Generalized Soft Breaking Leverage for the MSSM

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Abstract

In this work we study implications of additional non-holomorphic soft breaking terms ($\mu', A'_t, A'_b$, and $A'_\tau$) on the MSSM phenomenology. By respecting the existing bounds on the mass measurements and restrictions coming from certain B-decays, we probe reactions of the MSSM to these additional soft breaking terms. We provide examples in which some slightly excluded solutions of the MSSM can be made to be consistent with the current experimental results. During this, even after applying additional fine-tuning constraints the non-holomorphic terms are allowed to be as large as hundreds of GeV. Such terms prove that they are capable of enriching the phenomenology and varying the mass spectra of the MSSM heavily, with a reasonable amount of fine-tuning.

We observe that higgsinos, the lightest stop, the heavy Higgs boson states $A, H, H^\pm$, sbottom and stau exhibit the highest sensitivity to the new terms. We also show how the light stop can become nearly degenerate with top quark using these non-holomorphic terms.

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1 Introduction

Despite the excitement of the Higgs boson discovery in ATLAS [1] and CMS [2], the results from the experiments conducted at the Large Hadron Collider (LHC) have brought a severe pressure on the supersymmetric models. Indeed, there have been no signal from the supersymmetric partners of the Standard Model (SM) particles. While motivations for SUSY did not disappear, a 125 GeV SM-like Higgs boson requires rather heavy stops that leads to the fine-tuning problem in the minimal supersymmetric extension of the SM (MSSM). Additionally, the LHCb results for the rare decays of B meson have a significant impact on the parameter space of the supersymmetric models such as constrained MSSM (CMSSM) and non-universal Higgs mass models (NUHM). For instance the observation of $B_s \rightarrow \mu^+\mu^-$[3] and the updated range of $b \rightarrow s\gamma$[4] especially disfavor CMSSM.

The scrutiny within the supersymmetric models may consider the lack of evidences to be incompleteness of such models, since supersymmetry (SUSY) has strong motivations such as resolution of the gauge hierarchy problem [5], unification of the gauge couplings [6], radiative electroweak symmetry breaking (REWSB) [7], dark matter candidate under R-parity conservation and etc. Considering the strong impacts of the experimental results, extensions of the MSSM such as next to MSSM (NMSSM) [8], R-parity violation (RPV) [9] have been excessively investigated and it has been found that such extended models are capable of providing results at the low energy scale that are in much better fit to the experimental results.

Alternatively and arguably as a much simpler way to extend the MSSM, one also can examine the generalized MSSM by considering non-holomorphic (NH) terms in the soft supersymmetry breaking (SSB) sector of the theory [10]. For simplicity, we restrict our search to the MSSM domain, but the consideration can be enlarged to the extended models [11]. In addition to the MSSM superpotential, the following terms exist in the NH extension of MSSM (NHSSM).

$$L'_{soft} = \mu' H_u^I \cdot H_d^I + \bar{Q} H_d^I A'_u \hat{U} + \bar{Q} H_u^I A'_d \hat{D} + \bar{L} H_u^I A'_e \hat{E} \text{ + h.c.} \quad (1)$$

where $\mu'$ is NH mixing term of the Higgs doublets and $A'_{u,d,e}$ are NH trilinear scalar couplings. We use a similar notation in Eq.(1) to the holomorphic superpotential, but the NH terms; $\mu'$ and $A'_{t,b,\tau}$, are independent of the holomorphic terms and treated as the free parameters of NHSSM. This similar notation is based on the fact that we do not add any new particle to the MSSM content, but rather we assume only the existence of NH terms given above. During our numerical investigation, we also assume CP and the R-parity to be conserved and require our solutions to satisfy that the lightest supersymmetric particle (LSP) is the lightest neutralino.

As can be predicted, the additional terms given in Eq.(1) can result in quite different phenomenology at the low scale. Since the degree of freedom is greater than
the MSSM, the region of the parameter space consistent with the current experimental constraints can be found much larger in NHSSM than that found in MSSM. To see this, let us start with the NH contributions to the supersymmetric mass spectrum, which can be summarized for scalar fermions as follows [10]:

\[
M^2_{\tilde{f}} = \begin{pmatrix}
m^2_{\tilde{f}_L \tilde{f}_L^*} & X_f \\
X^*_f & m^2_{\tilde{f}_R \tilde{f}_R^*}
\end{pmatrix}
\]

(2)

