Constraints on the cosmic distance duality relation with simulated data of gravitational waves from the Einstein Telescope

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The cosmic distance duality relation has been test through several astronomical observations in the last years. This relation establishes a simple equation relating the angular diameter \( D_A \) and luminosity \( D_L \) distances at a redshift \( z \), \( D_L D_A^2 (1+z)^{-2} = \eta = 1 \). However, only very recently this relation has been observationally tested at high redshifts \( z \approx 3.6 \) by using luminosity distances from type Ia supernovae (SNe Ia) and gamma ray bursts (GRBs) plus angular diameter distances from strong gravitational lensing (SGL) observations. No significant deviation from the CDDR validity has been verified, although the results did not rule out \( \eta \neq 1 \) with high confidence level. In this work, we test the potentialities of future luminosity distances from gravitational waves (GWs) sources to impose limit on possible departures of the CDDR jointly with current SGL observations. The basic advantage of \( D_L \) from GWs is being insensitive to non-conservation of the number of photons. By simulating 600, 900 and 1200 data of GWs using the Einstein Telescope (ET) as reference, we derive limits on \( \eta(z) \) function and obtain that the results will be at least competitive with current limits from the SNe Ia + GRBs + SGLs analyses.

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I. INTRODUCTION

In recent years, the unprecedented quality and quantity of astronomical data has thrown cosmology into the age of precision. The main cosmological parameters have been obtained with errors of a few percent through measurements of the cosmic background radiation (CMB) anisotropies [1, 2], observations of type Ia supernovae (SNe Ia) [3, 4] and measurements of the clustering of galaxies at different stages of the universe evolution [5, 6, 7, 8] (we refer the reader to [9] for a recent review).

This set of observations has established the ΛCDM model as the standard scenario. However, there is still no satisfactory understanding or description of the main components of this model, namely, the cold dark matter (CDM) and dark energy (Λ) fields. This motivates the need to probe the foundations and fundamental hypotheses of the standard cosmology. In recent works, the assumption of homogeneity and isotropy of the Universe on large scales has been probed [10, 11, 12, 13, 14], as well as the constancy of the fine structure constant [15, 16, 17, 18] and the CMB temperature evolution standard law [19, 20, 21]. The validity of the cosmic distance duality relation (CDDR), which relates the luminosity distance \( D_L \) of an object at a given redshift \( z \) to its angular diameter distance \( D_A \) via \( D_L D_A^2 (1+z)^{-2} = 1 \) has also been investigated. The CDDR is the subject of the present work.

The cosmic distance duality relation (CDDR) is the astronomical version of the reciprocity theorem derived by [22], which is based on fact that many geometric properties are invariant when the roles of the source and observer in astronomical observations are transposed. The reciprocity relation holds when photons follow null geodesic and the geodesic deviation equation is valid, while the CDDR holds if the reciprocity relation is valid and the number of photon is conserved. Thus, the possibilities of the CDDR violation are: non-conservation of the number of photons or evidence for a non-metric theory of gravity, in which photons do not follow null geodesic [24] (see also [25]). In the former case, non-conservation of the total number of photons can be related to presence of some opacity source and non-standard mechanisms such as the axion-photon conversion induced by intergalactic magnetic fields [26, 27, 28, 29] or scalar fields with a non-minimal multiplicative coupling to the electromagnetic Lagrangian [30, 31, 32, 33]. As one may see, any observational deviation from the CDDR validity would be an indication for new physics.

