Muon Physics: A Pillar of the Standard Model

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Since its discovery in the 1930s, the muon has played an important role in our quest to understand the sub-atomic theory of matter. The muon was the first second-generation standard-model particle to be discovered, and its decay has provided information on the (Vector – Axial Vector) structure of the weak interaction, the strength of the weak interaction, $G_F$, and the conservation of lepton number (flavor) in muon decay. The muon’s anomalous magnetic moment has played an important role in restricting theories of physics beyond the standard model, where at present there is a 3.4 σ difference between the experiment and standard-model theory.

The muon on the atomic nucleus has provided valuable information on the modification of the weak current by the strong interaction, which is complementary to that obtained from nuclear $\beta$ decay.

KEYWORDS: muon, weak decay, muon capture, magnetic moment, lepton flavor violation

1. Introduction

The muon was first observed in a Wilson cloud chamber by Kunze in 1933, where it was reported to be “a particle of uncertain nature.” In 1936 Anderson and Neddermeyer reported the presence of “particles less massive than protons but more penetrating than electrons” in cosmic rays, which was confirmed in 1937 by Street and Stevenson, Nishina, Tekeuchi and Ichimiya, and by Crussard and Leprince-Ringuet. The Yukawa theory of the nuclear force had predicted such a particle, but this “mesotron” as it was called, interacted too weakly with matter to be the carrier of the strong force. Today we understand that the muon is a second generation lepton, with a mass about 207 times the electron’s. Like the electron, the muon obeys quantum electrodynamics, and can interact with other particles through the electromagnetic and weak forces. Unlike the electron which appears to be stable, the muon decays through the weak force.

The muon lifetime of 2.2 $\mu$s permits one to make precision measurements of its properties, and to use it as a tool to study the semileptonic weak interaction, nuclear properties, as well as magnetic properties of condensed matter systems. The high precision to which the muonium ($\mu^+e^-$) atom hyperfine structure can be measured and calculated makes it a significant input parameter in the determination of fundamental constants. In this review, I will focus on the role of the muon in particle physics.

A beam of negative muons can be brought to rest in matter, where hydrogen-like atoms are formed, with a nuclear charge of $Z$. The Bohr radius for a hydrogen-like atom is inversely proportional to the orbiting particle’s mass ($r_n = [n^2\hbar c]/[mc^2Z\alpha]$), so that for the lowest quantum numbers of high-$Z$ muonic atoms, the muon is well inside of the atomic electron cloud, with the Bohr radius of the 1S atomic state well inside the nucleus. The $2P \rightarrow 1S$ x-ray energies are shifted because of the modification of the Coulomb potential inside the nucleus, and these x rays have provided information on nuclear root-mean-square charge radii. The Lamb shift in muonic hydrogen, $\Delta E_{2P-2S}$, which is being measured at the Paul Scherrer Institut (PSI), is given by $\langle 209.974(6) - 5.226R_p^0 + 0.036R_p^3 \rangle$ meV, where $R_p$ is the proton rms charge radius. This experiment should provide a precise measurement of $R_p$. The weak nuclear capture, called ordinary muon capture (OMC), of the muon on the atomic nucleus following the cascade to the 1S ground state, $\mu^- + ZN \rightarrow Z-1N + \nu_\mu$, is the analog to the weak capture of a K-shell electron by the nucleus, and provides information on the modification of the weak interaction by the hadronic matter.

The muon mass of $\sim 106$ MeV restricts the muon to decay into the electron, neutrinos, and photons. Thus muon decay is a purely leptonic process, and the dominant decay mode is $\mu^- \rightarrow e^- + \nu_\mu + \bar{\nu}_e$. This three-body decay tells us that the individual lepton number, electron and muon, is conserved separately, and that the two flavors (kinds) of neutrinos are distinct particles. Here the $\mu^-$ and $e^-$ are “particles” and the $\mu^+$ and $e^+$ are the antiparticles. In the 1950s, it became possible to make pions, and thus muons, in the laboratory. The energetically favorable decay $\mu^+ \rightarrow e^+\gamma$ was searched for and not found to a relative branching ratio of $< 2 \times 10^{-5}$. Also searched for was the neutrinoless capture of a $\mu^-$ on an atomic nucleus, $\mu^- + N \rightarrow e^- + N$, which was not found at the level of $\sim 5 \times 10^{-4}$. Such processes are said to “violate lepton flavor,” and continue to be the object of present and planned studies reaching to sensitivities of $10^{-14}$ and $10^{-16}$, respectively.

