Barkhausen noise from zigzag domain walls

B Cerruti and S Zapperi

CNR-INFM, Dipartimento di Fisica, Università ‘La Sapienza’, Piazzale Aldo Moro 2, 00185 Roma, Italy
E-mail: benedetta.cerruti@gmail.com and stefano.zapperi@roma1.infn.it

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Abstract. We investigate the Barkhausen noise in ferromagnetic thin films with zigzag domain walls. We use a cellular automaton model that describes the motion of a zigzag domain wall in an impure ferromagnetic quasi-two dimensional sample with in-plane uniaxial magnetization at zero temperature, driven by an external magnetic field. The main ingredients of this model are the dipolar spin–spin interactions and the anisotropy energy. A power law behaviour with a cut-off is found for the probability distributions of size, duration and correlation length of the Barkhausen avalanches, and the scaling exponents are in agreement with the available experiments. The link between the size and the duration of the avalanches is analysed too, and a power law behaviour is found for the average size of an avalanche as a function of its duration.

Keywords: Barkhausen noise (theory)

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1. Introduction

Understanding the properties of ferromagnetic thin films is still one of the open questions in the physics of magnetic systems. An important issue is linking the domain and domain wall structure of a material with its hysteretic properties, like the Barkhausen noise (BN). The Barkhausen effect [1] consists in the irregularity of the magnetization variation while magnetizing a sample with a slowly varying external magnetic field, and is due to the jerky motion of the domain walls in a system with structural disorder and impurities. Once the origin of BN was understood [2], it was soon realized that it could be used as an effective probe to investigate the magnetization dynamics in magnetic materials. Furthermore, from a purely theoretical point of view, it is a good example of dynamical critical behaviour, as evidenced by experimental observation of power law distributions for the statistics of the avalanche size and duration [3]. Moreover, there is a growing evidence that soft magnetic bulk materials can be grouped in different classes according to the scaling exponents values [4], so that BN could be seen as a non-destructive experimental tool for the analysis of the properties of a material, and a similar feature could be expected to hold for two dimensional materials.

Until now, most of the models and experiments on BN have focused on three dimensional systems [5]. The difficulties for the study of two dimensional samples, in fact, concern both the theoretical and the experimental aspects of the problem. On the theoretical side, the topology of domains and domain walls is much more rich and complicated in two dimensions (parallel or head-on domains, charged and uncharged walls, magnetization in or out of the film plane, parallel or zigzag walls, labyrinthic domains, etc) than in the bulk case [6] (mainly parallel domains with uncharged walls), so it is not obvious how to generalize the well known three dimensional models. The models currently used for three dimensional materials could be classified in two main groups, namely spin models of the Ising type [7]–[10], and single domain wall models [11]–[16]. On the experimental side, the inductive experimental set-up commonly used for bulk samples [5] is usually not suitable for thin films, due to the low intensity of the magnetic flux variation signal, which is roughly proportional to the sample thickness, and thus tends to vanish for very thin films. Conversely, the magneto-optical [17]–[19] and magneto-optical microscope magnetometry [20] experiments, though able to supply the domain structure, that is not accessible via inductive measurements, provide only partial information about the probed zone, and not about the whole sample. So, despite the
increasing interest concerning ferromagnetic thin film applications in magnetic recording technology and spintronic devices, a complete understanding of two dimensional system behaviour is still lacking.

In this paper we focus on two dimensional systems with zigzag domain walls, which arise in thin films with head-on magnetization between nearest-neighbour domains, mainly due to the balance between the magnetostatic and the anisotropy contributions to the system total energy [22]. Such walls have been observed for the first time in thin film magnetic recording media, where head-on domains are induced by means of the application of a recording head field, and were then observed in films of several magnetic media such as iron [23], Co [24], Gd–Co [22], epitaxial Fe films grown on GaAs(001) [25], ferrite–garnet films with strong cubic anisotropy [26] and many others. Such walls have been observed too in ferroelectric materials, such as Gd$_2$(MoO$_4$)$_3$ crystals [27].

