Numerical study of water residence time in the Yueqing Bay based on the eulerian approach

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Abstract. The Yueqing Bay was a semi-enclosed bay located in the southeast of Zhejiang Province, China. Due to substantial anthropogenic influences since 1964, the water quality in the bay had deteriorated seriously. Thus urgent measures should be taken to protect the water body. In this study, a numerical model was calibrated for water surface elevation and tidal current from August 14 to August 26, 2011. Comparisons of observed and simulated data showed that the model reproduced the tidal range and phase and the variations of current at different periods fairly well. The calibrated model was then applied to investigate spatial flushing pattern of the bay by calculation of residence time. The results obtained from a series of model experiments demonstrated that the residence time increased from 10 day at the bay mouth to more than 70 day at the upper bay. The average residence time over the whole bay was 49.5 day. In addition, the adaptation of flushing homogeneity curve showed that the residence time in the bay varied smoothly. This study provides a numerical tool to quantify the transport timescale in Yueqing Bay and supports adaptive management of the bay by local authorities.

1. Introduction
The Yueqing Bay is located in the southeast of Zhejiang Province, China and is adjacent to the East China Sea (see Figure 1). The bay has been subjected to substantial anthropogenic influences since 1964, including reclamation and dam construction. In recent years, the water quality and ecosystem of the bay have been strongly affected by coastal industries and land-based pollution. Both the amount of discharged nutrients and their retention time contribute to eutrophic conditions in the bay. Environmental reports by the local government state that the water quality of the bay has been deteriorating since the 1980s[1]. Consequently, sufficient understanding of the hydrodynamics and the long-term water exchange characteristics of the bay are urgently required to promote comprehensive measures to protect its ecosystem.

Water quality models provide a useful tool to study eutrophication status in marine areas with high resolution. Water quality models should be able to simulate both the physical transport and the biochemical processes. Both coupled and non-coupled water quality models are available. Whereas these models are rather complex in terms of biochemical processes and the success of their predictions depends greatly on the long-term transport processes of nutrients and phytoplankton simulated by hydrodynamic transport models[2]. In fact, retention of pollutants and time required for the pollutants to reach given locations of the sea are critical to the water quality. From a management perspective, it is important to know the timescale for nutrients and pollutants discharged into the bay to be transported out of it through the bay mouth.
Various timescales have been developed to quantify the transport processes and to assess the assimilative capacity of a water body. These timescales include flushing time, residence time (RT), turn-over time, transit time, age, et al.\cite{3}, and they are often referred to different definitions and formulations in terms of calculations by different authors\cite{4,5}. Some researchers have paid attentions to this matter. Bolin and Rodhe\cite{6} summarized previous results and introduced a more rigorous definition for the timescales of age, transit time and turn-over time. Zimmerman\cite{7} introduced residence time and defined it as the remainder of the lifetime of a particle considered, which was actually complement of the age. Takeoka\cite{8} summarized previous works and introduced the residence time as analogous to the definition of age and transit time.

Figure 1. Map and bathymetry of the Yueqing Bay and the locations of the field measurement stations (yellow circles indicate the water surface elevation stations and the green circles indicate the tidal current stations. The red line at the bay mouth indicates the boundary for tracer release).

The Yueqing Bay is a semi-enclosed, calabash-shaped bay, which is relatively narrow in the middle (approximately 3.5 km wide) and relatively wide (16 km) at the bay mouth (Figure 1). The average water depth is approximately 6.3 m, and the maximum depth exceeds 70 m. The bay is a famous
macrotidal bay in China with a tidal range that increases from 4.66 m at the bay mouth to 8.53 m at the upper bay\(^9\).

2. Methods

2.1. Model description

Zimmerman\(^7\) defined the residence time as the time for each material element to reach the outlet since its initial location. Zimmerman’s definition is for each element and it can be applied to deal with spatially varying cases. According to Zimmerman’s definition, the residence time is the complement of the age. In analogy to the definition of age, Takeoka\(^8\) introduced the remnant function to define residence time. Consider a reservoir and the amount of material in it to be \(R_0\) at time \(t = 0\), and the residual amount of material at time \(t\) to be \(R(t)\). Then \(R(t)\) is the amount of material whose residence time is larger than \(t\). The residence time distribution function can be defined as:

\[
\phi = \frac{1}{R_0} \frac{dR(\tau)}{d\tau}
\]

(1)

It can be further assumed that:

\[
\lim_{\tau \to +\infty} R(\tau) = 0
\]

