TOPOLOGY OF NEUTRAL HYDROGEN WITHIN THE SMALL MAGELLANIC CLOUD

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ABSTRACT

In this paper, genus statistics have been applied to an H\textsc{i} column density map of the Small Magellanic Cloud in order to study its topology. To learn how topology changes with the scale of the system, we provide topology studies for column density maps at varying resolutions. To evaluate the statistical error of the genus, we randomly reassign the phases of the Fourier modes while keeping the amplitudes. We find that at the smallest scales studied (40 pc ≤ λ ≤ 80 pc), the genus shift is negative in all regions, implying a clump topology. At the larger scales (110 pc ≤ λ ≤ 250 pc), the topology shift is detected to be negative (a “meatball” topology) in four cases and positive (a “swiss cheese” topology) in two cases. In four regions, there is no statistically significant topology shift at large scales.

Subject headings: galaxies: individual (Small Magellanic Cloud) — galaxies: ISM

1. INTRODUCTION

The Small Magellanic Cloud (SMC) is an irregular dwarf galaxy. Although the SMC is believed to be gravitationally bound to the Milky Way, recent studies suggest that this may not be true (Besla et al. 2007). The SMC is relatively metal-poor and very gas-rich, and represents an ideal galaxy in which to study the formation of stars in low-metallicity environments. Its proximity to the Large Magellanic Cloud (LMC) and Milky Way has resulted in a turbulent history. Recent evidence points to a close encounter of stars in low-metallicity environments. Its proximity to the Milky Way, recent studies suggest that this may not be true (Mathewson et al. 1974; Murai & Fujimoto 1980; Stanimirovic et al. 2002). The interaction between the SMC and the LMC is still dynamic, with observations showing that metal-poor star clusters with ages <200 Myr and metallicity ratios [Fe/H] < −0.6 within the LMC originated from infalling SMC gas (Bekki & Chiba 2007).

Statistical techniques are indispensable for studying turbulence in the interstellar medium (ISM). Images of neutral hydrogen (H\textsc{i}) distribution are frequently used, as in most cases it is possible to ignore self-absorption (Lazarian 1995). H\textsc{i} also occupies a large portion of the Galactic disk (roughly a 20% filling factor), and its movements should reflect large-scale turbulence (Lazarian 1999). Furthermore, the prevalence of H\textsc{i} means that it can be studied not only in our galaxy, but in nearby galaxies as well.

Several studies have performed statistical analyses of the H\textsc{i} distribution in the SMC (Stanimirovic et al. 1999, 2002). These studies determined the spatial power spectrum of the H\textsc{i} intensity and employed the velocity channel analysis technique (Lazarian & Pogosyan 2000) to reveal shallower-than-Kolmogorov velocity and density spectra (see Beresnyak et al. 2005). However, power spectra, though informative about the distribution of energy with scale, are not sensitive to gas topology. For instance, H\textsc{i} surveys of the SMC have shown numerous filamentary structures and shells of expanding gas. These surveys have detected 501 shells dispersed throughout the SMC (see Fig. 1), six of which have radii >350 pc and are large enough to be classified as supergiant shells (SGSs) (Staveley-Smith et al. 1997). Many of these shells were created by massive associations of OB stars, but some show no spatial correlation to any young stellar population. These “orphan” shells may be a result of gamma-ray bursts or collisions between high-velocity clouds and the SMC (Hatzidimitriou et al. 2005). Alternatively, they could be produced by ISM turbulence.

This calls for employing other techniques for statistical studies of the SMC. Recent years have been marked by an increased interest in statistical studies of astrophysical turbulence (see Lazarian 2004 for a review). In particular, two techniques have been developed, velocity channel analysis (VCA) and velocity coordinate spectrum (VCS), which are capable of studying power spectra of velocity and density (Lazarian & Pogosyan 2000, 2004, 2006). However, the topology of the gas distribution cannot be described by a power spectrum. On the other hand, genus analysis, with its ability to accurately describe and quantify the topology, is a promising synergetic tool in the quest to better understand turbulence.

