Update-Efficient Regenerating Codes with Minimum Per-Node Storage

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Abstract—Regenerating codes provide an efficient way to recover data at failed nodes in distributed storage systems. It has been shown that regenerating codes can be designed to minimize the per-node storage (called MSR) or minimize the communication overhead for regeneration (called MBR). In this work, we propose a new encoding scheme for \([n, d]\) error-correcting MSR codes that generalizes our earlier work on error-correcting regenerating codes. We show that by choosing a suitable diagonal matrix, any generator matrix of the \([n, \alpha]\) Reed-Solomon (RS) code can be integrated into the encoding matrix. Hence, MSR codes with the least update complexity can be found. An efficient decoding scheme is also proposed that utilizes the \([n, \alpha]\) RS code to perform data reconstruction. The proposed decoding scheme has better error correction capability and incurs the least number of node accesses when errors are present.

I. INTRODUCTION

Cloud storage is gaining popularity as an alternative to enterprise storage where data is stored in virtualized pools of storage typically hosted by third-party data centers. Reliability is a key challenge in the design of distributed storage systems that provide cloud storage. Both crash-stop and Byzantine failures are likely to be present during data retrieval. A crash-stop failure makes a storage node unresponsive to access requests. In contrast, a Byzantine failure responds to access requests with erroneous data. To achieve better reliability, one common approach is to replicate data files on multiple storage nodes in a network. Erasure coding is employed to encode the original data and then the encoded data is distributed to storage nodes. Typically, more than one storage nodes need to be accessed to recover the original data. One popular class of erasure codes is the maximum-distance-separable (MDS) codes. With \([n, k]\) MDS codes such as Reed-Solomon (RS) codes, \(k\) data items are encoded and then distributed to and stored at \(n\) storage nodes. A user or a data collector can retrieve the original data by accessing any \(k\) of the storage nodes, a process referred to as data reconstruction.

Any storage node can fail due to hardware or software damage. Data stored at the failed nodes need to be recovered (regenerated) to remain functional to perform data reconstruction. The process to recover the stored (encoded) data at a storage node is called data regeneration. Regenerating codes first introduced in the pioneer works by Dimakis et al. in [1], [2] allow efficient data regeneration. To facilitate data regeneration, each storage node stores \(\alpha\) symbols and a total of \(d\) surviving nodes are accessed to retrieve \(\beta\) symbols from each node. A trade-off exists between the storage overhead and the regeneration (repair) bandwidth needed for data regeneration. Minimum Storage Regenerating (MSR) codes first minimize the amount of data stored per node, and then the repair bandwidth, while Minimum Bandwidth Regenerating (MBR) codes carry out the minimization in the reverse order. There have been many works that focus on the design of regenerating codes [3]–[10]. Recently, Rashmi et al. proposed optimal exact-regenerating codes that recover the stored data at the failed node exactly (and thus the name exact-regenerating) [10]; however, the authors only consider crash-stop failures of storage nodes. Han et al. extended Rashmi’s work to construct error-correcting regenerating codes for exact regeneration that can handle Byzantine failures [11]. In [11], the encoding and decoding algorithms for both MSR and MBR error-correcting codes were also provided. In [12], the code capability and resilience were discussed for error-correcting regenerating codes.

In addition to bandwidth efficiency and error correction capability, another desirable feature for regenerating codes is update complexity [13], defined as the maximum number of encoded symbols that must be updated while a single data symbol is modified. Low update complexity is desirable in scenarios where updates are frequent. Clearly, the update complexity of a regenerating code is determined by the number of non-zero elements in the row of the encoding matrix with the maximum Hamming weight. The smaller the number, the lower the update complexity is.

