On the selection of the best companding technique for PAPR reduction in OFDM systems

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ABSTRACT
Orthogonal frequency division multiplexing (OFDM) is used in high data rate applications due to its ability to cope with frequency-selective channels. However, OFDM suffers from the high peak-to-average power ratio (PAPR) problem, which reduces the power amplifier (PA) efficiency or otherwise degrades bit error rate (BER) and increases out-of-band (OOB) radiation. In the literature, there are various PAPR reduction techniques. Among them companding techniques have small computational complexity, which make them attractive to be used in mobile stations (MS). Generally, companding techniques expand small signals while compressing large signals or compress large signals without affecting small signals. In this paper, different PAPR reduction companding transforms are compared. Results showed that companding transforms that compress large signals without affecting small signals (such as, Log companding and Tanh companding) are better than the others from a BER point of view. Results also showed that the Log companding transform is better than the Tanh transform, in terms of PAPR reduction gain and OOB radiation reduction. So the Log companding transform can be considered as the best practical companding transform among others.

1. Introduction
Recent wireless communication applications require excessive high data rate, which make it more subspecies to frequency selectivity. Orthogonal frequency division multiplexing (OFDM) has been widely used in high-speed wireless communications, due to its capability to combat frequency selectivity. However, a serious disadvantage of OFDM is its high peak to average power ratio (PAPR), which significantly decreases the efficiency of power amplifier (PA) in order to meet the linearity requirements (in terms of bit error rate (BER) and out-of-band (OOB) radiation) of wireless communication standards. To enhance the efficiency of PA without degrading BER performance or increase OOB radiation, many PAPR reduction techniques are introduced in the literature (Zahra, Tarrad, & Mounir, 2014). Among them companding transforms have the lowest computational complexity regardless of the number of subcarriers which make companding transforms

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attractive techniques. Companding transforms are typically applied to speech signals to optimize the required number of bits per sample. Since OFDM and speech signals behave similarly in the sense that high peaks occur infrequently, the same companding transforms can also be used to reduce the OFDM signal’s PAPR (Rahmatallah & Mohan, 2013). However, the major drawback of companding techniques is that they achieve PAPR reduction at the cost of increasing the BER. This degradation in BER performance is due to two factors: First, companding distorts the modulating data symbols at the transmitter from their original constellation; second, the channel noise is expanded at the receiver by the decompanding process (Rahmatallah, Bouaynaya, & Mohan, 2011a, December, 2011b, April). For any companding technique, there is the PAPR reduction gain value at which the BER reaches its minimum. This is called efficient PAPR reduction gain. Efficient PAPR reduction gain is not necessarily the lowest possible value of PAPR. Finding efficient PAPR reduction gain would mean to find optimal companding transform parameters. Thus, performance of specific transform depends heavily on its parameters (Huang, Lu, Chuang, & Zheng, 2001).

In the literature there are different transforms used for companding such as μ-law companding transform (Wang, Tjhung, & Ng, 1999a, 1999b), exponential companding transform (Jiang, Yang, & Song, 2005a, December, 2005b), hyperbolic tangent (Tanh) companding transform (Lowe & Huang, 2007, August), threshold Log companding transform (Gong, Ye, Feng, & Ke, 2005, September), linear symmetric transforms (LST) (Rahmatallah et al., 2011a, December), and linear asymmetric transforms (LAST) (Huang, Lu, Zheng, Letaief, & Gu, 2004).

In this paper these six companding transforms are compared in terms of three main metrics:

1. **Complementary cumulative distribution function (CCDF):** is used to compare their PAPR reduction gains.
2. **BER:** is used to compare their performances in the presence of nonlinear PA.
3. **Power spectral density (PSD):** is used to compare the required Input Back-Off (IBO) by each one of them to follow the given regulation spectral mask.

The rest of this paper is organized as follows. In Section 2, related studies are presented. Section 3 provides a brief description of the OFDM system and the high PAPR problem. In Section 4 definitions of the six companding transforms – stated above – are briefly summarized. Performances of previously defined companding transforms are evaluated and compared by means of simulation in Section 5. Finally, the conclusions are drawn in Section 6.

