Diffractive vector meson production at HERA using holographic AdS/QCD wavefunctions

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We demonstrate another success of the AdS/QCD correspondence by showing [1, 2] that an AdS/QCD holographic light-front wavefunction for the $\rho$ meson generates predictions for the cross-sections of diffractive $\rho$ production that are in agreement with data collected at the HERA electron-proton collider [3, 4].

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1. Introduction

The AdS/QCD correspondence [5, 6, 7, 8] refers to the connection between QCD in physical spacetime and string theory in a higher dimensional anti-de Sitter (AdS) space. The precise nature of this connection has not yet been elucidated but there is growing evidence, to which we add here, that there exists such a connection. One particular realization of this connection is light-front holography [9] proposed by Brodsky and de Téramond. In light-front holography, the confining QCD potential at equal light-front time between a quark and antiquark in a meson is determined by the profile of the dilaton field which breaks conformal invariance of the higher dimensional AdS space in which strings propagate.

In a semi-classical approximation to light-front QCD, Brodsky and de Téramond derived a Schroedinger-like equation for mesons:

\[
\left(-\frac{d^2}{d\zeta^2} - \frac{1}{4\zeta^2} + U(\zeta)\right) \Phi(\zeta) = M^2 \Phi(\zeta) ,
\]

where \(\zeta = \sqrt{x(1-x)}r\) is the transverse separation between the quark and antiquark at equal light-front time \(^1\), \(L\) is the orbital quantum number, \(M\) is the mass of the meson and \(\Phi(\zeta)\) is the transverse mode of the light-front wavefunction which is itself given by

\[
\phi(x, \zeta, \phi) = \frac{\Phi(\zeta)}{\sqrt{2\pi\zeta}} f(x)e^{iL\phi}.
\]

It remains a challenge to derive the confining potential \(U(\zeta)\) from first-principles QCD but after identifying \(\zeta\) with the co-ordinate in the fifth dimension and angular momentum with the fifth dimensional mass\(^2\), equation (1.1) describes the propagation of spin-J string modes, in which case \(U(\zeta)\) is determined by the choice for the dilaton field. Remarkably, it can be shown [10] that the dilaton profile is constrained to be quadratic so that the resulting confining potential is given by

\[
U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2(J - 1). \tag{1.3}
\]

Solving equation (1.1) with this confining potential yields the eigenfunctions

\[
\Phi_{nL}(\zeta) = \kappa^{1+L} \sqrt{\frac{2n}{(n+L)!}} \zeta^{1+L} \exp\left(-\kappa^2 \zeta^2 / 2\right) L_n^L(\kappa^2 \zeta^2) \tag{1.4}
\]

with the corresponding eigenvalues

\[
M_{nL,S}^2 = 4\kappa^2 \left(n + L + \frac{S}{2}\right) \tag{1.5}
\]

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\(^1\) \(x\) is the fraction of light-front momentum carried by the quark and \(r\) is the transverse separation between the quark and the antiquark at equal ordinary time.

\(^2\) \((m_R R)^2 = -(2 - J)^2 + L^2\) where \(R\) is the radius of curvature in AdS space.
2. The $\rho$ meson wavefunction

For the $\rho$ meson, $n = 1, L = 0$ and $J = 1$ so that $\kappa = M_{\rho}/\sqrt{2} = 0.54$ GeV. In equation (1.2), $f(x)$ is fixed by comparing the expressions for the pion EM form factor in light-front QCD and in AdS space. This yields $f(x) = \sqrt{x(1-x)}$ so that the resulting AdS/QCD for the $\rho$ is then given by

$$\phi(x, \zeta) \propto \sqrt{x(1-x)} \exp \left(-\frac{k^2 \zeta^2}{2}\right) \exp \left(-\frac{m_f^2}{2\kappa^2 x(1-x)}\right)$$

(2.1)

where the dependence on the quark mass has been introduced according to the prescription by Brodsky and de Téramond [11]. Here we use a light quark mass $m_f = 0.14$ GeV [1].

