A FUZZY MULTI-ATTRIBUTE DECISION-MAKING METHOD UNDER RISK WITH UNKNOWN ATTRIBUTE WEIGHTS

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Abstract. This paper aims at solving hybrid multiple attributes decision-making problems under risk with attribute weight known and a new decision approach based on entropy weight and TOPSIS is proposed. First, the risk decision matrix is transformed into the certain decision matrix based on the expectation value. Then, the deviation entropy weight method is used to determine the attribute weights. And according to the definitions of the distance and the positive/negative ideal solutions for different data types, the relative closeness coefficients can be calculated by TOPSIS. Furthermore, the alternatives are ranked by the relative closeness coefficients. Finally, an application case is given to demonstrate the steps and effectiveness of the proposed approach.

Keywords: hybrid decision, risk decision, TOPSIS.

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1. Introduction

The hybrid multiple attribute decision making problems are the multiple attribute decision making problems where the attributes contain both the quantitative index and the qualitative index. The multiple attribute decision making is widely used in the field of society, economy, management, military affairs and engineering technology to solve the problems such as investment decision, project evaluation, economic benefit evaluation and personnel performance appraisal, etc. (Hwang and Yoon 1981; Zavadskas et al. 2010a, 2010b; Ginevičius et al. 2008; Ginevičius 2009; Liu 2009a, 2009b; Arslan and Aydin 2009). The quantitative indexes of these
problems are usually difficult to be quantified accurately, and they often take the fuzzy or incomplete form, so these problems are called the hybrid multiple attribute decision making problems and the attribute values are expressed by different data types, such as precision number, interval number, triangular fuzzy number, linguistic variable. Furthermore, for some decision making problems, the decision-makers often face an uncertain environment and the attribute values of the alternatives are the random variables which change as the natural state, and the decision-makers was uncertain of their real state in the future, but they can give all possible natural states, and they can quantify the randomness by setting the probability distribution. These above decision making problems are called the multiple attribute decision making under risk (Yu et al. 2003).

Some decision making problems are both the hybrid multiple attribute decision making problems and the multiple attribute decision making problems under risk, because of the complexity and uncertainty of the decision making problems, so we called these decision making problems the hybrid multiple attribute decision making problems under risk. So the researches on the hybrid multiple attribute decision making problems under risk have not only the important theory significance but also the strong practical value. Yu et al. (2003) researched on the hybrid multiple attributes decision making problem where the attribute weights are unknown and the attribute values are the real numbers, and they proposed a correlative mathematics model. Xia and Wu (2004) proposed the TOPSIS method based on hybrid multiple attribute decision making problem under the attribute weight known. Ding et al. (2007) researched on the hybrid multiple attribute decision making problems where the attribute values are the hybrid number, such as the real number, the interval number, the linguistic variable and the uncertain linguistic variable, and they proposed a decision making method based on the similarity degree under attribute weight known. Yan et al. (2008) also researched on the hybrid multiple attribute decision making problems under the attribute weight unknown, firstly, the maximizing deviation method was used to determine the attribute weight, and then the grey relation method was used to solve the ranking of the alternatives. Wang (2005), Bai et al. (2006), Wang and Cui (2007) researched on the hybrid multiple attribute decision making methods from the aspects of the connection number, the possibility degree and the entropy weight, respectively. However, the attribute index under risk wasn’t considered in these references (Xia and Wu 2004; Ding et al. 2007; Yan et al. 2008; Wang 2005; Bai et al. 2006; Wang and Cui 2007). Luo and Liu (2004) proposed a grey fuzzy relation method and the double base points method, based on the decision making problem under risk where the attribute weights are unknown and the attribute values are the interval numbers. Yao (2007) proposed an extended TOPSIS method, based on the multiple attribute decision making problems under risk with the continuous random variables. At present, the research on the hybrid multiple attribute decision making problems under risk is less. Rao and Xiao (2006) proposed a dynamic hybrid multiple attribute decision making method under risk based on the grey matrix relation degree, which was aiming at the hybrid multiple attribute decision making problems under risk where the attribute weights were unknown and the attribute values were the real numbers, the interval number and the linguistic fuzzy numbers.

This paper focuses on the hybrid multiple attribute decision making problems under risk with the attribute weight known. Firstly, the risk decision matrix is transformed into the
certain decision matrix based on the expectation value; then the deviation entropy weight method is used to determine the attribute weights; finally, the TOPSIS method is used to solve the hybrid decision making problems.

