Abstract

Recent observation of supernovae type Ia show clearly that there is a large scale repulsive force in the Universe. Neither of the four known fundamental interactions can account for this repulsive force. Gravity is known to be the interaction responsible for the large scale structure and evolution of the Universe. The problem with gravity is that it gives rise to a force which is attractive only. Gravity theories, including General Relativity, deals with gravity as an attractive force. Although this is consistent with our experience in the solar system and other similar astrophysical systems, gravity theories fail to account for SN type Ia observation. So, we are in a real problem concerning the interpretation of these observation. This problem is only ten years old. In order to go out of this problematic situation, scientists have suggested the existence of a type of energy in the Universe that is responsible for the above mentioned repulsive force. They have given this type of energy the exotic term "Dark Energy". Although this type of energy forms more than two thirds of the energetic contents of our Universe, its reasonable nature is missing in all gravity theories.

The aim of the present work is to review the present status of the problem of dark energy. Also, to suggest a new geometric solution for this problem.

1 Introduction

In the last ten years, astronomical observations show that some phenomena are not consistent with the present accepted physical theories. Among these observations:

1. The supernovae (SN) type Ia observations [1],which indicate very clearly that our Universe is in an accelerating expansion phase. The problem is that this type of expansion necessities the presence of a large scale repulsive force in the Universe. Assuming that gravity plays the main role in the structure and evolution of the Universe, one cannot interpret the above mentioned observations. The reason is that gravity, as known in the solar system, gives
rise to an attractive force only, which is treated as a physical fact. Gravity theories, including general relativity (GR), are constructed taking this fact into account. More precisely, the dynamical equations of FRW-models (with vanishing cosmological constant) give rise to a de-accelerating Universe. This is a real discrepancy between theory and observations.

2. The rotation curves of spiral galaxies [2], which concern observations of star rotational velocities in such type of galaxies. The curves resulting from observational (giving the relation between the distance of the star from the center of galaxy and its rotation velocity), show a flat behavior for the outer regions of the galaxy, while the curves resulting from theories, including the orthodox GR, are bent towards the x-axis (the axis giving the distance of the star from the center of the galaxy). This gives another discrepancy between observations and known physics.

3. The observation of the velocities of spacecrafts, Pioneer 10, 11 [3]. These two spacecrafts were launched in March 1972 and April 1973, respectively. Both vehicles have an apparatus emitting radio waves with certain wave length. The comparison between radio waves received from these vehicles and radio waves from an identical apparatus, on the Earth’s surface, gives the radial velocities of each vehicle, via the red-shift phenomena, for the two vehicles, are found to be different from those obtained from gravity theories. This represents a third discrepancy between experiment and gravity theories.

It seems that the above mentioned problems are closely connected to each other. All are on scales larger than the solar system scale and have some relation to gravity theories. These problems indicate clearly that an important ingredient is missing from gravity theories. The correct identification of this ingredient, leading to a solution of one of the above mentioned problems, would automatically lead to solutions of the other problems, if they are connected.

The aim of the present work is to throw some light on possible ingredients, missing from gravity theories, concentrating on the first problems. In section 2 we give brief survey of different interpretations of dark energy. In section 3 we introduce a principle leading our investigations. In section 4. we give a brief review on the geometry appropriate for the suggested solution. The paper is discussed and concluded in section 5.

2 Different Interpretations of Dark Energy

Since the discovery of the accelerating expansion of the Universe, many authors have suggested different solutions for this problem. The solutions suggested
imply the existence of some type of energy with negative pressure. This type of energy is known in the literature as "Dark Energy". We are going to review briefly different solutions, of this problem, proposed to probe the nature of such peculiar type of energy.

