Constraints on new interactions from neutron scattering experiments

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Constraints for the constants of hypothetical Yukawa-type corrections to the Newtonian gravitational potential are obtained from analysis of neutron scattering experiments. Restrictions are obtained for the interaction range between $10^{-12}$ and $10^{-7}$ cm, where Casimir force experiments and atomic force microscopy are not sensitive. Experimental limits are obtained also for non-electromagnetic inverse power law neutron-nucleus potentials. Some possibilities are discussed to strengthen these constraints.

1 Introduction.

Hypothetical new long-range interactions have been discussed from different points of view. First, developments in quantum field theory have lead to the possibility of the existence of number of light and massless elementary particles [1]. Exchange of such particles between two bodies can reveal itself as an additional interaction potential of Yukawa or power-type and result in a deviation of the gravitational force from Newtonian law. On the other hand the theoretical models have been developed recently with extra spatial dimensions [2] leading to an additional forces at short distances. The deviations of results of measurements of gravity forces at macroscopic distances from calculations based on Newtonian physics can be seen in the experiments of Galileo-, Eötvös- or Cavendish-type [3] performed with macro-bodies. At smaller distances $(10^{-7} - 10^{-2})$ cm the effect of these forces can be observed in measurements of the Casimir force between closely placed macro-bodies (for review see

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[4]) or in the atomic force microscopy experiments. At even smaller distances such experiments are not sensitive enough, and high precision particle scattering experiments may play their role. In view of absence of electric charge the experiments with neutrons are more sensitive than with charged particles, electromagnetic effects in scattering of neutrons by nuclei are generally known and can be accounted for with high precision. As an example of precision scattering experiments we present here the analysis of the measured total neutron scattering cross section by heavy nuclei in the eV-keV energy range, and consider some possibility to increase the sensitivity in inferring new interactions from the neutron scattering experiments.

2 Non-Newtonian gravity.

For the potential of the form

\[ U(r) = \frac{g}{r} e^{-r/\lambda} \]  

(1)

where \( g = -\alpha GMm \), \( G \) – the Newtonian gravitational constant, \( M \) and \( m \) are the masses of gravitating bodies (nuclear and neutron), the Born amplitude for the scattering by homogeneous gravitating ball (nucleus) with radius \( R \) is

\[ f(q) = \frac{2mg\lambda^2}{\hbar^2(q^2\lambda^2 + 1)}F(q). \]  

(2)

Here the nuclear formfactor is

\[ F(q) = \frac{3}{(qR)^3}[\sin(qR) - qR\cos(qR)], \]  

(3)

where \( R \) is nuclear radius.

Inverse power-(\(N+1\)) gravitation potentials

\[ U_{\text{extra}}(r) = \frac{g_N}{r} \left(\frac{r_0}{r}\right)^N, \]  

(4)

where \( g_N = -\alpha_N GMm \), \( r_0 \) is arbitrary length and may be taken 1 fm, and \( N=1,2,... \) appear [2] in models with large extra dimensions for \( r < R_c \), \( (R_c \) is the characteristic compactification radius).

The value of the interaction radius \( \lambda \) in the Yukawa-type potential of the Eq.(1), the power \( N \) in the power-law potential of the Eq.(4), the compactification radius \( R_c \) of extra dimensions and the strengths of these interactions \( \alpha \)
and $\alpha_N$ expressed here as their ratios to the newtonian interaction, are not predicted by theory and are the subject of experimental investigation. Restrictions for the Yukawa-type potential down to $10^{-8}$ cm range were obtained previously from experiments with macro-bodies. The bounds obtained in these experiments on the value $\alpha$ increase from $\sim 1$ at the millimeter distances to $10^{35}$ at the distances $\sim 10^{-8}$ cm. The summary of these results is shown in the Figure 1 with corresponding referencies.

