Toward Understanding the Anisotropic Point Spread Function of Suprime-Cam and Its Impact on Cosmic Shear Measurement

Takashi HAMANA, Satoshi MIYAZAKI, and Yuki OKURA
National Astronomical Observatory of Japan, 2-21-1 Osawa, Mitaka, Tokyo 181-8588

Tomohiro OKAMURA and Toshifumi FUTAMASE
Astronomical Institute, Tohoku University, Aramaki, Aoba-ku, Sendai 980-8578

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Abstract

We examined the anisotropic point spread function (PSF) of Suprime-Cam data utilizing dense star field data. We decomposed the PSF ellipticities into three components—the optical aberration, atmospheric turbulence, and chip misalignment in an empirical manner—and evaluated the amplitude of each component. We then tested a standard method for correcting the PSF ellipticities used in weak lensing analysis against a mock simulation. We found that, for long-exposure data, the optical aberration has the largest contribution to the PSF ellipticities, which could be modeled well by a simple analytic function based on the lowest-order aberration theory. The statistical properties of PSF ellipticities resulting from atmospheric turbulence were investigated by using numerical simulations. The simulation results are in a reasonable agreement with the observed data. It follows from these findings that the spatial variation of PSF ellipticities consists of two components: one is a smooth and parametrizable component arising from the optical PSF, and the other is a non-smooth and stochastic component resulting from the atmospheric PSF. The former can be well corrected by the standard correction method soling a polynomial fitting function. However, for the latter, its correction is affected by the common limitation caused by sparse sampling of PSFs due to a limited number of stars. We also examined the effects of the residual PSF anisotropies on Suprime-Cam cosmic shear data (5.6-degree$^2$ of $i'$-band data). We found that the shape and amplitude of the B-mode shear variance are broadly consistent with those of the residual PSF ellipticities measured from the dense star field data. This indicates that most of the sources of residual systematic are understood, which is an important step for cosmic shear statistics to be a practical tool of the precision cosmology.

Key words: cosmology: observations — dark matter — gravitational lensing: weak — large-scale structure of universe

1. Introduction

Weak gravitational lensing has now become a unique and practical tool to probe dark-matter distributions irrespective of its relation to luminous objects, thanks to great progress in techniques for weak-lensing analysis as well as in wide-field imaging instruments. For reviews see Mellier 1999; Bartelmann & Schneider 2001; Refregier 2003; Hoekstra & Jain 2008; Munshi et al. 2008). Measuring the statistics of weak lensing shear, also known as the cosmic shear statistics, has been recognized as one of the most powerful probes of cosmological parameters, and is employed as a primary science application by many ongoing/future wide-field survey projects [e.g., the Panoramic Survey Telescope and Rapid Response System (Pan-STARRS); Dark Energy Survey (DES); Hyper Suprime-Cam survey; the Kilo-Degree Survey (KIDS); the Large Synoptic Survey Telescope (LSST); Wide Field Infrared Survey Telescope (WFIRST); Euclid mission]. It should be, however, noted that the full potential of cosmic shear statistics to place a constraint on the cosmological parameters is archived only if systematic errors are reduced down to a required level (Huterer et al. 2006; Cropper et al. 2013; Massey et al. 2013).

In the process of weak-lensing analysis, the correction for the point-spread function (PSF) is of fundamental importance, because it has two serious effects on any galaxy shape measurement: One is circularization of the images, which systematically lowers the lensing shear. The other is a coherent deformation of the images caused by the anisotropy of the PSF, which may mimic a weak lensing shear. Much effort has been made to develop techniques and software to correct the PSF effects, and the weak-lensing community has conducted blind tests using mock data to evaluate their performance (Heymans et al. 2006a; Massey et al. 2007; Bridle et al. 2010; Kitching et al. 2012, 2013; and references therein). Most of the software is designed to correct the PSF, regardless of its origins. Major sources of PSF, recognized so far, except for diffraction, include atmospheric turbulence, optical aberration, the misalignment of CCD chips on a focal plane and pixelization. An alternative and complementary approach to this issue is to investigate the properties of the PSF for a specific instrument, while focusing on a specific cause(s), which may help to optimize the PSF correction scheme, and to understand a level of residual systematics (Hoekstra 2004; Jarvis & Jain 2004; Wittman 2005; Jarvis et al. 2008; Jee et al. 2007, 2011; Rhodes et al. 2007; Jee & Tyson 2011; Heymans et al. 2012; Chang et al. 2013).

The latter approach is exactly what we consider in the present paper. The purpose of this study is twofold: First, we...
look at the properties of anisotropic PSFs of the Subaru prime focus Camera (Suprime-Cam: Miyazaki et al. 2002). We pay a special attention to evaluating the amplitude and spatial correlation of any PSF anisotropies caused by different sources. To do this, we used a set of dense star field images from which we could sample the PSF densely, enabling us to investigate the spatial variation of the PSF in detail. Second, using mock simulation data of PSFs, we examined how well the PSF anisotropy can be corrected using the standard PSF correction scheme.

The motivation of this study concerns the existence of a non-zero B-mode in the cosmic shear correlation functions measured from Suprime-Cam data. The measurement was made by Hamana et al. (2003); they analyzed 2 degree², data and found statistically significant B-mode correlations (see section 5 for an updated, but similar result). Since the B-mode shear is not produced by gravitational lensing in the standard gravity theory at the lowest order of the perturbative treatment,¹ its existence indicates any non-lensing process(es) taking place. The origin(s) of the B-mode was unclear; it could be simply due to remaining systematics in the data reduction and analysis. Also, it may arise from an intrinsic alignment of galaxy shapes (e.g., Croft & Metzler 2000; Catelan et al. 2001; Crittenden et al. 2002; Jing 2002; Hirata & Seljak 2004; Heymans et al. 2006b). More importantly, the B-mode may be produced by a non-standard gravity (e.g., Yamauchi et al. 2012, 2013). Thus, the B-mode shear, in principle, provides us with valuable information on gravity theory. It is thus important to first understand the level of any residual systematic in the data analysis (see, for a similar approach, Hoekstra 2004; Van Waerbeke et al. 2005).

The outline of this paper is as follows; in section 2, data reduction and PSF measurements of dense star field data are described. The amplitude and spatial correlation of PSF anisotropies caused by different sources are evaluated separately in section 3. The PSF correction scheme adopted in our cosmic shear analysis is compared against the mock simulation data in section 4. In section 5, we assess the impact of the residual PSF anisotropy on the cosmic shear analysis. Finally, a summary and discussion are given in section 6. In appendix data in section 4. In section 5, we assess the impact of the residual PSF anisotropy on the cosmic shear analysis. Finally, a summary and discussion are given in section 6. In appendixes, we summarize the properties of PSF ellipticities caused by third-order optical aberrations (appendix 1), and describe a numerical study of PSF ellipticities caused by atmospheric turbulence (appendix 2).

