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Elastic buckling and plastic collapse during 3D concrete printing

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A B S T R A C T

This contribution studies failure by elastic buckling and plastic collapse during 3D concrete printing of wall structures. Four types of experiments were performed, which demonstrate the circumstances under which elastic buckling and plastic collapse occur, the effect of geometrical imperfections on the buckling response, the influence by the curing rate of the concrete material on the buckling stability, and the conditions leading to the successful printing of a complex, practical structure (a picnic table). The experimental results are compared to those computed by the parametric 3D printing model recently developed by Suiker (Int. J. Mech Sci, 137: 145–170, 2018), showing a very good agreement. The design formulas and design graphs deduced from the parametric model serve as a useful tool for accurately designing wall structures against failure during 3D concrete printing. Furthermore, they may be applied to optimize the process conditions during 3D printing, by providing the maximal printing velocity, the optimal geometrical characteristics, or the minimal amount of material required for accurately designing the structure.

1. Introduction

The application of extrusion-based 3D concrete printing eliminates the necessity for conventional casting, by accurately placing specific volumes of material in successive layers through a computer-controlled positioning procedure. Over the past decade, printing technologies based on extrusion have seen an enormous growth in popularity, which can be ascribed to the relatively low costs involved, the potential in producing complex geometrical shapes, the ease in operation, the suitability for production in small volumes, a high dimensional accuracy, and the straightforward integration of computer-aided design (CAD) software [1–3]. These advantages recently were utilized in successful applications in architectural and civil engineering, such as 3D concrete printed houses, bridges, architectural forms and building components [4–8].

Despite the success of 3D concrete printing, detailed knowledge about the influence of specific manufacturing parameters and conditions on the mechanical behavior of the object during the printing process is limited. This can be largely attributed to the complexity and diversity of the process parameters, which may result in unforeseen failures by a lack of mechanical performance or dimensional accuracy [9–12]. In fact, the low stiffness and strength of the fresh printing material may cause the objects’ resistance against structural failure during the printing process to be more critical than during the final application phase. To date, appropriate material and process parameters are commonly determined by trial and error, whereby it remains unclear whether an optimal set of parameters has been achieved for the object under consideration. Indeed, minimizing the amount of concrete material required for preserving the strength and stability of the object during printing, or maximizing the printing velocity, will further reduce the printing time and manufacturing costs.

In order to improve the above aspects, it is necessary to utilize physics-based modeling tools that are able to accurately predict the influence of individual process parameters on the failure behavior of the object during printing. Suiker [9] recently developed a mechanistic parametric model that can be applied for the prediction of failure of straight wall structures during extrusion-based 3D printing processes. Due to its fundamental character and its large potential for a range of 3D printing applications, the model has received ample media attention, see e.g., [13]. The parametric model is generic (i.e., independent of the printing material applied), and distinguishes between failure by elastic buckling (a stability mechanism) and plastic collapse (a strength mechanism), see Fig. 1. These two failure mechanism indeed have been recognized as highly relevant in the field of 3D concrete printing, see also [12,14–17]. The parametric model describes the structural failure behavior as a function of the main printing process parameters, which are the curing characteristics of the printing material, the printing speed, the geometrical characteristics of the printed object, its dead weight, its heterogeneous strength and stiffness characteristics, and the presence of geometrical imperfections. The various material and
process parameters governing the 3D printing process have been reduced to 5 unique, dimensionless failure parameters, namely 3 parameters defining elastic buckling and 2 parameters characterizing plastic collapse. The results computed for representative wall structures have been converted to practical design formulas and design graphs, see [9] for more details. Subsequently, in [11] it has been demonstrated that these modeling results are in excellent agreement with those obtained from advanced 3D finite element method simulations, and therefore may serve as a useful, practical tool for the prediction and analysis of structural failure during extrusion-based 3D printing processes.

In the present work, the practical applicability of the parametric model of Suiker [9] is demonstrated for various wall structures manufactured by 3D concrete printing. Through a detailed comparison with the results obtained from dedicated experiments, it is shown (i) how the design formulas and design graphs deduced in [9] can be applied to accurately predict and analyze experimental failure by elastic buckling and plastic collapse, and (ii) how they can be utilized for the optimization of 3D concrete printing processes. For this purpose, 4 types of printing experiments were performed, which refer to the design of a practical, complex wall structure - a picnic table - against failure during 3D concrete printing. For the analysis of elastic buckling, consider the three basic configurations of a printed wall of length $l$, width $b$ and thickness $h$ illustrated in Fig. 2, which have been taken from [9]. The three walls are fully-clamped at the bottom (the displacement in $x$-direction is prevented), and differ by the boundary conditions in the $y$-direction, i.e., the boundary conditions along the vertical boundaries. The wall configurations are designated as (i) a free wall (the vertical boundaries are kinematically unconstrained), (ii) a simply-supported wall (the displacement in $z$-direction is prevented and the rotation about the $x$-direction is free along the vertical boundaries), and (iii) a fully-clamped wall (both the displacement in $z$-direction and the rotation about the $x$-direction are prevented along the vertical boundaries). In the present communication the buckling responses derived in [9] for these three wall configurations will be used to predict structural failure responses observed during 3D concrete printing. The wall structures are printed by adding material in a layer-wise fashion, whereby it is assumed that during the printing of an individual layer the strength and stiffness properties in that layer do not substantially change. In other words, the characteristic time defining the curing

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**Fig. 1.** Wall failure by elastic buckling (left) and plastic collapse (right).

**Fig. 2.** Three basic wall configurations: a free wall (left), a simply-supported wall (middle), and a fully-clamped wall (right). The figure has been reprinted from [9].
The elastic buckling behavior of the wall is described by the out-of-plane deflection \( w_c \) evaluated along the vertical direction of the wall, at the wall symmetry line \( y = b/2 \) (i.e., at the half width of the wall, see Fig. 2). The vertical coordinate \( X \) along which \( w_c \) is evaluated refers to an Eulerian coordinate system that is connected to the printing nozzle, as illustrated in Fig. 3. Note from this figure that the Eulerian coordinate \( X \) is related to the Lagrangian coordinate \( x \) (with its origin at the bottom of the wall) and the time \( t \) via \( X = \tilde{X}(x,t) = x - l + it \), whereby the wall growth velocity \( l \) represents the time rate of change of the wall length during the printing process. The wall growth velocity can be obtained from printing process parameters as [9]

\[
l = \frac{Q}{v_n h_l} = \frac{\dot{l}}{\tilde{r}_l},
\]

with \( Q = v_n h_l \) the material volume discharged from the printing nozzle per unit time, \( v_n \) the horizontal speed of the printing nozzle, \( h_l \) the time required for printing an individual layer, and \( \dot{l} \) the thickness and height of the printed layer, respectively. For the description of elastic buckling, it is convenient to rewrite the Eulerian coordinate \( X \) and the out-of-plane deflection of the wall \( w_c \) in a dimensionless form, i.e., \( \tilde{X} = \chi g(t) \) and \( \tilde{w}_c = \omega^o(t) \). Note that the use of the Eulerian coordinate \( X \) allows for describing the time and spatial dependency of the wall buckling response by means of a single, dimensionless parameter. Accordingly, the dimensionless total out-of-plane deflection \( \Psi = \tilde{\Psi}(\tilde{X}) \) at the symmetry line \( y = b/2 \) of the straight wall can be decomposed as

\[
\tilde{\Psi}(\tilde{X}) = \tilde{\Psi}^{\omega^o}(\tilde{X}) + \tilde{\Psi}^{\omega^e}(\tilde{X}),
\]

where \( \tilde{\Psi}^{\omega^e} \) is the deflection generated under the applied dead weight loading \( \omega^e \) and \( \tilde{\Psi}^{\omega^o} = \omega^o(t) / h \) characterizes possible geometrical imperfections generated during the printing process, with \( \omega^o(t) = \omega^o(X(t)) \) representing the actual imperfection profile. Correspondingly, the equilibrium condition for elastic wall buckling can be expressed in terms of the displacements \( \tilde{\Psi}^{\omega^e} \) and \( \tilde{\Psi}^{\omega^o} \) by means of the following fourth-order differential equation [9]:

\[
\left( \tilde{w}_c \tilde{\Psi}^{\omega^e}(\tilde{X}) \right)_{\tilde{X}} - \left( \tilde{w}_c \tilde{\Psi}^{\omega^o}(\tilde{X}) \right)_{\tilde{X}} = \left( \tilde{w}_c \tilde{\Psi}^{\omega^o}(\tilde{X}) \right)_{\tilde{X}} - \tilde{w}_c \tilde{\Psi}^{\omega^o}(\tilde{X}),
\]

