Non-factorization contributions in $D \to \pi K, KK$ decay

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Abstract

We have analyzed the $D \to \pi K, KK$ decay with the naive factorization (NF), QCD factorization (QCDF) and QCD factorization including soft-gluon exchanges (QCDF+SGE). In these decay channels, the soft-gluon effects are firstly calculated with light cone QCD sum rules. Comparing the three kind approaches, we can find the calculation results have made much more improved QCD factorization (QCDF) than the naive factorization (NF), and the calculation results have also made improved QCD factorization including soft-gluon exchanges (QCDF+SGE) than the QCD factorization (QCDF) in the color-suppressed decay channels. In addition, we find the soft-gluon effects are larger than the leading order contributions, and the calculation results are close to the experimental data for the color-suppressed decay channels. In color-allowed decay channel $D^0 \to \pi^+ K^-$, the soft-gluon effects are small and we should consider other power terms, such as final state interaction and annihilation effects.

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1. Introduction

The study of heavy meson decays is important for understanding the standard model (SM) and search for the sources of $CP$ violation. However, The hadronic two-body weak decays of $D$ meson involve nonperturbative strong interactions and spoil the simplicity of the short distance behavior of weak interactions. Therefore, a simplified approach in which the amplitudes of these processes are given by a factorizable short distance current-current effective Hamiltonian is not expected to work well. Various approaches were employed to include long distance effects. The most commonly and very frequently used prescription, motivated by $\frac{1}{N_c}$ arguments [1], is to apply generalized factorization [2-3]. This phenomenological treatment works reasonably well in color-allowed $D$ decays [3], but it is failing in the color-suppressed $D \to \pi\pi, \pi K$ and $D \to K\bar{K}$ decays [4].

It is necessary that we study $D$ meson nonleptonic decays beyond the factorization approach. A few years ago, M. Beneke et al.[5] gave a NLO calculation the hadronic matrix element of $B \to \pi\pi, \pi K$ in the heavy quark limit. They pointed out that in the heavy quark limit the radiative corrections at the order of $\alpha_s$ can be calculated with perturbative QCD method. In $D \to \pi K$ decay, the momentum transition square is $q^2 = 1.7 GeV^2$, and the radiative corrections of the hard-gluon exchanges can also be calculated with perturbative QCD approach. So, the hadronic matrix elements for $D \to \pi K$ can be expanded by the powers of $\alpha_s$ and $\frac{\Lambda_{QCD}}{m_c}$ as follows:

$$\langle K\pi|O_i|D\rangle = \langle K|j_1|D\rangle\langle \pi|j_2|0\rangle[1 + \sum r_n\alpha_s^n + O(\frac{\Lambda_{QCD}}{m_c})],$$

where $O_i$ are some local four-quark operators in the weak effective Hamiltonian and $j_{1,2}$ are bilinear quark currents. In Eq. (1), the power correct term $O(\frac{\Lambda_{QCD}}{m_c})$ includes soft-gluon effects, final state interaction, which cannot be calculated in QCD factorization and perturbative QCD method. For the $B$ meson two-body decay, the term is small, but it is large and cannot be neglected in the $D \to \pi K$ decay. A few years ago, A. Khodjamirian [6] has presented a new method to calculate the hadronic matrix elements of nonleptonic $B$ meson decays within the framework of the light cone QCD sum rules, where the nonfactorizable soft-gluon contributions can effectively be dealt with. Obviously, this approach can be applied to $D \to \pi K$ decay.

The QCD factorization method can be applied to $D \to \pi\pi, \pi K$ and $K\rho$ decay, but we should calculate the contribution of power term $O(\frac{\Lambda_{QCD}}{m_c})$. The power term includes the
contributions of soft-gluon effect, final state interaction and annihilation effects, since the power term in $D \to \pi\pi, \pi K, K\rho$ decay is larger than $B \to \pi\pi, \pi K, K\rho$ decay. We firstly considered $D \to \pi\pi$ decay in QCD factorization and light cone QCD sum rules method [7]. We found either the hard-gluon effect ($O(\alpha_s)$ correction) or the soft-gluon effect is small, and only found the calculation result of $D^0 \to \pi^+\pi^-$ decay channel approaches the experiment data. It indicated that we should consider the contributions of final state interaction and annihilation effects in $D \to \pi\pi$ decay. In this paper, we apply the QCD factorization including light cone QCD sum rules method to study $D \to \pi K, KK$ decay and obtain new results. In $D^0 \to \pi^0\bar{K}^0$ decay, we find both hard gluon and soft gluon contributions exceed the leading order largely, and the calculation result is accordance with the experiment data. In other decay channels, we should calculate all power terms, which include soft-gluon exchanges, final state interaction and annihilation effects, and then we can compare the calculation results with the experiment data. However, the final state interaction and annihilation effects haven’t reliable method to calculate up to now.

