Dynamic Scalar Torsion and an Oscillating Universe

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For the Poincaré gauge theory of gravity we consider the dynamical scalar torsion mode in a cosmological context. We explore in particular the possibility of using dynamical torsion to explain the current state of the accelerating Universe. With certain suitable sets of chosen parameters, this model can give a (qualitatively) proper description of the current universe without a cosmological constant, and the universe described is oscillating with a period of the Hubble time.

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1. Introduction

The accelerating expansion of the Universe shown independently by two groups[12] established the existence of dark energy with a negative pressure. The idea of a dark energy becomes one of the greatest challenges for our current understanding of fundamental physics[34]. Among a number of possibilities to describe this dark energy component, the simplest and most theoretically appealing way may be by means of a cosmological constant $\Lambda$, which acts on the Einstein field equations as an isotropic and homogeneous source with a constant equation of state $w \equiv p/\rho = -1$. Another approach toward constructing a model for an accelerating universe is to work with the idea that the unknown, unclumped dark energy component is due exclusively to a minimally coupled scalar field $\Phi$ (quintessence field) which has not yet reached its ground state and whose current dynamics is basically determined by its potential energy $V(\Phi)$. This idea has received much attention over the past few years and a considerable effort has been made in understanding the role of quintessence fields on the dynamics of the Universe[07,8]. However, without a specific motivation from fundamental physics for the light scalar fields, these quintessence models can be constructed relatively arbitrarily, and thus there is some difficulty in relating any underlying theory to the observable structure of the Universe.
Here we consider an accounting for the accelerated universe in terms of an alternate gravity theory with an additional natural dynamic geometric quantity: torsion. Torsion has been used in several modern theories. The Einstein-Cartan theory has non-dynamic torsion driven by spin density. The dominant source is spin $1/2$ fermions, which would only produce axial torsion. The effect is expected to be small at ordinary densities, but it could have a major influence at high densities (e.g. beyond $10^{48}$ gm/cm$^3$), and thus it is expected to have important physical effects only in the early universe. Torsion cosmology investigations were initiated by Kopcynski.\textsuperscript{9} Some early investigations noted that torsion could prevent singularities, but soon it was argued that non-linear torsion effects were more likely to produce stronger singularities.\textsuperscript{10}

The Poincaré gauge theory of gravity (PGT)\textsuperscript{11,12} based on a Riemann-Cartan geometry, allows for dynamic torsion in addition to curvature. Because of its gauge structure and geometric properties it was regarded as an attractive alternative to general relativity. The various possible combinations of well behaved propagating modes in the linear theory were identified. Then it was noted that effects due to the non-linear constraints could be expected to render most of these cases physically unacceptable.\textsuperscript{13} An investigation showed that two particular “scalar torsion” cases had a well posed initial value problem.\textsuperscript{14} Later Hamiltonian investigations\textsuperscript{15,16} supported the conclusion that these two dynamic scalar torsion modes may well be the only acceptable dynamic PGT torsion modes.

In one scalar torsion mode only the axial vector torsion is dynamic (and it turns out to be dual to the gradient of a scalar field). Axial torsion is naturally driven by the intrinsic spin of fundamental fermions; in turn it naturally interacts with such sources. Thus for this mode one has some observational constraints. Note that except in the early universe one does not expect large spin densities. Consequently it is generally thought that axial torsion must be small and have small effects at the present time. This is one reason why we do not focus on this mode here.

The other scalar torsion mode is vector torsion (which turns out to depend on the gradient of a scalar field). There is no known fundamental source which directly excites this mode. Conversely this type of torsion does not interact in any direct dramatic fashion with any familiar type of matter. Hence we do not have much in the way of constraints as to its magnitude. We could imagine it as having significant magnitude and yet not being dramatically noticed except indirectly through the non-linear equations. This mode in particular has also attracted our interest because of a conspicuous consequence of the non-linear equations: in this case there is a critical non-zero value for the affine scalar curvature.

