ABSTRACT The wind energy conversion system (WECS) frequently operates under highly stochastic and unpredictable wind speed. Thus, the maximum power (MP) extraction, in such unpredictable scenarios, becomes a very appealing control objective. This paper focuses on the extraction of MP from a variable-speed WECS, which further drives a permanent magnet synchronous generator (PMSG). At the first stage, the dynamical model of PMSG is converted into Bronwsky form, which is comprised of both visible and internal dynamics. The first-order internal dynamics are proved stable, i.e., the system is in the minimum phase. The control of the second-order visible dynamics, to track a varying profile of the wind speed, is the main consideration. This job is accomplished via Backstepping-based robust Sliding Mode Control (SMC) strategy. Since the conventional SMC suffers from the inherited chattering issue, thus, the discontinuous control component in the SMC scheme is replaced with super-twisting and real-twisting control laws. In addition, the immeasurable states' information is estimated via gain-scheduled sliding mode observer. The overall closed-loop stability is ensured by analysing the quasi-linear form, which supports the separation principle. The theoretical claims are authenticated via simulation results, which are performed in Matlab/Simulink environment. Besides, a comparative analysis is carried out with the standard literature results, which quite obviously outshines the investigated control approaches in terms of varying wind profile tracking and the corresponding control input.

INDEX TERMS Wind energy conversion system, Maximum power tracking, Sliding mode control strategy, Permanent magnet Synchronous generator.

I. INTRODUCTION

The increasing demand for electrical energy is the foremost issue across the globe because of the environmental crisis such as the decline in the availability of fossil fuels, emission of greenhouse gases and the various pollution problems. To resolve the issue, the only reliable solution is the consideration of renewable resources, i.e., geothermal, hydro-power, solar, biomass and wind. Relatively, wind energy is the latest form of energy that is cost-friendly and having no undesirable impacts on the surrounding environment [1], [2]. Consequently, the harnessing of wind energy for power generation is an active research area in the last two decades. Researchers believe that investments in the aforementioned area may overtake the market in future.

Commercially available wind turbines are quite capable of conversion but with the addition of minor noise to the outside environment. However, the advanced technology have resulted in high quality sophisticated wind turbines. Generally, the WECS works either autonomously or in-grid connected mode. To be more precise, PMSG is more common in
power conversion system for the last few years due to its high efficiency, smaller size and reduced cost. Consequently, PMSG-based WECS has been used in wind energy conversion systems. Due to the intermittent and stochastic nature of variable wind speed, the most challenging task in WECS is to extract MP [3], [4]. Particularly, for partial load, the efficiency of WECS is more significant [5]. Therefore, to increase its efficiency, in partial load regime, the maximum power point tracking (MPPT) has been introduced.

Various attempts have been made to compose classical control strategies for WECS, to extract maximum power, but have not been satisfactory due to the uncertain and highly nonlinear dynamical structure of the wind turbine [6]. In model-based control design, the feedback linearization-based control law is convenient but its sensitive nature to various parametric uncertainties degrades its performance [7]. To address the issue, many nonlinear control schemes are introduced. The smart control techniques such as neural network control strategy [8], Takagi-Sugeno-Kang and Mamdani fuzzy logic control design [9] have been used for WECS. However, these control approaches suffer from long offline training periods and time-consuming computations. Consequently, the sliding mode controller (SMC) can be taken as an alternate option for the WECS owing to its simple design, robustness to parametric variations, insensitivity to an external perturbation. However, the inherited chattering is still an issue that needs to be settled. In the context of robust maximum power extraction from WECS, SMC having an exponential reaching law is proposed in [10], which reduces the adverse effects of the chattering across the switching manifold and improve the total harmonic distortion. Conventional SMC, with super-twisting control law, is proposed in [11], [12] to suppress the chattering phenomena while considering the availability of all the state variables. A feed-forward neural networks-based global SMC is proposed in [13]. The benefits of this method, over [10]–[12], were the uncertain dynamics, which give birth to substantial chattering issues, were estimated via neural networks and the robustness enhancement was claimed. Another very appealing strategy, while combining terminal SMC with neuro-fuzzy estimated parameters, was proposed in [14], which resulted in appealing enhancement was claimed. Another very appealing strategy, while combining terminal SMC with neuro-fuzzy estimated parameters, was proposed in [14], which resulted in appealing results. It is worthy to mention that state availability is assumed in all the aforesaid SMC strategies, which is somewhat impractical.

