Letter

Mass and radius formulas for low-mass neutron stars

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Neutron stars, produced at the death of massive stars, are often regarded as giant neutron-rich nuclei. This picture is especially relevant for low-mass (below about solar mass, $M_{\odot}$) neutron stars, where non-nucleonic components are not expected to occur. Due to the saturation property of nucleonic matter, leading to the celebrated liquid-drop picture of atomic nuclei, empirical nuclear masses and radii can be approximately expressed as a function of atomic mass number. It is, however, not straightforward to express masses and radii of neutron stars even in the low-mass range where the structure is determined by a balance between the pressure of neutron-rich nucleonic matter and gravity. Such expressions would be of great use given possible simultaneous mass and radius measurements. Here we successfully construct theoretical formulas for the masses and radii of low-mass neutron stars from various models that are consistent with empirical masses and radii of stable nuclei. In this process, we discover a new equation-of-state parameter that characterizes the structure of low-mass neutron stars. This parameter, which plays a key role in connecting the mass–radius relation of the laboratory nuclei to that of the celestial objects, could be constrained from future observations of low-mass neutron stars.

Subject Index E32, D41

1. Introduction Neutron stars have been serving as laboratories to probe the densest and most neutron-rich matter in the Universe. It is generally believed that the outer, low-density part of a neutron star (crust) consists of a body-center-cubic lattice of neutron-rich nuclei, embedded in a gas of electrons and, if any, dripped neutrons, and near normal nuclear density ($\rho_0$), the nuclei melt into uniform nucleonic matter, which mainly composes the star’s core [1]. The equation of state (EOS) of matter in the star, i.e., neutron star matter, has a one-to-one correspondence to the star’s mass ($M$) and radius ($R$) relation via hydrostatic equilibrium. Observational data for $M$ have been accumulated [1,2], whereas those for $R$ have been recently estimated from observations of thermonuclear X-ray bursts with photospheric radius expansion and thermal spectra from quiescent low-mass X-ray binaries [3–6].

In theoretically describing laboratory nuclei and neutron star matter, it is useful to consider the energy of “nuclear matter,” i.e., hypothetical infinite matter, composed of neutrons and of protons that have their electric charge switched off. For simplicity, as neutron star matter, we will consider
Table 1. Nuclear matter EOS parameters

| EOS      | $K_0$ (MeV) | $L$ (MeV) | $\eta$ (MeV) |
|----------|-------------|-----------|---------------|
| OI-EOSs  | 180         | 31.0      | 55.7          |
|          | 180         | 52.2      | 78.9          |
|          | 230         | 42.6      | 74.7          |
|          | 230         | 73.4      | 107           |
|          | 280         | 54.9      | 94.5          |
|          | 280         | 97.5      | 139           |
|          | 360         | 76.4      | 128           |
|          | 360         | 146       | 197           |
| Shen     | 281         | 114       | 154           |
| Miyatsu  | 274         | 77.1      | 118           |
| FPS      | 261         | 34.9      | 68.2          |
| SLy4     | 230         | 45.9      | 78.5          |
| BSk19    | 237         | 31.9      | 62.3          |
| BSk20    | 241         | 37.4      | 69.6          |
| BSk21    | 246         | 46.6      | 81.1          |

zero-temperature, $\beta$-equilibrated, charge-neutral matter made of real nucleons and electrons. The EOS of nuclear matter is still uncertain even near $\rho_0$, while it can be constrained from terrestrial nuclear experiments [7] and neutron star observations [4,5,8] via theoretical calculations. It is noteworthy that the candidates for low-mass neutron stars have been discovered in binary systems [9], which could give additional information on the EOS once $R$ is measured.