An earlier procedure to obtain the meson wavefunction is by boosting a non relativistic gaussian Schroedinger wavefunction [12, 13] which results in the so-called Boosted Gaussian (BG):

$$\phi_{BG}(x, \zeta) \propto x(1-x) \exp \left(-\frac{m_f^2 R^2}{8x(1-x)}\right) \exp \left(-\frac{2\zeta^2}{R^2}\right).$$

(2.2)

If $R^2 = 4/\kappa^2$ then the two wavefunctions differ only by a factor of $\sqrt{x(1-x)}$, which is not surprising given that in both cases confinement is modelled by a harmonic oscillator [1]. In what follows we shall consider a parameterization that accommodates both the AdS/QCD and the BG wavefunctions:

$$\phi(x, \zeta) \propto [x(1-x)]^\beta \exp \left(-\frac{k^2 \zeta^2}{2}\right) \exp \left(-\frac{m_f^2}{2\kappa^2 x(1-x)}\right).$$

(2.3)

The AdS/QCD wavefunction is obtained by fixing $\beta = 0.5$ and $\kappa = 0.55$ GeV where as the BG wavefunction is obtained by fixing $\beta = 1$ and treating $\kappa$ as a free parameter.

3. Results and conclusions

To compute the rate for diffractive $\rho$ production, we use the dipole model of high-energy scattering [14, 15, 16, 17] in which the scattering amplitude for diffractive $\rho$ meson production is a convolution of the photon and vector meson $q\bar{q}$ light-front wavefunctions with the total cross-section to scatter a $q\bar{q}$ dipole off a proton. QED is used to determine the photon wavefunction and the dipole cross-section can be extracted from the precise data on the deep-inelastic structure function $F_2$ [18, 19]. This formalism can then be used to predict rates for vector meson production and diffractive DIS [13, 20] or to to extract information on the $\rho$ meson wavefunction using the HERA data on diffractive $\rho$ production [21, 22]. Here we use it to test whether the HERA data prefer the AdS/QCD wavefunction given by equation (2.1). To do so, we compute the $\chi^2$ per data point in the $(\beta, \kappa)$ parameter space using the parametrization (2.3) for the $\rho$ wavefunction.3

Figure 1 confirms that the AdS/QCD prediction lies impressively close to the minimum in $\chi^2$. The best fit has a $\chi^2$ per data point equal to 114/76 and is achieved with $\kappa = 0.56$ GeV and $\beta = 0.47$ which should be compared with the AdS/QCD prediction: $\kappa = 0.54$ and $\beta = 0.5$ shown

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3 We include the electroproduction data and decay width datum in the fit.
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Figure 1: The $\chi^2$ distribution in the $(\beta, \kappa)$ parameter space. The AdS/QCD prediction is the white star.

as the white star on figure 1. Note that the BG prediction i.e. $\beta = 1, \forall \kappa$, is clearly further away from the minimum in $\chi^2$.

Finally, we note that these results are produced using a particular Color Glass Condensate dipole model [18] but that similar results are obtained by using other forward dipole models [19] that fit the $F_2$ structure function data. It remains to be seen how the $\chi^2$ distribution changes if a more sophisticated dipole model, such as the recent impact parameter saturation model [23] which fits the combined HERA $F_2$ data, is used.

References

[1] J. R. Forshaw and R. Sandapen, An AdS/QCD holographic wavefunction for the rho meson and diffractive rho meson electroproduction, Phys.Rev.Lett. 109 (2012) 081601, [arXiv:1203.6088].

[2] J. R. Forshaw and R. Sandapen, Diffractive rho production with an AdS/QCD holographic wavefunction for the rho meson, AIP Conf.Proc. 1523 (2012) 87–90, [arXiv:1211.4729].

[3] ZEUS Collaboration, S. Chekanov et. al., Exclusive $\rho^0$ production in deep inelastic scattering at HERA, PMC Phys. A1 (2007) 6, [arXiv:0708.1478].

