Mathematical model of rolling an elastic wheel over deformable support base

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Abstract. One of the main direction of economic growth in Russia remains to be a speedy development of north and northeast regions that are the constituents of the 60 percent of the country territory. The further development of these territories requires new methods and technologies for solving transport and technological problems when off-road transportation of cargoes and people is conducting. One of the fundamental methods of patency prediction is imitation modeling of wheeled vehicles movement in different operating conditions. Both deformable properties of tires and physical and mechanical properties of the ground: normal tire deflection and gauge depth; variation of contact patch area depending on the load and pressure of air in the tire; existence of hysteresis losses in the tire material which are influencing on the rolling resistance due to friction processes between tire and ground in the contact patch; existence of the tangential reaction from the ground by entire contact area influence on the tractive patency. Nowadays there are two main trends in theoretical research of interaction wheeled propulsion device with ground: analytical method involving mathematical description of explored process and finite element method based on computational modeling. Mathematical models of interaction tire with the ground are used both in processes of interaction individual wheeled propulsion device with ground and researches of mobile vehicle dynamical models operated in specific road and climate conditions. One of the most significant imperfection of these models is the description of interaction wheel with flat deformable support base whereas profile of real support base surface has essential height of unevenness which is commensurate with radius of the wheel. The description of processes taking place in the ground under influence of the wheeled propulsion device using the finite element method is relatively new but most applicable lately. The application of this method allows to provide the most accurate description of the interaction process of a wheeled propulsion devices and the ground, also this method allows to define tension in the ground, deformation of the ground and the tire and ground’s compression. However, the high laboriousness of computations is essential shortcoming of that method therefore it’s hard to use these models as part of the general motion model of multi-axis wheeled vehicles. The purpose of this research is the elaboration of mathematical model of elastic wheel rolling over deformable rough support base taking into account the contact patch deformation. The mathematical model of rectilinear rolling an elastic wheel over rough deformable support base, taking into account variation of contact patch area and variation in the direction of the radial and tangential reactions also load bearing capacity of the ground, is developed. The efficiency of developed mathematical model of rectilinear rolling an elastic wheel over rough deformable support base is proved by the simulation methods.
1. Introduction

One of the main direction of economical growth in Russia remains to be a speedy development of north and northeast regions that are the constituents of the 60 percent of the country territory. The further development of these territories requires new methods and technologies for solving transport and technological problems when off-road transportation of cargoes and people is conducting. In this way the movement of vehicle is difficult and impossible in some cases. The existing models of wheeled, tracked and rotary screw vehicles don’t correspond to present functional, efficiency, reliability and ecological requirements for wheeled propulsion devices when operating in north regions of the country where supporting surfaces are weak-bearing. In this way there are technical, economic and social need in creating and using vehicles with pneumatic wheeled propulsion devices including ultra-low pressure devices that satisfy the requirements.

It should be noted that our country doesn’t have necessary collection of energy efficient off-road vehicles currently. The existing off-road machines performed according to old traditional schemes and produced serially by industry don’t meet the requirements defining the efficiency and environmental friendliness of wheeled propulsion devices in difficult climatic operating conditions. In current situation the problem of wheeled vehicles patency prediction of movement over supporting surface with weak-bearing properties is relevant. [1,2]. One of the fundamental methods of patency prediction is simulation method of wheeled vehicle movement in different operational conditions that is based on mathematical models of interaction the elastic tire with deformable support base unevenness.

Established [3, 4] that the traction patency wheeled vehicle mutually affect both the deformation properties of tire and physical and mechanical characteristics of soil: normal deflection of the tire and the depth gauge; the change in the reference area of the contact patch depending on load and air pressure in the tire; the presence of hysteresis losses in the material of the tire that affect rolling resistance caused by friction in the contact patch on the ground; creating a tangent reactions of the soil over the entire area of contact.

Along with the definition of the dependencies to describe "load-deformation of the ground " and "load-deformation mover", not less important task is the selection of models reset vertical deformations of soil in contact with the pneumatic tire. On the correct choice of the type and kind of approximating dependence is largely determined by both qualitative and quantitative aspects of General solution of the problem of contact interaction of elastic mover with deformable ground [5].

