We study the Jacobian Poisson structures in any dimension invariant with respect to the discrete Heisenberg group. The classification problem is related to the discrete volume of suitable solids. Particular attention is given to dimension 3 whose simplest example is the Artin-Schelter-Tate Poisson tensors.