A simplified model for turbine thrust coefficient and experimental comparison

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Abstract. A simplified analytical model to estimate yawed-inflow wind turbine thrust coefficient ($C_T$) based on integrated blade element momentum theory is described and compared with experimental data from a wind tunnel-based wind turbine wake simulator platform. This model, named as I-BEM, requires general inputs such as, chord length at 2/3rd rotor radius from center (c), rotor diameter (D), rotor solidity ($\sigma$), tip speed ratio ($\lambda$), airfoil lift gradient with respect to angle of attack ($dC_L/d\alpha$), blade pitch angle ($\beta$), and inflow yaw angle ($\Upsilon$). The model program delivers results instantaneously and does not require high performance computing resources. The model-predicted and measured experimental data comparison showed that $C_T$ variation for various $\Upsilon$ and $\lambda$ can be predicted using a simple program. Primarily, no stall condition is assumed, leading prediction errors under such conditions. Additional model assumptions are outlined in the paper. This simplified analytical model could be utilized for thrust-based wake steering applications where $C_T$ measurements are required.

1. Introduction
A wind tunnel based “Hyper Accelerated Wind Farm Kinematic-control Simulator” (HAWKS) is being built at Texas Tech University (TTU) for real time emulation of wind farm control strategies. This wind tunnel-based simulator is a platform to measure wake flow characteristics and test wake control strategies. The HAWKS results will be implemented eventually at the Department of Energy (DOE)/Sandia Scale Wind Farm Technology (SWiFT) facility in Lubbock, Texas. Initially, the development of HAWKS involves the design of a single fully controllable model wind turbine capable of instantaneously altering the thrust coefficient ($C_T$) and power coefficient ($C_p$) through rapid changes in blade pitch, turbine yaw, and rotor speed. The wake flow field has been characterized by 2-D using particle image velocimetry (PIV), a well-known non-invasive optical measurement method. This platform will be augmented with multiple model turbines to test control strategies to improve overall wind farm power production. However, the HAWKS’s preliminary purpose has been to develop and test the control capabilities, as well as verifying fundamental wake characteristic predictions, given the lower Reynolds number (Re) in wind tunnel experiments compared to full-scale experiments.

In this regard, Ref. [1] demonstrated that the HAWKS’s wake deflections showed good agreement with previously described wake models, wind tunnel and field studies, implying the applicability of HAWKS to research wake control strategies in steady state. Most of the state of the art research in wake control for wind farm optimization is static in nature, i.e. assuming steady-state conditions [2]. However, static wake control lacks practicality due to instantaneous wake variation on account of inherent inflow.
variability. For example, dynamic wake deflection corresponding to continuous yaw misalignment change was demonstrated in [1]. An effective wake control strategy should accommodate for inflow variations and instantaneously guide the wake to its required position [3, 4].

Additionally, the physical explanations in literature for wake-deflection and wake-curl are evolving. Wake deflection can be primarily attributed to the transverse rotor thrust component and/or to induced flow at rotor. The wake deflection parametrization proposed by [5, 6] show direct proportionality between wake deflection and the rotor thrust coefficient $C_T$. This would be true either during intentional yaw-misalignment or inflow wind direction variations. Further, this will also hold if $C_T$ was altered due to yaw-misalignment angle or by changes in rotor speed $\omega$ [3]. Additionally, wake curling has been observed in simulation studies and is explained by a counter-rotating vortex pair that is formed at the top and bottom of the rotor [7, 8]. These structures warrant consideration during development of effective yaw-based wake steering systems [9, 10].

Obtaining $C_T$ at yawed inflow is particularly useful for estimating turbine loads under inevitable yaw misalignment and for wake steering control strategies. Field measurement of $C_T$ is not trivial. For instance, for thrust being measured by strain gauges at the tower base, it is difficult to isolate tower sway effects, introducing measurement noise [11]. Further, placing $C_T$ sensors for all turbines in a farm may not be always feasible. In an attempt to estimate $C_T$ without direct measurements, the wind industry could rely on high fidelity wind turbine analysis codes based on blade element method BEM to estimate $C_T$ (e.g. FAST [12]). In principle, BEM divides the rotor into small sections and induced velocity is independently calculated for each element across the rotor span using empirical airfoil data. This approach is computationally expensive and will have limited applicability for practical applications, since the overall response time to activate a control actuation (such as, yaw) will not occur within the inflow variability timescale.