The energy of uniform nuclear matter can be expanded around the saturation point of symmetric nuclear matter (SNM), i.e., nuclear matter made of the same number of neutrons and protons, with respect to the nucleon number density, $n_b$, and neutron excess, $\alpha$, defined as $\alpha \equiv (n_n - n_p)/n_b$, where $n_n$ and $n_p$ denote the neutron and proton number densities. In practice, in the vicinity of the saturation point of SNM at zero temperature, the energy per nucleon, $w$, of uniform nuclear matter can be written as a function of $n_b$ and $\alpha$ [10], i.e.,

$$w = w_0 + \frac{K_0}{18n_0^2}(n_b - n_0)^2 + \left[ S_0 + \frac{L}{3n_0}(n_b - n_0) \right] \alpha^2,$$

(1)

where $w_0$, $n_0$, and $K_0$ are the saturation energy, the saturation density, and the incompressibility of SNM, while $S_0$ and $L$ are associated with the symmetry energy coefficient $S(n_b)$. That is, $S_0 = S(n_0)$ is the symmetry energy coefficient at $n_b = n_0$, while $L$ characterizes the density dependence of the nuclear symmetry energy around $n_b = n_0$, defined as $L = 3n_0(dS/dn_b)|_{n_b=n_0}$. Among these five parameters in Eqn. (1), $w_0$, $n_0$, and $S_0$ can be relatively easier to determine from empirical data for masses and radii of stable nuclei, while the remaining two parameters, $K_0$ and $L$, are more difficult to fix [11]. This is why we focus on the various sets of $K_0$ and $L$ (Table 1) in analyzing neutron star matter.

These two parameters, $K_0$ and $L$, mainly determine the stiffness of neutron-rich nuclear matter, but have yet to be fixed. It is also suggested that $K_0$ is related to the giant resonances of stable nuclei [12], while $L$ is associated with the structure and reactions of neutron-rich nuclei [7,11,13] and the pressure of pure neutron matter at the saturation density of SNM. Additionally, one could constrain $L$ via quasi-periodic oscillations in giant flares observed from soft-gamma repeaters [14,15].

In contrast to the well-known empirical nuclear mass and radius formulas [16], the neutron star counterparts have to be theoretically given as a function of not only the central density ($\rho_c$), but such
EOS parameters as $K_0$ and $L$. So far, however, the dependence of low-mass neutron star models on $K_0$ and $L$ remains to be examined systematically. We thus start with construction of the neutron star models from various EOSs of neutron star matter that meet the following conditions:

1. Unified description of matter in the crust and core based on the same EOS of nuclear matter with specific values of $K_0$ and $L$.
2. Consistency of the masses and radii of stable nuclei calculated within the same theoretical framework with the empirical values.

Mass and radius formulas for low-mass neutron stars are finally obtained in such a way as to approximately reproduce the neutron star models thus constructed.

2. Adopted EOSs of neutron star matter

Among many available EOSs of neutron star matter, we adopt the EOSs that meet the above conditions, i.e., unified EOSs, which are categorized into three groups as in Table 1. The first is based on the phenomenological EOS of uniform nuclear matter that was constructed by two of us [11], using a simplified version of the extended Thomas–Fermi theory [17], in such a way as to reproduce empirical masses and radii of stable nuclei. They adopted the Padé-type potential energies with respect to the nucleon density $n_b$ for SNM and for neutron matter, respectively, and connected them in a quadratic approximation with respect to neutron excess $\alpha$. This form of the potential energy can well reproduce the variational calculations of [18], to which, in fact, the high-density behavior of neutron matter was adjusted. The $\alpha$ dependence of the potential energy is partially justified by the variational calculations of [19], and the expression for the total energy reproduces Eqn. (1) in the limit of $n_b \to n_0$ and $\alpha \to 0$. With such EOSs of uniform nuclear matter obtained for various sets of $(K_0, L)$, they constructed the EOSs of neutron star matter [20] by generalizing the above Thomas–Fermi theory as done by [17]. Hereafter, such EOSs of neutron star matter are referred to as the OI-EOSs. We remark that generally accepted values of $K_0$ lie in the range of $230 \pm 40$ MeV [21] or so, while the OI-EOSs include rather extreme cases of $K_0 = 180$ and 360 MeV, as shown in Table 1, to cover the large parameter space. The final mass and radius formulas would remain almost unchanged even if the OI-EOSs with $K_0 = 180$ and 360 MeV are not included in the fitting.

