Research on stability of robotic longitudinal-torsional ultrasonic milling with variable cutting force coefficient

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Abstract
With their successful applications in handling, spraying, arc welding and other processing fields, industrial robots are gradually replacing traditional CNC machine tools to complete machining tasks due to the wider working envelope and the higher flexibility. Aiming at the chatter problem, a robotic longitudinal-torsional ultrasonic milling method with variable force coefficient is proposed in this paper. Taking carbon fiber-reinforced plastics (CFRP) as the processing object, the influence of the fiber layup angle on the milling force is analyzed first; then, the robot milling force parameters are determined, and the robot milling kinematics model is established. Furthermore, the ultrasonic function angle is defined, and the cutting layer thickness model, the dynamic milling force model, and the dynamic differential equation under ultrasonic vibration are established to analyze the stability of robotic longitudinal-torsional ultrasonic milling of CFRP. Finally, the full discrete method is used to obtain stability lobe diagrams.

Keywords Industrial robots · Chatter · Longitudinal-torsional ultrasonic milling · Stability lobe diagrams

1 Introduction
Industrial robots are favored by various manufacturing industries because of their lower cost, higher flexibility, and stronger versatility, especially in the manufacturing fields of aerospace, automobiles, and electronic appliances [1–4]. With the technology advancement and wide application of industrial robots, they have gradually replaced traditional machine tools for manufacturing and processing in related fields, which is an important means and key link to promote the development of intelligent manufacturing. Carbon fiber-reinforced Plastics (CFRPs) are increasingly used in aerospace, defense and military industry, automobiles, high-speed trains, and sports and medical equipment. The use of CFRP in the processing and manufacturing of complex structural parts in the aerospace field put forward higher requirements on processing accuracy, quality, and efficiency so that traditional machining methods and processing tools cannot meet the accuracy requirements of CFRP. Ultrasonic-assisted machining has advantages in reducing cutting force, improving surface quality, and extending tool life, which is one of the most important method applied to the processing of CFRP [5, 6].

The application of ultrasonic-assisted processing technology to the composite materials was earlier researched in the 1980s. Japanese scholar Takeyama [7] studied the cutting force and surface quality and chip formation of ultrasonic-assisted cutting of glass fiber-reinforced resin-based composite materials. The research results showed that ultrasonic-assisted cutting has a significant effect on improving processing stability. Subsequently, Kim and Lee [8] studied the ultrasonic-assisted turning of CFRP under different processing parameters. The experimental results showed that, compared with ordinary turning, ultrasonic-assisted turning can obtain a higher quality of machined surface. Zhao et al. [9] carried out a comparative experiment between ultrasonic-assisted turning and ordinary turning of aluminum-based silicon carbide particle-reinforced composite materials, where the chip morphology was observed. The experimental results showed that the introduction of ultrasonic can effectively reduce the cutting process deformation to obtain a better surface quality. Existing studies
have shown that the use of ultrasound in the processing can significantly improve the processing stability and obtain a better surface quality than ordinary processing methods. Unfortunately, all the above studies apply ultrasonic-assisted techniques based on machine tools, and few studies involving ultrasonic-assisted approaches are conducted using industrial robots.

The related research applying ultrasonic technology to robotic milling processing has started in the past 10 years, and Guo et al. [10] carried out a comparative experiment of robotic rotary ultrasonic milling and ordinary milling to explore the feasibility of combining rotary ultrasonic machining with industrial robots. At the same time, the law of influence of ultrasonic technology on milling force, chatter, and surface accuracy was analyzed. The experimental results showed that ultrasonic milling can reduce the milling force by 22%, and the chatter amplitude can be reduced by more than 25%. Based on this, Sun et al. [11] studied the chatter problem during robotic ultrasonic-assisted machining immediately. A stability lobe diagram was obtained, and the accuracy of the established model is verified through experiments. Existing research shows that ultrasonic technology can be well integrated with industrial robots to improve the stability of robotic processing.

In this paper, the CFRP is regarded as the experimental workpiece. The longitudinal-torsional rotary ultrasonic technology is used to establish a dynamic milling force model to explore the influence of different force coefficients on the stability of robotic milling. The stability lobe diagram is obtained to analyze the influencing factors of the robotic longitudinal-torsional ultrasonic milling stability.