2. Data Reduction and PSF Measurement of Dense Star Field images

We used $i'$-band data of a dense stellar field taken with the Suprime-Cam on 2002/9/30, 2003/6/30, and 2003/7/1. The field is located at the galactic coordinates of $(l, b) = (38^\circ, -3^\circ)$. We collected 289 shots of 60 seconds exposure and 70 shots of 30 s exposure from the data archive SMOKA.² The seeing width (in FWHM) of those data range from 0.′46 to 0.′88 with a median of 0.′63.

Each CCD data was reduced using the SDFred³ software (Yagi et al. 2002; Ouchi et al. 2004). Note that we conservatively used only data within 15′ radius from the field center of Suprime-Cam, because at the outside of that, the point spread function (PSF) becomes elongated significantly, which may make the correction for the PSF inaccurate. Then, mosaicking of 10 CCDs was performed with SCAMP (Bertin 2006) and SWarp⁴ (Bertin et al. 2002). In addition to the individual exposure images, we also generated stacked images using SWarp. Note that the images obtained during a same night were taken with the same pointing (thus, no dithering was made). Since differences in the pointings of images taken on different nights are very small (less than the gaps between CCDs), no data from different CCDs was added.

Object detections were performed with SExtractor (Bertin & Arnouts 1996) and $h$findpeaks of IMCAT software (Kaiser et al. 1995), and two catalogs were merged by matching the positions with a tolerance of 1″. From the image of stars, we measured any anisotropy of the PSF. We used the following ellipticity estimator (exactly speaking the polarization, see Kaiser et al. 1995) to characterize the anisotropy of PSFs:

$$e = \begin{pmatrix} I_{11} - I_{22} & I_{12} \\ I_{11} + I_{22} & I_{11} + I_{22} \end{pmatrix},$$

$$I_{ij} = \int d^2\theta \ W_G(\theta)\theta_i\theta_j f(\theta),$$

where $f(\theta)$ is the surface brightness of an object and $W_G(\theta)$ is the Gaussian window function. We used getshapes of IMCAT for the actual computation of these quantities.

From catalogs of detected objects (almost all of them are stars), we generated two kinds of sub-catalogs by mimicking weak-lensing analysis: One is a “star-role” catalog that contains 700 (number density of 1 arcmin⁻²) of the highest $SN$ non-saturated stars. The other is a “galaxy-role” catalog that contains 30000 (42 arcmin⁻²) of the next highest $SN$ non-saturated stars. The minimum flux $SN$s for star-role and galaxy-role catalogs are sufficiently high: $SN_{\text{min}} > 330$ and > 40, respectively.

3. Analysis of PSF Ellipticities

An anisotropy of PSF results from several sources including the atmospheric turbulence, the optical aberration, the misalignment of CCD chips on a focal plane, the pixelization, and the tracking error of the telescope. Our attempt in this section is to examine the PSF on a source-by-source basis using physically motivated models. We evaluate the statistical properties of PSF anisotropies using a two-point correlation function related estimator, called the E- and B-mode aperture mass variances, defined by (Schneider et al. 1998; we follow the notation by Schneider et al. 2002)

¹ The B-mode shear can arise from higher order terms but their expected amplitude is much smaller than the observed signal (Schneider et al. 2002; Hilbert et al. 2009).
² (http://smoka.nao.ac.jp/).
³ SWarp was modified so that it could treat the bad pixel flag from the SDFred software properly.

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1. The B-mode shear can arise from higher order terms but their expected amplitude is much smaller than the observed signal (Schneider et al. 2002; Hilbert et al. 2009).
2. (http://smoka.nao.ac.jp/).
Fig. 1. Ellipticity maps, so-called whisker plot, showing illustrative examples of ellipticity patterns in the Suprime-Cam data. A 60-second exposure image is shown. The top-left panel shows the observed PSF ellipticities measured from high-SN star images (“star-role” stars), but after the mean component being subtracted. The mean component is displayed in the top-right corner of the upper panels. We fit the spatial variation of the observed PSFs (top-left panel) with the optical aberration model given by equation (7). The model consists of the symmetric (bottom-left), asymmetric (bottom-right), and constant components. A sum of the first two components is shown in the top-left panel, the constant component is shown in the top-right corner of upper panels.

\[
\langle M^2_{\text{ap}} \rangle (\theta) = \frac{1}{2} \int_0^{2\theta} \frac{d\phi}{\phi^2} \left[ \xi_+ (\phi) T_+ \left( \frac{\phi}{\theta} \right) + \xi_- (\phi) T_- \left( \frac{\phi}{\theta} \right) \right]
\]

(3)

and

\[
\langle M^2 \rangle (\theta) = \frac{1}{2} \int_0^{2\theta} \frac{d\phi}{\phi^2} \left[ \xi_+ (\phi) T_+ \left( \frac{\phi}{\theta} \right) - \xi_- (\phi) T_- \left( \frac{\phi}{\theta} \right) \right],
\]

(4)

where \( T_+ (\phi) \) and \( T_- (\phi) \) are defined in Schneider et al. (2002), and \( \xi_+ (\phi) \) and \( \xi_- (\phi) \) are the two-point correlation functions of the ellipticities, computed by

\[
\xi_+ (\theta) = \sum_{ij} (e_i e_{j+} + e_i e_{j-}) / N_p(\theta),
\]

(5)

\[
\xi_- (\theta) = \sum_{ij} (e_i e_{j+} - e_i e_{j-}) / N_p(\theta).
\]

(6)

In the above expressions, the summation is taken over all pairs of objects with a distance within the width of a bin considered \( \theta - \Delta_{\text{bin}}/2 < \phi < \theta + \Delta_{\text{bin}}/2 \). \( N_p(\theta) \) is the number of pairs in the bin, and \( e_t \) and \( e_s \) are the tangential and 45° rotated ellipticity components in the frame defined by the line connecting a pair of objects. Some authors adopt \( \xi^{+,-} \) instead of \( \langle M^2_{\text{ap}} \rangle \) as an estimator of the spatial correlation of PSF ellipticities and its residual, which remains after the correction (Jee & Tyson 2011; Chang et al. 2012, 2013). The reason for our choice of \( \langle M^2_{\text{ap},+} \rangle \) is as follows. In the case of an ellipticity field with power spectrum with a spectrum index more negative than \(-2\) (this is exactly our case as will be shown in subsequent subsections), \( \xi_+ \) becomes almost a flat shape, because the amplitude of not only the large-scale, but also the small-scale correlation function, is dominated by the power on larger scales.5 Also, even if only the larger scale PSF ellipticity correlation is corrected, not only the large-scale, but also the small-scale amplitude decreases. Therefore, \( \xi^{+,-} \) is not suited for quantifying the spatial correlation of our PSF ellipticity data. On the other hand, \( \langle M^2_{\text{ap},+} \rangle \) is a kind of bandpass filtered variance, and thus it is a more suitable estimator for our current purpose.

