Effect of the radial support stiffness of the ring gear on the vibrations for a planetary gear system

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Abstract
Planetary gear system is one of the critical components of various industrial transmission systems. In general, the ring gear is elastically fixed with the gearbox. The gearbox materials and their assembly relationships will affect the support stiffness of the ring gear and system vibrations. In this paper, a multi-body dynamic model for a planetary gear system with the elastic support of ring gear is developed to discuss the influence of the radial support stiffness of ring gear on the system vibrations. The planet bearings are also considered in the multi-body dynamic model. The rotational speed of the planet gear and carrier from the simulation and theoretical results are compared to validate the developed multi-body dynamic model. The influences of the radial support stiffness of the ring gear, carrier moment, and sun gear speed on the time- and frequency-domain vibrations of the planetary gear system are analyzed. The results denote that the waveform and amplitude of the time-domain vibration of the ring gear are greatly affected by the radial support stiffness of ring gear as well as the peak frequency amplitude and its sidebands. The peak frequency in the spectrum of ring gear is slightly affected by the radial support stiffness. It indicates that this study can give some guidance for the vibration control approaches for the planetary gear systems.

Keywords
Support stiffness, ring gear, vibrations, planetary gear

Introduction
Planetary gear system is one of the critical components of various industrial transmission systems, such as aero-engines and wind turbine gearboxes. The dynamic performances of the planetary gear system have a large influence on vibrations of the transmission systems. Moreover, the manufacturing errors and faults in the planetary gear system will greatly affect the vibrations of the transmission systems.¹⁻³ Thus, vibration analysis of the planetary gear system is very helpful for understanding the vibrations and noise of the transmission systems.

Many works have developed various methods including the lumped parameter modeling, finite element (FE) modeling, multibody dynamic (MBD) modeling, and experimental methods for studying the vibrations of planetary gear systems.⁴⁻⁵ For example, Ambarisha and Parker⁶ and Kahraman et al.⁷ introduced lumped-parameter and FE models to study the vibrations of a planetary gear system. Lethé et al.,⁸ Halsen et al.,⁹ Xing and Moan,¹⁰ Helsen et al.,¹¹ Jin et al.,¹² and Li et al.¹³ established different MBD models to analyze the vibrations of the planetary gearbox system. Guo and Parker¹⁴ presented a nonlinear dynamic model considering the bearing clearance and tooth backlash to discuss the nonlinear vibrations. Guo and Parker¹⁵ developed a lumped-parameter model
considering bearing clearances, back-side contact, tooth separation, and tooth wedging to analyze the translational vibrations of planetary gear system. Kim et al.\textsuperscript{16} studied the influences of the bearing deformations on the time-varying contact ratios and pressure angles of the planetary gear system as well as the vibrations. Chen et al.\textsuperscript{17,18} proposed a new method to study the effect of ring gear flexibility on the dynamic performance of planetary gear transmission set with a thin ring gear rim. Although various analytical, FE, and MBD methods have been utilized to describe the vibrations of planetary gear system, the influence of the support stiffness of ring gear on the vibrations of planetary gear system was not discussed in the above literature. However, in practice, the support stiffness of ring gear has a great influence on the system vibrations.\textsuperscript{19} On the other hand, Wu and Parker\textsuperscript{19} and Liu et al.\textsuperscript{20} studied the support stiffness of the ring gear on its vibrations. They only formulated the single ring gear models. The dynamic model for the planetary gear system considering both the support stiffness of the ring gear and planet bearings was not reported in the literature. Thus, the support stiffness of ring gear should be modeled for vibration analysis of the planetary gear systems, which is the goal of this paper.

In this paper, an MBD model for a planetary gear system with the elastic support of the ring gear is developed to discuss the influence of the radial support stiffness of the ring gear on the system vibrations. The planet bearings with the radial clearance are also considered in the MBD model. The meshing stiffness and damping for the meshing gears and bearing components are considered in the MBD model. A Coulomb friction model is utilized to model the frictions between the mating components of the system. A commercial MBD analysis software is applied to solve the developed MBD model. The rotational speed of the planet gear and carrier from the simulation and theoretical results are compared to validate the developed MBD model. The influences of the radial support stiffness of the ring gear, carrier moment, and sun gear speed on the time- and frequency-domain vibrations of the planetary gear system are analyzed.