To fulfill the need for fast, reliable $C_T$ estimation, a simplified analytical model based on helicopter aerodynamics theory is presented here. The model input parameters are: chord length at $2/3$rd rotor radius from center ($c$), rotor solidity ($\sigma$), tip speed ratio ($\lambda$), airfoil lift gradient with respect to angle of attack ($dC_L/d\alpha$), blade pitch angle ($\beta$), and inflow yaw angle ($\gamma$). The execution of the model described in the subsequent section provides results instantaneously and requires minimal inputs compared to BEM-based high-fidelity codes such as FAST. To validate the proposed simplified model, direct $C_T$ measurements are obtained from HAWKS for various operating rotor speeds and rotor yaw angles. Salient experimental setup details are discussed in the experimental setup section. The model $C_T$ prediction accuracy compared to HAWKS experimental data is discussed in the results and discussion section of the paper and is followed by conclusions.

2. Theoretical model development
Three turbine control variables are considered in the equation development for thrust coefficient. Rotor speed ($\omega$), Rotor blade pitch angle ($\beta$), and Rotor Yaw Angle with the wind inflow ($\gamma$). These are used along with air density ($\rho$), wind speed ($U$), rotor radius ($R$), and blade chord at the $2/3$ R radial station to calculate the required non-dimensional coefficients:

Tip speed ratio

$$\lambda = \frac{\omega R}{U}$$  \hspace{1cm} (1)

Rotor Solidity

$$\sigma = \frac{B c}{\pi R}$$  \hspace{1cm} (2)
where B is the number of blades and c is measured at 2/3R. It is assumed that the overall blade performance is represented by the characteristics at the 2/3R location.

Lift curve slope (pre-stall)

\[
\delta = \frac{dC_L}{d\alpha_{rad}}
\]  

where is the \(C_L\) lift coefficient and \(\alpha_{rad}\) is the angle of attack in radians.

The model uses typical airfoil characteristic lift data for the airfoil at the 2/3 radial station, with a correction for low Reynolds Number operation. Momentum Theory is used to develop an expression for rotor thrust coefficient as a function of average rotor induced velocity with a term added to account for the in-plane component of the inflow to account for the rotor operating in yawed flow.

\[
C_T = 4a\sqrt{1 - 2a\cos\gamma + a^2}
\]

Blade Element Theory is used to evaluate the thrust forces developed at the 2/3 radius station as a function of the rotor and wind inputs and the induced velocity. Integration along the blade span and around the rotor azimuth (necessary due to the yawed flow criteria), provides a second expression for thrust coefficient. To simplify the resulting equations, a number of assumptions are used and are summarized below. See Ref. [13] for details. The resulting expression for \(C_T\) is:

\[
C_T = 2\sigma\delta\lambda^2 \left[ \frac{-\beta}{6} + \frac{-\beta \sin^2\gamma}{4\lambda^2} + \frac{1}{4\lambda} (\cos\gamma - a) \right]
\]

Equating the thrust coefficients, rearranging terms, and simplifying as appropriate provides an expression for the induced velocity factor, \(a\), at the rotor disk. Since the resulting equation for induced velocity depends on the induced velocity itself, an iterative solution is required.

\[
a = \left( \frac{\sigma\delta}{24} \right) \left[ \frac{-\beta(2\lambda^2 + (-\beta)3\sin^2\gamma + 3\lambda(\cos\gamma - a))}{\sqrt{1 - 2a\cos\gamma + a^2}} \right]
\]

Once the induced velocity factor \(a\) has been determined, substitution into the momentum equation provides a value for \(C_T\). The authors have named this short code I-BEM, since it is based on both momentum theory and blade element theory, but uses integrated values along the blade span, hub to blade tip, and around the rotor azimuth, zero to 2\(\pi\) radians.

As mentioned above, in order to keep the equations relatively simple, a number of assumptions are used and must be considered when applying these equations:

- Blades are untwisted and untapered
- Radial flow along the blades is ignored
- Inflow angles are small
- The effect of the reversed flow region is small
- The blades continue to the center of rotation
- The lift curve slope is assumed constant and is a function of Re.
- The blades are rigid in all dimensions
- Tip losses are ignored
- Induced velocity is uniform over the rotor disk (see note on non-uniform inflow)

Note on non-uniform inflow:
In yawed flow there is a velocity component in the plane of the rotor disk as well as one normal to the disk. This fact causes the lift distribution that occurs at the leading edge of the disk to affect induced velocities over the whole disk. Therefore, the induced flow pattern over the disk varies such that the induced velocity $V_i$ increases with distance from the leading edge as defined by the in-plane component of rotor inflow, as shown in figure 1. For this analysis these effects are ignored in estimating the average thrust coefficient over the rotor. A more comprehensive discussion of this effect may be found in [13].

