HEATING CLUSTER GAS

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ABSTRACT

It is now generally agreed that some process prevents the diffuse gas in galaxy clusters from cooling significantly, although there is less agreement about the nature of this process. I suggest that cluster gas may be heated by a natural extension of the mechanism establishing the $M_{\text{BH}}$--$\sigma$ and $M_{\text{BH}}$--$M_{\text{bulge}}$ relations in galaxies, namely outflows resulting from super-Eddington accretion on to the galaxy’s central black hole. The black holes in cD galaxies are sporadically fed at unusually high Eddington ratios. These are triggered as the cluster gas tries to cool, but rapidly quenched by the resulting shock heating. This mechanism is close to the optimum efficiency for using accretion energy to reheat cluster gas, and probably more effective than “radio mode” heating by jets, for example. The excess energy is radiated in active phases of the cD galaxy nucleus, probably highly anisotropically.

Key words: accretion, accretion disks – black hole physics – cooling flows – galaxies: clusters: general

1. INTRODUCTION

Clusters of galaxies are the largest gravitationally bound objects in the universe. Assuming rough virial equilibrium between the component dark matter, gas, and galaxies, with velocity dispersion $\sigma \sim 1000 \text{ km s}^{-1}$, shows that within a core radius $R_{\text{core}} \sim 150 \text{ kpc}$ about the central cD galaxy the intercluster gas (total mass $\sim 10^{14} \text{ M}_\odot$) has a free–free cooling timescale shorter than the age of the universe (see Equation (3)). However, it is by now well established that there is no significant mass of cooling gas within $R_{\text{core}}$ flowing toward the cD galaxy, implying that some mechanism supplies energy to heat this gas. The most likely source of this energy is fairly clear: the cD implies that some mechanism supplies energy to heat this gas. Moreover, if the outer hole’s presence to the gas in the galaxy. Remarkably, this simple idea gives not only the observed proportionality $M_{\text{BH}} = C\sigma_4^4$, but also the quantitatively correct coefficient $C = f\kappa/\pi G^2$, where $f \approx 0.16$ is the gas fraction, $\kappa$ is the electron-scattering cross-section, and $G$ is the gravitational constant (King 2003, 2005). Further, at sufficiently large distances $R_c$ from the hole, Compton cooling is ineffective and the extra injection of thermalized kinetic energy accelerates the two shocks above the escape velocity $\sigma_3$, driving away the remaining gas, and fixing the baryonic mass $M_{\text{bulge}}$ of the galaxy bulge. In the limit of modest Eddington factors (which implies wind outflow speeds $v$ approaching $c$), one finds the relation $M_{\text{bulge}} \sim M_{\text{BH}}(m_p/m_e)\sigma_3/c \sim 10^3 M_{\odot}$ between bulge and black hole mass, very close to observation.

These results show that the outflows driven by super-Eddington accretion are very effective in communicating the hole’s presence to the gas in the galaxy. Moreover, if the outer (snowplow) shock reaches a large distance $R_c$ from the hole, it strongly heats this gas because the shock velocity exceeds the local velocity dispersion. Here we see a possible connection with the cluster gas cooling problem. These features of super-Eddington outflows in galaxies are obviously also desirable ingredients for any mechanism which might heat cluster gas. If the accretion energy of the central black hole could somehow drive a shell into the cluster gas, it could also reach a radius where Compton cooling of the reverse shock is ineffective. At this point the shock velocities would increase because some of the outflow kinetic energy is converted to heat and hence exerts pressure. This higher shock speed would exceed the local velocity dispersion in the cluster gas, heating the gas above the virial temperature and thus supplying heat as well as kinetic energy to the cluster gas.

