Mathematical Modeling of Survivability Function for Thermoelectric Module

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Abstract. Developing an optimized reliability model for thermoelectric module at the stress where the probability of module to functions without abruptive failure is a challenging aspect. One of the major reasons is the mismatch of thermal expansion coefficient, which has severe effects on segmented moduli compared to unsegmented moduli. The likelihood of a thermoelectric module to survive at certain level of thermo-mechanical stresses varies by varying number of component (layers) in thermoelectric leg. On another hand, selection of an adequate distribution model to predict reliability and sustainability of the thermoelectric module requires development of new optimized stress-strength-based model. In this paper the predictive reliability model for high temperature segmented module is derived from parametric Lognormal mean residual life and nonparametric Lognormal-kernel survival function to measure probability of module to survive at certain thermo-mechanical stress. A comprehensive comparative discussion has been done to illustrate the maximum likelihood based on Bayesian nonparametric lognormal-Kernel inference method regarding to Monte Carlo simulation, Weibull’s distribution, and Lognormal mean residual life for various shapes for the survival function. It has been demonstrated that nonparametric lognormal-kernel survival function has high ratio of probability to predict the survival of module at higher discrete thermo-mechanical stress data.

1. Introduction
Among the other feasible technologies for this purpose, thermoelectric (TE) energy is an interesting viable alternative because the TE module (shown in Figure 1) can convert given heat, through different sources, into electric power using the Seebeck affect without requiring moving components [1]. Although, this particular character provides a reliable source of electrical power for space applications but terrestrial applications, heat recovery and automobile industry, confront limitations. For terrestrial application the major drawbacks for TE module is the fragile reliability [2]. The major reason behind this vary fact is dynamic operational condition that subject module thermomechanical fatigue and structural failure [3]. Thermomechanical fatigue is a product of thermomechanical stress produced due to thermal expansion mismatch between different TE materials [4]. Another significant challenge is increase of temperature difference between thermoelements in order to increase the conversion efficiency. The increase in the temperature gradient intensifies operating conditions, which cause chemical and mechanical instability in TE devices [5].
Potential candidacy of TE module to be environmentally friendly technology have led to intensive work over the past 15-20 years [6]. Series of studies have been conducted to produce mechanically reliable high performance TE modules by evaluating and limiting the operational stresses. These studies indicated that high level of thermomechanical stresses comes from material’s properties, mechanical durability, and module’s geometry. Hee Seok Kim at el. [7] studied cumulative temperature dependence to reduce the gap between material and device by engineering thermal conductivity. The paper suggests that an accurate evaluation of p-n configuration and mechanical characteristics of interfacial structures can solve thermomechanical reliability of the module. Moo-Yeon Lee at el. [8] paper did a comprehensive discussion on leg’s geometry and different material’s compatibility with structure. Investigation suggests that cylindrical shape segmented leg provides 13.6% efficiency at thermal stress of 0.69 GPa. Whereas Naveen K. Karri at el. [3] and Suhir’s [9] mathematical model, on TE legs, gives insight to understand effect of shear stress between the layers and at the boundaries. Malzbender’s [10] work demonstrates that the stiffness, thickness and thermal expansion of a material are active variable to influence overall stress level of the module. Z.-H. Jin [11] continued Malzbender’s model and predicted that failure can occur in multilayer leg, if length to thickness ratio increases. G. Nikolova [12] developed a comprehensive mathematical model to study thermal and mechanical behavior of bonded layers. She has demonstrated that debonding (failure) of layers happen when module reaches at its critical shearing stress.

On another hand, modelling probabilistic reliability for TE module entails many challenges. Much of the existing experimental studies apply accelerated testing method to calculate mean-time-between-failure (MTBF) based on number of cycles to failures [13]. Whereas, the ceramics and glasses experimental studies apply Weibull’s distribution for reliability calculation [14]. Naveen K. Karri [3] and Andrew A. Wereszczak [15] studied structural reliability to evaluate thermomechanical stress using brittle material failure theory based on Weibull distribution. The paper described significant gain in reliabilities on small reduction in maximum principle stress. Consequently, the shape of TE leg has less effect on estimated probability of failure. Alternative to this notion, we published a paper [16], presenting simulative results, where lognormal distribution is used to calculate failure rate. Generally lognormal distribution is used to measure the rate of failure for micro-electric devices at high temperature stress. The distribution is based on the multiple failure model, which mean that at given temperature range the TE legs in a the given module undergoes a random increase of degradation, interlayer diffusion, sublimation or oxidation, leading to complete abruptive failure [17-19]. Therefore, the
use of lognormal distribution is mostly used to model components or devices that fail primarily due to stress or fatigue.

