Renormalization Group Effects on the Mass Relation Predicted by the Standard Model with Generalized Covariant Derivatives

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Abstract
Renormalization group analysis is made on the relation $m_H \simeq \sqrt{2} m_t$ for masses of the top quark and the Higgs boson, which is predicted by the standard model based on generalized covariant derivatives with gauge and Higgs fields. This relation is a low energy manifestation of a tree level constraint which holds among the quartic Higgs self-coupling constant and the Yukawa coupling constants at a certain high energy scale $\mu_0$. With the renormalization group equation at one-loop level, the evolution of the constraint is calculated from $\mu_0$ down to the low energy region around the observed top quark mass. The result of analysis shows that the Higgs boson mass is in $m_t \lesssim m_H \lesssim \sqrt{2} m_t$ for a wide range of the energy scale $\mu_0 \gtrsim m_t$ and it approaches to 177 GeV ($\approx m_t$) for large values of $\mu_0$.

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Confirmation of the existence of the top quark by the Collider Detector Fermilab (CDF and D0) groups [1, 2] has shown that the standard model is a consistent and correct theory of fundamental interactions. To enrich the model further, however, we must solve many basic problems remained concerning its scalar sector. In particular, it is expected to predict the Higgs boson mass for an experiment to observe it in near future.

Recently, one of the authors [3, 4] has reformulated the standard model by using the concept of generalized covariant derivatives with gauge and Higgs fields which act on a multi-spinor field consisting of all the chiral fermion fields. In the new model, the bosonic part of the Lagrangian is determined by field strengths of the gauge and Higgs fields which are constructed from the commutators of the generalized covariant derivatives. As one of interesting outcomes of this scheme, the strength of quartic self-interaction in the Higgs potential is fixed exclusively in terms of the Yukawa coupling constants and, as a result, the approximate relation \( m_H \approx \sqrt{2} m_t \) holds for the masses of the top quark and the Higgs boson at the tree level.

Unified description of the gauge and Higgs fields has been pioneered by Connes [3]. Using the noncommutative geometry, he introduced the Higgs field as a connection along the discrete direction in a doubly sheeted Minkowski spacetime. His theory predicts the tree level relation \( m_{\text{top}} = 2 m_W \) and \( m_{\text{Higgs}} = 3.14 m_W \). Álvarez et al. [6, 7] investigated the evolution of these relations under the one-loop renormalization group equations [8, 9]. Following their method, we analyze quantum effects on the restriction predicted by the standard model with the generalized covariant derivatives in this article.

Collecting three generations of the electroweak doublets \( \psi_{lj} \) and singlets \( \psi_{ej} \) of lepton fields (the doublets \( \psi_{qj} \), and singlets \( \psi_{uj} \) and \( \psi_{dj} \) of quark fields) into a multi-spinor field, we introduce the total fermion field

\[
\Psi(x) = \sum_{j=1}^{3} \sum_{\alpha=l,e,q,u,d} \psi_{\alpha j}(x) |\alpha j\rangle. \tag{1}
\]

