An N-atom Collective State Atomic Interferometer with Ultra-High Compton Frequency and Ultra-Short de Broglie Wavelength, with root-N reduction in Fringe Width

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We describe a collective state atomic interferometer with fringes as a function of phase narrowed by \(\sqrt{N}\) compared to a conventional interferometer, \(N\) being the number of atoms, without entanglement and violation of the uncertainty limit. This effect arises from the interferences among collective states, and is a manifestation of interference at a Compton frequency of ten nonillion Hz, or a de Broglie wavelength of ten attometer, for \(N = 10^6\). The detection process, being a measure of the amplitude of a collective state, can yield a net improvement of phase measurement by as much as a factor of 10.

PACS numbers: 06.30.Gv, 03.75.Dg, 37.25.+k

Matter wave interferometry has proven to be a potent technology in precision metrology. Atom interferometers have been successfully demonstrated as powerful gyroscopes [1], gravity gradiometers [2,3], matter-wave clocks [4] and may lead to a more accurate measurement of the fine structure constant than what is currently known [5, 6]. They also form strong test-beds for experiments to measure Newton’s gravitational constant [7], gravitational red-shift [8] and for inertial sensing [9].

The building block of a Conventional Raman Atom Interferometer (CRAI) is a three level atom in the \(\Lambda\)-configuration, with two metastable states, \(|g, p_z = 0\rangle \equiv |g, 0\rangle\) and \(|e, p_z = h(k_1 + k_2)\rangle \equiv |e, hk\rangle\) and an excited state \(|a, p_z = h k_1\rangle \equiv |a, hk_1\rangle\) coupled by two Raman-resonant counter propagating laser beams, with a single photon detuning \(\delta\) as shown in Fig. 1(a). One of the laser beams, with Rabi frequency \(\Omega_1\), couples \(|g, 0\rangle\) to \(|a, hk_1\rangle\), while the other laser, with Rabi frequency \(\Omega_2\), couples \(|a, hk_1\rangle\) to \(|e, hk\rangle\). In the limit of \(\delta \gg \Omega_1, \Omega_2\), the interaction can be described as an effective two level system excited by an effective traveling wave with a momentum \(hk = h(k_1 + k_2)\), with a Rabi frequency \(\Omega = \Omega_1\Omega_2/\delta\) (Fig. 1(b)) [10]. We assume that \(\Delta \gg \Gamma\), where \(\Gamma\) is the decay rate of \(|a\rangle\) so that the effect of \(\Gamma\) can be neglected. On being subjected to a sequence of \(\pi/2\)-dark-\(\pi\)-dark-\(\pi/2\) pulses as illustrated in Fig. 1(c), the atomic wavepacket first separates in to two components owing to the \(hk\) momentum transfer from the \(\pi/2\) pulse, then gets redirected and finally recombined to produce an interference which is sensitive to any possible phase difference, \(\Delta \phi\) between the two paths. The amplitude of \(|g\rangle\) at the end of the interferometric sequence varies as \(\cos^2(\Delta \phi/2)\). Further theoretical and experimental details of this can be found in ref. [11] and [12].

In the application of a CRAI as a gyroscope, \(\Delta \phi\) is the measure of its rotation sensitivity, arising due to the Sagnac effect. The \(\Delta \phi\) induced due to rotation at the rate of \(\Omega_G\) along an axis normal to the area, \(\Theta\) of the interferometer is given by, \(\Delta \phi = 4\pi \Theta m \Omega_G/\hbar\), where \(m\) is the mass of the atom [12, 13]. This expression can be derived by two different methods. In the first method, the path difference of the two counter-propagating waves arising due to rotation is multiplied by \(2\pi/\lambda_{dB}\), where \(\lambda_{dB}\) is the de Broglie wavelength of the atom, to arrive at the phase difference. The second method follows a more thorough derivation of the phase shift by invoking the relativistic addition of velocities to find the time lag, \(\Delta T\) in the arrival of the two branches of the wave. \(\Delta \phi\) is then the product of \(\Delta T = 2\Theta \Omega G/\epsilon^2\) and the wave frequency. In the case of CRAI, this frequency is the Compton frequency of the atom, \(\omega_C = \gamma mc^2/\hbar \approx mc^2/\hbar\), where \(c\) is the speed of light, and the relativistic time dilation factor, \(\gamma\), is close to unity for non-relativistic velocities. As discussed in greater detail in ref. [14] these approaches



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are equivalent due to the fact that the de Broglie wavelength is merely the laboratory frame manifestation of the Compton frequency induced phase variation in the rest frame of the atom [14,15,17].

