Multi-Graviton Theories in the Causal Approach

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Abstract

The system of multi-gravitons has been considered before in the framework of functional formalism. We consider here the system of multi-gravitons in the causal formalism of quantum field theory. We derive in this formalism the fact that distinct gravitons cannot interact. The proof is based on a careful analysis of the first two orders of the perturbation theory.

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1 Introduction

The general framework of perturbation theory consists in the construction of the chronological products such that Bogoliubov axioms are verified [1], [4], [3]; for every set of Wick monomials $A_1(x_1), \ldots, A_n(x_n)$ acting in some Fock space $\mathcal{H}$ one associates the operator

$$T(A_1(x_1), \ldots, A_n(x_n))$$

which is a distribution-valued operators called chronological product.

The construction of the chronological products can be done recursively according to Epstein-Glaser prescription [4], [10] (which reduces the induction procedure to a distribution splitting of some distributions with causal support) or according to Stora prescription [11] (which reduces the renormalization procedure to the process of extension of distributions). These products are not uniquely defined but there are some natural limitation on the arbitrariness. If the arbitrariness does not grow with $n$ we have a renormalizable theory. A variant based on retarded products is due to Steinmann [15].

Gravity is described by particles of helicity 2 (in the linear approximation). Theories of higher spin are not renormalizable. However, one can save renormalizability using ghost fields. Such theories are defined in a Fock space $\mathcal{H}$ with indefinite metric, generated by physical and un-physical fields (called ghost fields). One selects the physical states assuming the existence of an operator $Q$ called gauge charge which verifies $Q^2 = 0$ and such that the physical Hilbert space is by definition $\mathcal{H}_{\text{phys}} \equiv \text{Ker}(Q)/\text{Im}(Q)$. The space $\mathcal{H}$ is endowed with a grading (usually called ghost number) and by construction the gauge charge is raising the ghost number of a state. Moreover, the space of Wick monomials in $\mathcal{H}$ is also endowed with a grading which follows by assigning a ghost number to every one of the free fields generating $\mathcal{H}$. The graded commutator $d_Q$ of the gauge charge with any operator $A$ of fixed ghost number

$$d_Q A = [Q, A]$$

is raising the ghost number by a unit. It means that $d_Q$ is a co-chain operator in the space of Wick polynomials. From now on $[\cdot, \cdot]$ denotes the graded commutator. From $Q^2 = 0$ one derives

$$(d_Q)^2 = 0.$$  

A gauge theory assumes also that there exists a Wick polynomial of null ghost number $T(x)$ called the interaction Lagrangian such that

$$d_Q T = [Q, T] = i\partial_\mu T^\mu$$

for some other Wick polynomials $T^\mu$. This relation means that the expression $T$ leaves invariant the physical states, at least in the adiabatic limit. Indeed, if this is true we have:

$$T(f) \mathcal{H}_{\text{phys}} \subset \mathcal{H}_{\text{phys}}$$

up to terms which can be made as small as desired (making the test function $f$ flatter and flatter). We call this argument the formal adiabatic limit. It is an euristic way to justify from
the physical point of view relation (1.3). Otherwise, we simply have to postulate it. The preceding relation can be extended if we assume a polynomial Poincaré lemma as follows. One applies \( d_Q \) to (1.3) and obtains
\[
\partial_\mu d_Q T^\mu = 0
\]
so we expect that we have
\[
d_Q T^\mu = i\partial_\nu T^{\mu\nu}
\]
and so on. It turns out that there are obstructions to such a polynomial Poincaré lemma if we work with on-shell fields, so we must prove directly such type of identity.

One defines now the chronological products \( T(A_1(x_1),\ldots,A_n(x_n)) \) with \( A_1,\ldots,A_n \) of the type \( T, T^\mu, T^{\mu\nu}, \) etc. and formulates a proper generalization of (1.3). Such identity express, as (1.4), the fact that the scattering matrix leaves invariant the subspace of physical states, at least in some adiabatic limit sense. The analysis of these identities can be done by direct computations in lower orders of the perturbation theory, but a general proof in arbitrary orders is still an open problem in the general case, due to the quantum anomalies which do appear in the inductive procedure.

Our approach is purely quantum: we do not need a classical field theory to quantize. However, we mention that there is a variant of the perturbative quantum field theory where one keeps a closer connection with clasical theory. In this approach one can treat quantum fields on manifolds; see for instance [5] and the recent review [12].

We will consider a system of \( R \geq 1 \) distinct gravitions and prove that second order gauge invariance gives the following result: there is no interaction between distinct gravitons. Such a result was proved for the first time in the framework of the functional formalism in [2]. We provide here the proof in the causal formalism.
2 Quantum Gravity

The Fock space is generated by the fields $h^{\mu\nu}, u^\rho, \tilde{u}^\sigma$ of null mass i.e. we have the equations of motion:

$$\Box h^{\mu\nu} = 0 \quad \Box u^\rho = 0 \quad \Box \tilde{u}^\sigma = 0$$  \hspace{1cm} (2.1)

and we also assume the symmetry property

$$h^{\mu\nu} = h^{\nu\mu}$$  \hspace{1cm} (2.2)

and self-adjointness:

$$(h^{\mu\nu})^\dagger = h^{\mu\nu}, \quad (u^\rho)^\dagger = u^\rho, \quad (\tilde{u}^\sigma)^\dagger = -\tilde{u}^\sigma.$$

(2.3)

We denote

$$h \equiv \eta_{\mu\nu} h^{\mu\nu}. \hspace{1cm} (2.4)$$

The non-trivial 2-point functions are:

$$\langle \Omega, h^{\mu\nu}(x_1) h^{\rho\sigma}(x_2) \Omega \rangle = -\frac{i}{2} \left( \eta^{\mu\rho} \eta^{\nu\sigma} + \eta^{\nu\rho} \eta^{\mu\sigma} - \eta^{\mu\nu} \eta^{\rho\sigma} \right) D_0(x_1 - x_2),$$

$$\langle \Omega, u^\mu(x_1) \tilde{u}^\nu(x_2) \Omega \rangle = i \eta^{\mu\nu} D_0(x_1 - x_2),$$

$$\langle \Omega, u^\mu(x_1) u^\nu(x_2) \Omega \rangle = -i \eta^{\mu\nu} D_0(x_1 - x_2)$$  \hspace{1cm} (2.5)

