Spin current is a useful concept to describe the effect of the conduction electrons on the dynamics of the magnetic moments in a ferromagnet–normal-metal–ferromagnet trilayer. The interaction with the local exchange field causes the potential felt by the conduction electrons to be spin dependent. In the presence of a spin-dependent potential, the spin current carried by the conduction electrons is not conserved in the ferromagnetic layers. Since the total spin of the system is conserved, the missing spin current must be accounted for as a torque acting on the magnetic moments of the ferromagnets. The torque from non-conservation of the equilibrium spin currents is known as the magnetic exchange (or RKKY) interaction. (There is a direct analogy between the equilibrium spin current flowing in presence of a non-zero angle between the two magnetic moments and the persistent current flowing through a mesoscopic ring in presence of an Aharonov-Bohm flux.) An additional torque is exerted out of equilibrium when a current is passed through the system. Experimentally, this non-equilibrium torque has been shown to be large enough to switch the relative orientation of the two magnetic moments of the two magnetic layers from parallel to antiparallel to each other. In this paper, we report on our investigation of these torques when the trilayer is connected to a superconducting electrode. The non-equilibrium torque has been considered in Ref. where it is shown that the torque can drive the system to a configuration where the two magnetic moments are perpendicular to each other. Here, we address the question whether a similar effect can be expected for the (equilibrium) magnetic exchange interaction.

2 Definition of the spin torque

Following Ref. we consider a trilayer system consisting of two ferromagnetic layers and with a normal metal (N) spacer and one superconducting contact (S), see Fig. The magnetic moment of the layer adjacent to S is considered fixed, e.g., by anisotropy forces. In a lattice formulation, the trilayer is described by the the Bogoliubov-De Gennes (BdG) equation

\[ H_i \Psi_i + t \sum_{\langle ij \rangle} \Psi_j = \epsilon \Psi_i. \]  (1)

Here \( \Psi_i \) is the quasiparticle wavefunction amplitude on site \( i = (i_x, i_y, i_z) \), which has an electron-hole as well as a spin index, \( \Psi_i = (\psi_i^e \uparrow, \psi_i^e \downarrow, \psi_i^h \uparrow, \psi_i^h \downarrow)^T \). Further, \( t \) is the hopping amplitude, ...
The spin current $J_{i,x}$ carried by a quasi-particle at energy $\varepsilon$ and with wavefunction $\Psi$ reads

$$\vec{J}_{i,x,\varepsilon} = -t \ \text{Im} \ \sum_{i_0,i_x} \langle \psi_{i_0,i_x}^d | \vec{\sigma} | \psi_{i_0,i_x}^e \rangle - \langle \psi_{i_0,i_x}^d | \psi_{i_0,i_x}^{hT} | \psi_{i_0,i_x}^{h*} \rangle,$$

with $d = (1,0,0)$, while the equilibrium spin current $\vec{J}_{i,x}^\text{eq}$ is

$$\vec{J}_{i,x}^\text{eq} = -\frac{1}{2} \sum_{\varepsilon > 0} \vec{J}_{i,x,\varepsilon}.$$

Here, $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)^T$ is a vector of Pauli matrices acting on the spin index. Non-conservation of the spin current in $F_a$ and $F_b$ results in a torque $\vec{\tau}_a$ ($\vec{\tau}_b$) exerted on the magnetic moment of $F_a$ ($F_b$) (the origin is chosen in the normal metal spacer separating $F_a$ and $F_b$),

$$\vec{\tau}_a = \vec{J}_{-\infty} - \vec{J}_0, \quad \vec{\tau}_b = \vec{J}_0 - \vec{J}_{+\infty}.$$

### 3 Non-equilibrium torque

For a wide trilayer with a diffusive spacer layer, the main contribution to the non-equilibrium torque lies in the plane spanned by the two magnetic moments ($\vec{h}_a, \vec{h}_b$) (the $xz$ plane in Fig. 1). In the absence of the superconducting contact, the non-equilibrium torque $\tau_{a(b)}^{\text{ne}}$ induced by a current $I$ changes of sign at $\theta = 0, \pi$, thus stabilizing the configurations $\theta = 0$ or $\theta = \pi$ depending of the direction of the current. The presence of the superconductor, however, gives a geometric constraint that allows one to find a relation between $\tau_{a}^{\text{ne}}$ and $\tau_{b}^{\text{ne}}$, from which it follows that $\tau_{a}^{\text{ne}}$ also vanishes at $\theta = \pi/2$. To find this relation, note that the spin current vanishes inside $S$ (for bias voltage $eV < \Delta_0$). Although the ferromagnetic layers do not conserve spin current, they do conserve the majority and minority currents parallel to their exchange fields $\vec{h}_a(b)$, hence

$$\vec{\tau}_a \perp \vec{h}_a, \quad \vec{\tau}_b \perp \vec{h}_b.$$
Since $\vec{J}_x = 0$ and $\vec{h}_b$ points along the $z$-axis, we find from Eq. (3),

$$\vec{J}_0 = J_0 \vec{x}$$  \hspace{1cm} (7)

