String tension and monopoles in $T \neq 0$ SU(2) QCD

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Abstract

Monopole and photon contributions to abelian Wilson loops are calculated using Monte-Carlo simulations of finite-temperature SU(2) QCD in the maximally abelian gauge. Long monopole loops alone are responsible for the behavior of the string tension in the confinement phase up to the critical $\beta_c$. Short monopole loops and photons do not contribute to the string tension. The abelian and the monopole spacial string tensions (both of which agree with the normal ones for $\beta < \beta_c$) show a $g^4(T)T^2$ scaling behavior in the deconfinement phase. The abelian spacial string tension is in agreement with the full one even in the deconfinement phase.

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I. INTRODUCTION

The dual Meissner effect due to condensation of color magnetic monopoles is conjectured to be the color confinement mechanism in QCD [1,2]. The scenario is easily understood if we consider QCD after abelian projection [3]. The abelian projection of QCD is to extract an abelian theory performing a partial gauge-fixing such that the maximal abelian torus group remains unbroken. After the abelian projection, $SU(3)$ QCD can be regarded as a $U(1) \times U(1)$ abelian gauge theory with magnetic monopoles and electric charges. 'tHooft conjectured that the condensation of the abelian monopoles is the confinement mechanism in QCD [3].

There are, however, infinite ways of extracting such an abelian theory out of $SU(3)$ QCD. It seems important to find a good gauge in which the conjecture is seen clearly to be realized even on a small lattice. A gauge called maximally abelian (MA) gauge has been shown to be interesting [4–6].

Recently an effective $U(1)$ monopole action is derived from vacuum configurations in $SU(2)$ QCD [7,8]. Entropy dominance over energy of the monopole loops, i.e., condensation of the monopole loops is shown to occur always in the infinite volume limit if extended monopoles [8] are considered [9]. After the abelian projection in the MA gauge, infrared behaviors of $SU(2)$ QCD may be described by a compact-QED like $U(1)$ theory with the running coupling constant instead of the bare one and with the monopole mass on a dual lattice.

Moreover, it is shown that the string tension which is a key quantity of confinement is explained by monopole contributions alone [10]. This is realized also in compact QED [11].

The aim of this note is 1) to show that the same thing happens also in finite-temperature $SU(2)$ QCD by means of evaluating monopole and photon contributions to abelian Wilson loops, 2) to study what kind of monopole loops is responsible for the string tension and 3) to study the behaviors of another non-perturbative quantity, i.e., the spacial string tension both in the confinement and the deconfinement phases.
II. FORMALISM

We adopt the usual $SU(2)$ Wilson action. The maximally abelian gauge is given \cite{4} by performing a local gauge transformation $V(s)$ such that

$$R = \sum_{s,\hat{\mu}} \text{Tr} \left( \sigma_3 \tilde{U}(s, \hat{\mu}) \sigma_3 \tilde{U}^\dagger(s, \hat{\mu}) \right)$$

is maximized. Here

$$\tilde{U}(s, \hat{\mu}) = V(s)U(s, \hat{\mu})V^{-1}(s + \hat{\mu}). \tag{1}$$

After the gauge fixing is over, there still remains a $U(1)$ symmetry. We can extract an abelian gauge variable from the $SU(2)$ one as follows;

$$\tilde{U}(s, \hat{\mu}) = A(s, \hat{\mu})u(s, \hat{\mu}), \tag{2}$$

where $u(s, \hat{\mu})$ is a diagonal abelian gauge field and $A(s, \hat{\mu})$ has off-diagonal components corresponding to charged matters. It is to be noted that any $U(1)$ invariant quantity written in terms of the abelian link variables $u(s, \hat{\mu})$ after an abelian projection is $SU(2)$ invariant \cite{10}.

After an abelian projection in MA gauge, it has been shown that an abelian Wilson loop operator written in terms of $u(s, \hat{\mu})$ alone reproduce the full string tension in ($T = 0$) $SU(2)$ QCD \cite{5}. Hence we study here the abelian Wilson loops to derive the string tension in finite-temperature $SU(2)$ QCD.

