Novel method for identification of intrinsic vibration modes in piezoelectric crystals

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Abstract. Novel method for identification of intrinsic vibration modes in piezoelectric crystals is introduced. This method is based on recently proposed theoretical model of piezoelectric resonance frequencies dependence on temperature. Calculation of crystal sample eigenmodes is based on Lagrange-Hamilton variation principle. Comparison of temperature dependencies of calculated resonance frequencies with measured ones reveals reliable method for identification of vast majority of the experimentally observed resonances. Influence of sample sizes variation on eigenmode frequencies is also considered.

1. Introduction
Resonant ultrasound spectroscopy (RUS) technique has been developed since 1960-th [1, 2]. At present one of the main task of RUS is determination of elastic and piezoelectric constants of various materials [3-5] as well as internal friction [6]. Elastic constants can be associated with free energy, Debye temperature, specific heat capacity and with other important parameters of the material. Decay of acoustic waves, which can be described by implementation of imaginary parts in elastic constants, gives information about unharmonicity and other relaxation mechanisms that occur in the material.

Obviously elastic constants can be determined by measuring velocities of sound waves in the given material. This principle is used in methods where propagation time of sound pulse in the sample is determined. Measurements of intrinsic vibration modes of the samples have been conducted since 1935. At that time there were no accurate methods for numerical calculations of eigenfrequencies of samples with complex geometry. However for isotropic noncrystalline cubic and spherical bodies solutions were found in 1880. In 1964 year the technique for determination of elastic properties of solids together with its dependence on temperature was demonstrated [1]. In that paper elastic and anelastic properties of sphere samples were determined by measuring its intrinsic vibration spectrum. Later samples in the form of rectangular parallelepiped were usually used in RUS, mostly due to the relative simplicity of manufacturing. More attention was paid to the low symmetry crystals.

In RUS the elastic properties of material are determined by measuring resonance vibration frequencies of sample. Substantial advantage of RUS technique is that whole set of elastic constants can be determined from single measurement, provided that experiment is performed correctly. Usually sample in the form of rectangular parallelepiped is fixed along its diagonal between two piezoelectric transducers. One of the transducers is used for excitation of sample vibrations at certain frequency, which can be tuned, and another one acts as a receiver. When excitation frequency coincides with any
intrinsic vibration frequency of the sample the resonance peak can be observed in response. Intrinsic vibration frequencies of the sample depend on elastic constants, material density as well as sample form and sizes. Usually samples of millimeter dimensions are used and measured resonance frequencies lie in MHz range. Elastic constants can be determined from the resonance spectrum provided that all other necessary parameters are measured independently. It should be mentioned that mechanical contact of the transducers can considerably affect resonance frequencies. External influence is eliminated as much as possible in noncontact RUS [7]. For example if material possess piezoelectric properties the most convenient way to excite vibrations in it is to use external radiofrequency electric field, which can be applied avoiding mechanical contact between sample and electrodes.

From calculating point of view the problem of elastic constants determination is laborious enough. It includes both direct calculations of resonance frequencies for given set of elastic constants and inverse problem of identification of calculated eigenmodes with measured ones. That’s why RUS methods became popular only after development of powerful computers. Milestone works concerning RUS appeared in early 1970-th [8, 9]. It was demonstrated that exact analytical solutions of eigenmode problem can be obtained only for samples with simple geometry such as plate, sphere, etc.

Following iteration procedure is used for calculation of elastic constants from experimentally measured resonance spectrum. Resonance frequencies of the sample are calculated using some initial set of elastic constant values. Then calculated spectrum is compared with measured one. By varying values of elastic constants one try to attain exact agreement between measured and calculated resonance spectra. Accuracy of calculated values of elastic constants depends on the amount of measured resonances. Its number should exceed 5-10 times that of independent material constants to be determined. Still in case of large quantity of detected resonances the convergence of described iteration procedure and as follows correspondence of calculated intrinsic mode with particular mode, measured experimentally, is under question. Besides, it is often happens so that in certain frequency range the number of experimentally observed modes turns out to be much less in comparison with calculated ones. It means that for accurate mode identification additional criteria should be applied. The most robust method is laser interferometry technique used for visualization of resonance vibration amplitude distribution at sample surface [10]. Despite this identification method is reliable it is also laborious enough because usually resonance amplitude values are of several nanometers order.