Several tests have been proposed in the past years assuming a deformed CDDR relation, such as: \( D_L D_A^{-1} (1+z)^{-2} = \eta(z) \). The ideal way to observationally test the CDDR is via independent measurements of intrinsic luminosities and sizes of the same object, without using a specific cosmological model [44]. We may quote approaches involving: measurements of the angular diameter distance (ADD) of galaxy clusters, observations of SNe Ia, estimates of the cosmic expansion \( H(z) \) from cosmic chronometers, measurements of the gas mass fraction in galaxy clusters and observations of strong gravitational lensing [34, 35, 36, 37, 38, 39, 40, 41, 42, 43].
All these tests were performed using different sources for $D_A$ and $D_L$. On the other hand, the Ref. [43] proposed a test which uses exclusively $D_L$ and $D_A$ measurements of the very same object (massive galaxy clusters), obtained from their Sunyaev-Zel’’dovich and X-ray observations. Also, in order to test the CDDR at high-$z$, Refs. [45, 46] considered a sample of strong gravitational lensing system along with the latest gamma-ray burst distance modulus data [47]. No significant deviation from the CDDR validity was verified, although the results did not completely rule out $\eta \neq 1$. It is worth mentioning that a common limitation of all aforementioned tests is that, if $\eta \neq 1$ is obtained, the fundamental reason for the departure may not be known since the results of these tests are sensitive to both conditions for the CDDR violation.

On the other hand, the recent detection of an electromagnetic counterpart (GRB 170817A) to the gravitational wave signal (GW170817) from the merger of two neutron stars [51, 52, 53, 54] has opened the possibility to obtain not only important astrophysical information on the emitting sources, but also to probe cosmology [54, 55]. This makes the first time that a cosmic event has been viewed in both gravitational waves and light. Thus opens the windows of a long-awaited multi-messenger astronomy. The application in cosmology was first proposed by Schutz in 1986, who showed that it is possible to determine the Hubble constant from GW observations, by using the fact that GWs from stellar binary systems encode distance information [57]. Thus the inspiraling and merging compact binaries consisting of neutron stars (NSs) and black holes (BHs), can be considered analogously as the supernovae (SN) standard candles, namely the standard sirens. Unlike SNe Ia, from GWs one can measure the luminosity standard siren directly without the need of cosmic distance ladder: standard sirens are self-calibrating. This property can help us dodge the influence of the conservation of the number of photon on the test of CDDR, which should be considered when one uses the SNe Ia observations. Combining the measurements of the sources’ redshifts from, for example, the electromagnetic (EM) counterpart one can obtain the luminosity distance and redshift relationship.

In this paper, we explore the ability of the gravitational wave detections to constrain a possible departure from the CDDR. The analyses are performed by using luminosity distances of simulated data of gravitational waves while the angular diameter distances are from 118 strong gravitational lensing systems with redshift ranges 0.075 $\leq z_l \leq$ 1.004 and 0.20 $\leq z_s \leq$ 3.60. We estimate the constraints on $\eta(z)$ parameterisations from the simulation of 600, 900 and 1200 data of GWs using the Einstein Telescope (ET) as reference. The ET is a third-generation ground-based detector of GWs being ten times more sensitive in amplitude than the advanced ground-based detectors, covering the frequency range of $1 - 10^4$ Hz. We use Gaussian process (GP) to obtain the luminosity distances for each SGL system from simulated GW data. In order to compare our results with the previous one of Refs. [45, 46], which tested the CDDR using SGL, SNe Ia and GRBs, we consider three functional forms of $\eta(z)$: $\eta(z) = 1 + \eta_0 z$, $\eta(z) = 1 + \eta_0 z/(1 + z)$ and $\eta(z) = 1 + \eta_0 \ln(1 + z)$ (if $\eta_0 = 0$ the validity is verified). As result, we obtain that future results from GWs will be at least competitive with current limits from current analyses.

II. TESTING THE CDDR WITH STRONG GRAVITATIONAL LENSING

Recently, some works used strong gravitational lensing observations to test the CDDR in a cosmological model independent approach [45, 46, 48]. For a flat universe, the approach is as follows. Let us consider the following observational quantity obtained from strong gravitational lensing systems:

$$D = \frac{D_{A,s}}{D_{A,z}} = \frac{\theta_{E, c}^2}{4\pi \sigma_{SIS}^2},$$

where the subindices (ls), (s) and (l) correspond to lens-source, source and lens, respectively. The comoving distance $r_{ls}$ can be written as

$$r_{ls} = r_s - r_l.$$  \hspace{1cm} (2)

