A unified polar cap/striped wind model for pulsed radio and gamma-ray emission in pulsars

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ABSTRACT

Because of the recent discovery by Fermi of about 50 new gamma-ray pulsars, it has become possible to look for the statistical properties of their pulsed high-energy emission, especially their light curves and phase-resolved spectra. These pulsars emit, by definition, mostly gamma-ray photons, but some of them are also detected in the radio band. For those seen in these two extreme energies, the relation between the time lag of the radio/gamma-ray pulses and the gamma-ray peak separation, when both high-energy pulses are seen, helps to constrain the magnetospheric emission mechanisms and location. This idea is analysed in detail in this paper, assuming a polar cap model for the radio pulses and a striped wind geometry for the pulsed high-energy counterpart.

Combining the time-dependent emissivity in the wind, supposed to be inverse Compton radiation, with a simple polar cap emission model along and around the magnetic axis, we compute the radio and gamma-ray light curves, summarizing the results in several phase plots. We study the phase lag as well as the gamma-ray peak separation dependence on the pulsar inclination angle and on the viewing angle. Using the gamma-ray pulsar catalogue, compiled from the Fermi data, we are able to predict the radio lag/peak separation relation and to compare it with available observations taken from this catalogue.

This simple geometric model, combining polar cap and striped wind radiation, is satisfactory for explaining the observed radio/gamma-ray correlation. This supports the idea of distinct emission locations for the radio and gamma-ray radiation. Nevertheless, time retardation effects, such as curved space–time and magnetic field lines winding up close to the neutron star, can lead to a discrepancy between our predicted time lag and a more realistic relation as deduced from the gamma-ray catalogue. Moreover, as there is no accurate polar cap description available so far, large uncertainties remain on the altitude and geometry of the radio emission.

Key words: radiation mechanisms: non-thermal – relativistic processes – pulsars: general – stars: winds, outflows – gamma-rays: stars.

1 INTRODUCTION

A new catalogue on gamma-ray pulsars obtained by the Fermi Large Area Telescope (LAT; Abdo et al. 2010a) has increased the number of gamma-ray pulsars from seven to about 50. Now, new pulsars are discovered regularly. For the first time, this allows a reasonable statistical analysis on the high-energy emission properties of these objects, such as spectral shape, cut-off energy and a comparison between radio and gamma-ray radiation if available. For some high-luminosity pulsars, this analysis has been completed by a phase-resolved study, as for the Crab (Abdo et al. 2010b), Vela (Abdo et al. 2009) and Geminga (Abdo et al. 2010c). Radio pulses and gamma-ray photons are expected to be produced in different emission sites, probably close to the neutron star surface for the former, described by a polar cap model (Radhakrishnan & Cooke 1969), and in the vicinity of the light cylinder for the latter, explained by outer gaps (Cheng, Ho & Ruderman 1986) or alternatively by the striped wind (Pétri 2009). Polarization properties at multiwave-lengths would certainly help to constrain the geometry of the sites of emission (Dyks, Harding & Rudak 2004; Pétri & Kirk 2005; Pétri 2009). Gamma-ray light curves alone can already give good insight into the magnetosphere (Romani & Watters 2010).

The new sample of Fermi-LAT gamma-ray pulsars has increased interest in the modelling of gamma-ray emission. The detection of several millisecond gamma-ray pulsars was not expected and came as a real surprise. Thus, Venter, Harding & Guillemot (2009) have focused special attention on this class of millisecond pulsars to probe the geometry of the emission regions, taking into account relativistic effects.

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Very recently, gamma-ray light curves have been computed for the simple vacuum dipole model (Bai & Spitkovsky 2010b) or better for a realistic magnetospheric model based on three-dimensional magnetohydrodynamics (MHD) simulations of the near pulsar magnetosphere. This requires some post-processing prescription about the emission location and mechanism within the magnetosphere (Bai & Spitkovsky 2010a). In this model, gamma rays are expected close to the light cylinder.

An alternative site for the production of pulsed radiation was investigated a few years ago by Kirk, Skjæraasen & Gallant (2002). This model is based on the striped pulsar wind, originally introduced by Coroniti (1990) and Michel (1994). Emission from the stripped wind originates outside the light cylinder and relativistic beaming effects are responsible for the phase coherence of this radiation. It has already been shown that this model can satisfactorily fit the optical polarization data from the Crab pulsar (Pétri & Kirk 2005) as well as the phase-resolved high-energy spectral variability of Geminga (Pétri 2009).

In this paper, the aim is to complete the atlas of light curves and phase plots performed by Watters et al. (2009) based on the polar cap, the slot gap and the outer gap models. It furnishes an extended benchmark to test different scenarios for the physical processes occurring in the pulsar magnetosphere. We use an explicit asymptotic solution for the large-scale magnetic field structure related to the oblique split monopole (Bogovalov 1999) and responsible for the gamma-ray light curves combined with a simple polar cap geometry for the radio counterpart. Details are given in Section 2. We then compute the properties of the radio and high-energy light curves for the pulsed emission and compare our results with several gamma-ray pulsars extracted from the Fermi catalogue. Relevant results are discussed in Section 3 before this study is concluded.

