Ferromagnetic features of zero-bias conductance peaks in a ferromagnet/insulator/superconductor junction

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Abstract

We present a general formula for tunneling conductance in ballistic ferromagnet/ferromagnetic insulator/superconductor junctions where the superconducting state has the opposite spin pairing symmetry. The formula shows, correctly, that ferromagnetism has been induced by the effective mass difference between up- and down-spin electrons. This effectively mass mismatched ferromagnet and a standard Stoner ferromagnet have been employed in this paper.

As an application of the formulation, we have studied the tunneling effect for junctions including a spin-triplet p-wave superconductor, where we choose a normal insulator for the insulating region, although our formula can be used for a ferromagnetic insulator. Then, we have been able to devote our attention to features of a ferromagnetic metal. The conductance spectra show a clear difference between the two ferromagnets depending upon the method of normalization of the conductance. In particular, an essential difference is seen in the zero-bias conductance peaks, reflecting the characteristics of each ferromagnet. From the obtained results, we suggest that the measurements of the tunneling conductance in the junction provide us with useful information about the mechanism of itinerant ferromagnetism in metals.

(Some figures may appear in colour only in the online journal)

1. Introduction

Andreev reflection (AR), which occurs at the interface of superconducting junctions, is one of the most important elemental processes in transport through superconducting junctions [1]. A theory of transport taking into account the AR was formulated by Blonder, Tinkham and Klapwijk referred to as BTK theory [2]. The BTK theory enables us to probe the pairing state of superconductors. For example, for a junction consists of a normal metal and an unconventional superconductor, the quantum interference effect between the injected and the Andreev reflected particles, which mutually feel the different sign of the superconducting pair potential through the scattering event at the interface of the junction, forms the so-called zero-energy Andreev bound states (ZABS) [3, 4]. Indeed, the ZABS originating from the d-wave symmetry of superconducting pair potential have been observed as zero-bias conductance peaks (ZBCPs) in tunneling experiments of high $T_C$ cuprate superconductors [5–7] according to the theoretical prediction of Tanaka and Kashiwaya (TK) [4]. The ZBCPs reflecting the ZABS are the essential features of the electrical conduction in normal metal/insulator/unconventional superconductor junctions and provide us with information about the pairing symmetry of the superconductor [8–10]. On the other hand, in ferromagnet/insulator/conventional superconductor junctions, the measurements for low energy transport via AR offer the opportunity to probe the magnetic properties such as...
the polarization of ferromagnetic materials [11–13]. The AR in this junction is suppressed by the exchange field in the ferromagnetic layer. As a result, the conductance at low energy of the junction is suppressed responding to the polarization of the ferromagnet. The behavior of ZBCPs in ferromagnet/insulator/unconventional superconductor junctions has been studied to understand the characteristic properties of unconventional superconductors and to utilize the properties as applications of spintronics [14–16]. In these junctions, the ferromagnet has been described within the Stoner model based on a picture of free electrons. However, in some materials, other descriptions of ferromagnetism are required. The ferromagnetism kinetically driven by a spin-dependent bandwidth asymmetry, or, equivalently, by an effective mass splitting between ↑- and ↓-spin particles [17–22] is an interesting model giving itinerant ferromagnetism.

Recently, Annunziata et al analyzed the charge and spin transport in ferromagnet/insulator/superconductor (F/I/S) junctions taking into account the above-mentioned spin-dependent bandwidth asymmetry ferromagnet, making the effective masses have different values in the ferromagnet [23–25]. They clarified that, from the knowledge of the critical transmission angle, measurement of the effective mass difference among the particles was possible [23]. They also showed that the F/I/S junction is an effective probe to investigate the mechanism of ferromagnetism and the pairing symmetry of unconventional superconductors. Furthermore, they suggested that the F/I/S junction could be useful as a switching device using the spin current in the case of symmetry of the superconductor in the conventional s-wave case [24] and possibly as a spin-filtering device [25]. They studied F/I/S junctions with several types of superconducting symmetries such as the conventional s-wave, unconventional d_{x^2−y^2}-wave and the time reversal symmetry broken d_{x^2+y^2}±is or d_{x^2+y^2}±d_y states. Moreover, although the zero-bias anomaly of differential resistance has been shown experimentally in a Sr$_2$RuO$_4$–Pt point contact experiment [26], more recently, Kashiwaya et al [27] have shown ZBCPs, which are direct evidence of the ZABS, by a tunneling spectroscopic experiment of a Sr$_2$RuO$_4$–Au junction. There is much interest and importance in investigating problems of a spin-triplet p-wave symmetry nature because the p-wave pairing, in particular for a chiral charge $\pm \pm p_\parallel$, state breaking time reversal symmetry, is one of the best candidates for the bulk superconducting state of Sr$_2$RuO$_4$ [27–34].

