Quantum Computation with Quantum Dots and Terahertz Cavity Quantum Electrodynamics

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I. INTRODUCTION

A quantum computer processes quantum information which is stored in “quantum bits” (qubits). If a small set of fundamental operations, or “universal quantum logic gates,” can be performed on the qubits, then a quantum computer can be programmed to solve an arbitrary problem. The explosion of interest in quantum computation can be traced to Shor’s demonstration in 1994 that a quantum computer could efficiently factorize large integers. Further boosts came in 1996, with the proof that quantum error correcting codes exist. It has since been shown that if the quantum error rate is below an accuracy threshold, quantum information can be stored indefinitely.

The implementation of a large-scale quantum computer is recognized to be a technological challenge of unprecedented proportions. The qubits must be well-isolated from the decohering influence of the environment, but must also be manipulated individually to initialize the computer, perform quantum logic operations, and measure the result of the computation.

Implementations of universal quantum logic gates and quantum computers have been proposed using atomic beams, trapped atoms and ions, bulk nuclear magnetic resonance, nanostructured semiconductors, and Josephson junctions. In schemes based on trapped atoms and ions, qubits couple with collective excitations or cavity photons. Such long-range coupling enables two-bit gates involving an arbitrary pair of qubits, which makes programming straightforward. However, in the atomic and ionic schemes, the gates must be performed serially, whereas existing error correcting schemes require some degree of parallelism. In semiconductor and superconductor schemes which have been proposed, only nearest-neighbor qubits can be coupled, and significant overhead is involved in coupling distant qubits. However, some of these schemes have the important advantage that gate operations can be performed in parallel.

It is widely agreed that a solid-state quantum computer, if it can be realized, will be the only way to produce a quantum computer containing, for example, $10^3$ qubits. The remainder of this paper describes what is, to our knowledge, the first proposal for a semiconductor-based quantum computer in which quantum gates can be effected between an arbitrary pair of qubits. The qubits consist of the lowest electronic states of specially-engineered quantum dots (QDs) and are coupled by Terahertz cavity photons. The proposal combines ideas from the atomic and ionic implementations described above with recent developments in the spectroscopy of doped semiconductor nanostructures at Terahertz frequencies.

II. QUANTUM BITS AND FUNDAMENTAL QUANTUM LOGIC OPERATIONS

The fundamental building blocks of the proposed computer are the nanostructures shown in Fig. 1. Three disks of a semiconductor (e.g., GaAs) are embedded in a semiconductor with a larger band gap (e.g., AlGaAs). The central disk is taller than the outer two. The barriers between the disks are sufficiently thin to allow an electron to rapidly tunnel between them. A structure consisting of a set of three disks and the two intervening barriers is hereafter called a quantum dot (QD). Each QD which is to participate in the quantum computation must have one and only one electron. The potential and four lowest electronic energy levels for a particular realization of a QD are shown in Fig. 2. The lowest two energy levels, denoted $|0\rangle$ and $|1\rangle$, will form the qubits which store quantum information. The third energy level, labeled $|2\rangle$, will serve as an auxiliary state to perform conditional rotations of the state vector of the qubit, much like the auxiliary state in the ion trap computer. Below and above each QD is an electrical gate. Voltages applied to
these gates are used to control the spacing between and absolute values of the energy levels of the QDs via the Stark effect. A large number of individually-gated QDs is contained in a 3-D microcavity whose fundamental resonance has a wavelength \( \lambda_c \) much longer than a QD. A continuous-wave laser with a fixed wavelength different than \( \lambda_c \) is introduced through one side of the cavity.

Fig. 3a shows the energies \( E_{10} \) and \( E_{20} \) of the 0-1 and 0-2 transitions in a QD as a function of the electric field \( e \) applied via the gates. Also shown in Fig. 3a are the energies of a cavity mode photon \( h \omega_c \) a laser photon \( h \omega_l \) and the sum \( h \omega_l + h \omega_c \). The state of an electron in a QD can be coherently manipulated by tuning \( E_{10} \) and \( E_{20} \) into and out of resonance with \( h \omega_c, h \omega_l \), and the sum \( h \omega_l + h \omega_c \).

