Study of Double-Deck Vibrating Flip-Flow Screen Based on Dynamic Stiffness Characteristics of Shear Springs

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Abstract: Double-deck vibrating flip-flow screens have been widely used for the repurposing of decoration waste; however, the influence of shear spring stiffness on the screen’s vibration characteristics is under-researched. The shear spring stiffness affects the amplitude–frequency characteristics, phase–frequency characteristics, screening performance and processing capacity of the screen. In this paper, a mathematical model of the double-deck vibrating flip-flow screen is proposed based on a vibrating system with three degrees of freedom. Based on the experiments of the industrial screen, the amplitude–frequency and phase–frequency characteristics of the double-deck vibrating flip-flow screen were studied. Within the range of 25 to 75 rad/s, the amplitude of the main screen frame decreased gradually, the floating screen frames decreased at first and then increased and the amplitudes of the main and floating screen frames were dependent on the stiffness of the isolation springs and shear springs. When the frequency was 75 rad/s, the stiffness of the upper and lower shear springs was 11,440 kN/m, respectively, and the screening efficiency reached 97.09%.

Keywords: double-deck vibrating flip-flow screen; amplitude; shear spring; stiffness; screening efficiency

1. Introduction

The accumulation of decoration waste has created a severe pollution problem, and turning this urban blight [1] into recycled solid waste [2,3] has become a hot issue in Europe [4–6] and China [7–9]. Some of the legislation, such as The Waste Framework Directive (2008/98/EC) [10] of the EU and the 2015 Circular Economy Promotion Plan issued by the China National Development and Reform Commission, made clear requirements for the recycling and managing of construction waste. Figure 1 shows the process for crushing, screening and air separation as the particulate waste is reduced from 600 to 31.5 mm and material impurity in the recycled aggregate is reduced from 15.43% to less than 0.2%. The recycling of the decoration waste involves primary screening (roller screen) and secondary and tertiary screening (round vibrating screen); all the screening devices comprise 30–40% of the main equipment.

The structure of a D-VFFS (double-deck vibrating flip-flow screen) comprises a main screen frame, upper and lower floating screen frames, an exciter and driving system, the supporting structures, isolation springs, shear springs and polyurethane elastic screen plates (Figure 2). This vibrating system [11] has three degrees of freedom (in x direction). The main screen frame and floating screen frames are connected by shear springs [12], the main and the floating screen beams are alternately arranged, the polyurethane elastic screen plates are fixed on the two adjacent beams and the eccentric block of the exciter system is driven by a motor [13]. During operation, the main and floating screen frames move in opposite directions, and the beams on the frames cause the polyurethane plates to tense and relax, producing large deflection deformations. The materials on the screen
surface experience the bouncing–falling–bouncing cycle as the mesh of the polyurethane plates expands and contracts. It self-cleans the adhesion on the screen surface, overcomes mesh blockage and realizes the screening of fine and sticky materials [14].

**Figure 1.** Double–deck flip–flow screen in the decoration of waste recycling application.
Traditional vibrating screens [15] can experience defects such as screen blockage, hardening and low screening efficiency [16,17] when screening moist and fine materials [18]. The alternating movement of the flip-flow screen, its high-acceleration elastic screen plates (up to 50 g) and self-cleaning mechanism solved the problems of plugged holes and hardened screens and improved the efficiency [19,20].

The D-VFFS was designed to increase the processing capacity and reduce the equipment space, material grading, transportation and equipment investment. For accurate and improved screening efficiency, the stiffness [21] of the isolation springs and shear springs were carefully designed [9]. The stiffness of the isolation springs was calculated according to the amplitude and mass of the screen, whereas the stiffness of the shear springs was calculated according to the working characteristics in the near-resonance area of the screen. Their stiffness affects the characteristics of the vibrating floating screen frames [22], and adjusting it while in operation, according to the characteristics of the material and the distribution of the particle size, not only improves the screen’s motion characteristics but also improves the efficiency.

In this study, a three degrees of freedom vibrating system [11] mechanical model of a D-VFFS was established using the lumped-mass method, altering the parameters and solving the mechanical equations. The main screen frame had second-order resonance, and the floating screen frames had third-order resonance.