In the second group, there are two EOSs of neutron star matter calculated within the relativistic framework. One is the Shen EOS based on the relativistic mean field theory with the TM1 nuclear interaction [22], and the other is the Miyatsu EOS based on the relativistic Hartree–Fock theory with the chiral quark–meson coupling model [23]. In both EOSs, the same type of the Thomas–Fermi model as used for the OI-EOSs is used in describing neutron star matter in such a way as to reproduce empirical masses and radii of stable nuclei.

The third group is composed of the five EOSs of neutron star matter based on the Skyrme-type effective interactions: FPS [24], SLy4 [25], BSk19, BSk20, and BSk21 [26–28]. The FPS interaction, which was constructed by fitting the properties of uniform nucleon matter calculated by [18], well reproduces the empirical ground-state properties of doubly magic stable nuclei via the Hartree–Fock calculations. The SLy4 interaction was constructed by [29] in such a way as to reproduce the microscopic EOS of neutron matter calculated with the UV14+UVII nuclear force by [30] as well as the empirical ground-state properties of doubly magic stable nuclei within the Hartree–Fock approximation. The BSk19, BSk20, and BSk21 interactions are written in the form of the nuclear energy-density functionals, which are derived from generalized Skyrme interactions in such a way as to fit all the available nuclear mass data [26]. As a result, empirical charge radii were also well reproduced. These
3. Neutron star models  

Now, we construct nonrotating neutron stars by integrating the Tolman–Oppenheimer–Volkoff equations from the stellar center of density $\rho_c$ outward up to the position where the pressure vanishes. It is not clear up to what density the adopted unified EOSs are applicable. Nonetheless, one can expect that non-nucleonic components such as hyperons and quarks do not occur below $\sim 2\rho_0$ [1] and that the uncertainty from three-neutron interactions in the EOS of pure neutron matter becomes relevant above $\sim 2\rho_0$, as suggested by quantum Monte Carlo (QMC) calculations [35]. We thus examine the stellar models for $\rho_c \leq 2\rho_0$, where $\rho_0$ is set to $2.68 \times 10^{14}$ g cm$^{-3}$, and the resultant $M$–$R$ relations are plotted in Fig. 1(a).

To systematically describe various stellar models, we introduce a new auxiliary parameter $\eta$ defined as $\eta = (K_0L^2)^{1/3}$. The values of $\eta$ are shown in Table 1. Remarkably, the $M$–$R$ relation changes almost smoothly with $\eta$. Note that the OI-EOSs [20] with $L \lesssim 10$ MeV are too soft to keep the pressure positive and thus not used here. This implies the lower limit of $\eta$ of order 30 MeV. Meanwhile,
the EOS models used here cover the values of \( \eta \) up to \( \sim 200 \text{ MeV} \), which is significantly larger than expected from existing nuclear experiments. We remark that the powers of \( L \) and \( K_0 \) in \( \eta \) are chosen to be simple rational numbers in such a way that \( \eta \) has the same unit as \( L \) and \( K_0 \), i.e., MeV. If one considers arbitrary real numbers as the exponents, therefore, one could choose different kinds of \( \eta \) with which the \( M-R \) relation changes as smoothly as the present choice.

From the observational viewpoint, the radiation radius \( R_\infty = R/\sqrt{1 - 2GM/Rc^2} \) and the gravitational redshift \( z = 1/\sqrt{1 - 2GM/Rc^2} - 1 \) with the gravitational constant \( G \) and the speed of light \( c \) could be more relevant in describing the stellar properties than \( M \) and \( R \). The calculated \( z-R_\infty \) relation again shows a smooth change with \( \eta \) [Fig. 1(b)]. The photon flux, if detected, would be proportional to \((R_\infty/D)^2\), where \( D \) is the distance from the Earth, while the gravitational redshift could be determined from the possible shift of atomic absorption lines in spectra of the stars.

