CONFINEMENT IN PARTIALLY BROKEN
ABELIAN CHERN-SIMONS THEORIES

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Abstract

Planar Chern-Simons (CS) theories in which a compact abelian gauge group $U(1) \times U(1)$ is spontaneously broken to $U(1) \times \mathbb{Z}_N$ are investigated. Among other things, it is noted that the theories just featuring the mixed CS term coupling the broken to the unbroken $U(1)$ gauge fields in general exhibits an interesting form of confinement: only particles carrying certain multiples of the fundamental magnetic vortex flux unit and certain multiples of the fundamental charge of the unbroken $U(1)$ gauge field can appear as free particles. Adding the usual CS term for the broken $U(1)$ gauge fields does not change much. It merely leads to additional Aharonov-Bohm interactions among these particles. Upon introducing the CS term for the unbroken $U(1)$ gauge fields, in contrast, the confinement phenomenon completely disappears.
1 Introduction

The spectrum and the Aharonov-Bohm interactions described by all conceivable 2+1 dimensional Chern-Simons theories in which every compact $U(1)^k$ are broken down to a finite cyclic subgroup by means of the Higgs mechanism have recently been established in [1]. Although briefly mentioned, the interesting possibility that some $U(1)$ gauge groups remain unbroken was not explored there. The purpose of this paper is to give a complete description of these partially broken theories. We will exclusively work in 2+1 dimensional Minkowski space with signature $(+,−,−)$. Greek indices run from 0 to 2 and spatial components are labeled by Latin indices. Natural units in which $\hbar = c = 1$ are used throughout.

2 The model

For convenience, we will focus on the simplest example of a partially broken 2+1 dimensional abelian Chern-Simons (CS) theory, namely that in which the compact gauge group $U(1) \times U(1)$ is spontaneously broken down to $U(1) \times \mathbb{Z}_N$. The most general action we can write down for such a theory is of the form

$$S = \int d^3x \left( \mathcal{L}_{YMH} + \mathcal{L}_{\text{matter}} + \mathcal{L}_{\text{CS}} \right),$$

where $A^{(i)}_\kappa$ with $i = 1, 2$ denote two compact $U(1)$ gauge fields with coupling constant $e^{(i)}$. All repeated Greek and Latin indices are summed over. Except where otherwise stated this summation convention holds for the rest of the paper. The Higgs field $\Phi$ carries charge $Ne^{(2)}$, i.e. $\mathcal{D}_\kappa \Phi = \left( \partial_\kappa + iNe^{(2)}A^{(2)}_\kappa \right) \Phi$. It is endowed with a nonvanishing vacuum expectation value $v$ through the well-known potential $V(\Phi) = \frac{\lambda}{4}(|\Phi|^2 - v^2)^2$ with $\lambda, v > 0$. Thus the compact gauge group $U(1) \times U(1)$ is spontaneously broken down to $U(1) \times \mathbb{Z}_N$ at the energy scale $M_H = v\sqrt{2\lambda}$. To proceed, the charges introduced by the matter currents $j^{(1)}$ and $j^{(2)}$ in (2.3) are quantized as $q^{(1)} = \int d^2x j^{(1)} = n^{(1)}e^{(1)}$ and $q^{(2)} = \int d^2x j^{(2)} = n^{(2)}e^{(2)}$ with $n^{(1)}, n^{(2)} \in \mathbb{Z}$. We will also assume the presence of Dirac monopoles which implies that the topological masses in (2.4) satisfy the quantization conditions (see [1] and references therein)

$$\mu^{(i)} = \frac{p^{(i)}e^{(i)}e^{(i)}}{\pi} \quad \text{and} \quad \mu^{(12)} = \frac{p^{(12)}e^{(1)}e^{(2)}}{\pi} \quad \text{with} \quad p^{(i)}, p^{(12)} \in \mathbb{Z}. \quad (2.5)$$

As an exception to the rule, there is no summation over the repeated index $(i)$ in this case.

2.1 $\mu^{(1)} \neq 0$

In the following, the main emphasis will be on the effective low energy ($E \ll M_H$) or equivalently the effective long distance ($r \gg 1/M_H$) physics described by (2.1). We will
first take the topological mass $\mu^{(1)}$ to be nonzero and return to the special case $\mu^{(1)} = 0$ in section 2.2.

