Determination of the Thermal Diffusivity of a Multi-layered Soil Through a Conduction-Convection Algorithm

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R E S U M O
A temperatura do solo pode interferir em diversas atividades antrópicas. É de grande importância o conhecimento desse fenômeno para diversas áreas, como as ciências ambientais, a agricultura, a engenharia e a geografia física. A difusividade térmica é um dos parâmetros termodinâmicos mais relevantes do solo, tendo grande relevância na descrição deste fenômeno. Em São João, região Agreste do Estado de Pernambuco, o estudo teve como objetivo determiná-la através do algoritmo da condução-convecção, dividindo-a em camadas distintas, sendo avaliado seu desempenho nas profundidades de 0,08; 0,20 e 0,40 m através de perfis de temperaturas. A metodologia se mostrou eficaz e obteve alta correspondência estatística, com erros quadráticos médios percentuais inferiores a 1,9%, adquirindo poucas flutuações em picos máximos de temperaturas diárias para a camada mais superficial. O teste de Tukey não apresentou diferenças significativas entre os dados de campo e os valores simulados. A subdivisão do solo em camadas distintas mostrou-se necessária para se obter uma modelagem computacional eficiente e o método utilizado foi considerado adequado para uso na região estudo e para tipo do solo apresentado.

Palavras-chave: Difusividade térmica do solo, Algoritmo Condução-Convecção, Temperatura no solo, Modelagem computacional.

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A B S T R A C T
Soil temperature can interfere with various anthropogenic activities. The knowledge of this phenomenon is of great importance for several areas, such as the environmental sciences, agriculture, engineering and physical geography. Thermal diffusivity is one of the most relevant thermodynamic parameters of the soil, having great relevance in the description of this phenomenon. In São João, in the Agreste region of the State of Pernambuco, the objective of this study was to determine it through the conduction-convection algorithm, dividing it into distinct layers, and its performance at depths of 0.08; 0.20 and 0.40 m through temperature profiles. This methodology was effective and obtained a high statistical correspondence, with the average percentage of quadratic errors being lower than 1.9%, acquiring few fluctuations in maximum peaks of daily temperatures for the most superficial layer. The Tukey test did not present significant differences between the field data and the simulated values. The subdivision of
the soil into distinct layers was necessary to obtain an efficient computational modeling and the method used was considered suitable for use in the study region and for the soil type presented.

Key words: Thermal diffusivity soil, convection – conduction algorithm, soil temperature, computational modeling.

Introduction

Among the physical processes that occur in soil, thermodynamics is of essential relevance in several areas, such as Biology, Agriculture and Engineering. According to the study by Márquez et al. (2016), the geothermal interaction of this medium with the surface occurs in a substantial way from 6 to 15 m depth. Energy exchanges in structures, such as houses and buildings, were initially evaluated for thermal comfort in colder regions such as northern Europe, the USA and Canada. Subsequently, it proved suitable for refrigeration in low-latitude, increasing its profitability and study interest, since the heat switching is carried out in a way that thermal equilibrium enters the parts that constitute the system. Obtained from computational simulations, the heat interactions are the thermal balance of the buildings, affecting its internal temperature. The thermal performance of cooling and heating can be analyzed according to the sum of degrees-hours (Batista et al., 2011).

For Agreste Pernambucano, this type of research is interesting, since during the year, on average, the temperature of this region can vary from 14°C, during the winter, to 38°C, in the summer, as a result of its lower humidity and higher altitudes compared to other areas in the state (APAC, 2016).

According to authors such as Rao et al. (2005) and Oliveira et al. (2014) soil heat can be studied using several thermodynamic edaphic parameters, among them is thermal diffusivity, which expresses how heat diffuses through a material. For Silans et al. (2006), it is an important factor that indicates the capacity of transporting heat to its interior, or to its surface, depending on the direction of heat flow. It varies with time and space, depending among other factors, on the constitution, grain size, density and soil structure. These properties are variable with depth, and soil moisture, variable according to time, especially in the stratifications closest to the surface. Thus, it is of the utmost importance that these layers be studied separately from each other. Nowamooz et al. (2015) showed that the compounds of the medium directly influence the heat transfer, being essential, for a good modeling, to characterize these distinctions. To study other thermodynamic parameters associated with diffusivity, Kun and JieMin (2008) showed the direct dependence of the heat flux and the depth investigated for thermal energy of solar origin.

