RENORMALONS ON THE LATTICE AND THE OPE FOR THE PLAQUETTE: A STATUS REPORT

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The first ten coefficients in the perturbative expansion of the plaquette in Lattice SU(3) are computed both on a $8^4$ and on a $24^4$ lattice. They are shown to be fully consistent with the growth dictated by the first IR Renormalon and with the expected finite size effects on top of that. As already pointed out a few years ago, this leads to a puzzling result on the smaller lattice: when the contribution associated with the Renormalon is subtracted from Monte Carlo measurements of the plaquette, what is left over does not scale (as expected) as $a^4$, but as $a^2$. While the analysis is not yet complete on the larger lattice, the implications of such a finding is discussed.

1 Introduction: the Gluon Condensate in Lattice Gauge Theory

A longstanding problem in Lattice Gauge Theory (LGT) is that of a first principles determination of the Gluon Condensate (GC). In LGT the GC is given in terms of Wilson Loops, for instance the basic plaquette $W$, for which an Operator Product Expansion (OPE) can be written in terms of

$$W = W_0 + \frac{W_4 \Lambda^4}{Q^4} + \ldots$$  \hspace{1cm} (1)

Here $Q = 1/a$ is the inverse lattice spacing, acting as the scale needed in every definition of the GC, $W_0$ is the contribution associated with the Identity operator, while $W_4$ is associated with the “genuine” (i.e. dim = 4) condensate. In view of Eq. (1), a standard approach was to exploit the following formula

$$W_{MC}(\beta) - \sum_n c_n \beta^{-n} = c \ Z(\beta) \ G_2 \ (a\Lambda)^4 + \ldots$$  \hspace{1cm} (2)

which is to be understood as follows: $W_0$ is computed in Perturbation Theory and is subtracted from the Monte Carlo measurements at various values of $\beta$; the leading contribution that is left on the right is the second term in the OPE, which is the relevant one, whose signature is dictated by Asymptotic Scaling, that is $(a\Lambda)^4 \sim \exp(-\beta/(2b_0))$.

The former procedure is actually ill defined, since the definition of the perturbative contribution to be subtracted is plagued by Renormalons. In Ref. 1...
the first eight coefficients of the perturbative expansion of the plaquette were computed via Numerical Stochastic Perturbation Theory, the Renormalon factorial growth was singled out and shown to introduce an indetermination of order \( (\Lambda/Q)^4 \), which is just the order of the term one would be interested in. The situation is in a sense even worse. In Ref. 3 the Renormalon contribution was resummed and subtracted: to our astonishment what was left over did not scale as \( a^4 \), but as \( a^2 \), at least on the \( 8^4 \) lattice on which the computations were first performed on. Such a result is a challenge to our understanding of power effects: for a lucid discussion see Ref. 4.

2 Renormalons on the lattice: new results and confirmation

Ne now address the following questions: Was the leading behaviour correctly singled out? Are finite size effects under control (remember that the Renormalon growth impinges more and more on the IR region as higher order orders are computed)? Answers can be got (for details see Ref. 5) by considering the following formula for the Renormalon contribution

\[
W^\text{ren}_0(s, N) = C \int_{Q^2_0(N)}^{Q^2} \frac{k^2 dk^2}{Q^2} \alpha_s(sk^2) = \sum_n C^\text{ren}_n(C, s, N) \beta^{-n} \quad (3)
\]

The integral is simply the expected form (obtained from dimensional and Ren.G. considerations) for the condensate on a finite lattice: the lower limit of integration is the IR cutoff, function of the lattice size \( N \), while the scale \( s \) is in charge of matching from a continuum to a lattice scheme. The coefficients in the expansion (computable in terms of Incomplete Gamma functions) are functions of an overall constant, the scale \( s \) and the lattice size \( N \). In Ref. 3 the values for \( s \) and \( C \) were obtained by fitting \( C^\text{ren}_n(C, s, N) \) to the first eight coefficients on the \( N = 8 \) lattice. From \( C^\text{ren}_n(C, s, N) \) one can now infer higher orders on different lattice sizes (simply changing the parameter \( N \)). By doing this and actually computing the first ten coefficients of the expansion both on a \( 8^4 \) and on a \( 24^4 \) lattice, one gets for instance Fig (1). While the actual numbers will be published soon elsewhere, an impressive agreement is manifest between the first ten coefficients as inferred as above and as actually computed on the \( 24^4 \) lattice. Finite size effects are of order \( 3 - 4\% \) at tenth order.

3 Conclusions and perspectives

What about the subtraction? By repeating the procedure the bizarre result is actually stable on the \( 8^4 \) lattice, which leaves the puzzle still there. On the larger lattice work is still in progress: first indications are that there could be
space for recovering the standard \( (a^4) \) result, due to a tiny interplay between perturbative and non-perturbative finite size effects. This would of course still pose the question of what to blame for the dependence on finite size. Further study on the sensitivity to boundary conditions is also in progress.

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References

1. See for example B. Allés, M. Campostrini, A. Feo and H. Panagopoulos, \textit{Phys. Lett.} B \textbf{324}, 443 (1994) and references therein.
2. F. Di Renzo, G. Marchesini and E. Onofri, \textit{Nucl. Phys.} B \textbf{457}, 202 (1995).
3. G. Burgio, F. Di Renzo, G. Marchesini and E. Onofri, \textit{Phys. Lett.} B \textbf{422}, 98 (1998).
4. M. Beneke, \textit{Physics Reports} \textbf{317}, 1 (1999).
5. F. Di Renzo and L. Scorzato, to be issued soon.