Analysis of Adaptive Clamping Force of Fixture Based on Finite Element Method

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Abstract: The deformation of the workpiece under the action of clamping force is an important factor that affects the machining accuracy in the machining process. In order to deal with the deformation of such workpiece, the research of machining deformation under the adaptive clamping force of intelligent fixture was carried out based on genetic algorithm and finite element analysis method, and the synchronization and optimization of fixture layout and clamping force were realized. The optimum clamping force of each key clamping position of the fixture is calculated by MATLAB, and the adaptive clamping force optimization design model is established to solve the clamping force curve. Under the same fixture layout, the constant clamping force and the adaptive clamping force are used respectively as the simulation boundary conditions, and the simulation model of the workpiece milling is established. The stress values and deformation after the workpiece are processed under the two clamping forces. The results show that the stress value and the processing deformation under the adaptive clamping force are both smaller, which verifies the validity of the research method.

1. Introduction

In the machining process, when the workpiece is subjected to the cutting force and clamping force, the deformation will occur, causing machining errors. When the error exceeds the set tolerance range, the workpiece will not be used [1]. However, the error is caused to a great extent by improper positioning and clamping. Therefore, the correct positioning and clamping of the workpiece are crucial to improving the machining accuracy of the workpiece.

Hurtado et al. [2] established a stiffness matrix based on the static equilibrium conditions of the workpiece and the fixture and concluded that the positive stiffness of the stiffness matrix is a sufficient condition for stable clamping. This method must be based on solving the contact stress as a precondition, but the solution process is cumbersome and complicated and therefore affects the computational
efficiency to a great extent. Liao et al. [3] believed that the workpiece undergoes dynamic cutting forces that change with time in the machining process. Therefore, it is considered that the supporting reaction force of the fixture components also dynamically changes. After the deformation of each fixture component in the entire machining process is obtained by the finite element method. The deformation is less than zero as the basis for determining the stability of the workpiece but did not mention the workpiece clamping layout program. Qin Guohua et al. [4] proposed a processing technique based on the principle of minimum total residual energy and used the finite element method to simulate the contact force and the deformation of the workpiece. It was found that the size of the clamping force, the action point, and the tightening sequence have an important influence on the deformation of the workpiece. Tao et al. [5] viewed the fixture components as rigid and the workpiece as elastic. A stiffness model of the workpiece-fixture system was proposed to solve the contact stress between the workpiece and the fixture. However, the method is carried out without considering the contact strength of the workpiece and the fixture. Wu et al. [6] set out from the static stability, established the workpiece-fixture system elastic friction model, replaced contact between the workpiece and the fixture components with springs, optimized the layout of the fixture using genetic algorithms, but not from the perspective of the dynamic characteristics further analyzes and optimizes the fixture layout. The above theory is to optimize the size of the clamping force under the assumption that the point of application does not change. In this paper, according to the actual situation, the workpiece and the fixture components are regarded as the elastic body, and the mathematical model with friction and frictionless conditions is established. Based on the clamping scheme, an optimization model of the clamping force is established and the clamping force size and the point of action are optimized so that the clamping state of the workpiece is optimal.

2. Clamping scheme optimization

2.1. Static balance constraint:

As shown in Figure 1, the workpiece-fixture system has K positioning elements (triangles) and n clamping elements (shaded triangles). The workpiece is subjected to the combined action of cutting force rotation \( W_c \), gravity rotation \( W_g \), and clamp clamping force of the clamp [7]. It is assumed that the unit normal vector and the orthogonal unit cut vector of the clamping element at the i-th contact point \( r_i = [x_i, y_i, z_i]^T \) are respectively \( n_i = [n_{ix}, n_{iy}, n_{iz}]^T \), \( n_i = [n_{ix}, n_{iy}, n_{iz}]^T \), \( b_i = [b_{ix}, b_{iy}, b_{iz}]^T \). The static equilibrium equation of the workpiece is as follows:

\[
GF + W = 0
\]  

(1)
\[ G = \begin{bmatrix} n_1 & t_1 & b_1 & \cdots & n_{k+n} & t_{k+n} & b_{k+n} \\ r_1 \times n_1 \times t_1 \times b_1 & \cdots & r_{k+n} \times n_{k+n} \times t_{k+n} \times b_{k+n} \end{bmatrix} \in \mathbb{R}^{6 \times 3(k+n)} \]  
(2)

\[ F = \begin{bmatrix} F_{i_1}, F_{i_2}, \cdots, F_{(k+n)a}, F_{(k+n)b} \end{bmatrix} \in \mathbb{R}^{3(k+n) \times 1} \]  
(3)

\[ W = \begin{bmatrix} F_c + F_g \\ r_c \times F_c + r_g \times F_g \end{bmatrix} \in \mathbb{R}^{6 \times 1} \]  
(4)

Among them, \( G \) is the structural matrix of the clamping element, \( F \) is the contact force vector, and \( W \) is the external force rotation.

