SMALL-$x$ PHYSICS AND THE DETECTION OF UHE NEUTRINOS

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We evaluate both the tau lepton energy loss produced by photonuclear interactions and the neutrino charged current cross section at ultra-high energies, both relevant to neutrino bounds with Earth-skimming tau neutrinos.

Neutrino bounds at ultra-high energies (UHE) have been successfully established by the Pierre Auger Collaboration by looking for tau neutrinos that reach and exit the Earth. This Earth-skimming channel directly depends both on the tau range (the energy loss) and on the neutrino charged current cross section which determine the amount of matter with which the neutrino has to interact to produce an emerging tau. Contrary to the muon case, for tau leptons of energies above $E = 10^7$ GeV photonuclear interactions are responsible for the largest and the most uncertain contribution.

The $Q^2$ scale that contributes to the tau energy loss is low and moderate $Q^2$ at very low $x$, where perturbative and non perturbative QCD effects are mixed. The charged current (CC) neutrino cross section is produced by $W$-boson exchange what sets the relevant scale of $Q^2$ to values up to $M_W^2$ at low $x$, a region where perturbative QCD is expected to work. In both cases the relevant $x$ range lies well outside the regions where structure functions are measured, so one has to rely on extrapolations which contain significant uncertainties.

We present the combined analysis of both the tau energy loss and of the neutrino-nucleus cross section. Because of the large uncertainties in the existing models, the fact that none of them simultaneously covers the kinematical region relevant for both the tau energy loss and the neutrino-nucleus cross section, and the need of consistency in both calculations, we use and extend available models with the aim of estimating the theoretical uncertainty by considering extreme results. In this way, in the frame of the most relevant models, we scan the range of possible scenarios for the extrapolation of structure functions to the relevant $x$ and $Q^2$ range.
The contribution to the fractional average energy loss per unit depth of taus from photonuclear interactions above the scale of a few TeV,

$$b(E) = -\frac{1}{E} \left\langle \frac{dE}{dX} \right\rangle,$$

is obtained by integration of the lepton-nucleus differential cross section, $d\sigma^{lA}/dy$:

$$b(E) = \frac{N_A}{A} \int dy \int dQ^2 \frac{d\sigma^{lA}}{dQ^2dy},$$

where $N_A$ is Avogadro’s number, $A$ the mass number, and $y$ the fraction of energy lost by the lepton in the interaction. For the lepton-nucleus differential cross section we consider the general expression for virtual photon exchange in terms of structure functions.

The photonuclear contributions to $b(E)$ computed (for standard rock $A = 22$ throughout all this paper) with ALLM and with CKMT structure functions, and the same nuclear corrections, give very close results (see Fig. 1). The BB/BS calculation gives the largest of the predicted energy loss rates up to energies of the order $E = 10^7$ GeV. The BB/BS, ALLM, and CKMT calculations of the photonuclear contribution to tau energy loss, $b(E)$, agree within a 30% and go approximately parallel for all energies, which is an indication of a systematic normalization difference of the structure functions in each model. The lowest values of $b(E)$ at high energies is obtained with the ASW structure functions, which are based on the geometric scaling property that all data on $\sigma^{\gamma^*p}$ and on $\sigma^{\gamma^*A}$ lie on a single universal curve in terms of the scaling variable $\tau = Q^2/Q_{sat}^2$ whose form is inspired in saturation physics. Above the scale of $E = 10^7$ GeV the PT result exceeds all other existing predictions by at least a factor 2 already at $E = 10^9$ GeV (i.e. a factor 4 with respect the ASW prediction, see Fig. 1). Thus the PT prediction can be considered as an estimate of the upper limit of the tau energy loss at UHE.