One drawback of the decoding algorithms for MSR codes given in [11] is that, when one or more storage nodes have erroneous data, the decoder needs to access extra data from many storage nodes (at least \(k\) more nodes) for data reconstruction. Furthermore, when one symbol in the original data is updated, all storage nodes need to update their respective data. Thus, the MSR and MBR codes in [11] have the maximum possible update complexity. Both deficiencies are addressed in this paper. First, we propose a general encoding scheme for MSR codes. As a special case, least-update-complexity codes
are designed. Second, a new decoding algorithm is presented. It not only provides better error correction capability but also incurs low communication overhead when errors occur in the accessed data.

II. ERROR-CORRECTING MSR REGENERATING CODES

In this section, we give a brief overview of data regenerating codes and the MSR code construction presented in [11].

A. Regenerating Codes

Let $\alpha$ be the number of symbols stored at each storage node and $\beta \leq \alpha$ the number of symbols downloaded from each storage during regeneration. To repair the stored data at the failed node, a helper node accesses $d$ surviving nodes. The design of regenerating codes ensures that the total regenerating bandwidth be much less than that of the original data, $B$. A regenerating code must be capable of reconstructing the original data symbols and regenerating coded data at a failed node. An $[n, k, d]$ regenerating code requires at least $k$ and $d$ surviving nodes to ensure successful data reconstruction and regeneration [10], respectively, where $n$ is the number of storage nodes and $k \leq d \leq n - 1$.

The cut-set bound given in [2], [3] provides a constraint on the repair bandwidth. By this bound, any regenerating code must satisfy the following inequality:

$$B \leq \sum_{i=0}^{k-1} \min\{\alpha, (d-i)\beta\}. \quad (1)$$

From (1), $\alpha$ or $\beta$ can be minimized achieving either the minimum storage requirement or the minimum repair bandwidth requirement, but not both. The two extreme points in (1) are referred to as the minimum storage regeneration (MSR) and minimum bandwidth regeneration (MBR) points, respectively. The values of $\alpha$ and $\beta$ for the MSR point can be obtained by first minimizing $\alpha$ and then minimizing $\beta$:

$$\alpha = d - k + 1$$
$$B = (k - d + 1) = k\alpha, \quad (2)$$

where we normalize $\beta$ as 1.

There are two categories of approaches to regenerate data at a failed node. If the replacement data is exactly the same as that previously stored at the failed node, we call it the exact regeneration. Otherwise, if the replacement data only guarantees the correctness of data reconstruction and regeneration properties, it is called functional regeneration. In practice, exact regeneration is more desirable since there is no need to inform each node in the network regarding the replacement. Furthermore, it is easy to keep the codes systematic via exact regeneration, where partial data can be retrieved without accessing all $k$ nodes. The codes designed in [10], [11] allow exact regeneration.

B. MSR Regenerating Codes With Error Correction Capability

Next, we describe the MSR code construction given in [11]. In the rest of the paper, we assume $d = 2\alpha$. The information sequence $m = [m_0, m_1, \ldots, m_{B-1}]$ can be arranged into an information vector $U = [Z_1, Z_2]$ with size $\alpha \times d$ such that $Z_1$ and $Z_2$ are symmetric matrices with dimension $\alpha \times \alpha$. An $[n, d = 2\alpha]$ RS code is adopted to construct the MSR code [11]. Let $\alpha$ be a generator of $GF(2^\alpha)$. In the encoding of the MSR code, we have

$$U \cdot G = C, \quad (3)$$

where

$$G = \begin{bmatrix}
1 & 1 & \cdots & 1 & 1 \\
(a^0)^2 & (a^1)^2 & \cdots & (a^{n-1})^2 \\
(a^0)^{\alpha-1} & (a^1)^{\alpha-1} & \cdots & (a^{n-1})^{\alpha-1} \\
(a^0)^{1} & (a^1)^{1} & \cdots & (a^{n-1})^{1} \\
(a^0)^{0} & (a^1)^{0} & \cdots & (a^{n-1})^{0} \\
(a^0)^{2} & (a^1)^{2} & \cdots & (a^{n-1})^{2} \\
(a^0)^{\alpha} & (a^1)^{\alpha} & \cdots & (a^{n-1})^{\alpha} \\
\vdots & \vdots & \ddots & \vdots \\
\end{bmatrix}
\begin{bmatrix}
\bar{G} \\
\bar{G} \Delta
\end{bmatrix}, \quad (4)$$

