Cosmological Constant in a Regge State-sum Model of Quantum Gravity

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Abstract
We study the quantum contributions to the classical cosmological constant in a Regge state-sum model of quantum gravity in the effective action approach. We use a special path-integral measure and we include matter, in the form of a massive scalar field. The effective cosmological constant (CC) is given as a sum of 3 terms: the classical CC, the quantum gravity CC and the matter CC. Since observations can only measure the sum of these 3 terms, we can choose the classical CC to be equal to the negative value of the matter CC. Hence the effective CC is given by the quantum gravity CC, which is determined by the path-integral measure only. Since the path-integral measure depends on a free parameter, this parameter can be chosen such that the effective CC gives the observed value.

1. Introduction
The cosmological constant problem, for a review see [1], is the problem of explaining the presently observed value of the cosmological constant (CC) within a quantum theory of matter and gravitation. In any quantum gravity (QG) theory there should be a natural length scale, which is the Planck length $l_P \approx 10^{-35} \text{m}$. Consequently, the quantum correction to the classical
value of CC should be of order $l_p^{-2}$. However, this natural theoretical value is $10^{122}$ times larger from the observed value \cite{7}, and the problem is to explain this huge discrepancy. It is expected that an explanation should be provided by a well-defined QG theory. String theory has an explanation based on the landscape of string vacuua \cite{2}, but many people find this explanation unsatisfactory because it is a multiverse argument. Other QG theories, like loop quantum gravity \cite{3}, spin foams \cite{4} and casual dynamical triangulations \cite{5} have not been able to provide one.

Recently a generalization of spin-foam (SF) models of QG was proposed, under the name of spin-cube (SC) models \cite{6, 7}. The SC models were proposed in order to solve the two key problems of SF models: obtaining the correct classical limit and enabling the coupling of fermionic matter. This is achieved by introducing the edge lengths for a given triangulation of spacetime as independent variables and a constraint which relates the spins for the triangles with the corresponding triangle areas. A spin cube model is equivalent to a Regge state-sum model (RSS), and it has general relativity (GR) as its classical limit \cite{7}. A systematic study of the semiclassical approximation for RSS models was started in \cite{8}, by using the effective action approach. It was also shown in \cite{8} that an appropriate choice of the simplex weights, or equivalently by choosing the path-integral (PI) measure, one can obtain a naturally small CC, of the same order of magnitude as the observed value. However, the calculation in \cite{8} did not take into account the contribution from the matter sector, and as it is well known, the perturbative matter contributions to CC are huge compared to the observed value, see \cite{1}.

In this paper we are going to study the matter contributions to CC in a RSS model with a CC term, by using the effective action approach and the PI measure from \cite{8}. First we study the case of pure GR with a non-zero CC term and then we study the case with matter. We will show that the effective CC is a sum of 3 terms: the classical CC, the quantum gravity CC and the matter CC. Since observations can only measure the sum of these 3 terms, we can choose the classical CC to be equal to the negative value of the matter CC. Hence the effective CC will be given by the QG CC, which is determined by the PI measure. Since the PI measure depends on a free parameter, this parameter can be chosen such that the effective CC gives the observed value. Although our mechanism is independent of the value of the matter contributions to CC, we study in section 4 the form of the higher-loop matter contributions.
2. Effective action for gravity with a cosmological constant

We are going to study the effective action for a discrete QG theory based on the Regge discretization of GR with a CC term. Let $T(M)$ be a simplicial complex associated with a triangulation of a 4-manifold $M = \Sigma \times I$, where $\Sigma$ is a compact 3-manifold and $I$ is a compact interval. We restrict the topology of $M$ because we will consider only the semiclassical regime of QG. Let $L_\epsilon$, $\epsilon = 1, 2, ..., E$, be the edge lengths of $T(M)$, where $L_\epsilon$ satisfy the triangle inequalities\(^1\). The path integral of this theory, also known as the state sum, is given by the following integral

$$Z = \int_{D_E} \mu(L) dE L \exp \left( iS_{Re}(L)/l_p^2 \right), \quad (1)$$

where $D_E$ is a subset of $\mathbb{R}^E_+$ where the triangle inequalities hold and

$$S_{Re} = - \sum_{\Delta=1}^{F} A_{\Delta}(L) \theta_{\Delta}(L) + \Lambda_c V_4(L), \quad (2)$$

