The Higgs Sector and CoGeNT/DAMA-Like Dark Matter in Supersymmetric Models

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Abstract. Recent data from CoGeNT and DAMA are roughly consistent with a very light dark matter particle with \( m \sim 4 - 10 \) GeV and spin-independent cross section of order \( \sigma_{SI} \sim (1 - 3) \times 10^{-4} \) pb. An important question is whether these observations are compatible with supersymmetric models obeying \( \Omega h^2 \sim 0.11 \) without violating existing collider constraints and precision measurements. In this talk, I review the fact that the Minimal Supersymmetric Model allows insufficient flexibility to achieve such compatibility, basically because of the highly constrained nature of the MSSM Higgs sector in relation to LEP limits on Higgs bosons. I then outline the manner in which the more flexible Higgs sectors of the Next-to-Minimal Supersymmetric Model and an Extended Next-to-Minimal Supersymmetric Model allow large \( \sigma_{SI} \) and \( \Omega h^2 \sim 0.11 \) at low LSP mass without violating LEP, Tevatron, BaBar and other experimental limits. The relationship of the required Higgs sectors to the NMSSM “ideal-Higgs” scenarios is discussed.

1. Introduction
CoGeNT [1] and DAMA [2] both have hints of dark matter detection corresponding to a very low mass particle with very large spin-independent cross section. In [3], it is claimed that a consistent explanation for both hints is provided if \( \sigma_{SI} \sim (1.4 - 3.5) \times 10^{-4} \) pb, for \( m_{DM} = (9 - 6) \) GeV. Of course, the required \( \sigma_{SI} \) is dependent on the assumed local relic density and for example is reduced by \( \sim 60\% \) if \( \rho = 0.485 \) GeV/cm\(^3\) [4] is employed rather than the usual \( \rho = 0.3 \) GeV/cm\(^3\).

One would hope that CoGeNT/DAMA-like \( \sigma_{SI} \) at low \( m_{DM} \) could be consistent with simple supersymmetric models. However, the MSSM fails. For low \( m_{\tilde{\chi}^0_1} \), the MSSM generically predicts a value for \( \Omega_{\tilde{\chi}^0_1} h^2 \) that is far above the observed value; it is only for extreme choices that one can achieve \( \Omega_{\tilde{\chi}^0_1} h^2 \sim 0.11 \) [5]. For these same choices it turns out that \( \sigma_{SI} \) takes on its maximum possible value of \( \sim 1.7 \times 10^{-4} \) pb. This maximum value of \( \sigma_{SI} \) can be understood as follows. The cross section for \( \tilde{\chi}^0_1 \)-nucleon scattering is dominated by CP-even Higgs exchange and is given approximately by

\[
\sigma_{SI} \approx 0.17 \times 10^{-4} \text{ pb} \left( \frac{N_{43}^2}{0.1} \right) \left( \frac{\tan \beta}{50} \right)^2 \left( \frac{100 \text{ GeV}}{m_{H^0}} \right)^4 \cos^4 \alpha , \tag{1}
\]

where we have written \( \tilde{\chi}^0_1 = N_{11} \tilde{B} + N_{12} \tilde{W}^3 + N_{13} \tilde{H}_d + N_{14} \tilde{H}_u \) and referenced the most optimistic values of \( N_{43}^2 \), \( \tan \beta \) and \( m_{H^0} \). In the above, \( N_{43}^2 \) cannot be much larger than 0.1 because of limits on the Z invisible width, \( \tan \beta > 50 \) enters a non-perturbative Yukawa coupling domain.
and $m_{H^0} < 100$ GeV is not allowed by LEP limits. Further, to achieve even this most maximal value of $\sigma_{SI}$ in the MSSM, one must ignore the Tevatron limit, $B(B_s \to \mu^+\mu^-) \lesssim 5.8 \times 10^{-8}$. Once imposed, the largest $\sigma_{SI}$ for scenarios with $m_{\chi^0_1}$ in the CoGeNT/DAMA region and with $\Omega_{\chi_1^0}h^2 \sim 0.1$ is $\sigma_{SI} \sim 0.017 \times 10^{-4}$ pb, a factor of roughly 100 below the $\sigma_{SI}$ needed to explain the CoGeNT/DAMA events.

Thus, it is natural to turn to the even more attractive NMSSM model. The NMSSM is defined by adding a single SM-singlet superfield $\tilde{S}$ to the MSSM and imposing a $Z_3$ symmetry on the superpotential, implying

$$W = \lambda \tilde{S}H_u\tilde{H}_d + \frac{\kappa}{3} \tilde{S}^3 \quad (2)$$

The reason for imposing the $Z_3$ symmetry is that then only dimensionless couplings $\lambda, \kappa$ enter. All dimensionful parameters will then be determined by the soft-SUSY-breaking parameters. In particular, the $\mu$ problem is solved via $\mu_{\text{eff}} = \lambda(S)$ for which $\mu_{\text{eff}}$ is automatically of order a TeV (as required) since $\langle S \rangle$ is of order the SUSY-breaking scale, $m_{\text{SUSY}}$, which will be below a TeV.

The extra singlet field $\tilde{S}$ implies: 5 neutralinos, $\tilde{\chi}^0_{1-5}$ with $\tilde{\chi}^0_1 = N_{11}\tilde{B} + N_{12}\tilde{W}^3 + N_{13}\tilde{H}_d + N_{14}\tilde{H}_u + N_{15}\tilde{S}$ being either singlet or bino, depending on $M_1$; 3 CP-even Higgs bosons, $h_1, h_2, h_3$; and 2 CP-odd Higgs bosons, $a_1, a_2$.

