Ultra-Reliable and Low Latency Communication in mmWave-Enabled Massive MIMO Networks

Trung Kien Vu, Student Member, IEEE, Chen-Feng Liu, Student Member, IEEE, Mehdi Bennis, Senior Member, IEEE, Mérouane Debbah, Fellow, IEEE, Matti Latva-aho, Senior Member, IEEE, and Choong Seon Hong, Senior Member, IEEE

Abstract—Ultra-reliability and low-latency are two key components in 5G networks. In this letter, we investigate the problem of ultra-reliable and low-latency communication (URLLC) in millimeter wave (mmWave)-enabled massive multiple-input multiple-output (MIMO) networks. The problem is cast as a network utility maximization subject to probabilistic latency and reliability constraints. To solve this problem, we resort to the Lyapunov technique whereby a utility-delay control approach is proposed, which adapts to channel variations and queue dynamics. Numerical results demonstrate that our proposed approach ensures reliable communication with a guaranteed probability of 99.99%, and reduces latency by 28.41% and 77.11% as compared to baselines with and without probabilistic latency constraints, respectively.

Index Terms—5G, massive MIMO, mmWave, ultra-reliable low latency communications (URLLC).

I. INTRODUCTION

Currently, millimeter wave (mmWave) and massive multiple-input multiple-output (MIMO) techniques are investigated to provide reliable communication with an over-the-air latency of few milliseconds and extreme throughput [1]. While massive MIMO with large degrees of freedom provides high energy and spectral efficiency [2], mmWave frequency bands provide large bandwidth [3]. In addition, due to the short wavelength of mmWaves, large antenna array can be packed into highly directional beamforming, which makes massive MIMO practically feasible [4]. Thus far, most of existing works on mmWave-enabled massive MIMO systems focus mainly on providing capacity improvement [4], while latency and reliability are not addressed. Although latency and reliability are applicable to many scenarios (e.g. mission-critical applications), in this work, we are interested in the integration of mmWave communication and massive MIMO techniques, which holds the promise of providing great enhancements of the overall system performance [1]. Specifically, this letter is concerned with addressing the fundamental question in mmWave-enabled massive MIMO systems: “how to simultaneously provide order of magnitude capacity improvements and latency reduction?”

By invoking the Lyapunov optimization framework, an utility-optimal solution is obtained to maximize network throughput subject to queuing stability [5]. This solution establishes a utility-delay tradeoff, which achieves utility-optimality at the price of large queuing delays [5]. To cope with this shortcoming, in this letter the Lyapunov framework is extended to incorporate probabilistic latency and reliability constraints, which takes into account queue length, arrival rate, and channel variations with a guaranteed probability. To do so, the problem is formulated as a network utility maximization (NUM). By applying the drift-plus-penalty technique, the problem is decoupled into a dynamic latency control and rate allocation. Here, the latency control problem is a difference of convex (DC) programming problem, which is solved efficiently by the convex-concave procedure (CCP) [6]. Finally, a performance evaluation is carried out to show the latency reduction and the tradeoff between reliability, traffic intensity, and user density.

II. SYSTEM MODEL

Consider the downlink (DL) transmission of a single cell massive MIMO system consisting of one macro base station (MBS) equipped with $N$ antennas, and a set, $\mathcal{M} = \{1, \ldots, M\}$, of single-antenna user equipments (UEs). We assume that $N \geq M$ and $N \gg 1$. Further, the co-channel time-division duplexing (TDD) is considered in which the MBS estimates channels via the uplink phase. We denote the propagation channel between the MBS and the $m$th UE as $h_m = \sqrt{N} \Theta_m \tilde{h}_m$, where $\Theta_m \in \mathbb{C}^{N \times N}$ depicts the antenna spatial correlation, and the elements of $\tilde{h}_m \in \mathbb{C}^{N \times 1}$ are independent and identically distributed (i.i.d.) with zero mean and variance $1/N$. In addition, channels experience flat and block fading, and imperfect channel state information (CSI) is assumed. As per [9], the estimated channel can be modeled as $\hat{h}_m = \sqrt{1 - \tau_m^2} h_m + \tau_m \sqrt{N} \Theta_m \tilde{z}_m, \forall m \in \mathcal{M}$. Here, $\tilde{z}_m \in \mathbb{C}^{N \times 1}$ denotes the estimated noise vector which has i.i.d. elements with zero mean and variance $1/N$, and $\tau_m \in [0, 1]$ reflects the estimation accuracy; in case of perfect CSI, $\tau_m = 0$. Given the estimated channel matrix $\hat{H} = [\hat{h}_1, \ldots, \hat{h}_M] \in \mathbb{C}^{N \times M}$, the MBS utilizes the regularized zero-forcing (RZF) precoder with the precoding matrix, $V = [V_1, \ldots, V_M] \in \mathbb{C}^{N \times M}$, which is given by $V = (\hat{H}^\dagger \hat{H} + N_0 I_N)^{-1} \hat{H}^\dagger$. Note that $V_m$ is the precoding vector for UE $m$. In $V$, $\dagger$ denotes the conjugate transpose, and the regularization parameter $\alpha > 0$ is scaled by $N$ to ensure the matrix $\hat{H}^\dagger \hat{H} + N_0 I_N$ is well-conditioned as $N \to \infty$ [7]. Further, transmit power $\rho_m \geq 0$ is allocated to UE $m$. Denoting all allocated powers in the diagonal matrix $P = \text{diag}(p_1, \ldots, p_M)$, we have the constraint