is the Regge action for the Einstein-Hilbert action with the CC term, see [9]. $A_{\Delta}$ is the area of a triangle $\Delta$, $\theta_{\Delta}$ is the deficit angle and $V_4$ is the 4-volume of $T(M)$. We will also introduce a classical CC length scale $L_c$ such that

$$\Lambda_c = \pm \frac{1}{2L_c^2}. \quad (3)$$

We will choose the PI measure $\mu(L)$ as

$$\mu(L) = \exp \left( -V_4(L)/L_0^4 \right), \quad (4)$$

where $L_0$ is a new length scale. This type of measure insures the finiteness of $Z$ and generates a small quantum correction to the classical CC when $\Lambda_c = 0$ and $L_0 \gg l_p$, see [8]. This is a unique local measure which allows a perturbative effective action for large $L_\epsilon$ and is manifestly diffeomorphism invariant in the smooth limit ($E \to \infty$).

\(^1\)In the usual Regge calculus one considers triangulations of manifolds with Euclidean-signature metrics. We will consider the Lorentzian signature case, so that the triangle inequalities apply only to space-like triangles. Therefore we will use only the triangulations where all the triangles are spacelike.
The quantum effective action $\Gamma(L)$ associated to the theory defined by the path integral (1) is determined by the following integro-differential equation

$$e^{i\Gamma(L)/l_P^2} = \int_{D_E(L)} d^{E}l \mu(L + l) \exp \left( iS_{Re}(L + l)/l_P^2 - i \sum_{\epsilon = 1}^{E} \frac{\partial \Gamma}{\partial L_{\epsilon}} l_{\epsilon}/l_P^2 \right),$$

where $D_E(L)$ is a subset of $\mathbb{R}^E$ obtained by translating the region $D_E$ by the vector $-L$ [8].

When $L \to (\infty)^E$, then $D_E(L) \to \mathbb{R}^E$, and we can assume that the perturbative solution of (5) will be very-well approximated by the perturbative solution of the equation

$$e^{i\Gamma(L)/l_P^2} = \int_{\mathbb{R}^E} d^{E}l \exp \left( i\bar{S}_{Re}(L + l)/l_P^2 - i \sum_{\epsilon = 1}^{E} \frac{\partial \Gamma}{\partial L_{\epsilon}} l_{\epsilon}/l_P^2 \right),$$

where

$$\bar{S}_{Re}(L) = S_{Re}(L) + il_P^2 V_4(L)/L_0^4.$$  

This assumption is based on the results of [8], where it was shown that this is true for the exponentially damped PI measures.

The perturbative solution of (6) can be written as

$$\Gamma = \bar{S} + l_P^2 \bar{\Gamma}_1 + l_P^4 \bar{\Gamma}_2 + \cdots,$$

where $\bar{\Gamma}_n$ will be given by the EAD constructed for the action $\bar{S}_{Re}$, see [8]. Since

$$\bar{\Gamma}_n = \Gamma_{n,0} + l_P^2 \bar{\Gamma}_{n,1} + l_P^4 \bar{\Gamma}_{n,2} + \cdots,$$

we obtain

$$\Gamma = S_{Re} + l_P^2(-i \log \mu + \Gamma_{1,1}) + l_P^4(\Gamma_{2,0} + \bar{\Gamma}_{1,1}) + l_P^6(\Gamma_{3,0} + \bar{\Gamma}_{1,2} + \bar{\Gamma}_{2,1}) + \cdots.$$  

Hence

$$\Gamma_n(L) = D_n(L) + R_n(L),$$

where $D_n$ is the contribution from the n-loop EA diagrams for the action $S_{Re}$, while

$$R_n = Res_n \sum_{k=1}^{n-1} \bar{D}_k,$$
where
\[ Res_n f(l_P^2) = \lim_{l_P^2 \to 0} \frac{f^{(n)}(l_P^2)}{n!}. \] (13)
The \( \bar{D}_k \) terms are defined as
\[ \bar{D}_n(L) = D_n(L, \bar{L}_c^2), \] (14)
where
\[ \bar{L}_c^2 = L_c^2 \left(1 + \frac{i l_P^2 L_c^2 / L_0^4}{1 + i l_P^2 / L_0^2} \right)^{-1}. \] (15)