**Problem description**

A planetary gear system with the elastic support of the ring gear and planet bearings is plotted in Figure 1. In general, the ring gear is elastically fixed with the gearbox. The gearbox materials and the assembly relationship will affect the radial support stiffness of the ring gear. As the results in Wu and Parker,\textsuperscript{19} the dynamics of the ring gear can be greatly influenced by the radial support stiffness of the ring gear. The dynamics of the ring gear will affect the system vibrations.

To analyze the influence of the radial support stiffness of the ring gear on the vibrations of the system, a radial stiffness ratio is defined as

\[
\eta = \frac{K_{ri}}{K_{r}}
\]  

where \(K_r\) is the reference radial stiffness and \(K_{ri}\) is the designed radial stiffness. The simulation cases are given in Table 1, where \(n_{in}\) is the input speed of the sun gear and \(M_{p}\) is the moment of the carrier. Moreover, the circumferential torsion stiffness is \(1.67 \times 10^5\) N/mm.

![Figure 1. Schematics of a planetary gear system.](image-url)
Model formulation

The dynamic model for a planetary gear system is depicted in Figure 2. The model includes three planet gears, three planet bearings, one carrier, one ring gear, and one sun gear. The input speed and the moment are applied on the sun gear and carrier, respectively. The contacts between the gears and those in the planet bearings are modeled too. The mesh stiffnesses between the gears from equations (2) and (3) are considered in the MBD model as well as the contact stiffness between the planet-bearing components. The planet gears, planet bearings, carrier, ring gear, and sun gear are considered as rigid bodies. The discrete radial support stiffness and circumferential torsion stiffness of the ring gears are considered in the MBD model as shown in Figure 2.

Based on the potential energy principle, the total equivalent mesh stiffness between the sun and planet gear is calculated by\(^\text{21–23}\)

\[
1/K_{\text{eo}} = 1/K_{b1} + 1/K_{s1} + 1/K_{a1} + 1/K_{b2} + 1/K_{s2} + 1/K_{a2} + 1/K_h
\]  

where \(k_b, k_s, k_a, \) and \(k_h\) are bending, shear, axial compressive, and Hertzian contact stiffnesses, respectively. In equation (2), the subscript 1 denotes the sun gear and subscript 2 denotes the planet gear. The total equivalent mesh stiffness between the planet gear and ring gear is as follows\(^\text{21–23}\)

\[
1/K_{\text{ei}} = 1/K_{b1} + 1/K_{s1} + 1/K_{a1} + 1/K_{f1} + 1/K_{b2} + 1/K_{s2} + 1/K_{a2} + 1/K_h
\]

where \(k_f\) is the fillet-foundation stiffness. In equation (3), the subscript 1 denotes the planet gear and subscript 2 denotes the ring gear. The details for the stiffnesses in equations (2) and (3) are given in Tian,\(^\text{21}\) Chen and Shao,\(^\text{22,23}\) and Ma,\(^\text{24,25}\) In these references, Chen and Shao\(^\text{22,23}\) proposed the method for the involvement of the tooth fillet-foundation in the gear mesh stiffness calculation based on the potential energy principle and then applied to the gear mesh stiffness calculation of railway locomotive gear transmissions\(^\text{26,27}\) The bending stiffness \(K_b\) is

\[
1/K_b = \int_0^d (xcos\alpha - hsing\alpha)^2 dX
\]

| \(K_r\) (N/mm) | Stiffness ratio \(\eta\) |
|---------------|-------------------------|
| 1.65 × 10^6  | 0.5                     |
| 3.3 × 10^6   | 1                       |
| 6.6 × 10^6   | 2                       |
| 13.2 × 10^6  | 4                       |

Figure 2. A dynamic model for a planetary gear system.
where $x$, $h$, $z_1$, and $d$ are the distance between the mesh point and the a piece with the width of $dx$, the distance between the mesh point and the tooth center line, the pressure angle at the applied load point, and the distance between the mesh point and the root circle, respectively, and $E$ and $I_x$ are the Young’s modulus and the area moment of inertia where the distance between the section and the acting point of the applied force is $x$. The shear stiffness $K_s$ is

$$\frac{1}{K_s} = \int_0^d \frac{1.2 \cos^2 z_1}{G A_s} \, dx \tag{5}$$

where $G_s$ and $A_s$ are the shear Young’s modulus and the area of the section where the distance between the section and the acting point of the applied force is $x$. The axial stiffness $K_a$ is