A comprehensive survey of the PGT torsion cosmological models was presented some time ago by Goenner and Müller-Hoissen.\textsuperscript{17} Although this work only solved in detail a few particular cases, it developed the equations for all the PGT cases, including those for the particular model we consider here. Since this work was done prior to the discovery of the accelerating universe, they generally imagined torsion as perhaps playing a big role only at high densities in the early universe. More recently
investigators have begun to consider torsion as a possible cause of the accelerating universe (see e.g. [18, 19]) but the subject has not yet been explored in detail.

Our theoretical PGT analysis led us to consider just two dynamic torsion modes. An obvious place where we might see some physical evidence for these modes is in cosmological models. The homogeneous and isotropic assumptions of cosmology greatly restrict the possible types of non-vanishing fields. Curiously, for torsion there are only two possibilities: vector torsion which, moreover, is the gradient of a function (which depends only on time) and axial torsion which is dual to the gradient of a function (depending only on time). Thus the homogeneous and isotropic cosmologies are naturally very suitable for the exploration of the physics of the dynamic PGT scalar modes.

In this work, we take our first step to explore the possible evolution of the Universe with the scalar torsion mode of PGT. The main motivation is two-fold: (1) to have a better understanding of the PGT, in particular the possible physics of the dynamic scalar torsion modes; (2) to consider the prospects of accounting for the outstanding present day mystery—the accelerating universe—in terms of an alternate gravity theory, more particularly in terms of the PGT dynamic torsion. With the usual assumptions of isotropy and homogeneity in cosmology, we find that, under the model, the Universe will oscillate with generic choices of the parameters. The torsion field in the model plays the role of the imperceptible “dark energy”. With a certain range of parameter choices, it can account for the current status of the Universe, i.e., an accelerating expanding universe with a value of the Hubble constant which is approximately the present one. These promising results should encourage further investigations of this model, with a detailed comparison of its predictions with the observational data.

This report is organized as follows: We summarize the formulation of the scalar torsion mode of PGT in Sec. II and the expressions of this model in cosmology in Sec. III. We give a preliminary analytical analysis of the behavior of the field equations of this model in Sec. IV. In Section V we present the results of our numerical demonstrations for various choices of the parameters and the initial data. We discuss the implications of our findings in Sec. VI.

2. Torsion field equation

The affine connection of the Riemann-Cartan geometry has the form

$$\Gamma^k_{ij} = \Gamma^k_{ij} + \frac{1}{2}(T^k_{ji} + T^k_{ij} + T^k_{ij})$$

(1)

where $\Gamma^k_{ij}$ is the Levi-Civita connection and $T^k_{ij}$ is the torsion tensor. The Ricci curvature and scalar curvature of the Riemann-Cartan geometry can be expanded
in the form
\[ R_{ij} = \overline{R}_{ij} + \nabla_j T_i + \frac{1}{2} (\nabla_k - T_k)(T_{ji}^k + T^k_{ij} + T^k_{ji}) \]
\[ + \frac{1}{4} (T_{kmi} T^{km} + 2 T_{jkm} T^{mk}_i), \]  
\[ R = \overline{R} + 2 \nabla_i T^i + \frac{1}{4} (T_{ijk} T^{ijk} + 2 T_{ijk} T^{kji} - 4 T_i T^i), \]  
where \( \overline{R}_{ij} \) and \( \overline{R} \) are the Riemannian Ricci curvature and scalar curvature, and \( \nabla \) is the covariant derivative with the Levi-Civita connection.

For the case of interest here the torsion tensor has the restricted form
\[ T_{ijk} = \frac{2}{3} T_i [g_j g_k], \]  
where the vector \( T_i \) is the trace of the torsion. Consequently the affine Ricci curvature and scalar curvature become
\[ R_{ij} = \overline{R}_{ij} + \frac{1}{3} (2 \nabla_j T_i + g_{ij} \nabla^k T^k) + \frac{2}{9} (T_{ij} T_j - g_{ij} T_k T^k), \]  
\[ R = \overline{R} + 2 \nabla_i T^i - \frac{2}{3} T_i T^i. \]