The dependence of $\Delta \phi$ on $\omega_C$ (and therefore, $\lambda_{dB}$) of the atom has motivated the quest for matter wave interferometry with large molecules. To date, the largest molecule used for interferometry has a mass of $\sim 10,000$ atomic mass unit [18], which corresponds to the mass of about $75^{133}$Cs atoms. It is not obvious as to how one might be able to observe matter wave interference for particles with mass significantly larger than this. Furthermore, the minimum measurable phase variation (or, equivalently, rotation) for this interferometer, is not better than that of the largest area atomic interferometer based on $133$ Cs atoms, due to limitation of flux and the area enclosed. Here, we propose an experiment that would reveal evidence of matter wave interference where a collection of $N$ non-interacting, unentangled atoms (such as $^{85}$Rb or $^{133}$Cs) acts as a single particle, where $N$ can be as large as a million. For $^{85}$Rb, the corresponding Compton frequency will be $\sim 10^{11}$ (ten million) Hz, and the de Broglie wavelength will be $\sim 10^{-17}$ meter (ten attometer). Furthermore, it can improve the phase measurement sensitivity by a factor as much as 10. At the same time, this type of matter wave interferometry may open up new opportunities for sensitive measurement of gravitational wave redshift [8] or matter wave clocks [4].

Consider an assembly of $N$ identical independent atoms, simultaneously subjected to the $\pi/2$-$\pi$-$\pi/2$ sequence. If we imagine a situation where the ground state of the atoms, $|E_0\rangle \equiv |g_1, g_2, \ldots, g_N\rangle$ is coupled, directly and only, to the state where all the atoms in the ensemble were in the excited state, $|E_N\rangle \equiv |e_1, e_2, \ldots, e_N\rangle$, the resulting ensemble interferometer would experience a phase difference, $\Delta \phi_{EI} = N \Delta \phi$. However, existing technology does not allow such an excitation. Even if one were to use a pure Fock state of $N$ photons, the ensemble will evolve into a superposition of $N + 1$ symmetric collective states given by $|E_n\rangle = J(N,n)^{-1/2} \sum_k P_k |g^{\otimes(N-n)} e^{\otimes n}\rangle$, where $J(N,n) \equiv \left( \begin{array}{c} N \\ n \end{array} \right)$, $P_k$ is the permutation operator, and $n = 0, 1, 2, \ldots, N$ [19]. Since a laser is a superposition of many Fock states, the evolution of this system under laser excitation would produce a seemingly intractable superposition of these collective states. Modeling the laser field as a semi-classical one also does not simplify the picture much [23,24]. However, we show here that, by measuring the quantum state of a single collective state, it is possible to determine the effect of the interference among all the collective states, and describe how such a measurement can be done. Choosing this collective state to be one of the two extremal states (i.e., $|E_0\rangle$ or $|E_N\rangle$) also makes it possible to calculate this signal rather easily, due to the fact that the quantum state of the whole system can always be described as the tensor product of individual atomic states, assuming no interaction between these atoms. In particular, we show that for this signal, the fringe width is reduced by a factor of $\sqrt{N}$, without making use of entanglement. For the current state of the art of trapped atoms, the value of $N$ can easily exceed $10^6$, so that a reduction of fringe width by a factor of more than $10^3$ is feasible. We also show that the phase fluctuation of the CSAI can be significantly smaller, by as much as a factor of 10, than that for a conventional interferometer employing the same transition and same atomic flux. The extremely narrow resonances produced in the CSAI may also help advance the field of spin squeezing [20,23], which in turn is useful for approaching the Heisenberg limit in precision metrology.

