Ensuring the safety of industrial equipment by optimizing the maintenance and repair system

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Abstract. The determination of the standby condition reliability with various combinations of reliability indicators and parameters of the maintenance and repair system for steady and unsteady operating modes are considered. The obtained solutions make it possible to solve problems of managing the technical conditions, optimizing the reliability and safety of potentially dangerous technical objects.

1. Introduction
The peculiarity of industrial development in recent years is a fundamental change in its system properties, namely, the occurrence of risks caused by long causal relationships, the interdisciplinary nature of risks, global changes of anthropogenic nature, high sensitivity to “weak impacts”, reduced ability to predict the development of accidents and catastrophes by traditional methods etc. [1,2].

This inevitably leads to using a systematic approach to the design and operation of potentially hazardous technical objects, analyzing man-made and environmental risks, industrial and environmental safety using methods of system analysis, random process theory, reliability theory, operations research, decision theory, multi-criteria optimization and modern computing and information technologies [2-10].

Optimization of man-made risk and maintenance system optimal parameters are determined by the type and the complexity of an object itself, nature and importance of the functions performed the number and the type of its possible conditions, the severity of failures consequences, as well as the operation and maintenance strategy [11-13].

2. Materials and Methods
For potentially hazardous technical objects with periodic maintenance three states and four transitions are possible (Figure 1) [14,15]:
• from the working state (readiness state) 1 to the maintenance state 2 with frequency \( t_{TO} \) and, accordingly, the intensity of transitions \( \lambda_{12} = 1/t_{TO} \);
• from the readiness state 1 to the failure state 3 with transition rate equal to failure rate \( \lambda_{13} = \lambda_{32} \);
• from the maintenance state 2 to the readiness state 1 with intensity determined by the maintenance duration \( \lambda_{21} = 1/\tau_{TO} \);
• from the state of failure 3 to the maintenance state 2 (transition due to detection of a latent failure during maintenance) with transition rate \( \lambda_{32} \)
The only operational state of such an object is readiness state 1, and therefore the probability of this state \( P_1 \) can be considered as the main indicator of its reliability. In the safety analysis the main parameter is a failure probability (technical risk) \( Q = P_3 \).

![Graph of object states with periodic maintenance.](image)

Since the system of differential equations for the directed state graph in Figure 1 has the following form [14,15]:

\[
\begin{align*}
\frac{dP_1(t)}{dt} &= -\left(\lambda_{12} + \lambda_{13}\right)P_1(t) + \lambda_{21}P_2(t) = -\left(\lambda + \frac{1}{t_{TO}}\right)P_1(t) + \frac{1}{t_{TO}}P_2(t); \\
\frac{dP_2(t)}{dt} &= \lambda_{12}P_1(t) - \lambda_{21}P_2(t) + \lambda_{32}P_3(t) = \frac{1}{t_{TO}}P_1(t) - \frac{1}{t_{TO}}P_2(t) + \frac{1 + \lambda t_{TO}}{t_{TO}^2}P_3(t); \\
\frac{dP_3(t)}{dt} &= \lambda_{13}P_1(t) - \lambda_{32}P_3(t) = \lambda P_1(t) - \frac{1 + \lambda t_{TO}}{t_{TO}^2}P_3(t).
\end{align*}
\]

Since the equation system (5) is linearly dependent, one of the equations must be replaced by the normalizing condition.
In addition, it is necessary to set the initial conditions, for example \( P_1(0) = 1, P_2(0) = P_3(0) = 0 \). The solution of the equation system (5) regarding the probability of the readiness state \( P_1 \) can be obtained in the following form [16]:

