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On Robin’s inequality. (English) Zbl 07713337  
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Summary: Let \( \sigma(n) \) denote the sum of divisors function of a positive integer \( n \). Robin proved that the Riemann hypothesis is true if and only if the inequality \( \sigma(n) < e^{\gamma}n \log \log n \) holds for every integer \( n > 5040 \), where \( \gamma \) is the Euler-Mascheroni constant. In this paper we establish a new family of integers for which Robin’s inequality \( \sigma(n) < e^{\gamma}n \log \log n \) hold. Further, we establish a new unconditional upper bound for the sum of divisors function. For this purpose, we use an approximation for Chebyshev’s \( \theta \)-function and for some product defined over prime numbers.

MSC:  
11A25 Arithmetic functions; related numbers; inversion formulas  
11N56 Rate of growth of arithmetic functions

Keywords:  
Riemann hypothesis; Robin’s inequality; sum of divisor function

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