Experimental Entanglement and Nonlocality of a Two-Photon Six-Qubit Cluster State

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Introduction.—Progress in one-way quantum computing [1] requires the creation of \(n\)-qubit graph states [2] of high number of qubits. Graph states are also fundamental resources for quantum nonlocality [3, 4, 5, 6], quantum error correction [7], and quantum entanglement [2, 8]. In order to create multiquubit graph states it is possible to increase the number of entangled particles [8, 10, 11, 12, 13, 14, 15, 16] and to encode many qubits in each of them [11, 13, 14, 20]. Multiqubit graph states can be created by distributing the qubits between the particles so that each particle carries one qubit. This is the way in which four-qubit graph states with atoms [9] and photons [10, 11, 12, 13, 14], and six-qubit graph states with atoms [15] and photons [16] were created. A second strategy is to distribute the qubits so that each of the particles encodes two qubits. This has been used to create two-photon four-qubit graph states [17, 18, 19] and up to five-photon ten-qubit graph states [20]. By generalizing this strategy, we have created a six-qubit two-photon linear cluster state \(|LC_6\rangle\), by encoding three qubits in each particle: one qubit in the polarization and two qubits in the linear momentum degrees of freedom (DOFs). The \(|LC_6\rangle\) is the only distribution of six qubits between two particles whose perfect correlations have the same nonlocality as those of the six-qubit Greenberger-Horne-Zeilinger (GHZ) state [6], but only requires two separated carriers [5].

Consider the graph in Fig. 1 and associate a single qubit to each vertex. The linear cluster state \(|LC_6\rangle\) is defined as the only six-qubit state which satisfies \(g_i\big|LC_6\rangle = |LC_6\rangle, \forall i\), where \(g_i\) corresponds to the vertex \(i\) of the graph in Fig. 1 and is defined as \(g_i = X_i \bigotimes_{j \in \mathcal{N}(i)} Z_j\), where, e.g., \(X_i\) is the Pauli matrix of qubit \(i\) and \(\mathcal{N}(i)\) is the set of vertices which are connected to \(i\). An equivalent definition of graph states can be given in terms of Controlled-Z operations defined on qubits \(i\) and \(j\) as \(|\text{CZ}_{ij}\rangle = |0\rangle_i |0\rangle_j \otimes |1\rangle_j |1\rangle_i \otimes |Z_j\rangle_i\). The graph state \(|G\rangle\) associated to the \(N\)-vertex graph \(G\) can be written as

\[
|G\rangle = \left( \prod_{\{i,j\}} \text{CZ}_{ij} \right) |+\rangle_1, \quad (1)
\]

where \(\{i, j\}\) indicates the connected vertices in \(G\) and \(|+\rangle_i = \frac{1}{\sqrt{2}}(|0\rangle_i + |1\rangle_i)\).

The specific distribution of the six qubits between the two photons in Fig. 1 (qubits 1, 2, and 3 are carried by photon \(A\), and qubits 4, 5, and 6 by photon \(B\)) allows bipartite nonlocality [5] because, in this distribution, all the single-qubit Pauli observables satisfy EPR’s criterion for elements of reality [21]. Since the result of measuring any of the Pauli observables on qubits 1, 2, and 3 can be predicted with certainty from measurements on qubits 4, 5, and 6, and vice versa. This property is not satisfied by other methods of creating graph states using different DOFs of the same photon, where new qubits are added by local operations [20].

