Algorithms and Analysis for the SPARQL Constructs

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Abstract As Resource Description Framework (RDF) is becoming a popular data modelling standard, the challenges of efficient processing of Basic Graph Pattern (BGP) SPARQL queries (a.k.a. SQL inner-joins) have been a focus of the research community over the past several years. In our recently published work we brought community’s attention to another equally important component of SPARQL, i.e., OPTIONAL pattern queries (a.k.a. SQL left-outer-joins). We proposed novel optimization techniques – first of a kind – and showed experimentally that our techniques perform better for the low-selectivity queries, and give at par performance for the highly selective queries, compared to the state-of-the-art methods.

BGPs and OPTIONALs (BGP-OPT) make the basic building blocks of the SPARQL query language. Thus, in this paper, treating our BGP-OPT query optimization techniques as the primitives, we extend them to handle other broader components of SPARQL such as UNION, FILTER, and DISTINCT. We mainly focus on the procedural (algorithmic) aspects of these extensions. We also make several important observations about the structural aspects of complex SPARQL queries with any intermix of these clauses, and relax some of the constraints regarding the cyclic properties of the queries proposed earlier. We do so without affecting the correctness of the results, thus providing more flexibility in using the BGP-OPT optimization techniques.

1 Introduction

Resource Description Framework (RDF) is being used as the standard for representing semantically linked data on the web as well as for other domains such as biological networks, e.g., UniProt RDF network by Swiss Institute of Bioinformatics. RDF is a directed edge-labeled multi-graph, where each unique edge (S P O) is called a triple – P is the label on the edge from the node S to node O, and SPARQL is the standard query language for it.

SPARQL provides various syntactic constructs to form structured queries over RDF graphs. These constructs have a close similarity to their SQL counterparts. For instance, Basic Graph Patterns (BGP) of SPARQL (or TriplesBlock as referred to in the SPARQL grammar) are similar to the SQL inner-joins (∩). OPTIONAL patterns of SPARQL (OPTIONALGraphPattern in the SPARQL grammar) are similar to the left-outer-joins (∪). FILTERs of SPARQL makes up for the SQL LIKE clause and various other selection conditions. UNIONS (∪) and DISTINCTs of SPARQL are similar to their SQL counterparts. GroupGraphPattern of the SPARQL grammar consists of BGP, OPTIONAL, UNION, and FILTER components, and like SQL, SPARQL grammar too allows nested queries with a complex intermix of these query constructs. Since there is an equivalence between SPARQL and SQL constructs, we will use these terms interchangeably in the rest of the text.

BGP queries make the building blocks of SPARQL, and just like SQL inner-joins, they are associative and commutative, i.e., a change in the order of joins among the triple patterns does not change the final results –

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1 http://www.uniprot.org/format/uniprot_rdf
This query asks for all the friends and friends-of-
of-Jerry who have acted in any sitcom, and optionally it asks for the directors of the respective sit-
coms if their location was NYC. We have in all six triple
patterns in this query. Numbering them $T_1, T_2$ from top
to bottom, the query can be expressed as $Q = (T_1 \times
(T_2 \cup (T_3 \times T_4))) \bowtie (T_5 \times T_6)$. These triple patterns
form four BGP s in the query, which are as follows:
$P_1 = T_1, P_2 = T_2, P_3 = (T_3 \times T_4), P_4 = (T_5 \times T_6),$
and then the query can be expressed as $Q = (P_1 \times
(P_2 \cup P_3)) \bowtie (P_4)$. Note that we cannot do the
left-outer-join between $T_2$ and $T_3$, before evaluating $P_4 =
(T_5 \times T_6)$ and the UNION $P_2 \cup P_3$.

Analysis of the real world SPARQL queries shows
that queries with an internmix of BGP, OPTIONALs,
UNIONS, FILTERs indeed constitute over 94% the query
logs [14,22,26,16], and thus make these other constructs
like OPTIONAL, UNION, FILTER non-negligible from
the query processing and performance optimization
perspective. In our previous work [5] we focused on the
OPTIONAL pattern queries (referred to henceforth as
OPT queries), and proposed novel techniques for the
optimization of these queries. Our techniques extended
the ideas of nullification and best-match (or Generalized-
Outerjoin) operators as proposed in [24,25,13], and sho-
wed that for acyclic queries we can reduce the candidate
triples to minimal using the semi-join based pruning [8,
7,29], and avoid the nullification and best-match oper-
ations altogether (see the lemmas in [3]). Since BGP-
OPT make the building blocks of SPARQL queries, in
this paper we mainly show that our BGP-OPT optimi-
ization techniques can be used as primitives to eval-
uate queries with an internmix of the various SPARQL
query constructs. Much of the research based systems
developed for the optimization of SPARQL queries have
only handled the BGP component [6,20,10], and they
either do not handle other SPARQL constructs, or rely
on naive ways of processing them. The SPARQL pro-
cessing systems based on relational databases, such as
MonetDB or Virtuoso, just translate the SPARQL queries
into their SQL counterparts, by assuming that the RDF
graph is stored in the relational tables.

In the light of this, we propose to use our BGP-
OPT evaluation techniques as primitive building blocks
to cover a broader spectrum of the SPARQL queries.
While doing so, we focus on the procedural (algorithm-
ic) aspects of using BGP-OPT techniques than the empiri-
cal aspects, because our previous work has al-
ready established the usefulness of our BGP-OPT eval-
uation techniques – especially for the low-selectivity que-
In this paper, we make the following main contributions:

1. We propose a new method of forming the Graph of Supernodes (GoSN) that enhances our previously proposed method [5] (Section 2).
2. Using the above mentioned new way of constructing the GoSN, we show that the condition of acyclicity of Graph of Tables (GoT) of a BGP-OPT query can be relaxed in some cases in addition to the conditions given in [5] (Sections 1.1 and 1.2).
3. We propose a way of methodically using the BGP-OPT query optimization techniques for the queries with an intermix of UNION and FILTER clauses without evaluating each query in the UNION normal form (UNF) individually as proposed in [5] (Sections 9.1 and 9.2).
4. We also discuss handling of the DISTINCT clause with any intermix of these query clauses (Section 9.3).
5. In the context of UNION and FILTER, we bring to the light implications of “NULLs”, and their semantics for the nullification and best-match operators.
6. Since there is a close match between SPARQL query operators and SQL, our techniques and insights can be useful for their SQL counterparts too, with appropriate indexing and data representation methods in the relational setting.

2 Graph of Supernodes

In our previous work [5], we had outlined a way of capturing a SPARQL query with an intermix of BGP and OPTIONAL patterns using the Graph of Supernodes (GoSN). For the sake of completeness of the text, here we first describe the process of GoSN construction, and then elaborate on the new enhancements. These enhancements help in our propositions regarding the relaxation of the nullification and best-match operations, based on the cyclic properties of a query. For this construction of GoSN, we focus only on the BGP-OPT patterns without any other SPARQL constructs. They serve as the primitives for applying the BGP-OPT query processing techniques for a broader range of queries with any intermix of UNIONs, FILTERs, and DISTINCT as elaborated in Section 9.

A BGP-OPT pattern query connects multiple BGP patterns with each other using one or more OPTIONAL keywords. Connecting BGP patterns (⋈) with OPTIONAL clauses (⋈) introduces restrictions on the order of joining the triple patterns across various BGPs (refer to the example of reorderability given in Section 1.1). A GoSN is constructed from a BGP-OPT query using supernodes, and unidirectional or bidirectional edges, as follows.

Fig. 2.1: GoSN for Q1 in Section 1

Fig. 2.2: GoSN for ((P_a ⋈ P_b) ⋈ (P_c ⋈ P_d)) ⋈ (P_e ⋈ P_f)

Supernodes: In a BGP-OPT query of the form (P_1 ⋈ P_2), P_1 and P_2 are patterns that may in turn have nested OPTIONAL patterns inside them, e.g., P_1 = (P_2 ⋈ P_3), or either of P_1 and P_2 can be OPT-free. Generalizing it, if a pattern P_i does not have any OPTIONAL pattern nested inside it, it is an OPT-free BGP or simply a BGP. From the given nested BGP-OPT query, first we extract all such BGP, and construct a supernode (SN_i) for each P_i. The triple patterns in P_i are encapsulated in SN_i.

Since BGP are equivalent to SQL inner-joins, and OPTIONAL patterns are equivalent to SQL left-outer-joins, we serialize a nested BGP-OPT query considering its BGP and ⋈ (inner-join), ⋈ (left-outer-join) operators using proper parentheses. E.g., we serialize Q1 in Section 1 as (P_1 ⋈ P_2), where P_1 and P_2 are OPT-free BGP, SN_1 of P_1 encapsulates just T_1, and SN_2 of P_2 encapsulates T_2 and T_3 (see Figure 2.1).

Unidirectional edges: From the serialized query, we consider each OPT pattern of the type P_m ⋈ P_n. P_m or P_n may have nested OPT-free BGP inside them. Using the serialized-parenthesized form, we identify the leftmost OPT-free BGP nested inside P_m and P_n each. E.g., consider the example query given in Figure 2.2. If P_m = ((P_a ⋈ P_b) ⋈ (P_c ⋈ P_d)), and P_n = (P_e ⋈ P_f), P_a and P_e are the leftmost OPT-free BGP in P_m and

\[ P_a = (P_c ⋈ P_d) \]

queries which need to process a large amount of data have low selectivity and vice versa.

\[ P_e = (P_c ⋈ P_d) \]
we treat the very pattern as the leftmost for adding a directed edge. With this procedure, we can treat OPTIONAL patterns in a query in any order, but for all practical purposes, we start from the innermost OPTIONAL patterns, and recursively go on considering the outer OPTIONAL patterns using the parentheses in the serialized query. E.g., if a serialized query is \(((P_a \bowtie P_b) \bowtie (P_l \bowtie P_d)) \bowtie (P_i \bowtie P_l)\), with \(P_a...P_f\) as OPT-free BGPs, we add directed edges as follows: (1) \(S_{N_a} \rightarrow S_{N_b}\), (2) \(S_{N_c} \rightarrow S_{N_d}\), (3) \(S_{N_e} \rightarrow S_{N_f}\), (4) \(S_{N_a} \rightarrow S_{N_c}\).

