In this paper, we discuss the current status of core collapse supernova models and the future developments needed to achieve significant advances in understanding the supernova mechanism and supernova phenomenology, i.e., in developing a supernova standard model.

1. Introduction

Beginning with the first numerical simulations conducted by Colgate and White[1], three decades of supernova modeling have established a basic supernova paradigm. The supernova shock wave—formed when the iron core of a massive star collapses gravitationally and rebounds as the core matter exceeds nuclear densities and becomes incompressible—stalls in the iron core as a result of enervating losses to nuclear dissociation and neutrinos. The failure of this “prompt” supernova mechanism sets the stage for a “delayed” mechanism, whereby the shock is reenergized by the intense neutrino flux emerging from the neutrinospheres carrying off the binding energy of the proto-neutron star[2,3]. This heating is mediated primarily by the absorption of electron neutrinos and antineutrinos on the dissociation-liberated nucleons behind the shock. This past decade has also seen the emergence of multidimensional supernova models, which have investigated the role convection, rotation, and magnetic fields may play in the explosion[4–10], in some cases invoking new explosion paradigms.

Although a plausible framework is now in place, fundamental questions about the explosion mechanism remain: Is there more than one mechanism? For example, are some core collapse supernovae neutrino driven while others driven by magnetohydrodynamic jets? Is there one mechanism for one class of massive stars—e.g., stars with progenitor masses between 10 and 20 M⊙—and another for another class of massive stars—e.g., more massive progenitors? Or is the mechanism tailored to each progenitor? For neutrino-driven supernovae, is the neutrino heating sufficient, or are multidimensional effects such as convection and rotation necessary? In addition, any “standard model” for core collapse supernovae must reproduce observed supernova phenomenology, such as neutron star kicks[11] and the polarization of supernova emitted light[12]. To answer these questions and to develop a “standard model,” simulations in one, two, and three dimensions must be performed and coordinated.

2. One-Dimensional Supernova Models

The neutrino energy deposition behind the shock depends sensitively not only on the neutrino luminosities but also on the neutrino spectra and angular distributions in the postshock region, necessitating exact multigroup Boltzmann neutrino transport near and above the neutrinospheres or a very good approximation of it. [Multigroup, i.e., multi-neutrino energy, transport is necessary because the neutrino opacities are strongly energy dependent and low- and high-energy neutrinos may be transported in very different ways (e.g., diffusion versus free streaming) at any given spatial point in the core at any given time.] Ten percent variations in any of these quantities can make the difference between explosion and failure in supernova models[6,13]. Past simulations have implemented increasingly sophisticated approximations to multigroup Boltzmann transport, the most sophisticated of which is multigroup flux-limited diffusion[14,15]. A generic feature of this approximation is that it underestimates the isotropy of the neutrino...
angular distributions in the heating region and, thus, the heating rate\cite{16,17}. Therefore, the question arises whether or not failures to produce explosions in past one-dimensional models were the result of the transport approximations employed. It is important to note that, without invoking proto-neutron star (e.g., neutron finger) convection, simulations that implement multigroup flux-limited diffusion do not produce explosions\cite{14,15} (as we will discuss, the existence and vigor of proto-neutron star convection is a matter of debate\cite{7,18,19}).

To begin to address the question posed above, we have been simulating the core collapse, bounce, and postbounce evolution of 13, 15, and 20 M\odot stars, beginning with the precollapse models of Nomoto and Hashimoto\cite{20}, with a new neutrino radiation hydrodynamics code for both Newtonian and general relativistic spherically symmetric flows: AGILE–BOLTZTRAN. BOLTZTRAN is a three-flavor Boltzmann neutrino transport solver\cite{21,22}, now extended to fully general relativistic flows\cite{23}. In the 13 M\odot Newtonian (gravity) simulation we present here, it is employed in the $O(v/c)$ limit. AGILE is a conservative, adaptive mesh, general relativistic hydrodynamics code\cite{23,24}. Its adaptivity enables us to resolve and seamlessly follow the shock through the iron core into the outer stellar layers.