When imperfections vanish, \( \tilde{\Psi}^{\omega^o} = 0 \), the right-hand side of Eq. (4) becomes zero; this homogeneous form of the differential equation can be applied to solve for the critical bifurcation buckling length of a specific wall configuration. Furthermore, \( \tilde{w}_c = \tilde{w}_c(X) \) is the dimensionless curing function and \( \tilde{w}_c = \tilde{w}_c(X) \), \( \tilde{w}_c = \tilde{w}_c(X) \) and \( \tilde{w}_c = \tilde{w}_c(X) \) are dimensionless functions characterizing the geometrical characteristics of the wall structure and the characteristics of the printing material and printing process, see [9] for their specific forms. Together with the appropriate boundary conditions of the specific wall configuration under consideration, Eq. (4) is solved using a combined analytical-numerical procedure, the details of which can be found in [9]. The buckling response can be analyzed in a unique fashion using the following 3 dimensionless parameters [9]:

\[
\begin{align*}
I_r &= \left( \frac{\rho g h}{D_0} \right)^{\frac{1}{3}} I_w, \\
F &= \left( \frac{\rho g h}{D_0} \right)^{\frac{1}{2}} b, \\
\tilde{w}_c &= \left( \frac{D_0}{\rho g h} \right)^{\frac{1}{2}} \tilde{w}_c, \\
\end{align*}
\]

where \( \rho \) is the density of the printing material, \( g = 9.81 \text{ m/s}^2 \) is the gravitational acceleration, and \( D_0 \) is the initial wall bending stiffness, given by

\[
D_0 = \frac{E_0 h^3}{12(1 - \nu^2)},
\]

with \( \nu \) the Poisson’s ratio of the printing material (which is taken as constant during the printing process). In addition, the parameters \( I_r \) and \( b \) appearing in Eqs. (5)1,2 represent the actual critical buckling length and actual width of the wall, respectively. Note that the dimensionless curing rate \( \tilde{w}_c \) presented in Eq. (5)3 is proportional to the actual curing rate \( w^c(x) \) of the printing material, and inversely proportional to the wall growth velocity \( l \). Indeed, decelerating (accelerating) the printing process has the same effect on the stability (and strength) of a printed wall structure as increasing (decreasing) the curing rate of the printing material. In Suiker [9] the solution of the buckling equation, Eq. (4), has been computed for the three wall configurations depicted in Fig. 2, and for other, more advanced (rectangular) wall layouts, thereby
considering different types of curing processes (i.e., linear, exponentially-decaying). In Wolfs and Suiker [11] these solutions were subsequently compared to numerical results obtained from advanced 3D finite element method (FEM) simulations, showing an excellent agreement.

For a printing material that hardens in accordance with a linear curing process, the time-dependent stiffness evolution given by

$$E(t) = E_0(1 + \frac{t}{t_0})$$

is characterized by a linear curing function, i.e.,

$$E(t) = E_0(1 + \frac{t}{t_0})$$

(8)

with $t_0$ representing the linear curing rate. With this curing law, the critical dimensionless buckling length $l_{cr}$ for the free wall without imperfections sketched in Fig. 2 can be determined by solving the homogeneous part of the differential equation given by Eq. (4); as demonstrated in [9], this solution can be accurately approximated by the following closed-form expression:

$$l_{cr} = \frac{E(0)}{E_{cr}} = \frac{l_{cr,0}}{1 + \frac{t}{t_0}}$$

(9)

where $l_{cr}$ and $E_{cr}$ are given by Eqs. (5) and the lower limit $l_{cr,0} = 1.98635$ represents the rate-independent buckling length that corresponds to an infinitely slow curing process (or, equivalently, an infinitely fast wall growth). Eq. (9) illustrates that the critical buckling length of the free wall becomes larger under a higher dimensionless curing rate $E_{cr}$; in fact, a higher curing rate $E_{cr}$ (or a lower wall growth velocity $l$) enables the wall to better develop its bending stiffness during the printing process, which increases its buckling resistance.

Note from Eq. (9) that the buckling length of the free wall is independent of the dimensionless wall width $b$ presented in Eq. (5), which is due to the fact that the vertical sides of the free wall are kinematically unconstrained, thereby causing the buckling response to be uniform along the horizontal ($y$-)direction. Conversely, for the simply-supported wall and fully-clamped wall depicted in Fig. 2 the vertical sides are kinematically constrained, as a result of which the dimensionless buckling length $l_{cr}$ becomes dependent on both the dimensionless curing rate $E_{cr}$ and the dimensionless wall width $b$, i.e., $l_{cr} = \frac{l_{cr,0}}{1 + \frac{t}{t_0}}$. The solution of the buckling equation, Eq. (4), for these two configurations can be conveniently summarized in design graphs, in which the dimensionless buckling length $l_{cr}$ is plotted as a function of the dimensionless curing rate $E_{cr}$ for a broad selection of dimensionless wall widths $b$, see Figs. 4 (simply-supported wall) and 5 (fully-clamped wall). It can be seen in these figures that both for the simply-supported wall and fully-clamped wall the critical buckling length becomes larger at a smaller wall width $b$. This is, since the constraining effect induced by the boundary conditions along the vertical wall boundaries becomes stronger for a lower wall width, thereby increasing the buckling resistance. In addition, a fully clamped wall generally shows to have a larger buckling resistance than a simply-supported wall, which results from the relatively high constraint imposed by the clamped boundary conditions along the vertical wall boundaries.

Both for the simply-supported wall and the fully-clamped wall at very large wall width, $b \to \infty$, the constraining effect from the vertical wall boundaries on the buckling response vanishes, as a result of which the buckling curves asymptote towards the buckling curve for the free wall, Eq. (9), indicated in Figs. 4 and 5 by the dashed line. Observe further that at the domain bounds $l_{cr} = 10$ and $l_{cr} = 8$, chosen for the plotting of Figs. 4 and 5, the buckling curves for the shorter walls approach a vertical asymptote, implying that the buckling resistance at the corresponding curing rates $E_{cr}$ becomes infinitely large. This trend results from the fact that in the linear curing law provided by Eqs. (7) and (8), the elastic stiffness under continuous curing can grow unboundedly. When the stiffness development under curing has a limit (as, for example, is the case for an exponentially-decaying curing law), the buckling curves in

Fig. 4. Critical buckling length $l_{cr}$ versus curing rate $E_{cr}$ for a simply-supported wall printed under a linear curing process, see Eq. (8). The wall widths considered range from $b = 3$ to 20. The buckling curve for a free wall (which corresponds to the limit case $b \to \infty$), has been computed using Eq. (9) and is plotted for comparison (dashed line). Values for the rate-independent buckling length $l_{cr,0}$ at $E_{cr} = 0$ are plotted for a selection of wall widths $b$, and are presented as a reference. The figure has been reprinted from [9].

Fig. 5. Critical buckling length $l_{cr}$ versus curing rate $E_{cr}$ for a fully-clamped wall printed under a linear curing process, see Eq. (8). The wall widths considered range from $b = 5$ to 30. The buckling curve for a free wall (which corresponds to the limit case $b \to \infty$), has been computed using Eq. (9) and is plotted for comparison (dashed line). Values for the rate-independent buckling length $l_{cr,0}$ at $E_{cr} = 0$ are plotted for a selection of wall widths $b$, and are presented as a reference. The figure has been reprinted from [9].

Figs. 4 and 5 indeed become bounded, see [9] for more details. Nonetheless, since in a 3D printing process the stiffness (and strength) of concrete under early curing typically increases linearly [9–11], Eq. (9) and Figs. 4 and 5 can be applied for an adequate prediction of the experimental buckling responses presented in Section 3.

2.2. Plastic collapse

Instead of failing by elastic buckling, a wall structure during 3D concrete printing may fail by plastic collapse, see Fig. 1, whereby under the increasing dead weight loading the maximal stress at the wall bottom at a certain moment reaches the yield strength of the printing material. This behavior can be modeled by formulating the time
evolution of the material yield strength under curing as
\[
\tilde{\sigma}_y(t) = \tilde{h}_K(t) \sigma_{y,0},
\]  
(10)
where \(\sigma_{y,0}\) is the initial yield strength of the fresh printing material and \(\tilde{h}_K(t)\) is the curing function. When considering a linear curing process, the curing function has the form
\[
\tilde{h}_K(t) = 1 + \xi_l t, 
\]  
(11)
in which \(\xi_l\) represents the linear curing rate of the yield strength. Note that this curing rate generally may differ from the linear curing rate \(\xi_K\) of the elastic stiffness, presented in Eq. (8). The critical length at which the wall structure under linear curing experiences plastic collapse can be derived from the following yield criterion \([9]\)
\[
l_p = \left( \frac{\sigma_{y,0}}{\sigma_{y,z}} \right) \frac{1}{1 - \xi_l} \quad \text{with} \quad 0 \leq \xi_l < 1, 
\]  
(12)
with the dimensionless collapse length \(l_p\) and the dimensionless curing rate, \(\xi_l\), defined by
\[
\tilde{l}_p = l_p / \xi_l, \quad \xi_l = \frac{\xi_{K_e}}{\xi_{K_y}}.
\]  
(13)
As for the dimensionless curing rate of the elastic stiffness, Eq. (5)b, the dimensionless curing rate of the yield strength, \(\tilde{l}_p\), is proportional to the actual curing rate of the yield strength, \(l_p\), and inversely proportional to the wall growth velocity \(\bar{l}\) presented by Eq. (2). Note further from Eqs. (12) and (13) that the upper bound \(\xi_l = \xi_{K_y} = 1\) corresponds to the curing rate at which the growth of the yield stress, \(\sigma_{y,0} \xi_{K_y}\), becomes equal to the growth of the dead weight stress, ggl. Consequently, for curing rates above this upper bound the stress at the bottom of the wall is not able to reach the yield strength, so that plastic collapse can not occur and the actual plastic collapse length becomes infinitely large, \(l_p \to \infty\).

