Identity, Haecceity, and the Godzilla Problem*

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Abstract

In standard first order predicate logic with identity it is usually taken that \( a = a \) is a theorem for any term \( a \). It is easily shown that this enables the apparent proof of a theorem stating the existence of any entity whatsoever. This embarrassing result is a motivation for the construction of free logics, but in most orthodox treatments of first order logic with identity it is generally dealt with by being ignored. We investigate the possibility that this problem can be obviated by dropping the rule that \( a = a \) is a theorem and requiring instead that it be treated as a global but in principle defeasible assumption about the objects in the domain of discourse. We review some motivations in physics, philosophy, and literature for questioning the classical notion of self-identity, and we show that Carnap’s “null object” has a natural role to play in any system of predicate logic where self-identity can come into question.

1 Introduction: An Embarrassing Problem

If one’s only knowledge of logic came from standard university texts, one might think that elementary first order predicate logic with identity has all been worked out a long time ago and that there are thus no serious technical or conceptual problems lurking within it. We’re not sure that this comfortable view is right.

We’re going to begin by pointing to what we believe is an embarrassing problem for standard first-order predicate logic with identity. The usual approach is to take the

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self-identity of all objects in the universe of discourse as a logical truth or theorem; that is, it is taken that

$$\vdash \forall x (x = x);$$ (1)

or as

$$\vdash a = a$$ (2)

in a scheme allowing generalization to (1). The symbol $\vdash$ in these formulas is not meant to suggest that they are provable from the other resources of first-order logic; but rather that they are assertions that may be added without proof to first order predicate logic to give a theory of identity. In Lemmon’s system [11] the rule (1) is called Identity Introduction and symbolized $=I$. It is paired with Identity Elimination ($=E$), which says that if $a = b$ then $a$ may be substituted for $b$ or *vice versa* wherever they occur. Identity Elimination is the motor that drives virtually all of all applications of identity. Identity Intro, however, is rarely used, and as the following natural deduction proof shows, it allows consequences with which all logicians ought to be uncomfortable:

| Step | Formula          | Rule |
|------|------------------|------|
| 1    | $\forall x (x = x)$ | =I   |
| 2    | Godzilla = Godzilla | UE   |
| 3    | $\exists x (x = \text{Godzilla})$ | EI   |

So we have

$$\vdash \exists x (x = \text{Godzilla}).$$ (3)

We’ll call this argument pattern the *Categorical Godzilla*. (There is also a *Conditional Godzilla* which we will later introduce.) If $\forall x (x = x)$ is a logical truth, then we apparently have it as a *logical truth* that a certain city-trampling movie monster really does exist. This pattern could be repeated for any name whatsoever. *What is wrong with this picture?*

Of course, this problem is well known (or should be) and it is one of the motivations for the construction of various types of free logic, the defining characteristic of which is that names are not automatically assumed to refer [9, 2, 13]. We are sympathetic to free logic and we think that our observations here tend to give further motivation for its development. However, our primary aim in this note is the more limited goal of arguing against the theoremhood of $=I$. In doing so, we will show that there is reason to question $=I$ even as a universal assumption for at least some of the domains to which predicate logic might be applied. We will also show that a consequence of treating $=I$ as a global but defeasible assumption is, surprisingly perhaps, the resurrection of Carnap’s *null thing* [4], or something very much like it.
2 A Short History of Arguments for $=I$

Many currently used logic texts get around the awkward conclusion that $=I$ can be used to prove that anything whatsoever exists simply by not mentioning it at all. E. J. Lemmon’s widely used *Beginning Logic* [11] relegates it without comment to an exercise, and defends the theoremhood of $=I$ as follows:

For any term $t$, the rule $=I$ permits us to introduce into a proof at any stage $t = t$, resting on no assumptions. The idea should be clear: anything is itself, as a matter of logic; hence $t = t$ is logically true, and so can appear without assumptions. [11, p. 161]

It is by no means clear that everything being itself is a matter of logic. By contrast, the theorem $P \rightarrow P$ is a matter of logic, and indeed it is sometimes (confusingly) called Identity; it is almost as if Lemmon confused $=$ with the truth-functional connective $\rightarrow$, or perhaps $\equiv$.

A recent logic text by Paul Herrick trades on intuitions similar to Lemmon’s:

Each thing is identical with itself . . . Surely this needs no argument; certainly it is necessarily true. (How could something possibly not be identical to itself?) [7, p. 587]

One should be suspicious of arguments for $p$ of the form, “Surely $p$ . . .”. For surely (if we may) Lemmon and Herrick are appealing not to logical intuitions about identity, but *metaphysical* intuitions about identity. They are saying that it is a matter of *necessary fact* that every item is self-identical, and they have forgotten that pure logic as such expresses no facts. Facts, whether necessary or of the ordinary garden-variety, can be introduced into a logical problem only *by assumption*. The views of Lemmon and Herrick therefore seem to be part of a long tradition of mistaking presumed factual or metaphysical necessity for logical or mathematical necessity.

In *Principia Mathematica* Russell and Whitehead [17, *13] gave an apparently much more principled and precise defence of $=I$. They begin by defining $=$ by means of the Identity of Indiscernibles: $x = y$ means that if $\phi$ is a property of $x$ then $\phi$ is a property of $y$. (In this sketch of their exposition we gloss over niceties having to do with the Theory of Types.) The definition gets turned into a theorem by an application of universal instantiation: since it is true of any two arbitrarily selected entities that they are identical if and only if they share all properties, then that holds for all instances of $x = y$. Then as a special case of this result any arbitrarily selected entity is self-identical simply because any property of itself is a property of itself.
The Russell-Whitehead approach has the virtue of precision. However, modern authors tend to shy away from it because it requires second order logic. More important from the skeptical point of view we pursue here, the reliance on the Identity of Indiscernibles again amounts to building a metaphysical principle into formal logic. It could even be said (though no doubt contentiously) that Russell and Whitehead’s view, that all terms are self-identical because any arbitrary term would have the same properties as itself, borders on question-begging. Russell and Whitehead do not state the Categorical Godzilla, though it would be readily available in their system.

