Analysis of the Results of the Neutrino-4 Experiment on the Search for the Sterile Neutrino and Comparison with Results of Other Experiments

A. P. Serebrov* and R. M. Samoilov

Petersburg Nuclear Physics Institute, National Research Center Kurchatov Institute, Gatchina, 188300 Russia

* e-mail: serebrov_ap@pnpi.nrcki.ru

Received March 23, 2020; revised July 6, 2020; accepted July 7, 2020

New measurements of the flux and spectrum of reactor antineutrinos as functions of the distance from the center of the core of the SM-3 reactor (Dimitrovgrad, Russia) in the range of 6–12 m have been reported. Additional measurements have been performed. The amount of experimental data has been increased by almost a factor of 2. The model-independent analysis has been performed in order to determine the oscillation parameters $\Delta m^2_{14}$ and $\sin^2 2\theta_{14}$. The method of coherent addition of measurement results allows the direct demonstration of the effect of oscillations. The effect of oscillations is observed near the values $\Delta m^2_{14} = 7.25 \pm 0.13 \pm 1.08 \text{ stat} \pm 0.05 \text{ syst}$ and $\sin^2 2\theta = 0.26 \pm 0.08 \text{ stat} \pm 0.05 \text{ syst}$. This result has been compared to the results of other experiments on the search for the sterile neutrino. The joint analysis of the Neutrino-4 experiment with the gallium and reactor anomalies gives the value $\sin^2 2\theta_{14} = 0.19 \pm 0.04 (4.6\sigma)$. The results of the Neutrino-4 experiment have been compared to the results of the NEOS, DANSS, STEREO, and PROSPECT reactor experiments; MiniBooNE and LSND accelerator experiments; and IceCube experiment. According to the Neutrino-4 experiment (under the assumption that $m_4^2 = \Delta m^2_{14}$), the mass of the sterile neutrino is $m_4 = (2.68 \pm 0.13) \text{ eV}$. The calculations with the estimates of the mixing angles from other experiments give the values $m_{\nu_e}^{\text{eff}} = (0.58 \pm 0.09) \text{ eV}$, $m_{\nu_\mu}^{\text{eff}} = (0.42 \pm 0.24) \text{ eV}$, and $m_{\nu_\tau}^{\text{eff}} \leq 0.65 \text{ eV}$ for the masses of the electron, muon, and tau neutrinos, respectively. The extended Pontecorvo–Maki–Nakagawa–Sakata matrix for the $3 + 1$ model with one sterile neutrino is given.

DOI: 10.1134/S0021364020160122

1. INTRODUCTION

The experimental search for possible neutrino oscillations to the sterile state has been performed for many years, including experiments at accelerators, reactors, and artificial neutrino sources [1–24]. Sterile neutrinos are candidates for dark matter particles. The hypothesis of oscillations to the sterile state can be tested by the direct measurement of the dependence of the neutrino flux and neutrino energy spectrum on the distance in the range of 6–12 m. This method of relative measurements can be more accurate. To this end, a detector should be mobile and spectrally sensitive. To observe oscillations to the sterile state, it is necessary to detect the deviation of the distance dependence of the neutron flux from the $1/L^2$ law and the variation of the spectrum shape with the distance. If oscillations to the sterile state indeed occur, they can be described at short distances by the formula

$$P(\overline{\nu}_e \rightarrow \overline{\nu}_\nu) = 1 - \sin^2 2\theta_{14} \sin^2 \left( \frac{1.27 \Delta m^2_{14} \text{[eV}^2\text{]} L [m]}{E_\tau [\text{MeV}]} \right),$$

where $E_\tau$ is the energy of the antineutrino in MeV, $L$ is the distance in meters, $\Delta m^2_{14}$ is the difference of the squares of masses of the electron and sterile neutrinos, and $\theta_{14}$ is the mixing angle of the electron and sterile neutrinos. In the experiment, it is necessary to measure the flux and spectrum of antineutrinos as close as possible to the almost point source of antineutrinos.

This work continues the work “First Observation of the Oscillation Effect in the Neutrino-4 Experiment on the Search for the Sterile Neutrino” [24]. The aims of this work are to reveal the effect of doubling the amount of experimental data on the old result and to compare with other experiments in order to demonstrate the current status of the hypothesis of the sterile neutrino.
2. SCHEME OF THE DETECTOR

The scheme of the detector with passive and active shieldings is shown in Fig. 1. The detailed description of the detector used in the Neutrino-4 experiment, description of the preparation of the neutrino laboratory, the results of the measurement of the background, and the scheme of measurements with the full-scale detector can be found in [25].

3. MEASUREMENT MATRIX FOR THE ANTINEUTRINO FLUX AS A FUNCTION OF THE DISTANCE AND ENERGY

Below, we analyze all data collected from June 2016 to June 2019, when the reactor was stopped for modernization. From June 2019 to January 2020, the background was measured. The overall measurements with the operating (ON) and stopped (OFF) reactor lasted 720 and 417 days, respectively. The reactor was switched on and off 87 times. The ON–OFF difference is 223 events per day in the distance range from 6.5 to 9.0 m. The signal-to-background ratio is 0.54. Processes associated with the cosmic background are subtracted to obtain the ON–OFF antineutrino difference spectrum. We recall that the background of fast neutrons and gamma rays was measured as a function of the distance from the reactor and its power before placing the detector in the passive shielding [25, 26]. These measurements showed that any significant dependence of the background both on the power of the reactor and on the distance from it is absent. This makes it possible to expect that the ON–OFF difference signal is determined by the antineutrino flux when the reactor is stopped. Thus, here and below, the ON–OFF count means the antineutrino count.

