Computer simulation of ion beam-plasma interaction

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Abstract. In the paper a two-dimensional axisymmetric hybrid numerical model of the interaction of an ion beam with a plasma is presented. The model is based on the kinetic approximation for the ions whereas the electrons are assumed to be a fluid. To solve the Vlasov kinetic equation, the author's modification of the particle in cells method (PIC) is used. It was shown that the magnetic flux can be expelled from the volume filled by the plasma due to the plasma-beam interaction. The dynamics of the magnetic field cavity formation depending on the characteristics of the ion beam and the background plasma is investigated.

1. Introduction

The interaction of charged particle beams with plasma plays an important role in all types of gas discharge as well as in development of plasma injection methods in magnetic traps, measurements of plasma parameters and distribution functions. In particular, experiments on injection of beams, accumulation of fast ions and generation of a diamagnetic plasma confinement mode are carried out at the CAT facility (BINP SB RAS) [1]. The beam-plasma interaction plays a crucial role and determines the nature of physical processes also in a number of astrophysical phenomena, including interaction of the solar wind with the Earth’s magnetosphere, solar flares, supernova explosions. Despite numerous studies of beam-plasma interaction, many issues related to possible mechanisms for the development of turbulent regimes have not yet been resolved. The processes of beam-plasma interaction are non-linear, which makes it difficult to apply theoretical methods for solving the problem and leads to the need for numerical modelling.

The most complete description of plasma processes is possible on the basis of kinetic models. However, the large difference in the spatial and temporal scales of the ion and the electron components of the plasma imposes significant restrictions on the numerical implementation of these models even on modern supercomputer systems. The use of MHD models, where the plasma is represented by a conducting fluid, does not allow description of the processes taking into account the finite larmor radius of the ion component of the plasma, which play a crucial role in the development of the beam instabilities. At present, combined models are widely used, the ion component of the plasma is described by the Vlasov kinetic equations, and the electron component by the MHD equations [2, 3]. Hybrid models bridge the gap between MHD and kinetic approaches as well as spatial-time scales. The use of such models allows, in the case of the magnetization of electrons, to trace the dynamics of plasma flows for a wide range of plasma characteristics. In this paper, on the basis of the hybrid model [4], the process of injection of an ion beam into a magnetized plasma is considered. The created model made possible to take into account the large difference in the space-time scales that are specific for the problem under
consideration. Based on numerical simulation, the formation of a magnetic cavity, the characteristics of the generated plasma structures and their dependence on the parameters of the beam-plasma system and the external magnetic field are studied.

2. Statement of the problem

Let us consider the following problem. At the initial moment of time, the background hydrogen density plasma \( n_0 = \text{const} \) is located inside a cylindrical chamber of radius \( R_0 \) and length \( L \). The magnetic field \( \mathbf{B} = (B_r, 0, B_z) \) is created by a system of coils with currents on the border \( r = R_0 \). At the point with coordinates \( R = 0, z = L / 2 \) a beam of accelerated neutral atoms enters with a constant velocity \( \mathbf{V} = (V_r, 0, V_z) \). In the background plasma the particles decay into ions and electrons. The processes of decay of atoms in the problem are not considered. The injected beam has temperature \( T_b \), the stationary electron and ionic components of the background plasma are assumed to be cold \( (T_e = 0, T_i = 0) \). Due to axial symmetry, the problem is considered in a two-dimensional cylindrical formulation. To solve this problem, a hybrid numerical model applying the particle-in-cell method (PIC) is used. The original system of equations includes the Vlasov equations for the ion component of the plasma and the incoming beam, the equations of magnetic hydrodynamics for the electrons and Maxwell equations:

\[
\frac{\partial f_i}{\partial t} + \mathbf{v} \cdot \nabla f_i + \frac{\mathbf{F}_i}{m_i} = 0
\]

\[
\mathbf{F}_i = q_i \left( \mathbf{E} + \frac{1}{c} \mathbf{v} \times \mathbf{B} \right) + \mathbf{R}_i
\]

\[
m_e \left( \frac{\partial V_e}{\partial t} + (\nabla \cdot \mathbf{V}_e) \mathbf{V}_e \right) = -e \left( \mathbf{E} + \frac{1}{c} \mathbf{V}_e \times \mathbf{B} \right) - \nabla p_e \frac{n_e}{n_e} + \mathbf{R}_e
\]

