A variety of gravitational-wave transient sources can be modeled in the Fourier domain using a power law. This simple power-law model provides a reasonable approximation for gravitational-wave bursts from cosmic string cusps, cosmic string kinks, and the memory effect. Each of these sources is described using a different spectral index. In this work, we simulate interferometer strain data with injections of power-law and memory bursts to demonstrate parameter estimation, signal detection, and model selection. We show how Bayesian inference can be used to measure the power-law spectral index, thereby distinguishing between different astrophysical scenarios.

I. INTRODUCTION

The LIGO and Virgo Scientific Collaborations have observed and cataloged eleven significant gravitational-wave signals in the first two observing runs [1], each of them originating from the coalescence of binary black holes or neutron stars. Considerable effort is also undertaken to search for gravitational-wave “bursts”: unmodeled (or minimally-modeled) transients. Bursting signals have been proposed for a variety of mechanisms including pulsar glitches [2], neutron star collapse [3], core-collapse supernovae [4], cosmic-string interactions [5–7], and gravitational-wave memory [8,9]. Previous searches for gravitational-wave bursts include short-duration transients that arise from pulsar glitches and supernovae explosions [10,11], long-duration transients such as those from fallback accretion [12,13], cosmic strings [14], binary black hole mergers [15,16], intermediate mass black hole mergers [17], post-merger remnants from neutron star mergers [18], sub-solar mass binaries [19], eccentric binaries [20], those associated with magnetar bursts [21], those associated with neutrino emission [22], and those with electromagnetic counterparts [23,24]. Active pipelines that search for unmodeled burst transients include coherent WaveBurst (cWB) [25] and Omicron-LALInferenceBurst (oLIB) [26]. The possibility of an unexpected source is the most promising motivation for gravitational wave burst searches.

In the event of a gravitational-wave burst detection, a key question will be: what is the astrophysical source of this burst? In previous work, researchers have begun to assemble a set of tools to help answer this question, for example, developing principle component models of gravitational waves from supernovae [27,28] and performing source characterization on gravitational-wave bursts using wavelets [29,30]. In this work, we consider a power-law model, which can be used to describe a variety of gravitational-wave signals including bursts from cosmic string kinks, cosmic string cusps, and memory. Each of these signals is characterized by a unique spectral index. We demonstrate how Bayesian inference can be used to estimate the power-law index, thereby distinguishing between different astrophysical scenarios in this class of bursting signals. We also extend this analysis to show how we rule out power-law burst sources for gravitational-wave candidates that have their astrophysical source in question. By combining our method with the previously described techniques, it should be possible to construct a set of minimally-modeled bursts: supernovae bursts, cosmic string bursts, an arbitrary superposition of wavelets, etc. By comparing the Bayesian evidence from the different models in the catalog, it will be possible to determine which catalog entry best describes the burst.

This paper is structured as follows: Section II describes the set of power-law signal models used in the analysis. Section III reviews the fundamentals of Bayesian inference, establishes the prior distribution for each signal parameter, and lists the steps taken to simulate and analyze interferometer strain data. Section IV presents results from a set of injection simulations with discussions on their implications.

II. SIGNAL MODELS

We model signals with respect to phenomenological parameters, or signal characteristics, such that the dimensionality of each model remains low, ultimately reducing model complexity and improving statistical convergence. For example, rather than individually parameterizing the mass, distance, and inclination of an astrophysical system that emits memory, we just parameterize the “amplitude” of a memory signal. In all models, we use $t_A$ to denote the arrival time of a given burst signal.
A. Memory

The non-linear memory effect, a prediction of general relativity, was first derived in [31]. The memory effect is a linearly polarized, DC, gravitational-wave signal originating from an anisotropic gravitational-wave energy flux; see [32]. Memory signals have a characteristic rise time $\tau$ proportional to the total mass of the astrophysical system [9]. We model the time-domain waveform as,

$$ h_m(t) = A \tanh \left( \frac{t - t_A}{\tau} \right). $$

(1)

Here $t$ is time, and $A$ is a parameter describing the memory amplitude. In the Fourier domain, this memory model has the analytic form,

$$ \tilde{h}_m(f) = -i \pi A e^{-2\pi if\tau_A} (\sinh(\pi^2 \tau f))^{-1}. $$

(2)

In the limit of $\tau \to 0$, corresponding to an astrophysical system with minimal mass, this memory model approaches $-iAe^{-2\pi if\tau_A}/\pi f$, a power-law signal with a spectral index of one or equivalently, a step-function in the time domain.

