Analysis of the sliding parameters variation mechanism in engagement of gear hone teeth and parts with consideration of elastic deformation

Yu Bagaiskov
Volzhsky Polytechnic Institute (branch) of the Volgograd State Technical University, 43a Engelsa st., Volzhsky, Volgograd Region, 404130, Russia

E-mail: instra-ysb@rambler.ru

Abstract. Gear hones are used for finish machining of hardened gear teeth lateral faces applying the generating method. In service, due to penetration of abrasive grains into metal, wear and running-in of the tool material, availability of contact deformation and elastic strain of gear-hone teeth, especially in the case of elastic binding agents, the tooth contact takes place not in a point, but in an ellipse area. The gear hone tooth deformation value depends on the contact point position along the tooth height; it increases by ten folds from the lower contact point to the top. A model of engagement of gear hone teeth and parts that enables simulating the mobility of hone teeth against gear teeth due to elastic deformation of the former and making allowance for specific sliding variations in the contact point is suggested. Besides, the model tooth symmetry axis turns by a certain angle proportional to the deformation value, the contact points shift, and the design values of the center-to-center distance, total length of transmission, curvature radii, and profile pressure angles change. This allows to consider the influence of elastic strain for calculation of gear hone geometrics and therefore affect the metal removal intensity, as well as quality and accuracy of treatment.

A gear hone is an abrasive tool for finish machining of hardened gear teeth lateral faces. As a rule, it has the form of a helical gear and operates according to the generating method with reverse rotation and longitudinal feed [1].

Due to penetration of abrasive grains into metal, wear and running-in of the tool material (even with its absolute hardness) during operation, the contact between the hone teeth and a part takes place along a small ellipse-shaped area [3]. Besides, as distinct from engagement of regular gears, a hone and a processed gear have essentially different stress-strain performance. The elasticity modulus of the hone material is within the range of $E_h = 0.1 \cdot 10^3 - 6 \cdot 10^3$ MPa and it primarily depends on the binding agent (from the epoxy type to the flexible polyurethane type). It is 40 - 200 times lower than the elasticity modulus of the processed part material ($E_p = 2.15 \cdot 10^5$ MPa).

Total deformation $\omega_k$ of gear hone teeth in the engagement line direction, in a similar way to gears, consists of the contact deformation, the flexural deformation of teeth, and the tooth root deformation in the hone rim. The calculation results of contact deformation $\omega_k$ along the hone tooth height from the
lower contact point to the upper contact point depending on the value of $E_v$ made it clear that such deformation mainly varies rather evenly and depends significantly, almost proportionally, on $E_v$ [4].

The hone tooth flexural deformation value depends on the contact point position along the tooth height; it increases by a factor of 400 – 500 from the lower contact point to the top. Besides, in case of $E_v$ decrease from 6000 MPa to 100 MPa, $\omega_v$ increases by a factor of 60 [5]. The value of component $\omega_{vc}$ of the hone tooth deformation characterizes the tooth root displacement within the hone rim; in case the tooth height increases, this value increases by a factor of $4 - 20$ and escalates substantially at decreased elasticity modulus of the hone material [5].

The total deformation analysis revealed that contact deformation prevails in lower contact points. Within the pitch circle area, intensity of flexural deformation and root deformation increases: $\omega_{2v} = \omega_2 + \omega_{vc}$, approximately to the level of $\omega_k$. At the tooth top, total flexural deformation exceeds contact deformation by a factor of 4 already and amounts to 70% of total deformation [6]. The calculation results of gear hone teeth deformation are confirmed by experiments.

Comparing the elastic deformation values of gear hone teeth with the errors of the gears processed (profile error $f_p = 0.02 – 0.03$ mm for average-sized gears of accuracy degree 7 – 8) and the metal removal values ($0.005 – 0.020$ mm), it may be concluded that they are commensurate at $E_v < 6000$ MPa. It is assumed that the elastic value of hone tooth affects their performance.

In case gear hones with a higher degree of elasticity are used, the hone operation conditions, the sliding velocity, and the contact pattern sizes of hone teeth and machined gear surfaces change due to deformation of the hone teeth.

It is impossible to determine the sliding intensity parameters (specific sliding $\varrho_s$ and $\varrho_c$ and sliding velocity $V_s$) in the engagement of hone teeth and a processed gear with consideration of the elastic deformation values of the gear hone teeth applying the known recommendations for gears [7, 8] and hones [4]. It is therefore necessary to determine the values of the hone teeth and gears engagement parameters that affect their sliding intensity and depend on the elastic deformation values of hone teeth.

An engagement model of hone teeth and processed parts (figure 1) is suggested for analytical studies. The model enables simulating the mobility of hone teeth against gear teeth due to elastic deformation of the former.

