Gravity wraps Higgs boson

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It is shown that, under a conformal transformation with reference to the Higgs field, the Higgs boson can be completely decoupled from electroweak interactions with no apparent change in known properties of leptons, quarks and vector bosons. Higgs boson becomes part of a scalar-tensor gravity which can be relevant for Dark Energy. It interacts with matter sector via higher-dimensional terms (e.g. neutrino Majorana mass), and via the fields (of new physics) whose masses are not generated by the Higgs mechanism. Dark Matter and two-Higgs-doublet model are the simplest examples.

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The electroweak theory is a isospin SU(2)\textsubscript{L} times hypercharge U(1)\textsubscript{Y} gauge theory \[ SU(2)\textsubscript{L} \times U(1)\textsubscript{Y} \] spontaneously broken down to electric charge U(1)\textsubscript{e} via Higgs mechanism \[ \sum_{i}Y_{i}H_{i} \]. Higgs field, the order parameter of the electroweak transition, transforms as \[ \begin{pmatrix} 2 \ 0 \end{pmatrix} \] under \[ SU(2)\textsubscript{L}, U(1)\textsubscript{Y} \],

\[
H = \frac{\varphi}{\sqrt{2}} e^{i\frac{\varphi}{\sqrt{2}} \overline{T} \left( \begin{array}{c} 0 \\ v \end{array} \right)}
\]

where \( \overline{T} \) are the three \( SU(2)\textsubscript{L} \) generators weighted by the Goldstone boson fields \( \varphi \), and

\[
\varphi = 1 + \frac{h}{v}
\]

is the modulus field encoding the Higgs boson \( h \). The constant \( v \) is the vacuum expectation value (VEV) of the neutral component of the Higgs doublet.

Gauge invariance ensures that, it is always possible to perform an \( SU(2)\textsubscript{L} \) rotation to weed out the Goldstone bosons in \([4,5]\) so that Higgs field gets mapped into the representation in the unitary gauge

\[
H_{U} = \frac{\varphi}{\sqrt{2}} \left( \begin{array}{c} 0 \\ v \end{array} \right)
\]

everywhere in the electroweak Lagrangian \([6]\). The Higgs boson \( h \) is the remnant, and there can exist no gauge transformation that can erase it: It is in the spectrum as a real scalar particle, and it couples to all the particles whose masses are generated by the Higgs mechanism.

Inspired by the role of the gauge transformations in removing the Goldstone bosons, one wonders whether it is possible to remove the Higgs boson itself from electroweak interactions partially, if not entirely. This will be shown to be possible via the scalings of the fields by appropriate powers of the modulus \( \varphi \). In particular, as was first proposed in \([7]\), it will be proven that the Higgs boson can be wholly transferred from the matter sector to the gravity sector where it possesses only gravitational interactions and escapes detection in experiment. Though Higgs boson is completely decoupled from the electroweak sector, the leptons, quarks and vector bosons possess their already known properties: The observed masses, couplings, and mixings. However, there is no Higgs boson around to interact with. The Higgs boson gives rise to a scalar-tensor theory of gravity where strength of the gravitational interactions stays put at Newton’s constant (provided that Higgs boson stays perturbative \([3]\)). For realizing these novel features, it is beneficial to introduce

\[
H_{U} = \varphi^{a_{0}} H
\]

as a transformation rule which peals off \( \varphi \) from \( H_{U} \) to reduce it to

\[
H = \frac{1}{\sqrt{2}} \left( \begin{array}{c} 0 \\ v \end{array} \right)
\]

with \( a_{0} = 1 \).

The relation \([4]\) hints at a similar scaling transformation for fermions

\[
F = \varphi^{a_{1}/2} F
\]

where \( F \) denotes collectively the leptons \( \nu_{\alpha} \), quarks \( \sum_{i}Q_{i} \), and vector bosons \( W_{\alpha} \).

The \( U(1)\textsubscript{Y} \) gauge field \( Y_{\alpha} \) of \( SU(2)\textsubscript{L} \) gauge field \( \overline{W}_{\alpha} \) must be invariant

\[
Y_{\alpha} = Y_{\alpha}, \quad \overline{W}_{\alpha} = \overline{W}_{\alpha}
\]

as they pertain to gauge transformations.

