On the magnetic catalysis and and inverse catalysis of phase transitions in the linear sigma model

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Abstract
We consider the evolution of critical temperature both for the formation of a pion charged condensate as well as for the chiral transition, from the perspective of the linear sigma model, in the background of a magnetic field. We developed the discussion for the pion condensate in one loop approximation for the effective potential getting magnetic catalysis for high values of B, i.e. a raising of the critical temperature with the magnetic field. For the analysis of the chiral restoration, we go beyond this approximation, by taking one loop thermo-magnetic corrections to the couplings as well as plasma screening effects for the boson masses, expressed through the resumation of ring diagrams. Here we found the opposite behavior, i.e. inverse magnetic catalysis, i.e. a decreasing of the chiral critical temperature as function of the intensity of the magnetic field, which seems to be in agreement with recent results form the lattice community.

Keywords:

1. Introduction

Recently [1] we studied the formation of a pion charged condensate in the frame of the linear sigma model. The main idea was to consider the effective potential at the one loop level, taking the isospin chemical potential near the effective pion mass, varying then the intensity of the magnetic field in order to obtain the critical temperature. Defining

$$\int_\beta dx \equiv \int_0^\beta dx_3 \int d^3x,$$

where $\beta = 1/T$, being $T$ the temperature of the system, using a Lagrangian without fermionic sector but including isospin chemical potential $\mu_i$ a magnetic field, through the covariant derivative $D_\mu = \partial_\mu + i q A_\mu$, and expressed in terms of the charged pion fields $\pi^+\pi^-$ and $\pi^0$, the neutral pion field $\pi_0$ and the $\sigma$ field gives us the following action

$$S = \int_\beta dx \left[ (\partial_\mu - \mu_i)\pi^+_i (\partial_\mu + \mu_i)\pi^-_i + (\partial_i - ieA_i)\pi_0 (\partial_i + ieA_i)\pi_0 + \frac{1}{2}[(\partial\sigma)^2 + (\partial\pi_0)^2 + \mu_0^2(\sigma^2 + \pi_0^2 + 2\pi_+\pi_-)] + \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2) - c\sigma \right].$$

The term $c\sigma$ corresponds to the explicit chiral symmetry breaking term, being $c = f_\pi m_\pi^2$ and where $f_\pi$ is the pion decay constant. In the symmetric gauge, the external gauge field which produces a uniform magnetic field in the $z$ direction can be written as

$$A_z = \frac{1}{2}B \times \vec{r} = \frac{1}{2}B(-x_2, x_1, 0).$$

We may assume that the expectation value of the sigma field $\sigma$ has a non-vanishing value. This expectation

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value, if the explicit term $c \sigma$ is absent is responsible for the breaking of the chiral symmetry. Since the isospin symmetry is also broken, due to the formation of the charged pion condensate, we may expand both fields as quantum fluctuations around the classical fields

$$\sigma(x) = \bar{\sigma} + \sigma(x), \quad \pi(x) = \frac{1}{\sqrt{2}} \phi(x) + \bar{\pi}(x). \quad (4)$$

We proceed then to compute the effective potential, by considering that the order parameter $\bar{\sigma}$, a spatial average of the charged pion condensate, will be close to the normal phase, i.e. $\bar{\sigma} \approx 0$,

$$\bar{\sigma}(x) \equiv \left[ \frac{1}{V} \int d^3 \chi \phi^2 \right]^{1/2}, \quad (5)$$

where $V$ is the volume of the system. See [1] for technical details. $\phi_e$ satisfies a non-relativistic Schrödinger equation

$$[-(\nabla - eB)^2(x_1^2 + x_2^2)/4 + m_e^2 - \mu_e^2] \phi_e = E_e^2 \phi_e. \quad (6)$$

which reminds us the two-dimensional harmonic oscillator whose eigenvalues are given by

$$E_l^2(p_e) = p_e^2 + m_e^2 + (2l + 1)eB - \mu_e^2. \quad (7)$$

From these considerations, and restricting ourselves to the ground state, the classical field reads

$$\phi_e = \bar{\sigma} \left( \frac{1 - e^{-\Phi/2\Phi_0}}{\Phi/2\Phi_0} \right)^{1/2} e^{-eB(x_1^2 + x_2^2)/4}, \quad (8)$$

where $\Phi \equiv BA$ is the magnetic flux, $A$ is the area transverse to the external magnetic field and $\Phi_0 \equiv \pi/q$ is the quantum magnetic flux. With this definition of the order parameter $\bar{\sigma}$ it turns out that the tree-level effective mass is independent of the magnetic flux. A different prescription will produce a global flux dependent term. The relevant Feynman diagrams for the computation of the effective potential are shown in Fig. [1].

In this form we found, for high values of the magnetic field, magnetic catalysis as it is shown in Fig. [2]. For lower values of the magnetic field we found anticatalysis.

