EXPLAINING THE KNEE BY COSMIC RAY ESCAPE FROM THE GALAXY

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ABSTRACT

We study the escape of cosmic rays (CRs) with energies between $E/Z = 10^{14}$ eV and $10^{17}$ eV from our Galaxy, calculating the trajectories of individual CRs in recent models of the regular and turbulent Galactic magnetic field. Determining the average grammage $X(E)$ traversed by CRs, we find a knee-like structure of $X(E)$ around $E/Z = \text{few} \times 10^{15}$ eV for a coherence length $l_c \approx 5$ pc of the turbulent field. The resulting change in the slope of $X(E)$ is sufficiently strong to explain the proton knee observed by KASCADE: Thus the knee may, in this regard, be entirely explained by CR leakage from the Milky Way, rendering additional effects unnecessary. We find that the decrease of $X(E)$ slows down around $E/Z \approx 10^{16}$ eV in a model with a weak turbulent magnetic field, in agreement with the energy dependence of the proton flux as determined by KASCADE-Grande.

Subject headings: Cosmic ray theory, Galactic magnetic fields.

1. INTRODUCTION

The cosmic ray (CR) energy spectrum follows a power law on more than ten decades in energy, with only a few breaks, such as the knee at $E_0 \approx 4$ PeV which was first reported by Kulikov et al. (1958). It coincides with a suppression of the primary proton flux, and the composition becomes increasingly heavier in the energy range between the knee and $10^{15}$ eV [Apel et al. 2013; Abbasi et al. 2013]. A knee-like structure in the heavy component has been detected around $\approx 10^{15}$ eV by KASCADE-Grande (Apel et al. 2011), as well as features in the spectrum [Apel et al. 2013], see also [Abbasi et al. 2013]. Even if the elemental composition is not sufficiently well determined yet, these observations may be consistent with this region being a succession of rigidity-dependent knees corresponding to cutoffs at $E_0$ for individual particles with charges $Z$, as suggested first by Peters (1961), see also [Webber 1983; Stanev et al. 1993; Hörandel 2004; Berezinsky et al. 2004].

The origin of the knee is still unknown. Explanations fall into two main categories. First, it may correspond to the maximum rigidity to which CRs can be accelerated in supernova (SN) remnants, see for example [Stanev et al. 1993; Kobayakawa et al. 2003; Ptuskin & Zirakashvili 2003]. In practice, supernovae explode in different environments and should not all accelerate CRs to the same maximum rigidity $E_{\text{max}}/Z$, e.g. [Hillas 2005] and [Bell et al. 2013]. The knee may be the result of the superposition of different $E_{\text{max}}/Z$, e.g. [Sveshnikova 2003]. Elyakin & Wolfendale (1997, 2004) proposed that it may be the signature from a nearby source such as Monogem. Second, if the knee corresponds to the rigidity at which the CR Larmor radius $r_L$ starts to be of the order of the coherence length $l_c$ of the interstellar turbulent magnetic field, the behaviors of the CR diffusion coefficient and confinement time change, which in turn would induce a steepening in the spectrum. Hall diffusion has been proposed as a possible explanation e.g. by [Ptuskin et al. 1993; Candia et al. 2002, 2003]. Alternatively, the knee may be the superposition of the two effects discussed above, see [Hörandel 2004] for a review.

The goal of this Letter is to study CR escape from our Galaxy by propagating individual CRs in detailed models of the Galactic magnetic field (GMF) [Pshirkov et al. 2011, 2013; Jansson & Farrar 2012a,b]. We calculate two main observables: The time-averaged grammage $X(E)$ traversed by CRs before escape, and the time-dependent intensity of CR protons at the position of the Sun. The energy dependence of $X(E)$ allows us to decide if the knee can be caused solely by a break in the confinement time, or if an additional effect from the maximum energy $E_{\text{max}}$ of CR sources must be invoked. Moreover, we can constrain GMF models extrapolating $X(E)$ towards its measured value at low energies. Calculating the time-dependent intensity of CR protons, we determine likely values for the energy output and rate of Galactic CR sources that are required to explain measurements in the energy range $100$ TeV–$100$ PeV. Comparing these values at different energies, we can examine if the chosen scenario is consistent and verify if and above which energy an additional CR component may be needed. The main purpose of this Letter is to illustrate the power of the proposed approach, and a more complete study of the allowed parameter space will be presented in a future work.

