Generalized Space Time Autoregressive Integrated Moving Average with Exogenous (GSTARIMA-X) Models

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Abstract. Space time data is data that relates to events at previous times and locations. One of the models that used to analyze space time data is the Generalized Space Time Autoregressive Moving Average (GSTARMA) model. GSTARMA model uses parameters p and λk to represent the time order and spatial order respectively. Hence, this model can be called GSTARMA(p, λk). Because in fact there are more models with different parameters for different locations, GSTARMA(p, λk) model is more realistic. If space time data is not stationary, a different order is added so that the model becomes GSTARIMA(p, d, λk). In modelling sometimes the accuracy of the model can increase by other influence variables. These variables are named exogenous variables. The GSTARIMA model with exogenous variables is named the GSTARIMA-X(p, d, λk) model. This research aims to examine the model and its estimation of parameters. Model parameters are estimated using the Generalized Least Square (GLS) method. The method in this writing is literature study obtained from several articles, journals, and books that support in achieving research. The results of the study show that with the GLS method, parameter estimates can be obtained with the assumption of time order 1 and spatial order 1.

1. Introduction

Time series data is an observation’s series that are consecutive in time [1]. According to [2], time series is an observation of a variable taken from time to time sequentially in fixed time intervals. Time series analysis is a statistical procedure that is applied to predict the probabilistic structure of conditions that will occur in the future.

One of the developments of time series data is spatial time series data. In 1980, [3] developed a space time model called space time autoregressive moving average (STARMA) model. Because the parameters for all locations are assumed to be the same, the STARMA model is sometimes considered unrealistic. In 2008, [4] improved the STARMA model to become a generalized space time autoregressive moving average (GSTARMA) model with assumption that the observed locations have heterogeneous characteristics. Heterogeneous locations can be characterized by the characteristics of the location which are quantified by a weighted location matrix [7]. The model is considered more realistic because the parameters are assumed to be different for each location. The GSTARMA model with the zero moving average order is called the GSTAR model. In 2018, [5] reviewed the generalized least square (GLS) parameter estimation in the GSTAR model with the assumption of correlated residues. The GSTARMA model must meet the data stationary assumptions and must have an
autoregressive order, a moving average order, and a specified spatial order 1. The spatial order is determined to be 1 because spatial orders that are more than 1 are difficult to interpret in the model. Those who do not meet the data stationary assumption need to do a differencing process so that the model is developed into a generalized space-time autoregressive integrated moving average (GSTARIMA) [6].

In modeling sometimes, the accuracy of the model can increase by other influence variables. These variables are named exogenous variables. There are several ways to include exogenous variables into the model, one of which is by using the transfer function. The transfer function model is also called the ARIMAX model in time series modeling introduced by [1]. So far, the effective transfer function model is used for dynamic models with exogenous variables [8]. The GSTARIMA model that includes exogenous variables are called the GSTARIMA-X model. The GSTARIMA-X model using the transfer function was introduced by [8] in 2017. The results show that the model was decent in modeling rice price data in Indonesia with exogenous variables. Therefore, this study aims to examine the GSTARIMA-X model and its parameter estimation. Model parameters are estimated using the GLS method.

2. Method
This research is a theory-based study that examines the GSTARIMA-X model and its parameter estimates. The method in writing this is a literature study obtained from several articles, journals, and books that support the achievement of research objectives.

3. Result and Discussion
The GSTARIMA model is built from the GSTAR model with data that is not stationary and has a seasonal pattern. The addition of the influence of exogenous variables to the GSTARIMA model is carried out to raise the accurateness of the model. This model is called the GSTARIMA-X model. This section discusses stationarity, model identification, transfer function models, GSTARIMA-X with transfer function approach, GLS methods, and estimation of model parameters using GLS.

3.1. Stationarity
The multivariate time series data’s stationarity in variance can be seen through the Box-Cox plot, while the multivariate time series data’s stationary in the mean can be seen through the Matrix Autocorrelation Function (MACF) plot.