As pointed out in \([9]\), the identification of the yield strength \(\tilde{\sigma}_y\) from material experiments depends on the type of failure criterion adopted for the specific printing material. When assuming that plastic collapse occurs by means of compressive failure in accordance with the maximal stress theory, the yield strength is represented by the uniaxial compressive strength \(\sigma_c\),
\[
\tilde{\sigma}_y = \sigma_c. 
\]  
(14)
Conversely, under the assumption of pressure-dependent shear failure described by the Mohr-Coulomb theory, the yield strength is given by\(^1\)
\[
\tilde{\sigma}_y = \frac{2c \cos(\phi)}{1 - K - (1 + K) \sin(\phi)} \quad \text{with} \quad K = \min[K_y, K_z], 
\]  
(15)
in which \(\phi\) is the friction angle and \(c\) is the cohesion of the fresh printing material, and \(K\) is the coefficient of lateral stress at the wall bottom, which quantifies the proportion of the normal stresses in the two directions perpendicular to the vertical wall direction, i.e., the \(y\)- and \(z\)-directions designated in Fig. 2. As indicated in Eq. (15), the lowest of the two lateral normal stresses corresponds to the lowest, most critical value for the effective yield strength \(\tilde{\sigma}_y\), and therefore is decisive. For a printing material that lacks frictional resistance (\(\phi = 0\)), Eq. (15) reduces to
\[
\tilde{\sigma}_y = \frac{2c}{1 - K} \quad \text{with} \quad K = \min[K_y, K_z], 
\]  
(16)
which represents Tresca’s yield criterion that is based on reaching the maximal shear stress at the wall bottom.

When the contact between the wall bottom and the support structure is fully sticking, the displacements at the wall bottom are constrained in all three directions. For an elasto-plastic printing material characterized by a significant elastic response prior to plastic yielding, the lateral normal stresses at the wall bottom under this condition are equal to a factor \(\nu/(1 - \nu)\) times the vertical dead weight stress. Using a representative value for the Poisson’s ratio of fresh concrete of \(\nu = 0.3\) \([9,10]\), this sets the coefficient of lateral stress at the wall bottom as \(K = K_y = K_z = 0.3/0.7 = 0.43\). Conversely, if the contact between the bottom of the wall and the support structure is ideally smooth, the lateral normal stresses at the wall bottom will be equal to zero, in correspondence with \(K = K_y = K_z = 0\). Observations on the 3D printed concrete structures investigated in this study indicate that the contact conditions at the bottom layer typically are close to fully sticking, see Section 3.1 for more details. This is also the reason that a printed wall structure at its bottom may be modeled as fully clamped, see Fig. 2. Nonetheless, in the thickness direction of the wall (i.e., the \(z\)-direction in Fig. 2) the normal stress induced by this local constraint quickly reduces to zero with increasing vertical wall length (which in the field of applied mechanics is referred to as a “Saint-Venant’s effect”). This reduction is caused by the stress-free boundary conditions present at the vertical wall surfaces and the limited wall thickness. Detailed finite element simulations presented in \([11]\) indeed indicate that for representative 3D printed wall geometries the average normal stress in thickness direction is already close to zero at the second layer from the wall bottom. In other words, only the first layer fully experiences the kinematic constraint generated by the sticking contact conditions. Since this constraint lowers the effective shear stress required for catastrophic plastic collapse, the first layer develops limited plastic deformations; instead, the yield strength defining the critical moment of plastic collapse is reached first in the second layer from the wall bottom, see Section 3.1 for the experimental validation of this mechanism. In order to account for this effect, the yield stress in Eqs. (15) and (16) should be calculated with a coefficient of lateral stress equal to zero, \(K = K_y = 0\), whereby the plastic collapse length following from Eqs. (12) and (13) needs to be corrected by adding the height of the constrained, fully sticking bottom layer. Although this usually leads to a minor correction of the plastic collapse length, which may be ignored during the design phase of a 3D concrete printing process, it makes the prediction by the parametric model more accurate.

In contrast to an elasto-plastic printing material, for a viscous, fluidic printing material the elastic response prior to plastic yielding is minor. Consequently, during a 3D printing process the structure printed by such a material may only fail by plastic collapse, and not by elastic buckling. The yield strength \(\tilde{\sigma}_y\) for a viscous printing material is generally lower than for an elasto-plastic printing material, and the adhesion of the bottom layer to the support structure is smaller. Correspondingly, during the printing process the bottom layer experiences a low constraint against lateral deformation, and therefore will likely act as the most critical layer for reaching plastic collapse. Hence, for a viscous printing material the coefficient of lateral stress should be taken as zero, \(K = 0\), but the plastic collapse length following from Eqs. (12) and (13) does not need to be corrected for the (minor) lateral constraint of the bottom layer. Obviously, this only needs to be done if the concrete composition contains an accelerator that quickly transforms the viscous material into an elasto-plastic material during curing.

It further needs to be mentioned that the friction angle \(\phi\) of the fresh concrete used in the present study, as measured between 0 and 60 min. after printing, typically is very low, in the order of \(1^\circ\) to \(6^\circ\), see \([18]\) for more details. Accordingly, the difference between the yield strengths following from the Mohr-Coulomb criterion, Eq. (15), and the Tresca

\(^1\) Note that the third term in the denominator of Eq. (15) differs by a factor of \(-1\) from the corresponding term in the original expression, Eq. (79), presented in \([9]\). This is, because in the derivation of Eq. (79) the direction corresponding to the largest absolute value of the principal stress, which is the vertical wall direction, has been erroneously set equal to the horizontal width direction of the wall.
criterion, Eq. (16), is minor; hence, the relatively simple Tresca criterion given by Eq. (16) can be employed to validate the 3D printing experiments presented in Section 3. Note that with \( K = 0 \), Eq. (16) reduces to Eq. (14), considering that in a uniaxial compression test the critical normal stress (= uniaxial compressive strength) is equal to two times the critical shear stress (= cohesion), i.e., \( \sigma_c = 2\tau_c \).

Although in the above procedure the printing material is assumed to be characterized by a linear curing process, for alternative curing processes, e.g., an exponentially-decaying curing process, a yield function similar to Eq. (12) can be established, see \( [9] \) for more details. It should also be mentioned that Eq. (12) applies to printed structures of arbitrary geometry, since plastic collapse is considered to be a local, geometry-independent mechanism that takes place near the bottom of the printed structure.

2.3. Competition between elastic buckling and plastic collapse

Whether failure in a 3D printing process is governed by elastic buckling or plastic collapse is dependent on the minimal wall length at failure. Specifically, when the critical buckling length lies below the length required for plastic collapse, \( l_p < l_c \), elastic buckling is governing, while for \( l_p > l_c \) failure is driven by plastic collapse. Combining this criterion with Eqs. (5), and (13), allows to conveniently determine the operative failure mechanism as follows \( [9] \):

\[
\frac{l_c}{l_p} < \Xi: \quad \text{elastic buckling}, \quad \frac{l_c}{l_p} > \Xi: \quad \text{plastic collapse},
\]

with \( \Xi = \left( \frac{h}{h_0} \frac{\sigma_0}{(gR)^2} \right) \) (17).

Here, the dimensionless critical buckling length \( l_c \) for the free wall follows from Eq. (9), while the simply-supported wall and fully-clamped wall can be read off from the design graphs given by Figs. 4 and 5, respectively. Further, the dimensionless critical length for plastic collapse \( l_c \) is obtained from Eq. (12). The dimensionless parameter \( \Xi \) is the failure mechanism indicator, which, as Eq. (17) shows, is characterized by geometry and material characteristics and defines the failure mechanism transition point from elastic buckling to plastic collapse.

The 15 parameters characterizing a 3D printing process of straight wall structures have been conveniently summarized in Table 1, together with a reference to the model equation(s) in which they appear. In the parametric 3D printing model these parameters have been combined and reduced to only 5 dimensionless parameters, which are the 3 parameters in Eq. (5) that uniquely define elastic buckling, and the 2 parameters in Eq. (13) that characterize plastic collapse. Using these parameters, in Section 3 the model equations and graphs reviewed in this section will be applied to predict and analyze the failure responses observed in various 3D concrete printing experiments.