The equation of state of Lattimer and Swesty\cite{25} (LS EOS) is employed to calculate the local thermodynamic state and nuclear composition of the matter in nuclear statistical equilibrium (NSE). For matter initially in the silicon layer, the temperatures are insufficient to achieve NSE. In this region, the radiation and electron components of the LS EOS are used, while an ideal gas of $^{28}$Si is assumed for the nuclear component. For typical hydrodynamic timesteps ($\sim 1$ millisecond), silicon burning occurs within a single timestep for $T \sim 5$ GK\cite{26}; therefore, when a fluid element exceeds a temperature of 5 GK in our simulation, the silicon is instantaneously burned, achieving NSE and releasing thermal energy equal to the difference in nuclear binding energy between $^{28}$Si and the composition determined by the LS EOS.

Figure 1, taken from the simulation of Mezzacappa et al.\cite{27}, shows the radius-versus-time trajectories of equal mass (0.01M\odot) shells in the stellar iron core and silicon layer in a Newtonian simulation initiated from the 13 M\odot progenitor. Core bounce and the formation and propagation of the initial bounce shock are evident. This shock becomes an accretion shock, decelerating the core material passing through it. At $\approx 125$ ms after bounce, the accretion shock stalls at a radius $\approx 250$ km and begins to recede, continuing to do so during the first 500 ms of postbounce evolution. No explosion has developed in this model during this time. Similar behavior is exhibited in our 15 and 20 M\odot Newtonian models and in our 13 and 20 M\odot general relativistic models, although these have not yet reached 500 ms after bounce. We will continue these simulations and report on the final outcomes in subsequent papers. If explosions are consistently obtained in the more realistic general relativistic cases, this would imply that core collapse supernovae driven purely by neutrino heating are possible, although these same models would have to be considered in three dimensions to assess whether or not multidimensional effects alter this conclusion and, of course, to model any associated phenomenology, such as neutron star kicks.

3. Convection

Supernova convection falls into two categories: (1) convection near or below the neutrinospheres, which we refer to as proto-neutron star convection, and (2) convection between the gain radius and the shock, which we refer to as neutrino-driven convection. The gain radius is the radius at which neutrino heating and cooling via electron neutrino and antineutrino absorption and emission between the neutrinospheres and the shock balance. There is net neutrino heating above this radius and net neutrino cooling below it.

Proto-neutron star convection may aid the explosion mechanism by boosting the neutrinosphere luminosities. Hot, lepton-rich rich matter is convectively transported to the neutrinospheres. This mode of convection may develop owing to instabilities caused by lepton and entropy gradients established by the deleptonization of the proto-neutron star via electron neutrino escape near the electron neutrinosphere and by the weakening supernova shock (as the shock weakens, it causes a smaller entropy jump in the material flowing through it). Proto-neutron star convection is arguably the most difficult to investigate numerically because the neutrinos and the matter are coupled and, consequently, multidimensional simulations must include both multidimensional hydrodynamics and multidimensional, multigroup neutrino transport.

Neutrino-driven convection may aid the explosion mechanism by boosting the shock radius and the neutrino heating efficiency, thereby facilitating shock revival. It develops as the result of the entropy
Figure 1. We trace the shock, nuclear burning, and dissociation fronts (the shock and dissociation fronts are coincident), which carve out three regions in the \((r, t)\) plane. A: Silicon. B: Iron produced by infall compression and heating. C: Free nucleons and alpha particles.

gradient established as the shocked stellar core material infalls between the shock and the gain radius, being continually heated in the process.

3.1. Proto-Neutron Star Convection (1D): Neutron Fingers

The fundamental difficulty in modeling convection in spherically symmetric models is apparent: convection is a three-dimensional phenomenon, and spherically symmetric models can incorporate convection only in a phenomenological way (e.g., via a mixing-length prescription). Moreover, because convection is not admitted by the one-dimensional hydrodynamics equations, some imposed criterion for the existence of convection must be used.

