Chiral Quark Model with Configuration Mixing.

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Abstract

The implications of one gluon exchange generated configuration mixing in the Chiral Quark Model ($\chi$QM$_{gcm}$) with SU(3) and axial U(1) symmetry breakings are discussed in the context of proton flavor and spin structure as well as the hyperon $\beta$-decay parameters. We find that $\chi$QM$_{gcm}$ with SU(3) symmetry breaking is able to give a satisfactory unified fit for spin and quark distribution functions, with the symmetry breaking parameters $\alpha = .4$, $\beta = .7$ and the mixing angle $\phi = 20^\circ$, both for NMC and the most recent E866 data. In particular, the agreement with data, in the case of $G_A/G_V$, $\Delta_8$, $F$, $D$, $f_s$ and $f_3/f_8$, is quite striking.

It is well known that the chiral quark model ($\chi$QM) \cite{1, 2, 3} with SU(3) symmetry is not only able to give a fair explanation of “proton spin crisis” \cite{4} but is also able to account for the $\bar{u} - \bar{d}$ asymmetry \cite{3, 4, 7} as well as the existence of significant strange quark content $\bar{s}$ in the nucleon when the asymmetric octet singlet couplings are taken into account \cite{8}. Further, $\chi$QM with SU(3) symmetry is also able to provide fairly satisfactory explanation for various quark flavor contributions to the proton spin \cite{4}, baryon magnetic moments \cite{3, 8} as well as the absence of polarizations of the antiquark sea in the nucleon \cite{10, 11}. However, in the case of hyperon decay parameters the predictions of the $\chi$QM are not in tune with the data \cite{12}, for example, in comparison to the experimental numbers .21 and 2.17 the $\chi$QM with SU(3) symmetry predicts $f_3/f_8$ and $\Delta_3/\Delta_8$ to be $1\over 3$ and $5\over 3$ respectively. It has been shown \cite{10, 13} that when SU(3) breaking effects are taken into consideration within $\chi$QM, the predictions of the $\chi$QM regarding the above mentioned ratios have much better overlap with the data.

It is well known that constituent quark model (CQM) with one gluon mediated configuration mixing gives a fairly satisfactory explanation of host of
low energy hadronic matrix elements [14, 15, 16]. Besides providing a viable explanation for some of the difficult cases of photohelicity amplitudes [17], it is well known that one gluon generated configuration mixing is also able to provide viable explanation for neutron form factor [16, 18], which cannot be accommodated without configuration mixing in CQM. Therefore, it becomes interesting to examine, within the \( \chi \)QM, the implications of one gluon mediated configuration mixing for flavor and spin structure of nucleon. In particular, we would like to examine the nucleon spin polarizations and various hyperon \( \beta \)-decay parameters, violation of Gottfried sum rule, strange quark content in the nucleon, fractions of quark flavor etc. in the \( \chi \)QM with configuration mixing \( (\chi \text{QM}_{gcm}) \), with and without symmetry breaking. Further, it would be interesting to examine whether a unified fit could be effected for spin polarization functions as well as quark distribution functions or not.

For the sake of readability as well to facilitate the discussion, we detail the essentials of \( \chi \text{QM}_{gcm} \) discussed earlier by Harleen and Gupta [19]. The basic process, in the \( \chi \)QM, is the emission of a Goldstone Boson (GB) which further splits into \( q\bar{q} \) pair, for example,

\[
q_\pm \rightarrow GB^0 + q_\mp' \rightarrow (q\bar{q}') + q_\mp'.
\]

The effective Lagrangian describing interaction between quarks and the octet GB and singlet \( \eta' \) is

\[
\mathcal{L} = g_8 \bar{q} \phi q,
\]

where \( g_8 \) is the coupling constant,

\[
q = \begin{pmatrix} u \\ d \\ s \end{pmatrix}
\]

and

\[
\phi = \begin{pmatrix} \pi^0 + \beta \eta' + \zeta \eta' \\ \pi^- \\ -\pi^0 + \beta \eta' + \zeta \eta' \\ \alpha K^- \\ \alpha K^0 \\ -\beta \eta' + \zeta \eta' \end{pmatrix}.
\]