3.1. Optical Aberrations

Although PSF ellipticities arise from several causes, and vary temporally, there is a visibly identifiable component that

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5 See, e.g., the chapter 3 of the lecture note by N. Kaiser (http://www.ifa.hawaii.edu/kaiser/lectures/elements.pdf).
most likely results from optical aberrations. In the top-left panel of figures 1 and 2, we show two examples of whisker plots of PSF ellipticities where characteristic features of optical aberration can be clearly observed; we also show one case in figure 3 where an optical aberration is almost invisible. We fit the spatial variation of PSFs with the following fitting function of the optical PSF ellipticities, which is motivated from the lowest-order optical aberration theory (see appendix 1 for details):

\[
\begin{align*}
\left( e_1, e_2 \right) &= \left( c_1, c_2 \right) + \sum_{n_s} n_s a_{n_s} r_{n_s} \left( \cos 2 \psi \sin \psi \right) \\
&\quad + \sum_{n_a} n_a a_{n_a} r_{n_a} \left( \cos(\psi - \theta_{n_a}) \sin(\psi - \theta_{n_a}) \right),
\end{align*}
\]

where \( n_s = \{1, 2, 3, 4\} \) and \( n_a = \{1, 2\} \). We determine the model parameters \( s_{n_s}, a_{n_a}, \) and \( \theta_{n_a} \) by the standard least-squares method with observed PSF ellipticities. This model consists of the axi-symmetric, asymmetric, and constant components, which are displayed in the bottom-left panel, bottom-right panel, and top-right corner of the top-panels, respectively. Note that in the top-left panel, the PSF ellipticities measured from “star-role” stars, but after the constant component being subtracted, are plotted. It is clearly seen from figures 1 and 2 that the simple three-component model can well reproduce the observed PSFs. It may therefore be said that the optical aberration is one major origin of the PSF anisotropy. Note that the cause(s) of the constant component is uncertain. It arises not only from an optical aberration (the misalignment coma, see appendix 1), but also from, e.g., a tracking error. Since the constant ellipticity component does not contribute to the aperture mass variances, we do not consider it in the following discussion.

Having found that the optical aberration is one major source of the PSF ellipticities, and can be well modeled with one constant and two spatially variable components, we introduce estimators that quantify the amplitude of the symmetric and asymmetric components: \( \alpha_S = \text{sign}(\langle e_{\text{sym}}^2 \rangle^{1/2}) \) and \( \alpha_A = \langle e_{\text{sym}}^2 \rangle^{1/2} \), where the RMS scatter \( \langle e^2 \rangle^{1/2} \) is computed from the model ellipticities of 700 star-role PSFs for each exposure, and “sign” in \( \alpha_S \) is + or – for the tangentially (the mean tangential ellipticity has the plus sign) or radially elongated cases, respectively. The results for all exposures are plotted in figure 4. It can be observed from the figure that there is a “V-shaped” trend between the strengths of the symmetric component and the asymmetric component. In the lowest-order aberration theory, the symmetric aberration is caused by a shift in the focal-plane position from an ideal position, whereas the asymmetric one is caused by the decenter and/or tilt between the axes of the focal plane and the corrector (lenses). The result indicates that those two deviations happen in a mutually related manner. In figure 4 one may find that there are some exposures that are located somewhat outside of the “V-shaped” trend. It is speculated from a visual inspection that the aberration model fitting of those cases is affected due to the atmospheric PSF. In our model fitting procedure, such contamination is unavoidable. This issue should be noticed when one uses the model fitting for evaluating the optical PSF component from observed PSF ellipticities.

Next we examine the statistical properties of the PSF ellipticities using the aperture mass variance, equations (3) and (4), while especially focusing on the contribution from the optical aberration.
follows from a comparison between the left and right panels of the B-mode. It is argued that the residual variances mostly arise from atmospheric turbulence, whereas the central panels show the aberration model. The color scale of the central panels is same as that of the left panels. The right panels show the residual ellipticity ($\epsilon_{\mathrm{obs}} - \epsilon_{\mathrm{opt-model}}$). Note that the color scale of the right panels is shown in just bottom of each plot, and are different from the other panels.

aberration. To do so, we classified data using $\alpha_S$ and $\alpha_A$ into 9 groups as, shown in figure 4. We computed the average aperture mass variances for each group, and show the results in figure 5. The left panels of the figure show the results based on the observed data, from which it is found that the E-mode variance has a larger amplitude on larger scales, whereas in most cases the B-mode variance is smaller than the E-mode, especially on larger scales. If we assume a power-law spectrum for ellipticity power spectrum $\mathcal{P}(l) \propto l^n$, the measured E-mode variance indicates that $n \sim -3$.

The right panels of figure 5 show the aperture mass variances computed from the PSF ellipticities of the optical aberration model. Comparing those with the observed variance, plotted in the left panels, we can estimate the amount of the contribution from the optical aberration to the PSF ellipticity. We first look at the E-mode. It follows from the comparison between the aperture mass variance measured from the observed ellipticities and the model (the left and right panels figure 5) that for large aberration cases, such like G1–G3, G8, and G9, the model variance accounts for most of the observed variance, indicating that the optical aberration is the dominant component of the PSF ellipticities, as expected from the visual impression (figures 1 and 2). For small aberration cases (G4–G7), one may see that the aperture mass variances of the model are slightly smaller than the observed ones, except for G1. This also suggests the existence of other components than the optical one.

### 3.2. CCD Chip-Misalignment

Next, we examine the PSF ellipticities remaining after subtracting the optical aberration component. Let us start with a visual impression. In figures 6 and 7, the PSF ellipticities of the observed data (left), the optical aberration model (center), and the residual of those two ($\epsilon_{\mathrm{obs}} - \epsilon_{\mathrm{opt-model}}$) are plotted. Note that the mosaicked data consist of 2-row x 5-column CCD chips with a masked outer region (beyond 15’ radius from the field center). In those residual plots, there are two apparent features that should be noticed: one is discontinuities between chips, and the other is ripple-like patterns. The latter is a characteristic feature of PSF ellipticities caused by atmospheric turbulence, which was also observed in CFHT data (Heymans et al. 2012), and was also found in numerical simulations (Jee & Tyson 2011; Chang et al. 2012, 2013; also see appendix 2), which we examine in detail in the next subsection 3.3.

In this subsection, we focus on PSF ellipticities caused by misalignments of CCD chips. To do so, we first need to create data that are minimally affected by the optical aberration and atmospheric turbulence. The former could be minimized by using data with small values of both $|\alpha_S|$ and $\alpha_A$. The latter could be suppressed by stacking many images as the amplitude of the atmospheric PSF decreases, with the exposure time $\langle \epsilon^2 \rangle^{1/2} \propto T^{-1/2}$ (de Vries et al. 2007; Chang et al. 2012; Heymans et al. 2012; see also appendix 2). Thus, we generate a coadded image of 52 shots of 60-second exposures taken from the G5 group, where the effect of the optical aberration is at a minimum level, and the largest number of shots (52 shots) is contained. The stacking was done using Swarp, and the object detection and ellipticity measurement were performed in the same manner as that described in section 2.

