Revisiting $D_{s0}^{*}(2317)$ as a $0^+$ tetraquark state from QCD sum rules

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Stimulated by the renewed observation of $D_{s0}^{*}(2317)$ signal and its updated mass value $(2318.3 \pm 1.2 \pm 1.2)\text{ MeV}/c^2$ in the process $e^+e^- \rightarrow D_{s0}^{*+}D_{s0}^{*-} + \text{c.c.}$ by BESIII Collaboration, we devote to reinvestigate $D_{s0}^{*}(2317)$ as a $0^+$ tetraquark state from QCD sum rules. Technically, four different possible currents are adopted and high condensates up to dimension 12 are included in the operator product expansion (OPE) to ensure the quality of QCD sum rule analysis. In the end, we obtain the mass value $2.37_{-0.36}^{+0.50}\text{ GeV}$ for the scalar-scalar current, which agrees well with the experimental data of $D_{s0}^{*}(2317)$ and could support its explanation as a $0^+$ scalar-scalar tetraquark state. The final result for the axial-axial configuration is calculated to be $2.51_{-0.43}^{+0.61}\text{ GeV}$, which is still consistent with the mass of $D_{s0}^{*}(2317)$ considering the uncertainty, and then the possibility of $D_{s0}^{*}(2317)$ as an axial-axial tetraquark state can not be excluded. For the pseudoscalar-pseudoscalar and the vector-vector cases, their unsatisfactory OPE convergence makes that it is of difficulty to find rational work windows to further acquire hadronic masses.

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I. INTRODUCTION

Very recently, BESIII Collaboration announced the observation of the process $e^+e^- \rightarrow D_{s0}^{*+}D_{s0}^{*-} (2317) + \text{c.c.}$ for the first time with the data sample of 567 pb$^{-1}$ at a center-of-mass energy $\sqrt{s} = 4.6\text{ GeV}$. For the $D_{s0}^{*}(2317)$ signal, the statistical significance is reported to be $5.8\sigma$ and its mass is measured to be $(2318.3 \pm 1.2 \pm 1.2)\text{ MeV}/c^2$. Historically, $D_{s0}^{*}(2317)$ was first observed by BABAR Collaboration in the $D_{s}^{+}\pi^0$ invariant mass distribution $[2, 3]$, which was confirmed by CLEO Collaboration $[4]$ and by Belle Collaboration $[5]$. In theory, $D_{s0}^{*}(2317)$ could be proposed as a conventional $P$-wave $\bar{c}s$ meson with $J^P = 0^+$. However, one has to confront an approximate $150\text{ MeV}/c^2$ difference between the measured mass and the theoretical results from potential model $[6]$ and lattice QCD $[7]$ calculations. In addition, the absolute branching fraction $1.00_{-0.13}^{+0.09} \pm 0.14$ for $D_{s0}^{*}(2317)^{-} \rightarrow \pi^0 D_{s}^{-}$ newly measured by BESIII $[1]$ shows that $D_{s0}^{*}(2317)^{-}$ tends to have a significantly larger branching fraction to $\pi^0 D_{s}^{-}$ than to $\gamma D_{s}^{-}$, which differs from the expectation of the conventional $\bar{c}s$ state. As a feasible scenario resolving the above discrepancy, one can suppose $D_{s0}^{*}(2317)$ to be some multiquark system, such as a $DK$ molecule candidate $[8]$, a $\bar{c}sq\bar{q}$ tetraquark state $[9]$, or a mixture of a $\bar{c}s$ meson and a tetraquark state $[10]$. In a word, it is still undetermined and even unclear for the nature of $D_{s0}^{*}(2317)$.

