A Parameterized Neutrino Emission Model to Study Mass Ejection in Failed Core-collapse Supernovae

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Abstract

Some massive stars end their lives as failed core-collapse supernovae (CCSNe) and become black holes (BHs). Although in this class of phenomena the stalled supernova (SN) shock is not revived, the outer stellar envelope can still be partially ejected. This occurs because the hydrodynamic equilibrium of the star is disrupted by the gravitational mass loss of the protoneutron star (PNS) due to neutrino emission. We develop a simple parameterized model that emulates PNS evolution and its neutrino emission and use it to simulate failed CCSNe in spherical symmetry for a wide range of progenitor stars. Our model allows us to study mass ejection of failed CCSNe where the PNS collapses into a BH within \( \sim 100 \) ms and up to \( \sim 10^5 \) s. We perform failed CCSNe simulations for 262 different pre-SN progenitors and determine how the energy and mass of the ejecta depend on progenitor properties and the equation of state (EOS) of dense matter. In the case of a future failed CCSN observation, the trends obtained in our simulations can be used to place constraints on the pre-SN progenitor characteristics, the EOS, and on PNS properties at BH formation time.

Unified Astronomy Thesaurus concepts: Compact objects (288); Hydrodynamics (1963); Neutron stars (1108); Core-collapse supernovae (304); Nuclear astrophysics (1129); Stellar mass black holes (1611)

1. Introduction

Massive stars with zero-age main-sequence (ZAMS) mass \( M_{\text{ZAMS}} \geq 8 \, M_\odot \) undergo a core-collapse event once their nuclear fuel is exhausted. As the inert iron core collapses onto itself, its density and pressure increase by orders of magnitude within a fraction of a second. This occurs until densities reach nuclear saturation density, \( \rho_{\text{nuc}} \approx 2.7 \times 10^{14} \, \text{g cm}^{-3} \), and collapse is suddenly halted by the stiffening of nuclear interactions. At this moment, a protoneutron star (PNS) is born where the iron core of the massive star once was. Meanwhile, continuously infalling matter hits the hard PNS and rebounds, creating a shock wave that propagates outward through the still accreting outer layers of the star. Eventually this shock stalls, due to photodissociation of heavy nuclei and neutrino emission losses, and starts to recede. As matter continues to accrete onto the PNS, neutrino emission increases. If neutrinos leaving the PNS deposit enough energy behind the stalled shock, the shock is revived, unbinding most of the outer layers of the star and igniting a bright successful core-collapse supernova (CCSN).

However, a still unknown fraction of massive stars are expected to undergo a failed CCSN event, i.e., the shock wave is not revived and recedes as matter continues to accrete onto the PNS. Once enough matter is accreted, the PNS overcomes the maximum mass it can support and collapses into a black hole (BH) without a bright transient. This BH formation channel could occur for some massive stars and determining the landscape of which massive stars lead to successful explosions and which ones do not is an active topic of research (Sukhbold et al. 2016; Burrows et al. 2020; Couch et al. 2020; Boccioli et al. 2022b). Despite BH formation, mass ejection is still possible for failed supernovae (SNe) because of the hydrodynamic consequences of the neutrino emission that carries away \( \mathcal{O}(10\%) \) of the gravitational mass of the PNS. Although these neutrinos barely interact with the outer regions of the star as they fly by, their emission decreases the gravitational pull of the PNS on the outer layers of the collapsing star. The sudden disruption of the hydrostatic equilibrium creates a pressure wave deep within the star that travels outward, gaining or losing momentum depending on local stellar properties. If the pulse gains enough momentum it eventually forms a shock that becomes unbound and escapes to the interstellar medium (Nadyozhin 1980; Lovegrove & Woosley 2013; Fernández et al. 2018). In such a BH formation channel, the progenitor star would become momentarily brighter, for a few seconds up to a few days, before dimming significantly or even disappears altogether. Furthermore, ejection of the outer stellar envelope could lead to a bright transient, even if it is orders of magnitude less luminous than typical successful SNe. Although a failed SN has never been directly observed, there is some evidence that they do occur (Gerke et al. 2015; Adams et al. 2017; Allan et al. 2020; Murguía-Berthier et al. 2020; Basinger et al. 2021; Neustadt et al. 2021; Rodríguez 2022). However, interpretation of the supposed failed SNe transients and their progenitors is not without controversy and continuing observations are needed to settle the debate (Murphy et al. 2018; Humphreys 2019; Burke et al. 2020; Bear et al. 2022).

Historically, SN surveys seek for the sudden appearance of new light sources in the sky. However, to discover failed SNe one has to look for the opposite effect: a suddenly disappearing source. Such a survey was proposed by Kochanek et al. (2008) noticing that within a distance of 10 Mpc there are \( \sim 10^6 \) red supergiants (RSGs) and that these stars undergo a core-collapse event within \( \sim 10^5 \) yr. Thus, within a few years of observing stars within 10 Mpc, a survey would be likely to identify one or even a few stars that end their lives either with a bang

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(supernova (SN)) or a whimper (fall out of sight) (Kochanek et al. 2008). More recently, Tsuna (2021) proposed a survey to search for soft X-ray emission from collisions of the relatively slow, $\sim 600–800$ km s$^{-1}$, and low-mass, $\sim 0.1 M_\odot$, ejecta of failed CCSNe from blue supergiant stars (BSG) in the Large Magellanic Cloud (LMC) with the circumstellar medium. Even though the expected rate of BSG failed SNe in the LMC is of order $10^{-4}$ yr, these sources are expected to emit detectable X-rays for $10^7–10^8$ yr, depending on mass-loss rate before core collapse for typical BSG ejecta (Tsuna 2021). The identification of these CCSN progenitors and determination of their outcome would place new constraints on the rate of core-collapse events as well as the branching ratio of failed SNe. To date, the failed-SN rate has been estimated by Neustadt et al. (2021) to be $f = 0.16^{+0.21}_{-0.12}$ of all CCSNe and by Byrne & Fraser (2022) to be $f < 0.23$ for sources with absolute magnitude $< -14$.

Another reason to identify CCSNe progenitors, especially of failed CCSNe, is to shed light on the RSG problem. SN surveys by Smartt et al. (2009) and Williams et al. (2014) have shown that RSGs that are SN progenitors seem to be limited to $M_{\text{ZAMS}} \lesssim 16.5 M_\odot$ and $M_{\text{ZAMS}} \lesssim 20 M_\odot$, respectively, while some RSGs have $M_{\text{ZAMS}} \approx 25 M_\odot$. A possible solution to this problem is that stars in the range of $17–20 M_\odot \lesssim M_{\text{ZAMS}} \lesssim 25 M_\odot$ undergo failed CCSN events. However, detection selection effects or distinct stellar evolution pathways for stars in the $17–20 M_\odot \lesssim M_{\text{ZAMS}} \lesssim 25 M_\odot$ range could account for the discrepancy (Kochanek et al. 2008; Smartt et al. 2009; Bear et al. 2022). Therefore, unambiguous determination of a failed SN from a RSG progenitor could settle the debate around the RSG problem.

While direct observations are lacking, theoretical and computational works are necessary to evaluate the possible signatures of failed SN and to unequivocally match them to surveys. Analytic estimates by Piro (2013) show that the response of an RSG to the sudden loss of a few $0.1 M_\odot$ in gravitational mass by the PNS is a shock breakout with luminosity $\sim 10^{40}–10^{41}$ erg s$^{-1}$ that lasts $\sim 3–10$ days. Lovegrove & Woosley (2013), also studying RSG models, demonstrated that failed CCSNe are capable of unbinding the hydrogen envelope of RSGs; leading to faint, red, long-duration observable transients with luminosity $\sim 10^{38}–10^{39}$ erg s$^{-1}$ that last approximately a year. The main difference between the timescales of the events as predicted by Piro (2013) and Lovegrove & Woosley (2013) is that in the former the transient is powered by the kinetic energy of the ejected mass while in the latter the ejected envelope emits its energy via hydrogen recombination. Numerical simulations by Lovegrove et al. (2017) of the light curve and spectra of failed CCSNe from two RSG progenitors with $M_{\text{ZAMS}} = 15$ and $25 M_\odot$ considering a range of explosion energies show that the peak bolometric luminosity can be in the range of $10^{39}–10^{44}$ erg s$^{-1}$ depending on explosion energy, which is a function of the equation of state (EOS) and the pre-SN progenitor core compactness, structure, and mass. An increase in the maximum mass supported by PNSs before BH collapse, which is expected if the EOS of dense matter is stiff, results in a net increase of neutrino emission and a stronger shock. Also discussed by Lovegrove et al. (2017) is the importance of accurately modeling stellar atmosphere, opacity, and ambient medium to determine the observational prospects of failed SN with current and forthcoming missions.

The failed CCSN mass ejection mechanism has also been analyzed analytically by Coughlin et al. (2018) using linear perturbation theory. Their predictions for the stellar response to the sudden loss of gravitational mass in the core agree well with the spherically symmetric simulation results of Fernández et al. (2018) and Ivanov & Fernández (2021), respectively, F18 and I21 hereafter. While F18 used a parametric prescription to approximate the PNS neutrino mass loss using the nonrelativistic FLASH code (Fryxell et al. 2000; Dubey et al. 2009), I21 performed detailed numerical simulations of the PNS evolution up to BH formation using the general-relativistic GR1D code (O’Connor & Ott 2010; O’Connor 2015) and mapped the result to FLASH to gauge the ejecta properties. Both works showed that mass ejection in failed CCSNe is sensitive to pre-SN progenitor structure, the dense-matter EOS, and to a smaller degree, to the timescale of gravitational mass loss by neutrino emission, and might not occur at all for some progenitors.

