ROTATIONAL STABILIZATION OF MAGNETICALLY COLLIMATED JETS

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ABSTRACT

We investigate the launching and stability of extragalactic jets through nonlinear magnetohydrodynamic (MHD) simulation and linear eigenmode analysis. In the simulations of jet evolution, a small-scale equilibrium magnetic arcade is twisted by a differentially rotating accretion disk. These simulations produce a collimated outflow which is unstable to the current driven \( m = 1 \) kink mode for low rotational velocities of the accretion disk relative to the Alfvén speed of the coronal plasma. The growth rate of the kink mode in the jet is shown to be inversely related to the rotation rate of the disk, and the jet is stable for high rotation rates. Linear MHD calculations investigate the effect of rigid rotation on the kink mode in a cylindrical plasma. These calculations show that the Coriolis force distorts the \( m = 1 \) kink eigenmode and stabilizes it at rotation frequencies such that the rotation period is longer than a few Alfvén times.

Key words: galaxies: jets – galaxies: magnetic fields – instabilities – MHD – plasmas

Online-only material: color figures

1. INTRODUCTION

Large-scale, highly collimated energetic plasma outflows are observed in some active galactic nuclei (AGNs). Many models have been proposed for the formation of these jets (Ferrari 1998), but their launching, collimation, and stability remain open issues. Recent observations indicate that the magnetic field structure in AGN jets is helical in nature (Asada et al. 2005; Gabuzda et al. 2004; Marscher et al. 2008). This suggests that magnetic fields play a strong role in the collimation of AGN jets, as was proposed by Blandford and Payne (Blandford & Payne 1982), and that one can use a magnetohydrodynamic (MHD) model to describe their formation and evolution. However, both theory and laboratory experiments show that helical MHD equilibria can be unstable to current-driven kink modes. Understanding the effect of the kink mode on jet morphology is therefore critical to understanding their evolution. Here, we describe a computational MHD study of the stability of plasma jets relative to the kink mode and the effect that jet rotation has on the stability properties.

Many of the earlier computational efforts to model extragalactic jets concentrate on two-dimensional MHD models in which the accretion disk is treated as a boundary condition (Romanova et al. 1997; Ouyed & Pudritz 1997; Ustyugova et al. 2000). Even though each of the studies cited uses a different initial magnetic field, they all observe the formation of a steady outflow. More recent three-dimensional MHD simulations study the stability of the jet far from the galactic nucleus (Nakamura et al. 2001). These calculations inject flow and torsional Alfvén waves into an MHD equilibrium and show that wiggled structures form in the jet due to the kink mode. Similar calculations, which consider a more realistic atmosphere into which the jet expands, also examine the effect of the kink mode on the jet (Nakamura & Meier 2004). These calculations show that rapid rotation of the jet can have a stabilizing effect. The study discussed here aims to further examine the effect of equilibrium rotation on the stability of an expanding jet.

The effect of equilibrium flow on current-driven MHD instabilities has been investigated both in theory and laboratory experiments. Linear MHD calculations show that a sheared axial flow has no effect on the growth of the instability (Shumlak & Hartman 1995). This effect was confirmed experimentally (Shumlak et al. 2003). Later theoretical work studied the effect of sheared helical flow on the kink mode and showed that the sheared azimuthal flow stabilizes the mode by creating a phase shift in the plasma eigenfunctions (Wanex et al. 2005; Wanex & Tendeland 2007).

The work discussed here extends the two-dimensional simulations of jet launching (Romanova et al. 1997; Ouyed & Pudritz 1997; Ustyugova et al. 2000) to three-dimensions via nonlinear MHD calculations and considers the effect of jet rotation on the current-driven kink mode. By scanning the rotation of the disk, we scan jet rotation, and similar to previous results (Nakamura & Meier 2004), the rotation of the jet is observed to stabilize the column. To better understand the stabilizing mechanism of the rotation, we perform linear MHD analysis for a simple cylindrical plasma equilibrium with rigid rotation. These calculations show that the Coriolis force stabilizes the non-resonant kink by distorting the eigenmode.

The paper is organized as follows. Section 2 discusses the results of nonlinear simulations of extragalactic jet launching and evolution. The stability with regard to the kink mode is shown to depend on the rotational velocity of the accretion disk relative to the Alfvén speed of the initial magnetic arcade. Motivated by this result, Section 3 examines the linear stability of the kink mode in a cylindrical equilibrium with rigid rotation via initial-value MHD calculations. The results show that rigid rotation provides a stabilizing effect. In Section 4, ideal MHD eigenvalue calculations are used to confirm the results of Section 3 and to examine the effect of equilibrium rigid rotation on the unstable range of axial wave numbers. We also examine the physical mechanism of rotational stabilization using the eigenvalue calculations in Section 4 and show that the Coriolis force stabilizes the kink mode. Discussion of the results and conclusions are given in Section 5.

2. NONLINEAR CALCULATIONS OF JET PROPAGATION

To investigate jet propagation, we model the expansion of a magnetic arcade due to accretion disk rotation using a non-relativistic MHD model which ignores gravitational effects.
Similar to previous studies (Romanova et al. 1997; Ouyed & Pudritz 1997; Ustyugova et al. 2000), the accretion disk is treated as a boundary condition on the computational domain. The simulation is initialized with axisymmetric vacuum magnetic field that is tied to the disk and has zero net magnetic flux through the disk. Thus, both ends of all magnetic field lines are anchored to the accretion disk. The differential rotation of the accretion disk, which rotates with a Keplerian velocity profile, injects magnetic helicity and magnetic pressure into the magnetic field, causing it to coil and expand. The coiled magnetic field produces a hoop stress on the plasma that collimates it on the central axis. The effect of jet rotation on the stability of the column is explored by varying the rotation rate of the accretion disk in individual simulations.

