Initial state fluctuations and final state collectivity in high energy nuclear collisions: Status and Outlook

Björn Schenke
Physics Department, Brookhaven National Laboratory, Upton, NY 11973, USA
E-mail: bschenke@bnl.gov

Abstract. We review recent developments in the theoretical description of the initial state and collective dynamics in heavy ion and light-heavy ion collisions. After describing the current state of the art in event by event simulations of high energy nuclear collisions we will focus on open issues. These include early time non-equilibrium dynamics, small collision systems (p+p, p+A), hydrodynamic fluctuations, and lower beam energies (\(\sqrt{s} < 200\) GeV).

1. Introduction
A standard description of the bulk dynamics in heavy ion collisions has been established over the last several years. It includes the following components:

- An initial state description including fluctuations of nucleon positions and subnucleonic color charges
- A rapid transition to the applicability of viscous fluid dynamics
- Second order viscous relativistic fluid dynamics to describe the bulk of the evolution
- An equation of state provided by lattice Quantum Chromo Dynamics (QCD) calculations
- Description of the system at lower temperatures using microscopic transport simulations

We will discuss the various components in more detail below, focusing on the latest developments. We will then describe open issues and future directions.

2. Initial state
Over the last two years the modeling of the fluctuating initial state in high energy heavy ion collisions has advanced significantly. In the high energy limit, which is expected to be a good approximation at LHC energy (and possibly top RHIC energies), the initial energy momentum tensor can be computed from the gluon fields that are produced by two colliding sheets of color glass [1]. This description within an effective theory of quantum chromo dynamics (QCD) was established early on [2] and later implemented numerically [3, 4]. In [5, 6] it was significantly refined to cover finite extent and fluctuations in the transverse plane of the collision. Further, the color charge density distribution of the incoming nuclei was constrained using the IP-Sat model [7, 8] with parameters fit to HERA deeply inelastic scattering (DIS) data [9].

The big advantages of this IP-Glasma model are the reduction of free parameters compared to e.g. a Monte Carlo (MC) Glauber model [10] by using fits to DIS data, the inclusion of
subnucleonic fluctuations that naturally lead to negative binomial distributions of multiplicities [5, 11], and the inclusion of pre-equilibrium evolution via classical Yang-Mills dynamics.

The NeXus [12] and EPOS [13] models have similar physics included, do not, however, describe the initial stage of the collision by classical gluon fields but strings.

Unfortunately, for lower energy collisions, the use of the high energy limit is problematic. Currently, low energy collisions are possibly best described by the non-equilibrium initial conditions generated by the initial collisions and string fragmentations in the microscopic UrQMD model [14]. Also AMPT [15, 16] may be used to determine initial states for lower energy collisions [17]. These models are less directly connected to a fundamental theory, and generally at the lower energies more modeling is required.

In addition to continuous work on deriving appropriate initial state models, some research is focused on determining constraints on the initial state from experimental data. An important part of this is to find good estimators of final flow observables that can be computed from properties of the initial state alone. The simplest estimators are the corresponding eccentricities, however, the non-linearity of hydrodynamics requires the inclusion of nonlinear contributions to the estimators, particularly for the higher harmonics [18]. In [19] constraints on eccentricity values that are compatible with experimental data were derived. This can help to determine the quality of an initial state model without having to run many event by event simulations. Viscous effects on this mapping between the initial and final state were discussed in [20].

Related studies focus on the probability distributions of initial eccentricities and how they characterize final state flow distributions and their moments. It is found in [21] that the initial rms anisotropy in harmonic $n$ can be directly extracted from the measured ratio $v_n\{4\}/v_n\{2\}$, which gives direct access to a property of the initial density profile from experimental data. This holds for both heavy ion and p+A collisions.

3. Viscous fluid dynamics and transport
Following the success of ideal relativistic fluid dynamics [22] in describing the main characteristics of experimental data, second order† viscous relativistic fluid dynamics [23, 24, 25, 26, 27, 28, 29, 30, 31] allowed to refine simulations and improve the comparison with experimental data, in particular when also including realistic equations of state, determined from lattice QCD calculations [32, 33]. Most importantly, the combination of sophisticated initial state models and event by event viscous fluid dynamics allowed to determine the most precise constraints on the transport properties of the quark gluon plasma to date [34, 35, 36].

