PDF uncertainties in Higgs production at hadron colliders

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Abstract

Using the new schemes provided by the CTEQ and MRST collaborations and by Alekhin, we analyse the uncertainties due to the parton distribution functions (PDFs) on the next-to-leading-order cross sections of the four main production processes of the Standard Model Higgs boson at the LHC and the Tevatron. In the Higgs mass range where the production rates are large enough, the spread in the uncertainties when the three sets of PDFs are compared is of about 15\% in all processes and at both colliders. However, within one given set of PDFs, the deviations from the values obtained with the reference sets are much smaller, being of $O(5\%)$, except in the gluon–gluon fusion mechanism at relatively large Higgs boson masses, where they can reach the level of 10\% (15\%) at the LHC (Tevatron).
The discovery of the Higgs boson is the ultimate test of the Standard Model of the electroweak interactions, and the search for this particle is the major goal of the next round of high-energy experiments. If the Higgs boson is relatively light, $M_H \lesssim 200$ GeV, as is suggested by the electroweak precision measurements [1], it can be produced at the Tevatron Run II if enough integrated luminosity is collected [2, 3]. At the LHC, the Higgs boson can be produced over its entire mass range, $M_H \lesssim \mathcal{O}(1 \text{ TeV})$, in many and sometimes redundant channels [3, 4]. Once the Higgs boson is found, the next step would be to perform accurate measurements to explore all its fundamental properties. To achieve this goal in great detail, all possible Higgs cross sections and decay branching ratios should be measured in the most accurate manner. At the same time, we need very precise predictions and a good estimate of the various theoretical uncertainties that still affect these production cross sections and decay branching ratios, once higher effects are included.

Parton distribution functions (PDFs), which describe the momentum distribution of a parton in the proton, play a central role at hadron colliders. A precise knowledge of the PDFs over a wide range of the proton momentum fraction $x$ carried by the parton and the squared centre-of-mass energy $Q^2$ at which the process takes place, is mandatory to precisely predict the production cross sections of the various signals and background hard processes. However, they are plagued by uncertainties, which arise either from the starting distributions obtained from a global fit to the available data from deep-inelastic scattering, Drell–Yan and hadronic data, or from the DGLAP evolution [5] to the higher $Q^2$ relevant to the Tevatron and LHC scattering processes. Together with the effects of unknown perturbative higher order corrections, these uncertainties dominate the theoretical error on the predictions of the production cross sections.

PDFs with intrinsic uncertainties became available in 2002. Before that date, to quantitatively estimate the uncertainties due to the structure functions, it was common practice to calculate the production cross sections using the “nominal fits” or reference set of the PDFs provided by different parametrizations and to consider the dispersion between the various predictions as being the “uncertainty” due to the PDFs. However, the comparison between different parametrizations cannot be regarded as an unambiguous way to estimate the uncertainties since the theoretical and experimental errors spread into quantitatively different intrinsic uncertainties following their treatment in the given parametrization. The CTEQ and MRST collaborations and Alekhin recently introduced new schemes, which provide the possibility of estimating the intrinsic uncertainties and the spread uncertainties on the prediction of physical observables at hadron colliders\(^1\).

In this note, the spread uncertainties on the Higgs boson production cross sections at the LHC and at the Tevatron, using the CTEQ6 [7], MRST2001 [8] and ALEKHIN2002 [9] sets of PDFs, are investigated and compared.

The scheme introduced by both the CTEQ and MRST collaborations is based on the Hessian matrix method. The latter enables a characterization of a parton parametrization in the neighbourhood of the global $\chi^2$ minimum fit and gives an access to the uncertainty estimation through a set of PDFs that describes this neighbourhood. Fixed target Drell–Yan data as well as $W$ asymmetry and jet data from the Tevatron are used in the fit procedure.\(^2\)

\(^{1}\)Other sets of PDFs with errors are available in the literature [6], but they will not be discussed here.
The corresponding PDFs are constructed as follows:

