Simulation of mortality immunization for life insurance companies in Indonesia using duration and convexity approach

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Abstract. Duration and convexity are two important factors in interest rate immunization. These two factors used to be very closely related to financial assets immunization towards change in interest rate, but some of the latest studies had applied the concept of duration and immunization in terms of mortality immunization. This paper examines 24 mortality immunization strategies that applied the duration and convexity concept in insurance portfolios which consist of life insurance and annuity products. The outcome of this study is the optimal proportion of the life insurance and annuity products for life insurance companies in Indonesia. Numeric simulations had been done using Indonesia Mortality Table 2011 (TMI-2011) to obtain the value and characteristics of the optimal proportion for two portfolios which is affected by several factors, namely the implemented strategy, mortality model, mortality model change type, age of policy holder, year of application, payment paying period, and term policies.

Keywords: Immunization, duration, convexity, hedging

1. Introduction

Surplus and mortality rate have become substantial issue in insurance nowadays. Both are closely related to each other since mortality rate directly affects the surplus of the insurance company. In life insurance products, an increase in mortality rate causes a decrease in the company’s surplus, while decreasing the mortality rate causes an increase in the company’s surplus. The opposite occurs in the annuity product. Since the change in the mortality rate can affect the surplus that is essential for business, find ways to anticipate this change in mortality rate has been a matter of great urgency [1].

Research to anticipate the changes in mortality rates has grown in recent years and has been divided into two major strategies. The first strategy focuses on establishing models of the mortality rates to accurately predict mortality rates [2]. However, the uncertainty of movements in mortality rates cannot be fully illustrated by model. So there is a difference between the actual mortality rate and the mortality rate in the model, which is certainly risky for the company. To address this risk, a second strategy called immunization mortality was developed [1, 3, 4]. Immunization mortality is a strategy to protect the company’s surplus respect to the change of mortality rate. Immunization mortality ensures that changes in mortality rates do not adversely affect the surplus value of an insurance portfolio. This strategy has the advantage that this strategy does not focus on predicting mortality rates alone, thus avoiding the risk of differences in actual mortality rates with predicted mortality rates.
Duration and convexity are two essential things in mortality immunization. At the beginning, duration and convexity are closely related to the immunization of financial assets against changes in interest rates. Duration measures the sensitivity of asset prices respect to changes in interest rates, whereas convexity measures the sensitivity of duration to changes in interest rates [2]. However, recent studies have applied the concept of duration and convexity in mortality immunization. Many studies about mortality immunization using the concept of duration and convexity have been developed.

Well known strategies for immunization are duration matching and convexity matching strategy. In those strategies the financial asset, which in this case is the insurance portfolio, must fulfill a condition; that is the duration and the convexity of the portfolio must be zero. If the condition is met, then the change in mortality rate does not negatively affect the surplus of the portfolio, or at least the effect can be minimized.

Further studies have found that the duration and convexity of life insurance products and annuity products have an opposite sign. As also mentioned earlier, life insurance products and annuity products have the opposite effect on the company’s surplus when there is a change in mortality rate. Those facts give as an idea to determine an optimal proportion between life insurance products and annuity products in an insurance portfolio. This is commonly known as the natural hedging method.

In this paper, we will develop duration matching and convexity matching strategies in mortality immunization with natural hedging method. We will also simulate using Indonesia Mortality Table 2011 (TMI-2011) to calculate the optimal weights for two kinds of portfolio.

2. Mortality durations and convexities

Mortality rate is a measure of the number of deaths in a particular population, place, and certain of time. Rates of mortality could be presented in a varied expressions. There are four conventional forms that are \( \mu_x \) (the force of mortality), \( q_x \) (the one-year death probability), \( p_x \) (the one-year survival probability), and \( m_x \) (the central rate death). In addition to traditional forms, Lee et al. [5] and Cairns et al. [6] model’s the mortality rates in the form \( \mu_{x+k} \) and \( q_x/p_x \). Therefore, in this paper we will use six different forms of mortality rate. Denote:

- \( U_x = \{\mu_{x+k} : k = 1,2, ...\} \)
- \( Q_x = \{q_{x+k} : k = 1,2, ...\} \)
- \( P_x = \{p_{x+k} : k = 1,2, ...\} \)
- \( \ln U_x = \{\ln(\mu_{x+k}) : k = 1,2, ...\} \)
- \( Q_x/P_x = \{q_{x+k-1}/p_{x+k-1} : k = 1,2, ...\} \)
- \( \ln(Q_x/P_x) = \{\ln(q_{x+k-1}/p_{x+k-1}) : k = 1,2, ...\} \)

Further, we denote the six different forms with \( D_x \), which \( D_x \) can be one of the \( U_x, Q_x, P_x, \ln U_x, Q_x/P_x \), or \( \ln Q_x/P_x \).