For the PGT scalar torsion mode the gravitational Lagrangian density is
\[ L = -\frac{a_0}{2 R} + \frac{b}{24} R^2 + \frac{a_1}{8} (T_{ij} T^{ij} + 2 T_{ijk} T^{kji} - 4 T_i T^i). \]

Then the field equations of concern are
\[ a_0 \left( \overline{R}_{ij} - \frac{1}{2} g_{ij} R \right) - \frac{b}{6} R \left( R_{(ij)} - \frac{1}{4} g_{ij} R \right) \]
\[ + \frac{2 m}{3} \left( \nabla_i T_j - g_{ij} \nabla^k T^k \right) + \frac{m}{9} (2 T_i T_j + g_{ij} T_k T^k) = \mathcal{T}_{ij}, \]  
\[ \nabla_i R + \frac{2}{3} \left( R - \frac{6m}{b} \right) T_i = 0, \]  
with the appropriate assumption of the source spin density vanishing in Eq. (9).

Contracting Eq. (8) using Eq. (6) gives \( a_1 \overline{R} + m R + \mathcal{T} = 0 \). Equation (8) can now be re-written in an effective Einstein equation form:
\[ a_0 \left( \overline{R}_{ij} - \frac{1}{2} g_{ij} \overline{R} \right) = \tau_{ij} := \mathcal{T}_{ij} + \overline{\mathcal{T}}_{ij}, \]  
where the effective stress-energy tensor due to the scalar torsion is given by
\[ \overline{\mathcal{T}}_{ij} = \frac{b}{6} R (R_{(ij)} - \frac{1}{4} g_{ij} R) - \frac{2 m}{3} (\nabla_i T_j - g_{ij} \nabla^k T^k) - \frac{m}{9} (2 T_i T_j + g_{ij} T_k T^k). \]  
Relation (11) is quite useful both intuitively and practically, even though it has the shortcoming that all the second derivatives of the metric have not been isolated on one side of the equation.
3. Cosmology

With the cosmological assumption of isotropy and homogeneity, the spacetime metric takes on the FRW form,

\[ ds^2 = -dt^2 + a^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right] ; \]  

(12)

here we consider only the simplest case, the flat universe, \( k = 0 \). Then the nonvanishing Riemannian Ricci and scalar curvature are the well known

\[ \begin{align*}
R_{tt} &= 3(\dot{H} + H^2), \\
R_{rr} &= R_{\theta \theta} = R_{\phi \phi} = \dot{H} + 3H^2, \\
R &= 6(\dot{H} + 2H^2),
\end{align*} \]  

(13)

where \( H \) is the Hubble rate \( H = \partial_t (\ln a) \).

In accordance with the homogeneity and isotropy the torsion vector \( T \) depends only on time, and has just one non-vanishing component: \( \Phi(t) := T_t(t) \). Using (13) in the Field equations (8,9) we can finally give the necessary equations to integrate:

\[ \begin{align*}
\dot{a} &= aH, \\
\dot{H} &= -\frac{m}{6a_1}R - \frac{1}{6a_1}T - 2H^2, \\
\dot{\Phi} &= -\frac{a_0}{2a_1}R - \frac{1}{2a_1}T - 3H\Phi + \frac{1}{3}\Phi^2, \\
\dot{R} &= -\frac{2}{3}\left( R - \frac{6m}{b} \right)\Phi,
\end{align*} \]  

(14–17)

where

\[ \begin{align*}
3a_1H^2 + \frac{b}{24}R^2 - \frac{b}{18} \left( R - \frac{6m}{b} \right) (3H - \Phi)^2 &= T_{tt}, \\
T_{ij} &= \text{diag}(-\rho, p, p, p), \\
T &= g^{ij}T_{ij} = 3p - \rho, \\
p &= \omega \rho.
\end{align*} \]  

(18–20)

For a matter-dominated era, the pressure \( p \) is negligible, i.e., \( \omega \approx 0 \). For the stress-energy tensor from the scalar torsion the explicit expression is

\[ \begin{align*}
T_{tt} &= \frac{b}{24}R^2 - 3mH^2 - \frac{b}{18} \left( R - \frac{6m}{b} \right) (3H - \Phi)^2, \\
T_{\theta \theta} = T_{\phi \phi} &= \frac{1}{3}m(R - \overline{R}) - T_{tt},
\end{align*} \]  

(21–22)

and the off-diagonal terms vanish.

