A Neural Network based Frequency-domain Design Method for the Optimal Fractional Order PI\(^{\lambda}\)D\(^{\mu}\) Controller

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Abstract. A neural network (NN) based frequency-domain design method for the optimal PI\(^{\lambda}\)D\(^{\mu}\) controller is proposed in this paper, dividing the tuning process of the PI\(^{\lambda}\)D\(^{\mu}\) controller into two steps: the fractional orders of the controller are estimated by the NN based estimation models, and other controller parameters are calculated analytically using a modified frequency-domain method. A practical application of the proposed method on the permanent magnet synchronous motor (PMSM) speed control problem is studied. Motor speed control simulation is performed to verify the gain robustness, step response performance and anti-load disturbance performance of the obtained control system. Comparison with the typical frequency-domain method is performed and then the advantage of the proposed method is demonstrated.

1. Introduction

In recent years, as the increasing application of the fractional calculus in science and engineering, fractional calculus has drawn the interests of scholars in the modeling and control field \([1,2]\). In the field of system modeling, the characteristics of many real physical systems have been found to be well described by fractional order differential equations \([3,4]\). In the control theory and application field, fractional order controllers are being studied and used in order to achieve better control performance over the traditional integer order controllers \([5-8]\).

The proportional integral derivative (PID) controller, which is simple in structure and easy to implement, is the most widely used controller in industrial applications. As an extension of the PID controller, the fractional order PI\(^{\lambda}\)D\(^{\mu}\) controller provides more flexibility in the design and control scope than the traditional PID controller, because the integral order \(\lambda\) and derivative order \(\mu\) are extended to be real numbers \([9]\). Therefore, the fractional order PI\(^{\lambda}\)D\(^{\mu}\) controller has the potential to offer better control performance over the traditional PID controller. However, with two extra degrees of freedom, the tuning of the PI\(^{\lambda}\)D\(^{\mu}\) controller is more complicated.

A neural network (NN) based frequency-domain design method for the optimal PI\(^{\lambda}\)D\(^{\mu}\) controller is proposed in this paper. The tuning process of the PI\(^{\lambda}\)D\(^{\mu}\) controller is divided into two steps: the optimal fractional orders \(\lambda\) and \(\mu\) are estimated by two NN based estimation models, according to the design indices and two selected features of the plant model; then the other controller parameters are calculated analytically using a modified frequency-domain method, based on the estimated \(\lambda\) and \(\mu\). A practical application of the proposed method on the permanent magnet synchronous motor (PMSM) speed control problem is studied. Motor speed control simulation is performed to verify the gain robustness, step response performance and anti-load disturbance performance of the obtained control system. Comparison with the typical frequency-domain method is performed and then the advantage of the proposed method is demonstrated.
robustness, step response performance and anti-load disturbance performance of the control system obtained using the proposed method. Besides, the performance of the obtained control system is compared with that of the control system obtained using the typical frequency-domain design method, demonstrating the advantage of the proposed method.

2. Frequency-domain method based on fractional order estimation

Adopting the PI$^\lambda D^\mu$ controller, the typical feedback control system is shown in figure 1, where $r$ is the reference input, $y$ is the system output, $G(s)$ is the plant model and $C(s)$ is the PI$^\lambda D^\mu$ controller.

![Figure 1. The feedback control system with the PI$^\lambda D^\mu$ controller.](image)

The transfer function of the fractional order PI$^\lambda D^\mu$ controller is described by

$$C(s) = K_p \left(1 + \frac{K_i}{s^\lambda} + K_d s^\mu\right),$$  \hspace{1cm} (1)

where $K_p$, $K_i$, and $K_d$ are the proportional, integral and derivative gains, respectively; $\lambda$ and $\mu$ are the integral and derivative orders ranging from 0 to 2.

According to a typical frequency-domain design method, the control system should satisfy the “flat-phase” specification, namely, the phase characteristic curve should be flat at the gain crossover frequency,

$$\frac{d[\text{Arg}(G(j\omega)C(j\omega))]}{d\omega} = 0, \hspace{1cm} (2)$$

where $\omega_c$ is the given gain crossover frequency, satisfying

$$|G(j\omega_c)C(j\omega_c)| = 1, \hspace{1cm} (3)$$

The phase margin $\phi_m$ of the control system is also given, satisfying

$$\text{Arg}[G(j\omega)] + \text{Arg}[C(j\omega)] = -\pi + \phi_m. \hspace{1cm} (4)$$