The field equations of GR can be written as,

$$R_{\mu \nu} - \frac{1}{2} g_{\mu \nu} R = -\kappa T_{\mu \nu},$$  \hspace{1cm} (1)

where $R_{\mu \nu}$ is Ricci tensor, $R$ is Ricci scalar and $T_{\mu \nu}$ is the material energy tensor. In case of FRW-models with the metric $(x^1 \equiv r, x^2 \equiv \theta, x^3 \equiv \phi \ \& \ x^4 \equiv t),$

$$ds^2 = dt^2 - \frac{a^2(t)}{1 + \frac{k r^2}{4}} (dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2),$$  \hspace{1cm} (2)

where $a(t)$ is the scale factor and $k$ is the curvature constant, the field equations (1) give rise to the dynamical equations,

$$\frac{\dot{a}^2}{a^2} = \frac{8}{3} \pi \rho_o - \frac{k}{a^2},$$  \hspace{1cm} (3)

$$\frac{\ddot{a}}{a} = -\frac{4}{3} \pi (\rho_o + 3 p_o).$$  \hspace{1cm} (4)

It is clear from (4) that, for the proper density $\rho_o > 0$ and for the proper pressure $p_o \geq 0$, $\ddot{a}$ is negative (de-acceleration), which contradicts SN type Ia observations, as mentioned above. This problem may be solved if there is an additional term on the right hand side of (4) compensating the gravitational attraction. This can be achieved using one of the following suggestions:

1. By inserting a term, the cosmological term $(\varLambda)$, into the definition of the Einstein tensor as done by Einstein himself (cf. [4]). In this case, GR field equations will take the form

$$R_{\mu \nu} - \frac{1}{2} g_{\mu \nu} (R - 2 \varLambda) = -\kappa T_{\mu \nu},$$  \hspace{1cm} (5)

This will add a term in the dynamical equations compensating gravitational attraction to give, qualitatively, the observed phenomena.

2. By imposing an equation of state, with negative pressure, of the form,

$$p_o = \omega \rho_o,$$  \hspace{1cm} (6)
where $\omega$ is a negative parameter. It is to be considered that the use of (5), to construct world models, corresponds to $\omega = -1$. The cases with $\omega < -1$ are known as "phantom energy" (cf. [5]), while the cases with $\omega > -1$ are known as "quintessence energy" (cf. [6]). Belongs to this class of suggestions the use of an equation of state of the form,

$$p_o = \frac{-A}{\rho_o^\alpha},$$

where $A(> 0)$ and $\alpha(\geq 0)$ are parameters. This case is known in the literature as "Chaplygin gas" (cf. [7]).

3. "Modified Gravity" (cf. [8]): It is well known that in constructing the field equations of GR (4), using an action principle, the Lagrangian function contains a term linear in Ricci scalar, $R$. In the modified gravity theories, it is suggested that this term is replaced by any function $f(R)$ including quadratic and negative powers of $R$.

4. In this class of suggestions it is claimed that the introduction of extra dimension would solve the problem (cf. [9]). The resulting theory is of Kaluza-Klein type.

3 The Interaction Principle

In the previous section, a summary of different suggestions to understand the nature of dark energy are given. Some suggestions retain GR written in Riemannian geometry, while others modify GR, in Riemannian geometry also. The common feature in all attempts, summarized above, are carried out in the context of Riemannian geometry. The main assumption in this case is that:

"Riemannian space gives a complete representation of the physical world including space and time".

However, we are going to relax this assumption in the present work. Furthermore, in the present attempt, in order to explore what is missing behind the exotic term "dark energy" we are going to use the interaction principle [10]. This can be summarized as follows:

"Physical phenomena are just interactions between space-time properties and
the intrinsic properties of the elementary constituents of matter (energy)

Now assuming the validity of this principle and that the ingredient behind dark energy is an interaction of the type mentioned in this principle, one way to explore this interaction is to relax the assumption underlined above, in order to extend the geometric structure. This is because Riemannian space is limited and has no sufficient structure to apply the interaction principle satisfactorily. Its linear connection is symmetric which implies the vanishing of the torsion of space-time. The introduction of a non-symmetric linear connection will provide us with a geometric structure with a non-vanishing torsion. This will produce a wider space, more thorough than the Riemannian one, especially when it possesses a non-vanishing curvature. The extra structure of this space, can give more degrees of freedom which facilitates the application of the interaction principle. This would throw some light on the nature of dark energy. The solutions suggested in this case would be a pure geometric one.

4 An Appropriate Geometric Structure

In order to explore what is the missing ingredient behind the peculiar assumption of dark energy we are going to apply "the interaction principle". For this reason, it is preferable to use a complete geometric structure with the following general properties:

1. It should have a metric tensor since conventional gravity is proved to be a metric phenomena.

2. Its linear connection should be non-symmetric in order to have a non-vanishing torsion. This will extend the geometric structure.

