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A Control Theoretical Adaptive Human Pilot Model: Theory and Experimental Validation

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Abstract—This article proposes an adaptive human pilot model that is able to mimic the crossover model in the presence of uncertainties. The proposed structure is based on the model reference adaptive control, and the adaptive laws are obtained using the Lyapunov–Krasovskii stability criteria. The model can be employed for human-in-the-loop stability and performance analyses incorporating different types of controllers and plant types. For validation purposes, an experimental setup is employed to collect data and a statistical analysis is conducted to measure the predictive power of the pilot model.

Index Terms—Adaptive control, control of time-delay systems, crossover model, human decision-making, pilot modeling, uncertain systems.

I. INTRODUCTION

HUMANS’ unique abilities, such as adaptive behavior in dynamic environments, and social interaction and moral judgment capabilities, make them essential elements of many control loops. On the other hand, compared to humans, automation provides higher computational performance and multitasking capabilities without any fatigue, stress, or boredom [1], [2]. Although they have their own individual strengths, humans and automation also demonstrate several weaknesses. Humans may have anxiety and fear and may become unconscious during an operation. Furthermore, in the tasks that require increased attention and focus, humans tend to provide high-gain control inputs that can cause undesired oscillations. One example of this phenomenon, for example, is the occurrence of pilot-induced oscillations (PIOs), where undesired and sustained oscillations are observed due to an abnormal coupling between the aircraft and the pilot [3]–[6]. Similarly, automation may fail due to uncertainty, fault, or cyberattack [7]. Thus, it is more preferable to design systems where humans and automation work in harmony, complementing each other, resulting in a structure that benefits from the advantages of both.

To achieve a reliable human-automation harmony, a mathematically rigorous human operator model is paramount. A human operator model helps develop safe control systems and provides a better prediction of human actions and limitations [8]–[11]. Quasi-linear model [12] is one of the first human operator models, which consists of a describing function and a remnant signal accounting for nonlinear behavior. An overview of this model is provided in [13]. In some applications, where the linear behavior may be dominant, the nonlinear part of this model can be ignored, and the resulting lead-lag-type compensator is used in the closed-loop stability analysis [14]. The crossover model, proposed in [15], is another important human operator model in the aerospace domain. It is motivated from the empirical observations that human pilots adapt their responses in such a way that the overall system dynamics resembles that of a well-designed feedback system [16]. A generalized crossover model, which mimics human behavior when controlling a fractional-order plant, is proposed in [17]. In [18], a crossover model is employed to provide information about the human intent for the controller. In [19], the dynamics of the operator is represented as a spring–damper–mass system.

Control theoretical operator models employing optimal and adaptive control theories are also proposed by several authors. Optimal human models are based on the idea that a well-trained human operator behaves in an optimal manner [20]–[24]. On the other hand, adaptive models, such as the ones proposed in [25] and [26], aim to replicate the adaptation capability of humans in uncertain and dynamics environments. In [25] and [26], adaptation rules are proposed based on expert knowledge. The adaptive model proposed in [25] is applied to change the parameters of the pilot model based on force feedback from a smart inceptor [27]. A survey on various pilot models can be found in [28] and [29].

Several approaches are also developed for human model parameter identification. In [30], a two-step method using wavelets and a windowed maximum likelihood estimation are exploited for the estimation of time-varying pilot model parameters. In [31], a linear parameter-varying model identification framework is incorporated to estimate time-varying human state-space representation matrices. Subsystem identification is used in [32] to model human control strategies. In [33], a human operator model for preview tracking tasks is derived from the measurement data.

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In this article, we build upon the earlier successful pilot models and propose an adaptive human pilot model that modifies its behavior based on plant uncertainties. This model distinguishes itself from earlier adaptive models by having mathematically derived laws to achieve a crossover-model-like behavior, instead of employing expert knowledge. This allows a rigorous stability proof, using the Lyapunov–Krasovskii mathematical stability criteria, of the overall closed-loop system. Therefore, unlike earlier adaptive approaches, the design has formal guarantees to follow the crossover model in the presence of system uncertainty. Although expert knowledge can also be incorporated in the proposed design, it does not rely on this information to function. It is noted that in this article, we do not claim that the model we propose explains how the decisions are made by pilots. Rather, we propose an operator model that can make the overall system mimic the behavior of the crossover model. To validate the model, a setup, including a joystick and a monitor, is used. The participant data collected through this experimental setup are subjected to visual and statistical analyses to evaluate the accuracy of the proposed model. Initial research results of this study were presented in [34], where the details of the mathematical proof and human experimental validation studies were missing.

This article is organized as follows. In Section II, the problem statement is given. Obtaining reference model parameters, which determine the properties of the crossover model, is discussed in Section III. Section IV presents the human control strategy together with a stability analysis. Experimental setup, results, and a statistical analysis are provided in Section V. Finally, a summary is given in Section VI.

II. Problem Statement

According to McRuer’s crossover model [35], human pilots in the control loop behave in a way that results in an open-loop transfer function

$$Y_{OL}(s) = Y_h(s)Y_p(s) = \frac{\omega_c e^{-\tau s}}{s}$$  \hspace{1cm} (1)

near the crossover frequency, \(\omega_c\), where \(Y_h\) is the transfer function of the human pilot and \(Y_p\) is the transfer function of the plant. \(\tau\) is the effective time delay, including transport delays and high-frequency neuromuscular lags. The input and the output of the closed-loop transfer function, whose near-crossover-frequency open-loop behavior is given in (1), are the desired (reference) and achieved trajectories.

Consider the following plant dynamics:

\[
\begin{align*}
\dot{x}_h(t) &= A_h x_h(t) + B_h u(t - \tau) \\
y_h(t) &= C_h x_h(t) + D_h u(t - \tau)
\end{align*}
\]  \hspace{1cm} (3)

where \(x_h \in \mathbb{R}^{n_h}\) is the neuromuscular state vector, \(A_h \in \mathbb{R}^{n_h \times n_h}\) is the state matrix, \(B_h \in \mathbb{R}^{n_h \times n_u}\) is the input matrix, \(C_h \in \mathbb{R}^{1 \times n_h}\) is the output matrix, and \(D_h \in \mathbb{R}\) is the control output matrix. \(u \in \mathbb{R}\) is the neuromuscular input vector, which represents the control decisions taken by the human and fed to the neuromuscular system. \(y_h \in \mathbb{R}\) is the output vector, and \(\tau \in \mathbb{R}^+\) is a known, constant delay. The neuromuscular model parameters are assumed to be known and the output of the model, \(y_h\), is used as the plant input \(u_p\) in (2), that is, \(y_h = u_p\) (see Fig. 1).

By aggregating the human pilot and plant states, we obtain the combined open-loop human neuromuscular and plant dynamics as

\[
\begin{bmatrix}
\dot{x}_h(t) \\
\dot{x}_p(t)
\end{bmatrix} =
\begin{bmatrix}
A_h & 0_{n_h \times n_p} \\
B_p C_h & A_p
\end{bmatrix}
\begin{bmatrix}
x_h(t) \\
x_p(t)
\end{bmatrix} +
\begin{bmatrix}
B_h \\
B_p D_h
\end{bmatrix}
\begin{bmatrix}
\tau \\
u(t)
\end{bmatrix}
\]  \hspace{1cm} (4)

which can be written in the following compact form:

\[
\begin{align*}
\dot{x}_{hp}(t) &= A_{hp} x_{hp}(t) + B_{hp} u(t - \tau) \\
y_p(t) &= C_{hp} x_{hp}(t)
\end{align*}
\]  \hspace{1cm} (5)

where \(x_{hp} = [x_h^T \ x_p^T]^T \in \mathbb{R}^{(n_r+n_u)}\), \(A_{hp} \in \mathbb{R}^{(n_r+n_h) \times (n_r+n_h)}\), \(B_{hp} \in \mathbb{R}^{(n_r+n_u)}\), and \(C_{hp} = [0_{1 \times n_h} \ C_p] \in \mathbb{R}^{1 \times (n_r+n_u)}\).