The muon, like the electron, is a spin 1/2 lepton, with a magnetic moment given by

$$\mu_s = g_s \left( \frac{e}{2m} \right) \hat{s}; \quad \mu = (1 + a) \left( \frac{\hbar}{2m} \right); \quad a = \frac{(g_s - 2)}{2}$$

where the muon charge $q = \pm e$, and $g_s$, the Landé $g$-,
factor is slightly greater than the Dirac value of 2. The middle
equation above is useful from a theoretical point
of view, as it separates the magnetic moment into two
pieces: the Dirac moment which is unity in units of
the appropriate magneton, \(\frac{e\hbar}{2m}\), and is predicted
by the Dirac equation; and the anomalous (Pauli) moment,
where the dimensionless quantity \(a\) is referred to as the anomaly. The muon anomaly, like the electron’s, arises
from radiative corrections that are discussed below.

When the muon was discovered, it was an unexpected
surprise. Looking at this from our 21st century perspec-
tive, it is easy to forget how we reached what is now called
the “standard model” of subatomic physics, which incor-
porates three generations of leptons, \(e, \mu, \tau\) and their
neutrinos; three generations of quarks; the electro-weak
gauge bosons, \(\gamma\) and \(W\) and \(Z\); and the gluons that carry the
strong force. When this author joined the field as a grad-
uate student in the mid 1960s, none of this was clear.
Quarks were viewed by many as a mathematical device,
not as constituent particles. Even after quarks were in-
fed into the model, we only knew of the existence of three of them. While
the \(V-A\) structure of the weak interaction was first in-
fed from nuclear \(\beta\) decay, the study of muon decay has
provided a useful laboratory in which to study the purely
leptonic weak interaction, to search for physics beyond
the standard model, such as additional terms in the in-
teraction besides the standard-model \(V-A\) structure, as
well as looking for standard model forbidden decays like
\(\mu \rightarrow e\gamma\). For many years, the experimental value of the
muon’s anomalous magnetic moment has served to con-
strain physics beyond the standard model, and continues
that role today.

2. Muon Decay and \(G_F\)

The muon decay \(\mu^- \rightarrow e^-\nu_\mu\bar{\nu}_e\) is purely leptonic.
Since \(m_\mu << M_W\), muon decay can be described by
a local four-fermion (contact) interaction. While non-
renormalizable, at low energies it provides an excel-
 lent approximation to the full electroweak theory. The
weak Lagrangian is written as a current-current inter-
action, where the leptonic current is of the \((V - A)\) form,
\[\bar{u}\gamma(1 - \gamma_5)v\].

Michel\(^\text{11}\) first wrote down a parameterization of muon
decay, defining five parameters, \(\rho, \eta, \xi, \delta\) and \(h\), which are
combinations of the different possible couplings al-
lowed by Lorentz invariance in muon decay. The standard
model has clear predictions for these parameters and
they have been measured repeatedly over the intervening
years to search for physics beyond the standard model.
This tradition continues today, with the TWIST experi-
ment at TRIUMF, which is mid-way through a program
to improve on the precision of the Michel parameters by
an order of magnitude.\(^\text{12}\) While there are some scenarios
in which new physics would conspire to leave the Michel
parameters at their standard model value,\(^\text{13}\) a variance
from the standard-model values would be a clear sign of
new physics at work.

The muon lifetime, see Fig. 1 is directly related to the
strength of the weak interaction, which in Fermi theory
is described by the constant \(G_F\). The standard-model
electroweak gauge coupling \(g\) is related to \(G_F\) by\(^\text{14}\)
\[\frac{G_F}{\sqrt{2}} = g^2 \left(1 + \Delta r\right)\]
where \(\Delta r\) represents the weak boson mediated tree-level
process and its radiative corrections.\(^\text{15}\) In the standard
model, the Fermi constant is related to the vacuum
expectation value of the Higgs field by \(G_F = 1/(\sqrt{2} v^2)\).

