Synthesis of an algorithm for interference immunity

I N Kartsan\textsuperscript{1}, V N Tyapkin\textsuperscript{2}, D D Dmitriev\textsuperscript{3}, A E Goncharov\textsuperscript{4}, P V Zelenkov\textsuperscript{5}, I V Kovalev\textsuperscript{6}

\textsuperscript{1} Associate professor, Reshetnev Siberian State Aerospace University, Krasnoyarsk, Russia
\textsuperscript{2} Associate professor, Siberian Federal University, Krasnoyarsk, Russia
\textsuperscript{3} Associate professor, Siberian Federal University, Krasnoyarsk, Russia
\textsuperscript{4} Associate professor, Reshetnev Siberian State Aerospace University, Krasnoyarsk, Russia
\textsuperscript{5} Associate professor, Reshetnev Siberian State Aerospace University, Krasnoyarsk, Russia
\textsuperscript{6} Professor, Reshetnev Siberian State Aerospace University, Krasnoyarsk, Russia

E-mail: kartsan2003@mail.ru

Abstract. This paper discusses the synthesis of an algorithm for adaptive interference nulling of an 8-element phased antenna array. An adaptive beam-forming system has been built on the basis of the algorithm. The paper discusses results of experimental functioning of navigation satellite systems user equipment fitted with an adaptive phased antenna array in interference environments.

Keywords: adaptive phased antenna array, adaptive nulling algorithms for phased antenna arrays, navigation satellite systems

1. Introduction

Navigation satellite systems accommodating the demands of various customer groups have been increasingly gaining popularity in the last years. Among the most important fields is aviation where precise navigational flight data, landing and piloting are provided by satellites.

Spatial selection methods are considered to be the most promising for interference cancelling. These methods are based on the usage of adaptive phased antenna arrays [1]. The theoretical part of their usage had been thoroughly studied by scientists both in Russia and abroad. However, the usage of phased antenna arrays in user satellite navigation equipment has a number of specific features that require further research (exploitation in three frequency ranges, low desired signal level and receiving desired signals from various directions).

In this paper we shall discuss the method for synthesizing an adaptive interference nulling algorithm for an 8-element phased antenna array. The algorithm became the basis for building a device for noise compensating. The analog part of the adaptive phased antenna array comprises 8 antenna modules with 8 radio links connected to them. Signals from the radio links arrive at the analog-to-digital converter and further at the digital section; it contains 12 independent signal processing channels (GLONASS \(L_1, L_2\)). Each channel comprises a beam-
forming circuit and a digital signal processing channel. The beam-forming arrangement is a weighted adder for signals from 8 analog highways. The weighted coefficients are calculated by a signal processor in accordance with the formation of the required radiation pattern.

Performed tests of the 8-element adaptive phased antenna array have demonstrated that the synthesized adaptive interference nulling algorithm is highly effective and enables suppression of both narrowband and broadband interference.

The interference suppression coefficient using spatial filtering methods was:
- 40–45 dB (for harmonic interference);
- 35–40 dB (for noise interference with a limited spectrum);
- 30–35 dB (for broadband interference).

The widespread usage of various electronic devices and the stable increase in their number has become a major issue in terms of the exploited frequency range limitability and the electromagnetic environment becoming constantly more complicated. This has become a problem for the functioning of electronic devices. In such situations, improving interference immunity in the user satellite navigation equipment has become an utmost concern.

In [2] it had been demonstrated that the choice of the effectiveness criterion with consideration for the weighted vector of the phased antenna array does not matter significantly. The choice of the control algorithm for building the radiation pattern of the array – the algorithm has direct impact on the speed of the transition process and on the complexity of implementing the whole system. In [3] an adaptive phased antenna array with primary equal amplitude distribution had been discussed. The effectiveness of its adaptive algorithm using a basis in the shape of a partial exponential Fourier exponential series had been determined in comparison with the optimal algorithm.

2. Algorithm synthesis

In this paper a synthesis of an algorithm for the 8-element circular phased antenna array adaptive nulling system has been performed.

