Nuclear Stopping and Sideward Flow Correlation from 0.35A to 200A GeV

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In the recent years, the main aim of ultra-relativistic high energy heavy ion collisions (HICs) performed at SPS/CERN (\(\sqrt{s_{NN}} \sim 10.4\) GeV) and RHIC/BNL (\(\sqrt{s_{NN}} \sim 200.4\) GeV) is to search for new form of matter with partonic degrees of freedom, the so-called Quark-gluon plasma (QGP). Although great efforts have been made, no dramatic changes of experimental observables, such as jet-quenching, elliptic flow and strangeness enhancement, have been observed yet and it is hard to make a solid conclusion for the happening of the QGP phase transition \(\mathbb{E}\). Recently, the energy scan program is proposed for RHIC/BNL to perform HICs experiments with lower c.m. energy to search for the critical point \(\mathbb{[E]}\), which is as an endpoint of the first order phase transition line on the QCD phase diagram. If the critical point exists, it should appear on the QGP transition boundary at higher baryon chemical potential and lower colliding energy \(\mathbb{[E]}\). To extract the QGP phase transition signal, a large amount of possible experimental probes, such as particle ratio and collective flow \(\mathbb{etc}\) have been proposed. The time evolution of temperature and baryon chemical potential of the different colliding nuclei with various colliding energy would be mapping much broader \(T - \mu_B\) region on the QCD phase diagram than a single nuclei. However, it is complicated to uniformly and systematically obtain unambiguous experimental signal for QGP phase transition and also mark the location of the critical point on the QCD phase diagram, one of the possible choice is to keep insight in universal correlation pattern of two experimental observables for various colliding systems (system size and beam energy). Thus, the complicity of various colliding systems dependence of searching the phase transition signals can be reduced.

In this work, the correlation between the nuclear stopping and scale invariant nucleon sideward flow within the framework of UrQMD model from SIS/GSI to SPS/CERN energies for various scale impact parameters \(0 < b_0 = b/b_{max} < 1\) have been found, not just for global fixed impact parameters as in Ref.\([11]\). The scale invariant nucleon sideward flow is defined as \(\vec{F}(b_0) = \partial(<\frac{p_{cm}^x}{A}>/\partial(y_{cm}/y_{cm}^{proj})|_{[-1,1]}\), first proposed in Ref. \([14]\), the linear fitting slope of the normalized rapidity dependence of the normalized average in reaction plane transverse momentum with fitting range of \(-1 < y_{cm}/y_{cm}^{proj} < 1\), where \(<\frac{p_{cm}^x}{A}>\) is the average transverse momentum projected in the reaction plane per nucleon and \(p_{cm}^{proj}\) is projectile momentum in the center of mass system (c.m.s.), \(y_{cm}\) and \(y_{cm}^{proj}\) are the nucleon rapidity and projectile rapidity in c.m.s., respectively. The nuclear stopping ratio \(R\) as a measurement of degree of stopping of colliding nuclei at scale impact parameter \(b_0\), suggested in Ref. \([12, 13]\), is expressed as \(R(b_0) = \frac{\sum_i |p_{ti}|/\sum_i |p_{zi}|, where p_{ti} and p_{zi} are transverse and longitudinal momentum of the ith outgoing particle in the c.m.s., respectively. A colliding system dependence variable \(\rho_{mb}\) is also introduced to be a normalization factor for later calculations. It is defined as \(\rho_{mb}(A,E_{lab}) = MB(0) \times u_{cm}^{proj}/A^{1/3}\), where the \(MB(0)\) stands for the meson to baryon ratio for central collision \((b_0 = 0)\), \(u_{cm}^{proj} = \rho_{cm,jcm}^{proj}\) is the spatial component of four-velocity of the projectile in the c.m.s and \(A\) is the mass number of a nuclei in the symmetric colliding system. The overlapping volume of two collide nuclei and the nuclear passing time for central collisions respectively satisfy \(V \propto A\), and \(t_{pass} = r/u_{cm}^{proj} \propto A^{1/3}/u_{cm}^{proj}\), where \(r\) is the radius of nuclei. Thus, we have \(\rho_{mb} \propto MB(0)/(V \times t_{pass})\), standing for the meson to baryon ratio per unit volume, per passing time in the central collisions, which is used to characterize the strength of particle production at early stage \([15]\).