SU(3) symmetry breaking is introduced by considering different quark masses \( m_s > m_{u,d} \) as well as by considering the masses of Goldstone Bosons to be non-degenerate \( (M_{K,\eta} > M_\pi) \) [16, 13, 20], whereas the axial U(1) breaking is introduced by \( M_{\eta'} > M_{K,\eta} \) [3, 11, 13, 20]. The parameter \( a = |g_8|^2 \) denotes the transition probability of chiral fluctuation of the splittings \( u(d) \rightarrow d(u) + \pi^+(-) \),
whereas $\alpha^2 a$ denotes the probability of transition of $u(d) \rightarrow s + K^{-(0)}$. Similarly $\beta^2 a$ and $\zeta^2 a$ denote the probability of $u(d, s) \rightarrow u(d, s) + \eta$ and $u(d, s) \rightarrow u(d, s) + \eta'$ respectively.

The one gluon exchange forces [14] generate the mixing of the octet in $(56, 0^+)_{N=0}$ with the corresponding octets in $(56, 0^+)_{N=2}$, $(70, 0^+)_{N=2}$ and $(70, 2^+)_{N=2}$ harmonic oscillator bands [15]. The corresponding wave function of the nucleon is given by

$$|B> = (|56, 0^+ >_{N=0} cos\theta + |56, 0^+ >_{N=2} sin\theta)cos\phi$$
$$+ (|70, 0^+ >_{N=2} cos\theta + |70, 2^+ >_{N=2} sin\theta)sin\phi. \quad (3)$$

In the above equation it should be noted that $(56, 0^+)_{N=2}$ does not affect the spin-isospin structure of $(56, 0^+)_{N=0}$, therefore the mixed nucleon wave function can be expressed in terms of $(56, 0^+)_{N=0}$ and $(70, 0^+)_{N=2}$, which we term as non trivial mixing [21] and is given as

$$|8, \frac{1}{2} > = cos\phi|56, 0^+ >_{N=0} + sin\phi|70, 0^+ >_{N=2}, \quad (4)$$

where

$$|56, 0^+ >_{N=0,2} = \frac{1}{\sqrt{2}}(\chi' \phi' + \chi'' \phi'')\psi^s, \quad (5)$$

$$|70, 0^+ >_{N=2} = \frac{1}{2}[(\psi'' \chi' + \psi' \chi'')\phi' + (\psi' \chi' - \psi'' \chi'')\phi'']. \quad (6)$$

The spin and isospin wave functions, $\chi$ and $\phi$, are given below

$$\chi' = \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow), \quad \chi'' = \frac{1}{\sqrt{6}}(2 \uparrow\downarrow - \uparrow\downarrow - \downarrow\uparrow),$$

$$\phi'_p = \frac{1}{\sqrt{2}}(udu - duu), \quad \phi''_p = \frac{1}{\sqrt{6}}(2uud - udu - duu),$$

$$\phi'_n = \frac{1}{\sqrt{2}}(udd - dud), \quad \phi''_n = \frac{1}{\sqrt{6}}(uud + dud - 2duu).$$

For the definition of the spatial part of the wave function, $(\psi^s, \psi', \psi'')$ as well as the definitions of the spatial overlap integrals, we refer the reader to references [18, 21].

The contribution to the proton spin in the $\chi QM_{gcm}$, using the wavefunctions defined in Equations (4)-(6), can be written as
\[ \Delta u = \cos^2\phi \left[ \frac{4}{3} - \frac{a}{3} (7 + 4\alpha^2 + \frac{4}{3}\beta^2 + \frac{8}{3}\zeta^2) \right] + \sin^2\phi \left[ \frac{2}{3} - \frac{a}{3} (5 + 2\alpha^2 + \frac{2}{3}\beta^2 + \frac{4}{3}\zeta^2) \right], \]

\[ \Delta d = \cos^2\phi \left[ -\frac{1}{3} - \frac{a}{3} (2 - \alpha^2 - \frac{1}{3}\beta^2 - \frac{2}{3}\zeta^2) \right] + \sin^2\phi \left[ \frac{1}{3} - \frac{a}{3} (4 + \alpha^2 + \frac{1}{3}\beta^2 + \frac{2}{3}\zeta^2) \right], \]

and

\[ \Delta s = -a\alpha^2, \]