Using $r_s = (1 + z_s)D_{A,s}$, $r_l = (1 + z_l)D_{A,l}$ and $r_{ls} = (1 + z_s)D_{A,s}$, one may find

$$D = 1 - \frac{(1 + z_l)D_{A,l}}{(1 + z_s)D_{A,s}}.$$ \hspace{1cm} (3)

Finally, if one considers the deformed CDDR, such as, $D_A D_L^{-1}(1 + z)^{-2} = \eta(z)$, the above expression takes the form

$$\frac{(1 + z_s)\eta(z_s)}{(1 + z_l)\eta(z_l)} = (1 - D) \frac{D_{L,s}}{D_{L,l}}.$$ \hspace{1cm} (4)

III. LUMINOSITY DISTANCE FROM GRAVITATION WAVE SOURCES

In this section, we briefly introduce the concept of standard siren and summarize the method of [58] employed to simulate observations of GW standard sirens from the ET. The data produced in [58] has been used to forecast the constraints of cosmological parameters such as $H_0$, $\Omega_m$ and the equation of state $w$ (see also [59]). Here we will not present the details regarding the production of such data, referring the interested reader to [58] for the explanation of the whole process.

The Einstein Telescope is the third generation of the ground based GW detector. As proposed by the design document, it consists of three collocated underground detectors, each with 10 km arm and with a 60° opening angle. The ET is envisaged to be ten times more
FIG. 1. In all figures the luminosity distances and errors obtained from the simulated GW data for each SGL system are shown as red open circles. The luminosity distances and errors obtained for each SGL system by using Gaussian Process are shown as black filled circles. From left to right the number of the simulated GW data is 600, 900, and 1200.

FIG. 2. The likelihood functions from our analyses. In Figs. (a), (b) and (c) we plot the results by using 300, 600 and 900 GWs simulated. In all figures the black, red and blue curves are for P1, P2 and P3 $\eta(z)$ functions.

sensitive in amplitude than the advanced ground-based detectors, covering the frequency range of $1 - 10^4$ Hz. Here we use ET to simulate the GW observations. As for the sources’ redshifts, the binary merger of a NS with either a NS or BH is considered to be the progenitor of a short and intense burst of $\gamma$-rays (SGRB), an EM counterpart like SGRB can provide the redshift information if the host galaxy of the event can be pinpointed. The expected rates of BNS and BHNS detections per year for the ET$^2$ are about the order $10^5 - 10^7$. However, only a small fraction ($10^{-3}$) is expected to have the observation of SGRB. In this work we take the detection rate in the middle range ($10^5$), thus the order $10^5$ events with SGRB per year.

For the waveform of GW, we also apply the stationary phase approximation to compute the Fourier transform $\mathcal{H}(f)$ of the time domain waveform $h(t)$,

$$\mathcal{H}(f) = A f^{-7/6} \exp \left[ i \left( 2\pi f t_0 - \pi/4 + 2 \psi(f/2) - \varphi_{(2,0)} \right) \right],$$

where the Fourier amplitude $A$ is given by

$$A = \frac{1}{d_L} \sqrt{F_c^2 (1 + \cos^2(i))^2 + 4F_c^2 \cos^2(i)} \times \sqrt{5\pi/96\pi^{-7/6}M_c^{5/6}}.$$

Here $M_c$ denotes the observed chirp mass, $d_L$ is the luminosity distance which plays the most important role in our purpose. The definition of the beam pattern function $F$ for ET and the phase parameters such as the angle of inclination $i$, the functions $\psi$ and $\varphi_{(2,0)}$ can be found in $^58$, $^60$. Following the process of producing the GW mock data in $^56$, we simulate 600, 900 and 1200 GW events with the redshift information provided by the SGRB (see red open circles in Figs. 1a, 1b and 1c). The fiducial model we choose is the concordant model based on the most recent Planck results $^61$:

$$h_0 = 0.678, \quad \Omega_m = 0.308, \quad \Omega_K = 0, \quad w = -1, \quad (7)$$

here $H_0 = 100h_0$ km s$^{-1}$ Mpc$^{-1}$. The $d_L - z$ data sets from ET can extend the redshifts range up to $z \sim 5$, which can easily cover the redshift range of the SGL system. Thus it is suitable to combine these two data sets to test the CDDR.

