Pathways to non-sequential multiple ionization in strong laser fields

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The outgoing electrons in non-sequential multiple ionization in intense laser fields are strongly correlated. The correlations can be explained within a classical model for interacting electrons in the presence of the external field. Here we extend the previous analysis for two and three electrons to cases with up to eight electrons and identify the saddle configurations that guard the channels for non-sequential multiple ionization. For four and fewer electrons the electrons in the dominant configuration are equivalent, for six and more electrons this is no longer the case. The case of five electrons is marginal, with two almost degenerate transition configurations. The total number of configurations increases rapidly, from 2 configurations for three electrons up to 26 configurations for eight electrons.

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I. INTRODUCTION

Laser pulses with peak intensities of the order of $10^{14}$ W/cm$^2$ and wavelengths in the infrared can ionize several electrons simultaneously [2, 14]. Growing experimental evidence [2, 14, 15, 16] supports a rescattering process [2, 14, 15, 16] as the most likely mechanism for the rapid transfer of a sufficient amount of energy to the electrons. The interaction between the rescattered electron and the others takes place close to the nucleus, leading to the formation of a short lived, highly excited compound state with electrons near to the nucleus. This compound state will decay by emitting a single electron (the most likely decay mode) or several electrons. The highly excited electrons move so quickly that the changes in the field can be neglected: the electrons see an essentially static electric field. Experimental studies of the final momentum distribution for double ionization show that the electron momenta parallel to the field are strongly correlated [8, 9, 10, 11]. Building on the picture of an intermediate compound state we have proposed a classical origin for the electron correlations [14, 15, 16, 17, 18]. The aim is to identify the possible saddles, their critical energies and their threshold exponents. The number of configurations increases rapidly with the number of electrons. Beginning with five electrons also several nearby almost degenerate saddles appear. Most interestingly, for four and fewer electrons all electrons are equivalent in the dominant mode, but this is no longer true for six or more.

We conclude this introduction with notation and the formulation of the Hamiltonian. Since the electrons escape from the highly excited compound state so quickly, we may assume that the field is constant during the escape. Let the electric field point in the $z$-direction and let the electrons be labeled $i = 1, \ldots, N$ with positions $\mathbf{r}_i = (x_i, y_i, z_i)$. Then the Hamiltonian is, in atomic units,

$$H_N = \sum_{i=1}^{N} \frac{\mathbf{p}_i^2}{2} - \sum_{i=1}^{N} \frac{Z}{|\mathbf{r}_i|} + \sum_{i<j}^{N} \frac{1}{|\mathbf{r}_i - \mathbf{r}_j|} - F \sum_{i=1}^{N} z_i.$$  \hspace{1cm} (1)

Simple scaling shows that the field strength $F$ can be absorbed, so that without loss of generality we can take $F = 1$ [14, 17].

The aim now is to determine the stationary points of this Hamiltonian, where the derivatives with respect to momenta and positions vanish. For the most part this is not possible analytically. The numerical method of choice is the Newton-Raphson iteration [19]. The required calculation of the matrix of second derivatives can be turned to an advantage since it allows for an immediate determination of the stability properties of configuration. The main disadvantage of the method, but also of all other numerical methods, is that they do not provide any measure by which to judge whether all solutions have been found. On the practical side we can make sure that we have hit at least all the configurations with large domains.
of attraction in the Newton-Raphson method by starting with a sufficiently large number of randomly selected initial conditions. We used up to $10^6$ initial conditions. The fact that fairly similar but definitely different configurations could be detected may be interpreted as strong evidence that all configurations have indeed been found.

