System size dependence of hadron $p_T$ spectra in $p+p$ and Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV

P. K. Khandai$^1$, P. Sett$^2$, P. Shukla$^2$, V. Singh$^1$

$^1$Department of Physics, Banaras Hindu University, Varanasi 221005, India
$^2$Nuclear Physics Division, Bhabha Atomic Research Center, Mumbai 400085, India

E-mail: pshukla@barc.gov.in

Abstract. We make a systematic study of transverse momentum ($p_T$) spectra of hadrons produced in $p+p$ and in different centralities of Au+Au collisions at $\sqrt{s} = 200$ GeV using phenomenological fit functions. The Tsallis distribution gives a very good description of hadron spectra in $p+p$ collisions with just two parameters but does not produce the same in Au+Au collisions at intermediate $p_T$. To explain the hadron spectra in heavy ion collisions in a wider $p_T$ range we propose a modified Tsallis function by introducing an additional parameter which accounts for collective flow. The new analytical function gives a very good description of both mesons and baryons spectra at all centralities of Au+Au collisions in terms of parameters having a potential physics interpretation. With this modified Tsallis function we study the spectra of pions, kaons, protons, $\Lambda$ and $\Xi^-$ as a function of system size at $\sqrt{s_{NN}} = 200$ GeV. The parameter representing transverse flow increases with system size and is found to be more far baryons than that for mesons in central Au+Au collisions. The freeze-out temperature for baryons increases with centrality in Au+Au collisions more rapidly than for mesons.

QGP, Tsallis distribution, hadron spectra

Submitted to: J. Phys. G: Nucl. Phys.
1. Introduction

The heavy ion collisions at relativistic energies are carried out to study the properties of strongly interacting matter at high temperature, where a phase transition to a Quark Gluon Plasma (QGP) is expected. Measurements in Au+Au collisions, performed at the Relativistic Heavy Ion Collider (RHIC) already point to the formation of Quark Gluon Plasma (QGP) \[1, 2, 3\]. Experiments at RHIC continue to study the detailed properties of the strongly interacting matter using \( p+p \) and \( Au+Au \) systems at colliding energies with \( \sqrt{s_{NN}} \) ranging from 7.7 GeV to 200 GeV. The \( p+p \) collisions are used as baseline and are important to understand the particle production mechanism \[4\]. Transverse momentum (\( p_T \)) distributions of identified hadrons are the most common tools used to study the dynamics of high energy collisions. The high \( p_T \) hadrons are important for QGP studies as they measure the jet quenching \[5\] effect in QGP. The low \( p_T \) hadrons arise from multiple scatterings and follow an exponential distribution suggesting particle production in a thermal system \[6\]. In addition, the hadron spectra at intermediate \( p_T \) are sensitive to effects arising from quark recombination \[7\] in heavy-ion collisions.

The Tsallis distribution \[8, 9\] gives an excellent description of \( p_T \) spectra of all identified mesons measured in \( p+p \) collisions at \( \sqrt{s} = 200 \text{ GeV} \) \[10\]. It interprets the system in terms of temperature and a parameter which measures temperature fluctuation. In a recent work \[11\], we used the Tsallis distribution to describe the \( p_T \) spectra of identified charged hadrons measured in \( p+p \) collisions at RHIC (\( \sqrt{s} = 62.4, 200 \text{ GeV} \)) and at LHC (\( \sqrt{s} = 0.9, 2.76 \) and 7.0 TeV) energies. It has been shown \[9, 11\] that the functional form of the Tsallis distribution with thermodynamic origin is of the same form as the QCD-inspired Hagedorn distribution \[12, 13\]. The non-extensive parameter \( q \) of Tsallis is thus related to the power \( n \) of the Hagedorn function.