with $D_0(x_1 - x_2)$ the Pauli-Jordan distribution and $D_0^{(\pm)}(x_1 - x_2)$ the positive (and negative) frequency parts. It follows immediately:

$$\langle \Omega, h^{\mu\nu}(x_1) h^{\mu\nu}(x_2) \Omega \rangle = \langle \Omega, h(x_1) h^{\mu\nu}(x_2) \Omega \rangle = i \eta^{\mu\nu} D_0^{(\pm)}(x_1 - x_2)$$

$$\langle \Omega, h(x_1) h(x_2) \Omega \rangle = 4i D_0^{(\pm)}(x_1 - x_2)$$  \hspace{1cm} (2.6)

and the cannonical (anti)commutation relations are:

$$[h^{\mu\nu}(x_1), h^{\rho\sigma}(x_2)] = -\frac{i}{2} \left( \eta^{\mu\rho} \eta^{\nu\sigma} + \eta^{\nu\rho} \eta^{\mu\sigma} - \eta^{\mu\nu} \eta^{\rho\sigma} \right) D_0(x_1 - x_2),$$

$$\{u^\mu(x_1), \tilde{u}^\nu(x_2)\} = i \eta^{\mu\nu} D_0(x_1 - x_2)$$

(2.7)

We define the gauge charge according to

$$[Q, h^{\mu\nu}] = -\frac{i}{2} \left( \partial^\mu u^\nu + \partial^\nu u^\mu - \eta^{\mu\nu} \partial_\rho u^\rho \right), \quad \{Q, u^\mu\} = 0, \quad \{Q, \tilde{u}^\mu\} = i \partial_\mu h^{\mu\nu}, \hspace{1cm} (2.8)$$

so

$$[Q, h] = i \partial_\mu u^\mu, \quad [Q, \partial^\rho h_{\mu\nu}] = 0$$

(2.9)

and we can prove that the factor space $\text{Ker}(Q)/\text{Im}(Q)$ describes the many-body theory of gravitons i.e. it is isomorphic to the Fock space $\mathcal{F}(H^{[0,2]})$ associated to the Hilbert space $H^{[0,2]}$ of a particle of null mass and helicity 2.
We now describe the off-shell version of this construction [6], [7]. We consider the Grassmann algebra generated by the variables $h_{\mu\nu}, u_{\mu}, \tilde{u}_{\mu}$ of even parity and $u_{\mu}, \tilde{u}_{\mu}$ of odd parity. Next we consider the associated jet extension of order $r$
\[
h_{\mu\nu;\lambda_1...\lambda_p}, u_{\mu;\lambda_1...\lambda_p}, \tilde{u}_{\mu;\lambda_1...\lambda_p}, \quad p = 1, \ldots, r.
\]
There is no mass constrain in this algebra. Now we define the formal derivative $d_\lambda$ according to

\[
d_\lambda h_{\mu\nu;\lambda_1...\lambda_p} \equiv h_{\mu\nu;\lambda_0...\lambda_p},
\]

etc, and define the gauge charge $Q$ by the same formula as above but with $\partial_\mu \to d_\mu$.

The operator $d_Q$ does not square to zero anymore. Nevertheless we define

\[
\delta T^I \equiv d_\mu T^{I\mu}
\]

with $d_\mu$ the formal derivative and then

\[
s \equiv d_Q - i \delta.
\]

If we want to consider more than one specie of graviton, we only have to add a new index $A = 1, \ldots, R$ to the basic fields i.e. the Fock space is generated by the fields $h_{\mu\nu}^A, u_\rho^A, \tilde{u}_\sigma^A$, $A = 1, \ldots, R$. In the expressions for the 2-point functions, the cannonical (anti)commutators, etc. a factor $\delta_{AB}$ appears in the right-hand side i.e.

\[
< \Omega, h_{A}(x_1)h_{B}(x_2)\Omega > = -i \delta_{AB} \eta^{\mu\rho} \eta^{\nu\sigma} - \eta^{\mu\nu} \eta^{\rho\sigma} D_0(x_1 - x_2),
\]

\[
< \Omega, u_{A}(x_1)\tilde{u}_{B}(x_2)\Omega > = i \eta^{\mu\nu} \delta_{AB} D_0(x_1 - x_2)
\]

\[
< \Omega, \tilde{u}_{A}(x_1)u_{B}(x_2)\Omega > = -i \eta^{\mu\nu} \delta_{AB} D_0(x_1 - x_2)
\]

(2.13)

etc.
3 Perturbation Theory

We provide the necessary elements of (second order) of perturbation theory. Formally, we want to compute the scattering matrix

$$S(g) \equiv I + i \int dx g(x) T(x) + \frac{i^2}{2} \int dx dy g(x) g(y) T(x, y) + \cdots$$

(3.1)

where $g$ is some test function. The expressions $T(x, y)$ are called (second order) chronological products because they must verify the causality property:

$$T(x, y) = T(x) T(y)$$

(3.2)

for $x \succ y$ i.e. $(x - y)^2 \geq 0, x^0 - y^0 \geq 0$; in other words the point $x$ succeeds causally the point $y$. This is a relativistic generalization of the property

$$U(t, s) = U(t, r) U(r, s)$$

(3.3)

of the time evolution operator from non-relativistic quantum mechanics.

We go to the second order of perturbation theory using the causal commutator

$$D^{A,B}(x, y) \equiv D(A(x), B(y)) = [A(x), B(y)]$$

(3.4)

where $A(x), B(y)$ are arbitrary Wick monomials. These type of distributions are translation invariant i.e. they depend only on $x - y$ and the support is inside the light cones:

$$\text{supp}(D) \subset V^+ \cup V^-.$$ 

(3.5)

A theorem from distribution theory guarantees that one can causally split this distribution:

$$D(A(x), B(y)) = A(A(x), B(y)) - R(A(x), B(y)).$$

(3.6)

where:

$$\text{supp}(A) \subset V^+ \quad \text{supp}(R) \subset V^-.$$ 

(3.7)

The expressions $A(A(x), B(y)), R(A(x), B(y))$ are called advanced resp. retarded products. They are not uniquely defined: one can modify them with quasi-local terms i.e. terms proportional with $\delta(x - y)$ and derivatives of it.

There are some limitations on these redefinitions coming from Lorentz invariance and power counting: this means that we should not make the various distributions appearing in the advanced and retarded products too singular.

