Implications of Generalized $Z - Z'$ Mixing

K.S. Babu\[1\], Christopher Kolda\[2\] and John March-Russell\[3\]

School of Natural Sciences, Institute for Advanced Study, Princeton, NJ, USA 08540

Abstract

We discuss experimental implications of extending the gauge structure of the Standard Model to include an additional $U(1)$ interaction broken at or near the weak scale. We work with the most general, renormalizable Lagrangian for the $SU(2) \times U(1) \times U(1)$ sector, with emphasis on the phenomenon of gauge kinetic mixing between the two $U(1)$ gauge fields, and do not restrict ourselves to any of the “canonical” $Z'$ models often discussed in the literature. Low-energy processes and $Z^0$-pole precision measurements are specifically addressed.

---

1Email: babu@ias.edu
2Email: kolda@ias.edu
3Email: jmr@ias.edu
One of the simplest, and most well-motivated, extensions of the Standard Model (SM) is the addition of an extra $U(1)$ gauge factor to its $SU(3) \times SU(2) \times U(1)$ structure. Traditionally [1], the most studied such extensions were motivated by grand unified theories (GUT’s) of rank higher than that of the SM ($SO(10)$, $E_6$ or larger), or by geometric compactifications of heterotic string models which possessed a low-energy spectrum sharing many features with $E_6$ GUT’s. Examples of the resulting $Z'$ gauge bosons include the $\chi$ and $\psi$ models whose couplings to the SM fermions are defined by the decompositions $SO(10) \rightarrow SU(5) \times U(1)_\chi$ and $E_6 \rightarrow SO(10) \times U(1)_\psi$. However, it has recently become clear that these models are not especially favored, and that a larger class of $Z'$ models is more naturally considered.

There are two reasons for this change of view: first, even in “traditional” GUT-like models, the phenomenon of kinetic mixing [2, 3, 4] can significantly shift the predicted couplings of the $Z'$ to SM states away from their canonical values, as well as changing the relationship between other SM observables [5]; furthermore, kinetic mixing is in general generated by renormalization group (RG) running down from the high (i.e., GUT) scale to the weak scale [3, 4]. Second, from the string perspective, a much broader class of models with additional $U(1)$ factors now looks reasonable; this is due both to the construction of many non-geometrical string models in weak coupling perturbation theory (e.g., those arising from free-fermionic techniques) which share very little resemblance to GUT-like models, and to the recent developments in strongly-coupled string theory which show that additional gauge factors can arise non-perturbatively. It is also worth mentioning that string models naturally lead to non-zero kinetic mixing as a threshold effect at the string scale [6]. In either case the traditional parameterization in terms of the $U(1)'$ combinations in $E_6$ is inadequate.

Our intention in this paper is to explore the experimental consequences of the most general $U(1)$ extensions of the SM. Consistent with the effective field theory philosophy, we study the full set of additional renormalizable operators generated by the interactions of the $Z'$, and allowed by SM and $U(1)'$ gauge symmetries. One expects that additional matter fields would also arise in extended gauge models in order to cancel possible anomalies; however we will not study their effects here as the spectrum and charges of extra matter would be highly model-dependent.

For a generic $Z'$, one usually find that direct experimental searches, via, for example, Drell-Yan $Z'$ production at a $pp$ collider, will provide the strongest constraints on the existence of these new interactions. Such searches have been considered many times both in the theoretical and experimental literatures. However there exist a number of low-energy processes which are sensitive to extra gauge interactions, as well as $Z$-pole processes sensitive to $Z - Z'$ mixing. These processes can provide the most useful mechanisms for searching for and/or measuring $Z'$ physics, particularly if the extra $Z'$ is either leptophobic [4, 5] or close in mass to the $Z$. It is on these indirect and mixing bounds that we will concentrate in this work.

