Coherent quantum ratchets driven by tunnel oscillations

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received 7 June 2010; accepted in final form 20 July 2010

PACS 05.60.Gg – Quantum transport
PACS 72.70.+m – Noise processes and phenomena
PACS 73.23.Hk – Coulomb blockade; single-electron tunneling

Abstract – We demonstrate that the tunnel oscillations of a biased double quantum dot can be employed as driving source for a quantum ratchet. As a model, we use two capacitively coupled double quantum dots. One double dot is voltage biased and provides the ac force, while the other experiences the ac force and acts as coherent quantum ratchet. The current is obtained from a Bloch-Redfield master equation which ensures a proper equilibrium limit. We find that the two-electron states of the coupled ratchet-drive Hamiltonian lead to unexpected steps in the ratchet current.

The ratchet effect, i.e., the induction of a dc current by an ac force in the absence of any net bias, represents one of the most intriguing phenomena in the field of non-equilibrium transport [1,2]. Its quantum version [3], has been observed in nanostructured two-dimensional electron gases [4], double quantum dots [5], Josephson junctions [6,7], and Josephson junction arrays [8]. In all these experiments, spatially asymmetric potentials are driven by an external ac field stemming from a classical radiation source. We here address the question whether the tunnel oscillations of a single electron can be employed as ac-driving source that induces a sizable ratchet current.

Recently a quantum ratchet has been realised with a double quantum dot driven by the non-equilibrium noise of a close-by quantum point contact (“drive circuit”) [9]. The observed current exhibits characteristic ratchet features such as current reversals and a vanishing current at symmetry points. If the relevant energy levels of the double quantum dot are strongly detuned, inter-dot tunnelling is incoherent and occurs at a rate that can be derived from Fermi’s golden rule [10]. The resulting current is proportional to the noise correlation function and, thus, the ratchet can serve as a frequency-resolved noise detector [10–12]. However, when the detuning becomes of the order of the inter-dot coupling, coherent tunnel oscillations emerge and, thus, an advanced treatment becomes necessary.

In the scenario sketched so far, the drive circuit entails a force on the ratchet, while the corresponding backaction is ignored. Despite being a valid approach for classical driving fields, this becomes inadequate when the driving force stems from a single quantum-mechanical degree of freedom. Measurements of the drive current in the setup of ref. [9] indeed indicate a relevant backaction of the ratchet on the quantum point contact [13]. In turn, the driving can be considered as backaction of the point contact on the double dot [14]. Therefore, a consistent description requires including the drive circuit into the model. Despite the formal similarities with Coulomb drag in single quantum dots [15,16], we focus on the distinctly different regime with strongly biased drive circuit. We demonstrate that there the drive circuit acts as ac force.

We consider the setup sketched in fig. 1, where both the ratchet and the drive circuit are formed by capacitively coupled double quantum dots. Thereby we find even for small capacitive coupling a sizable ratchet current and, moreover, elucidate the role of eigenstates with one electron in the drive circuit and one in the ratchet.

The corresponding Hamiltonian reads\
\[ H = H_{\text{dots}} + \sum_{\ell} V_{\ell} + \sum_{\ell} H_{\ell}, \]

\[ H_{\text{dots}} = \sum_{\ell=1}^{4} \epsilon_{\ell} n_{\ell} - \frac{\Omega}{2} (c_{\ell}^{\dagger} c_{\ell} + c_{\ell}^{\dagger} c_{\ell}^{\prime}) - \frac{\Omega_{\text{fr}}}{2} (c_{\ell}^{\dagger} c_{3} + c_{\ell}^{\dagger} c_{4}) + U(n_{1} n_{3} + n_{2} n_{4}) \] (1)

refers to the two double quantum dots. Each of the four dots \( D_\ell, \ell = 1, \ldots, 4 \) is treated as a single level \( |\ell\rangle \) with onsite energy \( \epsilon_\ell \). The operators \( c_{\ell}^{\dagger} \) and \( c_{\ell} \) create and annihilate, respectively, an electron on dot \( D_\ell \), and \( n_{\ell} = c_{\ell}^{\dagger} c_{\ell} \) is the corresponding number operator. The second and third term constitute electron tunnelling between dots \( D_1 \)
Each dot is coupled to a lead. The tunnel oscillations with bias (chemical potential µ) provide a quantum ratchet (dots in Fig. 1: Capacitively coupled double quantum dots acting as a quantum ratchet (dots + drive circuit (dots + drive circuit) Fig. 1).)