A general Hamiltonian describing a QD interacting with cavity photons and the laser field is given by

\[
\hat{H} = h \omega_c \hat{a}_c^\dagger \hat{a}_c + E_{10}(e) \hat{\sigma}_{11} + E_{20}(e) \hat{\sigma}_{22} + h g_{01}(e) (\hat{a}_c^\dagger \hat{\sigma}_{01} + \hat{\sigma}_{10} \hat{a}_c) + H_{10}(e) (\hat{\sigma}_{01} \exp(i \omega_l t) + \hat{\sigma}_{10} \exp(-i \omega_l t)) + H_{21}(e) (\hat{a}_c^\dagger \hat{\sigma}_{12} \exp(i \omega_l t) + \hat{\sigma}_{21} \hat{a}_c \exp(-i \omega_l t))
\]

(1)

where \( \hat{a}_c \) denotes the cavity-mode annihilation operator, and \( \hat{\sigma}_{ij} = |i\rangle\langle j| \) is the projection operator from QD state \( |j\rangle \) to state \( |i\rangle \). The vacuum Rabi frequencies are

\[
ge_{\text{vac}} = \sqrt{\frac{h \omega_c}{2 e \epsilon_0 V}}
\]

(2)

is the amplitude of the vacuum electric field in the cavity, and \( e \) and \( V \) are the dielectric constant and volume of the cavity, respectively, \( q \) is the electronic charge and \( z_{ij} \) is the dipole matrix element of the \( |i\rangle \rightarrow |j\rangle \) transition. One step in the CNOT operation will be a Rabi oscillation between states \( |0\rangle \) and \( |2\rangle \) involving both cavity and laser photons at \( e = e_{t+c} \). An effective Hamiltonian describing these two-photon processes is given by replacing the last four terms of (1) with

\[
H_{2-\text{photon}} = \hbar \hat{\Omega}(e) (\hat{a}_c^\dagger \hat{\sigma}_{02} \exp(i \omega_l t) + \hat{\sigma}_{20} \hat{a}_c \exp(-i \omega_l t))
\]

(3)

where the two-photon effective Rabi frequency \( \hat{\Omega} \) is given by

\[
\hat{\Omega}(e) = \frac{g_{01}(e) \Omega_{10,1}(e) + g_{12}(e) \Omega_{01,1}(e)}{\omega_{21}(e) - \omega_l}
\]

(4)

The effective two-photon Hamiltonian neglects ac Stark shifts and terms which do not satisfy resonance conditions. In addition, we envision a scenario in which the first term of Eq. (4) dominates \( \omega_{21}(e_{t+c}) - \omega_l \ll \omega_{21}(e_{t+c}) - \omega_c \), and the conditional phase shift dominates over phase shifts induced by the cavity field alone \( (\Omega_{10,1}(e_{t+c}) \gg g_{01}(e_{t+c})) \).

During the operation of this quantum computer, a qubit which is simply storing quantum information is in state \( |0\rangle \) or state \( |1\rangle \), and the electric field across it is held at a fiducial value at which the energy levels of the qubit are not resonant with \( h \omega_c, h \omega_l \) or \( h \omega_l + h \omega_c \). For simplicity, we choose this fiducial field to be zero. For \( e = e_c \), the first interaction term dominates as \( |\omega_{10}(e_c) - \omega_c| < |\omega_{10}(e_c) - \omega_l| \). If the cavity contains one photon or the qubit state vector is in state \( |1\rangle \), then the qubit will execute vacuum Rabi oscillations with frequency \( g_{10} \), in which the probability of finding the electron in the excited state oscillates 90° out of phase with the probability of finding one photon in the cavity. For \( e = e_t \), the second interaction term has the resonant contribution. Here, the state vector of the qubit rotates between states \( |0\rangle \) and \( |1\rangle \) with laser Rabi frequency \( \Omega_{10,1} \). Finally, for \( e = e_{t+c} \), the \( H_{2-\text{photon}} \) dominates. If the cavity contains one photon and the qubit state vector begins in state \( |0\rangle \) then it rotates between states \( |0\rangle \) and the auxiliary \( |2\rangle \) with frequency \( \hat{\Omega}(e_{t+c}) \). If either the qubit is in state \( |1\rangle \) or the cavity does not contain a photon, then the qubit state vector is not rotated for \( e = e_{t+c} \).

