The Unruh Effect for Eccentric Uniformly Rotating Observers

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Abstract

It is common to use Galilean rotational transformation to investigate the Unruh effect for uniformly rotating observers. However, the rotating observer in this subject is an eccentric observer while Galilean rotational transformation is only valid for centrally rotating observers. Thus, the reliability of the results of applying Galilean rotational transformation to the study of the Unruh effect might be considered as questionable. In this work the rotational analog of the Unruh effect is investigated by employing two relativistic rotational transformations corresponding to the eccentric rotating observer, and it is shown that in both cases the detector response function is non-zero. It is also shown that although consecutive Lorentz transformations can not give a frame within which the canonical construction can be carried out, the expectation value of particle number operator in canonical approach will be zero if we use modified Franklin transformation. These conclusions reinforce the claim that correspondence between vacuum states defined via canonical field theory and a detector is broken for rotating observers. Some previous conclusions are commented on and some controversies are also discussed.

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I. **INTRODUCTION**

The Unruh effect predicts that linearly accelerated observer with constant proper acceleration in flat spacetime (Rindler observer) associates a thermal spectrum of particles to the no-particle state (Minkowski vacuum) of inertial observer. In other words, two vacuum states of these observers are not the same. So the particle content of field theory is observer-dependent and observers with different notions of positive and negative modes will disagree on the particle content of a given state; what we think of as an inert vacuum actually has the character of a thermal state. For a review on this effect and its applications and experimental proposals see [1]. First approach to this effect was based on canonical quantization of fields [2–4], which we will call ”**canonical approach**” and the second was based on excitation of a detector [4–6], which we will refer to it as”**detector approach**”. Despite the controversies and disagreements in the interpretations and relations between them, both approaches give the same mathematical result that an uniformly accelerated detector (observer) observes a thermal spectrum of particles and behaves as though it were placed in a thermal bath with temperature \( T = a/2\pi \), where \( a \) is the magnitude of the proper acceleration.

A special case of accelerated observers which can have more feasible experimental tests is eccentric uniformly rotating observer. In addition to the usual ambiguities related to the rotating observers [19], the particle detection due to acceleration of rotational motion is also controversial and the agreement between canonical approach and detector approach seems not to occur for this observer. The problem of finding true rotational transformation between the rotating and non-rotating frames is one important aspect of conflict. While most authors have used Galilean rotational transformation (GRT) to investigate the Unruh effect for rotating observer [7–13], some other have tried to use corresponding relativistic transformations [14–17]. As we briefly mention them below, their results are different.

All of those who use GRT, in canonical approach, obtain zero expectation value for particle number operator of rotating observer in vacuum state of inertial observer, but they do not have agreement on values (zero or nonzero) of detector response function [18]. This problem was named **the puzzle of rotating detector** [14]. A solution for this problem was introduced in [10] which states that ”confining the detector inside the limiting surface and imposing the speed of light restriction for detector, the rotating detector registers the absence of quanta and has vanishing response.” Another explanation is that ”the correspondence
between expectation value for particle number operator defined via canonical quantum field theory and detector response function is broken for general stationary motions”, and we must conclude that the two definitions are inequivalent [11]. On the other hand, some of those who use relativistic transformations conclude the particle detection both for canonical and detector approaches [14, 15] and the other claim that there is no particle detection because rotating observe does not have event horizon [16].

In this paper we will discuss that GRT is not applicable for eccentric uniformly rotating observers and we must replace it with the correct relativistic transformations between laboratory inertial observer and eccentric rotating observer. We will use two sets of relativistic transformation to investigate Unruh effect for eccentric uniformly rotating observer in canonical and detector approach. Also we will discuss other relativistic transformation mentioned above.

In section II we briefly mention the limitations and problems of GRT and introduce two types of relativistic transformations for eccentric uniformly rotating observer that can be replaced with GRT. In section III we use two sets of relativistic transformations introduced in section II to investigate the Unruh effect for eccentric uniformly rotating observer both in canonical and detector approaches. We conclude with a discussion section; comment on some former papers and discuss some controversies.

In this paper we use $S'$ for rest frame of laboratory observer and $S$ for accelerated observer’s frame. Greek letters take on the values 0,1,2,3. We use the metric with signature (+,-,-,-) and work in natural units.