and $C$ is the codeword vector with dimension $(\alpha \times n)$. $G$ contains the first $\alpha$ rows in $G$ and $\Delta$ is a diagonal matrix with $(a^0)^{\alpha}, (a^1)^{\alpha}, (a^2)^{\alpha}, \ldots, (a^{n-1})^{\alpha}$ as diagonal elements. Note that if the RS code is over $GF(2^\alpha)$ for $m \geq \lceil \log_2 {\alpha} \rceil$, then it can be shown that $(a^0)^{\alpha}, (a^1)^{\alpha}, (a^2)^{\alpha}, \ldots, (a^{n-1})^{\alpha}$ are all distinct. After encoding, the $i$th column of $C$ is distributed to storage node $i$ for $1 \leq i \leq n$.

III. ENCODING SCHEMES FOR ERROR-CORRECTING MSR CODES

RS codes are known to have very efficient decoding algorithms and exhibit good error correction capability. From [4] in Section II-B a generator matrix $G$ for MSR codes needs to satisfy:

1) $G = \begin{bmatrix} \bar{G} \\ \bar{G} \Delta \end{bmatrix}$, where $G$ contains the first $\alpha$ rows in $G$ and $\Delta$ is a diagonal matrix with distinct elements in the diagonal.
2) $\bar{G}$ is a generator matrix of the $[n, \alpha]$ RS code and $G$ is a generator matrix of the $[n, d = 2\alpha]$ RS code.

Next, we present a sufficient condition for $\bar{G}$ and $\Delta$ such that $G$ is a generator matrix of an $[n, d]$ RS code.

Theorem 1: Let $\bar{G}$ be a generator matrix of the $[n, \alpha]$ RS code $C_\alpha$ that is generated by the generator polynomial with roots $a^0, a^1, a^2, \ldots, a^{n-\alpha}$. Let the diagonal elements of $\Delta$ be $(a^0)^{\alpha}, (a^1)^{\alpha}, \ldots, (a^{n-1})^{\alpha}$, where $m \geq \lceil \log_2 {\alpha} \rceil$ and $\gcd(2^m - 1, \alpha) = 1$. Then $G$ is a generator matrix of $[n, d]$ RS code $C_d$ that is generated by the generator polynomial with roots $a^0, a^2, \ldots, a^{n-d}$.

Proof: We need to show that each row of $\bar{G} \Delta$ is a codeword of $C_d$, and all rows in $G$ are linearly independent.
Let \( c = (c_0, c_1, \ldots, c_{n-1}) \) be any row in \( \bar{G} \). Then the polynomial representation of \( c \bar{G} \) is
\[
\sum_{i=0}^{n-1} c_i(a^i)x^i = \sum_{i=0}^{n-1} c_i(a^\alpha x^i) = 0.
\]
Since \( c \in C_{\alpha} \), \( c \) has roots \( a^1, a^2, \ldots, a^{n-\alpha} \). Then it is easy to see that \( 5 \) has roots \( a^{\alpha+1}, a^{\alpha+2}, \ldots, a^{n-2\alpha} \) that clearly contain \( a^1, a^2, \ldots, a^{n-2\alpha} \). Hence, \( \alpha \Delta \in C_{\alpha} \).

In order to show that all rows in \( \bar{G} \) are linearly independent, it is sufficient to show that \( \alpha \Delta \notin C_{\alpha} \) for all nonzero \( c \in C_{\alpha} \). Assume that \( \alpha \Delta \in C_{\alpha} \). Then \( \sum_{i=0}^{n-1} c_i(a^i x^i) \) must have roots \( a^1, a^2, \ldots, a^{n-\alpha}. \) It follows that \( c(x) \) must have \( a^{\alpha+1}, a^{\alpha+2}, \ldots, a^\alpha \) as roots. Recall that \( c(x) \) also has roots \( a^1, a^2, \ldots, a^{n-\alpha} \). Since \( n - 1 \geq d = 2\alpha \), we have \( n - \alpha \geq \alpha + 1 \). Hence, \( c(x) \) has \( n \) distinct roots of \( a^1, a^2, \ldots, a^{n-\alpha} \). This is impossible since the degree of \( c(x) \) is at most \( n - 1 \). Thus, \( \alpha \Delta \notin C_{\alpha} \).