A controlled not (CNOT) operation is effected by a series of voltage pulses applied across the gates of a pair of qubits. The pulses always begin and end with the qubit at the fiducial electric field (\( e = 0 \)), and rise to a target value \( e_c \), \( e_t \), or \( e_{t+c} \). Figure 3b shows a sequence of voltage pulses which effects a two-qubit gate which is equivalent to a CNOT operation \([10]\). The cavity always begins with no photons. First, a \( \pi \)-pulse with height \( e_c \) and duration \( \delta t \) is applied to the control bit. If the control bit is in state \( |0\rangle \), it is unaffected. If it is in state \( |1\rangle \), it rotates into state \( |0\rangle \) and acquires a phase \( i \), and the cavity acquires a single photon. Next, a \( 2\pi \)-pulse with height \( e_{t+c} \) and duration \( \pi / (2 g_{10}) \) is applied to the target bit. If the target bit is in state \( |1\rangle \), it is unaffected. If it is in state \( |0\rangle \) and the cavity contains one photon, it acquires a phase \(-1\). Finally, a pulse with height \( e_c \), identical to the first pulse, is again applied to the control bit. If there is a photon in the cavity it is absorbed by the control bit, returning it to state \( |1\rangle \) while the control bit acquires another phase \( i \). The end result is a gate in which the state vector of the two-qubit system acquires a phase \(-i\) if and only if both control and target bits are initially \( 1 \). The sequence of state-vector rotations which is effected by the series of electric field pulses is identical to the sequence effected by a series of laser pulses applied to cold trapped ions in Ref. \([10]\). In order to effect a CNOT operation (inversion of the target bit if and only if the control bit is \( 1 \)), it is necessary to apply to the target bit \( \pi / 2 \) and \( 3\pi / 2 \) pulses with height \( e_L \) and durations \( \pi / (4 \Omega_{L,01}) \) and \( 3\pi / (4 \Omega_{L,01}) \), respectively, before and after the sequence shown in Fig. 3b \([10]\).

A few additional conditions are required to ensure the fidelity of CNOT operations. To ensure that nearly all of the state-vector rotation occurs while the electric field is at its target value, the rise and fall times \( \delta t \) of the
pulses must be short compared to the period of the Rabi oscillation at the target \( e \). At the same time, in order to minimize the probability of a transition between \( |0\rangle \) and \( |1\rangle \) induced by the ramping electric field, one requires the changes to the Hamiltonian to be adiabatic (\( \delta t \gg \hbar/E_{10} \)). As with other schemes for quantum computation, the timing between the successive pulses in the CNOT operation must be carefully adjusted to compensate for the quantum-mechanical phases accumulated by inactive qubits in their excited states. In order to achieve the sort of fidelity which is required for a CNOT operation in a quantum computer, it may be necessary to adjust the heights and durations of the electric field pulses to account for ac Stark shifts in the energy levels of the QDs which are induced by the laser field. These important details will be addressed in a future publication.

### III. REQUIREMENTS FOR QUANTUM COMPUTATION

The ability to effect a CNOT operation is one of several requirements for a universal quantum computer. Other requirements include:

1) Initializing the computer: Before a quantum computation begins, each qubit must be in a well-defined state. In the proposed computer, it suffices to wait, with all gate voltages at the fiducial voltage (\( e = 0 \)) and at a temperature \( T \ll E_{10}/k_B \), until each qubit relaxes to its ground state. For \( E_{10} \approx 10 \text{ meV} \), one requires a temperature \( T \ll 120 \text{ K} \). From calculations detailed below of the predicted energy relaxation times in QDs, a wait of less than 1 s will certainly ensure that all qubits are in state \( |0\rangle \).

2) Inputting initial data: At the beginning of a quantum computation, arbitrary rotations of the state vectors of qubits are required to load data into the qubit registers. Arbitrary one-bit rotations are effected using Rabi oscillations induced by the laser field, by applying pulses with height \( e_l \) and duration between 0 and \( 2\pi/(\Omega_{e,01}) \).

