Bosonic string quantization in a static gauge

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Abstract

The bosonic string in $D$-dimensional Minkowski space–time is quantized in a static gauge. It is shown that the system can be described by $D-1$ massless free fields constrained on the surface $L_m = 0$, for $m \neq 0$, where $L_m$ are the generators of conformal transformations. The free fields are quantized and the physical states are selected by the conditions $L_m |\psi_{ph}\rangle = 0$, for $m > 0$. The Poincaré group generators on the physical Hilbert space are constructed and the critical dimension $D = 26$ is recovered from the commutation relations of the boost operators. The equivalence with the covariant quantization is established. A possible generalization to the AdS string dynamics is discussed.

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Introduction

In this paper, we quantize the fundamental bosonic string propagating in flat $D$-dimensional Minkowski space–time in a static gauge. The static gauge relates the target spacetime coordinate $X^0$ to the evolution parameter $\tau$, and thus is the most natural gauge of particle or string dynamics. One would have thought that the subject of this paper was already settled in the 1970s and should be textbook material by now. However, to the best of our knowledge the quantization of the string in a static gauge has not been completely achieved to date (see [1–4] for the previous literature on the subject). In a sense the static gauge quantization is the least optimal route for quantizing the system that one would like to take: it neither allows for a solution of the constraint conditions, as is achieved in light-cone gauge (modulo the level-matching condition), nor is it manifestly covariant, leaving one with the need to demonstrate quantum Poincaré symmetry.

Hamiltonian reduction in this gauge leads to square root expressions for the energy $E$ and the other Poincaré symmetry generators and creates operator ordering ambiguities for them. A possible solution to this problem was proposed in [5] for the particle dynamics in a curved...
background. In this case, the static gauge naturally leads to the coordinate representation, where the energy square is quadratic in canonical momenta, which allows us to find a solution of the ordering problem for the operator $E^2$ up to a constant factor in front of the scalar curvature term. The constant can be fixed from the commutation relations of $E^2$ with other symmetry generators. The square root from the eigenvalues of $E^2$ then provides the energy spectrum. This quantization scheme in AdS spaces reproduces the well-known oscillator type spectrum of the AdS particle.

If one tries to apply a similar approach to the string dynamics, a complicated ordering ambiguity problem arises already in a flat background. Due to this problem, the static gauge was usually avoided in the literature and a consistency of the quantized string theory was analyzed in the light-cone gauge, where the form of the Poincare group generators is most simple [6] (see [7] for a textbook treatment).

We study quantization of the bosonic string dynamics in a static gauge and propose a solution of the ordering problem similar to the particle case 3. For simplicity, we consider an open string with worldsheet coordinates $(\tau, \sigma)$ given on the strip $\tau \in \mathbb{R}_1$, $\sigma \in (0, \pi)$, where the target space coordinates $X^\mu (\mu = 0, \ldots, D-1)$ satisfy the Neumann boundary conditions $X^\mu (\tau, 0) = 0 = X^\mu (\tau, \pi)$. The generalization of the obtained results to the closed string dynamics is straightforward. We use the Minkowski space metric $\eta_{\mu\nu} = \text{diag}(-1, 1, \ldots, 1)$.

**Hamiltonian reduction**

The open string dynamics is described by the following action in the first order formulation:

$$S = \int d\tau \int_0^\pi d\sigma \left( \mathcal{P}_\mu \dot{X}^\mu - \lambda_1 (\mathcal{P}_\mu \mathcal{P}^\mu + X^\mu \dot{X}_\mu) - \lambda_2 (\mathcal{P}_\mu X^\mu) \right).$$

(1)

Here $\mathcal{P}_\mu$ are the canonically conjugated variables to the target space coordinates $X^\mu$ and the Lagrange multipliers $\lambda_1, \lambda_2$ enforce the Virasoro constraints

$$\mathcal{P}_\mu \mathcal{P}^\mu + X^\mu \dot{X}_\mu = 0, \quad \mathcal{P}_\mu X^\mu = 0,$$

(2)

which are the generators of gauge transformations.

