Resonance nucleon cross section in the photonuclear reaction

Swapan Das

Nuclear Physics Division, Bhabha Atomic Research Centre, Trombay, Mumbai 400085, India
Homi Bhabha National Institute, Anushakti Nagar, Mumbai 400094, India

Abstract

The cross section of the photonuclear reaction in the resonance region is calculated to search the resonance-nucleon cross section in the nucleus. The latter cross section is used to evaluate the resonance-nucleus optical potential which can modify the cross section of the photonuclear reaction, arising due to the resonances produced in the reaction. The calculated photonuclear reaction cross section is compared with the data to explore the nuclear effect on the resonance-nucleon cross section.

1 Introduction

The energy distribution spectrum of the cross section measured for the photonuclear reaction in the resonance region ($\approx 0.2 - 1.2$ GeV) [1, 2] shows the disappearance of certain resonances which are distinctly visible in that measured for the photonucleon reaction [3, 4, 5]. The spectra measured for nuclei show the $\Delta(1232)$-peak is partially modified where as the peaks of higher resonances (e.g., $N(1520)$, ....) are vanished in the nucleus. The change in the spectral-shape of resonances in a nucleus occurs because of the nuclear medium effect on them. The Fermi-motion of the nucleon in a nucleus produces the suppression and broadening of the resonance peak. The resonance-nucleus interaction can also dampen and widen the peak of resonance, which is addressed as the collision broadening of the resonance. It linearly depends on the in-medium resonance-nucleon cross section, i.e., $\Gamma^c_R \propto \sigma^{*\text{RN}}_t$ [6].

The photonuclear reaction in the resonance region has been studied by Rapp et al., [7] using the vector-meson dominance model which states the photon is converted to the transverse $\rho$-meson. Hence, the $\gamma$-nucleon resonances have been interpreted as the $\rho$-nucleon resonances. The parameters appearing in this calculation are extracted by fitting the calculated photo-proton reaction cross section with the data. Using these parameters, the cross section of the photonuclear reaction is evaluated, and those are shown in well accord with the data reported for nuclei [7]. Kondratyuk et al., [8] has also calculated the cross section of the photonuclear reaction in the quoted region by evaluating the forward Compton scattering amplitude of the reaction. They have extracted the $\gamma$-nucleon

email: swapand@barc.gov.in
resonances’ parameters (i.e., mass and width) by fitting the calculated photonucleon reaction cross section with the data, and used those to evaluate the cross section of the photonuclear reaction.

The forward Compton scattering amplitude of the photonuclear reaction expressed in this paper differs from that done by Kondratyuk et al., [8]. The measured $\gamma$-nucleon resonance parameters [3], except for the mass of the $\Delta(1232)$ resonance, have been used to calculate the cross sections for the photonucleon and photonuclear reactions. As shown later, the calculated results due to $\Delta(1232)$-mass equal to 1220 MeV agree with the measured distributions (in the $\Delta(1232)$ resonance region) of the above reactions. The propagator of the resonance has been expressed by the eikonal form in the Glauber model [6]. The resonance-nucleus interacting (optical) potential appearing in the propagator is generated by folding the resonance-nucleon forward scattering amplitude with the nuclear density. The calculated results of the photonuclear reactions are fitted with the measured spectra [1] to extract the in-medium resonance-nucleon cross sections.

2 Formalism

The Lagrangian describing the resonance $R$ production due to the photon-nucleon $\gamma N$ coupling is categorized based on the positive or negative parity state of $R$. For the positive parity state, the Lagrangian describing the $R\gamma N$ coupling is given by [7, 9]

$$\mathcal{L}_{R\gamma N} = \frac{f_{R\gamma N}}{m_\pi} R^\dagger (s^\dagger \times k_\gamma) \cdot \epsilon(\lambda_\gamma) i_r^\dagger N. \quad (1)$$

