OpTopNET: A Learning Optimal Topology Synthesizer for Ad-hoc Robot Networks

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Abstract

In this paper, we synthesize a machine-learning stacked ensemble model a vector of which predicts the optimal topology of a robot network. This problem is technically a multi-task classification problem. However, we divide it into a class of multi-class classification problems that can be more efficiently solved. For this purpose, we first compose an algorithm to create ground-truth topologies associated with various configurations of a robot network. This algorithm incorporates a complex collection of nonlinear optimality criteria that our learning model successfully manages to learn. Then, we propose a stacked ensemble model whose output is the topology prediction for the particular robot associated with it. Each stacked ensemble instance constitutes three low-level estimators whose outputs will be aggregated by a high-level boosting blender. The results of the simulations, applying our model to a network of 10 robots, represents over 80% accuracy in the prediction of optimal topologies corresponding to various configurations of this complex optimal topology learning problem.

keywords: Ad-hoc robot networks, Topology learning, Stacked ensemble learning

1 Introduction

The integration of robotics and communication networks has been led to the realization of unprecedented technologies without which many wide-spread applications of robot networks could not be achieved. Robot networks contribute to a multitude of critical missions such as surveillance [16], search and rescue [18], explorations [15], collaborative communications [1], and mobilization of sensor networks [2]. Mobile networks are inherently ad-hoc, meaning one may exploit their communicational utilities without almost any need for immobile infrastructures. A robot network, similar to classical networks, requires a setting of communicational routes between its robots using which the passage of information can be feasible. The overall characterization of robots and their links involved in such routes is known as the topology of their network. A robot network shall constantly constitute a topology through which its robots are, either directly or indirectly, connected to each other. Topology dramatically impacts various aspects of communications in robot networks such as delay, fault tolerance, computation distribution, and connection reliability [3]. Thus, finding the optimal topology, based on specific criteria, associated with a particular configuration of a robot network is often of utmost importance.

Robot networks often enjoy high degrees of freedom associated with their robots which provide enough collective dexterity to perform their tasks. However, the cited rich dynamism must not jeopardize the connectivity of a their network. In particular, topology synthesis is essential in every motion planning step associated with the agents of a robot network. Accordingly, when

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some robots of a network need to move, their final locations have to be checked so that there is at least a topology that can
cover their connectedness in the new configuration of their network. So, topology synthesis is a continual requirement of
robot networks in the course of their operations. However, analytical synthesizes of optimal topologies demand labor-intensive
computations which are inefficient, should one take a massive network or complicated optimality criteria into account. In this
regard, computational complexity of topology checking can exponentially grow in the case of particular topology formations
\[26, 10, 14, 23, 11\], especially cycle topologies.

Chain topologies are a class of formations suitable for robot networks with a single actor robot while the remainder of its peers,
except a base robot, solely contribute to the maintenance of the communication flow between the actor and the base robots \[9\].
However, chain topologies are too restrictive in view of mission dexterity in many applications. Instead, cycle topologies are
legit options for various missions including surveillance, search and rescue, exploration/excavation, and so on. Nonetheless,
online computation of a cycle topology corresponding to a robot network essentially entails the computation of the largest cycle,
known as backbone, available among the robots of that network. This problem is indeed equivalent to checking whether the
graph of the network is Hamiltonian, which is an NP-hard problem \[8, 13\]. So, for a robot network with even a couple of robots,
their optimal topology cannot be analytically achieved efficiently, assuming a set of optimality measures.

To relax the necessity of direct cycle topology computation, we alternatively seek the synthesis of a data-driven predictor trained
by the data of various configurations of a robot network in addition to the optimal topology information associated with those
configurations. Given a new configuration of robots, one can efficiently feed that configuration to the predictor to yield a
predicted cycle topology of the network. We stress that such a topology prediction process may be extremely useful in the
motion planning of a robot network in a communicational perspective. Namely, when a motion step is planned for the robots of a
network, the result of those steps as a new configuration, can be assessed by the topology predictor. If the predictor returns any
(optimal) topology, then the maneuvers of the robots may be allowed. Otherwise, such a set of maneuvers shall be avoided as it
would be highly probable that their execution leads to various detachments of the robots from their network. Put differently, we
seek the solution to the following problem.

**Problem 1.** Suppose an algorithm, associated with a set of optimality criteria, using which one can yield the optimal topology
corresponding to an ad-hoc robot network. Then, given a dataset comprising the ground-truth results of the presupposed
algorithm, synthesize a data-driven model which can predict the optimal topology of an unknown configuration of the intended
ad-hoc robot network.

In the next section, we review the major results in topology learning for ad-hoc networks. Meanwhile, we highlight their
advantages and disadvantages in the context of robot networks.

### 1.1 Related Work

Various data-driven strategies have been employed to learn topological information of ad-hoc networks. As the first class of
methods, convolutional graph networks exhibit remarkable potential to learn graph structures in fully-connected networks using
convolutional graph layers \[5\]. However, these convolutional layers are not capable of properly learning topologies in incomplete
graphs which, are common in general robot networks. These layers are also not ideal artifacts to systematically apply optimality
constraints to learning models.

