Revisiting the transition $\Xi_{cc}^{++} \rightarrow \Xi_{c}^{(')} +$ to understand the data from LHCb

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Abstract

The LHCb collaboration newly measured the decay rate of doubly charmed baryon $\Xi_{cc}^{++} \rightarrow \Xi_{c}^{(')} + \pi^+$ and a ratio of its branching fraction with respect to that of the decay $\Xi_{cc}^{++} \rightarrow \Xi_{c}^{(')} + \pi^+$ is reported as $1.41 \pm 0.17 \pm 0.10$. This result conflicts with the theoretical predictions made by several groups.

In our previous work, following the prescription given in early literature where the $us$ diquark in $\Xi_{c}^{(')} +$ is assumed to be a scalar whereas in $\Xi_{c}^{(')} +$ is a vector i.e. the spin-flavor structure of $\Xi_{c}^{(')} +$ is $[us]_0c$ and that of $\Xi_{c}^{(')} +$ is $[us]_1c$, we studied the case of $\Xi_{cc}^{++} \rightarrow \Xi_{c}^{(')} +$ with the light front quark model. Numerically we obtained $\Gamma(\Xi_{cc}^{++} \rightarrow \Xi_{c}^{(')} + \pi^+) / \Gamma(\Xi_{cc}^{++} \rightarrow \Xi_{c}^{(')} + \pi^+) = 0.56 \pm 0.18$ which is about half of the data. While abandoning the presupposition, we suppose the spin-flavor structure of $us$ in $\Xi_{c}^{(')} +$ may be a mixture of scalar and vector, namely the spin-flavor function of $\Xi_{c}^{(')} +$ could be $\cos \theta [us]_0[c] + \sin \theta [us]_1[c]$. An alternative combination $-\sin \theta [us]_0[c] + \cos \theta [us]_1[c]$ would correspond to $\Xi_{c}^{(')} +$. Introducing the mixing mechanism the ratio $\Gamma(\Xi_{cc}^{++} \rightarrow \Xi_{c}^{(')} + \pi^+) / \Gamma(\Xi_{cc}^{++} \rightarrow \Xi_{c}^{(')} + \pi^+) = 0.56 \pm 0.18$ depends on the mixing angle $\theta$. With the mixing scenario, the theoretical prediction on the ratio between the transition rate of $\Xi_{cc}^{++} \rightarrow \Xi_{c}^{(')} +$ and that of $\Xi_{cc}^{++} \rightarrow \Xi_{c}^{(')}$ can coincide with the data as long as $\theta = 16.27^\circ \pm 2.30^\circ$ or $85.54^\circ \pm 2.30^\circ$ is set. Definitely, more precise measurements on other decay portals of $\Xi_{cc}^{++}$ are badly needed for testing the mixing mechanism and further determining the mixing angle.

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I. INTRODUCTION

In 2017, the LHCb collaboration observed the doubly charmed baryon Ξ⁺⁺ in the four-body final state Λ⁺⁺K⁻⁺π⁺π⁺ and the Ξ⁺⁺π⁺ portals successively. That baryon was expected for a long time by physicists of high energy physics because of its significance. The quark model predicted existence of baryons with two or three heavy quarks but they had evaded from experimental observation for long. With the great progress of detecting facilities and techniques, recently the LHCb collaboration observed the doubly charmed baryon via a decay portal Ξ⁺⁺→Ξ⁺⁺π⁺ with a branching fraction relative to that of the decay Ξ⁺⁺→Ξ⁺⁺π⁺ being 1.41±0.17±0.10.

On the theoretical aspect, some approaches have been applied to study the weak decay Ξ⁺⁺→Ξ⁺⁺. In Refs. 4,12 the predicted ratio Γ(Ξ⁺⁺→Ξ⁺⁺π⁺)/Γ(Ξ⁺⁺→Ξ⁺⁺π⁺) was not in keeping with the data. In our earlier work 13 the transition Ξ⁺⁺→Ξ⁺⁺ was explored within the light front quark model 14–38, where the three-quark picture of baryon was adopted. In that approach one needs to determine the vertex functions for the baryons by means of their inner structures. For Ξ⁺⁺ a naive and reasonable conjecture suggests that the two c quarks compose a physical subsystem (or a diquark) which serves as a color source for the light quark 39,40. The relative orbital angular momentum between the two c quarks is 0, i.e. the cc pair is in an S wave, and because it is in a color-anti-triplet 3, the spin of the cc pair must be 1 due to the symmetry requirement. In the works about single charmed-baryons, usually the two light quarks are supposed to reside in a subsystem as the light diquark 41,42. In those literatures, a presupposition Ref.42 suggests that the us diquark in Ξ⁺⁺ is a scalar whereas a vector in Ξ⁺⁺.

In the transition Ξ⁺⁺→Ξ⁺⁺ one c quark in the initial state would decay into an s quark via weak interaction while the other c quark and u quark are spectators in the process but the cu pair is not a diquark (or a physical subsystem) in the initial state, neither in the final state. To utilize the spectator scenario, the quark structure of [cc]₁u ([us]₁c or [us]₁c) should be mathematically rearranged into a sum of Σᵢ c[cu]ᵢ; (Σᵢ s[cu]ᵢ) where the sum runs over all possible spin projections via a Racah transformation. It is found that the spectator cu is independent of the quark (c or s) which is involved in the transition process. Thus the transition process can be divided into two steps: First, the physical structure [cc]₁u is rearranged into [cu]₁c by a Racah transformation and then the single c quark decays into s by emitting a gauge boson while the subsystem of [cu]₁ remains unchanged; Second, the [cu]₁s structure in the final state is re-ordered into the [us]₁c or [us]₁c structure through another Racah transformation. In our earlier work on the transition Ξ⁺⁺→Ξ⁺⁺ 13 the three quarks are treated as individual subjects i.e. possess their own momenta, and we obtained the branching ratio Γ(Ξ⁺⁺→Ξ⁺⁺π⁺)/Γ(Ξ⁺⁺→Ξ⁺⁺π⁺) as 0.56±0.18 which is almost half of the data.