The spin operator $s^\dagger = \sigma(S^\dagger)$ connects $N$ of $J^P = \frac{1}{2}^+$ to $R$ of $J^P = \frac{1}{2}^+(\frac{3}{2}^+)$, where $J^P$ denotes spin-parity of the particle. $i_r^\dagger = \tau_r(I_r^\dagger)$ does the isospin $\frac{1}{2} \rightarrow \frac{1}{2}(\frac{3}{2})$ transition of the particle. For $R$ of $J^P = \frac{5}{2}^+$, the Lagrangian can be expressed as $\mathcal{L}_{R\gamma N} = \frac{f_{R\gamma N}}{m_\pi} R^\dagger S^\dagger_{ij} k_\gamma i^\dagger_j N$, where $S^\dagger_{ij}$ elucidates the tensor coupling of the spin $\frac{1}{2}^+ \rightarrow \frac{5}{2}^+$ transition. It has been discussed elaborately in Ref. [10]. The Lagrangian describing the transition $N$ of $J^P = \frac{1}{2}^+$ to $R$ of $J^P = \frac{1}{2}^-(\frac{3}{2}^-)$ is given by [7, 9]

$$\mathcal{L}_{R\gamma N} = \frac{f_{R\gamma N}}{m_\pi} R^\dagger (E_\gamma s^\dagger \cdot \epsilon(\lambda_\gamma) - \epsilon^0(\lambda_\gamma)s^\dagger \cdot k_\gamma) i_r^\dagger N, \quad (2)$$

where $\epsilon^0$ (the time-component of photon polarization) is zero for the real photon, as it has only transverse polarizations [11]. The four-starred resonances [3] used in this calculation are listed in tables 1 and 2.

The nucleus $A$ can be assumed as a composition of the nucleon $N$ and nucleus $B$. The photon $\gamma$ couples to $N$ and produces the resonance $R$ which (after propagating certain
Table 1: Four-starred positive parity resonances [3]. $J^P$ denotes spin-parity of the resonance.

| $J^P$ | $\frac{1}{2}^+$ | $\frac{3}{2}^+$ | $\frac{5}{2}^+$ |
|-------|-----------------|-----------------|-----------------|
| Resonances | $N(1440)$ | $\Delta(1232)$ | $N(1680)$ |
|         | $N(1710)$ | $\Delta(1600)$ | $\Delta(1905)$ |
|         | $\Delta(1910)$ | $N(1720)$ | 
|         | $N(1900)$ | |

Table 2: Four-starred negative parity resonances [3].

| $J^P$ | $\frac{1}{2}^-$ | $\frac{3}{2}^-$ |
|-------|-----------------|-----------------|
| Resonances | $N(1535)$ | $N(1520)$ |
|         | $\Delta(1620)$ | $\Delta(1700)$ |
|         | $N(1650)$ | |
|         | $N(1895)$ | |

distance) decays to $\gamma'N'$. The nucleon $N'$ recombines with $B$ to form the nucleus $A'$ in the final state. Symbolically, it can be represented as $\gamma A \rightarrow RB \rightarrow \gamma' A'$.

The $T$-matrix of the reaction is given by

$$T_{fi} = \int \mathcal{A}_{r'dr} \sum_{R,B} <\gamma'A'|\Gamma_{R;N}^R|RB > G_R(r' - r) < BR|\Gamma_{R,N;N}^{-}\gamma A >, \quad (3)$$

where $\Gamma_{R,N}$ denotes the vertex operator. It can be expressed by the Lagrangians in Eqs. (1) and (2) without the wave-functions of the nucleon and resonance included in it.