We present a study of the Barkhausen noise at $T = 0$. We use a slightly modified version of a simple single wall discrete model for the motion of the zigzag wall that we recently proposed for the study of the dynamic hysteresis in ferromagnetic thin films [28]. Our model is based on the interplay between dipolar and anisotropy energy contributions, in the presence of structural disorder and an external magnetic field. Via cellular automaton simulations the model describes the motion, in a disordered ferromagnetic thin film, of a zigzag wall between two domains of opposite magnetizations meeting head-on at the wall, up to the saturation of the magnetization driven by the external magnetic field. We find that the probability distributions of the size, the duration and the correlation length of the Barkhausen avalanches show a power law behaviour with a cut-off, and the scaling exponents are in quantitative agreement with experimental data for a two dimensional sample, like for Co polycrystalline thin films [20,21]. Finally, in order to obtain a deeper insight into the link between the size and the duration of the avalanches, we study the behaviour of the average size of an avalanche as a function of its time duration, and find that it could be described too by a power law.

2. The model

Our purpose is to study the motion of a single domain wall driven by an external magnetic field, by discrete model simulations. To this end, we start from a model that we have recently introduced [28]: as we are interested in the macroscopic response, the aim of our model is to discretize the zigzag wall into minimum segments in order to map the quasi-two dimensional problem of the wall motion in a one dimensional model (figure 1), regardless of the details of the wall internal structure, that are not expected to influence the macroscopic length scale.

We calculate the total energy of an arbitrary zigzag wall configuration, considering only the magnetostatic, the anisotropy and the disorder contributions, and the interaction with the external magnetic field:

$$E = E_m + E_{\text{an}} + E_{\text{dis}} + E_{\text{ext}}. \quad (1)$$

In equation (1), the magnetostatic term $E_m$ takes into account the interaction between the magnetization and the stray field, the anisotropy $E_{\text{an}}$ is the energy cost of the deviations from the easy axis of the material, that are associated with the Néel tail surrounding the wall [28], and $E_{\text{dis}}$ models structural disorder, impurities, defects etc.
As we have discussed in our previous paper [28], the magnetostatic interaction energy between two segments $i$ and $j$ could be approximated as

$$E_{m,ij} = 8M_s^2\epsilon^2 p^2 \mu_0 \frac{1}{r_{ij}} ,$$

(2)

where $M_s$ is the saturation magnetization, $\epsilon$ is the sample thickness, $p$ is the minimal half-period of a zigzag configuration, $\mu_0$ is the vacuum permeability and $r_{ij}$ is the distance between the centres of mass of the two segments $i$ and $j$.

The anisotropy energy term in the simple case of a uniaxial crystal can be written as

$$E_{an} = \int d^3r K_u \sin^2 \phi,$$

(3)

where $K_u$ is the in-plane uniaxial anisotropy constant and $\phi$ is the angle between the easy axis and the magnetization vector. Assuming [22] that the magnetic charge associated with the magnetization rotation is uniformly distributed over the entire band containing the wall and a linear in-plane rotation of the magnetization vector, we obtain the anisotropy energy for unit length

$$E_{an} = \epsilon K_u h c(\theta),$$

(4)

where $K_u$ is the anisotropy constant of the material, $h$ is the zigzag amplitude and $c(\theta)$ is a constant function of the zigzag angle $\theta$ which could be evaluated numerically (see [28] for more details).

In our simulations, we have set the parameters so that $E_{m}'/E_{an} = 1/2$, where $E_{an}$ is defined in equation (3) and $E_{m}' \equiv 8M_s^2\epsilon^2 p \mu_0$ (see equation (2)). This choice leads to an estimate of the typical zigzag half-period $p_{eq}$ (see [28])

$$p_{eq} \simeq \frac{16\mu_0 M_s^2 \epsilon \tan \theta}{K_u c(\theta)} .$$

We can estimate the value of $p_{eq}$ using the parameters reported in the literature for typical ferromagnetic thin films. For example, for Fe/GaAs(001) [25], we can set $\mu_0 M_s = 2$ T and $K_u = 0.5 \times 10^5$ J m$^{-3}$ and consider a thickness $\epsilon = 25$ nm and an angle $\theta = 20^\circ$. We obtain

$$p_{eq} \simeq 200 \mu m ,$$

which is in good agreement with the typical length scale inferred from magneto-optical investigations [25].

