Effective medium theories for irregular fluffy structures: aggregation of small particles

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We study the extinction efficiencies as well as scattering properties of particles of different porosity. Calculations are performed for porous pseudospheres with small size (Rayleigh) inclusions using the discrete dipole approximation. Five refractive indices of materials covering the range from $1.20 + 0.00i$ to $1.75 + 0.58i$ were selected. They correspond to biological particles, dirty ice, silicate, amorphous carbon and soot in the visual part of spectrum. We attempt to describe the optical properties of such particles using Lorenz-Mie theory and a refractive index found from some effective medium theory (EMT) assuming the particle is homogeneous. We refer to this as the effective model.

It is found that the deviations are minimal when utilizing the EMT based on the Bruggeman mixing rule. Usually the deviations in extinction factor do not exceed $\sim 5\%$ for particle porosity $P = 0 – 0.9$ and size parameters $x_{\text{porous}} = 2\pi r_s, \text{porous}/\lambda \lesssim 25$. The deviations are larger for scattering and absorption efficiencies and smaller for particle albedo and asymmetry parameter. Our calculations made for spheroids confirm these conclusions. Preliminary consideration shows that the effective model represents the intensity and polarization of radiation scattered by fluffy aggregates quite well. Thus, the effective models of spherical and non-spherical particles can be used to significantly simplify computations of the optical properties of aggregates containing only Rayleigh inclusions.

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1. Introduction

Fluffy aggregate particles are encountered in the atmosphere and ocean, interstellar clouds, and biological and chemical media. Finding their optical properties is an important task for different fields of science and industry. Great progress in the theoretical study of the light scattered by small particles discerned in the last several years makes it possible to calculate the optical properties of arbitrary-shaped particles with anisotropic optical properties and inclusions.¹ However, a major part of the numerical techniques developed for aggregates is still computationally intensive. Moreover, the real structures of scatterers are poorly known, making detailed calculations often impossible. Therefore, it is attractive to find a way to treat the optics of large fluffy particles using simplified models; for example, to replace the aggregates by some simplified homogeneous particles with some average dielectric function (the approach is called the Effective Medium Theory EMT; see Refs. 2, 3 for discussion).

There are many different mixing rules for dielectric functions (see, e.g., Refs. 4–6). They are rediscovered from time to time and sometimes one effective medium expression can be derived from another one. The EMTs for mixtures of materials are traditionally considered in the framework of electrostatic fields.⁴ Evidently, this restricts the range of applicability of the EMTs. Note that previous considerations were given to small volume fractions of inclusions in particles (≤ 20 – 40%).

In this paper, we consider particles consisting of vacuum and some material. We analyze the optical properties of aggregate particles using the discrete dipole approximation (DDA⁷) and compare them with results using “effective” models; e.g., for porous pseudospheres, the scattering properties are determined assuming the sphere is homogeneous, and its refractive index is determined with an EMT. The porosity is varied up to 90% that corresponds to very fluffy particles resembling aggregates with fractal dimension < 2 (see, e.g., Ref. 8).

Some results for three-component composite particles (silicate, carbon and vacuum) were already presented by Voshchinnikov et al.⁹ They show that the EMT approach can give rather accurate results only if very porous particles have so called “Rayleigh” inclusions (small in comparison with the wavelength of incident radiation). At the same time, the optical properties of heterogeneous spherical particles having inclusions of various sizes (Rayleigh and non-Rayleigh) and very large porosity are found to resemble those of spheres with a large number (≥ 15 – 20) of different layers.

The particle models are described in Sect. 2. In Sect. 3 we present some illustrative results using the effective model, size and refractive index of inclusions, and particle shape variations. Concluding remarks are given in Sect. 4.
2. Models of Particles and Calculations

We consider spherical particles consisting of some amount of a material and some amount of vacuum. The amount of vacuum characterizes the particle porosity \( P \), where \( 0 \leq P < 1 \), which is introduced as

\[
P = \frac{V_{\text{vac}}}{V_{\text{total}}} = 1 - \frac{V_{\text{solid}}}{V_{\text{total}}},
\]

where \( V_{\text{vac}} \) and \( V_{\text{solid}} \) are the volume fractions of vacuum and solid material, respectively. If \( P = 0 \) the particle is homogeneous and compact and its optical properties are described by the Lorenz-Mie theory. If the porosity is small we can consider the particle as a solid matrix with vacuum inclusions. If the porosity is large (the case of very fluffy aggregates) the particle can be presented as a vacuum “matrix” with solid inclusions. For aggregates the porosity can be represented as unity minus the volume fraction of solid material in a sphere described around the aggregate. Fluffy particles also can be presented as homogeneous spheres of the same material mass with a refractive index found using an EMT. The size parameter of porous particles can be found as

\[
x_{\text{porous}} = \frac{2\pi r_{s,\text{porous}}}{\lambda} = \frac{x_{\text{compact}}}{(1 - P)^{1/3}} = \frac{x_{\text{compact}}}{(V_{\text{solid}}/V_{\text{total}})^{1/3}}.
\]