Genus statistics was developed to study the topology of the universe and the distribution of galaxies in three dimensions (Gott et al. 1986; 1987) and in two dimensions (Coles 1988; Coles & Plionis 1991; Melott et al. 1989; Plionis et al. 1992; Davis & Coles, 1993; Coles et al. 1993). Subsequent projects have used genus statistics to study the topology of the temperature variations within the cosmic microwave background (CMB) (Smoot et al. 1994; Colley & Gott 2003), and have also been applied to a systematic study of the variations of the two-dimensional (2D) genus with magnetohydrodynamic (MHD) simulations (Kowal et al. 2007). The use of genus statistics for the study of H\textsc{i} was first discussed in Lazarian (1999), and subsequent studies presented the first genus curves for the SMC (Lazarian et al. 2002; Lazarian 2004). A recent paper by Kim & Park (2007) provided a more thorough study of the topology of the H\textsc{i} peak brightness temperature distribution in the LMC.

The goal of the present paper is to extend the technique of Lazarian (2004) and Kim & Park (2007) by providing a quantitative measure of the uncertainties involved in the genus studies. We then apply the technique to the SMC H\textsc{i} data set to quantitatively describe the topology of the H\textsc{i} distribution. Kowal et al. (2007) have shown that genus statistics of a 3D density distribution from synthetic observations obtained with MHD simulations agrees with the genus results for the 2D integrated
column density of the same data set. Therefore, genus statistics complements the power-spectrum analyses in providing insights into the physical processes that shape the ISM.

In cosmological studies, the distributions in question, e.g., the distribution of CMB intensity, are nearly Gaussian, and genus is used to study small deviations from the Gaussian. Dealing with the ISM, in particular with the distribution of column densities of the SMC, one cannot expect deviations from symmetry to be small, a priori.

The structure of this paper is organized as follows. Section 2 discusses our approach to the analysis of data. Section 3 summarizes the observational procedure and subsequent data analysis of the H\(\text{I}\) column density map. Section 4 is an analysis and discussion of the cropped regions within the SMC, focusing on the genus shift and its topological implications. Section 5 summarizes the results and posits astrophysical connections between the genus analysis and the SMC.

2. GENUS: EXAMPLES AND MATHEMATICAL SETTINGS

Genus is a quantitative measure of topology. It can characterize both 2D and 3D distributions. The 2D genus can be represented as (Coles 1988; Melott et al. 1989)

\[ G \equiv \frac{\text{(number of isolated high-density regions)}}{\text{(number of isolated low-density regions)}} \]

where low- and high-density regions are selected with respect to a given density threshold. As a result, for a given 2D intensity map, a curve corresponding to different thresholds emerges (see Fig. 2).

For instance, a uniform circle would have a genus of 0 (one contiguous region of high density, i.e., an "island," and one contiguous region of low density, i.e., a "hole"), while a ring (an audio CD, for example) would have a genus of −1 (one contiguous region of high density and two contiguous regions of low density). Two separate circles, on the other hand, correspond to a genus of 1 (one hole and two islands), three separate circles correspond to a genus of 2, etc. Using the language of Richard Gott, one can say that genus can distinguish between "meatball" and "swiss cheese" topologies.

Furthermore, the genus can be represented mathematically as an integral using the Gauss-Bonnet theorem. In more specific terms, for the 2D case we have (Melott et al. 1989; Gott et al. 1990)

\[ G(\nu) = \frac{1}{2\pi} \int_{\mathcal{L}(\nu)} \kappa(x, y) \, dl, \]

where

\[ \mathcal{L}(\nu) \equiv \{(x, y) | I(x, y) = \nu\}, \]

\[ \kappa \equiv \frac{1}{r} \text{sign}(-\nabla I \cdot n), \]

\(I\) is an observed intensity, \(r\) is the principal radius of curvature, the integral follows a set of surface contours at given \(\nu\), and \(n\) is a normal vector, pointing outside of a contour. As seen from
equation (1), a contour enclosing a high-density region will give a positive contribution, while a contour enclosing a low-density one will give a negative contribution. Essentially, at a given threshold value \( \nu \), the genus value is the difference between the number of regions with a density higher than \( \nu \) and those with a density lower than \( \nu \).

