On Wilson Loops and Confinement without Supersymmetry from Composite Antisymmetric Tensor Field theories

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Abstract

A novel approach that does not rely on supersymmetry, nor in the AdS/CFT correspondence, to evaluate the Wilson loops associated with a gauge theory of area-preserving diffeomorphisms in terms of pure string degrees of freedom is presented. It is based on the Guendelman-Nissimov-Pacheva formulation of composite antisymmetric tensor field theories of volume-preserving diffeomorphisms. Such theories admit $p$-brane solutions. The quantum effects are discussed and we evaluate exactly the vev of the Wilson loops, in the large $N$ limit of quenched-reduced $SU(N)$ QCD, in terms of a path integral involving pure string degrees of freedom. It is consistent with the recent results based on the AdS/CFT correspondence and dual QCD models (dual Higgs model with dual Dirac strings). More general Loop wave equations in C-spaces (Clifford manifolds) are proposed in terms of holographic variables that contain the dynamics of an aggregate of closed branes ($p$-loops) of various dimensionalities.

I. Introduction

It has been believed for a long time that QCD confinement is supposed to be a non-perturbative solution to QCD in four dimensions; i.e to $SU(3)$ Yang-Mills theory [1]. A formal proof of the colour confinement amounts to a derivation of the area-law for a Wilson loop associated, for example, with the world lines of a quark-antiquark pair joined in by a string. The area law in the Euclidean regime is $W(C) \sim \exp(-TA)$ where $T$ is the string tension and the (Euclidean) area is $A = l t_E$. The colour-electric potential rises linearly with the length of the string separating the quark-antiquark and blows up in the $l \to \infty$. This would be a signal of (colour-electric field lines) confinement, an infinite amount energy would be required to separate the quarks. Many attempts have been explored to solve this problem, in particular those based on the so-called string ansatz [2-3]:

$$W[C] \sim \int_{\Sigma(C)} [DX] \exp(iS_{\text{string}}).$$

which says that the effective (collective) infrared degrees of QCD at strong coupling are given by string configurations whose worldsheets have for boundary the loop $C$. The Schwinger-Dyson equations for QCD can be reformulated as an infinite chain of equations for the Wilson loops that simplify drastically in the large $N$ limit giving the single equation known as the Makeenko-Migdal loop equation [4]. In light of the Maldacena AdS/CFT correspondence formulated by many authors [5] as a relation between partition functions, Maldacena and others proposed that the average value of a Wilson loop in the large $N$ limit, for $\mathcal{N} = 4$ $SU(N)$ SYM was given by the partition function of a world-sheet string action which ends along the loop $C$ in the four-dim boundary. Another approach has been based on the dual formulation of QCD [6] (in the infrared limit) given by a $U(1)$ gauge theory adjoined by a dual Higgs model with dual Dirac strings [7] (where the quarks live at their end-points). The average value of the Wilson loop in this dual phase obeys the area-law fall-off. For other approaches to solve the confinement problem based on Skyrmions and others methods see [8].

In this work we will present a novel approach that does not rely on supersymmetry nor the AdS/CFT correspondence, to evaluate the Wilson loop associated with a gauge theory of area-preserving diffeomorphisms, in terms of the (area) string degrees of freedom. It is based on the Guendelman-Nissimov-Pacheva formulation of composite antisymmetric tensor field theories of volume-preserving diffeomorphisms [9]. Such theories admit $p$-brane solutions after a dualization procedure [10]. Our first results are exact on-shell. The quantum effects are discussed next and we evaluate exactly the vev of the Wilson loops in the large $N$
limit of $SU(N)$ QCD in terms of a path integral involving pure string degrees of freedom. To achieve this goal we borrow extensively from our results based on the relationship among large $N$ quenched-reduced $SU(N)$ YM theories and strings/branes via the Moyal-Zariski deformation quantization [11, 16]. This average is consistent with the recent results based on the AdS/CFT correspondence and dual QCD models (dual Higgs model with dual Dirac strings). Finally we present more general Loop wave equations in C-spaces (Clifford manifolds) [12, 20] than those considered so far. These loop equations are given in terms of the holographic variables associated with an aggregate of closed branes (p-loops) of various dimensionalities.