### 3 Low energy neutron scattering – basic relations.

The problem of inferring the admixture of the weak long-range potentials from neutron cross section data is not trivial. During the years of attempts of the measurements the values of neutron-electron and neutron electric polarizability scattering amplitudes different approaches were used with different success (for review of some of these methods see [20]). The most precise data for these constants were obtained from the measurements of the neutron total scattering cross section by heavy nuclei, the differential cross section generally are measured with much lower precision. The contribution of the neutron-electron interaction to the neutron-nucleus scattering cross section is proportional to the electric charge of the nucleus, the contribution of the neutron electric polarizability is proportional to the square of the charge of the nucleus, therefore the measurements of scattering are performed for heavy nuclei. The contribution of non-Newtonian gravity should be proportional to the mass of the nucleus, it means that in this case the heavy nuclei should give the largest effect. On the other hand analysis of neutron scattering by heavy nuclei presents the most serious difficulties in view of great effect of resonances in neutron scattering. For light nuclei ($^4$He, C) the resonance effect and the contribution of electromagnetic interactions are much lower but there is no precision data on neutron scattering by these nuclei up to now.

In most publications the formalism used for the extracting the amplitudes of long-range interactions is described too shortly so that many important details are omitted. We use here the method close to proposed in [21] and most appropriate for analysis of cross sections for high enriched samples, in which the overwhelming contribution comes from one isotope. The experimental values of coherent scattering lengths are not used in this approach. The modification in
our approach consists in the following: we do not subtract the resonance contribution from experimental data before fitting the experimental data but include all resonance terms into the expression for the scattering amplitude and fit the experimental data to calculated cross section. Such procedure seems more reliable because resonance contribution to the scattering amplitude interferes with potential neutron-nucleus scattering amplitude and with the amplitudes of the long-range interactions.

Scattering amplitude

$$f(q) = f_0(q) + f_1(q) + f_{sch}(q) + f_{ne}(q) + f_p(q) + f_w(q),$$  \(5\)

where \(f_0(q)\) is the amplitude of nuclear s-scattering, \(f_1(q)\) is the amplitude of nuclear p-scattering, \(f_{sch}\) is the amplitude of electromagnetic (Schwinger) scattering, \(f_{ne}\) is the amplitude of \(n-e\) scattering, \(f_p\) is the amplitude of neutron-nucleus scattering due to neutron electric polarizability, \(f_w\) is the amplitude of hypothetical additional long-range neutron-nucleus interaction.

Transmission cross-section

$$\sigma(k) = \int |f(q)|^2 d\Omega \approx 4\pi |f_0(k)|^2 + |f_{ne}(k)|^2 + |f_p(k)|^2 + |f_w(k)|^2 + 12\pi |f_1(k)|^2 + \sigma_1p + \sigma_1w + \sigma_a + \sigma_{sch} + \sigma_{inc} + \Delta \sigma_{sol}.$$  \(6\)

In this expression \(f_i(k) = \int f_i(q)d\Omega/4\pi\), where ”i” stands for ”0”, ”ne”, ”p”, and ”w”; \(|f_1(k)|^2 = \int |f_1(q)|^2 d\Omega/12\pi\); instead of the terms \(\int |f_i|^2 d\Omega\) are used the terms \(\int f_i d\Omega|^2\). The latter is valid for the amplitude \(f_0\) which does not depend on the scattering angle, and for the amplitudes \(f_{n-e}, f_p,\) and \(f_w\) which are from three to four orders of magnitude lower than nuclear scattering amplitude \(f_0\).

The error induced by such an approximation as was shown by direct calculation does not exceed the tenth of millibarn that is significantly below the precision of the total neutron cross section by heavy nuclei, which in the best case is not better than several millibarns. \(\sigma_a\) is the capture cross section, \(\sigma_{sch}\) is the Schwinger scattering cross section due to the interaction of neutron magnetic moment with nuclear electric charge, \(\Delta \sigma_{sol}\) is the correction to the scattering cross section due to solid state structure and dynamics effects, and \(\sigma_{inc}\) is the incoherent cross section [22].

We do not neglect here small terms in the scattering cross section due to interference of p-scattering with the amplitudes \(f_p\), \((\sigma_{1p})\) and \(f_w\) \((\sigma_{1w})\).

$$f_0 = \frac{i}{2k}(1 - S_0),$$  \(7\)
\( k = 2.1968 \times 10^{-4} \frac{A}{A+1} E^{1/2} \) is the neutron wave vector (fm\(^{-1}\)) in center of mass system (neutron energy in the lab. system in eV).