the essential details of the derivation are presented in the Appendix A. The SU(3) symmetric calculations can easily be obtained from Equations (7), (8), (9) by considering \( \alpha, \beta = 1 \). The corresponding equations can be expressed as

\[ \Delta u = \cos^2\phi \left[ \frac{4}{3} - \frac{a}{9} (37 + 8\zeta^2) \right] + \sin^2\phi \left[ \frac{2}{3} - \frac{a}{9} (23 + 4\zeta^2) \right], \]

\[ \Delta d = \cos^2\phi \left[ -\frac{1}{3} - \frac{2a}{9} (\zeta^2 - 1) \right] + \sin^2\phi \left[ \frac{1}{3} - \frac{a}{9} (16 + 2\zeta^2) \right], \]

and

\[ \Delta s = -a. \]

After having examined the effect of one gluon exchange inspired configuration mixing on the spin polarizations of various quarks \( \Delta u, \Delta d \) and \( \Delta s \), we can calculate the following quantities

\[ G_A/G_V = \Delta_3 = \Delta u - \Delta d, \]

\[ \Delta_8 = \Delta u + \Delta d - 2\Delta s. \]

Similarly the hyperon \( \beta \) decay parameters \([22, 23, 24, 25]\) can also be expressed in terms of the spin polarization functions, for example,

\[ \Delta_4 = \Delta u - \Delta d = F + D, \]

\[ \Delta_8 = \Delta u + \Delta d - 2\Delta s = 3F - D. \]

Before we present our results it is perhaps desirable to discuss certain aspects of the symmetry breaking parameters employed here. As has been considered by Cheng and Li \([3]\), the singlet octet symmetry breaking parameter
ζ is related to $\bar{u} - \bar{d}$ asymmetry [3, 5, 6], we have also taken ζ to be responsible for the $\bar{u} - \bar{d}$ asymmetry in the χQM with SU(3) symmetry breaking and configuration mixing. Further the parameter ζ is constrained [5, 6, 20] by the expressions $\zeta = -0.7 - \beta/2$ and $\zeta = -\beta/2$ for the NMC and E866 experiments respectively, which essentially represent the fitting of deviation from Gottfried sum rule [3].

In Table 1, we have presented the results of our calculations pertaining to spin polarization functions $\Delta u$, $\Delta d$, $\Delta s$ and related parameters whereas, in Table 2, the corresponding hyperon β-decay parameters dependent on spin polarizations functions have been presented. First of all we have carried out a χ² fit for χQM with SU(3) symmetry breaking and configuration mixing for the spin polarization functions $\Delta u$, $\Delta d$, $\Delta s$, $G_A/G_V$ and other related parameters as well as quark distribution functions. The value of the mixing angle is taken to be $\phi \simeq 20^o$, a value dictated by consideration of neutron charge radius [16, 18]. In the table, however, we have considered a few more values of the mixing parameter $\phi$ in order to study the variation of spin distribution functions with $\phi$. The parameter $a$ is taken to be 0.1, as considered by other authors [3, 5, 11, 20]. The symmetry breaking parameters obtained from χ² fit are $\alpha = .4$ and $\beta = .7$, both for the data corresponding to most recent E866 [4] as well as NMC [3]. Further, while presenting the results of SU(3) symmetry breaking case without configuration mixing ($\phi = 0^o$), we have used the same values of parameters $\alpha$ and $\beta$, primarily to understand the role of configuration mixing for this case. The SU(3) symmetry calculations based on Equations (10), (11) and (12) are obtained by taking $\alpha = \beta = 1$, $\phi = 20^o$ and $\alpha = \beta = 1$, $\phi = 0^o$ respectively for with and without configuration mixing. For the sake of completion, we have also presented the results of CQM with and without configuration mixing. The spin polarization functions of which can easily be found from equations (7), (8) and (9), for example,

$$\Delta u = \cos^2\phi\frac{4}{3} + \sin^2\phi\frac{2}{3},$$  \hspace{0.5cm} (17)

$$\Delta d = \cos^2\phi\frac{1}{3} + \sin^2\phi\frac{1}{3},$$  \hspace{0.5cm} (18)

and

$$\Delta s = 0.$$  \hspace{0.5cm} (19)