Reliance on the Identity of Indiscernibles in order to define identity and justify theorems about it can be traced back through Frege [5] and Leibniz (discussed in Kneale [8]) to Aristotle. The latter seems to be the first to have introduced the concept of the Identity of Indiscernibles, though informally, in *De Sophisticis Elenchis* [1, Ch. 24 (179a37)]: “For only to things that are indistinguishable and one in essence is it generally agreed that all the same attributes belong.” His wording “it is generally agreed” suggests that this principle has a longer history, either orally or in writings now lost. Being “one in essence” is a metaphysical requirement for self-identity; Aristotle’s discussion surrounding the line quoted here explains why identity may otherwise be ambiguous if this high metaphysical standard is not met.

The reliance upon the Identity of Indiscernibles to get =I is therefore very old. Now =I, for our purposes, can be treated as either substantive or simply stipulative. If the latter, then it’s the kind of thing that we may choose to do without. If the former, then its place in the reasoning designed to justify logic is dubious at best. Logic should not be a substantive inquiry, but rather a formal one (this is how the field has been moving, and we think, for the better). What we should be interested in are extremely generalized relations between assertions about objects burdened with as few assumptions as possible. So how can we amend standard first order logic with identity in as conservative a way as possible, but so as to block the Categorical Godzilla?

3 Blocking the Godzilla Inference

There is no way to block the Godzilla proof by placing some sort of artificial restriction on EI without crippling or drastically reconstruing EI, which is not in line with our conservative approach. And the application of UE to line (1) seems to be entirely unobjectionable. One could simply introduce an *ad hoc* rule against making inferences of the Godzilla type; for instance, one might introduce a rule against applying EI to any formula of the form \(a = a\). This probably would be logically possible but it seems inelegant.
To begin with, we want to change as little as possible of classical first order logic, and only make such changes to identity theory as would be sufficient to block Godzilla in a natural way. (Further along we’ll suggest something more radical.) It is therefore much more pertinent to examine whether it really is reasonable to treat $\forall x(x = x)$ as a theorem.

The first point to note is that the Godzilla proof is simply an illustration of the rule that there is an existential claim built into any proposition of the form $Fa$. Suppose we assume or are given $Fa$, where $F$ is some property and $a$ is an individual. From this we conclude $\exists xFx$ by an immediate application of EI. So to assert that an individual does in fact possess a property is to imply the existence of an individual possessing that property, and this is true for any property $F$, including self-identity.

This suggests that another way of blocking the Categorical Godzilla might be to question whether self-identity really can be treated as if it were monadic property. If Godzilla is a city-trampler then something is a city-trampler; if Godzilla is self-identical then can we say that something is self-identical? It would be very odd if we could not, since self-identity certainly is something that pertains to individual entities when it pertains at all; hence, that approach does not seem promising either.

The key is that we do not want to build any assumptions about the existence of any entities whatsoever into our logic. Therefore, we can avoid the Categorical Godzilla if (i) we remind ourselves that $\forall xFx$ and $Fa$ can be introduced in a proof only as assumptions and (ii) insist that this rule be followed even when $F$ is self-identity—and even if we find it hard to imagine that any given entity could not be self-identical. On some metaphysical views it might indeed be the case that everything is self-identical, but this can’t be a matter of logic even if it is a necessary truth, whatever that might be. There are certainly some universes of discourse that contain only self-identical objects (such as the universe comprised of the set of natural numbers), but our choice of a universe of discourse is not a matter of logic either.

A possible defence of the orthodox view could be along the following lines: it could be said that in doing first-order logic we always take it for granted that the universe of discourse $U$ consists of objects that are already presumed to exist. We see two objections to this view.

First, even if we want to say that we take it from the outset that all items in $U$ exist, we do not mean to say that it turns out to be a theorem that some item in $U$, which might be, say, the city of Paris, France, happens to exist; rather, the existence of any item in the actual world is an empirical matter if we are talking about real-world entities (unless we could view the world from the perspective of the God of Leibniz for whom all apparently empirical truths are analytic), or a mathematical matter if we are talking about
mathematical entities such as sets or numbers.

Second, logic would not be very useful if we only allowed ourselves to talk about things that we already know or believe to exist. One of the most powerful tools of thought is the ability to consider hypothetical objects without existential commitment. This is just what we do in indirect proof in mathematics. Consider Euclid’s proof that there is no greatest prime: it begins by supposing hypothetically that there is a greatest prime, it gives this hypothetical number a name for convenience in calculation, and then shows that such a number must have contradictory properties. The universe of discourse should be precisely that—the collection of entities and subjects that we want to talk about and investigate, without necessarily having made a prior commitment to their existence. We only get existence out of a deduction if we have good reason to put it in as a premise. Of course, we may wish to do logic over specific sets of entities, such as the natural numbers or the items of furniture in someone’s office, which are already taken to exist and to have various definite properties. But logic should preserve a general freedom to talk of the hypothetical or the fictional, and to deny the existence of entities if the facts demand that we do so.