3. Experimental versus modeling results

The practical use of the parametric 3D printing model is demonstrated by means of a series of 4 different 3D concrete printing experiments, which refer to (i) elastic buckling and plastic collapse of a square wall layout, (ii) elastic buckling of a free wall with and without imperfections, (iii) elastic buckling of a free wall printed at different curing rates - or, equivalently, different wall growth velocities - and (iv) the design of a practical, complex structure - a picnic table - against failure during 3D concrete printing. The experiments were performed using the 3D concrete printing facility at the Eindhoven University of Technology, see Fig. 6, which consists of a gantry robot, a control unit, and a concrete mixer and pump (inset). The custom-designed concrete (Weber 3D 145–2) employed in the 3D printing experiments is composed of Portland cement (CEM I 52.5 R), a siliceous aggregate with a maximum particle size of 1 mm, rheology modifiers, limestone filler, additives, and a small amount of polypropylene fibers. The relatively small particle size of the aggregates creates a dense particle structure, which, together with the addition of a limestone filler, ensures that the porosity of the mixture remains small and its detrimental effect on the elastic stiffness (and strength) of the concrete is minor \( [19] \). Further, due to microstructural strength and stiffening effects, the addition of fibers to the concrete mixture in principle has a positive effect on the effective strength and stiffness properties of the concrete, although this influence may be insignificant for low fiber percentages \( [20] \). The composition was mixed with water into a homogeneous substance, after which it was pumped via a hose towards the printer head, where it was released from the printing nozzle to create a layer. The gantry robot controlling the calculated path of the printer head has four degrees of freedom, i.e., 3 mutually perpendicular translations and 1 rotation about the vertical axis. The setting of the appropriate process parameters, such as the printing velocity, the concrete viscosity, the height of the printer head above the printed layer, the printing rotation angle, and the properties of the nozzle opening, was accomplished through an extensive preliminary test program \( [4] \). An overview of the printing process parameters is given in Table 2.

At the start of the experimental program, the strength and stiffness characteristics of the concrete material were measured by means of uniaxial compression tests on cylindrical samples with a diameter and height of 70 mm and 140 mm, respectively, in agreement with the ASTM D2166 \( [21] \). The cylindrical samples were prepared at four different curing times, equal to 5, 15, 30 and 60 min., and were loaded in a displacement-controlled way in an Instron test rig. The loading rate imposed equals 42 mm/min., which corresponds to 30% linear strain per min. It is noted that the above range of curing times covers the printing times applied in most of the experiments. Following the procedure described in \( [18] \), the elastic stiffness \( E_c \) and the yield strength \( \sigma_y \) were deduced for the selected curing rates, with the results plotted in Figs. 7a and b, respectively. As explained in Section 2.2, the yield strength \( \sigma_y \), obtained from the uniaxial compression tests corresponds to the strength parameter \( \sigma_y \), in Eq. (14) and to the strength parameter \( \sigma_c \), in Eq. (16) (adopting \( K = 0 \)). The relation between the elastic stiffness (in kPa) and curing time (in min.) is obtained by applying a least-squares approach to the experimental data. Accepting a linear best fit with \( R^2 = 0.957 \) leads to the following relation:

\[
F(t) = 48.564 + 2.609t \quad \text{with} \quad E_c \text{in kPa and } t \text{ in min.} \quad (18)
\]

which has been indicated in Fig. 7a by the dashed line. A similar
procedure was followed for the determination of the yield strength (in kPa) as a function of the curing time (in min.), see Fig. 7b. Adopting a linear best fit with $R^2=0.999$ on the measured yield strength values results in

$$\sigma_p(t) = 2.835 + 0.329t \quad \text{with } \sigma_p \text{ in kPa and } t \text{ in min.} \quad (19)$$

The density $\rho$ provided in Table 2 appeared to be (virtually) insensitive to the curing time.

From Eq. (18), the initial stiffness characterizing the linear curing law in Eqs. (7) and (8) becomes $E_0 = 48.564$ kPa, while the curing rate for the elastic stiffness equals $\xi^E_0 = 2.609/48.564 = 0.0537 \text{ min}^{-1} = 8.954 \times 10^{-4} \text{ s}^{-1}$. Similarly, Eq. (19) provides the initial yield strength of the linear curing law in Eqs. (10) and (11) as $\sigma_{p,0} = 2.835$ kPa, with the corresponding curing rate equal to $\xi^\sigma_0 = 0.329/2.835 = 0.116 \text{ min}^{-1} = 1.934 \times 10^{-3} \text{ s}^{-1}$. Finally, the Poisson’s ratio of the fresh concrete was measured as $\nu = 0.3$. The above material parameters are conveniently summarized in Table 3.

### 3.1. Elastic buckling and plastic collapse of a square wall layout

The experimental program started by printing two square wall layouts of different wall width, i.e., a relatively small square wall layout with a wall width $b = 250$ mm and large square wall layout with $b = 500$ mm, see Fig. 8. In order to warrant a constant printing velocity during the printing process, the corners of the wall layouts were slightly rounded off, in accordance with a radius $r = 50$ mm.

The prediction of the operational failure mechanism can be done by using the model equations and design graphs outlined in Section 2. The procedure that needs to be followed has been summarized in Table 4. Starting at the top of the table, the model parameters listed columnwise are calculated consecutively, with a reference given to the corresponding design formula or design graph used, which eventually results in the operational failure mechanism indicated in the final row of the table. Specifically, from the wall width $b$ and the printing speed $v_n$, the period $T_l$ related to the printing an individual layer can be computed. This parameter, together with the printing process data presented in Table 2, is used to determine the wall growth velocity $l$ via Eq. (2). Subsequently, the initial wall bending stiffness $D_0$ given by Eq. (6) is

![Fig. 6. 3D concrete printing facility at the Eindhoven University of Technology, including a 4-axis gantry robot, a control unit, and a concrete mixer and pump (inset). The figure has been reprinted from [10].](image)
Table 3
Material parameters measured for the fresh concrete used in the 3D printing experiments. The material parameters serve as input for the linear curing laws for the elastic stiffness, Eqs. (7) and (8), and the yield strength, Eqs. (10) and (11).

| Parameter                        | Value                                      |
|----------------------------------|--------------------------------------------|
| Initial elastic stiffness        | $E_0 = 48.564$ [kPa]                        |
| Curing rate elastic stiffness    | $\varepsilon = 8.954 \times 10^{-4}$ [s$^{-1}$] |
| Poisson’s ratio                  | $\nu = 0.3$ [-]                            |
| Initial yield strength           | $\sigma_p = 2.635$ [kPa]                   |
| Curing rate yield strength       | $\dot{\sigma}_p = 1.934 \times 10^{-3}$ [s$^{-1}$] |

determined based on the printing process data and material data listed in Tables 2 and 3. This parameter serves as input for the dimensionless curing rate for the stiffness, $\dot{E}_l$, and the dimensionless wall width $b$ by Eqs. (5)_3 and (5)_2. As argued in [9], for the determination of the buckling response of a square wall layout, the individual walls may be considered as simply-supported, in correspondence with free bending rotations at the vertical boundaries, see Fig. 2. In fact, due to the symmetry of the wall layout, all four walls buckle simultaneously, a mechanism that can be referred to as global buckling. None of the walls can therefore provide rotational resistance to one of the two adjacent walls to which it is connected. Using the values for the dimensionless curing rate $\dot{E}_l$ and the dimensionless wall width $b$ calculated for each of the two square wall layouts, the corresponding dimensionless buckling length $l_c$, thus may be read off from the design graph presented in Fig. 4 for the simply-supported wall. This is done by interpolating between curves depicted for the selected $b$ values.

Regarding the mechanism of plastic collapse, the dimensionless curing rate for the yield strength, $\dot{E}_{pl}$, is computed from Eq. (13)$_b$, using the wall growth velocity $v$ calculated above and the printing process data and material data listed in Tables 2 and 3. The dimensionless curing rate $\dot{E}_{pl}$ subsequently specifies the dimensionless plastic collapse length $l_c$ via Eq. (12). In order to determine whether elastic buckling or plastic collapse occurs, the failure mechanism indicator $\lambda$ presented in Eq. (17) is computed, and compared against the ratio $l_c/l_p$ between the dimensionless critical lengths for elastic buckling and plastic collapse. For the small square wall layout $l_c/l_p > \lambda$, which, as indicated by Eq. (17), corresponds to failure by plastic collapse. Conversely, for the large square wall layout $l_c/l_p < \lambda$, implying failure by elastic buckling.

