A simple 5D SO(10) GUT and sparticle masses

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Abstract

Simple supersymmetric SO(10) GUT in five dimensions is proposed, in which the fifth dimension is compactified on the $S^1/(Z_2 \times Z'_2)$ orbifold with two inequivalent branes at the orbifold fixed points. In this model, all matter and Higgs multiplets reside on one brane (PS brane) where the Pati-Salam (PS) symmetry is manifest, while only the SO(10) gauge multiplet resides on the bulk. The supersymmetry breaking on the other brane (SO(10) brane) is transmitted to the PS brane through the gaugino mediation with the bulk gauge multiplet. We examine sparticle mass spectrum in this setup and show that the neutralino LSP as the dark matter candidate can be realized when the compactification scale of the fifth dimension is higher than the PS symmetry breaking scale, keeping the successful gauge coupling unification after incorporating threshold corrections of Kaluza-Klein modes of the bulk gauge multiplets.

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1 Introduction

Supersymmetric (SUSY) SO(10) Grand Unified Theory (GUT) is one of the most promising candidates beyond the Standard Model (SM). Among several models, the so-called renormalizable minimal SO(10) model has been paid a particular attention, where two kinds of Higgs multiplets \{10\oplus \overline{126}\} are utilized for the Yukawa couplings with matters \(16_i\) \((i = \text{generation})\) \[1, 2, 3\]. A remarkable feature of the model is its high predictivity. Indeed, it fixes all quark-lepton mass matrices including heavy right-handed neutrinos. So, it predicts a wide range of phenomena in low energy physics as well as new phenomena beyond SM, such as lepton-flavor violations, leptogenesis, proton decay etc. The recent reviews of the applications of minimal SO(10) to these wide range are given in \[4\]. However, after KamLAND data \[5\] was released, it entered to the precision measurements, and many authors performed new data fitting to match up these new data \[6, 7\].

On the other hand, there has been another theoretically important progress in the Higgs sector of the SO(10) model. The symmetry breaking pattern of the simple renormalizable Higgs superpotential \[8\] down to the SM was analyzed in detail \[9, 10\]. This gives the very unambiguous and detailed structures between the GUT and the Standard Model in unprecedented ways. However it accommodates conflicts on the gauge coupling unifications etc. \[7\].

In addition to the issue of the gauge coupling unification and proton decay, the minimal SO(10) model potentially suffers from the problem that the gauge coupling blows up around the GUT scale. This is because the model includes many Higgs multiplets of higher dimensional representations. In field theoretical point of view, this fact implies that the GUT scale is a cutoff scale of the model, and more fundamental description of the minimal SO(10) model would exist above the GUT scale.

Not only to solve these problems but also to give solutions to hitherto unsolved problems like the origin of SUSY breaking mediations, we have considered GUT in five dimensions (5D). As a simple realization of such a scenario, the minimal SO(10) model was considered in warped extra dimensions \[11\]. In this scenario, the Anti-de Sitter curvature and the fifth dimensional radius were chosen so as to realize the GUT scale as an effective cutoff scale in 4-dimensional effective theory. This idea has been utilized in an extended model proposed in \[12\], where the so-called type II seesaw mechanism dominates to realize the tiny neutrino masses through the warped geometry. In both models, the SO(10) gauge symmetry was considered to be broken by Higgs multiplets on branes as usual in 4-dimensional models.

Another possibility of constructing GUT models in extra-dimensions is to consider the so-called orbifold GUT \[13\], where the GUT gauge symmetry is (partly) broken by orbifold boundary conditions. A class of SO(10) models in 5D has been proposed \[14\], where SO(10) gauge symmetry is broken into the PS gauge group by orbifold boundary condition on \(S^1/(Z_2 \times Z_2')\) and further symmetry breaking into the SM gauge group is achieved by VEVs of Higgs multiplets on a brane. In this context, we have recently proposed a very simple SO(10) model \[15\]. In this model, all matter and Higgs multiplets reside only on a brane (PS brane) where the PS gauge symmetry is manifest, so that low energy effective description of this model is nothing but the PS model in 4D with a special set of matter
and Higgs multiplets. At energies higher than the compactification scale \(M_c\), the Kaluza-Klein modes of the bulk SO(10) gauge multiplet are involved in the particle contents and in fact, the gauge coupling unification was shown to be successfully realized by incorporating the Kaluza-Klein mode threshold corrections into the gauge coupling running \[15\]. More recently, it has been shown \[16\] that this orbifold GUT model is applicable to the smooth hybrid inflation \[17\]. Interestingly, the inflation model can fit the WMAP 5-year data \[18\] very well by utilizing the PS breaking scale \(v_{PS}\) and the gauge coupling unification scale predicted independently of cosmological considerations.