It is noted that the earlier calculation in 13 was based on the presupposition that the us diquark in Ξ⁺⁺ is a scalar whereas that in Ξ⁺⁺ is a vector 42. As is well known the flavor symmetry is broken so either the us diquark in Ξ⁺⁺ or in Ξ⁺⁺ could be a mixture of a scalar and a vector. In this new scenario a mixing angle θ (0<θ<π) should be
introduced for the mixing of flavor-spin wave functions i.e. \( \Xi^+_c = \cos \theta [us]_0[c] + \sin \theta [us]_1[c] \)
and \( \Xi'_{cc}^+ = -\sin \theta [us]_0[c] + \cos \theta [us]_1[c] \) where the subscript 0 or 1 represents the total spin of the \( us \) subsystem. When \( \theta \) is set to 0, i.e. \( \Xi^+_c = [us]_0[c] \) and \( \Xi'_{cc}^+ = [us]_1[c] \), the structures of \( \Xi^+_c \) and \( \Xi'_{cc}^+ \) restore the original setting supposed by the authors of Ref. \([42]\). In other words the transition matrix element \( \mathcal{A}(\Xi^{++}_{cc} \rightarrow \Xi^+_c) \) and \( \mathcal{A}(\Xi^{++}_{cc} \rightarrow \Xi^{(+)'}_{cc}) \) carried out in Ref. \([13]\) just correspond to the processes \( \mathcal{A}([cc]_1[u] \rightarrow [us]_0[c]) \) and \( \mathcal{A}([cc]_1[u] \rightarrow [us]_1[c]) \). Now as long as \( \theta \) is not equal to 0, the process for \( \mathcal{A}(\Xi^{++}_{cc} \rightarrow \Xi^+_c) \) should be replaced by \( \cos \theta \mathcal{A}([cc]_1[u] \rightarrow [us]_0[c]) + \sin \theta \mathcal{A}([cc]_1[u] \rightarrow [us]_1[c]) \), while for \( \mathcal{A}(\Xi^{++}_{cc} \rightarrow \Xi^{(+)'}_{cc}) \) the process is \( -\sin \theta \mathcal{A}([cc]_1[u] \rightarrow [us]_0[c]) + \cos \theta \mathcal{A}([cc]_1[u] \rightarrow [us]_1[c]) \). The simple extension means existence of new sub-transition matrix elements for \( \Xi^{++}_{cc} \rightarrow \Xi^+_c \) and \( \Xi^{++}_{cc} \rightarrow \Xi^{(+)'}_{cc} \). The mixture scenario changes the predicted rate from the old picture, obviously the newly obtained theoretical estimate on the rates depend on \( \theta \).

This paper is organized as follows: after the introduction, in section II we revisit the transition matrix element for \( \Xi^{++}_{cc} \rightarrow \Xi^{(+)'}_c \) in the light-front quark model. Our numerical results for \( \Xi^{++}_{cc} \rightarrow \Xi^{(+)'}_c \) are presented in section III. The section IV is devoted to our conclusion and discussions.

II. \( \Xi^{++}_{cc} \rightarrow \Xi^{(+)'}_c \) in the Light-Front Quark Model

A. The Structures of \( \Xi^{++}_{cc} \), \( \Xi^+_c \) and \( \Xi^{(+)'}_c \)

The spectator scenario may greatly alleviates the theoretical difficulties for calculating the hadronic transition matrix elements. However the diquarks (physical subsystems) \( cc \) and \( us \) in \( \Xi^+_c \) and \( \Xi^{(+)'}_c \) are not spectators, which means the spectator approximation cannot be directly applied. In fact the \( c \) and \( u \) quarks which do not undergo a transition in the process, i.e. the combination of \( cu \) is approximately regarded as a spectator (an effective subsystem).

As a three-body system, the total spin of a baryon can be realized by different constructing schemes and the Racah transformations can relate one to others. By the aforementioned rearrangement of quark flavors the physical states \( [cc]_1[u] \) and \( [us]_0[c] \) (or \( [us]_1[c] \)) are written into sums over the effective forms \( c[cu]_i \) and \( s[cu]_i \), respectively. The detailed transformations are \([3]\)

\[
[c^1c^2]_1[u] = \frac{\sqrt{2}}{2} (-\frac{\sqrt{3}}{2}[c^2][c^1]u_0 + \frac{1}{2}[c^2][c^1]u_1 \\
-\frac{\sqrt{3}}{2}[c^1][c^2]u_0 + \frac{1}{2}[c^1][c^2]u_1),
\]

(1)

\[
[us]_0[c] = \frac{1}{2}[s][cu]_0 + \frac{\sqrt{3}}{2}[s][cu]_1,
\]

(2)

\[
[us]_1[c] = \frac{\sqrt{3}}{2}[s][cu]_0 + \frac{1}{2}[s][cu]_1.
\]

(3)

In Ref. \([42]\) \( \Xi^+_c \equiv [c^1c^2]_1[u] \), \( \Xi^+_c \equiv [us]_0[c] \) and \( \Xi^{(+)'}_c \equiv [us]_1[c] \), instead, in this work we suppose
\[ \Xi^+ \text{ and } \Xi^{'+} \text{ to be mixtures of } [us]_0[c] \text{ and } [us]_1[c] \text{ i.e. } \Xi^+_c = \cos \theta [us]_0[c] + \sin \theta [us]_1[c] \text{ and } \\
\Xi^{'+}_c = -\sin \theta [us]_0[c] + \cos \theta [us]_1[c] \text{ where } \theta \text{ is the mixing angle (restricted in the first and second quadrants with } 0 < \theta < \pi). \]

**B. the form factors of } \Xi^{'+}_c \rightarrow \Xi^+_c \text{ and } \Xi^+_c \rightarrow \Xi^{'+}_c \text{ in LFQM}**

The leading Feynman diagram responsible for the weak decay \( \Xi^{'+}_c \rightarrow \Xi^{('})_c \) is shown in Fig. 1. Following the procedures given in Refs. [32–35] the transition matrix element can be computed with the vertex functions of \( | \Xi^{'+}_c(P, S, S_z) \rangle \text{ and } | \Xi^{(')}_c(P', S', S'_z) \rangle \). The \( cu \) subsystem stands as a spectator, i.e. its spin configuration does not change during the transition.

