Numerical analysis for transient processes of hydropower and water-supply systems sharing a headrace tunnel

J B Yang 1,3, J D Yang 1, W C Guo 2, A T Ma 1
1 State Key Laboratory of Water Resources and Hydropower Engineering Science, Wuhan University, Wuhan, 430072, China
2 School of Hydropower and Information Engineering, Huazhong University of Science and Technology, Wuhan, 430074, China
3 Zhongnan Engineering Corporation Limited, Power Construction Corporation of China, Changsha, 410014, China

Corresponding author: W C Guo, E-mail: wench@whu.edu.cn

Abstract: In order to achieve both hydropower generation and water supply, some hydropower stations with high enough water quality are able to provide water to downstream areas by drawing water directly from the headrace tunnel. Hydraulic interference can occur during these transient processes because the headrace tunnel is partially shared by both the hydropower and water-supply system. This paper uses numerical methods to examine the transient processes of the hydropower and water-supply systems sharing a headrace tunnel, using the Nanshui Reservoir Project as a case study. First, a mathematical model for the hydropower and water-supply system of the selected project is developed. Next, the influence of the closing and opening modes of the guide vanes and the water-supply system valves on the water conveyance system, the surge-chamber surge water levels and unit parameters is analyzed for the normal operating conditions of the hydropower and water-supply system. Results show that the water hammer wave generated by the closing and opening of the valve or the guide vane propagates upstream along the pipelines of its own, and then transmits into the opposite conduits at the intake point, ultimately inducing pressure fluctuations in the conduits and unit output swings. Nonetheless, the negative impacts can be minor. On the other hand, because of the transmission (as opposed to total reflection) of the water hammer wave at the intake point, a pressure relief role is exerted here to some extent.

1. Introduction

With diminishing potable water resources and concurrent increasing demands on high quality drinking water, some small and medium-sized rural reservoirs, that were previously used only for irrigation and hydropower generation purposes, are now being considered as possible drinking water sources for urban areas if their water quality is deemed high enough [1]. This new drinking water supply can be drawn directly from the headrace tunnel or the pressure conduit of a hydropower station in order to reduce financial investments in the project water intakes. However, consideration must be given to the fact that the shared headrace tunnel or pressure conduit in a combined hydropower and drinking water supply system will result in a water hammer wave that will inevitably transmit into the opposite channel system during the transient process, and incur unintended pressure fluctuations for the system not in use. Hydropower systems can also expect unit output swings or hydraulic interference.
phenomena during these dual operations. Thus, great importance should be attached to system safety and operating stability considerations during the renovation design of joint hydropower/drinking water supply projects.

Using the case study of the Nanshui Reservoir Project, this paper proposes a mathematical model for safely designing dual hydropower and drinking water-supply systems, focusing specifically on the influence of the closing and opening modes of the water-supply system valves and the guide vanes on the lineside pressure of the water conveyance system, the surge-chamber surge water level, and the unit parameters.

2. Mathematical Models

According to the layout characteristics of this project, the basic equation and characteristic line solving method of unsteady flow for one-dimensional pressure conduits are adopted to establish mathematical models in combination with reservoir, surge chamber, unit, bifurcated pipe, watershed, regulating valve, reducing valve and other boundaries.

2.1. Mathematical model of unsteady flow for one-dimensional pressure conduits

The characteristic line method is adopted to transform the continuity equation and momentum equation of the unsteady flow of pressure conduits into two ordinary differential equations in the characteristic direction, and then, under the condition of equal time interval grid, they are further transformed into a system of linear equations in two unknowns, i.e. \( Q_p \) and \( H_p \) [2].

\[
C^*: \quad Q_p = Q_{CP} - C_{QP} \cdot H_p \\
C^-*: \quad Q_p = Q_{CM} + C_{QM} \cdot H_p
\]

(1)\( (2)\)

where \( Q_p \) is the flow, \( H_p \) is the piezometric water head, \( Q_{CP}, \ C_{QP}, \ Q_{CM} \) and \( C_{QM} \) are all known quantities of the last moment. Through solving the system of equations, we obtain \( Q_p \) and \( H_p \) of the point in the conduit at the current moment.