Assumption 1: The pair \((A_{hp}, B_{hp})\) is controllable.

The goal is to obtain the input \(u(t)\) in (3), which is the output of the human decision-making process, such that the closed-loop system consisting of the adaptive human pilot model and the plant follow the output of a unity feedback reference model with an open-loop crossover model transfer function. We call this reference model as “crossover reference model” (see Fig. 1). It is noted that once \(u(t)\) is created, it is used as the input to the neuromuscular model, which creates the pilot motion. To summarize, assuming that the neuromuscular model, the parameters to determine the crossover model reference model, and the structure of the behavioral model are given.
plant with parametric uncertainties are available, we aim to find the evolution of the output of the human decision-making process, which is represented by the variable \( u(t) \), such that the closed-loop model (including the decision-making process, human neuromuscular model, and plant model) matches the crossover reference model.

The closed-loop transfer function of the reference model is therefore calculated as

\[
G_{cl}(s) = \frac{\omega_c e^{-\tau s}}{1 + \frac{\omega_c}{s} e^{-\tau s}} = \frac{\omega_c e^{-\tau s}}{s + \omega_c e^{-\tau s}}.
\]  

An approximation of (6) can be given as

\[
\hat{G}_{cl}(s) = \frac{b_m s^n + b_{m-1} s^{n-1} + \cdots + b_0}{s^n + a_{n-1} s^{n-1} + \cdots + a_0} e^{-\tau s},
\]

where \( n = n_h + n_p \) and \( m \) are positive real constants, and \( a_i \) and \( b_j \) for \( i = 0, \ldots, n-1 \) and \( j = 0, \ldots, m-1 \), are real constants to be estimated. The reference model then can be obtained as the state-space representation of (7) as

\[
\dot{x}_m(t) = A_m x_m(t) + B_m r(t - \tau)
\]

\[
y_m(t) = C_m x_m(t)
\]

where \( x_m \in \mathbb{R}^{(n_h+n_p)} \) is the reference model state vector, \( A_m \in \mathbb{R}^{(n_h+n_p) \times (n_h+n_p)} \) is the state matrix, \( B_m \in \mathbb{R}^{(n_h+n_p) \times m_p} \) is the input matrix, \( C_m \in \mathbb{R}^{1 \times (n_h+n_p)} \) is the output matrix, and \( r \in \mathbb{R}^{m_p} \) is the reference input. It is noted that \( C_m \) is selected to be equal to \( C_{hp} \).

III. REFERENCE MODEL PARAMETERS

The crossover transfer function (1) contains the crossover frequency, \( \omega_c \), which is not known \textit{a priori}. Experimental data, showing the reference input \( (r(t)) \) frequency bandwidth, \( \omega_r \), versus crossover frequency \( \omega_c \), is provided in [16] and [35], for plant transfer functions \( K \), \( K'/s \), and \( K'/s^2 \). We fit polynomials to these experimental results to obtain the crossover frequency of the open-loop transfer function given a reference input frequency bandwidth. These polynomials are given in Table I. It is noted that when the reference input has multiple frequency components, the highest frequency is used to calculate the crossover frequency.

Remark 2: In this work, we use the polynomial relationships provided in Table I for zero-, first-, and second-order plant dynamics with nonzero poles and zeros. Further experimental work can be conducted to obtain a more precise relationship between the crossover and reference input frequencies, but this is currently out of the scope of this work.

IV. HUMAN PILOT CONTROL DECISION COMMAND

The adaptive human pilot control decision command, \( u(t) \), is determined as

\[
u(t) = K_r \hat{x}_{hp}(t + \tau) + K_r r(t)
\]

where \( K_r \in \mathbb{R}^{1 \times (n_r+n_p)} \), \( K_r \in \mathbb{R}^{m_r \times m_p} \), and \( \hat{x}_{hp} \in \mathbb{R}^{(n_r+n_p)} \) is the predicted value for \( x_{hp} \). Using (5) and (9), the closed-loop dynamics can be obtained as

\[
\hat{x}_{hp}(t) = (A_{hp} + B_{hp} K_r K_x) \hat{x}_{hp}(t) + B_{hp} K_r r(t - \tau).
\]

Assumption 3: There exist ideal parameters \( K_r^* \) and \( K_x^* \) satisfying the following matching conditions:

\[
A_{hp} + B_{hp} K_r^* K_x^* = A_m
\]

\[
B_{hp} K_r^* = B_m.
\]

Equation (9) describes a noncausal decision command, which requires future values of the states. This problem can be eliminated by solving the differential equation (5) as a \( \tau \)-seconds ahead predictor as

\[
\hat{x}_{hp}(t + \tau) = e^{A_{hp} \tau} x_{hp}(t) + \int_{0}^{\tau} e^{-A_{hp} \eta} B_{hp} u(t + \eta) d\eta.
\]

By substituting (12) into (9), the human pilot control decision input can be written as

\[
u(t) = K_r \hat{x}_{hp}(t) + K_r \int_{-\tau}^{0} e^{-A_{hp} \eta} B_{hp} u(t + \eta) d\eta + K_r r(t).
\]

By defining \( \theta_i(t) \) and \( \lambda(t, \eta) \) as

\[
\theta_i(t) = K_r(t) K_x(t) e^{A_{hp} \tau}
\]

\[
\lambda(t, \eta) = K_r(t) K_x(t) e^{-A_{hp} \eta} B_{hp}
\]

(13) can be rewritten as (see Fig. 1)

\[
u(t) = \theta_i(t) x_{hp}(t) + \int_{-\tau}^{0} \lambda(t, \eta) u(t + \eta) d\eta + K_r(t) r(t).
\]

Since \( A_{hp} \) and \( B_{hp} \) are unknown, \( \theta_i \), \( \lambda \), and \( K_r \) need to be estimated. It is noted that (11) presents the equation that define the ideal value of \( K_r \), which is denoted as \( K_r^* \). Comparing (13) and (15), the ideal values of \( \theta_i \) and \( \lambda \) can be obtained as

\[
\theta_i^* = K_r^* K_x^* e^{A_{hp} \tau}
\]

\[
\lambda^*(\eta) = K_r^* K_x^* e^{-A_{hp} \eta} B_{hp}.
\]

The closed-loop dynamics can be obtained using (5) and (15) as

\[
\hat{x}_{hp}(t) = A_{hp} x_{hp}(t) + B_{hp} \theta_i(t - \tau) x_{hp}(t - \tau) + \int_{-\tau}^{0} B_{hp} \lambda(t - \tau, \eta) u(t + \eta - \tau) d\eta + B_{hp} K_r r(t - \tau).
\]

Defining the deviations of the adaptive parameters from their ideal values as \( \hat{\theta}_i = \theta_i - \theta_i^* \) and \( \hat{\lambda} = \lambda - \lambda^* \), adding and
Using (12), (18) is rewritten as

\[ e = \frac{\Phi_1}{\Phi} \]

Defining the tracking error as \( e(t) = x_{hp} - x_m \), subtracting (8) from (19), using (11), and following a similar procedure given in [38], it is obtained that:

\[
\dot{e}(t) = \dot{x}_{hp} - \dot{x}_m
\]

Using (12), (18) is rewritten as

\[
\dot{x}_{hp}(t) = A_m x_{hp}(t) - B_{hp} K^*_r K^*_x x_{hp}(t) + B_{hp} K_r (t - \tau) K_x (t - \tau) x_{hp}(t) + B_{hp} K_r (t - \tau) r(t - \tau).
\] (19)