The antineutrino flux measured as a function of the distance and energy can be represented in the form of a matrix containing 216 elements $N_{ik}$ each meaning the difference signal in the $i$th energy range and $k$th distance interval from the center of the reactor. The energy spectrum is divided into nine 500-keV ranges, which corresponds to the energy resolution of the detector of ±250 keV. The spatial interval corresponds to the sizes of the detector cell and is 23 cm. Thus, the antineutrino flux is measured at 24 positions in the range from 6.4 to 11.9 m. We also consider more detailed representations of the data with the division of the energy spectrum into 125- and 250-keV intervals.

4. SCHEME OF THE ANALYSIS OF THE EXPERIMENTAL DATA

The well-known problem of discrepancy between the measured and calculated antineutrino spectra is also manifested in our experiment [24]. Consequently, the analysis of experimental data should not be based on the exact determination of the energy spectrum. For this reason, we propose the model-independent analysis of the data with the formula

$$
\frac{N_{ik} \pm \Delta N_{ik}}{L_k^2} / K^{-1} \sum_k (N_{ik} \pm \Delta N_{ik}) L_k^2 = \frac{1 - \sin^2 \theta_{13} \sin^2 \left( \frac{1.27 \Delta m_{12}^2 L_k}{E_i} \right)}{K^{-1} \sum_k \left( 1 - \sin^2 \theta_{13} \sin^2 \left( \frac{1.27 \Delta m_{12}^2 L_k}{E_i} \right) \right)},
$$

(2)

where the numerator is the number of antineutrino events in $10^5$ s with the correction to the geometric factor $L_k^2$ and the denominator is the number of antineutrino events averaged over all distances.

Formula (2) allows the model-independent analysis of the data because the left-hand side includes only...
the experimental data \( k = 1, 2, \ldots, K \) for all distances in the range of 6.4–11.9 m, \( K = 24 \) and \( l = 1–9 \) specifies the 500-keV energy interval in the range of 1.5–6.0 MeV. The right-hand side of Eq. (2) is the same ratio expected in the presence of oscillations. The left-hand side of Eq. (2) is normalized to the spectrum averaged over all distances; therefore, the effect of oscillations in the denominator is significantly averaged if oscillations are sufficiently frequent for a given distance interval.

5. ANALYSIS OF THE EXPERIMENTAL RESULTS ON THE SEARCH FOR OSCILLATIONS

The measurement matrix includes the antineutrino flux as a function of the distance and energy. Each element \( N_{ik} \) of this matrix is the difference signal in the \( i \)th energy interval and the \( k \)th interval of distances from the center of the reactor. This matrix should be compared to the calculated matrix

\[
R_{ik}^{\text{exp}} = \frac{N(E_i, L_k) L_k^2 / K \sum_{k}(E_i, L_k) L_k^2}{1 - \sin^2 2\theta_{14} \sin^2 (1.27 \Delta m^2_{14} L_k / E_i)} = R_{ik}^{\text{th}}.
\]

If the measurement distance range is much longer than the characteristic oscillation period, the denominator in \( R_{ik}^{\text{th}} \) is significantly simplified:

\[
R_{ik}^{\text{th}} \approx \frac{1 - \sin^2 2\theta_{14} \sin^2 (1.27 \Delta m^2_{14} L_k / E_i)}{1 - 1/2 \sin^2 2\theta_{14}} \rightarrow 1.
\]

The experimental results can be compared to the matrix obtained from the Monte Carlo calculations using the \( \Delta \chi^2 \) method:

\[
\sum_{i,k}(R_{ik}^{\text{exp}} - R_{ik}^{\text{th}})^2 / (\Delta R_{ik}^{\text{exp}})^2 = \chi^2 (\sin^2 2\theta_{14}, \Delta m^2_{14}).
\]

The results of the \( \Delta \chi^2 \) analysis of the experiment data are shown in Figs. 2 and 3.

However, the effect of oscillations is observed in the region \( \Delta m^2_{14} = (7.26 \pm 0.07) \text{ eV}^2 \) and \( \sin^2 2\theta_{14} = 0.38 \pm 0.11 \) with a statistical significance of 3.5\( \sigma \) at the data processing with the division of the energy spectrum into 500-keV intervals. At the data processing with the division of the energy spectrum into 125-, 250-, and 500-keV intervals with averaging of three samples of the data, the effect of oscillations is observed in the region \( \Delta m^2_{14} = (7.25 \pm 0.13) \text{ eV}^2 \) and \( \sin^2 2\theta_{14} = 0.26 \pm 0.08 \) with a statistical significance of 3.2\( \sigma \).

![Figure 2](image1.png)

Fig. 2. (Color online) Regions of the allowed parameters of oscillations to the sterile state at (pink region) 99.95% C.L., (yellow region) 99.73% C.L., (green region) 95.45% C.L., and (blue region) 68.30% C.L. The presented results are obtained by the processing of data with the division of the energy spectrum into 500-keV intervals.

![Figure 3](image2.png)

Fig. 3. (Color online) (Left panel) Central region on a magnified scale for the processing of data with the division of the energy spectrum into 500-keV intervals. (Right panel) Same region for the processing of data with the division of the energy spectrum into 125-, 250-, and 500-keV intervals with averaging of three samples of data.
6. MONTE CARLO CALCULATIONS

In this section, we report the Monte Carlo calculations with the geometrical parameters of the source and detector with allowance for division into individual intervals.