The static (or time-like) gauge is introduced by the gauge fixing conditions

$$X^0 + \mathcal{P}_0 \tau = 0, \quad \mathcal{P}_0 = 0.$$

(3)

The action (1) in this gauge reduces to

$$S = \int d\tau \int_0^\pi d\sigma \left( \mathcal{P}_k \dot{X}^k - \frac{1}{2} \mathcal{P}_0^2 \right),$$

(4)

where $k = 1, \ldots, D-1$ and we have neglected the time derivative term $\frac{2}{\pi} (\frac{1}{2} \mathcal{P}_0^2 \tau)$.

With the help of (2) and (3), the term $\mathcal{P}_0^2$ can be expressed through the phase space variables ($\mathcal{P}_k, X^k$) and we find the free-field Hamiltonian in (4)

$$H = \frac{1}{2} \int_0^\pi d\sigma \left( \mathcal{P}^2 + \dot{X}^2 \right).$$

(5)

Thus, the reduced action (4) describes $D-1$ massless free fields with the constraints

$$\dot{X}^2 = 0, \quad \mathcal{P}_0 X^0 = 0,$$

(6)

which still remain from (2) and (3). At this stage we stop the Hamiltonian reduction and analyze the $(D-1)$-dimensional free-field theory with the constraints (6).

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3 We note that our work is distinct to the effective field theory studies of relativistic long strings as they appear, e.g. in QCD flux tubes. See the recent works [11] in this context as well as [12] where the quantum Lorentz symmetry in a static gauge was shown to hold in the effective theory.
The free fields on the $(\tau, \sigma)$ strip are given by

$$X_k(\tau, \sigma) = \frac{1}{2} \phi_k(\tau + \sigma) + \frac{1}{2} \phi_k(\tau - \sigma),$$

where the chiral components admit the mode expansion

$$\phi_k(z) = q_k + p_k z + \frac{1}{2 n} \sum_{n \neq 0} a_n \alpha^{n \cdot z}, \quad z \in \mathbb{R},$$

with the canonical Poisson brackets

$$\{p^i, q^j\} = \delta^{ij}, \quad \{a_m, a_n^\dagger\} = i n \delta_{m+n}.$$  \hspace{1cm} (7)

The generators of conformal transformations defined by

$$L_m = \frac{1}{2} \int_0^{2\pi} \frac{d\sigma}{\pi} \left( \frac{e^{i\alpha^m_n \phi}}{(\phi^2)^{1/2}} \right)^2 = \frac{1}{2} \sum_{n=-\infty}^{\infty} a_{m-n} a_n,$$

realize the Witt algebra

$$\{L_m, L_n\} = i (m - n) L_{m+n},$$

where $L_0$ coincides with the Hamiltonian (5).

The constraints (6) may then be seen to be equivalent to

$$L_m = 0, \quad \text{for} \quad m \neq 0,$$

and according to (10) they form a set of second class constraints.

The quantum theory of this system can be identified with a quantized $D-1$ component free-field theory restricted on the physical states, which satisfy the conditions

$$L_m |\Psi_{ph}\rangle = 0, \quad \text{for} \quad m > 0.$$  \hspace{1cm} (11)

We follow this scheme in the next section, but before quantization let us discuss the realization of the Poincare symmetry in a static gauge. To find the free-field form of the symmetry generators, one can start with the dynamical integrals of the initial system (1)

$$P^\mu = \int_0^{2\pi} \frac{d\sigma}{\pi} \mathcal{P}^\mu, \quad J^{\mu\nu} = \int_0^{2\pi} \frac{d\sigma}{\pi} \left( \mathcal{P}^{\mu} X^\nu - \mathcal{P}^{\nu} X^\mu \right)$$

and calculate them in the gauge (3), for example, at $\tau = 0$. The time component $X^0$ then vanishes and for $P^0$ only its zero mode $p^0$ survives, which can be expressed through the free-field variables from the first Virasoro constraint in (2). This simplification degenerates the boosts $J^{0k}$ and one has to deform them by the constraints (11), in order to keep the constraint surface Lorentz invariant. Taking a linear deformation only, we find

$$P^k = p^k, \quad J^{0k} = p^0 q^k + \frac{i}{p^0} \sum_{n \neq 0} a_n \alpha_n L_{-n},$$

$$P^0 = p^0 = \sqrt{2L_0}, \quad J^{0k} = p^0 q^k + \frac{i}{p^0} \sum_{n \neq 0} a_n \alpha_n L_{-n}.$$  \hspace{1cm} (12)