II

2.1 Branes as composite antisymmetric tensor field theories

In this section we will review the construction of $p'$-brane solutions to the rank $p+1$ composite antisymmetric tensor field theories [10] developed by Guendelman, Nissimov and Pacheva [9] when the condition $D = p + p' + 2$ is satisfied. These field theories possess an infinite-dimensional group of volume-preserving diffeomorphisms of the target space of the scalar primitive field constituents. The role of local gauge symmetry is traded over to an infinite-dimensional global Noether symmetry of volume-preserving diffs. The study of the Ward identities for this infinite-dim global Noether symmetry to obtain non-perturbative information in the mini-QED models (the composite form of QED) was analysed in [9].

The starting Lagrangian is defined [10]:

$$L = -\frac{1}{g^2} F_{\mu_1 \mu_2... \mu_{p+1}}^2, \quad F = dA = \epsilon_{a_1 a_2... a_{p+1}} \partial_{\mu_1} \phi^{a_1}... \partial_{\mu_{p+1}} \phi^{a_{p+1}}. \quad (1)$$

the rank $p + 1$ composite field strength is given in terms of $p + 1$ scalar fields $\phi^1(x), \phi^2(x),... \phi^{p+1}(x)$. Notice that the dimensionality of spacetime where the field theory is defined is not the standard YM type. We are going to find now $p'$-brane solutions to eq- (2), where $D = p + p' + 2$. These brane solutions obeyed the classical analogs of $S$ and $T$-duality [10]. Ordinary EM duality for branes requires $D = p + p' + 4$. The latter condition is more closely related to the EM duality among two Chern-Simons $p, p'$-branes which are embeddings of a $p, p'$-dimensional object into $p + 2; p' + 2$ dimensions. These co-dimension two objects are nothing but high-dimensional Knots. For the mathematical intricacies of Chern-Simons branes, high-dimensional knots and algebraic $K, L$ theory see [13]. A special class of (non-Maxwellian) extended-solutions to eqs- (2) requires a dualization procedure [10]:

$$G = \ast F \Rightarrow G^{\mu_1 \nu_2... \nu_{p'+1}}(\hat{\phi}(x)) = \epsilon^{\mu_1 \nu_2... \nu_{p'+1} \mu_{p+2} \nu_{p+3}... \nu_{2p+4}}F_{\mu_1 \nu_2... \nu_{p+1}}(\phi(x)) \quad (3)$$

After this dualization procedure the eqs- (2) are recast in the form:

$$G^{\mu_1 \nu_2... \nu_{p'+1}} \partial_{\mu_1} G_{\nu_2 \nu_3... \nu_{p'+1}}(\hat{\phi}(x)) = 0. \quad (4)$$

The dualized equations (4) have a different form than eqs-(2) due to the position of the indices (the index contraction differs in both cases). Extended $p'$-brane solutions to eqs- (4) exist based on solutions to the Aurilia-Smailagic-Spallucci local gauge field theory reformulation of extended objects given in [14]. These are [10]:

$$G^{\mu_1 \nu_2... \nu_{p'+1}}(\hat{\phi}(x))|_{x=X} = T_{X} \frac{\{X^{\nu_1}, X^{\nu_2},... , X^{\nu_{p'+1}}\}}{\sqrt{\frac{1}{(p'+1)!}[\{X^{\mu_1}, X^{\mu_2},... , X^{\mu_{p'+1}}\]}} \quad (5)$$

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where $T$ is the $p'$-brane tension and the Nambu-Poisson bracket w.r.t the $p'+1$ world-volume variables is defined as the ordinary determinant /Jacobian:

$$\{X^{\nu_1}, X^{\nu_2}, X^{\nu_3}, \ldots, X^{\nu_{p'+1}}\}_{NPB} = \epsilon^{\sigma_1 \sigma_2 \sigma_3 \ldots \sigma_{p'+1}} \partial_{\sigma_1} X^{\nu_1} \partial_{\sigma_2} X^{\nu_2} \ldots \partial_{\sigma_{p'+1}} X^{\nu_{p'+1}}. \quad (6)$$

All quantities are evaluated on the world-volume support of the $p'+1$-brane; i.e. one must restrict the field theory solutions to those points in the $D$-dimensional spacetime given by $x = X(\sigma^1, \sigma^2, \ldots)$. Solutions to all of the $D$-dim spacetime region can be extended simply by using delta functionals: $\delta(x - X(\sigma))$.