S-matrix

\[
S_0 = e^{2i\delta_0} \left( 1 - \sum_j \frac{ig_j \Gamma_{nj}}{(E - E_j) + i\Gamma_j/2} \right),
\]

where summation is performed over known \( s \)-resonances, \( \Gamma_{nj} = \Gamma_{nj}(E_j)k/k_j \), \( \Gamma_j = \Gamma_{nj} + \Gamma_{\gamma_j} \), and \( g_j \) are the neutron widths, the total widths, and the statistical weights of \( j \)th \( s \)-resonance.

Upon introducing

\[
\sum_{(1)} = \sum_j \frac{2}{k_j} \frac{g_j \Gamma_{nj}^0 (E - E_j)}{4(E - E_j)^2 + \Gamma_j^2},
\]

and

\[
\sum_{(2)} = \sum_j \frac{1}{k_j} \frac{g_j \Gamma_{nj}^0 \Gamma_j}{4(E - E_j)^2 + \Gamma_j^2},
\]

where \( \Gamma_{nj}^0 \) are the reduced neutron widths, the real and the imaginary parts of the nuclear scattering amplitude are

\[
\text{Re}f_0 = \frac{\sin(2\delta_0)}{2k} - \cos(2\delta_0) \sum_{(1)} - \sin(2\delta_0) \sum_{(2)}
\]

and

\[
\text{Im}f_0 = \frac{\sin^2\delta_0}{k} + \cos(2\delta_0) \sum_{(2)} - \sin(2\delta_0) \sum_{(1)}.
\]

For the distant and unknown resonances, when \( E \ll E_{j'} \), and \( \Gamma_{\gamma j'} \ll \Gamma_{nj'} \ll E_{j'} \), we use the expansion

\[
\sum_{(i')} \approx \sum_{(i')} (k = 0) + \sum_{(i')} (k = 0)k + \sum_{(i')} (k = 0)k^2/2,
\]

\((i' = 1, 2)\) so that:

\[
\sum_{(1')} \approx -\sum_{j'} \frac{g_{j'} \Gamma_{nj'} \Gamma_{nj}}{2k_{j'} E_{j'}} + \sum_{j'} \frac{g_{j'}^2 \Gamma_{nj'}^2 \Gamma_{\gamma j'}}{4k_{j'}^2 E_{j'}^3} k - \sum_{j'} \frac{g_{j'} \Gamma_{nj'} \Gamma_{\gamma j'}}{k_{j'} E_{j'}^2} E,
\]

and

\[
\sum_{(2')} = \sum_{j'} \frac{g_{j'}^2 \Gamma_{nj'} \Gamma_{nj'}}{4k_{j'}^2 E_{j'}^3} k + \sum_{j'} \frac{g_{j'}^3 \Gamma_{nj'}^3 \Gamma_{\gamma j'}}{8k_{j'}^3 E_{j'}^4} k^2.
\]
The contribution of these distant resonance terms to the cross section term proportional to $k$ is:

$$8\pi (R \sum'_{(1')} (0) - R^2 \sum'_{(2')} (0) + \sum'_{(1')} \sum'_{(1')} (0) + \sum'_{(2')} \sum'_{(2')} (0)),$$

and the contribution of these distant resonance terms to the cross section term proportional to $k^3$ is:

$$\frac{4\pi}{3} (2R^4 \sum'_{(2')} (0) - 3R^2 \sum''_{(2')} (0) - 4R^3 \sum'_{(1')} (0) +$$

$$+ 3 \sum'_{(1')} \sum''_{(1')} (0) + 3 \sum'_{(2')} \sum''_{(2')} (0)). \quad (17)$$

The calculation of these terms with sum over all known [23] resonances for the sample with isotopic content $^{208}\text{Pb}_{0.983} \quad ^{207}\text{Pb}_{0.011} \quad ^{206}\text{Pb}_{0.006}$ [18, 19] yields the contribution for $k$-proportional term less than $10^{-6}$ b, and for $k^3$-proportional term $\sim 10^{-5}$ b at the neutron energy about 20 keV.

