Dynamical Generation of Yukawa Interactions in Intersecting D-brane Models

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ABSTRACT: We construct a supersymmetric composite model in type IIA $T^6/(Z_2 \times Z_2)$ orientifold with intersecting D6-branes. Four generations of quarks and leptons are naturally emerged as composite fields at low energies. Two pairs of light electroweak Higgs doublets are also naturally obtained. The hierarchical Yukawa couplings for the quark-lepton masses can be generated by the interplay between the string-level higher dimensional interactions among “preons” and the dynamics of the confinement of “preons”. Besides having four generations of quarks and leptons, the model is not realistic in some points: some exotic particles, one additional U(1) gauge symmetry, no explicit mechanism for supersymmetry breaking, and so on. This model is a toy model to illustrate a new mechanism of dynamical generation of Yukawa couplings for the masses and mixings of quarks and leptons.

KEYWORDS: D-branes, Compactification and String Models, Quark Masses and SM Parameters, Technicolor and Composite Models
1. Introduction

The understanding of the masses and flavor mixings of quarks and leptons is one of the most important and difficult problems in particle physics. In the standard model these masses and mixings are described by Yukawa coupling constants which are free parameters in the model. In the field theory models beyond the standard model there are mainly two directions to derive these free parameters. One direction is the unification of Yukawa coupling constants in grand unified theories. The number of parameters decreases by the unification, and the hierarchical structure of Yukawa interactions is expected to be generated through the dynamics of spontaneous breaking of the unified gauge symmetry. Another direction is the compositeness of Higgs and/or quarks and leptons. The introduction of compositeness makes impossible to have Yukawa interactions in renormalizable way at the tree level, and the hierarchical Yukawa interactions are expected to be generated through the dynamics of strong coupling gauge interactions.

In string models we can expect that the Yukawa interactions for the masses of quarks and leptons are directly generated at the string level. The main difficulty of string models is to find the vacuum on which the particle spectrum of the standard model is realized without light exotic particles. Recent developments in the string models based on intersecting D-branes (see refs.[1, 2, 3, 4] for the essential idea) suggest a way to obtain realistic models. Especially, the models with low-energy supersymmetry [4, 5, 6, 7, 8, 9] are interesting, because these models are constructed as stable solutions of the perturbative string theory. Especially for the model of ref.[4], two important properties, the Yukawa interactions for quark-lepton masses[10] and supersymmetry breaking[11], are concretely discussed. In this paper we propose a new mechanism to generate Yukawa interactions for quark-lepton masses by explicitly constructing a string model.
Consider the type IIA superstring theory compactified on $\mathbb{T}^6/\langle \mathbb{Z}_2 \times \mathbb{Z}_2 \rangle$ orientifold, where $\mathbb{T}^6 = \mathbb{T}^2 \times \mathbb{T}^2 \times \mathbb{T}^2$. The type IIA theory is invariant under the $\mathbb{Z}_2 \times \mathbb{Z}_2$ transformation

$$
\theta : \quad X^k_\pm \to e^{\pm i 2\pi v} X^k_\pm, \quad (1.1)
$$

$$
\omega : \quad X^k_\pm \to e^{\pm i 2\pi w} X^k_\pm, \quad (1.2)
$$

where $v = (0, 0, 1/2, –1/2, 0)$ and $w = (0, 0, 1/2, –1/2)$ and

$$
X^k_\pm = \begin{cases} 
\frac{1}{\sqrt{2}} (X^{2k} \pm X^{2k+1}), & \text{for } k = 0, \\
\frac{i}{\sqrt{2}} (X^{2k} \pm iX^{2k+1}), & \text{for } k = 1, 2, 3, 4
\end{cases} \quad (1.3)
$$

with space-time coordinates $X^\mu$, $\mu = 0, 1, \cdots, 9$. The type IIA theory is also invariant under the $\Omega R$ transformation, where $\Omega$ is the world-sheet parity transformation and

$$
R : \quad \begin{cases} 
X^i \to X^i, & \text{for } i = 0, 1, 2, 3, 4, 6, 8, \\
X^j \to -X^j, & \text{for } j = 5, 7, 9.
\end{cases} \quad (1.4)
$$

We mod out the theory by the actions of $\theta$, $\omega$, $\Omega R$ and their independent combinations.

A D6$_a$-brane stretching over our three-dimensional space and winding in compact $\mathbb{T}^2 \times \mathbb{T}^2 \times \mathbb{T}^2$ space is specified by the winding numbers in each torus:

$$
[(n^1_a, m^1_a), (n^2_a, m^2_a), (n^3_a, m^3_a)]. \quad (1.5)
$$

A D6$_a$-brane is always accompanied by its orientifold image D6$_{a'}$ whose winding numbers are

$$
[(n^1_a, -m^1_a), (n^2_a, -m^2_a), (n^3_a, -m^3_a)]. \quad (1.6)
$$

The number of intersection between D6$_a$-brane and D6$_b$-brane is given by

$$
I_{ab} = \prod_{i=1}^3 (n^i_a m^i_b - m^i_a n^i_b). \quad (1.7)
$$

The intersecting angles, $\theta_a^i$, in each torus between D6$_a$-brane and $X^4$, $X^6$ and $X^8$ axes are given by

$$
\theta_a^i = \tan^{-1} \left( \frac{m^i_a}{\chi_i n^i_a} \right), \quad (1.8)
$$

where $\chi_i$ are the ratios of two radii of each torus: $\chi_i \equiv R^{(i)}_2 / R^{(i)}_1$. The system has supersymmetry, if $\theta_a^1 + \theta_a^2 + \theta_a^3 = 0$ is satisfied for all $a$. The configuration of intersecting D6-branes should satisfy the following Ramond-Ramond tadpole cancellation conditions.

$$
\sum_a N_a n^1_a n^2_a n^3_a = 16, \quad (1.9)
$$

$$
\sum_a N_a n^1_a m^2_a m^3_a = -16, \quad (1.10)
$$

$$
\sum_a N_a m^1_a n^2_a m^3_a = -16, \quad (1.11)
$$

$$
\sum_a N_a m^1_a m^2_a n^3_a = -16, \quad (1.12)
$$

\[ -2 - \]
where \( N_a \) is the multiplicity of D6\(_a\)-brane, and we are assuming three rectangular (untitled) tori. The Neveu-Schwarz-Neveu-Schwarz tadpoles are automatically cancelled, when both the Ramond-Ramond tadpole cancellation conditions and the supersymmetry conditions are satisfied.