CSAI description: We choose an ensemble of $N$ independent and non-overlapping atoms of the kind described above [24]. The ensemble is prepared such that initially the $i$-th atom is in its ground state, $|g_i\rangle$. The ensemble is assumed to be initially situated at $(x = 0, z = 0)$ and traveling along the $x$-direction with a velocity $V$. The ensemble undergoes the same $\pi/2$-$\pi$-$\pi/2$ sequence as described for the CRAI. Assuming resonant excitation, the Hamiltonian of the $i$-th atom after the dipole approximation, rotating-wave approximation, and rotating-wave transformation can be written as $H_i = \Omega_i |g_i\rangle \langle e_i|/2 + c.c.$ [25], where $\Omega_i$ is the Rabi frequency of the $i$-th atom. Here, a phase transformation on the Hamiltonian has also been applied to render $\Omega_i$ real. Therefore, the state of the atom initially in state $|\psi_i\rangle = c_{g_i}(0) |g_i\rangle + c_{e_i}(0) |e_i\rangle$ at a time, $t$, can be expressed as $|\psi_i\rangle = (c_{g_i}(0) \cos(\Omega_i t/2) - i c_{e_i}(0) \sin(\Omega_i t/2)) |g_i\rangle + (-i c_{g_i}(0) \sin(\Omega_i t/2) + c_{e_i}(0) \cos(\Omega_i t/2)) |e_i\rangle$. For the sake of simplicity and brevity, we consider only the case where the intensity profile of the beams are rectangular, so that $\Omega_i = \Omega$. In a real experiment, the Rabi frequencies of each atom depend on their positions relative to the Gaussian distribution of the beam intensity profile. Due to the non-zero temperature of the trapped atoms, they also experience Doppler shift arising from thermal motion. A detailed description of the effect of these inhomogeneities on the CSAI signal is presented in the accompanying Supplementary Material [13].

A $\pi/2$-pulse is applied to the ensemble at $t = 0$. The length of the $\pi/2$-pulse is such that $\Omega \tau = \pi/2$. Immediately after the $\pi/2$ pulse, each atom in the ensemble is in state $|\psi_i(\tau)\rangle = (|g_i\rangle - i|e_i\rangle) / \sqrt{2}$. At this point, the first dark-zone begins and lasts for a duration of $T_d$. By the end of this dark-zone, the component of the atom in state $|e_i\rangle$ drifts to $(x = \pm i T_d, z = h k T_d/m)$ due to an $\hbar k$ recoil received from the laser. The state $|g_i\rangle$ continues along the $x$-direction. We label the trajectories taken by $|g_i\rangle$ and $|e_i\rangle$, $A$ and $B$ respectively. The state of the atom at $t = \tau + T_d$ can thus be written as $|\psi(\tau + T_d)\rangle = |\psi(\tau + T_d)\rangle_A + |\psi(\tau + T_d)\rangle_B$, where $|\psi(\tau + T_d)\rangle_A = |g_i\rangle / \sqrt{2}$ and $|\psi(\tau + T_d)\rangle_B = -i |e_i\rangle / \sqrt{2}$. At the end of this dark zone, the ensemble encounters a $\pi$-pulse which causes the state $|g_i\rangle$ to evolve into $|e_i\rangle$ and vice-versa. The ensemble state at the end of this pulse is $|\psi(3\tau + T_d)\rangle = |\psi(3\tau + T_d)\rangle_A + |\psi(3\tau + T_d)\rangle_B$. The extremely narrow resonances produced in the CSAI may also help advance the field of spin squeezing [20,23], which in turn is useful for approaching the Heisenberg limit in precision metrology.
\[ |\psi(3\tau + T_d)\rangle_B, \text{ such that } |\psi(3\tau + T_d)\rangle_A = -i |e_i\rangle / \sqrt{2} \]
\[ \text{and } |\psi(3\tau + T_d)\rangle_B = - |g_i\rangle / \sqrt{2}. \]
\[ \text{Following this, the atoms are left to drift free for another dark-zone of duration } T_d \text{ at the end of which the two trajectories are redirected to converge, as shown in Fig. 1c and } \]
\[ |\psi(3\tau + 2T_d)\rangle = |\psi(3\tau + T_d)\rangle_B. \]
\[ \text{At } t = 3\tau + 2T_d, \text{ a third pulse of duration } T \text{ is applied to the atoms. If a relative phase difference of } \Delta \phi \text{ is introduced between the paths, the state of the atom at the end of the last } \pi/2\text{-pulse is } \]
\[ |\psi(4\tau + 2T_d)\rangle = |\psi(4\tau + 2T_d)\rangle_A + |\psi(4\tau + T_d)\rangle_B, \text{ where } \]
\[ |\psi(4\tau + 2T_d)\rangle_A = -i(-i \exp(-i \Delta \phi) |g_i\rangle + |e_i\rangle)/2 \text{ and } \]
\[ |\psi(4\tau + 2T_d)\rangle_B = -(|g_i\rangle - i \exp(i \Delta \phi) |e_i\rangle)/2. \]
\[ \text{This phase difference can be applied explicitly, or can occur, for example, due to a rotation of the entire system about an axis normal to the area carved by the trajectories.} \]