In our work, we calculate the leading order and $\alpha_s$ corrections in QCD factorization, and the soft-gluon effects in the light cone QCD sum rules for the $D \to \pi K, KK$ decay. In color-suppressed decay channels, we find the soft-gluon contributions are larger than the leading order contributions, and the calculation results are close to the experimental data for these decay channels. In color-allowed decay channels, the soft-gluon contributions are small and we should consider other power terms, such as the final state interaction and annihilation effects.

2. $D \to \pi K, KK$ in QCD Factorization

The low energy effective Hamiltonian for $D^0 \to \pi^0\bar{K}^0$ can be expressed as follows:

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} [(C_1(\mu)O_1(\mu) + C_2(\mu)O_2(\mu)],$$

where $C_i(\mu)$ are Wilson coefficients which have been evaluated to next-to-leading order. The four-quark operators $O_{1,2}$

$$O_1 = (\bar{u}d)V_{-A}(\bar{s}c)V_{-A},$$

$$O_2 = (\bar{u}_\alpha d_\beta)V_{-A}(\bar{s}_\beta c_\alpha)V_{-A},$$

(3)
the Wilson coefficients evaluated at $\mu = m_c$ scale are \[8\]
\[C_1 = 1.274, C_2 = -0.529. \quad (4)\]

In the following, we study $D \to \pi K, KK$ decay with the QCD factorization approach. This method is similar to that for $B \to \pi\pi, \pi K$ decay; see Ref. [5] for detail. As in Ref. [5], we obtain the QCD coefficients $a_i$ at next-to-leading order (NLO) and $O(\alpha_s)$ hard scattering corrections in naive dimension regularization (NDR) scheme. The coefficient $a_i(\pi K)(i = 1, 2)$ are split into two terms: $a_i(\pi K) = a_{i,I}(\pi K) + a_{i,II}(\pi K)$. They are given in Refs. [5, 14]. In $D \to \pi K, KK$ decay, the flavor structure is different from $B$ decays. When we replace the index $K$ and $\pi$ in the $B$ decays’ coefficients $a_i(\pi K)(i = 1, 2)$, we can obtain the coefficients $a_i(\pi K)(i = 1, 2)$ in $D$ decays. They are

\[a_{1,I} = C_1 + \frac{C_2}{N_c}(1 + \frac{C_F\alpha_s}{4\pi}V_\pi), a_{1,II} = \frac{C_2}{N_c}C_F\pi\alpha_sH_{\pi K}, \quad (5)\]
\[a_{2,I} = C_2 + \frac{C_1}{N_c}(1 + \frac{C_F\alpha_s}{4\pi}V_K), a_{2,II} = \frac{C_1}{N_c}C_F\pi\alpha_sH_{K\pi}. \quad (6)\]

Here $N_c = 3(f = 4)$ is the number of colors (flavors), and $C_F = \frac{N_c^2 - 1}{2N_c}$ is the factor of color. The functions in Eqs. (5) and (6) can be found in Ref. [5], which are

\[V_K = 12\ln \frac{m_c}{\mu} - 18 + \int_0^1 g(x)\phi_K(x)dx, \]
\[V_\pi = 12\ln \frac{m_c}{\mu} - 18 + \int_0^1 g(x)\phi_\pi(x)dx, \]
\[g(x) = 3\left(1 - \frac{2x}{1/x} \ln x - i\pi\right) + [2Li_2(x) - (\ln x)^2 + \frac{2\ln x}{1-x} - (3 + 2i\pi)\ln x - (x \leftrightarrow 1 - x)], \]
\[H_{\pi K} = \frac{f_D f_K}{m_D^2 F_D^{\to K}(0)} \int_0^1 \frac{d\phi_D(x)}{x} \int_0^1 \frac{d\phi_\pi(x)}{x} \int_0^1 \frac{dy}{y} \left[\phi_K (y) + \frac{2\mu_K \bar{x}}{m_c x}\right], \]
\[H_{K\pi} = \frac{f_D f_\pi}{m_D^2 F_D^{\to \pi}(0)} \int_0^1 \frac{d\phi_D(x)}{x} \int_0^1 \frac{d\phi_K(x)}{x} \int_0^1 \frac{dy}{y} \left[\phi_\pi (y) + \frac{2\mu_\pi \bar{x}}{m_c x}\right], \quad (7)\]