In this paper a comprehensive discussion is done around the fact that the module has survived some certain temperature gradient without any abruptive failure. An optimized survival function is presented for a segmented module that has survived a specific temperature range, to obtain mean residual life (MRL) for measuring the reliability of high temperature TE modules. This measure contains two aspects of information, the lifetime of a module and the temperature at which the module has ability to work operate without any failure.

A stress-strength model-based lognormal MRL function is obtained to differentiate various characteristics of segmented TE device at higher thermal stresses. This new model assists us to determine the temperature range at which the reliability of segmented device could be achieved above 90. Additionally, an optimized Bayesian nonparametric survival function is derived to measure the probability of segmented module to survive beyond the interface stress ($\sigma_y$).

2. Analytical model

For a given set of operating conditions, the reliability is defined as the probability that a system survives for some specific period of time. In our case, we drive our survival function on random variable “thermal stress ($\sigma$)” at specific temperature “T”. In order to operate efficiently, the generated thermal stress in TE module must be lower than the strength ($s$) of the overall module. The condition of reliability is that the TE module survives within modeled strength.

This condition can be mathematical expressed as $S = P(s > \sigma) = \int_{\sigma_0}^{\infty} f(s)ds$. Here $\sigma_0$ is initial mechanical stress before the temperature effect and $S$ is survival function. The survival function ($S$) of any TE module, with a probability that stress is less than the strength, where stress and strength are two independent variables, for all the possible values of the strength can be computed as

$$S = \int_{0}^{\infty} f_{S}(s) \left[ \int_{0}^{s} f_{\sigma}(\sigma) d\sigma \right] ds. \quad (1)$$

Then the Failure function can be given as case that the value of stress is higher than the strength of the device, that is, $F = 1 - S = P(s \leq \sigma)$. And for independent values of stress ($\sigma$) and strength ($s$) variables, the failure function is given as

$$F = \int_{0}^{\infty} F(s) \cdot f_{\sigma}(\sigma) d\sigma. \quad (2)$$

Since TE module is temperature dependent device, we assume here interface random real number ($y$) which is interim thermal stress at which module operates without any interruption, that is, $y = s - \sigma$. If we assume that $\sigma$ and $s$ are non-negative independent random variables then the ability to operate without interruption at interface temperature is given by $S (y > 0)$ and the P.D.F of the failure function ($y = 0$) can be determined through following solution

$$F_{y}(y)dy = f_{s}(s)ds \int_{s}^{\infty} f_{\sigma}(\sigma) d\sigma. \quad (3)$$

In equation (3) the value of strength is a fixed variable whereas the value of Stress has a random magnitude. In this scenario the sustainability of the device is assumed that all possible
value of $\sigma$ are less than the value of $s$ i.e. ($s > \sigma$). By assuming the range of $\sigma$ from $0 \to \infty$, the equation (3) can be solved as,

$$f_y(y)dy = f_S(s)ds \int_0^\infty f_S(y + \sigma) \cdot f_\sigma(\sigma)d\sigma. \quad (4)$$

Based on equation (4) reliability of the TE module can measured through survival function, given as $S = \int_0^\infty f_y(y)dy$, and the probability of failure can be written as

$$F = 1 - S = \int_{-\infty}^0 \int_0^\infty f_S(y + \sigma) \cdot f_\sigma(\sigma)d\sigma dy. \quad (5)$$

When we consider the reliability of segmented Module, the survival function of a system with multiple layers with multiple possible failure mode,

$$S = 1 - \prod_{i=1}^\varphi \left[ F_{n_i} \cdot F_{p_i} \right]. \quad (6)$$

Here, $S, \varphi$ and $F_{n_i} \cdot F_{p_i}$ present over all module survival function of TE module, number of significant failure mode and significant failure mode of $n$ and $p$ type TE leg with $i$ number of layers respectively.

