An efficient similarity-based level set model for medical image segmentation

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Abstract
It is usually difficult to correctly segment medical images with intensity inhomogeneity, which is of great significance in understanding of medical images. The local image intensity features play a vital role in accurately segmenting medical images with intensity inhomogeneity. Therefore, it is crucial to acquire the local intensity features for a deeper understanding of medical images. The main idea of this paper is to construct an efficient similarity-based level set model, which synthesizes the similarity theory, curve evolution and level set. Firstly, a local statistical function is modeled as different scales of Gaussian distributions to estimate bias fields, in which a real image can be approximately obtained for a more accurate medical image segmentation. Secondly, a new potential function is constructed to maintain the stability of the curve evolution, especially the signed distance profile in the neighborhood of the zero level set, which plays an important role in the correct segmentation. Thirdly, an adaptive condition criterion has been proposed to accelerate the convergence in the curve processing. Finally, the experiments on artificial and medical images and comparisons with the current well-known region-based models are discussed in details. Our extensive experimental results demonstrate that the proposed method can correctly segment medical images with intensity inhomogeneity in a few iterations and also is less sensitive to the initial contour.

Key words: Active contour, Intensity inhomogeneity, Bias fields, Similarity theory, Medical image segmentation, Level set

1. Introduction

Image segmentation is always one of the most crucial and fundamental task in computer vision. In the medical fields, many clinical and research applications rely on the intensity homogeneous regional segmentation. Unfortunately, it is a challenging problem due to poor resolution and intensity inhomogeneity between the ranges of the intensities in the regions to segmented(Li, et al., 2011),(Harders and Szkely 2003),(Ni, He and Yuan, 2015),(Ni, et all., 2016). Those widely used image segmentation algorithms(Samson, et al. 2000),(Chan and Vese, 2001),(Ranchin and Dibos, 2005) usually rely on intensity homogeneity are not suitable for images with intensity inhomogeneities. Over these decades, many approaches have been developed to segment images with intensity inhomogeneity. The well-known Mumford Shah model, called MS model(Mumford and Shah, 1989) is suitable to segment images with intensity inhomogeneity. However, because of its complex calculation, some simplified versions of MS model have been addressed, such as the PS model(Vese and Chan, 2002),(Chan and Vese, 2001). The PS model can achieve a favorable result for segmenting images with intensity inhomogeneity. However, it can not satisfy the actual application for its long iteration time.

Recently, some methods have been addressed to deal with images with intensity inhomogeneity, such as edge-based models and region-based models(Li, et all., 2011),(Melenk and Sauter, 2010),(Zhou, et al., 2015),(Wang, et al., 2015). These two types of models both have their pros and cons. Edge-based models utilize the image gradient to stop the
curve evolving process near the object boundaries, such as a new variational formulation (Pi et al., 2007), (Li et al., 2005), (Morel and Solimini, 2012) and a gradient flux flows method (Song, 2014) (Kolmogorov and Boykov, 2005), the edge-based models are sensitive to the initial contour that are easy to fall into local minimum (Yan et al., 2015). Region-based models superior to edge-based models in many ways. Region-based models and its variant methods are less sensitive to noise by utilizing the statistical information inside and outside the contour to control the evolution, such as local binary fitting model (LBF) (Li et al., 2007), (Li et al., 2008), local image fitting (LIF) (Zhang et al., 2010a), the local region-based Chan Vese model (LRCV) (Liu and Peng, 2012).

In this paper, we propose an efficient similarity-based level set model which is used to segment images with intensity inhomogeneity. We utilize different scales of Gaussian distributions to obtain the local image information. In the meanwhile, we can approximate the real image from the original image by utilizing bias fields. Furthermore, a double termination conditions and six order polynomial penalty energy function are constructed to accelerate the evolution time and to maintain stability of the curve evolution. The quantitative analysis and experimental results show that our proposed method is more efficient and less sensitive to the initial contour.

This paper is organized as follows: the previous works are reviewed in Section 2. Section 3 describes the proposed method with four subsections in details. Algorithms are proposed in Section 4. In Section 5, experimental results are validated to be effective by comparing with the representative models on both synthetic and medical images. Conclusions and future works are included in Section 6.

2. Related works

2.1. The CV model

Based on the MS models (Mumford and Shah, 1989), Chan and Vese proposed an active contour model using a piecewise constant function. Let \( I : \Omega \rightarrow \mathbb{R} \) be an input image and \( C \) be a closed curve, the CV model is formulated by minimizing the following energy function:

\[
E^{\text{CV}} = \mu \int_{\Omega} \delta(\phi) \nabla \phi \cdot dx + \nu \int_{\Omega} H(\phi) dx + \lambda_1 \int_{\text{inside}(C)} |I(x) - c_1|^2 dx + \lambda_2 \int_{\text{outside}(C)} |I(x) - c_2|^2 dx
\]

(1)

Where \( c_1 \) and \( c_2 \) denote the average intensities inside and outside of the curve \( C \), respectively. The first two terms on the right hand side are the weighted length of the curve and the weighted area of the region inside the zero level set of \( \phi \). And the coefficients \( \mu > 0, \nu > 0, \lambda_1 > 0 \) and \( \lambda_2 > 0 \) are fixed parameters. \( H(\cdot) \) is Heaviside function and \( \delta(\cdot) \) is Dirac function.

\[
H(x) = \frac{1}{2} \left[ 1 + \frac{2}{\pi} \arctan \left( \frac{x}{\epsilon} \right) \right]
\]

(2)

\[
\delta(x) = \frac{\partial H(x)}{\partial x} = \frac{1}{\pi} \frac{\epsilon}{\epsilon^2 + x^2}
\]

(3)

By minimizing Eq. (1) using the steepest descent method, we solve the \( c_1 \) and \( c_2 \) as follows:

\[
c_1 = \frac{\int_{\Omega} I(x) H(\phi) dx}{\int_{\Omega} H(\phi) dx}, \quad c_2 = \frac{\int_{\Omega} I(x)(1-H(\phi)) dx}{\int_{\Omega}(1-H(\phi)) dx}.
\]

(4)

From Eq. (4) we conclude that the two parameters can not reflect any local intensity information for images with intensity inhomogeneity. As a consequence, the CV model is not applicable for segmenting images such images.

