Primordial Black Holes from the QCD Axion

Francesc Ferrer,1, 2 Eduard Masso,3, 4 Giuliano Panico,1, 2 Oriol Pujolas2 and Fabrizio Rompineve2

1Department of Physics, McDonnell Center for the Space Sciences, Washington University, St. Louis, Missouri 63130, USA
2IFAE and BIST, Universitat Autònoma de Barcelona, E-08193 Bellaterra, Barcelona
3Departament de Física, Universitat Autònoma de Barcelona, E-08193 Bellaterra, Barcelona
4Laboratory of High Energy and Computational Physics, National Institute of Chemical Physics and Biophysics, Rävala pst. 10, 10143 Tallinn, Estonia

(Dated: July 6, 2018)

We propose a mechanism to generate Primordial Black Holes (PBHs) which is independent of cosmological inflation and occurs slightly below the QCD phase transition. Our setup relies on the collapse of long-lived string-domain wall networks and is naturally realized in QCD axion models with domain wall number $N_{DW} > 1$ and Peccei-Quinn symmetry broken after inflation. In our framework, dark matter is mostly composed of axions in the meV mass range along with a small fraction, $\Omega_{PBH} \gtrsim 10^{-4}\Omega_{CDM}$ of heavy $M \sim 10^{5} - 10^{7}M_{\odot}$ PBHs. The latter could play a role in alleviating some of the shortcomings of the ACDM model on sub-galactic scales. The scenario has distinct signatures in ongoing axion searches as well as gravitational wave observatories.

INTRODUCTION

The recent detection of gravitational waves emitted by the merging of relatively heavy black holes ($M \gtrsim O(10)M_{\odot}$)1 has revived interest in the proposal that the DM of the universe comprises Primordial Black Holes (PBHs)2-6. Although there are constraints on the abundance of PBHs for almost all viable masses (see e.g.7), a small relic abundance of heavy ($M \gtrsim 10^{5}M_{\odot}$) PBHs may play an important role in the generation of cosmological structures and alleviate shortcomings of the standard CDM scenario on sub-galactic scales8-9. Furthermore, such PBHs could shed light on the origin of the super-massive black holes (SMBHs) in the centers of most galaxies, some of which were already in place at very early times10,11.

Several fundamental physics scenarios may explain the existence of PBHs. Arguably, the most studied proposal relies on the gravitational collapse of density fluctuations generated during inflation (see e.g.7 and refs. therein). Nevertheless, it is interesting to understand whether PBHs could naturally arise in other contexts.

In this Letter we propose an alternative PBH formation mechanism, independent of inflationary physics, that relies on the collapse of axionic topological defects (see e.g.12 for an introduction). The generation of PBHs from defects has been investigated in different contexts including PBHs from the collapse of string loops13,14, and from domain walls (DWs) during inflation17. Here we discuss for the first time the formation of PBHs from long-lived string-DW networks15,19 (see20 for a setup closely related to ours) appearing in well-known realizations21-24 of the Peccei-Quinn (PQ) solution to the strong CP problem25-27. These so-called hybrid networks have multiple DWs attached to strings and can arise more generally from sequences of phase transitions in the early Universe.

When the PQ symmetry is broken after inflation, the axion abundance receives comparable contributions from the 1) misalignment mechanism, 2) radiation of quanta from string defects28-30, and 3) annihilation of the string-wall network31. We show in this Letter that there can be a fourth small contribution to the relic axion DM abundance in the form of heavy, $10^{5}-7M_{\odot}$, PBHs. Interestingly, this provides a concrete realization of the proposed role of massive PBHs in the early Universe19 in the context of QCD axion DM. Moreover, our scenario is not subject to some of the strong constraints arising from $\mu$-distortions in the CMB, which plague PBH formation mechanisms from gaussian inflationary fluctuations (see9 for a recent discussion).

The hybrid network dynamics is, of course, hard to analyze. However, for our purposes the essential features can be captured by focusing on the closed walls that arise in the network20.

COLLAPSE OF CLOSED DOMAIN WALLS

Once the Hubble length becomes comparable to the closed wall size $R_{s}$, the DW rapidly shrinks because of its own tension. This occurs at the temperature $T_{s}$ defined by $R_{s} \sim H_{s}^{-1} \approx g_{eff}(T_{s})^{-1/2}M_{p}/T_{s}^{2}$, where $M_{p} = (8\pi G_{N})^{-1/2}$ and $g_{eff}(T_{s})$ is the effective number of degrees of freedom at $T_{s}$. The total collapsing mass has two contributions: one induced by the wall tension $\sigma$, and another one coming from any possible difference in energy density between the two regions separated by the DW:

$M_{s} = 4\pi \sigma R_{s}^{2} + \frac{4}{3} \pi \Delta \rho R_{s}^{3} \sim 4\pi \sigma H_{s}^{-2} + \frac{4}{3} \pi \Delta \rho H_{s}^{-3}$ (1)

For closed DWs arising in the network $\Delta \rho \geq 0$ (see below), thus the wall bounds a region of false vacuum.