Using (12) and defining \( \Phi = K_r^{-1} - K_x^{-1} \), we can rewrite (20) as

\[
\dot{e}(t) = A_m e(t) + B_{hp} K_r^{-1} (K_x (t - \tau) - K^*_x K^*_r)
\]

\[
\times \left( e^{A_{hp} \tau} x_{hp}(t - \tau) + \int_{-\tau}^{0} e^{-A_{hp} \eta} B_{hp} u(t + \eta - \tau) d\eta \right)
\]

\[
+ B_{hp} \Phi(t - \tau) \left( K_r (t - \tau) K_x (t - \tau) e^{A_{hp} \tau} x_{hp}(t - \tau) + \int_{-\tau}^{0} e^{-A_{hp} \eta} B_{hp} u(t + \eta - \tau) d\eta \right)
\]

\[
+ K_r (t - \tau) r(t - \tau).
\] (21)

Using (16) and (21), we obtain that

\[
\dot{e}(t) = A_m e(t) + B_m \dot{\theta}_1(t - \tau) x_{hp}(t - \tau)
\]

\[
\times \left( (K_r^{-1}(t - \tau) \dot{\theta}_1(t - \tau) - K^{*-1}_{r} \theta^*_1) x_{hp}(t - \tau) + \int_{-\tau}^{0} (K_r^{-1}(t - \tau) \dot{\lambda}(t - \tau, \eta) - K^{*-1}_{r} \lambda^*_1(\eta)) u(t + \eta - \tau) d\eta \right)
\]

\[
+ B_m \Phi(t - \tau) u(t - \tau).
\] (22)

Using (14), (22) can be rewritten as

\[
\dot{e}(t) = A_m e(t) + B_m
\]

\[
\times \left( (K_r^{-1}(t - \tau) \dot{\theta}_1(t - \tau) - K^{*-1}_{r} \theta^*_1) x_{hp}(t - \tau) + \int_{-\tau}^{0} (K_r^{-1}(t - \tau) \dot{\lambda}(t - \tau, \eta) - K^{*-1}_{r} \lambda^*_1(\eta)) u(t + \eta - \tau) d\eta \right)
\]

\[
+ B_m \Phi(t - \tau) u(t - \tau).
\] (23)

Defining \( \theta_1 = K_r^{-1} \dot{\theta}_1 \) and \( \lambda_1 = K_r^{-1} \lambda \) and using their deviations from their ideal values, \( \dot{\theta}_1 = \theta_1^* - \dot{\theta}^*_1 \) and \( \lambda_1 = \lambda^*_1 - \lambda^*_1 \), (23) can be rewritten as

\[
\dot{e}(t) = A_m e(t) + B_m \dot{\theta}_1(t - \tau) x_{hp}(t - \tau)
\]

\[
+ B_m \int_{-\tau}^{0} \dot{\lambda}_1(t - \tau, \eta) u(t + \eta - \tau) d\eta
\]

\[
+ B_m \Phi(t - \tau) u(t - \tau).
\] (24)

The following lemma will be necessary to prove the main theorem of this article.

**Lemma 4:** Suppose that the continuous function \( u(t) \) is given as

\[
u(t) = f(t) + \int_{-\tau}^{0} \lambda(t, \eta) u(t + \eta - \tau) d\eta\]

where, \( f, : [\tau, \infty) \rightarrow R \) and \( \lambda : [\tau, \infty) \times [-\tau, 0] \rightarrow R \).

Then

\[
|u(t)| \leq 2(f + c_0 c_1) e^{c_2(t - \tau)} \quad \forall t \geq t' \]

if constants \( t', f, c_0, c_1 \in R^+ \) exist such that \( |f(t)| \leq f \)

\[
\int_{-\tau}^{0} \lambda^2(t, \eta) d\eta \leq c_0^2, \quad \text{for} \ t \in [t', t^*]
\]

and

\[
\int_{-\tau}^{0} u^2(t + \eta) d\eta \leq c_1^2 \quad \forall t \leq t'.
\] (28)

**Proof:** The proof of Lemma 4 can be found in [39]. \( \square \)

**Theorem 5:** Given the initial conditions \( \dot{\theta}_1(\zeta), \lambda_1(\zeta, \eta), \Phi(\zeta) \) and \( x_{hp}(\zeta) \) for \( \zeta \in [-\tau, 0] \), and \( u(\zeta) \) for \( \zeta \in [-2\tau, 0] \), there exists \( \tau^* \) such that for all \( \tau \in [0, \tau^*] \), the controller (15) with the adaptive laws

\[
\dot{\theta}_1(t) = -x_{hp}(t - \tau) e(t)^T P B_m
\]

\[
\dot{\lambda}_1(t) = -u(t - \tau) e(t)^T P B_m
\]

\[
\lambda_1(t, \eta) = -u(t + \eta - \tau) e(t)^T P B_m
\] (29)

\[30\]

where \( P \) is the symmetric positive definite matrix satisfying the Lyapunov equation \( A_m^T P + P A_m = -Q \) for a symmetric positive definite matrix \( Q \), which can be employed to obtain controller parameters using \( K_r = \text{Proj}(K, \Phi K_r), \theta_1(t) = K_r(t) \theta_1(t), \) and \( \lambda(t) = K_r(t) \lambda_1(t) \) and make the pilot neuro-muscular and plant aggregate system (S) follow the crossover reference model (8) asymptotically, i.e., \( \lim_{t \rightarrow \infty} x_{hp}(t) = x_m(t) \), while keeping all the signals bounded.
The reference signal \( r(t) \) is generated as a sum of the sinusoids with frequencies of 0.1, 0.3, 0.5, 0.7, 1, 1.3, and 1.5 rad/s with the same amplitude of 0.2 and without phase shift.

### A. Experimental Environment

In order to test the proposed adaptive human model against data, an experimental setup consisting of a Logitech Extreme 3D Pro joystick and a Toshiba Portege-Z30-B laptop with Intel Core i7 CPU is used (see Fig. 2).

The tracking task is performed by an operator monitoring the pursuit display, which provides information about the error between the target to be followed and follower, which is the output of the plant (see Fig. 3). The operator provides the input \( u_p \) (see Fig. 1) through the joystick, which is fed to the plant using MATLAB/Simulink (R2018b). In return, the response of the plant is calculated and shown on the laptop screen in real time.

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The tracking task is performed by an operator monitoring the pursuit display, which provides information about the error between the target to be followed and follower, which is the output of the plant (see Fig. 3). The operator provides the input \( u_p \) (see Fig. 1) through the joystick, which is fed to the plant using MATLAB/Simulink (R2018b). In return, the response of the plant is calculated and shown on the laptop screen in real time.

The reference signal \( r(t) \) is generated as a sum of the sinusoids with frequencies of 0.1, 0.3, 0.5, 0.7, 1, 1.3, and 1.5 rad/s with the same amplitude of 0.2 and without phase shift.

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Fig. 5. Time evolution of the error between the output of the plant controlled by the adaptive model and the reference model output.

Fig. 6. Evolution of human adaptive parameters $\theta_{x1}$ and $\theta_{x2}$.

The neuromuscular dynamics is taken as $Y_h(s) = \frac{(s + 3)}{(s + 2)}e^{-0.3s}$, where the time delay $\tau = 0.3$ is the effective time delay, including human decision-making delay and neuromuscular lags.

Remark 6: In this article, we assume that the neuromuscular dynamics are given. The procedure for finding the neuromuscular model can be found in [36] and [37].

Remark 7: In [26], where decoupled multiaxis tracking tasks are considered, a factor is introduced to represent the fact that the pilots are less aggressive in multiaxis tasks, compared to single-axis ones. In this work, although we cover the single-axis tracking task, an extension to two-axis tracking would not need any new model development or an increase in complexity, except using the same equations stated in the step-by-step implementation guide in Section V, for the additional axis, with different gain and frequency parameters. A detailed analysis of single- and double-axis tracking tasks can be found in [42].