In this paper, a formulation of the tunneling conductance for charge and spin currents in ferromagnet/ferromagnetic–insulator/superconductor (F/I/S) junctions will be presented by taking the effective mass difference leading to spin-band asymmetry between ↑- and ↓-spin particles in a ferromagnet [23–25] into our previous theory [14]. Although the formulation can be used for singlet and $S_z = 0$ triplet superconductors, we will study a chiral p-wave state. Our formula has a general form being able to include the ferromagnetic insulator, however, a normal insulator surrounded by the ferromagnet and the superconductor is considered for the insulating layer to get pure characteristic features of ferromagnetism in a ferromagnet. It is found that the normalized conductance spectra shows a clear difference between Stoner ferromagnets (STFs) and spin-band asymmetry ferromagnets (SBAFs). The present results may be helpful in investigations of the mechanism of ferromagnetism.

This paper is organized as follows. In section 2 we explain a theoretical model and derive a formulation following our previous method based on the BTK theory. The results for ferromagnet/insulator/chiral p-wave superconductor junctions are presented in section 3. Finally the results are summarized in section 4.

2. Model and formulation

For the model of formulation, we consider a two-dimensional ballistic F/I/S junction with semi-infinite electrodes as shown in figure 1. A flat interface is assumed to be located at $x = 0$, and the ferromagnetic insulator for up (down) spin is described by a potential $V_{f(\downarrow)}(x) = (V_0 + (-V_{\text{ex}}))\delta(x)$, where $\delta(x)$, $V_0$, and $V_{\text{ex}}$ are the $\delta$ function, a non-magnetic barrier amplitude and a magnetic barrier amplitude, respectively. For the ferromagnetism in the F electrode, we adopt two kinds of models of mechanisms, as shown in figure 2. One of these is the standard Stoner model in which the ferromagnetism is induced by the exchange potential, leading to the rigid energy shift between ↑-spin and ↓-spin bands. The other is a spin bandwidth asymmetry model proposed by Hirsch [18], in which the bandwidth is tuned relatively by the ratio of the effective masses between ↑- and ↓-spin particles. Although in the following free particle-like spectra of parabolic type are assumed as normal electronic dispersion relations, we need to define the concept of bandwidth for the above description. It implies that there can be some relations

![Figure 1. Schematic illustration of the scattering processes of an injected electron with ↑-spin at the F/F/I/S ballistic junction. Here, $\theta_1$, $\theta_{\text{AR}}$ and $\theta_0$ are injection, Andreev reflection and transmission angles, respectively. It is assumed that the normal reflection at the interface is totally specular, then the normal reflection angle is also given by $\theta_1$. For the case of a ↓-spin electron, it can be depicted by flipping ↑ to give ↓ in the figure.](image-url)
between this description and some effective one-band tight binding model permitting the effective masses of carriers to be proportional to the inverse of the width of the bands where the carriers get itinerancy. Hence, only giving different values of the masses for $\uparrow$- and $\downarrow$-spin electrons yields a bandwidth asymmetry.

The spatial dependence of the pair potential is taken as $\Delta(\mathbf{r}) = \Delta \Theta(x)$ for simplicity. In addition, we consider the $S_z = 0$ pairing states, where the elements of pair potential are given by $\Delta_{\uparrow \downarrow} = \Delta_{\downarrow \uparrow} = 0$ and $\Delta_{\uparrow \uparrow} = -\Delta_{\downarrow \downarrow}$ for the singlet pairing state or $\Delta_{\uparrow \uparrow} = \Delta_{\downarrow \downarrow}$ for the triplet pairing state. Thus, the effective Hamiltonian (Bogoliubov–de Gennes (BdG) equation) of the system can be reduced to the decoupled equation for the eigenstates $(u_{FS}(\uparrow \downarrow)(\mathbf{r}), v_{FS}(\downarrow \uparrow)(\mathbf{r}))^T$ and is given by

$$
\begin{pmatrix}
H_0^\sigma (\mathbf{r}) & \Delta(\mathbf{r}) \\
\Delta^*(\mathbf{r}) & -H_0^\sigma (\mathbf{r})
\end{pmatrix}
\begin{pmatrix}
u_{FS}(\mathbf{r}) \\
v_{FS}(\mathbf{r})
\end{pmatrix}
= E
\begin{pmatrix}
u_{FS}(\mathbf{r}) \\
v_{FS}(\mathbf{r})
\end{pmatrix} .
$$