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Concerning the disorder term $E_{\text{dis}}$, we consider only quenched (frozen) disorder, that does not evolve on the timescale of the magnetization reversal. We model the disorder by an energy contribution associated with each site of our discretized sample which may be occupied by a segment (our discrete unit length) of the zigzag wall. This term is extracted from an uncorrelated random Gaussian distribution with zero mean. The variance of the disorder distribution in the present simulations is chosen to be $\Delta = 5E_m'$. This value is chosen so that disorder has an appreciable effect on the length scales considered in the simulations. As in other interface depinning problems, disorder should always be relevant on large length scales, but if its value is too small its effect could be masked by the lattice.

Finally, the energy associated with the external magnetic field $H_{\text{ext}}$, set on the easy axis direction, is given by

$$E_{\text{ext}} = -\mu_0 H_{\text{ext}} M,$$

where $M$ is the magnetization of the system.

The dynamics of the model is implemented by switching on the external magnetic field $H_{\text{ext}}$, looking for all the pairs of segments with up–down slope in the wall, i.e. the void–particle pairs, and trying to exchange the positions in the pair. This rule allows only the forward motion of pairs of segments with up–down slope, and preserves the zigzag (solid-on-solid) structure of the wall. Once a possible displacement has been attempted, we calculate the total energy difference $\Delta E$ between the starting configuration and the new one by using equation (1). The move is accepted if $\Delta E \leq 0$, and in that case we update the configuration and continue the process; otherwise we reject it. The acceptance of the move corresponds to the starting of an avalanche, that goes on until the wall comes to rest, i.e. when the minimum difference of energy $\Delta E_{\text{min}}$ over all the void–particle pairs is bigger than zero. To restart the process and eventually trigger another avalanche, we increase the external field by an amount $\Delta H_{\text{ext}} = \Delta E_{\text{min}} / \mu_0 \Delta M$, where $\Delta M$ is the variation of the magnetization due to the flip of the void–particle pair, and continue the updating. For this parallel dynamics, we can identify the number of the iterations of the updating with the physical time. At the beginning of the simulation we let the system relax at $H_{\text{ext}} = 0$ starting from an $M = 0$ configuration and then we switch on the external field and drive the sample to the positive saturation (our sample has a finite height and we use longitudinal periodic boundary conditions).

### 3. Results

From experimental magneto-optical observations (figure 2) we can infer that the dynamics of the domain wall is jerky and proceeds by avalanches, preserving the value of the zigzag angle and with a gradual increase of the zigzag period and amplitude. These features are recovered in our simulations (see figure 3): the wall may be pinned by the disorder and the depinning of a void–particle pair driven by the external magnetic field could eventually trigger an avalanche. Moreover, when the field increases, the anisotropy and disorder energy terms become less important compared with $E_{\text{ext}}$. So the relevance of the magnetostatic interactions $E_m$ increases. Since $E_m$ tends to separate magnetostatic charges (all of the same sign at the wall), it drives a coarsening of the zigzag segments, whose period thus increases (figure 3).
An avalanche is defined as the event between two pinned configurations of the domain wall. Since in our model we only allow the forward motion of the wall, we can identify unambiguously the time duration and the spatial size of an avalanche: the time duration is the number of updates from the depinning of the wall to the new pinned configuration, and the size is defined as the area affected by the magnetization reversal during an avalanche. It is thus possible to analyse the statistics of the avalanches, by constructing the probability distributions of the size $S$, the duration $T$ and the correlation length $\xi$.

