Reynolds stress structures in a self-similar adverse pressure gradient turbulent boundary layer at the verge of separation.

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Abstract. Mean Reynolds stress profiles and instantaneous Reynolds stress structures are investigated in a self-similar adverse pressure gradient turbulent boundary layer (APG-TBL) at the verge of separation using data from direct numerical simulations. The use of a self-similar APG-TBL provides a flow domain in which the flow gradually approaches a constant non-dimensional pressure gradient, resulting in a flow in which the relative contribution of each term in the governing equations is independent of streamwise position over a domain larger than two boundary layer thickness. This allows the flow structures to undergo a development that is less dependent on the upstream flow history when compared to more rapidly decelerated boundary layers. This APG-TBL maintains an almost constant shape factor of $H = 2.3$ to $2.35$ over a momentum thickness based Reynolds number range of $Re_{\delta} = 8420$ to $12400$. In the APG-TBL the production of turbulent kinetic energy is still mostly due to the correlation of streamwise and wall-normal fluctuations, $\langle u'v' \rangle$, however the contribution from the other components of the Reynolds stress tensor are no longer negligible. Statistical properties associated with the scale and location of sweeps and ejections in this APG-TBL are compared with those of a zero pressure gradient turbulent boundary layer developing from the same inlet profile, resulting in a momentum thickness range of $Re_{\delta} = 3400$ to $3770$. In the APG-TBL the peak in both the mean Reynolds stress and the production of turbulent kinetic energy move from the near wall region out to a point consistent with the displacement thickness height. This is associated with a narrower distribution of the Reynolds stress and a 1.6 times higher relative number of wall-detached negative $uv$ structures. These structures occupy 5 times less of the boundary layer volume and show a similar reduction in their streamwise extent with respect to the boundary layer thickness. A significantly lower percentage of wall-attached structures is observed in the present case when compared with a similar investigation of a rapidly decelerating APG-TBL, suggesting that these wall-attached features could be the remanent from the lower pressure gradient domain upstream.

1. Introduction

In a desire to understand the complex dynamics that drive the turbulent boundary layer that develops when a flow passes over a solid surface, we often focus upon the canonical zero pressure gradient turbulent boundary layer (ZPG-TBL). This ZPG-TBL can originate from the leading
edge of a thin plate and retain a zero and hence constant non-dimensional pressure gradient as it develops along the plate. While examples of such a flow can be found in nature and industry, the occurrence of constant zero pressure gradient flow pales in comparison to the cases in which the pressure gradient varies in the streamwise direction. The case of an adverse pressure gradient is of particular interest owing to the resulting momentum deficit and if the pressure gradient is strong enough, the separation of the boundary layer, which can significantly affect both the mean and fluctuating forces that are experienced by the surface or in the wake of a body. Even in the absence of separation, the presence of a strong pressure gradient significantly alters the mean and fluctuating velocity profiles, resulting in peak turbulent activity and production moving from the vicinity of the wall to a location close to the displacement thickness [1]. In all boundary layers it is the product of the Reynolds stresses and the mean velocity gradients that drives the production of turbulent kinetic energy, with spatially and temporally coherent regions of intense Reynolds stress subsequently being responsible for the majority of the turbulence production and most of the wall-normal transfer of momentum. It is for this reason that such structures have formed the focus of numerous investigations [2, 3, 4].

Recently, following the quadrant analysis of Wallace et al. [3], Lozano-Durán et al. [5] performed a detailed statistical investigation of the extent, orientation, density and spacing of coherent three-dimensional Reynolds stress structures using data from direct numerical simulation (DNS) of turbulent channel flows at Reτ = 934 and 2003. This study found that most of the mean Reynolds stress was contained in wall-attached structures, which tended to be larger than the smaller, more isotropically aligned, wall-detached structures. Far less is known about how these structures behave in regions of adverse pressure gradient, which as the pressure gradient increases result in substantially different wall-normal Reynolds stress profiles [6, 7]. Unlike the ZPG-TBL, an adverse pressure gradient turbulent boundary layer (APG-TBL) is almost always associated with a change in pressure gradient, such as that over a curved surface, meaning the flow at a given point along the surface is generally a function of not only the local pressure gradient, but also the pressure distribution that precedes it. Using a series of large eddy simulations (LES) Bobke et al. [8] compared the turbulent statistics associated with a number of APG-TBLs under the influence of a near constant non-dimensional pressure gradient, with APG-TBL at the same displacement thickness based Reynolds number developing under a non-constant non-dimensional pressure gradient. Results demonstrated that APG-TBL over flat surfaces required a streamwise development distance of approximately 7 boundary layer thickness to overcome the history of their evolution and approach the local turbulent statistics associated with the equivalent constant non-dimensional pressure gradient.