Here $E$ is the energy of the quasiparticle and $H_0^\sigma (\mathbf{r})$ is the single particle Hamiltonian for $\sigma$-spin where $\bar{\sigma} = -\sigma$. On the ferromagnet side, the single particle Hamiltonian is given by $H_0^\sigma (\mathbf{r}) = -\hbar^2 \nabla^2/2m_\sigma - \rho U_{ex} - E_{FM}$ where $\sigma = \uparrow, \downarrow$, $m_\sigma$ is the effective mass for the $\sigma$-band particle, $\rho = +1(-1)$ for $\uparrow\downarrow$-spin. $U_{ex}$ is the exchange potential and $E_{FM}$ is the Fermi energy. The $H_0^\sigma$ on the superconductor side is given by $H_0^\sigma (\mathbf{r}) = H_0^\sigma (\mathbf{r}) = -\hbar^2 \nabla^2/2m_{FS} - E_{FS}$ where the $m_{FS}$ and $E_{FS}$ are the effective mass of the quasiparticle and the Fermi energy, respectively.

To describe the Fermi surface difference, we assume $E_{FS} = E_{F} = E_F$. In the following, we apply the quasi-classical approximation where $E_F \gg (E, \Delta(k))$ and the $k$-dependence of $\Delta(k)$ is replaced by the angle $\theta_s$ between the direction of the trajectory of quasiparticles in the superconductor and the interface normal. In the quasi-classical approximation, the wavevectors of $k_{\uparrow \downarrow}^{(1)}$ and $k_{ELQ(HLQ)}$ are given by $k_{\uparrow \downarrow}^{(1)} \equiv \sqrt{(2m_\downarrow U_{ex})/E_F}$ and $k_{ELQ(HLQ)} = k_s \equiv \sqrt{2m_s E_F/\hbar^2}$, respectively, where ELQ(HLQ) indicates electronlike(holelike) quasiparticles. For example, we assume the injection of $\uparrow\downarrow$-spin electrons from the ferromagnet at an angle $\theta_{\parallel}$ to the interface normal as shown in Figure 1. There are four possible scattering trajectories: AR with angle $\theta_{AR}$ as holes belonging to the $\downarrow\downarrow$-spin band, normal reflection (NR), transmission to superconductor as ELQ, and transmission as HLQ. These four processes are described in the same way for $\downarrow\down\down\down\down$-spin electrons by changing the scattering angle $\theta_{\parallel}$ to $\theta_{\uparrow\downarrow}$. Since the translational symmetry holds along the $y$-axis, the parallel momentum components of all trajectories are conserved $k_{\uparrow\uparrow} = k_{\downarrow\down\down\down\down}$, and $\theta_{\parallel}$ and $\theta_{\uparrow\down\uparrow}$ differ from each other except when $U_{ex} = 0$ and $m_\uparrow = m_\down\down\down\down\down$. Thus, the BdG equations are reduced to the effective one-dimensional equation due to the translational invariance along the $y$-axis of the Hamiltonian. Thus, the solutions of the BdG equations for $\sigma$-spin electron injections are described as

$$
\begin{pmatrix}
u_{FS} (x < 0) \\
v_{FS'} (x < 0)
\end{pmatrix} = \begin{pmatrix} 1 & 0 \\
0 & 1 \end{pmatrix}
\begin{pmatrix} \bar{e}^{ik_{\uparrow\uparrow}\cos \theta_\parallel} & a_\sigma \\
0 & b_\sigma
\end{pmatrix}
\begin{pmatrix} \bar{e}^{-ik_{\uparrow\uparrow}\cos \theta_\parallel} & 0 \\
a_\sigma & b_\sigma
\end{pmatrix}
\begin{pmatrix}
u_{FS} (x > 0) \\
v_{FS'} (x > 0)
\end{pmatrix} ,
$$

with

$$
u_{\pm} = \sqrt{\frac{1}{2} \left(1 + \frac{\Omega_{\pm}}{E}\right)},
\Omega_{\pm} = \sqrt{E^2 - \Delta_{\pm}^2},
\bar{e}^{i4\phi} = \frac{\Delta_{\pm}}{\Omega_{\pm}}
$$

where $\Delta_+ = \Delta(\theta_{\parallel})$, $\Delta_- = \Delta(\pi - \theta_{\parallel})$ and the probability coefficients $a_\sigma$, $b_\sigma$, $c_\sigma$ and $d_\sigma$ are for AR, NR, transmission ELQ and HLQ. These coefficients are calculated from the boundary conditions at $x = 0$

$$u(\nu)_{FS(\sigma)}(x = 0) = u(\nu)_{FS(\sigma)}(x = 0),
$$

(2.6)

$$\frac{\hbar^2}{2m_\sigma} \frac{du_{FS(\sigma)}}{dx}_{x = 0} = \frac{\hbar^2}{2m_\sigma} \frac{dv_{FS(\sigma)}}{dx}_{x = 0} = V_{\sigma} u_{FS(\sigma)}(x = 0)
$$