In any SUSY models, the origin of SUSY breaking and its mediation to the minimal SUSY SM (MSSM) sector is an important issue. Since the flavor-dependent soft SUSY breaking terms are severely constrained by the current experiments, a mechanism to transmit SUSY breaking naturally in a flavor-blind way is favorable. From this motivation, the SUSY breaking sector on the other brane is examined in Ref. \[15\]. In this setup, the bulk gauginos first obtain masses, and sfermion masses on the PS brane are automatically generated through the renormalization group equation (RGE) from the compactification scale to the electroweak scale. Importantly, the sfermion masses generated in this way are flavor-blind, because the interaction transmitting the gaugino mass to sfermion masses is the gauge interaction. This scenario is nothing but the gaugino mediation \[19\].

Unfortunately, the simple setup taken in Ref. \[15\] results in the right-handed slepton (normally, stau) being the lightest superpartner (LSP) and is disfavored in cosmological point of view. In this paper, we show that the previous conclusion is the result from the special condition, \(v_{PS} = M_c\), adopted in \[15\] and if we relax it to be \(v_{PS} < M_c\), bino-like neutralino arises as the LSP and so a usual dark matter candidate in SUSY models. We also show that even for the parameter choice, \(v_{PS} < M_c\), the gauge coupling unification is still successfully realized.

The paper is organized as follows: In the next section we briefly review the setup of our orbifold SO(10) GUT model. In Sec. 3, we investigate the gauge coupling unification for the case \(v_{PS} < M_c\). In Sec. 4, we examine the gaugino mediation in our model for \(v_{PS} < M_c\) and investigate sparticle masses, in particular, masses of the right-handed slepton (stau) and bino. We will see that bino becomes the LSP when \(M_c\) is sufficiently large. The last section is devoted for conclusion.

## 2 Model setup

Here we briefly review the orbifold SO(10) GUT model proposed in Ref. \[15\]. The model is described in 5D and the 5th dimension is compactified on the orbifold \(S^1/Z_2 \times Z'_2\) \[13\]. A circle \(S^1\) with radius \(R\) is divided by a \(Z_2\) orbifold transformation \(y \rightarrow -y\) \((y\) is the fifth dimensional coordinate \(0 \leq y < 2\pi R\)) and this segment is further divided by a \(Z'_2\) transformation \(y' \rightarrow -y'\) with \(y' = y + \pi R/2\). There are two inequivalent orbifold fixed points at \(y = 0\) and \(y = \pi R/2\). Under this orbifold compactification, a general bulk wave function is classified with respect to its parities, \(P = \pm\) and \(P' = \pm\), under \(Z_2\) and \(Z'_2\), respectively.
Assigning the parity \((P, P')\) to the bulk SO\((10)\) gauge multiplet as listed in Table 1, only the PS gauge multiplet has zero-mode and the bulk 5D N=1 SUSY SO\((10)\) gauge symmetry is broken to 4D N=1 SUSY PS gauge symmetry. Since all vector multiplets have wave functions on the brane at \(y = 0\), the SO\((10)\) gauge symmetry is respected there, while only the PS symmetry is on the brane at \(y = \pi R/2\) (PS brane).

| \((P, P')\) | bulk field | mass |
|-------------|-------------|------|
| (+, +) | \(V(15, 1, 1), V(1, 3, 1), V(1, 1, 3)\) | \(\frac{2n}{R}\) |
| (+, −) | \(V(6, 2, 2)\) | \(\frac{(2n+1)}{R}\) |
| (−, +) | \(\Phi(6, 2, 2)\) | \(\frac{(2n+1)}{R}\) |
| (−, −) | \(\Phi(15, 1, 1), \Phi(1, 3, 1), \Phi(1, 1, 3)\) | \(\frac{(2n+2)}{R}\) |

Table 1: \((P, P')\) assignment and masses \((n \geq 0)\) of fields in the bulk SO\((10)\) gauge multiplet \((V, \Phi)\) under the PS gauge group. \(V\) and \(\Phi\) are the vector multiplet and adjoint chiral multiplet in terms of 4D N=1 SUSY theory.