\[
P_1(t) = \frac{\lambda_{13}^2 + 2\lambda_{13}^2 \lambda_{12} + \lambda_{12}^2 - \lambda_{32} \lambda_{13} - \lambda_{32} \lambda_{12} - \lambda_{21} \lambda_{13} + \lambda_{21} \lambda_{12} -}{\left[ -\lambda_{32} - \lambda_{13} - \lambda_{21} - \lambda_{12} + \left( \lambda_{32}^2 - 2\lambda_{32} \lambda_{13} - 2\lambda_{32} \lambda_{12} - 2\lambda_{21} \lambda_{13} + \lambda_{21} \lambda_{12} + \lambda_{13} \right) \right]^{1/2}} \times \]

\[
-\frac{-\lambda_{13}^2 - 2\lambda_{32} \lambda_{13} - 2\lambda_{32} \lambda_{12} + \lambda_{13}^2 - 2\lambda_{21} \lambda_{13} + \lambda_{21} \lambda_{12} + \lambda_{13}^2}{-2\lambda_{21} \lambda_{13} + 2\lambda_{13} \lambda_{12} + \lambda_{21} \lambda_{12} + \lambda_{13}^2} \times \]

\[
-\frac{-\lambda_{12}^2 - 2\lambda_{32} \lambda_{13} - 2\lambda_{32} \lambda_{12} + \lambda_{12}^2 - 2\lambda_{21} \lambda_{13} + \lambda_{21} \lambda_{12} + \lambda_{12}^2}{-2\lambda_{21} \lambda_{13} + 2\lambda_{13} \lambda_{12} + \lambda_{21} \lambda_{12} + \lambda_{12}^2} \times \]

\[
\times \exp \left[ \frac{1}{2} \left[ -\lambda_{32} - \lambda_{13} - \lambda_{21} - \lambda_{12} + \lambda_{13}^2 \lambda_{12} \lambda_{13} - 2\lambda_{32} \lambda_{12} - 2\lambda_{32} \lambda_{12} + \lambda_{13} \right] \right. \]

\[
+ \frac{\lambda_{13}^2 + 2\lambda_{13} \lambda_{12} + \lambda_{12}^2}{\lambda_{32} + \lambda_{13} + \lambda_{21} + \lambda_{12} + \left( \lambda_{32}^2 - 2\lambda_{32} \lambda_{13} - 2\lambda_{32} \lambda_{12} - 2\lambda_{21} \lambda_{13} + \lambda_{21} \lambda_{12} + \lambda_{13} \right) \left. \right] \right] \times \]

\[
+ \frac{\lambda_{13}^2}{\lambda_{32} + \lambda_{13} + \lambda_{21} + \lambda_{12} + \left( \lambda_{32}^2 - 2\lambda_{32} \lambda_{13} - 2\lambda_{32} \lambda_{12} - 2\lambda_{32} \lambda_{12} + \lambda_{13} \right) \times \]

\[
\times \exp \left[ -\frac{1}{2} \left[ -\lambda_{32} - \lambda_{13} - \lambda_{21} - \lambda_{12} + \lambda_{13}^2 \lambda_{12} \lambda_{13} - 2\lambda_{32} \lambda_{12} - 2\lambda_{32} \lambda_{12} + \lambda_{13} \right] \right. \]

\[
+ \frac{\lambda_{13}^2}{\lambda_{32} + \lambda_{13} + \lambda_{21} + \lambda_{12} + \left( \lambda_{32}^2 - 2\lambda_{32} \lambda_{13} - 2\lambda_{32} \lambda_{12} - 2\lambda_{32} \lambda_{12} + \lambda_{13} \right) \times \]

\[
\times \exp \left[ -\frac{1}{2} \left[ -\lambda_{32} - \lambda_{13} - \lambda_{21} - \lambda_{12} + \lambda_{13}^2 \lambda_{12} \lambda_{13} - 2\lambda_{32} \lambda_{12} - 2\lambda_{32} \lambda_{12} + \lambda_{13} \right] \right. \]

\[
+ \frac{\lambda_{13}^2}{\lambda_{32} + \lambda_{13} + \lambda_{21} + \lambda_{12} + \left( \lambda_{32}^2 - 2\lambda_{32} \lambda_{13} - 2\lambda_{32} \lambda_{12} - 2\lambda_{32} \lambda_{12} + \lambda_{13} \right) \times \]