**Bidirectional edges:** Next we consider each inner-join of type \(P_x \bowtie P_y\) in a serialized query. If \(P_x\) or \(P_y\) has nested OPTIONALs inside, we add a bidirectional edge between the supernodes of leftmost OPT-free BGPs. E.g., if \(P_x = (P_a \bowtie P_b)\), and \(P_y = (P_c \bowtie P_d)\), we add a bidirectional edge \(S_{N_a} \leftrightarrow S_{N_c}\). If \(P_x\) or \(P_y\) does not nest any OPTIONALs inside it, we consider the very pattern to be the leftmost for adding a bidirectional edge. We add bidirectional edges starting from the innermost inner-joins (\(\bowtie\)) using the parentheses in the serialized query, and recursively go on considering the outer ones, until no more bidirectional edges can be added. Considering the same example given under unidirectional edges, we add a bidirectional edge between \(S_{N_a} \leftrightarrow S_{N_c}\). The graph of supernodes (GoSN) for this example is shown in Figure 2.2. Thus we completely capture the nesting of BGPs and OPTIONALs in a query using this GoSN.

**2.1 Nomenclature**

Next, we introduce nomenclatures with respect to the supernodes in a GoSN and the OPTIONAL patterns in a SPARQL query.

**Master-Slave:** In an OPTIONAL pattern \(P_x \bowtie P_l\), we call pattern \(P_l\) to be a master of \(P_d\), and \(P_d\) a slave of \(P_c\). This master-slave relationship is transitive, i.e., if a supernode \(S_{N_f}\) is reachable from another supernode \(S_{N_e}\) by following at least one unidirectional edge in GoSN \((S_{N_e} \rightarrow ...S_{N_f})\), then \(S_{N_e}\) is called a master of \(S_{N_f}\) (see Figure 2.2).

**Peers:** We call two supernodes to be peers if they are connected to each other through a bidirectional edge, or they can be reached from each other by following only bidirectional edges in GoSN, e.g., \(S_{N_a}\) and \(S_{N_e}\) in Figure 2.2.

**Absolute masters:** Supernodes that are not reachable from any other supernode through a path involving any unidirectional edge are called the absolute masters, e.g., \(S_{N_a}\) and \(S_{N_e}\) in Figure 2.2 are absolute masters.

These master-slave, peer, and absolute master nomenclatures and relationships apply to any triple patterns enclosed within the respective supernodes too.

**Well-designed patterns:** As per the definition given by Pérez et al, a well-designed OPT query is – for every subpattern of type \(P' = P_b \bowtie P_i\) in the query, if a join variable \(?j\) in \(P_i\) appears outside \(P'\), then \(?j\) also appears in \(P_b\). A query that violates this condition is said to be non-well-designed.

For the scope of the text in this paper, we mainly consider well-designed queries, because they occur most commonly for RDF graphs, and remain unaffected by the differences between SPARQL and SQL semantics over the treatment of NULLs. Our previous text [5] discusses non-well-designed queries and their effect on the treatment of NULLs. We request the interested readers to refer to those (please see Appendices B and C in [5]).

**2.2 Graph of Triple Patterns (GoT)**

During the GoSN construction, we only added connections between the supernodes formed out of the BGPs in a query based on the structural semantics of the given query. Next we add labeled undirected edges between triple patterns as follows. If two triple patterns share one or more join variables among them, and are in direct master-slave hierarchy, or are part of the same supernode, we add an undirected edge between them. For instance, let \(T_i\) and \(T_j\) share a join variable \(?i\). If \(T_i \in S_{N_a}, T_j \in S_{N_b}, S_{N_a} \rightarrow S_{N_b}\), or if \(T_i \in S_{N_a}, T_j \in S_{N_b},\) then we add an undirected edge between \(T_i\) and \(T_j\), with the edge label \(?j\). Recall that the triple patterns encapsulated in the supernodes share the same master-slave or peer hierarchy as their respective supernodes. These undirected edges among the triple patterns create a graph of triple patterns (GoT) [5]. The GoT for Q1 in Section 1 is shown by “red” connecting edges in Figure 2.1. The edge labels in the GoT are not shown to avoid cluttering the figure.

**Definition 2.1** If the graph of tables (GoT) of a BGP-OPT query is connected, then the query is free from any Cartesian joins, and is considered to be a connected query.

E.g., following is an example of a non-connected query (Cartesian join), because the triple pattern \(\langle factor :livesIn :LA\rangle\) does not have any shared variable with the other two triple patterns \(\langle Jerry :hasFriend ?friend\rangle\) and \(\langle ?friend :name ?name\rangle\).

**SELECT** ?friend ?name ?factor
Fig. 2.3: GoSN of Figure 2.2 after coalescing absolute masters

WHERE {
  :Jerry :hasFriend ?friend .
  ?friend :name ?name .
  OPTIONAL {
    ?actor :livesIn :LA .
  }
}

Let us consider a subgraph of this GoT consisting of only the triple patterns encapsulated inside all the absolute master supernodes and all the undirected edges incident on them.

**Property 2.1** If a query is well-designed and connected, then the subgraph of GoT consisting only of triple patterns within the absolute masters is always connected.

**Property 2.2** In a well-designed connected query, a slave supernode never has more than one incoming unidirectional edge.

Observing Properties 2.1 and 2.2, we coalesce all the absolute masters of GoSN to form a single absolute master supernode of the GoSN – \( SN_{abs} \). While doing so, we remove any bidirectional edges incident on the coalesced absolute master supernodes. For every unidirectional edge between a coalesced absolute master and its slave, a unidirectional edge is added between \( SN_{abs} \) and the corresponding slave. Figure 2.3 shows the transformed GoSN of the original GoSN shown in Figure 2.2 after coalescing absolute masters \( SN_a \) and \( SN_c \).

### 3 Acyclicity and Minimality

In this section, first we define the acyclicity of a SPARQL (equivalently SQL) query, and then discuss the minimality of triples (tuples), and the effect of the cyclic properties of a query on it.

#### 3.1 Acyclicity of Queries

For the concept of acyclicity of a query we take into consideration the graph of tables (GoT). We define the equivalence classes of edges of GoT as follows.

**Definition 3.1** For every triple pattern, its incident edges in GoT are put in equivalence classes such that all the edges in a given equivalence class have either same edge labels or their edge labels are subset of some other edge’s label in the same class. E.g., if \( T_i \) has three edges incident on it with labels \{?a\}, \{?a,?b\}, \{?c\}, then \{\{?a\},\{?a,?b\}\} make one equivalence class and \{\{?c\}\} in another.

A triple pattern is called a leaf if it has only one equivalence class among its incident edges. An acyclic query is then defined as follows. If we recursively remove leaf triple patterns, and edges incident on them from a GoT, and then if we are left with an empty GoT at the end, then the query is acyclic. The set of leaf triple patterns are chosen recursively in each round after previous leaves and their incident edges are removed. This process is reminiscent of GYO-reduction \[29\]. GYO-reduction assumes a hypergraph where each attribute in a table is a node, and a hyperedge represents a table. However, to be consistent with our representation of GoT and GoSN, we have formulated this definition of acyclicity instead of using GYO-reduction.

#### 3.2 Minimality of Triples

The triples associated with a triple pattern (or tuples in a table) are said to be minimal for the given join (BGP or BGP-OPT) query, if every triple (tuple) is part of one or more final join results of the query. There does not exist any triple that gets eliminated as a result of its join with another triple (associated with another triple pattern). Consider the same query given in Figure 2.1 along with the sample data associated with it in Figure 1.1. \( T_2 \) \(?friend :actedIn ?sitcom\) has five triples associated with it – (1) :Larry :actedIn :CurbYourEnthu, (2) :Julia :actedIn :Seinfeld, (3) :Julia :actedIn :Veep, (4) :Julia :actedIn :NewAdoOldChristine, (5) :Julia :actedIn :CurbYourEnthu. But they are not minimal for this query, because after \( T_2 \)’s join with \( T_3 \) (?sitcom :location :NYC), all tuples but :Julia :actedIn :Seinfeld associated with \( T_2 \) get eliminated.

### 4 Nullification and Best-match

SPARQL BGP queries are analogous to the SQL inner-join queries, and hence the joins over the triple patterns in BGP queries are associative and commutative, i.e., a change in the order of joins between the triple patterns does not change the query results. SPARQL queries with OPTIONAL patterns are analogous to the SQL left-outer-joins, and hence they are not associative.
or commutative. An example of such non-reorderable query is given in Section 4. However, reorderability of joins is a powerful feature that enables the query optimizer to explore many query plans. Hence, Rao et al. and Galindo-Legaria, Rosenthal proposed ways of reordering intermixed inner and left-outer joins by using additional operators nullification and best-match (or Generalized Outerjoin) [12,13,24,25].

For the completeness of the text, first we will briefly see how nullification and best-match operators work with the same example as given in [5]. For more details of these operators, we request the interested readers to refer to [25,24,12,13]. Consider the same query given in Figure 4.1 along with the sample data associated with it in Figure 4.1. :NYC has been the location for a lot of American sitcoms, and a lot of actors have acted in them (they are not shown in the sample data for conciseness). But, among all such actors, :Jerry has only two friends, :Julia and :Larry. Hence, T1 is more selective than T2 and T3. A left-outer-join reordering algorithm as proposed in [25,13] will typically reorder these joins as (T1 ⊥ T2) ⊥ T3. Due to this reordering, all four sitcoms that :Julia has acted in show up as the bindings of ?sitcom (see Res1 in Fig. 4.1), although only :Seinfeld was located in the :NYC. To fix this, nullification operator is used, which ensures that variable bindings across the reordered joins are consistent with the original join order in the query (see Res2 in Figure 4.1).

The nullification operation caused results that are subsumed within other results. A result r1 is said to be subsumed within another result r2 (r1 ⊆ r2), if for every non-null variable binding in r1, r2 has the same binding, and r2 has more non-null variable bindings than r1. Thus results 3–5 in Res2 are subsumed within result 2. The best-match operator (or minimum union as defined by Galindo-Legaria in [11]) removes all the subsumed results (see Res3). Final results of the query are given as best-match(nullification((T1 ⊥ T2) ⊥ T3)).

4.1 Nullification, Best-match, and Minimality of Triples

In [5], we made an important observation that if every triple pattern in an OPTIONAL pattern query has minimal triples associated with it, then nullification and best-match operations are not required (ref. Lemma 3.1 in [5]). We made yet another observation through Lemmas 3.3 and 3.4 in [5], that for the following two classes of the OPTIONAL pattern queries nullification and best-match is not required if we use Algorithm-1 in [5] to reduce the set of triples associated with each triple pattern in the query, prior to generating final results using multi-way pipeline join algorithm (Algorithm-3 in [5]).

