Magnetar oscillations pose challenges for strange stars

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ABSTRACT

Compact relativistic stars allow us to study the nature of matter under extreme conditions, probing regions of parameter space that are otherwise inaccessible. Nuclear theory in this regime is not well constrained: one key issue is whether neutron stars are in fact composed primarily of strange quark matter. Distinguishing the two possibilities, however, has been difficult. The recent detection of seismic vibrations in the aftermath of giant flares from two magnetars (highly magnetized compact stars) is a major breakthrough. The oscillations excited seem likely to involve the stellar crust, the properties of which differ dramatically for strange stars. We show that the resulting mode frequencies cannot be reconciled with the observations for reasonable magnetar parameters. Ruling out strange star models would place a strong constraint on models of dense quark matter.

Key words: stars: magnetic fields—stars: neutron—stars: oscillations—X-rays: stars—equation of state

The properties of compact stars offer the best opportunity to study the phase diagram of Quantum Chromodynamics (QCD) at extreme densities, a region that is poorly constrained. One hypothesis, based on the possibility that strange quark matter (SQM) could be absolutely stable, is that compact stars are made almost entirely of deconfined SQM (Witten 1984). In this scenario, terrestrial matter made of nuclei is only metastable, albeit with an essentially infinite lifetime because it would require multiple weak interactions to convert normal matter to SQM (Weber 2006). However, the extreme ambient conditions characteristic of compact stars can facilitate the conversion of ordinary matter to SQM on short enough time-scales so that some or all compact stars could be strange stars. Distinguishing strange stars from neutron stars, however, has proved difficult. Signatures such as potentially smaller radii, and a supposed inability to glitch, are fraught with serious observational and theoretical uncertainty.

New observations, however, could change this. Certain magnetars (compact stars with magnetic fields $\approx 10^{14}$ G), the Soft Gamma Repeaters (SGRs), exhibit gamma-ray flares powered by field decay (Duncan & Thompson 1992). Timing analysis of rare giant flares, which have peak luminosities $10^{38} - 10^{46}$ erg s$^{-1}$ and decaying tails lasting several minutes, has recently revealed high frequency Quasi-Periodic Oscillations (QPOs). The 2004 giant flare of SGR 1806-20 shows QPOs at 18, 26, 30, 92, 150, 625 Hz and higher (Israel et al. 2005; Watts & Strohmayer 2004; Strohmayer & Watts 2005), whilst the 1998 giant flare from SGR 1900+14 has QPOs at 28, 53, 84 and 155 Hz (Strohmayer & Watts 2005). The most promising model involves seismic vibrations, triggered by a starquake associated with the giant flare. The lowest frequency 18, 26 Hz QPOs fit predictions for Alfvén modes of the core. For the higher frequencies, attention has focused on toroidal shear modes of the crust (or their global magneto-elastic equivalent). These have frequencies that are, for neutron stars, a good match to the observations (McDermott, van Horn & Hansen 1988; Duncan 1998; Piro 2005; Glampedakis, Samuelsson & Andersson 2006; Samuelsson & Andersson 2007). The 28-155 Hz QPOs would be $n = 0$ modes (no radial node) with differing angular quantum number $l$. The 625 Hz QPO is consistent with being the $n = 1$ first radial overtone.

The likely dependence on crust properties is very exciting. Originally, strange stars were expected to be devoid of a solid crust and were characterized by an ultra-dense quark liquid extending up to the surface (Haensel, Zdunik & Pichon 1986). Such bare strange quark stars could not account for torsional shear oscillations since there is no solid region in the vicinity of the surface. However, strange stars can have solid crusts. One possibility (Alcock, Farhi & Olinto 1986) is that the strange star has a thin crust of normal nuclear material extending down to neutron drip at density $\rho \approx 4 \times 10^{14}$ g cm$^{-3}$ (for a neutron star the crust extends beyond neutron drip to $\rho \approx 10^{14}$ g cm$^{-3}$), suspended above liquid SQM by an enormous electric field. A more recent model (Jaikumar, Reddy & Steiner 2006)
posits a crust in which nuggets of SQM are embedded in a uniform electron background. The strange star crusts have different shear speeds and are thinner than neutron star crusts: both factors affect shear mode frequencies, crust thickness $R$ being of particular importance to the radial overtones (Hansen & Coffie 1986). Magnetar seismology may therefore offer a robust means of distinguishing strange stars from neutron stars.