2 MODEL

The model employed to compute the gamma-ray pulse shape emanating from the striped wind is briefly re-examined in this section. The geometrical configuration and emitting particle distribution functions follow the same lines as those described in Pétri (2009).

The magnetized neutron star rotates at an angular speed of Ωs (period P = 2π/Ωs) directed along the Oz-axis (i.e. the rotation axis is given by Ωs = (Ωs, eϕ)). We use a Cartesian coordinate system with coordinates (x, y, z) and orthonormal basis (eϕ, eψ, e). The stellar magnetic moment is denoted by µs. It is assumed to be dipolar and makes an angle χ with respect to the rotation axis such that

\[ \mu_s = \mu_s \sin(\cos(\Omega_s r)e_\psi + \sin(\Omega_s r)e_\phi) + \cos(\chi)e_z. \]

This angle is therefore defined by \( \cos \chi = \mu_s \cdot e_\psi/\mu_s \). The inclination of the line of sight with respect to the rotational axis, and defined by the unit vector \( n = \sin \xi e_\psi + \cos \xi e_z \).

Thus, we have \( \cos \chi = n \cdot e_\psi \). Moreover, the wind expands radially outwards at a constant velocity \( V \) close to the speed of light denoted by \( c \).

Because we are not interested in the phase-resolved spectra, but only in the light curves above 100 MeV, our model only involves geometrical properties related to the magnetic field structure and viewing angle. The particle distribution function does not play any role except for its density number. Dynamical properties (such as energy distribution) related to the emitting particles are unimportant in this case. In order to compute the light curves, we use a simple expression for the emissivity of the wind related to inverse Compton scattering. This is explained in the following subsections.

2.1 Magnetic field structure

The geometrical structure of the wind is based on the asymptotic magnetic field solution given by Bogovalov (1999). Outside the light cylinder, the magnetic structure is replaced by two magnetic monopoles with equal and opposite intensity. The current sheet sustaining the magnetic polarity reversal arising in this solution, expressed in spherical coordinates (r, θ, φ), is defined by

\[ r_c(\theta, \phi, t) = \beta_c r_L \left( \pm \arccos(\cot \phi \cot \chi) + \frac{ct}{r_L} - \phi + 2\pi \right). \]

Here, \( \beta_c = V/c, r_L = c/\Omega_s \) is the radius of the light cylinder, \( t \) is the time as measured by a distant observer at rest and \( f \) is an integer. Because of the ideal MHD assumption, this surface is frozen into the plasma and therefore also moves radially outwards at a speed \( V \). Strictly speaking, the current sheets are infinitely thin and the pulse width would be inversely proportional to the wind Lorentz factor \( \Gamma_w = (1 - \beta_c^2)^{-1/2} \) (Kirk, Lyubarsky & Pétri 2009). Here, as already done for the study of the synchrotron polarization of the pulsed emission (Pétri & Kirk 2005) and for Geminga (Pétri 2009), we release this restrictive and unphysical prescription. Indeed, the current sheets are assumed to have a given thickness, parametrized by the quantity \( \Delta_\psi \) (see equation 6 for an explicit expression of the thickness). Moreover, inside the sheets, the particle number density is very high while the magnetic field is weak. In whole space, the magnetic field is purely toroidal and given by

\[ B_\phi = B_L \frac{R_L}{r} \eta_\psi. \]

The strength of the magnetic field at the light cylinder is denoted by \( B_L \). In the original work of Bogovalov (1999), \( \eta_\psi \) is related to the Heaviside unit step function and can only have two values, \( \pm 1 \), leading to the discontinuity in magnetic field. In order to make the transition more smooth, we redefine the function \( \eta_\psi \) by

\[ \eta_\psi = \tanh(\Delta_\psi \psi) \]

\[ \psi = \cos \theta \cos \chi + \sin \phi \sin \chi \cos(\psi - \Omega_s \left(t - \frac{r_L}{c}\right)). \]

With these formulae, the physical length of the transition layer has a thickness of the order of

\[ 2\pi \beta_c r_L/\Delta_\psi. \]

2.2 Particle density number

The aim of this paper is to show the behaviour of the pulsed high-energy light curves emanating from the striped wind flow for different magnetic plus emitting particle configurations. We do not perform a detailed study of the phase-resolved spectral variability. Thus, for this purpose, it is sufficient to fix the particle density number without specifying the distribution in momentum space. We assume an e± pair mixture flowing dominantly within the current sheets defined by equation (3). We thus adopt the following expression for the total particle density number:

\[ N_\psi(r, t) = \frac{(N - N_0) \text{sech}^2(\Delta_\psi \psi) + N_0}{r^2}. \]
\text{N}_0 \text{ sets the minimum density in the stripes, between the current sheets, whereas } N \text{ defines the highest density inside the sheets. We refer to Pétri (2009) for more details about the justification of this choice.}

### 2.3 Inverse Compton emissivity in the wind

These particles are visible through their inverse Compton radiation. The corresponding total emissivity is denoted by \( f_{ic}^{obs} \). Strictly speaking, this radiated power should depend on the space location \( r \) and the time of observation \( t \), as well as on the energy of the detected photons. However, recall that a spectral study is out of the scope of this work, so the energy dependence can be dropped. We only retain variation with \( r \) and \( t \).

