Light deflection Phenomenon Simulation and Calibration of the Earth-Observation Satellite

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Abstract: The advance of the resolution of optical observation satellite requires more accurate image acquisition location and geometrical model, therefore, light deflection effect must be taken into account in the geometrical model. The deflection of light is a phenomenon of a deviation between the apparent position and the true position of the observed object, due to the relative velocity of the observer (the remote sensing satellite). This article firstly models the light deflection effect of the earth-observation process of satellite, using this model, the deviation of the "apparent" location and the "real" location of the ground target is calculated. Then the influence of the satellite observation pointing angle to light deflection effect is analyzed. In the end, the correction method of the light deflection effect is advised, and through emulation, we can see that the image acquisition location error can be greatly improved after the correction.

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1. Introduction
At present, agile satellites have become an important development direction of remote sensing satellites. Many countries have successfully launched or are studying agile satellites. Typically there are Worldview series in the United States, Pleiades Series in France and so on. Relevant research work is also being carried out in China. The agile satellite can realize large angle and fast maneuver in a short time. With its fast attitude maneuver ability, the agile satellite can quickly change the orientation of the on-board camera to the ground, and realize efficient and flexible observation of the ground targets[1] It is necessary to consider the influence of this agile imaging mode on the geometric model of satellite remote sensing photogrammetry positioning, and whether there are new factors to consider. This paper points out that for agile satellites, because the maneuvering angle of each image is not fixed, the photogrammetry positioning model needs to consider the light deflection phenomenon. The magnitude of light deflection effect is closely related to the maneuvering angle of the satellite, and needs to be considered in the later image processing process to achieve high-precision image positioning accuracy.

The ray deflection of remote sensing satellite during earth observation mainly comes from two aspects: atmospheric refraction and the deviation between the observed position and the actual position caused by the movement of the object relative to the observer. In order to distinguish them clearly, they are called atmospheric refraction effect and relative deviation effect. The atmospheric refraction effect is due to the change of refractive index of the earth's atmosphere medium with height. When the camera equipped on the remote sensing satellite observes the earth, the light carrying the
information of the ground objects will deflect before reaching the camera, resulting in the displacement of the image points. The relative deviation effect is due to the transverse (vertical to the observing light direction) velocity of the satellite relative to the observed target. It was first discovered by James Bradley in astronomical observations in 1725.\cite{2}\cite{3}\cite{4}.

In this article, we mainly consider the relative deviation effect. Firstly, the geometric model of ray deflection is established. Then, the simulation of typical scenes is carried out to obtain the magnitude of ray deflection caused by deviation effect and its impact on positioning accuracy. The simulation results show that the satellite maneuvering angle has a great influence on the ray deflection. Finally, some suggestions for correcting the ray deflection effect are given. By correcting the ray deflection effect, the positioning accuracy of the image can be greatly improved.

2. Geometric Model of Relative Deviation Effect
When the optical remote sensing satellite observes the earth, because the satellite has a certain orbital speed, the relative movement between the remote sensing satellite and the ground observation target will cause a difference between the observation position of the ground target and its actual position. As shown in Figure 1, the satellite’s “theory” observation light vector $\mathbf{U}$ pointing to the ground target $P$, but because the satellite moves fast at a speed of $\mathbf{V}$ relative to the ground target, the actual observation light of the satellite comes from $\mathbf{W}$, points to the ground target $P'$. The angle between $\mathbf{U}$ and $\mathbf{W}$ is defined as the deflection angle $\delta$. $\theta$ is defined as the angle between the satellite's velocity $\mathbf{V}$ and the direction of observation light $\mathbf{U}$, that is the angle between the x-axis and $\mathbf{U}$ in Figure 1.

Fig.1 Light deflection phenomenon during the earth-observation process of satellite

To describe this relative deviation effect, we define two coordinate systems: S (satellite coordinate system) and E (earth coordinate system). Each coordinate system uses its own Cartesian coordinate system to measure space and time: S (t, x, y, z), E (t, x', y', z'). The x-axis is parallel to the x'axis, the y-axis is parallel to the y'axis, and the z'axis is also parallel to the z'axis. The relative velocity between the two coordinate systems is $\mathbf{V}$ with the direction along x-axis.