Finally, we wish to make note of previous related works which considered many of these same issues in particular limits: Kim, et al. [8] on low-energy observables;
The most general renormalizable Lagrangian for the Standard Model with an additional $U(1)$ (denoted $U(1)'$) is given by (for convenience the QCD and scalar sectors have been omitted):

$$\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_{Z'} + \mathcal{L}_{mix}$$

$$\mathcal{L}_{SM} = -\frac{1}{4} \hat{B}_{\mu\nu} \hat{B}^{\mu\nu} - \frac{1}{4} \hat{W}_a^{\mu\nu} \hat{W}^{a\mu\nu} + \frac{1}{2} M_Z \hat{Z}_\mu \hat{Z}^{\mu}$$

$$- e \sum_i \bar{\psi}_i \gamma^\mu \left( \frac{1}{c_W} (Y_L^i P_L + Y_R^i P_R) \hat{B}_\mu + \frac{1}{s_W} P_L T^a \cdot \hat{W}_a^\mu \right) \psi_i$$

$$\mathcal{L}_{Z'} = -\frac{1}{4} \hat{Z}'_{\mu\nu} \hat{Z}'^{\mu\nu} + \frac{1}{2} M_{Z'}^2 \hat{Z}'_\mu \hat{Z}'^{\mu} - \frac{g'}{2} \sum_i \bar{\psi}_i \gamma^\mu (f_V^i - f_A^i \gamma^5) \psi_i \hat{Z}'_\mu$$

$$\mathcal{L}_{mix} = -\frac{\sin \chi}{2} \hat{Z}'_{\mu\nu} \hat{B}^{\mu\nu} + \delta M^2 \hat{Z}'_\mu \hat{Z}'^{\mu}$$

where: $\hat{B}_{\mu\nu}, \hat{W}_a^{\mu\nu}, \hat{Z}'_{\mu\nu}$ are the field strength tensors for $U(1)_Y, SU(2)_L$ and $U(1)'$ respectively; $\psi_i$ are the fermion fields; $f_V^i$ and $f_A^i$ are the vector and axial charges of the fermions under $U(1)'$; $Y_L^i$ and $Y_R^i$ are the hypercharges of the left- and right-handed components of the fermions with the normalization $Y_L^e = -\frac{1}{2}$ and $Y_R^e = 1$; $\hat{Z}_\mu = \hat{c}_W W_3^\mu - \hat{s}_W \hat{B}_\mu$ is the usual $Z$-boson of the Standard Model (the usual photon, $\hat{A}_\mu$ is orthogonal to $\hat{Z}_\mu$); $\hat{Z}'$ is the boson of the new $U(1)'$; $\hat{s}_W$ is the sine of the weak angle; and $P_{L,R} = (1 \mp \gamma^5)/2$. The mass terms are assumed to come from spontaneous symmetry breaking via scalar expectation values.

The most noticeable feature of the above Lagrangian is the presence of the mixing terms in $\mathcal{L}_{mix}$. The second (mass-mixing) term is familiar and generally arises when the Higgs bosons of one group are also charged under the second group. The first term is less familiar and is allowed for the case of two abelian groups only because $F_{\mu\nu}$ is gauge-invariant for abelian groups. (As an aside, though, we note that the above Lagrangian and all subsequent discussion do generalize to the case of new electrically neutral gauge bosons coming from nonabelian gauge groups, except that $\chi = 0$ there by gauge invariance.) Most previous analyses of extra $U(1)$’s have assumed $\chi = 0$ at the weak scale even though it is often generated by threshold corrections or via renormalization group flow. It is the purpose of the present analysis to pay special attention to the new contributions to physical processes that arise for $\chi \neq 0$.

In order to discuss the physical implications of the new gauge boson, it is necessary to work in the physical (or mass) eigenbasis for the $Z - Z'$ system. Going to the physical eigenbasis requires both diagonalizing the field strength terms and the mass
terms. This can be seen as a two-step process in which we first diagonalize the field strengths via a $GL(2,R)$ transformation:

$$
\begin{pmatrix}
\hat{B}_\mu \\
\hat{Z}'_\mu
\end{pmatrix}
= \begin{pmatrix}
1 & -\tan\chi \\
0 & 1/\cos\chi
\end{pmatrix}
\begin{pmatrix}
B_\mu \\
Z'_\mu
\end{pmatrix}.
$$

(5)

