Core-crust transition pressure evolution in post-glitch epoch for a Vela-type pulsar

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We propose that the post-glitch epoch in a Vela-type pulsar corresponds to a transition time in which the broken crust is readjusting its temperature and core-crust transition pressure after the changes produced by the glitch. To describe the evolution of the pulsar in this epoch we use a sequence of stationary and axisymmetric relativistic models of slowly rotating neutron stars with a surface layer crust. Previously, we have formulated the matching conditions on the surface of the star in terms of physical properties (total mass of the star, core-crust transition pressure,...).

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Many pulsars show sudden spin jumps, glitches, which are superimposed to the gradual spin down due to the continued loss of angular momentum suffered by the star. After a relaxation period of the order of hundreds of days (post-glitch epoch), the star almost recovers its usual rhythm of spin down. The study of the properties of both glitch and post-glitch epoch are essential to understand the structure of the neutron star that models the pulsar.

Glitches have been observed in most of the younger isolated pulsars. Particularly, more than a dozen glitches with period variation of the order of $10^{-5}$ and $10^{-6}$ have been observed in the Crab and Vela pulsars respectively since their discovery. Many mechanisms have been proposed to explain glitches and post-glitch epoch.

Most of the models that deal with glitches are associated with the layer structure that neutron stars are thought to possess. Basically two regions of the star interior can be differentiated: the core and the crust. The properties of the crust are very different to those of the core, and it is thought that this region has a solid crystalline structure similar to a metal.

Let us mention that in the thermal glitch model proposed by Link and Epstein the glitch is explained by a sudden deposition of thermal energy in the inner crust, energy that would come from a rupture of the solid crust. The inner crust superfluid is very sensitive to star quake generated temperature perturbations, affecting to the rotation behavior of the star. Furthermore, it is noteworthy that the dynamical properties of the neutron star depend strongly on the transition pressure between the star’s core and the crust as have been pointed out by Cheng, Yuan, and Zhang and Lattimer and Prakash.

According with those two ideas, we propose that the post-glitch epoch can be understood as a transition time in which the broken crust is readjusting its temperature. The temperature excess that the inner layers of the crust has just after a glitch, would gradually dissipate during the post-glitch epoch until the crust reaches its equilibrium temperature. Our hypothesis is that the cooling of the exterior layers of the star would lead to a sensitive variation of the transition pressure between the core and the crust. In consequence, the post-glitch epoch will correspond to a readjustment epoch for this transition pressure. Because many dynamical properties of the neutron star strongly depend on the core-crust transition pressure, the gradual cooling down of the inner layers of the crust would explain its evolution all along the post-glitch epoch.

In order to be able to control the core-crust transition pressure, we will consider that the crust is thin enough so that we can treat it as a surface layer that envelopes the star’s core. We will consider the case of a relativistic star with rigid and slow rotation, so we can make use of Hartle and Thorne formalism. The introduction of a surface layer of energy density will change the boundary conditions on the surface of the star. We will show that it is interesting to use the total mass, the central density and the core-crust transition pressure as the parameters that characterize a configuration.

When performing the simulation of the post-glitch epoch, we must take into account that there are two very different time scales in the evolution of the star: the post-glitch epoch duration, that is of the order of hundreds of days, in contrast with the rotation period of the star that is of the order of seconds or even milliseconds. This important feature allows us to approximate each evolution state of the star during the post-glitch epoch, using an hydrodynamic equilibrium configuration with the appropriate parameters (total mass, central density, and core-crust transition pressure). We will fit the evolution of the angular velocity of the star with a sequence of configurations of different core-crust transition pressure, assuming that neither the mass nor the central density changes during the post-glitch epoch.

Model of a slowly rotating neutron star with a surface layer crust

We assume that the neutron star is in permanent rigid rotation. The star rotates with constant angular velocity $\Omega$ around an axis, so the resulting space-time is ax-
isymmetric and stationary. Appropriate coordinates can be chosen so that the metric can be written as follows
\[ ds^2 = -H^2 dt^2 + Q^2 dr^2 + r^2 K^2 \left( d\theta^2 - \sin^2 \theta (d\phi - L dt)^2 \right). \]
We assume that the rotation of the star is sufficiently slow and, hence, the Hartle-Thorne perturbative solution can be used \[^8\]. To second order in the angular velocity of the star \( \Omega \), it can be proved that the components of the metric have the following expression:

\[
\begin{align*}
g_{tt} &= -e^{2\phi(r)} \left[ 1 + 2h_0(r) + 2h_2(r) P_2(\theta) \right] \quad (1a) \\
+ &r^2 \sin^2 \theta \omega^2(r) \\
g_{t\phi} &= g_{\phi t} = -r^2 \sin^2(\theta) \omega(r) \quad (1b) \\
g_{rr} &= e^{2\lambda(r)} \left[ 1 + 2m_0(r) + 2m_2(r) P_2(\theta) \right] \quad (1c) \\
g_{\theta\theta} &= r^2 \left[ 1 + 2k_2(r) P_2(\theta) \right] \quad (1d) \\
g_{\phi\phi} &= \sin^2(\theta) g_{\theta\theta} \quad (1f)
\end{align*}
\]
We have to solve the Einstein equations in the interior of the star, where the matter is described by a perfect fluid \( \Gamma_{\mu\nu} = (\rho + p) u_{\mu} u_{\nu} + p g_{\mu\nu} \) with equation of state \( \rho = \rho(p) \) (here \( u^\mu \) is the four-velocity, \( \rho \) is the proper energy density, and \( p \) is the pressure of the fluid) and in the outer region, where space is empty. Also, we have to impose appropriate boundary conditions in the surface of the star.