[4] The H1 Collaboration, Diffractive Electroproduction of $\rho$ and $\phi$ Mesons at HERA, JHEP 05 (2010) 032, [arXiv:0910.5831].

[5] J. Erdmenger, N. Evans, I. Kirsch, and E. Threlfall, Mesons in Gauge/Gravity Duals - A Review, Eur.Phys.J. A35 (2008) 81–133, [arXiv:0711.4467].

[6] J. Casalderrey-Solana, H. Liu, D. Mateos, K. Rajagopal, and U. A. Wiedemann, Gauge/String Duality, Hot QCD and Heavy Ion Collisions, arXiv:1101.0618.

[7] M. S. Costa and M. Djuric, Deeply Virtual Compton Scattering from Gauge/Gravity Duality, arXiv:1201.1307.

[8] G. F. de Teramond and S. J. Brodsky, Hadronic Form Factor Models and Spectroscopy Within the Gauge/Gravity Correspondence, arXiv:1203.4025.
[9] G. F. de Teramond and S. J. Brodsky, *Light-Front Holography: A First Approximation to QCD*, Phys.Rev.Lett. 102 (2009) 081601, [arXiv:0809.4899].

[10] S. J. Brodsky, G. F. de Tâl'ramond, and H. G. Dosch, *Conformal Symmetry, Confinement, and Light-Front Holographic QCD*, arXiv:1302.5399.

[11] S. J. Brodsky and G. F. de Teramond, *Light-Front Holography and Novel Effects in QCD*, AIP Conf.Proc. 1116 (2009) 311–326, [arXiv:0812.3192].

[12] J. Nemchik, N. N. Nikolaev, E. Predazzi, and B. G. Zakharov, *Color dipole phenomenology of diffractive electroproduction of light vector mesons at HERA*, Z. Phys. C75 (1997) 71–87, [hep-ph/9605231].

[13] J. R. Forshaw, R. Sandapen, and G. Shaw, *Colour dipoles and ρ, φ electroproduction*, Phys. Rev. D69 (2004) 094013, [hep-ph/0312172].

[14] N. N. Nikolaev and B. G. Zakharov, *Colour transparency and scaling properties of nuclear shadowing in deep inelastic scattering*, Z. Phys. C49 (1991) 607–618.

[15] N. N. Nikolaev and B. G. Zakharov, *Pomeron structure function and diffraction dissociation of virtual photons in perturbative QCD*, Z. Phys. C53 (1992) 331–346.

[16] A. H. Mueller, *Soft gluons in the infinite momentum wave function and the BFKL pomeron*, Nucl. Phys. B415 (1994) 373–385.

[17] A. H. Mueller and B. Patel, *Single and double BFKL pomeron exchange and a dipole picture of high-energy hard processes*, Nucl. Phys. B425 (1994) 471–488, [hep-ph/9403256].

[18] G. Soyez, *Saturation QCD predictions with heavy quarks at HERA*, Phys. Lett. B655 (2007) 32–38, [arXiv:0705.3672].

[19] J. R. Forshaw and G. Shaw, *Gluon saturation in the colour dipole model?*, JHEP 12 (2004) 052, [hep-ph/0411337].

[20] J. R. Forshaw, R. Sandapen, and G. Shaw, *Further success of the colour dipole model*, JHEP 11 (2006) 025, [hep-ph/0608161].

[21] J. R. Forshaw and R. Sandapen, *Extracting the rho meson wavefunction from HERA data*, JHEP 11 (2010) 037, [arXiv:1007.1990].

[22] J. R. Forshaw and R. Sandapen, *Extracting the Distribution Amplitudes of the rho meson from the Color Glass Condensate*, JHEP 1110 (2011) 093, [arXiv:1104.4753].

[23] A. H. Rezaeian, M. Siddikov, M. Van de Klundert, and R. Venugopalan, *Analysis of combined HERA data in the Impact-Parameter dependent Saturation model*, Phys.Rev. D87 (2013) 034002, [arXiv:1212.2974].