The Poisson brackets of these functions form the Poincare algebra on the phase of the free-field theory and the invariance of the constraint surface (11) is given by

$$\{J^{0k}, L_m\} = \frac{m}{p^0} \sum_{n \neq -m, 0} \frac{a_n^{k+m}}{n+m} L_{-n} - \left( \frac{i(mq^k + p^k)}{p^0} + \frac{m}{(p^0)^2} \sum_{n \neq 0} a_n^k L_{-n} \right) L_m.$$  \hspace{1cm} (13)

Note that the rhs of this equation contains $L_m$s with both positive and negative indices. The quantum version of (16), therefore, cannot provide the Lorentz invariance of the physical system.
we write the first level states in the form
\[ \text{onto the momentum-dependent ground state} \]

Higher powers of the constraints \( J \) in the Hilbert space defined by (12). Hence, one needs further deformation of the boosts in terms of the constraints \( J \).

We quantize the system by lifting the canonical Poisson brackets (8) to the commutation relations
\[ \{ p^i, q^j \} = -i \delta^{ij}, \quad \{ a^k_n, a^l_m \} = m \delta_{m+n} \delta^{kl}. \]
The unconstrained Hilbert space \( \mathcal{H} \) is generated by the action of the creation operators \( a^k_n \) onto the momentum-dependent ground state \( | \vec{p} \rangle \), which is defined by
\[ a^k_n | \vec{p} \rangle = p^k | \vec{p} \rangle, \quad a^k_n | \vec{p} \rangle = 0, \quad n > 0. \]

The operators \( L_0 \) have no ordering ambiguity, except \( L_0 \), and choosing \( L_0 = \frac{1}{2} \vec{p}^2 + N \), where \( N \) is the level operator,
\[ N = \sum_{n>0} a_{-n} a_n, \]
one gets the Virasoro algebra in the standard form
\[ \{ L_m, L_n \} = (m - n)L_{m+n} + \frac{D-1}{12} (m^3 - m) \delta_{m+n}. \]

Let us now describe the physical states defined by (12). Since the commutators of \( L_1 \) and \( L_2 \) generate all other \( L_n \)s with positive \( n \), it suffices to check only the two conditions
\[ L_1 | \Psi \rangle = 0, \quad L_2 | \Psi \rangle = 0, \]
to verify whether a state \( | \Psi \rangle \) is physical or not. The ground state \( | \vec{p} \rangle \) is obviously physical. If we write the first level states in the form \( | \Psi \rangle = \lambda^k a^k_0 | \vec{p} \rangle \), where \( \lambda^k \) are constants, from (25) we...
obtain $\lambda^k p^k = 0$. Thus, there are $D - 2$ independent components on the first exited level, which is in accordance with other quantization schemes. The conditions (25) for the second level physical states $|\Psi\rangle = (\Lambda^{kl} a^k_+ d^l_{-1} + \rho^k a^k_{-2}) |\vec{p}\rangle$ lead to $\Lambda^{kl} p^l + \rho^k = 0$, $\Lambda^{kk} + 2 \rho^k p^k = 0$, and the number of independent components here is $\frac{(D-2)(D+1)}{2}$. This is again consistent with the light-cone or covariant quantization schemes.

One can continue the description of higher level physical states as in the covariant quantization [7]. The difference here is that we do not have negative or zero norm states and no factorization over the zero norm states exists. A physical state with momentum $\vec{p}$ and level $N$ will be denoted by $|\vec{p}, N\rangle$.

Let us consider the Poincare symmetry generators. The translation and rotation generators have no ordering ambiguity and one can use their classical expressions (14) directly. The ordering freedom for the energy square operator can be expressed by a parameter $a$ in the form $(p^0)^2 = 2(L_0 - a)$ and we get the energy operator

$$p^0 = \sqrt{\vec{p}^2 + 2(N - a)},$$

which is diagonal on the states $|\vec{p}, N\rangle$. The mass operator then becomes $M^2 = 2(N - a)$.