The transformations \([4,5,6]\) are realized differently by different sectors of the electroweak theory \([7]\). For instance, the kinetic terms of the fermions

\[
- \frac{1}{2} \sum_{F=L,Q,E,U,D} \overline{F} \left( \begin{array}{c} \overline{n} \\ \overline{p} \end{array} \right) F
\]

scale as \( \varphi^{2a_{1}/2} \). The Yukawa interactions

\[
- \left[ h_{E} L H E + h_{D} Q H D + h_{U} U H^{+} U + H. C. \right]
\]

behave \( \varphi^{2a_{1}+a_{0}} \). The kinetic terms of the gauge fields

\[
- \frac{1}{4} g^{\alpha\beta} g^{\rho\sigma} Y_{\mu\nu} Y_{\alpha\beta} - \frac{1}{4} \phi^{\nu} g^{\rho\sigma} \overline{W}_{\mu\nu} \overline{W}_{\alpha\beta}
\]

are invariant under the conformal transformation.
are obviously invariant due to (7). In contrast, the Higgs sector

\[ -g^{\alpha\beta} (\mathcal{D}_\alpha H) \mathcal{D}_\beta H - m_H^2 H^\dagger H - \lambda (H^\dagger H)^2 \] (11)

behaves rather heterogeneously, since each term involves different powers of \( \varphi \).

The scaling characteristics of (8), (9), (10) and (11) do not accommodate an invariance principle. Indeed, imposing invariance directly gives \( a_{1/2} = 0 \) and \( a_0 = 0 \), which are trivial. Therefore, for achieving invariance, even at a partial level, an extra agent is needed; the metric tensor \( g_{\alpha\beta} \). Namely, \( g_{\alpha\beta} \) must be elevated to the status of a dynamical variable. It is to scale as \( \mathcal{O}(\sqrt{\varphi}) \)

\[ g_{\alpha\beta} = \varphi^{\alpha\beta} \mathcal{G}_{\alpha\beta} \] (12)

concurrently with (1), (3) and (7). Having metric introduced in the game, the total action becomes

\[ S_{\text{tot}} = \int d^4x \sqrt{-g} \left[ \mathcal{L}_{\text{GR}} - \lambda_R \mathcal{R} H^\dagger H + \mathcal{L}_{\text{EW}} \right] \] (13)

where, in the minimal case (not including higher-curvature terms or fields beside the metric tensor), its gravitational part

\[ \mathcal{L}_{\text{GR}} = \frac{1}{2} M_G^2 (\mathcal{R} - \Lambda) \] (14)

is the Einstein-Hilbert term with cosmological constant \( \Lambda \) and bare gravitational scale \( M_G \) such that

\[ M^2_{PL} = M_G^2 - \frac{1}{2} \lambda_R v^2 \] (15)

is the Planck scale induced by the non-minimal coupling in (13).

Part of the action (13) explicitly involves the Higgs boson \( \varphi \)

\[ \mathcal{L}(h) = -\frac{1}{2} v^2 \left[ g^{\alpha\beta} \partial_\alpha \varphi \partial_\beta \varphi + m^2_H \varphi^2 + \frac{\lambda}{2} v^2 \varphi^2 \right] \]

\[ - \frac{1}{2} \varphi^2 g^{\alpha\beta} \left[ M^2_W W^\dagger_a W^\alpha_\beta + M^2_Z Z^\alpha_\beta \right] \]

\[ + \varphi \sum_{F = \ell, u, d} m_F \mathcal{F} + \frac{1}{2} \lambda_R \varphi^2 \mathcal{R} \] (16)

while the rest is entirely free from the Higgs boson. Expanding this in powers of \( h \), at the lowest order, the vacuum energy becomes

\[ V(v) = M_G^2 \Lambda + \frac{1}{2} m^2_H v^2 + \frac{\lambda}{4} v^4 \] (17)

if the gravitational sector deposits only the cosmological term \( M_G^2 \Lambda \). The vacuum energy density \( V(v) \) is minimized for

\[ v^2 = -\frac{m^2_H}{\lambda} \] (18)

which makes sense if Higgs is tachyonic \( i. e. \) \( m^2_H < 0 \). En passant, one notices that \( V(v) \) can be tuned to vanish by taking

\[ \Lambda = \frac{\lambda}{4} \left( \frac{v^2}{M_G} \right)^2 \] (19)

with \( v \) is given by (13). With this tuning, because of vanishing \( V(v) \), the background metric \( g^{(0)}_{\alpha\beta} \) (which must nullify the Ricci scalar because of vanishing \( V(v) \)) can be identified with the flat Minkowski metric \( \eta_{\alpha\beta} \).

After transforming the action (13) according to the rules in equations (4), (6) and (7), one finds that part of the electroweak Lagrangian \( \mathcal{L}_{\text{EW}} \) becomes independent of \( \varphi \) while the gravity sector \( \mathcal{L}_{\text{GR}} \) metamorphoses into a scalar-tensor theory by swallowing \( \varphi \) [3, 8].