(2)

The average residence time \(\tau_r\) of the material is defined as:

\[
\tau_r = \int_0^{\infty} \tau \phi(\tau)d\tau
\]

(3)

Integrating equation (3) by parts gives:

\[
\tau_r = \int_0^{\infty} \frac{R(\tau)}{R_0} d\tau = \int_0^{\infty} r(\tau)d\tau
\]

(4)

where \(r(\tau) = R(\tau) / R_0\) is called the remnant function\(^8\). Since the remnant function is defined for an individual material considered, it can be directly applied to calculate the residence time for a pollutant that is discharged into a water body at a particular location and time. The remnant function can be obtained by integrating temporal pollutant concentration distributions over the model domain predicted by the following dye transport equation:

\[
\frac{\partial c(t, x)}{\partial t} + \nabla (uc(t, x) - K\nabla c(t, x)) = 0
\]

(5)

where \(c\) is the dye concentration, \(u\) is the velocity field, \(K\) is the diffusivity tensor, \(t\) is time and \(x\) is coordinate.

In addition, the hydrodynamic model is based upon a set of shallow water equations, including depth-integrated continuity equation and momentum equations. The equation set were discretized with finite volume method.

2.2. Model configuration

The computational domain consisted of 71715 triangle meshes, whose size ranged from 200 m in the bay to 8 km at the open boundary (Figure 2). The bathymetry data were interpolated onto the triangle nodes with bi-linear methods for the purpose of this study. The model was forced at the open boundary with twelve tidal constituent\(\text{i.e.,} four\) semi-diurnal tidal constituents: \(M_2, S_2, N_2, K_2\), four diurnal tidal constituents: \(K_1, O_1, P_1, Q_1\), two shallow water tidal constituents \(M_4, M_6\) and two long period tidal constituents \(M_6, M_8\). The tidal harmonic constants were obtained from TPXO8-atlas. Both the resolution of the semi-diurnal tidal constituents and diurnal tidal constituents were \(1/30^\circ \times 1/30^\circ\), whereas the resolution of the shallow water tidal constituents and long period tidal constituents were \(1/6^\circ \times 1/6^\circ\). Baroclinic pressure gradients are not included in the model. There exist temperature and salinity gradients in the bay, however, the barotropic pressure gradient is much stronger due to its macro-tidal characteristic.
3. Model calibration

To determine its capability in practical applications, the model was calibrated with a large set of observed data, including water surface elevation and tidal current. These data set will be used in the following sub sections separately. The main calibrated parameters included tidal harmonic constant at the open boundary, Manning’s $n$ related to bottom roughness and parameters related to eddy viscosities. These parameters will be tuned cautiously during model validation. The model performance was measured in terms of Mean Absolute Error (MAE), Root Mean Square Error (RMSE) and Index of Agreement (IA), which is given by Willmott et al.\textsuperscript{[10]}. Their definitions are as follows:

\begin{align}
MAE &= \left\langle |X_{obs} - X_{sim}| \right\rangle \\
RMSE &= \sqrt{\left\langle (X_{obs} - X_{sim})^2 \right\rangle} \\
IA &= 1 - \frac{\left\langle \left| X_{obs} - \left\langle X_{obs} \right\rangle \right| + \left| X_{sim} - \left\langle X_{sim} \right\rangle \right| \right\rangle}{\left\langle \left| X_{obs} - \left\langle X_{obs} \right\rangle \right| \right\rangle + \left\langle \left| X_{sim} - \left\langle X_{sim} \right\rangle \right| \right\rangle}
\end{align}

where $X_{obs}$ and $X_{sim}$ are the observed and model simulated data, respectively. $\left\langle \right\rangle$ is the Expectation Operator (or Averaging Operator). These parameters enable simple, straightforward comparisons of the simulated results with observations: The MAE and RMSE have the same units with the observed data and simulated data. The smaller MAE and RMSE are, the better the model results fit the observations. IA is a non-dimensional and bounded measure with values closer to 1 indicating better agreement.

