ROSAT X-ray sources and exponential field decay in isolated neutron stars

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Abstract. In this paper we semi-analytically evaluate influence of the exponential decay of magnetic field on the fate of isolated neutron stars. The fact of ROSAT observations of several X-ray sources, which can be accreting old isolated neutron stars gives us an opportunity to put some limits on the parameters of the exponential decay.

We argue, that, if most part of neutron stars have approximately the same decay and initial parameters, then the combinations of the bottom magnetic momentum, \( \mu_b \), in the range \( \sim 10^{28} - 10^{29.5} \text{ G cm}^3 \) and characteristic time scale, \( t_d \), in the range \( \sim 10^7 - 10^8 \text{ yrs} \) for standard initial magnetic momentum, \( \mu_0 = 10^{30} \text{ G cm}^3 \), can be excluded, because for that sets of parameters neutron stars never come to the stage when accretion of the interstellar medium on their surfaces is possible even for low velocity of neutron stars and relatively high density of the interstellar medium. The region of excluded parameters increases with \( \mu_0 \) decreasing.

Key words: neutron stars – magnetic fields – stars: magnetic field – X-rays: stars – accretion

1. Introduction

Evolution of neutron stars (NSs) can be called “magnetorotational”, because all main astrophysical manifestations of these objects are determined by their periods and magnetic fields.

Four main regimes, as described for example in Lipunov (1992), are possible for isolated NSs: ejector, when a star represents a radio pulsar, or a dead pulsar, spinning down due to magneto–dipole radiation; propeller, when surrounding captured matter cannot penetrate through the centrifugal barrier; accretor, when matter can reach the surface, and the NSs appear as an X-ray source; and georotator, when gravitation becomes insignificant, because magnetic pressure dominates everywhere over the gravitational pull, and geo-like magnetosphere is formed.

Magnetic field decay in NSs is an uncertain subject. Many models were suggested (see, for example, Ding et al., 1993; Jahan Miri & Bhattacharya, 1994). The only strong observational result is that for the exponential (or nearly exponential) decay characteristic time scale, \( t_d \), is longer than \( \sim 10^7 \text{ yrs} \), because no expected effects of decay are observed in radio pulsars (Lyne et al. 1998).

Field decay was used in the case of old accreting isolated NSs by Konenkov & Popov (1997) and Wang (1997) to explain properties of the source RX J0720-3125. And it seems, that if this source really is an accreting old NS and if it was born as a normal radio pulsar (with small period, \(< 1 \text{ s} \) and high magnetic field, \( \sim 10^{12} \text{ G} \)), its properties can be explained only if decay is working (or if this source was born with such unusual parameters). Recently the influence of the field decay in isolated NSs was investigated in Colpi et al. (1998) and Livio et al. (1998). An attempt to include field decay into population synthesis of isolated NSs was made in Popov et al. (1999).

Here we try to put some limits on the parameters of the exponential field decay, assuming, that old isolated NSs are observed as accreting X-ray sources (Walter, Wolk & Neihauser 1996; Haberl et al. 1996, 1998; Neihauser & Trümper 1999; Schwope et al. 1999), neglecting the possibility, that all of them can be highly magnetized NSs, “magnetars” (see a brief discussion on this assumption in Popov et al., 1999).

2. Calculations and results

The main idea of our work is to calculate the ejector time, \( t_E \), i.e. a time interval spent by a NS on the ejector stage, for different parameters of the field decay and standard assumptions on the initial parameters of a NS, and to compare that time with the Hubble time, \( t_H \).

For constant field \( t_E \) monotonically depends upon NS’s velocity and ISM density:

\[
t_E(\mu = \text{const}) \sim 10^9 \mu_{30}^{-1} n^{-1/2} v_{10} \text{ yrs.}
\]  

If \( t_E \) for decaying field for some sets of parameters is greater than \( t_H \sim 10^{10} \text{ yrs} \) even for relatively high concentration of interstellar medium (ISM), \( \sim 1 \text{ cm}^{-3} \), and

\[
t_E(\mu = \text{const}) \sim 10^9 \mu_{30}^{-1} n^{-1/2} v_{10} \text{ yrs.}
\]
For decaying magnetic field the situation can be different, 

\[ \mu = \mu_0 \cdot e^{-t/t_d}, \mu > \mu_b \]

(2)

where \( \mu_0 \) is the initial magnetic momentum, \( \mu = \frac{1}{2}B_p R_{NS}^2 \), here \( B_p \) - polar magnetic field, and \( R_{NS} \) - NS radius, \( t_d \) - characteristic time scale of the decay, and \( \mu_b \) - bottom magnetic momentum, which is reached in:

\[ t_{cr} = t_d \cdot \ln \left( \frac{\mu_0}{\mu_b} \right). \]

(3)

After that moment magnetic field is assumed to be constant.