(2.7)

$$\frac{\hbar^2}{2m_\sigma} \frac{dv_{FS(\sigma)}}{dx}_{x = 0} = \frac{\hbar^2}{2m_\sigma} \frac{dv_{FS(\sigma)}}{dx}_{x = 0} = V_{\sigma} v_{FS(\sigma)}(x = 0).
$$

(2.8)

As explained in our previous paper, the reflection process depends upon the size relation of the Fermi surfaces between FM and SC. In the following, we will consider a situation

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**Figure 2.** A sketch of the dispersion relation between energy and wavenumber, and the Fermi surface for an STF (left side) and for an SBAF (right side). A free electron model with rigid energy shift in an STF and different effective masses for each spin in an SBAF is assumed.
where \( k_\perp < k_\parallel < k_1 \) and \( m_1/m_S = m_S/m_\parallel \) with \( m_\parallel > m_S > m_1 \). Following the BTK theory and taking care of the probability conservation of quasiparticle flow
\[
|b_{\ell}|^2 + \frac{v_{f,\sigma}}{v_{f,\sigma}} |a_{\ell}|^2 + \frac{v_{S,\sigma}}{v_{f,\sigma}} |c_{\sigma}|^2 + \frac{v_{b,\sigma}}{v_{f,\sigma}} |d_{\sigma}|^2 = 1
\]
the conductance \( G_{S,\sigma}^{(\text{CS})} \) for the \( \sigma \)-spin charge (spin) current through the system can be calculated by
\[
G_{S,\sigma}^{(\text{CS})} = 1 + \left( -\frac{v_{f,\sigma}}{v_{f,\sigma}} |a_{\ell}|^2 - |b_{\ell}|^2 \right)
\]
where \( v_{f,\sigma} = h k_\sigma/m_\sigma \) is the group velocity of the \( \sigma \)-spin particles in the ferromagnet and \( v_{S,\sigma} = h k_S/m_S \) is that of the ELQ (HLQ) in the superconductor. It is worth noting that our conductance formula \( G_{S,\sigma}^{(\text{CS})} \) is different from that used in former works [23–25]. In our formulation the conductance is an extension of the previous formulation [8, 14] to the present situation; this is needed for the correct treatment of the mass mismatch in the same metal so that the coefficient of AR \( a_\sigma \) should be given by the ratio of group velocities rather than by that of wavenumbers as a consequence of the conservation law of particle flow. Using the obtained AR \( a_\sigma \) and NR \( b_\sigma \) coefficients in the same way as in the previous paper [14] based on the TK formula [4], the charge (spin) conductance for each spin \( \uparrow \) and \( \downarrow \) can be formulated by
\[
G_{S,\uparrow}^{(\text{CS})} = G_{N,\uparrow} \times \frac{1 - |\Gamma_{\uparrow,\uparrow}|^2 (1 - G_{N,\uparrow}) + (-)G_{N,\downarrow} |\Gamma_{\uparrow,\downarrow}|^2}{[1 - \Gamma_{\uparrow,\downarrow} - \Gamma_{\uparrow,\uparrow} - \Gamma_{\downarrow,\uparrow}] \exp[i(\psi_{\uparrow} - \psi_{\downarrow})] + \Theta(\theta_{\bar{\sigma}} - \theta_C) + [1 - \Theta(\theta_{\bar{\sigma}} - \theta_C)] G_{N,\uparrow}}
\times \frac{1 - |\Gamma_{\downarrow,\uparrow}|^2}{[1 - \Gamma_{\uparrow,\downarrow} - \Gamma_{\uparrow,\uparrow} - \Gamma_{\downarrow,\uparrow}] \exp[i(\psi_{\uparrow} - \psi_{\downarrow})] + \Theta(\theta_{\bar{\sigma}} - \theta_C)] G_{N,\uparrow}} (2.11)
\]
\[
G_{S,\downarrow}^{(\text{CS})} = G_{N,\downarrow} \times \frac{1 - |\Gamma_{\downarrow,\downarrow}|^2 (1 - G_{N,\downarrow}) + G_{N,\uparrow} |\Gamma_{\downarrow,\uparrow}|^2}{[1 - \Gamma_{\uparrow,\downarrow} - \Gamma_{\uparrow,\uparrow} - \Gamma_{\downarrow,\uparrow}] \exp[i(\psi_{\uparrow} - \psi_{\downarrow})] + \Theta(\theta_{\bar{\sigma}} - \theta_C)] G_{N,\uparrow}} (2.12)
\]