As at low energies $\delta = -kR'_n$, where $R'_n$ is the nuclear part of the scattering radius, expansion of the first term in $\text{Re}f_0$ gives:

$$\text{Re}f_0 = -R'_n + \frac{2}{3} R'^3 n^2 - \frac{2}{15} R'^5 k^4 + \frac{4}{315} R'^7 k^6 - \frac{2}{2835} R'n^9 k^8 -$$

$$- \cos(2\delta_0) \sum_{(1)} - \sin(2\delta_0) \sum_{(2)} + h_0 + h_1 k^2. \quad (18)$$

The last two terms represent the contribution from the tails of distant and unknown (including at negative energies) resonances, and comes from the expansion of the term $\cos(2\delta_0) \sum_{(1)}$. At low energies $E \ll |E_{j'}|$, and $\Gamma_{j'} \ll |E_{j'}|$ this contribution is:

$$- \sum_{j'} \frac{g_{j'} \Gamma_{nj'}}{2k_{j'} E_{j'}} - \sum_{j'} \frac{g_{j'} \Gamma_{nj'}}{2k_{j'} E_{j'}} E = h_0 + h_1 k^2. \quad (19)$$

The first term enters into unknown scattering radius $R'_n$, the second term we have to consider as unknown (fitted) parameter.

The amplitude of the neutron-atom scattering due to the neutron-electron interaction is

$$f_{\text{ne}} = -b_{\text{ne}} Z (\bar{f} - \bar{h}), \quad (20)$$

where $b_{\text{ne}}$ is neutron-electron scattering length,

$$\bar{f}(k) = \frac{2}{x} (\sqrt{1 + x} - 1), \quad x = 12(k/q_0)^2, \quad q_0 (\text{fm}^{-1}) = 1.9 \times 10^{-5} Z^{1/3} \quad (21)$$
is the atomic formfactor, and

\[
\tilde{h}(k) = 1 - \frac{(kR_N)^2}{5} + 2\frac{(kR_N)^4}{105} - \ldots
\]  

(22)

is the nuclear charge formfactor [24].

\[R_N = 1.2027A^{1/3} = 7.127 \text{ fm for } ^{208}\text{Pb}\] is the nuclear electric radius.

The neutron electric polarizability scattering amplitude

\[f_p = f_{p0} \cdot Q,\]  

(23)

where

\[f_{p0} = \alpha_n \left(\frac{Ze}{\hbar}\right)^2 \frac{m}{R_N} = 32.78 \text{ mfm}\]  

(24)

for Z=82 and the neutron electric polarizability \(\alpha_n = 1.2 \times 10^{-3} \text{ fm}^3\), the formfactor

\[Q = \frac{6}{5} - \frac{\pi}{3}(kR_N) + \frac{2}{7}(kR_N)^2 - \frac{4}{405}(kR_N)^4 - \ldots\]  

(25)

The \(p\)-scattering amplitude is

\[f_1 = \frac{i}{2k}(1 - S_1),\]  

(26)

\[S_1 = e^{2i\delta_1}(1 - \sum_j \frac{ig_j\Gamma_{nj}}{(E - E_j) + i\Gamma_j/2}).\]  

(27)

Here summation is performed over known \(p\)-resonances, \(\Gamma_j = \Gamma_{nj} + \Gamma_{\gamma j}\), and \(\Gamma_j, \Gamma_{nj}, \Gamma_{\gamma j}\), and \(g_j\) are the total width, the neutron width, the gamma-width and the statistical weight of \(j^{th}\) \(p\)-resonance.

\[\Gamma_{nj} = \frac{k}{k_j}\Gamma_{nj}(E_j)\frac{v_{1j}}{v_{0j}},\]  

(28)

where

\[v_{0j} = \frac{(k_jR)^2}{1 + (k_jR)^2}, \quad v_{1j} = \frac{(kR)^2}{1 + (kR)^2}.\]  

(29)

The phase of \(p\)-scattering

\[\delta_1 = -kR + \arctg(kR) + \arcsin\left[\frac{k}{3}(kR)^2\frac{(R - R_1')}{1 + (kR)^2}\right],\]  

(30)

where \(R\) is the channel radius, \(R_1'\) is the \(p\)-wave scattering radius.
Interference terms of the $p$-scattering amplitude and the polarization amplitude are:

$$\sigma_{1p} = 4\pi \text{Re} f_1(k)f_p\left(\frac{4\pi}{5}x - \frac{2}{3}x^2\right), \quad (31)$$

where $x = kR$.