As elaborated above that we study here segmented TE modules for high temperature operating atmosphere, we take lognormal distribution, instead of Weibull distribution. The strength-stress density function for lognormal distribution is derived [20] as

$$f_y = \frac{1}{y\delta\sqrt{2\pi}} \exp \left[ -\frac{1}{2\delta^2}(\ln y - \mu)^2 \right]. \quad (7)$$

Here, $y > 0$, and the parameters $\mu$ and $\delta$ are the mean and the standard deviation, respectively, of the variable $\ln y$. If we take log of interference condition, i.e. $\ln y = \ln s - \ln \sigma$, we get mean log values to get probability plot. The condition is $\bar{y} = \frac{\bar{s}}{\bar{\sigma}}$, where bar shows average log value for thermal stress and strength. Here the mean log value for strength is taken as ultimate stress at which the component materials in form of segmentation can survive at modeled temperature. The system is reliable if the probability of survival function is equal to 1 (see Figure 2) and probability of module to survive can be given as

$$S = P\left(\frac{s}{\sigma} > 1\right) = P(y > 1) = \int_1^\infty f_y(y)dy \quad (8)$$

The lognormal distribution provides, as shown in Figure 2, an Upside-Down Bathtub shape for MRL and survival function.

3. Mean Residual Life for Lognormal distribution

In reliability analysis, lifetimes are mostly taken as random variables when probability distributions are considered. In thermoelectric field most famous distribution used in reliability analysis is Weibull distribution, which is the most suitable model for modules operating $T >$
50°C [21]. But rising demand of high temperature modules limits Weibull’s distribution and requires new methods to predict accurately operating life of TE modules, especially segmented modules. In order to develop a mathematical model, there are five main characteristics which must be define by the reliability model and those are PDF, CDF, failure rate, survival function and MRL function. In this paper we focus on MRL function in order to develop a condensed information to measure precise reliability of the segmented TE module. The main purpose of MRL is to measure at which thermal stress value TE component works before the module faces abruptive failure. This give us two aspects of module, its lifetime and fact that module has been working to a certain thermal stress ($\sigma$) without halt.

Suppose $\sigma$ is a continuous non-negative random variable with CDF $F(\sigma_y)$, PDF $f(\sigma_y)$ and survival $S(\sigma_y) = 1 - F(\sigma_y)$. To define residual life random variable at threshold thermal stress ($\sigma_{Th}$), the life expectancy would be $\sigma_{Th} = \sigma - \sigma_y | \sigma > \sigma_y$ [22]. This implies that mean residual life (MRL) can be calculate as

$$M(\sigma_y) = E(\sigma - \sigma_y | \sigma > \sigma_y) = \frac{1}{S(\sigma_y)} \int_{\sigma_y}^{\infty} S(y) dy, \ \sigma_y \geq 0 \quad (9)$$

The lognormal distribution belongs to those distribution which have no closed form of survival function, so will use equation (9) to obtain the MRL function for lognormal distribution. In this regard, the PDF and CDF of a lognormal are given as

$$f(\sigma) = \frac{1}{\sigma \sqrt{2\pi}\delta^2} \exp \left[ -\frac{1}{2} \left( \frac{\ln \sigma_y - \mu}{\delta} \right)^2 \right] \quad (10)$$

$$F(\sigma_y) = \int_{0}^{\sigma_y} \frac{1}{\sigma_y \sqrt{2\pi}\delta^2} \exp \left[ -\frac{1}{2} \left( \frac{\ln \sigma_y - \mu}{\delta} \right)^2 \right] d\sigma_y \quad (11)$$

Therefore, the survival function TE module can be written as

$$S(\sigma) = 1 - \varphi \cdot \frac{\ln \sigma - \mu}{\delta} \quad (12)$$

Let $z_\sigma = \frac{\ln \sigma - \mu}{\delta}$, then $\sigma = \exp[z_\sigma \delta + \mu] \text{ and } \partial \sigma = \sigma \exp[z_\sigma \delta + \mu] dz_\sigma$. Main residual life for lognormal distribution will become [23]

$$M(\sigma) = \exp \frac{\mu + \delta^2}{2} \cdot \frac{1 - \varphi \cdot \frac{\ln \sigma_y - (\mu + \delta^2)}{\delta}}{1 - \varphi \cdot \frac{\ln \sigma_y - \mu}{\delta}} - \sigma \quad (13)$$

Figure 2 provides PDF (top left), CDF (bottom left), Survival function (Top right) and MRL function (bottom right). MRL function demonstrates reported number of TE modules that survive at specific thermal stress. As compare to Weibull survival function, where no transformation is necessary, both lognormal MRL and survival functions shows fitting graph for higher value of scale ($\delta$) and location ($\mu$) parameters. The domains of the survival functions, based on equation (13) for $\sigma \leq \sigma_y$ can take on values from 0 to infinity, however for $1 < \sigma_y < 0$ goes from 0 to $\sigma_{Th}/\sigma$. 


