Quantum-stochasticity-induced asymmetry in angular distribution of electrons in a quasi-classical regime

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Impacts of quantum stochasticity on the dynamics of an ultra-relativistic electron beam head-on colliding with a linearly polarized ultra-intense laser pulse are theoretically investigated in a quasi-classical regime. Generally, the angular distribution of the electron beam keeps symmetrically in transverse directions in this regime, even under the ponderomotive force of the laser pulse. Here we show that when the initial angular divergence $\Delta \theta \leq 10^{-6} \lambda_0$ with $\lambda_0$ being the normalized laser field amplitude, an asymmetric angular distribution of the electron beam arises due to the quantum stochasticity effect, via simulations employing Landau-Lifshitz, quantum-modified Landau-Lifshitz equations, and quantum stochastic radiation reaction form to describe the radiative electron dynamics respectively. The asymmetry is robust against a variety of laser and electron parameters, providing an experimentally detectable signature for the nature of quantum stochasticity of photon emission with laser and electron beams currently available.

Dynamics of an electron in electromagnetic fields is a fundamental issue in either classical electrodynamics [1] or quantum electrodynamics (QED) [2]. Apart from the main Lorentz force, the electron also suffers from the reaction force of radiation. In classical realm, the radiation reaction (RR) effect is taken as radiation damping stemming from the radiated electromagnetic fields coupling the external fields. The well-known Lorentz-Abraham-Dirac (LAD) equation [3-5] self-consistently describes the electron motion accounting for RR effects as an additional four-force. However, LAD equation gives unphysical solutions, such as the “runaway” solution. To fix it, Landau-Lifshitz (LL) equation, was developed through bringing a perturbative iteration in the RR terms in the LAD equation, and it is employed as the classical equation of electron motion at relatively low electromagnetic wave amplitude [6]. On the other hand, LL equation overestimates the radiative energy loss, since it unphysically includes the emission of photons with energy higher than the electron kinetic energy [7]. Recently, a quantum-modified LL equation is derived with quantum-recoil corrections through rescaling the RR force by a factor of $I_{QED}/I_c$ (i.e., the ratio of the radiation intensities within QED and classical approaches) [8, 9], avoiding the aforementioned classical overestimation. Another distinguishing property of radiation in QED is the nature of quantum stochasticity, i.e., the discrete and probabilistic character of photon emission [10][12]. The quantum stochasticity effect (QSE) would increase the yield of high-energy photons [13], cause the quantum quenching of radiation losses [14], alter the energy spectrum of emitted photons [12] or scattered electrons [10][15][16], reshape the space-distribution of photons [17] or electrons [18][19], etc [20][22].

Nowadays, the development of ultra-short ultra-intense laser techniques, particularly the application of PW lasers [23], has stimulated the research interests in confirmatory experiments on QED theory [24][26]. Recently, quantum RR was reported to be observed in experiments of laser-electron-beam interaction [25][26], via comparing the electron- and photon-spectra to be observed in experiments of laser-electron-beam interaction [25][26], via comparing the electron- and photon-spectra detected with those simulated in quantum theoretical model. Note that in these experiments, the QSE is detected mixed with other quantum properties, such as quantum recoil.