B. Behavior of the Adaptive Model

The error between the plant output and the reference model is shown in Fig. 5. The effect of uncertainty injection can be seen at $t = 70$ s. Figs. 6–8 show the adaptive human model parameters. To understand the amount of agreement between these results and the human experimental trials, visual and statistical analyses are provided in the following.

Remark 8: The model is developed based on adaptive control principles, and therefore, even when the plant model is not changing, the adaptive parameters may continue to vary based on the changes in the reference inputs. This parameter variance, however, is not a problem since the model’s tracking capability, as well as the boundedness of the adaptive parameters, is guaranteed by the Lyapunov analysis. We also know that unless the inputs are persistently exciting, the adaptive parameters may not converge to their ideal values. Therefore, different trajectories may result in different model parameters. This is not an issue since these parameters are tuned automatically by the adaptive process.

Remark 9: Adaptation rates are important in terms of transient behavior, and therefore, their selection is important to obtain a successful model. There are some guidelines that can be used in the implementation, at least to start at a good initial condition for the tuning process [43], [44]. Consider a generic adaptive law $\dot{\theta} = -\Gamma e_1 \Omega$, where $\Gamma$ is the adaptation rate matrix, $e_1$ is the tracking error, and $\Omega$ is the vector of corresponding system signals. Assuming that $e_1$ and elements of $\Omega$ are the same order of magnitude as the reference signal $r$, the adaptation rate for a parameter $\theta$ is chosen as $\Gamma = \theta^*/(3\tau_m\bar{r}^2)$, where $\theta^*$ is an estimate of the desired adaptive parameter, $\tau_m$ is the smallest time constant of the reference model, and $\bar{r}$ is the maximum possible amplitude of the reference signal.

C. Participants and Experimental Procedure

Eleven participants (six women and five men) from the graduate and undergraduate student pools of Bilkent University participated in the experiment. All of the participants read and signed the “informed consent to participate” document. This study is approved by the Bilkent University Ethics Committee for research with human participants. Before the experiments, to familiarize the participants with the experimental setup, and its environment, consisting of the display and the joystick, each participant was asked to follow a given reference via joystick inputs for the duration of 200 s. To prevent learning during these warm-up runs, the reference input, uncertainty injection times, and the uncertainty types were chosen differently from
the ones used in the real experimental runs. In particular, the reference signal for the warm-up runs consisted of the sum of the sinusoids with frequencies of 0.1, 0.5, 1, and 1.5 rad/s with the same amplitude of 0.2 and without phase shift. The plant dynamics at the beginning of the warm-up run was $(2/(s^2+3s+2))$. At $t = 45$ s, the dynamics changed to $(5/(s+2))$ in a step-like manner (suddenly). It is changed to $(3/(s+1))$ at around $t = 90$ s using a sigmoid function (gradually) and again changed to a zero-order dynamics at 150 s (suddenly).

D. Time- and Frequency-Domain Analyses of the Adaptive Model

Let $f_{p1}(t), f_{p2}(t), \ldots, f_{pk}(t)$ be the plant outputs when participants $p1, p2, \ldots, pk$ are in the loop, respectively. For each $f_{pi}(t)$, $t = T_1, T_2, \ldots, T_N$, where $T_j$, $j = 1, 2, \ldots, N$, represents a sampling instant, at each sampling instant $T_j$, the minimum, the maximum, and the mean values of the plant outputs when participants are in the loop can be obtained as

\begin{align*}
  f_{pmin}(T_j) &= \min_{i=1,2,\ldots,k} f_{pi}(T_j), \quad j = 1, \ldots, N \\
  f_{pmax}(T_j) &= \max_{i=1,2,\ldots,k} f_{pi}(T_j), \quad j = 1, \ldots, N \\
  f_{pmean}(T_j) &= \frac{\sum_{i=1}^{k} f_{pi}(T_j)}{k}, \quad j = 1, \ldots, N
\end{align*}

where $k = 11$ is the number of participants. Fig. 9 shows the evolutions of $f_{pmin}$ and $f_{pmax}$, together with $f_{ad}(t) \in \mathbb{R}^N$, which is the plant output when the adaptive human model is in the loop, where $t = T_1, T_3, \ldots, T_N$. It is seen that the plant output when adaptive human model is in the loop almost always stays between the maximum and the minimum values of the plant output when participants are in the loop. Furthermore, Fig. 10 shows that $f_{pmean}$ and $f_{ad}$ evolve reasonably close to each other. The adaptive model and the mean participant data are also compared in the frequency domain both before and after the uncertainty introduction, in Figs. 11 and 12, respectively. As seen from the figures, both the adaptive model and participant data show similar properties with the crossover model. However, the model starts to deviate from the human data for lower frequencies. The transfer functions used to plot these Bode plots are obtained using the time-response data provided in Fig. 10. For the systems run by the adaptive model and the participants, we obtained the open-loop transfer functions $\left(3.5/s\right)e^{-0.3s}$ and $\left(4/(s+0.8)\right)e^{-0.3s}$, respectively, before the uncertainty introduction, and $\left(4.5/s\right)e^{-0.3s}$ and $\left(3.5/(s+0.1)\right)e^{-0.3s}$, after the uncertainty introduction. Fig. 13 shows the close match between the closed-loop time responses of the estimated transfer functions and the data used to obtain them.

E. Comparison Between the Adaptive and Fixed Human Models

In this section, to demonstrate the advantages of having an adaptive human model instead of a fixed one, we compare the behavior of these two models.
As the fixed human model, we use the transfer function 
\[(1.125(s^2 + 3s + 2))/(s^2 + 3s))e^{-0.3s},\]
which makes the closed-loop transfer function equal to the crossover model given in (32).

Fig. 14(a) shows that before the occurrence of uncertainty at \(t = 70\) s, both the fixed and the adaptive human models present a reasonable agreement with the experimental results. However, after the uncertainty introduction, the fixed model becomes unstable, while the adaptive model continues to show acceptable performance. The result demonstrates the danger of using a fixed model in the presence of time delays. Using the method proposed in [45], one can find that the system after the uncertainty introduction becomes unstable for delay values larger than 0.22 s. To provide a comparison in the stable region, we plot the response of the fixed model for a delay value of 0.22 s in Fig. 14(b). The plot shows that even though the response is stable, it fails to provide a reasonable prediction of the human pilot response after the uncertainty introduction.

\section*{F. Statistical Analysis of the Adaptive Model Using Confidence Intervals}

The difference between the plant output when the \(i\)th participant is in the loop and when the adaptive human model is in the loop is defined as
\[d_i = f_{ad} - f_{pi}, \quad i = 1, \ldots, k\]  
(37)
where \(d_i = [d_i(T_1), \ldots, d_i(T_N)]^T \in \mathbb{R}^N, i = 1, \ldots, k,\)
is called the \(i\)th difference. The mean and the standard deviation of the \(i\)th difference is obtained as
\[\bar{d}_i = \frac{\sum_{j=1}^N d_i(T_j)}{N}, \quad i = 1, \ldots, k\]  
(38)
\[s_i = \sqrt{\frac{\sum_{j=1}^N (d_i(T_j) - \bar{d}_i)^2}{N - 1}}, \quad i = 1, \ldots, k.\]  
(39)

The normal-scores plot for \(\bar{d}_i\) is shown in Fig. 15. The figure does not show any significant deviation from the normal distribution. This shows us that the data do not suggest that the population of mean errors, \(\bar{d}_i\), deviates significantly from normal distribution. The sample mean and the sample standard deviation of \(\bar{d}_i\)’s can be obtained as
\[\bar{d} = \frac{\sum_{i=1}^k \bar{d}_i}{k}\]  
(40)
\[s = \sqrt{\frac{\sum_{i=1}^k (d_i - \bar{d})^2}{k - 1}}.\]  
(41)