The stability analysis of the stationary point in the full phase space allows us to determine the behavior of the cross-section for non-sequential escape close to the threshold [18]. Among all Lyapunov exponents of the saddle there is one, $\lambda_r$, whose eigenvector components point in the same direction along the field axis: it corresponds to a simultaneous motion of all electrons in the same direction away from the saddle. Borrowing terminology from chemical reactions, we call this subspace the reaction coordinate. Because of the repulsion between electrons all configurations have additional unstable eigenspaces, which enter in the threshold exponent. If the initial energy of the system equals the saddle energy only a trajectory living in the symmetric subspace can lead to a simultaneous escape of all electrons. This reduces the dimensionality of the problem and the cross-section vanishes. For higher energy some deviations from the symmetric motion are possible, giving a finite volume of initial conditions and a non-vanishing cross-section.

The dependence of the cross-section on energy $\sigma(E)$ close to the saddle energy $E_N$ can be obtained in the spirit of the Wannier analysis [20] [21] [22] [28], resulting in

$$\sigma(E) \sim (E - E_N)^\mu,$$

with an exponent

$$\mu = \frac{1}{\lambda_r} \sum \lambda_i. \quad (3)$$

The $\lambda_i$’s are the positive Lyapunov exponents of the saddle except for the reaction coordinate exponent $\lambda_r$. The cross section is large if the exponent is small, i.e. if the saddle is crossed quickly (large $\lambda_r$) or if the differences from the symmetric motion grow slowly (small $\lambda_r$). These cross section exponents are an additional characteristic of the multiple ionization process.

The remainder of the paper is organized as follows: in the next section we discuss a few obvious, and highly symmetric configurations. This is followed by our numerical results for up to eight electrons in section [II]. We conclude with a few remarks in section [IV].

II. SYMMETRIC CONFIGURATIONS

A. All electrons on a ring

We assume that all electrons are situated in a plane perpendicular to the field and that they obey a $C_{NV}$ symmetry. The reflection symmetry limits the momenta to be parallel to the symmetry planes and thus confines the motion to a dynamically allowed subspace in the high-dimensional $N$-body phase space. That is, if in the full phase space of the $N$-body problem initial conditions are prepared in this subspace they will never leave it. In the symmetry subspace we can use cylindrical coordinates for the electrons, $z_i = z$, $\rho_i = \rho$ and $\varphi_i = 2\pi i/N$. Then, for zero total angular momentum along the field axis and in the scaled variables where $F = 1$, the Hamiltonian can be reduced to

$$H(\rho, p, \rho, z) = \frac{p^2 + p_z^2}{2N} - \frac{N^2}{2\rho^2 + z^2} + \frac{W}{2\rho} - Nz, \quad (4)$$

with

$$W = \sum_{i < j} \frac{1}{\sin \left(\frac{\pi}{N} (j - i) \right)} = \sum_{k=1}^{N-1} \frac{N - k}{\sin \left(\frac{\pi k}{N} \right)}. \quad (5)$$

The Hamiltonian possesses a stationary point at $p_\rho = 0$, $p_z = 0$, $\rho = \rho_N$ and $z = z_N$ where

$$\rho_N = \sqrt{\frac{W}{2N} \left( \frac{2N^2}{W} \right)^{2/3} - 1}^{1/4}$$

$$z_N = \sqrt{\frac{W}{2N} \left( \frac{2N^2}{W} \right)^{2/3} - 1}^{3/4}. \quad (6)$$

and

$$E_N = -\left( \frac{2N^2}{W} \right)^{1/6} - \sqrt{2NW} \left[ \left( \frac{2N^2}{W} \right)^{2/3} - 1 \right]^{-1/4}. \quad (7)$$

The first few saddles with electrons on a ring are shown in Fig. 1.

For $k \ll N$ the repulsion energy $W$ behaves like $W = (N^2/\pi) \sum (1/k)$. This suggests that $W$ increases like $N^2 \ln N$, and indeed numerical evidence supports

$$W \approx (0.3N^2 + 0.3N - 3.1) \ln N. \quad (8)$$

Since $W$ increases faster than quadratically, the configurations exist for finite $N$ only. Specifically, we find that the energy of the configuration first decreases but then increases again, and that there is no configuration for $N = 473$ or more particles [32]. The properties of the configurations, including stability and cross section exponents, are listed in table [II].