![Diagram of modified induced flow pattern](image)

**Figure 1.** Modified Induced flow pattern where $V_i$ represents the local induced velocity.

### 3. Experimental setup

In the present study, the HAWKS platform developed in the National Wind Institute (NWI) closed-loop wind tunnel located at Texas Tech University was used to provide experimental data at controlled operating conditions to verify the $C_t$ predictions from the proposed, simplified analytical model I-BEM. The NWI wind tunnel has a rectangular test section of 1.2m high x 1.8m wide. The horizontal-axis fully controllable model wind turbine used in this study is 3-bladed, with a rotor diameter $D$ of 0.27m. The rotor blades are NACA 0012 airfoils with constant chord $c$ of 0.013m throughout the span of the blade. The general specifications are listed in Table 1.

| Table 1. Summary of HAWKS model turbine characteristics |
|--------------------------------------------------------|
| Number of blades ($B$) | 3 |
| Airfoil section | NACA 0012 |
| Rotor diameter ($D$) | 0.27 m |
| Chord ($c$) | 0.013 m |
| Solidity ($\sigma$) | 0.092 |

The rotor drove a small 15W AMETEK 8693S039-SP DC generator equipped with an encoder to measure the rotor speed $\omega$, which is controlled by a DC-DC buck converter connected to the generator output. The gate switching is controlled by pulse width modulation PWM signal of appropriate duty cycle to change the apparent load seen by the generator such that the desired rotor speed $\omega$ is achieved. The pitching mechanism was extracted from a WLtoys model V931 RC helicopter, which comprises a set of 3 servomotors to perform collective pitch. Both the generator and the pitching mechanism are located inside the nacelle of 0.07m x 0.038m cross-section and 0.13m length. To generate yaw misalignment, the tower base is coupled to a Zaber X-RSW60A-E03 servomotor that allows a variation of the yaw angle. The rotation angles of both pitch and yaw servomotors as well as the rotor speed are controlled through PWM signals generated from Texas Instruments TMS320F28335 digital signal processor (DSP) board in combination with in-house built Simulink code.
The thrust force exerted by the wind on the model turbine was measured by a high-sensitivity 6-component load transducer JR3 model 30E12A-140-EF-100N mounted directly at the bottom of the model turbine tower. Data acquisition is carried out with an analog input NI 9205 module coupled to a NI cRIO 9074 chassis and processed in an in-house LabVIEW FPGA virtual instrument VI. Figure 2 shows the turbine in the test section of the wind tunnel when observed from the upstream with all the components.

4. Results and discussion

In this section, the I-BEM code capabilities to predict the thrust coefficient are assessed using the HAWKS turbine characteristics as baseline. To begin comparison of the proposed, simplified analytical model described above, measured HAWKS model turbine $C_T$ curves are first considered. The stream-wise oncoming flow velocity (free-stream) at the hub height was kept constant at $U_\infty = 10.85$ m/s. The rotational speed of the rotor was varied from $\omega = 2500$ to 6000 RPM, which gave a tip speed ratio range of 3.3 to 8 based on the freestream velocity used for the measurements. $C_T$ measurements as a function of the turbine operating parameters $Y$ and $\lambda$ with blade pitch angle $\beta$ set at $4^\circ$ are shown in figure 3a.

![Figure 2. HAWKS turbine components](image)

**Figure 2.** HAWKS turbine components

![Figure 3a. Thrust coefficient $C_T$ comparison between HAWKS measurements](image)

![Figure 3b. Thrust coefficient $C_T$ comparison between I-BEM model predictions](image)

**Figure 3.** Thrust coefficient $C_T$ comparison between a) HAWKS measurements and b) I-BEM model predictions.
The thrust force due to the tower and nacelle was measured separately and subtracted from the whole system thrust measurements to isolate actual effective thrust acting of the rotor. In figure 3a, the effective rotor thrust coefficient $C_T$ is approximately linearly proportional to $\lambda$ and is marginally affected by changes in $Y$ for $\lambda<6$. For $\lambda>6$, $C_T$ is less sensitive to $\lambda$; however, $C_T$ difference is noticeable for different $Y$. These observations corroborate previous studies for $Y$ range considered here [14, 15].