2. Mode identification

2.1. Calculation of intrinsic vibration modes

For determination of resonance frequencies for bodies of different geometry most common approach is to use Lagrange-Hamilton variation principle. Eigenmodes of piezoelectric crystal can be found by varying spatial part of Lagrangian of the system that can be written as follows [3]

\[
J_S = \int\int\int_\Omega \left( \rho w^2 u_{ij}^2 - c_{ijkl} S_{ij} S_{ij} + \varepsilon_{ij} E_i E_j + 2 d_{ijkl} S_{ij} E_k \right) d\Omega.
\] (1)

Here \( F_j = \partial F_j / \partial x_j \) denotes derivative; \( c_{ijkl}, d_{ijkl}, \varepsilon_{ij} \) are elastic, piezoelectric and dielectric tensors respectively; \( \rho \) is density of the sample; \( w \) is vibration frequency; \( u_i(x_1,x_2,x_3) \) are the components of mechanical displacement of the sample points; \( S_{ij} = 0.5(u_{i,j} + u_{j,i}) \) is strain tensor; \( E_i = -\varphi_i \) are components of electric field vector; \( \Omega \) is the crystal volume.

In order to solve numerically problem (1) the Rayleigh-Ritz method is usually used. Following this method unknown functions of displacement \( u_i \) and potential \( \varphi \) are expanded using some set of orthonormal basis functions \( \{ \psi_{pq} \} \) of the total number \( N_b \). It means that equations for eigenmodes can be obtained by substitution of series expansions

\[
u_i(x_1,x_2,x_3) = \sum_{p=1}^{N_b} C^u_{i,p} \psi_{p}(x_1,x_2,x_3), \quad \varphi(x_1,x_2,x_3) = \sum_{q=1}^{N_b} C^\varphi_{q} \psi_{q}(x_1,x_2,x_3),
\] (2)

into Lagrangian (1). As a result we obtain function that depends on the expansion coefficients \( C \)

\[
J_S(C) = \omega^2 \sum_{pq} C^u_{i,p} \psi_{i}^{*} \psi_{p} - C^\varphi_{q} C^{\varphi}_{q} \sum_{pq} \psi_{i}^{*} \psi_{p} - 2 \sum_{pq} C^\varphi_{q} C^{\varphi}_{q} \Pi_{pq} + C^{\varphi}_{p} C^{\varphi}_{q} \psi_{pq},
\] (3)
Matrices $W, Y, \Pi$ and $V$ are

\[
W_{pq} = \iiint_{\Omega} \rho \psi_p \psi_q \, d\Omega, \quad Y_{pq}^{ijk} = \iiint_{\Omega} c_{ijkl} \frac{\partial \psi_p}{\partial x_j} \frac{\partial \psi_q}{\partial x_k} \, d\Omega, \\
\Pi'_{pq} = \iiint_{\Omega} d_{ik} \frac{\partial \psi_p}{\partial x_j} \frac{\partial \psi_q}{\partial x_k} \, d\Omega, \quad V_{pq} = \iiint_{\Omega} e_{ij} \frac{\partial \psi_p}{\partial x_j} \frac{\partial \psi_q}{\partial x_k} \, d\Omega.
\]

(4)

After differentiation (3) with respect to expansion coefficients $C$ the minimum of the functional (1) corresponds to the condition of equal to zero derivatives $\partial J_s(C)/\partial C = 0$. In matrix form equations for eigenmodes can be written as

\[
\begin{cases}
(Y - \Pi' V^{-1} \Pi') C^a = W^2 W C^a \\
C^a = -V^{-1} \Pi' T C^a.
\end{cases}
\]

(5)

Here ‘$T$’ and ‘$-1$’ designate transposed and inverse matrix respectively and $C^a = (C^a_1, C^a_2, C^a_3)^T$.

Solution of system (5) gives eigenfrequencies $w_n = 2\pi f_n$ and pertinent spatial distributions of the displacement and potential the piezoelectric resonance thermal coefficients $\kappa^a_{ijkl}$ can be calculated provided that temperature dependence of the elastic constants $c_{ijkl}(\theta)$ is known. It was estimated that contribution to resonance frequency shift due to other parameters variation with temperature $\theta$ is not determinative.