with
\[
G_{N,\uparrow(\downarrow)} = \frac{4 \lambda_{\uparrow(\downarrow)}}{(1 + \lambda_{\uparrow(\downarrow)})^2 + Z_{\uparrow(\downarrow)}},
\]
\[
\exp[i(\psi_{\uparrow(\downarrow)})] = \frac{1 + \lambda_{\uparrow(\downarrow)} + i Z_{\uparrow(\downarrow)}}{1 + \lambda_{\uparrow(\downarrow)} + i Z_{\uparrow(\downarrow)}}, \quad Z_{\uparrow(\downarrow)} = \frac{Z_{\uparrow(\downarrow)}}{\cos \theta_{S}},
\]
\[
\lambda_{\uparrow(\downarrow)} = \sqrt{\gamma^{-1}(-1) + \gamma^{-1/2(1/2)}(1 - \gamma^{-1/2(1/2)} + (-)\chi)},
\]
\[
\theta_C = \cos^{-1}\sqrt{-\gamma^{-1/2}(\chi - 1 + \gamma^{1/2})} \quad \text{or} \quad \sin^{-1}\sqrt{-\gamma^{-1/2}(\chi - 1 + \gamma^{1/2})}
\]
\( \chi = U_{c\alpha}/E_F \) (0 < \( \chi \) < 1) and \( \gamma = m_1/m_\parallel \geq 1 \). In the above, \( G_{N,\sigma} \) corresponds to the conductance when the superconductor is in the normal state.

**Figure 3.** The magnetization \( M \) as a function of \( \gamma \) in the pure SBAF, and the inserted panel is \( M \) as a function of \( \chi \) in the pure STF.

We calculate the normalized conductance defined by
\[
G_{T}^{(\text{CS})} = \frac{\int_{\frac{\pi}{2}}^{\pi} d\theta_0 \cos \theta_0 (P_{\uparrow} G_{S,\uparrow}^{(\text{CS})} + P_{\downarrow} G_{S,\downarrow}^{(\text{CS})})}{\int_{\frac{\pi}{2}}^{\pi} d\theta_0 \cos \theta_0 (P_{\uparrow} G_{S,\uparrow} + P_{\downarrow} G_{S,\downarrow})}
\]
where the polarization \( P_\sigma \) for \( \sigma \)-spin is expressed as
\[
P_{\uparrow} = \frac{\gamma(1 + \chi)}{\gamma(1 + \chi) + 1 - \chi},
\]
\[
P_{\downarrow} = \frac{1 - \chi}{\gamma(1 + \chi) + 1 - \chi}.
\]

It is noted, in general, that the normalized conductance is defined alternatively depending on the actual experiments.

The above formulas (2.11), (2.12) and (2.14) can reproduce former formulas of tunneling conductance for junctions including a triplet superconductor (TS). For \( m_\parallel = m_\perp \), these equations coincide with that of the STF/I/TS junction [14], and for \( m_\parallel = m_\perp \) and \( U_{c\alpha} = 0 \), the conductance formula for the N/I/TS junction [8] is reproduced.

**3. Results**

First, we notice the growth of the magnetization \( M \) for STFs or SBAFs. Using the polarization \( P_\sigma \), \( M \) is given by \( M = P_{\uparrow} - P_{\downarrow} \). For the pure STF case (\( \gamma = 1 \)), the magnetization is equal to the magnitude of exchange splitting \( M = \chi (\equiv U_{c\alpha}/E_F) \). For the pure SBAF case (\( \chi = 0 \)), \( M \) is given by \( M = (\gamma - 1)/(\gamma + 1) \). Thus, the half-metal state in the SBAF case is an unphysical situation because \( \gamma = \infty \). Figure 3 shows the \( M \) in the SBAF case as a function of \( \gamma \). It can be seen that the growth rate of \( M \) becomes very gradual over \( \gamma \approx 50 \). From this one can expect clear differences in the transport properties depending on \( M \) between STFs and SBAFs near the half-metallic limit. Hereafter, we refer to the region under and near the half-metallic limit as the ‘strong ferromagnetic regime’.