The mentioned “second order” refers to a gradient expansion of the kinetic equations. There exist different ways to perform this expansion in the literature. Israel and Stewart [37, 38, 39] used an expansion of the distribution function around its local equilibrium value in a series of reducible Lorentz tensors composed of particle four-momentum $k^\mu$. They then truncated the expansion at second order in momentum, which leaves 14 unknown coefficients. Matching these coefficients to the 14 components of the particle four-current and energy momentum tensor (using the zeroth, first, and second moment of the Boltzmann equation) leads to the fluid dynamic equations. This approach however lacks a parameter that allows a systematic power-counting of corrections to the local equilibrium distribution function.

In a more recent derivation from kinetic theory [40, 41] an orthonormal basis is used in the expansion and exact relations between the expansion parameters and irreducible moments of the deviations of the distribution function from equilibrium are obtained. All terms in the expansion are kept, and the reduction of the degrees of freedom is done by identifying the microscopic time scales of the Boltzmann equation and keeping only the slowest ones. In addition, the equations

† The first order, or Navier Stokes formalism is conceptually simpler but introduces unphysical super-luminal signals that lead to numerical instabilities.
of motion for the dissipative quantities are truncated according to a systematic power-counting scheme in Knudsen and inverse Reynolds numbers. The resulting fluid dynamic equations are the current state of the art for simulations of heavy ion collisions.

Another recently developed approach is anisotropic hydrodynamics, where the gradient expansion is not done around an isotropic thermal distribution but one that has an arbitrarily large momentum anisotropy. This allows to systematically take into account the initially large difference between longitudinal and transverse pressure in heavy ion collisions [42, 43, 44].

Employing fluctuating IP-Glasma initial conditions \(v_1 - v_5\) at both RHIC and LHC energies were computed using viscous relativistic fluid dynamics in Ref. [35]. Best agreement with experimental data is achieved for shear viscosity values \(\eta/s = 0.12\) at RHIC and 0.2 at LHC. This hints at a temperature dependence of \(\eta/s\), making the QGP a less perfect liquid at higher temperatures. A detailed study of the effect of a temperature dependent \(\eta/s\) was conducted in [45]. A specific parametrization of \((\eta/s)(T)\) in [35] lead to reasonable simultaneous agreement of computed \(v_n\) coefficients with both RHIC and LHC data.

Bulk viscosity is now more commonly being included in fluid-dynamic simulations. The simulation in [46] uses a constant \(\zeta/s\), while bulk viscosity has been studied in detail in Ref. [47, 48], where it was found that its inclusion increases all \(v_n\) coefficients, opposite to the effect of shear viscosity. For ultra central events, where hydrodynamic calculations typically over predict the ratio of \(v_2/v_3\), bulk viscous effects [49] and the usually neglected nucleon-nucleon correlations in the incoming nuclei [50] help to improve agreement with experimental data.

Finally, it is worth noting that new exact solutions including radial expansion [51] have now been used for detailed tests of numerical implementations in certain limits [52], giving additional validation to the numerical codes that are being actively used.

4. Open issues

In the following we discuss some pressing outstanding issues in the description of high energy nuclear collisions whose solution will lead to significant advances in our understanding of nuclear matter under extreme conditions.

4.1. Pre-equilibrium dynamics

In the discussion above, we have assumed that we can directly go from the initial state to a fluid dynamic description of the medium, which involves the assumption of local thermal equilibrium and momentum isotropy (up to viscous corrections). However, the initial state is far from equilibrated and the initial dynamics will be out of equilibrium. The question remains if and how quickly the system can become thermalized and whether one can describe the process in the weak coupling limit.

In this limit, instabilities [53], triggered by quantum fluctuations, and subsequent strong scattering of over-occupied fields, may lead to rapid isotropization and quenching of \(\Pi^{\mu\nu}\) to reasonable values justifying the use of viscous hydrodynamics already at early times. These unstable dynamics require a full 3+1 dimensional simulation including a realistic description of quantum fluctuations. Significant progress is being made in this direction [54, 55, 56, 57, 58], however, some disagreement remains about the correct procedure to obtain results for realistic couplings within classical statistical real time lattice simulations [59].