In this article, a backstepping-based SMC scheme is synthesized to accurately track the varying wind profile in the WECS. This synergistic control strategy is designed to capture the salient features of both strategies. The nonlinear backstepping-based control strategy allows a step by step procedure to design a stabilizing control law via the Lyapunov stability method. [15], [16]. While the SMC scheme alters its configuration according to the system dynamics, thus having the capability to counteract any match disturbances. As discussed in [17]. Supper-twisting (ST) and real-twisting (RT) control laws suppress the chattering issue with enough accuracy. In addition, both these variants can be used as discontinuous control laws. Therefore, supper-twisting (ST) and real-twisting (RT) control laws are used as discontinuous control laws instead of conventional signum functions, in the final control structure. This results in the suppressed chattering as compared to feedback linearization and classical SMC. In addition, the newly designed controllers portray robustness against the external disturbances [18]. Since, the final control law needs the states’ information, which is unavailable in a practical scenario. Thus, the missing states’ information is estimated via a gain-scheduled sliding mode observer. The overall contribution includes the synthesis of a Backstepping-based SMC scheme along with a gain-scheduled sliding mode observer. The control law, proposed in this paper, is quite different from [10]–[12] in terms of the sliding manifold and the corresponding control structure. In addition, all the system’s state variables are reconstructed via a gain-scheduled sliding mode observer, which is not used previously for this particular application in the existing literature. In comparison with the standard literature [7], [13], [14], the proposed technique has a quite efficient transient response and having zero steady-state error, which is sustained hereafter. Moreover, the corresponding control efforts are also practically feasible.

This paper is organized into the following sections: In section 2, the mathematical modelling of a PMSG is presented. Section 3 describes the input-output form and the investigation of zero-dynamics stability while the control strategy is developed in section 4. Section 5 and 6 describes the optimal linearized model of PMSG and the formation of gain-scheduled sliding mode observer, respectively. Section 7 covers a wind profiles generation and the simulation results. Finally, section 8 describes the conclusion of the current work.

II. MODELLING OF WIND ENERGY CONVERSION SYSTEM

The significant model of WECS includes the aerodynamic model of wind turbine (WT) and model of PMSG, which are connected to an external load.

A. AERODYNAMIC MODEL OF WIND TURBINE

The turbine captures the wind power and converts it into rotational energy. If the turbine rotor captures the wind energy, the actual mechanical power \( P_{mech} \), available at the PMSG rotor, is quite smaller than the total power due to the stochastic and non-stoppable speed of the wind, which can be expressed as [7]

\[
P_{mech} = \frac{1}{2} \rho \pi R_t^2 \nu_w^3 C_p(\lambda, \beta),
\]

where \( \rho \) is the density of air, \( R_t \) is radius of the WT blade and \( \nu_w \) is the speed of the wind. \( C_p \) defines the efficiency of the turbine rotor that is called WT power coefficient. \( \beta \) is assumed to be constant, i.e., \( (\beta = 0) \), so \( C_p \) becomes \( C_p(\lambda) \).
\( \lambda \) is the ratio between the blade’s speed and the wind’s speed, which is given as follows

\[
\lambda = \frac{R_t \Omega_l}{v_w},
\]

where \( \Omega_l \) is the blades rotational speed. Thus, the mechanical output power of WT significantly increases according to the wind speed as clearly seen in Fig. 1. A peak of power is available for every wind speed. These peaks join to form a curve known as optimal regime characteristics (ORC).

\[
\Omega_{ref} = \frac{\lambda_{opt} \nu_w}{R_t}
\]

The power of PMSG rotor can be written as

\[
P_{mech} = \Gamma_{wind} \Omega_l
\]

According to wind torque expression, the mechanical torque of the shaft is given as

\[
\Gamma_{wind} = 0.5 p \rho \pi R^2 v^2 C_T(\lambda)
\]

where \( C_T(\lambda) \) is the torque coefficient defined as

\[
C_T(\lambda) = \frac{C_p(\lambda)}{\lambda}
\]

Where \( C_p(\lambda) \), \( C_T(\lambda) \) and \( \lambda_{opt} \) are the design parameters, which are usually provided by the wind turbine manufacturer.

**B. DYNAMICAL MODEL OF PERMANENT MAGNET SYNCHRONOUS GENERATOR (PMSG)**

The \( dq \)-model of PMSG, by discarding the zero component, is as follows [19]

\[
\begin{align*}
\dot{i}_d &= -R_s i_d + L_q i_q + \nu_d \\
\dot{i}_q &= -R_s i_q - (L_d i_d + \Phi_m) + \nu_q \\
\dot{\Omega}_h &= \frac{1}{J_h} (\Gamma_{wind} - \Gamma_{em})
\end{align*}
\]

where \( \nu_d \) and \( \nu_q \) are the \( dq \)-axes voltages, \( R_s \) is the stator resistance, \( p \) is the pole pair number, \( \Phi_m \) is the permanent magnet flux, \( \Omega_h \) is the high-speed of the shaft, \( J_h \) is the moment of inertia, \( \psi_d = L_d i_d + \Phi_m \) and \( \psi_q = L_q i_q \) are the \( dq \) fluxes, respectively. The mathematical expression of electromagnetic torque is \( \Gamma_{em} = p \Phi_m i_q \). Furthermore, \( L_d \) and \( L_q \) are the rotor inductance, which are supposed equally to each other, i.e., \( L_d = L_q = L \). Thus, we are dealing with a non-salient synchronous generator.