An abelian Wilson loop operator is given by a product of monopole and photon contributions \cite{10}. Here we take into account only a simple Wilson loop of size $I \times J$. Then such an abelian Wilson loop operator is expressed as

$$W = \exp \{ i \sum J_\mu(s) \theta_\mu(s) \}, \tag{3}$$

where $J_\mu(s)$ is an external current taking $\pm 1$ along the Wilson loop and $\theta_\mu(s)$ is an angle variable defined from $u(s, \hat{\mu})$ as follows:
\[ u(s, \hat{\mu}) = \begin{pmatrix} e^{i\theta_\mu(s)} & 0 \\ 0 & e^{-i\theta_\mu(s)} \end{pmatrix}. \]  

(4)

Since \( J_\mu(s) \) is conserved, it is rewritten for such a simple Wilson loop in terms of an antisymmetric variable \( M_{\mu\nu}(s) \) as \( J_\nu(s) = \partial'_\mu M_{\mu\nu}(s) \), where \( \partial' \) is a backward derivative on a lattice. \( M_{\mu\nu}(s) \) takes \( \pm 1 \) on a surface with the Wilson loop boundary. We get

\[ W = \exp\{-\frac{i}{2} \sum M_{\mu\nu}(s) f_{\mu\nu}(s)\}, \]

(5)

where \( f_{\mu\nu}(s) = \partial_\mu \theta_\nu(s) - \partial_\nu \theta_\mu(s) \) and \( \partial_\mu \) is a forward derivative on a lattice. Using the decomposition of \( f_{\mu\nu}(s) \) into a quantum fluctuation and the Dirac string term, we get [10]

\[ W = W_1 \cdot W_2 \]

(6)

\[ W_1 = \exp\{-i \sum \partial'_\mu \tilde{f}_{\mu\nu}(s) D(s - s') J_\nu(s')\} \]

\[ W_2 = \exp\{2\pi i \sum k_\beta(s) D(s - s') \frac{1}{2} \epsilon_{\alpha\beta\rho\sigma} \partial_\alpha M_{\rho\sigma}(s')\}, \]

where a monopole current \( k_\mu(s) \) is defined as \( k_\mu(s) = (1/4\pi) \epsilon_{\mu\alpha\beta\gamma} \partial_\alpha \tilde{f}_{\beta\gamma}(s) \) following DeGrand-Toussaint [12]. \( D(s) \) is the lattice Coulomb propagator. Since \( \partial'_\mu \tilde{f}_{\mu\nu}(s) \) corresponds to the photon field, \( W_1(W_2) \) is the photon (the monopole) contribution to the abelian Wilson loop. To study the features of both contributions, we evaluate the expectation values \( \langle W_1 \rangle \) and \( \langle W_2 \rangle \) separately and compare them with those of \( \langle W \rangle \).

The Monte-Carlo simulations were performed on \( 24^3 \times 8 \) lattice from \( \beta = 2.3 \) to \( \beta = 2.8 \). All measurements were done every 50 sweeps after a thermalization of 2000 sweeps. We took 50 configurations totally for measurements. The gauge-fixing criterion is the same as done in Ref. [13]. Using gauge-fixed configurations, we derived monopole currents and obtained the ensemble of monopole currents. We evaluate the averages of \( W \) using abelian link variables (called abelian) on the original lattice, of \( W_1 \) (photon part), and \( W_2 \) (monopole part) on the lattice, separately.

Assuming the static potential is given by linear + Coulomb + constant terms, we try to determine the potential using the least square fit. There are various ways, but we adopt a method similar to that [10,14] using the Creutz ratios.
III. MONOPOLE AND PHOTON CONTRIBUTIONS TO THE STRING TENSION

First we have checked two points, i.e., 1) if abelian Wilson loop can reproduce the full string tension of $SU(2)$ QCD also in finite-temperature $SU(2)$ QCD and 2) if the photon contribution $W_1$ and the monopole contribution $W_2$ are uncorrelated, i.e., $\langle W_1 W_2 \rangle \simeq \langle W_1 \rangle \langle W_2 \rangle$.

Our data of the string tensions from abelian Wilson loops are plotted in Fig. 1. We calculated the physical string tension and the spacial string tension. The spacial string tension is calculated by Wilson loops composed of only spacial link variables. In the deep confinement region, both string tensions show the same unique value as that in $(T = 0)$ $SU(2)$ QCD, which is also the same as the full string tension [10]. Moreover, the physical string tension vanishes at the critical coupling $\beta_c$. However, the spacial string tension does not vanish and remains finite even in the deconfinement phase.

To investigate the correlation of $W_1$ and $W_2$, we have calculated the following quantity

$$\frac{\langle W_1 W_2 \rangle - \langle W_1 \rangle \langle W_2 \rangle}{\langle W_1 W_2 \rangle}.$$  

They are less than several percent for each size of Wilson loops at each $\beta$. Hence the correlation is negligible in the MA gauge.