The form factors for the weak transition \( \Xi^{'+}_c \rightarrow \Xi^+_c \) are defined in the standard way as

\[
\langle \Xi^+_c(P', S', S'_z) | \bar{s} \gamma_{\mu}(1 - \gamma_5)c | \Xi^{'+}_c(P, S, S_z) \rangle = \\
\bar{u}_{\Xi^{'+}_c}(P', S'_z) \left[ \gamma_{\mu} g_1(q^2) + i \sigma_{\mu\nu} \frac{q^\nu}{M_{|\Xi^{'+}_c|}} f_2(q^2) + \frac{q_{\mu}}{M_{|\Xi^{'+}_c|}} f_3(q^2) \right] u_{\Xi^+_c}(P, S_z) - \bar{u}_{\Xi^{'+}_c}(P', S'_z) \left[ \gamma_{\mu} g_1(q^2) + i \sigma_{\mu\nu} \frac{q^\nu}{M_{|\Xi^{'+}_c|}} g_2(q^2) + \frac{q_{\mu}}{M_{|\Xi^{'+}_c|}} g_3(q^2) \right] \gamma_5 u_{\Xi^+_c}(P, S_z).
\]

(4)

where \( q \equiv P - P' \). In terms of the spin-flavor structures of \( \Xi^+_c \) and \( \Xi^{'+}_c \) the matrix element \( \langle \Xi^{'+}_c(P', S', S'_z) | \bar{s} \gamma_{\mu}(1 - \gamma_5)c | \Xi^+_c(P, S, S_z) \rangle \) can be written as

\[
\cos \theta ([s][cu]_0 | \bar{s} \gamma_{\mu}(1 - \gamma_5)c | [c][cu]_0) + \sin \theta ([s][cu]_1 | \bar{s} \gamma_{\mu}(1 - \gamma_5)c | [c][cu]_1).
\]

For the transition matrix elements \( ([s][cu]_0 | \bar{s} \gamma_{\mu}(1 - \gamma_5)c | [c][cu]_0) \) and \( ([s][cu]_1 | \bar{s} \gamma_{\mu}(1 - \gamma_5)c | [c][cu]_1) \) the form factors are denoted to \( f_i^s, g_i^s \) and \( f_i^v, g_i^v \), so we have

\[
f_1 = (\frac{\sqrt{6}}{4} \cos \theta - \frac{3\sqrt{2}}{4} \sin \theta) f_1^s + (\frac{\sqrt{6}}{4} \cos \theta + \frac{\sqrt{2}}{4} \sin \theta) f_1^v.
\]
III. NUMERICAL RESULTS

The fitted values of \( f_i \) defined as done in Eq. (4). Here we just add a symbol “\( ^i \)”, on form factors, we re-estimate the decay rates of semi-leptonic and no-leptonic decays with the new scenario for the diquark structures.

\[ g_1 = \left( \frac{\sqrt{6}}{4} \cos \theta - \frac{3\sqrt{2}}{4} \sin \theta \right) g^i_1 + \left( \frac{\sqrt{6}}{4} \cos \theta + \frac{\sqrt{2}}{4} \sin \theta \right) g^v_1, \]
\[ f_2 = \left( \frac{\sqrt{6}}{4} \cos \theta - \frac{3\sqrt{2}}{4} \sin \theta \right) f^i_2 + \left( \frac{\sqrt{6}}{4} \cos \theta + \frac{\sqrt{2}}{4} \sin \theta \right) f^v_2, \]
\[ g_2 = \left( \frac{\sqrt{6}}{4} \cos \theta - \frac{3\sqrt{2}}{4} \sin \theta \right) g^i_2 + \left( \frac{\sqrt{6}}{4} \cos \theta + \frac{\sqrt{2}}{4} \sin \theta \right) g^v_2, \]

(5)

and \( f^i_1, g^i_1, f^v_1, g^v_1 \) can be found in our earlier paper [20].

For the transition \( \langle \Xi^+_c(0) | s_{\gamma \mu}(1 - \gamma_5)c | \Xi^+_c(P, S, S') \rangle \) the form factors are also defined as done in Eq. (4). Here we just add a symbol “\( ^i \)” on \( f_1, f_2, g_1 \) and \( g_2 \) to distinguish the quantities for \( \Xi^+_c \rightarrow \Xi_c \) and those for \( \Xi^+_c \rightarrow \Xi'_c \). They are

\[ f'_1 = \left( \frac{\sqrt{6}}{4} \sin \theta - \frac{3\sqrt{2}}{4} \cos \theta \right) f^i_1 + \left( \frac{\sqrt{6}}{4} \sin \theta + \frac{\sqrt{2}}{4} \cos \theta \right) f^v_1, \]
\[ g'_1 = \left( \frac{\sqrt{6}}{4} \sin \theta - \frac{3\sqrt{2}}{4} \cos \theta \right) g^i_1 + \left( \frac{\sqrt{6}}{4} \sin \theta + \frac{\sqrt{2}}{4} \cos \theta \right) g^v_1, \]
\[ f'_2 = \left( \frac{\sqrt{6}}{4} \sin \theta - \frac{3\sqrt{2}}{4} \cos \theta \right) f^i_2 + \left( \frac{\sqrt{6}}{4} \sin \theta + \frac{\sqrt{2}}{4} \cos \theta \right) f^v_2, \]
\[ g'_2 = \left( \frac{\sqrt{6}}{4} \sin \theta - \frac{3\sqrt{2}}{4} \cos \theta \right) g^i_2 + \left( \frac{\sqrt{6}}{4} \sin \theta + \frac{\sqrt{2}}{4} \cos \theta \right) g^v_2. \]