The remainder of the paper is organized as follows. The CFRP fiber layup angle is analyzed to describe the influence of milling force coefficient on the robotic milling stability conveniently in Sect. 2. In Sect. 3, the CFRP milling force coefficient in robotic milling is determined. The influence mechanism of the two-dimensional longitudinal-torsional ultrasonic vibration on the kinematics characteristics of the robotic milling tool is mainly elaborated from two aspects of kinematics trajectory and speed analysis in Sect. 4. Section 5 establishes the cutting layer thickness model and dynamic milling force model under ultrasonic vibration to analyze the stability of robotic longitudinal-torsional ultrasonic milling. Then, the stability lobe diagram is obtained in Sect. 6. Finally, the concluding marks are included in Sect. 7.

### 2 Analysis of CFRP fiber layup angle in robotic milling

Milling force coefficient is an important input parameter for stability domain simulation. The anisotropy of mechanical properties of CFRP makes the cutting force coefficient time-varying, which greatly increases the complexity of the stability analysis of robotic milling. The change of cutting force coefficient in robotic milling mainly comes from two aspects: (1) the change of tool rotation angle and (2) the variation of fiber layup angle. As shown in Fig. 1a, the tool rotation angle \( \phi \) is the included angle between the cutting edge and the direction perpendicular to the feed direction during one revolution of the tool, and the fiber layup angle...
\( \theta \) is defined as the included angle between the feed direction of the tool and the direction in which the fibers are laid.

For large and complex aviation parts, the sizes and contours are sophisticated and diverse; therefore, the cutting angle sometimes changes continuously during the robotic milling process. In order to describe the influence of milling force coefficient on the robotic milling stability conveniently, the fiber layup angle is divided into 4 cases (as shown in Fig. 1a): 0°, 45°, 90°, and 135°, respectively. For the milling part shown in Fig. 1b, workpiece contour that requires four sides. The fiber layup direction of side is the same as the feed direction of the tool; hence, the fiber layup angle is 0°. By analogy, the fiber layup angles associated with side, side, and side are 90°, 45°, and 135°, respectively.

The fiber layup angle of the milling part (Fig. 1b) mentioned above is the standard situation of division. In most cases, the fiber layup angle of the part contour is irregular. In this case, it can be processed in the manner shown in Fig. 1c. For the milling contour \( A_B a_b C_d D_f A_d r \), the fiber layup angle of side is obviously still 0°. Side is approximated by 90° because it is within 67.5° to 90°. Similarly, side is approximated by 45°, and side is approximated by 135°. It should be noted that when the edge contour angle is between 157.5° and 180° and is approximately 180°, the cutting situation is equivalent to that of the fiber layup angle of 0°. According to the equivalence principle, fiber layup angle at the contour of the part as shown in Fig. 1c can be equivalent to one or more of the conditions of 0°, 45°, 90° and 135°.

The contour edges of the above parts (Fig. 1c) are all straight lines, while the types of milling parts are complex and diverse, and in many cases, there exist curved contour edges, as shown in Fig. 1d. Sides, and can be treated equivalently in the manner shown in Fig. 1c. For the arc segment \( E_d a_d r \), based on the idea of differentiation, it can be considered to be composed of countless small straight line segments. When the tool cuts from point \( E_d r \), it can be considered that the fiber layup angle is 0°. With the milling process, the fiber layup angle is 45° when it reaches the side, 90° when it reaches the edge, and 135° when it reaches the edge. Therefore, the fiber layup angle in the curve section can be equivalent to four situations of 0°, 45°, 90°, and 135°, and when the part has a curve profile, the milling part has these four situations simultaneously.

### 3 Determination of CFRP milling force coefficient in robotic milling

For CFRP milling, the milling force coefficient is related to the cutting angle and the thickness of undeformed chip. In the process of robotic milling, the cutting angle \( \rho \) of the tool is determined by the tool rotation angle \( \phi \) and the fiber layup angle \( \theta \), which can be expressed as [12]:

\[
\rho = \begin{cases} 
\phi + \theta & \rho < \pi \\
\phi + \pi - \theta & \rho \geq \pi
\end{cases}
\]  

(1)

During one revolution of the robotic milling tool, the thickness of undeformed chip can be expressed as:

\[
t_c = f_z \cdot \sin \phi
\]  

(2)

where \( f_z \) is the feed per tooth.

