The perturbative QCD predictions for the rare decay $B_s^0 \rightarrow a_0(980) a_0(980)$

Ze-Rui Liang$^{1,*}$ and Xian-Qiao Yu$^{1,†}$

$^1$School of Physical Science and Technology, Southwest University, Chongqing 400715, China

(Dated: March 19, 2019)

In this work, we calculate the branching ratios and CP violations of the $B_s^0 \rightarrow a_0(980) a_0(980)$ decay modes with both charged and neutral $a_0(980)$ mesons for the first time in the pQCD approach. Considering the recent observation of the BESIII collaboration that report a direct evidence of the quark-antiquark structure about the scalar meson $a_0(980)$, we regard $a_0(980)$ as the $q\bar{q}$ quark component in our present work, and then make predictions of this decay model. The branching ratios of our calculations are $B(B_s^0 \rightarrow a_0^0 a_0^0) = (5.17^{+1.36}_{-1.94}) \times 10^{-6}$ and $B(B_s^0 \rightarrow a_0^0 a_0^0) = (2.58^{+1.18}_{-0.92}) \times 10^{-6}$. We also calculate the CP violation parameters of $B_s^0 \rightarrow a_0(980) a_0(980)$ decay modes. The relatively large branching ratios make it easily to be tested by the running LHC-b experiments, and it can help us to understand both the inner properties and the QCD behavior of the scalar meson.

I. INTRODUCTION

It is well-known that, the rare decays, which only have pure annihilation contributions in Standard Model(SM) due to the totally different quark components between the initial and final state mesons, can provide rich information of CP violations and signals of possible new physics beyond the SM. The QCD factorization approach [1, 2], where the non-factorizable spectator scattering contributions and the annihilation contributions are adjustable parameters, which make the prediction unreliable. However, in the perturbative QCD(pQCD) approach [3–5], many rare decay modes can been studied [6–9], where the theoretical results were coincident well with the experimental data and it proved that the successful application of pQCD approach to mesons rare decays. Since the first scalar meson $f_0(980)$ was observed by the Belle collaboration in the charged decay mode $B^\pm \rightarrow K^\pm f_0(980) \rightarrow K^\pm \pi^+ \pi^\pm$ [10], and afterwards confirmed by BaBar [11], a lot of other scalar mesons have been discovered in the experiment successfully, many researches have been done about light scalar mesons [12–19]. However, as far as we know, there are very few works about the $B \rightarrow SS$ decays(S denote the scalar mesons) to be studied in these general factorization approaches, besides the $B \rightarrow a_0 a_0$ [13] and $B_s(a_{u/d}) \rightarrow K_S^0(1430)\bar{K}_S^0(1430)$ [20].

For a long time, the scalar mesons, especially for the $a_0(980)$ and $f_0(980)$, which are important for understanding the chiral symmetry and confinement in the low-energy region, are one of the key problems in the nonperturbative QCD [21]. However, the inner structure of scalar mesons is still a contradiction in both the theoretical and experimental side, and many works have been done about the scalar meson in order to solve this problem. In Ref. [21], the authors list many evidences that sustain the four-quark model of the light scalar mesons based on a series of experimental data. In Ref. [22], the predicted result of $B \rightarrow a_0(980)K$ is 2 times difference from the experimental result, and the author conclude that $a_0(980)$ cannot be interpreted as $q\bar{q}$. In Ref. [23], the authors showed that the production of the $S^*$ and $\delta$ and of low-mass $K \bar{K}$ pair have properties of the $K \bar{K}$ molecules. Moreover, the scalar meson are identified as the quark-antiquark quark hybrid. Nevertheless, these interpretations of the scalar mesons make theoretical calculations difficult, apart from the ordinary $q\bar{q}$ model.

In theoretical side, there are two interpretations about light scalar mesons below 2 GeV in Review of Particle Physics [24], the scalars below 1 GeV, including $f_0(500)$, $K^*(700)$, $f_0(980)$ and $a_0(980)$, form a SU(3) flavor nonet, and $f_0(1370)$, $a_0(1450)$, $K^*(1430)$ and $f_0(1500)$ (or $f_0(1700)$) that above 1 GeV form another SU(3) flavor nonet. In order to describe the structure of these light scalar mesons, the authors of Ref. [17] presented two Scenarios to clarify the scalar mesons (here, we only focus on the flavor wave function of the $a_0(980)$ meson, which are given in Ref. [25]):

(1) Scenario 1, the light scalar mesons, which involved in the first SU(3) flavor nonet, are usually regarded as the lowest-lying $q\bar{q}$ states, and the other nonet as the relevant first excited states. In the ordinary diquark model, the quark components of $a_0(980)$ are

\[ a_0^+(980) = ud, a_0^-(980) = \bar{u}\bar{d}, a_0^0(980) = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}), \]  

(1)

(2) Scenario 2, the scalar mesons in the second nonet are regarded as the ground states($q\bar{q}$), and the mass between 2.0 $\sim$ 2.3 GeV is first excited states. This Scenario indicate that the scalars below or near 1 GeV are four-quark bound states, while other

* Electronic address: liangzr@email.swu.edu.cn
† Electronic address: yuxq@swu.edu.cn
scalars consist of $q\bar{q}$ in Scenario 1. So the quark components of $a_0(980)$ are

\[a_0^+(980) = u\bar{d}s\bar{s}, \quad a_0^- (980) = \bar{u}d\bar{s}s, \quad a_0^0(980) = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})s\bar{s}.\] (2)

Recently, BES III collaboration declare that the flavor wave function of $a_0^+(980)$ and $a_0^0(980)$ are two-quark component through the decays $D^0 \to d\bar{u}e^+\nu \to a_0^-(980)e^+\nu$ and $D^+ \to d\bar{d}e^+\nu \to a_0^0(980)e^+\nu$ (and the charge conjugated ones), the decay modes are direct probe of the quark components of $a_0^\pm(980)$ and $a_0^0(980)$ [26]. And in Ref. [27], BES III declare the $a_0^0(980)$-$f_0(980)$ mixing in the $J/\psi \to \phi f_0(980) \to \phi a_0^0(980) \to \phi \eta'\pi^0$ and $\chi_{c1} \to a_0^0(980)\pi^0 \to f_0(980)\eta \to \pi^+\pi^-\pi^0$ decay modes, which is the first observation of $a_0^0(980)$-$f_0(980)$ mixing in experiment. But in this work, we will let the mixing effect aside and want to make comprehensive research in the future work.