In the low energy regime the Higgs field $\Phi$ takes groundstate values everywhere, i.e. $\Phi(x) = v \exp(i\sigma(x))$. Thus the low energy action is obtained by the following replacement in (2.7)

$$\mathcal{L}_{\text{YMH}} \overset{\leftrightarrow}{\longrightarrow} \frac{-1}{4} F^{(i)\kappa\nu} F_{(i)\kappa\nu} + \frac{M_A^2}{2} \tilde{A}_\kappa \tilde{A}_\kappa, \quad (2.6)$$

with $\tilde{A}_\kappa := A^{(2)}_\kappa + \partial_\kappa \sigma / N e^{(2)}$ and $M_A := N e^{(2)} v \sqrt{\Sigma}$. Varying this effective low energy action w.r.t. the two gauge fields yields the field equations

$$\partial_\nu F^{(1)\kappa\nu} = j^{(1)\kappa} - \mu^{(1)} \epsilon^{\kappa\nu\tau} \partial_\nu A^{(1)}_{\tau} - \frac{\mu^{(12)}}{2} \epsilon^{\kappa\nu\tau} \partial_\nu A^{(2)}_{\tau}, \quad (2.7)$$

$$\partial_\nu F^{(2)\kappa\nu} = j^{(2)\kappa} - \mu^{(2)} \epsilon^{\kappa\nu\tau} \partial_\nu A^{(2)}_{\tau} - \frac{\mu^{(12)}}{2} \epsilon^{\kappa\nu\tau} \partial_\nu A^{(1)}_{\tau} + j^{\kappa}_{\text{scr}}, \quad (2.8)$$

with $j^{\kappa}_{\text{scr}} := - M_A^2 \tilde{A}_\kappa$. It is easily verified that these equations imply that both gauge fields in this CS Higgs medium are massive.

There are three independent particle-like sources in this theory, namely the quantized matter charges $q^{(1)} = n^{(1)} e^{(1)}$ and $q^{(2)} = n^{(2)} e^{(2)}$ and the magnetic vortices corresponding to topologically stable finite energy solutions. In the low energy regime such a field configuration can be idealized as a point $x_0$ in the plane where the Higgs field vanishes: $\Phi(x_0) = 0$. Around this point the Higgs field makes a noncontractible winding in the vacuum manifold. That is, $\Phi(x) = v \exp(i\sigma(x))$ for $x \neq x_0$ and $j^\gamma_{\gamma} dl \partial_\tau \sigma = 2 \pi a$ with $\gamma$ a loop enclosing $x_0$ and $a \in \mathbb{Z}$ to the render Higgs field single valued. For finite energy, the covariant derivative of the Higgs field should vanish away from $x_0$, i.e.

$$D_{\tau} \Phi(x \neq x_0) = 0 \implies \tilde{A}_{\tau}(x \neq x_0) = 0. \quad (2.9)$$

So the holonomy in the Goldstone boson field $\sigma$ is accompanied by a holonomy in the gauge field $A^{(2)}_{\kappa}$. The well-known conclusion is that the vortices carry a quantized magnetic flux $\phi^{(2)} = \oint_{\gamma} dl A^{(2)}_{\tau i} = \oint_{\gamma} dl \partial_\tau \sigma / N e^{(2)} = 2 \pi a / N e^{(2)}$, with $a \in \mathbb{Z}$, located at some point $x_0$ in the plane.

Both the matter charges and the magnetic vortices enter the effective field equations (2.7) and (2.8) describing the physics away from the locations of the vortices. The matter charges enter by means of the matter currents $j^{(1)\kappa}$ and $j^{(2)\kappa}$ and the vortices through the magnetic flux current $- \frac{1}{2} \epsilon^{\kappa\nu\tau} \partial_\nu A^{(2)}_{\tau}$. From these equations we learn that both these matter currents and this flux current generate electromagnetic fields, which are screened at large distances by an induced appropriate combination of the screening current $j^{\kappa}_{\text{scr}}$ and the flux current $- \frac{1}{2} \epsilon^{\kappa\nu\tau} \partial_\nu A^{(1)}_{\tau}$. To be specific, the Gauss’ laws following from (2.7) and (2.8) read