A general look at Table 1, makes it clear that we have been able to get an excellent fit to the spin polarization data for the values of symmetry breaking parameters $\alpha = .4$, $\beta = .7$ obtained by χ² minimization. It is perhaps desirable to mention that the spin distribution functions $\Delta u$, $\Delta d$, $\Delta s$ show
much better agreement with data when the contribution of anomaly \[26\] is included. Similarly in Table 2, we find that the success of the fit obtained with \(\alpha = .4, \beta = .7\) hardly leaves anything to desire. The agreement is striking in the case of parameters F and D. We therefore conclude that the \(\chi\)QM with SU(3) and axial U(1) symmetry breakings along with configuration mixing generated by one gluon exchange forces provides a satisfactory description of the spin polarization functions and the hyperon \(\beta\) decay parameters.

In order to appreciate the role of configuration mixing in affecting the fit, we first compare the results of CQM with those of CQM\(_{\text{gcm}}\) \([19]\). One observes that configuration mixing corrects the result of the quantities in the right direction but this is not to the desirable level. Further, in order to understand the role of configuration mixing and SU(3) symmetry with and without breaking in \(\chi\)QM, we can compare the results of \(\chi\)QM with SU(3) symmetry to those of \(\chi\)QM\(_{\text{gcm}}\) with SU(3) symmetry. Curiously \(\chi\)QM\(_{\text{gcm}}\) compares unfavourably with \(\chi\)QM in case of most of the calculated quantities. This indicates that configuration mixing alone is not enough to generate an appropriate fit in \(\chi\)QM. However when \(\chi\)QM\(_{\text{gcm}}\) is used with SU(3) and axial U(1) symmetry breakings then the results show uniform improvement over the corresponding results of \(\chi\)QM with SU(3) and axial U(1) symmetry breakings. In particular, the agreement with data, in the case of \(G_A/G_V\), \(\Delta_8\), F, D, \(f_s\) and \(f_3/f_8\), is quite striking. To summarize the discussion of these results, one finds that both configuration mixing and symmetry breaking are very much needed to fit the data within \(\chi\)QM.

In view of the fact that flavor structure of nucleon is not affected by configuration mixing, it would seem that the results of \(\chi\)QM with SU(3) breaking will be exactly similar to those of \(\chi\)QM\(_{\text{gcm}}\) with SU(3) breaking. However, as mentioned earlier, one of the purpose of the present communication is to have a unified fit to spin polarization functions as well as quark distribution functions, therefore we have calculated the quark distribution functions with the symmetry breaking parameters obtained by \(\chi^2\) fit in the case of \(\chi\)QM with symmetry breaking and configuration mixing both for NMC and E866 data. To that end, we first mention the quantities which we have calculated. The basic quantities of interest in this case are the unpolarized quark distribution functions, particularly the antiquark contents given as under \([20]\)

\[
\bar{u} = \frac{1}{12}[(2\zeta + \beta + 1)^2 + 20]a, \tag{20}
\]

\[
\bar{d} = \frac{1}{12}[(2\zeta + \beta - 1)^2 + 32]a, \tag{21}
\]

\[
\bar{s} = \frac{1}{3}[(\zeta - \beta)^2 + 9\alpha^2]a. \tag{22}
\]
The quark numbers in the proton are presented as

\[ u - \bar{u} = 2, \quad d - \bar{d} = 1, \quad s - \bar{s} = 0, \tag{23} \]

where the quark and the antiquark numbers of a given flavor, in the quark sea, are equal.

There are important experimentally measurable quantities dependent on the above distributions. The deviation from the Gottfried sum rule [7] is one such quantity which measures the asymmetry between the \( \bar{u} \) and \( \bar{d} \) quarks in the nucleon sea. In the \( \chi \)QM the deviation of Gottfried sum rule from 1/3rd is expressed as

\[ \left[ \int_0^1 dx \frac{F_2^p(x) - F_2^n(x)}{x} - \frac{1}{3} \right] = 2/3 (\bar{u} - \bar{d}). \tag{24} \]