B. Cosmic String Interactions

It has been theorized that topological defects during a symmetry-breaking phase transition in the early Universe can give rise to one-dimensional strings that expand to cosmological scales and form cosmic string networks [5]. A network of cosmic strings can produce gravitational-wave bursts, arising from processes such as cusps [7] and kinks through string interactions. Cusps are parts of a cosmic string that move at relativistic speeds, their gravitational-wave emission is modelled by,

$$ \tilde{h}_c(f) = Ae^{-2\pi if\tau_A} f^{-4/3}. $$

(3)

Kink signals are sourced from discontinuities in the string’s tangent vector, with their resulting waveform as,

$$ \tilde{h}_k(f) = Ae^{-2\pi if\tau_A} f^{-5/3}. $$

(4)

Both waveforms are linearly polarized and are valid under the condition the line of sight from the source to the observer lies inside the emission cone, see [7] for details.

C. Arbitrary Power Law

Inspired by the morphological similarity between memory and string waveforms we propose to search for a general power-law signal, which might capture a range of scale-invariant phenomena that emit gravitational waves. We define a general power-law model

$$ \tilde{h}_\alpha(f) = Ae^{-2\pi if\tau_A} f^{-\alpha}, $$

(5)

where $\alpha$ is the power-law spectral index. We assume linear polarization. In what follows, we include the cosmic string waveforms as subsets of this power-law model.

III. METHOD

A. Bayesian Inference

We use Bayesian inference to estimate parameters and calculate the significance of some signal model given observed data. We denote the complex, frequency-domain strain data to be $d$ and a given frequency-domain waveform model $\mu(\theta)$ as a function of signal parameters $\theta$. We denote the prior distribution on signal parameters as $\pi(\theta)$. We then construct the posterior probability distribution $p(\theta|d)$ using Bayes’ theorem

$$ p(\theta|d) = \frac{L(d|\theta)\pi(\theta)}{Z}. $$

(6)

We employ a Gaussian likelihood function,

$$ \ln L(d|\theta) = -\frac{1}{2} \left[ \sum_j \frac{|d_j - \mu_j(\theta)|^2}{\sigma_j^2} + 2 \ln (2\pi\sigma_j^2) \right]. $$

(7)

The summation in the likelihood function is taken over $j$ frequency bins and $\sigma_j$ is the noise amplitude spectral density of the detector in the $j$th frequency bin. We assume Gaussian noise for the remainder of our analysis. The evidence is

$$ Z = \int L(d|\theta) \pi(\theta) d\theta. $$

(8)

Given signal models $A$ and $B$, with the ratio of the prior odds set to unity, we compare models to noise $N$, and models to other models by computing Bayes factors $BF_N^A = Z_A/Z_N$ (signal versus noise) and $BF_B^A = Z_A/Z_B$ (model versus model). Following convention, we here consider $\ln(BF) > 8$ to be strong evidence for a particular hypothesis [33].

B. Prior Distributions

We choose our priors to be uninformative. For example, we set the prior for the amplitude parameter (controlling the loudness of the signal) to be log-uniform. We choose the prior for the rise time, $\tau$, to be uniformly distributed between 0.5 – 50 ms. Anything outside this range is difficult to resolve with audio-band (10–2000 Hz) detectors. We choose the prior for the spectral index in Eq. 5 $\pi(\alpha)$, to be uniform on the interval $[0, 2]$. Note, the cosmic string models which have spectral indices of $\alpha = 4/3$ (cusp) and $\alpha = 5/3$ (kink) are included in the parameter space of the arbitrary power-law model. We use standard priors for extrinsic parameters such as polarization angle and sky location. We explicitly marginalize over the arrival time, with a uniform distribution over the duration of the data segment, and we can numerically reconstruct its posterior distribution in post-processing; see, e.g., [34].
FIG. 1: Marginalized posterior distributions for each signal parameter of the memory model (left), with signal/noise log Bayes factor \( \ln(BF) = 68.8 \) and a cosmic string kink model (right), with signal/noise \( \ln(BF) = 60.8 \). The red lines show the values of the injected parameters and the dotted lines in each one-dimensional posterior distribution represent one standard deviation credible intervals. The contour levels of the two-dimensional posterior distributions are 0.5, 1.0, 1.5, and 2 standard deviations. These plots show how well we recover signal parameters of a given astrophysical model.

C. Procedure

We outline the steps taken to simulate interferometer strain data and to run our Bayesian inference analysis on a given signal injection. We set the total duration of the data segment to be 4 seconds at a sampling rate of 4096 Hz. All signals are injected with \( t_A = 2 \) s. We assume a three-detector network consisting of two Advanced LIGO detectors and one Advanced Virgo detector operating at design sensitivity [35]. We generate Gaussian noise and inject a power-law signal with a randomly chosen set of injection parameters. We set the minimum and maximum frequency of our detectors to be \([20\, \text{Hz}, 2000\, \text{Hz}]\), corresponding to the most sensitive region of our detectors.