While the Fermi theory is nonrenormalizable, the QED
radiative corrections are finite to first order in \(G_F\),
and to all orders in the fine-structure constant \(\alpha\). This
gives the relationship\(^\text{14}\) between \(G_F\) and the muon life-
time, \(\tau_\mu\),
\[\frac{1}{\tau_\mu} = \frac{G_F^2 m_\mu^5}{192\pi^3}(1 + \Delta q)\]
where \(\Delta q\) is the sum of phase space, and QED and
hadronic radiative corrections. More properly one should
write \(G_\mu\) since new physics contributions could make \(G\)
different for the three leptons.\(^\text{16}\)

The MuLan experiment at PSI has recently re-
ported a new measurement of the muon lifetime
2.197 013 (21) (11) \(\mu s\) (\(\pm 11\) parts per million (ppm)),\(^\text{17}\)
to be compared with the previous world average
2.197 03 (4) \(\mu s\) (19 ppm).\(^\text{18}\) The new world average
muon lifetime of 2.197 019 (21) \(\mu s\) gives, assuming
only standard-model physics in muon decay, \(G_F = 1.166 371(6) \times 10^{-5}\ GeV^2\) (5 ppm). This experiment
should eventually reach a precision of 1 ppm on \(\tau_\mu\).

3. Nuclear Muon Capture

The weak capture of a muon on a proton has much in
common with nuclear \(\beta\) decay. As for other low-energy
weak processes, the interaction can be described as a
current-current interaction with the \((V - A)\) leptonic
process and its radiative corrections. More properly one should
draw \(G_\mu\) since new physics contributions could make \(G\)
different for the three leptons.\(^\text{16}\)

The weak current allowed by Lorentz invariance is\(^\text{20}\)
\[\bar{u}_n(p') \left[ g_V(q^2)\gamma^\lambda + \frac{g_M(q^2)}{2m_N} \sigma^{\lambda\nu} q_\nu + \frac{g_S(q^2)}{m_\mu} T^\lambda\right] u_p(p).\]
(4)
The corresponding form of the axial-vector current is
\[\bar{u}_n(p') \left[ -g_A(q^2)\gamma^\lambda \gamma_5 - \frac{g_P(q^2)}{m_\mu} \gamma_5 \gamma^\lambda q_\nu + \frac{g_T(q^2)}{2m_N} \sigma^{\lambda\nu} q_\nu \gamma_5\right] u_p(p),\]
(5)
The MuCap experiment stops $\mu^-$ in a gaseous hydrogen target that functions as a time projection chamber (TPC), making it possible to determine where the muon stops in the target. A comparison of the $\mu^-$ lifetime in this protonium target to the free $\mu^+$ lifetime, gives the capture rate and determines $g_p$. The MuCap experiment has recently reported a first result, \cite{26}$g_p(q^2 = -0.88m_n^2) = 7.3 \pm 1.1$, consistent with the expectation from chiral perturbation theory. They have a factor of four more data which are being analyzed. While it is not clear what is wrong with the previous ordinary muon capture and radiative capture experiments, the MuCap result seems to indicate that a modern experiment, with a gaseous target and information from the TPC, has settled the long-standing discrepancy.

4. The Magnetic and Electric Dipole Moments

The electric and magnetic dipole moments have been an integral part of relativistic electron (lepton) theory since Dirac’s famous 1928 paper, where he pointed out that an electron in external electric and magnetic fields has “the two extra terms

$$\frac{e\hbar}{c} (\sigma, \mathbf{H}) + \frac{e\hbar}{c} \rho_1 (\sigma, \mathbf{E}), \tag{6}$$

... when divided by the factor $2m$ can be regarded as the additional potential energy of the electron due to its new degree of freedom. \cite{27} These terms represent the magnetic dipole (Dirac) moment and electric dipole moment interactions with the external magnetic and electric fields.

In modern notation, the magnetic dipole moment (MDM) interaction becomes

$$\bar{u}_\mu \left[ eF_1(q^2)\gamma_\beta + \frac{ie}{2m_\mu}F_2(q^2)\sigma_{\beta\delta}q^\delta \right] u_\mu \tag{7}$$

where $F_1(0) = 1$, and $F_2(0) = a_\mu$. The electric dipole moment (EDM) interaction is

$$\bar{u}_\mu \left[ \frac{ie}{2m_\mu}F_2(q^2) - F_3(q^2)\gamma_\gamma \right] \sigma_{\beta\delta}q^\delta u_\mu \tag{8}$$

where $F_2(0) = a_\mu$, $F_3(0) = d_\mu$, with

$$d_\mu = \left( \frac{\eta}{2} \right) \left( \frac{e\hbar}{2mc} \right) \simeq \eta \times 4.7 \times 10^{-14} \text{ e cm.} \tag{9}$$

(This $\eta$, which is the EDM analogy to $g$ for the MDM, should not be confused with the Michel parameter $\eta$.)

