Relation between Magnetic Properties of Stator Core and Cogging Torque in 8-pole 12-slot SPM Synchronous Motors

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This paper discusses the influence of the magnetic properties of the stator core on cogging torque in 8-pole 12-slot surface-mounted permanent-magnet synchronous motors from the perspective of magnetic energy. In the finite element analyses of magnetic fields with modeled stator BH curves, cogging torque decreases drastically when the shape of magnets is changed from an arc to a semi-cylindrical structure. The influence of the stator BH curve on cogging torque is dominant in the low-cogging-torque motor with semi-cylindrical magnets. This is because magnetic energies in the gap and the magnets cancel each other and the magnetic energy in the stator core is dominant in the low-cogging-torque motor. Low maximum permeability and low magnetizing force in a high-B region are preferable for stator magnetic properties to reduce cogging torque. This is explained by the behavior of harmonics in the stator magnetic energy variation. The influences of steel grade, compressive stress, and punching in the stator core on cogging torque analyses are consistent with the results for the modeled BH curves. Low grade electrical steel sheets, high compressive stress, and magnetic annealing are effective to reduce cogging torque in the described motor design.

Keywords: SPM synchronous motor, cogging torque, stator, magnetic property, electrical steel sheet, magnetic energy

1. Introduction

Permanent-magnet synchronous motors have been widely used because they have higher efficiency than induction motors. Surface-mounted Permanent-Magnet (SPM) synchronous motors are especially suitable for servo system requiring rapid response and accurate decision of position. When designing servo motors, reduction of cogging torque is one of the most important issues because cogging torque disturbs the control of their rotation.

Cogging torque consists of three components, namely a component which pulsates the times of the number of the least common multiple of pole and slot numbers per one rotation of the rotor (“theoretical-order component”), a component which pulsates the times of the number of poles per one rotation of the rotor (“pole-number component”), and a component which pulsates the times of the number of slots per one rotation of the rotor (“slot-number component”). To reduce the peak-to-peak value of cogging torque, these three components must be reduced simultaneously.

The pole-number component appears because of the magnetic asymmetry of the stator. The fluctuation of the inner diameter and stress distribution by shrink fitting are the main causes of magnetic asymmetry of the stator.

The slot-number component appears because of the magnetic asymmetry of the rotor. The dispersion of the magnetic remanence of the magnets is one of the main causes of magnetic asymmetry of the rotor.

The theoretical-order component appears even when the stator and the rotor are perfectly symmetrical. The amplitude of this component depends on the numbers of poles and slots and the structure nearby the gap. The shape of magnets is an important parameter for the amplitude of this component. In addition, this component depends on the magnetic properties of the stator core. This means that we may be able to control the amplitude of cogging torque by optimizing the magnetic properties of the stator core. We have experimentally demonstrated that magnetic annealing of the stator core reduces this component.

To the best of our knowledge, however, there are few reports about the relation between the magnetic properties of the stator core and the theoretical-order component of cogging torque. In what situation we can control the amplitude of cogging torque by optimizing the magnetic properties of the stator core and what kind of magnetic properties are suitable for the stator core to reduce cogging torque are topics that are still under discussion.

In this paper, the relation between the magnetic properties of the stator core and the theoretical-order component of cogging torque is quantitatively investigated. BH curves of various electrical steel sheets are measured under various conditions. Finite element analyses of 8-pole 12-slot SPM synchronous motors with various magnet shapes and various modelled BH curves are performed to investigate how the change of BH curve in the stator core influences the theoretical-order component of cogging torque. The relation between them is discussed in relation to the behavior of magnetic energy. The influences of steel grade, compressive

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2. Method of Measurement and Analysis

BH curves of electrical steel sheets were measured by a single sheet tester with a stress load system. The schematic picture of the measurement system is shown in Fig. 1. A magnetic field was applied along the longitudinal direction of the sample. Main stress was applied parallel to the magnetic field. Weak stress was also applied to the yoke to avoid buckling of the sample. This stress was so weak that it did not affect the measurement results.

The grades of electrical steel sheet samples used were 50A1300, 50A700, 50A400, and 50A270. The samples were sheared to a length of 200 mm and various widths. The longitudinal direction of each sample was set to the rolling direction or the transverse direction of the sheets.

Cogging torque calculations were performed by 2D finite element method analysis of the magnetic field. Figure 2 shows the structure of the analyzed SPM synchronous motors. Specifications of the motors are shown in Table 1. The shapes of the magnets were varied from Magnet A (arc structure) to Magnet D (semi-cylindrical structure). BH curve of the rotor core was fixed.

