On the nature of the Born rule

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1 Abstract

A physical experiment comprises along the time trajectory a start, a time evolution (duration), and an end, which is the measurement. In non relativistic quantum mechanics the start of the experiment is defined by the wave function at time 0 taking into account the starting conditions, the evolution is described by the wave function following the Schrödinger equation and the measurement by the Born rule. While the Schrödinger equation is deterministic, it is the Born rule that makes quantum mechanics statistical with all its consequences. The nature of the Born rule is thereby unknown albeit necessary since it produces the correct ensemble averaged measures of the experiment. Here, it is demonstrated that the origin of the Born rule is the projection from the quantum frame (i.e. wave description) to the classical mechanics frame (i.e. particle description) described by a Ehrenfest theorem-oriented Fourier transformation. The statistical averaging over many measurements is necessary in order to eliminate the unknown initial and end time coordinate of the experiment in reference to the beginning of the universe.

2 Significance Statement

One of the many counter-intuitive phenomena of quantum mechanics is the loss of a deterministic description into a statistical classical mechanics read-out by the measurement. The measurement appears thereby to be the bridge between the quantum mechanics and classical mechanics world mathematically formulated by the Born rule. However, the nature of the Born rule is unknown. Here, it is demonstrated that the Born rule originates from the experimental lack of knowledge on the time coordinates at the start of the experiment, while the measurement itself is a Fourier transformation of its kind from the quantum frame (i.e. wave description) to the classical mechanics frame (i.e. particle description) following the Ehrenfest theorem. This finding demystifies the measurement problem with the Born rule and quantum mechanics in general because it makes quantum mechanics genuinely deterministic as classical mechanics is.

3 Introduction

The measurement of a quantum mechanical system can be considered a transformation from the quantum mechanical description to its classical mechanics one [1-5]. While both descriptions within are both deterministic and time reversible (in quantum mechanics with the Schrödinger equation and in classical mechanics with the Newtonian laws) it is stated that both are lost with the measurement. Within the standard quantum mechanical frame work this odd phenomenon under some conditions also called the collapse of the wave function is usually explained by the involvement of the observer (i.e. measurement device) in the experiment, a requested quantum mechanical super position between measurement device
and system under study, entanglement between the system under study and the environment, by decoherence due to an interaction with the environment, or by our apparent limitation to be able to detect only at the classical limit (i.e. being unable to measure quantum mechanically) [6-12]. It also nourishes distinct ontologies of quantum mechanics starting from the Copenhagen interpretation, via the Bohm-de Broglie, and Everett’s many world to retrocausal interpretations and discrete approaches [13-24]. It further builds the basis for the non-locality of quantum mechanics highlighted prominently by the Einstein-Podolski-Rosen paradox (EPR) in combination with the Bell inequalities [25-30].

The mathematical description of the measurement is the Born rule [1-5]. It yields statistically the correct result of the experiment, but can not calculate with certainty the result of a single experiment. Einstein and others concluded therefore that the established formulation of quantum mechanics must be incomplete [25]. However, the success of quantum mechanics and the experimental evidence collected on the Bell inequalities as well as the measurement described by a decoherence phenomenon are in favor of quantum mechanics as is [26-32]. Nonetheless, due to the odd properties of quantum mechanics extensions thereof or other approaches are still discussed [for example 13-24] and in light of the fundamental inconsistencies between quantum mechanics and general relativity requested [33].

In the work presented, the measurement problem of an experiment is revisited. It is thereby assumed that the measurement itself is a transformation from the wave to a particle description by a Fourier transformation following the Ehrenfest theorem, which is deterministic. The statistical origin of the Born rule is due to the time unknown nature of the initial $t_i$ and end $t_f$ time point of the experiment in relation to the absolute start of time, which varies from measurement to measurement, while the experimental time is known $t_e = t_f - t_i$. By doing so, the formulation presented yields for each experiment a deterministic result which converges to the Born rule upon statistically averaging. It is thereby demonstrated that the statistical origin of the Born rule is the unknown initial and end time coordinates of the experiment, while nature is deterministic both at the quantum mechanical as well as at the classical mechanics level including the measurement.

