Quantum Origin of Noise and Fluctuations in Cosmology

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Abstract

We address two basic issues in the theory of galaxy formation from fluctuations of quantum fields: 1) the nature and origin of noise and fluctuations and 2) the conditions for using a classical stochastic equation for their description. On the first issue, we derive the influence functional for a $\lambda\phi^4$ field in a zero-temperature bath in de Sitter universe and obtain the correlator for the colored noises of vacuum fluctuations. This exemplifies a
new mechanism we propose for colored noise generation which can act as seeds for galaxy formation with non-Gaussian distributions. For the second issue, we present a (functional) master equation for the inflaton field in de Sitter universe. By examining the form of the noise kernel we study the decoherence of the long-wavelength sector and the conditions for it to behave classically.
1. Galaxy Formation from Quantum Fluctuations

A standard mechanism for galaxy formation is the amplification of primordial density fluctuations by the evolutionary dynamics of spacetime [1, 2]. In the lowest order approximation the gravitational perturbations (scalar perturbations for matter density and tensor perturbations for gravitational waves) obey linear equations of motion. Their initial values and distributions are stipulated—oftentimes assumed to be a white noise spectrum. In these theories, fashionable in the sixties and seventies, the primordial fluctuations are classical in nature. The Standard model of Friedmann-Lemaitre- Robertson-Walker with power-law dependence (on cosmic time) generates a density contrast which turns out to be too small to account for the observed galaxy masses. The observed nearly scale-invariant spectrum also does not find any easy explanation in this model [3, 4].

The inflationary cosmology of the eighties [5, 6, 7] is based on the dynamics of a quantum field $\phi$ undergoing a phase transition. The exponential expansion of the scale parameter $a(t) = a_0 \exp(\mathcal{H}t)$ gives a scale-invariant spectrum naturally. This is one of the many attractive features of the inflationary universe, particularly with regard to the galaxy formation problem. The primordial fluctuations are quantum in nature. They arise from the fluctuations of the quantum field which induces inflation, sometimes called the inflaton. The density contrast $\delta \rho / \rho$ can be shown to be related to the fluctuations of the scalar field $\Delta \phi$ approximately by [8]

$$\frac{\delta \rho}{\rho} \approx \frac{H \Delta \phi}{\langle \dot{\phi} \rangle}$$

(1)

Here $\mathcal{H} = \dot{a}/a$ is the Hubble expansion rate, assumed to be a constant for the de Sitter phase of the evolution, and $< >$ denotes average over some spatial range. For the density contrasts to be within $10^{-4}$ when the modes enter the horizon the coupling constant in the Higgs field (e.g. a $\lambda \phi^4$ theory) in the standard models of unified theories has to be exceedingly small ($\lambda \sim 10^{-12}$).
The main features of the inflationary cosmology are determined by the dynamics of different sectors of the normal modes of the scalar field in relation to the exponential Hubble expansion of the background spacetime. The scalar field $\Phi$ evolves according to the equation

$$\ddot{\Phi} + 3H\dot{\Phi} + V'(\Phi) = 0 \tag{2}$$

where the potential $V(\Phi)$ can take on a variety of forms. A common form for the discussion of the generic behavior of old $|5$ and chaotic $|9$ inflation is the $\phi^4$ potential

$$V(\Phi) = \frac{1}{2}m^2\Phi^2 + \frac{1}{4!}\lambda\Phi^4 \tag{3}$$

For new inflation $|6,7$ to work, the potential has to possess a flat plateau, as in the Coleman-Weinberg form. Another commonly used potential is the exponential form $|10$.

Consider a scalar inflation field in a de Sitter space. In this so-called ‘eternal inflation’ stage the horizon size $l_h = H^{-1}$ is fixed. The physical wavelength $l$ of a mode of the inflation field is $l = p^{-1} = a(t)/k$ where $k$ is the wave number of that mode. As the scale factor increases exponentially, the wavelengths of many modes can grow larger than the horizon size. After the end of the de Sitter phase, the universe begins to reheat and turns into a radiation-dominated Friedmann universe with power law expansion $a(t) \sim t^n$. In this phase, the horizon size expands much faster than the physical wavelength. Some inflaton modes that left the de Sitter horizon will later reenter the Friedman horizon, i.e., the physical wavelength becomes shorter than the horizon size in this radiation or matter-dominated phase. The fluctuations of these long-wavelength inflaton modes that had gone out and later come back into the horizon play an important role in determining the large scale density fluctuations of the early universe which later evolve to galaxies. A common assumption is that these long wavelength inflaton modes behave classically while the other short wavelength inflaton modes behave like quantum fluctuations $|11$. While this overall
picture is generally accepted, a fully quantum mechanical description of the evolution of the
inflaton field and its fluctuations undergoing phase transitions in the inflationary universe
is still lacking.

One suggestion made by Starobinsky [12] and Bardeen and Bublik [13] in what is known
as ‘stochastic inflation’ is to split the inflation field into two parts at every instant according
to their physical wavelengths, i.e.,

$$
\Phi(x) = \phi(x) + \psi(x).
$$

The first part $\phi$ (the ‘system field’) consists of field modes whose physical wavelengths are
longer than the de Sitter horizon size $p < \epsilon H$. The second part $\psi$ (the ‘environment field’) consists of field modes whose physical wavelengths are shorter than the horizon size $p > \epsilon H$.

At early times the modes in the system behave with little difference from that in Minkowsky
space. Here $\epsilon$ is a small parameter measuring their deviation from the Minkowsky behavior.

Inflation continuously shifts more and more modes of the environment field into the system
after their physical wavelengths exceed the de Sitter horizon size.

Starobinsky’s model treats a free, massless, conformally-coupled field. With $m = 0$
and $\lambda = 0$ in (1.3), substitution of (1.4) into (1.2) gives an equation of motion for the
system field $\phi$

$$
\ddot{\phi}(t) + 3H \dot{\phi} + V'(\phi) = \xi(t)
$$

where

$$
< \xi(t) > = 0, \quad < \xi(t)\xi(t') >= \delta(t - t')
$$

The common belief is that the bath field contributes a white noise source [12, 13, 14].
With this assumption, the system field equation is thus rendered into a classical Langevin
equation with a white noise source. A Fokker-Planck equation can also be derived which
depicts the evolution of the probability distribution of the scalar field $P(\phi, t)$ [15]. Much
recent effort is devoted to the solution of this stochastic equation for a description of the inflationary transition and galaxy formation problems.