### 2. Related works

Wang et al. (1999a) introduced the μ-law companding scheme, it was the first PAPR reduction companding scheme. It was inspired from the fact that the OFDM signal has high PAPR envelop like voice. Similarly, the A-law was used by the same author in Wang, Tjhung, and Wu (2003) for OFDM PAPR reduction. However, μ-law and A-law can produce the same companding profile as shown in Sakran, Shokair, and Elazm (2009). The first classification of all possible companding profiles was first introduced in Huang et al. (2001). In which companding techniques are classified according to linearity
and symmetry into linear symmetric transform (LST), linear quasi-symmetric transform (LQST), non-linear symmetric transform (NLST), and non-linear quasi-symmetric transform (NLQST). Huang et al. (2001) show that companding techniques belong to NLQST are the best in terms of BER performance, while LQST is the best in terms of PAPR reduction gain. Later, Huang et al. (2004) classify companding techniques into four types LST, linear non-symmetric transform (LNST), NLST, and non-linear non-symmetric transform (NLNST), where LNST and NLNST correspond to LQST and NLQST, respectively. LNST and NLNST are also called LAST and non-linear asymmetric transform (NLAST) in the classification introduced in Rahmatallah and Mohan (2013). Huang et al. (2004) show that LNST is the best in terms of PAPR reduction and BER performance in the presence of nonlinear PA. Aburakhia, Badran, and Mohamed (2009) prove that the performance of LNST depends on the number of inflexion points. Rahmatallah et al. (2011a, December) state that regardless of the number of inflexion points, BER performance of LAST (i.e. LNST) will be superior to that of LST, if the slope of LAST is larger than that of LST. Also, the LAST scheme can be better than any other NLAST schemes, in terms of OOB-radiation reduction and BER performance under certain conditions as illustrated in Rahmatallah et al. (2011a, December) and Rahmatallah, Bouaynaya, and Mohan (2013). However, LAST may be impractical as will be shown later in Section 4. In the literature, there are various NLAST rather than μ-law and A-law companding schemes. Jiang et al. (2005a, December, 2005b) introduce an NLAST called the Exponential companding scheme, and show that it has superior performance over the μ-law scheme, in terms of PAPR reduction and BER performance in the presence of nonlinear PA. The Piecewise Companding (PW) scheme is an NLAST similar to the Exponential scheme introduced in Hou, Ge, Zhai, and Li (2010), in which results showed that PW and Exponential schemes have similar PAPR reduction and BER performances, when decompanding is not been used in the receiver. In Gong et al. (2005, September) another NLAST called threshold Log was introduced and compared to the μ-law and the modified version of the μ-law (Huang, Lu, & Zheng, 2002, November), results showed that the threshold Log has a PAPR reduction gain and BER performance better than both of them. Unlike Exponential companding that adjusts both small and large signals without affecting average power, the erfc companding scheme (Jiang, Lu, Wu, & Zhu, 2006, November; Jiang, Xiang, Richardson, Qu, & Zhu, 2007) converts Gaussian distribution of OFDM into uniform distribution. However, Jiang and Wu (2008) showed that the Exponential companding transform has PAPR reduction gain and BER performance better than erfc transform. In fact, erfc transform (Jiang et al., 2006, November, 2007) is a special case of the more general NLAST transform presented in Jeng and Chen (2011). Wang, Wang, Ge, and Ai (2012) developed a sinh companding transform and compared it to other NLAST such as the μ-law companding transform (Wang et al., 1999a), Exponential companding transform (Jiang et al., 2005b), general erfc transform (Jeng & Chen, 2011), and tanh companding transform (Yang, Zhou, & Qian, 2007, October), it has different performances with its different parameters. In Rahmatallah et al. (2013) companding techniques are classified into three classes LST, LAST, and non-linear companding transforms (NLCT). In this work, LST and LAST are compared along with the most promising NLCT schemes (i.e. μ-law, Exponential, Tanh, and Log) altogether by using the main three metrics (i.e. CCDF, BER, and PSD) mentioned above. Here, the output of all the companding schemes is filtered to decrease OOB radiation and is consistent with practical consideration. However, filtering causes peak regrowth and influences the PAPR reduction performance of the schemes.
3. OFDM system and high PAPR problem

