Epistemic Model and Three-valued Interpretation

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Abstract

In this paper, I propose a general framework called Epistemic Logic (EL) which captures a full range of epistemic phenomena, and apply it to the three-valued interpretation of natural language sentences. Further, I mention the ability which EL implies, i.e. treatment of indexicals, relationship to Data Semantics etc.

1 Epistemic Logic

Epistemic Logic consists of the syntax and semantics of EL with respect to which natural language sentences are interpreted.

1.1 Syntax of Epistemic Logic

The syntax of Epistemic Logic consists of a normal version of first-order predicate logic with modal operators 'may' and 'must' which is defined as follows:

(1) Vocabulary of EL
   i) individual constants: a, b, c, ... ∈ Const.
   ii) individual variables: x, y, z, ... ∈ Var.
   iii) j(≥ 0)-ary predicate symbols: pred^i ∈ Pred^j. (i ∈ I(j). I(j) is a segment of the set of natural numbers.)
   iv) logical connectives: ¬, ∧, ∨, ⊃, ≡ ∈ LC.
   v) quantification symbols: ∀, ∃ ∈ Q.
   vi) epistemic modal symbols: must, may ∈ EM.
   vii) auxiliary symbols: (, ).

Term = Const U Var is called the set of terms of EL.
Pred = ∪^j Pred^j is called the set of predicate symbols of EL.
Voc = Term U Pred U LC U Q U EM U {(, )} is called the set of the vocabulary of EL.
Exp = Voc®1 is called the set of well-formed expressions of EL.

(2) Formation Rules of EL

Formulas: The set Form of EL is the least set which satisfies the following conditions:
   i) If α_1, ..., α_j ∈ Term, pred^i ∈ Pred^j, then pred^i(α_1, ..., α_j) ∈ Form.
   ii) If φ, χ ∈ Form, then (¬φ), (φ ∧ χ), (φ ∨ χ), (φ ⊃ χ), (φ ≡ χ) ∈ Form.
   iii) If φ ∈ Form, then (∀xφ), (∃xφ) ∈ Form.
   iv) If φ ∈ Form, then (must φ), (may χ) ∈ Form.

WExp = (Voc \ {(, )}) U Form is called the set of well-formed expressions of EL.
Auxiliary symbols of formulas are eliminated in obvious cases.

1.2 Semantics of Epistemic Logic

The Semantics of Epistemic Logic consists of the epistemic model which is constructed from extensional model structures via extensional models.
1.2.1 Extensional Model Structure

I start from the traditional well-known relational structure for the interpretation of first-order formulas which is found e.g. in Bridge(1977):

\[ \mathfrak{A} = (A, \{R^i_{j}\}_{i \in I(j), j \in J}, \{F^i_k\}_{k \in K(l), j \in L}, \{c_m\}_{m \in M}) \]

Here, \( R^i_{j} \) is the i-th j-ary relation, \( F^i_k \) is the k-th l-ary function, and \( c_m \) is the m-th constant. But to simplify (3), I omit functions and constants so that (3) looks like as follows:

\[ \mathfrak{A} = (A, \{R^i_{j}\}_{i \in I(j), j \in J}) \]

On the other hand, I complicate (4) by replacing relations with attributes and their extensional functions for reasons explained in 2, and call it an extensional model structure:

\[ \text{Extensional Model Structure:} \]

\[ MS = (\text{Ind}, \{\text{Attr}^i_j\}_{i \in I(j), j \in J}, \text{Ext}) \]

\( \text{Ind: domain, i.e. (non-empty) set of individuals} \)

\( \text{Attr}^i_j: \) the i-th j-ary attribute

\( \text{Attr}^i_j \overset{\text{def}}{=} \bigcup_{i \in I(j)} \text{Attr}^i_j: \) set of j-ary attributes

\( \text{Attr} \overset{\text{def}}{=} \bigcup_{j \in J} \text{Attr}^i_j: \) set of attributes

\( \text{Ent: extensional function} \)

\( \text{Attr}^n \rightarrow \text{Rel}^n \subseteq \text{Ind}^n \) \( (n \geq 0) \)

1.2.2 Extensional Model

An extensional model is defined as follows:

\[ M = (MS, f) \]

Here, \( MS: \) extensional model structure in (5)

\( f: \) value assignment is the following partial function:

\( \text{Const} \rightarrow \text{Ind}, \)

\( \text{Pred}^n \rightarrow \text{Attr}^n \)

1.2.3 Epistemic Model

The Epistemic Model is defined as follows:

\[ \mathcal{M} = \mathcal{P}(\mathcal{M}) \setminus \emptyset \]

Here, \( \mathcal{M} \) is the set of extensional models with the same domain and set of attributes.