\[
\times \exp \left[ -\frac{1}{2} \left[ -\lambda_{32} - \lambda_{13} - \lambda_{21} - \lambda_{12} + \lambda_{13}^2 \lambda_{12} \lambda_{13} - 2\lambda_{32} \lambda_{12} - 2\lambda_{32} \lambda_{12} + \lambda_{13} \right] \right. \]

\[
+ \frac{\lambda_{13}^2}{\lambda_{32} + \lambda_{13} + \lambda_{21} + \lambda_{12} + \left( \lambda_{32}^2 - 2\lambda_{32} \lambda_{13} - 2\lambda_{32} \lambda_{12} - 2\lambda_{32} \lambda_{12} + \lambda_{13} \right) \times \]

\[
\times \exp \left[ -\frac{1}{2} \left[ -\lambda_{32} - \lambda_{13} - \lambda_{21} - \lambda_{12} + \lambda_{13}^2 \lambda_{12} \lambda_{13} - 2\lambda_{32} \lambda_{12} - 2\lambda_{32} \lambda_{12} + \lambda_{13} \right] \right. \]

\[
+ \frac{\lambda_{13}^2}{\lambda_{32} + \lambda_{13} + \lambda_{21} + \lambda_{12} + \left( \lambda_{32}^2 - 2\lambda_{32} \lambda_{13} - 2\lambda_{32} \lambda_{12} - 2\lambda_{32} \lambda_{12} + \lambda_{13} \right) \times \]

\[
\times \exp \left[ -\frac{1}{2} \left[ -\lambda_{32} - \lambda_{13} - \lambda_{21} - \lambda_{12} + \lambda_{13}^2 \lambda_{12} \lambda_{13} - 2\lambda_{32} \lambda_{12} - 2\lambda_{32} \lambda_{12} + \lambda_{13} \right] \right. \]
3. Results
The results show that the probability of the readiness state and, accordingly, the technical risk is determined, first of all, by the object failure rate and the frequency of maintenance (Table 1).

In addition, the results make it possible to estimate the duration of transient processes (“relaxation period”) in which the state probabilities are stabilized to some “final” ones and consequently it is possible to use the stationary model for calculations (Figure 2).

Table 1. The probability of the object readiness state ($\tau_{TO} = 10$ h).

| $t_{TO}$, years | $\lambda$, $h^{-1}$ | Technical lifetime $t$, years | 0.1 | 0.5 | 1 | 2 | 5 | 10 | Final probability |
|-----------------|---------------------|-----------------------------|-----|-----|---|---|---|---|------------------|
| 0.5             | $10^4$              |                             | 0.9358 | 0.8826 | 0.8798 | 0.8797 | 0.8797 | 0.8797 | 0.8797 |
|                 | $10^5$              |                             | 0.9958 | 0.9958 | 0.9958 | 0.9958 | 0.9958 | 0.9958 | 0.9958 |
|                 | $10^6$              |                             | 0.9977 | 0.9977 | 0.9977 | 0.9977 | 0.9977 | 0.9977 | 0.9977 |
| 1               | $10^4$              |                             | 0.9232 | 0.7728 | 0.7228 | 0.7093 | 0.7086 | 0.7086 | 0.7086 |
|                 | $10^5$              |                             | 0.9938 | 0.9918 | 0.9918 | 0.9918 | 0.9918 | 0.9918 | 0.9918 |
|                 | $10^6$              |                             | 0.9988 | 0.9988 | 0.9988 | 0.9988 | 0.9988 | 0.9988 | 0.9988 |
| 2               | $10^4$              |                             | 0.9187 | 0.7020 | 0.5724 | 0.4914 | 0.4734 | 0.4724 | 0.4724 |
|                 | $10^5$              |                             | 0.9920 | 0.9785 | 0.9747 | 0.9739 | 0.9739 | 0.9739 | 0.9739 |
|                 | $10^6$              |                             | 0.9991 | 0.9991 | 0.9991 | 0.9991 | 0.9991 | 0.9991 | 0.9991 |

Figure 2. The dependence of the readiness state probability on the lifetime $\tau_{TO} = 10$ h, $\lambda = 10^4 h^{-1}$.

