A paradigm for entanglement and information swapping of two qubits

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Abstract. Based on a general unitary operation of two-qubit system, we investigate the dynamics of entanglement, entropy and purity. Distinctive features in the different initial states settings of an interacting bipartite system are explored. The striking effects of initial product states on entanglement shows that zeros entanglement can be obtained during the development of the interaction.
1. Introduction

To test fundamental quantum concepts and implement various potential applications, including sensitive detection and quantum information processing, the engineering of quantum states has attracted considerable attention in recent years [1]. On the other hand, entanglement takes an important part in manipulating with information, storing, as in quantum coding [2, 20], sending, as in teleportation [3]. So, generating and quantifying the entanglement is one of the most important tasks in quantum information context. In quantum computer, qubits are the arithmetical units [4], which makes a new type of dynamics able to solve different problems faster than classical computer. It is desirable then to discuss the behavior of these qubits when a switch on/of the computer is taken place.

Investigating the dynamics of the purity of quantum system is one of the most important topics in the context of quantum information theory [5, 6]. It can be used as a measure of the degree of entanglement for mixed state [7, 8]. Bagan et. al. have been estimated the purity of a large number N of copies of a qubit state [9]. Our aim in the present article is twofold: The first is to discuss the dynamics of two separable qubits under the effect of the unitary operation. The second is to treat the phenomenon of swapping information from a sender (Alice) to a receiver (Bob). We suggest that Alice has a completely polarized qubit in x-direction, while Bob’s qubit takes any form. We show that the purity of one qubit can purified on the expance of the other one.

The rest of the paper is arranged as follows: Section 2, is devoted to describe the general two-qubit pairs by means of its Bloch vectors and a cross dyadic, which is defined by $3 \otimes 3$ matrix. In Section 3, we obtain the density operator for the individual subsystems, which can be used to read out how much information lost or gained. Several examples are given in Section 4, in which the initial states are assumed to be product states. In section 5, we consider a class for entangled states. The paper ends with a conclusion in Sec.6.

2. Two qubit pairs

Assume that the user Alice and Bob are given a state of a two qubit pairs to perform any quantum information task. The general form of the 2-qubit state is

$$\rho_{ab} = \frac{1}{4}(1 + \vec{s} \cdot \sigma^+ + \vec{t} \cdot \tau^+ + \vec{s} \cdot \vec{C} \cdot \tau^1),$$

(1)

where $\vec{s}$ and $\vec{t}$ are the Pauli’s spin vector of the first and the second qubits respectively [10]. The statistical operator for the individual qubits are specified by their Bloch vectors, $\vec{s} = \langle \vec{\sigma} \rangle$ and $\vec{t} = \langle \vec{\tau} \rangle$. The cross dyadic $\vec{C}$ is represented by a $3 \times 3$ matrix. It describes the correlation between the first qubit, $\rho_a = \text{tr}_b\{\rho_{ab}\} = \frac{1}{2}(1 + \vec{s} \cdot \sigma^1)$ and the second qubit $\rho_b = \text{tr}_a\{\rho_{ab}\} = \frac{1}{2}(1 + \vec{t} \cdot \tau^1)$. The Bloch vectors and the cross dyadic are
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given by

\[ \vec{s} = (s_x, s_y, s_z), \quad \vec{t} = (t_x, t_y, t_z), \quad \text{and} \quad \vec{C} = \begin{pmatrix} c_{xx} & c_{xy} & c_{xz} \\ c_{yx} & c_{yy} & c_{yz} \\ c_{zx} & c_{zy} & c_{zz} \end{pmatrix} \] (2)

Now, we suppose that the interacting Hamiltonian of the two qubits is given by

\[ \mathcal{H} = \sum_{i=1}^{3} \alpha_i \sigma_i \otimes I + \sum_{j=1}^{3} \beta_j I \otimes \tau_i + \sum_{i,j=1}^{3} \alpha_{ij} \sigma_i \otimes \tau_j \] (3)