Unsupervised learning strategies have also been used to perform learning processes without any pre-defined topological
information. For example, in \[6\], an online incremental method was introduced in which agents are sequentially clustered,
but the efficiency of this scheme in the case of larger networks seems to be questionable. Moreover, due to the unsupervised
nature of this method, it only minimizes a distance-based error term associated with network nodes. In this regard, imposing
arbitrary optimality measures is not a strength of this algorithm. A more versatile unsupervised algorithm \[20\] extends detectable
topologies to the class of non-stationary network data. This technique, although, is mostly useful for the memory-driven networks
whose dynamics follow the adaptive resonance theory \[4\]. As well, its results are inherently sub-optimal due to the fuzzy nature
of its algorithm.
Literature also includes learning strategies that are hybrid in terms of the used data structures in the course of topology learning. Namely, a sensor network topology learning approach [12] assumes the availability of a Markovian model describing the communicational interactions between its nodes. This model infers a topology based on the Monte Carlo maximization of the attachment of its nodes to each other. However, the strongly-restrictive assumption of this scheme, say, assuming agents’ coordinates are all fixed, makes it practically inapplicable to robot networks.

Learning network topologies based on computationally-affordable procedures have been studied, e.g., [7, 22], at the cost of targeting sub-optimal topologies. Using reinforcement learning to converge to optimal topologies gave rise to slow and fluctuating convergence to other sub-optimal solutions [25]. Topology analysis in high-frequency networks may safely assume very dense but limited clusters of agents [19]. Thus, the number of clusters in topologies are trivial, and the communication ranges are relatively longer than the distance of agents. In the sequel, despite the ease of computation, there are many optimal solutions which are noticeably different in view of their node adjacency characteristics. So, such methods cannot be extended to the cases where communications are critical in view of link quality similar to robot networks.

Given the review above, we argue that the current topology learning approaches lack sufficient flexibility and generality to be applied to the optimal topology prediction for ad-hoc robot networks. Thus, in the next section, we elucidate our contributions in this paper to fill those gaps, thereby solving Problem 1.

1.2 Contributions

The contributions of this article, systematically aiming at solving Problem 1, are fivefold as follows.

1. The general topology prediction problem is a multi-task classification with which is difficult to deal in terms of achieving high prediction accuracies. So, we transform that problem to a set of multi-class classification problems based on the divide-and-conquer paradigm. So, the topology prediction is distributed among all robots by differently-tuned instances of our learning architecture.

2. We embed the topological information of a robot network in a scalable manner in such a way that the dimensions of the multi-class classification problems are fully specified only based on the number of robots in a network.

3. Topology is generally a complex graph-based notion which cannot be easily expressed without graph data structures. Such graph data structures may not be processed in machine-learning pipelines as efficient as simple vectors of data. Thus, we partition a topology to a backbone cycle and a branch set, so that their information can be efficiently encoded to integer vectors.

4. We preserve topological correlations of robots in the course of distributed topological cluster predictions for each robot despite the distribution of the main multi-task classification problem to a set of multi-class ones associated with different robots.

5. Our learning model successfully predicts the results of the optimal topological label generator procedure of ours which includes complex non-linear structural optimality criteria. So, our learning model exhibits remarkable capabilities of learning different optimality templates in deep representations of robot network data.

To summarize, we collect the spatial coordinates of the robots of a network and the data associated with the optimal topologies of some already-checked configurations of that network. Then, we generate local datasets each of which corresponds to one of the robots and its topological correlations with its peers. We synthesize (train, validate, and test) a set of stacked ensembles of classifiers to solve the multi-class classification tasks associated with all of the robots. This model is capable of predicting the optimal topology of a new unexplored configuration of that robot network given the coordinate of its robots in that configuration.

1.3 Organization

This article is organized as follows. In Section 2, we establish the grounds of the required data which will be later used to synthesize an optimal topology predictor. For this purpose, Section 2.1 includes the definitions associated with the formalism
based on which we model different entities of optimal topology computation and their relations. Using those definitions, Section 2.2 illustrates an algorithm to compute the optimal topology of a given configuration of an ad-hoc robot network. Section 2.3 embeds the topological information of that algorithm’s output to structure a dataset for our further machine learning processing. In Section 3, we illustrate our approach to solve the problem of optimal topology learning for robot networks. To be specific, Section 3.1 covers the methodology of ours to resolve the cited problem statement in the realm of machine learning. Then in Section 3.2, we propose a stacked ensemble network of multi-class classifiers to solve this multi-task problem in a divide-and-conquer manner. Section 4 displays the promising results of the application of our model to a typical ad-hoc robot network. We discuss the challenges and limitations of this research, and future trends of research stemmed from our contributions in Section 5. Finally, our conclusions are drawn in Section 6.

2 Data Engineering Methodologies

2.1 Ground-Truth Optimal Topology Establishment

The area around a typical robot may be radially partitioned into three different regions according to the quality of its communication with other peers which reside in those regions. In particular, communication quality of a robot is maximum with respect to its peers which are located at a distance not farther than a connectivity threshold \( \delta > 0 \). The quality of communication then decreases when the robot is supposed to communicate with a peer whose location is outside of the aforesaid region but closer than the distance \( \delta + \epsilon \), where \( 0 < \epsilon < \delta \) is a tension bound factor. Finally, the robot cannot communicate with its peers whose locations are beyond the boundary of the described region. In view of the discussion, we employ \( \delta \) and \( \epsilon \) to define a robot network.