4 Obtaining of constraints for hypothetical long-range interactions from neutron scattering experiment.

The method to search for the admixture of long-range interaction in the neutron-nucleus scattering was proposed by Thaler [25] in application to the measurement of the neutron electric polarizability. The idea consists in the search for non-even in the neutron wave vector terms in the power series of the experimental angular distribution or the total neutron cross sections in keV energy range.

Indeed, in result of interference of the Born scattering amplitudes for the long-range potentials of the Eq.(4) and the short range nuclear scattering amplitude, the following terms appear in the low energy wave vector $k$ expansion for the total cross section (Eq.(6)):

$$f_{1w} = \frac{\alpha_1 \pi b r_0}{2k}$$

for $N = 1$,  
$$f_{2w} = -\frac{1}{2} \alpha_2 b r_0^2 \ln(kR)$$

for $N = 2$,  
$$f_{3w} = -\frac{1}{3} \alpha_3 b \pi r_0^3 k$$

for $N = 3$,  
$$f_{4w} = \frac{1}{3R} \alpha_4 b r_0^4 k^2 \ln(kR)$$

for $N = 4$,  
$$f_{5w} = \frac{1}{15} \alpha_5 \pi b r_0^5 k^3$$

for $N = 5$,  
$$f_{6w} = -\frac{2}{45} \alpha_6 b r_0^6 \ln(kR) k^4$$

for $N = 6$. \quad (32)

Here $b = \frac{2m^2 M G}{k^2}$, $b = 1.3 \times 10^{-35}$ fm\(^{-1}\) for the nucleus $^{208}$Pb.

The potentials for gravitating balls with radius $R$ for $r > R$ may be obtained by integration of the potentials of the Eq.(4) over the ball volume and have the form:

$$N = 1, \quad U_1(r) = \frac{3g_1 r_0}{2R^3 r} \left[ \frac{R^2 - r^2}{2} \ln \frac{R + r}{R - r} + Rr \right]; \quad (33)$$

$$N = 2, \quad U_2(r) = \frac{3g_2 r_0^2}{R^3 r} \left[ \frac{r}{2} \ln \frac{R + r}{R - r} - R \right]; \quad (34)$$

$$N = 3, \quad U_3(r) = \frac{3g_3 r_0^3}{2R^3} \left[ -\frac{1}{2r} \ln \frac{R + r}{R - r} + \frac{R}{r^2 - R^2} \right]; \quad (35)$$
\[ N = 4, \quad U_4(r) = \frac{g_4 r_0^4}{r(r^2 - R^2)^2}; \quad (36) \]

\[ N = 5, \quad U_5(r) = \frac{g_5 r_0^5}{(r^2 - R^2)^3}; \quad (37) \]

\[ N = 6, \quad U_6(r) = \frac{g_6 r_0^6}{5r} \frac{R^2 + 5r^2}{(r^2 - R^2)^4}. \quad (38) \]

Born scattering amplitudes for these potentials

\[ f_N = b \alpha_N \int_{R+\epsilon}^{\infty} V_N(r) \sin(qr) r dr, \quad (39) \]

where \( V_N = U_N / g_N \), \( \epsilon \) is the cut-off parameter in extra-gravity interaction, it is taken at the level of electroweak scale \( \sim 10^{-4} \text{ fm} \) \([2]\). These amplitudes have very complicated view, we are interested in the low neutron energy expansions of these amplitudes.