The first two terms represent the free evolution of the individual two qubits, while the third term represents the evolution of the correlation part. It is described by a dyadic consists of 9 elements. By using a suitable local unitary transformation, we can always choose a reference system in which this dyadic can be reduced to a diagonal one. In this case we have only 3 elements. In our treatment we shall consider the effective Hamiltonian only. In the computational basis the unitary operator \([\mathbf{11}]\) is given by

\[ U = \Gamma_1(|00\rangle\langle00| + |11\rangle\langle11|) + \Gamma_2(|01\rangle\langle01| + |10\rangle\langle10|) \]
\[ + \Gamma_3(|10\rangle\langle01| + |01\rangle\langle10|) + \Gamma_4(|11\rangle\langle00| + |00\rangle\langle11|) \] (4)

where,

\[ \Gamma_1 = e^{-i\alpha t} \cos t(\alpha_1 - \alpha_2), \quad \Gamma_2 = e^{i\alpha t} \cos t(\alpha_1 + \alpha_2), \]
\[ \Gamma_3 = -i e^{-i\alpha t} \sin t(\alpha_1 + \alpha_2), \quad \Gamma_4 = i e^{i\alpha t} \sin t(\alpha_1 - \alpha_2), \] (5)

This is canonical unitary operator \([\mathbf{12}, \mathbf{13}]\). Under the canonical evolution the density matrix of the 2-qubit state transforms to

\[ \rho^{\text{new}} = U \frac{1}{2} (1 + \vec{s} \cdot \sigma^\dagger) \frac{1}{2} (1 + \vec{t} \cdot \tau^\dagger) U^\dagger \]
\[ = \frac{1}{4} (1 + \vec{s}^{\text{new}} \cdot \sigma^\dagger + \vec{t}^{\text{new}} \cdot \tau^\dagger + \vec{\sigma} \cdot \vec{C}^{\text{new}} \cdot \vec{\tau}^\dagger), \] (6)

where the new Bloch vectors and the dyadic are given by,

\[ \vec{s}^{\text{new}} = (s_1, s_2, s_3), \quad \vec{t}^{\text{new}} = (t_1, t_2, t_3), \quad \vec{C}^{\text{new}} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{pmatrix} \] (7)

with

\[ s_1 = \cos 2\alpha_3 (s_x \cos 2\alpha_1 + c_{yz} \sin 2\alpha_1) + \sin 2\alpha_3 (t_x \sin 2\alpha_1 - c_{yz} \cos 2\alpha_1), \]
\[ s_2 = (1 - 2 \cos^2 \alpha_3) (s_y \cos 2\alpha_2 - c_{zx} \sin 2\alpha_2) - \sin 2\alpha_3 (t_y \sin 2\alpha_2 + c_{zx} \cos 2\alpha_2), \]
\[ s_3 = s_z [\cos^2 (\alpha_1 + \alpha_2) + \cos^2 (\alpha_1 - \alpha_2) - 1] + t_z [\cos^2 (\alpha_1 + \alpha_2) - \cos^2 (\alpha_1 - \alpha_2)] \]
\[ - \frac{1}{2} (c_{xy} + c_{yx}) [\sin 2(\alpha_1 - \alpha_2) + \sin 2(\alpha_1 + \alpha_2)], \] (8)

\[ t_1 = \sin 2\alpha_3 (s_x \sin 2\alpha_1 - c_{yz} \cos 2\alpha_1) - (1 - 2 \cos^2 \alpha_3) (t_x \cos 2\alpha_1 + C_{yz} \sin 2\alpha_1), \]
\[ t_2 = -\sin 2\alpha_3 (s_y \sin 2\alpha_2 + c_{zx} \cos 2\alpha_2) + (1 - 2 \cos^2 \alpha_3) (t_y \cos 2\alpha_2 - c_{zx} \sin 2\alpha_2), \]
\[ t_3 = s_z [\cos^2 (\alpha_1 - \alpha_2) - \cos^2 (\alpha_1 + \alpha_2)] + t_z [\cos^2 (\alpha_1 - \alpha_2) + \cos^2 (\alpha_1 + \alpha_2) - 1] \]
\[ + \frac{1}{2} (c_{xy} + c_{yx}) [\sin 2(\alpha_1 - \alpha_2) - \sin 2(\alpha_1 + \alpha_2)], \] (9)
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In this section, we investigate how the density matrices of the two qubits evolve. Tracing out the second qubit (Bob’s qubit), we obtain the first qubit (Alice’s qubit), as