The fringe shift at the end of the \( \pi/2 \)-dark-\( \pi \)-dark-\( \pi/2 \) is the result of the interference of the states from the two trajectories. This is characterized by measuring the probability of finding the atom in either of the two states. The signal as a measure of the amplitude of \( |g_i\rangle \), is therefore, \( S_{\text{CRAI}} = |(1 + \exp(-i \Delta \phi))/2|^2 = \cos^2(\Delta \phi/2) \). At this point we remind ourselves that the state of an ensemble is given by the direct product of its constituent atoms, \( |\Psi\rangle = \prod_{i=1}^N |\psi_i\rangle \), where \( |\Psi\rangle \) is the state of the ensemble \([25, 26]\). The signal of the CSAI is a measurement of any of the arising collective states. We choose to measure the state \( |E_n\rangle \) which is a probability of finding all the atoms of the ensemble simultaneously in \( |g_i\rangle \). This choice of state will be explained later on when we discuss the detection system of the CSAI. The signal of the CSAI is thus the product of the signals from the constituent atoms, \( S_{\text{CRAI}} = \prod_{i=1}^N S_{\text{CRAI}} = \cos^2 N(\Delta \phi/2). \) It is evident from Fig. 2 that the fringe width as a function of \( \Delta \phi \) decreases with increasing \( N \). We define this linewidth as the full width at half maximum (FWHM) of the signal fringe, \( \phi(N) = 2 \cos^{-1}(2^{-1/2N}) \). We have verified that to a good approximation, \( (\phi(1)/\phi(N))^2 \approx \sqrt{N} \).

This narrowing can be explained by considering a composite picture of the collective excitations of the ensemble. If the ensemble in the ground state interacts with a single photon of momentum \( \hbar k \), it will oscillate between \( |E_0, 0\rangle \leftrightarrow |E_1, \hbar k\rangle \). Consequently, it will exhibit collective behavior such that its center of mass (COM) recoils with a velocity equal to \( \hbar k/Nm \) in the direction of the absorbed photon \([27]\). Thus, this ensemble can be viewed as a single entity with Compton frequency \( N \) times that of a single constituent atom despite no interactions between the atoms. Conversely, the ensemble can also be pictured as a wave of \( \lambda_{1B} = h/Nmv \) that is \( N \) times lower than that of a single atom as pictured in Fig. 3. In the ideal case of uniform Rabi frequencies and Doppler shift related detunings, the first \( \pi/2\)-pulse splits the ensemble into a superposition of \( N + 1 \) symmetric collective states. We have shown the corresponding interpretation of the other, more general cases in ref. [25]. The state \( |E_n\rangle \) receives a recoil of \( nhk \) due to the first \( \pi/2\)-pulse and is deflected in the \( z \)-direction by \( nhkT_d/Nm \) by the end of the first dark zone, making an angle \( \theta_n = \tan^{-1}(nhk/NmV) \) with the \( x \)-axis. We label the path taken by this state as Path-\( n \). The subsequent \( \pi \)-pulse causes \( |E_n\rangle \) to evolve to \( |E_{N-n}\rangle \). This results in the deflection of the trajectory of the states so that all the \( N+1 \) trajectories converge by the end of the second dark-zone. The third pulse causes each of the \( N+1 \) states to split further. The resulting CSAI is, thus, \( J(N+1, 2) \) CRAI’s working simultaneously. Of these, there are \( x \) interferometers of area \( (N-x+1)\Theta/N \), producing signal fringes equaling \( \cos^2(N-x+1)\Delta \phi/2 \). Here \( x \) assumes values \( 1, 2, \ldots, N \). The interference between these cosinusoidal fringes result in the narrowing of the total fringe width. For a detailed and explicit example, please see the Supplementary Material \([14]\).