Then we define the chronological product by:

$$T(A(x), B(y)) = A(A(x), B(y)) + B(y) A(x) = R(A(x), B(y)) + A(x) B(y).$$

(3.8)

The expression $T(x, y)$ corresponds to the choice

$$T(x, y) \equiv T(T(x), T(x)).$$

(3.9)
The “naive” definition
\[ T(A(x), B(y)) = \theta(x^0 - y^0)A(x)B(y) + \theta(y^0 - x^0)B(y)A(x) \] (3.10)
involves an illegal operation, namely the multiplication of distributions. This appears in some loop contributions (the famous ultraviolet divergences).

We will need in the following the causal commutator
\[ D^{I,J}(x, y) \equiv D(T^I(x), T^J(y)) = [T^I(x), T^J(y)] \] (3.11)
where \([\cdot, \cdot]\) is always the graded commutator.

The chronological products \(T(A(x), B(y))\) must satisfy some axioms (Bogoliubov):

- **The “initial condition”**: \(T(A(x)) = A(x)\). (3.12)

- **Skew-symmetry** in all arguments:
  \[ T(A(x), B(y)) = (-1)^{f(A)f(B)}T(B(y), A(x)) \] (3.13)
  where \(f(A)\) is the Fermi number of the Wick monomial \(A\).

- **Poincaré invariance**: we have a natural action of the Poincaré group in the space of Wick monomials and we impose that for all \(g \in inSL(2, \mathbb{C})\) we have:
  \[ U_gT(A(x), B(y))U_g^{-1} = T(g \cdot A(x), g \cdot B(y)) \] (3.14)
  where in the right hand side we have the natural action of the Poincaré group on Wick monomials.

- **Causality**: if \(x - y\) is in the upper causal cone then we denote this relation by \(x \geq y\). In this case we have the factorization property:
  \[ T(A(x), B(y)) = A(x)B(y) \] (3.15)

- **Unitarity**: We define the **anti-chronological products** is an ordered subset, we define
  \[ \bar{T}(A(x), B(y)) \equiv A(x)B(y) + (-1)^{f(A)f(B)}B(y)A(x) - T(A(x), B(y)) \] (3.16)
  Then the unitarity axiom is:
  \[ T = T^\dagger. \] (3.17)

- **Power counting**: We can also include in the induction hypothesis a limitation on the order of singularity of the vacuum averages of the chronological products associated to arbitrary Wick monomials; explicitly:
  \[ \omega(<\Omega, T(A(x), B(y))\Omega>) \leq \omega(A) + \omega(B) - 4 \] (3.18)
  where by \(\omega(d)\) we mean the order of singularity of the (numerical) distribution \(d\) and by \(\omega(A)\) we mean the canonical dimension of the Wick monomial \(W\).
• **Wick expansion property**: we refer to the literature for the formulation.

The axioms can be extended to arbitrary chronological products \( T(A_1(x_1), \ldots, A_n(x_n)) \). Now we can construct the chronological products \( T(T^{I_1}(x_1), \ldots, T^{I_n}(x_n)) \) according to the recursive procedure.

We say that the theory is gauge invariant in all orders of the perturbation theory if the following set of identities:

\[
d_Q T^{I_1}(x_1), \ldots, T^{I_n}(x_n)) = i \sum_{l=1}^{n} (-1)^{s_l} \frac{\partial}{\partial x^\mu_l} T(T^{I_1}(x_1), \ldots, T^{I_l\mu}(x_l), \ldots, T^{I_n}(x_n)) \tag{3.19}
\]

are true for all \( n \in \mathbb{N} \) and all \( I_1, \ldots, I_n \). Here we have defined

\[
s_l \equiv \sum_{j=1}^{l-1} f(T^{I_j}) = \sum_{j=1}^{l-1} |I_j| \tag{3.20}
\]

In particular, the case \( I_1 = \ldots = I_n = \emptyset \) it is sufficient for the gauge invariance of the scattering matrix, at least in the adiabatic limit: we have the same argument as for relation (1.4).

To describe this property in a cohomological framework, we consider that the chronological products are the cochains and we define for the operator \( \delta \) by

\[
\delta T^{I_1}(x_1), \ldots, T^{I_n}(x_n)) = i \sum_{l=1}^{n} (-1)^{s_l} \frac{\partial}{\partial x^\mu_l} T(T^{I_1}(x_1), \ldots, T^{I_l\mu}(x_l), \ldots, T^{I_n}(x_n)) \tag{3.21}
\]

It is easy to prove that we have:

\[
\delta^2 = 0 \tag{3.22}
\]

and

\[
[d_Q, \delta] = 0. \tag{3.23}
\]

Next we define

\[
s \equiv d_Q - i\delta \tag{3.24}
\]

such that relation (3.19) can be rewritten as

\[
s T^{I_1}(x_1), \ldots, T^{I_n}(x_n)) = 0. \tag{3.25}
\]

We note that if we define

\[
\bar{s} \equiv d_Q + i\delta \tag{3.26}
\]

we have

\[
ss = 0, \quad \bar{s}s = 0 \tag{3.27}
\]

so expressions verifying the relation \( sC = 0 \) can be called *cocycles* and expressions of the type \( \bar{s}B \) are the *coboundaries*. One can build the corresponding cohomology space in the standard way.
If we have (3.20) for \( n = 1, 2, \ldots, p - 1 \) then the relation (3.20) for \( n = p \) can be broken by anomalies i.e. we have:

\[
sT(T^{I_1}(x_1), \ldots, T^{I_p}(x_p)) = A^{I_1, \ldots, I_p}(x_1, \ldots, x_p)
\]  
(3.28)

where \( A^{I_1, \ldots, I_p} \) is a quasi-local expression, having support in

\[
D_p = \{ x_1 = x_2 = \ldots = x_p \}. \quad (3.29)
\]

The gauge theory is physically meaningful if one can remove the anomalies by a redefinition of the chronological products.
4 First-Order

Proposition 4.1 Suppose that $T$ is a Wick polynomial in the (quantum) fields $h^\mu_\nu, u^\rho_A, \tilde{u}^\sigma_A$ ($A = 1, \ldots, R$) which is: (a) Poincaré invariant; (b) of canonical dimension $\omega(T) \leq 5$; (c) trilinear in the fields; (d) self-adjoint; (e) gauge invariant in the sense (1.3). Then one can take

$$T = t^{ABC} (h^\mu_\nu \partial_\mu h_B \partial_\nu h_C - 2h^\mu_\nu \partial_\mu h_B \partial_\sigma h^\sigma_C - 4h^\mu_A \partial_\mu h^\rho_B \partial_\sigma h^\nu_C$$

$$+ 4\partial_\mu h^\mu_B u^\rho B \partial_\rho \tilde{u}^\sigma_C - 4h^\mu_A \partial_\mu u^\rho_B \partial_\nu \tilde{u}^\sigma_C$$

(4.1)

up to a coboundary. Here $t^{ABC}$, $A, B, C = 1, \ldots, R$ are some real constants with complete symmetry in the indexes.