$$Q^{(1)} = q^{(1)} + \mu^{(1)} \phi^{(1)} + \frac{\mu^{(12)}}{2} \phi^{(2)} = 0, \quad (2.10)$$

$$Q^{(2)} = q^{(2)} + \mu^{(2)} \phi^{(2)} + \frac{\mu^{(12)}}{2} \phi^{(1)} + q_{\text{scr}} = 0, \quad (2.11)$$

with $Q^{(i)} := \int d^2 x \, \partial_j F^{(i)j0}$ the Coulomb charges which both vanish since both $U(1)$ gauge fields are massive. The Gauss’ law (2.10) indicates that the long range Coulomb fields
generated by a particle \((q^{(1)}, q^{(2)}, \phi^{(2)})\) being a composite of the quantized matter charges \(q^{(1)}\) and \(q^{(2)}\) and a vortex \(\phi^{(2)}\) are screened by an induced screening flux \(\phi^{(1)}\). From (2.11), we subsequently infer that the long range Coulomb fields \(F^{(2)} j^0\) generated by this screening flux \(\phi^{(1)}\) and the charge \(q^{(2)}\) and flux \(\phi^{(2)}\) carried by this particle are screened by a screening charge \(q_{\text{scr}} := \int d^2 x j_{\text{scr}}^0\) induced in the Higgs condensate \([2]\). Note that although the matter charges \(q^{(1)}\) and \(q^{(2)}\) and the magnetic flux \(\phi^{(2)}\) are necessarily quantized in this spontaneously broken compact gauge theory, both the screening flux \(\phi^{(1)}\) and screening charge \(q_{\text{scr}}\) can in principle take any real value. In other words, for each particle \((q^{(1)}, q^{(2)}, \phi^{(2)})\) in the spectrum of this theory there always exists a screening flux \(\phi^{(1)}\) and screening charge \(q_{\text{scr}}\) such that the long range Coulomb fields indeed vanish. As we will see in section 2.2 for \(\mu^{(1)} = 0\) this is no longer the case.

Since the long range electromagnetic fields are completely screened there are no classical long range interactions among the particles \((q^{(1)}, q^{(2)}, \phi^{(2)})\). The long range interactions we are left with are the quantum mechanical Aharonov-Bohm (AB) interactions due to \(\mu\) and the matter couplings (2.3) and the CS couplings (2.4). Specifically, a counterclockwise monodromy \(\mathcal{R}^2\) of a particle \((q^{(1)}, q^{(2)}, \phi^{(2)})\) and a remote particle \((q^{(1)'}, q^{(2)'}, \phi^{(2)'})\) leads to the AB phase (e.g. [1])

\[
\mathcal{R}^2 = \exp \left( iq^{(i)} \phi^{(i)'}' + iq^{(i)'} \phi^{(i)} + i\mu^{(i)} \phi^{(i)'} \phi^{(i)'} + i\frac{\mu^{(12)}}{2} \left( \phi^{(1)'} \phi^{(2)'} + \phi^{(1)'} \phi^{(2)} \right) \right), \tag{2.12}
\]

whereas a counterclockwise braiding of two remote identical particles \((q^{(1)}, q^{(2)}, \phi^{(2)})\) gives rise to the AB phase

\[
\mathcal{R} = \exp \left( iq^{(i)} \phi^{(i)} + i\mu^{(i)} \phi^{(i)} \phi^{(i)} + i\frac{\mu^{(12)}}{2} \phi^{(1)'} \phi^{(2)} \right). \tag{2.13}
\]