2. The description of the decision making problems

In the hybrid multiple attribute decision making problems under risk, suppose that \( A = (a_1, a_2, \ldots, a_n) \) presents the set of the evaluation alternatives, and \( C = (c_1, c_2, \ldots, c_n) \) presents the set of the evaluation indexes (or attributes), and \( W = (w_1, w_2, \ldots, w_n) \) represent the attribute weight set, where \( w_j \) is the weight of the attribute \( c_j \), and \( 0 \leq w_j \leq 1, \sum_{j=1}^{n} w_j = 1 \), and the attribute weight values are unknown. For the attribute \( c_j \), there are \( l_j \) kinds of possible states \( \Theta_j = (\theta_{1j}, \theta_{2j}, \ldots, \theta_{lj}) \), and the probability of the attribute \( c_j \) under the state \( \theta_i \) is \( p_{ij} \), where \( 0 \leq p_{ij} \leq 1, \sum_{i=1}^{l_j} p_{ij} = 1 \). For the attribute \( c_j \) under the natural state \( \theta_n \), the attribute value of the alternative \( a_i \) is \( x_{ij} \), the data type of \( x_{ij} \) is one of the precision number, the interval number, the triangular fuzzy number, and the linguistic variables (the data is shown in Table 1). The alternatives of the hybrid multiple attribute decision making under risk will be evaluated comprehensively according to these conditions.

Table 1. The decision data of the hybrid multiple attribute decision making under risk

| \( c_1 \) | \( c_2 \) | \( \ldots \) | \( c_n \) |
| --- | --- | --- | --- |
| \( \theta_1 \) | \( \theta_2 \) | \( \ldots \) | \( \theta_{l_1} \) | \( \theta_1 \) | \( \theta_2 \) | \( \ldots \) | \( \theta_{l_2} \) | \( \ldots \) | \( \theta_1 \) | \( \theta_2 \) | \( \ldots \) | \( \theta_{l_n} \) |
| \( p_1^1 \) | \( p_1^2 \) | \( \ldots \) | \( p_1^{l_1} \) | \( p_1^1 \) | \( p_1^2 \) | \( \ldots \) | \( p_1^{l_2} \) | \( \ldots \) | \( p_1^1 \) | \( p_1^2 \) | \( \ldots \) | \( p_1^{l_n} \) |
| \( a_1 \) | \( x_{11}^1 \) | \( x_{11}^2 \) | \( \ldots \) | \( x_{11}^{l_1} \) | \( x_{12}^1 \) | \( x_{12}^2 \) | \( \ldots \) | \( x_{12}^{l_1} \) | \( \ldots \) | \( x_{1n}^1 \) | \( x_{1n}^2 \) | \( \ldots \) | \( x_{1n}^{l_1} \) |
| \( a_2 \) | \( x_{21}^1 \) | \( x_{21}^2 \) | \( \ldots \) | \( x_{21}^{l_1} \) | \( x_{22}^1 \) | \( x_{22}^2 \) | \( \ldots \) | \( x_{22}^{l_1} \) | \( \ldots \) | \( x_{2n}^1 \) | \( x_{2n}^2 \) | \( \ldots \) | \( x_{2n}^{l_1} \) |
| \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) |
| \( a_m \) | \( x_{m1}^1 \) | \( x_{m1}^2 \) | \( \ldots \) | \( x_{m1}^{l_1} \) | \( x_{m2}^1 \) | \( x_{m2}^2 \) | \( \ldots \) | \( x_{m2}^{l_1} \) | \( \ldots \) | \( x_{mn}^1 \) | \( x_{mn}^2 \) | \( \ldots \) | \( x_{mn}^{l_1} \) |

3. The decision making method and the steps

3.1. Preliminaries

3.1.1. The operational laws of the interval number (Rao and Xiao 2006)

Let \( a = [a_L, a_U] \) and \( b = [b_L, b_U] \) be two interval numbers, then the operational laws are shown as follows:

\[
\begin{align*}
a + b &= [a_L, a_U] + [b_L, b_U] = [a_L + b_L, a_U + b_U], \\
a - b &= [a_L, a_U] - [b_L, b_U] = [a_L - b_U, a_U - b_L],
\end{align*}
\]
\[ ab \approx [a^L, a^U] \times [b^L, b^U] = [a^L b^L, a^U b^U], \tag{3} \]

\[ a / b \approx [a^L, a^U] / [b^L, b^U] = [a^L / b^L, a^U / b^L], \tag{4} \]

\[ \lambda a = \lambda [a^L, a^U] = [\lambda a^L, \lambda a^U], \quad \lambda > 0. \tag{5} \]

3.1.2. The operational laws of the triangular fuzzy numbers

Definition 1 (Wang and Zhao 2006): Let \( \tilde{a} = (a^L, a^M, a^U) \) be the triangular fuzzy number, and its membership function \( a(x) : R \rightarrow [0,1] \) is shown as follows:

\[
a(x) = \begin{cases} 
\frac{x - a^L}{a^M - a^L}, & x \in (a^L, a^M) \\
\frac{x - a^U}{a^M - a^U}, & x \in (a^M, a^U) \\
1, & x = a^M \\
0, & x \in (-\infty, a^L) \cup (a^M, \infty). 
\end{cases}
\tag{6} \]

The element \( x \) of the triangular fuzzy number is a real number, and its membership function \( a(x) \) represents the degree that the element \( x \) belongs to the fuzzy set \( \tilde{a} \). \( a(x) \) is the regular, continued, and convex function, and it is composed of the linear non-increasing and non-decreasing part, and it forms a triangle. Generally, \( a^L < a^M < a^U \), where \( a^L \) and \( a^U \) represent the Lower Bounds element and Upper Bounds element of the fuzzy number, respectively, and the difference value between \( a^L \) and \( a^U \) represents the fuzzy degree; \( a^M \) is the primary element of \( \tilde{a} \), and its membership degree is the highest. Specially, if \( a^L = a^M = a^U \), then \( \tilde{a} = a^M \), thus the triangular fuzzy number degenerates into a real number.