The soft-SUSY-breaking terms corresponding to the terms in $W$ are:

$$\lambda A_\lambda SH_u H_d + \frac{\kappa}{3} A_\kappa S^3 \quad (3)$$

It is important to recall that when $A_\lambda, A_\kappa \to 0$, the NMSSM has an additional $U(1)_R$ symmetry, in which limit the $a_1$ is pure singlet and $m_{a_1} = 0$. If, $A_\lambda, A_\kappa = 0$ at $M_U$, RGE’s give $A_\lambda \sim 100$ GeV and $A_\kappa \sim 1 - 20$ GeV, resulting in $m_{a_1} < 2m_B$ (see later) being quite natural and not fine-tuned [7]. In this situation $a_1$ is still primarily singlet so that $\cos \theta_A$ as defined by $a_1 = \cos \theta_A A_{MSSM} + \sin \theta_A A_S$ is typically quite small. A light singlet-like $a_1$ also arises in the $U(1)_{PQ}$ symmetry limit of $\kappa, \lambda, \kappa A_\kappa = 0$.

As is well known, the NMSSM maintains all the attractive features (especially coupling constant unification and automatic electroweak symmetry breaking from renormalization group evolution of the soft SUSY-breaking stop masses) of the MSSM while avoiding important MSSM problems. In particular, the level of finetuning is greatly reduced in “ideal Higgs” scenarios [8] in which the $h_1$ has SM-like $WW, ZZ$ couplings and $m_{h_1} \lesssim 105$ GeV but escapes LEP limits via $h_1 \to a_1a_1 \to 4\tau (m_{a_1} < 2m_B)$. Further, $m_{h_1} \lesssim 105$ GeV implies excellent precision electroweak consistency and suitably strong baryogenesis. In addition, the long-standing LEP excess in the $Z + b\bar{b}$ final state near $M_{h_1} \sim 100$ GeV is well fit if $m_{h_1}$ is in the vicinity of 100 GeV and $B(h_1 \to b\bar{b}) \sim 0.1 - 0.25$, the latter being automatic when $B(h_1 \to a_1a_1) \sim 0.75 - 0.9$.

However, if $h_1$ is SM-like then the arguments regarding limitations on achieving large $\sigma_{SI}$ given above continue to apply. An alternative yielding much larger maximum $\sigma_{SI}$ [9] is to arrange for the lightest Higgs, $h_1$, to have enhanced coupling to down-type quarks while it is the $h_2$ that couples to $WW, ZZ$ in SM-like fashion. We term this kind of scenario an “inverted Higgs” (IH) scenario. Many large $\sigma_{SI}$ scenarios have $m_{h_1} < 90$ GeV and $m_{h_2} \lesssim 110$ GeV, and are thus still pretty ideal in the sense described in the previous paragraph. We call such scenarios “inverted ideal Higgs” (IIH) scenarios. In the general NMSSM context, it is straightforward [10] to adjust $m_{a_1}$ so as to obtain $\Omega_{\chi_1^0}h^2 \sim 0.1$ (using $\tilde{\chi}_1^0\tilde{\chi}_1^0 \to a_1 \to X$ with $m_{a_1}$ small). Further, in IH and IIH scenarios we found [9] that one can achieve $\sigma_{SI} \sim (0.1 - 0.2) \times 10^{-4}$ pb without

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1 Here, $A_{MSSM}$ is the usual doublet pseudoscalar of the MSSM two-doublet Higgs sector and $A_S$ is the CP-odd component of the complex scalar field residing in the singlet superfield.
violating the $B(B_s \rightarrow \mu^+\mu^-)$ bound, or any other bound. But, to get $\sigma_{SI}$ as large as $1 \times 10^{-4}$ requires violating $(g-2)_\mu$ quite badly, and having some enhancement of the $s$-quark content of the nucleon.

In a second paper [11], we showed that an extended version of the NMSSM would allow $\sigma_{SI}$ as large as needed for the CoGeNT/DAMA events while maintaining consistency with all constraints, including $\Omega_{\chi_1^0}h^2 \sim 0.11$. In particular, we considered the ENMSSM in which we only generalize the superpotential and soft-SUSY-breaking potential, keeping to just one singlet superfield. The extended superpotential is given by

$$v_0^2\hat{s} + \frac{1}{2}\mu S\hat{S}^2 + \mu \hat{H}_u\hat{H}_d + \lambda\hat{S}\hat{H}_u\hat{H}_d + \frac{1}{3}\kappa\hat{S}^3,$$

and the soft Lagrangian is

$$B_\mu\hat{H}_u\hat{H}_d + \frac{1}{2}m_S^2|S|^2 + BS^2 + \lambda A_S\hat{S}\hat{H}_u\hat{H}_d + \frac{1}{3}\kappa A_S\hat{S}^3 + H.c.$$  (5)

Note that the presence of explicit $\mu$ and $B_\mu$ terms. These reduce the appeal of the model somewhat, but there are string-theory-inspired sources for such explicit terms. We found that scenarios in the ENMSSM with the largest $\sigma_{SI}$ are ones in which the $\chi_1^0$ is singlino-like and the $h_1$ is largely singlet (rather than bino-like and mainly $H_d$, respectively, as in the IH NMSSM scenarios). To first approximation, $\Omega_{\chi_1^0}h^2$ is controlled by $\chi_1^0\chi_1^0 \rightarrow h_1 \rightarrow X$ and $\sigma_{SI}$ is determined by $h_1$ exchange between the $\chi_1^0$ and the down-type quarks in the nucleon, especially $s$ and $b$. We term this scenario the singlino-singlet (SS) scenario.

In a very recent paper [12], it is found that the SS type scenario can be realized in the NMSSM provided the $h_1$ has $m_{h_1} \lesssim 1$ GeV. They call their scenario the Dark Light Higgs (DLH) scenario. This scenario requires a considerable degree of finetuning for the couplings, but is consistent with current experimental constraints.

While we await confirmation of the CoGeNT/DAMA excesses, it is very interesting to consider the implications of all the above scenarios for Higgs physics at the LHC. That will be the topic of the remainder of this presentation.