In this work we perform simplified numerical simulations of failed CCSNe. First, we develop a model to estimate neutrino emission and BH formation times for CCSNe based on spherically symmetric simulations using FLASH (Fryxell et al. 2000; Dubey et al. 2009; Couch 2013; O’Connor & Couch 2018a) and M1 neutrino transport (O’Connor 2015). Extending the simulations of Schneider et al. (2020, $S20$ hereafter) to a few other progenitors, we show that (1) the evolution of the neutrino luminosity can be well fitted by a function of the accretion rate onto the PNS and its baryonic mass and that (2) the entropy inside the PNS, which determines BH formation for an EOS, is well approximated by a function of the mass of the PNS and the time since the PNS has formed. Together, these two approximations allow us to simulate failed CCSNe by replacing the PNS and BH physics by a simple model that includes accretion of the supersonically infalling outer layers of the star and the loss of gravitational mass due to neutrino emission without having to solve computationally expensive neutrino transport equations.

Employing our model for PNS neutrino emission and BH formation, we perform CCSNe simulations in spherical symmetry for a wide range of pre-SN progenitor stars found in the literature. We watch as a sound pulse forms in the outer layers of the star due to the loss in core gravitational mass and continue our simulations well past BH formation until the sound pulse either becomes a shock and unbinds from the star or falls back into the BH. By using a large set of simulations we determine the dependence of the mass ejecta and its energy as a function of the pre-SN progenitor properties and EOS.

We discuss our model of neutrino emission and PNS evolution during CCSNe in Section 2. To validate our model, in Section 3 we compare PNS evolution until BH formation using our template to results obtained with significantly more computationally expensive core-collapse simulations that use M1 neutrino transport. In Section 4, we use our parameterized model to predict the impact of progenitor structure and EOS on the properties of failed CCSNe. We conclude in Section 5.

## 2. Model

We first review the main result of $S20$, and then use the insights gained to parameterize the PNS entropy evolution and its neutrino emission.
2.1. BH Formation

In S20, we have shown that for a given pre-SN progenitor the PNS gravitational mass and its most common entropy,\(^3\) simply referred to as entropy from now on, evolve almost independently of the EOS up to the point of BH formation. The latter was hinted at by Hempel et al. (2012) and Steiner et al. (2013) discussing the core collapse of a single progenitor star and many EOSs.

For clarity and simplicity we show in Figure 1 a scheme with four PNS mass-entropy evolution tracks for the core collapse of two different pre-SN progenitor stars, each one evolving considering two different dense-matter EOSs. In the scheme, dashed and dotted–dashed tracks show the entropy evolution of the PNS as a function of the PNS gravitational mass up to the point of BH formation. The latter depends on the maximum mass supported by a PNS at a given entropy, and moves to higher mass and lower entropy as matter is accreted and neutrinos are emitted. The PNS collapses into a BH earlier (lower PNS mass) for the softer EOS 1 than the stiffer EOS 2.

As shown in Figure 1, the PNS mass-entropy evolution for a given progenitor does not depend significantly on the EOS up to the point of BH formation, shown by filled circles in the scheme. The moment the BH forms, however, is EOS dependent. In Figure 1, we compare two different EOSs: EOS 1 is a softer EOS than EOS 2 at lower temperatures (low entropy), while both EOSs have similar stiffness at higher temperatures (high entropy). Therefore, for the more (less) compact progenitor A (B) the difference in PNS gravitational mass at the moment of BH formation is relatively small (large), \(\approx 0.1 M_\odot (\approx 0.4 M_\odot)\). Although there are only a few constraints on the maximum mass supported by a PNS at a given entropy, most models predict a maximum supported mass that increases with entropy (Steiner et al. 2013; Schneider et al. 2020).

The picture outlined above highlights that it would be difficult to extrapolate zero temperature NS properties from the observation of the formation of a BH from a CCSN. Nevertheless, this picture is useful to simplify the study of failed CCSNe since we are able to create a template for neutrino luminosity and PNS entropy evolution from the outcome of only a few simulations. With a reasonable template we are able to determine time-dependent neutrino emission from a PNS as well as the BH formation time for different progenitors and EOSs without having to solve computationally expensive neutrino transport equations. We discuss this now.

2.2. PNS Entropy Model

To create a simple template for PNS entropy evolution we perform spherically symmetric simulations of CCSNe using the FLASH code (Fryxell et al. 2000; Dubey et al. 2009; Couch 2013; O’Connor & Couch 2018a). Our simulations employ the SRO baseline EOS of Schneider et al. (2019b), Eggenberger Andersen et al. (2021), and the neutrino transport library NULIB of O’Connor (2015) as discussed in detail in S20 and Section 2.3.

We simulate the core collapse of many pre-SN progenitors with core compactness \(\xi_{cc,1}\) spanning the range of \(\sim 10^{-4.5}\) to \(\sim 1\) and envelope compactness \(\xi_{env,1}\) in the range of \(\sim 0.01\)–3.0. Core compactness is defined as in O’Connor & Ott (2011),

\[
\xi_M = \frac{M/M_\odot}{R(M_{baryon} = M)/1 \text{ km}} \bigg|_{t = t_{cc}},
\]

and envelope compactness as in F18,

\[
\xi_{env} = \frac{M_{cc}/M_\odot}{R_{cc}/R_\odot} \bigg|_{t = t_{cc}},
\]

where \(M_{cc}, R_{cc},\) and \(t_{cc}\) are the progenitor mass, radius, and time at the start of core collapse. The list of progenitors used to create a PNS mass-entropy evolution template and their properties are shown in Table 1. For each simulation, we compute the entropy \(s\) inside the PNS for 100 equally separated time instants from core bounce until a BH forms, or in the case of low-compactness models, until \(t_{simulation} - t_{cc} = 10\) s. We then fit the entropy as a function of two variables using a second-order-spline interpolation scheme (Virtanen et al. 2020). The variables are (1) a base-10 logarithm of the simulation time after core bounce, \(\log_{10} t\) where \(t = t_{simulation} - t_{bounce}\), and (2) the baryonic mass inside a radius

\(3\) The most common entropy \(s\) is defined as the value of the approximately flat PNS entropy that contains the largest amount of mass.
Table 1

| Name               | $M_{ZAMS}$ (M$_{\odot}$) | $M_{cc}$ (M$_{\odot}$) | $M_{Fe}$ (M$_{\odot}$) | $\xi_{2.5}$ | $\xi_{env}$ (z) |
|--------------------|--------------------------|-------------------------|-------------------------|-------------|-----------------|
| Fernández et al. (2018) |                          |                         |                         |             |                 |
| R12z00             | 12.0                     | 10.0                    | 1.45                    | 0.151       | 0.099           |
| W25z00             | 26.0                     | 11.9                    | 1.59                    | 0.214       | 0.108           |
| Y2S5z-2            | 25.0                     | 23.0                    | 1.62                    | 0.256       | 0.024           |
| W40z00             | 40.0                     | 10.3                    | 1.78                    | 0.370       | 0.271           |
| R15z00             | 15.0                     | 10.8                    | 1.50                    | 0.239       | 0.010           |
| B25z00             | 25.0                     | 11.7                    | 1.60                    | 0.336       | 0.123           |
| B30z-2             | 30.0                     | 16.0                    | 1.72                    | 0.337       | 0.111           |
| Y22z00             | 22.0                     | 11.1                    | 1.85                    | 0.546       | 0.016           |
| W50z00             | 50.0                     | 9.2                     | 1.91                    | 0.554       | 0.219           |
| B80z-2             | 80.0                     | 55.2                    | 3.32                    | 0.972       | 0.793           |

Woosley & Heger (2007)

| Name     | $M_{ZAMS}$ (M$_{\odot}$) | $M_{cc}$ (M$_{\odot}$) | $M_{Fe}$ (M$_{\odot}$) | $\xi_{2.5}$ | $\xi_{env}$ (z) |
|----------|--------------------------|-------------------------|-------------------------|-------------|-----------------|
| a50      | 50.0                     | 9.76                    | 1.50                    | 0.216       | 0.301           |
| a25      | 25.0                     | 15.8                    | 1.60                    | 0.326       | 0.011           |
| a40      | 40.0                     | 15.3                    | 1.83                    | 0.534       | 1.400           |

Woosley et al. (2002)

| Name     | $M_{ZAMS}$ (M$_{\odot}$) | $M_{cc}$ (M$_{\odot}$) | $M_{Fe}$ (M$_{\odot}$) | $\xi_{2.5}$ | $\xi_{env}$ (z) |
|----------|--------------------------|-------------------------|-------------------------|-------------|-----------------|
| z11      | 11.0                     | 11.0                    | 1.25                    | 0.005       | 0.591           |
| z12      | 12.0                     | 12.0                    | 1.36                    | 0.011       | 0.997           |
| z13      | 13.0                     | 13.0                    | 1.45                    | 0.025       | 1.255           |
| z14      | 14.0                     | 14.0                    | 1.40                    | 0.041       | 1.354           |
| z25      | 25.0                     | 25.0                    | 1.81                    | 0.385       | 2.148           |
| u40      | 40.0                     | 40.0                    | 1.90                    | 0.633       | 0.457           |
| u75      | 75.0                     | 74.1                    | 2.03                    | 0.873       | 0.299           |

Limongi & Chiefi (2006) (LC06a)

| Name     | $M_{ZAMS}$ (M$_{\odot}$) | $M_{cc}$ (M$_{\odot}$) | $M_{Fe}$ (M$_{\odot}$) | $\xi_{2.5}$ | $\xi_{env}$ (z) |
|----------|--------------------------|-------------------------|-------------------------|-------------|-----------------|
| a60      | 60.0                     | 16.9                    | 1.63                    | 0.424       | 0.081           |
| a80      | 80.0                     | 22.4                    | 1.67                    | 0.481       | 0.111           |
| a120     | 120.0                    | 30.5                    | 1.91                    | 0.534       | 0.143           |

Sukhbold et al. (2016) [10^{-4}]

| Name     | $M_{ZAMS}$ (M$_{\odot}$) | $M_{cc}$ (M$_{\odot}$) | $M_{Fe}$ (M$_{\odot}$) | $\xi_{2.5}$ | $\xi_{env}$ (z) |
|----------|--------------------------|-------------------------|-------------------------|-------------|-----------------|
| a9.0     | 9.0                      | 8.75                    | 1.32                    | 0.382       | 0.021           |
| a10.0    | 10.0                     | 9.68                    | 1.31                    | 1.99        | 0.019           |

Note. Compactness, $\xi_{2.5}$, values are for pre-SN progenitors at the start of collapse, and thus, different from the ones of O’Connor & Ott (2011), which were computed for a single EOS at the moment of core bounce.