Let \(\mu_0\) be the mean value of the population of mean errors, which is given as
\[\mu_0 = \frac{\sum_{i=1}^K \bar{d}_i}{K}\]  
(42)
where \(K\) is the population size. Since normal-scores plot, given in Fig. 15, did not provide any counter evidence, assuming that the distribution of the set of data \([\bar{d}_1, \ldots, \bar{d}_K]\)
is normal with mean \(\mu_0,\) \(\mu_0\) satisfies the following
concluded using (43) that we are 95% confident that $\chi_0^2$ and obtaining $\chi_0^2$, where $\mu$ respectively, and substituting $\bar{s}$ and calculating $\bar{d}$ point of the $t$ distribution with degree of freedom $k - 1$, which can be obtained from the $t$-distribution table. Since the number of participants, $k = 11$, is less than 30, it is appropriate to use the $t$-distribution. Using $\alpha = 0.05$, obtaining $t_{0.025, k}$ from the $t$-distribution table as 2.228, and calculating $\bar{d}$ as $-0.068$ and $s$ as 0.0379, it can be concluded using (43) that we are 95% confident that $\mu_0 = 0$ is in the interval $(-0.0323, 0.0187)$. This shows that the mean $\mu_0$ of the population’s mean deviation from the adaptive human model is reasonably close to zero.

Similarly, the variance, $\sigma_0^2$, of the population’s mean deviation from the adaptive human model satisfies the following probability [46]:

$$P\left(\frac{(k-1)s^2}{\chi^2_{a/2, k}} < \sigma_0^2 < \frac{(k-1)s^2}{\chi^2_{1-a/2, k}}\right) = 1 - \alpha$$

where $\chi^2_{a/2, k}$ is the upper $a/2$ point of the $\chi^2$ distribution with degree of freedom $k - 1$ and can be obtained from the $\chi^2$ distribution table. Calculating $s$ from (41), using $\alpha = 0.05$, and obtaining $\chi^2_{a/2, k}$ and $\chi^2_{1-a/2, k}$ from the $\chi^2$ table with ten degrees of freedom, it can be concluded using (44) that we are 95% confident that $\sigma_0$ is in the interval $(0.0265, 0.0663)$. This shows that the standard deviation $\sigma_0$ of the population’s mean deviation from the adaptive human model is reasonably small.

G. Statistical Analysis of the Adaptive Model Using Hypothesis Testing

In this analysis, we test whether the hypothesis “the mean value of the population mean errors, or the mean deviations from the adaptive model,” is zero. In other words, our null hypothesis, $H_0$, is given as

$$H_0 : \mu_0 = 0$$

where $\mu_0$ is defined in (42). The alternative hypothesis, $H_1$, is given as $H_1 : \mu \neq 0$. Similar to the confidence interval analysis, assuming that $\mu_0$ is the mean of a normally distributed set of data $\{\bar{d}_1, \ldots, \bar{d}_K\}$, where $K$ is the population size, the hypothesis $H_0$ is rejected if

$$\left|\frac{(\bar{d} - \mu_0)\sqrt{k}}{s}\right| \geq t_{a/2}$$

where $\bar{d}$ and $s$ are obtained from (40) and (41), $k$ is the number of participants, and $t_{a/2}$ is the upper $a/2$ point of the $t$ distribution with degree of freedom $k - 1$ [46]. Using the significance level $\alpha = 0.05$ and degree of freedom $k - 1 = 10$, obtaining $t_{0.025, 10} = 2.228$ from the $t$-distribution table, calculating $\bar{d}$ as $-0.068$ and $s$ as 0.0379 using (40) and (41), respectively, and substituting $\mu_0 = 0$ and $k = 11$, the left-hand side of (46) can be calculated as 0.5935, which is less than $t_{0.025, 10}$.

Therefore, we cannot reject $H_0$. We retain $H_0$ and conclude that $H_1$ fails to be proved.

Since we are retaining the null hypothesis, we want to minimize the probability $\beta$ of incorrectly retaining the null hypothesis. This means that we want our test’s power, $1 - \beta$, to be large, such as 0.95. What is the minimum required deviation of the population mean from 0, represented as $\mu_1$, that would make our test to incorrectly retain the null hypothesis with 0.05 probability, i.e., $\beta = 0.05$? To calculate this, we first write the rejection region, $R$, using (46) as

$$R : \left|\frac{(\bar{d} - \mu_0)\sqrt{k}}{s}\right| \geq t_{a/2} \implies R : |\bar{d}| \geq 0.0255.$$  

Defining $T = ((\bar{d} - \mu_1)\sqrt{k})/s$, for $\beta = 0.05$, we need

$$P\left(\frac{(-0.0255 - \mu_1)\sqrt{k}}{s} < T < \frac{(0.0255 - \mu_1)\sqrt{k}}{s}\right) = \frac{\beta}{2} = 0.025.$$  

Using the $t$-table, it can be found that the minimum $|\mu_1|$ that satisfies (48) is 0.051. This means that our test can detect an 0.051 deviation from the mean value of the mean error between the adaptive human model and the participant data when the probability of the test to incorrectly conclude that the model and the data are compatible ($\mu_0 = 0$) is only 5%.

Analyses of the experimental results where a first-order plant dynamics is used with a sudden uncertainty injection is provided above. All of the results, including the ones for the other cases, where plants with different orders and sudden/gradual uncertainty injections, are summarized in Tables II and III. The data collected from the participants can be reached at http://www.syslab.bilkent.edu.tr/research.
VI. SUMMARY

In this article, an adaptive human pilot model based on model reference adaptive control principles is proposed. This model mimics the pilot decision-making process by making sure that the overall closed-loop system follows the crossover model in the presence of plant uncertainties. The stability of the system is shown using the Lyapunov–Krasovskii stability criteria. Furthermore, experiments with human operators are conducted to validate the model. The detailed visual and statistical analyses of the experimental results show that the adaptive model creates similar system responses as the human operators.

APPENDIX

Proof of Theorem 5

Consider a Lyapunov–Krasovskii functional [39]

\[ V(t) = e^T P e + \text{tr} (\Phi^T(t) \Phi(t)) + \text{tr} (\dot{\delta}_1^T(t) \dot{\delta}_1(t)) + \int_{-\tau}^{0} \int_{t+\tau}^{t} \text{tr} (\dot{\delta}_1^T(\zeta) \dot{\delta}_1(\zeta)) d\zeta dv + \int_{-\tau}^{0} \int_{t+\tau}^{t} \text{tr} (\Phi^T(\zeta) \Phi(\zeta)) d\zeta dv + \int_{-\tau}^{0} \text{tr} (\lambda_1^T(\tau, \eta) \lambda_1(\tau, \eta)) d\eta + \int_{-\tau}^{0} \int_{t+\tau}^{t} \text{tr} (\lambda_1^T(\tau, \eta) \lambda_1(\tau, \eta)) d\eta d\zeta dv. \] (49)

The derivative of \( V(t) \) can be calculated as

\[ \dot{V}(t) = e^T(t) P e(t) + e^T(t) P \dot{e}(t) + 2 \text{tr} (\dot{\delta}_1^T(t) \dot{\delta}_1(t)) + 2 \text{tr} (\Phi^T(t) \dot{\Phi}(t)) + \int_{-\tau}^{0} \int_{t+\tau}^{t} 2 \text{tr} (\lambda_1^T(\tau, \eta) \lambda_1(\tau, \eta)) d\eta dv + \int_{-\tau}^{0} \text{tr} (\Phi^T(t) \Phi(t)) d\eta + \int_{-\tau}^{0} \int_{t+\tau}^{t} \text{tr} (\lambda_1^T(\tau, \eta) \lambda_1(\tau, \eta)) d\eta d\zeta dv. \] (50)