The received signal in a general situation is described by a vector function that considers its description in time and space. Differences between the desired signal and the interfering signals are used to distinguish the former from the latter. From a mathematical position these differences are conveniently addressed through a dependence of signals and interference from parameters, which can be both spatial and time-frequent. Thus, the taken oscillation in the general case is

\[ y(t) = x(t, \alpha, \beta) + n(t, \nu), \]

where \( x(t, \alpha, \beta) \) is the received desired signal vector with parameters \( \alpha \) and \( \beta \); \( \alpha \) is the informative parameter vector (phase, delay time, Doppler frequency, etc.); \( \beta \) is non-informative parameter vector that are caused by signal fluctuation; \( n(t, \nu) \) is the interference oscillation vector; \( \nu \) is the interference parameter vector.

Let’s assume that the interference is a vector accidental stationary process. The typical law of interfering signal oscillation is physically justified for most situations, since the interference normalizes in the comparatively narrowband interference paths of receivers.

The received signal phase in each antenna element of the array will be determined by the coordinates of this element, the direction from which interference and signals are transmitted and the angle of the beam.
In order to determine the vector of the amplitude and phase signal distribution it is necessary to know the direction from which the signal comes and the coordinates of the antenna element and the radiation pattern.

The direction from which the signal comes can be supposed as a single vector from a selected antenna element or phase center of the array to the signal source [4]

\[ L(\psi, \gamma) = (\cos(\psi)\cos(\psi) \sin(\psi)\cos(\gamma) \sin(\psi))^T, \]  

(2)

where \( \psi \) is the azimuth to the signal source (navigation satellite); \( \gamma \) is the angular altitude to the source of interference. The structure of the phased antenna array is determined by matrix \( A \) in which the Cartesian axles \( x, y, z \) are written in the \( k \) column of the \( i \) antenna element in relation to the phase center of the array, which matches the circumference center of the radius \( (R) \) on which the antenna elements are placed.

\[
A = R \begin{bmatrix}
0 & \frac{\sqrt{2}}{2} & 1 & \frac{\sqrt{2}}{2} & 0 & -\frac{\sqrt{2}}{2} & -1 & -\frac{\sqrt{2}}{2} \\
1 & -\frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} & -1 & 0 & \frac{\sqrt{2}}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix},
\]  

(3)

Let’s suppose that the phased antenna array elements are not targeted; their position is shown in Fig. 1.

Figure 1. Distribution of antenna elements in the array

The vector from the phase center of the array (its coordinates for this case are \((0 \ 0 \ 0)^T\)) to the \( k \) element a difference in the coordinates of the circumference center and array elements. In this case, it is just the \( k \) column of matrix \( A \), i.e. \( A^{\text{ck}} \). Thus, the vector of the amplitude and phase distribution through phase elements and setting beam direction to the \( i \) signal source shall be determined by the following

\[
H(\psi, \gamma) = \left\{ F_i(\psi, \gamma) \cdot \exp\left\{ -2\pi i \cdot (RA^{\text{ck}}, L(\psi, \gamma)) \right\} \right\},
\]  

(4)

the \( k \) element of the array; \( (RA^{\text{ck}}, L(\psi, \gamma)) \) is the dot product.

Spatial processing algorithms (5) are typically modified in actual practice.
\[ \hat{Y}_\Sigma(t) = \hat{Y}^T \begin{bmatrix} \Phi^{-1} \hat{X}(\alpha) \end{bmatrix} = \hat{Y}^T(t) \hat{R}(\alpha), \]  

where \( \hat{Y}_\Sigma(t) \) is the output signal of the adaptive antenna array; \( \hat{Y}^T \) is the vector of the input signal of the array; \( \Phi^{-1} \) is the inverse correlative interference matrix; \( \hat{R}(\alpha) \) is the weighted coefficients vector. For this the requirements for a simpler construction of a spatial processing device are considered. The algorithm is modified in terms of multiplication of the vector-columns of the received \( \hat{Y}(t) \) and anticipated \( \hat{X}(\alpha) \) oscillations by a transforming matrix \( \hat{A} \) [5]. The transformation of the vector-column can be determined by the formula

\[ \hat{Y}_{np}(t) = \hat{A} \hat{Y}(t), \quad \hat{X}_{np}(\alpha) = \hat{A} \hat{X}(\alpha). \]  