The description of a hypothetical or fictional object may or may not include or imply self-identity. Logic does not rule out either of these possibilities. For instance, the notion of the largest prime implies self-identity by the Peano axioms, even though there is no such number. However, it is not at all clear whether the script writers who defined Godzilla intended to imply that the creature should be self-identical; that doesn’t seem to have been relevant. We are entirely at liberty to define anything with any putative properties whatsoever, be it Quine’s round square cupola or a non-self identical movie monster. As Gaunilo realized a long time ago, no definition by itself can have any bearing on what exists. A definition can neither make something exist in the real world (as supposed by the confident authors of various ontological proofs) nor can it, by itself, prevent something from existing (even if the description of the putative entity is contradictory). In the end, as Hume indicated, the test of existence outside the lands of mathematics is always empirical.

We suggest that we can avoid the Godzilla inference, as least so far as first order logic is concerned, by taking the following two steps:

- We entirely drop the idea that (1) is a theorem that applies to any arbitrarily selected class of entities. Instead, any suggestion that some entity or set of entities are self-identical must be treated as an assumption and introduced to a natural deduction proof accordingly, unless it is specified in advance that one is quantifying over sets of entities (such as the natural numbers) which are already known to be self-identical. In such a case, a line in a deduction that says (say) \(3 = 3\) is justified by reference to number theory, not to =I.
Within the object language of first order logic we treat identity in a purely syntactic manner: if \( a = b \) all that this means is that \( a \) can be freely substituted for \( b \) or vice versa at any point in a proof.

The key new idea is that we do not take it as a theorem that \( \forall x (x = x) \). Rather, we take it that identity has to be either postulated or established from other postulates for any entity or class of entities.

The purely syntactic reading of \( a = b \) reflects the fact that inter-substitution is all that matters from the syntactic point of view. Most important, we can define identity this way without having to worry about deep metaphysical questions about what it means for entities to be identical. This is hardly to suggest that one should not investigate the nature of identity or the various sorts of identity that might be tenable (for indeed, our minimal syntax could be compatible with several interpretations of identity); it is simply to insist that views about the factual or metaphysical nature of identity should not be surreptitiously built into first-order logic. So self-identity cannot be taken as a given for all possible objects. Instead, a self-identity claim is to be introduced into a proof if needed by an assumption (which could be either universally quantified or of the form \( a = a \) for some particular term \( a \)) or by specifying in advance that one is quantifying over a class of objects (such as natural numbers) that are already known to be self-identical.

If we want a system that works the same way as standard first-order predicate logic with identity, so that we can do all of the usual definite description problems and employ the other non-problematic applications of identity, we can take it as not a logical truth but a global assumption (which need not be stated explicitly in each deduction) that we reason over domains of objects that are presumed to be self-identical. The logical status of \( =I \) then becomes something like the logical status of the parallel postulate in geometry. The parallel postulate was once presumed to be either a priori or deducible from the other rules of Euclidean geometry, but by the early 19th century it was evident that it was a logically independent assumption about the kinds of spaces that one was dealing with. (From the Riemannian viewpoint it applies to spaces with zero intrinsic curvature.) Logics in which self-identity fails or could fail for some or all non-null entities in the domain of discourse can be called non-Aristotelian, by analogy with non-Euclidean geometry. A logic that differs from classical (Aristotelian) predicate logic with identity only in that \( =I \) is taken to be a global assumption rather than a logical truth we will call an open classical logic—open because it is open to the possibility that the self-identity assumption might fail for one or more members of the domain. We will show below that even open classical logics must be non-Aristotelian in one particular respect, but there could well be many possible non-Aristotelian logics, just as there are many possible non-Euclidean geometries. In this
respect, logic still seeks its Riemannian synthesis.

4 Haecceities, or the Lack Thereof

We now want to take a different tack and examine some other motivations for considering non-Aristotelian logics, apart from a desire to avoid the Godzilla problem. We began by noting that it is not good to build metaphysics into one’s logic any more than one should build facts of geography into trigonometry. On the other hand, it is also desirable to have logics that are adequate to the uses to which people frequently put language, and to the kind of natural world we seem to find ourselves in. We’ll point out that non-Aristotelian logics of some sort (not necessarily classical) could well have applications in the logic of fiction, in philosophy, and in physics itself.

4.1 In Fiction

We will not venture deeply into the logic and metaphysics of fiction, except to note that it is not at all clear that simplistic self-identity is deemed to hold for all entities treated in fiction, the movies, and literature. Walt Whitman, in his poem ‘Song of Myself’ [21, p. 96], famously stated, “I contradict myself... I contain multitudes.” And what of a character caught in a self-contradictory time loop in a bad science fiction story who succeeds in committing suicide by shooting his grandfather? At some points along his worldline he exists if and only if he does not exist. Only fiction, of course; but the point is simply that it is open to authors even to question the self-identity of their characters.

4.2 In Philosophy

Within the history of philosophy (and overlapping importantly into physics) there has been and continues to be a debate between two camps who have very different views about the metaphysics of time and change. The Parmenideans see the world as static, the Heracliteans see the world as inherently dynamic. Plato (to whose work all of Western philosophy consists of footnotes, according to Whitehead) proposed a synthesis of Parmenidean and Heraclitean views: he distinguished between the world of Becoming (the unstable physical or natural world) and the world of Being (the world of stable ideal objects grasped by the intellect). Plato stated that everything in the natural world, not only obviously changeable things such as fire and water but even more apparently permanent solid matter, was in a process of perpetual flux:

Whenever we see anything in process of change, for example fire, we should
speak of it not as being a thing but as having a quality... And in general we should never speak as if any of the things we suppose we can indicate by pointing and using the expressions ‘this thing’ or ‘that thing’ have any permanent reality: for they have no stability and elude the designation ‘this’ or ‘that’ or any other that suggests permanence. [15, p. 68]

Nothing in the world of Becoming is ever exactly a such-and-such and thus one can never hope to fully grasp what it is. Plato’s words are open to interpretation, of course, but his view seems to suggest that objects in the natural world do not have sharp self-identity because they are always in the process of becoming something else.