Another important parameter for the formation of PBHs is the ratio of the Schwarzschild radius $R_{S}$ of the collapsing wall to the initial size $R_{s}$:

$p \equiv \frac{R_{S}}{R_{s}} \sim \frac{2 G_{N} M_{s}}{H_{s}^{-1}} \sim \frac{\sigma H_{s}^{-1}}{M_{p}} + \frac{\Delta \rho H_{s}^{-3}}{3M_{p}^{2}}$ (2)
Intuitively, if $p$ is close to 1 then the DW rapidly enters its Schwarzschild radius and forms a BH. If $p \ll 1$, however, the wall has to contract significantly before falling inside $R_S$. It is then less likely to form a BH, since asphericities, energy losses and/or angular momentum may severely affect the dynamics of the collapse. We will thus refer to $p$ as the figure of merit for PBH formation from the collapse of DWs.

The temperature behavior of $p$ and $M_*$ is crucial to our proposal. Whenever the tension terms dominate in (1) and (2), we have $M_* \sim T_s^{-4}$ and $p \sim T_s^{-2}$. If, instead, the energy difference terms dominate, we have $M_* \sim T_s^{-6}$ and $p \sim T_s^{-4}$.

Therefore, the duration of the hybrid network has a huge impact on the likelihood of forming PBHs, as well as on their masses. The use of long-lived string-wall networks is the essential new idea of our proposal. This requires multiple DWs attached to each string [12]. Interestingly, this can be realized in QCD axion models with domain wall number larger than one.  

---

**AXION DARK MATTER FROM STRING-WALL NETWORKS**

Let us now embed the basic mechanism illustrated in the previous section in the cosmological history of the QCD axion. Consider a scalar field $\Phi$ with a $U(1)_{PQ}$ symmetry broken at some temperature $T_{QCD}$ after inflation. The field acquires a VEV while its phase is identified with the QCD axion, i.e. $\Phi = v e^{i a(x)/\alpha}$, and string defects are formed (see e.g. [12]). Below $T_{PQ}$, the evolution of the axion is:

1. Most of the energy density in the strings dilutes as $\rho_{strings} \sim \mu_s H^2$, where $\mu_s$ is the string tension.  
2. In addition, the strings radiate axions [28–31]. Away from the strings, the homogeneous axion field is frozen because of Hubble friction.

At $T \lesssim O(\text{GeV})$ the QCD phase transition occurs. Non-perturbative effects generate a periodic potential for $a$

$$V(a, T) = \frac{m^2(T) v^2}{N_{DW}} \left[ 1 - \cos \left( \frac{N_{DW} a}{v} \right) \right],$$  \hspace{1cm} (3)

where $N_{DW}$ is the model dependent color anomaly, also known as DW number. The periodicity of $V$ is given by $2\pi F = 2\pi v / N_{DW}$. The dependence of the axion mass $m(T)$ with temperature is usually parametrized as

$$m^2(T) = \begin{cases} m_0^2, & \text{if } T \ll T_0, \\ m_0^2 \left( \frac{T}{T_0} \right)^{-n}, & \text{if } T \gtrsim T_0, \end{cases}$$  \hspace{1cm} (4)

where $n \approx 7$, $T_0 \approx 100 \text{ MeV}$ are numerical parameters which we take from [31] (see also appendix). Here, $m_0 \approx 0.01 A^2_{\text{QCD}} / F$ is the zero-temperature axion mass, with $\Lambda_{\text{QCD}} \approx 400 \text{ MeV}$.

The potential in (1) leads to the existence of DWs, with tension

$$\sigma(T) \approx 8 m(T) F^2.$$  \hspace{1cm} (5)

These become relevant once Hubble friction is comparable to the axion mass, i.e. at the temperature $T_1 \sim \text{GeV}$ defined by $3H(T_1) = m(T_1)$ (see also appendix).

For topological reasons, each string gets attached to $N_{DW}$ DWs at $T_1$. Thus, a string-wall network is formed, which also contains closed structures. At the same time, the homogeneous component of the axion field starts to oscillate and generates CDM from the misalignment mechanism.