3. It should have a non-vanishing curvature, as well.

These properties characterize a wide class of geometric structures known in the literature as the "Riemannian-Cartan geometry". We are going to use a certain structure of this class, known as the "Parameterized Absolute Parallelism (PAP)-geometry" [11]. Calculations within the context of this structure are easy and the physical meaning are clear. The non-symmetric linear connection of this structure can be written as,

\[ \hat{\Gamma}^{\alpha}_{\mu\nu} = \left\{ \alpha \right\}_{\mu\nu} + b'^{\alpha}_{\mu\nu}, \] (8)
where \( \{ \alpha \}_\mu \) is the Christoffel symbol of the second kind defined in terms of the metric of the space, \( b \) is a parameter and \( \gamma^\alpha_{\mu\nu} \) is the contortion of AP-structure. The torsion of the PAP-structure can be defined by (AP-quantities are without hats)
\[
\hat{\Lambda}^\alpha_{\mu\nu} \overset{\text{def}}{=} \hat{\Gamma}^\alpha_{\mu\nu} - \hat{\Gamma}^\alpha_{\nu\mu} = b(\gamma^\alpha_{\mu\nu} - \gamma^\alpha_{\nu\mu}),
\]
i.e.
\[
\hat{\Lambda}^\alpha_{\mu\nu} = b\Lambda^\alpha_{\mu\nu}. \tag{9}
\]
The Curvature corresponding to the non-symmetric connection (8) is in general non-vanishing and given by,
\[
\hat{B}^\alpha_{\mu\nu\sigma} = \hat{\Gamma}^\alpha_{\mu\sigma,\nu} - \hat{\Gamma}^\alpha_{\mu\nu,\sigma} + \hat{\Gamma}^\epsilon_{\mu\sigma} \hat{\Gamma}^\alpha_{\epsilon\nu} - \hat{\Gamma}^\epsilon_{\mu\nu} \hat{\Gamma}^\alpha_{\epsilon\sigma}, \tag{10}
\]
which can be written in the form [12],
\[
\hat{B}^\alpha_{\mu\nu\sigma} = R^\alpha_{\mu\nu\sigma} + b\hat{Q}^\alpha_{\mu\nu\sigma}, \tag{11}
\]
where
\[
R^\alpha_{\mu\nu\sigma} = \left\{ \frac{\alpha}{\mu\sigma} \right\}_\nu - \left\{ \frac{\alpha}{\mu\nu} \right\}_\sigma + \left\{ \frac{\epsilon}{\mu\nu} \right\} \left\{ \frac{\alpha}{\epsilon\sigma} \right\} - \left\{ \frac{\epsilon}{\mu\sigma} \right\} \left\{ \frac{\alpha}{\epsilon\nu} \right\}. \tag{12}
\]
and
\[
\hat{Q}^\alpha_{\mu\nu\sigma} \overset{\text{def}}{=} \gamma^\alpha_{\mu\nu} + \gamma^\alpha_{\nu\sigma} - b(\gamma^\beta_{\mu\sigma} \gamma^\alpha_{\beta\nu} - \gamma^\beta_{\mu\nu} \gamma^\alpha_{\beta\sigma}). \tag{13}
\]
The general path equation for this structure can be written as [13],
\[
\frac{dZ^\mu}{d\tau} + \left\{ \mu_{\alpha\beta} \right\} Z^\alpha Z^\beta = -b \Lambda^\mu_{(\alpha\beta)} Z^\alpha Z^\beta, \tag{14}
\]
where \( \tau \) is a parameter characterizing the path and \( Z^\mu \) is its tangent.

**Comments on the PAP-structure:**

1. It represents a class of Riemann-Cartan geometry. It has simultaneously non-vanishing parameterized torsion (9) and curvature(10).