Nietzsche wrote in a similar vein:

Logic too depends on presuppositions with which nothing in the real world corresponds, for example on the presupposition that there are identical things, that the same thing is identical at different points of time... [12, §11].

On this view, even the claim that I am now sitting in the same chair that I sat in earlier this evening is (from the logical point of view) pure stipulation. The later chair is like enough to the chair I sat on earlier for all practical purposes, so I might as well call it the same chair—but this is purely a stipulation justifiable only by its practical utility.

4.3 In Physics

We are certainly not suggesting that something is so just because Plato or Nietzsche said it. But such philosophical views, although imprecise and open to interpretation, are not merely quaint relics of pre-scientific or 19th century romantic thought; current professional debates on the reality of time and change turn on the same Heraclitean/Parmenidean point of dispute.

The static, plenum, or block-universe view is probably the sentimental favourite of many recent physicists and philosophers of science. (Kurt Gödel was a notable block universe theorist [22].) However, because of the Indeterminacy Relations and the fundamental non-Booleanity of quantum mechanics (for an explanation of which see Bub [3]), it is not clear that the static view is consistent with quantum mechanics [14]. The distinguished theorist Lee Smolin, trying to understand the basis of the conceptual roadblocks which he insists dog modern theoretical physics, remarks,

I believe there is something basic we are all missing, some wrong assumption we are all making... I strongly suspect that the key is time. More and more, I have the feeling that quantum theory and general relativity are both deeply
wrong about the nature of time... We have to find a way to **unfreeze** time—to represent time without turning it into space. [19, p. 257–7]

There is still no generally agreed upon method of unfreezing time, but the questions of how to represent time, and whether or not time is real or merely a funny sort of spatial dimension, are central themes in current work in quantum gravity (the attempt to find a quantum theory of spacetime structure). If the logic of identity is to be of any use in talking about identity of objects in time and space, it needs to be flexible enough to accommodate our rapidly evolving picture of identity in the physical world. Conceivably, some sort of non-Aristotelian logic as we conceive of it here could be a useful tool in Smolin’s project to unfreeze physics.

Paul Teller [20] has made some very relevant observations about the way that haecceity—the suchness or thisness of an entity, that which presumably founds its identity—is affected by quantum mechanics. His explanation of haecceity is very helpful:

Traditionally, philosophy has talked about an object’s “haecceity” to mark the idea that an object is distinct from all others in some manner that transcends all properties in any usual sense of the word ‘property.’ ... let us take for granted some things that presuppose the applicability of strict identity: that names can refer “directly,” that is without operating as definite descriptions; **that repeated use of the same name picks out the same referent** [our emphasis]; that repeated use of the same variable bound by the same quantifier picks out the same referent; and that sets are defined extensionally... Now, a metaphysician might ask: in virtue of what does strict identity apply to an object? Haecceities... are supposed to be some metaphysical feature, principle, characteristic, or “non-qualitative property” which answers this question. [20, p. 117]

As Teller goes on to explain, in quantum statistics particles do not have identities that can be tracked. To adapt Teller’s example, if Bloggs has $1000 in his bank account, it does not make sense to ask, **which** monetary tokens (such as pennies) does he have $1000 worth of? All that matters is that he is good for $1000. Similarly, if there are six photons in a box, all this means is that we are good for six photons; it does not make sense to ask, **which six photons are in the box?** As Teller explains, it has been found that if one assumes that the photons have distinct, trackable identities the way pennies do, one will count them wrong (because quantum particles are permutation-invariant, unlike classical objects) and get the wrong statistical predictions. It is therefore highly questionable that quantum mechanics allows for the notion of haecceity (and thereby self-identity) in anything like the classical sense. (See [6] for a detailed exploration of this problem.)
There is another respect in which quantum mechanics suggests something like the Heraclitean view. Any assertion in quantum physics has to be operationally grounded; by what measurement procedure could we know that a particle is identical to itself? Well, we might have to interact with it twice, and quantum mechanics tells us that there is no clear meaning to saying that if we measure (say) an electron at a certain spot, and then a tiny fraction of a second later measure another electron at nearly the same spot, that we have detected the same electron that we detected in the first measurement. To adapt a famous phrase from Heraclitus, we do and do not observe the same electron.

At around this point in the discussion, classically-minded thinkers are sometimes moved to exclaim, “But dammit, everything just is identical to itself!” This is an example of what can be called the *table-pounding argument* because such statements are often accompanied by pounding on a convenient mid-sized object. Unfortunately for the classical realist, quantum mechanics remains unmoved by any amount of furniture-thumping. Modern physics certainly suggests, and arguably demands, that we live in an extreme Heraclitean world of flux where self-identity cannot be asserted, or at least cannot be asserted in a classical way—except as a convenient approximation at scales where quantum effects can be ignored.

One does not necessarily employ first order predicate logic to reason about quantum mechanics, but in order for logic to be as useful as possible, in the kind of physical world we live in, it should be equipped to express facts of quantum mechanics as required and should therefore not have in-built assumptions that would conflict with quantum mechanics. In the spirit of Putnam’s recommendations [16], it is desirable to seek a way of doing classical logic that would naturally generalize to quantum logic.