– Acyclic GoT: OPTIONAL pattern queries whose GoT is acyclic.
– Only one join variable per slave: OPTIONAL pattern queries where there are cycles in the GoT, but any slave triple pattern has only one join variable in it, and that join variable is shared with its master triple pattern.
These classes of OPTIONAL pattern queries are considered good because they can avoid the overheads of the nullification and best-match operations despite the reordering of inner and left-outer joins. The important premise of these observations is that Algorithm-1 reduces the triples associated with each triple pattern in the OPTIONAL pattern query in such a way that even if we reorder the inner and left-outer joins while doing the multi-way pipelined join, it does not generate spurious results, and thus avoids the necessity of nullification and best-match. We extend this class of good OPTIONAL pattern cyclic queries beyond the ones in which all slaves have only one join variable, and these are discussed in Section 5.2. Before that we revisit our pruning method and multi-way joins to work with just GoT, and obviate the need of graph of join variables (GoJ) that was used in our previous work [5].

5 Pruning Triples

SPARQL (in turn SQL) queries can be evaluated using different equivalent plans. All the plans output exact same results. Typically, a plan with the least cost is chosen for evaluation. In the previous sections, we established the relationship between structural properties of a BGP-OPT query, minimality of triples, and nullification, best-match operations. However, we get the benefit of avoiding nullification and best-match, only if the triples associated with the query before performing the reordered joins are in the minimal (or favorable) form. We ensure that by using the pruning step before performing joins. The pruning phase only prunes the triples associated with each triple pattern in the query using a series of semi-joins, and the multi-way pipelined joins then produce the join results in a pipelined fashion, followed by nullification and best-match if required.

5.1 Semi-joins

Pruning of triples without performing joins is achieved through semi-joins. Semi-joins can be notationally represented as follows. $T_2 \bowtie_? j T_1 = \{ t \mid t \in T_2, t.？ j \in (\pi_{？ j}(T_1) \cap \pi_{？ j}(T_2)) \}$. Here $t$ is a triple matching $T_2$, and $t.？ j$ is a variable binding (value) of variable $？ j$ in $t$. After this semi-join, $T_2$ is left with only triples whose $？ j$ bindings are also in $T_1$, and all other triples are removed.

Bernstein et al [8, 7] and Ullman [29] have proved previously that if the graph of tables (GoT) of an inner-join query is acyclic, a bottom-up followed by a top-down pass with semi-joins at each table in this tree, reduces the set of tuples in each table to a minimal. Note that for the discussion of minimality of triples and semi-joins, we focus on the GoT of a query, and not GoSN. GoSN inherently encapsulates GoT inside it, since GoSN maintains connections between BGP blocks, whereas GoT maintains connections between individual triple patterns.

In our previous work, we proposed a pruning algorithm for BGP-OPT queries that makes use of graph of join variables (GoJ) and clustered-semi-joins. However, in this paper we propose an improved algorithm. Our algorithm is reminiscent of the full reducer semi-join sequence as given in [29], that uses the concept of graphs with hyperedges to represent tables (triangle patterns for SPARQL). But full-reducers only addressed the inner-joins, and we address an intermix of inner and left-outer joins. We first discuss it in Algorithm 5.1 and then discuss its differences from our previous Algorithm-1 in [5].

![Algorithm 5.1: prune_triples](image-url)
of a query according to the master-slave hierarchy – masters before their respective slaves, and among any two supernodes not in the master-slave hierarchy, we randomly pick one over another. Note that since these are well-designed queries and we have coalesced all the absolute master supernodes into SN_{abs}, the relative ordering among two such supernodes not in the master-slave hierarchy does not matter. We call this ordering among the supernodes sn-order as given by the order-supernodes function (ln 11), and SN_{abs} appears first in this order. If GoT is cyclic and there are cycles in slave supernodes too, then we just do pruning by following a greedy order of semi-joins – doing semi-joins over highly selective triple patterns first – honoring the master-slave hierarchy among the triple patterns \( T \). In case of a tie, we pick one randomly (ln 24). In doing this for all the triple patterns in \( T \), we pick SN_{i} is a slave supernode, we create \( \text{sn-order} \) from \( \text{sn-order} \) in (ln 12). We consider all the triple patterns encapsulated within that supernode. For each triple pattern \( T_{i} \) in \( SN_{i} \), and its connected triple patterns in \( SN_{i} \) alone, we create equivalence classes of edges of GoT incident on \( T_{i} \) (recall the definition of equivalence class of edges given in Section 3). We do not consider edges that connect \( T_{i} \) with triple patterns outside \( SN_{i} \). After doing this for all the triple patterns in \( SN_{i} \), we pick the triple patterns that have only one equivalence class among its edges (ln 17). This triple pattern is a leaf node. Among all such leaf nodes, we pick one which has the least number of triples associated with it – most selective one. In case of a tie, we pick one randomly (ln 18).

Among leaf \( T_{i} \)'s connected triple patterns within \( SN_{i} \), we pick the neighbors with the least number of triples associated with it, say \( T_{j} \) (ln 22), and add a semi-join \( T_{j} \bowtie T_{i} \) to the queue \( \text{order}_{bu} \) (bottom-up semi-join order) (ln 26). If \( SN_{i} \) is a slave supernode, we ensure to fetch its master’s variable binding restrictions as follows. If \( SN_{i} \) is a master of \( SN_{j} \), such that \( SN_{i} \rightarrow SN_{j} \), we transfer the constraints on the variable bindings from \( SN_{j} \) to \( SN_{i} \) as follows. Without losing generality, while adding a semi-join between two triple patterns \( T_{j} \bowtie T_{i} \) in \( SN_{i} \) to \( \text{order}_{bu} \), we first check if \( T_{i} \) has a neighbor \( T_{m} \) in \( SN_{i} \). If it does, then we add \( T_{j} \bowtie T_{m} \) to \( \text{order}_{bu} \) before \( T_{j} \bowtie T_{i} \). This ensures that any variable bindings imposed by a master of the triple patterns are transferred to their respective slave triple patterns (ln 27). Next we remove \( T_{i} \) and all the edges incident on it from consideration, and repeat the same procedure described above with the rest of the triple patterns in \( SN_{i} \).

Notice that through this procedure we recursively define a spanning tree over the graph of triple patterns (GoT) encapsulated within \( SN_{i} \), and make a bottom-up pass over it – every semi-join \( T_{j} \bowtie T_{i} \) denotes \( T_{j} \) to be the parent of \( T_{i} \) in the spanning tree, and the semi-join denotes the direction of the walk from the leaves to the internal nodes and the root of the tree, e.g., semi-join \( T_{j} \bowtie T_{i} \) denotes a walk from \( T_{i} \rightarrow T_{j} \) on the spanning tree. For the top-down pass, we simply reverse the queue \( \text{order}_{bu} \) to get \( \text{order}_{td} \) (ln 27). That is for every semi-join of type \( T_{j} \bowtie T_{i} \) in \( \text{order}_{bu} \), we add \( T_{i} \bowtie T_{j} \) to \( \text{order}_{td} \). However, we omit all the semi-joins of type \( T_{m} \bowtie T_{i} \) where \( T_{m} \) is a master of \( T_{i} \) (ln 25).

When the GoT is acyclic, the very first \( SN_{i} \) is always \( SN_{abs} \).

The main differences between our previously proposed technique (ref Algorithms 3.1, 3.2 in [5]), and Algorithm 5.1 here are as follows:

- In the present technique, we form a rooted tree over GoT, whereas in [5], the rooted tree was formed over graph of join variables (GoJ).
- In our present technique the rooted tree over GoT is formed in a bottom-up fashion, where we first pick the most selective triple patterns as the leaves and recursively build the internal nodes and root of the tree. In [5], the rooted tree over GoJ was formed in a top-down fashion, where we first picked a join variable with lowest selectivity as the root of the tree, and then recursively picked the internal nodes and leaves.
- In [5], we first did a bottom-up pass over all the induced GoJs of individual supernodes, and then did the top-down passes. In the present technique, we do the bottom-up and top-down passes on the triple patterns of each supernode in one go. Thus we do the full possible pruning of the triples associated with the triple patterns in a supernode, before moving on to its slaves. This helps in better pruning of the slaves, because their respective masters are already in the pruned state.

Our discussion of the properties of nullification, cyclicity of queries, minimality of triples, and the pruning procedure until now has been agnostic to the lower level storage structure and indexes on the RDF graph. Indeed, our propositions and algorithms presented earlier can work on any storage and index structure – only the semi-joins procedure used in Algorithm 5.1 will change depending on the storage structure and indexes. Nevertheless, an efficient storage and index structure helps in achieving better performance. We
achieve this through the usage of compressed bitvector indexes on RDF graphs, and procedures that directly work on these compressed indexes without decompressing them as proposed in \[9\]. For the completeness of the text, these are described in Section 5.

6 BitMat Indexes

In an RDF graph, let \( V_s \), \( V_p \), and \( V_o \) be the sets of unique subject, predicate, and object values. Then a 3D bitcube of RDF data has \( V_s \times V_p \times V_o \) dimensions. Each cell in this bitcube represents a unique RDF triple formed by the coordinate values (S P O). If this (S P O) triple is present in the given RDF dataset, that bit is set to 1 in the bitcube. The unique values of subjects, predicates, and objects in the original RDF data are first mapped to integer IDs, which in turn are mapped to the bitcube dimensions. To facilitate joins on S-O indexes, the bitcube is conceptually sliced along each dimension, by doing a bitwise OR on the other dimensions, and set \( V_p \) is mapped to a sequence of integers 1 to \( |V_p| \). Set \( V_o - V_s \) is mapped to a sequence of integers \( |V_o| + 1 \) to \( |V_o| \), and set \( V_p \) is mapped to a sequence of integers 1 to \( |V_p| \).

This bitcube is conceptually sliced along each dimension, and the 2D BitMats are created. In general, four types of 2D BitMats are created: (1) S-O and O-S BitMats by slicing the P-dimension (O-S BitMats are nothing but a transpose of the respective S-O BitMats), (2) P-O BitMats by slicing the S-dimension, and (3) P-S BitMats by slicing the O-dimension. Altogether we store \( 2 \times |V_p| + |V_s| + |V_o| \) BitMats for any RDF data. Figure 4.1 shows 2D S-O BitMats that we can get by slicing the predicate dimension (others are not shown for conciseness).

Each row of these 2D BitMats is compressed as follows. In the run-length-encoding, a bit-row like “111000” is represented as “[1] 3 2 1 4”, and “0010010000” is represented as “[0] 2 1 2 1 4”. Notably, in the second case, the bit-row has only two set bits, but it has to use five integers in the compressed representation. So we use a hybrid representation in our implementation that works as follows – if the number of set bits in a bit-row are less than the number of integers used to represent it, then we simply store the set bit positions. So “0010010000” will be compressed as “3 6” (3 and 6 being the positions of the set bits). This hybrid compression fetches us as much as 40% reduction in the index space compared to using only run-length-encoding as done in [1].