4. Non-Parametric Lognormal Survival Function

Nonparametric models, compared to parametric modelling, are mostly used to analyze failure data to estimate the MRL function [24]. Under extreme right censorship, the nonparametric model can provide an innovative method to deal with discrete and nonlinear censoring data. The relationship between nonparametric MRL and failure rate function can assist us to develop a better reliability model and effective system. Consequently, nonparametric Survival function aims to measure sustainability of a device based on their failure behaviors.

We used a Dirichlet process for obtaining a common nonparametric lognormal survival function [25]. In this regard we take density of survival function ($\hat{S}$) at threshold stress ($\sigma_{Th}$) with the respect of interference and strength stress. i.e.

$$\hat{S}(\sigma_y, G) = \int K(\sigma_y, \sigma_S)dG(\sigma_{Th})$$

(14)

Here $K$ is a lognormal distribution for the kernel, $G(\sigma_{Th})$ is nonparametric vector defined at threshold stress where we can still find our TE modules operating, the moment before module confronts abruptive failure. The nonparametric lognormal-Kernel distribution, for the discrete data based on stress-strength data distribution, can be given as

$$K(\sigma_y; \sigma_S = (\mu, \delta^2)) = \frac{1}{\sigma_y \sqrt{2\pi \delta^2}} \exp \left[ -\frac{1}{2} \cdot \left( \frac{\ln \sigma_y - \mu}{\delta} \right)^2 \right]$$

(15)

And nonparametric vector for a module to operate at threshold stress (surviving stress) can be defined as

$$G(\sigma_{Th}) = \sum_{\sigma=1}^{\infty} \omega_\sigma \delta_{f(\sigma)}(\sigma_{Th})$$

(16)

Here $f(\sigma)$ presents independent stress distributed identically over the module. And $\delta_{f(\sigma)} = \{1, 2 \ldots \}$ is the parametric indicative function of stress, which is the baseline distribution. $\omega_\sigma$
Figure 3. Comparative Nonparametric Survival Function graph for Segmented and Unsegmented present stick-breaking of discreteness and explicitly used to give random probability of discrete distribution. Since the distribution is random itself, its precision is derived from location parameter, define as $\{\delta_i\}_{i=1}^{\infty}$ and shape of survival $S(\sigma_y, G)$. The expansion of equation [13], we can locate the survival function curve, at $\sigma_y$ and $G(\sigma_{Th})$

$$\hat{S}(\sigma_y G(\sigma_{Th}) = \sum_{l=1}^{N} \rho \sigma_{Th} LN(\sigma_y; \mu, \delta^2)$$

(17)

Here $\rho$ comes from Bayesian inference to include posterior probability, which presents maximum likelihood of survival function derived from failure rate data. It is given as $\rho = E_v \prod_{i=1}^{v} (1 - E_{\sigma_y})$, where $E_v$ volumetric elastic constant of $i^{th}$ layer at $\sigma_y$ and $v_{\sigma_y} = \beta(1, \alpha)$. $\beta$ and $\alpha$ are two positive shapes defined as per Dirichlet process [26] and $i$ is the total number of components in the model. The nonparametric lognormal distribution mixed with Kernel distribution as per Dirichlet process model, for positive real number, is transformed as $Y = \log(\sigma) - \log(\sigma_y)$. The nonparametric survival probability for unsegmented module, to survival within domain of threshold stress but higher than interference stress, can be given as

$$\hat{S}(\sigma_y \leq \sigma \leq \sigma_{Th}; G) = Pr(\sigma \leq e^Y; G)$$

(18)

Respectively for segmented TE modules, the nonparametric probability can be given as

$$\hat{S}(\sigma_y \leq \sigma \leq \sigma_{Th}; G) = Pr(\sigma \leq e^Y; G)$$

(19)

$$\hat{S}(\sigma_y \leq \sigma \leq \sigma_{Th}; G) = \sum_{i \geq 1} \rho \varphi \cdot \frac{Y - \mu_i}{\delta_i}$$

(20)