Especially inspired by the BESIII’s new experimental result on $D_{s0}^{*}(2317)$ $[1]$, we devote to study it in the tetraquark picture, which is also helpful to deepen one’s understanding on nonperturbative QCD. One reliable way for evaluating the nonperturbative effects is the QCD sum rule method $[11]$, which is an analytic formalism firmly entrenched in QCD and has been fruitfully applied to many hadrons $[12–16]$. Concerning $D_{s0}^{*}(2317)$, there have appeared several QCD sum rule works to compute its mass basing on a $\bar{c}s$ meson picture $[17, 24]$, or taking a point of tetraquark view from QCD sum rules in the heavy quark limit $[25]$ as well as from full QCD sum rules involving condensates up to dimension 6 or 8 $[26, 27]$. It is known that one key point of the QCD sum rule analysis is that both the OPE convergence and the pole dominance should be carefully inspected. It has already been noted that some high dimension condensates may play an important role in some cases $[29, 32]$. To say the least, even if high condensates may not radically influence the OPE’s character, they are still beneficial to stabilize Borel curves. Therefore, in order to further reveal the internal structure of $D_{s0}^{*}(2317)$, we endeavor to perform the study of $D_{s0}^{*}(2317)$ as a $0^+$ tetraquark state in QCD sum rules adopting four different possible currents and including condensates up...
The rest of the paper is organized as follows. In Sec. II, $D_{s0}^*(2317)$ is studied as a tetraquark state in the QCD sum rule approach. The last part is a brief summary.

II. QCD SUM RULE STUDY OF $D_{s0}^*(2317)$ AS A 0$^+$ TETRAQUARK STATE

A. 0$^+$ tetraquark state currents

As one basic point of QCD sum rules, hadrons are represented by their interpolating currents. For a tetraquark state, its current ordinarily can be constructed as a diquark-antidiquark configuration. Thus, one can present following forms of 0$^+$ tetraquark currents:

$$j_{(I)} = \epsilon_{abcde} (q_a^T C \gamma_5 s_b)(\bar{q}_d \gamma_5 C \bar{Q}_e^T)$$

for the scalar-scalar case,

$$j_{(II)} = \epsilon_{abcde} (q_a^T C s_b)(\bar{q}_d C \bar{Q}_e^T)$$

for the pseudoscalar-pseudoscalar case,

$$j_{(III)} = \epsilon_{abcde} (q_a^T C\gamma_\mu s_b)(\bar{q}_d \gamma_\mu C \bar{Q}_e^T)$$

for the axial vector-axial vector (shortened to axial-axial) case, and

$$j_{(IV)} = \epsilon_{abcde} (q_a^T C\gamma_5 \gamma_\mu s_b)(\bar{q}_d \gamma_\mu \gamma_5 C \bar{Q}_e^T)$$

for the vector-vector case. Here $q$ denotes the light $u$ or $d$ quark, $Q$ is the heavy flavor charm quark, and the subscripts $a$, $b$, $c$, $d$, and $e$ indicate color indices.

B. tetraquark state QCD sum rules

The two-point correlator

$$\Pi_i(q^2) = i \int d^4 x e^{iq \cdot x} \langle 0 | T[j_{(i)}(x)j_{(i)}^\dagger(0)] | 0 \rangle, \quad (i = I, II, III, or IV)$$

(1)

can be used to derive QCD sum rules.

Phenomenologically, the correlator can be written as

$$\Pi_i(q^2) = \frac{\lambda_H^2}{M_H^2 - q^2} + \frac{1}{\pi} \int_{s_0}^{\infty} \frac{\text{Im} \left[ \Pi_{i,\text{phen}}(s) \right]}{s - q^2} ds + \ldots$$

(2)

where $s_0$ is the continuum threshold, $M_H$ denotes the hadron’s mass, and $\lambda_H$ shows the coupling of the current to the hadron $\langle 0 | j | H \rangle = \lambda_H$.

Theoretically, the correlator can be expressed as

$$\Pi_i(q^2) = \int_{(m_c + m_s)^2}^{\infty} \frac{\rho_i(s)}{s - q^2} ds + \Pi_i^{\text{cond}}(q^2),$$

(3)

where $m_c$ is the mass of charm quark, $m_s$ is the mass of strange quark, and the spectral density $\rho_i(s) = \frac{1}{\pi} \text{Im} [\Pi_i(s)]$. 


After matching Eqs. (2) and (3), assuming quark-hadron duality, and making a Borel transform \( \hat{B} \), the sum rule can be

\[
\lambda_f^2 e^{-M_H^2/M^2} = \int_{(m_+ + m_-)^2}^{s_0} \rho_i(s)e^{-s/M^2} ds + \hat{B}\Pi_i^{\text{cond}},
\]

with \( M^2 \) the Borel parameter.