This happens for the specific values of the exponents

\[ a_{1/2} = \frac{3}{2}, \quad a_2 = -2 \] (20)

which are, together with \( a_0 = 1 \), nothing but the conformal weights of the fields arising in scale transformations [3, 7]. With these weights, the image of (13) becomes

\[ S_{\text{tot}} = \int d^4x \sqrt{-G} \left[ \mathcal{L}_{\text{EW}}(h = 0) + \mathcal{L}_{\text{ST}}(G, h) \right] \] (21)

wherein the first term is nothing but the usual electroweak Lagrangian \( \mathcal{L}_{\text{EW}} \) with Higgs boson \( h \) set to zero everywhere, and the second term refers to a scalar-tensor theory of gravity which involves only gravity and the Higgs boson:

\[ \mathcal{L}_{\text{ST}} = \frac{1}{2} \left[ 6 M_G^2 \varphi^4 - (1 - 6 \lambda_R) v^2 \varphi^2 - \frac{1}{4} \lambda_R v^4 \right] \mathcal{G}^{\alpha\beta} \partial_\alpha \varphi \partial_\beta \varphi \]

\[ - \frac{1}{4} \lambda_R v^4 - \frac{1}{2} m^2_H v^2 \varphi^2 - M^2_G \Lambda \varphi^4 \]

\[ + \frac{1}{2} \left[ M^2_G \varphi^2 - \lambda_R v^2 \right] \mathcal{R} \] (22)

where \( \mathcal{R} (G) \) is the image of \( \mathcal{R} (g) \) under (12).

Comparison of the two Lagrangians, (10) and (22), manifestly shows that the Higgs boson \( h \) is completely transferred from the matter sector to the gravity sector. The Higgs boson interactions with gauge bosons and fermions in (10) convert into pure mass terms.

For revealing the physics implications of the scalar-tensor gravity (22), it is convenient to expand (21) about the vacuum configuration \( \varphi = 1 \) to find

\[ \mathcal{L}_{\text{ST}} = -\frac{1}{2} \left[ \delta^2 - \lambda_1 P(h) - \lambda_2 P(h^2) \right] \mathcal{G}^{\alpha\beta} (\partial_\alpha h) (\partial_\beta h) \]

\[ - \frac{1}{2} \left[ m^2_H v^2 + 4 M^2_G \Lambda \right] P(h) - M^2_G \Delta P(h^2) \]

\[ + \frac{1}{2} \left[ M^2_{PL} + M^2_G P(h) \right] \mathcal{R} - V(v) \] (23)

where \( \lambda_1 = \lambda_2 - \delta^2 = 1 - 6 \lambda_R - 2 \delta^2 \) with

\[ \delta^2 = 1 - 6 \lambda_R - \frac{6 M^2_G}{v^2} \] (24)
which must be positive and nonvanishing for Higgs boson not to be a ghost. The function
\[ P(h) = \sum_{n=1}^\infty (-1)^n (n+1) \left( \frac{h}{v} \right)^n \]  
results from the expansion of \( \varphi^{-2} \). The vacuum energy is \( V(\nu) \) as in (17), and it vanishes upon \( G_{\alpha\beta}^{(0)} \) and \( \eta_{\alpha\beta} \) to enable identification of \( G_{\alpha\beta}^{(0)} \) with \( \eta_{\alpha\beta} \). In fact, \( \eta_{\alpha\beta} \) and \( G_{\alpha\beta}^{(0)} \) are expected to be identical from (12). Unlike its finite polynomial interactions in (10), the Higgs boson develops new infinite-order interactions in (23) yet its mass-squared, \( m_h^2 = -2m_H^2 \), stays put at its value in the electroweak theory. These features ensure that the vacuum structure of the theory does not change in passing from (13) to (21).

Having structured and audited the mechanism, it could be enlightening to highlight and discuss its certain salient features. This is done below.

1. Frames for Higgs Boson. Collider experiments of decades have determined almost all of the parameters of the electroweak theory. The only yet-to-be-observed piece is the Higgs boson \( h \). The analyses of the Tevatron and LHC data are continuing, and it is likely that Higgs boson will be found to have a mass within one of those narrow intervals indicated by the most recent searches (10). It is, however, also likely that the Higgs boson will not be discovered at all.

The main outcome of the mechanism is that, particle colliders may not discover Higgs boson not because the electroweak theory is not working but because gravity secludes the Higgs boson from leptons, quarks and vector bosons. Indeed, gravity is there to couple everything, and conformal transformations (4), (6) and (12) switch the interaction scheme invariably from (13) (to be called Standard frame) to (21) (to be called Gravic frame to mean the usual Einstein frame). These two frames, as was first noted in (6), give strikingly opposite predictions for Higgs boson. In particular, colliders cannot access the Higgs boson in the Gravic frame due to its secluded nature not due to weakening of its signal by light singlet scalars (11).