After a short summary on useful standard quantum mechanics (4.1), a single measurement is studied by assuming a frame change from the quantum mechanics frame to the classical mechanics frame by a Fourier transformation (4.2), followed in 4.3 by the description of the time evolution of a quantum mechanical system from its start to the measurement. In (5) the results are discussed.
4 Theory

4.1 Standard Quantum Mechanics

Within the non relativistic quantum mechanics the time evolution $t$ of the wave function $\Psi(\vec{r}, t)$ describing the system under study (with $\vec{r}$ being the space vector) is described by the time-dependent Schrödinger equation

$$\hat{H} \Psi(\vec{r}, t) = i \hbar \frac{\partial}{\partial t} \Psi(\vec{r}, t)$$  

(1)

with $\hat{H} = -\frac{\hbar^2}{2m} \nabla^2 + V(\vec{r})$ the Hamilton operator (with $m$ the mass of the particle and $V(\vec{r})$ the acting potential), $\hbar$ the reduced Planck constant with $\hbar = h/2\pi$, and $i = \sqrt{-1}$. In the following description translation along the time (by $\delta$) is required. The following transformation is then given:

$$\Psi(\vec{r}, t - \delta) = \hat{U}_t(\delta) \Psi(\vec{r}, t)$$  

(2)

with the unitary operator $\hat{U}_t(\delta) = e^{i \hat{E} \delta t}$ (with the energy tensor $\hat{E} = i \hbar \frac{\partial}{\partial t}$).

In the following the Hamilton operator is considered time-independent yielding

$$\Psi(\vec{r}, t) = e^{-\frac{i}{\hbar} \hat{H} t} \Psi(\vec{r}, 0_e)$$  

(3)

enabling a separation between space and time with time starting at $0$ (denoted $0_e$) for the beginning of the experiment. The solutions of the Schrödinger equation are then given by

$$\Psi_n(\vec{r}, t) = e^{-\frac{i}{\hbar} \hat{E}_n t} \Psi_n(\vec{r})$$  

(4)

with $n$ an integer, and $\hat{H} \Psi_n(\vec{r}) = E_n \Psi_n(\vec{r})$ with $E_n = \text{const.}$ (i.e. $E_n = \hbar \omega_n$).

The general solution of the Schrödinger equation is then given by the superposition of all the $\Psi_n(\vec{r}, t)$:

$$\Psi(\vec{r}, t) = \sum_n c_n(0_e) \Psi_n(\vec{r}, t) = \sum_n c_n(0_e) e^{-i \frac{\hbar}{\hbar} E_n t}$$  

(5)

with $c_n(0_e) = \int \Psi(\vec{r}, 0_e) \Psi_n^*(\vec{r}, 0_e) dV \geq 0$ and $\Psi_n(\vec{r}, t)$, which are orthonormal to each other (i.e. $<\Psi_n|\Psi_m> = \delta_{mn}$ using the Dirac notation and with $\delta_{mn}$ being the Kronecker’s symbol). $c_n(0_e)$ is thus independent of time and correspondingly $c_n(0_e) = c_n(0)$.

The mean value of a measurement on an observable described by the hermitian operator $\hat{A}$ is described by the Born rule
\[ < \hat{A} > = < \Psi(\vec{r}, t) | \hat{A} | \Psi(\vec{r}, t) > = \int \Psi(\vec{r}, t)^* \hat{A} \Psi(\vec{r}, t) \, dV = \int \Phi(\vec{r}, t)^* \hat{A} \Phi(\vec{r}, t) \, dV \]  