For the steady state of operation the system of differential equations (5) degenerates into a system of linear algebraic equations:

\[
\begin{align*}
-\left(\lambda_{12} + \lambda_{13}\right)P_1 + \lambda_{21}P_2 &= \left(\tau_{TO}^{-1} + \lambda\right)P_1 + \tau_{TO}^{-1}P_2 = 0, \\
\lambda_{12}P_1 - \lambda_{21}P_2 + \lambda_{32}P_3 &= t_{TO}^{-1}P_1 - \tau_{TO}^{-1}P_2 + \left(t_{TO} - \left(\tau_{TO}^{-1} + \lambda\right)^{-1}\right)^{-1}P_3 = 0, \\
\lambda_{13}P_1 - \lambda_{32}P_3 &= \lambda P_1 - \left(t_{TO} - \left(\tau_{TO}^{-1} + \lambda\right)^{-1}\right)^{-1}P_3 = 0.
\end{align*}
\]
When the system (8) is supplemented by the normative condition $P_1 + P_2 + P_3 = 1$, the solutions for the final probabilities have the following form:

$$
P_1 = \frac{t_{TO}(1+t_{TO}\lambda)}{\lambda^2 t_{TO}^2 \tau_{TO} + \tau_{TO} + 2\lambda t_{TO} \tau_{TO} + \lambda^2 t_{TO}^3 + t_{TO} + \lambda t_{TO}^2},
$$

(9)

$$
P_2 = \frac{\tau_{TO}(2\lambda t_{TO} + \lambda^2 t_{TO}^2 + 1)}{\lambda^2 t_{TO}^2 \tau_{TO} + \tau_{TO} + 2\lambda t_{TO} \tau_{TO} + \lambda^2 t_{TO}^3 + t_{TO} + \lambda t_{TO}^2},
$$

(10)

$$
P_3 = \frac{\lambda^2 t_{TO}^3}{\lambda^2 t_{TO}^2 \tau_{TO} + \tau_{TO} + 2\lambda t_{TO} \tau_{TO} + \lambda^2 t_{TO}^3 + t_{TO} + \lambda t_{TO}^2}.
$$

(11)

Some of predicted results by formulas (9)-(11) are shown on Figures 3 and 4 in the form of graphs of the dependence of the readiness state probability $P_1$ and the inoperative state probability $P_2 + P_3$ on the frequency of maintenance $t_{TO}$ for various values of the failure rate $\lambda$ and the duration of maintenance $\tau_{TO}$.

Figure 3. The dependence of the readiness state probability on the maintenance frequency.

Figure 4. The dependence of the inoperative state probability on the maintenance frequency.
The presence of extreme (maximum or minimum) is noteworthy i.e. for the known values $\lambda$ and $\tau_{T0}$ there is an optimal frequency of maintenance as well as the maximum value of readiness state probability and the minimum value of technical risk.

4. Conclusion
From the analysis of the obtained dependencies and the results of calculations, the following conclusions can be drawn:

- the failure rate and frequency of maintenance have the greatest impact on the level of safety;
- the maximum value of the availability coefficient with a decrease in the failure rate is shifted towards higher values maintenance frequency;
- the optimal frequency of maintenance is proportional to its duration;
- deviations of the maintenance frequency from the optimal value (especially in the direction of decreasing) significantly reduce the availability coefficient of the object, especially at high failure rates.

The obtained analytical solutions for unsteady and steady-state operating modes make it possible to solve problems of managing the technical condition and optimizing the reliability and safety of potentially hazardous technical objects [17].

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