In order for the measure contributions to be perturbative, we see from (15) that we need \( l_P^2 / L_{0c} < 1 \), which is equivalent to
\[ L_{0} > \sqrt{l_P L_c}. \] (16)
We will study the case \( L_\epsilon > L_c \), since the perturbative analysis is simpler than in the \( L_\epsilon < L_c \) case. The large-\( L \) asymptotics of \( \bar{\Gamma}_n(L) \) functions can be determined from
\[ S_n(L) = O(L^4^n) / L_c^2, \] (17)
and the formula for the EA diagrams, see (23). Consequently, for \( n > 1 \)
\[ D_n(L) = O \left( \left( \frac{L_c^2}{L_\epsilon^4} \right)^{n-1} \right), \] (18)
where the \( O \) notation is defined as
\[ f(L) = O(L^a) \Leftrightarrow f(\lambda L) \approx \lambda^a g(L) \] (19)
when \( \lambda \to \infty \). Since
\[ \bar{\Gamma}_n(L) = D_n(L, \bar{L}_c^2), \] (20)
we obtain
\[ \bar{\Gamma}_n(L) = O \left( \left( \frac{\bar{L}_c^2}{L_\epsilon^4} \right)^{n-1} \right). \] (21)
The asymptotics (18) can be derived by considering the one-dimensional \((E = 1)\) toy model
\[ S_{Rc} = \left( L_c^2 + \frac{L_\epsilon^4}{L_c^2} \right) \theta(L), \] (22)
where \( \theta(L) \) is a homogenious \( C^\infty \) function of degree zero. Consequently
\[ D_n(L) = \sum_{l \in \mathbb{N}} c_{nl} (G(L))^{k_l} S_{n_1}(L) \cdots S_{n_l}(L), \] (23)
where \( G = 1/S_{Re}'', S_n = S_{Re}^{(n)}/n! \), \( k_l \) is the number of edges of an \( n \)-loop EA graph with \( l \) vertices and \( c_{nl} \) are numerical factors.

The asymptotics (18) implies that there are no \( O(L^4) \) terms in \( D_n(L) \), and hence \( D_n(L) \) cannot contribute to the effective CC. This also happens for the \( R_n \) terms, which can be seen from the toy model, where

\[
\tilde{S}_{Re}'' = \theta_1(L)[1 + (L^2/\bar{L}_c^2) \theta_2(L)],
\]

and \( \theta_k \) are homogenous functions of degree zero. Consequently

\[
\log \tilde{S}_{Re}'' = \log(L^2/\bar{L}_c^2) + \log \theta_1(L) + \log [1 + O(\bar{L}_c^2/L^2)],
\]

while from (21) it follows that

\[
R_n(L) = O((L_{0c}^2)^{-n+1}) + O(L^{-2}(L_{0c}^2)^{-n+2}) + O(L^{-4}(L_{0c}^2)^{-n+3}) + \cdots.
\]

We then obtain

\[
\Gamma_1 = O(L^4/L_0^4) + \log O(L^2/L_c^2) + \log \theta_1(L) + O(L_c^2/L^2),
\]

and

\[
\Gamma_n = D_n + R_n = O((L_c^2/L_0^2)^{n-1}) + L_{0c}^{2-2n} O(L_c^2/L^2) = L_{0c}^{2-2n} O(L_c^2/L^2).
\]

Note that we have discarded the constant pieces in \( \Gamma_n(L) \).

Hence there are no \( O(L^4) \) terms in \( \Gamma_n \) for \( n > 1 \) and therefore the effective cosmological constant will be determined by the \( \log \mu \) term, so that

\[
\Lambda_g = \Lambda_c + \Lambda_\mu = \pm \frac{1}{2L_c^2} \pm \frac{L_c^2}{2L_0^4}.
\]

The formula (29) follows from the physical effective action, which is defined as

\[
S_{eff} = (Re \Gamma \pm Im \Gamma)/G_N.
\]

We have used in (30) the QG Wick rotation

\[
\Gamma \rightarrow Re \Gamma \pm Im \Gamma,
\]

in order to make the effective action a real function, see [10, 7]. The sign ambiguity in (30) will be fixed by requiring that \( \Lambda_\mu \) is positive, see the next section.
Note that the condition (16) and $L_\epsilon > L_c$ insure that the effective action is semiclassical, i.e. the quantum corrections to the classical action will be small for $L_0 \gg \sqrt{l_P L_c}$. In this case

$$|S_{Re}|/l_P^2 \gg |\Gamma_1| = |\log \mu - \frac{1}{2} Tr \log S''_{Re}|,$$

and

$$|\Gamma_n| \gg l_P^2 |\Gamma_{n+1}|,$$

for all $n$.