Duration matching and convexity matching are cornerstones of the interest rate immunization strategy where the duration or convexity of liabilities matches that of assets. Duration quantifies the price’s sensitivity of an asset or liability to a constant change in interest rate, and convexity calculates the curvature or second derivative of the price [7]. In this chapter we derive the mortality durations and convexities of the net single premium of general annuity product with respect to an instantaneously proportional change and parallel shift, in the one-year mortality rates of six different forms.

First, consider a general annuity product, the \( h \)-year deferred and \( j \)-year temporary life annuity-due; its actuarial present value (APV) of one unit issued to an insured age \( x \), is denoted as

\[
h[d_x;j] = \sum_{k=h}^{h+j-1} kP_x \cdot v^k
\]
where \( v = 1/(1 + i) \) and \( i \) is the interest rate [8]. We can write the APV of general annuity product in equation 1, expressed in terms of \( D_x = \{d_{x+k-1} : k = 1,2,...\} \) as,

\[
\hat{a}_{x:j}(D_x) = \sum_{k=h}^{h+j-1} kP_x \cdot v^k = \sum_{k=h}^{h+j-1} \left( \prod_{l=1}^{k} w_{x+l-1} \right) \cdot v^k
\]

(2)

where corresponding \( D_x \) and \( w_{x+k-1} \) as follows in table 1.

All annuity products are special case of general annuity product. For example, \( qE_n \) (the APV of the \( n \)-year endowment) are special case of \( \hat{a}_{x:j} \) when \((h,j) = (n,1)\). The APV of the discrete life insurance also can be expressed in terms of \( \hat{a}_{x:j} \).

Prior to define and derive the mortality durations and convexities of \( \hat{a}_{x:j} \), the traditional interest rate durations and convexities were initially reviewed. Consider \( P(i) = \sum_{k=1}^{n} v^k \cdot C_k \), the present value of a financial asset or liability at time 0 with cash flows \( C_k \) at time \( k, k = 1,2,...n \). The dollar duration (DD) and dollar convexity (DC) with respect to \( i \) are defined by [7]:

\[
DD[P(i)] = \lim_{h \to 0} \frac{P(i+h) - P(i)}{h} = \frac{d}{di} P(i)
\]

and

\[
DC[P(i)] = \lim_{h \to 0} \frac{DD[P(i+h)] - DD[P(i)]}{h} = \frac{d^2}{di^2} P(i)
\]

From the equation we can see that duration and convexity measure the sensitivity of the price of financial assets to the constant change of the interest rate. We want to apply the concept of duration and convexity in interest rate to the duration and convexity in mortality rate. Lin et al. proposed a route to obtain the closed-form formulas for mortality dollar durations and convexities of \( \hat{a}_{x:j} \) in regards to an instantaneously proportional and an instantaneously parallel change in \( D_x \) [1].

| Table 1. Corresponding \( D_x \) and \( w_{x+i-1} \) for APV of general annuity product. |
|-----------------|-------------------|
| \( D_x \)      | \( w_{x+i-1} \)   |
| \( P_x \)       | \( p_{x+i-1} \)   |
| \( Q_x \)       | \( 1 - q_{x+i-1} \) |
| \( U_x \)       | \( e^{-\mu_x} \)  |
| \( \ln(U_x) \)  | \( e^{-e^{\ln(p_x)}} \) |
| \( Q_x/P_x \)   | \( \frac{1}{1 + q_x/p_x} \) |
| \( \ln(Q_x/P_x) \) | \( \frac{1}{1 + e^{\ln(q_x/p_x)}} \) |
Let $\gamma$ be the change of mortality rate. When the change is proportional, then $D_x$ shifted proportionally to $D_x^* = (1 + \gamma) \cdot D_x$. When the change is parallel, then $D_x$ moved constantly to $D_x^* = D_x + \gamma$. Apply similar concept from dollar duration and dollar convexity in interest rate, we define mortality duration and convexity as