2.2. Mathematical models for the turbine generator unit

A schematic diagram of the turbine generator unit for the case study project is shown in Figure 1, while the associated mathematical models are shown below.

![Figure 1. Schematic diagram of turbine generator unit.](image)

(1) Mathematical models for the turbine

The mathematical models for the Francis turbine[3] are shown in Table 1

| Equation description                      | Equation                                      | No. |
|------------------------------------------|-----------------------------------------------|-----|
| Characteristic line equation:            | \( Q_p = Q_{CP} - C_{QP} \cdot H_p \)         | (3) |
| Continuity equation:                     | \( Q_s = Q_{CM} + C_{QM} \cdot H_s \)         | (4) |
| Characteristic equation of turbine:      | \( Q_p = Q_p' D_i \sqrt{H_p - H_s} \)         | (8) |

Table 1. Mathematical models for the Francis turbine.
Mathematical models for the guide vane motion or governor

As shown in Table 2, the actions of guide vanes or governor are simulated in three different cases. In the first case, the governor does not participate in regulation (the change process of guide vane opening $y$ with time is artificially given) (Equation 13). In the second case, the governor participates in frequency regulation (Figure 2a, Equation 14). In the third case, the governor participates in power regulation (Figure 2b, Equation 15) [4].

(a) Frequency regulation: parallel PID-type governor
(b) Power regulation: PI-type governor

Table 2. Mathematical models for the guide vane motion or governor.

| Equation description | Equation                      | No. |
|----------------------|-------------------------------|-----|
| Guide vane motion equation: $y = y(t)$ |                               | 13  |
| Governor participating in frequency regulation: $b_T T_s T_n \frac{d^3 y}{dt^3} + (b_T T_s + b_T T_n + b_T T_p) \frac{d^2 y}{dt^2} + (T_T T_s + b_T T_p) \frac{dy}{dt} + b_T y = - \left( T_T T_n \frac{d^2 x}{dt^2} + T_T dx + x \right)$ | 14  |
| Governor participating in power regulation: $b_T T_s T_n \frac{d^2 y}{dt^2} + b_T T_p \frac{dy}{dt} = b_T T_n \frac{d(p_s - p_p)}{dt} + b_T (p_s - p_e)$ | 15  |

(3) Mathematical models for the generator

A first-order equation is adopted for the generator. For the change process of the unit rotation speed, we assume that the electromagnetic power of the generator remains constant, such that the motion equation for the generator’s rotating part is simplified into a moment of momentum equation, i.e.:

$$J \frac{d\omega}{dt} = M - M_e = \frac{30e_p P}{n^2 \pi} \Delta n$$

According to the different operating modes of the unit, Formula (16) can be rewritten as the three forms, as shown in Table 3:

Table 3. Mathematical models for the generator.

| Equation description                                      | Equation                                                   | No. |
|-----------------------------------------------------------|------------------------------------------------------------|-----|
| Unit not connected to the power grid: $n = n_b + 0.1875(M_s + M_m)\Delta t/\gamma GD^2$ | $n = n_b + 0.1875(M_s + M_m)\Delta t/\gamma GD^2$ | 17  |
Unit participating in frequency regulation:

\[ n = (n_0 + 0.1875(M_1 + M_{\omega 0} - M_g - M_{g0} + 2e_s M_r)\Delta t/GD^2) / (1 + e_s \Delta t/T_a) \]  

(18)

Unit participating in power regulation:

\[ n = n_0 \]  

(19)