(6)

with \( \Phi(\vec{r}, t) = \sum_n b_n(0) \Phi_n(\vec{r}, t) \) if the set of \( \Phi_n(\vec{r}) \) is a complete orthonormal system of the operator \( \hat{A} \) with the classical observables \( a_m \) (which are the eigenvalues of the operator) and with \( b_n(0) = \int \Phi(\vec{r}, 0) \Phi_n^*(\vec{r}, 0) \, dV \)

\[ < \hat{A} > = \sum_m |b^*_m b_m| a_m \]  

(7)

with \( |b^*_m b_m| = |b_m^2| \) the probability that the value \( a_m \) is measured. In particular, \( |\Phi_n(\vec{r})^* \Phi_n(\vec{r})| \) describes the probability at position \( \vec{r} \). If \( \hat{A} \) commutes with \( \hat{H} \), \( \Phi_n(\vec{r}, t) = \Psi_n(\vec{r}, t) \) can be selected and \( c_n(0_e) = b_n(0_e) \), respectively.

4.2 Revisiting the concept of a measurement and an experiment

In contrast to standard quantum mechanics, we define the measurement as a change in the reference frame from a wave description following quantum mechanics to a particle/state description following classical mechanics orchestrated by a Fourier transformation of its kind following the Ehrenfest theorem. Furthermore, the read-out should comprise the entire measured information of the system under investigation which is preserved by the Fourier transformation. Let us describe this Ansatz by a classical analog of a sound wave \( \varphi(t) = \cos(\omega t) \) with its Fourier transform analog to be \( \omega \) (if infinitely long investigated) capturing the entire information. While there is a sound wave we hear the frequency \( \omega \). It is important to note that theoretically the Fourier transformation guarantees that no information is lost in the measurement and thus both descriptions are equivalent (please note, that a loss of information happens however due to the timing issues discussed below). With other words, the event of a single measurement at time \( t_f \) (\( f \) for final, and with the big bang as the reference time \( t = 0 \)) is regarded and defined here as a projection from the wave description of a quantum mechanical system to the particle presentation through a Fourier transformation without loss of information. For the description of such a measurement a system \( \Psi(\vec{r}, t) \) is selected that evolves under a non-time-dependent Hamilton and that the observable of interest is described by the hermitian operator \( \hat{A} \) yielding \( F \hat{A} \Psi \) with \( F \) being the Fourier transformation (please note, the normalisation of the Fourier transformation is omitted here).

In the most simple case of a free particle this reads as

\[ F \hat{x} \Psi(\vec{r}, t_f) = \int dV e^{-\frac{\hat{x} \vec{r}}{2}} \hat{A} \Psi(\vec{r}, t_f) \]  

(8)

if the position of the particle is of interest, while
\[
F \hat{\rho} \Psi(\vec{p}, t_f) = \int d\vec{p} e^{-\frac{i}{\hbar} \vec{p} \cdot \vec{p}} \hat{\rho} \Psi(\vec{p}, t_f)
\]

(9)

if the momentum is of interest. Please note, that both expressions are for a single measurement and not yet comparable with the equivalent Born descriptions \(<\hat{x}>\) and \(<\hat{p}>\), which is a statistical average.

It is critical to note here, that when the experiment is repeated \(t_f\) changes because time is ongoing with the origin of time assumed to be at the big bang. Obviously this contrasts standard quantum mechanics, which start of each experiment is defined as \(t_i = 0\) and the duration of the experiment is given by a defined value \(t_e\) with \(t_f = t_e\) and thus \(t_f\) is the same with each experiment. Under the assumption of an ongoing time with each experiment having a different \(t_i\) and \(t_f\) but the same experimental time \(t_e = t_f - t_i\) the evolution of the periodic wave function can be rewritten with the orthonormal basis \(\Psi_n\) into

\[
\Psi(\vec{r}, t_f) = \Psi(\vec{r}, t_i + t_e) = \hat{U}_t(-t_i) \Psi(\vec{r}, t_e) = \sum_n c_n(t_i) \hat{U}_t^n(-t_i) \Psi_n(\vec{r}, t_e)
\]