The PSF ellipticities of the coadded data are shown in figure 8. One can see PSF discontinuities between the chips, even in the observed PSF (top-left panel) as well as in the residual PSFs (top-right panel).
Fig. 8. Ellipticity magnitude for each component: the left and right set of panels are for $e_1$ and $e_2$, respectively. Results from a coadded data of 52 60-second exposures from G5 group are shown. For each set of panels, top-left panel shows an observed ellipticity, bottom-left panel is for the aberration model, top-right panel shows the residual of those two ($e_{\text{obs}} - e_{\text{opt-model}}$), and bottom-right panel shows the chip-misalignment model described in the subsection 3.2. Note that the color scale of each column is shown at the top of each column.

chip-misalignment component from the residual PSF ellipticities by fitting the residual PSFs with the 2nd-order bi-polynomial function on the chip-by-chip basis. The derived PSF models are shown in the bottom-right panel of figure 8, in which it can be seen that characteristic features of chip-misalignment PSF ellipticities (such as discontinuities between the chips and the chip-scale smooth variation) are well reproduced by the model. We measured the aperture mass variances of the PSF ellipticities (shown in figure 9). Note the B-mode variances are too small to properly analyze, and thus we do not present them. Although our model may not extract only the chip-misalignment PSFs, but may be affected by other sources of PSF ellipticity, it may still be reasonable to consider that the model allows us to assess an approximate amplitude of the chip-misalignment PSF ellipticities. We read from the figure 9 that the amplitude of the chip-misalignment PSF ellipticities is roughly $\langle M_2^2 \rangle \sim 10^{-7} \times \theta$. It should, however, be noted that the chip-misalignment PSF is a phenomenon related to the optical aberration, and thus its amplitude may vary with the deviation in the focal-plane position. Therefore, the above value should be considered as a characteristic size.

It is also found from figure 9 that the aperture mass variance of the residual PSFs ($e_{\text{obs}} - e_{\text{opt-model}}$) is slightly larger than that of the chip-misalignment PSF model. The amplitude of the excess is roughly $\langle M_2^2 \rangle \sim 10^{-7}$. The origin of this excess is not clear, though it could be due to some imperfection in the models, and/or due to atmospheric PSFs, and/or due to other causes, such as pixelization effects caused in the process of resampling (Hamana & Miyazaki 2008). If the latter is the case, it can be settled by recently developed shape measurements schemes that do not involve a resampling process (e.g., Miller et al. 2007, 2013; Miyatake et al. 2013). Thus, we leave it for future work.

3.3. Atmospheric Turbulence

Let us turn to the PSF ellipticity arising from atmospheric turbulence. The properties of the atmospheric PSF ellipticity has been examined experimentally (Wittman 2005; Heymans et al. 2012), or using numerical simulations (de Vries et al. 2007; Jee & Tyson 20011; Chang et al. 2012, 2013). A major characteristic of the atmospheric PSF ellipticity found from previous studies is that the amplitude of the PSF ellipticities decreases with the exposure time as $(e^2)^{1/2} \propto T^{-1/2}$ (de Vries et al. 2007; Chang et al. 2012; Heymans et al. 2012). Also, the shape of the power spectrum and the aperture mass variance of the atmospheric PSF ellipticities was
We investigated the atmospheric PSFs using data from the G5 group. In top-panels of figure 10, we show the aperture mass variances for different exposure times: from top to bottom, single 30-second, 60-second exposures, coadded 4, 13, and 52 60-second exposures. As is shown in the plots, the amplitude of the aperture mass variance decreases as the exposure time increases. However, the minimum amplitude of the E-mode is set by the optical aberration PSFs and chip-misalignment PSFs, as shown in the last two subsections, whereas the B-mode variance decreases down to $\propto T_{\exp}^{1/2}$. This indicates that the optical and chip-misalignment PSF largely contribute to the E-mode PSF ellipticity.

In order to evaluate the amplitude of the atmospheric PSF ellipticities separately from other components, we make a working hypothesis that the amplitude of the deepest coadded data represents the amplitude of (quasi-)static PSF components (such like the optical and chip-misalignment PSFs); the difference from that gives an estimate of the atmospheric PSF ellipticities. In the bottom-panels of figure 10 and figure 11, we plot the differences, from which one finds that the aperture mass variances have both an exposure time dependence and a sharp shape, similar to those obtained from the numerical simulation (appendix 2). Also, the E- and B-mode variances have similar amplitudes, as expected from the numerical simulation. Although the above results were derived while relaying on the working hypothesis, those nice agreements would support a reasonableness of the hypothesis. We may therefore say that we successfully captured characteristic features of the atmospheric PSF ellipticities in the Suprime-Cam data.

4. Testing the PSF Correction Scheme Using Numerical Simulations

Here we consider how well the PSF ellipticities can be corrected by the standard correction scheme, which we describe below. The aim of this analysis is twofold. One is to understand what sets a lower limit of the anisotropic PSF correction; the other is to evaluate the amplitude of residual PSF ellipticities. To do this, we use both the dense star field data and mock data that we describe below.

Before going into details, we briefly describe a basic procedure concerning the anisotropic PSF correction implemented in actual weak-lensing studies so far (see for details, Heymans et al. 2006a; Massey et al. 2007 and references therein). Firstly, PSF ellipticities (or other estimators of PSF) are measured from high-SN star images. In usual weak-lensing analyses, a typical number density of stars used for PSF sampling is ~ 1 arcmin$^{-2}$. Secondly, the spatial variation of PSF ellipticities is modeled by fitting the PSF ellipticities with an analytic function, such as a polynomial. Then finally, an artificial shape deformation in galaxy images caused by the PSF is corrected by using the PSF model delivered by the analytic function. Although an actual implementation of the above procedure depends on the technique of weak-lensing analysis (Heymans et al. 2006a, 2006b; Massey et al. 2007), there is one common limitation that comes from a sparse sampling of stars, namely, the spatial variation of PSFs on scales smaller than the mean star separation is poorly sampled, and as a result small-scale components in the PSF anisotropy are hardly corrected.

Our PSF correction scheme is based on the KSB algorithm (Kaiser et al. 1995; Hamana et al. 2003; Heymans et al. 2006). We refer the reader to the above references for details. Here, we...
only describe some details concerning implementation, which are specific to this study. We use a bi-variable polynomial function for modeling the spatial variation of PSF ellipticities. We divide data into $2 \times 2$ regions, or into each CCD chip, and a PSF model is generated for each sub-region. The order of the polynomial is set to 4th for a $2 \times 2$-division, whereas the dotted line is for the chip-basis. From left to right, the exposure time is 1, 5, and 25 min, respectively.