We place all the matter and Higgs multiplets on the PS brane, where only the PS symmetry is manifest so that the particle contents are in the representation under the PS gauge symmetry, not necessary to be in SO\((10)\) representation. For a different setup, see [14]. The matter and Higgs in our model is listed in Table 2. For later conveniences, let us introduce the following notations:

\[
H_1 = (1, 2, 2)_H, \quad H'_1 = (1, 2, 2)'_H, \\
H_6 = (6, 1, 1)_H, \quad H_{15} = (15, 1, 1)_H, \\
H_L = (4, 2, 1)_H, \quad \bar{H}_L = (\bar{4}, 2, 1)_H, \\
H_R = (4, 1, 2)_H, \quad \bar{H}_R = (\bar{4}, 1, 2)_H.
\] (1)

Superpotential relevant for fermion masses is given by\(^1\)

\[
W_Y = Y^{ij} F^i_{Li} F^c_{Rj} H_1 + \frac{Y^i_{15}}{M_5} F^i_{Li} F^c_{Rj} (H'_1 H_{15}) + \frac{Y^{ij}}{M_5} F^i_{Li} F^c_{Lj} (\bar{H}_L \bar{H}_L)
\] (2)

\(^1\) For simplicity, we have introduced only minimal terms necessary for reproducing observed fermion mass matrices.
where $M_5$ is the 5D Planck scale. The product, $H'_1 H_{15}$, effectively works as $(15, 2, 2)_H$, while $H_R H_R$ and $\bar{H}_L \bar{H}_L$ effectively work as $(10, 1, 3)$ and $(\bar{10}, 3, 1)$, respectively, and are responsible for the left- and the right-handed Majorana neutrino masses. Providing VEVs for appropriate Higgs multiplets, fermion mass matrices are obtained. There are a sufficient number of free parameters to fit all the observed fermion masses and mixing angles.

Table 2: Particle contents on the PS brane at $y = \pi R/2$

| Matter Multiplets | $\psi_i = F_{Li} \oplus F_{Ri}^c$ (i = 1, 2, 3) |
|--------------------|-----------------------------------------------|
| Higgs Multiplets   | $(1, 2, 2)_H, (1, 2, 2)_H', (15, 1, 1)_H, (6, 1, 1)_H$
|                    | $(4, 1, 2)_H, (\bar{4}, 1, 2)_H, (4, 2, 1)_H, (\bar{4}, 2, 1)_H$ |

Table 2: Particle contents on the PS brane. $F_{Li}$ and $F_{Ri}^c$ are matter multiplets of i-th generation in $(4, 2, 1)$ and $(\bar{4}, 1, 2)$ representations, respectively.

We introduce Higgs superpotential invariant under the PS symmetry such as $[15]$

$$W = \frac{m_1}{2} H_1^2 + \frac{m'_1}{2} H'_1^2 + m_{15} \text{tr} [H_{15}^2] + m_4 (\bar{H}_L H_L + \bar{H}_R H_R)$$

$$+ (H_L H_R + \bar{H}_L \bar{H}_R) (\lambda_1 H_1 + \lambda'_1 H'_1) + \lambda_{15} (H_R H_R + H_L H_L) H_{15}$$

$$+ \lambda \text{tr} [H_{15}^2] + \lambda_6 (H_L^2 + \bar{H}_L^2 + H_R^2 + \bar{H}_R^2) H_6. \quad (3)$$

Parameterizing $\langle H_{15} \rangle = \frac{v_{15}}{\sqrt{2}} \text{diag}(1, 1, 1, -3)$, SUSY vacuum conditions from Eq. (3) and the D-terms are satisfied by solutions,

$$v_{15} = \frac{2\sqrt{6}}{3\lambda_{15}} m_4, \quad \langle H_R \rangle = \langle \bar{H}_R \rangle = \sqrt{\frac{8m_4}{3\lambda_{15}^2} \left( m_{15} - \frac{\lambda}{\lambda_{15}} m_4 \right)} \equiv v_{PS} \quad (4)$$

and others are zero, by which the PS gauge symmetry is broken down to the SM gauge symmetry. We choose the parameters so as to be $v_{15} \simeq \langle H_R \rangle = \langle \bar{H}_R \rangle$. Note that the last term in Eq. (3) is necessary to make all color triplets in $H_R$ and $\bar{H}_R$ heavy.