Here \(M^2_{\tilde{f}}\) is the general form of the mass-squared mass matrices of sfermions written in basis \((\tilde{f}_L, \tilde{f}_R)\) where \(\tilde{f} = \tilde{u}, \tilde{d}, \tilde{e}\) stands for up-type squarks, down-type squarks and sleptons respectively. The masses and mixings of sfermions can be written as follows:

\[
m_{\tilde{u}_L} = -\frac{1}{24}(-3g_2^2 + g_1^2)(-v_u^2 + v_d^2) + \frac{1}{2}(2m^2_\nu + v_u^2Y_u^\dagger Y_u),
\]

\[
m_{\tilde{u}_R} = \frac{1}{2}(2m^2_\nu + v_u^2Y_u^\dagger Y_u) + \frac{1}{6}g_1^2(-v_u^2 + v_d^2),
\]

\[
X_{\tilde{u}} = -\frac{1}{\sqrt{2}}[-v_d(\bar{\mu}Y_u^\dagger + A_u^\dagger) - v_uA_u^\dagger],
\]

(3)

\[
m_{\tilde{d}_L} = -\frac{1}{24}(3g_2^2 + g_1^2)(-v_u^2 + v_d^2) + \frac{1}{2}(2m^2_\nu + v_u^2Y_d^\dagger Y_d),
\]

\[
m_{\tilde{d}_R} = \frac{1}{2}(2m^2_\nu + v_u^2Y_d^\dagger Y_d) + \frac{1}{12}g_1^2(-v_u^2 + v_d^2),
\]

\[
X_{\tilde{d}} = -\frac{1}{\sqrt{2}}[v_u(\bar{\mu}Y_d^\dagger + A_d^\dagger) - v_dA_d^\dagger],
\]

\[
m_{\tilde{e}_L} = \frac{1}{2}v_u^2Y_e^\dagger Y_e + \frac{1}{8}(-g_2^2 + g_1^2)(-v_u^2 + v_d^2) + m^2_\nu,
\]

\[
m_{\tilde{e}_R} = \frac{1}{2}v_u^2Y_e^\dagger Y_e + \frac{1}{4}g_1^2(-v_u^2 + v_d^2) + m^2_\nu,
\]

\[
X_{\tilde{e}} = \frac{1}{\sqrt{2}}[-v_u(\bar{\mu}Y_e^\dagger + A_e^\dagger) + v_dA_d^\dagger].
\]

where \(\bar{\mu} \equiv \mu - \mu'\). Even though the diagonal elements are well-known masses of sfermions, the NH terms appear in the off-diagonal elements and hence they can significantly change the sfermion masses by altering their mixings.

Similarly the square mass matrices for the neutralino and chargino can be written as:
\[ M_{\tilde{\chi}^0} = \begin{pmatrix} M_1 & 0 & -\frac{1}{2}g_1 v_d & \frac{1}{2}g_1 v_u \\ 0 & M_2 & \frac{1}{2}g_2 v_d & -\frac{1}{2}g_2 v_u \\ -\frac{1}{2}g_1 v_d & \frac{1}{2}g_2 v_d & 0 & -\bar{\mu} \\ \frac{1}{2}g_1 v_u & -\frac{1}{2}g_2 v_u & -\bar{\mu} & 0 \end{pmatrix} \] (4)

and

\[ M_{\tilde{\chi}^\pm} = \begin{pmatrix} M_2 & \frac{1}{\sqrt{2}} g_\sqrt{2} v_u \\ \frac{1}{\sqrt{2}} g_\sqrt{2} v_d & -\bar{\mu}' \end{pmatrix} \] (5)

where \( M_{\tilde{\chi}^0} \) is mass matrix for the neutralinos in the basis \((\tilde{B}, \tilde{W}_0^0, \tilde{H}_0^d, \tilde{H}_0^u)\), while \( M_{\tilde{\chi}^\pm} \) is for the charginos in the basis \((\tilde{W}^-, \tilde{H}_d^-)\) and \((\tilde{W}^+, \tilde{H}_u^+)\). While all the NH terms affect sfermion masses, only \( \mu' \)—term is effective in the neutralino and chargino sector at tree-level. It is easy to infer from Eqs.(4,5) that the lightest mass eigenvalues of neutralino and chargino mass matrices are to be very small when \( \mu' \approx \mu \) (\( \bar{\mu} \approx 0 \)).

In this context, the NH terms can yield almost massless higgsino-like LSP.

Besides the sfermion sector, the Higgs sector of NHSSM is affected by all the NH terms. Tree-level masses of Higgs masses can be simply obtained by replacing the \( \mu \)-term with \( \mu - \mu' \), or equivalently with \( \bar{\mu} \), since the mixing of the MSSM Higgs doublets adds the terms \(-\mu'\lvert H_d \rvert^2 - \mu'\lvert H_u \rvert^2\) to the scalar potential. In addition to \( \mu' \)-term, the NH trilinear scalar interaction terms, \( A'_{t,b,\tau} \) contribute to the Higgs masses at loop levels [12]. Such contributions can have important results for the fine-tuning [13], since the 125 GeV Higgs boson mass can be satisfied without having heavy stops or large mixing in contrast to the case of MSSM [14].