So, \( x_{\text{porous}} = x_{\text{compact}} \) if \( P = 0 \).

2.A. DDA Calculations

The calculations of the optical properties of particles with inclusions are performed with the discrete dipole approximation (DDA). We use the version DDSCAT 6.0 developed by Draine & Flatau.\(^{10} \) This technique can treat particles of arbitrary shape and inhomogeneous structure.

The particles (“targets” in the DDSCAT terminology) are constructed employing a special routine producing quasispherical targets with cubic inclusions of a fixed size. The sizes of the target \( d_{\text{max}} \) and of the inclusions \( d_{\text{incl}} \) are expressed in units of the interdipole distance \( d \).

In contrast to previous modeling efforts (e.g., Refs. 11–13), porous particles are not produced by removing dipoles or inclusions from a target but by attributing the refractive index \( m = 1.000001 + 0.0i \) to the vacuum.

For the purpose of treating very porous particles, the number of dipoles in the pseudospheres is taken to be quite large. In all cases considered, particles with maximum size \( d_{\text{max}} = 91 \) are studied. This value corresponds to the total number of dipoles in pseudospheres \( N_{\text{dip}} = 357128 - 381915 \) depending on the size of inclusions \( d_{\text{incl}} \). Thus, the criterion of the validity of the DDA for calculations of the extinction/scattering cross sections \( |m|kd < 1 \) (\( m = n + ki \) is the complex refractive index of the material, see Sect. 3 for its choice, \( k = 2\pi/\lambda \))
the wavenumber with \( \lambda \) being the wavelength in vacuum) of Draine & Flatau\(^{10} \) is satisfied up to size parameter \( x_{\text{porous}} \approx 27 - 40 \).

Targets with randomly distributed cubic inclusions with values of \( d_{\text{incl}} \) ranging from 1 to 5 are considered. Note that the inclusions of the size \( d_{\text{incl}} = 1 \) are dipoles, while the inclusions with \( d_{\text{incl}} = 3 \) and 5 consist of 27 and 125 dipoles, respectively. The optical characteristics of pseudospheres with inclusions are averaged over three targets obtained for different random number sets. The calculations show that in our case such an approach is practically equivalent to time-consuming numerical averaging over target orientations.

2.B. EMT Calculations

An EMT allows one to determine an effective dielectric function \( \varepsilon_{\text{eff}} \) (the dielectric permittivity is related to the refractive index as \( \varepsilon = m^2 \)) of any heterogeneous particle consisting of several materials with dielectric functions \( \varepsilon_i \). EMTs are utilized extensively in optics of inhomogeneous media (see discussion in Refs. 4, 14–17 and references therein). However, full systematic studies of the accuracy of different mixing rules are lacking.

In this work we study several EMTs including the two most often used, the Bruggeman and Garnett EMTs. The formulas of mixing rules are collected in Table 1 (\( f \) is the volume fraction of component “1”), and corresponding references can be found in Refs. 6, 18. We usually consider that \( f = V_{\text{solid}}/V_{\text{total}} \) and \( 1 - f = V_{\text{vac}}/V_{\text{total}} \). Note also that the Garnett rule assumes that one material is a matrix (host material) in which the other material is embedded. When the roles of the inclusion and the host material are reversed, the inverse Garnett rule is obtained.

3. Numerical Results and Discussion

In this section, we present the results illustrating the behaviour of the efficiency factors or cross sections. We consider primarily the extinction efficiency factor \( Q_{\text{ext}} = C_{\text{ext}}/\pi r_s^2 \), where \( C_{\text{ext}} \) is the extinction cross section and \( r_s \) the radius of spherical particle. The refractive indices of compact particles are chosen to be \( m_{\text{compact}} = 1.20 + 0.00i \), \( m_{\text{compact}} = 1.33 + 0.01i \), \( m_{\text{compact}} = 1.68 + 0.03i \), \( m_{\text{compact}} = 1.98 + 0.23i \), and \( m_{\text{compact}} = 1.75 + 0.58i \). These values are typical of refractive indices of biological particles, dirty ice, silicate, amorphous carbon and soot in the visual part of the spectrum, respectively. The refractive indices are taken from the Jena–Petersburg Database of Optical Constants (JPDWC) described in Refs. 19, 20 (for soot we used data published in Ref. 21).