In the following sections, we apply our conductance formula to a ferromagnet/insulator/triplet superconductor.
M is broken due to the induced \( \text{figure 4(a)} \). This indicates that the retro-reflectivity of the magnetization \( M \) is reduced due to the induced \( \text{figure 4(a)} \). This indicates that the retro-reflectivity of the magnetization \( M \) is increased for both STFs and SBAFs.

\[
\chi = \frac{U_{ex}}{E_{F}} \quad \text{and} \quad \gamma = m_2/m_1.
\]

For the value \( M = 0.25 \), there are two cases, one is the pure STF, \( \chi = 0.25 \) and \( \gamma = 1 \), and the other is the pure SBAF, \( \chi = 0 \) and \( \gamma = 5/3 \). All other values of \( M \) are given in the same way except for the case of \( M = 0 \).

| Magnetization \( M \) | Exchange interaction \( \chi \) | Mass mismatch \( \gamma \) |
|-----------------------|-----------------------|-----------------------|
| 0                     | 0                     | 1                     |
| 0.25                  | 0.25                  | 1                     |
| 0.25                  | 0                     | 5/3                   |
| 0.5                   | 0.5                   | 1                     |
| 0.5                   | 0                     | 3                     |
| 0.75                  | 0.75                  | 1                     |
| 0.75                  | 0                     | 7                     |
| 0.99                  | 0.99                  | 1                     |
| 0.99                  | 0                     | 200                   |

(F/I/T/S) junction (F referring to STF or SBAF), where \( V_{ex} = 0 \). As the pairing potential, a triplet p-wave state is employed by choosing \( \Delta_{\uparrow\uparrow}(\theta_S) = \Delta_{\downarrow\downarrow}(\theta_S) = \Delta_{0} \exp(\phi_S), \Delta_{\uparrow\downarrow}(\theta_S) = \Delta_{\downarrow\uparrow}(\theta_S) = 0 \) for opposite spin pairing. In addition, we choose some sets of parameters \( (\chi, \gamma) \) giving the same \( M = \{0, 0.25, 0.5, 0.75, 0.99\} \) shown in table 1 so as to get clear characteristics of each ferromagnet.

### 3.1. Distinction between STF and SBAF

To investigate the consequence of the different mechanisms of magnetization, avoiding any effects of the normal barrier, we consider the highly transparent junction in the metallic limit \( (Z_0 = 0) \). In this case, the normalized total conductances \( G^T_C(eV) \) show the same trend that conductance values inside the energy gap \( eV < \Delta_0 \) are reduced when the value of magnetization \( M \) is increased for both STFs and SBAFs (figure 4(a)). This indicates that the retro-reflectivity of the AR is broken due to the induced \( M \). However, the \( M \) dependence of reduction for \( G^T_C(eV < \Delta_0) \) is different for each of them. The difference can be seen more clearly in the \( M \)-dependence of conductance values at \( eV = 0 \), \( G^T_C(0) \), in figure 4(b). It is found that the suppression of \( G^T_C(0) \) for the SBAF case is weaker than that for the STF case without the weak magnetization regime, \( 0.0 \leq M \leq 0.2 \) and at the half-metallic limit, \( M = 1.0 \). To clarify the reason for the different \( M \) dependence of conductances for the SBAF case and STF case, we show the critical angle of AR as a function of \( M \) in figure 5. The \( \theta_C \) for both SBAF and STF cases decreases with increasing \( M \). It is found that the difference between angles is getting larger from \( M \sim 0.2 \) to \( \sim 0.9 \), and converges to zero at \( M = 1.0 \). For the nearly half-metallic limit \( M = 0.99 \), the \( \theta_C \) is almost suppressed in the STF case, while it still remains in the SBAF case. The critical angles for STF and SBAF are \( \theta_C = \cos^{-1}\sqrt{M(=\sin^{-1}\sqrt{1-M})} \) and \( \theta_C = \cos^{-1}\sqrt{1-(\frac{M}{1+M})^{1/2}}(=\sin^{-1}\sqrt{1-(\frac{M}{1+M})^{1/2}}) \), respectively. Then, it is clear that \( \theta_C \) in the SBAF case is larger.
than that in the STF case for the same $M$ except for the non-magnetic state, $M = 0$ and the half-metal state, $M = 1$. Consequently, as shown in figure 4, the $G^C_T(eV < \Delta)$ in the SBAF case is larger than that in the STF case.

### 3.2. Ferromagnetic features of ZBCPs

It has been shown theoretically that the ZBCPs in an F/I/S junction would be useful for measuring the magnetization of a ferromagnet [14, 15]. Here, we study the validity of the ZBCPs for the distinction of ferromagnets. Figure 6 shows the conductance $G^C_T(eV)$ for the junction in the tunneling limit $Z = 5$. The ZBCPs seen in both STF and SBAF cases are attributed to the anisotropy of the pair potential of a p-wave superconductor. For the STF case, the previous results [14] have been reproduced (figure 6(a)). In contrast, there are some differences for the SBAF case. In particular, it is found that the conductance near $eV = 0$ increases slightly with increasing $M$ (figure 6(b)). This opposite behavior can be seen more clearly in the $M$ dependence of ZBCPs figure 6(c). With increasing $M$, in contrast to the monotonically decreasing behavior of the STF case, the ZBCP in the SBAF case increases up to a certain value of $M$ in the strong ferromagnetic regime and then suddenly decreases toward the half-metallic limit where the ZBCPs in both cases are suppressed perfectly. The cause of this opposite behavior could be reduced to the definition of normalization since the magnitude of the ZBCP has a constant value in the non-normalized case and depends on the conductance when the superconductor is in the normal state. There is another definition of normalization using the AR critical angle measured on the ferromagnet side [14, 15]. However, in that case, the AR critical angle itself depends on and is controlled by the magnitude of $M$, as a result, even the normalization depends on $M$. Accordingly, in order to avoid the influence of $M$, we calculate an angle averaged conductance defined as