**Definition 1 [Robot Network].** Let \( \delta \) and \( \epsilon \) be connectivity threshold and tension bound factor, respectively, associated with a set of robots

\[
\mathcal{R} := \{ \mathcal{R}_i \mid i \in I \},
\]

(1)

where \( I \) is an index set to differentiate the elements of \( \mathcal{R} \). Then, 3-tuple \( \mathcal{N} := (\mathcal{R}, \delta, \epsilon) \) represents a robot network. We overload the notation \( \mathcal{R}_i := (x_i, y_i) \) to also denote the planar coordinate of robot \( \mathcal{R}_i \).

The two definitions below specify the robots which are candidates for reliable connections or critical ones to a typical robot. Critical connections are those whose quality is not ideal.

**Definition 2 [Reliable Set].** Let \( \mathcal{N} = (\mathcal{R}, \delta, \epsilon) \) be a robot network. Then, given a robot \( \mathcal{R}_i \in \mathcal{R} \), the reliable set \( \mathcal{R}_i^+ \) is defined as follows.

\[
\mathcal{R}_i^+ := \left\{ r \in \mathcal{R} \setminus \{ \mathcal{R}_i \} \mid \| \mathcal{R}_i - r \| \leq \delta \right\}
\]

(2)

**Definition 3 [Critical Set].** Let \( \mathcal{N} = (\mathcal{R}, \delta, \epsilon) \) be a robot network. Then, given a robot \( \mathcal{R}_i \in \mathcal{R} \), the critical set \( \mathcal{R}_i^- \) is defined as follows.

\[
\mathcal{R}_i^- := \left\{ r \in \mathcal{R} \setminus \{ \mathcal{R}_i \} \mid \delta < \| \mathcal{R}_i - r \| \leq \delta + \epsilon \right\}
\]

(3)

We define the auxiliary notions of link set, associated with a robot network, and connection relation between two robots, as below.

**Definition 4 [Link Set].** Given a robot network \( \mathcal{N} \), its link set reads as the following:

\[
\mathcal{L}_\mathcal{N} := \left\{ (x, y) \mid (\forall i, j \in I) \left[ x \in (\mathcal{R}_i^+ \cup \mathcal{R}_i^-) \land y \in (\mathcal{R}_j^+ \cup \mathcal{R}_j^-) \right] \right\}.
\]

(4)

**Definition 5 [Connection Relation].** If robots \( \mathcal{R}_i \) and \( \mathcal{R}_j \), belonging to robot network \( \mathcal{N} \), are connected via link \( l \in \mathcal{L}_\mathcal{N} \), then the binary connection relation \( \mathcal{C}(i, j) \) (or symmetrically, \( \mathcal{C}(j, i) \)) holds.

Based on these definitions, the graph of a robot network is formalized as follows.
Definition 6 [Robot Network Graph]. Let $\mathcal{N} = (\mathcal{R}, \delta, \epsilon)$ be a robot network. Then, $\mathcal{G}_\mathcal{N} := (V_{\mathcal{G}_\mathcal{N}}, E_{\mathcal{G}_\mathcal{N}})$ is the robot network graph associated with $\mathcal{N}$ such that
\[
\begin{align*}
V_{\mathcal{G}_\mathcal{N}} &:= \mathcal{R}, \\
E_{\mathcal{G}_\mathcal{N}} &:= \mathcal{L}_\mathcal{N},
\end{align*}
\] are the set of vertices and edges of the graph, respectively.

Now, we establish the definition of a backbone cycle of a robot network which is one of the two elements shaping the definition of a robot network topology, as we will later see.

Definition 7 [Backbone Cycle]. Let $G_\mathcal{N}$ be the graph of a robot network $\mathcal{N}$. Then, $\psi_\mathcal{N} := (V_{\psi_\mathcal{N}}, E_{\psi_\mathcal{N}})$ is a backbone cycle of $G_\mathcal{N}$ such that
\[
V_{\psi_\mathcal{N}} \subseteq \left\{ \bigcup_{i \in I} \mathcal{R}_i^{+} \right\},
\]
meaning all vertices of the backbone cycle are reliable with respect to each other.

The relation below paves the way for scoping the communicational reachability of a robot in a backbone cycle by another peer which is not an immediate neighbor of it outside of the backbone cycle.

Definition 8 [Indirect Reachability]. If robot $\mathcal{R}_i$ can communicationally reach backbone cycle vertex $v$ via a succession of its neighboring vertices, then we say $\mathcal{R}_i$ indirectly reaches $v$, i.e., $\mathcal{R}_i \leadsto v$.

The second element of a robot network topology is a branch set the atomic unit defined as follows.