In Fig. 1 we show, as an illustration, evolutionary tracks of NSs on \( P - B \)-diagram for \( v = 10 \text{ km s}^{-1} \) and \( n = 1 \text{ cm}^{-3} \). Tracks start at \( t = 0 \), when \( p = p_0 = 0.020 \text{ s} \) and \( \mu = \mu_0 = 10^{30} \text{ G cm}^3 \), and end at \( t = t_H = 10^{10} \text{ yrs} \) (for \( t_d = 10^{12} \text{ yrs} \), \( t_d = 10^9 \text{ yrs} \) and for constant magnetic field) or at the moment, when \( p = p_E (t_d = 10^8 \text{ yrs} \) and \( t_d = 10^7 \text{ yrs} \)). The line with diamonds shows \( p = p_E (B) \).

As far as the accretion rate from the ISM is small (even for our parameters), less than \( \sim 10^{12} \text{ g s}^{-1} \), no influence of accretion onto decay was taken into account (see Urpin et al., 1996).

The ejector stage lasts until the critical ejector period, \( p_E \), is reached:

\[ p_E = 11.5 \mu_0^{1/2} n^{-1/4} v_{10}^{3/2} \text{ s}, \]

(4)

where \( v_{10} = \sqrt{v_p^2 + v_s^2}/10 \text{ km s}^{-1} \). \( v_p \) - spatial velocity of a NS. Here the sound velocity, \( v_s \), was taken into account, but as far as normally NSs spatial velocities are higher than \( 10 \text{ km s}^{-1} \) and the sound velocity outside hot low density ISM regions is lower than \( 10 \text{ km s}^{-1} \), we have \( \sqrt{v_p^2 + v_s^2} \approx v_p \). \( n \) is a concentration of the ISM.

Initial period should be taken to be much smaller than \( p_E \). We used \( p_0 = 0 \text{ s} \). Variations of \( p_0 \), if it stays much
less than \( p_E \), have little influence on our results, i.e. in that case \( t_E \) is determined only by \( p_E \) and history of the decay. We calculated spin-down according to magneto–dipole formula (but other regimes are possible, see Beskin et al. 1993 for a review):

\[
\frac{dp}{dt} = \frac{2}{3} \frac{4\pi^2 \mu^2}{\mu_0 V M^3},
\]

where \( \mu \) can be a function of time.

For our estimates we assumed constant velocity of NSs, \( v \), equal to 10 km s\(^{-1} \) and constant ISM concentration, \( n \), equal to 1 cm\(^{-3} \). These conditions give us a lower limit on \( t_E \), because normally velocity is significantly higher (fraction of slow velocity NSs is less than few percents, see Popov et al., 1999), and ISM density is smaller than the specified values (partly because high velocity NSs spend most of their lives in low density regions far from the Galactic plane).

After simple algebra one can obtain a formula for \( t_E \), depending upon \( t_d \), \( \mu_0 \), \( v \), \( n \) and \( \mu_b \):

\[
t_E = \begin{cases} 
-t_d \cdot \ln \left[ \frac{A}{t_d} \left( 1 + \frac{t_E^2}{A^2} - 1 \right) \right], & t_E < t_{cr} \\
\frac{t_{cr}}{A \mu_0 - t_d \frac{1}{2} \left( \frac{\mu_0}{\mu_b} \right)^2 \left( 1 - e^{-2t_{cr}/t_d} \right)}, & t_E > t_{cr}
\end{cases}
\]

where coefficient \( A \) is determined by the formula:

\[
A = \frac{3Ic}{2 \mu_0 \sqrt{2vM}} \approx 10^{17} I_{45} \mu_0^{-1} v_{10}^{-1/2} M_{11}^{-1/2} \text{ s},
\]

where \( M \) can be formally determined as a combination of intrinsic NS’s parameters, its velocity and ISM concentration using the Bondi formula even if a NS is not on the accretor stage:

\[
\dot{M} \approx 10^{11} \frac{n v_{10}^{-3}}{\text{g s}^{-1}}.
\]