When all parameters (mainly temperature) are uniform across the sample, then all the material constants in (4) can be taken outside the integral sign, and calculations are greatly simplified. Evaluation of the integrals in this case is reduced to evaluation of the following one:

\[
B^{\beta}_{pq} = \iiint_{\Omega} \frac{\partial \psi_p}{\partial x_j} \frac{\partial \psi_q}{\partial x_k} \, d\Omega.
\]

(6)

This integral can be further split into 1-dimensional integrals. More specifically, if we denote

\[
g^0_{p,q} = \int_{-L/2}^{L/2} \psi_p(x) \psi_q(x) \, dx, \quad g^{10}_{p,q} = \int_{-L/2}^{L/2} \psi_p(x) \frac{\partial \psi_q}{\partial x} \, dx, \quad g^{11}_{p,q} = \int_{-L/2}^{L/2} \frac{\partial \psi_p}{\partial x} \psi_q(x) \, dx.
\]

(7)

Then

\[
B^{\beta}_{pq} = \begin{pmatrix}
g^{11}_{p,q} & g^0_{p,q} & g^{10}_{p,q} \\
g^{10}_{p,q} & g^{00}_{p,q} & g^{10}_{p,q} \\
g^{11}_{p,q} & g^{10}_{p,q} & g^{00}_{p,q}
\end{pmatrix}.
\]

(8)

Weak temperature change of the sample can be treated as a perturbation of the initial problem, in which we rewrite first equation in linear system (5) as

\[
\Gamma^{\text{piezo}} C^a = \rho w^2 C^a,
\]

(9)

with $\Gamma^{\text{piezo}} = Y - \Pi' V^{-1} \Pi'$. If we denote a parameter that induces perturbation as $\mu$, then first-order correction to the $n$-th eigenmode of (9) gives

\[
\frac{\partial w_n}{\partial \mu} = \frac{1}{2\rho w_n} \frac{\partial (\Gamma^{\text{piezo}} C^a) \partial \mu}{\partial \mu} C^a_n - \frac{\partial \rho}{\partial \mu} w_n^2.
\]

(10)

Relation (10) enables one to compare changes of the resonance frequency induced by the variation of elastic constants and thermal expansion (density change). If we denote average value of elastic constant as $\langle c \rangle$ and average sample size as $\langle L \rangle$ then

\[
\left\langle \frac{\partial w_n}{\partial \mu} \right\rangle = \frac{1}{\langle L \rangle^2} \int d\langle c \rangle \frac{d\langle L \rangle}{d\mu} - 2 \frac{\partial \langle c \rangle}{\partial \mu} \frac{d\langle L \rangle}{d\mu},
\]

(11)
Typically for crystals the following relation is valid

\[ \frac{1}{\langle c \rangle} \frac{d\langle c \rangle}{d\theta} \gg \frac{1}{\langle L \rangle} \frac{d\langle L \rangle}{d\theta}. \]  

(12)

Now taking into account (12) the equation (10) is reduced to

\[ \frac{\partial \psi_{\lambda}}{\partial \mu} = \frac{C_{\psi,\lambda}}{2\rho w_n} \int \int \int \frac{\partial \psi_{\mu}}{\partial x_j} \frac{\partial \psi_{\nu}}{\partial x_i} \frac{\partial \psi_{\mu}}{\partial x_j} \frac{\partial \psi_{\nu}}{\partial x_i} \, d\Omega \left[ \frac{\partial \psi_{\mu}}{\partial x_j} \frac{\partial \psi_{\nu}}{\partial x_i} \right] C_{\psi,\lambda}^{\mu,\nu}. \]  

(13)

It was demonstrated that for common values of material piezoelectric and dielectric coefficients the contribution of piezoelectric effect to intrinsic vibration mode frequencies doesn’t exceed 0.5% [11]. These frequency shifts occur mostly due to inevitable presence of external forces, conditioned by clamping of piezoelectric specimen between transducers in measurement procedure. Despite frequency shifts produced are small enough still they are several times higher than frequency measurement error typical for nonpiezoelectric crystals.

Dependence on temperature of each resonance of piezoelectric crystal can be characterized by its piezoelectric resonance thermal coefficient:

\[ K_n^{\text{pnt}} = \frac{1}{2\pi} \frac{d\psi_n}{d\theta}. \]  

(14)

In case crystal is uniformly heated its temperature derivatives of elastic coefficients can be taken out from integral sign in (13). Then piezoelectric resonance coefficients can be calculated as follows

\[ K_n^{\text{pnt}} = \frac{\partial C_{ijkl}}{\partial \theta} B_{ijkl} C_{ijkl}^{\psi,\lambda}. \]  

(15)

Consequently full set of elastic constants of the crystal specimen can be deduced from a single frequency scan via multidimensional minimization of the error between the measured and calculated spectra. Spectrum is calculated from the known sample dimensions, its mass, and a set of surmised elastic constants. As it was stated the main problem is that usually number of calculated modes in narrow frequency range greatly exceeds number of resonances that are experimentally observed. Moreover measured resonance amplitudes considerably differ from one another and mathematical criteria for theoretical analysis of amplitude values has not been elaborated yet.

