The generalized load statistical parameters’ assessment on the building structures

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Abstract. The distribution laws’ composition of the permanent, long-term temporary, snow and wind loads acting on the structure of buildings and structures is estimated. The following load distribution laws are respectively adopted: normal, logarithmically normal, Gumbel and Weibull. The calculation of the four main central statistical moments was carried out, the approximate distribution law for the generalized load was established by the expansion method in a Gram-Charlier series, as well as the magnitude of its statistical variability. The characteristics of the load variability are compared with the approximate solution of A.N. Dobromyslov. The results obtained are intended for the practical solution of the problems of assessing the safety and residual life of buildings and structures using the reliability theory methods.

1. Introduction
Currently, in connection with the enterprises fixed assets’ depreciation, the problem of assessing the residual resource and justifying the possibility of the buildings and structures’ safe operation beyond the standard service life is becoming increasingly important. Taking the high degree of the initial information uncertainty into consideration, the stochastic nature of the material of structures’ degradation processes and acting loads, as well as the human factor, the task of assessing the resource can be solved correctly only on the basis of the probabilistic approach and with the reliability theory methods.

Classical probabilistic methods for calculating building structures are complex for engineering applications and require a large amount of statistical information that is currently missing. Therefore, all modern practical methods for calculating the safety and durability of the structures are based on the concept of a reliability index, which was introduced by A.R. Rzhanitsyn [1]. Among the publications on this topic, the works of the following authors can be noted: [2,3,4].

The task of calculating structures for safety is formulated as:

\[ \tilde{S} = \tilde{R} - \tilde{F} \]  

where \( \tilde{R} \) – is the generalized structural strength (yield strength, tensile strength, plastic moment, etc.); \( \tilde{F} \) – is the generalized load (load effect); \( \tilde{S} \) – defines the safety margin. The sign \( \sim \) denotes a random variable.
The safety calculation is carried out in the correlation approximation taking into account two numerical parameters’ characteristics (1) - mathematical expectations $\bar{R}$, $\bar{F}$ and variances $\sigma_R^2$, $\sigma_F^2$. The level of security is determined by the reliability index

$$\beta = \frac{\bar{S}}{\sigma_S} = \frac{\bar{R} - \bar{F}}{\sqrt{\sigma_R^2 - \sigma_F^2}}$$

(2)

which is directly related to the failure probability $P(\bar{S} < 0)$.

Of the two generalized parameters of formula (1), the strength factor is most definite $\bar{R}$. The structures material strength distribution law is close to normal, and the statistical characteristics of the strength can be easily calculated by the standard strength value, with the security of 0.95, and the known variation coefficients. As for the load effect, in the general case it is necessary to take into account the combined effect of the loads, each of which has its own distribution law, and the coefficient of their combinations. This greatly complicates the solution [1,5].

In the present work, on the example of the loads acting on the covering the truss of an operated 9-story building [6], a practical method of calculating the statistical characteristics of the total generalized load is described.

2. Statistical characteristics of the loads acting on the structure

Constant actual load, snow and wind loads collected on the building’s covering truss are considered. The load sharing laws $p_j(x) (j = n, l, g, v)$, as well as their average values and magnitude of variability are taken according to Code of Practice 20.13330.2016 and the data given in [5,7,8,9] (Table 1).

| Load     | Statistical Characteristics | Distribution law |
|----------|-----------------------------|-------------------|
|          | Mathematical expectation,[ kN/m²] | The variation coefficient | Standard, [kN/m²] |         |
| Constant | $m_n = 0.674$                | $f_n = 0.030$ | $\sigma_n = 0.029$ | Normal       |
| Actual load | $m_l = 13.060$             | $f_l = 0.100$ | $\sigma_l = 2.620$ | Lognormal    |
| Snow     | $m_g = 0.604$                | $f_g = 0.040$ | $\sigma_g = 0.242$ | Normal, Gumbel |
| Wind     | $m_v = 0.236$                | $f_v = 0.370$ | $\sigma_v = 0.087$ | Normal, Weibull |

