Algorithm For Constructing Modified Relative Phase Permeability For The Average Model Of Three-Phase Filtration In A Multilayered Layer

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Abstract. The paper presents models of relative phase permeabilities for laboratory relative phase permeabilities. A technique has been developed for modifying the relative phase permeabilities taking into account the jet flow. Using a computational experiment, the applicability of the proposed averaged models was investigated. The models described in the article are applicable in numerical calculations for multilayered reservoirs in order to reduce the dimension of the problems under consideration.

Keywords: filtration, phase permeability, modified permeability, jet flow pattern, computational experiment.

1. Introduction
Oil fields have, as a given, a pronounced layered heterogeneity. This greatly complicates multivariate hydrodynamic three-dimensional calculations. The mathematical problem is much more complicated if we take into account the three-phase (gas, oil, water) flow. In this case, the calculations require detailed geological information about the formation and significant time calculations on computer. Therefore, in practice, models averaged over the thickness of the formation are often used, allowing reducing the dimension of the initial problem in certain areas of the multilayer field and thereby significantly simplifying numerical calculations [1-3]. It should be noted that despite the widespread use of foreign and domestic software systems created by Landmark Halliburton Int, Schlumberger, Roxar Software Solution, Tigers, CMG, LAURA, etc., the question of reducing the dimension of the problem remains relevant, especially when combining layers into bundles in some areas of deposits, as well as when solving multivariate problems of optimal development of real layers.
2. The purpose of the work

The purpose of the work is to construct mathematical relative phase permeabilities from laboratory (experimental) relative phase permeabilities, to create a modification technique for the constructed relative phase permeabilities taking into account the jet flow, using a computational experiment, investigate the applicability for calculating averaged models, including those with modified relative phase permeabilities, previously proposed for two-phase filtration in [1-3].

2.1 Algorithm for the experimental determination of relative phase permeabilities in the oil-water system.

To determine the experimental relative phase permeabilities by the method of unsteady displacement (unsteady-state displacement) perform the following operations [4] (in the oil-water system):

1. A core is made from a core.
2. An initial oil saturation was created in this sample. Steps for that are as follows:
   - Saturate the sample with a model of produced water, drive off the moving water using a centrifuge.
   - Then the sample is saturated with kerosene. Subsequently, oil is pumped in the unit, replacing kerosene. Carry out the displacement of oil by water. Water is pumped through this oil-saturated sample. During the experiment, the amount of water injected, the amount of oil displaced, and the pressure drop across the sample are recorded.

After data processing, relative phase permeabilities are obtained. Similarly, core research is carried out for the oil-gas system. To simulate three-phase filtration, empirical formulas for carbonate rocks were obtained from the relative phase permeabilities obtained in the laboratory. Fig. 1 shows graphs of laboratory relative phase permeabilities for various cores and comparison of the graphs obtained by the formulas for relative phase permeabilities [3] in the form of functions:

\[
\begin{align*}
  k_{ro}(S_o) &= k_{ro}^0 \left[ (S_w - S_{wc}) / (1 - S_{wc} - S_{or} - S_{gc}) \right]^\alpha \\
  k_{rw}(S_w) &= k_{rw}^0 \left[ (S_o - S_{or}) / (1 - S_{wc} - S_{or} - S_{gc}) \right]^\beta \\
  k_{rg}(S_g) &= k_{rg}^0 \left[ (S_w - S_{gc}) / (1 - S_{wc} - S_{or} - S_{gc}) \right]^\gamma \\
  k_{ro}(S_p) &= k_{ro}^0 \left[ (S_o - S_{or}) / (1 - S_{wc} - S_{or} - S_{gc}) \right]^\delta
\end{align*}
\]

Where ,

- \( k_{ro}^0 \) – maximum relative phase permeability of oil; \\
- \( k_{rw}^0 \) – maximum relative phase permeability of water; \\
- \( k_{rg}^0 \) – maximum relative phase permeability of gas; \\
- \( S_{or} \) – residual oil saturation; \\
- \( S_{wc} \) – saturation of bound water; \\
- \( S_w, S_o \) – water and oil saturation,

\( S_{wc} \leq S_w \leq 1 - S_{or} \),
To obtain empirical formulas, it is necessary to select the following coefficients $\gamma, \mu, \phi, Swc, Sgc, \alpha, \beta$, so that the graphs are as close to the laboratory as possible.

As a result, we obtained for the first sample

$$\gamma, \mu, Swc=0.286, Sgc=0; \alpha_w=2.3, \beta_w=1.3, \alpha_g=1.3, \beta_g=3,$$

And for the second sample

$$\gamma, \mu, Swc=0.247, Sgc=, \alpha_w=2.5, \beta_w=2.8, \alpha_g=1.3, \beta_g=3.$$

Basically, the obtained coefficients are close, except for the degrees. Degrees vary from 1 to 3. In Fig. 1 and Fig. 2 it is clearly seen that the obtained theoretical graphs are close to the laboratory graphs. From the mathematical two-phase relative phase permeabilities, three-phase relative phase permeabilities were obtained according to the Stone II model [4].