Neutron-finger convection has been invoked by Wilson et al.\cite{15} in their one-dimensional models and has been deemed necessary by them to obtain supernova explosions. This mode of proto-neutron star convection arises in the presence of a negative electron fraction gradient and a positive entropy gradient in the postshock stellar core, resulting in higher-entropy, neutron-richer matter above lower-entropy, neutron-poorer matter in the core. With the assumption that energy transport by neutrinos is more efficient than lepton transport, neutron fingers develop under these conditions, resulting (like salt fingers in the ocean) in finger-like downflows of neutron-rich matter that penetrate deep into the stellar core. The assumption that energy transport is more efficient than lepton transport is justified in the following way: Three flavors of neutrinos (electron, muon, and tau) can transport energy, whereas only one (electron) can transport lepton number. However, detailed neutrino equilibration experiments carried out by Bruenn and Dineva\cite{18} demonstrate that the muon and tau neutrinos do not couple strongly with the stellar core matter in energy, and therefore, there is only one flavor (electron) that transports both energy and lepton number efficiently. Given the outcome of these numerical experiments, the fundamental assumption made by Wilson et al. should be reexamined. However, it is also important to note that the equilibration
experiments carried out by Bruenn and Dineva have to be repeated in light of energy exchange channels between the muon and tau neutrinos and the stellar core matter that have recently been identified (e.g., see the contribution by Thompson and Burrows in this volume[28]).

3.2. Proto-Neutron Star Convection (2D): Ledoux Convection

In certain regions of the stellar core, neutrino transport can equilibrate a convecting fluid element with its surroundings in both entropy and lepton number on time scales shorter than convection time scales, rendering the fluid element nonbuoyant. This will occur in intermediate regimes in which neutrino transport is efficient but in which the neutrinos are still strongly enough coupled to the matter. Figures 2 and 3 from Mezzacappa et al.[7] demonstrate that this equilibration can in fact occur. Figure 2 shows the onset and development of proto-neutron star convection in a 25 M⊙ model shortly after bounce in a simulation that did not include neutrino transport, i.e., that was a hydrodynamics-only run. Figure 3 on the other hand shows the lack of any significant onset and development of convection when neutrino transport was included in what was otherwise an identical model. Transport’s damping effects are obvious. (The same result occurred in our 15 M⊙ model[7].)

![Figure 2](image.png)

Figure 2. Two-dimensional entropy plots showing the evolution of proto-neutron star convection in our hydrodynamics-only 25 M⊙ model at 12, 17, and 27 ms after bounce.

On the other hand, in the model of Keil et al.[19], vigorous proto-neutron star convection developed, which then extended deep into the core as a deleptonization wave moved inward owing to neutrinos diffusing outward. In this model, convection occurs very deep in the core where neutrino opacities are high and transport becomes inefficient in equilibrating a fluid element with its surroundings.

It is also important to note in this context that Mezzacappa et al. and Keil et al. used complementary transport approximations. In the former case, spherically symmetric transport was used, which maximizes lateral neutrino transport and overestimates the neutrino–matter equilibration rate; in the latter case, ray-by-ray transport was used, which minimizes (zeroes) lateral transport and underestimates the neutrino–matter equilibration rate.

These outcomes clearly demonstrate that to determine whether or not proto-neutron star convection (including neutron fingers) exists and, if it exists, is vigorous, will require simulations coupling three-dimensional, multigroup neutrino transport and three-dimensional hydrodynamics. Moreover, realistic high-density neutrino opacities will also be needed.
3.3. Neutrino-Driven Convection (2D)

This mode of convection occurs directly between the gain radius and the stalled shock as a result of the entropy gradient that forms as material infalls between the two while being continually heated from below. In Figure 4, a sequence of two-dimensional plots of entropy are shown, illustrating the development and evolution of neutrino-driven convection in our $15 \, M_\odot$ model\(^\text{[8]}\). High-entropy, rising plumes and lower-entropy, denser, finger-like downflows are seen. The shock is distorted by this convective activity.