where $Li_2(x)$ is the dilogarithm, $f_K(f_D)$ is the $K(D$ meson) decay constant, $m_D$ is the $D$ meson mass, $F_D^{\to K}(0)$ ($F_D^{\to \pi}(0)$) is the $D \to K(D \to \pi)$ form factor at zero momentum transfer, and $\xi$ is the light-cone momentum fraction of the spectator in the $D$ meson. $H_{\pi K}$ and $H_{K\pi}$ depend on the wave function $\phi_D$ through the integral $\int_0^1 d\xi \phi_D(x)/\xi \equiv m_D/\lambda_D = 6.23$, with $\lambda_D = (250 \pm 75) MeV$, $\mu_K = m_K^2/(m_d + m_s)$, $\mu_\pi = m_\pi^2/(m_u + m_d)$, $m_u = 3 MeV$, $m_d = 6 MeV$, $m_s = (150 \pm 20) MeV$, $m_c = 1.3 GeV$, $m_\pi = 0.139 GeV,$
\( m_K = 0.494 \text{GeV} \). We take \( f_\pi = (132 \pm 0.26) \text{MeV} \), \( f_K = (170 \pm 1.5) \text{MeV} \), \( f_D = (200 \pm 20) \text{MeV} \), \( f_{D_s} = (280 \pm 18) \text{MeV} \), \( F^{D \to \pi}(0) = (0.65 \pm 0.10) \), \( F^{D \to K}(0) = (0.73 \pm 0.07) \) \cite{8}, \( F^{D_s \to K}(0) = (0.82 \pm 0.15) \) \cite{15}, \( \alpha_s(m_c) = 0.353 \), \( m_D = 1.869 \text{GeV} \), \( m_{D_s} = 1.968 \text{GeV} \), and the asymptotic wave functions \( \phi_K = \phi_\pi = 6x(1 - x) \).
3. \( D \to \pi K, KK \) in the light-cone QCD sum rules

In the following, we calculate the soft-gluon contributions for \( D \to \pi K, KK \) decay. Firstly, we calculate the soft -gluon effects of \( D^0 \to \pi^0 K^0 \) channel, and the calculation of other decay channels are similar as the channel. To estimate the soft-gluon corrections for \( D^0 \to \pi^0 K^0 \) channel, it is useful to rewrite down the effective Hamiltonian with the help of the Fierz transformation. For example, applying the Fierz transformation to the operator \( O_2 = (\bar{u}\Gamma_\mu d)(\bar{s}\Gamma_\mu c) \), we have the effective Hamiltonian relevant to the tree operators, 

\[
\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} e^{-i(p-k)x} \int d^4xe^{-i(p-q)x} \int d^4ye^{-i(p-k)x} \mathcal{T} \{ j_{a5}^{(K)}(y) \bar{O}_1(y) j_D^D(x) \} |\pi^0(q)>,
\]

where

\[
\bar{O}_1 = (\bar{u}\Gamma_\mu \frac{\lambda^a}{2}d)(\bar{s}\Gamma_\mu \frac{\lambda^a}{2}c),
\]

In the above \( \Gamma_\mu = \gamma_\mu (1 - \gamma_5), \quad Tr(\lambda^a\lambda^b) = 2\delta^{ab}, \) and

\[
O_2 = \frac{1}{3} O_1 + 2\bar{O}_1.
\]

First of all, we calculate the nonfactorizable matrix elements induced by the operator \( \bar{O}_1 \). As a starting object for the derivation of LCSR we choose the following vacuum-pion correlation function:

\[
F_\alpha^{(\bar{O}_1)}(p, q, k) = -\int d^4 x e^{-i(p-q)x} \int d^4 y e^{-i(p-k)x} \mathcal{T} \{ j_{a5}^{(K)}(y) \bar{O}_1(y) j_D^D(x) \} |\pi^0(q)>,
\]

where \( j_{a5}^{(K)} = \bar{s}\gamma_\alpha \gamma_5 d \) and \( j_5^{(D)} = m_c \bar{c} \gamma_5 u \) are the quark currents interpolating \( K^0 \) and \( D \) meson, respectively. The decomposition of the correlation function (11) in independent momenta is straightforward and contains four invariant amplitudes:

\[
F_\alpha^{(\bar{O}_1)} = (p-k)_\alpha F^{(\bar{O}_1)} + q_\alpha F_1^{(\bar{O}_1)} + k_\alpha F_2^{(\bar{O}_1)} + \epsilon_{\alpha\beta\lambda\rho} q^\beta p^\lambda k^\rho F_3^{(\bar{O}_1)}.
\]