Let us start with a simpler situation using mock PSF ellipticity catalogs. The mock data have the same geometry and number of objects as those of the dense star field data, that is 700 “star-role” objects and 30000 “galaxy-role” objects (see section 2). Instead of generating mock images, we simply generate object catalogs. Each object has various items: the positions, and 3-component PSF ellipticities, including optical, atmospheric, and CCD-misalignment components, denoted by $\epsilon_{\text{opt}}$, $\epsilon_{\text{atm}}$, and $\epsilon_{\text{chip}}$, respectively. The optical component is modeled by using the optical aberration-motivated model, equation (7), introduced in appendix 1; we adopted model parameters for the G5 group in subsection 3.1. For the CCD-misalignment component, we adopt the 2nd-order polynomial models obtained in subsection 3.2. The atmospheric component is modeled by the random field with the power spectrum having a power-law index of $-3$ based on our finding (see section 2 and appendix 2). Its amplitude is set by the empirical relation of equation (A6) for exposure times of 1, 5, and 25 min. We compute the total ellipticity by simply adding the three components, instead of treating in the convolution manner. This is a reasonable approximation, because each ellipticity component is small. Then, the anisotropic PSF correction is made for mock catalogs in the procedure described above.

The results from a mock simulation are presented in figure 12, from which the following three points can be seen. First, as mentioned above, the mean separation of stars ($\sim 1'$), which are used to model the spatial variation of PSFs, sets a fundamental limitation on the PSF correction. In the case of the aperture mass, its window function is most sensitive to fluctuations on scales of $\sim 1/5$ of the aperture size (Schneider et al. 1998). This defines a characteristic aperture scale of $\theta \sim 5'$; on scales larger than that, the correction works very well, but on smaller scale it does very poorly (this was argued by Hoekstra 2004). This is especially the case for the atmospheric PSF. Second, in spite of the above limitation, the optical components are well corrected, even on smaller scales. The reason of this is the smoothness of the optical component; as shown in subsection 3.1, the spatial variation of the optical PSFs can be well fitted by a polynomial-based model. Thus, even with a sparse sampling, it can be well-modeled with the polynomial function. Third, for the $2 \times 2$-division case, the CCD-misalignment component combined with the optical component sets a limit on the anisotropic PSF correction. This is a natural consequence that even if each individual component is smooth and is modeled by a polynomial function, the mixed PSFs may no longer be simple, and thus may not be corrected by the polynomial model. This can be partly avoided by adopting the chip-basis correction, as can be seen in the right panel of figure 12, though the residual variance does not reach to the level of the atmospheric PSF. A further improvement of the PSF correction may be achieved by recently proposed correction schemes (e.g., Chang et al. 2012; Bergé et al. 2012; Gentile et al. 2013); we leave it for future study. The above three findings provide us with a clue to interpret a result obtained from an analysis of real data.

Let us turn to the analysis of dense star field data. In figure 13, the aperture mass variances measured from the G5 group data (defined in figure 4) are presented for different exposure times, where an anisotropic PSF correction was made on the basis of $2 \times 2$ sub-regions, except for the magenta curve, to which the chip-basis correction was applied. From the figure, the following four findings are obtained. Firstly, it is found that after the PSF correction, the E- and the B-modes are almost equally partitioned, regardless of the properties of raw PSF ellipticities. A plausible reason for this is that the E- and B-modes are mixed up in the process of the PSF correction. Secondly, the result of a 60-second exposure is in a good agreement with that of the mock simulation. But, at a closer look at larger scales, it is found that the residual aperture mass variance falls less quickly than the mock result, indicating that the spatial variation of real PSF ellipticities is more complex than that assumed in the mock simulation. Thirdly, as expected from the result of the mock simulation, the residual aperture mass variance decreases with increasing the exposure time. However, the decrement is less than what can be expected from the mock simulation. Finally, it is found from a comparison of two different correction schemes for 52-coadded data.
that the chip-basis correction makes only a small improvement over the other case. It is speculated from the last two points combined with the findings from the mock simulation (that is, an existence of a non-smooth component sets a lower limit of the correction on scales smaller than mean separation of stars) that there exists an unidentified PSF ellipticity component(s) whose amplitude is greater than the chip component mentioned in subsection 3.2 (see also figure 9). Although the origin of the unknown PSF component is not clear, the above result provides an empirical estimate of the “best performance” of the PSF correction scheme that we adopted in this paper.

5. Residual PSF Anisotropies in Suprime-Cam Cosmic Shear Data

Here, we examine the effects of residual PSF anisotropies on Suprime-Cam cosmic shear data. To do so, we measure the E- and B-mode shear aperture mass variance from 5.6-degree Suprime-Cam deep imaging data, and compare the B-mode shear variance with the residual PSF ellipticities measured from the dense star field discussed in section 3.

5.1. Suprime-Cam Data and Weak Lensing Analysis

We use the same data as those used in Hamana et al. (2012) in which basics of data and data analyses are described. Therefore here we only describe points specific to this work. Suprime-Cam $i'$-band data were collected from the data archive, SMOKA, under the following three conditions: data are contiguous with at least four pointings, the total exposure time for each pointing is longer than 1800 s, and the seeing width (in FWHM) is better than 0.65. Four data sets (named by SXDS, COSMOS, Lockman-hole, and ELAIS-N1) meet these requirements. The exposure times of individual exposures range from 120 and 420 s. Data reduction, mosaic stacking, and object detection were done using the same procedure described in section 2. The effective area after masking regions affected by bright stars is 5.57 degree$^2$. The depth of the coadded images varies among four fields, and it is found that the number counts of faint galaxies are saturated at AB magnitude of $i' = 25.2–25.5$.

For weak-lensing measurements, we adopt the so-called KSB method described in Kaiser, Squires, and Broadhurst (1995), Luppino and Kaiser (1997), and Hoekstra et al. (1998) with some modifications being made following recent developments (Heymans et al. 2006a), which we describe below. Stars are selected in a standard way by identifying the appropriate branch in the magnitude half-light radius ($r_h$) plane, along with the detection significance cut $S/N > 10$. The number density of stars is found to be $\sim 1$ arcmin$^{-2}$ for the four fields. We only use galaxies that meet the following three conditions: (i) a detection significance of $S/N > 3$ and $nu > 10$, where $nu$ is an estimate of the peak significance given by $h_{\text{findpeaks}}$, (ii) $r_h$ is larger than the stellar branch, and (iii) the AB magnitude is in the range of $22 < i' < 25$, where the galaxy number counts of four fields are almost the same. The number density of resulting galaxy catalog is $\sim 23.5$ arcmin$^{-2}$, and is quite uniform among four fields. We measured the shape parameters of objects by getshapes of IMCAT. The PSF correction was done on a sub-field basis, where the coadded data that were divided into sub-fields of about $15' \times 15'$ (approximately one fourth of the Suprime-Cam’s field-of-view). An anisotropic PSF correction was done following the implementations by MH, CH, and TS of Heymans et al. (2006a). The spatial variation of PSF ellipticities measured from stars were modeled with the 4th order bi-polynomial function; also, the PSF model combined with the smear polarizability tensor ($P_{sm}$) of each galaxy was used to correct the PSF ellipticities of the galaxies. In KSB formalism, the shear (we denote by $\gamma$) is related to the observed ellipticity through the shear polarizability tensor, $P_{\gamma}$, which is evaluated by a smoothing and weighting method developed by Van Waerbeke et al. (2000; and see section A5 of Heymans et al. 2006a).