The motion and amplitude characteristics of the main floating screen frame were investigated under different shear springs’ stiffness conditions to prove the consistency between the theoretical model and experimental conclusion. Furthermore, through the deep screening experiments of the recycled aggregate of decoration waste, the better screening performance of the D-VFFS was verified.

2. Theoretical Model

2.1. Mechanical Model

The mechanical model of the D-VFFS was based on the lumped-mass method, as shown in Figure 3. In $x_1y_1$, the direction $x_1$ is parallel to the upper and lower screen surfaces, and $y_1$ is perpendicular ($l = 1, 2, 3$ representing the upper, lower floating and main screen frames, respectively).
The shear springs arranged on the D-VFFS1861 (the manufacturer is Maanshan Tian-gong Technology Co., LTD, Ma’anshan, China) were thinner, but their stiffness in the $y$ direction was very large [23]. The upper and lower floating screen frames moved together with the main, and the relative motion was generated in the $x$ direction. The polyurethane screen plates relax when the main and floating screen frames move toward each other. Its relative stretch is small, and screen plates have no elastic deformation, so there was no elastic force exerted on them [23].

2.2. Dynamic Equation

The dynamic characteristics of the D-VFFS were studied by changing the number of shear springs (stiffness): $x_1$, $x_2$ and $x_3$ represented the displacements of the upper, lower floating and the main screen frames in the $x$ direction, and $y_1$, $y_2$ and $y_3$ represent the displacements in the $y$ direction, respectively.

The excitation force:

$$F = m_0 \omega^2 r$$  \hspace{1cm} (1)

The component forces in the $x$ direction:

$$F_x = F \cos \omega t = m_0 \omega^2 r \cos \omega t$$  \hspace{1cm} (2)

The component forces in the $y$ direction:

$$F_y = F \sin \omega t = m_0 \omega^2 r \sin \omega t$$  \hspace{1cm} (3)
The differential equations for the three degrees of freedom vibrating system \(x_1, x_2\) and \(x_3\) and \(y_1, y_2\) and \(y_3\) were:

\[
M\ddot{X} + K_x X = F_x \tag{4}
\]

\[
M\ddot{Y} + K_y Y = F_y \tag{5}
\]

where \(M\) and \(K\) are the mass matrix and stiffness matrix, respectively.

\[
M = \begin{pmatrix}
m_1 & 0 & 0 \\
0 & m_2 & 0 \\
0 & 0 & m_3
\end{pmatrix}
\]

\[
K_x = \begin{pmatrix}
k_{1x} & 0 & -k_{1x} \\
0 & k_{2x} & -k_{2x} \\
-k_{1x} & -k_{2x} & k_{1x} + k_{2x} + k_{3x}
\end{pmatrix}
\]

\[
K_y = \begin{pmatrix}
k_1 & 0 & -k_1 \\
0 & k_2 & -k_2 \\
-k_1 & -k_2 & k_1 + k_2 + k_3
\end{pmatrix}
\]

where \(X, Y\) and \(F\) are the displacement matrices and force matrix, respectively.

To express the displacement in the \(x\) and \(y\) directions (\(e^{i\omega t} = \cos \omega t + jsin \omega t\)), let the particular solution of the above equation be

\[
\begin{align*}
x_1 &= x_{10}e^{i\omega t} \\
x_2 &= x_{20}e^{i\omega t} \\
x_3 &= x_{30}e^{i\omega t} \\
y_1 &= y_{10}e^{i\omega t} \\
y_2 &= y_{20}e^{i\omega t} \\
y_3 &= y_{30}e^{i\omega t}
\end{align*}
\]  

(6)

The second derivative of the above equation was obtained as follows:

\[
\begin{align*}
\ddot{x}_1 &= -x_{10}\omega^2 e^{i\omega t} \\
\ddot{x}_2 &= -x_{20}\omega^2 e^{i\omega t} \\
\ddot{x}_3 &= -x_{30}\omega^2 e^{i\omega t}
\end{align*}
\]  

(7)

After substituting Equations (6) and (7) into Equations (4) and (5), the following solution can be obtained:

\[
\begin{align*}
x_1 &= x_{10}\cos \omega t - \frac{Bk_{1x}}{A_k} - \frac{-Bk_{1x} + ABC}{AB_k} m\omega^2 \cos \omega t \\
x_2 &= x_{20}\cos \omega t - \frac{Bk_{2x}}{A_k} - \frac{-Bk_{2x} + ABC}{AB_k} m\omega^2 \cos \omega t \\
x_3 &= x_{30}\cos \omega t - \frac{Bk_{3x}}{A_k} - \frac{-Bk_{3x} + ABC}{AB_k} m\omega^2 \cos \omega t
\end{align*}
\]  

(8)

where \(A = -m_1\omega^2 + k_{1x}, B = -m_2\omega^2 + k_{2x}\) and \(C = k_{1x} + k_{2x} + k_{3x} - m_3\omega^2\).