In what follows only the amplitude \( F^{(\bar{O}_1)} \) is relevant. The correlation function is calculated in QCD by expanding the T-product of three operators, two currents and \( \bar{O}_1 \), near the light-cone \( x^2 \sim y^2 \sim (x-y)^2 \sim 0 \). For this expansion to be valid, the kinematical region should be chosen as:

\[
q^2 = p^2 = k^2 = 0, \quad |(p-k)^2| \sim |(p-q)^2| \sim |P^2| \gg \Lambda_{QCD}^2,
\]
where $P \equiv p - k - q$. The correlation function (11) can be calculated employing the light-cone expansion of the quark propagator [6]:

$$S(x, 0) = -i(0)\langle T q(x)\bar{q}(0)|0\rangle = \Gamma(d/2)x^2/2\pi^2(-x^2)^{d/2} + \frac{\Gamma(d/2 - 1)}{16\pi^2(-x^2)^{d/2-1}} \int_0^1 dv ((1 - v)x^2 \sigma_{\mu\nu} G_{\mu\nu}^{\alpha}(vx) + v x^2 \sigma_{\mu\nu} G_{\mu\nu}^{\alpha}(vx))^2, \quad (14)$$

where $G_{\mu\nu} = g_s F^{a}_{\mu\nu}(\lambda^a/2)$, which is the gluonic field strength and the soft gluon effects are from the term, $d$ is the space-time dimension. Following the standard procedure for QCD sum rule calculation, we can obtain the hadronic matrix element of the operator $\tilde{O}_1$

$$A^{\tilde{O}_1}(D^0 \to \pi^0 K^0) = \langle K^0(\pi^0(p)|\tilde{O}_1|D^0(p - q)) \rangle = \frac{\pi^2 f_K f_D m_D^2}{s^K_0} \int_{m^2}^{s^K_0} d\epsilon \frac{\epsilon^{m^2 - s}}{\epsilon^{m^2}} \int_{m^2}^{s^K_0} d\epsilon' \frac{\epsilon'^{m^2 - s'}}{\epsilon'^{m^2}} \int_{s^K}^{s^D} ds \int_{s^K_0}^{s^D} ds' e^{-\frac{s^K_0 - s^D}{s^K}} Im_{s_0} Im_{s_0} F_QCD(s, s', m^2_D), \quad (15)$$

where $s^K_0$ and $s^D_0$ are effective threshold parameters for $K$ and $D$ meson.

A straightforward calculation shows that only the twist-3 wave function $\varphi_{3\pi}(\alpha_i)$ and the twist-4 ones $\varphi_{\parallel}(\alpha_i), \varphi_{\perp}(\alpha_i)$, whose definitions can be found in Ref. [6], contribute to the invariant function $F^{\tilde{O}_1}$. The results are:

$$F^{\tilde{O}_1}_QCD = F^{\tilde{O}_1}_{tw3} + F^{\tilde{O}_1}_{tw4}, \quad (16)$$

with

$$F^{\tilde{O}_1}_{tw3} = \frac{m_c f_{3\pi}}{4\pi^2} \int_0^1 dv \int D\alpha_i \varphi_{3\pi}(\alpha_i) \times \frac{(m^2_c - (p - q)^2(1 - \alpha_1))(p^2 - (p - k)^2(1 - \alpha_3))}{(2 - v)(q \cdot k) + 2(1 - v)(q \cdot (p - k))(q \cdot (p - k)). \quad (17)}$$

We get the same calculation results as Ref. [6] for Eq. (17) when it be substituted for $m_c \to m_b$, but the twist-4 contribution hasn’t been showed in Ref. [6]. Now, we give the invariant amplitude from the twist-4 term.

$$F^{\tilde{O}_1}_{tw4} = \frac{m^2_c f_{3\pi}}{4\pi^2} \int_0^1 dv \int D\alpha_i \varphi_{\perp}(\alpha_i) \frac{1}{m^2_c - (p - q + q\alpha_1)^2} \frac{(4v - 6)(p - k)q}{(p - k - q\alpha_3)^2} \quad (17)$$
In Eq. (17) and (18), we make use of the following nonlocal operator matrix elements:

\[
\begin{align*}
\langle 0 | \bar{u}(0) \sigma_{\mu\nu} \gamma_5 G_{\alpha\beta}(vy) u(x) | \pi^0(q) \rangle & = \left( \frac{m^2}{2\pi^2} \right) \phi_{\parallel}(\alpha_i, \mu) e^{-i(q(x_{\alpha_i} + y_{\alpha_3}))} \\
\langle 0 | \bar{u}(0) \gamma_\mu \bar{G}_{\alpha\beta}(vy) u(x) | \pi^0(q) \rangle & = \frac{q_\mu q_{\alpha_3}}{\sqrt{2}} \phi_{\perp}(\alpha_i, \mu) e^{-i(q(x_{\alpha_i} + y_{\alpha_3}))} + \frac{q_\mu q_{\alpha_3}}{\sqrt{2}} \phi_{\parallel}(\alpha_i, \mu) e^{-i(q(x_{\alpha_i} + y_{\alpha_3}))},
\end{align*}
\]