Proposals [10][18] for unambiguous identification of QSE mainly involves quantum radiation dominated regime (QRDR) characterized by the parameter of $R_s = \alpha \chi \lambda_0 \lesssim 1$ [24][27]. Here, $\alpha \approx 1/137$ is the fine-structure constant, indicating the order of photon-emission probability of the electron in a formation length ($l_f \sim \lambda_0 / \alpha_0$ with $\lambda_0$ being the laser wavelength [28]); $\alpha_0 \equiv e E_0 / (m \omega_0)$ is the normalized laser field parameter, $\omega_0$ the laser frequency, and $e$ (> 0), $m$ the electron charge and mass, respectively; and $\chi \equiv e \sqrt{-F_\mu \mu / m^2} \approx 2 \omega_0 \lambda_0 \gamma / m$ is the nonlinear electron quantum parameter corresponding to the ratio of the typical emitted photon energy with the initial electron kinetic energy as $\omega_\gamma / \omega_0 \sim \chi$, where $F_\mu$ is the field tensor and $p^\mu$ the is four-vector of electron momentum. Units $\hbar = c = 1$ are used throughout. Apparently, the QRDR represents a regime where the energy loss of an electron in a laser period due to RR is comparable with its initial energy. The QRDR can elicit remarkable impacts of the RR effects on electron dynamics even when $\chi \ll 1$, whereas it also requires super high electron energy and laser intensity in experiments. Moreover, signatures associated with electron- and/or photon-spectrum [10][15][16][29] would be submerged by the...
where,\( E \) and \( \omega \) distributed in temporal dimension at a pulse duration of \( \lambda \) with respect to deflection angles of \( \delta \). Two-dimension distributions of electron number density \( \chi_{\delta}(z)\) with \( K_0 \) being the n-order modified Bessel function of the second kind. In each time step \( \Delta t \), the probability for an electron to emit a photon with energy of \( \omega_{\gamma} = \delta \varepsilon_0 \) is calculated with Eq. \( 3 \), i.e. \( W_{\text{rad}}(\delta) = \frac{dW_{\text{rad}}}{d\varepsilon_0}d\varepsilon_0 \Delta t \). To avoid an infrared cutoff, we take \( \delta = \frac{\varepsilon_0}{\mu r_1} \) with \( r_1 \) being a random number in \([0,1] \). Another random number \( r_2 \) in \([0,1] \) is used to determine if a photon is emitted: if \( W_{\text{rad}}(r_1) < r_2 \), reject emitting a photon; otherwise, emit a photon of \( \omega_{\gamma} \). Given the smallness of the emission angle \( \gamma \) for an ultra-relativistic electron, the emitted photon is assumed to move along the electron velocity. Here, the \( \gamma \) is the electron Lorentz factor. More detailed information on this method and its accuracy have been shown in Ref.\([38]\). Between emissions, the electron dynamics in the laser field is governed by classical Lorentz equations of motion.

In our simulations, effects from electron spin or emitted photon polarization are ignored due to its negligible integrated influence in the nonlinear Compton scattering process.\([39,40]\). While similar classical\([41,42]\) and quantum\([36,38,43]\) simulation models have also been put forward, the signature of QSE, i.e. the differences between the classical and quantum models, are expected to be invariant for the scheme and parameters considered here, as shown below.

A typical simulation result, employing a feasible scenario involving an electron beam of \( \varepsilon_0 = 300 \text{ MeV} \) and a laser pulse with peak intensity of \( a_0 = 30 \) or \( I_0 = 1.2 \times 10^{21} \text{ W/cm}^2 \) (corresponding to the quantum parameters of \( \chi_{\text{max}} \approx 0.08 \) and \( R_c \approx 0.02 \)), is illustrated in Fig. 2. The electron bunch is set with features of laser-accelerated electron source\([44,46]\). \( N_e = 1 \times 10^7 \) electrons uniformly distributed longitudinally in a cylindrical form at length of \( L_e = 5 \mu \text{m} \) and normally distributed transversely in a radius of \( R_e = 1 \mu \text{m} \) with standard deviation of \( \sigma_{x,y} = 0.3 \mu \text{m} \). The angular divergence is \( \Delta \theta = 1 \) mrad and energy spread (FWHM) of \( \Delta \varepsilon = 42 \text{ MeV} \). The scattering laser pulse is linearly polarized along \( x \) direction, tightly focused at a waist radius of \( w_0 = 3 \mu \text{m} \), and Gaussian distributed in temporal dimension at a pulse duration of \( \tau = 87 \varepsilon_0 \). The laser wavelength is \( \lambda_0 = 1 \mu \text{m} \).