The correlative matrix \( \Phi \) and the inverted \( \Phi^{-1} \) are also transformed. The weighted vector-column \( \hat{R}(\alpha) \) undergoes transformation as well. The transformed values considering (6) shall equal:

\[ \Phi_{np} = M_{tt} \begin{bmatrix} \frac{Y_{np}(t) Y_{np}^T(t)}{2} \end{bmatrix} = \hat{A} \Phi \hat{A}^T, \]
\[ \Phi_{np}^{-1} = (\hat{A} \Phi \hat{A}^T)^{-1}, \quad \hat{R}_{np}(\alpha) = \Phi_{np}^{-1} \hat{X}_{np}(\alpha). \]  

The transformation structure of the spatial processing algorithm is identical to the transformed

\[ \hat{Y}_{\Sigma np}(t) = \hat{Y}_{np}^T(t) \hat{R}_{np}(\alpha) = \hat{Y}_{np}(t). \]  

When comparing algorithms (5) and (8) it is easy to see that they are totally equivalent providing that the transformation matrix \( \hat{A} \) has an inverted \( \hat{A}^{-1} \). So, if we substitute in (8) values \( \hat{Y}_{np}(t) \) and \( \hat{R}_{np}(\alpha) \) from (6) and (7), the result shall be

\[ \hat{Y}_{np}(t) = \hat{A}^T (\hat{A} \hat{A}^{-1} (\hat{A}^*)^{-1} A \hat{X}(\alpha) = \hat{Y}(t) \hat{R}^*(\alpha) = \hat{Y}_\Sigma(t). \]  

Matrix \( \hat{A} \) is used for building circuits with a main receiving channel and \((M-1)\) compensating receiving channel with low-directivity features of the directivity. When selecting a transforming matrix it is necessary to consider the requirement for decoupling of the main and compensating channels for the desired signal. This is one of the possible examples for such a matrix:

\[
\begin{bmatrix}
X_i(\alpha) & X_2^*(\alpha) & X_i^*(\alpha) & X_{m-1}(\alpha) & X_m^*(\alpha) \\
-X_i^*(\alpha) & X_2^*(\alpha) & 0 & \cdots & 0 & 0 \\
0 & -X_i^*(\alpha) & X_2^*(\alpha) & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & \cdots & -X_{m-1}^*(\alpha) & X_m^*(\alpha)
\end{bmatrix}.
\]  

The transformed vector-column \( \hat{Y}_{np}(t) = \parallel Y_{np}(t) \parallel \) of received oscillations \( \hat{Y}(t) \) switches on the voltage of the main channel

\[ \hat{Y}_{np1}(t) = \sum_{i=1}^{M} \hat{Y}_i(t) \hat{X}_i^*(\alpha). \]

And the voltage of the compensating channels

\[ \hat{Y}_{npk}(t) = -\hat{Y}_i(t) \hat{X}_i^*(\alpha) + \hat{Y}_{i+1}(t) \hat{X}_{i+1}^*(\alpha). \]
The expected oscillation \( X(\alpha) \) is transformed to
\[
\dot{X}_{np}(\alpha) = \dot{A}X(\alpha) = \begin{bmatrix} M & 0 & 0 & \cdots & 0 \end{bmatrix} \tag{13}
\]
where \( M = \sum_{j=1}^{M} \ddot{X}_j(\alpha)\ddot{X}_j(\alpha) \).

The zero value of the elements of vector-column \( \dot{X}_{np}(\alpha) \), beginning from the second indicate to the decoupling of the main and compensating channels for the desired signal. The latter is a mechanism for forming compensating channels with differential schemes. Due to this fact, a null is formed in the resulting directivity characteristics of the compensating channels. The angular position of this null matches the expected direction of the desired signal input \( \alpha \).

The expression for the transformed weighted vector-column of spatial processing \( \dot{R}_{np}(\alpha) \) can be found using the following matrix equation
\[
\Phi_{np} \dot{R}_{np}(\alpha) = \dot{X}_{np}(\alpha). \tag{14}
\]

It is convenient to solve, writing down \( \Phi_{np} \) and the vector-column \( \dot{R}_{np}(\alpha) \) in the following manner
\[
\Phi_{np} = \begin{bmatrix} \Phi_{np11} & \Phi_{np1}^T \\ \Phi_{np1} & \Phi_{npc} \end{bmatrix}, \quad \dot{R}_{np}(\alpha) = \begin{bmatrix} \dot{R}_1 \\ \dot{R}_2 \end{bmatrix} \tag{15}
\]

where \( \Phi_{np11} \) is the total interference and noise dispersion at output of the main receiving channel
\[
\Phi_{np11} = M_{II} \begin{bmatrix} \dot{Y}_{np1}(t)\ddot{Y}_{np1}(t) \\ \frac{1}{2} \end{bmatrix}; \tag{16}\]