The smooth change of the stellar properties with \( \eta \) suggests that not only future nuclear experiments but also simultaneous measurements of \( M \) and \( R \) or, equivalently, \( z \) and \( R_\infty \) could constrain \( \eta \), which could in turn lead to restriction of the stellar models. In particular, observations of low-mass neutron stars would be essential. For example, the radiation radius of the X-ray source CXOU 132619.7–472910.8 in the globular cluster NGC 5139 (\( \omega \) Cen) has been determined as \( R_\infty = 14.3 \pm 2.1 \text{ km} \) from the Chandra data [36]. The allowed region from this \( R_\infty \) is shown in Figs. 1(a) and (b) with the shaded region. This is still consistent with various values of \( \eta \), but future precise determination of \( R_\infty \) could constrain \( \eta \), if \( M \) is low enough. Additionally, thermal spectra detected from quiescent low-mass X-ray binaries are expected to give \( M \) and \( R \) simultaneously [3,4].

4. Mass and radius formulas  To examine the dependence of the stellar properties on \( \eta \) more clearly, we plot the stellar masses calculated for \( \rho_c = 2.0\rho_0 \), 1.5\( \rho_0 \), and 1.0\( \rho_0 \) [Fig. 2(a)]. From this figure, we find that the stellar masses for fixed \( \rho_c \) can be approximately expressed as a linear function of \( \eta \), \( M/M_\odot = c_0 + c_1(\eta/100 \text{ MeV}) \), where \( c_0 \) and \( c_1 \) are adjustable parameters that depend on \( \rho_c \). The validity of \( \eta \) is now evident. The deviation of the calculations from the linear fit at \( \rho_c = 2.0\rho_0 \) is larger than that at \( \rho_c = 1.0\rho_0 \), particularly for BSk20 and BSk21. Such deviation is of the order of uncertainties in \( M \) due to three-neutron interactions obtained from the QMC evaluations [35]. The parameters \( c_0 \) and \( c_1 \) can then be expressed as a quadratic function of \( u_\epsilon \equiv \rho_c/\rho_0 \) within the accuracy of errors less than a few percent (Fig. 3). Finally, we obtain the mass formula:

\[
\frac{M}{M_\odot} = 0.371 - 0.820u_\epsilon + 0.279u_\epsilon^2 - (0.593 - 1.25u_\epsilon + 0.235u_\epsilon^2) \left( \frac{\eta}{100 \text{ MeV}} \right), \tag{2}
\]

where we confine ourselves to \( \rho_c \gtrsim 0.9\rho_0 \); otherwise, the stellar models can become unstable with respect to decompression, depending on the EOS of neutron star matter.

We also find that the gravitational redshift calculated for fixed \( \rho_c \) can be approximately expressed as a linear function of \( \eta \) [Fig. 2(b)]. Then, just like the mass formula (2), we can obtain the theoretical formula for \( z \) as

\[
z = 0.00859 - 0.0619u_\epsilon + 0.0255u_\epsilon^2 - (0.0429 - 0.108u_\epsilon + 0.0120u_\epsilon^2) \left( \frac{\eta}{100 \text{ MeV}} \right). \tag{3}
\]

Using Eqs. (2) and (3), one could estimate the values of \( \eta \) and \( u_\epsilon \) from possible simultaneous measurements of \( M \) and \( z \). In general, Eqs. (2) and (3) can have as many as four sets of solutions \( (u_\epsilon, \eta) \) for given observational values of \( M/M_\odot \) and \( z \). As mentioned above, however, Eqs. (2) and (3) are valid in the range of \( 0.9 \lesssim u_\epsilon \lesssim 2.0 \). In this range, as shown in Fig. 2, the solution \( (u_\epsilon, \eta) \) has to be unique.
Fig. 2. Neutron star masses (a) and gravitational redshifts (b) as a function of $\eta$. The stellar models constructed from various unified EOSs are given for $\rho_c = 2.0\rho_0, 1.5\rho_0,$ and $1.0\rho_0$. The solid, broken, and dotted lines are the linear fitting to the cases of $\rho_c = 2.0\rho_0, 1.5\rho_0,$ and $1.0\rho_0$, respectively (see text for details).