Most problematic are the boosts. Using their classical expression (17), we represent the action of the boost operators on a physical state $|\vec{p}, N\rangle$ in the following form:

$$J^{0k} |\vec{p}, N\rangle = \left( p^0 q^k : + \frac{i}{p^0} \sum_{n=1}^{N} \sum_{(n_1, \ldots, n_j)} f^{(n_1, \ldots, n_j)} ( p^0 ) L_{-n_1} \cdots L_{-n_j} \frac{d^{nk}}{n} \right) |\vec{p}, N\rangle, \quad (27)$$

where $p^0 q^k := \frac{1}{2} ( q^k p^0 + p^0 q^k ) = q^k p^0 - \frac{i p^k}{p^0}$ ; $(n_1, \ldots, n_j)$ are the ordered partitions of an integer $n$, with $n_1 + \cdots + n_j = n$, $n \geq n_1 \geq \cdots \geq n_j > 0$ and the coefficients $f^{(n_1, \ldots, n_j)}$ are the quantum counterparts of $f_j$.

We allow quantum deformations for these coefficients and to find them we require the Lorentz invariance of the physical Hilbert space. This implies that the states $J^{0k} |\vec{p}, N\rangle$ are physical, i.e. they satisfy the conditions (25), and the Lorentz algebra

$$[J^{0k}, J^{0l}] |\vec{p}, N\rangle = i J^{kl} |\vec{p}, N\rangle \quad (28)$$

is fulfilled. Since the operators $J^{0k}$ preserve the level $N$, equation (28) is equivalent to the relation $\langle N| [J^{0k}, J^{0l}] |\vec{p}, N\rangle = \langle N| J^{kl} |\vec{p}, N\rangle$, where $\langle N|$ is a ‘bra’ physical state of level $N$.

Note that the calculation of these matrix elements can be simplified due to

$$\langle N| J^{0k} J^{0l} |\vec{p}, N\rangle = \langle N| : p^0 q^k : J^{0l} |\vec{p}, N\rangle, \quad (29)$$

which follows from the structure of the boost operators in (27). Using these conditions, one can fix the coefficients $f^{(n_1, \ldots, n_j)}$ level by level.

The Lorentz invariance of the vacuum state is obvious. On the first excited level we have

$$J^{0k} |\vec{p}, 1\rangle = \left( q^k p^0 - \frac{ip^k}{2p^0} + \frac{if^{(1)}}{p^0} L_{-1} d^k_1 \right) |\vec{p}, 1\rangle, \quad (30)$$

where $p^0 = \sqrt{\vec{p}^2 + 2(1 - a)}$. In this case, one has to check only the first condition (25)

$$L_1 J^{0k} |\vec{p}, 1\rangle = [L_1, J^{0k}] |\vec{p}, 1\rangle = 0, \quad (31)$$

and using that $[L_1, q^k] = -ia^k$, we find

$$f^{(1)} = \frac{p^2 + 2(1 - a)}{p^0^2}. \quad (32)$$

Due to (29), the check of (28) is reduced to

$$\langle 1| p^0 q^k \frac{if^{(1)}}{p^0} L_{-1} d^k_1 |\vec{p}, 1\rangle = -\langle 1| a^k d^k_1 |\vec{p}, 1\rangle, \quad (33)$$
which provides \( f^{(1)} = 1 \) and at the same time fixes the ordering constant \( a = 1 \). The corresponding mass operator \( M^2 = 2(N - 1) \) then reproduces the bosonic string spectrum.

On the second level equation (27) takes the form
\[
\mathcal{J}^{0k}|\vec{p}, 2\rangle = \left( q^2 q^0 - \frac{i p^k}{2p^0} + \frac{i}{p^0} L_{-1} d_1^k + \frac{i f^{(2)}}{2p^0} L_{-2} d_1^k + \frac{i f^{(1,1)}}{2p^0} L_{-1} L_{-1} d_1^k \right)|\vec{p}, 2\rangle,
\]
(34)
with \( p^0 = \sqrt{\vec{p}^2 + 2} \), and the conditions \( L_1 \mathcal{J}^{0k}|\vec{p}, 2\rangle = 0, L_2 \mathcal{J}^{0k}|\vec{p}, 2\rangle = 0 \) are equivalent to
\[
3f^{(2)} + 2(\vec{p}^2 + 1)f^{(1,1)} = 2, \quad (4\vec{p}^2 + D - 1)f^{(2)} + 6\vec{p}^2 f^{(1,1)} = 4(\vec{p}^2 + 5).
\]
(35)