2.2. Piezoelectric resonance frequencies temperature dependence for mode identification

Simple ways of the crystal intrinsic vibration modes identification can be realized by exploiting experimentally controlled variations of sample parameters or external factors that lead to change of resonance frequencies. We have recently introduced novel method for mode identification of piezoelectric crystals by analyzing piezoelectric resonance thermal coefficients \( K_n^{\text{pnt}} \) of the excited resonances [12]. In first approximation elastic constants temperature derivatives can be taken from literature. For each \( n \)-th resonance the value of \( K_n^{\text{pnt}} \) is measured in crystal uniform heating experiment. Then each experimentally observed piezoelectric resonance at \( Rf_n \) frequency is attributed to the calculated one \( Rf_n^{\text{calc}} \) that has closest value of \( K_n^{\text{pnt}} \), computed using equation (15).

Introduced algorithm allows closest fit of experimental vibration spectrum with calculated one because resonance frequency shift with temperature of the certain mode is directly connected with its dependence on each elastic constant. As follows frequency dependence on temperature is also specified by spatial configuration of displacement, i.e. by vibration mode structure.

Measure of difference between calculated and measured resonance spectra can be taken, for example, as follows

\[ M = \sum_{n=1}^{N} \sigma \left( Rf_n^{\text{calc}} - Rf_n \right)^2. \]  

(16)
Here \( N_e \) is total number of experimentally observed resonance modes and \( \sigma_i \) is weighting factor.

In detail algorithm of mode identification and elastic constants determination includes steps that are listed below:

- Processing of the piezoelectric resonance spectra measured for two crystal temperatures, i.e. determination of resonances (frequencies \( Rfn \) and corresponding amplitudes) at these temperatures and determination of piezoelectric resonance thermal coefficients \( K_{n}^{\text{pert}} \) of each mode;
- Calculation of crystal intrinsic vibration modes frequencies \( Rfn^{\text{calc}} \) and corresponding \( K_n^{\text{pert}} \) values using some set of elastic constants and its temperature derivatives;
- Selection of frequency interval \( \Delta f \) (usually about 50 kHz), in which correspondence of certain measured resonance to the calculated one will be sought for.
- Selection from measured spectrum the resonance with highest amplitude and putting it in accordance with calculated mode in region \( \Delta f \) that has closest calculated value of \( K_n^{\text{pert}} \) coefficient to that of the given resonance. This step is performed consequently for other measured resonances in descending order of their amplitudes;
- Calculation of identification error using (16), in which weighting factors correspond to resonance amplitudes;
- Variation of elastic constants for finding best agreement between measured and calculated modes, i.e. minimum of (16).

2.3. Resonance frequency dependence on sample sizes

As it was estimated in (12) the shift of eigenmode frequencies due to variation of sample sizes, induced by crystal temperature change, is much less in comparison with contribution of elastic constants alteration with temperature. However any imperfection of sample form and uncertainty of sample size absolute values may result in substantial difference between measured and calculated resonance spectra even if elastic properties of the material are known for sure. Dependence of intrinsic vibration mode frequencies on variation of sample sizes is considered below.

For estimation we can use equation (10) with perturbation parameter \( \mu \) representing any size \( L \) of the crystal. However in this case derivative of \( \Gamma_{\text{piezo}} \) in respect to size parameter, expressed using designations (4) as

\[
\frac{\partial \Gamma_{\text{piezo}}}{\partial L} = \frac{\partial Y}{\partial L} - \left( \frac{\partial \Pi}{\partial L} V^{-1} \Pi^T - \Pi V^{-2} \frac{\partial V}{\partial L} \Pi^T + \Pi V^{-1} \frac{\partial \Pi^T}{\partial L} \right),
\]

(17)
cannot be directly calculated because basis functions depend on crystal sizes itself. So \( B_{pq}^{L} \) derivatives with respect to \( L \) should be clarified.