For example, if to omit in these expansions the constant and even in the wave vector terms (which is impossible to distinguish from contribution from short-range nuclear interaction) we have the following contributions to the formfactor in the total scattering cross section:

\[ f_{4w} = \frac{1}{6} \alpha_4 b r_0^4 \ln(kR) + \frac{2}{7} \ln(k\epsilon) \cdot k^2 \quad (40) \]

for the potential of Eq.(36),

\[ f_{5w} = \frac{\pi}{15} \alpha_5 b r_0^5 [1 + \frac{2\pi}{7} (kR)^2] \cdot k^3 \quad (41) \]

for the potential of Eq.(37) and

\[ f_{6w} = \frac{1}{45} \alpha_6 b r_0^6 \ln(kR) + \ln(k\epsilon) \cdot k^4 \quad (42) \]

for the potential of Eq.(38). The first terms in the brackets correspond to the pointlike potentials of Eqs.(4), the remaining terms come from the fact that the sources of potentials are spherical bodies. For the potentials of Eqs.(33-35) with \( N \leq 3 \) no additional terms appear in these expansions for the scattering amplitudes for gravitating balls in comparison to the cases of the pointlike scatterers of Eqs.(32).
As an example of the measurement of the total neutron cross section in eV–keV energy range we used two sets of data from the publications [18, 19] and [26]. These groups measured the total cross section of highly enriched $^{208}$Pb samples with atomic concentrations: $^{208}$Pb$_{0.983}$ $^{207}$Pb$_{0.011}$ $^{206}$Pb$_{0.006}$ in the energy range from 1.26 eV up to 24 keV with declared precision from $\sim$ 20 mb to 2.5 mb in [18, 19] and $^{208}$Pb$_{0.9712}$ $^{207}$Pb$_{0.0179}$ $^{206}$Pb$_{0.0108}$ in [26] with similar precision.

The outlined formalism was used for the procedure of extracting the contribution of possible new long-range interactions to the total neutron cross section. The MINUIT program [27] was used with three fitted parameters: the $s$-scattering radius $R'_n$, the term corresponding to the contribution to the $s$-scattering amplitude from distant and unknown resonances $h_1$, and contribution from the unknown long-range potential $f_w$. The constraints obtained from both sets of data: [18, 19] and [26] were very close.

The neutron-electron scattering length was varied between $b_{ne} = -1.32 \times 10^{-3}$ fm and $b_{ne} = -1.59 \times 10^{-3}$ fm[28–32], the value of the neutron electric polarizability was varied in the limits $\pm 30\%$ around $\alpha_n = (1.22 \pm 0.19) \times 10^{-3}$ (fm)$^3$, the latter value being obtained as an averaged over the results from two recent measurements of the Compton scattering by deuteron [33, 34].

Figure 1 shows obtained limits for non-Newtonian gravity from these neutron scattering data in terms of relative potential strength $\alpha$ in Eq.(1) as a function of the Yukawa length scale $\lambda$. It shows also the present status of constraints from the previous larger scales experiments.

Figure 2 shows the limits on the value of the inverse power law potential of Eq.(4), parametrized in the form:

$$U(r) = U_0(1\text{fm})M\left(\frac{1\text{fm}}{r}\right)^{N+1},$$

where $M$ is the relative nuclear mass.

It is hardly believable that radical increase in precision of keV-neutron scattering cross section may be achieved in the forthcoming years. Analysis of systematic effects and uncertainties in the processing data of neutron scattering arising mostly from resonance structure of cross section shows that the limiting precision lies in the region 5 mb. Not the least is the problem of correct accounting the background in the time of flight measurements [18, 19, 26, 35]: different methods of measurement and subtracting the background yield significantly differing results.
Possibly some increase in sensitivity in the search for new interactions in the neutron-nuclei scattering may be achieved by measuring the energy dependence of the asymmetry of slow neutron scattering by noble gases - the method of Fermi–Marshall [36, 30].