Milling force coefficients \( K_r \) at different fiber layup angles can be determined by the following equation when carbide cutting tools are used in CFRP milling:

\[
K_r = (a_1 + a_2 t_c + a_3 t_c^2 + a_4 t_c^3) (a_5 + a_6 \rho + a_7 \rho^2 + a_8 \rho^3 + a_9 \rho^4)
\]  

(3)

where the coefficient \( a \) is shown in Tables 1 and 2 [12].

According to formula (3), the milling force coefficient can be obtained as shown in Fig. 2. It can be found that as the tool rotates, the cutting force coefficient changes significantly with the tool rotation angle, and there are significant differences in the milling force coefficient at different fiber layup angles. In the CFRP milling stability analysis process, the average milling force is generally taken as the external excitation force of the dynamic system. Here, the milling force coefficients of different fiber layer angles are averaged out and the results are shown in Table 3.

### 4 Kinematics analysis of robotic longitudinal-torsional ultrasonic milling

Longitudinal-torsional ultrasonic machining technology can suppress the chatter in robotic milling from the perspective of changing the acting direction of the external

| Table 1 | Thickness coefficients of undeformed chips |
|---------|-------------------------------------------|
| Coefficient | \( a_1 \) | \( a_2 \) | \( a_3 \) | \( a_4 \) |
| \( K_r \) | 93.8524 | -4937.97 | 42,553.5 | 5.22094e+06 |
| \( K_r \) | 53.5786 | -7056.16 | 446,076 | -1.00688e+07 |

| Table 2 | Tool cutting angle coefficients |
|---------|---------------------------------|
| Coefficient | \( a_4 \) | \( a_6 \) | \( a_7 \) | \( a_8 \) | \( a_9 \) | \( a_{10} \) |
| \( K_r \) | 3.34966 | -34.6409 | 117.3 | -106.092 | 38.0041 | -4.78616 |
| \( K_r \) | 1.2173 | -14.3331 | 39.3735 | -33.1359 | 11.2643 | -1.35615 |
excitation. Therefore, the kinematics analysis of the tool cutting edge is the premise and foundation to construct the stability analytical model of the robotic longitudinal-torsional ultrasonic milling system. As shown in Fig. 3, compared with the traditional milling method, the cutting edge’s motion trajectory of the robotic longitudinal-torsional ultrasonic milling tool is affected by both longitudinal vibration and torsional vibration simultaneously, and its kinematics is significantly different. In this section, the influence mechanism of longitudinal-torsional ultrasonic vibration on the kinematic characteristics of robotic milling tool is described from two aspects of kinematic trajectory and velocity analysis and definition of ultrasonic functional angle.

4.1 Analysis of tool kinematic trajectory and velocity

As shown in Fig. 3, taking the center point of the tool end as the origin point, the tool feed direction is defined as direction X, the direction perpendicular to the tool feed direction is defined as direction Y, and the tool axial direction is defined as direction Z. The kinematics equation of any point P on the cutting edge of the milling cutter can be expressed as follows:

\[
\begin{align*}
\begin{align*}
S_x(t) &= R \cos \left( \omega t + \frac{B \sin (2\pi f_{vb}t)}{1000 - R} + \varphi \right) + V_f t \\
S_y(t) &= R \sin \left( \omega t + \frac{B \sin (2\pi f_{vb}t)}{1000 - R} + \varphi \right) \\
S_z(t) &= h_0 + \frac{A}{1000} \cdot \sin(2\pi f_{va}t)
\end{align*}
\end{align*}
\]

(4)

\[
\begin{align*}
V_L &= 2\pi f_{va} \cdot \left( \frac{A}{1000} \right) \cdot \cos \left( 2\pi f_{va}t \right) \\
V_T &= 2\pi f_{vb} \cdot \left( \frac{B}{1000} \right) \cdot \cos \left( 2\pi f_{vb}t \right)
\end{align*}
\]

(5)

where R is the tool radius (mm); \( \omega \) is the angular velocity (rad/s) of tool rotation; \( V_f \) is the feed speed (mm/s); \( h_0 \) is the height (mm) of any point \( P \) on the cutting edge from the machined surface; \( A \) is the amplitude (μm) of longitudinal ultrasonic vibration; \( B \) is the amplitude (μm) of torsional ultrasonic vibration; \( f_{va} \) is the frequency of longitudinal ultrasonic vibration (Hz); \( f_{vb} \) is the frequency of torsional ultrasonic vibration (Hz); \( t \) is the processing time (s); \( S_x(t) \), \( S_y(t) \), and \( S_z(t) \) are the motion trajectories of the tool cutting edge in the X, Y, and Z directions, respectively.