As done in Pétri (2009), the inverse Compton light curves are obtained by integrating this emissivity over the whole wind region. This wind is assumed to extend from a radius \( r_0 \) to an outer radius \( r_s \), which can be interpreted as the location of the termination shock. Therefore, the inverse Compton radiation at a fixed observer time \( t \) is given by

\[
I_{ic}^{obs}(t) = \int_{r_0}^{r_s} \int_0^{2\pi} f_{ic}^{obs}(r, t_{ret}) r^2 \sin \theta \, dr \, d\theta \, d\phi.
\]

The retarded time is expressed as

\[
t_{ret} = t - \sqrt{(r_0 - r)^2/c^2} \approx t - \frac{(r_0 - r)}{c}.\]

Equation (8) is integrated numerically. We compute the inverse Compton intensity for several geometries. We are therefore able to predict the phase-resolved pulse shape, for any inclination of the line of sight and any obliquity of the pulsar. Results and applications to some gamma-ray pulsars are discussed in Section 3.

### 2.4 Polar cap emissivity

The polar caps or their close vicinity to an altitude up to a few stellar radii from the stellar crust are the favourite sites to explain the coherent pulsed radio emission. For our geometric model, both polar caps radiate most efficiently at their centre, which means along the magnetic axis. When moving away from the poles, staying on the stellar surface, the radio intensity should decrease. We therefore choose a smooth Gaussian decreasing with distance from the magnetic north or south pole, almost complete extinction occurring outside the polar caps. Their size is determined by the location of the foot of the last open magnetic field lines, on the stellar surface. For simplicity, bending of magnetic field lines, (general) relativistic effects and plasma effects are not included. The distortions implied by these effects are not included in our description. Moreover, in order to express the adjustment for the aligned rotator, the size of one polar cap, assumed to have a circular shape, is obtained from its angular opening angle \( \theta_{pc} \) leading to a radius \( R_{pc} \) such that

\[
\sin \theta_{pc} = \sqrt{\frac{R_s}{r_L}}.
\]

\[
R_{pc} = R_s \sin \theta_{pc}.
\]

The maximal intensity arises when the line of sight and magnetic moment are aligned or counter-aligned. For a sharp transition between on and off states, emission is expected only when the angle formed by \( \mathbf{\mu}_s \) and \( \mathbf{n} \) is less than the angular opening angle; thus \( (\mathbf{\mu}_s \cdot \mathbf{n}) \leq \sin \theta_{pc} \). In other words, a polar cap (either north or south) is only seen if

\[
\cos(\mu_s \cdot n) = \frac{\mu_s \cdot n}{\mu_s} \geq \cos \theta_{pc} = \sqrt{1 - \frac{R_s}{r_L}}.
\]

This is translated mathematically for our Gaussian by introducing the intermediate function \( \Phi_{pc} \), defined as

\[
\Phi_{pc}(t) = \frac{\mu_s \cdot n}{\mu_s} = \cos \chi \cos \phi + \sin \chi \sin \phi (\Omega_\phi t).
\]

Therefore, one polar cap is visible if

\[
\Phi^2_{pc}(t) \geq 1 - \frac{R_s}{r_L}.
\]

Expressed as a polar cap emissivity \( \epsilon_{pc} \) with a Gaussian similar shape, we write

\[
\epsilon_{pc} \propto \exp \left[ \alpha \left( \Phi^2_{pc} - 1 + \frac{R_s}{r_L} \right) \right],
\]

where \( \alpha \) is a positive constant controlling the extension of significant polar cap emission. \( \epsilon_{pc} \) is the maximum for \( \mu_s \) and \( \mathbf{n} \) colinear with a value close to unity and tends to zero for large distances from the magnetic poles. We stress that this description is purely phenomenological, to be included in the phase plot for gamma-ray light curves.

### 3 RESULTS

Our simple geometrical model allows us to perform some analytical calculations to deduce the relative lag in arrival time between radio and gamma-ray photons and the gamma-ray peak separation, if both radio emission and double peak structure are detectable. In the following subsections, we perform a detailed analysis of the light-curve properties and correlation with radio wavebands depending on \( \chi \) and \( \zeta \). We then close the discussion with application to some gamma-ray pulsars from the Fermi first-year catalogue.