* The MESA (Paxton et al. 2011, 2013, 2015, 2018, 2019) in lists to generate the CCSN progenitors were obtained from https://bitbucket.org/rafefman/bhsa_mesa_progenitors/src/master/.

of 500 km, $M_{500}$. Here, $t_{simulation}$ is the time from the start of the simulation and we approximate the bounce time as the time after the central density of the star reaches $10^{12}$ g cm$^{-3}$.

In Figure 2, we show the results of the entropy fit $s(t, M_{500})$, the tracks in $t-M_{500}$ space for each core-collapse simulation using the progenitors in Table 1, and the deviations between the fit and the simulation results. First, we observe that in the first second after bounce, the PNS entropy is, to first order, only a function of time after the core bounce, i.e., almost independent of progenitor compactness or PNS mass. Thus, for compact progenitors that form BHs in $t \lesssim 1$ s, it would have been sufficient to fit the entropy as a function of time. However, since the trend breaks down for PNSs that take longer than 1 s to collapse into a BH, we opt for a more robust two-variable fit. Note that the fit for massive PNSs when $M_{500} \gtrsim 2.5 M_{\odot}$ and $t \lesssim 1$ s predicts decreasing entropy with increasing mass. This may be unrealistic and simply an artifact of the fitting as these regions in phase space are not populated by our simulations. We also notice that the deviation in entropy between the fit and the simulations is often within 5% of each other, with the largest deviations taking place near the end of each simulation. Furthermore, changes of $\sim 10\%$ in the estimated PNS entropy do not alter significantly the BH formation time and its initial mass. For very-low-compactness progenitors, which take significantly longer than 1 s to form a BH, we expect changes in entropy to cause an even smaller relative error in the BH formation time, as the maximum mass supported by the EOS does not change more than 0.1 $M_{\odot}$ compared to that of a cold NS. In Section 2.4, we discuss how we approach neutrino emission for SN progenitors that take more than 10 s after core bounce to form a BH.

2.3. Neutrino Emission Model

Similar to what is done for the PNS entropy, we also fit the neutrino luminosity $L_{\nu}$ for the three neutrino species considered in our simulations, $\nu = \nu_e, \bar{\nu}_e, \nu_x$, where $\nu_x$ includes contributions from $x = \mu$, $\bar{\mu}$, $\tau$, and $\bar{\tau}$ neutrino species. The neutrino luminosity fits are done for each neutrino species independently and are interpolated as a function of (1) the mass inside a radius of 500 km from the center of the star, $M_{500}$, and (2) the accretion rate at 500 km, $M_{500}$, using a radial-basis function interpolation scheme. We chose the variables $M_{500}$ and $M_{500}$, instead of $\log_{10} t$ and $M_{500}$ as in the entropy template because they provide a considerably better fit to the neutrino luminosity and are similar to the variables used in the analytic neutrino
luminosity approximation of Lovegrove & Woosley (2013) and F18.

In Figure 3, we show the electron neutrino luminosity fit, \( L_{\nu_e} \), and the path traced in \( M_{500} - M_{600} \) by the PNSs generated by the collapse of the progenitors described in Table 1. The interpolated luminosity is usually within 20% of the simulated values although large deviations do occur at selected times. Specifically, during the neutronization burst, when the accretion rate is \( \mathcal{O}(10 M_\odot \text{ s}^{-1}) \), and at times where accretion has a sharp decrease due to the accretion of a shell boundary at 500 km. Since such moments are quite brief with respect to the BH formation time, their contributions to the total neutrino luminosity are small. We observe similar trends for the luminosity fits of the other two neutrino species, \( \bar{\nu}_e \) and \( \nu_x \), except for the lack of a neutronization burst.

Our method to predict neutrino luminosity during CCNSe has other limitations. First, we observe that the neutrino luminosity predicted using our simplified model can, in some instances, deviate significantly from the full M1 neutrino-transport scheme calculations. This effect is stronger for the heavy neutrino luminosity \( L_{\nu_x} \) and is especially true for stiff EOSs, which increase the time to form BHs. This is in part because to simulate core collapse with stiffer EOSs, where BH formation occurs for larger PNS masses, we have to extrapolate the neutrino luminosity fits into regions of the \( M_{600} - M_{500} \) parameter space that our template, based on results obtained with the SRO baseline EOS, does not cover. We discuss these effects further in Section 3.

Low-compactness progenitors accrete matter very slowly, and therefore, take a long time to collapse into a BH, \( t \gtrsim 10 \text{ s} \) after core bounce. Since our model is limited to fits where \( M_{500} > 0.01 M_\odot \text{ s}^{-1} \), extrapolation from this region may lead to relatively high neutrino luminosity at late times. In fact, while accretion leads to an increase in baryonic mass, extrapolating from the parameter space probed can result in unrealistically high or low neutrino luminosity, which can lead to BHs forming too fast (no neutrino emission) or never forming at all (neutrino luminosity larger than accretion rate). To prevent these unrealistic scenarios, we cap the neutrino luminosity \( L_\nu \) for each species to \( M_{500}/10 \). This only affects low-compactness progenitors several seconds after core bounce, when neutrino emission is expected to dissipate any thermal heat gain due to accretion in timescales much shorter than the accretion timescales. Thus, to a good approximation, we may consider such NSs to be cold, zero temperature, and slowly increasing their mass. We remark that for most EOSs and cold NSs with baryonic mass \( M_{\text{baryon}} \gtrsim 1.5 M_\odot \), 

\[
\frac{dE_{\text{bind}}}{dM_{\text{baryon}}} \approx 0.30 - 0.35, \quad \text{where} \quad E_{\text{bind}} = M_{\text{baryon}} - M_{\text{grav}}.
\]

This is the binding energy of the cold NS. Thus, by capping the neutrino emission rates to a combined luminosity of \( \sim 0.3 M_{500} \), we obtain a decent estimate of neutrino emission at late times.

2.4. Simulations

The parameterizations discussed above allow us to simulate the outcome of failed CCSNe without having to solve the computationally expensive neutrino transport equations. Thus, we are able to simulate failed SN from the start of core collapse until shock breakout, if it occurs, which may be minutes to years after a BH forms, depending on the progenitor star. The simulations are performed using the FLASH4 code (Fryxell et al. 2000; Dubey et al. 2009; Couch 2013; O’Connor & Couch 2018a) in spherical symmetry. The code is adapted to estimate the PNS entropy \( \tilde{s} \) and neutrino luminosity \( L_\nu \), using the parameterized templates described above. Our runs are divided into three stages discussed below. In every run, we use a spherical grid with adaptive mesh refinement that extends out to \( 2 \times 10^{15} \text{ cm} \) in the first two stages and then mapped onto a grid that extends to \( 2.048 \times 10^{15} \text{ cm} \) in the last stage. In all stages, on the coarsest level, there are 256 grid zones and we allow up to 20 total levels of refinement, resulting in the smallest grid zone with a length of 149 m for the first two stages and 153 km for the last stage.

2.4.1. Stage 1: Before Core Bounce

We evolve the core collapse of the pre-SN progenitor until the central density reaches \( \rho_c = 10^{12} \text{ g cm}^{-3} \). We do not use any neutrino transport and set \( t = 0 \) (core bounce) at the end of this stage. Simulating this short period before core bounce is necessary to accurately reproduce the mass accretion rate onto the PNS after core bounce, and it is crucial to use the neutrino luminosity template discussed in Section 2.3.

2.4.2. Stage 2: From Core Bounce to BH Formation

At core bounce, we replace the inner \( r = r_0 = 100 \text{ km} \) of the simulation volume by a hole with constant density \( \rho_{\text{hole}} \). The density \( \rho_{\text{hole}} \) is computed assuming that the hole contains a mass that is equal to half of the mass between \( r \) and \( 2r \), guaranteeing that \( \rho_{\text{hole}} \ll \rho(r) \) for \( r = r_0 \), and thus, the mass just outside the hole free falls onto the PNS. This reproduces the expected accretion rate for CCSNe that do not lead to successful explosion, as is the case for all spherically
symmetric simulations for the pre-SN progenitors probed in this work. The missing gravitational mass from decreasing the density in the region within the hole radius $r$ is placed as a point mass at the origin. In order to increase the time step of the simulation as the system evolves, the hole size is increased with a constant speed such that $v = r_0 + vr$, where we set $v = 10\text{ km s}^{-1}$. The point mass at the origin and the hole density are evolved according to the hole size evolution and the neutrino emission computed from our templates.