In this paper, we explore the low scale phenomenology in the NHSSM framework, and we consider effects of the NH terms by considering two benchmark points. We aim to probe allowed parameter space of the NHSSM in accord with the current experimental constraints. The outline of the rest of the paper is as follows. We explain the scanning procedure and the experimental constraints applied in our analysis in Section 2, where we also briefly describe the benchmark points and their implications in MSSM. We present the results and phenomenological determination of ranges of the NH terms in Section 3. We spare Section 4 on a few words on the fine-tuning in NHSSM, and finally; we summarize and conclude our results in Section 5.

## 2 Scanning Procedure

We employ state of the art codes which are the fortran code prepared by SARAH [15] for the use of SPheno [16]. This code does not include the NH terms in the renormalization group equations (RGEs) that are evaluated from the grand unification scale \( M_{\text{GUT}} \) down to the low energy scale. Rather, it has the pure MSSM RGEs
that allows us to cross-check the benchmark points with SPheno and other public spectrum generators (like SOFTSUSY [17] and ISAJET [18]) at the weak scale. The NH terms are defined at the low scale. Once the spectrum and the observables are obtained within the MSSM, the NH terms are imposed and the spectrum and observables are recalculated with the contributions from the NH Lagrangian given in Eq.(1). We choose two benchmark points which are currently excluded by the experimental constraints. These points provide solutions for which the lightest neutralino is LSP, and the radiative electroweak symmetry breaking (REWSB) is satisfied. Also we set $\mu > 0$ and $m_t = 173.3$ GeV [19], where $m_t$ is the mass of top quark. Note that one or two sigma variation in $m_t$ do not change the results too much [20]. Once we recalculate the low scale observables in our scan after taking into account the contributions from the NH terms, we require our solutions to satisfy the mass bounds [21], the constraints from the rare decays $B_s \to \mu^+\mu^-$ [3] and $b \to s\gamma$ [4]. These constraints can be summarized as follows:

$$m_h = (123 - 127) \text{ GeV}$$

$$m_\tilde{g} \leq 1.4 \text{ TeV}$$

$$0.8 \times 10^{-9} \leq \text{BR}(B_s \to \mu^+\mu^-) \leq 6.2 \times 10^{-9} \ (2\sigma)$$

$$2.99 \times 10^{-4} \leq \text{BR}(b \to s\gamma) \leq 3.87 \times 10^{-4} \ (2\sigma)$$

where we display the current mass bounds on the SM-like Higgs boson [1, 2] and gluino [23], because they have changed since the LEP era. We do not apply the Higgs mass bound strictly by taking it about 125 GeV, since the theoretical uncertainties in minimization of the scalar potential and the experimental uncertainties in measures of $m_t$ and $\alpha_s$ lead to about 3 GeV uncertainty in estimation of the Higgs boson mass. Note that the Higgs boson mass constraint has a strong impact on the stop sector, since it requires either heavy stops or large SSB trilinear $A_t$–term that lead to the stop masses at the order of TeV [24]. In addition to the constraints given in Eq.(6), we require our solutions to do no worse than the SM prediction for the muon anomalous magnetic moment $\Delta(g - 2)_\mu > 0$, and we also imposed chargino LEP bound $m_{\tilde{\chi}^\pm} > 105 \text{ GeV}$.

We present our benchmark points in Table 1 where all masses are given in GeV. Both points satisfy REWSB and neutralino being LSP condition and they have acceptable fine-tuning ($\Delta_{EW} \lesssim 10^3$) in the MSSM framework. Point 1 is taken from Ref. [25] and it is currently excluded by the constraint from the rare decay process $b \to s\gamma$, according to our criteria. Point 1 is taken as a sample to show contributions from the NH Lagrangian of Eq.(1) and explain the cuts which we apply to determine the ranges of the NH terms. In addition to Point 1, we consider also Point 2 that is obtained from our scan searching for light stops of mass about 500 GeV. It is excluded by the BR($b \to s\gamma$) constraint like Point 1. It also leads to the stop quark of 490
Table 1: Benchmark points excluded by the constraints from the decay process $b \to s\gamma$ in the MSSM. All masses are given in GeV. The first block at top represents the GUT scale parameters, while all other blocks list the parameters at the low scale. Point 1 displays a solution with stau NLSP, while it is the lightest chargino in Point 2. Point 2 also depicts a solution with the lightest stop of mass about 490 GeV. The fine-tuning measures are in acceptable range ($\Delta_{EW} \lesssim 10^3$) for both points.

GeV mass that is almost excluded for the LSP of mass about 180 GeV [26]. We aim for this point to lower the stop mass with contributions from the NH terms down to $\lesssim 200$ GeV whereby it is nearly degenerate with the top quark.