The threshold values in equation (2) are selected so that they represent area fractions \( f \). For a Gaussian field, they are defined as (Hoyle et al. 2002)

\[
f = \frac{1}{\sqrt{2\pi}} \int_{\nu}^{\infty} e^{-x^2/2} \, dx.
\] (3)

Raising the threshold level \( \nu \) from the mean value would cause the low-density regions to merge together, causing the genus to become positive, reach its maximum, and begin to decrease to zero, whereas positive regions begin to disappear with larger \( \nu \). Conversely, lowering the threshold level \( \nu \) from the mean value would cause the high-density regions to coalesce, resulting in a negative genus.

More importantly, however, the genus curve for a random Gaussian distribution is known (Coles 1988):

\[
G(\nu) = \frac{1}{(2\pi)^{3/2}} \frac{\langle \kappa^2 \rangle}{2} \nu e^{-\nu^2/2} = A \nu e^{-\nu^2/2}.
\] (4)

This particular form of the genus curve characterizes a Gaussian random field, whatever its power spectrum is. This is an extremely important point of the genus analysis, because it allows us to separate topology effects from the ones caused by the power-spectrum behavior.

We expect that the sign of the genus curve at the mean intensity level does describe the field topology. A positive genus will represent a clump-dominated field, while a negative one will mean the domination of holes. However, it is more convenient to work with the zero of the genus curve \( \nu_0 \), because it can be naturally normalized to the field variance. In this case, for the intensity with the subtracted mean value, a negative \( \nu_0 \) corresponds to the clumpy topology, while a positive \( \nu_0 \) indicates the “swiss cheese” topology.

An example of the genus curve is presented in Figure 2, which shows an example of clumpy topology as \( \nu_0 < 0 \). Additional information on the distribution topology can be obtained by comparing the genus curve for the given distribution with the one for a Gaussian distribution but the same dispersion (see eq. [4]). The maximum and minimum points of the genus curve correspond to percolations of the distribution (see Colombi et al. 2000). The fact that the observed genus falls more slowly at large thresholds of \( \nu \) than the genus of the Gaussian distribution indicates that the islands are more discrete and pronounced than for the Gaussian distribution.

As a Gaussian field always has a neutral topology, fitting the Gaussian genus for an estimation of \( \nu_0 \) cannot be an optimal choice. As we do not rely on a particular field statistics, we need a robust method for estimation of \( \nu_0 \) for any form of a genus curve. In this paper, we fit a polynomial between the global extrema of a genus curve while also requiring a zero derivative at the ends of the interval. Practical calculations show that the fifth-power polynomial is flexible enough to represent the varying shapes of non-Gaussian genera, and always has a single zero, which we interpret as an estimation of \( \nu_0 \). The higher polynomials tend to oscillate when applied to a noisy genus.

It can be shown with a simple example that large-scale trends, even linear trends, can significantly distort a genus curve, changing its shape and making it noisier. Such large-scale gradients usually do not carry any topological information, and should be removed before estimation of \( \nu_0 \). Here, we have options such as subtracting a polynomial background or Fourier filtering low harmonics in the whole map or a particular region.

Another possible source of contamination is the presence of nonabundant compact features, where the amplitude is high enough to affect the mean value. Such features should be either removed from the map or weakened by reducing the image contrast. Taking the median value instead of the mean one when calculating \( \nu_0 \) is also an option.