We estimate the gravitational mass inside the 500 km sphere used in our templates to be $M_{\text{grav}}(t) = M_{500}(t) - E_n(t)$, where $E_n(t) = \int_0^r r L_n(r')dr'$ is the total energy emitted in neutrinos and the sum runs over the three neutrino species considered. This approach is justified as, to a very good approximation, neutrinos are the only source of gravitational mass loss. By tracking the entropy and gravitational mass evolution we determine the moment a BH forms from $M_{\text{grav}}(t) = M_{\text{max}}(s(t))$, where $s(t)$ is the template estimate for the PNS entropy. The point of BH formation depends on $M_{\text{max}}(s)$, which can be set to some fixed value or be computed for a desired EOS, see Figure 1. Low-compactness progenitors may take significantly longer than 40 s to collapse into a BH, the time when the hole radius reaches the 500 km used in our templates. When this happens, we simply compute the baryonic mass and accretion rate at the hole radius $r$ instead of 500 km as the time for matter accreted at $r$ to reach 500 km is significantly shorter than the dynamical timescale of the system.

2.4.3. Stage 3: After BH Formation

Once a BH forms, the neutrino luminosity is set to zero and the total gravitational mass loss from neutrino emission is $\delta M_{\text{grav}} = E_n(t_{\text{BH}})$. Because of the change in gravitational mass of the inner core, hydrodynamic equilibrium in the outer layers of the pre-SN progenitor is disturbed (Nadyozhin 1980; Lovegrove & Woosley 2013; Coughlin et al. 2018). This perturbation creates a pressure wave that propagates outward toward the surface of the star and may result in some mass ejection, even in the case of failed SNe. To determine the mass ejection and its energy, we follow the evolution of the system until the pulse turns into a shock and leaves the star or the pulse velocity becomes negative, indicating the pulse will fall back into the BH. We limit the hole radius to $r_{\text{max}} = 2 \times 10^4 \text{ km}$, at which point we fix $r = r_{\text{max}}$. As in F18 and I21, we (1) fill the region outside the star with a constant-density ambient medium in hydrostatic equilibrium for numerical reasons and (2) map pressure, density, and proton fractions from the progenitor to recover the remaining thermodynamic variables using the Helmholtz EOS (Timmes & Swesty 2000) to minimize transients. The ambient medium is set to a hydrogen gas with density $\rho_{\text{amb}} = [10^{-18}, 10^{-16}, 10^{-14}, 5 \times 10^{-13}] \text{ g cm}^{-3}$ for stars with radii $R > [10^4, 10^5, 10^6, 10^7] \text{ km}$, respectively. To achieve densities below $10^{-6} \text{ g cm}^{-3}$, the Helmholtz EOS as implemented in FLASH is extended to $10^{-20} \text{ g cm}^{-3}$ using a version of the publicly available Timmes & Arnett (1999) EOS code. Similar to F18 and I21, simulations are terminated once the pulse reaches the atmosphere surrounding the star and its temperature drops to within 1% of the EOS table lower limit, $T_{\text{low}} = 10^4 \text{ K}$ or if the pulse leaves the simulation volume. The spatial resolution of our simulations in the outer regions of the star, $r > 10^5 \text{ cm}$, is $\Delta r/r = 4 \times 10^{-3}$, similar to the low-resolution runs of I21. We did not perform detailed resolution studies since I21 showed that resolution is a smaller source of uncertainty than changes in the EOS.

3. Template Accuracy

Using the methods described in Section 2, we assess the accuracy of our model in describing the core collapse of the 40 $M_\odot$ solar metallicity pre-SN progenitor of Woosley & Heger (2007), s40WH07, using a variety of EOSs. Besides the baseline EOS of S20, used to construct our PNS entropy and neutrino emission templates, we explore 21 other EOSs found in the literature. The full set of EOSs contains the baseline SRO EOS, SRO0.75, which has $m_s/m_n = 0.75$, and its stiff variant SRO0.55, which has $m_s/m_n = 0.55$ and soft variant SRO0.95 with $m_s/m_n = 0.95$ (Schneider et al. 2019a; Egggenberger Andersen et al. 2021). We also include the APR, APR14, NRAPR, and SKAPR EOSs (Akmal & Pandharipande 1997; Akmal et al. 1998; Steiner et al. 2005; Schneider et al. 2019b); two variants of the Lattimer and Swesty (LS) EOS with incompressibilities $K_{\text{sat}} = 180$ (LS180) and 220 MeV baryon$^{-1}$ (LS220) (Lattimer & Douglas Swesty 1991; O’Connor & Ott 2011); the DD2, FSU-Gold, TM1, and TMA EOSs (Hempel et al. 2012); the IU-FSU EOS (Fattoyev et al. 2010); and the two DD2 variants that include hyperons: BHB$^\Lambda$ and BHB$^{\Lambda\sigma}$ (Banik et al. 2014); the Shen EOS (Shen et al. 1998) and its variant including $\Lambda$ hyperons (Shen et al. 2011); the SFHo and SFHx EOSs (Steiner et al. 2013); the Togashi EOS (Togashi et al. 2017); and the Furusawa EOS (Furusawa et al. 2017).

In Figure 4, we compare the results of simulations that use the simplified templates, discussed in Section 2, to simulations performed using a full M1 neutrino-transport scheme for the three SRO EOS variants. We show that simulations using the simplified neutrino transport scheme predicted the evolution of the PNS entropy, the integrated neutrino luminosity $E_n$, and the PNS gravitational mass $M_{\text{grav}}$ within a few percent of their equivalent M1 neutrino-transport runs. However, since our template was built for a single EOS the end of the runs occurred earlier (later) by up to 20% of the runtime for EOSs softer (stiffer) than the baseline SRO0.75 EOS. This occurs because the stiffness of the EOS at a given entropy affects the neutrino luminosity: softer EOSs lead to faster PNS contraction, and therefore, faster heating and higher neutrino emission rates (Schneider et al. 2019b; Yasin et al. 2020). This effect, though, is not taken into account in our template, which is based solely on $M_{500}$ and $M_{600}$. Thus, the integrated neutrino luminosity by BH formation time predicted by our template can also differ by up to $\sim 20\%$ from the M1 neutrino-transport runs, depending on the progenitor and EOS used. We stress that this $\sim 20\%$ difference in BH formation time and total gravitational

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Footnotes:

4 This approach ignores that a shock wave is created at the surface of the PNS and travels outward before stalling and falling back onto the PNS. However, this is not relevant as this region is sonically disconnected from the outer layers by the supersonic accretion flow, where the sound pulse we are interested in forms.

5 To handle the accuracy needed at very-low densities we extend the Timmes & Arnett (1999) EOS code, timmes.tcbz found in https://cocobud.com/code_pages/eos.shtml, to quadruple precision.

6 The stiff NL3 (Hempel et al. 2012) and LS375 (Lattimer & Douglas Swesty 1991; O’Connor & Ott 2011) EOSs were not considered as they would require us to rely on our neutrino emission template in regions of $M_{500} - M_{600}$ parameter space where it becomes unreliable.
mass loss (integrated neutrino luminosity) is only a secondary source of uncertainty as the total gravitational mass loss due to neutrino emission by BH formation time can differ by a factor of $\sim 2$ for different EOSs for the same progenitor (S20, I21).

We extend the analysis above to the other 19 EOSs considered by comparing deviations in observables $A$, $\sigma_A = 1 - A^{\text{template}}/A^{(M1)}$, obtained from M1-transport simulations and their counterparts that use our parameterized model. In Figure 5, we plot the deviations between the two approaches for the BH formation time $t_{\text{BH}}$, gravitational mass $M_{\text{grav}}$, PNS gravitational mass $\delta M_{\text{grav}}$ (which is equal to the total integrated neutrino luminosity $E_\nu$), and PNS entropy $\tilde{s}$. The last three quantities are computed $\sim 1$ ms before BH formation time $t_{\text{BH}}$. Plots have the full transport results in the horizontal axis and include deviations for 22 EOSs found in the literature.
be up to a factor of ~10 larger, \( \sigma_{\text{neut}} \lesssim 20\% \) (\( \lesssim 100\text{ ms} \)) and \( \sigma_{\text{Mej}} \lesssim 25\% \).

Moreover, we note that the average neutrino emission rate, \( \delta M_{\text{grav}}/\tau_{\text{BH}} \), is also satisfactorily approximated by our template, even if the total emitted neutrino energy by BH formation time deviates by up to 20% in some cases due to difference in predicted \( t_{\text{BH}} \). In fact, except for EOSs that are very stiff in their temperature dependence (SRO0.55 EOS) or very soft (SRO0.95, FSU-GOLD, LS180, LS220, SFHo, and Togashi EOSs), the average neutrino emission rate computed from our template is within 5% of the one obtained from M1-transport simulations. Even EOSs that are very soft or very stiff, and thus, have a larger deviation in the predicted PNS binding energy, \( \left| \sigma_{\text{Mej}} \right| \sim 20\% \), have relative deviations in average neutrino emission rate a factor of 2–4 lower, \( \left| \delta M_{\text{grav}} \right|/\tau_{\text{BH}} \lesssim 5\%–10\% \). See, for example, the slopes of \( E_{\nu} \) for the SRO0.55 and SRO0.95 EOSs in Figure 4.

From the reasons stated above, we deem that our approximations yield acceptable results for BH formation time and gravitational mass loss from neutrino emission in CCSNe, at least for pre-SN progenitor stars that collapse into a BH within a few seconds. For PNSs that take much longer than 1 s to collapse into a BH, we expect that our model will be even more accurate, since most of the PNS gravitational mass loss occurs when the PNS is relatively cold, and thus, when an approximately linear relationship exists between change in baryonic mass and gravitational mass or PNS binding energy. In fact, for all EOSs considered in this work \( dE_{\text{bind}}/dM_{\text{baryon}} \simeq 0.30–0.35 \).