- a global fit of the data is performed using the free parameters \(N_{\text{PDF}} = 20\) for CTEQ and \(N_{\text{PDF}} = 15\) for MRST; this provides the nominal PDF (reference set) denoted by \(S_0\) and corresponding to CTEQ6M and MRST2001E, respectively;
- the global \(\chi^2\) of the fit is increased by \(\Delta \chi^2 = 100\) for CTEQ and \(\Delta \chi^2 = 50\) for MRST, to obtain the error matrix [note that the choice of an allowed tolerance is only intuitive for a global analysis involving a number of different experiments and processes];
- the error matrix is diagonalized to obtain \(N_{\text{PDF}}\) eigenvectors corresponding to \(N_{\text{PDF}}\) independent directions in the parameter space;
- for each eigenvector, up and down excursions are performed in the tolerance gap, leading to \(2N_{\text{PDF}}\) sets of new parameters, corresponding to 40 new sets of PDFs for CTEQ and 30 sets for MRST. They are denoted by \(S_i\), with \(i = 1, 2N_{\text{PDF}}\).

To built the Alekhin PDFs [9], only light-target deep-inelastic scattering data [i.e. not the Tevatron data] are used. This PDF set involves 14 parameters, which are fitted simultaneously with \(\alpha_s\) and the structure functions. To take into account the experimental errors and their correlations, the fit is performed by minimizing a \(\chi^2\) functional based on a covariance matrix. Including the uncertainties on the \(\alpha_s\) fit, one then obtains \(2N_{\text{PDF}} = 30\) sets of PDFs for the uncertainty estimation.

The three sets of PDFs discussed above are used to calculate the uncertainty on a cross section \(\sigma\) in the following way [10]: one first evaluates the cross section with the nominal PDF \(S_0\) to obtain the central value \(\sigma_0\). One then calculates the cross section with the \(S_i\) PDFs, giving \(2N_{\text{PDF}}\) values \(\sigma_i\), and defines, for each \(\sigma_i\) value, the deviations \(\Delta \sigma_i^\pm = |\sigma_i - \sigma_0|\) when \(\sigma_i \geq \sigma_0\). The uncertainties are summed quadratically to calculate \(\Delta \sigma^\pm = \sqrt{\sum_i \sigma_i^\pm^2}\).

The cross section, including the error, is then given by \(\sigma_0 \pm \Delta \sigma^\pm\).

This procedure is applied to estimate the cross sections for the production of the Standard Model Higgs boson in the following four main mechanisms [11]:

\[
\begin{align*}
\text{associate production with } W/Z : & \quad q \bar{q} \rightarrow VH \\
\text{massive vector boson fusion} : & \quad qq \rightarrow Hqq \\
\text{the gluon gluon fusion mechanism} : & \quad gg \rightarrow H \\
\text{associate production with top quarks} : & \quad gg, q\bar{q} \rightarrow t\bar{t}H
\end{align*}
\]

We will consider the NLO cross sections for the production at both the LHC and the Tevatron, and use the Fortran codes \texttt{V2HV}, \texttt{VV2H}, \texttt{HIGLU} and \texttt{HQQ} of Ref. [12] for the evaluation of the production cross sections of processes (1) to (4), respectively. A few remarks are to be made in this context:

- The NLO QCD corrections to the Higgs-strahlung processes [13, 14] are practically the same for \(WH\) and \(ZH\) final states; we thus simply concentrate on the process \(qq \rightarrow WH\), which has a larger cross section at the LHC and at the Tevatron.
- The vector boson fusion process, \(pp \rightarrow Hqq\), for which the NLO corrections have been calculated in [16, 14, 15], is only relevant at the LHC and will not be discussed in the case of the Tevatron, where the cross sections are too small to be relevant.
For the gluon fusion process, $gg \to H$, we include the full dependence on the top and bottom quark masses of the NLO cross section [17] and not only the result in the infinite top quark mass limit [18].

For the $pp \to Ht\bar{t}$ production process, the NLO corrections have been calculated only recently [19] and the programs [which are very slow because of the complicated final state] for calculating the cross sections are not yet publicly available. However, we choose a scale for which the LO and NLO cross sections are approximately equal and use the program HQQ for the LO cross section that we fold with the NLO PDFs.