4. A preliminary analysis of the equations

Equations (14–17) are the main equations for the integrations to evolve the system. Regarding the parameters in the field equations, the Newtonian limit requires \( a_0 \equiv (8\pi G)^{-1} \). We take \( a_1 > 0 \) and \( b > 0 \) to satisfy the energy positivity requirement. Furthermore, we assume \( m > 0 \) since \( m \) is the mass parameter of the dynamic torsion mode.
Before the detailed results are shown, we briefly analyze the equations, to obtain some insight about their behavior. Let us first study the behavior of the affine scalar curvature $R$. For convenience, we define $Y(t) \equiv R(t) - 6m/b$, then we can rewrite Eq. (17) as

$$\dot{Y} = -\frac{2}{3} Y \Phi .$$

(23)

The second derivative of $Y$ with respect to time can be obtained by operating a time derivative on Eq. (23) and using Eq. (16). From this analysis we find that the late-time behavior will be essentially affected by the initial value $Y(t_0)$. $Y$ would grow unboundedly if its initial value is chosen to be positive. In order to have a bounded value of $Y(t)$ at late time, the initial value of $Y$ is taken to be $Y(t_0) \leq 0$. Then it can be shown from Eq. (23) that $Y(t)$ will always be $Y(t) \leq 0$. Here we will restrict ourselves to the situation where $Y \leq 0 \Rightarrow R \leq \frac{6m}{b}$,

(24)

so that this model could be suitable for describing the current status of the Universe.

Now let us turn our attention to $\Phi$, the nonvanishing part of the torsion. An analysis of the approximate equation of motion leads to the conclusion that we can expect periodic behavior, with a period around $T = 2\pi \sqrt{b/2m}$ and an estimated maximal amplitude. With the same kind of reasoning, we can find that $R$ has a similar periodic behavior.

However there seems to be no way to figure out the behavior of the expansion factor $a$ with a similar analysis. The acceleration of the expansion factor $a$ can be derived by taking a further derivative on Eq. (14) and using Eq. (15):

$$\ddot{a} = -\frac{mR + T}{6a_1}a - \frac{\dot{a}^2}{a} .$$

(25)

We can see from this equation that there does not exist an obvious leading term. Therefore it is not easy to understand the behavior of $a$ with a suitable approximation. However, the period of $a$ and $H$, if it exists, should be the same as $\Phi$ and $R$, since the variables are all highly coupled to each other, and thus there should exist a common period in the solution. We demonstrate this point with the numerical analysis in the next section.

However, we need to look into the scaling features of this model before we can obtain the sort of results we seek on a cosmological scale. In terms of fundamental units we can scale the variables as

$$t \rightarrow t/\ell_p, \ a \rightarrow a, \ H \rightarrow \ell_p H, \ \Phi \rightarrow \ell_p \Phi, \ R \rightarrow \ell_p^2 R,$$

(26)

and the corresponding scaling of the parameters are

$$a_0 \rightarrow \ell_p^2 a_0, \ a_1 \rightarrow \ell_p^2 a_1, \ m \rightarrow \ell_p^2 m, \ b \rightarrow b,$$

(27)

where $\ell_p \equiv \sqrt{8\pi G}$. In such a case, all the variables, and the scaled parameters $a_0, a_1$, and $b$, become dimensionless, and $a_0 = 1$. Furthermore, equations (14,17) remain
unchanged under such a scaling. However, as we are interested in the cosmological scale, it is practical to use another scaling to turn the numerical values of the scaled variables “gentler” (i.e., not stiff) from the numerical integration. In order to achieve this goal, let us introduce a dimensionless constant $T_0$, which has the order of magnitude of the Hubble time. Then the scaling is

$$
t \rightarrow T_0 t, \quad a \rightarrow a, \quad H \rightarrow H/T_0, \quad \Phi \rightarrow \Phi/T_0, \quad R \rightarrow R/T_0^2, \quad a_0 \rightarrow a_0, \quad a_1 \rightarrow a_1, \quad m \rightarrow m, \quad b \rightarrow H_0^2 b,
$$

With this scaling, all the field equations are kept unchanged while the period $T \rightarrow T_0 T$.

