Central and peripheral hadron-nucleus collisions in the Additive Quark Model

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Abstract

Peripheral nucleon-nucleus collisions occur at the high energies mainly through the interaction with one constituent quark from the incident nucleon. The central collisions should involve all three constituent quarks and each of them can interact several times. We calculate the average number of quark-nucleus interactions for both the cases in good agreement with the experimental data on $\phi$-meson, $K^{*0}$ and all charged secondaries productions in $p + Pb$ collisions at LHC energy $\sqrt{s} = 5$ TeV.

1 Introduction

The Additive Quark Model (AQM) treats the nucleon as a system of three quasi free constituent quarks. They play the roles of incident particles in terms of which the hadron scattering is described. First phenomenological predictions of AQM have demonstrated a fairly good agreement with experimental data.

Basically, the scattering amplitude is presented in AQM as a sum over the terms with a given number of constituent quarks involved into the process. For the proton-nucleus scattering there are three classes of interactions depending on whether only one,
two or three incident constituent quarks participate. The sum of their probabilities is normalized to unity.

The probabilities of all three types of events are of the same order for heavy nuclear target at the fixed target energies. The probability of the three quark interaction grows at the LHC energies because of the growth of the interaction cross section thereby reducing two other probabilities due to the total normalization.

All three events are essentially dependent on the hadron-nucleus impact parameter. Three quark interaction dominates in the central collisions. The configuration with two interacting and one non-interacting quarks has a smaller probability which in turn is larger than the probability to have a single interacting quark only. The situation for the peripheral hadron-nucleus collisions is opposite. Here the one quark interaction is most probable while the probability of the three quark interaction is minimal. Thus the peripheral hadron-nucleus scattering looks more close to the hadron-nucleon one. As a result the multiplicity of the produced secondaries in the central collisions should be about three times larger than for the peripheral collisions.

In the present paper we calculate the total number of the incident constituent quarks interactions with the target nucleons for the all three event classes. The ratios of the numbers of the quark interactions in central and peripheral collisions is compared with the LHC experimental data at $\sqrt{s} = 5$ TeV.

2 Calculation of the probabilities and the number of quark interactions for various event classes.

The probabilities for the one, two and all three quarks from the fast nucleon to interact with the target nucleus were firstly calculated in ref. They have the form

\[
\begin{align*}
{v_1} &= \frac{3}{\sigma_{pA}^{inel}} \int d^2b e^{-2\sigma_q T(b)}(1 - e^{-\sigma_q T(b)}) \\
{v_2} &= \frac{3}{\sigma_{pA}^{inel}} \int d^2b e^{-\sigma_q T(b)}(1 - e^{-\sigma_q T(b)})^2 \\
{v_3} &= \frac{1}{\sigma_{pA}^{inel}} \int d^2b (1 - e^{-\sigma_q T(b)})^3.
\end{align*}
\]

Here the target nuclear profile function,

\[
T(b) = A \int_{-\infty}^{\infty} \rho(b, z) dz,
\]
is given by Fermi nuclear matter distribution

$$\rho(r) = \frac{\rho_0}{1 + e^{-c_2 r}}$$

with the parameters

$$c_1 = 1.15 A^{1/3} \text{ fm}, \quad c_2 = 0.51 \text{ fm}$$

and \(\rho_0\) value determined by the normalization \(\int d^3 r \rho(r) = 1\). The cross section of the proton-nucleus inelastic scattering,

$$\sigma^{\text{inel}}_{pA} = \int d^2 b \left(1 - e^{-\sigma^{\text{inel}}_{pN} T(b)}\right), \quad (2)$$

is expressed through the proton-nucleon inelastic cross-section \(\sigma_{pN}\). The constituent quark inelastic cross-section with the target nucleon is assumed to be \(1/3\) of the proton-nucleon one, \(\sigma_q = 1/3 \sigma^{\text{inel}}_{pN}\).

Given the factor \(e^{-\sigma_q T(b)}\) as a probability for a quark not to interact with the target at the distance \(b\) the expressions (11) are rather evident. The value \(v_1\) stands for the processes where one of the three quarks interacts with the nucleus while the other two quarks do not. The \(v_2\) value refers to the opposite situation, \(v_3\) is the probability for all three quarks to interact, \(v_1 + v_2 + v_3 = 1\).

The cross section (2) can be recast in the form

$$\sigma^{\text{inel}}_{pA} = \int d^2 b e^{-\sigma^{\text{inel}}_{pN} T(b)} \left[ e^{\sigma^{\text{inel}}_{pN} T(b)} - 1 \right] = \sum_{\nu=1}^{\infty} \int d^2 b e^{-\sigma^{\text{inel}}_{pN} T(b)} \frac{1}{\nu!} \left[ \sigma^{\text{inel}}_{pN} T(b) \right]^{\nu} = \sum_{\nu=1}^{\infty} \sigma^{(\nu)}_{pA}. $$

Each term comes to the sum from the interactions with \(\nu\) target nucleons. The average number of the collisions in the proton-nucleus scattering is therefore equal to

$$\langle \nu \rangle_{pA} = \frac{1}{\sigma^{\text{inel}}_{pA}} \sum_{\nu=1}^{\infty} \nu \sigma^{(\nu)}_{pA}$$

$$= \frac{1}{\sigma^{\text{inel}}_{pA}} \int d^2 b e^{-\sigma^{\text{inel}}_{pN} T(b)} \sum_{\nu=1}^{\infty} \frac{1}{(\nu - 1)!} \left[ \sigma^{\text{inel}}_{pN} T(b) \right]^{\nu} = A \frac{\sigma^{\text{inel}}_{pN}}{\sigma^{\text{inel}}_{pA}}.$$