2. Although the curvature (10) is non-linear in the connection (8), it can be slitted into two parts: the first is a pure function of Christoffel symbol only, \( R(\{\}) \), which is the Riemann-Christoffel curvature tensor; while the second is a fourth order tensor, function of the contortion (or the torsion) only, \( \hat{Q}(\gamma) \). This is an important property in physical applications which is discussed later.
3(a)- For $b = 0$ the PAP-structure becomes Riemannian. Equations (9), (11) and (14) will reduce respectively, to

$$\hat{\Lambda}^\alpha_{\mu\nu} = \Lambda^\alpha_{\mu\nu} = 0,$$

$$\hat{B}^\alpha_{\mu\nu\sigma} = R^\alpha_{\mu\nu\sigma},$$

$$\frac{dZ^\mu}{d\tau} + \left\{ \mu_{\alpha\beta} \right\} Z^\alpha Z^\beta = 0.$$  

So, any field theory constructed in the PAP-space can be easily compared with orthodox GR upon taking to $b = 0$. Furthermore, equation (14) is reduced to the ordinary geodesic equation (17), under the same condition. This equation gives rise to the attractive force of conventional gravity.

3(b)- For $b = 1$ the PAP-structure reduces to the conventional AP-structure (cf. [14]) with a non-vanishing torsion

$$\hat{\Lambda}^\alpha_{\mu\nu} \Rightarrow \Lambda^\alpha_{\mu\nu} = \Gamma^\alpha_{\mu\nu} - \Gamma^\alpha_{\nu\mu} = (\gamma^\alpha_{\mu\nu} - \gamma^\alpha_{\nu\mu}),$$

but with a vanishing curvature [14].

$$\hat{B}^\alpha_{\mu\nu\sigma} \Rightarrow B^\alpha_{\mu\nu\sigma} = R^\alpha_{\mu\nu\sigma} + Q^\alpha_{\mu\nu\sigma} \equiv 0.$$  

The last relation is very important. Although the curvature $B(\Gamma)$, given by (19), is an identically vanishing tensor, giving rise to a flat space, its constituents $R(\{\})$ and $Q(\gamma)$ are not vanishing objects. The tensor $Q(\Gamma)$, defined by (13) with $b = 1$, compensates the curvature $R(\{\})$ of the space-time in such a way that $B(\Gamma)$ vanishes, as shown by (19). For this reason the tensor $Q(\gamma)$ is called the anti-curvature or the additive inverse of the curvature tensors. Both tensors satisfy Bianchi differential identity, whose contracted form can be written, for these tensors, as [15]

$$(R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R)_{;\mu} \equiv 0,$$

$$(Q^{\mu\nu} - \frac{1}{2}g^{\mu\nu}Q)_{;\mu} \equiv 0.$$  

The identity (20) is interpreted, in the context of geometrization scheme, as a geometric representative of some conservation law, so is (21). For this reason we call the quantities between brackets in equations (20) and (21) the curvature energy and the torsion energy, respectively [15]. Torsion energy is a type of
energy that is mainly caused by torsion, since the anti-curvature tensor $Q(\gamma)$ is purely made of the contortion (or the torsion). It has been shown in [10] that torsion energy is associated with a repulsive force giving rise to anti-gravity.

4. The term on the right hand side of the PAP-path equations (14) has been interpreted physically as representing a type of interaction between the torsion of the background space-time and the quantum spin of the moving particle [13]. The linearized form of this equation gives rise to the generalized potential,

$$\Phi = \Phi_N + \Phi_T,$$

where $\Phi_N$ is the Newtonian gravitational potential and

$$\Phi_T = -b\Phi_N,$$

is the torsion anti-gradational potential. It is clear that, as $\Phi_N$ gives rise to an attractive force, $\Phi_T$ will give rise to a repulsive one.

5 Conclusion

1. The existence of torsion, in the space-time structure, gives rise to a repulsive force. Consequently torsion energy is an energy connected to this force, which gives rise to a negative pressure.

2. The missing physical ingredient is the spin-torsion interaction and consequently dark energy is nothing more than torsion energy.

3. Since electromagnetic phenomena are the results of interaction between the electromagnetic field and the electric charge (an elementary particle intrinsic property), then one can conclude, using the interaction principle, that the electromagnetic field is a space-time property. Some authors have succeeded in representing the electromagnetic field as a geometric property (cf.[16], [17]) with successful applications (cf. [18], [19], [20]).

4. The main assumption, given in section 3, should now be replaced by: “A space with simultaneously non-vanishing curvature and torsion gives a complete representation of the physical world including space and time”.

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