To conclude this section, we quote a favourite story from Bertrand Russell:

> It is obvious that, if you think of all the things that are in the world, they cannot be divided into two classes—namely, those that exist, and those that do not. Non-existence is, in fact, a very rare property. Everybody knows the story of the two Germanic pessimistic philosophers, one of whom exclaimed: ‘How much happier were it never to have been born.’ To which the other replied with a sigh: ‘True! But how few are those who achieve this happy lot.’ [18, p. 147]

We have a great deal of respect for Russell, but it is by no means obvious that existence is such a clear-cut concept in a quantum universe. Perhaps if Teller is right then non-existence is not such a difficult property to attain or at least to approximate after all, at least within the limits allowed by the Uncertainty Principle.

To summarize: self-identity holds for some idealized objects, such as the natural
numbers, and it is approximately enough true to not be misleading for many mid-sized physical objects such as tables and chairs. Physics tends to suggest that it could well be simply dead wrong at scales where quantum mechanics is important, although this remains an important open question. But again, if predicate logic is to be as widely applicable as possible to reasoning about things in the nature world, not to mention fictional objects, it must not be burdened with the presumption that everything that can be quantified over is necessarily self-identical.

5 The Null Object

Now we will show how non-Aristotelian even a nearly-classical open logic has to be if it is to have enough expressive power to be useful in mathematics and daily reasoning.

Let us consider the following variation on the Categorical Godzilla proof, which we dub the Conditional Godzilla:

\[ \forall x (x = x) \]

1 (1) \( \forall x (x = x) \) A

1 (2) Godzilla = Godzilla 1 UE

1 (3) \( \exists x (x = \text{Godzilla}) \) 2 EI

Thus by conditional proof we have

\[ \vdash \forall x (x = x) \rightarrow \exists x (x = \text{Godzilla}). \]  

(4)

Here we have assumed \( \forall x (x = x) \) rather than taken it as a theorem. We’re on firmer ground in that respect. But we still end up with a peculiar result: assuming the self-identity of all objects in the universe of discourse apparently also allows us to prove the existence of anything in that universe, only this time not as an ersatz theorem but as a consequence of the assumption on line (1). So if we give all entities in \( U \) the benefit of the doubt and grant them self-identity, we are still committed to their existence—but now the dependence of the existence result upon assumption is obvious.

In part this proof is simply an illustration of the point noted earlier, that in first order predicate logic, to posit any property (including self-identity) of an entity is to imply that the entity exists. So if we give all entities in \( U \) the benefit of the doubt and grant them self-identity—not as a presumed logical or metaphysical truth but simply for the sake of argument—that still implies that they exist. On the other hand, non-existent things are non-self-identical, simply because no properties of any sort can in fact be predicated of them at all (regardless of how they were defined). But what happens if the facts of a matter demand that we deny the existence of something that we had provisionally admitted to \( U \)?

Continue the above proof as follows:
∀x(x = x) → ∃x(x = Godzilla) 1–3 CP

5 (5) −∃x(x = Godzilla) A (An empirical given.)

5 (6) −∀x(x = x) 4,5 MT

5 (7) ∃x(x ≠ x) 6 Duality

By asserting on line (5) the empirical fact that a certain described entity does not exist, we seem to be forced into a bizarre existence claim anyway! One approach could be to simply not make assertions like (5) on the grounds that in classical predicate logic we hold or pretend that all names refer. But predicate logic would be greatly restricted in its usefulness if it could not assert the non-existence of a putative entity known only by a name or description. That is something that we are entirely free to do in ordinary language, as well as scientific and mathematical reasoning. As we have already noted, one of the most useful tools of reasoning is the ability to discuss something hypothetically, be it the largest prime or the gunman on the grassy knoll. What we need is a natural interpretation of the odd thing whose existence is cited in line (7). We suggest that it may be useful to think of this object, or “object”, as a null entity. In a logical system it acts in a way analogous to the ground in an electrical circuit; it is the elephant graveyard for all names and descriptions which fail to refer.

One can see a foreshadowing of this approach in Carnap’s interpretation of Frege’s solution to the problem of improper definite descriptions [4, 35–9]. As Carnap explains, Frege was concerned to construct his ideal logical language so that a definite description picked out a unique object. There is an obstacle in the cases of improper definite descriptions, terms which have the form of a description but which name nothing or many things. Carnap observes that a possible response is “to count among the things also the null thing, which corresponds to the null class of space-time points” [4, 36]. It is beyond the aim of this paper to go into more detail regarding the problem of improper definite descriptions, but it is well worth noting that there is a precedent in the literature for this kind of solution to problems not too dissimilar to those in which we are most interested here.

For Carnap, a null object $a_0$ is simply a name that is left free—it does not denote anything. Since names can denote anything we want, it is open to us to simply leave one name unassigned in the course of a piece of reasoning. Classically (i.e., in a logic with a sharp concept of identity), a null object can be naturally defined as follows:

$$a_0 := x(x ≠ x).$$  

("Let $a_0$ be an $x$ such that $x ≠ x$.")

It is not essential that $a_0$ be defined this way: that is, as whatever is non-self-identical. The notion of a null object could easily survive a

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1 Our notation here is non-standard and requires some explanation. It would be more common to write
loosening or broadening of the concept of self-identity. However, if we accept the classical notion of the null object, then lines (4)–(7) above do not in fact demonstrate the existence of anything at all—precisely because they demonstrate the existence of the null object, which is not anything at all. So we have not stumbled into an existence claim simply because we tried to deny the existence of something.

