D-brane States and Annulus Amplitudes in OSp Invariant Closed String Field Theory

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This talk is based on

Yutaka Baba (RIKEN), Nobuyuki Ishibashi (Tsukuba) and K.M.,

• JHEP 07 (2008) 046 [arXiv:0805.3744]

cf • JHEP 0710, 008 (2007) [arXiv:0706.1635]
• JHEP 0705, 020 (2007) [arXiv:hep-th/0703216]
• JHEP 0605, 029 (2006) [arXiv:hep-th/0603152]

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What we would like to study

D-branes in closed SFT

(cf. Hashimoto-Hata)

D-brane states in $OSp$ invariant SFT (Baba-Ishibashi-K.M)

\[ \Rightarrow \left\{ \begin{array}{l}
\bullet \text{coincident } N \text{ D-branes} \\
\bullet \text{including ghost D-branes} \\
\bullet \text{states for one D-brane in constant } B \text{ background}
\end{array} \right. \]

D-brane tension; disk amplitudes are correctly reproduced

What we did

• states for $N$ (ghost) D-branes separately located

• annulus amplitudes

→ reproduce usual amplitudes with correct normalizations

In particular, the moduli space of amplitude is covered completely and only once.
OSp invariant SFT (Siegel, Uehara, Neveu, West, Zwiebach, Kugo, Kawano) can be regarded as a light-cone gauge SFT with “transverse” coordinates

\[ X^M = (X^\mu; C, \bar{C}) \in \text{OSp}(25, 1|2) \quad \mu = 0, \ldots, 25 \quad C, \bar{C}: \text{ghosts (spin 0)} \]

\[
S = \int dt \left[ \frac{1}{2} \Phi \cdot \left( \alpha i \frac{\partial}{\partial t} - (L_0 + \tilde{L}_0 - 2) \right) \Phi + \frac{2g}{3} \Phi \cdot (\Phi \ast \Phi) \right]
\]

- “light-cone variables” \( t(= x^+) \) (proper time), \( \alpha (= 2p^+) \) (string length)
  \( \rightarrow \) unphysical ones canceled with ghosts via Parisi-Sourlas mechanism
- \( \ast \): joining-splitting type interaction

\[ \begin{array}{c}
\begin{array}{c}
\text{2} \\
\text{3}
\end{array} \\
\begin{array}{c}
\text{1}
\end{array}
\end{array} \quad \begin{array}{c}
\begin{array}{c}
\text{2} \\
\text{3}
\end{array} \\
\begin{array}{c}
\text{1}
\end{array}
\end{array} \]

- BRST invariance \( \leftarrow \) (non-linear) \( J^{-C} \) of OSp(25, 2|2)
  similar to \( J^{-i} \) of SO(25, 1) light-cone gauge SFT
State for one flat D$p$-brane sitting at $X^i = Y^i$ ($i = p + 1, \ldots, 25$)

$$|D(Y)\rangle = \lambda \int d\zeta \tilde{O}_D(\zeta, Y) |0\rangle,$$

$$\tilde{O}_D(\zeta, Y) = \exp \left[ a \int_{-\infty}^{0} d\alpha \, e^{\zeta \alpha} \epsilon \langle B(Y) | \bar{\psi} \rangle + b \zeta^2 \right],$$

$$|B(Y)\rangle^T = e^{-\frac{T}{|\alpha|}(L_0 + \tilde{L}_0 - 2)} |B(Y)\rangle,$$

$|B(Y)\rangle$: boundary state for D$p$-brane at $X^i = Y^i$

- $a$ and $b$ are determined by BRST invariance
- $|D\rangle \sim e^{aB \cdot \Phi} |0\rangle$
  
  $\rightarrow$ insert boundaries in the closed string worldsheet
  
  $\rightarrow a$ determines the D-brane tension

(Baba-Ishibashi-K.M.)

$$a = \frac{(2\pi)^{13}}{(8\pi^2)^{\frac{p+1}{2}} \sqrt{\pi}},$$

$$b = \frac{(2\pi)^{13} \epsilon^2 (-\ln \epsilon)^{\frac{p+1}{2}}}{16(\frac{\pi}{2})^{\frac{p+1}{2}} \sqrt{\pi} g}.$$
States for separately located $N$ flat D-branes simply given as a direct product of $\int d\zeta \bar{O}_D$

$$|D_N(Y(I))\rangle\rangle = \lambda_{N+} \prod_{I=1}^{N} \int d\zeta_I \bar{O}_D(\zeta_I, Y(I))|0\rangle$$

$\Leftarrow$ BRST invariant (in the leading order in $\epsilon$)