Let \( \tilde{a} = (a^L, a^M, a^U) \) and \( \tilde{b} = (b^L, b^M, b^U) \) be two triangular fuzzy numbers, then according to the extension principle of the fuzzy sets, the operational laws are shown as follows (Wang and Zhao 2006):

\[ \tilde{a} + \tilde{b} = [a^L, a^M, a^U] + [b^L, b^M, b^U] = [a^L + b^L, a^M, a^U + b^U], \tag{7} \]

\[ \tilde{a} - \tilde{b} = (a^L - b^U, a^M - b^M, a^U - b^L), \tag{8} \]

\[ \tilde{a} \tilde{b} = [a^L, a^M, a^U] [b^L, b^M, b^U] = [a^L b^L, a^M b^M, a^U b^U], \tag{9} \]

\[ \lambda \tilde{a} = [\lambda a^L, \lambda a^M, \lambda a^U], \lambda \geq 0, \tag{10} \]

\[ \frac{1}{\tilde{a}} = \left( \frac{1}{a^U}, \frac{1}{a^M}, \frac{1}{a^L} \right). \tag{11} \]

3.1.3. The transformation between the linguistic variable and the triangular fuzzy number

The linguistic assessment value is generally chosen from the predefined linguistic assessment set. The linguistic assessment set is an ordered set which is composed of the odd elements,
such as the linguistic assessment set \( S = \{ \text{very poor, poor, fair, good, very good} \} \) which is composed of five elements; the linguistic assessment set \( S = \{ \text{very poor, poor, moderately poor, fair, moderately good, good, very good} \} \) which is composed of seven elements. When the number of the elements is seven, the corresponding relation between the linguistic variable and the triangular fuzzy number is shown in Table 2.

\[ \begin{array}{ccc}
\text{Number} & \text{Linguistic valuation set } S & \text{Triangular fuzzy number} \\
1 & \text{very poor} & (0,0,0.1) \\
2 & \text{poor} & (0,0.1,0.3) \\
3 & \text{moderately poor} & (0.1,0.3,0.5) \\
4 & \text{fair} & (0.3,0.5,0.7) \\
5 & \text{moderately good} & (0.5,0.7,0.9) \\
6 & \text{good} & (0.7,0.9,1) \\
7 & \text{very good} & (0.9,1,1) \\
\end{array} \]

**Table 2.** The corresponding relation between the linguistic variable and the triangular fuzzy number (the number of the elements is seven)

3.2. The decision making method

3.2.1. Transformation of linguistic variables into triangle fuzzy numbers

According to the operational laws of the interval number and the triangular fuzzy number, solve the expectation value of each state in Table 1 in order to transform the risk decision matrix into a certain decision matrix \( Z = [z_{ij}]_{m \times n} \), where

\[
z_{ij} = \sum_{t=1}^{l_j} p_j^t x_{it}^t.
\]

3.2.2. The normalization of the decision making matrix

Normalize the decision making matrix, in order to eliminate the effect of the different physical dimensions on the decision making result. The most common index (attribute) type are the benefit index \( I_1 \) and the cost index \( I_2 \). The normalized methods are shown as follows:

(1) The normalization method of the real number:

\[
r_{ij} = z_{ij} / \left( \sum_{i=1}^{m} z_{ij}^2 \right) \quad j \in I_1, \\
r_{ij} = 1 / z_{ij} / \left( \sum_{i=1}^{m} (1 / z_{ij})^2 \right) \quad j \in I_2.
\]

(2) The normalization method of the interval numbers (Da and Xu 2002):

- suppose that the interval number is expressed by \( z_{ij} = [z_{ij}^L, z_{ij}^U] \), after being normalized, \( z_{ij} = [z_{ij}^L, z_{ij}^U] \) changes into \( r_{ij} = [r_{ij}^L, r_{ij}^U] \), then the normalization method is shown as follows:
\[
\left\{ \begin{array}{l}
\frac{r_{ij}^{-L}}{L} = z_{ij}^{-L} / \sqrt{\sum_{i=1}^{m} (z_{ij}^{U})^2} \\
\frac{r_{ij}^{-U}}{U} = z_{ij}^{-U} / \sqrt{\sum_{i=1}^{m} (z_{ij}^{-L})^2} \\
\end{array} \right. \\
\quad j \in I_1, \quad (14a)
\]

\[
\left\{ \begin{array}{l}
\frac{r_{ij}^{L}}{L} = (1 / z_{ij}^{U}) / \sqrt{\sum_{i=1}^{m} (1 / z_{ij}^{L})^2} \\
\frac{r_{ij}^{U}}{U} = (1 / z_{ij}^{L}) / \sqrt{\sum_{i=1}^{m} (1 / z_{ij}^{U})^2} \\
\end{array} \right. \\
\quad j \in I_1, \quad (14b)
\]