Other meta-information such as, the number of triples, and condensed representation of all the non-empty rows and columns in each BitMat, is also stored along with each BitMat. This information helps us in quickly determining the number of triples in each BitMat and its selectivity without counting each triple in it, while processing the queries. A 2D S-O or O-S BitMat of predicate :hasFriend represents all the triples matching a triple pattern of kind (\(?a :\text{hasFriend} \ ?b\)) , a 2D P-S BitMat of O-value :Seinfeld represents all triples matching triple pattern (\(?c \ ?d :\text{Seinfeld}\)), and so on.

Fig. 6.1: 3D Bitcube of RDF data in Figure 4.1

Query execution uses the fold and unfold primitives, which process the compressed BitMats without uncompressed them [9].

Fold operation is represented as ‘fold(BitMat, RetainDimension) returns bitArray’. It takes a 2D BitMat and folds it by retaining the RetainDimension. More succinctly, a fold operation is nothing but projection of the distinct values of the particular BitMat dimension, by doing a bitwise OR on the other dimension. It can be represented as:

\[
\text{fold}(BM_{Ti}, \text{dim}_{Tj}) \equiv \pi_{Tj}(BM_{Tj})
\]

\(BM_{Tj}\) is a 2D BitMat holding the triples matching \(T_j\), and \(\text{dim}_{Tj}\) is the dimension of BitMat that represents variable ?j in \(T_j\). E.g., for a triple pattern (?friend :actedIn ?sitcom), if we consider the O-S BitMat of predicate : actedIn, ?friend values are in the column dimension, and ?sitcom values are in the row dimension of the BitMat.

Unfold is represented as ‘unfold(BitMat, MaskBitArray, RetainDimension)’. For every bit set to 0 in the MaskBitArray, unfold clears all the bits corresponding to that position of the RetainDimension of the BitMat. Unfold can be simply represented as:

\[
\text{unfold}(BM_{Ti}, \beta_{Tj}, \text{dim}_{Tj}) \equiv \{t \mid t \in BM_{Tj}, t.j \in \beta_{Tj}\}
\]

\[\text{for the scope of this paper, we do not consider joins on S-P or O-P dimensions.\]
t is a triple in BM_{T_{i}} that matches T_{i}. \beta_{j} is the MaskBitArray containing bindings of ?j to be retained. dim_{j} is the dimension of BM_{T_{j}} that represents \beta_{j}, and t.?j is a binding of ?j in triple t. In short, unfold keeps only those triples whose respective bindings of ?j are set to 1 in \beta_{j}, and removes all other.

Until now we discussed semi-joins only notionally as T_{i} \bowtie T_{k} = \{t \mid t \in T_{i}, t.?j \in \{\pi_{T_{i}}(T_{i}) \cap \pi_{T_{k}}(T_{k})\}\}. Semi-joins can be implemented using the BitMat indexes and the fold, unfold primitives as given in Algorithm 6.1.

Algorithm 6.1: semi-join

\textbf{input}: T_{i}, T_{k}, ?j

// T \bowtie T_{k}
1. \beta_{j} = \text{fold}(BM_{T_{i}}, \text{dim}_{j}) \text{ AND } \text{fold}(BM_{T_{k}}, \text{dim}_{j});
2. unfold(BM_{T_{i}}, \beta_{j}, \text{dim}_{j});

7 Multi-way Pipelined Joins

In the previous sections, we established the algorithmic basis of our techniques for understanding the characteristics of a BGP-OPT query and pruning the triples associated with the each triple pattern in the query before generating the final results – Algorithm 5.1 only prunes the candidate RDF triples, but does not generate the final results. In Section 6, we took an overview of our storage and indexing structure for the RDF graphs. In this section, put the pruning algorithm and index structure together with our technique of \textbf{multi-way-pipelined joins} to generate the final results as given in Algorithm 7.1.

Algorithm 7.1: Query processing

\textbf{input} : Original BGP-OPT query
\textbf{output} : Final results
1. GoSN = get-GoSN(Orig BGP-OPT query);
2. \text{bool NB-reqd = decide-best-match-reqd(GoSN, GoJ);}
3. prune_triples(GoSN); // Alg 5.1
4. tporder = sort-tps-master-slave();
5. stps = spanning-tree-tps();
6. vmap = empty-map, size as num of vars in query;
7. \text{// Final result generation - Alg 7.2}
8. if NB-reqd then
9. \text{finalres = best-match(alternes);}
10. else
11. \text{finalres = alternes;}
12. return finalres;

In Algorithm 7.1 for query processing, we first construct the GoSN (ln 1). Next, we decide if nullification and best-match are required. This decision is based on the cyclic properties of the query, and we discuss them in Section 8 and Lemmas 5.1 and 5.2. We call \textbf{prune_triples} (Algorithm 5.1) to prune the unwanted triples. Pruning operation also on-the-fly loads the BitMat associated with each triple pattern containing only triples satisfying that triple pattern (we have not shown this operation explicitly in Algorithm 5.1 but is explained next).

We choose an appropriate BitMat for each triple pattern as follows. If the triple pattern in the query is of type (?var :fx1 :fx2), i.e., with two fixed positions, we load only one row corresponding to :fx1 from the P-S BitMat for :fx2. Similarly for a TP of type (:fx1 :fx2 ?var), we load only one row corresponding to :fx2 from the P-O BitMat for :fx1. E.g., for (?sitcom :location :NYC) we load only one row corresponding to :location from the P-S BitMat of :NYC. If the triple pattern is of type (?var1 :fx1 ?var2), we load either the S-O or O-S BitMat of :fx1. If ?var1 is a join variable and ?var2 is not, we load the S-O BitMat and vice versa. If both, ?var1 and ?var2, are join variables, we observe the order of semi-joins in order with, to check if the semi-join over the given triple pattern is over ?var1 or ?var2 first. If semi-join over ?var1 comes before ?var2, we load the S-O BitMat and vice versa.

While loading the BitMats, we do active pruning using the triple patterns whose BitMats are already initialized. E.g., if we first load BitMat BM_{T_{i}} containing triples matching (?:Jerry :hasFriend ?friend), then while loading BM_{T_{j}}, we use the bindings of ?friend in BM_{T_{i}} to actively prune the triples in BM_{T_{j}} while loading it. Then while loading BM_{T_{k}}, we use the bindings of ?sitcom in BM_{T_{j}} to actively prune the triples in BM_{T_{k}}. We check whether two triple patterns are joining with each other over an inner or left-outer join using GoSN with the \textbf{master-slave} or peer relationship, and then we decide whether to use other BitMat’s variable bindings.

Note that using \textbf{prune_triples}, we prune the triples in BitMats, but we need to actually “join” them to produce the final results. For that we use \textbf{multi-way-join} (ln 7 in Algorithm 7.1). This procedure is described separately in Section 7.1. After \textbf{multi-way-join}, we use \textbf{best-match} to remove any subsampled results if nullification was required as a part of \textbf{multi-way-join} (discussed in Section 8). In \textbf{best-match}, we externally sort all the results generated by \textbf{multi-way-join}, and then remove the subsampled results with a single pass over them.
### Algorithm 7.2: `multi-way-join`

```plaintext
input : vmap, stps, visited, nulreqd
output: all the results of the query

1. if visited.size == stps.size then
2. if nulreqd then
3. nullification (vmap);
4. output (vmap);  // generate a single result
5. return;
6. if visited is empty then
7. T_1 = first TP from stps;
8. visited.add(T_1);
9. for each triple t ∈ BM_{T_1} do
10. generate bindings for vars(T_1) from t, store in vmap;
11. multi-way-join(vmap, stps, visited, nulreqd);
12. else
13. T_i = next triple pattern in stps;
14. atleast-one-triple = false;
15. for t ∈ BM_{T_i} with same bindings do
16. atleast-one-triple = true;
17. store vars(T_i) bindings of t in vmap;
18. visited.add(T_i);
19. multi-way-join(vmap, stps, visited, nulreqd);
20. visited.remove(T_i);
21. if (atleast-one-triple == false) then
22. if T_i is an absolute master then
23. return;  // This means T_i is a slave
24. set all vars(T_i) to NULL in vmap;
25. visited.add(T_i);
26. multi-way-join(vmap, stps, visited, nulreqd);
27. visited.remove(T_i);
```

7.1 Multi-way Pipelined Join

Before calling `multi-way-join`, we first sort all the triple patterns in the query as follows. Considering the triple patterns in SN_{obs}, we sort them in the ascending order of the number of triples left in each triple pattern’s BitMat. Then we sort the remaining supernodes in the descending order of master-slave hierarchy. That is, among two supernodes connected as SN_1 → SN_2, triple patterns in SN_1 are sorted before those in SN_2. Among the triple patterns in the same supernode (peers), they are sorted in the ascending order of the number of triples left in their BitMats. Let us call this order `tporder` (ln 4 in Algorithm 7.1). From `tporder`, we construct a conceptual spanning tree as follows. The very first triple pattern is designated as the root of the tree, and added to a new sort order of triple patterns, called `stps`. We recursively pick the next triple pattern from `tporder` such that it is connected to at least one triple pattern in `stps` (ln 5 in Algorithm 7.1). This `stps` is used in `multi-way-join` to produce final results and decide the join order. In `multi-way-join` we also use at most ∑_{T_i ∈ Q} vars(T_i) additional memory buffer, where vars(T_i) are the variables in every triple pattern T_i in the query Q. This is `vmap` in Alg. 7.2. Thus we use negligible additional memory in `multi-way-join`.

At the beginning, `multi-way-join` gets `stps`, an empty `vmap` for storing the variable bindings, an empty `visited` list, and a flag `nulreqd` indicating if nullification is required. We go over each triple in BM_{T_1} of the first triple pattern in `stps`, generate bindings for the variables in T_1, and store them in `vmap`. We add T_1 to the `visited` list, and call `multi-way-join` recursively for the rest of the triple patterns (ln 6–11). In each recursive call, `multi-way-join` gets a partially populated `vmap` and a `visited` list that tells which triple pattern’s variable bindings are already stored in `vmap`. `Stps` order is formed in a way that from second position onward each triple pattern in `stps` has at least one connection in `stps` order before it. Thus, for the next recursive call of `multi-way-join`, we are expected to find at least one variable binding in `vmap` for the next non-visited T_i (ln 12). `Stps` order ensures that a master triple pattern’s variable bindings are stored in `vmap` before its slaves. If there exists one or more triples t in BM_{T_i} consistent with the variable bindings in `vmap`, then for each such t we generate bindings for all the variables in T_i, store them in `vmap`, and proceed with the recursive call to `multi-way-join` for the rest of the triple patterns (ln 13–20). Notice that, this way we `pipeline` all the BitMats, and do not use any other intermediate storage like hash-tables.