II. RELATIVISTIC TRANSFORMATIONS FOR ECCENTRIC UNIFORMLY ROTATING DETECTOR

As we have shown in [19, 20], Galilean rotational transformation (GRT)

$$
t = t' , \quad r = r' , \quad \phi = \phi' - \Omega t , \quad z = z'$$

(1)

for relation between centric inertial observer and eccentric uniformly rotating observer who rotates with constant angular velocity $\Omega$ at constant radius distance from the center of rotation is not true; Specially absoluteness of time and not distinguishing between observers at different radii in these transformations cause inconsistent kinematical interpretations
when we want to explain phenomenon such as transverse Doppler effect and Sagnac effect. This transformation is only applicable for relation between two centric observer that one rotates uniformly and has no translational motion and the other is a non-rotating inertial observer. So using GRT for an eccentric rotating detector is not true and the results that has been obtained by these transformations for the Unruh effect in rotating frames are not valid.

We assume a detector on the edge of a rigid disc that rotates uniformly counterclockwise with angular velocity $\Omega$ in the $X'Y'$ plane around its axis ($Z'$ axis). Such detectors are the ones which are related to the real experimental setups. As we have shown in [20], there are two type of relativistic rotational transformations to describe the relation between this rotating observer (detector) and laboratory inertial observer:

A. Special Relativistic Transformation (SRT)

SRTs are based on consecutive Lorentz transformations and Fermi coordinates. In [23, 24] these coordinate transformations between inertial laboratory (primed) and eccentric uniformly rotating (unprimed) frames are given as follows

\[
\begin{align*}
t &= \gamma^{-1}(t' - R\Omega \gamma y) \\
x &= x' \sin(\gamma \Omega t) + y' \cos(\gamma \Omega t) - R \\
y &= \gamma^{-1}[x' \cos(\gamma \Omega t) + y' \sin(\gamma \Omega t)] , \\
z &= z' 
\end{align*}
\]

in which $\gamma = (1 - R^2 \Omega^2)^{-1/2}$, $\Omega$ is the uniform angular velocity of the disk and $R$ is the radius of the circular path. In their setup the origin of the rotating frame is on the rim of the circular path. The metric components in such a rotating frame are given as follows

\[
ds^2 = -\gamma^2[1 - (R + x)^2\Omega^2 - \Omega^2 \gamma^2 y^2]dt^2 + dx^2 + dy^2 + dz^2 - 2y\Omega dx dt + 2x\Omega dy dt
\]  

B. Modified Franklin Transformations (MFT)

In [19], looking for a consistent relativistic rotational transformation between an inertial observer and an observer at a non-zero radius (eccentric observer) on a uniformly rotating disk, the following transformations (in cylindrical coordinates) were introduced

\[
t = \cosh(\Omega R/c)t' - \frac{R}{c} \sinh(\Omega R/c) \phi' ; \quad \rho = \rho'
\]
\[ \phi = \cosh(\Omega R/c)\phi' - \frac{c}{R} \sinh(\Omega R/c)t' ; \quad z = z', \quad (4) \]
in which \( \Omega \) is the uniform angular velocity of the disk and \( R \) is the radial position of the observer on the disk. Note that the origin of the rotating frame \( S \) is chosen to be at the center of the rotating disk so that both inertial and rotating frames assign the same radial coordinate to the events. The corresponding metric in the rotating observer’s frame is given by,

\[ ds^2 = -c^2 \cosh^2(\Omega) (1 - \rho^2 R^2 \tanh^2(\Omega)) dt^2 + d\rho^2 + \rho^2 \cosh^2(\Omega) (1 - \rho^2 R^2 \tanh^2(\Omega)) d\phi^2 - 2cR \sinh(\Omega) \cosh(\Omega) (1 - \rho^2 R^2) dtd\phi + dz^2. \quad (5) \]

III. PARTICLE DETECTION BY UNIFORMLY ROTATING ECCENTRIC OBSERVER

Solving Klein-Gordon equation for a massless scalar field in flat spacetime of laboratory inertial observer in cylindrical coordinate gives positive modes solution as

\[ f = \frac{1}{2\pi\sqrt{2\omega}} \exp(-i\omega t' + im\phi' + ikz') J_m(q\rho') \quad (6) \]
in which \( m \) is an integer, \( J_m \) is the Bessel function and \( \omega = \sqrt{q^2 + k^2} \). By expanding the field in term of a complete set of positive modes \( f_i \) and negative modes \( f_i^* \) and creation and annihilation operators (\( \hat{a}_i^\dagger \) and \( \hat{a}_i \)), we have

\[ \Phi = \sum_i (\hat{a}_i f_i + \hat{a}_i^\dagger f_i^*) \quad (7) \]

