On the Uplink Transmission of Multi-user Extra-large Scale Massive MIMO Systems

Xi Yang, Student Member, IEEE, Fan Cao, Student Member, IEEE, Michail Matthaiou, Senior Member, IEEE, and Shi Jin, Senior Member, IEEE

Abstract

With the inherent benefits, such as, better cell coverage and higher area throughput, extra-large scale massive MIMO has great potential to be one of the key technologies for the next generation wireless communication systems. However, in practice, when the antenna dimensions grow large, spatial non-stationarities occur and users would only see a portion of the base station antenna array, which we call visibility region (VR). To exploit the impact of spatial non-stationarities, in this paper, we investigate the uplink transmission of multi-user extra-large scale massive MIMO systems by considering VRs. In particular, we first propose a subarray-based system architecture for extra-large scale massive MIMO systems. Then, tight closed-form uplink spectral efficiency (SE) approximations with linear receivers are derived. With the objective of maximizing the system sum achievable SE, we also propose schemes for the subarray phase coefficient design. In addition, two statistical CSI-based greedy user scheduling algorithms are developed. Our results indicate that statistical CSI-based scheduling algorithms achieve great performance for extra-large scale massive MIMO systems, and it is not necessary to simultaneously turn on all the subarrays and radio frequency chains to serve the users.

Index Terms

Ergodic spectral efficiency, extra-large scale massive MIMO, scheduling, spatial non-stationarity, subarray design.

I. INTRODUCTION

Massive multiple-input multiple-output (MIMO), whose idea is to employ a large-scale antenna array at a base station (BS) to serve multiple users simultaneously, thus, achieving
great spatial multiplexing gains and better spectral efficiency, has been identified as one of the key technologies in fifth-generation wireless communication systems \[1\]-\[3\]. When the antenna dimension continues to increase, the arrays turn out to be physically very large and the benefits such as, channel hardening, asymptotic inter-user channel orthogonality, cell coverage, area throughput, etc., promised by massive MIMO can be fully harnessed from a theoretical point of view. These extra-large scale antenna arrays could be developed and be integrated into large infrastructures, such as, the roof of airports, the walls of stadiums, or large shopping malls \[4\]. With these enhanced benefits, extra-large scale massive MIMO is a very promising technology for the sixth-generation of wireless communications \[5\].

However, the reality is not so idealistic: based on some recent measurement results in \[6\], when the antenna dimension becomes large, spatial non-stationarities start to kick in. This arises from the fact that when the dimension of an antenna array is large, the far-field propagation assumption breaks down since the distances between the BS and scatterers or users are smaller than the Rayleigh distance \[7\], and users can only see a portion of the BS antenna array due to the energy-limited scattering propagation paths and the extra-large array size. The portion of the antenna array at BS seen by users is called visibility region (VR) \[8\]. Each user has its specific VR and the locations of VRs for different users can be separate, partially overlapped, or completely overlapped, depending on the surrounding environment and the users’ relative positions along the antenna array.

Due to the existence of VRs, the performance characterization of extra-large scale massive MIMO systems is different from that of stationary massive MIMO systems by simply letting their number of BS antennas go to infinity. There are several works exploiting the performance of extra-large scale massive MIMO systems. In \[9\], with the objective of improving the computational efficiency, a disjoint subarray-based receiver architecture and distributed linear data fusion receiver with bipartite graph-based user selection method were proposed. Moreover, \[10\] presented a capacity analysis of extra-large scale massive MIMO by introducing a tractable non-stationary channel model which divides the scattering clusters into two categories, i.e., wholly visible clusters and partially visible clusters, and regards these clusters as an array with virtual antennas. A simple non-stationary channel model, which has connections with the stationary massive MIMO channel, was proposed in \[11\]. A downlink performance analysis of multi-user massive MIMO systems with linear precoders was also provided. Finally, the results in \[11\] indicated that the VR significantly impacts the performance of linear precoders. Despite that, there is less of work that investigates the uplink performance of extra-large scale massive MIMO systems when taking the spatial non-stationarities into account.
Motivated by these existing works, we mainly focus on the uplink transmission of multi-user extra-large scale massive MIMO systems by considering VRs. In particular, we firstly propose a practical system architecture suitable for extra-large scale massive MIMO, where a subarray-based hybrid architecture is adopted to alleviate the overall hardware cost and complexity. Then, the uplink achievable spectral efficiencies (SEs) of the extra-large scale massive MIMO system with linear receivers are examined. Afterwards, we investigate the design of the subarray in order to maximize the achievable SE. Two statistical channel state information (CSI)-based greedy user scheduling algorithms are also proposed and numerical simulations are performed to validate their performance. The main contributions of this paper can now be summarized as follows:

- We derive tight closed-form ergodic uplink achievable SE approximations for the multi-user extra-large scale massive MIMO system with linear receivers, i.e., maximum ratio combining (MRC) receiver and linear minimum mean squared error (LMMSE) receiver. The ergodic achievable SE approximation for the MRC receiver shows that in order to maximize the system sum achievable SE, users with their VRs covering different subarrays or VRs with less overlap should be simultaneously scheduled. On the other hand, the ergodic achievable SE approximation for the LMMSE receiver indicates that we should simultaneously schedule as many users as possible.

- By considering two subarray architectures, i.e., the subarray with phase shifters and the on-off switch-based subarray, we investigate the design of the subarray for the extra-large scale massive MIMO system. For the subarray with phase shifters, the optimal phase coefficients are the phases of the eigenvectors corresponding to the maximum eigenvalues of the main block matrices of the channel correlation matrix. For the on-off switch-based subarray, the user who has larger sum energy radiated to the subarrays should be selected for communication to maximize the sum achievable SE.

- With the aim of maximizing the sum achievable SE, we propose two statistical CSI-based greedy scheduling algorithms, i.e., the statistical CSI-based greedy user scheduling algorithm and the statistical CSI-based greedy joint user and subarray scheduling algorithm. Numerical results manifest that in the extra-large scale massive MIMO regime, it is not necessary to simultaneously turn on all subarrays and radio frequency (RF) chains to serve the users. The introduction of dynamic subarray scheduling is beneficial to achieve better system performance with lower energy consumption.

The rest of this paper is organized as follows: In Section II, we present the system architecture and the signal model for the extra-large scale massive MIMO system. Section III investigates the uplink ergodic achievable SEs under the linear receivers as well as the phase
coefficient design of subarrays. The proposed statistical CSI-based user scheduling algorithms are provided in Section IV. Section V presents the numerical results and we conclude the paper in Section VI.

Throughout the paper, we use bold lowercase $\mathbf{a}$ and bold uppercase $\mathbf{A}$ to denote vectors and matrices, respectively. The superscripts $(\cdot)^\ast$, $(\cdot)^T$, and $(\cdot)^H$ represent the conjugate, transpose, and conjugate-transpose operations of matrix, respectively; $\mathbf{I}_N$ is an identity matrix with dimension $N \times N$; $\odot$ and $\otimes$ denote the element-wise product and the Kronecker product, respectively. Also, $\mathbb{E}\{\cdot\}$ is the expectation operation, $\|\mathbf{a}\|$ stands for the norm of the vector $\mathbf{a}$, $\text{tr} (\mathbf{A})$, $\det (\mathbf{A})$, and $\mathbf{A}^{-1}$ stand for the trace, determinant, and inverse of the matrix $\mathbf{A}$, respectively, $\text{diag}(x_1, x_2, \ldots, x_N)$ represents a diagonal matrix with diagonal elements $x_i$, $i = 1, \ldots, N$, while $\text{blkdiag}(\mathbf{X}_1, \mathbf{X}_2, \ldots, \mathbf{X}_N)$ represents a diagonal matrix with block diagonal matrices $\mathbf{X}_i$, $i = 1, \ldots, N$.