Figure 3 shows the evaluation of survival function based obtained results for segmented and unsegmented TE legs. TE module working on high and low temperature result different ratio of
reliability. Low temperature unsegmented TE modules demonstrated higher rate of survival on given standard (4 mm$^3$) leg volume. One of the major reasons is that unsegmented TE module has low adverse effect from thermal expansion compare to segmented TE module. At given leg volume unsegmented modules has 95% survival rate within range of $10 \leq \sigma < 60$ (MPa). Whereas the 95% of survival rate for low temperature segmented TE module exists only between $0 \leq \sigma < 30$ (MPa), due to mismatch of thermal expansion between different materials. The main difference between simple MRL and Survival function and nonparametric survival function is rapid decline in curve of Figure 3 compare to Figure 2. MRL has definite method to predict reliability of module based on failure rate data whereas nonparametric survival function can give us precise dimensions (volume) and thermally induce stress to produce reliable characteristics for segmented and unsegmented TE modules. Hence more intuitive conditions (especially boundary conditions of TE leg) are produced as alternatives on given survival curves. This makes easy for us to obtain graphical MRL function-based reliability to analyze survival function on sufficient conditions.

5. Comparative Discussion
The influence of temperature on thermoelectric module and its material properties plays a significant role to predict reliability as outlined in the introduction. The analytical model corresponds to stress- strength covariance to obtain probability of survival and MRL of the module. The temperature gradient along the TE leg, between cold and hot sides, leads significant changes by influencing thermal conductivity. Two main aspects that govern the reliability of the module are (i) amount of heat absorbed and (ii) material property changes due to heat distribution along the module. In our previous paper [16] we illustrated the linear relationship between temperature distribution and thermal stress generation, here we have notice that there is minor bow, shown in figure4 (a) in the temperature-stress relationship (in segmented cases). The inclusion of Thomson and joule heat [37] in temperature distribution profile leads to this bow in graph (shown in figure 4).

The Figure 4 (b) corresponding to temperature profiles by considering contribution of boundary conditions at each end (hot and cold) for the emergence of deviation from linear temperature profile at maximum efficiency. The simulated results show the temperature profile dependence on material properties, which contribute nonconstancy, leading temperature line bending, especially on hot side. Thomson heat generation (or absorption) gradient, in double integration $k(T) \frac{\partial^2T}{\partial y^2}$, fills the loss of heat, causing a bow on the gradient line [27].

For visualization, the effect of Thomson heat generation (absorption) on stress-strength covariance profile, based on temperature gradient, is obtained for nonparametric lognormal model and compared with obtain simple Monte Carlo simulation (see Figure 5).

The simulation results demonstrate that bending with torsion and phase displacement, varying with length to thickness ratio, have influence over the maximum thermal shearing and mechanical shearing stresses. Figure 5(a) shows positions of maximum covariance between stress (shearing) and strength for a stable (segmented and unsegmented) TE module using general stress-strength relationship through Monte Carlo simulation. Whereas Figure 5 (b) presents the covariance profile for high and low temperature segmented TE modules under highly induced thermal stresses, where the maximum covariance corresponds to the critical position. For the ratio of thermal shearing stresses to the strength, at significant bending stresses, the plane of maximum covariance occurs from 300 MPa onwards (specific case of segmentation using bismuth telluride material). The plane of maximum covariance varies from material to material and module to module. The resemblance between graph (a) and (b) shows the relevancy of nonparametric lognormal distribution to obtain critical stress for segmented TE module through covariance plane.
On further investigation, the demonstrated Figure 6 shows relevance vector regression on stress-strength covariance. The optimized nonparametric lognormal distribution model, developed to understand stress-strength covariance, was simulated under Fast Multi-output relevance vector regression (MRVR) [28] in MATLAB. Figure 6 illustrate the distribution of Survival function in domain of threshold stress. A noteworthy observation is that the probability of survival function encompasses characteristics of upside-down bathtub shape for individual module threshold stress discrete analysis. The distribution of failure data has same characteristics for shape of survival function as Figure 3. The deviations are different when compared with high temperature segmented (SKD) and relatively decreases after 300 MPa when compare to low temperature TE modules (bismuth telluride). This has significant implications on TE system design and optimization of discrete data distribution to apply nonparametric
Figure 6. Parametric and Nonparametric lognormal survival function.

