 7 + \frac{110f^2}{9} + O\left(f^4\right) ; \ r \to -18 - 12f^2 + O\left(f^4\right) ; \ r \to -2 - \frac{2f^2}{9} + O\left(f^4\right) \right)
\]

(36)

24. Three different results from the three solutions for the scale \( r \): So there are three different ways to obtain \( M_{K9} \) in (33) here. Any of the three solutions in (36) yield \( f^2 \) in (35) and so \( M_P f \) for (33). But they will not give the same values for \( M_{K1} \) and \( M_{K2} \) in (32). What they give, in the same order as in (36), is:

\[
\begin{align*}
\begin{cases}
M_{K1} \to -\frac{3M_P}{2}, \\
M_{K2} \to -\frac{\sqrt{15}M_P}{2}, \\
M_{K9} \to -fM_P
\end{cases}
\end{align*}
\]

(37)

\[
\begin{align*}
\begin{cases}
M_{K1} \to \frac{\sqrt{15}M_P}{2}, \\
M_{K2} \to \frac{3M_P}{2}, \\
M_{K9} \to -fM_P
\end{cases}
\end{align*}
\]

(38)

\[
\begin{align*}
\begin{cases}
M_{K1} \to -\frac{f^2M_P}{6\sqrt{10}}, \\
M_{K2} \to -\sqrt{6}M_P, \\
M_{K9} \to -fM_P
\end{cases}
\end{align*}
\]

(39)

Note that the value of \( M_{K1} \) is of order \( M_P \) for the first two cases (37) and (38). These come from the first two choices in (36). However the value of \( M_{K1} \) is of order \( f^2 M_P \) for the third case (39), which comes from

---

\(^7\)The three equations have 15 solutions [34]. Four of them lead to \( SU(5) \to SU(4) \) and four more lead to \( SU(5) \to SU(3) \times U(1) \). The other 7 have \( M_{K9} \to 0 \), which is less interesting. We choose one solution (Sol15) that leads to \( SU(5) \to SU(3) \times U(1) \) and we assume that the other three that lead to \( SU(5) \to SU(3) \times U(1) \) lead to similar results below.

\(^8\)See Paragraph 27 below.
the third choice in (36). Now we will look at the Vector Boson Masses, and we will see that the only case that has a chance of meeting experiment is the third case in (36), together with its consequence (39).

25. **Yang Mills Vector Fields:** The VEVs above determine the masses of the vector bosons. First we define:

\[
V = \begin{pmatrix}
V_3 + \frac{V_4}{\sqrt{5}} - \frac{2V_5}{\sqrt{15}} & V_4 + iV_5 & V_5 + iV_6 & V_6 + iV_7 & V_7 + iV_8 & V_8 + iV_9 & V_9 + iV_{10} \\
V_1 - iV_2 & -V_3 + \frac{V_4}{\sqrt{5}} - \frac{2V_5}{\sqrt{15}} & V_5 + iV_6 & V_6 + iV_7 & V_7 + iV_8 & V_8 + iV_9 & V_9 + iV_{10} \\
V_4 - iV_5 & V_6 - iV_7 & -\frac{2V_8}{\sqrt{5}} - \frac{2V_9}{\sqrt{15}} & V_{17} + iV_{18} & V_{18} + iV_{19} & V_{19} + iV_{20} \\
V_{13} - iV_{14} & V_{15} - iV_{16} & V_{17} - iV_{18} & V_{19} + iV_{21} & V_{21} + iV_{22} & V_{22} + iV_{23} \\
V_{19} - iV_{20} & V_{21} - iV_{22} & V_{23} - iV_{24} & V_9 - iV_{10} & \frac{5}{\sqrt{5}}V_{11} - V_{12} & -V_{11}
\end{pmatrix}
\]

(40)

26. **Mass Term for Vector Bosons:** These arise from the following terms in the action [32,33]:

\[
\frac{1}{2}g_5^2 \text{Tr}([V_S][V,S]) \rightarrow \left\langle \frac{1}{4}g_5^2 \text{Tr}([V,K][V,K]) \right\rangle
\]

\[-g_5^2 (\overline{P}V VH_L) \rightarrow \left\langle \frac{1}{2}g_5^2 (\overline{P}V VH_R) \right\rangle; -g_5^2 (\overline{P}V VH_R) \rightarrow \left\langle \frac{1}{2}g_5^2 (HVV\overline{P}) \right\rangle
\]

(41)

(42)