2.2. Local region-based Chan Vese (LRCV) model

Liu et al. (2012) proposed a local region-based Chan-Vese model (LRCV) which replace the two constants \( c_1 \) and \( c_2 \) by weighted averages of their own point neighborhood. The energy functional is defined as following:

\[
E^{\text{LRCV}}(c_1(x), c_2(x), C) = \lambda_1 \int_{\text{inside}(C)} |I(x) - c_1(x)|^2 dx + \lambda_2 \int_{\text{outside}(C)} |I(x) - c_2(x)|^2 dx
\]

(5)

More Specifically, for a point \( x \in \mathbb{R}^2 \), its intensity can be approximated by a weighted average of the image intensity \( I(y) \), where \( y \) is the neighborhood of \( x \). Therefore, the average intensity can be reformulated as follows:

\[
c_1(x) = \frac{\int_{\Omega} g_1(x-y)I(y)H(\phi(y)) dy}{\int_{\Omega} g_1(x-y)H(\phi(y)) dy}, \quad c_2(x) = \frac{\int_{\Omega} g_1(x-y)I(y)(1-H(\phi(y))) dy}{\int_{\Omega} g_1(x-y)(1-H(\phi(y))) dy}.
\]

(6)
Where \( g_i \) is a Gaussian kernel function, and \( g_i(x - y) \) can be defined as a weight parameter assigned to each intensity \( I(y) \) at \( y \). In the LRCV model, \( c_1(x) \) and \( c_2(x) \) are spatially varying, but \( c_1 \) and \( c_2 \) are constants in the CV model; To an extent, the LRCV can segment images with intensity inhomogeneity successfully. Nonetheless, the computational complexity is so high for computing four convolutions.

2.3. The region scale fitting model

To overcome segmentation difficulty caused by intensity inhomogeneities, Li et al. proposed a region scale fitting model (RSF), which utilized intensity information in local regions at a controllable scale (Li, et al., 2008) to segment images with intensity inhomogeneity. The region scalability of the RSF model is due to the kernel function with a scale parameter, which use intensity information in regions at a controllable scale, from small neighborhoods to the entire domain, then is incorporated into a variational level set formulation with a level set regularization term. The RSF energy is defined by as below:

\[
E_{RSF}(\phi, f_1, f_2) = \sum_{i=1}^{3} A_i \int_{\Omega} K_\sigma(x, y) |I(y) - f_1(x)|^2 M_1(\phi(y)) dy + v \int |\nabla H(\phi(x))| dx + \mu \mathcal{R}(\phi)
\]

Where \( M_1(\phi) = H(\phi) \) and \( M_2(\phi) = 1 - H(\phi) \), \( f_1(x), f_2(x) \) are two values for approximating image intensities in \( \Omega_1 \) and \( \Omega_2 \) respectively. The first term is the data fitting, which plays a key role in the RSF model; the second term is called length item to maintain the curve smoothness; the third one consists of a regularization term for keeping the regularity of curve evolution. And \( K \) is a truncated Gaussian Kernel function with the variance of \( \sigma^2 \).

\[
K_\sigma(x) = \frac{1}{(2\pi)^{3/2}\sigma^3} e^{-|x|^2/2\sigma^2}
\]

The standard deviation \( \sigma^2 \) is used to control the region scalability from small neighborhood to the whole image domain, and has desirable performance for images with weak object boundaries, besides, it has a strict requirement for the location of initial contour, which also limits its practical application (Wang, et al., 2015), (Zhang et al., 2015), (Huang et al., 2011), (Zhang et al., 2016).

3. The Proposed Model

In this section, we shall present our proposed model for segmenting medical images with the zero-level sets of an energy function. In the proposed model, we combine global image properties with local intensity information to solve the intensity inhomogeneity. Specifically, we construct a new potential function to ensure the curve smoothness in the curve evolution process.

Therefore, the overall energy function is roughly modeled as three parts in follows equation:

\[
E = \alpha \cdot E^L + \beta \cdot E^G + E^R
\]

Where \( E^L \) is the local term, \( E^G \) is the global term, and \( E^R \) is the potential regularization term.

3.1. Local similarity model

Due to the poor radio-frequency coil uniformity, static field inhomogeneity and the illumination, the images have the property of intensity inhomogeneity which can be generally modeled as spurious smoothly varying field named bias field and the noise(Zhang, et al., 2015), (Vovk, et al., 2007). The most general model in describing the acquired images with intensity inhomogeneity effect is described as follows:

\[
I(x) = b(x) J(x) + n(x), x \in \Omega
\]

Where \( I(x) : \Omega \rightarrow R \) is the acquired image; \( J(x) : \Omega \rightarrow R \) is the inhomogeneity free image; \( b(x) : \Omega \rightarrow R \) denotes the bias field that accounts for the intensity inhomogeneity; \( n(x) \) means the noise approximated by Gaussian distributed with zero mean and variance \( \sigma^2 \) (Vovk, et al., 2007), (Li, et al., 2013). The Model diagram is as shown in the Fig.1.