To find the characteristics of the total load, we find the parameters of the corresponding distribution laws and calculate 4 central moments according to the formula:

$$\mu_s = \int_{-\infty}^{\infty} (x - m_s)^s p(x) dx, s = 1, \ldots, 4$$

(3)

2.1 Permanent load

Constant load distribution density:

$$p_n(x) = \frac{1}{\sqrt{2\pi}\sigma_n} \exp \left( -\frac{(x - m_n)^2}{2\sigma_n^2} \right)$$

(4)

2.2 Payload (actual load)

For the payload, we use the lognormal distribution:
Lognormal law parameters $m_0$ and $\sigma_0$ we find by the formulas:

$$m_0 = \ln\left(\frac{m^2}{m^2 + D_i}\right) = 2.550$$  \hspace{1cm} (6)$$

$$\sigma_0 = \sqrt{\ln\left(1 + \frac{D_i}{m^2}\right)} = 0.198$$  \hspace{1cm} (7)$$

Using the analytical dependences given in [10], we find the central statistical moments of the random payload value:

$$\mu_{il} = m_i = \exp\left(\frac{\sigma^2}{2} + m_0\right)$$

$$\mu_{2l} = D_i = \exp\left(\sigma^2 + 2m_0\right) \cdot \left(\exp\sigma^2 - 1\right)$$

$$\mu_{3l} = \left(\exp\sigma^2 + 2\right) \sqrt{\exp\sigma^2 - 1}$$

$$\mu_{4l} = \exp\left(4\sigma^2\right) + 2\exp\left(3\sigma^2\right) + 3\exp\left(2\sigma^2\right) - 3$$ \hspace{1cm} (8)$$

2.3 Snow load
To approximate the snow load annual maximums random variable distribution, we use the Gumbel law:

$$p_s(x) = a_s \exp\left\{-a_s\left(x - u_s\right) - \exp\left[-a_s\left(x - u_s\right)\right]\right\}$$  \hspace{1cm} (9)$$

We calculate the parameters of the Gumbel law:

$$a_s = 1.28255$$

$$\sigma_s = 5.309$$

$$u_s = m_s - \frac{0.577216}{a_s} = 0.495$$

The statistical moments of the snow load maxima random value are calculated by the formula (3). The asymmetry coefficient and the reduced excess for the Humbel law are constant.

2.4 Wind load
To describe a random value of the maximum wind pressure, we use the Weibull law.

$$p_v(x) = \frac{\beta}{\alpha} \left(\frac{x}{\alpha}\right)^{\beta-1} \exp\left\[-\left(\frac{x}{\alpha}\right)^\beta\right\]$$  \hspace{1cm} (10)$$

The Weibull Law parameters $\alpha = 0.265; \beta = 2.941$ we find from the nonlinear algebraic equations system solution:
\[ \alpha G \left( \frac{1}{\beta} + 1 \right) = m_v \]
\[ \alpha^2 \left[ G \left( \frac{2}{\beta} + 1 \right) - \left( G \left( \frac{2}{\beta} + 1 \right) \right)^2 \right] = D_v \]  \hspace{1cm} (11)

where \( G \) is the gamma function.

To calculate the statistical moments of the Weibull law, it is possible to use the analytical dependences given in [10]

\[ \mu_{iv} = m_v \]
\[ \mu_{2v} = \alpha^2 \left[ G \left( \frac{2}{\beta} + 1 \right) - \left( G \left( \frac{2}{\beta} + 1 \right) \right)^2 \right] \]
\[ \mu_{4v} = a \alpha^4 \]  \hspace{1cm} (12)

The following notation is used in formulas (12):

\[ g_1 = G \left( 1 + \frac{1}{\beta} \right) ; g_2 = G \left( 1 + \frac{2}{\beta} \right) ; g_3 = G \left( 1 + \frac{3}{\beta} \right) ; g_4 = G \left( 1 + \frac{4}{\beta} \right) ; \]
\[ a = g_4 - 4g_1g_3 + 6g_2g_1^2 - 3g_1^4 \].