The existence of an EDM implies that both $P$ and $T$ are violated. \cite{28,29} This can be seen by considering the non-relativistic Hamiltonian for a spin one-half particle in the presence of both an electric and magnetic field: $\mathcal{H} = -\bar{\mu} \cdot \vec{B} - \bar{d} \cdot \vec{E}$. The transformation properties of $\vec{E}, \vec{B}, \vec{\mu}$ and $\vec{d}$ are given in the Table I, and we see that while $\bar{\mu} \cdot \vec{B}$ is even under all three, $\bar{d} \cdot \vec{E}$ is odd under both $P$ and $T$. While parity violation has been observed in many weak processes, direct $T$ violation has only been observed in the neutral kaon system. \cite{30} In the context of CPT symmetry, an EDM implies CP violation, which is allowed by the standard model for decays in the neutral kaon and $B$-meson sectors.

Observation of a non-zero electron or muon EDM would be a clear signal for new physics. To date no permanent EDM has been observed for the electron, the neutron, or an atomic nucleus, with the experimental limits given in Table II. It is interesting to note that
in his original paper Dirac stated “The electric moment, being a pure imaginary, we should not expect to appear in the model. It is doubtful whether the electric moment has any physical meaning, since the Hamiltonian ... that we started from is real, and the imaginary part only appeared when we multiplied it up in an artificial way in order to make it resemble the Hamiltonian of previous theories.” Even in the 4th edition of his quantum mechanics book from 1958, well after the suggestion of Purcell and Ramsey that one should search for a permanent EDM, Dirac held fast to this point of view.

While CP violation is widely invoked to explain the baryon-antibaryon asymmetry of the universe, the CP violation observed to date in the neutral kaon, and in the B meson sectors is too small to explain it. This CP deficit has motivated a broad program of searches for EDMs in a range of systems. Many extensions to the standard model, such as supersymmetry, do not forbid new sources of CP-violation, and the failure to observe it has placed severe restrictions on many models.

The magnetic dipole moment can differ from its Dirac value \( g = 2 \) for several reasons. Recall that the proton’s \( g \)-value is 5.6 \(( g_p = 1.79)\), a manifestation of its quark-gluon internal structure. On the other hand, the leptons appear to have no internal structure, and the EDMs are thought to arise from radiative corrections, i.e., from virtual particles that couple to the lepton. We would emphasize that these radiative corrections need not be limited to the standard-model particles, but rather the physical values of the lepton anomalies represent a sum-rule over all virtual particles in nature that can couple to the lepton, or to the photon through vacuum polarization loops.

The standard model value of a lepton’s anomaly, \( a_L \), has contributions from three different sets of radiative processes: quantum electrodynamics (QED) – with loops containing leptons (\( e, \mu, \tau \)) and photons; hadronic – with hadrons in vacuum polarization loops; and weak – with loops involving the bosons \( W, Z \), and Higgs. Examples are shown in Fig. 2. Thus

\[
a^{(\text{SM})}_{\mu} = a^{(\text{QED})}_{\mu} + a^{(\text{hadronic})}_{\mu} + a^{(\text{weak})}_{\mu}.
\]

The dominant contribution from quantum electrodynamics (QED), called the Schwinger term,\(^{36} \) \( a^{(2)} = \alpha/2\pi \), and is shown diagrammatically in Fig. 2(a). The QED contributions have been calculated through four loops, with the leading five-loop contributions calculated.\(^{37} \) Examples of the hadronic and weak contributions are given in Fig. 2(b)-(e).