By substituting (29)–(31) into (51), it is obtained that

\[ \dot{V}(t) = -e^T(t) Q e(t) - 2 \int_{-\tau}^{0} \text{tr} (x_{hp}(t - \tau) e^T(t) P B_m \dot{\delta}_1(t + \tau)) dv + 2 \int_{-\tau}^{0} \text{tr} (\dot{\delta}_1^T(t) \dot{\delta}_1(t)) + \int_{-\tau}^{0} \text{tr} (\Phi^T(t) \Phi(t)) d\eta \]

\[ + \tau \int_{-\tau}^{0} \text{tr} (\dot{\delta}_1^T(t) \dot{\delta}_1(t)) + \int_{-\tau}^{0} \text{tr} (\Phi^T(t) \Phi(t)) d\eta \]

\[ + \tau \int_{-\tau}^{0} \text{tr} (\dot{\delta}_1^T(t) \dot{\delta}_1(t)) - \int_{-\tau}^{0} \text{tr} (\dot{\delta}_1^T(t) \dot{\delta}_1(t)) dv + \int_{-\tau}^{0} \text{tr} (\Phi^T(t) \Phi(t)) d\eta \]

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\[ -e^T(t)Qe(t) + \int_{-\tau}^{0} \text{tr}\left(2\tilde{\Phi}^T(t)\tilde{\Phi}(t + \nu) + \Phi^T(t+\nu)\Phi(t) - \Phi^T(t + \nu)\Phi(t + \nu)\right) d\nu \\
+ \int_{-\tau}^{0} \text{tr}\left(2\tilde{\Phi}^T(t)\tilde{\Phi}(t + \nu) + \Phi^T(t+\nu)\Phi(t) - \Phi^T(t + \nu)\Phi(t + \nu)\right) d\nu \\
+ \int_{-\tau}^{0} \int_{-\tau}^{0} \text{tr}\left(2\tilde{\lambda}_1^T(t, \eta)\tilde{\lambda}_1(t, \eta) + \tilde{\lambda}_2^T(t, \eta)\tilde{\lambda}_2(t, \eta)\right) d\eta d\nu. \]  

Using the trace property $\text{tr}(A + B) = \text{tr}(A) + \text{tr}(B)$ and the algebraic inequality $a^2 \geq 2ab - b^2$ for two scalars $a$ and $b$, it can be shown that $\text{tr}(2A^T B + A^T A - B^T B) \leq 2\text{tr}(A^T A)$. Using these inequalities, (52) can be rewritten as

\[ \dot{V}(t) \leq -e^T(t)Qe(t) + \int_{-\tau}^{0} \text{tr}\left(2\tilde{\Phi}^T(t)\tilde{\Phi}(t + \nu) + \Phi^T(t+\nu)\Phi(t) - \Phi^T(t + \nu)\Phi(t + \nu)\right) d\nu \\
+ \int_{-\tau}^{0} \int_{-\tau}^{0} \text{tr}\left(2\tilde{\lambda}_1^T(t, \eta)\tilde{\lambda}_1(t, \eta) + \tilde{\lambda}_2^T(t, \eta)\tilde{\lambda}_2(t, \eta)\right) d\eta d\nu. \]  

Defining $q \equiv ((\lambda_{\min}(Q))/\|B_m^T P\|_F^2)$, the inequality

\[ q - 2\tau \left(\|x_{hp}(t-\tau)\|^2 + \|u(t-\tau)\|^2 + \int_{-\tau}^{0} \|u(t + \eta - \tau)\|^2 d\eta\right) > 0. \]  

needs to be satisfied for the nonpositiveness of $\dot{V}$. Assuming that $x_{hp}$ and $u$ are bounded in the interval $[t_0 - 2\tau, t_0)$, the rest of the proof is divided into the following four steps.

**Step 1:** In this step, the negative semidefiniteness of the Lyapunov–Krasovskii functional’s (49) time derivative in the interval $[t_0 - \tau, t_0)$ is shown, which leads to the boundedness of the signals in this interval. In addition, an upper bound for $u$ in the interval $[t_0 - 2\tau, t_0)$ is given.

Suppose that

\[ \sup_{\zeta \in [t_0 - \tau, t_0)} \|x_{hp}(\zeta)\|^2 \leq \gamma_1 \]

\[ \sup_{\zeta \in [t_0 - 2\tau, t_0)} \|u(\zeta)\|^2 \leq \gamma_2 \]  

for some positive $\gamma_1$ and $\gamma_2$, and $\tau_1$ is given such that

\[ 2\tau_1 (\gamma_1 + \gamma_2 + \tau_1 \gamma_2) < q. \]  

Then, the following inequality is satisfied:

\[ q - 2\tau \left(\|x_{hp}(\zeta - \tau)\|^2 + \|u(\zeta - \tau)\|^2 + \int_{-\tau}^{0} \|u(\zeta + \eta - \tau)\|^2 d\eta\right) > 0 \]

\[ \forall \zeta \in [t_0, t_0 + \tau) \quad \forall \tau \in [0, \tau_1]. \]  

It follows that $V(t)$, defined in (49), is nonincreasing for $t \in [t_0, t_0 + \tau)$. Thus, we have

\[ \lambda_{\min}(P)\|e(\zeta)\|^2 \leq e(\zeta)^T P e(\zeta) \leq V(t_0) \]

which leads to

\[ \|x_{hp}(\zeta)\| - \|x_{m}(\zeta)\| \leq \|e(\zeta)\| \leq \sqrt{\frac{V(t_0)}{\lambda_{\min}(P)}}. \]  

Then, we have

\[ \|x_{hp}(\zeta)\| \leq \sqrt{\frac{V(t_0)}{\lambda_{\min}(P)}} + \|x_{m}(\zeta)\| \]

for $\forall \zeta \in [t_0, t_0 + \tau)$. We also have the inequality

\[ \|\Phi(\zeta)\|^2 \leq V(t_0) \implies \|K_{r-1} - K_{r-1}(\zeta)\|^2 \leq V(t_0) \implies \|K_{r-1}(\zeta)\| \leq \sqrt{V(t_0)} + \|K_{r-1}\| \]