with \( D_\alpha = d\alpha_1 d\alpha_2 d\alpha_3 \delta(1 - \alpha_1 - \alpha_2 - \alpha_3) \). Finally, the LCSR for the \( D^0 \to \pi^0 K^0 \) matrix element of the operator \( \tilde{O}_1 \) from the soft-gluon exchange is obtained by applying to the duality approximation and Borel transformation in the \( D \) channel. The result can be written as:

\[
A_1 = A(\tilde{O}_1)(D^0 \to \pi^0 K^0) = \left( \frac{m^2}{4\pi^2 f_K} \right) \phi_{\parallel}(\alpha_i, \mu) e^{-i(q(x_{\alpha_i} + y_{\alpha_3}))} + \frac{q_\mu q_{\alpha_3}}{\sqrt{2}} \phi_{\perp}(\alpha_i, \mu) e^{-i(q(x_{\alpha_i} + y_{\alpha_3}))}.
\]

where \( u_0^D = m^2 c^2/s^D_0 \), and the following definitions are introduced:

\[
\begin{align*}
\frac{\partial \Phi_1(w, v)}{\partial w} & = \bar{\varphi}_{\parallel}(w, 1 - w - v, v) + \bar{\varphi}_{\perp}(w, 1 - w - v, v), \\
\frac{\partial \Phi_2(v)}{\partial v} & = \Phi_1(1 - v, v).
\end{align*}
\]
The asymptotic forms of the pion distribution amplitudes in Eqs. (21)-(22) are given by [6]:

\[ \varphi_{3\pi}(\alpha_i) = \varphi_{3K}(\alpha_i) = 360\alpha_1\alpha_2\alpha_3^2, \]

\[ \tilde{\varphi}_{\perp}(\alpha_i) = 10\delta^2\alpha_3^2(1 - \alpha_3), \]

\[ \tilde{\varphi}_{\parallel}(\alpha_i) = -40\delta^2\alpha_1\alpha_2\alpha_3. \] (23)

We find our calculation result (Eq. (21)) has a little difference with Eq. (30) in Ref. [6]. In Eq. (21), the final term includes function \((\frac{m_2^2}{u M^2} - 2)\tilde{\varphi}_2(u)\), and this function is corresponding to \((\frac{m_2^2}{u M^2} - \frac{s}{M^2} - 1)\tilde{\varphi}_2(u)\) in Ref. [6], which includes variation \(s\). We think the function including variation \(s\) should appear in the first integral \(\int_0^1 dse^{-\frac{s}{M^2}}\), and it should not be in the second integral \(\int_{u_0}^1 du\), so that the matrix element \(A\) is independent of variation \(s\). Our calculation result \(A_1\) (Eq. (21)) is a constant. In Ref. [6], the matrix element \(A\) (Eq. (30)) is a function of variation \(s\). It should print error in Ref. [6].

We write the decay amplitudes of \(D^0 \to \pi^0 K^0\), which include factorization and non-factorization parts:

\[ M_{f+a_{s}}(D^0 \to \pi^0 K^0) = i\frac{G_F}{\sqrt{2}} V_{cs} V_{ud} f_K F_{D^0 \to \pi^0}(0)(m_D^2 - m_\pi^2)a_2, \] (24)

\[ M_{a_{f}}(D^0 \to \pi^0 K^0) = \sqrt{2}G_F V_{cs} V_{ud} C_1 A_1. \] (25)

The matrix elements of other decay channels, can be calculated as similar as \(D^0 \to \pi^0 K^0\) channel, and we can write down them directly as follows:

\[ A_2 = A(\bar{O}_1)(D^0 \to \pi^+ K^-) \]

\[ = im_2^D(\frac{1}{4\pi f_\pi \int_0^{s_0} ds e^{-\frac{s}{M^2}}}(\frac{m_c^2}{2f_D m^4_D} \int_{u_0}^1 du \frac{m_D^2}{u} e^{-\frac{m_D^2}{u M^2}})
\times[m_c \frac{f_{3K}}{u} \int_0^u \frac{dv}{v} \varphi_{3K}(1 - u, v) + f_K \int_0^u \frac{dv}{v} [3\tilde{\varphi}_{\perp}(1 - u, v)]
- \frac{m_2^2}{u M^2} - 1) \frac{\Phi_1(1 - u, v)}{u}]
+ f_K(\frac{m_2^2}{u M^2} - 2) \frac{\Phi_2(u)}{u^2}). \] (26)

\[ A_3 = A(\bar{O}_2)(D^+_s \to K^+ K^0) \]