The Higgs doublet mass matrix is given by

$$\begin{pmatrix} H_1 & H'_1 & H_L \end{pmatrix} \begin{pmatrix} m_1 & 0 & \lambda_1 \langle H_R \rangle \\ 0 & m'_1 & \lambda'_1 \langle \bar{H}_R \rangle \\ \lambda_1 \langle \bar{H}_R \rangle & \lambda'_1 \langle \bar{H}_R \rangle & m_4 \end{pmatrix} \begin{pmatrix} H_1 \\ H'_1 \\ H_L \end{pmatrix} \ . \quad (5)$$

Requiring the tuning of parameters to satisfy

$$\det M = m_1 m'_1 m_4 - (m_1 \lambda'^2_1 + m'_1 \lambda^2_1) v_{PS}^2 = 0, \quad (6)$$

It is possible to consider a different superpotential by introducing a singlet chiral superfield $[16]$, so that this model is applicable to the smooth hybrid inflation scenario $[17]$. 

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only one pair of Higgs doublets out of the above three pairs becomes light and is identified as the MSSM Higgs doublets while the others have mass of the PS symmetry breaking scale.

In Ref. [15], assuming \( M_c = v_{PS} \) and imposing the left-right symmetry, the gauge coupling unification was examined. We relax this assumption and consider the case \( v_{PS} < M_c \) in the next section. However, note that \( v_{PS} = 1.19 \times 10^{16} \) GeV is fixed as the same as in Ref. [15] since the PS scale is determined as the scale where the SU(2)_L and SU(2)_R gauge couplings coincide with each other. The PS scale in our model is very high relative to other 5D orbifold SO(10) models [14]. The high value of \( v_{PS} \) is advantageous for dangerous proton decay due to dimension six operators. From Eq. (2), the right-handed neutrino mass scale is given by \( M_R \sim Y_R v_{PS}^2 / M_5 \). For \( M_5 \sim 10^{17} \) GeV (which can be estimated from the parameters obtained in the next section), the scale \( M_R = \mathcal{O}(10^{14} \) GeV) preferable for the seesaw mechanism can be obtained by a mild tuning of the Yukawa coupling \( Y_R \sim 0.1 \).

3 Gauge coupling unification

In the orbifold GUT model, we assume that a more fundamental extra-dimensional GUT theory takes place at some high energy beyond the compactification scale. For the theoretical consistency of the model, the gauge coupling unification should be realized at some scale after taking into account the contributions of Kaluza-Klein modes of the bulk gauge multiplet to the gauge coupling running [20].

In our setup, we take \( v_{PS} < M_c \) and the evolution of gauge couplings has three stages, \( G_{321} \) (SM+MSSM), \( G_{422} \) (the PS stage) and the PS stage with the Kaluza-Klein mode contributions. Since we have imposed the left-right symmetry, the SU(2)_L and SU(2)_R gauge couplings must coincide with each other at the scale \( \mu = v_{PS} \). As a consequence, the PS scale is fixed from the gauge coupling running in the MSSM stage.

In the \( G_{321} \) stage, we have

\[
\frac{1}{\alpha_i(\mu)} = \frac{1}{\alpha_i(M)} + \frac{b_i}{2\pi} \ln \left( \frac{M}{\mu} \right); \quad (i = 3, 2, 1),
\]

where \( M = M_Z \), and \( b_i \)s are

\[
b_3 = -7, \quad b_2 = -19/6, \quad b_1 = 41/10
\]

for \( M_Z \leq \mu \leq M_{SUSY} \). For \( M_{SUSY} \leq \mu \leq v_{PS} \), \( M = M_{SUSY} \) and

\[
b_3 = -3, \quad b_2 = 1, \quad b_1 = 33/5.
\]

At the PS scale, the matching condition holds

\[
\begin{align*}
\alpha_3^{-1}(v_{PS}) &= \alpha_4^{-1}(v_{PS}) \\
\alpha_2^{-1}(v_{PS}) &= \alpha_{2L}^{-1}(v_{PS}) \\
\alpha_1^{-1}(v_{PS}) &= \frac{2\alpha_4^{-1}(v_{PS}) + 3\alpha_{2R}^{-1}(v_{PS})}{5}
\end{align*}
\]