On the other hand, the presence of white Gaussian receiver noise would not change the topology, as it corresponds to a completely symmetric genus. We can substantiate this statement as follows. Let us consider some small region near the intersection of the plane at the level \( \nu \) and the map surface. If the map has positive curvature in the direction of the gradient, adding such noise would not change the topology, as it corresponds to a completely symmetric genus. However, if the map has positive curvature in the direction of the gradient, adding such noise would shift the genus count to the positive direction. If the curvature is negative, the shift is negative. On the other hand, the mean curvature at the mean level will be positive for clumps and negative for holes, which means that the genus count at this level will be shifted up for clumps and down for holes; i.e., it would not change its sign. This means that the topology type cannot be changed by adding such noise.

The analysis would be incomplete without an estimation of the variance of \( \nu_0 \). Our method for finding the variance in our present study differs from earlier ones (Lazarian 2004; Kim & Park 2007). Following suggestions from Peter Coles, a pioneer in genus analysis, we generated for each map a set of images with randomly shifted phases of individual harmonics. This procedure causes the field to take Gaussian statistics, and therefore zero \( \langle \nu_0 \rangle \), but allows us to effectively estimate its variance.
More specifically, we take a fast Fourier transform (FFT) of the region being studied and assign the phase of each harmonic to a random variable uniformly distributed in $[-\pi, \pi]$, keeping Hermitian conjugacy of the Fourier image. After the inverse FFT, we calculate the respective $\nu_0$. After repeating this procedure several times, we calculate the variance of $\nu_0$. In our case, 10 repetitions appeared to be enough to obtain a statistically relevant variance.

3. OBSERVATION, DATA ANALYSIS, AND RESULTS

The $\text{H} \text{I}$ column density image used in this study is a composite obtained with the Australia Telescope Compact Array (ATCA) and the Parkes Telescope in Australia (Fig. 3). ATCA, a radio interferometer, was used to observe 320 overlapping regions containing the SMC. These data were combined with observations from the 64 m Parkes radio telescope, which observed a $4.5^\circ \times 4.5^\circ$ region centered on R.A. $= 01^h01^m$, decl. $= -72^\circ56^\prime$ (Stanimirović et al. 1999). The data from these two telescopes were merged to create a complete image of the $\text{H} \text{I}$ column density of the SMC, with a continuous sampling of spatial scales from 30 pc to 4.5 kpc. For more information on the merging process, see Stanimirović et al. (1999). The effective angular resolution of the combined column density image is $98^\prime$, implying a spatial resolution of 30 pc at a distance of 60 kpc.

The effective smoothing scale $\lambda$ is given here in terms of the effective half-power beamwidth (HPBW) and is always greater than 30 pc. For the $150 \times 150$ pixel region ($40 \text{ pc} \leq \lambda \leq 250 \text{ pc}$), using a $\lambda$ larger than 15% of the image length resulted in a genus curve that was useless; both the high- and low-density regions coalesced together, resulting in too few regions for the genus statistic to analyze. The background was subtracted using a fifth-order polynomial, with subsequent filtering out of the first two Fourier harmonics.

An offshoot of the FFT package in IDL was also utilized to study the $\text{H} \text{I}$ column density image of the SMC. The SMC column density map was converted into Fourier space, and

![Fig. 4.—Snapshots of the Fourier-filtered SMC. Top left: The SMC with the high frequencies excluded; the general shape of the SMC is apparent, but no small-scale features are seen. Bottom left: The SMC with the low frequencies excluded; the numerous small-scale features of the SMC are readily apparent, and much more visible. The counterpart of each image (top and bottom right) is the absolute value of the natural log of the image, which results in a contour-like image.](image)

![Fig. 5.—Genus shift vs. smoothing radius for the entire SMC. The underlying astrophysical processes behind the genus shift are discussed in §§ 4 and 5.](image)
noninformative frequencies were removed (see Fig. 4). An inverse Fourier transform was then applied to recover the information. This methodology allows us to decompose the image into different frequency ranges, thereby probing the SMC at different scales. By removing the low frequencies, it is possible to focus on the small-scale structure of the SMC. Similarly, by removing the high frequencies we decrease the resolution, and focus on the large-scale structure of the SMC. This method works in conjunction with the smoothing scale methodology as described above.