Figure 7 illustrates the nonparametric model, at 200, 300, 500, 700 and 900 MPa stress data to approximate the appropriate priors for segmented and unsegmented modules. We used interference random variable $y$, under the random stress $\sigma$. The range of the unsegmented high temperature TE module on the log scale was $(3.69356, 8.2986)$, extending the prior variance about 0.85. The shape parameter was set 2 so that the unsegmented high temperature module could have finite variance. Whereas for segmented high temperature module, following the same approach, the range on the log scale was set on $(4.646774, 6.350078)$ so the expansion of prior variance is 0.55. Number of components varies from 4-9, depending on the module.

Posterior estimates for the densities for the segmented and unsegmented (low and high temperature) are shown in the Figure 7. Nonparametric lognormal function is able to describe the peaks and valleys that the parametric model can’t. There is a slight discrepancy from the point of estimate and the density of the data around 900-1000 (MPa).

But nevertheless, the data density remains within the estimated interval for given model. By comparing the densities regarding to different temperatures under nonparametric lognormal function gives us a clear insight to understand mean residual life. Figure 8 in that respect points interval estimates of the posterior density for survival function for segmented (Low and High Temperature) and unsegmented (high temperature).

Looking at the estimated densities we can see that segmented high temperature module has lowest surviving life compared to segmented low temperature module. The survival function estimates show that after 400 MPa the survival curve monotonically decreases. Subsequently the underlying explanation is that failure rate function at thermal stress ($\sigma \geq \sigma_y$) is finite for nonparametric survival model which distinguishes it from other existing models with infinite initial failure rates. If modules are stressed to failure lower than $\sigma_y$, then it is difficult to distinguish between lognormal and Weibull distribution. If Its above then $\sigma_y$, then the probability of making a correct choice is fair, especially for small sizes samples and nonparametric lognormal module becomes quite good. The subsequent a comparative investigation was conducted for the probability of failure for various thickness of segmented TE leg. The probability of failure for Weibull’s distribution, Monte Carlo simulation and nonparametric survival function, for both low and high temperature modules are shown in Figure 9. The Monte Carlo simulation-based technique assists to distinguish different distributions and predict
Figure 7. Relative frequency histogram and densities of survival function for segmented and unsegmented modules under the Nonparametric model

Figure 8. Probability of Survival for segmented (Low and High Temperature) and unsegmented (high temperature)
Figure 9. Comparative Probability Analysis between Weibull, Monte Carlo and Nonparametric distributions

reliability based on empirically determined failure data [29]. Especially distinguishing between nonparametric lognormal and Weibull distribution is of interest here because both are used to model probability of failure. The changing the in thickness corresponds to temperature very closely, producing varying probability map.

We used the posterior predictive survival approach, introduced by Gelfand and Gosh [30] to compare Weibull’s model to the nonparametric lognormal mixture model. The comparative method is used to minimize expectation of specified survival function under posterior predictive distribution model to replicate response observed in data. For both segmented HT and LT, the nonparametric lognormal mixture model performs significantly better than the Weibull’s model. The comparison of distribution regarding to thickness of the thermoelectric leg provides us insight for modeling devices. This aspect ultimately changes the prospect of sizing the legs. We have noticed that unsegmented TE legs can survive between 4-5 (mm) with probability of failure of 30% , whereas the segmented TE legs can survive between 6-7 (mm) with probability of failure of 35%. As Shown in graph that onwards, the probability of failure enhances up to 60% for nonparametric lognormal distribution whereas 45% for Weibull distribution. The result of the comparison supports our earlier argument [16] that nonparametric lognormal survival function is indeed a better model for these circumstances compared to Weibull model.

Conclusions
The article presents some basic properties and characteristics of nonparametric survival function, taking MRL function to define it. The provided method for obtain survival stress compared to different distribution allow us to study various factors to find definite shape of survival function. It has been shown that the optimal survival ability of a segmented module depends on stress-strength density function. The survival function is obtained through non-parametric lognormal distribution and an appropriate model for thermal stress, caused by change in temperature gradient, entails 80% accuracy to predict life of module for both high and low temperature. It has been shown that when using non-parametric lognormal distribution to extrapolate beyond
the range of critical thermal stress, the non-parametric lognormal survival function can predict, based on average failure rates, operating life of TE module better than parametric lognormal and Weibull distribution.

Consequently, the comparative discussion between non-parametric model, Weibull distribution and Monte Carlo simulation based on module area gives a meaningful insight over sustainability of module on different temperature gradient and thickness of each layer. The average thickness of the leg is 5 to 7 mm, the range between which leg can survive at critical thermal stress without failing.