In this present work, motivated by the uncertain inner structure of the $a_0(980)$, we explore the branching ratios and CP-violating asymmetries of rare decay mode $B_s^0 \to a_0(980)a_0(980)$ \(^1\) in perturbative QCD approach within the traditional two-quark model for the first time. Because the LHC-b collaboration are collecting more and more B mesons decays data, so we believe that our results can be testified by the experiment in the near future time.

This article is organized roughly in this order: in Section II, we give a theoretical framework of the pQCD, list the wave functions that we need in the calculations, and also the perturbative calculations; in Section III, we make numerical calculations and some discussions for the results that we get; and at last, we summary our work in the final Section.

II. THE THEORETICAL FRAMEWORK AND PERTURBATIVE CALCULATION

The pQCD approach have been widely applied to calculate the hadronic matrix elements in the B mesons decay modes, it is based on the $k_T$ factorization. The divergence of the end-point singularity can be safely avoided by preserving the transverse momenta $k_T$ in the valence quark, and the only input parameters are the wave functions of the involved mesons in this method. Then the transition form factors and the different contributions, whose may contain the spectator and annihilation diagrams, are all calculated in this framework.

A. Wave Functions and Distribution Amplitudes

In kinematics aspects, we adopt the light-cone coordinate system in our calculation. Assuming the $B_s^0$ meson to be rest in the system, we can describe the momenta of the mesons in light-cone coordinate system, where the momenta are expressed in the form of $(p^+, p^-, p_T)$ with the definition $p^\pm = \frac{p_0 \pm p_3}{\sqrt{2}}$ and $p_T = (p_1, p_2)$.

In our calculation, the wave function of the hadron $B_s^0$ can be found in Refs. [28–30]

\[\Phi_{B_s^0} = \frac{i}{\sqrt{2N_c}} (\hat{p}_B + m_{B_s}) \gamma_5 \phi_{B_s}(x_1, b_1),\] (3)

where the distribution amplitude(DA) $\phi_{B_s}(x_1, b_1)$ of $B_s^0$ meson is written as mostly used form, which is

\[\phi_{B_s}(x_1, b_1) = N_B x_1^2 (1 - x_1)^2 \exp\left[-\frac{m_{B_s}^2 x_1^2}{2\omega_{B_s}^2} - \frac{1}{2} (\omega_{B_s} b_1)^2\right],\] (4)

the normalization factor $N_B = 62.8021$ can be calculated by the normalization relation $\int_0^1 dx \phi_{B_s}(x_1, b_1 = 0) = f_{B_s}/(2\sqrt{2N_c})$ with $N_c = 3$ is the color number and decay constant $f_{B_s} = 227.2 \pm 3.4$ MeV. Here, we choose shape parameter $\omega_{B_s} = 0.50 \pm 0.05$ GeV [9].

For the scalar meson $a_0(980)$, the wave function can be read as [17, 31]:

\[\Phi_{a_0}(x) = \frac{1}{2\sqrt{2N_c}} [\hat{p} \phi_{a_0}(x) + m_{a_0} \phi_{a_0}^S(x) + m_{a_0} (\hat{p} \cdot \hat{n} - 1) \phi_{a_0}^T(x)],\] (5)

where $x$ denotes the momentum fraction of the meson, and $n = (1, 0, 0_T), v = (0, 1, 0_T)$ are light-like dimensionless vectors.

\(^1\) $a_0(980)$ will abbreviated as $a_0$ in the last part.
The $\phi_{a_0}$ is leading-twist distribution amplitude, the explicit form of which is expanded by the Gegenbauer polynomials [17, 31]:

$$\phi_{a_0}(x, \mu) = \frac{3}{\sqrt{2N_c}}x(1-x)\{f_{a_0}(\mu) + \bar{f}_{a_0}(\mu)\} \sum_{m=1,3}^{\infty} B_m(\mu) C_m^{3/2}(2x - 1),$$  \hspace{0.5cm} (6)

and the twist-3 DAs $\phi_{a_0}^S$ and $\phi_{a_0}^T$ are adopted the asymptotic forms in our predictive calculation,

$$\phi_{a_0}^S(x, \mu) = \frac{1}{2\sqrt{2N_c}} \bar{f}_{a_0}(\mu), \hspace{0.5cm} (7)$$
$$\phi_{a_0}^T(x, \mu) = \frac{1}{2\sqrt{2N_c}} \bar{f}_{a_0}(\mu)(1 - 2x), \hspace{0.5cm} (8)$$

where $f_{a_0}$ and $\bar{f}_{a_0}$ are the vector and scalar decay constants of the $a_0$ meson respectively, $B_m$ is Gegenbauer moment and $C_m^{3/2}(2x - 1)$ in DA of $\phi_{a_0}$ is Gegenbauer polynomials, these parameters are scale-dependent. A lot of calculations have been carried out about the light scalar mesons in various model [32–34]. In this article, we adopt the value for decay constants and Gegenbauer moments in the DAs of the $a_0$ as listed follow, which were calculated in QCD sum rules at the scale $\mu = 1$ GeV [17]:

$$\bar{f}_{a_0} = 0.365 \pm 0.020\text{GeV}, B_1 = -0.93 \pm 0.10, B_3 = 0.14 \pm 0.08.$$  \hspace{0.5cm} (9)

It’s noticeable that only the odd Gegenbauer moments are taken into account due to the conservation of vector current or charge conjugation invariance. And we also pay attention to only the Gegenbauer moments $B_1$ and $B_3$ because the higher order Gegenbauer moments make tiny contributions and can be ignored safely.

And the Gegenbauer polynomials are

$$C_1^{3/2}(2x - 1) = 3(2x - 1), \hspace{0.5cm} C_3^{3/2}(2x - 1) = \frac{35}{2}(2x - 1)^3 - \frac{15}{2}(2x - 1).$$  \hspace{0.5cm} (10)

The vector and scalar decay constants satisfy the relationship

$$\bar{f}_{a_0}(\mu) = \mu_{a_0} f_{a_0}(\mu)$$  \hspace{0.5cm} (11)

with

$$\mu_{a_0} = \frac{m_{a_0}}{m_d(\mu) - m_u(\mu)},$$  \hspace{0.5cm} (12)

and $m_{a_0}$ is the mass of the scalar meson $a_0$ and $m_d$ and $m_u$ are the running current quark masses in the $a_0$ meson. From the above relationship, it is clear to see that the vector decay constant is proportional to the mass difference between the $d$ and $u$ quark, the mass difference is so small after considering the $SU(3)$ symmetry breaking that would heavily suppress the vector decay constant, which lead to the vector decay constants of the scalar mesons are very small and can be negligible. Likewise, for the same reason that only the odd Gegenbauer moments are considered, the neutral scalar mesons can not be produced by the vector current, so in this work we adopt the vector constant $f_{a_0} = 0$.