To find the eigenvalues of the Vector Bosons, and their multiplicities, we add the right hand sides of the above together and we call the resulting expression \(-\frac{1}{2}(V \cdot M^2_V \cdot V)\). Then we implement the Exchange Transformations. Next we substitute the expressions (22) and (23). Then we perform the shift (26). Then we set \(K_i \rightarrow 0\). Then we take two derivatives to generate the \(24 \times 24\) matrix \((M^2_V)_{ab}\). The Mathematica calculation results in the following eigenvalues of \((M^2_V)_{ab}\), with the following multiplicities, evaluated to the lowest order of \(f\):

\[
r \rightarrow -\frac{18}{f^2} \Rightarrow \begin{pmatrix}
0 & 9 \\
64g_5^2 M^2_V & 6 \\
\frac{8}{7} f^2 g_5^2 M^2_P & 1 \\
36g_5^2 M^2_P & 2 \\
4g_5^2 M^2_P & 6
\end{pmatrix} ;
\]

\[
r \rightarrow 18 - 12f^2 \Rightarrow \begin{pmatrix}
0 & 9 \\
60g_5^2 M^2_V & 6 \\
\frac{8}{7} f^2 g_5^2 M^2_P & 1 \\
60g_5^2 M^2_P & 2 \\
f^2 g_5^2 M^2_P & 6
\end{pmatrix} ;
\]

(43)

\[
r \rightarrow -2 - \frac{2f^2}{9} \Rightarrow \begin{pmatrix}
0 & 40g_5^2 M^2_P & \frac{8}{7} f^2 g_5^2 M^2_P & f^2 g_5^2 M^2_P & 40g_5^2 M^2_P \\
9 & 6 & 1 & 2 & 6
\end{pmatrix}
\]

(44)

The nine zero eigenvalues \(0 \rightarrow 9\) in (43) and (44), confirm that we have \(SU(3) \times U(1)\) as our residual symmetry here. The two solutions in (43), which come from the first two solutions in (36), are unsatisfactory. They both\(^9\) yield totally unsatisfactory masses of order \(M_P\) for the \(W\) vector boson (the multiplicity 2 item). The only satisfactory solution is (44). It comes from \(r \rightarrow -2 - \frac{2f^2}{9}\), which is the third solution in (36).

27. **Renormalization Group Issues and a Value for \(g_5\):** Here are the squared masses of the \(Z\) and the \(W\) mass, from (44):

\[
M^2_Z = \frac{8}{7} g_5^2 M^2_P f^2; M^2_W = g_5^2 M^2_P f^2
\]

(45)

It follows that:

\[
\frac{M_W}{M_P} = f g_5 = 3.35 \times 10^{-17}; \cos^2 \theta_W = \frac{M^2_W}{M^2_Z} = \frac{5}{8} \Rightarrow \sin^2 \theta_W = \frac{3}{8}
\]

(46)

As is well known, the naive value of \(\sin^2 \theta_W\) in this \(SU(5)\) model does not agree with experiment. This feature has been much discussed in rigid versions of the \(SU(5)\) SUSY GUT and the renormalization group is

---

\(^9\)The choice \(r \rightarrow 18 - 12f^2\) also leads, in the second item in (43), to a low mass for one of the \(X, Y\) complex triplet vector bosons.
clearly relevant here \[8,20\]. Now we also have the problem of separating \(g_5 f\) into the two components \(g_5\) and \(f\). This separation is needed in order to convert the results into specific mass predictions. We get different values for \(g_5 f\), depending on whether we try to fit \(M_W\) or \(M_Z\). We cannot fit both. Here we choose to fit \(M_W\). In the Tables in Paragraph (37), we leave this question open. Our general assumption \[8,20\] is that \(g_5 \approx 1\). This calls for further analysis.

28. Scalar Field Mass Matrix: Consider the real symmetric matrix:

\[ M_{ij} = \left\langle \frac{\partial^2 V_R}{\partial K_i \partial K_j} \right\rangle ; \quad i = 1 \cdots 34, j = 1 \cdots 34 \quad (47) \]

This can be used to generate:

\[ M_{\text{Scalar}}^{2ij} = \left\langle \frac{\partial^2 V_R}{\partial K_i \partial K_j} \right\rangle = e^{\frac{1}{2} \kappa^2 (M_{K_0}^2 + M_{K_1}^2 + M_{K_2}^2)} M_{ik} M_{jk} \quad (48) \]

where we use (17) or (24) for \(V_R\). Note that for our chosen solution in (32) and (33) we have \[34\]

\[ e^{\kappa^2 K} \rightarrow e^{\frac{1}{2} \kappa^2 (M_{K_0}^2 + M_{K_1}^2 + M_{K_2}^2)} = e^3 \quad (49) \]

This mass squared for the scalar bosons (48) is necessarily positive semi-definite. It is easier to work out the real symmetric matrix \(M_{ij}\) in (47) than it is to work out (48) directly. We work out \(M_{ij}\) and then let Mathematica find its eigenvalues. Then we simply square those eigenvalues and multiply by the factor \(e^3\), which comes from (49). This yields the same result as taking (24), because of the equations (27).