Similarly the \( \bar{u}/\bar{d} \), which can be measured through the ratio of muon pair production cross sections \( \sigma_{pp} \) and \( \sigma_{pn} \), is also an important parameter which gives an insight into the \( \bar{u}, \bar{d} \) content [29]. The other quantities of interest is the quark flavor fraction in a proton, \( f_q \), defined as

\[ f_q = \frac{q + \bar{q}}{\sum_q (q + \bar{q})}, \tag{25} \]

where q’s stand for the quark numbers in the proton. Also we have calculated the ratio of the total strange sea to the light antiquark contents given by

\[ \frac{2\bar{s}}{\bar{u} + \bar{d}}, \tag{26} \]

and the ratio of the total strange sea to the light quark contents given by

\[ \frac{2\bar{s}}{u + d}. \tag{27} \]

The above mentioned quantities based on quark distribution functions have been calculated using the set of parameters, \( \alpha = .4 \) and \( \beta = .7 \), which minimizes the \( \chi^2 \) fit for the spin distribution functions and quark distribution functions in the \( \chi \)QM with SU(3) symmetry breaking as well as configuration mixing. The results of our calculations are presented in Table 3. The general survey of Table 3 immediately makes it clear that the success achieved in the case of spin polarization functions is very well maintained in this case also. Apparently it would seem that \( \chi \)QM_{gcm} with SU(3) symmetry breaking would not add anything to the success in \( \chi \)QM with SU(3) symmetry breaking. However, as has been mentioned earlier also that one of the purpose of the present communication is to have a unified fit to spin and quark distribution functions so we have presented the results of our calculations with same symmetry.
breaking parameters for quark distribution functions. The calculated values hardly leave anything to be desired both for the NMC and E866 data. Our results show considerable improvement in the case of ratio of the total strange sea to the light antiquark contents \(\frac{n_{3\bar{s}}}{n_{\bar{d}+u}}\) whereas there is a big improvement in the case of ratio of the total strange sea to the light quark contents \(\frac{n_{3\bar{s}}}{n_{\bar{u}+d}}\), the strange flavor fraction \(f_s\) and \(f_3/f_8\) in comparison to \(\chi_{QM}\) with SU(3) symmetry.

It is perhaps desirable to compare our results with those of Cheng and Li, derived by considering SU(3) and axial U(1) breakings \([13]\). Before we compare our results, it needs to be mentioned that Cheng and Li have considered the NMC data only. It may be mentioned that in the present calculations, for the NMC data, the parameter \(\zeta\) is constrained by the relation \(\zeta = -0.7 - \beta/2\), following from the fitting of \(\bar{u}/\bar{d}\), considered by other authors as well \([20]\), whereas Cheng and Li do not put any restriction on \(\zeta\). Therefore, from Table 4, it can be seen that except for \(\bar{u}/\bar{d}\), there is a broad agreement between the two models. However, one must keep in mind that the \(\chi_{QM_{gcm}}\) is able to give a good fit to the E866 data as well.

To summarize, we have investigated the implications of configuration mixing and SU(3) symmetry breaking, for proton spin and flavor structure. We find that \(\chi_{QM_{gcm}}\) with SU(3) symmetry breaking is able to give a satisfactory unified fit for spin and quark distribution functions, with the symmetry breaking parameters \(\alpha = .4\), \(\beta = .7\) and the mixing angle \(\phi = 20^\circ\), both for NMC as well as the most recent E866 data. In particular, the agreement in the case of \(G_A/G_V\), \(\Delta_8\), \(F\), \(D\), \(f_s\) and \(f_3/f_8\), is quite striking. For a better appreciation of the role of configuration mixing, we have also carried out corresponding calculations in the case of CQM with configuration mixing and also in \(\chi_{QM}\) with SU(3) symmetry and SU(3) symmetry breaking without configuration mixing, the latter being carried out with the same values of the symmetry breaking parameters, \(\alpha = .4\) and \(\beta = .7\). It is found that configuration mixing improves the CQM results, however in the case of \(\chi_{QM}\) with SU(3) symmetry the results become worse. The situation changes completely when SU(3) symmetry breaking and configuration mixing are included simultaneously. Thus, it seems that both configuration mixing as well as symmetry breaking are very much needed to fit the data within \(\chi_{QM}\).

