Abstract

Demagnetization, commonly employed to study ferromagnets, has been proposed as the basis for an optimization tool, a method to find the ground state of a disordered system. Here we present a detailed comparison between the ground state and the demagnetized state in the random field Ising model, combing exact results in $d = 1$ and numerical solutions in $d = 3$. We show that there are important differences between the two states that persist in the thermodynamic limit and thus conclude that AC demagnetization is not an efficient optimization method.

Key words: random magnets, optimization, hysteresis, demagnetization

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Disordered systems are widely studied both for their conceptual importance and because the presence of randomness provides prototypical examples of complex optimization problems [1]. A disordered system can be non-trivial even at zero temperature due to the presence of a complex energy landscape. The properties of the ground-state (GS) are often difficult to determine analytically, and exact numerical evaluation becomes computationally prohibitive for large systems. Thus one is lead to construct approximate schemes, typically based on a non–equilibrium dynamics to find low energy states. In this respect a recently proposed method is hysteretic optimization [2]. Its basis is an analogy to a ferromagnetic demagnetization procedure: an external oscillating field with decreasing amplitude and low frequency is applied to the system. In ferromagnetic materials, one obtains at zero field after this procedure the demagnetized state (DS), which is used as a reference state for material characterization.

Here we analyze the differences between the GS and the DS in the ferromagnetic random field Ising model (RFIM), which has been extensively studied in literature as a paradigmatic example of disordered system [3]. In the RFIM, a spin $s_i = \pm 1$ is assigned to each site $i$ of a $d$–dimensional lattice. The spins are coupled to their nearest-neighbors spins by a ferromagnetic interaction of strength $J$ and to the external field $H$. In addition, to each site of the lattice it is associated a random field $h_i$ taken from a Gaussian probability $\rho(h) = \exp(-h^2/2R^2)/\sqrt{2\pi R}$, with variance $R$. The Hamiltonian thus reads

$$ H = -J \sum_{\langle i,j \rangle} s_i s_j - H \sum_i s_i + \sum_i h_i s_i $$

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\[ F = -\sum_{(i,j)} J s_i s_j - \sum_i (H + h_i) s_i, \]  

(1)

where the first sum is restricted to nearest-neighbors pairs. The \( T = 0 \) equilibrium problem amounts to find the minimum of \( F \) for a given realization of the random-fields (i.e. the GS). This can be achieved for in a polynomial CPU-time [1], with exact combinatorial algorithms. The one dimensional case can instead be solved exactly [4].

For the out of equilibrium case, we consider the dynamics proposed in Refs. [5]: at each time step the spins align with the local field \( s_i = \text{sign}(J \sum_j s_j + h_i + H) \), until a metastable state is reached. This dynamics can be used to obtain the hysteresis loop. The system is started from a state with all the spin down \( s_i = -1 \) and then \( H \) is ramped slowly from \( H \rightarrow -\infty \) to \( H \rightarrow \infty \).

The main hysteresis loop selects a series of metastable states, which in principle are not particularly close to the ground state. To obtain low energy states, we perform a demagnetization procedure: the external field is changed through a nested succession \( H = H_0 \rightarrow H_1 \rightarrow H_2 \rightarrow \ldots \rightarrow H_n \rightarrow \ldots \rightarrow 0 \), with \( H_{2n} > H_{2n+2} > 0 \), \( H_{2n-1} < H_{2n+1} < 0 \) and \( dH \equiv H_{2n} - H_{2n+2} \rightarrow 0 \). In \( d = 3 \) simulations we perform an approximate demagnetization using \( dH = 10^{-3} \), while in \( d = 1 \) we obtain the exact solution in the limit \( dH/dt \rightarrow 0 \) [6].

In Fig. 1 we plot the energy difference between the GS and the DS in \( d = 3 \) as a function of disorder for different system sizes \( L \). While the curves clearly change with the system size, there is no indication that in the thermodynamic limit \( L \rightarrow \infty \) the two energies converge to the same value. This is confirmed by the exact result in \( d = 1 \), reported in the inset, which is obtained directly in the thermodynamic limit. Also in this case there is a region in which the energies of the GS and the DS differ substantially. The energy differences are small when disorder is large, since almost each spin trivially aligns with its local field. [3,5]. For low disorder, deep in the ferromagnetic state, which is only present in \( d = 3 \) \([3,5]\), the differences between GS and DS are again small, reflecting the fact that almost all spins point in one direction.

In conclusions, our analysis indicate that demagnetization is not an efficient optimization tool, apart from the cases in which the DS and GS are trivial. We have performed a detailed analysis of the domain structures in the DS and the GS and found that the domain structures are not easily related by local spin flips. This cast some doubt on the wide applicability of optimization algorithms based on demagnetization.

**References**

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