Currently in the theoretical study of the interaction of propellers with the ground formed two main directions: analytical method involving a mathematical description of the process under consideration [6 – 8] and finite element method (FEM), based on computer simulation [9 – 12]. In the study of the interaction of wheel elements with the soil, an analytical method has found wide application. Developed mathematical models of the interaction of the tyre with the ground allow to solve various tasks. These models are used as in the study of processes of interaction of a single wheel mover with the soil array and at research of dynamic models of mobile machines operating in specific road-soil (RSC) and climatic conditions. One of the most significant shortcomings of these models is the description of the interaction of the wheel with a smooth deformable base, while the real profile of the support surface has a significant height of irregularities is comparable with the radius of the wheel.

The study of the processes occurring in the soil mass under the influence of the wheel mover, the finite element method are relatively new, currently the largest application. This method is better than others provided numerical procedures for the study of mathematical models of objects. The most important advantage is the availability of implicit methods of integration of systems of differential equations. The application of this method is the most accurate to describe the process of interaction of wheel elements with the soil, to determine the stresses in the soil mass deformation of tire and soil compaction. In contrast to analytical methods form the contact patch of the tyre with an elastic support surface is the result of the simulation taking into account the independent soil characteristics and the constructive and operational parameters of the engine. However, a significant disadvantage of this
method is its high computational complexity, therefore, with currently available computers to use these models as part of the overall model of the motion of multi-wheeled vehicles difficult.

The aim of this study is to develop a mathematical model of rolling elastic wheels on deformable rough reference base, taking into account the deformation of the contact patch.

2. A mathematical model of the rolling

Key assumptions

- Normal pressure in the contact patch evenly distributed.
- The force of interaction of the wheel with its support base directed in the opposite direction from the slip speed.

Coordinate systems and basic calculation dependencies

The proposed model uses two different coordinate systems figure 1 due to the structure and form of the equations of motion of the object.

First, the stationary coordinate system (SCS) $O_2X_2Y_2Z_2$, is used to simulate the specified ground traffic conditions. The origin of the system of point $O_2$ coincides with the beginning of the simulated track.

To determine the forces acting on the car from the ground we introduce a micro mobile coordinate system (MMCS) under which we understand the system $O_TX_TY_TZ_T$, the center of which $O_T$ coincides with the geometric center of rotation, the axis of $O_TX_T$ coincides with the projection of the longitudinal axis of symmetry of the wheel on the support surface. Consider the scheme of movement of the wheel in figure 1.

Figure 1. A design scheme of wheel rolling over uneven sub-base:
1 – strain profile of soil; 2 – deformable soil profile; 3 – the undeformed profile of the soil.

For model development we use the results of [13]. On the lower semicircle of the undeformed profile of the wheel will select a certain number of points, $n$, the position of which will determine the angle $\alpha_i$ between the vertical line dropped from the center of the wheel on the axis $X_2$, and a ray connecting a point of the undeformed profile of the wheel with its center figure 1. The number of points is chosen based on a compromise between accuracy of the model and its performance. Define the coordinates $X_{2i}$ and $Y_{2i}$ of selected points of the profile at the SCS.
where $X_{20}, Y_{20}$ are the coordinates of the center of the wheel $O_B$ SCS. The vertical coordinate $Z_{2i}$ $i$-th point of the undeformed profile of the wheel in the SCS, we define by the formula

$$ Z_{2i} = Z_{20} - r_i \cos \alpha_i, $$

where $Z_{20}$ is the vertical coordinate of the wheel center at the SCS. The formation of the longitudinal profile $Z_{2gr}$ is carried out according to the following algorithm:

$$ Z_{2gr} = \begin{cases} 
Z_{2gr}^{undef}, & \text{if } X_i \geq X_{20} + X_r \\
(1 - \frac{h_r}{X_r})Z_{2i}^{undef}, & \text{if } X_{20} \leq X_i < X_{20} + X_r \\
Z_{2gr}^{undef} - h_r, & \text{if } X_i \leq X_{20} 
\end{cases} $$