2.2. Friction cone constraint: When the friction between the workpiece and the fixture element is considered, the combined force of the frictional force and the normal contact force between the workpiece and the fixture element must be within the friction cone. According to the Coulomb friction theorem:

\[ (F^i_n)^2 + (F^i_b)^2 \leq (\mu^i F^i_a)^2 (1 \leq i \leq k + n) \]  
(5)

Among them, \( \mu^i \) is the coefficient of friction between the workpiece and the part \( i \) fixture elements.

![Figure 2 Linearized friction cone](image)

As shown in Fig. 2, the friction cone constraint can be approximately expressed as a tangent positive 4K polyhedron (K is a natural number), so the above equation can be approximately converted linearly to:

\[ H_{4k,i} F_i \geq 0 (1 \leq i \leq k + n) \]  
(6)

Where, \( H_{4k,i} \) is a polyhedral matrix and can be expressed as:

\[ H_{4k,i} = \begin{bmatrix} \mu^i & -\cos \alpha^i & -\sin \alpha^i \\ \mu^i & -\cos \alpha^2 & -\sin \alpha^2 \\ \cdots & \cdots & \cdots \\ \mu^i & -\cos \alpha^s & -\sin \alpha^s \\ \cdots & \cdots & \cdots \\ \mu^i & -\cos \alpha^4k & -\sin \alpha^4k \end{bmatrix} \in \mathbb{R}^{4k \times 3} \]  
(7)

Thus, from the above equation, when the number of polyhedron edges is more positive, the degree of approximation is higher. It is the tilt angle of the long line perpendicular to the polygon, which is expressed as follows:
\[ \alpha_s = \frac{\pi}{4} + \frac{\pi}{2k} (s-1)(1 \leq s \leq 4k), (0 \leq \alpha_s \leq 2\pi) \] (8)

For workpiece-fixture systems, friction cone constraints can be expressed as:
\[ H_{4k} F \geq 0 \] (9)

Where, \[ H_{4k} = \text{diag}(H_{4K,1}, H_{4K,2}, \cdots, H_{4K,k+n}) \].

2.3. Unilateral contact constraint: In the actual clamping process, in order to ensure the accuracy of the clamping, it is required that the positioning element and the clamping element cannot be separated from the workpiece, that is, the contact normal force must point to the workpiece, that is, the following constraint conditions must be satisfied:
\[ F_{in} \geq 0 \] (10)

For workpiece-fixture systems, unilateral contact constraints can be expressed as:
\[ NTF \geq 0 \] (11)

Where, \[ N = \text{diag}(n_1^T, n_2^T, \cdots, n_{k+n}^T) \], \[ T = \text{diag}(T_1, T_2, \cdots, T_{k+n}) \], \[ T_i = [n_i, t_i, b_i] \].

2.4. Mathematical model of adaptive clamping force optimization: When external forces such as cutting force and gravity act on the workpiece being processed at the same time, as the machining continues, the rigidity of the workpiece is continuously reduced, and it is necessary to maintain the balance of the workpiece with adaptive clamping force. Not only that, but also to achieve the smallest possible adaptive clamping force, in order to be able to control the workpiece deformation [8]. This paper optimizes the adaptive clamping force on the premise that the position of the positioning element and the clamping element is known. Because there are still many unknowns in the optimization problem, the problem is static and indefinite, and the conventional static equilibrium equation cannot be used. For solving, the minimum norm principle should be used to solve [9]. Thus, combined with the appeal analysis, the following adaptive clamping force optimization model can be obtained:
\[
\min \left\{ \left\| F \right\| = (F^T F)^{1/2} \right\}
\sum_{i=1}^{n} F_{in}^T F
\] (12)

For the double-objective optimization problem, it can generally be solved by the \( \varepsilon \)-constraint method [10]. The \( \varepsilon \)-constraint method uses a function in a multi-objective optimization function as a basic objective function and converts another objective function into a constraint condition. So the above algorithm can be converted to:
\[
\min \left\{ \left\| F \right\| = (F^T F)^{1/2} \right\}
\sum_{i=1}^{n} F_{in}^T F \leq \varepsilon_{iter-1}, iter \geq 2
\] (13)