$\rho_m \geq 0$ is allocated to UE $m$. Denoting all allocated powers in the diagonal matrix $P = \text{diag}(p_1, \ldots, p_M)$, we have the constraint
Tr(PV^TV) \leq P$, with $P$ the maximum transmit power of the MBS. With the aid of the results in \[9\] Theorem 1], the transmit power allocation constraint can be expressed as

\[
\frac{1}{N} \sum_{m=1}^{M} p_m \leq P, \quad p_m \geq 0, \quad \forall m \in \mathcal{M},
\]

where the parameter $\Omega_m$ is the solution to $\Omega_m = \frac{1}{N} \text{Tr}(\Theta_m (\frac{1}{N} \sum_{m=1}^{M} \frac{\Theta_m + I_N}{\lambda_m} )^{-1})$. By designing the precoding matrix $\mathbf{V}$ and transmit power $\mathbf{P}$, the ergodic DL rate of $U_m$ in $\mathcal{M}$ is

\[
r_m(\mathbf{P}) = \mathbb{E} \left[ \log_2 \left( 1 + \frac{p_m |h_m|^2}{1 + \sum_{k \neq m} p_k |h_k|^2} \right) \right].
\]

We invoke results from random matrix theory in order to get the deterministic equivalence for (2) \[9\]. In particular, as $N \geq M$ and $N \gg 1$, for small $\alpha$, the ergodic DL rate almost surely converges to

\[
r_m(\mathbf{P}) \xrightarrow{a.s.} \mathbb{E} \left[ \log_2 \left( 1 + p_m(1 - \tau^2) \right) \right] = \lambda_m, \quad \forall m \in \mathcal{M},
\]

where $\Rightarrow l^2$ denotes almost sure convergence \[9\], \[Theorem 2\]. Moreover, we assume that the MBS has queue buffers to store UE data \[5\]. In this regard, we first index the coherence time block by slot $t \in \mathbb{Z}^+$. At the beginning of each slot $t$, the queue length for UE $m$ is denoted by $Q_m(t)$, which evolves as follows:

\[
Q_m(t+1) = [Q_m(t) - r_m(t)]^+ + a_m(t), \quad \forall m \in \mathcal{M},
\]

where $[x]^+ = \max(x, 0)$, and $a_m(t)$ is the data arrival rate of UE $m$. Further, we assume that $a_m(t)$ is i.i.d. over time slots with mean arrival rate $\lambda_m$ and upper bounded by $a_m^\text{max} \[5\].

III. PROBLEM FORMULATION

According to Little’s law \[11\], the average delay is proportional to $\text{lim}_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} \mathbb{E}[Q_m(t)]/\lambda_m$. Thus, we use $Q_m(t)/\lambda_m$ as a delay measure and enforce an allowable upper bound $d_m^\text{th}$. We further note that the delay (or queue length) bound violation is related to reliability. Thus, taking into account the latency and reliability requirements, we characterize the delay bound violation with a tolerable probability. Specifically, we impose a probabilistic constraint on the queue size length for UE $m \in \mathcal{M}$ as follows:

\[
\Pr \left\{ \frac{Q_m(t)}{\lambda_m} \geq d_m^\text{th} \right\} \leq \epsilon_m, \quad \forall t.
\]

In \[5\], $d_m^\text{th}$ reflects the UE delay requirement. Here, $\epsilon_m \ll 1$ is the target probability for reliable communication.

In order to reduce latency, the intuitive way is to send as many data as possible. However, this might over-allocate resources to UEs, i.e., $r_m(t) \gg Q_m(t)$. To handle this issue, we enforce a maximum rate constraint $r_m^\text{max}$ for each UE $m$. On the other hand, we enforce the MBS to guarantee for all UEs a certain level of QoS, i.e., the minimum rate requirement $r_m^\text{min}$, $\forall m \in \mathcal{M}$.