And the normalization relationship of the twist-2 and twist-3 DAs are

$$\int_0^1 dx \phi_{a_0}(x) = 0,$$
$$\int_0^1 dx \phi_{a_0}^S(x) = \bar{f}_{a_0},$$
$$\int_0^1 dx \phi_{a_0}^T(x) = \frac{\bar{f}_{a_0}}{2\sqrt{2N_c}}.$$  \hspace{0.5cm} (13)

B. Perturbative Calculations

For $B_s^0 \rightarrow a_0a_0$ decay mode, the relevant weak effective Hamiltonian can be written as [35]

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \left\{ V_{ub} V_{us}^* \left[ C_1(\mu)O_1(\mu) + C_2(\mu)O_2(\mu) \right] - V_{tb} V_{ts}^* \left[ \sum_{i=3}^{10} C_4(\mu)O_i(\mu) \right] \right\},$$  \hspace{0.5cm} (14)
where $G_F = 1.66378 \times 10^{-5}$ GeV$^{-2}$ is Fermi constant, and $V_{ud}V_{us}^*$ and $V_{tb}V_{ts}^*$ are Cabibbo-Kobayashi-Maskawa (CKM) factors, $O_i(\mu) \ (i = 1, 2, ..., 10)$ is local four-quark operator, which will be listed as follows, and $C_i(\mu)$ is corresponding Wilson coefficient.

(1) Current-Current Operators (Tree):

\begin{align}
O_1 &= (\bar{s}_\alpha u_\beta)_{V-A}(\bar{u}_\beta b_\alpha)_{V-A}, \\
O_2 &= (\bar{s}_\alpha u_\alpha)_{V-A}(\bar{u}_\beta b_\beta)_{V-A},
\end{align}

(2) QCD Penguin Operators:

\begin{align}
O_3 &= (\bar{s}_\alpha b_\alpha)_{V-A} \sum_q (\bar{q}_\beta q_\beta)_{V-A}, \\
O_4 &= (\bar{s}_\alpha b_\beta)_{V-A} \sum_q (\bar{q}_\beta q_\alpha)_{V-A}, \\
O_5 &= (\bar{s}_\alpha b_\beta)_{V-A} \sum_q (\bar{q}_\beta q_\beta)_{V-A}, \\
O_6 &= (\bar{s}_\alpha b_\beta)_{V-A} \sum_q (\bar{q}_\beta q_\alpha)_{V+A},
\end{align}

(3) Electroweak Penguin Operators:

\begin{align}
O_7 &= \frac{3}{2}(\bar{s}_\alpha b_\alpha)_{V-A} \sum_q e_q(\bar{q}_\beta q_\beta)_{V+A}, \\
O_8 &= \frac{3}{2}(\bar{s}_\alpha b_\beta)_{V-A} \sum_q e_q(\bar{q}_\beta q_\alpha)_{V+A}, \\
O_9 &= \frac{3}{2}(\bar{s}_\alpha b_\alpha)_{V-A} \sum_q e_q(\bar{q}_\beta q_\beta)_{V-A}, \\
O_{10} &= \frac{3}{2}(\bar{s}_\alpha b_\beta)_{V-A} \sum_q e_q(\bar{q}_\beta q_\alpha)_{V-A},
\end{align}

with the color indices $\alpha, \beta$ and $(qq)_{V\pm A} = \bar{q}\gamma_\mu(1 \pm \gamma_5)q$. The $q$ denotes the $u$ quark and $d$ quark, and $e_q$ is corresponding charge.

FIG. 1. The lowest order Feynman diagrams of the $\bar{B}_s^{0} \to a_0^+ a_0^-$ decays in pQCD approach.
The momenta of the $B_0^0, a_0^+, a_1^0$ meson in the light-cone coordinate read as

\[ p_B = p_1 = \frac{m_B}{\sqrt{2}} (1, 1, 0_T), \]
\[ p_2 = \frac{m_B}{\sqrt{2}} (r_{a_0}^2, 1 - r_{a_0}^2, 0_T), \]
\[ p_3 = \frac{m_B}{\sqrt{2}} (1 - r_{a_0}^2, r_{a_0}^2, 0_T), \]

with the $B_0^0$ mass $m_{B_0}$ and the mass ratio $r_{a_0} = \frac{m_{a_0}}{m_{B_0}}$.

And the corresponding light quark’s momenta in each meson read as

\[ k_1 = (x_1 p_1^T, 0, k_{1T}) = \left( \frac{m_B}{\sqrt{2}} x_1, 0, k_{1T} \right), \]
\[ k_2 = (0, x_2 p_2^T, k_{2T}) = (0, \frac{m_B}{\sqrt{2}} (1 - r_{a_0}^2) x_2, k_{2T}), \]
\[ k_3 = (x_3 p_3^T, 0, k_{3T}) = \left( \frac{m_B}{\sqrt{2}} (1 - r_{a_0}^2) x_3, 0, k_{3T} \right). \]

Then based on the pQCD approach, we can write the decay amplitude as

\[ A \sim \int dx_1 dx_2 dx_3 b_1 b_2 b_3 d_3 \times \text{Tr}[H(x_1, b_1, t) C_1 \Phi_B(x_1, b_1) \Phi_{a_0^+}(x_2, b_2) \Phi_{a_0^-}(x_3, b_3) S_{1}(x_1) e^{-S(t)}], \]

where $b_1$ is the conjugate momenta of $k_1$, and $t$ is the largest energy scale in hard function $H(x_1, b_1, t)$. The $e^{-S(t)}$ suppress the soft dynamics [36] and make a reliable perturbative calculation of the hard function $H$, which come from higher order radiative corrections to wave functions and hard amplitudes. $\Phi_M$ represent universal and channel independent wave function, which describes the hadronization of mesons.

Fig. 1 display the typical Feynman diagrams of the $B_0^0 \to a_1^0 a_0^-$ decays at the lowest order, and this decay only have pure annihilation topologies. We can find that this decay is similar to the $B_0^0 \to \pi^+ \pi^-$ [6, 8], which have four diagrams contributing to the $B_0^0 \to a_1^0 a_0^-$, (a),(b) are factorization annihilation diagrams, other two diagrams are non-factorization annihilation diagrams. As depicted in Fig. 1, we calculate the factorizable and non-factorizable annihilation diagrams respectively. We use $F$ and $M$ denote the factorizable and non-factorizable annihilation contributions respectively, and the subscript $a (c)$ denote the contributions of the Feynman diagrams Fig. 1(a) and (b) (Fig. 1(c) and (d)) and the superscript $LL, LR, SP$ is the $(V - A)(V - A)$, $(V - A)(V + A)$ and $(S - P)(S + P)$ vertex, respectively. The vertex $(S - P)(S + P)$ is the Fierz transformation of the $(V - A)(V + A)$.