Notationally, we express all parameters and fields in the original, mixed basis as hat-ted, and those in the physical basis without hats. So $(\hat{B}_\mu, \hat{Z}'_\mu)$ are the original $U(1)_Y$ and $U(1)'$ gauge fields with non-diagonal kinetic terms, while $(B_\mu, Z'_\mu)$ have canonical gauge kinetic terms. The process of diagonalization introduces new interactions among the gauge bosons and the matter fields. Specifically, the $U(1)'$ charge of all fields is shifted by an amount proportional to their hypercharge and $\sin\chi$; thus fields which were not charged under $U(1)'$ now have some non-zero $U(1)'$ charge due to the non-orthogonal rotation above.

Next we go to the physical eigenbasis (via an $O(3)$ rotation) by diagonalizing the mass terms which arise after both $U(1)'$-breaking and $SU(2) \times U(1)$-breaking. In the end, one mass eigenstate is massless (the photon, $A_\mu$), while the other two (denoted $Z_{1,2}$) receive masses. In terms of $(B_\mu, W^3_\mu, Z'_\mu)$, or alternatively $(\hat{A}_\mu, \hat{Z}_\mu, \hat{Z}'_\mu)$, one finds:

$$
A_\mu = \hat{c}_W B_\mu + \hat{s}_W W^3_\mu
= \hat{A}_\mu + \hat{c}_W \sin\chi \hat{Z}'_\mu
$$

$$
Z_{1,2\mu} = \cos\xi (\hat{c}_W W^3_\mu - \hat{s}_W B_\mu) + \sin\xi Z'_\mu
= \cos\xi (\hat{Z}_\mu - \hat{s}_W \sin\chi \hat{Z}'_\mu) + \sin\xi \cos\chi \hat{Z}'_\mu
$$

(6)

where

$$
\tan 2\xi = \frac{-2 \cos\chi (\delta \hat{M}^2 + \hat{M}_Z^2 \hat{s}_W \sin\chi)}{\hat{M}_Z^2 - \hat{M}_Z^2 \cos^2\chi + \hat{M}_Z^2 \hat{s}_W^2 \sin^2\chi + 2 \delta \hat{M}^2 \hat{s}_W \sin\chi}.
$$

(7)

In order to make contact with experiment, we must choose some definition for the couplings in terms of well-measured parameters. First we notice that the Lagrangian for the photon is the canonical one:

$$
\mathcal{L}_A = \overline{\psi}_i \gamma^{\mu} Q^i A_\mu \psi_i
$$

(8)

so we identify $\hat{e} = e$ where $e = \sqrt{4\pi\alpha}$ is the usual electric charge. Given that $G_F$, $\alpha$ and $M_{Z_1}$ are the best-measured parameters in the SM, we then make the conventional choice to define the “physical” weak angle via:

$$
\hat{s}_W^2 c_W^2 = \frac{\pi\alpha (M_{Z_1})}{\sqrt{2} G_F M_{Z_1}^2}.
$$

(9)

\footnote{This equation corrects Eq. (27) of Ref. \cite{ref} in which the last term in the denominator has the incorrect sign.}
However, Eq. (9) is also true with the replacements \( s_W \rightarrow \hat{s}_W, \ c_W \rightarrow \hat{c}_W \) and \( M_{Z_1} \rightarrow \hat{M}_Z \), leading to the identity \( s_W c_W M_{Z_1} = \hat{s}_W \hat{c}_W \hat{M}_Z \). Keeping only to leading order \( \xi \) in \( \xi \) and isolating the \( Z_1 \) interactions, the Lagrangian then takes the form:

\[
\mathcal{L}_{Z_1} = -\frac{e}{2s_Wc_W} \left( 1 + \frac{\xi^2}{2} \left( \frac{M_{Z_2}^2}{M_{Z_1}^2} - 1 \right) + \xi s_W \tan \chi \right) \\
\times \bar{\psi}_i \gamma^\mu \left\{ (T_3^i - 2Q_i s_w^2 + \xi \tilde{f}_V^i) - (T_3^i + \xi \tilde{f}_A^i) \gamma^5 \right\} \psi_i Z_{1\mu}
\]