The elements of \( \mathcal{M} \) are called situations.

1.3 Entity of EL

The set \( \text{Ent} \) of entities of EL is defined parallel to \( \text{Exp} \) as follows:

\[ \text{Atomic Entity of EL} \]

i) individuals: \( a, b, c, \ldots \in \text{Ind} \).

ii) \( \text{Par} = \{x \mid x \in \text{Var}\}: \) set of parameters of EL.

iii) \( \text{Attr}^n: \) set of n-ary attributes.

iv) \( \text{LO} = \text{LC}: \) set of logical operators of EL.

v) \( \text{Quant} = Q: \) set of quantifiers of EL.

vi) epistemic modal operators: \( \text{must}, \text{may} \in \text{EpMod} \)
vii) (, ) : parentheses.

\[ G\text{Ind} = \text{Ind} \cup \text{Par} \]

is called the set of general individuals of EL.

\[ \text{Attr} = \bigcup \text{Attr}^n \]

is called the set of attributes.

\[ A\text{Ent} = G\text{Ind} \cup \text{Attr} \cup \text{LO} \cup \text{Quant} \cup \text{EpMod} \cup \{(, )\} \]

is called the set of atomic entities of EL.

\[ \text{Ent} = A\text{Ent} \]

is called the set of entities of EL.

\[ \text{Infons} \]

The set \( \text{Inf} \subset \text{Ent} \) of infons of EL is the least set which satisfies the following conditions:

i) \( t \in \text{Inf} \) (\( t \) means the 'undefined' infon.)

ii) If \( t_1, \ldots, t_n \in G\text{Ind}, \text{Attr}^n \in \text{Attr}^n \), then \( \text{Attr}^n(t_1, \ldots, t_n) \in \text{Inf} \).

iii) If \( \text{inf}_1, \text{inf}_2 \in \text{Inf} \), \( \{\neg, \land, \lor, \exists, \equiv\} = \text{LO} \), then \( (\neg \text{inf}_1), (\text{inf}_1 \land \text{inf}_2), (\text{inf}_1 \lor \text{inf}_2), (\text{inf}_1 \equiv \text{inf}_2) \in \text{Inf} \).

iv) If \( \text{inf} \in \text{Inf} \), \( Q \in \text{Quant} \), \( x \in \text{Par} \), then \( (Qx \text{inf}) \in \text{Inf} \).

v) If \( \text{inf} \in \text{Inf} \), \( \text{epmod} \in \text{EpMod} \), then \( (\text{epmod inf}) \in \text{Inf} \).

\[ \text{WEnt} = (A\text{Ent} \setminus \{(, )\}) \cup \text{Inf} \]

is called the set of well-formed entities of EL.

Parentheses of infons are eliminated in obvious cases.

1.4 Interpretation

Natural language sentences are translated to the corresponding formulas of EL. However, the latter are not directly interpreted, but translated to the intermediate entity of EL, i.e. infons, which are then interpreted w.r.t. a situation, and their semantic behaviors are explained.

1.4.1 Interpretation of infons

(10) The truth value \( s(c, \text{inf}) \) of \( \text{inf} \in \text{Inf} \) w.r.t. a situation \( s \) and a context situation \( c \) is defined as follows:

i) \( s(c, \text{inf}) = 1 \) if for all \( M \in s, M(c, \text{inf}) = 1 \),

ii) \( s(c, \text{inf}) = 0 \) if for all \( M \in s, M(c, \text{inf}) = 0 \).

(1 and 0 mean 'true' and 'false' respectively.)

(11) \( \pi : \text{Par} \rightarrow \text{Ind} \)

is called a parameter assignment.

\( d \in \text{Ind}, \text{and } x \in \text{Par} \).

\( \pi_d^x = (\pi \setminus \{(x, \pi(x))\}) \cup \{(x, d)\} \)

is called the \( (x,d) \)-variant of \( \pi \).

(12) The truth value \( M(c, \text{inf}) \) of \( \text{inf} \in \text{Inf} \) w.r.t. an extensional model \( M \) and a context situation \( c \) is defined as follows:

i) \( M(c, \text{inf}) = 1 \) if for all \( \pi, M(c, \pi, \text{inf}) = 1 \),

ii) \( M(c, \text{inf}) = 0 \) if for all \( \pi, M(c, \pi, \text{inf}) = 0 \).