According to [2] the sample cross correlation for the \(i^{th}\) and \(j^{th}\) components in the time lag is stated in the following equation

\[
\hat{\rho}_{ij}(k) = \frac{\sum_{t=1}^{n-k} (x_{i,t} - \bar{Z}_i)(x_{j,t+k} - \bar{Z}_j)}{\sum_{t=1}^{n} (x_{i,t} - \bar{Z}_i)^2 \sum_{t=1}^{n} (x_{j,t} - \bar{Z}_j)^2}^{1/2}
\]

where \(\bar{Z}_i\) and \(\bar{Z}_j\) are the sample mean of the corresponding series component.

3.1.1. Stationarity in variance. Time series data is stationary in variance if the data fluctuations are constant or constant from time to time. If the data is not stationary in variance then a transformation is performed [9]. The transformation used is the power transformation. Box-Cox [10] denotes a power transformation as

\[
Z_t^* = \frac{Z_t^{(k)} - 1}{\lambda}
\]

where \(\lambda\) is the transformation parameter at the interval [-2, 2].
3.1.2. Stationarity in mean. Time series data is stationary in the mean if the data fluctuations are around the average value. According to [9], if the stationary conditions in the average are not met, it is necessary to do the differencing process with the following formula

\[ \nabla^d = (1 - B)^d Z_t. \]

3.2. Model identification

In general, the choice of spatial order in the GSTARIMA model is limited to 1, because orders greater than 1 will be complicated to interpret [11]. Thus, the spatial order in the GSTARIMA-X model is also limited to 1.

3.2.1. Location weight. Selection and determination of location weight is one of the fundamental problems in the GSTARIMA model. The weight of the location which gives the smallest forecast error in the model being made is the best location weight. In general, some statisticians in the space problems in the GSTARIMA model. The weight of the location which gives the smallest forecast error on kth lag is defined as follows

\[ \rho_{ij}(k) = \frac{\gamma_{ij}(k)}{\sigma_i \sigma_j}, \quad k = 0, \pm 1, \pm 2, \pm 3, \ldots \]  

where \( \gamma_{ij}(k) \) is the cross-covariance between observations in kth lag on ith and jth location, \( \sigma_i \) and \( \sigma_j \) is the standard deviation or standard deviation from the ith and jth observations. According to [14], a cross-correlation formula on data is written

\[ r_{ij}(k) = \frac{\sum_{t=k+1}^{n} \left[Z_i(t) - \bar{Z_i}\right]\left[Z_j(t-k) - \bar{Z_j}\right]}{\sqrt{\sum_{t=1}^{n} \left[Z_i(t) - \bar{Z_i}\right]^2 \left[Z_j(t-k) - \bar{Z_j}\right]^2}} \]  

where \( k = 1, 2, \ldots, p \), \( r_{ij}(k) \) is autocorrelation on ith and jth location in kth lag, \( Z_i(t) \) is data at ith location and ith time, \( n \) is quantity of data, \( \bar{Z_i} \), and \( \bar{Z_j} \) is mean of data on ith and jth location. In order to make the sum of each element in the row and column is 1, normalization is carried out. Each element of the normalized weighting matrix is expressed as

\[ W_{ij} = \frac{r_{ij}(1)}{\sum_{k=1}^{p} r_{ij}(k)} \text{ where } i \neq j \text{ and } \sum_{j} W_{ij} = 1 \]  

The weight of the location cross correlation normalization raises the possibility of a relationship between locations. Flexibility in sign and the magnitude of the relationship between different locations is an advantage of this weight [15].

3.3. Transfer function model

In an event there may be the influence of other variables. These variables are exogenous variables. The STARMA model with the addition of exogenous variables was first introduced by [16]. One way to import the exogenous variables into the model is the transfer function approach.