II. SYSTEM MODEL

In this section, we firstly describe the system architecture of the extra-large scale massive MIMO system, and then the signal model is provided.

A. System Architecture

Consider a multi-user extra-large scale massive MIMO system illustrated in Fig.1 where a BS equipped with an $M$-element large uniform linear array (ULA) serves $K$ single-antenna users simultaneously.

Since the massive number of antennas at the BS render independent RF chain per antenna element impractical in terms of hardware cost and system complexity, we propose a subarray-based hybrid architecture as presented in Fig.1. The BS consists of subarrays, an RF chain pool, and a baseband processing unit. Each subarray includes $M/N$ antenna elements and thus $N$ subarrays are configured. The RF chain pool contains RF chains and each RF chain can be statically or dynamically assigned to a dedicated subarray. Digital processing, such as, channel estimation, data detection, or user scheduling is performed in the baseband processing unit. Note that each subarray is connected with an RF chain and therefore can support one data stream. Due to existence of VR in extra-large scale massive MIMO systems, multiple consecutive subarrays could be covered by one user. Moreover, when overlapped VRs occur, the multiple consecutive subarrays may also support other users simultaneously, which inevitably creates inter-user interference.

To harvest the array gain provided by the large number of antennas, two different subarray architectures i.e., the subarray with phase shifters and the on-off switch-based subarray, are
considered. In former architecture, each antenna element in the subarray is connected with a phase shifter, while in the latter type, the phase shifter is replaced by a switch which is always turned on in the uplink. Hence the signals acquired by antennas in a on-off switch-based subarray are directly combined without programmed phase shifts before conveyed to the RF chain. It is worth noting that compared with the subarray with phase shifters and despite the anticipated performance loss, the on-off switch-based subarray is much cheaper and more hardware-implementation friendly, especially in extra-large scale massive MIMO regime. In addition, as will be presented in the numerical results, the on-off switch-based subarray also yields great performance when combined with the LMMSE receiver.

B. Signal Model

We focus on the uplink transmission of the multi-user extra-large scale massive MIMO system in this paper. Taking the spatial non-stationarity of the extra-large scale MIMO system channels into consideration, we model the channel \( h_k \in \mathbb{C}^{M \times 1} \) between the \( k \)-th user and the BS as

\[
h_k = \Theta_k^{1/2} g_k,
\]  

(1)
where \( g_k \sim \mathcal{CN}(0, I_M) \) and \( \Theta_k \) represents the correlation matrix at the BS for the \( k \)-th user, given as \( \Theta_k = D_k^{1/2}R_kD_k^{1/2} \) [11]. Note that \( R_k \) denotes the classical spatial correlation matrix of user \( k \) corresponding to the case of a stationary massive MIMO channel and the spatial non-stationarity, i.e., the user’s VR, in the extra-large scale massive MIMO channel is characterized by the real diagonal matrix \( D_k = \text{diag}(d_1(d_k), d_2(d_k), \ldots, d_M(d_k)) \), where \( d_m(d_k) \), \( m = 1, \ldots, M \), denotes the spatial non-stationarity for user \( k \) at the antenna element \( m \). We point out only a few diagonal elements of \( D_k \) are non-zero [11].

When a ULA is employed at BS, the \( R_k \) in \( \Theta_k \) can be expressed as [13]

\[
R_k = [a(\theta_k)a^H(\theta_k)] \odot P(\theta_k, \sigma_k),
\]

where \( a(\theta_k) \) is the steering vector of the ULA, defined as

\[
a(\theta_k) = [1, e^{j2\pi d \sin \theta_k}, \ldots, e^{j2\pi (M-1)d \sin \theta_k}]^T,
\]

where \( \theta_k \) represents the mean angle of arrival (AoA) of the \( k \)-th user and \( d \) denotes the antenna element spacing normalized by the carrier wavelength. In our simulations, we set \( d = 1/2 \). Most importantly, \( P(\theta_k, \sigma_k) \) captures the angular spectrum of AoA and its entries come from a Gaussian angular spread distribution with variance \( \sigma_k^2 \) [13]. The \( \{m,n\}^{\text{th}} \) entry of \( P(\theta_k, \sigma_k) \) can be given by

\[
\{P(\theta_k, \sigma_k)\}_{m,n} = e^{-2[\pi d(m-n)^2] \sigma_k^2 \cos^2 \theta_k}, \quad m, n = 1, \ldots, M.
\]

Therefore, in the uplink transmission of the multi-user extra-large scale MIMO system, the received signal at the BS can be written as

\[
y = \sqrt{p_u}Hx + n,
\]

where \( p_u \) is the transmit power of each user, \( H = [h_1, h_2, \ldots, h_K] \) represents the multi-user uplink channel matrix, and \( x = [x_1, x_2, \ldots, x_K]^T \) is the transmitted signal from \( K \) users with \( x_k \sim \mathcal{CN}(0, I) \) for \( k = 1, 2, \ldots, K \); \( n \) is the complex Gaussian noise satisfying \( n \sim \mathcal{CN}(0, \sigma^2 I) \). Without loss of generality, we set \( \sigma^2 = 1 \). Additionally, for ease of exposition, we assume that \( \mathbb{E}\{g_kg_k^H\} = 0, \forall i \neq k \), that is to say, there is no correlation between any pair of channels across different users.

Since we employ a subarray-based hybrid architecture in the multi-user extra-large scale MIMO system as presented in Fig. [1] the received signals at BS will be firstly combined in the

\footnote{Although the subsequent analysis is applicable for general cases, we set the non-zero diagonal elements of \( D_k \) to be 1 in the simulation section for simplicity.}
analog domain and then be linearly demodulated in the digital domain. Thus, the processing procedure can be formulated as

\[ r = \sqrt{p_u}A^H W^H x + A^H W^H n, \]  

(6)

where \( W = \text{blkdiag}(w_1, w_2, \ldots, w_N) \in \mathbb{C}^{M \times N} \) represents the combining matrix in the analog domain and can be expressed as

\[ W = \begin{pmatrix} w_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & w_N \end{pmatrix}, \]  

(7)

where \( w_i \in \mathbb{C}^{(M/N) \times 1} \) is a constant modulus vector, that is to say, all the elements of \( w_i \) have constant amplitude of \( \sqrt{N/M} \) and \( w_i^H w_i = 1 \) for \( i = 1, 2, \ldots, N \). Also, \( A \in \mathbb{C}^{N \times K} \) is the linear detection matrix in the digital domain, which has different expressions for different linear receivers such as, MRC receiver and LMMSE receiver. Here, we define \( F = W^H H \) and assume that perfect CSI is available at the BS\(^2\), then \( A \) can be given by

\[ A = \begin{cases} F, & \text{for MRC}, \\ F \left( F^H F + \frac{1}{p_u} I_K \right)^{-1}, & \text{for MMSE}. \end{cases} \]  

(8)

Hence, the signal of the \( k \)-th user at the BS can be expressed as

\[ r_k = \sqrt{p_u} a_k^H W^H h_k x_k + \sqrt{p_u} \sum_{i=1, i \neq k}^K a_k^H W^H h_i x_i + a_k^H W^H n, \]  

(9)

where \( a_k = A(:, k) \) is the \( k \)-th column of the matrix \( A \).