The results of the monopole and the photon contributions to the string tension are shown in Fig. 2 (monopole) and Fig. 3 (photon). The monopole contributions to Wilson loops are obtained with relatively small errors. The Creutz ratios of the monopole contributions having small errors are almost independent of the loop size. This means that the monopole contributions are composed only of an area, a perimeter and a constant terms without a Coulomb term. We find both physical and spacial string tensions from the monopoles almost agree with those from the abelian Wilson loops in the confinement phase. In the deconfinement phase, the monopole contributions to the physical string tension vanish, whereas those to the spacial one remain non-vanishing. This is consistent with the data of the asymmetry of the monopole currents running in the timelike and the spacelike directions in the
deconfinement region \cite{15}. The string tension in the photon part is negligibly small. But the photon spacial string tension seems to become finite as $\beta$ becomes larger. This may be due to that the linear + Coulomb fit to the spacial Wilson loops is not appropriate in the deconfinement phase.

**IV. A LONG MONOPOLE LOOP AND THE STRING TENSION**

The features of the abelian monopole currents were studied in \cite{16} through Monte-Carlo simulations. They obtained the followings \cite{16}: 1) In the confinement phase, about half of the monopole currents are connected into one long loop although the link number occupied by monopole currents is less than 5 percent of the total link number. Namely only one long monopole loop exists in one vacuum configuration in the confinement phase. All the other monopole loops are short. 2) In the deconfinement phase, long monopole loops disappear. All monopole loops are short. These data are shown in Table 1.

These results bring us an idea that the long monopole current plays an important role in the string tension, because the physical string tension exists only in the confinement phase and vanishes in the deconfinement phase. We have investigated the contributions from the longest monopole loops and from all other monopole loops to the string tension separately.

The data are shown in Fig. 4. Clearly, the contributions from the long loop alone reproduce almost the full value of the string tension. On the other hand, the short loop contributions are almost zero. The small finite value of the short loop contribution near $\beta_c$ can be understood as the contribution from the next longest loop appearing in some vacuum configurations near $\beta_c$.

As shown in Table 1, only a few percent of the total links are occupied by monopole currents belonging to the long loop. Nevertheless, it gives rise to the full value of the string tension.

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\[1\] Near the critical $\beta$, some vacuum configurations have two (or three) long loops, the sum of whose lengths is about the same as that of one long-loop case.
tension which is a key quantity of confinement. This suggests there are monopoles of the two types, one of which is long and is responsible for the string tension and the others are short and have no contributions to the string tension. The latter may correspond to a lattice artifact which exists also in compact QED. The importance of long loops is consistent with the role played by extended monopoles in the QCD vacuum.

V. SCALING BEHAVIORS OF THE SPACIAL STRING TENSION FOR $T > T_C$

As shown above, the spacial string tension remains non-vanishing in contrast with the physical one. It is shown that, at high temperature, four dimensional QCD can be regarded through dimensional reduction as an effective three dimensional QCD with $A^0$ as a Higgs field. The effective gauge coupling constant $g_3^2$ is given by $g^2(T)T$ in terms of the temperature and the four dimensional coupling $g(T)$. If the temperature dependence of the pure gauge term of the effective theory dominates the string tension, the spacial string tension corresponding to that in the effective three dimensional theory is expected to obey

$$\sqrt{\sigma_s(T)} = cg^2(T)T,$$

where the temperature dependent running coupling constant is determined at high temperature by the $\beta$ function. Up to the two loops, it is given by

$$g^{-2}(T) = \frac{11}{12\pi^2} \ln T/\Lambda_T + \frac{17}{44\pi^2} \ln(2 \ln T/\Lambda_T).$$

The scaling behavior of the spacial string tension (7) derived from the usual full Wilson loops is confirmed recently in Monte-Carlo simulations of $SU(2)$ QCD.