There are four sectors of open string depending on the D6-branes on which two ends of open string are fixed: \( aa, ab + ba, ab' + b'a, aa' + a'a \) sectors. Each sector gives matter fields in low-energy four-dimensional space-time. The general massless field contents are given in table 1. A common problem of the model building in this framework is the appearance of the massless adjoint fields in \( aa \) sector, since there are no massless matter field in the adjoint representation under the standard model gauge group in Nature. These fields are expected to be massive in case of the appropriate curved compact space, because these are the moduli fields of D6-brane configurations.

In the next section we construct a supersymmetric composite model. The standard model gauge groups are originated from the D6-branes which are not parallel to any O6-planes. Tadpole cancellation conditions can be satisfied by introducing several additional D6-branes which are parallel to certain O6-planes. Since the USp(\( N \)) gauge interactions on these D6-branes are stronger than the U(\( N \)) gauge interactions in general, it is natural to identify them to the confining dynamics among “preons”. Four generations of quarks and leptons are naturally emerged. Two pairs of light electroweak Higgs doublets are also naturally obtained. In section 3 the dynamical generation of Yukawa interactions for the quark-lepton mass and mixing is discussed. The hierarchical Yukawa coupling matrices can be generated by the interplay between the string-level higher dimensional interactions among “preons” and the dynamics of the confinement of “preons”. We are not going to calculate all the Yukawa coupling constants in this toy model, but instead we give a general scenario to have small Yukawa coupling constants and describe an example of having quark-lepton mass differences. In the last section we present a summary and conclusions.

### 2. The Model and Low-energy Dynamics

The D6-brane configuration of the model is given in table 2. Both tadpole cancella-

| sector     | field                                                                 |
|------------|----------------------------------------------------------------------|
| \( aa \)  | \( U(N_a/2) \) or USp(\( N_a \)) gauge multiplet. \( 3 U(N_a/2) \) adjoint or \( 3 \) USp(\( N_a \)) anti-symmetric tensor chiral multiplets. |
| \( ab + ba \) | \( I_{ab} (\square_a, \square_b) \) chiral multiplets.               |
| \( ab' + b'a \) | \( I_{ab'} (\square_a, \bar{\square}_b) \) chiral multiplets.        |
| \( aa' + a'a \) | \( \frac{1}{2} (I_{aa'} - \frac{1}{N} I_{aO6}) \) symmetric tensor chiral multiplets. |
|            | \( \frac{3}{2} (I_{aa'} + \frac{1}{N} I_{aO6}) \) anti-symmetric tensor chiral multiplets. |

Table 1: General massless field contents on intersecting D6-branes. In \( aa \) sector, the gauge symmetry is USp(\( N_a \)) or U(\( N_a/2 \)) corresponding to whether D6\(_a\)-brane is parallel or not to a certain O6-plane, respectively. In \( aa' + a'a \) sector, \( k \) is the number of tilted torus, and \( I_{aO6} \) is the sum of the intersection numbers between D6\(_a\)-brane and all O6-planes.
Table 2: Configuration of intersecting D6-branes. All three tori are considered to be rectangular (untilted). Three D6-branes, D6_4, D6_5 and D6_6, are on top of certain O6-planes.

| D6-brane | winding number | multiplicity |
|----------|---------------|--------------|
| D6_1     | [(1,−1),(1,1),(1,0)] | 4            |
| D6_2     | [(1,1),(1,0),(1,−1)]   | 6 + 2        |
| D6_3     | [(1,0),(1,−1),(1,1)]   | 2 + 2        |
| D6_4     | [(1,0),(0,1),(0,−1)]   | 12           |
| D6_5     | [(0,1),(1,0),(0,−1)]   | 8            |
| D6_6     | [(0,1),(0,−1),(1,0)]   | 12           |

Orientifold projection conditions and supersymmetry conditions are satisfied in this configuration under the condition of \( \chi_1 = \chi_2 = \chi_3 = \chi \). D6_2-brane consists of two parallel D6-branes of multiplicity six and two which are separated in the second torus in a consistent way with the orientifold projections. D6_3-brane consists of two parallel D6-branes of multiplicity two which are separated in the first torus in a consistent way with the orientifold projections. D6_1, D6_2 and D6_3 branes give gauge symmetries of \( U(2)_L = SU(2)_L \times U(1)_L \), \( U(3)_c \times U(1) = SU(3)_c \times U(1)_c \times U(1) \) and \( U(1)_1 \times U(1)_2 \), respectively. The hypercharge is defined as

\[
\frac{Y}{2} = \frac{1}{2} \left( \frac{Q_c}{3} - Q \right) + \frac{1}{2} (Q_1 - Q_2),
\]

where \( Q_c, Q, Q_1 \) and \( Q_2 \) are charges of \( U(1)_c, U(1), U(1)_1 \) and \( U(1)_2 \), respectively. There is another non-anomalous \( U(1) \) gauge symmetry, \( U(1)_R \), whose charge is defined as

\[
Q_R = Q_1 - Q_2.
\]

The remaining three \( U(1) \) gauge symmetries which are generated by \( Q_L \) (namely \( U(1)_L \)), \( Q_c + Q \) and \( Q_1 + Q_2 \) are anomalous, and their gauge bosons have masses of the order of the string scale. These three anomalous \( U(1) \) gauge symmetries are independent from the two non-anomalous \( U(1) \) gauge symmetries: \( \text{tr}((Y/2)Q_L) = 0 \), for example.