First, the total contribution of the Feynman diagrams Fig. 1 (a) and (b), which only involve the wave function of the final light scalar mesons, are

1. $(V - A)(V - A)$

\[ F_{a}^{LL} = 16\pi C_{FB} m_{B_0}^2 \int_0^1 dx_2 dx_3 \int_0^\infty b_2 b_3 dB_2 dB_3 \]
\[ \times \left\{ 2r_{a_0}^2 (1 + x_3) \phi_{a_0}^S(x_3) \phi_{a_0}^S(x_2) - 2r_{a_0}^2 (1 - x_3) \phi_{a_0}^T(x_3) \phi_{a_0}^S(x_2) - (r_{a_0}^2 + x_3 - 3r_{a_0}^2 x_3) \phi_{a_0}(x_3) \phi_{a_0}(x_2) \right\} \]
\[ \times h_a(x_2, x_3, b_2, b_3) E_{af}(t_a) S_{1}(x_3) \]
\[ - \left[ 2r_{a_0}^2 (1 + x_2) \phi_{a_0}^S(x_3) \phi_{a_0}^S(x_2) - (r_{a_0}^2 + x_2 - 3r_{a_0}^2 x_2) \phi_{a_0}(x_3) \phi_{a_0}(x_2) - 2r_{a_0}^2 (1 - x_2) \phi_{a_0}^S(x_3) \phi_{a_0}^T(x_2) \right] \]
\[ \times h_b(x_2, x_3, b_2, b_3) E_{af}(t_b) S_{1}(x_2) \right\}, \]

the evolution function $E_{af}(t_i)$ is defined by

\[ E_{af}(t_i) = \alpha_s(t_i) \exp[-S_{a_0^+}(t_i) - S_{a_0^-}(t_i)]. \]

where the largest energy scales $t_i (i = a, b)$ to eliminate the large logarithmic radiative corrections are chosen as:

\[ t_a = \max\{M_{B_0} \sqrt{x_i}, 1/b_2, 1/b_3\}, \]
\[ t_b = \max\{M_{B_0} \sqrt{x_i}, 1/b_2, 1/b_3\}. \]

2. $(V - A)(V + A)$

\[ F_{a}^{LR} = F_{a}^{LL}, \]
Then the total non-factorizable annihilation decay amplitudes for the Fig. 1 (c) and (d) diagrams are

$$M_{cL} = \frac{64\pi C_F m_B^2}{\sqrt{2} N_c} \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 b_2 db_1 db_2 \phi_B(x_1, b_1)$$

$$\times \{ [(r_{a0}^2 (x_1 - x_3 + 2 x_2) - x_2) \phi_{a0}(x_2) \phi_{a0}(x_3) - r_{a0}^2 (x_1 - x_3 - x_2) \phi_{a0}^S(x_2) \phi_{a0}^S(x_3)$$

$$- r_{a0}^2 (x_1 - x_3 - x_2) \phi_{a0}^T(x_2) \phi_{a0}^T(x_3)]$$

$$\times h_c(x_1, x_2, x_3, b_1, b_2) E_{naf}(t_c) \}$$

$$+ \{ [(r_{a0}^2 (x_2 - x_1 - 2 x_3 - 2) + x_1 + x_3) \phi_{a0}(x_2) \phi_{a0}(x_3) - r_{a0}^2 (2 + x_1 + x_3 + x_2) \phi_{a0}^S(x_2) \phi_{a0}^S(x_3)$$

$$+ r_{a0}^2 (x_2 - x_1 - x_3) \phi_{a0}^S(x_2) \phi_{a0}^T(x_3)$$

$$+ r_{a0}^2 (x_2 - x_1 - x_3) \phi_{a0}^T(x_2) \phi_{a0}^T(x_3)]$$

$$\times h_d(x_1, x_2, x_3, b_1, b_2) E_{naf}(t_d) \} \}$$

(25)

$$M_{cP} = -\frac{64\pi C_F m_B^2}{\sqrt{2} N_c} \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 b_2 db_1 db_2 \phi_B(x_1, b_1)$$

$$\times \{ [(x_1 - x_3 + r_{a0}^2 (x_1 - x_3 + x_2) \phi_{a0}(x_2) \phi_{a0}(x_3) + r_{a0}^2 (x_1 - x_3 - x_2) \phi_{a0}^S(x_2) \phi_{a0}^S(x_3)$$

$$+ r_{a0}^2 (x_1 + x_2 - x_3) \phi_{a0}^S(x_2) \phi_{a0}^T(x_3) + r_{a0}^2 (x_1 + x_2 - x_3) \phi_{a0}^T(x_2) \phi_{a0}^S(x_3)$$

$$+ r_{a0}^2 (x_1 - x_3 - x_2) \phi_{a0}^T(x_2) \phi_{a0}^T(x_3)]$$

$$\times h_c(x_1, x_2, x_3, b_1, b_2) E_{naf}(t_c) \}$$

$$+ \{ [-x_2 - r_{a0}^2 (x_1 + x_3 - 2 x_2 - 2) \phi_{a0}(x_2) \phi_{a0}(x_3) + r_{a0}^2 (2 + x_1 + x_3 + x_2) \phi_{a0}^S(x_2) \phi_{a0}^S(x_3)$$

$$- r_{a0}^2 (x_1 + x_2 - x_3) \phi_{a0}^S(x_2) \phi_{a0}^T(x_3) - r_{a0}^2 (x_1 + x_2 - x_3) \phi_{a0}^T(x_2) \phi_{a0}^S(x_3)$$

$$+ r_{a0}^2 (-2 + x_1 + x_3 + x_2) \phi_{a0}^T(x_2) \phi_{a0}^T(x_3)]$$

$$\times h_d(x_1, x_2, x_3, b_1, b_2) E_{naf}(t_d) \} \}$$

(26)

with the color factor $C_F = \frac{N_c^2 - 1}{2 N_c} = \frac{4}{3}$.