A hone tooth flexes under load: the tooth symmetry axis of the model turns about conditional center $C$ by angle $\theta$ proportional to its flexural deformation. A distorted hone tooth is shown by the dotted line in the figure. At $\omega_k + \omega_{2v} = \omega_2$, contact point $K (K \rightarrow K^\prime)$ shifts and conditional tooth arm $0_bC$ turns by angle $\Delta \varphi$ into position $0_bC^\prime$ due to simultaneous contact ($\omega_k$) and flexural ($\omega_{2v}$) deformation. At such $CC^\prime$, the value of displacement of the conditional hone-tooth turning center, due to deformation $\omega_{2v}$ in contact point $K$, is the projection of $\omega_2$ on the base circle chord: $CC^\prime = \omega_2 \cdot \cos(\alpha_{\omega_2} - \delta \alpha)$.

Due to availability of the hone teeth deformation non-uniform along the teeth height and contact point displacement, gear ratio $U$ of the hone-gear engagement becomes variable within the circular pitch. Therefore, the following allowances are made in order to enable using the common patterns of gear engagement in point $K$ of the hone teeth and gear contact. Hone tooth arm $0_bC^\prime$ is in coincidence with distorted tooth symmetry axis $0_bC^\prime$ provided that $0_bC^\prime = 0_bC^\prime = r_{b0}$. The geometric turning center is conditionally shifted from point $0_b$ into point $0_b^\prime$. New engagement line $N_0N_0^\prime$ is drawn though contact point $K^\prime$ at angle $\Delta \alpha$ to $N_0N_1$. In this case, considering that ratio $r_{b1} / r_{b0}$ of the base circle radii of the gear and hone remains unchanged, the value of gear ratio $U$ can be taken as constant for calculation of the sliding parameters.
In case of elastic deformation of hone teeth, the values of curvature radii $\rho_0$ and $\rho_1$, pressure angles $\alpha_0$ and $\alpha_1$ in the current contact point along the profile height, and engagement face angles $\alpha_{W0}$ and $\alpha_{W1}$ change due to displacement of the contact point of the hone teeth and gear, of the engagement line, and of the center-to-center distance.

The values of the engagement angles in the end sections of the hone and the gear ($\alpha'_{W0}$ and $\alpha'_{W1}$), changed due to hone teeth deformation, are determined for point $K'$ according to figure 1 and using the following formulas:

$$\alpha'_{W0} = \arccos \frac{d_{W0}}{d'_{W0K}}; \quad \alpha'_{W1} = \arccos \frac{d_{W1}}{d'_{W1K}}.$$  \hspace{1cm} (1)

In formulas (1), the diameter values of hone and gear’s pitch circles for point $K'$ at $d'_{W0K} + d'_{W1K} = 2a'_w$:

$$d'_{W0K} = \frac{2a'_w}{U+1}; \quad d'_{W1K} = \frac{2a'_wU}{U+1}.$$ \hspace{1cm} (2)

Changed center-to-center distance $0'_00'_1 = a_w$ at $a'_w \neq a_w$:

$$a'_w = \frac{d_{W0} \cos\left(\frac{\pi - \theta}{2}\right) \sin\left(\frac{(\pi - \theta)/2 - \delta \alpha + \Delta \varphi}{\sin \nu}\right)}{\sin \nu}.$$ \hspace{1cm} (3)

The values of angles $\theta$ and $\delta \alpha$ are determined in figure 2:
\[ \theta = \arctg \frac{DD'}{BC - BD'} = \]
\[ = \arctg \frac{2 \omega_{z_k} \cos(\alpha_{o_k} - \gamma)}{d_{o_k} \cos \gamma - d_{\omega_0} - S_k \tan(\alpha_{o_k} - \gamma) - 2 \omega_{z_k} \sin(\alpha_{o_k} - \gamma)}; \]  
\[ \delta \alpha = \alpha_{\omega_0} - \alpha_{o_k} - \gamma \]  
\[ \gamma = \arcsin \left( \frac{S_k}{d_{o_k}} \right); \]
\[ S_k = 2AK = d_{o_k} \left( \frac{S_{\omega_0}}{d_{\omega_0} \cos \beta_{\omega_0}} + \text{inv} \alpha_{\omega_0} - \text{inv} \alpha_{o_k} \right) \cos \beta_{o_k}; \]
\[ d_{o_k} = 2r_{o_k}. \]

Angle \( \Delta \phi \) is determined on the basis of figure 1 using triangle \( O_0CC' \):
\[ \Delta \phi = 2 \arctg \left( \frac{\omega_{z_k} \cos(\alpha_{\omega_0} - \delta \alpha)}{d_{\omega_0}} \right). \]

**Figure 2.** Diagram to determine rotation angle \( \theta \) of the hone tooth symmetry axis considering the hone tooth flexural deformation.