Where \( \text{iter} \) is, the number of iterations used to terminate the search process. It depends on the search accuracy \( \delta \), and its expression is as follows:
\[
\text{iter} = \left\lfloor \log_2 \left( \frac{\varepsilon_{iter-1}}{\delta} \right) \right\rfloor \]
(15)

Among them, the symbol "[\cdot]" indicates a rounding function,
3. Synchronization optimization of fixture layout and clamping force

As shown in FIG. 3, the overall flow of fixture layout and clamping force synchronization optimization based on a genetic algorithm and a finite element method is shown. Firstly, the workpiece is modeled by three-dimensional CAD software, and then the finite element model is established in the DM module of Ansys Workbench, the boundary condition, and the load are set; thirdly, the maximum machining deformation of the workpiece is obtained by fitting the spring contact stiffness by the least square method. Finally, on the premise that Isight software has been integrated into the Ansys Workbench module, the design scheme of the fixture is optimized by using the genetic algorithm provided by Isight software itself.

![Figure 3: Overall process of clamping optimization scheme](image)

3.1. Workpiece parameters and pass path: The dimensions of the workpiece are 149mm x 53mm x 20mm, the thickness of the side wall is 4mm, the thickness of the middle wall is 6mm, and the thickness of the bottom plate is 2mm. The geometry of the workpiece is shown in Figure 4. The material of the workpiece is 7050-T7451. The fixture component uses a ball-head fixture. The center point of the contact surface between the fixture and the workpiece is used as the position of the fixture component. The material properties of the fixture component are shown in Table 1. The coefficient of friction is 0.4. The tangential contact stiffness between the fixture and the workpiece is 5.84e7N/m, and the normal contact stiffness is 4.83e7N/m.

![Figure 4: Workpiece geometry](image)
### Table 1: Material properties

| Parameters            | Positioning clamping Element |
|-----------------------|------------------------------|
| Material              | AISI 1144                     |
| Density (kg/m³)       | 7800                         |
| Modulus of elasticity (Gpa) | 207                     |
| Poisson's ratio       | 0.292                        |

3.2. Workpiece parameters and tool path and processing parameters

The inner wall of the workpiece is milled to 2mm. The path of the tool is shown in Fig. 5. Taking into account the amount of calculation, the path of the tool path is simplified and discrete into 24 sub-steps. In any condition, the cutting force is the same size. The cutting forces in the X, Y, and Z directions are 584N, 827N, and 332N, respectively.

![Figure 5: Tool Path](image)

3.3. Geometric parameters of spiral end mills and cutting parameters for high speed machining:

Since the workpiece material used in this paper is aluminum alloy and high-speed machining technology is adopted, the geometric parameters of the spiral end mill and the cutting amount of high-speed machining are optimized for the machining conditions. Specific parameters are shown in Table 2 and Table 3.

### Table 2: Geometric parameters of spiral end milling cutter

| Diameter (mm) | Number of teeth | Rake angle (°) | Tool rear angle (°) | Helix angle (°) |
|---------------|-----------------|----------------|--------------------|-----------------|
| 10            | 4               | 12             | 10                 | 30              |

### Table 3: Cutting parameters of high speed machining

| Spindle speed (rpm) | Feed rate (mm/z) | Axial cutting depth (mm) | Radial cutting depth (mm) |
|---------------------|-------------------|--------------------------|---------------------------|
| 24000               | 0.2               | 6                        | 2                          |
3.4. Fixture element orientation: $L_1$-$L_6$ is the positioning element and $C_1$-$C_3$ is the clamping element. In the parametric model, the positioning and clamping elements are replaced by semi-elastic contact models. The supporting directions of the springs are shown in Table 4.

| Components | Unit normal vector $n_i$ | Unit tangent vector $t_i$ | Unit tangential $b_i$ |
|------------|--------------------------|--------------------------|-----------------------|
| L1         | [1,0,0]T                 | [0,1,0]T                 | [0,0,1]T              |
| L2         | [0,1,0]T                 | [0,0,1]T                 | [1,0,0]T              |
| L3         | [0,1,0]T                 | [0,0,1]T                 | [1,0,0]T              |
| L4         | [0,0,1]T                 | [1,0,0]T                 | [0,1,0]T              |
| L5         | [0,0,1]T                 | [1,0,0]T                 | [0,1,0]T              |
| L6         | [0,0,1]T                 | [1,0,0]T                 | [0,1,0]T              |
| C1         | [-1,0,0]T                | [0,1,0]T                 | [0,0,-1]T             |
| C2         | [0,-1,0]T                | [0,0,-1]T                | [1,0,0]T              |
| C3         | [0,-1,0]T                | [0,0,-1]T                | [1,0,0]T              |

3.5. Multi-objective optimization results: With Isight software integrated into the ANSYS Workbench module, using the Isight software to optimize the fixture design program, an optimized module from the parametric model file was obtained, and the structural parameters, force parameters, and overall deformation values in the ANSYS were extracted. Then set the parameters of the genetic algorithm, set the number of population 20, the number of iterations 200, the hybrid probability of 1.0, the mutation probability of 0.01, the remaining parameters to maintain the system default.