In order to illustrate the complete picture of the proposed experiment, we consider \(^{85}\text{Rb}\) as the atomic species without the loss of generality. One can start with trapping atoms in a magneto-optical trap (MOT) cooled down to the Doppler cooling limit of \( \sim 146 \mu \text{K} \) \([28]\). We have verified that the CSAI signal peak value is ultra-sensitive to the temperature of the trap \([14]\). Therefore, one can cool the MOT further down to about \( \sim 2 \mu \text{K} \) by the method of polarization gradient cooling \([28]\), for example. After capturing the atoms in a cloud of diameter \( \sim 0.1 \text{mm} \), the trapping magnetic fields and the repump beams are turned off. The trapping beams are kept on for an additional \( \sim 100 \mu \text{s} \) to repump nearly all the atoms to the \( F = 2 \) state. A bias magnetic field of \( \sim 1 \text{G} \), generated with a pair of Helmholtz coils, is turned on in the \( z \)-direction. After the trapping beams are turned off, the atoms begin to free fall and a pair of counter-propagating right circularly polarized \((\sigma+)\) beams are turned on in the \( z \)-direction. One of these beams is \( \sim 1.5 \text{GHz} \) red detuned from the \( F = 2 \rightarrow F' = 2 \) transition (D1 line), and the other is \( \sim 1.5 \text{GHz} \) red detuned from the \( F = 3 \rightarrow F' = 2 \) (D1 line). The second Raman beam is generated from the first one via an acousto-optic modulator (AOM). The AOM is driven to by a highly stable frequency synthesizer (FS), which is tuned close to \( \sim 3.034 \text{ GHz} \) corresponding to the frequency difference between the \( F = 2 \) and \( F = 3 \) states in the \( 5S_{1/2} \) manifold.

These beams would excite off-resonant Raman tran-
sitions between $F = 2, m_F = m$ and $F = 3, m_F = m$ levels for $m = \pm 2, \pm 1, 0$. With $g_F = -1/3$ for $F = 2$ and $g_F = -1/3$ for $F = 3$, each transition will be shifted by $\delta_0 = -m(g_F=3 - g_F=2)\mu_B B / h = -0.94\text{MHz}/G \cdot mB$, where $B$ is the applied magnetic field. The signal from the five transitions are resolved if the linewidth of the Raman transition is less than $\delta_2$. The $m = 0$ transition is the most insensitive to the external magnetic field and its fluctuations. This makes it the ideal transition for building a stable CSAI. Thus, the states $|g\rangle$ and $|e\rangle$ in Fig. 1(a) would correspond to the hyperfine ground state for building a stable CSAI. Thus, the states $|g\rangle$ and $|e\rangle$ in Fig. 1(a) would correspond to the hyperfine ground state. The Rabi frequencies, one involving the excited states together. The resulting system will be a million times that of a single Rb-85 atom. Therefore, the states $|g\rangle$ and $|e\rangle$ will exhibit the characteristics of a single entity of mass that is the same way as that for the $\Lambda$ system by adiabatically eliminating the excited states together. The resulting system has a coupling rate that is the same as that for the two Raman Rabi frequencies, one involving $F' = 2, m_{F'} = 1$ and the other involving $F' = 3, m_{F'} = 1$. The laser intensities at $\Omega_1$ and $\Omega_2$ are adjusted to ensure that the light shifts of $|g\rangle$ and $|e\rangle$ are matched.