The move to free logic provides another natural motivation for considering the null object. Again, a free logic is defined as a system of predicate logic which allows for the possibility of empty domains of discourse and predicates that do not refer. Consider $Fa \lor \neg Fa$. This is true even in $\emptyset$ (the null set, an empty universe) because if a term $a$ does not refer then $\neg Fa$ holds for any predicate $F$. (If the present King of France doesn’t exist then it is true that he is not bald.) However, the fact that $Fa \lor \neg Fa$ is true in an empty universe seems to license a dubious inference:

$$(1) \quad Fa \lor \neg Fa \quad \text{Theorem}$$

$$(2) \quad \exists x (Fx \lor \neg Fx) \quad 1 \text{ EI (???) }$$

That is, we seem to have once again infered a theorem asserting the existence of an entity—this time, an apparent element of $\emptyset$!

One way to deal with this is to not talk about empty universes; and this is what is usually done in elementary predicate logic texts, where the puzzle of the empty universe is either not mentioned or glossed over. Another way to block this inference is to not allow EI for empty universes, but this requires that one know in advance that a universe is empty, and we are supposed to be able to do logic without any existential presumptions at all. Here, again, a null object can help us. Classically, we have $\emptyset = \{ x \mid x \neq x \}$. Then $a_0 \in \emptyset$. (Indeed, it’s the only element of $\emptyset$.) Then line (2) above can only apply to $a_0$ (in $\emptyset$), and we have (in $\emptyset$) $Fa_0 \lor \neg Fa_0$. Now, $Fa_0$ is false for any predicate $F$; by bivalence $\neg Fa_0$ and therefore $Fa_0 \lor \neg Fa_0$ are true in $\emptyset$. So the above deduction is valid in $\emptyset$.

Please note: this bit of reasoning does not imply that there actually is anything at all in $\emptyset$!

Null objects thus allow us to extend the validity of certain puzzling arguments to empty universes in a natural way, and allows us to express the non-existence of named or described entities when the facts demand that we do so—so long as we decide that we can live with a definition such as this using Hilbert’s $\varepsilon$-symbol, as $a_0 := \varepsilon x (x \neq x)$. However, the usual reading of $\varepsilon x A$ commits us to more than we want when discussing null objects: Leisenring [10, p. 1] says, “Intuitively, the $\varepsilon$-term $\varepsilon x A$ says ‘an $x$ such that if anything has the property $A$, then $x$ has that property’.” Our simpler notation is inspired by the reading of $\exists x Fx$ as “There exists an $F$.” The symbol $\exists$ is “there exists,” and $xFx$ is “an $F$”. Any expression of the latter form can be called an indefinite denotator. Precisely because it is so thoroughly indefinite, it could (depending on the facts of the matter) denote nothing.
one more very odd sort of mathematical creature under the floorboards of everyday reasoning. Just as set theory would be hobbled without the formal device $\emptyset$, and arithmetic could not operate without 0, it could well be that predicate logic has been hobbled all along without a formal, placeholder referent for names and descriptions that do not refer.

One further point: is the null object self-identical? Clearly not, by its very definition, Eq. (5). So even if we want to keep our predicate logic as close to classical as possible (by adhering to $=I$ as a global assumption but making no other changes in our deductive methods), if we also want to be able to assert that some names or descriptions fail to refer there must be at least one non-self-identical (and therefore null) object in every domain of reference. To this extent, then, even an open classical logic is non-Aristotelian.

We were tempted to speak of the null object, but Gillman Payette (private communication) was quick to point out to us that even this would be saying too much about it. Suppose that we tried to define $a_0$ as follows:

$$a_0 := \iota x (x \neq x).$$

(“Let $a_0$ be the $x$ such that $x \neq x$.”) To speak of anything as the $F$ is to allow that it can be equal to something bearing a proper name. Suppose that $a_0 = b$. So long as we are allowed $=E$ (and we could not do much useful reasoning about identity without it), we can substitute $a_0$ for $b$ and get that $a_0$ is self-identical—precisely the thing we don’t want. So $a_0$ cannot even have the property of uniqueness. In this respect the analogy between null objects, and the null set and 0, breaks down because the latter entities can stand in identity relations. So the null object or objects must remain utterly indefinite; our $a_0$ is just a placeholder for the absence of all properties, demanded by the syntax of predicate logic.

There is one further intriguing observation to be made about null objects. The definition (5) is a very natural way to specify a null object in Aristotelian logic. But suppose we want to consider non-Aristotelian logics where classical identity is not always available. We would need a more general conception of null objects. If we are allowed to quantify over predicates, then we could define

$$a_0 := x(\forall F(-Fx)).$$

This has the advantage that it could apply to logics without classical identity. But one encounters a challenge that should by now be very familiar to logicians. Let us say that an object is prediphobic if it will admit of no predicates whatsoever:

$$P x := x \text{ is prediphobic}$$

$$:= \forall F(-Fx)$$

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Then clearly \( P \alpha \rightarrow \neg P \alpha \), and given \( P \alpha \) as well, we have detonation. This is a typical instance of the hazards of second-order logic. It also suggests that any logic in which classical identity fails, but in which a more general null object of the form (7) is desired, would have to be paraconsistent in some sense.