Similarly,

\[
\begin{align*}
y_1 &= y_{10}\sin \omega t - \frac{E_k}{D_k} - \frac{DE}{D_k} m\omega^2 \sin \omega t \\
y_2 &= y_{20}\sin \omega t - \frac{E_k}{D_k} - \frac{DE}{D_k} m\omega^2 \sin \omega t \\
y_3 &= y_{30}\sin \omega t - \frac{E_k}{D_k} - \frac{DE}{D_k} m\omega^2 \sin \omega t
\end{align*}
\]  

(9)

where \(D = -m_1\omega^2 + k_1, E = -m_2\omega^2 + k_2\) and \(F = k_1 + k_2 + k_3 - m_3\omega^2\).

2.3. The Parameters

The parameters of D-VFFS1861 are shown in Table 1. The amplitudes mentioned in this paper include those for \(X\) and \(Y\), which are the absolute values.
Table 1. Parameter table of D-VFFS1861.

| Symbol | Item | Value  |
|--------|------|--------|
| m      | The total mass of the exciter eccentric block (kg) | 345.04 |
| r      | Eccentric distance (mm) | 12.18 |
| B      | Width of plate (mm) | 1800 |
| L      | Length of plate (mm) | 6100 |
| m<sub>1</sub> | Upper floating screen frame mass (kg) | 1608.25 |
| m<sub>2</sub> | Lower floating screen frame mass (kg) | 1622.56 |
| m<sub>3</sub> | Main screen frame mass (kg) | 7339.42 |
| P      | Motor power (kw) | 22 |
| k      | Stiffness of shear spring (N/mm) | 220 |
| n      | The number of shear spring groups | 12–15 |
| ω      | Excitation frequency rad/s | 75 |
| K<sub>3</sub> | Stiffness of vibration isolation springs (N/m) | 5,231,730 |

3. Experimental

3.1. Experimental System

The high precision data acquisition instrument and acceleration sensors were connected; the parameters were set and the centers of mass for the upper, lower floating and main screen frame were selected as the testing points. The acceleration sensors were arranged vertically on the magnetic bridge, and the magnetic sensors were fixed on the testing points to ensure that the direction of the sensor was parallel to the screen surface, as shown in Figure 4.

Figure 4. Test and analysis system diagram: 1. Double-deck vibrating flip-flow screen; 2. ICP acceleration sensors; 3. Multi-channel strain conditioner; 4. High-precision data acquisition instrument; 5. Frequency converter; 6. Computer.

3.2. Materials

Figure 5a presents the samples used in the experiment. They were obtained from the decoration waste recycling project in Yixing, China. Figure 5b shows the cumulative particle size distribution curves of the raw materials. The main components of the sample included concrete particles, brick particles, mortar, soil, wood pieces, plastic pieces and paper pieces. The geometry was mainly irregularly lumpy, schistose and strip. When the designated size was 5 and 10 mm, the corresponding yields of the samples (in mass percentages) in 0–5 mm, 5–10 mm and >10 mm size fractions were 45.44%, 17.86% and 36.7%, respectively. The yield of fine particles (0–5 mm) was the largest, and they easily passed through the screen aperture. The yield of the coarse particles (>10 mm), which could not pass through the screen aperture, was the second-largest. Schistose particles (>10 mm) passed the 10-mm screen aperture with 5–10-mm particles but could not pass through the 5-mm screen aperture, which reduced the screening performance.
3.3. Motion Trail Test

In this section, the D-VFFS1861 centroid trajectory was determined. The upper and lower floating screen frames had 52 shear springs pieces (13 groups of 4 pieces arranged symmetrically. The operating frequency was 75 rad/s between the 50–51-s time domain signals, as shown in Figure 6a,b. The centroid motion trajectories are shown in Figure 7.
As can be seen from Figure 6a,b, the amplitudes of the upper and lower floating frames and the main screen frame were 7.8, 8.3 and 2.4 mm in the x direction and 4.7, 4.4 and 4 mm in the y direction, respectively.