Definition 9 [Branch]. Given a robot network $\mathcal{N}$, branch $b_{\psi_\mathcal{N}}$ with respect to backbone cycle $\psi_\mathcal{N}$ is a link $l \in \mathcal{L}_\mathcal{N}$ between $\mathcal{R}_i, \mathcal{R}_j \in \mathcal{R}$, i.e., preserving $C(i, j)$, if any of the following mutually-exclusive statements holds.
\[
\begin{align*}
(7a) & \quad (\mathcal{R}_i \in V_{\psi_\mathcal{N}}) \land (\mathcal{R}_j \not\in V_{\psi_\mathcal{N}}), \\
(7b) & \quad (\mathcal{R}_i \not\in V_{\psi_\mathcal{N}}) \land (\mathcal{R}_j \in V_{\psi_\mathcal{N}}), \\
(7c) & \quad (\exists v \in V_{\psi_\mathcal{N}}) \left[ (\mathcal{R}_i \leadsto v) \lor (\mathcal{R}_j \leadsto v) \right].
\end{align*}
\]

Remark 1. Relations (7a) and (7b) refer to the robots which are immediately adjacent to a backbone cycle. However, (7c) addresses the orphan robots which can only reach a backbone-cycle robot indirectly via their connections to other robots.

Given the required ingredients defined above, a robot network topology’s definition may read as below.

Definition 10 [Robot Network Topology]. Let $I$ be an index set whose cardinality determines the number of the robots in its associated robot network $\mathcal{N}$. Given, backbone cycle $\psi_\mathcal{N}$, fix a constant $\mathcal{K} \leq |I| - |V_{\psi_\mathcal{N}}|$. Then, considering the branch set $\mathcal{G}_\mathcal{N} \supset \mathcal{B}_\mathcal{N} := \left\{ b_{\psi_\mathcal{N}}^k \right\}_{k=1}^{\mathcal{K}}$, the robot network topology $\mathcal{T}_\mathcal{N} := (\psi_\mathcal{N}, \mathcal{B}_\mathcal{N})$ corresponds to $\mathcal{N}$ if both of the following conditions are simultaneously fulfilled.
\[
\begin{align*}
(8a) & \quad V_{\mathcal{G}_\mathcal{N}} \cup V_{\psi_\mathcal{N}} = V_{\mathcal{G}_\mathcal{N}} \\
(8b) & \quad (\forall v \not\in V_{\psi_\mathcal{N}}) \left( (\exists b_{\psi_\mathcal{N}}^k \in \mathcal{B}_\mathcal{N}) v \in V_{b_{\psi_\mathcal{N}}^k} \right)
\end{align*}
\]

Remark 2. Equation (8a) mandates that the unification of the robots belonging to the backbone cycle and all of the branches must cover all of the robots of the network. Equation (8b) requires that any robot which does not belong to the backbone cycle must be a part of one of the branches of the topology.

The auxiliary notion of robot degree, defined below, is the last required piece to formulate optimality for a robot network topology.

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2In a graph, a cycle is a closed path of successive vertices and links in which neither vertices (except the start/end one) nor links may be repeated [24].
Definition 11 [Robot Degree]. Given a robot \( R_i \), its degree, denoted by \( D_i \), is defined as the cardinality of the set
\[
\left\{ \forall j \in \arg \{ R \setminus \{ R_i \} \} \mid E(i,j) \right\}
\]
which refers to the number of the connections of \( R_i \) to other robots.

Definition 12 [Optimal Robot Network Topology]. A robot network topology is optimal, represented by \( T^* N : = (\psi^*_N, B^*_N) \), if the following conditions are met.

(i) \( \psi^*_N \) is the largest cycle of \( N \);

(ii) All branches of \( B^*_N \) are minimum spanning trees, with respect to the distance between robots, over their robot sets;

(iii) Branch links of \( B^*_N \) are as reliable as possible\(^3\);

(iv) Branch links of \( B^*_N \) are as structurally-distributed as possible, i.e.,
\[
\min_{i, j \in \arg \{ R_i, \arg \{ R_j \} \}} |D_i - D_j|
\]

2.2 Topology Computation Algorithm

In the previous section, we precisely defined what features characterize an optimal robot network topology. In this section, we further develop an algorithm, i.e., Algorithm 1, which computes the aforesaid desired topology associated with a typical robot network. This algorithm particularly provides the ground truth knowledge which we will later benefit from in the course of dataset preparation and predictor synthesis to solve Problem 1.

One needs to feed a set of inputs to the algorithm: first, an index set which distinguishes robots of a robot network; second, the configuration parameters of a network which are its zone range, connectivity threshold, and tension bound factor; and finally a random generator which uniformly generates the locations of the network’s robot. In line 1, a variable is created to contain the robots of the network. Namely, we assign two random numbers as the planar coordinate to each robot, in line 2, followed by a location validation in line 3. Here, if the generated coordinate provides the robot (at least critical) connectedness with respect to at least one of its neighboring peers, then we add the robot to the robot network in line 4. Otherwise, the coordinate is discarded, in line 5, and a new one has to be generated for the robot. Once all robots are assigned to communicationally valid coordinates, reliable and critical sets associated with them are computed, in line 6. In line 7, the robots are sorted based on the cardinalities of their reliable sets. Any equality of those measures, e.g., those expressed by Definition 12, between any pairs of robots makes the cardinalities of their critical sets as the tie-breaker in the course of the quoted sorting process. In the case of the equality of the latter factors, those robots are randomly lined in the current index of the sorted set. Then, the optimal robot network topology may be achieved by computing its constituents. In particular, in lines 8 and 9, we compute its backbone cycle and its branch set, respectively. In the end, we yield the desired optimal robot network topology, in line 10.