A couple of remarks are pertinent here. First of all, for each particle \((q^{(1)}, q^{(2)}, \phi^{(2)})\) in the spectrum there exists an anti-particle \((\bar{q}^{(1)}, \bar{q}^{(2)}, \bar{\phi}^{(2)})\) such that the pair may decay into the vacuum. In other words, the topological proof of the spin-statistics connection (see for instance [1] and references given there) applies to this theory. Thus the quantum statistics phase (2.13) is the same as the spin factor obtained by a counterclockwise rotation over an angle of \(2\pi\) of the particle \((q^{(1)}, q^{(2)}, \phi^{(2)})\). Secondly, a crucial observation [2] in the derivation of the AB phases (2.12) and (2.13) is that in contrast to the screening fluxes \(\phi^{(1)}\) the screening charges \(q_{\text{scr}}\) attached to the particles do not couple to the AB interactions. The point is that the screening charges \(q_{\text{scr}}\) do not only couple to the holonomy in the gauge field \(A^{(2)}_\kappa\) around a remote vortex \(\phi^{(2)}\), but also to the holonomy in the Goldstone boson field \(\sigma\). This is immediate from (2.3). Let \(j_{\text{scr}}^\kappa := -M^2 A^{(2)}\) be the screening current associated with some screening charge \(q_{\text{scr}}\). The second term at the l.h.s. of (2.6) couples this current to the combined field \(A^{(2)}_\kappa\) around the remote vortex: \(j_{\text{scr}}^\kappa \tilde{A}^{\kappa}\). As we have seen in (2.3), away from the location \(x_0\) of the vortex the holonomies in the gauge fields and the Goldstone boson are related such that \(\tilde{A}^{\kappa}\) strictly vanishes. Consequently, as long as the screening charge stays away from the location of the vortex, the interaction term \(j_{\text{scr}}^\kappa \tilde{A}^{\kappa}\) vanishes and therefore does not generate an AB phase in the process of taking a screening charge around a remote vortex.

Henceforth, the particles \((q^{(1)} = n^{(1)} e^{(1)}, q^{(2)} = n^{(2)} e^{(2)}, \phi^{(2)} = 2\pi a/Ne^{(2)})\) will be labeled as \((n^{(1)}, n^{(2)}, a)\). In terms of these integral quantum numbers the AB phases (2.12)
and (2.13) become

\[ R^2 = \exp \left( \frac{2\pi i}{N} (n^{(2)}a' + n^{(2)}a + 2p^{(2)}Naa') - \frac{\pi i}{p^{(1)}} (n^{(1)} + \frac{p^{(12)}}{N}a)(n^{(1)}' + \frac{p^{(12)}}{N}a') \right), \quad (2.14) \]

\[ R = \exp \left( \frac{2\pi i}{N} (n^{(2)}a + \frac{p^{(2)}}{N}aa) - \frac{\pi i}{2p^{(1)}} (n^{(1)} + \frac{p^{(12)}}{N}a)(n^{(1)} + \frac{p^{(12)}}{N}a') \right), \quad (2.15) \]

where we substituted the values of the screening fluxes \( \phi^{(1)} \) and \( \phi^{(1)'} \) following from the Gauss’ law (2.10) and the quantization of the topological masses (2.5). Note that under these long range AB interactions the integral charge label \( n^{(2)} \) becomes a \( Z_N \) quantum number.

Let us now turn to the Dirac monopoles that may be introduced in this compact gauge theory. There are two species carrying the quantized magnetic charges \( g^{(1)} = 2\pi m^{(1)}/e^{(1)} \) and \( g^{(2)} = 2\pi m^{(2)}/e^{(2)} \) with \( m^{(1)}, m^{(2)} \in Z \). In this 2+1 dimensional Minkowski setting these monopoles are instantons tunneling between states with flux difference \( \Delta \phi^{(1)} = 2\pi m^{(1)}/e^{(1)} \) and \( \Delta \phi^{(2)} = -2\pi m^{(2)}/e^{(2)} \). From the Gauss’ laws (2.10) and (2.10) we infer that these flux tunnelings are accompanied by charge tunnelings. Specifically, in terms of our favourite integral charge and flux labels the tunnelings induced by the two distinct minimal monopoles read

\[ \begin{align*}
\text{monopole (1):} & \quad \begin{cases}
n^{(1)} &\mapsto n^{(1)} + 2p^{(1)} \\
n^{(2)} &\mapsto n^{(2)} + p^{(12)}
\end{cases} \\
\text{monopole (2):} & \quad \begin{cases}
a &\mapsto a - N \\
n^{(1)} &\mapsto n^{(1)} + p^{(12)} \\
n^{(2)} &\mapsto n^{(2)} + 2p^{(2)}
\end{cases}
\end{align*} \]

(2.16) (2.17)