5. Numerical demonstration

In this section we would like to demonstrate two points: (i) In the degenerate case, i.e., $Y = 0$, the torsion in the system becomes kinetic instead of being dynamic, and the expansion is always decelerating; (2) In generic cases, i.e., $Y < 0$, the torsion in the system is dynamic, and the functional pattern of the expansion factor has a periodic feature, i.e., it could be accelerating for a while, and then be followed by a period of deceleration with the pattern repeating. With suitable choices of the parameters and the initial values of the fields involved, it is possible to change the period of the dynamic fields as well as their amplitudes; (3) In the model, with some choices of the parameters and the initial values of the fields, it is possible to mimic the main apparent dynamic features of the Universe, i.e., the value of the Hubble function is the current Hubble constant in an accelerating universe after a period of time on the order of the Hubble time. In such a case, this model will describe an oscillating universe with a period on the order of magnitude of the Hubble time. This allows us to constrain the parameters and/or the value of the torsion field by comparing the observed data with the result from this model.

The 4th-order Runge-Kutta method is applied for the integration of the field equations (14-17). The Universe is assumed to be matter-dominated, thus $\mathcal{T} \approx -\rho$. The mass density $\rho$ is determined from the fields via Eq. (18). For all cases, the fields and the parameters are scaled with Eq. (27) to be dimensionless. For the third case, the fields and the parameters are scaled further with Eq. (28) to achieve a realistic cosmology. In all cases we choose $Y(t_0) \leq 0$. The system will keep $Y(t) \leq 0$ as long as the initial value of $Y$ is so chosen.

5.1. Case I: constant $R$ case

In this case, the initial values of the fields are as follows: $a(t_0) = 10^5$, $H(t_0) = 10^{-5}$, $\Phi(t_0) = 2 \times 10^{-2}$, $Y(t_0) = 0$, and the parameters are taken to be $m = 10^{-6}$, $b = 8 \times 10^{-4}$. The result is shown in Fig. 1, where the evolved values of $H$, $\dot{a}$, $\Phi$, and $R$ are plotted in different panels.

It is obvious in the bottom-right panel of Fig. 1 that the affine scalar curvature $R$ remains constant, $6m/b$. In other words, $Y(t) = 0$. The behavior of the torsion
Fig. 1. Evolution of the Hubble function, the 2nd time derivative of the expansion factor, the temporal component of the torsion, and the affine scalar curvature as functions of time with the parameter choice and the initial data in Case I.

Φ can be understood through Eq. (16). Φ will increase (or decrease) until its value balances the rhs of Eq. (16); this mainly depends on the sign change of the term $3H\Phi$ provided $H > 0$ and $T > 0$. With the current initial choice in this case, Φ decreases promptly until the balancing point is reached, as seen in the bottom-left panel of Fig. 1. However, Φ will not be a constant since the rhs of Eq. (16) still changes with time. The Hubble function $H$ will always decrease, as shown in the upper-left panel of Fig. 1, since all the terms on the rhs of Eq. (15) are negative. Therefore, the acceleration of the expansion factor, $\ddot{a}$, will be always negative, as seen in the upper-right panel of Fig. 1. Obviously, the scenario described in this case cannot explain the acceleration of the current Universe. However, it could represent the very final stage of an oscillating universe since the amplitude of the affine scalar curvature is damped to $6m/b$, as described in the next two cases.

5.2. Case II: oscillating acceleration of $a$

For this case, we take the initial values of the field as follows: $a(t_0) = 10^8$, $H(t_0) = 10^{-8}$, $\Phi(t_0) = 10^{-4}$, $Y(t_0) = -10^{-8}$ and the parameters are chosen to be $m = 10^{-8}$, $b = 8 \times 10^{-4}$. The results plotted in Fig. 2 show that $\ddot{a}$, $\Phi$, and $R$ are damped
periodic.