Experimental preparation.—We create the state \(|LC_6\rangle\), equivalent up to single qubit unitary transformations to

![Graph](https://example.com/graph.png)

**FIG. 1:** Graph associated to a two-photon six-qubit entangled state. Each set represents a photon and each vertex corresponds to a qubit. Each link represents a CZ operation between the two connected qubits. Dashed lines represent links present in the \(|LC_6\rangle\) state and absent in the \(|HE_6\rangle\) state. In the experiment, qubits 1 and 4 are encoded into external/internal (E/I) modes, qubits 2 and 5 into horizontal/vertical (H/V) polarization, and qubits 3 and 6 into right/left (r/l) modes. See the text for details.
\( |\text{LC}_6\rangle \), in two steps: first, we prepare a six-qubit hyperentangled state \( (|\text{HE}_6\rangle) \) [cf. Fig. 1] by a triple entanglement of two photons. The quantum information is encoded in the polarization (qubits 2 and 5) and longitudinal momentum (qubits 1 and 4, and 3 and 6) photon DOFs. Then, we transform \( |\text{HE}_6\rangle \) into \( |\text{LC}_6\rangle \) by applying a sequence of unitary transformations which entangle qubits 1 and 2, and qubits 5 and 6.

The experimental setup used to create and measure the \( |\text{LC}_6\rangle \) is illustrated in Fig. 2. We used spontaneous parametric down-conversion (SPDC) in a single 0.5 mm thick Type I \( \beta \)-barium-borate (BBO) crystal excited by a continuous wave UV laser, following a scheme described in Fig. 2 [22] [23]. Precisely, four pairs of correlated spatial modes [24], labeled as \( (r) \) (r) side of the emission cone and as \( I (E) \) considering the internal (external) modes [cf. Fig. 2a)] were selected within the conical emission of the crystal. The starting point for the cluster state generation was the six-qubit \( |\text{HE}_6\rangle \), given by the product of one polarization and two longitudinal momentum entangled states.

\[
|\text{HE}_6\rangle = \frac{1}{\sqrt{2}} (|EE\rangle_{AB} + |II\rangle_{AB})
\]

\[
\otimes \frac{1}{\sqrt{2}} (|HH\rangle_{AB} - |VV\rangle_{AB}) \otimes \frac{1}{\sqrt{2}} (|rr\rangle_{AB} + |\ell\ell\rangle_{AB}),
\]

where \( A (B) \) corresponds to the up (down) side of the conical crystal emission.

By using the following correspondence between physical states and qubit states

\[
\begin{align}
|\text{HE}_6\rangle &= \{ |E\rangle_A, |I\rangle_B \} \rightarrow \{ |0\rangle_1, |1\rangle_1 \}, \quad (3a) \\
|H\rangle_A, |V\rangle_A \rightarrow \{ |0\rangle_2, |1\rangle_2 \}, \quad (3b) \\
|\ell\rangle_A, |r\rangle_A \rightarrow \{ |0\rangle_3, |1\rangle_3 \}, \quad (3c) \\
|E\rangle_B, |I\rangle_B \rightarrow \{ |0\rangle_4, |1\rangle_4 \}, \quad (3d) \\
|H\rangle_B, |V\rangle_B \rightarrow \{ |0\rangle_5, |1\rangle_5 \}, \quad (3e) \\
|\ell\rangle_B, |r\rangle_B \rightarrow \{ |0\rangle_6, |1\rangle_6 \}, \quad (3f)
\end{align}
\]

the hyperentangled state \( \{ |E\rangle \} \) is equivalent, up to single qubit unitary transformations, to the graph state \( |\text{HE}_6\rangle \) shown in Fig. 1. Specifically, \( |\text{HE}_6\rangle = H_2 X_3 H_3 H_4 Z_5 |\text{HE}_6\rangle \), where \( H_i \) denotes the Hadamard operation on qubit \( i \). By Eq. 1, the cluster state \( |\text{LC}_6\rangle \) is obtained from \( |\text{HE}_6\rangle \) by applying the CZ_{12} and CZ_{65} gates. Then, by applying the gates CX_{12} (a Controlled-X operation) and CZ_{65} on the hyperentangled state \( |\text{HE}_6\rangle \), we obtain

\[
|\text{LC}_6\rangle = \text{CX}_{12} \text{CZ}_{65} |\text{HE}_6\rangle = H_2 X_3 H_3 H_4 Z_5 |\text{LC}_6\rangle. \quad (4)
\]

