Dark Matter at the Center and in the Halo of the Galaxy

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Abstract. All presently known stellar-dynamical constraints on the size and mass of the supermassive compact dark object at the Galactic center are consistent with a ball of self-gravitating, nearly non-interacting, degenerate fermions with mass between 76 and 491 keV/c\(^2\), for a degeneracy factor \(g = 2\). Similar to the masses of neutron stars and stellar-mass black holes, which are separated by an Oppenheimer-Volkoff (OV) limit between 1.4 to 3 \(M_\odot\), the masses of the supermassive fermion balls and black holes are separated by an OV limit of \(1.1 \times 10^8 M_\odot\), for a fermion mass of 76 keV/c\(^2\) and \(g = 2\).

Sterile neutrinos of 76 keV/c\(^2\) mass, which are mixed with at least one of the active neutrinos with a mixing angle \(\theta \sim 10^{-7}\), are produced in about the right amount in the early Universe by incoherent resonant and non-resonant scattering of active neutrinos having an asymmetry of \(L \sim 10^{-2}\). The former process yields sterile neutrinos with a quasi-degenerate spectrum while the latter leads to a thermal spectrum. The mixing necessarily implies the radiative decay of the sterile neutrino into an active neutrino in about \(10^{19}\) years which makes these particles observable.

As the production mechanism of the sterile neutrino is consistent with the constraints from large scale structure formation, cosmic microwave background, big bang nucleosynthesis, core collapse supernovae, and diffuse X-ray background, it could be the dark matter particle of the Universe. At the same time, the quasi-degenerate components of this dark matter, may be responsible for the formation of the supermassive degenerate fermion balls and black holes at the galactic centers via gravitational cooling.

1 Introduction

In a recent paper Schödel et al. reported a new set of data \cite{1} including the corrected old measurements \cite{2} on the projected positions of the star S2(S0-2) that was observed during the last decade with the ESO telescopes in La Silla (Chile). The combined data suggest that S2(S0-2) is moving on a Keplerian orbit with a period of 15.2 yr around the enigmatic strong radiosource Sgr A* that is widely believed to be a black hole with a mass of about \(2.6 \times 10^6 M_\odot\) \cite{2,3}. The salient feature of the new adaptive optics data is that, between April and May 2002, S2(S0-2) apparently sped past the point of closest approach with a velocity \(v \sim 6000\) km/s at a distance of about 17 light-hours \cite{1} or 123 AU from Sgr A*.

Another star, S0-16 (S14), which was observed during the last few years by Ghez et al. \cite{4} with the Keck telescope in Hawaii, made recently a spectacular
U-turn, crossing the point of closest approach at an even smaller distance of 8.32 light-hours or 60 AU from Sgr A∗ with a velocity \( v \sim 9000 \text{ km/s} \). Ghez et al. [4] thus conclude that the gravitational potential around Sgr A∗ has approximately \( r^{-1} \) form, for radii larger than 60 AU, corresponding to 1169 Schwarzschild radii of 26 light-seconds or 0.051 AU for a \( 2.6 \times 10^6 M_\odot \) black hole. Although the baryonic alternatives are presumably ruled out, this still leaves some room for the interpretation of the supermassive compact dark object at the Galactic center in terms of a finite-size non-baryonic dark matter object rather than a black hole. In fact, the supermassive black hole paradigm may eventually only be proven or ruled out by comparing it with credible alternatives in terms of finite-size non-baryonic objects [5].

The purpose of this paper is to explore, using the example of a sterile neutrino as the dark matter particle candidate, the implications of the recent observations for the degenerate fermion ball scenario of the supermassive compact dark objects which was developed during the last decade [5–11].