There have been many attempts to use the Tsallis or Hagedorn distributions to fit the \( p_T \) spectra of hadrons produced in heavy ion collisions. To fit the \( \pi^0 \) spectrum measured in \( Au+Au \) collisions, the PHENIX collaboration used a function referred as the modified Hagedorn formula \[14, 15\]. The spectra of other mesons are then obtained by \( m_T \) scaling which were used to get the hadronic decay cocktails for single or dielectron spectra. In one of our works \[16\], we used this modified Hagedorn formula to test the \( m_T \) scaling for mesons extensively at \( \sqrt{s}=200 \text{ GeV} \) and for baryons in the work \[17\]. While the modified Hagedorn formula gives a very good description of hadron \( p_T \) distributions, its parameters lack a physics interpretation. There have been few other attempts to find a good fit function for hadron spectra measured in various colliding systems e.g. work in Ref. \[18\] adds another exponential term with Tsallis function to fit the data.

The shape of the \( p_T \) distributions of mesons and baryons resemble the \( p_T \) distribution of quarks as shown by recombination models \[19\]. Thus, the change in the \( p_T \) spectra of quarks due to collective flow etc. will be reflected in the measured \( p_T \) distribution of hadrons in heavy ion collisions which could be included in phenomenological fit functions such as the Hagedorn or Tsallis formula. To explain the hadron spectra in heavy ion collisions in the large \( p_T \) range, we propose a modified
Tsallis function by introducing an additional parameter which accounts for transverse flow. The new analytical function gives very good description of both mesons and baryons spectra at all centralities of Au+Au collisions in terms of parameters having potential physics interpretation. We make a systematic study of the parameters of the modified Tsallis function for pions, kaons, protons, \( \Lambda(1115) \), and \( \Xi^- \) as a function of system size at \( \sqrt{s_{NN}} = 200 \) GeV using measured hadron spectra from PHENIX and STAR experiments.

2. The Tsallis distribution and collective transverse flow

The Tsallis distribution \([8, 9]\), describes a thermal system in terms of two parameters \( T \) and \( q \), is given by

\[
E \frac{d^3N}{dp^3} = C_q \left( 1 + (q - 1) \frac{E}{T} \right)^{-1/(q-1)}. \tag{1}
\]

Here \( C_q \) is the normalization constant, \( E \) is the particle energy, \( T \) is the temperature and \( q \) is the so-called nonextensivity parameter which measures the temperature fluctuations \([20]\) in the system as: \( q - 1 = Var(T)/<T>^2 \). The values of \( q \) lie between \( 1 < q < 4/3 \).

Using the relation \( 1/(q - 1) = n \), Eq. 1 takes the form

\[
E \frac{d^3N}{dp^3} = C_n \left( 1 + \gamma \left( mT - \beta \vec{p}_T \right) \right)^{-n}. \tag{2}
\]

Here \( C_n \) is the normalization constant and \( m_T = \sqrt{p_T^2 + m^2} \). Larger values of \( q \) correspond to smaller values of \( n \) describing a system away from thermal equilibrium. In terms of QCD, smaller values of \( n \) imply dominant hard point-like scattering. Phenomenological studies suggest that, for quark-quark point scattering, \( n \sim 4 \) \([21, 22]\), and when multiple scattering centers are involved \( n \) grows larger and can go up to 20 for proton \( p_T \) spectra.

If there exists a transverse flow of particles in a co-moving frame or system, then we can replace the energy by the following four vector form

\[
E = v^\mu p_\mu = \gamma (mT - \vec{\beta} \vec{p}_T), \tag{3}
\]

where the factor \( \gamma = 1/\sqrt{1 - \beta^2} \), \( v^\mu = \gamma (1, \vec{\beta}, 0) \) and \( p_\mu = (m, -\vec{p}_T, 0) \) are four-velocity and four-momentum of particles in central rapidity region. Assuming, \( \vec{\beta} \) and \( \vec{p}_T \) to be collinear and denoting the average transverse velocity of the system by \( \beta \), the Tsallis distribution in Eq. 2 takes the form

\[
E \frac{d^3N}{dp^3} = C_n \left( 1 + \frac{\gamma (mT - \beta p_T)}{nT} \right)^{-n}. \tag{4}
\]
System size dependence of hadron $p_T$ spectra in $p+p$ and $Au+Au$ collisions at $\sqrt{s_{NN}} = 200$ GeV