2. SIMULATION PROCEDURE

For the propagation of CRs in the GMF, we use the code described and tested by Giacinti et al. (2012). For the present work, we have implemented more recent GMF models [Pshirkov et al. 2011, 2013; Jansson & Farrar 2012a,b], but the most important change is the use of a reduced coherence length, $l_c = 5$ pc, for the turbulent field. Such a value is in line with recent measurements of $l_c$ in some regions of the Galactic disk, see e.g. [Jacobelli et al. 2013]. We assume that the field fluctuations follows an isotropic Kolmogorov power-spectrum $P(k) \propto k^{-5/3}$ extending down to a minimal
length scale $l_{\text{min}} \lesssim 1$ AU.

We use the same value $l_c = 5$ pc for the coherence length in the whole Galaxy. The use of a too small coherence length at large Galactic latitudes will underestimate the confinement time of CRs from the Galactic halo, but has only a small influence on the two observables we use in this analysis, grammage and CR intensity. Since the interstellar gas is strongly concentrated within the thin Galactic disk, the grammage traversed by CRs is determined mainly by the time CRs spend within this thin layer. The same holds true for the observed CR intensity which is measured at $z = 0$.

We assume that the surface density of CR sources follows the distribution

$$n(r) = \left( \frac{r}{R_\odot} \right)^{0.7} \exp \left( -3.5 \frac{(r - R_\odot)}{R_\odot} \right)$$

with $R_\odot = 8.5$ kpc as the distance of the Sun to the Galactic center. This distribution was obtained by Green (2013), fitting the distribution of supernova remnants.

3. GRAMMAGE

An important constraint on CR propagation models comes from ratios of stable primaries and secondaries produced by CR interactions on gas in the Galactic disk. In particular, the B/C ratio has only a small influence on the two observables we use in this analysis, grammage and CR intensity. Since the interstellar gas is strongly concentrated within the thin Galactic disk, the grammage traversed by CRs is determined mainly by the time CRs spend within this thin layer. The same holds true for the observed CR intensity which is measured at $z = 0$.

In all cases, the grammage traversed by CRs at reference energies $E_0/Z = 5-15$ GeV was found to lie in the range $9-14$ g/cm$^2$. In their model with reacceleration, the grammage decreases as $X(E) = X_0(E/E_0)^{-0.3}$ above energies $E_0 > 20-30$ GeV per nucleon, while the best-fit value for the normalization constant $X_0$ of the grammage was determined to be $X_0 = 9.4$ g/cm$^2$.

We calculate the grammage $X = c \int dt \rho(x(t))$ by injecting CRs with the radial distribution $n(z) = N \exp(-z/z_{1/2})$ with $N = 0.3$ g/cm$^3$ at $R_\odot$ and $z_{1/2} = 0.21$ kpc, inspired by Nakamichi & Sofue (2003) and Evoli et al. (2007). We set $n = 10^{-4}$ g/cm$^3$ as minimum gas density up to the edge of the Milky Way at $|z| = 10$ kpc. Since the grammage $X(E) \propto E^{-\delta}$ scales as the confinement time $\tau(E) \propto E^{-\delta}$, we can use this quantity also to deduce possible changes in the CR intensity as a consequence of a change in the CR leakage rate.

In Fig. 1, we show the grammage traversed by CRs with energies $E/Z$ between $10^{14}$ eV and $10^{17}$ eV, propagated in the GMF model of Jansson & Farrar (2012a). The upper (green) line corresponds to computations using both the regular and the turbulent fields proposed by Jansson & Farrar (2012a), while for the lower (red) curve we rescaled the turbulent field strength by a factor 0.1 ($B_{\text{rms}} \rightarrow B_{\text{rms}}/10$). The two lowest-energy data points shown here are consistent with the $X(E) \propto E^{-1/3}$ behaviour expected for a turbulent magnetic field with a Kolmogorov power-spectrum. Around a few PeV, the grammage steepens to an approximate power-law $X(E) \propto E^{-1.3}$ which lies in-between the expectations for Hall diffusion ($X(E) \propto E^{-1}$) and small-angle scattering ($X(E) \propto E^{-2}$). Finally, the turn-over of the grammage which is visible in the lower curve at the highest energies corresponds to the approach of its asymptotic value obtained for straight line propagation in the limit $E \rightarrow \infty$. As a consequence, the predicted proton spectrum above $10^{16}$ eV should harden by approximately one power. $\Delta \delta \sim 1$, using the Jansson & Farrar (2012a) model with a reduced turbulent field. Interestingly, the proton flux deduced by the KASCADE-Grande collaboration displays such an energy dependence.