We use the Gaussian process method to obtain luminosity distances and their errors for each SGL system from the mock GW data (see black filled circles in figs. 1a, 1b and 1c). The luminosity distance errors from 1200 and 900 simulated points are around 15% and 30%, respectively, smaller than those from 600 points. The Gaussian process$^2$ is designed to use the supervised

$^1$http://www.et-gw.eu/

$^2$http://www.gaussianprocess.org/gpml/
learning to build a model (or function) from the training data and then use the model to forecast the new samples. In our works, it can reconstruct a function from data without assuming a parametrization for it. The GP method has been used in many works\cite{62,63,64,65}. We use the GaPP code developed in\cite{66} to derive our results. For detailed description of the Gaussian process, one can refer to\cite{66}, see also\cite{67,68,69}. The reconstructed $dL - z$ points from simulated GW data for each SGL system are plotted in Fig. 11.

### IV. DATA

The angular diameter distance data used in our analyses were reported in Ref.\cite{48} for 118 SGL systems observed by the Sloan Lens ACS survey (SLACS), BOSS Emission-Line Lens Survey (BELLS), Lenses Structure and Dynamics Survey (LSD) and Strong Legacy Survey SL2S surveys. In this compilation, the 118 lens and sources are in the redshift ranges: $0.075 \leq z_s \leq 1.004$ and $0.20 \leq z_l \leq 3.60$. Since several studies have shown that slopes of density profiles of individual galaxies show a non-negligible scatter from the singular isothermal sphere\cite{19}, we consider the approach of\cite{18} to describe the lensing systems called Power Law model. In the power law model (PLaw), a spherically symmetric mass distribution in lensing galaxies is assumed to obey a more general power-law index $\gamma$, $\rho \propto r^{-\gamma}$. By using the PLaw model, the Einstein radius is:

$$\theta_E = \frac{4\pi\sigma_{ap}^2 D_{ls}}{c^2} \left( \frac{\theta_{ap}}{\theta_{ap}} \right)^{2-\gamma} f(\gamma),$$

(8)

where $\sigma_{ap}$ is the stellar velocity dispersion inside the aperture of size $\theta_{ap}$ and

$$f(\gamma) = -\frac{1}{\sqrt{\pi}} \frac{(5 - 2\gamma)(1 - \gamma)}{3 - \gamma} \frac{\Gamma(\gamma - 1)}{\Gamma(\gamma - 3/2)} \times \left[ \frac{\Gamma(\gamma/2 - 1/2)}{\Gamma(\gamma/2)} \right]^2.$$

(9)

Therefore:

$$D = D_{ls}/D_{ls} = \frac{\theta_{ls}^2}{4\pi\sigma_{ap}^2} \left( \frac{\theta_{ap}}{\theta_{ap}} \right)^{2-\gamma} f^{-1}(\gamma).$$

(10)

As one may see, the distribution becomes a SIS for $\gamma = 2$. All relevant information necessary to obtain $D$ in Eq. (10) for both models and perform our fit are in Table 1 of\cite{18}. In our analyses (see next section), we marginalize over $\gamma$ by using the following flat prior: $\gamma: 1.5 < \gamma < 3.0$. Also following\cite{18}, we replace $\sigma_{ap}$ by $\sigma_0$ in the PLAW model.

In order to test the ability of the gravitational wave observations to impose limits on possible departures from the CDDR, we consider the following CDDR parameterisations:

1. P1: $\eta(z) = 1 + \eta_0 z/(1 + z)$;
2. P2: $\eta(z) = 1 + \eta_0 z/(1 + z)$;
3. P3: $\eta(z) = 1 + \eta_0 \ln(1 + z)$.