Taking the derivative of Eq. (4) with respect to \( -1/M^2 \) and then dividing by Eq. (4) itself, one can arrive at the hadron’s mass sum rule

\[
M_H = \left\{ \int_{(m_+ + m_-)^2}^{s_0} \rho_i(s)e^{-s/M^2} ds + \frac{d(\hat{B}\Pi_i^{\text{cond}})}{d(-1/M^2)} \right\} / \left\{ \int_{(m_+ + m_-)^2}^{s_0} \rho_i(s)e^{-s/M^2} ds + \hat{B}\Pi_i^{\text{cond}} \right\}.
\]

In detail, the spectral density \( \rho_i(s) = \rho_i^{\text{pert}}(s) + \rho_i^{(q\bar{q})}(s) + \rho_i^{(g^2 G^2)}(s) + \rho_i^{(g\bar{q}\sigma Gq)}(s) + \rho_i^{(g\bar{q}q)}(s) + \rho_i^{(g^3 G^3)}(s) + \rho_i^{(g^4 G^4)}(s) \)

and the term

\[
\hat{B}\Pi_i^{\text{cond}} = \hat{B}\Pi_i^{(q\bar{q})(g^2 G^2)} + \hat{B}\Pi_i^{(q\bar{q})(g\bar{q}\sigma Gq)} + \hat{B}\Pi_i^{(q\bar{q})}\hat{B}\Pi_i^{(g^2 G^2)} + \hat{B}\Pi_i^{(q\bar{q})(g\bar{q}\sigma Gq)} + \hat{B}\Pi_i^{(q\bar{q})(g\bar{q}\sigma Gq)} + \hat{B}\Pi_i^{(g^3 G^3)}(g\bar{q}\sigma Gq) + \hat{B}\Pi_i^{(g^4 G^4)}(g\bar{q}\sigma Gq) + \hat{B}\Pi_i^{(g^4 G^4)(g\bar{q}\sigma Gq)} + \hat{B}\Pi_i^{(g^4 G^4)(g\bar{q}\sigma Gq)},
\]

including condensates up to dimension 12 can be derived with the similar techniques as Refs. e.g. [16, 33]. In reality, their concrete expressions for \( \rho_i(s) \) and \( \hat{B}\Pi_i^{\text{cond}} \) are the same as our previous work [34] other than that \( m_Q \) should be replaced by the charm quark mass \( m_c \), which are not intended to list here for conciseness.

C. numerical analysis and discussions

To extract the numerical value of \( M_H \), one could perform the analysis of sum rule [15] and take the input parameters as \( m_c = 1.27 \pm 0.03 \) GeV, \( m_\pi = 96^{+4}_{-3} \) MeV, \( \langle q\bar{q} \rangle = -(0.24 \pm 0.01)^3 \) GeV, \( \langle ss \rangle = m_0^2 \langle q\bar{q} \rangle, \langle g\bar{q}\sigma Gq \rangle = m_0^2 \langle q\bar{q} \rangle, m_0^3 \approx 0.8 \pm 0.1 \) GeV\(^2\), \( \langle g^2 G^2 \rangle = 0.88 \pm 0.25 \) GeV\(^4\), and \( \langle g^3 G^3 \rangle = 0.58 \pm 0.18 \) GeV\(^6\) [11, 13, 33]. As a standard procedure, both the OPE convergence and the pole dominance should be considered to find proper work windows for the threshold \( \sqrt{s_0} \) and the Borel parameter \( M^2 \). Moreover, \( \sqrt{s_0} \) characterizes the beginning of continuum states and can not be taken at will.

Taking the scalar-scalar case as an example, its different dimension OPE contributions are compared as a function of \( M^2 \) in FIG. 1. Graphically, one can see that there are three main condensate contributions, i.e. the dimension 3 two-quark condensate, the dimension 5 mixed condensate, and the dimension 6 four-quark condensate. These condensates could play an important role on the OPE side. The direct consequence is that it is of difficulty to choose a so-called “conventional Borel window” namely strictly satisfying that the low dimension condensate should be bigger than the high dimension contribution. Coming to think of it, these main condensates could cancel each other out to some extent. Meanwhile, most of other high dimension condensates involved are very small, for which can not radically influence the character of OPE convergence. All of these factors make that the perturbative term could play an important role on the total OPE contribution and the convergence of OPE is still under control.

