Adaptive OFDM Synchronization Using Quadratic Search Step Sizes

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Abstract—Conventional blind joint symbol time offset (STO) and carrier frequency offset (CFO) estimators for orthogonal frequency division multiplexing (OFDM) systems are high in computational complexity, which increases power consumption and reduces the lifetime of the communications device. In this paper, a novel, real-time, low complexity, blind, and adaptive synchronization algorithm for OFDM systems is presented. It is shown through simulation that the proposed estimator performs better than previously proposed adaptive and conventional non-adaptive algorithms in terms of mean squared error (MSE) once the algorithm has converged, has quicker convergence times than previously proposed adaptive algorithms, and is robust to both small and large changes in STO.

Keywords—adaptive, OFDM, synchronization

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is a prevalent multicarrier modulation method that has been adopted in numerous standards such as IEEE 802.11a/g/p, LTE, and DVB-T/T2. However, OFDM is sensitive to time and frequency offsets which need to be estimated in order to demodulate the received data with low bit error rate [1]. Much research has focused on blindly estimating these offsets: blind estimators have the advantage of not requiring any known training data to estimate some desired parameter, thus preserving high bandwidth efficiency. Furthermore, they do not require any knowledge of the channel. A comprehensive survey of the previously proposed blind synchronization approaches that jointly estimate the symbol time offset (STO) and carrier frequency offset (CFO) in OFDM systems is presented in [2]. A major drawback of previously proposed methods is that they are high in computational complexity, which increases power consumption and reduces the lifetime of the communications device.

In order to reduce the high computational complexity, two adaptive real-time OFDM synchronization algorithms were presented in [3] and [4]. Both algorithms significantly reduce the computational complexity (and thus power consumption), however, each algorithm has its own drawbacks. In [3], the convergence time can be slow especially for large OFDM symbol periods when changes to the STO occur. In [4], the algorithm’s performance is dependent upon a parameter that varies with the channel.

The contribution of this paper is a novel, real-time, low complexity, blind, and adaptive synchronization algorithm for OFDM systems. The proposed algorithm utilizes a quadratically increasing search step size to track large changes in STO, and a joint estimate of the STO and CFO is produced as each symbol is received resulting in better tracking accuracy in terms of mean squared error (MSE) than existing adaptive and non-adaptive algorithms. The proposed algorithm also achieves more robust tracking with quicker convergence times than existing adaptive algorithms. The layout of the paper is as follows. Section II briefly reviews the two adaptive OFDM synchronization algorithms presented in the literature and presents the proposed algorithm. Section III presents simulation results that compare the performance of the proposed algorithm with the adaptive algorithms presented in [3]-[4] and the non-adaptive algorithm presented in [2]. The paper is concluded in Section IV.

II. ADAPTIVE SYNCHRONIZATION ALGORITHM

Before discussing the adaptive algorithms, a non-adaptive algorithm is discussed since it is used later in the simulation results and helps clarify the notation used in this paper.

A. Non-Adaptive Time and Frequency Estimation

The conditional maximum likelihood (CML) estimator from [2] is given by (1)-(3).

\[
\hat{\theta}^* = \arg \max_{\theta \in [0, N-1]} J(\hat{\theta}) \tag{1}
\]

\[
J(\hat{\theta}) = \sum_{m=1}^{M-1} \left\{ \frac{1}{2} \sum_{k=0}^{N-1} \left[ r(k + m(N_{cp} + N)) r^*(k + m(N_{cp} + N)) \right] + \left[ r(k + N + m(N_{cp} + N)) r^*(k + N + m(N_{cp} + N)) \right] \right\} \tag{2}
\]

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The integer STO is defined to be the first arrival path received and is assumed to be less than the number of subcarriers \( N \) (i.e., \( \theta \in [0, N-1] \)). \( N_{cp} \) is the length of the cyclic prefix, \( r(k) \) is the \( k \)th received sample of the critically sampled (i.e., \( N+N_{cp} \) samples per OFDM symbol) OFDM signal, \( M \) is the number of symbols used, and the normalized probability \( \epsilon \) is set to zero to prevent the algorithm from diverging.

