The Wendelstein Weak Lensing (WWL) pathfinder: Accurate weak lensing masses for Planck clusters

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ABSTRACT

We present results from the Wendelstein Weak Lensing (WWL) pathfinder project, in which we have observed three intermediate redshift Planck clusters of galaxies with the new 30’×30’ wide field imager at the 2m Fraunhofer Telescope at Wendelstein Observatory. We investigate the presence of biases in our shear catalogues and estimate their impact on our weak lensing mass estimates. The overall calibration uncertainty depends on the cluster redshift and is below 8.1–15 per cent for \( z \approx 0.27–0.77 \). It will decrease with improvements on the background sample selection and the multiplicative shear bias calibration.

We present the first weak lensing mass estimates for PSZ1 G109.88+27.94 and PSZ1 G139.61+24.20, two SZ-selected cluster candidates. Based on Wendelstein colors and SDSS photometry, we find that the redshift of PSZ1 G109.88+27.94 has to be corrected to \( z \approx 0.77 \). We investigate the influence of line-of-sight structures on the weak lensing mass estimates and find upper limits for two groups in each of the fields of PSZ1 G109.88+27.94 and PSZ1 G186.98+38.66. We compare our results to SZ and dynamical mass estimates from the literature, and in the case of PSZ1 G186.98+38.66 to previous weak lensing mass estimates. We conclude that our pathfinder project demonstrates that weak lensing cluster masses can be accurately measured with the 2m Fraunhofer Telescope.

Key words: gravitational lensing: weak – galaxies: clusters: general – cosmology: observations

1 INTRODUCTION

With masses above \( 10^{14} \, M_\odot \), clusters of galaxies are the massive end of the distribution of collapsed structures in the Universe. By measuring the abundance of clusters as a function of mass and redshift, the evolution of structure formation can be studied. Halo abundance experiments (for a review, cf. Allen et al. 2011) let us examine the underlying dark matter density field, as well as the growth rate of structure.

In order to exploit their full potential as cosmological probes, masses have to be determined accurately. Large surveys rely on inexpensive observables, such as richness, X-ray luminosity and SZ Compton parameter, which do not provide an absolute mass scale. The Mass-Observable Relation (MOR) relates the cosmology dependent theoretical cluster mass function to observables. Using samples of galaxy clusters for which weak lensing mass estimates are available, the MOR can be calibrated (e.g. Hoekstra et al. 2012; Marrone et al. 2012; Gruen et al. 2014; Mantz et al. 2016; Melchior et al. 2017).

In this work, we show that the Wendelstein Wide Field Imager (WWFI) installed at the 2m Fraunhofer telescope (Hopp et al. 2008, 2014; Kosyra et al. 2014) can be used to conduct cluster weak lensing studies and add to the list of clusters with accurate mass estimates. The paper is organized as follows. In Section 2, we introduce our data and describe the reduction procedure and the photometry. We elaborate on the shear measurement procedure in Section 3

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and address the possibility of systematic bias in our shape catalogues. Section 4 provides a description of our background sample selection procedure and a discussion of the impact of uncertainties in the redshift estimation on the derived cluster masses. We explain the weak lensing analysis of our clusters in Section 5. Our results are presented in Section 6 and compared to SZ and X-ray studies in Section 7. Concluding remarks are given in Section 8. Throughout, we use a flat standard ΛCDM model with ΩM = 0.27 and H0 = 72 km s⁻¹Mpc⁻¹.

2 OBSERVATIONS AND DATA REDUCTION

2.1 Instruments

We have observed our targets with the 2.0m Fraunhofer Telescope (see Hopp et al. 2014) using the Wendelstein Wide Field Imager (see Kosyra et al. 2014). The WWL field project has been among the first projects to provide science verification during telescope commissioning using the WWFI as the scientific first light instrument since 2014. The camera consists of a 2 × 2 mosaic of (4k)² pixel 15 μm e2v CCDs. Each of the four CCDs has four readout ports. The field of view is 27.6' × 29.0' ≈ 0.22 deg² with a pixel scale of 0.2 arcsec/pixel. The filter wheels are equipped with five optical SDSS-like broad-band filters ugriz. For this project, we have used three-band photometric gri data only. With sub-arcsecond median seeing at the telescope site and a design that aims to reduce the amount of ghost images, WWFI data are suitable for cluster weak lensing studies.

2.2 Cluster sample

For this work we have selected three Planck clusters of galaxies (Planck Collaboration et al. 2014b) in order to increase the MOR calibration sample. Thanks to the location of the telescope, clusters far up in the northern hemisphere can be included in the WWL sample. We are the first to target PSZ1 G109.88+27.94 and PSZ1 G139.61+24.20 for weak lensing studies. The large number of field stars allows for an exquisite test for how well PSF anisotropies of the camera can be modeled. This strategy can turn out to be problematic, however, when stars become too many or bright stars cause over-saturation and bleeding effects. We show the PSF modelling in Section 3.1.

Table 1 provides more information on our targets. Spectroscopic redshifts are available for all of these clusters. We note that the spectroscopic redshift of PSZ1 G109.88+27.94 that is referenced in Planck Collaboration et al. (2014b) could not be confirmed. Our data, as well as SDSS photometry (DR-14 Abolfathi et al. 2017) suggests, that the true redshift of this cluster is likely much larger (z ≈ 0.8). More details on the redshift estimation of this object are given in Sections 4 and 6.1.

We have obtained three band photometric data with the WWFI SDSS-like g, r, i filters. We use the r band as our lensing band. The good seeing in this band and the long integration times (up to 10.5 hours) make it the most useful for weak lensing analyses.

2.3 Reference field

In order to get a clean background sample selection for the weak lensing analysis, we need reliable redshift estimates. This can be achieved by comparing the flux of the observed galaxies in all available filters to galaxies with known redshifts (Gruen et al. 2014; Gruen & Brimioulle 2017). Those reference galaxies have to be observed with the same set of filters as has been used for the cluster fields. We call this reference field W-EGS, since we have chosen a sub-region of the extended groth strip, an extremely well studied patch on the sky, for our analysis. It overlaps with CFHTLS-D3 (the Canada-France-Hawaii Telescope Legacy Survey-Deep3) (Davis et al. 2007), which provides extremely deep u', g', r', i', z' data, and is also covered in the near-infrared with J, H, K (Bielby et al. 2012). Using this data set, we use a template-fitting approach to get good photometric redshift estimates for the field. In accordance to the Planck cluster fields, we have observed W-EGS in g, r, i and obtained a photometric catalogue including approximately 25 000 galaxies. Fig. 1 shows the footprints of W-EGS and CFHTLS-D3, as well as the region of the extended groth strip observed with Spitzer/IRAC in the mid-infrared (Barnby et al. 2008). The background sample selection is described in more detail in Section 4.

2.4 Data reduction

We perform de-biasing, flat-fielding and masking of cosmic rays (Gössl & Riffeser 2002) and charge persistences in the raw image frames. In a first step, the background subtraction, final astrometry and co-addition of the resampled images is done using SCAMP (Bertin 2006) and SWarp (Bertin et al. 2002). We exclude frames with too large PSF size and too low sky transparency (cf. equation 1).

In order to select the appropriate cut on the seeing, we consider the distribution of the FWHM of the PSF modeled
bands (for more details cf. e.g. Brimioulle et al. 2013).

The transparency of a frame

\[ T_{i,t} = \frac{t_i \times F_{SCAMP,i}}{\max(t_i \times F_{SCAMP,1})} \]

where \( t_i \) is the exposure time and \( F_{SCAMP,i} \) the flux scale of image \( i \) as calculated by SCAMP. The transparency threshold has been set to \( T_{i,t} \geq 50 \) per cent of the ideal transparency of all nights (\( \max(t_i \times F_{SCAMP,1}) \)). This quantity is a measure for the amount of absorption in the atmosphere, i.e. \( T_{i,t} \approx 100 \) per cent in a cloudless night. Although a stricter cut would be preferable in order to ensure an unbiased photometry, a 50 per cent-cut has turned out to sufficiently exclude any noisy exposures while not removing too many frames for the stacking. We do not rely on a constant photometric solution \( T_{i,t} = 100 \), as long as the transparency is comparable in the cluster fields and the W-EGS stacks.

### 2.5 Photometry

Due to a lack of standard star observations, we fix the \( r \) band zero-points by calibrating the fluxes of the field stars relative to the Pan-STARRS PV3 (Panoramic Survey Telescope And Rapid Response System Processing Version 3) catalogue (cf. Flewelling et al. 2016). We use SExtractor in dual image mode to detect objects in the \( r \)-band stacks and extract the flux from all filter bands. We use IRAF\(^1\) to convolve all images of an individual field to the same PSF and measure AB magnitudes in an aperture with a diameter of 8 pixels (1.6 arcsec). We perform Stellar Locus Regression (SLR) in order to find the zero-point offsets in the remaining \( g \) and \( i \) bands (for more details cf. e.g. Brimioulle et al. 2013). The minimization of the residuals in colour-colour diagrams is done with respect to the stellar library of Pickles (1998). Our zero-points have an uncertainty of only 1 per cent.

\(^1\) http://iraf.noao.edu/

### 3 PSF MODELING AND SHEAR MEASUREMENT

#### 3.1 PSF modeling

The surface brightness profiles of galaxies and stars alike are modified by the atmosphere, the telescope and the CCD. Only pre-seeing galaxy shapes can be used to perform a weak lensing analysis. The PSF is defined as the response of the Fraunhofer Telescope optics to a point source. The field stars can therefore be used to measure the PSF at the corresponding image positions. We show the ellipticity of the field stars in our stacked \( r \)-band images in Fig. 2. We verify that the selected stars are homogeneously distributed across the field of view. Gaps in the pattern shown in Fig. 2 are the results of masking. Usually the affected areas are centred on very bright stars, as the increased photon noise affects the photometry and the shape measurement. In the case of PSZ1 G109.88+27.94 however, part of the chip gaps had to be masked as well, since, as a result of the dithering of the camera, the stacks were much shallower in this region and the PSF model could not fit the data well.

We use the values of the measured stellar ellipticity in order to model the overall PSF distribution as a function of pixel coordinates. We filter the photometric catalogues according to the SExtractor star classifier, FLAG=0 and size larger than the stellar flux radius. We then use PSFEx (Bertin 2011) to model the spatial variation of the PSF as a polynomial function of position in the image plane. We find that the splitting of the stacked images into 16 subregions of equal area and running PSFEx on each field improves the quality of the modeled PSF ellipticities.

The order of the polynomial has to be chosen carefully. We use our PSF models to create mock star catalogues and visualize how well we can reproduce the PSF of our telescope optics. We consider the residuals of the field and modeled stars (Fig 3) and make whisker plots of the ellipticity residuals (Fig. 4). Although the WWFI PSF can be rather elliptical, our modeled stellar ellipticities fit the measured data well, with absolute value of the mean residual ellipticities between \( 10^{-5} \) and \( 1.6 \times 10^{-4} \). The error of the mean

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**Table 1.** The cluster sample investigated in this paper and the reference field W-EGS that is needed for the background sample selection (cf. Section 2.3). Columns from left to right: object name, right ascension, declination, spectroscopic redshift as given in the PSZ1 catalogue, WWFI exposure time and PSF FWHM in the \( g, r, i \) stacks respectively. The referenced redshifts are from (a) Photo-z from Abolfathi et al. (DR-14 2017), (b) Planck Collaboration et al. (2015b) and (c) Piffaretti et al. (2011).

| Object     | RA (J2000) | Dec (J2000) | z    | \( t_{exp} \) (h) | FWHM(\arcsec) | WWFI filter |
|------------|------------|-------------|------|-------------------|--------------|-------------|
| PSZ1 G109.88+27.94 | 18:23:15.1 | +78:24:27 | 0.77\(^1\) | 4.15              | 0.92         | \( g \)     |
| PSZ1 G139.61+24.20  | 06:22:13.9 | +74:41:39  | 0.267\(^1\) | 0.70              | 1.19         | \( g \)     |
| PSZ1 G186.98+38.66  | 08:50:12.0 | +36:03:36  | 0.378\(^1\) | 1.48              | 1.10         | \( g \)     |
| W-EGS          | 14:19:48.0 | +52:54:36  | 0.55  | 4.29              | 1.05         | \( g \)     |

\(^1\) http://iraf.noao.edu/
Figure 2. The pattern of the point spread function illustrated as whisker plots of the stellar ellipticities of the $r$ band stacked images. In the upper left corner of each image, we show a 10 per cent ellipticity for comparison. Calibration experiments conducted with the WWFI during the period of observations are the main reason why the PSF pattern is unstable over time. Bright stars have made it necessary for us to mask part of the field, which explains the regions of "missing data" especially for PSZ1 G109.88+27.94 and PSZ1 G139.61+24.20.