To simplify the computation, the noise is ignored. Therefore, if we decompose from the two the multiplicative components \( b(x) \) and \( J(x) \), we can obtain the real image. Accordingly, the real image will be segment correctly. However,
it is an ill-posed problem due to the lack of sufficient knowledge about the image. In this paper, we have proposed a new approach for approximating bias field by smoothly Gaussian functions with different scales.

Theoretically, a function can be approximated by a linear combination of a number of basis functions up to arbitrary accuracy (Powell, 1981), (Li, Gore, and Davatzikos, 2014), (Hedges and Olkin, 2014). In our method, the bias field is approximated by a linear combination of a given set of smooth basis functions. The mathematical model of bias field as follows:

$$
\hat{B}(x) = \sum_{k=1}^{N} w_k g_k
$$

Where $W = w_1, w_2, ..., w_N$ represents weight parameters, $G = g_1, g_2, ..., g_N$ means the basis functions. $k$ is the size of basis function. It is crucial to statistically analyze each pixel with respect to its local neighborhood. In addition, bias field slowly varies spatially, and it is smooth. Motivated by bias field correction method, we present an approximate method to estimation of bias field by using multiscale truncated Gaussian function. In practice, four zero mean and larger variance of the Gaussian functions are used to approximately estimate $\hat{B}(x)$ in our model. Accordingly, Eq.12 can be rewritten as the following formula:

$$
\hat{J}(x) = I(x) / \hat{B}(x)
$$

Where $\hat{B}(x)$ is the approximate value of bias field.

Accordingly, the image of the local energy function can be expressed as the following formula:

$$
E^L(d_1, d_2, C) = \int_{\text{inside}(C)} |\hat{J}(x) - d_1|^2 dx + \int_{\text{outside}(C)} |\hat{J}(x) - d_2|^2 dx
$$

Where $d_1, d_2$ represent the intensity averages of the image $\hat{J}(x)$ of inside $C$ and outside $C$, respectively. The curve $C$ is expressed as the following the zero level set outline of implicit function.

$$
\phi(x) = \begin{cases} 
> 0, & \text{inside}(C) = \omega \\
= 0, & C = \partial \omega \\
< 0, & \text{outside}(C) = \Omega \bar{\omega}
\end{cases}
$$

To solve this minimization problem, the contour $C$ is replaced by the implicit function $\phi(x)$. Considering that $\phi(x) > 0$ if the point $x$ is inside $C$, $\phi(x) < 0$ if the point $x$ is outside $C$, and $\phi(x) = 0$ if the point $x$ is on $C$. Hence, the local energy functional can be reformulated in terms of $\phi(x)$ as follows:

$$
E^L(d_1, d_2, \phi) = \int_{\Omega} |\hat{J}(x) - d_1|^2 H(\phi(x)) dx + \int_{\Omega} |\hat{J}(x) - d_2|^2 (1 - H(\phi(x))) dx
$$

$$
\begin{cases} 
\hat{d}_1(\phi) = \frac{\int_{\Omega} \hat{J}(x) H(\phi(x)) dx}{\int_{\Omega} H(\phi(x)) dx}, \\
\hat{d}_2(\phi) = \frac{\int_{\Omega} \hat{J}(x) (1 - H(\phi(x))) dx}{\int_{\Omega} (1 - H(\phi(x))) dx}.
\end{cases}
$$

In local similarity model,

### 3.2. Global statistical model

As described above, local statistical model can not take into consideration of the image noise, image texture information. And it is very difficult to achieve the accuracy of image segmentation. Therefore, we must consider the global image properties. Because the CV model can well deal with the intensity homogeneous image segmentation problem, the global term is directly derived from the CV model. Differs from this model, we replace the Heaviside function by a new one.

$$
E^G(c_1, c_2, \phi) = \int_{\Omega} |I(x) - c_1|^2 \hat{H}(\phi(x)) dx + \int_{\Omega} |I(x) - c_2|^2 (1 - \hat{H}(\phi(x))) dx
$$

![Fig. 1 Illustration of the image model with intensity inhomogeneity. Left to right: the acquired image $I(x)$, the bias field $B(x)$, the free image $J(x)$, and the noise $n(x)$.](image-url)
Where \( H(\cdot) \) in Eq.2 is an approximate value of Heaviside function. A Heaviside function has the value 0 for \( x < 0 \), 1 for \( x > 0 \), and 0.5 for \( x = 0 \). Strictly speaking, the Heaviside is not a continuous function but a unit step one, and it can be approximated by Eq.2 in the CV model. As shown in Fig.2(a), we conclude that the value of \( H(x) \) is less than 1 when \( x > 0 \) and larger than 0 when \( x = 0 \). As a result, the value is inaccurate when calculating the value of \( c_i \) (Chen et al., 2009).

The closer it gets to the Heaviside function, the better the smoothness effect. Therefore, it is of great significance in constructing a high fitting function of Heaviside. Motivated by the classical active contour models (Chan and Vese, 2001), (Yushkevich et al., 2006), (Zhang et al., 2010b) and optimization problem (Xia et al., 2012), we construct a better approximation function about Heaviside. The function is defined as follows:

\[
H'(x) = \frac{1}{2} \cos\left(\frac{\pi}{2} - \frac{\tan(x)}{\varepsilon}\right) + \frac{1}{2} \tag{19}
\]

Accordingly, as the derivative of \( H(x) \), the Dirac function is denoted as:

\[
\delta'(x) = \frac{1}{2} \sin\left(\frac{\pi}{2} - \frac{\tan(x)}{\varepsilon}\right), \frac{\varepsilon}{\varepsilon^2 + x^2} \tag{20}
\]

![Fig. 2 Function comparison chart of different epsilon parameters. (a) traditional Heaviside function, (b) our new Heaviside function, (c) traditional Dirac function, (d) our new Dirac function.](image)

The value of \( \varepsilon \) in the Heaviside function is directly related to the quality of the segmentation result. Therefore, it is important to select an appropriate value for \( \varepsilon \). As shown in Fig.2(b) and (d), the values of \( \delta'(x) \) tend to be near zero when \( \varepsilon \) is too small, the result will fall into a local minimum. However, if \( \varepsilon \) is large, although \( \delta'(x) \) tends to get a global minimum, the final contour location may not be accurate. Through quantitative analysis, we let \( \varepsilon = 1.5 \) in the experimental part.