3. Computational experiment results

As an example, a ten-layer square-shaped layer was considered, which was developed by different water flooding systems. The problem was solved numerically for a given pressure differential between injection and producing wells, the external boundary was impenetrable. In the calculations, we used grids of blocks shown in Fig. 4: 11x11x10 (x, y, z) for the reference three-dimensional case; and 11x11x1 for the averaged two-dimensional case (rescaled). In the calculations, a completely implicit scheme was used.

Fig. 1: Laboratory relative phase permeabilities RPP for carbonate rocks.
Figure 3 shows graphs of modified RPP (oil-water) versus water saturation $S_w$ for a uniform law with linear RPP. The graphs of the modified RPP for the oil-gas system were obtained in a similar form. The calculation formulas have the form [2,3]:

\[
k_{rw}^m = k_{rw} \cdot [1 + \nu \cdot \sqrt{3} \cdot (1 - S_p)],
\]

\[
k_{ro}^m = k_{ro} \cdot [1 - \nu \cdot \sqrt{3} \cdot S_p].
\]

Here the mobility of water $S_p$

The oil-water system is calculated

Figure 2: Mathematical relative phase permeabilities RPP for carbonate rocks.
128 – initial reservoir pressure, atm;
22 – formation temperature, C;
100 – temperature of injected water, C;
55 – bottom hole pressure at the producing well, atm.;
170 – bottom hole pressure at the injection well, atm.;
=0.5 – maximum relative phase permeability of water;
=0.7 – maximum relative phase permeability of oil;
=0.8 – maximum relative phase permeability of gas;
Sor=0.2 – residual oil saturation;
Swc=0.2 – saturation of bound water;
Sgc=0.1 – pinch gas saturation;
Sw,So,Sg – water, oil and gas saturation,

Swc ≤Sw≤ 1- Sor.- Sgc.

Tasks were solved with physical parameters [2]: 1230 - reservoir depth, m; 1000 - distance between producing wells for a 5-point element, m; 707 - between production and injection wells for a 5-point element, m; 500 - distance between producing wells for a 9-point element, m; 500 and 707 - between production and injection wells for a 9-point element, (in meters).
The numerical calculations of the process within the framework of the Musket - Moris three-phase filtration model for five-point and nine-point water flooding systems were carried out for four different mathematical models.

3.1. Two-dimensional models :

1. C - averaged model with mathematical relative phase permeabilities and average absolute permeability over the thickness of the reservoir.
2. B is an averaged model described in detail in [2,3], with modified relative phase permeabilities for the β-probability distribution law for parameters of the form γ = 1, η = 2. This is a well-known uniform law; it was applied at the highest value of the coefficient of variation of vertical heterogeneity equal to 0.56. In this case, the absolute permeability of the layers was set by table No. 1.

3.2. Three-dimensional models :

1. A8 – the reference three-dimensional model with ten isolated layers of the same thickness (no vertical flows);
2. A7 - the same as the previous model, but with uninsulated layers.

Table 1. The distribution of permeability in layers for the A7-model for a uniform law, m darcy

| K_1 | K_2 | K_3 | K_4 | K_5 | K_6 | K_7 | K_8 | K_9 | K_10 |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 4500| 5500| 3500| 6500| 2500| 7500| 1500| 8500| 500 | 9500 |

The calculations were performed for 8 different versions of the standards described in detail in [3], but Fig. 4 shows graphs of only two boundary models. All other graphs of standards are between the graphs of options A8, A7.
It is clearly seen in Fig. 5 that the averaged solution B with modified relative phase permeabilities has a smaller error with respect to the reference solution A8, and solution C with mathematical relative phase permeabilities has a smaller error with respect to the reference solution A7. Similar results were obtained for the nine-point water flooding system, and for other development indicators, both for gas and oil. Solutions B and C limit all three-dimensional reference solutions Ai for which calculations have been performed.

![Graph of the functions of the dependence of the total volume of extracted oil and gas Vp on the time date for a five-point water flooding system, and accordingly for solutions - A7, A8, C, B](image)

**Fig. 5.** Graphs of the functions of the dependence of the total volume of extracted oil and gas $V_p$ on the time date for a five-point water flooding system, and accordingly for solutions - A7, A8, C, B

### 4. Conclusions.

A comparative analysis of the graphs of the development indicator functions for various three-phase models revealed the correct construction of averaged modified relative permeabilities in three-phase models. Numerical calculations on three-phase models showed good results, - the graphs of model B from the lower limit for the extracted volumes of oil and gas, and the graphs of model C from the upper limit for all 8 standards considered in the work. This indicates the validity of applying this averaging technique for a three-phase filtration model. Similar positive results in the numerical calculations of two-phase filtration took place in [2,3].
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