As can be seen from figures 4 and 5, both the distributions show a power law behaviour for almost two decades, respectively $P(S) \sim S^{-\tau} f(S/S_0)$ for the size and $P(T) \sim T^{-\alpha} g(T/T_0)$ for the time duration probability function, and a cut-off, whose values are $S_0$ and $T_0$. The nature of the cut-off, which is an experimentally well known feature, is in our model due to finite size effects, as can be seen comparing the distributions for different system sizes. We find that the cut-off distributions scale respectively as $S_0 \sim L^D$ and $T_0 \sim L^\Delta$, where $L$ is the total length of the sample, $L = np$, and with $D \sim 2.2$ and $\Delta \sim 1.6$. In figures 4(b) and 5(b) we show that on rescaling the variables as $S/L^D$ and $T/L^\Delta$, the rescaled probability distributions $P(S)L^{-\tau}$ and $P(T)L^{-\alpha}$ obtained for the

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Figure 2. Magneto-optical experimental data: the coloured area represents the spatial difference between the two configurations before and after an avalanche (i.e. the size of the avalanche). Courtesy of Durin.

Figure 3. A zoom of four avalanches in a simulation. The coloured area represents the difference in magnetization between two successive configurations before and after an avalanche (in units of the half-period and amplitude of the zigzag).
Figure 4. (a) Probability distribution for the avalanche size $S$, for three different values of the total sample length $L$. (b) The probability distribution for the rescaled variable $S/L^D$, where $D$ is a fitted scaling exponent whose value is $\sim 2.2$. In the power law region of the plots, they collapse onto the same curve.

Figure 5. (a) Probability distribution for the avalanche duration $T$, for three different values of the total sample length $L$. (b) The probability distribution for the rescaled variable $T/L^\Delta$, where $\Delta$ is a fitted scaling exponent whose value is $\sim 1.6$. In the power law region of the plots, they collapse onto the same curve.

various system sizes collapse onto the same curve (the lack of good scaling for small values of $S$ and $T$ is due to effects of the discrete nature of the model). The power law behaviour could be characterized by the associated scaling exponents. For our model, the scaling exponents for the avalanche size and the duration distributions are respectively $\tau \sim 1.34$ and $\alpha \sim 1.55$, and do not essentially depend on the system size, or on the intensity of the anisotropy and the disorder, that may besides influence the very low $S$ or $T$ zones of the probability distributions. The value of $\tau$ is in good agreement with recent experimental results on Co polycrystalline thin films [20, 21], that give $\tau \sim 4/3$, although this reference does not provide information on the duration statistics. In these references [20, 21], the authors report direct, time-resolved domain observations of a Barkhausen avalanche obtained by a magneto-optical microscope magnetometer, which directly visualizes the

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microscopic behaviour of the avalanches in thin films. The observations are restricted to a limited zone of the sample for experimental reasons, and domain topology shows a system of tips, that could be interpreted as a part of a zigzag domain wall. Anyway, more experimental confirmations would be necessary.

From our simulations we can also study the statistics of the correlation length $\xi$, defined as the longitudinal size of an avalanche (see figure 6) and measuring the portion of the wall affected by the avalanche process. The probability distribution of the correlation lengths decays roughly as a power law with cut-off, namely $P(\xi) \sim \xi^{-\beta} h(\xi/\xi_0)$, where $\xi_0$ is the cut-off of the distribution (figure 7). Unfortunately, the quality of the correlation length statistics is not suitable for obtaining a reliable estimate of the exponent $\beta$, since even for the largest system size ($L = 400$) the scaling region spans at most one decade. This can be seen in figure 7(b) where the data do not collapse properly. The value $\beta \sim 0.4$ used to rescale the data is obtained from a fit of the $L = 400$ distribution and it has to be interpreted at most as a rough estimate. Simulations of larger system sizes would be necessary to determine more accurately the value of $\beta$.

The lack of scaling of figure 7 is probably related to the statistics of small avalanches, which display a sort of noise in the low $\xi$ region of figure 7. For small values of $\xi$, even

Figure 6. Definition of the correlation length in an avalanche. The grey zone represents the area affected by the avalanche.