3.A. Effect of the size of inclusions

The size of constituent particles (inclusions) is an important parameter influencing light scattering by aggregates. In Ref. 9 it was demonstrated that the Lorenz-Mie theory together with
the standard EMTs (Garnett or Bruggeman) reproduces the optical properties of aggregates for particles with small (Rayleigh) inclusions only. If the inclusions are not simple dipoles in the DDA terms, the scattering characteristics of aggregates are not well reproduced by the EMT calculations. This fact is illustrated in Fig. 1 where the size dependence of the extinction efficiencies is plotted for two values of particle porosity. For illustration we choose the Bruggeman EMT.

For particles consisting of cubes containing 27 and 125 dipoles, the difference between the DDA and Bruggeman-EMT calculations becomes quite large ($\gtrsim 20\%$) for size parameters $x_{\text{porous}} \gtrsim 10$. In this case the size parameter of the inclusions is 3 and 5 times larger than for simple dipoles. This is enough to modify the pattern of extinction. Larger inclusions produce curves having different slope than simple dipoles and the Bruggeman-EMT. This conclusion is valid for other factors and other refractive indices.

Note that the mixing rules with non-Rayleigh inclusions were developed in the context of the extended EMT theory (see, for example, the discussion in Ref. 3). For aggregates consisting of inclusions of various sizes (Rayleigh and non-Rayleigh), a model of layered particles can be applied (see discussion in Ref. 9). Below we consider particles with simple dipole inclusions only.

3.B. Choice of the EMT

Figure 2 shows the normalized extinction cross sections $C^{(n)}_{\text{ext}}$ for aggregates with small (Rayleigh) inclusions and the effective models based on the Lorenz-Mie calculations with five different EMTs. The normalized cross sections are calculated as

$$C^{(n)} = \frac{C(\text{porous particle})}{C(\text{compact particle of same mass})} = (1 - P)^{-2/3} \frac{Q(\text{porous particle})}{Q(\text{compact particle of same mass})}.$$  

They allow one to analyze the role of porosity in particle optics. The quantity $C^{(n)}$ shows how porosity increases or decreases the cross section. Three panels in Fig. 2 provide results for particles of different masses. For each panel the mass of the particle remains constant but its size increases according to Eq. (2). The refractive index of compact particles is equal to $m_{\text{compact}} = 1.330 + 0.010i$. The refractive indices of porous particles generally decrease with the growth of porosity. Their values are given in Table 2 for three values of $P$. As follows from Table 2 the difference between the values of $m$ is not large, but is enough to produce a noticeable difference of the extinction efficiencies especially at large porosity (see Fig. 2). The largest and smallest values of the effective refractive indices (both real and imaginary parts) are obtained from the Birchak and Lichtenecker mixing rules, respectively. Correspondingly, the properties calculated with these $m$s deviate most strongly from the properties for aggregates. We also find the relative deviations in the efficiency factors (in
percents) as
\[
\text{Deviation} = \frac{Q(\text{EMT-Mie}) - Q(\text{DDA})}{Q(\text{DDA})} \cdot 100\%.
\]  
(4)

Note that the deviations for particles of different mass and porosity \(\lesssim 5\%\) if the Bruggeman, Garnett or Looyenga mixing rule is used. However, the deviation becomes \(> 5\%\) for the Birchak and Lichtenecker rules (see Fig. 3). From Fig. 3 it is seen that the effective models based on the Bruggeman and Looyenga rules reproduce the extinction of aggregates (deviation \(\lesssim 1\%\)) rather well if \(P \lesssim 0.7\). For larger porosity the Bruggeman model works better. The usage of the Garnett rule leads to deviations within \(\sim 4\%\) yielding properties generally smaller than those for aggregates.