Note that two basic assumptions are made in transforming a quantum field theoretic problem to a classical stochastic mechanics problem as in the stochastic inflation program: 1) The low frequency scalar field modes (the system) behave classically. 2) The high frequency quantum field modes (the bath) behave like a white noise. Most previous researchers seem to hold the view that the first condition is obvious and the second condition is proven. In our view neither case is clear. We think that the first proposal is plausible, but the proof is non-trivial while the second proposal is dubious and a correct proof does not yet exist. Overall, a more rigorous treatment of the relation of quantum and classical fluctuations, and the source and nature of noise is needed before a sound foundation for this program can be established.

On the first issue one needs to consider the conditions upon which quantum fluctuations evolve to be classical. It requires first an understanding of quantum to classical transition, which involves the decoherence process [16] [17]. It also questions the conditions when a quantity (field or geometry) can be effectively viewed as fluctuation rather than part of the background. Both quantum field and statistical considerations are needed to give a clear picture of the relation of quantum to classical and background to fluctuations. In particular one needs to consider the decoherence of different histories of quantum fields in a given spacetime dynamics (in the context of semiclassical cosmology), and, more thoroughly, that of the histories of spacetimes themselves (in the context of quantum cosmology) [18]. Some work has appeared in addressing this aspect of the problem in inflationary cosmology [19]. Our current research on inflationary cosmology is directed towards clarifying these two issues. We are using different concepts and approaches in quantum mechanics [20] and quantum kinetic theory to explore the relation between quantum and classical fluctuations, and applying some of the techniques attempted earlier in quantum
cosmology to decoherence in inflationary cosmology, but we shall not discuss this issue here. Our concern here is mainly with the second proposal, although the two issues are related and the theoretical framework we use here can be used to address both.

On the issue of noise, note that for a free field the inflaton modes do not interact with the bath modes and they do not interact with one another. These field modes behave like a collection of non-interacting free particles in an ideal gas. The separation is like a sieving partition which moves in time. It is obvious that adding or taking away some particles (modes) from the system should not disturb the motion of other particles in the system. But the system as a whole may lose or gain energy through the exchange of particles with the environment, which itself is depleting in content. The common claim of researchers in stochastic inflation is that the effect of this infusion of modes on the system is like a noise source, in particular, a white noise source for free fields.

There are two problems with this view. Theoretically, a rigorous treatment of this problem requires a quantum field theory of open systems, which, contrary to what is commonly perceived and practised, is not a straightforward matter. What constitutes the system actually changes in time as it is constantly enhanced by modes from the environment and interacts with them. Physically, if one works in formalisms which deal only with pure states, as has been done so far in most papers written on this topic, it is difficult to understand how the concept of noise arise. Even if one forces in the identification of a noise source by splitting the fields and averaging part of them one cannot find a corresponding dissipation force. This is an unsatisfactory feature since physically noise and dissipation should always appear together according to the general fluctuation-dissipation relation. (Some authors misconstrue the red-shift term $3H\dot{\phi}$ in the Klein-Gordon equation as dissipation. It is a mistake). A correct treatment should use a formalism which can encompass the statistical nature of mixed states and the dynamics of reduced density matrices as we shall show below.
Here we seek a more basic approach to this issue which removes these two drawbacks. 1) We adopt a fully field-theoretical treatment of non-equilibrium quantum systems. We use the influence functional formalism to treat the system-bath interaction and show how noise arises from quantum fields when one field (or a sector therein) is coarse-grained, and how its averaged effect on another field (or sector) is described in a functional master equation, or a functional Fokker-Planck-Wigner or Langevin equation. We show how one can identify the nature of noise corresponding to different baths and system-bath couplings. 2) We discuss the more realistic albeit more difficult case of an interacting system field and propose a different mechanism for the generation of noise in the inflationary universe, viz., colored noise generation from the nonlinear interaction of the inflation quantum field. We take the usual $\lambda \phi^4$ potential assumed in most inflation models as example, although the mechanism of colored noise generation illustrated thus is generic in nature. The colored noise source produced in this way provides a natural mechanism for the generation of non-Gaussian spectrum of density perturbations.

From the general statistical physics point of view, the above issues which pervade in the problems of inflationary cosmology and quantum cosmology have their roots in problems of quantum open systems, many of them can be understood from simple examples in quantum mechanics.

We have studied these problems in the context of non-equilibrium statistical mechanics using the paradigm of quantum Brownian motion. We refer the reader to these papers for details and for a comparison with the present field-theoretical problem for the discussion of the same issues. In Sec. 2 we discuss the generation of colored noise from interacting quantum fields in Minkowsky spacetime, assuming for simplicity two scalar fields with the full range of modes and a bi-quadratic form of coupling. In Sec. 3 we discuss the corresponding problem in de Sitter spacetime. Once the noise source is derived, one can then solve the Langevin equation for the inflaton field, or the Fokker-Planck-Wigner equation for the distribution function of the scalar field. We only write
down the master equation here. In the discussion section (Sec. 4) we summarize our findings, and discuss how realistic our assumptions are, and project possible problems in its consequences. The main aim of this work, which is the first part of a project on noise, fluctuations and structure formation, is to show how noise arises from interacting quantum fields, or, more specifically, in a fully quantum field-theoretical context, how different noise sources (usually colored) can arise from different (nonlinear) interactions between the system and the environment fields. In the second part of this project currently under investigation, we shall describe from the stochastic dynamics of quantum fields in the early universe how structures are formed from general fluctuations described by colored noises.
2. Colored Noise from Interacting Quantum Fields in Minkowski Spacetime