Table 3  Average milling force coefficients at four fiber layup angles

| Coefficient | 0 (deg) | 45 (deg) | 90 (deg) | 135 (deg) |
|-------------|---------|---------|---------|-----------|
| \( K_t \)   | 771     | 497     | 747     | 999       |
| \( K_r \)   | 124     | 62      | 134     | 204       |

![Fig. 3](image)

Fig. 3  Schematic diagram of robotic longitudinal-torsional ultrasonic milling
$S_x(t)$, and $S_z(t)$ are the trajectory coordinates of point $P$ on the cutting edge in $X$, $Y$, and $Z$ directions, respectively; $\phi$ is the phase difference between longitudinal ultrasonic vibration and torsional ultrasonic vibration; $V_L$ and $V_T$ represent longitudinal velocity and torsional velocity respectively; and the value 1000 is the unit conversion between millimeter and micron ultrasonic amplitudes.

As shown in Fig. 4, take point $P$ in Fig. 3 as the origin of coordinates, the $OP$ direction is the $X_r$ direction, the cutting speed $V_c$ direction is the $Y_c$ direction, and the axis perpendicular to the $X_rY_c$ plane is the $Z_z$ direction. The feed speed can be decomposed into $X_r$ and $Y_c$ directions to obtain

$$
\begin{align*}
V_{xr} &= V_f \cdot \sin (\phi_j) \\
V_{yc} &= V_f \cdot \cos (\phi_j)
\end{align*}
$$

where $\phi_j$ is the radial immersion angle of the $j$-th tooth. Combined with tool helix angle $\lambda$, it can be defined as:

$$
\phi_j = (2\pi \Omega / 60) \cdot t + 2\pi (j - 1)/N_z - [a_p/(2R)] \cdot \tan \lambda
$$

where $\Omega$ is the spindle speed, $N_z$ is the number of tool teeth, and $a_p$ is the depth of cut of the tool.

4.2 Definition of ultrasonic functional angle

The determination of ultrasonic functional angle is the basis of calculating dynamic cutting layer thickness. As shown in Fig. 4, the longitudinal vibration effect makes the radial cutting speed $V_{xr}$ of conventional milling shift to $V_{Lr}$, and the radial cutting speed $V_{Lr}$ is further shifted to $V_r$ under the action of torsional vibration. Therefore, the ultrasonic functional angles $\gamma$ and $\alpha$ under longitudinal and torsional vibration can be defined as:

$$
\gamma = \begin{cases}
\arctan \left( \frac{V_{Lr}}{V_r} \right), & V_L \leq 0 \\
\pi - \arctan \left( \frac{V_{Lr}}{V_r} \right), & V_L > 0
\end{cases}
$$

$$
\alpha = \begin{cases}
\arccos \left( \frac{|V_{Lr} + V_{yc}|}{\sqrt{(V_{Lr} + V_{yc})^2 + (V_L)^2 + (V_{xr})^2}} \right), & V_T \leq 0 \\
\pi - \arccos \left( \frac{|V_{Lr} + V_{yc}|}{\sqrt{(V_{Lr} + V_{yc})^2 + (V_L)^2 + (V_{xr})^2}} \right), & V_T > 0
\end{cases}
$$

When the values of $\gamma$ and $\alpha$ are not zero, the processing type is robotic longitudinal-torsional ultrasonic milling (two-dimensional longitudinal-torsional vibration); when $\gamma$ is not zero and $\alpha$ is zero, the processing type is robotic rotating ultrasonic milling (one-dimensional axial vibration); and when the values of $\gamma$ and $\alpha$ are both zeros, the robotic rotating ultrasonic milling is transformed into ordinary robotic milling (no ultrasonic).

