Light hadron production in $B_c \to B_s^{(*)} + X$ decays

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The article is devoted to $B_c \to B_s^{(*)} + n\pi$ decays with $n = 1, 2, 3, 4$. In the framework of factorization theorem the branching fractions of these processes can be written as convolution of hard part, describing $B_c \to B_s^{(*)}W$ vertices, and spectral functions, that correspond to transition of virtual $W$-boson into a final $\pi$-meson system. These functions were obtained from the fit of experimental data on $\tau$-lepton decay and electron-positron annihilation. Using different sets of $B_c \to B_s^{(*)}W$ decay form-factors we present branching fractions and distributions over the invariant mass of the final $\pi$-meson system.

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I. INTRODUCTION

$B_c$-meson is the heaviest of the particles stable with respect to strong and electromagnetic interaction. The decays of the ground state of $(bc)$-system can be caused only by weak interaction, with either $c$-quark decays, $b$-quark decays or annihilation allowed. According to [1], the ratios of these processes are $\sim 45\%$, $37\%$ and $18\%$ respectively. Up to now only two decay modes, caused by $b$-quark decay, are observed: $B_c \to J/\psi \pi$ and semi-leptonic decay $B_c \to J/\psi \ell \nu$. With the help of the former mode $B_c$-meson mass was determined with pretty good accuracy: $m_{B_c} = 6275.6 \pm 2.9 \pm 2.5$ MeV (CDF collaboration [2]) and $m_{B_c} = 6300 \pm 14 \pm 5$ MeV (D0 collaboration [3]). The latter decay mode gives the opportunity to measure $B_c$-meson life time: $\tau_{B_c} = 0.448^{+0.038}_{-0.036} \pm 0.032$ ps (D0 collaboration [4]) and $\tau_{B_c} = 0.475^{+0.053}_{-0.049} \pm 0.018$ ps (CDF collaboration [5,6]). Both the mass of $B_c$-meson and its lifetime are in good agreement with theoretical predictions based on potential quark models and QCD sum rules [7-11].

According to perturbative QCD estimates [12], the cross section of $B_c$-meson production in hadronic experiments is about $10^{-3}$ of $B$-meson production cross section. As a result, one can expect about $10^9$ $B_c$-mesons at LHC collider luminosity $\sim 1$ fb$^{-1}$, so detailed investigation of ground $B_c$-meson and excited states of $(bc)$-family (there are 16 narrow states below $BD$-pair production threshold) would be possible.

In our previous work [13] we considered $B_c$-meson decays with $b$-quark as a spectator, namely the reactions $B_c \to J/\psi + X$, where $X$ stands for lepton pairs, light quarks $ud$ or $\pi$-meson system $n\pi$ for $1 \leq n \leq 4$. In the framework of factorization theorem the amplitude of this process can be written as a convolution of $B_c \to J/\psi W$-decay width and spectral functions that describe the transition of virtual $W$-boson into final $\pi$-mesons state. These spectral functions do not depend on $W$-boson production mechanism, so they can be determined from experimental and theoretical analysis of other reactions, for example $\tau$-lepton decay $\tau \to \nu_\tau + X$ or electron-positron annihilation $e^+e^- \to X$. In the current work we consider induced by $c$-quark weak decay reactions $B_c \to B_s^{(*)} + X$. In contrast to $B_c \to J/\psi + X$ decays, these processes are Cabibbo-allowed, so their branching fractions are about an order higher than the branching fractions of $B_c \to J/\psi + X$ decays. Form theoretical point of view these branching fractions are determined, from one side, by effective hamiltonian of weak $c \to s$ decay, where all higher-order QCD corrections are taken into account, and, on the other hand, by form-factors of $B_c \to B_s^{(*)}$ transitions. These form-factors can be obtained in different models, for example QCD sum rules [11,14,15], potential quark models [1,16,17,18], light-front approach [19,20]. Calculated with these form factors widths of $B_c \to B_s^{(*)} + X$ decays differ from one another and, since considered here decays are dominant, this leads to different predictions for $B_c$-meson lifetime. As we have mentioned above, the lifetime of $B_c$-meson is known experimentally with pretty good accuracy, so this difference can be used to select physical set of form-factors.

