Wood moisture accounting in creep equations

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Abstract. Wood as a structural material differs from other materials used in construction and industry, high variability of its properties. This is due, among other things, to the action of various variable factors – moisture content, temperature, inclination of fibers and others. The influence of wood moisture is especially strong – as a result of moistening the strength of wood considerably decreases in comparison with durability in absolutely dry condition. In addition, wood is a significantly nonlinear structural material. Non-linearity is manifested both in short-term and prolonged loading. Research of wood in time can be divided into two directions: longterm durability of wood and creep of wood. Modern models of wood creep are based on linear theories. Analysis of experimental data of various researchers on wood creep, has shown that the area of linear creep is observed at rather low levels of loading. At medium and high levels of experimental loading the existing models of linear wood creep theory do not correspond to experimental data. All this points to the actuality of the development of methods of calculating wooden structures for creep (linear and nonlinear), taking into account the wood moisture.

1. Introduction
The experimental studies of wood during the short-term and long-term loading showed its considerable nonlinear behaviour when different schemes of loading are applied. Non-linearity is found even when the strain is little. Taking this into account it should be noted that reconsidering the creep of wood is one of the key trends in improving the standards of wooden structures design.

Creep strains are influenced by several factors. One of these is wood moisture. Moisture increase causes creep strain speed.

The issue of wood moisture accounting in rheological models as well as in linear creep equations has been paid attention to by both Russian and foreign scientists. But up until now significant gaps still remain in studying the non-linear creep of wood.

A series of important experimental results are presented in A.S. Sogoyan’s thesis [1]. For instance, he has proved the dependence of the creep of wood and modulus of elasticity on wood moisture. Prokopovich I.E. and Zenderidze V.A. [2] noted that though the dependence between moisture and strength of wood has been thoroughly studied before, the effect of temporary moisture on the characteristics of long-term creep ability is less known. The existing experimental data are insufficient for a relatively complete description of the behaviour of wooden structure elements during different moisture regimes including such a natural one as drying. There being an appropriate external analogy, the authors found it quite reasonable to use an apparatus designed to account for the description of the
ageing effect upon the stress-strain state of a solid for the needs of describing the drying effect. This allowed to put forward an idea to justify the approach suggested by A.S. Sogoyan who proposes to consider the wood moisture effect as a result of a certain ageing connected with drying. Such an approach has found no proper practical application so far.

In A.M. Borovikov’s paper [3] it is shown that wood deformability and strength considerably depend on moisture \( w \) and temperature \( t \). Meanwhile the wood ageing effect is denied. The notion of standard moisture, which helps to recalculate deformability and temperature, has been introduced to compare the results. Continuing the research in this direction, R.B. Orlovich in his paper [4] writes a creep of wood integral equation taking into consideration equilibrium and relative moisture:

\[
\varepsilon(t) = \frac{\sigma(t)}{\varepsilon(\omega)} - \int_{\tau_1}^{t} \sigma(\tau) \frac{\partial S(\tau, \omega)}{\partial \tau} d\tau + \varepsilon_w(t) + \varepsilon_0(t),
\]

where \( \omega = F[\rho(t), T(t)] \) - equilibrium moisture; 
\( \varphi(t) \) – relative moisture; 
\( T(t) \) – temperature; 
\( S(t) = [1 + \psi(t)] \frac{1}{E} \);

\[
\psi(t, \tau, \omega) = \frac{\psi_w(\omega)[1-\omega]/[1-\psi(t)]}{t-\tau} \int_{0}^{t} F[\omega(t + \rho)] d\rho; \text{ when } \omega = 12; \psi_0 = 0.15.
\]

In the papers devoted to wood rheology different simplified mathematical formulas are used to describe experienced simple creep curves received under the constant experimental efforts. Y.M. Ivanov [5] added the data which helped to integrate the moisture effect on the creep strain:

\[
\varepsilon_\infty = \varepsilon(t_0) \frac{10^{-2}}{0.735-0.02086w} t^{0.21},
\]

where moisture \( \omega \) is measured in percent.

For the creep of wood ultimate characteristics Y.M. Ivanov’s empirical suggestion can be transformed into the following formula:

\[
\varphi_\infty(w) = \frac{\varphi_\infty}{0.0221} \frac{10^{-2}}{0.735-0.02086w},
\]

where \( w \) – wood moisture is measured in percent.

Then this dependence can be reduced to a specific type taking into account the level of loading \( \frac{\sigma}{R} \) for the case of non-linear creep, where \( R \) – wood strength:

\[
\varphi_\infty(\sigma, w) = \frac{\varphi_\infty}{0.0221} \frac{10^{-2}}{0.735-0.02086w},
\]

Eventually, the wood creep characteristic would look as:

\[
\varphi(t, \tau) = \varphi_\infty(\sigma, w) \cdot \varphi_1(t, \tau),
\]

where \( \varphi_\infty(\sigma, w) \) – the ultimate characteristic of the creep which does not depend on strain \( \varphi_\infty(\sigma) \), if the creep of wood is linear but it is affected only by wood moisture.