29. Eigenvalues of the Scalar Field Mass Matrix: There are six non-zero real eigenvalues for \(M_{\text{Scalar}}^{2ij}\). Firstly, there are 15 zero eigenvalues. Then there are masses for a real octet, a complex triplet and a complex singlet. Then there are 3 real Higgs boson masses which are the three cube roots of a complicated cubic equation. We can reduce this complicated set of masses to simple ones however. First we substitute \(g_2 \rightarrow r g_1\) in the expressions for the Masses in \(M_{ij}\). Then we substitute \(r \rightarrow -2 - 2 f^2 - \frac{23 f^4}{405} - \frac{887 f^6}{36450} + O(f^8)\) in accord with the results in Paragraph 26. The remaining dependence on \(g_1\) turns out to be exactly linear for the square of \(M_{ij}\), as seen below in Equation (50). We find \[34\] that the squared masses and their multiplicities reduce simply to the following, to lowest order in \(f\):

\[ M_{\text{Scalar}}^2 \rightarrow \begin{pmatrix} 0 & \frac{10}{15} \sqrt{5} M_{\text{SP}}^2 & \frac{5}{25} \sqrt{5} M_{\text{SP}}^2 & \frac{15}{15} \sqrt{5} M_{\text{SP}}^2 & \frac{15}{8} \sqrt{5} M_{\text{SP}}^2 & \frac{33 M_{\text{SP}}^2}{24 \sqrt{5}} & \frac{33 M_{\text{SP}}^2}{10 \sqrt{5}} & \frac{54 M_{\text{SP}}^2}{54 \sqrt{5}} & M_H^2 \end{pmatrix} \quad (50) \]

In the above we use an abbreviation \(M_{\text{SP}} = e^{3/4} \sqrt{-g_1 M_P^2}\) for the ‘SuperPlanck’ mass, which we define below in (52). The sum is 15 Goldstone+ 8 Octet + 6 Complex Triplet+ 2 Doublet + 3 Higgs = 34 real Higgs Bosons. Since we need the known Higgs Mass to be the lightest of the three neutral masses, it follows that, as shown in the last term in (50):

\[ M_{\text{Higgs}}^2 = h^2 M_P^2 = \frac{f^4 M_{\text{SP}}^2}{54 \sqrt{5}} = \frac{f^4}{54 \sqrt{5}} \left(e^{3/4} \sqrt{-g_1 M_P^2}\right)^2 = -\frac{f^4}{54 \sqrt{5}} e^{3/2} g_1 M_P^2 \quad (51) \]

Our unit of mass here is the SuperPlanck mass\(^{10}\). It takes the value

\[ M_{\text{SP}} = e^{3/4} \sqrt{-g_1 M_P^2} = 1.22 \times 10^{36} g_5^2 \text{ GeV} = 2.18 \times 10^{12} g_5^2 \text{ grams} = 2.18 g_5^2 \text{ Megatonnes} \quad (52) \]

where we determined the value of \(g_1\), using (46) and (51), to be:

\[ g_1 = -\frac{54 \sqrt{5} M_{\text{Higgs}}^2}{f^4 M_P^2} = -e^{-3/2} \frac{54 \sqrt{5} g_5^2}{(g_5 f)^4} h^2 = -5.81 \times 10^{34} g_5^4 \quad (53) \]

\(^{10}\)This closely resembles the neutrino see-saw mechanism \[35\]. The Higgs mass is tiny compared to the Planck mass, just as the neutrino mass is tiny compared to the Electroweak masses.
30. The Gravitino Mass Term: The Gravitino mass term can be found in [1]:

\[ \mathcal{L}^{(2)} = \frac{1}{2} \kappa^2 e^2 \Delta K W_R^\mu R \sigma^{\mu\nu} \psi_\nu R \]  

(54)

So the mass squared is, from (14)

\[ M_{\text{Gravitino}}^2 = (\kappa^4 e^2 K \Delta K)^2 R \]  

(55)

Now we put in the VEVs, depending on \( g_5 \), as noted in Paragraph 27, to get [34]:

\[ M_{\text{Gravitino}} = 2 \times 10^{36} g_5^2 \text{ GeV} \approx 3.57 g_5^2 \text{ Megatonnes} \]  

(56)