(10)
For $\mu \geq M_c$ in the PS stage, the threshold corrections $\Delta_i$ due to Kaluza-Klein modes in the bulk gauge multiplet are added,

$$\frac{1}{\alpha_i(\mu)} = \frac{1}{\alpha_i(v_{PS})} + \frac{b_i}{2\pi} \ln \left( \frac{v_{PS}}{\mu} \right) + \Delta_i. \quad (i = 4, 2_L, 2_R)$$

The beta functions from the matter and Higgs multiplets on the PS brane are

$$b_4 = 3, \; b_{2L} = b_{2R} = 6.$$  

The Kaluza-Klein mode contributions are given by

$$\Delta_i = \frac{1}{2\pi} b_i^{\text{even}} \sum_{n=0}^{N_i} \theta(\mu - (2n + 2)M_c) \ln \left( \frac{(2n + 2)M_c}{\mu} \right)$$

$$+ \frac{1}{2\pi} b_i^{\text{odd}} \sum_{n=0}^{N_i} \theta(\mu - (2n + 1)M_c) \ln \left( \frac{(2n + 1)M_c}{\mu} \right)$$

with

$$b_i^{\text{even}} = (-8, -4, -4),$$

$$b_i^{\text{odd}} = (-8, -12, -12)$$

under $G_{422}$.

![Figure 1: Gauge coupling unification in the left-right symmetric case. Each line from top to bottom corresponds to $g_3$, $g_2$ and $g_1$ for $\mu < v_{PS}$, while $g_3 = g_4$ and $g_2 = g_{2R}$ for $\mu > v_{PS}$. Here, we have taken $M_c = 2.47 \times v_{PS}$.](image)

Fig. 1 shows the gauge coupling evolutions for the left-right symmetric case. The PS symmetry breaking scale, $v_{PS}$, is determined from the gauge coupling running in the MSSM stage.
by imposing the matching condition, \( \alpha_2^{-1}(v_{PS}) = \alpha_{2R}^{-1}(v_{PS}) = (5\alpha_1^{-1}(v_{PS}) - 2\alpha_3^{-1}(v_{PS}))/3 \), and we find

\[
v_{PS} = 1.19 \times 10^{16} \text{ GeV},
\]

for the inputs, \((\alpha_1(M_Z), \alpha_2(M_Z), \alpha_3(M_Z)) = (0.01695, 0.03382, 0.1176)\) and \(M_{SUSY} = 1 \text{ TeV}\). For the scale \(\mu \geq v_{PS}\), there are only two independent gauge couplings \(\alpha_4\) and \(\alpha_2 = \alpha_{2R}\), so that the gauge coupling unification is easily realized with a suitable \(M_c\). In this figure, we have taken (corresponding to the result in the next section)

\[
M_c = 2.47 \times v_{PS} = 2.95 \times 10^{16} \text{ GeV}
\]

and after including Kaluza-Klein threshold contributions into the gauge coupling evolutions, the gauge coupling unification is realized at

\[
M_{GUT} = 7.54 \times 10^{16} \text{ GeV}.
\]

As \(M_c\) is raised, \(M_{GUT}\) becomes smaller. As mentioned before, we assume that a more fundamental SO(10) GUT theory takes place at \(M_{GUT}\).

4 Gaugino mediation and sparticle masses

The origin of SUSY breaking and its mediation to the MSSM sector is still a prime question in any phenomenological SUSY models. Since flavor-dependent soft SUSY breaking masses are severely constrained by the current experiments, a mechanism which naturally transmits SUSY breaking in a flavor-blind way is the most favorable one.

In higher dimensional models, the sequestering [21] is the easiest way to suppress flavor-dependent SUSY breaking effects to the MSSM matter sector. Since all matters reside on the PS brane in our model, the sequestering scenario is automatically realized when we simply assume a SUSY breaking sector on the brane at \(y = 0\). The SO(10) gauge multiplet is in the bulk and can directly communicate with the SUSY breaking sector through the higher dimensional operator of the form,

\[
\mathcal{L} \sim \delta(y) \int d^2 \theta \frac{X}{M_5^2} \text{tr} [W^\alpha W_\alpha],
\]

where \(X\) is a singlet chiral superfield which breaks SUSY by its F-component VEV, \(X = \theta^2 F_X\). Therefore, the bulk gaugino obtains the SUSY breaking soft mass,