In the next section we present analytical expressions for transferred momentum distributions of branching fractions of the decays $B_c \to B_s^{(*)} + X$, where $X$ is lepton pair $\ell\nu$ or light meson system. These distributions are expressed through spectral functions of the final state $X$. In section III explicit expressions of spectral functions, obtained from analysis of $\tau$-lepton decay $\tau \to \nu_\tau + X$ and electron-positron annihilation $e^+e^- \to X$, are given. Using these

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expressions we give predictions for branching fractions of the decays $B_c \rightarrow B_s^{(*)} + n\pi$ and distributions over the squared momentum of light mesons system. In the last section of our paper we give the conclusion.

II. ANALYTICAL RESULTS

Decays $B_c \rightarrow B_s^{(*)} + n\pi$ are caused mainly by weak $c$-quark decay $c \rightarrow sW^+ \rightarrow u\bar{d}$ (see diagram shown in fig. 1), while $b$-quark is a spectator. Effective Lagrangian of this process has the form

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cs} V_{ud}^* [C_+ (\mu) O_+ + C_- (\mu) O_-],$$

where $G_F$ is Fermi coupling constant, $V_{ij}$ are CKM mixing matrix elements and $C_\pm (\mu)$ are Wilson coefficients, that describes higher-order QCD corrections. The operators $O_\pm$ are defined according to

$$O_\pm = (\bar{d}_i u_j)_{V-A} (\bar{c}_i b_j)_{V-A} \pm (\bar{d}_j u_i)_{V-A} (\bar{c}_i b_j)_{V-A},$$

where $i, j$ are colour indexes of quarks and $(\bar{q}_1 q_2)_{V-A} = \bar{q}_1 \gamma_\mu (1 - \gamma_5) q_2$. Since in the considered here decays the light-quark pair is in colour-singlet state, the amplitude of these processes should be proportional to factor

$$a_1 (\mu) = \frac{1}{2N_c} [(N_c - 1) C_+ (\mu) + (N_c + 1) C_- (\mu)].$$

If QCD corrections are neglected, this function is equal to $a_1 (\mu) = 1$. Due to higher-order logarithmic corrections the dependence of this coefficient on renormalisation scale $\mu$ appears \[10\], and on $\mu \sim m_b$ it is equal to \[10\]

$$a_1 (m_b) = 1.2.$$

The matrix element of the decay $B_c \rightarrow B_c^{(*)} + \mathcal{R}$, where $\mathcal{R}$ is some set of light hadrons, has the form

$$\mathcal{M} \left[ B_c^+ \rightarrow B_c^{(*)} W^+ \rightarrow B_s^{(*)} + \mathcal{R} \right] = \frac{G_F V_{cs}}{\sqrt{2}} a_1 \mathcal{H}_\mu \epsilon_\mu^{\mathcal{R}},$$

where $\epsilon^{\mathcal{R}}$ is the effective polarization vector of light hadron system $\mathcal{R}$, and $\mathcal{H}$ vertex is
\[ \mathcal{H}_\mu = f_+ (q^2) p_\mu + f_- (q^2) q_\mu \]

for \( B_c \)-meson decay to pseudo-scalar \( B_s \)-meson and

\[ \mathcal{H}_\mu = \epsilon_\mu F_0^A (q^2) + (\epsilon p_1) p_\mu F_+^A (q^2) + (\epsilon_1 q_\mu F_-^A (q^2) - i\epsilon^{\mu \nu \alpha \beta} \epsilon_\nu p_\alpha q_\beta F_V (q^2) \]

for \( B_c \)-meson decay to vector \( B_s^* \)-meson. In these expressions \( p_{1,2} \) are the momenta of \( B_c \) and \( B_s^{(*)} \)-mesons respectively, \( p = p_1 + p_2 \), \( q = p_1 - p_2 \) is the momentum of virtual \( W \)-boson, and \( f_\pm (q^2) \), \( F_0^A (q^2) \) and \( F_V (q^2) \) are the form-factors of \( B_c \rightarrow B_s^{(*)} W^* \) transition.