In the Herant et al.\(^\text{[4]}\) simulations, large-scale convection developed beneath the shock, leading to increased neutrino energy deposition, the accumulation of mass and energy in the gain region, and a thermodynamic engine they claimed ensured explosion, although Herant et al. stressed the need for more sophisticated multidimensional, multigroup transport in future models.\(^\text{[4]}\) They used two-dimensional “gray” (neutrino-energy–integrated, as opposed to multigroup) flux-limited diffusion in neutrino-thick regions and a neutrino lightbulb approximation in neutrino-thin regions. In a lightbulb approximation, the neutrino luminosities and rms energies are assumed constant with radius.\(^\text{[4]}\) In the Burrows et al. simulations\(^\text{[5]}\), neutrino-driven convection in some models significantly boosted the shock radius and led to explosions. However, they stressed that success or failure in producing explosions was ultimately determined by the values chosen for the neutrino spectral parameters in their gray ray-by-ray (one-dimensional) neutrino diffusion scheme.\(^\text{[5]}\) In spherical symmetry (1D), all rays are the same. In a ray-by-ray scheme in axisymmetry (2D), not all rays are the same, although the transport along each ray is a 1D problem. In this latter case, lateral transport between rays is ignored.\(^\text{[5]}\) Focusing on the neutrino luminosities, Janka and Müller\(^\text{[6]}\) using an adjustable central neutrino lightbulb, conducted a parameter survey and concluded that neutrino-driven convection aids explosion only in a narrow luminosity window (±10%), below which the luminosities are too low to power explosions and above which neutrino-driven convection is not necessary to power explosions. In more recent simulations carried out by Swesty\(^\text{[29]}\) using two-dimensional gray flux-limited diffusion in both neutrino-thick and neutrino-thin regions, it was demonstrated that the simulation outcome varied dramatically as the matter–neutrino “decoupling point,” which in turn sets the neutrino spectra in the heating region, was varied within reasonable limits.\(^\text{[29]}\) The fundamental problem in gray transport schemes is that the neutrino spectra, which are needed for the heating rate, are not computed. The spectra are specified by choosing a neutrino “temperature,” normally chosen to be the matter temperature at decoupling. In a multigroup scheme, the spectra are computed dynamically.) In our two-dimensional models\(^\text{[8]}\), the angle-averaged shock radii do not differ significantly from the shock trajectories in their one-dimensional counterparts, and no explosions are
obtained, as seen in Figure 4. Neither the luminosities nor the neutrino spectra are free parameters. Our two-dimensional simulations implemented precomputed spherically symmetric (1D) multigroup flux-limited diffusion neutrino transport, compromising transport dimensionality to implement multigroup transport and a seamless transition between neutrino-thick and neutrino-thin regions, although without feedback between the hydrodynamics and the transport.

In light of the neutrino transport approximations made, the fact that none of the simulations have been three dimensional, and the mixed outcomes, next-generation simulations will have to reexplore neutrino-driven convection in the context of three-dimensional simulations that implement more realistic three-dimensional multigroup neutrino transport.

4. Rotation and Magnetic Fields

The observed polarization of spectra in a number of core collapse supernovae is a fingerprint of gross asymmetries in the explosions, with axis ratios as large as 2:1, which has lead to the suggestion that these explosions must have been jet-like[12]. If so, what is the origin of these jet-like explosions? Are the jets formed early on as part of the explosion mechanism, or do they form as the supernova shock wave propagates through an aspherical medium? Three classes of models that exhibit jet-like behavior have been investigated: neutrino-driven jet models after core collapse and bounce in a rotating stellar core[9], MHD-driven jet models from the core collapse and bounce of rotating, magnetized cores[30,31,10], and neutrino- or MHD-driven jet models around a newly formed black hole in a failed or successful supernova, respectively, in a rotating stellar core[32]. Two things of note: (1) Having jet-like explosions does not imply the explosions are not neutrino-driven. (2) There may be several explosion paradigms at work in the full range of observed supernovae of all progenitor stars.