Consider a sequence of \( N \) information symbols \( A_k \) (QAM symbol) modulating \( N \) subcarriers in the frequency domain. Then, an IFFT is used to convert the frequency domain vector \( \mathbf{A} = [A_0, A_1, \ldots, A_{N-1}] \) to a time-domain vector \( \mathbf{x} \) (OFDM symbol), conventionally written as \( \mathbf{x} = \text{IFFT} \left( \mathbf{A} \right) \). Time domain samples \( x_n \) of time-domain vector \( \mathbf{x} \) are given by

\[
x_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} A_k e^{j \frac{2 \pi n k}{N}}, \quad 0 \leq n \leq N - 1. \tag{1}
\]

For a large number of subcarriers \( N \), real and imaginary parts of the time-domain samples \( x_n \) of complex OFDM symbols (which are the summation of uniformly distributed frequency-domain symbols \( A_k \)) will be Gaussian distributed due to the central limit theorem. Assume statistical independence among frequency-domain symbols \( A_k \). Then, the amplitude of the OFDM signal will be Rayleigh distributed suffering from high PAPR.

The PAPR of one OFDM symbol is given by

\[
PAPR = \max_{n \in \{0, N-1\}} \left| x[n] \right|^2 \frac{E\{|x[n]|^2\}}{}, \tag{2}
\]

where \( E\{\} \) is the expectation operator. The PAPR is usually described by means of statistical distribution as PAPR itself seems to be a random variable. The probability that PAPR of the OFDM symbol going larger than a particular threshold \( \xi_o \) is called the complementary cumulative distributive function (CCDF) and written mathematically as

\[
\text{CCDF} = \text{prob}\{\text{PAPR} \geq \xi_o\}. \tag{3}
\]

Practically, CCDF is used to measure the PAPR reduction capability of any technique (Youssef, Tarrad, & Mounir, 2016, December).

4. Companding techniques

Companding transforms can be classified, in terms of linearity and symmetry, into four classes: LST, LAST, NLST, and NLAST. Figure 1 illustrates the profiles of them, while Figure 2 shows the OFDM system employing the companding transform (Rahmatallah & Mohan, 2013).

\( \mu \)-Law companding (Wang et al., 1999a, 1999b) is an NLAST that was the first companding transform used in PAPR reduction. Companded \( \hat{x}^{\mu\text{-law}} \) and de-companded signal \( \hat{r}^{\mu\text{-law}} \) by \( \mu \)-Law transform are given by

\[
\hat{x}^{\mu\text{-law}} = A \cdot \text{sgn}(x_n) \cdot \ln \left[ 1 + \mu \left| x_n / A \right| \right] \left/ \ln [1 + \mu] \right. \tag{4}
\]

\[
\hat{r}^{\mu\text{-law}} = A \cdot \exp \left[ \frac{r_n \cdot \ln [1 + \mu] - 1}{A \cdot \text{sgn}(r_n)} \right], \tag{5}
\]

where \( A \) is the normalization constant such that \( 0 \leq |x_n / A| \leq 1 \) and \( \text{sgn}(x_n) \) denote the sign of \( x_n \). In \( \mu \)-Law transform small signals are enlarged while the large signals remain
unchanged, thus it increases the average power of the amplifier input signal, hence reduces the PAPR. This makes the companding signal itself more sensitive to the nonlinearity of PA (Huang et al., 2001). However, Wang et al. (1999a) only considered the effect of quantization noise and ignored PA nonlinearity (Aburakhia et al., 2009).

Later, another NLAST namely exponential companding (Jiang et al., 2005a, December, 2005b) has been developed to overcome the problem of increasing average power and to have efficient PAPR reduction. This scheme transforms the Rayleigh distributed OFDM signal into a uniformly distributed (companded) signal without changing the average power level. Companded $\hat{x}_{\text{exp}}$ and de-companded signal $\hat{r}_{\text{exp}}$ by exponential companding.

