Local renormalizable gauge theories from nonlocal operators

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Abstract

The possibility that nonlocal operators might be added to the Yang-Mills action is investigated. We point out that there exists a class of nonlocal operators which lead to renormalizable gauge theories. These operators turn out to be localizable by means of the introduction of auxiliary fields. The renormalizability is thus ensured by the symmetry content exhibited by the resulting local theory. The example of the nonlocal operator $\text{Tr} \int A_\mu \overleftrightarrow{D^2} A_\mu$ is analysed in detail. A few remarks on the possible role that these operators might have for confining theories are outlined.

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1 Introduction

The understanding of the behavior of Yang-Mills theories in the nonperturbative infrared region is a great challenge in quantum field theory. Different approaches are currently employed to address this issue, namely: lattice gauge theories, study of the Schwinger-Dyson equations, duality mechanisms, restriction of the domain of integration in the Feynman path integral in order to take into account the existence of the Gribov copies, variational principles, condensates, exact renormalization group, Several results have been achieved so far, having received confirmation from the various approaches. This is the case, for example, of the infrared suppression of the two point gluon correlation function and of the infrared enhancement of the ghost propagator in the Landau gauge.

Nevertheless, a satisfactory description of the gluon and quark confinement is not yet at our disposal. One still has the feeling that much work is needed.

The aim of this paper is to call attention to the fact that there exist nonlocal operators which can be consistently added to the Yang-Mills action. This means that, for those specific operators, a renormalizable computational framework can be worked out. As is well known, adding a nonlocal term to the Yang-Mills action is a delicate operation. In most cases the requirement of renormalizability cannot be accomplished. However, in a few cases, the nonlocal term can be cast in local form through the introduction of additional localizing fields. Furthermore, the resulting local theory might exhibit a rich content of symmetries, enabling us to establish its multiplicative renormalizability to all orders. It is worth underlining that, being nonlocal, these operators can induce deep modifications on the large distance behavior of the theory. As such, they might be useful in order to investigate nonperturbative features, being of particular interest for confining theories.

As an explicit example of such nonlocal terms, we shall present a detailed analysis of the nonlocal operator

\[ \mathcal{O} = \frac{1}{2} \int d^4x \, A_\mu^a (\frac{1}{D^2})^{ab} A_\mu^b, \]  

(1)

where \( D^2 \) stands for the covariant Laplacian

\[ (D^2)^{ab} = D_{\mu}^{ac} D_{\mu}^{cb}, \]

\[ D_{\mu}^{ab} = \delta^{ab} \partial_{\mu} - g f^{abc} A_{\mu}^c. \]  

(2)

Through this example we shall be able to provide a general overview of what can be called a consistent framework for a nonlocal operator which can be added to the Yang-Mills action, namely:

- achievement of a localization procedure,
- investigation of the symmetry content of the resulting local action,
- proof of the multiplicative renormalizability of the theory.

In order to have an idea of the relevance of such nonlocal terms for the infrared behavior of Yang-Mills theories, let us spend a few words on two examples which have been analysed recently, and which fulfil the requirements of localizability and renormalizability. The first example is provided by the Zwanziger horizon term which implements the restriction of the domain of integration in the Feynman path integral
to the Gribov region $\Omega$ in the Landau gauge\(^2\)\(^\text{[12, 13]}\), namely
\[
S_H = -g^2 \gamma^4 \int d^4x \ f^{abc} A^b_\mu \left( \frac{1}{(D_\nu D_\nu)} \right)^{ad} f^{dec} A^e_\mu , \tag{3}
\]
where the parameter $\gamma$, known as the Gribov parameter, has the dimension of a mass. The second example is given by the gauge invariant nonlocal operator
\[
S_m = \frac{m^2}{2} \int d^4x \ F^a_{\mu\nu} \left( \frac{1}{D^2} \right)^{ab} F^{b}_{\mu\nu} , \tag{4}
\]
which, when added to the Yang-Mills action, yields an effective gauge invariant mass $m$ for the gluons\(^3\)[37, 38, 39], a topic which is receiving increasing attention in recent years. As shown in\(^\text{[12, 13, 40, 41, 38, 39]}\), both operators (3),(4) are localizable, the resulting local theories enjoy the property of being renormalizable. In particular, in\(^\text{[12, 43, 39]}\) one finds the two loop calculation of the anomalous dimensions corresponding to expressions (3),(4).

Concerning now the operator $O$, eq.(1), a few potential interesting features might be pointed out in order to motivate better its investigation. We observe that its introduction in the Yang-Mills action leads to a deep modification of the gluon propagator in the infrared. More precisely, as we shall see in the next section, the addition of the term (1) will give rise to a tree level gluon propagator which is of the Gribov type\(^\text{[11, 12, 13]}\), i.e. it is suppressed in the infrared, exhibiting positivity violation, a feature usually interpreted as a signal of confinement. This should be not surprising. Notice in fact that, in the quadratic approximation, both operators (1),(3) reduce to the same expression, thus yielding the same propagator. Also, we mention that expression (1) can be easily adapted to the lattice formulation, thus it could also be investigated through numerical simulations.

The present work is organized as follows. In section 2 we describe the localization procedure for the operator (1). Section 3 is devoted to the study of the symmetry content of the resulting local action. In section 4 we derive the set of Ward identities. In section 5 we present the algebraic characterization of the most general local invariant counterterm, and we establish the renormalizability of the model. Section 6 collects the conclusion.

## 2 The localization procedure

Let us start by considering the gauge fixed Yang-Mills action with the addition of the nonlocal operator $O$, eq.(1), namely
\[
S = S_{YM} + S_{gf} + \sigma^4 O , \tag{5}
\]
where $S_{YM}$ is the Yang-Mills action in four dimensional Euclidean space-time,
\[
S_{YM} = \frac{1}{4} \int d^4x \ F^a_{\mu\nu} F^{a}_{\mu\nu} , \tag{6}
\]
with
\[
F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu . \tag{7}
\]
The term $S_{gf}$ stands for the gauge fixing term, here taken in the Landau gauge, i.e.
\[
S_{gf} = \int d^4x \ \left( b^a \partial_\mu A^a_\mu + \bar{c}^a \partial_\mu D^a_\mu c^b \right) , \tag{8}
\]
---