The Monte Carlo simulation was performed for an antineutrino source in the form of a \(42 \times 42 \times 35\)-cm reactor core and for the antineutrino detector taking into account its geometrical sizes (50 \(22.5 \times 22.5 \times 85\)-cm sections). We used the antineutrino spectrum of \(^{235}\text{U}\) (although it is of no significance because the energy spectrum of antineutrinos in Eq. (2) is canceled) multiplied by the function \(1 - \sin^2(1.27\Delta m^2_{14}L_k/E)\) of the effect of oscillations. The energy resolution of the detector, which was \(\pm 250\) keV, was the most important parameter in this simulation. The right panel of Fig. 4 shows the dependence of the oscillation pattern on the energy resolution of the detector. The oscillation curve with the measured energy resolution of the detector of \(\pm 250\) keV should be the most appropriate to describe the experiment data.

The left panel of Fig. 4 shows the model matrix \((N_{ik} \pm \Delta N_{ik})L_k^2/K^{-1}\sum N_{ik}L_k^2\) for calculation, where \(\Delta N_{ik}/N_{ik} = 1\%\), which is much better than that in the experiment. The left and right panels of Fig. 4 demonstrate the oscillation pattern on the \((E, L)\) plane and as a function of the ratio \(L/E\), respectively.

The performed Monte Carlo simulation clearly demonstrates that the energy resolution of the detector is very important for detecting the effect of oscillations. We note that the energy resolution of the detector determines the number of observed oscillations rather than their amplitude. Furthermore, the effect of oscillations can be revealed only by plotting the experimental ratio \(N_{ik}L_k^2/K^{-1}\sum N_{ik}L_k^2\) as a function of the ratio \(L/E\). It is noteworthy that the summation of the matrix elements over the energy or the distance significantly reduces the possibility of revealing the effect of oscillations. In addition, measurements in the range of \(6–9\) m are particularly important, because measurements in the range of \(9–12\) m make an insignificant contribution to the sensitivity of the experiment, but serve for the correct general normalization of the results.

7. METHOD OF THE COHERENT ADDITION OF THE MEASUREMENT RESULTS

As mentioned above, the effect of oscillations can be revealed reliably from the plotted dependence of
the experimental ratio $N_{ikL_k}^2/K^{-1}\sum N_{ikL_k}2$ on $L/E$. The coherent summation of data with the same ratio $L/E$ allows the direct demonstration of the effect of oscillations. The $\Delta\chi^2$ method previously used to compare the experimental $(E, L)$ matrix to the calculated matrix makes it possible only to reveal the presence of oscillations and to determine the optimal parameters. Using these optimal parameters, we plot the experimental ratio $N_{ikL_k}^2/K^{-1}\sum N_{ikL_k}2$ as a function of $L/E$, compare it to the calculated dependence, and test the optimality of the parameters using the $\Delta\chi^2$ method.

The experimental data were analyzed in detail using the division of the energy spectrum into 125-, 250-, and 500-keV intervals. The aim of this processing of the data is to avoid fluctuations of the final result in the case of a single system of sampling of the data. To this end, we used 24 positions in distance (with an interval of 23 cm) and various divisions into energy intervals: 9 energy intervals (with a step of 0.5 MeV), 18 energy intervals (with a step of 0.25 MeV), and 36 energy intervals (with a step of 0.125 MeV). The corresponding matrices contained 216, 432, and 864 elements, respectively. To plot the dependence of $N_{ikL_k}^2/K^{-1}\sum N_{ikL_k}2$ on the ratio $L/E$, we joined the neighboring results by 8, 16, and 32 points, respectively. Further, the resulting $L/E$ dependences were averaged and, thereby, fluctuations were averaged at different samplings of the data.

To process the data with averaging over the 125-, 250-, and 500-keV intervals (black squares), fitting with the presented parameters gives a fit criterion of 28%, whereas the fit criterion with a constant is only 3%. For the hypotheses with and without oscillations, we obtained $\chi^2/\text{DoF} = 20/17$ and $32/19$, respectively. The corresponding confidence levels are shown in Fig. 3 (right panel).

The processed data with averaging are shown by black squares in Fig. 5 in comparison with the results of processing with an interval of 500 keV corresponding to the energy resolution of the detector (blue triangles). It is seen that squares and triangles are statistically consistent. Both sets of points can be satisfactorily described by the calculated curve with the parameters $\Delta m^2_4 = 7.25 \text{ eV}^2$ and $\sin^2 2\theta_{14} = 0.26$. Fitting with these parameters for data processed with an energy interval of 500 keV corresponding to the energy resolution of the detector (blue triangles) gives a fit criterion of 45%. At the same time, the fit criterion with a constant (absence of oscillations) is only 8%. For the hypotheses with and without oscillations, we obtained $\chi^2/\text{DoF} = 17.1/17$ and $30/19$, respectively.

For the reliability of the final result, we choose the processing of the data with averaging. In this case, the effect of oscillations is observed at the parameters $\Delta m^2_4 = (7.25 \pm 0.13) \text{ eV}^2$ and $\sin^2 2\theta_{14} = 0.26 \pm 0.08$ with a statistical significance of $3.2\sigma$.