The UrQMD model \([16]\) used here is a type of numerical transport models, which is based on the quark, di-quark, string and hadronic degrees of freedom. It includes 50 different baryon species(nucleon, hyperon and their resonances up to 2.11 GeV) and 25 different meson species. Two types of equation of state, the hard

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EoS with incompressibility $K = 380$ MeV (Only for beam energy up to 4 A GeV) and cascade are contained in the UrQMD model. The model has successfully been applied to reproduce the experimental results from SIS/GSI to SPS/CERN energies [17].

A group of symmetric colliding nuclei with five pairs: $^{197}$Au+$^{197}$Au, $^{129}$Xe+$^{129}$Xe, $^{96}$Ru+$^{96}$Ru, $^{58}$Ni+$^{58}$Ni, $^{40}$Ca+$^{40}$Ca are combined with 30 and 12 incident kinetic energies, respectively. The first combination including total $150 = 5 \times 30$ colliding systems with 30 beam energies per nucleon from 0.35 to 200 GeV (0.35, 0.5, 0.66, 0.83, 1.0, 1.5, 2.0, 3.0, 4.0, 5.3, 6.6, 8.0, 10.0, 10.93, 11.9, 12.9, 13.93, 15, 16.9, 17.92, 18.95, 20, 24.22, 36.0, 55.0, 76.92, 102.33, 131.33, 163.36, 200.0) for each pair of the colliding nuclei and the second one including total $60 = 5 \times 12$ colliding systems with 12 beam energies per nucleon from 0.35 to 3.9 GeV (0.35, 0.5, 0.66, 0.83, 1.0, 1.5, 2.0, 2.35, 2.7, 3.1, 3.5, 3.9) are researched with cascade and hard EoS of UrQMD, respectively.

Fig. 1 shows the beam energy and system size dependence of central ($b_0 = 0$) nuclear stopping ratio $R(0)$ and the predefined variable $\rho_{mb}$ as well as the semi-central ($0.3 < b_0 < 0.4$) scale invariant nuclear sideward flow $\tilde{F}(b_0)$ with the cascade EoS. In the left panels of Fig. 1, the $R(0)$ and $\tilde{F}(b_0)$ both decrease monotonously with the beam energy per nucleon from 0.35 to 200 GeV for three pairs of symmetric colliding nuclei: Au+Au, Ru+Ru and Ca+Ca, and larger nuclear stopping ratio is observed for heavier colliding nuclei than the lighter one for a fixed beam energy. More detailed information on the system size dependence of $R(0)$ and $\tilde{F}(b_0)$ are illustrated in right panels of Fig. 1. Both $R(0)$ and $\tilde{F}(b_0)$ increase monotonously with mass number $A$, which is proportional to the size of the colliding system. The defined variable $\rho_{mb}$, increasing with beam energy and decreasing with system size, is also shown in lower panel of the Fig. 1.

With cascade EoS in UrQMD, the observables: central ($b_0 = 0$) nuclear stopping ratio $R(0)$, $\rho_{mb}$ and scale invariant nuclear sideward flow $\tilde{F}(b_0)$, with non-zero $b_0$ satisfying $0 < b_0 < 0.8$ and with an interval of 0.1, are calculated for the mentioned 150 colliding systems with beam energy per nucleon from 0.35 to 200 GeV. After the logarithmic operations are performed on both normalized nuclear stopping $R(0)/\rho_{mb}$ and normalized scale invariant nuclear sideward flow $\tilde{F}(b_0)/\rho_{mb}$, the resulted two variables, $\ln[R(0)/\rho_{mb}]$ and $\ln[\tilde{F}(b_0)/\rho_{mb}]$ show strong universal correlation for various colliding systems with a given $b_0$ bin. For illustration, the correlation with the $b_0$ from 0.2 to 0.6 and the corresponding linear fit line are shown in Fig. 2. For the hard EoS case, the results of mentioned 60 colliding systems with beam energy per nucleon from 0.35 to 3.9 GeV are also shown in Fig. 3. The superposed solid line shown in Fig. 2 and Fig. 3 are the results of the linear fit for the corresponding correlation.

By the linear fit of the correlation in Fig. 2 and Fig. 3, the analytic relation between the two variables: the $R(0)$ and $\tilde{F}(b_0)$ can be expressed as:

$$\ln\left[\frac{R(0)}{\rho_{mb}}\right] = L \times \ln\left[\frac{\tilde{F}(b_0)}{\rho_{mb}}\right] + m$$

FIG. 1: (Color Online) Left panels: The excitation function of the central nuclear stopping ratio, the variable $\rho_{mb}$ and the semi-central ($0.3 < b_0 < 0.4$) scale invariant nuclear sideward flow. Right panels: System size dependence of the three experimental observables.

