Are the $\Theta^+(1540)$, $\Xi^{--}(1860)$ and $D^{*-}p(3100)$ Pentaquarks or Heptaquarks?

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We study the $\Theta^+(1540)$ discovered at SPring-8. We apply Quark Model techniques, that explain with success the repulsive hard core of nucleon-nucleon and kaon-nucleon exotic scattering, and the short range attraction present in pion-nucleon and pion-pion non-exotic scattering. We find a K-N repulsion which excludes the $\Theta^+$ as a $K^-N$ s-wave pentaquark. We explore the $\Theta^+$ as a crypto-heptaquark, equivalent to a $K^-\pi^-N$ borromean boundstate, with positive parity and total isospin $I=0$. The attraction is provided by the pion-nucleon and pion-kaon interactions. The other candidates to pentaquarks $\Xi^{--}(1860)$, observed at NA49, and $D^{*-}p(3100)$, observed at H1, are also studied as linear molecular heptaquarks.

1 Introduction

The $uudd\bar{s}$ pentaquark $\Theta^+(1540)$ was discovered at LEPS\cite{1} and DIANA\cite{2}. After the Jefferson Lab confirmation\cite{3}, it was observed in several different experiences, with a mass of $1540\pm10$ MeV and a decay width of $15\pm15$ MeV. Recently the $dsss\bar{u}$ pentaquark $\Xi^{--}(1860)$ was observed at NA49\cite{4} and the $uudd\bar{c}$ pentaquark $D^{*-}p(3100)$ was observed at H1\cite{5}. These are extremely exciting states because they may be the first exotic hadron to be discovered, with quantum numbers that cannot be interpreted as a quark and an anti-quark meson or as a three quark baryon. Exotic multiquarks are expected since the early works of Jaffe\cite{6,7}, and some years ago Diakonov, Petrov and Polyakov\cite{8} applied skyrmions to a precise prediction of the $\Theta^+$. The $\Xi^{--}(1860)$ and $D^{*-}p(3100)$ belong probably to the same family of exotic flavour pentaquarks.

We start in this talk by reviewing the Quark Model (QM) and the Resonating Group Method (RGM)\cite{9}, which are adequate to study states where several quarks overlap. First we apply the RGM to show\cite{10,11,12,13} that the exotic $N-K$ hard core s-wave interaction is repulsive, excluding the $\Theta^+$ as a bare s-wave pentaquark $uddu\bar{s}$ state or a tightly bound s-wave $N-K$ narrow resonance. However a $\pi$ could also be present in this system, in which case the binding energy would be of the order of $30 \ MeV$. Moreover this state of seven quarks would have a positive
parity, and would have to decay to a p-wave $N - K$ system, which is suppressed by angular momentum, thus explaining the narrow width of the $\Theta^+$. We then put together the $\pi - N$, $\pi - K$ and $N - K$ interactions to show that the $\Theta^+$ is possibly a borromean three body s-wave bound state of a $\pi$, a $N$ and a $K$, with positive parity and total isospin $I = 0$. Finally we extend the crypto-heptaquark picture to flavour $SU(4)$ and study the $Xi^{++}$ and $D^* - p$ multiquarks.

2 A Quark Model Criterion for Repulsion/attraction

We use a standard Quark Model Hamiltonian. The Resonating Group Method is a convenient method to compute the energy of multiquarks and to study hadronic coupled channels. The RGM was first used by Ribeiro to explain the $N - N$ hard-core repulsion. We compute the matrix element of the Hamiltonian in an antisymmetrised basis of hadrons,

$$V_{\text{meson } A \rightarrow \text{baryon } B} = \frac{\langle \phi_B \phi_A | (1 + P_{AB})(1 - V_{13} + V_{23} + V_{14} + V_{24})P_{13} + A_{123} + A_{14} | \phi_A \phi_B \rangle}{\langle \phi_B \phi_A | (1 + P_{AB})(1 - P_{13}) | \phi_A \phi_B \rangle}$$

where the exchange operator $P_{14}$ produces the states colour-octet x colour-octet, expected in multiquarks, and where $A_{23}$ is the quark-antiquark annihilation potential, crucial to the chiral symmetry of the interaction.\cite{15,16}

The exchange overlap results in a separable potential, and we arrive at the criterion for the interaction of ground-state hadrons:

- whenever the two interacting hadrons have a common flavour, the repulsion is increased,
- when the two interacting hadrons have a matching quark and antiquark the attraction is enhanced.

3 Why the $\Theta^+$ cannot be a simple $uudd\bar{s}$ or $K - N$ state

Applying the criterion to the S=1, I=0 pentaquark, arranged in the color singlet clusters $uud - d\bar{s}$ or $d\bar{u}u - u\bar{s}$ we find repulsion! Indeed we are able to reproduce the repulsive K-N exotic s-wave phase shifts, which have been understood long ago.\cite{10,11,12} Moreover all other $uudd\bar{s}$ systems are even more repulsive or unstable. Because we checked all our only approximations, say using a variational method, and neglecting the meson exchange interactions, we estimate that something even more exotic is probably occuring!