Proof: It follows easily that we can have terms with $\omega = 3, 5$.

(i) In the case $\omega = 3$ we can have the following terms:

$$T_1 = t_1^{ABC} h^\mu_B h_B h^\rho_C,$$

$$T_2 = t_2^{ABC} h^\mu_B h_B h_C,$$

$$T_3 = t_3^{ABC} h_B h_B h_C,$$

$$T_4 = t_4^{ABC} h^\mu_B u_B \tilde{u}^\nu_C,$$

$$T_5 = t_5^{ABC} h_B u_B \tilde{u}^\nu_C$$

(4.2)

where

$$t_j^{ABC} = A \leftrightarrow B = B \leftrightarrow C, \quad j = 1, 3,$$

$$t_2^{ABC} = A \leftrightarrow B.$$

(4.3)

By simple “partial integration” we can write

$$d_\Omega T = i\partial_\mu X^\mu + iu^\mu_A Y^A + iZ$$

(4.4)

with easily computable expressions $Y \sim hh, Z \sim uu\tilde{u}$. The equation (1.3) becomes

$$u^\mu_A Y^A + Z = \partial_\mu \tilde{T}^\mu, \quad \tilde{T}^\mu \equiv T^\mu - X^\mu.$$

(4.5)

The generic form for $\tilde{T}^\mu$ is:

$$\tilde{T}^\mu = u_{A\nu} T^\mu_A + S^\mu$$

(4.6)

with $T^\mu_A \sim hh, S^\mu \sim uu\tilde{u}$.

If we introduce in (4.5) then we easily obtain $T^\mu_A = 0$ and from here $Y^A = 0$. If we write explicitly this equation then it immediately follows that $t_j = 0, \quad j = 1, \ldots, 5$ so there is no solution with canonical dimension 3.
(ii) For \( \omega = 5 \) we have terms of the type \( hh \) and \( hu \) with two derivatives distributed on the three factors. We can simplify the list using the following observations (see [8]).

- We can consider for the first type only terms of the form \( h\partial h\partial h \) because we can eliminate the terms of the form \( hh\partial h \) subtracting a total derivative. In the same way we can consider only terms of the type \( u\partial uu, u\partial uh, uu\partial h \).

- We can use the identity:

\[
\Box f_j = 0, \quad j = 1, 2, 3 \implies (\partial_\mu f_1)(\partial_\mu f_2)f_3 = \frac{1}{2} \partial_\mu \left[ (\partial_\mu f_1)f_2f_3 + f_1(\partial_\mu f_2)f_3 - f_1f_2(\partial_\mu f_3) \right] \]

(4.7)
to eliminate some terms.

- We can list all possible expressions of the type \( d\Omega L \) with \( \omega(L) = 4 \) and \( gh(L) = -1 \) to eliminate another set of terms. We are left with the following list:

\[
T_1 = t^{1\mu\nu}_{ABC} h^\mu_A \partial_\mu h^\nu_B \partial_\nu C \quad T_2 = t^{2\mu\nu}_{ABC} h^\mu_A \partial_\mu h^\nu_B \partial_\nu C \quad T_3 = t^{3\mu\nu}_{ABC} h^\mu_A \partial_\mu h^\nu_B \partial_\nu C \quad T_4 = t^{4\mu\nu}_{ABC} h^\mu_A \partial_\mu h^\nu_B \partial_\nu C \rho \rho \\
T_5 = t^{5\mu\nu}_{ABC} h^\mu_A \partial_\mu h^\nu_B \partial_\nu C \quad T_6 = t^{6\mu\nu}_{ABC} h^\mu_A \partial_\mu h^\nu_B \partial_\nu C \rho \\
T_7 = t^{7\mu\nu}_{ABC} u^\mu_A \partial_\mu h^\nu_B \partial_\nu C \quad T_8 = t^{8\mu\nu}_{ABC} u^\mu_A \partial_\mu h^\nu_B \partial_\nu C \rho \\
T_9 = t^{9\mu\nu}_{ABC} u^\mu_A \partial_\mu h^\nu_B \partial_\nu C \quad T_{10} = t^{10\mu\nu}_{ABC} u^\mu_A \partial_\mu h^\nu_B \partial_\nu C \\
T_{11} = t^{11\mu\nu}_{ABC} u^\mu_A \partial_\mu h^\nu_B \partial_\nu C \quad T_{12} = t^{12\mu\nu}_{ABC} u^\mu_A \partial_\mu h^\nu_B \partial_\nu C \\
T_{13} = t^{13\mu\nu}_{ABC} u^\mu_A \partial_\mu h^\nu_B \partial_\nu C \quad T_{14} = t^{14\mu\nu}_{ABC} u^\mu_A \partial_\mu h^\nu_B \partial_\nu C \\
T_{15} = t^{15\mu\nu}_{ABC} u^\mu_A \partial_\mu h^\nu_B \partial_\nu C \quad T_{16} = t^{16\mu\nu}_{ABC} u^\mu_A \partial_\mu h^\nu_B \partial_\nu C \\
T_{17} = t^{17\mu\nu}_{ABC} u^\mu_A \partial_\mu h^\nu_B \partial_\nu C \quad T_{18} = t^{18\mu\nu}_{ABC} u^\mu_A \partial_\mu h^\nu_B \partial_\nu C \\
\]

(4.8)

We can proceed as above and obtain (4.14) and (4.15), but now the generic form of \( \tilde{T}^\mu \) is more complicated:

\[
\tilde{T}^\mu = u^\mu_A T_A^{\mu\nu} + (\partial_\rho u^\nu_A) T_A^{\rho\nu} + (\partial_\sigma u^\nu_A) T_A^{\mu\rho\sigma} + S^\mu
\]