Referring to [17], assuming that \( Z_i(t) \) influenced by \( X_i(t) \). The dynamic relationship of \( Z_i(t) \) and \( X_i(t) \) formulated by

\[ Z_{it} = v_i(B)X_{it} + \varepsilon_{it}. \]
According to [5], model on equation (6) is known as transfer function model where \( v(t) = \frac{\omega_{ik}(B)B^b_l}{\delta_{ik}(B)} \), so \( Z_i(t) \) can be written as

\[
z_{it} = \frac{\omega_{ik}(B)B^b_l}{\delta_{ik}(B)} x_{it} + \varepsilon_{it}. \tag{7}
\]

According to [17] equation (7) can be expressed in linear equations

\[
\left( 1 - \sum_{k=1}^{r} \delta_{ik} B^k \right) z_{it} = \left( \omega_{0} - \sum_{k=1}^{s} \omega_{ik} B^k \right) x_{i(t-b)} + \left( 1 - \sum_{k=1}^{r} \delta_{ik} B^k \right) \varepsilon_{it}
\]
or

\[
z_{it} = \sum_{k=1}^{r} \delta_{ik} B^k z_{it} + \omega_{0} x_{i(t-b)} - \sum_{k=1}^{s} \omega_{ik} B^k x_{i(t-b)} + \varepsilon_{it}^* \tag{8}
\]

3.4. GSTARIMA-X with transfer function approach

Still according to [17] the spatial lag operator when applied to transfer function model (8) formulated as

\[
z_{i(t)} = \sum_{k=1}^{r} \sum_{l=0}^{N} \delta_{ik} L^l z_{i(t-k)} + \sum_{l=0}^{N} \omega_{0l} L^l x_{i(t-b)} - \sum_{k=1}^{s} \sum_{l=0}^{N} \omega_{kl} B^k x_{i(t-b-k)} + \varepsilon_{i(t)}^* \tag{9}
\]

Based on the spatial lag operator’s definition, equation (9) can be expressed as

\[
z_{i(t)} = \sum_{k=1}^{r} \sum_{l=0}^{N} \delta_{ik} L^l z_{j(t-k)} + \sum_{l=0}^{N} \omega_{0l} L^l x_{j(t-b)} - \sum_{k=1}^{s} \sum_{l=0}^{N} \omega_{kl} B^k x_{j(t-b-k)} + \varepsilon_{j(t)}^* \tag{10}
\]

where \( \delta_{ikl} \) symbolize output series autoregressive for \( i \)th location, \( k \)th lag of time, and \( l \)th lag of spatial; \( \xi_k \) is autoregressive spatial order of \( z_{i(t)} \) at \( k \)th lag of time; \( \omega_{0l} \) is parameter that symbolize the length series \( x_{i(t-b)} \) affecting series \( z_{i(t)} \) for \( i \)th location, zero time lag, and \( l \)th lag of spatial; \( \xi_0 \) is spatial order of \( \omega_{0l} \) at zero time lag; \( \omega_{ikl} \) is parameter that symbolize the length of series \( x_{i(t-b)} \) affecting series \( z_{i(t)} \) for \( i \)th location, \( k \)th lag of time, and \( l \)th lag of spatial; \( \xi_k \) is spatial order of \( \omega_{kl} \) at \( k \)th lag of time; \( b \) symbolize order of time when the input series start affecting the output series; and \( \varepsilon_{i(t)}^* \) is noise series of \( i \)th location and \( t = 1, ..., T \). Referring to [17] the matrix form of equation (10) is

\[
Z_t = \sum_{k=1}^{r} \sum_{l=0}^{N} \delta_{ikl} W_l^t z_{i(t-k)} + \sum_{l=0}^{N} \Omega_{0l} W_l^t x_{t-b} - \sum_{k=1}^{s} \sum_{l=0}^{N} \omega_{kl} B^k x_{t-b-k} + \varepsilon_{t}^* \tag{11}
\]

where \( \delta_{ikl} \), \( \Omega_{0l} \), and \( \Omega_{kl} \) are \( N \times N \) diagonal matrix that representing the model parameters; \( z_{t} = (z_{1(t)}, z_{2(t)}, ..., z_{N(t)})^T \); \( x_{t} = (x_{1(t)}, x_{2(t)}, ..., x_{N(t)})^T \); and \( \varepsilon_{t}^* = (\varepsilon_{1(t)}^*, \varepsilon_{2(t)}^*, ..., \varepsilon_{N(t)}^*)^T \). Due to the spatial lag operator of the noise series, equation (11) becomes

\[
Z_t = \sum_{i=1}^{n} \sum_{k=0}^{\xi} \delta_{ikl} W_{i}^{t-k} z_{i-k} + \sum_{i=0}^{\xi} \Omega_{0i} W_{i}^{t-b} x_{i-b} - \sum_{k=1}^{s} \sum_{l=0}^{\xi} \omega_{ikl} B^k x_{i-b-k} + \varepsilon_{i}^* - \sum_{i=0}^{\xi} \sum_{l=0}^{\xi} \phi_{ii} W_{i}^{t} \varepsilon_{i}^* + \alpha_i - \sum_{i=0}^{\xi} \sum_{l=0}^{\xi} \Theta_{il} W_{i}^{t} \alpha_{i-k} \tag{12}
\]