Using the background of fast neutrons from cosmic rays, we tested possible systematic effects. The reason is that fast neutrons imitate the detection of antineutrinos because recoil protons from fast neutrons imitate a signal from a positron. To test systematic effects, we stopped the antineutrino flux (the reactor) and performed the same analysis of the data. As a result, the oscillation curve disappeared. Thus, it was shown that systematic instrumental effects are absent.

8. ADDITIONAL ANALYSIS OF THE RELIABILITY OF THE RESULT

It is often discussed that more stringent constraints on the reliability of the result can be obtained with the Feldman–Cousins method. According to Wilks’ theorem, the $\Delta\chi^2$ method can be successfully applied in the presence of an effect at a statistical significance of $3\sigma$ or larger. Processing of the sample only with an interval of 500 keV without systematic errors with gives $\sin^2 2\theta_{14} = 0.38 \pm 0.11 (3.5\sigma)$, whereas the averaging of three samples provides $\sin^2 2\theta_{14} = 0.26 \pm 0.08 (3.2\sigma)$. Since the statistical significance of the observed effect is above $3\sigma$, we believe that it is not necessary to apply the Feldman–Cousins method and propose to make another additional analysis of our data.
The initial distribution of the (ON–OFF) counts in the entire energy range is shown in the upper panel of Fig. 6. It is the deviation of the counts from the average value for different series of measurements, which is in each case normalized to its statistical error. This allows joining all measurements in order to reveal the additional spread of the data besides the statistical one. The upper panel of Fig. 6 demonstrates the statistically determined normal distribution. This means that instabilities additional to variations of the cosmic background are not observed [25].

We compare it to the distribution obtained for the ratio $R_{\text{exp}}^\Delta$ from the same sample of the data. This distribution, as well as the ON–OFF distribution, is normalized to the statistical error and is the deviation of $R_{\text{exp}}^\Delta$ from unity. The lower panel of Fig. 6 shows the distribution of all 216 $L/E$ points in the range from 0.9 to 4.7. It is seen that the distribution of $R_{\text{exp}}^\Delta$ already differs from the normal distribution because of the effect of oscillations. This comparison indicates that $\chi^2/\text{DoF} = 25.9/16$, which indicates that the distribution of the ratio $R_{\text{exp}}^\Delta$ cannot be described by this function because the validity of such a description is only 5%. The presence of oscillations should broaden the distribution of $R_{\text{exp}}^\Delta$.

To summarize, the effect of oscillations is manifested when the following three methods of processing are used.

(i) The $\Delta\chi^2$ method on the $(\sin^2 2\theta_{14}, \Delta m^2_{14})$ plane.
(ii) The method of coherent summation in the $L/E$ variable.
(iii) The analysis of the difference of the distribution $R_{\text{exp}}^\Delta$ from the normal one because of the effect of oscillations.

9. SYSTEMATIC EXPERIMENTAL ERRORS

One of the possible systematic errors in the oscillation parameter $\Delta m^2_{14}$ is determined by the accuracy of the energy calibration of the detector, which is estimated as $\pm 250$ keV. The relative accuracy of the ratio $L/E$ is determined by the relative accuracy of the energy measurement because the relative accuracy of the distance measurement is much higher. The relative accuracy of the energy measurement in the most statistical part 3–4 MeV of the measured neutrino spectrum is 8%. Consequently, the possible systematic error of the parameter $\Delta m^2_{14}$ is $\delta(\Delta m^2_{14})_{\text{syst}} \approx 0.6 \text{ eV}^2$. A different systematic error in the determination of the parameter $\Delta m^2_{14}$ can appear when the $\chi^2$ method is used to process the data because of the appearance of additional regions (satellites) around the optimal value $\Delta m^2_{14} = 7.25 \text{ eV}^2$. The nearest regions are at values of 5.6 and 8.8 $\text{eV}^2$. However, their probability of the appearance of such a value is no more than 9%. Therefore, the possible systematic error can be estimated as $\delta(\Delta m^2_{14})_{\text{syst}} \approx 0.9 \text{ eV}^2$. Thus, the total systematic error of $\Delta m^2_{14}$ is $\delta(\Delta m^2_{14})_{\text{syst}} \approx 1.08 \text{ eV}^2$; i.e.,

$$\Delta m^2_{14} = (7.25 \pm 0.13_{\text{syst}} \pm 1.08_{\text{syst}}) \text{ eV}^2 = (7.25 \pm 1.09) \text{ eV}^2.$$
The systematic error of the parameter $\sin^2 2\theta_{14}$ can appear when the optimal $\sin^2 2\theta_{14}$ value is obtained by the $\chi^2$ method. The above analysis has shown that such deviation is possible. It was removed by more detailed processing with the use of different energy intervals. This analysis with different energy intervals was expanded. As a result, the standard deviation in the samples does not exceed a value of 0.05, which should be considered as an additional systematic error of the parameter $\sin^2 2\theta_{14}$. Thus, $\delta(\sin^2 2\theta_{14})_{\text{syst}} = 0.05$ and the mixing parameter is $\sin^2 2\theta = 0.26 \pm 0.08_{\text{stat}} \pm 0.05_{\text{syst}}$. Thus, the statistical significance is $3.2\sigma$ and the square summation of the statistical and systematic errors gives the result $\sin^2 2\theta = 0.26 \pm 0.09$.

