Elastic and inelastic scattering of neutrinos and WIMPs on nuclei

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The event rates for WIMP-nucleus and neutrino-nucleus scattering processes, expected to be detected at ton-scale rare-event detectors, are investigated. We focus on nuclear isotopes that correspond to the target nuclei of current and future experiments looking for WIMP- and neutrino-nucleus events. The nuclear structure calculations, performed in the context of the Deformed Shell Model, are based on Hartree-Fock intrinsic states with angular momentum projection and band mixing for both the elastic and inelastic channels. Our predictions in the high recoil energy tail, show that detectable distortions of the measured/expected signal may be interpreted through the inclusion of the non-negligible incoherent channels.

1. INTRODUCTION

The recent observation of coherent elastic neutrino nucleus scattering (CEνNS) process by the COHERENT experiment [1, 2] has opened up a wide range of new opportunities to test the Standard Model (SM) predictions [3, 4] as well as to investigate possible new physics signatures [5] (for a recent review see Ref. [6]). Being a rapidly developing field, currently there are numerous projects aiming to measure this process at the Spallation Neutron Source (SNS) [7], at the European Spallation Source (ESS) [8] or near nuclear reactors such as CONUS [9], CONNIE [10], MINER [11], TEXONO [12], RED100 [13], RICOCHET [14], NUCLEUS [15]. Furthermore, there is an intimate connection between dark matter (DM) and neutrino experiments [16]. The Weakly Interacting Massive Particles (WIMPs) are probably the most promising nonbaryonic cold DM candidates [17]. The latter arise in various frameworks beyond the SM and various experiments [18] such as DarkSide [19], DEAP-3600 [20], CDEX, SuperCDMS [21], LUX [22], XENON1T [23], DARWIN [24] and PandaX-II [25] are looking for tiny WIMP signals. The most appealing WIMP candidate is the Lightest Supersymmetric Particle (LSP) which is expected to be stable and interacts very weakly with matter. In most cases, it is the lightest neutralino which is a linear combination

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of the four neutral fermions $\tilde{B}$, $\tilde{W}_3$, $\tilde{H}_1$ and $\tilde{H}_2$, with $\tilde{B}$ and $\tilde{W}_3$ being the supersymmetric (SUSY) partners of the $U(1)$ gauge field $B$ and the third component of the $SU(2)$ gauge field $W_3$, while $\tilde{H}_1$ and $\tilde{H}_2$ are the SUSY partners of the light and heavy Higgs scalars [26].

The early data from the Cosmic Background Explorer (COBE) [27] and Supernova Cosmology project [28] implied that most of the DM is cold. Moreover, the recent WMAP [29] and Planck satellite [30] data showed that about 26.8% mass of the universe is non-luminous DM, the luminous matter is forming only 4.9% of the mass with the rest 68.3% being dark energy. There are many experimental efforts to detect the elusive, yet to be observed WIMPs which interact weakly with matter. Occasionally, the latter will collide with the nuclei of the detector and the resulting recoil may provide the finger-prints regarding their existence. One such effort is the Super CDMS SNOLAB project (at the nickel mine, Sudbury, Canada) using Silicon and Germanium crystals. For details regarding other experimental attempts, see Refs. [22, 25, 31–36]. There are also efforts to search DM Axion candidates using low-noise superconducting quantum amplifiers, being hypothetical particles originally postulated to solve the strong CP problem. The Axion Dark Matter Experiment (ADMX), situated at the University of Washington, reported results [37] showing that “it is the world’s first and only experiment to have achieved the sensitivity” to hunt for the DM Axions.

The odd-$A$ isotopes $^{127}$I, $^{133}$Cs and $^{133}$Xe, studied in this work, are among the most popular nuclei, used in many experiments as detectors, for both WIMP-nucleus [38–41] and CE$\nu$NS searches [42]. It is worth noting, that the detection techniques of WIMP-nucleus scattering and CE$\nu$NS are closely related, i.e. in both cases only nuclear recoil events are required. On the other hand, a CE$\nu$NS event can perfectly mimic WIMP-nucleus scattering, thus being an irreducible background to direct detection DM searches, hence the neutrino-floor is an important source of background [43–46]. This work is an extension of Ref. [5] where we have investigated the impact of new physics interactions due to neutrino magnetic moments and $Z'$ mediators by evaluating the expected CE$\nu$NS event rates at ton-scale direct DM detection experiments. According to our findings, the latter processes could potentially constitute a major source of background events. Specifically, these results indicate that the novel contributions may lead to a distortion of the expected recoil spectrum that could limit the sensitivity of the direct search experiments. At the same time, given the low-threshold capabilities of the next generation DM experiments, the observation of solar CE$\nu$NS events at a DM experiment will be a direct confirmation of its low mass WIMP sensitivity [47]. While the potential of probing physics beyond the SM through CE$\nu$NS measurements at large-scale DM detectors has been exhaustively tested, nuclear physics inputs still remain a large source of uncertainty [48]. Regarding elastic processes, the latter is encoded in the nuclear form factor which if not properly addressed can lead to a misleading interpretation of a neutrino- or WIMP-induced signal.

In the literature there is a considerable number of theoretical calculations which describe
several aspects of CEνNS and direct detection of DM through nuclear recoils. For the case of CEνNS, realistic nuclear structure calculations have been previously performed, based on various models such as the Quasi-particle Random Phase Approximation (QRPA) [49], Microscopic Quasi-particle Phonon Model (MQPM) [50], coupled-cluster theory [51] and Hartree-Fock (HF) + Bardeen-Cooper-Schrieffer (BCS) calculations [52]. For elastic WIMP-nucleus scattering, apart from the dominant scalar interaction, one needs to consider spin-spin interaction coming from the axial current. On the other hand for the case of inelastic scattering, the scalar interaction practically does not contribute. The scalar interaction can arise from squark exchange, Higgs exchange, the interaction of WIMPs with gluons etc. Some theoretical calculations are described in Refs. [53–56], while recently in Ref. [57] the authors examined the possibility of detecting electrons in light WIMP mass searches. They considered a particle model involving WIMPs interacting with electrons through the exchange of Z-bosons and found that event rates of 0.5–2.5 per kg.yr would be possible in this scenario. Moreover, shell model calculations [58–61] have been performed to study DM event rates with $^{129,131}$Xe, $^{127}$I, $^{73}$Ge, $^{29}$Si, $^{27}$Al, $^{23}$Na and $^{19}$F as detectors (for heavier nuclei, a truncated shell model space is employed).

In recent years, the deformed shell model (DSM), based on HF deformed intrinsic states with angular momentum projection and band mixing, has been established for the reliable description of several nuclear properties. The model proved quite successful in the mass range $A = 60 - 90$ [62], in describing spectroscopic properties including spectroscopy of $N = Z$ odd-odd nuclei with isospin projection [63], in double beta decay half-lives [64, 65], in $\mu \rightarrow e$ conversion in the field of nuclei [66] etc. Recently it was employed to study the event rates for WIMP with $^{73}$Ge as the detector [33]. In addition to the energy spectra and magnetic moments, the model was used to calculate the spin structure functions, nuclear structure factors for the elastic and inelastic WIMP-nucleus scattering. Furthermore, within the framework of DSM we have recently calculated various new physics processes that could potentially contribute to CEνNS and the respective neutrino-floor [5], not only for $^{73}$Ge but also for other promising nuclei such as $^{71}$Ga, $^{75}$As and $^{127}$I.