1 MODEL

Most previous studies computing torsional shear modes of the stellar crust assume free slip over the fluid core. But if the strong magnetic field couples crust and core together then one should instead consider global magneto-elastic perturbations. Recent work on the coupled problem, however, suggests that the vibrations most likely to be observed have very similar frequencies to those computed in the uncoupled problem (Glampedakis, Samuelsson & Andersson 2006; Levin 2007). The natural crust frequencies are therefore physically relevant, so for simplicity we neglect coupling and apply zero traction boundary conditions (Carroll et al. 1986).

Following Piro (2005) we use a plane-parallel geometry with constant gravitational acceleration $g = GM/R^2$ in the vertical direction $z$, $M$ and $R$ being the mass and radius of the strange star. The Newtonian equations of hydrostatic equilibrium in the zero field limit (McDermott, van Horn & Hansen 1986) posits a crust in which nuggets of SQM are embedded in a uniform electron background. The strange star crusts have different shear speeds and are thinner than neutron star crusts: both factors affect shear mode frequencies, crust thickness $R$ being of particular importance to the radial overtones (Hansen & Coffie 1986). Magnetar seismology may therefore offer a robust means of distinguishing strange stars from neutron stars.

The parameter $\Gamma = (3/\pi n_i)^{1/3}$, $n_i$ the density of ions and $a = (3/4\pi n_i)^{1/3}$ the average inter-ion spacing. The parameter $\Gamma = (Ze)^2/\alpha k_B T$, where $T$ is the temperature and $\Gamma = \Gamma_0 = 173$ marks the point at which the solid lattice melts to form an ocean (Parouni & Hamaguchi 1993), and we use this to determine the upper boundary of the crust. Crust temperature in the tail of a giant flare is not well constrained: observations set a lower limit of $10^7$ K (Tinto et al. 2005), but theory suggests that it could be as high as $\sim 10^9$ K (Lyubarsky, Eichler & Thompson 2002), so we examine the range $T = 10^7 - 10^8$ K.

We consider stellar models with $M = 1.2 - 2.5M_\odot$ and $R = 8 - 15$ km, subject to causality constraints, to cover the full range of possible strange star parameter space (Page & Reddy 2006). Both thin nuclear and nugget crust models are studied (Alcock, Farhi & Olinto 1986; Jaikumar, Reddy & Steiner 2006). We model magnetic fields in the range $B = 10^{12} - 10^{15}$ G. The strong field affects only the very outermost low density layers of the crust (Harding & Lai 2005); we estimate the corrections to computed mode frequencies to be less than 1%. Energetic arguments show that global crust structure, including the suspended crust model, is unaffected.

For the thin nuclear crust there is some uncertainty in the composition of neutron-rich nuclei, which will be constrained by future rare isotope experiments. We therefore survey a range of equations of state (Haensel & Pichon 1995; Rüster, Hempel & Schaffner-Bielich 2006). The nugget crust is composed of a lattice of strange quark nuggets embedded in a background degenerate electron gas. The electrons contribute to the pressure, and the nuggets to the energy density. The density within the crust is given by $\rho = x\rho_0$, where $\rho_0 = n_{\text{quark}}\mu_q$ is the energy density of quark matter inside nuggets and depends on the density $n_{\text{quark}} \approx 1$ fm$^{-3}$ and chemical potential $\mu_q \approx 300$ MeV at which stable quark matter vanishes. $x$ is the volume fraction occupied by nuggets and is given by