Driven with no extra contribution from the kinetic energy randomized in the reverse shock (this would be an “energy driven” flow). The dynamics of this two-shock structure now fix the relation between the black hole mass and the galaxy properties. If the black hole mass is below a critical value $C\sigma_3^4$, the Eddington thrust $L_{\text{Edd}}/c$ is too weak to lift the interstellar gas against the galactic potential measured by $\sigma_3$. Thus, the shock does not propagate outside the hole’s immediate vicinity and accretion can continue. However, once $M_{\text{BH}}$ reaches the critical value $C\sigma_3^4$, the shock attains the escape velocity $\sim \sigma_3$ and expands to large radii, preventing further growth in $M_{\text{BH}}$. The most reliable source of this energy is the so-called cD galaxy, which communicates the hole’s presence to the intermediate gas. At modest Eddington factors $\dot{m}_p = M_{\text{BH}}/L_{\text{Edd}}$ the radiation field couples to the outflow via the single-scattering limit. This imparts momentum $L_{\text{Edd}}/c$ to it (King & Pounds 2003; King 2003, 2005), and simultaneously Compton-cools the reverse shock as the outflow sweeps up the galaxy’s interstellar gas. Thus, a forward shock is driven into the ambient gas purely by the momentum of the super-Eddington outflow (momentum-driven) with no extra contribution from the kinetic energy randomized in the reverse shock (this would be an “energy driven” flow). The dynamics of this two-shock structure now fix the relation between the black hole mass and the galaxy properties. If the black hole mass is below a critical value $C\sigma_3^4$, the Eddington thrust $L_{\text{Edd}}/c$ is too weak to lift the interstellar gas against the galactic potential measured by $\sigma_3$. Thus, the shock does not propagate outside the hole’s immediate vicinity and accretion can continue. However, once $M_{\text{BH}}$ reaches the critical value $C\sigma_3^4$, the shock attains the escape velocity $\sim \sigma_3$ and expands to large radii, preventing further growth in $M_{\text{BH}}$.
2. CLUSTER GAS

To fix ideas, I derive here the properties of the core gas in a typical cluster. For simplicity, I approximate this as an isothermal sphere characterized by the velocity dispersion \( \sigma_c \). Then the gas density at radius \( r \) is

\[
\rho = \frac{f_g \sigma_c^2}{2\pi G r^2},
\]

and the gas mass inside radius \( R \) is

\[
M(R) = 4\pi \int_0^R \rho r^2 dr = \frac{2f_g \sigma_c^2 R}{G} \simeq 8 \times 10^{13} M_\odot \sigma_{1000}^2 R_{\text{Mpc}}.
\]

where \( R_{\text{Mpc}} = R/1\text{Mpc}. \) Assuming virial equilibrium, the gas temperature is \( T \simeq 10^8 \sigma_{1000}^2 \text{K}. \) The dominant cooling process is free–free emission, with cooling time \( T^{1/2}/\rho. \) This is shorter than a Hubble time \( t_H \) for \( \rho < \rho_{\text{cool}} \simeq 10^{-26} \sigma_{1000} \text{g cm}^{-3}, \) i.e., within a core radius (using Equation (1))

\[
R_{\text{core}} = 150 \sigma_{1000}^{1/2} \text{ kpc}.
\]

From Equation (2), the mass of this core gas is

\[
M_{\text{core}} = 1.3 \times 10^{13} \sigma_{1000}^{5/2} M_\odot.
\]

To prevent significant cooling of this gas requires an energy input of about 1 keV per baryon, i.e., an energy \( E_k \sim (1\text{keV}/10^3 \text{MeV}) M_{\text{core}} c^2 \sim 10^{-6} M_{\text{acc}} c^2. \) The total gravitational binding energy released in accreting mass \( M_{\text{acc}} \) on to the central black hole of the cD galaxy is \( E_{\text{acc}} = \epsilon M_{\text{acc}} c^2, \) with \( \epsilon \sim 0.1. \) If \( \eta_{\text{heat}} \) denotes the efficiency with which this energy is used to heat the cluster gas, we see that the total accreted mass required to prevent cooling is

\[
M_{\text{acc}, h} \simeq 10^{-5} \frac{M_{\text{core}}}{(\epsilon/0.1) \eta_{\text{heat}}} \simeq 1.3 \times 10^{8} \sigma_{1000}^{5/2} (\epsilon/0.1) \eta_{\text{heat}} M_\odot.
\]

Clearly, heating by the central black hole cannot work if the required mass \( M_{\text{acc}, h} \) exceeds its total mass \( M_{\text{BH}}. \) If this is \( \sim 10^9 M_\odot, \) we need \( \eta_{\text{heat}} \gtrsim 0.1. \) The mechanism described below has \( \eta_{\text{heat}} \simeq 0.2. \) I compare this with other forms of heating in Section 5.

3. HEATING CLUSTER GAS

I suggest here that cluster gas may be heated by an extension of the process establishing the \( M_{\text{BH}} - \sigma_g \) and \( M_{\text{BH}} - M_{\text{bulge}} \) relations in individual galaxies, involving super-Eddington accretion on to the central black hole. The resulting outflow is roughly spherical (see below) and sweeps up the gas in an inner thin shell and tries to drive it out against gravity. Sijacki et al. (2007) have recently performed a cosmological simulation with a form of mechanical feedback on cluster gas, and indeed found that it could prevent cooling. However, much of the interaction between the outflow and the cluster gas necessarily occurs on scales not accessible to current numerical simulations. Here I adopt a simple analytical picture in the hope of getting some physical insight into the process.