One advantage of the proposed scheme is that it can now operate on a smaller finite field than that of the scheme in [11].

Another advantage is that one can choose \( G \) (and \( \Delta \) accordingly) freely as long as it is the generation matrix of an \([n, \alpha]\) RS code. In particular, as discussed in Section I, to minimize update complexity, it is desirable to choose a generator matrix where the row with the maximum Hamming weight has the least number of nonzero elements. Next, we present a least-update-complexity generator matrix that satisfies (4).

**Corollary 1**: Let \( \Delta \) be the one given in Theorem I. Let \( \bar{G} \) be the generator matrix of a systematic \([n, \alpha]\) RS code, namely,
\[
\bar{G} = [D/I]
\]
where
\[
D = \begin{bmatrix}
 b_{00} & b_{01} & b_{02} & \cdots & b_{0(n-\alpha-1)} \\
b_{10} & b_{11} & b_{12} & \cdots & b_{1(n-\alpha-1)} \\
b_{20} & b_{21} & b_{22} & \cdots & b_{2(n-\alpha-1)} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
b_{(\alpha-1)0} & b_{(\alpha-1)1} & b_{(\alpha-1)2} & \cdots & b_{(\alpha-1)(n-\alpha-1)}
\end{bmatrix}
\]
\(I\) is the \((\alpha \times \alpha)\) identity matrix, and
\[
x^{n-\alpha+i} = u_i(x)g(x) + b_i(x) \text{ for } 0 \leq i \leq \alpha - 1.
\]
Then, \( G = \begin{bmatrix} \bar{G} & \bar{G} \Delta \end{bmatrix} \) is a least-update-complexity generator matrix.

**Proof**: The result holds since each row of \( \bar{G} \) is a nonzero codeword with the minimum Hamming weight \( n - \alpha + 1 \).

**IV. EFFICIENT DECODING SCHEME FOR ERROR-CORRECTING MSR CODES**

Unlike the decoding scheme in [11] that uses \([n, d]\) RS code, we propose to use the subcode of the \([n, d]\) RS code, the \([n, \alpha = k - 1]\) RS code generated by \( \bar{G} \), to perform the data reconstruction. The advantage of using the \([n, k - 1]\) RS code is two-fold. First, its error correction capability is higher (namely, it can tolerate \( \lfloor \frac{d-k+2}{2} \rfloor \) instead of \( \lfloor \frac{d}{2} \rfloor \)) errors). Second, it only requires the access of two additional storage nodes (as opposed to \( d - k + 2 = k \) nodes) for the first error to correct.

Without loss of generality, we assume that the data collector retrieves encoded symbols from \( k + 2v \) (\( v \geq 0 \)) storage nodes, \( j_0, j_1, \ldots, j_{k+2v-1} \). We also assume that there are \( v \) storage nodes whose received symbols are erroneous. The stored information of the \( k + 2v \) storage nodes are collected as the \( k + 2v \) columns in \( Y_{x \times (k+2v)} \). The \( k + 2v \) columns of \( G \) corresponding to storage nodes \( j_0, j_1, \ldots, j_{k+2v-1} \) are denoted as the columns of \( G_{k+2v} \). First, we discuss data reconstruction when \( v = 0 \). The decoding procedure is similar to that in [10].

