Comment on ”Models of Intermediate Spectral Statistics”

T. Gorin\textsuperscript{1}, M. Müller\textsuperscript{2} and P. Seba\textsuperscript{3,4}

\textsuperscript{1} Centro de Ciencias Físicas, UNAM, Campus Morelos, Cuernavaca, México, 62251, A.P. 48-3
\textsuperscript{2} Facultad de Ciencias, UAEM, C.P. 62210, Cuernavaca, Morelos, México
\textsuperscript{3} Institute of Physics, Czech Academy of Science, Cukrovarnicka 10, Prague, Czech Republic
\textsuperscript{4} Pedagogical University, Department of Physics, Hradec Kralove, Czech Republic

PACS: 03.65, 05.45

Abstract

In \cite{1} it was proposed, that the nearest-neighbor distribution $P(s)$ of the spectrum of the Bohr-Mottelson model is similar to the semi-Poisson distribution. We show however, that $P(s)$ of this model differs considerably in many aspects from semi-Poisson. In addition we give an asymptotic formula for $P(s)$ as $s \to 0$, which gives $P'(0) = \pi\sqrt{3}/2$ for the slope at $s = 0$. This is different not only from the GOE case but also from the semi-Poisson prediction that leads to $P'(0) = 4$.

The motivation for this comment stems from the impression, that the article “models of intermediate spectral statistics” can easily be misinterpreted in two ways: One might be led to believe that (i) the semi-Poisson distribution is universal, and (ii) the universality class of “intermediate statistics” is as well defined and established as for example the Poisson ensemble or the GOE. In this comment, we will argue, that both statements are wrong.

The purpose of \cite{1} is to present models which could constitute a “third” universality class of systems which show so called “intermediate statistics”, previously introduced by Shklovskii in \cite{5}. The Poissonian and the Gaussian ensemble (for definiteness, consider orthogonal ensembles only) are considered as the first two universality classes in this list.

As in the Poissonian and in the GOE case, where the respective members have common and unique statistical properties, one would expect the same to hold for the models with intermediate statistics. In \cite{1} the authors concentrate on the distribution of nearest neighbor spacings. In the Poissonian case it is given by $P(s) = \exp(-s)$, in the GOE case it is close to the well known Wigner surmise $P(s) \approx (\pi/2)\exp(-\pi/4s^2)$, whereas in the case of the “intermediate statistics” the candidate proposed in \cite{1} is the semi-Poisson distribution $P(s) = 4s\exp(-2s)$.

In what follows we will show, that the level spacing distribution in the case of the Bohr-Mottelson model, that represents one of the candidate systems for the intermediate statistics mentioned in \cite{1} is in fact very different from the proposed semi-Poisson distribution. This discrepancy can be found in fact already on a figure published in \cite{1} but without discussing the problem. However, \cite{4} gives an overview over the statistical properties of different variants of the Bohr-Mottelson model.
In [1] the following matrix model (originally introduced by Bohr and Mottelson) is presented as a possible candidate showing statistical properties similar to the semi-Poisson distribution:

$$H_{mn} = e_n \delta_{mn} + t_m t_n .$$  \hspace{1cm} (1)

$H$ is a $N \times N$-matrix, $e_n$ are mutually independent random variables uniformly distributed over a finite interval, and the $t_n$ are chosen with equal absolute value squared $t_n^2 = r$. The authors of [1] sketch a procedure for calculating analytically the 2-point correlation function. For small distances it should agree with $P(s)$, so that one can derive the slope $P'(0) = \pi \sqrt{3}/2$ of the spacing distribution at $s = 0$. This slope is different from the GOE case, where it equals to $\pi^2/6$ as well as from the slope of the semi Poisson distribution that equals to 4. The difference to the GOE case is remarked, but a similar remark on the difference to the semi-Poisson is avoided.

An unprejudiced reader might believe, that the correlation properties of the matrix model are similar to semi-Poisson, even though that the spectral statistics of this model differ remarkably.

In Figure 1 we present the numerical result for $P(s)$ obtained for an ensemble of 1000 matrices of dimension 750. For the statistical analysis we used only one third of the states in the center of the spectral region. The numbers $e_n$ are uniformly distributed over an interval $[-1, 1]$ and the elements of the vector $\vec{t}$ are chosen as $t_i = \sqrt{\alpha/(\pi \rho N)}$, where $\rho$ is the level density in the center of the spectrum, $N$ is the dimension of the matrix and $\alpha = 10$ is the coupling constant. (We checked, that larger coupling does not change the numerical results). Figure 1.a demonstrates the qualitative differences in the behavior of $P(s)$ between the random matrix model and the semi-Poisson distribution. The slope at $s = 0$ is smaller, the maximum of $P(s)$ is slightly shifted to the right, and for values $s > 1$ it shows significant deviations well above of the statistical error. Hence the spacing distribution of the Bohr-Mottelson model is not close to semi-Poisson.

Figure 1.b. shows a magnification of the interval $0 \leq s \leq 1/2$ using the same data as in figure 1.a. In this figure we additionally plotted the asymptotic result \([1]\) for the present model as a dashed line. The basic idea of the approximation is the following: In order to get a short distance between two neighbored levels in the spectrum of $H$, three eigenvalues of $H_0$ have to come close together. Then the levels which are farther away can be neglected. Therefore we can
restrict the sum

\[ K(E) = \sum_{i=1}^{N} \frac{t_i^2}{E - e_i} \]  \hspace{1cm} (2)

whose roots define the eigenvalues of \( H \), to those terms with the three consecutive eigenvalues. Resolving for the two roots, calculating their distance, and averaging over the levels \( e_i \) leads to the following formula

\[ P(s) = \frac{9s}{4} \frac{\pi/2}{\int_0^{\pi/2} d\phi \exp \left[ -\frac{3s}{2} \left( \cos \phi + \sin \phi \right)/ \sqrt{1 + \sin(2\phi)/2} \right]}. \]  \hspace{1cm} (3)

The dashed curve in Figure 1.b. is obtained from a numerical integration of (3). At short distances, this approximation describes the numerical data much better than the semi-Poisson. A Taylor expansion of the integrand of (3) gives \( P'(0) = \pi \sqrt{3}/2 \) for the slope at \( s = 0 \), the same result as found in [1].

A detailed numerical investigation of several statistical properties of the type of models can be found also in [2, 3, 4].

To conclude, if one wants to insist on the introduction of a “third” universal ensemble, one should possibly use a criterium similar to the following (cited from [1]): “…the main features are (i) the existence of level repulsion (as in random matrix ensembles), and (ii) slow (approximately exponential) fall-off …”. Even though this definition is quite “spongy”, it seems to be the only way to make sure, that the systems discussed fit into this class.

ACKNOWLEDGEMENTS M. Müller acknowledges financial support from the CONNACyT (No.32101-E).

References

[1] E.B. Bogomolny, U. Gerland and C. Schmit, Phys. Rev. E 59 (1999) R1315
[2] F.-M. Dittes, I. Rotter and T.H. Seligman, Phys. Lett A 158 (1991) 14
[3] T. Gorin, F.-M. Dittes, M. Müller, I. Rotter and T.H. Seligman, Phys. Rev. E 56 (1997) 2481
[4] M. Pascaud and G. Montambaux, Ann. Phys. 7 (1998) 406
[5] B. I. Shklovskii, Phys. Rev. B 47 (1993) 11487