Restricts ourselves to spinless electrons. Further possible restrictions are minor, because the ratchet effect in tunnelling from a lead to a dot has twice the probability of the reversed process. The consequences for the ratchet current, however, are minor, because the ratchet effect in such systems is dominated by internal excitations [17]. Thus in order to focus on the main physics at work, we restrict ourselves to spinless electrons. Further possible charge ratchet effects stemming from spin-orbit coupling are relevant only in the presence of an additional magnetic field [18].

Each dot $D_\ell$ is coupled to a lead $\ell$ fully described by $H_\ell = \sum_q \epsilon_q c_\ell^q c_{\ell q} + \langle c_\ell^q c_{\ell q'} \rangle = \delta(\epsilon_q - \mu_x) \delta_{\ell \ell'} \delta_{qq'}$ with chemical potential $\mu_x$ and the Fermi function $f(x) = \frac{\exp(x/k_B T) + 1}{\exp(x/k_B T) - 1}$. The dot-lead contact is established by the tunnel Hamiltonian $V_\ell = \sum_q V_\ell q c_\ell^q c_{\ell q} + c_{\ell q}^\dagger c_{\ell q'} + c_{\ell q'}^\dagger c_{\ell q}$. We assume within a wide-band limit that all coupling strengths $\Gamma_\ell(\epsilon) = 2\pi \sum_q |V_\ell q|^2 \delta(\epsilon - \epsilon_q)$ are energy independent and symmetric, such that $\Gamma_1 = \Gamma_2 = \Gamma$ and $\Gamma_3 = \Gamma_4 = \Gamma_\ell$.

By established techniques [19], we derive for the reduced density operator of the dots, $\rho$, the Bloch-Redfield master equation (in units with $\hbar = 1$)

$$\dot{\rho} = -i[H_{\text{dots}}, \rho] - tr_{\text{leads}} \int_0^{\infty} d\tau \sum_\ell [V_\ell, [\hat{V}_\ell(-\tau), R]]$$

where $R = \rho \otimes \rho_{\text{leads}}$. The tilde denotes the interaction picture operator $\tilde{\hat{V}}(\tau) = U_0(\tau)^\dagger x U_0(\tau)$ with $U_0(\tau) = \exp[-i(H_{\text{dots}} + \sum_\ell H_\ell t)]$ the propagator in the absence of dot-lead tunnelling. The inter-dot tunnelling, however, has to be included in $U_0$ to ensure compliance with equilibrium conditions [20]. This is here crucial, because otherwise the master equation would predict a spurious current that exceeds the ratchet current by two orders of magnitude —to our knowledge, a so clear failure of neglecting inter-dot tunnelling has never been reported. The central quantities of interest are the currents through the dot-lead contacts defined as the time-derivative of the charge in the respective lead, $I_\ell = -e(d/dt)N_\ell$. This expression is evaluated like the second term of eq. (2).

For the numerical solution of the master equation (2), we need to cope with the interaction picture representation of the tunnelling operators $V_\ell$. For the lead operators, we readily insert $c_\ell q(t) = c_\ell q \exp(-i\epsilon_q t)$, while for the dot operators, we accomplish this task by decomposing both the master equation and the current operators into the eigenstates of $H_{\text{dots}}$. Thus, we have to solve the eigenvalue equation $H_{\text{dots}}|\psi^{(n)}\rangle = E^{(n)}|\psi^{(n)}\rangle$. The complementary index $(n)$ reflects the respective electron number $n = n_{e,\ell} = \langle \hat{n}_{e,\ell}^{(n)} \sum_\ell |n_{e,\ell}\rangle$. Then the master equation assumes the form $\dot{\rho}_{\alpha\beta} = -i[E^{(n)}_{\alpha} - E^{(n)}_{\beta}] \rho_{\alpha\beta} + \sum_{\gamma\delta} \mathcal{L}_{\alpha\beta,\gamma\delta} \rho_{\gamma\delta}$. The full expression for $\mathcal{L}_{\alpha\beta,\gamma\delta}$ is somewhat lengthy so that we do not write it explicitly.

**Stochastic ac driving.** — Before addressing the question how the ratchet acts back on the drive circuit, we work out the scenario in which electrons tunnelling through the drive circuit entail an effective ac force on the ratchet. In doing so, we generalise the treatment of ref. [10] to the case of delocalised ratchet electrons.

