The Deconfinement Transition in SU(4) Lattice Gauge Theory

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The deconfinement transition in SU(4) lattice gauge theory is studied on $N_s^3 \times N_t$ lattices with $N_s = 8-16$ and $N_t = 4-8$ using a modified Wilson action which is expected to have no bulk transitions. The susceptibility $\chi_{\text{max}} | L |$ is found to increase linearly with spatial volume for $N_t = 4, 5, \text{ and } 6$, indicating a first order deconfinement phase transition. The latent heat is estimated to be $\approx \frac{2}{3}$ of the corresponding ideal gas energy density at $T_c$.

1. INTRODUCTION

The nature of the phase transition(s) to Quark-Gluon Plasma, which we hope to see in the experiments of RHIC and later of LHC, and the physics driving it, have always been of great interest. While the real world has presumably two very light $(u,d)$ flavours of quarks and one somewhat heavier $(s)$ flavour, both analytical and numerical methods in lattice QCD begin from the limiting cases of either massless or infinitely massive quarks. One talks of the chiral symmetry restoring phase transition and the deconfinement phase transition in these two cases respectively and has suitable order parameters to investigate them. For quarks with $N$ colours and $N_f$ massless flavours, these transitions are related to spontaneous breaking of a global $Z(N)$ and $SU(N_f) \times SU(N_f)$ chiral symmetry. Since these symmetries are broken explicitly to various extents in the real world, which of them is more relevant is a priori not clear. The low masses of the light flavours suggest chiral symmetry to be the dominant one. However, it is seen in numerical simulations that the energy density shows a large change at the chiral transition and even the order parameter for the deconfinement transition, also rises to nonzero values there. These apparently mysterious observations can be explained using large $N$ arguments, if the deconfinement transition for $N \geq 4$ is of second order. SU(4) is clearly the simplest case to check this out.

Numerical simulations of SU(4) theory at finite temperatures have been done in the past and recently as well. All of them used the Wilson action, or the more general mixed action:

$$S = \sum_P \left[ \beta \left( 1 - \frac{\text{Re tr } U_P}{N} \right) + \beta_A \left( 1 - \frac{\text{tr } A U_P}{N} \right) \right] , \quad (2)$$

A well known problem in the simulations with these actions, especially for large $N$, is the presence of a bulk transition which is a lattice artifact. The phase diagram of the mixed action,

$$L(\vec{x}) = \frac{1}{N} \left( 1 - \prod_{t=1}^{N_t} U_4(\vec{x}, t) \right) , \quad (1)$$

also rises to nonzero values there. These apparently mysterious observations can be explained using large $N$ arguments, if the deconfinement transition for $N \geq 4$ is of second order. SU(4) is clearly the simplest case to check this out.

Figure 1. A schematic phase diagram in $(\beta, \beta_A)$-plane for the mixed action of eq. (2).
Wilson axis ($\beta_A = 0$). For $N \geq 4$, D is expected to be where E is shown, causing a first order bulk transition for the usual Wilson action. In order to avoid it, simulations at negative $\beta_A$ \cite{2} and for larger $N_t = 6$ were made. They lead to a first order deconfinement phase transition for $SU(4)$.

From our extensive studies\cite{4} of the deconfinement phase transition for the action above but for the $SU(2)$ theory, we know that bulk transitions affect the order and location of the deconfinement transition in subtle and inexplicable ways, even leading to apparent qualitative violations of universality. Universality was restored \cite{5} in that case only after eliminating the bulk transitions associated with the $Z(2)$ vortices and $Z(2)$ monopoles by adding large chemical potentials for them. It seems natural to expect that the bulk transitions for $N > 2$ can also be cleaned off by suppressing the corresponding $Z(N)$ objects. We pursue this idea here for $SU(4)$ to investigate its deconfinement phase transition.

2. SIMULATIONS AND RESULTS

Generalizing the idea of positive plaquette models\cite{6} in the literature, we use the action

$$S = \beta \sum_P \left(1 - \frac{\text{tr} \, U_P}{N}\right) \cdot \theta\left(\frac{\pi}{N} - |\alpha|\right),$$

where $-\pi < \alpha \leq \pi$ is the phase of $\text{tr} \, U_P$. By adding the adjoint term of eq. (2) to the action (3), one sees that the phase diagram of the resultant mixed should not have the bulk lines AC or BC and hence the endpoint D or E.

We have simulated the above action on $N_s^3 \times N_t$ lattices for $N_s = 8, 10, 12, 15, 16$ and $N_t = 4, 5, 6, 8$ using a 15-hit Metropolis et al. algorithm. The calculations were done on a cluster of pentiums. Typically short runs to look for points of rapid variations in $\langle |L| \rangle$ were followed by long runs (a few million sweeps) to determine the susceptibility $\chi_{|L|}$ using the histogramming technique. Usual finite size scaling techniques were used to determine the order of the transition and its exponents.

In simulations on $N_s^3 \times 4$ lattices, $N_s = 8, 10, 12$, one sees hot and cold starts converge quickly at couplings a little away from the transition point on its both sides but a clear co-existence of states is visible for all lattices at the transition point. The tunneling frequency goes down with spatial volume. The histograms of $|L|$ show peaks which become narrower with increasing volume and the gap between them remains unchanged. These classic signs of a first order phase transition are confirmed by a quantitative analysis of the linear growth of $\chi_{|L|}$ with volume, as seen in Fig. 2. The horizontal lines in each case are predictions obtained by scaling the $N_s = 8$ results linearly with volume.