\[ = im_2^D(\frac{1}{4\pi f_K} \int_0^{s_0} ds e^{-\frac{s}{M^2}}}(\frac{m_c^2}{2f_D m^4_D} \int_{u_0}^1 du \frac{m_D^2}{u} e^{-\frac{m_D^2}{u M^2}})
\times[m_c \frac{f_{3K}}{u} \int_0^u \frac{dv}{v} \varphi_{3K}(1 - u, v) + f_K \int_0^u \frac{dv}{v} [3\tilde{\varphi}_{\perp}(1 - u, v)]
- \frac{m_2^2}{u M^2} - 1) \frac{\Phi_1(1 - u, v)}{u}]
+ f_K(\frac{m_2^2}{u M^2} - 2) \frac{\Phi_2(u)}{u^2}). \] (27)
and

\[ A_4 = A(\tilde{O}_2)(D_s^+ \to \pi K) \]

\[ = i m_D^2 \left( \frac{1}{4\pi^2 f_\pi} \int_0^{s_0^u} ds \frac{m_c^2}{m_D^4} \int_0^1 \frac{du}{u} e^{\frac{m_D^2}{m_f^2} - \frac{m_c^2}{uM^2}} \right) \]

\[ \times \left[ \frac{m_c f_{3K}}{u} \int_0^u \frac{dv}{v} \varphi_{3K} (1 - u, u - v, v) + f_K \int_0^u \frac{dv}{v} \right] \]

\[ - (\frac{m_c^2}{uM^2} - 1) \Phi_1 (1 - u, v) + f_K (\frac{m_c^2}{uM^2} - 2) \Phi_2 (u) \],

(28)

where \( \tilde{O}_2 = (\bar{u} \Gamma_\mu \frac{\lambda^a}{2} c)(\bar{s} \Gamma_\mu \frac{\lambda^a}{2} d) \) and \( s_0^u \) is effective threshold parameters for \( \pi \) meson.

In the following, we give the decay amplitudes of other channels, which include factorization and non-factorization parts:

\[ M_{f+\alpha_s}(D^0 \to \pi^+ K^-) = i \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} f_\pi F^{D \to K} (0) (m_D^2 - m_K^2) a_1, \]  

(29)

\[ M_{nfg}(D^0 \to \pi^+ K^-) = \sqrt{2} G_F V_{cs}^* V_{ud} C_2 A_2. \]  

(30)

\[ M_{f+\alpha_s}(D^+ \to \pi^+ \bar{K}^0) = i \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} [f_\pi F^{D \to \bar{K}} (0) (m_D^2 - m_\pi^2) a_1 \]

\[ + f_K F^{D \to \pi} (0) (m_D^2 - m_\pi^2) a_2], \]  

(31)

\[ M_{nfg}(D^+ \to \pi^+ \bar{K}^0) = \sqrt{2} G_F V_{cs}^* V_{ud} (C_1 A_1 + C_2 A_2). \]  

(32)

\[ M_{f+\alpha_s}(D_s^+ \to K^+ \bar{K}^0) = i \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} f_K F^{D_s \to K} (0) (m_{D_s}^2 - m_K^2) a_2, \]  

(33)

\[ M_{nfg}(D_s^+ \to K^+ \bar{K}^0) = - \sqrt{2} G_F V_{cs}^* V_{ud} C_1 A_3. \]  

(34)

\[ M_{f+\alpha_s}(D_s^+ \to \pi^+ K^0) = i \frac{G_F}{\sqrt{2}} V_{cd}^* V_{ud} f_\pi F^{D_s \to K} (0) (m_{D_s}^2 - m_\pi^2) a_1, \]  

(35)

\[ M_{nfg}(D_s^+ \to \pi^+ K^0) = \sqrt{2} G_F V_{cd}^* V_{ud} C_2 A_4. \]  

(36)

where the amplitude \( M_{f+\alpha_s} = M_f + M_{\alpha_s} \) represents the sum of the leading order factorization \( M_f \) and non-factorization \( M_{\alpha_s} \) from the hard-gluon exchanges, the amplitude \( M_{nfg} \) is the non-factorization parts from soft-gluon exchanges. The total amplitude \( M \) is the sum of \( M_{f+\alpha_s} \) and \( M_{nfg} \).
4. Numerical calculation

In the numerical calculations we take \( s_0^\pi = 0.7 \text{GeV}^2 \)[6], \( s_0^K = 1.2 \text{GeV}^2 \)[9] and \( s_0^D = (6 \pm 1) \text{GeV}^2 \)[10]. In \( \mu_c = \sqrt{m_D^2 - m_c^2} \approx 1.3 \text{GeV} \), \( f_{3\pi}(\mu_c) = 0.0035 \text{GeV}^2 \), \( \delta^2(\mu_c) = 0.19 \text{GeV}^2 \) are nonperturbative parameters in light-cone wave functions [10], and the CKM matrix are \( V_{ud} = V_{cs} = 0.9734 \div 0.9749 \) and \( V_{cd} = 0.227 \)[11]. Having fixed the input parameters, one must find the range of the values \( M^2 \) and \( M^2 \) for which the sum rules (Eq. (21)) is reliable. At the interval \( M^2 = 8 - 12 \text{GeV}^2 \) and \( M^2 = 6 - 15 \text{GeV}^2 \), we find the value of \( A_1 \) (Eq. (21)) is quite stable. The \( D \) meson life time \( \tau(D^0) = (4.12 \pm 0.027) \times 10^{-13} \text{s} \), \( \tau(D^+) = (1.05 \pm 0.013) \times 10^{-12} \text{s} \), \( \tau(D_s^+) = (4.9 \pm 0.09) \times 10^{-13} \text{s} \). In the \( D \) rest frame, the two body decay width is