The created state, \( |\text{LC}_6\rangle \), is, up to a unitary transformation, equivalent to the two-photon six-qubit cluster state \( |\text{LC}_6\rangle \) by the correspondence (4). Specifically, the relation given in (4) between \( |\text{LC}_6\rangle \) and \( |\text{LC}_6\rangle \), implies that \( |\text{LC}_6\rangle \) is the only common eigenstate of the generators \( g_i \) obtained from \( g_i \) by changing \( X_2 \leftrightarrow Z_2, X_3 \rightarrow -Z_3, Z_3 \rightarrow X_3, X_4 \rightarrow Z_4, \) and \( X_5 \rightarrow -X_5 \). Qubits 1 and 4 are encoded by the \( E/I \) degree of freedom, qubits 2 and 5 by the \( H/V \) polarization, and qubits 3 and 6 by the \( r/\ell \) degree of freedom.
The state \( \lceil \text{LC}_6 \rceil \) can be written as
\[
| \lceil \text{LC}_6 \rceil \rangle = \frac{1}{2} \left[ |EE\rangle_{AB} |\phi^\pm\rangle_\pi |r\rangle_{AB} + |EE\rangle_{AB} |\phi^\pm\rangle_\pi |r\ell\rangle_{AB} - |II\rangle_{AB} |\psi^\pm\rangle_\pi |r\rangle_{AB} + |II\rangle_{AB} |\psi^\pm\rangle_\pi |r\ell\rangle_{AB} \right],
\]
where \( |\phi^\pm\rangle_\pi = \frac{1}{\sqrt{2}} (|HH\rangle_{AB} \pm |VV\rangle_{AB}) \) and \( |\psi^\pm\rangle_\pi = \frac{1}{\sqrt{2}} (|HV\rangle_{AB} \pm |VH\rangle_{AB}) \) are the standard polarization Bell states.

The transformation from the hyperentangled state to the cluster state was carried out by two wave-plates intercepting the \( |\text{HE}_6\rangle \)'s output modes. Precisely, since qubits 1 (\( E/I \)) and 2 (\( H/V \)) are encoded in photon A, the CX_{12} gate was obtained by applying a half wave-plate (WP) oriented at 45° on the internal A modes (\( a_2 \) and \( a_3 \) in Fig. 2b). Equivalently, the CZ_{65} was obtained by inserting a half WP oriented at 0° on the left B modes (\( b_3 \) and \( b_4 \)). In the actual experiment, we used one WP intercepting both \( a_2 \) and \( a_3 \) modes, while one WP was used for the \( b_3 \) mode and one for the \( b_4 \) mode [see Fig. 2b)].

The experimental setup sketched in Fig. 2b) and 2c) allows the simultaneous measurement of three single qubit compatible observables for each particle. It is given by two chained interferometers whose core elements are given by three symmetric (50/50) beam splitters BS_{1}, BS_{2A}, and BS_{2B}. In BS_{1}, the four \( r \) modes are made indistinguishable from the corresponding \( r \) modes both in space and time, while \( I \) and \( E \) modes belonging to the A (B) side are matched on BS_{2A} (BS_{2B}). Two pairs of single photon detectors detect the output modes \( A \) or \( B \), while polarization entanglement is measured by four polarization analyzers (not shown in the Figure), one for each detector. Nearly 500 coincidences per second were detected, which is 4 orders of magnitude larger than the rate of the six-photon linear cluster state [16].

