ON THE CHEMICAL POTENTIAL OF DARK ENERGY

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Abstract

It is widely assumed that the observed universe is accelerating due to the existence of a new fluid component called dark energy. In this article, the thermodynamics consequences of a nonzero chemical potential on the dark energy component is discussed with special emphasis to the phantom fluid case. It is found that if the dark energy fluid is endowed with a negative chemical potential, the phantom field hypothesis becomes thermodynamically consistent with no need of negative temperatures as recently assumed in the literature.

1 Introduction

The current idea of an accelerating Universe driven by dark energy is based on a large convergence of independent observational results, and its explanation constitutes one of the greatest challenges for our current understanding of fundamental physics [1, 2]. The origin and the nature of dark energy is still a mystery, however, there is no doubt that its existence is beyond the domain of the standard model of particle physics [3].

Among a number of possibilities to describe this dark energy component, the simplest and most theoretically appealing way is by means of a cosmological constant Λ, which acts on the Einstein field equations as an isotropic and homogeneous source with a constant equation of state parameter $p/\rho = -1$. On the other hand, although cosmological scenarios with a Λ term might explain most of the current astronomical observations, from the theoretical viewpoint they are plagued with some fundamental problems thereby stimulating the search for alternative dark energy models driven by different candidates [4, 5].

In 1995, Lima and collaborators [6, 7] analyzed several thermodynamic and statistical properties of a dark energy fluid. They assumed a dark energy fluid component phenomenologically described by an equation of state $p = \omega \rho$ with null chemical potential. Later on, Lima and Alcaniz (and independently Brevik et al. [8]) stressed that their theoretical thermodynamic treatment ruled out the case of phantom energy because the comoving entropy of a dark component with $\omega < -1$ is negative (see Reference [9] for other interesting discussions of phantom fluids). However, thermodynamic arguments in favor of the phantom hypothesis were put forward by González-Díaz and Sigüenza [10]. They claimed that the temperature of a phantomlike fluid is always negative in order to keep its entropy positive definite (as statistically required).

In this work we reanalyze the thermodynamics properties of an expanding universe filled with a dark energy fluid endowed with a non-zero chemical potential. As we shall see, the main effect of a chemical potential is that the phantom scenario becomes thermodynamically consistent with no need to assume negative temperatures.
2 Thermodynamics properties of the dark energy

Let us now consider that the Universe is described by the homogeneous and isotropic Friedmann-Robertson-Walker (FRW) geometry \((c = 1)\)

\[
ds^2 = dt^2 - a^2(t) \left( \frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right),
\]

where \(\kappa = 0, \pm 1\) is the curvature parameter and \(a(t)\) is the scale factor. In what follows we consider that the matter content is a fluid described by the equation of state

\[
p = \omega \rho,
\]

where \(p\) is the pressure, \(\rho\) is the energy density and \(\omega\) a constant parameter which may be positive (white energy) and negative (dark energy). The cases \(\omega = 1/3, 1,\) and \(-1\) characterizes, respectively, the blackbody radiation, a stiff-fluid and the vacuum state while \(\omega < -1\) stands to a phantomlike behavior.

Following standard lines, the equilibrium thermodynamic states of a relativistic simple fluid are characterized by an energy momentum tensor \(T^\alpha_\beta\), a particle current \(N^\alpha\) and an entropy current \(S^\alpha\) which satisfy the following relations

\[
T^\alpha_\beta = (\rho + p)u^\alpha u^\beta - pg^\alpha_\beta, \quad T^\alpha_\beta;_\beta = 0,
\]

\[
N^\alpha = nu^\alpha, \quad N^\alpha;_\alpha = 0,
\]

\[
S^\alpha = su^\alpha, \quad S^\alpha;_\alpha = 0,
\]

where \((;)_\beta\) means covariant derivative, \(n\) is the particle number density, and \(s\) is the entropy density. In the FRW background, the above conservation laws can be rewritten as (a dot means comoving time derivative)

\[
\dot{\rho} + 3(1 + \omega)\rho \frac{\dot{a}}{a} = 0, \quad \dot{n} + 3n \frac{\dot{a}}{a} = 0, \quad \dot{s} + 3s \frac{\dot{a}}{a} = 0,
\]

whose solutions can be written as:

\[
\rho = \rho_0 \left( \frac{a_0}{a} \right)^{3(1+\omega)}, \quad n = n_0 \left( \frac{a_0}{a} \right)^3, \quad s = s_0 \left( \frac{a_0}{a} \right)^3,
\]

where the positive constants \(\rho_0, n_0, s_0\) and \(a_0\) are the present day values of the corresponding quantities (hereafter present day quantities will be labeled by the index “0”). On the other hand, the quantities \(p, \rho, n,\) and \(s\) are related to the temperature \(T\) by the Gibbs law

\[
nT d\left( \frac{s}{n} \right) = d\rho - \frac{\rho + p}{n} dn,
\]

and from Gibbs-Duhem relation \([11]\) there are only two independent thermodynamic variables, say, \(n\) and \(T\). Now, by assuming that \(\rho = \rho(T, n)\) and \(p = p(T, n)\) and combining the thermodynamic identity \([12]\)

\[
T \left( \frac{\partial p}{\partial T} \right)_n = \rho + p - n \left( \frac{\partial \rho}{\partial n} \right)_T,
\]
with the conservation laws as given by (6), one may show that the temperature satisfies
\[
\frac{\dot{T}}{T} = \left( \frac{\partial p}{\partial \rho} \right)_n \frac{\dot{n}}{n} = -3\omega \frac{\dot{a}}{a}.
\] (10)

Therefore, assuming that \( \omega \neq 0 \) we obtain
\[
n = n_0 \left( \frac{T}{T_0} \right)^\frac{1}{2} \quad \Leftrightarrow \quad T = T_0 \left( \frac{a}{a_0} \right)^{-3\omega}.
\] (11)

The temperatures appearing in the above expressions are positive regardless of the value of \( \omega \). In particular, in the radiation case (\( \omega = 1/3 \)), one finds \( aT = a_0T_0 \) as should be expected. As compared to this case, the unique difference is that the dark energy fluid (even in the phantom regime) becomes hotter in the course of the cosmological adiabatic expansion since its equation of state parameter is a negative quantity. A physical explanation for this behavior is that thermodynamic work is being done on the system [6, 7].