Here we shall consider crystal sample in the form of rectangular parallelepiped with sizes \( L_1 \times L_2 \times L_3 \). It is common practice to use Legendre polynomials \( P_{pe}(x_i) \) as basis functions [8, 9]

\[
\psi_p(x_1, x_2, x_3) = \prod_{i=1}^{3} \left[ \frac{2p_{\mu i} + 1}{L_i} \right]^{1/2} P_{p_e}(x_i) \left( \frac{2x_i}{L_i} \right),
\]

(18)
because for such sample shape they form orthogonal system:

\[
\iint d\Omega \psi_p \psi_q d\Omega = \delta_{p_1,p_1} \delta_{p_2,p_2} \delta_{p_3,p_3}.
\]

(19)

Here \( p_{\mu i} \) is polynomial degree specified by \( i = 1,3 \) and \( p = 1, N_b \) where \( N_b \) is total number of basis functions.

It can be demonstrated that due to properties of Legendre polynomials the following expressions for derivatives of \( B_{pq}^{L/} \) can be obtained:
Components $g$ can be calculated using relations (7).

It can be demonstrated that contribution of the first term in (17) is several orders higher than that of the parenthesized sum. So from variation theory the resonance frequency change due to sample size alteration will be given by the following relation

$$\frac{\partial B_{py}}{\partial L_1} = -\frac{1}{L_1} \left[ \begin{array}{cccc} g_{11}^{11} & g_{00}^{00} & g_{10}^{01} & g_{10}^{00} \\ g_{01}^{01} & g_{11}^{11} & g_{01}^{00} & g_{01}^{00} \\ g_{10}^{10} & g_{00}^{00} & g_{00}^{00} & g_{00}^{00} \\ g_{10}^{01} & g_{01}^{00} & g_{01}^{00} & g_{01}^{00} \end{array} \right], \quad (20)$$

$$\frac{\partial B_{py}}{\partial L_2} = -\frac{1}{L_2} \left[ \begin{array}{cccc} 0 & g_{00}^{01} & g_{00}^{10} & g_{00}^{00} \\ 0 & g_{11}^{10} & g_{11}^{10} & g_{11}^{10} \\ 0 & g_{00}^{00} & g_{00}^{00} & g_{00}^{00} \\ 0 & g_{01}^{00} & g_{01}^{00} & g_{01}^{00} \end{array} \right], \quad (21)$$

$$\frac{\partial B_{py}}{\partial L_3} = -\frac{1}{L_3} \left[ \begin{array}{cccc} g_{00}^{00} & g_{00}^{00} & g_{00}^{10} & g_{00}^{00} \\ g_{00}^{00} & g_{00}^{00} & g_{00}^{00} & g_{00}^{00} \\ g_{01}^{00} & g_{01}^{00} & g_{01}^{00} & g_{01}^{00} \\ g_{01}^{00} & g_{01}^{00} & g_{01}^{00} & g_{01}^{00} \end{array} \right]. \quad (22)$$

Piezoelectric resonance dimensional coefficient $K_{p}^{\text{nd}}$ that determines resonance frequency shift with the change of crystal size in $x_i$ direction is

$$K_{p}^{\text{nd}} = \frac{1}{2\pi} \frac{\partial \omega}{\partial L_i}. \quad (24)$$

3. Experimental results

Simplified block-scheme of the experimental setup used for measurement of piezoelectric resonances in crystals is shown in figure 1. Low voltage from radiofrequency (RF) generator is applied to the crystal that is placed between two metallic electrodes forming capacitor. Load resistor $R$ is connected in series with the capacitor. For each value of the electric filed $E$ frequency $f$ the crystal response is analyzed by measuring complex voltage $U_R$ on the load resistor via lock-in amplifier.

![Figure 1. Block-scheme of experimental setup.](image-url)

Resonances correspond to specific peaks that are observed at certain frequencies $Rf_c$ in measured $U_R$ signal, which is directly proportional to the electric current flowing through the crystal. For crystal temperature stabilization the capacitor with crystal in it is placed inside thermostat. Radiofrequency power that affects crystal is of tens microwatts order and doesn’t change crystal temperature.

Experiment was performed with quartz crystal in the form of rectangular parallelepiped with dimensions $3 \times 3 \times 30$ mm³. Crystal is cut along crystallographic axes and its $z$-axis is directed along $x_3$. 
Response spectrum measured for two orientation of quartz crystal at temperature 293 K is introduced in figure 2.

![Figure 2](image_url)

**Figure 2.** Frequency dependencies of the $U_R$ voltage amplitude measured for two orientations of quartz crystal in respect to electrodes.