The neutron-atom scattering amplitude
\[ a(\theta) = a_N + a_{n-e}Z f(\theta) \]
in result of interference of the nuclear and electron scattering leads to the scattering cross section;
\[ \frac{d\sigma(\theta)}{d\Omega} = a_N^2 + 2a_Na_{n-e}Z f(\theta), \]
with the relative value of the anisotropic term
\[ \delta\left(\frac{d\sigma(\theta)}{d\Omega}\right) = 2 \frac{a_{n-e}}{a_N} Z f(\theta). \]
Atomic formfactor may be calculated with sufficient precision according to [24]
\[ f(q) = [1 + 3(\frac{q}{q_0})^2]^{-1/2}, \]
with
\[ q_0 = \beta Z^{1/3}(\AA^{-1}). \]
In these expressions \( a_N \) is the nuclear scattering amplitude, \( a_{n-e} \) is the amplitude of n-e scattering, \( Z \) is the charge number. The value of \( \beta \) is taken from the tables [37].

Significant diffraction effect appears in the neutron scattering by noble gases due to atom-atom interactions [38–43]. The static structure factor depends on interatomic interaction potential \( U(r) \) and atomic density \( n \):
\[ S(q) = 1 + n \int_0^\infty (e^{-U(r)/kT} - 1)e^{iqr}d^3r. \]
For the spherically symmetric potential \( U(r) \) the structure factor is:
\[ S(q) = 1 + n \frac{4\pi}{q} \int_0^\infty (e^{-U(r)/kT} - 1)\sin(qr)rdr. \]
The scattering asymmetry is introduced as the ratio of intensities of the scattered neutrons to the angles $\theta_1$ and $\theta_2$, (usually $\theta_1 + \theta_2 = \pi$).

$$\frac{S(\theta_1)}{S(\theta_2)}.$$  \hfill (51)

Figure 3 shows the results of the calculations of the effects of hypothetical interactions on the center-of-mass scattering asymmetry by Xe gas together with the effects of neutron diffraction and neutron-electron interaction. The curves show the results of calculation of asymmetry for the forward and backward angles, respectively, $30^\circ$ and $150^\circ$. For Xe several best approximations for the potential of interatomic interaction were taken from the literature.

5 Constraints for hypothetical long-range interactions from antiprotonic atoms.

Some constraints on long-range forces may be obtained from the measurements of energies of electric-dipole transitions in hadronic atoms. The average distance of hadron from nucleus is

$$< r > \simeq \frac{200 m_r}{Z m_r} \frac{3n^2 - l(l - 1)}{2} \text{(fm)},$$  \hfill (52)

where $n$ and $l$ are the quantum numbers of the state, and $m_r$ is the reduced mass of the bound hadron. This distance is in the range of dozens to thousand of fermies for studied antiprotonic atoms. The most convenient transitions between the states $(n, l)$ are of the type $(n, n - 1) \rightarrow (n - 1, n - 2)$ with $n$ large enough to decrease as much as possible the effects of strong interaction on the energy states of orbiting hadrons. To estimate the contribution to the energy shifts of these states from the potentials of Eq.(1) and Eq.(4) we have to calculate the expectation value in the hydrogenlike atomic states of these potentials: They are

$$\Delta E_n = \frac{gZ}{n^2 a_0} \left( \frac{2\lambda Z}{2\lambda Z + na_0} \right)^{2n},$$  \hfill (53)

where $Z$ is nuclear charge, $a_0 = \hbar^2 / \mu e^2$, for the potential of Eq. (1) and

$$\Delta E_n = \frac{gN}{r_0} \left( \frac{2Zr_0}{na_0} \right)^{(N+1)} \frac{(2n - N - 1)!}{(2n)!}.$$  \hfill (54)
for potentials of Eq.(4). The difference of experimental energies of transitions $E(n, l) - E(n-1, l-1)$ and the calculation of the QED predictions for these transition energies may be assumed to be the effect of long-range interaction. No real difference was observed in the most precise experiments we used for obtaining these restrictions. We used the reported values of experimental uncertainty $4 \times 10^{-7}$ eV for transitions $(32,31) \rightarrow (31,30)$ in antiprotonic helium [17] and the value 50 eV for transitions $(5,4) \rightarrow (4,3)$ in antiprotonic sulfur [16] as the boundaries for the difference in the energy shifts due to these long-range interactions. Figure 1 shows these constraints as a function of $\lambda$ for the potential of Eq.(1). For the potential of Eq.(4) similar constraints may be obtained assuming that the compactification radius $R_{\text{comp}} \gg < r >$ – the average distance of the orbiting antiproton from the nucleus. The latter is calculated according to Eq.(51) and is $\sim 0.2 \, \text{Å}$ for the case of antiprotonic helium [17], and $\sim 50$ fm for the case of antiprotonic sulfur [16]. The constraints for the potential of Eq.(4) are $\alpha_2 \geq 6 \times 10^{33}$ for the measurement [16] and $\alpha_2 \geq 6 \times 10^{32}$ for the measurement [17].