One of the purposes of the present paper is to describe in detail the results for $^{127}$I, $^{133}$Cs and $^{133}$Xe and also to study the coherent and incoherent event rates for both neutrino-nucleus and WIMP-nucleus scattering. We should stress that the DSM is applied to nuclei beyond $A = 90$ for the first time in the present analysis. For neutrino-nucleus scattering, in addition to the aforementioned isotopes we have also considered the $^{23}$Na, $^{40}$Ar and $^{73}$Ge nuclear targets (DSM results for the spectroscopic properties of $^{23}$Na and $^{40}$Ar will be presented elsewhere). The paper has been organized as follows: in Sec. 2 we summarize the most important features of the adopted nuclear DSM, while in Sec. 3 and Sec. 4 we discuss the formalism of coherent and incoherent WIMP-nucleus and neutrino-nucleus scattering, respectively. Finally, our results are discussed in Sec. 5 and our main conclusions in Sec. 6.
2. DEFORMED SHELL MODEL

The details of this model have been described in our earlier publications (for details see [62]). Assuming axial symmetry for a given nucleus, starting with a model space consisting of a given set of single particle (sp) orbitals and effective two-body Hamiltonian (TBME + spe), the lowest energy intrinsic states are obtained by solving the HF single particle equation self-consistently. Excited intrinsic configurations are obtained by making particle-hole excitations over the lowest intrinsic state. It is worth noting that these intrinsic states denoted by \( \chi_K(\eta) \) do not have definite angular momenta. Hence, states of good angular momentum, projected from an intrinsic state \( \chi_K(\eta) \), can be written in the form

\[
|\psi_{JM}(\eta)\rangle = \frac{2J+1}{8\pi^2\sqrt{N_{JK}}} \int d\Omega D_{JK}^*(\Omega) R(\Omega) |\chi_K(\eta)\rangle, \quad (1)
\]

where \( N_{JK} \) is the normalization constant given by

\[
N_{JK} = \frac{2J+1}{2} \int_0^\pi d\beta \sin \beta d\beta d_J^{\alpha K}(\beta) \langle \chi_K(\eta)|\psi_{JM}(\alpha)\rangle. \quad (2)
\]

Here, \( R(\Omega) = \exp(-i\alpha J_z) \exp(-i\beta J_y) \exp(-i\gamma J_z) \) denotes the general rotation operator and \( \Omega \) represents the Euler angles \((\alpha, \beta, \gamma)\). The good angular momentum states projected from different intrinsic states are not in general orthogonal to each other, hence, band mixing calculations are performed after appropriate orthonormalization. The resulting eigenfunctions are of the form

\[
|\Phi_{JM}(\eta)\rangle = \sum_{K,\alpha} S_{K\eta}^{JM}(\alpha)|\psi_{JK}(\alpha)\rangle, \quad (3)
\]

with \( S_{K\eta}^{JM}(\alpha) \) being the expansion coefficients. The nuclear matrix elements occurring in the calculation of magnetic moments, elastic and inelastic spin structure functions etc. are evaluated using the wave functions \( |\Phi_{JM}(\eta)\rangle \). DSM is well established to be a successful model for transitional nuclei (with \( A=60-90 \)) [62–64, 67, 68].

In order to evaluate the WIMP-nucleus scattering event rates, we first calculate the energy spectra and magnetic moments by employing the DSM wave functions and then we compare our results with available experimental data [69]. A good agreement with experiment is ensuring the reliability of our predictions on the event rates. In our nuclear structure calculations for \(^{127}\text{I}, ^{133}\text{Cs}\) and \(^{133}\text{Xe}\), the single particle orbits, their energies and the assumed effective interaction (obtained by re-normalizing the CD-Bonn potential) are taken from Ref. [70]. The model space consists of the orbitals \(0g_{7/2}, 1d_{5/2}, 1d_{3/2}, 2s_{1/2}\) and \(0h_{11/2}\) with the closed core \(^{100}\text{Sn}\). The single particle energies for the five orbitals are 0.0, 0.4, 1.4, 1.3 and 1.6 MeV for protons and 0.0, 0.7, 2.1, 1.9 and 3.0 MeV for neutrons as given in [70], while we neglect the coulomb term in our calculations. From the experimentally measured static quadrupole moment \(Q\) for the \(^{127}\text{I}\) nucleus, the experimental negative sign of
the $E2/M1$ mixing ratios for the lighter iodine isotopes and also from different theoretical analyses, Ding et al. [71] concluded that $^{127}\text{I}$ has a slightly oblate shape at least at low excitation. Hence we restrict ourselves to HF solutions with oblate shape for this nucleus. The lowest oblate HF single particle spectrum is shown in Fig. 1. Also, for $^{133}\text{Cs}$ and $^{133}\text{Xe}$ the lowest HF intrinsic state which is oblate is also shown in the same figure. As described before, we obtained the lowest HF configuration by performing an axially symmetric HF calculation for each nucleus. Then, various excited configurations are obtained by making particle-hole excitations over this lowest HF configuration. We have chosen a total of 6, 3 and 4 configurations for $^{127}\text{I}$, $^{133}\text{Cs}$ and $^{133}\text{Xe}$, respectively. These choices reproduce reliably the spectroscopic properties of the above nuclear isotopes. The calculated energy levels obtained from angular momentum projection and band mixing for each nucleus are classified into collective bands on the basis of the $E2$ transition probabilities between them. As an example, the calculated energy spectrum for $^{127}\text{I}$ is compared with the available experimental data in Fig. 2.

For the $^{127}\text{I}$ isotope, four low lying positive parity collective bands have been identified experimentally [71] and are reasonably well described within the DSM. First, DSM reproduces correctly the ground state $5/2^+$ level. Secondly, experimentalists have suggested that the low lying positive parity bands in this nucleus should be associated with the proton $d_{5/2}$ and $g_{7/2}$ configurations. An analysis of the DSM wave functions shows that these collective bands originate either from the lowest HF configuration given in Fig. 1 where the odd proton is in the $k = 5/2^+$ deformed orbit or from the first excited intrinsic state which mainly originates from the spherical $1d_{5/2}$ and $0g_{7/2}$ orbits. Thus, using DSM we also predict that the collective bands for this nucleus originate mainly from the $d_{5/2}$ and $g_{7/2}$ orbitals. Turning briefly to $^{133}\text{Cs}$, it can be seen that the DSM calculation reproduces correctly the ground state which is $J = 7/2^+$. In addition, a collective band built on this level has been observed for this nucleus and the calculated $K = 7/2^+$ band is found to be somewhat compressed compared to experiment. This collective band originates mainly from the lowest oblate intrinsic state. Finally, for the case of $^{133}\text{Xe}$, the experimental data shows that well defined collective bands are not present in this nucleus. DSM is found to reproduce the ground state spin ($3/2^+$) and also the ordering of the excited levels (spin of the first excited state is $1/2^+$). In conclusion, for the above three nuclei the DSM reproduces the spin of the ground state and the first excited level correctly, as well as the energy of the first excited level reasonably well. Note that, these two levels are important for the study of the elastic and inelastic scattering processes we are undertaking.

