Tensor supermultiplets and
toric quaternion-Kähler geometry†

Bernard de Wit and Frank Saueressig

Institute for Theoretical Physics and Spinoza Institute,
Utrecht University, Utrecht, The Netherlands

B.deWit@phys.uu.nl, F.S.Saueressig@phys.uu.nl

Abstract

We review the relation between 4n-dimensional quaternion-Kähler metrics with n + 1 abelian isometries and superconformal theories of n + 1 tensor supermultiplets. As an application we construct the class of eight-dimensional quaternion-Kähler metrics with three abelian isometries in terms of a single function obeying a set of linear second-order partial differential equations.

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1 Introduction

A fascinating feature of string theory is the rich interplay between its physical properties and mathematics. An example of this is provided by supersymmetric effective actions, where the scalar fields coordinatize a target space manifold whose geometry is of a restricted type. Based on this close link, it is not surprising that progress in understanding the low-energy limit of string theory often comes hand in hand with a better understanding of the corresponding geometries.

This paper focuses on the geometries arising in the hypermultiplet sector of $N = 2$ supergravity theories. Since a hypermultiplet contains four real scalars, the corresponding manifolds have dimension $4n$, where $n$ is the number of hypermultiplets. Supersymmetry restricts the underlying metric to be quaternion-Kähler \[1\]. In case of a single hypermultiplet, this is equivalent to the metric being Einstein with self-dual Weyl curvature. These spaces have been studied extensively by both mathematicians and physicists (see e.g. \[2, 3, 4\]) and it turned out that the most general four-dimensional quaternion-Kähler metric is determined by a single function obeying a non-linear partial differential equation \[4\]. When the metric is toric, i.e., when it has two commuting isometries, this equation can be linearized and reduces to a Laplace-like equation on the upper half-plane \[2\]. For $n > 1$ the quaternion-Kähler manifolds are characterized by their holonomy group $\text{Sp}(1) \times \text{Sp}(n)$. This makes the study of their metrics very involved since a generic quaternion-Kähler manifold does, for instance, not admit a Kähler potential.

From $N = 2$ supergravity theories in four space-time dimensions \[5\] it is clear that there exists a relation between quaternion-Kähler manifolds and hyperkähler cones. The latter provide the hypermultiplet target spaces of field theories that are invariant under the rigid $N = 2$ superconformal symmetry \[6\]. Such a cone has a homothetic Killing vector and three complex structures which rotate isometrically under the group $\text{Sp}(1)$ \[7, 8\]. It is a cone over a $(4n + 3)$-dimensional 3-Sasakian manifold, which in turn is an $\text{Sp}(1)$ fibration of a $4n$-dimensional quaternion-Kähler space. The quaternion-Kähler manifold is the $N = 2$ superconformal quotient of the hyperkähler cone and there is a one-to-one relation between quaternion-Kähler spaces and hyperkähler cones. This relation was worked out explicitly in \[9\].

Quaternion-Kähler geometries simplify further when the corresponding hyperkähler cone possesses $n + 1$ abelian symmetries. In this case one can use the (Hodge) duality between scalars and second rank tensor gauge fields to dualize some of the scalars into tensors and work with tensor supermultiplets instead of hypermultiplets. The conformal supergravity theories including an arbitrary number of tensor supermultiplets and their corresponding superconformal quotient have recently been worked out in \[10\]. Subsequently this formulation has led to major progress in understanding perturbative and non-perturbative properties of four-dimensional effective string actions \[11, 12\].

Here we focus on the mathematical implications of the superconformal quotient for superconformal tensor multiplets coupled to supergravity. In \[10\] this technique has already been used to give an elegant derivation of the four-dimensional quaternion-Kähler metrics with two commuting isometries, obtained originally in \[2\]. After reviewing the general framework, we will construct
the class of eight-dimensional quaternion-Kähler metrics with three abelian isometries. These metrics are described by a single function which obeys a simple set of linear partial differential equations. Further details and proofs will be presented in a forthcoming paper.