Also note that the effective action will remain semiclassical if $L_\epsilon$ is large and $L_\epsilon < L_c$, but in this case we need $L_\epsilon \gg l_P$ in addition to the condition (16). This can be seen from the asymptotics of $\bar{\Gamma}_n(L)$ terms when $L_\epsilon < L_c$, since

$$\log S''(L) = \log \theta_1(L) + \log \left[1 + O(L^2/\bar{L}_c^2)\right]$$

and

$$\bar{\Gamma}_{n+1}(L) = O(1/L^{2n}) \left[1 + O(L^2/\bar{L}_c^2)\right].$$

In the case when $L_\epsilon > L_c$, the condition $L_\epsilon \gg l_P$ can be satisfied if

$$L_\epsilon \geq L_K \gg l_P,$$

where $L_K > L_c$. The minimal triangulation length $L_K$ will serve as a QFT cutoff in the smooth-manifold approximation.

3. Effective action for gravity with a scalar field

In order to see what is the effect of matter on the value of CC we will consider a scalar field $\phi$ on a 4-manifold $M$ with a metric $g$ such that the scalar-field action is given by

$$S_s(g, \phi) = \frac{1}{2} \int_M d^4x \sqrt{|g|} [g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - U(\phi)],$$

where $U(\phi)$ is a polynomial of the degree greater or equal than 2.

When the metric $g$ is non-dynamical, the EOM of (37) are invariant under the constant shifts of the potential $U$. However, we know that the metric is dynamical, so that the constant shifts in $U$ will give contributions to the
cosmological constant term. These classical shifts of the potential will affect the value of $\Lambda_c$, so that we will assume that $\Lambda_c \neq 0$.

On $T(M)$ the action (37) becomes

$$S_{Rs} = \frac{1}{2} \sum_{\sigma} V_\sigma(L) \sum_{k,l} g^{kl}_\sigma(L) \phi_k' \phi_l' - \frac{1}{2} \sum_{\pi} V^*_\pi(L) U(\phi_\pi) ,$$

where $g^{kl}_\sigma$ is the inverse matrix of the metric in a 4-simplex $\sigma$

$$g^{(\sigma)}_{kl} = \frac{L^2_{0k} + L^2_{0l} - L^2_{kl}}{L^2_{0k} + L^2_{0l}} ,$$

$\phi_k' = (\phi_{\pi k} - \phi_{\pi 0})/L_{0k}$ and $V^*_\pi$ is the volume of the dual cell for a vertex point $\pi$ of $T(M)$, see [9].

The quantum corrections due to gravity and matter fluctuations can be described by the effective action based on the classical action

$$S(L, \phi) = \frac{1}{G_N} S_{Re}(L) + S_{Rs}(L, \phi) .$$

Since

$$S(L, \phi)/\hbar = S_{Re}(L)/l^2_P + G_N S_{Rs}(L, \phi)/l^2_P = S_{Rm}(L, \phi)/l^2_P$$

the EA equation becomes

$$e^{i\Gamma(L, \phi)/l^2_P} = \int_{D_E(L)} dE \prod_{\pi} \chi_\pi \exp \left[ i \tilde{S}_{Rm}(L + l, \phi + \chi)/l^2_P \right]$$

$$- i \sum_\epsilon \frac{\partial \Gamma}{\partial L_\epsilon} l_\epsilon/l^2_P - i \sum_\pi \frac{\partial \Gamma}{\partial \phi_\pi} \chi_\pi/l^2_P .$$

where $\tilde{S}_{Rm} = S_{Re} + G_N S_{Rs}(L, \phi)$.

Since we are using an exponentially damped PI measure for the $L$ variables, we can use the approximation $D_E(L) \approx \mathbb{R}^E$ when $L_\epsilon \to \infty$, see [8].

We can then solve (42) perturbatively in $l^2_P$ by using the EA diagrams for the action $\tilde{S}_{Rm}$.