(3) The normalization method of the triangular fuzzy numbers (Da and Xu 2002):

- suppose that the triangular fuzzy number is expressed by \((a_{ij}^{-L}, a_{ij}^{m}, a_{ij}^{U})\), after being normalized, \((a_{ij}^{-L}, a_{ij}^{m}, a_{ij}^{U})\) changes into \((b_{ij}^{-L}, b_{ij}^{m}, b_{ij}^{U})\), then the normalization method is shown as follows:

\[
\left\{ \begin{array}{l}
\frac{r_{ij}^{L}}{L} = z_{ij}^{-L} / \sqrt{\sum_{i=1}^{m} (z_{ij}^{U})^2} \\
\frac{r_{ij}^{M}}{M} = z_{ij}^{M} / \sqrt{\sum_{i=1}^{m} (z_{ij}^{M})^2} \\
\frac{r_{ij}^{U}}{U} = z_{ij}^{U} / \sqrt{\sum_{i=1}^{m} (z_{ij}^{L})^2} \\
\end{array} \right. \\
\quad j \in I_1, \quad (15a)
\]

\[
\left\{ \begin{array}{l}
\frac{r_{ij}^{L}}{L} = (1 / z_{ij}^{U}) / \sqrt{\sum_{i=1}^{m} (1 / z_{ij}^{L})^2} \\
\frac{r_{ij}^{M}}{M} = (1 / z_{ij}^{M}) / \sqrt{\sum_{i=1}^{m} (1 / z_{ij}^{M})^2} \\
\frac{r_{ij}^{U}}{U} = (1 / z_{ij}^{L}) / \sqrt{\sum_{i=1}^{m} (1 / z_{ij}^{U})^2} \\
\end{array} \right. \\
\quad j \in I_2, \quad (15b)
\]

3.2.3. The definition of the distance in different data types

(1) Real numbers.

Let \(a\) and \(b\) be two real numbers, then the distance between \(a\) and \(b\) is defined as follows:

\[d(a, b) = |a - b|. \quad (16a)\]

(2) Interval numbers.

Let \(a (a', a'')\) and \(b (b', b'')\) be two interval numbers, then the distance between \(a\) and \(b\) and is defined as follows:
\[ d(a,b) = \frac{\sqrt{2}}{2} \sqrt{(a^L - b^L)^2 + (a^U - b^U)^2}. \] 

(16b)

(3) Triangular fuzzy numbers.

Let \( a = (a^l, a^m, a^r) \) and \( b = (b^l, b^m, b^r) \) be two triangular fuzzy numbers, then the distance between \( a \) and \( b \) is defined as follows:

\[ d(a,b) = \frac{\sqrt{3}}{3} \sqrt{(a^L - b^L)^2 + (a^M - b^M)^2 + (a^U - b^U)^2}. \]  

(16c)

3.2.4. The attribute weight

The entropy method firstly appeared in the thermodynamics, and it was introduced into the information theory by Shannon (1948). Nowadays, it has been widely used in engineering, economy, finance, etc. Information entropy is the measurement of the disorder degree of a system (Meng 1989). It can measure the amount of useful information with the data provided. When the difference of the values among the evaluating objects on the same attribute is large, while the entropy is small, it illustrates that this attribute provides more useful information, and the weight of this attribute should be set larger. On the other hand, if the difference is smaller and the entropy is larger, the relative weight would be smaller (Qiu 2002). Hence, the entropy theory is an objective way for the weight determination, and it has been widely used to determine the weight in decision making problems (Hwang and Yoon 1981; Zeleny 1982; Qiu 2002; Liu 2010; Zou et al. 2006; Wang and Lee 2009). However, the entropy theory is only used in the classical multiple criteria decision making (MCDM) problems, among which the attribute value is measured in the crisp numbers, and this paper used the entropy method to determine the weight in hybrid types of the attribute value.

For the attribute \( c_j \), we defined that the deviation \( D_{ij} \) between the alternative \( a_j \) and all other deviation:

\[ D_{ij} = \sum_{k=1}^{m} d(r_{ij}, r_{kj}) \quad (i=1,2,\cdots,m; j=1,2,\cdots,n). \]  

(17)

For the attribute \( c_j \), we defined that the total deviation \( D_{ij} \) between each alternative and all other alternative:

\[ D_j = \sum_{i=1}^{m} D_{ij} = \sum_{i=1}^{m} \sum_{k=1}^{m} d(r_{ij}, r_{kj}) \quad (j=1,2,\cdots,n). \]  

(18)

For the attribute \( c_j \), the decision making information can be expressed by the following entropy \( E_j \):

\[ E_j = -K \sum_{i=1}^{m} \frac{D_{ij}}{D_j} \ln \frac{D_{ij}}{D_j} \quad (1 \leq j \leq n), \]  