We numerically evolve the visco-resistive non-relativistic MHD equations,

\[ \frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) - \nabla \times \frac{\eta}{\mu_0} (\nabla \times \mathbf{B}) \]  

(2)

\[ \rho \frac{\partial \mathbf{v}}{\partial t} + \rho (\mathbf{v} \cdot \nabla \mathbf{v}) = \frac{1}{\mu_0} (\nabla \times \mathbf{B}) \times \mathbf{B} - \nabla p + \nabla \cdot \nu \rho \nabla \mathbf{v} \]  

(3)

\[ \frac{n}{\gamma - 1} \left( \frac{\partial k_B T}{\partial t} + (\mathbf{v} \cdot \nabla) k_B T \right) = - \frac{1}{2} \rho \nabla \cdot \mathbf{v} + \nabla \cdot n K \nabla k_B T, \]  

(4)

where \( n \) is the particle density, \( \mathbf{B} \) is the magnetic field, \( \mathbf{v} \) is the flow velocity, \( p \) is the thermal pressure, \( T \) is the ion and electron temperature, \( K \) is the isotropic thermal diffusivity, \( \nu \) is the viscosity, \( \eta \) is the resistivity, and \( \gamma \) is the ratio of the specific heats chosen such that \( \gamma = 5/3 \). The particle density, \( n \), is related to the mass density, \( \rho \), by a factor of the ion mass. The pressure and temperature are related by the ideal gas relation, assuming that the electrons and ions have the same temperature, \( p = 2n k_B T \). There is an extra term added to the right-hand side of the continuity equation (Equation (1)), given by \( \nabla \cdot D \nabla n \). This diffusive term is added for numerical smoothing and the diffusivity coefficient, \( D \), is generally chosen to be small. The thermal diffusivity, \( K \), is chosen to be 100 times the electromagnetic diffusivity. The effect of the gravitational force due to the massive galactic central object has been ignored, so gravity does not appear in the momentum equation (Equation (3)).

The MHD equations are evolved in time using the NIMROD code (Sovinec et al. 2004). NIMROD is well benchmarked and has been used to model a wide array of plasma experiments (Sovinec et al. 2003) and magnetospheric physics (Zhu et al. 2006). A cylindrical computational domain with a cylindrical coordinate system given by \( (r, \theta, z) \) is used. The spatial discretization scheme combines two numerical methods. A mesh of high-order finite elements is used in the poloidal \((r,z)\) plane, where the degree of the polynomial basis functions is chosen by the user, and the azimuthal \( \theta \) direction is represented with finite Fourier series. The parameter \( m \) is used to identify Fourier components in the azimuthal direction. Convergence studies show that a resolution of \( 0 \leq m \leq 5 \) is sufficient for the dynamics of the expanding jet. Using logarithmic packing of the poloidal mesh on the central axis and disk boundary, we can resolve the jet dynamics in a large domain using a poloidal mesh of 48 by 48 fifth-order elements.

Previous studies searching for steady-state outflows have treated the outer boundaries of the domain with open boundary conditions, allowing kinetic and magnetic energy to flow out of the domain (Romanova et al. 1997; Ouyed & Pudritz 1997; Ustyugova et al. 2000). We use closed, perfectly conducting boundary conditions on the outer boundaries to avoid inward propagating wave characteristics. While this boundary condition is certainly unphysical, the outer boundaries are placed at a distance of \( r = z = 100 r_i \), where \( r_i \) is the inner radius of the accretion disk, which is far from the dynamic region of the calculation.

The model of the accretion disk/jet system treats the accretion disk as a boundary condition at \( z = 0 \), where a smoothed axisymmetric Keplerian velocity profile is applied to \( v_0 \):
creates a helical distortion to the magnetic structure. The initial flow velocity is set to zero, and the accretion disk flow is ramped from zero at \( t = 0 \) to a steady profile within one turn of the disk at \( r = r_i \). The Keplerian flow of the disk acts to twist the coronal magnetic field, building magnetic pressure above the disk which launches the outflow. This twisting of the magnetic field also creates a strong \( \theta \)-component to the field, causing a hoop stress which pinches the plasma on the central axis and collimates the outflow.

The results discussed here are given in units of the initial field quantities. All velocities are given in units of the Alfvén speed at the origin; the magnetic Prandtl number, \( \nu \mu \), is the thermal pressure and \( P_B \) is the magnetic pressure. The last dimensionless parameter, the drive parameter, \( \hat{V}_D \), is defined as

\[
\hat{V}_D = \frac{v_0(r = r_i, z = 0)}{v_A(r = r_i, z = 0)},
\]

where \( v_A \) is the Alfvén velocity. This parameter can be understood as the ratio of how fast the accretion disk twists coronal magnetic field lines to how fast the information of this twisting propagates through the corona. In order to maintain a constant resistive diffusion time relative to the rotation period of the accretion disk in different simulations, the parameter \( S \cdot \hat{V}_D \) is held constant as \( \hat{V}_D \) is varied. Three sets of parameters are considered: \( P_M, \beta \), and the product \( S \cdot \hat{V}_D \) are fixed at 1, 1, and 200\( \pi \), respectively, and \( \hat{V}_D \) is varied with the values 0.5, 1.0, and 4.0.

The values of \( \hat{V}_D \) are chosen to be similar to previous studies (Moll et al. 2008; Nakamura & Meier 2004; Ouyed et al. 2003) which consider sub-Alfvénic disk rotation, and to extend the disk rotation to the previously unstudied super-Alfvénic regime.