\( \dot{\Phi}_{np1} \) is the vector-column of relative correlative moments of interference voltage in the compensating channels and main channel
\[
\dot{\Phi}_{np1} = M_{II} \begin{bmatrix} \dot{Y}_{npk}(t)\ddot{Y}_{np1}(t) \\ \frac{1}{2} \end{bmatrix}, \quad k = 2M; \tag{17}\]

\( \Phi_{npc} \) is the correlative interference matrix of the compensating channels
\[
\Phi_{npc} = M_{II} \begin{bmatrix} \dot{Y}_{npk}(t)\ddot{Y}_{npk}(t) \\ \frac{1}{2} \end{bmatrix}; \tag{18}\]
\( \hat{\mathbf{R}}_1 \) and \( \hat{\mathbf{R}}_2 \) represent the element of the weighted vector of the main channel and the vector-column of the weighted coefficients of the compensating channels respectively. Substituting (15–18) in matrix equation (14) we produce a system of two equations

\[
\begin{align*}
\Phi_{np1} \hat{\mathbf{R}}_1 + \Phi_{np2}^T \hat{\mathbf{R}}_2 &= M, \\
\Phi_{np1} \hat{\mathbf{R}}_1 + \Phi_{npk} \hat{\mathbf{R}}_2 &= 0
\end{align*}
\]  

(19)

Solving this system in relation to \( \hat{\mathbf{R}}_1 \) and \( \hat{\mathbf{R}}_2 \), we shall get

\[
\begin{align*}
\hat{\mathbf{R}}_1 &= \frac{M}{(\Phi_{np1} - \Phi_{np}^T \Phi_{npk} \Phi_{np1})}, \\
\hat{\mathbf{R}}_2 &= -\Phi_{npk} \Phi_{np1} \hat{\mathbf{R}}_1
\end{align*}
\]  

(20)

From (20) we can see that the first element of the weighted vector is the real quality and functions as a normalization factor. Without quality loss for detection this factor can be taken as equal to zero, shifting to the normalizing weighted vector \( \frac{\mathbf{R}_{np}(a)}{\mathbf{R}_1} \). According to (20), the latter shall be equal

\[
\hat{\mathbf{R}}_\Delta = \frac{\hat{\mathbf{R}}_2}{\mathbf{R}_1} = -\Phi_{npk} \Phi_{np1}
\]  

(21)

The output voltage of the processing device with a designated main channel is determined by the formula

\[
\hat{Y}_\Sigma = \hat{Y}_{np1}(t) + \hat{Y}_{npk}^T(t) \hat{\mathbf{R}}_\Delta^*.
\]  

(22)

The device for compensating interference received by the side lobes of the antenna’s radiation pattern is built with a synthesized algorithm; the device is shown in Fig. 2.

Figure 2. Adaptive phased antenna array beam-forming system
The analog sector of the adaptive phased antenna array comprises 8 antenna modules with 8 connected radio paths. Signals from the radio paths are transmitted to the analog-to-digital converter and then to the digital sector. The digital sector comprises 12 independent channels for processing $L_1$, $L_2$, GLONASS signals. Each channel consists of a beam-forming network and a digital signal processing channel. The beam-forming network is a weighted signal adder for signal from 8 analog paths. Weighted coefficients are calculated by the signal processor according to the desired radiation pattern. Particularly, during a high-precision measuring mode the weighted coefficients are selected to compensate the difference in the desired signal passage between antennas: thus, desired signals arrive at the adder with one phase. An example of such a mode for adaptive phased antenna arrays is discussed in [6].

Let’s demonstrate the test results of a model of anti-interference navigation satellite systems user equipment with an 8-element phased antenna array. Comparison was performed on type MRK-33 user equipment.

The anti-interference test is performed in two modes of the navigation satellite system: signal grabbing and signal tracking of navigation satellite signals. It is necessary to calibrate the measuring circuits, using the method described in [7].