Fig. 9 shows that the model predictions of the operational failure mechanism - presented in the bottom row of Table 4 - indeed are confirmed by the 3D concrete printing experiments. The experimental failure response depicted in Fig. 9(a) illustrates that the small square wall layout with $b = 250$ mm fails by plastic collapse, as characterized by vertical deformations induced by a significant plastic broadening of the bottom layers. In contrast, Fig. 9(b) shows that the large square wall layout with $b = 500$ mm fails by elastic buckling, characterized by progressive, lateral deformations of one of the four walls of the square wall layout. Although from the aspect of symmetry the four walls theoretically should collapse simultaneously, the presence of small inhomogeneities in the actual geometry and concrete material will break this symmetry, such that buckling is initiated by the collapse of an individual wall.

In correspondence with the discussion in Section 2.2, for the small square wall layout the largest plastic deformations in the wall thickness direction indeed are observed to take place in the second layer from the wall bottom, see Fig. 10. The first layer is kinematically constrained from the sticking contact with the support structure below, such that the effective stress required for catastrophic plastic collapse remains relatively small, thereby limiting the plastic deformation of this layer. Due to a substantial plastic broadening of the second layer, its final thickness after plastic collapse is observed to be more than two times the thickness of the first, constrained layer.

As a next step, the model predictions for the number of printed layers leading to failure will be compared to the corresponding experimental values. For the small square wall layout with $b = 250$ mm, this is done by inserting the value for $l_p$ presented in Table 4 into Eq. (13)$_b$, which, together with the parameter values for $\sigma_p, \rho$ and $g$, allows to compute the actual plastic collapse length $l_p$, at which the structure fails. The plastic collapse length is divided by the layer height $t_l$ presented in Table 2, and then rounded up to obtain an integer value for the number of layers at failure. In correspondence with the argumentation in Section 2.2, the number of layers is subsequently raised by 1 in order to account for the presence of a constrained bottom layer with limited plastic deformation, leading to the value of $n_p = 21$ layers presented in Table 5. This prediction is in good agreement with the experimental value of $n_p = 23$ layers following from the 3D printing experiments. The reason that the model provides a slightly smaller number of layers for plastic collapse than the experiments can be ascribed to the gradual development of plastic deformations in the layers before the moment of catastrophic collapse. In fact, the actual average layer height, as measured close to the moment of plastic collapse from the ratio between the actual wall height and the number of printed layers, turned out to be 9.0 mm, which is 6.5% lower than the initial layer height of $t_l = 9.5$ mm listed in Table 2. Indeed, using a layer height of 9.0 mm increases the model prediction of the number of layers at plastic collapse to $n_p = 23$, which is in excellent correspondence with the experimental result.

For the large square wall layout with $b = 500$ mm, the number of
Table 4
Model prediction of the operational failure mechanism during 3D printing of a small ($b = 250$ mm) and large ($b = 500$ mm) square wall layout. Starting at the top of the table, the parameters listed columnwise are calculated consecutively, with a reference given to the corresponding equation or design graph used. The printing process data and material data required for the computation of specific parameters are listed in Tables 2 and 3. As indicated in the final row of the table, the small and large square wall layouts are predicted to fail by plastic collapse and elastic buckling, respectively.

| Parameter                                      | Small square wall layout | Large square wall layout |
|------------------------------------------------|--------------------------|--------------------------|
| Wall width                                      | $b = 250$ [mm]           | $b = 500$ [mm]           |
| Period of printing an individual layer          | $\tau_l = 48/\rho y = 9.6$ [s] | $19.2$ [s]               |
| Wall growth velocity, Eq. (2)                   | $l = 0.990$ [mm/s]       | $0.495$ [mm/s]           |
| Initial wall bending stiffness, Eq. (6)          | $D_0 = 0.740$ [Nm]       | $0.740$ [Nm]             |
| Dimensionless curing rate for stiffness, Eq. (5)$_a$ | $\xi_p^l = 0.08$ [-]    | $0.16$ [-]               |
| Dimensionless wall width, Eq. (5)$_a$           | $\xi_p^W = 5.76$ [-]    | $1.37$ [-]               |
| Dimensionless buckling length, Fig. 4           | $l_p = 2.88$ [-]         | $2.85$ [-]               |
| Dimensionless curing rate for strength, Eq. (13)$_a$ | $\xi_p^s = 0.54$ [-]    | $0.27$ [-]               |
| Dimensionless plastic collapse length, Eq. (12) | $l_p = 1.37$ [-]         | $2.16$ [-]               |
| Failure mechanism indicator, Eq. (17)           | $\xi / l_p = 1.59$ [-]  | $1.59$ [-]               |
| Ratio of failure lengths                        | $l_p / l_p = 6.29$ [-]  | $1.32$ [-]               |
| Operational failure mechanism, Eq. (17)         | Plastic collapse ($l_p / l_p > \xi$) | Elastic buckling ($l_p / l_p < \xi$) |

![Image](image1.png)
a) Plastic collapse of the small square wall layout ($b = 250$ mm).

![Image](image2.png)
b) Elastic buckling of the large square wall layout ($b = 500$ mm).

Fig. 9. Experimental failure responses of the small and large square wall layouts.

Fig. 10. Thickness of the first and second layer of the small square wall layout ($b = 250$ mm) after plastic collapse, seen from the bottom surface.

Table 5
Number of layers at failure by plastic collapse ($n_p$) and elastic buckling ($n_e$) during the 3D printing of small ($b = 250$ mm) and large ($b = 500$ mm) square wall layouts, as computed by the parametric 3D printing model (1st row) and measured in two individual 3D concrete printing (3DCP) experiments (2nd and 3rd row).  

| Parameter                                      | Small square wall layout | Large square wall layout |
|------------------------------------------------|--------------------------|--------------------------|
| # layers at failure from parametric model      | $n_p = 21$ (23)$^a$      | $n_e = 27$               |
| # layers at failure from 3DCP experiment 1     | $n_p = 23$               | $n_e = 27$               |
| # layers at failure from 3DCP experiment 2     | $n_p = 23$               | $n_e = 28$               |

$^a$ The layer height used for the calculation of the number of printed layers at plastic collapse, $h_p = 21$, is taken equal to the initial layer height of $h = 9.5$ mm presented in Table 2. When using the actual average height of 9.0 mm of the plastically deformed layers, as measured close to the moment of plastic collapse from the ratio between the actual wall height and the number of printed layers, this number increases towards $n_e = 23$, which turns out be in excellent agreement with the two experimental values.

layers corresponding to elastic buckling is calculated by first inserting the value for $l_p$ listed in Table 4 into Eq. (5)$_a$, which, together with the parameter values for $D_0$, $\rho y$, $g$ and $h$, leads to the actual buckling length $l_p$, at which the structure fails. Dividing the actual buckling length by the layer thickness $\tau_l$ indicated in Table 2, and rounding up the result in order to obtain an integer value for the number of layers, leads to $n_e = 27$ layers for elastic buckling. The vertical deformations generated in the printed layers prior to elastic buckling appeared to be minor, and therefore may be neglected in the calculation of the number of printed layers for buckling. Note that the modeling value is in excellent agreement with the experimental values of $n_e = 27$ layers and $n_e = 28$ layers measured in the two 3D concrete printing experiments of the large square wall layout, see Table 5. It may be further concluded from the
3.2. Elastic buckling of a free wall with and without imperfections

The effect of imperfections on the elastic buckling response is examined by considering the free wall configuration depicted in Fig. 2. The imperfection profile imposed during the 3D printing process is characterized by a periodic shift of a specific number of layers over a prescribed horizontal distance. In the parametric 3D printing model, the imperfection profile is idealized by a sinusoidal function \( w_{m,0} = \hat{w} \sin(2 \pi x / L) \), which is defined by the amplitude \( w_{m,0} \) and wavelength \( L = n \cdot t_i \) of the periodic shift, with \( t_i \) the height of an individual printed layer and \( n \) the number of layers. The left and right graphs illustrate the cases \( n = 2 \) and \( n = 4 \), respectively. The figure has been reprinted from [9].

The geometrical characteristics of the free walls with and without imperfections are summarized in Table 6. The printing process parameters and material parameters characterizing the 3D concrete printing experiments are in accordance with advanced 3D FEM simulations [11] has confirmed that a discrete spatial shift, periodically applied across a specific number of layers (see Fig. 11), can be indeed accurately represented by the continuous, sinusoidal approximation, Eq. (20), used in the parametric 3D printing model.

The buckling responses of these two free wall configurations will be compared to that of a free wall without imperfections; this configuration was printed twice to check the test reproducibility. Both the imperfection and perfect free walls the wall width equals \( b = 1 \) m. The geometrical characteristics of the free walls with and without imperfections are summarized in Table 6. The printing process parameters and material parameters characterizing the 3D concrete printing experiments are in accordance with the data presented in Tables 2 and 3, respectively.