(6)

III. NUMERICAL RESULTS

A. The form factors for \( \Xi^+_c \rightarrow \Xi^+_c \) and \( \Xi^+_c \rightarrow \Xi'_c \)

In Ref. [13] we used a polynomial to parameterize these form factors \( f^i_1, g^i_1, f^v_1 \) and \( g^v_1 \) \((i = 1, 2)\),

\[ F(q^2) = F(0) \left[ 1 + a \left( \frac{q^2}{M_{\Xi^+_c}} \right) + b \left( \frac{q^2}{M_{\Xi^+_c}} \right)^2 + c \left( \frac{q^2}{M_{\Xi^+_c}} \right)^3 \right]. \]

(7)

The fitted values of \( a, b, c \) and \( F(0) \) in the form factors are presented in Table I. With the form factors, we re-estimate the decay rates of semi-leptonic and no-leptonic decays with the new scenario for the diquark structures.

B. Non-leptonic decays of \( \Xi^+_c \rightarrow \Xi^+_c + M \) and \( \Xi^+_c \rightarrow \Xi'_c + M \)

The transition matrix element of the non-leptonic decay is complicated due to involving non-perturbative physics. We did the calculation in Ref. [13] by employing the factorization assumption,

\[ \langle \Xi^+_c(0') | \mathcal{H} | \Xi^+_c(P, S_z) \rangle \]

5
| $F$  | $F(0)$   | $a$   | $b$   | $c$   |
|------|----------|-------|-------|-------|
| $f_1^s$ | 0.586    | 1.57  | 1.59  | 0.704 |
| $f_2^s$ | -0.484   | 2.06  | 2.42  | 1.17  |
| $g_1^s$ | 0.420    | 0.983 | 0.692 | 0.258 |
| $g_2^s$ | 0.228    | 1.90  | 2.07  | 0.960 |
| $f_1^v$ | 0.610    | 2.04  | 2.27  | 1.06  |
| $f_2^v$ | 0.463    | 2.14  | 2.49  | 1.19  |
| $g_1^v$ | -0.140   | 0.422 | 0.0931| 0.00632|
| $g_2^v$ | 0.0673   | 0.925 | 0.245 | -0.0862|

**TABLE I: The form factors given in polynomial form.**

![Graph](image)

**FIG. 2:** The dependence of \( \Gamma(\Xi_{cc}^+ \to \Xi_{cc}^+ \pi)/\Gamma(\Xi_{cc}^+ \to \Xi_{cc}^+ \pi) \) on \( \theta \).

\[
\frac{\Gamma(\Xi_{cc}^+ \to \Xi_{cc}^+ \pi)}{\Gamma(\Xi_{cc}^+ \to \Xi_{cc}^+ \pi)} = \frac{G_F V_{cb} V_{bl}^*}{\sqrt{2}} f_M \langle \Xi_{cc}^+ (P', S'_z) | \bar{s} \gamma^\mu (1 - \gamma_5) c | \Xi_{cc}^+ (P, S_z) \rangle,
\]

where \( f_M \) is the decay constant of meson \( M \). Besides the decay rate, the up-down asymmetry parameter \( \alpha \) (its definition can be found in Appendix) is also an experimentally observable quantity which has obvious significance for understanding the governing mechanism (including information about the non-perturbative effects). In our following tables we explicitly offer the theoretically estimated values for \( \alpha \) corresponding to different mixing angles.

Using the Eq. (8) we show the dependence of the ratio \( \Gamma(\Xi_{cc}^+ \to \Xi_{cc}^+ \pi)/\Gamma(\Xi_{cc}^+ \to \Xi_{cc}^+ \pi) \) on \( \theta \) which is depicted in Fig. 2 (the horizontal band centered with the dotted line corresponds to the range allowed by the experimental error tolerance). With the data \( 1.41 \pm 0.17 \pm 0.10 \) we fix \( \theta \) to be \( 16.27^\circ \pm 2.30^\circ \) or \( 85.54^\circ \pm 2.30^\circ \). The mixing angle deviates from \( 0^\circ \), which might manifest the scale of the flavor SU(3) symmetry breaking for the concerned processes.

The mixing mechanism can change the predictions on the non-leptonic decays significantly.
We list the estimated decay rates and up-down asymmetries of those processes with the three different mixing angles.

Comparing the results shown in the three tables one can find:
1. The orders of magnitude for the channels are unchanged with or without the mixing;
2. \( \Xi^+_{cc} \rightarrow \Xi^+_{cc} \pi \) and \( \Xi^+_{cc} \rightarrow \Xi^+_{cc} \rho \) are the main two-body decay channels for \( \Xi^+_{cc} \);
3. The relative sizes between \( \Gamma(\Xi^+_{cc} \rightarrow \Xi^+_{cc} \pi) \) and \( \Gamma(\Xi^+_{cc} \rightarrow \Xi^+_{cc} \pi) \) are varied for the different mixing angles.