$$D^4_{[h][\hat{a}_{x:j}]}(D_x) = \lim_{\gamma \to 0} \frac{h[\hat{a}_{x:j}](D_x^*) - h[\hat{a}_{x:j}](D_x)}{\gamma} = \frac{\partial}{\partial \gamma} \left[ h[\hat{a}_{x:j}](D_x^*) \right]_{\gamma=0} \quad (3)$$

and

$$C^4_{[h][\hat{a}_{x:j}]}(D_x) = \lim_{\gamma \to 0} \frac{D[h[\hat{a}_{x:j}](D_x)] - D[h[\hat{a}_{x:j}](D_x)]}{\gamma} = \frac{\partial^2}{\partial \gamma^2} \left[ h[\hat{a}_{x:j}](D_x) \right]_{\gamma=0} \quad (4)$$

where ($\lambda = p$) is applied when the change is proportional and ($\lambda = c$) is applied when the change is parallel.

Consider that when $D_x$ is shifted proportionally or moved constantly to $D_x^*$, then $kP_x$ is changed to $kP_x^* = kP_x \cdot f_{D_x}^\lambda(k, \gamma)$ and $h[\hat{a}_{x:j}](D_x) = \sum_{k=h}^{h+j-1} kP_x \cdot v^k$ becomes $h[\hat{a}_{x:j}](D_x^*) = \sum_{k=h}^{h+j-1} kP_x^* \cdot v^k$ where $f_{D_x}^\lambda(k, \gamma)$ is an adjustment function and $\lambda = p$ or $c$ indicates a proportional or parallel change in $D_x$. We then expand $f_{D_x}^\lambda(k, \gamma)$ with respect to $\gamma$ to

$$f_{D_x}^\lambda(k, \gamma) = f_{D_x}^\lambda(k, 0) + \frac{\partial f_{D_x}^\lambda(k, \gamma)}{\partial \gamma} \bigg|_{\gamma=0} \times \gamma + \frac{\partial^2 f_{D_x}^\lambda(k, \gamma)}{\partial \gamma^2} \bigg|_{\gamma=0} \times \frac{\gamma^2}{2} \quad (5)$$

with $f_{D_x}^\lambda(0, \gamma) = 1$ and $f_{D_x}^\lambda(k, 0) = 1$. If we substitute equation 5 to equation 3 and equation 4 we define the mortality duration and convexity of $h[\hat{a}_{x:j}](D_x)$, as

$$D^4_{[h][\hat{a}_{x:j}]}(D_x) = \frac{\partial}{\partial \gamma} \left[ \sum_{k=h}^{h+j-1} f_{D_x}^\lambda(k, \gamma) \cdot kP_x \cdot v^k \right] = \sum_{k=h}^{h+j-1} d^\lambda_{D_x}(k) \cdot kP_x \cdot v^k \quad (6)$$

and

$$C^4_{[h][\hat{a}_{x:j}]}(D_x) = \frac{\partial^2}{\partial \gamma^2} \left[ \sum_{k=h}^{h+j-1} f_{D_x}^\lambda(k, \gamma) \cdot kP_x \cdot v^k \right] = \sum_{k=h}^{h+j-1} c^\lambda_{D_x}(k) \cdot kP_x \cdot v^k \quad (7)$$

respectively, where the slope function

$$d^\lambda_{D_x}(k) = \frac{\partial f_{D_x}^\lambda(k, \gamma)}{\partial \gamma} \bigg|_{\gamma=0} \quad (8)$$

and the curvature function

$$c^\lambda_{D_x}(k) = \frac{\partial^2 f_{D_x}^\lambda(k, \gamma)}{\partial \gamma^2} \bigg|_{\gamma=0} \quad (9)$$
Table 2 to table 4 gives the adjustment function $f_{D_x}^A(k, \gamma)$, the slope function $d_{D_x}^P(k)$, and the curvature function $c_{D_x}^\beta(k)$ for corresponding $D_x$.

### 3. Duration matching and convexity matching strategies for life insurance and annuity product
In this section we study about mortality duration convexity and strategy for a life insurance portfolio composed of a life insurance and an annuity product. Then, we study the effect to the surplus and how to determine the weights of portfolio when the strategy is applied.