The motivation for the stop mass nearly degenerate with the top quark comes
from the fact that the LHC has not excluded such light stop solutions yet [26] and the recent studies [27] show that $\bar{t}t^*$ cross section is less than the error in calculation of top pair production which is measured to be [28]

$$\sigma_{tt^*}^{\sqrt{s}=8 \text{ TeV}} = 241 \pm 2 \text{ (stat.)} \pm 31 \text{ (syst.)} \pm 9 \text{ (lumi.)} \text{ pb.}$$ (7)

When stop is almost degenerate with the top quark, decay products from $\bar{t}t$ and $\tilde{t}\tilde{t}^*$ are identical and it is challenging to distinguish stop and top quarks from each other [27]. It has been also shown that it is possible to obtain light stop masses about $\lesssim 200$ GeV in CMSSM, however; a huge amount of fine-tuning is required due to a large mixing between stop quarks in order to induce a 125 GeV Higgs boson mass [29]. There are exclusive studies which show that requiring acceptable fine-tuning measures bound the stop mass to about 500 GeV from below [30]. It is worth to study with Point 2 in the NHSSM framework, because contributions from the NH terms help to raise the Higgs boson mass and loose stress on the stop sector. We explore the NH parameter space in which the stop can be found to be nearly degenerate with top quark and consistent with the fine-tuning constraints.

3 Phenomenological Cut-offs for NH Contributions

We divided this section into pieces in order to emphasize the effects of NH terms separately. We start with probing the impact of the $\mu'$ term first by setting $A'_{t,b,\tau} = 0$. Then, the following subsection studies NH trilinear scalar interaction couplings.

3.1 $\mu'$ term

Let us start to investigate contributions from the non-holomorphic terms and phenomenological bounds on them by considering Point 1 of Table 1, which is already inconsistent with the constraints from the rare decays of B-meson at 2$\sigma$. Since the supersymmetric contributions to such rare decays come from the MSSM’s Higgs sector; one can expect that the NH mixing term, $\mu'$, can significantly change the B-physics implications.

Figure 1 displays the plots in $BR(B \to X_s\gamma)$, $BR(B_s \to \mu^+\mu^-)$ and $\Delta(g-2)_{\mu}$ versus $\mu'$ panels respectively. The plots are obtained for $A'_{t,b,\tau} = 0$. The red part of the curve represent the solutions which are consistent with the experimental constraints mentioned in Section 2, while the blue part stands for being excluded. The NH contribution to the process $B \to X_s\gamma$ can be written as $BR(B \to X_s\gamma) \propto A_t - (\bar{\mu} + A'_t) \cot \beta$ [12] and for $A'_t = 0$ we see that $\mu' \lesssim -400$ GeV can provide enough contribution to satisfy the constraint from $BR(B \to X_s\gamma)$ decay. The least $BR(B \to X_s\gamma)$ prediction is obtained when $\mu' \approx 1.3$ TeV which happens in the blue region excluded by also several constraints. On the other hand, one can obtain
The NH contribution to the $BR(B \rightarrow X_{s}\gamma)$ and $\Delta(g - 2)_{\mu}$ can be understood clearer if one considers the masses of neutralinos and charginos. As is mentioned above, when $\mu' \approx \mu = 1658$ for BMP1, the lightest neutralino mass tends to be zero as seen from plots in $m_{\tilde{\chi}_{0}^{3}} - \mu'$ and $m_{\tilde{\chi}_{1}^{0}} - \mu'$ planes of Figure 2. The color coding is the same as Figure 1. In the CMSSM framework, the lightest neutralino is usually mostly bino, and the Higgsino components of neutralino are found to be relatively heavier. The $m_{\tilde{\chi}_{0}^{3}} - \mu'$ plane of Figure 2 shows that the Higgsino mass linearly increases as $\mu'$.
increases in the red region. However in the blue region with \(1200 \lesssim \mu' \lesssim 2000\) GeV, \(m_{\tilde{\chi}^0_3}\) remains constant even if \(\mu'\) changes. It should be remembered that \(\mu'\) can drive masses of the lightest neutralino and chargino to zero when \(\mu' \approx \mu = 1658\) GeV for BMP1. We present lightest neutralino mass variation in the \(m_{\tilde{\chi}^0_1} - \mu'\) plane (right panel) of Figure 2 for BMP1. One can easily see that the \(\mu'\)-term has no effect on the lightest neutralino mass in red region at all. It is because, the lightest neutralino is mostly bino in this region. However, when \(\mu' \approx \mu\), the higgsinos become lighter than the bino and the lightest neutralino is formed mostly by the higgsinos. A similar mass pattern is obtained also for the chargino sector. While the lightest chargino is mostly wino in CMSSM, it is found to be mostly higgsino in our model when \(\mu' \approx \mu\). In this context, since the lightest chargino mass is close to zero, it is excluded by the LEP bound on chargino mass that is why it is observed in the blue part of the curves. As is also seen from \(BR(B \to X_s\gamma) - \mu'\), and \(\Delta(g-2)_{\mu} - \mu'\) planes of Figure 1, we obtain the steepest part of the curves in the same region with \(\mu' \approx \mu\). Since it is very light, the chargino channel dominates over the supersymmetric contribution to \(BR(B \to X_s\gamma)\) in this region. Similarly \(\Delta(g-2)_{\mu}\) receives the dominant contributions from the neutralino-smuon channel. Note that the sign of contributions to \(\Delta(g-2)_{\mu}\) is proportional to \(\text{sgn}(\mu M_2)\), and since \(\mu \equiv \mu - \mu'\) changes its sign from positive to negative, the implications for \(\Delta(g-2)_{\mu}\) become worse than the SM and hence it is excluded by our requirement that we assume the solutions to do no worse than the SM on \(\Delta(g-2)_{\mu}\). In this context our requirement can bound the NH \(\mu'\)-term range in a general scan as \(\mu' \lesssim \mu\). The situation is very similar as can be seen from the first panel of Figure 3 for our BMP2.