**Detection system:** At the end of the $\pi/2$-dark-$\pi/2$ sequence, a fourth probe beam is applied to the ensemble to measure the amplitude of one of the remaining collective states, via the method of zero photon detection. In order to explain this, we revert to the three-level model of the atom. In free space, a classical laser beam of Rabi frequency $\Omega_0$ coupling $|e, \hbar k\rangle$ to $|a, \hbar k_1\rangle$ will cause the atom to irreversibly emit a photon of momentum $\hbar k_1$, via the transition $|a, \hbar k_1\rangle \rightarrow |g, 0\rangle$. The emitted photon will move against the direction of the applied beam $^{[22]}$. The cavity is coupled to the atomic transition $|g\rangle \rightarrow |a\rangle$ with coupling rate $g_0 = |e.\langle r|E/h$ where $|e.\langle r| is the dipole moment of the atom and the field of the cavity is $E = \sqrt{2}\hbar \omega_1 \epsilon_0 \Phi T$, where $\Phi$ is the cross-sectional area of the atomic ensemble and $T$ the interaction time $^{[22]}$. If the probe beam is an off-resonant classical laser pulse with frequency $\omega_2$, the extreme bad cavity limit of free space will allow Raman transition to occur between the collective states $|E_n\rangle$ and $|E_{n-1}\rangle$ with coupling rate $\Omega_n = \sqrt{n(N - n + 1)}\Omega', \text{where } \Omega' = g_0 \Omega_2 / 2\delta$. Drawing analogy from the calculation in ref. $^{[30]}$ of the effective decay rate of $|E_1\rangle$, we conclude that the state $|E_n\rangle$ will decay at an effective rate $\gamma_n = n(N + 1 - n)\gamma_{sa}$, where $\gamma_{sa} \propto \Omega'^2$ is the single atom effective decay rate.

The emitted photons and the probe beam are recombined and sent to a high speed detector, which generates a DC voltage along with a signal at the beat frequency $\sim 3.034\text{GHz}$ with an unknown phase. This signal is bifurcated and one of the parts is multiplied by the FS signal, while the other is multiplied by the FS signal phase shifted by $90^\circ$. The signals are then squared before being combined and sent through a low pass filter (LPF) to derive the DC voltage. This DC voltage is proportional to the number of scattered photons. A lower limit is set for the voltage reading and any values recorded above it will indicate the presence of emitted photons during the probe period. The duration of the probe beam is set at $\gamma_N T = 10$, where $\gamma_N = N\gamma_{sa}$ is the slowest decay rate to ensure that even the longest lived state is allowed to decay completely. If no photon is emitted, the voltage will read below the limit, indicating that the ensemble is in state $|E_0\rangle$. If the ensemble is in any other collective state, at least one photon will be emitted. This process is repeated $M$ times for a given value of $\Delta \phi$. The fraction of events where no photons are detected will correspond to the signal for this value of $\Delta \phi$. This process is then repeated for several values of $\Delta \phi$, producing the signal fringe for a CSAI.

**FIG. 3.** $\lambda_{AB}$ of an Rb-85 atom moving at a constant velocity of 300 m/s is $1.56(10^{-11})$ m. In the rest frame of the atom, its characteristic $f_c = 1.91(10^{25})$ Hz. A cluster of $10^6$ such atoms will exhibit the characteristics of a single entity of mass that is a million times that of a single Rb-85 atom. Therefore, $\lambda_{AB}$ will be $1.56(10^{-17})$ m and $f_c = 1.91(10^{15})$ Hz.

**FIG. 4.** (a) Atomic Interferometer experiment for an ensemble of $A$-type atoms for detecting state $|E_0\rangle$. (b) Interaction between the collective states in the bad cavity limit, which is an irreversible process.
**Performance comparison:** In order to compare the performance of the CSAI to that of the CRAI, we analyze the stability of the phase difference measured by them by investigating the fluctuation that has both quantum mechanical and classical components, i.e. \[ \delta \Delta \phi_{\text{total}} = (\Delta S_{\text{QM}} + \Delta S_{\text{classical}})/|\partial S/\partial \Delta \phi| \], where \( S(\Delta \phi) \) is the signal and \( \Delta \phi \) is the phase difference introduced in the interferometer away from its center value. Since the signal depends on the phase, the fluctuations in an interferometer are not necessarily constant. Therefore, there is no unique value of signal to noise ratio (SNR) to compare unless the CSAI and the CRAI are compared at a particular value of the phase difference. Thus, the fluctuations in signal must be compared as a function of \( \Delta \phi \).

In ref. [14], we have given a detailed discussion of the quantum fluctuation due to quantum projection noise, \[ \Delta P = \sqrt{P(1-P)} \] [31], where \( P \) is the population of the state being measured, and the classical noise in the long term regime. The ratio of the phase fluctuations of the CSAI to that of the CRAI reveals that they perform comparably around \( \Delta \Phi = 0 \) if the interferometers have perfect collection efficiency. However, as shown in ref. [14], the CRAI suffers from imperfect collection efficiency due to the latter’s dependence on experimental geometry. On the other hand, the collection efficiency of the CSAI is close to unity owing to the fact that the fluorescence of photons is collected through coherent Raman scattering. As a result, for the same number of atoms detected per unit time, the CSAI is expected to outperform the CRAI by as much as a factor of 10.