In summary: Formulas (5) to (13) and Formula (17) constitute the mathematical models for the unit load rejection; Formulas (5) to (12), Formula (14) and Formula (18) constitute the mathematical models for the unit participating in frequency regulation; Formulas (5) to (12), Formula (15) and Formula (19) constitute the mathematical models for the unit participating in power regulation. In these equations: \( H_p \) and \( Q_p \) represent the piezometric water head and flow of the runner inlet; \( H_s \) and \( Q_s \) represent the piezometric water head and flow of the runner outlet; \( J \) represents the moment of inertia of the unit’s rotating part; \( GD^2 \) represents moment of inertia; \( \omega \) represents the angular speed of rotation of the unit; \( n \) represents the unit rotation speed; \( n_r \) represents the rated unit rotation speed; \( M_t \) represents the moment of the turbine; \( M_g \) represents the resistance moment of the generator; \( P_r \) represents the rated power; \( D_1 \) represents the diameter of runner; \( n_1, Q_1, M_1 \) represent unit rotation speed, unit discharge and unit moment, respectively; \( y \) represents guide vane opening; \( p \) and \( p_s \) represent the relative values of the unit’s given power and real power; \( Q_{CP}, C_{QP}, Q_{CM} \) and \( C_{QM} \) represent the coefficients of the characteristic line equation; \( b_p \) represents the permanent speed droop; \( b_t \) represents the momentary speed droop; \( T_p \) represents the time constant of buffer device; \( T_s \) represents the time constant of the frequency measurement differential; \( T_y \) represents the time constant of the main servomotor response; \( e_s \) represents the self-regulation coefficient of the power grid. Parameters with subscripts represent the values at the initial or last moment.

2.3. Mathematical models for the other hydraulic boundaries

In the pipe system, the mathematical models for the other hydraulic boundaries (see the schematic diagrams in Figure 3) can be established through simultaneously solving the positive characteristic line equation for the end-section of the preceding conduits and the negative characteristic line equation for the head-section of the downstream conduits, as well as the Continuity equation, the Energy equations for the other hydraulic characteristic. These equations are shown in Table 4.

![Figure 3. Schematic diagram of the other hydraulic boundaries.](image)

Table 4. Mathematical models for other hydraulic boundaries.

| Equation description | Equation | No. |
|----------------------|----------|-----|
| Regulating valve + lower reservoir | | |
| Reducing valve | | |
| Bifurcation point | | |
| Watershed | | |
| Restricted orifice surge chamber | | |

4
Characteristic line equation for the end-section of the preceding conduit:

\[ Q_{p1} = Q_{cp1} - C_{QP1} \cdot H_{p1} \] (20)

Characteristic line equation for the head-section of the downstream conduits:

\[ Q_{p2} = Q_{CM2} + C_{QM2} \cdot H_{p2} \] (21)
\[ Q_{p3} = Q_{CM3} + C_{QM3} \cdot H_{p3} \] (22)

Regulating valve

Energy equation:

\[ H_{p1} = Z_{p} + \xi \frac{Q_{p1}^2}{2gA_{p1}^2} \] (23)

Reducing valve

Continuity equation:

\[ Q_{p1} = Q_{p2} \] (24)

Energy equation:

\[ H_{p1} - H_{p2} = \zeta \frac{Q_{p1} |Q_{p1}|}{2gA_{p1}^2} \] (25)

Bifurcation point

Continuity equation:

\[ Q_{p1} = Q_{p2} + Q_{r3} \] (26)

Energy equation:

\[ H_{p1} = H_{p2} = H_{p3} \] (27)

Watershed

Continuity equation:

\[ Q_{p1} = Q_{r2} + Q_{rF} \] (28)

Surge chamber

Continuity equation:

\[ Q_{p1} = Q_{rF} + Q_{p2} \] (29)

Energy equation:

\[ H_{p1} = H_{p2} = H_{pF} \] (30)

Water level equation:

\[ Z = H_{pF} + ZZ2 - \varsigma Q_{rF} |Q_{rF}| \] (31)

\[ Z = Z_0 + \frac{Q_{rF} + Q_0 \Delta F}{2} \] (32)

In summary: Formulas (20) and (23) constitute the mathematical models for regulating valve + lower reservoir; Formulas (20), (21), (24) and (25) constitute the mathematical models for the reducing valve; Formulas (20) to (22), Formula (26) and (27) constitute the mathematical models for the bifurcation point (excluding the local head loss at the bifurcation point); Formulas (20), (21) and (28) constitute the mathematical models for the watershed (assuming that watershed flow \( Q_{p3} \) maintains a steady value in the transient process); Formula (20), Formula (21) and Formulas (29) to (32) constitute the mathematical models for the surge chamber (excluding the local head loss at the bottom of the surge chamber).