We define the total joint matched-filter signal-to-noise ratio (SNR) to be

\[
\rho = \frac{\langle d, \mu \rangle_{\text{tot}}}{\langle \mu, \mu \rangle_{\text{tot}}^{1/2}},
\]

(9)

where

\[
\langle a, b \rangle_{\text{tot}} = 4\Delta f \sum_k \sum_j R \left( \frac{a_{j,k}^* b_{j,k}}{\sigma_{j,k}^2} \right),
\]

(10)

Here, \( \Delta f \) is the frequency resolution and \( k \) is an index for the detector.

We carry out inference using the nested-sampling algorithm “dynesty” with 500 live points [36]. We use the Bayesian inference software package Bilby [37] to perform our analysis.

IV. SIMULATIONS

A. Parameter Estimation

To infer the properties of a gravitational-wave burst event, we calculate the posterior distributions for each parameter in a given signal model. We show the one-dimensional and two-dimensional posterior distributions for an injection of a memory and cosmic string kink signal in Fig. 1, quantifying how well we recover a set of injected parameters.

B. Signal Detection

We calculate how loud a memory and power-law event must be in order to detect the astrophysical signal from background noise. We show results from injection simulations of memory and power-law signals in Fig. 2 with injection parameters drawn from the chosen priors.
By using the posterior distribution of the spectral index to characterize a power-law burst, and calculating the model Bayes factor of a power-law model with other signal models, we differentiate between various astrophysical burst scenarios. Suppose we were to detect a cosmic string cusp signal, how do we establish that it is not a memory or kink burst? To demonstrate how we use the Bayes factor to select between signal models, we inject a cosmic string cusp signal, equivalent to a power law with $\alpha = 4/3$, and perform a search for a memory, kink, cusp and power-law burst. We incrementally increase the amplitude, while keeping all other parameters fixed, to observe how our confidence increases with the loudness of the injected signal, shown in Fig. 3.

We highlight important features of model selection, using the power law as an example. When comparing the power law to memory at a signal-to-noise ratio of $\sim 7$, the power-law hypothesis is more significant than the memory hypothesis. We note that the power-law model has a larger prior volume relative to the cosmic string models. As a result of Occam’s penalty, it is difficult to calculate overwhelming evidence in favor of a power-law model over a cosmic string model shown by the power law/cusp and power law/kink Bayes factors remaining close to $\ln(BF) = 0$ in Fig. 3.

D. Extension to Binary Black Hole Mergers

In the event of a detection of a massive binary black hole merger, one with large component spins, or one with a large mass ratio, the merger waveform may look quali-
tatively different than the “chirping” waveforms observed thus far [1]. In such situations, it will be natural to ask whether or not the candidate event is a binary coalescence, or some other type of burst source. We show how to answer this question by computing the black hole to power law Bayes factor to quantify which model supports the data best.

As an example, we simulate an equal-mass massive binary black hole merger signal with total mass $M_{\text{tot}} = 250 M_\odot$, at signal-to-noise $\rho = 14.5$, using the IMRPhenomPv2 waveforms [38, 39]. This results in a signal that is in-band for only the merger-ringdown portion of the coalescence. We show the similarity between cosmic string signals and a massive binary merger signal in Fig. 4. Next, we perform a search for a binary black hole, a cosmic string cusp (Eq. 3), and a cosmic string kink (Eq. 4) signal. We calculate the following Bayes factors: $\ln(\text{BF}_{\text{BBH cusp}}) = 84.6$ and $\ln(\text{BF}_{\text{BBH kink}}) = 84.4$. Given the magnitude of these Bayes factors, the black hole signal hypothesis is significantly more likely than either of the cosmic string burst signals. In this case, we are able to rule out cosmic strings as the astrophysical origin of the burst event.

**V. SUMMARY AND OUTLOOK**

In this paper, we present a unified framework for detecting gravitational-wave burst signals from memory, cosmic strings, and other bursts that are well approximated by a power law in the Fourier domain. Using simulations, we estimate the signal parameters of a gravitational-wave burst, identify astrophysical signals from interferometer noise, and characterize between power-law bursts by measuring their spectral index.

In testing our classification method we have assumed Gaussian noise. In reality, ground-based interferometer detector noise is populated by glitches, power lines, seismic activity, and other transient noise sources. In future work, we will investigate how to handle the non-Gaussian nature of noise, for example, using techniques such as the Bayes Coherence Ratio defined in [40].

Our signal models are of low dimensionality, especially when compared to the large parameter space of compact binary coalescence signals. Therefore, it takes only a few minutes to analyze a few seconds of data using a single core processor. We aim to perform a search for the memory effect and power-law bursts described in this paper using data from previous observing runs of the LIGO-Virgo network [41].

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