**Figure 1.** Profiles of the four companding transform classes (Rahmatallah & Mohan, 2013).

**Figure 2.** Block diagram of the OFDM system employing the PAPR reduction companding technique.
Transform are given by

\[ \hat{x}_n^{\text{exp}} = \text{sgn}(x_n) \cdot \sqrt{d/\alpha} [1 - e^{-|x_n|^2/\sigma^2}], \]

(6)

\[ \hat{r}_n^{\text{exp}} = \text{sgn}(r_n) \cdot \sqrt{-\sigma^2 \ln (1 - (r_n^2/\alpha))}, \]

(7)

where \( \alpha = \left( \frac{\mathbb{E}[|x_n|^2]}{\mathbb{E}\left[\sqrt{1 - e^{-|x_n|^2/\sigma^2}}\right]^2} \right)^{d/2} \) is a constant that used to keep input and output signals at the same average power and \( \sigma \) is the variance of \( x_n \). When \( d \geq 2 \) exponential companding transform compresses large input signals and expands small signals simultaneously. Contrary to \( \mu \)-law companding that only enlarges small signals (Jiang et al., 2005a, December, 2005b). However, the distribution of large amplitude signals is increased by uniform companding. Hence, we can predict that BER performance is degraded when the OFDM transmitters employ PA with heavy nonlinearity (Hou et al., 2010).

Rahmatallah and Mohan (2013) have shown, based on computer simulation, that LAST is the best among other companding schemes in terms of PAPR reduction and BER. These performance gains were achieved by introducing an inflexion point in LAST so that small and large signal's amplitudes could be treated with different scales. LAST Companded \( \hat{x}_{\text{LAST}} \) and de-companded \( \hat{r}_{\text{LAST}} \) signals are defined by

\[ \hat{x}_{n, \text{LAST}} = \begin{cases} 
\frac{1}{u} \cdot x_n, & |x_n| \leq x_{th} \\
- u \cdot x_n, & |x_n| > x_{th}
\end{cases} \]

(8)

\[ \hat{r}_{n, \text{LAST}} = \begin{cases} 
\frac{1}{u} \cdot r_n, & n \in \phi_1(x_{th}) \\
- u \cdot r_n, & n \in \phi_2(x_{th})
\end{cases} \]

(9)

where \( 0 < u < 1 \) is the piecewise slope parameter and \( 0 < x_{th} < \max (|x_n|) \) is the inflexion point. In order to keep the average power of the transmitted signal unchanged, average power may be set as the value of the inflexion point. \( \phi_1(x_{th}) \) and \( \phi_2(x_{th}) \) are the index sets of OFDM samples below and above the threshold \( x_{th} \), respectively. It is assumed that the receiver has the knowledge of the two sets (Aburakhia et al., 2009; Huang et al., 2004). This means SI of \( N \) bits are needed, but for complex input \( X_n \), the transform should be applied to real and imaginary separately, thus SI becomes \( 2N \) bits.

Rahmatallah et al. (2011a, December) proved that BER performance superiority of LAST depends on the slope rather than the number of discontinuity points. Also, Rahmatallah et al. (2011a, December) provide a sufficient but not necessary condition, for the superiority of LAST over LST, in terms of BER, that is the minimum piecewise slope of LAST \( u_{\text{min}} \) (regardless of the number of discontinuity points) must be larger than the slope of LST \( (a) \), where companded \( \hat{x}_{\text{LST}} \) and de-companded \( \hat{r}_{\text{LST}} \) signals are given as follows:

\[ \hat{x}_{n, \text{LST}} = (a \cdot x_n) + b, \]

(10)

\[ \hat{r}_{n, \text{LST}} = (r_n - b)/a, \]

(11)

where \( 0 < a < 1 \) is the slope of the LST and \( b > 0 \) is the offset.