(19)

where \( K = 1/\ln m \), and \( m \) is the number of the alternatives. Suppose that \( \frac{D_{ij}}{D_j} = 0 \), and \( \frac{D_{ij}}{D_j} \ln \frac{D_{ij}}{D_j} = 0 \).
The difference degree of attribute $c_j$ can be calculated as follows:

$$G_j = 1 - E_j \quad (1 \leq j \leq n).$$

(20)

The entropy weight $w_j$ can be calculated as follows:

$$w_j = G_j / \sum_{j=1}^{n} G_j(1 \leq j \leq n).$$

(21)

3.2.5. The weighting hybrid matrix

According to the entropy weight, calculate the weighting normalized matrix $V$:

$$V = (v_{ij})_{mn} = \begin{bmatrix}
w_1 r_{11} & w_2 r_{12} & \cdots & w_n r_{1n} \\
w_1 r_{21} & w_2 r_{22} & \cdots & w_n r_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
w_1 r_{m1} & w_2 r_{m2} & \cdots & w_n r_{mn}
\end{bmatrix}. \quad (22)

3.2.6. Use TOPSIS to evaluate the alternatives

Technique for Order Performance by Similarity to Ideal Solution (TOPSIS) is a practical and useful technique for ranking and selection of a number of possible alternatives through measuring Euclidean distances. TOPSIS was first developed by Hwang and Yoon (1981). It bases on the concept that the chosen alternative should have the shortest distance from the positive ideal solution (PIS) and the farthest from the negative ideal solution (NIS). TOPSIS is widely used in multiple attribute decision making, and it has been extended with respect to various attributes by using the fuzzy numbers instead of the precise numbers. Jahanshahloo et al. (2006a, 2006b) extended TOPSIS to solve the decision making problems where the attribute value take the from of the interval number and the fuzzy number. Chen and Tsao (2008) extended the TOPSIS method based on the interval-valued fuzzy sets in decision analysis. This paper is to extend the TOPSIS method to solve the decision making problems with the hybrid types of the attribute value.

(1) The positive / negative ideal solution of the alternative:

– suppose that $G^+$ and $G^-$ represent the positive and negative ideal solution, respectively. For the attribute $c_j$, $g_j^+$ and $g_j^-$ represent the attribute value of the positive and negative ideal solution, respectively. Then the positive and negative ideal solution for different data types is shown as follows:

(i) Real number type. If the attribute value of the attribute $c_j$ is the real number $v_{ij}$, then:

$$g_j^+ = \max_i (v_{ij}), \quad g_j^- = \min_i (v_{ij}). \quad (23a)$$

(ii) Interval number type. If the attribute value of the attribute $c_j$ is interval number $[v^L_{ij}, v^U_{ij}]$, then:

$$g_j^+ = [g^+_j, g^+_j] = [\max_i (v^L_{ij}), \max_i (v^U_{ij})]$$

$$g_j^- = [g^-_j, g^-_j] = [\min_i (v^L_{ij}), \min_i (v^U_{ij})]. \quad (23b)$$
(iii) Triangular fuzzy number type.

If the attribute value of attribute $c_j$ is the triangular fuzzy number $[v_{ij}^L, v_{ij}^M, v_{ij}^U]$, then

$$g_j^+ = [g_j^{iL}, g_j^{iM}, g_j^{iU}] = [\max_{i}(v_{ij}^L), \max_{i}(v_{ij}^M), \max_{i}(v_{ij}^U)]$$

$$g_j^- = [g_j^{iL}, g_j^{iM}, g_j^{iU}] = [\min_{i}(v_{ij}^L), \min_{i}(v_{ij}^M), \min_{i}(v_{ij}^U)].$$

(2) Calculate the distance between each alternative and the positive / negative solution, respectively:

$$L_i^+ = L(a_i, G^+) = \sqrt{(d(v_{i1}, g_1^+))^2 + (d(v_{i2}, g_2^+))^2 + \cdots + (d(v_{in}, g_n^+))^2},$$

$$L_i^- = L(a_i, G^-) = \sqrt{(d(v_{i1}, g_1^-))^2 + (d(v_{i2}, g_2^-))^2 + \cdots + (d(v_{in}, g_n^-))^2},$$

where $d(v_{i1}, g_1^+), d(v_{i2}, g_2^+), \ldots , d(v_{in}, g_n^+)$ and $d(v_{i1}, g_1^-), d(v_{i2}, g_2^-), \ldots , d(v_{in}, g_n^-)$ use the different data type of $(v_{i1}, g_1^+), (v_{i2}, g_2^+), \ldots , (v_{in}, g_n^+)$ and $(v_{i1}, g_1^-), (v_{i2}, g_2^-), \ldots , (v_{in}, g_n^-)$, respectively, and they are calculated with formula (16a),(16b),(16c).

(3) Determine the relative closeness degree.