2. The Inverted Higgs Scenarios

The largest elastic scattering cross sections arise in the case of large $\tan\beta$, significant $N_{13}$ (the Higgsino component of the $\chi_1^0$), and relatively light $m_{H_d}$, where $H_d$ is the Higgs with enhanced coupling to down quarks, $C_{H_d^0d^0} \sim \tan\beta$. In this limit, the relevant scattering amplitude is

$$\frac{a_d}{m_d} \approx \frac{-g_2g_1N_{13}N_{11}\tan\beta}{4m_Wm_{H_d}^2},$$

which in turn yields

$$\sigma_{SI} \approx \frac{g^2_2g^2_1N^2_{13}N^2_{11}\tan^2\beta m_{\chi_1^0}^2m_{p,n}^4}{4\pi m_W^2m_{H_d}^4(m_{\chi_1^0}^2 + m_{p,n}^2)^2}\left[f_{p,n}^{(p,n)} + \frac{2}{27}f_{TG}^{(p,n)}\right]^2 \approx 1.7 \times 10^{-5}\text{ pb}\left(\frac{N_{13}^2}{0.10}\right)\left(\frac{\tan\beta}{50}\right)\left(\frac{100\text{ GeV}}{m_{H_d}}\right)^4.$$

Constraints on the light $h_1 \sim H_d$ configuration are significant. We had to update NMHDECAY [13, 14] to include all the latest constraints. We then linked to micrOMEGAs [15] for the computation of $\Omega_{\chi_1^0}h^2$ and $\sigma_{SI}$ as in NMSSMTools [16]. The important constraints are:
(i) Constraints on the neutral Higgs sector from $Zh_2$ at LEP. These are important since $m_{\text{SUSY}}$ should be low in order to minimize $m_{h_2}$ and this keeps $m_{h_2}$ low. In these cases, the $h_2$ can be in the “ideal” zone ($m_{h_2} < 105$ GeV) and escapes LEP detection via $h_2 \to a_1 a_1 \to 4\tau$ decays with $m_{a_1} < 2m_B$ (but close to $2m_B$ in order to avoid BaBar limits on $\Upsilon_{3S} \to \gamma a_1 \to \gamma\tau^+\tau^-$). Of course, we also require compliance with the recent ALEPH limits [17] on the $e^+e^- \to Z4\tau$ channel.

(ii) LEP constraints on $h_1a_1$ and $h_1a_2$. The $h_1a_1$ cross section is $\propto \text{maximal} \times (\cos \theta_A)^2$. Thus, small $\cos \theta_A$ is desirable, which fits with both the approximate $U(1)_R$ limit mentioned earlier and the need to not have overly strong $\chi_1^0 \chi_1^0 \to a_1^* \to X$ annihilations (as required to achieve adequate $\Omega_{\chi_1^0} h^2$) when $m_{a_1}$ is small and, in particular, not far from the $2m_{\chi_1^0} \sim m_{a_1}$ resonance pole.

(iii) Tevatron direct Higgs production limits. There are two especially relevant limits given the need to focus on large $\tan \beta$ in order to achieve large $\sigma_{SI}$. The first is the limit on $b\bar{b}h_1$ with $h_1 \to \tau^+\tau^-$ associated production [18], which scales as $C^2_{h_1b\bar{b}} \sim \tan^2 \beta$, the latter being something we want to maximize. The second limits are those on $t \to h^+b$ with $h^+ \to \tau^+\nu_\tau$ [19] (dominant at large $\tan \beta$). These are critical to include since the $h^+$ tends to be quite light (e.g. $\sim 120 - 140$ GeV) when $h_1$ is $H_u$-like and the $h_2$ is SM-like.

(iv) Limits from $\Upsilon$ decays. These are especially crucial for constraining scenarios with $m_{a_1} < 2m_B$. We have included the latest $\Upsilon_{3S} \to \gamma\mu^+\mu^-$, $\gamma\tau^+\tau^-$ limits [20, 21], whose impacts on ideal Higgs scenarios were explored in [22].

(v) $B$-physics constraints. The most restricting constraint arises from the very strong Tevatron limit of $B(B_s \to \mu^+\mu^-) < 5.8 \times 10^{-8}$. At large $\tan \beta$, achieving a small enough value fixes $A_t$ as a function of $m_{\text{SUSY}}$. Next comes $b \to s\gamma$. The $\mu_{\text{eff}} > 0$ scenarios have roughly $1\sigma$ discrepancy with the $2\sigma$ experimental window. In contrast, the $\mu_{\text{eff}} < 0$ scenarios only rarely have a $b \to s\gamma$ problem. One must also check $B^+ \to \tau^+\nu_\tau$. $\mu_{\text{eff}} > 0$ scenarios pass easily, but $\mu_{\text{eff}} < 0$ scenarios with the largest $\sigma_{SI}$ have $1\sigma - 2\sigma$ deviations from the experimental $2\sigma$ window.

(vi) $(g-2)_\mu$. This is possibly crucial. For $\mu_{\text{eff}} < 0$, the largest $\sigma_{SI}$ values are achieved when $(g-2)_\mu$ is a few sigma outside the $2\sigma$ limits including theoretical uncertainties. If $(g-2)_\mu$ is strictly required to lie within the $2\sigma$ window, then the largest $\sigma_{SI}$ that can be achieved for $\mu_{\text{eff}} < 0$ is about a factor of 50 below that needed for the CoGeNT/DAMA events. For $\mu_{\text{eff}} > 0$, the largest $\sigma_{SI}$ points yield $(g-2)_\mu$ within the $2\sigma$ exp.+theor. window, but after including all other constraints the $\sigma_{SI}$ values for $\mu_{\text{eff}} > 0$ are not as large as those found with $\mu_{\text{eff}} < 0$ (before imposing the $(g-2)_\mu$ constraint).

(vii) $\Omega_{\chi_1^0} h^2$: Of course, we require that any accepted scenario have correct relic density within the experimental limits encoded in NMSSMTools, $0.094 \leq \Omega_{\chi_1^0} h^2 \leq 0.136$.