Soares et al. (2014) and Oliveira et al. (2015) used six methodologies to calculate thermal diffusivity, highlighting in both studies the conduction-convection methodology. These algorithms are very attractive, since only temperature measurements are required. However, they are limited to uniform soil layers. The apparent thermal diffusivity values can also be determined by the ratio of thermal conductivity to volumetric thermal capacity.

The use of methodologies that evaluate the thermal behavior of the soil is interesting because, if field trials were carried out directly, it would significantly increase the cost of the research, plus the lengthy time spent for execution and monitoring. Therefore, evaluating the performance of methods that evaluate and simulate this behavior is of extreme importance in the scientific environment, in areas such as environmental sciences, physical geography and agriculture.

The objective of this study was to evaluate the method of determining the thermal diffusivity of the conduction-
convection soil for single layers of a stratification in a region of the city of São João, Agreste Pernambucano, simulating daily temperature profiles for a week, to prove the effectiveness of the methodology.

**Materials and methods**

The data used in this study were obtained from an area located in the municipality of São João (Figure 1), 220 km from Recife, which is part of the Garanhuns microregion in the Southern Agreste of Pernambucano. The place has the following geographical coordinates: South, Latitude -08° 52' 32" and to the West, Longitude -36° 22' 00", with an altitude of 716 m. It presents the following climatic characteristics: annual average rainfall 579.1 mm, annual average temperature around 21ºC, rainy tropical type climate with dry summer (Beltrão et al., 2005).

The soil temperature was measured at depths of 0.02; 0.08; 0.10; 0.20; 0.30; 0.40; 0.50 m. In order to validate the obtained results, the curves of depths 0.08; 0.20 and 0.40 m were used.

![Figure 1. Location of the municipality of São João, in the Agreste Region of the State of Pernambuco.](image)

Measurements were performed using thermocouples (108-L, *Campbell Scientific, Inc.*, Logan, Utah, USA). These were collected at a frequency of one minute per minute and stored at a frequency of 30 minutes by the datalogger (CR 10x, *Campbell Scientific, Inc.*, Logan, Utah, USA) to minimize the error generated by natural factors that interfere with correct gauging values.

For this study, thermal diffusivity values vary only between the layers, being fixed for each stratification itself. For Oliveira et al., (2014) this consideration can...
be used to simplify the computational modeling, without significantly affecting the results generated at the end of the process.

The thermocouple calibration process consisted of measurements of the electromotive force generated by the different metals, for several known temperature values that have already been previously tabulated by the manufacturer.

The law of thermal conduction, also known as Fourier law, states that the flow of heat through a material is proportional to temperature. This can also be applied to the soil, considering it uniform in a limited proportion. Equation (1) defines the Fourier equation and can represent the diffusion of heat for this study.

\[
\frac{\partial T}{\partial t} = K \frac{\partial^2 T}{\partial z^2} \tag{1}
\]

Since \( K \) is the coefficient of thermal diffusivity, \( T \) is the temperature, \( t \) is the time and \( z \) the depth.

According to Nowamooz et al. (2015), the finite difference method is used to numerically solve the above equation, where the thermal diffusivity is considered constant over time, but variable across the depth layers. The stability of this method is guaranteed by Richtmyer and Morton, (1967):

\[
\frac{K \Delta t}{(\Delta z)^2} < \frac{1}{2} \tag{2}
\]

The method of conduction-convection (Gao et al., 2003) was used to determine the diffusivity during the soil stratification, and for obtaining coherent results (Oliveira et al., 2015). The temperature of the soil surface can be represented by a sine wave, described in:

\[
T(0, t) = \bar{T} + A \sin(\alpha t + \beta) \tag{3}
\]

In equation 3, \( T \) is the temperature, \( \bar{T} \) is the mean temperature, \( \alpha \) is the angular velocity of rotation of the earth, \( t \) is the time, \( \beta \) is the phase of the sinusoidal function and \( A \) is the oscillation amplitude.