$$x = \frac{\mu_q^3}{3\pi^2(n_Q - \chi_Q\mu_q)},$$

where $\mu_q$ is the electron chemical potential, $n_Q$ is the electric charge density of the quark nugget and $\chi_Q$ is its charge susceptibility (Jaikumar, Reddy & Steiner 2006). The quark matter parameters are poorly known and can only be determined within the context of specific models. In the Bag model, $n_Q = m_Q^2\mu_q/2\pi^2$ and $\chi_Q = \mu_Q^2/\pi^2$. Further, requiring SQM to be absolutely stable and simultaneously requiring normal nuclei to be metastable restricts $\mu_q$ to a narrow range centered around $\mu_q \approx 300$ MeV. Thus within the Bag model, the remaining uncertainty is parametrized through the effective strange quark mass $m_s$, which we expect to be in the range $150 - 250$ MeV.

The spherical nugget phase occupies most of the crust; we will neglect the small region at the base of the crust containing the pasta phase (see below). In computing shear modulus (Eqn. 2) the quantity

$$Z = \frac{4\pi}{3} f R^4_a(n_Q - \chi_Q\mu_q)$$

is the charge of the nugget where $R_a = y \lambda_d$ is the typical nugget size, $\lambda_d = 1/(4\pi\alpha\lambda_Q)^{1/3}$ being the Debye screening length and $\alpha$ the fine structure constant. The factor $f$ is a correction due to screening inside nuggets, $f \approx$
3(y − tanh y)/y³, where y ≈ 1.6 (Alford et al., 2006) gives f ≈ 0.5 for the larger droplets. The quantity a = Rₙ/x⁻¹/³ is the average inter-nugget distance, and n₁ = 3/(4πa³) the density of nuggets.

The variation of μₖ with depth is given by integrating the equations of hydrostatic equilibrium. Using the limits x = 0 and x = 1, Jaikumar, Reddy & Steiner (2006) followed this procedure to estimate ΔR. In reality the region with x > 0.5 that contains the pasta phase occupies a tiny region with thickness ≲ 1μm. Further, with increasing x the free energy gain of the heterogeneous state becomes negligible and even a small surface tension can disfavor the large x region. For these reasons we set x ≈ 0.5 at the base of the crust. The value of x at the top of the crust is set by the melting condition: for the parameters examined, it lies in the range x ≈ 10⁻¹² – 10⁻¹⁴.

Figure 2 shows the variation of vₙ and vₐ with depth for example crust models (compare to Fig. 1 of Piro (2003)). The shear speed in the nugget crust is smaller than in the thin-nuclear crust. This can be understood by noting that at constant pressure vₙ ∼ √Z²/³/A where A denotes the baryon number. Further, and unlike in the nuclear case, both Z and Z/A of the nuggets decrease rapidly with depth.

2 RESULTS

Our results are good agreement with analytic estimates (McDermott, van Horn & Hansen 1988; Duncan 1993; Piro 2003; Samuelsson & Andersson 2007). Overall, frequencies scale with vₙ. The n = 0 mode frequencies are almost independent of ΔR, and varying B and T changes frequency by no more than a few Hz. The frequencies scale as [(l+2)(l−1)]¹/², making it difficult to fit a mode sequence with an 18 or 26 Hz fundamental. Following previous studies we assume that these represent global Alfvén modes, and search for a fundamental at ≈ 30 Hz. The radial overtones vary little with l, but depend strongly on ΔR, frequencies tending to increase as the crust thins. Crust thickness is set in part by compactness. Increasing temperature also tends to push overtone frequencies up as the outer layers of the crust melt. The overtones also depend strongly on B, since vₐ exceeds vₙ at magnetar field strengths.

For the thin nuclear crust models the residual uncertainty in the equation of state leads to variations of at most a few percent, and in what follows we quote results based on Haensel & Pichon (1994). For the parameter space studied the frequency of the n = 0, l = 2 mode lies in the range 26 - 54 Hz. To obtain a fundamental ≤ 30 Hz requires high mass (M ≥ 2.4M⊙ for all radii, or M ≥ 2.2M⊙ and R ≥ 14 km), pushing the limits of strange star parameter space (Page & Reddy 2006).