(1)

where $X_i$ is the current coordinate of the $X_i$-th point of the profile gauge in the SCS; $h$ – depth gauge; $Z_{2gr}^{undef}$ - the vertical coordinate of the undeformed profile of the gauge (simulated in advance by a known method described, for example, in [14]). $X_r$ the amount determined by the formula

$$ X_r = \sqrt{r_k^2 - (Z_{20} - h_r)^2} $$

where $r_k$ – free radius of the wheel. The $d_{ri}$ deflection of the tire in the radial direction for the $i$-th point of the undeformed profile is determined from the following ratios

$$ d_{ri} = \begin{cases} 
0, & Z_{2gr} \leq Z_i \\
(Z_{2gr} - Z_{2i}) \cos \alpha_i, & Z_{2gr} \geq Z_{2i} 
\end{cases} $$

where $Z_{2gr}$ – vertical coordinate of the profile of the support base for the $i$-th point of the wheel. Thus, to determine the reactions of interaction of wheels with the supporting surface $R_x$ and $R_z$ in MMCS in the presence of several areas of "overlap" profile substructure undeformed contour of the wheel it is necessary to determine the equivalent angle $\alpha_{eq}$, the point of application of the total radial reaction $R_r$ in the radial direction and the tangential reaction $R_T$ figure 1.

Define $\alpha_{eq}$ as a weighted average value

$$ \alpha_{eq} = \frac{\sum_{i=1}^{n} \alpha_i d_{ri}}{\sum_{i=1}^{n} d_{ri}} $$

Radial reaction $R_r$ is the sum of two components: elastic $R_{ry}$ and damping $R_{rd}$: $R_r = R_{ry} + R_{rd}$. $R_{ry}$ depends on the equivalent deflection of the tire

$$ d_{eq} = \frac{\sum_{i=1}^{n} d_{ri}}{n_k} $$

where $n_k$ is the number of points in the undeformed profile in contact with the support surface. $R_{rd}$ depends on the speed of the deflection of the tire in the radial direction. We define the projection of velocity of points of the contour of the wheel on the axis $X_r$ and $Z_r$.
\[ V_{\alpha r} = \omega_k (r_k - dr) \cos \alpha_i + V_{0 X r}; \]
\[ V_{\alpha \alpha} = \omega_k (r_k - dr) \sin \alpha_i + V_{0 Z r}, \]

where \( \omega_k \) – angular velocity of rotation of the wheel; \( V_{0 X r} \) and \( V_{0 Z r} \) – projection of the velocity vector of the wheel center (point \( O \)) on the axis \( X_T \) and \( Z_T \), respectively. The vector of linear speed of the \( i \)-th point of the undeformed profile of the wheel in the radial direction

\[ V_{\alpha i} = V_{\alpha r} \sin \alpha_i + V_{\alpha \alpha} \cos \alpha_i. \]

The speed of profile deformation of the \( i \)-th point in the radial direction

\[ \frac{d}{dt} (dr_r) = \ddot{Z}_{2v} \cos \alpha_i - V_{\alpha i}. \]

Equivalent speed of deflection

\[ \frac{dr_{eqv}}{dt} = \sum_{i=1}^{n} \left( \frac{d}{dt} (dr_i) \right) \]

Further, knowing the elastic and damping characteristics of the tire in the radial direction, we find \( R_r \). Tangential reaction \( R_T = \mu_SR_r \), where - coefficient of friction-partial slip [15].

\[ \mu_s = \mu_{smax} \times \left( 1 - e^{-\frac{S_k}{S_0}} \right) \]

\( \mu_{smax} \)- coefficient of friction full slip for a given angle \( \alpha \) rotation of the velocity vector of sliding, \( S_k \) the coefficient of slipping, \( S_0 \) is a constant. This expression is valid for non cohesive soils. The value \( \mu_{smax} \) specifies the maximum value of the function \( \mu_s(S_k) \), and, combined with the constant \( S_0 \) is the gradient of the \( \mu_s(S_k) \) function at the origin, reflecting the properties of the soil at small slides.