$G_R(r' - r)$ in Eq. (3) represents the propagation of resonance from its production point $r$ to decay point $r'$. Since the forward $T$-matrix is related the cross section of the reaction, the eikonal approximation in the Glauber model is used to expressed $G_R(r' - r)$ [6]:

$$G_R(r' - r) = \delta(b' - b)\theta(z' - z)e^{ik_{R,r}(r'-r)}D_{k_R}(b, z', z). \quad (4)$$

The non-relativistic form of $D_{k_R}(b, z', z)$ is given by

$$D_{k_R}(b, z', z) = -\frac{i}{v_{R\parallel}}exp \left[ \frac{i}{v_{R\parallel}} \int_{z}^{z'} dz'' \{ \Delta(m) - V_{OR}(b, z'') \} \right] \quad (5)$$

with $\Delta(m) = m - m_R + \frac{i}{2}\Gamma_R$, where $m$ is the invariant mass of the decay-products of the resonance $R$. $m_R$ and $\Gamma_R$ are the resonant mass and width of $R$ [3]. $V_{OR}$ denotes the resonance-nucleus optical potential, elaborated later. $v_R$ is the velocity of the resonance.
After taking the average over the initial state and the summation over the final state, the $T$-matrix for the elastic photon scattering on a spin zero nucleus can be written as $\bar{T}_{ii} = \frac{1}{2} \sum \lambda_\gamma \lambda_{\gamma'} T_{ii}$, where $\lambda_\gamma(\gamma')$ denotes the polarization of the incoming (outgoing) photon. The scattering amplitude $F_{ii}$ is related to $\bar{T}_{ii}$ as $F_{ii} = \frac{m_A}{4\pi\sqrt{s}} \bar{T}_{ii}$ [12], where $m_A$ is the mass of the target and $\sqrt{s}$ denotes the total energy in the center of mass system. In the considered energy region (i.e., $\leq 1.2$ GeV), the above equation can be reduced to $F_{ii} = \frac{1}{4\pi} \bar{T}_{ii}$ for a nucleus [12]. Therefore, the total scattering cross section $\sigma_\gamma^A$ of the photonuclear reaction is

$$\sigma_\gamma^A = \frac{4\pi}{k_\gamma} \text{Im}[F_{ii}].$$

The above formalism can be modified to calculate the cross section of the photonucleon reaction, where the Fermi-motion and the resonance-nucleus potential do not exist. The spins of the nucleons in the initial and final states have to be accounted for taking the average over the initial state and the summation over the final state.

### 3 Result and Discussions

The cross section calculated for the photo-proton $\gamma p$ reaction is presented in Fig. 1(a). The short-dashed curves represent the cross sections due to $\Delta(1232)$, $N(1520)$, $N(1535)$, $N(1680)$ and $\Delta(1700)$ resonances, as indicated in the figure. Amongst them, the dominant contribution to the cross section distinctly occurs because of the $\Delta(1232)$ resonance. The cross section $< 15 \mu b$ due to other resonances is not shown explicitly. The calculated cross section arising because of all resonances, as listed in Tables 1 and 2, is denoted by the short-long-short dashed curve. The long dashed curve refers the back-ground of the reaction, taken from Ref. [8]. The dot-dashed curve arises due to the addition of the back-ground to the cross section, i.e., it is the summation of the short-long-short dashed curve and long dashed curve. As compared in Fig. 1(b), the calculated result (i.e., the dot-dashed curve) reproduces the measured energy distribution spectrum due to Zyla et al. [3].

The cross sections evaluated for the photo-neutron $\gamma n$ reaction are shown in Fig. 2(a). It describes the results qualitatively similar to those presented in Fig. 1(a) for the $\gamma p$ reaction, except that the contribution of $N(1680)$ resonance to the cross section of the $\gamma n$ reaction is negligibly small compared to that occurring in the $\gamma p$ reaction. The calculated result of the $\gamma n$ reaction (denoted by the dot-dashed curve) is compared with the data [5] in Fig. 2(b). As shown in the figure, the calculated result agrees with the measured distribution, but it overvalues the data by 10%.