Regarding the values of the harmonic oscillator size parameter $b$ used for different nuclei, they have been chosen assuming the well-known formula ($b \propto A^{1/6}$) and they are listed in Table I. In our earlier work [66] in the calculation of transition matrix elements regarding $\mu \rightarrow e$ conversion in $^{72}\text{Ge}$, we had taken the value $b = 1.90$ fm.
TABLE I: Value of the harmonic oscillator parameter $b$ (in units of fm) used for different nuclei.

| Nucleus | $^{23}\text{Na}$ | $^{40}\text{Ar}$ | $^{73}\text{Ge}$ | $^{127}\text{I}$ | $^{133}\text{Cs}$ | $^{133}\text{Xe}$ |
|---------|------------------|------------------|------------------|------------------|------------------|------------------|
| $b$ (fm)| 1.573            | 1.725            | 1.90             | 2.09             | 2.11             | 2.11             |

TABLE II: The calculated DSM magnetic moments for the different nuclear states of $^{127}\text{I}$, $^{133}\text{Cs}$ and $^{133}\text{Xe}$. The results refer to the bare gyro-magnetic ratios and are compared with the experimental ones from Ref. [69].

| Nucleus | $J$ | $<l_p>$ | $<S_p>$ | $<l_n>$ | $<S_n>$ | $\mu$ (nm) | Expt |
|---------|-----|--------|--------|--------|--------|------------|------|
| $^{127}\text{I}$ | $5/2^+$ | 2.395 | -0.211 | 0.313 | 0.002 | 1.207 | 2.813 |
|         | $7/2^+$ | 2.580 | -0.243 | 1.097 | 0.007 | 0.969 | 2.54  |
|         | $3/2^+$ | 1.542 | -0.181 | 0.148 | -0.008 | 0.560 | 0.97  |
| $^{133}\text{Cs}$ | $7/2^+$ | 3.40 | -0.34 | 0.49 | -0.048 | 1.69 | 2.582 |
|         | $5/2^+$ | 2.47 | -0.25 | 0.30 | -0.034 | 1.23 | 3.45  |
| $^{133}\text{Xe}$ | $3/2^+$ | 0.39 | -0.04 | 1.44 | -0.285 | 1.26 | 0.81  |
|         | $1/2^+$ | 0.30 | -0.03 | 0.004 | 0.230 | -0.75 |

We mention that in the calculation of WIMP-nucleus event rates, the nuclear spin plays an important role and, therefore, we also calculate the nuclear magnetic moments for the isotopes in question. The calculated magnetic moments, predicted in the framework of the DSM, for $^{127}\text{I}$, $^{133}\text{Cs}$ and $^{133}\text{Xe}$ are tabulated in Table II and compared with the experimental data [69]. The contribution of protons and neutrons to the orbital and spin parts are also given in this Table for the reader’s convenience. Focusing on $^{127}\text{I}$, the magnetic moments for the levels $5/2^+$, $7/2^+$ and $3/2^+$ are experimentally known, while the DSM calculated values with bare gyro-magnetic moments are off by about a factor of 2. A similar trend is also found for $^{133}\text{Cs}$ and $^{133}\text{Xe}$ isotopes. It should be noted, however, that shell model calculations by Coraggio et al. [70] using state-dependent effective charges for the B(E2) values and state-dependent quenching factors for Gamow-Teller transitions $GT^+$, obtained a much better agreement with the experimental data. The latter implies that a similar approach with state-dependent gyro-magnetic moments in our DSM calculations is expected reproduce the experimental magnetic moments for the nuclei in question much better. We furthermore note that, for $^{71}\text{Ga}$, $^{73}\text{Ge}$ and $^{127}\text{I}$, Holmlund et al. [54] have calculated the magnetic moments as well as their orbital and spin parts within the MQPM and compared the obtained results with the predictions of other models. For $^{127}\text{I}$, they concluded that,
FIG. 1: HF single-particle energy spectra for $^{127}$I, $^{133}$Cs and $^{133}$Xe. Circles (◦) represent protons and crosses (×) represent neutrons. The HF energy E (in MeV), mass quadrupole moment Q (in units of the square of the oscillator length parameter) and the total azimuthal quantum number K are also given.
FIG. 2: Comparison of the DSM energy spectrum with the experimental data for $^{127}\text{I}$. The experimental values are taken from [69].

Even though MQMP predictions were close to the single particle estimates, the experimental values of the magnetic moments could not be exactly reproduced quantitatively. Therefore, to achieve a better reproducibility of the experimental data, detailed shell model calculations are required.

3. EVENT RATES FOR WIMP-NUCLEUS SCATTERING

The WIMP flux coming from the galactic halo on the Earth is expected to be quite large, of the order of $10^5 \text{ cm}^{-2} \text{ s}^{-1}$. Even though the interaction of WIMP with matter is weak, this flux is sufficiently large for the galactic WIMPs to deposit a measurable amount of energy in an appreciably sensitive detector apparatus when they scatter off nuclei. Most of the experimental searches of WIMPs are based on the direct detection through their interaction with target nuclei. The relevant formalism of WIMP-nucleus scattering has been discussed in our earlier works [5, 33] (see also Refs. [17, 54, 55]). For completeness however, below
we summarize briefly the most important steps. Here, we consider the spin-spin interaction and the scalar interaction. In the case of the former, the WIMP couples to the spin of the nucleus, while the scalar interaction is proportional to the square of the mass number $A$. In the expressions for the event rates, the particle physics (SUSY) part is separated from the nuclear part so that the role played by the nuclear physics part becomes apparent.

### 3.1. Wimp-nucleus elastic scattering

The differential event rate per unit detector mass for a WIMP with mass $m_\chi$ can be written as \[18\]

$$dR = N_t \phi f \frac{d\sigma}{d|q|^2} |q|^2 d|q|^2,$$

(4)

where $\phi = \rho_0 v/m_\chi$ is the DM flux, with $\rho_0$ being the local WIMP density, $N_t$ stands for the number of target nuclei per unit mass and $f$ denotes the WIMP velocity distribution which is assumed to be of Maxwell-Boltzmann type. The latter takes into account the distribution of the WIMP velocity relative to the detector (or Earth) and also the motion of the Sun and Earth. Note that by neglecting the rotation of Earth around its own axis, then $v = |\mathbf{v}|$ corresponds to the relative velocity of WIMP with respect to the detector. Finally, $q \equiv |q|$ represents the magnitude of the 3-momentum transfer to the nuclear target which is related to the dimensionless variable $u = q^2 b^2/2$ with $b$ denoting the harmonic oscillator length parameter.