### C. Semi-leptonic decays of \( \Xi^+_{cc} \rightarrow \Xi^+_{cc} l \bar{\nu}_l \) and \( \Xi^+_{cc} \rightarrow \Xi^+_{cc} l \bar{\nu}_l \)

Pre-setting different mixing angles, we repeat the evaluations of the rates of \( \Xi^+_{cc} \rightarrow \Xi^+_{cc} l \bar{\nu}_l \) and \( \Xi^+_{cc} \rightarrow \Xi^+_{cc} l \bar{\nu}_l \). The dependent of the differential decay widths \( d\Gamma/d\omega (\omega = P/P^0) \) on \( \omega \) are depicted in Fig. 3 and 4. One can find that the curve shapes of Fig. 3 and Fig. 4 are

### TABLE II: The Widths (in unit 10^{10}s^{-1}) and up-down asymmetry of non-leptonic decays \( \Xi^+_{cc} \rightarrow \Xi^+_{cc} \pi \) in \([13]\) (\( \theta = 0^\circ \)).

| mode | width | up-down asymmetry | mode | width | up-down asymmetry |
|------|-------|-------------------|------|-------|-------------------|
| \( \Xi^+_{cc} \rightarrow \Xi^+_{cc} \pi \) | 13.6±1.8 | -0.441±0.009 | \( \Xi^+_{cc} \rightarrow \Xi^+_{cc} \pi \) | 7.68±0.92 | -0.982±0.005 |
| \( \Xi^+_{cc} \rightarrow \Xi^+_{cc} \rho \) | 11.0±1.5 | -0.429±0.016 | \( \Xi^+_{cc} \rightarrow \Xi^+_{cc} \rho \) | 13.9±1.2 | -0.111±0.034 |
| \( \Xi^+_{cc} \rightarrow \Xi^+_{cc} K \) | 1.03±0.14 | -0.402±0.008 | \( \Xi^+_{cc} \rightarrow \Xi^+_{cc} K \) | 0.492±0.059 | -0.998±0.002 |
| \( \Xi^+_{cc} \rightarrow \Xi^+_{cc} K^* \) | 0.414±0.055 | -0.422±0.021 | \( \Xi^+_{cc} \rightarrow \Xi^+_{cc} K^* \) | 0.623±0.052 | -0.014±0.030 |

### TABLE III: The widths (in unit 10^{10}s^{-1}) and up-down asymmetry of non-leptonic decays \( \Xi^+_{cc} \rightarrow \Xi^+_{cc} \pi \) with \( \theta = 16.27^\circ \pm 2.30^\circ \).

| mode | width | up-down asymmetry | mode | width | up-down asymmetry |
|------|-------|-------------------|------|-------|-------------------|
| \( \Xi^+_{cc} \rightarrow \Xi^+_{cc} \pi \) | 8.37±0.69 | -0.087±0.070 | \( \Xi^+_{cc} \rightarrow \Xi^+_{cc} \pi \) | 11.8±0.5 | -0.991±0.006 |
| \( \Xi^+_{cc} \rightarrow \Xi^+_{cc} \rho \) | 5.59±0.56 | -0.167±0.079 | \( \Xi^+_{cc} \rightarrow \Xi^+_{cc} \rho \) | 17.6±0.3 | -0.228±0.014 |
| \( \Xi^+_{cc} \rightarrow \Xi^+_{cc} K \) | 0.642±0.052 | -0.081±0.063 | \( \Xi^+_{cc} \rightarrow \Xi^+_{cc} K \) | 0.789±0.041 | -0.967±0.010 |
| \( \Xi^+_{cc} \rightarrow \Xi^+_{cc} K^* \) | 0.187±0.021 | -0.211±0.084 | \( \Xi^+_{cc} \rightarrow \Xi^+_{cc} K^* \) | 0.756±0.011 | -0.107±0.011 |

### TABLE IV: The Widths (in unit 10^{10}s^{-1}) and up-down asymmetry of non-leptonic decays \( \Xi^+_{cc} \rightarrow \Xi^+_{cc} \pi \) with \( \theta = 85.54^\circ \pm 2.30^\circ \).

| mode | width | up-down asymmetry | mode | width | up-down asymmetry |
|------|-------|-------------------|------|-------|-------------------|
| \( \Xi^+_{cc} \rightarrow \Xi^+_{cc} \pi \) | 8.36±0.69 | -0.952±0.023 | \( \Xi^+_{cc} \rightarrow \Xi^+_{cc} \pi \) | 11.8±0.6 | -0.507±0.032 |
| \( \Xi^+_{cc} \rightarrow \Xi^+_{cc} \rho \) | 16.8±0.9 | -0.106±0.023 | \( \Xi^+_{cc} \rightarrow \Xi^+_{cc} \rho \) | 8.23±0.67 | -0.456±0.004 |
| \( \Xi^+_{cc} \rightarrow \Xi^+_{cc} K \) | 0.554±0.049 | -0.977±0.039 | \( \Xi^+_{cc} \rightarrow \Xi^+_{cc} K \) | 0.869±0.038 | -0.455±0.029 |
| \( \Xi^+_{cc} \rightarrow \Xi^+_{cc} K^* \) | 0.834±0.042 | -0.005±0.013 | \( \Xi^+_{cc} \rightarrow \Xi^+_{cc} K^* \) | 0.269±0.027 | -0.419±0.008 |
FIG. 3: Differential decay rates $d\Gamma/d\omega$ for the decay $\Xi_{cc}^{+} \rightarrow \Xi_{c}^{+} l\bar{\nu}_{l}$ (a) and $\Xi_{cc}^{+} \rightarrow \Xi_{c}^{+} l\bar{\nu}_{l}$ (b) in [13] ($\theta = 0^\circ$).