From presented above amplitudes it is easy to calculate the widths of the \( B_c \rightarrow B_s^{(*)} + \mathcal{R} \) decays:

\[ \frac{d\Gamma}{dq^2} (B_c \rightarrow B_s^{(*)} + \mathcal{R}) = \frac{1}{2M} \frac{G_F^2 V_{cb}^2}{2} a_1^2 \mathcal{H}_\mu \mathcal{H}_\nu \epsilon_\mu \epsilon_\nu \mathcal{F} (B_c \rightarrow B_s^{(*)} + \mathcal{R} \rightarrow \mathcal{R}) \]

where \( M = M_{B_s} \) is the mass of initial \( B_c \)-meson, and Lorentz-invariant phase space is defined according to

\[ d\Phi (Q \rightarrow k_1 \ldots k_n) = (2\pi)^4 \delta^4 (Q - \sum_i k_i) \prod_i \frac{d^3 k_i}{(2\pi)^3 2E_i} . \]

The following recurrence relation holds for this expression:

\[ d\Phi (B_c \rightarrow B_s^{(*)} + \mathcal{R}) = \frac{d\Phi (B_c \rightarrow B_s^{(*)} W^*) \mathcal{F} (W^* \rightarrow \mathcal{R})}{d\Phi (W^* \rightarrow \mathcal{R})} , \]

Using it one can perform the integration over the phase space of light hadron system \( \mathcal{R} \):

\[ \frac{d\Gamma}{dq^2} (B_c \rightarrow B_s^{(*)} + \mathcal{R}) = \frac{1}{2\pi} \int d\Phi (W^* \rightarrow \mathcal{R}) \epsilon_\mu^R \epsilon_\nu^R (q_\mu q_\nu - q^2 g_{\mu \nu}) \mathcal{F} (q^2) + q_\mu q_\nu \rho_{R,L} (q^2) \right). \]

In the framework of factorization theorem introduced here spectral functions \( \rho_{R,L} (q^2) \) are universal, so they can be determined from theoretical and experimental analysis of other reactions, for example \( \tau \)-lepton decay \( \tau \rightarrow \nu + \mathcal{R} \) or electron-positron annihilation \( e^+ e^- \rightarrow \mathcal{R} \). Explicit expressions of these spectral functions for different final states \( \mathcal{R} \) are presented in the next section.

From presented above matrix elements it is easy to obtain squared transferred momentum distributions for the considered in our article decays. In the case of pseudo-scalar \( B_s \)-meson we have

\[ \frac{d\Gamma}{dq^2} (B_c \rightarrow B_s + \mathcal{R}) = \frac{G_F^2 V_{cb}^2 a_1^2}{32\pi M} \beta \left\{ |f_+|^2 \left[ M^4 \beta^2 + (M^2 - m^2)^2 \rho_T^R \right] + |f_-|^2 q^2 + 2Re \left[ f_+ f_-^* \right] q^2 \right\} , \]

where \( M \) and \( m \) are the masses of \( B_c \)- and \( B_s^{(*)} \)-mesons respectively, and

\[ \beta = \sqrt{\frac{(M - m)^2 - q^2}{M^2}} + \sqrt{\frac{(M + m)^2 - q^2}{M^2}} \]

is the velocity of \( B_s \)-meson in \( B_c \)-meson rest frame. In the case of vector \( B_s \)-meson in the final state the distribution has the form
for example quark models [10], covariant light-front models [26], etc. These form-factors can be parametrised in different forms, for example:

1. monopole expression

\[ F_i(q^2) = \frac{F(0)}{1 - q^2/m^2_{\text{pole}}}, \]