\[
n_e \left( \frac{\partial T_e}{\partial t} + (\nabla \cdot \mathbf{V}_e) T_e \right) + (\gamma - 1) p_e \nabla \cdot \mathbf{V}_e = (\gamma - 1) (Q_e - \nabla \cdot \mathbf{q}_e)
\]

\[
\nabla \times \mathbf{B} = \frac{4\pi}{c} \mathbf{j}
\]

\[
\nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t}, \quad \nabla \cdot \mathbf{B} = 0
\]

Here, \( f_i \) is the ion distribution function, \( \mathbf{B} \) and \( \mathbf{E} \) are the magnetic and the electric fields, \( \mathbf{V}_e \) is the electron velocity, \( \mathbf{V}_i = \frac{1}{m_i} \int f_i(r, \mathbf{v}, t) \mathbf{v} n_i \) is the average velocity of ions, \( p_e \) and \( T_e \) are the pressure and temperature of the electrons. The collision force \( \mathbf{R}_i = -\mathbf{R}_e \) takes into account the exchange of pulses between the electron and ion components of the plasma and beam, \( \mathbf{R}_e = -m_e (\mathbf{V}_e - \mathbf{V}) / \tau_{ei} \), where \( \tau_{ei} \) is characteristic ion-electron collision time. From the dissipation mechanisms, plasma conductivity and electronic thermal conductivity are taken into account. Heat generated in electrons is \( Q_e = j^2 / \sigma \), electron heat flux is \( q_e = \kappa \nabla T_e \). Here it is assumed that the coefficient of thermal conductivity \( \kappa \) and frequency of collisions \( \nu = 1 / \tau_{ei} \) caused by anomalous processes of charged particles scattering on fluctuations of electromagnetic fields and do not depend on the parameters of the plasma and magnetic field \( (\nu = \text{const}, \kappa = \text{const}) \). In the calculations, the adiabatic index \( \gamma = 5/3 \) is used. To solve the Vlasov kinetic equation, the author's modification of PIC is used [5]. To solve the MHD and Maxwell equations, finite-difference schemes of the first order of accuracy are used. To solve the heat balance equation for electrons the up-wind scheme is used. In hybrid models, the condition of plasma quasi-neutrality is assumed to be \( n_e = n_i \) (the scales under consideration are larger than the Debye radius). The displacement currents are also neglected, only low-frequency processes are considered. The electric field is determined from the equation of motion of electrons under the assumption \( m_e = 0 \). This approximation ignores the dispersion effects associated with the electronic component of the plasma that define structure of waves propagating across the magnetic field.
When solving the Vlasov equations by the PIC method, a transition to Cartesian coordinates and back to cylindrical is made. The cell volume is proportional to the distance to the axis of the cylinder, the mass and the charge of the model particle also depend on their position relative to the axis. The characteristic size of solving problem is the ion dispersion size $\lambda = c/\omega_{pi}$, where $\omega_{pi} = (4\pi ne^2/m_i)^{1/2}$ is the ion plasma frequency. The step of a uniform spatial grid used in solving the problem is $0.1 \lambda < h_i < 0.2 \lambda$.

The dynamics of the interaction of an ion beam with a background plasma in magnetic field is traced up to the time moments $t = 40\omega_{iH}^{-1}$, where $\omega_{iH} = eB / m_i c$ is the ion cyclotron frequency. In the computations we used up to $10^6$ particles and the time step $\tau = 10^{-4} \omega_{iH}^{-1}$.

The series of calculations required developing a parallel implementation of the created program and its application on supercomputer complex (SSCC ICMMG SB RAS). The parallel code is based on the decomposition of the computational domain and particles decomposition within each subdomain for the effective load balancing.

3. Results of numerical simulation
Let us consider the dynamics of the plasma flow and the structure of the beam-plasma configuration when the ion beam is injected into a magnetized uniform density plasma. The initial configuration of the magnetic field corresponds to the configuration of an open magnetic trap with a mirror ratio $B(0, L/2)/B(0, 0) = 2$ and is set by solving the Poisson equation. Continuous injection of the beam is accompanied by the displacement of the magnetic field and the formation of its cavity. Inside the cavity, the value of the magnetic field may be less than 1% of its original value. The evolution of the magnetic field at successive moments of time is shown in figure 1.