The coefficient of slipping

\[ S_k = \frac{V_{0 X r} - \omega_k (r_k - dr_{eqv})}{\omega_k (r_k - dr_{eqv})} \quad \text{for the traction mode of wheel rolling;} \]
\[ S_k = \frac{V_{0 X r} - \omega_k (r_k - dr_{eqv})}{V_{0 X r}} \quad \text{for brake or driven mode of wheel rolling.} \]

Reaction of interaction of wheels with the supporting surface \( R_{X r} \) and \( R_{Z r} \) in MMCS

\[ R_{X r} = R_T \cos \alpha_{eqv} - R_T \sin \alpha_{eqv} - R_{fr} \]
\[ R_{Z r} = R_T \sin \alpha_{eqv} + R_T \cos \alpha_{eqv} \]

where \( R_{fr} \) – force frontal ground resistance of rolling wheels (bulldozer effect), which is determined by the method described [16].

\[ R_{fr} = 0.5b_r (r_w - h_r - dr_{eqv})^2 q_p y, \]
\[ q_p = a_p \cdot (0.01 \cdot 90^\circ)^{b_p} \]
\[ a_p = 1 + a_{fa} \cdot (0.1 \cdot \beta_0^{a_{fa}})^{b_{fa}} \]
\[ b_p = a_{fb} \cdot (0.1 \cdot \varphi_S^*)^{b_{fb}} \]
\[ a_{fa} = 0.3082 - 0.0709 \cdot \frac{\varphi_a^*}{\varphi_S^*} \]
\[ b_{fa} = 2.0751 + 1.3354 \cdot \frac{\varphi_a^*}{\varphi_S^*} \]
\[ a_{fb} = 0.5756 + 0.1024 \cdot \frac{\varphi_a^*}{\varphi_S^*} \]
\[ b_{fb} = 1.0608 + 0.0619 \cdot \frac{\varphi_a^*}{\varphi_S^*} \]

where \( b \) - bus width, \( \gamma \) - is the specific weight of the soil, \( M_N/m^3 \); \( \varphi_S^* \), \( \varphi_a^* \) - respectively angle of internal thorns of the soil and the friction angle of the soil on the wall of tyre deg. Graphs to determine \( \varphi_S^* \), \( \varphi_a^* \) presented in [16]. Dynamics of rotation of the wheel shown in figure 1 and described by equations

\[ J_w \dot{\omega}_w = M_w - R_{X_T}(r_k - dr_{eqv}) - M_f - M_h - M_T; \]
\[ M_f = R_f(r_k - dr_{eqv})\sin\alpha_{eqv}\cos\alpha_{eqv}; \]
\[ M_h = R_{Z_T}h_r \]

where \( M_w \) - torque applied to the wheel; \( M_T \) - braking torque to the wheel; \( M_f \) - moment of rolling resistance of the wheel due to the shift reaction \( R_r \); \( R_{XT} \) is the projection of force interaction of wheel with grunting MMCS on the axis of the \( OX_T \); \( M_h \) - resistance moment caused by the vertical deformation of the soil [16].

3. Determination of the parameters of the contact patch and the track width \( h_T \)

The components of the length of the contact patch within the right \( (L_1) \) and left \( (L_2) \) lower quarters of the circular arc undeformed contour of the tire figure 1, respectively, can be defined as follows. Dependence \( Z_{2gr} = f(X) \) within the bottom right quarter of the circular arc undeformed contour of the tyre is described by the formula (1). Then

\[ L_1 = \int_0^{X_T} \sqrt{1 + (f'(X))^2} dX = \frac{1}{2A} \left[ AX_T \sqrt{AX_T^2 + 1 + \sqrt{A}\sinh^{-1}(X_T\sqrt{A})} \right] \quad (2) \]

where \( \sinh(x) = \frac{e^x - e^{-x}}{2} \) is the hyperbolic sine. To calculate expression (2) value \( h_r(t_{i-1}) \) needs to take from the previous integration step to avoid the "algebraic loop". The initial value can be taken \( h_r|_{t=0} = 0 \). Due to the lack of analytical expressions \( Z_{2gr} = f(X) \) within the lower left quadrant circular arc undeformed contour of the tire replace the integral (2) end for \( n \) amount of plots the undeformed profile of the tire for which \( dr_i \neq 0 \) when \( -\frac{\pi}{2} \leq \alpha_i \leq 0 \)

\[ L_2 = \sum_{i=1}^{n} \sqrt{(Z_{2gr i} - Z_{2gr i-1})^2 + (X_{2gr i} - X_{2gr i-1})^2} \]