(4.10)

where \( T_A^{\mu\nu}, T_A^{\mu\rho\nu}, T_A^{\mu\nu\sigma} \) are bilinear in \( h \) and \( S^\mu \sim uu\tilde{u}. \) Moreover we can assume that \( T_A^{\mu\rho\sigma} = \rho \leftrightarrow \sigma. \) By direct computations we get from (4.15)

\[
Y^\nu_A = \partial_\nu T_A^{\mu\nu} \\
T_A^{\rho\nu} = -\partial_\mu T_A^{\mu\rho} \\
\partial_\rho \partial_\sigma u^\nu_A (T_A^{\sigma\rho\nu} + \partial_\mu T_A^{\mu\rho\sigma}) = 0 \\
\partial_\mu \partial_\rho \partial_\sigma u^\nu_A T_A^{\mu\rho\sigma} = 0.
\]

(4.11)

From the first two equations we obtain

\[
Y^\mu_A = -\partial_\rho \partial_\sigma T_A^{\rho\sigma}
\]

(4.12)

so we can suppose that \( T_A^{\rho\sigma} = \rho \leftrightarrow \sigma. \) We write

\[
T_A^{\rho\mu\sigma} = t_A^{\rho\mu\sigma} + \eta^{\rho\sigma} t_A^\mu \\
T_A^{\mu\rho\sigma} = t_A^{\mu\rho\sigma} + \eta^{\rho\sigma} t_A^\mu
\]

(4.13)
where the expressions $t_{\rho \mu \sigma}^{\rho \mu \sigma}$, $t_A^{\rho \mu \rho \sigma}$ do not contain terms with the factor $\eta^{\rho \sigma}$ and we have the symmetry properties $t^{\rho \mu \sigma} = \rho \leftrightarrow \sigma$, $t_A^{\rho \mu \rho \sigma} = \rho \leftrightarrow \sigma$. From (4.12) we obtain

$$Y_A^\mu = -\partial_\rho \partial_\sigma t_A^{\rho \mu \rho \sigma} - \Box t_A^\mu$$

(4.14)

and from the third equation of the system (4.11) it follows

$$t_A^{\rho \mu \rho \sigma} = -\partial_\mu t_A^{\rho \mu \rho \sigma}$$

(4.15)

so the previous relation becomes:

$$Y_A^\mu = \partial_\rho \partial_\sigma t_A^{\rho \mu \rho \sigma} - \Box t_A^\mu.$$ (4.16)

But the last relation of the system (4.11) shows that the first term in the right hand side is null, so we have:

$$Y_A^\mu = -\Box t_A^\mu.$$ (4.17)

Because $t_A^\mu$ is bilinear in $h$ the expression $\Box t_A^\mu$ will be a sum of terms of the type $\partial_\mu f \partial^\mu g$. So we have to determine the expression $Y_A^\mu$ by direct computation: we compute the expressions $d_Q T_j$ and by “partial integration” put them in the standard form from (4.4). Then we must consider only the terms which are not of the form $\partial_\mu f \partial^\mu g$, group them and put the result to zero.

The solution of the system is the following:

$$

\begin{align*}
t_4^{ABC} & = -2 t^{ABC}, & t_2^{ABC} & = -4 t^{ABC}, & t_5^{ABC} & = t^{ABC}, \\
t_7^{ABC} & = 4 t^{ABC}, & t_8^{ABC} & = 4 t^{ABC}, & t_{13}^{ABC} & = 4 t^{ABC}
\end{align*}

(4.18)

with $t^{ABC}$ having the property of complete symmetry. We can rewrite $T_7 + T_{13}$ as the fourth term from the expression $T$ from the statement plus a total derivative which we can eliminate.

The expression $T$ cannot be easily compared to the classical expression derived from Hilbert-Einstein Lagrangian. As remarked in [3] we can add two terms of the type (4.7); also we can rewrite the ghost contribution, all amounting to the elimination of a total derivative. In this case we get the expression from [3]:

Proposition 4.2

$$

\begin{align*}
T & = t^{ABC} \left( \partial_\mu h_B \partial_\nu h_C - 2 h_A^{\mu \nu} \partial_\mu h_B \partial_\nu h_C^{\sigma} - 4 h_A^{\mu \nu} \partial_\rho h_B^{\alpha \sigma} \partial_\sigma h_C^{\rho} \\
& \quad - 2 h_A^{\mu \nu} \partial_\rho h_B \partial_\sigma h_C^{\rho} + 4 h_A^{\mu \nu} \partial_\rho h_B \partial_\sigma h_C^{\rho} \partial_\rho h_C^{\sigma} \\
& \quad - 4 h_A^{\mu \nu} \partial_\rho u_B^\rho \partial_\sigma u_C^\nu + 4 \partial^\rho h_A^{\mu \nu} u_B^\rho \partial_\sigma u_C^\nu + 4 h_A^{\mu \nu} \partial_\rho u_B^\rho \partial_\sigma u_C^\nu - 4 h_A^{\mu \nu} \partial_\rho u_B^\rho \partial_\sigma u_C^\nu \right).
\end{align*}

(4.19)

$$

\begin{align*}
T^\mu & = t^{ABC} \left[ u_A^\mu \left( 2 \partial^\rho h_B^{\alpha \sigma} \partial_\nu h_C^{\rho} - 4 \partial_\rho h_B^{\alpha \sigma} \partial^\rho h_C^{\sigma} - \partial_\mu h_B \partial_\nu h_C \right) \\
& \quad + \partial_\rho u_B^\rho \left( h_A^{\mu \nu} \partial_\rho h_B \partial_\sigma h_C^{\sigma} + 2 h_A^{\mu \nu} \partial_\rho h_B \partial_\sigma h_C^{\rho} + 2 h_A^{\mu \nu} \partial_\rho h_B \partial_\sigma h_C^{\rho} \partial_\rho h_C^{\sigma} \right) \\
& \quad - \partial_\rho u_B^\rho \left( h_A^{\mu \nu} \partial_\rho h_B \partial_\sigma h_C^{\sigma} - 4 \partial_\rho h_B \partial_\sigma h_C^{\rho} + 4 h_B^{\mu \nu} \partial_\rho h_C^{\sigma} \right) \\
& \quad - \partial_\rho u_B^\rho \left( 2 \partial_\rho h_B \partial_\sigma h_C^{\rho} - 4 \partial_\rho h_B \partial_\sigma h_C^{\rho} \partial_\rho h_C^{\sigma} \right) \\
& \quad + 2 u_A^\mu \left( \partial_\rho u_B^\rho \partial_\sigma u_C^\nu - \partial_\rho u_B^\rho \partial_\sigma u_C^\nu \partial_\rho u_B^\rho \partial_\sigma u_C^\nu \right) \\
& \quad + 2 u_A^\mu \left( \partial_\rho u_B^\rho \partial_\sigma u_C^\nu + \partial_\rho u_B^\rho \partial_\sigma u_C^\nu \right) \right].
\end{align*}