\(^2\)For the generalization of the horizon function (3) to the maximal Abelian gauge see\(^\text{[35, 36]}\).
where the auxiliary field $b^a$ is the Lagrange multiplier enforcing the Landau gauge condition, $\partial_\mu A_\mu^a = 0$, and $(\bar{c}^a, c^a)$ are the Faddeev-Popov ghost fields. Notice that, in order to have the correct dimensions, a parameter $\sigma$ with the dimension of a mass has been introduced in expression (5). As the purpose of the present work is that of showing that a local and renormalizable action can be obtained from the nonlocal expression (1), $\sigma$ will be treated as a free parameter. After having proven the renormalizability of the resulting local theory, one can address the issue of whether $\sigma$ could be generated in a dynamical way, being associated to a possible condensation of the operator $O$, i.e. $\langle O \rangle \neq 0$. As it happens in the case of the Gribov parameter $\gamma$ [11, 12, 13] of the horizon function, eq. (3), this would demand that the parameter $\sigma$ is a solution of a suitable gap equation, enabling us to express it as a function of the gauge coupling constant $g$ and of the invariant scale $\Lambda_{QCD}$. Although being out of the aim of the present work, we shall come back to this interesting point in the conclusion, where a possible strategy to face the dynamical generation of the parameter $\sigma$ will be outlined.

An interesting feature of the action (5) is that it gives rise to a tree level gluon propagator which displays the characteristic Gribov behavior [11, 12, 13], namely

$$\langle A_\mu^a(k)A_\nu(-k) \rangle = \delta^a_b \frac{k^2}{k^4 + \sigma^4} \left( \delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right).$$ (9)

As already remarked, this is a consequence of the fact that both operators (1), (3) reduce to the same expression in the quadratic approximation, thus leading to the same tree level propagator.

Although the operator (1) is nonlocal, it can be cast in local form by introducing a suitable set of auxiliary localizing fields. This is performed according to

$$e^{-\sigma^4 O} = \int D\bar{B}DBD\bar{G}DG \exp \left\{ - \int d^4x \left[ \frac{1}{2} (\bar{B}_\mu^a D_\mu^a D_\nu^b B_\nu^b - \bar{G}_\mu^a D_\mu^a D_\nu^b G_\nu^b) + \frac{\sigma^2}{2} (B - \bar{B})_\mu^a A_\mu^a \right] \right\},$$ (10)

where $(\bar{B}_\mu^a, B_\mu^a)$ are bosonic vector fields, while $(\bar{G}_\mu^a, G_\mu^a)$ are anticommuting fields. Thus, for the partition function $Z$ of the model we may write

$$Z = \int DADbDcD\sigma e^{-S} = \int DADbDcD\sigma D\bar{B}DBD\bar{G}DG e^{-S_{\text{Local}}},$$ (11)

where the local action $S_{\text{Local}}$ is now given by

$$S_{\text{Local}} = S_{YM} + S_{gf} + S_{\text{aux}} + S_{\sigma},$$ (12)

with

$$S_{\text{aux}} = \frac{1}{2} \int d^4x \left( \bar{B}_\mu^a D_\mu^a D_\nu^b B_\nu^b - \bar{G}_\mu^a D_\mu^a D_\nu^b G_\nu^b \right),$$

$$S_{\sigma} = \frac{\sigma^2}{2} \int d^4x \left( B - \bar{B} \right)_\mu^a A_\mu^a.$$(13)

3 Symmetry content

To analyze the symmetry content of our model we shall start first by considering the case in which the parameter $\sigma$ is set to zero, i.e. $\sigma = 0$, yielding

$$S_0 = S_{\text{Local}}|_{\sigma=0} = S_{YM} + S_{gf} + S_{\text{aux}}.$$(14)
The action (14) is completely equivalent to the Yang-Mills action, since the introduction of the auxiliary fields \((\bar{B}^a{}_{\mu}, B^a{}_{\mu})\) and \((\bar{G}^a{}_{\mu}, G^a{}_{\mu})\) amounts simply to inserting a unity factor, \(i.e.\)
\[
1 = \int DBDBD\bar{D}\bar{G}\exp \left[ -\frac{1}{2} \int d^4x \left( \bar{B}^a{}_{\mu} D^a_{\mu} D^b_{\nu} B^b{}_{\mu} - \bar{G}^a{}_{\mu} D^a_{\mu} D^b_{\nu} G^b{}_{\mu} \right) \right].
\] (15)

Furthermore, the action (14) enjoys the following symmetries

- The BRST symmetry:
  \[
  sA^a{}_{\mu} = -D^a_{\mu} c^b, \\
  sc^a = \frac{g^2}{2} f^{abc} c^a c^b, \\
  s\bar{c}^a = b^a, \\
  sb^a = 0, \\
  sB^a{}_{\mu} = g f^{abc} B^b{}_{\mu}, \\
  s\bar{B}^a{}_{\mu} = g f^{abc} \bar{B}^b{}_{\mu}, \\
  sG^a{}_{\mu} = g f^{abc} G^b{}_{\mu}, \\
  s\bar{G}^a{}_{\mu} = g f^{abc} \bar{G}^b{}_{\mu}.
  \] (16)

- The \(\delta\)-symmetry:
  \[
  \delta B^a{}_{\mu} = C^a{}_{\mu}, \\
  \delta G^a{}_{\mu} = 0, \\
  \delta \bar{G}^a{}_{\mu} = \bar{B}^a{}_{\mu}, \\
  \delta \bar{B}^a{}_{\mu} = 0.
  \] (17)

Evidently,
\[
sS_0 = \delta S_0 = 0.
\] (18)

Both operators \(s\) and \(\delta\) are nilpotent, obeying the following anticommutation relations
\[
s^2 = \delta^2 = \{s, \delta\} = 0.
\] (19)

In the same way as the BRST transformations allow us to introduce the ghost number operator \(N_{gh}\)
\[
N_{gh} S_0 = \int d^4x \left( c^a \frac{\delta}{\delta c^a} - \bar{c}^a \frac{\delta}{\delta \bar{c}^a} \right) S_0 = 0,
\] (20)
the \(\delta\)-transformations (17) enable us to introduce a second operator \(N_f\) associated to the anticommuting fields \((G^a{}_{\mu}, G^a{}_{\mu})\), namely
\[
N_f S_0 = \int d^4x \left( G^a{}_{\mu} \frac{\delta}{\delta G^a{}_{\mu}} - \bar{G}^a{}_{\mu} \frac{\delta}{\delta \bar{G}^a{}_{\mu}} \right) S_0 = 0.
\] (21)