2. Isgur-Wise function \( \xi(w) \)

3. exponential parametrization, suitable for potential models, where form-factors are determined by integral of initial and final quarkonia wave-functions, that have Gaussian from:

\[ F_i(q^2) = F(0) \exp \left\{ c_1 q^2 + c_2 q^4 \right\}. \]

All parametrizations are almost equivalent and in our article we use this the exponential parametrization \(^{[1]}\) for all sets of form-factors. Numerical values of the parameters \( F(0), c_{1,2} \) for different from-factors stets are given in table \([1]\)

### III. NUMERICAL RESULTS

When inclusive decays \( B_c \rightarrow B_s^{(*)} u \bar{d} \) are consideres, corresponding spectral functions are equal to

\[ \rho^{ud}_F(q^2) = 0, \quad \rho^{ud}_T(q^2) = \frac{1}{2\pi^2}. \]

Branching fractions of these decays for listed above form-factor sets are

\[ \text{B}_{\text{SR}}(B_s u \bar{d}) = 19\%, \quad \text{B}_{\text{PM}}(B_s u \bar{d}) = 13\%, \quad \text{B}_{\text{LF}}(B_s u \bar{d}) = 6.5\%, \]
\[ \text{B}_{\text{SR}}(B_s^{(*)} u \bar{d}) = 20\%, \quad \text{B}_{\text{PM}}(B_s^{(*)} u \bar{d}) = 23\%, \quad \text{B}_{\text{LF}}(B_s^{(*)} u \bar{d}) = 13\%. \]
The branching fractions of semileptonic decays \( B_c \to B_s^{(*)} \ell \nu \) can be obtained from these values by simple substitution

\[
\rho^\ell_\nu (q^2) = 0, \quad \rho_T^{\ell \nu} (q^2) = \frac{1}{N_e a_\ell^2} \rho_T^{ud} (q^2).
\]

Let us now consider \( B_c \to B_s^{(*)} \pi^+ \) decay. The \( W \to \pi \) vertex is written as

\[
\langle \pi^+ | J_\mu | W \rangle = f_\pi q_\mu,
\]

where \( q_\mu \) is \( \pi \)-meson momentum and constant \( f_\pi \) can be determined, for example, from the width of lepton decay \( \pi^+ \to e^+\nu + e^- \). Using experimental value for the width of this decay we obtain \( f_\pi \approx 130 \text{ MeV} \). Spectral functions, that correspond to vertex (3) are

\[
\rho_L^\pi (q^2) = f_\pi^2 \delta (q^2 - m_\pi^2), \quad \rho_T^\pi (q^2) = 0.
\]

The values of \( B_c \to B_s^{(*)} \pi \) branching fractions for listed in the previous section form-factor sets are

\[
\text{Br}_{SR} (B_s \pi) = 18\%, \quad \text{Br}_{PM} (B_s \pi) = 12\%, \quad \text{Br}_{LF} (B_s \pi) = 5.5\%,
\]

\[
\text{Br}_{SR} (B_s^* \pi) = 7\%, \quad \text{Br}_{PM} (B_s^* \pi) = 9.4\%, \quad \text{Br}_{LF} (B_s^* \pi) = 6.2\%.
\]

It is clearly seen, that the difference in form-factors leads difference in the branching fractions of these decays. All these branching fractions, in the other hand, are about an order of magnitude higher, that the branching fractions of \( B_c \to J/\psi \pi \) decay [13]. The reason is that on the quark level \( B_c \to B_s^{(*)} + R \) decays are caused by Cabbibo-allowed \( c \to s u d \) decay, while \( B_c \to J/\psi + R \) decays are Cabbibo-suppressed.