2.3 Topology Embedding and Dataset Formation

Algorithm 1, described in the previous section, computes the optimal topology associated with a particular configuration of a robot network. We need to embed the minimum data corresponding to various optimal topology computation scenarios in a dataset for the purpose of further machine-learning-based topology prediction syntheses. In this regard, robot locations of an intended network are the key pieces of information according to which topologies may vary from one configuration of the robot network to another. So, given an index set \( J \) associated with a robot set \( R \), the following row-vector \( \Xi^d \) represents the location information of \( R \).
\[
\Xi^d := \begin{bmatrix} \arg \min_j R^x_j & \arg \min_j R^y_j & \cdots & \arg \max_j R^x_j & \arg \max_j R^y_j \end{bmatrix}_{1 \times 2|J|}
\]

\(^3\)The more reliable connections are embedded into a topology, the more reliable it is.
Algorithm 1: Optimal Topology Computer

**Inputs:** Robot network index set $\mathcal{I}$
- Zone range $Z$
- Connectivity threshold $\delta$
- Tension bound factor $\epsilon$
- Uniform random generator $U(0,Z)$

**Output:** The optimal topology $T^*_N$ of a robot network considering $|\mathcal{I}|$ robots given $Z$, $\delta$, and $\epsilon$

1. $\mathcal{R} \leftarrow \emptyset$
2. **foreach** $i \in \mathcal{I}$ **do**
3. 
4. 
5. 
6. 
7. $\mathcal{R} \leftarrow \text{Sort } \mathcal{R} \text{ based on the cardinalities of } \mathcal{R}_i^+ \text{ and } \mathcal{R}_i^-$ in the priority according to their order ($\forall i \in \mathcal{I}$)

Given $N = (R, \delta, \epsilon)$ and considering the conditions of Definition 12

8. Compute $\psi^*_N$ based on the condition (i)
9. Compute $B^*_N$, fulfilling the conditions (ii), (iii) (if not violating (ii)), and (iv) (if not violating (ii) and (iii))
10. **return** $T^*_N \leftarrow (\psi^*_N, B^*_N)$

We also have to embed the topological information of the algorithm’s output as the labels associated with a robot network configuration in $(1 \times |\mathcal{I}|)$-row-vector $\Xi^t$. Namely, we assign each robot $\mathcal{R}_i$ to a cluster $\Xi^t_i$ such that the following sentences hold.

\[
\begin{align}
\left( \forall \mathcal{R}_i \in \mathcal{R} \right) & \left( (\mathcal{R}_i \in V_{\psi^*_N}) \Rightarrow (\Xi^t_i := i) \right) \tag{12a} \\
\left( \forall \mathcal{R}_i \in \mathcal{R} \right) & \left( (\mathcal{R}_i \notin V_{\psi^*_N}) \land (\exists \mathcal{R}_j \in V_{\psi^*_N})(\mathcal{R}_i \leadsto \mathcal{R}_j) \right) \Rightarrow (\Xi^t_i := j) \tag{12b}
\end{align}
\]

In other words, if a robot belongs to the backbone cycle of its topology, its cluster entry equals its index. Otherwise, its index is that of the backbone-cycle robot which is indirectly reachable by the intended robot. Thus, an augmented data record $\Xi$ may be obtained as follows.

\[
\Xi := \left[ \Xi^d | \Xi^t \right]_{1 \times 3|\mathcal{I}|}
\]

Finally, the overall dataset is a $(n \times 3|\mathcal{I}|)$-matrix including $n$ records of $\Xi$ associated with different configurations of its corresponding robot network.

3 Optimal Topology Predictor Synthesis

3.1 Approach

The problem of predicting the optimal topology of a robot network may be transformed to a classification problem, in which each robot shall be assigned to a particular cluster associated with the overall clusters of its network topology. The dataset structure, introduced in Section 2.2, illustrates that the cluster ID of each robot is its own ID if the robot belongs to the backbone cycle of its topology. Otherwise, its cluster ID shall be that of its nearest indirect neighbor which is a part of the backbone cycle. According to this formulation, one notes that cluster values are non-binary and their majority equals the number of robots in a desired robot network. In this sense, this problem is inherently a multi-task classification problem which is notoriously difficult when the number of labels and their possible values are both relatively large [21]. The optimal topology learning problem belongs to this class of problems because, assuming a robot network including $n$ robots, its dataset has $n$ columns each of which may
Figure 1: Interrelated correlations among robot coordinates in the view of each other’s classifiability. For example, the scattering dependency of the classifiability of the horizontal coordinate component of robot 5, i.e., X5, is displayed with respect to the coordinate components of robots 1 and 2. One can observe that if the coordinates of a particular robot, say, X1-Y1, are taken into account, the separation boundary to classify X5 is different than the cases of considering correlated coordinates, such as X1-Y2 or X2-Y1. In these cases, the classifiability of X5 exhibits more convoluted trends. As we already discussed, optimal topology prediction includes various optimality criteria which can impose complicated constraints to the validity of a separation boundary. So, one can preserve the information regarding such complicated patterns in the course of robot cluster classifications by presenting the total robot network information to each robot’s multi-class classifier. (The numerical specification of the dataset including the data of this analysis is presented in Section 4.1.)