The fermionic Lagrangian density is given by

\[
\mathcal{L}_f = i \sum_{j=1}^{3} \sum_{\alpha=l,e,q,u,d} \bar{\psi}_{\alpha j} \gamma^\mu D_\mu \psi_{\alpha j} \\
+ \sum_{i,j=1}^{3} \left\{ a_{ij}^{(e)} \bar{\psi}_{li} \phi \psi_{ej} + a_{ij}^{(u)} \bar{\psi}_{qi} \tilde{\phi} \psi_{uj} + a_{ij}^{(d)} \bar{\psi}_{qi} \phi \psi_{dj} + \text{h.c.} \right\}, \tag{2}
\]
where
\[ D_\mu = \partial_\mu - ig_c A^{(3)}_\mu \frac{1}{2} \lambda_a \sum_{\alpha=q,u,d} |\alpha\rangle\langle\alpha| \]
\[ - ig A^{(2)}_\mu \frac{1}{2} \tau_a \sum_{\alpha=q,l} |\alpha\rangle\langle\alpha| \]
\[ - ig' A^{(1)}_\mu \frac{1}{2} \gamma_5 \sum_{\alpha=l,e,q,u,d} y_\alpha |\alpha\rangle\langle\alpha| \]
is the ordinary covariant derivatives of the gauge group SU(3)_c \times SU(2)_L \times U(1)_Y, \phi is the Higgs doublet and \( a^{(s)}_\mu \) \((s = e, u, d)\) are the Yukawa coupling constants. By factorizing \( L_f \) as
\[ L_f = \bar{\Psi} i\gamma_\mu D_\mu \Psi = \bar{\Psi} \left( i\gamma_\mu D_\mu \Psi \right) \]
we determine the generalized covariant derivative operator \( D_\mu \) acting upon \( \Psi(x) \) in the form
\[ D_\mu = \partial_\mu - ig_c A^{(3)}_\mu - ig A^{(2)}_\mu - ig' A^{(1)}_\mu - i \frac{1}{4} \gamma_5 A^{(0)}_\mu. \]
Here \( A^{(k)}_\mu \) \((k = 1, 2, 3)\) are the operator-valued gauge fields defined by
\[ A^{(3)}_\mu = A^{(3)a}_\mu \frac{1}{2} \lambda_a \sum_{\alpha=q,u,d} |\alpha\rangle\langle\alpha|, \]
\[ A^{(2)}_\mu = A^{(2)a}_\mu \frac{1}{2} \tau_a \sum_{\alpha=q,l} |\alpha\rangle\langle\alpha|, \]
\[ A^{(1)}_\mu = A^{(1)}_\mu \frac{1}{2} \sum_{\alpha=l,e,q,u,d} y_\alpha |\alpha\rangle\langle\alpha| \]
with \( y_l = -1, y_e = -2, y_q = 1/3, y_u = 4/3 \) and \( y_d = -2/3 \), and \( A^{(0)}_\mu \) is the operator-valued Higgs fields
\[ A^{(0)} = \sum_{ij} (\phi a^{(e)}_{ij} |li\rangle\langle ej| + \bar{\phi} a^{(u)}_{ij} |qi\rangle\langle uj| + \phi a^{(d)}_{ij} |qi\rangle\langle dj|) \]
\[ + \sum_{ij} (\phi^* a^{(e)\ast}_{ij} |ei\rangle\langle li| + \bar{\phi}^* a^{(u)\ast}_{ij} |uj\rangle\langle qi| + \phi^* a^{(d)\ast}_{ij} |dj\rangle\langle qi|) \]
\[ + (c + c_5 \gamma^5) \]
where \( c \) and \( c_5 \) are real constants. For both the gauge and Higgs fields, the field strengths \( F^{(k)}_{\mu\nu} \) \((k = 0, 1, 2, 3)\) are introduced into the theory through the commutator of the covariant derivatives as
\[ [D_\mu, D_\nu] = -i g_c F^{(3)}_{\mu\nu} - i g F^{(2)}_{\mu\nu} - i g' F^{(1)}_{\mu\nu} - i \frac{1}{4} \partial_\rho F^{(0)}_{\mu\nu}, \]
where the factors $g_k$ are specified by normalizing the kinetic parts of the bosonic Lagrangian derived below.

The bosonic Lagrangian density $L_b$ is determined by the field strengths as

$$L_b = -\frac{1}{4!} \frac{1}{4} \sum_{k=0}^{3} \text{Tr}(\gamma^5 F^{(k)}_{\mu \nu} \gamma^5 F^{(k)\mu \nu})$$

$$= -\frac{1}{4} \sum_{k=1}^{3} F^{(k)\alpha}_{\mu \nu} F^{(k)\mu \nu} + (D_\mu \phi)^\dagger (D^\mu \phi) + \mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2. \quad \text{(9)}$$

Namely, the bosonic part of the Lagrangian of the standard model is naturally reproduced from the information of its fermionic part in this scheme. The coefficient of the quartic term $(\phi^\dagger \phi)^2$ of Eq.(9) is, in definition, different from that of Refs. [7] and [9] by the factor 2. Furthermore, the constants in the Higgs potential are determined as functions of parameters appearing in the generalized covariant derivatives $D_\mu$ as follows:

$$\mu^2 = c_0^2 - 3c^2 \quad \text{(10)}$$

and

$$\lambda = \frac{1}{2} \frac{1}{\text{tr}} \left\{ (A^+_e A_e)^2 + 3(A^+_u A_u)^2 + 3(A^+_d A_d)^2 \right\}, \quad A_s = (a^{(s)}_{ij}), \quad s = e, u, d. \quad \text{(11)}$$

We interpret this relation between the quartic Higgs self-coupling $\lambda$ and the Yukawa coupling constants as the constraint at a certain high energy scale $\mu_0$.