It should be stressed that the derivation of the temperature evolution law presented here is fully independent of the entropy function, as well as of the chemical potential \( \mu \). The above expressions also imply that for any comoving volume of the fluid, the product \( T^{\frac{1}{2}} V \) remains constant in the course of expansion and must also characterize the equilibrium states (adiabatic expansion) regardless of the value of \( \mu \). Further, by inserting the temperature law into the energy conservation law (7), one obtains the energy density as function of the temperature
\[
\rho = \rho_0 \left( \frac{T}{T_0} \right)^{\frac{1+\omega}{\omega}}.
\] (12)

Now, in order to determine the chemical potential and its influence on the thermodynamic of dark energy, we consider the Euler relation [11]
\[
Ts = p + \rho - \mu n,
\] (13)

where \( \mu \) in general can also be a function of \( T \) and \( n \) [13, 14]. By combining the above expression with equations (2), (7) and (11) we obtain:
\[
\mu = \mu_0 \left( \frac{a}{a_0} \right)^{-3\omega} = \mu_0 \left( \frac{T}{T_0} \right),
\] (14)

where
\[
\mu_0 = \frac{1}{v_0} [(1 + \omega)\rho_0 - T_0s_0].
\] (15)

This straightforward thermodynamic result has some interesting consequences. Remember that all the present day quantities labeled by the index “0” are positives. In principle, the chemical potential may be either positive or negative, and it also depends on the values of the \( \omega \)-parameter. In particular, \( \mu \) is always negative (\( \mu_0 < 0 \)) in the case of phantom energy, and becomes even more negative in the course of time (\( T \) grows with
the scale factor during the cosmic evolution). It is also known that $\mu$ is zero in the case of photons ($\omega = 1/3$) because they are their own antiparticles. In this case, (15) yields correctly that $s_0T_0 = (4/3)\rho_0$ as should be expected. In general, if $\mu = 0$, necessarily the relation $s_0T_0 = (1 + \omega)\rho_0$ must be obeyed, which is just the present day expression of $sT = (1 + \omega)\rho$ as required by (13).

At this point, the fundamental question is: How the chemical potential modifies the entropy constraints derived in the previous papers [6, 7]?

In order to show that we compute explicitly the entropy of dark energy for a comoving volume $V$. As remarked before, the entropy function should scale as $S \propto T^\omega V$. Actually,

$$S(T, V) \equiv sV = \left[\frac{(1 + \omega)\rho_0 - \mu_0 n_0}{T_0}\right] \left(\frac{T}{T_0}\right)^{1/\omega} V = s_0V_0,$$

which remains constant as expected (see discussion below Eq.(11)). However, in order to keep the entropy $S \geq 0$ (as statistically required), the following constraint must be satisfied:

$$\omega \geq \omega_{\text{min}} = -1 + \frac{\mu_0 n_0}{\rho_0},$$

which introduces a minimal value to the $\omega$-parameter, below which the entropy becomes negative. This is a remarkable expression and its consequences are apparent. For instance, consider that $\mu_0 = 0$ (no chemical potential). In this case, the smallest value of the $\omega$-parameter is $\omega_{\text{min}} = -1$ and the previous analysis by Lima and Alcaniz [7] is fully recovered, that is, the phantom domain ($\omega < -1$) is thermodynamically forbidden. However, for a negative chemical potential, the phantomlike regime becomes thermodynamically allowed thereby recovering the hypothesis of phantom energy without appealing to negative temperature as proposed in the literature [10]. Note also that for a positive chemical potential not even a cosmological constant ($\omega = -1$) is possible.

In Figure 1, we summarize the basic thermodynamic results.

3 Concluding Remarks

In this paper we have investigated the thermodynamics properties of a dark energy fluid with equation of state, $p = \omega\rho$, by assuming that its chemical potential is different from zero.

In Figure 1, we summarize the main results of the present analysis including a chemical potential to the dark energy fluid. As discussed in previous section, the regions with $S < 0$ are thermodynamically forbidden. Note also that many dark energy fluids satisfy the combined constraints regardless of the $\mu$ sign, that is, a large interval of negative $\omega$ values is allowed from thermodynamic considerations. However, a phantom like behavior ($\omega < -1$) is permitted only for $\mu < 0$. It should be stressed that for $\mu = 0$ one finds $\omega_{\text{min}} = -1$ (see Eq. (17)) in accordance to the results previously derived by Lima and Alcaniz [7]. The present analysis with $\mu \neq 0$ also opens the possibility for an equation of state parameter $\omega < -1$, thereby recovering the idea of a phantom dark energy with no need of negative temperatures. Therefore, as far as we known, the inclusion of a negative chemical potential to the dark energy fluid is the only way to save the phantom hypothesis without to violate basic thermodynamic properties.
Figure 1: The allowed intervals of $\omega$ values (heavy lines) and forbidden (dashed lines) for null, positive and negative chemical potentials. Note that a large portion of the dark branch $\omega < 0$ is always thermodynamically permitted. However, for $\mu \geq 0$, the phantomlike behavior ($\omega < -1$) is thermodynamically forbidden.

Acknowledgments

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