By modeling the noise-plus-interference term as additive Gaussian noise independent of \( x_k \) with zero mean and variance of \( p_u \sum_{i=1, i \neq k}^K |a_k^H W^H h_i|^2 + \|a_k^H W^H h_k|^2 \)\(^2\), we obtain the ergodic achievable SE of the \( k \)-th user in the uplink data transmission as

\[ R_k = \mathbb{E}_h \left\{ \log_2 \left( 1 + \frac{p_u |a_k^H W^H h_k|^2}{p_u \sum_{i=1, i \neq k}^K |a_k^H W^H h_i|^2 + \|a_k^H W^H h_k|^2} \right) \right\}, \]  

(10)

and the sum uplink achievable SE of the multi-user extra-large scale massive MIMO system

\(^2\)Channel state information can be obtained by various methods, such as, by sending orthogonal pilots from users. Note that only the low-dimension effective channel \( F \) is needed to be estimated in the uplink. In this paper, we assume perfect CSI in order to assess in detail the impact of non-stationarities without introducing complicated notation. Note also that our results can be regarded as upper bounds of what will be achieved in practice.
becomes
\[ R = \sum_{i=1}^{K} R_i. \]  

(11)

In the next section, we aim to examine the ergodic achievable SEs in (10) and (11) under two different types of receivers, i.e., MRC receiver and LMMSE receiver, and then identify the influence of specific VR distributions on the system ergodic achievable SE.

III. UPLINK ACHIEVABLE SPECTRAL EFFICIENCY ANALYSIS

In this section, we investigate the uplink achievable SEs of linear receivers, i.e., MRC receiver and LMMSE receiver, for extra-large scale massive MIMO systems under perfect CSI. The design of the subarray is also discussed corresponding to the hardware architecture illustrated in Section II.

A. MRC Receiver

When the MRC receiver is employed at the BS, the approximation of the ergodic achievable SE is provided in the following theorem.

**Theorem 1:** When the MRC receiver is adopted in the uplink of the multi-user extra-large scale massive MIMO system, the ergodic achievable SE of the \( k \)th user can be approximated by

\[ R_{\text{MRC}, \text{app}}^k = \log_2 \left( 1 + \frac{\text{tr}(B\Theta_k B\Theta_k) + \text{tr}^2(B\Theta_k)}{\sum_{i=1, i \neq k}^{K} \text{tr}(B\Theta_i B\Theta_k) + \frac{1}{p_u} \text{tr}(BB^H\Theta_k)} \right). \]  

(12)

**Proof:** When the MRC receiver is employed at the BS, we have the linear detection matrix \( A = F \), and

\[ a_k = W^H h_k. \]  

(13)

Substituting (13) into (10), the ergodic uplink achievable SE for the \( k \)th user can be written as

\[ R_k^{\text{MRC}} = \mathbb{E}_h \left\{ \log_2 \left( 1 + \frac{p_u |h_h^H W W^H h_k|^2}{p_u \sum_{i=1, i \neq k}^{K} |h_h^H W W^H h_i|^2 + \|h_h^H W W^H\|^2} \right) \right\}, \]  

\[ = \log_2 \left( 1 + \frac{p_u \mathbb{E}_h \{|h_h^H B h_i|^2\}}{p_u \sum_{i=1, i \neq k}^{K} \mathbb{E}_h \{|h_h^H B h_i|^2\} + \mathbb{E}_h \{|h_h^H B|^2\}} \right), \]  

(14)
where (a) applies the approximation $\mathbb{E}\{\log_2(1 + X/Y)\} \approx \log_2(1 + \mathbb{E}\{X\}/\mathbb{E}\{Y\})$ from [14] and we define $B = WW^H$. Note that $W$ is a block diagonal matrix owing to the subarray-based hardware architecture, thus $B$ is also a block diagonal matrix. Assume now that $B = \text{blkdiag}(\bar{B}_1, \bar{B}_2, \ldots, \bar{B}_N)$, then $\bar{B}_i = w_iw_i^H, i = 1, \ldots, N$. The results can be derived directly by calculating the terms in the numerator and the denominator of (14) as following:

$$
\mathbb{E}_h \{ h_k^H Bh_k \} = \text{tr}(B \mathbb{E}_h \{ h_k^H h_k^H \}) = \text{tr}(B \Theta_k), \quad (15)
$$

$$
\mathbb{E}_h \{ |h_k^H Bh_k|^2 \} = \text{tr}(B \Theta_k B \Theta_k) + \text{tr}^2(B \Theta_k), \quad (16)
$$

$$
\mathbb{E}_h \{ |h_k^H Bh_i|^2 \} = \mathbb{E}_h \{ \text{tr}(B h_i^H B^H h_k h_k^H) \} = \text{tr}(B \Theta_i B \Theta_k), \quad (17)
$$

$$
\mathbb{E}_h \{ \|h_k^H B\|^2 \} = \text{tr}(BB^H \Theta_k). \quad (18)
$$

Substituting (15)-(18) into (14) and (11), we obtain (12).

Observe from Theorem 1 that, to maximize the ergodic achievable SE per user and consequently maximize the system sum achievable SE, $\sum_{i=1,i \neq k}^K \text{tr}(B \Theta_i B \Theta_k)$ should be minimized in (12). Since $\text{tr}(B \Theta_i B \Theta_k) \geq 0$, $\Theta_i = D_i^{1/2} R_i D_i^{1/2}$ and $\Theta_k = D_k^{1/2} R_k D_k^{1/2}$ are block diagonal matrices and $B$ is a block diagonal matrix corresponding to the subarray architecture as well, we can obtain $\text{tr}(B \Theta_i B \Theta_k) = 0$ when $1(D_i) \cap 1(D_k) = 0$, in which $1(D_i)$ denotes the $N$-dimension indicator function with its $i$th element calculated by

$$
[1(D_i)]_n = \begin{cases} 
1, & D_i \cap \text{blkdiag}(0, \ldots, \bar{B}_n, \ldots, 0) \neq 0, \\
0, & D_i \cap \text{blkdiag}(0, \ldots, \bar{B}_n, \ldots, 0) = 0.
\end{cases} \quad (19)
$$

Therefore, for the MRC receiver, in order to maximize the system sum achievable SE, users with their VRs covering different subarrays or VRs with less overlap should be scheduled simultaneously in the extra-large scale massive MIMO system. By recalling that the MRC receiver has no capability of cancelling inter-user interference which becomes more problematic in the high signal-to-noise ratio (SNR) regime, we exploit the LMMSE receiver in the next subsection.
B. LMMSE Receiver

Similarly, when the LMMSE receiver is employed at BS, we have the linear detection matrix \( A = F \left( F^H F + \frac{1}{p_u} I_K \right)^{-1} \). Substituting \( A \) into (10), the ergodic achievable SE of the \( k \)th user under LMMSE receiver can be expressed as

\[
R_{k}^{\text{LMMSE}} = \mathbb{E}_h \left\{ \log_2 \left( \frac{1}{\left( I_K + p_u F^H F \right)^{-1}} \right) \right\} .
\]  

(20)

Since \( [M^{-1}]_{kk} = \frac{\det(M^{kk})}{\det(M)} \), where \( M^{kk} \) is the \((k, k)\)th minor of the matrix \( M \) ([15]), combining \((F^H F)^{kk} = F^H_{(k)} F_{(k)}\) where \( F_{(k)} \) corresponds to \( F \) with the \( k \)th column removed ([16]), we can rewrite (20) as

\[
R_{k}^{\text{LMMSE}} = \mathbb{E}_h \left\{ \log_2 \det \left( I_K + p_u F^H F \right) \right\} - \mathbb{E}_h \left\{ \log_2 \det \left( I_{K-1} + p_u F^H_{(k)} F_{(k)} \right) \right\} .
\]  

(21)

Note that it is greatly challenging, if not impossible, to directly evaluate (21) under general cases, hence in what follows we analyze the ergodic achievable SE by giving a separate treatment for two cases, i.e., (i) completely overlapped VR case, (ii) partially overlapped VR case.