Another two NLAST based on the hyperbolic tangent (Tanh) and the threshold Log are introduced in Lowe and Huang (2007, August) and Gong et al. (2005, September),
respectively. Companded $\hat{x}_{\text{Tanh}}$ and de-companded $\hat{r}_{\text{Tanh}}$ signals of the hyperbolic tangent (Tanh) transform are given by

$$\hat{x}_{\text{Tanh}}^n = C_1 \cdot \tanh \left( \frac{x_n}{C_2} \right),$$  \hspace{1cm} (12)

$$\hat{r}_{\text{Tanh}}^n = C_2 \cdot \arctan \left( \frac{r_n}{C_1} \right),$$  \hspace{1cm} (13)

and de-companded $\hat{r}_{\text{Log}}$ signals of the threshold Log companding transform are given by

$$\hat{x}_{\text{Log}}^n = \begin{cases} x_n, & |x_n| \leq x_{\text{th}}, \\ K_1 \ln \{ 1 + (x_nK_2) \}, & |x_n| > x_{\text{th}}, \end{cases}$$  \hspace{1cm} (14)

$$\hat{r}_{\text{Log}}^n = \begin{cases} r_n, & |r_n| \leq x_{\text{th}}, \\ (e^{r_n / K_1} - 1) / K_2, & |r_n| > x_{\text{th}}, \end{cases}$$  \hspace{1cm} (15)

where $x_{\text{th}}$ is the threshold, after which compression is done. $K_1$ and $K_2$ are positive numbers controlling the compression level, with $(0 \leq k_1, k_2 \leq 1)$. The slope of Tanh and Log transforms are given by $(C_1 \cdot C_2)$ and $(K_1 \cdot K_2)$, respectively (Rahmatallah et al., 2011b, April). Setting $C_1 = 1/C_2$ and $K_1 = 1/K_2$ is the required condition to maintain the average power of the OFDM signal unchanged after companding (Rahmatallah et al., 2011b, April; Rahmatallah & Mohan, 2013).

Finally, companding techniques are nonlinear processes that lead to in-band noise and OOB radiation. OOB radiation can be reduced by filtering after companding; however, this will cause peak regrowth (Jiang & Wu, 2008). Companding with filtering will be used throughout this paper.

5. Simulation and results

In this section computer simulations are performed to compare the performance of the six companding transforms introduced in the previous section. Performance evaluation is done using the three main metrics CCDF, BER, and PSD. Simulation parameters are listed in Table 1.

Based on simulation parameters listed in Table 1, optimum parameters that achieve efficient PAPR reduction for these companding transforms are found by means of simulation; $\mu = 2$ is the optimum value for $\mu$-Law transform, increasing $\mu$ will increase compression and average power, thus increasing PAPR reduction gain, but on the other hand lead to higher BER. $d = 2$ is the optimum value that achieves efficient PAPR reduction for exponential companding transform as per (Jiang et al., 2005b; Rahmatallah & Mohan, 2013), increasing $d$ performance of exponential transform become as clipping as for the

| Table 1. Simulation parameters. |
|----------------------------------|
| Simulation parameter | Value |
| Number of subcarriers | $N = 256$ |
| Number of data subcarriers | 192 |
| Oversampling factor | $L = 4$ |
| PA model | SL (linearized PA) |
| Modulation scheme | 16-QAM |
| IBO | 4dB |
| PSD regulation mask | ETSI EN 302 021 Type-G (ETSI EN 301 021, 2003) |
large amplitudes of the signal. Inflection point $x_{in}$ in the LAST companding transform is set to the square root of the average power, in order to keep the average power of the compressed signal unchanged, while $u = 0.8$ achieves the efficient PAPR reduction. The optimum parameters of LST are $a = 0.8$ and $b = 0.4$, these values achieve efficient PAPR reduction. Likewise Tanh with $C_1 = 1/C_2 = 1.2$ and Log with $K_1 = 1/K_2 = 1$.

Input/output profiles of the six transforms are depicted in Figure 3, using optimum parameters that achieve the efficient PAPR reduction per transform.

In Figure 4 the CCDF of the six companding transforms are compared with the case of no PAPR reduction, it can be noted that $\mu$-Law companding and Log companding have the largest PAPR reduction gain, while Tanh companding has the lowest PAPR reduction gain. Also it can be noted that LAST and LST have the same performance due to the same slope (i.e. $\mu = 0.8$ and $a = 0.8$).