Hence, we can use our model to study EOS effects in failed CCSNe for different progenitors without having to resort to simulations that include complex detailed and computationally expensive neutrino transport. As discussed above, this should work even for very-low-compactness progenitors, which may take weeks after core bounce to accrete enough matter to collapse into a BH.

### 3.1. Computational Considerations

Due to the simplicity of our PNS model we are able to evolve a pre-SN progenitor star from core collapse until material is ejected from its surface using significantly less computational resources than are needed to evolve the same progenitor from core collapse to BH formation considering neutrino transport. Typically, each of our simulations costs between 32 and 128 CPU hr on the TETRALITH supercomputer using 8 CPU cores. This compares favorably to approximately 50 CPU hr per second of PNS evolution when using M1 neutrino transport with FLASH as in S20. Furthermore, the computational cost is expected to be significantly larger in fully relativistic simulations, such as those of I21 employing the GR1D code. This shows that even detailed nonrelativistic simulations exploring detailed neutrino transport would be unfeasible with limited resources for stars that take \( \gtrsim 10\text{ s} \) to collapse into BHs. However, as I21 showed, detailed simulations may not be necessary to determine mass ejecta from failed CCSNe as long as the mass-loss rate and timescale in the PNS are reasonably well described. As we have argued, our approach does just that.

### 3.2. Comment on Multidimensional Effects

It is well established that the dimensionality of a CCSN simulation can completely alter its outcome (Bethe 1990; Janka et al. 2007; Couch & Ott 2015). Thus, since only 3D simulations can fully describe all the relevant processes occurring during a CCSN, we briefly address how multidimensional effects could affect our results. Their quantification, however, is beyond the scope of this work. First, negative entropy and lepton fraction gradients that appear in spherically symmetric simulations trigger turbulent convection in multidimensional cases. Convection redistributes matter within the PNS and alters both the PNS entropy and its composition profile (Nordhaus et al. 2010; Pan et al. 2018). Furthermore, since convection adds an extra source of pressure to the post-shock region, it may enlarge the maximum stable PNS mass and delay BH formation (Pan et al. 2018). Moreover, rising fluid elements transfer energy to the shock and facilitate its expansion (Bethe 1990; Couch & Ott 2015; Radice et al. 2018), modifying the neutrinosphere’s properties, and therefore, neutrino luminosity (Pan et al. 2016; O’Connor & Couch 2018b; Radice et al. 2018; Couch et al. 2020; Boccioli et al. 2022b). Second, in multidimensional simulations the shock could stall at a larger distance from the PNS and for a longer time than in the spherically symmetric runs we used to create the PNS templates (Schneider et al. 2019b). Finally, the rates of neutrino emission and accretion may also be modified by other hydrodynamical instabilities, such as the standing accretion shock instability or lepton emission self-sustained asymmetry (Walk et al. 2020). Each of these effects could delay BH formation and enhance and lengthen neutrino luminosity, leading to a higher energy deposition into the sound pulse and to an increase in both the mass and energy of the outgoing ejecta. This indicates the need to study failed CCSNe in 2D and/or 3D, or at least, create PNS parameterizations based on the many simulations already available. Alternatively, one could estimate multi-dimensional effects by comparing our PNS parameterizations to ones built from those that use spherically symmetric simulations that employ the SN Turbulence in Reduced-dimensionality formalism (Couch et al. 2020; Warren et al. 2020; Boccioli et al. 2021, 2022a).

### 4. Results

#### 4.1. Overview of the Pulse Dynamics

Using the model described above we can estimate the neutrino emission from a PNS up to the point of BH formation and explore the aftermath of the BH-enveloping star. Due to neutrino emission, a pressure wave appears deep within the star and propagates outward. We now discuss the dynamics of this pulse. To facilitate direct comparisons to the work of F18, we first describe the same progenitors plotted in their Figures 4 and 5, i.e., an RSG with \( M_{\text{ZAMS}} = 15 \, M_\odot \) (R15z00), a blue supergiant (BSG) with \( M_{\text{ZAMS}} = 25 \, M_\odot \) (B25z00), and a Wolf–Rayet (W-R) star with \( M_{\text{ZAMS}} = 40 \, M_\odot \) (W40z00), all with solar metallicity. EOS effects are gauged by comparing \( M_{\text{Mej}}^{\text{max}} = 2.0 \, M_\odot \) proxy for a soft EOS, to \( M_{\text{Mej}}^{\text{max}} = 2.5 \, M_\odot \) proxy for a stiff EOS. While the 2.0 \( M_\odot \) limit is likely a physical lower limit for BH formation from PNS collapse, 2.5 \( M_\odot \) should be close to the upper limit for BH formation and is the same limit set by F18.
In Figure 6, we plot stellar profiles for velocity, Mach number, density, and temperature from core bounce until the simulations are stopped. Qualitatively, and to a degree, quantitatively, our results agree well with those of F18. For the progenitors shown in Figure 6, a sound pulse is created deep inside the star and first acquires a positive velocity in the carbon-oxygen shell, which extends from $10^4 \text{ cm} \lesssim r \lesssim 3 \times 10^{10} \text{ cm}$. The pulse propagates outward and speeds up or slows down according to local stellar structure, accelerating to $v^\text{max} \approx 500$–$1000 \text{ km s}^{-1}$ for PNSs that collapse into a BH when the PNS mass reaches $M^\text{grav} = 2.5 \ M_\odot, 2.0 \ M_\odot$. As this occurs, the pulse speed becomes comparable to the local speed of sound, Mach number $Ma \gtrsim 0.1$, and the pulse develops clearly defined leading and trailing edges (F18).

W-R stars, here represented by the W40z00 pre-SN progenitor (right plots of Figure 6), have their carbon-oxygen core exposed as they lose both their hydrogen and helium envelopes prior to core collapse. Thus, due to the relatively small size of these stars and the sharp density gradient near the stellar surface, the pulse accelerates quickly and its speed increases $\approx 10$-fold, reaching $\approx 100 \ Ma$ as it leaves the star.

Meanwhile, BSG stars (represented by the B25z00 progenitor in the center plots of Figure 6) still retain most of their helium envelopes by core collapse. As the pulse propagates through the helium envelope, both its mass and its velocity increase or decrease by a factor of a few, depending on the EOS and progenitor structure. Finally, the pulse speeds up significantly as it crosses the surface of the star, reaching speeds $\approx 100 \ Ma$.

RSG stars (represented by the R15z00 progenitor in the left plots of Figure 6), on the other hand, still maintain both their helium and hydrogen shells until core collapse. In these progenitors, the propagating pressure wave can lose up to 90\% of its velocity as it crosses the outermost layers of the star before picking up speed again and crossing the stellar surface at a few $Ma$.

Simulations using the soft and stiff EOS proxies differ in that for stiff EOSs the pulse is both faster and more extended, indicating that it carries away more mass and energy. Besides the pulse formed deep within the star, we also note that a second pulse appears in the stellar surface-atmosphere interface prior to the pulse arrival. These surface pulses were predicted by Coughlin et al. (2018) and also appear in the simulations of I21, although it is difficult to estimate their significance as the atmosphere thermodynamics are not fully consistent due to the lower temperature limit of our EOS tables, $T_{\text{low}} = 10^4 \text{ K}$.

We note that for pre-SN progenitors that make BHs quickly, i.e., have large core-compactness parameter $\xi_{2.5}$, the accretion rate onto the PNS within the timescale of BH formation may be curbed by the stalled or the still expanding SN shock, an effect not captured in our simplified simulations. As in the case of multidimensional effects, this could delay BH formation, and thus, allow for a larger PNS gravitational mass loss due to neutrino emission. This would lead to larger energy deposition into the sound pulse, increasing both its mass and energy. However, if the SN shock were revived it would likely overwhelm any sound pulse formed, since a revived SN shock has significantly higher energy than any of the sound pulses studied here.
2.0grav row for four values of smaller than the escape velocity. Here, we de...

4.2. Ejecta Properties

In Figure 7, we plot the time evolution of pulse properties as they propagate outward through the star for the same three progenitors discussed in Section 4.1. Here, we define the pulse as the region of the star between the trailing edge, the stellar zone where velocity first becomes larger than 1% of the maximum speed, and the front edge, the furthest radial coordinate that has velocity over half of the maximum speed. This is a good approximation when the shock has developed, although it underestimates the size of the pulse early on, when the front edge is not as well defined. The properties are shown for four distinct values of the maximum mass supported by PNSs at the time of BH formation: $M_{\text{grav}}^{\text{max}}/M_\odot = 2.0, 2.2, 2.4, \text{and } 2.6$. The $M_{\text{grav}}^{\text{max}}$ limits were chosen based on the most physically sound EOSs available that were still close to the bounds of our neutrino emission model.

In the first row of Figure 7, we plot the evolution of internal energy, $E_{\text{int}}$, gravitational energy, $E_{\text{grav}}$, kinetic energy, $E_{\text{kin}}$, and total energy, $E_{\text{tot}}$, contained in the pulse. Depending on the EOS proxy, i.e., the value of $M_{\text{grav}}^{\text{max}}$, the maximum kinetic energy imparted to the pulse due to loss of core gravitational mass changes by a factor of $\approx 10$ to $\approx 30$ for a given CCSN progenitor. Although the variation in $E_{\text{kin}}$ depends on the progenitor, it is a monotonically increasing function of the PNS mass $M_{\text{grav}}^{\text{max}}$ at BH formation time. Also, the EOS-dependent variations in $E_{\text{kin}}$ lead to pronounced differences in pulse properties that persist as the pulse crosses the surface of the star and part of it becomes unbound. The choice of EOS impacts the kinetic energy of the ejecta by as much as a factor of $\approx 20$ for the RSG and $\approx 40$ for the BSG and W-R progenitors.