The possibility of probing additional space dimensions with fast neutron scattering experiments was considered recently in [44]. I am grateful to Dr. W. M. Snow (Indiana University) for attracting my attention to this paper. The author is grateful also to Dr. A. B. Popov for important comment and to Dr. G. S. Samosvat for consultations and discussions.

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List of Figures

Fig. 1. Bounds $\alpha_G$ vs. $\lambda$ (90% C.L.) from different non-Newtonian gravity experiments. The area above the lines is excluded: 1 – Cavendish-type experiment of Seattle group[5]; 2 – microcantilever Cavendish-type experiment of Stanford[6]; 3 – Casimir force measurements; constraints from[7, 4], based on earlier experiments of[8, 9]; 4 – Casimir force measurement[10], the bound from[11]; 5 – Casimir force measurements with atomic force microscope[12], bounds from[4]; 6 – neutron quantum levels in the Earth gravitational field, experiment[13], bounds from[11]; 7 – van der Waals force measurements[9], bounds from[14, 4]; 8 – atomic-force microscopy experiment[15]; 9 – bounds obtained in this work from X-ray energy measurements in antiprotonic sulfur atoms[16]; 10 – bounds obtained in this work from the measurements of the transition frequencies between large $n$ levels in antiprotonic He-atoms[17]; 11 – bounds obtained in this work from analysis of neutron total cross section scattering by $^{208}$Pb nuclei[18, 19];

Fig.2. Limits (90% C.L.) on the value of the interaction potential $U_0(1\text{ fm})$ vs $N$ in the expression $U(r)=U_0(1\text{fm})M\left(\frac{1\text{fm}}{r}\right)^{N+1}$: 1 – from X-rays in $\tilde{p}-^4\text{He}$ - atoms; 2 – from X-rays in $\tilde{p}-^8\text{S}$ – atoms; 3 – from the measurement of the total $n-^{208}\text{Pb}$ – scattering; 4 - the same as 3, but additional interference terms for scattering amplitudes for gravitating balls of Eqs. (40-42) were included to the expression for the total scattering cross section of Eq.(6).

Fig. 3. The calculated neutron energy dependence of the center-of-mass scattering asymmetry ($\theta_1 = \pi/6, \theta_2 = 5\pi/6$) from 1 atm pressure Xe gas target in result of different effects: 1 and 2 – $n-\bar{e}$ interaction with the values of $n-\bar{e}$ scattering lengths $-1.32\cdot10^{-3}\text{fm}$ and $-1.59\cdot10^{-3}\text{fm}$; 3, 4, 5, and 6 – diffraction effect for different approximations of the interatomic Xe–Xe interaction: 3 and 4 - two approximations of Lennard-Jones potential for Xe[38, 39], 5 and 6 - potentials of more complicated form from[40]; 7, 8, 9 and 10 - effect of hypothetical potential of the form Eq.(1) for $\lambda$ equal to $10^5\text{fm}$, $10^4\text{fm}$, $10^3\text{fm}$ and $10^2\text{fm}$ and $\alpha$ equal to $10^{23}$, $10^{25}$, $10^{27}$.
and $10^{29}$ respectively; 11, 12 and 13 - effect of hypothetical potential of the form Eq. (4) for $N = 1$ and compactification radius $10^5$fm, $10^4$fm and $10^4$fm and $\alpha_1$ equal $10^{29}$, $10^{30}$ and $10^{31}$ respectively.