5 Stability analysis of robotic longitudinal-torsional ultrasonic milling

5.1 Cutting layer thickness model

The cutting layer thickness of the robotic longitudinal-torsional ultrasonic milling includes dynamic and static parts. For CFRP milling, the two-degree of freedom model is usually used for stability analysis, as shown in Fig. 4. The dynamic cutting layer thickness is measured along the direction of the radial cutting speed $V_{xr}$, which can be expressed as:

$$
dh_0 = \Delta x \cdot \sin (\phi_j) + \Delta y \cdot \cos (\phi_j)
$$

where $\Delta x$ and $\Delta y$ represent the dynamic displacement along directions $X$ and $Y$. 

Fig. 4  Vector diagram of cutting edge velocity in robotic longitudinal-torsional ultrasonic milling
Under the effect of axial ultrasonic vibration, the direction of radial cutting speed changes from \(V_{x'}\) to \(V_{y'}\). At this time, the measurement of dynamic cutting layer thickness along the radial direction under the effect of one-dimensional axial ultrasonic vibration can be expressed as:

\[
dh_1 = (dh_0) \cdot \sin \gamma - \Delta z \cdot \cos \gamma \quad (11)
\]

As shown in Fig. 5, the combined effect of longitudinal vibration and torsional vibration makes the radial cutting direction move from \(V_{y'}\) to \(V_{y}\), and the dynamic cutting layer thickness should be measured along the direction of radial cutting speed. Based on the ultrasonic functional angles \(\gamma\) and \(\alpha\), the dynamic cutting layer thickness under the action of two-dimensional longitudinal-torsional ultrasonic vibration can be expressed as:

\[
dh_2 = (dh_1) \cdot \sin \alpha \quad (12)
\]

### 5.2 Dynamic milling force model

During the process of robotic longitudinal-torsional ultrasonic milling, the cutting forces on a single cutting edge \(j\) include radial cutting force \(F_{rj}\) and tangential cutting force \(F_{yj}\). In a single ultrasonic vibration cycle, the magnitude and direction of dynamic milling force change along with ultrasonic vibration. Its value is expressed as:

\[
\begin{bmatrix}
    dF_{rj} \\
    dF_{yj}
\end{bmatrix} = g(\phi_j) \begin{bmatrix} K_r \\ K_y \end{bmatrix} \cdot b \cdot dh_2
\]

where \(g(\phi_j)\) is a window function and the value can be 1 or 0. When the value is 1, it means that tooth \(j\) is involved in cutting, and when it is 0, it means that tooth \(j\) is not involved in cutting. \(K_r\) and \(K_y\) represent the average shear force coefficient, and \(b\) is the actual cutting depth that can be expressed as \(b = a_p + A_s s'\cdot s' = \sin (2\pi f_{ul})\).

The dynamic cutting force along directions \(X\) and \(Y\) at any point \(P\) on a single cutting edge can be expressed as:

\[
\begin{bmatrix}
    dF_{xj} \\
    dF_{yj}
\end{bmatrix} = \begin{bmatrix}
    -ss_1 s_2 - cc_1 - c \\
    -s_1 s_2 c + sc_1 - s
\end{bmatrix} \cdot \begin{bmatrix}
    dF_{rj} \\
    dF_{yj}
\end{bmatrix}
\]

\(\text{where} \ s = \sin(\phi_j), \ c = \cos(\phi_j), \ s_1 = \sin(\alpha), \ c_1 = \cos(\alpha), \ \text{and} \ s_2 = \sin(\gamma).\)

Combine Eqs. (13) and (14), the expression of dynamic milling force of robotic longitudinal-torsional ultrasonic milling can be obtained by summing all cutter teeth as:

\[
\begin{bmatrix}
    dF_x \\
    dF_y
\end{bmatrix} = a_p \cdot \alpha(t) \cdot \{\Delta r\} + A \cdot \beta(t) \cdot \{\Delta r\}
\]

\(\text{where} \ \alpha(t) \ \text{and} \ \beta(t) \ \text{are} \ 2 \times 2 \ \text{matrices and} \ \{\Delta r\} \ \text{is dynamic displacement. They can be expressed as:}\)

\[
\begin{bmatrix}
    a_{xx} & a_{xy} \\
    a_{yx} & a_{yy}
\end{bmatrix} \cdot \beta(t) = \begin{bmatrix} \beta_{xx} \\ \beta_{yy} \end{bmatrix}
\]