We first consider quantum fields in a Minkowski spacetime. The separation of a single field into the high and low momentum sectors are rather cumbersome to carry out, so for simplicity we will consider two independent self-interacting scalar fields $\phi(x)$ depicting the system, and $\psi(x)$ depicting the bath. The physics is expected to be similar to the partitioned case. The classical action for these two fields are given respectively by:

\begin{align}
S[\phi] &= \int d^4x \left\{ \frac{1}{2} \partial_{\nu}\phi(x) \partial^{\nu}\phi(x) - \frac{1}{2} m_{\phi}^2 \phi^2(x) - \frac{1}{4!} \lambda_{\phi} \phi^4(x) \right\} \\
S[\psi] &= \int d^4x \left\{ \frac{1}{2} \partial_{\mu}\psi(x) \partial^{\mu}\psi(x) - \frac{1}{2} m_{\psi}^2 \psi^2(x) - \frac{1}{4!} \lambda_{\psi} \psi^4(x) \right\} = S_0[\psi] + S_I[\psi]
\end{align}

where $m_{\phi}$ and $m_{\psi}$ are the bare masses of $\phi(x)$ and $\psi(x)$ fields respectively. Both fields have a quartic self-interaction with the bare coupling constants $\lambda_{\phi}$ and $\lambda_{\psi}$. In (2.2) we have written $S[\psi]$ in terms of a free part $S_0$ and an interacting part $S_I$ which contains $\lambda_{\psi}$. We assume that these two scalar fields interact via a bi-quadratic coupling

\begin{equation}
S_{int} = \int d^4x \left\{ -\lambda_{\phi\psi} \phi^2(x) \psi^2(x) \right\}
\end{equation}

and also that all three coupling constants $\lambda_{\phi}$, $\lambda_{\psi}$ and $\lambda_{\phi\psi}$ are small parameters of the same order. The total classical action of the combined system plus bath field is then given by

\begin{equation}
S[\phi, \psi] = S[\phi] + S[\psi] + S_{int}[\phi, \psi]
\end{equation}

The total density matrix of the combined system plus bath field is defined by

\begin{equation}
\rho[\phi, \psi, \phi', \psi', t] = <\phi, \psi | \hat{\rho}(t) | \phi', \psi '>
\end{equation}

where $|\phi >$ and $|\psi >$ are the eigenstates of the field operators $\hat{\phi}(x)$ and $\hat{\psi}(x)$, namely,

\begin{align}
\hat{\phi}(\vec{x})|\phi > &= \phi(\vec{x})|\phi >, \\
\hat{\psi}(\vec{x})|\psi > &= \psi(\vec{x})|\psi >
\end{align}
Since we are primarily interested in the behavior of the system, and of the environment only to the extent in how it influences the system, the quantity of relevance is the reduced density matrix defined by

$$\rho_{\text{red}}[\phi, \phi', t] = \int d\psi \rho[\phi, \psi, \phi', \psi, t]$$  \hspace{1cm} (12)

For technical convenience, let us assume that the total density matrix at an initial time is factorized, i.e., that the system and bath are statistically independent,

$$\hat{\rho}(t_0) = \hat{\rho}_\phi(t_0) \times \hat{\rho}_\psi(t_0)$$  \hspace{1cm} (13)

where \(\hat{\rho}_r(t_0)\) and \(\hat{\rho}_\psi(t_0)\) are the initial density matrix operator of the \(\phi\) and \(\psi\) field respectively, the former being equal to the reduced density matrix \(\hat{\rho}_r\) at \(t_0\) by this assumption.

The reduced density matrix of the system field \(\phi(x)\) evolves in time following

$$\rho_r[\phi_f, \phi'_f, t] = \int d\phi_i \int d\phi'_i \ J_r[\phi_f, \phi'_f, t | \phi_i, \phi'_i, t_0] \ \rho_r[\phi_i, \phi'_i, t_0]$$  \hspace{1cm} (14)

where \(J_r\) is the propagator of the reduced density matrix:

$$J_r[\phi_f, \phi'_f, t | \phi_i, \phi'_i, t_0] = \int \frac{d\phi}{\phi_i(x)} \int \frac{d\phi'}{\phi'_i(x)} \ \rho_{\phi}[\phi_i, \phi'_i, t_0] \ J_r[\phi_f, \phi'_f, t | \phi_i, \phi'_i, t_0]$$  \hspace{1cm} (15)

The influence functional \(F[\phi, \phi']\) is defined as

$$F[\phi, \phi'] = \int d\psi_f(x) \int d\psi_i(x) \int \rho_{\psi}[\psi_i, \psi'_i, t_0] \ J_r[\phi_f, \phi'_f, t | \phi_i, \phi'_i, t_0] \ J_r[\phi_f, \phi'_f, t | \phi_i, \phi'_i, t_0]$$  \hspace{1cm} (16)

$$\times \ \exp \left\{ S[\psi] + S_{\text{int}}[\phi, \psi] - S[\psi'] + S_{\text{int}}[\phi', \psi'] \right\}$$

which summarizes the averaged effect of the bath on the system. The influence action \(\delta A[\phi, \phi']\) and the effective action \(A[\phi, \phi']\) are defined as

$$F[\phi, \phi'] = \exp i\delta A[\phi, \phi']$$  \hspace{1cm} (17)
\[ A[\phi, \phi'] = S[\phi] - S[\phi'] + \delta A[\phi, \phi'] \] (18)

The above is the formal framework we shall adopt. Let us now begin the technical discussion of how to evaluate the influence action perturbatively. If \( \lambda_{\phi \psi} \) and \( \lambda_{\psi} \) are assumed to be small parameters, the influence functional can be calculated perturbatively by making a power expansion of \( \exp i \{ S_{\text{int}} + S_I \} \). Up to the second order in \( \lambda \), and first order in \( \hbar \) (one-loop), the influence action is given by

\[
\delta A[\phi, \phi'] = \left\{ < S_{\text{int}}[\phi, \psi] >_0 - < S_{\text{int}}[\phi', \psi'] >_0 \right\} \\
+ \frac{i}{2} \left\{ < S_{\text{int}}[\phi, \psi] >_0^2 - < S_{\text{int}}[\phi, \psi] >_0^2 \right\} \\
- i \left\{ < S_{\text{int}}[\phi, \psi] S_{\text{int}}[\phi', \psi'] >_0 - < S_{\text{int}}[\phi, \psi] >_0 < S_{\text{int}}[\phi', \psi'] >_0 \right\} \\
+ \frac{i}{2} \left\{ < S_{\text{int}}[\phi', \psi'] >_0^2 - < S_{\text{int}}[\phi', \psi'] >_0^2 \right\} 
\] (19)

where the quantum average of a physical variable \( Q[\psi, \psi'] \) over the unperturbed action \( S_0[\psi] \) is defined by

\[
<Q[\psi, \psi'] >_0 = \int d\psi_f(\vec{x}) \int d\psi_1(\vec{x}) \int d\psi'_1(\vec{x}) \rho_{\psi}[\psi_i, \psi'_i, 0] \\
\times \int \psi_j(\vec{x}) \psi_j(\vec{x}) D\psi \int \psi'_j(\vec{x}) \psi'_j(\vec{x}) D\psi' \exp i\left\{ S_0[\psi] - S_0[\psi'] \right\} \times Q[\psi, \psi'] \\
\equiv Q \left[ \frac{\partial}{i \partial J_1(x)}, \frac{\partial}{i \partial J_2(x)} \right] F^{(1)}[J_1, J_2] \bigg|_{J_1=J_2=0} 
\] (20)