Before presenting the shear aperture mass variance measured from coadded data, it is worth looking into PSF ellipticities of individual exposures. To do so, we generated mosaicked data of individual exposures, and selected stars in the same procedures as described above. The spatial variation of PSF ellipticities measured from stars were fitted to the optical model, equation (7), and the amplitude of the symmetric and asymmetric components of the model were evaluated (see subsection 3.1 and appendix 1 for details). The results are plotted in figure 14, in which one may see a “V-shaped” trend similar to results obtained from the dense star field data (figure 4), though the scatter is somewhat larger for this case. This similarity implies that knowledge of the PSF anisotropy learned from the dense star field analysis (section 3) is applicable to cosmic shear analyses. It should, however, be noticed that there is one component that is not taken into consideration in the dense star field analysis, namely the stacking of multiple dithered exposures. It is obvious that the stacking of dithered exposures makes the spatial variation of PSFs complicated, which may result in a poorer anisotropic PSF correction. We return to this point later.
5.2 Cosmic Shear Aperture Mass Variance

We compute the E- and B-mode aperture mass variance of galaxy shears through the two-point correlation functions using equations (3)–(5) with the summations in equation (5) being replaced with a weighted summation, \( \sum w_i w_j y_i y_j / \sum w_i w_j \), where \( w_i \) is the weight for the \( i \)-th galaxy. In addition to the shear, we computed the uncorrected-shear, that is, the galaxy shear to which the correction for the PSF anisotropy is not applied. The results are presented in figure 15. The error bars indicate the root-mean-square among 100 randomized realizations, in which the orientations of galaxies are randomized, and presumably represent the statistical error coming from the galaxy shape noise. It is found from figure 15 that the B-mode shear variance is non-zero on the aperture scale of \( \theta \lesssim 10' \).

It should be noted that adjacent bins are correlated. In order to compare the shear variances with those of the residual PSF ellipticities, we transformed the star ellipticity to the galaxy aperture mass variances measured from faint galaxies. The error bars in the case of the dense star field data, the turnaround aperture scale is about 5', which is naturally set by the mean separation of stars (~1') (see section 4), whereas it is about 10' for the cosmic shear data. The reason for this is unclear, though it could be related to an excess B-mode uncorrected shear variance observed at ~10'.

Finally, we evaluated the influence of the residual PSF anisotropies on the cosmic shear E-mode measurement. We plot in the left panel of figure 15 the ΛCDM prediction based on the 7-year WMAP cosmological model (Komatsu et al. 2011) as a basis for comparison. In computing the ΛCDM prediction, we used the HALOFIT model by Smith et al. (2003), and we used the redshift distribution of source galaxies inferred by the following manner. We utilized the COSMOS field data. We merged the galaxy catalog used for the cosmic shear measurement with the COSMOS photometric redshift catalog (Ilbert et al. 2009), and computed the redshift distribution by adopting the photometric redshift, which is presented in figure 16. We fit the redshift distribution with the following function (Fu et al. 2008):

\[
  n(z) = A \frac{a^d + b^e}{z^b + c},
\]

where the normalization, \( A \), was determined by imposing \( \int dz \ n(z) = 1 \) within the integration range \( 0 < z < 6 \). We found that a parameter set \( (a, b, c) = (0.6, 7, 0.6) \) gives a reasonably good fit to the data, as shown in figure 16. It is found from a comparison between the ΛCDM prediction and the residual PSF shear variance evaluated from the dense star field analysis (figure 15) that on scales below 10', the residual shear variance is more than 10% of the expected cosmic shear variance.
The comparison between the measured E- and B-mode shear variances on those scales results in even worse figures. On the other hand, on larger scales ($\theta > 15'$), where the anisotropic PSF correction works well, the residual variance is well less than ten percent of the measured and expected E-mode variance. This is one of the important findings of this paper. Before closing, we comment on an effect of the noise bias (Refregier et al. 2012; Melchior & Viola 2012; Okura & Futamase 2012) on the above analysis. The noise bias introduces a systematically lower ellipticity value with its amplitude depending on the SNR of the model is 1.07 which is in a very good agreement with the value derived from the photometric redshift data ($\langle z_p \rangle = 1.06$). Refregier et al. (2012) found that the amplitude of noise bias is a few percent for SNR = 10 objects. Since the threshold SNR's adopted in the above analysis are $SN > 10$ for the cosmic shear galaxy selection and $SN > 40$ for the dense star field analysis, the expected amplitude of the noise bias on the above shear variance measurements might be less than 5%. Therefore, the conclusions from the above analysis are, at least qualitatively, not affected by the noise bias.

6. Summary and Discussion

We analyzed the anisotropic PSF of Suprime-Cam data utilizing the dense star field data. We decomposed the PSF ellipticities into three components (the optical aberration, the atmospheric turbulence, and the chip-misalignment) in an empirical manner, and assessed the amplitude of each component. We then tested a standard method for correcting the PSF ellipticities against a mock simulation based on models of the PSF ellipticities obtained from the dense star field data analysis. Finally we examined the impact of the residual PSF ellipticities on the cosmic shear measurement.

We found that the optical aberration has the largest contribution to the PSF ellipticities of long-exposure (say, longer than 10 min) data. We also found that the spatial variation of the optical PSFs can be modeled by a simple three-component model, which is based on the lowest-order aberration theory described in appendix 1. It was also found that the optical PSF ellipticities vary smoothly on the focal plane, and thus are well corrected by the standard correction method in which the spatial variation is modeled by a polynomial function.

The PSF ellipticities after subtracting the optical PSF model ellipticities show discontinuities between CCD chips, which indicates that the misalignment of chips on the focal plane induces an additional PSF ellipticity. In order to make a crude estimation of the amplitude of the chip-misalignment PSF ellipticities, we fitted the residual PSF ellipticities to the 2nd-order bi-polynomial function on a chip-by-chip basis. It turns out that the amplitude of the chip-misalignment PSF ellipticities is lower than the optical component, though it is not negligible. It was found from the mock simulation that the chip-misalignment component combined with the optical component puts a lower limit on the capability of the PSF correction, which can be, in principle, avoided by employing a chip-basis correction scheme.