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APPENDIX

The spin structure of a nucleon, following Linde et al. [20], is defined as

\[ B \equiv \langle B | N | B \rangle, \]

where \( |B\rangle \) is the nucleon wavefunction and \( N \) is the number operator given by

\[ N = n_u u^\uparrow + n_d d^\uparrow + n_s s^\uparrow, \]

where the coefficients of the \( q^{\uparrow \downarrow} \) are the number of \( q^{\uparrow \downarrow} \) quarks. The spin structure of the 'mixed' nucleon, defined through the Equation (4), is given by

\[ \langle 8, \frac{1}{2}^+ | N | 8, \frac{1}{2}^+ \rangle = \cos^2 \phi < 56, 0^+ | N | 56, 0^+ > + \sin^2 \phi < 70, 0^+ | N | 70, 0^+ >, \quad (A1) \]

\[ < 56, 0^+ | N | 56, 0^+ > = \frac{5}{3} u^\uparrow + \frac{1}{3} u^\downarrow + \frac{1}{3} d^\uparrow + \frac{2}{3} d^\downarrow, \quad (A2) \]

and

\[ < 70, 0^+ | N | 70, 0^+ > = \frac{4}{3} u^\downarrow + \frac{2}{3} u^\uparrow + \frac{2}{3} d^\downarrow + \frac{1}{3} d^\uparrow, \quad (A3) \]

where we have used Equations (5) and (6) of the text.

In the \( \chi \)QM, the basic process is the emission of a Goldstone Boson which further splits into \( q\bar{q} \) pair as mentioned in Equation (1) of the text. Following Linde et al. the spin structure after one interaction can be obtained by substituting for every quark, for example,

\[ q^\uparrow \rightarrow P_q q^\uparrow + |\psi(q^\uparrow)|^2, \]

where \( P_q \) is the probability of no emission of GB from a \( q \) quark and the probabilities of transforming a \( q^{\uparrow \downarrow} \) quark are \( |\psi(q^\uparrow)|^2 \), given as

\[ |\psi(u^\uparrow)|^2 = \frac{a}{6} (3 + \beta^2 + 2 \zeta^2) u^\uparrow + a d^\uparrow + a \beta s^\downarrow, \quad (A4) \]

\[ |\psi(d^\uparrow)|^2 = a u^\downarrow + \frac{a}{6} (3 + \beta^2 + 2 \zeta^2) d^\uparrow + a \beta s^\downarrow, \quad (A5) \]

\[ |\psi(s^\downarrow)|^2 = a \beta^2 u^\uparrow + a \beta d^\downarrow + \frac{a}{3} (2 \beta^2 + \zeta^2) s^\downarrow. \quad (A6) \]

For the definitions of \( \alpha, \beta \) and \( \zeta \) we refer the readers to the text.

Using Equations (A1)-(A6), we obtain
\[
\hat{B} = \cos^2 \phi \left[ \frac{5}{3} (P_u u^\dagger + |\psi(u^\dagger)|^2) + \frac{1}{3} (P_u u^\dagger + |\psi(u^\dagger)|^2) + \frac{1}{3} (P_d d^\dagger + |\psi(d^\dagger)|^2) + \frac{2}{3} (P_d d^\dagger + |\psi(d^\dagger)|^2) \right] \\
+ \sin^2 \phi \left[ \frac{4}{3} (P_u u^\dagger + |\psi(u^\dagger)|^2) + \frac{2}{3} (P_u u^\dagger + |\psi(u^\dagger)|^2) + \frac{2}{3} (P_d d^\dagger + |\psi(d^\dagger)|^2) + \frac{1}{3} (P_d d^\dagger + |\psi(d^\dagger)|^2) \right]. 
\] 
(A7)

The spin polarization for any quark is defined as \(\Delta q = n_{q^\dagger} - n_{q^\dagger} - n_{\bar{q}^\dagger}\). Using the spin structure from Equation (A7) we can calculate the spin polarizations, which come out to be

\[
\Delta u = \cos^2 \phi \left[ \frac{4}{3} - \frac{a}{3} (7 + 4\alpha^2 + \frac{4}{3} \beta^2 + \frac{8}{3} \zeta^2) \right] + \sin^2 \phi \left[ \frac{2}{3} - \frac{a}{3} (5 + 2\alpha^2 + \frac{2}{3} \beta^2 + \frac{4}{3} \zeta^2) \right], 
\]
(A8)

\[
\Delta d = \cos^2 \phi \left[ -\frac{1}{3} - \frac{a}{3} (2 - \alpha^2 - \frac{1}{3} \beta^2 - \frac{2}{3} \zeta^2) \right] + \sin^2 \phi \left[ \frac{1}{3} - \frac{a}{3} (4 + \alpha^2 + \frac{1}{3} \beta^2 + \frac{2}{3} \zeta^2) \right], 
\]
(A9)

and

\[
\Delta s = \cos^2 \phi [-a\alpha^2] + \sin^2 \phi [-a\alpha^2] = -a\alpha^2. 
\]
(A10)