Maciel et al. [9] applied the same methodology as Lozano-Durán et al. [5] to investigate Reynolds stress structures in a DNS of a ZPG-TBL and in DNS of a rapidly decelerating non-self-similar APG-TBL, just upstream of a thin separation bubble. Sweeps and ejections were found to be longer in the ZPG-TBL, ejections extending to streamwise lengths of 5 boundary layer thicknesses δ, yet were both shorter and more inclined with respect to the wall, but occupied a larger proportion of the overall volume in the APG-TBL, with respect to the local boundary layer thickness. Near wall and large scale wall-attached sweeps and ejections were found to be far less numerous in the APG-TBL, consistent with the turbulent activity being lifted away from the wall. In the case of the APG-TBL, statistics were accumulated over a domain spanning five local boundary thickness (5δ), over which the shape factor $H = δ_1/δ_2$ demonstrated a substantial variation from $H = 1.97$ to 3.42. It is unclear what influences this rapid deceleration might have on the identified structures, the extent of which can cover this entire domain.

One way to separate the influence of upstream flow evolution is to consider self-similar APG-TBLs, where the flow evolves over a domain in which the relative contribution from each term in the governing equations remains approximately independent of streamwise position [10, 11, 12]. As a consequence the statistics in these self-similar boundary layers collapse under a local outer
velocity $U_e$ and displacement thickness scaling $\delta_1$, with the exception of an exceedingly smaller near wall region which continues to follow a viscous scaling until separation, analogous to the dual inner and outer scaling that exists in a ZPG-TBL. A range of self-similar APG-TBL can also be found in which the non-dimensional pressure gradient, $\beta = \delta_1 P_e/\tau_w$, where $P_e$ is the far-field pressure gradient and $\tau_w$ is the mean wall shear stress, remains constant. These particular cases allow for the exploration of the structure of an APG-TBL that experiences the influence of a constant contribution from the pressure gradient term in the governing equations.

In the present study, DNS of both a ZPG-TBL over a domain of 8.8$\delta$ with momentum thickness based $Re_{\delta_2} = 3400$-3770 and a self-similar APG-TBL at the verge of separation over a domain of 2.6$\delta$ with $Re_{\delta_2} = 8430$-12400 [13], are used to investigate the influence of a strong constant non-dimensional pressure gradient on the instantaneous Reynolds stress structures that are present in a flow near the point of separation. To be consistent with previous studies [5, 9], detection of these instantaneous Reynolds stress structures is performed based on a threshold of

$$\langle \Omega_z \rangle$$

and wall-normal $v$ velocity fluctuations and the use of a three-dimensional clustering algorithm.

2. Boundary layer datasets

The ZPG-TBL and APG-TBL data sets were computed by solving the incompressible Navier-Stokes equations in a three-dimensional rectangular volume, with constant kinematic viscosity $\nu$ and a fluid density of unity. The three flow directions are streamwise ($\nu$), wall-normal ($y$) and spanwise ($z$), with mean velocity components in these directions denoted by $\langle U \rangle$, $\langle V \rangle$ and $\langle W \rangle$, respectively, with velocity fluctuations represented by $u$, $v$, $w$. The desired pressure gradient is set by applying a wall-normal velocity at the upper surface of the simulations, combined with an irrotationality condition. The distribution of wall-normal velocity required to simulate an APG-TBL at the verge of separation was determined based on the potential flow solution in an expanding duct, corrected for the growth of the boundary layer and was chosen to satisfy an expected mean outer streamwise velocity distribution following Mellor & Gibson [11]. An inlet recycling and rescaling was used for both boundary layers. Further details of the boundary condition and recycling can be found in Kitsios et al. [13]. For details of the numerical methodology, parallelism and inlet recycling the reader is referred to Borrell, Sillero & Jiménez [14] and Sillero [15]. Numerical details associated with the data sets are given in table 1. Note the domain of interest (DoI) used for the detection of structures in the APG-TBL is slightly larger than the self-similar domain to ensure that the full streamwise extent of the structures within the domain of interest is captured. The domain and detailed evaluation of this self-similar APG-TBL can be found in Kitsios et al. [13].