We investigated the properties of PSF ellipticities resulting from atmospheric turbulence using a numerical simulation of wave propagation through atmospheric turbulence under a typical weather condition of the Subaru telescope on Maunakea (appendix 2). From the simulation results, we evaluated the power spectrum and aperture mass variance of the atmospheric PSF ellipticities, and derived power-law fitting functions of them. As was already pointed out in literature (Wittman 2005; de Vries et al. 2007; Jee & Tyson 20011; Chang et al. 2012, 2013; Heymans et al. 2012), the RMS amplitude of the atmospheric PSF ellipticities decreases as $T_{\text{exp}}^{-1/2}$. We computed the aperture mass variance of the dense star field data for various exposure times (from 30-second to 52 × 60-second). The amplitude of the atmospheric PSF ellipticities was evaluated by assuming that the deepest data represents the amplitude of (quasi-)static PSF components, and the difference from that is due to the atmospheric PSFs ellipticities. The results are found to be in reasonable agreement with the simulation results. Therefore, we may conclude that the aperture mass variance of the atmospheric PSF ellipticities for a long-exposure data (say, longer than 10 minutes), is at least one order of magnitude smaller than that of the optical PSF ellipticities. Since the atmospheric PSFs are not a smoothly varying component, its correction is affected by the common limitation that the PSF correction on scales smaller than the mean star separation works very poorly because of the poor sampling of the spatial variation of the PSFs on those scales.

The above findings provide us a clue to develop an optimal PSF interpolation scheme. It is found that the spatial variation of PSF ellipticities consists of two components: one is a smooth and parameterizable component arising from the optical aberration and chip-misalignment; the other is a non-smooth and stochastic component arising from the atmospheric
PSFs. The former can be modeled with a parametric model, as shown in this paper. Also it has been argued that an interpolation scheme based on the principle-component analysis is effective for such a case (Jarvis & Jain 2004; Jee & Tyson 2011; see also Miyatake et al. 2013 and Lupton et al. 2001 for an actual implementation for Suprime-Cam data reduction pipeline). On the other hand, it is shown in Bergé et al. (2012) and Gentile et al. (2013) that local-type correction schemes, such like the radial basis functions and Kriging work well for atmospheric PSFs. Apparently, a hybrid interpolation scheme, in which the above two types of interpolation schemes are optically incorporated, is a strong candidate for achieving a better PSF correction.

We examined the effects of the residual PSF anisotropies on Suprime-Cam cosmic shear data. We also compared the B-mode shear variance measured from 5.6-degree$^2$ $i'$-band data with the residual PSF ellipticities (but being properly transformed into shear) measured from the dense star field data, which can be considered as the “best performance” of our PSF ellipticity correction scheme. It is found that the shape and amplitude of the B-mode shear variance are broadly consistent with those of the residual PSF ellipticities. This indicates that most of the sources of residual systematic are understood, which is an important step for cosmic shear statistics to be a practical tool of precision cosmology. However, it is also found that the B-mode shear amplitude at scales of $\sim 10'$ are systematically larger than the residual PSF. The reason for this excess is unclear; one possible reason is the stacking of dithered multiple exposures, which is not involved in the dense star field analysis. Such stacking-related issues may be avoided by employing a weak-lensing shape measurement scheme on the basis of individual exposures (e.g., Miller et al. 2007, 2013; Miyatake et al. 2013), which combined with a hybrid interpolation scheme will be addressed in future work.

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Appendix 1. PSF Ellipticity Originated from the Third-Order Optical Aberrations

We summarize the properties of ellipticity in a PSF caused by third-order optical aberrations with and without rotational symmetry. We consider a system with a circular pupil. We follow the notation by Thompson (2005, see figure 1 of his paper): $\vec{H}$ represents the position in the image field; without loss of generality we can choose $(H_x, H_y) = (0, h)$. $\vec{p}$ represents the position in the exit pupil with $(\rho_x, \rho_y) = (\rho \sin \phi, \rho \cos \phi)$.

Let us start with a rotationally symmetric system. In this case, the wavefront distribution is given by (Thompson 2005)

$$W = W_{040}\rho^4 + W_{131}\rho^3 h \cos \phi + W_{222}\rho^2 h^2 \cos^2 \phi + W_{220}\rho^2 h^2 + W_{311}\rho h^2 \rho \cos \phi.$$  \hspace{1cm} (A1)

Among those five Seidel aberrations, those that make an ellipticity parameter $e$ non-zero are the Seidel coma (the second term), and astigmatism (the third term). The Seidel coma results in an ellipticity whose major axis oriented toward radial direction with $|e| \propto r^2$ where $r$ is the distance from the center of the image field. On the other hand, a combination of the astigmatism and the field curvature (the fourth term) results in a radially elongated (if $|W_{222} + W_{220}| > |W_{220}|$) or tangentially elongated (if $|W_{222} + W_{220}| < |W_{220}|$) ellipticity parameter with $|e| \propto r^4$ (see figure 17). The last term, the distortion, does not generate ellipticity for a point source, because it only introduces a shifting of the image position on a focal plane. However, for an extended source, its differential effect causes a radially or tangentially elongated ellipticity with $|e| \propto r^2$. It should, however, be noticed that unlike other aberration terms, the image deformation caused by the distortion is due to mapping, and thus this should be treated separately from the PSF.
We determine the model parameters \( \{s, a, \theta\} \) by the standard least-squares method with the observed PSF ellipticities. In doing so, we take \( n = \{1, 2, 3, 4\} \) and \( n = \{1, 2\} \). As shown in section 3, this simple model succeeds very well in fitting the global pattern of PSF ellipticities in Suprime-Cam data.

**Appendix 2. A Numerical Study of PSF Ellipticities Caused by Atmospheric Turbulence**

We examined the statistical properties of PSF ellipticities caused by atmospheric turbulence using a numerical simulation. The aim of this study is to investigate the shape and amplitude of the power spectrum of atmospheric PSF ellipticities, which is essential knowledge for understanding the statistical properties of PSF ellipticities. Our approach is similar to one taken by Jee and Tyson (2011), that is to utilize a numerical simulation of wave propagation through atmospheric turbulence for evaluating atmospheric PSFs.

Here, we briefly describe our numerical simulation. Since we heavily used publicity open software, ARROYO (Britton 2004), we refer the reader to the above reference and software documents\(^7\) for details of models, computational methods and their implementation. The atmospheric turbulence was modeled by eleven-layer frozen Kolmogorov screens with the atmospheric model at Mauna-Kea proposed by Ellerbroek and Rigaut (2000). The strength of the total atmospheric turbulence was set by the Fried length, \( r_0 \), for which we adopted a value proposed by Ellerbroek and Rigaut (2000): \( r_0 = 0.23 \) m at a wavelength of 0.5 \( \mu \)m. Neither a large scale nor small scale cutoff (the, so-called, outer and inner scale) on the Kolmogorov power spectrum was imposed. The wind model given by equation (3.20) of Hardy (1998) was adopted with random wind directions. Having the atmospheric model ready, an atmospherically disturbed phase function of a wavefront was computed. We considered a monochromatic electromagnetic wave with a wavelength of 0.8 \( \mu \)m. Then, an instantaneous PSF was obtained by Fourier transforming the phase function within a pupil, for which we assumed a circular pupil with 8.2 m diameter for simplicity. All of the above models and computations were implemented in ARROYO subroutines. Finally, a sequence of instantaneous PSFs were added to obtain a long-exposure PSF. We computed PSFs of 1, 10, and 60-seconds exposures on 64 \( \times \) 64 regular grids over a 30’ \( \times \) 30’ focal plane.