In the phenomenological side, a comparison between pole contribution and continuum contribution of sum rule [11] for the threshold \( \sqrt{s_0} = 2.8 \) GeV is shown in FIG. 2, which manifests that the relative pole contribution is about 50% at \( M^2 = 1.6 \) GeV\(^2\) and decreases with \( M^2 \). In a similar way, the upper bounds of Borel parameters are \( M^2 = 1.5 \) GeV\(^2\) for \( \sqrt{s_0} = 2.7 \) GeV and \( M^2 = 1.7 \) GeV\(^2\) for \( \sqrt{s_0} = 2.9 \) GeV. Thereby,
Borel windows for the scalar-scalar case are taken as 0.8 ~ 1.5 GeV$^2$ for $\sqrt{s_0} = 2.7$ GeV, 0.8 ~ 1.6 GeV$^2$ for $\sqrt{s_0} = 2.8$ GeV, and 0.8 ~ 1.7 GeV$^2$ for $\sqrt{s_0} = 2.9$ GeV. In FIG. 3, the mass value $M_H$ as a function of $M^2$ from sum rule (5) for the scalar-scalar case is shown and one can visually see that there are indeed stable Borel plateau. In the chosen work windows, $M_H$ is calculated to be 2.37 ± 0.33 GeV. Furthermore, in view of the uncertainty due to variation of quark masses and condensates, we have 2.37 ± 0.33$^{+0.17}_{-0.03}$ GeV (the first error is resulted from variation of $\sqrt{s_0}$ and $M^2$, and the second error reflects the uncertainty rooting in the variation of QCD parameters) or briefly 2.37$^{+0.30}_{-0.17}$ GeV for the scalar-scalar tetraquark state.

For the axial-axial case, its OPE contribution in sum rule (4) for $\sqrt{s_0} = 2.8$ GeV is shown in FIG. 4 by comparing various dimension contributions. Similarly, the dimension 3, 5, and 6 condensates could cancel each other out to some extent and most of other dimension condensates are very small. On the other hand, the phenomenological contribution in sum rule (4) for $\sqrt{s_0} = 2.8$ GeV is pictured in Fig. 5. Eventually, work windows for the axial-axial case are chosen as 0.9 ~ 1.5 GeV$^2$ for $\sqrt{s_0} = 2.7$ GeV, 0.9 ~ 1.6 GeV$^2$ for $\sqrt{s_0} = 2.8$ GeV, and 0.9 ~ 1.7 GeV$^2$ for $\sqrt{s_0} = 2.9$ GeV. The corresponding Borel curves for the axial-axial case are displayed in FIG. 6 and its mass is evaluated to be 2.51 ± 0.41 GeV in the chosen work windows. With an eye to the uncertainty from the variation of quark masses and condensates, for the axial-axial tetraquark state we achieve 2.51$^{+0.20}_{-0.02}$ GeV (the first error reflects the uncertainty from the variation of $\sqrt{s_0}$ and $M^2$, and the second error roots in the variation of QCD parameters) or shortly 2.51$^{+0.61}_{-0.43}$ GeV.

For the pseudoscalar-pseudoscalar case, its various dimension OPE contribution in sum rule (4) for $\sqrt{s_0} = 2.8$ GeV is shown in Fig. 7. One may see that there are also three main condensates, i.e. the dimension 3, 5, and 6 condensates. However, what apparently distinct from the foregoing two cases is that two main condensates (i.e. the dimension 3 and 6 condensates) have a different sign comparing to the perturbative term, which leads that the perturbative part and the total OPE even have different signs at length. The dissatisfaction OPE property causes that related Borel curves are rather unstable visually, and it is difficult to find reasonable work windows for this case. Accordingly, it is not advisable to continue extracting a numerical result.

For the vector-vector case, its different dimension OPE contribution in sum rule (4) for $\sqrt{s_0} = 2.8$ GeV is shown in Fig. 8. There appears the analogous problem as the pseudoscalar-pseudoscalar case, and the most direct consequence is that corresponding Borel curves are quite unstable. Hence it is hard to find appropriate work windows to grasp an authentic mass value for the vector-vector case.

FIG. 1: The various dimension OPE contribution as a function of $M^2$ in sum rule (4) for $\sqrt{s_0} = 2.8$ GeV for the scalar-scalar case.
FIG. 2: The phenomenological contribution in sum rule (1) for $\sqrt{s_0} = 2.8$ GeV for the scalar-scalar case. The solid line is the relative pole contribution (the pole contribution divided by the total, pole plus continuum contribution) as a function of $M^2$ and the dashed line is the relative continuum contribution.

