Information content of option-implied probabilities

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Abstract. In this work, we examine the usefulness of option-implied probabilities for the purpose of predicting the size and direction of the underlying S&P 500 index. To this end, we extract at-the-money, weekly probabilities from interpolated call option values and examine their predictive value for the period 2004–2013. Our findings indicate that option prices contain statistically significant, forward looking information about the size and direction of the weekly changes in the underlying index. Moreover, we find that the methodology proposed in this paper is more effective at predicting the probability of large weekly decreases than the popular VIX index.

1. Introduction
Since options are forward-looking instruments, one would expect that they contain valuable information about investors’ expectations and the underlying asset’s returns. Typically, practitioners and academicians use information obtained from implied-volatilities, but several researchers have found that the risk-neutral density function (RND) is also valuable. Recent, excellent reviews of these two approaches can be found in [1] and [2], respectively. Studies carried out in [3] and [4] find that the parameters obtained from RNDs contain predictive values concerning the returns of the underlying.

In this study, we take a slightly different approach. We use at-the-money probabilities implied by and extracted from quoted options. Then, we examine whether changes in the option-implied probabilities contain information about the size and direction of future index returns. This is accomplished by fitting an interpolant to the available call option prices, and then extracting the required weekly option-implied probabilities. Moreover, we compare the performance of this simple procedure to the performance of the widely used VIX index.

The rest of the paper is organized as follows. In Section 2, we describe our methodology. In Sections 3, we present the results of our empirical study. In Section 4, we discuss our findings. Section 5 concludes the paper.

2. Option-implied probabilities
Because options are quoted at a discrete set of expiries and strike prices, an interpolant has to be fitted to the available data. This is commonly accomplished by the following procedure [5]. First, option prices are converted to implied volatilities using the Black and Scholes formula [6]. Then, an interpolant is fitted to the implied. Finally, the implied volatility values are converted back to option prices. The interpolant employed is typically a low-order polynomial, as in [7,8], or a type of spline, as in [5,8]. In this study, we use the thin-plate, spline-based interpolant studied in [8] for several reasons. First, the method is straightforward to implement. Moreover, its empirical performance has been tested using S&P 500 index
options, and the model provides an excellent fit to observed prices. Finally, the technique is capable of generating option prices at arbitrary expiries. The latter is necessary to extract probabilities at a given maturity.

2.1. Extracting option-implied probabilities
Given a call option \( C(K, T) \) with strike price \( K \) and maturity \( T \), the price of binary option \( B(K, T) \) can be obtained from

\[
B(K, T) = e^{-rt} P(S_T > K) = \frac{dC(K, T)}{dK},
\]

where \( r \) is the interest rate [4]. Hence, given a continuous set of call options, the probability that the stock will finish above a certain level, \( K \), at time \( K \), can be approximated using

\[
P(S_T > K) \approx e^{rt} \frac{C(K + \Delta K, T)}{\Delta K}.
\]

2.2. Forecasting with option-implied probabilities
Our approach is based on a simple idea. Although there is a difference between real-world and risk-neutral probabilities, changes in option-implied probabilities often lead to corresponding changes in physical probabilities. Specifically, if the weekly change in risk-neutral probability is positive or negative, then investors expect the underlying index to rise or fall, respectively.

A widely used indicator of market risk implied by options is the VIX index constructed by the Chicago Board Options Exchange (CBOE) [9]. VIX is calculated from S&P 500 stock index options as a weighted average of quoted option prices to provide an estimate for the model-free volatility. Hence, it represents the option market’s expectation of the volatility of the S&P 500 index over the next 30 days. Since a large increase in the VIX typically causes the left tail of the distribution to be fatter and the skew to be more negative, the median of the distribution likely increases as a result. Moreover, one would expect that changes in the VIX can predict the probability of large increases and decreases. Therefore, because of its widespread use and properties, the VIX index can be used as an appropriate benchmark against our approach.

3. Empirical analysis
3.1. Data
To assess the performance of the proposed approach, we use S&P 500 Index call option prices. We obtained the data from the IVolatility database, and we use daily closing prices for each Wednesday between January 1, 2004, and December 31, 2013. Moreover, the data is filtered for possible biases, as described in [4].

3.2. Methodology
For each of the 518 trading days in the sample, following [8], a thin-plate spline-based interpolant was fitted to all available implied volatilities. Then, using this interpolant and Equation (2), we calculated the one-week option-implied probabilities.

3.3. Forecasting the direction of large changes
To assess the information content of call option prices, we record the following after a positive and negative change in the weekly option-implied probabilities: (i) the percentage of times the index is positive, (ii) the percentage of times the index is higher than 1 per cent, and (iii) the percentage of times the index is lower...
than 1 per cent. The results are presented in Table 1. We also record these three types of changes following positive or negative weekly changes in the VIX index. The results are reported in Table 2. We also include the results of chi-square tests and P-values in Tables 1 and 2 to assess the statistical significance of our findings.