The values of \( n \) and \( t_i \), the factor \( \omega \) in the expression for \( \tau \) determines the contribution of the exponential term in the imperfection profile, Eq. (20), which is kept relatively small by specifying \( \omega \) such that \( \tau = 0.1 \). The dimensionless wavenumber \( \hat{w} \) given by Eq. (21) can be converted to the dimensionless wavelength \( \Gamma \) as

\[
\Gamma = \frac{2 \pi}{\hat{w}} = \frac{n_i t_i \xi_0}{l}.
\]

Note that the imperfection profile is essentially characterized by two lengthscale parameters, which are the dimensionless amplitude \( \hat{w}_{m,0} \) and the dimensionless wavenumber \( \hat{w} \) (or, alternatively, the dimensionless wavelength \( \Gamma \)). As indicated by Eqs. (20) and (21), the specific values for \( \hat{w} \) and \( \Gamma \) follow from printing process data and material data listed in Tables 2 and 3, and the number of layers \( n_i \) across which the periodic horizontal shift takes place. Finally, a detailed comparison with advanced 3D FEM simulations [11] has confirmed that a discrete spatial shift, periodically applied across a specific number of layers (see Fig. 11), can be indeed accurately represented by the continuous, sinusoidal approximation, Eq. (20), used in the parametric 3D printing model.
imperfections are computed by inserting the imperfection profile Eq. (20) into the right-hand side of the buckling equation, Eq. (4), and solving this equation together with the appropriate boundary conditions of the free wall in a numerical fashion, the details of which can be found in [9]. The values for \( w_{m,0} \), \( \xi_L \), and \( \xi_T \) defining the imperfection profile in Eq. (20) are computed as explained above, and are summarized in Table 7.

Since the period \( T_l \) of the printing of an individual layer for a free wall with a width \( b = 1 \) [m] is the same as for the small square wall layout with width \( b = 0.25 \) [m], the dimensionless curing rates for the elastic stiffness and the plastic strength of these two configurations are equal, and correspond to the values listed in the second column of Table 4, i.e., \( \xi_E = 0.08 \) and \( \xi_L = 0.27 \). Inserting the value for \( \xi_E \) into Eq. (9) leads to a dimensionless buckling length for the free wall without imperfections of \( I_{cr} = 2.12 \). In addition, substituting the value for \( \xi_T \) into Eq. (12) results in a dimensionless plastic collapse length of \( I_p = 1.37 \). In accordance with the criterion presented in Eq. (17), this leads to \( I_{cr}/I_p = 1.55 \), which is smaller than the failure mechanism indicator \( \Lambda = 1.59 \) listed in Table 4, so that it may be concluded that the free wall indeed fails by elastic buckling.

The above value of \( I_p = 2.12 \) for the free wall without imperfections is sketched in Fig. 13 (horizontal dashed line), together with the simulated buckling responses for the free walls with imperfection profiles of short wavelength \( (n_l = 2) \) and long wavelength \( (n_l = 10) \). The figure depicts the dimensionless wall length \( I \), defined in accordance with Eq. (5), as a function of the dimensionless total deflection \( w_c \) evaluated at the top \( (X = 0) \) of the free wall. The free walls with imperfections are assumed to fail when the wall top deflection exceeds a value of 2 times the initial imperfection amplitude, \( \Xi = 2w_{m,0} = 0.2 \), as indicated by the vertical dashed line. This criterion is motivated from the experimental observation that at this eccentricity individual layers are no longer printed coherently on top of each other, thereby precluding buckling of the free wall structure. Indeed, after surpassing this deformation limit, the wall top deflection of the imperfect walls grows unboundedly, whereby the wall length \( I \) approaches the critical buckling length \( I_{cr} = 2.12 \) of the free wall without imperfections. The critical values for the dimensionless wall length at \( \Xi = 0.2 \) are represented in Fig. 13 by black dots, and are equal to \( I = 1.99 \) and \( I = 1.92 \) for the free walls with

\[
\begin{align*}
\text{Table 6} \\
\text{Geometrical characteristics of the free walls printed with and without imperfections.} \\
\hline
\text{# layers characterizing the wavelength of the periodic horizontal shift of 0.1h} & \text{Wall width} \\
\hline
\text{Free wall with imperfection profile of short wavelength} & n_l = 2 & b = 1 \text{ [m]} \\
\text{Free wall with imperfection profile of long wavelength} & n_l = 10 & b = 1 \text{ [m]} \\
\text{Free wall without imperfections} & - & b = 1 \text{ [m]} \\
\hline
\end{align*}
\]
Imperfection profile with short wavelength \( (n_t = 2) \) | Imperfection profile with long wavelength \( (n_t = 10) \)  
---|---  
Dimensionless imperfection amplitude, Eq. (20) | \( m_0^{*} = 0.1 \) [\(-\)] | 0.1 [\(-\)]  
Dimensionless wavenumber, Eq. (21) | \( \xi_w = 365.5 \) [\(-\)] | 73.1 [\(-\)]  
Dimensionless boundary factor, Eq. (21) | \( \tau = 0.1 \) [\(-\)] | 0.1 [\(-\)]

![Fig. 13. Buckling responses of free wall configurations with imperfection profiles of short wavelength \((n_t = 2)\) and long wavelength \((n_t = 10)\), computed by the parametric model. The dimensionless wall length \( I \) defined in accordance with Eq. (5) is plotted versus the dimensionless deflection \( \bar{m}^* \) evaluated at the top \((X = 0)\) of the free wall. The walls are assumed to fail when the wall top deflection exceeds a value of 2 times the initial imperfection amplitude, \( \bar{m}^* = 2m_0^{*} = 0.2 \), as indicated by the vertical dashed line. The corresponding values for the dimensionless wall length are represented by the black dots, and equal \( I = 1.99 \) \((n_t = 2)\) and \( I = 1.92 \) \((n_t = 10)\). The buckling load of the free wall without imperfections equals \( I = I_c = 2.12 \), which is computed using Eq. (9), and is designated by the horizontal dashed line.](image)

Imperfection profiles of short \((n_t = 2)\) and long \((n_t = 10)\) wavelength, respectively. These dimensionless critical wall lengths provide the corresponding actual critical wall lengths via Eq. (5). After dividing the actual critical wall length by the layer thickness \( t_l \) and rounding the result up to an integer value, the number of printed layers \( n_c \) is obtained at which the free wall is considered to buckle.

Table 8 lists the predicted number of printed layers \( n_c \) at elastic buckling for the free walls with and without imperfections, together with the corresponding values measured in the 3D concrete printing experiments. The three free walls tested buckled in a similar fashion, in accordance with the response presented in Fig. 14 for the free wall with imperfections of short wavelength \((n_t = 2)\). It can be concluded that the buckling behavior of straight wall structures built by 3D concrete printing is not that sensitive to imperfections. It can be further confirmed that the ratios \( n_c/n_t = 18/10 = 1.8 \) and \( n_c/n_t = 19/2 = 9.5 \), which follow from the parametric model data presented in Table 8 for, respectively, the free walls with long and short imperfection wavelengths, correctly reflect the number of sine waves of the corresponding imperfection profiles shown in Fig. 13.

3.3. Elastic buckling of a free wall printed at different curing rates

The effect of the curing rate - or, equivalently, the wall growth velocity - on the elastic buckling behavior of a free wall is explored by printing walls with three different widths, namely \( b = 1 \) m, 2.5 m and 5 m. These three wall configurations will be referred to as the “short free wall”, “intermediate free wall”, and “long free wall”, respectively. The experimental buckling response of the free wall configurations turned values also indicate the same trend for the three configurations tested, in a sense that the free wall with the imperfections of long wavelength \((n_t = 10)\) buckles first, followed by the free wall with imperfections of short wavelength \((n_t = 2)\), and finally the free wall without imperfections. Furthermore, despite that the imperfection amplitude of \( w_{0}^{c,h} = 0.1h = 5.5 \) mm is relatively large (i.e., considerably larger than the natural imperfections typically encountered in 3DCP wall structures), the difference in the number of layers at failure between the perfect and imperfect free walls is rather small, from which it may be concluded that the buckling behavior of straight wall structures built by 3D concrete printing is not that sensitive to imperfections. It can be further confirmed that the ratios \( n_c/n_t = 18/10 = 1.8 \) and \( n_c/n_t = 19/2 = 9.5 \), which follow from the parametric model data presented in Table 8 for, respectively, the free walls with long and short imperfection wavelengths, correctly reflect the number of sine waves of the corresponding imperfection profiles shown in Fig. 13.