Fig. 3. Values (marks) of the adjustable parameters $c_0$ and $c_1$ in the mass formula. The corresponding quadratic fitting curves (solid and broken lines) are also shown as a function of $\rho_c/\rho_0$. Here we consider the stellar models only for $\rho_c \gtrsim 0.9\rho_0$ to avoid unstable neutron star models.

It is straightforward to obtain the formula for $R$ from Eqs. (2) and (3). The obtained formula can be compared with the calculations of $R$ for $\rho_c = 1.0\rho_0, 1.5\rho_0,$ and $2.0\rho_0$ (Fig. 4). We confirm a good agreement between those two except for $\eta \lesssim 70$ MeV. The mass and radius formulas could help to constrain not only the nuclear matter parameter $\eta$ but also a star’s $\rho_c$ via possible simultaneous measurements of the star’s $M$ and $R$. If such measurements are precise, $\eta$ could be deduced to within an accuracy of $\pm 20$ MeV, which would provide a basis for analyzing more massive neutron stars.
Fig. 4. Neutron star radii as a function of $\eta$. The stellar models constructed from various unified EOSs are given for $\rho_c = 1.0 \rho_0$ (black), $1.5 \rho_0$ (red), and $2.0 \rho_0$ (blue). The solid, broken, and dotted lines are the formula values for the cases of $\rho_c = 2.0 \rho_0$, $1.5 \rho_0$, and $1.0 \rho_0$, respectively, obtained from Eqs. (2) and (3). The thick straight line denotes the converging behavior expressed by Eq. (4).

From Fig. 4, one can also observe that the calculated $R$ depends nonlinearly on $\rho_c$ at small values of $\eta$, while converging on an approximately linear function of $\eta$ at sufficiently large values of $\eta$:

$$R = 10.32 + 2.57 \left( \frac{\eta}{100 \text{ MeV}} \right) \text{ km.}$$

Note that such nonlinear dependence at small values of $\eta$ arises from the flattened behavior of the corresponding $M–R$ relations that can be seen from Fig. 1(a), while such convergence at large values of $\eta$ is related to the vertically straightened behavior of the corresponding $M–R$ relations.

5. Conclusion In this paper, we have succeeded in constructing the theoretical formulas for the masses, gravitational redshifts, and radii of low-mass neutron stars as functions of the star’s central density and the new EOS parameter $\eta$ in a manner that is consistent with empirical masses and radii of stable nuclei. The value of $\eta$, which characterizes the stiffness of neutron star matter, remains unknown, but could be deduced from possible simultaneous $M$ and $R$ measurements via comparison with our formulas if the star observed is light enough. Thus, firm evidence for the presence of low-mass neutron stars is first of all desired. One promising candidate is the neutron star in the high-mass X-ray binary 4U 1538-52, of which the mass could be significantly low or even the lowest among stars with known mass if the binary orbit is eccentric [37,38]. The X-ray burster 4U 1724-307 in the globular cluster Terzan 2 is even more interesting because the X-ray data from the cooling phase of photospheric radius expansion bursts apparently allow the object to have a relatively low mass and still a significantly large radius [39]. Such conclusions are tentative partly because of the dependence on the atmosphere models adopted and partly because of uncertainties in the distance to the object, but, if valid, might eventually suggest the $\eta$ value of the order of, or even larger than, 130 MeV.

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