The relative closeness degree between each alternative and ideal solution is shown as follows:

$$C_i = \frac{L_i^-}{L_i^+ + L_i^-} \quad (i = 1, 2, \cdots , m).$$

(4) Rank the order of the alternatives.

The evaluation alternatives can be ranked according to the value of the relative closeness degree, and the bigger the relative closeness degree is, the better the alternative is.

### 4. Application case

An enterprise plans to set a new factory. Suppose that the enterprise will choose an optimized alternative from three alternatives $a_1$, $a_2$, and $a_3$. Suppose that there are four attributes $c_1$, $c_2$, $c_3$ and $c_4$: the direct benefits $c_1$, the indirect benefits $c_2$, the social benefits $c_3$ and the pollution loss $c_4$. Market forecasts that direct benefits $c_1$ and indirect benefits $c_2$ have four natural states: very good $(\theta_1)$, good $(\theta_2)$, fair $(\theta_3)$ and poor $(\theta_4)$; social benefits $c_3$ and pollution loss $c_4$ have three natural states: very good $(\theta_1)$, good $(\theta_2)$, fair $(\theta_3)$. Where the direct benefits $c_1$ is expressed by the real number; indirect benefits $c_2$ is expressed by the interval number; social benefits $c_3$ is expressed by the linguistic variable shown in Table 2; pollution loss $c_4$ is expressed by the triangular fuzzy number. The decision data of each attribute is shown in Table 3.

| $c_1$ | $c_2$ |
|-------|-------|
| $\theta_1$ | $\theta_1$ |
| 0.1 | 0.1 |
| $\theta_2$ | $\theta_2$ |
| 0.3 | 0.2 |
| $\theta_3$ | $\theta_3$ |
| 0.4 | 0.2 |
| $\theta_4$ | $\theta_4$ |
| 0.2 | 0.4 |

Table 3. The decision data of each attribute
Table 4. The decision data of each attribute (cont.)

|   | \(c_3\) |   | \(c_4\) |
|---|---------|---|---------|
|   | \(\theta_1\) | \(\theta_2\) | \(\theta_3\) | \(\theta_1\) | \(\theta_2\) | \(\theta_3\) |
| \(a_1\) | good | good | fair | [190,200,210] | [195,205,215] | [200,210,220] |
| \(a_2\) | very good | fair | poor | [210,220,230] | [215,225,235] | [185,195,205] |
| \(a_3\) | M-good* | good | M-poor** | [175,195,205] | [235,245,255] | [195,215,230] |

* M-good = moderately good, ** M-poor = moderately poor.

The decision steps are shown as follows:

1. Transform the risk matrix into the certain matrix

\[
Z = \begin{bmatrix}
26.6, [114.0, 126.0] & [0.54, 0.74, 0.88] & [196.0, 206.0, 216.0] \\
23.9, [115.7, 135.9] & [0.36, 0.49, 0.63] & [198.5, 208.5, 218.5] \\
23.4, [120.6, 136.0] & [0.40, 0.60, 0.77] & [197.0, 215.0, 230.0]
\end{bmatrix}
\]

2. Normalize the decision matrix

\[
R = \begin{bmatrix}
0.6224, [0.4959, 0.6228] & [0.4066, 0.6907, 1.1543] & [0.5107, 0.5667, 0.6325] \\
0.5593, [0.5033, 0.6718] & [0.2710, 0.4574, 0.8264] & [0.5172, 0.5736, 0.6398] \\
0.5476, [0.5246, 0.6723] & [0.3012, 0.5601, 1.0100] & [0.5133, 0.5915, 0.6735]
\end{bmatrix}
\]

3. Calculate the entropy weight

\[w = (0.3870, 0.1692, 0.1639, 0.2800);\]

4. Calculate the weighting hybrid matrix

\[
V = \begin{bmatrix}
0.2409, [0.0839, 0.1054] & [0.0666, 0.1132, 0.1891] & [0.1430, 0.1587, 0.1771] \\
0.2164, [0.0852, 0.1137] & [0.0444, 0.0749, 0.1354] & [0.1448, 0.1606, 0.1791] \\
0.2119, [0.0888, 0.1137] & [0.0493, 0.0918, 0.1655] & [0.1437, 0.1656, 0.1886]
\end{bmatrix}
\]

5. Solve the positive and negative ideal solution

\[G^+ = (0.2409, [0.0888, 0.1137], [0.0666, 0.1132, 0.1891], [0.1448, 0.1656, 0.1886]);\]
\[G^- = (0.2119, [0.0839, 0.1054], [0.0444, 0.0749, 0.1354], [0.1430, 0.1587, 0.1771]);\]

6. Calculate the distances between each alternative and the positive/negative ideal solution

\[L^+ = (0.0147, 0.0733, 0.0506);\]
\[L^- = (0.0692, 0.0124, 0.0347);\]

7. Calculate the relative closeness degree

\[C = (0.8252, 0.1445, 0.4070);\]

8. Rank the order of the alternatives

According to the values of relative closeness degree, the ranking of the alternatives is:

\[a_1 \succ a_3 \succ a_2.\]
(9) Verify the effectiveness of this method

In order to verify the effectiveness of this method, we use the gray correlation method proposed by Rao and Xiao (2006) to re-rank the alternatives, and the ranking result is: 

\[ a_1 > a_3 > a_2 \]. It is the same as the result ranked by the method proposed in this paper, so we think the method proposed in this paper is effective. In addition, compared with method proposed by Rao and Xiao (2006), the TOPSIS method proposed in this paper is simpler in computing, and this is also the reason that TOPSIS method is more commonly used in the decision making problems.