In the second row of Figure 7, we plot the evolution of the total pulse mass $M_{\text{pulse}}$ and the total unbound mass $M_{\text{grav}}^{\text{esc}}$. The latter quantity is obtained comparing the local pulse and escape velocities. We expect the true total unbound mass to be located between these two extrema as the pulse mass is increasing and the unbound mass is decreasing by the end of the simulations. Simulations that employed softer EOSs, i.e., smaller $M_{\text{grav}}^{\text{max}}$, showed a larger gap between $M_{\text{pulse}}$ and $M_{\text{grav}}^{\text{esc}}$ at the end of the runs than simulations that used stiffer EOSs, larger $M_{\text{grav}}^{\text{max}}$. For example, $M_{\text{pulse}}$ is larger by a factor of 10 compared to the simulations of the R15z00 progenitor employing $M_{\text{grav}}^{\text{max}} = 2.0 M_\odot$, since this run ends at about the same time that the leading edge of the pulse becomes unbound. The difference is much smaller for simulations of the W40z00 progenitor since the pulse can propagate for a long time outside of the star before simulations are terminated, allowing the necessary time for $M_{\text{pulse}}$ and $M_{\text{grav}}^{\text{esc}}$ to converge. The unbound mass by the end of the simulation is at least 0.1 $M_\odot$ for the R15z00 progenitor, but may be as high as 5 $M_\odot$ when considering the stiffest EOSs in our runs. Meanwhile, a progenitor such as B25z00 (W40z00) is not likely to eject more than 0.1 $M_\odot$ ($10^{-3} M_\odot$), but ejects at least $\approx 10^{-3} M_\odot$ ($\approx 10^{-6} M_\odot$) through the failed CCSN mechanism. Lovegrove & Woosley (2013), F18, and I21 pointed out that a significant uncertainty in the mass ejected and its energy is due to pre-SN progenitor structure, while I21 showed that EOS effects may lead to a factor of a few uncertainty in the total mass ejected during a failed CCSN. However, our results show that this uncertainty may be even larger and span a few orders of magnitude for some progenitors.

Finally, we discuss the third row of Figure 7. We note that the qualitative behavior of the maximum velocity and the escape velocity where speed is at a maximum is the same for every progenitor, regardless of EOS. Quantitatively, however, the highest pulse velocities for each progenitor occur for EOSs that take the longest to form BHs. Since high outgoing velocities mean that the pulse travels faster throughout the star,

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5 We exclude the pulses that develop at the stellar surface-atmosphere interface by setting a cap on the lowest density considered as part of the pulse to be at least half of the lowest density in the pulse in the previous step. We evaluate pulses starting at 10 s after core bounce and increase the analysis time by $\sim 15\% - 2\%$ every analysis step until any part of the pulse has $T < 1.01 \times 10^9$ K or the pulse leaves the simulation volume.
and furthermore, the local escape velocity decreases with the distance to the center of the star, this means that the more time a BH takes to form the more mass becomes unbound and the earlier this happens. We find no cases where an increase in time to form a BH results in increased ejecta mass and lower ejecta velocity or vice versa.

In Table 2 we compare some of our predictions for ejecta properties and BH formation for the three progenitors discussed above considering \( M_{\text{max}}^{\text{grav}} = 2.0 \text{--} 2.6 M_\odot \) in 0.1 \( M_\odot \) steps. In Table 3, we also include results from the interpolation mass-loss model (Model I) of I21 for SFHo, LS220, and DD2 EOSs, and the results from the high-resolution exponential mass-loss model, eHR, in Table 2 of F18. This discrepancy may be due to the mass offset necessary to keep consistency between the inner region of the simulations of I21 performed with GR1D and the outer regions that are computed using FLASH; see their Equation (5) and Figure 2 where the mass loss due to neutrino emission for the LS220 EOS runs has a 0.02 \( M_\odot \) shift added before core bounce. Finally, our results for the RSG and BSG progenitors are comparable to the eHR model of F18, where BHs always form with a mass of \( M_{\text{max}}^{\text{grav}} = 2.5 M_\odot \), except for the total energy carried away by neutrinos \( \delta M_{\text{grav}} \) (total energy of the ejecta \( E_{\text{tot}} \)) in the RSG (BSG) case. Meanwhile, for the more compact progenitor star B25z00, the F18 results are more in line with our \( M_{\text{max}}^{\text{grav}} = 2.6 M_\odot \) EOS proxy.

Besides the progenitors discussed above, we simulate failed CCSN for the other progenitors in F18, see Table 1, incrementing \( M_{\text{max}}^{\text{grav}} \) from 2.0--2.6 \( M_\odot \). In Figure 8, we summarize our results for the distribution of total pulse kinetic energy \( E_{\text{kin}}^{\text{pulse}} \) and mass \( M_{\text{ejecta}}^{\text{pulse}} \) as well as its average velocity \( \langle v_{\text{ejecta}} \rangle = \sqrt{2 E_{\text{kin}}^{\text{pulse}}/M_{\text{ejecta}}} \) at the time the pulse reaches the atmosphere. Simulations with different \( M_{\text{max}}^{\text{grav}} \) are plotted in a blue to red color scale. We do not include results for the B80z00 progenitor because, unlike the results from F18 and I21, none of our simulations for this progenitor produced any unbound material. The gray symbols in the plot are the results of CCSNe simulations of 262 progenitors found in the literature setting \( M_{\text{max}}^{\text{grav}} = 2.5 M_\odot \), and will be discussed in more detail in Sections 4.3 and 4.4.

For the quantities plotted, we consider the entire pulse at the end of the run as an upper limit of possible ejected mass, while the mass of the pulse that is unbound is taken as a lower limit. We also estimate a value between these two extremes considering how much mass would be ejected if all hydrogen recombination energy were converted into kinetic energy (Lovegrove & Woosley 2013). The recombination energy is computed as in F18 and I21

\[
E_{\text{rec}} = \frac{\chi_H}{m_p} \int_{M_{\text{p}}} X_H(M) dM, \tag{3}
\]

where \( \chi_H = 13.6 \text{ eV} \) is the energy emitted by a hydrogen nucleus as it binds an electron, \( m_p \) is the proton mass, and the integral over the hydrogen mass fraction \( X_H(M) \) is performed over the different zones that form the pulse. A key difference between how F18 and I21 determine the amount of unbound...
mass and how we do is that we opt to compute whether any part of the pulse is locally unbound instead of determining if the entire pulse is unbound.

First, we notice that for a given progenitor uncertainty in the EOS, probed by different values of $M_{\text{grav}}^\text{max}$, can lead to changes of a few orders of magnitude in the predicted pulse ejecta energy and mass. This is true regardless of whether we consider the whole pulse mass or only the unbound mass, and whether or not we include hydrogen recombination energy. These are much wider ranges than observed by I21 comparing the soft DD2 EOS and the stiff SFHo EOS. This result is not surprising given that for these two EOSs, BHs should form with initial mass $M_{\text{BH}}$ that is a few orders of magnitude higher than the one we explore here. In most cases, though, much of the spread in pulse mass $M_{\text{ej}}$ and kinetic energy $E_{\text{kin}}$ results from simulations employing the softest EOSs. Since these predict a short time until BH formation, the pulses formed may not even acquire enough energy to become unbound by the time they reach the atmosphere, even when we consider the energy input from hydrogen recombination. Nevertheless, despite large variations in predictions of pulse unbound mass and kinetic energy, the dispersion in the average ejected material velocity $\langle v_{\text{ej}}\rangle$, whenever some material is unbound, is significantly smaller. Note that $\langle v_{\text{ej}}\rangle^2/2 = \langle E_{\text{kin}}\rangle/M_{\text{ej}}$ is a measure of average energy per unit of mass in both the pulse and the ejected material. This average velocity, and similarly, the kinetic energy per unit mass are well correlated to envelope compactness $\xi_{\text{env}}$, provided that we ignore results for very soft EOSs for some of the progenitors, but mostly uncorrelated with core compactness $\xi_{2.5}$.

Figure 8. Estimates for ejecta kinetic energy $E_{\text{kin}}$ (top row), mass $M_{\text{ej}}$ (middle row), and average velocity $\langle v_{\text{ej}}\rangle$ (bottom row) for failed CCSNe simulations for progenitors with core compactness $\xi_{2.5}$ (left column) and envelope compactness $\xi_{\text{env}}$ (right column). Results are shown for our simulations using the F18 progenitors and different mass limits for BH formation between 2.0 and 2.6 $M_\odot$ (blue to red color scale) and for another 262 progenitors found in the literature setting $M_{\text{grav}}^\text{max} = 2.5 M_\odot$. Crosses indicate estimates of ejecta mass considering hydrogen recombination, while triangles pointing up consider unbound mass ignoring hydrogen recombination and triangles pointing down consider all matter in the pulse at the end of the simulation.