### 2. Critical temperature for chiral restoration as function of the magnetic field

We also explored the chiral symmetry restoration, as function of the temperature and the intensity of the magnetic field, which is of course related to evolution of

$$\bar{\sigma}(T, B)$$

but going beyond the usual mean field approximation. In fact the essential new points of our calculation are the introduction of thermo-magnetic corrections to the vertices, both for the bosonic as well as for the fermionic sector, and the resumation of the self energy corrections for the propagators of the bosonic field in the evaluation of the effective potential. The charged pions and the quark propagators were handled according to Schwinger’s proper time representation. This analysis has shown that an inverse magnetic catalysis behavior, i.e the critical temperature decreases as function of the magnetic field strength, emerges already for small values of the magnetic field. In the next section we just mention some details. The reader should go to the original reference [2] for a complete description of the technical details. Here we added the fermionic sector, ne-
glecting density effects. Our model is given then by

\[
\mathcal{L} = \frac{1}{2} (\partial_\mu \sigma)^2 + \frac{1}{2} (D_\mu \vec{\pi})^2 + \frac{\mu^2}{2} (\sigma^2 + \vec{\pi}^2) - \frac{\lambda}{4} (\sigma^2 + \vec{\pi}^2)^2 + i \bar{\psi} \gamma^\mu D_\mu \psi - g \bar{\psi} (\sigma + i \gamma^5 \vec{\tau} \cdot \vec{\pi}) \psi. \tag{9}
\]

\(\psi\) is an \(SU(2)\) isospin doublet. We have computed the effective potential going beyond the strictly one-loop approximation by considering self energy leading high temperature corrections to the boson propagators and one loop thermo-magnetic corrections to the bosonic and fermionic couplings as well. The self energy corrections were inserted later, according to the spirit of the ring resumation, as plasma screening effects for the boson’s mass squared \([3]\). The same philosophy was adopted previously in a discussion of the abelian Higgs model \([4]\) were we also found magnetic anticalysis. In Fig. 3 we show the relevant diagrams for the one loop boson self energies. Only the charged pions feel the external magnetic field, being then handled as Schwinger propagators.

\[\text{(a)} \quad \text{(b)} \quad \text{(c)}\]

Figure 3: One-loop Feynman diagrams contribution to the boson’s self energies. The dashed line denotes the charged pion, the continuous line is the sigma and the double line represents the neutral pion and the continuous line with arrows represents the quarks.

The leading order at high temperature from these diagrams is given by

\[
\Pi = \lambda \frac{T^2}{2} + N_f g^2 \frac{T^2}{6}, \tag{10}
\]

where \(N_f\) is the number of quark species.

In Fig. 4 we show the relevant one loop diagrams for the thermo-magnetic corrections to the coupling \(\lambda\). In Fig. 5 the relevant diagrams for the thermo-magnetic corrections to the coupling \(g\) are shown.

The calculation for the coupling corrections were carried on in the weak field limit approximation where well known expressions for the bosonic \([5]\) and the fermionic propagators \([6]\) were used.

We refer the reader to \([2]\) for the explicit expression of the effective coupling \(\lambda_{\text{eff}}\) that emerges form this analysis. It diminishes as function of the external magnetic field. Contrary to the effective bosonic coupling, the triangle corrections to the fermionic coupling \(g\) produce an effective coupling \(g_{\text{eff}}\) that increases in a very mild way with the external field.

The philosophy behind our calculation was to find the effect of the magnetic field on the critical temperature where the curvature of the effective potential vanishes. In Fig. 6 we show the critical temperature behavior, normalized by the critical temperature in the absence of an external field, including the full thermo-magnetic dependence of the couplings. Here we have set the tree level coupling \(\lambda\) to fixed value and vary the tree level coupling \(g\).

A similar behavior was found in the complementary case where we set the tree-level coupling \(g\) to a fixed value and vary the tree-level coupling \(\lambda\). These results seem to be in agreement with recent lattice QCD results \([7–9]\) which indicate that the transition temperature for chiral restoration with \(2 + 1\) quark flavors, as measured from the behavior of the chiral condensate and susceptibility as well as from other thermodynamic observables, significantly decreases with increasing magnetic field.
Figure 6: Color on-line Effect of the full thermomagnetic dependence of couplings on the critical temperature for a fixed value of the tree-level $\lambda = 0.225$ and different values of the tree-level $g$ as function of $b = qB/\mu^2$. In all cases the critical temperature is a decreasing function of $b$.

It is interesting to mention in this context a different approach. Recently, Refs. [10, 11] have postulated an ad hoc magnetic field and temperature dependent running coupling, inspired by the QCD running of the coupling with energy, in the Nambu-Jona-Lasinio model, which makes the critical temperature decrease with increasing magnetic field.

3. Conclusions

We have discussed the magnetic evolution of two different phase transitions that may occur in hadronic physics: the formation of a charged pion condensate triggered by the presence of a non-vanishing isospin chemical potential, which is usually refer as the pion superfluid phase and the restoration of chiral symmetry breaking. The analysis was done in the frame of the linear sigma model. Our results, from a strict one-loop analysis indicate magnetic catalysis for the formation of the pion condensate for high values of the magnetic field. The discussion of chiral symmetry breaking was much more involved, since we incorporate in the effective potential thermo-magnetic corrections to the couplings as well as plasma screening effects emerging from a resumation of one loop self energy boson corrections. Here we found antica catalysis, i.e. a decreasing behavior of the critical temperature as function of the external magnetic field. This result seems to be in agreement with recent results form the lattice QCD community.

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