To illustrate the impact of some of the constraints discussed above, I give three figures. In the first, Fig. 1, representative maximal values of $\sigma_{SI}$ (averaged over protons and neutrons) are plotted after: a) imposing LEP limits; b) requiring $0.094 < \Omega_{\chi_1^0} h^2 < 0.136$; c) imposing BaBar limits from $\Upsilon_{3S}$ decays; and d) requiring $B(B_s \to \mu^+\mu^-) < 5.8 \times 10^{-8}$. These are termed “level I” constraints. Note that $\sigma_{SI} \sim 6 \times 10^{-5}$ pb ($2 \times 10^{-5}$ pb) can be achieved for $\mu_{\text{eff}} < 0$ ($\mu_{\text{eff}} > 0$). In Fig. 2, the masses of the CP-even Higgs bosons are plotted for $\mu_{\text{eff}} > 0$ cases. Note how low all these masses are. These same plots for $\mu_{\text{eff}} < 0$ would be quite similar.

The low masses imply that Tevatron limits on direct Higgs production could be important. In Fig. 3, we plot the points from Fig. 1 that are fully consistent with Tevatron limits on $b\bar{b}Higgs$ with $Higgs \to \tau^+\tau^-$ and on $t \to h^+b$ with $h^+ \to \tau^+\nu_\tau$. Typically, one finds maximal $\sigma_{SI}$ values in the $(1 - 2) \times 10^{-5}$ pb range (a factor at least 5 below the $\sigma_{SI}$ needed for CoGeNT/DAMA). Many of the points of Fig. 1 with larger $\sigma_{SI}$ would survive if we relaxed these direct production
Figure 1. All points obtained after imposing level-I constraints (see text) but without imposing either Tevatron direct Higgs production limits or \((g-2)_\mu\) limits.

Figure 2. \(m_{h_2}\) and \(m_{h^+}\) vs. \(m_{h_1}\) for \(\mu_{\text{eff}} = +200\) GeV points. Only level-I (see text) constraints are imposed. There is a great amount of point overlap in these plots.

limits by 1\(\sigma\) (combined experimental and theoretical error). The points of the \(\mu_{\text{eff}} < 0\) plot in Fig. 3 would all be eliminated if the predicted \((g-2)_\mu\) is required to be within 2\(\sigma\) of the observed value, whereas all the \(\mu_{\text{eff}} > 0\) points are very consistent with the observed \((g-2)_\mu\).

To further illustrate the nature of the Higgs sector for a \(\mu_{\text{eff}} > 0\) scenario, I give details of a sample \(\tan \beta = 40\) inverted-ideal Higgs point in Table 1. Let me make a few remarks. For this somewhat unusual point, the \(h_1\) is fairly singlet, the \(h_2\) and \(a_2\) are largely \(H_d\) and it is the \(h_3\) that is most SM-like. Nonetheless, the effective precision electroweak mass (defined by \(\ln m_{\text{eff}} \equiv \sum_{i=1,2,3} \left[ g_{ZZ h_i}^2 / g_{ZZ h_i}^{\text{SM}} \right] \ln m_{h_i} \)) receives substantial contributions from the low mass Higgses, \(h_1\) and \(h_2\), and lies below 105 GeV and is thus in the range that is ideal for precision electroweak data. Next, note that Higgs decays to \(\tilde{\chi}^0_1\tilde{\chi}^0_1\) are unimportant, but that Higgs to Higgs pair decays are often significant. Prospects for LHC detection of some of the Higgs are very good. In particular, \(b\bar{b}h_2 + b\bar{a}_2\) with \(a_2, h_2 \rightarrow \tau^+\tau^-\) should be readily observable. Even \(a_1\) discovery via \(gg \rightarrow a_1 \rightarrow \mu^+\mu^-\) looks promising [23] because \(C_{a_1b_5} \sim 6\) and \(m_{a_1}\) is not directly under the \(\Upsilon_{3S}\) peak. The SM-like \(h_3\) is possibly the most difficult to detect because: a) its \(\gamma\gamma\) decay mode is suppressed by \(C_V(1) < 1\) and mass \(m_{h_3} \sim 126\) GeV that is above the mass where the branching ratio to \(\gamma\gamma\) is maximal; and, b) its decays to Higgs pairs will reduce all standard
Figure 3. $\sigma_{SI}$ vs. $m_{\chi_1^0}$ for points fully consistent with Tevatron limits on $b\bar{b} + \text{Higgs}$ and $t \rightarrow h^+ b$. Level-I constraints are imposed. $(g - 2)_{\mu}$ is still terrible (perfectly ok) for the surviving $\mu_{\text{eff}} < 0$ ($\mu_{\text{eff}} > 0$) points.

detection modes.

Finally, let me note that in the NMSSM study of [24] cross sections as large as those found here are not achieved; rather, they find $\sigma_{SI}^{\text{Max}} \sim (1 - 1.5) \times 10^{-6}$ pb (without enhancing the $s$-quark content of the nucleon). The smaller $\sigma_{SI}$ is largely because they did not seek scenarios with $h_1 \sim H_d$. Ref. [24] also considers the possibility of enhancing $\sigma_{SI}$ by a factor of $\sim 3$ by enhancing the $s$-quark content of the nucleon. If we adopted this same enhancement, our $\sigma_{SI}$ values would approach the lowered CoGeNT/DAMA $\sigma_{SI}$ range applicable if we also employed the higher local density of $\rho \sim 0.485$ GeV/cm$^2$ mentioned earlier. However, current analyses do not appear to allow for such a large $s$-quark enhancement [25].

3. The Singlino-Singlet Scenarios

In these SS scenarios, the LSP is primarily singlino and the Higgs responsible for large $\sigma_{SI}$ is mainly singlet. In [11], we pursued the extended NMSSM as defined earlier and looked for scenarios of the SS type. What we found was a kind of see-saw balance between $\Omega_{\chi_1^0} h^2$ and $\sigma_{SI}$ such that when $\Omega_{\chi_1^0} h^2 \sim 0.1$ then $\sigma_{SI}$ is very naturally in the CoGeNT/DAMA preferred zone. Below, I provide a few details.