6 Summing Up

The notion that \( =I \) is a theorem or logical truth leads to the unacceptable result that the existence of any named or described entity can be proven as a matter of logic. The most natural way out of this embarrassment is to think of \( =I \) as an assumption, not a “logical truth”. Indeed, there are ample reasons within fiction, philosophy, and physics why we might want to speak of entities whose self-identity is in doubt, and we should have logics that are open to this possibility. We suggest that a logic in which \( =I \) holds for all non-null entities in its domain be called Aristotelian; otherwise, non-Aristotelian. If \( =I \) is taken to hold as a logical truth, we’ll call that a classical Aristotelian logic. (We expect that such logics will sooner or later become historical curiosities.) If \( =I \) is taken to hold merely as an assumption, but still an assumption applying to all objects in the domain of discourse, we’ll say that such a logic is an open Aristotelian logic. We conjecture—though this remains to be shown in full rigour—that an open Aristotelian logic can do all the deductive work that classical Aristotelian logic can do, without falling into the absurdity of the Categorical Godzilla. Beyond this, an important research project is to explore possible non-Aristotelian logics.

Our attempt to de-ontologize logic (by removing \( =I \) as a theorem) ironically forces us to include null objects in any domain of reference, even when the logic is Aristotelian. But this is not an expansion of our ontology (except for a modest addition to our collection of symbols) because the null object is not any thing, even though it may turn out to be just as indispensable as \( \emptyset \) and 0.

One result seems clear: we can no more take it to be a logical truth that everything is self-identical than we can take it to be a logical truth that everything is green. If we do impute self-identity to all non-null members of a domain of discourse, it is only by courtesy or because it is a domain (such as \( \mathbb{N} \) or the furniture in someone’s office) for which we have good reason to think that self-identity holds throughout. And any logic that hopes to be adequate to the ordinary demands of discourse in the real world must always allow for the possibility that self-identity fails for some entities of interest.
APPENDIX: Some Properties of an Open Classical Logic

We have a lot of work to do in order to clarify the properties of open classical logic, let alone explore other non-Aristotelian logics that might be feasible and link them with free logics. Here we list without proof some immediate consequences and properties of an open classical logic.

It is simply a result of classical first order logic that we have \(a = a \rightarrow \exists x(x = a)\). For a natural deduction system, this is very easily provable by the rules of EI and conditional introduction, with \(a = a\) as an assumption. If the system is stated in a normal axiomatic presentation, we have as an axiom that \(B \rightarrow \exists x(B)\), where some instances of a particular name \(a\) occurring in \(B\) can be replaced by the variable \(x\), bound by the existential quantifier. The identity \(a = a\) is such a formula, and so \(a = a \rightarrow \exists x(x = a)\) is just an instance of this axiom. A result of this with a classical theory of identity is that \(\exists x(x = b)\) is provable for any name \(b\), whereas in our open logic, \(\exists x(x = b)\) follows only in cases where we explicitly assume \(b = b\).

We retain the provability of sequents of the form \(a = b, Pa \vdash Pb\). In the natural deduction setting, this is enforced by a rule of identity elimination. More generally, it is a form of Leibniz’s law. For axiomatic and sequent calculus purposes, we can simply include this sequent as a primitive rule, as is common in the proof theory literature.

In a sequent system our proposal amounts to rejecting \(\vdash a = a\) as an axiom. However, given the inclusion of the identity elimination rule, all one need to do to reason more or less usually with identity is to include a premise of the form \(a = a\); that is to include \(a = a\) on the left-side of the \(\vdash\). Including this extra premise is always admissible, by the rule of thinning, and so, just as in the natural deduction case, we demand only that one make one’s extra-logical assumptions about identity explicit.

A result of this rule is that \(a = b \vdash a = a\) is provable. So, under the assumption that \(a\) is equal to anything, we have that \(a = a\), and if \(a \neq a\), then \(a\) is identical to nothing. With identity elimination, we can also easily prove that \(a = b, b = c \vdash a = c\), where \(b = c\) is the formula in which \(a\) is substituted for \(b\) to attain the conclusion. Hence, we can clearly prove that \((a = b \land b = c) \rightarrow a = c\). Similarly, we can show that \(a = b \equiv b = a\). So, as a result, = enjoys symmetry and transitivity in all cases, and reflexivity in those cases where it applies at all. While this ‘conditional’ reflexivity is strictly weaker than reflexivity, it guarantees that in contexts where we have assumed self-identity to hold of the names we reason with, identity behaves classically. The differences are, of course, with those names of which we make no such assumption.

Of course, since we reject that \(a = a\) is a theorem, we do not have that the formula
$a \neq a$ allows for the proof of any formula whatsoever. In general, the assumption $a \neq a$ will only generate triviality when we explicitly assume some other formula which implies $a = a$, because we retain the rule of explosion (ex falso quodlibet).

We also retain the law of excluded middle, because the propositional fragment of our logic is purely classical. So, we have that $\vdash a = a \lor a \neq a$; however, as we do not have that $\vdash a = a$ or $\vdash a \neq a$, the logic is not prime. This is just to say that it is not universally true that $\vdash A \lor B$ holds only when either $\vdash A$ or $\vdash B$ holds. This leaves a potentially interesting avenue to intuitionist open logic available. It strays beyond our aims to investigate such a logic, but we note here that such an approach may have interesting consequences for common subjects which motivate intuitionistic logic, for instance mathematics\(^2\) and other areas where epistemic restrictions on our knowledge are salient.

No proposal to amend an established logic can be taken seriously until the metatheory of the amended logic is worked out. We have yet to do this. It seems very likely that dropping a rule of inference leaves us on safe ground with regard to soundness. In particular, dropping $=I$ as a theorem makes no difference at all to what can actually be deduced with first order predicate logic with identity except that we lose certain inferences that (for reasons we have explained) we would like to lose anyway.