where we have used

\[
\hat{M}_{Z_1}^2 = M_{Z_1}^2 \left( 1 + \sin^2 \frac{\xi^2}{2} \left( M_{Z_2}^2 - 1 \right) \right)
\]

and defined

\[
\tilde{f}_{V,A}^i = \frac{g'c_W s_W}{e \cos \chi} f_{V,A}^i
\]

\[
s_s^2 = \sin^2 \theta_s
\]

\[
= s_w^2 + c_w^2 s_W \xi \tan \chi - \frac{c_w^2 s_w}{c_w^2 - s_w^2} \left( \frac{M_{Z_2}^2}{M_{Z_1}^2} - 1 \right) \xi^2
\]

The last equation defines yet another weak angle which appears only in the vector interaction vertices.

This Lagrangian has a very familiar form and can be taken over directly to the effective Lagrangian formulation of the \( S, T, U \) parameters \([5,10,11]\). In that formulation, the model-independent (i.e., \( g' \rightarrow 0 \)) part of the corrected \( Z_1 \) interaction Lagrangian has the form:

\[
\mathcal{L}_{Z_1} = -\frac{e}{2s_Wc_W} \left( 1 + \frac{\alpha T}{2} \right) \bar{\psi}_i \gamma^\mu \left\{ (T_3^i - 2Q_i s_w^2) - T_3^i \gamma^5 \right\} \psi_i Z_{1\mu}.
\]

where

\[
s_s^2 = s_w^2 + \frac{1}{c_w^2 - s_w^2} \left( \frac{1}{4} \alpha T - c_w^2 s_w^2 \alpha S \right)
\]

Comparing Eqs. (10) and (12) to Eqs. (13) and (14) we can identify the \( Z - Z' \) contributions to \( S, T \) to be:

\[
\alpha S = 4 \xi c_w^2 s_w \tan \chi
\]

\[
\alpha T = \xi^2 \left( \frac{M_{Z_2}^2}{M_{Z_1}^2} - 1 \right) + 2 \xi s_w \tan \chi
\]

which hold to lowest order in \( \xi \). Several comments are in order: First, \( Z - Z' \) mixing without kinetic mixing (i.e., \( \chi = 0 \)) always shifts \( T \) to larger values since the \( Z_1 \) mass will always be smaller than the pure SM \( Z \) mass (assuming \( \tilde{M}_{Z'} > \tilde{M}_Z \)). However in

\[\text{It is often the case that } \xi^2 (M_{Z_2}^2/M_{Z_1}^2) \sim O(\xi) \text{ so we will keep it when working at } O(\xi).\]
the presence of kinetic mixing, $T$ can have either sign even though $M_{Z_1}$ is still smaller than $M_Z$. Second, non-zero $S$, of either sign, can be generated only in the presence of kinetic mixing. Third, although $U$ does not appear in the above formulation, it can be explicitly calculated and one finds it remains zero to leading order in $\xi$.

The $Z_1 \bar{\psi} \psi$ interaction Lagrangian can then be given in final form:

$$L_{Z_1} = -\frac{e}{2s_W c_W} \left( 1 + \frac{\alpha T}{2} \right) \bar{\psi}_i \gamma^\mu \left\{ \left( g_V^i + \xi f_V^i \right) - \left( g_A^i + \xi f_A^i \right) \gamma^5 \right\} \psi_i Z_{1\mu}$$

(16)

with the following identifications:

$$g_A^i = T_3^i$$
$$g_V^i = T_3^i - 2Q_i s^2_\star$$

(17)

and $s^2_\star$ as defined in Eq. (12). The same procedure can also produce the $Z_2 \bar{\psi} \psi$ interaction Lagrangian:

$$L_{Z_2} = -\frac{e}{2s_W c_W} \bar{\psi}_i \gamma^\mu \left\{ \left( h_V^i - g_V^i \xi \right) - \left( h_A^i - g_A^i \xi \right) \gamma^5 \right\} \psi_i Z_{2\mu}$$