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References

[1] D Thesis 2009 Modelling of thermoelectric devices for electric power generation.
[2] S Baschel, E Koubli, J Roy, and R Gottschal 2018 Impact of Component Reliability on Large Scale Photovoltaic Systems’ Performance doi: 10.3390/en1016579.
[3] N K Karri and C Mo 2018 Metalization and Processing Temperatures 47(10) doi: 10.1007/s11664-018-6505-1.
[4] G Li et al. 2016 Acta Materialia Atoministic explanation of brittle failure of thermoelectric 103 775–780 doi: 10.1016/j.actamat.2015.11.021.
[5] S Twaha, J Zhu, Y Yan, and B Li 2016 Renew. Sustain. Energy Rev. 65 698–726 doi: 10.1016/j.rser.2016.07.034.
[6] C Gayner and K K Kar 2016 Prog. Mater. Sci. 83 330–382 doi: 10.1016/j.pmatsci.2016.07.002.
[7] H S Kim, W Liu, and Z Ren 2017 Energy Environ. Sci. 10(1) 69–85 doi: 10.1039/c6ee02488b.
[8] M Y Lee, J H Seo, H S Lee, and K S Garud 2020 Symmetry (Basel) 12(5) doi: 10.3390/sym12050786.
[9] A Ziahari, E Suhir, and A Shakouri 2014 Microelectronics J. 45(5) 547–553 doi: 10.1016/j.mejo.2013.12.004.
[10] J Malzbender 2004 J. Appl. Phys. 95(4) 1780–1782 doi: 10.1063/1.1642289.
[11] Z H Jin 2014 Microelectron. Reliab. 54(6–7) 1363–1368 doi: 10.1016/j.microrel.2014.02.028.
[12] N Geargana and I Jordika 2013 Structure Subjected To Monotanically Increasing 937–947.
[13] A Note 2004 Reliability and the Electronic Engineer 1 1–4.
[14] Z Bertalan, A Shekhawat, J P Sethna, and S Zapperi 2014 Fracture strength: Stress concentration, extreme value statistics and the fate of the Weibull distribution 1–8.
[15] O M Jadaan and A A Wereszczak 2009 Probabilistic Mechanical Reliability Prediction of Thermoelectric Legs Under contract DE-AC05-00OR22725.
[16] S Sattar 2020 J. Phys. Conf. Ser. 1560(1) doi: 10.1088/1742-6596/1560/1/012025.
[17] T Torstensson 2004 Reliability in fatigue.
[18] A D Telang and V Mariappan 2008 Hazard Rate of Lognormal Distribution: An Investigation 4(2) 103–108.
[19] E Bridget and M Abiodun 2017 Inference on Stress-Strength Reliability for Log-Normal Distribution based on Lower Record Values 22 77–97.
[20] M Baro-tijerina and G Duran-medrano 2020 March.
[21] B Baru, D Tiwari, D Kundu, and R Prasad Distribution for Brittle Materials 1–25.
[22] D Banjovic 2009 Metrika 69(2–3) 337–349 doi: 10.1007/s00184-008-0220-5.
[23] R C Gupta and S Lvin 2005 Math. Comput. Model. 42(9–10) 939–946 doi: 10.1016/j.mcm.2005.06.005.
[24] V Poynor and A Kottas 2019 Biostatistics 20 (2) 240–255 doi: 10.1093/biostatistics/kxx075.
[25] G S Mudholkar, D K Srivastava, and M Freimer 1995 Technometrics 37(4) 436–445 doi: 10.1080/00401706.1995.10484376.
[26] J Sethuraman 1994 Statistica Sinica 4 639–650 [Online] http://www3.stat.sinica.edu.tw
[27] P Ponnusamy, J de Boor, and E Müller 2020 Appl. Energy 262 doi: 10.1016/j.apenergy.2020.114587.
[28] Y Ha and H Zhang 2019 Econ. Model. 81 217–230 doi: 10.1016/j.econmod.2019.04.007.
[29] E Environ, M Zebarjadi, K Esfarjani, M S Dresselhaus, Z F Ren, and G Chen 2012 Environmental Science Perspectives on thermoelectrics: from fundamentals to device applications pp. 5147–5162 doi: 10.1039/c1ee02497c.
[30] A E Gelfand and S K Ghosh 1998 Biometrika 85(1) 1–11 doi: 10.1093/biomet/85.1.1.