\[
\begin{align*}
c_{11} &= c_{xx}, \\
c_{12} &= -\frac{1}{2}(s_z + t_z) \left[ \sin 2(\alpha_1 - \alpha_2) - \sin 2(\alpha_1 + \alpha_2) \right], \\
&\quad - (c_{xy} + c_{yx}) \left[ \cos^2(\alpha_1 - \alpha_2) + \cos^2(\alpha_1 + \alpha_2) - 1 \right], \\
c_{13} &= (s_y - c_{xz}) \sin 2\alpha_3 \sin 2\alpha_2 - (1 - 2 \cos^2 \alpha_3)(t_y \sin 2\alpha_2 + c_{xz} \cos 2\alpha_2), \\
c_{21} &= -\frac{1}{2}(s_z + t_z) \left[ \sin 2(\alpha_1 - \alpha_2) - \sin 2(\alpha_1 + \alpha_2) \right] \\
&\quad - (c_{xy} + c_{yx}) \left[ \cos^2(\alpha_1 - \alpha_2) - \cos^2(\alpha_1 + \alpha_2) \right], \\
c_{22} &= c_{yy} \\
c_{23} &= -\sin 2\alpha_3(s_x \cos 2\alpha_1 - c_{xy} \sin 2\alpha_1) - (1 - 2 \cos^2 \alpha_3)(t_x \sin 2\alpha_1 - c_{yz} \cos 2\alpha_1) \\
c_{31} &= -\sin 2\alpha_3(t_y \cos 2\alpha_2 - c_{xz} \sin 2\alpha_2) - (1 - 2 \cos^2 \alpha_3)(s_y \sin 2\alpha_2 + c_{xz} \cos 2\alpha_2) \\
c_{32} &= \sin 2\alpha_3(t_x \cos 2\alpha_1 - c_{yz} \sin 2\alpha_1) - (1 - 2 \cos^2 \alpha_3)(s_x \sin 2\alpha_1 - c_{xy} \cos 2\alpha_1) \\
c_{33} &= c_{zz}.
\end{align*}
\] (10)

3. The dynamics of the individual qubits

In this section, we investigate how the density matrices of the two qubits \( \rho_a \) and \( \rho_b \) evolve. Tracing out the second qubit (Bob’s qubit), we obtain the first qubit (Alice’s qubit), as

\[
\tilde{\rho}_a = \frac{1}{2}(1 + \tilde{s} \cdot \sigma^1),
\] (11)

where

\[
\begin{align*}
\tilde{s}_1 &= \cos 2t\alpha_3(s_x \cos 2t\alpha_1 + c_{xy} \sin 2\alpha_1) + \sin 2t\alpha_3(t_x \sin 2t\alpha_1 - c_{yz} \cos 2t\alpha_1), \\
\tilde{s}_2 &= \cos 2t\alpha_3(s_x \sin 2t\alpha_2 + c_{xy} \sin 2t\alpha_2) - \sin 2t\alpha_3(t_y \cos 2t\alpha_2 + c_{xz} \cos 2t\alpha_2), \\
\tilde{s}_3 &= \frac{1}{2}(s_z + t_z)(\cos 2t(\alpha_1 + \alpha_2) + \cos 2t(\alpha_1 - \alpha_2)) \\
&\quad - \frac{1}{2}(c_{xy} + c_{yx})(\sin 2t(\alpha_1 + \alpha_2) + \sin 2t(\alpha_1 - \alpha_2)).
\end{align*}
\] (12)

Similarly, if we trace out the first qubit we obtain the density matrix for the second qubit. In this context, we can write \( \rho_b \) in the following form,

\[
\tilde{\rho}_b = \frac{1}{2}(1 + \tilde{t} \cdot \tau^1),
\] (13)

where,

\[
\begin{align*}
\tilde{t}_1 &= \sin 2t\alpha_3(s_x \sin 2t\alpha_1 - c_{xy} \cos 2t\alpha_1) + \cos 2t\alpha_3(t_x \cos 2t\alpha_1 + c_{yz} \sin 2t\alpha_1), \\
\tilde{t}_2 &= -\sin 2t\alpha_3(s_y \sin 2t\alpha_2 + c_{xz} \cos 2t\alpha_2) - \cos 2t\alpha_3(t_y \cos 2t\alpha_2 + c_{xz} \sin 2t\alpha_2),
\end{align*}
\]
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\[ t_3 = \frac{1}{2}(s_z + t_z)[\cos 2t(\alpha_1 + \alpha_2) + \cos 2t(\alpha_1 - \alpha_2)] \]
\[ + \frac{1}{2}(c_{xy} + c_{yx})[\sin 2t(\alpha_1 + \alpha_2) - \sin 2t(\alpha_1 - \alpha_2)], \]

(14)