Throughout this paper the outer velocity $U_e$, displacement thickness $\delta_1$ and momentum thickness $\delta_2$ are defined as by Spalart & Watmuff [16] using the spanwise vorticity based velocity proposed by Lighthill [17]. Under this definition the outer velocity is given by

$$U_e(x) = U_\Omega(x, y_\Omega), \quad \text{where}$$

$$U_\Omega(x, y) = -\int_0^y \langle \Omega_z \rangle(x, y) \, dy,$$  

(1)

(2)

$\langle \Omega_z \rangle$ is the mean spanwise vorticity and $y_\Omega$ is the distance from the wall at which this vorticity has decayed to 0.2% of the mean vorticity at the wall. This definition was used to account for the absence of a uniform free stream velocity in the case of the strong adverse pressure gradient. Following this definition the displacement and momentum length scales are given by

$$\delta_1(x) = -\frac{1}{U_e} \int_0^{y_{\Omega}} y \langle \Omega_z \rangle(x, y) \, dy,$$  

and

$$\delta_2(x) = -\frac{2}{U_e} \int_0^{y_{\Omega}} y U_\Omega \langle \Omega_z \rangle(x, y) \, dy - \delta_1(x),$$  

(3)

(4)
Table 1. Numerical details of the ZPG and APG-TBL DNS data sets. Domain sizes are non-dimensionalized by the displacement thickness \((\delta_1)\) at the position \(x_\ast\) where \(Re_\delta = 4800\) and by the displacement thickness at the center of the domain of interest \(x_{DoI}\).

|                         | ZPG     | APG     |
|-------------------------|---------|---------|
| Nominal pressure gradient | \(\beta\) | 0       | 39      |
| Streamwise data points   | \(N_x\) | 8193    | 8193    |
| Wall-normal data points  | \(N_y\) | 315     | 1000    |
| Spanwise data points     | \(N_z\) | 1362    | 1362    |
| Streamwise domain size   | \(L_x/\delta_1(x_\ast)\) | 480    | 303     |
| Wall-normal domain size  | \(L_y/\delta_1(x_\ast)\) | 22.7   | 73.4    |
| Spanwise domain size     | \(L_z/\delta_1(x_\ast)\) | 80.1   | 50.7    |
| Streamwise grid spacing  | \(\Delta x/\delta_1(x_\ast)\) | 0.0585 | 0.0370  |
| Wall-normal spacing at wall | \(\Delta y_{wall}/\delta_1(x_\ast)\) | \(1.53 \times 10^{-3}\) | \(9.71 \times 10^{-4}\) |
| Wall-normal spacing free stream | \(\Delta y_\infty/\delta_1(x_\ast)\) | 0.0992 | 0.254   |
| Spanwise grid spacing    | \(\Delta z/\delta_1(x_\ast)\) | 0.0585 | 0.0370  |
| Displacement thickness ratio | \(\delta_1,DoI/\delta_1,DoI,APG\) | 0.098  | 1       |
| Reynolds number range in DoI | \(Re_\delta\) | 4460 \(\rightarrow\) 5140 | 18450 \(\rightarrow\) 30200 |
| Reynolds number range in DoI | \(Re_\delta\) | 3400 \(\rightarrow\) 3770 | 8430 \(\rightarrow\) 12400 |
| Streamwise length of DoI | \(L_{DoI}/\delta_1(x_{DoI})\) | 54     | 11      |

In the case of the ZPG-TBL this outer velocity and length scales are consistent with classical definitions.