D6_4-brane, which consists of twelve D6-branes, gives the gauge symmetry of \( \text{USp}(12)_{D6_4} \), if all the twelve D6-branes are on top of one O6-plane. Since there are eight O6-planes with the same winding numbers, there are some ambiguities to set the place of twelve D6-branes, and the actual gauge symmetry is the subgroup of \( \text{USp}(12)_{D6_4} \) in general. Figure 1 gives one configuration which we take in this paper. The resultant gauge symmetry is

\[
\text{USp}(2)_{D6_4,1} \times \text{USp}(2)_{D6_4,2} \times \text{USp}(2)_{D6_4,3} \times \text{USp}(2)_{D6_4,4} \times \text{USp}(2)_{D6_4,5} \times \text{USp}(2)_{D6_4,6}. \tag{2.3}
\]

This is the same for D6_5 and D6_6 branes. The resultant gauge symmetries by D6_5 and D6_6 branes are

\[
\text{USp}(2)_{D6_5,1} \times \text{USp}(2)_{D6_5,2} \times \text{USp}(2)_{D6_5,3} \times \text{USp}(2)_{D6_5,4} \tag{2.4}
\]

and

\[
\text{USp}(2)_{D6_6,1} \times \text{USp}(2)_{D6_6,2} \times \text{USp}(2)_{D6_6,3} \times \text{USp}(2)_{D6_6,4} \times \text{USp}(2)_{D6_6,5} \times \text{USp}(2)_{D6_6,6}. \tag{2.5}
\]
Figure 1: Configurations of twelve, eight and twelve D6-branes of D6$_4$, D6$_5$ and D6$_6$ branes, respectively. The numbers in brackets are multiplicities of D6-brane stacks, and the numbers without brackets specify one of the USp(2) gauge groups in eqs. (2.3), (2.4) and (2.5), respectively.

The value of the coupling constant of U($N$) gauge interaction at the string scale $M_s$ is determined by

$$g_U^2 = \sqrt{4\pi \kappa_4 M_s \frac{\sqrt{V_6}}{V_3}}, \quad (2.6)$$

where $\kappa_4 = \sqrt{8\pi G_N}$, $M_s = 1/\sqrt{\alpha'}$, $V_6$ is the volume of compact six-dimensional space and $V_3$ is the volume of corresponding D6-brane in compact six-dimensional space[11]. In case of that the D6-brane is parallel to a certain O6-plane, the corresponding USp($N$) gauge coupling constant is given by

$$g_{USp}^2 = 2\sqrt{4\pi \kappa_4 M_s \frac{\sqrt{V_6}}{V_3}}, \quad (2.7)$$

which has an extra factor 2[12]. In our model, all the USp(2) gauge interactions of eqs. (2.3), (2.4) and (2.5) naturally have larger gauge coupling constants than the gauge coupling constants of any other gauge interactions, because of smaller $V_3$ and the extra factor 2.
We call these USp(2) gauge interactions “hypercolor” interactions. These “hypercolor” interactions give the confining force of “preons”. The actual values of the gauge coupling constants at the string scale in our model are given by

$$\alpha_U = \frac{2}{\sqrt{4\pi}} \kappa_4 M_s \frac{\chi^{3/2}}{1 + \chi^2},$$

(2.8)

$$\alpha_{\text{USp}} = \frac{4}{\sqrt{4\pi}} \kappa_4 M_s \frac{1}{\sqrt{\chi}}$$

(2.9)

for all U(N) and USp(N) type gauge groups, respectively. We have a relation between three gauge couplings of the standard model at the string scale as

$$\alpha_3 = \alpha_2 = \frac{7}{9} \alpha_Y.$$  

(2.10)

The value of the U(1)$_R$ gauge coupling constant at the string scale is given by $\alpha_R = \frac{7}{18} \alpha_Y$. If we choose $\kappa_4 M_s \sim 1$ and $\chi \sim 0.1$, the values of the scales of dynamics of all USp(2) gauge interactions are of the order of $M_s$, and the values of the standard model gauge coupling constants are reasonably of the order of $1/100$ at the string scale.

A schematic picture of the configuration of intersecting D6-branes of this model is given in figure 2. There are no $ab' + b'a$, $aa' + a'a$ sectors of open string in this configuration. The massless particle contents are given in table 3.

From figure 2 we see that there are the following Yukawa interactions at the string level.

$$W_{\text{left}} = \bar{q}_i C D + \bar{l}_i N D,$$

(2.11)

$$W_{\text{right}} = \bar{u}_i C \bar{D}^{(-)} + \bar{d}_i C \bar{D}^{(+)} + \bar{\nu}_i \bar{N} \bar{D}^{(-)} + \bar{\epsilon}_i \bar{N} \bar{D}^{(+)},$$

(2.12)

$$W_{\text{higgs}} = H_i^{(1)} TT^{(-)} + \bar{H}_i^{(2)} TT^{(+)}$$

(2.13)

$$W_{\text{massive}} = \bar{q}_i \bar{u}_j \bar{H}_k^{(2)} + \bar{q}_i \bar{d}_j \bar{H}_k^{(1)} + \bar{l}_i \bar{\nu}_j \bar{H}_k^{(2)} + \bar{l}_i \bar{\epsilon}_j \bar{H}_k^{(1)},$$

(2.14)

where $i, j, k = 1, 2$. Here, we just notice the existence of these Yukawa interactions without specifying the value of their coupling constants. The Yukawa interactions of eqs. (2.11) and (2.12) are the necessary ingredients to give masses to some exotic fields.

We also see from figure 2 the existence of the following higher dimensional interactions at the string level.

$$W_{\text{yukawa}} = C D \bar{C} \bar{D}^{(-)} TT^{(+)} + C D \bar{C} \bar{D}^{(+)} TT^{(-)} + N D \bar{N} \bar{D}^{(-)} TT^{(+)} + N D \bar{N} \bar{D}^{(+)} TT^{(-)}.$$  

(2.15)

Here, we also just notice their existence without specifying the value of their coupling constants. These interactions give Yukawa interactions for the quark-lepton mass after the “hypercolor” confinement at low energies.

In the following, we call the sectors of D6$_{1}$-D6$_{2}$-D6$_{4}$, D6$_{2}$-D6$_{3}$-D6$_{6}$ and D6$_{1}$-D6$_{3}$-D6$_{5}$, as left-handed, right-handed and Higgs sectors, respectively. Each sector gives left-handed quarks and leptons, right-handed quarks and leptons and Higgs doublets, respectively. We explain the result of the confinement by the strong USp(2) “hypercolor” dynamics in each sector in order in the following.
Figure 2: Schematic picture of the configuration of intersecting D6-branes. This picture describes only the situation of the intersection of D6-branes, and the relative place of each D6-brane has no meaning. The number at the intersection point between D6\textsubscript{a} and D6\textsubscript{b} branes denotes intersection number \( I_{ab} \) with \( a < b \).