Two-dimension distributions of electron number density with respect to deflection angles of \( \theta_x \) and \( \theta_y \), corresponding to the intuitive image from the detector of electronic deposition, are shown in the left column of Fig. 2. An asymmetry dominated by QSE appears in the electron distribution: in Fig. 2a,
the electron distribution is oval-shaped, with the major axis along the x direction and the minor axis along the y direction; while it is round-shaped in Figs. 2(c), 2(e) and 2(g) without QSE. For a more quantitative analysis we integrate the electron differential angular distributions in $\theta_x$ or $\theta_y$, and obtain the one-dimension distribution curves of electron density with respect to $\theta_x$ or $\theta_y$, respectively, as shown in the right column of Fig. 2. In Fig. 2(b), the angle spreads (FWHM) in x and y directions are $\Delta \theta_x = 4.28$ mrad and $\Delta \theta_y = 1.46$ mrad, respectively, resulting in an asymmetry of $\delta \sim \Delta \theta_y/\Delta \theta_x \approx 3$; in both Fig. 2(d) and 2(f), $\Delta \theta_x = \Delta \theta_y = 1.46$ mrad; and in Fig. 2(h), $\Delta \theta_x = \Delta \theta_y = 1.12$ mrad. With the initial $\Delta \theta_y = 1$ mrad before interaction, the broadening of $\Delta \theta_{x,y}$ (i.e. 0.12 mrad) in Fig. 2(h) results from the ponderomotive force of $F_p = -\gamma a_0^2 \mathbf{E}/(2\gamma)$ [47].

When RR is included, the electrons also suffer from radiative energy loss of $\Delta \gamma \sim a_0 k_L \gamma$ [28], leading to the wider $\Delta \theta_y \sim \Delta p_y/p_y \propto 1/\gamma$ in Fig. 2(b) and wider $\Delta \theta_{x,y}$ in Figs. 2(d) and 2(f). In this scheme of $\chi \ll 1$, $R_e \ll 1$ and $\alpha_0 \ll \gamma$, the electron deflection angle stemming from radiation loss and ponderomotive force is far less than that from QSE. With imaging, such as a Lanex film [44], the asymmetric distribution can be recorded to identify the QSE role.

![FIG. 2. Two-dimension distribution of electron number density of $d^2N_x/d\theta_x d\theta_y$ (mrad$^{-2}$) (left column), vs deflection angles of $\theta_x$ or $\theta_y$, and integrated one-dimension distribution of electron density of $dN_x/d\theta_x, (\text{mrad})^{-1}$ (right column), vs $\theta_x$ (red-solid) or $\theta_y$ (blue-dash-dotted). Rows from top to bottom are the simulated results calculated with RR in MCM, LLLM and LLM, respectively; and without RR.](image)

![FIG. 3. The evolution of $p_x$ (a) and $p_y$ (b) with respect to the laser phase $\eta=\omega_0 t - kz$, calculated in LLM (magenta-solid), LLM (blue-dash-dotted) and without RR effects (black-dashed). (c): The evolution of $p_x$ for 10 sample electrons in MCM. (d): The final electron energy spectrum in MCM (red-solid), LLM (magenta-dotted), LLM (blue-dash-dotted) and without RR effects (black-dashed).](image)