As shown in Figure 6, it is the convergence process of fixture layout and clamping forces synchronization optimization. The optimal fitness value $\sigma$ is 0.0852. For the convenience of observation, 13 generations are taken and individual optimal values are taken in each generation.

Table 5 shows the optimized results for each design variable, which is the position of the fixture components after optimization. The maximum deformation value obtained by using the multi-objective optimization method is 0.2929 mm less than that of the experiential design, which is a drop of 66.2%. Fig. 7 was the corresponding workpiece deformation cloud diagram before and after optimization. The uniformity of the deformation distribution obtained by calculation, is increased by 0.0579mm, which is increased by 58.5%. The maximum required clamping force is reduced by 584.8N, which is a decrease of 22%. This shows that the multi-objective optimization method has a significant effect on optimization. Among them, $F_1$, $F_2$, and $F_3$ are the magnitudes of the clamping force measured in the three directions of X, Y, and Z, respectively.
Table 5: Optimized position of fixture components

| Multi-objective optimization | x/mm | y/mm | z/mm |
|----------------------------|------|------|------|
| L1                         | 0    | 28.6 | 10.7 |
| L2                         | 120  | 0    | 13.1 |
| L3                         | 27.4 | 0    | 13.7 |
| L4                         | 12.2 | 40.4 | 0    |
| L5                         | 13.9 | 8.3  | 0    |
| L6                         | 137.5| 24.4 | 0    |
| C1                         | 149  | 24.1 | 11.1 |
| C2                         | 101.8| 53   | 10.4 |
| C3                         | 41.7 | 53   | 12.6 |
| F₁/N                       |      | 1427.7|
| F₂/N                       |      | 557.9 |
| F₃/N                       |      | 655.3 |
| F/mm                       |      | 0.0852|
| σ/mm                       |      | 0.0411|

(a) Before optimization

(b) After optimization

Figure 7: Before and after the optimization of the workpiece deformation cloud

The external dimensions of the workpiece are shown in Fig. 4, the path of the cutting tool is shown in Fig. 5, the geometric parameters of the spiral end mill are shown in Table 2, the processing parameters are shown in Table 3, the orientation of the fixture components is shown in Table 4, and the positions of the fixture components are shown in Table 5. Milling forces in all directions were calculated using ABAQUS simulations. Because the weight of the workpiece is too small and the change is not significant, it is negligible. The fixture coefficients of all the fixture elements and workpiece contact
points are equal to 0.4. Table 6 shows the tool point and its corresponding milling force at key positions of the workpiece.

| Tool point (mm) | Milling force (N) x y z | Tool point (mm) | Milling force (N) x y z |
|----------------|-------------------------|----------------|-------------------------|
| (142,9,17)    | -827 584 -332           | (61,9,17)      | -827 584 -332           |
| (142,46,17)   | -827 584 -332           | (61,46,17)     | -827 584 -332           |
| (107.5,46,17) | -584 -827 -332          | (34,46,17)     | -584 -827 -332          |
| (73,46,17)    | -584 -827 -332          | (7,46,17)      | -584 -827 -332          |
| (73,7,17)     | 827 -584 -332           | (7,7,17)       | 827 -584 -332           |
| (107.5,7,17)  | 584 827 -332            | (34,7,17)      | 584 827 -332            |
| (142,9,11)    | -827 584 -332           | (61,9,11)      | -827 584 -332           |
| (142,46,11)   | -827 584 -332           | (61,46,11)     | -827 584 -332           |
| (107.5,46,11) | -584 -827 -332          | (34,46,11)     | -584 -827 -332          |
| (73,46,11)    | -584 -827 -332          | (7,46,11)      | -584 -827 -332          |
| (73,7,11)     | 827 -584 -332           | (7,7,11)       | 827 -584 -332           |
| (107.5,7,11)  | 584 827 -332            | (34,7,11)      | 584 827 -332            |
| (142,9,5)     | -827 584 -332           | (61,9,5)       | -827 584 -332           |
| (142,46,5)    | -827 584 -332           | (61,46,5)      | -827 584 -332           |
| (107.5,46,5)  | -584 -827 -332          | (34,46,5)      | -584 -827 -332          |
| (73,46,5)     | -584 -827 -332          | (7,46,5)       | -584 -827 -332          |
| (73,7,5)      | 827 -584 -332           | (7,7,5)        | 827 -584 -332           |
| (107.5,7,5)   | 584 827 -332            | (34,7,5)       | 584 827 -332            |