$$\frac{1}{K_a} = \int_0^d \frac{\sin^2 z_1}{E A_s} \, dx \tag{6}$$

The Hertzian contact stiffness $K_h$ is

$$K_h = \frac{4(1 - \nu^2)}{\pi E w} \tag{7}$$

where $\nu$ is the Poisson’s ratio and $w$ is the tooth width. Here, the calculation method for the $K_f$ is complex and is given in Chen and Shao.\textsuperscript{22} The contact damping for the gears is given by\textsuperscript{28}

$$c_{\text{max}} = 2 \zeta \sqrt{\frac{K_1 I_2}{I_1 R_2^2 + I_2 R_1^2}} \tag{8}$$

where $\zeta$ is the damping ratio, which is 0.1, $K$ is the mesh stiffness, $I_1$ and $I_2$ are the inertia of the meshing gears, and $R_1$ and $R_2$ are the radii of the meshing gears.

The contact stiffness between the ball and races is as follows\textsuperscript{29,30}

$$K_{br} = \left( \frac{\pi^2 e^2 E_{eq} e}{4.5 \tau^3 \sum \rho} \right)^{0.5} \tag{9}$$

where $E_{eq}$ is equivalent modulus of elasticity, $\sum \rho$ is the curvature sum, $e$ is the elliptical eccentricity parameter, and $\tau$ and $e$ are the first and second kind of elliptical integral. The details for the parameters in equation (9) are discussed in Harris and Kotzalas.\textsuperscript{29} Furthermore, the bearing damping is written as

$$c_{br} = \lambda K_{br} \tag{10}$$

where the range of $\lambda$ is from 0.25 to $2.5 \times 10^{-5}$.\textsuperscript{31,32}

Moreover, the governing equation of the MBD model is defined as

$$\mathbf{M} \ddot{\mathbf{q}} + (\mathbf{C} + \mathbf{C}') \dot{\mathbf{q}} + (\mathbf{K} + \mathbf{K}') \mathbf{q} = \mathbf{T} + \mathbf{E} + \mathbf{F} \tag{11}$$

where $\mathbf{M}$, $\mathbf{K}$, and $\mathbf{C}$ are the mass, support stiffness, and support damping matrices for the MBD model of the planetary gear system, respectively; $\mathbf{K}'$ and $\mathbf{C}'$ are the contact stiffness and damping matrices, respectively; $\mathbf{q}$ is the generalized coordinates vectors for the system; $\mathbf{T}$ is the external system load vector; $\mathbf{E}$ is the fault excitation vector; and $\mathbf{F}$ is the internal system load vector. Moreover, the friction force between the gears is included in the MBD model. The friction force is included in the internal system load vector $\mathbf{F}$. 
**Results and discussion**

The simulation cases are given in Table 2, where \( n_{\text{in}} \) is the input speed of the sun gear and \( M_p \) is the moment of the carrier. The dynamic model in “Model formulation” section is solved in a commercial MBD analysis software MSC. ADAMS. The influence of the radial support stiffness of the ring gear is given in the following sections. Tables 3 and 4 give the geometrics for the planetary gear system and planet bearing, respectively. According to the parameters, the meshing stiffness and damping between sun and planet gears are \( 4.71 \times 10^5 \) N/mm and \( 86.62 \) N s/mm, respectively, and those between the planet and ring gears are \( 9.28 \times 10^5 \) N/mm and \( 157.59 \) N s/mm, respectively. The contact stiffness and damping between the ball and inner race are \( 2.49 \times 10^7 \) N/mm and \( 6228 \) N s/mm, respectively, and those between the ball and outer race are \( 1.94 \times 10^7 \) N/mm and \( 4858 \) N s/mm, respectively. The time-varying mesh stiffness is calculated in equations (2) and (3). In this paragraph, only the maximum meshing stiffness of the single tooth meshing case is given. The meshing stiffness for the multi-tooth meshing cases is determined by the contact relationships between the teeth of gears in the MBD model. This method means that a rectangular profile of total meshing stiffness between the gears is used in the MBD model.