2.1 Dynamics of the Left-handed Sector

The dynamics in this sector is very similar to the one in the model of ref.\cite{9}. The field contents of this sector are given in table 4. For USp(2)\textsubscript{1} gauge interaction, for example, there are three massless flavors, and the low-energy effective fields after the confinement are as follows (see ref.\cite{13} for the dynamics of the supersymmetric USp gauge theory).

\[
M_{L,1} = \begin{pmatrix} C_1 \\ D_1 \\ N_1 \end{pmatrix} \begin{pmatrix} C_1 & D_1 & N_1 \end{pmatrix} = \begin{pmatrix} [C_1C_1] & [C_1D_1] & [C_1N_1] \\ [D_1D_1] & [D_1N_1] & [N_1N_1] \end{pmatrix},
\]

(2.16)

where square brackets denote the anti-symmetric contraction of the indices of the fundamental representation of USp(2)\textsubscript{1}. The field contents after the “hypercolor” confinement are given in table 5. The following superpotential among the composite fields is dynamically generated.

\[
W_{\text{left}}^{\text{dyn}} = -\sum_{\alpha=1}^{6} \frac{1}{\Lambda_{L,\alpha}^3} \text{Pf} M_{L,\alpha} = -\sum_{\alpha=1}^{6} \frac{1}{\Lambda_{L,\alpha}^3} (\bar{d}_\alpha q_\alpha l_\alpha + \bar{d}_\alpha \Phi_\alpha S_\alpha + q_\alpha q_\alpha \Phi_\alpha),
\]

(2.17)

where \( \Lambda_{L,\alpha} \) is the scale of dynamics of each USp(2)\textsubscript{\alpha}. Since the values of all gauge coupling constants of USp(2)\textsubscript{\alpha} are equal at the string scale, all the scales of dynamics are equal to
a single $\Lambda_L$. The dynamically generated superpotential is invariant under all the gauge symmetry transformations once the moduli field dependence of $\Lambda_L$ is considered. The component of the Ramond-Ramond field which acts an important role for the anomaly cancellation is also the imaginary part of the moduli field in the scale of dynamics.

If the fields $S_\alpha$ have vacuum expectation values, the exotic fields of $\tilde{d}_\alpha$ and $\Phi_\alpha$ become massive and only $q_\alpha$ and $l_\alpha$ remain massless. The expected mechanism for the vacuum expectation value is the emergence of the Fayet-Iliopoulos term to the anomalous U(1)$_L$ in some general compact space, blown-up orientifold\cite{14}, for example. In this paper we simply assume large vacuum expectation values of all $S_\alpha$.

The Yukawa interactions of eq.\eqref{2.11} give masses for two of six fields of each $q_\alpha$ and $l_\alpha$. The values of the masses are determined by the values of the Yukawa coupling constants and the scale of dynamics $\Lambda_L$. The actual values of the Yukawa coupling constants are determined by the D-brane configuration. If we take the configuration of D6$_1$ and D6$_2$ branes as figure\ref{fig:D6}, $q_3$ and $q_5$ become massive pairing with $\tilde{q}_1$ and $\tilde{q}_2$, respectively, and $l_2$ and $l_4$ become massive pairing with $\tilde{l}_1$ and $\tilde{l}_2$, respectively.\footnote{These one-to-one pairings are satisfied in good approximation when the radii of all three tori are large: $R_1^{(i)}$, $R_2^{(i)} > \sqrt{\alpha'}$ for all $i = 1, 2, 3$. When the radii of the first torus are small, as we will take in the next}

| sector | SU(3)$_c \times$ SU(2)$_L \times$ USp(8) $\times$ USp(12)$_{D6_4} \times$ USp(12)$_{D6_6}$ | field |
|--------|--------------------------------------------------------------------------------|-------|
| D6$_1 \cdot$ D6$_2$ | $(3^*, 2, 1, 1, 1)(-1/6, 0)(+1, -1, 0) \times 2$ | $\tilde{q}_i$ |
|        | $(1, 2, 1, 1, 1)(+1/2, 0)(+1, -1, 0) \times 2$ | $\tilde{l}_i$ |
| D6$_1 \cdot$ D6$_4$ | $(1, 2, 1, 12, 1)(0, 0)(-1, 0, 0)$ | $D$ |
| D6$_2 \cdot$ D6$_4$ | $(3, 1, 1, 12, 1)(+1/6, 0)(0, +1, 0)$ | $C$ |
|        | $(1, 1, 12, 1)(-1/2, 0)(0, +1, 0)$ | $N$ |
| D6$_1 \cdot$ D6$_3$ | $(1, 2, 1, 1, 1)(+1/2, +1)(-1, 0, +1) \times 2$ | $H_i^{(1)}$ |
|        | $(1, 2, 1, 1, 1)(-1/2, -1)(-1, 0, +1) \times 2$ | $\tilde{H}_i^{(2)}$ |
| D6$_1 \cdot$ D6$_5$ | $(1, 2, 8, 1, 1)(0, 0)(+1, 0, 0)$ | $T^{(+)}$ |
|        | $(1, 2, 8, 1, 1)(+1/2, 0)(0, -1, 0)$ | $T^{(-)}$ |
| D6$_2 \cdot$ D6$_3$ | $(3, 1, 1, 1, 1)(-1/3, -1)(0, +1, -1) \times 2$ | $d_i$ |
|        | $(3, 1, 1, 1, 1)(+2/3, +1)(0, +1, -1) \times 2$ | $\bar{u}_i$ |
|        | $(1, 1, 1, 1, 1)(-1, -1)(0, +1, -1) \times 2$ | $\bar{e}_i$ |
|        | $(1, 1, 1, 1, 1)(0, +1)(0, +1, -1) \times 2$ | $\nu_i$ |
| D6$_2 \cdot$ D6$_6$ | $(3^*, 1, 1, 1, 12)(-1/6, 0)(0, -1, 0)$ | $C$ |
|        | $(1, 1, 1, 1, 12)(+1/2, 0)(0, -1, 0)$ | $N$ |
| D6$_3 \cdot$ D6$_6$ | $(1, 1, 1, 1, 12)(+1/2, +1)(0, 0, +1)$ | $D^{(+)}$ |
|        | $(1, 1, 1, 1, 12)(-1/2, -1)(0, 0, +1)$ | $D^{(-)}$ |