The cross sections of the photonuclear reactions have been calculated for $^{12}$C and $^{208}$Pb nuclei. The Fermi-motion of the nucleon and the back-ground in the cross section for nuclei
are taken from Ref. [8]. The interaction of the resonance with the nucleus is addressed by the resonance-nucleus optical potential $V_{OR}$, which is generated by folding the forward resonance-nucleon scattering amplitude $f_{RN}$ (in the free space) with the density of nucleus [13], i.e.,

$$-\frac{1}{v_R}V_{OR}(r) = \frac{2\pi}{k_R}f_{RN}(0)\varrho(r) = \frac{1}{2}[\alpha_{RN} + i]\sigma_{i}^{RN}\varrho(r),$$

where $\varrho(r)$ denotes the spatial distribution of the nuclear density, extracted from the electron scattering data [14].

$\alpha_{RN}$ in Eq. (7) represents the ratio of the real to imaginary parts of $f_{RN}$. $\sigma_{i}^{RN}$ is the total resonance-nucleon scattering cross section: $\sigma_{i}^{RN} = \frac{2\pi}{k_R}Im[f_{RN}]$. Assuming the elastic scattering of a resonance is not much differing from that of a nucleon [15], it can be taken as $\alpha_{RN} \approx \alpha_{NN}$ and $\sigma_{el}^{RN} \approx \sigma_{el}^{NN}$. The energy dependent measured values of $\alpha_{NN}$ and $\sigma_{el}^{NN}$ are listed in Ref. [3, 16]. For the reactive part, i.e., $\sigma_{r}^{RN} = \sigma_{i}^{RN} - \sigma_{el}^{RN}$, the resonance can be considered as the nucleon with excess energy $\Delta m$ (i.e., the mass difference between the resonance and nucleon) and $\sigma_{r}^{RN}$ can be estimated as [15]

$$\sigma_{r}^{RN}(T_{RN}) \approx \sigma_{r}^{NN}(T_{RN} + \Delta m),$$

where $T_{RN}$ is the total kinetic energy in the $RN$ center of system. $\sigma_{r}^{NN}$ denotes the reactive cross section of the nucleon-nucleon scattering. It can be determined from the measured $\sigma_{i}^{NN}$ and $\sigma_{el}^{NN}$ [3, 16].

The cross section per nucleon $\sigma_{i}^{GC}/A$ of the photo-carbon $\gamma C$ reaction has been evaluated without including the Fermi-motion of the bound nucleon and the resonance-nucleus optical potential in the calculation. The calculated results vs. the beam energy $E_{\gamma}$ are presented in Fig. 3. The curves appearing in the figure qualitatively illustrate those explained in Fig. 1(a) for the $\gamma p$ reaction.

The sensitivities of the Fermi-motion ($FM$) and the resonance-nucleus potential ($V_{OR}$) to the cross section $\sigma_{i}^{GC}/A$ are exhibited in Fig. 4. The dot-dashed curve (also shown in Fig. 3) represents $\sigma_{i}^{GC}/A$ (including the back-ground) of the $\gamma C$ reaction, where $FM$ and $V_{OR}$ are not considered in the calculation. The long-dashed curve results because of the inclusion of the Fermi-motion, taken from Ref. [8], in the calculated cross section. The broadening of the resonance peaks due to the Fermi-motion is distinctly visible. In fact, the peaks in the region of $N(1520)$ and $\Delta(1700)$ resonances, see Fig. 3, are smereed out drastically due to this motion. The dot-dot-dashed curve denotes $\sigma_{i}^{GC}/A$ due to $FM$ and $V_{OR}$ incorporated in the calculation. As shown in Fig. 4, the change in the spectral shape because of $V_{OR}$ is insignificant beyond the $\Delta(1232)$ resonance region.