If there are two \( \pi \)-mesons in the final state, the main decay mode would be \( B_c \to B_s^{(*)} \rho^+ \to B_s^{(*)} \pi^+ \pi^0 \). The vertex of \( \rho \)-meson interaction with \( W \)-boson has the form

\[
\langle \rho^+ | J_\mu | W \rangle = f_\rho m_\rho \epsilon_\mu,
\]

where \( m_\rho \) and \( \epsilon_\mu \) are \( \rho \)-meson mass and polarization vector, while \( f_\rho \)-constant is \( f_\rho \approx 210 \text{ MeV} \). In the limit of zero \( \rho \)-meson widths the spectral functions \( \rho_L^{\rho T} (q^2) \) are equal to

\[
\rho_L^{\rho} (q^2) = 0, \quad \rho_T^{\rho} (q^2) = f_\rho^2 \delta (q^2 - m_\rho^2).
\]

These spectral functions lead to following branching fractions of \( B_c \to B_s^{(*)} \rho \) decays:

\[
\text{Br}_{SR} (B_s \rho) = 7.6\%, \quad \text{Br}_{PM} (B_s \rho) = 5.4\%, \quad \text{Br}_{LF} (B_s \rho) = 3.1\%,
\]

\[
\text{Br}_{SR} (B_s^* \rho) = 21\%, \quad \text{Br}_{PM} (B_s^* \rho) = 22\%, \quad \text{Br}_{LF} (B_s^* \rho) = 15\%.
\]

As in previous case, these values exceed significantly the branching fractions of \( B_c \to J/\psi \rho \) decay.

It should be noted, however, that in contrast to \( B_c \to J/\psi + 2\pi \) decay, for \( B_c \to B_s^{(*)} + 2\pi \) decays it is not valid to neglect the width of \( \rho \)-meson. Because of the small mass difference \( M_{B_s} - M_{B_s} \approx 1 \text{ GeV} \), \( \rho \)-meson is almost at the end of the phase space, so its non-zero width changes significantly \( q^2 \)-distributions for these decays. For this reason one should use more realistic parametrization for spectral functions \( \rho_{L,T}^{2\pi} (q^2) \). Due to vector current conservation the longitudinal spectral function \( \rho_L^{2\pi} (q^2) = 0 \). The information on transverse spectral function can be obtained from analysis of \( \tau \)-lepton decay \( \tau \to \nu_\tau + 2\pi \). For this decay \( q^2 \)-distribution has the form

\[
\frac{d\Gamma (\tau \to \nu_\tau R)}{dq^2} = \frac{G_F^2}{16\pi m_\tau} \left( \frac{m^2_{\tau} - q^2}{m^2_\tau} \right) \left( \frac{m^2_\tau + q^2}{m^2_\tau} \right) \rho_T^{2\pi} (q^2),
\]

where \( m_\tau \) is \( \tau \)-lepton mass, and, in the framework of factorization theorem, spectral function \( \rho_T^{2\pi} (q^2) \) is universal, that is independent on \( \pi \)-pair production dynamics. The authors of paper [27] give the parametrization for this
Because of partial conservation of axial current longitudinal spectral function $\tau$, the reason for large difference between (4), (5) and (7), (8) branching fractions. In our article we use a simpler parametrisation, obtained from fit of experimental distributions of the branching fractions of $B_c \rightarrow B_s + \pi$ decay, however, could be interesting from experimental point of view, spectral function with $\rho$, $\rho'$- and $\omega$-meson contributions taken into account. In our article we use a more simple parametrisation

$$\rho^{2\pi}_T(s) = 1.35 \times 10^{-3} \left( \frac{s - 4m^2}{2} \right)^2 \frac{1 + 0.64s}{(s - 0.57)^2 + 0.013}$$

where $s$ is measured in GeV$^2$ (the spectral function itself is dimensionless).