Since $Y(t_0)$ is negative, $Y(t)$ remains negative. Thus $R(t) < 6m/b = 7.5 \times 10^{-5}$ as shown in the bottom-right panel of Fig. 2. Its period is about $T \approx 3500$, different from $2\pi \sqrt{b/2m} \approx 1256$ by about a factor of 3. As explained in the previous section, the difference comes from the choice of the initial values. $\Phi$ has the same period with its magnitude close to the maximal possible magnitude: $\sqrt{9m/b} \approx 1.6 \times 10^{-2}$. The most interesting part is the behavior of $\ddot{a}$, which is periodic with the same period as $\Phi$ and $R$. As shown in the top-right panel of Fig. 2, $\ddot{a}$ could be positive as well as being negative and the pattern of its function is similar to the pattern of $R$ with a minus sign. Therefore the behavior of $H$ is declining with a zig-zag pattern, as shown in the top-left panel of Fig. 2.

In a broader viewpoint of the evolution of this system, $\ddot{a}$, $\Phi$, and $R$ will be slowly damped and the zig-zag part of $H$ will be also smoothed after a long time. Eventually, the final stage of this case will tend to be similar with Case I, i.e., $\ddot{a} \leq 0$, $R = \text{const}$. However, the important point of this case is that the universe could oscillate due to dynamic torsion. In this scenario the present day acceleration of the Universe is not so strange, $\ddot{a}$ is oscillating and it happens to be increasing at this time. Furthermore, the period of the oscillation of a universe could be determined.
5.3. Case III: mimicking an accelerating universe at present

In this case, the initial values of the field are chosen as follows: $a(t_0) = 10$, $H(t_0) = 4/T_0$, $\Phi(t_0) = 5/T_0$, $Y(t_0) = -24/H_0^2$ and the parameters are taken to be $m = 0.78$, $b = 0.19T_0^2$, to obtain a presently accelerating universe after a time on the scale of the Hubble time.

In the top-left panel of Fig. 3, the Hubble function $H$ evolution is fast-damped and oscillating, with the magnitude being on the order of $1/T_0$ at a time near $T_0$ (the scale of the Hubble time). Moreover, $H$ is increasing at that time epoch. We can see this more clearly from the top-right panel of Fig. 3 where $\ddot{a}$ is plotted. It is obvious that $\ddot{a}$ is damped and oscillating during the evolution and is positive around $t \approx T_0$. In this setting, $R$, shown in the bottom-right panel of Fig. 3 is smaller than $6m/b = 24/T_0^2$, as stated in Eq. (24). Although the initial values of $\Phi$ and $H$ are similar in this case, the magnitude of $H$ is decreasing faster than that of $\Phi$. This
verifies that we can usually assume $\Phi \gg H$. Here the behavior of all the fields at the very beginning of the time should not be taken too seriously since in the early universe our matter-dominated era assumption breaks down. We hope to discuss a radiation-dominated universe with this model in the future.

Although we cannot know if the initial data and parameter values chosen here are realistic without a further investigation, this case does demonstrate the possibility of explaining the accelerating expansion of the universe with the dynamic scalar torsion PGT model. It should be noted that the accelerating expansion simply comes about because of the current phase of the oscillation of the universe. Such an oscillation mechanism may offer a good answer to the related well known coincidence problem. Hence it will be very interesting to see if this model can be consistent with up-to-date data from astronomical observations.