Figure 5 shows the genus shift as a function of smoothing radius $\lambda$ for a $400 \times 300$ pixel region enclosing the majority of the SMC. At all smoothing scales (40 pc $\leq \lambda \leq 270$ pc), the SMC shows negative shift, being statistically significant at small (40 pc $\leq \lambda \leq 80$ pc) and medium ($\lambda \sim 170$ pc) scales.

The results from the $150 \times 150$ pixel regions can be seen in Figure 6 and Table 1. At scales below 70–80 pc, every sampled region shows a genus curve with apparent negative $\mu$. Furthermore, at these scales each region exhibits asymmetry, although this varies from region to region. Five of the nine surveyed regions have a larger amplitude on the negative (low-density) side of the genus curve. We can infer from this asymmetry that the low-density holes are more isolated than expected, while the high-density clumps are more contiguous than expected. Combined with the negative genus shift, these two statistics show that there are merged high-density clumps surrounded by isolated

![Fig. 6.—Genus shift for the nine 150 x 150 pixel surveyed regions of the SMC. The underlying astrophysical processes behind the genus shift are discussed in §§ 4 and 5. The effective resolution, accounting for the instrument HPBW, is used. The plots are ordered by region number.](image)

| Region ID | $X$ (pc) | $Y$ (pc) | Scales (pc) | $\mu_{10}/\sigma$ |
|-----------|----------|----------|-------------|-------------------|
| 1.........  | 1500–3000 | 3000–4500 | 35–70       | $-0.10 \pm 0.02$  |
| 2.........  | 2000–3500 | 2000–3500 | 35–70       | $-0.09 \pm 0.02$  |
| 3.........  | 2800–4300 | 3600–5100 | 35–70       | $-0.09 \pm 0.03$  |
| 4.........  | 3000–4500 | 2000–3500 | 35–50       | $-0.06 \pm 0.03$  |
| 5.........  | 3300–4800 | 1900–3400 | 35–40       | $-0.07 \pm 0.04$  |
| 6.........  | 4000–5500 | 2000–3500 | 35–40       | $-0.07 \pm 0.04$  |
| 7.........  | 4000–5500 | 4000–5500 | 35–100      | $-0.09 \pm 0.04$  |
| 8.........  | 4000–5500 | 500–2000  | 35–50       | $0.13 \pm 0.07$   |
| 9.........  | 500–2000  | 2000–3500 | 35–50       | $-0.08 \pm 0.02$  |

**Table 1.** Statistically Significant Genus Shifts

*Note.—* Confidence intervals are given here for confidence probability 0.67.
low-density holes. The negative shift can be attributed to the numerous small clumps of gas that compose the ISM of the SMC. At the small scales at which the genus is probed, the numerous knots and filaments that compose the SMC are not seen. The genus shift curves for the individual regions diverge as the smoothing scale increases. At scales of $120 - 150$ pc, two of the regions show a positive genus, implying "swiss cheese" topology. We can conclude from the rising genus shift that the small clumps are merging together, while the holes and SGSs are coming into focus. Four regions have a negative shift at large scales ($150 - 250$ pc), and one of them shows mixed behavior (a positive genus at $120 - 150$ pc and a negative genus at $220 - 270$ pc). The error bars obtained by randomly applying phases to Fourier modes as described in § 2 show that our results are more reliable for some scales and for some parts of the SMC than for others. The example of genus curves for an individual region with mixed topological behavior for different $\lambda$ is shown in Figure 7.