**Model validation**

To validate the developed model, the rotational speed of the planet gear and carrier from the simulation and theoretical results are compared. Here, the input speed of the sun gear is 387 r/min, and the moment applied on the carrier is 350 Nm. Figure 3 gives the simulation rotational speed of the planet gear #1 and carrier. In Figure 3, the simulation mean values for the planet gear #1 and carrier are 1779.84 and 1116.28 r/min, respectively. According to the theoretical method in Lei et al., their theoretical values are 1779.84 and 1116.28 r/min, respectively. The above discussions may show some validation for the developed modeling approach. Note that

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**Table 2.** Input speed and applied moment used in the simulation cases.

| Simulation cases | Input speed \( n_{\text{in}} \) (r/min) | Applied moment \( M_p \) (Nm) |
|------------------|----------------------------------------|-------------------------------|
| 1                | 4000                                   | 50                            |
| 2                | 6000                                   | 50                            |
| 3                | 8000                                   | 50                            |
| 4                | 6000                                   | 25                            |
| 5                | 6000                                   | 100                           |

**Table 3.** Geometrics for the planetary gear system.

|                      | Planet | Sun | Ring |
|----------------------|--------|-----|------|
| Normal module (mm)   | 1.5    | 1.5 | 1.5  |
| Number of teeth      | 19     | 32  | 83   |
| Normal pressure angle (°) | 22.5  | 22.5| 22.5 |
| Pitch radius (mm)    | 28.5   | 48  | 117  |
| Face width (mm)      | 30     | 30  | 30   |
| Root radius (mm)     | 26.68  | 45.11| 124.5|
| Number of planet gears | 3     |     |      |

**Table 4.** Geometrics for the planet bearing.

| Parameters             | Value |
|------------------------|-------|
| Ball radius (mm)       | 3     |
| Pitch radius (mm)      | 12.65 |
| Diametric clearance (mm) | 0.01 |
| Number of ball         | 8     |
| Number of rows         | 1     |
the fluctuation of the calculation speed of the component can be observed. The fluctuation maybe caused by some clearances or gaps between the gear pairs or in the rolling bearings. Although the fluctuation occurs in the calculation speed, the mean value of the calculation speed is a constant value as shown in Figure 3. Thus, the mean value of the calculation speed from the simulation results is compared with the theoretical results.

The influence of the radial support stiffness on the vibrations

Time-domain vibrations. Figure 4 gives the influences of the stiffness ratios on the time-domain accelerations of the ring gear under the simulation case 1 \((n_i = 4000 \text{ r/min} \text{ and } M_p = 50 \text{ Nm})\). It depicts that the radial support stiffness has a large influence on the profile and amplitude of the acceleration for the ring gear. The maximum amplitude will decrease with the radial support stiffness. In Figure 4, the maximum acceleration amplitudes of the ring gear with different stiffness ratios are 68.08, 57.54, 48.67, and 40.68 m/s², respectively.

To evaluate the radial support stiffness of the ring gear on the time-domain vibration, three statistical parameters, such as maximum, peak to peak (PTP), and root mean square (RMS) values, are utilized. Figure 5 introduces the influence of the radial support stiffness on the RMS, maximum, and PTP values of the accelerations of the ring gear for different moments. As shown in Figure 5, the input speed is 6000 r/min. Figure 6 describes the influence of the radial support stiffness on the RMS, maximum, and PTP values of the vibrations of the ring gear for different input speeds. As shown in Figure 6, the moment is 50 Nm. As plotted in Figures 5 and 6, the maximum, PTP, and RMS values always decrease with the increase of the stiffness ratio; they always increase with the increase with the carrier moment when the input speed of the sun gear is fixed; and they always increase with the increase with the input speed of the sun gear when the carrier moment is a constant.

Furthermore, the influence of the radial support stiffness on the orbit plots of the ring gear and planet gear #1 is given in Figures 7 and 8, respectively. Since the orbit plot profiles of three planet gears are similar, only the orbit
Figure 4. Time-domain accelerations of the ring gear for simulation case 1: (a) $\eta_1 = 0.5$, (b) $\eta_2 = 1$, (c) $\eta_3 = 2$, and (d) $\eta_4 = 4$.

Figure 5. The influence of the radial support stiffness on the (a) RMS, (b) maximum, and (c) PTP values of the accelerations of the ring gear for different moments.

Figure 6. The influence of the radial support stiffness on the (a) RMS, (b) maximum, and (c) PTP values of the accelerations of the ring gear for different input speeds.
Figure 7. The influence of the radial support stiffness on the orbit plots of the ring gear for (a) $\eta = 0.5$, (b) $\eta = 1$, (c) $\eta = 2$, and (d) $\eta = 4$.

