Constraint Monotonicity, Epistemic Splitting and Foundedness Are Too Strong in Answer Set Programming

Yi-Dong Shen\textsuperscript{1} and Thomas Eiter\textsuperscript{2}

\textsuperscript{1}State Key Laboratory of Computer Science, Institute of Software, Chinese Academy of Sciences
Beijing 100190, China (E-mail: ydshen@ios.ac.cn)

\textsuperscript{2}Vienna Institut f"{u}r Logic and Computation, Technische Universit"{a}t Wien
Favoritenstrasse 9-11, A-1040 Vienna, Austria (E-mail: eiter@kr.tuwien.ac.at)

Abstract

Recently, some researchers \cite{9,1,2} introduced the notions of subjective constraint monotonicity, epistemic splitting, and foundedness for epistemic logic programs, aiming to use them as main criteria/intuitions to compare different answer set semantics proposed in the literature on how they comply with these intuitions. In this note we demonstrate that these three properties are too strong and may exclude some desired answer sets/world views. Therefore, such properties should not be used as necessary conditions for answer set semantics.

1 Introduction

Gelfond \cite{6} introduced the notion of epistemic specifications which are disjunctive logic programs extended with two epistemic modal operators $K$ and $M$. Informally, for a formula $F$ and a collection $\mathcal{A}$ of interpretations, $KF$ is true in $\mathcal{A}$ if $F$ is true in every $I \in \mathcal{A}$, and $MF$ is true in $\mathcal{A}$ if $F$ is true in some $I \in \mathcal{A}$. An epistemic specification/program $\Pi$ consists of rules of the form

\begin{equation}
L_1 \mid \cdots \mid L_m \leftarrow G_1 \land \cdots \land G_n
\end{equation}

where each $L$ is an object literal that is either an atom $A$ or its strong negation $\neg A$, and each $G$ is an object literal, a default negated literal of the form $\neg L$, or a modal literal of the form $KL$, $\neg KL$, $ML$ or $\neg ML$. A rule (1) is called a constraint if its head is $\bot$, and called a subjective constraint if additionally each $G$ is a modal literal. $\Pi$ is a non-epistemic program if it contains no modal literals.

Gelfond \cite{6} defined the first answer set semantics for an epistemic program $\Pi$ as follows. Given a collection $\mathcal{A}$ of interpretations as an assumption, $\Pi$ is transformed into a modal reduct $\Pi^\mathcal{A}$ w.r.t. $\mathcal{A}$ by first removing all rules with a modal literal $G$ that is not true in $\mathcal{A}$, then removing the remaining modal literals. The assumption $\mathcal{A}$ is defined to be a world view of $\Pi$ if it coincides with the collection of answer sets of $\Pi^\mathcal{A}$ under the GL-semantics defined in \cite{8}.
It turns out that the above semantics for epistemic programs has both the problem of unintended world views with recursion through $K$ and the problem due to recursion through $M$ \cite{7,11}. For the first problem, an illustrative example is $\Pi = \{p \leftarrow Kp\}$; under the above semantics $\Pi$ has two world views $A_1 = \{\emptyset\}$ and $A_2 = \{\{p\}\}$, where as commented in \cite{7}, $A_2$ is undesired. For the second problem, a typical example is $\Pi = \{p \leftarrow Mp\}$; by the above semantics $\Pi$ has two world views $A_1 = \{\emptyset\}$ and $A_2 = \{\{p\}\}$, where as commented in \cite{11}, $A_1$ may be undesired.

To address the two problems, several approaches have been proposed \cite{11,10,8,16}. In particular, Shen and Eiter\cite{16} presented an approach that significantly differs from the others in the following aspects. (i) They introduced the modal operator $\text{not}$ to directly express epistemic negation, where $\text{not} F$ expresses that there is no evidence proving that $F$ is true. Modal formulas $KF$ and $MF$ are viewed as shorthands for $\neg \text{not} F$ and $\neg \text{not} \neg F$, respectively. (ii) Due to having the modal operator $\text{not}$ to express epistemic negation, they further proposed to apply epistemic negation to minimize the knowledge in world views, a novel principle they named knowledge minimization with epistemic negation. It is based on the principle of knowledge minimization with epistemic negation that they presented a completely new definition of world views, which are free of both the problem with recursion through $K$ and the problem through $M$. (iii) Their approach is general and can be used to extend any existing answer set semantics for non-epistemic programs, such as those defined in \cite{14,15,20,3,5,18,17}, to one for epistemic programs.

Very recently, some researchers \cite{9,1,2} introduced the notions of subjective constraint monotonicity, epistemic splitting, and foundedness for epistemic programs, aiming to use them as main criteria/intuitions to compare different answer set semantics proposed in the literature on how they comply with these intuitions. Specifically, they criticized the semantics defined in \cite{11,10,3,16}, saying that these semantics do not satisfy the three properties.