It is straightforward to calculate the $\beta$ functions of the standard model at the one-loop level [3]. By rewriting the SU(3)$_c$, SU(2)$_L$ and U(1)$_Y$ gauge coupling constants ($g_c$, $g$ and $g'$) as

$$g_3 = g_c, \quad g_2 = g, \quad g_1 = \sqrt{\frac{5}{3}} g' \quad \text{(12)}$$

and defining the $\beta$ functions by

$$\beta_i = \mu \frac{\partial \alpha_i}{\partial \mu}, \quad \alpha_i = \frac{g_i^2}{4\pi} \quad (i = 1, 2, 3), \quad \text{(13)}$$
we get

\[4\pi\beta_3 = -2 \left( \frac{33}{3} - \frac{4}{3}N_f \right) \alpha_3^2,\]  
(14)

\[4\pi\beta_2 = -2 \left( \frac{22}{3} - \frac{4}{3}N_f - \frac{1}{6} \right) \alpha_2^2,\]  
(15)

\[4\pi\beta_1 = -2 \left( -\frac{4}{3}N_f - \frac{1}{10} \right) \alpha_1^2,\]  
(16)

where \(N_f\) is the number of fermion generations. Note the difference in definitions of gauge coupling constants \(g_3, g_2\) and \(g_1\) in this article and in Ref. [3].

The matrix \(A_u = (a_{ij}^{(u)})\) of the Yukawa coupling constants for the up quark sector satisfies the evolution equation

\[4\pi\frac{\partial A_u}{\partial \mu} = A_u \left[ \frac{1}{4\pi} \text{tr} \left\{ (A_e^\dagger A_e) + 3(A_u^\dagger A_u) + 3(A_d^\dagger A_d) \right\} \right. \]

\[\left. + \frac{3}{2} \frac{1}{4\pi} \left( A_u^\dagger A_u - A_d^\dagger A_d \right) - \frac{1}{2} \left( 16\alpha_3 + \frac{9}{2}\alpha_2 + \frac{17}{10}\alpha_1 \right) \right].\]  
(17)

For the \(\beta\) function of the quartic Higgs self-coupling \(\lambda\) defined by

\[\beta_H = \mu \frac{\partial \alpha_H}{\partial \mu}, \quad \alpha_H = \frac{\lambda}{4\pi},\]  
(18)

we get

\[4\pi\beta_H = 24\alpha_H^2 - \frac{9}{5}\alpha_H\alpha_1 - 9\alpha_H\alpha_2 + \frac{27}{200}\alpha_1^2 + \frac{9}{20}\alpha_1\alpha_2 + \frac{9}{8}\alpha_2^2 \]

\[\left. + \frac{1}{16\pi^2} \text{tr} \left\{ (A_e^\dagger A_e) + 3(A_u^\dagger A_u) + 3(A_d^\dagger A_d) \right\} \right\} \lambda \]

\[\left. - \frac{2}{16\pi^2} \text{tr} \left\{ (A_e^\dagger A_e)^2 + 3(A_u^\dagger A_u)^2 + 3(A_d^\dagger A_d)^2 \right\} \right].\]  
(19)

In view of the large top quark mass in the low energy region, it is not unnatural to assume that the Yukawa coupling constants are subject to

\[|a_{33}^{(u)}| \gg |a_{ij}^{(s)}|, \quad s = c, d; \quad (ij) \neq (33)\]  
(20)

for other energy scales also. This approximation simplifies the renormalization group equations for \(N_f = 3\) as

\[4\pi\beta_3 = -14\alpha_3^2 = 4\pi \frac{d\alpha_3}{dt},\]  
(21)
\[ 4\pi\beta_2 = -\frac{19}{3}\alpha_2^2 = 4\pi \frac{d\alpha_2}{dt}, \quad (22) \]
\[ 4\pi\beta_1 = \frac{41}{5}\alpha_1^2 = 4\pi \frac{d\alpha_1}{dt}, \quad (23) \]
\[ 4\pi\beta_t = \alpha_t \left( 9\alpha_t - 16\alpha_3 - \frac{9}{2}\alpha_2 - \frac{17}{10}\alpha_1 \right) = 4\pi \frac{d\alpha_t}{dt}, \quad (24) \]

and
\[ 4\pi\beta_H = 24\alpha_H^2 + 12\alpha_H\alpha_t - \frac{9}{5}\alpha_H\alpha_1 - 9\alpha_H\alpha_2 + \frac{27}{200}\alpha_1^2 + \frac{9}{20}\alpha_1\alpha_2 - \frac{9}{8}\alpha_2 - 6\alpha_t^2 \quad (25) \]

where \( t = \ln \mu \), and the \( \beta \) function for the Yukawa coupling constant of the top quark was introduced by
\[ \beta_t = \mu \frac{\partial \alpha_t}{\partial \mu}, \quad \alpha_t = \frac{(a^{(u)}_{33})^2}{4\pi}. \quad (26) \]