3. Flow description and comparison

The substantial differences in the flow between the ZPG-TBL and the APG-TBL at the verge of separation are demonstrated by the mean velocity and the \(\langle uv \rangle\) Reynolds stress profiles in figure 1. Profiles are shown for both local outer velocity \(U_e\) and displacement thickness \(\delta_1\) scaling and inner viscous scaling based on the local friction velocity \(u_\tau\) and kinematic viscosity \(\nu\), denoted by the \(^+\) superscript. Profiles are presented for the beginning, middle and end of the domain that will be used for the structural detection. The ZPG-TBL shows a good collapse under both scalings over this range, while the APG-TBL profiles remain reasonably consistent under the outer scaling. As discussed in section 2 the domain used for the present structural detection in the APG-TBL is larger than the self-similar domain investigated in Kitsios et al. [13], resulting in the small variation in the present profiles when compared to those of Kitsios et al. [13]. Unlike the ZPG-TBL, which possess a fairly broad mean Reynolds stress profile, the peak Reynolds stress in the APG-TBL occurs further from the wall and is confined to a much narrower region of the flow in regards to both inner and outer length scales.

The contribution of the \(\langle uv \rangle\) component of the mean Reynolds stress to the production of turbulent kinetic energy is shown in figure 2. In the ZPG-TBL the Reynolds stress results in a single peak production in the vicinity of \(y^+ = 14\) viscous units above the wall. The APG-TBL maintains a similar peak that move slightly closer to the wall in terms of both inner and outer units, however this pales in comparison to the peak that develops in the vicinity of \(y = \delta_1\). Given the logarithmic axis used for these plots the relative contribution of these peaks is better illustrated using the premultiplied profile in figure 3, which highlight the fairly even contribution to the total kinetic energy production in the ZPG-TBL and the negligible role of
the inner production peak in the APG-TBL.

In the ZPG-TBL, the negligible mean streamwise velocity gradients \( \frac{\partial \langle U \rangle}{\partial x} \), and negligible mean wall-normal and spanwise velocities \( \langle V \rangle, \langle W \rangle \), leaves the \( \langle uv \rangle \) Reynolds stress component as the only component that makes any significant contribution to the production, given by

\[
P = -\langle u_i u_j \rangle S_{ij}, \quad \text{where}
\]

\[
S_{ij} = \frac{1}{2} \left( \frac{\partial \langle U_i \rangle}{\partial x_j} + \frac{\partial \langle U_j \rangle}{\partial x_i} \right).
\]

This is demonstrated by the comparison of the total production and the \( \langle uv \rangle \) associated production shown in figure 3(b). For the ZPG-TBL the difference is indeed negligible, however in the APG-TBL the deceleration of the flow in the streamwise direction and the larger wall-normal velocity component result in a noticeable contribution from \( \langle uu \rangle \) and \( \langle vv \rangle \) components.

**Figure 1.** Profiles of mean streamwise velocity (a,b) and \( \langle uv \rangle \) Reynolds stress (c,d) non-dimensionalized by the local mean outer velocity \( U_e \) and displacement thickness \( \delta_1 \) (a,c) and by viscous units based on local friction velocity \( u_\tau \) and kinematic viscosity \( \nu \) (b,d). Individual profiles are plotted for the beginning (——), middle (– – –) and end (— · —) of the domain of interest of the ZPG-TBL, and the beginning (——), middle (– – –) and end (— · —) of the domain of interest of the APG-TBL.
Figure 2. Production of turbulent kinetic energy associated with the $\langle uv \rangle$ Reynolds stress component, non-dimensionalized by the local mean outer velocity $U_e$ and displacement thickness $\delta_1$ (a) and by viscous units based on local friction velocity $u_\tau$ and kinematic viscosity $\nu$ (b). Individual profiles are plotted for the beginning (---), middle (----) and end (-----) of the domain of interest of the ZPG-TBL, and the beginning (---), middle (----) and end (-----) of the domain of interest of the APG-TBL.