Let us now try to take into account the term \(S_{\sigma}\) in (13). As is easily seen, this term breaks both \(s\) and \(\delta\) symmetries, in fact
\[
sS_{\sigma} = -\frac{\sigma^2}{2} \int d^4x (B - \bar{B})^a{}_{\mu} \partial_{\mu} c^a, \\
\delta S_{\sigma} = \frac{\sigma^2}{2} \int d^4x G^a{}_{\mu} A^a{}_{\mu}.
\] (22)
Moreover, the breaking terms (22) can be kept under control by embedding the action (12) in a more
general model with exact invariance, a strategy already successfully employed in the case of Zwanziger’s
horizon function $S_H$ [12, 13], eq.(3), and of the nonlocal mass term $S_m$ [38, 39], eq.(4). To this purpose
we introduce a set of external sources, namely
\[
\{X_{\mu\nu}, \bar{X}_{\mu\nu}; Y_{\mu\nu}, \bar{Y}_{\mu\nu}; U_{\mu\nu}, \bar{U}_{\mu\nu}; V_{\mu\nu}, \bar{V}_{\mu\nu}\}
\]
which enable us to introduce the composite operators $B^a_A, \bar{B}^a_A, G^a_A, \bar{G}^a_A$. Requiring that
the sources $X_{\mu\nu}, \bar{X}_{\mu\nu}, Y_{\mu\nu}, \bar{Y}_{\mu\nu}, U_{\mu\nu}, \bar{U}_{\mu\nu}, V_{\mu\nu}, \bar{V}_{\mu\nu}$ transform as
\[
sX_{\mu\nu} = Y_{\mu\nu}, \quad sY_{\mu\nu} = 0,
\]
\[
s\bar{X}_{\mu\nu} = \bar{Y}_{\mu\nu}, \quad s\bar{Y}_{\mu\nu} = 0,
\]
\[
sV_{\mu\nu} = U_{\mu\nu}, \quad sU_{\mu\nu} = 0,
\]
\[
s\bar{V}_{\mu\nu} = \bar{V}_{\mu\nu}, \quad s\bar{V}_{\mu\nu} = 0,
\]
and
\[
\delta V_{\mu\nu} = -X_{\mu\nu}, \quad \delta X_{\mu\nu} = 0,
\]
\[
\delta \bar{V}_{\mu\nu} = X_{\mu\nu}, \quad \delta \bar{X}_{\mu\nu} = 0,
\]
\[
\delta U_{\mu\nu} = Y_{\mu\nu}, \quad \delta Y_{\mu\nu} = 0,
\]
\[
\delta \bar{U}_{\mu\nu} = \bar{Y}_{\mu\nu}, \quad \delta \bar{Y}_{\mu\nu} = 0,
\]
it is apparent that the action $\tilde{S}_\sigma$
\[
\tilde{S}_\sigma = s\delta \int d^4x \left( U_{\mu\nu}B^a_A - V_{\mu\nu}G^a_A \right)
= \int d^4x \left( -X_{\mu\nu}B^a_A + Y_{\mu\nu}G^a_A + U_{\mu\nu}B^a_A - V_{\mu\nu}G^a_A \right)
\]
is left invariant by both $s$ and $\delta$ operators
\[
s\tilde{S}_\sigma = \delta \tilde{S}_\sigma = 0.
\]
Furthermore, it turns out that the original term $S_\sigma$ is recovered from $\tilde{S}_\sigma$ when the external sources attain
their physical values, defined as
\[
X_{\mu\nu}|_{\text{phys}} = U_{\mu\nu}|_{\text{phys}} = \frac{\alpha^2}{2} \delta_{\mu\nu},
\]
\[
Y_{\mu\nu}|_{\text{phys}} = Y_{\mu\nu}|_{\text{phys}} = Y_{\mu\nu}|_{\text{phys}} = V_{\mu\nu}|_{\text{phys}} = \bar{V}_{\mu\nu}|_{\text{phys}} = 0.
\]
Thus, we have
\[
\tilde{S}_\sigma|_{\text{phys}} \rightarrow S_\sigma.
\]
The previous equation allows us to introduce a more general action
\[
\tilde{S}_{\text{Local}} = S_0 + \tilde{S}_\sigma,
\]
where $S_0$ and $\tilde{S}_\sigma$ are respectively given by (13) and (20), which is left invariant by both $s$ and $\delta$ operators,
\[
s\tilde{S}_{\text{Local}} = \delta \tilde{S}_{\text{Local}} = 0.
\]
while reducing to the action $S_{\text{Local}}$, eq. [12], when the sources attain their physical values

$$\tilde{S}_{\text{Local}}_{\text{phys}} \to S_{\text{Local}}.$$  

(32)

We see thus that the action $S_{\text{Local}}$ has been embedded in a more general action, $\tilde{S}_{\text{Local}}$, exhibiting exact invariance. Moreover, $\tilde{S}_{\text{Local}}$ turns out to display a further global symmetry $U(4)$:

$$Q_{\mu\nu} \tilde{S}_{\text{Local}} = 0,$$

(33)

where

$$Q_{\mu\nu} \equiv \int d^4x \left( B^a_{\mu\nu} \delta B^a_{\mu\nu} - B^a_{\mu\nu} \delta B^a_{\mu\nu} + G^a_{\mu\nu} \delta G^a_{\mu\nu} - \tilde{G}^a_{\mu\nu} \delta \tilde{G}^a_{\mu\nu} + X_{\mu\nu} \delta X_{\mu\nu} - \tilde{X}_{\mu\nu} \delta \tilde{X}_{\mu\nu} + Y_{\mu\nu} \delta Y_{\mu\nu} - \tilde{Y}_{\mu\nu} \delta \tilde{Y}_{\mu\nu} + U_{\mu\nu} \delta U_{\mu\nu} - \tilde{U}_{\mu\nu} \delta \tilde{U}_{\mu\nu} + V_{\mu\nu} \delta V_{\mu\nu} - \tilde{V}_{\mu\nu} \delta \tilde{V}_{\mu\nu} \right).$$

(34)

This symmetry can be associated to a new quantum number whose generator is the trace of $Q_{\mu\nu}$, i.e., $Q_4 \equiv Q_{\mu\nu}$. As already noticed in [12, 13, 38, 39], the existence of this global invariance allows us to differentiate between the indices which refer to $U(4)$ and the remaining Lorentz indices. Denoting by $i, j, k, \ldots$, the indices corresponding to the $U(4)$ invariance, expressions (33) and (34) can be rewritten as

$$Q_{ij} \tilde{S}_{\text{Local}} = 0,$$

(35)

and

$$\tilde{S}_{\text{Local}} = S_{\text{YM}} + S_{\text{gf}} + S_{\text{aux}} + \tilde{S}_{\sigma},$$