It is convenient to introduce a dimensionless field $\sqrt{G_N} \phi$, so that $\sqrt{G_N} \phi \to \phi$ and $S_{Rm} = S_{Re} + S_{Rs}$. The perturbative solution will be given by

$$\Gamma(L, \phi) = S_{Rm}(L, \phi) + l^2_P \Gamma_1(L, \phi) + l^4_P \Gamma_2(L, \phi) + \cdots ,$$
where $\Gamma_n$ are given by the EA diagrams corrected by the measure contributions, see section 2. It is not difficult to see that

$$\Gamma(L, \phi) = \Gamma_g(L) + \Gamma_m(L, \phi),$$

and that for constant $\phi$ configurations

$$\Gamma_m(L, \phi) = V_4(L) U_{eff}(\phi).$$

We expect that the expansion (43) will be semiclassical for $L \gg l_P$ and $\phi \ll 1$. This can be verified by studying the one-dimensional ($E = 1$) toy model for the potential

$$U(\phi) = \frac{\omega^2}{2} \phi^2 + \frac{\lambda}{4!} \phi^4,$$

where $\hbar \omega = m$ is the matter field mass and $\lambda$ is the matter self-interaction coupling constant. The toy-model classical action can be taken to be

$$S_{Rm}(L, \phi) = \left( L^2 + \frac{L^4}{L_c^2} \right) \theta(L) + L^2 \left[ \phi^2 + \frac{L^2}{L_m^2} (\phi^2 + a\phi^4) \right] \theta(L),$$

where $L_m = 1/\omega$, $\lambda/4! = a/L_m^2$ and the PI measure $\mu = \exp(-L^4/L_0^4)$.

The first-order quantum correction to the classical action (40) is determined by

$$\Gamma_1 = i \frac{V_4}{L_0^4} + \frac{i}{2} Tr \log \left( \begin{array}{cc} S_{LL} & S_{L\phi} \\ S_{L\phi} & S_{\phi\phi} \end{array} \right),$$

where $S_{xy}$ are the submatrices of the Hessian matrix for $S_{Rm}$. Since

$$S_{LL} = O(L^2), \quad S_{L\phi} = O(L^3)O(\phi), \quad S_{\phi\phi} = O(L^4)[1 + O(\phi^2)],$$

for $L$ large, then

$$\Gamma_1 = i \frac{V_4(L)}{L_0^4} + \frac{i}{2} Tr \log S_{LL} + \frac{i}{2} Tr \log S_{\phi\phi} + O(\phi^2).$$

The first term in (50) is the QG correction to the classical CC, while the matter sector will give a quantum correction to CC from the third term. This can be seen by considering the smooth manifold approximation, i.e. when $E \gg 1$. In this case the third term in (50) can be calculated by using the continuum approximation

$$S_{Rs}(L, \phi) \approx S_s(g, \phi).$$
and the corresponding QFT in curved spacetime.

Let us consider an edge-length configuration which satisfies (36). The condition (36) ensures that the QG corrections are small and if $L_K \ll L_m$, we can calculate $Tr \log S_{\phi \phi}$ by using the Feynman diagrams for $S_s$ with the UV momentum cutoff $\hbar/L_K = \hbar K$. Consequently the corresponding CC contribution will be given by the flat space vacuum energy density, since

$$Tr \log S_{\phi \phi} \big|_{\phi=0} \approx V_M \int_0^K k^3 dk \log(k^2 + \omega^2) + \Omega_m(R, K) \equiv \delta \Gamma_1(L), \quad (52)$$

and

$$\Omega_m(R, K) = a_1 K^2 \int_M d^4x \sqrt{|g|} R$$
$$\quad + \log(K/\omega) \int_M d^4x \sqrt{|g|} [a_2 R^2 + a_3 R^\mu_\nu R_{\mu\nu} + a_4 R^\mu_\nu\rho\sigma R_{\mu\nu\rho\sigma} + a_5 \nabla^2 R]$$
$$\quad + O(L_K^4/L^2), \quad (53)$$

where $a_k$ are constants, see [11]. Therefore the only $O(L^4)$ term in $\delta \Gamma_1$ is

$$c_1 V_M K^4 \log(K/\omega) = c_1 \frac{V_M}{L_K^4} \log(L_m/L_K), \quad (54)$$

where $c_1$ is a numerical constant.

The physical effective action is given by the formula (30), so that the one-loop CC is given by

$$\Lambda_1 = \pm \frac{1}{2L_c^2} + \Lambda_\mu + c_1 \frac{l_P^2}{2L_K^4} \log(K/\omega), \quad (55)$$

where $c_1$ is a numerical constant of $O(1)$. We can write this as

$$\Lambda_1 = \Lambda_\mu + \Lambda_c + \Lambda_m, \quad (56)$$

and it is not difficult to see that the higher-loop matter contributions to CC will preserve this structure, due to (44) and (45). Consequently

$$\Lambda = \Lambda_\mu + \Lambda_c + \Lambda_m, \quad (57)$$

where

$$\Lambda_m = \frac{l_P^2}{L_K^4} f(\lambda, K^2/\omega^2), \quad (58)$$
\[ \bar{\lambda} = \lambda l_P^2 \] and \( f(x, y) \) is a \( C^\infty \) function, see the next section. We can then choose the free parameter \( L_c \) such that

\[ \Lambda_c + \Lambda_m = 0, \quad (59) \]

so that

\[ \Lambda = \Lambda_\mu = \frac{l_P^2}{2L_0}. \quad (60) \]

Note that \( \Lambda_\mu > 0 \) if we choose the + sign in (30).