Figure 3. Production of turbulent kinetic energy associated with the $\langle uv \rangle$ Reynolds stress component non-dimensionalized by the local mean outer velocity $U_e$ and displacement thickness $\delta_1$ premultiplied by the height above the wall (a) and relative to the total production associated with all terms of the Reynolds stress tensor $P = -\langle u_i u_j \rangle S_{ij}$ where $S_{ij}$ is the mean strain rate tensor (b). Profiles are plotted for the middle (-----) of the domain of interest of the ZPG-TBL, and the middle (-----) of the domain of interest of the APG-TBL for the contribution of the $\langle uv \rangle$ component. The profiles for the corresponding total production are given by (- - - - -) and (- - - - -).
that reduces the total production at the inner peak and increase the peak in the outer region. Nevertheless, the majority of the contribution is still associated with \( \langle uv \rangle \).

The degree to which the streamwise and wall-normal components remain correlated is shown in figure 4. In both boundary layers these fluctuations remains negatively correlated with a peak correlation occurring at around \( y^+ = 10 \) in the ZPG-TBL, moving to around \( y^+ = 4 \) in the APG-TBL. In the APG-TBL this correlation decreases slightly through the buffer layer before increasing again through the outer layer, consistent with the influence of near wall streaks and large scale low-speed and high-speed structures in the outer layer. Despite this dip the correlation remains in the vicinity of 0.4 for most of the ZPG-TBL. The APG-TBL involves a much lower correlation of around 0.2 in the near wall region, rising to a correlation of 0.4 at the outer production peak. A similar trend was observed for the rapidly decelerating APG [6], indicating that in both cases the adverse pressure gradient reduced the orderly structure of the flow, particularly in the near wall region, while a high correlation remains in the outer region.

![Figure 4.](image)

**Figure 4.** Correlation of streamwise \( u \) and wall-normal \( v \) velocity fluctuations non-dimensionalized by the local displacement thickness \( \delta_1 \) (a) and by viscous units based on local friction velocity \( u_\tau \) and kinematic viscosity \( \nu \) (b). Individual profiles are plotted for the beginning (---), middle (-- --) and end (---) of the domain of interest of the ZPG-TBL, and the beginning (---), middle (-- --) and end (---) of the domain of interest of the APG-TBL.

### 4. Structure detection methodology

From section 3 it is clear that despite significant differences in the Reynolds stress profiles, the individual structures that result in the mean \( \langle uv \rangle \) Reynolds stress, continue to play a significantly role in APG-TBLs. The structures that provide the greatest contribution can be defined as spatially coherent regions in which the product of the local fluctuating streamwise and wall-normal velocity components, \( uv \), is large with respect to the background fluctuations at that location. Following Lozano-Durán et al. [5] these structures can be identified by first satisfying the local condition

\[
\frac{|uv(x, y, z)|}{u_{\text{rms}}(x, y)v_{\text{rms}}(x, y)} > H^*,
\]

where \( u_{\text{rms}} \) and \( v_{\text{rms}} \) represent the root mean square velocity fluctuations and \( H^* \) is the hyperbolic hole size associated with a percolation analysis that is performed to determine the value of \( H^* \) that provides an appropriate balance between the volume of the largest detected
Structure with respect to the volume of the domain of interest and the number of individual detected structures. The effect of varying $H^*$ is demonstrated in figure 5 with a sudden drop in the largest volume $V_{\text{largest}}$ indicating the level at which the largest structure begins to be broken into separate structures. In the ZPG-TBL this crisis occurs for $H^* = 0.9$ compared to $H^* = 1.75$ for the APG-TBL. In order to be consistent with previous studies [5, 9] a decision was made to use $H^* = 1.75$ for both flows.

Individual structures are computed by clustering adjacent cells in the dataset where equation 7 is satisfied, cells share a common face, and all cells fall into the same $uv$ quadrant. Quadrants are defined following Wallace et al. [3] where $Q_1$ represents the quadrant where $u > 0$, $v > 0$, $Q_2$ where $u < 0$, $v > 0$, $Q_1$ where $u < 0$, $v < 0$ and $Q_4$ where $u > 0$, $v < 0$. $Q_2$ structures are commonly referred to as ejections due to their role in transporting low momentum flow away from the wall, while $Q_4$ structure are referred to as sweeps, which transport high momentum flow towards the wall. It is these sweeps and ejections that provide the negative $uv$ correlation associated with the mean Reynolds stress and the production of turbulent kinetic energy. We will refer to this combination of $Q_2$ and $Q_4$ structures as $Q_\pm$.