As shown in Figure 7, the motion trail at the center of mass of the main, upper and lower floating screen frames was elliptical. The maximum single amplitudes were 2.4, 7.8 and 8.3 mm in the x direction and 4, 4.7 and 4.4 mm in the y direction.

The maximum amplitudes in the time domain curves were equal to the motion trajectory curves, and the amplitudes of the floating screen frames were larger than for the main screen frame; the amplitude of the upper floating screen frame was slightly higher than for the lower; the mass of the upper floating screen frame was slightly lower than that of the lower. Furthermore, the upper and lower floating screen frames had the same motion characteristics.

4. Results and Discussion

4.1. Theoretical Dynamic Amplitude–Frequency Characteristics

4.1.1. In the x Direction

Substituting the parameters (Table 1) into Equation (8), the first, second and third-order resonances occurred at \( \omega_{01} = 12.80 \), \( \omega_{02} = 84.15 \) and \( \omega_{03} = 101.36 \) rad/s. The amplitudes of the upper and lower floating and main screen frames reached the maximum values. The amplitude–frequency characteristics curves of the frames in the x direction are shown in Figure 8.

![Figure 8. Amplitude–frequency characteristic curves in the x direction.](image)

When \( \omega < \omega_{01} \), the amplitude \( X \) in the x direction of the floating screen frames and the main screen frame increased with the \( \omega \). At \( \omega_{01} = 12.80 \) rad/s, the maximum values of \( X \) were \(-120.45, -120.25\) and \(-117.38\) mm. When \( \omega_{01} < \omega < \omega_{02} \), the amplitudes of the floating screen frames first decreased rapidly and then increased slowly. When \( \omega \) increased to \( \omega_{02} = 84.15 \) rad/s, the maximum values of \( X \) were \(-18.18\) and \(-15.91\) mm. The amplitude of the main screen frame decreased rapidly at first and decreased slowly with an increase in the \( \omega \) at \( \omega_{02} = 84.15 \) rad/s. The maximum value of \( X \) was \(-100.9\) mm. At \( \omega_{02} < \omega < \omega_{03} \), the amplitudes of the three screen frames increased rapidly with an increase in the \( \omega \), showing an inverse-phase characteristic compared to the floating screen frames. At \( \omega_{03} = 101.36 \) rad/s, the maximum values of \( X \) were \(-1269.70, -1238.89\) and \(-236.46\) mm. At \( \omega > \omega_{03} \), the amplitudes of the screen frames decreased rapidly with an increase in the \( \omega \), (more than 120 rad/s), decreased slowly until \( X \) approached \( X = 1 \) and \(-6\) mm, and the main screen frame showed antiphase characteristics with the floating screen frame.
4.1.2. In the $y$ Direction

The shear springs had the same stiffness specification: $k = 220$ N/mm. $K_1$ and $K_2$ represented the shear spring stiffness of the floating screen frames in the $y$ direction and satisfied the equation $K_j = \sum_{j=1}^{n} k_j$, where $n$ was the number of springs that determined the values of $K_1$ and $K_2$. In the screen design, the isolation spring stiffness $K_3$ was determined by the mass and amplitude of the screen. According to the traditional designs, the stiffness of the shear springs was generally three times greater than it was in the $x$ direction. To verify this, $3K_{ix}$, $5K_{ix}$, and $10K_{ix}$, respectively, were selected to analyze the amplitude–frequency characteristics of the three screen frames in the $y$ direction. The characteristics curves are shown in Figure 9a–c.

![Amplitude–frequency characteristics curves in the $y$ direction](image)

**Figure 9.** Amplitude–frequency characteristics curves in the $y$ direction: (a) the upper floating screen frame, (b) the lower floating screen frame and (c) the main screen frame.

At $K_i = 3K_{ix}$, $5K_{ix}$ and $10K_{ix}$, the floating screen frames had a third-order resonance, and the main screen frame had a second-order resonance. The amplitudes of the three screen frames showed the same change and phase in the $y$ direction with the increase in $\omega$. The first-order resonance frequency was the same, and the amplitudes were similar, which indicated that the first-order resonance frequency was caused by the characteristics of the isolation springs.

