Interplay between social influence and competitive strategical games in multiplex networks

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We present a model that takes into account the coupling between evolutionary game dynamics and social influence. Importantly, social influence and game dynamics take place in different domains, which we model as different layers of a multiplex network. We show that the coupling between these dynamical processes can lead to cooperation in scenarios where the pure game dynamics predicts defection. In addition, we show that the structure of the network layers and the relation between them can further increase cooperation. Remarkably, if the layers are related in a certain way, the system can reach a polarized metastable state. These findings could explain the prevalence of polarization observed in many social dilemmas.

INTRODUCTION

Social dilemmas are situations where individual interests are in conflict, like sharing resources or generating common goods. These situations are commonly modeled by the Prisoner’s Dilemma [1] or the Stag Hunt game. Remarkably, cooperation in situations where individual interest are in conflict is surprisingly common in reality, although defection has been shown to prevail in many theories and controlled experiments [2–4]. Several mechanisms have therefore been proposed to explain the emergence of cooperation in these scenarios, for example direct and indirect reciprocity (image scoring/reputation) [5], kin and group selection [6,7], success-driven migration [8], or punishment [9].

Another mechanism responsible for the emergence of cooperation in social dilemmas could be the fact that strategic interactions between individuals or institutions do not occur in isolation. In particular, individuals that engage in strategic interactions are simultaneously exposed to social influence and, consequently, the spread of opinions. Following this line of reasoning, we assume that social influence impacts the decisions of the players [10,11], and that, vice versa, the decisions of the players impact the opinions that are propagated in the system. This consideration naturally raises the following questions: Can the interplay between social influence and game theoretical decisions enable cooperation in social dilemmas? And, what is the impact of the network topology and the relation between the structure of the social and strategical domain?

To answer these questions, we present a model where game theoretical decisions and social influence take place in different layers of a multiplex network [12–18]. In such systems, each layer contains the same set of nodes, but links are usually different in different layers. However, we find that indeed the coupling between evolutionary game dynamics and social influence can sustain partial cooperation in the Prisoner’s Dilemma, where total defection prevails in the isolated game dynamics, and partial or even full cooperation in the Stag Hunt game. In both cases, for appropriate parameters, the state of full defection can be avoided entirely. The role of the relation between the different layers of the system is especially interesting. In particular, only certain relations between the layers give rise to a metastable state in which nodes that adopt the same strategy self-organize into local clusters. This state could explain the emergence and prevalence of polarization observed in many situations that resemble social dilemmas.
In reality, individuals or institutions interact in strategic games via a contact network, like a network between firms or countries. We will discuss the influence of the
structure of the contact network and of the correlations between different networks later. For now, we study the model on a mixed population, in other words, we assume a homogeneous and infinite population in the absence of dynamical correlations and noise.

The mixed population (meanfield) assumption allows us to derive differential equations for the evolution of the density of cooperators $c_I$ in the game layer and $c_{II}$ in the opinion layer (see Supplementary Materials),

\[
\begin{align*}
\partial_t c_I &= (1 - \gamma) c_I (1 - c_I) \tanh (k (c_I (1 - T) + S (1 - c_I))) + \gamma (c_{II} - c_I), \\
\partial_t c_{II} &= (1 - \gamma)(2 \beta - 1) c_{II} (1 - c_{II}) + \gamma (c_I - c_{II}),
\end{align*}
\]

where $S$ and $T$ denote the parameters from the payoff matrix, equation (1), $\gamma \in [0, 1]$ controls the strength of the coupling between the opinion and game dynamics and $\beta \in [0, 1]$ is the bias of the opinion dynamics. Finally, $\langle k \rangle$ denotes the mean degree of the contact network.