It is interesting to note that, in the case of \(\Delta s\), both \(|56\rangle\) and \(|70\rangle\) wavefunctions are contributing in the same manner. It can be easily understood when one considers Equations (A4) and (A5). As is evident, in these equations both \(u^\dagger\) and \(d^\dagger\) contribute to \(s^\dagger\) in the same manner, similarly \(u^\dagger\) and \(d^\dagger\) contribute to \(s^\dagger\) in the same manner. When this is used along with Equations (A2) and (A3), one can immediately find out that \(\Delta s\) comes out to be \(-a\alpha^2\).

It is interesting to compare the spin polarizations given by Equations (A8) and (A9) to similar ones derived by Linde et al. \cite{20} (Equations B1 and B2) in the case of wavefunction with quark-gluon mixing (Equations 17 and 18 of reference \cite{20}). Since the \(|56\rangle\) part of the wavefunction is same in both the cases, we compare the term corresponding to the \(\sin^2 \phi\) coefficients. In Equations (A8) and (A9), in comparison to the coefficients of \(\sin^2 \phi\), the corresponding term in the case of Linde et al. has proportionality factor of -1/3 despite a different corresponding spin structure. This can be easily understood when one recognises that the contributions to \(\Delta u\) and \(\Delta d\) are proportional to \([\frac{4}{3} u^\dagger - \frac{2}{3} u^\dagger]\) and \([\frac{2}{3} d^\dagger - \frac{1}{3} d^\dagger]\) for the \(|70\rangle\) wavefunction. Interestingly the spin polarizations for the gluonic wavefunction considered by Linde et al., are proportional to \([\frac{8}{9} u^\dagger - \frac{10}{9} u^\dagger]\) and \([\frac{4}{3} d^\dagger - \frac{5}{3} d^\dagger]\), which has a proportionality factor of -1/3 as compared to our \(|70\rangle\) wavefunction. Thus, the contributions of the spin polarizations corresponding to the coefficients of \(\sin^2 \phi\) are proportional.
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Without configuration mixing | With configuration mixing
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| | CQM with SU(3) symmetry | CQM with SU(3) symmetry breaking | CQM\(_{\text{gcm}}\) with SU(3) symmetry | CQM\(_{\text{gcm}}\) with SU(3) symmetry breaking

| \(\Delta u\) | 0.85 ± 0.05 | 1.33 | 0.79 | 0.81 | 0.96 | 0.99 | 20° | 1.26 | 1.27 | 1.28 | 1.29 | \(\chi^2\) \text{minimization in} \(\chi_{\text{QM}}\) with one gluon generated configuration mixing (\(\chi_{\text{QM}}\text{gcm}\)) and SU(3) symmetry breaking

| \(\Delta d\) | -0.41 ± 0.05 | -0.33 | -0.35 | -0.37 | -0.40 | -0.41 | 20° | -0.25 | -0.31 | -0.32 | -0.33 | -0.37 | -0.41

| \(\Delta s\) | -0.07 ± 0.05 | 0.05 | 0.01 | -0.12 | -0.02 | -0.02 | 20° | 1.26 | 1.66 | 1.14 | 1.18 | 1.35 | 1.49

| \(G_A/G_V\) | 1.267 ± 0.0035 | 1.66 | 1.14 | 1.18 | 1.35 | 1.49 | 20° | 1.54 | 1.06 | 1.08 | 1.11 | 1.26 | 1.28

| \(\Delta_8\) | 0.58 ± 0.025 | 1.00 | 1.07 | 1.11 | 1.26 | 1.28 | 1.32 | 1.32

* Values after inclusion of the contribution from anomaly \[26\].