If we do not find any triple t in BM_{T_i} consistent with the existing variable bindings in `vmap`, then – (1) if T_i is an absolute master, we `rollback` from this point, because an absolute master triple pattern cannot have NULL bindings (ln 23), else (2) we map all the variables in T_i to NULLs, and proceed with the recursive call to `multi-way-join` (ln 24–27). When all the triple patterns in the query are in the `visited` list, we check if we require `nullification` to ensure consistent variable bindings in `vmap` across all the slave triple patterns, and output one result (ln 28–30). We continue this recursive procedure till triples in BM_{T_i} are exhausted (ln 31–33). Intuitively, `multi-way-join` is reminiscent of a relational join plan with reordered left-outer-joins – that is, in `stps` we sort master triple patterns before their slaves, and masters generate variable bindings before slaves in `vmap`.

8 Cycles in the Queries

In this section we discuss the cyclic properties of the queries and how they affect the requirement of nullification and best-match operations after reordering the inner and left-outer-joins in a query, that makes an important part of reorderability of joins.
8.1 Acyclic Queries

In [5], we proposed our pruning algorithm for OPTIONAL pattern queries by constructing a graph of join variables (GoJ) as follows. Every join variable in the query is a node in GoJ, and two join variables have an undirected edge between them, if both the join variables co-occur in a triple pattern in the query. In [5], we proposed that if the GoT of an OPT query is acyclic then the GoJ is acyclic too. However, there exists queries where this equivalence may not hold. An example of such a query is given below.

```sparql
SELECT ?a ?b ?c
WHERE {
  ?a :p1 ?b .
  ?b :p2 ?c .
  ?c :p3 ?a .
  ?a ?b ?c .
}
```

Note that in this query the triple pattern (?a ?b ?c) joins with every other triple pattern over two join variables. Per our definition of acyclicity given in Section 3.1, this query is indeed acyclic. However, its GoJ is cyclic as shown in Figure 8.1. Also note that in this query there is a join on the object-predicate position ($T_1$ and $T_3$) which is very uncommon in the RDF data. Most SPARQL queries have triple patterns joining over only one variable, and these joins are on subject-object, subject-subject, or object-object positions (since they naturally indicate edges incident on the nodes in the graph). Hence our original proposition and the technique of clustered-semi-joins on GoJ in [5] still work correctly for the SPARQL queries where any two triple patterns always join only over one join variable (and all the queries used in our experiments in [5] satisfied this condition). Nevertheless, our improved `prune_triples` method (Algorithm 5.1) in this article works correctly for such corner case queries too.

**Lemma 8.1** Nullification and best-match can be avoided for an OPTIONAL pattern query with an acyclic GoT.

*Proof* Nullification and best-match processes are described in Section 4. In that, it can be observed that nullification is required when some variable in a triple pattern $T_i$ is bound to a value that does not exist in another triple pattern $T_j$ that is either $T_i$’s master or peer. When a BGP-OPT query is completely acyclic – individual GoTs of each OPT-free BGP component as well as the entire GoT of the query across all the triple patterns is acyclic – pruning (Algorithm 5.1) ensures that each triple pattern is left with minimal number of triples – there does not exist any binding for a variable in $T_i$ that does not exist in its master or peer triple pattern $T_j$.

This minimality is ensured as follows. When we do pruning of triples in $SN_{abs}$, the triples associated with triple patterns in $SN_{abs}$ are reduced to minimal. This follows directly from the proofs of Bernstein et al. and Ullman [8,7,29]. When we do pruning of triples in slave supernodes, the bindings in its acyclic master supernode are already minimalized, and we propagate those on the present slave supernode’s variable bindings (lines 19–21 in Algorithm 5.1). Thus after completion of `prune_triples`, each triple pattern in the query has minimal triples associated with its triple patterns. Thus nullification and best-match can be avoided. □

8.2 Cyclic Queries

For OPT-free BGPs, i.e., pure inner-joins, with a cyclic GoT, minimality of triples cannot be guaranteed using the procedure of bottom-up followed by top-down pass of semi-joins on the spanning tree over GoT as described in in Section 8.1 [8,7,29]. This result carries over immediately to the queries with an intermix of BGP and OPT patterns too whose GoT is cyclic. Thus, we simply prune the triples by following a greedy order of semi-joins, while adhering to the master-slave hierarchy among the triple patterns in the query. The greedy order of semi-joins is determined by the relative selectivity of the triple patterns, and the master-slave hierarchy between them (ln 7 in Algorithm 5.1). Since minimality of triples in each TP is not guaranteed, we need to use the nullification and best-match operations in a reordered query to ensure consistent variable bindings, and to remove subsumed results.

This observation in general holds for all cyclic BGP-OPT queries, but we identify a subclass of cyclic BGP-OPT queries that can avoid nullification and best-match – in such queries the following conditions hold:
1. A subgraph of GoT representing only the triple patterns in any slave supernode is acyclic.

2. There is only equivalence class of edges connecting triple patterns in the master-slave hierarchy between two supernodes. That is, if $SN_A \rightarrow SN_B$ and we put all GoT edges of type $\langle T_m, T_s \rangle$, $T_m \in SN_A, T_s \in SN_B$ in equivalence classes, then there is only one equivalence class. Generalizing, this property holds for each and every pair of supernodes $SN_A \rightarrow SN_B$ in master-slave hierarchy that have an directed edge among them in GoSN. Recall our definition of equivalence classes of edges given in Definition 3.1.

For such queries, we do greedy way of pruning using semi-joins over the triple patterns in $SN_{abs}$, and after that for each slave supernode, we follow the same procedure of bottom-up and top-down pruning as described in Algorithm 5.1 (ref. ln 7–11 and ln 12–30).

In Figure 8.2, we have given an example of a query where there are two equivalence classes of GoT edges running between $SN_{abs}$ and the slave supernode $SN_B$, and hence it cannot avoid nullification and best-match. Notice carefully that in this query although the individual GoTs of $SN_{abs}$ and the slave supernode are acyclic, the GoT over all the triple patterns is cyclic.

**Lemma 8.2** For a BGP-OPT query, if (1) there is only one equivalence class of edges across any pair of master-slave supernodes, and (2) each subgraph of GoT representing the triple patterns in each slave supernode is acyclic, then nullification and best-match can be avoided if the triples associated with the triple patterns are pruned with prune_triples (Algorithm 5.7), and results are produced using multi-way-join (Algorithm 7.2). This holds even if the GoT of the triple patterns in $SN_{abs}$ is cyclic.

**Proof** In Lemma 8.1 we saw that if each OPT-free BGP component of a query has triple patterns with minimal triples associated with it, nullification and best-match can be avoided. When $SN_{abs}$ has a cyclic GoT, it cannot be guaranteed that the triple patterns in it have minimal triples associated with them after prune_triples method. Note that we prune the triples in $SN_{abs}$ before any other slave supernodes in the query, and then do not revisit it again in prune_triples (lines 1 and 12).

Thus when a slave supernode fetches the variable bindings from its master (line 10–21) in Algorithm 5.1, the master’s variable bindings have already been frozen (irrespective of whether minimal or not). During multi-way-join, we again visit triple patterns in their master-slave hierarchy, i.e., we start with the triple patterns in $SN_{abs}$, create variable bindings for all the triple patterns in it, and then move on to other slave supernodes, in their respective master-slave hierarchy.

If the GoT of just triple patterns in $SN_{abs}$ has cycles, then we backtrack while generating variable bindings for its triple patterns whenever there is a mismatch (ln 23 in Algorithm 7.2). Thus when we start processing the slave triple patterns from an acyclic slave supernode, all the variable bindings of $SN_{abs}$ have been decided, and are consistent across all the triple patterns of $SN_{abs}$. Having only one equivalence class of edges between $SN_{abs}$ and its slave, say $SN_i$, means that there is always one triple pattern, say $T_m$, in $SN_{abs}$ and respectively $T_s$ in $SN_i$, which have one or more shared variables between them that cover all the shared variables between $SN_{abs}$ and $SN_i$. Then in multi-way-join, whenever we start processing the triple patterns from $SN_i$ for generating variable bindings, we first ensure that $T_s$ in $SN_i$ has the exact same bindings for all the variables shared between it and $T_m$. It may happen that such a (composite) binding does not exist in $T_s$, then we immediately set all the variables in $SN_i$ to NULL bindings, and do the same for all of slaves of $SN_i$ too. Thus at the end of one iteration of multi-way-join, we do not have any inconsistent variable bindings in vmap that necessitate nullification, followed by best-match. We assume that the BGP-OPT queries are well-designed, which ensures that no master-slave supernode that is not connected with a directed edge share variables among them, that are not shared between any intermediate master supernode.

Note that this is possible only because each slave supernode has an acyclic GoT, and the triple patterns in it have minimal triples, that are consistent across the same slave supernode. Since there is only one equivalence class edge between any master-slave pair of supernodes, a unique triple pattern in the slave can determine if the respective bindings exist in the slave.
UNIONs, FILTERs, DISTINCT

Basic Graph Patterns (BGPs) connect the triple patterns through inner-joins, and the OPTIONAL patterns connect two BGPs through left-outer-joins. BGP-OPT patterns make the basic building blocks of the SPARQL query language, as we will see in this section later through the query rewriting rules in UNIONs and FILTERs. SPARQL, like SQL, has other non-join constructs too such as UNIONs, FILTERs, DISTINCT. SPARQL also has additional constructs like ORDER BY, FROM, FROM NAMED, REDUCED, OFFSET, LIMIT, ASK, CONSTRUCT etc. SPARQL grammar is recursive, and it can be pictorially represented as shown in Figure 9.1. In our work we have mainly focused on the SPARQL components that constitute the recursive part of the language, and thus make the performance intensive components of the query evaluation strategies. Having done the BGP-OPT component analysis in the previous sections, further in this section, we analyze UNIONs, FILTERs, and DISTINCT clauses, and their interaction with the BGP-OPT component.

9.1 UNIONs

The SPARQL grammar does not enforce the two patterns being UNIONed to be union compatible (SQL does). However, for our discussion, we assume well-designed UNIONs - for every subpattern $P' = (P_1 \cup P_2)$ in a query, if a variable $?j$ appears outside $P'$, then it must appear both in $P_1$ and $P_2$. This assumption avoids having to deal with a query with “dangerous” variables, Cartesian products, and disconnected GoT [23].

For any BGP-OPT-UNION query, first we identify UNION-free BGP-OPT subcomponents of the query. E.g., in a query of the form $P_1 \cup ((P_2 \bowtie P_3) \cup (P_4 \bowtie P_5))$, where $P_{1...5}$ are all OPT-free BGPs, and following are the UNION-free components - (a) $P_1$, (b) $P_2 \bowtie P_3$, (c) $(P_4 \bowtie P_3)$. For each such UNION-free BGP-OPT sub-component of the query, we prune the triples associated with each triple pattern in that component using our prune_triples procedure (Algorithm 5.1). Note that for this pruning procedure, we ignore all the other sub-components of the query and the respective triple patterns in those sub-components. We only consider the given sub-component as an independent BGP-OPT query.