The nonlinear dynamical equations of the PMSG-WECS connected to the load, reported in (7), can be expressed as follows

\[
\begin{align*}
\dot{x}_1 &= -R_s x_1 + p(L_q - L_{ch}) x_2 x_3 - R_{ini} x_1 \\
\dot{x}_2 &= -R_s x_2 - p(L_d + L_{ch}) x_1 x_3 - R_{ini} x_2 + p \Phi_m x_3 \\
\dot{x}_3 &= \frac{d_1 v^2}{t} + \frac{d_2 v^2 x_1}{t^2} + \frac{d_3 x^2}{t^3} - p \Phi_m x_2
\end{align*}
\]

where \( [x_1, x_2, x_3] = [i_d, i_q, \Omega_h] \), are the system’s states, which represents currents along \( d \)-axis, \( q \)-axis and the rotational speed of the blades, respectively. In this case, the \( \Omega_h = \Omega_l \times i, \) with \( i \) as the gear ratio. \( L_{ch} \) is the equivalent chopper inductance and \( R_{ini} \) is initial value of the chopper equivalent resistance. WECS has a fixed efficiency for the entire speed range, i.e., low speed shaft power, \( P_i \) is equal to high speed shaft power, \( P_h \). WECS modeling can be clearly seen from Fig. 2.

**Remark 1:** The dynamics of power electronics are neglected because of being more rapid than the PMSG-VSWT dynamics.
III. INPUT-OUTPUT FORM

The nonlinear PMSG-WECS model (8) can be expressed in general form as follows

\[
\begin{align*}
\dot{x} &= f(x) + g(x)u \\
y &= h(x),
\end{align*}
\]

(9)

where \(x \in \mathbb{R}^m\) represents the state vector, \(u \in \mathbb{R}^n\) is the control input, while \(f(x)\) and \(g(x)\) are nonlinear smooth vector fields which have the following expressions.

\[
f(x) = \begin{bmatrix}
-R_s x_1 + p(L_s - L_{ch}) x_2 x_3 \\
-R_s x_2 - p(L_s + L_{ch}) x_1 x_3 + p\Phi_m x_3 \\
\frac{d_1 x_1^2}{q_{ch}} + \frac{d_2 x_2^2}{q_{ch}} - p\Phi_m x_2
\end{bmatrix},
\]

\[
g(x) = \begin{bmatrix}
-x_1 \\
\frac{x_1}{L_s + L_{ch}} \\
0
\end{bmatrix}
\]

where

\[u = R_{ch}\]

The output, \(y = h(x) = x_3 = \Omega_h\) is the angular speed of the rotor shaft. Since, our objective is to control \(\Omega_h\), therefore, (8) can be transformed into input-output form by defining the following transformation.

\[
\begin{align*}
z_1 &= y = h(x) = x_3 = \Omega_h \\
z_2 &= L_f h(x) = \frac{\partial h(x)}{\partial x} f(x) \\
z_3 &= L_f^2 h(x) = \frac{x_1}{x_2}
\end{align*}
\]

(10)

Since the relative degree 'r' of the system \((r = 2)\) is one less than the system order \(n\), i.e., \((r < n)\) as \(n = 3\). So the input-output Bronswey form appears as follows

\[
\begin{align*}
\dot{z}_1 &= z_2 \\
\dot{z}_2 &= L_f^2 h(x) + L_f L_f h(x)u
\end{align*}
\]

(11)

\[
\dot{z}_3 = -\frac{m_4}{m_1} \left( k_1 z_3 m_1 + k_2 z_1 m_1 + k_3 z_3 m_1 u \right) + \left( \frac{z_3 m_1}{m_4} \right) \left( \frac{m_2^2}{m_1^2} \right) - \frac{1}{m_4} l_3 m_1 z_2 - l_3 z_1 + \frac{l_4 m_1 u}{m_4}
\]

(12)

So, one of the transformed dynamics, i.e., \(z_3\) represents the internal dynamics. The detailed expressions of the Lie-derivatives are given by the following equations.

\[
\begin{align*}
L_f^2 h(x) &= -m_4 f_2(x) - (m_2 + 2m_3 x_3) f_3(x) \\
L_f L_f h(x) &= l_4 m_4 x_2
\end{align*}
\]

(13)

Now, it is necessary to discuss the zero-dynamics stability.

A. STABILITY ANALYSIS OF THE ZERO-DYNAMICS

The nonlinear system’s dynamics are divided into 2 parts i.e., an internal part and an external (input-output) part when performing the input-output conversion. Since, the external dynamic states i.e., \((z_1, z_2)\) are controllable states and are directly controlled by \(u\) while the stability of the internal dynamic state i.e., \((z_3)\) is simply determined by the location of zeros called zero-dynamics for a nonlinear system.

To calculate the zero dynamics, the following variables should be set to zero, i.e., \(z_1 = z_2 = u = 0\) in (12). By simplifying (12), finally one gets

\[
\dot{z}_3 = -z_3 (k_1 - l_1),
\]

(14)

where \(k_1 > l_1\), so

\[
\dot{z}_3 = -K z_3,
\]

(15)

where \(K\) is a positive integer. So, the zero-dynamic state, \(z_3\) is stable as long as \(k_1 > l_1\).

