Comment on “Cooling by Heating: Refrigerator Powered by Photons”

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Results obtained recently in Phys. Rev. Lett. 108, 120603 (2012) by Cleuren et al. apparently contradict to the third law of thermodynamics. We discuss a scenario for resolving this contradiction, and show that this scenario is pertinent for clarifying the general message of the third law.

1. A recent interesting letter by Cleuren et al. presents a model for a refrigerator operating between two thermal baths at temperatures $T_c$ and $T_h$, $T_c < T_h$, [1]. Among other findings, Cleuren et al. report on a low-$T_c$ regime, where the heat $\dot{Q}_c$ taken per time-unit from the low-temperature bath scales as

$$\dot{Q}_c \sim T_c. \quad (1)$$

If [1] is assumed to hold for all temperatures down to $T_c \to 0$, it would invalidate (for this model example) the third law of thermodynamics [2]. This law states that the rate of cooling of the low-temperature bath goes to zero.

2. The purpose of the present note is to explain why to our opinion the proposal by Levy et al. cannot be accepted as a fundamental reason for saving the third law, and then make our own attempt of arguing against the validity of [1] at very low temperatures. We stress that we do not make any claim on the invalidity of the results by [1] within their model. It is the applicability of the model for low temperatures that is questioned.

3. The contradiction between [1] and the third law is deduced via routine thermodynamic considerations: (i) the low-temperature thermal bath stays in overall equilibrium despite of its interaction with the refrigerator. Hence it will respond to the refrigerator by lowering its temperature. (ii) The rate at which its temperature $T_c$ is lowered can be evaluated within the linear response, since the energy taken out due to refrigeration is much smaller than the energy of the bath. Hence [2]

$$\dot{T}_c = \frac{\dot{Q}_c}{c(T_c)}, \quad (2)$$

and taking into account that for $T_c \to 0$ the constant-volume heat capacity $c(T_c)$ behaves at least as $c(T_c) \sim T_c$ for reasonable thermal baths (including the electron bath studied in [1]), one concludes that $T_c$ will be at least constant for $T_c \to 0$, which contradicts to the third law.

4. To save the third law, Levy et al. propose that the Hamiltonian of the refrigerator employed in [1] is supplemented by another, physically well motivated term that invalidates [1] for a low $T_c$. [2]. This salvation of the third law is not satisfactory for the following reason.

Any model of refrigerator must describe its specific function. This description necessarily involves taking “limits”, i.e. letting certain parameters in the respective Hamiltonian go to zero. We distinguish two types of such asymptotic behavior:

1. “Circumstantial limits” strengthen the functional characteristics of the model. Applying such limits is a natural desire of building better devices. Indeed, good devices do have rather special Hamiltonians. As the evolution of room refrigerators shows, even when their theoretical operating principles are clear, it still takes many years and substantial engineering efforts to built good devices, precisely because many unwanted terms in their Hamiltonians are to be eliminated.

2. “Dysfunctional” limits would suppress the desired function of the device (asymptotically).

Now Levy et al. based their arguments on an circumstantial limit as a potential reason of violating the third law. It may be difficult to implement this limit in practice, but nothing in the analysis by Levy et al. shows that the term they propose cannot in principle be made as small as desired. This viewpoint on the salvation of the third law would suggest that this law is not fundamental, but it holds due to imperfections present in the Hamiltonian.

In contrast, we are going to argue that the violation of the third law by Cleuren et al. relates to a dysfunctional limit: if it is applied down to very low temperatures, the basic functional characteristics of the model (its power of refrigeration) will be harmed.

5. Now we explain why the weak-coupling master-equation-based refrigerator model by Cleuren et al. gets limited at low $T_c$. An essential feature of such Markov models is the detailed balance with respect to each thermal bath. Due to this, for $T_c = T_h = T$ (equal temperature baths) the refrigerator density matrix $\rho$ has the Gibbs form $\rho \propto e^{-H/T}$, where $H$ is the refrigerator’s Hamiltonian. For $T \to 0$ this predicts that the refrigerator itself will be in its pure ground state. Such a conclusion is impossible for a system permanently coupled to a thermal bath, provided that both the system-bath interaction Hamiltonian and its commutator with the full Hamiltonian stay finite (non-zero); see e.g. [3].
The model studied in [1] belongs to this class.

The usual way of understanding the low-temperature limit of the Gibbs density matrix for an open system is to assume that simultaneously with $T \to 0$ the coupling to the bath is made progressively smaller. However, for the present situation this limiting process for justifying the Gibbs density matrix down to low temperatures does not work, since any refrigerator should have a finite coupling to the baths for ensuring a finite power of its operation. While the argument strictly speaking applies only for $T_c = T_h = T$, it should be clear that there are low-$T_c$ validity limits of the weak-coupling master equation also for $T_c < T_h$. Hence if the weak-coupling master equation is forced to apply for all $T_c \to 0$, its coupling to the low-temperature bath should be made progressively weaker nullifying $Q_c/T_c$ in (1).

6. The above argument on the inapplicability of the usual Markov models at low temperatures suggests that

the analysis of the low-temperature refrigeration will certainly benefit from being carried out in a set-up, where the refrigerator bath interaction is treated exactly, without any assumption on progressively weak refrigerator-bath interactions. Now one should and can ensure that the power of refrigeration stays finite down to $T_c = 0$. Such a model-dependent analysis has been carried out recently showing that although the regime (1) is reproduced by sufficiently good devices at moderate and low values of $T_c$, it is broken down for very low temperatures holding the third law [3].

Our conclusion is that a proposition like (1) should always be supplemented by demanding that the power of refrigeration stays finite. Then presumably it cannot hold down to very low temperatures, as the model studied in [1] shows. If such a proposition could be shown to hold down to $T_c = 0$ with only circumstantial limits involved, it would constitute a a “real” violation of the third law.

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