(37)

with $S_{\text{aux}}$, $\tilde{S}_{\sigma}$ given by

$$S_{\text{aux}} = \frac{1}{2} \int d^4x \left( \tilde{B}^a_i D^{ab}_{\mu} B^b_i - \tilde{G}^a_i D^{ab}_{\mu} G^b_i \right),$$

$$\tilde{S}_{\sigma} = \int d^4x \left[ -\tilde{X}_{\mu\nu} A^a_{\mu\nu} + Y_{\mu\nu} A^a_{\mu\nu} - U_{\mu\nu} B^a_{\mu\nu} + V_{\mu\nu} G^a_{\mu\nu} \right] \partial_{\mu} c^a.$$

(38)

### 3.1 Identification of the final complete classical action

We can now identify the complete classical action to start with. To this purpose, the action $\tilde{S}_{\text{Local}}$ has to be supplemented by three additional terms given, respectively, by

$$S_{\text{ext}} = \frac{s}{2} \int d^4x \left( -\Omega^a_{\mu} A^a_{\mu} + L^a c^a \right) + s \delta \int d^4x \left( \tilde{N}^a_i B^a_i + M^a_i \tilde{G}^a_i \right)$$

$$= \int d^4x \left( -\Omega^a_{\mu} D^{ab}_{\mu} c^b + \frac{1}{2} f^{abc} L^a c^b c^c + g f^{abc} \tilde{M}^a_i c^b B^c_i + g f^{abc} M^a_i c^b \tilde{B}^c_i + g f^{abc} \tilde{N}^a_i c^b G^c_i \right),$$

(39)
\[ S_\lambda = \delta \int d^4x \frac{\chi^{abcd}}{16} G_i^a B_i^b \left( B_j^c B_j^d - G_j^c G_j^d \right) = \int d^4x \frac{\chi^{abcd}}{16} \left( B_i^a B_i^b - G_i^a G_i^b \right) \left( B_j^c B_j^d - G_j^c G_j^d \right), \] (40)

and

\[ S_\zeta = s \delta \left( \zeta \int d^4x \ V_{i\mu} V_{i\mu} \right) = \zeta \int d^4x \left( \bar{X}_{i\mu} U_{i\mu} - \bar{V}_{i\mu} Y_{i\mu} \right). \] (41)

Let us analyze each term separately. The first one, eq. (39), is needed in order to take into account the nonlinear BRST transformations of the fields, see eq. (16). In this term we have introduced new external sources \( \Omega_{i\mu}^a, L^a, M_i^a, \bar{M}_i^a, N_i^a, \bar{N}_i^a \), which transform as

\[ s \Omega_{i\mu}^a = s L^a = s M_i^a = s \bar{M}_i^a = s N_i^a = s \bar{N}_i^a = 0, \] (42)

and

\[ \delta \bar{N}_i^a = -\bar{M}_i^a, \quad \delta \bar{M}_i^a = 0, \]
\[ \delta M_i^a = N_i^a, \quad \delta N_i^a = 0, \]
\[ \delta \Omega_{i\mu}^a = 0, \quad \delta L^a = 0, \] (43)

ensuring both \( s \) and \( \delta \) invariance of \( S_{\text{ext}} \). The second term, \( S_\lambda \), is a quartic term in the auxiliary fields, allowed by power counting. As such it has to be introduced from the beginning. The \( \delta \) invariance of \( S_\lambda \) is manifest. Its BRST invariance is achieved by demanding that the quartic coupling \( \chi^{abcd} \) is an invariant tensor in the adjoint representation, namely

\[ f^{man} \chi^{abcd} + f^{mnb} \chi^{acmd} + f^{mcn} \chi^{abmd} + f^{mnd} \chi^{abcd} = 0. \] (44)

Also, from expression (40) one easily infers that \( \chi^{abcd} \) possesses the following symmetry properties

\[ \chi^{abcd} = \chi^{cda} = \chi^{bacd}. \] (45)

Finally, the third term, \( S_\zeta \), obviously invariant under \( s \) and \( \delta \), contains terms which depend only on the external sources. This term is allowed by power counting, being in fact needed for the renormalizability of the model. The parameter \( \zeta \) in expression (41) is a constant dimensionless parameter.

Therefore, for the complete starting classical action \( \Sigma \), we obtain

\[ \Sigma = S_{YM} + S_{gt} + S_{\sigma} + S_{\text{ext}} + S_\lambda + S_\zeta \]
\[ = S_{YM} + \int d^4x \left[ b^a \partial_\mu A_{\mu}^a + e^a \partial_\mu D_{\mu}^{ab} c^b + \frac{1}{2} \bar{B}_i^a D_{\mu}^{ab} B_i^b - \frac{1}{2} \bar{G}_i^a D_{\mu}^{ab} B_i^b - \bar{X}_{i\mu} B_i^a A_\mu^a \
+ Y_{i\mu} \bar{G}_i^a A_\mu^a + U_{i\mu} \bar{B}_i^a A_\mu^a + \bar{V}_{i\mu} G_i^a A_\mu^a - \bar{Y}_{i\mu} B_i^c - X_{i\mu} \bar{G}_i^a - V_{i\mu} \bar{B}_i^a - \bar{U}_{i\mu} G_i^a \right] \partial_\mu c^a \
+ \zeta \left( \bar{X}_{i\mu} U_{i\mu} - \bar{V}_{i\mu} Y_{i\mu} \right) - \Omega_{\mu}^{abc} \partial_\mu c^c + \frac{1}{2} f^{abc} L^a c^b c^c + gf^{abc} M_i^a c^b B_i^c + gf^{abc} M_i^a c^b \bar{B}_i^c \
+ gf^{abc} \bar{N}_i^a c^b G_i^c + gf^{abc} N_i^a c^b \bar{G}_i^c + \frac{\chi^{abcd}}{16} \left( B_i^a B_i^b - G_i^a G_i^b \right) \left( B_j^c B_j^d - G_j^c G_j^d \right). \] (46)

Let us also display, for further use, all quantum numbers of the fields and sources, given in tables 1 and 2.