2 Superconformal tensor multiplet Lagrangians

The bosonic fields of an \( N = 2 \) tensor supermultiplet consist of three scalar fields, an antisymmetric tensor gauge field \( E_{\mu\nu} \), and a complex auxiliary field (the latter will be ignored in the following). The scalar fields transform in the vector representation of the SU(2) R-symmetry group, and can be described conveniently in terms of symmetric tensors \( L^{ij} \), satisfying

\[
L^{ij} = (L^{ij})^* = \varepsilon^{ik} \varepsilon^{jl} L^{kl}.
\]

In [10] we proved that superconformal Lagrangians of \( n+1 \) tensor multiplets are encoded in a "potential" \( \chi(L) \), which depends on the fields \( L_{ij}^I \) labelled by indices \( I = 1, 2, \ldots, n+1 \). Conformal invariance requires this potential to be SU(2) invariant and homogeneous of degree +1, which implies

\[
\frac{\partial \chi(L)}{\partial L_{ij}^I} L^{kl} = \frac{1}{2} \delta^{ij}_k \chi(L). \tag{1}
\]

Supersymmetry requires the second derivative of \( \chi(L) \) to satisfy the condition,

\[
\varepsilon_{kl} \frac{\partial^2 \chi(L)}{\partial L_{ik}^I \partial L_{jl}^J} = 2 F_{IJ}(L) \varepsilon^{ij}, \tag{2}
\]

which defines a function \( F_{IJ}(L) \), symmetric in \( I \) and \( J \). As it turns out \( F_{IJ}(L) \) plays the role of the target space metric of the \( 3(n+1) \) tensor multiplet scalars. One can prove that (1) and (2) imply two more equations,

\[
\varepsilon_{ij} \varepsilon_{kl} \frac{\partial^2 F_{IJ}(L)}{\partial L_{ik}^I \partial L_{jl}^J} = 0, \quad \chi(L) = 2 F_{IJ}(L) L_{ij}^I L^{ij} \tag{3},
\]

the first of which appeared originally in [13, 14]. As a by-product of the above equations one deduces that the \( k \)-th derivative of \( \chi(L) \) is symmetric in both the multiplet indices \( I_1, I_2, \ldots, I_k \) and in the SU(2) indices \( i_1, i_2, \ldots, i_2k \), and can thus be denoted unambiguously by \( \chi_{I_1 \ldots I_k i_1 \ldots i_2k} \).

Because of SU(2) invariance, the potential \( \chi(L) \) depends on only \( 3n \) independent variables and is subject to \( \frac{3}{2}(n-1)(n+2) \) second-order differential equations (for \( n \geq 2 \) ) corresponding to (2). For \( n = 1 \), \( \chi(L) \) depends on 3 variables and is subject to a single differential equation, while, for \( n = 0 \), \( \chi(L) \) is proportional to \( \sqrt{L_{ij} L^{ij}} \).

The general Lagrangian coupling an arbitrary number of tensor multiplets to conformal supergravity has recently been worked out in [10]. Here we display its bosonic part without the auxiliary tensor multiplet fields,

\[
e^{-1} \mathcal{L} = \chi \left( \frac{1}{6} R + \frac{1}{2} D \right) + F_{IJ} \left[ -\frac{1}{2} D_{\mu} L_{ij}^I D^{\mu} L^{ij} + E_{\mu}^I E^{ij} + E_{\mu}^J \right] + \frac{i}{2} e^{-1} \varepsilon^{\mu \rho \sigma} F_{IJK} L_{ij}^I \partial_{\rho} L_{kl}^J \partial_{\sigma} L_{lm}^K \varepsilon^{kl} E_{\mu}^I, \tag{4}
\]

where the \( E_{\mu}^I \) denote the field strengths associated with the tensor gauge fields. We also note the presence of some of the conformal supergravity fields contained in the Weyl multiplet, such as
the vierbein field \( e_\mu^a \), the SU(2) gauge fields \( V_\mu^{ij} \) and an auxiliary scalar field \( D \). Furthermore, \( D_\mu L_{ij} \) denotes a derivative covariant with respect to local dilatations and local SU(2), and \( R \) is the Ricci scalar associated with a spin-connection field \( \omega_\mu^{ab} \).

An intricate term of the Lagrangian (4) is the last one, which is not manifestly invariant under tensor gauge transformations. Tensor gauge invariance requires that \( E_\mu^I \) couples to a tensor which is the pull back of a closed two-form. In addition, supersymmetry requires the supersymmetry variation of this form to be exact. The relevant two-form, \( F_I \equiv F_{IJK}^{ij} (L) dL_{ik}^J \wedge dL_{jl}^K \varepsilon^{kl} \), is indeed closed, as well as exact under arbitrary variations \( L^I \rightarrow L^I + \delta L^I \). This implies that it can locally be expressed in terms of a one-form, \( F_I = dA_I \). This one-form is, however, no longer invariant under SU(2).