TABLE V: The width (in unit $10^{12}$s$^{-1}$) and the ratio $R$ of $\Xi_{cc}^{+} \rightarrow \Xi_{c}^{+} l\bar{\nu}_{l}$ (left) and $\Xi_{cc}^{+} \rightarrow \Xi_{c}^{+} l\bar{\nu}_{l}$ (right).

| $\theta$   | $\Gamma$ | $R$     | $\Gamma$ | $R$     |
|------------|----------|---------|----------|---------|
| 0$^\circ$  | 0.100±0.015 | 7.14±0.61 | 0.0995±0.0091 | 1.34±0.07 |
| 16.27$^\circ$±2.30$^\circ$ | 0.0522±0.0051 | 46.6±0.5  | 0.130±0.003 | 1.63±0.05 |
| 85.54$^\circ$±2.30$^\circ$ | 0.143±0.009 | 1.22±0.04 | 0.0732±0.0051 | 6.14±0.57 |

similar for $\Xi_{cc}^{+} \rightarrow \Xi_{c}^{+} l\bar{\nu}_{l}$ and $\Xi_{cc}^{+} \rightarrow \Xi_{c}^{+} l\bar{\nu}_{l}$. By contrast, the curve tendencies for $\Xi_{cc}^{+} \rightarrow \Xi_{c}^{+} l\bar{\nu}_{l}$ and $\Xi_{cc}^{+} \rightarrow \Xi_{c}^{+} l\bar{\nu}_{l}$ in Fig. 3 and in Fig. 5 are just left to right reversed from each other while the integrated quantities (decay widths) are close. The total decay widths and the ratio of the longitudinal to transverse decay rates $R$ corresponding to the three mixing angles are all listed in table V.

IV. CONCLUSIONS AND DISCUSSIONS

In Ref. [13] we calculated the transition rate of $\Xi_{cc}^{+} \rightarrow \Xi_{c}^{(i)}$ in the light front quark model with a three-quark picture of baryon. To calculate the transition $\Xi_{cc}^{+} \rightarrow \Xi_{c}^{(i)}$ we need know the inner spin-flavor structures of all the concerned baryons. Generally the two charm quarks constitute a diquark which joins the light quark to make the baryon $\Xi_{cc}^{+}$. Because two $c$ quarks are identical heavy flavor fermions in a color anti-triplet, it must be a vector boson.
In Ref. [42] the scenario that light $u_s$ pair in $\Xi_c^+$ ($\Xi_c^{'+}$) is pre-set as a scalar (vector) diquark, was employed in our previous study [13]. With the prescription we calculate the form factors of the transition $\Xi_{cc}^+ \to \Xi_c^{(')}$ and the decay rates of $\Xi_{cc}^+ \to \Xi_c^{(')} l \bar{\nu}_l$ and $\Xi_{cc}^+ \to \Xi_c^{(')} M$.

However, we notice that the ratio $\Gamma(\Xi_{cc}^+ \to \Xi_c^+ \pi)/\Gamma(\Xi_{cc}^+ \to \Xi_{c}^{'} \pi)$ obtained with that prescription was $0.56 \pm 0.18$ [13] which does not agree with the data newly observed by the LHCb collaboration. To reconcile our theoretical result and data, we should find what was wrong and how to remedy the theoretical framework. One possible pitfall might be the spin-flavor structure of $\Xi_c^+$ ($\Xi_c^{'+}$) pre-set in Ref. [42] which was based on a precise SU(3) flavor symmetry. However, in fact the symmetry is upset by the difference between the mass of $s$ quark and those of $u$ and $d$ quarks, to manifest this breaking we are motivated to suggest that the spin of $u_s$ pair in $\Xi_c^+$ ($\Xi_c^{'+}$) is the mixture of the scalar and vector diquarks. By introducing a mixing angle $\theta$ the amplitudes become $A(\Xi_{cc}^{+} \to \Xi^{+}) = \cos \theta A([cc]_1[u] \rightarrow [us]_0[c]) + \sin \theta A([cc]_1[u] \rightarrow [us]_1[c])$ and $A(\Xi_{cc}^{'+} \to \Xi^{'+}) = -\sin \theta A([cc]_1[u] \rightarrow [us]_0[c]) + \cos \theta A([cc]_1[u] \rightarrow [us]_1[c])$. Apparently the newly achieved ratio $\Gamma(\Xi_{cc}^+ \to \Xi_c^+ \pi)/\Gamma(\Xi_{cc}^+ \to \Xi_c^{'} \pi)$ depends on the parameter $\theta$. We fix $\theta = 16.27^\circ \pm 2.30^\circ$ or $85.54^\circ \pm 2.30^\circ$ by fitting the data of LHCb.

Using the mixing angles we calculate the rates of semileptonic decays $\Xi_{cc}^+ \to \Xi_c^+ l \bar{\nu}_l$ and $\Xi_{cc}^+ \to \Xi'_c l \bar{\nu}_l$. We find that the shapes of Fig. 3 and Fig. 4 are similar for $\Xi_{cc}^+ \to \Xi_c^+ l \bar{\nu}_l$ and $\Xi_{cc}^+ \to \Xi'_c l \bar{\nu}_l$, respectively. By contrast, the curve tendencies for $\Xi_{cc}^+ \to \Xi_c^+ l \bar{\nu}_l$ and $\Xi_{cc}^+ \to \Xi'_c l \bar{\nu}_l$ in Fig. 3 and in Fig. 4 are just left to right reversed from each other. With the same theoretical framework, we also evaluate the rates of several non-leptonic decays. Our numerical results indicate that the order of magnitudes of these decays are unchanged.

FIG. 4: Differential decay rates $d\Gamma/d\omega$ for the decay $\Xi_{cc}^+ \to \Xi_c^+ l \bar{\nu}_l$ (a) and $\Xi_{cc}^+ \to \Xi'_c l \bar{\nu}_l$ (b) with $\theta = 16.27^\circ \pm 2.30^\circ$. 

In Ref. [42] the scenario that light $u_s$ pair in $\Xi_c^+$ ($\Xi_c^{'+}$) is pre-set as a scalar (vector) diquark, was employed in our previous study [13]. With the prescription we calculate the form factors of the transition $\Xi_{cc}^+ \to \Xi_c^{(')}$ and the decay rates of $\Xi_{cc}^+ \to \Xi_c^{(')} l \bar{\nu}_l$ and $\Xi_{cc}^+ \to \Xi_c^{(')} M$. 