Here, \( F^{(1)}[J_1, J_2] \) is the influence functional of the free bath field, assuming a linear coupling with external sources \( J_1 \) and \( J_2 \).

\[
F^{(1)}[J_1, J_2] = \int d\psi_f(\vec{x}) \int d\psi_1(\vec{x}) \int d\psi'_1(\vec{x}) \rho_{\psi}[\psi_i, \psi'_i, t_0] \int \psi_j(\vec{x}) \psi_j(\vec{x}) D\psi \int \psi'_j(\vec{x}) \psi'_j(\vec{x}) D\psi' \\
\times \exp i\left\{ S_0[\psi] + \int d^4 x J_1(x) \psi(x) - S_0[\psi'] - \int d^4 x J_2(x) \psi'(x) \right\} 
\] (21)
Let us define the following free propagators of the $\psi$ field

\begin{align}
<\psi(x)\psi(y)>_0 &= iG_{++}(x,y) \\
<\psi'(x)\psi'(y)>_0 &= -iG_{--}(x,y) \\
<\psi(x)\psi'(y)>_0 &= -iG_{+-}(x,y) \\
<\psi'(x)\psi(y)>_0 &= -iG_{-+}(x,y)
\end{align}

Then the influence action is given by

\begin{align}
\delta A[\phi,\phi'] &= \int d^4x \left\{ -\lambda_\psi \phi G_{++}(x,x) \phi^2(x) \right\} \\
&- \int d^4x \left\{ -\lambda_\psi \phi G_{++}(x,x) \phi'^2(x) \right\} \\
+ \int d^4x \int d^4y \lambda_\psi^2 \phi^2(x) \left\{ -iG_{++}^2(x,y) \right\} \phi^2(y) \\
- 2 \int d^4x \int d^4y \lambda_\psi^2 \phi^2(x) \left\{ -iG_{+-}^2(x,y) \right\} \phi'^2(y) \\
+ \int d^4x \int d^4y \lambda_\psi^2 \phi^2(x) \left[ -iG_{-+}^2(x,y) \right] \phi'^2(y)
\end{align}

Note that if the bath is at zero temperature, i.e., if the bath field $\psi$ is in a vacuum state,

\begin{equation}
\hat{\rho}_b(t_0) = |0><0|
\end{equation}

then the influence functional (2.16) is the so-called Schwinger- Keldysh or closed-time-path (CTP) or ‘in-in’ vacuum generating functional |25 and the influence action (2.20) is the usual CTP or in-in vacuum effective action. In such cases, the propagators (2.17)-(2.19) are just the well known Feynman, Dyson and positive-frequency Wightman propagators of a free scalar field given respectively by,

\begin{align}
G_{++}(x,y) &= G_F(x-y) = \int \frac{d^n p}{(2\pi)^n} e^{ip(x-y)} \frac{1}{p^2 - m_\psi^2 + i\epsilon} \\
G_{--}(x,y) &= G_D(x-y) = \int \frac{d^n p}{(2\pi)^n} e^{ip(x-y)} \frac{1}{p^2 - m_\psi^2 - i\epsilon}
\end{align}
\[ G_{+-}(x, y) = G^+(x - y) = \int \frac{d^np}{(2\pi)^2} e^{ip(x-y)} 2\pi i\delta(p^2 - m_\psi^2)\theta(p^0) \]  

(29)

The perturbation calculation for \(\lambda \phi^4\) theory in the CTP formalism has been carried out before for quantum fluctuations [26] and for coarsed-grained fields [27, 28]. We find the effective action for this biquadratically-coupled system-bath scalar field model to be

\[
A[\phi, \phi'] = \left\{ S[\phi] + \delta S_1[\phi] + \delta_2[\phi] \right\} - \left\{ S[\phi'] + \delta S_1[\phi'] + \delta_2[\phi'] \right\} + \delta A[\phi, \phi']
\]

\[
= S_{\text{ren}}[\phi] + \int d^4x \int d^4y \frac{1}{2} \lambda_{\phi\psi}^2 \phi^2(x)V(x-y)\phi^2(y)
\]

\[
- S_{\text{ren}}[\phi'] - \int d^4x \int d^4y \frac{1}{2} \lambda_{\phi\psi}^2 \phi'^2(x)V(x-y)\phi'^2(y)
\]

\[
- \int_{t_0}^t ds_x \int d^3\vec{x} \int_{t_0}^{s_y} ds_y \int d^3\vec{y} \lambda_{\phi\psi}^2 \left[ \phi^2(x) - \phi'^2(x) \right]
\]

\[
\times \eta(x - x') \left[ \phi^2(y) + \phi'^2(y) \right]
\]

\[
+ i \int_{t_0}^t ds_x \int d^3\vec{x} \int_{t_0}^{s_x} ds_y \int d^3\vec{y} \lambda_{\phi\psi}^2 \left[ \phi^2(x) - \phi'^2(x) \right]
\]

\[
\times \nu(x - y) \left[ \phi^2(y) - \phi'^2(y) \right]
\]

(30)

Here \(S_{\text{ren}}[\phi]\) is the renormalized action of the \(\phi\) field, now with physical mass \(m_{\phi r}^2\) and physical coupling constant \(\lambda_{\phi r}\), namely,

\[
S_{\text{ren}}[\phi] = \int d^4x \left\{ \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m_{\phi r}^2 \phi^2 - \frac{1}{4!} \lambda_{\phi r} \phi^4 \right\}
\]

(31)

and the kernel for the non-local potential in (2.25) is

\[
V(x - y) = \mu(x - y) - \text{sgn}(s_x - s_y)\eta(x - y)
\]

(32)

which is symmetric.