Our calculations made for other mixing rules (e.g., quasi-crystalline, coherent potential, see expressions in Refs. 5,18) show that these rules cannot reproduce even the general behaviour of the extinction (e.g., \(C^{(n)}_{\text{ext}}\) increase with the growth of porosity for \(x_{\text{compact}} = 1\)). Based on the data presented in Figs. 2 and 3, the three best effective models (with Bruggeman, Garnett and Looyenga mixing rules) are chosen for further analysis. The results for these three models and aggregates are shown in Fig. 4. This Figure is plotted for one value of \(x_{\text{compact}} = 3\) and two values of \(m_{\text{compact}}\) corresponding to silicate and carbon in the visible part of spectrum. It is seen that the best results are obtained if the model based on the Bruggeman rule is applied. It provides extinctions resembling those of aggregates with small inclusions for particles of different size parameters, porosity and refractive indices of inclusions. So, further considerations are made on the models with the Bruggeman rule.

3.C. Effect of the refractive index of inclusions

The discussion above is mainly relevant to porous water ice in the visible part of spectrum. Now we consider particles with inclusions of different refractive indices. The comparison between the DDA and Bruggeman calculations is made in Figs. 5 and 6 for two particle porosity \(P = 0.33\) and 0.9. The upper panels show the extinction efficiency factors dependence on the size parameter \(x_{\text{porous}}\). Five different refractive indices have been considered. The effective refractive indices found with the Bruggeman rule are indicated in Table 3. It is seen that the effective models describe the general behaviour of extinction rather well. In all cases the deviations between the factors \(Q_{\text{ext}}\) found for the aggregate and for the effective model do not exceed \(\sim 5\%\) (see lower panels of Figs. 5 and 6). The exception is the case of silicate particles \((m_{\text{compact}} = 1.680 + 0.030i)\) and the porosity \(P = 0.33\). The Lorenz-Mie theory produces the ripple structure of the extinction for these particles. Such structure does not appear in our DDA calculations. This is because our targets are not smooth spheres but pseudospheres whose cubic inclusions effectively destroy the resonances.
3. D. Other factors

We also consider how well the effective model reproduces the scattering \(Q_{\text{sca}}\) and absorption \(Q_{\text{abs}}\) efficiencies, the particle albedo

\[
\Lambda = \frac{Q_{\text{sca}}}{Q_{\text{ext}}} \quad (5)
\]

and the asymmetry parameter of the phase function \(F(\Theta, \Phi)\)

\[
g = \langle \cos \Theta \rangle = \frac{\int_{4\pi} F(\Theta, \Phi) \cos \Theta \, d\omega}{\int_{4\pi} F(\Theta, \Phi) \, d\omega}. \quad (6)
\]

These quantities are plotted in Fig. 7. The comparison is made for the refractive indices of inclusions \(m_{\text{compact}} = 1.33 + 0.01i\) and particle porosity \(\mathcal{P} = 0.9\). It is seen that the agreement of results of the DDA and the Bruggeman–Mie computations is rather good. Our calculations performed for other values of \(\mathcal{P}\) and \(m_{\text{compact}}\) show that the effective models better reproduce the extinction properties than the scattering and absorption properties. In the latter case the relative deviation usually does not exceed 10\% (in comparison with 5\% for extinction). At the same time albedo and the asymmetry parameter are reproduced by the effective models with high accuracy: the relative deviation usually does not exceed 2\%.

3. E. Effect of the particle shape

All previous results have been obtained for fluffy spherical particles that can serve as an approximate model of aggregate particles randomly oriented in space (3D orientation). If the aggregates have a preferential axis of rotation (2D orientation) they can be considered as fluffy axisymmetric particles (e.g., prolate or oblate spheroids).

We perform DDA calculations of the efficiency factors for targets having the shape of prolate spheroids with Rayleigh inclusions. The results are compared with those calculations performed using the separation of variables methods (SVM, see Ref. 22) for homogeneous spheroids whose effective refractive index is found from the Bruggeman EMT. Figure 8 shows the size dependence of the extinction efficiencies for prolate spheroids with aspect ratio \(a/b = 2\) for the case of the incident radiation propagating along the rotation axis of the spheroid \((\alpha = 0^0)\). Note that for the considered case, the agreement between the DDA and the Bruggeman–SVM computations is even better than for spheres (cf. Fig. 6): the relative deviations \(\lesssim 4\%\) for \(x_{\text{porous}} \lesssim 40\). So, the effective models of non-spherical particles seems to improve the accuracy of the effective model computations for aggregates containing small size inclusions.