It is very desirable to introduce a new radial coordinate \( R \). We can define a coordinate transformation \( r \rightarrow r(R, \theta) = R + \xi(R, \theta) \) \[^4\], so that surfaces of \( R = \text{const.} \) has constant density, i.e., this coordinate is adapted to the surface of the star.

We assume that the core of the star is surrounded by a thin surface layer (simplified crust model). Then we find that in the surface of the star \( R = a \), the stress-energy tensor is non null and it can be written as follows:

\[
T^\mu_\nu(R) = \rho_c(\theta) u^\mu u^\nu(\theta) \delta(R - a) \quad (2)
\]
Hence, we also assume that the matter of the surface layer can be described as a perfect fluid. We allow this fluid to rotate with its own angular velocity \( \Omega_c \). For the surface energy densities we will consider, we can despise the contribution of the pressure to the stress-energy tensor. Therefore the surface energy density can be written to second order as follows:

\[
\rho_c(\theta) = \varepsilon + \delta \varepsilon(\theta) = \varepsilon + \delta \varepsilon_0 + \delta \varepsilon_2 P_2(\theta) \quad (3)
\]
where \( \varepsilon, \delta \varepsilon_0 \), and \( \delta \varepsilon_2 \) are constants.

We must impose certain junction conditions on the interior and exterior solutions taking into account the introduction of a surface layer of matter on the border of the star. We use and intrinsic formulation of these conditions: the first fundamental form of the hypersurface \( R = a \) has to be continuous and the second fundamental form has to have a jump given by the stress-energy tensor on the surface of the star \[^{10}\]. Expanding these junction conditions in terms of \( \Omega_c \), we find the conditions for each one of the metric functions

\[\begin{align*}
\omega(a)_{\text{ext}} &= \omega(a)_{\text{int}} \equiv \omega(a) \quad (5a) \\
J &= \frac{a^4}{6} e^{\lambda(a)_{\text{ext}}} \left( e^{\lambda(a)_{\text{int}}} \left[ \partial R^2(a) \right]_{\text{int}} + 16 \pi \varepsilon \right) \quad (5b)
\end{align*}\]
where \( \Omega_c = \Omega - \omega(a) \).

It can be seen \[^8\] that the first term of expression \( 5b \) has the same sign as \( \Omega_c \) (in our case always positive). However, the second term depends on the sign of \( \Omega_c \). Hence, contra-rotating configuration are possible (the core and the crust contra-rotate), and even, we
could find certain critical configuration in which the total angular momentum is null.

**Second order:** we obtain the following conditions.

$$\Delta [r^*(a, \theta)] = 0$$  \hspace{1cm} (6)

where $r^* = r + \xi_0 + (\xi_2 + r k_2) F_2(\theta)$ is defined as in [5], so both the mean radius $r^*$ and the eccentricity $e = [−3(k_2 + \xi_2/r)]^{1/2}$ are continuous. We also obtain the next conditions

$$\Delta \left[ e^{\lambda(a)} m_0(a) \right] = 4\pi a^2 \delta \xi_0 + \frac{8\pi}{3} a^4 e^{-2\psi(a)} \varepsilon_{cc} +$$

$$- \frac{1}{2} \xi_0(a) \Delta \left[ 3e^{-\lambda(a)} + e^{\lambda(a)}(8\pi a^2 \rho(a) - 1) \right]$$  \hspace{1cm} (7a)

$$\Delta \left[ \frac{e^{\lambda(a)} F_1(a) k_2(a)}{F_1(a)} \right] = \frac{1}{12} a^2 \omega^2 e^{-2\psi(a)} \Delta \left[ \frac{F_2(a)}{F_1(a)} \right]$$

$$+ \frac{1}{24} a^4 e^{-2\psi(a)} \Delta \left[ \frac{e^{-\lambda(a)}}{F_1(a)} \right] \left( \partial_R \omega(a) \right)^2 =$$

$$+ \frac{2\pi}{3} e^{-2\psi(a)} a^3 \varepsilon_{cc}^2 - \pi a \delta \xi_2$$  \hspace{1cm} (7b)

$$\Delta \left[ e^{\lambda(a)} \xi_2(a) \right] = -\frac{8\pi}{3} e^{-2\psi(a)} a^4 \varepsilon_{cc}^2 -$$

$$- \frac{\delta a}{6} e^{-2\psi(a)} \Delta \left[ e^{-\lambda(a)} \left( \partial_R \omega(a) \right)^2 \right]$$