3. Measurement and Modelling of BH Curves

Figure 3 shows the BH curves of various electrical steel sheets. The width of all samples is 30 mm. In this paper, all the BH curves are averaged data for the rolling and transverse directions. This figure indicates that high-grade electrical steel sheets such as 50A270 have high maximum permeability and low saturated flux density.

Figure 4 shows the BH curves of 50A400 under compressive stress. The width of the sample is 30 mm. This figure indicates that compressive stress drastically reduces the maximum permeability of electrical steel sheets (10) (11). In contrast, saturated flux density is hardly affected by compressive stress less than 30 MPa (11).

Figure 5 shows the BH curves of 50A400 with various sample widths. This figure indicates that shearing deteriorates magnetic properties of electrical steel sheets. Especially, the magnetizing force in a region of relatively high-B (1.2∼1.5 T) is increased (12).

These measurement results of BH curves show that maximum permeability and/or magnetizing force in a high-B region (over 1.2 T) are influenced by the change of steel grade,
compressive stress or shearing. To simplify the discussion about BH curve dependence of cogging torque, BH curves are modelled as shown in Fig. 6. Curve 1 is the standard BH curve. The variations of maximum permeability are expressed with Curves 1, 2, and 3. On the other hand, the variations of magnetizing force in a relatively high-B region and with saturated flux density are expressed with Curves 1, 4, and 5.

4. Results of Cogging Torque Analysis with Modelled BH Curves

Figure 7 shows the peak-to-peak values of cogging torque for various magnet shapes and modelled stator BH curves. Here, the values along the vertical axis are normalized by the results for the case of Magnet A and Curve 1. The change of magnet shape from Magnet A to Magnet D drastically reduces cogging torque. On the other hand, the changes in cogging torque by stator BH curve are 15–30% for all magnet shapes. In other words, magnetic properties of stator core are important to further reduce the cogging torque of low-cogging-torque motors with semi-cylindrical magnets. In addition, the tendency of stator BH curve dependence is the same, namely, that low maximum permeability and low magnetizing force in a high-B region are preferable to reduce cogging torque. Curve 3 and Curve 5 are the respective best and the worst BH curves for stator core to obtain low-cogging-torque motors. The peak-to-peak value of cogging torque is about 1–3% of the rated torque in the motor with Magnet D. In some cases, 1% or less of cogging torque to the rated torque is required for servo systems. This means that magnetic properties of the stator core are practically important parameters to control cogging torque.

To understand these characteristics, magnetic energy is investigated. Cogging torque $T_{\text{cog}}$ is equal to the differential of magnetic energy $E$ by mechanical angle $\theta$ of the rotor as shown below.

$$T_{\text{cog}} = -\frac{\partial E}{\partial \theta} \tag{1}$$

The equivalence of the left and right sides of Eq. (1) in all the analyses were confirmed. Some examples are shown in Fig. 8. This means that the magnetic field analyses are accurate enough to describe the behavior of cogging torque by magnetic energy. Because magnetic energy can be calculated for each part such as the stator core, magnet, and so on, we can identify the reason of cogging torque variation by investigating the magnetic energy for each part.

Figure 8 also shows that the number of pulsation times of cogging torque per one rotation of the rotor is 24, which is equal to the least common multiple of the number of the poles and slots.

Figure 9 shows the magnetic energy variations for each part. Here, Curve 1 is used for all analyses. DC components of magnetic energies are subtracted from the results. Magnetic energy for each part pulsates 24 times per one rotation of the rotor, which is consistent with Fig. 8.

In the case of Magnet A, magnetic energy variations in the gap and the magnets have the opposite phases but do not cancel each other. On the other hand, they cancel each other in

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**Fig. 6.** Modelled BH curves of electrical steel sheets

**Fig. 7.** Cogging torque analysis results for various magnet shapes and various BH curves for stator core

**Fig. 8.** Comparison between cogging torque and the differential of magnetic energy by the mechanical angle
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Fig. 9. Magnetic energy variations for each part

(a) Magnet A (b) Magnet B
(c) Magnet C (d) Magnet D

Fig. 10. The sum of the magnetic energy variations of the gap and the magnets

(a) Magnet A (b) Magnet B
(c) Magnet C (d) Magnet D

Fig. 11. Magnetic energy variations for the part of stator core

(a) Magnet A (b) Magnet B
(c) Magnet C (d) Magnet D

Fig. 12. The case of Magnet D. Therefore, the dominant component is that in the stator core. The magnetic energy variations in the rotor core are negligible in all cases. The cause of the inverted magnetic energy variation in the gap will be discussed in another paper.