(10)

with \(\hat{U}_t^n(-t_i) = e^{-\frac{i}{\hbar} t_i E_n}\) yielding with a time-independent Hamiltonian

\[
\Psi(\vec{r}, t_f) = \sum_n c_n(t_i) e^{-\frac{i}{\hbar} t_i E_n} \Psi_n(\vec{r}, t_e)
\]

(11)

In a first step towards the measurement the observable operator is added at time point \(t_f\).

\[
\hat{A} \Psi(\vec{r}_f, t_f) = \sum_n c_n(t_i) \hat{A} e^{-\frac{i}{\hbar} t_i E_n} \Psi_n(\vec{r}, t_e) = \sum_n c_n(t_i) e^{-\frac{i}{\hbar} t_i E_n} \hat{A} \Psi_n(\vec{r}, t_e)
\]

(12)

Eq. 10 indicates that due to the start of the experiment with \(t_i\) each orthonormal state described with \(\Psi_n\) has its own phase (i.e. \(-\frac{i}{\hbar} t_i E_n\) in the exponent) and by a repetition of the experiment this phase alters because the initial time \(t_i\) alters every time. Thus, albeit deterministically a repetition of the experiment does not yield the same wave function after evolution during time \(t_e\). This interpretation requests the definition of a statistical measure of the observable to be dependent on available information-only, which is \(t_e\) as given successfully (while ad hoc) by the Born rule (i.e. \(<\hat{A}> = \int \Psi(\vec{r}, t_e)^* \hat{A} \Psi(\vec{r}, t_e) dV = \int \Phi(\vec{r}, t)^* \Phi(\vec{r}, t) dV = \sum_n |b_n^* b_n| a_n\)).

Hence, in the next steps the dependence on the start of the experiment with \(t_i\) is described by \(t_e\). This is possible since the exponents in eq. 10 are of periodic nature with time periodicity \(\tau_n = \frac{\hbar}{E_n} = \frac{2\pi}{E_n}\) such that

\[
t_i = \alpha_n i \tau_n + \Delta t_{n,i}
\]

(13)
and
\[ t_e = \alpha_{n,e} \tau_n + \Delta t'_{n,e} + \Delta t''_{n,e} \]  

(14)

with \( \alpha \)'s integers and \( 0 \leq \Delta t_{n,i} \leq \tau_n \), \( 0 \leq \Delta t'_{n,e} + \Delta t''_{n,e} \leq \tau_n \) (Figure 1). It is
\[ \Delta t_{n,i} + \Delta t'_{n,e} = \tau_n \]  

(15)

Next, these relations are expanded to \( t_i \) and \( t_e \) using the periodicity of the wave function
\[ \Delta t_{n,i} = \tau_n - \Delta t'_{n,e} \]  

(16)

\[ t_i = \Delta t_{n,i} + \alpha_{n,i} \tau_n = \tau_n - \Delta t'_{n,e} - (\alpha_{n,e} + 1) \tau_n = -t_e + \Delta t''_{n,e} \]  

(17)

without loss of information within the formulas needed to describe an experiment. This leads to
\[ \hat{A} \Psi(\vec{r}, t_f) = \sum_n c_n(t_i) e^{-\frac{i}{\hbar} \Delta t''_{n,e} E_n} e^{+\frac{i}{\hbar} t_e E_n} \hat{A} \Psi_n(\vec{r}, t_e) \]  

(18)

with the following further boundary conditions (with \( \Delta t''_{n,e} < \tau_n - \Delta t'_{n,e} \) and \( \Delta t_{n,i} + \Delta t'_{n,e} = \tau_n \)):
\[ \Delta t''_{n,e} < \Delta t_{n,i} < \tau_n \]  

(19)

With other words, from experiment to experiment \( \hat{A} \Psi(\vec{r}, t_f) \) varies because \( \Delta t''_{n,e} \) varies.