10. DEPENDENCE OF THE ANITNEUTRINO FLUX ON THE DISTANCE FROM THE REACTOR IN THE RANGE OF 6–12 m

The left panel of Fig. 7 shows the difference between the antineutrino fluxes measured the operating and stopped reactor versus the distance from the center of the reactor core normalized to the $A/L^2$ law. In the case of this normalization, it should be taken into account that the effect of oscillations for the integral of the energy spectrum is averaged already at a distance of 6 m from the reactor core. This leads to the well-known deficit of the antineutrino flux that is $1 - 1/2 \sin^2 2\theta_{14} = 0.87$ for $\sin^2 2\theta_{14} = 0.26$. Thus, without absolute measurements of the antineutrino flux from the reactor, we know the deficit at large distances under the hypothesis of oscillations. The fit criterion for the approximation of the experiment dependence by a constant is 22%. Corrections caused by the finite dimensions of the reactor core and sections of the detector have a negligibly small value of 0.3%. Corrections caused by the difference between the axis of the detector motion and the direction to the center of the reactor core are also negligibly small, about 0.6%. Corrections caused by the signals of fast neutrons from the reactor are approximately 3%.

The right panel of Fig. 7 shows the dependence of the integral of the energy spectrum of the antineutrino flux calculated with the parameters $\Delta m^2_{41} = 7.25$ and $\sin^2 2\theta_{14} = 0.26$ on the distance from the reactor core. Four experimental points on this dependence correspond to the intervals $6–7.5, 7.5–9, 9–10.5,$ and $10.5–12$ m. Their ordinate position is $1 - 1/2 \sin^2 2\theta_{14} = 0.87$, which corresponds to the deficit for the integral spectrum.

11. COMPARISON ON THE RESULTS OF THE NEUTRINO-4 EXPERIMENT WITH THE REACTOR AND GALLIUM ANOMALIES

The oscillation parameter $\sin^2 2\theta_{14}$ measured in the Neutrino-4 experiment is twice as large as the deficit of the antineutrino flux from the reactor at large distances. To compare the results of the Neutrino-4 experiment to the reactor and gallium anomalies, the oscillation parameter $\sin^2 2\theta_{14}$ can be recalculated to the deficit and vice versa. The further comparison will
be made in terms of the oscillation parameter $\sin^2 2\theta_{14}$.

Figure 8 demonstrates the famous oscillation curve for reactor antineutrinos and the oscillation curve obtained in the Neutrino-4 experiment with the parameter $\sin^2 2\theta = 0.26 \pm 0.09 (2.9\sigma)$. The deficit of neutrinos called the gallium anomaly (GA) [8, 9] is $\sin^2 2\theta_{14} = 0.32 \pm 0.10 (3.2\sigma)$. The measurement of the reactor antineutrino anomaly (RAA) [27–29] gives $\sin^2 2\theta_{14} = 0.13 \pm 0.05 (2.6\sigma)$. The combination of these results gives the estimate of the mixing angle $\sin^2 2\theta_{14} = 0.19 \pm 0.04 (4.6\sigma)$.

12. COMPARISON WITH THE RESULTS OF OTHER EXPERIMENTS PERFORMED ON RESEARCH AND INDUSTRIAL REACTORS

Figure 9 (left panel) illustrates the sensitivities of the Neutrino-4 experiment, as well as the DANSS [17], NEOS [18], PROSPECT [19], and STEREO
experiments. The sensitivity to oscillations with large parameters $\Delta m_{4}^{2}$ in experiments performed on industrial reactors is strongly suppressed because of a large size of the reactor core. The Neutrino-4 experiment provides a higher sensitivity to oscillations with large parameters $\Delta m_{4}^{2}$ because of a compact reactor core, the possibility of measurements at small distances from the reactor, and a wide range of displacements of the detector.

The PROSPECT and STEREO experiments are next in the sensitivity to large parameters. These experiments are currently a factor of two less sensitive than the Neutrino-4 experiment. The acquisition of data has been recently begun in these experiments; hence, the continuation of these experiments will possibly confirm our result. The BEST experiment started in August 2019 at the Baksan Neutrino Observatory has a high sensitivity in the region $\Delta m_{4}^{2} > 5 \text{ eV}^{2}$ [22].

It is noteworthy that the method of coherent summation of results in the parameter $L/E$ is necessary for the demonstration of the real effect of oscillations. The method of coherent summation of results in the parameter $L/E$ at short distances for now has been actively used only in the Neutrino-4 experiment. The $(E, L)$ parameter planes for the Neutrino-4, STEREO, and PROSPECT experiments are shown in the right panel of Fig. 9, where the difference in the sensitivity between these experiments is seen.

13. STRUCTURE OF THE 3 + 1 NEUTRINO MODEL AND REPRESENTATION OF THE PROBABILITY OF VARIOUS OSCILLATIONS

To compare with muon experiments, we recall the structure of the 3 + 1 neutrino model and representation of the probabilities of various oscillations in the form

$$
|U_{e4}|^2 = \sin^2(\theta_{14}),
|U_{\mu4}|^2 = \sin^2(\theta_{24}) \cos^2(\theta_{14}),
|U_{\tau4}|^2 = \sin^2(\theta_{34}) \cos^2(\theta_{24}) \cos^2(\theta_{14}),
$$

$$
P_{\nu_{e} \nu_{e}} = 1 - 4|U_{e4}|^2(1 - |U_{e4}|^2) \sin^2 \left( \frac{\Delta m_{12}^2 L}{4 E_{\nu_e}} \right) = 1 - \sin^2 2\theta_{ee} \sin^2 \left( \frac{\Delta m_{12}^2 L}{4 E_{\nu_e}} \right),
$$