Also we can solve Klein-Gordon equation for a massless scalar field in the rotating observers frame and expand the field in term of a new complete set of positive and negative modes \( (g_i, g_i^*) \) and new creation and annihilation operators (\( \hat{b}_i, \hat{b}_i^\dagger \))

\[ \Phi = \sum_i (\hat{b}_i g_i + \hat{b}_i^\dagger g_i^*) \quad (8) \]

If we show the vacuum state of inertial observer by \( |0_f\rangle \) and the rotating observer’s particle number operator by \( \hat{n}_g \) then we have

\[ \langle 0_f | \hat{n}_g | 0_f \rangle = \sum_j |\beta_{ij}|^2 \quad (9) \]
in which the Bogolyubov coefficient $\beta$ is defined as

$$\beta_{ij} = -(g_i, f_j^*)$$  \hspace{1cm} (10)$$

and definition of inner product is given by

$$(\varphi_1, \varphi_2) = -i \int_{\Sigma} (\varphi_1 \nabla_\mu \varphi_2^* - \varphi_2^* \nabla_\mu \varphi_1) n^\mu \sqrt{h} dx^{n-1}$$  \hspace{1cm} (11)$$

where $\Sigma$ is hypersurface that we integrate over it, $h$ is the determinant of $h_{ij}$ which is the induced metric on $\Sigma$ and $n^\mu$ is the normal vector to $\Sigma$. Since the inner product is independent of the hypersurface over which the integral is taken, we can take the integral over $t = 0$ hypersurface. According to (9) non-zero value for coefficient $\beta$ means non-zero expectation value of particle number operator of the rotating observer in the vacuum state of laboratory observer.

On the other hand, according to [18] the detector response function is given by

$$F(E) = \int_{-\infty}^{\infty} d\Delta \tau e^{-iE\Delta \tau} G^+(x(\tau_1), x(\tau_2))$$  \hspace{1cm} (12)$$

where $G^+$ is the positive Wightman function defined as

$$G^+(x'_1, x'_2) = \frac{-1}{4\pi^2[(t'_1 - t'_2 - i\epsilon)^2 - |x'_1 - x'_2|^2]}$$  \hspace{1cm} (13)$$

and $x(\tau)$ is the worldline of detector and $\tau$ is its proper time.

Now we calculate Bogolyubov coefficient $\beta$ and detector response function using two relativistic rotational transformations introduced in previous section.

**A. Special Relativistic Transformation (SRT)**

1. Canonical Approach

By special relativistic transformations [2] and corresponding metric [3] and make two simplifying assumptions on the values of $R$ and $\Omega$ that $R = 1$ and $\Omega = \frac{1}{2}$ (which are not important in our discussion) we have

$$g_{00} = 1 - x^2/3 - 2x/3 - 4y^2/9 , \quad g_{01} = 2y/3 , \quad g_{02} = -2x/3 , \quad g_{03} = 0 , \quad g_{ij} = -\delta_{ij}$$ (14)
and so $g = -(x - 3)^2/9$ and

$$g^{\mu\nu} = \frac{1}{(x - 3)^2} \left( \begin{array}{cccc} 9 & 6y & -6x & 0 \\ 6y & -x^2 + 6x + 4y^2 - 9 & -4yx & 0 \\ -6x & -4yx & 3(x^2 + 2x - 3) & 0 \\ 0 & 0 & 0 & -1 \end{array} \right)$$

(15)

and so the corresponding Klein-Gordon equation for massless scalar field

$$\left( \frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu \Phi) \right) = 0$$

is given by

$$\frac{9}{(x - 3)^2} \frac{\partial^2 \Phi}{\partial t^2} + \frac{12y}{(x - 3)^2} \frac{\partial^2 \Phi}{\partial t \partial x} - \frac{12x}{(x - 3)^2} \frac{\partial^2 \Phi}{\partial t \partial y} - \frac{6y}{(x - 3)^3} \frac{\partial \Phi}{\partial t} + \frac{4y^2 - (x - 3)^2}{(x - 3)^2} \frac{\partial^2 \Phi}{\partial x^2} + \frac{12y^2 - 3(x - 3)^2}{3(x - 3)^3} \frac{\partial \Phi}{\partial x} + \frac{3(x^2 + 2x - 3)}{(x - 3)^2} \frac{\partial^2 \Phi}{\partial y^2} = 0$$

(16)

Although it seems necessary to obtain the analytic solution for this partial differential equation to continue and calculate the Bogolyubov coefficient, but it is not possible. As it mentioned very shortly in [26], this is the reason why rotational transformations based on consecutive Lorentz transformations can not give a coordinate system within which the canonical approach of a quantum field can be carried out.