Assuming that the fractional orders $\lambda$ and $\mu$ have been obtained, the pending parameters of the controller are reduced from five to three. According to (4), an equation about $K_i$ and $K_d$ is derived,

$$K_d = s_i K_i + s_0, \hspace{1cm} (5)$$

where

$$s_i = \frac{T \omega_c^{-\lambda} \cos\left(\frac{\pi}{2} \lambda\right) + \omega_c^{-\lambda} \sin\left(\frac{\pi}{2} \lambda\right)}{\omega_c^{\mu} \sin\left(\frac{\pi}{2} \mu\right) - T \omega_c^{\mu} \cos\left(\frac{\pi}{2} \mu\right)}, \hspace{1cm} (6)$$

$$s_0 = \frac{T}{\omega_c^{\mu} \sin\left(\frac{\pi}{2} \mu\right) - T \omega_c^{\mu} \cos\left(\frac{\pi}{2} \mu\right)}, \hspace{1cm} (7)$$

$$T = \tan(-\pi + \phi_m - \text{Arg}[C(j\omega_c)]), \hspace{1cm} (8)$$
According to (2), another equation about $K_i$ and $K_d$ is also obtained,
\[
\mu_\omega^{-1} \sin \left( \frac{\pi}{2} \mu \right) K_d + (\lambda + \mu) \omega \mu^{-1} \sin \left( \frac{\pi}{2} (\lambda + \mu) \right) K_i + \lambda \omega^{-1} \sin \left( \frac{\pi}{2} \lambda \right) K_i
\]
\[
+ 2M \omega_\mu^{-1} \cos \left( \frac{\pi}{2} (\lambda + \mu) \right) K_d K_i + 2M \omega_\mu \cos \left( \frac{\pi}{2} \mu \right) K_d + M \omega^{-2} \lambda^2 K_i
\]
\[
+ \omega_\mu^{-2} K_d^2 + 2M \omega_\mu^{-1} \cos \left( \frac{\pi}{2} \lambda \right) K_i + M = 0,
\]
where
\[
M = \frac{d[\text{Arg}(G(j\omega))]}{d\omega} \bigg|_{\omega=\omega_c}.
\]

Therefore, the values of $K_i$ and $K_d$ can be calculated according to (5) and (9). Besides, $K_p$ can be obtained according to (3).

3. NN based estimation models

The fractional orders $\lambda$ and $\mu$ should be determined before other controller parameters ($K_i$, $K_d$ and $K_p$) are calculated. In order to ensure good dynamic performance of the control system, the NN is used to model the optimal distribution of $\lambda$ and $\mu$ with respect to plant model’s features and design indices.

3.1. Samples collection

The input vector of the estimation model input is determined as $[\varphi(\omega_c), d\varphi(\omega_c), \omega_c, \varphi_m]^T$, where $\omega_c$ and $\varphi_m$ are the given gain crossover frequency and phase margin, $\varphi(\omega_c)$ represents the phase of the model at $\omega_c$, while $d\varphi(\omega_c)$ represents the derivative with respect to phase at $\omega_c$. The estimation model of the fractional order is constructed within a subspace specified by $\varphi(\omega_c)$, $d\varphi(\omega_c)$, $\omega_c$ and $\varphi_m$. In this paper, the estimation models of $\lambda$ and $\mu$ are studied focusing on their actual applications on a class of the PMSM speed control system. Therefore, the range of $\varphi(\omega_c)$ is set from -90° to -180°. Besides, the range of $\omega_c$ is set from 30 rad/s to 70 rad/s and that of $\varphi_m$ is set from 30° to 70°, covering the general design requirements of this class of motion control systems [10].

According to the ranges of $\omega_c$ and $\varphi_m$, several values of $\omega_c$ and $\varphi_m$ are selected to be the given design indices. Besides, the typical second-order and third-order systems are selected to be the test models to construct the training samples. Therefore, the input vector can be obtained according to the design indices ($\omega_c$, $\varphi_m$) and test models' features ($\varphi(\omega_c)$, $d\varphi(\omega_c)$).

The optimal values of $\lambda$ and $\mu$ for each test model and design index pair ($\omega_c$, $\varphi_m$) are selected according to the dynamic performance of the corresponding control systems. A loss function is introduced to evaluate the dynamic performance of the control system,
\[
J = \kappa_1 \int_0^t \|e(t)\|^2 dt + \kappa_2 \int_0^t \|u(t)\|^2 dt,
\]
where $e(t)$ represents the deviation between the desired and actual output of the control system, $u(t)$ represents the controller’s output, $\kappa_1$ and $\kappa_2$ are the weights balancing the dynamic performance and the energy consumption of the control system.

The optimal values of $\lambda$ and $\mu$ are obtained for each test model and design index pair ($\omega_c$, $\varphi_m$) using an enumeration method. A training sample is constructed by combining the input vector $[\varphi(\omega_c), d\varphi(\omega_c), \omega_c, \varphi_m]^T$ and the output ($\lambda$, $\mu$). The steps of the enumeration method are listed below.

1) Select several values of $\lambda$ and $\mu$ uniformly among their value ranges.
2) Calculate the PI'D' controller parameters for each ($\lambda$, $\mu$) pair, based on the design indices and plant model’s features, using the modified frequency-domain method.
3) Perform the step response simulation and calculate the loss functions of the control systems corresponding to different values of $\lambda$ and $\mu$.

4) Select the $(\lambda, \mu)$ pair with the smallest loss function to be the optimum for each test model and design index pair.