$$+ \frac{1}{2} \xi_0(a) \Delta \left[ F_3(a) \right] = - \frac{16\pi}{3} a^4 e^{-2\psi(a)} \varepsilon_{cc}^2$$  \hspace{1cm} (7c)

where we have defined $\Delta [f(a)] = f|_{\Sigma_{ext}} - f|_{\Sigma_{int}}$ and

$$F_1(a) = e^{2\lambda(a)}(1 + 8\pi a^2 \rho(a)) - 1$$  \hspace{1cm} (8a)

$$F_2(a) = e^{\lambda(a)}(7 + 8\pi a^2 \rho(a)) - 3 e^{-\lambda(a)}$$  \hspace{1cm} (8b)

$$F_3(a) = 64\pi a^4 e^{3\lambda(a)} \rho(a) + 8\pi a^4 e^{-3\lambda(a)} (\rho(a) - p(a))$$

$$+ 8\pi a^2 e^{3\lambda(a)} (3p(a) - \rho(a)) - e^{3\lambda(a)} - 3 e^{-\lambda(a)}$$  \hspace{1cm} (8c)

Basically, this conditions fix the jump in the second order perturbation of the mass function, and also fix the second order perturbation on the surface density ($\delta \xi_0$ and $\delta \xi_2$). Finally the last of the second order conditions fixes the value of the angular velocity of the crust:

$$\Omega_c = \omega(a) \pm (\Omega - \omega(a))$$  \hspace{1cm} (9)

We obtain two possible configurations for the same star core and transition pressure: one with the crust co-rotating with the interior fluid and with identical velocity, and a special configuration with contra-rotating crust. Current research on this strange contra-rotating configurations is under development, and it will be presented elsewhere. In what follows in this paper we will use the co-rotating configuration.

In a summary, to obtain a model of a neutron star with a surface layer crust in permanent slow rotation, for a given equation of state of the core, we have to fix three parameters. We choose the total mass of the star, the central density, and the core-crust transition pressure. Then, we integrate the Einstein equations inside the star (using a realistic equation of state for the matter) and we match this solution with the exterior solution found in [5] using the junction conditions explained above. In fig. 2 we show multiple configurations for both static stars and rotating stars, with a crust of approximately 10% of the total mass. The angular velocity displayed is $\Omega_c = \frac{M_{crust}}{\pi r_{crust}^2} \left( \text{mass shedding angular velocity} \right)$. On the static configurations without crust we write the value log 10($\rho_c$). We have used an equation of state for high pressures due to Glendenning [11] (K=240 MeV, $\frac{m}{\pi} = 0.78$ and $\sigma = 0.6$) and the BPS equation of state for the lower pressures. Similar results have been obtained for the analitical fit of the Sly equation of state presented in [5].

**Post-glitch epoch of a Vela-type pulsar**

Now we are going to study the post-glitch epoch of a Vela-type pulsar. The typical evolution of the angular velocity for a Vela-type pulsar during glitch time and post-glitch epoch is presented in fig. 2 [11]. It is interesting to note that in this problem there are two different scales of time, one related with the rotation period of the pulsar, and the other one with the duration of the post-glitch epoch. This fact allow us to describe the evolution of the pulsar in the post-glitch epoch using a sequence of stationary and axisymmetric configurations with the appropriate parameters to follow the evolution of the angular velocity of the pulsar. We assume that during the post-glitch epoch neither the total mass nor, the central density of the star are changed. Hence, in our description, the changes in the pulsar during this epoch are due to readjustment of the core-crust transition pressure.

In order to control the core-crust transition pressure, we assume that the crust is thin enough to be treated as a surface layer crust. Then to obtain the sequence of stationary and axisymmetric configurations mentioned above, we use our model of slowly rotating neutron star with superficial crust.
Let us describe our analysis. We assume that the neutron star has 1.44\(M_\odot\) and central density 1.279 \(10^{15}\text{gcm}^{-3}\). At the beginning of the \textit{post-glitch} epoch this star has a crust with the 10\% of the total mass. Since then, we adjust the transition pressure to fit the angular velocity of the model to the evolution shown in fig. 4. The points in this figure correspond to the elements of the sequence of models used to describe the evolution of the pulsar in the \textit{post-glitch} epoch.

We present our results in fig. 5 that shows the evolution of the core-crust transition pressure and eccentricity, and fig. 6 that shows the evolution of the surface matter distribution and quadrupolar moment. We obtain that the core-crust transition pressure decreases with time and causes a progressive reduction in the eccentricity, and a movement of matter on the surface of the star, from the equator to the poles. Also, the quadrupolar moment decreases in time all along the \textit{post-glitch}.

In consequence, we think that the \textit{post-glitch} epoch could be understood as a readjustment epoch for the core-crust transition pressure. Our conjecture is that the temperature excess that the inner layers of the crust have just after a \textit{glitch} would gradually dissipate during \textit{post-glitch} epoch until the crust reaches its equilibrium temperature. These changes of temperature produce the evolution of the core-crust transition pressure and the rest of the properties of the star.

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