B. One electron in the center

The simplest extension from the symmetric ring is to place one electron on the field axis and the others on a ring as shown in Fig. 1. Such configurations have three characteristic lengths, the radius of the ring $\rho_{N-1}$, the position of the ring $z_{N-1}$ and the position of the electron on the axis $z_e$. No analytical solutions have been found. The numerically determined characteristics are collected in table [II].
Shown are cases on a ring. The electron in the center is shown shaded.

**FIG. 2:** Saddles with one electron on the field axis and the other $N - 1$ electrons on a ring. The columns are the number $N$ of electrons, the energy $E_N$ at the saddle, the number $n_u$ of unstable directions (excluding the one along the reaction coordinate), the Lyapunov exponent $\lambda$, along the reaction coordinate, the critical exponent $\mu$ near threshold, the position $z_c$ of the electron on the axis and the radius $\rho_{N-1}$ and the distance $z_{N-1}$ from the nucleus for the ring of $N - 1$ electrons.

| $N$ | $E_N$ | $n_u$ | $\lambda$ | $\mu$ | $z_c$ | $\rho_{N-1}$ | $z_{N-1}$ |
|-----|-------|-------|----------|-------|------|------------|--------|
| 4   | -10.9398 | 3     | 1.0271   | 4.2423 | 1.6543 | 1.2996     | 1.2718 |
| 5   | -14.8001 | 4     | 0.9750   | 5.6633 | 1.8199 | 1.4324     | 1.3950 |
| 6   | -18.8975 | 5     | 0.9345   | 7.2035 | 1.9690 | 1.5583     | 1.4960 |
| 7   | -23.1867 | 6     | 0.9015   | 8.8984 | 2.1054 | 1.6775     | 1.5813 |
| 8   | -27.6372 | 9     | 0.8739   | 10.9698 | 2.2317 | 1.7905     | 1.6553 |

**TABLE II:** Saddles with all electrons in a plane through the field axis. The columns are the number $N$ of electrons, the energy $E_N$ at the saddle, the number $n_u$ of unstable directions (excluding the one along the reaction coordinate), the Lyapunov exponent $\lambda$, along the reaction coordinate, the critical exponent $\mu$ near threshold, and the minimal and maximal positions along the field axis, $z_{\text{min}}$ and $z_{\text{max}}$, respectively.

| $N$ | $E_N$ | $n_u$ | $\lambda$ | $\mu$ | $z_{\text{min}}$ | $z_{\text{max}}$ |
|-----|-------|-------|----------|-------|-----------------|-----------------|
| 3   | -7.3902 | 3     | 1.0981   | 3.7040 | 1.1143 | 1.4666         |
| 4   | -10.3975 | 5     | 1.0177   | 6.8831 | 1.0295 | 1.5699         |
| 5   | -13.5302 | 7     | 0.9568   | 10.7953 | 0.9170 | 1.7755         |
| 6   | -16.7566 | 9     | 0.9082   | 15.4282 | 0.7898 | 1.8639         |
| 7   | -20.0549 | 11    | 0.8680   | 20.7764 | 0.6543 | 2.0199         |
| 8   | -23.4091 | 13    | 0.8339   | 26.8366 | 0.5141 | 2.0986         |

**C. All electrons on a line**

A third class of configurations has all electrons in a plane through the field axis. The electrons are placed like beads on a string. With the confinement of electrons to an almost linear arrangement the saddle energies are higher than in the previous cases and the cross section exponents are also much larger. The properties of the saddles are listed in table III and their positions are shown in Fig. 3.