**Completely Overlapped VR Case:** In this case, users are closely distributed in a relatively small region in front of the extra-large scale massive MIMO system, therefore, the VRs of different users completely overlap. To further simplify the problem, we assume that \( \Theta_1 = \cdots = \Theta_K = \Theta \), then

\[
H = \Theta^{1/2}[g_1, g_2, \ldots, g_K]
= \Theta^{1/2} G,
\]  

(22)

where \( G \triangleq [g_1, g_2, \ldots, g_K] \) and \( G \sim \mathcal{CN}(0, I_M \otimes I_K) \). Therefore,

\[
F^H F = G^H \tilde{\Theta} G,
\]  

(23)

where we define \( \tilde{\Theta} \triangleq \Theta^{1/2} WW^H \Theta^{1/2} \). Substituting (23) into (21), we have the ergodic achievable SE of the \( k \)th user under the completely overlapped VR case as

\[
R_{k}^{\text{LMMSE, Com}} = \mathbb{E}_h \left\{ \log_2 \det \left( I_K + p_u G^H \tilde{\Theta} G \right) \right\} - \mathbb{E}_h \left\{ \log_2 \det \left( I_{K-1} + p_u G^H_{(k)} \tilde{\Theta} G_{(k)} \right) \right\}
= (a) \frac{K \log_2 e}{\Pi_{m<n} \left( \beta_n - \beta_m \right)} \left( \sum_{i=M-K+1}^{M} \det E_{K,M}(i) - \sum_{i=M-K+2}^{M} \det E_{K-1,M}(i) \right),
\]  

(24)

where (a) comes from [16] Proposition 4], \( \beta_1 \geq \ldots \geq \beta_M \) are the eigenvalues of \( \tilde{\Theta} \), \( E_{K,M}(i) \)
and $E_{K-1,M}(i)$ are $M \times M$ matrices with their $(s,t)$th element being

\[
[E_{p,M}(i)]_{s,t} = \begin{cases} 
\beta_s^{t-1}, & t \neq i, \\
\beta_s^{t-1} e^{rac{1}{\beta_s p}} \sum_{h=1}^{p-M+t} E_h \left( \frac{1}{\beta_s p} \right), & t = i,
\end{cases}
\tag{25}
\]

where $E_h(\cdot)$ denotes the exponential integral function. Note that (24) should be a lower bound of the ergodic achievable SE of the completely overlapped VR case since the multi-user interference is maximized when $\Theta_1 = \cdots = \Theta_K = \Theta$. In addition, stationary massive MIMO is actually a special case of the extra-large scale MIMO where the VRs of different user completely overlap.

**Partially Overlapped VR Case:** In this case, users are randomly and relatively sparsely distributed along the whole extra-large scale antenna array and the VRs of different user partially overlap. Theorem 2 analyzes the ergodic achievable SE for this partially overlapped VR case.

**Theorem 2:** When the LMMSE receiver is adopted in the uplink of the multi-user extra-large scale massive MIMO system, the ergodic achievable SE of the $k$th user can be approximated by

\[
R_{k_{LMMSE,app}} = \log_2 \left[ 1 + p_u \text{tr} \left( B^{\Theta_k} \right) \right].
\tag{26}
\]

**Proof:** See Appendix A.

Note that in the partially overlapped VR case, there exists a special scenario that only a few users are sparsely distributed in front of the extra-large scale massive MIMO system and thus no overlapped VRs appear. The analysis for this special scenario is also presented in Appendix A. Furthermore, since there is less possibility that VRs of different users completely overlap, especially when cooperated with user scheduling algorithms, we mainly focus on the partially overlapped VR case in the subsequent analysis.

Theorem 2 indicates that, for the purpose of maximizing the system sum achievable SE with the LMMSE receiver, we should simultaneously schedule as many users as possible who have larger $\text{tr} \left( B^{\Theta_k} \right)$. In addition, the subarray architecture including the number of antennas per subarray, the phase coefficients of the phase shifter network, and so on, should also be well designed to match the correlation matrix $\Theta$ such that $\text{tr} \left( B^{\Theta} \right)$ is maximized. In the next subsection, we precisely elaborate on the phase coefficient design of the subarray with a phase shifter network. Another architecture, i.e., the on-off switch-based subarray, is also investigated to provide insights into the corresponding user scheduling.

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1In fact, when we approximate $(I_K + p_u F^H F)$ with $\text{diag}(I_K + p_u F^H F)$, the ergodic achievable SE for the completely overlapped VR case can also be approximated by (26) in Theorem 2 with a looser tightness.
C. Design of Subarray

Referring back to the hardware architecture illustrated in Section II, the design of two subarray architectures i.e., the subarray with phase shifters and the on-off switch-based subarray, are considered in this subsection.

1) Subarray with Phase Shifters: When subarrays with phase shifters are deployed, every antenna at the BS is connected with an independent phase shifter. In what follows, we consider high precision phase shifters, nevertheless, low resolution phase shifters can also be employed to further reduce the hardware cost which is, however, beyond the scope of this paper. On the basis of (12) in Theorem 1 and (26) in Theorem 2 in order to maximize the ergodic achievable SE for each user and thus for the whole system, in the next analysis we propose a phase coefficient design from the aspect of maximizing \( \text{tr}(B\Theta_k) \).

Since \( B = \text{blkdiag}(\bar{B}_1, \bar{B}_2, \ldots, \bar{B}_N) \) and \( \bar{B}_i = w_i w_i^H, i = 1, \ldots, N \), we define

\[
\Theta_k = \begin{pmatrix}
\bar{\Theta}_{k,11} & \ldots & \bar{\Theta}_{k,1N} \\
\vdots & \ddots & \vdots \\
\bar{\Theta}_{k,N1} & \ldots & \bar{\Theta}_{k,NN}
\end{pmatrix},
\]

(27)

where \( \bar{\Theta}_{k,ij} \in \mathbb{C}^{(M/N) \times (M/N)} \), \( \forall i, j = 1, \ldots, N \) denotes the \( i \)th row \( j \)th column block matrix of \( \Theta_k \), then

\[
B\Theta_k = \begin{pmatrix}
\bar{B}_1\bar{\Theta}_{k,11} & \ldots & \bar{B}_1\bar{\Theta}_{k,1N} \\
\vdots & \ddots & \vdots \\
\bar{B}_N\bar{\Theta}_{k,N1} & \ldots & \bar{B}_N\bar{\Theta}_{k,NN}
\end{pmatrix}.
\]

(28)

Hence,

\[
\text{tr}(B\Theta_k) = \text{tr} \left( \sum_{i=1}^{N} \bar{B}_i \bar{\Theta}_{k,ii} \right)
\]

\[
= \sum_{i \in S_k} \text{tr}(w_i^H \bar{\Theta}_{k,ii} w_i)
\]

\[
= \sum_{i \in S_k} w_i^H \bar{\Theta}_{k,ii} w_i,
\]

(29)

where \( S_k \) represents the ensemble of the non-zero block matrices \( \bar{\Theta}_{k,ii} \) for \( \Theta_k \) and (a) utilizes the trace property \( \text{tr}(AB) = \text{tr}(BA) \). Consequently, based on (29), to maximize the ergodic achievable SE for user \( k \), \( w_i \) should be chosen as the eigenvector of \( \bar{\Theta}_{k,ii} \) corresponding to

\footnote{For MRC receiver in (12), because \( \text{tr}^2(B\Theta_k) > \text{tr}(B\Theta_k B\Theta_k) \), we also concentrate on maximizing \( \text{tr}(B\Theta_k) \) as with the LMMSE receiver in (26).}
the maximum eigenvalue. Considering that \( w_i \) is realized by phase shifters with constant-modulus constraints, we design
\[
w_i = \frac{N}{M} e^{j\angle v_{k,i}},
\]
where \( N/M \) is introduced for normalization and \( v_{k,i} \) is the eigenvector of \( \Theta_{k,ii} \) corresponding to the maximum eigenvalue. It is important to note that the phase coefficient design of each subarray in (30) is designed in terms of the low-dimension matrix \( \Theta_{k,ii} \in \mathbb{C}^{(M/N) \times (M/N)} \), instead of the correlation matrix \( \Theta_k \in \mathbb{C}^{M \times M} \) across the whole extra-large scale antenna array. Therefore, the calculation of \( w_i \) for different subarrays can be executed in parallel and the computation complexity can also be greatly reduced.