Distributions of $B_c \rightarrow B_s^{(*)} + 2\pi$-decays branching fractions over squared transferred momentum $q^2$ for listed in the previous section form-factor sets are presented in fig.2. Solid, dashed and dash-dotted lines in this figure correspond to form-factor sets “SR”, “QM” and “LF” respectively. Corresponding branching fractions are

$$\text{Br}_{SR} (B_s^{(*)} \pi) = 6.1\%, \quad \text{Br}_{PM} (B_s^{(*)} \pi) = 4.3\%, \quad \text{Br}_{LF} (B_s^{(*)} \pi) = 2.4\%,$$

(7)

$$\text{Br}_{SR} (B_s^{(*)} \pi) = 13\%, \quad \text{Br}_{PM} (B_s^{(*)} \pi) = 14\%, \quad \text{Br}_{LF} (B_s^{(*)} \pi) = 8.3\%.$$  

(8)

One can easily see that these branching fractions are smaller than presented above values (4), (5). In fig.3 we present $q^2$-distributions of $B_c \rightarrow B_s^{(*)} + 2\pi$ decay (solid line) and $B_c \rightarrow B_s^{(*)} + u\bar{d}$ (dashed line. In this case the spectral function does not depend on $q^2$, see eq. (2)). Experimental value of $\rho$-meson mass is shown on this figure by vertical lines. From this figure it is clear, that in the region $q^2 \approx (m_\rho + \Gamma^2/2)^2$, where spectral function is significantly non-zero, the distributions $d\Gamma (B_c \rightarrow B_s^{(*)} + u\bar{d})/dq^2$ vary strongly, so one cannot neglect $\rho$-meson width in this case. This is the reason for large difference between (4), (5) and (7), (8) branching fractions.

If there are three $\pi$-mesons in the final state the main production mode would be $a_1 \rightarrow \rho \pi \rightarrow 3\pi$. There are two possible charge configurations ($\pi^+ \pi^- \pi^+$ and $\pi^+ \pi^- \pi^0$), and in our article $3\pi$ stands for sum of these final states. Because of partial conservation of axial current longitudinal spectral function $\rho_T^{3\pi}(q^2)$ could be set equal to zero. In paper [27] the parametrization of transverse spectral function $\rho_T^{2\pi}(q^2)$, expressed through mass and width of $a_1$-meson is proposed. In our article we use a simpler parametrisation, obtained from fit of experimental $q^2$ distribution of $\tau \rightarrow \nu_{\tau} + 3\pi$ decay:

$$\rho^{3\pi}_T(s) = 5.86 \times 10^{-5} \left( \frac{s - 9m^2}{8} \right)^4 \frac{1 + 190s}{(s - 1.06)^2 + 0.48}.$$

where $s$ is measured in GeV$^2$. Distributions of $B_c \rightarrow B_s^{(*)} + 3\pi$ branching fractions over $q^2$ for different stets of form-factors are shown in fig.3. Corresponding branching fractions are equal to

$$\text{Br}_{SR} (B_s^{(*)} 3\pi) = 0.096\%, \quad \text{Br}_{PM} (B_s^{(*)} 3\pi) = 0.068\%, \quad \text{Br}_{LF} (B_s^{(*)} 3\pi) = 0.039\%,$$

$$\text{Br}_{SR} (B_s^{(*)} 3\pi) = 0.23\%, \quad \text{Br}_{PM} (B_s^{(*)} 3\pi) = 0.24\%, \quad \text{Br}_{LF} (B_s^{(*)} 3\pi) = 0.16\%.$$  

One can clearly seen, that these values are significantly smaller, than presented above branching fractions of the decays $B_c \rightarrow B_s^{(*)} + \pi$, $B_c \rightarrow B_s^{(*)} + 2\pi$. This decays, however, could be interesting from experimental point of view.

![Figure 2: Distributions over $q^2$ of the branching fractions of $B_c \rightarrow B_s + 2\pi$ decay (left figure) and $B_c \rightarrow B_s^{(*)} + 2\pi$ decay (right panel). Solid, dashed and dash-dotted lines in these figure correspond to “SR”, “QM” and “LF” form-factor sets respectively.](image-url)
Figure 3: Distributions of branching fractions of $B_c \rightarrow B_s + 2\pi$, $B_c \rightarrow B_s + u\bar{d}$ decays (left panel) and $B_c \rightarrow B_s^* + 2\pi$, $B_c \rightarrow B_s^* + u\bar{d}$ (right panel) over the squared transferred momentum. On both figures solid and dashed lines line correspond to $\pi\pi$ and $u\bar{d}$ final states respectively. Experimental value of $\rho$-meson mass is shown by vertical lines.