This blind estimator has low computational complexity and is similar to the widely adopted estimator [1] without the signal-to-noise ratio (SNR) factor; its performance agrees with the widely adopted estimator for high SNR. Conventionally, the cost function (2) is evaluated for each \( \hat{\theta} \in [0, N-1] \), which is computationally expensive for large \( N \) or \( M \).

In order to reduce the computational complexity and make the synchronization algorithm suitable for real-time tracking of the STO and CFO, an adaptive algorithm is desirable. An estimate of the STO and CFO is produced with every symbol received, and it does not require unnecessary computations at each of the \( N \) possible STO estimates; only a small subset of them. Before presenting the proposed algorithm, the previous two approaches are now briefly reviewed.

### B. Krishnamurthy Approach

In the pioneering work of [3], two adaptive algorithms are proposed: 1) locally convergent designed for constant STO and 2) globally convergent designed for time varying STO. Both algorithms utilize an empirical measure of the occupation probability \( \alpha \). In the locally convergent algorithm, only one of the adjacent STO estimates is compared with the previous estimate; and in the globally convergent algorithm, one of the \( N-1 \) possible STO estimates is compared with the previous estimate.

The drawback to the locally convergent algorithm is that it cannot track changes in the STO. The drawback to the globally convergent algorithm is that the candidate STO estimate is selected according to a uniform random variable, so if any change in the STO occurs, the convergence time can be slow especially for large \( N \).

### C. Chen Approach

In [4], the algorithms from [3] are combined to achieve higher tracking accuracy and speed. It combines a locally convergent and globally convergent algorithm into one algorithm based on the use of a gate value. In order to decrease the convergence time, a bidirectional search method is used with increasing search step size.

As mentioned by Chen, a drawback to this algorithm is that the gate value \( \alpha_{\text{gate}} \) needs to be chosen carefully since it depends on the maximum value of \( \alpha \) which varies with the channel, step-size, and mode the algorithm is in.

### D. Proposed Algorithm

The proposed adaptive synchronization algorithm is shown in Fig 1. The algorithm is initialized using the STO and CFO estimates provided by the non-adaptive approach from Section II.A on the first \( 2N+N_{cp} \) received samples. It is assumed that the initial STO estimate is close (i.e., within 10 samples) to the true STO. Note that the non-adaptive approach from Section II.A is not considered to be part of the proposed algorithm. Any blind estimator can be used to initialize the algorithm (and as the cost function), but this one is used for the reasons mentioned earlier. The \( N \times 1 \) vector of occupation probabilities \( \alpha \) is initialized accordingly based on this initial STO estimate and the search step size (in samples) \( s \) is set to one. As will be discussed shortly, \( \eta \) is used to keep track of the intervals (in symbols) when the STO estimate has not yet converged; it is initially set to zero.

For each subsequent symbol \( m \), the STO estimate is determined by selecting the STO estimate from the set \( \{ \hat{\theta}_{m-1}^*, \hat{\theta}_{m-1}^* - s, \hat{\theta}_{m-1}^* + s \} \) or \( \{ \hat{\theta}_1^*, \hat{\theta}_1^* - s, \hat{\theta}_1^* + s \} \) yielding the maximum cost function, depending on whether or not \( s > 0 \). \( \hat{\theta}_1^* \) and \( \hat{\theta}_2^* \) are uniformly distributed random STO estimates on \( [0,N-1] \) where \( \hat{\theta}_1^* \neq \hat{\theta}_2^* \). As will be explained later, \( s \) is set to zero to prevent the algorithm from diverging. Another precaution that is taken in order to prevent divergence is that each STO estimate from the set \( \{ \hat{\theta}_{m-1}^* - s, \hat{\theta}_{m-1}^* + s \} \) is always checked to ensure it is within the interval \( [0,N-1] \). If either estimate is less than \( 0 \), it is set equal to \( 0 \); and if either estimate is greater than \( N-1 \), it is set equal to \( N-1 \).