(13) Let \( M = (MS, f) (MS = (\text{Ind}, \{\text{Attr}^i\}_{i \in I(J)}, J, \text{Ext})) \) be an extensional model, \( \pi \) be a parameter assignment, and \( c \) be a context situation. Then, the function \( M(c, \pi, .) \) with the domain \( \text{WEnt} \) is defined as follows:

i) a) \( M(c, \pi, .) \upharpoonright \text{Ind} \) is constant function.

b) \( M(c, \pi, .) \upharpoonright \text{Par} = \pi \).

c) \( M(c, \pi, .) \upharpoonright \text{Attr} = \text{Ext} \).

d) \( M(c, \pi, .) \upharpoonright \text{LO} \) is the function such that, for an arbitrary \( l \in \text{LO} \), \( M(c, \pi, l) \) is the correspondidng truth function of \( lo \) upon \( \{1, 0\} \) defined in the first-order logic in a familiar manner.

e) \( M(c, \pi, .) \upharpoonright \text{Quant} \cup \text{EpMod} \) is a function such that \( \forall \mapsto \wedge, \exists \mapsto \lor, \text{must} \mapsto \wedge, \text{and } \text{may} \mapsto \lor. \) (\( \wedge, \lor \) are suprema and infimum function respectively.)

ii) In the following, \( \text{inf}, \text{inf}_1, \text{inf}_2 \in \text{Inf} \).

\( M(c, \pi, .) \upharpoonright \text{Inf} \) is the partial function \( \text{Inf} \rightarrow \{1, 0\} \) defined as follows:
a) $M(c, \pi, \top)$ is undefined.

b) If $t_1, \ldots, t_n \in GInd$, $Attr^n \in Attr^n$, then

\[
M(c, \pi, Attr^n(t_1, \ldots, t_n)) = M(c, \pi, Attr^n)(M(c, \pi, t_1), \ldots, M(c, \pi, t_n)).
\]

c) $M(c, \pi, \neg (inf)) = M(c, \pi, \neg (M(c, \pi, inf))).$

d) $M(c, \pi, (inf_1 \land inf_2)) = M(c, \pi, inf_1)M(c, \pi, \land)M(c, \pi, inf_2).

e) $M(c, \pi, (inf_1 \lor inf_2)) = M(c, \pi, inf_1)M(c, \pi, \lor)M(c, \pi, inf_2).

f) $M(c, \pi, (inf_1 \supset inf_2)) = M(c, \pi, inf_1)M(c, \pi, \supset)M(c, \pi, inf_2)$.

g) $M(c, \pi, (\equiv inf_1 \equiv inf_2)) = M(c, \pi, inf_1)M(c, \pi, \equiv)M(c, \pi, inf_2)$.

In the following, $x \in Par.$

h) If $\text{quant} \in \text{Quant}$, then

\[
M(c, \pi, \text{quant} \ x \text{ in } f) = M(c, \pi, \text{quant})\{M(c, \pi, f) \mid d \in \text{Ind}\}.
\]

i) If $\text{epmod} \in \text{EpMod}$, then

\[
M(c, \pi, \text{epmod} \text{ in } f) = M(c, \pi, \text{epmod})\{M(c, \pi', f) \mid M(c, \pi', f) \in f\}.
\]

($\pi'$ is an arbitrary parameter assignment.)

In (13iii), if $M(c, \pi, inf)$ is undefined, $M(c, \pi, -\text{inf})$ is also undefined. Likewise for (13iid-i).

c in $M(c, \pi, \cdot)$, $s(c, \cdot)$ etc. is omitted in unnecessary cases.

The interpretation of $\text{inf} \in \text{Inf}$ always begins with $c(c, \text{inf})$.

### 1.4.2 Interpretation of EL formulas

(14) \[
\Phi_M : \text{Exp} \to \text{Ent}
\]

is called the translation function of $\text{Exp}$ to $\text{Ent}$ w.r.t. $M$, and defined as follows:

Let $\alpha_1, \ldots, \alpha_n \in \text{Voc}$. Then,

\[
\Phi_M(\alpha_1, \ldots, \alpha_n) = \begin{cases} 
\text{If } \Phi_M(\alpha_1), \ldots, \Phi_M(\alpha_n) \text{ are all defined: } \Phi_M(\alpha_1) \cdots \Phi_M(\alpha_n), \\
\text{otherwise: } \top.
\end{cases}
\]

Here,

a) $\Phi_M \mid \text{Const} = \top \mid \text{Const}$.