Table 1: The calculated values of spin polarization functions \(\Delta u\), \(\Delta d\), \(\Delta s\), and quantities dependent on these: \(G_A/G_V\) and \(\Delta_8\) both for NMC and E866 data with the symmetry breaking parameters obtained by \(\chi^2\) minimization in the \(\chi_{\text{QM}}\) with one gluon generated configuration mixing (\(\chi_{\text{QM}}\text{gcm}\)) and SU(3) symmetry breaking.
Table 2: The calculated values of hyperon $\beta$ decay parameters in the $\chi$QM with and without configuration mixing as well as with and without SU(3) symmetry breaking for the values of $\alpha$ and $\beta$ obtained by $\chi^2$ minimization in the case of $\chi_{QM_{gem}}$ with SU(3) symmetry breaking for NMC and E866 data.
| Parameter     | Expt value | CQM with SU(3) symmetry | χQM with SU(3) symmetry breaking |
|--------------|------------|-------------------------|--------------------------------|
|               |            | NMC | E866 | NMC | E866 |
| \( \bar{u} \) | .168 .21   | .168 | .21  | .168 | .21  |
| \( \bar{d} \) | .315 .33   | .315 | .33  | .315 | .33  |
| \( \bar{s} \) | .46 .45    | .15  | .10  | .15  | .10  |
| \( \bar{d} - \bar{u} \) | .147 ± .024 [5] | 0    | .147 | .12  | .147 | .12  |
| \( \bar{u}/\bar{d} \) | .51 ± 0.09 [24] | 1    | .53  | .63  | .53  | .63  |
| \( I_G \)     | .235 ± .005 | .33  | .235 | .253 | .235 | .253 |
| \( \frac{2\bar{s}}{\bar{u}+\bar{d}} \) | .477 ± .051 [3] | .9  | 1.66 | .62  | .38  |
| \( \frac{2\bar{s}}{u+d} \) | .099 ± .009 [30] | 0    | .26  | .25  | .09  | .06  |
| \( f_u \)     | .48 .49    | .55  | .56  | .38  | .39  |
| \( f_d \)     | .33 .33    | .38  | .39  | .38  | .39  |
| \( f_s \)     | .10 ± 0.06 [30] | 0    | .19  | .18  | .07  | .05  |
| \( f_3 = f_u - f_d \) |              | .15  | .15  | .17  | .18  |
| \( f_8 = f_s \) |              | .43  | .46  | .79  | .86  |
| \( f_u + f_d - 2f_s \) |              | .21  | .21  | .21  | .21  |
| \( f_3/f_8 \) | .21 ± .005 [3] | .33  | .33  | .33  | .21  | .21  |

Table 3: The calculated values of quark distribution functions and other dependent quantities as calculated in the χQM with and without SU(3) symmetry breaking both for NMC and E866 data, with the same values of symmetry breaking parameters as used in spin distribution functions and hyperon β decay parameters.
### Table 4: Comparison of the results of $\chi QM_{gcm}$ for the quark spin and flavor distribution functions at the mixing angle $\phi = 18^\circ$ with those of Cheng and Li.

| Parameter       | Expt value | Cheng and Li [13] | Present results |
|-----------------|------------|-------------------|-----------------|
| $\Delta u$      | $0.85 \pm 0.05$ [24] | .87               | .87             |
| $\Delta d$      | $-0.41 \pm 0.05$ [24] | -0.41             | -0.37           |
| $\Delta s$      | $-0.07 \pm 0.05$ [24] | -0.05             | -0.06           |
| $G_A/G_V$       | $1.267 \pm 0.0035$ [22] | 1.28              | 1.24            |
| F/D             | .575       | .57               | .60             |
| 3F-D            | .60        | .57               | .62             |
| $\bar{d} - \bar{u}$ | .147 ± .024 [4] | .15               | .147            |
| $\bar{u}/\bar{d}$ | $0.51 \pm 0.09$ [29] | .63               | .53             |
| $\bar{u}/\bar{u}$ | $0.67 \pm 0.06$ [8] | .60               | .62             |
| $\frac{2\bar{s}}{\bar{u} + \bar{d}}$ | $.477 \pm .051$ [30] | .60              | .62             |
| $f_s$           | $.10 \pm 0.06$ [30] | .09               | .07             |
| $f_3/f_8$       | $.21 \pm 0.05$ [3] | .20               | .21             |