Dependence of average inter-particle distance upon the temperature of neutrals in dusty plasma crystals

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Abstract. It is often suggested that inter-particle distance in stable dusty plasma structures decreases with cooling as a square root of neutral gas temperature. Deviations from this dependence (up to the increase at cryogenic temperatures) found in the experimental results for the pressures range \(0.1–8.0\) mbar and for the currents range \(0.1–1.0\) mA are given. Inter-particle distance dependences on the charge of particles, parameter of the trap and the screening length in surrounding plasma are obtained for different conditions from molecular dynamics simulations. They are well approximated by power functions in the mentioned range of parameters. It is found that under certain assumptions thermophoretical force is responsible for inter-particle distance increase at cryogenic temperatures.

1. Introduction
Plasma with micron-sized condensed particles shows properties, which are uncharacteristic to other systems. Dusty particles acquire a high fluctuating electric charge (up to \(10^4 e\), where \(e\) is elementary charge) due to the different mobility of electrons and ions constituting plasma. Charge strongly depends on the surrounding plasma parameters and the presence of other charged particles. High charges lead to a strong particles interaction. Being placed into a trap compensating mutual repulsion and the Earth gravity, particles form structures that are called “dusty plasma crystals”. These crystals can consist of single particles or particle clusters, form different lattices, rotate, carry shock waves and melt [1–3]. Full theory of these crystals is not finished for a few reasons:

(i) the potential describing highly charged particles interactions in plasma is in question (usually Yukawa potential is used);
(ii) external forces acting on dust particles are numerous and not easy to be taken into account together;
(iii) a medium containing dust particles (glow discharge, for example) is also poorly understood in terms of its plasma characteristics.

In this work, dusty plasma is studied by the method of molecular dynamics which is already developed for this system [4]. Inter-particle distance is chosen as a convenient parameter which
allows analyzing influence of main forces and effects on the properties of dusty plasma structures. This choice is made due to the discovered deviation of inter-particle distance dependence on temperature from the square root of temperature in many experiments. In the second section these experiments are described shortly and their results are given. The third section presents the model used for molecular dynamics simulations. The fourth section is devoted to the obtained results for inter-particle distance dependence on the particle charge, the trap parameter and the screening length. The qualitative explanation of the effect of inter-particle distance increase at cryogenic temperatures (about 100 K and lower) is presented.

2. Reference experiments
Most experiments with dusty plasma are conducted at a room temperature. Studying dusty plasma in cryogenic conditions started in the beginning of this century [5]. Degree of non-ideality of this system sharply increases with temperature decrease that leads to the formation of unusual types of structures such as vibrating and rotating spheres, structures with screw rotation and vortex motion [6]. Conducting such experiments is complicated by the cost of cooling technologies and by the requirement to provide an opportunity of visual observation of dust structures. Usually cryogenic dusty plasma is obtained in glow discharges [7]. Elementary processes and plasma kinetics in them are not fully understood hence it is a relevant field of research.

This work is based on the results of two experimental groups which measured inter-particle distance in their cryogenic experiments.

Data of the first experiment is analyzed together with the authors of this work [8]. Glow discharge is ignited in helium in a vertical tube placed into a double Dewar flask. The temperature range is 4.2–295 K. Dusty structures are recorded on a camera, the recordings are synchronized with the values of discharge parameters (current, voltage, gas pressure and tube walls temperature). Inter-particle distance dependence on temperature for different values of discharge parameters (pressures range 0.1–8.0 mbar and currents range 0.1–1.0 mA) are obtained by the authors. Different types of structures (anisotropic crystals consisting of vertical chains, oscillating crystals) observed in this experiment are identified. It is found that the functional form of the dependence of inter-particle distance upon temperature is convex. If it was completely defined by the ion Debye length, given by the formula

$$\lambda_D = \sqrt{\frac{kT}{8\pi n_{ion} e^2}},$$

as it was suggested by some authors [1, 7], it would have been proportional to the square root of temperature therefore would have been concave. Moreover at about 100 K for most values of discharge current and pressure inter-particle distance starts to grow that stays unexplained.