### 2.2 Wilson Loops and Confinement

In this section we are going to study the string solutions ($p' = 1$) to the rank two ($p + 1 = 2$) composite antisymmetric tensor field theories of area-preserving diffs in $D = 4 = p + p' + 2 = 2 + 2$. The Wilson loop associated with the composite gauge field is defined:

$$exp \left[ i \oint_C A_\mu(\phi^a) \, dx^\mu \right]. \quad (7)$$

Due to the Abelian-looking form of the composite field strength (as we said earlier, the algebra of volume-preserving diffs is not abelian) one can nevertheless use Stokes law:

$$F = dA \Rightarrow F_{\mu\nu}(\phi) \equiv \{\phi^1, \phi^2\} = \epsilon_{ab} \partial_\mu \phi^a \partial_\nu \phi^b. \quad (8)$$

after using Stokes law the exponential can be written as:

$$exp \left[ i \int \int_{\Sigma(C)} F_{\mu\nu}(\phi^a) dx^\mu \wedge dx^\nu \right]. \quad (9)$$

where the flux is evaluated through a surface $\Sigma(C)$ whose boundary is $C$. If one evaluates all these quantities along the points $x$ whose support lie on the string-world sheet $x = X$ one may use the string solutions above to the composite antisymmetric tensor field theory given by the previous equations (5):

$$G(\tilde{\phi}) = \Pi = \ast F(\phi) \Rightarrow$$

$$G^{\nu_1 \nu_2}(\tilde{\phi})|_{x = X} = \Pi^{\nu_1 \nu_2}(X) = \frac{T\{X^{\nu_1}, X^{\nu_2}\}}{\sqrt{-\frac{1}{2}(X^\mu, X^\nu)\{X_\mu, X_\nu\}}} = \epsilon^{\nu_1 \nu_2 \mu_1 \mu_2} F_{\mu_1 \mu_2}(\phi)|_{x = X}. \quad (10)$$

where $T$ is the string’s tension and one is using now ordinary Poisson brackets. The quantity $\Pi^{\mu\nu}$ is the area-conjugate momentum of the string obeying the Hamilton-Jacobi equation for the string analog of a point particle momentum. Hamilton-Jacobi equations for strings and branes have been given in [14]. Using these relations above (10) allows one to rewrite the flux (after inserting the product of two spacetime epsilon tensors $\epsilon_{\mu_1 \mu_2 \mu_3 \mu_4}$) as:

$$\frac{1}{4!} \epsilon^{\mu_1 \mu_2 \mu_3 \mu_4} F_{\mu_1 \mu_2}(\phi) \epsilon_{\mu_1 \mu_2 \mu_3 \mu_4} dx^{\mu_1} \wedge dx^{\mu_2} = G^{\mu_3 \mu_4}(\tilde{\phi}) d\tilde{\Sigma}_{\mu_3 \mu_4}. \quad (11)$$

For those self dual string configurations, the following relations among the Poisson brackets are obeyed:

$$Self Dual Strings \Rightarrow d\Sigma = \ast d\Sigma \Rightarrow \{X_{\mu_3}, X_{\mu_4}\}_{PB} = \epsilon_{\mu_1 \mu_2 \mu_3 \mu_4} \{X^{\mu_1}, X^{\mu_2}\}_{PB}. \quad (12)$$

Self dual strings automatically obey the string equations of motion as a result of the Jacobi identities for the Poisson brackets:

$$\{X^\nu, \{X_\mu, X_\nu\}\} = \epsilon_{\mu \nu \rho \tau} \{X^\nu, \{X^\rho, X^\tau\}\} = 0. \quad (13)$$