**No Error**: In this case, \( v = 0 \) and there is no error in \( Y \). Then,
\[
Y = UG_k = [Z_1 Z_2] \begin{bmatrix} \bar{G}_k \\ \bar{G}_k \Delta \end{bmatrix} = [Z_1 \bar{G}_k + Z_2 \bar{G}_k \Delta].
\]

Multiplying \( \bar{G}_k^T \) and \( Y \) in [7], we have [10].
\[
\bar{G}_k^T Y = \begin{bmatrix} \bar{G}_k^T Z_1 \bar{G}_k + \bar{G}_k^T Z_2 \bar{G}_k \Delta \end{bmatrix} = P + Q \Delta.
\]

Since \( Z_1 \) and \( Z_2 \) are symmetric, \( P \) and \( Q \) are symmetric as well. The \((i, j)\)th element of \( P + Q \Delta \), \( 1 \leq i, j \leq k \) and \( i \neq j \), is
\[
p_{ij} + q_{ij} a^{(j-1)\alpha},
\]
and the \((j, i)\)th element is given by
\[
p_{ji} + q_{ji} a^{(i-1)\alpha}.
\]
Since \( a^{(j-1)\alpha} \neq a^{(i-1)\alpha} \) for all \( i \neq j \), \( p_{ij} = p_{ji}, \) and \( q_{ij} = q_{ji} \), combining (9) and (10), the values of \( p_{ij} \) and \( q_{ij} \) can be obtained. Note that we only obtain \( k - 1 \) values for each row of \( P \) and \( Q \) since no elements in the diagonal of \( P \) or \( Q \) are the components in each row of \( P \), it is possible to decode \( \bar{G}_k^T Z_1 \bar{G}_k \) by the error-and-erasure decoder of the \([n, k - 1]\) RS code.

Since one cannot locate any erroneous position from the decoded rows of \( P \), the decoded \( \alpha \) codewords are accepted as \( \bar{G}_k^T Z_1 \bar{G}_k \). By the construction of \( G \), it is easy to see that \( G \) is a generator matrix of the \([n, k - 1]\) RS code. Hence, each row in the matrix \( \bar{G}_k^T Z_1 \bar{G}_k \) is a codeword. Since we have known \( k - 1 \) components in each row of \( P \), it is possible to decode \( \bar{G}_k^T Z_1 \bar{G}_k \) by the error-and-erasure decoder of the \([n, k - 1]\) RS code.

\(2\) The error-and-erasure decoder of an \([n, k - 1]\) RS code can successfully decode a received vector if \( s + 2v < n - k + 2 \), where \( s \) is the erasure (no symbol) positions, \( v \) is the number of errors in the received portion of the received vector, and \( n - k + 2 \) is the minimum Hamming distance of the \([n, k - 1]\) RS code.
Multiple Errors: Before presenting the proposed decoding algorithm, we first prove that a decoding procedure can always successfully decode $Z_{1}$ and $Z_{2}$ if $v \leq \left\lfloor \frac{n-k+1}{2} \right\rfloor$ and all storage nodes are accessed. Due to space limitation, all proofs are omitted in this section.

Assume the storage nodes with errors correspond to the $\ell_{0}$th, $\ell_{1}$th, . . . , $\ell_{v-1}$th columns in the received matrix $Y_{n \times n}$. Then,

\[
\frac{G^{T}}{Y_{n \times n}} = \frac{G^{T}}{U} + \frac{G^{T}}{E} = \left[ \frac{G}{G_{A}} \right] + \frac{G^{T}}{E} = \left[ \frac{\hat{G}^{T}}{Z_{1}} \frac{\hat{G}^{T}}{\hat{G}^{T}} \right] + \frac{G^{T}}{E} ,
\]

where

\[
E = \left[ G_{0 \times (\ell_{1} - t_{0} - 1)}^{T} | \cdot \cdot \cdot | G_{0 \times (n - \ell_{v - 1})}^{T} \right] .
\]

**Lemma 1:** There are at least $n-k+2$ errors in each of the $\ell_{0}$th, $\ell_{1}$th, . . . , $\ell_{v-1}$th columns of $G^{T}Y_{n \times n}$.