The calculated $\sigma_{i}^{GC}/A$ represented by the dot-dot-dashed curve, explained in Fig. 4, is presented with the data [1] in Fig. 5. This figure shows that the calculated result reproduces well the shape of the measured spectrum in the $\Delta(1232)$ resonance region within a factor of $\approx 1.4$, but it does not agree with the quoted distribution in the higher
resonance region. It should be mentioned that the measured resonances’ parameters used to calculate the $\gamma C$ reaction cross section are those used to evaluate the photonucleon reaction cross sections. The calculated results of the photonucleon reactions, as shown in Figs. 1(b) and 2(b), are in accord with the data. Therefore, it can be assumed that the in-medium resonance-nucleon cross section $\sigma^{\ast RN}_t$, except for the $\Delta(1232)$ and $N(1440)$ resonances, is larger than its free space value $\sigma^{RN}_t$ used in Eq. (7). To disentangle it, the cross sections of the $\gamma C$ reaction have been evaluated replacing $\sigma^{RN}_t$ by $\sigma^{\ast RN}_t (> \sigma^{RN}_t)$ in Eq. (7), and the calculated results are compared with the data [1] in Fig. 5. It shows that the calculated cross section due to $\sigma^{\ast RN}_t \approx 50\sigma^{RN}_t$, excluding for the $\Delta(1232)$ and $N(1440)$ resonances, reproduces well the energy distribution of the measured spectrum for the $\gamma C$ reaction [1]. However, the magnitude of the calculated cross section is approximately 1.4 times larger than the data.

The cross section for $^{208}$Pb nucleus has been evaluated using $\sigma^{\ast RN}_t \geq \sigma^{RN}_t$, and the calculated results are compared with the data [1] in Fig. 6. The results in this figure describe qualitatively those shown in Fig. 5 for $^{12}$C nucleus. The average of the cross sections evaluated for $^{12}$C and $^{208}$Pb nuclei, as shown in Fig. 7, agrees with the measured distribution of the average cross section [1]. The magnitude of the calculated results overestimate the data by a factor of 1.3.

4 Conclusions

The cross section of the photonuclear reaction has been evaluated to explore the scattering parameters of the resonances in the nucleus. The calculated results are compared with the data to extract the in-medium resonance-nucleon cross section. It shows the resonance-nucleon cross section in the nucleus, except for the $\Delta(1232)$ and $N(1440)$ resonances, is increased by the factor about 50 in compare to its free-space value.

5 Acknowledgement

The author thanks A. K. Gupta and S. M. Yusuf for their encouragement to work on theoretical nuclear physics.
References

[1] N. Bianchi, V. Muccifora, E. De Sanctis, A. Fantoni, P. Lavi Sandri et al., Phys. Rev. C 54 (1996) 1688.

[2] V. Muccifora, N. Bianchi, A. Deepman, E. De Sanctis, M. Mirazita et al., Phys. Rev. C 60 (1999) 064616; M. Anghinolfi, V. Lucherini, P. Rossi, N. Bianchi, P. Corvisiero et al., Phys. Rev. C 47 (1993) R922; N. Bianchi et al. Phys. Lett. B 299 (1993) 219; 309 (1993) 5; 325 (1994) 333; Th. Frommhold, F. Steiper, W. Henkel, U. Kneissl, J. Ahrens et al., Z. Phys. A 350 (1994) 249; J. Ahrens, Nucl. Phys. A 446 (1985) 229c.

[3] P. A. Zyla et al., (Particle Data Group), Prog. Theor. Exp. 2020 (2020) 083C01; https://pdg.lbl.gov./2020/hadronic-xsections/hadron.html;

[4] T. A. Armstrong, W. R. Hogg, G. M. Lewis, A. W. Robertson et al., Phys. Lett. B 34 (1971) 535; Phys. Rev. D 5 (1972) 1640.

[5] T. A. Armstrong, W. R. Hogg, G. M. Lewis, A. W. Robertson et al., Nucl. Phys. B 41 (1972) 445.

[6] S. Das, Phys. Rev. C 103 (2021) 035205.

[7] R. Rapp, M. Urban, M. Buballa and J. Wambach, Phys. Lett. B 417 (1998) 1.

[8] L. A. Kondratyuk, M. I. Krivoruchenko, N. Bianchi, E. De Sanctis, and V. Muccifora, Nucl. Phys. A 579 (1994) 453.

[9] W. Peters, M. Post, H. Lenske, S. Leupold and U. Mosel, Nucl. Phys. A 632 (1998) 109.