By substituting (2.16) and (2.17) in (2.14) and (2.15), respectively, we learn that these local tunneling events are invisible to the long range monodromies with the other particles \( (n^{(1)'}, n^{(2)'><a') \) in the spectrum of this theory and that two particles connected by either one of these monopoles have the same quantum statistics phase or equivalently the same spin factor. The conclusion is that for fixed \( p^{(1)} \neq 0 \) the effective low energy spectrum of the theory compactifies to \( (n^{(1)}, n^{(2)}, a) \) with \( n^{(2)}, a \in 0, 1, \ldots, N-1 \) and \( n^{(1)} \in \{ 0, 1, \ldots, 2p^{(1)} - 1 \} \). Here, it is of course understood that the modulo \( N \) calculus for the flux quantum number \( a \) and the modulo \( 2p^{(1)} \) calculus for the charge quantum numbers \( n^{(1)} \) involve the charge jumps displayed in (2.16) and (2.17) respectively. Thus all in all there are just a finite number \( 2p^{(1)}N^2 \) of different stable particles in this theory.

The different 2+1 dimensional CS actions for a compact gauge group \( G \) are known [3] to be classified by the cohomology group \( H^4(BG, Z) \). A straightforward calculation [4] for the compact gauge group \( G \simeq U(1) \times U(1) \) for example reveals the isomorphism \( H^4(B(U(1) \times U(1)), Z) \simeq Z \times Z \times Z \). This is in agreement with the fact that the most general CS action we can write down for two compact \( U(1) \) gauge fields is of the form (2.4). That is to say, the integral CS parameters \( (p^{(1)}, p^{(2)}, p^{(12)}) \) in (2.3) label the different elements of \( H^4(B(U(1) \times U(1)), Z) \). For the compact gauge group \( G \simeq U(1) \times Z_N \), in turn, we arrive [5] at the identity \( H^4(B(U(1) \times Z_N), Z) \simeq Z \times Z_N \times Z_N \) which indicates that

\[ \footnote{It should be noted that the Dirac monopoles in this CS theory actually only appear as monopole/anti-monopole pairs linearly confined by a string representing the wordline of the particle created/annihilated by the monopole/anti-monopole in these pairs [4, 5].} \]
two of the three integral CS parameters in our spontaneously broken model (2.12) become cyclic with period $N$. An evaluation of the AB phases (2.14) and (2.15) shows that this is indeed the case. To be specific, the result of shifting $p^{(2)} \mapsto p^{(2)} + N$ in (2.14) and (2.15) is the same as keeping $p^{(2)}$ fixed and replacing $(n^{(1)}, n^{(2)}, a)$ by $(n^{(1)}, [n^{(2)} + a], a)$ where the rectangular brackets denote modulo $N$ calculus in the range $0, 1, \ldots, N - 1$. Something similar holds for the integral CS parameter $p^{(12)}$. The effect of shifting $p^{(12)} \mapsto p^{(12)} + N$ in (2.14) and (2.15) is the same as keeping $p^{(12)}$ fixed and replacing $(n^{(1)}, n^{(2)}, a)$ by $([n^{(1)} + a], n^{(2)}, a)$ where the rectangular brackets denote modulo $2p^{(1)}$ calculus in the range $0, 1, \ldots, 2p^{(1)} - 1$. In other words, if we just look at the long distance physics, then the CS parameters $p^{(2)}$ and $p^{(12)}$ are indeed cyclic with period $N$. That is, up to a relabeling of the particles the theories corresponding to the integral CS parameters $(p^{(1)}, p^{(2)} + N, p^{(12)})$ and that defined by the CS parameters $(p^{(1)}, p^{(2)}, p^{(12)} + N)$ describe the same spectrum and the same AB interactions as that with CS parameters $(p^{(1)}, p^{(2)}, p^{(12)})$. As an aside, since the number of particles in the spectrum is proportional to $p^{(1)}$ it is clear that this CS parameter does not exhibit any kind of periodicity.