With the increase in $K$, the amplitudes gradually decreased, and the second-order resonance frequency increased; that is, the resonance curves shifted to the right. The amplitude of the floating screen frame increased gradually, and the greater the $K$, the greater the third-order resonance frequency. The amplitudes of the first-order resonance frequency region of the screen frames were close to each other. The amplitude differences between the floating screen frames was especially small, which indicated that they have similar amplitude–frequency characteristics near the first-order resonance frequency region. The second-order resonance frequency region of the floating screen frames was different from that of the main screen frame. The floating screen frames had similar second-order
resonance frequencies and maximum amplitudes, whereas the second-order resonance frequency of the main screen frame was greater. There was a third-order resonance frequency in the floating screen frame but not in the main screen frame. This indicated that the second-order and third-order resonance frequencies of the floating screen frames were determined by the stiffness characteristics of the shear springs. With the increase of $\omega$, the amplitudes of the floating screen in the $y$ direction tended to be $Y = 0$ mm the main screen tended to be $Y = -6$ mm and the phases of the main and floating screens were opposites.

4.2. Experimental Dynamic Amplitude–Frequency Characteristics

By adjusting the number of shear springs, changing their stiffness and analyzing the screen’s amplitude–frequency characteristics, the number of shear springs were 13, 14 and 15 groups, the size of each shear spring was 280 mm $\times$ 80 mm $\times$ 54 mm and they had an operating frequency of 25–75 rad/s.

4.2.1. $K_1$ Was Constant, $K_2$ Was Variable

The number of the upper and lower floating screen frame shear springs were 13, 14 and 15 groups, and the corresponding number of lower floating screen frames were 13, 14 and 15 groups, respectively. An analysis of the amplitude–frequency characteristics of the main and floating screen frames is shown in Figure 10a–c.

![Figure 10](image)  
(a) Influence of stiffness variation on amplitude–frequency characteristic curves: (a) 13, X; (b) 14, X and (c) 15, X.

With the increase of $\omega$, the amplitude of the main screen frame decreased gradually; however, the amplitudes of the floating screen frames decreased then increased. Since the number of shear springs in the upper screen frame was constant, when the number of springs in the lower screen frame increased, the main screen frame amplitude increased about 8%, the amplitude of the upper floating screen frame decreased by about 2% and
the amplitude of the lower screen frame decreased about 36%. It was observed that the stiffness and amplitude of the lower screen frame changed a great deal, but those for the main and upper screen frames changed slightly.

4.2.2. $K_1$ Was Variable, $K_2$ Was Constant

The number of the lower floating screen frame shear springs were 13, 14 and 15 groups, and the corresponding number of upper floating screen frame were 12, 13 and 14 groups, respectively. An analysis of the amplitude–frequency characteristics of the main and floating screen frames is shown in Figure 11a–c.

With the increase of $\omega$, the amplitude of the main screen frame decreased gradually; those for the floating screen frames decreased at first and then increased. The number of shear springs in the lower floating screen frame was constant, so there was an increase in the number of springs in the upper frame, the main screen frame amplitude increased by about 4%, the upper screen frame amplitude decreased by about 37% and the lower screen frame amplitude decreased by about 8%. It could be seen that, when the stiffness of the upper screen frame changed significantly, the upper screen frame amplitude did as well, whereas the amplitude for the main and lower floating screen frames changed slightly.

4.2.3. $K_1$ and $K_2$ Were Variable

The number of the upper and lower floating screen frame shear springs were 13, 14 and 15 groups to analyze the amplitude–frequency characteristics of the main and floating screen frames, as shown in Figure 12.
Figure 12. Influence of stiffness variations on amplitude–frequency characteristic curves X, X.

Figure 12 shows that, with an increase in $\omega$, the amplitude of the main screen frame decreased gradually, while the floating screen frames decreased first and then increased; as the stiffness of the upper and lower floating screen frames increased, their amplitudes decreased by about 40% and 33%, respectively, while the amplitude of the main screen frame increased by nearly 10%. The change in amplitude occurred at a high frequency.