Figure 3. The residuals of measured and modeled ellipticities of the field stars in the $r$ band stacked images. The residuals $res = (e^\text{measured} - e^\text{PSF})$ scatter around zero and are nearly symmetrically distributed. Mean values for the components of $res$, standard deviations $\sigma_{res}$ and the number of stars used for the fit $n$ are given for the individual fields.
residual ellipticities is well below 0.004. Comparing these results to previous weak lensing studies using instruments like MegaCam (Brimiou et al. 2013), the WFI on MPG/ESO (Gruen et al. 2013) and SuprimeCam (von der Linden et al. 2014), we conclude that the WWFI PSF is well behaved and that we can model its spatial behaviour very accurately.

A metric to identify the best fitting PSF model has been proposed by Rowe (2010). The ellipticity residual auto-correlation function $D_1$ and the cross-correlation function of residuals and and measured ellipticities $D_2$, defined as

$$D_1(\bar{\theta}) \equiv \left\langle \left(e^\star - e^\text{PSF}\right) \cdot \left(e^\star - e^\text{PSF}\right) \right\rangle (\bar{\theta}),$$

$$D_2(\bar{\theta}) \equiv \left\langle e^\star \cdot \left(e^\star - e^\text{PSF}\right) + \left(e^\star - e^\text{PSF}\right) \cdot e^\star \right\rangle (\bar{\theta}),$$

(2)

can be used as a measure of the quality of the fit. Naturally, a perfect model means $D_1, 2 = 0$ on all scales. The closer to zero $D_1(\bar{\theta})$ and $D_2(\bar{\theta})$ are, the better the reconstructed PSF pattern. For our weak lensing analysis, separation angles of $\bar{\theta} \lesssim 60$ arcsec are of interest. We reject PSF models, for which $|D_1(\bar{\theta})| > 10^{-5}$. For a perfect PSF model both, $D_1$ and $D_2$, are equal to zero. For each cluster field, we model the PSF as polynomials with increasing order. We then compare the corresponding $D_1, 2(\bar{\theta})$ and select the model being closest to zero while not showing signs of overfitting. The best fitting Rowe statistics for our cluster sample are presented in Fig. 5. The best-fitting PSF orders are 5, 6, 6 for PSZ1 G109.88+27.94, PSZ1 G139.61+24.20 and PSZ1 G186.98+38.66, respectively.

Again comparing to literature, we find our Rowe statistics to be very similar to the findings of Gruen et al. (2014), both in amplitude of $D_{1, 2}$ and general trend in that the offset of $D_2$ from zero can be as large as $3 \times 10^{-5}$. They also find $D_1$ to be much better behaved than $D_2$ and show a consistency with zero on even the smallest scales for their cluster sample. Gruen et al. (2014) do not see an offset of $D_2$ to zero in their data.

Fig. 5 gives the first indication, that our PSF model for the $r$-band stacked image of PSZ1 G109.88+27.94 might be biased, as $D_2(\bar{\theta} \gtrsim 1$ arcmin) $\approx 2 \times 10^{-5}$. We investigate this possibility and the implications on the quality of our mass estimate for PSZ1 G109.88+27.94 further in Section 3.4.2.3 and Section 3.4.2.3.

3.2 Shape measurement

We prepare the galaxy catalogue for the actual shape measurement procedure by preselecting unsaturated sources with flux radii larger than the stellar flux radius. Using our PSFEx model and an implementation of the KSB+ (Kaiser et al. 1994; Luppino & Kaiser 1997; Hoekstra et al. 1998) pipeline of Gruen et al. (2013), we calculate pre-seeing galaxy shapes.

We run KSB+ on the prepared postage stamps and the PSFEx PSF model simultaneously. Polarizations (Kaiser et al. 1994) are calculated from the second moments of the surface brightness distribution $I(\theta)$ of the galaxies. These are measured within an aperture weighted with a Gaussian weight function $w(\theta)$ centred on the galaxy centroid. We scale $w(\theta)$ with the measured half-light radius of the observed galaxy (Gruen et al. 2013).

Given weighted second moments of an object with sur-
The auto-correlation function of the ellipticity residuals $D_1$ and the cross-correlation function of the residuals and the measured ellipticities $D_2$ of the field stars in the $r$-band, as defined in equation (2) (cf. Rowe 2010). The angular separation of the objects is denoted by $\bar{\theta}$. The $\bar{\theta}$-statistics for PSZ1 G109.88+27.94, PSZ1 G139.61+24.20 and PSZ1 G186.98+38.66 are presented in purple (circles), black (squares) and red (triangles) respectively. On large scales, PSZ1 G109.88+27.94, PSZ1 G139.61+24.20 and PSZ1 G186.98+38.66 are presented in purple (circles), black (squares) and red (triangles) respectively. On large scales, $\bar{\theta}$ is consistent with zero for all clusters. With the exception of the innermost datapoint of PSZ1 G109.88+27.94, all values of $D_2$ are smaller than $5 \times 10^{-3}$. None of the correlation functions show signs of overfitting, as the cross-correlation function of the residuals and the measured ellipticities is mostly non-negative on large scales. For small angular separations, the error bars grow increasingly large, yet $D_2$ is consistent with zero.

3.3 Shear measurement

Polarizations are defined as weighted second moments of the image intensities. The observed post-seeing polarization $e_{\text{obs}}$ does not only depend on $e_{\text{int}}$ but also on the polarization of the PSF image $p$ and the reduced shear $g$ distorting the intrinsic galaxy shape. Introducing tensors to describe the linear response of $e_{\text{obs}}$ to $p$ and $g$, the observed polarization can be expressed as:

$$ e_{\text{obs}} = e_{\text{int}} + \hat{p}^m p + \hat{P}^v g, $$

where $\hat{p}^m$ is the shear responsivity tensor and $\hat{P}^v$ denotes the shear responsivity tensor. Point-like sources have $e_{\text{int}} = g = 0$, which means that, in the absence of PSF distortions, the ellipticity of a star is zero. The atmospheric seeing is described as a large circularly symmetric disc convolved with a small, highly anisotropic distortion. The PSF anisotropy corrected ellipticity is thus given by:

$$ e_{\text{cor}} = e_{\text{obs}} - \hat{p}^m p. $$

where the position dependent vector $p$ is estimated from the PSF image (Luppino & Kaiser 1997; Hoekstra et al. 1998).

Now taking the response of the ellipticity to the shear into account, while assuming the galaxies to be randomly oriented, we obtain an estimate for the ensemble reduced shear:

$$ \langle g \rangle = \langle e \rangle = \frac{2}{\mu P} e_{\text{cor}}, $$

where $e$ is the measured galaxy ellipticity. In equation 7, we have approximated the inverted shear responsivity tensor as $(P^v)^{-1} \approx 2/\mu P$. In accordance to Gruen et al. (2013), we only include objects with successful KSB+ shape measurements with $\mu P \geq 0.1$ in our shape catalogues.

The reduced shear $g$ combines the effects on the galaxy images induced by the convergence $\kappa$ and the shear $\gamma$:

$$ g = \frac{\gamma}{1 - \kappa}. $$

In polar coordinates, where the lens is at the origin, the tangential and cross component of the gravitational shear can be written in terms of the polar angle $\varphi$:

$$ g_t = - [g_1(\varphi) \cos(2\varphi) + g_2(\varphi) \sin(2\varphi)] $$

$$ g_\times = -g_1(\varphi) \sin(2\varphi) + g_2(\varphi) \cos(2\varphi). $$
3.4 Impact of biases on the WWL shapes

The FWHM of the PSF in our lensing band is in the sub-arcsecond regime and the PSF ellipticity can be as large as \( \sim 15 \) per cent in some areas of the stacked images (cf. Fig. 2). The question arises whether a weak lensing measurement is actually feasible with our current WWFI data. In order to verify the quality of our shape catalogues, we have to test whether our implementation of KSB+ recovers unbiased galaxy shapes in the presence of large PSF ellipticity, or if our PSF models are an insufficient description of the true PSF in our stacks.

Calibration bias is caused by a poor correction for the effects of atmospheric seeing on galaxy shapes. Heymans et al. (2006) describe the effect on the measured shear as a multiplicative and additive bias,

\[
\langle e^\text{obs} - e^\text{true} \rangle = m \times \langle e^\text{true} \rangle + c. \tag{10}
\]

In the presence of a calibration bias, \( m \) is expected to be non-zero, while PSF systematics also imply \( c \neq 0 \). If the response of \( e^\text{obs} \) to shear is non-linear, a third term has to be introduced (Heymans et al. 2006, their equation 11), which we do not consider in our analysis.

3.4.1 Multiplicative shear bias

The multiplicative shape measurement bias \( m \) influences the amplitude of the measured ellipticities, which translates into a change of the shear amplitude. As the mass measurement scales with the shear, \( m \) biases weak lensing mass estimates. We determine the multiplicative bias by repeating the shape measurement on simulated galaxy images. We correct our shape catalogues by applying a S/N-calibration of the galaxy ellipticities. Gruen et al. (2013) determined the dependency of \( m \) in our pipeline on the signal-to-noise ratio of a galaxy by fitting a functional form of \( m \),

\[
m \approx -0.025 - 0.17 \exp \left( -\frac{(S/N)_{\text{gal}}}{17} \right), \tag{11}
\]

with a minimum \((S/N)_{\text{gal}}\) of 10. They define \( m \) as the deviation of the ratio of mean shapes as measured on simulated cluster fields using their implementation of KSB+ to the true shapes from 1. As expected, the absolute value of the multiplicative bias decreases as a function of signal-to-noise until it reaches an almost constant value for large \((S/N)_{\text{gal}}\).

The simulations that have been used are not a perfect representation of the WWFI data. One of the main problems is likely the large PSF ellipticity. A dependency of \( m \) on the profiles of the galaxies and the distribution of their sizes and ellipticities has also not been taken into account. For these reasons we expect a residual multiplicative shear bias. We conservatively estimate this bias to be up to 5 per cent.

In order to get a feeling for how realistic this value might be, we make use of the fact that one cluster in our sample (PSZ1 G186.98+38.66) has also been observed with Subaru and is part of the WtG project. We match the shape catalogues and calculate the tangential shear signals using the different data sets. Our mean tangential shear in 5 radial bins from the cluster centre is proportional to the shear signal calibrated by WtG with a proportionality constant that is consistent with 1, i.e. \( \sigma_m = 0.03 \pm 0.07 \). Selection effects and biases in the WtG galaxy shape catalogue aside, this test shows that \( \sigma_m = 0.05 \) is an adequate budget for the residual multiplicative shear bias.

3.4.2 Additive shear biases

We consider three different kinds of additive systematics in our shape catalogues, one of which is constant over the whole fields and two of which are related to the PSF. We investigate their impact on our cluster masses in Section 3.4.2.1, 3.4.2.2 and 3.4.2.3, respectively.