V.N. Volynskij in his paper [6] also points out that wood moisture is the most influential factor among other ones which affect wood physicomechanical characteristics. He also mentions that though many papers have been devoted to the study of this factor and wide statistics have been gathered, a new summary is required. When dealing with this problem one should define the type of stress state of wood, its temperature, density and other parameters.

The scientist has analysed the available data about the research carried out in the studies of the wood moisture effect on its performance. It has been found out that there are certain differences in tensile strengths on moisture during stretching, compression and curving of pure wood. In standards and reference books the basis of calculating the moisture conversion indexes is the dependency that works in the moisture range from 5 to 30 %.

The introduced conversion indexes are differentiated according to the wood species and density. Though, the latter has not been verified experimentally. There have been worked out some algorithms of calculating this index which shows the strength fraction of raw wood in correlation to the strength of absolutely dry wood according to the known reference data. It is demonstrated that these indexes do not depend on wood density.
Moreover, the scientist has proved that moisture affects the strength of lumber less than that of pure wood.

2. Accounting for the effect of wood moisture on its creep

Analysing the results of the mentioned authors’ research as well as the papers of the foreign scientists, one can say there exist three different principles which allow to account the wood moisture effect on the creep of wood.

The first principle of the moisture-time analogy introduces the notion of the given time suggested by H. Leaderman [7]. This principle was not meant to be applied to solving design calculation problems, but to gain the decrease in the period the creep and relaxation of wood indexes defining experiments lasted. This proposition found in A. Martensson’s paper [8] implies that with varying moisture and temperature characteristics, the rheological behaviour of wood can satisfy an equation of exactly the same structure as at constant moisture and temperature, but with a different time scale.

The given time $\xi$ is determined by the ratio:

$$d\xi = \frac{dt}{a(u)},$$

where $a(u)$ is the function decreasing to zero, the initial value of which is given at $\alpha = 68.5277$ found when $u = 0$, and the final value at moisture $u = 0.3159$.

The modulus of wood elasticity at such a ratio does not depend on moisture, which is a certain disadvantage of the theory.

It is known that the data obtained by this method testifies to some qualitative confirmation of such an approach but fails to testify to any reliable quantitative predictions [9].

According to the second principle, the physicochemical dependences of moisture are used to describe the process of deforming wood together with viscoelastic bodies. The construction of complex models of a large number of springs and viscous resistances greatly increases the order of the resolving differential creep equation of wood, creates major problems in the numerical implementation of the problems of calculating wooden structures. To make it simpler, all the major components of the model are treated separately in the research; their speeds are integrated автономно, which introduces an error in the results. The results of the model descriptions received analytically do not match the range of viscoelastic and humid elements, selected during the initial model compiling and are unable to describe the elastic properties of wood.

The third principle of wood moisture accounting is borrowed from the experimental works of A.M. Borovikov and Y.M. Ivanova who suggested taking it into account whenever the changes in wood moisture affect its mechanical characteristics such as its strength index, modulus of elasticity and the creep of wood index. For these mechanical characteristics of wood the authors mentioned above suggested their own variants of regression equations when indexes depend on the wood moisture. Under the well-known law of time changing the wood moisture introduced by certain known scientists it becomes possible to detest the process of changing long-term deformations of wood and take this process into account in concrete calculations of the stress-strain state of wooden structures based on appropriate methods of the theory of creep and plasticity.

These arguments indicate the relevance of improving the creep models of wood, taking into account the change in the effect of wood moisture, in the framework of the theory of creep developed by N. Arutyunyan, V. B. Kolmanovsky, Yu.N. Rabotnov, A.R. Rzhanitsyn.

3. Creep equations accounting for wood moisture

According to the results of research, the authors of the article obtained refined and new creep models of compressed wood using various creep theories, which allow to take into account the nonlinear creep of wood and its moisture.

After analyzing the linear creep equations of wood proposed by various scientists, the integral creep equation of wood was converted to a differential form. This allows writing a generalized equation in the form corresponding to the equation of A.R. Rzhanitsyn:
Voigt model with nonlinear force: simplifications can be made. The nonlinear creep equation for wood will correspond to the use of the equation of nonlinear creep of wood in the form of a differential equation:

$$\varepsilon(t) = \frac{1}{\gamma(1+k_{def}(w))} \frac{1}{E_0(w)} \frac{1}{1+K_{def}(w)} \frac{E_0(w)}{1+K_{def}(w)} \varepsilon(t)$$

where $\gamma$ - the coefficient for wood in the equations of Sogoyan A.S., Prokopovich I.E., Zelenidnev V.A., Yatsenko V.F., Orlovich R.B., Roschina S.I.; $k_{def}$ - Eurocode 5 coefficient of deformation; $w$ - wood moisture; $E_0$ - elasticity modulus. Experimental and analytical data on dependencies $E_0(w)$ and $k_{def}(w)$ is advisable to use from the works of A. Borovikov and Ivanova Yu.M.