**III. ALL CONFIGURATIONS**

A summary of the symmetric configurations discussed in the previous section is provided in Fig. 4, which shows the saddle energies for different $N$. For the range of $N$ included in the figure the saddle energies for all configurations decrease, but as the example of the ring configurations shows, they can be expected to increase for sufficiently large $N$. There is a crossover in the config-
FIG. 3: Saddles with all electrons in a plane. Shown are cases \(N = 3, 4, 5, 6, 7\) and \(8\), projected onto the \(x\)-\(z\)-plane, with the field pointing along \(z\) axis.

FIG. 4: Energy of the saddles divided by \(N\) in the different symmetry classes for increasing number of electrons. The open circles are for electrons on a ring (\(C_{N\nu}\) symmetry), the crosses for configurations with one electron on the axis and the others on a ring (\(C_{(N-1)\nu}\) symmetry) and full circles are for the configurations with all electrons on a line (\(C_\nu\) symmetry).

A. Four electrons

For four electrons there are 4 configurations, shown in Fig. 5 Their properties are listed in table IV. The dominant non-sequential ionization takes place in the vicinity of the saddle corresponding to all electrons on a ring in a plane perpendicular to the field axis, as in the cases for two and three electrons. This saddle possesses both the lowest energy and the smallest critical exponent and thus the most favorable dependence of the cross-section on energy.

The next two configurations are almost degenerate in energy and have similar critical exponents. They form a triangle with an additional electron placed in the center. The saddle \(\nu = 2\) possesses a \(C_{3\nu}\) symmetry, but the state \(\nu = 3\) has a \(C_\nu\) symmetry only. The last \(\nu = 4\) configuration, with all electrons in a plane, has significantly higher energy and larger critical exponent compared to the previous states.

TABLE IV: Saddles for four electrons, ordered by increasing saddle energy. The columns give the number \(\nu\) of the state, the energy \(E_\nu\) at the saddle, the number \(n_u\) of unstable directions (excluding the one along the reaction coordinate), the Lyapunov exponent \(\lambda_\nu\) along the reaction coordinate, the critical exponent \(\mu\) near threshold, and some comment on the states.

| \(\nu\) | \(E_\nu\) | \(n_u\) | \(\lambda_\nu\) | \(\mu\) | Comment           |
|-------|---------|--------|------------|-------|------------------|
| 1     | -11.1059| 3      | 1.0340     | 4.0971| all on a ring    |
| 2     | -10.9398| 3      | 1.0271     | 4.2423| a ring plus center|
| 3     | -10.9398| 4      | 1.0271     | 4.2708| —                |
| 4     | -10.3975| 5      | 1.0177     | 6.8831| all on a line    |
For five electrons we find 5 configurations shown in Fig. 6 with properties given in Table V. The five electron problem constitutes actually a marginal case where the configuration of all electrons on a ring starts losing its dominance. The energy of this configuration is only slightly lower than the energy of the state $\nu = 2$ with one electron in the center and the others on a ring. The state $\nu = 2$ possesses, however, a smaller critical exponent than the $\nu = 1$ configuration. Therefore, unless there is a wide disparity in prefactors in the cross section law or residual correlations in the compound state that prefer one state over the other, we expect that both will contribute significantly to the process of non-sequential five electron escape.

The configurations $\nu = 3$ and $\nu = 4$ have the $C_v$ symmetry and reveal higher energy and larger critical exponents than the previous ones. The last $\nu = 5$ configuration with all electrons on a line is the least significant because of its high energy and its high critical exponent.