Fidelity.—We measured the fidelity of our preparation by measuring the 64 stabilizers \( \bar{s}_i \) of \( |\text{LC}_6\rangle \), i.e., all the products of the generators \( g_i \). We obtained (see Table I)
\[
F = \frac{1}{64} \sum_{i=1}^{64} \langle \bar{s}_i \rangle = 0.6350 \pm 0.0008, \tag{6}
\]
which constitutes an improvement of 7% with respect to the best previous fidelity for six-qubit graph states with six particles [15, 16]. The fidelity value is limited by imperfections in phase and polarization settings, such as the two controlled operations (CX and CZ), and mainly by non perfect mode matching on the three beam splitters (BSs). Note that the measurements on the second momentum (I/E qubit) are naturally affected by imperfections of the first momentum setup. Using single mode fibers combined with integrated quantum optical circuits in the experimental setup would allow to largely restore the state fidelity [25]. Other DOFs, such as time-energy and orbital angular momentum, could be adopted to increase the number of qubits. However, this imposes the use of optical components of high quality to preserve the fidelity.

Entanglement witness.—We tested whether or not the created state has genuine six-qubit entanglement (i.e., inexplicable by five or less qubit entanglement). For that purpose, we measured an entanglement witness specifically designed [26] to detect genuine six-qubit entanglement around the \( |\text{LC}_6\rangle \),
\[
W_F = 1 - 2|\text{LC}_6\rangle \langle \text{LC}_6| = 1 - \frac{1}{32} \sum_{i=1}^{64} \bar{s}_i, \tag{7}
\]
where \( \mathbb{1} \) is the identity operator. There is entanglement whenever
\[
\langle W_F \rangle = 1 - 2F < 0. \tag{8}
\]
We obtained,
\[
W_F = -0.270 \pm 0.002, \tag{9}
\]
which is negative by 135 standard deviations and thus proves the existence of a genuine six-qubit entanglement.

Quantum nonlocality.—The specific state we have created is the only distribution of six qubits between two particles whose perfect correlations have the same nonlocality as those of the six-qubit Greenberger-Horne-Zeilinger (GHZ) state [4] and, instead of requiring six separated carriers to show nonlocality, it only requires two [3]. In any local theory in which all the single-qubit Pauli observables can be regarded as elements of reality in the sense of EPR [22], the following Bell inequality [6] must hold:
\[
B \leq 4 \equiv B_{\text{LHV}}, \tag{10}
\]
where
\[
B = |g_1 (\mathbb{1} + g_2)(\mathbb{1} + g_3)(\mathbb{1} + g_4)(\mathbb{1} + g_5)g_6|. \tag{11}
\]
This inequality is the optimal one to detect nonlocality even when the \( |\text{LC}_6\rangle \) has a maximum amount of white noise [6]. EPR’s assumption is that single-qubit observables on photon A (B) are elements of reality (i.e., have pre-assigned outcomes) when their outcomes can be predicted with probability 1 from measurements on photon B (A). However, in our experiment, the single-qubit observables on photon A (B) in the inequality (10) can be predicted from measurements on photon B (A) with probabilities ranging from 0.78 to 0.94. Therefore, we need to relax EPR’s assumption and assume that single-qubit Pauli observables are elements of reality if they can be predicted with probability higher than 0.77. For example, if \( \langle X_3 Z_5 X_6 \rangle = 1 - \epsilon \), with \( 0 \leq \epsilon < 1 \), then a fraction \( \epsilon (1 - \epsilon) \) of the pairs are uncorrelated (perfectly correlated). Therefore, the outcome of \( X_3 \) in photon A can be correctly predicted from the outcome of \( Z_5 X_6 \) in photon B.

\[\text{RAW TEXT ENDS}\]
with probability $\frac{1}{2}$ for the the correlated pairs and with probability $\frac{1}{2}$ for the uncorrelated pairs. Thus the outcome of $X_3$ can be predicted with probability $1(1-\epsilon) + \frac{1}{2}\epsilon = 1 - \frac{1}{2}\epsilon$.