Next, we need to resolve the potential \( t_i \) dependency of the coefficients \( c_n(t_i) \). In the standard description of quantum mechanics \( c_n(t = 0) \) and thus independent of time, while the experimental time dependency of the system is only within the wave function. This is also true for the presented discussion since \( c_n(t_i) = \int \Psi(\vec{r}, t_i) \Psi^*_n(\vec{r}, t_i) dV = \int \Psi(\vec{r}, 0) \Psi^*_n(\vec{r}, 0) dV \), which yields \( c_n(t_i) \equiv c'_n \) and thus is time independent.

Eq. 18 reads now
\[ \hat{A} \Psi(\vec{r}, t_f) = \sum_n c'_n e^{-\frac{i}{\hbar} \Delta t''_{n,e} E_n} e^{+\frac{i}{\hbar} t_e E_n} \hat{A} \Psi_n(\vec{r}, t_e) \]  

(20)

When averaged over many experiments the following is obtained by integrating over \( \Delta t_{n,i} \) from \( -\frac{\tau_n}{2} \) to \( \frac{\tau_n}{2} \). These integration borders are necessary in order to have both the mean \( \Delta t_{n,i} \) and the mean \( t_i \) modulo \( \tau_n \) equal to 0 (please note, an integration from \( -\tau_n \) to \( \tau_n \) would result in mean \( \Delta t_{n,i} \) and the mean \( t_i \) modulo \( \tau_n \) of \( \frac{\tau_n}{2} \) and would result in a non exactly defined \( t_i \) as each \( \tau_n \) is different).
\[
\{ \hat{A} \Psi(\vec{r}, t_f) \}_\text{time average} = \sum_n c'_n \frac{1}{\tau_n} \int_{-\tau_n/2}^{\tau_n/2} d\Delta t_{n,i} \frac{1}{\Delta t_{n,i}} \int_0^{\Delta t_{n,i}} d\Delta t'_{n,e} e^{-\frac{i}{\hbar} \Delta t'_{n,e} E_n} e^{\frac{i}{\hbar} t_e E_n} \hat{A} \Psi_n(\vec{r}, t_e)
\]

(21)

\[
\{ \hat{A} \Psi(\vec{r}, t_f) \}_\text{time average} = \sum_n c'_n \frac{1}{\pi} \int_0^{\pi} \frac{\sin(t)}{t} dt e^{\frac{i}{\hbar} t_e E_n} \hat{A} \Psi_n(\vec{r}, t_e)
\]

(22)

with \( \frac{1}{\pi} \int_0^{\pi} \frac{\sin(t)}{t} dt \) having a real value and can be absorbed into the coefficients \( c'_n \frac{1}{\pi} \int_0^{\pi} \frac{\sin(t)}{t} dt = c_n(0) \).

Please also note, that the normalisation factor for the Fourier transformation could be incorporated here as well.

\[
\{ \hat{A} \Psi(\vec{r}, t_f) \}_\text{time average} = \sum_n c_n(0) e^{\frac{i}{\hbar} t_e E_n} \hat{A} \Psi_n(\vec{r}, t_e)
\]

(23)

The Fourier transformation of the measurement is applied to the system. In the most simple case of a free particle’s position this reads as

\[
\Gamma \{ \hat{x} \Psi(\vec{r}, t_f) \}_\text{time average} = \int dV e^{-\frac{i}{\hbar} \vec{p} \cdot \vec{r}} \sum_n c_n(0) e^{\frac{i}{\hbar} t_e E_n} \hat{x} \Psi_n(\vec{r}, t_e)
\]