The effective ac driving can be obtained from Hamiltonian (1) as follows. An electron on dot $D_1$ shifts the onsite energy $\epsilon_1$ by $U_1$, while $\epsilon_2$ is shifted by $U_2$ if an electron resides on dot $D_4$, i.e., the ratchet detuning $\epsilon = \epsilon_2 - \epsilon_1$ changes by $U_2$, where $\epsilon = n_2 - n_1$. Hence the ratchet acquires the stochastic Hamiltonian $H_{\text{noise}} = \frac{U_2}{2} \xi(n_2 - n_1)$. For its treatment with Fermi’s golden rule, we need to compute the Fourier transformed $\tilde{C}(\omega)$ of the correlation function $C(t) = \langle \xi(t)\xi(0) \rangle$. For large bias voltage $\mu_x - \mu_y \gg \Omega_\ell$, all levels of the drive circuit lie within the voltage window. Then the corresponding Bloch-Redfield equation assumes the Lindblad form $\dot{\rho} = -\frac{i}{\hbar}[H_{\text{dots}} \rho] + \sum_{\alpha\beta,\gamma\delta} \mathcal{L}_{\alpha\beta,\gamma\delta} \rho_{\gamma\delta}$, where $H_{\text{dots}} = -\frac{i}{\hbar} \Omega_\ell c_2^\dagger c_3 + \frac{i}{\hbar} \Gamma_\ell (c_2^\dagger + c_3^\dagger) c_1^\dagger c_1 + \Gamma_\ell (c_1^\dagger + c_1) c_2 c_3^\dagger + \Gamma_\ell (c_2 c_3^\dagger + c_3 c_2^\dagger)$ with $\mathcal{L}_{\alpha\beta,\gamma\delta} = \Gamma_\ell (c_2 c_3^\dagger + c_3 c_2^\dagger)$ and $\mathcal{L}_{\alpha\beta,\gamma\delta} = \Gamma_\ell (c_2^\dagger c_3 + c_3^\dagger c_2)$, where $\xi$ are independent and identically distributed random variables.

By analogy to the pure-ratchet case, we require that $\tilde{C}(\omega) = \frac{\Gamma_\ell}{\hbar} e^{-\omega^2/\hbar^2} (\omega - \Omega_\ell)^{-1} + \Gamma_\ell/\hbar (\omega - \Omega_\ell)^{-1}$, because for the lead to have zero current, pairing scattering to the drive has to be included. The static contribution $\Gamma_\ell/\hbar$ vanishes for $\Omega_\ell \ll \Gamma_\ell$. The ratchet current $I_\ell$ in response to the driving is $I_\ell = -e \langle \xi(t)\xi(0) \rangle$. The resulting ratchet current $I_\ell$ in response to the driving is $I_\ell = -e \frac{\Gamma_\ell}{\hbar} e^{-\omega^2/\hbar^2} (\omega - \Omega_\ell)^{-1} + \Gamma_\ell/\hbar (\omega - \Omega_\ell)^{-1}$, because for the lead to have zero current, pairing scattering to the drive has to be included. The static contribution $\Gamma_\ell/\hbar$ vanishes for $\Omega_\ell \ll \Gamma_\ell$. The ratchet current $I_\ell$ in response to the driving is $I_\ell = -e \langle \xi(t)\xi(0) \rangle$.

The Hamiltonian $H_{\text{noise}}$ induces transitions between the one-electron eigenstates of the ratchet Hamiltonian, $|g\rangle = \cos \theta |1\rangle + \sin \theta |2\rangle$ and $|e\rangle = -\sin \theta |1\rangle + \cos \theta |2\rangle$ with 20007-p2
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Fig. 2: Transitions between the one-electron states $|g\rangle$, $|e\rangle$ and the empty state $|0\rangle$ of the stochastically ac-driven ratchet.

\[
\cos(2\theta) = \frac{\epsilon}{E} \quad \text{and the level splitting} \quad E = (\epsilon^2 + \Omega^2)^{1/2}.
\]

For sufficiently small $U$, a golden-rule calculation with the transition matrix element $\langle e|H_{\text{noise}}|g\rangle = \frac{1}{2} U \xi \sin(2\theta)$ yields the rate $\gamma = \frac{1}{2} U^2 \sin^2(2\theta) C(E)$. Once the electron is in the excited state $|e\rangle$, it may tunnel to lead 1 or to lead 2. In turn, if the ratchet double dot is in the empty state $|0\rangle$, an electron may tunnel from one of the leads to the ground state $|g\rangle$. Since incoherent tunnel rates are proportional to the overlaps $|\langle \ell | s \rangle|^2$, $\ell = 1, 2, s = e, g$, we obtain for transitions between the ratchet states the stochastic process sketched in fig. 2 with the rates $\Gamma^+ = \cos^2 \theta$ and $\Gamma^- = \sin^2 \theta$.