be occupied by any integer of the range \([1, n]\). One also have to encode integer values of cluster IDs to binary vectors before training any classifier. Such an encoding process by itself multiplies the number of columns of the dataset. A multi-task problem based on such a noticeable number of features may require a massive number of records to cover various permutations of all those integer entries so that under-fitting would be less likely to happen. Alternatively, we take a divide-and-conquer strategy into account to transform the multi-task topology prediction of robot network to a set of multi-class classification problems each of which corresponds to the cluster prediction for only one of the robots of a network.

Based on this divide-and-conquer strategy, we replicate the main dataset of the multi-task problem, as many as the number of the robots in a network, such that each replicated dataset only includes the target label columns of the main dataset, but the input data of the main dataset is totally copied to the replicated datasets. We preserve the total input data of the main dataset in all replica datasets because topology synthesis process has to consider the correlations between the robots of a network. In particular, Fig. 1 shows a sample set of correlations between coordinate components of two typical robots. As we already observed in the formulation of Algorithm 1, a robot cannot be efficiently assigned to a cluster solely based on its own coordinate. In the sequel, all robot location data have to be incorporated into the multi-class classifier syntheses for all of the robots of a robot network. Overall, we seek the generation of \(n\) multi-class classifiers each of which associates with a particular robot of a network. Finally, the accumulation of the predictions of all those distributed classifiers is known as the solution to the topology prediction with respect to a configuration of a robot network.

Ensemble learning is a technique to exploit the prediction power of various types of estimators to predict a particular feature. The more complex a machine learning problem is, the more significant the impact of ensemble learning will be. Thus, we propose OpTopNET, a particular network of stacked ensembles of multi-class classifiers each of which performs the optimal topology prediction corresponding to one of the multi-class classification problems described above.
3.2 Architecture

The architecture of OpTopNET is depicted in Fig. 2. This computational pipeline first replicates the main dataset to \( n \) local instances each of which includes the full input data associated with the locations of robots. However, they differ in that each one comprises the cluster labels of only one robot of that network. The label IDs are inherently integers, nonetheless there is no order-wise semantic relation between them. To address this issue, binarized versions of label IDs of the main dataset are taken into account in the local replicas. Then, each local dataset is used to train a particular stacking ensemble of classifiers. In each instance of these aggregated classifiers, the local dataset is, in parallel, fed into three diverse types of classifiers such as a random forest classifier, a \( k \)-neighbors classifier, and a deep neural network classifier. This type selection seeks a sufficiently-diverse set of classifiers whose error types are potentially very different. Thus, this diversity of error types may increase the overall stacked ensemble’s accuracy [27, 17].

Given a classification step, the predictions of those low-level classifiers are employed to train a XGBoost-based blender deep network at the top of each stacked ensemble. Since each high-level blender collects and aggregates the predictions of its low-level classifying peers in the course of a classification task, one expects that the blender’s prediction outperforms those of the low-level classifiers. The output of each stacked ensemble is a predicted cluster associated with the robot which is represented by that ensemble. So, the set including all those local cluster predictions constitutes the overall predicted optimal topology of the network of those robots. Moreover, the overall accuracy of OpTopNET is the average accuracy of its underlying stacked ensembles.

The planned \( k \)-neighbors classifier, random forest classifier, and XGBoost blender are standard artifacts of Scikit-Learn library whose optimally-tuned hyper-parameters are specified in Section 4.2. The deep neural network models of OpTopNET can be efficiently formulated by the sequential models Keras library. The internal structure of the employed deep neural network is rendered in Fig. 3. Namely, a series of dense layers are sequentially connected to each other each of which is batch normalized. We also further regularize the model by adding \( l_2 \) kernels to all of its dense layers. A softmax layer, including 10 neurons, provides the output of each multi-class classification subproblem of ours.
4 Simulations

We exhibit numerical simulations representing the effectiveness of OpTopNET in the prediction of a robot network including 10 robots. For this purpose, we first characterize the computational aspects of the dataset encompassing the data of the intended robot network in Section 4.1. Section 4.2 elaborates on the tuning procedures applied to the hyper-parameters of our model. Put differently, we illustrate the optimization of the OpTopNET model in the course of randomized $k$-fold cross-validations. We finally report the results of our simulations in terms of prediction accuracies.

4.1 Dataset Characterization

Our dataset includes 2000 records of various configurations of a 10-robot network as well as their optimal topologies, computed by Algorithm 1 in the format presented in Section 2.3. We set the zone range as 1, while the values corresponding to connectivity threshold and tension bound factor are 0.5 and 0.1, respectively. We first shuffle the dataset before forking the local datasets associated with each of the multi-class classification problems. As already explained in Section 3.2, each local dataset is associated with the target labels of a particular robot of the network. The label contents are indeed integer cluster IDs which are encoded to binary vectors. We also decrease the index of each robot by one unit so that the ID range [1,10] is substituted with [0,9] to be compatible with the numbering convention of output layers of deep neural networks under Keras framework.