The \( z \)-component of the fluid velocity for the \( \hat{V}_D = 0.5 \) and 4.0 calculations at \( t = 65.6 \) and 121.7 \( T_i \), respectively, is shown in Figure 1. While a collimated outflow is produced for both values of \( \hat{V}_D \), non-axisymmetric structure forms in the column for the \( \hat{V}_D = 0.5 \) case, due to the presence of an MHD instability. The effect of the instability on the magnetic structure of the jet can be seen in Figure 2. Here, the magnitude of the magnetic field is shown for \( \hat{V}_D = 0.5 \) and 1.0 at \( t = 43 \ T_i \). While the jet has expanded to a similar length for both values of \( \hat{V}_D \) at these times, the modification of the magnetic structure is more significant for \( \hat{V}_D = 0.5 \). For both cases, an \( m = 1 \) kink mode creates a helical distortion to the magnetic structure.

To confirm the source of the asymmetry in the \( \hat{V}_D = 0.5 \) simulation, we plot the energy of individual Fourier components in Figure 3. The \( m = 1 \) component is the first to become unstable, and it nonlinearly drives the \( m > 1 \) components when it reaches a significant level. The nonlinear drive is confirmed by artificially resetting the dependent fields in the \( m = 1 \) component to zero during the course of a simulation. As can be seen from the dashed traces in Figure 3, removing the \( m = 1 \) component causes the larger \( m \) components to decay, until the \( m = 1 \) returns to a significant level. Thus, the \( m = 1 \) component nonlinearly channels energy into the \( m > 1 \) components.

A plot of the magnetic energy of the \( m = 1 \) Fourier component for all three jet simulations is shown in Figure 4. The jet is unstable to an \( m = 1 \) mode for \( \hat{V}_D = 0.5 \) and 1.0, while it remains nearly stable for \( \hat{V}_D = 4.0 \). We calculate the linear growth rate of the \( m = 1 \) mode by making a linear fit to the magnetic energy of the \( m = 1 \) mode when it is in the linearly growing phase. The growth rates for \( \hat{V}_D = 0.5, 1.0, 4.0 \)...
and the boundary at \( r = r_0 \) is treated as a perfect conductor, where \( r_0 \) is much smaller than the radius of the domain of the nonlinear simulations described in Section 2. The results are given in terms of the Alfvén propagation time across the radius of the cylinder, \( \tau_A = r_a \nu_A^{-1} \), where \( \nu_A \) is the Alfvén speed at \( r = 0 \). Here, we solve a linear version of Equations. (1)–(4) for perturbations to MHD equilibria, with an arbitrary perturbation included in the initial velocity field. The dissipation coefficients are chosen to give a Lundquist number of \( S = 1 \times 10^6 \) and a magnetic Prandtl number of \( P_M = 1 \). If an MHD equilibrium is unstable, the solution obtained will be the most unstable linear eigenmode, and the growth rate is determined from the resulting exponential growth.

Our MHD equilibria are based on the paramagnetic pinch (Bickerton 1958), which is a one-dimensional Ohmic equilibrium with uniform axial electric field. The equilibrium is characterized by the parallel current profile, \( \lambda(r) \), defined as

\[
\lambda(r) = \frac{\mu_0 J_0(r) \cdot B_0(r)}{B_0(r)^2} = \frac{E_{r0} B_{z0}(r)}{\eta B_0(r)^2},
\]

where a subscript 0 is used to represent equilibrium fields. The profile discussed here is defined in terms of the on-axis parallel current, \( \lambda_{\|} = \lambda(r = 0) \), and the width of the equilibrium current profile decreases with increasing \( \lambda_{\parallel} \). Given that the \( -1/2 \int \delta E \cdot \delta B \, d\mathbf{x} \) term is the only potentially destabilizing term in the linear ideal potential energy that is independent of \( \nabla p_v \) (Freidberg 1987a), the parallel current is related to the free magnetic energy available to drive the kink mode. Moreover, for the paramagnetic pinch, \( \lambda_{\parallel} \) serves as a stability parameter for the mode. The equilibrium magnetic field is found by choosing a value for \( \lambda_{\parallel} \) and numerically integrating Ampère’s Law, \( \nabla \times \mathbf{B}_0 = \mu_0 J_0 \), using Equation (10) for the parallel component of \( J_0 \).

A plot of radial profiles of \( \lambda \) from the nonlinear jet calculation with \( \dot{V}_D = 4.0 \) at \( t = 121.7 \, T_i \) is shown in Figure 5 for \( z = 20.25, 30.14, 40.41, \) and \( 50.23 \, r_i \). The curves overlap since there is not a significant gradient in \( \lambda \) in the \( z \)-direction. Thus, a one-dimensional equilibrium for the linear calculations is a good approximation of the \( \lambda \) profiles in the nonlinear jet calculations. For comparison, the \( \lambda \) profile for the paramagnetic pinch with \( \lambda_{\parallel} = 5.0 \) is also plotted in Figure 5.