The occupation probability is then updated according to

\[
\alpha_m = \alpha_{m-1} + \mu \left( e_{\hat{\theta}_m} - \alpha_{m-1} \right)
\]

where \( e_{\hat{\theta}_m} \) is a \( N \times 1 \) unit vector with a one in the \( \hat{\theta}_m \) th position and \( \mu \) is the step-size. Note the distinction between the step-size \( \mu \) and the search step size \( s \). Finally, the STO and CFO estimates are produced from the STO estimate with the highest occupation probability, denoted by \( \alpha_{\text{peak}} \).

Unlike [4], where the step-size was determined by comparing \( \alpha_{\text{peak}} \) to a gate value \( \alpha_{\text{gate}} \), the proposed algorithm uses the cost function of the STO estimate to determine the search step size. As seen in Fig. 2, when there is a large change in STO and the algorithm has not yet converged, the cost function is less than when it is tracking the STO. Therefore, large changes in STO can be detected by monitoring the cost function of the STO estimate. The current cost function is compared to a threshold \( \lambda \) to determine whether a large change in STO has occurred and if the algorithm has converged yet. A good value for \( \lambda \) was empirically determined to be four times the average of the first 20 symbols’ cost functions, i.e.,

\[
\lambda = 4 \times \frac{1}{20} \sum_{m=0}^{19} j(\hat{\theta}_m)
\]
It is assumed that the STO is somewhat constant (i.e., $|\theta_m - \theta_{m-1}| \leq 1$) for the first 20 symbols so that no large change in STO occurs. This assumption is reasonable since the STO varies slowly with time \[5\].

If the current cost function is greater than $\lambda$, the algorithm has converged, so $s$ is set to one and $\eta$ is set to zero. If the current cost function is less than $\lambda$, the algorithm has not converged yet, so $\eta$ is incremented by one and $s$ is set according to Fig 1. This results in a quadratically increasing search step size with the goal of decreasing the convergence time (note that this is a larger increase in search step size when compared to \[4\] which only increases its search step size by 1 sample per symbol). However, if $s > N/2$, $s$ is set to 0, which results in random candidate estimates in order to prevent permanent divergence (since two of the three candidate estimates could be wasted by investigating the STO estimates 0 and $N-1$ if $s$ is too large).

Utilizing the cost function to make the proposed algorithm’s performance independent of the channel is a key novel aspect of this work. As long as the channel is not rapidly
changing (i.e., its power delay profile is constant over multiple OFDM symbols), $\lambda$ can be used to determine when a large change in STO has occurred to switch the convergence mode from local to global; if the channel is rapidly changing, $\lambda$ can be updated periodically. What constitutes a good value for $\lambda$ is that only large changes, not small changes, in STO should result in the cost function falling below $\lambda$. [4] is the only previously proposed approach that combines the local and global convergent modes into one algorithm. However, as mentioned earlier, which mode the algorithm is in is determined by comparing $\alpha_{peak}$ to a gate value $\alpha_{gate}$; the author found that a good value for $\alpha_{gate}$ is difficult to determine and varies case-by-case since it depends on the maximum value of $\alpha$, which varies with the channel, step-size, and mode the algorithm is in.

### E. Computational Complexity

Following [3] and [4], the number of complex multiplications performed is used to measure the computational complexity. The computational complexity of the non-adaptive estimator given by (1)-(3) is $3(N+N_{cp})$ for each symbol if three buffers of size $N_{cp}$ are used.