Remark 2: The dynamic model presented in (7) (adopted from (19)) is equivalently represented, in state space form, in (8). The (9) is also the most general form of (8) with vector fields \(f(x)\) and \(g(x)\) and \(u = R_{ch}\) as an affine control input to the system.

IV. BACKSTEPPING-BASED SMC STRATEGIES USING DIFFERENT REACHABILITY LAWS

In this section, the design procedure of Backstepping-based SMC strategy, while using different Reachability Laws, is comprehensively demonstrated.

A. BACKSTEPPING-BASED SMC STRATEGY: USING CONVENTIONAL REACHABILITY LAW (BSMC)

The nonlinear dynamics of the model, given in (11), can be steered to a desired reference by minimising the error between the actual and reference point. Owing to this concept, the error is defined as follows

\[
\begin{align*}
e_1 &= z_1 - z_{1\text{ref}} \\
e_f &= \int_0^t e_1 d\tau
\end{align*}
\]

(16)

Now, the design of control law is pursued by defining a Lyapunov function as \(V_1 = 1/2 e_1^2\) and its time derivative along (16) and (11), it becomes

\[
\dot{V}_1 = e_1 (z_2 - \dot{z}_{1\text{ref}})
\]

(17)

By selecting \(z_2\) as virtual control law, which is given as follows

\[
z_{2\text{ref}} = \dot{z}_{1\text{ref}} - K_1 e_1
\]

(18)

The differential equation (17) is exponential stable, i.e., \(\dot{V}_1 = -K_1 e_1^2 = -K_1 V_1\), where \(K_1\) is positive constant. To proceed to the next step, we define a new error variable
as \( e_2 = z_2 - z_{2\text{ref}} \). By putting (18), one can obtain \( z_2 = e_2 - K_1e_1 + \dot{z}_{1\text{ref}} \), with this (17), which can be expressed as

\[ \dot{V}_1 = -K_1e_1^2 + e_1e_2 \]  

(19)

Since, all the error variables are defined. Therefore, a novel sliding surface, in terms of error variables, is defined as follows

\[ s = c_1e_1 + e_2 + c_2e_1, \]

(20)

where \( c_1 \) and \( c_2 \) are positive parameters. Before proceeding to the control design, it is suitable to make a remark.

Remark 3: It is to be noted that the presented sliding surface is quite novel which is of proportional-integral (PI) type in the conventional error variable \( e_1 \) and cumulatively a proportional integral derivative (PID) type surface in the backstepping variable \( e_2 \). This kind of sliding surface is not yet used in the existing literature, which makes our design quite novel. The advantage of this surface is that it helps in the elimination of steady-state errors for such stochastic nature desired outputs.

By taking the time derivative of \( s \) along (16), and (11), one get the following expression.

\[ \dot{s} = c_1\dot{e}_1 + c_2\dot{e}_2 + \frac{L_f}{L_g}h(x) + L_gL_fh(x)u - \dot{z}_{1\text{ref}} + K_1\dot{e}_1 \]

(21)

To calculate the equivalent control law, which drives the system trajectories in sliding mode, posing \( \dot{s} = 0 \) and calculating for the control component, one gets

\[ u_{\text{equ}} = \frac{1}{L_gL_fh(x)}[-c_1\dot{e}_1 - c_2e_1 - \frac{L_f}{L_g}h(x) - K_1\dot{e}_1 + \dot{z}_{1\text{ref}}] \]

(22)

The discontinuous control component is based on the conventional reachability, therefore, \( u_{\text{dis}} \) is defined as follows

\[ u_{\text{dis}} = -K_2(s), \]

(23)

where \( K_2 \) is the design parameter. Finally, the overall control input, is given as

\[ u = u_{\text{equ}} + u_{\text{dis}} \]

(24)

To prove the sliding mode enforcement, a Lyapunov function in terms of the sliding surface is defined as \( V = \frac{1}{2}s^2 \). The time derivative of this energy function, along (21) becomes

\[ \dot{V} = s(\dot{c}_1\dot{e}_1 + c_2\dot{e}_2 + \frac{L_f}{L_g}h(x) + L_gL_fh(x)u - \dot{z}_{1\text{ref}} + K_1\dot{e}_1) \]

(25)

Using \( u \) from (24), the above expression reduces to

\[ \dot{V} = -K_2s \]  

(26)

This equation can also be written as follows

\[ \dot{V} = -K_1\sqrt{2V} \]

This differential inequality confirms the finite time enforcement of sliding mode along the designed sliding manifold, i.e., \( s = 0 \) is achieved in finite time. Consequently, one get the following tracking error dynamics

\[ e_2 + c_1e_1 + c_2\int_0^t e_1d\tau = 0 \]

(27)

This expression shows that \( e_1 \to 0 \) as \( t \to \infty \). As conventional reachability based law is used, therefore, the control input will exhibit chattering across the manifold in sliding mode. To get rid of these unwanted effects, it is recommended to use the saturation function instead of the signum function. However, the response may be a bit slower. Thus, in the following study, it is suggested to use the super-twisting law as reachability to alleviate the chattering.