We tested the Bell inequality \cite{10} and obtained

$$B_{\exp} = 7.018 \pm 0.028,$$

equivalent to a degree of nonlocality $D = \frac{B_{\exp}}{B_{\min}}$ of $1.7545 \pm 0.0070$, which is a considerable improvement compared to previous violations of Bell inequalities only involving perfect correlations and using four-qubit states: $(2.59 \pm 0.08)/2 = 1.29$ \cite{12}, $(2.73 \pm 0.12)/2 = 1.36$ \cite{13}, $(3.4145 \pm 0.0095)/2 = 1.70$ \cite{17} and $(2.50 \pm 0.04)/2 = 1.25$ \cite{27}. A higher value of $D$ has been reached for a Bell inequality not involving perfect correlations \cite{10,22,29}. To our knowledge, the result of Eq. \cite{12} represents the first nonlocality test with a six-qubit graph state. The fact that we have obtained a higher degree of nonlocality than with simpler systems is an experimental confirmation that quantum nonlocality can increase as the complexity of the system grows in spite of the decrease of the fidelity \cite{3}.

**Persistency of entanglement.**—Linear cluster states are particular entangled states that, when some qubits are lost, still present some entanglement and nonlocality \cite{11}. Here we can check that, by tracing qubits 3 and 6 or, alternatively, qubits 1 and 4, the remaining four qubits are still entangled and violate a Bell inequality. Indeed, we observed the violation of the following Bell inequalities:

$$\beta \leq 2,$$

$$\beta' \leq 2,$$

where,

$$\beta = \left| \tilde{g}_1 (I + \tilde{g}_2) (I + \tilde{g}_4) \right|,$$

$$\beta' = \left| (I + \tilde{g}_3) (I + \tilde{g}_5) \tilde{g}_6 \right|.$$  \hspace{1cm} (14a)

\hspace{1cm} (14b)

The first is a 2-2-0-2-1-0-setting Bell inequality (i.e., it only involves measurements on qubits 1, 2, 4, and 5); the second is a 0-1-2-0-2-2-setting Bell inequality (i.e., it only involves measurements on qubits 2, 3, 5, and 6). We tested these two Bell inequalities, obtaining

$$\beta_{\exp} = 2.325 \pm 0.014,$$

$$\beta'_{\exp} = 2.881 \pm 0.012.$$  \hspace{1cm} (15a)

\hspace{1cm} (15b)

These results correspond, respectively, to a violation of 23 and 73 standard deviations. The fact that the violation in (15a) is lower than that in (15b) is due to the critical E/I mode matching occurring on BS$_{2A}$, and BS$_{2B}$. We attribute this to the angular divergence of the selected modes that enhances their transverse size in the measurement setup.

**Conclusions.**—In this Letter we have presented the first experimental demonstration of a six-qubit linear cluster state built on a two-photon triple entangled state. An entanglement witness has been measured for this state and its persistency of entanglement and quantum nonlocality properties have been characterized in detail. Cluster states built on two photons and more DOFs present both advantages and disadvantages with respect to multi-photon cluster states. On one side, no more than few pairs of photons at a time are created by SPDC, due to the probabilistic nature of this process; then, multi-photon detection is seriously affected by the limited quantum efficiencies of detectors; finally, an entangled state built on a large number of particles is more affected by decoherence because of the increased difficulty of making photons indistinguishable. On the other side, increasing the number of DOFs implies an exponential requirement of resources, for instance, $2^N$ k-modes per photon must be selected within the emission cone to encode N qubits in each photon. Despite that, working with a limited number of DOFs (up to four) is still more convenient than increasing the number of photon pairs. Hence a hybrid approach (i.e., multi-DOF/multi-photon states) can be conceived in a medium-term time scale to overcome the structural limitations in generation/detection of quantum photon states.