3.4.2.1 Mean ellipticity

We find a spatially constant mean ellipticity of \( \langle \epsilon_1 \rangle = (-2.8,-2.4) \pm (1.7,1.7) \times 10^{-3} \) in our cluster fields. In the presence of a radially non-symmetric mask of random orientation, this causes a mean tangential shear component that affects the recovered weak lensing cluster mass. The impact of mean ellipticity bias on the mass depends on the applied mask in the individual field. Consequently, it is really a “statistical” uncertainty and will decrease with increasing WWL sample size. We estimate the uncertainty on the cluster mass by adding a constant offset of \( \epsilon \) to the observed galaxy ellipticity, running our two-parameter NFW fitting code (cf. Section 5.3) and comparing the results to the mass profile that best fits our original data.

In this way we find the statistical uncertainty on the cluster mass caused by the mean ellipticity in the data to be smaller than \( \sigma_\epsilon \leq 9.0\%,7.2\%,0.3\% \) for PSZ1 G109.88+27.94, PSZ1 G139.61+24.20 and PSZ1 G186.98+38.66, respectively. We show the impact of a hypothetical additive shear bias within \( 3\sigma \) from the measured mean ellipticity in Fig. 6. As expected, the conservative masking in the stacks of PSZ1 G109.88+27.94 and PSZ1 G139.61+24.20 causes an impact of a non-zero \( \epsilon \) on the cluster mass. The weak lensing mass of PSZ1 G186.98+38.66, however, changes even less significantly. Even if we assume an additive shear bias that is 3\( \sigma \) larger than \( \epsilon \), the uncertainty due to this on the cluster mass is less than 2 per cent for this field.

Heymans et al. (2012) find a mean ellipticity of \( \langle \epsilon_{1,2} \rangle = (0.1,2.0) \pm (0.1,0.1) \times 10^{-3} \) in the CFHTLenS shape catalogues. The value scales with galaxy size and signal-to-noise. Even surveys as large as the Dark Energy Survey (DES) still find a non-zero mean ellipticity. Jarvis et al. (2016) find mean ellipticities of \( \langle \epsilon_{1,2} \rangle = (0.1,6.8) \times 10^{-4} \) in their IM3SHAPE and \( \langle \epsilon_{1,2} \rangle = (-0.4,10.2) \times 10^{-4} \) in their NGMIX shape catalogue for the Science Verification data. The DES Year 1 survey shows a mean ellipticity of \( \langle \epsilon_{1,2} \rangle = (3.5,2.8) \times 10^{-4} \) and \( \langle \epsilon_{1,2} \rangle = (0.4,2.9) \times 10^{-4} \) using METACALIBRATION and IM3SHAPE, respectively (Zuntz et al. 2017). This shows that the mean ellipticity can be different if another shape measurement technique is applied to the same data and that \( |\epsilon| \) might decrease if the survey area increases.

The reason for this phenomenon is not known but does not depend on the PSF model. The linear model introduced in Section 3.4.2.3 disentangles the PSF dependent from the spatially constant part of the galaxy ellipticities.

3.4.2.2 Model bias

An insufficiently modeled PSF can cause a position dependent additive shear bias, which we call the PSF model bias. Our PSF models were designed to minimize the PSF model bias (cf. Section 3.1). We use
We find \(|\sigma_\text{ter} mass \) equal to \(t\) respec-

\(\text{tially constant mean ellipticity, this bias is subdominant for}

\(\text{PSZ1 G109.88+27.94,}

\text{in the field of PSZ1 G186.98+38.66, the effect of PSF model bias can be as large as the mean

\text{ellipticity bias. As } e_{\text{PSF}} \text{ could be negative, these two ef-

\text{fects could also cancel out and leave our mass constraints unaffected by these additive biases.}

3.4.2.3 PSF leakage Even if there is only a small PSF model bias in our shape catalogues, there is a sec-

\text{ond type of position dependent additive bias. PSF leakage occurs, if the PSF is not deconvolved properly from

\text{the source images or if selection depends on the alignment of PSF and galaxy ellipticity. This causes the observed grav-

\text{itational shear and the PSF ellipticity to be correlated. We can use this effect to confirm the presence of some leakage

\text{in our shape catalogues by calculating the cross-correlation function of PSF model and galaxy ellipticity}

\begin{equation}
\xi_{\text{PSF,gal}}^+ = \langle e_{\text{PSF}}^+ \cdot e_{\text{gal}}^+ \rangle + \langle e_{\text{PSF}}^- \rangle.
\end{equation}

\text{In the equation above, } e_{\text{gal}}^+ \text{ denotes the galaxy ellipticities, as measured by our KSB+ pipeline. At each galaxy position, we also have an estimate for the value of the PSF ellipticity predicted by our PSFEx model } e_{\text{PSF}}^+ . \text{As can be seen in Fig. 7, the correlation between galaxy and PSF ellipticities is small (} \lesssim 0.0006) \text{ and strongest for PSZ1 G109.88+27.94.}

\text{On large scales, PSZ1 G186.98+38.66, too, shows some correlation (} \xi_{\text{PSF,gal}}^+ \approx 0.0001), while the PSZ1 G139.61+24.20 galaxy shapes seem to be unbiased by PSF leakage.}

\text{Linear fit: Classical approach The amount of leakage is generally described as a linear dependency of the galaxy shapes on the PSF ellipticities. We can then rewrite equation 10 in the following way:}

\begin{equation}
\langle e_{\text{obs}}^+ - (1-m)e_{\text{true}}^+ \rangle = c = a^+ (e_{\text{PSF}}^+) + b.
\end{equation}

\text{Under the assumption that the galaxies are randomly ori-

\text{ented (i.e. } \langle e_{\text{true}}^+ \rangle = 0), we can find the linear leakage factor } a \text{ and the additive leakage bias } b \text{ by assuming a linear model for the observed mean galaxy ellipticities } \langle e_{\text{obs}}^+ \rangle .

\begin{equation}
\langle e_{\text{obs}}^+ \rangle = a \cdot \langle e_{\text{PSF}}^+ \rangle + b.
\end{equation}
The open green symbols show the results using our method from $b$ values of the first component of the additive leakage bias line, respectively. The same colour scheme is used to show the mean value of $a_1, a_2$ and $\bar{\alpha}$ (dashed, dotted and dotted-dashed line, respectively). Right panel: The dark blue symbols show the values of the first component of the additive leakage bias $b_1$, the second component $b_2$ is shown in light blue. The dark (light) blue dashed (dotted) line shows the mean value of $b_1$ ($b_2$). The mean ellipticities and their $1\sigma$ intervals are shown in pink ($c_{1}^{(1)}$) and purple ($c_{2}^{(1)}$).

where we average over galaxies with similar PSF ellipticities. The additive leakage bias $b$ is a noisy estimate for the mean ellipticity $c^{(1)}$ (cf. Section 3.4.2.1), as it is constant over the whole field.

The $c_{PSF,gal}^e$ of the cluster fields suggests that, out of all cluster fields in our WWL pathfinder sample, the linear leakage factor $a$ should be largest for PSZ1 G109.88+27.94 (cf. Fig. 7). We fit our model of $\langle e^{obs} \rangle$ (cf. equation 14) to the data to find estimates on $a$ and $b$. The results of our linear fit can be seen in Fig. 8. Our estimated components of the linear leakage factor $a_{1,2}$ are all consistent with zero but have large uncertainty due to shape noise. We can neither confirm that $a_1 = a_2$, nor can we make any statements on the variation of $a$ in the different fields. The same holds for the additive leakage bias. The error bars are much larger than the error on the mean of the galaxy ellipticities in the fields $c_{1,2}^{(e)}$. Though there is no indication that $c_{1}^{(e)} \neq c_{2}^{(e)}$, we find a $2\sigma$ evidence that $b_1 \neq b_2$ for PSZ1 G139.61+24.20.

If we average $b_1$ and $b_2$ over all fields, we find consistent results compared to the mean ellipticity. The mean leakage factor in the fields is $a_{1,2} \approx (0.06, 0.03) \pm (0.02, 0.06)$. The strategy to fit a linear relation between $e$ and $e^{PSF}$ does not yield results we can use to correct our ellipticities. Below, we present a new approach to correct for the effect of the additive shear bias $\sigma e^{PSF}$ on cluster weak lensing mass estimates despite the low number of galaxies in our catalogues.

Leakage correction for small shape catalogues For our cluster weak lensing analysis, we are only interested in the azimuthally averaged component of the additive shear bias $e^{PSF}$. We introduce a method to model the additive bias on the mean tangential shear averaged in circular annuli around the cluster centre $\langle \epsilon \rangle \equiv \epsilon^{PSF}$. We make the assumption that the additive shear bias caused by leakage is linearly dependent on the PSF ellipticity and that a constant contribution is negligibly small (i.e., $\langle b \rangle = 0$). We further assume that $a_1 \approx a_2$ to write the observed tangential shear signal measured in radial bins as

$$
\langle e^{obs}_t \rangle = -\langle e^{true}_t + \alpha e^{PSF} \rangle \cos(2\theta) - \langle e^{true}_2 + \alpha e^{PSF}_2 \rangle \sin(2\theta) \\
= \langle e^{true}_t \rangle + \alpha \langle e^{PSF}_1 + e^{PSF}_2 \rangle + \bar{\alpha} \langle e^{PSF}_2 \rangle,
$$

where we have defined the tangential PSF leakage factor $\bar{\alpha}$. In order to calculate the systematic tangential shear profile $\langle \epsilon^{PSF} \rangle$, we first have to estimate $\bar{\alpha}$ and then measure the tangential shear profile of our model PSF ellipticities. Using our definition of $\bar{\alpha}$, we can write $\langle e_t^* + e^{PSF} \rangle = \tilde{\epsilon}^{PSF,gal}(\bar{\theta})$ (cf. Fig. 7).

The $\epsilon^{PSF,gal}(\bar{\theta})$ is the autocorrelation function of the PSF ellipticity $\tilde{\epsilon}^{PSF,gal}(\bar{\theta})$ and is shown in Fig. 9 for the cluster fields. The angular galaxy separation is once again denoted by $\bar{\theta}$. We calculate $\tilde{\epsilon}^{PSF,gal}(\bar{\theta})$ and $\epsilon^{PSF,PSF}(\bar{\theta})$ in 10 bins of angular galaxy separation and fit the linear leak-
age factor $\tilde{\alpha}$ to the functional $\xi_{\text{PSF,gal}}^{\epsilon\ast}(\theta) = \tilde{\alpha} \xi_{\text{PSF}}^{\epsilon\ast}(\theta)$. The standard errors are estimated using bootstrapping. We can now compare the results of the fit with the values for the components of $\alpha$ we have obtained in Section 3.4.2.3 (cf. Table 2). $\xi_{\text{PSF,gal}}$ and $\xi_{\text{PSF,PSE}}$ are not proportional in all cases. This is due to varying patterns of the PSF in the fields. While the increase of $\xi_{\text{PSF,gal}}$ is proportional to the amount of PSF leakage into the galaxy shapes, $\xi_{\text{PSF,PSE}}$ is simply a measure for the spatial variation of the PSF pattern. The PSF patterns of PSZ1G109.88+27.94 and PSZ1G186.98+38.66 are very homogenous in large parts of the images (cf. Fig. 2), which means that $\xi_{\text{PSF,gal}}$ is large on small and intermediate scales and then decreases faster for PSZ1G186.98+38.66 than for PSZ1G109.88+27.94. The whisker plot of PSZ1G139.61+24.20, however, shows that the PSF ellipticities are not correlated with each other to the same degree and not at all on the largest scales. More complex PSF patterns are more difficult to correct, and consequently the amount of PSF leakage increases with decreasing $\xi_{\text{PSF,gal}}$.