31. The Master Equation: The origin of "Suppressed SUSY" is in the Master Equation for SUSY. As in any gauged quantum field theory, the Master equation generates all the Ward identities of the theory that arise from its symmetries. This Master equation takes the form:

\[ \mathcal{M}[A] \equiv \int d^4 x \left\{ \delta A \delta B_I + \delta A \delta \Sigma_I + \delta A \delta F_I + \delta A \delta S_I \right\} = 0 \]  

(57)

Here \( B_I \) are (Grassmann even) 'Bosons' and \( F_I \) are (Grassmann odd) 'Fermions'. Then \( \Sigma_I \) are Grassmann odd Zinn sources\(^11\) and \( S_I \) are Grassmann even Zinn sources. One must include all the fields here, and \( I \) is a collective sort of index. The Physical Bosons are \( z^\alpha, A^A, e^a \) and the Physical Fermions are \( \chi, \lambda, \psi_\mu \). Many kinds of ghosts and antighosts and gauge fixing terms are also present, and auxiliaries are integrated.

32. The Master Equation requires Supergravity: The easiest way to see this is to recall that one needs to integrate over all fields to get the Master Equation, and it is essential that the starting transformations be nilpotent. Ghosts and antighosts are fields for this purpose, as is well known from the Yang Mills theory. Any SUSY theory requires closure using the fact that two susy transformations yield a translation. For rigid SUSY, this means that the translation and SUSY ghosts are space-time constants. But an attempt to keep the supergravity ghosts constant means that one cannot integrate them as fields. So they need to be spacetime dependent to formulate the Master Equation. But that means that Supergravity is present.

33. The Nilpotent Operator: Becchi, Rouet and Stora, with help from Zinn-Justin and Tyutin, and many others, noted that there is a fundamental feature here [17,18]. The form above gives rise to a nilpotent operator as follows:

\[ \delta \equiv \int d^4 x \left\{ \delta A \delta B_I + \delta A \delta \Sigma_I + \delta A \delta F_I + \delta A \delta S_I \right\} \]  

(58)

Because (57) yields zero, it follows that:

\[ \delta^2 = 0 \]  

(59)

34. Generating Functionals and Counterterms are Cocycles of \( \delta \): It is easy to show that the one-loop contribution to the 1PI Generating Functional \( \mathcal{G}^1 \), and the local one-loop contribution \( \mathcal{A}^1 \) to the renormalized action should satisfy:

\[ \delta \mathcal{G}^1 = 0; \delta \mathcal{A}^1 = 0 \]  

(60)

The solution of the second equation above has the form

\[ \mathcal{A}^1 = \mathcal{H}^1 + \delta \mathcal{B}^1 \]  

(61)

---

\(^{11}\)Zinn sources' have sometimes been called 'Antifields', but they are sources, not fields, and they are not anti anything, so we call them Zinn sources instead. These play a crucial role in Suppressed SUSY, and their source character is crucial.
where \( H^i \) are local invariants in the cohomology space of \( \delta \), and the local ‘coboundaries’ \( B^i \) are basically terms that yield field and source renormalizations. One can then iterate this situation to any order using results like those in [37]. Modulo interesting issues like anomalies, the result of this is that the action that renormalizes the theory to all orders has a simple relation with the original action. It is generally the same as regards the field and Zinn source content and structure, but there is room for renormalization of the coefficients. For the non-renormalizable theory here, there are new invariants too.

### 35. The Generating Functional for the Higgs Sector

For the Higgs sector, the old field variables and the old Zinn sources are: Fields: \( H^i_L, H^i_R, S^a \); Zinns: \( \Gamma^i_L, \Gamma^i_R, \Gamma^a_s \); (\( i = 1 \cdots 5, a = 1 \cdots 24 \)). These are all complex. They appear in the old action in \( A_{\text{old Zinn}} = \int d^4x \left\{ \Gamma^i_L \delta H^i_R + \Gamma^i_R \delta H^i_L + \Gamma^a_s \delta S^a \right\} + * + \cdots \) where the missing terms include non-linear functions of the sources resulting from integration of auxiliaries.

The new Field variables, Zinns and antighosts are: Fields: \( H^i, K^a \); Zinns: \( \Gamma^i, \Gamma^a, J^i, J^a \); Antighosts: \( \eta, \eta^a \); (\( i = 1 \cdots 5, a = 1 \cdots 24 \)). \( H^i, \Gamma^i, J^i, \eta^i \) are complex. \( K^a, \Gamma^a, J^a, \eta^a \) are real. These appear in the new action in \( A_{\text{new Zinn}} = \int d^4x \left\{ \Gamma^i \delta H^i + J^i \delta \eta^i + * + \Gamma^a \delta K^a + J^a \delta \eta^a + \cdots \right\} \) where the missing terms include non-linear functions of the Sources resulting from integration of auxiliaries.