In the two-dimensional axisymmetric model of Fryer and Heger[9], the first to include the effects of both convection and rotation, a weak, jet-like explosion along the rotation axis was obtained. Unfortunately, all axisymmetric simulations must contend with reflecting boundary conditions on the rotation axis, which numerically bias on-axis outflow, not allowing the flow to turnover and thereby break axisymmetry. Moreover, in three dimensions, the interaction of convection and rotation would result in “tornadic” flows not admitted by the imposed axisymmetry, which implies that the flow should be qualitatively different in three dimensions. In the two-dimensional models of Leblanc and Wilson[30] and Symbalisty[31], it was concluded that unusually high rotation rates and magnetic field strengths were required to generate explosions, requiring, for example, magnetic field strengths as large as $10^{17}$ Gauss. In the model of
MacFadyen and Woosley [2], although a plausible neutrino energy deposition mechanism was invoked, the energy deposition was parameterized and not modeled in detail. Thus, a full exploration of these possibilities will require three-dimensional (neutrino) radiation hydrodynamics, magnetohydrodynamics, and magneto-radiation hydrodynamics simulations.

5. Outlook

Fundamental questions in supernova theory remain, which can only be answered via systematic and coordinated simulations in one, two, and three dimensions.

We have shown results from the first 500 ms of a one-dimensional (spherically symmetric) Newtonian simulation with Boltzmann neutrino transport initiated from a 13 $M_\odot$ progenitor. In light of our implementation of Boltzmann transport, if we do not obtain explosions in this model, or other models we have initiated from different progenitors (see also Rampp and Janka [3]), it would suggest that improvements in our initial conditions (pre-collapse models) and/or input physics are needed, and/or that the inclusion of multidimensional effects such as convection, rotation, and magnetic fields are required ingredients in the recipe for explosion. In the past, it was not clear whether failure in spherically symmetric models was the result of transport approximations or the lack of inclusion of important physics. With the implementation of Boltzmann transport, this conclusion can be made unambiguously. We will report on the continued evolution of our 13 $M_\odot$ model and on our other models in subsequent papers.

Potential improvements in our initial conditions and input physics include: improvements in precollapse models [34–37]; the use of ensembles of nuclei in the stellar core rather than a single representative nucleus; computing the neutrino–nucleus cross sections with detailed shell model computations [38]; and the inclusion of nucleon correlations in the high-density neutrino opacities [39,40]. These improvements all have the potential to quantitatively, if not qualitatively, change the details of our simulations. Thus, it is important to note that the conclusions drawn here are drawn considering the initial conditions and input physics used.

To accurately investigate multidimensional effects such as convection, rotation, and magnetic fields, future simulations must be carried out in three dimensions and must implement realistic, three-dimensional, multigroup neutrino transport. Three-dimensional simulations will be necessary to assess, for example, the vigor of convection in the proto-neutron star, where the neutrinos and the matter are strongly coupled and the flow is three-dimensional, and to assess the character of neutrino-driven convection behind the shock in a stellar core that is both rotating and convecting. Certainly, three-dimensional simulations are required to study the development of MHD jets in stellar cores, and given that the development of such jets depends in some scenarios on the convection in the proto-neutron star, which in turn will depend on the neutrino transport, all of these studies must be strongly coupled.

Acknowledgments

A.M. is supported at the Oak Ridge National Laboratory, managed by UT-Battelle, LLC, for the U.S. Department of Energy under contract DE-AC05-00OR22725. He would like to thank the organizers, Joergen Christensen-Dalsgaard and Karlheinz Langanke, for a delightful conference, and Raph Hix for valuable comments on the manuscript.