The singlino coupling to down-type quarks is given by:

$$a_d = \frac{g_2 \kappa N_c^2 \tan \beta F_u(h_1) F_d(h_1)}{8 m_W m_{h_1}^2}$$

where $h_1 = F_d(h_1) H_0^0 + F_u(h_1) H_0^0 + F_s(h_1) H_S^0$ and the crucial trilinear coupling that couples a singlino pair to the singlet Higgs $H_S^0$ is proportional to $\kappa$. This leads to

$$\sigma_{SI} \approx 2.2 \times 10^{-4} \text{ pb} \left( \frac{\kappa}{0.6} \right)^2 \left( \frac{\tan \beta}{50} \right)^2 \left( \frac{45 \text{ GeV}}{m_{h_1}} \right)^4 \left( \frac{F_u^2(h_1)}{0.85} \right) \left( \frac{F_d^2(h_1)}{0.15} \right),$$

which is consistent with the value required by CoGeNT and DAMA/LIBRA for the indicated $\kappa$, $m_{h_1}$ and $h_1$ component values. (Of course, one really sums coherently over all the CP-even Higgs bosons.) Furthermore, the large singlet fraction $F_s^2(h_1) \sim 0.85$ of the $h_1$ will allow it evade the constraints from LEP II and the Tevatron.
Table 1. Properties of a particularly attractive but phenomenologically complex NMSSM point with \( \mu_{\text{eff}} = +200 \) GeV, \( \tan \beta = 40 \) and \( m_{\text{SUSY}} = 500 \) GeV. All Tevatron limits ok. \( h_3 \) is the most SM-like. In the last row, the brackets give the range of \( B \)-physics predictions for this point after including theoretical errors as employed in NMHDECAY.

| \( \lambda \) | \( \kappa \) | \( A_\lambda \) | \( A_\kappa \) | \( M_1 \) | \( M_2 \) | \( M_3 \) | \( A_{\text{soft}} \) |
|---|---|---|---|---|---|---|---|
| 0.081 | 0.01605 | −36 GeV | −3.25 GeV | 8 GeV | 200 GeV | 300 GeV | 479 GeV |
| \( m_{h_1} \) | \( m_{h_3} \) | \( m_{\tilde{h}_2} \) | \( m_{\tilde{h}_3} \) | \( m_{\tilde{a}_1} \) | \( m_{\tilde{a}_2} \) |
| 53.8 GeV | 97.3 GeV | 126.2 GeV | 10.5 GeV | 98.9 GeV | 128.4 GeV |
| \( C_{V(h_1)} \) | \( C_{V(h_3)} \) | \( C_{V(h_3)} \) | \( m_{\chi^0_1} \) | \( C_{h_1d\tilde{b}_s} \) | \( C_{h_1b\tilde{d}_s} \) | \( C_{h_3d\tilde{b}_s} \) | \( C_{h_3b\tilde{d}_s} \) |
| −0.505 | 0.137 | 0.852 | 0.24 | 39.7 | −5.1 | 6.7 | 39.4 |
| \( m_{\tilde{\chi}^0_1} \) | \( \tilde{N}_{11} \) | \( \tilde{N}_{13} \) | \( m_{\chi^0_1} \) | \( m_{\tilde{\chi}^0_1} \) | \( \sigma_{SI} \) | \( \sigma_{SD} \) | \( \Omega_{\tilde{\chi}^0_1} h^2 \) |
| 7 GeV | −0.976 | −0.212 | 79.1 GeV | 153 GeV | 0.93 \times 10^{-5} \text{ pb} | 0.45 \times 10^{-5} \text{ pb} | 0.12 |

Table 2. The \( \pm 2\sigma \) experimental ranges for the \( B \) physics observables tabulated in the last row of Table 1.

| \( B(B_s \rightarrow \mu^+\mu^-) \) | \( B(b \rightarrow s\gamma) \) | \( B(h^+ \rightarrow \tau^+\nu\tau) \) | \( (g-2)_\mu \) |
|---|---|---|---|
| \( < 5.8 \times 10^{-8} \) (95\% CL) | \( [3.03 - 4.01] \times 10^{-4} \) | \( [0.34 - 2.3] \times 10^{-4} \) | \( [0.88 - 4.6] \times 10^{-9} \) |

Table 3. LHC Neutral Higgs Discovery Channels (\( b\bar{b}h_2, b\bar{b}a_2 \rightarrow b\bar{b}2\tau \) absent since \( m_{h_2} \sim m_{a_2} < 100 \) GeV, the lower limit of the studies used — this should be a highly viable mode) (also \( t\bar{t} \rightarrow b\bar{b}h^+ \rightarrow \tau^+\nu X = \text{excellent channel at LHC} \)

\begin{align*}
| L = 30 \text{ fb}^{-1} | & | L = 300 \text{ fb}^{-1} |
\begin{array}{cccc}
WW \rightarrow h_3 \rightarrow 2\tau & bbh_3 \rightarrow b\bar{b}2\tau & gg \rightarrow h_3 \rightarrow 4\ell & gg \rightarrow h_3 \rightarrow 2\ell 2\nu
\end{array}
\begin{array}{c}
3.8\sigma
2\sigma
1.4\sigma
1.1\sigma
\end{array}
\begin{array}{c}
WW \rightarrow h_3 \rightarrow 2\tau
\end{array}
\begin{array}{c}
14\sigma
\end{array}
\end{align*}

Meanwhile, the thermal relic density of neutralinos is determined by the annihilation cross section and the \( \chi^0_1 \) mass. In the mass range we are considering here, the dominant annihilation channel is to \( bb \) (or, to a lesser extent, to \( \tau^+\tau^- \)) through the s-channel exchange of the same scalar Higgs, \( h_1 \), as employed for elastic scattering, yielding:

\[
\sigma_{\chi^0_1\chi^0_1} = \frac{N_{e2}^2 m_b^2 m_{h_1} E_s^2(h_1) E_d^2(h_1)}{64\pi m_W^2 \cos^2 \beta} \frac{m_{\chi^0_1}^2 (1 - m_{h_1}^2 / m_{\chi^0_1}^2)^{3/2}}{m_{\chi^0_1}^2 (1 - m_{h_1}^2 / m_{\chi^0_1}^2)^{3/2}} \frac{v^2}{m_{\chi^0_1}^2 (1 - m_{h_1}^2 / m_{\chi^0_1}^2)^{3/2}}.
\]
Table 4. Properties of a typical ENMSSM point with \( \tan \beta = 45 \) and \( m_{\text{SUSY}} = 1000 \text{ GeV} \).