4.3. Effect of Core Compactness

We now fix the mass at BH formation to $M_{\text{grav}}^\text{max} = 2.5 M_\odot$ and discuss how progenitor properties affect the ejecta kinetic energy, mass, and average velocity. To obtain a clearer picture we increase the set of progenitors studied so that we explore thoroughly the range of core and envelope compactness of pre-SN progenitors found in the literature. Thus, besides the 10 progenitors from F18, we also simulate 52 progenitors from Sukhbold et al. (2016) with solar metallicity, 36 progenitors from Sukhbold et al. (2018) (12 progenitors with the standard mass loss of Nieuwenhuijzen & de Jager (1990), 12 with half of the standard mass loss, and 12 with one-tenth of the standard mass loss), 126 progenitors from Woosley et al. (2002) (37 with solar metallicity, 59 with ultralow metallicity, and 30 with zero metallicity), 27 rotating progenitors from Woosley & Heger (2006) where rotation velocities were set to zero before the start of our simulations, and 22 progenitors from Laplace et al. (2021) (12 stars evolved isolated and 10 stars evolved in binary systems).

For this set of progenitors, the ejecta kinetic energy $E_{\text{kin}}$ depends on core compactness $\xi_{2.5}$, top-left plot of Figure 8. For low-compactness cores, $\xi_{2.5} \lesssim 0.2$, the ejecta kinetic energy is in the range of $10^{46}$ erg $\lesssim E_{\text{kin}} \lesssim 10^{47.5}$ erg. Meanwhile, as the core compactness increases the kinetic energy of the ejected material decreases on average. Finally, for compact cores, $\xi_{2.5} \gtrsim 0.45$, the average kinetic energy is considerably lower, $10^{42}$ erg $\lesssim E_{\text{kin}} \lesssim 10^{46}$ erg.

The ejected mass $M_{\text{ej}}$, however, has a different behavior than the kinetic energy $E_{\text{kin}}$. We observe two different bands for the ejected mass, center-left plot of Figure 8. For one set of
overall progenitor stars $0.1 \, M_\odot \lesssim M_{\text{ejecta}} \lesssim 1 \, M_\odot$ and looks independent of compactness, even for very compact progenitor cores. For another set of stars, though, $M_{\text{ejecta}} \sim 0.1 \, M_\odot$ for low-compactness cores and decreases as the compactness increases. For this second set, progenitors with compact cores, $\xi_{2.5} \gtrsim 0.5$ eject at most $\sim 10^{-3} \, M_\odot$. As we discuss below, what distinguishes the two bands is the envelope compactness, $\xi_{\text{env}}$.

Finally, the average speed of the material ejected, $\langle v_{\text{ejecta}} \rangle$, shows three groups, which are mostly independent of core compactness. In fact, the groups are well separated by mass ejected. For ejecta with $M_{\text{ejecta}} \approx 1 \, M_\odot$, the material has an average speed of order $30$–$100 \, \text{km} \, \text{s}^{-1}$, regardless of core compactness. Stars that eject $0.1 \, M_\odot$ or less usually eject material with $300$–$1000 \, \text{km} \, \text{s}^{-1}$, again independent of progenitor core compactness. A third pattern is seen for stars that eject very little mass, $\lesssim 10^{-3} \, M_\odot$, as we will show in more detail below. In this case, ejecta speeds exceed $2000 \, \text{km} \, \text{s}^{-1}$ except for low core-compactness stars, $\xi_{2.5} \lesssim 0.1$, where speeds are $500$–$1000 \, \text{km} \, \text{s}^{-1}$.

4.4. Effect of Envelope Compactness

Low envelope compactness pre-SN progenitors, $\log 0 \xi_{\text{env}} \lesssim -1$, such as RSGs and YSGs, can produce a lot of ejecta from $\sim 0.1 \, M_\odot$ up to a few $M_\odot$, although this amount can be much smaller or nonexistent for certain combinations of soft EOSs and progenitors. Whenever there is ejecta, however, it travels somewhat slowly, $\langle v_{\text{ejecta}} \rangle \approx 10$–$100 \, \text{km} \, \text{s}^{-1}$. Thus, if most BHs form with masses close to that of the maximum currently known mass of a cold NS, $M_{\text{max}}^{\text{grav}} \approx 2.0$–$2.2 \, M_\odot$ (Antoniadis et al. 2013; Cromartie et al. 2020), then failed SNe from RSGs probably do not lead to detectable electromagnetic signatures, as their ejecta are slow and have low energy. Nevertheless, these systems could still shine if a disk forms due to progenitor rotation and power other types of transients (Woosley & Heger 2012).

Failed CCSNe of medium envelope compactness pre-SN progenitors, $\xi_{\text{env}} \sim 0.1$–$1$, a range that includes BSGs, often unbind $\gtrsim 0.01$–$0.1 \, M_\odot$ of material. Although, again, certain combinations of soft EOSs and progenitors result in lower ejecta mass or even no ejecta at all. Qualitatively similar variations to the RSG case are seen in both the energy of the ejecta and their average velocity, which usually range from $\langle v_{\text{ejecta}} \rangle \approx 100$–$1000 \, \text{km} \, \text{s}^{-1}$.

Progenitors with very compact envelopes, such as W-R stars, always eject some of their outer layers in the EOS limits we tested, albeit never more than $0.01 \, M_\odot$ according to our simulations. Furthermore, for a given progenitor the ejecta always has a similar average velocity in the range of $\langle v_{\text{ejecta}} \rangle \approx 1500$–$2500 \, \text{km} \, \text{s}^{-1}$, regardless of the EOS and how much mass is ejected. If the expected ejecta velocity for this class of progenitors is indeed almost EOS independent, this would help constrain the scope of surveys similar to the one proposed by Tsuna (2021) to search for SN remnants of failed CCSNe in the LMC.

4.4. Effect of Envelope Compactness

In Figure 9, we couple the plots in Figure 8 for the PNSs that form BH with $M_{\text{max}}^{\text{grav}} = 2.5 \, M_\odot$. S+16 are progenitors from Sukhbold et al. (2016), S+18 from Sukhbold et al. (2018), L+21 from Laplace et al. (2021), F+18 from F18, WH06 from Woosley & Heger (2006), and WH02 from Woosley et al. (2002) with $s$, $u$, and $z$ representing solar, ultralow ($10^{-5}$), and zero metallicity. Red symbols with black contours represent failed CCSNe that did not eject any material.

Specifically, the progenitors are more likely to cluster according to whether or not they retain their hydrogen ($\log 0 \xi_{\text{env}} \sim -2$ and often type II SN progenitors), helium...
(log $0_{\text{env}}^c \sim 0$ and often type Ib SN progenitors), and carbon-oxygen (log $0_{\text{env}}^c \sim 1.3$ and often type Ic SN progenitors) envelopes. Envelope properties depend mostly on ZAMS mass and metallicity, and by core collapse, are mostly independent of core properties. The exception is that pre-SN progenitor stars that form extended cores, $\xi_{2.5} \lesssim 0.2$, hardly ever have their carbon-oxygen envelopes exposed, log $0_{\text{env}}^c \gtrsim 1$.

As discussed above, stars with extended cores and extended envelopes generally eject more mass, although this is not always the case. As a matter of fact, it is clear from Figure 9 that using only the core and envelope compactness parameters $\xi_{2.5}$ and $\xi_{\text{env}}$ is not enough to determine the range of possible outcomes of a failed collapse event, even when considering only a single EOS proxy. For example, for $\xi_{\text{env}} \sim 0.01$ and $\xi_{2.5} \sim 0.5$–0.6, we find both progenitor stars that manage to eject a few $0.1 M_\odot$ at slow speeds as stars that fail to unbind any material at all. A similar statement is true for progenitors with intermediate envelope compactness, $\xi_{\text{env}} \sim 0.3$, and compact cores, $\xi_{2.5} \gtrsim 0.5$: while some progenitors do eject fast material, $\langle v_{\text{ejecta}} \rangle \sim 1000 \text{ km s}^{-1}$, others do not produce any ejecta.

In Figure 10, we show the unbound mass as a function of the progenitor mass $M_{cc}$ and radius $R_{cc}$ at the start of core collapse. Although these quantities are combined to determine envelope compactness $\xi_{\text{env}}$, it is clear that they contribute separately to whether material is ejected or not. For example, we notice that progenitors that appear clustered in the region $\xi_{2.5} \gtrsim 0.5$ and $\xi_{\text{env}} \approx 0.01$ in the plots of Figure 9 are now well separated by masses in the range of $15$–$45 M_\odot$ and a large radius of $R_{cc} \approx 10^9 \text{ km}$. Finally, from this plot we can also observe that progenitors that can unbind more material through the failed-SN mechanism are those that have a lower mass as their core starts to collapse and a large radius, such as RSGs and YSGs.

4.5. Effect of BH Formation Time

In Figure 11, we plot the same quantities as in Figure 8, but now as a function of BH formation time $t_{\text{BH}}$ and the total gravitational mass loss due to neutrino emission $\delta M_{\text{grav}}$.

In the top-left plot of Figure 11, we can see that there is an overall scaling in the energy imparted to the pulse due to gravitational mass loss in the PNS with BH formation time that saturates if a BH takes too long to form, $t_{\text{BH}} \gtrsim 10 \text{ s}$, as is the case for low core compactness progenitors. Therefore, an observational estimate of the kinetic energy of the ejecta of a failed CCSN would allow us to place some constraints on the time between the start of core collapse and BH formation. Such observation on the Milky Way or a nearby galaxy, where neutrino signals will also be measured (Sumiyoshi et al. 2006; O’Connor & Ott 2011, 2013), could be used to impose further constraints on the EOS.