(10) The sensitivity analysis

When we add or reduce the number of alternatives, original ranking results may be changed in TOPSIS method, that is, the reverse order problem is produced (Li 2008). In order to analysis the sensitivity of this sample which is to identify the optimal alternative, we remove the worst alternative \( a_2 \), and re-rank for \( a_1 \) and \( a_3 \) by method proposed in this paper, then check whether there is the reverse order problem.

After removing the worst alternative \( a_2 \), we get the entropy weight

\[ w = (0.25, 0.25, 0.25, 0.25). \]

Then we get the relative closeness degree after a series of calculating steps

\[ C = (0.7379, 0.2621). \]

So, the ranking of the alternatives is: 

\[ a_1 > a_3. \]

Obviously, before and after removing the worst alternative \( a_2 \), they are the same as ranking result, that is, the reverse order problem does not exist in this sample.

5. Conclusions

The hybrid multiple attribute decision making problems under risk are more consistent with the realistic situation, and they are widely applied. In this paper, the hybrid multiple attribute decision making method under risk based on entropy weight and TOPSIS is presented, and the decision making steps are given. The definition of the method is definite and it is easy to understand. The method can solve the risk decision making problems based on many data types, including the precision number, the interval number, the fuzzy number and the linguistic variable. Compared with the method proposed by Rao and Xiao (2006), the method proposed in this paper is simpler in computing, and it enriches and develops the theory and method of the hybrid decision making under risk. But in this paper, the expectation value method is adopted to transform the risk decision making problems into the certain ones when solving risk decision making problem, and it is a very simple method. So other transform methods will be researched continuously in the future.

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References

Arslan, G.; Aydin, O. 2009. A new software development for fuzzy multicriteria decision making, Technological and Economic Development of Economy 15(2): 197–212. doi:10.3846/1392-8619.2009.15.197-212

Bai, M. G.; Zhu, J. F.; Yao, Y. 2006. Multiple Mixed-attribute Decision Making Method Based on Possibility Degree, Commercial Research (14): 19–21.

Chen, T. Y.; Tsao, C. Y. 2008. The interval-valued fuzzy TOPSIS method and experimental analysis, Fuzzy Sets and Systems 15: 1410–1428. doi:10.1016/j.fss.2007.11.004

Da, Q. L.; Xu, Z. S. 2002. Single-objective optimization model in uncertain multi-attribute decision making, Journal of Systems Engineering 17(2): 50–55.

Ding, C. M.; Li, F.; Qi, H. 2007. Technique of hybrid multiple attribute decision making based on similarity degree to ideal solution, Systems Engineering and Electronics 29(5): 737–740.

Ginevičius, R.; Podvezko, V.; Raslanas, S. 2008. Evaluating the alternative solutions of wall insulation by multi-criteria methods, Journal of Civil Engineering and Management 14(4): 217–226. doi:10.3846/1392-3730.2008.14.20

Ginevičius, R. 2009. Quantitative evaluation of unrelated diversification of enterprise activities, Journal of Civil Engineering and Management 15(1): 105–111. doi:10.3846/1392-3730.2009.15.105-111

Hwang, C. L.; Yoon, K. 1981. Multiple Attribute Decision Making: Methods and Application. Springer, New York.

Jahanshahloo, G. R.; Hosseinzadeh Lotfi, F.; Izadikhah, M. 2006a. An algorithmic method to extend TOPSIS for decision making problems with interval data, Applied Mathematics and Computation 175: 1375–1384. doi:10.1016/j.amc.2005.08.048

Jahanshahloo, G. R.; Hosseinzadeh Lotfi, F.; Izadikhah, M. 2006b. Extension of the TOPSIS method for decision making problems with fuzzy data, Appl. Math. Comput. 181: 1544–1551. doi:10.1016/j.amc.2006.02.057

Liu, P. D. 2009a. Multi-Attribute Decision making Method Research Based on Interval Vague Set and TOPSIS Method, Technological and Economic Development of Economy 15(3): 453–463. doi:10.3846/1392-8619.2009.15.453-463

Liu, P. D. 2009b. A novel method for hybrid multiple attribute decision making, Knowledge-Based Systems 22(5): 388–391.

Liu, P. D. 2010. Research on the Supplier Selection of Supply Chain Based on Entropy Weight and Improved ELECTRE-III Method, International Journal of Production Research. First published on: 17 February 2010 (iFirst).