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TABLE I: Experimental results: measurement of the 64 stabilizers $\tilde{s}_i$ of $|\text{LC}_6\rangle$, i.e., all the products of the generators $\tilde{g}_i$. Last three columns indicate in which Bell inequality test each experimental value was used.

| Stabilizer | Experimental value $B_{\text{exp}}$ | $\beta$ | $\beta'$ |
|------------|------------------------------------|--------|--------|
| $\tilde{g}_1$ | $0.5928 \pm 0.0075$ ✓ |        |        |
| $\tilde{g}_2$ | $0.8788 \pm 0.0053$ |        |        |
| $\tilde{g}_3$ | $0.9984 \pm 0.0005$ ✓ |        |        |
| $\tilde{g}_4$ | $0.9973 \pm 0.0008$ ✓ |        |        |
| $\tilde{g}_5$ | $0.7905 \pm 0.0057$ ✓ |        |        |
| $\tilde{g}_6$ | $0.8310 \pm 0.0062$ ✓ |        |        |
| $g_{1}\tilde{g}_2$ | $0.5657 \pm 0.0059$ ✓ |        |        |
| $g_{1}\tilde{g}_3$ | $0.5930 \pm 0.0075$ ✓ |        |        |
| $g_{1}\tilde{g}_4$ | $0.5602 \pm 0.0076$ ✓ |        |        |
| $g_{1}\tilde{g}_5$ | $0.5872 \pm 0.0076$ ✓ |        |        |
| $g_{1}\tilde{g}_6$ | $0.4653 \pm 0.0095$ ✓ |        |        |
| $g_{2}\tilde{g}_3$ | $0.8586 \pm 0.0062$ ✓ |        |        |
| $g_{2}\tilde{g}_4$ | $0.8775 \pm 0.0053$ ✓ |        |        |
| $g_{2}\tilde{g}_5$ | $0.7042 \pm 0.0066$ ✓ |        |        |
| $g_{2}\tilde{g}_6$ | $0.8288 \pm 0.0062$ ✓ |        |        |
| $g_{3}\tilde{g}_4$ | $0.9970 \pm 0.0099$ ✓ |        |        |
| $g_{3}\tilde{g}_5$ | $0.7896 \pm 0.0057$ ✓ |        |        |
| $g_{3}\tilde{g}_6$ | $0.7484 \pm 0.0056$ ✓ |        |        |
| $g_{4}\tilde{g}_5$ | $0.7393 \pm 0.0084$ ✓ |        |        |
| $g_{4}\tilde{g}_6$ | $0.8312 \pm 0.0062$ ✓ |        |        |
| $g_{5}\tilde{g}_6$ | $0.6392 \pm 0.0060$ ✓ |        |        |
| $g_{5}\tilde{g}_6$ | $0.4504 \pm 0.0092$ ✓ |        |        |
| $g_{6}\tilde{g}_6$ | $0.6063 \pm 0.0074$ ✓ |        |        |
| $g_{6}\tilde{g}_6$ | $0.5378 \pm 0.0086$ ✓ |        |        |
| $g_{1}g_{2}\tilde{g}_6$ | $0.4169 \pm 0.0065$ ✓ |        |        |
| $g_{1}g_{3}\tilde{g}_6$ | $0.5603 \pm 0.0076$ ✓ |        |        |
| $g_{1}g_{4}\tilde{g}_6$ | $0.5874 \pm 0.0075$ ✓ |        |        |
| $g_{1}g_{5}\tilde{g}_6$ | $0.4651 \pm 0.0063$ ✓ |        |        |
| $g_{1}g_{6}\tilde{g}_6$ | $0.5882 \pm 0.0074$ ✓ |        |        |