We define the network utility as $\sum_{m=1}^{M} \omega_m f(\bar{r}_m)$ where the time average expected rate $\bar{r}_m = \text{lim}_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} \mathbb{E}[r_m(t)]$ and the non-negative weight $\omega_m$ for each UE $m$. Additionally, we assume that $f(\cdot)$ is a strictly concave, increasing, and twice continuously-differentiable function. Taking into account these constraints presented above yields the following network utility maximization:

\[
\text{OP:} \quad \max_{\mathbf{P}(t)} \sum_{m=1}^{M} \omega_m f(\bar{r}_m)
\]

subject to $r_m^\text{min} \leq r_m(t) \leq r_m^\text{max}$, $\forall m \in \mathcal{M}$, $\forall t$, \(1\) and \(5\).

Our main problem involves a probabilistic constraint \[5\], which cannot be addressed tractably. To overcome this challenge, we apply Markov’s inequality \[11\] to linearize \[5\] such that

\[
\Pr \left\{ \frac{Q_m(t)}{\lambda_m} \geq d_m^\text{th} \right\} \leq \mathbb{E}[Q_m(t)]/\lambda_m d_m^\text{th}. \quad \text{Then,} \quad (5) \quad \text{is satisfied if}
\]

\[
\mathbb{E}[Q_m(t)] \leq \lambda_m d_m^\text{th}, \quad \forall m \in \mathcal{M}, \forall t.
\]

Therefore, we consider \(7\) to represent the latency and reliability constraint. Assuming that \(\{a_m(t)\}/\lambda_m \geq 1\) is a Poisson arrival process \[11\], we have that \(\mathbb{E}[Q_m(t)] = t\lambda_m - \sum_{\tau=1}^{t} r_m(\tau)\) which is plugged into \(7\). Subsequently, we obtain

\[
r_m(t) \geq t\lambda_m - \lambda_m d_m^\text{th}, \quad \forall m \in \mathcal{M}, \forall t,
\]

which represents the minimum rate requirement in slot $t$ for UE $m$ for reliable communication. Here, we transform the probabilistic latency and reliability constraint \[5\] into one linear constraint \[3\] of instantaneous rate requirements, which helps to analyse and optimize the URLLC problem. In particular, if the delay requirement/reliability constraint is looser (i.e., larger $d_m^\text{th}$ or $\epsilon_m$), the instantaneous rate requirement is reduced. In contrast, if we have a tighter constraint for reliable communication or delay requirement, then the instantaneous rate requirement is higher. Combining \(6\) and \(8\), we rewrite OP as follows:

\[
\max_{\mathbf{P}(t)} \sum_{m=1}^{M} \omega_m f(\bar{r}_m)
\]

subject to $r_m^\text{min} \leq r_m(t) \leq r_m^\text{max}$, $\forall m \in \mathcal{M}$, $\forall t$, \(9\a\) and \(9\b\), with $r_m^\text{th}(t) = \max\{r_m^\text{min}, t\lambda_m - \lambda_m d_m^\text{th}, t\lambda_m(\bar{r}_m - \sum_{\tau=1}^{t} \varphi_m(\tau))\}$.

IV. LYAPUNOV OPTIMIZATION FRAMEWORK

To tackle \(9\), we resort to Lyapunov optimization techniques \[5\]. Firstly, for each DL rate $r_m(t)$, we introduce the auxiliary variable vector $\varphi_m(t) = (\varphi_m(t))/\forall m \in \mathcal{M}$ that satisfies

\[
\varphi_m(t) = \lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} \mathbb{E}[\varphi_m(t)] \leq \bar{r}_m, \quad \forall m \in \mathcal{M},
\]

\[
\varphi_m(t) \leq \mathbb{E}[\varphi_m(t)] \leq \bar{r}_m, \quad \forall m \in \mathcal{M}, \forall t, \quad \text{subject to} \quad (1), \quad (10), \quad \text{and} \quad (11).
\]