Here is the Generating Functional [21,22,23,24] for the Higgs Sector that we will use here. It is a function of non-linear functions of the Sources resulting from integration of auxiliaries.

\[
\mathcal{G}_{\text{Higgs}} = \int d^4x \left\{ \frac{1}{\sqrt{2}} (H^i_L + \bar{\Pi}^i_R) \Gamma^i + \frac{i}{\sqrt{2}} (H^i_L - \bar{\Pi}^i_R) \eta^i + S^a \frac{1}{\sqrt{2}} (\Gamma^i + i\eta^i) \right\} + * \tag{62}
\]

The new fields and the new J-type sources are obtained as functions of the old fields as follows:

\[
H^i = \frac{\delta \mathcal{G}}{\delta \Gamma^i} = \frac{1}{\sqrt{2}} (H^i_L + \bar{\Pi}^i_R); \quad J^i = \frac{\delta \mathcal{G}}{\delta \eta^i} = \frac{i}{\sqrt{2}} (H^i_L - \bar{\Pi}^i_R); \tag{63}
\]

\[
K^a = \frac{\delta \mathcal{G}}{\delta \Gamma^a} = \frac{1}{\sqrt{2}} (S^a + \bar{S}^a); \quad J^a = \frac{\delta \mathcal{G}}{\delta \eta^a} = \frac{i}{\sqrt{2}} (S^a - \bar{S}^a) \tag{64}
\]

We can combine these to write the old fields in terms of the new fields and the new J-type sources:

\[
H^i_L = \frac{1}{\sqrt{2}} \left( H^i - iJ^i \right); \quad \bar{\Pi}^i_R = \frac{1}{\sqrt{2}} \left( H^i + iJ^i \right); \quad S^a = \frac{1}{\sqrt{2}} (K^a - iJ^a); \quad \bar{S}^a = \frac{1}{\sqrt{2}} (K^a + iJ^a) \tag{65}
\]

Note that for the construction of the quantized action, the two complex fields \( H_L, H_R \) are replaced by one complex field \( H \), and the complex Field \( S \) in the adjoint representation of the SU(5) gauge theory, is replaced by a real field \( K \). The old Zinn sources are obtained from:

\[
\Gamma_R^i = \frac{\delta \mathcal{G}}{\delta H^i_L} = \frac{1}{\sqrt{2}} (\Gamma^i + i\eta^i); \quad \Gamma_R^i = \frac{\delta \mathcal{G}}{\delta H^i_R} = \frac{1}{\sqrt{2}} (\bar{\Gamma}^i - i\bar{\eta}^i); \quad \Gamma_R^a = \frac{\delta \mathcal{G}}{\delta S^a} = \frac{1}{\sqrt{2}} (\Gamma^a + i\eta^a) \tag{66}
\]

The latter are relevant to the cohomology etc. but we do not need them here for the present Lagrangian construction.

### 36. The Generating Functional for the Gauginos, and the Meaning of the Master Equation

The Generating Functionals for the other scalar and spin \( \frac{1}{2} \) sectors are simpler, at least for now. For example if the old Gauginos and Zinn sources are \( \lambda^a, Y^a \) and the new Zinn Sources and antighosts are \( \Lambda^a, M^a \) we could take: \( \mathcal{G}_{\text{Gauginos}} = \int d^4x \left\{ \lambda^a M^a \right\} \); \( \Lambda^a = \frac{\delta \mathcal{G}}{\delta \lambda^a} = \lambda^a; \quad Y^a = \frac{\delta \mathcal{G}}{\delta \Lambda^a} = M^a \), where all the sources and fields are Majorana spinors (\( Y^a, M^a \) are Grassmann even and \( \lambda^a, \Lambda^a \) are Grassmann odd). Note in particular that:

\[
\int d^4x Y^a \delta \lambda^a = \int d^4x \frac{d^4y}{N} \partial_\mu V^\gamma_{\mu\nu} C + \cdots \rightarrow \int d^4x M^a \partial_\mu V^\gamma_{\mu\nu} C + \cdots \tag{67}
\]

The last term \( \int d^4x M^a \partial_\mu V^\gamma_{\mu\nu} C \) does not do very much. There is no kinetic term here for the antighost \( M^a \), so it does not propagate. Gauge fields are not so simple. Here is a useful remark [36]:

---

10
The Master Equation is the most general quantum mechanical solution for the problem of unphysical degrees of freedom, whenever they appear.