This type of approach is described in detail in King (2003, 2005). The second of these papers gives the equations of motion of the swept-up gas shell and shows that this clears the accreting gas away from the central black hole once the hole mass reaches the critical value

\[
M_\sigma = \frac{f_g k}{\pi G^2 \sigma_g^4}.
\]

One could follow the same procedure in considering the effects on cluster gas, but for our purposes a simpler method is adequate. We note that the weight of the shell of swept-up gas cluster is \( W(R) = GM(R)[M_{\text{total}}(R)]/R^2; \) here \( M(R) \) is the mass of the shell, and \( M_{\text{total}}(R) \) is the total mass inside cluster radius \( R, \) including dark matter. Neglecting the contribution of the cD galaxy mass, which is small for \( R \sim R_{\text{core}}, \) this is simply \( M(R)/f_g, \) since \( M(R) \) is just the gas mass originally inside \( R \) before the passage of the shock. (Note that in King 2005 the second term of Equation (2) should read \( GM(R)[M + M_{\text{total}}(R)]/R^2, \) and the correct definition of \( M_\sigma \) immediately below Equation (3) is \( f_g k/\pi G^2 \sigma_g^4. \) Using Equation (2), we see that the weight \( W(R) \) is independent of \( R \) for large \( R, \) and has the constant value

\[
W = \frac{4 f_g \sigma_c^4}{G}
\]

(the shell and total mass each increase as \( R, \) so their product exactly cancels the inverse-square weakening of gravity). We can now decide whether the shell reaches large \( R, \) and so heat the cluster gas, by comparing the weight \( W \) with the thrust produced by the super-Eddington accreting black hole in the center of the cD galaxy (this procedure does not give the time dependence of the motion, which requires one to solve the shell’s equation of motion taking account of its increasing inertia, compared to King 2005).

In the single-scattering limit expected for modest Eddington ratios \( f_g, \) this thrust is simply \( L_{\text{edd}}/c = 4\pi GM/R. \) This gives the expected result that the shell would reach large \( R, \) and thus heat the cluster gas, if and only if the black hole mass exceeded the value in Equation (6) with \( \sigma_c \) in place of \( \sigma_g, \) which as we have seen in Section 1 is unrealistically large (~6 \times 10^{10} M_\odot). We would expect instead that as in other galaxies, the hole would have only reached the smaller value, Equation (6), given by the galaxy’s internal velocity dispersion \( \sigma_c. \)

Now let us consider the effects of an accretion episode with higher Eddington ratios than supermassive black holes in galaxy centers (see Equation (12)). For the well-known system SS433, which has
\( \dot{m}_E \sim 5000 \), Begelman et al. (2006) and Poutanen et al. (2007) show that the features anticipated by Shakura & Sunyaev appear. In particular, the total accretion luminosity is \( \sim L_{\text{Edd}}[1 + \ln \dot{m}_E] \), and is almost entirely channeled by scattering into a narrow pair of funnels around the disk axis, so that the outflow is essentially spherical apart from these two funnels. These two results suggest that highly super-Eddington accretion on to stellar-mass compact objects offers a plausible explanation for most if not all ultraluminous X-ray sources (ULXs; for example, King et al. 2001; King 2009). Most importantly for our purposes, most of the super-Eddington mass inflow is blown away from a radius \( R_{\text{circ}} \sim 9 \dot{m}_E R_{\text{in}} / \epsilon \) (where \( R_{\text{in}} \) is the inner disk radius near the black hole) with mechanical luminosity

\[
\frac{1}{2} \dot{M} v^2 \sim L_{\text{Edd}}. \tag{8}
\]

This resulting relation \( v = (2L_{\text{Edd}}/\dot{M})^{1/2} \) allows us to estimate the thrust exerted by the accreting hole on its surroundings, i.e.,

\[
\dot{M} v = (2 \dot{M} L_{\text{Edd}})^{1/2} = \left( \frac{2 \dot{M}}{L_{\text{Edd}}} \right)^{1/2} L_{\text{Edd}} = \left( \frac{2 \dot{m}_E}{\epsilon} \right)^{1/2} \frac{L_{\text{Edd}}}{c}. \tag{9}
\]

This exceeds the single-scattering estimate by the factor \( \sim (2 \dot{m}_E/\epsilon)^{1/2} \). This is potentially a lower limit to the increase, as there is a thermal pressure contribution if the external shock cannot cool. However, Compton cooling still operates on the outflow, since a luminosity \( \sim L_{\text{Edd}} \) escapes isotropically through the outflow rather than via the funnels along the disk axis. This also means that the temperature profile remains flat or decreasing radially inward in the cluster center.