In order to ensure the inequality constraint \(11\), a virtual queue length tradeoff. This control parameter is conventionally chosen to be static and identical for all UEs \[5\]. However, this setting does not hold for system dynamics (e.g., instantaneous data arrivals) and the diverse system configuration.
(i.e., different delay and QoS requirements). Thus, we dynamically design these control parameters. From the analysis in the Lyapunov optimization framework [5], we can find
\[ Y_m(t) \leq \nu_m(t) \omega_m \pi_m + a_{m_{\text{max}}} \] with \( \pi_m \) being the largest first-order derivative of \( f \). Letting \( \omega_m = 1, \forall m \in \mathcal{M} \), we have the lower bound
\[ \pi_m \nu_m(t) \geq \nu_m(t), \forall m \in \mathcal{M} \] for selecting the control parameters, where \( \nu_m(t) = \max[Y_m(t) - a_{m_{\text{max}}}, 1] \). Subsequently, following the straightforward calculations of the Lyapunov drift-plus-penalty technique which are omitted for space, we obtain
\[ 13a \leq \mathbb{E} \left[ \sum_{m=1}^{M} (Y_m(t) \varphi_m(t) - \nu_m(t) \omega_m f(\varphi_m(t))) - \sum_{m=1}^{M} Y_m(t) r_m(Y(t)) + C | Y(t) \right]. \] (14b)

Due to space limitation, we omit the details of the constant value \( C \) which does not influence the system performance [5]. We note that the solution to LP is acquired by minimizing the right-hand side (RHS) of (14a) and (14b) in every slot \( t \). Further, (14a) is related to the reliability and QoS requirements while (14b) reflects optimal power allocation to UEs.

A. Auxiliary Variable and Control Parameter Selection

Considering the logarithmic fairness utility function, i.e., \( f(x) = \log(x) \), minimizing the RHS of (14a) for each \( m \in \mathcal{M} \) is formulated as:

\[ \min_{\varphi_m(t), \nu_m(t)} \quad Y_m(t) \varphi_m(t) - \nu_m(t) \log(\varphi_m(t)) \] subject to
\[ \pi_m \nu_m(t) \geq \nu_m(t), \] \[ \nu_m(t) \leq \nu_m(t) \leq \nu_m(t), \] (15b) (15c)

Before proceeding with problem (15), we rewrite \( -\nu_m(t) \log(\varphi_m(t)) \) in (15a), for any \( \varphi_m(t) \geq 0 \) and \( \nu_m(t) > 0 \), as
\[ \nu_m(t) \log \left( \frac{\nu_m(t)}{\varphi_m(t)} \right) - \nu_m(t) \log(\nu_m(t)), \]

in which both \( h_0(\varphi_m(t), \nu_m(t)) \) (i.e., relative entropy function) and \( g_0(\nu_m(t)) \) (i.e., negative entropy function) are convex functions. Since (15a) is the difference of convex functions while constraints (15b) and (15c) are affine functions, problem (15) belongs to DC programming problems [12], which can be efficiently and iteratively addressed by the CCP [6]. The CCP algorithm to obtain the solution to problem (15) is detailed in Algorithm 1. We note that the CCP provably converges to the local optima of DC programming problems [6]. However, due to space limitation, we omit the convergence proof of Algorithm 1 (please refer to [6] for the formal proof).

B. Power Allocation

The optimal transmit power in (14b) is computed by
\[ \min_{P(t)} \quad \sum_{m=1}^{M} Y_m(t) r_m(P(t)) \] subject to
(1). Here, the objective function is strictly convex for \( p_m(t) \geq 0, \forall m \in \mathcal{M} \), and the constraints are compact. Therefore, the optimal solution of \( P^*(t) \) exists and is efficiently reached by numerical methods.

After obtaining the optimal auxiliary variable and transmit power, we update the queues \( Q_m(t + 1) \) and \( Y_m(t + 1) \) as per (4) and (12), respectively.

Algorithm 1 CCP algorithm for solving sub-problem (15c)

while \( m \in \mathcal{M} \) do
initially \( i = 0 \) and a feasible point \( \nu_m^i \) in (15c)
repeat
Convexify \( \tilde{g}_0(\nu_m(t), \nu_m(t)) \) (15d)
-find \( \min_{\varphi_m(t), \nu_m(t)} h_0(\varphi_m(t), \nu_m(t)) - \tilde{g}_0(\nu_m(t), \nu_m(t)) + \nu_m(\varphi_m(t)) \) subject to (15b) and (15c)
find the optimal \( \varphi_m(t) \) and \( \nu_m(t) \)
update \( \nu_m(t+1) := \nu_m(t) \) and \( i := i + 1 \)
until Convergence
end while