2) On-Off Switch-Based Subarray: At the expense of performance degradation, a on-off switch-based subarray requires much lower hardware complexity and hardware cost when compared with a subarray with phase shifters. Without phase shifters, \( B \) for the on-off switch-based subarray becomes
\[
B = \frac{N}{M} \text{diag}(1_{M/N}, 1_{M/N}, \ldots, 1_{M/N}),
\]
where \( 1_{M/N} \) denotes the all-ones matrix, i.e.,
\[
1_{M/N} = \begin{pmatrix}
1 & \ldots & 1 \\
\vdots & \ddots & \vdots \\
1 & \ldots & 1
\end{pmatrix}_{M/N}.
\]
Suppose \( \Lambda = \text{blkdiag}(1_{M/N}, 1_{M/N}, \ldots, 1_{M/N}) \), then \( B = N\Lambda/M \), and (12) and (26) can be simplified to
\[
R_{k,\text{MRC,app}} = \log_2 \left( 1 + \frac{\text{tr}(\Lambda \Theta_k \Lambda \Theta_k) + \text{tr}^2(\Lambda \Theta_k)}{\sum_{i=1,i\neq k}^K \text{tr}(\Lambda \Theta_i \Lambda \Theta_k) + \frac{M}{p_u N} \text{tr}(\Lambda \Theta_k)} \right)
\]
and
\[
R_{k,\text{LMMSE,app}} = \log_2 \left[ 1 + \frac{N_p u \text{tr}(\Lambda \Theta_k)}{M} \right]
\]
respectively. As with the case of subarray with phase shifters, to maximize the ergodic achievable SE, we pay attention to the analysis of \( \text{tr}(\Lambda \Theta_k) \) as well. In the on-off switch-
based subarray, we have
\[
\text{tr}(\Lambda \Theta_k) = \text{tr} \left( \sum_{i=1}^{N} 1_{M/N} \bar{\Theta}_{k,ii} \right) = \sum_{i \in S_k} \sum_{m<n} 2 \text{Re} \left( (\bar{\Theta}_{k,ii})_{mn} \right) + \text{tr}(\Theta_k),
\]
(35)
where \((\bar{\Theta}_{k,ii})_{mn}\) denotes the \(m\)th row \(n\)th column element of \(\bar{\Theta}_{k,ii}\) and \(\text{Re}( (\bar{\Theta}_{k,ii})_{mn} )\) represents the real part of \((\bar{\Theta}_{k,ii})_{mn}\). Hence, \(\sum_{i \in S_k} \sum_{m<n} 2 \text{Re} \left( (\bar{\Theta}_{k,ii})_{mn} \right) + \text{tr}(\Theta_k)\) is the sum of the real parts of the \(M/N\)-dimension non-zero main block diagonal matrices of the \(k\)th user’s correlation matrix. This indicates that \(\text{tr}(\Lambda \Theta_k)\), to some extent, reflects the sum power that has been radiated on the on-off switch-based subarrays at the BS by the \(k\)th user. As a consequence, when on-off switch-based subarrays are deployed at the extra-large scale massive MIMO system, the user who has larger sum energy radiated to the subarrays should be scheduled for communication in order to maximize the system sum achievable SE.

IV. USER SCHEDULING

Scheduling is of great significance in multi-user communication systems, especially for multi-user extra-large scale massive MIMO because of the existence of VRs, which could be used for further improving the spectral and energy efficiency. However, instantaneous CSI-based scheduling is impractical for extra-large scale massive MIMO due to the unconventionally large number of antenna elements and relatively large number of users to be served. As such, the acquisition of instantaneous CSI for all users will result in unaffordable computation complexity and training overhead.

To tackle this problem, we propose two statistical CSI-based greedy scheduling schemes with the aim of maximizing the system sum achievable SE with linear receivers. Given that the MRC receiver is adopted, as we can see from (12), users whose VRs cover different subarrays or those with fewer overlapped VRs should be scheduled so as to maximize the sum achievable SE. In the other case where a LMMSE receiver is employed, (26) showcases that, to obtain the maximum of the achievable SE, as many users as possible with larger \(\text{tr}(B\Theta_i)\) should be scheduled. In the following, we firstly schedule users utilizing statistical CSI in a greedy manner. Then, the algorithm investigating the feasibility of jointly scheduling users and subarrays after taking energy consumption into consideration is provided.

A. Statistical CSI-based Greedy User Scheduling

Generally, utilizing exhaustive search could reach the optimal solution in scheduling problems, however, it may be not appropriate for multi-user extra-large scale massive MIMO
due to the extremely large computational complexity and long runtime. Therefore, a sub-optimal user scheduling algorithm, i.e., the greedy user scheduling algorithm, whose main idea is to achieve an optimal result during each scheduling step and, thus, greatly reduce the algorithm complexity, is proposed. We summarize our proposed statistical CSI-based greedy user scheduling algorithm in Algorithm 1.

**Algorithm 1** Statistical CSI-based Greedy User Scheduling Algorithm

| Input: | $U_s = \emptyset$, $U_n = \{1, 2, \ldots, K\}$, $N_s = 0$, $N_u$, $R = 0$, $R_{temp} = 0$. |
|--------|--------------------------------------------------------------------------------------------------|
| 1:     | while $N_s < N_u$ do \ |
| 2:     | for each $u_i \in U_n$ do \ |
| 3:     | calculate the system sum achievable SE $R_{U_s \cup u_i}$; \ |
| 4:     | end for \ |
| 5:     | select $u_i$ with the largest $R$ among $R_{U_s \cup u_i}$ as a newly scheduled user candidate $u_{sel}$; \ |
| 6:     | if $R_{temp} \leq R$ then \ |
| 7:     | $U_s = U_s \cup u_{sel}$, $U_n = U_n \setminus \{u_{sel}\}$, $R_{temp} = R$, $N_s = N_s + 1$; \ |
| 8:     | else \ |
| 9:     | break; \ |
| 10:    | end if \ |
| 11:    | $R = R_{temp}$; \ |
| 12:    | end while \ |
| Output: | $U_s$, $R$. |

In Algorithm 1, firstly, we initialize all the system parameters, including the scheduled user set $U_s = \emptyset$, the number of scheduled users $N_s = 0$, and the unscheduled user set $U_n = \{1, 2, \ldots, K\}$. The total number of users to be scheduled and served is $N_u$, and the system sum achievable SE is initialized as $R = 0$.

Next, we select the users in the unscheduled user set $U_n$ one by one and calculate their corresponding updated system sum achievable SEs based on the scheduling results of the previous iteration. A user will be added to the scheduled user set only if it reaches the maximum of the updated system sum achievable SEs among all unscheduled users from $U_n$, as well as produces a positive gain compared with the last iteration results. To update the system sum achievable SE in this step, (12) and (26) are leveraged when MRC and LMMSE receivers are adopted respectively. Note that in the phase shifter-based subarray, if a subarray is not covered by any user’s VR, the phase coefficients of the subarray would be set to the default value zero, i.e., $\angle v_{k,i} = 0$; if a subarray is covered by multiple users simultaneously, then the phase coefficients of the subarray would be set to the sum of the phase corresponding to the multiple users.

Then, the algorithm keeps running until $N_s = N_u$ or there is no SE gain when adding a new user. Finally, the algorithm outputs the final scheduling results i.e., the scheduled user set $U_s$ and the system sum achievable SE $R$. 
Note that the proposed greedy user scheduling algorithm exploits only statistical CSI, i.e.,
the knowledge of the channel correlation information instead of the instantaneous channel
gains. This is beneficial and more practical especially for extra-large scale massive MIMO
systems. Moreover, considering the existence of VR and the cases that some subarrays may be
covered by no user, we further examine the possibility to jointly schedule users and subarrays
and propose a statistical CSI-based greedy joint user and subarray scheduling scheme in the
next subsection.