By calculating the optimal clamping force for each key position in MATLAB, a set of adaptive clamping force curves for the workpiece machining process can be obtained. As shown in Figure 8, it can be seen from the figure that the clamping forces F1, F2, and F3 have their own changes in the process of processing, such as in the first frame in the process of F1 first sharp reduction, and then gently lowered, while processing the second frame F1 basically remain unchanged. In addition, it can be seen from Figure 8 that the clamping force in the three directions of most key positions is less than the value in Table 3, but due to the use of a layered optimization design method for the adaptive clamping force, the clamping force with a few key positions is greater than the value in Table 3, but the adaptive clamping force is still considered to be the research aim.
4. Machining deformation analysis

After the simulation of the workpiece machining process is completed, the stress state of the workpiece under constant clamping force and adaptive clamping force is shown in Fig. 9. It can be seen that the internal stress of the workpiece is in an unbalanced state at this time and Fig. 9(a) is higher than Fig. 9(b) under the action of a constant clamping force under an adaptive clamping force. This is because the finite element calculation of the machining simulation is performed in the time domain using the center difference method. Although the dynamic solver can effectively simulate the milling process of the workpiece, the calculation result cannot determine whether the ideal equilibrium state is achieved. Therefore, the stress state of the workpiece can only be within the artificially set time zone.

![Stress state of workpiece before stress balance](image)

(a) Constant clamping force

(b) Adaptive clamping force

Figure 9: Stress state of workpiece before stress balance

In order to solve the above problems, it is necessary to obtain the stress re-equilibrium state of the workpiece based on the dynamics solution. Firstly, the boundary constraint conditions of the finite element model must be removed. Then the static analysis of the model is used to obtain the true processing distortion of the workpiece status. The iterative method used in static analysis can directly obtain the stress-equilibrium state of the finite element model and does not consider the effect of load time. After the static analysis, the stress state of the workpiece under constant clamping force and adaptive clamping force is shown in Figure 10 below. It can be seen that the stress value and the stress balance of the workpiece have been reduced before, and the stress value of Fig. 10(b) under the action of adaptive clamping force is lower than that in the constant clamping force. Therefore, it can be considered that the adaptive clamping force has a certain control effect on the stress state of the workpiece after processing.
Under the action of constant clamping force and adaptive clamping force, the machining deformation after the workpiece milling is completed as shown in Fig. 11. To make the observation more convenient, Fig. 11(a) and 11(b) have enlarged the workpiece deformation of the same multiples, it can be seen that the whole workpiece is warped and deformed. The processing deformation mainly concentrates on the wall plate out, and the deformation of the wall plate under the constant clamping force of Fig. 11(a) is more obvious than that of Fig. 11(b) under the effect of the adaptive clamping force. It should be particularly noted that although the workpiece processing, distortion in Figure 11 and the actual workpiece processing deformation under experimental conditions must have errors, it can still be considered that the adaptive clamping force at this time has a certain degree of control over the processing distortion of the workpiece effect.
5. Conclusion
In this paper, based on the finite element method, the workpiece deformation under the action of the adaptive clamping force of the intelligent fixture is studied. Through the analysis of the processing deformation, the deformation law of the workpiece is obtained, which provides a reference for the future intelligent fixture clamping research. Through the study of this paper, the main work and conclusions are as follows:

The genetic algorithm and the finite element method were applied to the synchronous optimization of the fixture layout and clamping force of the aircraft structural parts. The contact between the fixture components and the workpiece was replaced by a spring. The machining trajectory was multi-step discrete, and the optimization results showed that not only the reduction the machining deformation, but also improves the uniformity of machining deformation.

Based on the optimized fixture layout, the clamping force curve was obtained by establishing an adaptive clamping force optimization design model. Through an example analysis, the effectiveness of the research method was verified.

The workpiece finishing processes were simulated. Under the same fixture layout, the deformation of the workpiece under the action of the adaptive clamping force and the constant clamping force was predicted. The former work quality is higher, which also provides a new method for the control of workpiece deformation.

Competing interests
Huang Qi and Yadav Srijana have contributed equally. The authors show no conflict of interests.

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