![Figure 3: Lightest neutralino and light stop masses against \(\mu'\) for BMP2. The color coding is the same as Figure 1.](image)

As bounding the \(\mu'\)-term from above, one can also bound it from below. As a comparison we present the lightest neutralino and light stop masses against \(\mu'\) for BMP2 in Figure 3. The color coding is the same as Figure 1. A similar curve for the lightest neutralino mass is obtained when \(\mu' \approx \mu = 1478\) GeV. As shown in \(m_{\tilde{t}_1} - \mu'\)
plane, $\mu'$ leads to relatively lighter stop masses, and while the stop mass is about 500 GeV in the CMSSM framework, it can be as light as $\sim 180$ GeV in NHSSM. However, the blue curve takes over the red one when $\mu' \lesssim 1400$ GeV. The stop becomes lighter than the lightest neutralino and it is excluded by our requirement that allows only the solutions for which the lightest neutralino is the LSP. While the LSP stop bounds the $\mu'-$term from below as $\mu' \gtrsim -\mu$, this bound can be found different if some other sparticles become LSP.

![Figure 4](image_url)

Figure 4: Plots in $m_h - \mu'$ and $m_{H^\pm} - \mu'$ planes for BMP1. The color coding is the same as Figure 1.

Before concluding this section, sensitivity of the Higgs sector to the $\mu'-$term should be investigated. As emphasized above the $\mu'$-term dominantly controls the Higgs and Higgsino masses, and hence one can naturally expect a different phenomenology associated with the physical Higgs states of MSSM. Figure 4 displays the results in $m_h - \mu'$ and $m_{H^\pm} - \mu'$ planes for BMP1. The color coding is the same as Figure 1. In contrast to the expectation, the SM-like Higgs boson mass decreases only $\sim 0.5$ GeV as $\mu'$ increases in its negative values in the red region. On the other hand, the other Higgs states, which are rather heavy, seem more sensitive to the $\mu'$-term. Related with heavy higgses, $m_A$, $m_H$ and $m_{H^\pm}$ exhibit similar behavior, and hence we present our results only in $m_{H^\pm} - \mu'$ plane. According to the plot obtained, masses of these heavy Higgs states increase with $\mu'$, and it is possible to rise their masses up about 400 GeV in the red region for BMP1.

We have observed very similar behavior of the low scale observables under the presence of NH terms, and hence we do no repeat all the results for BMP2.

In Figure 5, in order to sum up our findings, we present two charts which show the changes in supersymmetric mass spectra for both BMP1 and BMP2. We use the same color coding as that we use in the plots. While the bars show the total changes in masses, red represents the masses consistent with the experimental constraints including those from the rare decays of B-meson. The left chart represents BMP1, while the right one displays BMP2. These two charts clearly exemplify the similar
behavior under the presence of the NH terms. The small charts at the right top of
the big ones represent the scan over the NH parameters with the same color coding.
A larger range for \( \mu' \) is found for BMP1 than BMP2, since the LSP stop is excluded
in the case of BMP2. As mentioned above, masses of the heavy Higgs boson states change
with the \( \mu' \)—term in the same amount, while the change in the SM-like Higgs boson
is negligible in the charts. The neutralino and chargino sector represent the
interchange between the higgsinos and bino-wino. The red region in the two lightest
neutralinos and similarly in the lightest chargino is not visible, since their masses
are not changed by \( \mu' \) in the red region. A small change in the lightest sbottom is observed, while it is at the order of a few hundred GeV in the lightest stop. Besides
this, masses of heavy sbottom and stop states negligibly changes. Finally gluino mass
receives no contribution at all, as should be expected.

