Precise measurements of electromagnetic transitions in light nuclei: what we can learn and why it matters

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Abstract. A new generation of calculations for light nuclei based on realistic two and three-nucleon interactions is being developed. The project is very much a “work in progress” and experimental tests of the predicted wave functions help refine the computational methods and better constrain the modeling of important three-body correlations. However, to be useful, the data need to be both accurate and precise. I will present new measurements of electromagnetic matrix elements of the A = 10 nuclei $^{10}$C, $^{10}$B and $^{10}$Be as an example of the interplay between measurement and calculation, and discuss some successes and open challenges.

More than half a century ago Denys Wilkinson warned us of the danger that the science of nuclear physics was breaking up into subfields. Up to that point, all aspects of subatomic physics were frequently discussed together. But there were so many new emerging fields of endeavour; studying the structure of nucleons, the forces between nucleons, how the nuclear mean field emerged from the forces, and trying to understand the multitude of individual states being found in each isotope, that the researchers were drifting apart, developing their own language, and having their own meetings. Denys’s warning was that these issues were so intimately linked that the trend of increasing fragmentation would not lead to greater wisdom. By and large, I think it is fair to say his warnings were ignored, and the fields drifted very far apart, particularly in the 1970’s and 80’s. It is so refreshing to have a conference now, one hundred years after the discovery of nuclei, where we try to pull these threads back together. It is heartening to see such spectacular advances in all our subfields, and on all energy scales.

This talk is about making progress in understanding light nuclei in a way which address some of Denys’s concerns. The underlying concept is to take the bare forces between nucleons (that we know quite well) and then try to calculate nuclear states in complex nuclei, without any need to resort to introducing any arbitrary potential or renormalization of the effective forces, or introducing a spin-orbit effect, or have any need for effective charges. It is computationally enormously challenging and is only possible because of the progress in massively parallel computing. Currently, the computation is so difficult that even $^{12}$C is a huge challenge. However, even in these light nuclei many questions can be addressed on a more fundamental level: What is the mean field? Where does the spin-orbit come from? What is the influence of the tensor force? What drives alpha-clustering? What is pairing, etc? Hopefully, in the future, with this increased insight, we will develop more physics-based effective interactions that can be used in the traditional Shell Model. I should comment that the shell model is also evolving rapidly and is reaching towards mass A = 100, again because of increased computational power. There are not many parameters in these new ab-initio models, so the prospects of extrapolating to unknown nuclear matter, like very neutron-rich halo nuclei, neutron drops, neutron stars, or astrophysically important nuclear reaction rates at very low energies, are much improved.
However, the first step towards these goals is to be able to calculate all the properties of the light nuclei really well. The devil is in the detail, and there are several similar computational efforts heading in the same general direction. I am only going to talk about our experimental tests of *ab-initio* calculations that were made using the Variational and Greens Function Monte Carlo methods at Argonne [1,2]. This is because this research was originally motivated by discussions with Steven Pieper and Bob Wiringa, so I know their work best and talk to these guys every day. It is not in any way meant to disparage the many other excellent and important complementary efforts using the No Core Shell Models (NCSM) [3,4] or Effective Field Theories (EFT) [5,6] which are all aimed in the same direction, but using slightly different philosophies and tools. The Argonne approach to calculating light nuclei is to construct a properly symmetrized wave function for the particles involved then take the V18 parameterization of the nucleon-nucleon force and use Variational and Greens Function Monte Carlo methods to minimize the binding energy of each nuclear state. It rapidly becomes apparent that two-body-forces are not the whole story and a two-body calculation neither provides enough binding nor does it calculate the right sequence states in key isotopes. Clearly, higher order correlations involving more nucleons or/and the exchange of more mesons must come into play. A whole series of “three body” forces have been investigated during the last three decades, starting from the simplest two-pion exchange model which was proposed in the 1950’s [7]. However, more diagrams must be included, including repulsive terms to discourage the nucleons from collapsing and occupying the same space. With these forces, a really good representation of almost all nuclear states up to 12C emerges. The binding energies are predicted to a level which is as good as any of the empirical mass models (i.e. precise to ~500 keV). In addition, all the sequences of states seem to be well reproduced in spin and parity. In truth, the Illinois and Argonne three-body forces which have evolved are a representation of “the rest of the interaction” and may contain four-body and other correlations. It is a fair criticism that this approach will only ever produce a “effective” 3-body force which is not universal and is intimately only linked to the V18 two-body force. However, it is a pragmatic approach which has taken us far. Eventually, a more rigorous hierarchy of forces will emerge from Effective Field Theories which are more universal and the contribution from higher correlations will be explicitly delineated.