Now, we have all the details to study the evolution of any physical property of the total system which is represented by \( \rho_{ab} \) and its individual systems \( \rho_a \) and \( \rho_b \). For example to quantify the amount of entanglement contained in the entangled states, we shall use a measurement introduced by K. Zyczkowski [14, 15, 16]. This measure states that if the eigenvalues of the partial transpose are given by \( \lambda_j, j = 1, 2, 3, 4 \), then the degree of entanglement, DOE is defined by

\[ DOE = \sum_{i=j}^4 |\lambda_j| - 1 \]

(15)

Also, we can study the evolution of the purity of the individual subsystems which is given by,

\[ \eta_i = tr\rho_i^2, \]

(16)

where \( i = a, b \) (see for example [17, 18]). Further, the amount of information contained in a density operator \( \rho \) is defined by the Von Neumann entropy \( S = -\sum_j \lambda_i \ln \lambda_i \), where \( \lambda_i \) are the eigenvalues of the density operator [19]. If we start with an entangled initial state \( \rho_{ab} \), then we search for the survival amount of entanglement under the effect of the Hamiltonian. In what follows we shall consider two classes: The first for product states and the second for entangled pure state.

4. Product Class

The first class: In this section, we consider the evolution of the two qubits, where the initial two states are given by

\[ \rho_a = \frac{1}{2}(1 + s_x \sigma_x) \quad \text{and} \quad \rho_b = \frac{1}{2}I. \]

(17)

Under the unitary transformation (14), the product state \( \rho_a \otimes \rho_b \) of the two qubits (17) evolves to its final state with density operator,

\[ \rho_{ab} = \frac{1}{4}(1 + s_x \cos(2t\alpha_1) \cos(2t\alpha_3)\sigma_x + s_x \cos(2t\alpha_1) \sin(2t\alpha_3)\tau_x) \]
\[ - s_x \cos 2t\alpha_1 \sin 2t\alpha_3 \sigma_y \tau_z - s_x(1 - 2 \cos^2 t\alpha_3) \sin 2t\alpha_1 \sigma_z \tau_y) \]

(18)

In Fig. (1a), we plot the degree of entanglement as a function of the scaled time. In these calculations we assume that Alice and Bob perform the same local unitary operation. Also the unitary operator is defined where \( \alpha_1 = \alpha_2 = \alpha_3 = \frac{\pi}{6} \) and \( s_x = 1 \). It is clear that as soon as the interaction is switched on, an entangled state is generated. For any value of identical \( \alpha_i \), this state swap from the entangled into separable states. But as \( \alpha_i \) decreases, the robust of remaining in entangled or separable state is decreased.
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Figure 1. (a) shows the degree of entanglement, (b), shows the purity for the output states $\tilde{\rho}_a$ (solid-line) and $\tilde{\rho}_b$ (the dot-line). (c), shows the entropy for $\tilde{\rho}_a$ (solid-line) and $\tilde{\rho}_b$ (the dot-line). The parameters are, $\alpha_i = \frac{\pi}{6}$; $s_x = 1, s_y = s_z = 0$ and $t_x = t_y = t_z = 0$.

Since, we are interested in the behavior of the individual qubits through the evolution, we calculate the density operator for each qubit. Consequently,

$$\tilde{\rho}_1 = \frac{1}{2} (1 + \cos 2t\alpha_1 \cos 2t\alpha_3 \sigma_x), \quad \tilde{\rho}_2 = \frac{1}{2} (1 + \sin 2t\alpha_1 \sin 2t\alpha_3 \tau_x).$$  (19)

For these new states of the two qubits, we investigate how the purity and entropy evolve. In Fig.(1b), we consider the purity as a function of the scaled time, for the same parameters as in Fig.(1). From this figure we see how the purity is completely swapped from one pair to another. This phenomena is very important in quantum information context. It is possible to Alice to send all the information she has. We assume that Alice wants to distribute a quantum key to Bob for performing a quantum cryptography. She will code the secure key in her qubit by applying any coding protocol. Then she agrees with Bob on the unitary operators which they have to do. Fig.(1c), shows the behavior of the entropy for each qubit. It is clear that, the phenomena of swapping is satisfied for the entropy, where as the entropy of one qubit vanish the purity of the second qubit is maximum [20].