By taking \( L_0 \approx 10^{-5} m \) we obtain the observed value of CC, which is

\[ l_P^2 \Lambda_\mu \approx 10^{-122}. \quad (61) \]

Note that \( L_0 \gg l_P \), which is consistent with the condition \( L_0 \gg \sqrt{l_P L_c} \) for the validity of the semiclassical approximation. This is important because CC can be observed only in the semiclassical regime of a QG theory.

4. Higher-loop matter contributions to CC

The structure of the perturbative solution is such that at higher-loops, the only contributions to CC are from the matter sector and are given by the sum of \( n \)-loop 1PI QFT FD with no external legs with a momentum cut-off \( hK \). This is because the \( \phi \)-independent terms in the EA are determined by the non-zero diagrams when the \( \phi \to 0 \) limit of the propagator and the vertex functions is used to evaluate the EAD. This leaves only the matter vacuum-energy diagrams.

Let us consider

\[ U(\phi) = \epsilon \frac{\omega^2}{2} \phi^2 + \frac{\lambda}{4!} \phi^4, \quad (62) \]

where \( \epsilon = 1 \) for the usual matter, while \( \epsilon = -1 \) for a Higgs field and \( \lambda > 0 \).

In the \( \epsilon = 1 \) case we have that

\[ \delta_n \Lambda_m = \langle \text{chain} \rangle_n + \langle \text{flower} \rangle_n + \langle \text{mellon} \rangle_n + \langle \text{polygon in circle} \rangle_n + \cdots, \quad (63) \]

where the mellon graphs appear for \( n \geq 3 \), the polygon and the flower graphs appear for \( n \geq 4 \) and so on.

We would like to determine the large-\( K \) behaviour of these graphs. This asymptotics is generically given by \( O(K^D) \), where \( D \) is the degree of the
superficial divergence of the graph. However, there are exemptions, and we will show that this happens in the case of flower graphs.

The 2-loop matter contribution to $CC$ is given by the chain graph

\[ \frac{d_2 \Lambda_m}{\Lambda} = c_2 \lambda l_P^4 \left( \int_0^K \frac{k^3 dk}{k^2 + \omega^2} \right)^2 \approx c_2 \lambda l_P^4 \frac{K^4}{L_L^4 L_K^4}. \] (64)

since $K \gg \omega$. This agrees with $D = 4$ for the 2-loop chain graph.

At 3 loops we have the chain graph contribution

\[ \frac{d_3 \Lambda_m}{\Lambda} = c_3 \lambda^2 l_P^6 \left( \int_0^K \frac{k^3 dk}{k^2 + \omega^2} \right)^2 \int_0^K \frac{q^3 dq}{(q^2 + \omega^2)^2} \approx c_3 \lambda^2 l_P^6 K^4 \ln(K^2/\omega^2). \] (65)

This graph has $D = 4$ and the asymptotics (65) is consistent with this value of $D$.

For the 3-loop mellon graph we obtain

\[ \frac{d_3^M \Lambda_m}{\Lambda} = m_3 \lambda^2 l_P^6 \int_0^K \frac{k^3 dk}{k^2 + \omega^2} \int_0^K \frac{q^3 dq}{q^2 + \omega^2} \int_{r \leq K} \frac{d^4\vec{r}}{(\vec{r} - \vec{r} - \vec{q})^2 + \omega^2} \approx m_3 \lambda^2 l_P^6 K^4 \ln(K^2/\omega^2), \] (66)

which again agrees with the corresponding $D$.