Here \(\eta\) and \(\nu\) and \(\mu\) are real nonlocal kernels

\[
\eta(x - y) = \frac{1}{16\pi^2} \int d^4p \frac{e^{ip(x-y)}}{(2\pi)^4} \pi \sqrt{1 - \frac{4m_\psi^2}{p^2}} \theta(p^2 - 4m_\psi^2) \times \text{sgn}(p_0)
\]

(33)
\[
\nu(x-y) = \frac{2}{16\pi^2} \int \frac{d^4p}{(2\pi)^4} e^{ip(x-y)} \pi \sqrt{1 - \frac{4m^2_\psi}{p^2}} \theta(p^2 - 4m^2_\psi) 
\]

(34)

\[
\mu(x-y) = -\frac{2}{16\pi^2} \int \frac{d^4p}{(2\pi)^4} e^{ip(x-y)} \frac{1}{p^2} d\alpha \ln \left| 1 - i\epsilon - \alpha(1 - \alpha) \frac{p^2}{m^2_\psi} \right| 
\]

(35)

The imaginary part of the influence functional can be viewed as arising from a noise source \(\xi(x)\) whose distribution functional is given by

\[
P[\xi] = N \times \exp \left\{ -\frac{1}{2} \int d^4x \int d^4y \xi^2(x) \lambda^{-2}_\phi \nu^{-1}(x-y) \xi^2(y) \right\} 
\]

(36)

where \(N\) is a normalization constant. The action describing the noise \(\xi(x)\) and system field \(\phi(x)\) coupling is

\[
\int d^4x \left\{ \xi(x)\phi^2(x) \right\} 
\]

(37)

In the associated functional Langevin equation for the field, the corresponding stochastic force arising from the biquadratic coupling we have assumed is

\[
F_\xi(x) \sim \xi(x)\phi(x) 
\]

(38)

which constitutes a multiplicative noise |24.

From the influence action (2.25), it is seen that the dissipation generated in the system by this noise is of the nonlinear non-local type. If we define the dissipation kernel \(\gamma(x-y)\) by

\[
\eta(x-y) = \frac{\partial}{\partial(s_x - s_y)} \gamma(x-y) 
\]

(39)

then

\[
\gamma(x-y) = \frac{1}{16\pi^2} \int \frac{d^4p}{(2\pi)^4} e^{ip(x-y)} \pi \sqrt{1 - \frac{4m^2_\psi}{p^2}} \theta(p^2 - 4m^2_\psi) \frac{1}{|p_0|} 
\]

(40)

In the Langevin field equation, the dissipative force is
\[ F_\gamma(x) \sim \left\{ \int d^4y \, \eta(x-y)\phi^2(y) \right\} \phi(x) \]  \hspace{2cm} (41)

As discussed in Ref. 24, we find that a fluctuation-dissipation relation exists between the dissipation kernel (2.34) and the noise kernel (2.29):

\[ \nu(x) = \int d^4y \, K(x-y)\eta(y) \]  \hspace{2cm} (42)

where

\[ K(x-y) = \delta^3(\vec{x} - \vec{y}) \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} e^{i\omega(s_x-s'_x)} |\omega| \]

\[ = \delta^3(\vec{x} - \vec{y}) \int_0^{+\infty} \frac{d\omega}{\pi} \omega \cos \omega(s_x-s_y) \]  \hspace{2cm} (43)

Apart from the delta function \( \delta^3(\vec{x} - \vec{x}') \), the convolution kernel for quantum fields has exactly the same form as for the quantum Brownian harmonic oscillator with linear or nonlinear dissipations at zero temperature.

Thus we have given an explicit first-principle derivation of noise from quantum fluctuations of interacting quantum fields. We want to make three comments before closing this section. First, note that here, as distinct from the free field case of Ref. 12, the noise arises only because the coupling \( \lambda_{\phi\psi} \) between the system and the environment field is non-zero. Second, it would be of interest to find the conditions upon which the colored noise appears as white, i.e., \( \nu(s) \to \delta(s) \) independent of the detailed form of nonlinear coupling. This is possible from the quantum mechanical cases studied in Ref. 24. It could be at high temperature, or by a proper choice of the form of the spectral density of the bath. But in field theory the second alternative is not obviously implementable. Third, we have discussed a zero-temperature bath here, where the noise is of purely quantum nature, i.e., arising from vacuum fluctuations. One can easily include finite temperature baths and
deduce the noise from thermal fluctuations of the bath. This is similar to the attempts of Ref. 29. Noises in finite temperature fields are discussed in Ref. 30 for both Minkowski and de Sitter spacetimes.
3. Master Equation with Colored Noise in de Sitter Universe

We shall now proceed to calculate the influence functional for an interacting field in de Sitter universe and identify the noise source. Following Ref. 30 we shall derive the master equation from this influence functional for a special case and use it to examine the issue of decoherence. This equation and its associated Langevin or Fokker-Planck equation would enable one to calculate the fluctuation spectrum as a classical stochastic dynamics problem.

Consider a real, gauge singlet, massive, $\lambda \Phi^4$ self-interacting scalar field in a de Sitter spacetime with metric

$ds^2 = g_{\mu \nu} dx^\mu dx^\nu = dt^2 - a^2(t) d\vec{x}^2$ (44)

In the inflationary regime of interest, the scalar factor $a(t)$ expands exponentially in cosmic time $t$

$a(t) = a_0 \exp Ht$ (45)

The classical action of the inflaton field $\Phi(x)$ is

$S[\Phi] = S_0[\Phi] + S_I[\Phi]$ (46)

where

$S_0[\Phi] = \int d^n x \sqrt{-g(x)} \left\{ \frac{1}{2} g^{\mu \nu} \partial_\mu \Phi \partial_\nu \Phi + \frac{1}{2} \xi_n R(t) \Phi^2 \right\}$ (47)

is that part of the classical action describing a free, massless, conformally coupled scalar field, and

$S_I[\Phi] = \int d^n x \sqrt{-g(x)} \left\{ \frac{1}{2} m^2 \Phi^2 + \frac{1}{2} \xi_0 \xi R(x) \Phi^2 - \frac{1}{4!} \lambda \Phi^4 \right\}$ (48)
is the remaining (interactive) terms with contributions from nonzero $m, \lambda$, and $\xi$, i.e., massive, self-interacting, or non-conformal coupling. Here we use $\xi = 0$ for conformal coupling and $\xi = 1$ for minimal coupling in four dimensions and $\xi_n = \frac{(n-2)}{4(n-1)}$ is a constant which is equal to $1/6$ in 4-dimensions.