V. NUMERICAL RESULTS

We consider a single-cell massive MIMO system [3] in which the MBS, with \( N = 32 \) antennas and \( P = 38 \) dBm, is located at the center of the \( 0.5 \times 0.5 \) km\(^2 \) square area. UEs from 8 to 60 UEs per km\(^2 \) are randomly deployed within the MBS’s coverage with a minimum MBS-UE distance of 35 m. Data arrivals follow a Poisson distribution with different means, and the rate requirements are specified as \( r_{m_{\text{max}}} = 1.2 \lambda_m, r_{m_{\text{min}}} = 0.8 \lambda_m, \forall m \in \mathcal{M} \). The system bandwidth is 1 GHz. The path loss is modeled as a distance-based path loss with the line-of-sight (LOS) model [8] for urban environments at 28 GHz [13]. The maximum delay requirement \( d_{\text{th}} \) and the target probability reliability \( \epsilon \) are set to 10 ms and 5%, respectively. The numerical results are obtained via Monte-Carlo simulations over 10000 realizations with different channel realizations and UE locations. Furthermore, we compare our proposed scheme with the following baselines:

- **Baseline 1** refers to the Lyapunov framework in which the probabilistic latency constraint (5) is considered.
- **Baseline 2** is a variant of **Baseline 1** without the probabilistic latency constraint (5).

A. Impact of Arrival Rate

In Fig. 1 we report the average latency versus the mean arrival rates \( \lambda = E[\lambda(t)] \) for \( M = 16 \). At low \( \lambda \), all schemes do not violate latency constraints, and our proposed algorithm outperforms other baselines with a small gap. At higher \( \lambda \), the average delay of **baseline 2** increases dramatically as \( \lambda > 1.8 \) Gbps, since **baseline 2** does not incorporate the

1\footnote{The multi-cell scenario raises a problem of additional delay due to the need of information exchange among base stations, which is required by either the coordination scheme or distributed approach. This problem is also a very interesting open topic for future work.}

2\footnote{We assume that the probability of LOS communication is very high, while the impact of other channel models is left for future works.}
delay constraint, whereas our proposed scheme reduces latency by 28.41% and 77.11% as compared to baselines 1 and 2, respectively, when $\lambda = 2.4 \text{ Gbps}$. When $\lambda > 2.4 \text{ Gbps}$, the average delay of all schemes increases, violating the delay requirement of 10 ms. It can be observed that under limited maximum transmit power, at very high traffic demand, the latency requirement could not be guaranteed. This highlights the tradeoff between the mean arrival rate and latency. In Fig. 2, we report the tail distribution (complementary cumulative distribution function (CCDF)) of latency to showcase how often the system achieves a delay greater than target delay levels. In particular, at $\lambda = 2.4 \text{ Gbps}$, by imposing the probabilistic latency constraint $[5]$, our proposed approach and baseline 1 ensure reliable communication with better guaranteed probabilities, i.e., $\Pr(\text{delay} > 7.5 \text{ms}) < 10^{-4}$ and $\Pr(\text{delay} > 9.4 \text{ms}) < 10^{-4}$, respectively. In contrast, baseline 2 violates the latency constraint with a high probability, where $\Pr(\text{delay} > 10 \text{ms}) = 74.75\%$.

B. Impact of User Density

In Fig. 3 we compare the average user throughput (avgUT) and average latency of our proposed approach with the two baselines under the impact of user density. Additionally, we consider the weighted sum rate maximization (WSRM) case without considering queue dynamics, i.e., problem $[6]$ without the constraints $[5]$ and $[6]$. The WSRM case is the conventional way to find the system throughput limit but suffers from higher latency. Since all users share the same resources, the average delay (“solid lines”) increases with the number of users $M$, whereas the avgUT (“dash lines”) decreases. Fig. 3 further shows that when $M > 24$, the delay of all schemes increases dramatically and is far-above the latency requirement. Hence, only a limited number of users can be served to guarantee the delay requirement, above which, a tradeoff between latency and network density exists. Our proposed approach achieves better throughput and higher latency reduction than baselines 1 and 2, while the WSRM case has the worst delay performance as expected. Compared with WSRM, our proposed approach maintains at least 87% of the throughput limit, while achieving up to 80% latency reduction. Moreover, our proposed approach reaches Gbps capacity, which represents the capacity improvement brought by the combination of mmWave and massive MIMO techniques. Numerical results show that our approach simultaneously provides order of magnitude capacity improvements and latency reduction.

VI. CONCLUSION

In this letter, we have investigated the problem of mmWave-enabled massive MIMO networks from a latency and reliability standpoint. Specifically, the problem is modeled as a NUM problem subject to the probabilistic latency/reliability constraint and QoS/rate requirement. By incorporating these constraints, we have proposed a dynamic Lyapunov control approach, which adapts to channel variations and system dynamics. Numerical results show that our proposed approach reduces the latency by 28.41% and 77.11% as compared to current baselines.

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