3. Below $T_1$, the energy density of the network is quickly dominated by horizon-size DWs. The subsequent evolution crucially depends on the DW number (see e.g. [12]). If $N_{DW} = 1$ the network is unstable and rapidly decays, generating an additional contribution to the axion DM abundance. If instead $N_{DW} > 1$, the network is stable because strings are pulled in different directions by the DWs. The network dilutes more slowly than radiation and matter, and one faces the so-called DW problem [33]. To avoid this cosmological catastrophe, a bias term can be added to the axion potential [34], [31, 35] of the form:

$$V_B(a) = A_B^1 \left[ 1 - \cos \left( \frac{a}{v} + \delta \right) \right].$$  \hspace{1cm} (6)

Notice that the periodicity of (6) is different from (3), in such a way that there is only one global minimum per period $2\pi v$. Furthermore, the phase $\delta$ represents a generic offset between the bias term and the QCD potential. The addition of (6) to (3) leads to an energy difference between the false and true minima, $\Delta \rho \approx A_B^1$. This generates pressure which competes against the wall tension and renders the network unstable [34]. Balance between the two competing effects is obtained when $\sigma \approx A_B^1 H^{-1}$, which is confirmed by detailed computations and the appendix for more details). Here, $A_B$ is the model dependent color anomaly, also known as DW number.

---

1. The original DFSZ [23, 24] axion has $N_{DW} = 6$, while the simplest KSVZ realization [21, 22] has $N_{DW} = 1$. However, generalizations of the latter with $N_{DW} > 1$ can be considered.

2. See however [29–30] for recent claims of small logarithmic deviations from such scaling regime.

3. For the time being, we consider a bias term which switches on at $T_0$ and remains constant thereafter. We discuss more about this point later on.
numerical analysis of the network evolution in the presence of a bias term [31]. Most of the network disappears at a temperature:

\[ T_2 = \epsilon \left( \frac{M_p \Delta \rho}{\sigma} \right)^{1/2} \left( \frac{90}{\pi^2 g_{\text{eff}}(T_2)} \right)^{1/4}, \]

where \( \epsilon \sim O(0.1 - 1) \) is a parameter which increases with \( N_{\text{DW}} \) and has been numerically determined in [31]. Depending on their initial size, most of the closed DWs in the network will collapse at different temperatures between \( T_1 \) and \( T_2 \). In the process, axions are radiated in such a way that the total axion DM abundance today is given by \( \Omega_a = \Omega_{\text{min}} + \Omega_{\text{strings}} + \Omega_{\text{mw}} \). Here, we have included contributions from misalignment, strings and the network. The axion DM abundance has been numerically studied in [31] (see however [28–30] for more recent estimates of the string contribution) and we review the dependence of \( \Omega_a \) on \( F \) and \( T_2 \) in the appendix.

There are two crucial points to take from the discussion above: 1) for \( N_{\text{DW}} > 1 \) there can be a significant separation between \( T_1 \), the temperature of network formation, and \( T_2 \), the temperature at which its annihilation is efficient, and 2) since (6) lifts the degeneracy of the \( N_{\text{DW}} \) vacua, closed structures surrounding regions with energy \( A^4_B \) can exist in the network. Therefore, from now on we assume \( N_{\text{DW}} > 1 \).

In Fig. 1 we plot the constraints on the \( F-T_2 \) plane for the case \( N_{\text{DW}} = 2 \). The blue-shaded region is excluded because of DM overproduction. The region in parameter space that is in conflict with constraints from supernovae cooling according to the standard analysis in [35] is shown in gray, while the orange-shaded region displays a more conservative recent estimate [37]. The thick black lines signal the largest allowed value of the offset phase \( \delta \) in (6) that does not spoil the axion solution to the strong CP problem. We thus conclude that a viable region of parameter space exists, around \( T_2 \approx 5 \text{ MeV} \) and corresponding to \( A_B \sim 10^{-3} \Lambda_{\text{QCD}} \), where no tuning of \( \delta \) is required. This is in contrast with the conclusion reached in [31], where an overconservative bound on \( \theta_{\text{QCD}} \) was assumed. The untuned region of parameter space is slightly reduced as \( N_{\text{DW}} \) increases. Nonetheless, even for \( N_{\text{DW}} = 6 \) only a mild tuning \( \delta \sim 0.1 \) is required.

In Fig. 1 we also show the relevant would-be BH masses (red lines) and the figure of merit (dashed lines) for closed DWs which collapse at \( T_\ast \approx T_2 \). In the most interesting region of parameter space, we find \( p \sim 10^{-6} \), five orders of magnitude larger than for \( T_\ast \sim T_1 \). This shows the advantage of considering \( N_{\text{DW}} > 1 \). Nevertheless, \( p \) remains quite small and at this point it is unclear whether this leads to a significant fraction of PBHs.

**PBHs from Late Collapses**

In the previous section we have seen that most of the axionic string-wall network disappears around \( T_2 \). Crucially, \( T_2 \) roughly coincides with the temperature when the vacuum energy contribution to \( M_\ast \) and \( p \) starts dominating over the wall tension, as dictated by (1) and (2). Therefore, for closed DWs collapsing at \( T_\ast < T_2 \), \( p \) increases more steeply, as \( T_\ast^{-4} \).