The check of (28) now is reduced to
\[
(2|p^q q^k \left( \frac{i f^{(2)}}{2p^0} L_{-2} d_1^k + \frac{i f^{(1,1)}}{2p^0} L_{-1} L_{-1} d_1^k \right)|\vec{p}, 2\rangle = -\frac{1}{2} (2|d_2 d_1^k |\vec{p}, 2\rangle,
\]
(36)
and it gives the additional equation \( f^{(2)} - f^{(1,1)} = 1 \). These three equations define the coefficients of the second level
\[
f^{(1,1)} = \frac{1}{e + 1}, \quad f^{(2)} = \frac{e}{e + 1}, \quad \text{with} \quad e = 2(\vec{p}^0)^2,
\]
(37)
and also fix the critical space–time dimension \( D = 26 \).

Using the coefficients of the first two levels and the obtained values of the parameters \( a \) and \( D \), for the third level coefficients we similarly obtain
\[
f^{(1,1,1)} = \frac{2}{(e + 1)(e + 4)}, \quad f^{(2,1)} = -\frac{e}{(e + 1)(e + 4)}, \quad f^{(3)} = \frac{e^2 + e}{(e + 1)(e + 4)}.
\]
(38)
This procedure can be continued step by step and if one knows all coefficients up to the level \( N - 1 \), one can calculate the coefficients on the level \( N \). This construction provides the following structure:
\[
f^{(n_1, \ldots, n_j)} = \frac{\mathcal{P}^{(n_1, \ldots, n_j)}(e)}{\prod_{m=1}^{n-j}(e + m^2)},
\]
(39)
where \( \mathcal{P}^{(n_1, \ldots, n_j)}(e) \) is a polynomial of the degree \( n - j \) with a partition-dependent coefficient.

To prove (39), one can project the ‘ket’ state \( \mathcal{J}^{0k}|\vec{p}, N\rangle \) onto a ‘bra’ state of the level \( N \) \( (L_{-m_1} \cdots L_{-m_j} 0) \), defined by a partition \( (m_1, \ldots, m_j) \). This projection has to vanish, since the state \( \mathcal{J}^{0k}|\vec{p}, N\rangle \) has to be physical. Hence, the coefficients of the level \( N \) satisfy the equation
\[
\mathcal{M}(m_1, \ldots, m_j, n_1, \ldots, n_j)f^{(n_1, \ldots, n_j)} = F_{(m_1, \ldots, m_j)}.
\]
(40)
Here \( \mathcal{M} \) is the matrix with the coefficients \( (0) \mathcal{J} L_{-m_1} \cdots L_{-m_j} 0 |0\rangle \) and \( F_{(m_1, \ldots, m_j)} \) is obtained from the lower level coefficients. The determinant of the matrix \( \mathcal{M} \) is just the Kac determinant, and its simple form [8] for the central charge \( c = D - 1 = 25 \) provides the denominator in (39).

We could not find a closed form of the numerator for an arbitrary partition, though in some cases they are calculable exactly by recurrence relations. For example, in the case of \( f^{(1,\ldots,1)} \) the polynomial in the numerator of (39) degenerates to a constant and, as in the classical case, it is given by the Catalan number
\[
\mathcal{P}^{(1, \ldots, 1)}_0 = (-1)^{N-1} C_{N-1}.
\]
(41)
Indeed, we explicitly computed the \( f^{(n_1, \ldots, n_j)} \) coefficients up to the level 8.
Connection to the covariant quantization

The covariant quantization [7] contains additional ‘oscillator’ degrees of freedom created by the time-component operators \( \hat{a}^0_{-n} \), with \( n > 0 \). The norm of the corresponding states is indefinite due to the commutation relations \( [\hat{a}^0_m, \hat{a}^0_n] = -m \delta_{m+n} \).