3. F. Intensity and polarization

We also perform illustrative calculations of the intensity and polarization of scattered radiation (see Fig. 9). It is seen that satisfactory agreement between the effective model and
DDA computations is obtained for small and intermediate scattering angles \((\Theta \lesssim 60^\circ)\) only. For larger scattering angles the difference becomes rather large, especially for the second and third minima. This is not unexpected, since diffraction plays a major role for small scattering angles, and this depends primarily on the external morphology of the particle. At larger scattering angles, the internal composition plays a larger role. However, the deviations in reproducing these minima are a small concern when we consider a natural polydispersion of particles. In this case, the minima become washed out due to the polydispersion.

4. Conclusions

We study the general optical behaviour of aggregate particles when the porosity increases.

The main results of the paper are the following:

1. Extinction produced by porous pseudospheres with small size (Rayleigh) inclusions can be calculated employing the Lorenz-Mie theory with the refractive index found using an EMT. The deviations that arise using the Bruggeman effective model do not exceed \(\sim 5\%\) for particle porosity \(P = 0 - 0.9\) and size parameters \(x_{\text{porous}} \lesssim 25\).

2. The effective models represent the behaviour of other properties (scattering and absorption efficiencies, particle albedo, asymmetry parameter) quite well and can be used for calculations of the intensity and polarization of radiation scattered by fluffy aggregates under certain conditions. Preliminary consideration shows that the above conclusions are also valid for spheroidal particles.

3. The effective models can significantly simplify computations of the optical properties of aggregates containing only Rayleigh inclusions.

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Table 1. Mixing rules for the refractive indices.

| Mixing rule     | Formula                                                                 |
|-----------------|--------------------------------------------------------------------------|
| Bruggeman       | \[ f \left( \frac{\varepsilon_1 - \varepsilon_{\text{eff}}}{\varepsilon_1 + 2\varepsilon_{\text{eff}}} \right) + (1 - f) \left( \frac{\varepsilon_2 - \varepsilon_{\text{eff}}}{\varepsilon_2 + 2\varepsilon_{\text{eff}}} \right) = 0 \] |
| Garnett         | \[ \varepsilon_{\text{eff}} = \varepsilon_2 \left[ 1 + \frac{3f \frac{\varepsilon_1 - \varepsilon_2}{\varepsilon_1 + 2\varepsilon_2}}{1 - f \frac{\varepsilon_1 - \varepsilon_2}{\varepsilon_1 + 2\varepsilon_2}} \right] \] |
| Inverse Garnett | \[ \varepsilon_{\text{eff}} = \varepsilon_1 \left[ 1 + \frac{3(1 - f) \frac{\varepsilon_1 - \varepsilon_2}{\varepsilon_1 + 2\varepsilon_2}}{1 - (1 - f) \frac{\varepsilon_1 - \varepsilon_2}{\varepsilon_1 + 2\varepsilon_2}} \right] \] |
| Looyenga        | \[ \varepsilon_{\text{eff}}^{1/3} = f \varepsilon_1^{1/3} + (1 - f) \varepsilon_2^{1/3} \] |
| Birchak         | \[ \varepsilon_{\text{eff}}^{1/2} = f \varepsilon_1^{1/2} + (1 - f) \varepsilon_2^{1/2} \] |
| Lichtenecker    | \[ \log \varepsilon_{\text{eff}} = f \log \varepsilon_1 + (1 - f) \log \varepsilon_2 \] |
Table 2. Effective refractive indices \( m = n + ki \) of porous particles calculated using different EMTs presented in Fig. 2 \( (m_{\text{compact}} = 1.330 + 0.010i) \).

| Mixing rule  | Porosity \( P = 0.3 \)    | Porosity \( P = 0.5 \)    | Porosity \( P = 0.9 \)    |
|--------------|----------------------------|----------------------------|----------------------------|
| Bruggeman    | 1.2284 + 0.0069i           | 1.1611 + 0.0048i           | 1.0310 + 0.0009i           |
| Garnett      | 1.2247 + 0.0066i           | 1.1579 + 0.0045i           | 1.0308 + 0.0008i           |
| Looyenga     | 1.2277 + 0.0068i           | 1.1611 + 0.0048i           | 1.0316 + 0.0009i           |
| Birchak      | 1.2310 + 0.0070i           | 1.1650 + 0.0050i           | 1.0330 + 0.0010i           |
| Lichtenecker | 1.2210 + 0.0064i           | 1.1533 + 0.0043i           | 1.0289 + 0.0008i           |
Table 3. Effective refractive indices \( m = n + ki \) of porous particles calculated using the Bruggeman EMT presented in Figs. 5 and 6.