In the satellite coordinate system S (a moving coordinate system), the direction of light transmission can be expressed as $\mathbf{U}$, and there is $\|\mathbf{U}\| = c$. In the earth coordinate system E (a fixed coordinate system), the direction of light transmission is assumed to $\mathbf{W}$, and can be deduced from $\mathbf{U}$ by Lorenz transformation\cite{5}. The Lorenz transformation guarantees a constant modulus magnitude of
and \( \mathbf{W} \), because the light transmission speed remains constant as \( c \) in any coordinate system.

\[
\begin{align*}
W_x &= K \cdot (U_x + V) \\
W_y &= K \cdot \beta \cdot U_y \\
W_z &= K \cdot \beta \cdot U_z
\end{align*}
\] (1)

among them \( |\mathbf{W}| = c \), \( \beta = \frac{1}{\sqrt{1 + \frac{V^2}{c^2}}} \), \( K = \frac{1}{1 + \frac{V^2}{c^2}} \).

The observing angle of the optical remote sensing satellite can be expressed as \((\Psi_x, \Psi_y)\), and \( \Psi_x \) is the rolling angle of the satellite, \( \Psi_y \) is the pitch angle of the satellite. In the satellite coordinate system \( S \), the theoretical point vector \( \mathbf{U} \) of the satellite can be expressed as:

\[
\mathbf{U} \approx c \cdot f(\Psi_x, \Psi_y) \cdot \left[ \begin{array}{c} \tan(\Psi_y) \\ -\tan(\Psi_x) \\ 1 \end{array} \right]
\] (2)

There is \( f(\Psi_x, \Psi_y) = \frac{1}{\sqrt{1 + \tan^2(\Psi_x) + \tan^2(\Psi_y)}} \).

The "actual" pointing of a satellite can be expressed as:

\[
\mathbf{W} = \mathbf{U} + \hat{\mathbf{W}} = c \cdot f(\Psi_x, \Psi_y) \cdot \left[ \begin{array}{c} \tan(\Psi_y) \\ -\tan(\Psi_x) \\ 1 \end{array} \right] + \left[ \begin{array}{c} V_x \\ V_y \\ 0 \end{array} \right]
\] (3)

The deviation angle \( \delta \) is:

\[
\sin^2 \delta = \frac{V_x^2 + V_y^2 + (V_x \cdot \tan(\Psi_y) + V_y \cdot \tan(\Psi_x))^2}{c^2 \cdot (1 + \tan^2(\Psi_x) + \tan^2(\Psi_y))}
\]

3. Theoretical simulation of light deflection effect

Assume the range of satellite rolling and pitching angle is between -45 degree and 45 degree. The deflection angles of satellite light under different rolling and pitching angles are calculated. The calculation results are shown in the figure below.

![Fig.2 Relationship between the light deflection angle and the satellite observation angle](image)

When the satellite observes at the sub-satellite point \((\Psi_x, \Psi_y = 0)\), the deflection angle of light \( \delta \) is the largest, approaching \( 23 \mu \text{rad} \). The deflection angle varies greatly with the maneuvering angle \( \delta \) of the satellite, and decreases as the pitch angle increases. This is because of \( V_x \gg V_y \), so the
deflection angle varies greatly with the change of the pitch angle.

The deflection effect of light will lead to the deviation of satellite pointing angle, which will lead to the error of satellite positioning accuracy.

For the mainstream commercial remote sensing satellites such as WV series and Pleiades satellite, which have great agile imaging capabilities, they mostly imaging at a certain angle. They not only maneuver at the rolling direction, but also at the pitching direction. They have a large range of maneuvering angle, frequent maneuvering ability, and even real-time change of maneuvering angle during the imaging process.

The following figure shows the positioning accuracy error of satellite because of the relative deviation effect under different maneuvering angles.

![Fig.3 Image acquisition location error caused by light deflection effect (m)](image)

The simulation results show that the error of satellite positioning accuracy due to the relative deviation effect varies with the satellite maneuvering angle. With the increase of maneuvering angle, the positioning accuracy error of satellite increases, and the change rate increases. This is because with the increase of the satellite maneuvering angle, the observation oblique distance of the satellite increases, and the curvature effect of the earth becomes larger.