FIG. 3: The mass of $0^+$ tetraquark state with the scalar-scalar configuration as a function of $M^2$ from sum rule (5). The continuum thresholds are taken as $\sqrt{s_0} = 2.7 \sim 2.9$ GeV. The ranges of $M^2$ are $0.8 \sim 1.5$ GeV$^2$ for $\sqrt{s_0} = 2.7$ GeV, $0.8 \sim 1.6$ GeV$^2$ for $\sqrt{s_0} = 2.8$ GeV, and $0.8 \sim 1.7$ GeV$^2$ for $\sqrt{s_0} = 2.9$ GeV.

FIG. 4: The various dimension OPE contribution as a function of $M^2$ in sum rule (1) for $\sqrt{s_0} = 2.8$ GeV for the axial-axial case.
FIG. 5: The phenomenological contribution in sum rule (1) for $\sqrt{s_0} = 2.8$ GeV for the axial-axial case. The solid line is the relative pole contribution (the pole contribution divided by the total, pole plus continuum contribution) as a function of $M^2$ and the dashed line is the relative continuum contribution.

FIG. 6: The mass of $0^+$ tetraquark state with the axial-axial configuration as a function of $M^2$ from sum rule (5). The continuum thresholds are taken as $\sqrt{s_0} = 2.7 \sim 2.9$ GeV. The ranges of $M^2$ are $0.9 \sim 1.5$ GeV$^2$ for $\sqrt{s_0} = 2.7$ GeV, $0.9 \sim 1.6$ GeV$^2$ for $\sqrt{s_0} = 2.8$ GeV, and $0.9 \sim 1.7$ GeV$^2$ for $\sqrt{s_0} = 2.9$ GeV.

FIG. 7: The various dimension OPE contribution as a function of $M^2$ in sum rule (1) for $\sqrt{s_0} = 2.8$ GeV for the pseudoscalar-pseudoscalar case.
III. SUMMARY

Triggered by the new observation of $D_{s0}^*(2317)$ by BESIII Collaboration, we investigate that whether $D_{s0}^*(2317)$ could be a $0^+$ tetraquark state employing QCD sum rules. In order to insure the quality of sum rule analysis, contributions of condensates up to dimension 12 have been computed to test the OPE convergence. We find that some condensates, i.e. the two-quark condensate, the mixed condensate, and the four-quark condensate are of importance to the OPE side. Not bad for the scalar-scalar and the axial-axial cases, their main condensates could cancel each other out to some extent. Most of other condensates calculated are very small, which means that they could not radically influence the character of OPE convergence. All these factors bring that the OPE convergence for the scalar-scalar and the axial-axial cases is still controllable.

To the end, we gain the following results: firstly, the final result for the scalar-scalar case is $2.37^{+0.50}_{-0.36}$ GeV, which is in good agreement with the experimental value of $D_{s0}^*(2317)$. This result supports that $D_{s0}^*(2317)$ could be deciphered as a $0^+$ tetraquark state with the scalar-scalar configuration. Secondly, the eventual result for the axial-axial case is $2.51^{+0.43}_{-0.44}$ GeV, which is still coincident with the data of $D_{s0}^*(2317)$ considering the uncertainty although its central value is somewhat higher. In this way, one could not preclude the possibility of $D_{s0}^*(2317)$ as an axial-axial configuration tetraquark state. Thirdly, the OPE convergence is so unsatisfying for the pseudoscalar-pseudoscalar and the vector-vector cases that one can not find appropriate work windows to acquire reliable hadronic information.

In the future, with more data accumulated at BESIII or a fine scan from PANDA[36], experimental observations may shed more light on the nature of $D_{s0}^*(2317)$. Besides, one can also expect that the inner structure of $D_{s0}^*(2317)$ could be further uncovered by continuously theoretical efforts.

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[1] M. Ablikim et al. (BESIII Collaboration), arXiv:1711.08293 [hep-ex].
[35] C. Patrignani et al. (Particle Data Group), Chin. Phys. C 40, 100001 (2016).
[36] E. Prencipe et al. (PANDA Collaboration), EPJ Web Conf. 95, 04052 (2015).