#### 3.1 High-energy peak separation

In the striped wind model, the light curve for the pulsed emission shows usually (in a favourable configuration) a double-peak structure. In this case, there exists a simple relation between this double gamma-ray peak separation, denoted by \( \Delta \), and the geometry of the system, assuming a rotating dipole with magnetic obliquity \( \chi \) and line-of-sight inclination \( \zeta \). Indeed, to understand the physical mechanism, let us assume that the wind radiates mostly when crossing a spherical shell located at a radius \( R_0 \). Because of the strong relativistic beaming effect for ultrarelativistic expansion, the observer only sees light travelling towards him [i.e. in the direction \((\vartheta = \chi, \varphi = \pi/2)\)]. For particles located in the current sheets, this corresponds to an observer time \( t_s \) satisfying

\[
R_0 = r_s(\vartheta, \varphi, t_s) = \beta_r r_L \left[ \pm \arccos(-\cot \chi \cot \zeta) + \frac{c t_s}{r_L} - \frac{\pi}{2} + 2\pi \right].
\]

where \( l \in \mathbb{Z} \) is a natural integer. Solving for this time \( t_s \) at which the photon leaves the current sheet, we find

\[
t_s = \frac{R_0}{c} \pm \frac{\arccos(-\cot \chi \cot \zeta) + \frac{P}{4} - l P}{\Omega_s}.
\]

The \( \pm \) sign labels the arm of the double spiral structure (seen in the equatorial plane) responsible for the radiation. Recall that the double-peak light curves follow from this double spiral shape.
Therefore, the time separation between two consecutive pulses becomes
\[
\Delta t_{\text{con}} = t^+_\text{con} - t^-_\text{con} = 2 \frac{\arccos(-\cot \chi \cot \zeta)}{\Omega}. \tag{17}
\]

Note that this is not necessarily the high-energy peak separation because it can, in principle, vary between zero and a full period \( P \). To be more precise, we have to compare one specified pulse with its two neighbours in time, one coming earlier to the observer and the other later. Doing so, we obtain
\[
\Delta t_{\text{peak}} = \min \left[ |t^+_\text{con} - t^-_\text{con}|, |t^+_\text{con} - (t^-_\text{con} + P)| \right] = \min(\Delta t_{\text{con}}, |\Delta t_{\text{con}} - P|). \tag{18}
\]

Normalizing to the period of the pulsar \( P \), the phase separation reads
\[
\Delta = \frac{\Delta t_{\text{peak}}}{P} = \min \left[ \frac{\arccos(-\cot \chi \cot \zeta)}{\pi}, \frac{\arccos(-\cot \chi \cot \zeta)}{\pi} - 1 \right]. \tag{19}
\]

From the definition of the \( \arccos \) function, we obtain
\[
\Delta = \begin{cases} \frac{\arccos(-\cot \chi \cot \zeta)}{\pi} & \text{if } \cot \chi \cot \zeta \leq 0, \\ 1 - \frac{\arccos(-\cot \chi \cot \zeta)}{\pi} & \text{if } \cot \chi \cot \zeta \geq 0. \end{cases} \tag{20}
\]

Finally, the relation between obliquity \( \chi \), line-of-sight inclination angle \( \zeta \) and peak separation \( \Delta \) can be rearranged into
\[
\cos(\pi \Delta) = |\cot \chi \cot \zeta|. \tag{21}
\]

To summarize, we find
\[
\cos(\pi \Delta) = |\cot \chi \cot \zeta|. \tag{22}
\]

This relation is shown in Fig. 1 where \( \cos \chi \) is plotted versus \( \cos \zeta \) for a constant value of the separation \( \Delta \) starting from zero to 0.5 with a step of 0.02.

A few special cases are worth mentioning. First, zero separation or overlapping of both pulses, corresponding to \( \Delta = 0 \), implies \( \zeta = \pi/2 - \chi \). For the chosen variables in Fig. 1, this corresponds to the quarter circle of radius unity centred at the origin. Secondly, a separation of half a period, \( \Delta = 0.5 \), implies a line of sight containing the equatorial plane of the pulsar, \( \zeta = \pi/2 \). This is independent of the obliquity. This corresponds to the straight horizontal line along the \( x \)-axis in Fig. 1. The relation equation (22) was already noticed by Kirk (2005, see figure therein). For symmetrical reasons, we assume that \( \chi \in [0, \pi/2] \), and therefore \( \cot \chi \geq 0 \).

### 3.2 Radio lag–peak separation (\( \delta \)–\( \Delta \)) relation

Next, we show that the combination of polar cap/striped wind allows us to derive a simple analytical relation between the lag of radio versus gamma-ray arrival time \( \delta \) and the high-energy peak separation \( \Delta \). Because the two sites of emission are very distinct, inside and outside the light cylinder, the lag is interpreted as a time-of-flight delay between the region of radio radiation and that for gamma-ray photon production.

The choice of the origin of time and line of sight located in the plane (\( Oy\zeta \)) (therefore \( \varphi = \pi/2 \)) implies that the radio photon emitted towards the observer will occur at periodic dates with period \( P \), for one pole, given by
\[
t_{\text{pc}}^n = \frac{\pi}{2 \Omega_1} + \frac{2\pi}{\Omega_2} = \frac{P}{4} + kP. \tag{23}
\]

The gamma-ray photon from the current sheets arrives earlier because, being closer to the observer, it needs a shorter flying time given by \( \Delta t = (D - R_\gamma)/c \), leading to a time of detection given by
\[
t_\gamma = t_{\text{pc}}^n + \frac{D - R_\gamma}{c}. \tag{24}
\]