(18)

with the following additional definitions:

$$h_V^i = f_V^i + \bar{s}(T_3^i - 2Q^i) \tan \chi$$
$$h_A^i = f_A^i + \bar{s} T_3^i \tan \chi$$
$$\bar{s} \equiv \sin \theta = s_W + \frac{s^4_W}{c^2_W - s^2_W} \left( \frac{1}{4c^2_W} \alpha S - \frac{1}{2} \alpha T \right)$$

(19)

where the last equation defines yet another weak angle. If $\chi$ is very small, then the shift $\bar{s}^2 - s^2_V$ is higher order in small $\chi$ and $\xi$; however, there is no reason given current data to exclude the possibility of small $\xi$ but large $\chi$, since $\chi$ always appears suppressed by $\xi$ in the $Z_1$ interaction Lagrangian.

2 Observables

There are two sets of constraints on the existence of a $Z'$ which will be considered here: precision measurements of neutral current processes at low energies, and $Z^{-}$pole constraints on $Z - Z'$ mixing. In principle, one usually expects other new states to appear at the same scale as the $Z'$, including its symmetry-breaking sector and any additional fermions necessary for anomaly cancellation. However, because these states are highly model-dependent, we will not include their effects in the expressions that follow.
2.1 Low-energy observables

At energies far below $M_Z$, the effects of an additional gauge field with mass $M'_Z \sim M_Z$ can easily be comparable to those of the electroweak $Z$. Therefore in processes sensitive to off-shell $Z$-exchange, signals for $Z'$-exchange may also be constrained. We consider several such processes in this section. Earlier discussions of these processes can be found in Refs. [8, 12].

Whether or not the new gauge interactions are parity violating, stringent constraints can arise from atomic parity violation (APV) experiments. At low energies, the effective Lagrangian for electron-quark interactions can be written:

$$\mathcal{L}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{q=u,d} \left\{ C_{1q} (\overline{q}(\gamma_{\mu}\gamma^5)q)(\overline{e}(\gamma_{\mu}e)) + C_{2q} (\overline{q}(\gamma_{\mu}q)(\overline{e}(\gamma_{\mu}\gamma^5)e)) \right\}$$  \hspace{1cm} (20)

where $C_1$ and $C_2$ are given in terms of the underlying physics by

$$C_{1q} = 2(1 + \alpha T)(g_A^e + \xi f_A^e)(g_V^q + \xi f_V^q) + 2r(h_A^e - \xi g_A^e)(h_V^q - \xi g_V^q)$$ \hspace{1cm} (21)

$$C_{2q} = 2(1 + \alpha T)(g_V^e + \xi f_V^e)(g_A^q + \xi f_A^q) + 2r(h_V^e - \xi g_V^e)(h_A^q - \xi g_A^q)$$

with $r = (M_{Z_1}/M_{Z_2})^2$. The second ($r$-dependent) term come from $Z_2$ exchange and is often non-negligible given that $\xi \sim (M_{Z_1}/M_{Z_2})^2$ in many cases.

One then defines the “weak charge,” $Q_W$, of an atom:

$$Q_W = -2[C_{1u}(2Z + N) + C_{1d}(Z + 2N)]$$  \hspace{1cm} (22)

where $Z$ ($N$) is the number of protons (neutrons) in the atom. We can express the effects of $Z - Z'$ mixing as a shift in $Q_W$ away from the (one-loop corrected) SM prediction:

$$\Delta Q_W \simeq -\frac{Z}{e_w^2 - s_w^2} \alpha S + \left( Q_W + \frac{4c_w^2 s_w^2 Z}{e_w^2 - s_w^2} \right) \alpha T$$

$$- 4(2Z + N) \left\{ \xi g_A^e (\tilde{f}_V^u - rh_a^u) + \xi g_V^u (\tilde{f}_A^e - rh_V^e) + rh_A^e h_V^u \right\}$$

$$- 4(Z + 2N) \left\{ \frac{a}{1 - (1 - y)} \right\}$$  \hspace{1cm} (23)

to lowest order in $\xi$.