Let us focus on the region around \( F \lesssim 10^8 \text{ GeV} \) and \( T_2 \approx 7 \text{ MeV} \) in Fig. 1 which leads to the best case scenario for PBH formation. In Fig. 2 we plot the figure of merit (dashed lines) and PBH masses (red lines) for DWs collapsing at \( T_\ast < T_2 \).

Fig. 2 shows that DWs that collapse roughly when \( T_\ast < 0.1 \) \( T_2 \) are quite likely to lead to the formation of PBHs. These structures only have to contract by one order of magnitude before entering their Schwarzschild radius. Energy losses via radiation of axions as well as the growth of asphericities can be neglected for such short contractions. Indeed, the radiation of energy from a closed spherical Sine-Gordon DW was studied in [38] and shown to become relevant only once the wall has contracted to a size \( R \sim R_{\ast}^{1/3} m^{-1/3} \ll 0.1 \text{ } H_{\ast}^{-1} \).

Similarly, in the thin wall approximation asphericities do not spoil the formation of PBHs for large \( p \) [39]. Furthermore, we have numerically simulated the collapse of Sine-Gordon non-spherical DWs and checked that they can indeed contract down to \( r_{\text{min}} \lesssim 0.1 \text{ } R_\ast \) [40]. The resulting PBHs would

---

4 We have neglected angular momentum in the numerical simulation. We
have masses $M_\star \sim 10^4 - 10^7 M_\odot$.

Let us now estimate the fraction $f \equiv \Omega_{PBH}/\Omega_{CDM}$. After $T_2$, the energy density of the network is dominated by the bias contribution. However, at any given $T_\star < T_2$ only a small fraction $P_{nw}$ of the original network survives. Therefore,

$$\rho_{nw}(T_\star) \sim P_{nw}(T_\star) \Delta \rho. \quad (8)$$

Assuming that the formation of PBHs occurs mostly at a single temperature $T_\star$, the actual fraction $f$ is then given by:

$$f \sim p^N \times \frac{\rho_{nw}(T_\star)}{\rho_{CDM}(T_\star)}, \quad (9)$$

where $N$ takes into account the effects of asphericities and angular momentum. One should keep in mind that $f$ might be further suppressed by the probability of finding closed structures in the network. In $\Omega_{CDM}(T_\star) \sim \rho_{CDM}(T_2)(T_2/T_\star)^3$ is the energy density of CDM at $T_\star$. In the most interesting region of parameter space in Fig. 1 $\rho_{CDM}(T_2)$ is dominated by the contribution from axion quanta radiated by the network. Hence, $\rho_{CDM}(T_2) \approx \rho_{nw}(T_2) \sim \Delta \rho$. Putting everything together, we find:

$$f \sim p^N P_{nw}(T_\star) \left( \frac{T_2}{T_\star} \right)^3. \quad (10)$$

To estimate the actual value of $f$ requires knowledge of $P_{nw}$.

In this respect, the simulations of $\Omega_{PBH}$ show that, for $\epsilon \simeq 0.5$ in $[7]$, only 10% of the original network survives at $T_2$, while for $\epsilon \simeq 0.3$ the percentage is reduced to 1%. We do not know the subsequent evolution of the network. Nevertheless, let us assume for simplicity that the network decay follows a power law beyond $T_2$:

$$P_{nw}(T_\star) \sim \left( \frac{T_2}{T_\star} \right)^{-\alpha}. \quad (11)$$

Fitting (11) to the aforementioned results of $\Omega_{PBH}$ gives $\alpha \approx 7$. The final fraction $f$ then depends significantly on $N$, which we do not expect to be large for $p$ close to one. For example, for $N \approx 2$ the right hand side of (10) peaks at $T_\star \sim T_2/30$, corresponding to $p \sim 1$ and giving $f \sim 10^{-6}$ and $M_\star \sim 10^6 M_\odot$.

However, this result is sensitive to the precise numerical scaling of $P_{nw}$ after $T_2$. In this regard, it is interesting to notice that numerical simulations hint at slight deviations from the scaling regime $[31]$. The decay of the network can then be slower, resulting in smaller $\alpha$ and larger $f$.

Observations require that $f \lesssim 10^{-4}$ for PBHs with $M \sim 10^6 M_\odot$ (see also [9] for a more recent discussion). As we have seen, this constraint is easily satisfied in our scenario.

Nevertheless, to confidently estimate the actual fraction requires additional numerical studies, which we leave for future work. Let us remark that if a larger $f$ can be obtained with our mechanism, then (non)observations of PBHs may actually give additional constraints on axion models with $N_{DW} > 1$.