The further explanation of the asymmetric electron distribution from QSE is analyzed in Fig. 3. The momenta of a number of sample electrons are calculated. For simplicity, we set the initial electron momentum along $-z$ direction and the initial position of $(0, 0, 0)$, with $t_0$ defined as the time when the electron reaches the laser focus. Apart from RR, the increment of electron momentum should be zero, as one can see from the Lorentz equation for electron transverse motion of $d\mathbf{p}_\perp/d\tau = -e\mathbf{E}_\perp$ with the integral value of $\mathbf{E}_\perp$ being zero in a normal symmetric laser pulse. As $\gamma \gg 1$, the RR force could be estimated by the leading order of $\gamma^2$ in Eq. 1 as $F_{RR} \approx -2e^2/(3\pi^2)\mathbf{E} \cdot \mathbf{B}$, with $\delta = 2e^2/(3\pi^2)\mathbf{E} \cdot \mathbf{B}$ [48]. With $|\mathbf{v}| \approx \gamma_v \approx 1$, it can be written as $F_{RR} \approx -8e^2/(3\pi^2)\mathbf{E} \cdot \mathbf{B}$. In this case that $\mathbf{p}$ evolves according to $\mathbf{E}$ with a phase delay of $\pi/2$, the integral of the RR force in $x$ direction is nearly zero. Correspondingly, the net transverse momentum increment of an electron passing through the symmetric laser field should also be zero with RR included, in coincidence with the numerical result in LLM. With $g(x) \in [0.96, 1]$, the electron deflection angle in LLM is close to that in LLM. Above all, whenever RR is considered or not, in our regime, the final $\Delta p_x$ in Fig. 3(a) should be zero, and naturally $\Delta p_y$ in Fig. 3(b) should also be zero due to the linear polarization of the laser pulse. Without QSE, the electron distribution should be symmetric transversely, as shown in Fig. 2.

The evolutions of $p_x$ of 10 sample electrons in MCM is elaborated in Fig. 3(c). Each electron experiences a stochastic radiation process, resulting in a randomly distributed final $p_x$, causing a broadened $\Delta \theta_y$ in Fig. 2(a). The broadening effect in one emission could be estimated from $\Delta \theta_y \sim (p_y - p_y^0)/p_y = -p_y^0/p_y \leq -\chi a_0/\gamma \approx -2\omega_0 a_0^2/m \approx 4.36$ mrad, where $p_y^0$ and $p_y$ are the initial electron momentum component and emitted photon momentum component in the x direction. The number of photons emitted by one electron should be $N_y \sim \alpha_0(\gamma)/T_0 = \ldots$
The angle spread caused by QSE obtained numerically with \( \Delta \theta - \Delta \theta_t \) (red-solid) and analytically with \( \Delta \theta_{\phi 1} = 1.75 \times (2m/\omega_0) \alpha_0^2 \) (blue-dashed) and \( \Delta \theta_{\phi 2} = 2(\omega_0/m) \alpha_0^2 \) (black-dotted).

The overlap of effects from multiple emission lead to a total enlargement of \( \Delta \theta_t \) around 3 times larger than \( \Delta \theta_\phi \) in Fig. 2(a). Therefore, even in quasi-classical regime of \( \chi \ll 1 \) and \( R \ll 1 \), we can obtain an obvious angle broadening of \( \Delta \theta_t \) dominated by QSE.

As investigated previously [10, 15–16], the final electron energy spectrum can also spread due to the QSE role, see Fig. 3(d). The spectrum curve of MCM occupies the same mean electron energy of 240 MeV with that in MLLM, but with a wider spread of \( \Delta \varepsilon = 75 \) MeV than that of \( \Delta \varepsilon = 28 \) MeV in MLLM. In LLM, a similar energy spread \( \Delta \varepsilon = 24 \) MeV is obtained, but the mean electron energy is lower (226 MeV) due to the overestimation of radiation loss. The result that the energy spread is reduced by the RR classically, whereas enlarged by QSE significantly, is consistent with the conclusion of Ref. [10]. The larger energy loss in LLM, cannot lead to a distinguishable difference in electron distribution from MLLM owing to the equivalent overestimation of the transverse momentum loss, as shown in Eq. 2. Comparatively, the asymmetric electron angular distribution can be measured more easily than the electron energy spread, particularly considering the fluctuation and statistical uncertainty in experiments.