(12) that known as the GSTARIMA-X model with the Transfer Function approach. If the data is not stationary, equation (12) becomes GSTARIMA-X with the transfer function approach as in Equation (13)

\[
\nabla Z_t = \sum_{i=1}^{n} \sum_{k=0}^{\xi} \delta_{ikl} W_{i}^{t-k} \nabla z_{i-k} + \sum_{i=0}^{\xi} \Omega_{0i} W_{i}^{t-b} x_{i-b} - \sum_{k=1}^{s} \sum_{l=0}^{\xi} \omega_{ikl} B^k x_{i-b-k} + \varepsilon_{i}^* - \sum_{i=0}^{\xi} \sum_{l=0}^{\xi} \phi_{ii} W_{i}^{t} \varepsilon_{i}^* + \alpha_i - \sum_{i=0}^{\xi} \sum_{l=0}^{\xi} \Theta_{il} W_{i}^{t} \alpha_{i-k} \tag{13}
\]

3.5. Generalized least square (GLS) method

The GLS method is the development of OLS method that is used to estimate regression parameters by considering the autocorrelation error [18]. The GLS parameter estimate considers the error covariance
variance matrix. Minimizing the number of generalized error squares is the way to estimate model parameters using GLS method [19]. Suppose that the following regression equation is given

\[ Z = X\beta + \varepsilon. \]

According to [19], the estimator properties with the GLS method are described as follows.

1) The unbiased estimator \( \hat{\beta} \) for \( \beta \) is \( E(\hat{\beta}) = \beta \) when \( E(Z) = X\beta. \)

2) Variance-covariance matrix \( \hat{\beta} \) is \( \text{cov}(\hat{\beta}) = (X'\Omega^{-1}X)^{-1} \) when \( \text{cov}(Z) = \Omega. \)

3) If \( \varepsilon \sim N(0, \Omega_0 \Omega_1), \) then estimator \( \hat{\beta} \) is \( \hat{\beta} \sim N(\beta, \text{cov}(\hat{\beta})). \)

3.6. Estimation of model parameters using GLS

3.6.1. Estimation of GSTARIMA model parameters. According to GSTARIMA model which has been discussed in 3.2., we obtained the order of \( \lambda_k \) is 1, and \( \phi_k \) can be written as \( \phi_k^{(i)} \) for \( k = 0,1 \). Hence GSTARIMA formulated by

\[
\nabla Z_i(t) = \phi_{0i}^{(i)} \nabla Z_i(t-1) + \phi_{1i}^{(i)} w_j \nabla Z_j(t-1) + \varepsilon_i(t) - \sum_{k=1}^{q} (\Theta_{ik}^{(i)} \varepsilon_i(t-k) + \Theta_{jk}^{(i)} w_j \varepsilon_j(t-k))
\]

(14)

where \( \nabla Z_i(t) = (1 - B^d) Z_i \) express the observations at \( t=0,1,2,\ldots,T \), and location \( i = 1,2,3,\ldots, N \).

According to [6], autoregressive and moving average parameters estimation in the GSTARIMA model is done by using the least squares method. This statement is corroborated by [20] which says that the GSTARIMA parameter estimation is done by estimating the GSTARI parameter, then estimating the GSTIMA parameter.