The vanishing of the second term of the last equation is due to the Jacobi identities of the Poisson bracket. Upon evaluation of the flux through the (self-dual) string world sheet, whose boundary is $C$, and restricting
to self dual string configurations allows finally to yield the explicit relationship between the Wilson loop for the field $A_{\mu}(\phi)$ and the Dirac-Nambu-Goto string action, in terms of the string coordinates $X^{\mu}(\sigma, \tau)$, and whose worldsheet boundary is $C$:

\[
W(C) = \exp \left[ i \oint_{C} A_{\mu}(\phi)dx^{\mu} \right] \big|_{x=X} = \exp \left[ iT \int \int_{\Sigma(C)} d\sigma d\tau \sqrt{-\{X^\mu, X^\nu\}\{X^\mu, X^\nu\}} \right].
\]  

(14)

since the determinant of the induced worldsheet metric as a result of the string’s embedding onto the (flat) target spacetime is:

\[
det [h_{ab}] = det [\eta_{\omega\nu} \partial_\omega X^\mu \partial_\nu X^\nu] = \{X^\mu, X^\nu\}\{X^\mu, X^\nu\}.
\]

(15)

Therefore, we have proven, on-shell, that the Wilson loop associated with the composite antisymmetric tensor field theory of area-preserving diffeomorphisms, after using Stokes law, equals the exponential of the self-dual string action whose worldsheet boundary is $C$.

Some important remarks are in order. Firstly, one must not confuse the physical closed string, a loop, with the rectangular Wilson loop associated with the static quark-antiquark world lines at the ends of an open string of length $l$. It is not difficult to verify that the rectangular (Euclidean) Wilson loop spanned by the quark-antiquark world lines in a (Euclidean) time $t_E$, the string of length $l$ and of area $lt_E$, does not obey the self-dual string equations of motion. Secondly, the physical loop $C$ coincides with the boundary of a closed-string world sheet associated with a closed string history. For the confinement of 3 quarks located inside a closed string using these composite models (composite measures) based on a dynamical tension generation for strings and branes see [15].

Notice the importance of the self-dual string condition and that no supersymmetry was required nor invoking the AdS/CFT correspondence was necessary. The only remnants of the AdS/CFT duality were that the l.h.s of eq- (14) involves integrating over the closed-string worldsheet boundary terms, whereas the r.h.s of eq-(14) involves a pure stringy bulk term (closed strings contain gravity) and that a dualization procedure was necessary in order to find $p'$-brane solutions to composite antisymmetric tensor field theories of rank $p + 1$. The fact that no supersymmetry and that no AdS/CFT was necessary to obtain the on-shell relation for the Wilson loop in terms of self-dual string configurations is the first important result of this work.

This has been so far a classical (on-shell) result. It is warranted to study the full quantum theory (off-shell extension). In particular to evaluate the average Wilson loop $W(C)$ and show, in fact, that it can be expressed in string variables. This we shall do next.

2.3 The Average Wilson loop via Quenched-Reduced large N QCD

Hoppe long ago [22] proved that the large $N$ limit (which is basis dependent, not unique) of $SU(N)$ is isomorphic to the algebra of area-preserving diffs of a sphere. The topology of the surface is important since other algebras like $w_{\infty}$ and $w_{1+\infty}$ associated with $w_{\infty}$ strings, higher conformal spins, are the area-preserving diffs of a plane and cylinder respectively. Since the GNP formulation of antisymmetric tensor field theories involve area-preserving (volume) diffs, it is natural to study now the $SU(N)$ YM theories in the large $N$ limit.

In this last section we will compute the vacuum expectation value of the Wilson loop in the infinite colour limit via a Moyal deformation quantization procedure. We have shown in [11] that a Moyal deformation quantization allows to study the large $N$ limit of $SU(N)$ YM theories. $SU(N)$ reduced-quenched gauge theories admit hadronic bags and Chern-Simons (dynamical boundaries) membranes excitations in the large $N$ limit. This Moyal deformation approach also furnishes dynamical membranes (a QCD membrane) when the quenching is performed along a line, instead of a point.