4.3. Phase–Frequency Characteristics

An operating frequency range of 25–75 rad/s was chosen to analyze the phase–frequency characteristics. With the increase in $\omega$, the maximum phase angle of the floating screen frames relative to the main, and the upper floating screen frame relative to the lower, were $0.1\pi$. The analysis was conducted when $K_u < K_l$, $K_u = K_l$, $K_u > K_l$. The phase–frequency characteristic curves are shown in Figure 13a–c.

Figure 13 shows the influence of stiffness changes on the phase–frequency. Whether $K_u < K_l$, $K_u = K_l$ or $K_u > K_l$, in the low-frequency region, the motion of the floating screen frames lagged behind that of the main screen frame; in the high-frequency region, the upper motion was ahead of the main. The maximum phase angle occurred at the maximum operating frequency; the maximum phase angle of the upper and lower floating screen frames were 0.7 and 0.95$\pi$, respectively; the motion of the upper floating screen frame was ahead of the lower at 25–75 rad/s, and the phase angle decreased as the stiffness increased.

As the stiffness of the floating screens was different, it influenced the phase of the floating and the main screens. The greater the difference of the stiffness between the upper and lower floating screen frames, the greater the phase angle of the floating screen frames relative to the main. The maximum phase angle of the upper was less than the lower, and they were obtained at a different frequency and in an opposite phase, but the values were nearly the same.
4.4. Comparison of Theoretical and Experimental Amplitude–Frequency Characteristics

To verify that the theoretical amplitude–frequency characteristics were consistent with the experiment, the equivalent stiffness of the upper and lower floating screen frames were 11,440 kN/m at 25–75 rad/s when compared, as shown in Figure 14.
With the increases in $\omega$, the theoretical and experimental amplitudes of the main screen gradually increased. The theoretical was greater than the experimental, and the difference was larger at 25–40 rad/s but near-stable at 40–75 rad/s. The difference in amplitudes was caused by the shear springs’ damping characteristics. The theoretical and experimental amplitudes decreased at first but then increased with $\omega$; the theoretical increased slowly at first and then decreased gradually but faster than the experimental. The theoretical amplitude–frequency characteristic curves had a higher similarity to the experimental curves; thus, the theoretical model had to be corrected.

5. Screening Experiments

The best operation came not only from the larger amplitudes of the floating screen frames but, also, from the smaller amplitude of the main frame. From the experimental amplitude–frequency characteristics, we obtained the best optimum operation (13, 13): 13 groups shear springs were arranged on the upper and lower floating screen frames, and the amplitudes of the main and upper and lower floating screen frames were 2.35, 6.72 and 7.25 mm, respectively. To study the screening performance of a double-deck vibrating flip-flow screen for recycling fine aggregate decoration waste, the screen apertures were 5 mm and 10 mm. Figure 15 shows the raw material and oversized, middle and undersized products. Figure 16 shows the distribution of the particle sizes. The size fractions were 0–3, 3–5, 5–10, 10–16, 16–25 and 25–50 mm.

![Figure 15](image-url)

Figure 15. Samples of (a) the raw material, (b) oversized, (c) middle and (d) undersized products.

As shown in Figure 16, the distribution of each size of the raw material was 30.68% (0–3 mm), 14.76% (3–5 mm), 17.86% (5–10 mm), 18.25% (10–16 mm), 17.86% (16–25 mm) and 0.58% (25–50 mm). The distribution of the oversized product was 0.78% (0–3 mm), 0.19% (3–5 mm), 0.19% (5–10 mm), 22.14% (10–16 mm), 50.87% (16–25 mm) and 25.83% (25–50 mm). The effective placement efficiency of the coarse particles was up to 98.83%, and the content of mismatches on the screen (10 mm) was only 1.17%, because fine particles adhered to the surfaces of coarse particles and passed through the screen mesh, so there were some fine particles in the oversized product. The distribution of each size of the middle product was 1.55% (0–3 mm), 5.05% (3–5 mm), 71.46% (5–10 mm), 21.94% (10–16 mm) and 0% (16–50 mm). The effective placement efficiency of the middle product was 71.46%, and the contents of mismatches on and under the screen were 21.94% and 6.60% when the upper mesh size was 10 × 20 mm; the distribution of each size of the undersized product was 53.79% (0–3 mm), 44.47% (3–5 mm), 1.75% (5–10 mm) and 0% (10–50 mm). That means the effective placement efficiency
of fine particles was up to 98.25%, and the content of the mismatches under the screen (5 mm) was only 1.75% due to the mesh shape (5 × 12-mm rectangle).