2 Stellar-dynamical constraints for fermion balls

In a self-gravitating ball of degenerate fermionic matter, the gravitational pressure is balanced by the degeneracy pressure of the fermions due to the Pauli exclusion principle. Nonrelativistically, this scenario is described by the Lane-Emden equation with polytropic index \( p = \frac{3}{2} \). Thus the radius \( R \) and mass \( M \) of a ball of self-gravitating, nearly non-interacting degenerate fermions scale as [7]

\[
R = \left[ \frac{91.869 \, h^6 \, g^2 \, M}{m^8 \, G^3} \right]^{1/3} = 3610.66 \, \text{ld} \left( \frac{15 \, \text{keV}}{m c^2} \right)^{8/3} \left( \frac{2}{g} \right)^{2/3} \left( \frac{M_\odot}{M} \right)^{1/3}.
\]

(1)

Here \( 1.19129 \, \text{ld} = 1 \, \text{mpc} = 206.265 \, \text{AU} \), and \( m \) is the fermion mass. The degeneracy factor \( g = 2 \) describes either spin 1/2 fermions (without antifermions) or spin 1/2 Majorana fermions. (\( \equiv \) antifermions). For Dirac fermions and antifermions, or spin 3/2 fermions (without antifermions), we have \( g = 4 \). Using the canonical value \( M = 2.6 \times 10^6 M_\odot \) and \( R \leq 60 \, \text{AU} \) for the supermassive compact dark object at the Galactic center, we obtain a minimal fermion mass of \( m_{\text{min}} = 76.0 \, \text{keV}/c^2 \) for \( g = 2 \), or \( m_{\text{min}} = 63.9 \, \text{keV}/c^2 \) for \( g = 4 \).

The maximal mass for a degenerate fermion ball, calculated in a general relativistic framework based on the Tolman-Oppenheimer-Volkoff equations, is the Oppenheimer-Volkoff (OV) limit [8]

\[
M_{\text{OV}} = 0.38322 \, \frac{M_\odot^3}{m^2} \, \left( \frac{2}{g} \right)^{1/2} = 2.7821 \times 10^9 M_\odot \left( \frac{15 \, \text{keV}}{m c^2} \right)^2 \left( \frac{2}{g} \right)^{1/2},
\]

(2)
where $M_{\text{Pl}} = (\hbar c/G)^{1/2} = 1.2210 \times 10^{19}$ GeV is the Planck mass. Thus, for $m_{\text{min}} = 76.0$ keV/c$^2$ and $g = 2$, or $m_{\text{min}} = 63.9$ keV/c$^2$ and $g = 4$, we obtain

$$M_{\text{OV}}^{\text{max}} = 1.083 \times 10^8 M_\odot.$$  \hfill (3)

In this scenario all supermassive compact dark objects with mass $M > M_{\text{OV}}^{\text{max}}$ must be black holes, while those with $M \leq M_{\text{OV}}^{\text{max}}$ are fermion balls.

Choosing as the OV limit the canonical mass of the compact dark object at the center of the Galaxy, $M_{\text{OV}}^{\text{min}} = 2.6 \times 10^6 M_\odot$, yields a maximal fermion mass of $m_{\text{max}} = 491$ keV/c$^2$ for $g = 2$, or $m_{\text{max}} = 413$ keV/c$^2$ for $g = 4$. In this ultrarelativistic limit, there is little difference between the black hole and degenerate fermion ball scenarios, as the radius of the fermion ball is 4.45 compared to 3 Schwarzschild radii for the radius of the event horizon of a non-rotating black hole of the same mass. In fact, varying the fermion mass between $m_{\text{min}}$ and $m_{\text{max}}$, one can smoothly interpolate between a fermion ball of the largest acceptable size and a fermion ball of the smallest possible size, at the limit between fermion balls and black holes.

The masses of the supermassive compact dark objects discovered so far at the centers of both active and inactive galaxies are all in the range $10^6 M_\odot \lesssim M \lesssim 3 \times 10^9 M_\odot$.  \hfill (4)