However, we notice that the ratio $\Gamma(\Xi_{cc}^+ \to \Xi_c^+ \pi)/\Gamma(\Xi_{cc}^+ \to \Xi_{c}^{'} \pi)$ obtained with that prescription was $0.56 \pm 0.18$ [13] which does not agree with the data newly observed by the LHCb collaboration. To reconcile our theoretical result and data, we should find what was wrong and how to remedy the theoretical framework. One possible pitfall might be the spin-flavor structure of $\Xi_c^+$ ($\Xi_c^{'+}$) pre-set in Ref. [42] which was based on a precise SU(3) flavor symmetry. However, in fact the symmetry is upset by the difference between the mass of $s$ quark and those of $u$ and $d$ quarks, to manifest this breaking we are motivated to suggest that the spin of $u_s$ pair in $\Xi_c^+$ ($\Xi_c^{'+}$) is the mixture of the scalar and vector diquarks. By introducing a mixing angle $\theta$ the amplitudes become $A(\Xi_{cc}^{+} \to \Xi^{+}) = \cos \theta A([cc]_1[u] \rightarrow [us]_0[c]) + \sin \theta A([cc]_1[u] \rightarrow [us]_1[c])$ and $A(\Xi_{cc}^{'+} \to \Xi^{'+}) = -\sin \theta A([cc]_1[u] \rightarrow [us]_0[c]) + \cos \theta A([cc]_1[u] \rightarrow [us]_1[c])$. Apparently the newly achieved ratio $\Gamma(\Xi_{cc}^+ \to \Xi_c^+ \pi)/\Gamma(\Xi_{cc}^+ \to \Xi_c^{'} \pi)$ depends on the parameter $\theta$. We fix $\theta = 16.27^\circ \pm 2.30^\circ$ or $85.54^\circ \pm 2.30^\circ$ by fitting the data of LHCb.

Using the mixing angles we calculate the rates of semileptonic decays $\Xi_{cc}^+ \to \Xi_c^+ l \bar{\nu}_l$ and $\Xi_{cc}^+ \to \Xi'_c l \bar{\nu}_l$. We find that the shapes of Fig. 3 and Fig. 4 are similar for $\Xi_{cc}^+ \to \Xi_c^+ l \bar{\nu}_l$ and $\Xi_{cc}^+ \to \Xi'_c l \bar{\nu}_l$, respectively. By contrast, the curve tendencies for $\Xi_{cc}^+ \to \Xi_c^+ l \bar{\nu}_l$ and $\Xi_{cc}^+ \to \Xi'_c l \bar{\nu}_l$ in Fig. 3 and in Fig. 4 are just left to right reversed from each other. With the same theoretical framework, we also evaluate the rates of several non-leptonic decays. Our numerical results indicate that the order of magnitudes of these decays are unchanged.
FIG. 5: Differential decay rates $d\Gamma/d\omega$ for the decay $\Xi_{cc}^+ \rightarrow \Xi_c^+ l\bar{\nu}_l$ (a) and $\Xi_{cc}^+ \rightarrow \Xi_c^{' +} l\bar{\nu}_l$ (b) with $\theta = 85.54^\circ \pm 2.30^\circ$

but the relative sizes between $\Gamma(\Xi_{cc}^+ \rightarrow \Xi_c^+ M)$ and $\Gamma(\Xi_{cc}^+ \rightarrow \Xi_c^{'} M)$ are varied for the different mixing angles.

We hope that the experimentalists can make more precise measurements on those relevant decay channels of $\Xi_{cc}^+$. The new data would tell us whether our mechanism can survive, then one can pin down the right one from the two possible mixing angles we fixed. Definitely, the theoretical studies on the double-heavy baryons would be helpful for getting a better understanding about the quark model and the non-perturbative QCD effects.

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Appendix A: Semi-leptonic decays of $B_1 \rightarrow B_2 l\bar{\nu}_l$

The helicity amplitudes are related to the form factors for $B_1 \rightarrow B_2 l\bar{\nu}_l$ through the following expressions $^{[43, 45]}$

$$H_{\pm,0}^V = \frac{\sqrt{Q}}{\sqrt{q^2}} \left( M_{B_1} + M_{B_2} \right) f_1 - \frac{q^2}{M_{B_1}} f_2,$$

$$H_{\pm,1}^V = \sqrt{2Q} \left( -f_1 + \frac{M_{B_1} + M_{B_2}}{M_{B_1}} f_2 \right),$$
\[ H^{A}_{\pm,0} = \frac{\sqrt{Q_+}}{\sqrt{q^2}} \left( (M_{B_1} - M_{B_2}) g_1 + \frac{q^2}{M_{B_1}} g_2 \right), \]
\[ H^{A}_{\pm,1} = \sqrt{2Q_+} \left( -g_1 - \frac{M_{B_1} - M_{B_2}}{M_{B_1}} g_2 \right). \]  

(A1)

where \( Q_\pm = 2(P \cdot P' \pm M_{B_1}M_{B_2}) \) and \( M_{B_1} (M_{B_2}) \) represents \( M_{B_{cc}}^\pm (M_{B_{cc}}^\pm) \). The amplitudes for the negative helicities are obtained in terms of the relation

\[ H^{V,A}_{-\lambda, -\lambda_w} = \pm H^{V,A}_{\lambda, \lambda_w}, \]  

(A2)

where the upper (lower) index corresponds to \( V (A) \). The helicity amplitudes are

\[ H_{\lambda, \lambda_w} = H^{V}_{\lambda, \lambda_w} - H^{A}_{\lambda, \lambda_w}. \]  

(A3)

The helicities of the \( W \)-boson \( \lambda_w \) can be either 0 or 1, which correspond to the longitudinal and transverse polarizations, respectively. The longitudinally \( (L) \) and transversely \( (T) \) polarized rates are respectively \([43,45] \).

\[ \frac{d\Gamma_L}{d\omega} = \frac{G_F^2 |V_{cb}|^2}{q^2 p_c M_{B_2}} \frac{q^2 p_c M_{B_2}}{12 M_{B_1}} \left[ |H_{\frac{1}{2},0}|^2 + |H_{-\frac{1}{2},0}|^2 \right], \]
\[ \frac{d\Gamma_T}{d\omega} = \frac{G_F^2 |V_{cb}|^2}{q^2 p_c M_{B_2}} \frac{q^2 p_c M_{B_2}}{12 M_{B_1}} \left[ |H_{\frac{1}{2},1}|^2 + |H_{-\frac{1}{2},-1}|^2 \right]. \]  

(A4)

where \( p_c \) is the momentum of \( B_2 \) in the rest frame of \( B_1 \).