According to Gao et al. (2003), the one-dimensional equation of conduction and thermal convection of the soil is solved numerically by applying the traditional harmonic model (HM) and the Laplace transform method. Thus, the method proposed in equation 4, serves the purpose of calculating the apparent thermal diffusivity, estimating the heat transport through conduction and convection, using the amplitude \( A \) referring to \( z \) (depth), \( \beta \) is the referring phase, shown below:

\[
K = \frac{(z_1 - z_2)^2 \alpha \ln\left(\frac{A_1}{A_2}\right)}{(\beta_1 - \beta_2) \left( \beta_1 - \beta_2 \right)^2 + \ln^2\left(\frac{A_1}{A_2}\right)} \tag{4}
\]

In order to evaluate the agreement between the calculated and observed values, different statistical criteria were evaluated: i) The Mean Squared Error (EQM), which indicates the degree of deviation between the experimental determinations and the values calculated by the corresponding theoretical model. It is expressed as a percentage, and tends to zero when the estimated and theoretical values tend to be the same. This test provides easy-to-understand information on model performance, as well as allowing a forward-looking comparison of the actual deviation between the calculated value and the measured value; ii) The deviation ratio (RD), which describes the ratio between the spread of the experimental determinations and the scattering of the values calculated by the corresponding theoretical model, tends to 1 (um) when the estimated values, and those of the theoretical model are consistent; iii) The modeling efficiency (EM) that indicates if the theoretical model gives a better estimate of the experimental determinations than the average value of these determinations, tends to 1 (um).

\[
EQM = \left[ \frac{1}{N} \sum_{i=0}^{N} (T_i - M)^2 \right] * 100 \frac{M}{M} \tag{5}
\]
According to Antonino et al. (2004), one should consider $T_i$ the values calculated by the model, $M_i$ the experimental values and $\bar{M}$ mean of the experimental values, and $N$ is the number of determinations.

The coefficient of determination, also called $R^2$, is a measure of adjustment of a generalized linear statistical model, such as Linear Regression, in relation to the observed values. The $R^2$ varies between 0 and 1, and indicate in percentage, how much the model can explain the observed values. The higher the $R^2$, the more explanatory the model is, the better it fits the sample, defined in equation 8 (Antonino et al., 2004).

$$R^2 = \frac{ \left( \sum_{i=1}^{n}(x_i - \bar{x})y_i \right)^2 } { \sum_{i=1}^{n}(x_i - \bar{x})^2 \sum_{i=1}^{n}(y_i - \bar{y})^2}$$  \hspace{1cm} (8)

The significance of the modeling obtained was performed through the Tukey test, between the simulated values and the field results found respectively for each depth presented in Table 2.

**Results and Discussion**

The graphs with the evolution of the temperature in the different depths can be seen in Figure 2. This is presented as characteristic for the studied region of the Agreste Pernambucano (Santos, 2011).

![Figure 2. Temperatures in the different depths studied for a period of one week.](image)

The temperatures have different amplitudes and phases along the depths of the soil. The most superficial (0.02 m and 0.10 m) presents a much higher amplitude compared to the others. The mean depths (0.20 m and 0.30 m) have similar amplitudes, numerically lower than the more superficial and higher than the deeper ones. It is...
perceptible that these are similar to those of 0.40 m and 0.50 m in comparison with the others. The apparent thermal diffusivity was calculated through these temperature data, considering a layer (two depths) for each parameter. Table 1 shows the correspondence between the layers and the depths that constitute it.

The values found for the thermal diffusivity corroborate with the graphs presented in Figure 2. The layers 2 and 5, composed by the depths 0.20 and 0.30 m; 0.40 and 0.50 m, respectively, presented the highest values for the thermal parameter. Thus, they transferred heat more quickly to the adjacent stratifications, where they are presented in Figure 2 in a more distinct way. The other diffusivities found were smaller, because the measurements of the temperatures found had less variation in relation to their adjacent layer, so the thermal transfer processes occurred more slowly.

In order to prove the efficiency of the method, the temperatures were measured at depths of 0.08 m; 0.20m and 0.40m, since it is possible to perform statistical analysis through equations 5, 6, 7 and 8, on the performance obtained by the convection-conduction algorithm. In Table 2, it can be observed that the results found have an average quadratic error lower than 1.87% and a modeling efficiency superior to 0.94; indicating that the methodology was highly satisfactory.