To find the inclusive density of the secondaries in the central (midrapidity) region it is necessary to calculate the average number of collisions with the target nucleus.
This number gets separate contributions from the three classes of events specified by equations (1). Taking the probability for the first class,

\[ v_1 = \frac{3}{\sigma_{pA}^{inel}} \int d^2 b e^{-3\sigma_q T(b)} (e^{\sigma_q T(b)} - 1), \]

the relevant collisions number \( \langle \nu_1 \rangle \) is obtained similar to Eq.(3),

\[ v_1 \cdot \langle \nu_1 \rangle = \frac{3}{\sigma_{pA}^{inel}} \int d^2 b e^{-3\sigma_q T(b)} \sum_{\nu=1}^{\infty} \nu \frac{[\sigma_q T(b)]^\nu}{\nu!} \]

\[ = \frac{3}{\sigma_{pA}^{inel}} \int d^2 b e^{-2\sigma_q T(b)} \sigma_q T(b). \] (4)

The second class probability,

\[ v_2 = \frac{3}{\sigma_{pA}^{inel}} \int d^2 b e^{-2\sigma_q T(b)} (e^{\sigma_q T(b)} - 1)^2, \]

is worked out in the same manner. The parenthesis is expanded and the terms with \([\sigma_q T(b)]^\nu\) are regarded as arising from \(\nu\) collisions. Multiplying them by \(\nu\) and summing up the series one gets

\[ v_2 \cdot \langle \nu_2 \rangle = \frac{6}{\sigma_{pA}^{inel}} \int d^2 b e^{-2\sigma_q T(b)} (e^{\sigma_q T(b)} - 1) \sigma_q T(b). \] (5)

Repeating these steps for the third class,

\[ v_3 = \frac{1}{\sigma_{pA}^{inel}} \int d^2 b e^{-3\sigma_q T(b)} (e^{\sigma_q T(b)} - 1)^3, \]

returns the value

\[ v_3 \cdot \langle \nu_3 \rangle = \frac{3}{\sigma_{pA}^{inel}} \int d^2 b e^{-2\sigma_q T(b)} (e^{\sigma_q T(b)} - 1)^2 \sigma_q T(b). \] (6)

All three classes yield in aggregate the average proton-nucleus collision number,

\[ \nu_{pA} = v_1 \cdot \langle \nu_1 \rangle + v_2 \cdot \langle \nu_2 \rangle + v_3 \cdot \langle \nu_3 \rangle = \frac{1}{\sigma_{pA}^{inel}} \int d^2 b \sigma_{pN} T(b) = A_{pN} \sigma_{pN}^{inel}. \]

The equation (3) gives for the proton-lead collision at the LHC energy \(\sqrt{s} = 5\) TeV (taking \(\sigma_{pN}^{inel} = 69.86\) mb and \(\sigma_{pA}^{inel} = 1965\) mb)

\[ \langle \nu \rangle_{pA} = 7.36. \] (7)
When only one incident constituent quark interacts with the lead nucleus one obtains from the equations (1) and (4) \( v_1 = 0.19, v_1 \cdot \langle \nu_1 \rangle = 0.26 \), while for the two or three interacting quarks the equations (1) and (5),(6) give \( v_2 = 0.20, v_3 = 0.61, v_2 \cdot \langle \nu_2 \rangle = 0.82, v_3 \cdot \langle \nu_3 \rangle = 6.297 \), that means

\[
\langle \nu_1 \rangle = 1.28, \quad \langle \nu_2 \rangle = 4.16, \quad \langle \nu_3 \rangle = 10.31.
\]

### 3 Numerical results and conclusion

As has been mentioned above \( \nu_1 \) is related to the peripheral proton-lead collisions, \( \nu_3 \) to the central collisions and \( \nu_2 \) to the intermediate type. The experimental data on the inclusive secondaries density in the midrapidity region are presented in refs.\(^{7,8}\) for \( p + Pb \) scattering at \( \sqrt{s} = 5 \) TeV. In these papers the events are divided into several classes with respect to the mean multiplicities. It is reasonable to assume that the smallest and the highest multiplicities refer to the peripheral and the central collisions whereas the intermediate interactions occur mainly in the collisions of two constituent quarks. However the intermediate region should include also the tails from the single and three quarks interactions that makes it more complicated to analyze. The outcome should be somewhere between the two limiting cases but we do not consider it.

The ratio of the inclusive densities of any secondaries in the central and peripheral collisions is compared below with the obtained \( \nu_3/\nu_1 \) ratio.

Table. Comparison of the calculated \( \nu_3/\nu_1 \) values with the experimental data for the ratio of the inclusive densities in the central (event classes 0–20%) and peripheral (event classes 80–100%) collisions for the all charged secondaries, \( K^{*0} \) and \( \phi \) (see refs.\(^{7,8}\) for details).

| \( \nu_3/\nu_1 \) | \( \frac{dN}{dy} \) (central)/\( \frac{dN}{dy} \) (peripheral) |
|------------------|---------------------------------|
| 8.05             | all charged                     |
|                  | \( K^{*0} \)                     |
|                  | \( \phi \)                       |
| 8.3 ± 0.4        | 7.4 ± 1.2                       |
| 9.7 ± 1.4        |                                 |

The presented table shows unexpectedly good agreement of our calculations with the experimental data. The important point is that the experimental ratios for the different secondaries coincide inside the error bars. The theoretical uncertainties can be estimated at the level 10-15%, this level is confirmed by the validity of another AQM predictions.\(^{11}\)

It will be interesting to investigate the same ratios for another secondary particles as well as another nuclear targets.
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