To deal with this problem one decomposes the scalar fields \( L_{ij} \) according to a U(1) subgroup of SU(2) into a real and a complex field, \( x \) and \( v \), respectively [13, 14] (see also, [9, 10]), and we define \( L_{12}^I = -\frac{1}{2} i x^I \), \( L_{22}^I = v^I \). In these variables (2) takes the form \( \chi x^I x^J + \chi \bar{v}^I \bar{v}^J = 2 F_{IJ} \). Furthermore, one can show that \( F_{IJ} \) can be written as the second derivative of another, homogeneous, function \( L(x, v, \bar{v}) \), i.e., \( F_{IJ} = L_{x^I x^J} = - L_{\bar{v}^I \bar{v}^J} \). The function \( L \) is only invariant under a U(1) subgroup of SU(2), and its multiple derivatives are again symmetric in the multiplet indices \( I, J, K, \ldots \). The potential \( \chi \) can be expressed in terms of \( L \) by
\[
\chi = L - x^I L_{x^I} .
\]

Another relation is provided by the second equation of (3).

In terms of \( L \) we can write down an explicit representation for the one-form \( A_I \),
\[
A_I = i \left( L_{x^I \bar{v}^J} d\bar{v}^J - L_{x^I v^J} dv^J \right) .
\]
The coefficients \( L_{x^I v^J} \) can be found from the potential \( \chi(L) \) by integration, as their first derivative with respect to \( x^K \) is symmetric in \( I, J, K \) and equal to \( L_{x^I x^J v^K} = \frac{1}{2} (\chi x^I x^J v^K + \chi \bar{v}^I \bar{v}^J v^K) \).

We stress that the Lagrangian (4) can be expressed in terms of both \( L \) and \( \chi \). Which formulation is more convenient depends on the problem to be addressed.

### 3 The superconformal quotient

The superconformal Lagrangians considered in the previous section are invariant under local superconformal symmetries. These theories are gauge equivalent to matter-coupled supergravity of the standard Poincaré type. The conversion, which is known as the superconformal quotient, can be effected either by imposing gauge conditions or by employing gauge invariant variables so that the gauge degrees of freedom will eventually decouple from the Lagrangian. Part of the

\[ L(x, v, \bar{v}) = \Im \int \frac{d\zeta}{2 \pi i} \chi(\eta^I), \]

where \( \eta^I = v^I / \zeta + x^I - \bar{v}^I \zeta \) and \( H(\eta^I) \) is an arbitrary function homogeneous of degree one.
superconformal symmetries reflect themselves in the Lagrangian (4), namely the dilatations and local SU(2) transformations. They imply that the target space for the scalar fields is a cone. After performing the quotient and upon dualizing the tensor fields to scalars, one ends up with a 4n-dimensional quaternion-Kähler space.\footnote{Without the fields of the Weyl multiplet, the superconformal symmetries are only realized in a rigid way. The resulting target spaces, after a tensor-scalar duality transformation, are 4(n + 1)-dimensional hyperkähler cones \[9, 6\].} For the Lagrangian (4) the superconformal quotient has been carried out in \cite{10} and we will briefly summarize the result. In the next section we will demonstrate the results for \(n = 2\). The case \(n = 1\) was already treated in \cite{10}.

To carry out the superconformal quotient in the tensor sector, one first converts to scale invariant variables by rescaling the \(L_{ij}^I\) by the inverse of \(\chi\) and switches from the symmetric tensor to a vector notation according to,

\[ L_{ij}^I = -i\chi \vec{L}^I \cdot (\vec{\sigma})_i^k \varepsilon_{jk}. \] (8)

The scale invariant fields are then subject to a non-trivial constraint,

\[ F_{IJ}(\vec{L}) \cdot \vec{L}^I = \frac{1}{4}, \]

where \(F_{IJ}(\vec{L}) = \chi F_{IJ}(L)\). Here we make use of the homogeneity of \(F_{IJ}\).

Subsequently, the SU(2) gauge fields \(V_{\mu}^i\) are eliminated through their equation of motion and the Lagrangian (4) takes the form,

\[ e^{-1}L = \chi \left[ \frac{1}{6}R + \frac{1}{2}D - \frac{1}{2}(\partial_\mu \ln \chi)^2 \right] - \chi \left[ G_{ij}^{(1)} \partial_\mu \vec{L}^i \partial^\mu \vec{L}^j + G_{ij,KL}^{(2)} (\vec{L}^i \cdot \partial_\mu \vec{L}^j) (\vec{L}^K \cdot \partial_\mu \vec{L}^L) \right] + \chi^{-1} \left[ \mathcal{H}_{ij}^{(1)} E_{\mu}^i E^{\mu j} \right] + E_{\mu i} \left[ \mathcal{H}_{ij}^{(2)} \vec{L}^i \cdot (\vec{L}^K \times \partial_\mu \vec{L}^L) F_{KL} + A_{\mu i} \right]. \] (9)

Note that in this notation, the scalar fields obviously parameterize a cone with radial variable \(\chi\). Eliminating the SU(2) gauge fields does not affect the local SU(2) invariance, so that the fields \(\vec{L}^i\) comprise \(3n - 1\) degrees of freedom (for \(n \geq 2\)). Note that we have dropped a total derivative as compared to (4), by making use of (7). As a result the tensor gauge fields appear exclusively through their corresponding field strengths. The local SU(2) invariance of the last term is realized because \(A_{\mu i}\) transforms nontrivially under SU(2).