The WIMP-nucleus differential cross section in the laboratory frame is then given by [17, 33, 54–56]

$$\frac{d\sigma(u,v)}{du} = \frac{1}{2} \sigma_0 \left( \frac{1}{m_{p,b}} \right)^2 \frac{c^2}{v^2} \frac{d\sigma_A(u)}{du},$$

(5)

with

$$\frac{d\sigma_A(u)}{du} = (f^0_A)^2 F_{00}(u) + 2 f^0_A f^1_A F_{01}(u) + (f^1_A)^2 F_{11}(u)$$

$$+ [Z (f^0_S + f^1_S)]^2 |F_Z(u)|^2$$

$$+ [(A-Z) (f^0_S - f^1_S)]^2 |F_N(u)|^2$$

$$+ 2Z(A-Z) [(f^0_S)^2 - (f^1_S)^2] |F_Z(u)||F_N(u)|,$$

(6)

where $F_Z(u)$ and $F_N(u)$ denote the nuclear form factors for protons and neutrons respectively. In the latter expression, the first three terms correspond to spin contribution coming from the axial current and the last three terms account for the coherent part coming mainly from the scalar interaction. Here, $f^0_A$ and $f^1_A$ represent isoscalar and isovector parts of the axial vector current and similarly $f^0_S$ and $f^1_S$ represent isoscalar and isovector parts of the scalar current. The nucleonic current parameters $f^0_A$ and $f^1_A$ depend on the specific SUSY model assumed in this work for the WIMP (LSP). However, $f^0_S$ and $f^1_S$ depend on the hadron model used to embed quarks and gluons into nucleons [72].
The normalized spin structure functions $F_{\rho \rho'}(u)$ with $\rho, \rho' = 0, 1$ of Eq.(6) are defined as

$$F_{\rho \rho'}(u) = \sum_{\lambda, \kappa} \frac{\Omega_{\rho}^{(\lambda, \kappa)}(u)\Omega_{\rho'}^{(\lambda, \kappa)}(u)}{\Omega_{\rho}\Omega_{\rho'}},$$

(7)

$$\Omega_{\rho}^{(\lambda, \kappa)}(u) = \sqrt{\frac{4\pi}{2J_{\rho} + 1}} \times \langle J_{\rho}\| \sum_{j=1}^{A} [Y_{\lambda}(\Omega_{j}) \otimes \sigma(j)]_{\lambda, j}\rangle_{u} r_{j} \omega_{\rho}(j)\| J_{\rho} \rangle,$$

(8)

where $\omega_{0}(j) = 1$ and $\omega_{1}(j) = \tau(j)$ with $\tau = +1$ for protons and $\tau = -1$ for neutrons, $j_{\lambda}$ is the spherical Bessel function, while the static spin matrix elements are defined as $\Omega_{\rho}(0) = \Omega_{\rho}^{(0,1)}(0)$. The reduced matrix element appearing in Eq.(8) is then evaluated within the framework of DSM. For this purpose, we need the sp matrix elements of the operator of the form $t_{l}^{(l,s), J}$, given by

$$\langle n_{i} l_{i} j_{i} \| \hat{t}^{(l,s)} \| n_{k} l_{k} j_{k} \rangle = \sqrt{(2j_{k} + 1)(2j_{j} + 1)(2J + 1)(s + 1)(s + 2)} \left\{ \begin{array}{ccc} l_{i} & 1/2 & j_{i} \\ l_{k} & 1/2 & j_{k} \\ l & s & J \end{array} \right\} \langle l_{i}\| \sqrt{4\pi} Y_{l}^{l}\| l_{k} \rangle \langle n_{i} l_{i} j_{i}(kr)\| n_{k} l_{k} \rangle,$$

(9)

where $\{\}$ denotes the 9-$j$ symbol. Assuming that the polar axis is aligned along the direction of $v_{1}$ (velocity of Earth with respect to the Sun) and converting the integration variables into dimensionless form, the event rate is obtained by integrating Eq.(4) with respect to $u$, velocity $v$ and the scattering angle $\theta$, as

$$\langle R \rangle_{el} = \int_{-1}^{1} d\xi \int_{\Psi_{\text{min}}}^{\Psi_{\text{max}}} d\psi \int_{u_{\text{min}}}^{u_{\text{max}}} d\sigma_{A}(u) \frac{d\sigma_{A}(u)}{du} \cdot$$

(10)

In the above expression, $G(\psi, \xi)$ is given by

$$G(\psi, \xi) = \frac{\rho_{0}}{m_{\chi} A m_{p}} \frac{\sigma_{0}}{m_{p} b} \left( \frac{1}{m_{p} b} \right)^{2} \frac{c^{2}}{\sqrt{\pi} v_{0}} \psi e^{-\lambda^{2} - \psi^{2} e^{-2\lambda^{2}}},$$

(11)

with $\psi = v/v_{0}$, $\lambda = v_{E}/v_{0}$, $\xi = \cos(\theta)$. For our calculation we employed the following parameters: WIMP density $\rho_{0} = 0.3$ GeV/cm$^{3}$, $\sigma_{0} = 0.77 \times 10^{-28}$cm$^{2}$, proton mass $m_{p} = 1.67 \times 10^{-27}$ kg. The velocity of the Sun with respect to the galactic center is taken to be $v_{0} = 220$ km/s and the velocity of the Earth relative to the Sun is taken as $v_{1} = 30$ km/s. The velocity of the Earth with respect to the galactic center $v_{E}$ is given by $v_{E} = \sqrt{v_{0}^{2} + v_{1}^{2} + 2v_{0}v_{1}\sin(\gamma)\cos(\alpha)}$ where $\alpha$ is the modulation angle which stands for the phase of the Earth on its orbit around the sun and $\gamma$ is the angle between the normal to the ecliptic and the galactic equator [26] which is taken to be $\simeq 29.8^\circ$. 
For simplicity, by writing \( X(1) = F_{00}(u) \), \( X(2) = F_{01}(u) \), \( X(3) = F_{11}(u) \), \( X(4) = |F_Z(u)|^2 \), \( X(5) = |F_N(u)|^2 \), \( X(6) = |F_Z(u)||F_N(u)| \) the event rate per unit mass of the detector of Eq.\((10)\) can be cast in the form

\[
\langle R \rangle_{el} = (f_1^0)^2 D_1 + 2f_0^0 f_1^1 D_2 + (f_1^1)^2 D_3 \\
+ [Z (f_0^0 + f_3^0)]^2 D_4 \\
+ [(A - Z) (f_0^0 - f_3^0)]^2 D_5 \\
+ 2Z(A - Z) [(f_0^1)^2 - (f_3^1)^2] D_6 ,
\]

where \( D_i \) being the three dimensional integrations of Eq.\((10)\), defined as

\[
D_i = \int_{-1}^{1} d\xi \int_{\psi_{\text{min}}}^{\psi_{\text{max}}} d\psi \int_{u_{\text{min}}}^{u_{\text{max}}} G(\psi, \xi) X(i) \, du , \quad i = 1, \ldots, 6.
\]

