TESTING GAUSSIANITY IN THE WMAP DATA OF OT FOREGROUND REDUCED MAP

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A considerable effort has recently gone into the study of Gaussianity of cosmic microwave background (CMB) data. Among such attempts, there is one with two non-Gaussianity indicators, proposed by the authors, and used in a search for significant deviation from Gaussianity in the WMAP internal linear combination (ILC) and in the single frequency WMAP maps with the \( KQ75 \) mask. Here we extend and complement these results by performing a similar analysis for the de Oliveira-Costa and Tegmark (OT) WMAP three-year \( KQ75 \) masked map, in which the foreground is reduced through a different statistical cleaning procedure.

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1. Introduction

A key prediction of the so-called slow-roll inflationary models is that they cannot generate detectable non-Gaussianity of the cosmic microwave background (CMB) temperature fluctuations within the accuracy of the Wilkinson Microwave Anisotropy Probe (WMAP).\(^1\) There are, however, a number of inflationary models that predict non-Gaussianity at a level detectable by the WMAP experiment (see the reviews Refs.\(^2\)). Thus, non-Gaussianity in the CMB data may be potentially important to discriminate classes of inflationary models.

The WMAP team have found that the CMB data are consistent with Gaussianity.\(^3\) However, a number of recent analyses of CMB data performed with different statistical estimators have provided indications of deviations from Gaussianity in the CMB temperature fluctuations (for a list of references see, for example, Ref\(^4\) and references therein). On the other hand, one does not expect that a single statistical estimator can be sensitive to all possible forms of non-Gaussianity that may be present in WMAP data. Thus, it is important to employ alternative indicators in order to shed light on the possible causes for the reported non-Gaussianity of CMB data.
Recently we proposed two new non-Gaussianity indicators, based on skewness and kurtosis of large-angle patches of CMB maps, which provide a directional measure of deviation from Gaussianity on large angular scales. A distinctive feature of these indicators is that they make it possible to construct sky maps of Gaussianity from the CMB temperature data. In Ref. 4 we also built and studied in details such maps of Gaussianity generated from the data of WMAP three and five-year internal linear combination (ILC) maps with $KQ75$ mask, and found no statistically significant deviation from Gaussianity. Here we extend and complement these studies by using our indicators to carry out a new analysis of Gaussianity with the CMB data of the so-called OT map three-year $KQ75$ masked map, in which the Galactic foregrounds are cleaned with the TOH procedure.

2. Indicators and Maps

The steps in the construction of our non-Gaussianity indicators and the associated maps, are the following:

i. Take a discrete set of points \{$j = 1, \ldots, N_c$\} homogeneously distributed on the CMB celestial sphere $S^2$ as the centers of spherical caps of a given aperture $\gamma$;

ii. Calculate for each cap $j$ the skewness and kurtosis given, respectively, by

\[
S_j = \frac{1}{N_p \sigma_j^3} \sum_{i=1}^{N_p} (T_i - \overline{T})^3 \quad \text{and} \quad K_j = \frac{1}{N_p \sigma_j^4} \sum_{i=1}^{N_p} (T_i - \overline{T})^4 - 3, \quad (1)
\]

where $N_p$ is the number of pixels in the $j^{th}$ cap, $T_i$ is the temperature at the $i^{th}$ pixel, $\overline{T}$ is the CMB mean temperature, and $\sigma$ is the standard deviation. Clearly, the numbers $S_j$ and $K_j$ obtained in this way for each cap can be viewed as a measure of non-Gaussianity in the direction of the center of the cap $(\theta_j, \phi_j)$.

iii. Patching together the $S_j$ and $K_j$ values for each cap, one obtains our indicators, i.e., discrete functions $S = S(\theta, \phi)$ and $K = K(\theta, \phi)$ defined over the celestial sphere, which can be used to measure the deviation from Gaussianity as a function of $(\theta, \phi)$.