The overtone frequencies are all high. Even at B = 10¹² G, the lowest frequency is ≈ 725 Hz, for the model with the thickest crust (M = 2.2M⊙, R = 15 km, ΔR/R = 8%). For B > 10¹⁴ G, the lower limit on overtone frequency is higher still, at ≈ 1100 Hz. There is no model that would allow an overtone at 625 Hz. Figure 2 illustrates the effects of varying B and T on the frequencies of both the n = 0 and n = 1 modes.

For the nugget crust models there is additional uncertainty in the value of μₙ. Increasing μₙ increases shear speed, and for the n = 0 modes frequency scales directly with μₙ. However, in general vₙ is lower than for the nuclear crust models. For the parameter space studied the frequency of the n = 0, l = 2 fundamental lies in the range ≈ 1 - 21 Hz. There is no model that would permit a fundamental in the range 28-30 Hz.

The range of possible frequencies for the n = 1 overtones is extremely large. Crust thickness increases dramatically with μₙ, and this effect can in fact compensate for the change in vₙ. For the highest value of μₙ, ΔR/R is comparable to or thicker than the thin nuclear crust; for the lowest values, however, it can be an order of magnitude thinner. Temperature dependence is complex. As T increases from 10⁷ K frequency drops as vₙ drops, the effect being more pronounced than for the nuclear crust. However at high enough T crust thinning accelerates dramatically. This can offset the effects on vₙ and cause frequency to rise again. At B = 10¹² G, frequencies for the n = 1 overtone
Nugget crust mode frequencies (for a star with $M = 1.4 M_\odot$, $R = 12$ km) for strange quark mass $m_s = 150$ MeV (solid) and $m_s = 250$ MeV (dotted). Left: effect of varying magnetic field, for fixed $T = 10^8$ K. Right: effect of varying temperature, for fixed $B = 10^{12}$ G. The minimum in frequency for the overtone occurs at higher temperatures for $m_s = 250$ MeV.

can be as low as $\approx 260$ Hz. Frequencies do however rise as $B$ increases, and by $B = 10^{14}$ G only a few very extreme models permit an overtone frequency as low as 625 Hz ($M = 1.2 M_\odot$, $R = 15$ km, $m_s \approx 250$ MeV, $T \gtrsim 9 \times 10^8$ K). Figure 3 illustrates the effects of varying $B$, $T$ and $m_s$ on the $n = 0$ and $n = 1$ modes.

3 DISCUSSION

For neutron star models torsional shear modes, or their global magneto-elastic equivalents, are a good fit for the observations. For strange stars, the situation is much more difficult. A thin nuclear crust model can give a fundamental in the right range if stellar mass is large, but the overtone frequencies are far too high. The nugget crust model permits a wider range of frequencies due to the uncertainty in the strange quark mass $m_s$. The lowest order $n = 0$ modes could explain some of the QPOs in the range 18-150 Hz, but because the fundamental frequencies are so low it is difficult to fit a mode sequence, given the expected scaling with $l$ (Samuelsson & Andersson 2007). For the radial overtones there are regions of parameter space at high temperature with modes at the right frequency, but only for magnetic fields at least an order of magnitude lower than those inferred for magnetars. At magnetar field strengths only models at the very limits of parameter space permit an overtone frequency as low as 625 Hz. The other constraint on the nugget crust is the sensitivity of the overtones to temperature fluctuations (greater for this model than for the thin nuclear crust or neutron star crusts). The observations indicate that the 625 Hz QPO has high coherence and lasts for several hundred seconds, a period during which temperature could vary substantially (Watts & Strohmayer 2006).