TABLE V: Saddles for five electrons, ordered by increasing saddle energy. The columns give the number $\nu$ of the state, the energy $E_{\nu}$ at the saddle, the number $n_u$ of unstable directions (excluding the one along the reaction coordinate), the Lyapunov exponent $\lambda_{\nu}$ along the reaction coordinate, the critical exponent $\mu$ near threshold, and some comment on the states.

| $\nu$ | $E_{\nu}$ | $n_u$ | $\lambda_{\nu}$ | $\mu$ | comment |
|-------|----------|-------|------------------|-------|---------|
| 1     | -14.8004| 4     | 0.9817           | 5.7342| all on a ring |
| 2     | -14.8001| 4     | 0.9750           | 5.6633| ring plus center |
| 3     | -14.7763| 5     | 0.9766           | 6.0823| —        |
| 4     | -14.3922| 6     | 0.9684           | 7.0893| —        |
| 5     | -13.5302| 7     | 0.9568           | 10.7953| all on a line |

B. Five electrons

For six electrons we find 11 saddles, with properties listed in Table VI. The first five configurations are shown in Fig. 7. The state with all particles on a ring is no longer dominant, it appears in the middle of Table VI. The states have $C_{3v}$ ($\nu = 1$), $C_v$ ($\nu = 2$), $C_{4v}$ ($\nu = 3$) and $C_{2v}$ ($\nu = 5$) symmetries.

C. Six electrons

For six electrons we find 11 saddles, with properties listed in Table VI. The first five configurations are shown in Fig. 7. The state with all particles on a ring is no longer dominant, it appears in the middle of Table VI. The state with an electron in the center and the others on a ring dominates the non-sequential ionization process. It has both the lowest energy and the smallest critical exponent. The states $\nu = 2$ and $\nu = 3$ look similar, but differ in their symmetries: state $\nu = 2$ has a $C_v$ symmetry, state $\nu = 3$ has a $C_{3v}$ symmetry.
TABLE VII: Saddles for seven electrons, ordered by increasing saddle energy. The columns give the number $\nu$ of the state, the energy $E_\nu$ at the saddle, the number $n_u$ of unstable directions (excluding the one along the reaction coordinate), the Lyapunov exponent $\lambda_\nu$ along the reaction coordinate, the critical exponent $\mu$ near threshold, and some comment on the states.

| $\nu$ | $E_\nu$ | $n_u$ | $\lambda_\nu$ | $\mu$ | Comment |
|------|--------|------|---------------|------|---------|
| 1    | -23.1867 | 6    | 0.9015        | 8.8984 | ring plus center |
| 2    | -23.0773 | 7    | 0.8980        | 9.3618 | —        |
| 3    | -22.9904 | 8    | 0.8991        | 10.1259 | —        |
| 4    | -22.8409 | 8    | 0.8980        | 10.7743 | —        |
| 5    | -22.8360 | 8    | 0.8967        | 10.7509 | —        |
| 6    | -22.8126 | 9    | 0.9036        | 11.3732 | —        |
| 7    | -22.7859 | 10   | 0.9077        | 12.0849 | all on a ring |
| 8    | -22.7451 | 9    | 0.8932        | 11.2920 | —        |
| 9    | -22.5550 | 9    | 0.8904        | 12.3631 | —        |
| 10   | -22.4109 | 9    | 0.8905        | 12.8292 | —        |
| 11   | -22.0806 | 10   | 0.8861        | 13.8150 | —        |
| 12   | -22.0804 | 11   | 0.8861        | 13.8850 | —        |
| 13   | -21.5215 | 10   | 0.8805        | 16.1329 | —        |
| 14   | -20.5494 | 11   | 0.8680        | 20.7764 | all on a line |

FIG. 8: The lowest lying saddles for seven electrons, projected onto the $x$-$z$-plane with the field axis (top row) and onto the $x$-$y$-plane perpendicular to the field. Some electrons are shaded to help identification in the two sets of frames. The states possess $C_{6\nu}$ ($\nu = 1$) and $C_\nu$ ($\nu = 2, 3, 4, 5$) symmetries.

D. Seven electrons

For seven electrons we have 14 states, with the properties listed in table VII. In Fig. 8, the first five states are shown. The configuration with one electron in the center and the others on a ring dominates the simultaneous electron escape, just as for the six electron problem.

E. Eight electrons

For eight electrons we have 26 configurations, listed in table VIII. The dominant configuration has two electrons in the center and the other six around it, see Fig. 9. The $\nu = 2$ saddle also has two electrons in the center, but different arrangements of the outer electrons.