The numerical values of the four statistical moments for the random loads, as well as the asymmetry coefficients and reduced excess are given in Table 2.

| Type of load | Statistical moments | Coefficient asymmetries | Cited excess |
|--------------|---------------------|-------------------------|--------------|
|              | \( \mu_1 \)         | \( \mu_2 \) | \( \mu_3 \) | \( \mu_4 \) | \( A_4 \) | \( E_4 \) |
| Constant     | 0.674               | 8.180 \times 10^{-4}  | 0.000        | 2.010 \times 10^{-6} | 0.000 | 3.000 |
| Actual load  | 13.060              | 6.820                  | 10.830       | 170.570         | 0.608 | 3.664 |
|              | [kN/m^2]            | [kN/m^2]               | [kN/m^2]     | [kN/m^2]        | [kN/m^2] | [kN/m^2] |
| Snow         | 0.604               | 0.058                  | 0.016        | 0.018           | 1.140 | 5.400 |
| Wind         | 0.236               | 7.650 \times 10^{-3}  | 1.260 \times 10^{-6} | 1.600 \times 10^{-4} | 0.190 | 2.740 |
| Total        | 14.570              | 6.890                  | 10.850       | 173.320         | 0.600 | 3.652 |
| generalized  | [kN/m^2]            | [kN/m^2]               | [kN/m^2]     | [kN/m^2]        | [kN/m^2] | [kN/m^2] |

2.5 Evaluation of the composition of the laws of distribution

The random loads considered are the independent quantities with different distribution laws. We find the approximate distribution law of the total generalized load using the Gram-Charlier series expansion.
\[
p_{Q}(x) = \Phi(u) - \frac{1}{6} \frac{\mu_{Q}}{\mu_{Q}^2} \Phi^{(3)}(u) + \frac{1}{24} \frac{\mu_{Q}^2}{\mu_{Q}^4} \Phi^{(4)}(u) + \frac{1}{72} \frac{\mu_{Q}^3}{\mu_{Q}^5} \Phi^{(5)}(u) + \ldots
\]

(13)

Designation of parameters in the formula (13):

\( \Phi(u) \) – is the probability density of the standard normal distribution \( u = \frac{x - m_Q}{\sigma_Q} \);

\( F^{(k)}(u) \) – is the \( k \)-th derivative of the function \( \Phi(u) \);

\[
\begin{align*}
F^{(3)}(u) &= (u^2 - 1) \Phi(u); \\
F^{(4)}(u) &= (-u^3 + 3u) \Phi(u); \\
F^{(5)}(u) &= (-u^5 + 10u^3 - 5u) \Phi(u).
\end{align*}
\]

Figure 1. The total generalized load distribution law

The graph in Figure 1 shows that the distribution law of the total load is close to normal. The coefficient of the load variation is 0.18.

3. Discussions and results

The results can be compared with the load values adopted when assessing the reliability of buildings and structures according to the method of A. N. Dobromyslov [11]. With an unknown coefficient of variation in the load on the structure, the author proposes to take its mathematical expectation equal to the standard value as a safety factor \( m_Q = Q_n \). The reliability coefficient for load is equal to \( \gamma_f = 1.2 \).

Figure 2. Graphical representation of the calculated load value
Figure 2 shows two graphs of the load distribution law: the original with the mathematical expectation \( m_Q = 14.57 \text{kN/m}^2 \) and the shifted to the right by \( Q_n - m_Q \). The analysis showed that the value of the load with security three standards is \( Q_p = m_Q + 3 \sigma_Q = 22.45 \text{kN/m}^2 \). The calculated load according to the method [11] is equal to \( Q_p' = 1.2 Q_n = 22.65 \text{kN/m}^2 \), that is, almost complete coincidence of numerical values is obtained.

4. Summary
A solution that allows to reduce the permanent, temporary long-term, snow and wind loads with different distribution laws to one random total load, is obtained.

It was found that the total load with sufficient accuracy for practical calculations can be approximated by the normal distribution law with the following parameters:

\[
m_Q = \sum_i m_i; \sigma_Q = \sqrt{\sum_{j=1}^l \sigma_{ij}^2}; (i = n, l, g, v); f_Q = 0.18.
\]

The results obtained are intended to be used for the practical solution of the problems of assessing the safety and residual buildings and structures’ life using the reliability theory probabilistic-statistical models.

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