The hadronic contribution cannot be calculated directly from QCD, since the energy scale is very low \((m_\mu c^2)\), although Blum has performed a proof of principle calculation on the lattice.\(^{44} \) Fortunately dispersion theory\(^{36} \) gives a relationship between the vacuum polarization loop and the cross section for \( e^+ e^- \to \) hadrons,

\[
a_{\mu}(\text{Had}; 1) = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_0^\infty \frac{ds}{s^2} K(s) R(s);
\]

\[
R = \{\sigma_{\text{tot}}(e^+ e^- \to \text{hadrons})\}/\{\sigma_{\text{tot}}(e^+ e^- \to \mu^+ \mu^-)\},
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and experimental data are used as input.\(^{38,39} \)

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The muon anomaly is sensitive to a number of potential candidates for physics beyond the standard model.\(^{41} \)

1. muon substructure, where the contribution depends on the substructure scale \( \Lambda \) as

\[
\delta a_{\mu}(\Lambda) \simeq \frac{m_\mu^2}{\Lambda^2};
\]

2. W-boson substructure,

3. new particles that couple to the muon, such as the supersymmetric partners of the weak gauge bosons,

4. extra dimensions

The potential contribution from supersymmetry has generated a lot of attention.\(^{42,43} \) The relevant diagrams are shown in Fig. 3 below. A simple model with equal masses gives

\[
a^{(\text{SUSY})}_{\mu} \simeq \frac{\alpha(M_Z)}{8\pi\sin^2\theta_W m_\mu^2} \tan\beta \left(1 - \frac{4\alpha}{\pi} \frac{m_\mu}{\tilde{m}_\mu}\right);
\]

\[
\simeq (\text{sgn}\mu) \times 10^{-10} \frac{\tan\beta}{100\text{ GeV}}^2
\]

where \( \tan\beta \) is the ratio of the two vacuum expectation values of the two Higgs fields. If the SUSY mass scale were known, then \( a^{(\text{SUSY})}_{\mu} \) would provide a clean way to determine \( \tan\beta \).
When a single energy threshold is placed on the decay electrons, the number of high-energy electrons is modulated by the spin precession frequency, Eq 17, producing the time distribution

$$N(t, E_{th}) = N_0(E_{th})e^{-\frac{t}{\tau(E_{th})}}[1 + A(E_{th})\cos(\omega_a t + \phi(E_{th}))].$$

(18)
as shown in Fig. 4.\textsuperscript{40} The value of $\omega_a$ is obtained from a least-squares fit to these data. The five-parameter function (Eq. 18) is used as a starting point, but many additional small effects must be taken into account.\textsuperscript{40}

In E821, both $\mu^+$ and $\mu^-$ were measured, and assuming CPT invariance, the final result obtained by E821,\textsuperscript{40} $a^\exp_{\mu} = 116\,592\,080 (63) \times 10^{-11}$, is shown in Fig. 5, along with the individual measurements and the standard-model value. The present standard-model value $a^\text{SM}(00) = 116\,591\,785 (61) \times 10^{-11}$, and one finds $\Delta a_{\mu} = 295(88) \times 10^{-11}$, a 3.4 $\sigma$ difference.

Fig. 3. The lowest-order supersymmetric contributions to the muon anomaly. The $\chi$ are the superpartners of the standard-model gauge bosons.

$$\omega_S = -\frac{q_B}{2m}q_B(1 - \gamma) - \omega_C = -\frac{q_B}{m\gamma}.$$  

(15)
The magnetic field in Eq. 16 is the average field seen by the ensemble of muons. This technique has been used in all but the first experiments by Garwin, et al.,\textsuperscript{45} which used stopping muons, to measure the anomaly. After Garwin, et al., made a 12\% measurement of the anomaly, a series of three beautiful experiments at CERN culminated with a 7.3 ppm measure of $a_{\mu}$,\textsuperscript{34}

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4.1 Measurement of the Anomalous Magnetic Dipole Moment

Measurement of the magnetic anomaly uses the spin motion in a magnetic field. For a muon moving in a magnetic field, the spin and momentum rotate with the frequencies:

$$\omega_s = -\frac{q_B}{2m}q_B(1 - \gamma); \quad \omega_C = -\frac{q_B}{m\gamma}.$$  

(15)
The spin precession relative to the momentum occurs at the difference frequency, $\omega_a$, between the spin and cyclotron frequencies, Equation 15:

$$\omega_a = \omega_S - \omega_C = -\left(\frac{g - 2}{2}\right)\frac{q_B}{m} = -a_{\mu}q_B.$$  

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In the context of a constrained minimal supersymmetric model (CMSSM), \((g - 2)_\mu\) provides an orthogonal constraint on dark matter from that provided by the WMAP survey, as can be seen in Fig. 6.