Using the classical approach to estimate $\alpha_{1,2}$, the components of the galaxy shapes $\epsilon_{1,2}$ could already be corrected directly but the errors are usually ~ 2-4 times larger than the absolute values of $\alpha_{1,2}$. With our new model for the leakage factor, we get $\tilde{\alpha} \simeq (15 \pm 3 \pm 2,3 \pm 1) \times 10^{-2}$ for PSZ1G109.88+27.94, PSZ1G139.61+24.20 and PSZ1G186.98+38.66, respectively. Confirming our prediction from Section 3.4.2.3, we find that the tangential leakage factor is largest for PSZ1G109.88+27.94. It is more than four times larger than the estimated $\tilde{\alpha}$ in the field of PSZ1G139.61+24.20. This strong field dependency of $\tilde{\alpha}$ could be explained by the choice of our model, i.e. on the underlying assumption $b = 0$.

We cannot disentangle any contribution from a possible PSF model bias from $\tilde{\alpha}$, which could affect our estimates for the tangential leakage factor. This effect varies locally in each field, since the PSF patterns are not the same and the quality of our PSF models might be very different. As mentioned in Section 3.4.2.2, the Rowe statistics are very sensitive to the amount of PSF model bias in our stacks. While the PSF model bias is negligibly small for PSZ1G186.98+38.66, it might become important for PSZ1G109.88+27.94 and PSZ1G139.61+24.20. Indeed, the residuals $\tilde{\alpha} - a_1$, $\tilde{\alpha} - a_2$, $a_1 - a_2$ are smallest for PSZ1G186.98+38.66. Our estimates of $\tilde{\alpha}$ for PSZ1G109.88+27.94 and PSZ1G139.61+24.20, however, might be biased low or high, since we have not corrected for the effect of $\epsilon_{\text{PSF}}$.

We can now use our estimates of $\tilde{\alpha}$ to measure the systematic tangential shear in radial bins of $\theta$

$$\langle \epsilon_t^{\text{sys}}(\theta) \rangle = \tilde{\alpha} \langle \epsilon_t^{\text{PSF}}(\theta) \rangle. \quad (16)$$

where $\theta$ is the distance of the galaxy from the cluster centre (cf. Fig. 10). We find that $\langle \epsilon_t^{\text{sys}} \rangle$ is negative and that the effect is larger in the outskirts of the stacks. While the overall shape of the systematic tangential shear seems to be the same in all cluster fields, PSZ1G109.88+27.94 possesses the largest additive shear bias with $\epsilon_t^{\text{sys}} = -3.6 \pm 0.2 \times 10^{-3}$ at a distance of 18.9 arcmin from the centre of the cluster. This shows that the tangential alignment of the PSF shapes is the same in all stacks.

**Figure 10.** Our model for the systematic tangential (filled symbols) and cross (open symbols) shear in the cluster fields as a function of distance from the cluster centre. A negative systematic shear means that the observed reduced shear in the cluster fields will be biased low. We use our calculated values of $\langle \epsilon_t\rangle$ and $\langle \epsilon_{\text{PSF}} \rangle$ to apply a leakage correction to the measured weak lensing shear signal.

We use 10 radial bins in the distance range of $0' < \theta < 20'$ to measure the systematic tangential shear and correct for the effect of leakage on the galaxy shapes by subtracting $\langle \epsilon_t^{\text{sys}} \rangle$ from $\langle \epsilon_{\text{obs}} \rangle$. Accordingly, we can define the systematic cross shear for all cluster fields and correct the measured B-modes. We also apply a leakage correction to the data in our weak lensing analysis.

There is no impact of a model error for $\tilde{\alpha} = \text{const}$. on our estimates of the position dependent $\epsilon_{\text{PSF}}^{\epsilon\ast}$. If we assume a false systematic tangential shear profile, we bias our NFW cluster mass estimate of PSZ1G109.88+27.94 by 2 per cent and those of PSZ1G139.61+24.20 and PSZ1G186.98+38.66 by 1 per cent. Compared to the mean ellipticity bias and in some fields compared to the PSF model bias discussed in Sections 3.4.2.1-3.4.2.2, this PSF leakage correction bias $\sigma_{\text{PSF}}^{\epsilon\ast}$ is subdominant.

### 3.4.3 Star-galaxy cross-correlation

Finally, we consider the cross-correlation functions of the stellar and the galaxy ellipticities

$$\xi_{\ast, \text{gal}}^{\epsilon\ast} = \langle \epsilon_{\ast} \ast \epsilon_{\text{gal}} \rangle \pm \langle \epsilon_{\text{gal}} \ast \epsilon_{\ast} \rangle. \quad (17)$$

An obviously different behaviour of $\xi_{\ast, \text{gal}}^{\epsilon\ast}$ compared to $\xi_{\text{PSF,gal}}^{\epsilon\ast}$ would indicate PSF model bias. Consequently, we use the star-galaxy cross-correlation function as a consistency check to test whether our leakage model in Section 3.4.2.3 sufficiently describes the data. We calculate $\xi_{\ast, \text{gal}}^{\epsilon\ast}(\theta)$ and present the results in Fig. 11. As expected, the
Table 2. Field name, the leakage factor $\tilde{\alpha}$ from equation 16 and its bootstrap error, the first and second component of the leakage factor $\alpha$ (cf. equation 14) and their $1\sigma$ errors.

| Object            | $\tilde{\alpha}$ | $\sigma_{\tilde{\alpha}}$ | $\alpha_1$ | $\sigma_{\alpha_1}$ | $\alpha_2$ | $\sigma_{\alpha_2}$ |
|-------------------|-------------------|-----------------------------|-------------|----------------------|-------------|----------------------|
| PSZ1 G109.88+27.94 | 0.15              | 0.01                        | 0.03        | 0.13                 | 0.11        | 0.17                 |
| PSZ1 G139.61+24.20 | 0.03              | 0.02                        | 0.07        | 0.23                 | -0.07       | 0.23                 |
| PSZ1 G186.98+38.66 | 0.03              | 0.01                        | 0.08        | 0.06                 | 0.06        | 0.13                 |

Figure 11. The cross-correlation function of the ellipticities of field stars and galaxies as a function of angular separation for PSZ1 G109.88+27.94 (violet circles), PSZ1 G139.61+24.20 (black squares) and PSZ1 G186.98+38.66 (red triangles). The star-galaxy cross-correlation shows the same behavior as $\xi_{\text{PSF,gal}}^+$ (cf. Fig. 7).

star-galaxy cross-correlation function $\xi_{\text{star,gal}}^+$ is consistent with $\xi_{\text{PSF,gal}}^+$ but with larger errors. This is also the case for $\xi_{\text{star,gal}}^-$ and $\xi_{\text{PSF,gal}}^-$ with $\xi_{\text{PSF,gal}}^-$. On the smallest scales with $\theta \lesssim 30$ arcsec, we cannot measure the star-galaxy cross-correlation well and the errorbars in Fig. 11 are accordingly large. $\xi_{\text{star,gal}}^+$ and $\xi_{\text{PSF,gal}}^+$ show that stellar and galaxy ellipticities and also the ellipticities of the modeled PSF and the measured galaxies ellipticities are not, or only minimally correlated on any scale for PSZ1 G139.61+24.20. The other two clusters of our sample, however, show a small correlation between galaxy shapes and the shapes of stars (and PSF model). As discussed in the previous section, this is due to systematics in the PSF modeling and can be accounted for by applying a leakage correction if the leakage factor $\tilde{\alpha}$ is known.

4 BACKGROUND SAMPLE SELECTION

This section describes our method to select a sample of lensed background galaxies for weak lensing analysis from our three-band WWFI photometry. The lensing signal scales as

$$\beta_{\pm}(z_s, z_d) \equiv D_{\Delta}(z_s) \left\{ \frac{D_{\Delta}}{D_{\Delta}} \right\} \begin{cases} z_s - z_d, & z_s \leq z_d, \\ 0, & \text{otherwise} \end{cases} .$$  \hspace{1cm} (18)

We know the spectroscopic cluster redshift $z_d$, so the lensing strength $\beta_{\pm}$ is simply a function of source redshift $z_s$. For foreground galaxies and cluster members, $\beta_+ = 0$. It rises steeply with increasing source redshift until it reaches an asymptotic value, $\beta_+ < 1$, as $z_s \to \infty$ (cf. Fig. 12).

In weak lensing surveys, photometric redshifts are often calculated using template fitting algorithms (e.g. Brimiouelle et al. 2008) or machine learning approaches (e.g. Rau et al. 2015). However, photometric redshifts cannot be well constrained using either method, if only three-band photometric information is available. Due to non-linear error propagation from equation (18), imprecise redshift point estimates bias $\beta_{\pm}$ estimates and make a clean background sample selection impossible. We use the probabilistic method of Gruen et al. (2014) to determine source redshift probability distributions for the galaxies, despite the limited photometric information. The position of a galaxy in magnitude space can be

$\beta_+ = 0.267, 0.3774, 0.77$ for PSZ1 G139.61+24.20, PSZ1 G186.98+38.66 and PSG1 G109.88+27.94, respectively.
compared to that of reference galaxies with accurate redshift information. Using this approach, we consider the full colour-magnitude distribution of the reference galaxies empirically and gain information on $p(z)$ of each source galaxy (cf. Section 4.2).

### 4.1 Photometric redshifts for EGS

As a first step, we obtain precise photometric redshifts for the galaxies in CFHTLS-D3. As mentioned in Section 2.3, we have eight-band $u^*, g', r', i', z', J, H, K$ photometric data at our disposal (Davis et al. 2007; Bielby et al. 2012). We detect sources on the deepest ($r$ band) W-EGS co-added image, extract fluxes, magnitudes and errors from the CFHTLS-D3 pointing and create a photometric catalogue as described in Section 2.5. We use the photometric template-fitting Photz code of Bender et al. (2001) and follow the approach of Brimioulle et al. (2013). The overlap of the field with the Deep Extragalactic Evolutionary Probe-2 (DEEP2) survey (Newman et al. 2013) allows for a spectroscopic calibration and validation of our achieved photometric redshift estimates. The spectroscopic sample has a limiting apparent magnitude of $R_{AB} = 24.1$ and contains $\sim 2\%$ of all galaxies with photometric information up to $z_{spec} \approx 2$. We obtain a photometric redshift accuracy of $\sigma_{z_{corr}}/(1+z_{spec}) = 0.026$ and an outlier rate of $\eta = 1.5$ per cent (cf. Fig. 13).

We match our CFHTLS-D3 photometric redshifts to the sources detected on our stacked W-EGS images. We obtain a catalogue, which contains photometric information using WWFI $g, r, i$ filter bands and reliable CFHTLS $u^*, g', r', i', z', J, H, K$ photometric redshift estimates. This data set is used as a reference field and connects the redshift of a galaxy to its WWFI magnitudes/colors.

### 4.2 Using three band photometry to estimate galaxy redshifts

We now use the three-band WWFI photometry of our cluster fields and our redshift catalogue for W-EGS to estimate the $\beta_z$ of each background galaxy. Our background selection procedure is based on the fact that galaxies with similar magnitudes belong to the same distribution of morphological classes and redshifts. Here, we only give a brief overview, as the method is described in detail in Gruen et al. (2014).

Our aim is to get an estimate on the lensing strengths $\beta(z)$ of each galaxy in the cluster fields. For each cluster field, we have a catalogue of $i$ galaxies with a set of apparent magnitudes $m_i^k = \{m_{ij}^k\}$, where $j = g, r, i$. For W-EGS, we have a similar catalogue with $m_i^{egs} = \{m_{ij}^{egs}\}$, ($k>i$).

We apply a cut on the flux radius of the reference galaxies that corresponds to the cut we make on the galaxies in our shape catalogues. Galaxies are considered to be comparable, if $|\Delta m| = |m_i^k - m_{ij}^{egs}| \leq 0.1$. In this way, a reference sample of galaxies $k'$ can be assigned to each galaxy in the cluster fields. The redshift probability distribution $p(z | m_i^k)$ is then given by the distribution of the reference redshifts $p(z | m_i^{egs})$. Thanks to the overlap of W-EGS and CFHTLS D3, we have been able to assign a reliable photometric redshift estimate to each source in this reference field (Section 4.1). The lensing strength for a given galaxy $i$ can then be estimated to be

$$\beta_z(m_i^k) = (\beta_z(m_i^{egs}))_{k'}.$$  

In order for the source catalogues not to be contaminated by cluster member galaxies, we apply a cluster member correction (cf. Gruen et al. 2014, their Section 3.1.3). Naturally, the excess of cluster members is a function of position, with the maximum found in the cluster center. If we were to neglect this effect, the estimated $\beta_z$ would be affected in such a way that $\beta_z$ near the cluster center would be overestimated the strongest.