2.2 $\mu^{(1)} = 0$

The partially broken CS theories (2.1) with $\mu^{(1)} = 0$ and $\mu^{(12)} \neq 0$ (or equivalently $p^{(1)} = 0$ and $p^{(12)} \neq 0$) are special. Due to the fact that the screening flux $\phi^{(1)}$ disappears from the Gauss’ law (2.10), in general part of the naively expected spectrum becomes confined. To be concrete, after substituting $\mu^{(1)} = 0$ in (2.10) along with the quantization (2.3) of the topological mass $\mu^{(12)}$ and the quantizations $q^{(1)} = n^{(1)} e^{(1)}$ and $\phi^{(2)} = 2 \pi a / Ne^{(2)}$ with $n^{(1)}, a \in \mathbb{Z}$ of the matter charges and the magnetic flux carried by the vortices, respectively, we end up with the condition

$$n^{(1)} = - \frac{p^{(12)} a}{N}.$$  (2.18)

So in contrast to the case $\mu^{(1)} \neq 0$ the integral charge and flux quantum numbers $n^{(1)}$ and $a$ cease to be independent in this theory. It turns out to be most transparent to keep the flux quantum number $a$ and to consider $n^{(1)}$ as an ‘induced’ screening charge. Since all the variables in (2.18) are integers, it is not guaranteed that there exists a screening charge $n^{(1)} \in \mathbb{Z}$ for every flux $a \in \mathbb{Z}$ such that this relation is satisfied. In fact, only if the flux quantum number $a$ is a multiple of $N/ \gcd(N, p^{(12)})$ (with $\gcd(N, p^{(12)})$ denoting the greatest common divisor of $N$ and $p^{(12)}$) there exists an integral screening charge $n^{(1)}$ such that the Gauss law (2.18) can be obeyed. In other words, only particles carrying magnetic flux $a$ being a multiple of $N/ \gcd(N, p^{(12)})$ appear as free particles in the theory. For particles carrying flux $a$ different from multiples of $N/ \gcd(N, p^{(12)})$, in turn, it is impossible to satisfy the Gauss’ law (2.18). From (2.10) it then follows that these particles would be surrounded by unscreened long range Coulomb fields $F^{(1)}$ corresponding to diverging Coulomb energy in this massive gauge theory. The conclusion is that particles carrying fluxes different from multiples of $N/ \gcd(N, p^{(12)})$ are confined, i.e. they do not appear as free particles.

Besides our favourite flux quantum number $a$ the particles in this theory may in principle be endowed with two other independent internal quantum numbers, namely the charge quantum number $q^{(2)} = n^{(2)} e^{(2)}$ with $n^{(2)} \in \mathbb{Z}$ and the flux quantum number $\phi^{(1)}$ which can take any real value as long as the Gauss’ law (2.11) is satisfied. However, upon
plugging the Gauss’ law (2.10) with $\mu^{(1)} = 0$ into (2.12) and (2.13) with $\mu^{(1)} = 0$, we see that the long range AB interactions are actually completely independent of the flux $\phi^{(1)}$ carried by the particles. So if we are just interested in the long distance physics of the theory we may nicely forget about possible fluxes $\phi^{(1)}$ attached to the particles and simply label these as $(n^{(2)}, a)$. With (2.11), it is then readily checked that in terms of these two independent integral quantum numbers the AB phases (2.12) and (2.13) can be cast in the form

$$\mathcal{R}^2 = \exp \left( \frac{2\pi i}{N} (n^{(2)} a' + n^{(2)' a} + \frac{2p^{(2)}}{N} a a') \right),$$

(2.19)

$$\mathcal{R} = \exp \left( \frac{2\pi i}{N} (n^{(2)} a + \frac{p^{(2)}}{N} a a) \right).$$

(2.20)

Since the flux quantum number $a$ is a multiple of $N/\gcd(N, p^{(12)})$, the integral charge label $n^{(2)}$ becomes a $\mathbb{Z}_{\gcd(N, p^{(12)})}$ quantum number under these long range interactions. Furthermore, in the presence of the minimal Dirac monopole (2.17) the flux quantum number $a$ is conserved modulo $N$. As it should, the local combined tunnelings (2.17) induced by this monopole are again unobservable to the monodromies (2.19) with all the particles in the spectrum and two particles connected by this monopole carry the same quantum statistics phase or equivalently spin factor (2.20). The same obviously holds for the tunneling (2.11) with $p^{(1)} = 0$ induced by the other minimal monopole. Thus the spectrum of this theory only consists of a total number of $(\gcd(N, p^{(12)}))^2$ different stable particles which can be labeled as $(n^{(2)}, a)$ with $n^{(2)} \in \{0, 1, \ldots, \gcd(N, p^{(12)}) - 1\}$ and $a \in 0, N/\gcd(N, p^{(12)}), \ldots, N - N/\gcd(N, p^{(12)}).