Figure 8. The influence of the radial support stiffness on the orbit plots of the planet gear #1 for (a) $\eta = 0.5$, (b) $\eta = 1$, (c) $\eta = 2$, and (d) $\eta = 4$. 
The orbit ranges of three planet gears are still analyzed. The orbit ranges of the system bodies are described by four range parameters $a$, $b$, $c$, and $d$. The influence of the radial support stiffness on the above four range parameters is plotted in Figure 9. It gives that the orbits of ring and planet gears are greatly influenced by the radial support stiffness, especially for that of ring gear. The orbit ranges of ring and planet gears decrease with the radial support stiffness as shown in Figure 9. The orbits for three planet gears are different, which may be produced by the clearances or elastic ring foundations, etc. As shown in Figure 9(b) to (d), although the behaviors of planet gear #1 are different from those of planet gear #2 and #3 when $g$ is increasing, their differences are very small; the reason may be that the meshing positions of the three planet gears are different due to the vibrations caused by the clearances of the planet bearings, clearances between the gears, and the elastic support of ring gear, etc.

Frequency-domain vibrations. Figure 10 gives the influence of the radial support stiffness on the frequency-domain vibrations of the ring gear for simulation case 1 ($n_m = 4000$ r/min and $M_p = 50$ Nm). In Figure 10(a), for $g_1 = 0.5$, a peak frequency $f_m - f_c = 1016$ Hz is observed, and there are some sidebands around the peak frequency, which include $f_m - 2f_s(r) - 3f_c = 862$ Hz, $f_m - f_s(r) - 3f_c = 922$ Hz, $f_m - f_s(r) - f_c = 949$ Hz, $f_m + f_c = 1041$ Hz, $f_m + 6f_c = 1098$ Hz, $f_m + f_s(r) + 2f_c = 1134$ Hz, and $f_m + 2f_s(r) = 1161$ Hz. In Figure 10(b), for $g_2 = 1$, a peak frequency $f_m - f_c = 1016$ Hz is also observed, and the sidebands are $f_m - 2f_s(r) - 8f_c = 821$ Hz, $f_m - 2f_s(r) = 903$ Hz, $f_m - f_s(r) - f_c = 949$ Hz, $f_m + f_c = 1041$ Hz, $f_m + 6f_c = 1098$ Hz, and $f_m + f_s(r) + 6f_c = 1175$ Hz. In Figure 10(c), for $g_3 = 2$, a peak frequency $f_m - f_c = 1016$ Hz is observed too, and the sidebands include $f_m - 3f_s(r) = 839$ Hz, $f_m - 2f_s(r) = 903$ Hz, $f_m - f_s(r) + 2f_c = 995$ Hz, $f_m + f_c = 1041$ Hz, and $f_m + 6f_c = 1109$ Hz. In Figure 10(d), for $g_4 = 4$, a peak frequency $f_m - f_c = 101$ Hz is also observed, and the sidebands are $f_m - 3f_s(r) = 842$ Hz, $f_m - 2f_s(r) - f_c = 932$ Hz, $f_m - f_s(r) - 3f_c = 932$ Hz, $f_m + f_c = 1041$ Hz, and $f_m + f_s(r) + 4f_c = 1150$ Hz. Table 5 gives comparisons of the peak frequency, sidebands, and their amplitudes for different stiffness ratio cases. Although the four stiffness ratio cases have the same peak frequency, as given in Table 5, their amplitudes are very different. The amplitudes for the above four cases are 1.897, 1.425, 0.9162, and 0.4861 m/s², respectively. It denotes that the peak frequency amplitude decreases with the radial support stiffness.
Figure 10. Frequency-domain accelerations of the ring gear for simulation case 1: (a) $\eta_1 = 0.5$, (b) $\eta_2 = 1$, (c) $\eta_3 = 2$, and (d) $\eta_4 = 4$. 
The sidebands include frequencies, the spectrum waveforms for the simulation cases 2, 3, 4, and 5 are not shown as follows. However, since the spectrum waveforms for different simulation cases are similar and their differences are the peak frequencies, the spectrum waveforms for the simulation cases 2, 3, 4, and 5 are not shown as follows. However, their peak frequencies, sidebands, and relative amplitudes will be discussed in the next paragraphs.