### 4 Ward identities.

The action (46) enjoys a large set of Ward identities. In fact, it turns out that \( \Sigma \) fulfills:
A | b | c | B | B | G | G
---|---|---|---|---|---|---
dimension | 1 | 2 | 2 | 0 | 1 | 1 | 1
ghost number | 0 | 0 | −1 | 1 | 0 | 0 | 0
N_f number | 0 | 0 | 0 | 0 | 0 | 1 | −1
Q_4-charge | 0 | 0 | 0 | 1 | −1 | 1 | −1

Table 1: The quantum numbers of the fields.

| Ω | L | X | X | Y | Y | U | V | V | M | M | N | N |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
dimension | 3 | 4 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 3 | 3 | 3 | 3
ghost number | −1 | −2 | −1 | 0 | 0 | −1 | 0 | −1 | −1 | 0 | −1 | −1 | −1
N_f number | 0 | 0 | 1 | 0 | 1 | 0 | 0 | −1 | 0 | −1 | 0 | 0 | 1
Q_4-charge | 0 | 0 | 1 | −1 | 1 | −1 | 1 | −1 | 1 | −1 | 1 | −1 | −1

Table 2: The quantum numbers of the sources.

• The Slavnov-Taylor identity

\[ S(\Sigma) = 0 \, , \quad (47) \]

\[
S(\Sigma) \equiv \int d^4x \left( \frac{\delta \Sigma}{\delta \Omega^a_\mu} \frac{\delta \Sigma}{\delta A^a_\mu} + \frac{\delta \Sigma}{\delta L^a} \frac{\delta \Sigma}{\delta c^a} + \frac{\delta \Sigma}{\delta L^a} \frac{\delta \Sigma}{\delta c^a} + \frac{\delta \Sigma}{\delta M^a_i} \frac{\delta \Sigma}{\delta B^a_i} + \frac{\delta \Sigma}{\delta M^a_i} \frac{\delta \Sigma}{\delta B^a_i} + \frac{\delta \Sigma}{\delta N^a_i} \frac{\delta \Sigma}{\delta G^a_i} + \frac{\delta \Sigma}{\delta N^a_i} \frac{\delta \Sigma}{\delta G^a_i} + b^a \frac{\delta \Sigma}{\delta c^a} + Y_{i\mu} \frac{\delta \Sigma}{\delta X_{i\mu}} + \bar{X}_{i\mu} \frac{\delta \Sigma}{\delta Y_{i\mu}} + U_{i\mu} \frac{\delta \Sigma}{\delta V_{i\mu}} + \bar{V}_{i\mu} \frac{\delta \Sigma}{\delta U_{i\mu}} \right) \, , \quad (48) \]

• The Landau gauge-fixing condition

\[
\delta \frac{\Sigma}{\delta b^a} = \partial_\mu A^a_\mu \, , \quad (49) \]

• The anti-ghost equation

\[
\frac{\delta \Sigma}{\delta c^a} + \partial_\mu \frac{\delta \Sigma}{\delta \Omega^a_\mu} = 0 \, , \quad (50) \]

• The ghost equation

\[ G^a(\Sigma) = \Delta^a_{\text{class}} \, , \quad (51) \]

where

\[ G^a \equiv \int d^4x \left( \frac{\delta \Sigma}{\delta c^a} + g f^{abc} \frac{\delta \Sigma}{\delta b^c} \right) \, , \quad (52) \]

and

\[ \Delta^a_{\text{class}} = \int d^4x g f^{abc} \left( \Omega^b_\mu A^c_\mu - L^b_\mu c^c + \bar{M}^b_i B^c_i + M^b_i B^c_i - \bar{N}^b_i G^c_i - N^b_i \bar{G}^c_i \right) \]  

Notice that the breaking term \( \Delta^a_{\text{class}} \) is linear in the quantum fields. It is thus a classical breaking, not affected by the quantum corrections [44].
The ghost number Ward identity
\[ N_{gh}(\Sigma) = 0 , \] (54)
with
\[ N_{gh} \equiv \int d^4x \left( c^a \frac{\delta}{\delta c^a} - c^a \frac{\delta}{\delta c^a} \tilde{c} \delta y_{ij} \frac{\delta}{\delta y_{ij}} - x_{ij} \frac{\delta}{\delta x_{ij}} - v_{ij} \frac{\delta}{\delta v_{ij}} \tilde{u}_{ij} \frac{\delta}{\delta u_{ij}} \right. \\
\left. - \Omega^\alpha \frac{\delta}{\delta \Omega^\alpha} - 2L^a \frac{\delta}{\delta L^a} \tilde{m}^a \frac{\delta}{\delta m^a} - M^a \frac{\delta}{\delta m^a} \tilde{n}^a \frac{\delta}{\delta n^a} - n^a \frac{\delta}{\delta n^a} \right) , \] (55)

The Ward identity corresponding to the fermionic invariance
\[ N_f(\Sigma) = 0 , \] (56)
where \( N_f \) stands for the operator
\[ N_f \equiv \int d^4x \left( G^a_i \frac{\delta}{\delta G^a_i} - \tilde{G}^a_i \frac{\delta}{\delta \tilde{G}^a_i} + Y^\mu \frac{\delta}{\delta Y^\mu} + U^\mu \frac{\delta}{\delta U^\mu} \tilde{u}^\mu \frac{\delta}{\delta \tilde{u}^\mu} \right. \\
\left. - \tilde{V}^\mu \frac{\delta}{\delta \tilde{V}^\mu} - \tilde{U}^\mu \frac{\delta}{\delta \tilde{U}^\mu} + \tilde{N}^a \frac{\delta}{\delta \tilde{N}^a} - N^a \frac{\delta}{\delta N^a} \right) . \] (57)

The global \( U(4) \) invariance
\[ Q_{ij}(\Sigma) = 0 , \] (58)
where
\[ Q_{ij} \equiv \int d^4x \left( B^a_i \frac{\delta}{\delta B^a_i} + G^a_i \frac{\delta}{\delta G^a_i} \tilde{G}^a_i \frac{\delta}{\delta \tilde{G}^a_i} - \tilde{V}^\mu \frac{\delta}{\delta \tilde{V}^\mu} - \tilde{U}^\mu \frac{\delta}{\delta \tilde{U}^\mu} + \tilde{N}^a \frac{\delta}{\delta \tilde{N}^a} - N^a \frac{\delta}{\delta N^a} \right) , \] (59)

The rigid symmetries
\[ R^{(A)}_{ij}(\Sigma) = 0 , \] \( A = 1, 2, 3, 4 \), (60)
where
\[ R^{(1)}_{ij} \equiv \int d^4x \left( B^a_i \frac{\delta}{\delta B^a_i} + G^a_i \frac{\delta}{\delta G^a_i} \right. \\
\left. \tilde{G}^a_i \frac{\delta}{\delta \tilde{G}^a_i} - \tilde{V}^\mu \frac{\delta}{\delta \tilde{V}^\mu} - \tilde{U}^\mu \frac{\delta}{\delta \tilde{U}^\mu} + \tilde{N}^a \frac{\delta}{\delta \tilde{N}^a} - N^a \frac{\delta}{\delta N^a} \right) , \] (61)