The gamma-ray photon from the current sheets arrives earlier because, being closer to the observer, it needs a shorter flying time given by \( \Delta t = (D - R_\gamma)/c \), leading to a time of detection given by
\[
t_\gamma = t_{\text{pc}}^n + \frac{D - R_\gamma}{c}. \tag{25}
\]

As a consequence, the difference in arrival time between both types of photon, focusing on the radio emission coming out of the north pole (visible for \( 0 < \zeta < \pi/2 \)), is
\[
\Delta t = t_\gamma - t_{\text{pc}}^n + \frac{R_\gamma - R_0}{c} = \frac{1 - \beta_\gamma}{\beta_\gamma} \frac{R_0}{c} \pm \frac{\arccos(-\cot \chi \cot \zeta)}{\Omega_1} + \frac{R_\gamma}{c} = -(k + l)P. \tag{27}
\]

By normalizing this time to the period of the pulsar \( P \) and using equation (20) with \( \cot \chi \cot \zeta \geq 0 \), we arrive at
\[
\delta = \frac{t_\gamma}{P} = \frac{1 - \beta_\gamma}{\beta_\gamma} \frac{R_0}{2\pi r_L} \pm \frac{1 - \Delta}{2} + \frac{R_\gamma}{2\pi r_L} - (k + l). \tag{28}
\]
The same procedure applied to the south pole with $\pi/2 < \zeta < \pi$ implies $\cot \zeta \cot \chi \leq 0$, leading to the same conclusion because of symmetry.

Recall that in order to observe pulsed emission from the radially expanding wind, the flow has to be relativistic with $\Gamma_\gamma \gg 1$ and $R_0$ must lie close enough to the light cylinder such that the underlying condition is fulfilled:

$$R_0 \lesssim 2\pi \Gamma_\gamma^2 \chi L_\ast. \tag{29}$$

Thus, the upper limit to the first term in equation (28) is

$$1 - \frac{R_0}{2\pi \chi L_\ast} - \frac{1 - \frac{\beta}{\Gamma_\gamma^2}}{2} \lesssim \frac{1}{2}. \tag{30}$$

Moreover, for all the pulsars in the catalogue, even for the millisecond pulsars, the light-cylinder radius is much larger than the stellar radius, $r_c \gg R_*$. To a good approximation, we can neglect the term $R_0/(2\pi r_c)$ in equation (28); the neutron star is located well within its light cylinder. The remaining expression for a positive lag is

$$\delta \approx \frac{1 - \Delta}{2}. \tag{31}$$

This has to be compared with the $\delta-\Delta$ diagram published in Abdo et al. (2010a). The data points, as well as the relation equation (31), are shown in Fig. 2. All the points but one lie below the line given by equation (31). The spread in radio lag along this line can be understood as a fluctuation in the precise location of the most significant emission in the wind.

Finally, we give an expression for the linear fit to the sample of pulsars showing double-peaked gamma-ray light curves simultaneously with radio emission by

$$\Delta_{\text{GR}} \approx 0.59 - 1.06\delta, \tag{32}$$

shown by the blue line in Fig. 2.

### 3.3 Other retardation effects

Including general relativistic effects does not change much the time lag between radio and gamma-ray photons. Indeed, using the Schwarzschild geometry, for a photon with radial motion, the time required to reach a distant observer located at a distance $D$, starting on the stellar surface is

$$c\Delta_{\text{GR}} = D - R_* + R_0 \ln \left( \frac{D - R_*}{R_0 - R_*} \right). \tag{33}$$

where $R_0 = 2GM/c^2$ is the Schwarzschild radius. A similar expression applies for gamma-rays

$$c\Delta_{\gamma} = D - R_0 + R_0 \ln \left( \frac{D - R_*}{R_0 - R_*} \right). \tag{34}$$

Therefore, the time of flight difference compared to Newtonian gravity is

$$\Delta t_{\text{GR}} = \Delta t_{\gamma} - \Delta t_{\ast} = \frac{R_\ast - R_0}{c} + \frac{R_\ast}{c} \ln \left( \frac{R_\ast - R_*}{R_0 - R_*} \right). \tag{35}$$

Normalized to the pulsar period we obtain an increase by an amount of

$$\delta_{\text{GR}} = \frac{R_\ast}{2\pi \chi L_\ast} \ln \left( \frac{R_\ast - R_*}{R_0 - R_*} \right). \tag{36}$$

For neutron star parameters, this remains negligible compared to unity because $R_\ast \ll R_\ast$. Typical values are $\delta_{\text{GR}} \approx 0.01$.

### 3.4 Phase-inclination diagram

Next, we show the phase-inclination diagram for different inclinations of the line of sight $\zeta$ as well as different magnetic obliquities $\chi$ of the pulsar. This is done for the high-energy photons as well as for the radio emission coming from the polar cap (see Fig. 3). In order to point out the relative phase shift between both emission processes, we normalized independently their respective maximum intensity to unity. The continuous rotated S-shaped structure is a characteristic of the striped wind whereas the two spots correspond to a mapping of the two polar caps. Both structures are symmetric with respect to the equatorial plane (i.e. compared to an inclination $\zeta = 90^\circ$), as expected from the geometry.