Finally, we note that the NNLO corrections are also known in the case of the Higgs-strahlung $q\bar{q} \to HV$ [20] and fusion $gg \to H$ [in the infinite top quark mass limit] [21, 22] processes. We do not consider these higher order corrections since the CTEQ and MRST PDFs with errors are not available at this order. The errors for the $gg \to H$ process at NNLO, including soft-gluon resummation, have been discussed in Ref. [22] using an approximate NNLO PDF set provided by Alekhin [9].

The expected NLO Higgs boson production cross sections at the LHC and the Tevatron, as a function of the Higgs mass, are shown in Fig. 1, using the CTEQ6M reference set for the PDFs. As can be seen, at the LHC, the cross sections for $gg \to H$ and $qq \to qqH$ are above the 0.1 pb level for Higgs masses up to 1 TeV, while for the $q\bar{q} \to HW$ and $q\bar{q}/gg \to t\bar{t}H$ processes they become of the order of 0.1 pb for Higgs masses around 200 GeV. At the Tevatron, only the processes $gg \to H$ and $q\bar{q} \to HW$ have sizeable cross sections [$\gtrsim 0.01$–0.1 pb] for Higgs boson masses below 200 GeV. We will therefore simply concentrate on these particular processes in the Higgs boson mass range where they are relevant.

Before analysing the uncertainties on the production cross sections, let us first discuss and compare the various PDFs. In Fig. 2, the MRST and Alekhin densities for the gluon and for the up and down quarks and antiquarks, normalized to the CTEQ 6 ones, are displayed for a wide range of $x$ values and for a fixed c.m. energy $Q^2 = (100 \text{ GeV})^2$. One notices the following main features [the same gross features are observed for $Q^2 = (500 \text{ GeV})^2$]:

- The MRST gluon PDF is smaller than the CTEQ one, except for $x$ values around $x \sim 0.1$; in contrast, the Alekhin gluon PDF is larger than the CTEQ one for all $x$ values, except for values of $x$ around $x \sim 0.01$ and for very high $x$.
- The MRST (anti)quark PDFs are practically equal in magnitude and are smaller than the CTEQ ones for low $x$ values, while they are in general slightly larger for higher $x$, except for values near unity; in the Alekhin case, all (anti)quark PDFs are larger than the CTEQ ones, except for the $\bar{u}$ density above $x \sim 0.05$. For values, $x \gtrsim 10^{-4}$, the differences between the Alekhin and the CTEQ6 PDFs are more pronounced than the differences between the MRST and the CTEQ ones.

Let us now comment on the intrinsic uncertainties of the PDF sets of the CTEQ and MRST collaborations, which follow the same approach. As discussed in Refs. [7] and [8],

\textbf{In fact, even the nominal PDFs are not known completely at NNLO since the full Altarelli-Parisi splitting functions are not yet available at this order of perturbation theory. However, the MRST collaboration and Alekhin have approximate solutions; only the Alekhin set includes uncertainty estimates, though.}
Figure 1: The NLO cross sections for Higgs production at the LHC (left) and the Tevatron (right) as a function of the Higgs mass. The reference CTEQ6M set is used.

Figure 2: MRST and Alekhin densities for the gluon, up quark/down quark and antiquarks, normalized to the CTEQ6 ones, as a function of $x$ and for $Q^2 = (100 \text{ GeV})^2$. 

\[ f_{\text{PDF}}(x, Q^2) \]
\[ Q^2 = (100 \text{ GeV})^2 \]
\[ \text{MRST}/\text{CTEQ} \]
\[ \text{Alekhin}/\text{CTEQ} \]
three different behaviours of the uncertainty bands can be distinguished, according to three different ranges of the variable $x$: decreasing uncertainties at low $x$, constant or slightly oscillating ones at intermediate $x$, and increasing ones at high $x$. The magnitude of these uncertainties depends on the considered parton and on the c.m. energy $Q^2$. In the case of quarks, the three behaviours are observed: the low-$x$ behaviour extends up to $x \sim \text{few } 10^{-3}$, and the high-$x$ one starts in the neighbourhood of $x = 0.7$. At high $Q^2$, the uncertainties at high and low-$x$ values exceed a few tens of a per cent and in the intermediate regime, they are less than a few per cent. In the gluon case and at high $Q^2$, the low-$x$ and the intermediate-$x$ bands are not as well separated as in the case of quarks; the uncertainty band reaches also the few per cent level. The high-$x$ regime starts in the neighbourhood of $x \sim 0.3$, i.e earlier than in the case of quarks.