In the strong coupling limit, which is accessible for a conformal field theory via the conjectured AdS/CFT correspondence [60], it was found that the system can be described by viscous hydrodynamics for times as early as \(\tau \sim 0.25\text{ fm}\) [61]. Unfortunately, the detailed QCD process that leads to such “hydrodynamization” is completely inaccessible in this approach.
4.2. Small collision systems

High-multiplicity events of proton-lead, deuteron-gold, and $^3$He-gold collisions produce $v_n$ values strikingly similar to those in heavy-ion collisions of comparable final-state particle multiplicity [62, 63, 64, 65, 66]. This observation has triggered strong interest in determining whether the same mechanism as in heavy ion collisions, namely collective flow, is responsible for generating the measured $v_n$, or whether multi-particle correlations from the initial state can explain the phenomenon.

Several calculations that use fluctuating initial states and final state collective effects can successfully explain the observed $v_n$ in p+A collisions [67, 68, 69, 70]. In particular the observed mass hierarchy of identified particle elliptic flow is naturally reproduced by models based on fluid dynamics [71, 69]. Viscous hydrodynamic calculations using the IP-Glasma model under-predict $v_2$ and $v_3$ significantly [72]. The main reason for this is the assumption of a mostly spherical proton which leads to insufficient initial shape fluctuations. This becomes even clearer when studying d+Au and $^3$He+Au collisions in the same model [73]. Here the obtained $v_2$ and $v_3$ are significantly larger. Introducing a more granular substructure of the proton as suggested in [74] could help improve the agreement with experimental data in p+Pb collisions.

One should question whether the system size and viscous corrections in such small systems allow for the application of hydrodynamics. In [75] it was found that in p+Pb collisions Knudsen numbers are beyond the applicability limit almost during the whole evolution even when using a small constant $\eta/s = 0.08$. This does not mean that the main physics driving the measured anisotropies is not that of final state effects. It means, however, that results obtained using viscous fluid dynamic simulations are not necessarily to be trusted.

There are alternative theoretical explanations for the measured multi-particle correlations and momentum anisotropies. Saturation models have reproduced correlation data over a wide kinematic range, [76, 77] showing that additional hydrodynamic flow is in fact required to describe the data from heavy-ion collisions, but has no room in smaller systems like p+p or p+A [76]. Various other calculations [78, 79, 80] also indicate that momentum anisotropies can be generated without the need of final state collective effects.

It is still not settled what effect mainly produces the observed long range correlations and momentum anisotropies in small systems like p+p and p+A. It is however clear that the study of light-heavy ion collisions is a rich subject that will lead to deep insight into the mechanism of particle production in QCD.

4.3. Hydrodynamic fluctuations

Due to the finite number of particles, additional fluctuations should occur during the hydrodynamic evolution. Owing to the fluctuation-dissipation theorem, the amplitude of these hydrodynamic fluctuations is governed by the viscosities [81, 82]. This offers the possibility to constrain the viscosity of the QGP independently from the discussed anisotropic flow analysis.

Recently, hydrodynamic fluctuations have been implemented in a fully 3+1 dimensional viscous fluid dynamic simulation [83]. The results obtained in this work show that the contribution of hydrodynamic fluctuations to event by event fluctuations of observables in most situations is sub-leading compared to the contribution from initial state fluctuations. However, in ultra central collisions the average values and variances of computed $v_n$ including only hydrodynamic fluctuations are on the order of the values obtained experimentally. Hence, a calculation combining initial state fluctuations and thermal fluctuations for ultra central events, could potentially explain the data and provide an alternative measurement of $\eta/s$ [83].

4.4. Low energy collisions

Describing lower energy collisions using fluid dynamics comes with several challenges. First, the assumption of zero baryon chemical potential $\mu_B$, which is good at high energies, does no
longer hold. It is necessary to include baryon number conservation and an equation of state with finite $\mu_B$. The latter cannot be directly computed on the lattice because of the sign problem. However, at small values of the baryon chemical potential, the construction of such an equation of state using a Taylor expansion of the pressure \cite{84} is possible \cite{85, 86, 87}. Another challenge is the determination of a realistic initial state. Most likely the high energy limit and hence the color glass condensate and glasma framework will not provide a good description for relatively low energy collisions. Thus, one will have to resort to string models as mentioned above. Further, baryon diffusion can play an important role in addition to shear and bulk viscosity \cite{88} and microscopic hadronic descriptions \cite{89, 90, 91, 92, 93} potentially gain in importance relative to the hydrodynamic stage. All of these aspects make the description of low energy collisions a complex task that needs significant theoretical development. Comprehensive simulations including all relevant physics and sources of fluctuations will be necessary to determine the detailed phase structure of QCD by comparison of observables with experimental data.