3.1. Water level.

![Figure 2. Mesh, boundary and computational domain of the model.](image-url)
Among the tidal constituents mentioned above, $M_2$ is the dominant one, which is followed by $S_2$. For simplicity, only the amplitudes and phase lags of $M_2$ and $S_2$ were tuned to calibrate the water surface elevation (the stations were shown in Figure 1). The model simulated water surface elevations along with the observed ones at these stations were presented in Figure 3. The calibrated time span lasted from August 14 to August 26, 2011. It can be seen that the modeled surface elevations compare favorably with the observations except for a relatively larger discrepancy occurred between August 22 and August 25, 2011 during the neap tide period. Furthermore, it can be seen from the surface elevations that the tidal range increases as the tide propagates from the outer bay to the inner bay. The statistical indicators of model results were presented in Table 1. The MAEs between the observed hourly surface elevations and the simulated ones from August 14 to August 26, 2011 at the DST, DSW, DM, DMD and HH station were 0.14, 0.18, 0.13, 0.12 and 0.13 m respectively. The corresponding RMSEs were 0.18, 0.22, 0.16, 0.15 and 0.17 m, respectively. The IAs for six stations were all 1.00.

Table 1. MAE, RMSE and IA between observed and simulated water surface elevation.

| Site | MAE  | RMSE | IA  |
|------|------|------|-----|
| DSW  | 0.14 | 0.18 | 1.00|
| DST  | 0.18 | 0.22 | 1.00|
| DMY  | 0.13 | 0.16 | 1.00|
| DMD  | 0.12 | 0.15 | 1.00|
| HH   | 0.13 | 0.17 | 1.00|

Table 2. MAE, RMSE and IA between observed and simulated current velocity and current direction.

| Site | Velocity MAE | RMSE | IA  | Direction MAE | RMSE | IA  |
|------|--------------|------|-----|---------------|------|-----|
| A2   | 0.12         | 0.14 | 0.91| 8             | 18   | 0.99|
| A3   | 0.10         | 0.12 | 0.92| 21            | 40   | 0.94|
| A5   | 0.09         | 0.13 | 0.92| 7             | 16   | 0.99|

3.2. Current

For the current calibration, an intensive field measurement was conducted at seven stations (their locations were shown in Figure 1) during a discontinuous time span from August 14 to August 26, 2011, which included three typical periods: the spring tide period, the medium tide period and the neap tide period. Each period lasted about 27 continuous hours with an interval of half-hour. The spring tide period lasted from August 14 to August 15, 2011. The medium tide period lasted from August 18 to August 19, 2011. The neap tide period lasted from August 23 to August 24, 2011. The current was calibrated by adjusting the Manning coefficient $n$ to make the modeled current velocity and current direction agree with the observations. For brevity, three typical stations (A2, A3, A5) were chosen to represent the inner bay, the lower bay and the outer bay, respectively. Figure 4 compared the modeled current velocity and current direction with the observations at these three stations. It can be seen that the modeled current agreed well with the observations in both magnitude and direction. The statistical indicators of modeled current results were presented in Table 2. The IAs of current velocity at three stations were 0.92, 0.92 and 0.91, respectively. The IAs of current direction at three stations were 0.94, 0.99 and 0.99, respectively. Overall, the model reproduced the variations of current at different periods satisfactorily.
Figure 3. Comparison of water surface elevation at each stations. The lines indicate the simulated values and the circles indicate the observed ones.

Figure 4. Comparison of current velocity and current direction at each stations. The lines indicate the simulated values and the circles indicate the observed ones.

Figure 5 presented the velocity distribution at the time of maximum flood and maximum ebb, respectively. At the time of maximum flood, the northwestern tidal current from the East China Sea divides into three branch when it travels through the waterways between Damen Island, the Luxi Island and the Yueqing Peninsula. One branch pours into the Yueqing Bay. The second branch travels northwestern and disappears when it approaches the mainland. The third branch flows southwestern and finally merges with the tidal current from the open sea. The branch flowing into the Yueqing Bay divides into two parts around the Maoyan Island and travels to the upper bay separately, which is
regulated by the topography in the bay. The tidal current at the time of maximum ebb travels in the way almost opposite to that at the time of maximum flood and will not be detailed anymore.

![Figure 5](image.png)

**Figure 5.** Horizontal distribution of velocity at the time of (a) maximum flood and (b) maximum ebb.

4. **Summary of methodology**

Passive tracer simulations were carried out based on the validated barotropic model above to track spatial and temporal distribution of the conservative tracer. The tracer was released instantaneously in the bay (the regions above the red line in Figure 1) with a uniform concentration of one after a spin-up time of 15 days. To investigate the variability of the flushing pattern induced by the current pattern and tidal strength, the tracer was released at 13 selected time points during both spring and neap tidal cycles, thus forming 26 different scenarios. Scenarios names for spring tide and neap tide were presented in panel(b) and panel(c) of Figure 6, respectively. Finally, the average residence time for each individual mesh was computed and analyzed according to equation (4).