| \( \lambda \) | \( \kappa \) | \( \lambda_b \) | \( A_\lambda \) | \( A_b \) | \( M_1 \) | \( M_2 \) | \( M_3 \) | \( A_{\text{soft}} \) |
|---|---|---|---|---|---|---|---|---|
| 0.011 | 0.596 | -0.026 GeV | 3943 GeV | 17.3 GeV | 150 GeV | 300 GeV | 900 GeV | 679 GeV |

| \( B_S \) | \( \mu_S \) | \( v_S^3 \) | \( \mu \) | \( B_\mu \) | \( \mu_{\text{eff}} \) | \( B_\mu^{\nu \nu} \) |
|---|---|---|---|---|---|---|
| 0 | 7.8 GeV | 4.7 GeV | 164 GeV | 668 GeV | 164 GeV | 178 GeV |
| \( m_{h_1} \) | \( m_{h_2} \) | \( m_{h_3} \) | \( m_{a_1} \) | \( m_{a_2} \) | \( m_{h^+} \) |
| 82 GeV | 118 GeV | 164 GeV | 82 GeV | 164 GeV | 178 GeV |
| \( F^2_S(h_1) \) | \( F^2_S(h_2) \) | \( F^2_S(h_3) \) | \( F^2_S(a_1) \) | \( F^2_S(a_2) \) |
| 0.86 | 0.14 | 0.0 | 0.996 | 0.14 | 0.86 | 0.86 | 0.14 |

For the range of masses and cross sections considered here, we find \( m_{\chi_1^0}/T_{\text{FO}} \approx 20 \), yielding a thermal relic abundance of

\[
\Omega_{\chi_1^0} h^2 \approx 0.11 \left( \frac{0.6}{\kappa} \right)^2 \left( \frac{50 \tan \beta}{45 \text{ GeV}} \right)^2 \left( \frac{m_{h_1}}{7 \text{ GeV}} \right)^2 \left( \frac{0.85}{F^2_S(h_1)} \right) \left( \frac{0.15}{F^2_S(a_2)} \right),
\]

i.e. naturally close to the measured dark matter density, \( \Omega_{\text{CDM}} h^2 = 0.1131 \pm 0.0042 \) for the same choices for \( \kappa, m_{h_1} \), and composition fractions as give CoGeNT/DAMA-like \( \sigma_{\text{SI}} \). The only question is can we achieve the above situation without violating LEP and other constraints. Basically, one wants a certain level of decoupling between the singlet sectors and the MSSM sectors, but not too much. To find out, we performed parameter scans with an extended version of NMHDECAY and micrOMEGAs that includes both the non-NMSSM parameters of Eqs. (4) and (5) as well as the latest B-physics and Tevatron constraints. We find points for \( 15 < \tan \beta < 45 \) that are consistent (within the usual \( \pm 2\sigma \) combined theory plus experimental windows – excursions in \( b \to s \gamma \) and \( b b h, h \to \tau^+ \tau^- \) that fall slightly outside this window are present at high \( \tan \beta \)) with all collider and B-physics constraints having the appropriate thermal relic density and \( \sigma_{\text{SI}} \) as large as few \( \times 10^{-4} \) pb.

The complete framework has contributions to \( \sigma_{\text{SI}} \) and \( \Omega_{\chi_1^0} h^2 \) beyond Eqs. (9) and (10) and high-\( \sigma_{\text{SI}} \) points typically have large contributions from the non-singlet Higgses. I confine myself to discussing one ‘typical’ point that does the job. Its properties are tabulated in Table 4.

Let us note the following regarding this particular point.

(i) What you see is that the \( h_1, a_1 \) have separated off from something that is close to an MSSM-like Higgs sector with \( h_2 \sim h^0 \) being SM-like and \( h_3 \sim H^0, a_2 \sim A^0 \) and \( h^+ \sim H^+ \).

(ii) Detection of the \( h_2 \) would be possible via the usual SM-like detection modes planned for the MSSM \( h^0 \).

(iii) There are some \( h_2, a_2 \to \tilde{\chi}_1^0 \tilde{\chi}_1^0 \) decays, but at such a low branching ratio level that detection of these invisible decay modes would be unlikely, even if very interesting.

(iv) Decays to pairs of Higgs of any of the heavier Higgs bosons are not of importance. Of course, by choosing \( m_{\text{SUSY}} = 1000 \text{ GeV} \) so that \( m_{a_2} > 114 \text{ GeV} \) (beyond the LEP limits), we have not forced the issue. It will be interesting to look for SS scenarios that are ideal-Higgs-like with \( m_{h_2} < 110 \text{ GeV} \).
(v) One sees that $h_1$ and $a_1$ decay primarily to $\tilde{\chi}_1^0\tilde{\chi}_1^0$ but that there also decays to $b\bar{b}$ and $\tau^+\tau^-$ with reduced branching ratios of 0.33 and 0.03 compared to the normal $B(b\bar{b}) \sim 0.85$ and $B(\tau^+\tau^-) \sim 0.12$.

(vi) $h_1$ and $a_1$ do have somewhat enhanced couplings to $b\bar{b}$ (in this example $C_{h_1b\bar{b}}, C_{a_1b\bar{b}} \sim \sqrt{F^2_y(h_1,a_1) \tan \beta \sim 17}$) and so the rates for $gg \rightarrow b\bar{b}h_1 + gg \rightarrow b\bar{b}a_1$ will be quite substantial. However, the reduced $B(h_1,a_1 \rightarrow \tau^+\tau^-) \sim 0.03$ implies that detection of such production in the $b\bar{b} + \tau^+\tau^-$ final state might prove challenging, probably requiring very high $L$ at the LHC.