Next we apply following three conversion rules on the BGP-OPT-UNION query and convert the query in the UNION Normal Form (UNF) [21]. Note that for the notations given below a pattern $P_i$ need not be a BGP, it can be a pattern with other sub-patterns nested inside it, and the $\equiv$ symbol indicates that the results generated by the queries on the either side of $\equiv$ are always the same for a well-designed union compatible query.

1. $(P_1 \cup P_2) \bowtie P_3 \equiv (P_1 \bowtie P_3) \cup (P_2 \bowtie P_3)$.
2. $(P_1 \cup P_2) \bowtie P_3 \equiv (P_1 \bowtie P_3) \cup (P_2 \bowtie P_3)$.
3. $P_1 \bowtie (P_2 \cup P_3)$ is rewritten as $(P_1 \bowtie P_2) \cup (P_1 \bowtie P_3)$. However, $P_1 \bowtie (P_2 \cup P_3) \neq (P_1 \bowtie P_2) \cup (P_1 \bowtie P_3)$ for conventional union. We will elaborate on this rewrite after presenting our query evaluation technique below.

We convert any BGP-OPT-UNION query in the UNF by applying the above three conversion rules, such that the resulting query looks like $P_i \cup P_{i+1} \ldots \cup P_{i+k}$ where each $P_{i+j}, 0 \leq j \leq k$ is a UNION-free BGP-OPT pattern. Note that we have deliberately used suffixes $(i + j), 0 \leq j \leq k$ for this representation, to avoid confusion with the patterns in a BGP-OPT-UNION query before bringing it in the UNF. Now each of these subqueries can be treated independently as BGP-OPT queries for the purpose of entire query evaluation. For each such subquery, we run multi-way-join as given in Algorithm 7.2. Note that multi-way-join needs to know if it has to do nullification on each generated two patterns would have the same size and the same variables in them.

5 For the clarity and conciseness, in Figure 9.1 we have shown the core recursive performance intensive components of SPARQL. SPARQL 1.1 grammar has other syntactic components that are not shown in the figure.

6 Two patterns are union compatible, if they have the same arity and same attributes, e.g., for SPARQL it means that the

7 However, if we do a minimum-union the results will be the same as elaborated later.
result. Just like we do for any UNION-free BGP-OPT query, for each $P_{i+j}$ subquery in the UNF, nullification is required if it violates any of the following conditions:

a. Considering the GoSN of $P_{i+j}$, GoT of triple patterns enclosed in each slave of GoSN is acyclic.

b. For every pair of master-slave supernodes such as $SN_a → SN_b$, we consider equivalence classes of GoT edges $⟨T_m, T_s⟩, T_m ∈ SN_a, T_s ∈ SN_b$ (recall our definition of equivalent classes of GoT edges from Section 5). Then for every pair of master-slave supernodes, there is only one equivalence class of GoT edges between master-slave triple patterns.

Once the final results are generated for each $P_{i+j}$ subquery in the UNF using the multi-way-join and nullification (wherever required), we do a “union all” of results of all the subqueries – all the results are compiled together along with any duplicates as well. Next, we decide when we need to use the best-match operation over the unioned results, and after that we elaborate why we do this. Best-match is required if –

- nullification operation was used in at least one subquery in the UNF.
- while bringing the original query in UNF, at least once pattern $P_1 ⊙ (P_2 ∪ P_3)$ was rewritten as $(P_1 ⊙ P_2) ∪ (P_1 ⊙ P_3)$.

For the first condition – nullification used in any subqueries – it is straightforward to see that best-match is required to remove the subsumed results (ref. Section 4). For the second condition, recall that for the third union expansion rule, we noted that $P_1 ⊙ (P_2 ∪ P_3) ≠ (P_1 ⊙ P_2) ∪ (P_2 ⊙ P_3)$, and now we elaborate on this. Per SPARQL grammar, the UNION operation does a “union all” of the results produced by the unioned patterns, without removing the duplicates. Also when the unioned elements have $> 1$ arity, the set union operation does not remove subsumed results (ref. Section 4) for the definition of subsumed results.

**Definition 9.1** A union operation that removes subsumed results is called minimum union.

Let two sets of bindings for variable pairs $⟨?a, ?b⟩$ be unioned as $\{(p1, NULL)\} ∪ \{(p1, p2)\}$. The result of this union is $\{(p1, NULL), (p1, p2)\}$. However, if the sets of variable bindings being unioned are $\{(NULL, NULL)\} ∪ \{(p1, p2)\}$, then the result is $\{(p1, p2)\}$. This means that union of NULL co-existing with another non-null value stays in the final results despite being subsumed by another result, but not by itself. Also in SPARQL, unlike most SQL systems, the joins are null-compatible, i.e., the join of variable bindings of $⟨?a, ?b⟩$, $\{(p1, NULL)\} × \{(p1, p2)\}$ is $\{(p1, p2)\}$, and $\{(NULL, NULL)\} × \{(p1, p2)\} = \{(p1, p2)\}$. Because of the above two important observations, if we rewrite a pattern $P_1 ⊙ (P_2 ∪ P_3)$ as $(P_1 ⊙ P_2) ∪ (P_1 ⊙ P_3)$, it may generate different results than $P_1 ⊙ (P_2 ∪ P_3)$, if either $P_2$ or $P_3$ or both generate all null results.

Thus, for $P_1 ⊙ (P_2 ∪ P_3)$ pattern rewritten as $(P_1 ⊙ P_2) ∪ (P_1 ⊙ P_3)$, the final results may not match. However the original query and its UNF rewrite are semantically the same, and a query execution method that employs minimum-union instead of union-all or set-union generates all the non-duplicate and non-subsumed results correctly.

Thus we conclude this discussion that minimum union instead of set-union or union-all, allows the third rewrite rule for converting a BGP-OPT-UNION query into UNF while keeping the patterns before and after the rewrite equivalent with respect to the generated results. As a result, it allows any BGP-OPT-UNION query to be brought in the UNF allowing us more options for query optimization strategies such as the ones described as a part of this article.

In our previous work [5], we had proposed to convert a BGP-OPT-UNION query into the UNF and process each of the subqueries in the UNF independently. This involves duplication of efforts to prune the triples associated with a triple pattern that appears in multiple UNF subqueries (due to rewrite rules). Through the method presented in this section, we avoid this by treating each union-free BGP-OPT component of the query independently, pruning it before bringing the query in UNF, and use normal sequence of multi-way-join, followed by best-match wherever required by the structure of the query.

**Lemma 9.1** Nullification and best-match can be avoided if each subquery $P_{i+j}, 1 ≤ j ≤ k$ in the Union Normal Form of a BGP-OPT-UNION query satisfies the following two conditions:

a. Considering only the GoSN of $P_{i+j}$, GoT of triple patterns enclosed in each slave supernode of this GoSN is acyclic.

b. For every pair of master-slave supernodes such as $SN_a → SN_b$ in the GoSN of $P_{i+j}$, there is only one equivalent class of GoT edges.

**Proof** This lemma is obtained by combining the results of Lemmas 9.1 and 9.2 and hence the proof follows from the proof of those lemmas. This is because each $P_{i+j}$ subquery in UNF can be treated as a BGP-OPT query independently for the purpose of generating the final results of the query. □
9.2 FILTERs

SPARQL FILTER construct can have a complex Boolean expression that is supposed to be evaluated for every answer/result generated for the pattern over which it is applied. Below we have given an example of a BGP-OPT-FILTER query.

```sparql
SELECT ?friend ?sitcom ?dir
WHERE {
    :Jerry :hasFriend ?friend .
    ?friend :age ?age .
    ?friend :actedIn ?sitcom .
    OPTIONAL {
        ?sitcom :hasDirector ?dir .
        ?sitcom :location :NYC .
    }
    FILTER(?age < 60 && ?dir != :Jerry)
}
```

In essence, this query is asking for all the friends of :Jerry who have acted in a ?sitcom, and optionally information of the ?dir of the ?sitcom too, if it was located in :NYC. Notice that the FILTER condition further emphasizes that the ?friend must be younger than “60” years of age, and the ?sitcom’s director (?dir) cannot be :Jerry himself.

In general, FILTER expression of a SPARQL query can be notationally represented as \( P_2 F(R) \), where \( P_2 \) is a SPARQL query pattern – BGP, OPTIONAL, UNION or any combination of these – and \( R \) is a Boolean valued filter condition to be applied on \( P_2 \). For the scope of our discussion, we assume safe filters \(^{[21,22]}\), i.e., for \( P_2 F(R) \), all the variables in \( R \) appear in \( P_2 \), \( vars(R) \subseteq vars(P_2) \). This is in line with our previous discussion of well-designed OPTIONAL and UNION patterns. Unsafe (non-well-designed) filters can alter the semantics of the OPTIONAL patterns (ref \([21]\) for more detailed discussion). The FILTER pattern evaluation techniques presented here can work with any combination of BGP, OPTIONAL, and FILTER, because BGP-OPT remain as the building blocks of a SPARQL query, and our query evaluation techniques are built for these basic blocks.

For a SPARQL query with FILTERs, first we push-in the filter conditions as much as possible using the following rewrite rule on each UNION-free subcomponent of the query without bringing the query in the UNF (the first three rewrite rules are described in Section 9.1).

4. \( (P_1 \bowtie P_2) F(R) \equiv (P_1 F(R)) \bowtie P_2 \).

This rule can be applied as our queries are assumed to be well-designed. After pushing-in the filters, we run prune_triples on each UNION-free subcomponent of the query same as described in Section 9.1. We have an option to apply the FILTER conditions while loading the BitMat associated with each triple pattern. However, we can do so only if the respective FILTER expression satisfies following constraints.

1. FILTER expression is composed of conjunctive expression (only “\&\&” and no “||”), and
2. each subexpression in the conjunction consists of only one variable (e.g., \( FILTER(?a > 60 \&\& ?b != 10) \)).

Then each subexpression of FILTER can be applied while loading the BitMat associated with the triple pattern that contains the respective variable in the FILTER subexpression. However if a FILTER expression consists of disjunction (“||”), it can only be applied after multi-way-join, once each result of the query is fully constructed. The decision of applying a filter during BitMat loading will also depend on the selectivity of the filter subexpression, and the available indexes. If these constraints are not satisfied, FILTER can be applied only in multi-way-join. For simplicity, and the scope of this article, we assume that all the FILTER expressions in a BGP-OPT-UNION-FILTER query are applied only in the multi-way-join procedure, as a part of the nullification process.