There have been many attempts to use Tsallis distributions in hydrodynamical models [e.g. 23, 24]. These formalisms are used to explain the hadronic spectra produced in heavy ion collisions in the $p_T$ range of 0 to 3 GeV/c. Our goal is to find an analytical fit function which works in a wider $p_T$ range. We propose a modified Tsallis formula given by

$$E \frac{d^3N}{dp^3} = C_n \left( \exp\left( -\frac{\gamma \beta p_T}{nT} \right) + \frac{\gamma m_T}{nT} \right)^{-n}. \quad (5)$$

The low and high $p_T$ limits of this formula are given by

$$E \frac{d^3N}{dp^3} \approx \exp\left( -\frac{\gamma (m_T - \beta p_T)}{T} \right) \quad \text{for} \quad p_T \to 0 \quad (6)$$

$$\approx \left( \frac{\gamma m_T}{nT} \right)^{-n} \quad \text{for} \quad p_T \to \infty. \quad (7)$$

Thus at low $p_T$, it represents a thermalized system with collective flow and at high $p_T$ it becomes a power law which mimics "QCD inspired" a quark interchange model [13].

Figure 1 gives a comparison of different fit functions namely the Tsallis by Eq. 2, Tsallis including radial flow by Eq. 4, modified Tsallis by Eq. 5, Boltzmann distribution including flow by Eq. 6, and power law by Eq. 7 along with the blast wave model from Ref [25] using Boltzmann distribution. The modified Tsallis function is fitted on charged and neutral pions measured in 10-20 % central Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV to get the fit parameters which are then used to calculate all other functions given in the figure. The normalizations for all functions are arbitrary as our aim is to compare the shapes of different functions. On comparing the modified Tsallis with the original Tsallis we can study the effect of parameter $\beta$ which enhances the contribution in intermediate $p_T$ range. In the low $p_T$ range the modified Tsallis tends to an exponential distribution given by Eq. 6 and to a power law in high $p_T$ range. Thus it preserves the well known property of the Tsallis distribution which at low $p_T$ tends to an exponential of form Eq. 6 (with $\beta = 0$) and at high $p_T$ is consistent with a power law. Another interesting thing we note is, both the blast wave model and the distribution given by Eq. 6 including average radial flow velocity are similar describing the data in low $p_T$ range.

The Tsallis including radial flow by Eq. 4 is also shown in the figure. This does not give the both low and high $p_T$ limits for the same set of parameters. It is possible to fit the data with this function using a very large value of $n$. It is expected that the blast wave model using Tsallis [23, 24] has similar behavior which is indicated by large values of $n$ (smaller values of $q - 1$) obtained in their work.

We test the formula (Eq. 5) using a large amount of data on transverse momentum spectra measured in $p+p$ and $Au+Au$ collisions at $\sqrt{s_{NN}} = 200$ GeV, which are listed in Table 1 along with their references. Here we use all the data from PHENIX ($|y| < 0.35$) [26, 27, 28, 29] and STAR ($|y| < 0.5$, $|y| < 0.75$) [30, 31, 32, 33] experiments. The errors on the data are quadratic sums of statistical and uncorrelated systematic errors wherever available. The particle spectra are studied as a function of system size using $p+p$ collisions and $Au+Au$ collisions of four centralities; 10-20 %, 20-40 %, 40-60 % and 60-80 %.
System size dependence of hadron $p_T$ spectra in $p+p$ and Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV

We employ the $\chi^2$ method to fit the data and Table 1 gives the $\chi^2$ and number of degrees of freedom. During our fits we consider a large $p_T$ range and do not assume that the parameters for all particles are the same in a particular collision system. The values of $n$ are decided by the initial hard scatterings which involves point like qq or multiple scattering centers and is different for different particles and they depend on energy of the colliding system [11].
System size dependence of hadron $p_T$ spectra in $p+p$ and Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV

Table 2. Values for $n$, obtained from the fits in $p+p$ system at $\sqrt{s} = 200$ GeV.

| Particle | Values of $n$ |
|----------|---------------|
| $\pi$    | 9.55 ± 0.13   |
| $k$      | 8.61 ± 0.24   |
| $p$      | 11.95 ± 0.10  |
| $\Lambda$ | 13.24 ± 5.36 |
| $\Xi$    | 13.24 ± 5.18  |

