Optical Quantum Random Number Generator

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Abstract

A physical random number generator based on the intrinsic randomness of quantum mechanics is described. The random events are realized by the choice of single photons between the two outputs of a beamsplitter. We present a simple device, which minimizes the impact of the photon counters' noise, dead-time and after pulses.

Random numbers are employed today as well for numerical simulations as for cryptography. Unfortunately computers are not able to generate true random numbers, as they are deterministic systems. Numerical pseudo-random generators rely on complexity [1]. Although such pseudo-random numbers can generally be employed for numerical computation, their use in cryptography, for example to generate keys, is more critical. The only way to get true random numbers, hence true security for crypto-systems, is to build a generator based on a random physical phenomenon [2,3,4]. As quantum theory is intrinsically random, a quantum process is an ideal base for a physical random number generator.

The randomness of a sequence of numbers can be extensively tested, though not proven. It is thus of interest to thoroughly understand the behavior of the random process, so as to gain confidence in its proper random operation. A statistical process, however, is generally hard to analyze because it involves a lot of variables. Fortunately, some quantum processes can be well described with only a few variables, like, for example, the random choice of a single photon between the two outputs of a beamsplitter [5,6,7]. In this paper, we present a simple, easy to use and potentially cheap random number generator based on this quantum process and on the technique of single photon counting. It fulfills the two major requirements for a physical random number generator: low correlations between successive outputs and stability to external perturbations.

The principle of the generator is illustrated in the figure 1. Weak pulses of a 830 nm LED are coupled into a monomode fibre. At the output of the 2 meter long monomode fibre all photons are in the same mode, therefore indistinguishable, irrespective of any thermal fluctuation of the LED. They then impinge on two multimode fibres glued together some mm away from the monomode fiber. Both multimode fibers are coupled to the same photon counter, one of the multimode fiber introducing a 60 ns delay. By detecting the time of arrival of the photon one can determine which path it took. Labeling the short path by '0' and the long by '1' one obtain a sequence of random bits. The generation rate is of approximately 100kHz, corresponding to 0.1 photon per pulse as the LED is pulsed at 1MHz. Note that a Poissonian photon number distribution with mean number 0.1 is a good approximation to the ideal single photon delta-distribution. A FPGA circuit [Xilinx XC 3130] is used to pulse the LED and to detect the coincidences. It features three counters, one for the '0' bits, one for the '1', defined by two 10 ns large time windows corresponding to the two different arrival times. The third counter
measures the rate of thermal noise thanks to a time window outside the photon arrival times. The USB port interface to the computer offers sufficient speed, Plug & Play support and also the necessary power supply. The generator fits in a box of small size (68 x 150 x 188 mm).

As photon counter we use a passively quenched Si-APD [EG&G C30902S] in the Geiger mode [8]. The limited efficiency of the detector is not an issue since the photons which are not detected do not influence the output of the generator. The thermal noise is not troublesome as it should be random, but our goal is to avoid such type of statistical random process. By using a 10 ns coincidence window the contribution of thermal counts is reduced below 0.5% even without cooling the detector. More critical are fast changes of the detector efficiency due to e.g. a ripple on the bias voltage. Recombining the two optical paths on one detector rather than two makes the generator much less sensitive to variabilities of the detectors. Indeed, most of these variations will cancel since they affect in the same way the ‘0’ and ‘1’ events separated by only 60 ns. However, we have to take into account the fact that the detector is not in the same state after a detection as before. Immediately after a detection the bias voltage goes below breakdown and the efficiency of the detector is zero (dead-time). It then increases gradually and reaches its original value only after 1 µs. Hence, for pulse frequencies ≥ 1MHz a two detector scheme would reveal strong correlations, the probability to detect a ‘1’ after a ‘0’ being greater than the probability to detect a ‘0’. In our one detector scheme this effect is mostly eliminated, but to a small correlation due to the difference in detection time of 60 ns. This correlation is limited to the first adjacent bit. It affects only 10% of the bits, as we have adjacent detection only 10% of the time. The correlation can be further reduced by decreasing the pulse rate or by electronically rejecting adjacent detections. We also have to consider the phenomenon of after-pulses: an increased probability of darkcounts immediately after a detection [8]. After-pulses decrease with temperature and as we work at room temperature this effect is not significant.

The raw bits at the output of the generator are not equiprobable, because it is impossible to achieve a perfect 50/50 coupling between the two optical paths. But, in order to obtain a 50/50 distribution, one can unbias the bits by appropriate mathematical procedures. The simplest procedure is that of von Neumann [9], but it’s efficiency is limited to 25%. Fortunately, there are much more effective procedures, in particular that of Y. Peres [10] which achieves the maximal efficiency given by the entropy per bit of the sequence. In our prototype the bits are approximately 40/60 distributed and the unbiasing procedure efficiency is greater than 90%.

In order to check the randomness of the output we applied the autocorrelation test, which measure the correlation between bits at a distance n:

$$
\Gamma(n) = \frac{1}{N} \sum_{i=0}^{N-1} X_i \oplus X_{(i+n) \mod N}
$$

where \{X_i\}_{i=0}^{N-1} is a sequence of N bits. Applying this test on 10^9 raw bits with 1 ≤ n ≤ 2000, we found no particular correlation apart from the case n = 1, which is 5σ bellow the mean. All the other points are normally distributed around the mean value. The correlation between adjacent bits is very small, on the order of 2 · 10^{-4}. It can be explained by the dead time of the detector, as discuss above. It disappears when an unbiasing procedure is applied. Other tests, like frequency, serial and run [11,12], entropy [13] and Maurer’s [14,15], did not reveal any deviation from a perfect random source.

In conclusion, we demonstrated a random number generator using a basic quantum process. Apart from a small correlation between successive bits which is explained and can be eliminated, the generator behaves like a perfect random source. As the time delay between the detections corresponding to a ‘0’ or to a ‘1’ is very small, external perturbations hardly influence the output
of the generator. The prototype is small, potentially cheap, easy to use and fast enough for cryptographic applications.

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**Figure Caption**

Figure 1: Schematic diagram of the random number generator