2. Detector Approach

With the assumption that the detector is at the origin of rotating frame and using transformations (2), the detector’s trajectory in the laboratory frame is given by

$$x' = R \cos(\gamma \Omega t) , \quad y' = R \sin(\gamma \Omega t) , \quad z' = 0 , \quad t' = \gamma t$$

(17)

and by (13) we have

$$G^+(x',x'_2) = \frac{-1}{4\pi^2[(\gamma \Delta \tau - i\epsilon)^2 - 2R^2(1 - \cos(\gamma \Omega \Delta \tau))]}$$

(18)

Inserting (18) in (12) the detector response function is given by

$$F(E) = \int_{-\infty}^{\infty} d\Delta \tau \frac{e^{-iE\Delta \tau}}{(\gamma \Delta \tau - i\epsilon)^2 - 4R^2 \sin^2(\gamma \Omega \Delta \tau)}.$$  

(19)

Except in some constant coefficients this is the same as obtained and numerically evaluated in [8] and so has non-zero value.
B. Modified Franklin Transformations (MFT)

1. Canonical Approach

For eccentric rotating observer using modified Franklin transformations \[4\] and corresponding metric \[5\] we have \( g = -\rho^2 \) and

\[
g^{\mu\nu} = \begin{pmatrix}
\cosh^2(R\Omega) - \frac{R^2 \sinh^2(R\Omega)}{\rho^2} & 0 & \frac{R^2 - \rho^2}{R^2} \cosh(R\Omega) \sinh(R\Omega) & 0 \\
0 & -1 & 0 & 0 \\
\frac{R^2 - \rho^2}{R^2} \cosh(R\Omega) \sinh(R\Omega) & 0 & -\frac{\cosh^2(R\Omega)}{\rho^2} + \frac{\sinh^2(R\Omega)}{R^2} & 0 \\
0 & 0 & 0 & -1
\end{pmatrix}
\] \[20\]

So the Klein Gordon equation is as below

\[
(\cosh^2 \beta - \frac{R^2}{\rho^2} \sinh^2 \beta) \frac{\partial^2 \Phi}{\partial t^2} - \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho \frac{\partial \Phi}{\partial \rho} \right) - \frac{1}{\rho^2} \frac{(\cosh^2 \beta - \sinh^2 \beta)}{R^2} \frac{\partial^2 \Phi}{\partial \phi^2} \\
+ \frac{2(R^2 - \rho^2)}{R^2} \sinh \beta \cosh \beta \frac{\partial^2 \Phi}{\partial t \partial \phi} - \frac{\partial^2 \Phi}{\partial z^2} = 0.
\] \[21\]

Assuming a trial solution as

\[
g = \exp(-i\omega' t + im' \phi + ik' z)R(\rho)
\] \[22\]

and inserting in \[21\], the radial part equation is

\[
\frac{d^2 R(\rho)}{d\rho^2} - \frac{1}{\rho} \frac{dR(\rho)}{d\rho} + \left[ (m' + \sqrt{2}\omega')^2 + \frac{-\omega'^2 + 2m'^2 - 2\sqrt{2}\omega'm'}{\rho^2} \right] R(\rho) = 0
\] \[23\]

in which we set \( R = 1 \) and \( \cosh^2(\beta) = 2 \) for simplicity. (These assumptions do not affect the results.) This equation is a cylindrical Bessel equation and so the positive mode solution corresponding to it is

\[
g = N \exp(-i\omega' t + im' \phi + ik' z)J_m(q' \rho)
\] \[24\]

in which \( N \) is a normalization factor. As in the case of GRT \[7\] we can see that the Bogolyubov coefficient \( \beta \) is zero here, so using MFT the canonical approach concludes the absence of particle in the vacuum state of laboratory observer for the rotating observer.
2. Detector Approach

On the other hand the detector’s trajectory in the rotating frame is

$$\rho = R , \phi = 0 , \ z = 0$$

(25)

Using MFT to obtain the trajectory for the laboratory observer we have

$$t_2' - t_1' = \cosh(R\Omega)\Delta\tau , \ \varphi_2' - \varphi_1' = \frac{\sinh(R\Omega)}{R}\Delta\tau$$

(26)

in which $$\Delta\tau = t_2 - t_1$$. Wightman function in cylindrical coordinate is as below

$$G^+(x_1', x_2') = -\frac{1}{4\pi^2} \frac{1}{(t_2' - t_1' - i\epsilon)^2 - \left[\rho_2'^2 + \rho_1'^2 - 2\rho_1'\rho_2'\cos(\varphi_2' - \varphi_1') + (z_2' - z_1')^2\right]}$$

(27)

so we have

$$G^+(x_1', x_2') = \frac{-1}{4\pi^2 \left[(\cosh(\beta)\Delta\tau - i\epsilon)^2 - 2R^2(1 - \cos(\frac{\sinh(\beta)}{R}\Delta\tau)\right]$$

(28)

inserting in (12) the detector response function is given by

$$F(E) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} d\Delta\tau \frac{e^{-iE\Delta\tau}}{(\cosh(\beta)\Delta\tau - i\epsilon)^2 - 4R^2\sin^2(\frac{\sinh(\beta)}{2R}\Delta\tau)}$$

(29)

so in comparison with (19) it has non-zero value.