#### Table 2. Adjustment Function $f_{D_x}^A(k, \gamma)$

| $D_x$ | $f_{D_x}^P(k, \gamma)$ | $f_{D_x}^\beta(k, \gamma)$ |
|-------|-------------------------|---------------------------|
| $U_x$ | $\prod_{i=1}^k [p_{x+i-1}^\gamma] = (q_{px})^\gamma$ | $\prod_{i=1}^k [e^{-\gamma}] = e^{-k\gamma}$ |
| $Q_x$ | $\prod_{i=1}^k \left[1 - \gamma \cdot \left(\frac{1}{p_{x+i-1}} - 1\right)\right]$ | $\prod_{i=1}^k \left(1 - \frac{\gamma}{p_{x+i-1}}\right)$ |
| $P_x$ | $\prod_{i=1}^k (1 + \gamma) = (1 + \gamma)^k$ | $\prod_{i=1}^k \left(1 + \frac{\gamma}{p_{x+i-1}}\right)$ |
| $\ln(U_x)$ | $\prod_{i=1}^k \left(p_{x+i-1}\right)^{(-\ln(p_{x+i-1}))^{\gamma}}$ | $\prod_{i=1}^k \left(p_{x+i-1}\right)^{\gamma - 1} = (q_{px})^{\gamma - 1}$ |
| $Q_x/P_x$ | $\prod_{i=1}^k \left[\frac{1}{p_{x+i-1} + q_{x+i-1} \cdot (q_{x+i-1}/p_{x+i-1})^\gamma}\right]$ | $\prod_{i=1}^k \left[\frac{1}{1 + \gamma \cdot p_{x+i-1}}\right]$ |
| $\ln(Q_x/P_x)$ | $\prod_{i=1}^k \left[p_{x+i-1} + q_{x+i-1} \cdot (q_{x+i-1}/p_{x+i-1})^\gamma\right]$ | $\prod_{i=1}^k \left[1 + \gamma \cdot p_{x+i-1}\right]$ |

#### Table 3. Slope Function $d_{D_x}^P(k)$

| $D_x$ | $d_{D_x}^P(k)$ | $d_{D_x}^\beta(k)$ |
|-------|---------------|-------------------|
| $U_x$ | $\sum_{i=1}^k \left[\ln(p_{x+i-1})\right] = \ln(q_{px})$ | $-\sum_{i=1}^k [1] = -k$ |
| $Q_x$ | $-\sum_{i=1}^k \left[\frac{1}{p_{x+i-1}} - 1\right]$ | $-\sum_{i=1}^k \left[\frac{1}{p_{x+i-1}}\right]$ |
| $P_x$ | $\sum_{i=1}^k [1] = k$ | $\sum_{i=1}^k \left[\frac{1}{p_{x+i-1}}\right]$ |
| $\ln(U_x)$ | $\sum_{i=1}^k \left[\ln(p_{x+i-1}) \times \ln(-\ln(p_{x+i-1}))\right]$ | $\sum_{i=1}^k \left[\ln(p_{x+i-1})\right] = \ln(q_{px})$ |
| $Q_x/P_x$ | $-\sum_{i=1}^k [q_{x+i-1}]$ | $-\sum_{i=1}^k \left[p_{x+i-1}\right]$ |
| $\ln(Q_x/P_x)$ | $-\sum_{i=1}^k [q_{x+i-1} \cdot \ln\left(q_{x+i-1}/p_{x+i-1}\right)]$ | $-\sum_{i=1}^k [q_{x+i-1}]$ |
Table 4. Curvature Function $c_D^2(k)$

| $D_k$   | $c_D^P(k)$ | $c_D^c(k)$ |
|---------|------------|------------|
| $U_x$   | $[d_{U_x}^P(k)]^2$ | $[d_{U_x}^c(k)]^2$ |
| $Q_x$   | $[d_{Q_x}^P(k)]^2 - \sum_{i=1}^k \left( \frac{1}{p_{x+i-1}} - 1 \right)^2$ | $[d_{Q_x}^c(k)]^2 - \sum_{i=1}^k \left( \frac{1}{p_{x+i-1}} \right)^2$ |
| $P_x$   | $[d_{P_x}^P(k)]^2 - \left( \sum_{i=1}^k 1^2 \right)$ | $[d_{P_x}^c(k)]^2 - \sum_{i=1}^k \left( \frac{1}{p_{x+i-1}} \right)^2$ |
| $\ln(U_x)$ | $[d_{\ln(U_x)}^P(k)]^2 + \left[ \sum_{i=1}^k \ln(p_{x+i-1})[\ln\ln(p_{x+i-1})] \right]^2$ | $[d_{\ln(U_x)}^c(k)]^2 + \sum_{i=1}^k \ln(p_{x+i-1})$ |
| $Q_x/P_x$ | $[d_{Q_x/P_x}^P(k)]^2 + \sum_{i=1}^k (q_{x+i-1})^2$ | $[d_{Q_x/P_x}^c(k)]^2 + \sum_{i=1}^k (p_{x+i-1})^2$ |
| $\ln(Q_x/P_x)$ | $[d_{\ln(Q_x/P_x)}^P(k)]^2 - \sum_{i=1}^k \left[ q_{x+i-1}p_{x+i-1}\left(\ln\frac{q_{x+i-1}}{p_{x+i-1}}\right)^2 \right]$ | $[d_{\ln(Q_x/P_x)}^c(k)]^2 - \sum_{i=1}^k \left[ q_{x+i-1}p_{x+i-1} \right]$ |