B. Backstepping-Based SMC Strategy: Using Super-Twisting Reachability Law (BSTSMC)

As reported earlier, the objective is to reduce the chattering phenomenon, therefore, the following super-twisting algorithm is used as reachability law.

\[ u_{\text{dis}} = -\alpha|s|^\frac{3}{2} - \beta\int_0^t (s)d\tau \]

(28)

Consequently, the overall control law will becomes

\[ u_{\text{super}} = \frac{1}{L_gL_fh(x)}[-c_1\dot{e}_1 - c_2e_1 - \frac{L_f}{L_g}h(x) - K_1\dot{e}_1 + \dot{z}_{1\text{ref}}] - \alpha|s|^\frac{3}{2} - \beta\int_0^t (s)d\tau, \]

(29)

where \( \alpha \) and \( \beta \) are the constant parameters and \( s \) is the sliding manifold based on backstepping variables \( e_1 \) and \( e_2 \). Note that, the use of super-twisting reachability does not alter the order of sliding modes, i.e., we are still dealing with first order SMC. However, the benefit gained is the suppression of chattering.

C. Backstepping-Based SMC Strategy: Using Real-Twisting Reachability Law (BRtSMC)

At this stage, the main interest is to suppress the chattering and to observe accurate tracking performance. Therefore, a real-twisting-based reachability law is defined as

\[ u_{\text{dis}} = -\alpha e_1 - \beta e_2 \]

(30)

The expression of the controller with this reachability looks as follows

\[ u_{\text{real}} = \frac{1}{L_gL_fh(x)}[-c_1\dot{e}_1 - c_2e_1 - \frac{L_f}{L_g}h(x) - K_1\dot{e}_1 + \dot{z}_{1\text{ref}}] - \alpha(e_1) - \beta(e_2) \]

(31)

It is necessary to mention that the stability analysis from (25)-(27) remains valid for the super-twisting as well as real-twisting sliding mode control law.

Remark 4: In practical implementations, the system (9) may only be available with output. Since, the control algorithm is depending on the output state \( x_3 \) as well as \( x_1 \) and \( x_2 \). Therefore, a state observer will be needed to estimate \( x_1 \) and \( x_2 \). Furthermore, to make simple the stability analysis,
one needs to have the separation principle. For this purpose, system (9) in quasi-linear form can be expressed via the following procedure.

V. OPTIMAL LINEARIZATION OF PMSG-WECS

The nonlinear system (9) which is affine in control, at point $x_{opt}$, can be described as follow

$$\dot{x}_{opt} = f(x_{opt}) + g(x)u = A(x_{opt})x_{opt} + g(x)u$$

(32)

At this stage the objective is to get a state-dependable matrix $A(x)$, one may have

$$f(x) = A(x)x$$

(33)

or

$$f(x_{opt}) = A(x_{opt})x_{opt}$$

(34)

So, following the procedure outlined in [20], assume $a_i^T$ to be the $i$th row of the matrix $A(x)$. For this purpose (33) and (34) can be rewritten as

$$f_i(x) = a_i^T(x)x, i = 1, 2, ..., n$$

(35)

and

$$f_i(x_{opt}) = a_i^T(x_{opt})x_{opt}$$

(36)

By expanding the left hand side of (35) at $x_{opt}$ and discarding the higher order terms, one gets

$$f_i(x) = f_i(x_{opt}) + \nabla^T f_i(x_{opt})(x-x_{opt}) = a_i^T(x)x$$

(37)

where $\nabla^T f_i(x) \in \mathbb{R}^{n \times 1}$ is the gradient of $f_i$ evaluated at $x$. Using (36), the above equation can be restated as

$$\nabla^T f_i(x_{opt})(x-x_{opt}) = a_i^T(x_{opt})(x-x_{opt})$$

(38)

In order to get $a_i$, a constrained minimization problem is expressed as

$$\min_{a_i} \ j = \frac{1}{2} \|\nabla f_i(x_{opt}) - a_i(x_{opt})\|^2$$

(39)

The first-order optimality criterion for the augmented cost function $j$ is $j = \frac{1}{2} \|\nabla f_i(x_{opt}) - a_i(x_{opt})\|^2 + \lambda_i f_i(x_{opt}) - a_i^T(x_{opt})x_{opt}$ with $\lambda_i$ as Lagrange-Multiplier, results in $\nabla a_i = 0$, i.e.,

$$a_i = \nabla f_i(x_{opt}) - \lambda_i x_{opt}$$

(40)

The Lagrange-Multiplier, $\lambda_i$ is determined from (40) with pre-multiplied $x_{opt}$ and substituted in (36), the expression of $\lambda_i$ comes out as follow

$$\lambda_i = \frac{x_{opt}^T \nabla f_i(x_{opt}) - f_i(x_{opt})}{\|x_{opt}\|^2}; \quad x_{opt} \neq 0$$