b) $\Phi_M \mid \text{Var}$: the function such that, for an arbitrary $x \in \text{Var}$, $x \mapsto x \in \text{Par}$

c) $\Phi_M \mid \text{Pred} = \top \mid \text{Pred}$.

d) $\Phi_M \mid \text{LC} \cup \text{LO} \cup \text{QU} \cup \text{EM} \cup \{, , \}$: the function such that, for an arbitrary $\alpha \in \text{LC} \cup \text{LO} \cup \text{QU} \cup \text{EM} \cup \{, , \}$,

\[
\alpha \mapsto \alpha \in \text{LO} \cup \text{QU} \cup \text{LO} \cup \text{QU} \cup \text{EM} \cup \{, , \}.
\]

According to the above definition of $\Phi_M$, elements of $W\text{Exp}$ and $W\text{Ent}$ correspond to each other as follows:

| $\text{WExp}$ | $\text{WEnt}$ |
|----------------|----------------|
| individual constant | individual |
| individual variable | parameter |
| n-ary predicate symbol | n-ary attribute |
| logical connective | logical operator |
| quantification symbol | quantifier |
| auxiliary symbol | parentheses |
| formula | infon |

We represent $\Phi_M(a)$ with a ($a \in \text{Const}$), and $\Phi_M(\text{pred})$ with pred ($\text{pred} \in \text{Pred}$). So, the notational difference between an expression of EL and the corresponding entity is that terms and predicates in the former are represented in roman characters, whereas the corresponding general individuals and attributes in the latter are represented in italicized characters. But the former is a string of symbols, the latter is a semantic entity.

$\text{form} \in \text{Form}$ is interpreted w.r.t. $s(c, \cdot)$, $M(c, \cdot)$ and $M(c, \pi, \cdot)$ as follows:

(16) i) $s(c, \text{form}) = 1 \iff \text{for all } M \in s, M(c, \text{form}) = 1$.

ii) $s(c, \text{form}) = 0 \iff \text{for all } M \in s, M(c, \text{form}) = 0$.

iii) $M(c, \text{form}) = 1 \iff \text{for all } \pi, M(c, \pi, \text{form}) = 1$.

iv) $M(c, \text{form}) = 0 \iff \text{for all } \pi, M(c, \pi, \text{form}) = 0$.

v) $M(c, \text{form}) = 1 \iff M(c, \pi, \Phi_M(\text{form})) = 1$.

vi) $M(c, \text{form}) = 0 \iff M(c, \pi, \Phi_M(\text{form})) = 0$. 

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1.4.3 \(\models\text{-notation}

(17) Let \(A\) be a formula or an infon. Then, \(M(c, \pi, A) = 1\), \(M(c, A) = 1\), and \(s(c, A) = 1\) are also represented by \(M, \pi \models_c A\), \(M \models_c A\), and \(s \models_c A\) respectively.

(18) i) \(M, \pi \not\models_c A\) \iff It is not the case that \(M, \pi \models_c A\), 
   ii) \(M \not\models_c A\) \iff It is not the case that \(M \models_c A\), 
   iii) \(s \not\models_c A\) \iff It is not the case that \(s \models_c A\).

c in \(M(c, \cdot), \models\text{-etc. is omitted in unnecessary cases.}

1.4.4 Translation of natural language sentences to formulas of EL

For the interpretation of natural language sentences, we only assume an intuitive translation function \(Tr\) which translates natural language sentences to their corresponding formulas of EL. I.e.,

(19) \(Tr(sent) = form\),

\(sent\) is a natural language sentence, and \(form \in Form\.)

whose truth value is then determined by \(c(c, Tr(sent))\) w.r.t. the context situation \(c\), i.e. the situation where \(sent\) is uttered.

2 Infon as an Intermediate Entity

The reason for assuming the intermediate entity "attribute" is the following: Let \(M = (\text{Ind, Attr, Ext}, f)\) be an extensional model. \(f\) represents the rule of an epistemic subject with which semantic entities are assigned to expressions of EL. But which semantic entities are they? \(f(\text{pred}_1)\) is an attribute which is then extensionalized by \(Ext\), but \(f(\alpha)\) (\(\alpha\) is a term of EL.) is an individual. \(\Phi_M(form)\), i.e. an infon, could be thought of as the intension of a sentence in the traditional sense. But there are following differences. First, the semantic correspondence of terms appears as individuals in infons, which represents the (alternative) opinion of Kripke(1972), Kaplan(1977, 1978, 1989) that proper names and indexicals directly denote individuals without intermediate entities such as intension. Second, attributes \(Attr^f_1\) (which correspond to traditional intensions of predicates) are introduced as undefined elements, and their extensions are one of their properties induced by \(Ext\). \(\Phi_M(form)\) is an semantic entity written in a general concept language which can be compared and possessed by different epistemic subjects. So, we could interpret that \(form\) expresses the Kaplanian character, \(\Phi_M(form)\) the Kaplanian content, and \(M(\pi, \Phi_M(form))\) represents the extension.