The behaviour of the Higgs production cross sections and their uncertainties depends on the considered partons and their $x$ regime discussed above. In Figs. 3 and 4, we present the cross sections in, respectively, the case of the $q\bar{q} \rightarrow HW$ and $gg \rightarrow H$ processes at both the LHC and the Tevatron, while in Fig. 5, we show the cross sections in the case of the $qq \rightarrow qqH$ and $pp \rightarrow ttH$ processes at the LHC only. The central values and the uncertainty band limits of the NLO cross sections are shown for the CTEQ, MRST and Alekhin parameterizations. In the insets to these figures, we show the spread uncertainties in the predictions for the NLO cross sections, when they are normalized to the prediction of the reference CTEQ6M set. Note that the three sets of PDFs do not use the same value for $\alpha_s$: at NLO, the reference sets CTEQ6M, MRST2001C and A02 use, respectively, the values $\alpha_s^{\text{NLO}}(M_Z) = 0.118$, 0.119 and 0.117.

By observing Figs. 3–5, we see that the uncertainties for the Higgs cross sections obtained using the CTEQ6 set are two times larger than those using the MRST2001 sets. This is mainly due to two reasons: first, as noted previously, the CTEQ collaboration increased the global $\chi^2$ by $\Delta \chi^2 = 100$ to obtain the error matrix, while the MRST collaboration used only $\Delta \chi^2 = 50$; second, $2 \times 20$ parameter uncertainties are summed quadratically in CTEQ6, while only $2 \times 15$ are used in the MRST case. The uncertainties from the Alekhin PDFs are larger than the MRST ones and smaller than the CTEQ ones. In the subsequent discussion, the magnitude of the uncertainty band is expressed in terms of the CTEQ6 set.

- $q\bar{q} \rightarrow VH$: at the LHC, the uncertainty band is almost constant and is of the order of 4% [for CTEQ] over a Higgs mass range between 100 and 200 GeV. At the Tevatron, the uncertainty band increases with the Higgs mass and exceeds 6% at $M_H \sim 200$ GeV. To produce a vector plus a Higgs boson in this mass range, the incoming quarks originate from the intermediate-$x$ regime at the LHC, at Tevatron energies, however, some of the participating quarks originate from the high-$x$ regime. This explains the increasing behaviour of the uncertainty bands observed in the Tevatron case. The different magnitude of the cross sections, $\sim 12\%$ ($\sim 8\%$) larger in the Alekhin case than for CTEQ at the LHC (Tevatron), is due to the larger quark and antiquark densities; see Fig. 2. For this particular PDF set, the difference in the shifts of the central values in the LHC and Tevatron cases, is due to the different initial states at the two machines: in $pp$ collisions, the antiquark comes from the sea, while in $p\bar{p}$ collisions, it is a valence+sea antiquark and the sea quark shift compared to the CTEQ case is more important than the valence+sea one; see Fig. 2.
Figure 3: The CTEQ, MRST and Alekhin PDF uncertainty bands for the NLO cross sections for the production of the Higgs boson at the LHC (left) and at the Tevatron (right) in the $q \bar{q} \rightarrow HW$ process. The insets show the spread in the predictions, when the NLO cross sections are normalized to the prediction of the reference CTEQ6M set.