We generated 20 realizations. The mean width in FWHM of PSFs among the realizations was 0.65 for a 60-second exposure, which is in a good agreement with the observed value at the Subaru telescope (Miyazaki et al. 2002). As an

\(^7\) [http://eraserhead.caltech.edu/arroyo/arroyo.html](http://eraserhead.caltech.edu/arroyo/arroyo.html).
Ellipticity magnitude obtained from the numerical simulation described in appendix 2: upper and lower panels are for $e_1$ and $e_2$, respectively, and the color scales are displayed on the top of those panels. The exposure times are, from left to right, 1-second, 10-seconds, and 60-seconds, respectively.

Power spectrum of PSF ellipticities measured from the simulation results. The dashed, long-dashed, and solid lines are for 1-second, 10-second, and 60-second exposures, respectively. The error bars, plotted only for the 60-second result for clarify, show the RMS among 20 realizations.

Aperture mass variance of PSF ellipticities measured from the simulation results. The dashed, long-dashed and solid lines are for 1-second, 10-second, and 60-second exposures, respectively. The error bars, plotted only for the 60-second result for clarify, show the RMS among 20 realizations.

Illustrative example, one realization is shown in figure 19, where the ripple like feature appears. Similar features were observed in CFHT data (Heymans et al. 2012) and Suprime-Cam data (see figures 6 and 7). It should, however, be noted that the appearance varied widely between realizations, depending on turbulence and wind properties.

We measured the power spectrum and the aperture mass variance from the simulation data, and present them in figures 20 and 21, respectively. The following three major findings were derived from the figures: First, the amplitude of the power spectrum decreases as $P_e \propto T_{\text{exp}}^{-1}$, as was already pointed out in previous studies (Wittman 2005; de Vries et al. 2007; Jee & Tyson 2001; Chang et al. 2012, 2013; Heymans et al. 2012). Second, the power spectrum is approximated by a power-law model with a power index of $-3$. A plausible reason for the power-law shape is our assumption of the power-law turbulence power spectrum without a cutoff. In reality, a large-scale cutoff (the, so-called, outer scale) at around 10–100 m is expected (e.g., Hardy 1998), which may result in a cutoff in the PSF ellipticity power spectrum at a scale of around 1°. Third, the atmospheric turbulence results in E/B-mode almost equally partitioned PSF ellipticities. Through a closer look at figure 21, one may however find that the E-mode amplitude is slightly, but systematically, larger than the B-mode. The reason for this is unclear, and we leave it for future work.

In addition to the above findings, we also find from figures 20 and 21 that models $P_e(l) \sim 0.8l^{-3}$ and $(M_{\text{ap,\perp}}^2)(\theta) \sim 4 \times 10^{-7} \theta$ give a good fit to the 60-second result. In what follows, we discuss how those correlation amplitudes depend on other observational parameters, including the exposure time and the atmospheric seeing width in FWHM ($\theta_{\text{atm}}$). Consider the case $D \gg r_0$, where $D$ denotes the diameter of a telescope, and the monochromatic radiation with a wavelength $\lambda$ from a point source. The wavefront from a point source is disturbed by atmospheric turbulence, and we consider, for a simple approximation, the disturbed wavefront to be segmented $r_0$-sized patches of constant phase and random phases between the individual patches (Saha 2007, and see e.g., figure 5.3 of that textbook for an illustration). The number of patches within a pupil is approximately $N_{\text{patch}} \sim (D/r_0)^2$. A wavefront from each patch results in a sharp speckle PSF on a focal plane, and a total PSF is viewed as a superposition of randomly distributed speckles over an extent of the atmospheric seeing.
size $\theta_{\text{atm}} \sim \lambda/r_0$ (Hardy 1998; Saha 2000; see for illustrative examples, figure 3 of Kaiser et al. 2000; and figure 5 of Jee & Tyson 2011). In this picture, the PSF anisotropy is understood as a natural consequence of a finite number of speckles. Since the disturbed wavefront changes quickly, many different speckles are accumulated as $N_{\text{spec}} \propto T^{-1/2}$, and thus the PSF becomes rounder. It is found from the above numerical simulation that the RMS of PSF ellipticities decreases as $(e^2)^{1/2} \propto T^{-1/2}$, which combined with the above mentioned relationships leads to the following relationships:

$$\langle e^2 \rangle^{1/2} \propto N_{\text{spec}}^{-1/2} \times T^{-1/2} \times N_{\text{patch}}^{-1/2} \times T^{-1/2} r_0 \propto T^{-1/2} \theta_{\text{atm}}. \tag{A5}$$

We tested this relationship against numerical simulations with different values of $r_0$, from which the above dependences of $r_0$ and $\theta_{\text{atm}}$ on $(e^2)^{1/2}$ were verified. Combining the last relationship with the power-law fitting models, we have

$$P_{e}(l) \sim 0.8 \times 10^{-3} \times \left( \frac{T_{\text{exp}}}{1 \text{ min}} \right)^{-1} \left( \frac{\theta_{\text{atm}}}{0.65} \right)^{-2} \tag{A6}$$

and

$$\langle M_{\text{ap,1}}^2 \rangle(\theta) \sim 4 \times 10^{-7} \times \theta \left( \frac{T_{\text{exp}}}{1 \text{ min}} \right)^{-1} \left( \frac{\theta_{\text{atm}}}{0.65} \right)^{-2}. \tag{A7}$$

Although the above relationships are quite crude, they are useful to quickly evaluate the magnitude of atmospheric PSF ellipticities for a given set of observational parameters. Note that these relationships are valid only for a telescope with a diameter of 8.2 m. It may be worth pointing out that it is found from numerical simulations with different telescope diameters that the amplitude of PSF ellipticities is insensitive to $D$. This differs from a scaling relationship of $P_{e} \propto D^2$ expected from the above consideration (likewise for $\langle M_{\text{ap,1}}^2 \rangle$). This difference may be explained by the diameter dependence of the PSF, which is not taken into account in the above consideration. According to the diffraction theory (e.g., Hardy 1998), the PSF is the inverse Fourier transform of the optical transfer function (OTF) which is the autocorrelation function of the product of the telescope pupil function with the atmospheric phase function. In that relationship, the diameter of the telescope enters through the pupil function. Therefore, roughly and intuitively speaking, the power spectrum of PSF ellipticities relates to the OTF, and the diameter acts as a large-scale cutoff. Since the turbulence power spectrum has a larger power on a larger scales (Kolmogorov power spectrum has the shape of $k^{-11/3}$), the large-scale cutoff imposed by the telescope diameter may have a great impact on the amplitude of the PSF ellipticities.

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