W-EGS has to contain a sufficient amount of galaxies with similar photometric properties as those of objects in the cluster fields. Fig. 14 shows the distribution of the 1" aperture limiting $r$-band magnitudes for each field, for which $S/N > 10$. PSZ1 G186.98+38.66 is the shallowest cluster field in the lensing band with an $r$-band limiting magnitude of $\sim 25$ mag. The $r$-band stack of W-EGS is as deep as the other cluster fields. With $\sim 25.5$ mag in $g$ and $\sim 24.8$ mag in $i$, it is deeper than the cluster fields by $\leq 0.5$ mag.

Table 3 provides basic properties of the background sample with successful shape measurement. Compared to Gruen et al. (2014), we have higher number densities of background galaxies at comparable redshifts, which is due to the fact that our stacks are slightly deeper than theirs. Furthermore, they require a higher minimum lensing strength $\beta_{min}$ for their source galaxies. We have found that a cut on the lensing strength of $\beta_{min} = 0.05$ is conservative enough to remove the remaining contamination by cluster members from the source catalogues but is still low enough that not too many background galaxies are lost for the analysis.

### 4.3 Photometric redshift uncertainty

Due to the proportionality of the lensing signal and $\beta_z$, biases in our catalogue of lensing strengths have a direct im-
Table 3. Basic properties of the background samples. Columns from left to right: Object name, threshold \( \beta_{\text{min}} \), corresponding threshold redshift \( z(\beta_{\text{min}}) \), mean lensing strength \( \langle \beta \rangle \), the corresponding redshift \( z(\langle \beta \rangle) \), total number and number density of background galaxies (i.e. with \( \beta > \beta_{\text{min}} \)) and at a distance \( r > 500 \) kpc from the cluster centre, and the fraction of masked area in the images.

| Object            | \( \beta_{\text{min}} \) | \( z(\beta_{\text{min}}) \) | \( \langle \beta \rangle \) | \( z(\langle \beta \rangle) \) | \( N_{\text{bg}}^{\text{KSB}} \) | \( n_{\text{bg}}^{\text{KSB}} \) [arcmin\(^{-2}\)] | \( A_{\text{masked}}/A \) |
|-------------------|-------------------------|-----------------------------|-----------------------------|-----------------------------|-------------------------|-----------------|----------------|
| PSZ1 G109.88+27.94| 0.05                    | 0.82                        | 0.08                        | 0.85                        | 6992                    | 9               | 0.114          |
| PSZ1 G139.61+24.20| 0.05                    | 0.28                        | 0.41                        | 0.48                        | 7293                    | 9               | 0.118          |
| PSZ1 G186.98+38.66| 0.05                    | 0.40                        | 0.27                        | 0.54                        | 9739                    | 12              | 0.112          |

Figure 14. Distribution of 1'' aperture \( r \)-band magnitudes of galaxies with \( S/N > 10 \). The blue histogram shows the distribution of apparent brightnesses of W-EGS \( r \)-band stacked images. The limiting \( r \)-band magnitude is largest for W-EGS (blue), PSZ1 G109.88+27.94 (purple) and PSZ1 G139.61+24.20 (black) and is smallest for PSZ1 G186.98+38.66 (red).

The WWL pathfinder

5. WEAK LENSING ANALYSIS

The surface mass density of a foreground object acting as a gravitational lens determines the distortion of the background images. The mean tangential component of the shear on a circle \( \langle \gamma_t(\theta) \rangle \) is given by the convergence \( \kappa \) inside the circle subtracted by the convergence at its edge,

\[
\langle \gamma_t(\theta) \rangle = \langle \kappa(\theta) \rangle - \langle \kappa(\theta) \rangle.
\]

The reduced shear is given by \( g = \gamma/(1-\kappa) \). Equation (20) implies that the measurement of the tangential shear averaged over all galaxies in a circular annulus around the cluster centre can be used to directly estimate the azimuthally averaged mass profile of a cluster of galaxies. Yet, in order to retain the full information on the surface mass density, we need more information about the system than just the reduced shear alone. If only the gravitational shear is measured, one cannot distinguish between models for \( \kappa \) that are similar except for an additional sheet of mass. A possible way to break this mass sheet degeneracy is to assume a functional form of \( \kappa(\theta) \). In order to constrain weak lensing cluster masses, we consider two model profiles: the Singular Isothermal Sphere (SIS) and the NFW profile.

5.1 Fit of a Singular Isothermal Sphere

The simplest model to describe the density profile of a cluster of galaxies is the Singular Isothermal Sphere (e.g. Binney & Tremaine 1998). In this model, the galaxy cluster is thought of as a spherically symmetric, self-gravitating ideal gas cloud consisting of collisionless particles. The surface...
mass density at a distance \( r \) from the cluster centre with a constant velocity dispersion \( \sigma_v \) is given by

\[
\Sigma(r) = \frac{\sigma_v^2}{2\pi G} \int_{-\infty}^{\infty} \frac{dz}{r^2 + z^2} = \frac{\sigma_v^2}{2Gr},
\]

where \( G \) is the gravitational constant. Using the definition of the convergence \( \kappa \), we can show that

\[
\kappa(r) = \frac{\sigma_v^2}{2G \Sigma_{\text{m}}(r)} r^2 = 2\pi D_L \beta_L \left( \frac{\sigma_v}{c} \right)^2 \frac{1}{r} = \gamma(r).
\]

We average the reduced tangential shear in eight radial distance bins from \( \theta = 2 - 20 \) arcmin and fit equation (22) to the signal. We use the average lensing strength of the sample \( (\beta_L) \) in this one-parameter fit and constrain the velocity dispersion of the WL galaxy clusters. We exclude galaxies, for which \( \theta = r/D_L \leq 2 \) arcmin and apply a cut on the distance fraction with \( \theta_{\text{min}} = 0.05 \). The mass of an SIS out to a radius \( r \) can then be calculated as

\[
M_{\text{SIS}}(r) = \frac{2\sigma_v^2}{G} r.
\]

5.2 Significance map

We estimate the significance of the detected tangential alignment of background galaxies by using the aperture mass statistic (Schneider 1996). The aperture mass significance allows us to visualize the two-dimensional weak lensing signal and helps to identify neighbouring mass distributions acting as lenses themselves. We calculate the aperture mass as the weighted sum over all background galaxies in a circular aperture

\[
M_{\text{ap}}(\theta) = \sum_i w(\theta - \theta_i) g_{\text{L}}. \tag{24}
\]

Here, \( g_{\text{L}} \) denotes the reduced tangential shear of a galaxy \( i \) at a position \( \theta \) with respect to the centre of mass. The uncertainty of this aperture mass is then given by

\[
\sigma_{M_{\text{ap}}}(\theta) = \left( \frac{1}{2} \sum_i w^2(\theta - \theta_i) |g_{\text{L}}|^2 \right)^{1/2}.
\]

In our analysis, we use a Gaussian weight function

\[
w(\theta) = \begin{cases} \exp\left[ -|\theta|^2/(2\sigma_w^2) \right] & |\theta| < 3\sigma_w, \\ 0 & \text{otherwise} \end{cases}
\]

and choose \( \sigma_w = 3 \) arcmin as the width of the aperture. The ratio of the aperture mass and its uncertainty gives the significance \( M_{\text{ap}}/\sigma_{M_{\text{ap}}} \), which is positive, where a tangential shear signal of an overdensity has been detected. It is strongest at the centre of a cluster of galaxies and grows weaker with the distance to the core of the dark matter halo.

In order to check for systematics, we define the cross aperture in analogy to equation (24) by replacing the tangential shear with the cross component \( g_x \).

5.3 NFW model

We further use the two-parametric density profile of a dark matter halo (Navarro & White 1996; Navarro et al. 1997) to perform a likelihood analysis. Numerical simulations (Navarro et al. 1995) have shown that the 3-dimensional density of dark matter as a function of the radius \( r \) is best described by

\[
\rho_{\text{NFW}}(r) = \frac{\rho_0}{r/r_s(1 + r/r_s)^2}, \tag{27}
\]

where the scale radius \( r_s \) and \( \rho_0 \) are parameters describing the density profile of the individual dark matter halo. The dimensionless concentration parameter is related to the virial radius and the scale radius via \( c = r_{\text{vir}}/r_s \). We use the virial radius \( r_{200\text{m}} \) at which the enclosed average density \( \bar{\rho} \) reaches a value that is 200 times that of the mean matter density for our fit. In order to compare our results to literature, we also express this quantity in terms of the critical density \( r_{200\text{m}} \) (and accordingly \( r_{500\text{c}} \)).

The virial mass \( M_{200\text{m}} \) and concentration parameter \( c_{200\text{m}} \) can be used to predict the signal shear \( \gamma_{\text{NFW}}(\theta) \) and convergence \( \kappa_{\text{NFW}}(\theta) \) a dark matter halo would cause for a galaxy at infinite redshift at a given position \( \theta \) (cf. Bartelmann 1996; Wright & Brainerd 2000). We use our distance fraction estimates from Section 4 to compute the theoretical value of the reduced shear as described in Seitz & Schneider (1997).

\[
\left\langle \chi_{\text{NFW}}^2(\theta) \rightangle = \frac{\langle \beta_z \rangle \gamma_{\text{NFW}}^2}{1 - \langle \beta_z \rangle^2}.
\]

Assuming that the convergence is negligibly small, this relation is reduced to \( \left\langle \chi_{\text{NFW}}^2(\theta) \rightangle = \langle \beta_z \rangle \gamma_{\text{NFW}}^2 \), which gives consistent results. Equation (28) includes a correction for the linear response of the reduced shear to the dispersion of \( \beta_z \).

We fit the theoretical reduced shear of the observed galaxies to their tangential shear profiles (cf. equation 9) using the minimum \( \chi^2 \) method of Avni (1976).

The log-likelihood is proportional to \( \chi^2(M, c) \),

\[
\mathcal{L}(M, c | \epsilon_{\text{data}}) \propto c^{-\chi^2(M, c)/2},
\]

and depends on mass and concentration of the cluster of galaxies. The \( \chi^2 \) can easily be calculated as

\[
\chi^2 = \sum_{i,j} \left( \gamma_{\text{NFW},i} - \epsilon_{\text{data},i,j} \right)^2 \sigma_{\text{int}} \sigma_{\text{int}} + \sigma_{\text{data}}^2
\]

for a set of given parameters \( M_{200\text{m}} \) and \( c_{200\text{m}} \). Here, \( \epsilon_{\text{data}} \) describes the \( i^{th} \) component of the ellipticity of the \( j^{th} \) galaxy in the data set. The predicted ellipticities are expressed by \( \epsilon_{\text{NFW}} \). The uncertainty \( \sigma_{\text{data}} \) of the shear is given by the quadratic average of the ellipticity errors. The intrinsic ellipticity dispersion is set to \( \sigma_{\text{int}} = 0.25 \).

The maximization of the likelihood \( \mathcal{L} \) is equivalent to the minimization of equation (30). We fit \( c \) and \( M \) simultaneously and calculate the confidence intervals according to Avni (1976).