In the above,

$$R(t) = \frac{6}{a^2(t) a(t)} \ddot{a}(t)$$

(49)

is the scalar curvature, and $\sqrt{-g(x)} = a^{n-1}(t), .$

In the Starobinsky scheme, one makes a system-bath field splitting

$$\Phi(\vec{x}, t) = \phi(\vec{x}, t) + \psi(\vec{x}, t)$$

(50)

such that the system field is defined by

$$\phi(\vec{x}, t) = \int_{|\vec{k}|<\Lambda} \frac{d^3\vec{k}}{(2\pi)^3} \Phi(\vec{k}, t) \exp ip \cdot x$$

(51)

and the bath field is defined by

$$\psi(\vec{x}, t) = \int_{|\vec{k}|>\Lambda} \frac{d^3\vec{k}}{(2\pi)^3} \Phi(\vec{k}, t) \exp ip \cdot x$$

(52)

where $\Lambda$ is the cutoff wavenumber determined by the horizon size. The system field $\phi(x)$ contains the long wavelength modes, which undergoes a slow-rollover phase transition in the inflation period, while the bath field $\psi$ contains the short wavelength modes, which are the quantum fluctuations. With this splitting, the classical action (3.3) can be written as

$$S[\Phi] = S[\phi] + S[\psi] + S_{int}[\phi, \psi]$$

(53)
\[
S_{\text{int}}[\phi, \psi] = \int d^n x \sqrt{-g(x)} \left\{ -\frac{1}{6} \lambda \phi^3 \psi - \frac{1}{4} \lambda \phi^2 \psi^2 - \frac{1}{6} \lambda \phi \psi^3 \right\} 
\tag{54}
\]

One can define the influence functional in de Sitter space in a similar way as in the Minkowski space. Assuming that \( S_I[\psi] \) and \( S_{\text{int}}[\phi, \psi] \) are small perturbations, then up to the second order in \( \lambda \) and to the first loop, one can show that the influence action of the system field is given by

\[
\delta A[\phi, \phi'] = \int d^n x a^{n-1}(t) \left\{ -\frac{1}{2} \lambda < \psi^2(x) >_0 \phi^2(x) + \frac{1}{2} \lambda < \psi'^2(x) >_0 \phi'^2(x) \right\} 
+ \int d^n x a^{n-1}(t) \int d^n x' a^{n-1}(t') \frac{i}{4} \lambda M^2(x) \phi^2(x') 
\times \left\{ \left[ < \psi(x) \psi(x') >_0 \right]^2 - \left[ < \psi(x) \psi'(x') >_0 \right]^2 \right\} 
+ \int d^n x a^{n-1}(t) \int d^n x' a^{n-1}(t') \frac{i}{4} \lambda M^2(x) \phi'^2(x') 
\times \left\{ \left[ < \psi'(x) \psi(x') >_0 \right]^2 - \left[ < \psi'(x) \psi'(x') >_0 \right]^2 \right\} 
+ \int d^n x a^{n-1}(t) \int d^n x' a^{n-1}(t') \frac{i}{16} \lambda^2 \phi^2(x) \left[ < \psi(x) \psi(x') >_0 \right]^2 \phi^2(x) 
- 2 \int d^n x a^{n-1}(t) \int d^n x' a^{n-1}(t') \frac{i}{16} \lambda^2 \phi^2(x) \left[ < \psi(x) \psi'(x') >_0 \right]^2 \phi'^2(x') 
+ \int d^n x a^{n-1}(t) \int d^n x' a^{n-1}(t') \frac{i}{16} \lambda^2 \phi'^2(x) \left[ < \psi'(x) \psi'(x') >_0 \right]^2 \phi'^2(x') 
\tag{55}
\]

where \( M^2(x) = m^2 + \xi_n \xi R(x) \) and the quantum average over a conformally-coupled massless free field \(< >_0\) is defined similar to (2.17-19). Note at the one loop level, (3.12) is similar to (2.20), apart from the mass coupling and the non-conformal coupling terms in the original classical action (3.5). The other two interaction terms in (3.11) do not contribute at the one loop level.

Since the bath field \( \psi(\vec{x}) \) only contains high momentum (long wavelength) modes, when we calculate the Feynman diagrams of (3.12) by dimensional regularization, the momentum space integrations of Feynman diagrams are restricted to the region which is outside of the sphere with radius \( \Lambda \). This incomplete integration region in momentum space creates
some technical difficulty. For simplicity, we extend the range of all integrations in the Feynman diagrams to cover the whole momentum space. That is equivalent to assuming that $\phi$ and $\psi$ are two independent fields. Making such an assumption does not change the effect of the bath field greatly because in a realistic setting there exists other environmental fields which the system field interacts with (e.g., heat bath). Under such an approximation the system field is enhanced over the stochastic scheme in the high frequency sector, but the overall behavior of galaxy spectrums will not be affected significantly, because it is determined mainly by the low-frequency sector anyway.

Since the de Sitter space (3.1) is conformally-flat, a changeover to conformal time and conformally-related fields

$$\tau = \int dt \frac{1}{a(t)}$$  \hspace{1cm} (56)

$$\tilde{\psi}(\vec{x}, \tau) = a^{1-\tilde{\beta}}(\tau)\psi(\vec{x})$$  \hspace{1cm} (57)

can simplify the calculations. Let us also define the conformal mass by

$$\tilde{M}^2 = a^2 M^2$$  \hspace{1cm} (58)