Finally, we use that, for wide trilayers, $\vec{J}_{-\infty}$ is polarized along $\vec{h}_a$ and therefore, by Eq. (8) gives no contribution to the torque. The torque $\tau_a$ is thus given by the projection of $\vec{J}_0$ to the unit vector perpendicular to $\vec{h}_a$ and hence,

$$\tau_a^{\text{ne}} = \tau_b^{\text{ne}} \cos \theta.$$ \hspace{1cm} (8)

Equation (8) shows that the non-equilibrium torque $\tau_a^{\text{ne}}$ can stabilize the $\theta = \pi/2$ configuration. (Note, however, that the presence of the S contact creates an asymmetry between the torques $\tau_a$ and $\tau_0$, so that it is necessary for this effect that the magnetic moment of F$_b$ is held fixed.)

4 Equilibrium torque

The previous section dealt with the non-equilibrium torque, which lies in the plane spanned by the two magnetic moments $\vec{h}_a$ and $\vec{h}_b$. It was found that $\tau_a^{\text{ne}} = 0$ for $\theta = \pi/2$. We now ask whether the equilibrium torque has the same property.

The answer is negative. Again, we can see this by a geometrical argument. In equilibrium, no spin current flows outside the trilayer on either side, and thus $\vec{\tau}_a^{\text{eq}} = -\vec{\tau}_b^{\text{eq}} = \vec{J}_0$. In combination with Eq. (3), one then finds that $\tau_b^{\text{eq}}$ and $\tau_a^{\text{eq}}$ point out of the plane spanned by $\vec{h}_a$ and $\vec{h}_b$. There is no relation as simple as Eq. (8) for the out-of-plane component of the torque, hence no special behavior at $\theta = \pi/2$ is expected.

There are two ways to compute the equilibrium torque: either by a direct calculation of the equilibrium spin currents in the trilayer, cf. Eq. (4), or from the derivative of the ground state energy with respect to the angle $\theta$,

$$\tau_a^{\text{eq}} = \frac{\partial E}{\partial \theta}.$$ \hspace{1cm} (9)

It should be emphasized that the equilibrium torque involves contributions from energies in the entire conduction band, as opposed to the non-equilibrium torque where only quasiparticle states near the Fermi level play a role.

To illustrate the $\theta$ dependence of $\tau_a^{\text{eq}}$, we have performed a numerical exact diagonalization of the BdG equation (1). We have considered the special case of a one-dimensional trilayer, $i = i_x$ in Eq. (1). In Fig. 2, $\tau_a^{\text{eq}}$ is plotted as a function of $\theta$. We notice that the equilibrium torque (exchange interaction) depends strongly on the value of the superconducting gap $\Delta_0$. This implies that both the direct spin exchange between the two magnets and other contributions that involve the superconductor play a role. For a very large gap $\Delta_0 \gg t$, where $2t$ is the width of the conduction band in $N$, the superconductor acts as a hard wall and the results are qualitatively the same as without superconductivity. However, an interesting regime appear for relatively small values of the gap $\Delta_0 < t$ where the equilibrium torque depends strongly on $\Delta_0$, although the actual $\theta$-dependence of $\tau_a^{\text{eq}}$ is very sensitive to the particular choice of parameters and, for a realistic calculation, would require knowledge of the detailed band structure of the different materials. In the particular example shown in Fig. 2, for $\Delta_0 = 0.1t$ the biquadratic harmonic dominates the exchange coupling, showing that, after eventual fine tuning of parameters, nontrivial values of $\theta$ can be stabilized. (A similar effect may also occur without a superconducting contact, see, e.g., Ref. 8.)

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a This is the case if the majority and minority spins are incoherently transmitted or reflected by the ferromagnetic layers, which is a good assumption for a wide trilayer with many propagating channels at the Fermi level. However, the conclusion (8) also holds for coherent transmission or reflection Ref. 5.
Figure 2: Equilibrium torque $\tau_{eq}$ for a one-dimensional FNF trilayer with one superconducting contact. Torque is shown as a function of the angle $\theta$ between the moments of the two ferromagnetic layers, and for various values the superconducting gap $\Delta_0$, or without superconducting contact, see figure. The system consists of 80 sites, the relative sizes of the different layers being respectively for $N$, $F_a$, $N$, $F_b$, $N$ and $S$: 9, 11, 9, 11, 4 and 36. We set $v_i = \epsilon_F = 0$ and $h_i = 0.8$. All energies are measured in units of $t$.

Acknowledgments

We thank P. Chalsani, A. A. Clerk, E. B. Myers, and D. C. Ralph for useful discussions. This work was supported by the NSF under grant no. DMR 0086509 and by the Sloan foundation.

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