FIG. 2: (Color Online) The correlation between the central nuclear stopping ratio and scale invariant nucleon sideward flow, with $b_0$ varying from 0.2 to 0.6 and an interval of 0.1, are calculated for the mentioned 150 colliding systems within the cascade EoS. The solid line in each panel is the linear fit of the corresponding correlation.
where the two fitting parameters $L$ and $m$ are introduced to represent the slope and intercept, respectively. Generally speaking, there is nothing particular for any of the two variables $R$ and $F$, and they are of equal importance. Actually, it is found that not only central stopping ratio, but also non-central nuclear stopping ratio is correlated with the scale invariant nucleon flow $F(b_0)$, which means for two independent scale impact parameters $b_0^F$ and $b_0^R$, the relation between the corresponding $R(b_0^F)$ and $F(b_0^R)$ can be expressed as:

$$\ln\frac{R(b_0^R)}{\rho_{mb}} = L \times \ln\frac{F(b_0^F)}{\rho_{mb}} + m \quad (2)$$

The fitting parameters $L$ and $m$, depending on both $b_0^F$ and $b_0^R$ for the cascade and hard nuclear EoS cases, are illustrated in Fig. 4. In the two upper panels of Fig. 4, the $L$ shows almost no dependence on the $b_0^F$ and $b_0^R$, and it is larger for hard EoS than the cascade one. Thus, it can be regarded as a constant parameter to characterize the nuclear EoS. The $b_0^F$ and $b_0^R$ dependence of $m$ are also illustrated in the two lower panels of the Fig. 4, which is strongly affected by $b_0^F$, $b_0^R$ and also by the different nuclear EoS. The two parameters are both colliding systems independence, as they are both the universal fitting parameters for various colliding systems. The parameter $m = m(b_0^F, b_0^R)$ is only as a function of $b_0^F$ and $b_0^R$, and the so-called correlation function $C(b_0^F, b_0^R) = e^{-m(b_0^F, b_0^R)}$ is defined to describe the correlation strength between the nuclear stopping ratio $R(b_0^R)$ and scale invariant nucleon sideward flow $F(b_0^F)$. Two new variables: $R^* (b_0^F) = R(b_0^F)/\rho_{mb}$ and $\tilde{F}^* (b_0^F) = F(b_0^F)/\rho_{mb}$ are defined for the simplification of eqn. (2). Then, it can be rewritten as:

$$\tilde{F}^* (b_0^F) = \left( \frac{R^* (b_0^F) C(b_0^F, b_0^R)}{L} \right)^{\pm} \quad (3)$$

The colliding system as well as scale impact parameter dependence of single experimental observable is further investigated for any specific colliding system. From the eqn. (3), for any given $b_0^R$ and $b_0^F$, the two correlate observables are respectively calculated with two different scale impact parameters, $(b_0^{E1}, b_0^{E2})$ and $(b_0^{R1}, b_0^{R2})$, then we have:

$$\tilde{F}(b_0^{E1}) = \frac{e^{m(b_0^{E1}, b_0^R)}/L}{e^{m(b_0^{R1}, b_0^R)/L}}; \quad \frac{R(b_0^{R1})}{R(b_0^{R2})} = e^{-m(b_0^{E1}, b_0^{R2})} \quad (4)$$

The terms at the right side of the two equations in (4) are colliding systems independence and the two equations are satisfied for any fixed $b_0^R$ and $b_0^F$, respectively, which indicate that the variables of the two observables $\tilde{F}(b_0^F)$ and $R(b_0^R)$ can be separated as colliding system dependent term multiplied by scale impact parameter dependent term as:

$$R(A, E_{lab}, b_0^R) = \xi^R(A, E_{lab}) \times \eta^R(b_0^R) \quad (5)$$

$$\tilde{F}(A, E_{lab}, b_0^F) = \xi^F(A, E_{lab}) \times \eta^F(b_0^F) \quad (6)$$

With the variable separable property, which is nontrivial and not common for all the experimental observables,
the excitation properties of the correlative observables for any scale impact parameter are the same. For better understanding the fitting parameters $L$ and $m$, as well as the normalization factor $\rho_{mb}$, the equ. (5) and equ. (6) are introduced into equ. (2), then we obtain:

$$\ln\left[\frac{\xi^R(A, E_{lab})}{\rho_{mb}}\right] = L \ln\left[\frac{\xi^F(A, E_{lab})}{\rho_{mb}}\right] + m(b_0^F, b_0^R) + L \ln[\eta^F(b_0^R)] - \ln[\eta^R(b_0^R)]$$

