Detection of gravitational waves with quantum encryption technology

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Abstract

We propose a new technique for detecting gravitational waves using Quantum Entangled STate (QUEST) technology. Gravitational waves reduce the non-locality of correlated quanta controlled by Bell’s inequalities, distorting quantum encryption key statistics away from a pure white noise. Gravitational waves therefore act as shadow eavesdroppers. The resulting colour distortions can, at least in principle, be separated from noise and can differentiate both deterministic and stochastic sources.

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Quantum cryptography provides a stunning application of Einstein-Podolsky-Rosen (EPR) correlations and Bell’s inequalities [1–7]. Not only does it promise perfectly secure key distribution but we argue that it may also allow the detection of the shadowy traces of gravitational waves whose existence is the most important outstanding test of Einstein’s General Relativity and the subject of massive current and next-generation experiments [8–11].

A general quantum encryptographic scheme consists of key generation using entangled quantum states, by two parties (Alice and Bob) interested in communicating securely. An attack may be made by an eavesdropper (Eve) who secretly attempts to determine the key as the pairs of entangled EPR quanta travel to Alice and Bob. By performing a sequence of measurements on these entangled pairs of photons, Alice and Bob determine the key they will use to encrypt their message. The vital advantage quantum mechanics provides lies in the impossibility that Eve can intercept the secret key, made up of individual quanta, without giving away her presence to Alice and Bob, since such interception unavoidably alters the entanglement of the EPR pairs, as measured by violations of Bell’s inequalities.

Variants of the standard BB84 protocol [1,2] based on the transmission of single pairs of EPR photons have been used recently in practical quantum key distribution over optic fiber networks up to 48 km in length [12]. Similar experiments [7,13,14] have illustrated the feasibility of quantum encryption in practical situations and the field is now sufficiently mature to be a tool in fundamental research beyond the foundations of quantum mechanics.

In this letter we propose a simple gedanken experiment to detect the effects of gravitational waves through the distortions they cause in the statistics of the quantum keys determined by Alice and Bob. The use of quantum encryption technology may be implemented in at least two ways: one based on randomly swapping polarisers, the second based on laser interferometry.

Consider the Ekert protocol [3] in which Alice and Bob are equipped with randomly swapping polarizers. Entangled pairs of photons are emitted in the singlet state

$$\lvert \Psi \rangle = \frac{1}{\sqrt{2}} (\lvert H \rangle_A \lvert V \rangle_B + \lvert V \rangle_A \lvert H \rangle_B),$$

(1)
where the photon $A$ is sent to Alice, and the photon $B$ to Bob. $H$ and $V$ denote the horizontal and vertical polarizations, prepared by a laser coupled to a parametric down-conversion device. The arrival time of the photons at the polarizers is synchronized with their random swapping.

If a polarizer happens to be correctly oriented, the incident photon is detected, and a “1” is recorded. Otherwise a “0”. Repetition generates two equal length binary strings $A$ and $B$, corresponding to the measurements of Alice’s and Bob’s detectors (see Fig. (1)).

Alice and Bob then publicly announce the orientations of their polarizers corresponding to each element in $A$ and $B$. They then eliminate the elements of $A$ and $B$ corresponding to non-coincident orientations of the two polarizers. The string entries of the remaining subsets of $A$ and $B$ form the two quantum keys, $K_A$ and $K_B$. In the absence of gravitational waves and noise the two keys coincide, $K_A = K_B$, since the photon pairs were perfectly entangled.

With the keys determined Alice proceeds with the transmission of a message encrypted with her key, using e.g. logical AND or XOR, to Bob, who decodes it with his key, $K_B$. However, this is not of interest to us. Instead, cross-correlation of the keys $K_A$ and $K_B$ allows, in principle, the detection of gravitational waves.

This detection proceeds thanks to a fundamental property of quantum cryptography: the keys derived from an ideal experiment are Markovian, pure white noise random strings of “0”s and “1”s. The presence of a gravitational wave colours the cross-correlation statistics so they are no longer white (see Figures 1 and 2).