for $\forall \zeta \in [t_0, t_0 + \tau)$. It is noted that the boundedness of $\Phi = K_{r-1} - K_{r-1}$ does not guarantee the boundedness of $K_r$. In order to guarantee the boundedness of $K_r$
of the boundedness of $\Phi$, the projection algorithm [40] is employed as
\[ \dot{K}_r = \text{Proj}(K_r, -K_rB_m^TPe(t)u^T(t - \tau)K_r) \]  
with an upper bound $K_{\max}$, that is, $\|K_r\| \leq K_{\max}$. Thus, a lower bound for $\|K_r^{-1}(\xi)\|$ can be calculated using the following algebraic manipulations:
\[
\begin{align*}
K_r(\xi)K_r^{-1}(\xi) &= I \\
\Rightarrow \|K_r(\xi)K_r^{-1}(\xi)\| &= 1 \\
\Rightarrow 1 &\leq \|K_r(\xi)\|\|K_r^{-1}(\xi)\| \leq K_{\max}\|K_r^{-1}(\xi)\| \\
\Rightarrow \frac{1}{K_{\max}} &\leq \|K_r^{-1}(\xi)\|. 
\end{align*}
\]  
Defining $k_1 = \sqrt{V(\bar{t})} + \|K_r^{-1}\|$ and using (63), it is obtained that
\[
\frac{1}{K_{\max}} \leq \|K_r^{-1}(\xi)\| \leq k_1, \: \xi \in [t_0, t_0 + \tau). 
\]  
Therefore, $K_r$ is always bounded and $K_r^{-1}(\xi)$ is bounded for $\forall \xi \in [t_0, t_0 + \tau)$. Furthermore, using the definitions of $\theta_1, \theta_2, \lambda$, and $\lambda_1$ given in Theorem 5, and the nonincreasing Lyapunov functional (49), it can be concluded that
\[
\begin{align*}
\|\bar{\theta}_1(\xi)\|^2_F &\leq V(t_0) \quad \Rightarrow \quad \|K_r^{-1}(\xi)\|d(\xi,T)\|^2_F \leq V(t_0) \\
\int_{-\tau}^{0} \|\bar{\lambda}(\xi, \eta)\|^2_F d\eta &\leq V(t_0) \\
\Rightarrow \int_{-\tau}^{0} \|K_r^{-1}(\xi)\|\lambda(\xi, \eta)\|^2_F d\eta &\leq V(t_0) 
\end{align*}
\]  
for $\forall \xi \in [t_0, t_0 + \tau)$. Using (67) and (68), it can be obtained that
\[
\begin{align*}
\|\bar{\theta}_1(\xi)\|^2_F &\leq K_{\max}^2 V(t_0) \\
\int_{-\tau}^{0} \|\lambda(\xi, \eta)\|^2_F d\eta &\leq K_{\max}^2 V(t_0) 
\end{align*}
\]  
for $\forall \xi \in [t_0, t_0 + \tau)$. To simplify the notation, we define
\[
I_0 \equiv \max\left(\frac{\sqrt{V(t_0)}}{\lambda_{\min}(P)} + \sup_{[t_0,t_0+\tau]} \|x_m(\xi)\|, \frac{K_{\max}\sqrt{V(t_0)}}{K_{\max}V(t_0)} \right) 
\]  
where $R_{\max}$ is the upper bound of the reference input $r(t)$. An upper bound on the control signal $u(t)$ for $t \in [t_0, t_0 + \tau)$ can be derived by using Lemma 4 and (15). In particular, setting $t'_i = t_0$, $t'_j = t_0 + \tau$, and $\xi_{c_i} = V(t_0)$, we obtain that
\[
\|u(\xi)\| \leq 2\left(\int_{-\tau}^{0} u^2(t_0 + \eta)d\eta\right)^{1/2} I_0 e^{b\tau} 
\]  
for $\forall \xi \in [t_0, t_0 + \tau)$, where $\bar{f}$, which is the upper bound of $\theta_1(t)x_{hp}(t) + K_r(t)r(t)$, depends only on $I_0$. Defining $g(\gamma_2, I_0, \tau) \equiv 2\left(\int_{-\tau}^{0} u_0^2\sqrt{T}e^{b\tau}\right)^{1/2}, (71)$ can be rewritten as
\[
\|u(\xi)\| \leq g(\gamma_2, I_0, \tau), \: \forall \xi \in [t_0, t_0 + \tau). 
\]  
\]  
The rest of the proof is similar to the one given in [39]. In the following, a summary of the next steps are given.

**Step 2:** A delay range $[0, \tau_2]$ is found, which satisfies the condition (56) over the interval $[t_0, t_0 + 2\tau]$ as
\[
2\tau_2(\bar{I}_0^2 + (\max(\gamma_2, g(\gamma_2, I_0, \tau_2)))^2(1 + r_2)) < q 
\]  
which leads to $\|x_{hp}(\xi)\| \leq I_0, \: \forall \xi \in [t_0, t_0 + 2\tau), \forall \tau \in [0, \bar{\tau}_2), \bar{\tau}_2 = \min(\tau_1, \tau_2)$.

**Step 3:** It is shown in this step that the bound on $u$ over the interval $[t_0, t_0 + \tau]$ depends only on $A_{hp}, B_{hp}, T$, and $\tau$, where $T$ is a value between $t_0$ and $\tau$. Denoting this upper bound as $U(I_0)$, we have $|u(t)| \leq U(I_0), t \in [t_0, t_0 + \tau]$.

**Step 4:** Using the calculated upper bound for $u$ in the previous step, a delay range $[0, \tau_2]$ is found, which satisfies the condition
\[
2\tau_2(\bar{I}_0^2 + (\max(U(I_0), g(U(I_0), I_0, \tau_2)))^2(1 + r_3)) < q. 
\]  
For $\tau_4 = \min(\bar{\tau}_2, \tau_2), \|x_{hp}(\xi)\| \leq I_0$ and $|u(\xi)| \leq U(I_0)$ for all $\xi \in [t_0, t_0 + \tau], \forall \tau \in [0, \tau_4]$. The above four steps show that $x_{hp}(\xi)$ and $u(\xi)$ are bounded for $\forall \xi \in [t_0, t_0 + k\tau], k \geq 1 \forall \tau \in [0, \tau_4]$. By assuming that $x_{hp}$ and $u$ are bounded for a given $k$, the rest of the proof consists of showing that the boundedness of these variables hold for $k + 1$. Using this assumption and repeating steps 1–4, which leads to satisfying (74), we conclude that the Lyapunov function is nonincreasing and $\|x_{hp}(\xi)\| \leq I_0$, and $|u(\xi)| \leq g(U(I_0), I_0, T) \leq 0 \: \forall \xi \in [t_0, t_0 + (k + 1)\tau], \tau \leq \tau_4 \leq \tau_3$. This completes the boundedness proof by induction. Then, using Barbalat’s Lemma, it can be shown that the error between the human-in-the-loop system output $x_{hp}$ and the reference model output $x_m$ converges to zero.

**REFERENCES**

[1] W. D. Nothwang, M. J. McCourt, R. M. Robinson, S. A. Burden, and J. W. Curtis, “The human should be part of the control loop?” in Proc. Resilience Week (RWS), Aug. 2016, pp. 214–220.

[2] M. Korber, W. Schneider, and M. Zimmermann, “Vigilance, boredom proneness and detection time of a malfunction in partially automated driving,” in Proc. Int. Conf. Collaboration Technol. Syst. (CTS), Jun. 2015, pp. 70–76.

[3] Y. Yildiz and I. V. Kolmanovsky, “A control allocation technique to recover from pilot-induced oscillations (capio) due to actuator rate limiting,” in Proc. Amer. Control Conf., Jun. 2010, pp. 516–523.

[4] Y. Yildiz and I. Kolmanovsky, “Stability properties and cross-coupling performance of the control allocation scheme CAPIO,” J. Guid., Control, Dyn., vol. 34, no. 4, pp. 1190–1196, Jul. 2011.

[5] D. M. Acosta et al., “Pilot evaluation of a control allocation technique to recover from pilot-induced oscillations,” J. Airvec, vol. 52, no. 1, pp. 130–140, Jan. 2015.

[6] S. S. Tohidi, Y. Yildiz, and I. Kolmanovsky, “Pilot induced oscillation mitigation for unmanned aircraft systems: An adaptive control allocation approach,” in Proc. IEEE Conf. Control Technol. Appl. (CCTA), Aug. 2018, pp. 343–348.

[7] W. Li, D. Sadigh, S. S. Sastry, and S. A. Seshia, “Synthesis for human-in-the-loop control systems,” in Proc. Int. Conf. Tools Algorithms Construct. Anal. Syst., Cham, Switzerland: Springer, 2014, pp. 470–484.

[8] T. Hulin, A. Albu-Schaffer, and G. Hirzinger, “Passivity and stability boundaries for haptic systems with time delay,” IEEE Trans. Control Syst. Technol., vol. 22, no. 4, pp. 1297–1309, Jul. 2014.

[9] T. Yucelen, Y. Yildiz, R. Sipahi, E. Yousefi, and N. Nguyen, “Stability limit of human-in-the-loop model reference adaptive control architectures,” Int. J. Control, vol. 91, no. 10, pp. 2314-2331, Oct. 2018.
[10] E. Eraslan, Y. Yildiz, and A. M. Annaswamy, “Shared control between pilots and autopilots: An illustration of a cyberphysical human system,” IEEE Control Syst., vol. 40, no. 6, pp. 77–97, Dec. 2020.