Table 3: Low-energy particle contents before “hypercolor” confinement. Here, we use the representations of USp(12) and USp(8) groups instead of using the representations of many USp(2) groups, for simplicity. The fields from $\alpha\alpha$ sectors are neglected for simplicity.
(SU(3)_c x SU(2)_L) x (USp(2)_1 x USp(2)_2 x USp(2)_3 x USp(2)_4 x USp(2)_5 x USp(2)_6)_{D64} (Y/2, Q_R)(Q_L, Q_c + Q, Q_1 + Q_2)

| sector     | field contents | field |
|------------|----------------|-------|
| D6_1 x D6_2 | (3^*, 2)(1, 1, 1, 1, 1, 1)(-1/6, 0)(+1, -1, 0) x 2 | \(\tilde{q}_i\) |
|            | (1, 2)(1, 1, 1, 1, 1, 1)(+1/2, 0)(+1, -1, 0) x 2 | \(\tilde{l}_i\) |
| D6_1 x D6_4 | (1, 2)(1, 1, 1, 1, 1, 1)(0, 0)(-1, 0, 0) | \(D_\alpha\) |
|            | (1, 2)(1, 2, 1, 1, 1, 1)(0, 0)(-1, 0, 0) | |
|            | (1, 2)(1, 1, 2, 1, 1, 1)(0, 0)(-1, 0, 0) | |
|            | (1, 2)(1, 1, 1, 2, 1, 1)(0, 0)(-1, 0, 0) | |
|            | (1, 2)(1, 1, 1, 1, 2, 1)(0, 0)(-1, 0, 0) | |
| D6_2 x D6_4 | (3, 1)(1, 1, 1, 1, 1, 1)(+1/6, 0)(0, +1, 0) | \(C_\alpha\) |
|            | (3, 1)(1, 2, 1, 1, 1, 1)(+1/6, 0)(0, +1, 0) | |
|            | (3, 1)(1, 1, 2, 1, 1, 1)(+1/6, 0)(0, +1, 0) | |
|            | (3, 1)(1, 1, 1, 2, 1, 1)(+1/6, 0)(0, +1, 0) | |
|            | (3, 1)(1, 1, 1, 1, 2, 1)(+1/6, 0)(0, +1, 0) | |
|            | (1, 1)(2, 1, 1, 1, 1, 1)(-1/2, 0)(0, +1, 0) | \(N_\alpha\) |
|            | (1, 1)(1, 2, 1, 1, 1, 1)(-1/2, 0)(0, +1, 0) | |
|            | (1, 1)(1, 1, 2, 1, 1, 1)(-1/2, 0)(0, +1, 0) | |
|            | (1, 1)(1, 1, 1, 2, 1, 1)(-1/2, 0)(0, +1, 0) | |
|            | (1, 1)(1, 1, 1, 1, 2, 1)(-1/2, 0)(0, +1, 0) | |

**Table 4:** Field contents of the left-handed sector. Here, \(i = 1, 2\) and \(\alpha = 1, 2, \ldots, 6\).

| field          | SU(3)_c x SU(2)_L                                                                 |
|----------------|-----------------------------------------------------------------------------------|
| \(\tilde{q}_i\) | \((3^*, 2)(-1/6, 0)(+1, -1, 0) x 2\)                                               |
| \(\tilde{l}_i\) | \((1, 2)(+1/2, 0)(+1, -1, 0) x 2\)                                                |
| \([C_\alpha C_\alpha] \sim d'_\alpha\) | \((3^*, 1)(+1/3, 0)(0, +2, 0) x 6\)                                              |
| \([C_\alpha D_\alpha] \sim q_\alpha\) | \((3, 2)(+1/6, 0)(-1, +1, 0) x 6\)                                              |
| \([C_\alpha N_\alpha] \sim \Phi_\alpha\) | \((3, 1)(-1/3, 0)(0, +2, 0) x 6\)                                              |
| \([D_\alpha D_\alpha] \sim S_\alpha\) | \((1, 1)(0, 0)(-2, 0, 0) x 6\)                                              |
| \([D_\alpha N_\alpha] \sim l_\alpha\) | \((1, 2)(-1/2, 0)(-1, +1, 0) x 6\)                                              |

**Table 5:** Field contents of the left-handed sector after the “hypercolor” confinement. Here, \(i = 1, 2\) and \(\alpha = 1, 2, \ldots, 6\).

which are exponentially suppressed as an approximation. As the result, we have four generation fields of left-handed quarks and left-handed leptons.

section, \(\tilde{q}_1\) becomes massive by pairing with a linear combination of \(q_3\) and \(q_5\), and \(\tilde{q}_2\) becomes massive by pairing with another linear combination of \(q_2\) and \(q_5\). The similar occurs for \(\tilde{l}_1\) and \(\tilde{l}_2\).
2.2 Dynamics of the Right-handed Sector

Almost the same mechanism in the left-handed sector occurs in the right-handed sector. Here, we are not going to describe all the details, but simply present the results. The low-energy effective fields after the “hypercolor” confinement are

\[
M_{R,\alpha} = \begin{pmatrix}
\bar{C}_\alpha \\
\bar{N}_\alpha \\
\bar{D}_\alpha^+(+1) \\
\bar{D}_\alpha^-(+1)
\end{pmatrix}
\begin{pmatrix}
\bar{C}_\alpha \\
\bar{N}_\alpha \\
\bar{D}_\alpha^+(+1) \\
\bar{D}_\alpha^-(+1)
\end{pmatrix} = \begin{pmatrix}
\bar{C}_\alpha \bar{C}_\alpha [\bar{C}_\alpha \bar{N}_\alpha] [\bar{C}_\alpha \bar{D}_\alpha^+(-1)] [\bar{C}_\alpha \bar{D}_\alpha^-(-1)] \\
\bar{N}_\alpha \bar{N}_\alpha [\bar{N}_\alpha \bar{D}_\alpha^+(+1)] [\bar{N}_\alpha \bar{D}_\alpha^-(-1)] \\
\bar{D}_\alpha^+(+1) \bar{D}_\alpha^+(+1) [\bar{D}_\alpha^+(+1) \bar{D}_\alpha^-(+1)] \\
\bar{D}_\alpha^-(-1) \bar{D}_\alpha^-(-1) [\bar{D}_\alpha^-(-1) \bar{D}_\alpha^+(+1)]
\end{pmatrix},
\]