At 4 loops the flower graph appears, and it gives

\[ \frac{d_4^F \Lambda_m}{\Lambda} = f_4 \lambda^3 l_P^6 \left( \int_0^K \frac{k^3 dk}{k^2 + \omega^2} \right)^3 \int_0^K \frac{q^3 dq}{(q^2 + \omega^2)^6}. \] (67)

This integral has $D = 4$, but its asymptotics is given by $D = 6$. The reason is that the second integral is not asymptotic to $K^{-2}$ but it is asymptotic to a non-zero constant, so that

\[ \frac{d_4^F \Lambda_m}{\Lambda} \approx f_4 l_P^2 K^4 \lambda^3 (K/\omega)^2. \] (68)

An $n \geq 3$ chain graph gives

\[ \frac{d_n^C \Lambda}{\Lambda} = c_n \lambda^{n-1} l_P^{2n} \left( \int_0^K \frac{k^3 dk}{k^2 + \omega^2} \right)^2 \left( \int_0^K \frac{k^3 dk}{(k^2 + \omega^2)^2} \right)^{n-2} \approx c_n \lambda^{n-1} l_P^{2n} K^4 \left( \ln(K^2/\omega^2) \right)^{n-2}, \] (69)
while an \( n \geq 4 \) mellon graph gives

\[
\delta^F n \Lambda_{\phi} = p_n \lambda^{n-1} l_P^{2n} \int_0^K \frac{k^3 dk}{k^2 + \omega^2} \int_0^K \frac{q^3 dq}{q^2 + \omega^2} \left( \int_{r \leq K} \frac{d^4 \vec{r}}{(r^2 + \omega^2)((\vec{r} - \vec{k} - \vec{q})^2 + \omega^2)} \right) \left( \ln(K^2/\omega) \right)^{n-2} \approx p_n \lambda^{n-1} l_P^{2n} K^4 \left( \ln(K^2/\omega) \right)^{n-2}.
\]

A flower graph gives for \( n \geq 4 \)

\[
\delta^F n \Lambda_m \approx f_n l_P^2 K^4 \bar{\lambda}^{n-1} (K^2/\omega)^{n-3}.
\]

As far as the other 1PI vacum graphs are concerned, their \( D < 4 \), and consequently the main contribution for large \( K \) is given by

\[
\Lambda_m \approx l_P^2 K^4 \left[ c_1 \ln(K^2/\omega^2) + \sum_{n \geq 2} c_n \bar{\lambda}^{n-1} \left( \ln(K^2/\omega^2) \right)^{n-2} + \sum_{n \geq 4} d_n \bar{\lambda}^{n-1} (K^2/\omega^2)^{n-3} \right],
\]

where \( \bar{\lambda} = \lambda l_P^2 \) is dimensionless. Since \( K \gg \omega \), we get

\[
\Lambda_m \approx l_P^2 K^4 \sum_{n \geq 4} d_n \bar{\lambda}^{n-1} (K^2/\omega^2)^{n-3},
\]

so that the flower graphs have a dominant contribution.

This expansion will be perturbative if

\[
\bar{\lambda}K^2/\omega^2 < 1.
\]

Since \( \bar{\lambda} = 1/8 \) and from

\[
K \gg \omega,
\]

we get \( K/\omega = 10^k \) where \( k \geq 2 \). Hence \( 10^{2k-1} < 1 \), which is not possible for \( k \geq 2 \). Therefore for a given \( K \) we have to calculate \( \Lambda_m \) for a large number of loops in order to obtain an accurate value.

Hence (73) is a perturbative approximation of an exact non-perturbative value for \( \Lambda_m \), valid for \( L_K \gg l_P \). We can write \( \Lambda_m \) as

\[
\Lambda_m = l_P^2 K^4 f(\bar{\lambda}, K^2/\omega^2).
\]

Then whatever is the value of \( \Lambda_m \) we can choose \( L_c \) such that the equation (59) is satisfied.
In the case of the Higgs field, $\epsilon = -1$, and the perturbation expansion is made around the minimum $\phi_0$ of the potential $U(\phi)$, so that

$$U(\phi) = U(\phi_0) + 2\omega^2\phi^2 + 2\omega\sqrt{2}\lambda\phi^3 + \lambda\phi^4.$$  \hfill (77)

Due to the $\phi^3$ term in $U(\phi)$, the set of diagrams which contribute to CC will be larger, but again $D \leq 4$ for all graphs except the flower graphs.

5. Conclusions

We have shown that a discrete QG theory based on the Regge formulation of GR with a CC term can solve the problem of CC. In this case the QG contributions to CC can be calculated explicitly, and they are given by a simple expression (29). The matter contributions to CC are given by the EA loop diagrams for the matter QFT with a physical momentum cut-off $\bar{h}/L_K$, where $L_K \gg l_P$. This contribution cannot be calculated explicitly, but it will have a definite value $\Lambda_m$. Due to (57), we can choose $\Lambda_c$ such that $\Lambda_c = -\Lambda_m$ so that $\Lambda = \Lambda_c = l^2_P/2L_0^4$, where $L_0$ is a free parameter entering the QG PI measure. By choosing $L_0 \approx 10^{-5} m$ we obtain the presently observed value of the CC. This value of $L_0$ is natural for our approach, because it satisfies $L_0 \gg l_P$, and hence $L_0$ belongs to the region of validity of the semiclassical approximation.