In this note, we clarify the matter by demonstrating that these three properties are too strong and may exclude some desired answer sets/world views. Therefore, such properties should not be used as necessary conditions for answer set semantics.

\section{Subjective constraint monotonicity is too strong, while the requirement of epistemic splitting is even more restrictive}

A semantics is said to satisfy subjective constraint monotonicity if for any epistemic program $\Pi$ and subjective constraint $C$, a world view of $\Pi \cup \{C\}$ is also a world view of $\Pi$; in other words, adding any constraint $C$ to $\Pi$ would never introduce new world views. The epistemic splitting property is even more restrictive in the sense that every semantics satisfying epistemic splitting also satisfies subjective constraint monotonicity \cite{1}.

As a typical example, let $\Pi = \{p \mid q\}$, which has a unique world view $\{\{p\}, \{q\}\}$. Then subjective constraint monotonicity requires that for any subjective constraint $C$, $\Pi \cup \{C\}$ should either have a unique world view $\{\{p\}, \{q\}\}$ or have no world view. For example,
under subjective constraint monotonicity the following program

\[ \Pi_1: \quad \begin{array}{c}
p \mid q \\
\bot \leftarrow \neg K p \\
\end{array} \quad r_1 \quad C \]

has no world view, as the only world view \( \{ \{p\}, \{q\} \} \) of \( \Pi = \{ p \mid q \} \) is not a model of \( \Pi_1 \). Note that under the semantics of \( [11, 10, 3, 16] \), \( \Pi_1 \) has a world view \( \mathcal{A} = \{ \{p\} \} \). It is argued in \( [9, 19] \) that \( \{ \{p\} \} \) should not be a world view of \( \Pi_1 \) because it violates subjective constraint monotonicity.

We comment that the requirement of constraint monotonicity (resp. epistemic splitting), i.e., adding constraints to a logic program should not introduce new answer sets/world views, is too strong and may exclude some desired answer sets/world views, as demonstrated in the following examples.

1. For a non-epistemic program \( \Pi \), the GL-semantics \( [8] \) satisfies the constraint monotonicity property that adding a constraint \( \bot \leftarrow \text{body}(r) \) to \( \Pi \) may rule out some answer sets of \( \Pi \), but would never introduce new answer sets \( [13] \). However, very recent research \( [17] \) reveals that the GL-semantics may miss some desired answer sets that violate constraint monotonicity (see Section 4.1 in \( [17] \)). As an example, consider the following non-epistemic program

\[ \Pi_2: \quad \begin{array}{c}
a \mid b \\
a \leftarrow b \\
\bot \leftarrow \neg b \\
\end{array} \quad r_1 \quad r_2 \quad C \]

where \( C \) is a constraint. Intuitively, the rule \( r_1 \) presents two alternatives for answer set construction, namely \( a \) or \( b \), and the rule \( r_2 \) infers \( a \) if \( b \) has already been derived. We distinguish between the following two cases.

First, suppose that we choose \( a \) from \( r_1 \). As \( b \) is not inferred from \( r_1 \), the rule \( r_2 \) is not applicable; so rules \( r_1 \) and \( r_2 \) together infer a possible answer set \( I_1 = \{ a \} \). As \( I_1 \) does not satisfy the constraint \( C \), it is not a candidate answer set for \( \Pi_2 \).

Alternatively, suppose that we choose \( b \) from \( r_1 \); then by \( r_2 \) we obtain a possible answer set \( I_2 = \{ a, b \} \). \( I_2 \) satisfies the constraint \( C \), so it is a candidate answer set for \( \Pi_2 \).

As \( I_2 = \{ a, b \} \) is the only model of \( \Pi_2 \), it is the only candidate answer set and thus we expect \( I_2 \) to be an answer set of \( \Pi_2 \). However, as \( \Pi_2 \setminus \{ C \} \) has only one answer set \( \{ a \} \), this desired answer set \( I_2 \) for \( \Pi_2 \) violates the constraint monotonicity property.

2. For epistemic programs, the requirement of subjective constraint monotonicity (resp. epistemic splitting) may also exclude some world views that are reasonably acceptable. As an example, consider the above program \( \Pi_1 \) again. As the rule \( r_1 = p \mid q \) offers
two alternatives for answer set construction, namely \( p \) or \( q \), we can generate from \( r_1 \) two possible answer sets: \( \{p\} \) and \( \{q\} \). Then we can construct from the two possible answer sets three possible world views: \( \mathcal{A}_1 = \{\{p\}\} \), \( \mathcal{A}_2 = \{\{q\}\} \) and \( \mathcal{A}_3 = \{\{p\}, \{q\}\} \). As \( \mathcal{A}_2 \) and \( \mathcal{A}_3 \) do not satisfy the constraint \( \bot \leftarrow \neg Kp \), \( \mathcal{A}_1 \) is the only candidate world view and thus we expect it to be a world view of \( \Pi_1 \). However, this desired world view will be excluded if we enforce subjective constraint monotonicity.