In this paper, we take this statement farther, to conjecture that those Exchange Transformations of the Master Equation which change the functionality of scalar bosons and spin 1/2 fermions, also satisfy the quantum mechanical requirements. Note that the full Exchange Transformations are linear, homogeneous and invertible, so that the full Action, with Zinn sources, simply gets various terms renamed. These formal aspects of Suppressed SUSY clearly need careful attention.

37. Conclusion: Suppressed SUSY is a very simple idea, but it has complicated consequences. Here is a summary. In accord with Paragraph 27, \( g_5 \approx 1 \) is left undetermined:

| Parameters | Scales | Mass Names (GeV) |
|------------|--------|-----------------|
| \( f g_5 = M_W/M_P \) | \( h = M_H/M_P \) | \( g_1 \) | \( r \approx g_2/g_1 \) | \( 2.4 \times 10^{18} \) | \( M_{SP} \) |
| \( 3.35 \times 10^{-17} g_5 \) | \( 5.2 \times 10^{-17} \) | \(-5.81 \times 10^{14} g_5^4 \) | \(-2 - 2 f^2/9 \) | \( 1.22 \times 10^{36} g_5^2 \) |

| VEVs (GeV) | Electroweak Masses (GeV) |
|------------|-------------------------|
| \( M_{K0} \) | \( M_Z = \sqrt{2} f g_5 M_P \) |
| \( -f M_P \) | \( M_W = f g_5 M_P \) |
| \(-\sqrt{6} M_P \) | \( M_H = h M_P \) |

| Planck Masses (GeV) | Super Planck Masses (GeV) |
|---------------------|---------------------------|
| X                   | H_{octet}                 |
| Y                   | H_{triplet}               |
| \( 2\sqrt{10} g_5 M_P \) | \( 2.05 M_{SP} \) |
| \( 2\sqrt{10} g_5 M_P \) | \( 0.68 M_{SP} \) |
| \( H^+ \)           | \( Higgs_2 \)             |
| \( 2.05 M_{SP} \)   | \( 81 M_{SP} \)           |
| \( Higgs_3 \)       | \( 2.05 M_{SP} \)         |
| Gravitino            | \( 1.64 M_{SP} \)         |

After the Exchange Transformations, the Action, written in terms of the remaining quantized fields alone, looks smaller and simpler. There is a natural way to get a zero cosmological constant (at tree level), as was demonstrated using the ‘square root’ mass style superpotential above in (8), which generated the scalar potential (17), while (18) is identically zero. This depends on \( g_1, g_2 \) and \( M_P \). After the Exchange Transformation here, this model has many possible vacuum states, all with zero energy and zero cosmological constant. We chose to examine one where SU(5) breaks down to SU(3) \( \times U(1) \). There is a natural see-saw which creates three levels of mass, and all the masses in the Higgs/ Gauge sector are determined from the \( g_1, g_2 \) in (14), \( M_P = \frac{1}{2} f M_P \) and \( g_5 \). The VEV equations here that generate the Vector Boson mass matrix are functions only of the ratio \( r = \frac{g_2}{g_1} \) for this model.

As shown in Paragraphs 24 and 26, there are three possible solutions of the VEV equations that arise for the theory, but only one of them has a W mass comparable to the Z mass. This one requires that \( r \rightarrow -2 - 2 f^2/9 + O(f^4) \), where \( g_5 f = \frac{M_W}{M_P} \approx 10^{-17} \), and then the masses of the X and Y vector bosons are of order \( M_P \). The Scalar mass squared matrix is a linear function of \( g_1 \), multiplied by a complicated function of \( r \). Inserting \( r \rightarrow -2 - 2 f^2/9 + O(f^4) \) yields a mass matrix for the six scalar boson masses. As shown in Paragraph 29, setting the lowest mass neutral scalar to have the mass of the Higgs determines the value \( g_1 \) in terms of \( f \) and \( g_5 \), and it then follows that the five other scalar boson multiplets, and the Gravitino (Paragraph 30), all have masses of the SuperPlanck order \( M_{SP} \approx 1\text{Megatonne} \). It is simple to remove or add scalar or spinor fields, to vary and test the model. Here we removed half the scalar Higgs, and all of the Squarks, Sleptons, Gauginos and Higgsinos. We need to add some of them back to deal with the remaining issue of the weak angle etc. With Suppressed SUSY, spontaneous breaking of SUSY is not necessary because superpartners can be removed, and so the need for the SSM [9], with its invisible sector and explicit soft SUSY’ breaking is gone. Once some Suppressed SUSY is present, one could expect mass splitting everywhere from loop effects. The masses found here seem encouraging for the ideas of Suppressed SUSY. A priori, it might have been impossible to get the masses of the W, Z and Higgs sufficiently smaller than those of the other particles, given that there are very few parameters here. Nucleon decay has not
been found at the level indicated by the SU(5) GUT as predicted by the older methods [33]. However the relevant masses (X, Y, Heavy Higgs) are predicted to be heavier using Suppressed SUSY, so this question is re-opened here. Another feature that is quite strange here, and encouraging, is that it is possible to fit the Higgs mass with the seesaw mechanism in Paragraph 29. This arises naturally from the form of the superpotential. The relevant matrix is very complicated, and the reason that this happens is not clear to the author. It is also possible of course to take more complicated versions of the superpotential as noted in footnote 6 above. What changes arise from that?