6. Discussion

In this work we introduce into the evolution of a universe without a cosmological constant a certain dynamical PGT scalar torsion mode taken from our earlier work. From the assumption of the homogeneity and isotropy of the universe, only the temporal component of the torsion $\Phi$ will survive and affect the evolution of the universe at late times. With the field equations (14–17), we analyzed analytically and numerically the evolution of the system. We found that there is a critical value of the affine curvature $R$, i.e., $R = 6m/b$, which divides the system into two different behaviors. If the initial value $R(t_0) > 6m/b$, then $R(t)$ is always greater than $6m/b$ and the amplitudes of all the variables may grow unboundedly. If $R(t_0) < 6m/b$, then $R(t)$ will be always less than $6m/b$ and $\ddot{a}, \Phi, R$, tend to have a damped periodic behavior with the same period. With certain choices of the parameters of $m$ and $b$, and of the initial data of $a, H, \Phi$, and $R$, like Case III in the previous section, this model can describe an oscillating universe with an accelerating expansion at the present time. If we consider instead the spacetimes as Riemannian, by absorbing the contribution of the torsion of this model into the stress-energy tensor on the rhs of the Einstein’s equation, then this contribution will act like an exotic fluid with its mass density $\rho_T$ and pressure $p_T$ varying with respect of time. Moreover, it presently has a negative pressure and consequently a negative parameter in the equation of state, i.e., $\omega_T$, which drives the universe into accelerating expansion.

Before we can give an adequate discussion of the viability of this model as an explanation of the accelerating universe, we should check whether this model can survive under the constraints of the theoretical and experimental tests. There have been numerous investigations on the existence of torsion since this geometric quantity entered the realm of gravity (see 20, 21, 22, and the references therein.) As mentioned above, this model has not only passed the important classical tests — “no-ghosts” and “no-tachyons” — it is one of the two scalar torsion modes, the only PGT cases which are known to have a well posed initial value problem and which may well be the only viable dynamic PGT torsion modes that can evade the
non-linear constraint problems. There have been a lot of laboratory tests in search of torsion. The main idea among these experiments is the spin interaction between matter and torsion. The cosmological tests on torsion investigate the effect of torsion-induced spin flips of neutrinos in the early Universe which could alter the helium abundance and have other effects on the early nucleosynthesis. However, Dirac fermions interact only with the totally antisymmetric pseudo-scalar part of the torsion. Thus these tests can only consider the pseudo-scalar-mode/axial-vector torsion, not the scalar mode torsion used in our model. The type of torsion used in our model does not interact directly with any known matter. Thus, these tests cannot really give a serious constraint on the amplitude of our scalar-mode torsion.

Among the models in which torsion is applied to the cosmological problem, Capozziello et al. have done a serious study on replacing the role of the cosmological constant in the accelerating Universe. With a totally antisymmetric torsion without dynamical evolution, their model is consistent with the observational data by tuning the amount of the torsion density, although this model cannot solve the coincidence problem. On the other hand, the oscillating universe models with a designed mechanism: an oscillating potential, an oscillating parameter of the equation of state, etc., aim to solve the coincidence problem. Here we found that our model takes some virtues from both kind of models, i.e., our model is capable of solving the coincidence problem of an accelerating universe with a dynamical scalar-mode torsion, which is naturally obtained from the Geometry of the Riemann-Cartan spacetimes, instead of an exotic scalar field or a designed mechanism.

In this work we proposed the scalar torsion mode of the PGT as a viable model for explaining the current status of the Universe. The source of the torsion could come from the huge density of the particles with the aligned spins in the early universe. This scalar mode of torsion could be considered as a “phantom” field, at least in the matter-dominated epoch, since it will not interact directly with matter; it only interacts indirectly via gravitation. Then the dynamics of the scalar torsion mode could drive the Universe in an oscillating fashion with an accelerating expansion at present. However, there are also some points which need to be studied in much more detail before this model can more closely conform to reality. The model in Case III of the previous section, suggests that the mass parameter of the torsion, $m$, might be close to $a_0$, and the parameter for the “kinetic” energy density of the torsion, $b$, may need to be as huge as $T_0^2$ to achieve an accelerating universe. The restricted window of the parameter choices which allows a behavior like that of our universe might render the model less favored, even though the matter in the universe is not able to directly interact with the torsion. Meanwhile, the required choice of initial data and the values of the parameters may make this model unsuited to solving the fine-tuning problem. However, these dark sides will not be able to diminish the possibility of the scalar mode of the torsion in this model playing a magnificent role in the evolution of the Universe. Moreover, we only used one of the viable modes of torsion in PGT, and the argument may be more general if it can be extended to all the viable PGT torsion modes. Further studies on model building,
a comparison between the observational data and the predictions of this model, and parameter/initial data determination are currently under investigation.31

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