### ORIGIN OF THE BIAS TERM

Let us now discuss some ideas related to the bias term. The possibility that $[7]$ arises from an explicit breaking of the PQ symmetry due to Planck-suppressed effective operators has been considered in the literature $[41,42]$. However, the lore is that gravitational effects would break the PQ symmetry only at the non-perturbative level (i.e. via gravitational instantons, see $[43]$ and $[44]$ for a recent discussion). In this case, the size of the induced potential would be of order $\mathcal{A}_B \sim M_p e^{-\delta M_\star/P}$. The corresponding $T_2$ would then be too small and fall out of the allowed region of parameter space in Fig. 1.

Here, we would like to point out an alternative possibility to generate the bias term. Consider a dark gauge sector, which also breaks the $U(1)_{B-L}$ via anomalies and has DW number $N_{DW} = 1$. The specific matter spectrum and couplings of this hidden sector are not crucial to our discussion, even though cosmological and collider constraints should be checked in any concrete realization. Such a dark sector would then precisely generate a contribution to the axiom potential of the form $[6]$, with $\mathcal{A}_B$ related to the scale of dark gluon condensation, and $\delta$ containing the dark sector $\beta$-term.

Interestingly, this naturally allows for a non-trivial temperature dependence of the bias term. Indeed, the scale $\mathcal{A}_B$ will now have a temperature dependence analogous to the
QCD axion potential in [5]:

\[
A_B(T)^4 = m_B^2(T) n^2 = m_B^2(T) N_{DW}^2 F^2,
\]

with (see also appendix)

\[
m_B^2(T) = d_T^2 \left( \frac{\Lambda_B}{F} \right)^{-n'}, \quad \text{if } T > T_{0,B}.
\]

The natural expectation is that \( m_B \) will increase as temperature decreases until \( T_{0,B} \sim \Lambda_B \), and remain constant afterwards. Here, \( \Lambda_B \) is the dark confinement scale and \( d_T, n' \) depend on the dark spectrum.

These parameters have an important impact on PBH formation. For instance, for \( T_2 \sim 5 \) MeV and \( d_T, n' \sim 1 \) the bias term has not yet reached its asymptotic value at \( T_2 \). Therefore, \( p \) and \( M_\star \) now scale as \( T_{4-4-n'}^\ast \) and \( T_{6-6-n'}^\ast \) respectively from \( T_2 \) to \( T_\ast \sim T_{0,B} \). A large figure of merit can then be attained in less than one order of magnitude in \( T_\ast \) and lighter PBHs may be generated (down to \( \sim 10^4 M_\odot \)). Alternatively, if \( d_T \ll 1 \) and/or \( n' \gtrsim 6 \), \( \Lambda_B \) roughly coincides with \( T_2 \) and we recover the previous case. We leave a more detailed investigation of the dark sector for future work.

**CONCLUSIONS**

We have discussed a new mechanism to generate PBHs in the context of QCD axion realizations. It proceeds by the late collapses of closed DWs in a long-lived string-DW network, which arises in QCD axion realizations with \( N_{DW} > 1 \) and PQ symmetry broken after inflation.

Lacking accurate knowledge of the network evolution and collapse, we cannot give precise predictions for the fraction and masses of the PBHs. However, under reasonable assumptions, depending on the temperature behavior of the bias term, PBHs with masses in the range \( M \sim 10^4 - 10^7 M_\odot \) and representative fraction \( f \gtrsim 10^{-6} \) can be created. Interestingly, such heavy PBHs can play an important role as seeds for the formation of cosmological structure, alleviating several problems of the CDM scenario on sub-galactic scales, and providing an avenue to explain the origin of the SMBHs [8][9].

Our proposal appears to prefer small values of the axion decay constant, \( F \lesssim 10^9 \) GeV, corresponding to axion masses in the meV range. These values are close to the lower bounds from the cooling of supernovae [36] [37] [55], which are however subject to astrophysical uncertainties and are not universal (see e.g. [46]). On the other hand, small values of \( F \) might be observationally interesting. In this respect, it is intriguing that several stellar systems show a mild preference for a non-standard cooling mechanism, which can be interpreted in terms of a DFSZ QCD axion [47] [48].

In addition, several experiments will be probing this region of QCD axion masses in the near future. In particular: axion helioscopes IAXO [49] and TASTE [50], light-shining-through-walls experiments such as ALPS II [51], and the long-range force experiment ARIADNE [52].

Our mechanism might be probed at gravitational wave observatories via the detection of gravitational radiation from: SMBH binaries at LISA [53], the annihilation of the string-wall network [54] at aLIGO (O5) [55], LISA, ET [56] and SKA [57].

Finally, let us mention that similar dynamics to the one described in this Letter can occur for generic Axion-Like-Particles (ALPs), not necessarily coupled to QCD gluons. In this case, it is possible to raise the figure of merit \( p \) by considering very light ALPs and thereby delaying the network collapse to temperatures \( T_\ast \lesssim 100 \) keV. This scenario may be strongly constrained by the production of extremely heavy PBHs.