![Figure 1: The photonuclear energy loss rate, $b(E)$, computed in different models.](image)

Much of the uncertainty in the tau energy loss is actually due to nuclear effects. The choice of nuclear corrections translates into differences in the calculated value of $b(E)$ by a factor rising from 1.5 to 2.5 as the tau energy increases in the range $E = 10^6$-$10^9$ GeV when using the ALLM structure function. This energy range corresponds to the region of very low $x$ where differences in the nuclear correction factor are large. In Fig. 2 it is shown how the small $x$ contribution becomes more and more important as energy increases.

We have also studied how the uncertainties in the $F_2$ structure function at low $x$ affect the CC neutrino deep inelastic cross section that is expressed in terms of the structure function $F_2$. 
as follows:

$$\frac{d\sigma^{\nu N}_{CC}}{dQ^2 dy} = \frac{G_F^2}{4\pi} \left( \frac{M_W^2}{M_W^2 + Q^2} \right)^2 \frac{F_2^{\nu N}}{y} \left[ 1 + (1 - y)^2 \right],$$

(3)

where $E$ is the neutrino energy and $y$ the fraction of energy lost by the neutrino in the interaction. In this expression $F_L$ and $xF_3$ contributions are neglected since $F_L$ tends to zero as $Q^2$ rises and $xF_3$ deals basically with the valence partons which hardly contribute at the low $x$ values relevant for the cross section.

The $F_2$ structure function for neutrino interaction is related to the $F_2$ structure function for charged lepton interactions by the ratio of the weak and electromagnetic couplings through $F_2^{\nu N} = \frac{18}{5} F_2^{lN}$ (assuming a symmetric sea). Then, to calculate the neutrino-nucleon cross section at high energies, the structure function $F_2$ for charged lepton interaction valid up to very low $x$ and high $Q^2$ must be used.

The neutrino-nucleon cross sections from ALLM and CKMT structure functions are clearly below predictions from modern parton densities\(^\text{12}\), so to discuss the theoretical uncertainties in the estimation of the CC neutrino-nucleon cross section we have taken the parameterization of $F_2$ à la BCDMS obtained by the SMC Collaboration\(^\text{13}\), which correctly represents the existing experimental data at high $Q^2$ and provides a smooth connection at neutrino energies around $E = 10^7$ GeV with the parton density prediction.

We have performed three different extrapolations at low $x$ of the $F_2$ parameterization à la BCDMS, one following the ASW structure function, a second one from the phenomenological parameterization fitting low $x$ HERA data\(^\text{14}\), and the third one which corresponds to the double logarithmic approximation (DLA) in QCD\(^\text{15}\) (KOPA). The ASW and KOPA structure functions are valid at low $x$, $x < 0.01$, i.e. at high energies.

We can see in Fig. 3 that in comparison with the prediction obtained with evolved QCD parton densities\(^\text{12}\), both KOPA and ASW estimations are below at high energies. On the other hand the extrapolation of the HERA based parameterization with the exponent $\lambda = 0.0481 \ln(Q^2/0.292^2)$ ($F_2 \sim x^{-\lambda}$), produces an extremely fast increase of the cross section with energy, since this exponent rises to values above $\lambda \sim 0.5$ when $Q^2$ becomes large, in contradiction with perturbative calculations. For the more realistic scenarios, when the rise of the exponent freezes to smaller values $\lambda < 0.4$, our prediction supports the result obtained in previous detailed analysis\(^\text{12}\). When considering only physically motivated extrapolations, the theoretical uncertainty at $E = 10^9$ GeV is a factor 2. In the case of the CC neutrino-nucleon cross section the importance of nuclear effects at high energies is expected to be small\(^\text{16}\).

The detection of UHE $\tau$-neutrinos is dominated by small $x$ physics both at low and high $Q^2$. The effects discussed in this paper should be accounted for in the process of optimizing the determination of a bound from Earth-skimming $\nu_\tau$. 

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**Figure 2:** The relative contribution of $x < x_{cut}$ to the photonuclear energy loss rate, $b(E)$. 

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Figure 3: The neutrino-nucleon CC cross section as a function of the neutrino energy, $E$.

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