Thus, as $M_{\text{OV}}^{\text{max}}$ falls into this range as well, we need both supermassive fermion balls ($M \leq M_{\text{OV}}^{\text{max}}$) and black holes ($M > M_{\text{OV}}^{\text{max}}$) to describe the observed phenomenology. At first sight, such a hybrid scenario does not seem to be particularly attractive. However, it is important to note that a similar scenario is actually realized in Nature, with the co-existence of neutron stars which have masses $M \leq M_{n}^{\text{OV}}$, and stellar-mass black holes with mass $M > M_{n}^{\text{OV}}$, as observed in stellar binary systems in the Galaxy [13]. Here the OV limit $M_{n}^{\text{OV}}$, which includes the nuclear interaction of the neutrons, is somewhat uncertain due to the unknown equation of state. But the consensus of the experts [13] is that it must be in the range

$$1.4 M_\odot \leq M_{n}^{\text{OV}} \lesssim 3 M_\odot.$$  \hfill (5)

None of the observed neutron stars have masses larger than $1.4 M_\odot$, while there are at least nine candidates for stellar-mass black holes larger than $3 M_\odot$ [13]. It is thus conceivable that Nature allows for the co-existence of supermassive fermion balls and black holes as well. Of course, we would expect characteristic differences in the properties of supermassive fermion balls and black holes. Similarly, pulsars and stellar-mass black holes are quite different, as pulsars have a strong magnetic field and a hard baryonic surface, while black holes are surrounded by an immaterial event horizon instead. However, one may also argue that the astrophysical differences between supermassive black holes and fermion balls close to the OV limit are not so easy to detect because both objects are of non-baryonic nature.
3 Cosmological constraints for sterile neutrino dark matter

If the supermassive compact dark object at the Galactic center is indeed a degenerate fermion ball of mass $M = 2.6 \times 10^6 M_\odot$ and radius $R \leq 60$ AU, the fermion mass must be in the range

$$76.0 \text{ keV}/c^2 \leq m \leq 491 \text{ keV}/c^2 \quad \text{for} \quad g = 2$$
$$63.9 \text{ keV}/c^2 \leq m \leq 413 \text{ keV}/c^2 \quad \text{for} \quad g = 4.$$  \hspace{1cm} (6)

It would be most economical if this particle could represent the dark matter particle of the Universe, as well. In fact, it has been recently shown [9] that an extended cloud of degenerate fermionic matter will eventually undergo gravitational collapse and form a degenerate supermassive fermion ball in a few free-fall times, if the collapsed mass is below the OV limit. During the formation, the binding energy of the nascent fermion ball is released in the form of high-energy ejecta at every bounce of the degenerate fermionic matter through a mechanism similar to gravitational cooling that is taking place in the formation of degenerate boson stars [9]. If the mass of the collapsed object is above the OV limit, the collapse inevitably results in a supermassive black hole.

The conjectured fermion could be a sterile neutrino $\nu_s$ which does not participate in the weak interactions. We will now assume that its mass and degeneracy factor is $m_s = 76.0 \text{ keV}/c^2$ and $g_s = 2$, corresponding to the largest fermion ball that is consistent with the stellar-dynamical constraints. In order to make sure that this fermion is actually produced in the early Universe it must be mixed with at least one active neutrino, e.g., the $\nu_e$. Indeed, for an electron neutrino asymmetry

$$L_{\nu_e} = \frac{n_{\nu_e} - n_{\overline{\nu}_e}}{n_\gamma} \sim 10^{-2}$$  \hspace{1cm} (7)

and a mixing angle $\theta_{es} \sim 10^{-7}$ [14], incoherent resonant and non-resonant active neutrino scattering in the early Universe produces sterile neutrino matter amounting to the required fraction $\Omega_m h^2 = (0.135^{+0.008}_{-0.009})$ [15] of the critical density of the Universe today. Here $n_{\nu_e}$, $n_{\overline{\nu}_e}$ and $n_\gamma$ are the electron neutrino, electron antineutrino and photon number densities, respectively. An electron neutrino asymmetry of $L_{\nu_e} \sim 10^{-2}$ is compatible with the observational limits on $^4$He abundance, radiation density of the cosmic microwave background at decoupling, and formation of the large scale structure [16,17] which constrain the electron neutrino asymmetry to the range

$$-4.1 \times 10^{-2} \leq L_{\nu_e} \leq 0.79.$$  \hspace{1cm} (8)

Within these limits, $L_{\nu_e}$ must be currently regarded as a free parameter which may be determined by future observations, in a similar way as the baryon asymmetry

$$\eta = \frac{n_B - n_{\overline{\nu}_e}}{n_\gamma}$$  \hspace{1cm} (9)
has been determined by big bang nucleosynthesis to $\eta = (2.6 - 6.2) \times 10^{-10}$ [18] and by the cosmic microwave background radiation for $\eta = (6.1^{+0.3}_{-0.2}) \times 10^{-10}$ [15]. There is no reason to expect that $L_{\nu_e}$ and $\eta$ should be of the same order of magnitude.

