Pseudo-involutions in the Riordan group. (English) [Zbl 1490.05009]
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Summary: We consider pseudo-involutions in the Riordan group where the generating function \( g \) for the first column of a Riordan array satisfies a palindromic or near-palindromic functional equation. For those types of equations, we find, for very little work, the pseudo-involutionary companion of \( g \) and have a pseudo-involution in a \( k \)-Bell subgroup. There are only slight differences in the ordinary and exponential cases. In many cases, we also develop a general method for finding \( B \)-functions of Riordan pseudo-involutions in \( k \)-Bell subgroups, and show that these \( B \)-functions involve Chebyshev polynomials. We apply our method for many families of Riordan arrays, both new and already known.

We also have some duality and reciprocity results. Since many of the examples we discuss have combinatorial significance, we conclude with a few remarks on the general framework for a combinatorial interpretation of some of the generating function results we obtain.

MSC:

05A15 Exact enumeration problems, generating functions
20H99 Other groups of matrices
11B83 Special sequences and polynomials

Keywords:

Riordan group; Riordan array; pseudo-involution

Software:

OEIS

Full Text: arXiv Link

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