The corresponding $C_{2q}$ couplings are measured in combination with the $C_{1q}$ couplings in polarized electron-nucleon scattering experiments. The left-right asymmetry, $A_{LR}$, can be expressed in the quark model as:

$$\frac{A}{Q^2} = a_1 \frac{1 + (1 - y)^2}{1 - (1 - y)^2}$$  \hspace{1cm} (24)

where $Q^2 > 0$ is the momentum transfer in the scattering, and $y$ is the fractional energy loss of the electrons in the nucleon rest frame. The $a_i$ are defined via:

$$a_1 = \frac{3G_F}{\sqrt{2\pi\alpha}} \left( C_{1u} - \frac{1}{2} C_{1d} \right)$$

$$a_2 = \frac{3G_F}{\sqrt{2\pi\alpha}} \left( C_{2u} - \frac{1}{2} C_{2d} \right).$$  \hspace{1cm} (25)
We can again expand $\Delta a_1$ and $\Delta a_2$ to lowest order in $\xi$, keeping all orders in $r$ and $\chi$:
\[
\Delta a_1 = a_1 \alpha T + \frac{3G_F}{5\sqrt{2}\pi\alpha} \left\{ 2\xi \left( \frac{3}{4} - \frac{5}{3}s_W^2 \right) \left( f^e_A + rh^e_A \right) - \xi \left( f^u_A - \frac{1}{2} f^d_A \right) \right. \\
\left. + 2r \left( h^e_A - \frac{1}{2} \xi \right) \left( h^u_A - \frac{1}{2} h^d_A \right) + \frac{5}{3(c_W^2 - s_W^2)} \left( \frac{1}{4} \alpha S - c_W^2 s_W^2 \alpha T \right) \right\} 
\]
\[
\Delta a_2 = a_2 \alpha T + \frac{3G_F}{5\sqrt{2}\pi\alpha} \left\{ 3\xi \left( f^e_A + rh^e_A \right) + 2\xi \left( 2s_W^2 - \frac{1}{2} \right) \left( f^u_A - \frac{1}{2} f^d_A \right) \right. \\
\left. + 2r \left( h^e_A + \left( 2s_W^2 - \frac{1}{2} \right) \xi \right) \left( h^u_A - \frac{1}{2} h^d_A \right) \right\}. 
\] 

Stringent limits on $Z - Z'$ mixing also arise from neutrino-hadron scattering experiments. These experiments parametrize their results in terms of the effective Lagrangian:
\[
\mathcal{L}_{\nu\text{Hadron}} = -\frac{G_F}{\sqrt{2}} \bar{\nu} \gamma^\nu (1 - \gamma^5) \nu \sum_q \left[ \epsilon_L(q) \bar{\psi} \gamma_\mu \left( 1 + \gamma^5 \right) q + \epsilon_R(q) \bar{\psi} \gamma_\mu \left( 1 + \gamma^5 \right) q \right]. 
\] 

The $\epsilon$-parameters can be expressed as:
\[
\epsilon_{L,R}(q) = \frac{1}{2} (1 + \alpha T) \left\{ (g_Y^q \pm g_A^q) \left[ 1 + \xi (\tilde{f}_V^q \pm \tilde{f}_A^q) \right] + \xi (\tilde{f}_V^q \pm \tilde{f}_A^q) \right\} 
\]
\[
+ \frac{r}{2} \left\{ (h_Y^q \pm h_A^q) (h_Y^q \pm h_A^q) - \xi (g_Y^q \pm g_A^q) (h_Y^q \pm h_A^q) - \xi (h_Y^q \pm h_A^q) \right\}. 
\] 

This can be expanded, keeping only terms linear in $\xi$, to determine a shift $\Delta \epsilon_{L,R}(q)$, but it is trivial to do and we will abstain from showing it here.

### 2.2 $Z$-pole observables

Electroweak measurements made at LEP and SLC while sitting on the $Z$-resonance have greatly suppressed sensitivities to $Z'$ physics except through the mixing with the $Z$. This is because the production rate for $Z_1 \equiv Z'$ would overwhelm most $Z_2$ direct production signals, and at the pole, $Z - Z'$ interference effects vanish. (We ignore in this section the possibility that the $Z_2$ could be nearly degenerate with the usual $Z_1$.)