We take separate disjoint splits of local datasets into account for training, validation, and testing purposes of OpTopNET estimators, say, %72, %18, and %10 of the overall number of the available records, respectively.

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$^4$The simulations are performed on a Windows 10 x64 machine supported by a Core i7 1.80 GHz processor, 8GB RAM, and an Intel UHD Graphics 620. The following specific libraries are employed in the course of all performed simulations on Python 3.7.3: Tensorflow and Keras 2.5.0, Scikit-Learn 0.24.2

$^5$The implementation of the dataset generator routine, i.e., Algorithm 1 by which the dataset of this study is generated, may be found in https://git.io/JXKSb.
Table 1: Hyperparameter settings and performance report associated with the synthesized OpTopNET.

| Classifier type | Classifier type | optimal parameters and measures | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | Average accuracy (%) |
|-----------------|-----------------|---------------------------------|---|---|---|---|---|---|---|---|---|---|----------------------|
| Random Forest   | Number of estimators | 100                           |   |   |   |   |   |   |   |   |   |   | 76.2                 |
|                 | Depth            | 5                              |   |   |   |   |   |   |   |   |   |   |                      |
|                 | Accuracy (%)     | 89.0 90.0 88.5 85.0 75.0 74.5 72.5 61.5 64.0 62.0 |   |   |   |   |   |   |   |   |   |   |                      |
| k-Neighbors     | Number of neighbors | 6 5 9 9 9 9 9 9 9 9 |   |   |   |   |   |   |   |   |   |   |   | 76.4                 |
|                 | Weights          | Uniform                        |   |   |   |   |   |   |   |   |   |   |                      |
|                 | Algorithm        | Ball tree Brute Auto Auto Auto Auto Auto Auto Auto |   |   |   |   |   |   |   |   |   |   |   |                      |
|                 | p                | 3 3 1 2 2 2 2 2 2 2 |   |   |   |   |   |   |   |   |   |   |   | 76.4                 |
|                 | Accuracy (%)     | 88.5 93.5 85.0 81.5 76.0 76.5 72.5 64.5 66.5 59.0 |   |   |   |   |   |   |   |   |   |   |   |                      |
| Deep Neural Network | Number of hidden layers | 3 3 3 1 3 4 3 3 4 |   |   |   |   |   |   |   |   |   |   |   | 76.7                 |
|                 | Number of neurons | 149 255 219 145 236 284 296 136 190 239 |   |   |   |   |   |   |   |   |   |   |   |                      |
|                 | Learning rate    | 0.0001                         |   |   |   |   |   |   |   |   |   |   |   |                      |
|                 | Accuracy (%)     | 89.7 91.1 89.5 80.2 78.2 76.2 70.0 65.9 64.1 62.0 |   |   |   |   |   |   |   |   |   |   |   |                      |
| Stacked XGBoost Blender | Number of estimators | 100 100 130 130 130 130 130 180 180 180 |   |   |   |   |   |   |   |   |   |   |   | 81.3                 |
|                 | Max depth        | 4 3 4 3 4 3 4 4 4 5 |   |   |   |   |   |   |   |   |   |   |   |   |                      |
|                 | Booster          | gbtree                          |   |   |   |   |   |   |   |   |   |   |   |                      |
|                 | Learning rate    | 0.0002                          |   |   |   |   |   |   |   |   |   |   |   |                      |
|                 | Accuracy (%)     | 92.4 94.0 91.3 86.1 82.1 80.2 75.2 74.1 68.1 69.0 |   |   |   |   |   |   |   |   |   |   |   |                      |

4.2 Results

In the course of the optimization of the hyperparameters of OpTopNET, selected batch size is 32. In the case of random forest classifiers and k-neighbors classifiers, we use 10 splits of input data to randomly spot the hyperparameters which maximize the accuracy metrics of those estimators. In particular, we sweep the number of estimators and the depth corresponding to random forest classifiers in integer ranges [100, 500] and [3, 10], respectively. For k-neighbors classifiers, number of neighbors and the power parameter p are swept within integer ranges [1, 10] and [1, 3], respectively. Algorithm hyperparameter options are ball tree, kd tree, brute, and auto. Moreover, we assess uniform and distance options for the weights hyperparameter. After performing randomized cross-validations, as Table 1 expresses, the average accuracy performances of the optimally-tuned random forest classifiers and k-neighbors classifiers are %76.2 and %76.4, respectively.

Deep neural network elements of OpTopNET are initialized based on the following specification. Batch normalization momentum associated with the batch normalization layers is 0.999. We take two strategies into account to properly regularize our model. First, for each deep neural network, we apply an $l_2$ regularization factor of 0.01 to every dense layer of that model. Second, we schedule the learning rate dynamics of that model based on the following exponential decay profile

$$\eta(t) := \eta_0 \cdot 10^{-t/s},$$

where $s$ is the number of steps per epoch.