\[
M_\lambda \sim \frac{F_X M_c}{M_5^2} \simeq \frac{F_X}{M_P} \left( \frac{M_5}{M_P} \right),
\]

where \(M_c\) comes from the wave function normalization of the bulk gaugino, and we have used the relation between the 4D and 5D Planck scales, \(M_5^3/M_c \simeq M_P^2 \) \((M_P = 2.4 \times 10^{18} \text{ GeV})\) in the last equality. As usual, we take \(M_\lambda = 100 \text{ GeV}-1 \text{ TeV}\). Once the gaugino obtains
non-zero mass, SUSY breaking terms for sfermions are automatically generated through the RGE from the compactification scale to the electroweak scale. This scenario is nothing but the gaugino mediation \cite{19} and flavor-blind sfermion masses are generated through the gauge interactions. In this setup, a typical gaugino mass in Eq. (19) is smaller than the gravitino mass $m_{3/2} \simeq F_X/M_P$ by a factor $M_5/M_P < 1$.

As discussed in Ref. \cite{15}, for $M_c = v_{PS}$, we find that the right-handed slepton (normally, stau) is the LSP, because the sfermion mass spectrum is obtained from the boundary condition with vanishing soft masses at $M_c = v_{PS} = 1.19 \times 10^{16}$ GeV. This result is problematic for cosmology. As pointed out in Ref. \cite{22}, this stau LSP problem is cured by the soft mass RGE running from the compactification scale to the GUT scale in a GUT model. In the following, we apply this idea to our model with $M_c > v_{PS}$.

For the scale, $v_{PS} \leq \mu \leq M_c$, we are in the PS stage and the RGEs of gaugino and sfermion masses are given by

\[
\frac{d}{dt} \left( \frac{M_4}{\alpha_4} \right) = \frac{d}{dt} \left( \frac{M_{2L}}{\alpha_{2L}} \right) = \frac{d}{dt} \left( \frac{M_{2R}}{\alpha_{2R}} \right) = 0,
\]

\[
\frac{dm_{F_c}^2}{dt} = -\frac{15}{4\pi} \alpha_4 M_4^2 - \frac{3}{2\pi} \alpha_{2L} M_{2L}^2,
\]

\[
\frac{dm_{F_c}^2}{dt} = -\frac{15}{4\pi} \alpha_4 M_4^2 - \frac{3}{2\pi} \alpha_{2R} M_{2R}^2,
\]  

where $t = \ln(\mu/M_c)$, $\alpha_4$ and $\alpha_{2L} = \alpha_{2R}$ are the PS gauge coupling of the corresponding gauge groups (whose RGE solutions are obtained in the previous section), and $M_4$, $M_{2L}$, and $M_{2R}$ are the corresponding gaugino masses. Sfermion mass spectrum is obtained by solving the RGEs with the boundary conditions, $m_{F_c}(M_c) = m_{F_c}(M_c) = 0$. Analytic solutions of Eq. (20) at $\mu = v_{PS}$ are easily found:

\[
m_{F_c}(v_{PS}) = \frac{5}{4} M_4^2(v_{PS}) \left[ \left( \frac{\alpha_4(M_c)}{\alpha_4(v_{PS})} \right)^2 - 1 \right] + \frac{1}{4} M_{2L}^2(v_{PS}) \left[ \left( \frac{\alpha_{2L}(M_c)}{\alpha_{2L}(v_{PS})} \right)^2 - 1 \right],
\]

\[
m_{F_c}(v_{PS}) = \frac{5}{4} M_4^2(M_c) \left[ \left( \frac{\alpha_4(v_{PS})}{\alpha_4(v_{PS})} \right)^2 - 1 \right] + \frac{1}{4} M_{2R}^2(v_{PS}) \left[ \left( \frac{\alpha_{2R}(M_c)}{\alpha_{2R}(v_{PS})} \right)^2 - 1 \right].
\]  