(7)

As for various colliding systems $(A, E_{lab})$ and scale impact parameters $(b_0^F, b_0^R)$, the equ. (7) are always satisfied, we have:

$$\ln\left[\frac{\xi^R(A, E_{lab})}{\rho_{mb}}\right] = L \ln\left[\frac{\xi^F(A, E_{lab})}{\rho_{mb}}\right] + m(b_0^F, b_0^R)$$

(8)

$$m(b_0^F, b_0^R) = \ln[\eta^R(b_0^R)] - L \ln[\eta^F(b_0^F)]$$

(9)

The equ.(8) demonstrates that the colliding system dependent terms of the two correlative observables have been connected by introducing a proper normalization factor $\rho_{mb}$, which is also colliding system dependent and may be not unique for present correlation or even not necessary for other correlative analysis. In equ. (8), the universal fitting parameter $L$ is uniquely determined by the colliding system dependent properties of the two observables, that is the reason the $L$ is not related to the $b_0^R$ and $b_0^F$ for a given nuclear EoS (See Fig. 4). Thus, it is supposed to be a constant characteristic parameter to characterize the nuclear intrinsic properties. The variable separable properties of the two correlative observables in equ. (5) and (6), and the analytic relation between colliding system dependent terms in equ. (8) are the origin of the correlation presented by equ. (2). As a consequence of the correlation for various scale impact parameters, the intercept parameter $m$ can be expressed as equ. (9), which is the combination of the scale impact parameter dependent terms of the two correlative observables and without the cross terms. Derived from equ. (5), (6) and (9), the differential of the parameter $m(b_0^F, b_0^R)$ can be written as:

$$\frac{\partial m}{\partial b_0^R} = \frac{\partial \ln[\eta^R(b_0^R)]}{\partial b_0^R} = \frac{\partial \ln[R(A, E_{lab}, b_0^R)]}{\partial b_0^R}$$

(10)

$$-\frac{1}{L} \frac{\partial m}{\partial b_0^F} = \frac{\partial \ln[\eta^F(b_0^F)]}{\partial b_0^F} = \frac{\partial \ln[R(A, E_{lab}, b_0^F)]}{\partial b_0^F}$$

(11)

The differential of $m(b_0^F, b_0^R)$ in equ.(10) and (11) are only related to $b_0^F$ and $b_0^R$, respectively (See Fig. 4) and uniquely determined by the corresponding experimental observable. The colliding system dependent properties of the differential of experimental observables in equ. (10) and (11) can be used to validate whether the observables are variable separable or not, which is a necessary and not sufficient condition for the present correlation. Because the equ. (2) is only the fitting equation for the two correlative observables and not all of the colliding systems exactly satisfy the equ. (2), the differential of $m$ may be weakly dependent on specific colliding system.

We have performed correlative analysis between the nuclear stopping and scale invariant nucleon sideward flow for various colliding systems and scale impact parameters. The complication of colliding system dependence of single observable can be separated from the scale impact parameter dependence, and has been reduced to two universal fitting parameters $L$ and $m$, which can be used to determine the nuclear EoS or other intrinsic properties. The essential of the universal correlation behavior between nuclear stopping and scale invariant nucleon sideward flow from SIS to SPS energies may result from the pressure of the matter in HICs, which is dominated by nuclear EoS, in-medium $NN$ cross section etc. and intimately connected to the nuclear stopping and nucleon sideward flow$^{13, 15, 18, 19, 20, 21, 22}$. The strong correlation may indicate that the pressure produced in HICs may be also with the variable separable property as in equ. (8) and (9), and it may be broken down by the sudden change of the nuclear bulk properties, such as phase transition etc. As the phase transition mechanism is not explicitly involved in UrQMD model and it is also found in Ref. $^{23}$ that the collapse of excitation function of the sideward flow and elliptic flow can be used to probe the first order QGP phase transition. Thus, for qualitative analysis, it is predicted that the universal correlation may be broken down at different sets of beam energies for various colliding nuclei, which could serve as signals for the first order QGP phase transition. Furthermore, if the universal correlation pattern restores at much higher energies, where the crossover from hadronic phase to partonic phase would happen, the location of the critical point can be unitedly restricted by the mapping of different colliding nuclei with the corresponding lower limit of the restoration energies on the QCD phase diagram. The real experimental data as well as the theoretical calculation are expected to be compared with the UrQMD simulation results and the detailed correlation mechanism of the two experimental observables should be further studied.

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