Theoretically speaking, the integral in equation (4) should be conducted until the remnant function approached zero, which was impracticable in reality. A stopping criteria was thus adopted. For each simulation the model was run until the relative error of the accumulative residence time \( \tau^n_{Err} \) for each mesh in the bay was less than \( \tau_{cr} \), where \( \tau^n_{Err} \) is defined as follows:

\[
\tau^n_{Err} = \frac{\tau_{Err}^{(n+1)T} - \tau_{Err}^{nT}}{\tau_{Err}^{nT}}
\]

where \( T \) is period of a tidal cycle. In this study \( \tau_{cr} \) was set to 0.001 [3,11,12].
5. Results and Discussion
Spatial distribution of residence time in the bay obtained averaging the whole set of simulations results were presented in Figure 7(a). Overall, the distribution was mainly dependent on the relative distance to the bay mouth. The residence time increased from 10 day at the bay mouth to more than 70 day at the upper bay. Specifically, the RT around the Ximen Island and in the Qingjiang River were more than 70 day. The 60 day and 50 day isoline lay around Maoyan Island and Jiangyan Island, respectively. The region above Dashiuiwan-Lianyu had RT more than 40 day and the region below the cross section had RT less than 40 day. The average residence time over the whole bay was 49.5 day. In addition, the RT above Dashiuiwan-Lianyu was relatively uniform along the transverse section whereas the RT on the east was larger than that on the west below the cross section. This distribution pattern was regulated by the topography of the bay. As shown in Figure 1, the depth on the east of the lower bay was larger than 10 m and the depth on the west was less than 5 m, thus forming a tidal channel along the east coast and vast tidal flat on the west side of the lower bay. Therefore the tidal current was stronger on the east than that on the west(as shown in Figure 5) and more material at the upper bay and from outside the bay was transported through the deep channel.

In order to better understand the influence of tidal forcing on the water residence time, a further analysis had been carried out. The standard deviation(STD) and the coefficient of variation(CV) of residence time for each individual mesh was computed and presented as contour plots in panel(b) and panel(c) of Figure 7. The CV was defined as the ratio between the standard deviation and the average value of the residence time of the 26 scenarios. The STD at most region of the bay was around 6.5 day and only the region near bay mouth had STD ranging from 3 day to 6 day. The CV ranged from 0.1 at the upper bay to more than 0.25 at the bay mouth, indicating that the RT at the bay mouth was more
sensitive than that at the upper bay to the phase of the tide at the moment of tracer release. This sensitivity should be taken into account while performing the computation of flushing times.

Figure 7. Horizontal distribution of (a) residence time, (b) standard deviation of residence time and (c) coefficient of variation.

To understand the spatial characteristic of flushing, the flushing homogeneity curve (FHC) was adopted based on the spatial distribution of residence time through summarizing percentage area of it. In general, the steeper the FHC the less variation in the values of residence time throughout the examined region. A hypothetical completely mixed bay would yield vertical FHC. Figure 8 presented the FHC for this study. As can be seen in Figure 8, 11% of the bay had the values of residence time greater than 70 day and 23% of the bay had the values less than 20 day. On the other hand, about 14 day were required to renew 10% water area of the bay and about 70 day were required to flush 90% of the bay. Generally speaking, the curve is flat, which indicated there no exist sharp gradients of residence time in the bay and this could be proofed by Figure 7(a). The percentage corresponding to the bay-average residence time was 45%.
6. Conclusions
A spatial approach of water residence time was presented and applied in this paper to study the flushing characteristics in Yueqing Bay. The results obtained from a series of simulations showed that the residence time increased from 10 day at the bay mouth to more than 70 day at the upper bay. The average residence time over the whole bay was 49.5 day. Statistical indicator of CV revealed that the RT at the bay mouth was more sensitive than that at the upper bay to the phase of the tide at the moment of tracer release. The concept of FHC was applied to demonstrate flushing properties, which provided useful information about the variation characteristic of flushing. The curve for this study was flat, indicating that the residence time in the bay varied smoothly. About 14 day were required to renew 10% water area of the bay and about 70 day were required to flush 90% of the bay. Furthermore, both baroclinic and wind induced current may have certain impact on the flushing properties of the bay and should be taken into consideration in the future study.

Acknowledgments
The study was financially supported by the Science and Technology Plan for Zhejiang Province 2017F30007 and Science and Technology Project of The Bureau of Water Resources of Zhejiang Province (RC1604).

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