\[
\Gamma(D \to P_1P_2) = \frac{1}{8\pi} |M(D \to P_1P_2)|^2 \left| \frac{P}{m_D^2} \right|, \tag{37}
\]

where \( P_1 \) and \( P_2 \) are two pseudoscalar meson (\( \pi \) and \( K \)), and the momentum of the \( P_1 \) meson is given by

\[
|P| = \frac{[(m_D^2 - (m_{P_1} + m_{P_2})^2)(m_D^2 - (m_{P_1} - m_{P_2})^2)]^{\frac{1}{2}}}{2m_D}, \tag{38}
\]

The corresponding branching ratio is given by

\[
Br(D \to P_1P_2) = \frac{\Gamma(D \to P_1P_2)}{\Gamma_{total}}. \tag{39}
\]

where \( \Gamma_{total} \) denotes the total decay width of \( D \) meson. The total decay width of one meson is related to its mean life time \( \tau \) by \( \Gamma_{total} = \hbar/\tau \). With the above parameters and formulae, we can get the branching ratios of \( D \to \pi K, KK \) decay obtained in some approaches with that of the experiment.

Table 1: The branching ratios of \( D \to \pi K, KK \) decay obtained in some approaches together with experimental result.

| Decay channel | NF | QCDF | QCDF+SGE | Experiment |
|---------------|----|------|----------|------------|
| \( D^0 \to \pi^0 K^0 \) | \( 2.4 \times 10^{-3} \) | \( (3.66 \pm 0.55) \times 10^{-2} \) | \( (2.20 \pm 0.11) \times 10^{-2} \) | \( (2.28 \pm 0.22) \times 10^{-2} \) |
| \( D^0 \to \pi^+ K^- \) | \( 5.63 \times 10^{-2} \) | \( (7.18 \pm 1.01) \times 10^{-2} \) | \( (6.15 \pm 0.40) \times 10^{-2} \) | \( (3.80 \pm 0.09) \times 10^{-2} \) |
| \( D^+ \to \pi^+ K^0 \) | \( 6.55 \times 10^{-2} \) | \( (1.66 \pm 0.23) \times 10^{-2} \) | \( (2.70 \pm 0.17) \times 10^{-2} \) | \( (2.77 \pm 0.18) \times 10^{-2} \) |
| \( D_s^+ \to K^+ K^0 \) | \( 2.15 \times 10^{-3} \) | \( (4.27 \pm 0.68) \times 10^{-2} \) | \( (3.86 \pm 0.26) \times 10^{-2} \) | \( (3.6 \pm 1.1) \times 10^{-2} \) |
| \( D_s^+ \to \pi^+ K^0 \) | \( 3.49 \times 10^{-3} \) | \( (5.64 \pm 0.97) \times 10^{-3} \) | \( (5.07 \pm 0.35) \times 10^{-3} \) | \( < 8 \times 10^{-3} \) |
The branching ratios of $D \rightarrow \pi K, KK$ decay channels are presented in Table 1, where the second column is the result of naive factorization (NF) and the total amplitudes $M_{f+\alpha}$ in Eqs. (24), (29), (31), (33) and (35) corresponding to different decay channels of $D \rightarrow \pi K, KK$, $a_1$ and $a_2$ are calculated in the leading order, i.e. the parameters $a_1 = C_1 + \frac{C_2}{3}$ and $a_2 = C_2 + \frac{C_1}{3}$, the third column is the result of QCD factorization (QCDF), the amplitudes are also calculated by $M_{f+\alpha}$ in Eqs. (24), (29), (31), (33) and (35) but $a_1$ and $a_2$ are calculated by QCD factorization approach from Eqs. (5)-(7), which include the leading order and $O(\alpha_s)$ corrections, the fourth column is the result of QCD factorization including soft-gluon exchanges (QCDF+SGE) which are our results, the total amplitude is the sum of $M_{f+\alpha}$ and $M_{ngf}$ in Eqs. (24)-(25) and (29)-(36), and the final column is the experimental data [12]. From Table 1, we can find that the prediction of naive factorization is far from the experimental data and the QCD factorization method have improved the calculation results. In our approach (QCD factorization including soft-gluon effects), the calculation results are close to the experimental data in $D^0 \rightarrow \pi^0 K^0$, $D^+ \rightarrow \pi^+ K^0$ and $D_s^+ \rightarrow K^+ \overline{K}^0$ decay channels. We find the soft-gluon corrections are rather large, which are larger than the leading order contributions in $D^0 \rightarrow \pi^0 K^0$ and $D_s^+ \rightarrow K^+ \overline{K}^0$ decay channels. For $D^0 \rightarrow \pi^+ K^-$ decay, the results from the three approaches do not agree with the experimental data, and we think the reason is that we only calculate the soft-gluon contributions and do not consider the final state interaction and the annihilation effects in the power term $O(\Lambda_{QCD}/m_c)$. The theoretical uncertainties