It is clear that all the Feynman diagrams in (3.12) after the conformal transformation are identical to those in (2.25) which we have calculated before. We find the following effective action (henceforth $t$ will denote the conformal time $\tau$)
\[ A[\phi, \phi'] = S_r[\phi] - \int d^4x a^3(t) \frac{1}{2} \delta m^2(t) \phi^2 \]
\[ + \int d^4x a^3(t) \int d^4x' a^3(t') \frac{1}{2} \lambda^2 \phi^2(x) V(x - x') \phi^2(x') \]
\[ - S_r[\phi'] + \int d^4x a^3(t) \frac{1}{2} \delta m^2(t) \phi'^2 \}
\[ - \int d^4x a^3(t) \int d^4x' a^3(t') \frac{1}{2} \lambda^2 \phi'^2(x) V(x - x') \phi'^2(x') \]
\[ - \int_{t_0}^{t_f} dt \int d^3\vec{x} a^3(t) \int_{t_0}^{t} dt' \int d^3\vec{x}' a^3(t') \lambda^2 \left[ \phi^2(x) - \phi'^2(x) \right] \]
\[ \times \eta(x - x') \left[ \phi^2(x') + \phi'^2(x') \right] \]
\[ + i \int_{t_0}^{t_f} dt \int d^3\vec{x} a^3(t) \int_{t_0}^{t} dt' \int d^3\vec{x}' a^3(t') \lambda^2 \left[ \phi^2(x) - \phi'^2(x) \right] \]
\[ \times \nu(x - x') \left[ \phi^2(x') - \phi'^2(x') \right] \]

where

\[ V(x - x') = \mu(x - x') - \text{sgn}(t - t') \eta(x - x') \] (60)

is the kernel of the non-local potential. Here we have introduced the counter terms for
the mass, the field-geometry coupling constant, and the self-interaction coupling constant
renormalization of the \( \phi \) and \( \phi' \) fields respectively, and with them the corresponding physical parameters.

As before, we see that the last two terms in (3.16) are the dissipation and noise terms
whose kernels are given by (2.28) and (2.29) respectively with conformal time here replacing
cosmic time in the Minkowsky space results. The dissipation is of a nonlinear non-local
type. The noise is coupled to the system with an action in the form

\[ \int d^4x \sqrt{-g(x)} \left\{ \xi(x) \phi^2(x) \right\} \] (61)

The stochastic force (noise) \( \xi(x) \) has the following functional distribution

22
\[ P[\xi] = N \times \exp \left\{ -\frac{1}{2} \int d^4x \int d^4x' \, \xi(x) \left[ \frac{\nu^{-1}(x-x')}{\lambda^2 a^3(t) a^3(t')} \right] \xi(x') \right\} \] (62)

One can show from this that
\[
\left\{ \begin{array}{l}
<\xi(x)>_\xi = 0 \\
<\xi(x)\xi(x')>_\xi = \nu(x-x')
\end{array} \right. \tag{63}
\]

So this is a nonlinearly-coupled colored noise. The fluctuation-dissipation relation for this field model in de Sitter space is exactly the same as that in Minkowski space (2.37) and (2.38).

This sample calculation shows the origin and nature of noise from a quantum field in a cosmological setting. We can now turn to the second issue raised at the beginning, i.e., decoherence in the long wave-length sector. To analyse this problem we need to know the master equation, at least the form of the diffusion terms in that equation.

The functional quantum master equation for this field-theoretical model with general nonlinear non-local dissipation and non-linearly coupled colored noise has a complicated form in cosmic time (denoted before as \( t \)). However, in conformal time (in these equations also denoted as \( t \)), it is similar to that in Minkowski spacetime, which has been derived in [24]. We will not repeat that derivation here, but just mention a simple case to end our discussion. This is the case in cosmic time where one can get an explicit form of the functional quantum master equation, i.e., by making a local truncation in the effective action (3.16). Setting

\[
V(x-x') = v_0(t)\delta^4(x-x') \tag{64}
\]

\[
\eta(x-x') = \frac{\partial}{\partial(t-t')} \left\{ \gamma_0(t)\delta(x-x') \right\} \tag{65}
\]

\[
\nu(x-x') = \nu_0(t)\delta(x-x') \tag{66}
\]
we get the effective action
\begin{equation}
A[\phi, \phi'] = \int_0^t ds \int d^3\vec{x} \left\{ \frac{1}{2} \dot{\phi}^2 - \frac{1}{2} a^2(t) [\nabla \phi]^2 - \frac{1}{2} \left[ m^2 + \frac{1 + \xi r}{6} R(x) \right] \phi^2 - \frac{1}{4!} \lambda_r \phi^4 \\
- \frac{1}{2} \dot{\phi'}^2 + \frac{1}{2} a^2(t) [\nabla \phi']^2 - \frac{1}{2} \left[ m^2 + \frac{1 + \xi r}{6} R(x) \right] \phi'^2 + \frac{1}{4!} \lambda_r \phi'^4 \\
- \frac{1}{2} \delta m^2(t) \phi^2 + \frac{1}{2} \delta m^2(t) \phi'^2 + \frac{1}{2} \lambda^2 v(t) \phi^4 - \frac{1}{2} \lambda^2 v(t) \phi'^4 \\
- 2 \lambda^2 a^3(t) \gamma_0(t) (\phi^2 - \phi'^2) (\phi' - \phi'') - 3 \lambda^2 a^2(t) \dot{\alpha}(t) \gamma_0(t) (\phi^4 - \phi'^4) \\
+ i \lambda^2 v_0(t) (\phi^2 - \phi'^2)^2 \right\}
\end{equation}

From this we can derive the functional quantum master equation in the local truncation approximation
\begin{equation}
i \frac{\partial}{\partial t} \rho_r[\phi, \phi', t] = \hat{H}_\rho[\phi, \phi', t] \rho_r[\phi, \phi', t]
\end{equation}

where
\begin{equation}
\hat{H}_\rho[\phi, \phi', t] = \int d^3\vec{x} a^3(t) \left\{ \hat{h}_r(\phi) - \hat{h}_r(\phi') \\
+ 3 \lambda^2 a^2(t) \dot{\alpha}(t) \gamma_0(t) \left[ \phi^4(\vec{x}) - \phi'^4(\vec{x}) \right] \\
+ 2 \lambda^2 \gamma_0(t) \left[ \phi^2(\vec{x}) - \phi'^2(\vec{x}) \right] \left[ \phi(\vec{x}) \frac{\delta}{\delta \phi(\vec{x})} - \phi'(\vec{x}) \frac{\delta}{\delta \phi'(\vec{x})} \right] \\
- i \lambda^2 v_0(t) \left[ \phi^2(\vec{x}) - \phi'^2(\vec{x}) \right] \right\}
\end{equation}

and
\begin{equation}
\hat{h}_r(\phi) = - \frac{1}{2} a^3(t) \frac{\delta^2}{\delta \phi^2(\vec{x})} + \frac{1}{2} a(t) [\nabla \phi(\vec{x})]^2 \\
+ \frac{1}{2} \left[ m^2 + \frac{1 + \xi r}{6} R(t) \right] \phi^2(\vec{x}) \\
+ \frac{1}{4!} \lambda_r \phi^4(\vec{x}) + \delta m^2(t) \phi^2(\vec{x}) - \frac{1}{2} \lambda^2 v(t) \phi^4(\vec{x})
\end{equation}

