Fermion mass and mixing patterns from a rotating mass matrix

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Abstract

It is shown that all existing data on mixing between up and down fermion states (i.e. CKM matrix and neutrino oscillations) and on the hierarchical quark and lepton mass ratios between generations are consistent with the two phenomena being both consequences of a mass matrix rotating in generation space with changing energy scale. As a result, the rotation of the mass matrix can be traced over some 14 orders of magnitude in energy from the mass scale of the $t$-quark at 175 GeV to below that of the atmospheric neutrino at 0.05 eV. This is a summary of recent work done in collaboration with Chan Hong-Mo and José Bordes.
By a rotating mass matrix we mean that the mass matrix of a fermion species, for example any one of $U$ the up quarks ($t, c, u$), $D$ the down quarks ($b, s, c$), $L$ the charged leptons ($\tau, \mu, e$) and $N$ the neutrinos ($\nu_\tau, \nu_\mu, \nu_e$), not only has scale-dependent eigenvalues but also eigenvectors which depend on scale. This last property means that the eigenvectors change orientation or rotate in generation space.

This is not just a hypothetical situation, because already in the Standard Model the renormalization group equation for the $U$ mass matrix has a term

$$16\pi^2 \frac{dU}{dt} = -\frac{3}{2} DD^\dagger U + \ldots, \tag{1}$$

where the factor $D^\dagger U = V$ is the CKM quark mixing matrix. Now this matrix is experimentally found to be non-diagonal, meaning that even if $U$ is diagonal at a certain scale, it will become non-diagonal as a result of running to a different scale, at which when it is re-diagonalized the eigenvector will have rotated from its previous orientation. However, given that the off-diagonal elements of the CKM matrix are small, this effect can usually be neglected in practice in the Standard Model. This may very well not be the case in extensions of the Standard Model, where other forces may contribute to the mass matrix rotation to an observable extent.

In this note we shall turn the argument around and seek experimental evidence of a rotating mass matrix, in the manner we shall now detail.

There are two well established experimental facts which the Standard Model does not explain, namely fermion mass hierarchy and fermion mass mixing. The masses of the up and down quarks, and the charged fermions differ by more than 1–2 orders of magnitude as one goes from one generation to the next \cite{2}, as can be seen in Table 1. The absolute values of the various CKM matrix elements have been measured to a good accuracy \cite{2}:

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 0.9742 - 0.9757 & 0.219 - 0.226 & 0.002 - 0.005 \\ 0.219 - 0.225 & 0.9734 - 0.9749 & 0.037 - 0.043 \\ 0.004 - 0.014 & 0.035 - 0.043 & 0.9990 - 0.9993 \end{pmatrix}$$

The absolute values of the leptonic MNS mixing matrix, on the other hand, are not so well determined at present, but from neutrino oscillation experiments \cite{3} we know that the mixing is in general considerably larger than the corresponding CKM mixing. The experimental estimates are:

$$\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} = \begin{pmatrix} \ast & 0.4 - 0.7 & 0.0 - 0.15 \\ \ast & \ast & 0.56 - 0.83 \\ \ast & \ast & \ast \end{pmatrix}$$

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Since the Standard Model gives no explanation for these two very remarkable experimental facts, one is tempted to seek other origins for them, and if one could find a single mechanism which could give rise to both, it would certainly provide a very interesting and promising avenue of study \[4\].

We now make the following rotating hypothesis:

The rotating mass matrix gives rise to all observed mixing and lower generation masses.

We note as a side remark that this hypothesis is made and implemented in the Dualized Standard Model \[5\], from which we obtain good agreement with experiment\[4\].

So we start with a mass matrix which is of rank 1, and which remains factorized on running, so that at any scale we have only one non-zero eigenvalue

1\[Specifically, with 3 real parameters fitted to data, it gives correctly to within present experimental bounds the following measured quantities: the mass ratios \(m_c/m_t\), \(m_s/m_b\), \(m_\mu/m_\tau\), all 9 elements \(|V_{rs}|\) of the CKM matrix, plus the 2 elements \(|U_{\mu3}|\) and \(|U_{e3}|\) of the MNS matrix. It gives further by interpolation sensible though inaccurate estimates for the following: the mass ratios \(m_u/m_t\), \(m_d/m_b\), \(m_e/m_\tau\) and the solar neutrino angle \(U_{e2}\). Moreover, numerous detailed predictions have been made, and tested against data, in flavour-violation effects over a wide area comprising meson mass differences, rare hadron decays, \(e^+e^-\) collisions, and muon-electron conversion in nuclei, with further predictions on effects as far apart in energy as neutrinoless double-beta decays in nuclei and cosmic ray air showers beyond the GZK cut-off of \(10^{20}\) eV at the extreme end of the present observable energy range.\]
Thus the whole mass matrix can be encapsulated by one single normalized rotating vector $v = (\xi, \eta, \zeta)$ (without loss of generality we may assume $\xi \geq \eta \geq \zeta$). Note that the eigenvalue $m_T$ depends on the fermion species $T = U, D, L, N$.