where square brackets denote the anti-symmetric contraction of the indices of the fundamental representation of USp(2) \(\alpha\) in eq. (2.18). The field contents after the “hypercolor” confinement is given in Table 6. The following superpotential among the composite fields

\[
\begin{align*}
\bar{u}_\iota &\sim [d'_\alpha]^3(1,0,0) \\
\bar{d}_\iota &\sim [d_\alpha]^3(1,0,0) \\
\bar{\nu}_\iota &\sim [e_\alpha]^3(1,0,0) \\
\bar{e}_\iota &\sim [e_\alpha]^3(1,0,0)
\end{align*}
\]

\[
\begin{align*}
[\bar{C}_\alpha \bar{C}_\alpha] &\sim (1,1)(-2,0,0,0) \\
[\bar{C}_\alpha \bar{N}_\alpha] &\sim (3,1)(-1,0,0,0) \\
[\bar{C}_\alpha \bar{D}_\alpha^+(+1)] &\sim (3,1)(-1,0,0,0) \\
[\bar{C}_\alpha \bar{D}_\alpha^-(+1)] &\sim (3,1)(-1,0,0,0) \\
[\bar{N}_\alpha \bar{D}_\alpha^+(+1)] &\sim (3,1)(-1,0,0,0) \\
[\bar{N}_\alpha \bar{D}_\alpha^-(+1)] &\sim (3,1)(-1,0,0,0) \\
[\bar{D}_\alpha^+(+1) \bar{D}_\alpha^+(+1)] &\sim (3,1)(-1,0,0,0) \\
[\bar{D}_\alpha^-(-1) \bar{D}_\alpha^-(-1)] &\sim (3,1)(-1,0,0,0)
\end{align*}
\]

Table 6: Field contents of the right-handed sector after the “hypercolor” confinement. Here, \(i = 1, 2\) and \(\alpha = 1, 2, \cdots, 6\).
is dynamically generated.

\[ W_{\text{dyn}}^{\text{left}} = - \sum_{\alpha=1}^{6} \frac{1}{\Lambda_{R,\alpha}} \text{Pf} M_{R,\alpha} = - \sum_{\alpha=1}^{6} \frac{1}{\Lambda_{R,\alpha}} \left( d'_\alpha \tilde{\Phi}_\alpha \tilde{S} \alpha + d'_\alpha \nu_\alpha d_\alpha + \Phi_\alpha d_\alpha u_\alpha \right), \]

(2.19)

where \( \Lambda_{R,\alpha} \) is the scale of dynamics of each USp(2)\( \alpha \). Since all the coupling constants of USp(2)\( \alpha \) are equal at the string scale, all the scales of dynamics are equal: \( \Lambda_{R,\alpha} = \Lambda_R \). This dynamically generated superpotential is invariant under all the gauge symmetry transformations considering the moduli dependence of \( \Lambda_R \).

If all \( \tilde{S}_\alpha \) have vacuum expectation values, the fields \( d'_\alpha \) and \( \tilde{\Phi}_\alpha \) become massive and only \( u_\alpha, d_\alpha, \nu_\alpha \) and \( e_\alpha \) remain massless. The expected mechanism for the vacuum expectation value is the emergence of the Fayet-Iliopoulos term to the anomalous U(1)\( Q_1 + Q_2 \) in some more general compact space. In this paper we simply assume large vacuum expectation values of all \( \tilde{S}_\alpha \).

The Yukawa interactions of eq.(2.12) give masses to two of six fields of each \( u_\alpha, d_\alpha, \nu_\alpha \) and \( e_\alpha \). The values of the masses are determined by the values of the Yukawa coupling constants and the scale of dynamics \( \Lambda_R \). The actual value of the Yukawa coupling constants are determined by the D-brane configuration. If we take the configuration of D6\(_2\) and D6\(_3\) branes as figure [Fig. 4] and take the radii of the first torus are not so large: \( R_1^{(1)}, R_2^{(1)} \gg \sqrt{\alpha'} \), a linear combination of \( u_1 \) and \( u_3, u_5 \), a linear combination of \( d_1 \) and \( d_3, d_5 \), \( \nu_2 \), a linear combination of \( \nu_4 \) and \( \nu_6 \), \( e_2 \) and a linear combination of \( e_4 \) and \( e_6 \) become massive by pairing with \( \bar{u}_1, \bar{u}_2, \bar{d}_1, \bar{d}_2, \bar{\nu}_1, \bar{\nu}_2, \bar{e}_1 \) and \( \bar{e}_2 \), respectively. The radii of other tori are assumed to be large in comparison with \( \sqrt{\alpha'} \), and we neglect small contributions from the exponentially suppressed Yukawa interactions at the string level. The values of the masses are of the order of \( \Lambda_R \). As the result, we have four generation fields of right-handed quarks and right-handed leptons.

**Figure 4:** Actual D6-brane configuration of D6\(_2\) and D6\(_3\) branes. Gray lines with arrow denote D6-branes of D6\(_3\) and the solid lines with arrow denote D6-branes of D6\(_2\). Two stacks of D6-branes of D6\(_3\) in the first torus result the difference between the fields with weak isospin 1/2 and -1/2.

### 2.3 Dynamics of the Higgs Sector

The dynamics of the Higgs sector is different from that in other sectors. The field contents in this sector are given in table [Table 5]. The dynamics of the gauge interaction is that of
USp(2) with two massless flavors. The low-energy effective fields after the “hypercolor” confinement are

\[ V_a = \begin{pmatrix} T_{a}^{(+)T_{a}^{(-)}} \\ T_{a}^{(-)}T_{a}^{(+)T_{a}^{(-)}} \end{pmatrix} = \begin{pmatrix} [T_{a}T_{a}] & [T_{a}T_{a}^{(+)T_{a}^{(-)}}] & [T_{a}T_{a}^{(-)}] \\ [T_{a}^{(+)T_{a}^{(-)}}] & [T_{a}^{(+)T_{a}^{(-)}}] & [T_{a}^{(+)T_{a}^{(-)}}] \\ [T_{a}^{(-)}] & [T_{a}^{(-)}T_{a}^{(+)T_{a}^{(-)}}] & [T_{a}^{(-)}T_{a}^{(+)T_{a}^{(-)}}] \end{pmatrix}, \quad (2.20) \]

where square brackets denote the anti-symmetric contraction of the indices of the fundamental representation of USp(2)\(_a\) in eq. (2.4). The field contents after the “hypercolor” confinement are given in table 8.