$$Q_S = Q_S,\uparrow + Q_S,\downarrow, \quad Q_{S,\sigma} = \frac{\int_{\pi/2}^{\pi/2} d\theta_S \cos \theta_S P_{\sigma} G^C_{S,\sigma}}{\int_{\pi/2}^{\pi/2} d\theta_S \cos \theta_S}.$$

We show the calculated results of the angle averaged conductance $Q_S$ in figure 7 which, in both the STF and SBAF cases, shows the same tendency to decrease with increasing $M$ (figure 7(a)). Similarly, the ZBCP is a decreasing function of $M$ (figure 7(b)). It is also shown that the reduction ratio differs in each case, just as in the metallic limit. Thus, the opposite behavior seen in normalized conductance would reduce to the conductance in the normal state. Therefore, it is noticed that the conductance of the junction for the superconductor in the normal state plays an important role when considering two different ferromagnetisms.

In order to clarify the difference between the STF case and the SBAF case, we calculate the conductance in a ferromagnet/normal metal (F/I/N) junction for both metallic and tunneling limits. The angle averaged conductance in the F/I/N junction $Q_N = \sum_\sigma Q_{N,\sigma}$ is defined in a similar way to that in an F/I/S junction, replacing $G^C_{S,\sigma}$ by $G_{N,\sigma}$. The calculated results of $Q_{N,\sigma}$ for both $Z_0 = 0$ and 5 are shown in figure 8. The angle resolved conductance $G^C_{N,\sigma}$ for $\sigma$-spin is rewritten as $G^C_{N,\sigma} = 4 \cos \theta_S \tilde{\lambda}_{\sigma}/((\cos \theta_S + \tilde{\lambda}_{\sigma})^2 + Z^2_{0,\sigma})$, where $\tilde{\lambda}_{\sigma} = \sqrt{\cos^2 \theta_S + \rho \chi}$ in STF/I/N and $\tilde{\lambda}_{\sigma} = \gamma^{-1/2} \sqrt{\cos^2 \theta_S + (\gamma^2/2 - 1)}$ in SBAF/I/N junctions. Here,
Figure 7. Angle averaged conductance spectra $Q_5$ as a function of magnetization (a) and ZBCPs versus magnetization strength (b) in the tunneling limit $Z = 5$. Here, the conductance for the SBAF case is indicated as a solid line and for the STF case as a dotted line.

Figure 8. Magnetization dependence of angle averaged normal conductance $Q_N$, $Q_{N,\uparrow}$ and $Q_{N,\downarrow}$ for the SBAF case (solid line) and the STF case (dotted line) in the tunneling limit for the case of the superconductor being in the normal state (a) for $Z = 0$, (b) for $Z = 5$. We mention the properties of the $M$-dependence of $G_{N,\sigma}^C$ through $\chi$ or $\gamma$ in advance of descriptions about $Q_N$. In an STF/I/N junction, the $G_{N,\uparrow}^C$ increases following growth of the magnetization, i.e. with increasing $\chi$, since the gain in Fermi energy due to the band shift is larger than the Fermi surface effect [14] acting as an effective barrier between the STF and normal metal, under conservation of momentum along the $y$-direction. On the other hand, because there is no Fermi energy gain from the spread of the bandwidth due to the effective mass mismatch in the SBAF, the influence of the effective barrier arising from the Fermi surface effect becomes stronger with the increase of $\gamma$ and the $G_{N,\uparrow}^C$ in the SBAF/I/N junction decreases with increasing $\gamma$ and becomes zero in the limit of $\gamma \to \infty$. $G_{N,\downarrow}^C$ for both STF and SBAF cases decreases with increasing the magnetization caused by $\chi$ or $\gamma$.