Note from Section 9.1 that if a query contains UNIONed patterns, we first prune the triples in the UNION-free subcomponents of the query using prune_triples (Algorithm \(3.1\)), then bring the query in the UNF, and then evaluate each subquery independently using multi-way-join. For a query with UNIONs and FILTER conditions, after prune_triples, we bring the query in UNF by applying rewrite rules 1–3 described in Section 9.1 rule 4 described earlier in this section, and additionally applying rule 5 below.

5. \( (P_1 \cup P_2) F(R) = (P_1 F(R)) \cup (P_2 F(R)) \).

Any FILTER expressions that were not applied during BitMat loading are applied as a part of the nullification procedure, when multi-way-join produces each result. Nullification not only ensures consistent variable bindings (for any reordered inner and left-outer joins), but also evaluates the filter conditions. If the filter condition fails for the given pattern, we nullify all the variable bindings of that pattern and any slaves of the pattern. Recall that here the GoSN of the query comes in handy for quickly determining the nullification of any slave patterns. Once the results of all the subqueries on UNF are generated, we union them all, and apply best-match (minimum union) iff \( (P_1 \bowtie (P_2 \cup P_3)) \) was rewritten as \( (P_1 \bowtie P_2) \cup (P_1 \bowtie P_3) \) for some subquery, or at least one variable binding was nullified during the nullification operation in multi-way-join,
either as a result of a cyclic subquery or the filter condition.

9.3 DISTINCT

A DISTINCT keyword in the SELECT clause eliminates duplicates from the results. Consider the following BGP query over an RDFized version of a movie database like IMDB, which is asking for all the distinct pairs of the actors (?a) and their directors (?d).

```sql
SELECT DISTINCT ?a ?d
WHERE{
    ?m rdf:type :Movie .
    ?m :hasActor ?a .
    ?m :hasDirector ?d .
}
```

:UmaThurman has acted in three movies directed by :QuentinTarantino, they are :PulpFiction, :KillBillVol1, and :KillBillVol2. Without the DISTINCT clause, this query will generate three copies of (:UmaThurman, :QuentinTarantino) as the variable bindings for (?a, ?d) in the results. With the DISTINCT clause we will get only one copy.

SPARQL algebra allows an arbitrary number of variables in the DISTINCT clause (just like SQL). When there are multiple variables in the DISTINCT clause (like in the example above), the distinct values are composite of the bindings of the respective variables, e.g., (:UmaThurman, :QuentinTarantino) is distinct from (:UmaThurman, :WoodyAllen), although they both share :UmaThurman. If the variables in the DISTINCT clause appear in different triple patterns, like in our example, we have to generate intermediate variable bindings of the other variables not in the DISTINCT clause, and discard them later. This may create a memory overhead. E.g., we first have to generate bindings of all three variables (?m, ?a, ?d), project out only bindings of (?a, ?d), and then pass these (?a, ?d) binding pairs through the DISTINCT filter to remove duplicates. Hence for an arbitrary number of variables in the WHERE and DISTINCT clauses, evaluation of a query may become memory intensive if the DISTINCT clause is composed of many variables that do not appear in the same triple pattern.

For a SPARQL query with any intermix of BGP, OPTIONAL, UNION, FILTER, we employ different techniques for the evaluation of the DISTINCT clause depending on the other clauses in the query. Since BGP-OPT patterns are the basic building blocks of a SPARQL query, we begin with how to handle the DISTINCT clause with BGP and OPTIONAL patterns so as to avoid using extra memory overhead for the removal of non-essential variables, e.g., ?m in our example above.

9.3.1 Acyclic BGP

For a SPARQL query with just a basic graph pattern, i.e., acyclic per the definition of acyclicity as presented in Section 5, we prune the triples using `pruneTriple` (Algorithm 5.1), which leaves minimal triples in each BitMat associated with the triple patterns. `Multi-way-join` projects out all the variable bindings without duplicate removal. If we project out only some variables, we may get duplicates (in composite value form), as seen in the example elaborated at the beginning of this section. Hence we need a way to remove the bindings of non-essential variables, i.e., those not appearing in the DISTINCT clause, and remove duplicates from the composite bindings of the DISTINCT variables. For this discussion we assume the query to not have OPTIONAL, UNION, or FILTER patterns. Thus the GoSN of the query is going to have only one supernode, $S_{NaS}$, and all the triple patterns are going to be encapsulated inside it. Considering the graph of triple patterns (GoT) with undirected edges between the triple patterns, we identify a Minimal Covering Subgraph (MCS) such that triple patterns in MCS cover all the variables appearing in the DISTINCT clause, and no other strict subgraph of this MCS covers DISTINCT variables – in short we identify the minimum triple patterns from GoT that are required to project out the DISTINCT variable bindings.

![Fig. 9.2: BGP and GoT of the example query](image-url)

MCS can be identified methodically as follows. First identify all the triple patterns such that it has one or more DISTINCT variables in it, and then identify a minimal subgraph that connects all these triple patterns. Eliminate a $T_i$ from this subgraph, if it has at least one neighbor $T_j$ in this subgraph such that the dist vars($T_i$) $\subseteq$ dist vars($T_j$), i.e., the DISTINCT variables appearing in $T_i$ also appear in $T_j$, and elimination
of $T_i$ does not disconnect the rest of the subgraph. We continue this process until we do not find any triple pattern to eliminate. This makes the minimal covering subgraph (MCS). This MCS is acyclic, because the original GoT from which the MCS is carved out is acyclic too. Thus conceptually this MCS represents a subquery of the original BGP query. Now we will operate on this MCS to eliminate non-essential variables – those not appearing in the DISTINCT clause but appearing in the MCS because they connect one or more DISTINCT variables. Before that we review some important properties of Boolean matrix multiplication (BMM) of BitMats. BitMats conceptually represent an RDF graph’s adjacency matrices.

Figure 9.2 shows the Basic Graph Pattern (BGP) and the graph of triple patterns (GoT) of the query given at the beginning of this section. Triple patterns $?m :\text{hasActor} \ ?a$ and $?m :\text{hasDirector} \ ?d$ are connected to each other with label $?m$ in the GoT. Note that these two triple patterns are part of the MCS due to $?a$ and $?d$ variables, but the triple pattern $?m \text{ rdf:type} :\text{Movie}$ is not. The BitMats associated with these triple patterns contain all the triples which have predicates (edge-labels) :\text{hasActor} and :\text{hasDirector} respectively, and they in turn represent the adjacency matrices of two subgraphs of the original RDF graph, with only :\text{hasActor} and :\text{hasDirector} edge labels respectively. The transpose of the BitMat of :\text{hasDirector} conceptually reverses the direction of edges between the respective RDF nodes and represents a triple pattern $?d :\text{hasDirector} \ ?m$. Thus if we do a BMM of the transpose of BitMat of :\text{hasDirector} with the BitMat of :\text{hasActor}, the resultant matrix represents all the distinct pairs of nodes that have at least one undirected path with edge labels :\text{hasDirector} :\text{hasActor} between them, and eliminates the bindings of $?m$. It, in turn, eliminates the RDF nodes representing $?m$, which is a common variable between the two triple patterns. Figure 9.3 pictorially represents this concept of Boolean Matrix Multiplication. Note that since this query is acyclic, and we have already done prune_triple, $\text{BitMat}_1$ and $\text{BitMat}_2$ have minimal triples left in it, thus their BMM gives us the exact distinct pairs of ($?a, ?b$) in $\text{BitMat}_{12}$.

Property 9.1 An edge between two triple patterns in a minimal covering subgraph signifies a potential Boolean matrix multiplication between them to eliminate the variables common between them, which appear as the edge label.

We make use of this property to methodically eliminate non-essential variables in the MCS to shrink it further, and get a set of BitMats absolutely required to project the DISTINCT variable bindings. The MCS shrinking process is carried out as follows.

1. For every pair of triple patterns $T_i$ and $T_j$ that have an undirected edge between them with label, say $?m$, such that $?m$ is a non-essential variable, do a BMM of $T_i$ and $T_j$ such that bindings for $?m$ get eliminated. Whether we need to take a transpose of the BitMat or not depends on whether we have loaded S-O or O-S BitMat for the respective triple pattern (ref Section 5). E.g., in our example if we have loaded O-S BitMat of ($?m :\text{hasDirector} \ ?d$) and S-O BitMat of ($?m :\text{hasActor} \ ?a$), then we do a BMM of $\text{BM}_{\text{hasDirector}} \times \text{BM}_{\text{hasActor}}$ with no change in the respective BitMats. However if we have loaded S-O BitMat of ($?m :\text{hasDirector} \ ?d$) or O-S BitMat of ($?m :\text{hasActor} \ ?a$), we take the transpose of them before doing the BMM. Let us denote the resultant BitMat as $T_{ij}$.

2. $T_{ij}$ can connect to any neighbor of $T_i$ or $T_j$ (excluding $T_i$ and $T_j$), if it shares the exact same variables with the respective neighbor as done by $T_i$ or $T_j$. If $T_{ij}$ can thus connect to all of $T_i$ and $T_j$’s neighbors, we remove both $T_i$ and $T_j$ by connecting $T_{ij}$ to their neighbors.

3. $T_{ij}$ may not be able to connect to all of the $T_i$ and $T_j$’s neighbors, if $T_i$ or $T_j$ connects with other triple patterns over the same edge label between $T_i$ and $T_j$, e.g., $?m$. In that case we keep either of $T_i$ or $T_j$, eliminate the other. We connect $T_{ij}$ to the preserved triple pattern and eliminated triple pattern’s neighbors (whichever it can connect to). This procedure of preserving either of $T_i$ or $T_j$ is explained further. If $T_i$ is connected to $T_j$ over, say $?s$, $T_{ij}$ cannot connect to any neighbors of $T_i$ and $T_j$ that are connected over $?s$. By preserving either of $T_i$ or $T_j$, we ensure that the remaining MCS remains connected.

4. We continue this process of eliminating triple patterns and their respective BitMats, until we have an optimal set of BitMats.
MCS where each edge label contains a DISTINCT variable, or it has only one BitMat.

In Figure 9.4, we have shown a sample query where we need to eliminate the bindings of ?s, while preserving correlations between the bindings of ?x, ?y, ?z, ?t. The figure also shows an evolution of this MCS to eliminate ?s using the above algorithm. Intuitively, we eliminate all the intermediate non-required variables, by establishing a direct correlation between the bindings of the required variables. E.g., when ?x and ?t are part of two different triple patterns connected over ?s, bindings of ?x and ?t are correlated through ?s. When we do a BMM, BM(?x, ?s) x BM(?s, ?t) = BM(?x, ?t), we establish a direct correlation between the bindings of the (?x, ?t) pair, and eliminate the need of having ?s as an intermediary.