(41)

Substitution of (41) into (40) leads to

$$a_i = \nabla f_i(x_{opt}) + \frac{f_i(x_{opt}) - x_{opt}^T \nabla f_i(x_{opt})}{\|x_{opt}\|^2} x_{opt}$$

(42)

Using the above formulation, the nonlinear control-focused model of the PMSG-WECS can be expressed as a quasi-linear model of the following form

$$\dot{x} = A_{sys}(x) + B_{sys}u$$

$$y = C_{sys}x$$

(43)

where $A_{sys} = [\phi_{11}, \phi_{12}, \phi_{13}, \phi_{21}, \phi_{22}, \phi_{23}, \phi_{31}, \phi_{32}, \phi_{32}]$, $B_{sys} = [-\frac{x_1}{L_2+L_{ch}}, -\frac{x_2}{L_2+L_{ch}}, 0]^T$ and $C_{sys} = [0 \ 0 \ 1]$. Now, the states estimator, for the above-formulated system will be studied in the subsequent section.

**Remark 5:** Since a quasi linearized model, which supports the separation principles, is obtained for the PMSG-WECS. Therefore, it is quite suitable to design again scheduled robust sliding mode observer for this newly constructed linearized form to provide us with the estimated measurements of the unavailable states of the system.

VI. GAIN-SCHEDULED UTKIN OBSERVER: A SLIDING MODE OBSERVER

The conventional Luenberger observer shows high sensitivity to disturbance throughout the estimation process. In order to make a robust estimation of the states, an observer based on the concept of the sliding mode is investigated in the following study. The observer which will be dealt with is reduced order in nature and it demonstrates gain scheduling property which is quite appealing in practical scenarios when one deals with a linear model of the plant.

To transform the system into two subsystems i.e., system with available states and system with non-available states, a similarity transformation of the following form is carried out. Let $T = [N^T \ C]^T$(where $N^T$ generates the null space of $C$) be the transformation which transforms (43) to the following form [20]

$$\begin{bmatrix} \dot{z} \\ \dot{y} \end{bmatrix} = TA(x)T^{-1}\begin{bmatrix} z \\ y \end{bmatrix} + TBu$$

(44)

and

$$\begin{bmatrix} \dot{z} \\ \dot{y} \end{bmatrix} = CT^{-1}x$$

(45)

where $z = [x_1, x_2]^T \in \mathbb{R}^{2 \times 1}$ and $y = [x_3] \in \mathbb{R}^{1 \times 1}$ are unavailable and available information, respectively. The system (44) in more explicit form looks as follows:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = A_{sys}(x)x + A_{sys}(x)y + B_{sys}u$$

(46)

An observer of the following form is defined to provide the unavailable states.

$$\begin{bmatrix} \dot{z} \\ \dot{y} \end{bmatrix} = A_{sys}(x)x + A_{sys}(x)y + B_{sys}u$$

(47)

Note that $G_1 \in \mathbb{R}^{2 \times 1}$ and $G_2 \in \mathbb{R}^{1 \times 1}$ represents the gain matrices which improve the performance and show robustness against certain uncertainties. In addition, $L \in \mathbb{R}^{2 \times 1}$ represents the gain of the discontinuous term $v$ which is
introduced, which leads (48) to the subsequent form. The corresponding error dynamics are expressed as follows:

\[
\begin{align*}
\dot{e}_x &= A_{11}(x)e_x + A_{12}(x)e_y + Lu - G_1\epsilon_y, \\
\dot{e}_y &= A_{21}(x)e_x + A_{22}(x)e_y - v - G_2\epsilon_y,
\end{align*}
\]

where \(e_x = \hat{z} - z\) and \(e_y = \hat{y} - y\). Now, to prove further the stability, some new transformation \(e_x = e_x + Le_y\) are introduced, which leads (48) to the subsequent form.

\[
\begin{bmatrix}
\dot{e}_x \\
\dot{e}_y
\end{bmatrix} =
\begin{bmatrix}
A_{11}(x) & A_{12}(x) \\
A_{21}(x) & A_{22}(x)
\end{bmatrix}
\begin{bmatrix}
e_x \\
e_y
\end{bmatrix} +
\begin{bmatrix}
0 \\
-I
\end{bmatrix} v,
\]

with the sub-matrices

\[
\begin{align*}
A_{11} &= A_{11}(x) + LA_{21}(x), \\
A_{12} &= A_{12}(x) - A_{11}L - G_1 + L(A_{22}(x) - G_2), \\
A_{22} &= A_{22}(x) - G_2 - A_{21}(x)L.
\end{align*}
\]

(50)