In the following, we discuss the dynamical properties of the system described by equation (3) for fixed values of $\beta$ and $\gamma$. We find three regions of different qualitative behavior, depending on the values of parameters $T$ and $S$. In particular, we find a region in which the system effectively behaves like the harmony game (red region in Fig. 2a), which means that only full cooperation in both layers is a stable solution (see Fig. 2b). Furthermore, we find a region where the system effectively behaves like the snowdrift game (blue region in Fig. 2b). In this region, the only stable solution is a mixed state, where a finite fraction of the population cooperates (see Fig. 2c). In this region, in general, the density of cooperators in the game dynamics and those who proclaim cooperation are not the same. Finally, there is a region which can be described as a mixture of the two above cases (green region in Fig. 2b). In this region, the system exhibits a bistable behavior. Full cooperation in both layers is a stable solution as well as a mixed state as described above (see Fig. 2d). The bistable region emerges as the system undergoes a transcritical bifurcation and the solution which corresponds to full cooperation becomes unstable. In the supercritical regime, the mixed state is stable.

To sum up, we have shown that the coupling to the opinion dynamics shifts the effective behavior of the game dynamics compared to the isolated case. The coupled system exhibits effectively a Harmony-like behavior, a Snowdrift-like behavior, or a mixture of both. Interestingly, the coupling to the biased opinion dynamics successfully avoids the situation of complete defection. So far, we have considered a fully mixed, homogeneous population. In the following, we discuss the impact of the topology of the underlying contact networks as well as the relationship between the two layers of the system.

Impact of the structural organization of the multiplex

Using the assumption of a fully mixed, homogeneous population we have shown how the coupling to the biased opinion dynamics can effectively transform the behavior of the system. However, in reality, networks are heterogeneous and highly clustered, which can have a significant effect on the outcome of dynamics taking place on the network.

Furthermore, in reality, the social influence layer and the strategic game layer are neither independent nor identical. In other words, real multiplex networks are not random combinations of their constituent layer’s topologies. Hence, the contexts—or domains—in which individuals make strategic decisions and by whom they are influenced are related. In [19] the authors have shown that these relations are given by geometric correlations in hidden metric spaces underlying each layer of the system. These correlations come in two flavors: popularity correlations, which are correlations between the degrees of nodes, and similarity correlations, which determine how likely an individual is to connect to the same nodes in different layers. In simple terms, these correlations control how “similar” the different contexts represented by the layers of the system are. For further details on geometric correlations between layers of real multiplex networks we refer the reader to [19]. Here, we focus on the impact of these structural properties on the dynamics of our model. What is, in general, the impact of geometric correlations on the behavior of the system? In particular, do stronger correlations favor or hinder cooperation? To answer these questions, we perform numerical simulation using the geometric multiplex model (GMM) developed in [19]. The model generates networks with a power-law degree distribution and a tunable level of mean local clustering. Furthermore, we can control the popularity correlations (by tuning parameter $\nu \in [0, 1]$) as well as the similarity correlations (by tuning parameter $\varphi \in [0, 1]$) independently from the individual layer topologies, which allows us to study their impact in isolation. We calculate approximated phase diagrams similar to Fig. 2 using the generated networks by performing numerical simulations. In particular, to capture the bistable region of the system, we perform simulations starting from different initial conditions, in particular $C_{I,II} = 0.01$ and $C_{I,II} = 0.99$ respectively. For $\beta = 0.7$ and $\gamma = 0.2$, the system either reaches full cooperation and consensus (“harmony state”), i.e. $C_{I}^{\text{final}}, C_{II}^{\text{final}} = 1$, or a state where a mixed strategy prevails and full consensus is not reached (“snowdrift state”). We find that there is a sharp transition between these regimes (see Fig. 1 in the Supplementary Materials). We define a critical line, above which the harmony state is reached with a probability of more than 50% (dashed black lines in Fig. 2 in the Supplementary Materials). The difference between the critical lines for the different initial conditions is an approximation of the bistable region, in which both the
observe that heterogeneous and clustered topologies in single layers increase cooperation (compare Figs. 3a and b). The presence of correlations between the layers increases the region in the parameter space where the harmony solution is approached, and hence further increases
Figure 4. Polarization of the system in the presence of angular correlations between the layers ($g = 1, \nu = 0$) for a multiplex with $N = 5000$ nodes, a power-law exponent 2.9, temperature 0.2, and mean degree 6 in both layers. Parameters of the game are $T = 0.8$ and $S = -0.4$, the bias $\beta$ is 0.7 and the coupling strength is 0.2. Results are for a single realization of our model starting with a density of cooperators of 0.1 in each layer. A visualization of this realization is shown in the Supplementary Video. The top row shows visualizations of the network layers. Color coded is the mean state of the each node, averaged over time. Each time step denotes 1000 update steps of each node. The bottom row shows the evolution of the density of cooperators in each angular bin. Numbers indicate selected clusters of nodes that tend to adopt the same strategy. Each timestep $t$ denotes $10^3$ rounds.