It is easily verified that the result of shifting $p^{(1)} \mapsto p^{(1)} + N$ in (2.19) and (2.20) is the same as keeping $p^{(2)}$ fixed and replacing $(n^{(2)}, a)$ by $([n^{(2)} + a], a)$ where the rectangular brackets denote modulo $\gcd(N, p^{(12)})$ calculus in the range $0, 1, \ldots, \gcd(N, p^{(12)}) - 1$. Hence, as in the previous section for $p^{(1)} \neq 0$ we infer that also for $p^{(1)} = 0$ the integral CS parameter $p^{(2)}$ is cyclic with period $N$ if we are only concerned with the long distance physics described by the theory. To proceed, the AB interactions (2.13) and (2.20) are obviously independent of the integral CS parameter $p^{(12)}$. In fact, the only difference between two theories labeled by different values for $p^{(12)}$ is the number $\gcd(N, p^{(12)})$ of unconfined fluxes in the spectrum. Since $\gcd(N, p^{(12)} + N) = \gcd(N, p^{(12)})$ we see that the theory with CS parameter $p^{(12)}$ and that with $p^{(12)} + N$ describe the same spectrum and AB interactions. Thus although the argument is somewhat different as that for the case $p^{(1)} \neq 0$ in the previous section, also for $p^{(1)} = 0$ the CS parameter $p^{(12)}$ is cyclic with period $N$ if we just consider the long distance physics.

For completeness’ sake, for $p^{(1)} = 0$ and $p^{(12)} = 0$ the two compact $U(1)$ gauge theories obviously decouple. In the absence of monopoles the unbroken $U(1)$ gauge theory is in the Coulomb phase, so besides the screened $\mathbb{Z}_N$ charge/flux composites $(n^{(2)}, a)$ with $n^{(2)}, a \in 0, 1, \ldots, N - 1$ of the broken $U(1)$ (CS) gauge theory, the spectrum now also consists of quantized charges $q^{(1)} = n^{(1)} e^{(1)}$ with $n^{(1)} \in \mathbb{Z}$ carrying long range Coulomb fields $F^{(1)j0}$. In the presence of monopoles for the unbroken $U(1)$ gauge field, in turn, the charges $q^{(1)} = n^{(1)} e^{(1)}$ are linearly confined \(\bar{\mathbb{Z}}\). Hence, the spectrum of free particles in this case just consists of the $\mathbb{Z}_N$ charge/flux composites $(n^{(2)}, a)$ with $n^{(2)}, a \in 0, 1, \ldots, N - 1$.\]
3 Concluding remarks

In this paper we have established the spectrum and Aharonov-Bohm interactions for all possible 2+1 dimensional Chern-Simons (CS) theories in which the compact gauge group $U(1) \times U(1)$ is broken down to the subgroup $U(1) \times \mathbb{Z}_N$ via the Higgs mechanism. Among other things, we found that the theories just featuring the mixed CS term in which the broken and unbroken $U(1)$ gauge fields are coupled generally exhibit an interesting form of confinement. Only particles carrying certain multiples of the fundamental vortex flux unit and certain multiples of the fundamental charge coupling to the unbroken $U(1)$ gauge field appear as free particles. Adding the standard CS term for the broken $U(1)$ gauge fields does not change much. It just leads to additional Aharonov-Bohm phases among these unconfined particles. Upon adding the CS term for the unbroken $U(1)$ gauge fields, in contrast, the foregoing confinement phenomenon completely disappears from the theory. Along the way the tunneling properties of the Dirac monopoles that we may add to these compact theories were also addressed. Moreover, it has been argued that the integral CS parameters labeling the mixed CS term and the standard CS term for the broken $U(1)$ gauge fields are cyclic with period $N$ if we are only considering the long distance physics described by these theories. No such periodicity arises for the integral CS parameter labeling the CS terms for the unbroken $U(1)$ gauge fields.

To conclude, with the results reported in [1] the generalization of the above discussion to planar CS theories with more than two (un)broken compact $U(1)$ gauge fields is straightforward. An interesting question remains whether these (partially) broken compact abelian CS theories play a role in the setting of the fractional quantum Hall effect.

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Note added

When this work was completed a paper [7] appeared which has some overlap with the discussion of section 2.2.

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