The area of the contact patch \( F_t \) figure 2 define by the formula
\[ F_t = 2 \int_0^\pi b(\alpha)[r_k - dr(\alpha)] \, d\alpha, \]
\[ b(\alpha) = b_t \left(1 + \frac{dr(\alpha)}{r_k}\right) \]

(3)

Where \( b_t \) the width of the tyre.

The discrete analogue of the formula (3) can be written as

\[ F_t = b_t \sum_{i=1}^{n_k-1} \left(2 + \frac{dr_{i+1} + dr_i}{r_k}(r_k - dr_i)(\alpha_{i+1} - \alpha_i) \right) \]

Define \( p_{oz} \) normal pressure in the contact patch. By the formula of Bernstein-Letoshnev [16]

\[ p_{oz} = c_s h^\mu, MPa \]

where \( c_s \) MPa – coefficient of soil deformation; \( \mu \) - is the density of the soil. On the other hand

\[ p_{oz} = \frac{p_z}{F_t 10^6}, MPA \]

where \( p_z \) - vertical wheel load. Then

\[ h^\mu = \left(\frac{p_z}{c_s F_t 10^6}\right)^\frac{1}{\mu} \]

4. The influence of lugs on the parameters of the motion of a wheel

In the presence of the cleats must also calculate the shear forces in the areas of the projections and depressions, as well as the removal of soil from the zone of contact in case of intense slipping of the wheel. Additional vertical penetration \( d_h_{vp} \) the center of the wheel in the ground caused by the excavation of soil from the area of contact is calculated by the formula [16]

\[ d_h_{vp} = \frac{t_{it} h_{gr{o}s_k}}{t_{gr{o}} (1 - s_k)} \]

\[ t_{gr{o}} = \frac{2\pi r_k}{n_{gr{o}}} \]

where \( t_{it} \)– length trench lug; \( h_{gr{o}} \)– grouser height; \( t_{gr{o}} \)– step lug; \( n_{gr{o}} \) the total number of lugs on the tyre circumference. Then the total vertical depth \( h_{\Sigma} \) wheel center

\[ h_{\Sigma} = h_r + d_h_{vp} \]

We assume that if \( h_{\Sigma} \geq r_k \), you lose the mobility of the machine due to the hanging of the machine body on the ground. To determine the additional shear forces \( R_{tj}^{gro} \) in the areas of the projections and depressions of the cleats figure 3 use the results obtained in [16]:
Figure 2. Calculation scheme for defining the area of the contact patch.

\[ R_{tj}^{gro} = F_{gro} c_s 10^6 \exp \left[ -\frac{(|e_x| - e_{xm})^2}{0.05 e_{xm}} \right], \]

\[ e_x = S(\Delta t) - \int_{t_1}^{t_2} \omega_k(t) (r_k - dr_{eqv}) dt, \]

\[ \Delta t = t_2 - t_1. \]

where \( F_{gro} \) – frontal area of the projection of the lug; \( e_{xm} \) – the maximum shift of the soil in which the cohesion of soil particles is not broken; \( e_x \) – current shift of the soil; \( S(\Delta t) \) is the path traversed by the center of the wheel during the time \( \Delta t \); \( \omega_k(t) \) is the current angular speed of rotation of the wheel.