Figure 7. (a) Probability distribution for the avalanches correlation length (in units of the minimum zigzag half-period $p$, for three different values of the total sample length $L$). (b) The probability distribution for the rescaled correlation length $\xi/L$. Due to the finite size of the simulations, the scaling region is to small and the three curves do not collapse properly onto the $y$ axis.
values of the correlation length $\xi = 2kp$, where $k$ is an integer, are more likely than odd values $\xi = (2k + 1)p$. This is because the elementary move in the model involves the displacement of two segments. Since avalanches are not necessarily locally connected, owing to the long range interactions, the correlation length is often obtained as the sum of distant elementary events. Therefore small avalanches will more probably involve a number of segment pairs, rather than a single cluster of adjacent reversed segments, which could possibly lead to an odd value for the correlation length. These problems stemming from the discrete nature of the model are related to the deviations observed in the data collapse of the probability distributions $P(S)$ and $P(T)$ for small $S$ and $T$ (see figures 4 and 5).

For very high values of $\xi$, we notice another deviation from the power law behaviour, this time due to the finite size of the sample. In fact the avalanches that span the whole sample could not have a correlation length bigger than the sample length $L$. Thus the probability distributions show peaks for $\xi$ values around the system size, which are just an artefact of finite simulations, such as of course the cut-off at the system length value. Thus the cut-off distribution scales as $\xi_0 \sim L$.

Another interesting issue that could be studied is the correlation between the size and the duration of the avalanches. Since avalanches with the same duration could show quite different sizes, this feature could be quantified by addressing the link between the mean size $\langle S(T) \rangle$ of an avalanche and its duration $T$ (figure 8). This function follows a power law behaviour $\langle S(T) \rangle \sim T^\gamma$ with an exponent close to $\gamma \sim 1.5$ for all the three sample sizes that we have investigated ($L = 100$, 200 and 400). We can check the consistency of the exponent $\gamma$ considering that it must be $P(S)\,dS \sim P(T)\,dT$. Using the power laws $P(S) \sim S^{-\tau}$, $P(T) \sim T^{-\alpha}$ and $\langle S(T) \rangle \sim T^\gamma$, it can be easily derived that it must be $[12, 29]$

$$\alpha = \gamma(\tau - 1) + 1.$$ 

Using $\gamma = 1.45$ and $\tau = 1.34$ we would obtain $\alpha = 1.49$ that is in reasonable agreement with the measured value $\alpha = 1.55$. Notice that $\gamma$ should also describe the scaling of the
high frequency part of the BN power spectrum [30, 31]. We could not check whether this result is valid also for our model because the signal for each loop is non-stationary and too short, being composed typically of less than ten avalanches.

4. Conclusions

The Barkhausen noise is known to be due to the jerky motion of the domain walls in a disordered material during the magnetization process. Even if the essential physics of the problem is well understood, many questions are still open. One of the most challenging related topics is the physics of ferromagnetic thin films, which displays new features with respect to bulk materials, and still remains to be fully explored both experimentally and theoretically. In this work we have applied a slightly modified version of a model that we have recently proposed [28] for the study of dynamic hysteresis for systems with zigzag domain walls. This model takes into account the contribution to the total energy of the dipolar long range interactions, the anisotropy and the disorder. The dynamics of the model describes qualitatively the experimentally observed features of the domain wall motion, like the jerky nature of the motion and the coarsening of the zigzag amplitude. We have studied the size, the duration and the correlation length of the avalanches by means of their probability distributions, via cellular automaton simulations. All these distributions show a power law behaviour and a cut-off due to finite size effects. The scaling exponents of the three distributions are derived and we find a good agreement for the value of $\tau \sim 1.34$, associated with the size statistics, with recent magneto-optical measurements on Co polycrystalline thin films [20]. Anyway, more experimental confirmations are needed, especially for the duration and the correlation length distributions, for which no experimental data are available up to now. Finally, we have investigated the link between the size and the duration of the Barkhausen avalanches, by studying the average size of an avalanche as a function of its duration $T$. Even this function follows a power law, with a scaling exponent close to 1.5.

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