Basically, a Moyal quantization takes the Lie-algebra valued operator $\hat{A}(x)$ into a $c$-number $A_{\mu}(x; q, p)$. Quenching to a point, and reduction, brings the quantity $A_{\mu}(x; q, p)$ to depend solely on the $(q, p)$ coordinates that subsequently are identified with the internal string/bag coordinates after the gauge field-target spacetime coordinate correspondence $A_{\mu}(q, p) \leftrightarrow X_\mu(\sigma)$ is made. Quenching to a spatial line yields $A_{\mu}(t; q, p)$ which are subsequently identified with the membrane coordinates $X_\mu(t, \sigma_1, \sigma_2)$. Lie-algebra commutators are mapped via the Weyl-Wigner-Groenowold-Moyal correspondence to Moyal brackets. The classical limit $\hbar \to 0$ is
related to the large $N$ limit via the identification $\hbar = 2\pi a_s$. Moyal brackets collapse to Poisson-brackets in that limit. The Lie-algebra trace operation is corresponds to an integration w.r.t the string/bag internal coordinates.

These results can be extended to more general $p$-brane actions given by Dolan-Tchrakian (Skyrme type actions) starting from Generalized Yang Mills theories in the large $N$ limit; i.e. branes are roughly speaking Moyal deformations of Generalized YM theories [11]. Deformation quantization beyond the Moyal procedure exists for more generalized Poisson brackers, the so-called Nambu-Possion brackets which are the Jacobians described earlier. Its deformation quantization requires the use of the Zariski star product [16]. It turns out that all $p$-brane actions, including those for Chern-Simons $p$-branes and Kalb-Ramond couplings to $p$-branes, can be obtained via a Zariski deformation quantization of the generalized Matrix models constructed in [16]. In particular, it was shown how Nambu-Goto strings can be obtained directly from $SU(N)$ Born-Infeld models in the large $N$ limit [17].

To illustrate the power of these approaches we will show how one can obtain the celebrated Maldacena relation relating the size of the $AdS_5$ throat $\rho^4$ to the 't Hooft coupling $Ng_Y^2M$ and the Planck scale $L_{Planck}^4 \sim (\alpha)^2$ (the inverse string tension squared) from a Moyal deformation approach to quenched-reduced large $N$ QCD. The bag constant, $\mu$, of mass dimension, was related to the bag tension as [11]:

$$T_{bag} = \mu^4 \sim \frac{1}{a^4g_Y^2}.$$  \hspace{1cm} (16)

where $a$ was related to the lattice spacing of the large $N$ quenched, reduced QCD given by $(2\pi/a) = \Lambda_{QCD} = 200$ Mev. Based on the known result that a stack of $N$ coincident $D3$ branes (whose world volume is four-dimensional) in the large $N$ limit is related to black $p = 3$ branes solutions to closed type IIB string theory in $D = 10$, and whose near-horizon geometry is given by $AdS_5 \times S^5$, one may set the lattice spacing $a$ associated with large $N$ quenched, reduced $SU(N)$ YM in terms of the Planck scale $L_P$ to be $a^4 = NL_P^4$. This merely states that we are setting the hadronic bag scale to be $a = N^{1/4}L_P$. Inserting this relationship into the expression for the bag tension gives:

$$T_{bag} = \mu^4 \sim \frac{1}{a^4g_Y^2} = \frac{1}{NL_P^4g_Y^2} \Rightarrow \mu^{-4} \sim (Ng_Y^2L_P^4).$$ \hspace{1cm} (17)

which has a similar form as the Maldacena relation if one identifies the size of the $AdS_5$ throat to the bag scale $\mu^{-1}$. We believe this is not a mere numerical coincidence but stems from the Moyal-Zariski deformation quantization of the generalized Matrix models [16, 17] which furnishes all of the known $p$-brane actions. Since strings/branes contain gravity then it is not surprising to see a connection between large $N$ QCD and gravity.