Figure 4: Distributions of $B_c \rightarrow B_s^{(*)} + 3\pi$ branching fractions over squared momentum of $3\pi$-meson system. Notations are same as in fig.2.

since in $\pi^+\pi^-\pi^+$ charge configuration $\pi^0$-meson, whose registration could be problematic, is absent. Moreover, in contrast to $B_c \rightarrow B_s^{(*)} + \pi$, $B_c \rightarrow B_s^{(*)} + 2\pi$ decays, the branching fractions of $B_c \rightarrow B_s^{(*)} + 3\pi$ decay are smaller than the corresponding branching fractions of $B_c \rightarrow J/\psi + 3\pi$ decay. The reason is that for $B_s^{(*)}$-meson in the final state the masses of initial and final mesons are rather close, so phase-space suppression compensates the enhancement caused by CKM matrix element.

If there are four $\pi$-mesons in the final state two charge configurations are possible: $\pi^+\pi^-\pi^0\pi^0$ and $\pi^+\pi^0\pi^0\pi^0$, in what follows 4$\pi$ stands for sum of these configurations. Due to vector colour conservation the longitudinal spectral function $\rho^{4\pi}_L(q^2) = 0$. Information about traverse spectral function $\rho^{4\pi}_T(q^2)$ is available from experimental data on $\tau \rightarrow \nu_\tau + 4\pi$ decay (see eq.(6) ) or electron-positron annihilation $e^+e^- \rightarrow 4\pi$. Cross section of the latter reaction is

$$\sigma(e^+e^- \rightarrow 4\pi) = \frac{4\pi\alpha^2}{s} \rho^{4\pi}_T(s).$$

In both cases we have similar form of the spectral function, that can be parametrized by the expression

$$\rho^{4\pi}_T(s) \approx 1.8 \times 10^{-4} \left( \frac{s - 16m_\pi^2}{s} \right) \frac{1 - 5.07s + 8.63s^2}{[(s - 1.83)^2 + 0.61]^2}.$$

In fig.5 $q^2$-distributions of $B_c \rightarrow B_s^{(*)} + 4\pi$ decay branching fractions for different form-factor sets are shown. Numerical
place between charmonia (¯c¯c-mesons) and bottomonia (¯b¯b-mesons), so they can be used for independent check of

These values are significantly smaller then presented above branching fractions of one, two, or three π-meson production, as well as branching fractions of $B_c \rightarrow J/\psi + 4\pi$ decay. The reason is mentioned above phase-space suppression in $B_c \rightarrow B_s^{(*)} + 4\pi$ reaction. Thus we can see, that for all form-factor sets dominant decay mode $c \rightarrow s$ is saturated by $B_c \rightarrow B_s^{(*)} \pi, \rho$ decays.

In table IV branching obtained in our article fractions of $B_c \rightarrow B_s^{(*)} + \mathcal{R}$ decays for different form-factor sets are listed. It is interesting to compare these results with simple estimates based on duality relations. According to paper [1] $\bar{c}$-meson decays with spectator $b$-quark takes about 45% of total $B_c$-meson width. These decays are almost saturated by $B_c \rightarrow B_s^{(*)} + X$ decays, so the sum of $B_c \rightarrow B_s^{(*)} + n\pi$ branching fractions should be about this value. It can be easily seen, that this is true for “SR” and “PM” form-factor sets (the corresponding sum for them is 44% and 40% respectively). For “LF” form-factors set this sum is significantly smaller (∼23%). From table IV it can be seen also, that for every decay mode the branching fraction obtained with “LF” form-factors sets is smaller then those obtained with “SR” and “PM” form-factors sets. As a result, the total width of $B_c \rightarrow B_s^{(*)} + X$ inclusive decay (and, hence, lifetime of $B_c$-meson) would contradict experimental value.