The general behaviour of the grammage as a function of energy agrees well with the theoretical expectations: A transition from large-angle to small-angle scattering or Hall diffusion is expected at the characteristic energy $E_c$ where the Larmor radius $r_L$ equals the coherence length $l_c$. For $l_c = 5$ pc and $B = 3 \mu G$, the value of the critical energy is $E_c/Z \approx 1 \times 10^{16}$ eV, which is slightly higher than what we find numerically. This small difference is however not unexpected, because this simple estimate does not take into account the fact that the magnetic field strength is non-uniform, and consists of a turbulent and a regular component.

In addition to the data points, we show in Fig. 1 with a dotted (blue) line the extrapolation of the grammage to lower energies, assuming that $X(E) \propto E^{-1/3}$ as expected for a Kolmogorov power-spectrum. Based on this extrapolation, the Jansson & Farrar (2012a) model with full turbulence leads to a grammage at $E \lesssim 100$ GeV which is a factor $\sim 10$ above the determinations from e.g. B/C measurements (blue cross). This discrepancy is in line with our determination of the diffusion coefficient in a purely turbulent magnetic field with strength $B_{\text{rms}} = 4 \mu G$ (Giacinti et al. 2012), which also disagreed by an order of magnitude with the extrapolation of the diffusion coefficient phenomenologically determined from the ratios of secondary to primary nuclei. Consistency with these measurements could be achieved, if either i) the grammage and thus the diffusion coefficient have an energy dependence sufficiently flatter than $\delta = 1/3$, ii) the coherence length $l_c$ is similar to or below the parsec scale, iii) the gas density is scaled down, or iv) the energy density of the turbulent magnetic field is reduced.
Such a reduction may be compatible with determinations of the turbulent field strength, if it is done asymmetrically. We have verified that using anisotropic turbulence where only the turbulent field components perpendicular to the regular field are decreased by a factor 10 leads to approximately the same grammage as a reduction of all three components. This means that results presented in this Letter are similar to those for such an anisotropic turbulent field.

Option i) could be realized if the magnetic turbulence has a power spectrum close to $P(k) \propto k^{-2}$; such a behaviour is in principle not excluded, though also not favoured above 200 GeV by the AMS-02 data (Evoli & Yan 2013). Measurements of the coherence length $l_c$ have been performed only along a few selected directions which contain regions emitting large synchrotron fluxes; if these directions are not representative of the Galactic disk as a whole, then it might be possible that the average value of $l_c$ in the disk is below $\sim 1$ pc; however, this would be incompatible with the observed position of the knee. Option iii) amounts simply to a rescaling of the grammage; however, the relatively large factor required to achieve an agreement of the grammage with low-energy measurements is in conflict with observations (Nakanishi & Sofue 2003; 2006). Option iv) corresponding to a rescaling of all three components of the turbulent field is displayed as the lower (red) line in Fig. 1. In the following, we consider this case, postponing a study of other options to a later work.

We have calculated the grammage also in the GMF model of Pshirkov et al. (2011, 2013). The CR confinement time was found to be twice as large as for the GMF model of Jansson & Farrar (2012a,b), leading to an even stronger discrepancy between the extrapolation of the model of Jansson & Farrar (2012a,b) and the observed position of the knee. Option iii) could be a rescaled GMF model with Kolmogorov turbulence $\sim N_{10^{15}}$ eV the intensity of protons from KASCADE data given in Apel et al. (2013), while we have employed at lower energies data from the GRAPES-3 experiment (Tanaka et al. 2007). We have chosen these two data sets, because they agree well in the energy range where the two experiments overlap. As GMF model, we use the Jansson & Farrar (2012a,b) model with a rescaled turbulent field.

Note the difference with Section 3. Here, $I(E, t)$ is predicted as a function of time. Since $I(E, t)$ fluctuates due to the discreteness of the sources, we calculate the probability distributions of how well the predicted and measured CR intensities agree. In contrast, the grammage is measured at relatively low energies, $E \lesssim 1$ TeV, where fluctuations due to the discrete source distribution play only a minor role and therefore only average values are relevant.