Results of Vasilyak’s group [9, 10] confirm this result. Their experimental data is obtained in neon glow discharge at gas pressure $P = 0.63$ Torr and discharge current $I = 0.61$ mA. A dependence of inter-particle distance on temperature is presented in the range of temperatures 77–295 K. The minimal distances of 25–40 µm that they measure are close to the value of the Debye radius. Their functional dependence is also convex. Inter-particle distance growth at about 100 K is also observed.

3. Computer simulations
Computer simulations are based on the method of molecular dynamics. The model of dusty plasma corresponds to a typical experiment in glow discharge ignited in a vertical tube [8]. Dust particles are located randomly at the beginning of numerical experiment. The following effects taking place in dusty plasma are considered:

(i) Dust particles interaction.

It is usually suggested that Coulomb field of charged particles in a plasma is screened by surrounding electrons and ions distributed around a particle according to the Boltzmann
law. This suggestion is not always correct in dusty plasma due to the high value of the charge of dust particles, ions and electrons drift in both horizontal and vertical direction and their collisions with each other. However it provides a good first approach in case the density of dusty structures is relatively low. Hence the used potential for particle interaction is Yukawa-like:

$$\varphi_{ij} = \frac{q}{r_{ij}} \exp \left( -\frac{r_{ij}}{\lambda} \right),$$  \hspace{1cm} (1)

where $r_{ij}$ is a distance between two particles (numbered $i$ and $j$) and $\lambda$ is the screening length in plasma (which may and may not coincide with the Debye screening length [1]).

(ii) Electrostatic parabolic trap.

It attracts dust particles to the center of the tube and is described by the formula:

$$\varphi_{\text{trap}} = \frac{1}{2} \alpha \vec{r}_i^2,$$  \hspace{1cm} (2)

where $\alpha$ is the trap parameter and $\vec{r}_i$ is the radius-vector of a particular particle from the center of the tube. In most works it is considered that this trap appears due to the different radial distributions of ions and electrons densities caused by ambipolar diffusion of charged plasma components to the walls of the tube. It is given by the formula [11]:

$$\alpha = -\frac{1}{r_i} \frac{n_{i\text{on}}}{e n_{i\text{on}}(r_i) \frac{d}{dr_i}},$$  \hspace{1cm} (3)

where $T_e$ is the electron temperature in plasma, $n_{i\text{on}}$ is the ion concentration which has the distribution close to the Bessel function $J_0(r_i/\Lambda)$, $\Lambda = 2.4/R$, $R$ is the tube radius.

Assuming the structure is formed near the axis (where $r \ll R$ so that Bessel function may be approximated by its first Taylor series term) the final formula for the attracting trap potential is:

$$\varphi_{\text{trap}} = 1.44 \frac{T_e}{e R^2} \vec{r}_i^2.$$  \hspace{1cm} (4)

(iii) Thermophoretical force.

Thermophoretical force appears due to the presence of temperature gradients in the positive column which lead to the asymmetry in the momentum transfer from neutrals hitting the particles. The force is directed towards lower gas temperatures. In order to describe it in this work the formula derived in [12] is used:

$$F_{\text{thermo}} = -3.33 \frac{k_B a^2}{\sigma} \frac{dT}{dr_i},$$  \hspace{1cm} (5)

where $k_B$ is the Boltzmann constant, $a$ is the particle radius, $\sigma$ is the gas kinetic cross-section for atomic scattering, $dT/dr_i$ is the temperature gradient in the radial direction of the tube.

The functional dependence of gas temperature on the distance from the center of the tube is obtained in [11]:

$$T(r_i) = \frac{iE}{8\lambda} J_0(\frac{r_i}{\Lambda}) + T_w,$$  \hspace{1cm} (6)

where $i$ is the discharge current, $E$ is the longitudinal electric field in the area of the existence of the structure, $\lambda$ is the thermal conductivity coefficient of gas, $T_w$ is the tube wall temperature.