\(f\) and \(Ext\) enables the following distinction:

(20) i) \(Ext(f_1(\text{red})) = \{a, b\}\)
    ii) \(Ext(f_2(\text{red})) = \{c, d\}\)
    iii) \(Ext(f_1(\text{red})) = \{c, d\}\)

(20i) is the extension of 'red' in the actual world \((Ext_1)\) by means of the 'correct' linguistic rule \(f_1\) in a community. (20ii) is its extension in another world, but the linguistic rule as such is correct. But in (20iii), the linguistic rule is false. For example, the epistemic subject mistakes 'red' for yellow. And \(\{c, d\}\) could be the set of yellow things.

Now, consider the following example:

(21) i) John: "I'm going to the party."
    ii) Marry (to John): "You are going to the party."

In (21), the character is different between i) and ii):

(22) i) go-to-the-party(I)
    ii) go-to-the-party(you)

But the content is the same between them:

(23) i) go-to-the-party(john).
assuming that \( \{M_j\} \) represents John’s context situation of (21i) with \( f_j(I) = john \), and \( \{M_m\} \) represents Marry’s context situation of (21ii) with \( f_m(you) = john \).

3 Three-valued Interpretation of Natural Language Sentences

3.1 Classification of Three-valuedness

Here, I distinguish the following three types of three-valuedness:

(24) Type I: category error, failure of existence presupposition,
Type II: undefined cases,
Type III: failure of information,

which are exemplified as follows:

(25) i) The capitalism is yellow.
   ii) Naomi is XYZ.
   iii) It’s gold. (In the situation where it’s not certain if it’s gold.)

Intuitively, the third truth value ‘neutral’ appears just in the case that an utterance is interpreted as neither true nor false. But the meaning of ‘false’ is not so clear. As to Type I, it should be understood as the predicate negation, so that the three-valuedness of (25i) w.r.t. \( M, \pi \) is formulated as

(26) i) \( M, \pi \models \neg\text{yellow(capitalism)} \)
   ii) \( M, \pi \models \text{yellow(capitalism)} \)

Here, we assume

(27) \( M(\pi, \text{yellow}) = \text{Ext(yellow)} = \text{yellow}^+ \subseteq \text{Ind} \)
    \( M(\pi, \text{yellow}) = \text{Ext(yellow)} = \text{yellow}^- \subseteq \text{Ind} \)
    \( \text{Ext(yellow)} \cap \text{Ext(yellow)} = \phi \)
    \( \text{Ind} \setminus (\text{Ext(yellow)} \cup \text{Ext(yellow)}) = \text{yellow}^N \)
    \( \text{yellow} = \text{yellow} \)

and call \( \text{yellow}^+, \text{yellow}^-, \text{yellow}^N \) the positive, negative, and neutral domain of \( \text{yellow} \) respectively. And the individual capitalism belongs to \( \text{yellow}^N \) so that the truth value ‘neutral’ is assigned to (25i).

As to (25ii), we assume that \( f(XYZ) \) is undefined. Then, \( \Phi_M(XYZ(\text{naomi})) = \top \). But, according to (13iia), \( M(c, \pi, \top) \) is undefined, i.e. neither true nor false.

As to (25iii), suppose that \( s = \{M_1, M_2\} \) such that (25iii) is true w.r.t. \( M_1 \), but it’s false w.r.t. \( M_2 \). Then,

(28) i) \( s \not\models \text{gold(it)} \)
   ii) \( s \not\models \neg\text{gold(it)} \)

And, (25iii) is interpreted as neutral w.r.t. \( s \).

There is a certain difference between the three-valuedness of Type I, II and Type III. The former appears w.r.t. an extensional model, but the latter appears w.r.t. a situation. As we see from (10), \( s(c, \text{inf}) \) is interpreted in a modal fashion, i.e. in order to determine the truth value of \( \text{inf} \) w.r.t. \( s \), we must refer to several extensional models. For this reason, we call the former types of three-valuedness the non-modal three-valuedness, the latter the modal three-valuedness. But this modality should not be understood in the traditional modal logical manner, because \( s(c, \text{inf}) \) is not determined w.r.t. other “possible worlds”, but w.r.t. extensional models in \( s \). So, we call this kind of modality “epistemic modality”.

