Systematic study of autocorrelation time in pure SU(3) lattice gauge theory

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Results of our autocorrelation measurement performed on Fujitsu AP1000 are reported. We analyze (i) typical autocorrelation time, (ii) optimal mixing ratio between overrelaxation and pseudo-heatbath and (iii) critical behavior of autocorrelation time around cross-over region with high statistic in wide range of $\beta$ for pure SU(3) lattice gauge theory on $8^4$, $16^4$ and $32^4$ lattices. For the mixing ratio $K$, small value (3-7) looks optimal in the confined region, and reduces the integrated autocorrelation time by a factor 2-4 compared to the pseudo-heatbath. On the other hand in the deconfined phase, correlation times are short, and overrelaxation does not seem to matter. For a fixed value of $K (=9$ in this paper), the dynamical exponent of overrelaxation is consistent with 2.

Autocorrelation measurement of the topological charge on $32^3 \times 64$ lattice at $\beta = 6.0$ is also briefly mentioned.

1. Aims of our study

Aims of our autocorrelation measurement are to investigate the followings.
1) typical autocorrelation time $\tau$
2) operator dependence of $\tau$
3) $\beta$ dependence of $\tau$
4) how overrelaxation(OR) reduces $\tau$ or not?
5) optimal mixing ratio between OR and pseudo-heatbath
6) lattice size dependence of $\tau$
7) origin of large error of $\tau$
8) critical exponent of $\tau$
9) $\tau$ for topological object on large($32^3 \times 64$) lattice

Points 1 to 5 were previously reported upon at LAT92. Points 6 - 8 are new, as well as some preliminary results about 9.

2. Notations and Parameters

Notations and parameters in this paper are as follows.
1) OBSERVABLE
We adopt the $1 \times 1$ wilson loop on $2^4$ lattice blocked from original lattice as an observable of our autocorrelation measurement.
2) MIXING PARAMETER $K$
To check the efficiency of overrelaxation, we introduce the mixing parameter $K$ which means that the ratio between the Brown-Woch microcanonical updating and Cabibbo-Marinari pseudo heat bath updating is set to be $K : 1$. 

LAT92[1]. Points 6 - 8 are new, as well as some preliminary results about 9.
3) AUTOCORRELATION FUNCTION
The autocorrelation function \( \rho(t) \) of observable \( O \) (= 1 \times 1 \) wilson loop) is defined as:

\[
\rho(t) = \frac{\langle (O(i) - O_A)(O(i + t) - O_B) \rangle}{\sqrt{\langle (O(i) - O_A)^2 \rangle \langle (O(i + t) - O_B)^2 \rangle}} \quad (1)
\]

where \( \langle \rangle \) denotes average over \( i \) and \( O_A = \langle O(i) \rangle \) and \( O_B = \langle O(i + t) \rangle \).

4) AUTOCORRELATION TIME
The autocorrelation time is defined in this paper as:

\[
\tau_{\text{int}} = \rho(0) + 2 \sum_{t=1}^{N} \rho(t) \frac{N - t}{N} \quad (2)
\]

where \( N \) is determined so that \( \tau_{\text{int}} \) is maximized, but \( N < 10 \% \) of the total sample and \( N < 3\tau_{\text{int}} \).

5) LATTICE SIZE and \( \beta \)

\( 8^4 \): 29 \( \beta \) values for \( K=9 \), 12 \( \beta \) values for \( K=0 \) and 4 \( \beta \) values for \( K=0-20 \)

\( 16^4 \): 45 \( \beta \)s for \( K=9 \) and 2 \( \beta \)s for \( K=0-12 \)

\( 32^4 \): 4 \( \beta \)s for \( K=9 \)

3. RESULTS
In Fig.1 and Fig.2 we show the mixing parameter dependence of the autocorrelation time \( \tau \) on \( 8^4 \) and \( 16^4 \) lattice respectively. In both cases, 2 \( \beta \) values are presented, just below the crossover corresponding to confinement (a), and above it (b). Note the expanded scale and short autocorrelation times in (b). In the confined phase, a small value of \( K \) (3-7) seems to reduce the autocorrelation time by a factor 2-4 compared to pseudo-heatbath. Above deconfinement, no clear \( K \)-dependence can be seen.

\( \tau \) is affected by large errors in these figures. To check the origin of this error, we prepare 40000 sweeps on \( 16^4 \) lattice at \( \beta=6.40 \) after thermalization. We divide 40000 sweeps into 4 bins of 10000 sweeps each. Next we calculate \( \rho(t) \) in each bin. The result is shown in Fig.3. We can see large discrepancy between the results obtained from these 4 bins. Clearly 40000 sweeps are not sufficient to obtain precise values for the autocorrelation function and time in this rather extreme case, very close to the deconfinement transition.

Next we show the results of \( \tau \) on different lattice sizes in Fig.3. In this figure we plot \( \tau \) on \( 8^4 \),
16^4 and 32^4 lattices as a function of the inverse lattice spacing a^{-1}. In order to convert the lattice coupling β (= 6/g^2) to the lattice spacing a, we use results of ref.4, obtained by a Monte Carlo Renormalization Group analysis. From Fig.4 we conclude that the critical exponent z defined by

\[
\frac{\tau_L}{\tau_{aL}^z} = \left(\frac{a\alpha L}{a L}\right)^z
\]

is consistent with 2. A detailed analysis is now in progress.

Finally we mention briefly our measurement of topological charge on 32^3 × 64 lattice. This analysis is performed on Fujitsu AP1000 with 1024 processors. We prepare 830 configurations separated 100 updating sweeps at β = 6.00 on 32^3 × 64 lattice, and then perform blocking twice to obtain blocked configurations of size 8^3 × 16. Topological quantities are measured using the cooling method on 8^3 × 16. Fig.5 shows the typical cooling history of topological charge \(Q\), the sum of the absolute value of the local topological density \(I\), and the normalized action \(\tilde{S}\), rescaled so that an instanton configuration has \(\tilde{S} = 1\). We use the value after 50 cooling sweeps as indicated by the arrow in Fig.5. As for the integrated autocorrelation time of the topological charge, \(\tau\) turns out to be quite small compared to 100 sweeps which is the interval between measurements. In this case we have tried to fit \(\rho(t)\) by the exponential function of \(t\) to get so called exponential autocorrelation time, but we can not extract reliable value at this stage since \(\rho(t)\) is fluctuating around 0 from \(t = 200\).

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Fig. 5 cooling history of $Q, I$ and $\tilde{S}$. 

![Graph showing cooling history of $Q, I$ and $\tilde{S}$](image-url)