We use the notation \( || \cdot \rangle \rangle \) for the ‘bra’ states of the covariant quantization, to distinguish them from the static gauge states. The physical states of the covariant quantization satisfy the conditions

\[
\hat{L}_m || \psi_{ph} \rangle = 0, \quad \text{for} \quad m > 0 \quad \text{and} \quad (\hat{L}_0 - 1) || \psi_{ph} \rangle = 0, \tag{42}
\]

where

\[
\hat{L}_m = L_m - L^0_m, \quad \text{with} \quad L^0_m = \frac{1}{2} \hat{a}^0_{m-n} \hat{a}^0_n. \tag{43}
\]

The physical states of the static gauge quantization, therefore, are the physical states of the covariant quantization with non excited time-component degrees of freedom. In particular, the state \( |\vec{p}, N \rangle \) can be identified with the states \( || p^0, \vec{p}; 0, N \rangle \), where 0 denotes the vacuum state in the time-component sector and \( p^0 \) is given by (26) at \( a = 1 \).

The boost operators in the covariant quantization

\[
f^{0k} = p^0 q^k - p^k q^0 + i \sum_{n>0} \left( \frac{\hat{a}^0_n \hat{a}^k_n}{n} - \frac{\hat{a}^k_n \hat{a}^0_n}{n} \right) \tag{44}
\]

have no ordering ambiguity and their action on the physical state \( || p^0, \vec{p}; 0, N \rangle \) is given by a finite sum in (44) with \( n \leq N \). Let us take the \( n \)th term of this sum, change the ordering in \( \hat{a}^0_n \hat{a}^k_n \) and consider the action of \( \hat{a}^0_n \) on \( || p^0, \vec{p}; 0, N \rangle \). The operator \( \hat{a}^0_n \) then creates an \( n \)-level state in the time-component sector and one can write the expansion

\[
\hat{a}^0_n || p^0, \vec{p}; 0, N \rangle = \sum_{(n_1, \ldots, n_j)} \tilde{f}^{(n_1, \ldots, n_j)}(p^0) L^0_{-n_1} \cdots L^0_{-n_j} || p^0, \vec{p}; 0, N \rangle, \tag{45}
\]

where \( \tilde{f}^{(n_1, \ldots, n_j)}(p^0) \) are the expansion coefficients and \((n_1, \ldots, n_j)\) is an ordered partitions of \( n \) as in (27). However, note that we now chose the opposite ordering. The action of the operator \( L^0_{-n_1} \) on the physical state \( || p^0, \vec{p}; 0, N \rangle \) can be replaced by \( L_{-n_1} \), and since \( L_{-n_1} \) commutes with the \( L^0_{-n} \) operators, we can move \( L_{-n_1} \) to the left in (45). With this replacement procedure in (45), the action of the operator (44) onto the physical state \( || p^0, \vec{p}; 0, N \rangle \) takes the structure (27), and comparing these two expressions we conclude that \( f^{(n_1, \ldots, n_j)}(p^0) = p^0 \tilde{f}^{(n_1, \ldots, n_j)}(p^0) \). This shows the equivalence between the two quantizations.

To our knowledge, the expansion coefficients in (45) are not known in a closed form, even for the space-component excited states.

Conclusion

We performed a quantization of the bosonic strings in a static gauge. It was shown that the string dynamics in \( D \)-dimensional Minkowski space can be described by \( D - 1 \) component conformal free-field theory, restricted on the constraint surface (11).

Most problematic in this approach are the boost operators. Their structure has been found on the basis of classical calculations. This structure defines the boosts up to some energy-dependent coefficients. These coefficients can be calculated to any desirable level, but their closed form is still missing. Low level calculations of the commutation relations of the boost operators provide the string mass spectrum and the space–time critical dimension in a very simple manner.
We have obtained the equivalence between the static gauge and the covariant quantization. This equivalence shows that the coefficients we were looking for in the static gauge quantization are just the expansion coefficients of the oscillator excitations in terms of the Virasoro excitations. A solution of this problem could be useful for the calculation of the Liouville $S$-matrix [9], where the Virasoro generators are deformed by the terms linear in the creation–annihilation operators.

The initial motivation of this work was a continuation of the paper [5] and the investigation of the AdS string in a static gauge. Even though the static gauge is not a conformal gauge for AdS strings, the Hamiltonian reduction can be done similarly to the flat background. The reduced picture here exhibits a new coset WZW structure, which differs from the Pohlmeyer reduction [10] and we plan to investigate it in the future.

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