Comparison of BER performances of the six companding transforms is shown in Figure 5. It can be noted that LAST companding transform has the best BER performance, although it has PAPR reduction gain similar to LST (due to the same slope) and worse than $\mu$-Law and Log. However, LAST is not practical due to the large number of SI, in our case 256 bits are required, using 16 QAM modulation, then 64 subcarrier are needed, i.e. 25% of subcarriers. On the other hand, exponential companding has the worst BER performance among others, as it becomes as Clipping for large signal in addition to make small signal vulnerable to nonlinearity. Also it can be noted that both Tanh and Log have the same BER performance that is better than others, except LAST.

It is worth mentioning that Tanh and Log transforms are better than LST in terms of BER, as the slopes of Tanh and Log transforms (i.e. $(C_1 \cdot C_2)$ and $(K_1 \cdot K_2)$, respectively) are larger than the slope of LST ($a$), the condition stated in Rahmatallah and Mohan (2013). But, Tanh has the worst PAPR reduction gain among others.

Figure 3. Profiles of different companding techniques.
It can be concluded that companding transforms keep small signals unchanged, while compressing large signals (i.e. Tanh and Log) are better than others in terms of BER. However, Log companding also has PAPR reduction gain better than that of Tanh, which has the worst PAPR reduction gain among others, as shown Figure 4.

**Figure 4.** PAPR reduction performance of different companding transforms.

**Figure 5.** BER performance of different companding transforms.
In addition to that Log transform requires a moderate IBO value to follow the PSD regulation mask, as shown in Figure 6. Transforms with high PAPR reduction (i.e. μ-law, LAST, LST, and Log) require low IBO to comply with the required constraint, while transforms with low PAPR reduction (i.e. Exponential and Tanh) require larger IBO to comply with the same constraint.

By using the three aforementioned metrics (i.e. CCDF, BER, PSD), Table 2 compares the performance of the six companding transforms in terms of PAPR reduction gain at certain CCDF (Figure 4), the required $E_b/N_0$ to achieve certain BER (Figure 5), and the required IBO to follow the given ETSI Mask (Figure 6). Neglecting the impractical LAST transform, it can be noted that Log and μ-law transforms have the highest PAPR reduction gain among others and reduces the required IBO by the same amount to follow the given regulation mask, while Log transform requires $E_b/N_0$ less than the μ-law transform by 1.5 dB to achieve the same BER.

Finally, it can be said that Log companding transform is the best practical companding transform among the others.

![Figure 6. PSD of different companding transforms complying with ETSI EN 301 021 Mask type-G, after passing through SL.](image)

**Table 2.** Comparison of the performance of the six companding transforms.

| Companding technique | PAPR reduction gain @ CCDF = 10^{-4} (dB) | Required $E_b/N_0$ @ BER = 10^{-6} | Required IBO to follow ETSI mask (dB) |
|---------------------|------------------------------------------|-----------------------------------|--------------------------------------|
| Tanh                | 0.77                                     | 18                                | 5                                    |
| Exponential         | 1.34                                     | 22                                | 4.8                                  |
| Log                 | 3.68                                     | 18.3                              | 4                                    |
| LST                 | 2.06                                     | 19.1                              | 4.1                                  |
| μ-Law               | 3.71                                     | 19.8                              | 3.9                                  |
| LAST (Impractical)  | 1.98                                     | 15.2                              | 3.9                                  |
6. Conclusion

In this paper, the performance of the six companding transforms (μ-law, exponential, Tanh, Log, LST, and LAST) are compared in terms of CCDF, BER, and PSD. Simulation is done by using optimum parameters that achieve efficient PAPR reduction for each companding transform.

Results show that LAST has the best BER performance among others and requires the lowest IBO to follow the given PSD regulation mask. It is an impractical technique as it requires an extra-large number of SI bits (which can be reduced to occupy at least 25% of subcarriers in our case).

Also, results observe that companding transforms keep small signals unchanged, while compressing large signals (i.e. Tanh and Log) are better than others from a BER point of view. Furthermore, Log companding also has PAPR reduction gain better than that of Tanh, and requires lower IBO than that of Tanh to follow the given PSD regulation mask.

Thus, it can be concluded that Log companding transform is the best practical companding transform among the others.

Disclosure statement

No potential conflict of interest was reported by the authors.

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