We next have the main theorem to perform data reconstruction.

**Theorem 2:** Let $\hat{G}^{T}Y_{n \times n} = \hat{P} + \hat{Q}\Delta$. Furthermore, let $\hat{P}$ be the corresponding portion of decoded codeword vector to $\hat{P}$ and $E_{P} = \hat{P} \oplus \hat{P}$ be the error pattern vector. Assume that the data collector accesses all storage nodes and there are $v$, $1 \leq v \leq \left\lfloor \frac{n-k+1}{2} \right\rfloor$, of them with errors. Then, there are at least $n-k+2-v$ nonzero elements in $\ell_{v}$th column of $E_{P}$, $0 \leq j \leq v-1$, and at most $v$ nonzero elements in the rest of columns of $E_{P}$.

The above theorem allows us to design a decoding algorithm that can correct up to $\left\lfloor \frac{n-k+1}{2} \right\rfloor$ errors. In particular, we need to examine the erroneous positions in $\hat{G}^{T}Z_{1}E$. Since $1 \leq v \leq \left\lfloor \frac{n-k+1}{2} \right\rfloor$, we have $n-k+2-v \geq \left\lfloor \frac{n-k+1}{2} \right\rfloor + 1 > v$. Thus, the way to locate all erroneous columns in $\hat{P}$ is to find out all columns in $E_{P}$ where the number of nonzero elements in them are greater than or equal to $\left\lfloor \frac{n-k+1}{2} \right\rfloor + 1$. After we locate all erroneous columns we can follow a procedure similar to that given in the no error case to recover $Z_{1}$ from $\hat{P}$.

The above decoding procedure guarantees to recover $Z_{1}$ when all $n$ storage nodes are accessed. However, it is not very efficient in terms of bandwidth usage. Next, we present a progressive decoding version of the proposed algorithm that only accesses enough extra nodes when necessary. Before presenting it, we need the following corollary.

**Corollary 2:** Consider that one accesses $k+2v$ storage nodes, among which $v$ nodes are erroneous and $1 \leq v \leq \left\lfloor \frac{n-k+1}{2} \right\rfloor$. There are at least $v+2$ nonzero elements in $\ell_{v}$th column of $E_{P}$, $0 \leq j \leq v-1$, and at most $v$ among the remaining columns of $E_{P}$.

Based on Corollary 2, we can design a progressive decoding algorithm that retrieve extra data from remaining storage nodes when necessary. To handle Byzantine fault tolerance,
failure patterns cannot be reconstructed using the previous algorithm in [11]. The advantage of the proposed algorithm is also overwhelming in the average number of accessed nodes for data reconstruction. Due to space limitation, the simulation results are omitted.

V. Conclusion

In this work we proposed a new encoding scheme for the \([n, 2\alpha]\) error-correcting MSR codes from the generator matrix of any \([n, \alpha]\) RS codes. It generalizes the previously proposed MSR codes in [11] and has several salient advantages. It allows the construction of least-update-complexity codes with a properly chosen systematic generator matrix. More importantly, the decoding scheme leads to an efficient decoding scheme that can tolerate more errors at the storage nodes, and access additional storage nodes only when necessary. A progressive decoding scheme was thereby devised with low communication overhead.

Possible future work includes extension of the encoding and decoding schemes to MBR points, and the study of encoding schemes with optimal update complexity and good regenerating capability.

ACKNOWLEDGMENT

This work was supported in part by CASE: The Center for Advanced Systems and Engineering, a NYSTAR center for advanced technology at Syracuse University; the National Science Council (NSC) of Taiwan under grants no. 99-2221-E-011-158-MY3 and NSC 101-2221-E-011-069-MY3; US National Science Foundation under grant no. CNS-1117560 and McMaster University new faculty startup fund.

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Node Failure Probability

Average Number of Accesses (Nodes)

[20,10,18] MSR Codes for Data Reconstruction

Previous Algorithm [14]
Proposed Algorithm