The Gravitino in this model is extremely massive (≈ 1 Megatonne), and stable too. An improved model would need to re-insert some fields to deal with the the weak angle, and also to keep the cosmological constant reasonable at loop order, if possible. That would probably create a decay mechanism for the Gravitino, maybe giving rise to a lighter LSP candidate for dark matter [38]. At any rate, this theory needs examination at one loop, and perhaps, ultimately, an examination from the superstring point of view.

Acknowledgments

I thank Doug Baxter, Carlo Becchi, Philip Candelas, James Dodd, Mike Duff, Richard Golding, Dylan Harries, Chris Hull, Sergei Kuzenko, Pierre Ramond, Graham Ross, Peter Scharbach, Kelly Stelle, Xerxes Tata, J.C. Taylor, Steven Weinberg, and Peter West for stimulating correspondence and conversations.

References

[1] D. Z. Freedman and A. Van Proeyen, ‘Supergravity’, Cambridge, UK (2012): ISBN: 9781139368063 (eBook), 9780521194013 (Print) (Cambridge University Press). Page 371 summarizes the potential we use. See also pages 119, 276, 370, 388, 389, 390 for useful information about normalizations etc.

[2] Many of the original papers on SUSY and Supergravity are collected in “Supersymmetry”, Vols. 1 and 2, ed. Sergio Ferrara, North Holland, World Scientific, (1987).

[3] S. J. Gates, M. T. Grisaru, M. Rocek and W. Siegel, “Superspace”, Benjamin, 1983.

[4] J. Wess and J. Bagger, “Supersymmetry and Supergravity”, Second Edition, Princeton University Press (1992).

[5] I. L. Buchbinder and S. M. Kuzenko, “Ideas and methods of supersymmetry and supergravity: Or a walk through superspace,” Bristol, UK: IOP (1998) 656 p

[6] H. Baer and X. Tata, “Weak scale supersymmetry: From superfields to scattering events,” Cambridge, UK: Univ. Pr. (2006)

[7] P. C. West, “Introduction to supersymmetry and supergravity,” Singapore, Singapore: World Scientific (1990) 425 p

[8] Steven Weinberg: “The Quantum Theory of Fields” Volume 3, Cambridge University Press, ISBN 052155602. This contains a readable brief history, a summary of the problems with SUSY, and a useful summary dealing with the renormalization group in the context of SUSY GUT.

[9] Howard E. Haber, The Review of Particle Physics (2017) C. Patrignani et al. (Particle Data Group), SUPERSYMMETRY, PART I (THEORY), Revised September 2015 http://pdg.lbl.gov/

[10] O. Buchmueller and P. de Jong, The Review of Particle Physics (2017) C. Patrignani et al. (Particle Data Group), SUPERSYMMETRY, PART II (EXPERIMENT) Updated September 2015. http://pdg.lbl.gov/
There was even a conference last year (2016) in Madrid entitled ‘Is SUSY alive and well?’ https://workshops.ift.uam-csic.es/susyaaw. The author thanks the organizers and participants for an interesting conference.

The Master Equation goes back to [13] and Zinn-Justin’s early contribution is set out in a later textbook [14]. An early and pithy introduction was in [15]. A bit of history and Zinn Justin’s involvement is in [16]. A more recent treatment is in [20].

[13] C. Becchi, A. Rouet and R. Stora, “Renormalization of Gauge Theories,” Annals Phys. 98, 287 (1976).

[14] J. Zinn-Justin, “Quantum Field Theory and Critical Phenomena”, Oxford Science Publications, Reprinted 1990.

[15] J. C. Taylor, “Gauge Theories of Weak Interactions,” Cambridge 1976, 167p

A summary and some history can be found in J. Zinn-Justin, “From Slavnov-Taylor identities to the ZJ equation,” Proc. Steklov Inst. Math. 272, 288 (2011).