$$
P_{\nu_{\mu} \nu_{e}} = 1 - 4|U_{\mu4}|^2(1 - |U_{\mu4}|^2) \sin^2 \left( \frac{\Delta m_{12}^2 L}{4 E_{\nu_e}} \right) = 1 - \sin^2 2\theta_{\mu e} \sin^2 \left( \frac{\Delta m_{12}^2 L}{4 E_{\nu_e}} \right),
$$

$$
P_{\nu_{\tau} \nu_{e}} = 4|U_{e4}|^2 |U_{\mu4}|^2 \sin^2 \left( \frac{\Delta m_{12}^2 L}{4 E_{\nu_e}} \right) = \sin^2 2\theta_{\mu e} \sin^2 \left( \frac{\Delta m_{12}^2 L}{4 E_{\nu_e}} \right),
$$

$$
\sin^2 2\theta_{ee} = \sin^2 2\theta_{14},
\sin^2 2\theta_{\mu e} = 4 \sin^2 \theta_{24} \cos^2 \theta_{14}(1 - \sin^2 \theta_{24} \cos^2 \theta_{14}) = \sin^2 2\theta_{24},
\sin^2 2\theta_{\mu e} = 4 \sin^2 \theta_{14} \sin^2 \theta_{24} \cos^2 \theta_{14} = \frac{1}{4} \sin^2 2\theta_{14} \sin^2 2\theta_{24}.
$$

These relations between the parameters of various oscillations are necessary for the comparative analysis of experimental results presented in Figs. 10 and 11.

An important relation for the experimental verification of the 3 + 1 model is that the amplitudes of electron and muon oscillations in the processes of disappearance determine the amplitude $\sin^2 2\theta_{\mu e}$ in the process of appearance of electron neutrinos in the beam of muon neutrinos.

Effects indicating the process of oscillations to the sterile neutrino were detected in the Neutrino-4, reactor anomaly, gallium anomaly, MiniBooNE, LSND, and IceCube experiments.

14. COMPARISON ON THE RESULTS OF THE NEUTRINO-4 EXPERIMENT WITH THE RESULTS OF THE IceCube EXPERIMENT

Figure 10 shows the results of the Neutrino-4 experiment in comparison with the IceCube experiment. The best fitting of the oscillation parameters in the IceCube experiment [31] is obtained with the parameters

$$
\Delta m_{14}^2 = 4.4^{+13.53}_{-2.08} \text{ eV}^2,
\sin^2(2\theta_{24}) = 0.10^{+0.10}_{-0.07}.
$$

The $\Delta m_{14}^2$ values from both experiments are in agreement within one standard deviation, and the
and values are in agreement within 1.3σ, although the 3 + 1 model does not require this.

15. COMPARISON ON THE RESULTS OF THE NEUTRINO-4 EXPERIMENT WITH THE RESULTS OF THE MiniBooNE AND LSND ACCELERATOR EXPERIMENTS

In addition, it is of interest to compare the results of the Neutrino-4 experiment with the LSND [1] and MiniBooNE [2] accelerator experiments. We compared the results of these experiments [32] with the results of the Neutrino-4 experiment on the (sin²θ₁₄, sin²θ₂₄) plane in comparison with the MiniBooNE and LSND experiments. It is seen that the distribution of Δχ² has a local minimum in the region of large Δm² values, which coincides with the region of the minimum Δm² ≈ 7.25 eV² in the distribution of Δχ² for the Neutrino-4 experiment.

16. COMPARISON WITH THE KATRIN EXPERIMENT ON THE MEASUREMENT OF THE NEUTRINO MASS

The mass of the electron neutrino can be estimated using the oscillation parameters obtained in the Neutrino-4 experiment and the well-known formulas for the neutrino model [33, 34] with the expansion to the 3 + 1 model:

\[ m_\nu^{\text{eff}} = \sqrt{\sum m_i^2 |U_{ei}|^2} \]

\[ \sin^2 \theta_{14} = 4|U_{14}|^2 (1 - |U_{14}|^2) \]

\[ |U_{14}|^2 \ll 1; \quad |U_{14}|^2 = \frac{1}{4} \sin^2 2\theta_{14} \]

The cosmological constraints on the sum of the masses of active neutrinos \( \sum m_\nu = m_1 + m_2 + m_3 \) are in the range of 0.11–0.54 eV [35]. Since \( \Delta m_{14}^2 = 7.25 \) eV², we have \( m_4 \geq 7.25 \) eV and \( m_4, m_2, m_1 \ll m_4 \). Thus, the effective mass of the electron neutrino can be calculated by the formula

\[ m_\nu^{\text{eff}} = \sqrt{\sum m_i^2 |U_{ei}|^2} \]
The upper bound on the accuracy of the result for more accurate consideration of the approximation using the Pontecorvo–Maki–Nakagawa–Sakata (PMNS) matrix is no more than 10%.

It is necessary to briefly discuss this result in context of the known cosmological constraints on the number of types of neutrinos and on the sum of the masses of active neutrinos.

Depending on the scale of masses, sterile neutrinos can affect the development of the Universe and can be responsible for the baryon asymmetry of the Universe and for dark matter [36]. However, sterile neutrinos with a small mass and a small mixing angle that hardly affect cosmology are also allowable [36]. Such sterile neutrinos are hardly thermalized in the primary plasma and leave it at the early stage.