We conclude that the frequencies of toroidal shear modes in strange star crusts have serious difficulty explaining the QPO frequencies observed during magnetar hyperflares. If the results of Glampedakis, Samuelsson & Andersson (2006) and Levin (2007) hold true, this conclusion will not change greatly when coupling to core is included in the calculation. However, these results must now be verified using more sophisticated models that include a realistic stellar geometry, field configuration, and general relativistic corrections. There may also be other types of modes in the right frequency range (Glampedakis 2006), although these should be harder to excite and detect. We note that there are alternative non-seismic models for the QPOs, but that these models have serious difficulties (for a discussion see Watts & Strohmayer 2006).

The clear distinction between the theoretical predictions for neutron star and strange star crust models is extremely promising. It offers a robust, largely model independent means of distinguishing strange stars from neutron stars, something that has in the past been lacking. This type of study should lead to rapid progress in constraining the equation of state of compact stars. Ruling out the strange star hypothesis would directly impact the phase diagram of QCD at finite chemical potential. It would offer a strong constraint on models of dense quark matter, indicating that the deconfinement transition is not significantly lowered by the dynamics of the strange quarks. One important implication is that multiply strange hadronic states such as the H-dibaryon in the terrestrial context are less likely. In addition, constraints on the models at finite chemical potential should be relevant for finite temperature extensions of these models which have been employed to describe heavy-ion experiments such as the Relativistic Heavy-Ion Collider (RHIC). Perhaps most importantly, ruling out strange stars directly from observations is the only way to ascertain that terrestrial matter is stable.

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REFERENCES

Alcock C., Farhi E., Olinto E., 1986, ApJ, 310, 261
Alford M.G., Rajagopal K., Reddy S., Steiner A.W., 2006, Phys. Rev. D, 73, 114016
Carroll B.W., Zweibel E.G., Hansen C.J., McDermott P.N., Savedoff M.P., Thomas J.H., van Horn H.M., 1986, ApJ, 305, 767
Chugunov A.I., 2006, MNRAS, 371, 363
Duncan R.C., 1998, ApJ, 498, L45
Duncan R.C., Thompson C., 1992, ApJ, 392, L9
Farouki R.T., Hamaguchi S., 1993, Phys. Rev. E, 47, 4330
Glampedakis K., Samuelsson L., Andersson N., 2006, MNRAS, 371, L74
Harding A.K., Lai D., 2006, Rep. Prog. Phys., 69, 2631
Israel G. et al., 2005, ApJ, 628, L53

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Jaikumar P., Reddy S., Steiner A.W., 2006, Phys. Rev. Lett., 96, 041101
Levin Y., 2007, MNRAS, 377, 159
Lyubarsky Y., Eichler D., Thompson C., 2002, ApJ, 580, L69
McDermott P.N., van Horn H.M., Hansen C.J., 1988, ApJ, 325, 725
Page D., Reddy S., 2006, Ann. Rev. Nucl. Part. Sci., 56, 327
Piro A.L., 2005, ApJ, 634, L153
Rüster S.B., Hempel M., Schaffner-Bielich J., 2006, Phys. Rev. C, 73, 035804
Samuelsson L., Andersson N., 2007, MNRAS, 374, 256
Strohmayer T.E., van Horn H.M., Ogata S., Iyetomi H., Ichimaru S., 1991, ApJ, 375, 679
Strohmayer T.E., Watts A.L., 2005, ApJ, 632, L111
Strohmayer T.E., Watts A.L., 2006, ApJ, 653, 593
Tiengo A., Esposito P., Mereghetti S., Rea N., Stella L., Israel G.L., Turolla R., Zane S., 2005, A&A, 440, L63
Watts A.L., Strohmayer T.E., 2006, ApJ, 637, L117
Watts A.L., Strohmayer T.E., to appear in the Proceedings of the 36th COSPAR Assembly, Advances in Space Research, astro-ph/0612252
Weber F., 2006, Prog. Part. Nucl. Phys., 54, 193
Witten E., 1984, Phys. Rev. D, 30, 272