**Table 1.** Percentage of positive weekly changes, large decreases, and large increases in the price of the S&P 500 index following positive and negative weekly changes in the option-implied probabilities.

|                                | Number of observations | Percentage of future up-movement | Percentage of changes larger than plus 1 per cent | Percentage of changes smaller than minus 1 per cent |
|--------------------------------|------------------------|----------------------------------|--------------------------------------------------|--------------------------------------------------|
| Positive weekly change in P(S_\text{T}>K) | 269                    | 63.57%                           | 39.78%                                           | 19.70%                                           |
| Negative weekly change in P(S_\text{T}>K) | 247                    | 50.20%                           | 24.70%                                           | 27.53%                                           |
| Chi-squared statistic          | 9.3953                 | 13.3367                          | 4.3953                                           |                                                  |
| P-value                        | 0.0022                 | .00026                           | 0.0360                                           |                                                  |

4. Results and discussion

The results presented in Table 1 show that following positive weekly changes in the option-implied probabilities: (i) the likelihood of a large future up movement in the underlying increases, (ii) the likelihood of the index going up also increases, and (iii) the likelihood of a large future down movement in the underlying decreases. Additionally, the P-values indicate that all results are statistically significant. Moreover, it can be observed from Table 2 that the probability of a future up-movement larger than 1 per cent in the underlying is more likely after a positive weekly change in the VIX index.

However, the P-values presented in Table 2 show that weekly changes in the VIX index are a poor predictor for the probability of increase or decrease larger than 1 per cent. Although market participants typically track the absolute value of the VIX index to estimate the probability of large movements, our findings are still somewhat surprising. Specifically, our results indicate that weekly changes in the option-implied probabilities are much more effective at predicting the probability of large changes in the underlying than weekly changes in the VIX index. Therefore, our findings should be of high interest to portfolio managers, risk managers, and other practitioners.

5. Conclusion and future research

In this study, we present evidence that option-implied probabilities contain information about the future weekly returns of the S&P 500 index. Moreover, we find that this information is not included in the VIX index. Hence, our results are highly relevant for practitioners and academics. Additionally, in our empirical study, we have only examined the information content implied by at-the-money probabilities. Future studies could explore the information content of other probabilities extracted from the call option surface. Another potential avenue of research is to examine the ability of our approach to predict future volatility. Subsequently, the results could be compared to the approach presented in [10]. Finally, in our future work, we plan to investigate whether our findings hold true in other markets.
Table 2. Percentage of positive weekly changes, large decreases, and large increases in the price of the S&P 500 index following positive and negative weekly changes in the VIX index.

|                              | Number of observations | Percentage of future up-movement | Percentage of changes larger than plus 1 per cent | Percentage of changes smaller than minus 1 per cent |
|------------------------------|------------------------|---------------------------------|--------------------------------------------------|---------------------------------------------------|
| Positive weekly change in VIX| 252                    | 58.73%                          | 39.29%                                           | 22.22%                                            |
| Negative weekly change in VIX| 264                    | 55.68%                          | 26.14%                                           | 24.24%                                            |
| Chi-squared statistic        |                        | 0.48932                         | 10.1525                                          | 0.2948                                            |
| P-value                      |                        | 0.4842                          | 0.0014                                           | 0.5871                                            |

References

[1] Fu X, Arisoy Y E, Shackleton M B and Umutlu M 2016 Option-Implied volatility measures and stock return predictability. *The Journal of Derivatives*, 24(1), pp. 58–78.

[2] Chen R R, Hsieh P L, Huang J 2018 Crash risk and risk neutral densities. *Journal of Empirical Finance*, 47(1), pp. 162–189.

[3] Hamidieh K 2017 Estimating the tail shape parameter from option prices. *The Journal of Risk*, 19(6), pp. 85–110.

[4] Orosi G 2017 Information content of right option tails: Evidence from S&P 500 index options. *Journal of Asset Management*, 18(7), 516-526.

[5] Figlewski S 2010 Estimating the Implied Risk-Neutral Density for the US Market Portfolio Volatility and Time Series Econometrics: Essays in Honor of Robert Engle ed T. Bollerslev, J.R. Russell and M.W. Watson (Oxford, UK: Oxford University Press) pp. 323–53.

[6] Black F and Scholes M 1973 The pricing of options and corporate liabilities. *Journal of Political Economy*, 81(3), pp. 637–654.

[7] Bahaludin H and Abdullah M H 2017 March. Empirical performance of interpolation techniques in risk-neutral density (RND) estimation. *Journal of Physics: Conference Series*, 819(1), p. 012026). IOP Publishing.

[8] Orosi G 2012 Empirical performance of a spline-based implied volatility surface. *Journal of Derivatives & Hedge Funds*, 18(4), pp. 361–376.

[9] Exchange C B O 2009 The CBOE volatility index-VIX. White Paper, pp. 1–23.

[10] Biktimirov E N and Wang C 2017 Model-Based versus model-free implied volatility: Evidence from North American, European, and Asian index option markets. *The Journal of Derivatives*, 24(3), pp. 42–68.