The evaluation criterion for interference immunity during natural tests is such a level of the interfering signal during which there is an absence of a solution to the navigation problem for at least four satellites that are in tracking and grabbing modes. Interference immunity is evaluated with and without using spatial filtering methods.

Increasing the interference level with a spacing of 1 dB from the original value, a value was recorded during which there was a loss in tracking (navigation data output stopped) – the level of suppressing the tracking scheme. Counting is begun from the point when one satellite is excluded from processing; it is continued until all prior visible satellites are excluded.

To evaluate the interference immunity of a phased antenna array in a grabbing mode, the interfering signal level is set at 10 dB higher than the level of tracking loss. An attenuator is used to gradually decrease the interference level with a spacing of 1 dB at output; grabbing of the navigational signal and navigational sightings were recorded.

In Figs. 3–5 the output spectra of signals at an intermediate frequency without any spatial processing (blue highlighting) is demonstrated; spatial filtering is shown in red. Approximately 100 000 samples were used to build the spectra.

![Output spectra of signals under narrowband interference attack](image)

**Figure 3.** Output spectra of signals under narrowband interference attack: blue – without spatial selection; red – with spatial selection
For the experiment on the interference immunity of the user navigation equipment under harmonic interference attack, the initial level of interference power was set at 10 dB lower than the minimal level for narrowband interference for devices lacking any interference protection. This issue was discussed in [8].

Fig 4 demonstrates the output spectra of signals attacked by noise interference with a limited spectrum.

![Figure 4. Output spectra of signals under noise interference attack with a limited spectrum: blue – without spatial selection; red – with spatial selection](image)

Employing spatial selection methods during noise interference attack with limited spectrum (Fig. 4) provides an interference suppression coefficient of 30 – 35 dB for user equipment.

In Fig 5 the input spectra of signals attacked by broadband noise interference is demonstrated.

![Figure 5. Input spectra of signals under broadband noise interference attack: blue – without spatial selection; red – with spatial selection](image)

The analysis of the graphs in Fig. 5 shows that spatial filtering sustains the interference suppression coefficient for broadband interference at 25–30 dB higher for user navigation equipment with an adaptive phased antenna array than for equipment without any interference protection.

3. Conclusion

Thus, the performed natural experimentations of an 8-element adaptive phased antenna array demonstrate that the synthesized algorithm for adaptive nulling of phased antenna arrays
is highly effective and is capable of suppressing both narrowband and broadband interfering signals.

The interference suppression coefficient using spatial filtering methods was:
- 40–45 dB for harmonic interference attack;
- 30–35 dB for noise interference attack with a limited spectrum;
- 25–30 dB for broadband interference attack.

References

[1] Electronic systems: basics of assembly and theory (Radioelektronnye sistemy: osnovy postroenii i teorii) 2007 Ed. 3 Ia D Shirman Ed. Moscow: Radiotekhnik

[2] Tyapkin V N, Dmitriev D D, Konnov V G, Fomin A N 2012 Method of determination of spectrum factors vector due to the likelihood ratio criterion Vestnik SibGAU Issue 3 (43) P 76–79

[3] Tyapkin V N, Dmitriev D D, Garin E N, Sokolovskiy A V 2013 Operation of amplitude-phase distribution of adaptive phased array Industrial Automatic Control Systems and Controllers No. 4 P 66–71

[4] GLONASS. Basic concepts and operation [GLONASS. Printsipy postroeniia i funktsionirovaiia] 2010 4th ed., Perov A I, Kharisov V N Eds. Moscow: Radiotekhnik

[5] Almazov V B, Manzhos V N 1985 Gathering and processing radar data [Poluchenie i obrabotka radiolokatsionnoi informatsii]. Kharkov

[6] Sharfunova T G, Tyapkin V N, Dmitriev D D 2013 The measurement accuracy of navigation parameters in user equipment GLONASS satellite navigation system, equipped with an antenna array Radiotekhnik No. 6 P 22–25

[7] Tyapkin V N, Fateev Yu L, Dmitriev D D, Konnov V G 2012 Calibration of measuring path for ECM test of navigation equipment of customers of satellite navigation systems Vestnik SibGAU Issue 4 (44) P 139–142

[8] Tyapkin V N, Dmitriev D D, Moshkina T G 2012 Potential interference immunity of navigation equipment of customers of satellite radio navigational systems Vestnik SibGAU Issue 3 (43) P 113–119