$\Leftarrow |B(Y)\rangle^\epsilon \ast |B(Y')\rangle^\epsilon$ is suppressed by $\epsilon \frac{(\Delta Y)^2}{4\pi^2}$ with $\Delta Y^i = Y^i - Y'^i$

(cf. Kishimoto-Matsuo-Watanabe)

cf. Coincident case: (Baba-Ishibashi-K.M.)

$$|D_N\rangle = \lambda_{N+} \int d^N \zeta \Delta_N^2(\zeta_1, \ldots, \zeta_N) \prod_{i=1}^{N} \bar{O}_D(\zeta_i)|0\rangle$$

$\Delta_N$: Vandermonde det $\Rightarrow \zeta_i$ as eigenvalues of a matrix $T$

i.e. $|D_N\rangle = \int dT \exp \left[ a \int d\alpha \text{Tr} T^{\alpha \epsilon} \langle B|\bar{\psi}\rangle + b \text{Tr} T^2 \right] |0\rangle$

$\rightarrow$ non-abelian nature of coincident D-branes: $U(1)^N \rightarrow U(N)$
Annulus Amplitudes

To verify our construction of D-brane states further
⇒ Evaluate annulus amplitudes
  ◦ with one closed string external line
  ◦ the annulus is suspended between two parallel Dp-branes sitting at $X^i = Y^i$ and $Y'^i$

How to read the S-matrix elements (proposal)
(i) evaluate the correlation function at $O(g^1)$

$$\langle O_\phi(t, k) \rangle_{D_2(Y; Y')} \equiv \frac{\langle 0 | O_\phi(t, k) | D_2; Y, Y' \rangle}{\langle 0 | D_2; Y, Y' \rangle}$$

$$\simeq -3! a^2 \int_{\alpha_1, \alpha_2 > 0, \alpha_3 < 0} d\alpha_1 d\alpha_2 d\alpha_3 \int_0^t dT \frac{2g}{3} \langle V^0_3(1, 2, 3) | B(Y) \rangle_T^1 | B(Y') \rangle_T^2 e^{-\frac{T-t}{\alpha_3} H^{(3)}} | DDF_\phi ; k \rangle_3$$

with $O_\phi(t, k) = \int d\alpha \left( \text{ghost} \langle 0 | \otimes x \langle DDF_\phi ; k | \right) | \Phi(t) \rangle$: observable
changing the integration variables \((\alpha_3, \alpha_1, T) \rightarrow (\alpha, \varrho, \tilde{\tau})\)
mapping to a cylinder of circumference \(2\pi\) and length \(-i\pi\tilde{\tau}\) \((\tilde{\nu}-plane)\)

\[
\rho(\nu) = \alpha \ln \frac{\vartheta_1 \left( \frac{i}{2\pi} (\tilde{\nu} + i\pi\tilde{\tau} \varrho) \right)}{\vartheta_1 \left( \frac{i}{2\pi} (\tilde{\nu} - i\pi\tilde{\tau} \varrho) \right)} + \alpha \varrho \tilde{\nu}, \quad \varrho = \frac{\alpha_1}{\alpha}, \quad \alpha = -\alpha_3
\]

(ii) residue of the one-shell pole at \(k^2 + M^2 = 0\) for the closed string external state \(\rightarrow\) \(S\)-matrix element

\[
S_{\phi DD} = -4\pi^2 i g a^2 \int_0^1 d\varrho \int_0^{i\infty} d\tilde{\tau} \tilde{\tau} \prod_{n=1}^\infty (1 - e^{2\pi i n\tilde{\tau}})^2 \times X \left[ B(Y') \right] e^{i\pi\tilde{\tau} \varrho (L_0^X + \tilde{L}_0^X - 2)} V_\phi e^{i\pi\tilde{\tau} (1-\varrho) (L_0^X + \tilde{L}_0^X - 2)} \left[ B(Y) \right]_X
\]
This coincides with the S-matrix in the 1st quantization

\[ \prod_{n=1}^{\infty} (1 - e^{2\pi in\tilde{\tau}})^2 \] coincides with the ghost contribution

- as an integral over the moduli space of the one-punctured cylinder

\[
\left\{ \begin{array}{ll}
\text{correct measure} \\
\text{moduli space is covered completely and only once}
\end{array}\right.
\]

\[ \frac{1}{2} \]

**Summary**

Using the OSp invariant SFT for bosonic strings,

- construct 2nd quantized states corresponding to \( N \) parallel D-branes separately located \( \leftrightarrow \) BRST invariant (in the leading order of \( \epsilon \))

- show that above states reproduce usual results with the correct integral region and measure over the moduli space

Here we just present the annulus amplitude involving one external closed string state.

The extension with more general case is possible: with \( N \) external legs, annulus ending on the same D-brane....