Constraints on the allowed mixing angle and $U(1)'$ charges arise by fitting all data simultaneously to the ansatz of $Z - Z'$ mixing. For any observable, $\mathcal{O}$, the shift in that observable, $\Delta \mathcal{O}$, can be expressed (following the procedure of Refs. [3, 10]) as:
\[
\frac{\Delta \mathcal{O}}{\mathcal{O}} = A_S^\mathcal{O} \alpha S + A_T^\mathcal{O} \alpha T + \xi \sum_i B_i^\mathcal{O} \tilde{f}_i 
\] 

where $i$ runs over the independent $Z' \bar{\psi} \psi$ couplings. We have of course assumed that the shifts on the right-hand side of Eq. (30) are small so that the equation can be linearized.
### Table 1: Strength of dependence of observables on parameters of $Z_1$ Lagrangian. See Eq. (30).

| $O$ | $A_O^O$ | $A_O^I$ | $B_O^{V_u}$ | $B_O^{A_u}$ | $B_O^{V_d}$ | $B_O^{A_e}$ | $B_O^{A_e}$ |
|-----|---------|---------|-------------|-------------|-------------|-------------|-------------|
| $\Gamma_Z$ | -0.49 | 1.35 | -0.89 | -0.40 | 0.37 | 0.37 | 0 |
| $R_\ell$ | -0.39 | 0.28 | -1.3 | -0.56 | 0.52 | 0.30 | 4.0 |
| $\sigma_h$ | 0.046 | -0.033 | 0.50 | 0.22 | -0.21 | -1.0 | -4.0 |
| $R_b$ | 0.085 | -0.061 | -1.4 | -2.1 | 0.29 | 0 | 0 |
| $R_c$ | -0.16 | 0.12 | 2.7 | 4.1 | -0.59 | 0 | 0 |
| $\overline{A_e}$ | -24.9 | 17.7 | 0 | 0 | 0 | -26.7 | 2.0 |
| $\overline{A_b}$ | -0.32 | 0.23 | 0.71 | 0.71 | -1.73 | 0 | 0 |
| $\overline{A_c}$ | -2.42 | 1.72 | 3.89 | -1.49 | 0 | 0 | 0 |
| $M^2_W$ | -0.93 | 1.43 | 0 | 0 | 0 | 0 | 0 |

3 Conclusions

In this paper we have presented general expressions for the shifts in precision measurements at low energies and at the $Z^0$ pole due to the presence of an additional $U(1)'$ interaction. In doing so, we have always used the most general parametrization of an $SU(2) \times U(1) \times U(1)'$ model, including gauge kinetic mixing. We have made no expansions in small parameters apart from the assumption that the mixing angle $\xi$ is small; in particular the expressions herein do not assume a large mass hierarchy between the $Z$ and $Z'$. We have not attempted to fit the present data to any particular model because the Standard Model as is works quite nicely. However, if deviations from the Standard Model are observed, we propose to use the framework presented here as a guide for understanding the underlying physics.

If the $U(1)'$ charges are generation-dependent, there exist severe constraints in the first two generations coming from precision measurements such as the $K_L - K_S$ mass splitting and $B(\mu \to 3\tau)$ owing to the lack of GIM suppression in the $Z'$ interactions; however, constraints on a $Z'$ which couples only to the third generation are somewhat weaker. In any case, per generation there are only five independent $Z' \overline{\psi} \psi$ couplings; we can choose them to be $\tilde{f}_u V$, $\tilde{f}_u A$, $\tilde{f}_d V$, $\tilde{f}_d A$, $\tilde{f}_\nu V$. All other couplings can be determined in terms of these, e.g., $\tilde{f}_V = (\tilde{f}_u + \tilde{f}_d)/2$, $\tilde{f}_A = (\tilde{f}_u - \tilde{f}_d)/2$ and $\tilde{f}_d = \tilde{f}_u + \tilde{f}_d$. Thus in Eq. (30), the variable $i$ runs over these 5 (per generation) independent variables.