Following Maciel et al. [9] individual structures were rejected using the criteria

$$V_{\text{struct}} < 3\Delta x^3.$$  \hspace{1cm} (8)

To prevent possible biasing of the statistics, structure that cross the boundaries of the domain of interest were also rejected. In the case of the ZPG-TBL structures were detected using 50 statistically independent snapshots of the domain of interest and 39 independent snapshots for the APG-TBL.

5. Results

The number of identified structures in each quadrant $N_Q$ relative to the total number of identified structures $N_{\text{total}}$ and the percentage of the total volume occupied by the combined volume of different quadrant structures $V_Q$ are presented in table 2. The total volume $V_{\text{total}}$ was specified as the volume from the wall up to the mean boundary layer thickness. The ZPG-TBL shows a propensity for $Q_2$ ejections followed by $Q_4$ sweeps, combining to make up 60% of the detected structures. $Q_2$ and $Q_4$ events also occupy a much larger proportion of the volume indicating that

![Figure 5](image-url)

_Figure 5._ Percolation analysis for the identification of $uv$ structures in the ZPG-TBL (a) and the APG-TBL (b) showing the effect of chosen hyperbolic size $H^*$ on the volume ratio of the largest structure $V_{\text{largest}}$ and on the number of detected structures $N_{\text{objects}}$ relative to the maximum number of detected structures $N_{\text{max}}$. 

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they are not only more numerous but also larger than both the $Q_1$ and $Q_3$ structures. In the case of this strong APG-TBL the relative number of structures in each quadrant are far more even with the $Q_-$ structures now only making up just over 50% of the detected structures. Similar difference were observed by Maciel et al. [9] between the relative number of $Q_-$ structures in ZPG-TBL and rapidly decelerating non-self-similar APG-TBL, however the individual number of sweeps and ejects were not reported. The relative number of different quadrant structures in the present ZPG-TBL are similar to the those reported in DNS of a fully developed channel flow [5].

Unlike the previous APG-TBL study, which reports an increase in the relative volume that is occupied by $Q_-$ structures from 8.4% in the ZPG-TBL to 10.6% equivalent in the rapidly decelerating APG-TBL, the present study shows a significant decrease in the volume occupied by all the different quadrant structures from 33% in the ZPG-TBL to 6.3% in the strong self-similar APG-TBL. This is consistent with the narrower peak in the Reynolds stress away from the wall in the APG-TBL, compared to the more even distribution across the ZPG-TBL. It is not clear why the volume occupied by $Q_-$ structures in the present ZPG-TBL is so much higher than that reported by Maciel et al. [9], however it maybe associated with smaller domain used in their analysis of the ZPG-TBL, with their PDFs of the streamwise extent of detected structure indicating that a number of structure reach the bounds of their domain of interest.

The wall-normal extent of the $Q_-$, $Q_2$ and $Q_4$ structures can be examined by considering the joint probability density function (PDF) of the nearest and furthest point from the wall for the structures. In the ZPG-TBL the peak in the near wall region indicates that the majority of structures extend down to the wall, with heights up to the boundary layer thickness. The clustering along the diagonal line represents the extent of structures that begin at a fixed height $y_{min}$ above the wall. The averaged values of the lower boundary of each structure $y_{min}$, upper bound $y_{max}$ and centroid $y_{cent.}$ are presented in table 4. PDFs for the APG-TBL demonstrate a considerably larger percentage of structures that begin above the wall, referred to here as a wall-detached structures. The relative number and volume of wall-attached structures, where $y_{min} < 0.2 \delta_1$, and near wall structures, where $y_{cent.} < 0.2 \delta_1$, are given in table 3. The percentage of attached structures is approximately 4 times higher for the ZPG-TBL with 3 times as many near wall structures. Maciel et al. [9] report a slightly higher percentage of wall-attached $Q_-$ and near wall structures in their ZPG-TBL but also report a percentage of attached structures in their rapidly decelerating APG-TBL roughly 3 times higher than that observed in the present self-similar case.