This algorithm is monotonic – at the end of one iteration, the edges, nodes, and BitMats in an MCS remain the same or become fewer than before. It gradually eliminates the edges with join variables that do not appear in the DISTINCT clause. At the end of the procedure, we are left with an MCS with BitMats – either original or new ones created in the process of BMM – and edges with only DISTINCT variables. This algorithm always converges when all the non-required variables are eliminated from the MCS. Also note that the total BitMats at the end of the algorithm are always fewer than the original BitMats in the query – note that in step 2, we remove two BitMats while creating a new one, and in step 3 we remove one BitMat and create one. Hence eventually we are left with fewer BitMats – thus reducing the memory requirements.

We join these BitMats with each other using the same multi-way-join procedure (Algorithm 7.2). Note that we can carve out an MCS from the original GoT, because the query is acyclic, and each triple pattern in the query has minimal triples after prune_triples (Algorithm 5.1).

9.3.2 Acyclic BGP-OPT queries

For the queries with an intermix of BGP and OPTIONAL patterns, we consider queries whose (a) GoT is completely acyclic, and (b) queries with cyclic GoT.

Like an acyclic BGP query, for an acyclic BGP-OPT query, prune_triples ensures minimal triples associated with each triple pattern in the query. We eliminate non-essential variables before doing multi-way-join as follows. Starting with SNabs, we go over all the supernodes in the master-slave hierarchy (masters before their respective slaves). We mark a supernode if it contains one or more variables appearing in the DISTINCT clause that do not appear in any of its masters. This is because a variable’s binding from the master triple pattern always dominates the binding from a slave triple pattern. So there is no need to consider a slave supernode if all of its DISTINCT variables are covered by one or more of its masters. After this, we carve out a Minimal Covering GoSN (MCGoSN) such that all the marked nodes are in a minimal connected subgraph of GoSN.

An MCGoSN is connected if there is an undirected path from one supernode to another disregarding the directionality of the edges connecting the supernodes (recall our GoSN construction from Section 2). Since we have a unique SNabs in our GoSN, any connected MCGoSN always contains SNabs. Now there are two cases – (1) SNabs and every supernode in MCGoSN contains at least one DISTINCT variable, (2) all the DISTINCT variables are contained in one or more slaves. For the second case, we cannot use the Boolean Matrix Multiplication technique to eliminate non-essential variables, and thus for this type of query
we have to resort to standard way of listing all the bindings of DISTINCT variables, and then removing duplicates.

For the first case however, we can use the BMM technique as follows. We order all the supernodes in MCGoSN per master-slave hierarchy. We start with $SN_{abs}$ and the triple patterns in it, and carve out a Minimal Covering Subgraph (MCS) from the GoT of only these triple patterns that cover the DISTINCT variables that appear in $SN_{abs}$. Next we iteratively go over the next supernode in the master-slave order, say $SN_i$. We extend the previously carved MCS to minimally cover the triple patterns in $SN_i$ for the DISTINCT variables that appear only in $SN_i$, but not in any of its masters. At the end of this exercise we get an MCS that contains triple patterns from different supernodes.

Next, we need to eliminate the non-essential join variables and triple patterns from this MCS in the same way we did for pure BGP queries. However, in this MCS, we may have a master-slave hierarchy between the two neighboring triple patterns. We take care of this hierarchy as follows. Pairs of connected triple patterns, $(T_i, T_j)$ can be categorized as – (a) both $T_i$ and $T_j \in SN_{abs}$, (b) $T_i$ is a master of $T_j$ or vice versa, (c) $T_i$ and $T_j$ are slave peers contained in the same slave supernode, (d) $T_i$ and $T_j$ are contained in different slave supernodes.

Starting with type (a) pairs first, we completely reduce the part of MCS contained in $SN_{abs}$. This reduction will cause some changes in the MCS nodes and connections. Next we consider all (b) type pair of nodes in MCS. Let us assume $T_i$ is the master of $T_j$. If the edge-label between $(T_i, T_j)$ is a non-essential variable, we do a BMM, remove $T_j$, and reconnect $T_i$ to the newly created BitMat. Once we are done with all type (a) and (b) pairs, we shrink (c) type pairs same as the acyclic BGP technique. We ignore the (d) type edges. That is because recall that we assume OPTIONAL pattern queries to be well-designed, so even if there are (d) type edges in the MCS, $T_i$ and $T_j$ always have an indirect path connecting them through their masters. Thus we gradually shrink this MCS to leave only the BitMats containing the bindings of the DISTINCT variables. Then we run multi-way-join on these BitMats in the standard way in the master-slave hierarchical order.

### 9.3.3 Cyclic BGP-OPT queries

For the cyclic BGP-OPT queries, the minimality of triples after prune_triples cannot be guaranteed, so we cannot use this technique. For such queries, we have to enumerate the results with duplicates, and then remove of them using a naïve method.

#### 9.3.4 UNION, FILTER

For queries with UNION or FILTER clauses along with BGP and OPT patterns, we cannot use the optimization technique of carving out an MCS from the entire query. The reason is – we can carve out an MCS for DISTINCT variable binding projections only if each triple pattern in the query has minimal triples associated with it. In a BGP-OPT query with UNION pattern, we do the pruning of UNION-free BGP-OPT sub-patterns in the given query (as described in Section 9.1). Then get the query in the UNF $P_1 \cup P_2 \cup \ldots \cup P_k$, and perform multi-way-joins on each $P_i$ in the UNF. However, since we did the pruning only on the UNION-free BGP-OPT subparts of the original query, the triples associated with each triple pattern in $P_i$ may not be minimal.

The method presented in Section 9.2 for handling FILTERs in DISTINCT-free queries shows that an arbitrary FILTER condition can cause nullification while doing multi-way-joins, thereby altering the minimality of triples associated with the triple patterns on-the-fly. Hence for the SPARQL queries asking for DISTINCT projection of some variables, where the query body has UNIONS or FILTER conditions, we use the naïve way of projecting out all the variable bindings of the DISTINCT variables, and then sorting to remove any duplicates.

### 10 Related Work

SPARQL BGP queries are similar to SQL inner-joins, and thus naturally a lot of SQL inner-join optimization techniques have been applied to SPARQL BGP query optimization. While the BGP query optimization has got a lot of attention, the discussion about optimization of other SPARQL components such as OPTIONAL patterns, UNIONS, FILTERs, DISTINCT clauses is quite sparse. This is despite the fact that in the context of SPARQL queries, these other components do make as large as 94% of the queries [14,22,26,16]. Previous work [4,10,21,28], has extensively analyzed the semantics of well-designed OPTIONAL patterns from the perspective of tractability properties. However, these texts have not focused on the discussion of other SPARQL components that are covered in this article. The idea of query graph of supernodes (GoSN) presented in this paper is reminiscent of well-designed pattern trees (WDPT) [18,19], but WDPTs are undirected and unordered, whereas GoSN is directed, and establishes an order among the
patterns (master-slave, peers), which is an integral part of our optimization techniques. Also while previous discussion has focused on WDPTs and tractability results, they have not taken into consideration the aspects of optimization techniques from the point of view of minimality of triples, and the order of processing semi-joins and multi-way-joins, which make the performance intensive components of query evaluation techniques.

Galindo-Legaria, Rosenthal [11,12,13] and Rao et al [24,25] have proposed ways of achieving SQL left-outer-join optimization through reordering inner and left-outer joins. Their work is closest to the work in this article. Rao et al have proposed nullification and best-match operators to handle inconsistent variable bindings and subsumed results respectively (see Section 4). In their technique, nullification and best-match are required for each reordered query, as the minimality of tuples is not guaranteed. They do not use methods like prune_triples to eliminate unwanted tuples before joins. Bernstein et al and Ullman [7,8,29] have proved the properties of minimality for acyclic inner-joins only. Through our work, we have taken a major step forward by extending these properties in the context of SPARQL OPTIONAL patterns (SQL left-outer-join), and finding ways to avoid overheads like nullification and best-match operations. We have also extended previous results presented in [5] about the class of queries that can avoid nullification and best-match despite reordering of inner and left-outer joins.

Additionally, in this article we have extensively analyzed the UNION, FILTER, and DISTINCT clauses of SPARQL from the point of view of minimality of triples, ways of unifying the results (minimum-union versus standard union-all), and structural aspects of queries (cyclicity). With this analysis we have shown that a large number of join patterns, as well as some (good) cyclic – SPARQL queries can be optimized by using our techniques of BGP-OPT query processing as building blocks, and can avoid the additional overheads of processing and indexing for the correctness of the results. Our article also throws a new light on the treatment of NULL values in the RDF and SPARQL context, and the implications of various SPARQL components – other than Basic Graph Patterns (inner-joins) – in the presence of NULL values in the query results. To the best of our knowledge, this topic, which directly affects implementation and optimization methods, has not been handled before.

For inner-join optimization, RDF engines like TriAD [15], RDF-3X [20], gStore [31], take the approaches like graph summarization, sideway-information-passing etc for an early pruning of triples. Systems like TripleBit [30] use a variable length bitwise encoding of RDF triples, and a query plan generation that favors queries with “star” joins, i.e., many triple patterns joining over a single variable. RDF engines built on top of commercial databases such as DB2RDF [9] propose creation of entity-oriented Flexible schemas and better data-flow techniques through the query plan to improve the performance of “star” join queries. Along with this, there are distributed RDF processing engines such as H-RDF-3X [17] and SHARD [27].

While many of these engines mainly focus on efficient indexing of RDF graphs, BGP queries (inner-joins), and exploiting “star” patterns in the queries, we have focused on the broader components of SPARQL patterns such as OPTIONALs, UNIONs, FILTERs, and DISTINCT, which cannot always exploit the benefits of inner-join focused query optimizers.

11 Conclusion

In this article we have done an extensive analysis of a wide range of SPARQL components such as OPTIONAL patterns, UNIONs, FILTERs, DISTINCTs, and proposed that they can be evaluated by simply using our BGP-OPT pattern’s optimization techniques as building blocks. We have extended the previously proposed concepts of minimality of triples (tuples), cyclicity of queries, and nullification, best-match operations. With this first of a kind analysis of a wide range of SPARQL components, we hope to create novel optimization techniques for performance intensive SPARQL components. Our analysis shows that this is be possible by simple semantic manipulation of various intermixed query components. Our article also elaborately discusses the treatment of NULL values in the RDF and SPARQL context.

Since there is a very close resemblance between SPARQL components and SQL queries, our proposed techniques, as well as observations about query’s structural properties and optimization opportunities, can be directly applicable to the respective SQL components too.

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