This functional quantum master equation and its associated Langevin equation or Fokker-Planck-Wigner equation can be used to analyze the dynamics of the system field (long wavelength modes in the stochastic inflation scheme) for studying the decoherence and structure formation processes in the early universe. We have only begun this investigation and details will be made available in a later publication. Here, as a preliminary
result, we can get some qualitative information on how the system decoheres by analyzing the behavior of the diffusion term in the master equation. Diffusive effects are generated by the last term in the effective action (3.16) that produces the following contribution on the right hand side of the master equation for $\rho[\phi, \phi']$:

$$
\dot{\rho}[\phi, \phi', t] \propto - (\phi^2 - \phi'^2) \ast D(t) \ast (\phi^2 - \phi'^2) \times \rho[\phi, \phi', t]
$$

(71)

Here the symbol $\ast$ denotes the convolution product and $\phi$ represents a configuration of the scalar field in a surface of constant conformal time. The diffusion "coefficient" $D$ is therefore a nonlocal kernel that can be written in terms of its spatial Fourier transform as

$$
D(\vec{x}, \vec{y}, t) = \int \frac{d\vec{k}}{(2\pi)^3} \nu_k(t) \exp(-i\vec{k}(\vec{x} - \vec{y}))
$$

(72)

It is not easy to analyze the effect produced by a term like the one appearing in (3.28). However, we can use the following argument (see Ref. 31) to qualitatively investigate if the diffusive effects are stronger for long wavelength modes than they are for short ones. Note that the coefficient in (3.28) can be written in terms of the product of the Fourier transform (3.29) and that of the field $Q(x) = \phi^2$:

$$
(\phi^2 - \phi'^2) \ast D(t) \ast (\phi^2 - \phi'^2) = \int d\vec{k}(Q - Q') \hat{k} D_k(Q - Q') \hat{k}
$$

(73)

Let us now examine the dependence on $k = |\vec{k}|$ of the function $D_k$ entering in (3.29). Using our previous results, it is not hard to prove that this function can be written in terms of the physical wave vector $p = k/a$ as

$$
D_k(t) = \frac{a^4}{4\pi} \lambda^2 \left(1 - \frac{H}{p} f\left(\frac{p}{H}\right) + g\left(\frac{p}{H}\right)\right)
$$

(74)

where

$$
f(x) = \frac{1}{2\pi} \int_0^{2x} dx \left[-\sin x Ci(x) + \cos x Si(x)\right]
$$

(75)

$$
g(x) = \frac{1}{2\pi} \int_0^{2x} dx \left[\cos x Ci(x) + \sin x Si(x)\right]
$$

(76)
and $Si(x), Ci(x)$ are the usual integral trigonometric functions. In Figure 1 we have plotted $D_k(t)$ for a fixed value of the conformal time as a function of $\frac{p}{H}$, i.e., the ratio between the horizon size and the physical wavelength. The function has a strong peak in the infrared region of the spectrum suggesting that diffusion effects (decoherence is one of them) are indeed more pronounced for long wavelength modes and weaker for wavelengths shorter than the horizon size.
4. Discussions

We have outlined the first part of a program to describe structure formation from primordial quantum fluctuations via stochastic dynamics. To end, we briefly summarize our findings and discuss the feasibility of our mechanism and its implications.

A. What is new?

What we have accomplished here are: 1) supply a quantum field-theoretical definition and derivation of noise; 2) relate different types of noise to different couplings of the system and environment; and 3) derive an equation of motion—the master equation for the reduced density matrix or the Fokker-Planck equation for the associated Wigner functional. In this process 4) we showed from first principles how one can derive the equations of stochastic dynamics from interacting quantum field theory, both in flat and curved spacetimes; and 5) we proposed a new scheme of noise generation based on nonlinear coupling which is different from the Starobinsky mechanism (which assumes a free field with a moving partition). Since an interacting field (e.g., a $\lambda \phi^4$ or Coleman-Weinberg potential) is what is usually used for generating inflation anyway our mechanism is rather natural.

B. How realistic are the conditions?

The noise in our scheme arises from the system field (the inflaton) coupling nonlinearly to an environment field. What in a realistic situation could play the role of the environment field? One can assume as in the stochastic inflation scheme that the system field consists of the low frequency modes and the environment field that of the high frequency modes of one single inflaton field. The model we have studied, which has two separate self-interacting scalar fields coupled biquadratically each assuming a full spectrum of modes, can be viewed as an approximation to this scheme. The environment field can also be referring to other fields present besides the inflaton field. Only the quantum fluctuations of such fields need be present in our scheme to generate the noise which seeds the galax-
ies. Even if one assumes nothing, there is always the gravitational field itself which the
inflaton field is coupled to, and the vacuum gravitational fluctuations can equally seed the
structures in our universe |32, 33. (Note that in such cases the coupling is of a derivative
form rather than the polynomial form in this example. Noise arising from a derivative
type of coupling has been studied in connection with the issue of gravitational entropy in
minisuperspace quantum cosmology |34.)

C. Physical consequences

Noises arising from nonlinear couplings are under general circumstances colored. They
generate fluctuations which could give rise to non-Gaussian galaxy distributions (NGD).
There are, of course, simpler ways to generate NGD. A changing Hubble rate $H = \dot{a}/a$
as in a ‘slow-roll’ transition, or an exponential potential $V(\phi)$ |10 will do. However, such
mechanism only generates NGD at very long wavelengths, much longer than the horizon
size to be relevant to the observable spectrum.

As for the present scheme, since the value of $\lambda$ is restricted to be very small ($< 10^{-12}$)
in the standard GUT inflationary models (so that the magnitude of the density contrast
is compatible with the observed value $\delta \rho/\rho \approx 10^{-4}$ when the fluctuation mode enters the
horizon), the constituency of the colored portion of the noise is accordingly small. Whether
a nonlinear coupling will generate excessive inhomogeneities is an open question. It is still
too premature for us to speculate on the general behavior. Details of galaxy formation
analysis from the stochastic equations of motion derived here with different types of colored
noise and realistic physical parameters will be reported in a later publication.
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