Figure 2. The susceptibility $\chi_{|L|}$ as a function of $\beta$ for $N_s^3 \times 4$ lattices.

For larger $N_t$, we used many longer runs in the region of strong variation of $\langle |L| \rangle$ to obtain the susceptibility directly and used the histogramming only for the finer determination of the critical coupling. Our results for $\langle |L| \rangle$ as a function of $\beta$ clearly show the expected shift for a deconfinement phase transition for $N_t = 5, 6$ and 8. This is evident in the $\beta_c$ determinations from the $\chi_{|L|}^{\text{max}}$, as seen in Table 1 for $N_t = 6$ for two different spatial volumes. Again, using the peak height for the smaller spatial volume, the $\chi_{|L|}^{\text{max}}$ on the bigger lattice can be predicted, assuming a linear growth with volume. The prediction listed in the table can be seen to be in very good agreement with the direct Monte Carlo determination. Along with the shifts in $\beta$, it confirms that the same physical deconfinement phase transition is
two different observables $\Delta$, the latent heat can be obtained from a growth in $\chi_{L}$-susceptibility peak, $\chi_{L}^{\max}$.

| $N_{t}$ | $\beta_{c,N_{t}}$ | $\chi_{L}^{\max}$ | $\Delta_{\text{predicted}}$ |
|--------|-----------------|-----------------|-----------------|
| 12     | 10.675          | 10.686(5)       | 4.36(35)        |
| 16     | 10.675          | 10.676(5)       | 10.3(95)        |

The values of $\beta$ at which long simulations were performed on $N_{t}^{3} \times 6$ lattices, $\beta_{c}$ and the height of the $|L|$-susceptibility peak, $\chi_{L}^{\max}$.

Table 1

Table 2

| $N_{t}$ | 4    | 5    | 6    | 8    |
|---------|------|------|------|------|
| $\Delta_{1}$ | 21.03(5) | 11.02(6) | 8.31(5) | 6.57(16) |
| $\Delta_{2}$ | 9.89(14) | 7.77(40) | 6.04(60) | 6.45(99) |

Both the latent heat estimates of eq.(4) as a function of $N_{t}$.

being simulated on these lattices thus approaching the continuum limit of $a \rightarrow 0$ in a progressive manner by keeping the transition temperature $T_{c}$ constant in physical units.

3. LATENT HEAT

While the results above for the deconfinement order parameter $\langle |L| \rangle$ and the corresponding susceptibility, $\chi_{L}$, are indicative of a first order deconfinement phase transition, one needs to make sure that they indeed are not due to a coincident first order bulk transition. Apart from the characteristic (logarithmic) shift of the transition point with $N_{t}$, seen above, the latent heat of a first order phase transition should also remain constant as $N_{t} \rightarrow \infty$. Requiring the pressure to be continuous at the deconfinement phase transition, the latent heat can be obtained from two different observables $\Delta_{1} \equiv \Delta(\epsilon - 3p)/T_{c}^{4}$, and $\Delta_{2} \equiv \Delta(\epsilon + p)/T_{c}^{4}$, where

$$\Delta_{1} = -48N_{t}^{4}a\frac{\partial g^{-2}}{\partial a}\Delta P ,$$
$$\Delta_{2} = 32N_{t}^{4}C(g^{2})\frac{\partial P_{t}}{g^{2}}(\Delta P_{t} - \Delta P_{s}) ,$$

and $C(g^{2}) = (1 - 0.2366g^{2} + O(g^{4}))$ for $SU(4)$. $\Delta$ denotes discontinuities across the transition in the respective variables. In order to obtain the $\Delta P$, $\Delta P_{s}$ and $\Delta P_{t}$, the minimum of the histogram $N(|L|)$ was used to separate the two phases in each case. From eq. (4), it is clear that the plaquette discontinuity $\Delta P \propto N_{t}^{-4}$ in order to obtain the same latent heat in physical units, as $N_{t} \rightarrow \infty$. Indeed, its decrease with $N_{t}$ was seen to be consistent with expectations for $N_{t} \geq 5$. Furthermore, both estimates must agree in this limit, as the neglected cut-off corrections become then insignificant. Table 3 suggests this to be the case, leading to an estimate which is $\approx \frac{1}{3}$ of the ideal gas energy density at $T_{c}$ and agrees with earlier results.

4. SUMMARY

The linear growth of $\chi_{L}^{\max}$ with volume for $N_{t} = 4$, suggests a first order deconfinement phase transition for $SU(4)$. Various indicators, such as, histograms and evolutions, are in accord with this. Increasing $N_{t}$ to 5, 6 and 8 shows the expected shift of the deconfinement transition, with a growth in $\chi_{L}^{\max}$ that is consistent with being linear in volume. The plaquette discontinuity $\Delta P$ decreases as the fourth power of $N_{t}$, indicating both a lack of a bulk transition and a first order deconfinement phase transition. The large estimated latent heat, being about 2/3 of the corresponding ideal gas energy density, suggests the deconfinement transition to grow stronger in nature as the number of colours in increased.

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