With a real theory, and now with good predictive power, we must turn back to experimental data to appraise how well the model is doing, and to improve our knowledge of 3-body correlations. The data on nuclear masses is excellent, far better that anything that can be calculated, and masses in general are what was used for constraining the few parameters in the three-body forces. However, beyond that, the model is sufficiently good that a new generation of precise experiments have been developed to challenge and improve the Hamiltonians used, the trial wave functions, and the computational methods. I will only mention a few of them as this is becoming a cottage industry. One key area of experiment, which really gave this whole *ab-initio* approach impetus, was measuring RMS radii. At Argonne, Lu and Mueller’s magneto-optical trap (MOT) group measured 6He [8], then 8He [9] at GANIL. Using atomic physics techniques, they could measure the “smearing” of the 4He core in space, arising from recoil effects caused by the valence neutrons. Other groups precisely measured lithium and beryllium isotopes [10,11]. The excellent GFMC prediction of the spatial distribution of nucleons in nuclei has been used to predict direct-reaction transfer cross-sections and absolute spectroscopic factors. At higher energies, 100’s MeV/u, proton knockout reactions at NSCL have probed the momentum of nucleons in particular orbits. Even higher, on the GeV scale at JLAB, electron scattering has probed the full momentum distribution of nucleons in a nucleus, through measuring (e,e’p) form factors. All these new measurements rely on GFMC wave functions to interpret the data. In complete contrast, at extremely low energies, reaction rates can now be better extrapolated to stellar energies far below the Coulomb barrier. The extrapolation from what can be measured terrestrially down to the important true stellar energies now has a much sounder theoretical basis based on reaction modeling with GFMC wave functions.
I am going to discuss electromagnetic decays between states in light nuclei. These transition rates probe the ab-initio calculations in a different way to the “Static” measurements of mass or radial extent. The electromagnetic matrix elements depend on the overlap of initial and final state and have been found to be extremely sensitive to small admixtures in the wave functions. It has been a surprise to all of us how sensitive these transition rates are to 3-body forces, which appear to be drivers of configuration mixing. It may eventually emerge that the most sensitive probe we can have of some aspects of the 3-body and higher correlations come from transition rate studies. If so, it is a good question to ask “which nuclei will be most sensitive to 3-body forces?” as it is these we should focus on. One may suggest 3-body correlations are most important when nucleons are close together, so alpha-clustered nuclei like helium, beryllium, carbon and oxygen are the best nuclei to study. Practically, helium isotopes have no excited bound states, and oxygen cannot be calculated in detail, so we are left with beryllium and carbon. In fact, A = nuclides $^{10}$Be, $^{10}$B, and $^{10}$C form a fascinating triplet which we will probe. A good way to think of them is that they consist of two alpha particles (which in $^{8}$Be are not bound at all) but here, in A = 10, they are bound by two valence nucleons; two neutrons in beryllium (giving it a half-life of 1.51 million years), a proton and neutron in boron (giving total stability) or two protons in carbon (with a half-life of 19.3s). Many questions might be asked; how does this two-nucleon glue work? How far apart are the alpha particles? Where are the valence particles? Are the valence neutrons in $^{10}$Be in the same spatial place as their proton counterparts in $^{10}$C (i.e. is there mirror symmetry of the wave functions)? These are not new questions, and attempts were made to address them in the 1960’s [12]. However, new precise measurements, which I will discuss, and modern calculations, can now allow us to address them with some confidence. Before I start on the actual measurements, it is an interesting exercise to think about these small objects as radiating antennae. A rotating two-alpha particle cluster is a textbook stick antenna with two protons at each end separated by a distance 2r. In fact, it can be easily shown that the radiation rate for simple end-over-end molecular-like tumbling is proportional to $r^4$, so is very sensitive to the separation of alpha clusters This motion is called “Isoscalar” motion as the protons and neutrons all move coherently. It is the major radiation-emitting mode. What about the valence spectators? They could contribute nothing at all. This is easy to think about this in $^{10}$Be as they are neutrons, and so uncharged and not involved in radiation. In fact, the second excited state in $^{10}$Be arises not from tumbling, but from the two neutrons re-coupling from angular momentum zero to two, they start rotating around the waist of the two-alpha core. This motion has a projection of its angular momentum along the axis of the di-alpha cluster, $K = 2$. In heavier nuclei it would be called the “gamma-bandhead”. Perhaps not surprisingly, this motion emits radiation very slowly, nearly 100 times more slowly than tumbling. What about the two valence protons in $^{10}$C? Even if they have exactly the same spatial distribution as the neutrons in $^{10}$Be, the fact that they are charged means they may alter the radiation rates. A difference between $^{10}$Be($T_z = +1$) and $^{10}$C($T_z = -1$) radiation rates reflects the fact that there is an Isovector contribution to the radiation, something that is different for protons and neutrons. Finally, if the $^{10}$B($T_z = 0$) radiation rate is not the average of $^{10}$Be and $^{10}$C, then this would reflect an Isotensor contribution to the radiation which can only happen if the wave functions are not symmetric. All these effects can be calculated, as the Hamiltonian has many charge non-conserving terms. It is expected that Isoscalar contribution dominate, so to determine the Isovector and Isotensor contribution we need a new level of experimental precision, at the few percent level. This was our experimental goal. The state of play in experiment, and theory when we started on this challenge is shown in figure 1. The data are so scattered that they span nearly a factor of two in range, and it is not even clear what the sign of the Isovector term has. No matrix element had been measured for the equivalent transition in $^{10}$B, so we had no knowledge about any possible Isotensor contribution.