| sector | \((SU(3)_c \times SU(2)_L) \times (USp(2)_1 \times USp(2)_2 \times USp(2)_3 \times USp(2)_4)_{D65} \) | field |
|--------|-------------------------------------------------------------------------------------------------|-------|
| \(D6_1 \cdot D6_3\) | \((1, 2)(1, 1, 1, 1)_{(+1/2, +1)}(-1, 0, +1) \times 2\) | \(H_{i}^{(1)}\) |
| \(D6_1 \cdot D6_5\) | \((1, 2)(1, 2, 1, 1)_{(0, 0)}(+1, 0, 0)\) | \(T_{a}\) |
| \(D6_3 \cdot D6_5\) | \((1, 1)(2, 1, 1, 1)_{(+1/2, +1)}(0, 0, -1)\) | \(T_{a}^{(+)}\) |
| | \((1, 1)(1, 2, 1, 1)_{(+1/2, +1)}(0, 0, -1)\) | \(T_{a}^{(-)}\) |
| | \((1, 1)(1, 2, 1, 1)_{(+1/2, +1)}(0, 0, -1)\) | \(T_{a}^{(+)}\) |
| | \((1, 1)(1, 2, 1, 1)_{(-1/2, -1)}(0, 0, -1)\) | \(T_{a}^{(-)}\) |
| | \((1, 1)(1, 1, 2, 1)_{(-1/2, -1)}(0, 0, -1)\) | \(T_{a}^{(+)}\) |
| | \((1, 1)(1, 1, 2, 1)_{(-1/2, -1)}(0, 0, -1)\) | \(T_{a}^{(-)}\) |

**Table 7:** Field contents of the Higgs sector. Here, \(i = 1, 2\) and \(a = 1, 2, 3, 4\).

No superpotential is dynamically generated, but the following constraints have to be
imposed.

\[ PfV_a = \Lambda_{H,a}^4, \]  

(2.21)

where \( \Lambda_{H,a} \) is the scale of dynamics of USp(2) \(_a\). Since the gauge coupling constants of USp(2) \(_a\) are equal at the string scale, the scales of dynamics are also equal: \( \Lambda_{H,a} = \Lambda_H \). The constraints of eq. (2.21) mean non-zero vacuum expectation values of some low-energy effective fields. We take the following vacuum expectation values which keep maximal symmetry.

\[ \langle V_{a12} V_{a34} \rangle = \langle S_{H2,a} S_{H2,a} \rangle = \Lambda_H^4. \]  

(2.22)

This vacuum expectation values do not break any gauge symmetries including anomalous U(1) gauge symmetries, because of the moduli dependence of \( \Lambda_H \). If it is possible to factorize the bilinear operator and

\[ \langle V_a \rangle = \Lambda_H^2 \begin{pmatrix} i\sigma_2 & 0 \\ 0 & i\sigma_2 \end{pmatrix}, \]  

(2.23)

the anomalous U(1)\(_L\) and U(1)\(_Q_1+Q_2\) gauge symmetries are spontaneously broken. Here, we do not discuss this problem in detail, and we leave it for future works.

The Yukawa interactions in eq. (2.13) give masses to two of four fields of each \( \bar{H}_a^{(1)} \) and \( H_a^{(2)} \). The values of the masses are determined by the values of the Yukawa coupling constants and the scale of dynamics \( \Lambda_H \). The composite fields \( \bar{H}_1^{(1)} \) and \( \bar{H}_3^{(1)} \) become massive with elementary fields \( H_1^{(1)} \) and \( H_2^{(1)} \), respectively, and the composite fields \( H_2^{(2)} \) and \( H_4^{(2)} \) become massive with elementary fields \( \bar{H}_2^{(2)} \) and \( \bar{H}_1^{(2)} \), respectively. Here, we are neglecting the contributions from the exponentially suppressed Yukawa coupling constants as an approximation. All the massless composite Higgs fields \( \bar{H}_1^{(1)}, \bar{H}_4^{(1)}, H_1^{(2)} \) and \( H_3^{(2)} \) have Yukawa interactions with composite quarks and leptons.

3. Dynamical Generation of Yukawa Coupling Constants

The higher dimensional interactions of eq. (2.15) in superpotential come from the recombination processes among open strings at six intersection points: (D6\(_2\)-D6\(_4\)) - (D6\(_4\)-D6\(_1\)) - (D6\(_1\)-D6\(_5\)) - (D6\(_5\)-D6\(_3\)) - (D6\(_3\)-D6\(_6\)) - (D6\(_6\)-D6\(_2\)). A schematic picture of a recombination process is given in figure 5.

These interactions give Yukawa interactions for the quark-lepton mass and mixing after the “hypercolor” confinement.

\[ \sum_{\alpha,\beta=1}^{6} \sum_{a=1}^{4} \frac{g_{\alpha\beta a}}{M_s^3} [C_{a} D_{a}] [\bar{C}_{\beta} \bar{D}_{\beta}^{(-)}] [T_{a} T_{a}^{(+)}] \sim \sum_{\alpha,\beta=1}^{6} \sum_{a=1}^{4} \frac{\Lambda_{LR} \Lambda_{H} q_{\alpha} u_{\beta} H_{a}^{(2)} M_s^3}{M_s^3}, \]  

(3.1)

\[ \sum_{\alpha,\beta=1}^{6} \sum_{a=1}^{4} \frac{g_{\alpha\beta a}}{M_s^3} [C_{a} D_{a}] [\bar{C}_{\beta} \bar{D}_{\beta}^{(+)}] [T_{a} T_{a}^{(-)}] \sim \sum_{\alpha,\beta=1}^{6} \sum_{a=1}^{4} \frac{\Lambda_{LR} \Lambda_{H} q_{\alpha} d_{\beta} \bar{H}_{a}^{(1)} M_s^3}{M_s^3}, \]  

(3.2)

\[ \sum_{\alpha,\beta=1}^{6} \sum_{a=1}^{4} \frac{g'_{\alpha\beta a}}{M_s^3} [N_{a} D_{a}] [\bar{N}_{\beta} \bar{D}_{\beta}^{(-)}] [T_{a} T_{a}^{(+) \prime}] \sim \sum_{\alpha,\beta=1}^{6} \sum_{a=1}^{4} \frac{\Lambda_{LR} \Lambda_{H} l_{\alpha} \nu_{\beta} H_{a}^{(2)} M_s^3}{M_s^3}, \]  