To investigate this result more precisely, the sum of the magnetic energy variations of the gap and the magnets for all cases are displayed in Fig. 10. As the magnet shape changes from Magnet A to Magnet D, the sum of them decreases drastically. In the case of Magnet D, it is negligible. These characteristics have little stator BH curve dependence.

In contrast, magnetic energy variations in the stator core show similar stator BH curve dependence for all magnet shapes as displayed in Fig. 11. The ranges of magnetic energy variations are less than 20 mJ and the magnetic energy is high in the stator cores with high maximum permeability and high magnetizing force in a high-B region at the mechanical angle of 7.5° for all magnet shapes. In addition, maximum permeability and magnetizing force in high-B region dependencies of magnetic energies are monotonic. We should note that, however, the magnet shape has a non-zero influence on the magnetic energy variation of the stator core. Especially, the influence of the structure on the inversion of the magnetic energy variation for the motor with Curve 3 and arc magnets such as Magnet A will be discussed later.

These results indicate that magnetic energy is simply divided into a gap and magnets component and a stator core component, and that they are almost independent of each other. Magnetic energy variation in the gap and the magnets is affected by the shape of the magnets, but not so much by the stator BH curve. In contrast, magnetic energy variation in the stator core is affected by the stator BH curve, but not so much by the shape of the magnets. Therefore, even if the magnet shape is optimized as the motor with Magnet D, where magnetic energy variation of the gap and the magnets is negligible as shown in Fig. 10(d), the magnetic energy of the stator core will remain, resulting in relatively large stator BH curve dependence of cogging torque.

The above discussion clarified why low-cogging-torque motors are sensitive to the magnetic properties of stator BH curves. To understand the relation between the stator BH curve and cogging torque in low cogging-torque-motors, the stator core is divided into three groups as shown in Fig. 12. Here, the number three is chosen because the least common multiple of pole and slot numbers divided by pole number is three. The magnetic energy of each group in the stator core is shown in Fig. 13 for Magnet D and Curve 1. This figure clearly indicates that the third harmonic order component of
magnetic energy in each group is the cause of the magnetic energy variation in the whole stator core. Although the order of the harmonic causing magnetic energy variation in the whole stator core depends on the numbers of poles and slots, the magnetic energy variation in the whole stator core is zero when the waveform of magnetic energy in each group is sinusoidal and the phase angle is shifted relatively 120 degrees in mechanical angle.

Figure 14 shows the stator core BH curve dependence of magnetic energy in Group 1 for Magnet D. As the maximum permeability decreases, the waveform of magnetic energy becomes sinusoidal, leading to the decrease of the third order harmonic. Increase of the DC and fundamental components does not contribute to the magnetic energy variation in the whole stator core. In contrast, as the magnetizing force in a high-B region increases, the third order harmonic of magnetic energy increases.

These behaviors mean that non-linearity of the BH curve caused by high maximum permeability and/or high magnetizing force in a high-B region increases the third harmonic of magnetic energy in the stator core.

If the B waveform in the stator core is sinusoidal and the BH curve is perfectly linear, the magnetic energy variation in each group is sinusoidal, because magnetic energy is calculated by Eq. (2).

$$E = \int H dB$$

This leads to no third order harmonic of magnetic energy in each group for the abovementioned case. The causes of the appearance of the third order harmonic are the deviation of the B waveform in the stator core from sinusoidal and/or the non-linearity of the BH curve.

Steep increase of magnetizing force in a high-B region leads to the steep increase shown by a dotted arrow in a high-E (namely high-B) region in Fig. 14. Because the third harmonic is the least order component, it is highly probably that the third harmonic of magnetic energy in the stator core will increase as a result of an increase in non-linearity of the stator BH curve even in other 8-pole 12-slot SPM synchronous motors. This discussion is also valid for SPM synchronous motors in which the least common multiple of pole and slot numbers divided by the pole number is three, such as 6-pole 9-slot SPM synchronous motors.