The two parameters of the fit are, in fact, correlated. The dependency of concentration at a given cluster redshift \( z_{\text{cl}} \) on the halo mass has been investigated using simulations. We adopt the mass-concentration relation of Duffy et al. (2008)

\[
c(M, z_{\text{cl}}) = A \left( \frac{M}{M_{\text{pivot}}} \right)^B (1 + z_{\text{cl}})^C, \tag{31}
\]

where we use \( M_{\text{pivot}} = 2 \cdot 10^{12} \, h^{-1} M_\odot \), \( A = 10.14, B = -0.081 \) 

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where \( P \) the likelihood with the concentration term \( \beta \) of the cluster and, if available, results from previous studies. For each object, we provide: 

\[
\mathcal{M}_{\text{Planck}} = 1 - 2\pi \int_{\bar{A}} \left[ \frac{1}{(1 + \bar{y})} \right] d\bar{y}
\]

Combining confidence regions in the parameter space of the maximum-likelihood solutions for virial mass \( M \) are able to reduce the uncertainty on \( c \) and \( M \). We find maximum-likelihood solutions for virial mass \( M_{200m} \) and concentration parameter \( c_{200m} \) and determine projected and combined confidence regions in the parameter space of the model.

6 RESULTS

In this section, we present our results of the weak lensing analysis of three Planck SZ clusters of galaxies. For each object, we provide:

(i) A color image, a description of the visual appearance of the cluster and, if available, results from previous studies.

(ii) The mass significance map, calculated using equation (24-26). By applying a cut on \( \rho_{\text{min}}^{\text{SIS}} = 0.05 \), we try to optimize the lensing signal. If this threshold had been chosen lower, the small lensing signal of objects would increase the shape noise, as the scatter of intrinsic ellipticities is large compared to the weak lensing shear. At the same time, a threshold too high would remove too many lensed objects which, too, would increase the noise. For each of the clusters, we show the significance of non-zero aperture masses as a function of position in the field.

(iii) The tangential shear profile and the results of a fit of an SIS model. The virial masses are computed as

\[
M_{\text{SIS}}^{200c} = \frac{800\pi}{3} \rho_{\text{crit}}(z) \left[ \frac{\Delta_{c}}{10H(z)} \right]^3,
\]

\[
M_{\text{SIS}}^{200m} = \frac{800\pi}{3} \rho_{\text{m}}(z) \left[ \frac{\sqrt{2} \sigma^2_{\text{c}}} {10H_0 \Omega_{\text{m},0}(1 + z)^3} \right]^3,
\]

where we have used equation (23) and the definitions of the critical and the mean matter density.

(iv) The results of our NFW fit. In order to stay in the weak lensing regime and reduce the likelihood to include cluster member galaxies in the background sample, we exclude galaxies at a distance \( r < 500 \) kpc from the fitting procedure. We do not exclude galaxies in the outer regions of the field, as that would remove more sources from the analysis, which would result in an increase of the shot noise.

Whenever we expect line-of-sight structures to be present

\[ \log \sigma_c = 0.18 \] (Bullock et al. 2001), we use equation (31) to define a concentration prior

\[
P_c(c, M, z) = \exp \left[ \frac{-\left( \log(c) - \log(c(M, z)) \right)^2}{2\sigma_c^2} \right],
\]

where \( \log \) denotes the logarithm to base 10. Multiplying the likelihood with the concentration term \( P_c(c, M, z) \), we are able to reduce the uncertainty on \( c \) (and \( M \)). We find maximum-likelihood solutions for virial mass \( M_{200m} \) and concentration parameter \( c_{200m} \) and determine projected and combined confidence regions in the parameter space of the model.

6.1 PSZ1 G109.88+27.94

PSZ1 G109.88+27.94 is an SZ selected cluster candidate that has been discovered by the Planck satellite with a relatively low S/N of 5.3 (5.8) (Planck Collaboration et al. 2014a, 2015a). A group of elliptical galaxies is clearly visible in the gr-i image we show in Fig. 15. While the SZ signal is centred on the brightest cluster galaxy, the X-ray centroid is shifted to the south-west relative of the cluster. We cannot verify the spectroscopic redshift of \( z = 0.47 \) given in the Planck catalogues (Planck Collaboration et al. 2014b). Thanks to the coverage of the field by SDSS DR-14 data, we use their photometric redshift estimate of the BCG (Brightest Cluster Galaxy) for our analysis. A comparison of the \( r-i \) colours of our WWFI observations to those of the ref-

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3 For more details on the clusters, SZ and X-ray footprints, see [http://szcluster-db.ias.u-psud.fr/sitools/client-user/SZCLUSTER_DATABASE/project-index.html](http://szcluster-db.ias.u-psud.fr/sitools/client-user/SZCLUSTER_DATABASE/project-index.html).
ference galaxies in W-EGS support the assumption that the cluster is at a redshift of $z_{\text{phot}} = 0.77 \pm 0.04$.

Our deep colour image reveals a set of two arc candidates towards the south of the BCG. The arcs seem to lie on opposite sides of the critical line and are distorted by two nearby yellow galaxies in the foreground of the cluster. We use their projected distance from the cluster centre to get a rough estimate of the Einstein radius. $\theta_E \approx 14''$ from the BCG, which corresponds to a physical distance of $\sim 100$ kpc at the cluster redshift.

Wen & Han (2015) have identified two groups in the field. Fg1 (Foreground group 1) has a photometric redshift of $z_{\text{phot}} = 0.22$, Fg2 (Foreground group 2) is at $z_{\text{phot}} = 0.29$.

### 6.1.1 Significance map

The significance map for PSZ1 G109.88+27.94 is shown in Fig. 16. The cluster has been detected with an aperture mass significance of $3.5\sigma$. The $\sigma_{\text{ap}}$-peak is centred on the BCG. There is another $3\sigma$ that seems to be correlated to the central peak at $(\Delta R A, \Delta Dec) \sim (-2, -4)$. This peak could be due to the imprint of Fg2 on the background galaxy shapes. It does not coincide with the BCG of Fg2 but is shifted by $2.7'$ towards the center of the main cluster. The aperture mass peaks with $|\Delta R A| > 10'$ are close to the image boarder and are thus likely due to noise.

![Figure 16. Aperture mass significance map of PSZ1 G109.88+27.94 centred on the brightest cluster galaxy. The background color represents the significance calculated according to equation (24). Galaxies with $\beta_k < 0.05$ have been excluded from the analysis. The dotted box encompasses the inner $3 \times 3$ arcmin$^2$ region of the cluster, for which we present a colour image in Fig. 15. The positions of the foreground groups Fg1 and Fg2 are marked by the open and solid green square symbols.](image)

### 6.1.2 Tangential shear profile and fit of an SIS

The tangent alignment of background galaxies around PSZ1 G109.88+27.94 is shown in Fig. 17. We have applied a leakage correction to the data as explained in Section 3.4.2.3. As the systematic tangential shear increases with increasing distance from the centre of the image, neglecting this effect would cause $g_t$ to be biased low for large $\theta$.

The fit of an SIS profile to the measured tangential shear implies a velocity dispersion of $\sigma_v = (1800 \pm 200)$ km s$^{-1}$ (blue line). The mass is estimated to be $M_{\text{SIS}} = 4.4^{+16}_{-13} \times 10^{14}$ M$_\odot$.

![Figure 17. Tangential alignment of PSZ1 G109.88+27.94. The black (grey) circles (crosses) show the tangential (cross) reduced shear $g_t$ ($g_x$). The blue line shows the fit of an SIS density profile to the binned reduced shear $g_t$. An inner region with a radius of 2' (dashed line) has been excluded from the analysis. The cross shear is mostly consistent with zero. A leakage correction has been applied to the shear (cf. Section 3.4.2.3). The orange and pink lines show the theoretical tangential shear profile of one and three dark matter NFW halos with centres and redshifts as used in the one-, two- and three-halo NFW fits (cf. Section 6.1.3) and the best fitting masses (and concentration parameters).](image)

### 6.1.3 NFW fit

We perform four different NFW fits. In our first two-parameter fit, we assume a single dark matter halo at $z = 0.77$ and fit for the virial mass $M_{200m}$ and concentration parameter $c_{200m}$. While the cluster mass is estimated to be $M_{200m} = 46_{-19}^{+22} \times 10^{14}$ M$_\odot$ with concentration $c_{200m} = 2.0^{+1.8}_{-1.5}$ (Fig. 18, black contours). The use of a concentration prior reduces the uncertainty on $c_{200m}$ and we obtain $M_{200m} = 42_{-11}^{+17} \times 10^{14}$ M$_\odot$ and $c_{200m} = 2.9_{-0.9}^{+1.4}$. This fit yields $\chi^2_{\text{min}} = 1.3$ (Fig. 18, orange contours).

We try to find the masses of the two group candidates identified by Wen & Han (2015) by assuming the mass-concentration relation of Duffy et al. (2008).
and performing a three parameter fit of the halo masses. We find $M_{200m}^{NFW} = (4.0^{+2.3}_{-2.5} \times 10^{14}) \, M_\odot$ for the main component, $Fg1$ and $Fg2$ respectively. Our three-halo fit shows the cluster at $z = 0.77$ is clearly the dominant mass component in the field. Our analysis suggests that $Fg1$ and $Fg2$ have a comparable mass but we can only claim a 1σ detection of these group candidates. The $\chi^2_{min}$ is approximately 1.3. The tangential shear signal predicted by our best fitting three-halo model is shown in pink in (Fig. 17).

### 6.2 PSZ1 G139.61+24.20

PSZ1 G139.61+24.20 is at a spectroscopic redshift of $z_{cl} = 0.267$ (Planck Collaboration et al. 2015b). Fig. 19 shows the central region of the cluster, which coincides with both, the SZ footprint and the X-ray centroid. We have chosen the diffuse brightest cluster galaxy as the centre of the dark matter halo of PSZ1 G139.61+24.20 in our analysis. Giacintucci et al. (2017) detect a radio minihalo in the core of PSZ1 G139.61+24.20, which implies that there is a diffuse radio source in the centre of the cluster. We confirm PSZ1 G139.61+24.20 to be a massive cluster of galaxies and present the first weak lensing mass estimate for this object.

#### 6.2.1 Significance map

The significance map (Fig. 20) shows a peak at the position of the BCG at a significance of ~ 4σ. The SZ footprint of the cluster shows the same orientation as our 2d-lensing signal. Another small 3.5σ peak can be seen towards the North-East of the cluster but is likely due to noise.

#### 6.2.2 Tangential shear profile and fit of an SIS

The measured shear profile of PSZ1 G139.61+24.20 is shown in Fig. 21. The velocity dispersion is estimated to be $\sigma_v = (800 \pm 100) \, \text{km} \, \text{s}^{-1}$, which corresponds to a mass of $M_{200m}^{SIS} = 6.3^{+2.7}_{-2.1} \times 10^{14} M_\odot$. The cross component of the shear is mostly consistent with zero, except for the second bin, in which we measure a negative $g_\times$ at a ~ 2σ level.

#### 6.2.3 NFW fit

Without making any assumptions about the concentration of the cluster, the two-parametric NFW fit (Fig. 22) yields $M_{200m}^{NFW} = 2.2^{+2.2}_{-1.0} \times 10^{14} M_\odot$. Again, the concentration that we get from this fit is low with $c_{200m} = 1.6^{+1.9}_{-1.3}$. Upper limits of the projected intervals of 68%, 90% and 99% confidence are indicated by the grey lines. Using the concentration prior of Bullock et al. (2001) and Duffy et al. (2008) gives $M_{200m}^{NFW,P} = 13.5^{+6.9}_{-4.8} \times 10^{14} M_\odot$, a concentration parameter of $c_{200m}^{NFW,P} = 3.7^{+1.8}_{-1.2}$ and $\chi^2_{min} = 1.4$. 

---

**Figure 18.** We perform a maximum likelihood estimation assuming an NFW profile of the main cluster component in the field of PSZ1 G109.88+27.94. We present the likelihood contours of the fit of an NFW shear profile with virial mass $M_{200m}$ and concentration $c_{200m}$ to the data. The solid contours show the combined 1 and 2σ confidence regions of the two-parametric fit, whereas the dashed lines mark the projected confidence intervals. The orange contours have been obtained using the concentration prior of Bullock et al. (2001) and Duffy et al. (2008), whereas the black lines show the NFW fit without any further assumptions about $c_{200m}$.