Cooperation is common in reality in social dilemmas where many theories predict the prevalence of defection. This contradiction could be resolved by taking into account further domains of interactions between individuals, in particular social influence.

We have presented a model based on multiplex networks with two layers. One layer represents the domain in which individuals engage in repetitive strategical games. The second layer corresponds to the domain of social influence, which we model using a biased opinion dynamics. In particular, we consider a bias towards cooperative attitudes. We have shown that the coupling between these dynamics can lead to cooperation in scenarios where the pure game dynamics predicts the prevalence of defection. Furthermore, we have shown that the coupling of these dynamics in combination with geometric correlations between the layers of the system can
lead to a metastable state of high polarization, in which nodes that adopt the same strategy self-organize into local clusters. These findings could explain the emergence and prevalence of polarization observed in many social dilemmas.

Real social and strategic interaction networks evolve in time, and their evolution could depend on the strategic choices of individuals [27, 28]. Hence, the inclusion of an evolving and adaptive topology constitutes an interesting task for future work. Furthermore, one could include the competition between different strategic networks, or incorporate external noise [8]. Finally, our findings suggest that hidden geometric correlations between different layers of multiplex networks can alter the behavior of the dynamics taking place of top of them significantly, and hence such correlations should be taken into account in future research on dynamical processes on multiplex networks [20].

METHODS

Geometric multiplex model (GMM)

The geometric multiplex model is based on the (single-layer) network construction procedure of the newtonian $S^1$ [31] and hyperbolic $H^2$ [32] models. The two models are isomorphic and here we present the results for the $H^2$ version. The construction of a network of size $N$ proceed firsts by assigning to each node $i = 1, \ldots, N$ its popularity and similarity coordinates $r_i, \theta_i$ and subsequently, connecting each pair of nodes $i, j$ with probability $p(x_{ij}) = 1/(1 + e^{-(x_{ij} - \theta_0)})$, where $x_{ij}$ is the hyperbolic distance between the nodes and $R \sim \ln N$ (see Supplementary Materials). The connection probability $p(x_{ij})$ is the Fermi-Dirac distribution where the temperature parameter $T_{\text{GMM}}$ controls the level of clustering in the network [33]. The average clustering $\bar{c}$ is maximized at $T = 0$, linearly decreases to zero with $T \in (0, 1)$, and is asymptotically zero if $T > 1$. As $T \rightarrow 0$ the connection probability becomes the step function $p(x_{ij}) \rightarrow 1$ if $x_{ij} \leq R$, and $p(x_{ij}) \rightarrow 0$ if $x_{ij} > R$. It has been shown that the $S^1$ and $H^2$ models can build synthetic networks reproducing a wide range of structural characteristics of real networks, including power law degree distributions and strong clustering [31, 32]. The use of these models for the single-layer networks allows for radial and angular coordinate correlations across the different layers. The level of these correlations can be controlled by model parameters $\nu \in [0, 1]$ and $g \in [0, 1]$, without affecting the topological structure of the single layers. The radial correlations, related to the node’s degree, increase with parameter $\nu$—at $\nu = 0$ there are no radial correlations, while at $\nu = 1$ radial correlations are maximized. Similarly, the angular correlations increase with parameter $g$—at $g = 0$ there are no angular correlations, while at $g = 1$ angular correlations are maximized. See [19] for details.

Finally, we extract the mutually connected component of the system to avoid disconnected nodes.

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