(24)

\[
\Gamma \{ \hat{x} \Psi(\vec{r}, t_f) \}_\text{time average} = \int dV \Psi^*(\vec{r}, 0) \sum_n e^{\frac{i}{\hbar} t_e E_n} \hat{x} c_n(0) \Psi_n(\vec{r}, t_e) = \int dV \sum_m \sum_n c^*_m(0) \Psi^*_m(\vec{r}, 0) e^{\frac{i}{\hbar} t_e E_n} \hat{x} c_n(0) \Psi_n(\vec{r}, t_e)
\]

\[
= \int dV \sum_m \sum_n c^*_m(0) \Psi^*_m(\vec{r}, 0) e^{\frac{i}{\hbar} t_e E_n} \hat{x} c_n(0) \Psi_n(\vec{r}, t_e) = < \Psi(\vec{r}, t_e) | \hat{x} | \Psi(\vec{r}, t_e) > = \langle \hat{x} \rangle
\]

because \( < \Psi_n | \Psi_m > = \delta_{mn} \), \( \hat{x} \) and \( \Psi_n \) commute and \( \Psi^*_m(\vec{r}, 0) = e^{-\frac{i}{\hbar} \vec{p} \cdot \vec{r}} \). Thus, the combination of the Fourier transformation with experimental repetition yielding time averaging yields the Born rule for the simple case discussed.

For a more general case with \( \Phi(\vec{r}, t_f) = \sum_m b_m(0) \Phi_m(\vec{r}, t_f) \) with \( \Phi_m \) being the orthonormal basis of \( \hat{A} \) (with \( b_m(0) = \int \Phi(\vec{r}, 0) \Phi^*_m(\vec{r}, 0) dV \))

the Fourier transformation for the measurement is then given by

\[
\Gamma \{ \hat{A} \Phi(\vec{r}, t_f) \} = \int dV \Phi^*(\vec{r}, 0) \hat{A} \Phi(\vec{r}, t_f) = \int dV \sum_m b^*_m(0) \Phi^*_m(\vec{r}, 0) \hat{A} \sum_k b_k(0) \Phi_k(\vec{r}, t_f)
\]

(25)
with $\hat{A}\Phi_k = a_k\Phi_k$ due to the orthonormal property of $\Phi_k$ in respect to $\hat{A}$.

$$F\{\hat{A}\Phi(\vec{r}, t_f)\} = \int dV \sum_m b_m(0)\Phi^*_m(\vec{r}, 0) \sum_k a_k b_k(0)\Phi_k(\vec{r}, t_f)$$  \hspace{1cm} (26)

with $\Phi_k(\vec{r}, t) = \sum_l <\Psi_l(\vec{r}, t)|\Phi_k(\vec{r}, t) > \Psi_l(\vec{r}, t)$

$$F\{\hat{A}\Phi(\vec{r}, t_f)\} = \int dV \sum_m b_m(0) \sum_l <\Psi^*_l(\vec{r}, 0)|\Phi^*_m(\vec{r}, 0) > \Psi^*_l(\vec{r}, 0)$$  \hspace{1cm} (27)

$$\sum_k a_k b_k(0) \sum_s <\Psi_s(\vec{r}, t_f)|\Phi_k(\vec{r}, t_f) > \Psi_s(\vec{r}, t_f)$$

$$F\{\hat{A}\Phi(\vec{r}, t_f)\} = \int dV \sum_m b_m(0) \sum_l <\Psi^*_l(\vec{r}, 0)|\Phi^*_m(\vec{r}, 0) > \Psi^*_l(\vec{r}, 0)$$  \hspace{1cm} (28)

$$\sum_k a_k b_k(0) \sum_s e^{+\frac{\pi}{\hbar} E_s} e^{-\frac{\pi}{\hbar} \Delta t''_{s,e} E_s} <\Psi_s(\vec{r}, t_f)|\Phi_k(\vec{r}, t_f) > \Psi_s(\vec{r}, t_e)$$

because of $<\Psi_n|\Psi_m> = \delta_{mn}$ $l = s$, which yields

$$F\{\hat{A}\Phi(\vec{r}, t_f)\} = \int dV \sum_m \sum_k b^*_m(0) a_k b_k(0) \sum_l <\Psi^*_l(\vec{r}, 0)|\Phi^*_m(\vec{r}, 0) >$$  \hspace{1cm} (29)

$$<\Phi^*_k(\vec{r}, t_f)|\Psi^*_l(\vec{r}, t_f) > \Psi^*_l(\vec{r}, t_e)\Psi_l(\vec{r}, t_e) e^{-\frac{\pi}{\hbar} \Delta t''_{\pi E}}$$