Total tangent force \( R_{tj}^{gro} \) grs projection on the axis \( X_T \) MMSC for all for all \( n \) grouser in the zone of contact of the wheel with its support base

\[ R_{tj}^{gro} = \sum_{j=1}^{n} R_{tj}^{gro} \cos \beta_j, \]

\[ n = \frac{L_1 + L_2}{2\pi r_k} n_{gro}. \]

Then the expression for determining the longitudinal reaction \( R_{X_T} \) wheels with a support base in a projection on the axis \( X_T \) MMSC

\[ R_{X_T} = R_t \cos \alpha_{eqv} - R_t \sin \alpha_{eqv} - R_{fr} + R_{tj}^{gro}. \]

5. Account the bearing capacity of the Foundation soil in horizontal direction under the action of vertical loads

The magnitude of the horizontal reaction \( R_{X_T} \) may be limited by two factors [16]: a wheel slip on the surface of the compacted soil after overcoming the forces of adhesion and loss of bearing capacity from the shear weight of the soil in the direction of action of the horizontal reaction. And if the coupling properties are accounted for by the factor \( \mu_s \), the loss of bearing capacity can be assessed by comparing
the existing shear stresses \( \tau \) with the maximum allowable stresses \( \tau_{\text{max}} \). The maximum shear stress in the contact patch is calculated using expression pendant

\[
\tau_{\text{max}} = p_{oo} t g \varphi_s^* + c_s 10^6.
\]

The current tangent stress \( \tau \) define by the formula [16]:

\[
\tau = p_{oo} t g \varphi_s^* \left[ 1 - \exp \left( - \frac{|e_x|}{0.1 e_{xm}} \right) \right] + c_s 10^6 \exp \left[ - \frac{(|e_x| - e_{xm})^2}{0.05 e_{xm}} \right] t g \varphi_s^*.
\]

Finally, the expression for \( R_{\chi \tau} \) takes the form

\[
R_{\chi \tau} = k_{\tau} (R_{\tau \cos \alpha_{eqv}} - R_{\tau \sin \alpha_{eqv}} - R_{fr} + R_{\tau g_{gro}}),
\]

\[
k_{\tau} = \begin{cases} 
1, & \text{if } \tau \leq \tau_{\text{max}} \\
\frac{\tau_{\text{max}}}{\tau}, & \text{if } \tau > \tau_{\text{max}}.
\end{cases}
\]

6. Verifying mathematical models

To verify the developed methodology was developed by the program model the motion of a single wheel in the software package MATLAB/SIMULINK. Parameters of the Foundation soil according to [16] are given in table 1.

| Parameters of the Foundation soil according | Marking | Value          |
|-----------------------------------------------|---------|----------------|
| The coefficient of deformation of the soil    | \( c_s \) | 0,18 MPa       |
| The density of the soil                       | \( \mu \) | 0,77           |
| The specific gravity of the soil              | \( \gamma \) | 0,0145 MN/m³ |
| The angle of internal friction of soil        | \( \varphi_s^* \) | 37⁰           |
| The maximum shift of the soil in which the cohesion of soil particles is not broken | \( e_{xm} \) | 0,05 m        |
| The coefficient of friction full slip         | \( \mu_{s \alpha_{\text{max}}} \) | 0,8           |
| Constant                                       | \( s_0 \) | 0,05           |

The main characteristics of the single wheel is presented in table 2.

| Characteristics of the single wheel | Marking | Value     |
|-------------------------------------|---------|-----------|
| Weight                             | \( m \) | 600 kg    |
| Free radius                        | \( r_k \) | 0,7 m    |
| Width                              | \( b_t \) | 0,6      |
| The total number of tire grousers  | \( n_{gro} \) | 60     |
| Tire grouser height                | \( h_{gro} \) | 0,015 m |
| The length of the bottoms of the tire grousers | \( t_{lt} \) | 0,015 m |
| The angular velocity of rotation of the wheel | \( \omega_k \) | 4 c⁻¹ |

In figure 4 shows the dependence of the height profile \( Z_{2gr} \) subgrade from the longitudinal coordinate \( X_2 \) of the fixed coordinate system. In figures 5 – 9 show, respectively, depending on the time of
movement \( t \) of the following parameters: depth gauge, \( h_c \), the height of the wheel center \( z_c \) equivalent to the deflection of the tire \( d\tau_{ecv} \), the length of the contact patch \( L = L_1 + L_2 \) equivalent angle \( \alpha_{ecv} \).