Fig. 6. Limits on dark matter placed by various inputs in CMSSM, with \(\tan \beta = 10\). The \(\Delta\) between experiment and standard-model theory is from Ref. [38], see text. The brown wedge on the lower right is excluded by the requirement the dark matter be neutral. Direct limits on the Higgs and chargino \(\chi^\pm\) masses are indicated by vertical lines. Restrictions from the WMAP satellite data are shown as a light-blue line. The \((g - 2)\) 1 and 2-standard deviation boundaries are shown in purple. The region “allowed” by WMAP and \((g - 2)\) is indicated by the ellipse, which is further restricted by the limit on \(M_{\chi}\). (Figure courtesy of K. Olive)

With the apparent 3.4 \(\sigma\) difference between theory and experiment, a new experiment to improve the error by a factor of 2 to 2.5 has been proposed to Brookhaven laboratory, but at present it is not funded. The theoretical value will continue to be improved, both with the expected availability of additional data on \(e^+e^-\) annihilation to hadrons, and with additional work on the hadronic light-by-light contribution.

4.2 The Search for a Muon Electric Dipole Moment

With an EDM present, the spin precession frequency relative to the momentum must be modified. The total frequency becomes \(\tilde{\omega} = \omega_a + \omega_\eta\), where

\[
\omega_\eta = -\frac{q}{m} \left[ \frac{\eta}{2} \left( \frac{E}{c} + \vec{\beta} \times \vec{B} \right) \right],
\]

(19)

with \(\eta\) defined by Eq. 9, and \(\omega_a\) by Eq. 17. The spin motion resulting from the motional electric field, \(\vec{\beta} \times \vec{B}\) is the dominant effect, so \(\omega_\eta\) is transverse to \(\vec{B}\). An EDM would have two effects on the precession, there would be a slight tipping of the precession plane, which would cause a vertical oscillation of the centroid of the decay electrons that out of phase with the \(\omega_a\) precession; and the observed frequency \(\omega\) would be larger,

\[
\omega = \sqrt{\omega_a^2 + \left( \frac{q\eta^3B}{2m} \right)^2}. \tag{20}
\]

The muon limit in Table II, placed by the non-observation of the vertical oscillation, is dominated by systematic effects. The limit obtained by this method in the CERN experiment,\(^{34}\) and likely to be obtained by ES21, cannot directly exclude the possibility that the entire difference between the measured and standard-model values of \(a_\mu\) could be caused by a muon EDM. Such a scenario would imply that the EDM would be \(d_\mu = 2.4(0.4) \times 10^{-19} \text{ e-cm}\), a factor \(\approx 10^8\) larger than the current limit on the electron EDM. While this would be a very exciting result, it is orders of magnitude larger than that expected from even the most speculative models.\(^{47–50}\)

To reduce systematic errors in the muon EDM measurement, a “frozen spin” technique has been proposed\(^{51}\) which uses a radial electric field in a muon storage ring, operating at \(\gamma << \gamma_{\text{magic}}\) to cancel the \((g - 2)\) precession. The EDM term, Eq. 19, would then cause the spin to steadily move out of the plane of the storage ring. Electron detectors above and below the storage region would detect a time-dependent up-down asymmetry that increased with time. As in the \((g - 2)\) experiments, detectors placed in the plane of the beam would be used, in this case to make sure that the radial-\(E\)-field cancels the normal spin precession exactly. Adelmann and Kirsh\(^{52}\) have proposed that one could reach a sensitivity of \(5 \times 10^{-23} \text{ e-cm}\) with a small storage ring at PSI. A letter of intent at J-PARC\(^{53}\) suggested that one could reach \(< 10^{-24} \text{ e-cm}\) there. The ultimate sensitivity would need an even more intense muon source, such as a neutrino factory.

5. The Search for Lepton Flavor Violation

The standard-model gauge bosons do not permit leptons to mix with each other, unlike the quark sector where mixing has been known for many years. Quark mixing was first proposed by Cabibbo,\(^{54}\) and extended to three generations by Kobayashi and Maskawa,\(^{55}\) which is described by a mixing \(3 \times 3\) matrix now universally called the CKM matrix. With the discovery of neutrino mass, we know that lepton flavor violation (LFV) certainly exists in the neutral lepton sector, with the determination of the mixing matrix for the three neutrino flavors having become a world-wide effort.