According to the above consideration, the mass of the sterile neutrino can be estimated as $m_s = (2.68 \pm 0.13) \text{ eV}$. Using the parameters $\sin^2 2\theta_{14} = 0.19 \pm 0.04 (4.6\sigma)$ obtained from the Neutrino-4 experiment jointly with the reactor and gallium anomalies and, primarily, the value $\Delta m^2_{14} = (7.2 \pm 1.09) \text{ eV}^2$ obtained for the first time in the Neutrino-4 experiment, the mass of the electron neutrino is estimated as $m_{\nu_e}^{\text{eff}} = (0.58 \pm 0.09) \text{ eV}$. The calculated mass of the neutrino is consistent with the constraint on the mass of the neutrino $m_{\nu_e}^{\text{eff}} \leq 1.1 \text{ eV}$ at 90% C.L. obtained in the KATRIN experiment [37]. Furthermore, the determined parameters of the sterile neutrino make it possible to predict a value that can be obtained in the KATRIN experiment. Figure 12 [38] shows the constraints on the sterile neutrino from the KATRIN experiment at the achieved accuracy level and prospects of its increase.

Thus, using the data of the IceCube experiment for $\sin^2 2\theta_{24}$, the mass of the muon neutrino can be estimated as $m_{\nu_\mu}^{\text{eff}} = (0.42 \pm 0.24) \text{ eV}$.

Finally, using the upper bound $\sin^2 2\theta_{14} \leq 0.21$, the upper bound on the mass of the tau neutrino can be obtained in the form $m_{\nu_\tau} \leq 0.65 \text{ eV}$.

### 17. COMPARISON WITH EXPERIMENTS ON THE MEASUREMENT OF THE NEUTRINO MASS FROM THE DOUBLE NEUTRINOLESS BETA DECAY

In experiments on the double beta decay, the Majorana neutrino mass is determined by the expression

$$m_{\nu}^{\text{eff}} = \sqrt{m_s^2 |U_{ei}|^2} = \frac{1}{2} \sqrt{m_t^2 \sin^2 2\theta_{14}}.$$

The best constraints on the Majorana mass were obtained in the GERDA experiment [39] on the measurement of the half-life of an isotope, which depends on the Majorana mass as

$$m(0\nu\beta\beta) = \sum_{i=1}^{4} |U_{ei}|^2 m_i.$$

This expression for the $3 + 1$ model and $m_1, m_2, m_3 \ll m_4$ can be simplified to the form $m(0\nu\beta\beta) = m_1 U_{14}^2$, which gives

$$m(0\nu\beta\beta) = (0.13 \pm 0.03) \text{ eV}.$$

The lower limit on the half-life gives the upper limit on the Majorana mass: the lower limit for the half-life is $T_{1/2}^{0\nu} > 1.8 \times 10^{26} \text{ yr}$ (90% C.L.) and the upper limit for the Majorana mass is $m_{\beta\beta} < 80–182 \text{ meV}$.

The further improvement of the accuracy of the experiment on the double beta decay can allow detecting the Majorana mass or excluding the existence of the Majorana neutrino. It is noteworthy that the results depend on the hierarchy of neutrino masses.
18. PMNS MATRIX IN THE 3 + 1 MODEL

The PMNS matrix for four states together with the sterile neutrino, whose parameters are determined in our Neutrino-4 experiment, in experiments on the reactor and gallium anomalies, and in the IceCube experiment, has the form

$$U^{(3+1)}_{\text{PMNS}} = \begin{pmatrix}
0.824_{-0.008}^{+0.007} & 0.547_{-0.011}^{+0.003} & 0.147_{-0.006}^{+0.003} & 0.224_{-0.025}^{+0.025} \\
0.409_{-0.036}^{+0.036} & 0.634_{-0.006}^{+0.022} & 0.657_{-0.014}^{+0.044} & 0.160_{-0.05}^{+0.08} \\
0.392_{-0.048}^{+0.025} & 0.547_{-0.028}^{+0.056} & 0.740_{-0.048}^{+0.012} & <0.229 \\
<0.24 & <0.30 & <0.26 & >0.93
\end{pmatrix}.$$  

Restrictions on the $U_{ei}$ values were obtained from unitarity relations for columns under the condition that the sum of squares of all four elements of a column is below unity plus one standard deviation. The scheme of mixing of neutrino flavors with the sterile neutrino for the direct and inverse mass hierarchies is shown in Fig. 13.

![Fig. 13. (Color online) Scheme of mixing of neutrino flavors with the sterile neutrino for the (left panel) direct and (right panel) inverse mass hierarchies.](image)

19. CONCLUSIONS

The comparative analysis of the Neutrino-4 data and other experiments on the search for the sterile neutrino provides the following conclusions.

First, the region of the reactor and gallium anomalies with the parameters $\Delta m^2_{14} < 3 \text{ eV}^2$ and $\sin^2 2\theta_{14} > 0.1$ is excluded with the C.L. higher than 99.7% ($>3\sigma$).

Second, however, the effect of oscillations is observed in the region of the parameters $\Delta m^2_{14} = (7.25 \pm 1.09) \text{ eV}^2$ and $\sin^2 2\theta_{14} = 0.26 \pm 0.08_{\text{stat}} \pm 0.05_{\text{syst}}$.

Third, the result can be compared to the results of other experiments on the search for the sterile neutrino. Effects indicating oscillations to the sterile neutrino were detected in the following five types of experiments:

(i) the Neutrino-4 experiment;
(ii) reactor experiments, the so-called reactor anomaly;
(iii) experiments with a Cr-51-based neutrino source (gallium anomaly);
(iv) MiniBooNE and LSND accelerator experiments;
(v) the IceCube experiment.