The system (49) shows that states observation problem is now appearing as states regulation problem under the action of the discontinuous control \(v\). To prove that the state converges to zero, the stability analysis is outlined. Consider a Lyapunov function of the following form \(V_o = \frac{1}{2}e^2_y\). The time derivative of \(V_o\) along (49) becomes

\[
V_o = e_y(\dot{A}_{21}(x)e_x + \dot{A}_{22}e_y - v),
\]

\[
\leq -|e_y| (M_1 - |A_{21}(x)e_x + A_{22}e_y|) 
\leq -\eta |e_y|
\]

where \(\eta\) is a positive constant and satisfy the following inequality.

\[
M_1 - |A_{21}(x)e_x + A_{22}e_y| \geq \eta.
\]

(52)

The inequality (51), in alternate form, appears as \(V_o \leq -2\eta V_o^{\frac{1}{2}}\) which has the same form as in Lemma 2 of [21]. Thus the error variable \(e_y\) converges to origin in a finite-time which is given by

\[
t_x \leq \frac{1}{\eta} \sqrt{2V_o(0)}. 
\]

(53)

Note that, the gain \(L\) should be selected in such a way that the matrix \(\dot{A}_{11} = |A_{11}(x) + LA_{21}(x)|\) becomes Hurwitz at each iteration. Similarly, \(G_2\) should be chosen to make \(A_{22}(x) - G_2 - A_{21}(x)L = \dot{A}_{22}\) Hurwitz.

It is quite worthy to report that \(L\) and \(G_2\) are designed via the conventional linear quadratic regulator (LQR) strategy to place the poles of \(\dot{A}_{11}\) and \(\dot{A}_{22}\) in the left half-plane (LHP). In the final step, \(G_1\) must be chosen to satisfy \(A_{11}(x) = 0\). The suitable selection of these gains i.e., \(L_1, G_1\) and \(G_2\) will result infinite time convergence of \(e_y\) to zero, where, the state \(e_x\rightarrow 0\) asymptotically. Hence, the stability of the observer is proved.

Remark 6: It is necessary to look into the overall closed-loop stability. Since, the system can be expressed in the quasi-linear form, which supports the separation principle. Therefore, in this article, the stability of the controller and observer are performed separately which in the final stage proves the overall closed-loop stability, i.e., the stability of the closed-loop plant is subjected to the controller and observer’s stability, simultaneously.

VII. SIMULATION RESULTS AND DISCUSSION

In this section, the core objective is to present and discuss the simulation results of the maximum power extraction from the WECS under the action of the control algorithms devised in the aforesaid study. The overall study is done by considering two reference wind profiles which mainly vary because of the variations in the parameters. The closed-loop study, in the presence of the controller and observers, is illustrated in Fig. 3.

At this stage, it is convenient to outline the wind profile generation. Since natural wind exhibit irregular variations in wind speed over a long period. It is because of the environmental conditions such as weather, trees, buildings and areas of the sea. Normally, wind speed, which is highly stochastic in nature, can be modelled as follows [7]

\[
V(t) = V_o(t) + V_f(t),
\]

(54)

where \(V_o(t)\) is a slowly varying component, which is obtained from the measured data, while \(V_f(t)\) is a rapidly varying turbulence component. The turbulence component variates, typically within 10 minutes, and can be described by power spectrum (von Karman’s spectra). The transfer function of the shaping filter is as follow

\[
H_i(j\omega) = \frac{K_F}{(1 + j\omega T_F)^{5/6}},
\]

(55)

where \(K_F\) and \(T_F\) depend upon low-frequency wind speed, \(V_o(t)\). The non-stationary wind speed can be obtained by the block diagram displayed in Fig. 4. Now, the wind profile-1 is generated by setting the parameters \(K_F = 1\) and \(T_F = 0.2\) with the shaping filter given in (55). In Matlab/Simulink environment, simulations are carried out for 3 kW PMSG-based WECS. The numerical solver, used for the simulation, was Euler method with a step size of 0.001 seconds. The other parameters of the system were set as: the maximum power coefficient \(C_{p_{max}} \approx 0.476\), optimal tip speed ratio \(\lambda_{opt} \approx 7\lambda_{opt}\), the average speed of wind is about \(7m/s\) and a medium turbulence intensity (using von Karman spectrum) is \(\sigma = 0.15\). The closed-loop simulation, according to block diagram reported in Fig. 3, are performed over a period of 50 seconds. Note that all the aforesaid three controllers, i.e., BSMC, BSTSMC and BRTSMC were tested one by one in the closed-loop structure and their comparative results are developed.
Initially, the extraction of maximum power is made possible by operating the turbine at optimum TSR \((\lambda_{opt})\) that will ensure \(C^*_p\) under the action of designed three control schemes. These optimum values are achieved by controlling the rotational shaft speed of the PMSG. Figs. 5, 6 and 7 ensure the tracking of rotational speed by keeping TSR and power coefficient at its optimum values. While comparing the reference tracking, BSMC exhibits high-frequency oscillations as compared to BSTSMC. Moreover, BRTSMC undergoes oscillatory tracking around the reference with comparatively lower amplitude than the other two. Furthermore, one can also describe the superiority of the proposed technique by observing their settling times, i.e., BRTSMC converges to zero at 1 second in a zoomed section of Fig. 5, whereas, BSTSMC converges to zero in 0.3 seconds while BSMC converges to zero in 1 second. Therefore, it is observed that MPPT is more effective in BRTSMC.