To give the explicit expressions for the functions appearing above it is useful to introduce the expression \(N_{ij} = 4F_{IK}\vec{L}^K \cdot \vec{L}^j\), so that \(\text{tr}(N) = 1\). When lowering the upper index of the \(k\)-th power of this matrix by contraction with \(F_{IJ}\), one obtains a symmetric matrix \(N^k_{IJ}\). Using this notation we obtain,

\[
\mathcal{H}_{ij}^{(1)} = F_{IJ} + F_{IK} L^K (M^{-1})^{rs} L_s^L F_{IL},
\]

\[
\mathcal{H}_{ij}^{(2)} = \frac{1}{16 \det(M)} \left[ (1 - \text{tr}(N^2)) F_{IJ} + 2N^2_{IJ} \right],
\]

\[
G_{ij}^{(1)} = F_{IJ} - \frac{1}{128 \det(M)} \left[ (1 + \text{tr}(N^2)) N_{IJ} - 2N^3_{IJ} \right],
\]

\[
G_{ij,KL}^{(2)} = \frac{1}{16 \det(M)} \left[ N_{IK} N_{JL} + \frac{1}{2}(1 + \text{tr}(N^2)) F_{IL} F_{JK} + (N^2_{IL} F_{JK} + N^2_{JK} F_{IL}) \right].
\]
The $3 \times 3$ matrix $M$ arises naturally when eliminating the SU(2) gauge fields and is given by

$$[M]_{rs} = \left( \frac{1}{4} \delta^{rs} - L^I J L^J \right). \tag{11}$$

Its determinant and inverse are equal to

$$\det(M) = \frac{1}{192} \left[ 1 - \text{tr}(\mathcal{N}^3) \right],$$

$$[M^{-1}]_{rs} = \frac{1}{32 \det(M)} \left[ (1 - \text{tr}(\mathcal{N}^2)) \delta_{rs} + 8 L^I \mathcal{N}^I L^J \right]. \tag{12}$$

4 The case of three tensor multiplets

We now use the formalism of the previous section to construct eight-dimensional quaternion-Kähler metrics with three abelian isometries. These isometries originate from the tensor fields, which are dualized to scalars. This implies that we should start from (9) with $n + 1 = 3$ tensor supermultiplets.

The natural starting point of the construction is the most general potential $\chi$ for three tensor multiplets, $L_{ijI}$, where the indices $I, J = 1, 2, 3$ enumerate the tensor multiplets. The potential must be invariant under SU(2) rotations. In order to make this invariance manifest, we introduce the SU(2) invariant fields,

$$\phi_{IJ} = L_{ijI} L_{ijJ}, \tag{13}$$

so that we are dealing with symmetric $3 \times 3$ tensors $\phi_{IJ}$. In terms of these fields the constraints (1) and (2) reduce to

$$\phi^{IJ} \partial \chi(\phi) \partial \phi_{IJ} = \frac{1}{2} \chi(\phi), \quad \partial \chi(\phi) \partial \phi_{I[K} \partial \phi_{L]J} = 0. \tag{14}$$

Furthermore we derive,

$$F_{IJ}(\phi) = \frac{\partial \chi(\phi)}{\partial \phi^{IJ}}. \tag{15}$$

Using these results we can evaluate the general formulae (9), (10) and (11) for the case of three tensor multiplets. As before we introduce scale invariant variables, $\hat{\phi}^{IJ} = \chi^{-2} \phi^{IJ}$, so that $\mathcal{N}^{IJ} = 2 F_{IK} \hat{\phi} K J$. With respect to these, it is straightforward to evaluate (10) for $n = 2$. In particular one can prove that $\mathcal{H}_{IJ} = \mathcal{H}^{(1)}_{IJ} = \frac{1}{8} \mathcal{H}^{(2)}_{IJ}$. To write down explicit Lagrangians, one may use an explicit coordinatization of the $\phi^{IJ}$. This can, for instance, be done by using the local SU(2) invariance to bring the matrix $L^{rI}$ into lower triangular form,

