The fractal analysis of the topography and gravitational field of Venus

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Abstract. The purpose of this paper is to present the results of structural analysis of gravitational and topographic parameters of Venus using the data from space missions, including “Magellan”. The model gravitational potentials are presented as analytical functions of coordinates. The model is constructed on the basis of variations of Venus’ artificial satellites orbits. As a result, fractal correlations of Venus’ geoid anomalies and gravitational potential in both longitude and latitude as well as the mean value of fractal dimensions are calculated. The mean fractal dimension of Venus topographic model in latitude is $\bar{D} = 1.061$, in longitude is $\bar{D} = 1.037$; the mean fractal dimension of Venus gravitational potential model in latitude is $\bar{D} = 0.96$, in longitude is $\bar{D} = 1.053$.

1. Introduction
Currently, the basic methods of analyzing processes in complex systems are statistical ones [1–5]. They have restrictions of probabilistic and statistical approaches, since not every process and not every result (form and structure of system) is of probabilistic nature. In contrast to this, fractal analysis allows studying structure of complex objects taking into account their qualitative specificity. Determination of fractal dimension allows studying both the structure itself and the connection between the structure and processes of its formation. The problem of developing methods of determining fractal structures of complex objects is relevant in this regard. Thus, anomalies of Venus’ gravitational field and variations of Venus’ physical surface are multi-parametric systems whose analysis should be conducted using the methods of physics of complex systems, and fractal analysis is one of them [6]. In this paper, fractal structures of Venus were studied on the basis of NASA “Magellan” space mission data [7]. Scientific purposes of “Magellan” space mission were studying chemical processes as well as modelling the internal structure of Venus. The equipment of the artificial satellite allowed scanning almost entire surface of Venus using theradar system with synthetic aperture of S-range (12 cm) and microwave radiometer. These devices also allowed investigating topography on the basis of measurements taken by the special radar – altimeter [7]. Inaccuracies in some data obtained by “Magellan” were taken into account using the information which had been collected during “Venera 15”, “Venera 16”, and “Pioneer Venus 1” space missions. The data on gravitational field of Venus obtained during the space missions was used as well. In astrophysics, gravitational potentials presented as analytical functions of coordinates are used. They are determined by analyzing celestial bodies’ artificial satellites orbits evolution and gravitational effects that may be measured. Such models allow studying gravitational field at some distance from celestial body and judging mass distribution in its interior. It is also necessary to consider the fact that choice of basic reference surface on Venus is specified by certain value of potential or point on the surface through which the geoid passes. To construct models of gravitational potential anomalies we used the data from [8].

2. The method of Venusian structures fractal analysis
The models of Venus’ physical surface and gravitational field anomalies are of complex structure, and to study them the methods of multi-parametric analysis are required. Such models are fractal structures. Thus, the best method of analyzing such systems is fractal geometry.
differences between fractal dimension for the Venus’ surface and gravitational field anomalies models and their real physical values show that there is some complex distribution of model structure in space. In this paper, while constructing a model of fractal object the Weierstrass-Mandelbrot fractal function was used. On the one hand, complex physical systems cannot be described as a single fractal and represent multifractals consisting of a set of interconnected fractals of their own dimensions. On the other hand, in a fractal model, each part of it repeats the entire model on structure and does not change at various scales, that is recursive. Use of multifractal analysis allows studying a system as a spectrum of fractal dimensions. Such method provides high accuracy when describing complex fractal structures by investigating local areas. In the present paper, for determining and analyzing fractal dimensions the Minkowski mathematical algorithm, which is a simplified option of Hausdorff-Besicovitch dimension and provides high reliability and accuracy, was used.

Let $M$ be a limited set in the plane and let us define $N(\sigma)$ as a minimal number of an area being investigated covers whose combination covers $M$ [9]. In general case, the set $M$ has a dimension of $D=\dim M, 0\leq D\leq 2$, if at $\sigma\to 0$ the number of covers $N(\sigma)$ increases as $R/\sigma^D$, where $R$ is a positive constant called $D$-measure of the set $M$. For $R$ in the general case we may write:

$$R = \lim_{\sigma\to 0} \sigma^D N(\sigma).$$

Expression (1) is not convenient when calculating, since it contains the value of $D$-measure. The fractal dimension $D$ may be determined according to the following expression [9]:

$$D = \dim M = \lim_{\sigma\to 0} \frac{\ln N(\sigma)}{\ln \frac{1}{\sigma}},$$

as

$$\lim_{\sigma\to 0} \frac{\ln N(\sigma)}{\ln \frac{1}{\sigma}} = \lim_{\sigma\to 0} \frac{\ln R}{\ln \frac{1}{\sigma}} = \lim_{\sigma\to 0} \frac{D \ln \frac{1}{\sigma} + \ln R}{\ln \frac{1}{\sigma}} = D.$$  

To analyze Venus’ fractal structure the data obtained during “Magellan” space mission [7] was used. The model of gravitational potentials presented as analytical functions of coordinates was used. The model included Venus’ artificial satellite orbit variations and other measurements of its gravitational field. The constructed gravimetric model of Venus showed the difference between real and normal gravity force variations. Using this model, one may study mass distribution in Venus interior and investigate gravitational field near the surface of the planet. The basic reference surface is specified by the point on its surface coinciding with the certain value of potential. Depending on gravimetric potential, the reference surface may be both under and above the physical surface of Venus similarly to terrestrial potential [8]. The obtained topographic and gravimetric models were investigated using the Minkowski algorithm. The study of the obtained fractal dimensions allowed conducting comparative analysis of various geophysical parameters of Venus.