Hence, it is obvious from the reference track, shown in Fig. 5, that all the three controllers display very good responses with considerably negligible steady-state errors. On the other hand, the tip speed ratio (TSR) in Fig. 6, the maximum power coefficient in Fig. 7, and the aerodynamic power in Fig. 6 of the BRTSMC controller are quite appealing, as compared to BSMC and BSTSMC. Hence, it comes out that the performance of the BRTSMC is comparatively better than BSMC and BSTSMC. Note that it is not necessary that the performance of the BRTMC will always be better than the other two if tested on other applications. Their performances vary from system to system. It is worthy to note that BSTSMC is easy to implement as compared to BRTSMC because it doesn’t require the derivative of the output variable.
Since, in the above study, we highlighted that BRTSMC performs better than some standard controllers [7]. The comparison is carried out in tracking performances, TSR, mechanical power coefficient and ORC. It is evident from Fig. 9 that the tracking of BRTSMC is better than FBLC. The BRTSMC tracks the reference very closely while that of FBLC exhibits steady-state error, which can be seen from the zoomed picture. Similarly, the other performance parameters are very nicely followed by BRTMC while FBLC lacks in all. See for a detailed look at Figs. 10, Figs. 11 and 12. Thus, it is determined that BRTSMC outshines the other counterparts.

Remark 7: In this work, the performance of the BRTSMC is compared with FBLC. However, if one compares the performance of BSMC and BSTSMC, then it is still confirmed that these two controllers also perform better than FBLC. For the sake of shortness, the detailed presentation is avoided.

To authenticate the performance of the proposed controller, a second wind profile is generated by setting the $K_F = 4$ and $T_F = 10$, while using the same shaping filter $H_t(j\omega)$, which is given (55). Furthermore, to make the scenario of the implementation more practical, it is assumed that two states $x_1$ and $x_2$ are not available. So, a virtual sensor, as outlined in the aforementioned theory, is designed via a gain-scheduled sliding mode observer and then the virtually measured states are used in the proposed controller. Figs. 13 and 14 show that the missing states are exactly estimated via the gain-scheduled sliding mode observer the so-called gain-scheduled Utkin observer.
Based on the observed states, the simulations are performed and the results are recorded for the proposed three controllers. Fig. 15 illustrates the generator reference speed $\Omega_{ref}$ versus the actual speed of the shaft, $\Omega_h$, where the controller ensures good reference tracking. Having looked at the figure, it is clear that BRTSMC tracks the reference trajectory with minimum steady-state error (see the zoomed picture) as compared to BSTSMC and BSMC. Fig. 16 shows the nonlinear plot between wind turbine power versus generator actual speed where it can be seen that the BRTSMC graph lies closer to ORC as compared to BSTSMC as well as BSMC. Similarly, Fig. 17 shows the TSR $\lambda$, which is exactly 7, closed to its optimal value in the case of BRTSMC, while the BSTSMC and BSMC oscillate around 7. The average wind speed is 7 m/sec.

Finally, Fig. 18 shows the evolution of the power coefficient $C_p$. It can be seen that the value of $C_p$ is held at or near $C_{P_{max}}$ despite all variations in the wind and other parameters. The desirable maximum power coefficient $C_{P_{max}}$ for the VSWT system is 47% and the BRTSMC lies exactly on 0.47 without oscillations while excursion occurs in BSTSMC and BSMC. Thus, it is confirmed the newly proposed controller, based on the now PID surface, is an appealing candidate for the MPPT-WECS. Its implementation, in other energy applications is highly suggested because of its benefits over FBLC.

**VIII. CONCLUSION**

In this article, the model-based synergistic control laws with gain-scheduled sliding mode observers are presented for tracking the varying wind profile in WECS. The WECS further drives PMSG, which converts wind energy into electrical energy. The core schemes of the synergistic control laws are Backstepping and SMC approaches, which, when employed in the conventional structure, give us a BSMC strategy. Subsequently, In the aforesaid approach, the discontinuous control law in SMC is replaced by super-twisting and then real-twisting control law, which gives us BSTSMC and BRTSMC, respectively. Since, the designed control strategies, for successful operation, depending on the system’s states information. Therefore, a gain-scheduled sliding mode observer (GSSMO) is designed to reconstruct the immeasurable states’ information after transforming the nonlinear system into quasi-linear form. The effectiveness of the closed-
loop systems, i.e. which include the proposed control strategies and GSSMO, are confirmed via Simulink/MATLAB environment. The results in the presence of load and disturbances were quite appealing and the newly investigated laws proved to be a more practical and appealing candidate for the aforesaid energy system. It is worthy to mention that the simulation results of the proposed strategies are compared with standard literature results. In nutshell, the new strategies especially BSTSMC outshines all the employed designed strategies. The overall closed-loop stability was claimed while taking support of the separation principle.

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