### 3.3. A new potential function

During the curve evolution of traditional, there are some problems such as re-initialization, irregularity curve evolution, which can lead to numerical errors and even destroy the stability of the curve evolution. To compensate for the lack of existing methods, we need to keep the signed distance property and the curve smoothness in the evolution process. Based on the above problems, the following penalty energy functional was proposed in 2005 and the enhanced method in...
2010 to describe the deviation of the level set function from a signed distance function. The penalty energy functional is described as follows:

\[ R_{p2}(\phi) = \int_{\Omega} p_2(|\nabla \phi(x)|)dx \]  

(21)

Li et al. construct a kind of trigonometric function as a penalty term and prove the effectiveness of the energy function in theory. And the experimental results have demonstrated that it can keep the stability in the evolution process. Because the trigonometric function is computed by Taylor expansion, which is represented a linear combination of high order polynomial function. The low-order polynomial function has the characteristics of simple operation and fast computing speed. Then, we construct a new potential function as the following:

\[ p_2(s) = \begin{cases} \frac{1}{2}s^2(\frac{1}{2} - 1)^2, & 0 \leq s \leq 1 \\ \frac{1}{3}(s - 1)^2, & s \geq 1 \end{cases} \]  

(22)

To illustrate the benefit of the new potential function, we define a temporary function as follows:

\[ dp(s) = \frac{p_2'(s)}{s} = \begin{cases} (s^2 - 1)^2 + 2s^2(s - 1), & 0 \leq s \leq 1 \\ 1 - \frac{1}{s}, & s \geq 1 \end{cases} \]  

(23)

Fig. 3 shows that the potential function \( p_2(s) \) has two minimum points at \( s=0 \) and \( s=1 \), which has the same characteristic as the reference (Li, et al., 2010). However, the difference is that our potential function has lower computation complexity. The new potential function is defined as follows:

\[ R_{p2}(\phi) = \int_{\Omega} p_2(|\nabla \phi(x)|)dx \]  

(24)

Where \( p_2(\cdot) \) is a double-well potential function. By using Gateaux derivative, we obtain the gradient flow:

\[ \frac{\partial \phi}{\partial t} = -\mu \frac{\partial R_p}{\partial \phi} = \mu div(d_p|\nabla \phi|\nabla \phi) = div(D\nabla \phi) \]  

(25)

Where \( div(\cdot) \) is the divergence operator. \( D = \mu d_p(|\nabla \phi|) \) is called diffusion rate (Li, et al. 2010), (Morel and Quinn, 2004). Fig.3(b) demonstrates the property of the diffusion in the following conclusion:

\[ \lim_{s \to 0} d_p(s) = \lim_{s \to 0} d_p(s) = 1 \Rightarrow |\mu d_p(|\nabla \phi|)| \leq \mu \]  

(26)

(1) for \( |\nabla \phi| > 1 \), the diffusion rate \( D \) is positive, and it will forward diffusion for decreasing \( |\nabla \phi| \);

(2) for \( \sqrt{1/3} < |\nabla \phi| \leq 1 \), \( D \) is negative, and it will backward diffusion for increasing \( |\nabla \phi| \);

(3) for \( |\nabla \phi| < \sqrt{1/3} \), \( D \) is positive, and it will forward diffusion for decreasing \( |\nabla \phi| \) down to zero.

According to the above analysis, we get the mathematical definition of energy regularization model:

\[ E^R = \lambda \int_{\Omega} \delta(\phi(x))|\nabla \phi(x)|dx + \mu \int_{\Omega} p_2(|\nabla \phi(x)|)dx \]  

(27)
3.4. Level set formulation

In this section, the overall energy functional can be further described as follows:

\[
E = \sum_{i=1}^{2} \left( \alpha \int_{\Omega} |J(x) - d_i|^2 M_i(\phi(x)) dx + \beta \int_{\Omega} |H(x) - c_i|^2 M_i(\phi(x)) dx + \lambda \int_{\Omega} \delta(\phi) \nabla \phi(x) dx + \mu \int_{\Omega} p_2(|\nabla \phi(x)|) dx \right)
\]  

(28)

By calculus of variations, and keeping \( d_1, d_2, c_1, c_2 \) fixed and minimizing \( E \) with respect to \( \phi \), we can deduce the associated Euler-Lagrange equation for \( \phi \). The following variational formulations can be obtained:

\[
\frac{\partial \phi}{\partial t} = \delta(\phi) [-\beta(I(x) - c_1)^2 + \alpha (J(x) - d_1)^2 + \beta (I(x) - c_2)^2 + \alpha (J(x) - d_2)^2] + \mu \delta(\phi) \nabla |\nabla \phi| + \delta(\phi) \nabla (d_2 \nabla \phi) \nabla \phi)
\]  

(29)

In the above equation, we use the Neumann boundary condition as the boundary condition. Because the boundary has the characteristics of smoothness, the spatial partial derivatives by the forward difference and the temporal partial derivatives are obtained. Thus, Eq.29 can be approximately expressed as follows:

\[
\frac{\phi_{i,j}^{k+1} - \phi_{i,j}^k}{\Delta t} = L(\phi_{i,j}^k)
\]  