The estimate in Equation (9) shows that the likely black hole mass results in a large enough thrust to heat the cluster gas if a significant Eddington ratio holds for some fraction of a cluster dynamical time \( R_{\text{circ}}/\sigma_c \sim 1.5 \times 10^8 \) yr. Equating the value in Equation (9) to the weight \( 4 f_g \sigma_c^4 / G \) of the cluster gas shows that the minimum black hole mass needed to get the shock out to the core radius is

\[
M_{\text{BH}} = \left( \frac{\epsilon}{2 \dot{m}_E} \right)^{1/2} f_g \kappa \pi G^2 \sigma_c^4. \tag{10}
\]

Put another way, a super-Eddington accretion episode can successfully heat the cluster gas provided the Eddington ratio exceeds the critical value

\[
\dot{m}_E(\text{crit}) = \frac{180 \epsilon_0.1}{M_q^2} \sigma_{1000}^4. \tag{11}
\]

for a fraction of the cluster dynamical time, where \( \epsilon_0.1 = \epsilon/0.1 \) and \( M_q = M_{\text{BH}}/10^9 M_\odot \).

4. ACCRETION

We have seen that a sufficiently high Eddington ratio for the central black hole in the cD galaxy is required to heat cluster gas, albeit for a relatively short timescale. The accretion rate specified by Equation (11) is extreme—close to the dynamical rate \( f_g \sigma_c^4 / G \) for the host cD galaxy. If the black hole in the latter obeys the \( M_{\text{BH}}-\sigma_g \) relation, this rate implies an Eddington ratio

\[
\dot{m}_E \sim \frac{\epsilon c}{4 \sigma_g} \sim 30, \tag{12}
\]

where I have taken \( \epsilon = 0.1, \sigma_g = 300 \) km s\(^{-1}\) at the last step. This is in one sense reassuring, as it shows that most supermassive black holes in galaxy centers do not experience very high Eddington ratios, justifying the use of the single-scattering limit in deriving the \( M_{\text{BH}}-\sigma \) relation for them. Conversely, if cluster gas is heated by the process discussed here, the central regions of the cD galaxy must experience gas inflow rates of order \( 10^3 M_\odot \) yr\(^{-1}\), which come close to destabilizing them, at least for a short time.

There is an obvious candidate for this very rapid accretion—the onset of the cooling catastrophe. If nothing acted to reheat the cluster gas, this would eventually begin to flow in toward the cD galaxy at rates \( \sim M_{\text{core}}/t_H \sim 10^8 \sigma_{1000}^2 M_\odot \) yr\(^{-1}\). The cD galaxy must react long before such rates are reached. Its central black hole drives a snowplow shock out through the cluster gas, reaching the core radius and reheating the enclosed gas to the virial temperature in a dynamical time \( \sim 10^8 \) yr. This stabilizes the cluster gas and stops further inflow. The duty cycle of the cooling-infall phase is thus of order \( 10^{-2} \). Only a small amount of cluster can cool before being reheated, so clusters typically appear to be in virial equilibrium.

5. ENERGY BUDGET

During an active phase of the type described above, the central black hole gains mass at about its Eddington rate for some \( 10^8 \) yr, and thus typically grows by \( \sim 10^9 M_\odot \). In return, it reheats \( \sim 10^{13} M_\odot \) of cluster gas. However, several observational surveys put strict limits on the ratio of total jet to radiative output by active galactic nuclei, or equivalently \( \eta_r M_r / \dot{E}_r \). For example, Cattaneo & Best (2008; see also Merloni & Heinz 2008) find this ratio is \( \lesssim 0.1 \). Accordingly, we find from Equation (16) that

\[
\frac{E_r}{\dot{E}_q} \lesssim \eta_r < 1. \tag{17}
\]

for the heat input into the cluster in this form of the “quasar” mode from accreting mass \( M_q \) on to the black hole. Discussions of cluster heating (e.g., Sijacki et al. 2007) contrast the quasar mode with the “radio” mode. This is motivated by observations (e.g., Birzan et al. 2004; Rafferty et al. 2006) which suggest that radio-loud FR I sources can inflate X-ray cavities. The accretion of gas mass \( M_q \) is assumed to produce jet kinetic energy

\[
\dot{E}_j = \eta_r c^2 M_r. \tag{14}
\]