The $A$ and $B$ coefficients are given numerically in Table 1 where we assume generation-independent $Z'$ interactions. To lowest order, the coefficients in the table depend only on the measured SM parameters. The observables we consider here include those measured directly at LEP/SLC: $\Gamma_Z$, $R_\ell = \Gamma_{\text{had}}/\Gamma_{\ell+\ell^-}$, $\sigma_h = 12\pi \Gamma_e \Gamma_{\text{had}}/M^2_{Z_1} \Gamma^2_Z$, $R_b = \Gamma_b/\Gamma_{\text{had}}$ and $R_c = \Gamma_c/\Gamma_{\text{had}}$. We also include the parameters $\overline{A}_e$, $\overline{A}_b$ and $\overline{A}_c$ which are extracted from the corrected asymmetries: $A_{FB}^{(0, f)} = 2 \overline{A}_e \overline{A}_f$ and $A_{LR}^0 = \overline{A}_e$.
Model begin to appear and/or direct observations of a new gauge boson are made, such fits will play an important part in extracting the physics of the new interactions. In particular, it would be important to extract from the experimental data a measurement of the couplings \( f_{V,A}^i \) and \( \chi \) independently. This is a considerable challenge without foreknowledge of the underlying gauge structure; the question of how to go about doing this is currently under study \[13\].

Acknowledgements

This research was supported in part by US Department of Energy contract #DE-FG02-90ER40542, the Alfred P. Sloan Foundation and the W.M. Keck Foundation. CK also wishes to acknowledge the generous support of Helen and Martin Chooljian. Our gratitude goes out to the Theory Group at the ICTP, Trieste for their hospitality during the extended workshop on highlights in astroparticle physics, the Aspen Center for Physics, and the Department of Theoretical Physics, Oxford University where parts of this work were completed. Finally we would like to thank C. Carone and T. Trippe for encouraging us to put these results together in a concise form.

References

[1] See J. Hewett and T. Rizzo, Phys. Rep. 183 (1989) 193 for a review.

[2] B. Holdom, Phys. Lett. 166B (1986) 196.

[3] F. del Aguila, Acta Phys. Polon. B25 (1994) 1317;
F. del Aguila, M. Cvetić and P. Langacker, Phys. Rev. D52 (1995) 37.

[4] K.S. Babu, C. Kolda and J. March-Russell, Phys. Rev. D54 (1996) 4635.

[5] B. Holdom, Phys. Lett. B259 (1991) 329.

[6] K. Dienes, C. Kolda and J. March-Russell, Nucl. Phys. B492 (1997) 104.

[7] C. Carone and H. Murayama, Phys. Rev. D52 (1995) 484;
P. Chiappetta, J. Layssac, F. Renard and C. Verzegnassi, Phys. Rev. D54 (1996) 789;
G. Altarelli, et al., Phys. Lett. B375 (1996) 292;
E. Ma, hep-ph/9709474

[8] J. Kim, P. Langacker, M. Levine and H. Williams, Rev. Mod. Phys. 53 (1981) 211;
P. Langacker, M. Luo and A. Mann, Rev. Mod. Phys. 64 (1992) 87.

[9] G. Altarelli, et al., Mod. Phys. Lett. A5 (1990) 495.
[10] C. Burgess, et al., Phys. Rev. D49 (1994) 6115.

[11] M. Peskin and T. Takeuchi, Phys. Rev. Lett. 65 (1990) 964;
    G. Altarelli and R. Barbieri, Phys. Lett. B253 (1990) 161;
    B. Holdom and J. Terning, Phys. Lett. B247 (1990) 88;
    M. Golden and L. Randall, Nucl. Phys. B361 (1991) 3.

[12] W. Marciano and J. Rosner, Phys. Rev. Lett. 65 (1990) 2963; Erratum: 68 (1992) 898;
    K. Mahanthappa and R. Mohapatra, Phys. Rev. D43 (1991) 3093; Erratum: D44 (1991) 1616.

[13] K.S. Babu, K. Dienes, C. Kolda and J. March-Russell, in preparation.