### V. ANALYSES AND RESULTS

The constraints on the $\eta_0$ parameter are derived by evaluating the likelihood distribution function, $\mathcal{L} \propto e^{-\chi^2/2}$, with

$$\chi^2 = \sum_{i}^{118} \frac{\left[ (1 + z_{i,ls})\eta(z_{i,ls}) - (1 - D_i)/(D_{ls}/D_{ls}) \right]^2}{\sigma_i^2},$$

(11)

where $\sigma_i^2$ stands for the statistical errors associated to the $D_L(z)$ of the GWs sources and the gravitational lensing observations and it is obtained by using standard propagation errors techniques. For the gravitational lensing error one may show that:

$$\sigma_D = D\sqrt{4(\delta\sigma_0)^2 + (1 - \gamma)^2(\delta\theta_E)^2},$$

(12)

Our results for $\eta_0$ by using the 600, 900 and 1200 simulated luminosity distance data are plotted in figures (2a), (2b) and (2c). In all figures, the black, red and blue solid lines correspond to P1, P2 and P3, respectively. The black solid horizontal lines correspond to 1 and 2 $\sigma$ (C.L.). As one may see, the analyses are consistent with the CDDR validity within 1 $\sigma$ (C.L.).

At this point, it is interesting to compare our forecast results with some actual tests involving the angular diameter distances from SGL systems. In Ref.\cite{16} the authors combined the 118 SGL measurements with luminosity distances from the Union2.1 compilation and the latest Gamma-Ray Bursts (GRBs) data. Two cosmological model-independent methods were considered to associate the redshifts of SNe Ia and GRBs with the redshifts of the lens and source of observed from SGL systems: In method A the SNe Ia + GRBs data with $|z_{SN,e}/GRB - z_l| < 0.005$ and $|z_{SN,e}/GRB - z_s| < 0.005$ were binned; in method B a reconstruction of the luminosity distance with the smoothing method by combining it with the Cross Statistic was performed. Other analysis were also performed in Ref.\cite{15}, by fitting the luminosity distance of SNe Ia and GRBs with a second degree polynomial function.

By comparing the results at 1 $\sigma$ c.l., we obtain error bars 40% smaller than that from the method A of the Ref.\cite{16}, regardless the $\eta(z)$ functions considered. By considering the method B of this same reference, we obtain larger error bars when the P1 and P2 functions are considered, more precisely, 40% and 30%, respectively. For the P3 function the error bars are similar. Finally, by considering our results and those from Ref.\cite{15}, we obtain that our error bars are 20%, 80% and 90% smaller when the P1, P2 and P3 functions are considered, respectively. The results did not depend on the number.
GW simulated, showing that the errors of the SGL systems dominate the fits. As one may see, in most case tighter limits are obtained by using luminosity distance of GW sources. As commented earlier, the fundamental advantage here is not to use different set data for the luminosity distances at high redshifts but the insensitive of the GWs to non-conservation of the number of photons.

VI. CONCLUSIONS

The direct observations of gravitational waves made by LIGO/Virgo observatories opened up a new window to observational cosmology. More precisely, for this kind of event it is possible to measure the luminosity distance from waveform and amplitude of the gravitational waves observations, being this distance insensitive to a possible non-conservation of the number of photons, unlike other sources such as SNe Ia and GRBs.

In this work, we have used simulations based on the Einstein telescope and tested the potentialities of future measurements of the luminosity distances from gravitational waves sources. We have place limits on possible departures from the cosmic distance duality relation (CDDR) at high redshifts (z ≈ 3.6) jointly with current estimates of the angular diameter distances from strong gravitational lensing systems. Simulating 600, 900 and 1200 events, we obtained the luminosity distances for each SGL system via Gaussian processes and put limits on η(z) functions by considering three different parameterisations.

By comparing our results with previous ones (e.g., [45, 46]) which reported tests at high redshifts using current data of SNe Ia, GRBs and SGL systems, we have obtained that future measurements of the luminosity distances of gravitational waves sources will be at least competitive with the current analyses. However, unlike SNe Ia and GRBs observations, distance measurements from GWs observations are insensitive to non-conservation of the number of photons and, then, a non-standard physical can be identified if possible departure from the CDDR validity is observationally verified.

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