![Image](image_url)

**Figure 4.** The dependence of the height profile \( Z_{2gr} \) subgrade from the longitudinal coordinate \( X_2 \).

![Image](image_url)

**Figure 5.** The dependence of the depth gauge \( h_c \) time of motion \( t \).

![Image](image_url)

**Figure 6.** The dependence of the height of the wheel center \( z_c \) time of motion \( t \).
Figure 7. The dependence of the equivalent deflection of the tire $d_{eq}$ on the time of motion $t$.

Figure 8. The dependence of the length of contact patch $L$ on the time of motion $t$.

Figure 9. The dependence of the equivalent angle $\alpha_{eq}$ on the time of motion $t$.

Shown in figures 5 – 9 dependence allow to judge about the efficiency of the proposed mathematical model of the elastic wheel movement on uneven deformable support base.

7. Conclusions
- The mathematical model of rectilinear elastic wheel rolling on a rough support deformable base, taking into account the change in the area of the contact patch, the direction of the radial and tangential reactions, and the bearing capacity of the soil.
• Methods of simulation prove the efficiency of the developed mathematical model of rectilinear rolling elastic wheels on uneven deformable support base.

References
[1] Kotliarenko V I 2008 The main directions of improving the patency of wheeled vehicles (Moscow: Publisher MGIY)
[2] Shukhman S B, Pliev I A and Malyarevich V E 2008 Ways to improve the environmental properties of multi-wheel drive vehicles operated in the far North The Automotive Industry, No 10 pp 15-7
[3] Wong J Y 2001 Theory of Ground Vehicles (Wiley IEEE)
[4] Pryadkin V I 2011 Assessment of the impact of the highly flexible tyre on the rolling surface Forestry No 5 pp 42-3
[5] Pirkowskij J V 2001 The theory of the motion of four-wheel drive vehicle (applied problems of optimization of chassis design) proc. manual for schools (Moscow)
[6] Ageikin Y S, Volska N S and Chicheken I V 2010 The flow of cars (Moscow: Publisher MGIY) pp 34-56
[7] Babijchuk A E, Ageikin Y S and Volska N S 2013 The technique of determination of power losses in the rolling of the wheel mover with the type of drivetrain and the air pressure in the tires of the car AAI No 3 pp 24–7
[8] Lepeshkin A V and Petrov S E 2012 Mathematical model of interaction between elastic wheels with deformable bearing surface in the case of steady rectilinear rolling Conf. AAI "Car and tractor const. in Russia: devel. t prior. and training" (Moscow) Publisher MSTU «MAMI» pp 141 – 9
[9] Shoop S, Kestler K and Haehnel R 2006 Finite element modeling of tires on snow Tire Science and Technology V 34 No 1 p 2–37
[10] Botero J, Gobbi M and Mastinu G 2005 A new mathematical model of the traction force in pneumatic tire snow chain systems AssociazioneItalianaPerL’AnalisiDelleSollecitazioni / Dipartimento di Meccanica, Politecnico di Milano p 10 www.aiasonline.org/AIAS2005/Articoli/art084.pdf
[11] Belkin A E and Narsky N L 2004 Finite element analysis of the contact of the tires with the supporting surface on the basis of the shell model Vestnik MSTU. Series Engineering No 3 pp 14–28
[12] Odintsov O A 2008 Development of a method of solving nonlinear contact problems of stationary rolling tires Diss. kand. tech. sciences. Publisher BMSTU
[13] Zhileikin M M and Padalkin B V 2016 The mathematical model of rolling elastic wheels on rough rigid base Izvestiya vuzov. Mechanical engineering No 3 pp 24–9
[14] Polungian A A 2008 Design of four-wheel drive wheeled vehicles (Moscow: Publisher BMSTU) V. 1 p 496
[15] Ellis J R 1975 Vehicle Handling (Moscow: Mashinostroenie) p 216
[16] Larin V V 2010 Theory of motion of four-wheel drive wheeled vehicles (Moscow: Publisher BMSTU) p 391