As we shall see in the next section, this set of Ward identities will enable us to prove the renormalizability of the complete action\(^{[40]}\).
5 Algebraic characterization of the invariant counterterm and renormalizability

Having established all Ward identities fulfilled by the complete action, eq. [46], we can now turn our attention to the characterization of the most general invariant counterterm \( \Sigma_{\text{CT}} \). Following the algebraic renormalization procedure [44], \( \Sigma_{\text{CT}} \) has to be an integrated local polynomial in the fields and sources with dimension bounded by four, with vanishing ghost and \( N_i \) numbers as well as \( Q_4 \)-charge, and obeying the following constraints

\[
B_{\Sigma} \Sigma_{\text{CT}} = 0, \\
\frac{\delta}{\delta \mu} \Sigma_{\text{CT}} = 0, \\
\left( \frac{\delta}{\delta \overline{\epsilon}^a} + \partial_{\mu} \frac{\delta}{\delta \Omega_{\mu}^a} \right) \Sigma_{\text{CT}} = 0, \\
G^a \Sigma_{\text{CT}} = 0, \\
N_{\text{gh}} \Sigma_{\text{CT}} = 0, \\
N_i \Sigma_{\text{CT}} = 0, \\
Q_{ij} \Sigma_{\text{CT}} = 0, \\
R_{ij}^{(A)} \Sigma_{\text{CT}} = 0, \tag{62}
\]

where \( B_{\Sigma} \) is the nilpotent linearized Slavnov-Taylor operator

\[
B_{\Sigma} \equiv \int d^4x \left\{ \frac{\delta \Sigma}{\delta \Omega_{\mu}^a} \delta \frac{\delta}{\delta A_{\mu}^a} + \frac{\delta \Sigma}{\delta A_{\mu}^a} \delta \frac{\delta}{\delta \Omega_{\mu}^a} + \frac{\delta \Sigma}{\delta \Omega_{\mu}^a} \delta \frac{\delta}{\delta \Omega_{\mu}^a} + \frac{\delta \Sigma}{\delta L^a} \delta \frac{\delta}{\delta \Omega_{\mu}^a} + \frac{\delta \Sigma}{\delta \Omega_{\mu}^a} \delta \frac{\delta}{\delta \Omega_{\mu}^a} + \frac{\delta \Sigma}{\delta \Omega_{\mu}^a} \delta \frac{\delta}{\delta \Omega_{\mu}^a} \\
+ \frac{\delta \Sigma}{\delta M_i^a} \delta \frac{\delta}{\delta B_i^a} + \frac{\delta \Sigma}{\delta B_i^a} \delta \frac{\delta}{\delta M_i^a} + \frac{\delta \Sigma}{\delta N_i^a} \delta \frac{\delta}{\delta G_i^a} + \frac{\delta \Sigma}{\delta G_i^a} \delta \frac{\delta}{\delta N_i^a} + \frac{\delta \Sigma}{\delta G_i^a} \delta \frac{\delta}{\delta G_i^a} + \frac{\delta \Sigma}{\delta G_i^a} \delta \frac{\delta}{\delta G_i^a} \\
+ b^a \frac{\delta}{\delta \epsilon^a} + Y_i \frac{\delta}{\delta X_i} + \bar{X}_i \frac{\delta}{\delta \bar{X}_i} + U_{\mu} \frac{\delta}{\delta U_{\mu}} + \bar{V}_{\mu} \frac{\delta}{\delta \bar{V}_{\mu}} \right\}, \tag{63}
\]

\[
B_{\Sigma} B_{\Sigma} = 0. \tag{64}
\]

After a rather lengthy analysis, the most general allowed counterterm \( \Sigma_{\text{CT}} \) compatible with all Ward identities is found to be

\[
\Sigma_{\text{CT}} = a_0 S_{\text{YM}} + \int d^4x \left\{ a_1 A_{\mu}^a \frac{\delta S_{\text{YM}}}{\delta A_{\mu}^a} + a_1 \left( \Omega_{\mu}^a + \partial_{\mu} \epsilon^a \right) \partial_{\mu} \epsilon^a + \frac{a_2}{2} \left( B_i^a \partial^2 B_i^a - \bar{G}_i^a \partial^2 G_i^a \right) \\
- (a_1 + a_2) \left[ \frac{g}{2} f^{abc} \left( B_i^a B_i^b - \bar{G}_i^a \bar{G}_i^b \right) \partial_{\mu} A_{\mu}^c + g f^{abc} \left( \bar{B}_i^a \partial_{\mu} B_i^b - \bar{G}_i^a \partial_{\mu} G_i^b \right) A_{\mu}^c \right] \\
+ (2a_1 + a_2) \left[ \frac{g}{2} f^{abc} f^{cde} \left( B_i^a B_i^b - \bar{G}_i^a \bar{G}_i^b \right) A_{\mu}^d A_{\mu}^e \right. \\
\left. - (a_1 + a_3) \left( \bar{X}_i \partial_{\mu} B_i^a + \bar{V}_i \partial_{\mu} G_i^a \right) \partial_{\mu} \epsilon^a \right] \\
\left[ 2a_2 + a_4 \right] \frac{\lambda^{abcd}}{16} + a_4 \frac{N^{abcd}}{16} \right\} \left( B_j^a B_j^b - \bar{G}_j^a \bar{G}_j^b \right) \left( B_j^a B_j^b - \bar{G}_j^a \bar{G}_j^b \right) \right] + a_5 \zeta \left( \bar{X}_i \partial_{\mu} U_i - \bar{V}_i \partial_{\mu} Y_i \right) \right\}. \tag{65}
\]