Results of \( t_E \) calculations for several values of \( \mu_0 \) and \( t_d \) are shown in Fig. 2. Horizontal regions in the left parts of all curves appear because there a NS reaches the critical period \( p = p_E \) before it reaches bottom magnetic momentum at \( t = t_{cr} \), and \( t_E \) is not depended upon the rest field decay history. On the right side from the maximum on the curves a NS reaches \( \mu = \mu_b \) with period significantly less than \( p_E(\mu_b) \), and it takes a long time after \( t = t_{cr} \) to reach \( p = p_E \) (this time is longer for lower \( \mu_b \)). On the left side from the maximum, but before the horizontal region, a NS reaches \( \mu = \mu_b \) with a period close to \( p_E \), and it is closer if \( \mu_b \) is lower, so very quickly after \( t = t_{cr} \) a NS can spin down to \( p_E \). The first point at the right corresponds to the bottom momentum equal to initial momentum, i.e. to the case without decay.

One can see from that figure, that for some combination of parameters \( t_E \) is longer than the Hubble time.

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**Fig. 3.** Characteristic time scale of the magnetic field decay, \( t_d \), vs. bottom magnetic momentum, \( \mu_b \). In the filled region \( t_E \) is greater than \( 10^{10} \text{ yrs} \). Dashed line corresponds to \( t_H = t_d \cdot \ln(\mu_0/\mu_b) \), where \( t_H = 10^{10} \text{ years} \). Solid line corresponds to \( p_E(\mu_b) = p(t = t_{cr}) \), where \( t_H(t) = t_d \cdot \ln(\mu_0/\mu_b) \). Both lines and filled region are plotted for \( \mu_0 = 10^{30} \text{ G cm}^{-3} \). Dot-dashed line is the same as the dashed one, but for \( \mu_0 = 5 \cdot 10^{29} \text{ G cm}^{-3} \). Dotted line is a border of the “forbidden” region for \( \mu_0 = 5 \cdot 10^{29} \text{ G cm}^{-3} \).

**Fig. 4.** Characteristic time scale of the magnetic field decay, \( t_d \), vs. bottom magnetic momentum, \( \mu_b \). In the filled region \( t_E \) is greater than \( 10^{10} \text{ yrs} \). Dashed line corresponds to \( t_H = t_d \cdot \ln(\mu_0/\mu_b) \), where \( t_H = 10^{10} \text{ yrs} \). Solid line corresponds to \( p_E(\mu_b) = p(t = t_{cr}) \), where \( t_{cr} = t_d \cdot \ln(\mu_0/\mu_b) \). Both lines and region are plotted for \( \mu_0 = 10^{29} \text{ G cm}^{-3} \).
It means, that NSs never evolve further than the ejector stage.

We argue, that as far as accreting isolated NSs are observed, combinations of \( t_d \) and \( \mu_0 \) for which no accreting isolated NS appear can be excluded. We plotted it in Figs. 3 and 4.

Filled regions represent space of the parameters where \( t_E \) is longer than \( 10^{10} \) yrs, so in that region a NS never come to the accretor stage, and doesn’t appear as accreting X-ray source. With the fact of observations of accreting old isolated NSs by ROSAT this region can be called “forbidden” for selected parameters of the exponential field decay (and for specified \( \mu_0 \)).

In the “forbidden” region in Fig. 3, which is plotted for \( \mu_0 = 10^{30} \) G cm\(^3\), all NSs reach the bottom field in a Hubble time or faster, and evolution on the late stages of their lives goes on with the field equal to the bottom. The left side is determined approximately by the condition:

\[
p_E(\mu_b) = p(t = t_{cr}). \tag{9}
\]

Small difference between the line, correspondent to the condition above, and the left side of the “forbidden” region appears because a NS can slightly change its period even with the momentum \( \mu = \mu_0 \), but due to small value of the field angular momentum losses are also small.

The right side of the region is roughly determined by the value of \( \mu_b \), with which a NS can reach the ejector stage with any \( t_d \), i.e. this \( \mu_b \) corresponds to the minimum value of \( \mu_0 \) with which a NS reach the ejector stage without field decay.