![Fig. 14. Buckling response of a free wall with imperfections of short wavelength \((n_t = 2)\).](image)

Table 8
Number of layers at elastic buckling \((n_c)\) during the 3D printing of the free walls with and without imperfections, as computed by the parametric 3D printing model (1st row) and measured by 3D concrete printing (3DCP) experiments (2nd row).

| | Free wall with long imperf. wavelength \((n_t = 10)\) | Free wall with short imperf. wavelength \((n_t = 2)\) | Free wall without imperf. \((n_t = 20)\) |
|---|---|---|---|
| # layers at buckling from parametric model \( n_c \) | 18 | 19 | 20 |
| # layers at buckling from 3DCP experiments \( n_c \) | 18 | 20 | 21* |

* The same value of \( n_c = 21 \) for the free wall without imperfections was measured in two separate 3DCP experiments, indicating a perfect test repeatability.
Table 9
Period $T_l$ for the printing of an individual layer and the wall growth velocity $l$ for the short ($b = 1$ m), intermediate ($b = 2.5$ m) and long ($b = 5$ m) free wall configurations.

| Wall growth velocity, Eq. (2) | Short free wall ($b = 1$ m) | Intermediate free wall ($b = 2.5$ m) | Long free wall ($b = 5$ m) |
|-------------------------------|-----------------------------|--------------------------------------|---------------------------|
| $T_l = b/v_o$ = 9.6 s         | 24.0 s                      | 48.0 s                               |
| $l = 0.996$ mm/s              | 0.396 mm/s                  | 0.198 mm/s                           |

Fig. 15. Dimensionless buckling length $l_c$, of the free wall configuration as a function of the dimensionless curing rate $E$. Experimental results (black dots) from 3D concrete printing experiments of free walls with 3 different wall widths versus the response computed by the parametric model (solid line) with linear curing, Eq. (9).

out to be similar to that shown in Fig. 14 for the free wall with imperfections of short wavelength, and therefore will not be depicted here. Using the printing process data listed in Table 2, the values calculated for the wall growth velocity $l$ and the period for the printing of an individual layer, $T_l$, are summarized in Table 9. Unfortunately, the 3D concrete printing experiments of the three free wall configurations had to be scheduled on a different day than the preceding experiments presented in Sections 3.1 and 3.2, whereby it was observed that the material consistency of the fresh concrete applied was higher than that of the mixture in the preceding experiments, defined in Table 3. This observation can be confirmed in the analysis of the dimensionless buckling lengths $l_c$, measured for the three free wall configurations, see Fig. 15, which were computed by multiplying the number of layers $n_l$ at buckling by the layer thickness $t_o$, after which the resulting buckling length $l_c$ was converted to its dimensionless value $l_c$, using Eq. (5). In order for Eq. (9) to adequately describe the trend indicated by the three experimental points (black dots) in Fig. 15, the material parameters $E_o$ and $E_l$ listed in Table 3 needed to be increased towards values representative of a printing material with a higher consistency. Specifically, the results from the experiments and parametric model depicted in Fig. 15 are based on the elastic material properties given in Table 10, which indeed leads to a good mutual agreement. Variations measured in the elastic properties of different samples of fresh concrete, such as those presented in Tables 2 and 3, were also reported in previous works [9–11], and may be partly due to the strong sensitivity of the effective properties of concrete to changes in the heterogeneous, microstructural characteristics. In addition, the effect of accelerated curing, initiated during the 3D printing of relatively large structures with a long printing time (such as the long free wall with $b = 5$ m), increases the calibrated value for the curing rate $E_c$, as described below in more detail.

The experimental and modeling results depicted in Fig. 15 clearly illustrate that the resistance against wall buckling increases with increasing curing rate. This can be ascribed to the fact that for a larger wall width the wall growth velocity decreases, see Table 9; consequently, during the printing process the concrete material has more time to harden, as a result of which the overall elastic stiffness, and thus the buckling resistance of the wall, becomes larger.

Table 10
Elastic material parameters of the fresh concrete used in the 3D printing experiments of short ($b = 1$ m), intermediate ($b = 2.5$ m) and long ($b = 5$ m) free walls. The material parameters were calibrated by applying Eq. (9) to the experimental buckling lengths measured in the 3D concrete printing experiments, see Fig. 15.

| Parameter                      | Value              |
|--------------------------------|--------------------|
| Initial elastic stiffness      | $E_0 = 55.0$ [kPa] |
| Curing rate elastic stiffness  | $E_c = 7.0 \times 10^3$ [kPa] |
| Poisson’s ratio                | $\nu = 0.3$ [-]    |

3.4. Design of a complex structure - a picnic table - against failure during 3D concrete printing

The practical use of the parametric model is further demonstrated by the printing of a picnic table with a relatively complex geometry, of which the wall layout is illustrated in Fig. 16. The layout of the picnic table is based on a design made by the Dutch engineering and
consultancy firm Witteveen+Bos. The total circumference of the wall layout equals \( C = 9167 \text{ mm} \). The picnic table is built up by printing 80 layers, which, with a layer height of 9.5 mm given in Table 2, leads to a total height of \( H = 760 \text{ mm} \), see also Table 12. The corresponding printing time of the picnic table is 117.3 min. (almost 2 hours).

In order to validate that the picnic table does not fail during the 3D printing process, it is necessary to check the following two conditions: (i) plastic collapse of the structure should be prevented, and (ii) the most critical wall in the structural layout may not buckle. For the picnic table depicted in Fig. 16, the most critical wall for buckling is the longest wall with an effective width of \( b = 690 \text{ mm} \), located in between the table top and bottom. Note that at the connections with the table top and bottom the in-plane rotation of the critical wall is constrained by the bending resistance of the remaining structure. As pointed out in [9], a finite rotational constraint at the connections with the remaining structure causes the buckling length of the critical wall to fall in between that of a wall without a rotational constraint at the connections, i.e., a simply-supported wall, and that of a wall with full rotational constraint at the connections, i.e., a fully-clamped wall. In other words, the dimensionless buckling length of the critical wall in Fig. 16 is bounded by the range \( l_{oc,s}^c < l_{oc} < l_{oc,f} \), whereby the buckling lengths \( l_{oc,s}^c \) and \( l_{oc,f}^c \) for the simply-supported and fully-clamped walls can be read off from the corresponding design graphs presented by Figs. 4 and 5, respectively. However, in order to be on the safe side, instead of establishing the above range for the critical buckling length, the conservative, lower bound value, \( l_{oc} = l_{oc,f}^c \), may be used in the design against buckling. Elastic buckling will be prevented if this lower bound exceeds the (dimensionless) structural height, \( \Pi > \Pi_c \). Here, the value of the dimensionless structural height \( \Pi \) is determined in the same fashion as the dimensionless buckling length \( l_{oc} \) in Eq. (5):

\[
\Pi = \left( \frac{\phi_k}{D_0} \right)^{\frac{1}{4}} H, \tag{23}
\]

with \( H (= 760 \text{ mm}) \) the actual height of the structure.

Since the picnic table is characterized by a relatively long printing time and was printed on the same day as the three free wall configurations presented in Section 3.3, it is reasonable to adopt the same elastic parameters for the linear curing law in Eqs. (7) and (8), i.e., the values listed in Table 10. Due to a total printing time of almost 2 hours, it may be expected that the curing process of the fresh concrete at some stage significantly accelerates, also stimulated by the heating up of the 3D printing facility. As discussed in the previous section, the increased material stiffness caused by substantial accelerated curing is underestimated under the assumption of a linear curing process, so that the buckling length obtained from the parametric model is expected to lie below the value that would follow from incorporating accelerated curing in the model, see [11] for a detailed comparison on this aspect. Hence, the critical buckling length computed here is conservative, and thus on the safe side for the structural design against elastic buckling.

For the design against plastic collapse, the plasticity parameters \( \phi_0 \) and \( \hat{\varepsilon}_c \) presented in Table 3 are adopted, which, for similar reasons as mentioned above, are expected to provide a conservative, safe value for the plastic collapse length.

In accordance with the format of Table 4, the model parameters used for predicting whether or not the picnic table fails are listed consecutively in Table 13. Starting with elastic buckling, the value given for the dimensionless curing rate, \( \hat{\varepsilon}_c = 5.86 \), and the dimensionless width of the critical wall, \( \hat{s} = 7.63 \), are used in the design graph of

Table 11
Number of layers at elastic buckling for free walls with 3 different wall widths, \( b = 1 \text{ m}, 2.5 \text{ m} \) and \( 5 \text{ m} \), following from the parametric 3D printing model (first row), Eq. (9), and 3D concrete printing experiments (second row).

| # layers at buckling from parametric model | Short free wall \((b = 1 \text{ m})\) | Intermediate free wall \((b = 2.5 \text{ m})\) | Long free wall \((b = 5 \text{ m})\) |
|------------------------------------------|-------------------------------|---------------------------------|-----------------|
| \( n_{oc} = 26 \)                        | \( n_{oc} = 33 \)            | \( n_{oc} = 43 \)              |
| \( n_{oc} = 23 \)                        | \( n_{oc} = 32 \)            | \( n_{oc} = 46 \)              |

Table 12
Circumference \( C \) and height \( H \) of the printed picnic table.

| Parameter | Value |
|-----------|-------|
| Circumference \( C \) | \( 9167 \text{ mm} \) |
| Height \( H \) | \( 760 \text{ mm} \) (80 layers) |