Figure 3b shows $C_T$ characteristic derived from I-BEM using HAWKS turbine geometry and corresponding turbine operating conditions as inputs. Qualitatively, the $C_T$ computed with I-BEM agree well with the HAWKS $C_T$ experimental data trends. Quantitatively, the I-BEM predicts the HAWKS $C_T$ measurements with sufficient accuracy for $5.5<\lambda<8$ range, with a difference no larger than 0.1. Below $\lambda=5.5$, I-BEM overestimates the experimental values. This is due to the HAWKS turbine operating in stall regime with a corresponding loss in lift and increase in drag.

Figure 4 shows the HAWKS blade average angle of attack $\alpha$ and chord-based Reynolds ($Re_c$) number variation estimate as a function of $\lambda$ for various $Y$. $\alpha$ was estimated by accounting for blade pitch, $\beta=4^\circ$ for a given $\lambda$. $Re_c$, was based on the local velocity estimate. These estimates were obtained by considering The angle of attack seems to exponentially decrease with increase in tip speed ratio. This is justified since the angle of attack is inversely proportion to $\lambda$. Additionally, the axial induction factor is directly proportional to the $\lambda$, decreasing the angle of attack further with increasing $\lambda$. Further, the angle of attack marginally decreases with $Y$ increase, with the difference decreasing as $\lambda$ increases. The variation of the angle of attack will depend on the yaw and rotation direction in practice. However, the angle of attack difference due to $Y$ is less significant compared to change in $\lambda$. Further, as expected there is a linear increase in $Re_c$ with increase in $\lambda$, since $\lambda$ is directly proportional to the local tangential velocity. As a result, for $\lambda<5.5$, the $Re_c<50\times10^4$ and the angle of attack>$8^\circ$ over the blade length. However, Xfoil prediction shown in figure 4, for coefficient of lift $C_l$ with respect to angle of attack (see [16]) point that the NACA 0012 airfoil stalls at $\alpha>7.5$. As a result, the HAWKS turbine blade is likely operating in the stall regime for $\lambda<5.5$. The I-BEM code assumes $dC_l/d\alpha=5.9$, which is in the normal operation range of the airfoil. Under these conditions, the prediction accuracy of the I-BEM code decreases as a result. Additionally, it is practically difficult to measure the HAWKS model pitch and claim it to be constant at high rotor speeds that are required to operate in a meaningful $\lambda$ range. The high rotor speed also introduces vibrations and $C_T$ measurement noise. This may further affect the accuracy of the I-BEM predictions even for $5.5<\lambda<8$ range. Regardless, the I-BEM code predicts the $C_T$ with reasonable accuracy for no stall conditions that usually are the case for practical operating range of $5.5<\lambda<8$ with modest computing and input parameter requirements.

Figure 4. HAWKS Turbine blade operation parameters a) Angle of attack and Reynolds number as a function of yaw and tip speed ratio and b) Xfoil $C_L$ vs. $\alpha$ estimate as function of chord Reynolds number ($50\times10^4$-$1\times10^6$) [16].
5. Conclusion

In this work, a simplified model to estimate $C_T$ based on integrated blade element/momentum theory is presented. This model, named as I-BEM, incorporates $C_T$ estimation capability in yawed inflow conditions. The I-BEM model is compared with HAWKS experimental data and shows good agreement for practical tip speed ratio range. The model assumes that the flow is not stalled and seems to lose prediction accuracy for stalled flow. At full-scale, the operating Reynolds number will be higher than the HAWKS model turbine, likely improving the model accuracy. The model error causing separation at relatively lower angle of attacks will not occur at higher Reynolds number.

Additionally, proposed model execution converges after a few iterations to an approximate value of $C_T$. Speed of execution, desktop-level computing resources, and limited input requirement compared to high fidelity sophisticated BEM-based tools such as FAST are the main attractive features of the proposed model. The I-BEM model can be used for cases where rotor-average flow quantity estimates, such as induced velocity are required without installation of additional probes and high-performance computational resources.

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