Table 3. $\chi^2$/ndf values for the fits in $p+p$ and Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV.

| Particle | $\chi^2$/NDF values for fits $p+p$ system | Au+Au system |
|----------|-------------------------------------------|---------------|
|          | $10 - 20\%$                             | $20 - 40\%$  | $40 - 60\%$  | $60 - 80\%$  |
| $\pi$    | 28.2/27                                  | 11.7/35       | 34.2/35       | 31.4/35       | 24.1/35       |
| $k$      | 7.4/10                                   | 20.2/13       | 30.0/13       | 20.6/13       | 7.0/13        |
| $p$      | 21.8/16                                  | 11.3/21       | 12.7/21       | 10.5/21       | 8.1/21        |
| $\Lambda$ | 14.2/18                                  | 14.3/14       | 11.5/14       | 3.6/14        | 17.0/14       |
| $\Xi$    | 4.5/8                                    | 10.7/10       | 3.9/10        | 7.2/10        | 0.6/3         |

The three parameters $T$, $n$ and $\beta$ are not completely independent of each other and effect of one parameter can be absorbed in the other to certain extent. Thus in our fit procedure first we fit the measured spectra of pions, kaons, protons, $\Lambda$ and $\Xi$ in $p+p$ collisions using the modified Tsallis distribution (Eq. 5) with the same temperature to obtain different values of $n$ (Table 2) for different particles. The values of $\beta$ are small for $p+p$ collisions as is expected. The spectra of different particles in Au+Au collisions for all centralities are fitted to obtain $\beta$ and $T$ as a function of system size. When going from $p+p$ collisions to Au+Au collisions, the initial hard scattering is assumed to be the same and hence the value of $n$ for each particle is kept fixed to values obtained from $p+p$ collisions. At high $p_T$ the shape of particle spectra in Au+Au collisions remains a power law without a noticeable increase in value of $n$ obtained from $p+p$ spectra so this assumption is good and helps us studying the behavior of the other two parameters as we increase the system size.

3. Results and Discussions

Figure 2(a) shows the transverse mass spectra of pions [26, 27], kaons [26], protons, $\Lambda$ and $\Xi$ [30] in $p+p$ collisions at $\sqrt{s} = 200$ GeV fitted with the modified Tsallis function [Eq. 5] with the same temperature to obtain different values of $n$. The fit quality is good for all particles in all $p_T$ range (0.3 to 6 GeV/$c$) shown by ratio of data points to
System size dependence of hadron $p_T$ spectra in $p+p$ and Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV

Figure 2. (Color online) (a) The invariant yields of pions ($\pi^+ + \pi^-)/2$, $\pi^0$) [26, 27], kaons ($K^+ + K^-)/2$ [26], protons $(p + \bar{p})/2$ [30], $\Lambda$ ($(\Lambda + \bar{\Lambda})/2$) [31] and $\Xi$ ($(\Xi^- + \Xi^+$/2) [31] as a function of $m_T$ for $p+p$ collision at $\sqrt{s} = 200$ GeV. The solid lines are the modified Tsallis distribution (Eq. 5). (b) Shows the Data/fit.

Figures 3 (a) and (c) show transverse mass spectra of pions [28, 29] and kaons [28], respectively in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV for all four centralities fitted with the modified Tsallis function. Here the values of $n$ for pions and kaons are fixed to the values obtained from $p+p$ collisions. Figures 3(b) and (d) give the ratio of data points to the fit function for pions and kaons respectively which show the quality of fit. One can notice that the modified Tsallis gives a very good fit for pions in all centralities of Au+Au collisions from very low $p_T$ to high $p_T$. The kaon spectrum, available only at low $p_T$, is also well described.

Figure 4(a) shows transverse mass spectra of protons [32] in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV for different centralities. Figure 4(b) shows the ratio of data points to the fit function. Except for the last two points for most central collisions, the data of all centralities are well reproduced. Figure 5 is same as Fig. 4 but for $\Lambda$ [33]. Figure 6 is as Fig. 4 but for $\Xi$ [33]. The modified Tsallis function gives very good quality of fit for all hadrons measured in Au+Au collisions at all centralities. The $\chi^2/NDF$ values for fits in $p+p$ and Au+Au system are listed in Table 3.