3) Readout: At the end of a quantum computation, the state of each qubit must be measured. During the read-out phase, we propose that a narrow-band detector with high quantum efficiency and the sensitivity to detect single THz photons be tuned to the frequency of the cavity mode \( \omega_c \). The qubits can then be read out sequentially by tuning them to \( \omega_c \). If the qubit is in state \( |1\rangle \), it will emit a photon which will be detected. For the parameters discussed below, the rate at which qubits can be read out will be roughly \( g_{01} \geq 3 \times 10^8 \text{ Hz} \). A detector is required which has a noise equivalent power \( N\text{E}\text{P} \approx E_{10}/g_{01} = 10^{-77} \text{ W/Hz}^{1/2} \) with a bandwidth greater than \( g_{01} \). While such a detector is not currently available, one of us has proposed a tunable antenna-coupled intersubband terahertz (TACIT) \( [21,22] \) detector which will be fabricated from semiconductor quantum wells, could be monolithically-integrated into the quantum computer, and has been modeled to achieve the required speed and sensitivity.

4) Error correction: Existing schemes for error correction require the execution of quantum logic gates in parallel. One can imagine parallelizing the scheme proposed here by enlarging the cavity to create several cavity modes in the frequency range over which QD energy level spacings are tunable. This would come at the cost of slowing down gate operations by reducing the vacuum Rabi frequency, and hence the vacuum Rabi frequency \( g_{01} \alpha V^{-1/2} \). A more intriguing possibility is to somehow marry a nearest-neighbor-coupled semiconductor scheme for quantum computation like \( [12,14] \) with a nonlocal scheme like the one proposed here. In this case, logic gates would be effected in parallel in clusters of qubits coupled with nearest-neighbor interactions, while qubits in distant clusters could communicate serially via long-range interactions mediated by cavity photons.

5) Decoherence: This is the most problematic issue pertaining to most quantum computers. In the computer proposed here, decoherence of the electronic state of the QD as well as of the cavity photons must be considered.

There are no experimental data on the decoherence of electronic intraband excitations in isolated QDs loaded with a single electron. Dephasing in “open” quantum dots defined by gate electrodes in a 2-DEG has been studied, yielding dephasing times \( t_0 \leq 2 \text{ ns} \). The studied dots have \( E_{10} \leq 20 \text{ meV} \). The times are consistent with those predicted for disordered 2-D systems. The rate of spontaneous emission of acoustic phonons in \( \approx 200 \text{ nm} \) double QD devices containing 15-25 electrons has recently been deduced. From the transport currents of order \( I = 10^{-12} \text{ A} \), one deduces an energy relaxation time \( g/I = 10^{-7} \text{ s} \) for transitions with energies near \( 50 \text{ meV} \). However, the transition energies, QD geometry, and number of electrons in the experiments \( [23,24] \) are very different from those of Fig. 1, and it is thus impossible to draw conclusions about decoherence in the QDs envisioned in this proposal.

Many interactions will potentially cause decoherence of electrons in the computer proposed here. Some can be mitigated by clever engineering, including

(a) the emission of freely-propagating photons which is eliminated because the QDs are in a 3-D cavity with a very high quality factor:

(b) the interaction with fluctuations in the potentials of the two gate electrodes associated with each QD. Both cross-talk from switching voltages on distant QDs and thermal fluctuations (i.e., Johnson noise) on a QD’s gate electrodes can contribute to fluctuating gate potentials with frequencies much lower than \( E_{10}/\hbar \). Such low frequency noise causes adiabatic changes \( \delta \bar{E}_N(t) \) to the energy of levels \( E_N \), which lead phase errors \( \delta \phi_{e_n}(t) = -1/\hbar \int \delta E_N(e_N(t')d't) \) (here, \( n = 0,1 \)). Such phase errors can be restricted to occur only during the time of a logic operation. The gate electrodes are made out of a superconductor. When a QD is not involved in a logic
ooperation, its two gates are connected to a superconducting ground by a superconducting path. Since there is no dissipation, there are no thermal fluctuations. Furthermore, electric fields generated far from the QD are screened by the gate electrodes. While a QD is being switched, the connection to the superconducting ground must be broken. Low-frequency noise which occurs during this time will contribute to an error in the accuracy of a CNOT operation. These and other possible errors in CNOT operation, as well as possible ways to correct them, will be analyzed in a future publication.

(c) the interaction with metastable traps in the semiconductor. Metastable traps in the semiconductor are a source of extremely slow time-varying electric fields ($f < 100$ Hz). If the fluctuating traps are far from a given gate electrode, they will cause a slowly fluctuating electric field near that electrode, which is screened as described in (b). Hence, it is only the traps which are fluctuating in the tiny volume between the two gate electrodes that pose a serious problem. The density of such traps in semiconductor nanostructures is constantly being reduced with advances in processing.