Let us here study the behavior of the spacial string tension derived from the abelian Wilson loops. Since the relation between the lattice spacing $a(\beta)$ and the coupling $\beta$ is not known sufficiently well, we have performed additional Monte-Carlo simulations varying $N_T$ on $24^3 \times N_T$ lattices to get information at different temperatures following Bali et al. We have measured the string tension for $N_T = 2, 4, 6, 8$ and $12$ at $\beta = 2.30, 2.51$ and $2.74$ which are the critical points for $N_T = 4, 8$ and $16$, respectively.
The data are plotted in Fig. 5 and Fig. 6. In the case of string tension derived from abelian Wilson loops, we get almost the same behaviors as those of the normal ones denoted by cross points. The latter is cited from Ref. [18]. $\sqrt{\sigma}/T_c$ is independent of $\beta$ and so the spacial string tension is expected to be a physical quantity remaining in the continuum limit. To study the scaling behavior in more details, we show in Fig. 7 and Fig. 8 the data $T/\sqrt{\sigma_s}$ versus $T/T_c$. Both data seem to satisfy the scaling behavior. We get $c = 0.357(19)$ and $\Lambda_T = 0.073(8)T_c$ which is almost equal to $c = 0.369(14)$ and $\Lambda_T = 0.076(13)T_c$ obtained in [18]. When we consider the spacial string tension derived from monopole Wilson loops, we find the value is a little bit lower: $c = 0.326(48)$ and $\Lambda_T = 0.053(31)T_c$. We have not yet known if the discrepancy between the abelian and the monopole spacial string tensions is real or due to systematic errors coming from the fitting. Actually it is not certain whether the linear + Coulomb fit to the spacial Wilson loops adopted here is correct or not. Hence we have not a definite conclusion whether monopoles alone can reproduce also the spacial string tension at high temperature or not. Further studies are needed to clarify the point.

This work is financially supported by JSPS Grant-in Aid for Scientific Research (B)(No.06452028).
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TABLE I. The total monopole loop length, the density, i.e., the ratio of the total loop length to the number of total links, the longest loop length and the ratio of the longest loop length to the total loop length on $24^3 \times 8$ lattice.

| beta | total loop length | (density) | the longest loop length | ratio of the longest loop |
|------|------------------|-----------|-------------------------|--------------------------|
| 2.30 | 22525            | (5.09%)   | 16990                   | 75.4%                    |
| 2.35 | 16832            | (3.80%)   | 11267                   | 66.9%                    |
| 2.40 | 12327            | (2.79%)   | 7091                    | 57.5%                    |
| 2.45 | 8575             | (1.94%)   | 3591                    | 41.9%                    |
| 2.48 | 7014             | (1.59%)   | 2278                    | 32.5%                    |
| 2.51 | 5248             | (1.19%)   | 1101                    | 21.0%                    |
| 2.53 | 4467             | (1.01%)   | 579                     | 13.0%                    |
| 2.57 | 3140             | (0.71%)   | 267                     | 8.5%                     |
| 2.63 | 1973             | (0.45%)   | 138                     | 7.0%                     |
| 2.70 | 1103             | (0.25%)   | 75                      | 6.8%                     |
FIG. 1. Physical string tensions (circle) and spacial string tensions (square) from abelian Wilson loops on $24^3 \times 8$ lattice.
FIG. 2. Physical string tensions (circle) and spacial string tensions (square) from monopoles on $24^3 \times 8$ lattice.
FIG. 3. Physical string tensions (circle) and spacial string tensions (square) from photons $24^3 \times 8$ lattice.
FIG. 4. The contributions from the longest monopole loop to string tensions (circle) and from all other monopole loops to string tensions (square) on $24^3 \times 8$ lattice.
FIG. 5. Square root of the spatial string tensions from abelian Wilson loops versus temperature on $24^3 \times N_T$ lattice ($N_T = 2, 4, 6, 8$ and $12$) at $\beta = 2.30, 2.51$ and $2.74$. The cross symbol denotes the normal spatial string tensions. The broken line shows the value of the physical string tension in the confinement phase.
FIG. 6. Square root of the spacial string tensions from monopoles versus temperature on $24^3 \times N_T$ lattice ($N_T = 2, 4, 6, 8$ and 12) at $\beta = 2.30, 2.51$ and $2.74$. The cross symbol denotes the normal spacial string tensions. The broken line shows the value of the physical string tension in the confinement phase.
FIG. 7. The ratio of the temperature and square root of the spacial string tensions from abelian Wilson loops versus temperature on $24^3 \times N_T$ lattice ( $N_T = 2, 4, 6, 8$ and $12$ ) at $\beta = 2.30, 2.51$ and $2.74$. The solid line shows a fit to the data in the region $1.33 \leq T/T_c \leq 8$, using the two-loop relation for $g(T)$. 
FIG. 8. The ratio of the temperature and square root of the spacial string tensions from monopoles versus temperature on $24^3 \times N_T$ lattice ($N_T = 2, 4, 6, 8$ and $12$) at $\beta = 2.30, 2.51$ and $2.74$. The solid line shows a fit to the data in the region $1.33 \leq T/T_c \leq 8$, using the two-loop relation for $g(T)$. 