The lower and upper limits of integrations given in Eq.\((10)\) and \((13)\) are taken from Ref. \([56]\) and read

\[
\psi_{\text{min}} = \frac{c}{v_0} \left( \frac{A m_p T_{\text{thres}}}{2 \mu_r^2} \right)^{1/2},
\]

\[
\psi_{\text{max}} = -\lambda \xi + \sqrt{\lambda^2 \xi^2 + \frac{v_{\text{esc}}^2}{v_0^2} - 1 - \frac{v_1^2}{v_0^2} - 2 \frac{v_1}{v_0} \sin(\gamma) \cos(\alpha)},
\]

\[
u_{\text{min}} = A m_p T_{\text{thres}} b^2 ,
\]

\[
u_{\text{max}} = 2(\psi \mu_r b n_0/c)^2 .
\]

Assuming the escape velocity from our galaxy to be \( v_{\text{esc}} = 625 \text{ km/s} \), the quantity \( v_{\text{esc}}^2/v_0^2 - 1 - v_1^2/v_0^2 \) appearing in the definition of \( \psi_{\text{max}} \) is equal to 7.0525 and similarly \( 2v_1/v_0 \sin(\gamma) = 0.135 \) while, \( T_{\text{thres}} \) denotes the detector threshold energy and \( \mu_r \) is the reduced mass of the WIMP-nucleus system.

### 3.2. WIMP-nucleus inelastic scattering

In the case of inelastic WIMP-nucleus scattering, the initial and final nuclear states do not coincide and the corresponding cross section due to the scalar current is considerably smaller with respect to the elastic case. We then, focus on the spin dependent scattering and the inelastic event rate per unit mass of the detector can be written as

\[
\langle R \rangle_{\text{inel}} = (f_1^0)^2 E_1 + 2f_0^0 f_1^1 E_2 + (f_1^1)^2 E_3 , \quad (15)
\]

where \( E_1, E_2 \) and \( E_3 \) are the three dimensional integrals

\[
E_i = \int_{-1}^{1} d\xi \int_{\psi_{\text{min}}}^{\psi_{\text{max}}} d\psi \int_{u_{\text{min}}}^{u_{\text{max}}} G(\psi, \xi) X(i) \, du . \quad (16)
\]
The integration limits in the latter expression read [55, 56]

\[ u_{\text{min}} = \frac{1}{2} b^2 \mu_r^2 \frac{v_0^2}{c^2} \psi^2 \left[ 1 - \sqrt{1 - \frac{\Gamma}{\psi^2}} \right]^2, \]

\[ u_{\text{max}} = \frac{1}{2} b^2 \mu_r^2 \frac{v_0^2}{c^2} \psi^2 \left[ 1 + \sqrt{1 - \frac{\Gamma}{\psi^2}} \right]^2, \]  

where

\[ \Gamma = \frac{2 E^*}{\mu_r c^2}, \]

with \( E^* \) being the energy of the excited nuclear state. Here, \( \psi_{\text{max}} \) is same as in the elastic case and the lower limit \( \psi_{\text{min}} = \sqrt{\Gamma} \). The rest of the parameters, e.g. \( \rho_0, \sigma_0 \) etc. take the same values as in the elastic case.

4. NEUTRINO-NUCLEUS SCATTERING

In this Section we consider neutrino-nucleus scattering focusing on both coherent (\( g.s. \rightarrow g.s. \) transitions) and incoherent channels (\( g.s. \rightarrow \text{excited-state} \) transitions). In the low energy regime of SM electroweak interactions, the latter can be well studied through an effective Lagrangian written in the approximation of four-fermion contact interaction and normalized to the Fermi coupling constant \( G_F \), as

\[ \mathcal{L}(x) = \frac{G_F}{\sqrt{2}} L_\mu(x) H^\mu(x), \]

where the corresponding leptonic and hadronic currents are given through the expressions

\[ L_\mu(x) = \overline{\nu}(x) \gamma_\mu (1 - \gamma_5) \nu(x), \]

\[ H^\mu(x) = \sum_{f=n,p} \overline{\psi}_f(x) \gamma^\mu \left( g^f_L - g^f_A \gamma_5 \right) \psi_f(x). \]

The left- and right-handed couplings are written in terms of the usual vector (axial-vector) \( g^f_V \) (\( g^f_A \)) couplings as functions of the weak mixing angle \( \sin^2 \theta_W \equiv s^2_W = 0.2386 \)

\[ g^f_L = \frac{1}{2} \left( g^f_V + g^f_A \right), \quad g^f_R = \frac{1}{2} \left( g^f_V - g^f_A \right), \]

where the fundamental SM couplings of the nucleon \( f = \{n, p\} \) to the \( Z_0 \) boson read

\[ g^n_V = \frac{1}{2} - 2 s^2_W, \quad g^n_A = \frac{1}{2}, \]

\[ g^p_V = \frac{1}{2} \left( 1 - 2 s^2_W \right), \quad g^p_A = -s^2_W, \]

\[ g^n_L = -\frac{1}{2}, \quad g^n_R = -\frac{1}{2}, \]

\[ g^p_L = -1, \quad g^p_R = 0. \]
In what follows, the neutrino-nucleus cross sections and the corresponding event rates rely on the formalism of Bednyakov and Naumov (BN) [3], while the nuclear physics effects are incorporated by employing the DSM formalism described in Sec. 2. To set up the notation, we provide below the most important features of the BN formalism. We start by denoting the 4-momentum of incoming and outgoing neutrinos as $k = (E_\nu, k)$ and $k' = (E'_\nu, k')$, respectively, and the 4-momentum of the initial (final) nuclear state as $P_n$ ($P'_n$). Then, the energy of the outgoing neutrino is written in terms of the incident neutrino energy, the scattering angle $\theta$ between $k$ and $k'$, the nuclear excitation energy $\Delta \varepsilon_{mn}$ (i.e. the energy difference between the initial and final nuclear states) and the nuclear mass $m_A$, as

$$E'_\nu = \frac{m_A(E_\nu - \Delta \varepsilon_{mn}) - E_\nu \Delta \varepsilon_{mn} + \Delta \varepsilon_{mn}^2/2}{m_A + E_\nu (1 - \cos \theta) - \Delta \varepsilon_{mn}},$$

while the energy of the recoiling nucleus reads

$$T_A = \sqrt{m_A^2 + q^2 - m_A},$$

where $q$ is the 3-momentum transfer with magnitude

$$q \equiv |q| = \left( E'^2_\nu + E'^2_\nu - 2 E_\nu E'_\nu \cos \theta \right)^{1/2} \simeq \left( 2m_A T_A \right)^{1/2}. $$

Finally, it is worth noting that the nuclear recoil energy given in Eq.(25) can be adequately approximated by keeping only the first term in $1/m_A$ expansion, as

$$T_A \approx \frac{E_\nu (E_\nu - \Delta \varepsilon_{mn})(1 - \cos \theta) + \Delta \varepsilon_{mn}^2/2}{m_A}. $$