3. The above defined constraint monotonicity, which requires world views of \( \Pi \cup \{C\} \) to be world views of \( \Pi \) satisfying \( C \), amounts in essence to *interpreting constraint \( C \) as a query*. Let \( S \) be the collection of world views of \( \Pi \). A query \( C \) to \( \Pi \) is to find in \( S \) all world views that satisfy \( C \). Note that query \( C \) is not involved in the computation of any world view. This essentially differs from adding a constraint \( C \) to \( \Pi \), which aims to computing the collection of world views of \( \Pi \cup \{C\} \); due to that \( C \) is directly involved in the computation of every world view, a world view of \( \Pi \cup \{C\} \) is not necessarily a world view of \( \Pi \).

## 3 The foundedness requirement is too strong to characterize self-supported-free answer sets/world views

The foundedness property is defined in [2], which extends the notion of *foundedness* introduced in [12] from non-epistemic programs to epistemic programs. The GL-semantics [8] for non-epistemic programs also has the foundedness property. We use examples to demonstrate that the foundedness requirement is too strong and may exclude some desired answer sets/world views. For simplicity, we do not reproduce the definition of foundedness here; the reader is referred to [2].

1. Consider again the above non-epistemic program \( \Pi_2 \), which has only one model \( I = \{a, b\} \). Note that \( b \) is established (and also well-supported) in \( r_1 \). Then once \( b \) is selected from \( r_1 \), \( a \) is well-supported in \( r_2 \). By *well-supported* we mean *self-supported free*. Therefore, \( I \) is well-supported in \( r_1 \) and \( r_2 \). As \( I \) satisfies the constraint \( C \), it is the only candidate answer set for \( \Pi_2 \) and thus is a desired answer set of \( \Pi_2 \). However, this desired answer set, though being self-supported free, violates the foundedness property. (It is easy to check that \( \langle \{b\}, I \rangle \) is an unfounded set.) This example shows that the foundedness property cannot characterize self-supported-free answer sets.

2. Consider the following epistemic program:

\[
\Pi_3 : \quad p \mid q \\
p \leftarrow Kq \\
q \leftarrow Kp \\
\bot \leftarrow \neg Kp
\]

\[ r_1 \]
\[ r_2 \]
\[ r_3 \]
\[ C \]
As \( p \mid q \) offers two alternatives for answer set construction, namely \( p \) or \( q \), we can generate from \( r_1 \) two possible answer sets: \( \{p, \cdots \} \) and \( \{q, \cdots \} \), where “\( \cdots \)” stands for possible atoms that would be derived from the rules \( r_2 \) and \( r_3 \). Then we can construct from the two possible answer sets three possible world views: \( A_1 = \{\{p, \cdots \}\} \), \( A_2 = \{\{q, \cdots \}\} \), and \( A_3 = \{\{p, \cdots \}, \{q, \cdots \}\} = \{\{p\}, \{q\}\} \). Note that the two answer sets in \( A_3 \) must be different and no one is a proper subset of the other. We distinguish among the following three cases.

First, suppose that we choose \( A_1 = \{\{p, \cdots \}\} \). Note that \( p \) in \( A_1 \) is established in \( r_1 \). Then, as \( A_1 \) satisfies \( K_p \), \( q \) is well-supported in \( r_3 \) and thus \( A_1 = \{\{p, q\}\} \). \( A_1 \) also satisfies \( r_2 \) and \( C \), so it is a candidate world view for \( \Pi_3 \).

Second, suppose that we choose \( A_2 = \{\{q, \cdots \}\} \). Note that \( q \) in \( A_2 \) is established in \( r_1 \). Then, as \( A_2 \) satisfies \( K_q \), \( p \) is well-supported in \( r_2 \) and thus \( A_2 = \{\{p, q\}\} \). \( A_2 \) satisfies \( r_3 \) and \( C \), so it is further shown that \( \{\{p, q\}\} \) is a candidate world view for \( \Pi_3 \).

Finally, suppose that we choose \( A_3 = \{\{p\}, \{q\}\} \). \( A_3 \) does not satisfy \( C \), so it is not a candidate world view for \( \Pi_3 \).

Consequently, \( \{\{p, q\}\} \) is the only candidate world view for \( \Pi_3 \), so we expect it to be a world view of \( \Pi_3 \). However, this desired world view, though being self-supported free, violates the foundedness property. (It is easy to check that \( \{\{p\}, \{p, q\}\}, \{\{q\}, \{p, q\}\}\) is an unfounded set.) This example shows that the foundedness property cannot characterize self-supported-free world views.