3.2 \( A'_{t,b,\tau} \) terms

In the previous section, we have considered the NH contributions only from \( \mu' \). It
is because the most significant contributions to the B-physics observables come from
\( \mu' \). Even though NH \( A'_{t,b,\tau} \) terms are effective, their contributions are not enough to
correct the results for the targeted decays of B-meson, at least for the selected values
of our parameters in BMP1. This is not a must and the situation might be different
in alternative selections.

Let us start with Figure 6 where we present our results in \( m_{\tilde{t}_1} - A'_{t}, m_{\tilde{b}_1} - A'_b \)
and \( m_{\tilde{\tau}_1} - A'_{\tau} \) planes. The curves are all in blue, since all results are excluded by
the constraints from the rare decays of B-meson and higgs mass measurements. Each
plot is obtained by varying only a single parameter that is represented on the x-axis
of the planes. The \( m_{\tilde{t}_1} - A'_{t} \) plane shows that the effect of \( A'_{t} \) is rather increasing
the stop mass. The stop mass curve becomes steeper for negative values of \( A'_{t} \). On

Figure 5: Mass spectrum of the MSSM against \( \mu' \) for BMP1 (left) and BMP2 (right)
panels. Our color coding is as in Figure 1.
the other hand, sbottom and stau masses exhibits opposite behavior under the NH effects. Sbottom mass is almost constant for the positive $A'_b$, and it decreases with increasing $A'_b$ in its negative values, while the stau mass decreases with both negative and positive values of $A'_\tau$. BMP1 predicts the LSP neutralino mass to be about 425 GeV, and as is seen from the $m_{\tilde{\tau}_1} - A'_\tau$ plane, stau becomes lighter than the LSP neutralino when $A'_\tau \gtrsim 700$ GeV that is excluded by our requirement that the lightest neutralino is always LSP.

Figure 6: Plots in $m_{\tilde{t}_1} - A'_t$, $m_{\tilde{b}_1} - A'_b$ and $m_{\tilde{\tau}_1} - A'_\tau$ planes. The curves are all in blue, since all results are excluded by some of the constraints. Each plot is obtained by varying only a single parameter that is represented on the x-axis of the planes.

Since one of the important and strict constraints comes from the observation of 125 GeV Higgs boson, one should also consider the NH trilinear impact on the Higgs mass. As is well-known, the SM-like Higgs boson mass is bounded by $M_Z$ from above, and one needs to use the two-loop level contributions in order to raise the Higgs boson mass up to 125 GeV. In the loop contributions, the third family of charged sfermions have a special importance, since their couplings to the Higgs boson are large in comparison to the first two families. However, the constraints from the vacuum stability of the Higgs potential allow only minor contributions from sbottom and stau, so the 125 GeV Higgs boson constrains rather the stop masses and mixings. From Eqs.(3), the NH trilinear couplings, $A'_t$, $A'_b$ and $A'_\tau$ contribute respectively $-v_dA'_t$ to the stop mixing, $v_uA'_b$ to the sbottom mixing, and $v_uA'_\tau$ to the stau mixing to be consistent with the 125 GeV Higgs boson mass. These contributions may relax the requirement of heavy sfermions or large mixings. Figure 7 shows the impact of the trilinears $A'_t$, $A'_b$ and $A'_\tau$ on the lightest Higgs mass $m_h$ in BMP1. The results for the NH trilinear contributions to the SM-like Higgs boson mass show that the significant contributions come from $A'_t$. The $m_h - A'_t$ plane shows a linear correlation between the SM-like Higgs boson mass and $A'_t$. In addition, $A'_b$ has nonzero contribution, but its contribution is minor compared to $A'_t$. The $m_h - A'_\tau$ represents an interesting curve. The contribution from $A'_\tau$ is negligible for $-2000 \lesssim A'_\tau \lesssim 700$ GeV, and afterwards the mass curve makes a steep fall to $m_h \approx 90$ GeV. Recall that the stau becomes LSP in this region and hence it is excluded. Therefore, $A'_\tau$ has almost zero
contribution to the SM-like Higgs boson mass in its allowed range.

\[ m_h \] 
\[ A'_{\tau} \] 
\[ m_h \] 
\[ A'_{t} \] 
\[ m_h \] 
\[ A'_{b} \] 
\[ m_h \] 

Figure 7: Impact of the trilinears \( A'_t, A'_b \) and \( A'_\tau \) on the lightest Higgs mass \( m_h \) in BMP1.

In order to explicitly show the allowance and exclusion in ranges of the NH trilinear couplings, it is better to set \( \mu' \) nonzero such that the results become consistent with all the experimental constraints mentioned in Section 2. For this purpose we choose a moderate value for \( \mu' \) and set it to \(-750 \text{ GeV}\) which contributes enough to satisfy all the constraints. We sum up our findings for the NH trilinear couplings in mass charts for \( A'_t, A'_b \) and \( A'_\tau \) with \( \mu' = -750 \text{ GeV} \) respectively from top to bottom for BMP1 given in Figure 8. The color coding and explanation of the charts are same as in Figure 5.