The stability of diffuse pinches, such as the paramagnetic pinch, without equilibrium flow relative to the ideal kink mode has been well studied and is known to depend on the pitch.
of the magnetic field, \( P(r) = r B_z(r) B_\theta(r)^{-1} \). Considering eigenfunctions of the form \( e^{i m \theta - k z} \), energy analysis shows that for \( m = 1 \) and \( \frac{\partial P}{\partial r} = 0 \), the plasma is stable if \( k P(r) > 1 \) or \( k P(r) < (k^2 r^2 - 1) (3 + k^2 r^2)^{-1} \) for \( r \geq 0 \) and all values of \( k \) (Robinson 1971). When there is a region in the plasma where \( k P(r) \leq 1 \) and \( k P(r) \geq (k^2 r^2 - 1) (3 + k^2 r^2)^{-1} \), there is a source of free energy for the \( m = 1 \), \( k = k' \) kink mode, and it may be unstable. When \( k P(r) < 1 \) in the entire plasma, the mode is non-resonant. If there is a radius, \( r_s \), in the plasma where \( k P(r_s) = 1 \), \( r_s \) divides the plasma into two regions; one where there is free energy for the kink, and one where there is not; and the mode is called resonant. For the paramagnetic pinch, \( P(r) \) decreases monotonically, and there is free energy for the kink in the region with \( r > r_s \). Since the free energy for the kink is at radii larger than \( r_s \), the stabilizing effect of the conducting boundary at \( r = r_s \) affects both resonant and non-resonant modes. The magnetic pitch profile of the equilibrium used is shown in Figure 6. Since \( P(r = 0) = 2.0 \lambda_0^{-1}, k > 0.5 \lambda_0 \) modes are resonant and \( k < 0.5 \lambda_0 \) modes are non-resonant.

To examine the effect of jet rotation, we consider MHD equilibria with rigid rotation in the \( \theta \)-direction, and use \( \Omega \) to denote the rotation frequency. While previous studies have shown that sheared flow is more efficient at stabilizing the kink mode (Wanex et al. 2005), our nonlinear computations show little azimuthal shear in the vicinity of the jet. Radial profiles of the jet rotation frequency from the nonlinear jet simulation with \( \dot{V}_D = 4.0 \) at various times and axial positions are shown in Figure 7. As time increases, the jet rotation frequency reaches a steady state at higher azimuthal positions along the length of the column. For all values of \( z \), the rotation frequency is uniform to within 20% across the radius of the jet, which has a width of \( r \leq 1 \), and as the column propagates, the rotation frequency flattens. Thus, rigid rotation is a reasonable simplification.

The paramagnetic pinch is often considered to be a force-free equilibrium in which the current is purely parallel to the magnetic field. However, equilibrium azimuthal flow breaks the force-free nature, since the centrifugal force of the flow must be balanced by another MHD force. Two choices of force balance are considered here. The first, labeled “magnetic-balance,” balances the centrifugal force against the force from the perpendicular current,

\[
J_0 \times B_0 = -\rho_0 \Omega^2 \mathbf{r},
\]

and the parallel current is unchanged. Thus, while the current profile is modified by the introduction of the rotation, the \( \lambda(r) \) profile, which is related to the free-energy source for the kink mode, is unaffected. The second force-balance model, labeled “pressure-balance,” balances the centrifugal force against the equilibrium pressure,

\[
\nabla p_0 = \rho_0 \Omega^2 \mathbf{r}.
\]

For this case, the current profile is unchanged by the introduction of the rotation. However, as the rotation increases, the pressure profile becomes increasingly hollow in the sense that it peaks on the edge of the plasma, which can have a stabilizing effect (Freidberg 1987a). The equilibrium pressure is characterized by the plasma \( \beta \) on the central axis. The choice of the values for \( \lambda_0 \) and \( \beta \) is motivated by our nonlinear jet calculations, giving \( \lambda_0 = 5.0 \) and \( \beta = 1.0 \).

Our numerically computed growth rate of the \( m = 1, k = 0.4 \lambda_0 \) kink mode as a function of equilibrium rotation frequency, for both force balance models, is plotted in Figure 8. The results show that the growth rate of the mode decreases as rotation increases for both force balance models. The growth rate decreases somewhat faster in the pressure-balance model than in the magnetic-balance model, which we surmise is a result of the additional stabilizing effect of the hollow pressure profile in the pressure-balance model. While the results point to rotation as the important stabilizing mechanism, force-balance requires changes to the pressure profile or the perpendicular current profile as rotation is increased. To examine the effect of modifying the equilibrium forces to balance the centrifugal force from the rotation, a plasma which has the same equilibrium current as the magnetic-balance model, but without rotation, is considered. Here, the equilibrium pressure gradient replaces the centrifugal force by defining a profile which is peaked on the central axis. The resulting growth rate is also plotted in Figure 8. As the pressure gradient increases, the growth rate of the kink mode increases. This result confirms that rotation is the stabilizing influence in the \( \Omega \)-scans.
Previous theoretical and experimental studies show that sheared axial flow can stabilize the kink mode in a cylindrical plasma (Shumlak & Hartman 1995; Shumlak et al. 2003). Thus, we consider what effect axial flow has on jet stability in the nonlinear simulations via linear initial value calculations with equilibrium axial flow. Non-rotating force-free paramagnetic pinch equilibria with Gaussian axial flow profiles, given by \( v_A(r) = v_M e^{-(2r/r_0)^2} \), are considered. Motivated by the axial flow profiles in the nonlinear jet simulations, we choose \( v_M = 0.3 \ v_A \) and consider a range of \( \omega_k \) from 5.0 to 50.0 \( \lambda_0^{-1} \), where smaller values of \( \omega_k \) correspond to larger flow shear.

Axial flow profiles from the stable \( V_D = 4.0 \) jet simulation and the Gaussian profile used for the linear calculation with \( w_g = 25.0 \lambda_0^{-1} \) are shown in Figure 9. Growth rates of the kink mode as a function of \( \omega_k \) are plotted in Figure 10. A flow shear range comparable to that considered by Shumlak & Hartman (1995) is considered, but the change in the kink growth rate is less than 6.0%. We attribute this to the difference in the equilibrium described in Section 3.

Based on these results, we conclude that axial flow does not significantly influence the stability of the magnetic column in our nonlinear jet simulations.