Table 6 plots the influence of the radial support stiffness on the frequency-domain vibrations of the ring gear for simulation case 2 ($n_{m}=6000$ r/min and $M_{p}=50$ Nm). For $\eta_1=0.5$, a peak frequency $f_{m}-f_{c}=1532$ Hz is observed, and the sidebands include $f_{m}-f_{s,1}^{(r)}-2f_{c}=1405$ Hz, $f_{m}+f_{c}=1569$ Hz, $f_{m}+f_{s,1}^{(r)}=1641$ Hz, and $f_{m}+f_{s,1}^{(r)}+3f_{c}=1701$ Hz. For $\eta_2=1$, a peak frequency $f_{m}-f_{c}=1532$ Hz is also observed, and the sidebands are $f_{m}-f_{s,1}^{(r)}-f_{c}=1426$ Hz, $f_{m}+f_{c}=1569$ Hz, $f_{m}+f_{s,1}^{(r)}=1641$ Hz, and $f_{m}+f_{s,1}^{(r)}+3f_{c}=1701$ Hz. For $\eta_3=2$, a peak frequency $f_{m}-f_{c}=1532$ Hz is observed too, and the sidebands include $f_{m}-f_{s,1}^{(r)}-2f_{c}=1405$ Hz, $f_{m}-3f_{c}=1491$ Hz, $f_{m}+f_{c}=1569$ Hz, $f_{m}+f_{s,1}^{(r)}=1641$ Hz, and $f_{m}+f_{s,1}^{(r)}+3f_{c}=1701$ Hz. For $\eta_4=4$, a peak frequency $f_{m}+f_{c}=1569$ Hz is also observed, and the sidebands are $f_{m}-f_{s,1}^{(r)}-2f_{c}=1405$ Hz, $f_{m}-3f_{c}=1491$ Hz, $f_{m}-f_{c}=1532$ Hz, $f_{m}+f_{s,1}^{(r)}=1641$ Hz, and $f_{m}+f_{s,1}^{(r)}+3f_{c}=1701$ Hz. Table 6 shows that the peak frequencies of the first three stiffness ratio cases are at $f_{m}-f_{c}$, but that of the fourth stiffness ratio case is at $f_{m}+f_{c}$. Although the first three stiffness ratio cases have a same peak frequency, as given in Table 6, their amplitudes are very different. The peak frequency amplitudes for the above first three cases are 2.209, 1.166, and 1.281 m/s², respectively, and that...
for the fourth case is 0.5774 m/s². Similarly, it shows that the peak frequency amplitude decreases with the radial support stiffness.

Table 7 plots the influence of the radial support stiffness on the frequency-domain vibrations of the ring gear for simulation case 3 (n_in = 8000 r/min and M_g = 50 Nm). For \( \eta_1 = 0.5 \), a peak frequency \( f_m - f_c = 2041 \) Hz is observed, and the sidebands are \( f_m - 3f_c - 4f_c = 1565 \) Hz, \( f_m - 3f_c = 1669 \) Hz, \( f_m - f_c = 1932 \) Hz, \( f_m + f_c = 2090 \) Hz, \( f_m + f_c + 4f_c = 2299 \) Hz, and \( f_m + f_c + 8f_c = 2392 \) Hz. For \( \eta_2 = 1 \), a peak frequency \( f_m - f_c = 2041 \) Hz is observed too, and the sidebands include \( f_m - 3f_c - 4f_c = 1565 \) Hz, \( f_m - 3f_c = 1669 \) Hz, \( f_m - f_c = 1932 \) Hz, \( f_m + f_c = 2090 \) Hz, \( f_m + f_c + 4f_c = 2299 \) Hz, and \( f_m + f_c + 8f_c = 2392 \) Hz. For \( \eta_3 = 2 \), a peak frequency \( f_m - f_c = 2041 \) Hz is observed too, and the sidebands include \( f_m - 3f_c - 4f_c = 1565 \) Hz, \( f_m - 3f_c = 1669 \) Hz, \( f_m - f_c = 1932 \) Hz, \( f_m + f_c = 2090 \) Hz, \( f_m + f_c + 4f_c = 2299 \) Hz, and \( f_m + f_c + 8f_c = 2392 \) Hz. For \( \eta_4 = 4 \), a peak frequency \( f_m - f_c = 2041 \) Hz is also observed, and the sidebands include \( f_m - 3f_c - 4f_c = 1565 \) Hz, \( f_m - 3f_c = 1669 \) Hz, \( f_m - f_c = 1932 \) Hz, \( f_m + f_c = 2090 \) Hz, \( f_m + f_c + 4f_c = 2299 \) Hz, and \( f_m + f_c + 8f_c = 2392 \) Hz. Although the four stiffness ratio cases have a same peak frequency, as given in Table 7, their amplitudes are very different. The peak frequency amplitudes for the above four cases are 1.597, 1.795, 1.181, and 0.8976 m/s², respectively. Similarly, it shows that the peak frequency amplitude decreases with the radial support stiffness.