(d) inhomogeneity of quantum dots. Different dots will vary slightly in their energy levels and matrix elements. This inhomogeneity can arise from geometrical variations between the quantum dots, and also from the presence of quenched disorder (static charged defects). Inhomogeneity and static disorder do not contribute to decoherence of quantum bits. However, to perform accurate one- and two-bit operations in an inhomogeneous population of quantum dots, each quantum dot in the quantum computer will need to be calibrated (by performing a CNOT operation, for example) before a quantum computation is run. Note that all solid-state implementations of quantum computers will share the need to calibrate in order to overcome disorder.

The lifetime of a cavity photon must be sufficiently long to enable many CNOT operations with high fidelity. This will require the development of few-node THz cavities with extremely low loss. The expected cavity losses cannot be analyzed without details of the quantum computer’s architecture which are beyond the scope of this paper, and materials properties which have not yet been measured. It is likely that cavities made from conventional metals will introduce losses which are unacceptable. One attractive possibility is dielectric cavities, made, for example from ultrapure Si. Existing measurements of optical loss in Si at THz frequencies are dominated by free carrier absorption [27], which can be eliminated by purifying and cooling the Si. Residual losses in Si are due to a process in which a THz photon creates 2 phonons [28]. These minuscule THz losses have not been measured or realistically computed to our knowledge. A second promising possibility is to use quantum dots with $E_{01}$ smaller than the energy gap of a s-wave superconductor (3 meV for Niobium, 9 meV for $Rb_3C_{60}$ [29]). A cavity with volume $< \lambda^3$ and extremely low loss could then be made of a segment of superconducting transmission line.

IV. TIMES TO DECOHERE, PERFORM CNOT

Consider now a specific GaAs/AlGaAs QD and lossless dielectric cavity designed to minimize the time required for a CNOT operation, while at the same time avoiding the emission of longitudinal optical (LO) phonons ($\hbar \omega_{LO} \approx 36$ meV in GaAs) and also minimizing the rate of acoustic phonon emission. Cavity and laser photon energies are chosen to be 11.5 and 15 meV. These energies are sufficiently large to enable an adequate vacuum electric field $e_{vac}$ while their sum is still comfortably smaller than $\hbar \omega_{LO}$. Assuming perfect cylindrical symmetry, the states are labeled with quantum numbers $|l, m, n\rangle$, associated with the radial, azimuthal and axial degrees of freedom, respectively. The potential along the cylindrical axis of the QD (z-direction) and the numerically-computed four lowest energy levels are depicted in Fig. 2.

Figure 3a shows the transitions $E_{10}$ and $E_{20}$ vs. electric field $E$. Assuming infinite walls in the radial direction, the radial wavefunctions are given by Bessel functions. The difference between the energy of the ground and first radial excited states is $\Delta E_r = 30$ meV for radius $a = 13$ nm, assuming $m^* = m_e/15$. This is higher than the highest energy reached by an electron during a CNOT operation (26.5 meV = $\hbar \omega_{LO} + \hbar \omega_e$), eliminating decoherence arising from coupling between radial and axial excited states of the QD. The growth of QDs similar to those in Fig. 1 is currently being attempted. One method is to grow stacked self-assembled QDs [30]. A second method is to make QDs made by growing GaAs/AlGaAs quantum wells with the conduction band profile tailored to give the desired potential in the z-direction (for example, that shown in Fig. 2), depositing small islands on top of the quantum well to serve as an etch mask, etching through the quantum well layers which are not protected by the islands, and then regrowing AlGaAs [31].

Decoherence due to the emission of longitudinal acoustic (LA) phonons would be difficult to mitigate given a particular QD structure. It can be computed using the deformation potential approximation, in which electrons scatter from potential fluctuations arising from local volume compressions and dilations induced by LA phonons. Piezoelectric coupling between electrons and transverse acoustic phonons exists in III-V semiconductors, but is thought to be weaker than deformation potential coupling [32,33].