### 4 Conclusions

The above examples demonstrate that the properties of subjective constraint monotonicity, epistemic splitting and foundedness are too strong and may exclude some desired answer sets/world views. It was specifically emphasized in [7, 11, 16] that the focus of research on answer set semantics for epistemic programs is how to handle the two basic problems:

1. The problem of unintended world views caused by recursion through \( K \);
2. The problem of unintended world views caused due to recursion through \( M \).

In fact, by introducing the epistemic negation operator \( \text{not} \) and applying the principle of knowledge minimization with epistemic negation, Shen and Eiter [16] has presented a principled way to handle the two problems. For example, the desired answer sets/world views of the above programs \( \Pi_1 \) \( \Pi_3 \) can all be obtained by applying the general semantics defined in Definition 8 of [16], where the base answer set semantics \( \mathcal{X} \) for a non-epistemic program is that defined in Definition 10 of [17].
References

[1] P. Cabalar, J. Fandinno, and L. del Cerro. Splitting epistemic logic programs. In 15th International Conference on Logic Programming and Nonmonotonic Reasoning (LPNMR), pages 120–133, 2019.

[2] P. Cabalar, J. Fandinno, and L. F. del Cerro. Founded world views with autoepistemic equilibrium logic. In 15th International Conference on Logic Programming and Nonmonotonic Reasoning (LPNMR), pages 134–147, 2019.

[3] L. F. del Cerro, A. Herzig, and E. I. Su. Epistemic equilibrium logic. In Proc. 24th International Joint Conference on Artificial Intelligence (IJCAI-15), pages 2964–2970. AAAI Press/IJCAI, 2015.

[4] W. Faber, G. Pfeifer, and N. Leone. Semantics and complexity of recursive aggregates in answer set programming. Artificial Intelligence, 175(1):278–298, 2011.

[5] P. Ferraris, J. Lee, and V. Lifschitz. Stable models and circumscription. Artificial Intelligence, 175(1):236–263, 2011.

[6] M. Gelfond. Strong introspection. In Proceedings of the 9th National Conference on Artificial Intelligence, pages 386–391, 1991.

[7] M. Gelfond. New semantics for epistemic specifications. In Logic Programming and Nonmonotonic Reasoning - 11th International Conference LPNMR, pages 260–265, 2011.

[8] M. Gelfond and V. Lifschitz. Classical negation in logic programs and disjunctive databases. New Generation Computing, 9:365–385, 1991.

[9] P. Kahl and A. Leclerc. Epistemic logic programs with world view constraints. In Technical Communications of the 34th International Conference on Logic Programming (ICLP), pages 1–17, 2018.

[10] P. Kahl, R. Watson, E. Balai, M. Gelfond, and Y. Zhang. The language of epistemic specifications (refined) including a prototype solver. Journal of Logic and Computation, 2015.

[11] P. T. Kahl. REFINING THE SEMANTICS FOR EPISTEMIC LOGIC PROGRAMS. PhD thesis, Texas Tech University, USA, 2014.

[12] N. Leone, P. Rullo, and F. Scarcello. Disjunctive stable models: Unfounded sets, fixpoint semantics, and computation. Information and Computation, 135(2):69–112, 1997.

[13] V. Lifschitz, L. R. Tang, and H. Turner. Nested expressions in logic programs. Annals of Mathematics and Artificial Intelligence, 25(1-2):369–389, 1999.

[14] D. Pearce. Equilibrium logic. Annals of Mathematics and Artificial Intelligence, 47(1-2):3–41, 2006.

[15] W. Pelov, M. Denecker, and M. Bruynooghe. Well-founded and stable semantics of logic programs with aggregates. Theory and Practice of Logic Programming, 7(3):301–353, 2007.

[16] Y. D. Shen and T. Eiter. Evaluating epistemic negation in answer set programming. Artificial Intelligence, 237:115–135, 2016.

[17] Y. D. Shen and T. Eiter. Determining inference semantics for disjunctive logic programs. Artificial Intelligence, 277:1–28, 2019.

[18] Y. D. Shen, K. Wang, T. Eiter, M. Fink, C. Redl, T. Krennwallner, and J. Deng. FLP answer set semantics without circular justifications for general logic programs. Artificial Intelligence, 213:1–41, 2014.

[19] E. I. Su. Epistemic answer set programming. In 16th European Conference on Logics in Artificial Intelligence (JELIA), pages 608–626, 2019.

[20] M. Truszczynski. Reducts of propositional theories, satisfiability relations, and generalizations of semantics of logic programs. Artificial Intelligence, 174(16-17):1285–1306, 2010.