In the present APG-TBL a larger relative volume is occupied by a smaller fraction of attached structure in the APG-TBL, when compared to the ZPG-TBL, indicating that the structures

| Table 2. Number of structures identified in each quadrant $N_Q$ for the ZPG-TBL and the APG-TBL relative to the total number $N_{total}$ of structures identified and the combined volume occupied by structures of each quadrant $V_Q$ relative to the total volume $V_{total}$. |
|----------|--------|--------|--------|--------|
| ZPG-TBL  | $N_Q/N_{total}$ | 0.202  | 0.327  | 0.195  | 0.276  | 0.603  |
|          | $V_Q/V_{total}$  | 0.051  | 0.197  | 0.048  | 0.136  | 0.333  |
| APG-TBL  | $N_Q/N_{total}$ | 0.253  | 0.257  | 0.237  | 0.253  | 0.510  |
|          | $V_Q/V_{total}$  | 0.010  | 0.039  | 0.010  | 0.024  | 0.063  |
Figure 6. Joint PDFs of the wall-normal extent of the $Q_-$ structures in the ZPG-TBL (a) and APG-TBL (b), the $Q_2$ structures in the ZPG-TBL (c) and APG-TBL (d), and the $Q_4$ structure in the ZPG-TBL (e) and APG-TBL (f). Contour levels are shown for 1, 0.1, 0.01, and 0.001.

that remain must be significantly larger. In both boundary layers the $Q_4$ structures show a greater probability of being wall-attached and for a given $y_{\text{min}}$ tend to have a larger $y_{\text{max}}$ value, indicating that the detached $Q_4$ structures tend to be taller than their $Q_2$ counterparts.

Figure 7 shows the joint PDFs associated with the streamwise $L_x$ and wall-normal $L_y$ extent of the $Q_-$ structures and those of the streamwise and spanwise extent $L_z$. The dashed-lines represent the correlation between the spatial extent of the structures, indicating that for both boundary layers the height of the $Q_-$ structures tend to be 0.65 times their length and 1.1 times their span. Despite this similarity, the ZPG-TBL also demonstrates a tendency towards longer structures with a shorter wall-normal extent. This trend is reversed in the APG-TBL as a tendency develops towards taller structures, consistent with the increased Reynolds stress.
Table 3. Number and volume of wall-attached \( Q_{\text{attached}} \) and near wall \( Q_{\text{nearwall}} \) structures for the ZPG-TBL and the APG-TBL relative to the number \( N_{Q_-} \) and volume \( V_{Q_-} \) of \( Q_- \) structures.

|          | \( Q_{\text{attached}} \) | \( Q_{\text{nearwall}} \) |
|----------|---------------------------|---------------------------|
| ZPG-TBL  | \( N_Q/N_{Q_-} \) 0.444   | \( V_Q/V_{Q_-} \) 0.59    |
|          |                           |                           |
| APG-TBL  | \( N_Q/N_{Q_-} \) 0.115   | \( V_Q/V_{Q_-} \) 0.68    |

away from the wall. The mean dimensions associated with the \( Q_- \) structures are listed in table 4. On average the spatial extent of these structures are around 5 times shorter and 3 times shorter and narrower, consistent with \( Q_- \) structures occupying 5 times less volume than in the ZPG-TBL. The mean structure is also far more isotropic in extent as the streamwise length is reduced.