(30)

In our experiments, initialization of level set function is defined as follows:

\[
\phi_0(x) = \begin{cases} 
2, & x \in C_0 \\
-2, & \text{otherwise}
\end{cases}
\]  

(31)

4. Description of algorithm

4.1. An adaptive condition criterion

The curve evolution process will change with the topology of the target contour. Thus, it will stop when the curve is close to target border. If the execution is terminated earlier, we can not get the correct segmentation results; Conversely, if the evolution curve is close to the target contour, and the iterative process is still going, the oscillation phenomenon occurs. Reasonable termination condition criteria will help improve the execution rate algorithm. In theory, the total energy reaches a minimum when the internal and external forces tend to balance, the contour length will remain a constant. Wang et al. proposed a termination criterion which based on the length change of evolving curve. But for some complex images, use the criterion can’t stop the iteration, and even cause the program to enter infinite loop state. Therefore, we make further improvements based on the algorithm. To avoid an infinite loop, we add a fixed number of iterations on the basis of the original termination conditions. We call it a double termination criterion as Algorithm 1.

4.2. The proposed segmentation algorithm

In this section, combing with local information model, global information model and double termination condition criteria, we shall describe our proposed algorithm as Algorithm 2.

5. Experimental results

In this section, we will conduct the experiments to verify the effectiveness of our proposed model for segmenting synthetic and real images as shown in Fig.4. Then we shall compare our proposed method with the following four models: the CV model (CV), the region scale fitting model (RSF), the local region-based Chan Vese model (LRCV) as shown in Fig.5-Fig.8. Fig.9 shows the comparison chart of convergence time for four medical images. All the experiments are run in Matlab code on a computer with Intel Core i3 3.30GHz CPU and RAM 8.00G.

It should be noted that the proposed model is efficient for two-phase medical images, which usually generate foreground part and background part. It belongs to bimodal model, which cannot simultaneously detect different intensity multiple objects.
Algorithm 1 Double termination conditions criterion

Input:
Initial contour \( \text{InitialLSF} = \text{Len}(C(t)) \), Threshold of curve length \( \text{LenThreshold} \), Iteration maximum times \( T_{\text{max}} \), Iteration boolean \( \text{bool} = \text{true} \), Iteration step \( t \), Inner iteration controller \( k \).

Iteration:
1: while \( \text{bool} \) do
2: \( t \leftarrow t + 1 \);
3: compute \( \text{Len}(C(t + 1)) \);
4: \( \text{curLen} = |\text{Len}(C(t + 1)) - \text{Len}(C(t))| \)
5: if \( \text{curLen} < \text{LenThreshold} \) then
6: if \( k = T_{\text{max}} \) then
7: \( \text{bool} = \text{false} \)
8: else
9: \( k \leftarrow k + 1 \)
10: end if
11: else
12: \( k = 1 \)
13: end if
14: if \( t = T_{\text{max}} \) then
15: \( \text{bool} = \text{false} \)
16: return \( \text{bool} \)
17: end if
18: end while

Output:
Termination conditions \( \text{bool} \).

5.1. Application on test images
We use three representative test images to demonstrate the robustness of our method. And there are a synthetic image, a texture image and a noise image respectively. In Fig.4, the first row shows two different initial contours of three test images, respectively. The second row displays the corresponding segmentation results of our method. And their final level set functions are achieved in the third row.

Due to the local similarity model, our model can extract the texture features and intensity inhomogeneous features, so can be applied to texture images and inhomogeneous images as shown in Fig.4. We observe that the variances of object and background are different, so that our method can accurately extract the object, and is less sensitive to the initial contour. In addition, it can also effectively keep the stability of the curve in the process of its evolution.

Algorithm 2 Our Proposed Segmentation Algorithm

Input: Image \( I \).
1: Initial contour \( C_0 \), Initialize level set according to Eq.31, Time step \( \Delta t = 0.1, \epsilon = 1.5 \), Penalty parameters \( \mu = \lambda \times 255^2, \lambda \in (0, 1) \), Local and global control parameters \( \alpha = 0.8 \) and \( \beta = 1 \).
2: Compute the value according to Eq.29: \( \phi_{k+1}^{i,j} = \phi_{k}^{i,j} + \Delta t L(\phi_{k}^{i,j}) \).
3: Extract zero level set contour \( C \).
4: Judge double termination conditions using Algorithm.1.
5: If satisfied, then exit the program; otherwise, jump to 2.

Output:
The curve contour \( C \).

5.2. Comparisons with three competitive algorithms
Fig.5 applies the four models on a magnetic resonance image of a human heart with different initial contours. The first column shows the initial contours of the left ventricle of a human heart, and the second to the fifth column shows the segmentation results by the CV model, the RSF, the LRCV and our model, respectively. Due to the complicated characteristics of medical images, the CV model, the RSF and the LRCV fail to segment the object while our method succeeds. For the computational time in the LRCV model, it should be noticed that the CV model outperforms the LRCV in terms of the iteration time as shown in Fig.9. Nonetheless, the proposed method is proved to be more efficient.

Fig.6 compares our proposed method to the other three models by applying them to a microscope image of cells. The first column shows the initial contours close to the desired object. We can conclude it is an overfitting problem in the CV model, the RSF and the LRCV. Obviously, the results show that our proposed method is more accurate than others as shown in column 2-5.

Fig.7 shows the results on a brain MR image by CV model, the RSF, the LRCV and our method respectively. The
Fig. 4 Segmentation results of three test images. Top to bottom: initial contours, segmentation results of our method, final level set functions.

