A computational procedure to predict the load-settlement behavior of axially loaded piles in sandy soils

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ABSTRACT

In the early stages of design and analysis of structures that are supported by piled foundation, the stiffness of piles is often required to be evaluated. Without going through the complication of pile-soil system, piles can be simulated as springs if the load-settlement behavior of each pile is accurately determined.

In this paper, the governing differential equation of pile displacement problem is implicitly solved through a sequence of computation steps. A simple Matlab M-file is built up to perform the computations and present the analysis results. The proposed computation scheme accounts for the nonlinearity of soil mechanical properties and for the non-homogeneity of the soil profile along the pile depth. Consequently, an iterative loop of recalculation and tolerance checking is used and convergence is achieved very rapidly.

The procedure is then applied to calculate the pile-settlement behavior of an 1800 mm diameter and about 18 m length concrete pile installed in dense to very dense sandy soil. The pile which has a working load of about 6670 kN, is one of the New Hindiya railway bridge piles located in the central part of Iraq. The pile was in situ tested under an axial load of about twice the working load.

The results obtained by the proposed computation method compared well (the deviation is less than 10%) with the test results in terms of pile load settlement behavior. This may enhance the reliability of the proposed method.

Keywords: pile, displacement, axial loading, soil nonlinearity, computational procedure

1 INTRODUCTION

Piled foundation is a common practice in geotechnical engineering and is often recommended when the upper soil strata are relatively weak or compressible. Piles are also used to limit total and differential settlement of super structures to the allowed limits.

The applied loads on piles are transferred to the supporting soil through two components, namely; the skin or pile shaft resistance and the tip or pile end resistance. Both of these components are displacement dependent and depend on the mobilized portion of the surrounding soil shear strength. Therefore, the real behaviour of pile soil system is one of the complicated soil structure interaction problems that is thought to be still in need of more investigation. Due to the obvious difference in the mode soil shearing around the pile from that at the pile tip, the shaft—base load sharing and the interaction between these two components depend on the amount of displacement at each level along pile body relative to that of soil in the vicinity. While the shaft resistance need only a tendency of movement to be mobilised, the base resistance need a relatively large (about 0.05% to 1% of the pile diameter) displacement to be fully mobilised.

In the following paragraph, a brief review to some of the available modelling and analysis methods of the pile displacement problems in literature.

2 MODELING AND ANALYSIS OF PILE-SOIL SYSTEM

As a pile is axially loaded, Cooke (1974) considered the surrounding soil is displaced in a manner as if the soil consists of a series of co-centered cylinders with inner one is in full contact with the pile shaft. The most outer one which has a radius ($r_o$) is assumed to exhibit no vertical displacement as the pile effect vanishes at that radial distance. He assumed also that the shear stress ($\tau$) varies inversely with the radial distance ($r$) as follows:

$$\tau = \frac{\tau_o r_o}{r}$$  \hspace{1cm} (1)

where $\tau_o$ and $r_o$ are the shear stress at the pile soil interface and the pile radius, respectively.

Since the vertical component of soil displacement ($w$) is considered, the shear strain $\gamma$ is approximated as:
\[ \gamma = \frac{dw}{dr} \]  

(2)

Then, by introducing the soil shear modulus, the pile displacement \( w_o \) is calculated as follows:

\[ w_o = \int \frac{G_r'}{Gr} dw = \frac{G_r'}{G} \ln \left( \frac{r_m}{r_n} \right) \]  

(3)

and the shaft stiffness \( K_s \) will be:

\[ K_s = \frac{2\pi G L}{\zeta} \]  

(4)

where \( G \) is the average shear modulus along the pile and \( \zeta = \ln (r_m / r_n) \) which is ranging from 3 to 5. Considering the pile as a rigid element, the base stiffness \( K_b \) can be calculated basing on the theory of elasticity as:

\[ K_b = \frac{P_b}{w_o} = \frac{4r_i G_b}{(1-\nu)} \]  

(5)

where \( P_b \) is the base resistance, \( G_b \) and \( \nu \) is shear modulus of the bearing layer and its Poisson’s ratio, respectively (Fleming et al, 2009).

This approach which usually works at low loading levels, gives a linear load displacement relation. At higher loading levels, the pile displacement is no longer governed by the elastic properties of surrounding soil only, rather by the behavior of pile-soil interface also. Incorporating the elastic deformation of pile body, the problem gets more complicated especially when soil non-homogeneity along the pile depth is considered.