The top chart represents the effects of \( A'_t \), and it seems that once the constraints are satisfied, the contributions from \( A'_t \) does not violate them despite its wide range. On the other hand, the contributions from \( A'_b \) can contradict with the B-physics observables even if its range is not as wide as \( A'_t \). In the case of \( A'_\tau \), the blue part is excluded by the LSP neutralino requirement as mentioned above. It is a peculiar feature of our BMP1 that the stau and neutralino can be made nearly degenerate. If we consider BMP2 instead of BMP1, LSP neutralino requirement would exclude some contributions from \( A'_t \), since it leads to LSP stop at some point.

While contribution to the SM-like Higgs boson mass is not visible in the charts, the heavy Higgs boson states exhibit the same behavior as obtained in the chart given in Figure 5 for \( \mu' \). As expected, each NH trilinear coupling has a straightforward effect on the related particle. Namely, the impact of \( A'_t \) on the stop mass, the impact of \( A'_b \) on the sbottom mass, and the impact of \( A'_\tau \) on the stau mass can be seen straightforwardly from the charts. However, they might behave differently. The stop tends to be heavier with the contributions from \( A'_t \), while sbottom and stau become lighter in the case of nonzero \( A'_b \) and \( A'_\tau \) respectively. It is interesting to note that sbottom mass receives some contributions from \( A'_t \) as well as from \( A'_b \). It is because the threshold corrections to \( Y_b \) partly depends on the stop mass at \( M_{\text{SUSY}} \) [31], where \( M_{\text{SUSY}} \) is the scale at which the supersymmetric particles decouple. Similarly, the stop mass can be changed with the contributions from \( A'_b \) because of the threshold corrections.
Figure 8: Mass charts for $A'_t$, $A'_b$ and $A'_\tau$ with $\mu' = -750$ GeV respectively from top to bottom for BMP1. The color coding and explanation are the same as Figure 5.
corrections to $Y_t$, but its change is not as much as that in the sbottom mass [31].

As can be predicted from the presented examples, besides the stop-top degeneracy, one can predict novel sfermion decay patterns which may be subject of future studies. It should be stressed for our NH terms that we assumed third family dominance i.e. $A_u = A_t$, in this work, which is in fact a $3 \times 3$ matrix with 9 entries in the CP conserving case. On the other hand, by considering nonzero values for all families, one can study enhanced flavor phenomenology, too.

4 Note on Fine-Tuning

The NH terms mingle the sparticles such that $H_u$ can couple to d-type quarks and charged leptons at tree level, while it also provides a vertex that $H_d$ couples to up-type quarks and we saw in previous sections that they could significantly change the phenomenology at the low energy scale. In addition to the experimental constraints, one could define also some phenomenological conditions such as LSP neutralino applied in our analysis. Besides the experimental constraints and phenomenological conditions, one can also consider the fine-tuning in NHSSM, since it has more parameters which are involved in calculation of the low scale observables.

The measure of fine-tuning can be defined by considering the mass of Z-boson. Even though it is measured experimentally, it can be written in terms of the fundamental parameters obtained by minimizing the Higgs potential in NHSSM, as follows:

$$\frac{1}{2} M_Z^2 = -(\mu - \mu')^2 + \frac{(m_{H_d}^2 + \Sigma_d^d) - (m_{H_u}^2 + \Sigma_u^u) \tan^2 \beta}{\tan^2 \beta - 1}$$

where $\mu$ and $\mu'$ are respectively the holomorphic and NH bilinear Higgs mixing terms, $\tan \beta = \langle H_u \rangle / \langle H_d \rangle$, $m_{H_u,d}^2$ are the SSB mass terms of the Higgs doublets $H_u,d$, $\Sigma_u^u$ and $\Sigma_d^d$ are radiative contributions to masses of the Higgs doublets. Amount of the fine-tuning required to be consistent with the electroweak scale ($M_{EW} \sim 100$ GeV) can be calculated by defining [32]

$$\Delta_{EW} \equiv \text{Max}(C_i)/(M_Z^2/2)$$

where

$$C_H \equiv \left\{ \begin{array}{rl} C_{H_d} & = | m_{H_d}^2 / (\tan^2 \beta - 1) | \\ C_{H_u} & = | m_{H_u}^2 \tan^2 \beta / (\tan^2 \beta - 1) | \\ C_\mu & = | -(\mu - \mu')^2 | \end{array} \right.$$  