4. LINEAR EIGENVALUE CALCULATIONS

The results of the linear initial value calculations indicate that the nonlinear simulations remain robust to the kink mode for high rotation rates of the accretion disk because of the rotation of the jet itself. To further examine the effect of azimuthal rotation on the kink mode, we investigate the linear ideal MHD spectrum for rotating paramagnetic equilibria. This eigenmode analysis helps us develop physical insight into the effect of rotation, which is difficult to obtain from the initial value calculations. The theory considers a cylindrical domain which is periodic in the \( z \)-direction with the one-dimensional rigid-rotation equilibria described in Section 3.

4.1. Linear Eigenvalue Theory

The simplest approach in considering an MHD equilibrium with flow is to work in a Lagrangian representation. Assuming perturbations to the equilibrium depend on time as \( e^{-i \omega t} \), the linearized MHD equation of motion is given by

\[
\begin{align*}
- \rho_0 \omega^2 \xi - 2i \rho_0 \omega \mathbf{v}_0 \cdot \nabla \xi \\
+ \rho_0 \mathbf{v}_0 \cdot \nabla (\mathbf{v}_0 \cdot \nabla \xi) = F(\xi),
\end{align*}
\]

where from this point, fields without subscripts represent perturbations and are assumed to be small (Freiman & Rotenberg 1960; Waelbroeck 1996). The plasma displacement, \( \xi \), is defined by

\[
\mathbf{v} = \frac{\partial \xi}{\partial t}.
\]

The linear force operator, \( F(\xi) \), is given by

\[
F(\xi) = - \nabla p + \frac{1}{\mu_0} \mathbf{J}_0 \times \mathbf{B} + \frac{1}{\mu_0} (\nabla \times \mathbf{B}) \times \mathbf{B}_0 \\
+ \nabla \cdot (\rho_0 \mathbf{v}_0 \cdot \nabla \mathbf{v}_0),
\]

where the perturbed magnetic field and the perturbed pressure are given by

\[
\mathbf{B} = \nabla \times (\xi \times \mathbf{B}_0),
\]


\[ p = -(\xi \cdot \nabla p_0 + \gamma p_0 \nabla \cdot \xi). \] (17)

As a consistency check, we also evaluate the spectra derived from an Eulerian frame of reference by including equilibrium flow in the definition of the Lagrangian displacement vector, \( \xi \), which satisfies (Chandrasekhar 1961)

\[ v = \frac{\partial \xi}{\partial t} + \nabla \times (\xi \times v_0). \] (18)

Linearizing the MHD equations in the Eulerian frame with rigid equilibrium rotation gives the following momentum equation, force operator, induction equation, and pressure equation, respectively,

\[ -\omega_D^2 \xi - 2i \Omega \omega_D (\xi \times \xi) + r \Omega^2 (\nabla \cdot \xi) \]
\[ = \left[ \left( 3 + \frac{m \Omega}{\omega_D} \right) \hat{r} + i \frac{\omega_D}{\Omega} \right] \frac{1}{\rho_0} \mathbf{F}(\xi), \] (19)

\[ \mathbf{F}(\xi) = \frac{1}{\mu_0} (\mathbf{B}_0 \cdot \nabla \mathbf{B} + \mathbf{B} \cdot \nabla \mathbf{B}_0) - \nabla \left( p + \frac{1}{\mu_0} \mathbf{B} \cdot \mathbf{B}_0 \right). \] (20)

\[ \mathbf{B} = \nabla \times (\xi \times \mathbf{B}_0) + \frac{\Omega \mathbf{B}_0}{\omega_D} (\nabla \cdot \xi) (r k \hat{\theta} - m \hat{z}), \] (21)

\[ p = -(\xi \cdot \hat{r}) \frac{dp_0}{dr} - \gamma p_0 \left( 1 + \frac{m \Omega}{\omega_D} \right) (\nabla \cdot \xi), \] (22)

where \( \omega_D = -\omega - m \Omega \) is the Doppler shifted eigenfrequency.

Generalizing the analysis in Freidberg (1987b), the linearized equations in either reference frame are reduced to a pair of coupled first-order differential equations for the radial plasma displacement, \( \xi_r \), and the total perturbed plasma pressure, \( \tilde{P} = p + B \mathbf{B}_0 / \mu_0 \). Assuming spatial dependence of the perturbed fields of the form \( e^{i m \phi - i k z} \), Equations (13)-(17) and Equations (19)-(22) become systems of ordinary differential equations (ODE’s) with respect to the \( r \)-coordinate. By considering the projection of Equations (13) and (19) in the \( \hat{b} \) and \( \hat{h} \) directions, where \( \hat{b} = \frac{\mathbf{B}_0}{|\mathbf{B}_0|} \) and \( \hat{h} = \hat{b} \times \hat{r} \), the \( \hat{b} \) and \( \hat{h} \) components of the plasma displacement can be solved analytically. Substituting these results into Equations (13) and (17) and Equations (19) and (22) produces sets of coupled ODE’s with the same general form,

\[ A(r, \omega) \frac{d}{dr} \left( r \xi_{tr} \hat{r} \right) = B(r, \omega) \left( r \xi_{tr} \hat{r} \right). \] (23)

in both reference frames.

We consider the plasma to be surrounded by a conducting shell at the radius \( r = r_a \) by defining \( \xi_{tr}(r_a) = 0 \). The regularity condition at \( r = 0 \) is imposed by the cylindrical geometry of the domain. Expansion of \( \xi_{tr} \) in a power series for small values of \( r \) shows that regular solutions satisfy \( \xi_{tr} \propto r \lambda^{-1} \). Equation (23) coupled with these boundary conditions defines an eigenvalue problem with \( \omega \) as the eigenvalue.

It should be noted that while the form of this eigenvalue equation is the same in both reference frames, the ODE coefficients matrices \( A \) and \( B \) are unique to each frame. Equation (23) is derived for a general equilibrium flow in a Lagrangian frame, and the coefficients can be found in Bondeson et al. (1987). The ODE coefficients for a plasma equilibrium with rigid rotation and uniform axial flow in an Eulerian frame can be found in Appl & Camenzind (1992).