In order to enable an three-valued interpretation, we must a bit modify (10), (12) and (13ii) in the following manner:

(29) I) To (10) and (12),
   iii) Otherwise, \( s(c, \text{inf}) = N \) (\( N \) means ‘neutral’) and
iii) Otherwise, \( M(c, inf) = N \) are added respectively.

II) (13ia) is altered to \( 'M(c, \pi, \uparrow) = N' \).

III) (13ib) is altered to:

1. \( M(c, \pi, Attr^i_{\pi}(t_1, \cdots, t_n)) = 1 \), if \( M(c, \pi, Attr^i_{\pi})(M(c, \pi, t_1), \cdots, M(c, \pi, t_n)) = 1 \).
2. \( M(c, \pi, Attr^i_{\pi}(t_1, \cdots, t_n)) = 0 \), if \( M(c, \pi, Attr^i_{\pi})(M(c, \pi, t_1), \cdots, M(c, \pi, t_n)) = 1 \).
3. \( M(c, \pi, Attr^i_{\pi}(t_1, \cdots, t_n)) = N \), otherwise.

IV) An appropriate three-valued interpretation are given to \( \neg, \land, \lor, \exists, \forall \). (See e.g. Prior(1968) etc.)

Now, we can summarize the above-mentioned three-valued interpretation as follows:

(30) \( s \models^{TP} PP_{inf} \)

\( PP \) and \( TP \) represents the truth state of Type I, II, and Type III respectively, i.e., \( TP, PP \in \{T, \bar{T}, N, \bar{N}, F, \bar{F}\} \).

\( T, \bar{T}, N, \bar{N}, F, \bar{F} \) mean ‘true’, ‘not true’, ‘neutral’, ‘not neutral’, ‘false’, ‘not false’ respectively.

The interpretation of \( inf \) which (30) presents is defined in the following manner:

(31) i) \( M(\pi, PP_{inf}) = 1 \iff M(\pi, inf) = PP \) (I.e., ‘neutral’ if \( PP = N \), etc.)
ii) \( M(\pi, PP_{inf}) = 0 \iff M(\pi, inf) \neq PP \).

(32) i) \( s \models^{T} PP_{inf} \iff \) for all \( M \in s, M \models PP_{inf} \).
ii) \( s \models^{\bar{T}} PP_{inf} \iff \) for some \( M, \pi (M \in s), M, \pi \not\models PP_{inf} \).
iii) \( s \models^{N} PP_{inf} \iff \) for one \( M, \pi (M \in s), M, \pi \models PP_{inf} \), and for another \( M', \pi' (M' \in s), M', \pi' \not\models PP_{inf} \).
iv) \( s \models^{F} PP_{inf} \iff \) for all \( M \in s, M \not\models PP_{inf} \), or for all \( M \in s, M \models PP_{inf} \).
vi) \( s \models^{\bar{F}} PP_{inf} \iff \) for some \( M, \pi (M \in s), M, \pi \models PP_{inf} \).

3.2 Hierarchy of Epistemic Modality

But (30) is not the complete form of three-valued interpretation yet. There remains a possibility to extend its epistemic modal part, which is recognized by the following comparison of (simplified two-valued) interpretations:

(33) Extensional Model: \( M \not\models^{\neg} p \) (\( \neg \) means ‘false’.)

Situation Semantics: \( M \not\models^{\neg, p; 0} \)

Signed Formula of Hintikka: \( M \not\models^{F, \neg} p \)

Here, we find three kinds of negation: \( (\#), (\neg, 0, F), \) and \( (\neg) \). And this corresponds to the tripartite distinction of neustic, tropic, and phrastic part of a sentence utterance discussed in Hare(1970) and Lyons(1977:749pp.). According to them, an utterance of a sentence can be analyzed as follows:

(34) I-say-so[it-is-so[prop]]

And I-say-so part, it-is-so part, and prop part (i.e. the propositional content of the sentence) are called neustic part, tropic part, and phrastic part respectively. The three kinds of negation are applied to these respective parts, i.e.:

| Hare       | neustic negation | tropic negation | phrastic negation |
|------------|------------------|-----------------|-------------------|
| Epistemic Model | \( \# \)       | 0               | \( \# \)          |
| Situation Semantics | \( \# \)       | 0               | \( \# \)          |
| Signed Formula      | \( \# \)       | \( F \)         | \( \# \)          |

Then the extended form of (30) so as to include the neustic part looks like as follows:

(36) \( NP(S \models^{TP} PP_{inf}) \)

Here, \( NP, TP, PP \in \{T, \bar{T}, N, \bar{N}, F, \bar{F}\}. NP, TP, \) and \( PP \) are truth values of neustic, tropic, and phrastic part respectively, and \( S \in \text{Pow} (\text{Pow}(M)) \).