**Figure 1.** The magnetic field lines at $t = 5, 10, 20$ (from left to right).

Hereinafter, the spatial dimensions are expressed in units of $c/\omega_0$, the time – in $\omega_{iH}^{-1}$, the velocity is normalized to the Alfvén speed $V_A = B_0/(4\pi n_0 m_i)^{1/2}$, where $B_0 = B(0, L/2)$, the value of the magnetic field on the axis of the system, $n_0$ – the density of the background plasma at the initial time moment. The size of the magnetic cavity at a given time $t$ depends on the density of the beam $N$ and its velocity $V$. So at the time $t = 10$, when the injection velocity is $V = 0.1$ for the beam density $N = 4$ and $N = 100$ the transverse size of the cavity is 0.24 and 0.63, respectively ($z = L/2$). With a constant beam density of $N = 100$ and velocity values $V = 0.1, 0.2, 0.4$, the transverse size of the cavity is 0.63, 1.01, 1.38, respectively. The dependence of the transverse size of the magnetic field cavity on the time, shown in figure 2, demonstrates the formation of the quasi-stationary mode. This mode is characterized by a nearly constant transverse dimension of the magnetic cavity at the moments of time $t > 20$ for $V = 0.1, N = 100$.

The amplitude of the magnetic field at the time $t = 20$ is $A = B(t = 20)/B_0 = 1.6$. The beam deceleration in a magnetic field is accompanied by a change of the ion trajectory due to larmor rotation and the appearance of the $\varphi$-component of its velocity $V_{\varphi}$. The collisionless laminar mechanism of the effective energy exchange of the beam-plasma is provided by the generation of vortex electric field $E_\varphi$. This mechanism was studied in detail in [6]. The map of the $\varphi$-component of the electric field $E_\varphi$ is shown in figure 3 at the time $t = 20$. From this figure one may observe that the maximum field values are concentrated in the region of the beam braking at the boundary of the magnetic cavity. The density distribution of the injected beam in the central section of the simulation box, $z = L/2$, at successive times is shown in figure 4. The label 1 corresponds to the time $t = 6$, label $2 - t = 14$, label $3 - t = 18$, label $4 - t = 20$. The self-consistent interaction of the injected beam with the magnetic field and the background
plasma leads to the displacement of the magnetic field in the injection region and, accordingly, to an increase in the gyroradius of the ions there. The size of the beam penetration is defined by the larmor radius in the magnetic field of the cavity.

Figure 2. The cavern radius time evolution.

Figure 3. The electric field $E_\phi$ at $t = 20$.

Figure 4. The beam density distribution.

Figure 5. The background density distribution.

As a result of the interaction of plasma flows, plasma disturbances are generated, which propagate in the background plasma in the form of a compression wave. The structure of the compression waves in the central section $z = L/2$ at successive times is shown in figure 5. The times $t = 6$ is marked with the label 1, $t = 14$ – with the label 2. The spatial size of the wave propagating with the velocity $U$ is $d = c^2/(4\pi\sigma U)$ and it depends on the value of the magnetic conductivity $\sigma = ne^2/m_e\nu$. The size $d \approx 0.2$ at $\nu = 0.2$, $U = 0.1$. The beam propagation is accompanied by the displacement of the background plasma from the region of the magnetic cavity, which is filled with the plasma of the beam being injected (figure 4 and 5).

The results of numerical simulation were obtained for the plasma and injected beam parameters applied to the conditions of laboratory experiments at BINP SB RAS: $B_0 = 0.2$ T, $n_0 = 10^{12}$ cm$^{-3}$, $T_b = 10$ eV, $V_i = 4 \times 10^7$ cm s$^{-1}$, $N = 10^{14}$ cm$^{-3}$.

4. Conclusion

On the basis of numerical modeling, nonlinear processes of continuous injection of an ion beam into the magnetic system of an open plasma trap are investigated. The self-consistent beam-plasma interaction leads to the formation of a magnetic cavity, the size of which determines the volume of accumulated plasma.
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