5) Check the terminal condition: if the accuracy of the obtained $\lambda$ and $\mu$ is smaller than a threshold $\delta$, go to step 6; otherwise, build a smaller value range around the current optimal value, go to step 1.

6) Set the optimal fractional orders pair $(\lambda, \mu)$ for each test model and design index pair as the output of the input vector $[\varphi(\omega_c), d\varphi(\omega_c), \omega_c, \varphi_m]^T$.

3.2. Estimation model establishment

The obtained samples are divided into three groups: the training set, the validation set and the test set, which are used for network weights training, stopping point determination and generalization ability testing.

A three-layer perceptron network (input layer, hidden layer and output layer) is adopted to build the estimation model. The sigmoid function is selected to be the activation function of the hidden layer, while the pure linear function is used in the output layer. The mean squared error (MSE) is selected as the optimization objective function and the Levenberg-Marquardt (LM) algorithm is adopted to train the network weights.

The NN based estimation model of $\mu$ is shown in figure 2, with $[\varphi(\omega_c), d\varphi(\omega_c), \omega_c, \varphi_m]^T$ as the input vector, $\mu$ as the output. In order to improve the estimation accuracy of $\lambda$, $\mu$ is added into the input vector of the estimation model of $\lambda$, namely, $[\varphi(\omega_c), d\varphi(\omega_c), \omega_c, \varphi_m, \mu]^T$. Thus, the estimation model of $\lambda$ is shown in figure 3.

4. Application to PMSM speed control

The NN based frequency-domain method is applied to design the PI'D' controller for a PMSM speed control system. The model of a PMSM can be converted into a DC motor model via the coordinate transformation from the static three-phase coordinates to the rotary d-q coordinates. Adopting the parameters obtained using an output error based modelling method [3], the transfer function of the PMSM plant model is obtained,

$$G(s) = \frac{47979.26}{s^3 + 127.38s^2 + 9995.68s}.$$  \hspace{1cm} (12)

4.1. Gain robustness study

Given the gain crossover frequency $\omega_c = 30\text{rad/s}$, the phase margin $\varphi_m = 53^\circ$, the fractional orders $\lambda$ and $\mu$ are estimated by the NN based estimation model: $\lambda = 1.0074$, $\mu = 1.0553$. Therefore, the fractional order PI'D' controller is obtained applying the modified frequency-domain method,
The open-loop Bode plot of the control system is shown in figure 4. According to figure 4, the gain crossover frequency $\omega_c$ is 30rad/s and the phase margin $\phi_m$ is 53°, satisfying the design indices. Besides, the phase characteristic curve of the control system is flat at $\omega_c$, satisfying the flat-phase specification. Thus, the overshoots of the step responses under gain variations should be close to each other. In order to verify the gain robustness, the gain $K$ of the plant model is multiplied by 1.2 and 0.8, respectively, to simulate tiny plant model gain variations.

$$C_1(s) = 6.055 \left(1 + \frac{10.659}{s^{0.0074}} + 0.0027s^{1.0551}\right).$$ (13)

Setting the reference speed to be 1000rpm, the motor speed step response simulation is performed on the control systems with gain variations. The step responses are shown in figure 5. According to figure 5, the overshoots of the response of the control system with gain variations are close to each other. Therefore, the gain robustness requirement of the control system is satisfied.

4.2. Comparison with the typical frequency-domain method

In order to verify the advantage of the proposed method, a fractional order PI$^\lambda$ controller is designed using the typical flat-phase frequency-domain method [11]. Given the same design indices ($\omega_c = 30\text{rad/s}$, $\phi_m = 53^\circ$), the fractional order PI$^\lambda$ controller is obtained,

$$C_2(s) = 7.336 \left(1 + \frac{40.391}{s^{0.465}}\right).$$ (14)
Setting the reference speed to be 1000rpm, PMSM speed step response and load disturbance simulation is performed, using the PI^D^μ controller and PI^e^ controller to control the motor speed, respectively. The step responses of two control systems are shown in figure 6. The load disturbance responses are shown in figure 7.

According to figure 6, the step response of PI^D^μ control system has smaller overshoot and shorter settling time. Therefore, the PI^D^μ control system obtained using the proposed method achieves better step response performance. Besides, according to figure 7, the speed drop of two control systems is close to each other, but the response of the PI^e^ control system has obvious oscillation and longer recovery time. Therefore, the PI^D^μ control system obtained using the proposed method achieves better anti-load disturbance response performance.

5. Conclusions
A frequency-domain method for the optimal PI^D^μ controller based on NN estimation is proposed. The NN based estimation models for the optimal fractional orders are established. The controller parameters can be calculated analytically using a modified frequency-domain method, based on the estimation of λ and μ. The proposed method is applied to design the PI^D^μ controller for the PMSM speed control system. Simulation is performed to verify the gain robustness, step response performance and anti-load disturbance performance of the control system obtained using the proposed method. The advantage of the proposed method is demonstrated by the comparison with the existing frequency-domain method.

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