As seen from Eq.(10) above, the $\mu'$-term has a direct impact on the fine-tuning, and if one considers the same signs of $\mu$ and $\mu'$, its effect is to reduce the required
amount of fine-tuning until $C_{H_u}$ or $C_{H_d}$ takes over by definition of $\Delta_{EW}$ given in Eq.(9). Recall that the $\mu'-$term is bounded from above as $\mu' \approx \mu$ in order to avoid from the dangerously light neutrinos and charginos. In addition, it can be bounded from below by requiring the lightest neutralino being LSP. In our analysis, we see that the $\mu'-$term is mostly negative and it rather gives rise to the amount of fine-tuning as shown for BMP1 and BMP2 in Figure 9. The plots are obtained for $A_{t,b,\tau} = 0$. The color coding is the same as Figure 1. If one applies the fine-tuning condition as $\Delta_{EW} \lesssim 10^3$ (equivalently $\gtrsim 0.1$), then it provides a very stringent constraint on the $\mu'-$term and with the other constraints there remain a very narrow range in which the $\mu'$ can be varied for the chosen points BMP1 and BMP2 in our paper.

![Figure 9: EW fine tunings for BMP1(left) and BMP2 (right). The plots are obtained for $A_{t,b,\tau} = 0$. The color coding is the same as Figure 1.](image)

Even though the fine-tuning condition seems strictly constraining NHSSM, it should be considered with mass spectrum of the supersymmetric particles. As shown in Figure 3, the stop mass of BMP2 can be found as low as 180 GeV for $\mu' \sim 1200$ GeV that results in the fine-tuning measured by $\Delta_{EW} \sim 1800$ read from Figure 9. At this point the stop is nearly degenerate with the top quark and distinguishing $\tilde{t}_1 \tilde{t}_1^*$ events at LHC is challenging, since such events can result in the identical final states with $t\bar{t}$, and the cross section of $\tilde{t}_1 \tilde{t}_1^*$ is found to be less than the error in calculation of top pair production whose measure is given in Eq.(7). A recent study has shown that the region of the CMSSM parameter space which yield the light stop of mass $\lesssim 200$ GeV needs to be highly fine-tuned ($\Delta_{EW} \sim 10000$) [29]. In comparison to the results obtained for CMSSM, $\Delta_{EW} \sim 1800$ indicates a huge improvement in respect of the fine-tuning. Even though it is still excluded by the fine-tuning condition, the NH terms can provide much better results for the fine-tuning in some extended models such as NH non-universal Higgs models (NH/NUHM) or those with non-universal gaugino masses etc.
5 Conclusion

In this work, we studied the mass spectrum of MSSM with new NH soft breaking terms. In doing this we respected the experimental constraints especially from the rare decays of B-meson and the mass bounds on the supersymmetric particles. We have chosen two benchmark points from the CMSSM parameter space that are currently excluded by the experimental results on the B-physics observables. By probing the impact of the NH terms based on these two benchmark points we have deduced that the enlarged soft supersymmetry breaking sector with the NH terms has many advantages.

First of all, the B-physics predictions of CMSSM can be corrected with the contributions from the NH terms and hence the CMSSM parameter space allowed by the current experimental constraints can be found significantly larger, if one performs a more detailed scan over its fundamental parameters. Their contributions also change the mass spectrum of the supersymmetric particles. We have find that the Higgs sector except the SM-like Higgs boson exhibits a large sensitivity to the NH terms. While the effects on the SM-like Higgs boson is negligible, the masses of heavy Higgs states can differ up to 400 GeV. Among the NH terms, $\mu'$ strongly controls the Higgsino masses and it leads to Higgsino-like neutralino LSP whose mass is almost zero when $\mu' \approx \mu$. This region also results in almost massless chargino which is excluded by the LEP mass bound on the chargino. Besides the Higgs sector, also the lightest stop, sbottom and stau are sensitive to the NH contributions, while the heaviest states of them are totally blind to the NH terms. Changes in the mass spectra can yield different NLSP species such as stop and stau as we obtained for BMP1 and BMP2 and each NLSP has its own phenomenology.

In addition to NH enrichment in the low scale phenomenology, NHSSM can also be strictly constrained by the fine-tuning condition if one demands. On the other hand, NHSSM gives much better results for regions of CMSSM such as stop-top degeneracy region which requires a huge amount of fine-tuning. We found that one can obtain $\Delta_{EW} \sim 1800$ which indicates a significant improvement in comparison with the CMSSM results that imply $\Delta_{EW} \sim 10000$ without NH contributions. Moreover, the allowed ranges for some of the NH terms are striking since they can be as large as hundreds of GeV and satisfy all the criteria we have considered.

Under the pressure from the current experimental results, it seems crucial to consider MSSM and its alternative extensions, and the results presented in our study is an existential example of additional NH soft breaking terms possibility, which could be improved with a more thorough analysis.
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