The state with one electron in the center and the others on a symmetric ring is the fifth configuration, the one with all electrons on a ring the 17th configuration. The states possess $C_{2\nu}$ ($\nu = 1, 2$), $C_\nu$ ($\nu = 3, 4$) and $C_{7\nu}$ ($\nu = 5$) symmetries.

The state with all particles on a line has the highest energy and the largest critical exponent. This is the case not only for the eight electron problem but for all $N$ values analyzed here.

IV. CONCLUDING REMARKS

The analysis presented here confirms a trend anticipated from our analysis of triple ionization: that with an increasing number of electrons more and more saddle configurations and thus channels for non-sequential multiple ionization have to be considered. But the analysis also shows that, except for the somewhat marginal case of fivefold ionization, there is a well separated lowest lying saddle that dominates non-sequential multiple ionization. Table IX collects the symmetries and the threshold exponents of this lowest lying state.

A few experiments with multiple ionization of atoms have already been performed. However, momenta of the ionization products have been measured for double and triple ionizations only. Currently, measurements of momenta of more than two electrons are not available, not even of a single component, say the one along the field.
TABLE VIII: Saddles for eight electrons, ordered by increasing saddle energy. The columns give an index \( \nu \) of the state, the energy \( E_\nu \) at the saddle, the number \( n_\nu \) of unstable directions (excluding the one along the reaction coordinate), the Lyapunov exponent \( \lambda_\nu \) along the reaction coordinate, the critical exponent \( \mu \) near threshold, and some comment on the states.

| \( \nu \) | \( E_\nu \) | \( n_\nu \) | \( \lambda_\nu \) | \( \mu \) | comment |
|---|---|---|---|---|---|
| 1 | \(-27.6592\) | 7 | 0.8699 | 10.6553 | two in the center |
| 2 | \(-27.6523\) | 8 | 0.8698 | 10.9875 | |
| 3 | \(-27.6373\) | 7 | 0.8737 | 10.7629 | |
| 4 | \(-27.6373\) | 8 | 0.8737 | 10.7908 | |
| 5 | \(-27.6372\) | 9 | 0.8739 | 10.9698 | ring plus center |
| 6 | \(-27.6363\) | 8 | 0.8724 | 10.9400 | |
| 7 | \(-27.5495\) | 9 | 0.8679 | 11.5053 | |
| 8 | \(-27.5125\) | 8 | 0.8683 | 11.4266 | |
| 9 | \(-27.5122\) | 9 | 0.8684 | 11.5393 | |
| 10 | \(-27.4741\) | 9 | 0.8667 | 12.1438 | |
| 11 | \(-27.2884\) | 10 | 0.8677 | 12.9280 | |
| 12 | \(-27.2785\) | 10 | 0.8692 | 13.1026 | |
| 13 | \(-27.2537\) | 10 | 0.8650 | 13.4763 | |
| 14 | \(-27.2015\) | 10 | 0.8665 | 13.4391 | |
| 15 | \(-27.1801\) | 11 | 0.8629 | 13.4369 | |
| 16 | \(-27.0793\) | 11 | 0.8741 | 14.7803 | |
| 17 | \(-27.0208\) | 12 | 0.8799 | 15.9050 | all on a ring |
| 18 | \(-26.9869\) | 10 | 0.8639 | 14.3969 | |
| 19 | \(-26.9786\) | 10 | 0.8652 | 14.4260 | |
| 20 | \(-26.9774\) | 11 | 0.8659 | 14.7100 | |
| 21 | \(-26.9033\) | 11 | 0.8608 | 14.9062 | |
| 22 | \(-26.5081\) | 11 | 0.8568 | 16.7984 | |
| 23 | \(-26.3255\) | 11 | 0.8567 | 17.2700 | |
| 24 | \(-26.0390\) | 12 | 0.8534 | 18.2821 | |
| 25 | \(-25.1700\) | 12 | 0.8463 | 21.3654 | |
| 26 | \(-23.4091\) | 13 | 0.8339 | 26.8366 | all on a line |