The proposed algorithm utilizes three buffers of size $N+N_{cp}$ in order to avoid doing the same complex multiplication more than once. When the algorithm is tracking small changes in the STO (i.e., $s = 1$), only the cost functions of adjacent STO estimates are evaluated, so only $3(N_{cp}+2)$ complex multiplications are performed. In the rare occurrence of large changes in STO, the algorithm can perform up to $3(3N_{cp}) = 9N_{cp}$ complex multiplications since the sets of STO estimates (i.e., $\{\hat{\theta}_{m-1}^s - s, \hat{\theta}_{m-1}^s, \hat{\theta}_{m-1}^s + s\}$ and $\{\hat{\theta}^{(1)}, \hat{\theta}^{(2)}\}$) might not share any complex multiplications.

As mentioned in [5], since the STO varies slowly with time, changes in the STO are somewhat rare events that occur on the order of hundreds of symbols, so the computational complexity of the proposed estimator is approximately a factor of $3(N_{cp}+2) / (N+N_{cp}) = (N_{cp}+2) / (N+N_{cp})$ of the non-adaptive estimator. For the OFDM signal specifications given in Table I, this factor is $(16+2) / (64+16) = 0.225$ for IEEE 802.11a and $(10+2) / (128+10) = 0.087$ for the smallest bandwidth LTE signal.

### III. SIMULATION RESULTS

The performance of the proposed algorithm is compared to the global algorithm from [3], the algorithm from [4], and the non-adaptive estimator given by (1)-(3) through Monte Carlo simulations using 10,000 realizations. Generic OFDM signals are generated similar to those of the IEEE 802.11a and smallest bandwidth LTE signal standards in terms of bandwidth, $N$, and $N_{cp}$ (see Table I). BPSK modulation and Rayleigh fading channels are used where the channel coefficients are normalized to unit power. The channel remains constant for the first two symbols and then changes subsequently every few symbols; and the CFO is held fixed throughout at $\varepsilon = 0.3$. To be fair, all three algorithms are initialized using the STO and CFO estimates provided by the non-adaptive approach from Section II.A on the first $2N+N_{cp}$ samples and (2) is used as the cost function for subsequent symbols. $\mu = 0.9$ for the algorithm from [3] and the proposed algorithm, whereas $\mu = 0.6$ and $\alpha_{gate} = 0.5$ for the algorithm from [4]. In all of the simulations, the initial STO estimate is within $\pm$10 samples of the true STO.

In the first scenario, the performances of the algorithms for small changes in STO are simulated using the 3GPP Rural Area channel (Rax) [7] for IEEE 802.11a, which is effectively a twelfth order channel. The true STO is given by $\theta = [30 \ u \ 31 \ u \ 29 \ u \ 32 \ u \ 28 \ u \ 33 \ u \ 27 \ u]^T$ where $\ u$ is a 1x100 vector of ones. Fig. 3 and 4 show the STO estimates of 10 realizations of each algorithm and the MSE of the 10,000 STO and CFO estimates for each algorithm, respectively for SNR = 5dB. From Fig. 3, the proposed algorithm tracks the true STO the best whereas the other algorithms fluctuate more. Looking at Fig. 4, the proposed algorithm performs the best followed by the non-adaptive estimator and algorithm from [4], and the algorithm from [3] in terms of MSE. Compared to the non-adaptive estimator, the proposed algorithm’s STO and CFO estimation performance is over three times better in terms of MSE once the algorithm has converged. The reason for this significant improvement in performance is that when the proposed algorithm has converged, it restricts the set of candidate STO estimates to the previous STO estimate along with the two adjacent ones. This differs from the non-adaptive estimator which investigates all $N$ candidate estimates. Especially for low SNR, the non-adaptive estimator is susceptible to STO estimates maximizing (2) that differ largely from the true STO. This can be seen in Fig. 5 which depicts a histogram of the timing error (defined to be $\hat{\theta} - \theta$) at the 50th received symbol. 81% of the non-adaptive estimates

| Table I. OFDM Signal Specifications |
|-------------------------------------|
| **IEEE 802.11a** | **LTE** |
| Channel Bandwidth | 20 MHz | 1.4 MHz |
| FFT Size $N$ | 64 | 128 |
| Cyclic Prefix Length $N_{cp}$ (in FFT samples) | 16 | 10 |

[6], Section E.5.1.
result in an error of less than four samples whereas the percentage is 95% for the proposed algorithm. Furthermore, not shown in Fig. 5 is that the largest error for the non-adaptive estimator is 33 samples whereas the largest error is 7 samples for the proposed algorithm.