Displacement angle \( \nu \), resulting from elastic deformation of the hone tooth from center-to-center distance \( \Delta O_0O'_1F \), is determined as follows:
\[
\nu = \arctg \left[ \frac{d_{\alpha} \cos \left( \frac{\pi - \theta}{2} \right) \sin \left( \frac{\pi - \theta}{2} - \delta \alpha + \Delta \phi \right)}{a_{\theta} \pm d_{\theta} \cos \left( \frac{\pi - \theta}{2} \right) \cos \left( \frac{\pi - \theta}{2} - \delta \alpha + \Delta \phi \right)} \right].
\]

(10)

In formula (10), the negative sign is used at angle \(O_1O_oO'_o \leq \pi/2\), the positive sign is used at angle \(O_1O_oO'_o > \pi/2\).

The changed value of the gear tooth profile’s curvature radius in point \(K'\) is determined on the assumption of a low value of angle \(\Delta \alpha = \nu + \alpha'_{w1} - \alpha_{w1}\) and equation \(N'K = N'_1K'\):

\[
\rho'_1 = N'_1K' = \rho_{ik} + N_iN'_1 = \rho_{ik} + \frac{d_{\alpha}}{2} \sin \Delta \alpha.
\]

(11)

The value of the gear tooth profile’s curvature radius, changed due to deformation, in point \(K'\):

\[
\rho'_{ik} = N'_iK' = \left( N'_iN'_i - \frac{\rho'_1}{\sin \tau_i} \right) \sin \tau_o.
\]

(12)

The changed length of engagement line \(N'\), \(N'_1\) is determined on the basis of \(\Delta O_iN'_iP'\) and \(\Delta O'_iN'_iP'\):

\[
N'_iN'_1 = O'_iL = \frac{d_{\alpha} \tan \alpha'_{w1}}{2 \sin \tau_o} + \frac{d_{\alpha} \tan \alpha'_{w1}}{2 \sin \tau_1}
\]

(13)

Pressure angles of hone teeth and gear profiles in current point \(K'\), with consideration of the hone tooth deformation:

\[
\alpha'_o = \arctg \frac{2\rho'_0}{d_{\alpha}}; \quad \alpha'_i = \arctg \frac{2\rho'_i}{d_{\alpha}}.
\]

(14)

Thus, the suggested model enabled development of the diagram for analytical determination of the values of hone-gear engagement geometric parameters changed due to elastic deformation of hone teeth: \(\rho'_{ik}, \rho'_{i}, \alpha'_{w1k}, \alpha'_{w1i}, \alpha'_{ik}, \alpha'_{i}\). Using these values in the known formulas for calculation of sliding parameters allows to consider the influence of both geometric parameters and elastic behavior of the hone material, load, pairing of contacts on the value and pattern of variation along the hone teeth height and on the value components of sliding parameters; it also means the influence on the metal removal intensity and tool material wear, as well as the quality and accuracy in processing of the hardened tooth evolvement surface.

References

[1] Garshin A and Fedotova S 2008 Abrasive Materials and Tools. Manufacturing Procedure: Instructional Aid (SPB: Polytechnic University Press) p 1010
[2] Taramykin Yu 1966 Momentary Tooth Contact Shape and Area in Case of Gear Honing( Collection of Works by ENIMS Postgraduates) pp 188-197
[3] Schegolev V and Ulanova M 1977 Elastic Abrasive Tools (L.: Mashinostroenie Leningrad Division) p 184
[4] Bagaiskov Yu and Shumyacher V 2005 Performance Increase of Devices Made of Abrasive Composite Materials (Volgograd: Volzhsky Institute of Civil Engineering and Technologies (branch) of Volgograd State University of Architecture and Civil Engineering) p 200
[5]  Bagaiskov Yu 2018 *Bending Deformation Analysis of Gear Hone Tooth Lateral Faces* (MATEC Web of Conferences vol. 224: International Conference on Modern Trends in Manufacturing Technologies and Equipment) (Sevastopol, Russia) p 5

[6]  Bagaiskov Yu 2018 *Study of Total Elastic Deformation of Gear Hone Teeth* (Novokuznetsk: Fundamentals of Mechanics: proceedings of the international research to practice conference MS RDC, No. 3) pp 126-9

[7]  E. Airapetov, Genkin M and Kolin I 1971 *Flexibility of Straight-Tooth Spur Gearing* (M.: Nauka Vibroacoustic Activity of Gear Mechanisms) pp 13-59

[8]  Zablonsky K and Filipovich S 1976 *Tooth Stiffness Analysis of Spiral Gears* (News of Higher Institutions. Engineering) pp 75-9