**Figure 19.** RGB image using WWFI $g$, $r$ and $i$ band stacks. The dispyed 6 × 6 arcmin$^2$ region is centred on the brightest cluster galaxy of PSZ1 G139.61+24.20. The cyan line indicates a distance of 500 kpc at the cluster redshift $z_{cl} = 0.267$. This corresponds to the physical distance of the radius, that has been excluded from the NFW analysis described in the subsection below. North-east is towards the upper left in the image.
6.3 PSZ1 G186.98+38.66

PSZ1 G186.98+38.66 has an assigned spectroscopic redshift of $z_{\text{cl}} = 0.378$ (Piffaretti et al. 2011) in the Planck catalogues. The field contains a massive cluster of galaxies at (RA, Dec) = (08 : 50 : 11.2, +36 : 04 : 21) that has first been visually identified by Zwicky et al. (1961) and is commonly referred to as Zwicky 1953, ZwCl 0847+3617, RXC J0850.2+3603, or MACS J0850.1+3604. The cluster shows a prominent SZ imprint and X-ray signal which are aligned with the optical cluster centre.

According to the high concentration of luminous red galaxies (LRGs) at a redshift of about 0.35-0.4, this cluster field belongs to the 200 most massive lines of sight in the SDSS (Wong et al. 2013). Ammons et al. (2013) have studied this field with Hectospec at the MMT telescope on Mt. Hopkins, Arizona. Using their spectroscopic catalogues, they have identified two groups in the field, which are traced by LRGs. They find that the pointing is dominated by a massive cluster at redshift $z = 0.3774$ with a velocity dispersion of $\sigma = 1300$ kms$^{-1}$. A second, smaller group at redshift $z = 0.2713$ has a velocity dispersion of $\sigma = 300$ kms$^{-1}$. There is another LRG at a spectroscopic redshift of $z = 0.563$ in the field. Ammons et al. (2013) do not find evidence for the presence of a third group at this redshift, though this could be due to the limited depth of their spectroscopic sample.

Ammons et al. (2013) also use six-band $B, V, R_{\text{c}}, I_{\text{c}}, i', z'$ Subaru/Suprime-Cam data to search for strong lensing features. They have found a candidate multiply-imaged source at a photometric redshift of $z = 5.03^{+0.21}_{-0.17}$. This source galaxy has been used by Wong et al. (2013) to perform a joint weak and strong lensing analysis of the field using the same data as Ammons et al. (2013).

We choose the centre of the cluster at $z = 0.3774$ to be the brightest cluster galaxy (cf. Fig. 23). The field is very crowded with yellow cluster member galaxies.

PSZ1 G186.98+38.66 is part of the WtG sample of 51 massive galaxy clusters (Applegate et al. 2014). They only consider one halo at $z = 0.378$ to find a virial mass of $M(<1.5\text{Mpc}) = (15.8 \pm 2.6) \times 10^{14} \text{M}_\odot$ using their photometric redshift estimates $M(<1.5\text{Mpc}) = (16.9 \pm 3.2) \times 10^{14} \text{M}_\odot$ by applying a colour-cut.

6.3.1 Significance map

The significance map of PSZ1 G186.98+38.66 (Fig. 24) is centred on the cluster position. The main component of the cluster at $z = 0.3774$ is detected at a significance of $4\sigma$. The foreground group (Fg) leaves an imprint on the 2d-lensing signal at a significance of $3.5\sigma$ at $(\Delta RA, \Delta Dec) = (2, -2.5)$. The centroid position of this group is indicated by the open green symbol. The projected distance of the LRG at $z = 0.563$ to the main halo centre is very small. There is another $3.5\sigma$ peak of the aperture mass but we cannot find a corresponding overdensity of red galaxies at the designated position of the background group candidate (Bg).

Figure 20. Aperture mass significance map of PSZ1 G139.61+24.20 centred on the brightest cluster galaxy. The background color represents the significance calculated according to equation (24). Galaxies with $\beta_t < 0.05$ have been excluded from the analysis. The dotted box encompasses the inner $6 \times 6$ arcmin$^2$ region of the cluster, for which we present a colour image in Fig. 19.

Figure 21. Tangential alignment of PSZ1 G139.61+24.20. The black (grey) circles (crosses) show the tangential (cross) reduced shear $\gamma_t$ ($\gamma_x$). The blue line shows the fit of an SIS density profile to the binned reduced shear $\gamma_t$. An inner region with a radius of 2" (dashed line) has been excluded from the analysis. The cross shear is mostly consistent with zero. A leakage correction has been applied to the shear (cf. Section 3.4.2.3). The orange line shows the best fitting model of our two-parameter NFW fit (cf. Section 6.2.3).
Figure 22. We perform a maximum likelihood estimation assuming an NFW profile of the main cluster component in the field of PSZ1 G139.61+24.20. We present the likelihood contours of the fit of an NFW shear profile with virial mass $M_{200m}$ and concentration $c_{200m}$ to the data. The solid contours show the combined 1 and 2$\sigma$ confidence regions of the two-parametric fit, whereas the dashed lines mark the projected confidence intervals. The orange contours have been obtained using the concentration prior of Bullock et al. (2001) and Duffy et al. (2008), whereas the black lines show the NFW fit without any further assumptions about $c_{200m}$.

6.3.3 NFW fit

Fitting an NFW density profile to the measurement gives a mass of $M^{\text{NFW}}_{200m} = 32^{+13}_{-15} \times 10^{14} M_\odot$ with a concentration of $c^{\text{NFW}}_{200m} = 2.8^{+1.9}_{-1.2}$ and a minimum $\chi^2$ of $\chi^2_{\text{min}} = 1.3$. Using a prior on the concentration, we find $M^{\text{NFW,p}}_{200m} = 30^{+10}_{-8} \times 10^{14} M_\odot$ and $c^{\text{NFW,p}}_{200m} = 3.5^{+1.9}_{-1.0}$, where $\chi^2_{\text{min}}$ does not change significantly (cf. Fig. 26).

The three-halo fit reveals the foreground group to have a rather low mass of $M^{\text{NFW}}_{200m} \sim 2.5^{+2.5}_{-1.5} \times 10^{14} M_\odot$ to which we are not sensitive in our weak lensing study. The fit prefers a mass of $M^{\text{NFW}}_{200m} = 2.5 \times 10^{14} M_\odot$ for the background cluster candidate. The presence of the two groups implies a lower mass of $M^{\text{NFW}}_{200m} = 20^{+5}_{-4} \times 10^{14} M_\odot$ compared to the one-halo fit for the main cluster component. The minimum $\chi^2$ of this fit is slightly smaller compared to the one-halo NFW model with $\chi^2_{\text{min}} = 1.2$. We show the best fit model tangential shear signal in Fig. 25 in pink.

Figure 23. RGB image using WWFI $g$, $r$ and $i$ band stacks. The displayed $6 \times 6$ arcmin$^2$ region region is centred on the BCG of PSZ1 G186.98+38.66. The cyan line indicates a distance of 500 kpc at the cluster redshift. This corresponds to the physical distance of the radius, that has been excluded from the NFW analysis described in the subsection below. The cluster has a group redshift of 0.3774 (Ammons et al. 2013). North-east is towards the upper left in the image.

7 DISCUSSION AND COMPARISON TO LITERATURE

7.1 Comparison to SZ mass estimates

Having determined weak lensing masses for the WWL pathfinder sample, we compare our estimates to the SZ masses reported by Planck Collaboration et al. (2014b, 2015b) and Planck Collaboration et al. (2015c). We discuss the results of this work and suggest strategies to improve the weak lensing analysis of future WWL precision measurements.

Mass estimates based on Planck measurements of the Compton parameter $Y$ can be obtained via

$$D^2\chi^2 = \frac{\sigma_T}{m_e c^2} \int P_e dV,$$

(34)

where $\sigma_T$ is the Thompson scattering cross section, $m_e$ is the electron mass and $c$ is the speed of light. As the volume integral of the electron pressure $P_e$ equals the thermal energy of the electron gas, the Compton parameter is very closely correlated with both the temperature and the mass of the...
gas and thus, as both of these properties depend on the cluster mass, with the total mass $M_{500}$. The two Planck SZ cluster catalogues offer mass estimates within a radius of $R_{500}$, where

\[
M_{500} = 500 \times \frac{4\pi \rho_{\text{crit}}}{3} R_{500}^3.
\]  

(35)

The angular size $\theta_{500}$\(^4\) is the aperture used to extract the integrated Compton parameter $I_{500}$. Consequently, there is a degeneracy between cluster size and SZ flux, which has to be broken in order to derive the cluster mass (Planck Collaboration et al. 2011). This is accomplished by using the $M_{500}$-$D_A^2 Y_{500}$ relation between $\theta_{500}$ and $Y_{500}$ (Arnaud et al. 2010).

The catalogue of Planck Collaboration et al. (2014b) (hereafter PSZ1) includes 1 227 SZ selected clusters of galaxies, some of which have not yet been verified. PSZ1 has been created using the first 15.5 months of Planck observations. $M_{500}$ is based on X-ray calibrations scaling relations under the assumption of a flat universe with $\Omega = 0.7$, $\Omega_m = 0.3$ and $\Omega_{\Lambda} = 0.7$ (Planck Collaboration et al. 2015c). Planck Collaboration et al. (2015b), hereafter PSZ2) derive their mass proxy $M_{SZ}$ in a similar fashion.

We convert the weak lensing masses $M_{\text{WL}}^{500}$ to $M_{500}$ and find an overall agreement with the SZ masses within the errors (cf. Table 4) for the single-halo fits of PSZ1 G139.61+24.20 and PSZ1 G186.98+38.66. In the case of PSZ1 G109.88+27.94, the SZ mass is severely underestimated by PSZ1 and PSZ2, as a wrong redshift of $z = 0.4$ was assumed in their analysis.

Compared to the Planck weak lensing masses $M_{\text{WL}}^{500}$, have very large uncertainties. The errors of the SZ masses presented in Table 4 are purely measurement uncertainties. Neither intrinsic, statistical, nor systematic errors on the scaling relations have been taken into account. An intrinsic scatter of the SZ mass arises from a scatter in the SZ signal at fixed halo mass. Planck Collaboration et al. (2014a, 2015a) have found a value of $\sigma_{\text{int}} = 0.127 \pm 0.023$ for their data. As the weak lensing shear is sensitive to all of the matter along the line of sight to the cluster, a scatter into lensing mass has to be considered as well.