B. Statistical CSI-based Greedy Joint User and Subarray Scheduling

**Algorithm 2** Statistical CSI-based Greedy Joint User and Subarray Scheduling Algorithm

| Input: | \( U_s = \emptyset, S = \emptyset, U_n = \{1,2,\ldots,K\}, S_n = \{1,2,\ldots,N\}, N_s = 0, N_u, R = 0, \) \( Sub_{\text{max}}, Sub_{\text{min}}, R_{\text{temp}} = 0 \). |
|-------|------------------------------------------------------------------------------------------------------------------|
| 1:    | **while** \( N_s < N_u \) **do** |
| 2:    | **for each** \( u_i \in U_n \) **do** |
| 3:    | **for each** \( S_{u_i} \subset S_n \) **do** |
| 4:    | **if** \( Sub_{\text{min}} \leq |S_{u_i}| \leq Sub_{\text{max}} \) **then** |
| 5:    | calculate the system sum achievable SE \( R_{U_s \cup u_i, S_{u_i}} \); |
| 6:    | **else** |
| 7:    | continue; |
| 8:    | **end if** |
| 9:    | **end for** |
| 10:   | select \( S_{s_{\text{sel},u_i}} \) with the largest \( R \) among \( R_{U_s \cup u_i, S_{u_i}} \) as \( u_i \)’s subarray candidate; |
| 11:   | **end for** |
| 12:   | **select** \( u_{\text{sel}} \) with the largest \( R \) among \( R_{U_s \cup u_{\text{sel}}, S_{s_{\text{sel},u_{\text{sel}}}}} \) as a newly scheduled user candidate; |
| 13:   | **if** \( R_{\text{temp}} \leq R_{U_s \cup u_{\text{sel}}, S_{s_{\text{sel},u_{\text{sel}}}}} \) **then** |
| 14:   | \( U_s = U_s \cup u_{\text{sel}}, S = S \cup S_{s_{\text{sel},u_{\text{sel}}}}, U_n = U_n \setminus \{u_{\text{sel}}\}, S_n = S_n \setminus S_{s_{\text{sel},u_{\text{sel}}}}, R_{\text{temp}} = R, N_s = N_s + 1 \); |
| 15:   | **else** |
| 16:   | break; |
| 17:   | **end if** |
| 18:   | \( R = R_{\text{temp}} \); |
| 19:   | **end while** |
| Output: | \( U_s, S, R. \) |

Similar to **Algorithm 1**, we initialize all the system parameters at the first step in **Algorithm 2**
including the scheduled user set \( U_s = \emptyset \), the scheduled subarray set \( S = \emptyset \), the
unscheduled user set \( U_n = \{1,2,\ldots,K\} \), the unscheduled subarray set \( S_n = \{1,2,\ldots,N\} \),
the number of scheduled users \( N_s = 0 \), the total number of users to be scheduled and served
\( N_u \), and the system sum achievable SE \( R = 0 \). The maximum and minimum number of
scheduled subarrays per user are set as \( Sub_{\text{max}} \) and \( Sub_{\text{min}} \), respectively.

Next, for each user in the unscheduled user set \( U_n \), we select its best subarray set from
the unscheduled subarray set \( S_n \) (the number of selected subarrays must not be greater than
Table I

| Parameter | Fig. 2 | Fig. 3 | Fig. 4 | Fig. 5 | Fig. 6 | Fig. 10 |
|-----------|--------|--------|--------|--------|--------|--------|
| $M$       | 1024   | 1024   | 1024   | 1024   | 1024   | 1024   |
| $N$       | 128    | 128    | 128    | 128    | 128    | 128    |
| $K$       | 20     | 10     | 5      | 5      | 22     | 22     |
| $E$       | 160    | 128    | 160    | 160    | 128    | 128    |

$Sub_{\text{max}}$ and less than $Sub_{\text{min}}$) for transmission and receive combining, so that the system sum achievable SE is maximized after the current user is added. Note that (12) and (26) are utilized to calculate the system sum achievable SE when MRC and LMMSE receivers are adopted respectively. If the updated system sum achievable SE is larger than its counterpart, then we record the corresponding user index with its selected subarray set and the updated system sum achievable SE. As a result, this user becomes a candidate.

Based on the obtained candidates, the user who contributes with the strongest gain to the sum achievable SE is finally selected and added to $U_s$, with its corresponding selected subarray set $S_{sel,u_{sel}}$ added to $S$. At the same time, its user index $u_{sel}$ and selected subarray set $S_{sel,u_{sel}}$ are removed from $U_n$ and $S_n$ respectively. After that, the number of scheduled users, i.e., $N_s$, increases by one. When $N_s = N_u$ or there is no SE gain when adding a new user, the scheduling algorithm terminates and outputs $U_s$, $S$ and $R$.

Taking into consideration that each user only covers a limited portion of antenna arrays of BS and that some BS subarrays are possibly covered by no user, the proposed statistical CSI-based greedy joint user and subarray scheduling algorithm significantly enhances the energy efficiency by turning off uncovered subarrays, thereby facilitating the practical implementation of extra-large scale massive MIMO systems.

V. NUMERICAL RESULTS

In this section, the tightness of the approximated uplink ergodic achievable SEs under both MRC and LMMSE receivers is firstly investigated. Then, we verify the effectiveness of the proposed phase coefficient design in Section III.C. The performance of these proposed two statistical CSI-based user scheduling algorithms, i.e., the statistical CSI-based greedy user scheduling algorithm and the statistical CSI-based greedy joint user and subarray scheduling algorithm, are also evaluated.
A. Tightness of the Approximated Uplink Ergodic Achievable SE

We first verify the diagonal-dominant property of the matrix $Z$ with the LMMSE receiver. Fig. 2 provides the amplitudes of all the elements in $Z$ when $K = 20$ and all the users are randomly located along the extra-large antenna array with each user’s VR covering 160 antenna elements, i.e., $E = 160$, where $E$ denotes the number of antenna elements each user’s VR covers. Table I summarizes the values of the main parameters used in the numerical simulations for each figure. As can be seen from Fig. 2, the diagonal elements of $Z$ are apparently larger than the off-diagonal ones, which verifies the conclusion we drew in Section III.B.

Next, we investigate the tightness of the approximated uplink achievable SEs. Fig. 3 presents the uplink sum achievable SEs under the architectures of the phase shifter-based subarray and the on-off switch-based subarray. Users are randomly located along the extra-large antenna array without user scheduling. Note that in the phase shifter-based subarray, if a subarray is not covered by any user’s VR, the phase coefficients of the subarray would be set to

![Fig. 2](image)

Fig. 2. The amplitudes of all the elements in $Z$ when $K = 20$ and all the users are randomly located along the extra-large antenna array with each user’s VR covering 160 antenna elements, i.e., $E = 160$.

For simplicity, we assume that each user’s VR covers the same number of antenna elements. However, the simulation methodology also supports the general case, i.e., different $E$ for different users.
zero by default; while if a subarray is covered by multiple users simultaneously, then the phase coefficients of the subarray would be set to the sum of the phase corresponding to the multiple users.

![Graph showing uplink sum achievable SEs under the architectures of the phase shifter-based subarray and the on-off switch-based subarray.](image)

From Fig. 3, the proposed achievable SE approximations match well with the Monte-Carlo results, which indicates that the proposed achievable SE approximations in (12) and (26) are inherently useful for the subsequent user scheduling to maximize the system sum achievable SE. In addition, as the power of transmitted signal increases, the system sum achievable SEs continuously increase with the LMMSE receiver since it can effectively eliminate the interference between different users. However, for the MRC receiver, the system sum achievable SEs rapidly tend to saturation due to the persistent inter-user interference.