Suppose that a $q - \bar{q}$ pair is added to the system. Then the new system may bind. Moreover the heptaquark had a different parity and therefore it is an independent system (a chiral partner). Here we propose that the $\Theta^+$ is in fact a heptaquark with the strong overlap of a $K - \pi - N$, where the $\pi$ is bound by the I=1/2 $\pi - K$ and $\pi - N$ attractive interactions.
Only the I=1 elements are pentaquarks, or equivalently overlapping \( I = 1 \).

Then the isospin couplings in the \( \Theta^+ \). In (b) we exhibit, in a three dimensional strangeness-flavour-mass plot, the expected masses of the exotic anti-decuplet.

4 the \( K - N, \pi - K, \pi - N \) and \( K - \pi - N \) systems

We now investigate the borromean binding of the exotic \( \Theta^+ \) constituted by a \( N, K \) and \( \pi \) triplet. We arrive at the separable potentials for the different two-body potentials [11],

\[
V_{K-N} = \frac{2}{5} \bar{\tau}_A \cdot \bar{\tau}_B \left( M_\Delta - M_N \right) \left( \frac{2\sqrt{\pi}}{\alpha} \right)^3 e^{-\frac{\mu^2}{2\beta^2}} \int \frac{d^3p_\Lambda}{(2\pi)^3} \frac{e^{\frac{\beta^2}{2}}} \]

\[
V_{\pi-N} = \frac{2}{9} (2M_N - M_\Delta) \bar{\tau}_A \cdot \bar{\tau}_B N_\alpha^{-2},
\]

\[
V_{\pi-K} = \frac{8}{27} (2M_N - M_\Delta) \bar{\tau}_A \cdot \bar{\tau}_B N_\alpha^{-2},
\]

where \( \bar{\tau} \) are the isospin matrices.

Because the pion is quite light we start by computing the pion energy in an adiabatic \( K-N \) system. Our parameter set, tested in 2-body channels, is presented in Table I. The only favourable flavour combination is shown in Fig. 1 (a). Indeed we get quite a bound pion, but it only binds at very short \( K - N \) distances. However when we remove the adiabaticity, by allowing the \( K \) and \( N \) to move in the pion field, we find that the pion attraction overcomes the \( K - N \) repulsion but not yet the the \( K - N \) kinetic energy. Other effects may further increase attraction. We are planning to include full three-body Fadeev equations, the coupling to the \( K - N \) p-wave channel and the short-range two-pion-exchange-interaction.

5 \( SU(4) \) flavour: the \( \bar{K} - N - \bar{K} \) and anti-charmed systems

Extending the pentaquark and the molecular heptaquark picture to the full \( SU(3) \) anti-decuplet we arrive at the picture shown in Fig. 1 (b), where,

- The \( \Xi^- (1860) \) cannot be a \( ddss\bar{s} \) pentaquark because it would suffer from repulsion.
- Adding a \( q \) - \( \bar{q} \) pair we arrive at a \( I = 1/2 \) \( \bar{K} - N - \bar{K} \) linear molecule where the the \( \bar{K} - N \) system has isospin I=1, and it is an attractive system. We find that the \( \bar{K} - N - \bar{K} \) molecule is bound, although we are not yet able to arrive at a binding energy of \( \approx 60 \) MeV.
- Then the \( I = 1/2 \) elements of the exotic anti-decuplet are \( K - \bar{K} - N \) molecules.
- Only the I=1 elements are pentaquarks, or equivalently overlapping \( \bar{K} - N \) systems.

In what concerns anti-charmed pentaquarks like the very recently observed \( D^{*-}p \), or anti-bottomed ones, this extends the anti-decuplet to flavour \( SU(4) \) or \( SU(5) \). Anti-charmed pentaquarks were predicted by many authors, replacing the s by a c. Again the pentaquark \( uudd\bar{c} \) is unbound, and we are researching the possible \( D(D^*) - \pi - N \) molecular heptaquarks.
6 Conclusion

We conclude that the $\Theta^+(1540)$, $\Xi^{--}(1860)$ and $D^{*+}p(3100)$ hadrons very recently discovered cannot really be s-wave pentaquarks.

- We also find that they may be a heptaquark states, with two repelled $K$ and $N$ clusters bound third $\pi$ cluster.
- More effects need to be included, say exact Fadeev equations, the K-N p-wave coupled channel, and medium range interactions.
- This is a difficult subject with the interplay of many effects. The theoretical models should not just explain the pentaquarks, they should also comprehend other hadrons.

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