The differential coherent elastic neutrino-nucleus scattering (CE\nu NS) cross section in the BN formalism has been written as [3]

$$\frac{d\sigma_{coh}}{dT_A} = \frac{4 G_F^2 m_A}{\pi} \sum_{f=n,p} \sqrt{g_{coh}^f F_f} \left( A_+^f \left[ g_L^f - g_R^f ab(1 - b) \right] + A_-^f g_R^f \left[ 1 - ab(1 - b) \right] \right)^2, $$

while the corresponding differential incoherent neutrino-nucleus scattering (I\nu NS) cross section reads [3]

$$\frac{d\sigma_{inc}}{dT_A} = \frac{4 G_F^2 m_A}{\pi} \sum_{f=n,p} g_{inc}^f \left( 1 - |F_f|^2 \right) \times \left[ A_+^f \left( (g_{L,f} - g_{R,f} ab^2)^2 + g_{R,f}^2 ab^2(1 - a) \right) + A_-^f g_R^2 (1 - a) \left( 1 - a + ab^2 \right) \right]. $$

In the latter two expressions the parameters $a$ and $b$ are defined as

$$a = \frac{q^2}{q_{min}^2} \simeq \frac{T_A}{T_A^{max}}, \quad b^2 = \frac{m_A^2}{s}. $$
Here, \( A_{\pm}^p \equiv Z_\pm \) and \( A_{\pm}^n \equiv N_\pm \) represent the number of protons (neutrons) with spin \( \pm 1/2 \) and \( s = (p + k)^2 \) is the total energy squared in the center of mass frame (\( p \) denotes an effective 4-momentum of the nucleon). The correction factors \( g'_{\text{coh}} \) and \( g'_{\text{inc}} \) for coherent and incoherent processes, respectively, are of the order of unity and become more important for very low incoming neutrino energy (see the Appendix of Ref. [3]). We have furthermore verified that, for CE\( \nu \)NS the impact of \( g'_{\text{coh}} \) is less important while for the I\( \nu \)NS channel the corresponding corrections due to \( g'_{\text{inc}} \) become more significant. Finally, it is interesting to remind that in the case of zero momentum transfer it holds \( F_f(q = 0) = 1 \), and therefore, the incoherent cross section vanishes due its dependence on the quantity \( 1 - F_f(q) \).

5. RESULTS AND DISCUSSION

The performed nuclear structure calculations refer to the nuclear matrix elements for g.s. \( \rightarrow \) g.s. transitions (elastic channel) and for transition matrix elements in the case of inelastic scattering, for both neutrino-nucleus and WIMP-nucleus processes. For the elastic and inelastic WIMP-nucleus event rates given in Eqs.(12) and (15), the nucleonic current part has been separated from the nuclear part. We note that, \( D_i \) and \( E_i \) depend on the nuclear structure parameters as well as on the kinematics, assumption on the WIMP velocity and modulation effect while \( X(i) \) depend on the spin structure functions and the nuclear form factors. Thus, the nuclear structure calculations are needed for the evaluation of \( X(i) \).

5.1. Results for WIMP-nucleus scattering

| Nucleus | elastic | inelastic |
|---------|---------|-----------|
|         | \( \Omega_0 \) | \( \Omega_1 \) | \( \Omega_0 \) | \( \Omega_1 \) |
| \( ^{127}\text{I} \) | -0.494 | -0.505 | -0.276 | 0.019 |
| \( ^{133}\text{Cs} \) | -0.878 | -0.660 | 0.020 | 0.041 |
| \( ^{133}\text{Xe} \) | -0.835 | 0.636 | -0.031 | 0.013 |

In our calculations, we first evaluate the static spin matrix elements \( \Omega_0 \) and \( \Omega_1 \) of Eq.(8) for both elastic and inelastic scattering (see Table III). For the ground state of \( ^{127}\text{I} \) these values are -0.494 and -0.505, while for \( ^{133}\text{Cs} \) they are -0.878 and -0.660 and similarly, for \( ^{133}\text{Xe} \) the corresponding quantities are -0.835 and 0.636. Comparing with other works, in Ref. [55] the authors by using effective \( g \)-factors evaluated the elastic and inelastic event
rates for $^{127}$I and found that for elastic scattering, the values of $\Omega_2^0$ and $\Omega_2^1$ are 0.350 and 0.226, i.e. of the same order of magnitude as in our case. They also found that $\Omega_0$ and $\Omega_1$ have the same sign. The DSM wave functions given by Eq.(3) are used to calculate the normalized spin structure functions $F_{\rho\rho'}(u)$ and the squared nuclear form factors $|F_{Z,N}(u)|^2$ for the chosen set of nuclei. The results are shown in Fig. 3 as a function of $u$. As can be seen, for the case of $^{127}$I, the structure functions $F_{00}$, $F_{01}$ and $F_{11}$ show little variation up to about $u = 10$. However, for $^{133}$Cs the spin structure functions show variations from each other above $u = 1$ (a similar trend was found in Ref. [55]). Also, the DSM results for $^{133}$Xe show a similar trend. The proton and neutron nuclear form factors for $^{127}$I have same values up to about $u = 0.8$, while beyond this value the minima of the neutron form factors are shifted towards smaller $u$ in comparison to those of protons. The form factors for $^{133}$Cs and $^{133}$Xe show a similar trend as in $^{127}$I.

In Fig. 4, the nuclear structure dependent coefficients $D_i$, $i = 1, \ldots, 6$ given in Eqs.(13) are plotted for $^{127}$I, $^{133}$Cs and $^{133}$Xe, respectively as function of the WIMP mass $m_\chi$ for different values of the detector threshold. The peaks of these nuclear structure coefficients occur at around $m_\chi \sim 30$ GeV in the case of zero threshold energy and they shift towards higher values of $m_\chi$ as we go to larger threshold energy. In this work, we studied also
FIG. 4: Nuclear structure coefficients $D_i$, $i = 1, \ldots, 6$ as functions of the WIMP mass $m_x$ for the $^{127}$I, $^{133}$Cs and $^{133}$Xe isotopes. The results are evaluated for three values of the detector threshold $T_{\text{thres}} = 0, 10, 20$ keV, while the thickness of the graphs represents the annual modulation effect. Note that, for the case of $^{133}$Xe the coefficient $D_2$ is negative, while the coefficients $D_5$ are shown with dashed thick curves.
FIG. 5: The event rates due to elastic WIMP-nucleus scattering as a function of the DM mass $m_\chi$ for $^{127}$I, $^{133}$Cs and $^{133}$Xe at detector threshold $T_{\text{thres}} = 0, 10$ keV. The curve thickness represents the annual modulation effect.