The neustic part of (36) is interpreted as follows:

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Theoretically, we can further relativize the neustic modality, and construct an infinite hierarchy of higher order neustic modalities:

$$(38) \quad \ldots \text{NP}_2(S \models \text{NP}(S \models^{TP} PP A)) \ldots$$

(Here, $\text{NP}_2$ is analogous to $\text{NP}, TP,\text{ and PP}. S = \text{Pow(\text{Pow(\text{Pow}(M))))}$.) And this would be the full-fledged form of three-valued interpretation of natural language sentences. But linguistically, I believe that it's enough with the epistemic modalities up to neustic part.

### 3.3 Contraction of Neustic and Phrastic Part to Two-valuedness

In (30), the three-valuedness proper to EL is the tropic part, i.e. the three-valuedness caused by epictemic modality. So, we focus on the tropic three-valuedness, omit higher-order neustic parts, and contract the neustic and phrastic part to two-valued interpretation. But then, the tree-valued interpretation of the tropic part is expressed using neustic and phrastic negation as follows:

The phrastic part is contracted to two-valued form as follows. First, we treat the neutral case in Type I and II in (29) as false. Next, we omit the phrastic $N, \overline{N}$, identify $T, \overline{F}$ and $F, \overline{T}$ on the right hand side of (31) with 1 and 0 respectively, further, $T, \overline{F}$ and $F, \overline{T}$ on the left hand side with $\epsilon$ (empty string) and $\neg$ respectively. Then we can totally eliminate the phrastic part.

The two-valued interpretation of neustic part consists of (37i,ii). Further, we assume that $S = \{s\}$. Then (37i) and ii) amount to

$$(39) \quad \begin{align*}
\text{i) } & T(S, \models^{TP} P P inf) \iff s \models^{TP} P P inf, \\
\text{and} \\
\text{ii) } & \overline{T}(S, \models^{TP} P P inf) \iff s \not\models^{TP} P P inf.
\end{align*}$$

respectively. I.e., the neustic $T$ and $\overline{T}$ are identified with $\models$ and $\not\models$ respectively.

But then, considering (18), (32) and the two-valuedness of $M(\pi, inf)$, the interpretation of infons is expressed using $\models, \not\models$ and $\neg$ without tropic part as follows:

$$(40) \quad \begin{align*}
\text{i) } & s \models T A: s \models A \\
\text{ii) } & s \models \overline{T} A: s \not\models A \\
\text{iii) } & s \models N A: s \not\models A \text{ and } s \not\models \neg A \\
\text{iv) } & s \models \overline{N} A: s \models A \text{ or } s \models \neg A \\
\text{v) } & s \models F A: s \models \neg A \\
\text{vi) } & s \models \overline{F} A: s \not\models \neg A
\end{align*}$$

### 3.4 Data Semantic Modality and Epistemic Logic

In (25iii) and (28), we could neither say that it was gold, nor it wasn't gold. Here, let $s = c$ for simplicity. Then, according to the definition in 1.4.1, we certainly say that

$$(41) \quad \text{It may be gold,}$$

w.r.t. $s$. Further, we can say that

$$(42) \quad \text{He must be a honest guy.}$$

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w.r.t. the situation $s$ such that, for every $M \in s$, (42) without ‘must’ is true. Such ‘may’ and ‘must’ reduce the interpretation to two-valuedness, insofar as $inf$ in $epmodinf$ is defined.

But, as (13.3i.i) indicates, they do not represent traditional possible world semantic modalities, but ‘data semantic’ modalities discussed in Veltman (1981, 1985), Landman (1986).

I shortly consider the relationship between Data Semantics and EL.

3.4.1 Disjunctive sentences

In the situation of (25iii), we certainly say

(43) It’s gold, or it’s not gold.

In general, we can have the following situation:

(44) i) $s \not\in \; inf$,
    ii) $s \not\in \; \neg\; inf$,
    iii) $s \models inf \vee \neg\; inf$.

But the interpretation of disjunctive sentences poses a problem on Data Semantics. Because, in Data Semantics, the disjunction is interpreted in the following manner:

(45) $s \models inf \vee \neg\; inf \iff s \models inf$ or $s \models \neg\; inf$.