We employ scaled exponential linear unit as the activation function of each deep neural network model, whose kernel is initialized by lecun_normal method to maximize the compatibility between each pair of activation function and kernel initializer. Our Adam optimizer is supported by momentum decay parameter $\beta_1 = 0.9$ and scaling decay parameter $\beta_2 = 0.999$. A sparse categorical cross entropy is considered as the loss function of each deep neural network model. We perform cross-validation over 5 folds of data within 10 iterations for 90 epochs within which hyperparameter optimization is based on accuracy metric. In the course of the hyperparameter tuning, number of hidden layers may very between 1 to 4. Number of the neurons in each dense layer can be any integer between 1 and 300. The lower-bound and the upper bound of the initial learning rate are 0.0001 and 0.03,
respectively. Table 1 illustrates the found optimal hyperparameters as well as the overall average accuracy of the optimally-tuned deep neural network which is 76.7.

Finally, we construct stacked ensembles of each multi-class classifier to boost the performance of the single estimators investigated above. For this purpose, we use the XGBoost library, each instance of which is fed by the optimally-tuned estimators of a specific multi-class classification sub-problem. Max depth hyperparameter may vary between 3 and 10. Booster elements gbtree instances, and the learning rate range are from 0.0001 to 0.5. We then cross-validate the ensemble classifiers over 5 folds of data in the course of 10 iterations. The results are reflected in Table 1 indicating that the overall average accuracy of all stacked ensembles are boosted to 81.1. Accordingly, Fig. 4 specifically demonstrates how the XGBoost-based blender outperforms its low-level estimators in terms of accuracy.

One also observes that the applied regularizations to both deep neural network models and the stacked ensembles are efficient. As an instance for the proper regularization of the optimally-tuned deep neural networks, Fig. 5 renders the validation loss and validation accuracy associated with the deep neural network of robot 2, in which trends of validation loss and validation accuracy are descending and ascending, as expected.

As a validation-based performance example of the synthesized stacked ensembles, one may note Fig. 6 which shows the dynamics of loss and accuracy associated with the stacked ensemble’s blender of robot 7 in the course of both training and cross validation. In particular, the model exhibits no signs of overfitting given the ascending performance dynamics of validation accuracy and also the eventual reduction of validation loss.

We stress that the superior performance of the stacked ensembles compared to deep neural network components may be concluded based on the observation of the convergence of the validation accuracy metrics of stacked ensembles which yield within less number of epochs compared to what deep neural networks need to optimally converge. Also, the loss and accuracy trajectories of stacked ensembles exhibit smoother trajectories compared to those of the deep neural networks, as Fig. 6 typically illustrates. That is indeed because of the constructive contributions of the other components of the stacked ensembles, i.e., random forest classifiers and $k$-neighbors classifiers, to their gradual convergence.

Figure 4: Classification accuracy of stacking classifier and those of its constituent classifiers. (The vertical dashed line represents the overall average accuracy of the ensembles.)
5 Discussion

In the previous section, we displayed the effectiveness of the proposed solution of this paper in predicting optimal topology of ad-hoc robot networks in a data-driven manner. However, it would be beneficial to further investigate the solutions which may ameliorate the current challenges and the limitations of our strategy.

First, as noted in Section 2.3, out method needs to assign a label column to a network dataset as the container of the cluster ID’s of each robot associated with different network configurations. In other words, the more the robots of a network are, the larger the target column cardinality of its corresponding dataset will be. Increasing the number of robots may, then, make the prediction problem extremely challenging because correct topology predictions of all robots in a particular configuration of theirs become practically difficult. In that case, one may achieve sub-optimal topologies. So, a useful improvement would be a topology encoding scheme whose impact on the size of a final dataset less depends on the number of robots in a network. In a similar point of view, one may explore the applicability of OpTopNET on the topology predictions in higher-order spatial dimensions, particularly 3 dimensions. Such a generalization, on the same footing as the increment of the number of robots, increases the size of a dataset which may be a source of computational complications.

Second, this research mainly takes cycle topologies into account because of their prevalence in robotic applications. However, other topologies, in particular those based on chain backbones, may also be beneficial to be studied. Namely, the assessment of OpTopNET on predictions of those topologies may give rise to some amendments to its current architecture.

Third, our definition of optimal topology may be extended to the incorporation of more constraints and/or different optimality criteria. Such a potential variation may lead to new results regarding the adaptivity of OpTopNET with various metrics governing the optimality of topologies.

6 Conclusion

Optimal topology prediction for ad-hoc robot network mandates the learning and inference of complex spatial correlations between robots of a network and intended optimality criteria. In this paper, this complex problem is transformed into a multi-task classification problem in which each class is a cluster ID associated with the topology of a network. Inspired by the divide-and-conquer philosophy in algorithm design, we partition the total multi-task prediction problem corresponding to a whole robot network into a set of multi-class classification problems each of which associates with the prediction of the topological characteristics of its corresponding robot. A stacked ensemble architecture is proposed to solve each of those partitioned multi-class classification problems. Our conducted simulations portrays the effectiveness of the proposed learning methodology.

Figure 5: Accuracy dynamics corresponding to the deep neural network model of robot 2’s of the stacked ensemble.
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