The influences of laser and electron beam parameters on the QSE signature are discussed below to examine its robustness and to clarify the requirements for experimental observation. In Fig. 4 increasing of \( \alpha_0 \) from 10 to 100, \( \Delta \theta_t \) grows from 1 mrad to 43.1 mrad, and \( \Delta \theta_\phi \) from 1 mrad to 128.6 mrad in a faster pattern. The increment of \( \Delta \theta_t \) indicates the tendency of the growing transverse pondermotive force as the corresponding deflection angle of \( \theta_\parallel = (\alpha_0^2/\gamma^2)(\tau/\omega_0) \) [18, 47]. The \( \Delta \theta_t \) results from a combination of pondermotive force effect and QSE, sensitively dependent on \( \alpha_0 \) under the domination of the latter. The calculation about the broadening of deflection angle by QSE is performed, in Fig. 4(c), numerically and analytically. While analytical results taking into account one or 1.75 emission times are calculated, the latter matches
well with the numerical one even with an increasing number of emission $N_\gamma \propto a_0$, which can be explained in Fig. 5. It should be noted that the requirement crucial to observe the QSE signature in this quasi-classical regime reads

$$\Delta \theta \approx 2(\omega_0/m)a_0^2 \sim 10^{-6}a_0^2.$$  

Otherwise, the $\Delta \theta'$ would be submersed by the initial angular divergence $\Delta \theta$. This requirement gives a minimum of $a_0 \approx 20$ (or $I_0 \approx 5.5 \times 10^{20}$W/cm$^2$) for currently available angular divergence of the electron beam [46].

The impacts of laser pulse duration and initial electron mean energy are illustrated in Figs. 3(a) and 3(b). With the duration $\tau$ changing from 4 $T_0$ to 40 $T_0$, the asymmetry $\delta$ rises from 1.96 to 3.15 at $\tau = 11.5 T_0$, and then declines to 1.89 at $\tau = 40 T_0$. It is also demonstrated from the curve in Fig. 5(a) that the deflection effect from QSE (corresponding to $\Delta \theta'$) is strengthened at a decreasing speed with the growth of radiation number, which indicates a reasonable estimation of $\Delta \theta' \approx 1.75 \times 2(\omega_0/m)a_0^2$ as shown in Fig. 3(c). While the deflection effect resulting from transverse ponderomotive force (corresponding to $\Delta \theta$) is $\theta \propto \tau \propto N_\gamma$ [47], the asymmetry tends to be counteracted and weakened with abundant photon emissions. To get an apparent asymmetry $\delta$, a moderate initial electron energy $e_0$ is necessary, even though $\Delta \theta' \approx 2(\omega_0/m)a_0^2$ is independent of $e_0$, see Fig. 5(b). On the one hand, $e_0$ should be large enough to make $\chi > 0.01$ for a photon spectrum wide enough, which is necessary for QSE. On the other, it should also be not too large to ensure $\chi \ll 1$ to avoid momentous radiative loss.

For experimental feasibility, we also consider a case with a larger initial electron energy spread (150 MeV) and show the results in Fig. 5. The asymmetry of the electron distribution is stable with respect to that in Fig. 2 since the condition of $\Delta \theta \approx 2(\omega_0/m)a_0^2$ is fulfilled.

In conclusion, we have investigated the QSE effects of photon emission on the dynamics of an electron beam head-on colliding with a linearly polarized laser pulse in a quasi-classical regime of $\chi \ll 1$ and $R_\perp \ll 1$. Under the condition of $\Delta \theta \approx 10^{-6}a_0^2$, even when the radiation loss is far less than the electron kinetic energy, the QSE could be elicited and distinguished by the asymmetry of the final electron angular distribution between the laser polarization direction and the other orthogonal direction. This QSE signature could be observed intuitively on the image from detector of electron deposition, only by a single-shot. It provides a feasible scheme to test one of the fundamental quantum properties, the stochasticity nature of photon emission, with laser intensity $I_0 \lesssim 10^{21}$ and electron energy of hundreds of MeV currently available in experiments.

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