[17] C. Becchi, “Slavnov–Taylor and Ward identities in the electroweak theory,” Theor. Math. Phys. 182, no. 1, 52 (2015) [Teor. Mat. Fiz. 182, no. 1, 65 (2014)] doi:10.1007/s11232-015-0244-8 [arXiv:1407.3960 [hep-th]].

[18] C. Becchi, “Becchi-Rouet-Stora-Tyutin symmetry”, http://www.scholarpedia.org/article/Becchi-Rouet-Stora-Tyutin

[19] I. A. Batalin and G.A. Vilkovisky, “Gauge Algebra and Quantization”, Physics Letters B 102.1 (1981): 27-31

[20] Steven Weinberg: “The Quantum Theory of Fields” Volume 2, Cambridge University Press, ISBN 052155002. This contains an introduction to the Master Equation and BRS and a useful summary dealing with the renormalization group in the context of non-SUSY GUT and the weak angle.

[21] J. Dixon, “Chiral SUSY Theories with a Suppressed SUSY Charge,” Phys. Lett. B 760, 788 (2016) doi:10.1016/j.physletb.2016.07.060 [arXiv:1604.06442 [hep-th]].

[22] J. Dixon, “The SSM with Suppressed SUSY Charge,” Phys. Lett. B 761, 253 (2016) doi:10.1016/j.physletb.2016.07.059 [arXiv:1604.06396 [hep-th]].

[23] Herbert Goldstein, “Classical Mechanics” (Second Edition) Addison Wesley ISBN 0-201-02918-9

[24] L.D. Landau and E.M. Lifschitz, “Mechanics”, Pergamon Press (1960)

[25] Suppressed SUSY does not, in general, respect the non-renormalization theorems, which are discussed in all of the SUSY literature. These go along with the features in Paragraph 3. It has often been argued that the non-renormalization theorems help to explain the ‘hierarchy problem’, and that this supports the SSM. This is an issue that needs careful attention in the future, supposing that Suppressed SUSY yields some interesting results.

[26] Pran Nath, “Supersymmetry, Supergravity, and Unification” (Cambridge Monographs on Mathematical Physics) Dec 15, 2016

[27] G. Moultenka, M. Rausch de Traubenberg and D. Tant, “Low Energy Supergravity Revisited (I),” arXiv:1611.10327 [hep-th].

[28] S. Ferrara and A. Sagnotti, “Supergravity at 40: Reflections and Perspectives,” Riv. Nuovo Cim. 40, no. 6, 1 (2017) doi:10.1393/ncr/i2017-10136-6 [arXiv:1702.00743 [hep-th]].
[29] O. Lahav and A.R. Liddle, The Review of Particle Physics (2017) C. Patrignani et al. (Particle Data Group), 25. THE COSMOLOGICAL PARAMETERS Updated November 2015. http://pdg.lbl.gov/

[30] M. J. Mortonson, D. H. Weinberg, and M. White; revised by D. H. Weinberg and M. White; The Review of Particle Physics (2017) C. Patrignani et al. (Particle Data Group), 27. DARK ENERGY Revised November 2013 http://pdg.lbl.gov/

[31] S. Weinberg, “The Cosmological Constant Problem,” Rev. Mod. Phys. 61, 1 (1989). This summarizes problems that are still outstanding after 28 years.

[32] Ross, Graham G., ‘GRAND UNIFIED THEORIES’, Reading, USA: Benjamin/Cummings (1984) 497 P. (Frontiers In Physics, 60), (1985)

[33] A. Hebecker and J. Hisano, The Review of Particle Physics (2017) C. Patrignani et al. (Particle Data Group), 16. GRAND UNIFIED THEORIES Revised January 2016 by http://pdg.lbl.gov/

[34] The Mathematica Notebook will be placed on the arXiv as a pdf supplement to this article in arXiv.

[35] K. Nakamura (Kavli IPMU (WPI), U. Tokyo, KEK), and S.T. Petcov, The Review of Particle Physics (2017) C. Patrignani et al. (Particle Data Group), 14. NEUTRINO MASS, MIXING, AND OSCILLATIONS Updated June 2016. http://pdg.lbl.gov/

[36] The author got this remark from Carlo Becchi. See also [17,18] for further details.

[37] J. A. Dixon, “Field Redefinition and Renormalization in Gauge Theories,” Nucl. Phys. B 99, 420 (1975).

[38] M. Drees (Bonn University) and G. Gerbier, The Review of Particle Physics (2017) C. Patrignani et al. (Particle Data Group), 26. DARK MATTER Revised September 2015; http://pdg.lbl.gov/