In Fig. 2 we plot the probability distribution $p(\dot{N})$ for seven energies between 100 TeV and 100 PeV. The three distributions $p(\dot{N})$ for CR proton energies $E \leq 1$ PeV overlap very well, having a peak around the source rate $\dot{N} \sim 7 \times 10^{-3}$ yr. At 3 and 10 PeV, the peak moves to $\dot{N} \sim 2 \times 10^{-2}$ yr, while the maxima of the distributions for 30 and 100 PeV move back to the best-fit value found at lower energies. Generally, the width of the probability distribution $p(\dot{N})$ increases with increasing energy, indicating a larger variability of the intensity due to the stochastic nature of the source distribution.

We conclude from Fig. 2 that the assumptions used, a rescaled GMF model with Kolmogorov turbulence and a $1/E^{2.4}$ injection spectrum, lead to a consistent CR proton flux in the energy range around the knee, $E \sim (10^{14} - 10^{17})$ eV. The small difference in the best-fit value of the source rate found at different energies may be caused by deviations of the chosen GMF model from the true GMF: For instance, the shape of the two curves in Fig. 1 differ, because the two GMF models lead to a different behaviour of the confinement time $\tau(E)$ as a function of energy.

The obtained source rate $\dot{N} \sim 10^{-2}$ yr corresponds approximately to the Galactic SN rate and makes a GRB origin of Galactic CRs unlikely in the considered energy

![Figure 2](image-url)
range. Taken at face value, these numbers would indicate that a too large fraction of SNe can accelerate protons to \( E \sim 10^{17} \text{ eV} \). However, one should keep in mind that these conclusions hold only within the specific assumptions used: In particular, a change of the injection spectrum of CRs and a population dependent maximal energy as well as changes in the GMF model will modify the estimated source rate.

Finally, we note that the behavior of the source rate as a function of energy can be used as an indicator for a new (extragalactic) CR component. As one expects generically a smaller number of Galactic sources being able to accelerate to higher energies, an increase of the source rate as a function of energy would signal the onset of a new, additional CR component.

5. Conclusions and Perspectives

The two main explanations for the knee are i) a change in the CR confinement time in the Galaxy when their Larmor radius starts to be of the order of the coherence length \( \ell_c \) of the interstellar turbulence, and ii) a change in the number of sources able to accelerate CRs above \( \approx 4 \text{ PeV} \). We have shown that, if the coherence length in the Galactic disk is of the order of \( \ell_c = 5 \text{ pc} \) as suggested by e.g. [Iacobelli et al. (2013)], the CR confinement time \( \tau(E) \sim X(E) \) steepens in the correct energy range, around \( E_k \). Moreover, we find that the steepening in CR confinement time is sufficiently large to explain entirely the reduction of the proton flux measured by KASCADE in this energy range.

On the contrary, with a coherence length \( \ell_c \) of the order of \( \ell_c = 50 \text{ pc} \) in the Galactic disk, the change in the slope of \( \tau(E) \sim X(E) \) would be shifted to energies above \( 10^{16} \text{ eV} \). In this case the knee would have to be explained by the second possibility ii), which would yield precious information on Galactic CR accelerators. More measurements of the coherence length and strength of the turbulence should solve this crucial question.

A large coherence length \( \ell_c \) would however worsen the tension reported in the computations of the grammage. This tension is reduced if the coherence length is reduced, and/or the power-spectrum is flatter \( (\mathcal{P}(k) \propto k^{-\alpha} \text{ with } \alpha \to 2) \). However, we consider as most promising an approach to the case of anisotropic turbulence where only the strength of the turbulent field perpendicular to the regular field is reduced. With such a turbulence, the search for ultra-high energy CR sources may be facilitated by a weaker spread of their images on the sky.

We have calculated the time-dependent intensity of CR protons produced by discrete sources with prescribed rates and energy outputs in a specific GMF model. We have shown that the predicted distribution of CR intensities can be used to constrain a given parameter set. In the specific case of the GMF model considered here and with a weak turbulent field we have found that the decrease of \( X(E) \) stops around \( E/Z \approx 10^{16} \text{ eV} \). As a result, no substantial change in the source rate was required in the range up to 100 PeV for sources with an \( E^{2.4} \) injection spectrum and maximum energy \( E_{\text{max}} = 10^{17} \text{ eV} \). More generally, the method presented here can be used to search for the onset of a new (extragalactic) proton component: An increase in the required source rate beyond a given energy would hint at the beginning of a new component.

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