Due to the complexity of the ODE coefficients in Equation (23), we use a shooting method to solve the eigenvalue problem. A value is chosen for \( \omega \), and Equation (23) is numerically integrated from \( r = 0 \) to \( r = r_a \) using fourth-order Runge–Kutta integration. The choice of \( \omega \) is varied until the eigenfunction satisfies \( \xi_{tr}(r_a) = 0 \). A Newton–Raphson method is used to search the \( \omega \)-parameter space for functions that satisfy this boundary condition.

In the absence of equilibrium flow, the MHD force operators in Equations (15) and (20) are self-adjoint, and \( \omega \) is either purely real or purely imaginary (Freidberg 1987c). With the introduction of equilibrium flow, the force operator is no longer self-adjoint, and \( \omega \) and \( \xi_{tr}(r_c) \) can be complex (Freiman & Rotenberg 1960). The real component of the eigenvalue, \( \text{Re}[\omega] \), gives the oscillation frequency of the eigenmode, and the imaginary component, \( \text{Im}[\omega] \), determines its growth or decay rate. The Newton–Raphson method employed here is generalized to search the complex parameter space (Press et al. 2007). While Newton–Raphson readily generalizes to multiple dimensions, it converges only if the initial guess for the root is in the vicinity of the actual root. Since \( \omega \) is either purely real or purely imaginary without the equilibrium flow, Newton–Raphson is used in a one-dimensional space to find \( \text{Im}[\omega] \) for a given mode. The \( \Omega = 0 \) result is then used as an initial guess for a nearby equilibrium with flow, and that result is used as an initial guess for a slightly larger value of \( \Omega \). This process is repeated for increasing values of \( \Omega \).

4.2. Linear Eigenvalue Results

Results of the eigenmode analysis and growth rates from the initial value formulation of Section 3 for the non-resonant \( m = 1, k = 0.4 \lambda_o \) kink mode can be seen in Figure 11. Here, calculations are shown for both both force-balance models in both reference frames. The curves from the two reference frames are indistinguishable in this plot, and comparison of the eigenvalue formulation and the initial value formulation of the problem are shown to be in agreement. These results show that as \( \Omega \) increases, the growth rate of the kink mode decreases and is stable with sufficient rotation. We note that the marginal rotation
Figure 12. Growth rates of the resonant $m = 1, k = 0.6 \lambda_o$ kink mode as a function of the equilibrium rotation frequency, $\Omega$.
(A color version of this figure is available in the online journal.)

Figure 13. Critical rotation frequency for stabilization of the kink mode (solid line), and growth rate of the kink mode for $\Omega = 0$ (dotted line), as a function of the outer radial boundary, $r_a$, for the $m = 1, k = 0.4 \lambda_o$ kink mode.
(A color version of this figure is available in the online journal.)

period is larger than the Alfvén propagation time, i.e., Alfvénic flow within the cylinder is not required for stabilization.

We also examine the effect of rotation on resonant kink modes via the eigenvalue formulation. Growth rates for the $m = 1, k = 0.6 \lambda_o$ mode can be seen in Figure 12. While equilibrium rotation fully stabilizes the non-resonant kink mode described previously, rotation only reduces the growth rate of the resonant mode and does not completely stabilize it.

The eigenmode solutions treat the radial boundary at $r = r_a$ as a solid wall by setting $\xi(r_a) = 0$. However, there is no close boundary surrounding the plasma column in the nonlinear jet simulations. To evaluate the influence of the wall location, we recompute the eigenvalues as $r_a$ is varied. The critical rotation frequency, $\Omega_c$, for stabilization of the $m = 1, k = 0.4 \lambda_o$ kink mode as a function of $r_a$ is plotted in Figure 13. The resulting critical rotation frequency asymptotically approaches the value $\Omega_c \approx 0.24 (k v_A)^{-1}$, indicating that the stabilizing effect of the rotation remains as $r_a \to \infty$. The growth rate with $\Omega = 0$, $\gamma_o(r_a)$, as a function of $r_a$ is also plotted in Figure 13. The $\Omega_o(r_a)$ and $\gamma_o(r_a)$ curves follow the same asymptotic trend, implying that the dependence of $\Omega_o$ on $r_a$ is related to the free energy of the kink mode and not due to any changes in the stabilizing influence of rotation.

The linearized eigenvalue formulation allows for the examination of a range of axial wave numbers. The growth rate of the $m = 1$ kink mode as a function of $k$ for various values of $\Omega$ is calculated, and the results are shown in Figure 14. Without equilibrium rotation, there are lower and upper bounds on the unstable values of $k$. Both the upper and the lower bound increase with increasing rotation. For the equilibria considered here, modes with $k < 0.5 \lambda_o$ are non-resonant, and modes with $k \geq 0.5 \lambda_o$ are resonant. While the range in $k$-space of unstable non-resonant kink modes decreases with increasing rotation, the range of unstable resonant modes broadens with small growth rates on the order of $10^{-3} \tau_A^{-1}$.

We have explored a range of $\beta$ values to examine the effect of equilibrium thermal pressure on rotational stabilization. For moderate values of $\beta$, rotational stabilization is observed to be independent of $\beta$. However, for low $\beta$-values ($\beta \leq 0.06$) rigid rotation destabilizes the kink mode for $\Omega \gtrsim 0.4 \tau_A^{-1}$. The destabilized modes are compressible with a $\theta$-component of $\xi$ that is much larger than the other components. Thus, these modes are stabilized by equilibrium pressure for the moderate values of $\beta$ relevant to extragalactic jet systems.