First column shows the initial contours of the image, the RSF model fails to segment the object as shown in Fig. 7. Column 3. Whatever the initial contour is placed inside and outside of the objects, the CV model, the LRCV and the proposed method model have succeeded in the segmentation task as shown in Fig. 7. Column 2, 4, 5.

Fig. 8 shows the results on a brain 3T MRI image with intensity inhomogeneity, our method and RSF model can segment part of four classes successfully, and the rest of the two methods can roughly segment the image. In general, our method is more efficient as shown in Fig. 8. It can be seen from a quantitative comparison in Fig. 9 that the iteration time of the our model is less than that of the CV model, the RSF and the LRCV for above medical image segmentation.

In summary, the comparison results show that our proposed model is outperformed other three classical models in segmentation accuracy and efficiency. However, the model also has some limitations and drawbacks: Firstly, low accuracy approximation for bias field leads to under segmentation of images with intensity inhomogeneity. For example, in Fig. 5, segmentation results show that there are some fragments. Secondly, local similarity weight just comes from approximation theory rather than deterministic method. Therefore, for severely intensity inhomogeneous images (Eg. Fig. 6), it is necessary to further improve the segmentation accuracy and efficiency. Finally, unlike natural images, there is no ground truth for not existing medical image data sets, we usually rely on visual observation to judge the accuracy of medical image segmentation. As a result, the lack of quantitative analysis in experiments is one of our limitations.

Fig. 5 Comparisons of segmenting a magnetic resonance image of a human heart. The first column: initial contours. The second to fifth column: final segmentation results by the CV model, the RSF, the LRCV and our model, respectively.

5.3. Comparison of computational complexities

From the view point of computational time complexity, the main computational cost in image segmentation is closely related to the size of the image. For the four algorithms as mentioned in this paper, assuming the number of iterations is N, the complexity of the four competitive algorithms is O(N).
Fig. 6 Comparisons of segmenting a microscope cells image. The first column: initial contours. The second to fifth column: final segmentation results by the CV model, the RSF, the LRCV and our model, respectively.

Fig. 7 Comparisons of segmenting a brain MRI image. The first column: initial contours. The second to fifth column: final segmentation results by the CV model, the RSF, the LRCV and our model, respectively.

As shown in Fig. 9, the computation cost of CV model is time-consuming for the reinitializing process, which limits the practical applications. In addition, a numerical remedy for ensuring the stability of evolution can also result in time-consuming. Compared with CV model, the LRCV model reduces the dependency of manual initialization, and improves the segmentation accuracy at the expense of the improved single iteration time. In the RSF model, the model adopts the regularization strategy without expensive re-initialization, so the RSF model shows the advantage in terms of computational efficiency. However, the number of executions of the RSF model requires manual setup, which reduces the efficiency of the algorithm. In addition, there are four convolution operations in each iteration, so that the process greatly improves the computational complexity. Our proposed model has the advantage on the computational time, which lies in three aspects: firstly, the proposed model needs to perform two convolution operations in each iteration, which outperforms the RSF model. Secondly, an efficient function named potential function has been constructed to guarantee the smoothness of curve and the signed distance property, which accelerate the convergence of the curve. Thirdly, as described in Algorithm 1, our algorithm provides double termination mechanism, which ensures the curve evolution to terminate adaptively.

6. Conclusions and future works

The paper presents an efficient similarity-based level set method for medical image segmentation. The proposed method utilizes the bias field and global similarity compatibility as well as the local image information to efficiently segment medical images with intensity inhomogeneity. Moreover, the proposed method construct an adaptive condition and a polynomial penalty energy function to accelerate the evolution process in addition to keeping the stability of the curve evolution. Compared with the well-known CV model, the region scale fitting model (RSF) and the local region-based CV model (LRCV), the proposed method not only segments medical images of intensity inhomogeneity effectively, but also is less sensitive to the initial contour.

In future work, we mainly focus on three directions but not limited: Firstly, we will extend the current model using multi-phase segmentation method(Qiu, et al., 2015),(Kato, et al., 2015) or region-based convolutional networks(Girshick, et al., 2016). Secondly, we will further research the set of control parameters and improve the efficiency by multi-core CPU(Stegmaier, et al., 2016) or GPU acceleration technology(Zhou, He and Qiu, 2016). Thirdly, we will try to extend the
Fig. 8 Comparisons of segmenting a brain 3T MRI image. The first column: initial contours. The second to fifth column: final segmentation results by the CV model, the RSF, the LRCV and our model, respectively.

Fig. 9 Comparison results of convergence time.
ideas to other related fields in CAD/Graphics/Image/Vedio (Jing, et al., 2009)(Cai, et al., 2015)(Wu, et al., 2015),(Cheng, et al., 2016),(Sun, et al., 2016),(Li, He and Chen, 2016). Fourthly, we can refer to nature/image/vedio process (Rhemann, 2009),(Wu, Lim and Yang, 2013) and explore quantitative methods of medical image segmentation accuracy.