IV. DISCUSSION AND CONCLUSIONS

Here the Unruh effect for eccentric uniformly rotating observers was investigated by two relativistic rotational transformations corresponding to the eccentric rotating observer: consecutive Lorentz transformation and modified Franklin transformation. It was shown that the detector response function is non-zero in both cases. We also showed that although consecutive Lorentz transformations lead to calculational problem and give a frame within which the Klein-Gordon equation does not have an analytic solution, but if we use modified Franklin transformation, we obtain that the Bogolyubov coefficient related to number operator and so the expectation value of particle number operator is zero. This conclusions reinforce the claim that correspondence between vacuum states defined via canonical field theory and via a detector is broken for rotating observers [9, 11]. Following our comparative study in [20], here we showed that employing MFT instead of the SRT helps to investigate
the Unruh effect in canonical approach. It must be emphasized that in these relativistic transformations the upper limit for the velocity of the disk points (speed of light) is considered and unlike [10] there is no need to confine the detector inside a light cylinder. In order to answer this question that if particle distribution is characteristic of the thermal blackbody radiation with the finite temperature, we need analytic solution of coefficient $\beta$ and detector response function. Then we can judge the claim stated in [16] that the Stationary detector will not show an excitation spectrum which can be expressed simply in terms of Boltzmann factor.

There are two important issues we face in investigation of the unruh effect which seem to be the source of this effect: the acceleration and the event horizon. Is the existence of an event horizon a necessary condition for the Unruh effect? What about acceleration? In [16] following up [9] the existence of horizon is assumed as a necessary condition for creation of the Unruh effect; When there is an event horizon we can define two different Fock space and mixing creation and annihilation operators and will expect to have nonzero Bogolyubov coefficients. Also they argue that "for a rotating observer there is no event horizon since the orbit is restricted to a bounded region of space, so that a signal from an event anywhere in space will be able to reach the spacetime curve and any spacetime point can be reached by a light signal from a point on the curve." and conclude that for a uniformly rotating observer there is no corresponding unruh effect. The observer in our special relativistic approach which is the same as Mashhoon observer [23], has the $a/c < \omega$ condition and is the same as uniformly rotating observer in [16] and so, according to it's result, should not observe Unruh effect. But as mentioned in [25] if the existence of horizon is necessary then even for linear accelerating detector the particle detection will be impossible, unless there is a detector with constant acceleration from the past infinity to future infinity and this situations is practically inaccessible. On the other hand if the acceleration is the necessary and sufficient condition, then since the eccentric rotating observer has centripetal acceleration, particle detection can be expected. The remaining point is that the work done by the centripetal force in the case of uniform circular motion is zero and it can be an important differentiation between Rindler and uniformly rotating observer.

In [14, 15] using Trocheries-Takeno transformations, which we called Franklin transformations (FT) [19], it is shown that the rotating observer defines a vacuum state which is different from the Minkowski one. But as we have discussed [19], FTs have all kinematical
problems of GRT and can not be applied for relating eccentric rotating detector to centric laboratory observer. In addition, in [15] the Klein-Gordon’s solution that has given in rotating frame is coordinate transformed solution of the inertial one. But it is easy to show that by this assumption, unlike their conclusion, always we will have zero Bogolyubov coefficient. If \( g(x') \) in (12) is coordinate transformed of \( f(x) \), when we calculate integral [13] we need to express \( g \) and \( f \) in the same coordinates and need to apply inverse transformation on \( g \), so we will have

\[
\beta = -(g(x'), f^*(x)) = -(f(x), f^*(x)) = i \int \sum (f \nabla_\mu f - f \nabla_\mu f n^\mu \sqrt{-h} dx^{n-1}) = 0 \tag{30}
\]

and so it is impossible to obtain nonzero coefficient \( \beta \) by that suggested solution.

Acknowledgments

The author thanks University of Gonabad for supporting this project under the grants provided by the research council. Also thanks Prof. M. Nouri-Zonoz for primary valuable suggestion.

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