For the estimation of the SZ cluster masses, only one halo at the designated cluster redshift was assumed. If we compare the results of our one-halo NFW masses $M_{\text{WL}}^{500}$ to the results from Planck, we find that they agree within the errors for PSZ1 G139.61+24.20. The ~ 1.3σ discrepancy between $M_{\text{WL}}^{500}$ and $M_{500}^{\text{SZ}}$ ($M_{500}^{\text{SZ}}$) implies a larger cluster mass for PSZ1 G186.98+38.66 than the SZ signal suggests.
Table 4. Planck name, line-of-sight structure, right ascension, declination and the redshifts we used in our weak lensing analysis. Our weak lensing mass estimates \( M^\text{WL}_{200m} \) from the NFW fits assuming one and three dark matter halos along the line-of-sight and the SZ mass proxies of PSZ1 and PSZ2, where the masses are given in units of \( 10^{14} M_\odot \).

| Field name | LSS | RA (J2000) | Dec (J2000) | \( z_{\text{analysis}} \) | \( M^\text{WL}_{200m}^1 \) | \( M^\text{WL}_{200m}^3 \) | \( M^\text{WL}_{200c} \) | \( M^\text{SZ}_{200} \) | \( M^\text{SZ}_{500} \) |
|------------|-----|------------|-------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| PSZ1 G109.88+27.94 | Main | 18:23:23.0 | +78:23:13 | 0.77 | 4^{+27}_{-19} | 21^{+16}_{-10} | 40^{+10}_{-7} | 5.6^{+0.6}_{-0.7} | 5.2^{+0.4}_{-0.5} |
| | FG1 | 18:25:09.4 | +78:23:37 | 0.22 | - | - | 4.0^{+1.4}_{-1.3} | - | - |
| | FG2 | 18:22:02.8 | +78:17:43 | 0.29 | - | - | 4.0^{+1.4}_{-1.3} | - | - |
| PSZ1 G139.61+24.20 | Main | 6:21:48.9 | +74:42:04 | 0.267 | 13.5^{+6.9}_{-4.8} | 6.0^{+4.7}_{-2.9} | - | 7.1^{+0.6}_{-0.6} | 7.6^{+0.5}_{-0.5} |
| PSZ1 G186.98+38.66 | Main | 8:50:07.9 | +36:04:13 | 0.3774 | 30^{+10}_{-8} | 13.7^{+7.5}_{-5.1} | 20.0^{+3.4}_{-2.7} | 7.4^{+0.7}_{-0.8} | 6.8^{+0.5}_{-0.5} |
| | FG | 8:50:17.1 | +36:01:13 | 0.2713 | - | - | 2.5^{+2.2}_{-2.5} | - | - |
| | BG | 8:49:56.8 | +36:03:33 | 0.563 | - | - | 2.5^{+2.2}_{-2.5} | - | - |

Figure 26. We perform a maximum likelihood estimation assuming an NFW profile of the main cluster component in the field of PSZ1 G186.98+38.66. We present the likelihood contours of the fit of an NFW shear profile with virial mass \( M_{200m} \) and concentration \( c_{200m} \) to the data. The solid contours show the combined 1 and 2\( \sigma \) confidence regions of the two-parametric fit, whereas the dashed lines mark the projected confidence intervals. The orange contours have been obtained using the concentration prior of Bullock et al. (2001) and Duffy et al. (2008), whereas the black lines show the NFW fit without any further assumptions about \( c_{200m} \).

7.2 Comparison to weak lensing mass estimates

As mentioned before, only for PSZ1 G186.98+38.66 previous weak and strong lensing mass estimates do exist. The first weak lensing mass estimate from Applegate et al. (2014) predicts a total mass of \( M(<1.5\text{Mpc}) = (16.9 \pm 3.2) \times 10^{14} M_\odot \) in a sphere at redshift 0.378 with radius 1.5 Mpc from the cluster centre. We conclude, that our weak lensing mass estimates for this field agree well with the findings of Applegate et al. (2014).

7.3 Comparison to dynamical mass estimates

Finally, the velocity dispersions of PSZ1 G139.61+24.20 and PSZ1 G186.98+38.66 are known from literature. Amodeo et al. (2017) have identified 20 cluster member galaxies of PSZ1 G139.61+24.20 and 41 cluster member galaxies for PSZ1 G186.98+38.66 using spectra obtained with the GMOS multi-object spectrograph at Gemini Observatory. They measure a velocity dispersion of \( \sigma_{200c} = 1052^{+390}_{-273} \text{kms}^{-1} \) for PSZ1 G139.61+24.20 and \( \sigma_{200c} = 1432^{+166}_{-166} \text{kms}^{-1} \) for PSZ1 G186.98+38.66. Our SIS fits suggest velocity dispersions of \( \sigma_r = 800 \pm 100 \text{kms}^{-1} \) for PSZ1 G139.61+24.20 and \( \sigma_r = 1100 \pm 200 \text{kms}^{-1} \) for PSZ1 G186.98+38.66. Even with this crude model of the density profiles of the clusters, our \( \sigma_r \) estimates are in agreement with the measurements of Amodeo et al. (2017).

Ammons et al. (2013) have used a dynamical model to constrain the NFW mass and concentration of PSZ1 G186.98+38.66 from more than 500 galaxy spectra. They find a \( \sigma_{200c} = 1300^{+60}_{-60} \text{kms}^{-1} \) for the main cluster component at \( z = 0.3774 \) and \( \sigma_{500} = 300^{+90}_{-90} \text{kms}^{-1} \) for the group at \( z = 0.3774 \). They estimate the virial cluster mass to be equal to \( M_{200} = 3^{+2}_{-1} \times 10^{14} M_\odot \) and the mass of the smaller group to be equal to \( M_{200} = 0.6^{+0.4}_{-0.3} \times 10^{14} M_\odot \). Their mass estimate for the components of PSZ1 G186.98+38.66 agrees well with our results from the multiple halo fits. Though we cannot constrain the mass of the foreground group well, their prediction is well below our upper mass limit of \( M_{200} \lesssim 0.8 \times 10^{14} M_\odot \).

7.4 Impact of biases on our mass estimates

On average, weak lensing masses should be unbiased \( (M^\text{true} \approx M^\text{WL}) \). We have carefully investigated sources of multiplicative and additive bias that could affect our weak lensing mass estimates. We present an overview of the bias budget in Table 5.

While we do correct for multiplicative shape bias \( b \) by applying a \( S/N \)-calibration (cf. Section 3.4.1), we do not take a dependency of \( b \) on the galaxy profiles or on the distributions of sizes and ellipticities into account. Moreover, the simulations used to constrain the multiplicative shape bias might not be a good enough replication of our WWL data. A better calibration of \( b \) for future WWL projects will be
necessary. We estimate the residual multiplicative shear bias to be less than 5 per cent.

Usually, additive shear biases are neglected in cluster weak lensing studies, since the shear is measured in circular apertures and additive offsets cancel out. However, since our masks are not radially symmetric, additive shape biases can still affect the measured tangential shear signal. We consider three different types of additive shape bias. The mean ellipticity of our galaxies is not zero but equal to $\epsilon_{\text{sys}}^e$ (Section 3.4.2.1). As it depends on the random orientation of the applied masks, this constant additive shape bias causes a statistical uncertainty on the cluster mass. The PSF model bias is different in each cluster field. We use the Rowe statistics to constrain an upper limit of $c_{\text{PSF}}^e \lesssim 4.3 \times 10^{-3}$ for this bias in the field of PSZ1 G109.88+27.94, PSZ1 G139.61+24.20 and PSZ1 G186.98+38.66, respectively. The PSF model bias may be correlated between different pointings, in which the cluster is always near the centre. It will not decrease by taking more data. The third source of additive systematics in the shape catalogues is PSF leakage, which arises when the deconvolution of the PSF from the source images is not perfect. This type of additive shape bias is not spatially constant over the fields but should be the same for all observed fields. PSF leakage can be approximated as a linear dependency of the galaxy shapes on the PSF ellipticity, i.e. $\alpha e_{\text{PSF}}$. We calibrate the tangential shear signal by modeling the systematic tangential shear signal $\epsilon_{\text{sys}}^\gamma$ as a function of distance from the cluster centre. We estimate the remaining PSF leakage calibration bias on the cluster mass to be less than 2 per cent. The PSF leakage calibration bias $\epsilon_{\text{PSF}}^\gamma$ has been estimated using the statistical uncertainty of our leakage correction and will decrease with increasing cluster sample size.

We have also considered biases in our background sample selection. The photometric calibration is very precise with negligible errors on the zero-point and the photometric redshifts of the reference galaxies. The only significant contribution to the systematics budget comes from the cosmic variance and depends only on the cluster redshift. It is smallest for PSZ1 G139.61+24.20 at $z = 0.267$ and largest for PSZ1 G109.88+27.94 at $z = 0.77$ and is equal to $\sigma_{\text{PSF}} = 0.023, 0.006, 0.009$ for PSZ1 G109.88+27.94, PSZ1 G139.61+24.20 and PSZ1 G186.98+38.66, respectively. For clusters with high redshifts, such as PSZ1 G109.88+27.94, our applied background sample selection technique does not perform well using only gri photometry.

We give upper limits of the total bias on the cluster mass: 15% for PSZ1 G109.88+27.94, 11% for PSZ1 G139.61+24.20 and 8.1% for PSZ1 G186.98+38.66. Note that some biases are expected to cancel each other out, so the true mass bias might be significantly smaller than the values given in Table 5. The statistical uncertainties are almost of the same order as the systematical uncertainties and will decrease with increasing cluster sample size and the observation of additional reference fields. The residual multiplicative shear bias is the dominant source of systematics. An accurate calibration of $m$, or a shape measurement technique that performs better than KSB+, will be needed in order to bring this bias down in future studies. The large scatter of the PSF model bias upper limit shows that the performance of our PSF modeling technique is field dependent but performs well with $\sigma_{\text{PSF}} < 0.5\%$ in the case of PSZ1 G186.98+38.66.

### 8 CONCLUSIONS

We present the results of the first cluster weak lensing study using only data obtained at the Wendelstein Observatory in Bavaria, Germany. Our pathfinder sample consists of three massive SZ-selected clusters of galaxies. We obtain shape catalogues using an implementation of the KSB code and determine lensing strengths from our deep $g, r, i$ band photometric data by following the approach of Gruen et al. (2014). We carefully test for the impact of biases on our cluster mass estimates and present a simple method to approximate and correct for PSF leakage in weak lensing data where the large statistical uncertainty makes a precise estimation of the additive shear bias difficult. We use the mass-concentration relation of Duffy et al. (2008) as a prior and

| Bias | PSZ1 G109.88+27.94 at $z = 0.77$ | PSZ1 G139.61+24.20 at $z = 0.267$ | PSZ1 G186.98+38.66 at $z = 0.3774$ |
|------|---------------------------------|----------------------------------|-----------------------------------|
| **Statistical uncertainties** | | | |
| Mean ellipticity mass bias $\sigma_{\text{e}M}$ (Section 3.4.2.1) | 0.09 | 0.07 | 0.03 |
| PSF leakage calibration bias $\sigma_{\text{e}sys}^\gamma$ (Section 3.4.2.3) | 0.002 | 0.001 | 0.001 |
| **Total** | $< 9.0\%$ | $< 7.0\%$ | $< 3.0\%$ |
| **Systematic uncertainties** | | | |
| Residual multiplicative shape bias $\sigma_{\text{PSF}}$ (Section 3.4.1) | 0.07 | 0.07 | 0.07 |
| PSF model bias $\sigma_{\text{PSF}}$ (Section 4.3) | 0.085 | 0.035 | 0.005 |
| Cosmic variance $\sigma_{\beta}/\beta$ (§4.3) | 0.04 | 0.03 | 0.03 |
| **Total** | $< 12\%$ | $< 8.4\%$ | $< 7.6\%$ |
| **Total bias** | $< 15\%$ | $< 11\%$ | $< 8.1\%$ |
perform an NFW likelihood analysis to estimate the mass and concentration of the objects.

We present the first weak lensing mass estimates for the massive SZ-selected galaxy clusters PSZ1 G109.88+27.94 and PSZ1 G139.61+24.20. We correct the redshift estimate from PSZ1 and PSZ2 for PSZ1 G109.88+27.94 from $z = 0.4$ to $z_{phot} = 0.77$.

A two-parameter NFW-fit for the mass and concentration of a single dark matter halo in the field of PSZ1 G186.98+38.66 yields results, which are consistent with the weak lensing mass constraints from WtG. Assuming the presence of a foreground group at $z = 0.2713$ in the field of PSZ1 G186.98+38.66 we try to constrain its mass but are not sensitive to such low mass halos. We cannot exclude the existence of the background group candidate at a redshift of $z = 0.563$ in the field of PSZ1 G186.98+38.66, which would lower the mass of the main cluster significantly. Our findings confirm that the presence of line of sight structures can have a significant impact on recovered weak lensing cluster masses.

We conclude that we can use multi-band WWFI data to perform independent weak lensing studies of good quality for small samples of individual clusters. We plan to further improve our analysis and target a new sample of relaxed clusters of galaxies and individual galaxy clusters we deem worthy to be studied in more detail.

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