Following Bockelmann [34], assuming zero temperature, the rate at which electrons relax between QD states by emitting LA phonons is given by Fermi’s golden rule

$$\tau^{-1}_{\psi_{f} \rightarrow \psi_{i}} = \frac{2\pi}{\hbar} \sum_{j} |\langle \psi_{j} | W | \psi_{i} \rangle |^{2} \delta(E_{f} - E_{i} - E_{k})$$

Here, the deformation potential interaction is given by
\[ W = \sqrt{\frac{\hbar K}{2 \rho c_i}} D e^{i k \cdot \hat{x}}, \tag{6} \]

where \( K = |\vec{k}|, \rho = 5300 \text{ kg/m}^3, c_s = 3700 \text{ m/s}, \) and the deformation potential \( D = 8.6 \) eV. We approximate the eigenfunctions associated with motion in the \( z \)-direction by those for an infinite square well of the width 40 nm, which fit the exact wavefunctions reasonably well. As the volume \( V \) of the crystal is taken to infinity, it can be shown that the expression for \( \tau_{10} \) becomes

\[
\tau_{10}^{-1} = \frac{D^2 k^3}{4 \pi \hbar c n^2} \int_0^1 dq' \frac{q'}{1-q^2} \left| \int J_0(\alpha q r') J_0(\beta r') r' \right|^2 \times \frac{2}{\pi} \int_{-\pi/2}^{\pi/2} dz' \cos(z') \sin(2z') e^{i \beta \sqrt{1-q^2} z'} \tag{7} \]

Here, \( K_{10} = E_{10}/h c_s, q' = q/K_{10}, \) where \( q \) is the radial phonon wave-vector. \( N_{10}^{-1} = \int_{q_0}^{q_{01}} dq' \int J_0(\alpha q r') J_0(\beta r') r' \right|^2 \times \frac{2}{\pi} \int_{-\pi/2}^{\pi/2} dz' \cos(z') \sin(2z') e^{i \beta \sqrt{1-q^2} z'} \tag{7} \]

The time required to execute a CNOT operation for the particular QD structure is now estimated [37]. For a dielectric cavity resonating at \( h \omega_c = 11.5 \text{ meV} \) with an index of refraction \( n = 3.6 \), the maximum vacuum electric field \( e_{vac} \approx 49 \text{ V/m} \) is achieved for a cavity with minimal volume \( (\lambda_c/2)^3 \), where \( \lambda_c = c/n \omega_c = 30 \mu \text{m} \) is the wavelength of the resonant radiation inside the cavity. This vacuum electric field, together with the matrix elements \( z_{10}(e_c) = 1.177 \text{ MV/m} = 0.32 \text{ nm}, z_{10}(e_{l+c}) = 0.7668 \text{ MV/m} = 6.95 \text{ nm}, z_{10}(e_l) = 1.682 \text{ MV/m} = 4.97 \text{ nm}, \) and \( z_{21}(e_{l+c}) = 0.7668 \text{ MV/m} = 6.52 \text{ nm}, \) and a laser electric field of \( 30.7 \text{ kV/m} \), enable one to compute the time required for a CNOT operation. The \( 2\pi \) pulse applied to the target bit requires interaction with both a laser and a cavity photon, and hence is by far the longest operation, requiring 25 ns. The \( \pi \) pulses applied to the control bit require 3.3 ns each. Unconditional 1-bit rotations which occur at \( e = e_{l+c} \) take only a few ps for a laser electric field of 30 kV/m. It is likely the laser would need to be attenuated for these rotations, in order to satisfy the requirement that the transition time for the electrical pulse is much shorter than the period of the Rabi oscillation at the target electric field. If the only mechanism for decoherence is given by acoustic phonon emission, then the above calculations suggest that several thousand CNOT operations can be performed before the computer decoheres.

\section*{V. CONCLUSIONS}

We have proposed a quantum computer in which quantum information is stored in the lowest electronic levels of doped quantum dots. The energy levels in each dot are controlled by dedicated gate electrodes. THz photons in a cavity act as a data bus which can couple an arbitrary pair of quantum dots. A sequence of adiabatic voltage pulses applied to individual quantum dots can effect a CNOT operation involving any two quantum bits in the computer. We hope that this concrete proposal for a quantum information processor will stimulate theoretical and experimental activity. As with all proposals for quantum computation, the obstacles to implementing this one are formidable. Among the most important challenges, new types of QDs must be constructed, gated and loaded with single electrons [38]; few-mode THz cavities with extremely high Q must be fabricated; and single THz photons must be detected. Although each of these worthy challenges is beyond today’s state of the art, the rapid pace of progress in materials science and THz technology makes us optimistic that these obstacles will be overcome in the not-too-distant future.