Table 1. Values obtained through the conduction-convection method of the mean diffusivity coefficient (K̅) for layers of the studied soil.

| Layer | Depth          | K̅ (10−6m²s⁻¹) |
|-------|----------------|---------------|
| 1     | 0.02m a 0.10m  | 2.98          |
| 2     | 0.10m a 0.20m  | 8.32          |
| 3     | 0.20m a 0.30m  | 7.97          |
| 4     | 0.30m a 0.40m  | 3.64          |
| 5     | 0.40m a 0.50m  | 21.2          |

From the ratio of deviations, the largest difference was obtained as a statistical parameter, since the value that comprises the most coherent is the closest to the unit. In Figure 3, scatter plots were elaborated, in order to analyze the correspondence between the measured and the estimated data. The simple linear regression formed by these elements presented the coefficient of determination very close to one, reaffirming the modeling efficiency of the methodology used. Neto et al. (2015) showed that the temperature varies considerably between the surface and about the first 0.70 m of depth, and also, over time. The thermal diffusivity was analyzed in two soil types, Yellow Latosol and Neolithic Regolithic, showing changes, corroborating with what was examined in this work. Danelichen and Biudes (2011) studied the variation of K over a year, considering a soil in Cuiabá, MT, as uniform. Their statistical analysis showed that for the most superficial layer there was a good representation, with coefficients of determination higher than 0.85. At the other depths, the results were lower, with R² being presented in a range of 0.56 to 0.63.

According to Table 2, it can be seen that when considering the uniqueness of each portion of the medium the results may be higher than 0.94. The mean squared error has low numerical significance in all studied depths, being closest to the surface responsible for the highest error value presented for this statistical parameter. The ratio of deviations is directly linked to Figure 3, where the dispersion plots are presented at the 3 depths in question. Similar to the results

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of the EQM, RD achieves better performance at the deepest layer, decreasing as the depth increases. Already in the modeling efficiency, it is indicated that the middle layer (20cm) obtained better performance, with little variation in relation to the others. In relation to the coefficient of determination, the values obtained indicate better results from the layer closest to the surface, decreasing according to the increment of soil depth.

All the statistical parameters presented have relevant functions in the statistical analysis of this study. Although they present better efficiency results ($R^2 \times$ EQM / RD), the differences found are, numerically, of low significance. The Tukey test was performed to evaluate the difference in significance between the results found, where, for $P < 0.05$, no relevant differences were found.

Table 2. Statistical parameters (EQM, RD, EM, $R^2$) obtained by comparing measured and estimated values of soil temperatures at depths of 0.08 m, 0.20 m and 0.40 m, characterizing the layers tested for the efficiency of the method used.

| Statistical parameter | 0.08m       | 0.20m       | 0.40m       |
|------------------------|-------------|-------------|-------------|
| EQM(%)                 | 1.868±1     | 0.85±1      | 0.54±1      |
| RD                     | 1.444±1     | 1.21±1      | 1.05±1      |
| EM                     | 0.906±1     | 0.95±1      | 0.89±1      |
| $R^2$                  | 0.99±1      | 0.96±1      | 0.94±1      |

In another study, Danelichen et al. (2013) did a similar analysis for a soil in a Brazilian Pantanal, in a smaller period, considering the variation of the soil thermodynamics among its surface layers, and considering the absolute mean error and the quadratic error mean of this study, was presented as almost null, lower than 2%, reaffirming the importance of disintegrating the medium in different strata.

With the contrast of results obtained in Table 2, it can be seen that, the mean square errors resulted in percentages below 1.87, although it was almost full at a depth of 0.40 m. At a depth of 0.08 m, the greatest deviation ratio was recorded, due to the differences between the maximum peaks of temperature. In the modeling efficiency, the lowest efficiency was identified at 0.40 m; since despite the low fluctuations, due to their low amplitude, these differences directly influence this statistical parameter. The indicators showed that the simulations were very close to the one analyzed in the field. In comparison with the study carried out by Oliveira et al. (2015), it is possible to observe the evolution of the simulations, when the thermal diffusivity of each layer is taken into account, and the method used has performed better than the others.

These results were superior compared to that obtained with other methods, such as the amplitude and logarithm algorithms used by Diniz et al. (2013), whose results ranged from 0.74 to 0.92, in the $R^2$ coefficient. Soares et al. (2014) compared six (6) methodologies to determine the diffusivity, where the conduction-convection method presented good results. In a study by Oliveira et al. (2015), this algorithm more accurately represented the temperature profile before the others, for soil types similar to the one studied in this study.

As can be seen in Figure 4, the method corresponded to the observed temperatures, in their mean values and lower peaks. The highest differences observed were in the peaks of higher temperatures, especially in the closest layer of the surface,
where those peaks that were estimated had higher maximum values, reaching variations up to 1.2°C. The largest discrepancies can be identified in the first 4 days of modeling. The fifth day was simulated very closely while the sixth and seventh days showed differences below 0.6°C.