The observer will see different types of light curves depending on the viewing angle. First, radio photons are observable only when the line of sight intersects at least one polar cap, in the vicinity of the magnetic poles (i.e. when $\zeta \approx \chi \pm \theta_{p\bar{c}}$). Secondly, to observe the gamma-ray pulsation, this same line of sight should intersect the current sheets, or in other words $\pi/2 - \chi \leq \zeta \leq \pi/2 + \chi$. Conversely, no gamma-ray pulses are seen when $\zeta \leq \pi/2 - \chi$ or $\zeta \geq \pi/2 + \chi$. Thus, a necessary condition to detect simultaneously radio and gamma-ray photons is $\chi \geq \pi/4$ and $\zeta \approx \chi$. For these particular pulsars, possessing a double gamma-ray peak and radio emission, we are able to find the geometrical parameters from the peak separation (see Fig. 1). Thirdly, a single pulse in the high-energy light curve occurs when the line of sight passes just through the edge of the striped part of the wind; this can be interpreted as the special case $\zeta \approx \pi/2 - \chi$. In the very special case of one gamma-ray pulse and observable radio emission, this leads immediately to the geometrical parameters $\chi \approx \zeta \approx \pi/4 = 45^\circ$. This seems to be the case for PSR J0437-4715 and PSR J2229+6114.

The phase plots shown in Fig. 3 have been computed for different inclination angles $\zeta$ and obliquities $\chi$ but with constant current sheet thickness, parametrized by $\Delta_{\ast}$, and constant particle density contrast, which means constant ratio $N/N_0$. More precisely, we adopted $\Delta_{\ast} = 10$ and $N/N_0 = 10$. This last value explains the factor of around 10 in intensity between the off-pulse and on-pulse phases. Indeed, by decreasing the fluctuation in particle density between the hot unmagnetized and cold magnetized part of the wind, as done in Fig. 4, we recognize a similar trend in the intensity diagram. If the ratio $N/N_0 = 5$, the luminosity varies between 0.22 $\approx 1/5$ and 1 in normalized units whereas for $N/N_0 = 2$, it lies between 0.54 $\approx 1/2$ and 1.

Moreover, the current sheet thickness directly impacts on the duty cycle of the light curve, or in other words, on the width of
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Figure 3. Phase plot of the pulsed gamma-ray and radio emission components for a full period of the pulsar (phase ∈ [0, 1]) for an inclination of the line of sight ζ between 0° and 180° and magnetic obliquities χ = 18°, 36°, 54° and 72°, from top left to bottom right. Note the colour-coded range shown in the legends of each phase plot: the white colour corresponds to the faintest phase (off-pulse) with an intensity around 0.11 whereas the red colour corresponds to the brightest luminosity (i.e. 1, in normalized unities).

The gamma-ray pulses. This has been checked by changing the parameter Δφ to 5 or 2 instead of the previous value of 10. The results are shown in Fig. 5.

To better assess the differences, we summarize all the light curves for χ = 72° and ζ = 90° in two plots, as depicted in Fig. 6.

Finally, there are two limiting cases: (i) the aligned rotator shows no pulsed emission; (ii) the perpendicular rotator emits pulsed high-energy radiation over the whole sky and both its polar caps are visible. A direct consequence is a double-peak structure in both radio and gamma-ray light curves, with a separation of Δ = 0.5. PSR J0030+0451 is a typical example.

3.5 Flux correction factor

The photon flux $F_{\text{obs}}$ measured by an observer located on Earth is biased because of anisotropic emission from the wind depending on the viewing angle ζ. This fact is clear from the aforementioned phase-inclination plots shown in the previous subsections.
Figure 4. Phase plot of the pulsed gamma-ray and radio emission components for a full period of the pulsar (phase $\in [0, 1]$) for an inclination of the line of sight $\zeta$ between $0^\circ$ and $180^\circ$ and a magnetic obliquity $\chi = 72^\circ$. Only the ratio $N/N_0$ changes.

Figure 5. Phase plot of the pulsed gamma-ray and radio emission components for a full period of the pulsar (phase $\in [0, 1]$) for an inclination of the line of sight $\zeta$ between $0^\circ$ and $180^\circ$ and a magnetic obliquity $\chi = 72^\circ$. Only the parameter $\gamma$ varies.

The observed gamma-ray flux has to be corrected to obtain the true gamma-ray luminosity by introducing a correction factor $f_{\Omega}$ defined by

$$L_{\gamma} = 4\pi f_{\Omega} F_{\text{obs}} D^2.$$  

Here, $D$ is the distance of the pulsar to the observer and $F_{\text{obs}}$ is the observed flux. As in the polar cap and outer/slot gap models (Watters et al. 2009), the correction implied here by a relativistic beaming effect is given by

$$f_{\Omega}(\chi, \zeta_E) = \frac{\int_0^\pi \int_0^{2\pi} F_{\gamma}(\chi, \zeta, \varphi) \sin \zeta \, d\zeta \, d\varphi}{2 \int_0^{2\pi} F_{\gamma}(\chi, \zeta_E, \varphi) \, d\varphi}. \quad (38)$$