axis. But already the measurement of parallel momenta for just two electrons would suffice for a preliminary test of the ionization pathways considered here. In particular, the breaking of symmetry between the outgoing electrons should be accessible. As long as the electrons are interchangeable, the distribution of parallel electron momenta should be like that for double ionization, with a clear preference for similar momenta. For \( N = 5 \) and more escaping electrons additional structures in the momentum distribution should appear, since the lowest lying saddle has non-equivalent electrons, see Figs. 9a to 9b. The momentum distributions for the two \( N = 3 \) saddles are discussed in 17, and similar modifications in shape can be expected for more electrons.

The analysis presented here assumes simultaneous escape of all electrons in a multiple ionization event. It is of course also possible that first one or a few electrons escape, then the next batch of electrons leaves and so on, before one arrives at the final multiply charge ion. Sequential ionization becomes significant when the field intensity is high enough to ionize electrons from a highly excited state of the ion left behind when the first electrons are gone. Nevertheless, close to the threshold for non-sequential multiple ionization the characteristics of the non-sequential process as described here should be identifiable.

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[1] T. S. Luk, H. Pummer, K. Boyer, M. Shakidi, H. Egger, and C. K. Rhodes, Phys. Rev. Lett. 51, 110 (1983).
[2] A. L’Huillier, L. A. Lompre, G. Mainfray, and C. Manus, Phys. Rev. A 27, 2503 (1983).
[3] K. Boyer, H. Egger, T. S. Luk, H. Pummer, and C. K. Rhodes, J. Opt. Soc. Am. B 1, 4 (1984).
[4] B. Walker, B. Sheehy, L. F. DiMauro, P. Agostini, K. J. Schafer, and K. C. Kulander, Phys. Rev. Lett. 73, 1227 (1994).
[5] R. Moshammer, B. Feuerstein, W. Schmitt, A. Dorn, C.D. Schäfer, H. Rottke J. Ullrich, C. Trump, M. Wittmann, G. Korn, K. Hoffmann, and W. Sandner, Phys. Rev. Lett. 84, 447 (2000).
[6] Th. Weber, M. Weckenbrock, A. Staudte, L. Spiel-
berger, O. Jagutzki, V. Mergel, F. Afaneh, G. Urbasch, M. Vollmer, H. Giessen, and R. Dörner, Phys. Rev. Lett. 84, 443 (2000).

[7] Th. Weber, M. Weckenbrock, A. Staudte, L. Spielberg, O. Jagutzki, V. Mergel, F. Afaneh, G. Urbasch, M. Vollmer, H. Giessen, and R. Dörner, J. Phys. B: At. Mol. Opt. Phys. 33, L128 (2000).

[8] Th. Weber, H. Giessen, M. Weckenbrock, G. Urbasch, A. Staudte, L. Spielberg, O. Jagutzki, V. Mergel, M. Vollmer, and R. Dörner, Nature 405, 658 (2000).

[9] M. Wechenbrock, M. Hattass, A. Czasch, O. Jagutzki, L. Schmidt, T. Weber, H. Roskos, T. Lößler, M. Thomson, and R. Dörner, J. Phys. B: At. Mol. Opt. Phys. 34, L449 (2001).

[10] B. Feuerstein, R. Moshammer, D. Fischer, A. Dorn, C. D. Schröter, J. Deipenwisch, J. R. Crespo Lopez-Urrutia, C. Höhr, P. Neumayer, J. Ullrich, H. Rottke, C. Trump, M. Wittmann, G. Korn, and W. Sandner, Phys. Rev. Lett. 87, 043003 (2001).