Figure 4: Same as Fig. 3, but for the $gg \rightarrow H$ production process.
• $gg \rightarrow H$: at the LHC, the uncertainty band for the CTEQ set of PDFs decreases from the level of about 5% at $M_H \sim 100$ GeV, down to the 3% level at $M_H \sim 300$ GeV. This is because Higgs bosons with relatively small masses are mainly produced by asymmetric low-$x$–high-$x$ gluons with a low effective c.m. energy; to produce heavier Higgs bosons, a symmetric process in which the participation of intermediate-$x$ gluons with high density, is needed, resulting in a smaller uncertainty band. At higher masses, $M_H > \sim 300$ GeV, the participation of high-$x$ gluons becomes more important, and the uncertainty band increases, to reach the 10% level at Higgs masses of about 1 TeV. At the Tevatron, because of the smaller c.m. energy, the high-$x$ gluon regime is already reached for low Higgs masses and the uncertainties increase from 5% to 15% for $M_H$ varying between 100 GeV and 200 GeV. As discussed above and shown in Fig. 2, the MRST gluon PDF is smaller than the CTEQ one for low $x$ and larger for relatively high $x$ ($\sim 0.1$): this explains the increasing cross section obtained with MRST compared to the one obtained with CTEQ, for increasing Higgs boson mass at the LHC. At the Tevatron the gluons are already in the high-$x$ regime.

• $gg/q\bar{q} \rightarrow t\bar{t}H$: at the LHC, the associated production of the Higgs boson with a top quark pair is dominantly generated by the gluon–gluon fusion mechanism. Compared with the process $gg \rightarrow H$ discussed previously and for a fixed Higgs boson mass, a larger $Q^2$ is needed for this final state; the initial gluons should therefore have higher $x$ values. In addition, the quarks that are involved in the subprocess $q\bar{q} \rightarrow t\bar{t}H$, which is also contributing, are still in the intermediate regime because of the higher value [$x \sim 0.7$] at which the quark high-$x$ regime starts. This explains why the uncertainty band increases smoothly from 5% to 7% when the $M_H$ value increases from 100 to 200 GeV.

• $qq \rightarrow Hqq$: in the entire Higgs boson mass range from 100 GeV to 1 TeV, the incoming quarks involved in this process originate from the intermediate-$x$ regime and the uncertainty
band is almost constant, ranging between 3% and 4%. [This behaviour agrees with the one discussed in Ref. [15], where a uniform 3.5% uncertainty using the CTEQ PDF has been found.] When using the Alekhin set of PDFs, the behaviour is different, because the quark PDF behaviour is different, as discussed in the case of the \( q\bar{q} \rightarrow HV \) production channel. The decrease in the central value with higher Higgs boson mass [which is absent in the \( q\bar{q} \rightarrow HV \) case, since we stop the \( M_H \) variation at 200 GeV] is due to the fact that we reach here the high-\( x \) regime, where the Alekhin \( \bar{u} \) PDF drops steeply.

Finally, it should be noted that, besides the uncertainties on the PDFs discussed here, which can be viewed as “experimental uncertainties” since they concern the systematic and statistical uncertainties of the data included in the global fits for a given set of PDFs, there are several other sources of uncertainties on the PDFs, which are associated with the global parton analysis, which can be viewed as “theoretical errors”. Among these are the uncertainties due to the input assumptions, the selection of the fitted data, the truncation of the DGLAP perturbative series, and theoretical effects such as higher twist effects, etc. The impact of these errors, for instance in the case of the MRST PDF set, has been discussed recently [23]. The discussion of these errors is beyond the scope of the present note. However, in our analysis, we have used three different sets of PDFs in which many of the previous items are treated differently. One could, therefore, consider the spread in the predictions given by the three (reference) sets of PDFs as a rough measure of these theoretical uncertainties.

In summary, we have considered three sets of PDFs with uncertainties provided by the CTEQ and MRST collaborations and by Alekhin. We evaluated their impact on the total cross sections at next-to-leading-order for the production of the Standard Model Higgs boson at the LHC and at the Tevatron. Within a given set of PDFs, the deviations of the cross sections from the values obtained with the reference PDF sets are rather small, \( \mathcal{O}(5\%) \), in the case of the Higgs-strahlung, vector boson fusion and associated \( ttH \) production processes, but they can reach the level of 10% (15%) at the LHC (Tevatron) in the case of the gluon–gluon fusion process for large enough Higgs boson masses, \( M_H \sim 1 \) TeV (\( \sim 180 \) GeV). However, the relative differences between the cross sections evaluated with different sets of PDFs can be much larger. Normalizing to the values obtained with the CTEQ6M set, for instance, the cross sections can be different by up to 15% for the four production mechanisms.

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