In these equations: \( A \) represents the conduit area; \( H_{p1} \) and \( Q_{p1} \) represent the piezometric water head and the flow of the conduit end sections preceding the hydraulic boundary; \( H_{p1}, H_{p3}, Q_{p2} \) and \( Q_{p3} \) represent the piezometric water heads and the flows of the conduit head sections following the hydraulic boundary; \( Q_{rF} \) represents the watershed flow; \( H_{rF} \) and \( Q_{rF} \) represent the piezometric water head at the bottom of the surge chamber and the flow entering the surge chamber; \( Z \) represents the surge-chamber water level; \( Z_0 \) represents the lower reservoir water level; \( ZZ2 \) represents the datum elevation of downstream water levels; \( \xi \) represents the valve loss coefficient; \( \zeta \) represents the reducing valve loss coefficient; \( \varsigma \) represents the surge chamber impedance coefficient. The parameters with subscript represent the values at the initial or last moment.

### 3 Case Study

#### 3.1 Project overview

The Nanshui Hydropower Station is the first large-scale hydropower station built on the Nanshui River. During its initial construction in the 1960s, the project adopted a layout using one upstream surge chamber and three Francis turbine generator units (each with a standalone capacity of 25MW). In 2000,
The Nanshui Reservoir drinking water supply project plans to draw water from the 4th construction adit of the headrace tunnel of the hydropower station, and supply the water to Shaoguan by gravity flow. The water conveyance and distribution conduits will have a total length of about 37.948km. The project is planned in two stages. Stage-I planning includes a design water draw rate of 4.1m$^3$/s for the main conduit. After the water conveyance system bifurcates at stake mark K0+505, some of the water will enter the Ruyuan Water Plant with a design water diversion rate of 0.8 m$^3$/s, while the rest of the water will be conveyed to the Xihe No.2 Water Plant with a design water diversion rate of 3.3 m$^3$/s. The plane layout of the water conveyance system for hydropower generation and water supply is shown in Figure 4. In this article, the way of expression the stake mark of the water supply object is as follows: the initial letter K is the abbreviation of the ‘Kilometer’, and the numbers following K show the distance from the intake point of the water supply object in which the number before ‘+’ indicates how many kilometers and the number behind ‘+’ indicates how many meters. For example, K0+000 indicates the intake point of the water supply object; K0+291 indicates the location 291m from the intake point; and K37+944 indicates the location 37km plus 944m from the intake point.

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**Figure 4.** Schematic diagram of the water conveyance system of the Nanshui Reservoir hydropower and water-supply project.

Using the mathematical models in Section 2, modeling is performed in the transient process calculation software TOPsys [6]; The calculation diagram for the software is shown in Figure 5, where J1 represents the Nanshui Reservoir; J4 represents the upstream surge chamber; J17 represents the intake point; J34 represents the reducing valve; J6 represents the watershed before the Ruyuan Water Plant; J36 represents the regulating valve before the Xihe No.2 Water Plant; J7 represents the service...
reservoir of the Xihe No.2 Water Plant; J8 represents the Ruyuan Water Plant (virtual); J13 and J14 respectively represent 1st and 2nd small units; J23, J24 and J25 respectively represent 1st, 2nd and 3rd large units; J15, J16, J29, J30 and J31 all represent the lower reservoirs of the hydropower station.

Figure 5. Transient process calculation diagram for the water conveyance system for the Nanshui Reservoir hydropower and water-supply project.