The detection event rates for the studied nuclei have been calculated at a particular WIMP mass by reading out the corresponding values of $D_i$s from Fig. 4 for each nucleus and by using Eq. (10) assuming the nucleonic current parameters $f^0_A = 3.55 \times 10^{-2}$, $f^1_A = 5.31 \times 10^{-2}$, $f^0_S = 8.02 \times 10^{-6}$ and $f^1_S = -0.15 \times f^0_S$. Figure 5 illustrates the present results as a function of the WIMP mass $m_\chi$ for $^{127}$I, $^{133}$Cs and $^{133}$Xe. The following conclusions can be drawn: for $T_{\text{thres}} = 0$ keV, the highest event rate occurs for $^{127}$I at $m_\chi \sim 35$ GeV, while for $^{133}$Cs and $^{133}$Xe, the highest event rates occur at $m_\chi = 40$ GeV. The event rate decreases at higher detector threshold energy but the peak shifts to the higher values of $m_\chi$.

We now turn our attention to the inelastic channels of WIMP-nucleus scattering. For $^{127}$I, the first excited state $7/2^+$ appears at 57.6 keV above the ground state. It is thus interesting to notice that exotic WIMPS can potentially lead to large nucleon spin induced cross sections which in turn can lead to relatively large probability for inelastic WIMP-$^{127}$I scattering. The latter emphasizes the significance of our present results on inelastic scattering and motivates...
The static spin matrix elements $\Omega_0$ and $\Omega_1$ have values -0.276 and 0.019, respectively. Note that unlike the elastic case, $\Omega_0$ and $\Omega_1$ have opposite sign. Again the magnitude of $\Omega_0$ is smaller by a factor of 2 compared to the elastic case whereas the value of $\Omega_1$ is appreciably smaller (by a factor of 50). The spin structure functions $F_{\rho\rho'}$ relevant to inelastic scattering are plotted as functions of the dimensionless momentum transfer $u$ in Fig. 6. It becomes evident from the plots that the values of $F_{00}$, $F_{01}$ and $F_{11}$ are quite different from each other. Then, the nuclear coefficients $E_i$ are plotted in Fig. 7. At this point it is worth mentioning that $E_i$ do not depend on the detector threshold energy. As for the case of elastic scattering, the corresponding event rates due to inelastic transitions can be obtained by reading out the values of $E_i$ from the figure and using the nucleonic current parameters and Eq. (15). Note that the calculated nuclear structure coefficients are almost
of the same order of magnitude as compared to the results given by Ref. [55], however, we predict much smaller modulation effect. Similarly, the peak values of the nuclear structure coefficients occur at around $m_\chi = 70–80$ GeV, while in Ref. [55] it was found that the peaks occur at around $m_\chi \sim 200$ GeV.

For $^{133}$Cs, the first excited state, having $J = 5/2^+$, is at energy 81 keV. Because of the closeness of this state to the ground state, for this nucleus also one may expect large nucleon spin induced cross section leading to a non-negligible probability for inelastic scattering to occur. The calculated static spin matrix elements $\Omega_0$ and $\Omega_1$ have values 0.020 and 0.041, respectively. These values are almost 40 times smaller than the elastic case. The spin structure functions in the inelastic channel $7/2^+ \rightarrow 5/2^+$ are plotted as functions of $u$ in Fig. 6. The graphs for $F_{00}$, $F_{01}$ and $F_{11}$ are quite different from each other as in the case of $^{127}$I. As can be seen in Fig. 7, the corresponding nuclear coefficients $E_i$ for $^{133}$Cs are in general smaller compared to those of $^{127}$I. Finally, for $^{133}$Xe, the lowest excited state is $J = 1/2^+$ and appears at 262 keV. The calculated static spin matrix elements $\Omega_0$ and $\Omega_1$ for the transition $3/2^+ \rightarrow 1/2^+$ are $-0.031$ and $0.013$, i.e. more than 30 times smaller than the elastic case in this nucleus. The spin structure functions for this inelastic channel are plotted in Fig. 6, from where one sees that the values of $F_{00}$, $F_{01}$ and $F_{11}$ are quite different and show peaks. Finally, the nuclear coefficients $E_i$ are plotted in Fig. 7 from where it is shown that $E_1$, $E_2$ and $E_3$ are larger than those of $^{133}$Cs, while the modulation effect is similar.

5.2. Results for neutrino-nucleus scattering

There are many interesting applications of low energy neutrino scattering with nuclei that are of key importance to direct detection DM searches as shown in Ref. [5]. Apart from the fact that astrophysical neutrinos pose a significant background to DM searches, neutrino-nucleus scattering experiments are also relevant and complementary to the latter for a number of reasons. Among those, we mention that neutrino-nucleus experiments have opened a novel avenue to probe physics beyond the SM with phenomenological impact to DM (see Ref. [6] and references therein). Moreover, it has been recently shown that accelerator-produced DM can be detected in CE\nuNS experiments [73]. Finally CE\nuNS experiments can be used to probe the nuclear structure as well as to characterize the nuclear responses of the same target material employed in direct detection DM experiments.

In this Section, our calculations refer to laboratory $\nu$-sources (reactor and pion decay at rest $\pi$-DAR) and astrophysical (solar) neutrino sources having an evident connection to our previous results regarding WIMP-nucleus scattering [5]. We focus on popular nuclear targets employed in current and future neutrino-nucleus and DM scattering experiments. To this end, using the DSM we calculate the differential and integrated event rates due to
CEνNS and IνNS assuming SM interactions only, as
\[
\frac{dR_x}{dT_A} = \mathcal{K} \int_{E_{\nu}^{\text{min}}}^{E_{\nu}^{\text{max}}} \frac{d\sigma_x}{dT_A}(E_{\nu}, T_A) \lambda_{\nu}(E_{\nu}) \, dE_{\nu}, \quad x = \text{coh, inc}, \quad (31)
\]
where \(\lambda_{\nu}(E_{\nu})\) represents the relevant neutrino energy distribution function characterizing the neutrino source. The normalization factor \(\mathcal{K} = t_{\text{run}} \Phi_{\nu} N_{\text{targ}}\) depends on the exposure time, the neutrino flux and the number of target nuclei. Our goal is to compare the nuclear responses between the various chosen isotopes for CEνNS and IνNS processes. To achieve a more direct comparison, we relax ourselves from being experiment-specific but rather in our calculations we assume the same detector mass and exposure time for all nuclear targets. Specifically, for reactor and π-DAR based neutrino experiments we consider 1 kg of detector mass and 1 year of data taking period, while for the case of direct DM detection experiments we assume 1 ton of detector mass and 1 year of running time.