Note that the PS model is unified into a more fundamental SO(10) model and this unification leads to the well-known relation,

\[
\frac{M_4}{\alpha_4} = \frac{M_{2L}}{\alpha_{2L}} = \frac{M_{2R}}{\alpha_{2R}} = \frac{M_{1/2}}{\alpha_{GUT}},
\]  

where $M_{1/2}$ is the universal gaugino mass at the unification scale. Thus, the formulas for sfermion masses are simplified as

\[
m_{F_c}^2(v_{PS}) = m_{F_c}^2(v_{PS}) = \left( \frac{M_{1/2}}{\alpha_{GUT}} \right)^2 \left[ \frac{5}{4} (\alpha_4^2(M_c) - \alpha_4^2(v_{PS})) + \frac{1}{4} (\alpha_{2L}^2(M_c) - \alpha_{2L}^2(v_{PS})) \right].
\]
Solving the RGEs in the MSSM with the universal boundary condition $m_{\tilde{F}}^2(v_{PS}) = m_{\tilde{F}}^2(v_{PS}) = m_0^2$ at $\mu = v_{PS}$, we obtain the sfermion masses at the electroweak scale. In our model, the PS scale is almost the same as the usual SUSY GUT scale in the MSSM ($M_{GUT} \simeq 2 \times 10^{16}$ GeV) and the gauge couplings are roughly unified at the PS scale, $\alpha_4(v_{PS}) \simeq \alpha_2(v_{PS}) = \alpha_{2R}$ (see Fig. 1). Therefore, our study on the sfermion masses are almost the same as the one usual in the constrained MSSM. For a small $\tan \beta$ (say, $\tan \beta = 10$), we neglect Yukawa coupling contributions to the soft masses of right-handed sleptons, and the analytic solutions of the MSSM RGEs are found to be

$$M_1(\mu) = \alpha_1(\mu) \left( \frac{M_{1/2}}{\alpha_{GUT}} \right),$$

$$m_{\tilde{e}_c}^2(\mu) = \left( \frac{M_{1/2}}{\alpha_{GUT}} \right)^2 \frac{2}{\Pi} \left[ \alpha_1^2(v_{PS}) - \alpha_1^2(\mu) \right] + m_0^2. \quad (24)$$

If $m_0$ is large enough, the slepton (stau) mass is bigger than the bino mass ($M_1$). In our model, $m_0$ is given as a function of $M_\epsilon$. Fig. 2 shows the ratio,

$$R \equiv \frac{m_{\tilde{e}_c}^2}{M_1^2}, \quad (25)$$

as a function of $M_\epsilon/v_{PS}$. We can obtain $R \geq 1$ for $M_\epsilon/v_{PS} \geq 2.47$.

Now, for $M_\epsilon/v_{PS} \geq 2.47$, the bino-like neutralino will be the LSP and a good candidate for the cold dark matter in cosmology [23]. For a small $\tan \beta$, the annihilation processes of the bino-like neutralino are dominated by p-wave and are not so efficient. As a result, the neutralino relic density tends to exceed the upper bound of the observed dark matter density. This problem can be avoided if the neutralino is quasi-degenerate with the next LSP slepton.
(stau), and the co-annihilation process with the next LSP can lead to the right dark matter relic density. In our result, such a situation appears for $M_c \simeq 2.47 \times v_{PS} \simeq 2.95 \times 10^{16}$ GeV.

It would be interesting to note that the discrepancy of the abundance of $^7$Li between the observed values in WMAP and in metal poor halo stars may be explained the degeneracy between the LSP neutralino and stau [21].

5 Conclusion

We have considered SO(10) GUT in 5D, where the fifth dimension is compactified on the $S^1/(Z_2 \times Z_2')$ orbifold with two inequivalent branes at the orbifold fixed points. All the matter and Higgs multiplets reside on the PS brane, while the SUSY breaking sector is on the other brane. In this setup, we have two independent energy scales; $v_{PS}$ at which the PS symmetry is broken on the PS brane and $M_c$, the inverse of radius of the fifth dimension. Requiring the left-right symmetry in the model, the PS symmetry breaking scale is determined from the MSSM gauge coupling runnings. For the case with $M_c > v_{PS}$, we have investigated the gauge coupling unification and the sparticle mass spectrum through the gaugino mediation. We have found that after incorporating threshold corrections of Kaluza-Klein modes of the bulk gauge multiplets, the gauge couplings can be successfully unified at $M_{GUT}$, whose scale is determined once $M_c$ is fixed. Also, we have found that an appropriate choice of $M_c$ leads to the bino-like LSP neutralino quasi-degenerating with the next LSP slepton (stau), so that the co-annihilation between the LSP and next LSP provides the right relic abundance of the neutralino dark matter.

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