Except for \( \alpha_H(t) \), these differential equations are analytically solved as follows:
\[ \frac{1}{\alpha_3(t)} - \frac{1}{\alpha_3(t_0)} = \frac{7}{2\pi}(t - t_0), \quad (27) \]
\[ \frac{1}{\alpha_2(t)} - \frac{1}{\alpha_2(t_0)} = \frac{19}{12\pi}(t - t_0), \quad (28) \]
\[ \frac{1}{\alpha_1(t)} - \frac{1}{\alpha_1(t_0)} = -\frac{41}{20\pi}(t - t_0), \quad (29) \]

and
\[ \alpha_t(t) = \frac{F_t^{213}(t)}{F_t^{213}(t_0)} \left[ \frac{\alpha_t(t_0)}{1 - \alpha_t(t_0) \frac{9}{4\pi} \int_{t_0}^t \frac{F_t^{213}(t)}{F_t^{213}(t_0)} dt} \right]. \quad (30) \]

In the last expression for \( \alpha_t(t) \), the function \( F_t^{213}(t) \) was defined by
\[ F_t^{213}(t) = \alpha_3(t)^{8/7}\alpha_2(t)^{27/38}\alpha_1(t)^{-17/82}. \quad (31) \]

The scale relation in Eq.(24) for the Yukawa coupling constants enables us to represent the masses of the Higgs boson and the top quark as
\[ m_H^2 = 2\mu^2 = 2\lambda v^2, \quad m_t = a^{(u)}_{33} \frac{v}{\sqrt{2}} \quad (32) \]
in terms of the vacuum expectation value \( v = \sqrt{2} \langle \phi^0 \rangle \) and to approximate the constraint in Eq. (11) by

\[
\lambda = \frac{1}{2} \left( a_{33}^{(u)} \right)^2.
\]  

(33)

In order to investigate the deviation of this relation due to quantum effects, we impose the initial condition

\[
\alpha_{H}(\mu_0) \approx \frac{1}{2} \alpha_t(\mu_0)
\]  

(34)

at a high energy scale \( \mu = \mu_0 \) and calculate the Higgs boson mass \( m_H(\mu) \) at a lower energy scale \( \mu \) by solving the renormalization group equation. It is the tree level approximation that Eq. (33) leads to the mass formula

\[
m_H \approx \sqrt{2} m_t.
\]  

(35)

The CDF collaboration [1] and the D0 collaboration [2] have reported the value of the top quark to be

\[
m_t = 180 \pm 12\text{(stat)}^{+19}_{-15}\text{(syst)} \text{ GeV}
\]  

(36)

from the data of \( p\bar{p} \) collisions at \( \sqrt{s} = 1.8 \) TeV. As the first step to solve the renormalization group equation, let us adopt the value \( m_t = 180 \) GeV for the top quark mass and decide the values of the input parameters at the scale \( \mu = m_Z = 91.2 \) GeV. The gauge coupling constants take the values

\[
\alpha_3(\mu = m_Z) = 0.12, \quad \alpha_2(\mu = m_Z) = 0.034, \quad \alpha_1(\mu = m_Z) = 0.017,
\]  

(37)

and the vacuum expectation value is estimated to be

\[
\bar{v} = v(m_Z) = \frac{M_Z(m_Z) \cos \theta_W(m_Z)}{[\pi \alpha_2(m_Z)]^{1/2}} = (\sqrt{2} G_F)^{-1/2} = 246 \text{ GeV}.
\]  

(38)

Integrating the renormalization group equations from \( m_t = m_t(m_t) = 180 \) GeV to \( m_Z \), we get

\[
m_t(m_Z) \approx a_{33}^{(u)}(m_Z) \frac{\bar{v}}{\sqrt{2}} = 187.9 \text{ GeV}, \quad \alpha_t(m_Z) = 0.09283.
\]  

(39)
Figure 1: Running behaviors of the Yukawa coupling constant squared
\( \alpha_t(\mu) = (a^{(u)}_{33}(\mu))^2/4\pi \) and the quartic Higgs self-coupling constant \( \alpha_H(\mu) = \lambda(\mu)/4\pi \) versus the scale \( \mu \). The thick solid line representing the running of \( \alpha_t(\mu) \) is uniquely fixed by the experimental condition \( m_t(180 \text{ GeV}) = 180 \text{ GeV} \). The dashed line is for \( \alpha_t(\mu)/2 \). The thin solid lines showing the running of \( \alpha_H(\mu) \) have dependence on the initial scale \( \mu_0 \). The constraint \( \alpha_H = \alpha_t/2 \) at an initial scale \( \mu_0 \) determines the evolution curve \( \alpha_H(\mu) \). Three evolution curves \( \alpha_H(\mu) \) are drawn for the initial scales \( \mu_0 = 10^4, 10^8 \) and \( 10^{12} \) GeV.