Rejecting the last term in equation (1), we obtain the structure of the wood creep equation proposed by E.N. Kvasnikov. Using this equation, it is impossible to obtain the value of the long-term critical force of a compressed wooden column. The structure of equation (1) explains the features of changes in the relaxation time $(n = \frac{1}{\gamma(1+k_{def}(w))})$ and the long-term modulus of wood deformation $(H = \frac{E_0(w)}{1+K_{def}(w)})$, which is important when processing experiments on the long-term loading of compressed wood samples.

To describe the non-linear creep of wood we use the method suggested in the paper [10]. The non-linear creep of wood equation is written as follows:

$$\varepsilon(t) = \frac{\sigma(t)}{E_0} - \int_{t_0}^{t} \left[\sigma + \beta\sigma^2\right] \frac{1}{E_0} \frac{\partial}{\partial\varepsilon(t)} \varphi(t,\varepsilon) d\varepsilon,$$

where $\varphi(t,\varepsilon) = \varphi_0(\varepsilon) \cdot \left(1 - e^{-\gamma(t-t_0)}\right)$.

Taking the value $\varphi_0(\varepsilon)$ is not dependent on time ($w$ - moisture in %), we have the basic equation of nonlinear creep of wood:

$$\varepsilon(t) = \frac{\sigma(t)}{E_0} - \int_{t_0}^{t} \left[\sigma + \beta\sigma^2\right] \frac{1}{E_0} \varphi_0(\varepsilon) \frac{\partial}{\partial\varepsilon(t)} \left(1 - e^{-\gamma(t-t_0)}\right) dt.$$  (2)

In this equation, it is possible to take into account the dependence of the modulus of elasticity of wood on moisture $w$, substituting into it the appropriate empirical formula:

$$\varepsilon(t) = \frac{\sigma(t)}{E_0} - \int_{t_0}^{t} \left[\sigma + \beta\sigma^2\right] \frac{1}{E_0} \varphi_0(\varepsilon) \frac{\partial}{\partial\varepsilon(t)} \left(1 - e^{-\gamma(t-t_0)}\right) dt.$$  (3)

Non-linear equations (3) and (4) can be applied to woodwork when compressed, stretched or curved. They will differ in the numerical values of the coefficients $\beta$ and $\varphi_0(\varepsilon)$. These equations do not consider wood ageing. To do so, one should use the ageing Functions.

It is known that the expression $e^{-\gamma(t-t_0)}$ under the integral in equation (3) allows to convert it into a differential equation, more convenient in solving a number of practical problems of calculating wooden structures.

The equation (3) is differentiable in time $t$:

$$\dot{\varepsilon}(t) = \frac{\dot{\sigma}(t)}{E_0} - \gamma \int_{t_0}^{t} \left[\sigma + \beta\sigma^2\right] \frac{1}{E_0} \varphi_0(\varepsilon) e^{-\gamma(t-t_0)} dt + \gamma \sigma(t) + \beta\sigma^2(t) \frac{1}{E_0} \varphi_0(\varepsilon).$$  (5)

From the result obtained, we find the value of the integral in square brackets:

$$\gamma[1] = \frac{\dot{\sigma}(t)}{E_0} - \dot{\varepsilon}(t) + \gamma \sigma(t) + \beta\sigma^2(t) \frac{1}{E_0} \varphi_0(\varepsilon).$$

Substituting this value into the original equation (3), we get rid of the integral, and we get the basic equation of nonlinear creep of wood in the form of a differential equation:

$$\dot{\varepsilon}(t) + \gamma \dot{\varepsilon}(t) = \frac{\dot{\sigma}(t)}{E_0} + \gamma \sigma(t) + \beta\sigma^2(t) \frac{1}{E_0} \varphi_0(\varepsilon) + \gamma \sigma(t) \frac{1}{E_0}.$$  (6)

At low loading levels ($\frac{1}{R} \leq 0.38$ for compressed wood) let us write the equation of linear creep of wood, assuming:

$$\dot{\varepsilon}(t) + \gamma \dot{\varepsilon}(t) = \frac{\dot{\sigma}(t)}{E_0} + \gamma \sigma(t) \frac{1}{E_0} \varphi_0(\varepsilon) + \gamma \sigma(t) \frac{1}{E_0}.$$  (7)

To solve the simplest problems of bending under non-linear creep conditions, a number of simplifications can be made. The nonlinear creep equation for wood will correspond to the use of the Voigt model with nonlinear force:

$$\dot{\varepsilon}(t) + \gamma \dot{\varepsilon}(t) = \gamma \sigma(t) + \beta\sigma^2(t) \frac{1}{E_0} \varphi_0(\varepsilon).$$  (8)
This equation is also convenient for solving problems of cyclic loading of wooden structures, including those of oscillations.

4. Conclusion

Thus, the design models proposed in the article account for the rheological features of wood more accurately compared to existing theories, and also allow taking into account the influence of moisture on the mechanical characteristics of wood.

As a result, universal equations of linear and nonlinear creep of wood were obtained, taking into account its moisture. Substituting into these equations the values of the coefficients obtained experimentally, we can describe the work of wood in compression, tension and curving.

On the basis of the obtained creep equations for wood, it is possible in the future to conduct a study of the stress-strain state of compressed wooden structures that are in buckling conditions, with subsequent suggestions for refining the methodology for regulatory calculation.

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