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[37] The gate electrodes can be engineered to act as antennas, enhancing the vacuum electric field at the QDs and hence shortening the time required for a CNOT operation. This effect will be considered in a future publication.
[38] Quantum dots have been successfully loaded with a single mobile electron. See R. C. Ashoori, *Nature* **379**, 413 (1996); L. P. Kouwenhoven, T. H. Oosterkamp, M. W. S. Danoesastro, M. Eto, D. G. Austing, T. Honda, and S. Tarucha, *Science* **278**, 1788 (1997).

**FIG. 1.** Fundamental elements of the proposed quantum computer. Each set of quantum dots (QDs) contains 1 electron, and is individually addressable by a pair of gate electrodes. One QD is chosen to be a control bit, the other a target bit for a controlled-not (CNOT) operation. Many fundamental elements are embedded in a single-mode cavity.

**FIG. 2.** Potential and energy level diagram for the lowest energy levels of a set coupled QDs which is suitable for a qubit. The ground ($|0\rangle$) and first excited ($|1\rangle$) states are used to store quantum information. The second excited state ($|2\rangle$) is an auxiliary state which is used to effect a controlled-not operation, but does not store quantum information. The height of the QD is 41 nm, and the potential inside is 0 except for two 2 nm barriers with 65 meV potential which separate the central 17 nm well from the outer 10 nm wells.

**FIG. 3.** (a) Transition energies between states $|0\rangle$ and $|1\rangle$ ($E_{10}$) and between $|0\rangle$ and $|2\rangle$ ($E_{20}$) vs applied electric field, and photon energies of a cavity mode ($\hbar\omega_c$), a laser ($\hbar\omega_l$), and the sum $\hbar\omega_l + \hbar\omega_c$. The $E_{10}$ transition resonates with $\hbar\omega_c$ and $\hbar\omega_l$ at electric fields $e_c$ and $e_l$, respectively. The $E_{20}$ transition resonates with the two-photon transition with energy at electric field $e_{l+c}$. (b) A sequence of electric field pulses to a control and a target bit which are used in a CNOT gate. First, a “$\pi$” pulse is applied to the control bit, transferring a photon to the cavity and multiplying the state vector by $i$ if and only if the control bit is 1. Then, a “2$\pi$” pulse is applied to the target bit, multiplying the state vector by -1 if and only if there is a photon in the cavity and the target bit is in its ground state. Finally, a second “$\pi$” pulse is applied to the control bit, removing the photon from the cavity, returning the control bit to the excited state, and again multiplying the state vector by $i$. The state vectors in which the control bit is 0 are unaffected by the sequence of electric field pulses, and thus are not shown. One-bit rotations can be effected by applying an appropriately- timed pulse with amplitude $e_l$. As shown by Cirac and Zoller [10], the gate shown here, together 1-bit rotations on the target bit, result in a CNOT operation.
Fig. 1

cavity e, laser e
control bit
control e

target bit
target e
Fig. 2
State vector evolution

| Energy (meV) | Time (ns) | e (MV/m) |
|-------------|-----------|----------|
| $h\omega_l$  | 0 | 0  |
| $h\omega_c$  | 5 | 0.5 |
| $h\omega_l + h\omega_c$ | 10 | 1.0 |

(a)

(b)

Control bit $e$

Target bit $e$

State vector evolution

| Time (ns) | e (MV/m) |
|-----------|----------|
| 0 | 0  |
| 5 | 0.5 |
| 10 | 1.0 |
| 50 | 1.5 |
| 60 | 2.0 |

| $|1>_{c} 0>_{t} 0>$ | $|1>_{c} 1>_{t} 0>$ |
|----------------|----------------|
| $i|0>_{c} 0>_{t} 1>$ | $i|0>_{c} 1>_{t} 1>$ |
| $-i|0>_{c} 0>_{t} 1>$ | $i|0>_{c} 1>_{t} 1>$ |
| $|1>_{c} 0>_{t} 0>$ | $-|1>_{c} 1>_{t} 0>$ |

\[ \text{Energy: } E_{20}, E_{10} \]

\[ \text{Time: } t \]

\[ \text{Control bit: } e \]

\[ \text{Target bit: } e \]