Table 8 plots the influence of the radial support stiffness on the frequency-domain vibrations of the ring gear for simulation case 4 (n_in = 6000 r/min and M_g = 25 Nm). For \( \eta_1 = 0.5 \), a peak frequency \( f_m - f_c = 1532 \) Hz is observed, and the sidebands include \( f_m - 2f_c - 2f_c = 1311 \) Hz, \( f_m - 2f_c = 1411 \) Hz, \( f_m - 4f_c = 1445 \) Hz, \( f_m + f_c = 1569 \) Hz, \( f_m + 4f_c = 1656 \) Hz, and \( f_m + f_c + 2f_c = 1697 \) Hz. For \( \eta_2 = 1 \), a peak frequency \( f_m - f_c = 1532 \) Hz is also observed, and the sidebands are \( f_m - 2f_c = 1311 \) Hz, \( f_m - 2f_c = 1411 \) Hz, \( f_m + 4f_c = 1656 \) Hz, and \( f_m + f_c + 2f_c = 1697 \) Hz. For \( \eta_3 = 2 \), a peak frequency \( f_m - f_c = 1532 \) Hz is observed too, and the sidebands include \( f_m - 2f_c = 1311 \) Hz, \( f_m - 2f_c = 1411 \) Hz, \( f_m + 4f_c = 1656 \) Hz, and \( f_m + f_c + 2f_c = 1697 \) Hz. For \( \eta_4 = 4 \), a peak frequency \( f_m - f_c = 1532 \) Hz is also observed, and the sidebands are \( f_m - 2f_c = 1311 \) Hz, \( f_m - 2f_c = 1411 \) Hz, \( f_m + 4f_c = 1656 \) Hz, and \( f_m + f_c + 2f_c = 1697 \) Hz. Although the four stiffness ratio cases have a same peak frequency, as given in
The sidebands around the peak frequency include the radial support stiffness. The sidebands around the peak frequency show that the peak frequency amplitude decreases with the radial support stiffness.

Table 9 plots the influence of the radial support stiffness on the frequency-domain vibrations of the ring gear for simulation case 2 (n_m = 6000 r/min and M_m = 100 Nm). For η_1 = 0.5, a peak frequency f_m - f_c = 2041 Hz is observed, and the sidebands are f_m - 2f_s^{(r)} - 2f_c = 1311 Hz, f_m - f_s^{(r)} - 2f_c = 1411 Hz, f_m - 4f_c = 1445 Hz, f_m + f_c = 1569 Hz, f_m + 4f_c = 1656 Hz, f_m + f_s^{(r)} + 2f_c = 1697 Hz, and f_m + 2f_s^{(r)} = 1748 Hz. For η_2 = 1, a peak frequency f_m - f_c = 2041 Hz is also observed, and the sidebands include f_m - 2f_s^{(r)} - 2f_c = 1311 Hz, f_m - f_s^{(r)} - 2f_c = 1411 Hz, f_m - 4f_c = 1445 Hz, f_m + f_c = 1569 Hz, f_m + 3f_c = 1625 Hz, f_m + 4f_c = 1656 Hz, and f_m + f_s^{(r)} + 2f_c = 1697 Hz. For η_3 = 2, a peak frequency f_m - f_c = 2041 Hz is observed too, and the sidebands are f_m - 2f_s^{(r)} - 2f_c = 1311 Hz, f_m - f_s^{(r)} - 2f_c = 1411 Hz, f_m + f_c = 1569 Hz, f_m + 3f_c = 1625 Hz, f_m + 4f_c = 1656 Hz, and f_m + f_s^{(r)} + 2f_c = 1697 Hz. For η_4 = 4, a peak frequency f_m - f_c = 2041 Hz is also observed, and the sidebands include f_m - 2f_s^{(r)} - 2f_c = 1311 Hz, f_m - f_s^{(r)} - 2f_c = 1411 Hz, f_m + f_c = 1569 Hz, f_m + 4f_c = 1656 Hz, and f_m + f_s^{(r)} + 2f_c = 1697 Hz. Although the four stiffness ratio cases have a same peak frequency, as given in Table 9, their amplitudes are very different. The peak frequency amplitudes for the above four cases are 2.368, 1.922, 1.495, and 0.7283 m/s², respectively. Similarly, it shows that the peak frequency amplitude decreases with the radial support stiffness.