Acknowledgments

BPS is supported under DOE Contract No. DE-AC02-98CH10886 through a DOE Office of Science Early Career Award.

References

[1] Gelis F, Iancu E, Jalilian-Marian J and Venugopalan R 2010 Ann. Rev. Nucl. Part. Sci. 60 463–489
[2] Kovner A, McLerran L D and Weigert H 1995 Phys. Rev. D52 6231–6237
[3] Krasnitz A and Venugopalan R 2000 Phys. Rev. Lett. 84 4309–4312
[4] Lappi T 2003 Phys. Rev. C67 054903
[5] Schenke B, Tribedy P and Venugopalan R 2012 Phys. Rev. Lett. 108 252301
[6] Schenke B, Tribedy P and Venugopalan R 2012 Phys. Rev. C86 034908
[7] Bartels J, Golec-Biernat K J and Kowalski H 2002 Phys. Rev. D66 014001
[8] Kowalski H and Teaney D 2003 Phys. Rev. D68 114005
[9] Rezaeian A H, Siddikov M, Van de Klundert M and Venugopalan R 2013 Phys. Rev. D87 034002
[10] Miller M L, Reygers K, Sanders S J and Steinberg P 2007 Ann. Rev. Nucl. Part. Sci. 57 205–243
[11] Schenke B, Tribedy P and Venugopalan R 2014 Phys. Rev. C89 024901 (Preprint 1311.3836)
[12] Drescher H, Ostapchenko S, Pierog T and Werner K 2002 Phys. Rev. C66 044902 (Preprint hep-ph/0011219)
[13] Werner K, Karpenko I, Pierog T, Bleicher M and Mikhailov K 2010 Phys. Rev. C82 044904
[14] Petersen H, Steinheimer J, Burau G, Bleicher M and Stocker H 2008 Phys. Rev. C78 044901
[15] Zhang B, Ko C M, Li B A and Lin Z W 2000 Phys. Rev. C61 067901
[16] Pang L, Wang Q and Wang X N 2014 Phys. Rev. C89 064910 (Preprint 1309.6735)
[17] Solanki D, Sorensen P, Basu S, Raniwala R and Nayak T K 2013 Phys. Lett. B720 352–357
[18] Gardim F G, Grassi F, Luzum M and Ollitrault J Y 2012 Phys. Rev. C85 024908 (Preprint 1111.6538)
[19] Retinskaya E, Luzum M and Ollitrault J Y 2014 Nucl. Phys. A926 152–158 (Preprint 1401.3241)
[20] Gardim F G, Noronha-Hostler J, Luzum M and Grassi F 2014 (Preprint 1411.2574)
[21] Yan L and Ollitrault J Y 2014 Phys. Rev. Lett. 112 082301 (Preprint 1312.6555)
[22] Kolb P F, Solfrank J and Heinz U W 2000 Phys. Rev. C62 054909
[23] Baier R, Romatschke P, Son D T, Starinets A O and Stephanov M A 2008 JHEP 04 100
[24] Romatschke P and Romatschke U 2007 Phys. Rev. Lett. 99 172301
[25] Dasu T D and Teaney D 2008 Phys. Rev. C77 034905
[26] Luzum M and Romatschke P 2008 Phys. Rev. C78 034915
[27] Song H and Heinz U W 2009 J. Phys. G36 064033 (Preprint 0812.4274)
[28] Song H, Bass S A, Heinz U, Hirano T and Shen C 2011 Phys. Rev. Lett. 106 192301
[29] Schenke B, Jeon S and Gale C 2011 Phys. Rev. Lett. 106 042301
[30] Schenke B, Jeon S and Gale C 2011 Phys. Lett. B702 59–63
[31] Bozek P 2012 Phys. Rev. C85 034901
[32] Huovinen P and Petreczky P 2010 Nucl. Phys. A837 26–53
[33] Bazavov A, Bhattacharya T, Cheng M, DeTar C, Ding H et al. 2012 Phys. Rev. D85 054503
[34] Luzum M and Ollitrault J Y 2013 Nucl. Phys. A904-905 377c–380c (Preprint 1210.6010)
[35] Gale C, Jeon S, Schenke B, Tribedy P and Venugopalan R 2013 Phys. Rev. Lett. 110 012302
[36] Gale C, Jeon S and Schenke B 2013 Int. J. Mod. Phys. A28 1340011 (Preprint 1301.5893)
[37] Israel W 1976 Ann. Phys. 100 310–331