But then, in the situation of (44i,ii), (44iii) cannot be the case.

In order to avoid this, Data Semantics assumes that the disjunctive sentence in the above situation is interpreted with may. I.e.:

(46) $s \models may\; (inf \vee \neg \; inf)$.

Then (44i,ii) and (46) are all true, because the data semantic ‘may’ is interpreted in the same manner as EL. On the other hand, Data Semantics interprets conjunctive sentences as ‘$inf_1 \wedge inf_2$’ without modality. So, the treatment of disjunctive sentences remains unnatural.

But EL treats this problem in a straightforward manner. I.e., according to (10),

(47) $s \models inf \vee \neg\; inf \iff$ for all $M \in s, M \models inf \vee \neg\; inf$.

Because ‘$inf \vee \neg\; inf$’ is a logical truth, $M \models inf \vee \neg\; inf$ for all $M \in s$. So, $s \models inf \vee \neg\; inf$. This is an advantage of constructing a situation out of extensional models.

The whole story is illustrated as Fig.1. Here, the whole figure represents the lattice structure of $\mathfrak{M}$ with the top element $M$. And,

(48) $inf \in Inf, s \in \mathfrak{M}$. Then,

i) $sit(\; inf\; ) = \{ M \in \mathfrak{M} \mid M \models inf \}$

ii) $PI(\; inf\; ) = \{ s \mid s \subseteq sit(\; inf\; ) \}$

iii) $PI(\; s\; ) = \{ s' \mid s' \subseteq s \}$

$sit(\; inf\; )$ is called the situation generated by $inf$. $PI(\; inf\; )$ and $PI(\; s\; )$ are called the pseudo-ideal generated by $inf$ and $s$ respectively.

$PI(\alpha)$ is represented by the triangle with the top element $sit(\alpha)$ or $\alpha$. Here, it is the case that

(49) i) ‘$s \models inf \vee \neg\; inf$’ with Data Semantic ‘$\vee$’ $\iff s \in PI(\; inf\; )$ or $s \in PI(\; \neg\; inf\; )$,
    i) ‘$s \models inf \vee \neg\; inf$’ with Epistemic Logical ‘$\vee$’ $\iff s \in PI(\; inf \vee \neg\; inf\; )$.

The crucial defect of Data Semantics consists in the non-existence of ‘disjunctive’ situations such as $sit(\; inf \vee \neg\; inf\; )$ which exist in Epistemic Model that enable the latter to treat disjunctive sentences straightforwardly.

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3.4.2 Data Semantic modality and tropic interpretation

Considering (13) and (40), it is clear that

\[(50)\]

\begin{enumerate}[i)]
\item \(s \models \text{must inf} \iff s \models^{	ext{T}} \text{inf},\)
\item \(s \models \text{may inf} \iff s \models^{	ext{F}} \text{inf}.\)
\end{enumerate}

(Here, we assume that \(\text{inf}\) is defined.) i.e., ‘may’ and ‘must’ transfer the tropic interpretation to the phrastic part. In this sense, Data Semantic modality is a kind of tropic modality.

4 Conclusive Remarks

In this paper, I proposed a treatment of three-valuedness in natural language sentences using EL. But, the present framework of EL is a small portion of its full-fledged form. In this paper, ‘may’ and ‘must’ are always interpreted w.r.t. the context situation. The context situation corresponds to the utterance situation in Situation Semantics. However, if we want to treat tense, indexicals, belief sentences etc., we need another index corresponding to described (or focal) situation in Situation Semantics w.r.t. which ‘must inf’ and ‘may inf’ are interpreted more naturally.

Further, ‘must’ does not have the non-monotonicity which is captured by Data Semantics, although ‘may’ has it in the present framework too. In order to capture the non-monotonicity, we must introduce e.g. ‘background of speech’ (Germ.: Redehintergrund, see Kratzer(1978)).

But they do not mean defects of EL, but are all solved in an extended version of it.

Notes

1 Generally, \(A^*\) represents the set of non-empty concatenation of elements of \(A\).

2 A similar interpretation w.r.t. a situation is found in Fagin et al.(1995). But they apply it to the interpretation of the knowledge operator ‘\(K\)’.

3 According to the present definition, \(\not\models A \iff B\) means \(\neg\not\models (A \models B)\). So, \(s \not\models \text{inf}\) does not mean that, for all \(M \in s\), \(M \not\models \text{inf}\), which is normally the case in elementary logic books. Likewise for \(M \not\models \text{inf}\) etc.

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