The evolution function is

$$E_{naf}(t_i) = \alpha_s(t_i) \exp[-S_{B_i}(t_i) - S_{a0}(t_i) - S_{a0}(t_i)]_{b_2=b_3}$$

(27)

with the hard scales

$$t_c = \max\{M_B, \sqrt{x_1 x_2 x_3}, M_B, \sqrt{|x_1 x_2 - x_2 x_3|}, 1/b_1, 1/b_2\}$$

(28)

$$t_d = \max\{M_B, \sqrt{x_1 x_2 x_3}, M_B, \sqrt{|x_1 + x_2 + x_3 - x_1 x_2 - x_2 x_3|}, 1/b_1, 1/b_2\}$$

The hard scattering kernels function $h_i (i = a, b, c, d)$ involved in the above expression are written as:

$$h_a(x_2, x_3, b_2, b_3) = \frac{\pi i}{2} H_0^{(1)}(M_B b_2 \sqrt{x_2 x_3}) \times [\theta(b_2 - b_3) J_0(M_B b_3 \sqrt{x_1 x_3}) \frac{\pi i}{2} H_0^{(1)}(M_B b_2 \sqrt{x_3}) + (b_2 \leftrightarrow b_3)]$$

(29)

$$h_b(x_2, x_3, b_2, b_3) = \frac{\pi i}{2} H_0^{(1)}(M_B b_3 \sqrt{x_1 x_3}) \times [\theta(b_2 - b_3) J_0(M_B b_3 \sqrt{x_2 x_3}) \frac{\pi i}{2} H_0^{(1)}(M_B b_2 \sqrt{x_3}) + (b_2 \leftrightarrow b_3)]$$

(30)

$$h_c(x_1, x_2, x_3, b_1, b_2) = [\theta(b_2 - b_1) J_0(M_B b_1 \sqrt{x_1 x_2}) \frac{\pi i}{2} H_0^{(1)}(M_B b_2 \sqrt{x_2 x_3}) + (b_2 \leftrightarrow b_1)]$$

(31)

$$h_d(x_1, x_2, x_3, b_1, b_2) = [\theta(b_2 - b_1) J_0(M_B b_1 \sqrt{x_1 x_2 x_3}) \frac{\pi i}{2} H_0^{(1)}(M_B b_2 \sqrt{x_2 x_3}) + (b_2 \leftrightarrow b_1)]$$

(32)
where $J_0$ is the Bessel function and $K_0$, $I_0$ are modified Bessel function with $H_0^{(1)}(x) = J_0(x) + iY_0(x)$.

The $S_B(x_1)$, $S_{a_0}(x_i)$ used in the decay amplitudes are defined as:

$$S_B(x_1) = s(x_1p_1^+, b_1) + \frac{5}{3} \int_{1/b_1}^{t} \frac{d\bar{\mu}}{\mu} \gamma_q(\alpha_s(\bar{\mu})),
$$

$$S_{a_0}(x_2) = s(x_2p_2^+, b_2) + s(\bar{x}_2p_2^+, b_2) + 2 \int_{1/b_2}^{t} \frac{d\bar{\mu}}{\mu} \gamma_q(\alpha_s(\bar{\mu})),
$$

$$S_{a_0}(x_3) = s(x_3p_3^-, b_3) + s(\bar{x}_3p_3^-, b_3) + 2 \int_{1/b_3}^{t} \frac{d\bar{\mu}}{\mu} \gamma_q(\alpha_s(\bar{\mu})),
$$

(33)

where $\bar{x}_i = 1 - x_i$ and $\gamma_q = -\alpha_s/\pi$ is the anomalous dimension of the quark, and the Sudakov factor $s(Q, b)$ are resulting from the resummation of double logarithms and can be found in Ref. [37].

$$s(Q, b) = \int_{\mu}^{Q} \frac{d\bar{\mu}}{\mu} \left[ \ln\left(\frac{Q}{\bar{\mu}}\right) A(\alpha(\bar{\mu})) + B(\alpha(\bar{\mu})) \right]
$$

(34)

with

$$A = C_F \frac{\alpha_s}{\pi} + \left[ \frac{67}{9} - \frac{\pi^2}{3} - \frac{10}{27} n_f + \frac{3}{2} \beta_0 \ln\left(\frac{e^{\gamma_E}}{2}\right) \right] \frac{(\alpha_s)}{\pi}^2,
$$

$$B = \frac{2}{3} \frac{\alpha_s}{\pi} \ln\left(\frac{e^{2\gamma_E-1}}{2}\right),
$$

(35)

where $\gamma_E$ and $n_f$ are Euler constant and the active flavor number, respectively.

The threshold resummation factor $S_t(x)$ have been parameterized in [38], which is:

$$S_t(x) = \frac{2^{1+2c} \Gamma\left(\frac{3}{2} + c\right)}{\sqrt{\pi^c \Gamma(1+c)}} [x(1-x)]^c
$$

(36)

with the fitted parameter $c = 0.3$.

By adding the above contributions from the all of the Feynman diagrams in Fig. 1, the decay amplitude of the decay $B_s^0 \rightarrow a_0^+ a_0^-$ is then

$$\mathcal{A}(B_s^0 \rightarrow a_0^+ a_0^-) = V_{ub}V_{us}^* [C_2 M_{LL}^c] - V_{ub}V_{ts}^* ((2C_4 + \frac{1}{2} C_{10}) M_{LL}^c + (2C_6 + \frac{1}{2} C_8) M_{LL}^c)
$$

(37)

and the corresponding decay width is

$$\Gamma(B_s^0 \rightarrow a_0^+ a_0^-) = \frac{G_F^2 m_{B_s}^3}{128\pi} (1 - 2r_{a_0}^2)|\mathcal{A}(B_s^0 \rightarrow a_0^+ a_0^-)|^2.
$$

(38)

Here, it is noticeable that the contribution from the factorizable annihilation diagrams is very small and can be safely neglected due to the isospin symmetry.

Meanwhile, the decay amplitude for $B_s^0 \rightarrow a_0^0 a_0^0$ decay is

$$\sqrt{2}|\mathcal{A}(B_s^0 \rightarrow a_0^0 a_0^0)| = \mathcal{A}(B_s^0 \rightarrow a_0^+ a_0^-)
$$

(39)

and the decay width is

$$\Gamma(B_s^0 \rightarrow a_0^0 a_0^0) = \frac{G_F^2 m_{B_s}^3}{256\pi} (1 - 2r_{a_0}^2)|\mathcal{A}(B_s^0 \rightarrow a_0^0 a_0^0)|^2.
$$