In the last expression the coefficients \( a_k, k = 0, \ldots, 5 \), are free parameters and \( N^{abcd} \) is an invariant tensor with the same properties of \( \lambda^{abcd} \), eqs. [41], [45]. As discussed in [38, 39], the tensor \( N^{abcd} \) represents the contribution of quartic counterterms whose group structure does not allow to express them directly in terms of \( \lambda^{abcd} \).
It remains now to show that the invariant counterterm \((65)\) can be reabsorbed through a redefinition of the parameters, fields and sources of the classical starting action \(\Sigma\), according to
\[
\Phi^0 = Z\Phi, \\
J^0 = ZJ, \\
\lambda^{abcd}_0 = Z\lambda^{abcd} + Z^{abcd},
\]
where
\[
\Phi \equiv (A, b, c, \bar{c}, B, \bar{B}, G, \bar{G}), \\
J \equiv (g, \zeta, \Omega, L, X, \bar{X}, Y, \bar{Y}, U, \bar{U}, V, \bar{V}, M, \bar{M}, N, \bar{N}),
\]
so that
\[
\Sigma(\Phi^0, J^0, \lambda^{abcd}_0) = \Sigma(\Phi, J, \lambda^{abcd}) + \epsilon \Sigma_{CT} + O(\epsilon^2).
\]
By direct inspection, the renormalization constants are found to be
\[
Z_b = Z_L^2 = Z_A, \\
Z_c = Z_{\bar{c}} = Z_{\bar{\Omega}} = Z_g^{-1}Z_A^{-1/2}, \\
Z_G = Z_{\bar{G}} = Z_{\bar{B}} = Z_B, \\
Z_{\bar{X}} = Z_Y = Z_{\bar{V}} = Z_U, \\
Z_X = Z_V = Z_{\bar{Y}} = Z_{\bar{G}} = Z_g^{-1}Z_A^{1/2}Z_U, \\
Z_M = Z_{\bar{M}} = Z_N = Z_{\bar{N}} = Z_A^{1/2}Z_B^{-1/2},
\]
with
\[
Z_A = 1 + \epsilon (a_0 + 2a_1), \\
Z_g = 1 - \epsilon \frac{a_0}{2}, \\
Z_B = 1 + \epsilon a_2, \\
Z_U = 1 - \frac{\epsilon}{2} (a_0 + a_2 - 2a_3), \\
Z_\lambda = 1 + \epsilon a_4, \\
Z^{abcd} = \epsilon a_4 N^{abcd}, \\
Z_\zeta = 1 + \epsilon (a_0 + a_2 - 2a_3 + a_5).
\]
Equations \((68), (69), (70)\) show that the counterterm \(\Sigma_{CT}\) can be reabsorbed by means of a redefinition of the fields, sources and parameters of the starting action \(\Sigma\), establishing thus the renormalizability of the theory.

6 Conclusion

Nonlocal operators are known to play an important role in Yang-Mills theories. For example, in the absence of quarks, the vacuum expectation value of the Wilson loop proves to be an order parameter for the confining and deconfining phases of Yang-Mills theories. Also, for a large class of loops, the Wilson operator is renormalizable.

In this work we call attention to the existence of a slightly different class of nonlocal operators which can be added to the Yang-Mills action, while leading to a local and renormalizable theory. These features
are encoded in the possibility of achieving a rather simple localization procedure for these operators. The renormalizability is thus guaranteed due to the symmetry content of the resulting local theory. This framework has been illustrated through the example of the nonlocal operator of expression (1). Although many aspects related to these operators deserve a better understanding, let us spot here a few remarks which might be useful for further investigation.

- The first observation is related to the nonlocal character of these operators which, when added to the Yang-Mills action, can induce deep modifications on the infrared behavior of the theory. These operators could thus be useful in order to investigate nonperturbative aspects of confining theories. This is best illustrated by the example of Zwanziger’s horizon function (3), which implements the restriction of the domain of integration in the Feynman path integral up to the first Gribov horizon. In particular, the resulting tree level gluon propagator turns out to be suppressed in the infrared, according to \[ \langle A^a_\mu(k)A^b_\nu(-k) \rangle = \delta^{ab} \frac{k^2}{k^4 + \gamma^4} \left( \delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right). \] (71)

This propagator exhibits complex poles, a feature which is interpreted as a signal of gluon confinement. In other words, the gluon is destabilized by the presence of the Gribov horizon, so that it does not belong anymore to the physical spectrum.

- A second remark follows by noting that, due to dimensional reasons, these operators require the introduction of dimensionful parameters, i.e. \( \gamma \) for the horizon function (3), \( m \) for the nonlocal mass operator (4), and \( \sigma \) for the expression (1). This naturally rises the question of whether these parameters could be generated in a dynamical way, reflecting the possibility that the corresponding operators might condense, acquiring a nonvanishing vacuum expectation value. This could result in the lowering of the vacuum energy of the theory, signalling that the aforementioned condensates are in fact energetically favoured. This would require that these parameters are determined in a self-consistent way through suitable gap equations. Once again, this point can be illustrated by the example of the horizon term (3). From [11, 12, 13], one learns that the Gribov parameter \( \gamma \) is not a free parameter, being determined self-consistently through the gap equation

\[ \frac{\delta \Gamma}{\delta \gamma^2} = 0, \] (72)

where \( \Gamma \) stands for the quantum 1PI effective action evaluated with the Yang-Mills action supplemented by the horizon term (3). Equation (72) enables one to express \( \gamma \) as a function of the gauge coupling \( g \) and of the invariant scale \( \Lambda_{QCD} \).

- Notice that the gap equation (72) can be seen as a variational minimizing condition, stating that the quantum action \( \Gamma \) depends minimally from \( \gamma \). The same variational principle could be employed in order to investigate the dynamical origin of the gluon mass parameter \( m \) as well as to study the operator (1). It is worth mentioning that this variational principle has been in fact already employed in the study of the dimension two condensate \( \langle A^a_\mu A^a_\mu \rangle \) [12] in the Landau gauge which, due to the relationship

\[ -\frac{1}{4} \int d^4x F^a_{\mu\nu} \frac{1}{D^2} F^a_{\mu\nu} = \frac{1}{2} \int d^4x A^a_\mu A^a_\mu + \text{higher order terms}. \] (73)

can be seen as evidence in favour of the possible existence of a nonvanishing condensate \( \langle F \frac{1}{D^2} F \rangle \).

- One further aspect to be investigated is whether two or more nonlocal operators could be simultaneously added to the Yang-Mills action in such a way that the resulting theory preserves
renormalizability. This would require the absence of possible mixing among the various nonlocal operators which could jeopardize the renormalizability. Let us mention here that, so far, this issue has been investigated by considering the inclusion in the Yang-Mills action of both Zwanziger’s horizon function \[9\] and the nonlocal mass term \[11\]. Thanks to the rich symmetry content, it turns out that there is no mixing between the two operators, so that the resulting local theory can be proven to be renormalizable \[10\]. This result could allow us to investigate the effects of the gauge invariant nonlocal mass operator \[11\] in the presence of the Gribov horizon.

- Finally, it would be interesting to look at a more systematic way in order to search for other nonlocal operators which might lead to renormalizable theories. The inclusion of matter fields could also be exploited. For instance, the investigation of a possible nonlocal spinor mass term preserving chiral invariance could be of a certain interest.

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References.