Fig. 16. Wall layout of the picnic table. The layout of the picnic table is based on a design made by the Dutch engineering and consultancy firm Witteveen+Bos. The most critical wall for buckling, indicated in dark grey, has an effective width of 690 mm. All measures are in [mm].
Table 13
Model prediction of the occurrence of failure during the 3D printing of the picnic table illustrated in Fig. 16. Starting at the top of the above table, the parameters listed columnwise are calculated consecutively, with a reference given to the corresponding equation or design graph used. The printing process data required for the computation of specific parameters are listed in Table 2. The material data is obtained from Tables 10 (elastic parameters) and 3 (plastic parameters). The main dimensions of the picnic table are provided in Table 12. As indicated in the final row of the table, the picnic table will not fail during the 3D printing process.

| Picnic table          |
|-----------------------|
| Width critical wall   |
| Period of printing an individual layer |
| Wall growth velocity, Eq. (2) |
| Initial wall bending stiffness, Eq. (6) |
| Dimensionless curing rate for stiffness, Eq. (5)_1 |
| Dimensionless wall width, Eq. (5)_2 |
| Dimensionless buckling length, Fig. 4 |
| Dimensionless structural height, Eq. (23) |
| Dimensionless curing rate for strength, Eq. (13)_2 |
| Dimensionless plastic collapse length, Eq. (12) |
| Does failure occur?   |

Instead of checking for failure of structures under specific, pre-defined printing conditions, the model may also be applied to a-priori determine the optimal printing settings. For example, for the geometry and material specifications characterizing the picnic table, the model is able to determine the maximal horizontal printing velocity \( v_h \) at which the criterion for plastic collapse or elastic buckling is just satisfied. Regarding the mechanism of plastic collapse, first the value of the total height, \( H = 760 \text{ mm} \), needs to be lowered by the thickness of the kinematically constrained bottom layer (9.5 mm), see the discussion in Section 2.2, whereby the result is set equal to the plastic collapse length, \( l_p = 750.5 \text{ mm} \) (thus reflecting plastic collapse in the second layer from the bottom of the picnic table). This value is subsequently inserted into Eq. (13)_1 to compute the dimensionless plastic collapse length, yielding \( \ell_p = 5.45 \). Substituting this result into the yield criterion, Eq. (12), leads to a dimensionless curing rate of \( \ell_p = 0.817 \), which, after insertion into Eq. (13)_2, provides a wall growth velocity of \( l = 0.326 \text{ mm/s} \). Finally, via Eq. (2) the wall growth velocity is translated to the horizontal printing velocity at which the picnic table will plastically collapse, i.e., \( v_h = 18870 \text{ mm/min} \).

As a next step, the maximal horizontal printing velocity is computed from the criterion for elastic buckling. The height \( H = 760 \text{ mm} \) of the picnic table is set equal to the critical elastic buckling length, \( l_{cr} = H \). This buckling length is transferred to its dimensionless value via Eq. (5)_1, leading to \( l_{cr} = 8.40 \). Together with the dimensionless value of the width of the critical wall, \( \bar{b} = 7.63 \) (see Table 13), the dimensionless buckling length leads to the corresponding dimensionless curing rate, \( \ell_{cr} = 2.19 \), via Fig. 4. Inserting this value into Eq. (5)_2 results in the wall growth velocity, \( l = 0.289 \text{ mm/s} \), which, via Eq. (2), finally provides the horizontal printing velocity at which the critical wall buckles, \( v_h = 16760 \text{ mm/min} \). Note that the printing velocity associated to elastic buckling is lower than that for plastic collapse, and therefore decisive. The maximal horizontal printing velocity is a factor of 16760/6250 = 2.7 times larger than the horizontal printing velocity used for the 3D printing of the picnic table, see Table 2, thus leading to a substantial decrease of the manufacturing time. It needs to be checked though, that the increase in printing velocity does not lead to a violation of other performance criteria not considered here, such as those related to mass inertia effects generated at sharp corners of the printed object, or to the continuity in material flow from the printing nozzle.

4. Conclusions

This contribution analyzes failure by elastic buckling and plastic collapse during 3D concrete printing of wall structures. Four types of experiments were performed, which refer to (i) elastic buckling and plastic collapse of a square wall layout, (ii) elastic buckling of wall structures with and without geometrical imperfections, (iii) elastic buckling of wall structures printed at different curing rates - or equivalently, different wall growth velocities -, and (iv) the design of a practical, complex wall structure - a picnic table - against failure during 3D concrete printing. The experimental results are compared to the results obtained by the parametric 3D printing model recently developed by Suiker [9], and the main conclusions from this comparison study are summarized pointwise below:

- The type of failure mechanism and the number of layers to failure observed in the 3D printing experiments are in very good agreement with the predictions following from the parametric 3D printing model.
- The parametric model uses only 5 independent, dimensionless parameters for describing failure by elastic buckling and plastic collapse, see Eqs. (5) and (13). These parameters have proven to be
useful to efficiently and objectively analyze the failure responses obtained from 3D concrete printing experiments.

- The design formulas and design graphs obtained from the parametric model serve as a useful tool for accurately designing straight wall structures against failure during 3D printing. Furthermore, they may be applied to optimize the process conditions during 3D printing, by providing the maximal printing velocity, the optimal geometrical characteristics, or the minimal amount of material required for successfully printing the structure. The excellent correspondence between the model predictions and the results from 3D printing experiments demonstrates the suitability of these design tools for being incorporated in building codes and guidelines on 3D concrete printing. Note that the parametric model is not suitable for the analysis of advanced geometries that are curved, have a variable thickness or are inclined, such as shell-type or overhanging structures; the analysis of the 3D printing process of such geometries requires the use of the FEM modeling techniques described in [10,11].

- The effect of the curing behavior of fresh concrete on the time evolution of its elastic and plastic material properties are accurately described by the parametric model via linear curing laws with a total of only 5 material parameters, see Table 3. These material parameters can be obtained from standard uniaxial compression tests.

- If the results from uniaxial compression tests are not available, or the printing material is rather viscous such that a self-standing specimen required for a uniaxial compression test is difficult to construct, the material parameters for the curing law(s) may be calibrated from 3D printing experiments - see e.g., the values listed in Table 19 - by using the design formulas and/or the design graphs derived with the parametric 3D printing model.

- The parametric 3D printing model has been based on basic laws of physics, i.e., the failure criteria established for elastic buckling and plastic collapse were derived by starting from Newton’s first law that defines the equilibrium conditions under quasi-static loading, see [9] for more details. The model equations developed in [9] are therefore fundamental and widely applicable, e.g., the differential equation for buckling of a 3D printed wall, Eq. (4), in this respect is analogous to the classical differential equation for Euler buckling of a column under a point load. Since the model results are generic, they can be applied to various materials used in extrusion-based 3D printing processes. The present study focuses on a concrete material characterized by a linear curing law; for model results related to other types of curing laws, such as an exponentially-decaying or a quadratic curing law, the reader is referred to [9,11].

- The structural performance, or buildability, of a specific 3D printing material may be experimentally validated by using wall configurations that are similar to those constructed with the concrete printing material used in the present study. As demonstrated, for the validation of elastic buckling, the relatively simple free wall configuration is very suitable, see Fig. 14. In addition, the transition in failure mechanism from elastic buckling to plastic collapse is adequately identified from the 3D printing of square wall layouts with different wall widths, see Fig. 9. Circular wall layouts are less suitable for this purpose, due to the strong imperfection sensitivity of cylindrical shells under elastic buckling, which undermines the test reproducibility.

- During the 3D printing of straight wall structures, the imperfection sensitivity related to elastic buckling turns out to be minor. For many practical wall structures, the effect by possible imperfections on the critical buckling length can therefore be ignored. It is emphasized that this conclusion only holds for straight wall structures; for shell-type wall structures the imperfection sensitivity for buckling typically is significant, and should be accounted for in order to accurately determine its knock-down effect on the critical buckling length.

- Wall structures printed at a higher curing rate - or equivalently, a lower wall growth velocity - are characterized by a higher failure length. This is, because a higher curing rate increases the effective strength and stiffness of the wall, by which its resistance against failure, and thus its failure length, becomes larger.

- For practical structures with a complex geometry containing straight wall elements, the printing process can be adequately designed and optimized against structural failure by validating with the parametric model that (i) plastic collapse of the structure is prevented, and (ii) the most critical straight wall in the structural layout does not buckle. The most critical straight wall usually corresponds to the wall with the largest effective width, whereby a conservative, practical value for the critical buckling length is obtained by assuming rotation-free connections with the remaining part of the structure, i.e., a simply-supported wall.

Author statement

ASJS has conceived the experiments, performed the simulations, and has written the manuscript. RJMW and SML have conceived and performed the experiments, and have reviewed the manuscript. TAM has reviewed the manuscript.

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