The choice of initial and disk boundary conditions in simulations of jet formation can have a profound effect on the magnetic pitch profile, $P(r)$ (Moll et al. 2008). Thus far, we have considered only equilibria with monotonically decreasing $P(r)$ as is observed in the nonlinear jet simulations discussed in Section 2. To check the effect of rotation on a monotonically increasing pitch profile we consider equilibria with $P(r) = 1/2 + r^2/2$. The growth rate of the $m = 1, k = 1.0 \tau_A^{-1}$ mode as a function of equilibrium rotation frequency is plotted in Figure 15 for both force-balance models. Similar to the decreasing $P(r)$ cases, rigid rotation is shown to stabilize the kink mode, and we conclude that the rotational stabilization mechanism is not sensitive to the shape of the $P(r)$ profile.

We also use the eigenmode calculations to investigate the physical mechanism for the rotational stabilization. The linearized momentum equation in the Eulerian frame is given by

$$\frac{\partial \mathbf{v}}{\partial t} + \rho \mathbf{v} \cdot \nabla \mathbf{v}_0 + \rho_0 \mathbf{v}_0 \cdot \nabla \mathbf{v} + \rho \mathbf{v}_0 \cdot \nabla \mathbf{v}_0 = \mathbf{F}(\mathbf{v}),$$  

and the growth rate of the $m = 1, k = 0.4 \lambda_o$ kink mode is calculated as a function of $\Omega$, removing one equilibrium flow term from the left side at a time. The results are plotted in Figure 16, but results with the $\rho \mathbf{v}_0 \cdot \nabla \mathbf{v}_0$ term removed are not shown, as this term does not have a significant effect.
the computations without the $\rho_{\theta} v_0 \nabla v_0$ term, the growth rate increases with increasing $\Omega$, so this term must play a central role in the stabilization. With rigid rotation, this inertial term is

$$ (v \cdot \nabla) v_0 = -i \Omega \omega_D (\hat{z} \times \xi) + \Omega^2 (\nabla \cdot \xi) r. \quad (25) $$

By individually removing each of the two terms on the right side of Equation (25) at a time, we have determined that it is the first term which provides the stabilization. This term contributes to the Coriolis force in the frame of the plasma.

Plots of the $\xi_r$ and $\xi_\theta$ components of the eigenfunction for various equilibrium rotation rates are shown in Figures 17 and 18. While there is a slight change in Re$[\xi_r]$ and Im$[\xi_\theta]$ as $\Omega$ is varied, the change in Im$[\xi_r]$ and Re$[\xi_\theta]$ is more apparent. We note that the Coriolis term locally couples the radial and azimuthal components of translation due to the kink. This distorts the mode giving a radially dependent phase shift in $\xi_r$, and a corresponding change in the real part of $\xi_\theta$. This result is similar to that described in Wanex et al. (2005), where a radially dependent phase shift in the eigenmode due to a sheared equilibrium flow is shown to stabilize the kink mode. Here, we find that a rotational flow without shear introduces a stabilizing distortion of the mode via the Coriolis force.

It should be noted that the Coriolis term also appears in the $\rho_{\theta} v_0 \nabla v$ term in Equation (24):

$$ (v_0 \cdot \nabla) v = -i \Omega \omega_D (\hat{z} \times \xi) + m \Omega \omega_D \xi + \Omega^2 (\nabla \cdot \xi) \hat{r} - i m r \Omega^2 (\nabla \cdot \xi) \hat{\theta}, \quad (26) $$

but when the $(v_0 \cdot \nabla) v$ term is removed, the stabilization effect is not lost. Equation (26) contains another term which is first order in $\Omega$ given by, $m \Omega \omega_D \xi$. This term provides the Doppler shift in the frequency $\omega$. This Doppler shift appears in the other MHD equations as well. Thus, removing the $\rho_{\theta} v_0 \nabla v$ term temporally decouples the velocity field from the magnetic field, reducing the growth rate of the instability, as shown in Figure 16.

We also examine the effect of rotation on the resonant eigenmodes. Plots of $\xi_r$ for $m = 1, k = 0.6 \lambda_o$ kink mode, for various equilibrium rotation rates, are shown in Figure 19. Similar to the non-resonant case, the rotation introduces a significant phase shift in the radial component of the eigenfunction. However, for the resonant case, there is also a significant change in the real part of $\xi$, near the rational surface.

To assess the rotation in the simulated magnetic columns described in Section 2, we calculate the rotation frequencies at different values of $\tau$. The rotation frequencies plotted in Figure 20 are determined by making linear fits to the $\theta$-component of the fluid velocity over the radial coordinate. The simulation times chosen for these profiles are such that the kink mode is in the linear phase for the $V_D = 0.5$ and 1.0 calculations, as can be seen in Figure 4. It is clear that angular momentum injected by the accretion disk is transported axially by the jet as it expands. As $V_D$ increases, the rotation rate of the jet increases, providing greater stability for the kink mode.

For comparison to the results of the linear MHD calculations, we examine the $m = 1$ kink mode in the nonlinear jet simulations when it is in the linear phase. The $m = 1$ Fourier component of $v_r$ is plotted in Figure 21 for the unstable $V_D = 0.5$ and 1.0 jet simulations at times $t = 8.76$ and 10.26$T_j$. 

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**Figure 15.** Growth rate of the $m = 1, k = 1.0 \lambda_r$ non-resonant kink mode for monotonically increasing magnetic pitch equilibria as a function of equilibrium rotation.

(A color version of this figure is available in the online journal.)