Note that in the standard approach to the CC problem, see [1], it is assumed that the CC value is given by $\Lambda_m$. This is a bad assumption from the point of view of quantum gravity, since it is unnatural to assume that there is no a QG contribution to CC. In addition, $\Lambda_c$ is set to zero and consequently one encounters the problem of how to arrange the cancellation of the matter contributions to 122 decimal places, since the agreement with the observed CC value requires that $l^2_P\Lambda_m \approx 10^{-122}$ while the quantum matter corrections to CC are of $O(1)$, since the natural cut-off in the corresponding QFT is $L_K = l_P$. In our approach, we also use a QFT, but our QFT is an effective QFT, see [12], since it is an approximation for a discrete fundamental theory. We have shown that the QG semiclassical approximation is valid for $L_K \gg l_P$ and therefore $l^2_P\Lambda_m = O(l^4_P/L^4_K) \ll 1$. However, our $\Lambda_m$ is still much bigger than the observed value, since $L_K < 10^{-20} m$. This is because $L_K$ is a scale where the QG corrections are still small and the usual perturbative QFT is still valid, and from the LHC experiments we know that QFT is valid at the length scales of the order of $10^{-20} m$. This bound gives $l^2_P\Lambda_m \approx 10^{-52}$
which is still much greater than the observed CC. However, when we take into account the QG contributions to CC and a non-zero classical CC, this problem is solved by cancelling the matter contribution by appropriately choosing the value of the classical CC.

In the smooth-manifold approximation, the QG effective action $\Gamma_g(L)$ has the following semiclassical expansion

$$L_g = R + \Lambda + \frac{l_p^2}{l_p^4} \left[ K^2 R + a(L_c, L_m, L_K, L_0) R^2 + b(L_K, L_m) \nabla^2 R \right] + \frac{l_p^4}{l_p^4} \left[ c(L_K, L_c, L_m, L_0) R^3 + \cdots \right] + O(l_p^0), \quad (78)$$

where

$$\Gamma_g(L) \approx \int_M \sqrt{|\det g(x)|} \mathcal{L}_g(g(x)) \, d^4x.$$ 

Here $g(x)$ is a smooth metric on $M$, $R$ is the Reimann tensor and $L_K > L_c$. The dots in (78) indicate $\nabla^2 R^2$ and $\nabla^4 R$ terms and

$$aR^2 = a_1 R^2 + a_2 R_{\mu\nu} R^{\mu\nu} + \cdots.$$ 

The same notation applies to $b\nabla^2 R$ and $cR^3$ terms. The functions $a(L_k), b(L_k), \ldots$ are dimensionless, i.e. they are homogenous functions of degree zero.

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References

[1] J. Martin, *Comptes Rendus Physique* 13 (2012) 566-665, arXiv:1205.3365.
[2] S. Kachru, R. Kalosh, A. Linde and S. Trivedi, *Phys. Rev.* D68 (2003) 046005.
[3] K. Giesel and H. Sahlmann, *PoS QGQGS2011* (2011) 002, arXiv:1207.0359.
[4] A. Perez, *Living Rev. Rel.* 16 (2013) 3.
[5] J. Ambjorn, A. Gorlich, J. Jurkiewicz and R. Loll, *Int. J. Mod. Phys.* D22 (2013) 1330019.
[6] A. Miković and M. Vojinović, *Class. Quant. Grav.* 29 (2012) 165003.
[7] A. Miković, Rev. Math. Phys. 25 (2013) 10, 1343008.

[8] A. Miković, Effective action for Regge state-sum models of quantum gravity, arXiv:1402.4672

[9] H. W. Hamber, Gen. Rel. Grav. 41 (2009) 817-876.

[10] A. Miković and M. Vojinović, Class. Quant. Grav. 28 (2011) 225004.

[11] N.D. Birell and P.C.W. Davies, Quantum Fields in Curved Space, Cambridge University Press, 1982.

[12] S. Weinberg, PoS CD 09 (2009) 001, arXiv:0908.1964.