If these jets convert their energy into cluster heating with efficiency \( \eta_j \), we get heat input

\[
\eta_r \eta_j c^2 M_r \tag{15}
\]

into the cluster gas. Hence

\[
\frac{E_r}{\dot{E}_q} \sim \eta_r \eta_j M_r. \tag{16}
\]

However, several observational surveys put strict limits on the ratio of total jet to radiative output by active galactic nuclei, or equivalently \( \eta_r M_r / \dot{E}_r \). For example, Cattaneo & Best (2008; see also Merloni & Heinz 2008) find this ratio is \( \leq 0.1 \). Accordingly, we find from Equation (16) that

\[
\frac{E_r}{\dot{E}_q} \lesssim \eta_j < 1. \tag{17}
\]
This suggests that outflows of the type considered here likely to be more effective than jets in heating cluster gas, i.e., require less black hole mass growth to produce the same heating effect. Thus, for heating by an outflow, the total increase $\Delta M_{\text{BH}}$ in the mass of the central black hole is controlled by the rate at which cluster gas cools, i.e., $\Delta M_{\text{BH}} \simeq 10^{-4} M_{\text{cool}}$, so if $M_{\text{cool}} \sim M_{\text{core}}$ we expect $\Delta M_{\text{BH}} \sim 10^9 M_\odot$. In principle, $M_{\text{cool}}$ might exceed $M_{\text{core}}$ if the reheated gas cools more quickly than before, i.e., in less than a Hubble time. This would then require multiple heating events, and thus black hole mass growth $M_{\text{BH}} > 10^9 M_\odot$. In the simple spherically symmetric picture adopted here this does not happen, but this conclusion should be checked by numerical simulations allowing for deviations from this symmetry and thus local cooling instabilities. In view of Equation (17), if this picture requires excessive black hole mass growth, this problem is likely to be worse for radio mode heating.

6. DISCUSSION

I have suggested that cluster gas is heated by a natural extension of the process establishing the $M_{\text{BH}}-\sigma$ and $M_{\text{BH}}-M_{\text{bulge}}$ relations in galaxies. The privileged position of the central cD galaxy means that it is intermittently subject to extremely high gas inflow rates. These trigger highly super-Eddington accretion on to the central black hole, which reacts by driving a shock into the infalling gas, efficiently reheating it and stabilizing the cluster gas. The duty cycle of an active phase of this type is about $10^{-2}$, so most clusters appear to be stably in virial equilibrium.

During an active phase, the accreting central black hole of the cD galaxy emits $\sim 4L_{\text{Edd}} \sim 4 \times 10^{47}$ erg s$^{-1}$ into a narrow pair of cones, and $\sim L_{\text{Edd}} \sim 10^{47}$ erg s$^{-1}$ isotropically. If the scaling of the beaming factor with the Eddington ratio derived by King (2009) for ULXs holds here too, an observer situated in these cones would infer a still higher apparent luminosity $\sim 0.1 m_\odot^2 \times 4L_{\text{Edd}} \sim 4 \times 10^{48}$ erg s$^{-1}$. However, as the cone solid angle is only $\sim 10^{-2}$, and the duty cycle of active phases is also $\sim 10^{-2}$, it is unsurprising that such luminosities are not observed. An active phase in a cD galaxy would be observable through the isotropic Eddington emission. The dense outflowing wind implies a large photosphere, shifting the emission into the infrared, and a systematic search here might prove interesting.

Comparison of the simple treatment given here with observation requires care. In particular, a more realistic cluster potential must affect the ability of the swept-up gas shell to escape, and thus the duration of the active phases and the temperature structure of the cluster gas. Local density perturbations and the resulting cooling will have similar effects. These could produce shocked bubbles whose cooling times are significantly shorter than a Hubble time, as appears to be true of at least some observed cases. One would then require multiple heating events, totaling a much larger fraction of the cluster lifetime, in order to stave off catastrophic cooling, rather than the $\sim 1\%$ total heating time envisaged here. Answering these questions requires numerical simulation.

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