Figure 7 shows the variation of Tsallis parameters: (a) the transverse flow velocity $\beta$ and (b) the freeze-out temperature $T$ for pions, kaons, protons, $\Lambda$ and $\Xi$ for $p+p$ and different centralities of Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV. The errors on the parameters include the error on data and possible correlations among the parameters. The power $n$ for each particle are obtained by its $p_T$ spectrum in $p+p$ collisions. The flow velocities ($\beta$) for all particles are very small in $p+p$ collisions and increases with centrality in Au+Au collisions which is expected. For central collisions, there is
clear separation of flow velocity between mesons and baryons showing a dependence on number of constituent quarks. For the most central Au+Au collisions, the values of $\beta$ for protons is around 0.6 and for pions it is 0.4. Thus, when analyzing the spectra it is more important to group the particles as baryons and mesons than to apply any other criteria e.g. one based on strangeness. The behavior of freeze-out temperatures $T$ can also be grouped into mesons and baryons. With increasing system size, $T$ is almost constant for pions and increases weakly for kaons. For baryons the increase of temperature is more rapid as compared to mesons. There is also a clear separation of temperature between baryons and mesons towards most central Au+Au collisions.

Figure 3. (Color online) The invariant yields of (a) pions ($(\pi^+ + \pi^-)/2$, $\pi^0$) and (c) kaons ($(K^+ + K^-)/2$) as a function of $m_T$ for Au+Au collision at $\sqrt{s_{NN}} = 200$ GeV. The solid lines are the modified Tsallis distribution (Eq. 6). (b) and (d) show the Data/fit.
Figure 4. (Color online) (a) The invariant yield of proton \((p + \bar{p})/2\) as a function of \(m_T\) in Au+Au collisions at \(\sqrt{s_{NN}} = 200\) GeV for different centralities. The solid lines are the modified Tsallis distribution (Eq. 5). (b) Shows the Data/fit.

Figure 5. (Color online) (a) The invariant yield of \(\Lambda\) \(((\Lambda + \bar{\Lambda})/2)\) as a function of \(m_T\) in Au+Au collisions at \(\sqrt{s_{NN}} = 200\) GeV for different centralities. The solid lines are the modified Tsallis distribution (Eq. 5). (b) Shows the Data/fit.
System size dependence of hadron $p_T$ spectra in $p+p$ and $Au+Au$ collisions at $\sqrt{s_{NN}} = 200$ GeV

Figure 6. (Color online) (a) The invariant yield of $\Xi ((\Xi^- + \Xi^+)/2)$ as a function of $m_T$ in $Au+Au$ collisions at $\sqrt{s_{NN}} = 200$ GeV for different centralities. The solid lines are the modified Tsallis distribution (Eq. 5). (b) Shows the Data/fit.

Figure 7. (Color online) The variation of modified Tsallis parameters (a) the transverse flow velocity $\beta$, (b) the freeze-out temperature $T$ as a function of collision system size at $\sqrt{s_{NN}} = 200$ GeV for pions, kaons, protons, $\Lambda$ and $\Xi$. 
System size dependence of hadron $p_T$ spectra in $p+p$ and $Au+Au$ collisions at $\sqrt{s_{NN}} = 200$ GeV means that baryons freezeout earlier than mesons.

4. Conclusion

In this work we present a modified Tsallis function to describe the hadron spectra in heavy ion collisions. The function is analytic and which makes it convenient to use for fitting experimental hadronic spectra. The additional parameter we introduced accounts for transverse flow at low $p_T$. In low $p_T$ range the modified Tsallis tends to an exponential distribution with radial flow and to a power law in high $p_T$ range. Thus it preserves the well known property of the Tsallis distribution which at low $p_T$ tends to an exponential form with zero flow velocity and a power law at high velocities. With this modified Tsallis function we study the spectra of pions, kaons, protons, $\Lambda$(1115), and $\Xi^-$ as a function of system size at $\sqrt{s_{NN}} = 200$ GeV. This new analytical function gives a very good description of both meson and baryon spectra in all centralities of $Au+Au$ collisions in terms of parameters namely, temperature, power and transverse flow. We observe that the flow velocity extracted for all particles increases with centrality in $Au+Au$ collisions. There is a clear separation of flow between mesons and baryons in the most central $Au+Au$ collisions showing a dependence on number of constituent quarks. The behavior of freeze-out temperatures $T$ can also be grouped into mesons and baryons. For baryons the increase of temperature is more rapid than for mesons. The baryons in general freeze out earlier than the mesons.