Care has to be taken in defining physical masses and states when the mass matrix runs with scale. In our case when the mass matrix is given by a single vector we can adopt the following procedure [6] which guarantees that:

- lepton state vectors are always orthogonal,
- mixing matrix is always unitary,
- mass hierarchy is automatic.

First we run $m$ to a scale $\mu$ such that $\mu = m_T(\mu)$; this value we can reasonably call the mass of the highest generation. To fix ideas, let us concentrate on the $U$ type quarks $t, c, u$. The corresponding eigenvector $v_t$ is then the state vector of the $t$ quark. Having fixed this, we now know that the $c$ and $u$ lie in the 2-dimensional subspace $V$ orthogonal to $v_t$. As we go down in scale the eigenvector $v$ rotates so that $V$ is no longer the null eigenspace, and the projection of $m$ onto $V$ is a $2 \times 2$ matrix, again of rank 1. We now repeat the procedure for this submatrix and determine both the mass and the state of the $c$ quark (Figure 1). Once the $c$ state is determined, we know the $u$ state as well, as being the third vector of the orthonormal triad $(t, c, u)$. The $u$ mass is similarly determined.

The above procedure can be repeated for the other three types of fermions.$^2$ The direction cosines of the two triads $(t, c, u)$ and $(b, s, d)$, in other words, the 9 inner products, will give us the CKM matrix, Figure 2. Similarly for the MNS mixing matrix of the leptons.

The above procedure takes a much simpler form when we have only two generations, and we shall in fact study this case first, as a planar approximation for the full three generation case, to better illustrate the methodology and the results [4].

$^2$Neutrinos will need further special treatment which we shall omit here. Instead see reference [4].
Figure 1: The state vectors of the 3 physical states of the $U$ type quark.

Figure 2: Two triads of state vectors for the quarks on the trajectory.
Consider the up quarks first. When applied to this simple case, the procedure detailed above for defining masses and state vectors of flavour states gives the state vector $v_t$ of $t$ as the single massive eigenstate $v$ of the $U$-quark mass matrix at the scale $\mu = m_t$, and the vector $v_c$ as a vector orthogonal to $v_t$, as depicted in Figure 3. But this should not be interpreted to mean that $c$ has zero mass, because $m_c$ is given by the nonzero eigenvalue of the mass submatrix, now only $1 \times 1$, and is hence just the expectation value of $m$ in the state vector $v_c$, namely

$$m_c = m_t \sin^2 \theta_{tc} \neq 0,$$

where $\theta_{tc}$ is the rotation angle between the scales $\mu = m_t$ and $\mu = m_c$. Similarly, although the mass matrices of the $U$ and $D$ quarks are aligned in orientation at all scales, one sees from Figure 4 that by virtue of the rotation of the vector $v$ from the scale $\mu = m_t$ to the scale $\mu = m_b$ where the state vectors $v_t$ and $v_b$ are respectively defined, the two state vectors will not point in the same direction, having rotated by an angle $\theta_{tb}$ between the two scales:

$$V_{tb} = v_t \cdot v_b = \cos \theta_{tb} \neq 1.$$

Other fermion species are similar.

Using (3) and (4), and Figures 3 and 4 and similar ones for the other fermions, we obtain the following relations:

$$V_{tb} = \cos \theta_{tb}, \quad |V_{ts}| = |V_{cb}| = \sin \theta_{tb},$$

Figure 3: Masses for lower generation fermions via the “leakage” mechanism.
Figure 4: Mixing between up and down fermions.

from the mixing, and from the masses

\[ m_c/m_t = \sin^2 \theta_{tc}, \quad m_s/m_b = \sin^2 \theta_{bs}, \quad m_\mu/m_\tau = \sin^2 \theta_{\tau\mu}. \]  

(6)

Inputting from data \[2\] on mixing

\[ |V_{tb}| = 0.9990 - 0.9993 \quad |V_{ts}| = 0.035 - 0.043, \quad |V_{cb}| = 0.037 - 0.043, \]  

(7)

and on masses for up quarks

\[ m_t = 174.3 \pm 5.1 \text{ GeV}, \quad m_c = 1.15 - 1.35 \text{ GeV}, \]  