Completeness is a more difficult question: to show that open classical first order logic with identity is complete we would have to show that any formula we can no longer prove (by having removed $=I$ as a rule of deduction) is false in some models that leave true all the theorems that did not require $=I$. To put it another way, completeness is all about whether one can prove all of the tautologies in a system with the resources of the system. The Categorical Godzilla shows us that if the standard approach is complete and sound, $\exists x(x = a)$ must be a tautology for any term $a$ in every possible model; and as we have noted this could make sense only if it is somehow known that every term in the language refers. In an open classical logic $\exists x(x = a)$ is most certainly not a tautology in general, but rather a statement that depends upon the facts of a case. So again, we think that dropping $=I$ as a rule of inference will only prevent one from being able to prove formulas that have no business being tautologies anyway. But this question requires a more thorough study.

If nothing else, this appendix makes it clear that this work is a very first step into an interesting new territory, about which almost everything is as yet unknown. Our mere hope at this stage is that the reader is intrigued by our proposal, and convinced enough by our

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\(^2\)Consider the fact that from ZFC alone we can prove neither $\aleph_1 = 2^{\aleph_0}$ nor $\aleph_1 \neq 2^{\aleph_0}$. Of course, giving a set theoretic analysis of $=$ which matches our assumptions about this predicate goes beyond our purposes, but it may be a valuable way forward. This is in contrast to the well-understood notion of bijection underwriting $=$ in ZFC, and other common set theories.
philosophical argumentation to think that it may be worth developing in more detail.

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References

[1] Aristotle. *De Sophisticis Elenchis*. Online at
http://etext.lib.virginia.edu/toc/modeng/public/AriSoph.html; retrieved June, 2012. 4

[2] Bell, John L., D. DeVidi, and G. Solomon, *Logical Options: An Introduction to Classical and Alternative Logics*. Broadview, 2001. 2

[3] Bub, Jeffrey. *Interpreting the Quantum World*. Cambridge: Cambridge University Press, 1997. 9

[4] Carnap, Rudolf *Meaning and Necessity: A Study in Semantics and Modal Logic*. Chicago: University of Chicago Press, 1947. 2, 13

[5] Frege. G. ‘Begriffshift, a formula language, modeled upon that of arithmetic, for pure thought.’ First publication 1879. In J. van Heijenoort (ed.), *From Frege to Gödel: A Source Book in Mathematical Logic, 1879–1931*. Cambridge, MA: Harvard University Press, 1967, pp. 1–82. 4

[6] French, Steven, and Krause, Décio. *Identity in Physics: A Historical, Philosophical, and Formal Analysis*. Oxford: Clarendon Press, 2006. 10

[7] Herrick, Paul. *Introduction to Logic*. New York: Oxford University Press, 2013. 3

[8] Kneale, Martha and William Kneale. *The Development of Logic*. Oxford: Clarendon Press, 1962. 4

[9] Lambert, Karl. *Philosophical Applications of Free Logic*. New York & Oxford: Oxford University Press, 1991. 2

[10] Leisenring, A. C. *Mathematical Logic and Hilbert’s ε-Symbol*. New York: Gordon and Breach, 1969. 14

[11] Lemmon, E. J. *Beginning Logic*. Indianapolis, IN: Hackett, 1978. 2, 3

[12] Nietzsche, Friedrich. *Human, All-too-Human: A Book for Free Spirits*. Part I (1878). Translated by Helen Zimmern. In *The Complete Works of Friedrich Nietzsche*, Vol. 6. New York: Russell & Russell, 1964. 9

[13] Nolt, John. ‘Free Logic.’ In E. N. Zalta (ed.). *The Stanford Encyclopedia of Philosophy*. Spring 2011 Edition. 2

[14] Peacock, Kent A. “Temporal Presentness and the Dynamics of Spacetime.” In D. Dieks (ed.), *The Ontology of Spacetime*. Amsterdam: Elsevier, 2006, pp. 247–61. 9
[15] Plato. *Timaeus and Critias*. Translated by Desmond Lee. Harmondsworth, UK: Penguin, 1971. 9

[16] Putnam, Hilary. ‘Is Logic Empirical?’ In Robert S. Cohen and Marx W. Wartofsky (eds.), *Boston Studies in the Philosophy of Science*, Vol. 5. Dordrecht: D. Reidel, 1968, pp. 216–41. 11

[17] Russell, Bertrand, and A. N. Whitehead. *Principia Mathematica*. Second Edition. Cambridge: Cambridge University Press, 1927. 3

[18] Russell, Bertrand. *Portraits From Memory and Other Essays*. London: George Allen & Unwin, 1956. 11

[19] Smolin, Lee. *The Trouble With Physics: The Rise of String Theory, the Fall of a Science, and What Comes Next*. Boston & New York: Houghton Mifflin, 2006. 10

[20] Teller, Paul. ‘Quantum Mechanics and Haecceities.’ In Elena Castellani (ed.), *Interpreting Bodies: Classical and Quantum Objects in Modern Physics*. Princeton, NJ; Princeton University Press, 1998, pp. 114–41. 10

[21] ‘Song of Myself,’ in *The Portable Walt Whitman*. Revised and enlarged edition, selected & with notes by Mark van Doren. New York: Penguin, 1977, pp. 32–97. 8

[22] Yourgrau, Palle. *A World Without Time: The Forgotten Legacy of Gödel and Einstein*. New York: Basic/Perseus, 2005. 9