Assuming the structure is formed near the axis (where $r \ll R$) the final formula for the thermophoretical force used in simulations is:

$$F_{\text{thermo}}^r = 1.67 \frac{k_B a^2}{8\sigma} \frac{iE}{\lambda\Lambda^2} r_i.$$  \hspace{1cm} (7)
Table 1. Values of parameters of particles, discharge plasma and discharge used in simulations.

| Parameter                                | Value       |
|------------------------------------------|-------------|
| Mass \(m\) (g)                          | \(10^{-10}\) |
| Particle radius \(a\) (\(\mu\)m)        | 1.5         |
| Particle charge \(q\) (e)               | \(5 \times 10^3\) |
| Electrons temperature \(T_e\) (eV)      | 3           |
| Ion concentration \(n_{ion}(0)\) (cm\(^{-3}\)) | \(10^{10}\) |
| Helium thermal conductivity \(\lambda\) (W/m/K) | \(2.685 \times 10^{-3} T^{0.71}\) |
| Neutral-neutral collisions cross-section \(\sigma\) (10\(^{-20}\) m\(^2\)) | 15          |
| Discharge current \(i\) (mA)            | 0.4         |
| Longitudinal electric field \(E\) (V/cm) | 15          |
| Tube radius \(R\) (cm)                  | 0.8         |
| Step of numerical integration \(dt\) (s) | \(1.8 \times 10^{-4}\) |
| Friction coefficient \(\gamma\) (s\(^{-1}\)) | 3           |

(iv) Friction from the environment.
Dust particles move in a viscous medium consisting mostly of neutral gas atoms. Hence their interaction with the environment is well described by the Langevin equation and the corresponding force is:

\[
F_{\text{Lang}} = -2m\gamma \dot{r}_i + \sqrt{2m\gamma k_B T} \dot{h}(t),
\]

where \(m\) is the mass of a particle, \(\gamma\) is the friction coefficient, \(T\) is the temperature of the environment, \(dt\) is the step of numerical integration and \(\dot{h}(t)\) is a normally distributed random value.

The final equation for a particle acceleration looks as follows:

\[
a_i = q \frac{\partial \phi_{\text{trap}}}{\partial r_i} + q \frac{\partial \phi_{ij}}{\partial r_{ij}} + 1.67 \frac{k_B a^2}{8\sigma} \frac{iE}{\lambda^2 \sigma^2} \dot{r}_i - 2\gamma \dot{r}_i + \sqrt{\frac{2\gamma k_B T}{mdt}} \dot{h}(t). \tag{9}
\]

Values of parameters used in simulations with changing temperature are given in table 1. They coincide with the ones that are usually observed in real experiments but do not correspond to a particular one as not all of these parameters are always measured simultaneously. These assumptions allow to obtain the qualitative, not the quantitative explanation of anomalous increase of inter-particle distance at low temperatures.

4. Results

4.1. Without thermophoretical force
In the first part of simulations no assumptions are made relatively to the particle charge, the trap parameter and the screening length dependences on temperature. They are varied in the ranges which cover most typical experimental conditions and thermophoretical force (considered as a component of the trap) is not taken into account directly:

(i) trap parameter: \(\alpha \in [1 \times 10^{-3}; \, 15 \times 10^{-2}]\) SGS units;
(ii) screening length: \(\lambda \in [30; \, 1000]\) \(\mu\)m;
(iii) particle charge: \(q \in [300; \, 60000]\) e.
Figure 1. Inter-particle distance as a function of the particle charge at $\alpha = 0.01$ SGS units, $\lambda = 100$ $\mu$m in double logarithmic scale. The line is approximated by the formula $\ln r = -1.08 + 0.16 \ln q$.

Figure 2. Inter-particle distance as a function of the trap parameter at $q = 5000e$, $\lambda = 100$ $\mu$m in double logarithmic scale. The line is approximated by the formula $\ln r = -0.18 - 4.04 \ln \alpha$.

In each numerical experiment certain values of these three parameters are chosen as reference ones (for example, $\alpha = 0.01$ SGS units, $q = 1000e$, $\lambda = 100$ $\mu$m). During the run two of three parameters are fixed, the third one is varied in the range corresponding to it. Average inter-particle distance dependence on this parameter is obtained and analyzed. Dependences for all sets of values of the parameters are well approximated with power functions (the graphs in double logarithmic scale are given in figures 1–3).
Figure 3. Inter-particle distance as a function of the trap parameter at $q = 5000e$, $\alpha = 0.01$ SGS units in double logarithmic scale. The line is approximated by the formula $\ln r = -6.07 + 0.63 \ln \lambda$.