(40)
III. NUMERICAL RESULTS AND DISCUSSIONS

In this section, we will calculate the CP-averaged branching ratios and CP-violation asymmetries for the $B_s^0 \to a_0a_0$ decays and make some analyses about the results. First, we list the input parameters used in the calculations below. The masses and decay constant of the mesons, the lifetimes of the $B_s$ are [24],

$$m_{B_s} = 5.367 \text{GeV}, \bar{m}_b(m_b) = 4.2 \text{GeV}, m_{a_0} = 0.98 \pm 0.02 \text{GeV},$$

$$f_{B_s} = 227.2 \pm 3.4 \text{MeV}, \tau_{B_s} = 1.509 \text{ps}.$$  \hspace{1cm} (41)

and in the CKM matrix elements, the involved Wolfenstein parameters are

$$\lambda = 0.22453 \pm 0.00044, A = 0.836 \pm 0.015,$$

$$\bar{\rho} = 0.122^{+0.018}_{-0.017}, \bar{\eta} = 0.355^{+0.012}_{-0.011}.$$  \hspace{1cm} (42)

with the relations $\bar{\rho} = \rho (1 - \frac{1}{3} \frac{\Delta m}{\Delta s})$ and $\bar{\eta} = \eta (1 - \frac{1}{3} \frac{\Delta m}{\Delta s}).$

A. Branching Ratios

In SM, the $\gamma$ angle is associated with the CKM matrix element $V_{ub}$, which have the relationship $V_{ub} \sim |V_{ub}| e^{-i\gamma}$. So we can leave the the CKM phase angle $\gamma$ as a unknown parameter, and write the decay amplitude of the $B_s^0 \to a_0a_0$ decay, which is based on the Eq. (37),

$$\bar{A} = V_{ub}V_{us}^* T - V_{tb}V_{ts}^* P = V_{ub}V_{us}^* T (1 + z e^{i(\delta + \gamma)}),$$  \hspace{1cm} (43)

where the ratio $z = |V_{tb}V_{ts}^* / V_{ub}V_{us}^*| \cdot |P/T|$, and $\delta$ is the relative strong phase between the tree amplitudes($T$) and penguin amplitudes($P$). The value of $z$ and $\delta$ can be calculated from the pQCD.

Meanwhile, the decay amplitude of the conjugated decay mode $B_s^0 \to a_0a_0$ can be written by replacing $V_{ub}V_{us}^*$ with $V_{ub}V_{us}$ and $V_{tb}V_{ts}^*$ with $V_{tb}V_{ts}$ as

$$A = V_{ub}V_{us}^* T - V_{tb}V_{ts}^* P = V_{ub}V_{us}^* T (1 + z e^{i(\delta - \gamma)}).$$  \hspace{1cm} (44)

Then from the Eq. (43) and (44), the CP-averaged decay width of $B_s^0(B_s^0) \to a_0 a_0$ is

$$\Gamma(B_s^0(B_s^0) \to a_0^+ a_0^-) = \frac{G_F^2 m_{B_s}^3}{256 \pi} (1 - 2 r_{a_0}) (|A|^2 + |\bar{A}|^2)$$

$$= \frac{G_F^2 m_{B_s}^3}{128 \pi} (1 - 2 r_{a_0}) V_{ub}V_{us}^* T^2 (1 + 2 z \cos(\gamma) \cos(\delta) + z^2).$$  \hspace{1cm} (45)

In Fig. 2, we plot the average branching ratio of the decay $B_s^0 \to a_0^+ a_0^-$ and $B_s^0 \to a_0^- a_0^0$ about the parameter $\gamma$ respectively. Since the CKM angle $\gamma$ is constrained as $\gamma$ around 73.5$^\circ$ in Review of Particle Physics [24],

$$\gamma = (73.5 \pm 1.2)^\circ$$  \hspace{1cm} (46)

we get from Fig. 2 when we take $\gamma$ as $70^\circ \sim 80^\circ$,

$$5.08 \times 10^{-6} < B(B_s^0 \to a_0^+ a_0^-) < 5.34 \times 10^{-6};$$  \hspace{1cm} (47)

$$2.54 \times 10^{-6} < B(B_s^0 \to a_0^- a_0^0) < 2.67 \times 10^{-6}.$$  \hspace{1cm} (48)

The value of $z = 6.67$ indicate that the amplitude of the penguin diagrams is almost 6.67 times of that of tree diagrams. Therefore the main contribution come from the penguin diagrams in this decays, which enhance the results of the branching ratios.

When we utilize the input parameters and decay amplitudes and leave the phase angle $\gamma$ aside, it is easy to get the CP-average branching ratios for both containing the charged and neutral scalar mesons decay modes, which are

$$B(B_s^0 \to a_0^+ a_0^-) = 5.17^{+1.62}_{-1.39} (B_1)^{+0.24}_{-0.09} (B_3)^{+1.23}_{-1.03} (f_{a_0})^{+0.63}_{-0.55} (\omega_b)^{+0.99}_{-0.87} (t_e) \times 10^{-6},$$  \hspace{1cm} (49)
By adding all of the vital uncertainties in quadrature, we get neglected. It is apparent that the main errors are mainly caused by the non-perturbative input parameters, which we need more precise experimental data to determine. By adding all of these vital uncertainties in quadrature, we get $\mathcal{B}(\bar{B}_s^0 \to a_0^+ a_0^-) = (5.17^{+2.36}_{-1.94}) \times 10^{-6}$ and $\mathcal{B}(\bar{B}_s^0 \to a_0^0 a_0^0) = (2.58^{+1.18}_{-0.92}) \times 10^{-6}$.

In our previous work of $B_s^0 \to \pi^+ \pi^-$ [8] (one of the author have recalculated the $B_s^0 \to \pi^+ \pi^-$ and $B^0 \to K^+ K^-$ in 2012 [6]), the theoretical results of these two decay modes are $\mathcal{B}(B_s^0 \to \pi^+ \pi^-) = 5.10 \times 10^{-7}$ and $\mathcal{B}(B^0 \to K^+ K^-) = 1.56 \times 10^{-7}$, where the corresponding experimental results [39, 40] of these two decay modes have the branching ratios approximately at the order of $10^{-7} \sim 10^{-8}$. The predicted results of $B_s^0 \to a_0 a_0$ for both charged and neutral $a_0$ mesons, however, are at the order of $10^{-6}$ although these decay modes have the same quark components for both initial and final state mesons and the only pure annihilation contributions. So this results push us to make some comment about why the branching ratio of the $B_s^0 \to a_0^+ a_0^-$ is more large than the results of the $B_s^0 \to \pi^+ \pi^-$ decay and $B^0 \to K^+ K^-$ decay. By comparison, we can first find that the main underlying reason is that the QCD dynamics of the scalar meson $a_0$ is different from that of the pseudoscalar meson $\pi$ and $K$, where at the leading twist the scalar meson $a_0$ is dominated by the odd Gegenbauer polynomials but the pseudoscalar mesons both $\pi$ and $K$ are governed by the even Gegenbauer polynomials. Second the decay constant $f_{a_0}$ is about two times than the decay constants of the $f_{\pi}$ and $f_K$ [6, 41]. These two reasons lead to the non-factorizable annihilation contribution is more large in the $B_s^0 \to a_0 a_0$ mode. In Tab. I, we list the decay amplitudes of the $B_s^0 \to a_0 a_0$ for different distribution amplitudes of twist-2 or twist-3, and also we list the results of Ref. [6] about the decay mode $B^0 \to K^+ K^-$ for contrast. From Tab. I, it is obvious that the twist-2 DA make dominant contribution, and the decay amplitudes of the $B_s^0 \to a_0 a_0$ decay is approximately one order of the magnitude larger than that of the $B^0 \to K^+ K^-$.