It is straightforward [22] to find for the occupation probabilities the master equation

\[
\frac{d}{dt} \begin{pmatrix} p_0 \\ p_g \\ p_e \end{pmatrix} = \begin{pmatrix} -\Gamma & \Gamma & 0 \\ -\gamma & -\gamma & \gamma \\ 0 & -3\gamma & -(\Gamma + \gamma) \end{pmatrix} \begin{pmatrix} p_0 \\ p_g \\ p_e \end{pmatrix},
\]

and the current $I = e(\Gamma^+ p_e - \Gamma^- p_0)$, where $\Gamma = \Gamma^+ + \Gamma^-$. From the stationary solution $(p_0, p_g, p_e) \propto (\gamma, \Gamma + \gamma, \gamma)$ follows $I = e\gamma(\Gamma^+ - \Gamma^-)/(\Gamma^+ + \Gamma^- + 3\gamma)$. We insert the above expressions for $\gamma$ and $\Gamma^\pm$ and express the mixing angle $\theta$ in terms of $\epsilon$ and $\Omega$ to obtain in the limit $\gamma \ll \Gamma$ the ratchet current

\[
I = \frac{eU^2}{4} \left( \frac{\epsilon^2 + \Omega^2}{\epsilon^2 + \Omega^2} \right)^{3/2} C \left( \sqrt{\epsilon^2 + \Omega^2} \right)
\]

Notice that the second factor represents the essential difference to ref. [10]. Its origin is the delocalisation of the ratchet eigenstates for $\epsilon \lesssim \Omega$. In the present context, this regime is the most intriguing one, because it contains both the current maximum at $\epsilon \approx \Omega$ and the main current reversal at $\epsilon = 0$.

The analytical result (5) already allows an estimate for the size of the ratchet current. The Lorentzian $C$ assumes its maximum $\sim 1/\Gamma_{\text{dr}}$ when the ratchet and the drive circuit are in resonance, $E = \Omega_{\text{dr}}$. The corresponding condition on the second factor of this expression is $\epsilon = \Omega/\sqrt{2}$, so that the maximal ratchet current is roughly $I_{\text{max}} = eU^2/5\Gamma_{\text{dr}}$. Interestingly enough, for the operating regime considered here, this value depends only on the parameters of the drive circuit and on the coupling strength, but not on the ratchet parameters.

In the experiment of ref. [9], the dot-lead coupling is $40 \mu eV$, while the capacitive coupling $U$ is significantly smaller. Assuming $U = 0.2 \mu eV$, we obtain $I_{\text{max}} \approx 0.1 \mu A$, i.e., the appreciable value measured for driving with a quantum point contact [9]. Two double quantum dots with similar geometry and similar tunnel couplings but with the much stronger interaction $U = 20 \mu eV$ have already been realised [23], such that a considerably larger ratchet current should be achievable as well.

Backaction on the drive circuit. – Let us now turn to the treatment of the drive circuit as a quantum system that is affected by the coupling to the ratchet. For this purpose, we compute the currents numerically by solving master equation (2). Figure 3 demonstrates that for the large drive circuit bias $V_{\text{dr}} = 10\Omega/e$, the ratchet current agrees quite well with our prediction (5). In particular, it exhibits a current reversal close to $\epsilon = 0$, while the current maximum is obtained for $\epsilon \approx \pm \Omega$, i.e., in the coherent regime. The lack of perfect symmetry is due to the fact that on average, dot $D_3$ is slightly stronger populated than dot $D_4$. This causes a tiny additional bias, such that the current reversal is shifted from $\epsilon = 0$ to a slightly negative value. The drive current (inset of fig. 3) is influenced by the interaction close to $V_{\text{dr}} \approx \Omega_{\text{dr}}/e$ only. For larger $V_{\text{dr}}$, it stays practically constant, so that within the picture of stochastic ac driving, no change of the ratchet current is expected.

In contrast to this expectation, however, e.g. for $V_{\text{dr}} = 2\Omega/e$ (fig. 3), the ratchet current exhibits steps similar to those of Coulomb blockade. The location of these steps is best visible in the differential transconductance $\partial I/\partial V_{\text{dr}}$, shown in fig. 4(b). They are based on the fact that transitions between states with different electron number require the corresponding energy difference to lie within the voltage window. Thus, we can identify the states that govern the transport yielding a full quantum-mechanical picture of the ratchet mechanism. By investigating the location of the steps upon variation of $\mu_3$ and $\mu_4$, we find that they relate to the transitions marked
in fig. 4(a). All relevant transitions involve two-electron states, since the ratchet current is interaction induced.