(4.20)
\[ T^{\mu\nu} = 2 \, t^{ABC} \left\{ \left[ \left( -u^\rho_A \partial_\rho u_B \partial^\nu h^C_{\mu} - u^\rho_A \partial_\nu u_B \partial^\mu h^C_{\rho} \right) + u^\rho_A \partial_\rho u_B \partial^\nu h^C_{\mu} \right] \right\} - 4 \partial^\rho u A^\mu \partial_\rho u B^\nu h^C_{\mu\rho} \] (4.21)

\[ T^{\mu\nu\rho} = t^{ABC} \left[ u_A \lambda \left( \partial^\mu u^\nu B \partial^\lambda u^\rho C + \partial^\nu u^\rho B \partial^\lambda u^\mu C + \partial^\rho u^\mu B \partial^\lambda u^\nu C \right) \right. \]
\[ - \partial^\nu u A^\mu \partial_\lambda u B^\rho u C^\lambda \left. - \partial^\rho u A^\nu \partial_\lambda u B^\mu u C^\nu - \partial^\mu u A^\rho \partial_\lambda u B^\nu u C^\nu \right] - (\mu \leftrightarrow \nu) \] (4.22)

In the off-shell formalism [6] we have the following expressions for

\[ S^I \equiv sT^I = d_Q T^I - i\partial_\mu T^{I\mu}. \] (4.23)

\[ S = it^{ABC} \left( -2u_A^\mu d_\mu h_B A_\beta \Box h_{C}^{\alpha\beta} - 2d_\mu u_A^\mu h_B A_\beta \Box h_{C}^{\alpha\beta} + u_A^\mu d_\mu h_B \Box h_C + d_\mu u_A^\mu h_B \Box h_C \right. \]
\[ - 2d_\alpha u_A^\beta h_B A_\beta \Box h_C + 4d_\rho u_A^\mu h_B u_B^\rho \Box h_{C}^{\rho\nu} - 2u_A^\mu d_\mu u_B^\nu \Box \tilde{u}_C \nu \] (4.24)

\[ S^\mu \equiv 2i \ u_A^\mu d_\rho u_B^\nu \Box h_{C}^{\mu\nu} \] (4.25)

\[ S^{\mu\nu} = i \ (u_A^\rho d_\rho u_B^\nu \Box u_C^\mu - \Box u_A^\mu d_\rho u_B^\nu \Box u_C^\rho) \] (4.26)

Here \( \Box \) is the formal d’Alembert operator build from formal derivatives:

\[ \Box \equiv d_\mu d^\mu. \] (4.27)
5 Second Order

We compute the gauge variation in the off-shell formalism. This means that we compute off-shell the expression \( sT(T(x), T(y)) \). The result is:

\[
sT(T(x), T(y)) = \Box D^F(x - y)[A(x, y) + (x \leftrightarrow y)] + \partial_\mu \Box D^F(x - y)[A^\mu(x, y) - (x \leftrightarrow y)] \tag{5.1}
\]

where

\[
A = A_h + A_{gh}, \quad A^\mu = A_h^\mu + A_{gh}^\mu \tag{5.2}
\]

The explicit expressions are:

\[
A_h(x, y) = 2 \, t^{ABCD} \left[ (u_A^\lambda \partial_\lambda h_B^{\mu\nu})(x) \ (h_C^{\mu\nu} \partial_\rho h_D^{\rho\sigma}) + 4 \partial^\rho h_C^{\mu\rho} \partial^\sigma h_D^{\rho\sigma} + 2 \partial_\rho h_C^{\mu\rho} \partial^\rho h_D^{\rho\sigma} + 4 \partial_\rho h_C^{\mu\rho} \partial^\rho h_D^{\rho\sigma} \right] \tag{5.3}
\]

\[
A_{gh}(x, y) = 8 \, t^{ABCD} \left[ (u_A^\lambda \partial_\lambda h_B^{\mu\nu})(x) \ (h_C^{\mu\nu} \partial_\rho h_D^{\rho\sigma}) + 4 \partial^\rho h_C^{\mu\rho} \partial^\sigma h_D^{\rho\sigma} + 2 \partial_\rho h_C^{\mu\rho} \partial^\rho h_D^{\rho\sigma} + 4 \partial_\rho h_C^{\mu\rho} \partial^\rho h_D^{\rho\sigma} \right] \tag{5.4}
\]

\[
A_{gh}^\mu(x, y) = 8 \, t^{ABCD} \left[ (u_A^\lambda \partial_\lambda h_B^{\mu\nu})(x) \ (h_C^{\mu\nu} \partial_\rho h_D^{\rho\sigma}) + 4 \partial^\rho h_C^{\mu\rho} \partial^\sigma h_D^{\rho\sigma} + 2 \partial_\rho h_C^{\mu\rho} \partial^\rho h_D^{\rho\sigma} + 4 \partial_\rho h_C^{\mu\rho} \partial^\rho h_D^{\rho\sigma} \right] \tag{5.5}
\]
The next step is to make an off-shell renormalization i.e. to consider the expressions:

\[ T^R(T^\mu(x), T(y)) \equiv T(T^\mu(x), T(y)) - i \Box D^F(x - y) A^\mu(y, x) \]
\[ T^R(T(x), T^\mu(y)) \equiv T^R(T^\mu(y), T(x)) \]
\[ T^R(T(x), T(y)) \equiv T(T^\mu(x), T(y)). \] (5.8)

For these new expressions we have:

\[ s T^R(T(x), T(y)) = \Box D^F(x - y)[A(x, y) + (x \leftrightarrow y)] \] (5.9)

where

\[ A = A_h + A_{gh}. \] (5.10)