Furthermore, in order to reduce the uncertainties from the input non-perturbative parameters, such as the decay constant and the Gegenbauer moments on the theoretical side, we can define the ratio of the branching ratios of this two decay modes. From the numerical results of the considered decays, we get the ratio is

$$R = \frac{\mathcal{B}(\bar{B}_s^0 \to a_0^+ a_0^-)}{\mathcal{B}(\bar{B}_s^0 \to a_0^0 a_0^0)} \approx 2.0.$$  \hspace{1cm} (51)

The value 2.0 is mainly ascribed to the relationship between the decay amplitudes that $\sqrt{\lambda} A(\bar{B}_s^0 \to a_0^0 a_0^0) = A(\bar{B}_s^0 \to \pi^+ \pi^-)$.
TABLE I. The different source of twist-2 and twist-3 contribution.

| decay mode              | twist-2 $\phi_{\alpha \lambda}(\phi_{\alpha \lambda}^P)$ | twist-3 $\phi_{\alpha \lambda}^P(\phi_{\alpha \lambda}^P)$ | twist-3 $\phi_{\alpha \lambda}^T(\phi_{\alpha \lambda}^T)$ |
|-------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|
| $A(\bar{B}_s^0 \to a_0^+ a_0^-)$ | $(-2.0 - 2.1i) \times 10^{-4}$                          | $(+4.2 + 4.1i) \times 10^{-5}$                          | $(-2.27 - 0.79i) \times 10^{-6}$                         |
| $A(\bar{B}_s^0 \to K^+ K^-)$ [6] | $(-0.31 - 2.2i) \times 10^{-5}$                          | $(-0.61 - 0.55i) \times 10^{-5}$                        | $(-0.06 - 0.27i) \times 10^{-5}$                         |

$a_0^+ a_0^-$ and it can not be affected by other factors. Therefore, the relation $Br(\bar{B}_s^0 \to a_0^+ a_0^-) \sim 2Br(\bar{B}_s^0 \to a_0^0 a_0^0)$ is understandable and acceptable. Once the predicted results of this decay mode is confirmed by the collaborations, we will get more information about scalar mesons’s structure and its QCD behavior as well.

B. CP Violation Parameters

Now, we will calculate the CP violation parameters of the $\bar{B}_s^0 \to a_0 a_0$ decays in this subsection. The CP violation parameters of the $\bar{B}_s^0 \to a_0 a_0$ for both charged and neutral $a_0$ mesons are same because the decay amplitude of these two decay modes are similar and the factor in the front of the decay width formula can be reduced. In SM, CP violation originated from the CKM weak angle. For the neutral $B_s^0$ meson decays, we should take the effect of $B_s^0 - \bar{B}_s^0$ mixing into account, and the time dependent CP violation parameters of the two $B_s^0 \to a_0 a_0$ decays with charged and neutral scalar mesons and can be defined as

$$A_{\text{CP}} = \frac{\Gamma (B_s^0(\Delta t) \to a_0 a_0) - \Gamma (\bar{B}_s^0(\Delta t) \to a_0 a_0)}{\Gamma (B_s^0(\Delta t) \to a_0 a_0) + \Gamma (\bar{B}_s^0(\Delta t) \to a_0 a_0)}$$

$$= A_{\text{CP}}^{\text{dir}} \cos(\Delta m \Delta t) + A_{\text{CP}}^{\text{mix}} \sin(\Delta m \Delta t),$$

where $\Delta m$ is the mass difference between the two neutral $B_s^0$($\bar{B}_s^0$) mass eigenstates, and $\Delta t = t_{CP} - t_{tag}$ is the time difference between the tagged $B_s^0$($\bar{B}_s^0$) and the accompanying $B_s^0$($\bar{B}_s^0$) with opposite $b$ flavor decaying to the final CP eigenstate $a_0 a_0$ at the time $t_{CP}$.

From Eqs. (43) and (44), the direct CP violation parameter $A_{\text{CP}}^{\text{dir}}$ can be parameterized as

$$A_{\text{CP}}^{\text{dir}} \sim \frac{2 z \sin(\delta) \sin(\gamma)}{1 + 2 z \cos(\delta) \cos(\gamma) + z^2};$$

where $A_{\text{CP}}^{\text{dir}}$ is approximately proportional to CKM angle $\sin(\gamma)$, strong phase $\sin(\delta)$, and the relative size $z$ between the penguin contribution and tree contribution. We plot the direct CP violation parameter $A_{\text{CP}}^{\text{dir}}$ as the function of the weak angle $\gamma$ in Fig. 3, and one can see that the $A_{\text{CP}}^{\text{dir}}$ is approximately $-11.4\%$ at the peak when the $\gamma$ is $70^\circ < \gamma < 80^\circ$. The relative small direct CP asymmetry is also a result of the main contributions coming from penguin diagrams in this decays.

The involved mixing-induced CP violation parameter $A_{\text{CP}}^{\text{mix}}$ can be written as

$$A_{\text{CP}}^{\text{mix}} = -\frac{2 \text{Im}(\lambda_{CP})}{1 + |\lambda_{CP}|^2},$$

with the CP violation parameters $\lambda_{CP}$

$$\lambda_{CP} = \eta_{CP} \frac{V_{ub}^{*} V_{tb} (a_0 a_0 | H_{eff} | \bar{B}_s^0)}{V_{ub}^{*} V_{tb} (a_0 a_0 | H_{eff} | B_s^0)} = \frac{e^{-2i\gamma} (1 + z e^{i(\delta + \gamma)})}{1 + z e^{i(\delta - \gamma)}},$$

in which $\eta_{CP}$ is the CP-eigenvalue of the final state.