NSs to the right from the “forbidden” region leave the ejector stage, because their field cannot decay down to low values, and spin-down is fast enough during all their lives as ejectors, because the bottom magnetic momentum there is relatively high. To the left from the “forbidden” region the situation is different. Spin-down of NSs is very small and they leave the ejector stage not because of spin-down, but due to decreasing of \( p_E \), which depends upon the magnetic momentum.

Dashed line in Fig. 3 shows, that for all interesting parameters a NS with \( \mu_0 = 10^{30} \) G cm\(^3\) reach \( \mu_0 \) in less than \( 10^{10} \) yrs. Dot-dashed line shows the same for \( \mu_0 = 0.5 \cdot 10^{30} \) G cm\(^3\).

On Fig. 3 we also show the “forbidden” region and the line of reaching \( \mu_b \) for \( \mu_0 = 0.5 \cdot 10^{30} \) G cm\(^3\).

Fig. 4 is plotted for \( \mu_0 = 10^{29} \) G cm\(^3\). For long \( t_d > 4 \cdot 10^9 \) yrs, a NS again is not able to leave the ejector stage. It happens because the magnetic momentum can’t decrease down to small value of \( \mu \) (nearly \( \mu_b \)), and \( p_E \) is not decreasing enough.

3. Discussion and conclusions

We tried to evaluate the region of parameters, forbidden for models of the exponential magnetic field decay in NSs using the fact of observations of old accreting isolated NSs in X-rays.

If the main fraction of NSs have nearly the same initial parameters and parameters of the decay, then the intermediate values of \( t_d \) (\( \sim 10^7 - 10^8 \) yrs) in combination with the intermediate values of \( \mu_b \) (\( \sim 10^{28} - 10^{29.5} \) G cm\(^3\)) for \( \mu_0 = 10^{30} \) G cm\(^3\) can be excluded, because for that sets of parameters NSs spend all their lives on the ejector stage, never coming to the accretor stage.

As one can see in Fig. 2, for higher \( \mu_0 \) NSs should reach \( t_E \) even for \( t_d < 10^8 \) yrs, for smaller – the “forbidden” region should become wider. Results are depended on the initial magnetic field, \( \mu_0 \), ISM concentration, \( n \), and NS velocity, \( v \), so one can say, that the observed accreting isolated NSs come from, for example, the objects with high initial magnetic field, and the rest are not visible because they are in the forbidden region. To explore this idea in details population synthesis of NSs for realistic distributions of \( v, \mu_0 \) and \( n \) is needed. But we can say immediately, that the idea of obtaining accreting old isolated NSs from initially high field objects is not very promising, because the fraction of high field NSs can not be large (basing on radio pulsars observations), and as far as the fraction of low velocity NSs is not more than several percents (Popov et al. 1999) and the volume fraction filled with relatively high density ISM is also small, accreting old isolated NSs should come from “typical” population, i.e. from NSs with \( \mu_0 \) about \( 10^{30} \) G cm\(^3\) or less.

Actually, limits that we obtained are even stronger than they are in nature, because we didn’t take into account, that some significant time (in the case of field decay) a NS can spend on the propeller stage (spin-down rate at this stage is very uncertain, see the list of formulae, for example, in Lipunov & Popov 1995 or Lipunov 1992). Calculations of this effect, and calculations for different models of non-exponential field decay are subjects for future work.

We cannot say anything about parameters of field decay in the case of accretion in close binaries, because there situation is completely different, and our results cannot be applied to millisecond radio pulsars or other objects in close binary systems.

We note, that there is another reason due to which very fast decay down to small values of \( \mu_b \) also can be excluded, because it leads to huge amount of accreting isolated NSs. This situation is similar to “turning-off” the magnetic field of a NS (i.e., quenching any magnetospheric effect on the accreting matter), and for any velocity and density distributions we should expect significantly more accreting isolated NS than we have from ROSAT observations (of course, for high velocities X-ray sources will be very dim, but close NSs can be observed even for velocities \( \sim 100 \) km s\(^{-1}\)).

So, the existence of several old isolated accreting NSs, observed by ROSAT (if it is the correct interpretation of observations), can put important limits on the models of
the magnetic field decay for isolated (without influence of accretion, which can stimulate field decay) NSs, and models, from their side, should explain the fact of observations of $\sim 10$ accreting isolated NSs in the solar vicinity. We cannot discuss numerous details of connection between decay parameters and X-ray observations of isolated NSs without detailed calculations, we just tried to show, that this connection should be taken into account and made some illustrations of it, and future investigations in that field are wanted.

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