There are two approaches to determining the transition rates from the excited states of $^{10}$Be and $^{10}$C: Coulomb Excitation yield or lifetime measurement. We have focused on the latter, though an experiment has recently been done at TRIUMF [13] to try to determine the lifetime and quadrupole moment of the first excited state in $^{10}$Be through Coulomb Excitation. The lifetime is ~200 fs, so the Doppler Shift Attenuation Method (DSAM) [14] is appropriate.
Measurements from the 1970’s on the $B(E2; 2^+ \rightarrow 0^+)$ in $A = 10$ nuclides. The blue points are from the pioneering study of Fisher, Hanna and Paul [12]. Other measurements are compiled in [15]. It is clear that systematic uncertainties are so large that even the dominant Isoscalar contribution is not well determined.

The concept of the method is this. The excited state of interest can be formed in a nuclear reaction in which the parent nucleus is produced with a well determined velocity. When the state decays by gamma emission, the radiation is Doppler Shifted. In the laboratory, the measured energy of the gamma ray depends on the intrinsic energy, on the detection angle and on the velocity. If the state decayed instantly, the velocity inferred from the Doppler shift would match the production velocity (which can be inferred from reaction kinematics). However, if the nucleus is decelerating, then the Doppler Shift is reduced. The reduction depends on a convolution of the slowing history and on the lifetime of the state. If the slowing history is known, then the lifetime can be inferred. Our experimental method is simply to do this measurement with careful attention to detail.

1) We use 2-body reactions, in inverse kinematics and with large $Q$-values to make the nuclei recoil fast, thus increasing the size of Doppler Shifts. In our $^{10}\text{C}$ project the initial recoil velocity was $-12\% \text{c}$.  
2) We detect the recoiling nucleus in a spectrometer, the Argonne Fragment Mass Analyzer (FMA) in a small recoil cone near zero degrees. This helps determine the recoil-gamma opening angle, removes backgrounds from other reactions, and allows us to select reactions that directly populated our state of interest.  
3) We detect the gamma rays at 16 different angles, using Gammasphere. This allows many cross-checks and tests for systematic uncertainties. For example, we can detect a misalignment of the target of $<1 \text{ mm}$.  
4) We use many different slowing materials, and of different thicknesses, to cross check our modeling of slowing.
Figure 2. A key enabling technology relies on “Gammasphere” [16] which allows the measurement of very precise Doppler shifts at many angles simultaneously.

With all these tricks we can improve both precision and accuracy [17]. Figure 3 shows the Doppler Shift measured for $^{10}\text{C}$. The mean velocity of emission can be measured at the 0.3% level, but translating this into a lifetime is more difficult, as it needs data on the slowing profile and target thickness. We believe we can now infer lifetimes at the 3% level due to fitting (statistics) and a similar uncertainty due to systematic uncertainty. The leading uncertainty arises from not knowing exactly how thick the slowing medium is, as this determines how long the decelerating mechanism is working for. At these high velocities, this uncertainty is bigger than that of the actual stopping power. We hope in the future to use nanotechnology to fabricate more uniform slowing media of exactly known thickness.

Figure 3. The Doppler shift extracted for the 3358 keV decay from the only bound state in $^{10}\text{C}$. Nuclei are recoiling at ~12% c. The curvature due to relativistic effects is apparent.