For the striped wind model, this correction factor is shown in Fig. 7 with the full dependence on obliquity $\chi$ and inclination of Earth line of sight $\zeta_E$. We can approximately separate the correction factor into two regions of constant value. In the first region, for an obliquity $\chi < \pi/2 - \zeta_E$, the correction is close to uniform and equal roughly to 1.90. This case corresponds to a line of sight not crossing the
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3.6 Light-curve fitting and gamma-ray luminosity

We conclude our study by fitting the light curves of a small sample of pulsars. The only relevant free parameters in our model are the geometry of the wind, the particle density number and the size of the current sheets. In this section, we specialize our results to some Fermi-detected gamma-ray pulsars and show the best parameters fitting their high-energy light curves above 100 MeV. Therefore, knowledge of the viewing angle and the obliquity allows an estimation of the flux correction via the beaming factor. Eventually, a true gamma-ray luminosity versus spin-down luminosity can be plotted.

We start with an estimate of the peak separation when one radio pulse is detected. In this case, \( \zeta \approx \chi \) as explained above and the knowledge of \( \Delta \) immediately implies a solution for \( \zeta \). This has been done for several pulsars, as listed in Table 1. In all the results shown here, for simplicity, we have taken a constant Lorentz factor of the wind equal to \( \Gamma = 10 \).

Let us have a more in-depth look at a representative sample of Fermi-detected gamma-ray pulsars.

| Pulsar   | \( \delta \) (obs) | \( \Delta \) (obs) | \( \chi \) deg | \( \xi \) deg | \( \delta \) (model) | \( \Delta \) (1 - \( \Delta \))/2 | \( N/N_0 \) | \( f_\Omega \) | \( L_\gamma \) \( 10^{26} \) W | \( L_\gamma \) (corrected) \( 10^{26} \) W | Efficiency \( \eta \) |
|---------|-------------------|-------------------|--------------|--------------|-------------------|------------------------|--------|----------|-----------------|------------------|-------------|
| J0030+0451 | 0.18              | 0.44              | 67           | 85           | 0.28              | 5                      | 10     | 1.17     | 0.57            | 0.49             | 0.16        |
| J0218+4232 | 0.32              | 0.36              | 57           | 75           | 0.32              | 3                      | 3      | 1.10     | 27–69           | 24–62             | 0.10–0.26    |
| J0437–4715 | 0.43              | –                 | 45           | 40           | 0.5               | 10                     | 10     | 1.07     | 0.054           | 0.050             | 0.016       |
| J1124–5916 | 0.23              | 0.49              | 80           | 85           | 0.255             | 5                      | 3      | 1.20     | 100             | 83               | 0.007       |
| J2021+3651 | 0.17              | 0.47              | 73           | 65           | 0.265             | 10                     | 10     | 1.14     | 250             | 220              | 0.065       |
| J2032+4127 | 0.15              | 0.50              | 89           | 70           | 0.25              | 5                      | 3      | 1.18     | 34–170          | 29–145           | 0.11–0.55    |
| J2043+2740 | 0.20              | 0.36              | 57           | 65           | 0.32              | 5                      | 3      | 1.05     | 6               | 5.7              | 0.10        |
| J2229+6114 | 0.49              | –                 | 45           | 40           | 0.5               | 10                     | 10     | 1.07     | 17–1100         | 16–1026          | 0.0007–0.045 |

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Table 1. Geometry of the pulsar wind for eight gamma-ray pulsars. The angle of inclination and obliquity are given in degrees. The Lorentz factor of the wind is the same for the whole sample and is taken to be \( \Gamma = 10 \).
3.6.1 PSR J0030+0451

PSR J0030+0451 is a millisecond pulsar, $P = 4.87$ ms, showing a double-pulse structure in the radio band. This may suggest that both its magnetic poles are visible or, in other words, it is almost a perpendicular rotator with $\chi \approx 90^\circ$. Nevertheless, the maximal intensities of both radio pulses are noticeably different; this is interpreted as the line of sight passing closer to one magnetic pole than to the other. An alternative would be to explain this by a process occurring in the vicinity of the polar caps with variable efficiency. However, the polar cap emission is not the main purpose of this work so we simply assume identical shapes for both radio pulses. Therefore, an obliquity close to $90^\circ$ but less matches the right geometry. Setting $\chi \approx 67^\circ$ satisfactorily agrees with the radio light curve (see Fig. 8, top-left plot). To have the right gamma-ray peak separation, we have to adopt $\zeta \approx 85^\circ$. Moreover, the radio pulses are very broad, each of them having a duty cycle of roughly 0.2 in phase. Therefore, to reconcile our model with the data, we have to extend the polar cap region to a sizeable fraction of the whole neutron star surface, (Fig. 8, top-left plot). Emission starts right after the light-cylinder radius. There is still an excess of 0.1 in the phase delay compared to observations. This has to be explained by some other retardation effects of the radio pulse, such as the strong gravitational field regime (which we have shown to be negligible) or by the magnetic field bending as a result of charges flowing within the magnetosphere and disturbing the closed field-line structure taken to be an exact dipole.