The GSTARI model is represented in the matrix as

\[
\begin{pmatrix}
\nabla Z_1(t) \\
\nabla Z_2(t) \\
\vdots \\
\nabla Z_N(t)
\end{pmatrix}
= 
\begin{pmatrix}
\phi_{00}^{(0)} & 0 & \cdots & 0 \\
0 & \phi_{01}^{(1)} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \phi_{0N}^{(N)}
\end{pmatrix}
\begin{pmatrix}
\nabla Z_1(t-1) \\
\nabla Z_2(t-1) \\
\vdots \\
\nabla Z_N(t-1)
\end{pmatrix}
+ 
\begin{pmatrix}
\phi_{10}^{(0)} & 0 & \cdots & 0 \\
0 & \phi_{11}^{(1)} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \phi_{1N}^{(N)}
\end{pmatrix}
\begin{pmatrix}
\nabla Z_1(t) \\
\nabla Z_2(t) \\
\vdots \\
\nabla Z_N(t)
\end{pmatrix}
+ 
\begin{pmatrix}
\varepsilon_1(t) \\
\varepsilon_2(t) \\
\vdots \\
\varepsilon_N(t)
\end{pmatrix}
+ 
\begin{pmatrix}
\varepsilon_1(t) \\
\varepsilon_2(t) \\
\vdots \\
\varepsilon_N(t)
\end{pmatrix}
+ 
\begin{pmatrix}
\varepsilon_1(t) \\
\varepsilon_2(t) \\
\vdots \\
\varepsilon_N(t)
\end{pmatrix}
\]

(15)

where

\[
\begin{pmatrix}
\nabla Z_1(t) \\
\nabla Z_2(t) \\
\vdots \\
\nabla Z_N(t)
\end{pmatrix}
= 
\begin{pmatrix}
\phi_{00}^{(0)} & 0 & \cdots & 0 \\
0 & \phi_{01}^{(1)} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \phi_{0N}^{(N)}
\end{pmatrix}
\begin{pmatrix}
\nabla Z_1(t-1) \\
\nabla Z_2(t-1) \\
\vdots \\
\nabla Z_N(t-1)
\end{pmatrix}
+ 
\begin{pmatrix}
\phi_{10}^{(0)} & 0 & \cdots & 0 \\
0 & \phi_{11}^{(1)} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \phi_{1N}^{(N)}
\end{pmatrix}
\begin{pmatrix}
\nabla Z_1(t) \\
\nabla Z_2(t) \\
\vdots \\
\nabla Z_N(t)
\end{pmatrix}
+ 
\begin{pmatrix}
\varepsilon_1(t) \\
\varepsilon_2(t) \\
\vdots \\
\varepsilon_N(t)
\end{pmatrix}
+ 
\begin{pmatrix}
\varepsilon_1(t) \\
\varepsilon_2(t) \\
\vdots \\
\varepsilon_N(t)
\end{pmatrix}
+ 
\begin{pmatrix}
\varepsilon_1(t) \\
\varepsilon_2(t) \\
\vdots \\
\varepsilon_N(t)
\end{pmatrix}
\]

Equation (15) can be formed into linear models \( Z = Y\phi + \varepsilon \) where
\begin{equation}
\mathbf{Z} = \begin{bmatrix}
\nabla z_1(t) \\
\nabla z_2(t) \\
\vdots \\
\nabla z_N(t)
\end{bmatrix},
\mathbf{Y} = \begin{bmatrix}
\nabla z_1(t-1) & v_1(t-1) & 0 & \cdots & 0 \\
0 & 0 & \nabla z_2(t-1) & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & v_N(t-1)
\end{bmatrix},
\mathbf{e} = \begin{bmatrix}
\varepsilon_1(t) \\
\varepsilon_2(t) \\
\vdots \\
\varepsilon_N(t)
\end{bmatrix}.
\end{equation}

\[ \mathbf{Y} \begin{bmatrix}
\sigma_{11}I_T & \sigma_{12}I_T & \cdots & \sigma_{1N}I_T \\
\sigma_{21}I_T & \sigma_{22}I_T & \cdots & \sigma_{2N}I_T \\
\vdots & \vdots & \ddots & \vdots \\
\sigma_{N1}I_T & \sigma_{N2}I_T & \cdots & \sigma_{NN}I_T
\end{bmatrix} \otimes I_T = \Sigma \otimes I_T = \Omega \]