3.2. Calculation results and analysis of the transient process
The transient process calculation in this paper mainly focuses on the following tasks: selecting water level combinations in the upper and lower reservoir (provided in Table 6), assessing the influence of the terminal regulating valve closing/opening on the water-supply system under normal operating conditions, assessing the influence of the load rejection/increase of the turbine in the hydropower station on the lineside pressure of the entire water conveyance system, assessing the surge-chamber surge water level and unit regulating guarantee parameters, and assessing the influence of the terminal regulating valve for the water-supply system on the rotation speed and output swing of a unit under normal service conditions (participating in frequency regulation or power regulation).

Table 6. Working conditions for the calculation.

| Working condition No. | Water level combinations in the upper and lower reservoir |
|-----------------------|----------------------------------------------------------|
| D1                    | Normal water level of the upper reservoir=216.644m; design water level of the Xihe No.2 Water Plant=128.00m; downstream operating water level of the power house=88.454m |
| D2                    | Limit water level for power generation of the upper reservoir=194.744m; design water level of the Xihe No.2 Water Plant=128.00m; downstream operating tailwater level of the power house=88.104m |
| D3                    | Check flood level of the upper reservoir=226.644m; design water level of the Xihe No.2 Water Plant=128.00m; check flood level of the power house=89.944m |

(1) Parameters set for steady flow
Before calculating the unsteady flow in the system, it’s necessary to calculate the steady flow. For the hydropower system, these calculations require only the adjustment of the unit’s guide vane opening such that its output, water head, flow and other parameters can meet calculation conditions. However, for the water-supply system, it is also necessary to adjust the openings of the reducing valve and the terminal regulating valve concurrent with adjustments for the guide vane opening. The reducing valve opening adjustment is used to control the pressure distribution in the pipelines downstream, while the terminal regulating valve opening adjustment is used to regulate the water draw rate in the conduits. The primary problem to be solved, therefore, is how to adjust the reducing valve opening. If this opening is too large, high initial pressure in the downstream conduits will occur, and the maximum pressure in the unsteady transient process may not be able to meet system requirements. Conversely, if the opening of the reducing valve is too small, the initial pressure of the downstream conduits will also be low and the minimum pressure in the unsteady transient process may also not be adequate for meeting relevant requirements.
The water level combinations in the upper and lower reservoir are also dependent on finding a suitable opening width for the reducing valve. Under high water level and high water head conditions upstream, the opening of the reducing valve should be set relatively small. On the other hand, under low water level and low water head conditions upstream, the opening of the reducing valve should be set relatively large. Nonetheless, if the minimum pressure in the conduits under unsteady flow occurs upstream of the reducing valve at the upper bend of the conduits in the water-supply system (stake mark K0+166), it will be impossible to increase the initial site pressure through adjusting the reducing valve opening. This case is depicted in Figure 6, with initial openings for the reducing valve and the terminal regulating valve, and the initial pressure distribution line of the water-supply system illustrated.

**Figure 6.** Lineside initial pressure at steady flow of the Nanshui Reservoir water-supply project.
(from the upper reservoir to the Xihe No.2 Water Plant)

(2) Calculation results and analysis of unsteady flow

In calculating the transient process of unsteady flow, a number of conditions are incorporated: the reducing valve always maintains its initial opening, the unit follows the 8s linear guide vane closing law formerly adopted by the hydropower station, and the opening/closing law of the terminal regulating valve is optimized according to control conditions. Optimization results show that the linear closing law, from 100% opening to full vane closure, is 1,298.42 s. Optimization of the linear opening law, from full closure to 100% opening, is 580.79 s. Table 7 shows the extreme value results for the lineside pressure of the water conveyance system, the surge-chamber surge water levels, and the unit regulating guarantee parameters.