3. Harmonic models of topography and gravitational field of Venus

Using harmonic analysis of expansion in spherical functions of the topographical data received from boards of «Magellan» a topographical model of Venus was constructed [5, 10]:

$$h(\varphi, \lambda) = \sum_{n=0}^{N} \sum_{m=0}^{n} (\tilde{C}_{nm} \cos m\lambda + \tilde{S}_{nm} \sin m\lambda) \cdot \tilde{P}_{nm}(\cos \varphi) + \varepsilon,$$

where $h(\varphi, \lambda)$ – function of altitude;
$\varphi, \lambda$ – latitude, longitude (known parameters);
$\tilde{C}_{nm}, \tilde{S}_{nm}$ – normalized harmonic amplitudes;
$\tilde{P}_{nm}$ – normalized associated Legendre functions;
ε – random regression error.
This equation was also used for the analysis of positional observations taken at Engelhardt Astronomical Observatory [11].

On the basis of gravimetric measurements a harmonic model of Venus’ gravitational potential was constructed [12]:

\[ U(r, \varphi, \lambda) = \frac{GM}{r} \left[ 1 + \sum_{n=2}^{\infty} \left( \frac{R_0}{r} \right)^n \left( \bar{C}_{nm} \cos m\lambda + \bar{S}_{nm} \sin m\lambda \right) \cdot P_{nm}(\sin \varphi) \right] + \epsilon, \] (5)

where \( G \) – gravitational constant;
\( GM \) – the product of the Gravitational constant and the Mass of Venus (324 858.601 km\(^3\)/sec\(^2\)) [13];
\( r, \varphi, \lambda \) – spherical coordinate of reference point;
\( R_0 \) – equatorial radius of planet.

4. Study of Venusian fractal structures and the results of analysis
The topographical model of Venus was cut by planes on meridians with longitudes: 0, 20, 40, 60, 80, 100, 120, 140, 160, 180, 200, 220, 240, 260, 280, 300, 320, 340 degrees in order to create macroprofiles of the planet’s surface. As an example, at longitude of 180 degrees this may be seen in figure 1. The system of units on both \( X \) and \( Y \) axis correspond to the measured data and are not critical, since the profiles are considered as self-similar structures.

![Figure 1. Macroprofile of Venus topography at longitude of 180 degrees.](image)

X-axis corresponds to latitude (\( \varphi \)) in degrees, Y-axis corresponds to altitude in meters (\( h \)).

Similarly to this, the model of gravity anomalies was cut by planes on meridians with longitudes: 0, 20, 40, 60, 80, 100, 120, 140, 160, 180, 200, 220, 240, 260, 280, 300, 320, 340 degrees in order to create macroprofiles of the planet’s gravity. As an example, at longitude of 300 degrees this may be seen in figure 2.
Figure 2. Macroprofile of Venus gravity anomalies on the longitude of 300 degrees. X-axis corresponds to latitude in degrees (φ), Y-axis corresponds to units of acceleration mGal (0.001 cm/sec²).

For each cut, fractal dimensions were calculated according to equation (2), where \( N \) – number of covers in the area investigated. The division into covers starts with value of 24, for number of covers to be integer. The obtained values for the topographical model and the model of gravitational field anomalies are presented in figure 3.

Figure 3. Fractal dependence of topographical model and gravity anomalies in Venus latitude. X-axis corresponds to latitudes in degrees (φ), Y-axis corresponds to fractal dimension values \( D \). Line 1 corresponds to fractal dimensions of Venus topographical model, line 2 corresponds to fractal dimensions of gravity anomalies on Venus.

Analysis of figure 3 shows that for the topographical data the variations of fractal dimensions vary between 0.90 and 1.20. Considering the fact that the more significantly topological dimensions of a
structure differ from fractal ones, the more complex formation of the system is, the similarity of topographical formations on Venus is sufficiently defined. The variations of gravity anomalies fractal dimensions change between 0.76 and 1.23. Thus, the gravitational field of Venus may be described as a topological dissimilar system with a complex structure in latitude measurements.

The next stage is determination of fractal dimensions in longitude. To conduct this, cuts of Venus models by Venusian meridians in certain Venusian longitudes within 0–350 degrees with a step of 25 degrees were made. As a result, the curve in figure 4 was plotted.

![Figure 4](image)

**Figure 4.** Dependence of fractal dimension of topographical model and gravity anomalies on longitude of Venus. X-axis corresponds to longitude in degrees ($\lambda$), Y-axis corresponds to values of fractal dimension $D$. Line 1 corresponds to fractal dimensions of Venus topographical model; line 2 corresponds to fractal dimensions of gravity anomalies on Venus.

It should only be noted that topographical fractal dimensions behave more evenly while gravitational fractal dimensions show more anomalies, especially in figure. Analysis of figure 4 shows that for the topographical data the variations of fractal dimensions vary between 0.82 and 1.16. The variations of gravitation anomalies fractal dimensions change between 0.68 and 1.42. The latter indicates that gravitational field in Venusian longitude is of heterogeneous structure.

5. **Summary and conclusions**

In this work, the fractal dependencies for Venusian topographic model and anomalies of gravitational potential in both longitude and latitude are determined. The values of mean fractal dimensions are obtained: mean fractal dimensions of Venusian topographic model in latitude $D = 1.061$, in longitude $D = 1.037$; mean fractal dimensions of Venusian gravitational potential anomalies in latitude $D = 0.96$, in longitude $D = 1.053$.

As seen from the analysis of fractal dimensions for Venusian topographic model, the physical surface of Venus is close to sphere. Venusian gravitational field has a more heterogeneous and complex structure.

It should be noted that the method of fractal analysis allows introducing independent estimates of Venusian macrostructure, which leads to the new approaches to interpreting physical processes taking place on Venus. The further use of fractal comparative analysis when processing the data of space
observations allows obtaining the important results that will help to solve many problems of space investigations.

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