$$L^{rI} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 0 & w_3 \\ 0 & u_2 & u_3 \\ x_1 & x_2 & x_3 \end{bmatrix}, \tag{16}$$

\[\text{For compact spaces the maximal number of commuting isometries of a 4n-dimensional quaternion-Kähler space equals n + 1. In principle, the superconformal quotient can be applied to compact spaces, although only the non-compact spaces are physically relevant in supergravity. It is possible that non-compact spaces have more than n + 1 abelian isometries, and these are not obvious in the context of our construction. Nevertheless we expect that the results of this paper cover the full class of quaternion-Kähler spaces with at least n + 1 abelian isometries. This has been shown to be the case for n = 1 [10, 2].}\]
so that
\[
\phi^{11} = x_1^2, \quad \phi^{22} = u_2^2 + x_2^2, \quad \phi^{33} = w_3^2 + u_3^2 + x_3^2, \\
\phi^{12} = x_1 x_2, \quad \phi^{13} = x_1 x_3, \quad \phi^{23} = u_2 u_3 + x_2 x_3.
\] (17)

Note that the values of the scalars \(\phi^{IJ}\) are constrained to the hypersurface \(\text{tr}(\mathcal{N}) = 1\). This constraint can locally be solved for one of the coordinates defined in (17) which is then expressed in terms of the other scalars.

Using these results, the evaluation of the general tensor multiplet Lagrangian (9) yields
\[
e^{-1} \mathcal{L} = \chi \left[ \frac{1}{8} R + \frac{1}{2} D - \frac{1}{4} (\partial_{\mu} \ln \chi)^2 \right] \\
- \chi \left[ G^{(1)}_{IJ} \partial_{\mu} \tilde{L}^I \cdot \partial_{\mu} \tilde{L}^J + G^{(2)}_{IJ,KL} (\tilde{L}^I \cdot \partial_{\mu} \tilde{L}^J) (\tilde{L}^K \cdot \partial_{\mu} \tilde{L}^L) \right] \\
+ \chi^{-1} \mathcal{H}_{IJ} E_{\mu}^I E^{\mu J} + E_{\mu I} \left[ 8 \mathcal{H}_{IJ} \tilde{L}^J \cdot (\tilde{L}^K \times \partial_{\mu} \tilde{L}^L) F_{KL} + A_{\mu I} \right].
\] (18)

In the final step we introduce Lagrange multipliers \(s_I\) to impose the Bianchi identity on the field strength \(E_{\mu I}\) and eliminate these field strengths as unconstrained fields through their equations of motions. This results in the following eight-dimensional quaternion-Kähler metric,
\[
ds^2 = \mathcal{K}^{(1)}_{IJ} d\tilde{L}^I \cdot d\tilde{L}^J + \mathcal{K}^{(2)}_{IJ,KL} (\tilde{L}^I \cdot d\tilde{L}^J) (\tilde{L}^K \cdot d\tilde{L}^L) \\
+ 4 (A_I + ds_I) \tilde{L}^I \cdot (\tilde{L}^K \times d\tilde{L}^L) F_{KL} + \frac{1}{4} [\mathcal{H}^{-1}]^{IJ} (A_I + ds_I) (A_J + ds_J),
\] (19)

with the matrices
\[
\mathcal{K}^{(1)}_{IJ} = F_{IJ} - \frac{1}{384 \det(M)} \left[ 1 + 3 \text{tr}(\mathcal{N}^2) - 4 \text{tr}(\mathcal{N}^3) \right] \mathcal{N}_{IJ}, \\
\mathcal{K}^{(2)}_{IJ,KL} = 4 F_{IL} F_{JK} - \frac{1}{64 \det(M)} \left[ (1 - \text{tr}(\mathcal{N}^2)) (F_{IK} \mathcal{N}_{JL} + F_{JL} \mathcal{N}_{IK}) \\
+ 2 (\mathcal{N}_{IK}^2 \mathcal{N}_{JL} + \mathcal{N}_{JL}^2 \mathcal{N}_{IK}) - 4 \mathcal{N}_{IK} \mathcal{N}_{JL} \right].
\] (20)

Here we suppressed the overall factor of \(\chi\) which, when frozen to a constant value, sets the curvature scale of the metric. In view of the constraint, the \(\phi^{IJ}\) depend on 5 coordinates. The extra 3 coordinates are provided by the fields \(s^I\) originating from the tensor sector of the model. The line-element (19) thus provides the extension of the classification of four-dimensional toric quaternion-Kähler manifolds by Calderbank and Pedersen [2] to eight dimensions.

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