[11] R. Moshhammer, B. Feuerstein, J. Crespo Lopez-Urrutia, J. Deipenwisch, A. Dorn, D. Fischer, C. Höhr, P. Neumayer, C. D. Schröter, J. Ullrich, H. Rottke, C. Trump, M. Wittmann, G. Korn, and W. Sandner, Phys. Rev. A 65, 035401 (2002).

[12] P. B. Corkum, Phys. Rev. Lett. 71, 1994 (1993).

[13] K. C. Kulander, K. J. Schafer, and J. L. Krause, in Super-Intense Laser-Atom Physics, Proceedings of the NATO Advanced Research Workshop, Han-sur-Lesse, Belgium, 1993, edited by B. Piraux, A. L’Huillier, and K. Rzążewski (Plenum Press, New York, 1993).

[14] B. Eckhardt and K. Sacha, Physica Scripta T90, 185 (2001).

[15] K. Sacha and B. Eckhardt, Phys. Rev. A 63, 043414 (2001).

[16] K. Sacha and B. Eckhardt, in Super-Intense Laser-Atom Physics, Proceedings of the NATO Advanced Research Workshop, Han-sur-Lesse, Belgium, 2000, edited by B. Piraux and K. Rzążewski (Kluwer Academic Publishers, Dordrecht, 2001), pp. 79–83.

[17] K. Sacha and B. Eckhardt, Phys. Rev. A 64, 053401 (2001).

[18] B. Eckhardt and K. Sacha, Europhys. Lett. 56, 651 (2001).

[19] W. H. Press, S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, Numerical Recipes in FORTRAN The art of Scientific Computing (Cambridge University Press, New York, 1995).

[20] G. H. Wannier, Phys. Rev. 90, 817 (1953).

[21] A. R. P. Rau, Phys. Rep. 110, 369 (1984).

[22] J. M. Rost, Phys. Rep. 297, 271 (1998).

[23] J. M. Rost, Physica E 9, 467 (2001).

[24] P. Lambropoulos, P. Maragakis, and J. Zhang, Phys. Rep. 305, 203 (1998).

[25] Super-Intense Laser- Atom Physics, Proceedings of the NATO Advanced Research Workshop, Han-sur-Lesse, Belgium, 1993, edited by B. Piraux, A. L’Huillier, and K. Rzążewski (Plenum Press, New York, 1993).

[26] Super-Intense Laser-Atom Physics, Proceedings of the NATO Advanced Research Workshop, Han-sur-Lesse, Belgium, 2000, edited by B. Piraux and K. Rzążewski (Kluwer Academic Publishers, Dordrecht, 2001).

[27] A. Becker and F. H. M. Faisal, Phys. Rev. Lett. 84, 3546 (2000).

[28] R. Kopold, W. Becker, H. Rottke, and W. Sandner, Phys. Rev. Lett. 85, 3781 (2000).

[29] M. Lein, E. K. U. Gross, and V. Engel, Phys. Rev. Lett. 85, 4707 (2000).

[30] B. Feuerstein, R. Moshammer, and J. Ullrich, J. Phys. B: At. Mol. Opt. Phys. 33, L823 (2000).

[31] L.-B. Fu, J. Liu, and S.-G. Chen, Phys. Rev. A 65, 011404(R) (2001).

[32] L.-B. Fu, J. Liu, J. Chen, and S.-G. Chen, Phys. Rev. A 63, 043416 (2001).

[33] L.-B. Fu, J. Liu, and S.-G. Chen, Phys. Rev. A 65, 021406(R) (2002).

[34] A. Becker and F. H. M. Faisal, Phys. Rev. Lett. 89, 193003 (2002).

[35] This discussion corrects the one given in [14], where a much smaller number of possible configurations was obtained using $W = N(N-1)/\sin(\pi/N)$, with the large $N$ behavior $W \sim N^2$. This overestimate of the repulsion between the electrons is responsible for the smaller number of possible configurations.