2. Renormalizability and Unitarity. Electroweak theory is renormalizable in Standard frame in the absence of gravity. In fact, it is gravity which renders theory nonrenormalizable in the Standard frame, it is gravity which facilitates the Gravic frame, and hence, it must wholly be gravity which causes non-renormalizability in the Gravic frame. Scattering amplitudes of longitudinal weak bosons grow as \( g^2 E^2 / M_W^2 \) with the center-of-mass energy \( E \). Thus, unitarity is violated at the scale \( E_{\text{crit}} \approx 1.2 \text{ TeV} \). In Standard frame, Higgs boson restores unitarity if \( m_h < 1 \text{ TeV} \). In Gravic frame, unitarity is lost at \( E_{\text{crit}} \) yet Higgs boson couples to gravity and fields whose masses are not generated by the Higgs mechanism (supersymmetric partners, Kaluza-Klein levels or technicolor fields which can exist in the TeV domain).

3. Higher-Dimensional Operators. These are best exemplified by massive neutrinos. A Dirac neutrino acquires mass via the Yukawa interaction
\[ \mathcal{L}_{\text{EW}} \ni -h_\nu^D \bar{\nu} H \nu + \text{H. C.} \]  

which, under the scalings (4), (6) and (12) from the Standard frame (13), reduces to a pure neutrino mass term
\[ h_\nu^D \frac{v^2}{\sqrt{2}} \nu R + \text{H. C.} \]  

in the Gravic frame of (21). Therefore, Dirac neutrinos, like all charged leptons and quarks, exhibit no interactions with the Higgs boson in the Gravic frame.

A Majorana neutrino, on the other hand, acquires mass from the seesaw term
\[ \mathcal{L}_{\text{EW}} \ni -\frac{1}{M_R} h_\nu^M \left( \bar{L} H^c \right) \left( H^T L^c \right) + \text{H. C.} \]  

which, under the scalings (4), (6) and (12) from the Standard frame (13), becomes
\[ h_\nu^M \frac{v^2}{2M_R} \nu_L \nu_R^c + \text{H. C.} \]  

in the Gravic frame of (21). Therefore, a Majorana neutrino interacts with the Higgs boson in both Standard and Gravic frames. The two neutrino types are strikingly different. This is because Majorana neutrinos get their masses from a dimension-5 operator.

Higgs boson contributions to higher-dimensional operators cannot be pared off due to the presence of a mass scale. Therefore, Higgs boson can interact with all the matter species even in the Gravic frame in case Standard frame involves appropriate higher-dimensional operators. Higher-dimensional operators are generated also by quantum gravitational effects. Indeed, small perturbations about the vacuum configuration (about \( \eta_{\alpha\beta} \) or \( G_{\alpha\beta}^{(0)} \) in the two frames) form gravitational waves, whose quantization, if can ever be done, creates loops of gravitons. Thus, in the Gravic frame, the Higgs boson and matter sector, which communicate with each other through the metric
teractions with no changes in known properties of quarks, boson can be completely decoupled from electroweak
In the Gravic frame, excepting higher-dimensional operators, the Higgs boson interacts only with gravity. This immu-
ity of the Higgs boson to all forces but gravity is precisely what is required of the models of Dark Energy. The reason is that, Dark Energy, an example of which is the vacuum energy, participates only in gravitational interactions. It is thus highly likely that Higgs boson in Gravic frame pertains to Dark Energy, and it may cause an accelerated expansion for the Universe. This is expected on the basis of its Lagrangian which generalizes the non-minimally coupled scalar field theories which are known to yield accelerated expansion. Besides this, the non-standard Majorana neutrino coupling to Higgs boson in (29) can model the suggested role of neutrinos in forming Dark Energy.

The Higgs boson in Gravic frame, as it stands in (22) or (23), interacts only with gravity. It is thus immune to various bounds from solar system and other astrophysical structures [19]. Even if it interacts, via higher-dimensional operators, the Gravic Lagrangian (22) can be put in the Brans-Dicke form with which is too massive (around neutrino mass) to have observable effects on the solar system [20]. This can also be seen by reverting to the Standard frame.

One also notes that, instead of the minimal gravitational sector in [14], it is possible to consider a \( F(R) \) gravity or a dilatonic gravity or a more general structure. The essence of the mechanism does not change.

The present work has thus established that, the Higgs boson can be completely decoupled from electroweak inter-

leptons and vector bosons. Higgs boson becomes part of a scalar-tensor gravity which can be relevant for Dark Energy. In this frame, Higgs boson does not couple to fields whose masses are generated by the Higgs mecha-
nism. It couples to matter sector via higher-dimensional terms or via the fields of new physics (related to Dark Matter, multi-Higgs-doublet models, supersymmetry, extra dimensions or technicolor).

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