[1] J. Greensite, Prog. Part. Nucl. Phys. 51, 1 (2003) [arXiv:hep-lat/0301023].
[2] R. Alkofer, C. S. Fischer and F. J. Llanes-Estrada, Phys. Lett. B 611, 279 (2005) [arXiv:hep-th/0412330].
[3] L. von Smekal, R. Alkofer and A. Hauck, Phys. Rev. Lett. 79, 3591 (1997) [arXiv:hep-ph/9705242].
[4] L. von Smekal, A. Hauck and R. Alkofer, Annals Phys. 267, 1 (1998) [Erratum-ibid. 269, 182 (1998)] [arXiv:hep-ph/9707327].
[5] R. Alkofer and L. von Smekal, Phys. Rept. 353, 281 (2001) [arXiv:hep-ph/0007355].
[6] J. M. Maldacena, Phys. Rev. Lett. 80, 4859 (1998) [arXiv:hep-th/9803002].
[7] J. Polchinski and M. J. Strassler, Phys. Rev. Lett. 88, 031601 (2002) [arXiv:hep-th/0109174].
[8] H. Boschi-Filho, N. R. d. Braga and H. L. Carrion, Phys. Rev. D 73, 047901 (2006) [arXiv:hep-th/0507063].
[9] S. J. Brodsky and G. F. de Teramond, Phys. Rev. Lett. 96, 201601 (2006) [arXiv:hep-ph/0602252].
[10] O. Andreev and V. I. Zakharov, [arXiv:hep-ph/0703010].
[11] V. N. Gribov, Nucl. Phys. B 139 (1978) 1.
[12] D. Zwanziger, Nucl. Phys. B 323, 513 (1989).
[13] D. Zwanziger, Nucl. Phys. B 399, 477 (1993).
[14] H. Reinhardt and C. Feuchter, Phys. Rev. D 71, 105002 (2005) [arXiv:hep-th/0408237].
[15] C. Feuchter and H. Reinhardt, Phys. Rev. D 70, 105021 (2004) [arXiv:hep-th/0408236].
[16] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. B 147, 385 (1979).
[17] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. B 147, 448 (1979).
[18] F. V. Gubarev, L. Stodolsky and V. I. Zakharov, Phys. Rev. Lett. 86, 2220 (2001) arXiv:hep-ph/0010057.
[19] F. V. Gubarev and V. I. Zakharov, Phys. Lett. B 501, 28 (2001) arXiv:hep-ph/0010096.
[20] H. Verschelde, K. Knecht, K. Van Acoleyen and M. Vanderkelen, Phys. Lett. B 516, 307 (2001) arXiv:hep-th/0105018.
[21] D. Dudal, H. Verschelde, R. E. Browne and J. A. Gracey, Phys. Lett. B 562, 87 (2003) arXiv:hep-th/0302128.
[22] D. Dudal, H. Verschelde and S. P. Sorella, Phys. Lett. B 555, 126 (2003) arXiv:hep-th/0212182.
[23] J. A. Gracey, Eur. Phys. J. C 39, 61 (2005) arXiv:hep-ph/0411169.
[24] D. Dudal, H. Verschelde, J. A. Gracey, V. E. R. Lemes, M. S. Sarandy, R. F. Sobreiro and S. P. Sorella, JHEP 0401, 044 (2004) arXiv:hep-th/0311194.
[25] D. Dudal, J. A. Gracey, V. E. R. Lemes, M. S. Sarandy, R. F. Sobreiro, S. P. Sorella and H. Verschelde, Phys. Rev. D 70, 114038 (2004) arXiv:hep-th/0406132.
[26] J. M. Pawlowski, D. F. Litim, S. Nedelko and L. von Smekal, Phys. Rev. Lett. 93, 152002 (2004) arXiv:hep-th/0312324.
[27] C. S. Fischer and J. M. Pawlowski, Phys. Rev. D 75, 025012 (2007) arXiv:hep-th/0609009.
[28] A. Cucchieri, T. Mendes and A. R. Taurines, Phys. Rev. D 71, 051902 (2005) arXiv:hep-lat/0406020.
[29] J. C. R. Bloch, A. Cucchieri, K. Langfeld and T. Mendes, Nucl. Phys. B 687, 76 (2004) arXiv:hep-lat/0312036.
[30] A. Cucchieri, Nucl. Phys. B 508, 353 (1997) arXiv:hep-lat/9705005.
[31] S. Furui and H. Nakajima, Phys. Rev. D 73, 074503 (2006).
[32] S. Furui and H. Nakajima, Phys. Rev. D 73, 094506 (2006) arXiv:hep-lat/0602027.
[33] I. L. Bogolubsky, G. Burgio, M. Muller-Preussker and V. K. Mitrjushkin, Phys. Rev. D 74, 034503 (2006) arXiv:hep-lat/0511056.
[34] P. O. Bowman et al., arXiv:hep-lat/0703022.
[35] M. A. L. Capri, V. E. R. Lemes, R. F. Sobreiro, S. P. Sorella and R. Thibes, Phys. Rev. D 72, 085021 (2005) arXiv:hep-th/0507052.
[36] M. A. L. Capri, V. E. R. Lemes, R. F. Sobreiro, S. P. Sorella and R. Thibes, Phys. Rev. D 74, 105007 (2006) arXiv:hep-th/0609212.
[37] R. Jackiw and S. Y. Pi, Phys. Lett. B 403, 297 (1997) arXiv:hep-th/9703226.
[38] M. A. L. Capri, D. Dudal, J. A. Gracey, V. E. R. Lemes, R. F. Sobreiro, S. P. Sorella and H. Verschelde, Phys. Rev. D 72, 105016 (2005) arXiv:hep-th/0510240.
[39] M. A. L. Capri, D. Dudal, J. A. Gracey, V. E. R. Lemes, R. F. Sobreiro, S. P. Sorella and H. Verschelde, Phys. Rev. D 74, 045008 (2006) [arXiv:hep-th/0605288].

[40] N. Maggiore and M. Schaden, Phys. Rev. D 50, 6616 (1994) [arXiv:hep-th/9310111].

[41] D. Dudal, R. F. Sobreiro, S. P. Sorella and H. Verschelde, Phys. Rev. D 72, 014016 (2005) [arXiv:hep-th/0502183].

[42] J. A. Gracey, JHEP 0605, 052 (2006) [arXiv:hep-ph/0605077].

[43] J. A. Gracey, [arXiv:hep-th/0701139]

[44] O. Piguet and S. P. Sorella, Lect. Notes Phys. M28 (1995) 1.

[45] S. P. Sorella, Annals Phys. 321 (2006) 1747.

[46] D. Dudal et al., to appear.