All elements in the matrix can be expressed as:

\[
a_{xx} = \sum_{j=1}^{N} g(\phi_j) \cdot (-ss_1 s_2) \cdot [K_r \cdot (ss_1 s_2 + cc_1) + K_r \cdot c]
\]

\[
a_{yy} = \sum_{j=1}^{N} g(\phi_j) \cdot (cs_1 s_2) \cdot [K_r \cdot (ss_1 s_2 + cc_1) + K_r \cdot c]
\]

\[
a_{xy} = \sum_{j=1}^{N} g(\phi_j) \cdot (-ss_1 s_2) \cdot [K_r \cdot (-s_1 s_2 c + sc_1) + K_r \cdot s]
\]

\[
a_{yx} = \sum_{j=1}^{N} g(\phi_j) \cdot (cs_1 s_2) \cdot [K_r \cdot (-s_1 s_2 c + sc_1) + K_r \cdot s]
\]

where \(\alpha(t)\) and \(\beta(t)\) satisfy \(\beta(t) = s' \alpha(t)\) with \(s' = \sin (2\pi f_{ul})\).

### 5.3 Dynamic differential equations under ultrasonic vibration

Combined with the traditional two-degree of freedom dynamic differential equation of milling and considering the effect of ultrasonic vibration, the dynamic delay differential equation of robotic longitudinal-torsional ultrasonic milling is constructed as:

\[
\begin{bmatrix}
    m_{ix} & m_{iy} \\
    m_{iy} & m_{iy}
\end{bmatrix} \begin{bmatrix}
    \Delta x(t) \\
    \Delta y(t)
\end{bmatrix} + \begin{bmatrix}
    2m_{ix} \omega_{nx} \omega_{nx} & 2m_{iy} \omega_{ny} \omega_{ny}
\end{bmatrix} \begin{bmatrix}
    \Delta x(t) \\
    \Delta y(t)
\end{bmatrix}
\]

\[
+ \begin{bmatrix}
    m_{ix} \omega_{nx}^2 \\
    m_{iy} \omega_{ny}^2
\end{bmatrix} \begin{bmatrix}
    \Delta x(t) \\
    \Delta y(t)
\end{bmatrix} = \begin{bmatrix}
    a_p \alpha_{xx} + A \beta_{xx} \\
    a_p \alpha_{yy} + A \beta_{yy}
\end{bmatrix} \begin{bmatrix}
    -\Delta x(t) + \Delta x(t - T) \\
    -\Delta y(t) + \Delta y(t - T)
\end{bmatrix}
\]

\(\text{(17)}\)
where $\xi_{nx}$, $\xi_{ny}$ represent damping ratios; $\omega_{nx}$, $\omega_{ny}$ represent angular frequencies; and $m_{nx}$, $m_{ny}$, represent modal masses.

Then, by applying Cauchy transform to Eq. (17), the first-order dynamic delay differential equation for robotic longitudinal-torsional ultrasonic milling can be obtained

$$\dot{\Delta}(t) = A_0 \Delta(t) + B(t) \Delta(t) - B(t) \Delta(t - T)$$

(18)

Here, time period $T$ can be divided into $m$ intervals $\tau$, namely, $T = m \tau$. And $A_0 = \begin{bmatrix} J & H \\ L & J \end{bmatrix}$, $B(t) = \begin{bmatrix} 0 & 0 \\ U & 0 \end{bmatrix}$, where,

$$J = \text{diag}(-\xi_{nx} \omega_{nx}, -\xi_{ny} \omega_{ny}), H = \text{diag}(1/m_{nx}, 1/m_{ny})$$

$$L = \text{diag}\left[m_{nx} \alpha_{nx}^2 (\xi_{nx}^2 - 1), m_{ny} \alpha_{ny}^2 (\xi_{ny}^2 - 1)\right]$$

$$U = \begin{bmatrix} -\alpha_{p} \alpha_{xx} - A_{p} \beta_{xx} - \alpha_{p} \alpha_{xy} - A_{p} \beta_{xy} \\ -\alpha_{p} \alpha_{yx} - A_{p} \beta_{yx} - \alpha_{p} \alpha_{yy} - A_{p} \beta_{yy} \end{bmatrix}$$

For a given speed of spindle and axial depth of cut, when the norms of all eigenvalues of the transfer matrix $\Phi$ are less than 1, the robotic longitudinal-torsional ultrasonic milling system is stable. Otherwise, the system is unstable. Finally, use full discrete method [13] to calculate the transfer matrix $\Phi$ of robotic longitudinal-torsional ultrasonic milling. In the literature, the entire calculation program of stability simulation has been provided. The stability curves of robotic longitudinal-torsional ultrasonic milling can be obtained by substituting matrices $A_0$ and $B(t)$ of Eq. (18) into the program. Thus, considering the simplicity of expression, the specific simulation process will not be repeated in this paper.