(vii) Further work is needed to quantify discovery prospects in the $gg \rightarrow b\bar{b} + (h_1,a_1) \rightarrow b\bar{b} + E_T$ channel.

(viii) At this large $\tan \beta$, detection of the $h_3$ and $a_2$ would certainly be possible in $gg \rightarrow b\bar{b}h_3 + b\bar{b}a_2$ in the $h_3, a_2 \rightarrow \tau^+\tau^-$ decay channel.

(ix) For this sample case, the charged Higgs is just too heavy to allow $t \rightarrow h^+b$ decays and so one would have to turn to $gg \rightarrow t\bar{b}h^+ + t\bar{b}h^-$ with detection of the charged Higgs in the $\tau \nu_\tau$ final state. Further investigation is needed to assess the feasibility of such detection, but at least the cross section is very enhanced by virtue of the large $\tan \beta$ value.

A few notes regarding this scenario. First, it is the very large value of $A_3$ and the very small $\lambda$ that keep the singlet and MSSM Higgs sectors fairly separate. Second, the new parameters of the ENMSSM, $\mu$ and $B_\mu$ must be substantial. This is generally the case if you desire an SS scenario with $m_{h_1} > f_{ew}$ GeV.

Table 5. Properties of the SS DLH NMSSM point with $\tan \beta = 13.77$, $m_\tilde{\tau} = 1000$ GeV and $m_T = 200$ GeV.

| $\lambda$ | $\kappa$ | $\lambda s$ | $A_\lambda$ | $A_\kappa$ | $M_1$ | $M_2$ | $M_3$ | $A_{sof}$ |
|-----------|-----------|-------------|-------------|-----------|-------|-------|-------|-----------|
| 0.1205    | 0.00272   | 168 GeV     | 2661 GeV    | -24.03 GeV| 100 GeV | 200 GeV | 660 GeV | 750 GeV   |
| $m_{h_1}$ | $m_{h_2}$ | $m_{h_3}$   | $m_{a_1}$   | $m_{a_2}$ | $m_{h^+}$ |
| 0.811 GeV | 116 GeV   | 244 GeV     | 16.7 GeV    | 244 GeV   | 244 GeV  |
| $F_2^2(h_1)$ | $F_2^2(h_2)$ | $F_2^2(h_3)$ | $F_2^2(h_4)$ | $F_2^2(h_5)$ | $F_2^2(h_6)$ |
| 0.997      | 0.00017   | 0.0036      | 0.99        | 0.0       | 0.994    |
| $C_Y(h_1)$ | $C_Y(h_2)$ | $C_Y(h_3)$  | $C_{h_1b\bar{b}}$ | $C_{h_2b\bar{b}}$ | $C_{h_3b\bar{b}}$ | $C_{h_4b\bar{b}}$ | $C_{h_5b\bar{b}}$ |
| 0.06       | 0.998     | 0.0         | 0.183       | 0.994     | 13.77   | -0.12  | 13.77  |
| $m_{\tilde{\tau}}$ | $N_{12}$ | $N_{12} + N_{14}$ | $N_{15}$ | $\sigma_{SI}$ | $\Omega_{\tilde{\chi}_1^0}h^2$ |
| 7.2 GeV    | 0.0036    | 0.017       | 0.98        | 2.34 x 10^{-1} pb | 0.112   |
| $B(h_1 \rightarrow \tilde{\chi}_1^{0}(\tilde{\chi}_1^{0}))$ | $B(h_1 \rightarrow 2s, 2g, 2\mu)$ | $B(h_2 \rightarrow \tilde{\chi}_1^{0}(\tilde{\chi}_1^{0}))$ | $B(h_2 \rightarrow \tilde{\chi}_1^{0}(\tilde{\chi}_1^{0}))$ | $B(h_2 \rightarrow 2b, 2\tau)$ |
| 0.027      | 0.833     | 0.14, 0.027 | 0.05        | 0.45      | 0.37    | 0.038  |
| $B(h^+ \rightarrow tb)$ | $B(h^+ \rightarrow \tilde{\chi}_1^{1,2}\tilde{\chi}_1^{1,2,3,4,5})$ | $B(a_1 \rightarrow \tilde{\chi}_1^{0}(\tilde{\chi}_1^{0}))$ | $B(a_1 \rightarrow 2b, 2\tau, 2\mu)$ |
| 0.138      | 0.80      | 0.25        | 0.70, 0.042 | 0.00015  |
| $B(a_2, h_3 \rightarrow \tilde{\chi}_1^{1}(\tilde{\chi}_1^{1}))$ | $B(a_2, h_3 \rightarrow 2t, 2b, 2\tau)$ | $B(a_2, h_3 \rightarrow \tilde{\chi}_1^{1,2,3,4,5} \tilde{\chi}_1^{1,2,3,4,5})$ | $B(a_2, h_3 \rightarrow \tilde{\chi}_1^{1,2} \tilde{\chi}_1^{1,2})$ |
| 0.00       | 0.013, 0.126, 0.023 | 0.32         | 0.48         |

As noted earlier, in [12] an alternative SS scenario can be realized in the strict NMSSM, but only if $m_{h_1} \lesssim 1$ GeV. The properties of their representative point are tabulated in Table 5. Some observations regarding this scenario are the following.

(i) The $h_1$ is very light and very singlet. It is so weakly coupled to the down and up quarks that it can probably only be detected directly via $Y_{3S} \rightarrow \gamma h_1$ with $h_1 \rightarrow \mu^+\mu^-$. For
current data from BaBar and using $B(h_1 \rightarrow \mu^+\mu^-) \sim 0.027$ (see the Table), the limit from $\Upsilon_{3S} \rightarrow \gamma h_1 \rightarrow \gamma\mu^+\mu^-$ is $C_{h_1\mu\mu} \sim 0.2 - 0.3$ for $m_{h_1} \sim 1$ GeV (the limit fluctuates very rapidly). For this scenario the value of $C_{h_1\mu\mu} = 0.183$ (see the Table) is thus just below the BaBar limit. This indicates that increased statistics could very well reveal the light $h_1$ since $C_{h_1\mu\mu}$ cannot be much below this value and still provide a large enough $\sigma_{SI}$ to explain the CoGeNT/DAMA events.