(3.3)
\[ \sum_{\alpha, \beta=1}^{6} \sum_{a=1}^{4} \frac{g_{\alpha \beta a}^e}{M^3} \left[ N^e_{\beta} \tilde{D}^{(+)}_{\beta} \right] \left[ T_{a} T_{a}^{(-)} \right] \sim \sum_{\alpha, \beta=1}^{6} \sum_{a=1}^{4} g_{\alpha \beta a}^e \frac{\Lambda_L \Lambda_R \Lambda_H}{M^3} l_{a \beta} \tilde{H}_a (1)^\dagger. \] (3.4)

Since the scales $\Lambda_L$, $\Lambda_R$ and $\Lambda_H$ are the same order of magnitude of $M$, the low-energy Yukawa interactions for the quark-lepton mass are not trivially suppressed. The values of the elements of the Yukawa coupling matrices of $g^u$, $g^d$, $g^\nu$ and $g^e$ are determined by the D6-brane configuration. Namely, they depend on the places of six intersection points in each three torus. If the separation of intersection points is large, the value of the corresponding coupling constant is small, and vice versa. In case that the separation of intersecting points is large especially in a large torus, the value of the corresponding coupling constant is exponentially small. Therefore, we can expect some non-trivial structure in Yukawa coupling matrices.

It does not always happen that all the six intersection points coincide in all three tori. Therefore, almost all the values of elements in Yukawa coupling matrices are small. This could be a reason why the masses of quarks and leptons are smaller than the electroweak scale. Top quark is accidentally heavy in this scenario.

We are not going to explicitly calculate the Yukawa coupling matrices. Instead, we describe one specific example which gives a quark-lepton mass difference. Here, we investigate the coupling constants of the following Yukawa interactions among massless fields.

\[ W_{144} = g_{144}^d q_1 d_4 \tilde{H}_4 (1) + g_{144}^e l_1 e_4 \tilde{H}_4 (1). \] (3.5)

The relevant configuration of D6-branes is given in figure 5.

In the first torus six intersection points do not give finite area, and no exponential suppression factor emerges from the first torus. In the third torus six intersecting points are on the apexes of the hatched triangle, which results the same exponential suppression factors for each Yukawa coupling constant, $g_{144}^d$ and $g_{144}^e$.

The quark-lepton mass difference is realized by the two different D6-branes of D6$^2$ in the second torus. The cross-hatched triangular area in the second torus in figure 5 determines an exponential suppression factor to the down-type quark Yukawa coupling constant $g_{144}^d$, and the hatched quadrilateral area (including the cross-hatched triangular area) determines another exponential suppression factor to the charged lepton Yukawa

\[ \text{Figure 5: A schematic picture of a recombination process of open strings. The apices of the hexagon correspond to six intersection points.} \]
coupling constant $g_{144}$. Since the quadrilateral area is three times larger than the triangular area, the charged lepton Yukawa coupling is exponentially smaller than the down-type quark Yukawa coupling constant: $g_{144}^e \ll g_{144}^d$. The concept to have hierarchical Yukawa couplings utilizing the exponential suppression by the area has already been proposed in ref.[3]. The above scenario is a non-trivial realization of this concept.

To investigate the Yukawa coupling matrices further, we have to treat the problem much more precisely. The mass mixings in the left-handed sector, right-handed sector and Higgs sector should be solved including exponentially suppressed Yukawa interactions among “preons” at the string level. The normalization of composite fields in the Kähler potential (the normalization of the kinetic terms of composite fields) should also be considered. The full structure of Yukawa coupling matrices should be investigated in some specific D6-brane configurations which give some intuitive idea to explain the realistic quark-lepton mass spectrum and flavor mixings.

The electroweak symmetry breaking by the vacuum expectation values of Higgs fields is required to generate the masses of quarks and leptons as well as weak bosons. Although supersymmetry breaking is necessary for non-zero vacuum expectation values of Higgs fields in this model (of course, supersymmetry breaking is also required to give masses to superpartners), there are no explicit mechanism of supersymmetry breaking in this model. There is no hidden sector which can be realized by some D6-brane with no intersection with any other D6-branes. If there is a hidden sector D6-brane on which super-Yang-Mills theory is realized, supersymmetry could be dynamically broken by gaugino condensation with supergravity effects. If gauginos can condense in super-Yang-Mills theories with massless matter as assumed in ref.[11] (this means supersymmetry breaking through the Konishi anomaly relation without supergravity effects), there is a possibility of the dynamical supersymmetry breaking by the “hypercolor” dynamics in this model.

Figure 6: Configuration of D6-branes to investigate of Yukawa coupling constants.
4. Summary and Conclusions

We have constructed a supersymmetric composite model in type IIA $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifolds with intersecting D6-branes. Four generations of quarks and leptons are composite particles in this model. Two pairs of electroweak Higgs fields are also composite. The expected mechanisms to give masses to exotic particles should be further investigated. There is an additional non-anomalous U(1) gauge symmetry which should be spontaneously broken above the electroweak scale. This model is a toy model to illustrate a new mechanism of dynamical generation of Yukawa couplings for the masses and mixings of quarks and leptons.

The Yukawa interactions for the quark-lepton mass can be naturally generated by the interplay between higher dimensional interactions at the string level (six-dimensional operators in the superpotential in the four-dimensional effective theory) and “hypercolor” dynamics. The actual value of the Yukawa coupling constants are determined by the D6-brane configuration in three tori. Although the full structure of the Yukawa coupling matrices have not yet been investigated, a general mechanism to have small Yukawa coupling constants and a specific example which gives quark-lepton mass differences have been introduced.

There is no hidden sector in this model, and the explicit mechanism of supersymmetry breaking is absent. Although the “hypercolor” dynamics might break supersymmetry, the concrete analysis of the dynamics and the mediation of supersymmetry breaking is left for future works.

Introduction of the compositeness of Higgs fields and/or quark and lepton fields is one of the natural directions in the model building with intersecting D-branes. Many additional D-brane are required in addition to the D-branes for the gauge symmetry of the standard model to satisfy the tadpole cancellation conditions. It is natural to consider that these additional D-branes act some important roles in Nature.

It would be very interesting to explore more realistic model in this framework.

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