This work has been supported by the NSF grants number DGE-0801685 and DMR-1121262, and AFOSR grant number FA9550-09-1-0652.

[1] T. L. Gustavson, P. Bouyer and M. A. Kasevich, Phys. Rev. Lett. 78, 2046-2049 (1997).
[2] A. Peters, K. Y. Chung and S. Chu, Nature 400, 849-852 (1999).
[3] M. J. Snadden et al., Phys. Rev. Lett. 81, 971 (1998).
[4] S.-Y. Lan et al., Science 339, 554-557 (2013).
[5] R. Bouchendira, P. Cladé, S. Guellati-Khélifa, F. Nez, F. Biraben, Phys. Rev. Lett. 106, 080801 (2011); Ann. Phys. (Berlin) 525, 484492 (2013).
[6] M. Cadoret, et al. Phys. Rev. Lett. 101, 230801 (2008).
[7] J. B. Fixler et al., Science 315, 74 (2007); G. Lamporesi, et al., Phys. Rev. Lett. 100, 050801 (2008).
[8] H. Müller, A. Peters and S. Chu, Nature 463, 926-929 (2010).
[9] B. Camuel et al., Phys. Rev. Lett. 97, 010402 (2006); R. Geiger et al., Nature Communications 2, 474 (2011).
[10] M. S. Shahriar, M. Jheeta, Y. Tan, P. Pradhan, and A. Gangat, Opt. Comm. 243, 183 (2004).
[11] M. Kasevich and S. Chu, Phys. Rev. Lett. 67, 181-184 (1991).
[12] C.J. Bordé, Phys. Lett. A 140, 10 (1989).
[13] F. Richle, Th. Kisters, A. Witte, J. Helmcke and Ch. J. Bordé, Phys. Rev. Lett. 67, 177 (1991).
[14] Supplement to this paper, to be posted on arXiv shortly.
[15] M. O. Scully and J. P. Dowling, Phys. Rev. A 48, 3186 (1993).
[16] G. B. Malykin, Phys. Usp. 43, 1229 (2000).
[17] L. De Broglie, Thesis, University of Paris, Paris, France (1924).
[18] S. Eibenberger, S. Gerlich, M. Arndt, M. Mayor and J. Tüxen, Phys. Chem. Chem. Phys. 15, 14696 (2013).
[19] D. B. Hum, C. W. Chou, T. Rosenband, and D. J. Wineland, Phys. Rev. A 80, 052302 (2009).
[20] M. Kitagawa and M. Ueda, Phys. Rev. A 47, 5138 (1993).
[21] J. Hali, J. L. Sorensen, C. Schori, and E. S. Polzik, Phys. Rev. Lett. 83, 1319 (1999).
[22] A. Kuzmich, L. Mandel, and N. P. Bigelow, Phys. Rev. Lett. 85, 1594 (2000).
[23] K. Hammerer, A. S. Sorensen, and E. S. Polzik, Rev. Mod. Phys. 82, 1041 (2010).
[24] R. H. Dicke, Phys. Rev. 93, 99 (1954).
[25] R. Sarkar, M. E. Kim, R. Fang, Y. Tu, and S. M. Shahriar, arXiv preprint arXiv:1408.2296 (2014).
[26] F. T. Arecchi, E. Courtens, R. Gilmore and H. Thomas, Phys. Rev. A 6, 2211 (1972).
[27] M. O. Scully, E. S. Fry, C. H. Raymond Ooi, and K. Wódkiewicz, Phys. Rev. Lett. 96, 010501 (2006).
[28] C. Foot, Atomic physics (Oxford University Press, New York, 2008).
[29] L. M. Duan, M. D. Lukin, J. I. Cirac, and P. Zoller, Nature (London) 414, 413 (2001).
[30] C. W. Gardiner and P. Zoller, Quantum Noise (Springer-Verlag, Berlin, 1999).
[31] W. M. Itano, J. C. Bergquist, J. J. Bollinger, J. M. Gilligan, D. J. Heinzen, F. L. Moore, M. G. Raizen, and D. J. Wineland, Phys. Rev. A 47, 3554-3570 (1993).