So after all this, what has been learned? Figure 4 should be compared with figure 2. The data are certainly improved to a point we can really test models. For the tumbling mode we measure B(E2:2$\rightarrow$0) of 9.2(3) e$^2$fm$^4$ in $^{10}\text{Be}$ [17] and 8.8(3) e$^2$fm$^4$ in $^{10}\text{C}$ [18]. In $^{10}\text{Be}$ we measure the K = 2 mode has B(E2) = 0.11(2) e$^2$fm$^4$ for the ground state decay. Clearly, as expected, the Isoscalar motion is dominant. The two-cluster nature of the $^{10}\text{Be}$ core makes it far more collective and deformed than
any heavier nucleus, in liquid drop parlance it is beyond “hyperdeformed”. But beyond that we get insight into the Isovector world. Clearly, the Isovector contribution is very small and acts destructively to make the more charged nucleus, $^{10}$C have a less collective tumbling mode than $^{10}$B. Interestingly, this trend was predicted by the traditional shell model of Cohen and Kurath in the 1950’s [19]. Using those old wave functions, evaluated in [20], but substituting modern effective charges, we get almost perfect reproduction of the data. But what do the \textit{ab-initio} calculations add to this, and help us understand the empirical parameters fitted by Cohen and Kurath?

For $^{10}$Be many of the experimental features are very well reproduced. The alpha-clustered nature of the core is distinct, though never “built in” to the initial trial wave functions. With most three-body forces the separation of the alpha cores is also well reproduced, as evidenced by a predicted $B(E2:2^-\rightarrow0^-)$ of 8.8$(2)\ e^2fm^4$ in $^{10}$Be using the IL7 force [17]. What is less well reproduced is the mixing between to two quadrupole modes; tumbling and neutron re-coupling. In experiment we find the ratio of modes is 85, but the calculation always mixes things up to much and has a ratio of about 5. Somehow, nature likes to preserve symmetry and conserve the $K$-quantum number better than the full \textit{ab-initio} calculations. Part of this may be attributed to the prediction of how bound these two configurations are calculated to be; with most three body forces they are too close in binding energy, so perhaps not surprisingly they mix too much. Only a small admixture of the tumbling wave function is needed in the upper state to enhance the decay.

The $^{10}$C result is more challenging theoretically. Our first calculations took isospin symmetric wave functions. One could take the $^{10}$Be spatial distributions then recalculate the matrix elements after substituting two protons for two neutrons. In most calculations the $B(E2)$ for $^{10}$C was calculated to be $\sim30\%$ higher than the equivalent transition in $^{10}$Be. This is shown in figure 4 and can be seen to be sensitive to what three body force is used, but IL7, the “best” current force was best for transition matrix elements, but still far too high. A big hope was that this result implied true isospin symmetry breaking and it told us the protons in $^{10}$C were indeed spatially different from the neutrons in $^{10}$Be. However, we have done a proper minimization for $^{10}$C and this does not seem to fix the problem (the pink dashed line marked “cor” in figure 4). We are still struggling to understand the issues. One possibility goes back to mixing of the $K = 0$ and $K = 2$ modes of motion. In $^{10}$Be we know the calculation failed to reproduce the distribution of strength. Perhaps, in $^{10}$C, all the $B(E2)$ strength was calculated to lie in the tumbling mode, whereas in nature $\sim30\%$ of it really belongs in the $K = 2$ proton

![Figure 4. A synopsis of the new measurements and various GFMC calculations using different Hamiltonians (see text).](image)
re-coupling mode. This suggestion is consistent with cluster model predictions [21]. Unfortunately, for $^{10}$C, the second $J = 2$ state is unbound and it will be very hard to measure its transition rate.

So we have a lot of work before us. Our immediate challenge is to try to measure the equivalent transition in $^{10}$B. This is a very different measurement, in the odd-odd nucleus the $T = 1$, $J = 2$ state is high in excitation and unbound. The equivalent gamma branch is very weak, <0.5% of the decays. A first trial experiment at WNSL Yale [22] indicates that this branch can be measured accurately and is ~0.2%, though we need to reach <10% precision to achieve a satisfactory matrix element. In the longer term, pushing out to interesting neutron-rich nuclei like $^{11}$Be and $^{10}$C, and investigating magnetic radiation are challenging and appealing. This is the beginning of a long journey, but I hope Denys would agree that at last we are now travelling in the right direction.

I would like to thank Bob Wiringa and Steven Pieper for starting us on this endeavor, and staying with us when the outcomes were not as initially expected. They have done many extra calculations in order to gain more insight into the underlying physics. It is amusing that their big calculations take as long and are as expensive as the actual experiments. I would also like to thank Libby McCutchan who took over the experimental side of the project and made sure it was done properly. My co-authors in [17] all contributed enormously to reaching the final results and the experiments could not have been done without them. This work was supported by the DOE Office of Nuclear Physics under Contract No. DE-AC02-06CH11357, DEFG02-94ER40834, and under SciDAC Grant No. DEFC02-07ER41457.

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