Figure 16. Comparison of the screening percentages of various sizes of raw material, oversized, middle and undersized products.

The screening efficiency was used to assess the performance, which was calculated by the equation \( \eta = E_c + E_f - 100 \), where \( \eta \) is the screening efficiency (%), \( E_c \) represents the effective placement efficiency of the coarse particles (%) and \( E_f \) stands for the effective placement efficiency of fine particles (%). According to the above analysis, the screening efficiency \( \eta \) of D-VFFS1861 can reach 97.09%, thus, the screening performance was significantly superior to that of the general screening devices.

6. Conclusions

In this study, the effects of the stiffness of the shear springs on the amplitude–frequency characteristics, phase–frequency characteristics, screening performance and processing capacity of the screen were investigated. A three degrees of freedom vibrating system (in the \( x \) direction) mechanical model and dynamic equation were established using the lumped-mass method and obtained first-order, second-order and third-order resonances of 12.80, 84.15 and 101.36 rad/s, respectively. As the stiffness of the shear springs increased, the amplitudes of the main and floating screen frames changed. With the increase of \( \omega \), the amplitude of the main screen frame decreased gradually, and the amplitude of the upper and lower floating screen frames decreased and then increased. As the stiffness of the upper or lower floating screen frames changed, the corresponding amplitude had a 36% variation (compared to an amplitude variation of only 4% for the unchanged stiffness), and the amplitude of the main screen frame was 8%. Compared to the experimental curves, the theoretical amplitude–frequency characteristics had a high similarity to the experimental curves. The motions of the upper and lower floating screen frames lagged behind that of the main screen frame in the low-frequency region, and the upper motion was ahead of the main screen frame in the high-frequency region. The optimum conditions of the frequency were 75 rad/s, the stiffness of the upper and lower shear springs were 11,440 kN/m, respectively, and the screening efficiency reached 97.09%.

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Nomenclature

\[ m_1 \] the mass of the upper floating screen frame and half of the screen plate of the upper floating screen frame, kg.
\[ m_2 \] the mass of the lower floating screen frame and half of the screen plate of the lower floating screen frame, kg.
\[ m_3 \] the mass of the double-deck vibrating flip-flow screen box side plates, fixed beams, vibrator, strengthening beams, half of the upper and lower floating screen frame plates, kg.
\[ K_{x1} \] stiffness of the upper floating screen in the \( x \) direction, N/m.
\[ K_{x2} \] stiffness of the lower floating screen in the \( x \) direction, N/m.
\[ K_{x3} \] stiffness of the main screen frame in the \( x \) direction, N/m.
\[ K_{y1} \] stiffness of the upper floating screen in the \( y \) direction, N/m.
\[ K_{y2} \] stiffness of the lower floating screen in the \( y \) direction, N/m.
\[ K_{y3} \] stiffness of the main screen frame in the \( y \) direction, N/m.
\[ a \] distance between the center of the right spring of the upper floating screen frame and the center of the main screen frame in the \( x \) direction, mm.
\[ b \] distance between the center of the left spring of the upper floating screen frame and the center of the main screen frame in the \( x \) direction, mm.
\[ d \] distance between the center of the right spring of the lower floating screen frame and the center of the main screen frame in the \( x \) direction, mm.
\[ e \] distance between the center of the left spring of the lower floating screen frame and the center of the main screen frame in the \( x \) direction, mm.
\[ f \] distance between the center of the left spring of the main screen frame and the center of the main screen frame in the \( x \) direction, mm.
\[ g \] distance between the center of the right spring of the main screen frame and the center of the main screen frame in the \( x \) direction, mm.
\[ h_1 \] distance between the center of the upper floating screen frame and the center of the main screen frame in the \( y \) direction, mm.
\[ h_2 \] distance between the center of the lower floating screen frame and the center of the main screen frame in the \( y \) direction, mm.
\[ h_3 \] distance between the center of the spring in the \( x \) direction of the main screen frame and the center of the main screen frame in the \( y \) direction, mm.
\[ l_x \] distance between the vibration exciter center and main screen center in the \( x \) direction, mm.
\[ l_y \] distance between the vibration exciter center and main screen center in the \( y \) direction, mm.

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