We begin our calculational procedure by considering the CEνNS and IνNS channels for the case of neutrino experiments utilizing π-DAR neutrinos, like the COHERENT at the SNS (the ESS [8] is another promising possibility). It has been recently shown that a better agreement between the SM expectation and the COHERENT data is possible through DSM calculations [4], while reasonable constraints on nuclear physics parameters have been placed from the COHERENT data [74, 75]. The relevant nuclear isotopes are the \(^{133}\text{Cs}\) and \(^{127}\text{I}\), i.e. the detector materials used during the first run of COHERENT with a CsI detector that led to the first observation of CEνNS [1, 76]. By assuming a typical SNS neutrino flux of \(\Phi_{\nu} \sim 10^7 \, \nu \text{cm}^{-2}\text{s}^{-1}\) and a threshold of 100 eV we calculate the differential event rates and the number of events above threshold. The corresponding results are presented in Fig. 8 for \(^{127}\text{I}\), \(^{133}\text{Cs}\) and \(^{133}\text{Xe}\). From this figure it can be seen that the CEνNS channel dominates the expected signal by at least 1 order of magnitude. However, the IνNS channel can be detectable in the high energy tail of the recoil spectrum, i.e. a possible kink in the spectral shape distribution for high recoil energies is due to potential IνNS contributions. We also consider the \(^{40}\text{Ar}\) nucleus that has been employed by the CENNS-10 liquid Argon (LAr) detector subsystem [77] and recently detected CEνNS with more than 3σ significance [2] leading to improved constraints on various parameters with regards to neutrino physics within and beyond the SM [75]. Finally our calculations also include the \(^{73}\text{Ge}\) and \(^{23}\text{Na}\) nuclei which are the target material of the next generation detectors that will be deployed in the Neutrino-Alley at the SNS [7]. The respective results for \(^{23}\text{Na}\), \(^{40}\text{Ar}\) and \(^{73}\text{Ge}\) are depicted in Fig. 9 leading to the same general conclusions as for the heavier isotopes.

We are now turning our discussion to promising reactor based experiments looking for neutrino-nucleus events. Recently, there has been a very active experimental effort with new experiments aiming to measure CEνNS [42]. Novel developments and instrumentation based on cutting edge technologies are currently pursued in order to reduce the detection threshold in the sub-keV region. The most interesting nuclear isotope is the \(^{73}\text{Ge}\) which constitutes
FIG. 8: Differential (left panel) and integrated (right panel) event rates as a function of the nuclear recoil energy for $^{127}$I, $^{133}$Cs and $^{133}$Xe. The results are presented for CE$\nu$NS and Iv$\nu$NS processes with $\pi$-DAR neutrinos.
The main target material of the CONUS [9], CONNIE [10], MINER [11], NU-CLEUS [15], Ricochet [14], vGEN [78], TEXONO [79] experiments. It is worth mentioning that the $^{133}$Xe isotope is another interesting target being the target nucleus of the RED100 [80] experiment. By assuming a reactor antineutrino flux of $\Phi_\nu \sim 10^{13} \, \text{cm}^{-2}\,\text{s}^{-1}$ we illustrate the corresponding differential and integrated event rates as a function of the nuclear recoil energy in Figs. 10 and 11. Contrary to the SNS case discussed previously, it becomes evident

FIG. 9: Same as Fig. 8 but for $^{23}$Na, $^{40}$Ar, and $^{73}$Ge.
FIG. 10: Differential (left panel) and integrated (right panel) event rates as a function of the nuclear recoil energy for $^{127}$I, $^{133}$Cs and $^{133}$Xe. The results are presented for CE$\nu$NS and I$\nu$NS processes with reactor neutrinos.
that the CE$\nu$NS rate dominates over the I$\nu$NS channel by 2–3 orders of magnitude. As expected, the I$\nu$NS channel is not relevant due to the very low momentum transfer occurred when low energy reactor antineutrinos scatter off nuclei, i.e. the $1 - F_f(|q|)$ factor becomes tiny leading to an appreciable suppression of the incoherent cross section.

Finally, we study CE$\nu$NS and I$\nu$NS processes due to nuclear interactions of neutrinos at

FIG. 11: Same as Fig. 10 but for $^{23}$Na, $^{40}$Ar and $^{73}$Ge.
FIG. 12: Differential (left panel) and integrated (right panel) event rates as a function of the nuclear recoil energy for $^{127}$I, $^{133}$Cs and $^{133}$Xe. The results are presented for CE$\nu$NS and I$\nu$NS processes with solar neutrinos.
it has been recently noticed that existing uncertainties on the SM event rate, mainly those coming from the nuclear form factor, require further attention \cite{5, 48}. For this reason, in this work a comprehensive calculation that takes into account realistic nuclear structure calculations through the DSM as well as the nuclear responses from both CE$\nu$NS and I$\nu$NS interactions, is performed. Furthermore, since we are interested in low-threshold detectors we restrict ourselves to solar neutrinos only, without including the contributions due to At-
mospheric and Diffuse Supernova Neutrino Background (DSNB) neutrinos. The combined
effect of the large energy-related uncertainties characterizing the Atmospheric and DSNB
neutrino spectra in conjunction with the corresponding ones regarding the form factors at
large momentum-transfer, stand out as another motivation of this assumption. The target
nuclei of interest are $^{40}$Ar (DarkSide [19], DEAP-3600 [20]), $^{73}$Ge (CDEX, SuperCDMS [21])
and $^{133}$Xe (LUX [22], XENON1T [23], DARWIN [24], PandaX-II [25]). Our DSM results
are presented in Figs. 12 and 13. As expected, the I$\nu$NS rate coming from the low-energy
solar neutrino spectra is subdominant with respect to the CE$\nu$NS one. On the other hand,
this may not be the case when Atmospheric and DSNB neutrinos are taken into account.
As mentioned previously, the expected I$\nu$NS rates for high energy neutrinos and at high
momentum-transfer require special attention that goes beyond the scope of this study. Such
results are in progress and will be published in a future work.

6. SUMMARY AND CONCLUSIONS

In this study we have presented the expected neutrino-nucleus and WIMP-nucleus events
at prominent rare-event detectors, calculated in the framework of the deformed shell model
(DSM). The chosen nuclear isotopes, $^{23}$Na, $^{40}$Ar, $^{73}$Ge, $^{127}$I, $^{133}$Cs and $^{133}$Xe, were carefully
selected to correspond to present and future experiments looking for CE$\nu$NS events and
WIMP candidates at direct detection dark matter detectors.

The nuclear effects being probably the largest source of theoretical uncertainty are limit-
ing the potential of the relevant experiments in placing constraints on physics beyond the SM.
In this work, the addressed corrections coming from nuclear structure are adequately taken
into consideration through the DSM and are essential to accurately simulate the expected
event rates. In particular, on the basis of Hartree-Fock nuclear states, we have conducted
a comprehensive study by taking also into account the deformation of the studied nuclear
isotopes, leading to a more accurate determination of the spin nuclear structure function
and the nuclear form factors for protons and neutrons.

We have, furthermore, illustrated a comparison of the relevant magnitude between the
coherent and incoherent processes as well as their recoil energy dependence assuming various
neutrino sources. Our obtained results indicate that incoherent neutrino-nucleus processes
can lead to an observable enhancement of the expected signal that is well-above the energy
threshold at multi-ton direct detection dark matter detectors. Similarly a corresponding en-
hancement was found for the case of WIMP-nucleus scattering. We have finally emphasized
that in both cases a signal coming from the de-excitation gammas in the aftermath of an
incoherent processes can be erroneously misinterpreted as a possible new physics signature.
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