In this section, the sidebands around the peak frequency are clearly observed in the spectrum of the ring gear. The sidebands around the peak frequency include f_m - f_s^{(r)} - 2f_c, f_m + f_c, f_m + f_s^{(r)}, f_m + f_s^{(r)} + 3f_c, f_m - f_s^{(r)} - f_c, f_m + f_s^{(r)}, f_m + f_s^{(r)} + 3f_c, and f_m - 3f_c. It shows that the vibrations of planetary gear system are mainly caused by the meshing vibration between sun gear and planet gear, the rotational vibrations of sun gear, and the spin vibrations of carrier. It seems that the passing effects of planet gears will mainly modulate the amplitude of meshing vibration of planetary gears.

### Conclusions

This paper develops a MBD model for a planetary gear system with an elastic ring gear support to discuss the influence of the radial support stiffness of the ring gear on the system vibrations. The rotational speed of the planet gear and carrier from the simulation and theoretical results are compared to validate the developed MBD model. The influences of the radial support stiffness of the ring gear, carrier moment, and sun gear speed on the time- and frequency-domain vibrations of the planetary gear system are analyzed. The following conclusions are given as

1. The waveform and amplitude of the time-domain vibration of the ring gear are greatly affected by the radial support stiffness of the ring gear as well as the peak frequency amplitude and its sidebands.
2. The maximum, PTP, and RMS values always decrease with the increase of the stiffness ratio; they always increase with the increase of the carrier moment when the input speed of the sun gear is fixed, and they always increase with the increase of the input speed of the sun gear when the carrier moment is a constant.
3. The peak frequency (f_m - f_c) in the spectrum of the ring gear is slightly affected by the radial support stiffness. The peak frequency amplitude decreases with the radial support stiffness. The sidebands around the peak frequency are very different. The peak frequency amplitudes for the above four cases are 1.87, 1.887, 1.408, and 0.6242 m/s², respectively. Similarly, it shows that the peak frequency amplitude decreases with the radial support stiffness.

| Peak frequency f (Hz) | Amplitude (m/s²) | η_1 = 0.5 | η_2 = 1 | η_3 = 2 | η_4 = 4 |
|-----------------------|-----------------|-----------|--------|--------|--------|
| f_m - 2f_s^{(r)} - 2f_c | 1311 | 0.3798 | 0.2167 | 0.2187 | 0.2627 |
| f_m - f_s^{(r)} - 2f_c | 1411 | 0.2483 | 0.1954 | 0.2603 | 0.1811 |
| f_m - 4f_c | 1445 | 0.3385 | 0.1744 | 0.1950 | – |
| f_m - f_c | 1532 | 2.3680 | 1.9220 | 1.4950 | 0.7283 |
| f_m + f_c | 1569 | 2.0300 | 1.4670 | 1.1880 | 0.5714 |
| f_m + 4f_c | 1656 | 0.2809 | 0.2682 | 0.1913 | 0.1487 |
| f_m + f_s^{(r)} + 2f_c | 1697 | 0.2134 | 0.2490 | 0.1784 | 0.1565 |
| f_m + 2f_s^{(r)} | 1748 | 0.3083 | – | – | – |

Table 9. Peak frequencies and their amplitudes for case 5.
frequency include \( f_m - f_s^{(r)} - 2f_c \), \( f_m + f_c \), \( f_m + f_s^{(r)} + 3f_c \), \( f_m - f_s^{(r)} - f_c \), \( f_m + f_s^{(r)} \), \( f_m + f_s^{(r)} + 3f_c \), and \( f_m - 3f_c \).

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