The explicit expressions are:

\[ A_h(x, y) = 2 t^{ABCD} [2(u_A^\rho \partial_\lambda h_B^\rho)(x) (\partial_\mu h_\rho^\omega \partial_\nu h_D^\epsilon + h_\rho^\omega \partial_\nu \partial_\mu h_D^\epsilon - \partial_\mu h_{C\rho\sigma} \partial^\nu h_D^{\epsilon\sigma})(y) + (u_A^\rho \partial_\lambda h_B^\rho)(x) (-4 \partial_\mu h_\rho^\omega \partial_\nu h_D^\epsilon - 4 h_\epsilon^\rho \partial_\nu \partial_\sigma h_D^{\rho\omega} - 8 \partial_\rho h_{C\mu\sigma} \partial_\nu \partial_\mu h_D^{\epsilon\sigma} - 8 h_{C\mu\sigma} \partial_\nu \partial_\mu h_D^{\epsilon\rho})(y)
+ 2(\partial_\mu u_A^\rho h_B^\rho)(x) (4 \partial_\mu h_\rho^\omega \partial_\nu h_{D\lambda} - 4 h_\epsilon^\rho \partial_\nu \partial_\mu h_{D\lambda} + 4 \partial_\mu h_{C\nu\lambda} \partial_\sigma h_\rho^\omega + h_{C\lambda} \partial_\sigma h_\rho^\omega - 4 \partial_\mu h_{C\nu\lambda} \partial_\sigma h_\rho^\omega - 8 h_{C\nu\lambda} \partial_\mu \partial_\sigma h_\rho^\omega]
- 2 \partial_\lambda h_{C\mu\nu} \partial_\sigma h_\rho^\omega - 4 \partial_\mu h_{C\lambda} \partial_\sigma \partial_\rho h_\rho^\omega)(y)
+ 4(\partial_\mu u_A^\rho h_B^\rho)(x) (-\partial_\mu h_\rho^\omega \partial_\nu h_D^\epsilon - 4 \partial_\mu h_{C\nu\lambda} \partial_\sigma h_\rho^\omega - 4 \partial_\mu h_{C\lambda} \partial_\sigma h_\rho^\omega - 8 h_{C\lambda} \partial_\mu \partial_\sigma h_\rho^\omega)(y)
+ (\partial_\lambda u_A^\rho h_B^\rho)(x) (-\partial_\mu h_{C\nu\lambda} \partial_\sigma h_\rho^\omega - 2 \partial_\mu h_{C\nu\lambda} \partial_\sigma h_\rho^\omega + 4 \partial_\sigma h_{C\lambda\mu} \partial_\rho h_\rho^\omega + 4 \partial_\rho h_{C\lambda\mu} \partial_\rho h_\rho^\omega - 4 \partial_\rho h_{C\lambda\mu} \partial_\rho h_\rho^\omega - 8 h_{C\lambda\mu} \partial_\rho \partial_\sigma h_\rho^\omega)] (5.11)

\[ A_{gh}(x, y) = 8 t^{ABCD} [(u_A^\rho \partial_\lambda h_B^\rho)(x) (\partial_\rho u_\sigma^\lambda \partial_\nu \tilde{u}_D^\sigma + \partial_\sigma u_\nu^\lambda \partial_\rho \tilde{u}_D^\sigma + u_\epsilon^\lambda \partial_\nu \partial_\rho \tilde{u}_D^\sigma)(y)
- (\partial_\rho u_A^\sigma h_B^\rho)(x) (\partial_\rho u_\sigma^\lambda \partial_\nu \tilde{u}_D^\sigma + \partial_\sigma u_\nu^\lambda \partial_\rho \tilde{u}_D^\sigma + u_\epsilon^\lambda \partial_\nu \partial_\rho \tilde{u}_D^\sigma)
+ \partial_\rho u_\nu^\lambda \partial_\nu \tilde{u}_D^\sigma + u_\epsilon^\lambda \partial_\nu \partial_\rho \tilde{u}_D^\sigma + u_\epsilon^\rho \partial_\nu \partial_\sigma \tilde{u}_D^\rho)(y)
+ (\partial_\rho u_A^\sigma h_B^\rho)(x) (\partial^\rho u_\sigma^\lambda \partial_\nu \tilde{u}_D^\sigma + \partial^\rho u_\nu^\lambda \partial_\rho \tilde{u}_D^\sigma + u_\epsilon^\lambda \partial_\nu \partial_\rho \tilde{u}_D^\sigma)
+ \frac{1}{2} (\partial_\rho u_A^\rho h_B^\rho)(x) (\partial_\rho u^\nu_\nu \partial_\sigma \tilde{u}_D^\nu)(y)
- (u_A^\sigma \partial^\sigma u_B^\rho)(x) (\partial_\rho h_\mu^\nu \partial_\nu \tilde{u}_D^\rho + \partial_\mu h_\nu^\mu \partial_\nu \tilde{u}_D^\rho + h_\mu^\nu \partial_\nu \partial_\rho \tilde{u}_D^\rho)] (5.12)

In the on-shell limit we have

\[ \Box D^F(x - y) \to \delta(x - y) \] (5.13)

so the formula (5.9) becomes

\[ s T^R(T(x), T(y)) = \delta(x - y) A(x) \] (5.14)

where

\[ A(x) \equiv 2 A(x, x). \] (5.15)
Then remaining anomaly $\mathcal{A}$ can be eliminated iff it can be written under the form
\[
\mathcal{A} = sN = d_Q N + i \partial_\mu N^\mu \tag{5.16}
\]
i.e. it is a coboundary. We consider only the first contribution and note that it has the structure:
\[
\mathcal{A}_h = u^\mu H^A_\mu + \partial^\nu u^\mu H^A_\mu \tag{5.17}
\]
where $H^A_\mu$ and $H^A_{\mu\nu}$ are expressions tri-linear in $h^\alpha_{\beta B}$. From (5.16) it follows that the terms of the first kind (i.e. without derivatives on $u^\mu_A$) can appear only from $N^\mu$ more precisely we must have:
\[
N^\mu = u_\nu t^\mu_\nu + \cdots \tag{5.18}
\]
where $t^\mu_\nu$ are expressions tri-linear in $h^\alpha_{\beta B}$.