Substitution of the input values in Eqs. (37) \( \sim (39) \) into Eqs. (27) \( \sim (30) \) determines uniquely the evolutions of \( \alpha_k (k = 1, 2, 3) \) and \( \alpha_t \). Fig. 1 shows the behaviors of the \( \alpha_H(\mu) \) curves are plotted for \( \mu_0 = 10^4, 10^8 \) and \( 10^{12} \) GeV.

We estimate here the running effect of the vacuum expectation value \( v \). The renormalization group equation for \( v \) is obtained [9] by

\[
\frac{d \ln v}{dt} = \frac{1}{16\pi^2} \left[ \frac{9}{4} \left( g_1^2 + g_2^2 \right) - \text{tr} \left\{ (A_e^\dagger A_e) + 3(A_u^\dagger A_u) + 3(A_d^\dagger A_d) \right\} \right].
\]

(40)

Using Eq. (20), this is approximated by

\[
4\pi \frac{dv}{dt} = -v \left( 3\alpha_t - \frac{9}{4}\alpha_2 - \frac{9}{20}\alpha_1 \right)
\]

(41)

which has the solution

\[
v(t) = v(t_0) \left( \frac{\alpha_t(t)}{\alpha_t(t_0)} \right)^{-1/3} \left( \frac{\alpha_3(t)}{\alpha_3(t_0)} \right)^{8/21} \left( \frac{\alpha_2(t)}{\alpha_2(t_0)} \right)^{-9/76} \left( \frac{\alpha_1(t)}{\alpha_1(t_0)} \right)^{-7/492}.
\]

(42)
Figure 2: Dependence of the Higgs boson mass on the initial scale $\mu_0$. The dashed line represents the $\mu_0$ dependence of the Higgs mass $m_H(\mu)$ at the scale $\mu = m_t = 180$ GeV. Likewise the solid line is for the Higgs mass defined by the condition $m_H(\mu = m_H) = m_H$. The relation $m_t \leq m_H(m_H)$, $m_H(m_t) \leq 2^{1/2}m_t$ holds for a wide range of the scale $\mu_0 \geq m_t$. With the increase of the scale $\mu_0$, the Higgs mass approaches to 177 GeV ($\approx m_t$).

This running effect is confirmed numerically to be negligible\footnote{Note that the masses $m_H$ and $m_t$ are not physical pole masses. In Ref.\cite{11}, the relationship between the running mass and the physical pole mass is given and the difference between them is proved to be negligible.} for the range of mass scale considered here. Therefore, we use the following formulas as

$$m_t(\mu) = \sqrt{2\pi \alpha_t(\mu)} \, v(\mu) \simeq \sqrt{2\pi \alpha_t(\mu)} \, \bar{v}$$ \hspace{1cm} (43)

and

$$m_H(\mu) = \sqrt{8\pi \alpha_H(\mu)} \, v(\mu) \simeq \sqrt{8\pi \alpha_H(\mu)} \, \bar{v}$$ \hspace{1cm} (44)

for the running masses of the top quark and the Higgs boson for an arbitrary scale $\mu$.

Fig. 2 represents the dependence of the Higgs boson mass on the initial scale $\mu_0$. The dashed line is for the Higgs mass $m_H(\mu)$ at the scale $\mu = m_t = 180$ GeV. The solid line is for the Higgs mass defined by the condition

$$m_H(\mu = m_H) = m_H.$$ \hspace{1cm} (45)