4. DISCUSSION

The genus shift estimated in the previous section can give us insight into the underlying physical processes of the SMC. Referring to the genus shift of the entire SMC (see Fig. 5), it is readily apparent that the shift varies according to the smoothing scale $\lambda$. At the smallest scales studied, the genus shift has a negative value, implying a strong clump topology. We can infer that the clumps are caused by clouds of H\textsc{i} gas, as well as the numerous knots and filaments that compose the SMC. At medium scales ($120 - 200$ pc), the genus shift takes on a neutral or slight positive value. This increase in the genus shift can be connected to the abundance of shells that comprise the SMC, as the $\lambda$ = 100 pc (Stanimirović et al. 2007). This is consistent with our results, as the largest positive values of the genus shift occur between $120$ and $150$ pc. At the largest scales studied ($\lambda$ > 170 pc), the genus shift takes on a slight negative value; this is due to the prominent "wing" and "bar" features (Stanimirović et al. 1999), indicating that at even the largest scales, (large) clumps dominate the topology.

From our results, we reached the conclusion that the SMC tends to exhibit a clump topology. The dominance of the "meatball" topology is expected in the case of supersonic turbulence (Kowal et al. 2007). Several of the surveyed regions display characteristics similar to those of the supersonic (low-\beta) case, e.g., an asymmetrical genus curve with a tail that extends into the high-density portion of the plot (see Fig. 18 of Kowal et al. 2007 for more information). However, other possible reasons for a clump topology could be cooling instability and self-gravity of the H\textsc{i} gas.

However, our results differ from the results in Kim & Park (2007), who found that the H\textsc{i} distribution in the LMC shows mainly hole topology at intermediate scales, despite the fact that fewer shells are present in the LMC (124 as compared to 500 in the SMC). There are several important factors that could contribute to this difference. First, Kim & Park (2007) performed a genus analysis on the H\textsc{i} peak brightness temperature image, while our work used the H\textsc{i} column density distribution. Peak brightness images emphasize more small-scale and shell structure, while the column density distribution emphasizes density distribution, washing out small-scale fluctuations that are caused both by density and velocity fluctuations.

Second, while the LMC has an almost face-on H\textsc{i} disk, the SMC has a larger inclination and a non-disklike morphology. In fact, several authors have claimed a large line-of-sight depth for the SMC (see Stanimirović et al. 2004 for details). Therefore, any line of sight through the SMC integrates over a longer physical depth. It would be interesting to apply our procedure to the LMC H\textsc{i} column density image for a more direct comparison.

4 The isothermal equation of state used in this work may not be valid for all the scales studied here, but we can still use its results as a guide.
Interestingly, for several regions where a hole topology is detected, the estimated shell size is comparable to the one from Kim & Park (2007). A possible reason for a hole topology in regions 7 and 9 could be related to the shells created by supernovae explosions and stellar winds. It is interesting to note that those two regions are both off the main “bar” of the SMC, and most likely correspond to areas of low line-of-sight depth. Nigra et al. (2008) study the eastern wing region (our region 9) and find a small line-of-sight thickness. Our region 7 has been identified in Stanimirović et al. (1999) and Hatzidimitriou et al. (2005) as containing several “orphan” shells with high luminosities. Hatzidimitriou et al. (2005) hypothesize that these shells may be associated with an ancient chimney.

We stress that it is important to gauge the statistical techniques against numerical simulations. Kowal et al. (2007) recently undertook an extensive investigation of density statistics in MHD turbulence, which included different statistical tools, including genus. They concluded that the genus statistic is sensitive to the sonic Mach number $M_s$. In the case where the magnetic pressure dominates, i.e., the high-$\beta$ case with a subsonic Mach number (e.g., $M_s \approx 0.3$), the genus curve is highly symmetrical. For the low-$\beta$ cases and the supersonic case (e.g., $M_s \approx 2.1$ or 6.5), the curve stretches into the positive (high-density) side and becomes increasingly nonsymmetrical as $M_s$ increases. The end result is that it is possible to obtain $M_s$ from the plot of the genus curve, as the Mach number is directly related to the length of the high-density tail. Furthermore, the genus statistic also gives topological information. For values of low $M_s$, the genus curve is symmetrical, implying that there are equal numbers of high-density clumps and low-density holes. As $M_s$ increases, turbulence creates more high-density structures, which causes the tail of the genus curve to extend toward the high-density side. This corresponds well to the results of studying fractal dimensions of density while varying the density threshold in Kowal & Lazarian (2007).