In the center-left plot, we show the mass of the ejecta and notice two distinct trends for the ejecta, both of which saturate if collapse takes $t_{\text{BH}} \gtrsim 10 \text{ s}$. This is expected since the BH formation time is approximately a function of core compactness, $t_{\text{BH}} \propto \xi_{2.5}^{-3/2}$ (O’Connor & Ott 2011), and as we showed in Figure 8, two trends emerge for the ejected or pulse mass as a function of the core compactness $\xi_{2.5}$. In fact, the two trends are separated by whether or not the progenitor still has its hydrogen envelope at core collapse, as shown in the center plot of Figure 9. Similarly, for the speed of the ejecta we observe trends that mostly depend on envelope compactness, with little dependence on BH formation time, which is a function of core compactness.

4.6. Effect of Gravitational Mass Loss

In the top right side plot of Figure 11 we observe that, in general, ejecta kinetic energy increases with gravitational mass loss. However, the trend saturates for $\delta M_{\text{grav}} \gtrsim 0.3 M_\odot$ and is broken for some pre-SN progenitors. We also observe a weak correlation between ejecta mass and total gravitational mass loss, although the scatter is a few orders of magnitude. Finally, for the average speed of the ejecta, we notice three strips that are mostly uncorrelated with the PNS binding energy, each one related to the progenitor envelope properties. Thus, it is hardly possible to constrain the binding energy of the PNS at the time of collapse from measurements of the velocity of the ejecta alone.

4.7. Mass Ejected in the Hertzsprung–Russell Diagram

When observing a distant star we can measure neither its mass nor its radius directly. Instead, we have to rely on stellar evolution codes to infer those observables from the surface...
temperature and luminosity of the star. Here, we approximate the pre-SN stellar luminosity using the fit of Sukhbold et al. (2018),

\[ L_{\text{pre}} \approx 5.77 \times 10^{38} \left( \frac{M_{\text{He}}}{6M_\odot} \right)^{1/2} \text{erg s}^{-1}, \]  

where \( M_{\text{He}} \) is the mass bound by the helium core. For stars that have lost their helium shell, we use the carbon-oxygen core mass \( M_{\text{CO}} \). The effective surface temperature \( T_{\text{eff}} \) is estimated from the luminosity \( L_{\text{pre}} \) and the pre-SN progenitor radius \( R_{\text{cc}} \). For pre-SN progenitors that have their luminosity and surface temperatures available, such as those of F18 and Laplace et al. (2021), we obtain deviations in the 10% range using this approximation and opt to use it for all other stars. A few exceptions do occur, but deviations are at most a factor of two from the approximation of Equation (4).

Using the approximations above, we plot the ejected mass as a function of the observable quantities \( L_{\text{pre}} \) and \( T_{\text{eff}} \) in Figure 12. The diagram shows that the hottest progenitors at core collapse, \( T_{\text{eff}} \lesssim 10^{4.2} \text{K} \), usually collapse into a BH without a significant transient as they eject little to no mass, \( M_{\text{ejecta}} \lesssim 0.1M_\odot \). These progenitors are often the evolutionary endpoints of massive stars with low or zero metallicity (Woosley et al. 2002), stars in binary systems (Laplace et al. 2021), or fast rotating stars that mix their shells while in the main sequence (Woosley & Heger 2006). Recall that we removed the angular momentum to simulate the latter. Because these progenitors have lost their hydrogen shells by core-collapse time, in the case of a successful SN explosion they are observed as type Ib or type Ic SNe, stars that have their strongly bound helium or carbon-oxygen cores exposed, respectively.

Stars on the cold end of the diagram, RSGs or YSGs, and \( T_{\text{eff}} \lesssim 10^{3.7} \text{K} \), are more likely to unbind their outer layers in the case of a failed SN. This is expected, as the hydrogen envelopes that these colder stars retain until their core starts to collapse are loosely bound. When these stars explode successfully, they are observed as type II SNe due to their hydrogen-rich ejecta. Progenitors with luminosity \( L \lesssim 10^{5.1}L_\odot \) have been associated with type II SN while, to date more luminous RSGs and YSGs have not (Rodríguez 2022). Thus, these stars are reasonable candidates to be progenitors of failed SNe.
We developed simple templates to model the entropy evolution and neutrino emission rates of PNSs formed in CCSNe. The templates were built based on FLASH simulations that used M1 neutrino transport for 25 selected progenitors using the baseline SRO EOS of S20. Simulations using our templates reproduce the BH formation time and the PNS binding energy at BH formation time with a relative error \( \lesssim 20\% \) for an intermediate compactness progenitor and different hadronic EOSs. The template also reproduces the PNS gravitational mass and entropy as well as the average neutrino emission rate of PNSs within a few percent.

Once built, the templates allow us to carry out the evolution of massive stars that undergo failed core collapse, i.e., an SN where the shock is not revived, beyond the moment a PNS becomes a BH and until material is ejected from the surface of the star. The ejecta results from the disturbance in the hydrodynamic equilibrium in the outer layers of the star due to PNS gravitational mass loss by neutrino emission (Nadyozhin 1980; Lovegrove & Woosley 2013). Due to the simplicity of the emission model, we can simulate core-collapse events where the PNS takes less than a second to up to weeks to accrete enough material to collapse into a BH.

With this in mind, we explored how the EOS can affect failed CCSNe ejecta for different progenitor stars. In our model, EOS effects are gauged by setting the BH formation at different PNS masses. Despite the many simplifications in the core-collapse physics, our results expand upon previous works. The most significant improvement being the inclusion of a wide range of pre-SN progenitor stars with different ZAMS masses and core and envelope compactness.

As I21, we also observe that EOS stiffness can significantly alter the total amount of mass ejected and its average energy, sometimes by a few orders of magnitude. However, the average energy per unit of mass, probed by the average ejecta speed, is often not considerably affected by changes in EOS. This is especially true for stars with compact envelopes, such as W-R stars, where only a small fraction of the envelope, \( M_{\text{ejecta}} \ll 10^{-4} M_\odot \), is ejected with speeds in excess of \( 1000 \text{ km s}^{-1} \). Direct comparison of our results to those of F18 and I21 shows that changes in how the collapse is modeled can modify the predicted properties of the ejected mass in a failed core collapse in ways that are difficult to predict. Nevertheless, the overall qualitative picture is always the same, stiffer EOSs predict longer times to form BHs, which in turn, results in higher neutrino emissions, and thus, larger ejecta mass with higher average kinetic energy.

Our results also show that, even when considering that all PNSs form BHs with the same gravitational mass, core, and envelope compactness, \( \xi_{2.5} \) and \( \xi_{\text{env}} \), respectively, are not enough parameters to predict the ejecta properties of a failed CCSN. Although these two parameters do allow us to make educated guesses of the ejecta mass and energy for most stars, in some cases it is also necessary to consider the total mass of the star and/or its radius at the moment of core collapse separately.

An assumption made in this work and facilitated by the fact that we only perform spherically symmetric CCSNe simulations, where the SN shock is rarely revived, was that every progenitor could collapse into a BH. However, successful SNe are routinely observed and the pre-SN progenitor envelope properties can be inferred from the emission lines from the SN explosion. Sometimes, even the pre-SN progenitors themselves are identified by direct comparison of SN images to previous observations of the same region of the sky. In fact, pre-SN progenitors that retain their hydrogen envelopes evolve into RSGs and YSGs and have been reported to explode as type II SNe (Smartt et al. 2009; Williams et al. 2014; Rodríguez 2022).

At core-collapse time, these stars have an effective surface temperature of \( \log_{10}(T_{\text{eff}}/\text{K}) \approx 3.6 \) and, although their luminosity spans the range of \( \log_{10}(L_{\text{pre}}/L_\odot) \approx 4.5 - 6.5 \text{ dex} \), to date only progenitors with \( \log_{10}(L_{\text{pre}}/L_\odot) \lesssim 5.1 \text{ dex} \) have been directly associated with successful SN explosions (Rodríguez 2022). Thus, many of the stars in our data set, particularly those with \( \log_{10}(T_{\text{eff}}/\text{K}) \approx 3.6 \) and \( \log_{10}(L_{\text{pre}}/L_\odot) \gtrsim 5.1 \text{ dex} \), as can be seen in Figure 12, could be progenitors of failed SNe as currently suggested by observations. On the other hand, spherically symmetric simulations by Boccioli et al. (2022b) employing neutrino-driven turbulent convection using a time-dependent mixing length theory model (Couch et al. 2020; Boccioli et al. 2022a), suggest that some low-mass low-compactness progenitors, \( M_{\text{ZAMS}} \gtrsim 15 M_\odot \) and \( \xi_{2.5} \lesssim 0.25 \) and which we find to have luminosity \( L \lesssim 10^5 L_\odot \), could also lead to failed CCSNe.

Another limitation of our work is that we did not include rotation. Angular momentum effects are very important when simulating CCSNe (O’Connor & Ott 2011; Dessart et al. 2012; Richers et al. 2017) and may lead to the development of accretion disks around the PNS, which may facilitate mass ejection (Feng et al. 2018; Batta & Ramirez-Ruiz 2019; Murguia-Berthier et al. 2020). Thus, with respect to rotation, our results could be taken as lower limits to the amount of mass ejected and energy for a particular combination of progenitor and EOS. Our model could be extended to add rotation; however, this would result in an added dimension to the PNS physics and its evolution and neutrino emission templates would have to be updated accordingly.

From a practical perspective, one could compare our results to those of a failed CCSNe from a slowly rotating or nonrotating progenitor. Then, if a failed CCSN was observed, its progenitor identified, and total ejected mass and velocity estimated, one could, in principle, validate stellar evolution models as well as place limits on the EOS of hot-dense matter and PNS properties at BH formation time based on the trends observed in the results presented here. Ideally, this would be done using a more complete picture of failed core collapse; one built from a set of simulations, including more progenitors, other EOS templates validated by multidimensional simulations, other EOS proxies, and including rotation.

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