While the mixing observed in neutrinos does predict some level of charged lepton mixing, it is many orders of magnitude below present experimental limits.\(^{13}\) New dynamics,\(^{57–66}\) e.g. supersymmetry, do permit leptons to mix, and the observation of standard-model forbidden processes such as

\[
\mu^+ \rightarrow e^+\gamma; \quad \mu^+ \rightarrow e^+e^-; \quad \mu^- N \rightarrow e^- N; \quad \mu^+e^- \rightarrow \mu^- e^-; \quad \mu^- N \rightarrow e^+ + N' \]

(21)

(22)

would clearly signify the presence of new physics. The present limits on lepton flavor violation are shown in Fig. 7.

If lepton mixing occurs via supersymmetry, there will be a mixing between the supersymmetric leptons (sleptons) which would also be described by a \(3 \times 3\) mixing matrix. The schematic connection between lepton flavor violations and the dipole moments is shown in Fig. 8, and there are models that try to connect these processes.\(^{67}\)
In a large class of models, if the $\Delta \ell = 1$ LFV decay goes through the transition magnetic moment, one finds \[^{13}\]

$$B(\mu N \rightarrow eN) / B(\mu \rightarrow e\gamma) = 2 \times 10^{-3}B(A, Z), \quad (23)$$

where $B(A, Z)$ is a coefficient of order 1 for nuclei heavier than aluminum. \[^{68}\] For other models, these two rates can be the same, \[^{13}\] so in the design of new experiments, the reach in single event sensitivity for the coherent muon conversion experiments needs to be several orders of magnitude smaller than for $\mu \rightarrow e\gamma$ to probe the former class of models with equal sensitivity. Detailed calculations of $\mu - e$ conversion rates as a function of atomic number have also been carried out \[^{69}\] and if observed, measurements should be carried out in several nuclei.

From the experimental side, the next generation $\mu \rightarrow e\gamma$ experiment, MEG, is now under way at PSI, \[^{70}\] with a sensitivity goal of $10^{-13} - 10^{-14}$. Since the decay occurs at rest, the photon and positron are back-to-back, and share equally the energy $m_\mu c^2$. This experiment makes use of a unique “COBRA” magnet which produces a constant bending radius for the mono-energetic $e^+$, independent of its angle. The photon is detected by a large liquid Xe scintillation detector as shown in Fig. 9.

Of the various lepton-flavor violating reactions, only coherent muon conversion does not require coincidence measurements. The decay $\mu \rightarrow 3e$, while theoretically appealing, requires a triple coincidence and sensitivity to the whole phase space of the decay, and thereby is experimentally more challenging. It is the coherent muon to electron conversion, where with adequate energy resolution, the conversion electron can be resolved from background, that with adequate muon flux can be pushed to the $10^{-18}$ or $10^{-19}$ sensitivity. Such a program has been proposed for J-PARC. \[^{71}\]

The muonium to antimuonium conversion (left-hand process in Eq.22) represents a change of two units of lepton number, analogous to $K^0 \rightarrow \bar{K}^0$ oscillations. This process was originally proposed by Pontecorvo. \[^{72}\] An experiment at PSI \[^{73}\] obtained a single event sensitivity of $P_{MM} = 8.2 \times 10^{-11}$ which implies a coupling $G_{MM} \leq 3 \times 10^{-3}G_F$ at 90% C.L., where $G_F$ is the Fermi coupling constant. A broad range of speculative theories such as left-right symmetry, R-parity violating supersymmetry, etc., \[^{74}\] could permit such an oscillation.

### 6. Summary and Conclusions

Since its discovery, the muon has provided an important tool to study the standard model, and to constrain its extensions. Experiments in the planning stage for $(g - 2)$, the search for an electric dipole moment and lepton flavor violation in muon decay or conversion will continue this tradition. Research and development for new more intense muon sources, such as the muon ionization cooling experiment (MICE), \[^{75}\] will further propel increases in sensitivity. Muon experiments form an important part of the precision frontier in particle physics, which will continue to provide vital information complementary to that from the highest energy colliders.

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