In our analysis, we assumed that the H i gas is optically thin. This is an important assumption, as the analysis of the effects of absorptions in turbulent gas in Lazarian & Pogosyan (2004) suggests that absorption introduces a critical spatial scale for plane-of-sky statistics, with fluctuations larger than this critical scale being strongly affected by absorption effects. This means that the way in which we use the genus above is not applicable to $^{12}$CO data, but should be applicable to $^{13}$CO data with $^{12}$CO data in limbo, as this isotope is frequently thick. Potentially, the topology of gas at scales less than the critical scale is also of interest. However, Lazarian & Pogosyan (2004) suggest that fluctuations of the integrated self-absorbing spectral line may be dominated by velocity caustics, provided that the spectrum of density is sufficiently steep, e.g., $E(k) \sim k^{-\delta}$, where $\delta > 1$. The Kolmogorov spectrum corresponds to $E(k) \sim k^{-5/3}$ and is steep according to the aforementioned definition. The topology studies are, as discussed above, different from the studies of the spectrum. However, we believe that the criteria for density fluctuations being observable is the same. Therefore, we expect that for $\delta < 1$, one can analyze genus for the scales less than the critical one. Note that in terms of the present study, the constancy of the spectral indexes observed in Stanimirović & Lazarian (2001) provides additional evidence that H i is not substantially affected by absorption.

5. SUMMARY

In this paper, we have analyzed the H i column density map of the SMC in an attempt to elucidate its topological features. A brief summary of our results is as follows.

1. We have extended the genus analysis for column densities of diffuse gas by presenting a new procedure for estimating the topology indicator $\nu_0$ and its variance.

2. For small-scale smoothing (35 pc $\lesssim \lambda \lesssim 120$ pc), the SMC exhibits a negative shift, indicating a clump or “meatball” topology. We conjecture that this is due to the numerous clumps of gas created by supersonic turbulence. We know from numerical simulations that numerous high-contrast clumps are produced by such turbulence.

3. As the smoothing scale increases (120 pc $\lesssim \lambda \lesssim 150$ pc), the shift of the genus curve becomes less negative, trending toward a slight positive shift. This can be attributed to the averaging of small clumps, while larger shell and SGS structures throughout the SMC are less affected by smoothing. At these medium scales, the smaller gas clumps are less important, while the shells come into focus. These shells are potentially a result of stellar winds and supernovae from OB associations.

4. For larger regions with scales $\geq 100$ pc, the genus curve becomes noisier; however, in four cases the corresponding negative shift may indicate that most of the shells have sizes smaller than the smoothing scale.

5. The nine $150 \times 150$ pixel regions of the SMC exhibit slightly different trends. Although they all possess a clump topology at small scales, the curves at larger scales are rather different. Some trend toward a hole topology at larger $\lambda$, while others exhibit no positive genus shift. A possible reason for hole topology in regions 7 and 9 could be related to the shells created by supernovae explosions and stellar winds. We hope that in the future, regions with a particular topology can be identified with regions of physically distinct behavior. We may infer that the SMC is somewhat heterogeneous from region to region.

6. Genus analysis is an effective complementary tool in the study of turbulence. The power spectrum contains information about velocity fluctuations but does not possess topological information. Combined with genus statistics, both the velocity statistics and topological information can be obtained for a selected object.

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