Then it easily follows that we must have:
\[
H^A_\mu = \partial^\nu t^\mu_\nu. \tag{5.19}
\]

It is sufficient to consider the terms with two factors $h_B$ and one factor $h^\alpha_{\beta B}$ from $H^A_{\mu\nu}$ and write the generic ansatz for the corresponding sector of $t^\mu_\nu$:
\[
\begin{align*}
 t^{A(1)}_{\alpha\lambda} &= t^1_{ABCD} h_B \partial^\alpha h_D \partial^\lambda h_C \\
 t^{A(2)}_{\alpha\lambda} &= t^2_{ABCD} h_B \partial^\alpha h_C \partial^\lambda h_D \\
 t^{A(3)}_{\alpha\lambda} &= t^3_{ABCD} h_B \partial^\lambda h_D \partial^\alpha h_C \\
 t^{A(4)}_{\alpha\lambda} &= t^4_{ABCD} \eta_\alpha \partial^\lambda h_B \partial^\alpha h_C \\
 t^{A(5)}_{\alpha\lambda} &= t^5_{ABCD} \partial^\alpha h_B \partial^\lambda h_D \partial^\alpha h_C \\
 t^{A(6)}_{\alpha\lambda} &= t^6_{ABCD} \partial^\lambda h_B \partial^\alpha h_D \partial^\alpha h_C \\
 t^{A(7)}_{\alpha\lambda} &= t^7_{ABCD} \partial^\alpha h_D \partial^\lambda h_B \partial^\alpha h_C \\
 t^{A(8)}_{\alpha\lambda} &= t^8_{ABCD} \partial^\alpha h_B \partial^\lambda h_D \partial^\alpha h_C \\
 t^{A(9)}_{\alpha\lambda} &= t^9_{ABCD} \partial^\beta h_B h_C \partial^\gamma h_D \\
 t^{A(10)}_{\alpha\lambda} &= t^{10}_{ABCD} \eta_\alpha \partial^\gamma h_B h_C \partial^\beta h_D \\
 t^{A(11)}_{\alpha\lambda} &= t^{11}_{ABCD} \eta_\alpha h_B h_C \partial^\alpha \partial^\beta h_D \\
 t^{A(12)}_{\alpha\lambda} &= t^{12}_{ABCD} \eta_\alpha h_B h_C \partial^\alpha \partial^\beta h_D \\
 t^{A(13)}_{\alpha\lambda} &= t^{13}_{ABCD} \eta_\alpha h_B h_C \partial^\beta h_D \\
 t^{A(14)}_{\alpha\lambda} &= t^{14}_{ABCD} \partial^\beta h_B h_C \partial_\alpha h_D \\
 t^{A(15)}_{\alpha\lambda} &= t^{15}_{ABCD} \partial^\beta h_B h_C \partial_\alpha h_D \\
 t^{A(16)}_{\alpha\lambda} &= t^{16}_{ABCD} \eta_\alpha \partial^\beta h_B h_C h_D
\end{align*}
\tag{5.20}
\]
where
\[
t^A_{\mu\nu} = C \leftrightarrow D, \quad j = 1, 4, 14, 15, 16. \tag{5.21}
\]

We insert everything in the relation (5.19) and obtain a linear system. An easy consequence of this system is
\[
t^{ABCD} = A \leftrightarrow C. \tag{5.22}
\]
Now the proof goes as in [2]: we define the $R \times R$ matrices
\[
(T_A)_{BC} \equiv t^{ABC} \tag{5.23}
\]
and the previous relation can be written as:
\[
T_A T_B = T_B T_A \tag{5.24}
\]
Because the matrices $T_A$ are real, symmetric and commute they can be diagonalized simultaneously i.e in a convenient base we have:

$$t^{ABC} = \lambda_{AB} \delta_{BC}. \quad (5.25)$$

From here it follows that only the expressions $t^{AAA}$ can be non-zero. It means that there are no cross terms of interactions between two distinct gravitons. This is our main result. The case of massive gravitons [8] leads to the same no-interaction theorem. Indeed, in the massive case the terms of top canonical dimension $\omega = 5$ from all expressions are the same as in the massless case.

For completeness we finish the analysis in the case $R = 1$ i.e. a single graviton, so we do not need the indices $A, B, \ldots$. It can be proved rather easily that the ghost part of the anomaly $A_{gh}$ is a total derivative. So the relation (5.16) must be imposed only on $A_h$. First, we rewrite it in the form

$$A_h = u_\mu A^\mu_h + i \partial_\mu N^\mu_h \quad (5.26)$$

with $A^\mu_h \sim hhh$. Then the following result can be proved by some computations (see also [14]):

**Theorem 5.1** The following formula is true

$$A_h = d_Q N + \partial_\mu N^\mu \quad (5.27)$$

where:

$$N = 4i(2h^{\mu\nu}h^{\rho\sigma} \partial_\mu h_{\nu\sigma} \partial_\nu h_{\rho\mu} - h^{\mu\nu}h^{\rho\sigma} \partial_\lambda h_{\nu\rho} \partial_\lambda h_{\mu\sigma} - 4h^{\mu\nu}h^{\rho\sigma} \partial_\nu h_{\lambda\rho} \partial_\lambda h_{\mu\sigma} - 4h^{\mu\nu}h^{\rho\sigma} \partial_\nu h_{\lambda\rho} \partial_\lambda h_{\mu\sigma} + 2h^{\mu\nu}h^{\rho\sigma} \partial_\lambda h_{\mu\rho} \partial_\lambda h_{\nu\sigma} - 2h^{\mu\rho}h^{\nu\sigma} \partial_\lambda h_{\mu\nu} \partial_\lambda h_{\rho\sigma}). \quad (5.28)$$

The idea of the proof is similar to previous one. We compute the expressions of the type $d_Q N_j$ for various Wick monomials $N_j \sim hhhh$ and of canonical dimension $\omega = 6$. Next, “by partial integration” we exhibit them in the form

$$d_Q N_j = \partial_\mu M^\mu_j + u_\mu N^\mu_j. \quad (5.29)$$

If we succeed to fix the coefficients of these monomials such that

$$\sum a_j N^\mu_j = A^\mu_h \quad (5.30)$$

then the proof is finished. The monomials from the statement do the job. We can eliminate the anomaly $A$ by obvious redefinitions of the chronological products. We remark that the redefinition $N$ of $T(T(x), T(y))$ does not contain ghost terms. This seems to be the main advantage of the choice (4.2).
6 Conclusions

We have derived the no-interaction theorem for the multi-graviton system in the framework of the causal formalism of perturbative quantum field theory. However, interaction between two distinct species of gravitons might be possible mediated by matter fields, in higher orders of perturbation theory. This is a subject of further investigation.

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