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**Figure 16.** Growth rates of the $m = 1, k = 0.4 \lambda_o$ kink mode as a function of rotation frequency with individual inertial terms removed from the linearized momentum equation. For the solid curve all of the terms are present, for the dashed curve the $\rho_{\theta} v_1 \nabla v_0$ is removed, and for the dot–dashed curve the $\rho_{\theta} v_0 \nabla v_1$ is removed.

(A color version of this figure is available in the online journal.)

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**Figure 17.** $\xi_r$ component of eigenfunctions of non-resonant $m = 1, k = 0.4 \lambda_o$ kink modes for various equilibrium rotation rates. The eigenmodes are normalized to the maximum value of Re$[\xi]$.

(A color version of this figure is available in the online journal.)
respectively. For these times, the kink mode is in its linearly growing phase. Since the modes plotted in Figure 21 extend across the entire width of the jet, we conclude that the kink mode observed in the nonlinear simulations is a non-resonant mode. According to our linear results, these modes would be stable with increased rotation, as is the case in the $\hat{V}_D = 4.0$ simulation. Similar to the eigenmodes from the linear analysis shown in Figure 17, the distortion of the linear eigenmodes in the jet simulations (Figure 21) is due to a radially dependent phase shift in $v_r$.

5. DISCUSSION AND CONCLUSIONS

Nonlinear non-relativistic MHD simulations of jet evolution, starting from an equilibrium coronal plasma with zero net magnetic flux through the accretion disk, show the formation of a collimated outflow. This outflow is unstable to the current driven $m = 1$ kink mode for low rotation velocities of the accretion disk relative to the Alfvén speed of the coronal plasma. As it saturates, the kink mode broadens the outflow, but does not destroy the collimation. Similar to previous results (Nakamura & Meier 2004), for large rotation velocities of the accretion disk, the outflow is shown to be stable against the kink mode. Moreover, the growth rate of the $m = 1$ kink mode is shown to be inversely related to the rotation rate of the accretion disk. This result is counterintuitive in the sense that as the accretion
disk rotates faster, the collimating magnetic field in the jet coils tighter. As the coiling of the magnetic field increases, the current increases. Since the current is the source of free energy for the kink mode, one would expect that the jet would be more unstable for high rotation rates of the accretion disk. However, we observe that it is stable in this regime.

Motivated by the result of the nonlinear jet simulations, we explore the effect of rigid rotation on the $m = 1$ kink mode in a periodic cylindrical plasma via linear MHD calculations. The linear calculations are treated as an initial value problem in an Eulerian reference frame and as eigenvalue problems in Eulerian and Lagrangian reference frames. The results from all three methods are in agreement. While previous studies have shown that sheared flow is more efficient at stabilizing the kink mode (Wanex et al. 2005), we show that rigid equilibrium rotation stabilizes the non-resonant $m = 1$ kink mode via the Coriolis effect. The Coriolis effect links radial and azimuthal motions of the plasma, which distorts the kink eigenmode and reduces its growth rate.

The MHD equations used to model the jet propagation discussed in Section 2 include dissipative terms, and we should consider what effect dissipation has on the rotational kink stabilization. In order to obtain smooth numerical solutions, the values chosen for the resistivity and the viscosity in the nonlinear jet simulations are much larger than that of any astrophysical jet system. However, we use the dissipationless ideal MHD equations for the eigenvalue analysis discussed in Section 4. While dissipation certainly affects the energy densities in the outflow in the jet simulations, the rotational stabilization is an ideal effect and robust to the choice of the dissipation coefficients.

Our choice of initial conditions in the nonlinear jet simulations discussed in Section 2 has a significant effect on the shape of the magnetic pitch profile, $P(r)$, in the simulated jet. The combination of inertia in the initial coronal plasma and a rapidly decreasing magnetic field acts as a background which the magnetic flux can push against. This allows for the buildup of a large $B_0$, producing a monotonically decreasing $P(r)$. In contrast, the simulations of Moll et al. (2008) produce jets with a monotonically increasing $P(r)$. While these differences affect the shape of the linear eigenfunctions, the eigenvalue calculations discussed in Section 4.2 show that the rotational stabilization is insensitive to the shape of the $P(r)$ profile.

With a decreasing $P(r)$ profile and no equilibrium rotation, there are lower and upper bounds on unstable values of $k$ for the kink mode, and the growth rate, $\gamma(k)$, is a function of $k$. This can have a profound effect on the evolution of an expanding jet. The linear rigid rotation calculations discussed in this paper apply only to static equilibria. However, we contend that the results of these calculations can be used as a guide for considering the stability of the time-dependent equilibrium of an expanding jet. As the jet expands, the $k$-value of any given harmonic decreases in time, i.e., the harmonic is stretched by the jet expansion. If we consider an equilibrium that is expanding at a constant rate $s$ with an initial length $L$ and a mode with $k = k'$ at time $t = 0$, the total energy gained by the harmonic over time $t$ can be estimated as

$$\Delta E(t, k') = E' \int_0^t e^{2\gamma(k')\tau} d\tau,$$

where $E'$ is some initial energy in the mode. As we increase $s$, i.e., with faster expansion, the mode spends less time in the unstable range of $k$, and $\Delta E$ decreases. Moreover, equilibrium rotation acts to decrease the area under the $\gamma(k)$ curve, decreasing $\Delta E$ as well. Clearly, this a nonlinear process, and the qualitative description given here motivates further study.

While current-driven instabilities may play a role in the wiggled structures which are observed in some outflows (Reipurth et al. 2002; Worrall et al. 2007), other explanations for these structures have been presented, such as precession of the source object (Masciadri & Raga 2002). In general, a combination of these effects could contribute to the formation of these structures. Since rotation is shown to stabilize the kink mode, knowledge of the jet rotation velocity relative to the AlfVén velocity is critical for understanding the degree to which the kink plays a role.

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