\begin{table}[!h]
\centering
\begin{tabular}{|c|c|c|c|c|}
\hline
 & $\mathcal{R}$ & SR [10] & PM [10] & LF [26] \\
\hline
$\pi$ & 18 & 12 & 5.5 & 18 & 12 & 5.5 \\
$\rho$ & 7.6 & 5.4 & 3.1 & 7.6 & 5.4 & 3.1 \\
$2\pi$ & 6.1 & 4.3 & 2.4 & 6.1 & 4.3 & 2.4 \\
$3\pi$ & 0.096 & 0.068 & 0.039 & 0.096 & 0.068 & 0.039 \\
$4\pi$ & 0.0064 & 0.0045 & 0.0026 & 0.0064 & 0.0045 & 0.0026 \\
\hline
$ud$ & 19 & 13 & 6.5 & 19 & 13 & 6.5 \\
\hline
\end{tabular}
\caption{Branching fractions of $B_c \rightarrow B_s^{(*)} + \mathcal{R}$ decays}
\end{table}

values of these branching fractions are

\begin{align*}
\text{Br}_{\mathcal{R}} (B_s 4\pi) &= 0.0064\%, \quad \text{Br}_{\mathcal{PM}} (B_s 4\pi) = 0.0045\%, \quad \text{Br}_{\mathcal{LF}} (B_s 4\pi) = 0.0026\%, \\
\text{Br}_{\mathcal{SR}} (B_s^{(*)} 4\pi) &= 0.015\%, \quad \text{Br}_{\mathcal{PM}} (B_s^{(*)} 4\pi) = 0.016\%, \quad \text{Br}_{\mathcal{LF}} (B_s^{(*)} 4\pi) = 0.01\%.
\end{align*}

IV. CONCLUSION

The article is devoted to study of exclusive $B_c$-meson decays with production of $B_s^{(*)}$-meson an light hadron system $n\pi$ with $n = 1, 2, 3$ or 4.

$B_c$-mesons, that is particles that in valence approximation are build from $c$- and $b$-quarks, take the intermediate place between charmonia ($\bar{c}\bar{c}$-mesons) and bottomonia ($\bar{b}\bar{b}$-mesons), so they can be used for independent check of
Previous works were mainly devoted to two-particle decays of $B_c$-meson (see, for example, [1, 2, 8, 12, 23, 31]). In our recent article [13] we, on the contrary, consider decays $B_c \rightarrow J/\psi + n\pi$ with $n = 1, 2, 3$ or 4. In the framework of factorization theorem the branching fractions of these decays are written as convolution of hard part, describing $B_c \rightarrow J/\psi W$ decay and spectral functions, responsible for $W \rightarrow n\pi$ transition. Form factors of $B_c \rightarrow J/\psi W$ vertex can be determined from different theoretical models (QCD sum rules, potential models, light-front covariant quark models, etc), spectral functions — from analysis of $\tau$-lepton decays and electron-positron annihilation.

In the present article we consider in the same approach the decays $B_c \rightarrow B_s^{(*)} + n\pi$ with $n = 1, \ldots , 4$. Using different sets of $B_c \rightarrow B_s^{(*)} W$ vertex form-factors we calculated branching fractions of these decays and distributions over the squared momentum of $\pi$-mesons system. In contrast to $B_c \rightarrow J/\psi + n\pi$ decays these reactions are Cabibbo-allowed, so for one and two $\pi$-mesons in the final state their branching fractions are greater, than the branching fractions of corresponding $B_c \rightarrow J/\psi + n\pi$ decays. If the number of $\pi$-mesons in the final state is larger, the suppression caused by small phase-space in the reaction $B_c \rightarrow B_s^{(*)} + X$ is important and $B_c \rightarrow J/\psi + X$ decays dominate.

We would like to note, that considered in our article decays are suitable for $B_c$-meson study at hadron colliders. Besides large branching fractions there are also small background processes to these decays.

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