Consider an insurance portfolio $F^{LA}$ consisting of life insurance and annuity product with surplus $S^{LA}$. We can write $S^{LA}$, in terms of $D_x$, as

$$S^{LA}(D_x) = \sum_{k=0}^{\infty} C_k \cdot kp_x \cdot v^k \quad (10)$$

where $C_k$ is cash flow at time $k$. When $D_x$ is changed to $D_x^*$, substitute $kp_x^* = kp_x \cdot f_{D_x}^A(k, \gamma)$ to (3.1) then the surplus become

$$S^{LA}(D_x^*) = \sum_{k=0}^{\infty} C_k \cdot kp_x^* \cdot v^k = \sum_{k=0}^{\infty} C_k \cdot kp_x \cdot f_{D_x}^A(k, \gamma) \cdot v^k. \quad (11)$$

From equation 10 and equation 11, the change of the surplus is

$$\Delta S^{LA}(D_x) = S^{LA}(D_x^*) - S^{LA}(D_x) = \sum_{k=0}^{\infty} C_k \cdot kp_x \cdot [f_{D_x}^A(k, \gamma) - 1] \cdot v^k. \quad (12)$$

Substitute equation 5 to equation 12, we get

$$\Delta S^{LA}(D_x) = \gamma \times \sum_{k=0}^{\infty} d_{D_x}^A(k) \cdot C_k \cdot kp_x \cdot v^k + \frac{\gamma^2}{2} \times \sum_{k=0}^{\infty} c_{D_x}^A(k) \cdot C_k \cdot kp_x \cdot v^k \quad (13)$$
When duration matching strategy or convexity matching strategy is adopted, then the duration or convexity of the present value of assets matches that of the liability cash flows. In other words, the duration or convexity of the present value of the net cash flows is zero, or equivalently

\[ B^A[S^{LA}(D_x)] = 0 \iff \sum_{k=0}^{\infty} b^A_{D_x}(k) \cdot C_k \cdot r_k p_x \cdot v^k = 0 \]

where \( B = D \) and \( b^D_{D_x} = d^D_{D_x} \) for duration matching strategy, and \( B = C \) and \( b^C_{D_x} = c^C_{D_x} \) for convexity matching strategy. When the duration \((B = D)\) matching strategy is applied, equation 13 turns out to be

\[ \Delta S^{LA}(D_x) = \frac{y^2}{2} \times \sum_{k=0}^{\infty} c^C_{D_x}(k) \cdot C_k \cdot r_k p_x \cdot v^k \]  (14)

whereas when the convexity \((B = C)\) matching strategy is applied, equation 13 turns out to be

\[ \Delta S^{LA}(D_x) = \gamma \times \sum_{k=0}^{\infty} d^C_{D_x}(k) \cdot C_k \cdot r_k p_x \cdot v^k \]  (15)

Now, we study how to determine the optimal weights with each strategy for portfolio consisted of an annuity and a life insurance product. Consider an insurance portfolio \( F^{LA} \) containing of life insurance product and annuity product with weights \( w_L \) and \( w_A = 1 - w_L \), respectively. The weighted surplus of the portfolio is

\[ S^{LA} = w_L \cdot S^L + w_A \cdot S^A = w_L \cdot S^L + (1 - w_L) \cdot S^A. \]  (16)

We want to determine the weight that fulfill the duration matching strategy or convexity matching strategy condition, that is

\[ B^A[S^{LA}(D_x)] = 0. \]  (17)

or equivalently,

\[ B^A[w_L \cdot S^L(D_x) + (1 - w_L) \cdot S^A(D_x)] = 0 \iff w_L \cdot B^A[S^L(D_x)] + (1 - w_L) \cdot B^A[S^A(D_x)] = 0 \]

which leads to

\[ w_L = \frac{B^A[S^A(D_x)]}{B^A[S^A(D_x)] - B^A[S^L(D_x)]} \]

and

\[ w_A = 1 - w_L = \frac{-B^A[S^L(D_x)]}{B^A[S^A(D_x)] - B^A[S^L(D_x)]}. \]  (18)
Equation 18 gives the weight of the life insurance and annuity product in portfolio, which is determine by $B^A(D_x)$, called the mortality duration ($B = D$) or convexity ($B = C$) matching strategy with respect to an proportional ($\lambda = p$) or parallel ($\lambda = c$) change in $D_x$.