In a global analysis of the results presented in Figure 4, the behavior composed of the simulated values followed the ones found in the field, both in frequency and in amplitude, presenting minimum variations for this one, easily identified in part a) of Figure 4 and on the fifth day of part (b). This analysis corroborates with the graphs of Figure 3 a), b) and c) and with Table 2, which details the statistical results for each layer presented.

In the most superficial layer presented below, the variations of the vertical axis (temperature amplitudes), are numerically superior to the other two, and can directly influence the temperatures corresponding to the simulation performed. The difference with the deepest soil stratum is notorious, since as the temperature variation is lower than the surface, the computational results are more stable, reducing the simulation error.
Figure 3. Observed versus estimated temperature data at depth of a) 0.08 m; b) 0.20 m; and c) 0.40 m.

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It is possible to notice that the modeling followed the data measured in the field, since the greatest distinctions can be seen in Figure 4-a, where the maximum peaks simulated are numerically higher than those observed in the field. It can also be seen in Figure 4-b, on the 5th day, that the maximum simulated peak was numerically lower than the real one. Figure 4-c shows the minimum fluctuations, since the amplitude of this depth is low, minimizing the errors generated due to the low thermal diffusivity ($3.64 \times 10^{-6} \text{m}^2\text{s}^{-1}$). The largest discrepancies were observed around 1°C. Except for these cases, the convection-conduction algorithm proved to be quite satisfactory.

It can be observed in the linear regression (Figure 3-a), that in spite of the disagreements pointed out previously, the simulation, in the majority, agreed with the other values obtained and was characterized by the higher coefficient of determination (0.99).

Comparing the scatter plots shown in Figure 3, it can be seen that in b) and c) the results are close to each other, considering the amplitude of each temperature profile shown in Figure 4. In comparative analysis to Figure 3-a, it is assumed that the simulated points are more linear, the closer the profile is to the boundary condition. That is, the boundary conditions considered for each layer are derived from the previous simulation. A of 0.08 m was based on the values measured in loco, while 0.20 and 0.40 m were derived from the results already simulated computationally, accumulating the errors associated with this for each progress of depth.

The dispersion graphs corroborate with the results presented in Table 2, since the coefficient of determination is obtained through the linear regressions presented in Figure 3.

The uniformity of the points in relation to the linear function exposed in the scatter plot indicates the most accurate representation of the model in relation to the phenomenon occurring in reality (in the field), in which Table 2 has a great correlation with Figure 3, previously presented.

It can be identified in Figure 3, that the proximity of the simulated points and the straight bisector, which indicates the statistical distribution reproduces the studied phenomenon more efficiently.
a) Medido  —  Estimado

Temperature (°C) vs. Time (h)

b) Medido  —  Estimado

Temperature (°C) vs. Time (h)
Figure 4. Measurements of estimated and observed temperatures throughout the study period, at depths of a) 0.08 m; b) 0.20 m; and c) 0.40 m.

Conclusions

As predicted, the Conduction-Convection algorithm obtained efficient results to estimate the apparent thermal diffusivity at the place where the study was performed. The soil in question was considered to be composed of layers, subdividing it for better representation, so it was necessary to represent each layer with different thermal conductivities. The temperature simulation charts, along the period proposed in this study, followed the values measured in the field, with few fluctuations being identified, although they overestimated the maximum values of the daily measurements in the most superficial layer studied (0.08 m).

Nonetheless, statistical analyses and scatter plots have shown that the method was well developed. Tukey test showed that there are no significant differences between the data collected in the field and those simulated in the computer. It is worth emphasizing the importance of distinguishing the diffusivity, in the same soil, at different depths, showing that this is quite unique for each layer along this medium. The statistical parameters confirm the high efficiency of the method, with mean square error of less than 1.9%, modeling efficiency of more than 0.89 and coefficients of determination exceeding 0.94. In addition, it can be concluded that it is necessary to divide the soil into layers, with specific values of thermal diffusivity, to have a good thermodynamic study, and it is not very efficient to consider it uniform. This separation guarantees a closer study of the real phenomena, considering the different types, properties and physical indexes for each soil stratification.
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References

Antonino, A. C. D., Soares, W. A., Silva, E. B., Lima, J. R. S., Netto, A. M., Lima, C. A. B. O., 2004. Utilização do método inverso para a caracterização hidrodinâmica de um neossolo flúvico. Revista Brasileira de Recursos Hídricos 9, 81-87.