in the table 1 are estimated by the some parameters. We can give the uncertainties from QCD factorization and soft-gluon effects, respectively. In QCD factorization, the important uncertainties are from the input parameters: form factors $F_{D \rightarrow K}$, $F_{D_s \rightarrow K}$, $F_{D \rightarrow \pi}$, decay constants $f_D$, $f_{D_s}$, $f_K$, $f_\pi$ and the parameters $a_1$ and $a_2$. From Eqs. (5)-(7), we can find the uncertainties of $a_1$ and $a_2$ are from: (1) The Wilson coefficients: The coefficients $C_1$ and $C_2$ are uncertain, and they are in the ranges of: $1.216 \sim 1.274$ and $-0.415 \sim -0.53$ respectively. (2) The vertex corrections $V_M(M = \pi, K)$: we need to input the light-cone distribution functions $\phi_M(M = \pi, K)$. We take the asymptotic form of pion and kaon light-cone distribution functions $\phi_\pi(x) = \phi_K(x) = 6x(1-x)$, but the accurate form should be Gegenbauer polynomials. (3) The hard-scattering terms $H_{K\pi}$ and $H_{\pi K}$: we need to calculate the moment $\int_0^1 \frac{d\xi}{\xi} \phi_D(\xi) = \frac{m_D}{\lambda_D}$, the value of $\lambda_D$ at present is uncertain, a typical range being $\lambda_D = (250 \pm 75) MeV$. The uncertainties from
QCD factorization less than 18%. In soft-gluon effects, the uncertainties are from the input parameters: the decay constants $f_D$, $f_{D_s}$, $f_K$, $f_\pi$, the Wilson coefficients $C_1$ and $C_2$, and the Borel parameters $M'^2$ and $M^2$. The uncertainties from soft-gluon effects less than 7%, and the total uncertainties less than 25%. In $D \to \pi K, KK$, we find the QCD factorization (QCDF) and QCD factorization including soft-gluon exchanges (QCDF+SGE) have improvement on the calculation results. So the QCD factorization method can be applied to $D \to \pi K, KK$ decay, but the power term $O(\Lambda_{QCD}/m_c)$ should be included. In order to get accurate calculation results in $D \to \pi K, KK$ decay, the soft-gluon, the final state interaction and the annihilation effects should be calculated, and the calculation results can be improved. Now, the final state interaction and the annihilation effects have many models, but there is not a reliable method. So, it is necessary to study the $D \to \pi K, KK$ decay further.

5. Summary

In $D \to \pi K, KK$ decay, We find the prediction of naive factorization (NF) is far from the experimental data, the QCD factorization (QCDF) and QCD factorization including soft-gluon exchanges (QCDF+SGE) have improvement on the calculation results. In color-suppressed decay channels, such as $D^0 \to \pi^0\overline{K}^0$ and $D^+_s \to K^+\overline{K}^0$ decay, the soft-gluon corrections are rather large, which are larger than the leading order contributions. In color-allowed decay channel $D^+ \to \pi^+\overline{K}^0$, the soft-gluon corrections are rather large also, but the soft-gluon corrections are small in the color-allowed decay channel $D^0 \to \pi^+ K^-$. From Refs.[6, 13], we can find the soft-gluon contributions are different in different $B \to \pi\pi$ decay channels. For example, in $B^0 \to \pi^+\pi^-$ decay, the soft-gluon effects are smaller than the hard-gluon contributions because of the small Wilson coefficient $C_2$ in the decay amplitude. However, the soft gluon effects and the hard gluon are on the same order in $B^0 \to \pi^0\pi^0$ decay since the Wilson coefficient $C_1$ is large in this decay amplitude. In $D \to \pi K, KK$ decay, we can obtain the similar results. The QCD factorization method can be applied to $D \to \pi K, KK$ decay, but the power term $O(\Lambda_{QCD}/m_c)$ should be included. In order to get accurate calculation results in $D \to \pi K, KK$ decay, the soft-gluon, the final state interaction and the annihilation effects should be calculated, and then the calculation results can be improved.
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