On the other hand, the little non-linearity of Curve 3 does not contribute to the third order harmonic of magnetic energy in each group because the magnetic energy variation in the whole stator core is negligible for Magnet D and Curve 3 (see Fig. 11(d)). Here, we should note that the B waveform of the stator core is almost sinusoidal for Magnet D (not shown) because the shape of Magnet D is semi-cylindrical. Therefore, the main cause of the inverted magnetic energy variation for Curve 3 (especially in the case of Magnet A) is the deviation of the B waveform in the stator core from sinusoidal. Actually, the B waveform in the stator core for Magnet A is rectangular rather than sinusoidal (not shown). In other words, the deviation of the B waveform in the stator core from sinusoidal “distorts” the BH curve dependence of magnetic energy variation in the stator core.

This discussion means that the inverted magnetic energy variation for Curve 3 is attributed to the structure of the motor. The detailed relation between the magnetic energy variation in the stator core and the structure of the motor will be discussed in another paper.

The relation between non-linearity of the stator BH curve and higher order harmonics of magnetic energy may be more complicated. The comparison of stator BH curve dependence in cogging torque analyses among motors of various pole and slot numbers is an interesting future research subject.

5. Results of Cogging Torque Analysis with Measured BH Curves

In this chapter, the influence of steel grade, compressive stress, and punching are investigated for the motor with Magnet D and compared with the results in the preceding chapter.

Figure 15 shows the peak-to-peak values of cogging torque with various electrical steel sheets for the stator core. The data in Fig. 3 are used as the stator BH curve. The values along the vertical axis are normalized by the results for 50A400. Figure 15 indicates that low-grade electrical steel
Sheets are preferable to obtain low-cogging-torque motors. This is consistent with Fig. 7, because the BH curve of low-grade electrical steel sheets has lower maximum permeability and lower magnetizing force in a high-B region as shown in Fig. 3. This result tells us that the grade of electrical steel sheets for stator cores should be selected in consideration of not only iron loss but also linearity of the BH curve.

Figure 16 shows the peak-to-peak values of cogging torque with various compressive stresses along the circumference direction in the yoke. The values along the vertical axis are normalized by the results without stress. BH curves under stress are applied only to the yoke. This type of compressive stress often appears when a stator core is shrink-fitted by a frame. Figure 16 indicates that high compressive stress along the circumference direction is preferable to obtain low-cogging-torque motors. However, we should note that non-uniform high compressive stress may increase the pole-number component of cogging torque (1). This result is consistent with Fig. 7, because the BH curve under compressive stress has low maximum permeability as shown in Fig. 4.

Figure 17 shows the peak-to-peak values of cogging torque with and without considering the effect of punching. It is said that the effect of punching on electrical steel sheets is similar to that of shearing. In this analysis, only the effect of punching at the edge of regions A and B in Fig. 2 are considered. Because the width of region A is 10 mm, the BH curve with 10 mm width in Fig. 5 is applied to the whole of region A. Although there is a flux density distribution in region A, this simple application of BH curve to the teeth is a good first-order approximation. This type of application of BH curves to teeth was also reported in Ref. (13). In region B, it is desirable to consider the effect of punching at all the edges, including the edge next to the gap, but it is difficult to do so with the data in Sect. 3. Accordingly, the BH curve with 15 mm width in Fig. 5 is applied to the whole of region B to consider only the effect of punching at the edge nearby the slot. Figure 17 indicates that punching increases cogging torque. Therefore, magnetic annealing of the stator core is effective to reduce cogging torque, as has already confirmed experimentally (10). The influence of punching in region B is small because only the effect of punching at the edge nearby the slot is considered in this study. These results are consistent with Fig. 7, because the BH curves of narrow samples have high magnetizing force in a relatively high-B region as shown in Fig. 5. The development of a method for considering the effect of punching at the edge next to the gap, where the flux is perpendicular to the edge, is a subject for future study.

6. Conclusion

In this paper, the relation between magnetic properties of the stator core and cogging torque in 8-pole 12-slot surface-mounted permanent-magnet synchronous motors has been investigated by 2D finite element method analysis of the magnetic field. Analyses with modelled stator BH curves resulted in finding the following two characteristics.

• In low-cogging-torque motors, cogging torque is sensitive to the magnetic properties of the stator core.
• Low maximum permeability and low magnetizing force in the high-B region of a stator core are preferable to reduce cogging torque.

These characteristics are well explained by the fact that magnetic energies in the gap and the magnets cancel each other and the magnetic energy in the stator core is dominant in the low-cogging-torque motor.

The influence of the change of steel grade, compressive stress, and punching for the stator core has been investigated with measured BH curves. The results are consistent with that for modelled stator BH curves and indicates that the use of low grade electrical steel sheets, high compressive stress, and magnetic annealing are effective to reduce cogging torque in the case of the described motor design.

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