Table 1 contains partial list of measured resonance parameters (frequencies and piezoelectric resonance thermal coefficients $K_{prt}$) together with calculated ones exploiting introduced identification method, based on analysis of piezoelectric resonance thermal coefficients.

**Table 1.** Parameters of piezoelectric resonances determined for quartz crystal

| Resonance number | Measured $R_f$, kHz | Calculated $R_f^{calc}$, kHz | Disagreement $\Delta R_f$, kHz | Measured $K_{prt}$, Hz/K | Theoretical $K_{prt}$, Hz/K | $\Delta R_f$ induced by 3% increase of $L_3$, kHz |
|------------------|---------------------|-----------------------------|-----------------------------|--------------------------|-----------------------------|----------------------------------|
| 1                | 645.377             | 642.1                       | 3.28                        | -30.0                    | -32.5                       | -21.1                            |
| 2                | 655.485             | 578.1                       | 77.39                       | -50.4                    | -50                         | -29.1                            |
| 3                | 660.002             | 700.7                       | 40.7                        | -30                      | -31.1                       | -23.1                            |
| 4                | 663.0               | 630.3                       | 32.7                        | 12.5                     | 18.4                        | -29.4                            |
| 5                | 804.758             | 805.3                       | 0.54                        | 20                       | 23.6                        | -26.8                            |
| 6                | 909.592             | 897.1                       | 12.49                       | -45                      | -49.7                       | -33.3                            |
| 7                | 911.181             | 909.2                       | 1.98                        | 10                       | 7.9                         | -36.2                            |
| 8                | 926.8               | 905.9                       | 20.9                        | -27.5                    | -26.5                       | -30.2                            |
| 9                | 935.333             | 948.3                       | 12.96                       | -13.3                    | -12.5                       | -30.1                            |
| 10               | 939.548             | 871.3                       | 68.24                       | -23.3                    | -23.4                       | -36.2                            |
| 11               | 950.605             | 949.8                       | 0.81                        | -27.5                    | -27.6                       | -0.2                             |
| 12               | 982.016             | 977.2                       | 4.82                        | -56.7                    | -56.6                       | -5.9                             |
| 13               | 982.338             | 988.5                       | 6.16                        | -36.7                    | -36.4                       | -0.05                            |
| 14               | 1011.315            | 1018.3                      | 6.98                        | 0.5                      | 6.6                         | -1.26                            |

It should be stressed that resonance frequencies are measured with high precision. However accuracy of sample geometry measurement is low enough and doesn’t exceed 5%. Figure 3 represents calculated resonance frequency changes due to increase of crystal size $L_3$ by 3%. Corresponding values of frequency shift induced by $L_3$ change are listed in the last column of Table 1. It can be seen that in this case frequency shifts of certain resonances reach several tens of kilohertz. As an example,
A comparison between measured and calculated frequency values for four modes is introduced in Figure 3 inset. Error bars indicate calculated frequency intervals corresponding to 3% sample size $L_3$ uncertainty.

![Figure 3.](image-url)

**Figure 3.** Calculated resonance frequency change induced by 3% increase of crystal sample size $L_3$. Stars and circles represent $R_f^{\text{calc}}$ initial and resulted values respectively. Inset shows measured (triangles) and calculated frequency values with error bars indicating interval produced by 3% sample size $L_3$ uncertainty.

This means that correct determination of sample geometry parameters is very important for RUS measurements. In our case main disagreement of measured and calculated resonance frequencies is explained by the high uncertainty of crystal sizes.

### 4. Conclusions

Two ways of identification of the sample intrinsic vibration modes are introduced. First one relies on association of measured and calculated piezoelectric resonance thermal coefficients $K_w^{\text{pt}}$ of vibration modes. These coefficients are easily measured. However for $K_w^{\text{pt}}$ numerical calculation the temperature dependence of elastic constants is required. Also sample geometry should be determined with high accuracy. It is demonstrated that even small uncertainty of sample sizes can result in substantial disagreement of absolute values of measured and calculated resonance frequency values. Calculations reveal that for certain vibration modes the frequency change of tenth kilohertz order is produced by 3% variation of sample size. In turn, frequency disagreement of measured and calculated modes, exploiting its frequency dependence on temperature, can be of the same order.

Second method of mode identification that supposes variation of sample sizes can be used without predetermination of elastic constants dependence on external parameters such as temperature, pressure, humidity etc. However, main disadvantage of the last method is irreversible change of sample sizes.

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