The average Wilson loop is defined:

$$<W_A[C] >_{vev} = \int [DA] W_A(C) e^{iSyM[A]}.$$ \hspace{1cm} (18)

The Wilson loop for $SU(N)$ YM is:

$$W[C] = \frac{1}{N}trace~ Path~ exp [i \int_C A_\mu dx^\mu].$$ \hspace{1cm} (19)

In the quenched-reduced approximation, defined at a point, the Wilson loop shrinks to zero size and hence the exponential reduces to unity since the integral has collapsed to zero. So then we get $W[C] \rightarrow \frac{1}{N}trace\rightarrow 1 = 1$. The quenched-reduced YM action in the large $N$ limit becomes the Eguchi-Schild action for the string after using the $A_\mu(\sigma) \rightarrow X_\mu(\sigma)$ correspondence. This has been known for some time [18].

We have then the following results:

$$[DA] \rightarrow [DX] \quad W(C) \rightarrow 1. \quad e^{iSyM} \rightarrow e^{iS_{string}}.$$ \hspace{1cm} (20)

Under these conditions the quenched-reduced $SU(N)$ QCD, in the large $N$ limit, allows to compute exactly the vev of the Wilson loop purely in terms of string degrees of freedom given in terms of the Eguchi-Schild action for the string, the square of the Poisson brackets, which is area-preserving diffs invariant:
\[ <W[C]>_{vev} = \int_{\Sigma(C)} [DX]e^{iS_{string}} \equiv \Psi_o[C]. \]  

(21)

The physical meaning of this relation can be envisaged as follows. As we shrink the Wilson loop to a point, the subsequent large \( N \) limit procedure amounts to introducing an extra dependence on the phase space variables \((q, p)\), that later are identified as the string coordinates. The \( SU(N) \) fiber sitting at the point \( P \) becomes the area world-sheet of the string in the large \( N \) limit. Hence the Wilson loop which had initially shrunk to a point re-emerges as an internal loop living in the \( SU(N) \) fiber that was sitting at the point \( P \). This is compatible with the area-preserving diffs invariant nature of the Eguchi-Schild action. Roughly speaking, since areas are preserved, as we shrink the Wilson loop to a point (to zero) it must re-emerged along the fibers in order to preserve the area.

Hence we have obtained an exact result consistent with those given in the literature since (by definition) the vacuum wave-functional \( \Psi_o[C] \), appearing in the r.h.s, is defined by a path integral over all world-sheets whose boundary is \( C \). The latter is the quantum amplitude for a closed string to emerge from the vacuum (a "point") and sweep a world-sheet whose boundary is \( C \). The topology is given by a disc. A perturbative evaluation of the path integral requires summing over surfaces of all genera. For more general actions one must restrict the measure of integration modulo the volume of the world-sheet diffs group and the group of Weyl diffs for Polyakov-Howe-Tucker type of actions.

Nonperturbative effects are never seen in perturbation theory, like the contribution of self-dual string configurations (Euclideanized world-sheet). The latter have a dominant weight in the path integral since they saturate the lower bound of the (Euclidean) action. Hence it is not too surprising that the on-shell value of the Wilson loop for composite antisymmetric tensor field theories was given by the exponential of the self-dual string action.

The propagator from one loop configuration \( C_1 \) to another \( C_2 \) is given by the path integral:

\[ \Delta(C_1, C_2) = \int_{C_1}^{C_2} [DX] \exp[iS_{string}]. \]  

(22)

where the path integral involves summing over all surfaces \( \Sigma(C_1, C_2) \) bounded by the two loops \( C_1, C_2 \).

In [19] an explicit expression for the string representation of a quantum loop \( C \), in terms of a full phase space string path integral based on the Eguchi-Schild string action, was given. The wavefunctional was a complicated expression:

\[ \Psi[x, A, \sigma^{\mu\nu}], \quad \text{where} \quad \sigma^{\mu\nu}(C) \quad \text{were the holographic area projections of a loop onto the coordinate planes;} \quad A \quad \text{was the Eguchi temporal-area variable associated with the closed loop and} \quad x \quad \text{were the coordinate variables of the loop boundary.} \]

What was left open was to see what type of Loop wave equations the functional \( \Psi[x, A, \sigma^{\mu\nu}] \) obeyed.