Clearly, the discrete functions $S = S(\theta, \phi)$ and $K = K(\theta, \phi)$ can be expanded into their spherical harmonics in order to have their power spectra $S_\ell$ and $K_\ell$. Thus, for example, for the skewness one has $S(\theta, \phi) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} b_{\ell m} Y_{\ell m}(\theta, \phi)$ and $S_\ell = (2\ell + 1)^{-1} \sum_m |b_{\ell m}|^2$. Similar expressions obviously hold for the kurtosis $K = K(\theta, \phi)$.

\[a\]We have also used our indicators to test for Gaussianity the single frequency foreground unreduced WMAP three and five-year maps, but here for comparison we focus on the foreground-reduced ILC and OT maps.

\[b\]For a detailed discussion of the indicator briefly presented here we refer the readers to Ref. 4.
3. Main Results and Conclusions

In this section we shall report the results of our calculation of \( S = S(\theta, \phi) \) and \( K = K(\theta, \phi) \) indicators for the WMAP three-year data of the OT with \( KQ75 \) mask, and compare with the results of Ref. 4 for the ILC WMAP three-year map with the same mask.

![Monopole removed Skewness and Kurtosis maps](image)

Fig. 1. Monopole removed Skewness (left) and monopole removed Kurtosis (right) maps generated from the WMAP three-year OT map with mask \( KQ75 \). Colored high resolution versions of these figures are available in the arXiv version.

In order to minimize the statistical noise, in the calculations of skewness and kurtosis indicator maps (\( S - \)map and \( K - \)map) we have scanned the celestial sphere with spherical caps of aperture \( \gamma = 90^\circ \), centered at \( N_c = 12,288 \) points on the sphere homogeneously generated by using HEALPix package. 7

Figure 1 shows the Mollweide projection of the monopole removed \( S \) (left) and \( K \) (right) maps generated from the OT three-year map with a \( KQ75 \) mask. These maps show spots with higher and lower values of indicators \( S(\theta, \phi) \) and \( K(\theta, \phi) \), thus suggesting at first sight non-Gaussianity of the CMB as given by OT temperature map. Fig. 1 is also suggestive of large-scale dominant components (low \( \ell \) components) in the maps of both indicators.

![Power spectra of skewness and kurtosis](image)

Fig. 2. Low \( \ell \) differential power spectra of skewness \( |S_\ell - \overline{S_\ell}| \) (left) and kurtosis \( |K_\ell - \overline{K_\ell}| \) calculated from the monopole removed maps of Fig. 1 which were generated from the WMAP three-year OT map with mask \( KQ75 \). The 68\% and 95\% confidence levels (obtained from MC simulated random maps) are indicated, respectively, by the dashed and dash-dotted lines.

In order to obtain quantitative information about the observed inhomogeneous
distribution for these non-Gaussianity indicators, we have calculated the power spectrum of both the \( S \)–map and \( K \)–map for the three-year OT \( KQ75 \) cut-sky map. These power spectra allow to estimate the statistical significance of the \( S \) and \( K \) multipole values, by comparing their power spectra with the averaged power spectrum of the \( S \) and \( K \) maps obtained by averaging over a set of 1 000 Monte-Carlo-generated statistically Gaussian CMB maps. In the left panel of Fig. 2 we depict differential power spectrum \(| S_\ell - \overline{S}_\ell | \) (overline denotes the mean) are obtained from the Monte-Carlo simulations with each map being a stochastic realization of WMAP best-fitting angular power spectrum of the \( \Lambda \)CDM model, obtained by randomizing the multipole temperature components \( a_{\ell m} \) within the cosmic variance limits. The right panel shows the corresponding plot \(| K_\ell - \overline{K}_\ell | \) for the \( K \)–map. For a comparison the panels of Fig. 2 also show the differential power spectrum obtained from the WMAP three-year ILC \( KQ75 \) cut-sky map.

Figure 2 shows the deviation of \( S_\ell \) and \( K_\ell \) values (\( \ell = 1, \cdots, 10 \)) from the mean values of \( \overline{S}_\ell \) and \( \overline{K}_\ell \) are within (less than) 95% the mean values (for each \( \ell \)) obtained from 1 000 Monte-Carlo-generated statistically Gaussian maps. This clearly indicates how likely is the occurrence of the multipole values obtained from the OT data in the set of MC multipole values (from MC maps), giving therefore a clear indication of consistency with Gaussianity for the OT \( KQ75 \) masked map, in agreement with previous analyses performed with different statistical tools (see, e.g. Refs. 3–4).

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