At this stage it is interesting to note that incoherent resonant scattering of active neutrinos produces quasi-degenerate sterile neutrino matter, while incoherent non-resonant active neutrino scattering yields sterile neutrino matter that has approximately a thermal spectrum [14]. Quasi-degenerate sterile neutrino matter may contribute towards the formation of the supermassive compact dark objects at the galactic centers, while thermal sterile neutrino matter is mainly contributing to the dark matter of the galactic halos.

4 Observability of degenerate sterile neutrino balls

The mixing of the sterile neutrino with at least one of the active neutrinos necessarily causes the main decay mode of the $\nu_s$ into three active neutrinos [18] with a lifetime of

$$
\tau (\nu_s \rightarrow 3\nu) = \frac{192 \pi^3}{G_F^2 m_s^5 \sin^2 \theta_{es}} = \frac{\tau (\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu)}{\sin^2 \theta_{es}} \left( \frac{m_\mu}{m_s} \right)^5,
$$

which is presumably unobservable as the available neutrino energy is too small. Here $\tau (\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu)$ and $m_\mu$ are the lifetime and mass of the muon. However, there is a subdominant radiative decay mode of the sterile into an active neutrino and a photon with a branching ratio [20]

$$
B (\nu_s \rightarrow \nu \gamma) = \frac{\tau (\nu_s \rightarrow 3\nu)}{\tau (\nu_s \rightarrow \nu \gamma)} = \frac{27 \alpha}{8\pi} = 0.7840 \times 10^{-2},
$$

where $\alpha = e^2/\hbar c$ is the fine structure constant. The lifetime of this potentially observable decay mode is thus

$$
\tau (\nu_s \rightarrow \nu \gamma) = \frac{8\pi}{27 \alpha} \frac{1}{\sin^2 \theta_{es}} \left( \frac{m_\mu}{m_s} \right)^5 \tau (\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu),
$$

yielding, for $\theta_{es} = 10^{-7}$ and $m_s = 76.0$ keV/c$^2$, a lifetime of $\tau (\nu_s \rightarrow \nu \gamma) = 0.46 \times 10^{19}$ yr.

Although the X-ray luminosity due to the radiative decay of diffuse sterile neutrino dark matter in the Universe is presumably not observable, because it is well below the X-ray background radiation at this energy [14], it is perhaps possible to detect such hard X-rays in the case of sufficiently concentrated dark matter objects. In fact, this could be the smoking gun for both the existence of the sterile neutrino and the fermion balls. For instance, a ball of $M = 2.6 \times 10^6 M_\odot$ consisting of degenerate sterile neutrinos of mass $m_s = 76.0$ keV/c$^2$ [10], degeneracy factor $g_s = 2$, and mixing angle $\theta_{es} = 10^{-7}$ would emit 38 keV photons with a luminosity

$$
L_X = \frac{M c^2}{2\tau (\nu_s \rightarrow \nu \gamma)} = 1.6 \times 10^{34} \text{ erg/s},
$$

where $M$ is the mass of the ball.
within a radius of 60 AU, 8.32 light hours or $7.6 \times 10^{-3}$ arcsec of Sgr A*, assumed to be at a distance of 8 kpc. The current upper limit for X-ray emission from the vicinity of Sgr A* is $\nu L_\nu \sim 3 \times 10^{35}$ erg/s, for an X-ray energy of $E_X \sim 60$ keV [21], where $L_\nu = dL/d\nu$ is the spectral luminosity. Thus the X-ray line at 38 keV could presumably only be detected if either the angular or the energy resolution or both, of the present X-ray detectors are increased.

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