Continuing with the first scenario, Fig. 6 and 7 show the MSE of the 10,000 STO and CFO estimates for each algorithm and the mean STO estimate of 10,000 realizations, respectively for SNR = 10dB. Looking at Fig. 6, the proposed algorithm performs the best followed by the non-adaptive estimator, the algorithm from [4], and the algorithm from [3] in terms of MSE. The difference between the proposed algorithm’s and non-adaptive estimator’s STO and CFO estimation performances is less than before due to the increase in SNR, which makes the non-adaptive estimator less susceptible to STO estimates that result in large timing errors. From Fig. 7, the proposed algorithm has the smallest steady-state error; however, note that all of the algorithms exhibit a
bias in that the steady-state error is more than a sample. This is explained by the fact that the CML cost function was derived under the assumption of a single path channel; however, in multipath channels, the intersymbol interference occurring in the cyclic prefix causes the STO estimate $\hat{\theta}$ that maximizes (2) to be greater than the true STO (i.e., $\hat{\theta} > \theta$) [8]. Hereafter, the algorithms are defined to have converged once the steady-state values are within ±0.1 samples of each other. The convergence time (in symbols) for the adaptive algorithms is shown in Table II at each change in STO. Except for the first two changes, the proposed algorithm has a shorter convergence time than the algorithm from [4] for all changes in STO, both of whom have shorter convergence times than the algorithm from [3].

In the second scenario, the performances of the algorithms for large changes in STO are simulated using the 3GPP Typical Urban channel (Tux) [7] for LTE, which is effectively a fourth order channel. The true STO is given by $\theta = [60 \hat{u}, 20 \hat{u}, 100 \hat{u}]^T$. Fig. 8 shows the STO estimates of 100 realizations of each algorithm for SNR = 20dB. From Fig. 8, the proposed algorithm tracks the true STO the best and converges for every realization. In 96 of the 100 realizations, the proposed algorithm reaches within one sample of its steady-state estimate in less than 40 symbols. The algorithm from [3] fluctuates and there are many realizations where the algorithm from [4] permanently diverges. The reason the proposed algorithm takes longer to converge in the second scenario than in the first scenario is that the large changes in STO cause the cost function to drop below the threshold $\lambda$, resulting in larger (and sometime random) search step sizes. In the first scenario, the algorithm rarely used search step sizes larger than 1.

The above results demonstrate that the proposed algorithm is robust to both small and large changes in STO, has quicker convergence times than previously proposed adaptive algorithms, and performs better in terms of MSE once the algorithm has converged. The algorithm from [3] suffers from the candidate STO estimate being selected randomly which can lead to large fluctuations and long convergence times, and the algorithm from [4] suffers from the difficulty in finding a value of $\alpha_{	ext{goe}}$ that robustly tracks large changes in STO.

### IV. Conclusion

In this paper, a novel, real-time, low complexity, blind, and adaptive synchronization algorithm for OFDM systems was presented. It used the cost function of the STO estimate to help determine the search step size, which could be one, quadratically increasing, or random depending on whether the algorithm has converged, detected a large change in STO, or to prevent divergence, respectively. It was shown through simulation that the proposed estimator performs better than the existing adaptive [3]-[4] and non-adaptive [2] algorithms in Rayleigh fading multipath channels in terms of MSE once the algorithm has converged, it converges faster than [3]-[4], and it is robust to both small and large changes in STO. Compared to the conventional non-adaptive estimator, the proposed algorithm’s STO and CFO estimation performance is over three times better for low SNR at only a fraction of the computational complexity.

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