Loop wave equations for strings and \( p \)-branes were given in [20]. Dirac-like wave equations were also obtained by a suitable "square-root" procedure which generalized the Hosotoni string Dirac-like equations [21]. One can extend the loop wave equations [20] to C-spaces (Clifford manifolds) [12] by writing the more general loop equations for a nested family of \( p \)-loops, \( p = 0, 1, 2.. \) where the maximum value of \( p \) corresponds to a spacetime filling brane \( p + 1 = D \):

\[ \Psi[\Omega, x^\mu; \sigma^{\mu\nu}; \sigma^{\mu\nu\rho}, ....] \]  

(23a)

the Clifford-algebra valued object (a polyvector) is given by (setting the Planck scale to unity):

\[ X = \Omega I + x^\mu \gamma^\mu + \sigma^{\mu\nu} \gamma^\mu \wedge \gamma^\nu + .... \]  

(23b)

the loop wave equations is:

\[ \frac{\delta^2 \Psi}{\delta \Omega^2} + \frac{\delta^2 \Psi}{(\delta t x_\mu)^2} + \frac{\delta^2 \Psi}{(\delta \sigma_{\mu\nu})^2} + \frac{\delta^2 \Psi}{(\delta \sigma_{\mu\nu\rho})^2} + \ldots \ldots \ldots \ldots \ldots \ldots = \mathcal{L}^2 \Psi. \]  

(23c)

The more fundamental problem is to see if these C-space loop equations have a direct relation to the Maakenko-Migdal loop equations in the infinite colour limit of YM. The fact the the loop equations in C-spaces incorporate automatically the closed \( p \)-brane \( p \)-loops holographic coordinates is compatible with the AdS/CFT correspondence.
The loop transform (the analog of the Fourier transform) for the full-fledged \( SU(N) \) YM theory, in the large \( N \) limit, is:

\[
\Psi[C] = \int [DA] \left[ \int d^2 \sigma \exp \left[ \oint_C A_\mu(x^\mu; \sigma) dx^\mu \right] \right] \Psi[A].
\]

(24)

where now the gauge field \( A_\mu \) depends on the \( x^\mu \) coordinates as well, in addition to the string variables \( \sigma^1, \sigma^2 \). The theory is now an effective higher dimensional \( D = 6 \) one. No quenching nor reduction has taken place in the more general case. Conversely, the inverse loop transform will yield \( \Psi[A] \) in terms of \( \Psi[C] \). The main question is to see if the loop equation obeyed by \( \Psi[C] \) agrees in with those given in [20].

One can generalized these results to \( p \)-loops \( C_p \); i.e a closed \( p \)-brane enclosing a \( p + 1 \)-dimensional region \( \Sigma_{p+1}(C_p) \). The \( p \)-loop transform is defined in terms of an integral involving the rank \( p \) antisymmetric tensor field \( A_p \):

\[
\Psi[C_p] = \int [DA_p] \left\{ \int d^{p+1} \sigma \exp \left[ \oint_{C_p} A_{\mu_1, \mu_2, ..., \mu_p}(x^{\mu_1, \sigma^1, \sigma^2, ....) d\Sigma^{\mu_1, \mu_2, .... \mu_p} \right] \right\} \Psi[A_p].
\]

(25)

The integration w.r.t the internal \( p + 1 \) variables of the closed \( p \)-brane history (a \( p + 1 \) worldvolume) is the analog of the trace operation. The antisymmetric tensor field depends on the \( x^\mu \) spacetime coordinates and on the internal \( \sigma \) variables. It is the generalization of the large \( N \) limit of \( SU(N) \) YM given by the \( c \)-number field \( A_\mu(x, \sigma) \). One can follow a similar procedure to evaluate the average of the generalized Wilson loop \( W_{A_p}[C_p] \) in the quenched-reduced approximation and express it as a path integral over a \( p \)-brane action whose boundary is given by \( C_p \). More details about this will be given in a future publication.

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