A more profound discussion of the transport process requires knowledge about the structure of the eigenstates of Hamiltonian (1). For $U = 0$, they are given by the direct products of the ratchet states $|0\rangle$, $|g\rangle$, $|e\rangle$ and the respective eigenstates of the drive dots, $|0\rangle_d$, $|g\rangle_d$, $|e\rangle_d$. Thus, the one-electron states read $|s,0\rangle = |s\rangle |0\rangle_d$, and $|0, s\rangle = |0\rangle |s\rangle_d$, while the two-electron states are $|s,s'\rangle = |s\rangle |s'\rangle_d$, where $s, s' \in \{g, e\}$. Obviously, the interaction does not affect the one-electron states, while any two-electron state acquires for finite $U$ an admixture of all other two-electron states. From standard perturbation theory follows that the admixture is of the order $U^2$. For the small values of $U$ considered here, the admixture is small as well and, thus, it is appropriate to keep the notation $|s, s'\rangle$. The consequence of the interaction is that an electron moves from $|0\rangle_d$ and $|e\rangle_d$ to the drive dots, while the charge in the ratchet grows by one, i.e., the drive current induces a transition of the type $|g\rangle \rightarrow |g\rangle + \lambda |e\rangle$, where $\lambda = \mathcal{O}(U^2)$. So far, we have assumed that the setup is perfectly symmetric. In an experiment, however, the capacitive couplings are not precisely equal, but only of the same order. The same holds true for the dot-lead tunnel rates. Within the picture of stochastic ac driving, it is obvious that only the relative level shift is relevant, i.e., the sum of the left and the right interaction constant. In order to test how well this holds when backaction is included, we have performed numerical calculations with the interaction constants $U_{12} = 0.2\Omega$ and $U_{23} = 0.3\Omega$ (the indices refer to the dot numbers), which have been chosen such that their sum equals the value of $2U$ in figs. 3 and 4. The result (not shown) is that the position of the current reversal and the height of the steps change by a tiny amount which is not visible on the scales used in the plots. Moreover, we performed calculations with one dot-lead rate of the ratchet increased by 10%, while the other is reduced by the same amount, i.e., with $\Gamma_1 = 0.33\Omega$ and $\Gamma_2 = 0.27\Omega$. Again, all results remain practically the same. The physical reason for this inertia behaviour is that the changed parameters do not affect the excitation within the ratchet circuit, which represents the bottleneck of the transport process. This underlines that our predictions are not sensitive to moderate experimental imperfections.

**Conclusions.** – We have studied a coherent double-dot quantum ratchet similar to that of recent experiments [9,12], but with a driving that stems from tunnel oscillations in a close-by further double quantum dot. These tunnel oscillations turned out to be sufficiently strong and stable to induce ratchet currents of the order of those in related experiments. Moreover, we have found

\[ \text{Fig. 4: (Colour on-line) (a) Eigenenergies of the n-electron states } |\phi_n^{(s)}\rangle \text{ of the coupled double quantum dots for } U = 0.25\Omega. \text{ The arrows mark the transitions that govern the ratchet current. (b) Differential transconductance } \partial I/\partial V_{dr} \text{ highlighting the steps of the ratchet current as a function of the detuning and the bias at the drive circuit. All other parameters are as in fig. 3.} \]
characteristic features such as current reversals which allow steering the current into a direction of choice. Since our model includes both the ratchet and the drive circuit, we were able to investigate how the ratchet acts back on the tunnel oscillations and to draw a full quantum-mechanical picture of the physics at work. This revealed that the states of the drive circuit acquire components with high energies, which represents the relevant back-action by the ratchet. The measurable consequences are unexpected steps in the ratchet current for intermediate bias voltage applied to the drive circuit. Only when the drive voltage exceeds all relevant energy scales of the ratchet, the backaction becomes negligible, such that the picture of “stochastic ac driving” holds.

The ratchet effect is only one phenomenon from the broad field of ac-driven quantum mechanics. We expect the proposed driving mechanism to be so generic that other ac effects can be induced in the same way. This means that tunnel oscillations can generally be employed as on-chip ac-driving source. Thus we are confident that our results will trigger experimental effort in this direction as well as further theoretical studies.

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We thank G. Platero, F. Dominguez, and M. Lunde for helpful discussions. MS acknowledges funding by DAAD.

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