**Table 7.** Extreme values of the major-fluctuation transient process parameters.

| Category of calculation | Water conveyance system | Surge chamber | Unit |
|------------------------|-------------------------|--------------|------|
|                        | Lineside maximum pressure/m | Lineside minimum pressure/m | Highest surge water level /m | Lowest surge water level /m | Maximum pressure at spiral case end /m | Minimum pressure at draft tube inlet /m | Maximum rising rate of unit rotation speed/% |
| Extreme value/m Position | 187.73 K23+112 | 17.37 K0+166 217.58 | 189.79 K0+313 182127.14 | 1.96 -0.74 | / | / |
| Calculated value (Regulating valve action) | 158.63 K0+291 | 11.19 K0+166 248.92 | 183.85 K0+313 167.54159.03 | -6.20 -2.08 | 49.71 10.89 |

In the regulating the valve opening/closing processes, it is expected that the water hammer wave propagates to upstream along the hydropower system pipelines. This water hammer wave partially
transmits into the hydropower system pipelines at the watershed, and partially transmits upstream along the main conduit to the surge chamber, incurring fluctuations in surge-chamber water levels. The water hammer wave, as transmitted in both the hydropower generation pipelines and the surge chamber, ultimately gives rise to pressure fluctuations in the hydropower system pipelines, resulting in changes in the water head, flow, output and other parameters of the unit. The amplitude of this pressure fluctuation is highest at the regulating valve, but gradually decreases upstream along the water-supply system pipelines, and also gradually decreases downstream from the watershed along the hydropower system pipelines.

Similar to the above results, in the guide vane opening/closing process, the water hammer wave reflects at the guide vanes, propagates upstream along the hydropower system pipelines, and then partially transmits into the water-supply system pipelines at the watershed while also partially propagating upstream along the main conduit. In the main conduit, the water hammer wave reflects in the surge chamber, resulting in fluctuations in surge-chamber water levels. The water hammer wave is then transmitted into the water-supply pipelines, while the water level fluctuations in the surge chamber give rise to pressure fluctuations in the hydropower system pipelines. Thus, the amplitude of the pressure fluctuation is highest at the guide vanes, gradually decreases upstream along the hydropower system pipelines, and also gradually decreases downstream from the watershed along the water-supply system pipelines.

Figure 7 provides the changes in the surge-chamber surge water level, the pressure at spiral case end, the rotation speed (unit participating in frequency regulation), and the output (unit participating in power regulation) for the 3rd large unit caused by pressure fluctuations upstream of the regulating valve closing process for the Xihe No.2 Water Plant under working condition D3 (the 5 units operating normally). Figure 8 compares between the distribution lines of the lineside maximum and minimum pressures for the water-supply system when the regulating valve or guide vanes act alone.

Figure 7. Changing processes of the parameters in the closing process of the regulating valve under working condition D3.
When the hydropower and water-supply system run at the same time, it is clear that hydraulic connections are established via the intake point, and that the action of the terminal regulating valve and the guide vanes both give rise to pressure fluctuations in the system. These pressure fluctuations impact flow and other parameters in the opposing system, but in a limited way. The control conditions of the transient process of the two systems do not actually change in the presence of the opposite system. But within each system, the transmission (as opposed to the total reflection) of the water hammer wave at the intake point does exert a pressure relief role to some extent. Figure 9 provides the fluctuation processes of surge-chamber water level and pressure at spiral case end for the 3rd large unit after the load rejection of the five units of the hydropower station under working condition D3 in two cases (that is, when the hydropower station runs alone and when the hydropower and water-supply system run at the same time).

4 Conclusions
This paper takes Nanshui Reservoir Project as an example to establish a mathematical model for projects where the hydropower and water-supply system share a headrace tunnel, analyze the method of determining the steady flow parameters of the system and investigate the mutual influence of the two systems in the transient process. The results indicate that the water hammer wave generates by the closing and opening of the valve or the guide vane propagates upstream along the pipelines of its own, and then transmits into the opposite conduits at the intake point, ultimately inducing pressure fluctuations and unit output swings in the conduits. Nonetheless, the negative impacts can be minor. On the other hand, because of the transmission (as opposed to total reflection) of the water hammer wave at the intake point, a pressure relief role is exerted here to some extent.

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