If $z$ is a very small number, i.e., the penguin diagram contribution is suppressed comparing with the tree diagram contribution, the mixing induced CP asymmetry parameter $A_{\text{CP}}^{\text{mix}}$ is proportional to $\sin 2\gamma$, which will be a good place for the CKM angle $\gamma$ measurement. However as we have already mentioned, $z(=6.67)$ is large. We give the mixing CP asymmetry in Fig. 4, one can see that $A_{\text{CP}}^{\text{mix}}$ is not a simple $\sin 2\gamma$ behavior because of the so-called penguin pollution. It is close to $25.9\%$ when the angle $\gamma$ is constrained as $\gamma$ around $73.5^\circ$. At present, there are no CP asymmetry measurements in experiment but the possible large CP violation we predict for $\bar{B}_s^0 \to a_0 a_0$ decays might be observed in the coming LHC-b experiments.
In this paper, we make predictions of the rare decay $\bar{B}_s^0 \rightarrow a_0 a_0$ within the pQCD approach for the first time. Basing on the recently experimental results of the $q\bar{q}$ structure of the scalar meson $a_0$, we calculate the branching ratios and CP violation parameters of the decay $\bar{B}_s^0 \rightarrow a_0 a_0$ for both charged and neutral $a_0$ states. Our calculations show that the decay modes have relative large branching ratios, which are $B(\bar{B}_s^0 \rightarrow a_0^+ a_0^-) = (5.17^{+2.36}_{-1.94}) \times 10^{-6}$ and $B(\bar{B}_s^0 \rightarrow a_0^0 a_0^0) = (2.58^{+1.18}_{-0.92}) \times 10^{-6}$, and there is also large CP violation in the decay model, which can be tested by the running LHC-b experiments in the near future, and, of course, helping us to get better understanding of the QCD behavior of the scalar mesons.
ACKNOWLEDGMENTS

The authors would like to thank Dr. Ming-Zhen Zhou for some valuable discussions. This work is supported by the National Natural Science Foundation of China under Grant No.11047028 and No.11875226, and by the Fundamental Research Funds of the Central Universities, Grant Number XDJK2012C040.

[1] M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, Nucl. Phys. B 591, 313 (2000).
[2] M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, Phys. Rev. Lett. 83, 1914 (1999).
[3] H. n. Li and H. L. Yu, Phys. Rev. Lett. 74, 4388 (1995).
[4] H. N. Li and H. L. Yu, Phys. Lett. B 353, 301 (1995).
[5] H. n. Li and H. L. Yu, Phys. Rev. D 53, 2480 (1996).
[6] Z. J. Xiao, W. F. Wang and Y. y. Fan, Phys. Rev. D 85, 094003 (2012).
[7] X. Q. Yu, Y. Li and C. D. Lu, Phys. Rev. D 73, 017501 (2006).
[8] Y. Li, C. D. Lu, Z. J. Xiao and X. Q. Yu, Phys. Rev. D 70, 034009 (2004).
[9] A. Ali, G. Kramer, Y. Li, C. D. Lu, Y. L. Shen, W. Wang and Y. M. Wang, Phys. Rev. D 76, 074018 (2007).
[10] K. Abe et al. [Belle Collaboration], Phys. Rev. D 65, 092005 (2002).
[11] B. Aubert et al. [BaBar Collaboration], Phys. Rev. D 70, 092001 (2004).
[12] X. Liu, Z. J. Xiao and Z. T. Zou, Phys. Rev. D 88, 094003 (2013).
[13] D. Dou, X. Liu, J. W. Li and Z. J. Xiao, J. Phys. G 43, 045001 (2016).
[14] C. D. Lu, Y. M. Wang and H. Zou, Phys. Rev. D 75, 056001 (2007).
[15] F. Colangelo, F. De Fazio and W. Wang, Phys. Rev. D 81, 074001 (2010).
[16] W. Wang, Y. L. Shen, Y. Li and C. D. Lu, Phys. Rev. D 74, 114010 (2006).
[17] H. Y. Cheng, C. K. Chua and K. C. Yang, Phys. Rev. D 73, 014017 (2006).
[18] X. Liu and Z. J. Xiao, Phys. Rev. D 82, 054029 (2010).
[19] Z. T. Zou, Y. Li and X. Liu, Phys. Rev. D 97, 053005 (2018).
[20] X. Liu, Z. J. Xiao and Z. T. Zou, J. Phys. G 40, 025002 (2013).
[21] N. N. Achasov and A. V. Kiselev, Phys. Rev. D 97, 036015 (2018).
[22] Y. L. Shen, W. Wang, J. Zhu and C. D. Lu, Eur. Phys. J. C 50, 877 (2007).
[23] J. D. Weinstein and N. Isgur, Phys. Rev. D 41, 2236 (1990).
[24] M. Tanabashi et al. [Particle Data Group], Phys. Rev. D 98, 030001 (2018).
[25] R. L. Jaffe, Phys. Rev. D 15, 267 (1977).
[26] N. N. Achasov and A. V. Kiselev, Phys. Rev. D 98, 096009 (2018).
[27] M. Ablikim et al. [BESIII Collaboration], Phys. Rev. Lett. 121, 081802 (2018).
[28] C. D. Lü, K. Ukai and M. Z. Yang, Phys. Rev. D 63, 074009 (2001).
[29] Y. Y. Keum, H. N. Li and A. I. Sanda, Phys. Rev. D 63, 054008 (2001).
[30] Y. Y. Keum, H. n. Li and A. I. Sanda, Phys. Lett. B 504, 6 (2001).
[31] H. Y. Cheng and K. C. Yang, Phys. Rev. D 71, 054020 (2005).
[32] T. V. Brito, F. S. Navarra, M. Nielsen and M. E. Bracco, Phys. Lett. B 608, 69 (2005).
[33] K. Maltman, Phys. Lett. B 462, 14 (1999).
[34] C. M. Shakim and H. Wang, Phys. Rev. D 63, 074017 (2001).
[35] G. Buchalla, A. J. Buras and M. E. Lautenbacher, Rev. Mod. Phys. 68, 1125 (1996).
[36] H. n. Li and B. Tseng, Phys. Rev. D 57, 443 (1998).
[37] H. n. Li and K. Ukai, Phys. Lett. B 555, 197 (2003).
[38] T. Kurimoto, H. n. Li and A. I. Sanda, Phys. Rev. D 65, 014007 (2002).
[39] T. Aaltonen et al. [CDF Collaboration], Phys. Rev. Lett. 108, 211803 (2012).
[40] R. Aaij et al. [LHCb Collaboration], Phys. Rev. Lett. 118, 081801 (2017).
[41] K. Nakamura et al. [Particle Data Group], J. Phys. G 37, 075021 (2010).