| $g_{2}g_{3}\tilde{g}_6$ | $0.4148 \pm 0.0075$ ✓ |        |        |
| $g_{2}g_{4}\tilde{g}_6$ | $0.4450 \pm 0.0061$ ✓ |        |        |
| $g_{2}g_{5}\tilde{g}_6$ | $0.8592 \pm 0.0062$ ✓ |        |        |
| $g_{2}g_{6}\tilde{g}_6$ | $0.7036 \pm 0.0066$ ✓ |        |        |
| $g_{3}g_{5}\tilde{g}_6$ | $0.7468 \pm 0.0056$ ✓ |        |        |
| $g_{3}g_{5}\tilde{g}_6$ | $0.7038 \pm 0.0066$ ✓ |        |        |
| $g_{3}g_{6}\tilde{g}_6$ | $0.8285 \pm 0.0062$ ✓ |        |        |
| $g_{3}g_{6}\tilde{g}_6$ | $0.6861 \pm 0.0058$ ✓ |        |        |
| $g_{3}g_{6}\tilde{g}_6$ | $0.7357 \pm 0.0083$ ✓ |        |        |
| $g_{4}g_{5}\tilde{g}_6$ | $0.7484 \pm 0.0056$ ✓ |        |        |
| $g_{4}g_{6}\tilde{g}_6$ | $0.6625 \pm 0.0051$ ✓ |        |        |
| $g_{4}g_{6}\tilde{g}_6$ | $0.6394 \pm 0.0060$ ✓ |        |        |
| $g_{4}g_{6}\tilde{g}_6$ | $0.6067 \pm 0.0074$ ✓ |        |        |
| $g_{4}g_{6}\tilde{g}_6$ | $0.5391 \pm 0.0086$ ✓ |        |        |
| $g_{4}g_{6}\tilde{g}_6$ | $0.4334 \pm 0.0063$ ✓ |        |        |
| $g_{4}g_{6}\tilde{g}_6$ | $0.4247 \pm 0.0093$ ✓ |        |        |
| $g_{4}g_{6}\tilde{g}_6$ | $0.3960 \pm 0.0077$ ✓ |        |        |
| $g_{4}g_{6}\tilde{g}_6$ | $0.4435 \pm 0.0076$ ✓ |        |        |
| $g_{4}g_{6}\tilde{g}_6$ | $0.5897 \pm 0.0074$ ✓ |        |        |
| $g_{4}g_{6}\tilde{g}_6$ | $0.4349 \pm 0.0080$ ✓ |        |        |
| $g_{4}g_{6}\tilde{g}_6$ | $0.4465 \pm 0.0061$ ✓ |        |        |
| $g_{4}g_{6}\tilde{g}_6$ | $0.4465 \pm 0.0061$ ✓ |        |        |
| $g_{4}g_{6}\tilde{g}_6$ | $0.7037 \pm 0.0066$ ✓ |        |        |
| $g_{4}g_{6}\tilde{g}_6$ | $0.7465 \pm 0.0056$ ✓ |        |        |
| $g_{4}g_{6}\tilde{g}_6$ | $0.6113 \pm 0.0063$ ✓ |        |        |
| $g_{4}g_{6}\tilde{g}_6$ | $0.6860 \pm 0.0058$ ✓ |        |        |
| $g_{4}g_{6}\tilde{g}_6$ | $0.6624 \pm 0.0051$ ✓ |        |        |
| $g_{4}g_{6}\tilde{g}_6$ | $0.4235 \pm 0.0093$ ✓ |        |        |
| $g_{4}g_{6}\tilde{g}_6$ | $0.3735 \pm 0.0078$ ✓ |        |        |
| $g_{4}g_{6}\tilde{g}_6$ | $0.4071 \pm 0.0077$ ✓ |        |        |
| $g_{4}g_{6}\tilde{g}_6$ | $0.5059 \pm 0.0052$ ✓ |        |        |
| $g_{4}g_{6}\tilde{g}_6$ | $0.4884 \pm 0.0057$ ✓ |        |        |
| $g_{4}g_{6}\tilde{g}_6$ | $0.6112 \pm 0.0063$ ✓ |        |        |
| $g_{4}g_{6}\tilde{g}_6$ | $0.4046 \pm 0.0060$ ✓ |        |        |