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References

Arruda, J. R. F., Surface smoothing and partial spatial derivatives computation using a regressive discrete Fourier series, Mechanical Systems and Signal Processing, Vol.6, No.1 (1992), pp.41-50.
Cai, X. T., He, F. Z., Li, W. D., et al., Encryption based partial sharing of CAD models, Integrated Computer-Aided Engineering, Vol.22, No.3 (2015), pp.243-260.
Chan, T. F. and Vese L., Active contours without edges, IEEE Transactions on Image processing, Vol.10, No.2 (2001), pp.266-277.
Chen, Y., Zhang, J. and Macione, J. An improved level set method for brain MR images segmentation and bias correction, Computerized Medical Imaging and Graphics, Vol.33, No.7 (2009), pp.510-519.
Cheng, Y., He, F., Wu, Y., et al., Meta-operation conflict resolution for humanChuman interaction in collaborative feature-based CAD systems, Cluster Computing, Vol.19, No.1 (2016), pp.237-253.
Girshick, R., Donahue, J., Darrell, T., et al., Region-based convolutional networks for accurate object detection and segmentation. IEEE Transactions on Pattern Analysis and Machine Intelligence, Vol.38, No.1 (2016), pp.142-158.
Harders, M. and Szkeley, G., Enhancing human-computer interaction in medical segmentation, Academic Press, Vol.91, No.9 (2003), pp.1430-1442.
Hedges, L. V. and Olkin, I., Statistical method for meta-analysis, CAcademic press (2014).
Huang, Z., He, F. Z., Cai, X. T. et al., Efficient random saliency map detectionion, Science China Information Sciences, Vol.54, No.6 (2011), pp.1207-1217.
Ido, Y., Basic equations and constitutive equations of micropolar magnetic fluids with EB analogy and the Abraham expression of electromagnetic momentum, JSME International Journal Series B Fluids and Thermal Engineering, Vol.48, No.3 (2005), pp.488-493.
Jiang, Z., Arabshahi, S. A. and Watanabe, T., A study on design concept for a braille tactile sensor segment using softness parameters. JSME International Journal Series C Mechanical Systems, Machine Elements and Manufacturing, Vol.49, No.2 (2006), pp.480-487.
Jing, S., He, F., Han, S., et al., A method for topological entity correspondence in a replicated collaborative CAD system, Computers in Industry, Vol.60, No.7 (2009), pp.467-475.
Kato, M., Kaneko, K., Takahashi, M., et al., Segmentation of multi-phase X-ray computed tomography images, Environmental Geotechnics, Vol.2, No.2 (2015), pp.104-117.
Kolmogorov, V. and Boykov, Y., What metrics can be approximated by geo-cuts, or global optimization of length/area and flux, Tenth IEEE International Conference on Computer Vision (ICCV2005), Beijing, China(2005).
Li, C. M., Huang, R., Ding, Z. et al., A level set method for image segmentation in the presence of intensity inhomogeneities with application to MRI, IEEE Transactions on Image Processing, Vol.20, No.7 (2011), pp.2007-2016.
Li, C., Xu, C., Gui, C., et al., Level set evolution without re-initialization: a new variational formulation, IEEE Computer Society Conference on Computer Vision and Pattern Recognition(CVPR2005), Vol.1, (2005), pp.430-436.
Li, C., Kao, C. Y., Gore, J. C., et al., Implicit active contours driven by local binary fitting energy, IEEE Conference on Computer Vision and Pattern Recognition, (2007), pp.1-7.
Li, C., Kao, C. Y., Gore, J. C., et al., Minimization of region-scalable fitting energy for image segmentation, IEEE Transactions on Image Processing, Vol.17, No.10 (2008), pp.1940-1949.
Li, C. M., Gore, J. C. and Davatzikos, C., Multiplicative intrinsic component optimization (MICO) for MRI bias field estimation and tissue segmentation, Magnetic resonance imaging, Vol.32, No.7 (2014), pp.913-923.
Li, C. M., Xu, C., Gui, C. et al., Distance regularized level set evolution and its application to image segmentation, IEEE Transactions on Image Processing, Vol.19, No.12 (2010), pp.3243-3254.
Li, K., He, F. and Chen, X., Real-time object tracking via compressive feature selection, Frontiers of Computer Science, Frontiers of Computer Science, Vol.10, No.4 (2016), pp. 689-701.
Li, X. X., He, F. Z., Cai, X. T. et al., A method for topological entity matching in the integration of heterogeneous CAD systems, Integrated Computer-Aided Engineering, Vol.20, No.1 (2013), pp.15-30.
Lie, J., Lysaker, M. and Tai, X. C., A binary level set model and some applications to Mumford-Shah image segmentation, IEEE Transactions on Image Processing, Vol.15, No.5 (2006), pp.1171-1181.
Liu, S. and Peng, Y., A local region-based Chan–Vese model for image segmentation, Pattern recognition, Vol.45, No.7 (2012), pp.2769-2779.
Melenk, J. and Sauter, S., Convergence analysis for finite element discretizations of the Helmholtz equation with Dirichlet-to-Neumann boundary conditions, Mathematics of Computation, Vol.79, No.272 (2010), pp.1871-1914.
Morel, J. M. and Solimini, S., Variational methods in image segmentation: with seven image processing experiments, Springer Science(2012).
Morel, V. and Quinn, T. M., Cartilage injury by ramp compression near the gel diffusion rate. Journal of orthopaedic research, Vol.22, No.1 (2004), pp.145-151.
Mumford, D. and Shah, J., Optimal approximations by piecewise smooth functions and associated variational problems, Communications on pure and applied mathematics, Vol.42, No.5 (2000), pp.577-685.
Ng, M. K., Chan, R. H. and Tang, W. C., A fast algorithm for deblurring models with Neumann boundary conditions, SIAM Journal on Scientific Computing, Vol.21, No.3 (1999), pp.851-866.
Ni, B., He, F. and Yuan, Z. Y., Segmentation of uterine fibroid ultrasound images using a dynamic statistical shape model in HIFU therapy, Computerized Medical Imaging and Graphics, Vol.46, No.3 (2015), pp.302-314.
Ni, B., He, F., Pan, Y., et al. Using shapes correlation for active contour segmentation of uterine fibroid ultrasound images in computer-aided therapy, Applied Mathematics-A Journal of Chinese Universities, Vol.31, No.1 (2016), pp.37-52.
Pham, H and Hasegawa, H., Adaptive Plan System of Swarm Intelligent using Differential Evolution with Genetic Algorithm, Journal of Advanced Mechanical Design, Systems, and Manufacturing, Vol.7, No.3 (2013), pp.458-473.
Pi, L., Shen, C., Li, F., et al., A variational formulation for segmenting desired objects in color images, Image and Vision Computing, Vol.25, No.9 (2007), pp.1414-1421.
Powell, M. J. D., Approximation theory and methods, Cambridge university press, (1981).
Qiu, W., Yuan, J., Rajchl, M., et al., 3D MR ventricle segmentation in pre-term infants with post-hemorrhagic ventricle dilatation (PHVD) using multi-phase geodesic level-sets, NeuroImage, Vol.118, (2015), pp.13-25.
Ranchin, F. and Dibos, F., Variational level set methods: from continuous to discrete setting, applications in video segmentation and tracking, IEEE International Conference on Image Processing, Vol.1 (2005), pp.I-273.
Rhemann, C., Rother, C., Wang, J., et al., A perceptually motivated online benchmark for image matting, IEEE Conference on Computer Vision and Pattern Recognition, (2009), p.1826-1833.
Samson, C., Blanc-Fraud, L., Aubert, G., et al., A variational model for image classification and restoration, IEEE Transactions on Pattern Analysis and Machine Intelligence, Vol.22, No.5 (2000), pp.460-472.
Saravanan, A., Balamurugan, C., Sivakumar, K., et al., Optimal geometric tolerance design framework for rigid parts with assembly function requirements using evolutionary algorithms, The International Journal of Advanced Manufacturing Technology, Vol.73, (2014), pp. 1219-1236.
Song, H., Active contours driven by regularised gradient flux flows for image segmentation, Electronics Letters, Vol.50, No.14 (2014), pp.992-994.
Steigmaier, J., Amat, F., Lemon, W. C., et al., Real-time three-dimensional cell segmentation in large-scale microscopy data of developing embryos. Developmental cell, Vol.36, No.2 (2016), pp.225-240.
Sun, J., He, F., Chen, Y., et al., A multiple template approach for robust tracking of fast motion target, Applied Mathematics-A Journal of Chinese Universities, Vol.31, No.2 (2016), pp.177-197.
Vese, L. A. and Chan, T. F., A multiphase level set framework for image segmentation using the Mumford and Shah model, International journal of computer vision, Vol.50, No.3 (2002), pp.271-293.
Vovk, U., Pernuš, F. and Likar, B., A review of methods for correction of intensity inhomogeneity in MRI, IEEE Transactions on Medical Imaging, Vol.26, No.3 (2007), pp.405-421.
Wang, X. F., Min, H. and Zhang, Y. G., Multi-scale local region based level set method for image segmentation in the presence of intensity inhomogeneity, Neurocomputing, Vol.151 (2015), pp.1086-1098.