Li, W. 2008. Evaluation and Sensitivity analysis Based on Multiple Attribute Decision-Making. The master thesis of Donghua university, 33–63.

Luo, D.; Liu, S. F. 2004. Research on grey multi-criteria risk decision making method, Systems Engineering and Electronics 26(8): 1057–1060.

Meng, Q. S. 1989. Information theory. Xi'an Jiaotong : Xi'an Jiaotong University Press.

Qiu, W. H. 2002. Management decision and applied entropy. Beijing: China Machine Press.

Rao, C. J.; Xiao, X. P. 2006. Method of grey matrix relative degree for dynamic hybrid multi-attribute decision making under risk, Systems Engineering and Electronics 28(9): 1353–1357

Shannon, C. E. 1948. A mathematical theory of communications, Bell Systems Technical Journal 27(3): 379–423.

Wang, T. C.; Lee, H. D. 2009. Developing a fuzzy TOPSIS approach based on subjective weights and objective weights, Expert Systems with Applications 36: 8980–8985. doi:10.1016/j.eswa.2008.11.035

Wang, W.; Cui, M. M. 2007. A Technique of Entropy for Hybrid Multiple Attribute Decision Making Problems, Mathematics in Practice and Theory 37(3): 64–68.

Wang, X. 2005. Study for Multi-attribute Mixed Decision Making Model Basing on the Connection Number, Journal of Tianjin University of Science and Technology 20(3): 50–53.
Wang, X. Z.; Zhao, Y. Q. 2006. A New Operator for Triangular Fuzzy Numbers with Application in Weighted Fuzzy Reasoning, *Fuzzy Systems and Mathematics* 20(1): 67–71.

Xia, Y. Q.; Wu, Q. Z. 2004. A technique of order preference by similarity to ideal solution for hybrid multiple attribute decision making problems, *Journal Of Systems Engineering* 19(6): 630–634.

Yan, S. L.; Yang, W. C.; Xiao, X. P. 2008. The Hybrid Multiple Attributes Decision Making Method with Attribute Weight Unknown, *Statistics and Decision* (1): 16–18.

Yao, S. B. 2007. TOPSIS of Continuous Multiple Attribute Decision making Problems under Risk, *Statistics and Decision* (7): 10–11.

Yu, Y. B.; Wang, B. D.; Liu, P. 2003. Risky Multi-objective Decision making Theory and Its Application, *Chinese Journal of Management Science* 11(6): 9–13.

Zavadskas, E. K.; Kaklauskas, A.; Turskis, Z.; Tamošaitienė, J. 2008a. Selection of the Effective Dwelling House Walls by Applying Attributes Values Determined at Intervals, *Journal of Civil Engineering and Management* 14(2): 85–93. doi:10.3846/1392-3730.2008.14.3

Zavadskas, E. K.; Turskis, Z.; Tamošaitiene, J. 2010a. Risk assessment of construction projects, *Journal of Civil Engineering and Management* 16(1): 33–46. doi:10.3846/jcem.2010.03

Zavadskas, E. K.; Viliutiene, T.; Turskis, Z.; Tamosaitiene, J. 2010b. Contractor selection for construction works by applying SAW-G and TOPSIS grey techniques, *Journal of Business Economics and Management* 11(1): 34–55. doi:10.3846/jbem.2010.03

Zou, Z. H.; Yun, Y.; Sun, J. N. 2006. Entropy method for determination of weight of evaluating in fuzzy synthetic evaluation for water quality assessment indicators, *Journal of Environmental Sciences* 18(5): 1020–1023. doi:10.1016/S1001-0742(06)60032-6

NERAIŠKUSIS MAŽESNĖS RIZIKOS DAUGIATIKSLIS SPRENDIMŲ PRIĖMIMO METODAS SU NEŽINOMAIS PRISKIRIAMAIS REIKŠMINGUMAISS

Z. Han, P. Liu

Santrauka. Šiame straipsnyje siekiama išspręsti mišrias mažesnės rizikos daugiatiškus sprendimų priėmimo problemas su žinomu priskiriamu reikšmingumu bei yra siūlomas naujas sprendimų priėmimo metodas grindžiamas entropijos reikšmingumo ir TOPSIS. Pirmiausia, rizikos sprendimų matrica yra transformuojama į tam tikrą sprendimų matricą, grindžiama galimybes verte. Tuomet yra naudojamas entropijos reikšmingumo nuokrypio metodas norint nustatyti priskiriamą reikšmingumą. Atsižvelgiant į atstumo apibrėžimus ir teigiamus / neigiamus idealius sprendimus skirtingiems duomenų tipams, santykinio artumo koeficientas gali būti apskaičiuojami remiantis TOPSIS. Be to, alternatyvos yra reitinguojamos pagal santykinio artumo koeficientus. Galiausiai, yra pateiktas pritaikymo atvejis, siekiant parodyti visus žingsnius ir siūlomą metod veiksmingumą.

Reikšminiai žodžiai: mišrus sprendimas, rizikos sprendimas, TOPSIS.

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