(ii) Meanwhile, the $h_2$ is completely SM-like and its discovery at the LHC or Tevatron would be possible in the usual channels for a SM Higgs of the same mass.

(iii) The $a_1$ has a very small branching ratio to $\mu^+\mu^-$ (since $m_{a_1} > 2m_B$) and would have to be searched for in the $b\bar{b}$ or $\tau^+\tau^-$ decay mode. Since the $a_1$ is very singlet its production cross sections would be so small that this would likely be an impossible task.

(iv) The $h_3$, $a_2$, $h^+$ form a decoupled degenerate doublet with common mass of around 244 GeV. The $b\bar{b}h_3$ and $b\bar{a}_2$ couplings are both enhanced by a factor of $\tan^2 \beta = 13.77$. The most promising LHC signal would be $gg \rightarrow b\bar{b}h_3 + b\bar{a}_2$ with decay $h_3, a_2 \rightarrow \tau^+\tau^-$. Of course, $B(h_3, a_2 \rightarrow \tau^+\tau^-) \sim 0.023$ is uncomfortably small and even this signal would be quite weak (and does not emerge in the NMHDECAY LHC estimates as viable).

4. Conclusions

The CoGeNT/DAMA data suggests a large spin-independent cross section for dark matter scattering on nucleons, $\sigma_{SI} \sim (1 - 3) \times 10^{-4}$ pb, at low dark matter mass, $m_{DM} \sim 4 - 9$ GeV. This cannot be achieved for the lightest neutralino in the minimal supersymmetric model (MSSM) after imposing all constraints, including, in particular, $\Omega_{\chi^0_1} h^2 \sim 0.11$ and the Tevatron limit of $B(B_s \rightarrow \mu^+\mu^-) < 5.8 \times 10^{-8}$. It is then natural to ask if the simplest extension of the MSSM obtained by adding a singlet superfield to the MSSM (the NMSSM and ENMSSM models) can allow simultaneous compatibility between CoGeNT/DAMA events and all other constraints, or must one turn to more exotic supersymmetric or other models. As reviewed here, the NMSSM and ENMSSM can achieve large $\sigma_{SI}$ at low $m_{\chi^0_1}$ while satisfying all constraints, but only if the Higgs sector has the appropriate structure and properties. Indeed, given the LEP, Tevatron, BaBar and other constraints, only a limited number of possibilities within the NMSSM and ENMSSM have been delimited to date. These include:

(i) The “inverted-Higgs” (IH) scenarios where the lightest CP-even Higgs, $h_1$, is $H_d$-like while the $h_2$ has SM-like couplings to $WW, ZZ$, but might decay via $h_2 \rightarrow a_1a_1$.

After imposing $\Omega_{\chi^0_1} h^2 \sim 0.11$, and all other constraints, the value of $\sigma_{SI}$ that can be achieved falls short by something like a factor of 5 - 10 (assuming a reasonable $(g - 2)_\mu$ is required) in comparison to the value of $\sigma_{SI} \sim (1 - 3) \times 10^{-4}$ pb that is apparently required by CoGeNT/DAMA. However, this kind of scenario could become interesting if:

a) the relevant $\sigma_{SI}$ values at small $m_{\chi^0_1}$ turn out to be somewhat smaller (e.g. new data or increased local density $\rho$); b) the $(g - 2)_\mu$ restrictions employed turn out to be incorrect; and/or c) the s-quark content of the nucleons has been underestimated.

All Higgs bosons in the IH scenarios are quite light and discovery prospects are good. The $\chi^0_1$ is primarily bino. One can even have an “inverted-ideal-Higgs” (IIH) scenario in which the $ZZ$ coupling squared weighted Higgs mass, $m_{eff}$, is below 105 GeV and is thus in the ideal range for precision electroweak data, finetuning and electroweak baryogenesis. We focused on describing such an IIH scenario in our discussion.

(ii) The singlet-singlino (SS) scenarios in which the $h_1$ and $\chi^0_1$ are primarily singlet and singlino, respectively, with $m_{h_1}$ fairly small ($m_{h_1} \sim 40 - 70$ GeV) in the ENMSSM case and very small ($m_{h_1} \lesssim 1$ GeV) in the NMSSM DLH case.
In both these cases, one can achieve the required $\sigma_{SI} \sim 2 \times 10^{-4}$ pb while maintaining $\Omega_{\chi_1^0} h^2 \sim 0.11$ and obeying all constraints.

In the ENMSSM case, detecting the $h_1$ directly at the LHC in $gg \rightarrow b\bar{b}h_1$ with $h_1 \rightarrow \tau^+\tau^-$ might prove possible.

In the DLH case, detection of the $h_1$ would require a significant (but not enormous) increase in statistics relative to current BaBar data for $T_{3S} \rightarrow \gamma \mu^+\mu^-$. In both cases, most of the other Higgs bosons would be readily detectable.

In general, there is an intimate connection between achieving large $\sigma_{SI}$ at small $m_{\chi_1}$ and a relatively unusual Higgs sector structure. In the SS cases, detection of the singlet $h_1$ will be highly non-trivial, but absolutely necessary if we are to understand the source of the large $\sigma_{SI}$ and achieve a quantitative understanding of $\Omega_{\chi_1^0} h^2$. If at least some Higgs bosons are discovered and their properties measured and if the CoGeNT/DAMA events are confirmed as dark matter detection, then it is also possible that the NMSSM and ENMSSM supersymmetric models will be ruled out, requiring that more exotic possibilities be considered.

5. Acknowledgments

This work was supported by US DOE grant DE-FG03-91ER40674. I wish to thank my collaborators for their contributions to our joint work. I also wish to thank the organizers of PASCOS 2010 for their generous support and the Aspen Center for Physics for support during the course of this project. I also wish to thank C. Wagner for several useful communications.

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We wish to thank H. Leutwyler for very useful communications regarding these issues.