Variation in the wall-normal extent of \( Q_- \) structures with the height of the structure centroid \( y_{\text{cent}} \) above the wall can be observed in the joint PDF of \( L_y \) and \( y_{\text{cent}} \). (see figure 8). The dashed-
Table 4. Average wall-normal extent, centroid, length, span and height of \( Q_- \) structure in the ZPG-TBL and APG-TBL.

|               | ZPG-TBL                  | APG-TBL                 |
|---------------|--------------------------|-------------------------|
| \( \langle y_{min} \rangle \) | 1.02\( \delta_1 \) (0.24\( \delta \)) | 0.75\( \delta_1 \) (0.18\( \delta \)) |
| \( \langle y_{cent} \rangle \) | 1.20\( \delta_1 \) (0.28\( \delta \)) | 0.79\( \delta_1 \) (0.19\( \delta \)) |
| \( \langle y_{max} \rangle \) | 1.39\( \delta_1 \) (0.32\( \delta \)) | 0.83\( \delta_1 \) (0.20\( \delta \)) |
| \( \langle L_x \rangle \) | 0.66\( \delta_1 \) (0.11\( \delta \)) | 0.09\( \delta_1 \) (0.02\( \delta \)) |
| \( \langle L_y \rangle \) | 0.37\( \delta_1 \) (0.06\( \delta \)) | 0.08\( \delta_1 \) (0.02\( \delta \)) |
| \( \langle L_z \rangle \) | 0.35\( \delta_1 \) (0.06\( \delta \)) | 0.08\( \delta_1 \) (0.02\( \delta \)) |

The line represents the wall-attached structures with the region near the horizontal representing short wall-detached structures. The strong APG-TBL involves a large number of detached \( Q_- \) structures as previously discussed, however this also shows that the majority of these structures have a short wall-normal extent. At the location of peak Reynolds stress, \( y_{cent} = \delta_1 \), figure 8b indicates that this is near the highest point above the wall where \( Q_- \) structures have a reasonable probability of extending down to the wall.

**Figure 8.** Joint PDFs of the structure wall-normal centroid \( y_{cent} \) and the wall-normal extent \( L_y \) of the \( Q_- \) structures in the ZPG-TBL (a) and APG-TBL (b). Contour levels are shown for 1, 0.1, 0.01, and 0.001.

6. Conclusions

Mean Reynolds stress profiles and instantaneous Reynolds stress structures have been investigated in a self-similar APG-TBL at the verge of separation using data from DNS. Comparisons are made with the DNS of a ZPG-TBL developing from the same inlet boundary layer. Structures are detected based on a threshold of the local product of the \( uv \) velocity fluctuations relative to the local background turbulence level, and a clustering of adjacent points in the domain that reside in the same \( uv \) quadrant. The Reynolds stress in the APG-TBL shows a narrower peak located further from the wall which is responsible for the majority of the production. Unlike the ZPG-TBL the \( uv \) component of the Reynolds stress tensor in the APG-TBL is no longer the only component that is responsible for the production of turbulent kinetic energy, although it is still the primary contributor. The correlation between the \( u \) and \( v \) velocity fluctuations in the APG-TBL is half that of the ZPG-TBL in the near wall region, yet increases to a similar level in the outer region. This indicates less organised near wall ejections.
and sweeps, and is consistent with an increase in the relative number of $Q_1$ and $Q_3$ events.

ZPG-TBLs contain $Q_2$ ejections and $Q_4$ sweeps that together represent 60% of the detected $\langle uv \rangle$ structures, where as in the APG-TBL, ejections and sweeps only represent just over 50%. In the APG-TBL these structures are found to occupy 5 times less of the boundary layer volume. This difference is associated with a reduction in height and span, and a significantly shorter streamwise extent. The increase in turbulent activity in the outer flow correlates with the increase in the number of wall-detached $Q_-$ structures. The location of the peak Reynolds stress is found to be roughly the furthest distance from the wall that is occupied by a combination of attached and detached structures.

Comparison between the present self-similar APG-TBL and the investigation of the rapidly decelerating APG-TBL of Maciel et al. [9] following the same method of $uv$ structure detection, reveal a significantly lower percentage of wall-attached $Q_-$ and near wall structures in the present self-similar APG-TBL, along with a smaller streamwise extent of the present structures. Given the tendency for the wall-attached structures to be larger in both studies the smaller streamwise extent of the $Q_-$ structures in the self-similar APG-TBL is consistent with the reduced number of wall-attached structures. It is postulated that this difference is due to upstream flow history of the rapidly decelerated APG-TBL, which owing to the sudden onset of the pressure gradient, continues to maintain a number of wall-attached structures from upstream.

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