The ratio of the longitudinal to transverse decay rates \( R \) is defined by

\[ R = \frac{\Gamma_L}{\Gamma_T} = \frac{\int_{\omega_{\text{max}}}^{\omega_{\text{min}}} d\omega \ q^2 p_c \qquad |H_{\frac{1}{2},0}|^2 + |H_{-\frac{1}{2},0}|^2 \quad \left[ |H_{\frac{1}{2},1}|^2 + |H_{-\frac{1}{2},-1}|^2 \right]}{\int_{\omega_{\text{min}}}^{\omega_{\text{max}}} d\omega \ q^2 p_c \qquad |H_{\frac{1}{2},1}|^2 + |H_{-\frac{1}{2},-1}|^2 \quad \left[ |H_{\frac{1}{2},0}|^2 + |H_{-\frac{1}{2},0}|^2 \right]}. \]  

(A5)

Appendix B: \( B_1 \rightarrow B_2 M \)

In general, the transition amplitude of \( B_1 \rightarrow B_2 M \) can be written as

\[ \mathcal{M}(B_1 \rightarrow B_2 P) = \bar{u}_{B_2}(A + B\gamma_5)u_{B_1}, \]
\[ \mathcal{M}(B_1 \rightarrow B_2 V) = \bar{u}_{B_2} \epsilon^{\mu} [A_1\gamma^{\mu}\gamma_5 + A_2(p_c)\mu\gamma_5 + B_1\gamma_\mu + B_2(p_c)\mu]u_{B_1}, \]  

(B1)

where \( \epsilon^{\mu} \) is the polarization vector of the final vector or axial-vector mesons. Including the effective Wilson coefficient \( a_1 = c_1 + c_2/N_c \) (in Ref. \([46]\) \( a_1 = 1.05 \pm 0.10 \)), the decay amplitudes in the factorization approximation are \([47]\)

\[ A = \lambda f_P(M_{B_1} - M_{B_2}) f_1(M^2), \]
\[ B = \lambda f_P(M_{B_1} + M_{B_2}) g_2(M^2), \]
\[ A_1 = -\lambda f_V M \left[ g_1(M^2) + g_2(M^2) \frac{M_{B_1} - M_{B_2}}{M_{B_1}} \right], \]

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\[ A_2 = -2\lambda f_VM \frac{g_2(M^2)}{M_{B_1}}, \]
\[ B_1 = \lambda f_VM \left[ f_1(M^2) - f_2(M^2) \frac{M_{B_1} + M_{B_2}}{M_{B_1}} \right], \]
\[ B_2 = 2\lambda f_VM \frac{f_2(M^2)}{M_{B_1}}, \]

where \( \lambda = \frac{G_F}{\sqrt{2}} V_{cs} V_{q_1 q_2}^* a_1 \) and \( M \) is the meson mass. Replacing \( P, V \) by \( S \) and \( A \) in the above expressions, one can easily obtain similar expressions for scalar and axial-vector mesons.

The decay rates of \( B_1 \to B_2 P(S) \) and up-down asymmetries are

\[ \Gamma = \frac{p_c}{8\pi} \left[ \left( M_{B_1} + M_{B_2} \right)^2 - M^2 \right] |A|^2 + \frac{\left( M_{B_1} - M_{B_2} \right)^2 - m^2}{M_{B_1}^2} |B|^2, \]
\[ \alpha = -\frac{2\kappa \text{Re}(A^*B)}{|A|^2 + \kappa^2 |B|^2}, \]

where \( p_c \) is the \( B_2 \) momentum in the rest frame of \( B_1 \), \( m \) is the mass of pseudoscalar (scalar), and \( \kappa = \frac{p_c}{M_{B_2} + \sqrt{p_c^2 + M_{B_2}^2}} \). For \( B_1 \to B_2 V(A) \) decays, the decay rate and up-down asymmetries are

\[ \Gamma = \frac{p_c}{4\pi M_{B_1}} \left[ 2 \left( |S|^2 + |P_2|^2 \right) + \frac{\varepsilon^2}{m^2} \left( |S + D|^2 + |P_1|^2 \right) \right], \]
\[ \alpha = \frac{4m^2 \text{Re}(S^*P_2) + 2\varepsilon^2 \text{Re}(S + D) \text{Re}(P_1)}{2m^2 \left( |S|^2 + |P_2|^2 \right) + \varepsilon^2 \left( |S + D|^2 + |P_1|^2 \right)^2}, \]

where \( \varepsilon \) (\( m \)) is energy (mass) of the vector (axial vector) meson, and

\[ S = -A_1, \]
\[ P_1 = -\frac{p_c}{\varepsilon} \left( \frac{M_{B_1} + M_{B_2}}{E_{B_2} + M_{B_2}} B_1 + M_{B_1} B_2 \right), \]
\[ P_2 = \frac{p_c}{E_{B_2} + M_{B_2}} B_1, \]
\[ D = -\frac{p_c^2}{\varepsilon(E_{B_2} + M_{B_2})} (A_1 - M_{B_1} A_2). \]

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