Furthermore, comparing these results in Fig. 3, we find that, although the on-off switch-based subarray has an apparent performance loss in comparison to the structure of subarray with phase shifters, it can still achieve nearly 70% spectral efficiency performance with the MRC receiver and 80% with the LMMSE receiver. For example, with the MRC receiver, the saturated sum achievable SE under the phase shifter-based subarray architecture is 60 bits/s/Hz, while the saturated sum achievable SE with the on-off switch-based subarray is about 40 bits/s/Hz; for LMMSE receiver at the transmit SNR of 36 dB, the system sum...
achievable SEs are 142.7 bits/s/Hz and 156.3 bits/s/Hz under the on-off switch-based subarray and phase-shifter-based subarray, respectively. Moreover, the on-off switch-based subarray architecture can effectively reduce the hardware cost of the system by replacing the expensive phase shifters with low-cost switches. Hence, it would be a more practical hardware solution to apply the on-off switch-based subarray architecture in extra-large scale massive MIMO.

Fig. 4 presents the sum achievable SE under the special scenario of no overlapped VR, namely users are far apart from each other and the signal radiated by a different user covers different portions of the antenna array. The on-off switch-based subarray architecture is considered. As can be observed in Fig. 4, the proposed achievable SE approximations (12) and (26) yield again great tightness. Moreover, since users’ VRs do not overlap and thus no interference exists between users, the system sum achievable SE under the MRC receiver continuously increases with an increasing transmit power. Consequently, the MRC receiver achieves the same spectral efficiency as the LMMSE receiver. Hence, when there are fewer users to be serviced or when the scheduled users have no overlapped VR, the hardware-friendly MRC receiver should be considered, thereby achieving lower computational complexity with satisfactory performance.
Fig. 5. Comparison of the different subarray phase coefficient design under the scenario of no VR overlapping: \( M = 1024, N = 128, K = 5 \), and \( E = 100 \).

B. Comparison of Different Subarray Phase Coefficient Design

The sum achievable SEs under the special scenario of no overlapped VR for both the phase shifter-based subarray architecture and the on-off switch-based subarray architecture are provided in Fig. 5. LMMSE receiver is employed and two phase coefficient designs i.e., the proposed eigenvector-based phase coefficient design in Section III.C and the random phase coefficient design, are considered for the phase shifter-based subarray. The results in Fig. 5 indicate that the proposed eigenvector-based phase coefficient design achieves the best performance and reaps about 9 bits/s/Hz and 14 bits/s/Hz sum achievable SE gains over the random phase coefficient design and the on-off switch-based subarray design, respectively. What is more, due to the lack of phase alignment, the on-off switch-based subarray has the lowest system sum achievable SE. However, the on-off switch-based subarray induces the lowest hardware cost and computation complexity, therefore offering a low-cost alternative. Additionally, even if we adopt the on-off switch-based subarray architecture, 48 bits/s/Hz system sum achievable SEs can still be achieved at the SNR of 20 dB, which means that the averaged achievable SEs per user are 9.6 bits/s/Hz.
Fig. 6. The uplink sum achievable SEs under the architecture of the phase shifter-based subarray and the on-off switch-based subarray respectively. The parameters $M = 1024$, $N = 128$, $K = 22$, and $E = 128$ are set and users are randomly located along the extra-large antenna array. The statistical CSI-based greedy user scheduling algorithm is utilized and the number of users to be scheduled and served is $N_u = 12$.

C. User Scheduling

Two user scheduling algorithms, i.e., the statistical CSI-based greedy user scheduling algorithm and the statistical CSI-based greedy joint user and subarray algorithm, were proposed in Section IV. We firstly investigate the performance of the statistical CSI-based greedy user scheduling algorithm. Fig. 6 presents the system sum achievable SEs for the MRC and the LMMSE receivers under the statistical CSI-based greedy user scheduling algorithm. Users are randomly distributed along the extra-large antenna array as shown in Fig. 7 and the number of users to be served is $N_u = 12$. The proposed eigenvector-based phase coefficient design is leveraged in the phase shifter-based subarray architecture.

As can be observed from Fig. 6, regardless of the type of linear receivers (i.e., MRC receiver or LMMSE receiver), the phase shifter-based subarray provides an apparent performance improvement compared to the on-off switch-based subarray; especially with the MRC receiver, nearly 30 bits/s/Hz sum achievable SE gains are achieved. At low SNR, the LMMSE receiver does not apparently outperform the MRC receiver. However, as the transmit SNR increases, inter-user interference becomes stronger owing to the large number of scheduled users and the overlapped VRs. Therefore, the LMMSE receiver begins to exhibit its superiority. Note
that nearly 90 bits/s/Hz sum achievable SE gains can be acquired by the LMMSE receiver at high SNR.

It is also important to mention that, with the increasing transmit SNR and, thus, stronger inter-user interference, the number of users finally scheduled to be served under the MRC receiver does not always reach the target number of scheduled users, i.e., $N_u$. For example, only 7 users are scheduled when using the MRC receiver at the transmit SNR of 36 dB under the architecture of the phase shifter-based subarray. The index vector of the finally scheduled users is $[1, 6, 7, 8, 12, 20, 21]$ and Fig. 8 plots their positions and corresponding VRs’ coverings. Nevertheless, the LMMSE receiver shows its superiority in supporting more users to be served. The scheduled 12 users at the transmit SNR of 36 dB when using the LMMSE receiver is presented in Fig. 9 with their index vector being $[1, 4, 6, 8, 9, 10, 12, 13, 17, 18, 20, 22]$. Additionally, based on Figs. 8 and 9, it has also been verified that, to maximize the system sum achievable SE, for the MRC receiver, users whose VRs cover different subarrays or those with fewer VR overlaps should be scheduled, while for the LMMSE receiver, users with larger $\text{tr}(B\Theta_i)$ should be scheduled as many as possible.

Next, we exploit the performance of the statistical CSI-based greedy joint user and subarray scheduling algorithm in Fig. 10. The maximum and minimum number of subarrays that each user can be allocated to in the joint user and subarray scheduling algorithm are $Sub_{max} =$
Fig. 8. The scheduled 7 users when using the MRC receiver at the transmit SNR of 36 dB under the architecture of the phase shifter-based subarray. The index vector of the scheduled users is \([1, 6, 7, 8, 12, 20, 21]\).

8 and \(Sub_{\text{min}} = 6\) respectively and we set \(N_u = 12\). Hence, the number of subarrays (namely RF chains) for each user in the joint user and subarray scheduling algorithm is much less than that in the greedy user scheduling algorithm. Nevertheless, compared with Fig. 6, Fig. 10 indicates that the performance of these two linear receivers in the on-off switch-based subarray is only slightly deteriorated, and the performance loss with the phase shifter-based subarray is not obvious and even can be neglected. Based on these results, we find that, in extra-large scale massive MIMO systems, it is not necessary to simultaneously turn on all subarrays and RF chains to serve the users. The introduction of dynamic subarray scheduling is beneficial to achieve better system performance with lower system energy consumption. Besides, the statistical CSI-based greedy joint user and subarray scheduling algorithm collaborating with the on-off switch-based subarray architecture and the LMMSE receiver is a promising practical solution for extra-large scale massive MIMO.