A gravitational wave introduces a discoloration by changing the arrival time of the photons at Alice and Bob, by altering the detectors’ local time and the path length travelled by the photon. This implies that, in the key strings $K_A$ and $K_B$, the probability of a “1” (a detection) is no longer equal to the probability of a “0” (a non-detection). In addition the two strings will no longer coincide element by element: $K_A \neq K_B$. In order to analyse this effect one may construct the cross-correlation matrix between $K_A$ and $K_B$ (see Fig. 2) and search for off-diagonal power.

Alternatively it is convenient to consider the string $K \equiv K_A \otimes K_B$ formed using an
appropriate operator $\otimes$, such as logical AND. We then define the accumulated fluctuation $\xi(t)$ as the absolute value of the difference, for a given temporal length $t$, of the number of non-detections, $N_{[0]}$, and detections, $N_{[1]}$, in $K$, viz. $\xi(t) \equiv |N_{[1]} - N_{[0]}|$.

$\xi(t)$ then obeys the stochastic differential equation

$$\frac{d \xi}{dt} = \frac{\Gamma_{ph}}{2}(w(t) + h(t)),$$

where $\Gamma_{ph}$ is the photons pair rate, $w(t)$ is a stochastic process which describes the intrinsic noise of the system and $h(t)$ is the strain produced by the gravitational wave. The key feature of this equation is that, since the intrinsic noise $w(t)$ is due only to the effective randomness of the polarizer, it has the ideal statistical properties

$$\langle w(t) \rangle = 0, \quad \langle w(t)w(t') \rangle = D \delta(t - t').$$

In the idealized case where complex and experiment-specific noise sources (such as thermal and seismic fluctuations) are neglected, the intrinsic fluctuations in the time series $\xi$ extracted from the polarizers are described by a frequency-independent random process characterized by the noise spectral density $D$ - the noise-induced mean square fluctuations per unit frequency.

Since QUEST detectors use single photon pairs there is no shot noise and if $h(t) = 0$, the accumulated fluctuation is a random process with zero mean and a linearly increasing variance

$$(\text{Var } \xi)^2 = \langle \xi(t)\xi(t) \rangle = \frac{\Gamma_{ph}^2}{4}Dt.$$
\[
\langle \xi(t) \rangle = \frac{\Gamma_{ph}}{2} \int_0^t d\tau h(\tau).
\]

If instead a stochastic background of gravitational waves is present, the accumulated fluctuations are still described by a zero mean variable, but the gravitational wave affects the variance of \( \xi \), which for a stationary gravitational wave background is

\[
(\text{Var} \, \xi)^2 = \frac{\Gamma_{ph}^2}{4} D t \left[ 1 + \frac{1}{D} \int_0^t d\tau H(\tau) \right]
\]

where \( H(\tau) = \langle h(t)h(0) \rangle \).

In order to assess the sensitivity of our *gedanken* experiment to a gravitational wave we need to estimate the noise background induced by the effective randomness of the polarizer. Consider the data set collected by one observer divided into sub-sets of \( N \) points. Since each point corresponds to a photon, \( N = (\Gamma_{ph}/2)\tau_N \), where \( \tau_N \) is the temporal length of the sub-set. For each data sub-set, the background noise due to the polarizer is then

\[
S_N = D/N \sim 2 \cdot 10^{-43} \text{ Hz}^{-1} \times \left( \frac{10^8 \text{s}^{-1}}{\Gamma_{ph}} \right) \left( \frac{\Theta_{sw}}{10^{-10}} \right)^2 \left( \frac{\tau_{coh}}{10^{-12} \text{s}} \right) \left( \frac{10^3 \text{s}}{\tau_N} \right),
\]

where \( \Theta_{sw} \) is the percentage error in the swapping of the random polarizers and \( \tau_{coh} \) is the photon coherence time. At a given frequency \( f \) and for a data sub-set of about 20 minutes, the characteristic amplitude of the noise induced by the polarization swapper \( h_{rms} = (2fS_N)^{1/2} \) is then \( \sim 6.3 \times 10^{-22}(f/1\text{Hz})^{1/2} \). For comparison, the expected characteristic noise amplitude for the LIGO interferometer around 200 Hz is \( \sim 2 \times 10^{-22} \). The noise associated with the polarizers is therefore minimized at frequencies lower than the typical frequencies where large scale earth-based interferometers reach their best sensitivity.