REFERENCES

1. S. A. Colgate and R. H. White, Astrophysical Journal 143, 626 (1966).
2. J. R. Wilson, in Numerical Astrophysics, edited by J. M. Centrella, J. M. LeBlanc, and R. L. Bowers (Jones and Bartlett, Boston, 1985), pp. 422–434.
3. H. H. Bethe and J. R. Wilson, Astrophysical Journal 295, 14 (1985).
4. M. Herant et al., Astrophysical Journal 435, 339 (1994).
5. A. Burrows, J. Hayes, and B. A. Fryxell, Astrophysical Journal 450, 830 (1995).
6. H.-T. Janka and E. Müller, Astronomy and Astrophysics 306, 167 (1996).
7. A. Mezzacappa et al., Astrophysical Journal 493, 848 (1998).
8. A. Mezzacappa et al., Astrophysical Journal 495, 911 (1998).
9. C. L. Fryer and A. Heger, astro-ph/9907433.
10. A. M. Khokhlov et al., Astrophysical Journal 524, L107 (1999).
11. C. L. Fryer, A. Burrows, and W. Benz, Astrophysical Journal 496, 333 (1998).
12. J. C. Wheeler, in Cosmic Explosions, A Conference Proceedings, edited by S. S. Holt (AIP, Melville, 2000).
13. A. Burrows and J. Goshy, Astrophysical Journal Letters 416, L75 (1993).
14. S. W. Bruenn, in First Symposium on Nuclear Physics in the Universe, edited by M. W. Guidry and M. R. Strayer (IOP Publishing, Bristol, 1993), pp. 31–50.
15. J. R. Wilson and R. W. Mayle, Physics Reports 227, 97 (1993).
16. H.-T. Janka, Astronomy and Astrophysics 256, 452 (1992).
17. O. E. B. Messer, A. Mezzacappa, S. W. Bruenn, and M. W. Guidry, Astrophysical Journal 507, 353 (1998).
18. S. W. Bruenn and T. Dineva, Astrophysical Journal Letters 458, L71 (1996).
19. W. Keil, H.-T. Janka, and E. Müller, Astrophysical Journal Letters 473, L111 (1996).
20. K. Nomoto and M. Hashimoto, Physics Reports 163, 13 (1988).
21. A. Mezzacappa and S. W. Bruenn, Astrophysical Journal 405, 669 (1993).
22. A. Mezzacappa and O. E. B. Messer, Journal of Computational and Applied Mathematics 109, 281 (1998).
23. M. Liebendörfer, Ph.D. thesis, University of Basel, Basel, Switzerland, 2000.
24. M. Liebendörfer and F.-K. Thielemann, in Nineteenth Texas Symposium on Relativistic Astrophysics, edited by E. Aubourg, T. Montmerle, J. Paul, and P. Peter (Elsevier Science B. V., Amsterdam, 2000).
25. J. Lattimer and F. D. Swesty, Nuclear Physics A535, 331 (1991).
26. W. R. Hix and F.-K. Thielemann, Astrophysical Journal 511, 862 (1999).
27. A. Mezzacappa et al., Physical Review Letters, submitted (2000).
28. T. A. Thompson and A. Burrows, astro-ph/0009447.
29. F. D. Swesty, in Stellar Explosions, Stellar Evolution, and Galactic Chemical Evolution, edited by A. Mezzacappa (IoP Publishing, Bristol, 1998), pp. 539–548.
30. J. M. LeBlanc and J. R. Wilson, Astrophysical Journal 161, 541 (1970).
31. E. M. D. Symbalisty, Astrophysical Journal 285, 729 (1984).
32. A. I. MacFadyen and S. E. Woosley, Astrophysical Journal 524, 262 (1999).
33. H.-T. Janka and M. Rampp, Astrophysical Journal 539, L33 (2000).
34. G. Bazan and W. D. Arnett, Astrophysical Journal 496, 316 (1998).
35. H. Umeda, K. Nomoto, and T. Nakamura, in The First Stars, edited by A. Weiss (Springer, Berlin, 2000).
36. A. Heger, N. Langer, and S. Woosley, Astrophysical Journal 528, 368 (2000).
37. A. Heger, K.-H. Langanke, G. Martínez-Pinedo, and S. Woosley, astro-ph/0007412.
38. K.-H. Langanke and G. Martínez-Pinedo, Nuclear Physics A673, 481 (2000).
39. A. Burrows and R. F. Sawyer, Physical Review C58, 554 (1998).
40. S. Reddy, M. Prakash, J. M. Lattimer, and J. A. Pons, Physical Review C59, 2888 (1999).