| \( \mathcal{P} = 0 \)       | \( \mathcal{P} = 0.33 \)         | \( \mathcal{P} = 0.9 \)         |
|----------------------------|----------------------------------|---------------------------------|
| 1.2000 + 0.0000i           | 1.1328 + 0.0000i                 | 1.0193 + 0.0000i                |
| 1.3300 + 0.0100i           | 1.2183 + 0.0066i                 | 1.0310 + 0.0009i                |
| 1.6800 + 0.0300i           | 1.4471 + 0.0196i                 | 1.0588 + 0.0022i                |
| 1.9800 + 0.2300i           | 1.6431 + 0.1507i                 | 1.0795 + 0.0137i                |
| 1.7500 + 0.5800i           | 1.4916 + 0.3781i                 | 1.0707 + 0.03932                |
Fig. 1.
Fig. 2.
Fig. 3.
Fig. 4.
Fig. 6.
Fig. 7.
Fig. 8.

Prolate spheroid, $m_{\text{compact}}=1.33+0.01i$,

$a/b=2, \alpha=0^\circ$,

$\mathcal{P}=0.9$

--- Bruggeman effective model

*** DDA: $n_{\text{inp}}=1$
$m_{\text{compact}} = 1.33 + 0.01i$

$x_{\text{compact}} = 3, \quad p = 0.9$

Fig. 9.
Fig. 1. Size dependence of the extinction efficiency factors calculated for spheres with inclusions of different sizes (DDA computations) and with the Lorenz-Mie theory using the Bruggeman EMT. The refractive index of inclusions is $m_{\text{compact}} = 1.33 + 0.01i$. The effective refractive indices of porous particles are indicated in Table 3. The porosity of particles is $P = 0.33$ (upper panel) and $P = 0.9$ (lower panel). For given porosity the particles of the same size parameter $x_{\text{porous}}$ have the same mass. The effect of variations of the size of inclusions is illustrated.

Fig. 2. Porosity dependence of the normalized extinction cross sections calculated for spheres with small inclusions (DDA computations) and with the Lorenz-Mie theory using different EMTs ($m_{\text{compact}} = 1.33 + 0.010i$). The effects of variations of the EMT and particle size are illustrated.

Fig. 3. Dependence of the relative deviations of the extinction cross sections calculated with the DDA and EMT (see Eq. (4)) on the particle porosity. The particle parameters are the same as in Fig. 2 (middle panel).

Fig. 4. Porosity dependence of the normalized extinction cross sections calculated for spheres with small inclusions (DDA computations) and with the Lorenz-Mie theory using the three EMTs and two values of $m_{\text{compact}}$. The effect of variations of the EMT and refractive index is illustrated.

Fig. 5. Size dependence of the extinction efficiency factors (upper panel) calculated for spheres with small inclusions (DDA computations) and with the Lorenz-Mie theory using the Bruggeman EMT. The porosity of the particles is $P = 0.33$. The effective refractive indices of the porous particles are indicated in Table 3. The lower panel shows the percent difference between DDA results and Bruggeman EMT calculations as defined by Eq. (4). The effect of variations of the refractive indices of the inclusions is illustrated.

Fig. 6. The same as in Fig. 5 but now for porosity $P = 0.9$.

Fig. 7. Size dependence of the scattering ($Q_{\text{sca}}$) and absorption ($Q_{\text{abs}}$) efficiency factors, albedo $\Lambda$ and the asymmetry parameter $g$ for pseudospheres with small inclusions using DDA computations and the Bruggeman effective model. The refractive indices of inclusions are $m_{\text{compact}} = 1.33 + 0.01i$. The porosity of particles is $P = 0.9$.

Fig. 8. Size dependence of the extinction efficiency factors calculated for prolate spheroids with small inclusions using DDA computations and the Bruggeman effective model. The
refractive indices of inclusions are \( m_{\text{compact}} = 1.33 + 0.01i \), and the porosity of particles is \( \mathcal{P} = 0.9 \). The effect of variations of the particle shape is illustrated.

Fig. 9. Intensity and polarization of the scattered radiation calculated for pseudospheres with small inclusions (DDA computations) and effective models (Bruggeman–SVM computations). The refractive indices of the inclusions are \( m_{\text{compact}} = 1.33 + 0.01i \), the porosity of particles is \( \mathcal{P} = 0.9 \).