In the GSTARI model the correlated residuals between the covariance variance matrix equation is

\[ \mathbf{Y} \begin{bmatrix}
\sigma_{11} & \sigma_{12} & \cdots & \sigma_{1N} \\
\sigma_{21} & \sigma_{22} & \cdots & \sigma_{2N} \\
\vdots & \vdots & \ddots & \vdots \\
\sigma_{N1} & \sigma_{N2} & \cdots & \sigma_{NN}
\end{bmatrix} \otimes I_T = \Sigma \otimes I_T = \Omega \]

Furthermore, the GSTIMA parameter estimation is carried out. The steps for estimating the GSTIRA parameter are carried out the same as the estimation of the GSTARI parameter by changing the matrix from \( \mathbf{Z} \) to \( \mathbf{X} \). Hence \( \Theta \) and \( \mathbf{Y} \) written as

\[ \mathbf{Y} = \mathbf{X}\Phi + \mathbf{e} \]

Thus equation (13) is expressed in a matrix, that is

\[ \mathbf{Y} = \mathbf{X}\Phi + \mathbf{e} \]
$$\begin{align*}
\{ \Delta Z(t) \} &= \{ \Delta_{10} Z_1(t-1) \} + \{ \Delta_{11} Z_1(t-1) \} + \{ w_{12}(1) \Delta Z_1(t-1) + \cdots + w_{1n}(1) \Delta Z_n(t-1) \} \\
\{ \Delta Z_1(t) \} &= \{ \Delta_{10} Z_1(t-1) \} + \{ \Delta_{11} Z_1(t-1) \} + \{ w_{21}(1) \Delta Z_1(t-1) + \cdots + w_{2r}(1) \Delta Z_r(t-1) \} \\
\{ \Delta Z_n(t) \} &= \{ \Delta_{10} Z_1(t-1) \} + \{ \Delta_{11} Z_1(t-1) \} + \{ w_{n1}(1) \Delta Z_1(t-1) + \cdots + w_{nn}(1) \Delta Z_n(t-1) \}
\end{align*}$$

or

$$Z = XA + Y\Omega b + A\Omega_1 + B\Phi + \sum_{k=1}^{q} C_k \theta_k + \varepsilon$$

Previously, GSTARI-X model has been studied by [22]. In his research he stated that GSTARI-X parameter estimation is performed by using GSTIMA parameter for data, estimate GSTARI parameter for exogenous noise series. Therefore, in this research, the GSTARIMA-X parameter was estimated by estimating the GSTARI parameter for data, estimating GSTARIMA parameter for exogenous data, then estimating GSTIMA parameter for noise series. Hence obtained parameter estimation GSTARIMA-X model

$$\hat{\Lambda} = (X'\Omega^{-1}X)^{-1}X'\Omega^{-1}Z, \hat{\Omega}_b = (Y'\Omega^{-1}Y)^{-1}Y'\Omega^{-1}Z, \hat{\Omega}_1 = (A'\Omega^{-1}A)^{-1}A'\Omega^{-1}Z,$$

$$\hat{\Phi} = (B'\Omega^{-1}B)^{-1}B'\Omega^{-1}Z, \hat{\theta}_1 = (C_1'\Omega^{-1}C_1)^{-1}C_1'\Omega^{-1}Z, \cdots \hat{\theta}_k = (C_k'\Omega^{-1}C_k)^{-1}C_k'\Omega^{-1}Z.$$
\[ \hat{\alpha} = (X'\Omega^{-1}X)^{-1}X'\Omega^{-1}Z, \hat{\Omega}_0 = (Y'\Omega^{-1}Y)^{-1}Y'\Omega^{-1}Z, \hat{\Omega}_1 = (A'\Omega^{-1}A)^{-1}A'\Omega^{-1}Z, \]
\[ \hat{\phi} = (B'\Omega^{-3}B)^{-1}B'\Omega^{-1}Z, \hat{\theta}_1 = (C_1'\Omega^{-3}C_1)^{-1}C_1'\Omega^{-1}Z, \ldots \hat{\theta}_k = (C_k'\Omega^{-3}C_k)^{-1}C_k'\Omega^{-1}Z. \]

with assumption spatial order 1 and time order 1.

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