Second class: For this class, we assume that Alice and Bob both have the same qubit but on a different polarization as:

$$\rho_a = \frac{1}{2} (1 + \sigma_x), \quad \rho_b = \frac{1}{2} (1 - \tau_x).$$  (20)

Namely $s_x = 1, t_x = -1$ and the other components vanish. After the evolution, the joined density operator is defined by the Bloch vectors

$$\vec{s} = (\cos 2t\theta_1 \cos 2t\theta_3 - \sin 2t\theta_1 \sin 2t\theta_3, \quad 0, \quad 0),$$
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Figure 2. The same as Fig.(1), with \( t_x = -1 \) and \( t_y = t_z = 0 \).

\[
\vec{t} = (\cos 2t\theta_1(1 - 2\cos^2 t\theta_3), -\cos 2t\theta_2(1 - 2\cos^2 t\theta_3), 0),
\]

(21)
in addition to the dyadic \( \vec{C} \), which is defined by

\[
c_{xx} = c_{xy} = c_{xz} = 0,
\]
\[
c_{yx} = c_{yy} = 0, c_{yz} = \sin 2t\theta_1(1 - 2\cos^2 2t\theta_3) - \cos 2t\theta_1 \sin 2t\theta_3,
\]
\[
c_{zx} = 0, c_{zy} = -\sin 2t\theta_1(1 - 2\cos^2 2t\theta_3) - \cos 2t\theta_1 \sin 2t\theta_3, c_{zz} = 0,
\]

(22)

Also, the individual qubits are specified by their new Bloch vectors

\[
\vec{s} = (\cos 2t\theta_1 \cos 2t\theta_3, 0, 0),
\]
\[
\vec{t} = (-\cos 2t\theta_1 \cos 2t\theta_3 + \sin 2t\theta_1 \sin 2t\theta_3, 0, 0).
\]

(23)

In Fig.(2), we plot the degree of entanglement against the scaled time. From this figure, it is clear that the generated state is entangled most of the time except at specific points. At these points the two qubits are completely separated. This is clear from Fig.(2b), where we plot the purity of the two qubits. We see when the purities of the two qubits are completely swapped, the two qubits become separable states. This occurs at normalized \( t = 1.5, 2.5, 3.5 \) etc. On the other hand, when the purities of the two qubits coincide, the generated entangled state has a large degree of entanglement and it reaches its maximum value when the purities are maximum. This phenomena is seen once the interaction starts as well as at normalized \( t = 3n, n = 1, 2, 3, \ldots \). This feature is confirmed from Fig(2c), where the entropies of the two qubits are plotted. In some intervals neither Alice nor Bob know any information about the other. At these instance all the information has been transformed from one pair to each other. Alice can employ this technique to send information to Bob.
Third class: In this class, we assume that Alice has the same qubit as before, while Bob’s qubit is polarized along $x$ and $y$ axis. In explicit form

$$\rho_a = \frac{1}{2}(1 + s_x \sigma_x), \quad \rho_b = \frac{1}{2}(1 + t_x \tau_x + t_y \tau_y).$$

(24)

After the evolution the joined density operator $\rho_{ab}$ is defined by the following Bloch vectors

$$\vec{s} = (s_x \cos 2t \theta_1 \cos 2t \theta_3 + t_x \sin 2t \theta_1 \sin 2t \sin \theta_3, -t_y \sin 2t \theta_1 \sin \theta_3, 0),$$

$$\vec{t} = (-t_x \cos 2t \theta_1 (1 - 2 \cos^2 t \theta_3) + s_x \cos 2t \theta_1 \sin 2t \sin \theta_3,$$

$$-t_x \cos 2t \theta_2 (1 - 2 \cos^2 \theta_3), 0),$$

(25)

and the cross dyadic $\vec{C}$

$$c_{xx} = c_{xy} = 0, c_{xz} = -t_y \sin 2t \theta_2 (1 - 2 \cos^2 t \theta_3),$$

$$c_{yx} = c_{yy} = 0, c_{yz} = -t_x \sin 2t \theta_1 (1 - 2 \cos^2 t \theta_3) - s_x \cos 2t \theta_1 \sin 2t \theta_3,$$

$$c_{zx} = -t_y \cos 2t \theta_2 \sin 2t \theta_3,$$

$$c_{zy} = -s_x \sin 2t \theta_1 (1 - 2 \cos^2 t \theta_3) + t_x \sin 2t \theta_3 \cos 2t \theta_1, c_{zz} = 0.$$

(26)