The formula uniting all obtained results looks as follows:

$$ r(q, \lambda, \alpha) = r_0 q^a \lambda^b \alpha^c, \quad (10) $$

where $r_0$ is a constant and the powers $a, b, c$ depend on the set of reference values and change in the range:

$$ a \in [0.10; 0.29], \quad b \in [0.55; 0.83], \quad c \in [-0.36; -0.09]. \quad (11) $$

In the conditions of a typical experiment with helium ($\alpha \approx 0.04$ SGS units, $q \approx 4000e$, $\lambda \approx 40 \mu m$) the average inter-particle distance dependence on these parameters is given by the following formula:

$$ r(q, \lambda, \alpha) = (3.5 \pm 0.2) q^{0.15 \pm 0.01} \lambda^{0.55 \pm 0.01} \alpha^{-0.17 \pm 0.01}. \quad (12) $$

This formula may be used for the prediction of common aspects of structural properties behavior in dusty plasma. It is clear from the kind of this dependence that in an assumption the screening length decreases with temperature decrease (being proportional to the square root of temperature is not essential) and the particle charge weakly depends on the temperature of the neutrals (it is mainly defined by electrons temperature) the only parameter which may lead to the inter-particle distance increase in cryogenic area is the trap.

4.2. With thermophoretical force

Hence in the second part of computer simulations the effect produced on the trap by the thermophoretical force is taken into account. Created dusty plasma structures are two-dimensional because the considered temperature gradient appears between the axis of the tube and walls of the tube. It always exists due to the heat emission caused by the current flow in the discharge. As helium thermal conductivity depends on temperature, thermophoretical force also depends on temperature. During the numerical experiment temperature changes with the step 2 K and inter-particle distance dependence on temperature is obtained for different values of discharge current.
Inter-particle distance, m

225 230 235 240 245 250 255 260 265

Temperature, K

0 50 100 150 200 250 300 350

Figure 4. Inter-particle distance dependence on temperature for different values of discharge current: 0.2 (top line), 0.15, 0.10 and 0.001 mA (bottom line). In this model current influences only the value of thermophoretic force and the problem of discharge existence is beyond the scope of the model. It is visible that at high values of discharge current inter-particle distance starts to grow at about 100 K.

The results are presented in the figure 4. It is visible that when thermophoretical force is off (discharge current is 0.001 mA) inter-particle distance decreases with temperature gradually. In this model current influences only the value of thermophoretic force and the problem of discharge existence is beyond the scope of the model. Turning thermophoretical force on at relatively low discharge currents leads to common distance increase due to the trap potential decrease but the tendency stays the same. Further increasing the discharge current causes appearance of the effect of inter-particle distance growth at the temperature about 100 K. This proves that thermophoretical force is responsible for this phenomenon and the effect can be explained qualitatively. In order for the distance to grow there must be an increasing repulsive force or a decreasing attractive force. Thermophoretical force is of the first kind. At high temperatures its values are low because of high neutral gas thermal conductivity and it cannot oppose inter-particle distance falling due to the screening length decrease. At low temperatures its values and temperature derivative are high and produce larger effect on the inter-particle distance.

This result does not pretend to be precise quantitatively due to the lack of the experiment in which all the necessary parameters were measured simultaneously but shows the role of thermophoretical force in dusty plasma structures formation and explains the effect of inter-particle distance growth at cryogenic temperatures.

5. Conclusions
The inter-particle distance dependences on the particle charge, the trap parameter and the screening length in plasma are well approximated by power functions. Obtained formulas may be used to predict structural properties of dusty plasma crystals and to study applicability of different models for particle charging and interaction, trap formation and plasma properties. It is qualitatively proved that thermophoretical force is responsible for the anomalous inter-particle
distance growth at low temperatures (approximately 100 K) in two-dimensional structures in an assumption that discharge current, longitudinal electric field and gas composition stay constant in the course of an experiment.

These results are important for understanding structural properties of dusty plasma crystals and development of methods allowing to control them.

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