Wang, X. F., Huang, D. S. and Xu, H., An efficient local ChanCVese model for image segmentation, Pattern Recognition, Vol.43, No.3 (2010), pp.603-618.

Wu, Y., He, F., Zhang, D., et al., Service-Oriented Feature-Based Data Exchange for Cloud-Based Design and Manufacturing, IEEE Transactions on Services Computing, (online), available from [http://ieeexplore.ieee.org/xpls/abs_all.jsp?arnumber=7331652&tag=1], (accessed on Nov. 19, 2015).

Wu, Y., Lim, J. and Yang, M. H., Online object tracking: A benchmark, Proceedings of the IEEE conference on computer vision and pattern recognition, (2013), pp.2411-2418.

Xia, Q., Shi, T., Liu, S., et al., A level set solution to the stress-based structural shape and topology optimization. Computers & Structures, Vol.90 (2012), pp.55-64.

Yan, X. H., He, F. Z., Chen, Y. L, et al. An efficient improved particle swarm optimization based on prey behavior of fish schooling. Journal of Advanced Mechanical Design, Systems, and Manufacturing, Vol.9, No.4 (2015), JAMDSM0048-JAMDSM0048.

Yushkevich, P. A., Piven, J., Hazlett, H. C., et al., User-guided 3D active contour segmentation of anatomical structures: significantly improved efficiency and reliability. Neuroimage, Vol.31, No.3 (2006), pp.1116-1128.

Zhang, D.J., He, F. Z., Han, S. H. et al., Quantitative optimization of interoperability during feature-based data exchange, Integrated Computer-Aided Engineering, Vol.23, No.1 (2016), pp.31-50.

Zhang, K., Song, H. and Zhang, L., Active contours driven by local image fitting energy, Pattern recognition, Vol.43, No.4 (2010a), pp.1199-1206.

Zhang, K., Zhang, L., Song, H., et al., Active contours with selective local or global segmentation: a new formulation and level set method, Image and Vision computing, Vol.28, No.4 (2010b), pp.668-676.

Zhang, K., Zhang, L. K. M. Lam, et al., A level set approach to image segmentation with intensity inhomogeneity, IEEE Transactions on Cybernetics (2015), pp.1-12.

Zhou, Y., Shi, W. R., Chen, W., et al., Active contours driven by localizing region and edge-based intensity fitting energy with application to segmentation of the left ventricle in cardiac CT images, Neurocomputing, Vol.156 (2015), pp.199-210.

Zhou, Y., He, F and Qiu, Y., Optimization of parallel iterated local search algorithms on graphics processing unit, The Journal of Supercomputing, Vol.72, No.6 (2016), pp.2394-2416.