In the metallic limit $Z_0 = 0$ (figure 8(a)), it is found that the $Q_{N,\uparrow}$ in the STF/I/N junction increases with increasing $M$ in contrast to $Q_{N,\downarrow}$ decreasing toward zero in the half-metal state. In this case, $M$ is given directly as $M = \chi$. Thus, the total conductance $Q_N = Q_{N,\uparrow} + Q_{N,\downarrow}$ is reduced slightly by the Fermi surface effect with increasing $M$ up to $\sim 0.7$. In the SBAF/I/N junction, we can see similar behavior in $Q_{N,\uparrow(\downarrow)}$. The increase of $Q_{N,\uparrow}$ is due to $P_\uparrow$ which is an increasing function of $\gamma$. However, near the half-metallic limit, $Q_{N,\uparrow}$ reduces rapidly reflecting the behavior of $G_{N,\uparrow}$ which is a decreasing function of $M$ toward zero at $M = 1(\gamma = \infty)$, as mentioned above. Thus, as shown in figure 4, the $G_{ST,\uparrow}^S(eV)$ in the SBAF/I/S junction decreases slowly with increasing $M$ compared to that in the STF/I/S junction. The difference between STFs and SBAFs becomes clearer in the tunneling limit $Z = 5$ (figure 8(b)). With increasing $M$, $Q_{N,\sigma}$ in the STF case varies rapidly compared with that in the SBAF case. This is a difference in the barrier effect felt by particles with $\sigma$-spin in each case. The barrier potential simply becomes lower for particles with $\uparrow$-spin and higher for particles with $\downarrow$-spin in the STF case due to the rigid Fermi energy shift. However, the particles in SBAFs directly feel the barrier potential because there is no shift of the Fermi energy. Thus, in SBAF/I/N junctions, the increase of the magnitude of $Q_{N,\uparrow}$ due to $P_\uparrow$ is suppressed by the Fermi surface effect and the barrier potential. Then, $Q_{N,\uparrow}$ gets lower with increasing $M$ in contrast to the STF case. Therefore, the $Q_N$ in the SBAF/I/N junction shows the opposite behavior to that in the STF/I/N junction. As a result, the normalized conductance $G_{ST,\uparrow}^S(eV)$ in the SBAF/I/S junction decreases due
to the reduction of the $Q_N$ depending on $M$ (figures 6(b) and (c)). Indeed, as shown in figure 7, the angle averaged conductance $Q_\theta(eV)$ for both STF and SBAF cases show the same trend on varying $M$. Thus, it can be concluded that the measurement of $Q_N$ will also be useful to identify STFs and SBAFs. However, we emphasize that the measurement of ZBCPs originating from ZABS is a more powerful probe to investigate a ferromagnet than that of $Q_N$. Because two $Q_N$s seemingly show drastically different behavior depending on $M$ for large enough $Z_0$ (figure 8(b)), by carefully looking at the figure, differences of each value of $Q_N$ are not so large for the same $M$ except in the strong ferromagnetic regime. Therefore, it seems that an experimental distinction will become more difficult in measurements of $Q_N$. The ZBCP becomes clearer for larger $Z_0$, which can be expected to play a role in utilizing the differences between STFs and SBAFs.

4. Summary

In summary, we have derived a formula for the tunneling conductance in a ferromagnet/ferromagnetic insulator/superconductor with an antiparallel spin pairing junction by extending our previous theory for a standard Stoner ferromagnet (STF) so as to include a spin-band asymmetry ferromagnet (SBAF) originating from effective mass mismatch between particles with opposite spins. Applying the formulation for ferromagnet/insulator/p-wave superconductor junctions, differences between pure STFs and pure SBAFs have been extensively investigated. We found that, with increasing magnetization, the difference in tunneling conductance becomes clear. The clarity in the difference between STFs and SBAFs depends on the way in which the conductance is normalized and shows up more clearly in the ZBCP near the half-metallic limit. The obtained results suggest that the measurement of ZBCPs may be useful for discriminating between the mechanisms of ferromagnetism.

Although our formulation includes a ferromagnetic insulator, we have studied only the normal insulating barrier case in this paper. The spin-filtering effect is expected in ferromagnetic insulators [14] and in ferromagnets given by the effective mass mismatch [25]. As an interesting future project we will study extensively the spin-filtering effect in junctions including both ferromagnetic insulators and mass mismatched ferromagnets connected to superconductors of s-, d-wave and broken time reversal symmetry pairing states. Moreover, it will be important that the proximity effect is taken into account in the present formulation by carrying out a self-consistent calculation of the pairing potential in order to analyze the actual experiments. Indeed, ZBCPs have been observed in tunneling experiments of F/I/d-wave superconductor junctions [35]. Also, a ZBCP in a Sr$_2$RuO$_4$ junction has been observed [27], so it seems likely that tunneling spectroscopy of an F/I/Sr$_2$RuO$_4$ junction will be realized in the near future. Our conductance formula can easily be applied to such situations to give results comparable to the experimental ones.

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