Acknowledgments

We acknowledge the financial support from Board of Research in Nuclear Sciences (BRNS) for this project.

References

[1] Adams J et al STAR Collaboration 2005 *Nucl. Phys. A* **757** 10
[2] Adcox K et al PHENIX Collaboration 2005 *Nucl. Phys. A* **757** 184
[3] Adler S S et al PHENIX Collaboration 2005 *Phys. Rev. Lett.* **94** 082302
[4] Becattini F and Heinz U 1997 *Z. Phys. C* **76** 269
[5] Wang X 2004 *Phys.Lett.* B **579** 299
[6] Gatoff G and Wong C Y 1992 *Phys. Rev. D* **46** 997
[7] Fries R, Greco V and Sorensen P 2008 *Ann. Rev. Nucl. Part. Sci.* **58** 177
[8] Tsallis C 1988 *J. Stat. Phys.* **52** 479
[9] Biro T S, Purcsel G, Urmossy K 2009 *Eur.Phys.J.* A **40** 325
[10] Adare A et al PHENIX Collaboration 2011 *Phys.Rev. D* **83** 052004
[11] Khandai P K, Sett P, Shukla P and Singh V 2013 *Int. J. Mod. Phys.* **28**, 1350066
[12] Hagedorn R 1984 *Rev. del Nuovo Cim.* **6N** 101
[13] Balnkenbecler R and Brodsky S J 1974 *Phys. Rev. D* **10** 2973
[14] Adare A et al PHENIX Collaboration 2010 *Phys. Rev. C* **81** 034911
[15] Adare A et al PHENIX Collaboration 2011 *Phys. Rev. C* **84** 044905
[16] Khandai P K, Shukla P and Singh V 2011 *Phys. Rev. C* **84** 054904
System size dependence of hadron $p_T$ spectra in $p+p$ and $Au+Au$ collisions at $\sqrt{s_{NN}} = 200$ GeV

[17] Khandai P K, Sett P, Shukla P and Singh V 2012 arXiv:1205.0648 [hep-ph]
[18] Bylinkin A A and Rostovtsev A A 2010 arXiv:1008.0332 [hep-ph]
[19] Greco V, Ko C M and Levai P 2003 Phys. Rev. C 68 034904
[20] Wilk G and Wlodarczyk Z 2000 Phys. Rev. Lett. 84 2770
[21] Blankenbecler R, Brodsky S J and Gunion J 1975 Phys. Rev. D 12 3469
[22] Brodsky S J, Pirner H J and Raufeisen J 2006 Phys. Lett. B 637 58
[23] Shao M, Yi L, Tang Z, Chen H, Li C and Xu Z 2010 J. Phys. G 37 085104
[24] Tang Z, Xu Y, Ruan L, van Buren G, Wang F, Xu Z 2009, Phys. Rev. C 79, 051901
[25] Schnedermann E, Sollfrank J and Heinz U 1993, Phys. Rev. C 48, 2462
[26] Adare S S et al PHENIX Collaboration 2006 Phys. Rev. C 74 024904
[27] Adare A et al PHENIX Collaboration 2007 Phys. Rev. D 76 051106
[28] Adler S S et al PHENIX Collaboration 2004 Phys. Rev. C 69 034909
[29] Adare A et al PHENIX Collaboration 2008 Phys. Rev. Lett. 101 232301
[30] Adams J et al STAR Collaboration 2006 Phys. Lett. B 637 161
[31] Abelev B I et al STAR Collaboration 2007 Phys. Rev. C 75 64901
[32] Abelev B I et al STAR Collaboration 2006 Phys. Rev. Lett. 97 152301
[33] Adams J et al STAR Collaboration 2007 Phys. Rev. Lett. 98 62301