The shapes of both lines are almost the same with each other and both lines tend to converge to the common value $m_t = 180$ GeV as the scale $\mu_0$ increases. From this result we find that the relation $m_t \lesssim m_H(m_H)$, $m_H(m_t) \lesssim \sqrt{2}m_t$ holds for a wide range of the scale $\mu_0 \gtrsim m_t$ and that, with the increase of the scale $\mu_0$, the Higgs mass approaches to 177 GeV ($\approx m_t$).\footnote{Note that the masses $m_H$ and $m_t$ are not physical pole masses. In Ref.\cite{11}, the relationship between the running mass and the physical pole mass is given and the difference between them is proved to be negligible.}
In the analysis of Álvarez et al. [7] which has two constraints \( m_{\text{top}} = 2m_W \), \( m_{\text{Higgs}} = 3.14m_W \) as initial conditions for the renormalization group equation, both the value of the Higgs boson mass and the value of the initial mass scale \( \mu_0 \) are determined at the same time. For example if we use the value \( m_t = 186 \text{ GeV} \) as the top quark mass at the scale \( m_Z \), we get \( \mu_0 \sim 10^4 \text{ GeV} \) and \( m_H \sim 223 \text{ GeV} \) from Table 1 in Ref. [7]. By contrast we have only one constraint in Eq.(34) in our model. This means that, even when the value of the top quark mass is given, \( \mu_0 \) remains as free parameter. However, it is natural to assume that the scale \( \mu_0 \) at which the relation in Eq.(34) holds in the original Lagrangian density for local fields takes a sufficiently large value. Therefore the results of our analysis show that the Higgs boson has the mass being close to that of the top quark, \( i.e., m_H \approx m_t \).

Experiments at LEP exclude a large range of Higgs masses. Currently, the LEP precision tests fixed the lower bound to be \( m_H = 58.4 \text{ GeV} \) at 95% confidence level [12]. On the other hand, a theoretical constraint of the Higgs mass can be obtained from the vacuum stability requirement that our universe is in the true minimum of the Higgs potential [13]. The constraint depends upon the top quark mass and upon the scale \( \Lambda \) up to which the Standard Model remains valid. In case where the constraint is severest, \( i.e., \Lambda = 10^{19} \text{ GeV}, m_H > 135 \text{ GeV} + 2.1(m_{\text{top}} - 174 \text{ GeV}) \) [14, 15]. By non-perturbative calculations using lattice field theory, an upper bound on the Higgs mass is obtained as \( m_H < 710 \pm 60 \text{ GeV} \) [16]. Thus our predictions of the Higgs mass obtained in this letter are within the allowed bound for both experiment and theory.

Eq.(25) shows that the top quark Yukawa coupling constant \( a_{33}^{(u)} \) gives a negative contribution to the \( \beta \) function \( \beta_H \) of the Higgs self-coupling \( \lambda \). Owing to the large top quark mass, the value of \( \beta_H \) is always negative at low energy scale. Since the top quark Yukawa coupling constant itself decreases with scale, the value of \( \beta_H \) at high energy scale is positive. Eventually, \( \lambda \) falls with scale until some minimization is reached, and then rise. If this minimum is above zero, the standard model vacuum is stable [13, 14, 17]. As shown in Fig. 1, the minimum of \( \lambda \) is positive for all values of \( \mu_0 \) under our initial condition (34). Therefore, the vacuum is always stable in our model.

After \( \lambda \) reaches to some minimization, the \( \beta \) function of the Higgs self-coupling \( \beta_H \) is positive, and thus the Higgs self-coupling \( \lambda \) will eventually diverge, reaching to the Landau pole (the Landau ghost) [18]. Once \( \lambda \) exceeds unity it will diverge rapidly. In other words there is no practical difference
between the scale where \( \lambda \) tends to diverge and the non-perturbative scale corresponding to \( \lambda \geq 1 \). In Table 1 we show the scale where \( \lambda \) diverges for each \( \mu_0 \). The scale giving a Landau pole is not directly proportional to \( \mu_0 \). It increases approximately as \( \mu_0^{2.2} \) as \( \mu_0 \) increases. This means that the large value of \( \mu_0 \) leads to the small masses of the top quark and the Higgs boson, so that the increase of \( \lambda \) becomes slow. For low energy scale, \( a_{33} \) and \( \lambda \) take finite values.

In this way we have analyzed the effects of one loop quantum corrections on the constraint among the Yukawa coupling constants and the quartic Higgs self-coupling constant predicted by the new scheme of the standard model based on the generalized covariant derivatives. Numerical analysis of the renormalization group equation has shown that the masses of the Higgs boson and the top quark satisfy the relation \( m_H \approx m_t \) which deviates markedly from the tree level prediction \( m_H \approx \sqrt{2} m_t \). It is necessary to investigate the effects of quantum corrections on various constraints among coupling constants which are obtained in the grand unified theory \([19]\) based on the generalized covariant derivatives.

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