[11] J. Zhao and T. Iwasaki, “CPC control for harmonic motion of assistive robot with human motor control identification,” IEEE Trans. Control Syst. Technol., vol. 28, no. 4, pp. 1323–1336, Jul. 2020.

[12] D. T. McRuer and E. S. Krendel, “Dynamic response of human operators,” Tech. Rep. WADC-TR-56-524, 1957.

[13] D. T. McRuer and E. S. Krendel, “Mathematical models of human pilot behavior,” Tech. Rep. AGARD-AG-188, 1974.

[14] T. P. Neal and R. E. Smith, “A flying qualities criterion for the design of fighter flight-control systems,” J. Aircr., vol. 8, no. 10, pp. 802–809, Oct. 1971.

[15] D. Mencner and D. Graham, “Pilot-vehicle control system analysis,” in Proc. Guid. Control Conf., Aug. 1963, p. 310.

[16] G. Beerens, H. Danneweld, M. Mulder, and M. Van Paassen, “An investigation into crossover and pattern parameter adjustment,” in Proc. AIAA Model. Simul. Control. Conf., Aug. 2008, p. 7112.

[17] M. Martinez-Garcia, T. Gordon, and L. Shu, “Extended crossover model for human-control of fractional order plants,” IEEE Access, vol. 5, pp. 27622–27635, 2017.

[18] R. B. Warrier and S. Devasia, “Inferring intent for novice human-in-the-loop iterative learning control,” IEEE Trans. Control Syst. Technol., vol. 25, no. 5, pp. 1698–1710, Sep. 2017.

[19] J. J. Gil, A. Rubia, and J. Savall, “Decreasing the apparent inertia of an impedance haptic device by using force feedforward,” IEEE Trans. Control Syst. Technol., vol. 17, no. 4, pp. 833–838, Jul. 2009.

[20] R. D. Wierenga, “An evaluation of a pilot model based on Kalman filtering and optimal control,” IEEE Trans. Man-Mach. Syst., vol. MMS–10, no. 4, pp. 108–117, Dec. 1969.

[21] D. L. Kleinman, S. Baron, and W. H. Levison, “An optimal control model of human response—Part I: Theory and validation,” Automatica, vol. 6, no. 3, pp. 357–369, May 1970.

[22] X. Na and D. J. Cole, “Modelling and identification of a driver controlling a vehicle equipped with active steering where the driver and vehicle have different target paths,” in Proc. 11th Int. Symp. Adv. Vehicle Control (AVEC), 2012.

[23] M. M. Lone and A. K. Cooke, “Pilot-model-in-the-loop simulation environment to study large aircraft dynamics,” Proc. Inst. Mech. Eng., G, J. Aerosp. Eng., vol. 227, no. 3, pp. 555–568, Mar. 2013.

[24] W.-L. Hu, C. Rivetta, E. MacDonald, and D. P. Chassin, “Operator training reference models for human-in-the-loop systems,” in Proc. Annu. Hawaii Int. Conf. Syst. Sci., 2019, pp. 1–8.

[25] R. A. Hess, “Modeling pilot control behavior with sudden changes in vehicle dynamics,” J. Aircr., vol. 46, no. 5, pp. 1584–1592, Sep. 2009.

[26] R. A. Hess, “Modeling human pilot adaptation to flight control anomalies and changing task demands,” J. Guid., Control, Dyn., vol. 39, no. 3, pp. 655–666, Mar. 2016.

[27] S. Xu, W. Tan, and X. Qu, “Modeling human pilot behavior for a smart inceptor,” IEEE Trans. Man-Hum. Syst., vol. 49, no. 6, pp. 661–671, Dec. 2019.

[28] M. Lone and A. Cooke, “Review of pilot models used in aircraft flight dynamics,” Aerosp. Sci. Technol., vol. 34, pp. 55–74, Apr. 2014.

[29] S. Xu, W. Tan, A. V. Efremov, L. Sun, and X. Qu, “Review of control models for human pilot behavior,” Annu. Rev. Control, vol. 44, pp. 274–291, Jan. 2017.

[30] P. Zaal and B. Sweet, “Estimation of time-varying pilot model parameters,” in Proc. AIAA Modeling Simulation Technol. Conf., Aug. 2011, p. 6474.

[31] R. F. M. Duarte, D. M. Pool, M. M. van Paassen, and M. Mulder, “Experimental scheduling functions for global LPV human controller modeling,” IFAC-PapersOnLine, vol. 50, no. 1, pp. 15853–15858, Jul. 2017.

[32] X. Zhang, T. M. Seigler, and J. B. Hoagg, “Modeling the control strategies that humans use to control nonminimum-phase systems,” in Proc. Amer. Control Conf. (ACC), Jul. 2015, pp. 471–476.

[33] K. van der El, D. M. Pool, H. J. Danneweld, M. R. M. Van Paassen, and M. Mulder, “An empirical human controller model for preview tracking tasks,” IEEE Trans. Cybern., vol. 46, no. 11, pp. 2609–2621, Nov. 2016.

[34] S. S. Tohidi and Y. Yildiz, “Adaptive human pilot model for uncertain systems,” in Proc. 18th Eur. Control Conf. (ECC), Jun. 2019, pp. 2938–2943.

[35] D. T. McRuer and H. R. Jex, “A review of quasi-linear pilot models,” IEEE Trans. Hum. Factors Electron., vol. HFE-8, no. 3, pp. 231–249, Sep. 1967.

[36] R. E. Magdaleno, Experimental Validation and Analytical Elaboration for Models of the Pilot’s Neuromuscular Sub-System in Tracking Tasks, vol. 1757, Washington, DC, USA: NASA, 1971.

[37] M. M. van Paassen, J. C. van der Vaart, and J. A. Mulder, “Model of the neuromuscular dynamics of the human pilot’s arm,” J. Aircr., vol. 41, no. 6, pp. 1482–1490, Nov. 2004.

[38] K. S. Narendra and A. M. Annaswamy, Stable Adaptive Systems. Chelmsford, MA, USA: Courier Corporation, 2012.

[39] Y. Yildiz, A. Annaswamy, I. V. Kolmanovsky, and D. Yanakiev, “Adaptive posicast controller for time-delay systems with relative degree $n \leq 2$,” Automatica, vol. 46, no. 2, pp. 279–289, 2010.

[40] E. Lavretsky and K. A. Wise, Robust Adaptive Control. Springer, 2013, pp. 317–353.

[41] S. S. Tohidi and Y. Yildiz, “Handling actuator magnitude and rate saturation in uncertain over-actuated systems: A modified projection algorithm approach,” Int. J. Control, vol. 95, pp. 1–14, Sep. 2020.

[42] S. Barendswaard, D. M. Pool, M. M. Van Paassen, and M. Mulder, “Dual-axis manual control: Performance degradation, axis asymmetry, crossfeed, and intermittency,” IEEE Trans. Human-Mach. Syst., vol. 49, no. 2, pp. 113–125, Apr. 2019.

[43] Y. Yildiz, A. M. Annaswamy, D. Yanakiev, and I. Kolmanovsky, “Sparkignition-engine idle speed control: An adaptive control approach,” IEEE Trans. Control Syst. Technol., vol. 19, no. 5, pp. 990–1002, Sep. 2011.

[44] A. Alan, Y. Yildiz, and U. Poyraz, “High-performance adaptive pressure control in the presence of time delays: Pressure control for use in variable-thrust rocket development,” IEEE Control Syst., vol. 38, no. 5, pp. 26–52, Oct. 2018.

[45] N. Olgar and R. Sipahi, “An exact method for the stability analysis of time-delayed linear time-invariant (LTI) systems,” IEEE Trans. Autom. Control, vol. 47, no. 5, pp. 793–797, May 2002.

[46] R. A. Johnson and G. K. Bhattacharyya, Statistics: Principles and Methods. Hoboken, NJ, USA: Wiley, 2019.