VI. CONCLUSION

This paper has investigated the uplink transmission of multi-user extra-large scale massive MIMO systems. In order to perform this task, a subarray-based system architecture was firstly proposed. Then, we derived tight closed-form uplink achievable SE approximations for the extra-large scale massive MIMO system under linear receivers. Based on these
approximations, users with their VRs covering different subarrays or VRs with less overlap should be scheduled simultaneously under MRC receiver, while as many users as possible with larger $\text{tr}(\mathbf{B}\Theta)$ should be selected under LMMSE receiver. The design of the subarray with the objective of maximizing the system sum achievable SE has also been investigated. Our results indicate that for the subarray with phase shifters, the optimum phase coefficient design is the phases of the eigenvectors corresponding to the maximum eigenvalues of the main block matrices of $\Theta$. Afterwards, we proposed two statistical CSI-based greedy user scheduling algorithms. Numerical results manifest that in the extra-large scale massive MIMO system, it is not necessary to simultaneously turn on all subarrays and RF chains to serve the users. There is a tradeoff between the hardware cost and the system performance. Specifically, the statistical CSI-based greedy joint user and subarray scheduling algorithm collaborating with the on-off switch-based subarray architecture and the LMMSE receiver is a promising practical solution for extra-large scale massive MIMO systems.
Fig. 10. The uplink sum achievable SEs under the architecture of the phase shifter-based subarray and the on-off switch-based subarray respectively. The parameters $M = 1024$, $N = 128$, $K = 22$, and $E = 128$ are set and users are randomly located along the extra-large antenna array. The statistical CSI-based greedy joint user and subarray scheduling algorithm is utilized. The maximum and minimum number of subarrays that each user can be allocated are $Sub_{\text{max}} = 8$ and $Sub_{\text{min}} = 6$ respectively and the number of users to be served is $N_u = 12$.

APPENDIX A

PROOF OF THEOREM 2

When VRs of different user partially overlap, we have

$$\mathbf{F}^H \mathbf{F} = \mathbf{H}^H \mathbf{B} \mathbf{H}$$

$$= \begin{pmatrix}
g_1^H \Theta_1^{1/2} \mathbf{B} \Theta_1^{1/2} g_1 & \cdots & g_1^H \Theta_1^{1/2} \mathbf{B} \Theta_K^{1/2} g_K \\
\vdots & \ddots & \vdots \\
g_K^H \Theta_K^{1/2} \mathbf{B} \Theta_1^{1/2} g_1 & \cdots & g_K^H \Theta_K^{1/2} \mathbf{B} \Theta_K^{1/2} g_K
\end{pmatrix}. \quad (36)$$

Since $\mathbb{E}\{g_i g_k^H\} = 0, \forall i \neq k$, we obtain

$$\mathbb{E}\{g_i^H \Theta_i^{1/2} \mathbf{B} \Theta_k^{1/2} g_k\} = 0, \forall i \neq k,$$ hence,

$$\mathbb{E}\{\mathbf{I}_K + p_u \mathbf{F}^H \mathbf{F}\} = \text{diag}(1 + p_u g_1^H \Theta_1^{1/2} \mathbf{B} \Theta_1^{1/2} g_1, \ldots, 1 + p_u g_K^H \Theta_K^{1/2} \mathbf{B} \Theta_K^{1/2} g_K). \quad (37)$$

Furthermore, since VRs of different users only partially overlap, thus we can safely draw a conclusion that $(\mathbf{I}_K + p_u \mathbf{F}^H \mathbf{F})$ is a diagonal-dominant matrix. This diagonal-dominant property has been verified in the numerical results in Section V. Additionally, from (31) and
where we leverage the inequality of arithmetic and geometric means in (a) and the Jensen’s equality in (b). Define $Z \triangleq I_K + p_u F^H F$ and $\Lambda = \text{diag}(1/z_{11}, 1/z_{22}, \ldots, 1/z_{KK})$, then $Z$ is a diagonal-dominant matrix. Therefore, according to the Neumann Series [17], for a diagonal-dominant matrix $Z$, its inverse can be expressed as

$$Z^{-1} \approx \sum_{n=0}^{L} (I_K - \Lambda Z)^n \Lambda,$$

where $L$ represents the number of terms used in the Neumann Series. For simplicity, we set $L = 1$ and thus

$$Z^{-1} \approx 2\Lambda - \Lambda Z\Lambda.$$

Applying (40) into (38), we obtain

$$R_{L_{\text{MMSE}}}^{\text{LMMSE}} \geq -K \log_2 \left( \frac{1}{K} \mathbb{E}_h \left\{ \text{tr}(Z^{-1}) \right\} \right)$$

$$\approx -K \log_2 \left( \frac{1}{K} \mathbb{E}_h \left\{ \text{tr}(2\Lambda - \Lambda Z\Lambda) \right\} \right)$$

$$= -K \log_2 \left( \frac{1}{K} \text{tr} \left( \mathbb{E}_h \{ \Lambda \} \right) \right).$$

Moreover, $z_{ii} = 1 + p_u g_i^H \Theta_i^{1/2} B \Theta_i^{1/2} g_i$ and

$$\text{tr} \left( \mathbb{E}_h \{ \Lambda \} \right) \geq \frac{1}{\mathbb{E}_g \{ z_{ii} \}}$$

$$= \frac{1}{\sum_{i=1}^{K} 1 + p_u \text{tr}(B \Theta_i^i)}.$$
Substituting (42) into (41), we have

\[
R_{LMMSE} \approx -K \log_2 \left( 1 + \frac{1}{\sum_{i=1}^{K} \left[ 1 + p_u \text{tr}(B\Theta_i) \right]} \right)
\]

\[\leq \sum_{i=1}^{K} \log_2 \left[ 1 + p_u \text{tr}(B\Theta_i) \right], \quad (43)\]

where (a) utilizes the inequality of arithmetic and geometric means. Hence, the approximated ergodic system sum achievable SE under partially overlapped VR scenario can be expressed as

\[
R_{LMMSE,\text{PartialApp}} = \sum_{i=1}^{K} \log_2 \left[ 1 + p_u \text{tr}(B\Theta_i) \right], \quad (44)
\]

with each user contributing

\[
R_{k,\text{LMMSE,PartialApp}} = \log_2 \left[ 1 + p_u \text{tr}(B\Theta_k) \right]. \quad (45)
\]

The proof is concluded.

**Special Scenario:** When VRs of different users do not overlap, we have \(\Theta_1 \odot \Theta_2 \cdots \odot \Theta_K = 0\), thus

\[
F^H F = H^H B H
\]

\[= \text{diag}(g_1^H \Theta_1^{1/2} B \Theta_1^{1/2} g_1, \ldots, g_K^H \Theta_K^{1/2} B \Theta_K^{1/2} g_K), \quad (46)\]

and

\[
\mathbb{E}_h \left\{ \left[ (I_K + p_u F^H F)^{-1} \right]_{kk} \right\} = \mathbb{E}_h \{ (1 + p_u g_k^H \Theta_k^{1/2} B \Theta_k^{1/2} g_k)^{-1} \}
\]

\[\geq (\mathbb{E}_h \{ 1 + p_u g_k^H \Theta_k^{1/2} B \Theta_k^{1/2} g_k \})^{-1}
\]

\[= [1 + p_u \text{tr}(B\Theta_k)]^{-1}, \quad (47)\]

where (a) applies Jensen’s equality \(\mathbb{E}\{1/x\} \geq 1/\mathbb{E}\{x\}\) for \(x > 0\) and (b) comes from \(\mathbb{E}_h \{ p_u g_k^H \Theta_k^{1/2} B \Theta_k^{1/2} g_k \} = p_u \text{tr}(B\Theta_k)\). From (20), we have

\[
R_{k,\text{LMMSE}} = \mathbb{E}_h \left\{ \log_2 \left( \frac{1}{[ (I_K + p_u F^H F)^{-1} ]_{kk} } \right) \right\}
\]

\[\geq \log_2 \left( \frac{1}{\mathbb{E}_h \{ [ (I_K + p_u F^H F)^{-1} ]_{kk} \} } \right), \quad (48)\]

where Jensen’s equality \(\mathbb{E}\{\log_2(1/x)\} \geq \log_2(1/\mathbb{E}\{x\})\) for \(x > 0\) is applied in (a). Combining (48) with (47), the approximated ergodic achievable SE of the \(k\)th user under no overlapped
VR scenario can be given by

\[ P_{k}^{\text{LMMSE,NoApp}} = \log_2 \left[ 1 + p_u \text{tr} \left( \mathbf{B} \Theta_k \right) \right], \tag{49} \]

which is consistent with (45) as expected.

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