3.6.2 PSR J0218+4232

PSR J0218+4232 is another millisecond pulsar, $P = 2.32$ ms, showing a less clear double-pulse structure in gamma-rays. Its radio-pulse shape looks much more complicated with something like conal and core component, over a wide range of the pulse period. The best fit is obtained for an obliquity $\chi = 57^\circ$ and an inclination of $\zeta = 75^\circ$. The gamma-ray light curve adjusts well to the Fermi data. Moreover, the gamma-ray pulse time lag, compared to the middle of the radio pulse, matches precisely the measurements. Note that the gamma-ray off-pulse emission remains at an appreciable level over the full period (see Fig. 8, top-right plot).

3.6.3 PSR J0437–4715

PSR J0437–4715 is a special case of a millisecond pulsar, $P = 5.76$ ms, showing only one gamma-ray pulse combined with a sharp radio pulse. As explained in a previous discussion, the unique geometry allowing such a behaviour needs an obliquity almost equal to the inclination of the line of sight, namely $\zeta \approx 45^\circ$. Indeed, we take $\chi = 45^\circ$ and $\zeta = 40^\circ$ and arrive at the light curve presented in Fig. 8 (second row, left plot). For one gamma-ray pulse, our model predicts a delay of 0.5 in phase between radio and gamma-ray pulsars, in relatively good agreement with the Fermi measurement, 0.43. The overestimate is about 0.07.

3.6.4 PSR J1124–5916

PSR J1124–5916 shows a sharp double gamma-ray peak with significant off-pulse emission in combination with a single radio pulse. Adopting the parameters $\chi = 80^\circ$ and $\zeta = 85^\circ$, our model is compared with the observation in Fig. 8 (second row, right plot). The expected time lag 0.255 is not very different from the measured 0.23.

3.6.5 PSR J2021+3651

The PSR J2021+3651 gamma-ray light curve shows a narrow double-pulse structure and a single radio pulse. The peak separation is close to 0.5. Here again, for the best-fitting parameters $\chi = 73^\circ$ and $\zeta = 65^\circ$, the predicted lag of 0.265 overshoots the observation by roughly 0.1 (see Fig. 8, third row, left plot).

3.6.6 PSR J2032+4127

PSR J2032+4127 is an example of half-a-period peak separation, implying a line-of-sight inclination of $\zeta \lesssim 90^\circ$. Moreover, because we see a radio pulse it should be almost a perpendicular rotator. However, this would permit us to detect both radio components, which is not the case. So we must conclude that the inclination is slightly different from an orthogonal rotator with a small polar cap reproducing a width of 0.1 in the radio pulse (Fig. 8, third row, right plot). We found the best agreement for $\chi = 89^\circ$ and $\zeta = 70^\circ$.

3.6.7 PSR J2043+2740

It seems that PSR J2043+2740 shows evidence of three gamma-ray pulses (Fig. 8, bottom-left plot). The two extreme ones are significant whereas the middle one is less significant. We tried to fit the light curve according to the two well-separated pulses. The time delay is overestimated by 0.12.

3.6.8 PSR J2229+6114

Finally, PSR J2229+6114 is another example of a single-pulse gamma-ray pulsar. The best-fitting parameters correspond to the same values as for PSR J0437–4715. Now, the time lag is even better: again the model predicts 0.5 compared to the observed 0.49 (Fig. 8, bottom-right plot).

Details of the parameters used to fit the data are summarized in Table 1. This ends the sample of fitted gamma-ray pulsars. We have demonstrated that our striped wind/polar cap model implies a well-defined relation between gamma-rays and radio pulses. Single- and double-pulse light curves are expected. For some pulsars, the model explains this relationship well. However, it seems that some others pulsars possess a time lag slightly less than the expected value, on average smaller by an amount of 0.1 in phase. This probably finds its root in the geometry of the polar cap or in retardation effects not taken into account in this investigation.

3.6.9 Corrected gamma-ray luminosity

Finally, knowing the precise geometry from the light curves and time lag, we computed the flux correction factor for each of the above-mentioned pulsars. We summarize our results in Table 1. The ‘true’ efficiency is also given therein.

4 CONCLUSION

By combining the striped wind model with a simple prescription for the polar cap geometry and emission, we have been able to derive...
the time lag between radio and gamma rays according to the high-energy pulse separation, if available, in agreement with recent Fermi observations from the first gamma-ray pulsar catalogue. According to our composite model, it seems that the observed gamma-ray pulsation is generated just outside the light cylinder and radio pulses come from low-altitude polar cap locations.

A further careful inspection of individual pulsar light curves, the correlation between their radio and gamma-ray peaks, will allow us to severely constrain the geometry, deducing the obliquity and inclination of line of sight of the system.

The overestimate of time lag by roughly 0.1 in phase for many pulsars suggests that another ingredient is still missing if we are to fit the data properly. As shown, curved space–time is negligible, so we expect magnetic field line bending and/or plasma flow within the magnetosphere and other plasma effects to give us some clue to this enigma.
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