**Acknowledgments.** We thank J.J. Blanco-Pillado, J. Garriga, J. Redondo, K. Sasaki, G. Servant and T. Vachaspati for useful discussions. We acknowledge support by the Spanish Ministry MEC under grant FPA2014-55613-P and the Severo Ochoa excellence program of MINECO (grant SO-2012- 0234, SEV-2016- 0588), as well as by the Generalitat de Catalunya under grant 2014-SGR-1450. F. F. was also supported in part by the U.S. Department of Energy, Office of High Energy Physics, under Award Number No. DE-FG02-91ER40628.

**APPENDIX: QCD AXION DARK MATTER**

The aim of this appendix is to review the relevant formulae for the total axion dark matter abundance. The material presented here can be partially found in [31] (and refs. therein), together with more detailed explanations.

The total axion dark matter abundance is given by

\[
\Omega_a = \Omega_{\text{mis}} + \Omega_{\text{strings}} + \Omega_{\text{nw}},
\]

where the three terms on the right hand side of (14) represent respectively the contribution from: the misalignment mechanism, the radiation from axionic strings and the radiation from the string-wall network.

Let us first provide formulae for the axion mass. Following [31], we have

\[
m^2(T) = \begin{cases}
    c_0 \frac{\Lambda_{QCD}^4}{F^2}, & \text{if } T \lesssim T_0, \\
    c_T \frac{\Lambda_{QCD}^4}{F^2} \left( \frac{T}{\Lambda_{QCD}} \right)^{-n}, & \text{if } T \gtrsim T_0.
\end{cases}
\]

The parameters \( c_0, c_T \) and \( n \) can be determined semi-analytically using the Dilute Instanton Gas Approximation (DIGA) [58], valid at high temperatures. This approach gives \( c_0 \approx 10^{-3}, c_T \approx 10^{-7} \) and \( n \approx 7 \) [22] (see also lattice QCD)

---

[5] The lines of constant \( \delta \) in Fig. get modified, but viable regions with \( \delta \gtrsim 0.1 \) will persist.
results which agree or deviate from these values. From one finds \( T_0 \approx 100 \text{ MeV}. \)

Let us now move to the relic abundance. Firstly, let us focus on the contribution from the misalignment mechanism \( \Omega_{\text{mis}}. \) The QCD axion starts to oscillate at the temperature \( T_1 \) given by \( 3H(T_1) = m(T_1). \) By means of (15), we find

\[
T_1 \approx A_n \left( \frac{g_{aT}}{80} \right)^{-\frac{1}{\pi n}} \left( \frac{F}{10^9 \text{ GeV}} \right)^{-\frac{1}{F}} \Lambda_{\text{QCD}},
\]

where \( A_n = (7.5 \cdot 10^{16} \text{ eV})^{-\frac{1}{n}}. \) For \( c_T \approx 10^{-7} \) and \( n \approx 7, \) this gives \( T_1 \approx 3 \text{ GeV} \) for \( F \approx 10^9 \text{ GeV}. \) The relic abundance is given by

\[
\Omega_{\text{mis}} h^2 \approx B_n \sqrt{c_0 c_T} \left( \frac{F}{10^9 \text{ GeV}} \right)^{\frac{6+n}{4+n}} \left( \frac{\Lambda_{\text{QCD}}}{400 \text{ MeV}} \right)
\]

\( B_n \approx 0.8 \cdot 10^9 \times (2.2 \cdot 10^{10})^{-\frac{1}{2+n}}. \) Using the DIGA values for these parameters, the misalignment contribution saturates the observed dark matter abundance for \( F \approx 10^{11} \text{ GeV}. \)

Let us now move on to \( \Omega_{\text{strings}}. \) At the moment, there is some controversy in the literature regarding the magnitude of this contribution (see [28–30] for recent estimates with a different take on previous calculations). Without entering into details, we focus on the parametric dependence of \( \Omega_{\text{strings}} \) on \( F \)

\[
\Omega_{\text{strings}} h^2 \approx C_n \left( \frac{F}{10^9 \text{ GeV}} \right)^{\frac{6+n}{4+n}} \left( \frac{\Lambda_{\text{QCD}}}{400 \text{ MeV}} \right)
\]

where \( C_n \) is a numerical prefactor. In order to produce Fig. [4] we have used the formulae provided in [31], where \( C_n \approx 10^{-3}. \) In this case the contribution from strings is generically larger or comparable to the contribution from the misalignment angle, and the sum of the two contributions saturates the observed dark matter abundance for \( F \approx 2 \times 10^{10} \text{ GeV}. \) The precise behavior of \( \Omega_{\text{strings}} \) is not crucial to our proposal, since we are especially interested in the region of small \( F, \) where \( \Omega_{\text{mis}} \) and \( \Omega_{\text{strings}} \) represent a subdominant contribution to the total axion abundance.