It is also important to stress that, unlike large scale interferometers, this device effectively operates when the gravitational wavelength \( \lambda_{GW} \) is less than the distance \( d_{AB} \) between the two receivers \( A, B \). In particular, the probability of “detection” and “non-detection” become equal when \( \lambda_{GW} \gg 2d_{AB} \) and therefore there is a low frequency cut-off around \( f \sim c/2d_{AB} \). The precise response of such an experiment will depend on seismic and thermal noise which in turn depend on the exact experimental set-up. Since we are interested in general issues we do not address this important issue here.
An alternative to the swapping polarizer set-up is to exploit a quantum cryptographic protocol based on the continuous detection of photons in interferometers, relying on energy-time correlations rather than on polarization correlations [15]. We shall discuss in detail this implementation and noise-related issues in future work.

To clarify our proposal consider the effects of gravitational waves on the famous Bell inequalities [5] describing quantum non-locality. Both the polarizer and interferometer implementations can be unified in the following formalism. Let \( R_{ij}(\delta_A, \delta_B), i, j = 0, 1 \) be the number of time-correlated events detected by Alice (A) and Bob (B) as a function of instrument parameters \( \delta_i \), which will be polarization orientations or phase shifts in the case of an interferometer setup. The normalized correlation coefficient of the measurements made by the detectors A and B is then [3]

\[
E(\delta_A, \delta_B) = \frac{R_{00}(\delta_A, \delta_B) - R_{01}(\delta_A, \delta_B) - R_{10}(\delta_A, \delta_B) + R_{11}(\delta_A, \delta_B)}{R_{00}(\delta_A, \delta_B) + R_{01}(\delta_A, \delta_B) + R_{10}(\delta_A, \delta_B) + R_{11}(\delta_A, \delta_B)}.
\]

Following [16], one may then define the composite operator

\[
S \equiv |E(\delta'_A, \delta''_B) + E(\delta''_A, \delta'_B) + E(\delta''_A, \delta'_B) - E(\delta'_A, \delta''_B)|
\]

where \( \delta'_i \) and \( \delta''_i \) represent specific values of the parameters \( \delta_i \). The quantity \( S \) allows one to test the degree of violation of Bell’s inequalities; in particular, the value \( S = 2\sqrt{2} \) is achieved for maximal entanglement of the states. Gravitational waves reduce \( S \), as will a general eavesdropper.

We have outlined how quantum encryption technology may be exploited to yield a potentially sensitive detector of weak cosmic gravitational waves. These QUantum Entangled STate (QUEST) detectors are complementary to current and planned interferometric detectors such as LIGO and LISA. While QUEST detectors are not affected by shot noise, detailed understanding of all other relevant noise sources is lacking and depends on exact details of the detector set-up. Hence whether QUEST detectors can reach the sensitivity of the interferometric detectors by exploiting the non-locality fundamental to quantum mechanics is still unknown. What is certain is that gravitational waves will act as shadow
eavesdroppers, reducing the degree of entanglement between quantum states controlled by Bell’s inequalities, which is precisely how they would be detected.

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Captions

Figure 1

Schematic illustration of the proposed experiment: a source of EPR photons repeatedly sends a single pair of polarization-entangled photons to Alice and Bob who are equipped with randomly swapping polarizers. A gravitational wave, by affecting the path length and local proper time of the observer (here Bob) will reduce the probability of detection of the photon causing distortions of the detection statistics used to build the quantum keys (see Figure 2).

Figure 2

The averaged cross-correlation matrix of sample 50-element keys $K_A$ and $K_B$. Inset: The idealized white-noise case (without gravitational waves). The diagonal dominates in the large key length limit where the cross-correlation is simply $\propto \delta_{ij}$. The main figure schematically shows the effects of a deterministic gravitational wave which induces off-diagonal power representing non-white correlations.
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