APAC. Agência Pernambucana de Águas e Clima, 2016. Disponível: http://www.apac.pe.gov.br/meteorologia/. Acesso: 13 mar. 2016.

Batista, J. O., Lamberts, R., Güths, S., 2011. Influência dos algoritmos de condução e convecção sobre os resultados de simulações do comportamento térmico de edificações. Ambiente Construído 11, 79-97.

Beltrão, B. A., Mascarenhas, J. C., Miranda, J. L. F., Junior, L. C. S., Galvão, M. J. T. G., Pereira, S. N., 2005. Projeto cadastro de fontes de abastecimento por água subterrânea: Estado de Pernambuco. Diagnóstico do Município de São João. Disponível:
http://rigeo.cprm.gov.br/xmlui/bitstream/handle/doc/16626/Rel_S%C3%A3o%20J.pdf?sequence=1. Acesso: 10 mar. 2016.

Danelichen, V. H. M., Biudes, M. S., 2011. Avaliação da difusividade térmica de um solo no norte do pantanal; Ciência e Natura 33, 227-240.

Danelichen, V. H. M., Biudes, M. S., Souza, M. C., Machado, N. G., Curado, L. F. A., Nogueira, J. S., 2013. Soil Thermal Diffusivity of a Gleyic Solonetz Soil Estimated by Different Methods in the Brazilian Pantanal. Open Journal of Soil Science 3, 15-22.

Diniz, J. M. T., Aranha, T. R. B. T., Souza, E. P., Wanderley, J. A. C., Souza, E. P., Maracajá, P. B., 2013. Avaliação da difusividade térmica do solo de Campina Grande-PB – Brasil. Agropecuária Científica no Semi-Árido 9, 55-60.

Gao, Z., Fan, X., Bian, L., 2003. An analytical solution to one-dimensional thermal conductionconvective in soil, Soil Science 168, 99–107.

Kun, Y., Jiemin, W., 2008. A temperature prediction-correction method for estimating surface soil heat flux from soil temperature and moisture data; Sci China Ser D-Earth Sciences 51, 721-729.

Márquez, J. M. A., Bohórquez, M. Á. M., Melgar, S. G., 2016. Ground Thermal Diffusivity Calculation by Direct Soil Temperature Measurement. Application to very Low Enthalpy Geothermal Energy Systems. Sensors 16, 1-13.

Neto, J. A. M., Antonino, A. C. D., Lima, J. R. S., Souza, E. S., Soares, W. A., Alves, E. M., Almeida, C. A. B., Neto, J. A. S., 2015. Caracterização Térmica de Solos no Agreste Meridional do Estado de Pernambuco, Brasil. Revista Brasileira de Geografia Física 8, 167-178.

Nowamooz, H., Nikoosokhan, S., Lin, J., Chazallon, C., 2015. Finite differences modelling of heat distribution in multilayer soils with time-spatial hydrothermal properties. Renewable Energy 76, 7-15.

Oliveira, D. B., Albuquerque Neto, N.A., Soares, W. A., 2014. Análise de metodologias na determinação da difusividade térmica do solo. Diálogos:

Oliveira, D. B. C., Soares, W. A.
Oliveira, D. B., Albuquerque Neto, N.A., Soares, W. A., 2015. Estimates of Thermal Diffusivity and Heat Flow of a Soil in Agreste Pernambucano. Revista Brasileira de Geografia Física 8, 1053-1067.

Rao, T. V. R., Silva, B. B., Moreira, A. A., 2005. Características térmicas do solo em Salvador, BA. Revista Brasileira de Engenharia Agrícola e Ambiental 9, 554-559.

Richtmyer, R. D., Morton, K. W., 1967. Difference methods for initial-value problems. 2 ed. Interscience Publishers, New York.

Santos, J. C. B., 2011. Caracterização de Neossolos Regolíticos da Região Semi-Árida do Estado de Pernambuco. Tese (mestrado). Recife, UFRPE.

Silans, A. M. B. P., Silva, F. M., Werlang, L. M., Barbosa, F. A. R., 2006. Determinação in loco da difusividade térmica num solo da região de Caatinga - PB. Revista Brasileira de Ciência do Solo 30, 41-48.

Soares, W. A., Antonino, A. C. D., Lima, J. R. S., Lira, C. A. B. O., 2014. Comparação de Seis Métodos para a Determinação da Difusividade Térmica de um Latossolo Amarelo. Revista Brasileira de Geografia Física 7, 146-154.