The governing differential equation of the problem will be:

\[ EA \frac{d^2 w}{dz^2} - C f(z, w) = 0 \]  

(6)

in which \( E \) is the pile Young’s modulus, \( A \) is pile cross sectional area and \( C \) is its perimeter. The function \( f \) which represents the pile shaft resistance has great influence on the equation solution. For sandy soils where the shear stiffness increased almost linearly with depth, the equation will be:

\[ \frac{d^2 w}{dz^2} + \frac{ck}{EA} zw = 0 \]  

(7)

where \( k \) is the rate of increase of the soil shear stiffness with depth. By introducing a characteristic length \( \lambda \), equation (7) can be re-written in a non-dimensional form as:

\[ \frac{d^2 w^*}{dz^2} + \lambda z w^* = 0 \]  

(8)

in which \( z^* = \lambda z \), \( w^* = \lambda w \) and \( \lambda = \sqrt{ck / EA} \).

Scott (1981) showed that the analytical solution of equation (8) can be expressed in terms of a pair of Airy function \( Ai \) and \( Bi \) as follows:

\[ w^* = C_1 Ai(z^*) + C_2 Bi(z^*) \]  

(9)

\( C_1 \) and \( C_2 \) can be obtained from the pile boundary conditions.

This solution will be used later to verify the proposed computational procedure.

To account for the nonlinearity of soil mechanical properties, several models for pile-soil interface are available in the literature.

Desai and Siriwadane (1984) proposed an elastic-plastic model in which the shear resistance \( (r_s) \) increases linearly with displacement until a certain value \( (w_c) \) is reached. After that, it becomes constant \( (r_s - \mu \sigma) \).

As cited by Shen and Kushwaha (1998), Bekker (1966) proposed an exponential relation of the form:

\[ r_s = r_{\text{max}} [1 - \exp \left( -\frac{w}{m} \right)] \]  

(10)

where \( m \) is a curve fitting constant related to soil properties.

Clough and Duncan (1971) presented an extended interface hyperbolic model by making use of Janbu (1963) experimental work:

\[ r_s = \frac{w}{1/G_i + \frac{w R_f}{c_i + \sigma_s \tan \delta}} \]  

(11)

in which \( G_i \) is the initial shear modulus which is related to confining pressure \( (\sigma_i) \) as follows:

\[ G_i = K_i \gamma_w \left( \frac{\sigma_3}{P_u} \right)^n \]  

where \( K_i \) and \( n_i \) are fitting constants, \( P_u \) is the atmospheric pressure and \( \gamma_w \) is water unit weight.

Mosher (1984) suggested a hyperbolic model for the unit skin friction \( (f_s) \) and presented charts to obtain its maximum value at different depths of pile embedment for a variety of sandy soils. The model is:

\[ f_s = \frac{w}{1/k_f + w/f_{\text{max}}} \]  

(12)

where \( k_f \) is the initial shear stiffness and can be obtained from Table 1 below and \( f_{\text{max}} \) is the maximum value of unit skin friction.

| Table 1. Values of initial shear stiffness. |
|-------------------------------------------|
| Angle of internal friction (\( \varphi \)) | \( k_f \) (kPa/m) |
|-------------------------------------------|
| 28-31                                     | 8500 - 14500     |
| 32-34                                     | 14500 - 20500    |
| 35-38                                     | 20500 - 26500    |
Fig. 1. Variation of ultimate unit skin friction with pile depth ratio. (After Mosher, 1984).

Regarding pile tip displacement model, Vijayvergiya (1977) proposed the following relation:

\[ \frac{q}{q_{\text{max}}} = \left( \frac{w}{w_{c}} \right)^{n} \]  

(13)

In which, \( q \) and \( q_{\text{max}} \) are pile tip resistance and its ultimate value, respectively, \( w \) is pile tip displacement at which ultimate resistance is reached. The value of \( w_{c} \) is ranging from 4% to 9% of the pile diameter. \( n \) is a fitting constant ranging from 2 for loose sand to 4 for dense sand. The value of \( q \) can be obtained from a chart similar to that shown in Fig. 1 and presented by Mosher (1984). The last two models will be adopted in the present computation procedure to account for soil nonlinearity and nonhomogeneity with depth.

3 THE PROPOSED COMPUTATION PROCEDURE AND VERIFICATION

Analytical solution to Eq. 6 cannot be obtained, especially when soil nonlinearity and non-homogeneity are both considered. For this reason, the following computational procedure is proposed aiming at predicting the pile load-settlement behavior and axial load distribution along the pile. The procedure will be presented in algorithmic steps as follows:

- Input the pile geometric parameters and soil and pile mechanical properties including those varying with depth.
- Discretize the pile into the desired number of segments \( n \) and locate calculation \( (n+1) \) nodes.
- For each load increment, enter the calculation with an initial guess for pile tip contribution as a ratio of the load increment.
- Iteration starts by calculating base displacement by using Eq. 13. For the current displacement vector, compute the shaft resistance by using Eq. 12 and check the equilibrium to decide the need for the next round of calculations.
- If equilibrium is not satisfied with acceptable tolerance, calculate the axial force at each pile segment by subtracting the shaft friction at each level from the relevant (top) load increment, obtain then each segment deformation, accumulate them from bottom to top and update the shaft displacement by adding pile body deformation to the current displacement vector. Obtain also the new pile tip contribution and go to the next iteration until getting convergence. This procedure is transformed into a simple Matlab M-file by which the analysis results can be presented as curves that demonstrate the distribution of pile displacement and axial load along the pile depth. It is worth mentioning that convergence is usually reached very rapidly whatever the value of initial guess is. Before going further, it is necessary to verify the reliability of the computing procedure. This is done by comparing its results with a closed form analytical solution of the differential equation (Eq. 7). As mentioned earlier, the solution of this equation is expressed as a pair of Airy functions \( (Ai \text{ and } Bi) \). Since this solution does not account for soil nonlinear behavior, soil elastic parameters such as shear modulus, \( G \) and Poisson’s ratio, \( v \) are used obtain the values of pile shaft and base stiffness.

A concrete pile 500 mm in diameter and 10 m in length is analyzed. The pile is installed in a sandy soil with shear modulus varying linearly with depth \( (G = 1000z) \). Three different values for the shear modulus of the bearing layer below the pile tip \( (G_{b}) \) are examined to investigate the effect of its stiffness on the pile base contribution. The first is equal to shear modulus at pile length \( (G_{l}) \), the second and third are 4 times and 8 times that value, respectively. The analysis results are shown in Fig. 2. The load in this figure is nondimensionalized by dividing it by \((EA)\) of the pile and the depth and displacement are divided by the pile diameter. The continuous solid lines are relevant to the analytical solution while those shown as discrete points are obtained by the current computational procedure. It is obvious that the results obtained by the procedure compare very well with those of the analytical solution. In spite of the completely different computational approach, the deviation is limited to less than 3%. This
may confirm the reliability of the proposed procedure.

Fig. 2. Comparison between axial displacement and axial load along the pile as obtained by the current computation procedure with those relevant to the analytical (exact) solution.

4 CASE STUDY

The current procedure is applied to analyze a large scale pile loading test. The test pile is one of the railway bridge piles that is relevant to the New Hindiya Barrage and related structures project. The Barrage is located at the central part of Iraq, about 85 km south of Baghdad city. The test pile is 1800 mm in diameter and about 18 m in length, cast in situ concrete pile. It was installed in dense to very dense sandy soil whereas its tip was resting on calcareous sandstone. From in situ tests (CPT and SPT), it is typically found that the soil stiffness varies almost linearly with depth. The values of angle of internal friction ($\phi$) are ranging from $36^\circ$ to $40^\circ$. The pile working load is 6670 kN and the pile was loaded to double that value during the test. During the test, the axial load was increased in four equal increments up to the pile working load then unloaded in three decrements. During the following four loading-unloading cycles, the axial load beyond the working load increased in another four equal increments, one at each cycle, up to double the working load. The pile was then unloaded through three equal load decrements.

The following soil parameters are used as input data to apply the current computational procedure:

- $k_f = 26000$ kPa,
- $q_{max} = 6000$ kPa,
- $n = 4$,
- $w_c = 4\%$ of pile diameter and the unit shaft resistance $f_s$, relevant to $\phi = 38^\circ$ is approximated as $f_s = 95.76 \times [0.2 (z/D)^{0.5}]$ kPa from Fig. 1. Since the procedure does not account for permanent soil deformation upon unloading, the loading increments phase of pile test is considered in the analysis.

Fig. 3 displays the load settlement relationship obtained by the current computational procedure as a solid line. In addition, the measured values are denoted as discrete point.

Fig. 3. Pile load- Settlement relationship.

This figure clearly demonstrates the capability of the present computational procedure to account for the nonlinear behavior of soil mechanical properties. In addition, the good correspondence between measured and calculated values of pile settlement (the deviation is limited to less than 10%) enhances the reliability of the procedure.
5 CONCLUSIONS

Analytical closed form solutions to pile displacement problems can be obtained only to limited cases such as the surrounding soil is assumed to behave as an elastic material and its stiffness is constant or linearly increasing with depth. In order to account for soil nonlinearity and non-homogeneity, a computational procedure is suggested in this paper, verified then applied to a practical case study. The procedure which is based on the load transfer mechanism is an iterative one and satisfies displacement compatibility and forces equilibrium to the required tolerance. From the verification example and case study analysis results, it can be concluded that:

- Considering the soil as a linear elastic material gives an overestimated value to the shaft resistance component so that the load transfer to the pile tip is not more than 10% of the total load even if the bearing layer stiffness is eight times that relevant to the surrounding soil.
- Incorporating nonlinear models for unit shaft and unit base resistances with suitable parameters makes it possible to evaluate the load-settlement behavior with a good precision. The differences between measured and calculated values are limited to less than 10%.
- The proposed computational method converges rapidly (within 20 iterations) to the required tolerance, whatever the value of the initial guess is.

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