Table 1 summarizes the results of the reactor anomaly, Neutrino-4, and gallium anomaly experiments. The distributions of the parameter $\sin^2 2\theta_{14}$ corresponding to these experiments are shown in Fig. 14.

Fourth, the combination of these results gives the estimate of the mixing angle $\sin^2 2\theta_{14} = 0.19 \pm 0.04$ ($4.6\sigma$). The validity of the combination of the Neutrino-4 result and the reactor anomaly result can be questionable, but the difference between these results is $0.13 \pm 0.09$, which is only $1.4\sigma$ and, in addition, the error of the reactor anomaly result does not include the systematic error of the reactor calculations, which are still being discussed.

Fifth, the comparison of the results of the Neutrino-4 experiment with the results of the IceCube experiment indicates the possible consistency of the oscillation parameter $\Delta m^2_{14} = 7 \text{ eV}^2$ in the Neutrino-4 experiment and the oscillation parameter $\Delta m^2_{14} = 4.5 \text{ eV}^2$ in the IceCube experiment within the existing accuracy of the IceCube experiment.

Sixth, the comparison of the results of the Neutrino-4 and IceCube experiments with the results of MiniBooNE and LSND accelerator experiments on the $(\sin^2 2\theta_{\mu e}, \Delta m^2_{14})$ plane also indicates the possible coincidence of the oscillation parameter $\Delta m^2_{14} = 7 \text{ eV}^2$. The value $\sin^2 2\theta_{\mu e} = 0.002–0.013$ calculated from

Table 1. Results of the reactor anomaly, Neutrino-4, and gallium anomaly experiments

| Reactor anomaly | Neutrino-4 experiment | Gallium anomaly |
|-----------------|-----------------------|---------------|
| $0.13 \pm 0.05$ | $0.26 \pm 0.09$ | $0.32 \pm 0.10$ |
| $(2.6\sigma)$  | $(2.9\sigma)$         | $(3.2\sigma)$ |
| $0.29 \pm 0.07$ |                        |               |
| $(4.3\sigma)$  |                        |               |
| $0.19 \pm 0.04$ |                        |               |
| $(4.6\sigma)$  |                        |               |
the Neutrino-4 and IceCube experiments is in agreement with the value $\sin^2 2\theta_{14} = 0.002 - 0.006$ from the MiniBooNE and LSND experiments.

Seventh, finally, the analysis of the results of the Neutrino-4 experiment and experiments discussed above indicates the possibility of existence of the sterile neutrino with the parameters $\Delta m^2 \approx (7.25 \pm 1.09) \text{ eV}^2$ and $\sin^2 2\theta_{14} = 0.19 \pm 0.04 (4.6\sigma)$. In this case, under the assumption that $m^s_3 = \Delta m^2_{34}$, the mass of the sterile neutrino can be estimated as $m^s_3 = (2.68 \pm 0.13) \text{ eV}$.

Eighth, from these oscillation parameters, the mass of the electron neutrino can be estimated as $m_{\nu_1}^{\text{eff}} = (0.58 \pm 0.09) \text{ eV}$.

Ninth, using the estimate of $\sin^2 2\theta_{24}$ from the IceCube experiment and $\Delta m^2_{34} \approx (7.25 \pm 1.09) \text{ eV}^2$ from the Neutrino-4 experiment, the mass of the muon neutrino can be estimated as $m_{\nu_2}^{\text{eff}} = (0.42 \pm 0.24) \text{ eV}$, and the upper limit $\sin^2 2\theta_{34} \leq 0.21$ makes it possible to estimate the upper bound on the mass of the tau neutrino as $m_{\nu_3}^{\text{eff}} \leq 0.65 \text{ eV}$.

The estimates of the masses of the electron, muon, tau, and sterile neutrinos are illustrated in Fig. 15. It is seen that the sterile neutrino determines the masses of the other neutrinos in terms of the mixing angles $\theta$ of about $0.1 - 0.2$ and smaller.

It is noteworthy that the sum of the effective masses of the active neutrinos $m_{\nu_1}^{\text{eff}} + m_{\nu_2}^{\text{eff}} + m_{\nu_3}^{\text{eff}}$ is not directly related to the cosmological estimates of the sum of masses $m_1 + m_2 + m_3$.

Tenth, the PMNS matrix is presented for four states together with the sterile neutrino whose parameters are determined in our Neutrino-4 experiment, in experiments on the reactor and gallium anomalies, and in the IceCube experiment.

The final confirmation of the existence of the sterile neutrino requires the result at a C.L. of 5$\sigma$. We are going to create the second neutrino laboratory based on the SM-3 reactor and a new detector with a sensitivity higher by a factor of 3.

**ACKNOWLEDGMENTS**

We are grateful to M.V. Danilov, V.B. Brudanin, V.G. Egorov, Yu.A. Kamshykov, V.A. Shegelsky, V.V. Sinev, D.S. Gorbunov, and particularly to Yu.G. Kudenko for stimulating discussions. The delivery of the liquid scintillator from the laboratory headed by Prof. Jun Cao (Institute of High Energy Physics, Beijing, People’s Republic of China) has made a considerable contribution to this study.

**FUNDING**

This work was supported by the Russian Science Foundation (project no. 20-12-00079).

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