Let us finally discuss the contribution from the string-wall network, which is especially important for our proposal. The crucial difference with respect to the abundances from the misalignment mechanism and from strings is that the network radiates axions at \( T_3 < T_1. \) Thus, their abundance is less diluted and for small values of \( F \) it dominates over the other contributions. Assuming a so-called exact scaling regime for the evolution of the network, i.e. \( \rho_{\text{mw}} \sim \sigma H, \) we have

\[
\Omega_{\text{mw}} h^2 \approx 0.14 \times \left( \frac{F}{10^9 \text{ GeV}} \right) \left( \frac{\Lambda_{\text{QCD}}}{400 \text{ MeV}} \right)^2 \times \left( \frac{g_{aT}(T_2)}{10.75} \right)^{-1/4} \left( \frac{10 \text{ MeV}}{T_2} \right).
\]

Our expression for \( \Omega_{\text{mw}} \) differs from the one presented in [31] in that we keep the dependence on \( T_2, \) rather than trading it for \( A_B \) according to [7]. Furthermore, there are numerical prefactors in [18] which we have fixed according to the results of [31]. The sum of [16], [17] and [18] generates the solid blue curve in Fig. [4].

Let us now discuss the solid lines of constant \( \delta \) in Fig. [1]. The addition of the bias term misaligns the axion from the CP conserving minimum determined by the QCD potential [3]. In particular, the QCD angle \( \theta \equiv a/F \) at the minimum is approximately given by:

\[
\delta_{\text{min}} \approx \frac{A_B N_{\text{DW}} \sin \delta}{m^2 N_{\text{DW}}^2 F^2 + A_B^2 \cos \delta}.
\]

Inverting [7] and using \( \Delta \sigma \approx A_B^2 \sigma [1 - \cos(2\pi/N_{\text{DW}})], \) we find:

\[
A_B^2 \approx \frac{\pi^2 g_{aT}(T_2)}{90} \frac{T_2^2}{\epsilon [1 - \cos(2\pi/N_{\text{DW}})]} \frac{\sigma}{M_p}.
\]

In order to preserve the solution to the strong CP problem, we require \( \delta_{\text{min}} \lesssim 10^{-10} \) [36]. At constant \( \delta, \) (19) corresponds to a line in the logarithmic \( F - T_2 \) plane, as shown in Fig. [1]. Notice that the position of these lines depends on \( N_{\text{DW}}: \) in particular, the region of phenomenologically viable parameter space where \( \delta \approx 1 \) shrinks as we increase \( N_{\text{DW}}. \) Nevertheless, even for \( N_{\text{DW}} = 6 \) there is an allowed region of parameter space where only \( \delta \approx 0.1 \) is required.

[1] B. P. Abbott et al. (Virgo, LIGO Scientific), Phys. Rev. Lett. 116, 061102 (2016), 1602.03837.
[2] S. Hawking, Mon. Not. Roy. Astron. Soc. 152, 75 (1971).
[3] B. J. Carr and S. W. Hawking, Mon. Not. Roy. Astron. Soc. 168, 399 (1974).
[4] S. Bird, I. Cholis, J. B. Muñoz, Y. Ali-Haïmoud, M. Kamionkowski, E. D. Kovetz, A. Raccanelli, and A. G. Riess, Phys. Rev. Lett. 116, 201301 (2016), 1603.00464.
[5] S. Clesse and J. García-Bellido, Phys. Dark Univ. 15, 142 (2017), 1603.05234.
[6] M. Sasaki, T. Suyama, T. Tanaka, and S. Yokoyama, Phys. Rev. Lett. 117, 061101 (2016), 1603.08338.
[7] S. Clesse and J. García-Bellido, Phys. Rev. D94, 083504 (2016), 1607.06077.
[8] S. Clesse and J. García-Bellido, Phys. Rev. D92, 023524 (2015), 1501.07565.
[9] B. Carr and J. Silk (2018), 1801.00672.
[10] X.-B. Wu et al., Nature 518, 512 (2015), 1502.07418.
[11] E. Banados et al., Nature 553, 473 (2018), 1712.01860.
[12] A. Vilenkin and E. P. S. Shellard, *Cosmic Strings and Other Topological Defects* (Cambridge University Press, 2000), ISBN 9780521654760.

[13] A. Vilenkin, Phys. Rev. Lett. **46**, 1169 (1981), [Erratum: Phys. Rev. Lett.46,1496(1981)].

[14] S. W. Hawking, Phys. Lett. **B246**, 36 (1990).

[15] J. Fort and T. Vachaspati, Phys. Lett. **B311**, 41 (1993), hep-th/9305081.

[16] J. Garriga and M. Sakellariadou, Phys. Rev. **D48**, 2502 (1993), hep-th/9303024.