4. Numerical simulation of duration matching and convexity matching strategy in Indonesia
In this section we give numerical simulation to calculate the optimal weights for portfolio composed of life insurance and annuity product. We adopt data for Indonesian males from Indonesia Mortality Table 2011 (TMI-2011). From the data we obtain the value of $\delta p_x$ for $x \in [1, 111]$ which is needed to calculate the optimal weights. The mortality rate at age above 111, is set equal to 1. We also assume the premiums is set equal to 120 % of the net premiums.

Equation 18 give the optimal weights for general portfolio composed of life insurance product and annuity product. Specifically, in this paper we concentrate on two kinds of insurance portfolio:

- $F_{TP}$: the m-payment and n-year term life insurance and pure endowment with premiums $P^{TL}$ and $P^{PE}$, respectively, for insured age $x$. For this portfolio, we divide the condition in two cases:
  (i) For $x = 20 - 70$ and $m = n = 20$
  (ii) For $x = 20 - 60$ and $m = n = 65 - x$
- $F_{WA}$: the m-payment whole life insurance and the m-payment and n-year deferred whole life annuity-due with premiums $P^{WL}$ and $P^{DA}$, respectively, for insured age $x$. For this portfolio, we divide the condition in two cases:
  (i) For $x = 40 - 70$ and $m = n = 20$
  (ii) For $x = 40 - 60$ and $m = n = 65 - x$

Figure 1 and figure 2 gives the result for portfolio $F_{TP}$, while figure 3 and figure 4 gives the result for portfolio $F_{WA}$.

From the graph we can get some conclusion, as follows:

1. The weights from duration matching strategy and convexity matching strategy have similar value and characteristic. It can be seen from:
   - The optimal weights from duration matching strategy and convexity matching strategy in each case, are in the same range.
   - The optimal weights from duration matching strategy and convexity matching strategy in each case have similar characteristic, that is increasing or decreasing, respect to the age $x$.

2. Twenty four optimal weights from duration matching strategy and convexity matching strategy can be classified into seven groups, with the value in each group are very close. The groups are:

Group 1: $(D^P, U_x), (D^P Q_x), (D^C, \ln(Q_x/P_x)), (D^P, Q_x/P_x), (D^C, \ln(U_x))$
Group 2: $(D^P, \ln(U_x)), D^P(\ln(Q_x/P_x))$
Group 3: $(D^C, U_x), (D^C, Q_x), (D^P, P_x), (D^C, Q_x/P_x)$
Group 4: $(C^P, U_x), (C^P, Q_x), (C^P, Q_x/P_x)$
Group 5: $(C^C, U_x), (C^C, Q_x), (C^C, P_x), (C^C, Q_x/P_x)$
Group 6: $(C^C, \ln(U_x)), (C^C, \ln(Q_x/P_x))$
Group 7: $(C^P, \ln(U_x)), (C^P, \ln(Q_x/P_x))$
Figure 1. The optimal weights $w_{TL}$ for case (a) and (b) using duration matching strategy.

Figure 2. The optimal weights $w_{TL}$ for case (a) and (b) using convexity matching strategy.

Figure 3. The optimal weights $w_{WL}$ for case (a) and (b) using duration matching strategy.
5. Conclusion
The change in the mortality rate can affect the surplus that is essential for business. Therefore, find ways to anticipate this change in mortality rate has been a matter of great urgency. As a result, adopting immunization respect to the mortality rate changes is essential. In this paper, we examine 24 duration/convexity matching strategies for mortality immunization to determine the optimal weights of the portfolios $F_{TP}$ and $F_{WA}$. We also provide numerical simulation using Indonesia Mortality Table 2011 (TMI-2011). From the simulation we found that the weights from duration matching strategy and convexity matching strategy have similar value and characteristic. The optimal weights have classified into seven groups. For further analysis, we can use the classification to determine which strategy that gives the best result. However, determine the best strategy is not the scope of this paper.

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