(8)

for down quarks

\[ m_b = 4.0 - 4.4 \text{ GeV}, \quad m_s = 75 - 170 \text{ MeV}, \]  

(9)

and for charged leptons

\[ m_\tau = 1.777 \text{ GeV}, \quad m_\mu = 105.66 \text{ MeV}; \]  

(10)

we obtain compatible values for

\[ \theta_{tb} = 0.0374 - 0.0447, \quad 0.0350 - 0.0430, \quad 0.0370 - 0.0430 \]  

(11)

and

\[ \theta_{tc} = 0.0801 - 0.0894, \quad \theta_{bs} = 0.1309 - 0.2076, \quad \theta_{\tau\mu} = 0.2463. \]  

(12)
Figure 5: The rotation angle changing with scale as extracted from data, in the planar approximation.
These values suggest a smooth curve, which will be the case if these angles are swept out by a rotating vector $v$. This is clearly borne out by Figure 5, where the dotted curve represents the best-fit to data using MINUIT:

$$\theta = \exp(-2.267 - 0.509 \ln \mu) - 0.0075$$  \hspace{1cm} (13)

$\mu$ in GeV, with an excellent $\chi^2$ of 0.21 per degree of freedom. The solid curve shown is an earlier calculation [7, 8] to 1-loop in the Dualized Standard Model (see Footnote 1). The two curves are almost indistinguishable, and hence fit the data points equally well.

The positive result from the two-generation case is encouraging, but below the scale of roughly the $s$ quark mass, nonplanar effects begin to be appreciable, as can be estimated by the square of the Cabibbo angle which gives about 4%. Therefore to go further down the scale it is necessary to study the full three-generation case [4]. Writing

$$v(\mu) = (\xi(\mu), \eta(\mu), \zeta(\mu)),$$  \hspace{1cm} (14)

we shall present the extracted angles by plotting $\eta(\mu), \zeta(\mu)$ against scale $\mu$.

First we fix the $U$ triad to be

$$U: (1, 0, 0), \ (0, 1, 0), \ (0, 0, 1).$$  \hspace{1cm} (15)

Then the $D$ triad is just given by the elements of the CKM matrix:

$$D: (V_{tb}, V_{cb}, V_{ub}), \ (V_{ts}, V_{cs}, V_{us}), \ (V_{td}, V_{cd}, V_{ud}).$$  \hspace{1cm} (16)

We thus obtain the points $t$ and $b$ on the plot (Figure 6).

Next we consider $v(\mu = m_c)$. Since the mass $m_c$ comes from ‘leakage’ from $m_t$, we can write

$$v(\mu = m_c) = \cos \theta_{tc} v_t + \sin \theta_{tc} v_c = \sqrt{1 - m_c/m_t v_t} + \sqrt{m_c/m_t v_c},$$  \hspace{1cm} (17)

which gives the point $c$ in Figure 6. Note that the existence of another disjoint branch of the square root does not affect the question of interest to us here, namely, whether the allowed region is consistent with data lying on a smooth rotation curve, so long as the first branch already does, and can therefore be ignored.

Similar considerations give us a line as the allowed region for the $s$ quark. We shall leave open the question of the light quarks $u$ and $d$, since they are
Figure 6: A plot of the rotating vector $v(\mu)$ with its second and third components, i.e. $\eta(\mu)$ and $\zeta(\mu)$, as functions of $\ln \mu$, $\mu$ being the energy scale. The experimentally allowed values at any one scale are represented as an allowed region on a plaquette, with the scale corresponding to a plaquette being given by the intersection, denoted by a small circle, of its left-most boundary with the $\mu$-axis. The curve represents the result of a DSM one-loop calculation from an earlier paper [8] which is seen to pass through the allowed region on every plaquette except that for the electron $e$. For further explanation of details, please see text.

Tightly bound and both the definition and the value of their masses are quite uncertain.

In principle the leptons are not linked to the quarks, but the planar approximation above (Figure 5) and the DSM calculations [7, 8] both suggest that they can be put on the same trajectory as the quarks. In fact, by putting the $\tau$ point by interpolation between the $b$ and $c$ points, we can easily continue the rotation curve with the leptons, as is again evident from Figure 6.

We have not given in any detail on how the actual angles are extracted from the quark and lepton data, only the theory of it. Full details can be found in [4].

In conclusion we can say that, using the rotation hypothesis, and all the fermion mass and mixing data (apart from $m_u$ and $m_d$), we exhibit a smooth curve in 3-d space, passing through all the allowed regions, and spanning some 14 orders of magnitude in energy. Moreover, this curve coincides with the DSM curve [8] where we expect it to do, that is, where the 1-loop approximation is good.

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