[10] B. Friman and H. J. Pirner, Nucl. Phys. A 617 (1997) 469.

[11] F. Halzen and A. D. Martin, Quarks and Leptons, (John Wiley & Sons, New York, 1984) p. 134.

[12] T. Ericson and W. Weise, Pions and Nuclei, (Clarendon Press, Oxford, 1988) page nos. 23 and 431.

[13] R. J. Glauber, in Lectures in Theoretical Physics, edited by W. E. Brittin et al. (Interscience, New York, 1959), Vol. I, p. 315; J. M. Eisenberg and D. S. Kolton, Theory of Meson Interaction with Nuclei (John Wiley & Sons, New York, 1980) p. 158.
[14] C. W. De Jager, H. De Vries and C. De Vries, At. Data and Nucl. Data Tables 14 (1974) 479; H. De Vries, C. W. De Jager and C. De Vries, ibid, 36 (1987) 495.

[15] B. K. Jain, N. G. Kelkar and J. T. Londergan, Phys. Rev. C 47 (1993) 1701; S. Das, Phys. Lett. B 737 (2014) 75; Phys. Rev. C 92 (2015) 014621.

[16] C. Lechanoine-LeLuc and F. Lehar, Rev. Mod. Phys. 65 (1993) 47; D. V. Bugg, D. C. Salter, G. H. Stafford et al., Phys. Rev. 146 (1966) 980; S. Barshay, C. B. Dover and J. P. Vary, Phys. Rev. C 11 (1975) 360.
Figure 1: (color online). Upper part (a): The calculated total cross sections of the photo-proton $\gamma p$ reaction. The short-dashed curves represent the cross section $\sigma^p_t(R)$ arising due to the individual resonance $R$. The short-long-short-dashed curve occurs because of the contribution of all resonances, i.e., $\sum R \sigma^p_t(R)$, as listed in Tables 1 and 2. The long-dashed curve represents the back-ground ($BG$) contribution to the cross section [8]. The overall cross section $\sigma^p_t$, i.e., $\sum R \sigma^p_t(R) + BG$, is expressed by the dot-dashed curve.

Lower part (b): The calculated result (i.e., the dot-dashed curve) are compared with the data are due to Zyla et al., [3].
Figure 2: (color online). As in Fig. 1 but for the photo-neutron $\gamma n$ reaction. The data are taken from Ref. [5].
Figure 3: (color online). Total cross-section per nucleon $\sigma_t^{\gamma C}/A$ of the photo-Carbon $\gamma C$ reaction calculated without considering the Fermi-motion and the resonance-nucleus interacting potential. The curves appearing in the figure qualitatively represent those explained in Fig. 1(a).
Figure 4: (color online). The dot-dashed curve represents the calculated total cross section per nucleon $\sigma_{t}^{\gamma C}/A$, same as that shown in Fig. 3. The long-dashed curve denotes $\sigma_{t}^{\gamma C}/A$ evaluated incorporating the Fermi-motion ($FM$) of the bound nucleon, whereas the dot-dot-dashed curve represents that calculated including both $FM$ and the resonance-nucleus optical potential $V_{OR}$. 
Figure 5: (color online). The cross section/nucleon $\sigma_t^{C}/A$ calculated using the resonance-nucleon cross sections in the resonance-nucleus potential, see Eq. (7). The dot-dot-dashed curve, also appearing in Fig. 4, occurs because of the in-medium resonance-nucleon cross section $\sigma_t^{RN}$ taken equal to its free-space value $\sigma_t^{RN}$. Other curves arise due to $\sigma_t^{*RN} > \sigma_t^{RN}$, except for the $\Delta(1232)$ and $N(1440)$ resonances. The data are taken from Ref. [1].
Figure 6: (color online). As in Fig. 5 but for $^{208}$Pb nucleus. The data are taken from Ref. [1].
Figure 7: (color online). The average of the cross sections evaluated for $^{12}$C and $^{208}$Pb nuclei are compared with the data of the average cross section [1].