[17] H. Deng, J. Garriga, and A. Vilenkin, JCAP **1704**, 050 (2017), 1612.03753.

[18] T. W. B. Kibble, G. Lazarides, and Q. Shafi, Phys. Rev. **D26**, 435 (1982).

[19] A. Vilenkin and A. E. Everett, Phys. Rev. Lett. **48**, 1867 (1982).

[20] T. Vachaspati (2017), 1706.03868.

[21] J. E. Kim, Phys. Rev. Lett. **43**, 103 (1979).

[22] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. **B166**, 493 (1980).

[23] M. Dine, W. Fischler, and M. Srednicki, Phys. Lett. **104B**, 199 (1981).

[24] A. R. Zhitnitsky, Sov. J. Nucl. Phys. **31**, 260 (1980), [Yad. Fiz.31,497(1980)].

[25] R. D. Peccei and H. R. Quinn, Phys. Rev. Lett. **38**, 223 (1977).

[26] F. Wilczek, Phys. Rev. Lett. **40**, 279 (1978).

[27] S. Weinberg, Phys. Rev. Lett. **40**, 223 (1978).

[28] V. B. Klaer and G. D. Moore, JCAP **1708**, 030 (2011), 1012.4558.

[29] O. Wantz and E. P. S. Shellard, Phys. Rev. **D82**, 123508 (2010), 0910.1066.

[30] M. Kawasaki, K. Saikawa, and T. Sekiguchi, Phys. Rev. **D91**, 065014 (2015), 1412.0789.

[31] M. Kawasaki, K. Saikawa, and T. Sekiguchi, Phys. Rev. **D91**, 065014 (2015), 1412.0789.

[32] L. M. Widrow, Phys. Rev. **D40**, 1002 (1989).

[33] L. M. Widrow, Phys. Rev. **D39**, 3576 (1989).

[34] F. Ferrer, E. Massó, G. Panico, O. Pujolàs, and F. Rompineve, to appear (2018).

[35] B. Rai and G. Senjanovic, Phys. Rev. **D49**, 2729 (1994), hep-ph/9301240.

[36] A. Ringwald and K. Saikawa, Phys. Rev. **D93**, 085031 (2016), [Addendum: Phys. Rev.D94,no.4,049908(2016)], 1512.06436.

[37] R. Kallosh, A. D. Linde, D. A. Linde, and L. Susskind, Phys. Rev. **D52**, 912 (1995), hep-th/9502069.

[38] R. Alonso and A. Urbano (2017), 1706.07415.

[39] K. Hamaguchi, N. Nagata, K. Yanagi, and J. Zheng (2018), 1806.07151.

[40] L. Di Luzio, F. Mescia, E. Nardi, P. Panci, and R. Ziegler (2017), 1712.04940.

[41] M. Giannotti, I. Irastorza, J. Redondo, and A. Ringwald, JCAP **1605**, 057 (2016), 1512.08108.

[42] M. Giannotti, I. G. Irastorza, J. Redondo, A. Ringwald, and K. Saikawa, JCAP **1710**, 010 (2017), 1708.02111.

[43] M. Kawasaki, K. Saikawa, and T. Sekiguchi, Phys. Rev. **D91**, 065014 (2015), 1412.0789.

[44] E. Armengaud et al., JINST **9**, T05002 (2014), 1401.3233.

[45] V. Anastassopoulos et al. (TASTE), JINST **12**, P11019 (2017), 1706.09378.

[46] R. Bähre et al., JINST **8**, T09001 (2013), 1302.5647.

[47] A. Arvanitaki and A. A. Geraci, Phys. Rev. Lett. **113**, 161801 (2014), 1403.1290.

[48] A. Klein et al., Phys. Rev. **D93**, 024003 (2016), 1511.05581.

[49] K. Saikawa, Universe **3**, 40 (2017), 1703.02576.

[50] J. Aasi et al. (LIGO Scientific), Class. Quant. Grav. **32**, 074001 (2015), 1411.4547.

[51] M. Punturo et al., Class. Quant. Grav. **27**, 194002 (2010).

[52] G. Janssen et al., PoS **AASKA14**, 037 (2015), 1501.00127.

[53] S. R. Coleman, Subnucl. Ser. **15**, 805 (1979), [382(1978)].

[54] S. Borsanyi, M. Dierigl, Z. Fodor, S. D. Katz, S. W. Mages, D. Nogradi, J. Redondo, A. Ringwald, and K. K. Szabo, Phys. Lett. **B752**, 175 (2016), 1508.06917.

[55] C. Bonati, M. D’Elia, M. Mariti, G. Martinelli, M. Mesiti, F. Negro, F. Sanfilippo, and G. Villadoro, EPJ Web Conf. **137**, 08004 (2017), 1612.06269.