6 Stability lobe simulation of robotic longitudinal-torsional ultrasonic milling

The type of the industrial robot is KUKA KR210—R2700 EXTRA. The structure diagram of the robotic longitudinal-torsional ultrasonic milling system is built as shown in Fig. 6. Its specific components include robot body, longitudinal-torsional ultrasonic milling device, ultrasonic generator, and end effector. The amplitude and frequency of ultrasonic vibrations were measured by the laser vibrometer (Polytec, CLV2534), while ultrasonic device was mounted on the robotic spindle. Its frequency is 30000 Hz, the longitudinal vibration amplitude is 10 μm, and the torsional vibration amplitude is 4.5 μm.

| Tool material | Teeth number $N_z$ | Diameter $D$ (mm) | Helical angle $\beta$ (°) |
|---------------|-------------------|-------------------|------------------------|
| Cemented carbide | 2                 | 6                 | 35                     |

Figure 7 shows that the first-order mode of the robotic longitudinal-torsional ultrasonic milling, the motor in the end effector drives the spindle to move to realize cutting feed. The modal parameters of the robot in $X$ and $Y$ directions can be obtained through experimental modal analysis method. The modal test results are shown in Table 5.

Due to the weak rigidity of the robot structure, the influence of multiple order modes on its stability should be considered in the stability analysis. This research analyzed the first five order modes of the robotic longitudinal-torsional ultrasonic milling system to solve the stability lobe of the system as shown in Fig. 7, and the stability domains at different fiber layup angles are shown in Fig. 8. Figure 7 shows that the first-order mode of the robotic longitudinal-torsional ultrasonic milling system has no influence on the stable processing interval greater than 2000 r/min, so it does not appear in the stable domain diagram. Figure 8 shows that there are significant differences in the stability of robotic milling at different fiber layup angles. When the layup angle is 45°, the stability is the best, 0° and

| Modal | $f_{\omega}/f_{\omega_temp}$ (Hz) | $\xi_{nx}/\xi_{ny}$ (%) | $m_{nx}/m_{ny}$ (kg) |
|-------|---------------------------------|------------------------|----------------------|
| 1     | 15/18                           | 1.82/1.59              | 180.127/125.088      |
| 2     | 85/81                           | 2.01/2.00              | 5.609/6.177          |
| 3     | 165/185                         | 2.52/2.34              | 1.489/1.184          |
| 4     | 385/322                         | 2.11/2.55              | 0.273/0.391          |
| 5     | 720/675                         | 2.22/1.96              | 0.083/0.089          |

...
90° are close to each other, and 135° is the worst. When the milling part has a curve section (including four conditions of 0°, 45°, 90°, and 135°), the machining parameters should be set according to the stability domain at 135°.

7 Conclusion

In this paper, the stability of the robot longitudinal-torsional ultrasonic milling is analyzed. First, the CFRP fiber layup angle is analyzed to describe the influence of milling force coefficient on the robotic milling stability. Then, the kinematics analysis is carried out on the robot longitudinal-torsional ultrasonic milling. Based on the cutting layer thickness model, the dynamic milling force model, and the ultrasonic vibration dynamic differential equation, the stability of the milling is analyzed. Finally, the stability lobe diagram under different
milling force coefficients is obtained through numerical simulation, and the results show that the milling stability is the best when the fiber layup angle is 45°.

It is worth noting that there is still a lot of work to be considered in the future. Fiber layup angles are only considered to be approximately 0°, 45°, 90°, and 135°, while the influence of other angles on the stability of robot milling can be further studied and demonstrated. The research results of this paper prove that the variable milling force coefficient is effective in improving the stability of the robot longitudinal-torsional ultrasonic milling. However, how to further improve the stability of robot milling is worthy of further exploration.

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Declarations

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Competing interests The authors declare no competing interests.

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