Because of the unitary property of the time operator with $<\Phi(0)|\Psi(0)> = <\Phi(0)|\hat{U}^+_t(\delta)\hat{U}_t(\delta)|\Psi(0)> = <\Phi(\delta)|\Psi(\delta)>$, eq. 30 can be simplified to

$$F\{\hat{A}\Phi(\vec{r}, t_f)\} = \int dV \sum_m \sum_k b^*_m(0) a_k b_k(0) \sum_l <\Psi^*_l(\vec{r}, 0)|\Phi^*_m(\vec{r}, 0) > <\Phi^*_k(\vec{r}, 0)|\Psi^*_l(\vec{r}, 0) >$$  \hspace{1cm} (30)

which for each measurement differs and can not be simplified further as $\Delta t''_{\pi E}$ is different for each $l$ and changes from measurement to measurement.

After time averaging (see above and by incorporating the time averaging value $\frac{1}{\pi} \int_0^\pi \frac{\sin(t)}{t} dt$ into the term $b^*_m(0)b_m(0)$) we obtain
\[
F \{ \hat{A} \Phi(\vec{r}, t_f) \}_{time \ average} = \int dV \sum_m \sum_k b^*_m(0) a_k(0) \sum_l <\Psi^*_l(\vec{r}, 0)|\Phi^*_m(\vec{r}, 0)>
\]

\[
<\Phi^*_k(\vec{r}, 0)|\Psi^*_l(\vec{r}, 0) > \Psi_l(\vec{r}, t_e)\Psi^*_l(\vec{r}, t_e)
\]

\[
F \{ \hat{A} \Phi(\vec{r}, t_f) \}_{time \ average} = \sum_m b^*_m(0) a_m b_m(0) = <\hat{A}>
\]

Figure 1: The dependence between the various $\Delta$'s are illustrated here for the time with $\Delta t_{n,e} = \Delta t'_{n,e} + \Delta t''_{n,e}$ and $\tau_n = \Delta t'_{n,e} + \Delta t_{n,i}$ within a periodic wave function (i.e. $\cos(t/\tau_n)$).

5 Discussion

Assuming that the measurement of a (quantum mechanical) system is a Ehrenfest theorem-type Fourier transformation and that time is universal starting with time 0 at the beginning of the universe, the Born
rule has been derived here. It thereby not only links the wave and particle description in quantum mechanics with its classical analog, but highlights the origin of the statistical nature of the Born rule and thus quantum mechanics to be the unknown absolute starting time \( t_i \) and ending time \( t_f \) of an experiment, that change with each experiment. Thus, quantum mechanics including the measurement is genuinely deterministic, but not physical experiments as they have to be repeated and must be repeatable while the experimenter lacks information on the absolute time. While experimental physics is therefore restrained to the Born rule, nature is not and thus deterministic both at the quantum mechanical as well as at the Newtonian level and connected by a Fourier transformation. Interestingly, if experiments can be designed with \( t_e << \tau \) then the quantum mechanical system should behave deterministic. With other words, with increasing the time resolution of experiments, quantum mechanics may get deterministic again to be demonstrated. In return, a classical object can be regarded having a \( \tau \approx 0 \) (or \( \Delta t''_{n,e} \approx \text{const} \) and \( \Delta t_{n,i} \approx \text{const} \) for all \( n \)) and thus no statistical averaging is needed, which yields also a deterministic result. While these claims may be valuable for experimental physics, the presented approach to derive the origin of the Born rule is also relevant for the ontology of physics opening another approach than the Copenhagen interpretation, the Bohm-de Broglie, Everett’s many world as well as retrocausal interpretations of quantum mechanics [13-24] by giving the rather obscure entity and variable time more weight.

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