On the other hand after the evolution, the two qubits evolve as

$$\tilde{\rho}_a = \frac{1}{2}(1 + s_x \cos 2t \theta_1 \cos 2t \theta_3 \sigma_x - t_y \sin 2t \theta_3 \sigma_y),$$

$$\tilde{\rho}_b = \frac{1}{2}(1 + (t_x \cos 2t \theta_1 \cos 2t \theta_3 + s_x \sin 2t \theta_1 \sin 2t \theta_3) \tau_x$$

$$-t_y \cos 2t \theta_2 \cos 2t \theta_3 \tau_y),$$

(27)

In Figs.3a, we have plotted the amount of entanglement contained in the generated state $\rho_{ab}$. We consider $s_x = 1$ while $t_x = t_y = 0.5$, and assume that Alice and Bob...
both perform the same unitary operator with \( \frac{\pi}{6} \). We can show that the amount of entanglement is larger than the entanglement which is depicted in the first class, but less than the second class. Also the purity of the initial state of Bob is larger than the first class and smaller than the second one. In Fig.3c, the behavior of the entropies of the individual qubit is shown. This figure depicts a partially swapping of the purity. This is clear where the purity of Alice’s qubit never reaches zero. It decreases with amount enough for Bob’s qubit to reach the maximum purity. The amount of information which is known by Alice and Bob about the qubit of each other is plotted in Fig.3c. It is clear that in this class, Bob is not able to know all the information contained in Alice’s qubit.

5. Entangled Class

We consider a class of pure entangled states defined by its joint density operator

\[
\rho_{ab} = \frac{1}{4} (1 + p \sigma_x - p \tau_x - c_{xx} - q(c_{yy} + c_{zz})) ,
\]

where \( q = \sqrt{1 - p^2} \). This class of entangled state is very important, where for \( p = 0 \), one get the singlet state and separable for \( p = 1 \). This class has been studied in [21], where we quantify the amount of exchange information between this state and environment. Under the effect of the unitary operator, this state is transformed such that,

\[
\bar{s} = (p \cos 2t(\theta_1 + \theta_3), 0, 0),
\]

\[
\bar{t} = (p \cos 2\theta_1(1 - 2 \cos^2 2t\theta_3) + p \cos 2t\theta_1 \sin 2t\theta_3, -p \cos 2t\theta_2(1 - 2 \cos^2 t\theta_3), 0),
\]

and

\[
c_{xx} = -1, c_{xy} = c_{xz} = 0,
\]

\[
c_{yx} = 0, \quad c_{yy} = -1, \quad c_{yz} = p \sin 2t\theta_1(1 - 2 \cos^2 t\theta_3) - p \cos 2t\theta_1 \sin 2t\theta_3,
\]

\[
c_{zx} = 0, \quad c_{zy} = -p \cos 2t(\theta_1 + \theta_3), \quad c_{zz} = -q .
\]

On the other hand, if we trace out one qubit, we can obtain the other, so one can obtain the density operators for Alice and Bob qubits as

\[
\tilde{\rho}_a = \frac{1}{2} (1 + p \cos 2t\theta_1 \cos 2t\theta_3 \sigma_x), \quad \tilde{\rho}_b = \frac{1}{2} (1 - p \cos 2t(\theta_1 - \theta_3) \tau_x).
\]

The behavior of the amount of entanglement contained in the entangled state, which is defined by (29) and (30), is shown in Fig(4a). We consider that the initial state has \( p = 0.7 \) as an initial state, where this parameter completely defines the state. First of all the state is always entangled and the degree of entanglement increases and decrease but never reach zero. In Fig.4b and 4c, we plot the behavior of the purities and entropies of the individual systems, \( \tilde{\rho}_a \) and \( \tilde{\rho}_b \), respectively. From these two figures, we can see that the purity of one qubit may be increased on the expense of the other qubit. On the other hand the entropy of each of them can not reach zero. This means that, the two qubits still always have some information about each other.
6. Conclusion

In this contribution, we study the interaction of a two separable qubits by means of the canonical unitary operator. An analytical expression of the joint density operator is obtained. We calculate explicitly the reduced density operator for each qubit. Different classes of initial density operators are considered. The amount of entanglement contained in the joint entangled state is quantified by using the negativity measurement. The swapping phenomena of purity and entropy is investigated for the reduced density operator. It is clear that the unitary operator parameters $\alpha_i, i = 1, 2, 3$ play the central role in the interaction process. If the state of one qubit is different from the other qubit or polarized in an opposite direction, then the information is completely transformed from one qubit to the other.

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