Under typical calculation conditions, the positioning accuracy error caused by the relative deviation effect is about 15 meters. an error within 16 meters can be guaranteed by rolling pitch angle of 10 degrees; an error within 20 meters can be guaranteed by rolling pitch angle of 25 degrees; and an error within 30 meters can be guaranteed by rolling pitch angle of 35 degrees.

4. Correction of light deflection effect

In general, there is no maneuvering or small maneuvering angle in the imaging process of traditional civil remote sensing satellites. In the geometric correction process of images on ground, the deviation of the light deflection angle is usually eliminated in the calibration process of internal and external azimuth elements. So in these situations, this effect had been corrected.

But for the satellite with agile maneuvering imaging capability, the above analysis shows that the positioning accuracy error of the satellite is related to the maneuvering angle of the satellite. In this case, the traditional calibration method is no longer suitable and the correction matrix under different maneuvering angles should be considered to correct the geometric positioning model.

This correction can be done by rotating from $\mathbf{U}$ to $\mathbf{W}$, which can be done by quaternion conversion. The unit vector of the rotation axis can be expressed as:
The rotation angle, also the deflection angle of light, is \( \delta(\psi_x, \psi_y) \). The tectonic correction quaternion is: \( q(\psi_x, \psi_y) = \{q_x, q_y, q_z\} \cdot \sin(\delta/2), \cos(\delta/2) \} \).

Rotation unit vector, rotation angle and rotation conversion quaternion are functions of rolling angle and pitch angle of satellite. Each attitude parameter corresponds to a set of correction quaternions.

The following table gives the maximum, minimum and average values of the rotation unit vector and the rotation angle of the satellite in the range of rolling and pitching angle \((\pm 45^\circ)\) (assuming that the orbit altitude of the satellite is 643 km and the orbit inclination angle is 98°):

Table 1 The maximum, minimum and mean value of the rotation unit vector and the rotation angle (satellite's roll angle and pitch angle limited within ±45°)

|       | Maximum (max) | Minimum value (min) | average value (mean) |
|-------|---------------|---------------------|----------------------|
| \( R_x \) | -0.0312       | -0.0451             | -0.0402              |
| \( R_y \) | 0.9990        | 0.6910              | 0.8891               |
| \( R_z \) | 0.7222        | -0.7222             | 0                    |
| \( \delta \) | 23.0168       | 16.2588             | 20.8760              |

Next, the simulation analysis of the average correction results of optical deflection is given by using the average construction of rotation unit vector and rotation angle to correct quaternion.

The average correction results are as follows:

![Corrected positioning accuracy error curve (m)](image)

Fig.4 Image acquisition location error after a mean correction (m)

It can be seen that after average correction, the deflection angle of light decreases greatly. The deflection angle of light decreases to about 4\( \mu \)rad when viewing sub-satellite; when the rolling angle 0°, the pitch angle around ±35°, the deflection angle of light decreases to less than 1\( \mu \)rad. Similarly, the positioning accuracy error caused by deflection effect is greatly reduced. The error within 3m can be guaranteed in rolling pitch angle of 10°. The error within 6m can be guaranteed in rolling pitch angle of 25°. The error within 12m can be guaranteed in rolling pitch angle of 35°.

Average calibration has obvious positioning accuracy correction effect for small-angle maneuvering imaging situations, but limited effect for large-angle maneuvering imaging situations. For large-angle maneuvering imaging of agile satellite, it is necessary to apply corresponding
correction quaternion for each attitude parameter to correct the optical deflection effect in the optical imaging model.

5. Conclusion
When the optical remote sensing satellite observes the earth, the light deflection effect will cause the error of satellite positioning. In this paper, a geometric model of light deflection is established, and the relationship between light deflection and satellite attitude angle is studied. The simulation results show that under the typical calculation condition, the deflection angle of light will result in the positioning accuracy error of about 15 meters. When the light deflection effect is averaging corrected, the light deflection angle can be greatly reduced and the satellite positioning accuracy is greatly improved.

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