The remote Maxwell demon as energy down-converter

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Abstract

It is demonstrated that Maxwell’s demon can be used to allow a machine to extract energy from a heat bath by use of information that is processed by the demon at a remote location. The model proposed here effectively replaces transmission of energy by transmission of information. For that we use a feedback protocol that enables a net gain by stimulating emission in selected fluctuations around thermal equilibrium. We estimate the down conversion rate and the efficiency of energy extraction from the heat bath.

1 Introduction

Maxwell’s demon [1, 2] uses information to do work. It is by now understood that to store, manipulate and erase information the demon either has to give up its own low entropy state, or it must have access to an energy source [3, 4, 5, 6, 7, 8]. The demon’s function thus, while paradoxical at first sight, is in perfect agreement with the laws of thermodynamics, but it held and continues to hold many lessons about the relation between information and energy.

Far from being a mere thought experiment, by now Maxwell’s demon can and has been realized in the laboratory. It has been demonstrated that information can be converted into work indeed and Maxwell’s demon has become reality [9, 10]. The study of these systems has been very fruitful for the understanding of non-equilibrium statistical mechanics and the energy cost of information. More recently, the quantum version of Maxwell’s demon has spurred interest [11], which highlights the relation between statistical mechanics, thermodynamics and quantum information [12], research that bears relevance not only for quantum computing but also for quantum gravity.

Here, we are not concerned with the quantum version, but focus on the classical Maxwell demon from a new perspective, that of the local flow of energy.

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There is, in principle, no reason why the demon has to do its work the system that eventually generates work from the demon’s information, hereafter referred to as ‘the machine’. The machine can take energy from a heat bath it is embedded in, but without information this energy is not useful and cannot be exploited to do work. However, this opens up the possibility that the demon can work remotely, powered by some source of energy, and transmit to the machine only the information that allows to extract energy from the heat bath.

A similar idea was exploited for the recently proposed quantum energy teleportation [13, 14], which uses entanglement of the quantum vacuum. Since we are not dealing with quantum information but with classical information, we cannot, on the one hand, exploit entanglement. On the other hand, there is nothing that prevents us from copying classical information so it can be processed elsewhere. The system of the remote demon and the machine then essentially work as an energy down-converter: Instead of transmitting energy directly, only the information necessary to extract the energy is transmitted.

We will here lay out a model for how such a system can operate and estimate its efficiency.

2 Model for the remote demon

The machine is placed in a heat bath with temperature $T$. The demon has an energy-reservoir and is not in equilibrium with the heat bath; it has its own heat bath at temperature $T_D < T$. We are in the following not interested to construct a demon able to efficiently use the heat bath. Instead, we are interested in using the demon to allow the machine to efficiently use the heat bath by means of a feedback protocol exploiting statistical fluctuations, similar in spirit to the protocol used in [9].

In order to keep track of the flow of energy in the system, we use a four-level system (figure 1, left). The machine is composed of elements that each constitute such a four-level system.

In the four-level system, the ground level is $E_0$, and $E_1$ is a long-lived state that only slowly decays to $E_0$ with decay time $\tau_{10} \gg 1/T$. The level $E_1$ can be excited into the level $E_2$ which decays very quickly into $E_1$ with decay time $\tau_{21} \ll \tau_{10}$. The fourth and highest level $E_4$ can decay into $E_0$ or $E_1$. We denote the energy differences as $E_1 - E_0 = \Delta E$ and $E_2 - E_1 = \Delta \varepsilon$, where we assume $\Delta E \gg \Delta \varepsilon$.

This setting is suggestive of atomic energy levels, but the model depends only on the general level structure. Another way to realize such a four-level scheme is when the upper level $E_3$ is unstable and represents instead a potential barrier between the ground state $E_0$ and the levels $E_2$ and $E_3$ (figure 1, right). These two scenarios have the common properties that the number of states in $E_1$ can be measured with an energy smaller than $E_1 - E_0$. Note that it has recently become technologically feasible to create ‘artificial atoms’ with custom designed potential levels that could find an application here [15].

The purpose of the remote demon is to extract energy from the machine’s heat bath
by use of information. This information can only be in deviations from thermal equilibrium; we will here use temperature fluctuations. To make use of the thermal fluctuations, we divide the machine up into \( N \) cells small enough so that statistical deviations from thermal equilibrium can be detected. Each cell contains \( n \) of the elements that each are one of the four-level systems, so the demon consists of \( Nn \) elements. Since convergence to the mean goes with \( 1/n \), i.e., the average fluctuations decrease the larger the size of the cell, we will want the cells to be as small as possible.

We will assume that the exchange of information and energy between \( D \) and \( M \) is very fast and the signal extremely unlikely to interact with the heat bath, thus not in thermal equilibrium.

### 2.1 Simplified Case

We will first discuss a single instance of the case with \( n = 1 \), then turn to the cyclic case with arbitrary \( n \). The level configuration is akin that of the 3-level laser, in which stimulated emission can take place between the levels \( E_1 \) and \( E_0 \) at the frequency \( \Delta E \), just that we have an extra resonance added between \( E_1 \) and \( E_2 \). Suppose then we measure whether a state is in \( E_1 \) by use of the resonance between \( E_1 \) and \( E_2 \) at energy \( \Delta \varepsilon \), and then induce emission of the excited states with energy \( \Delta E \). In this case we will have invested the energy \( \Delta E + \Delta \varepsilon \) but are returned the energy \( 2\Delta E \). This emitted energy can be measured as pressure and does work \( W = 2\Delta E \). We assume that the energy \( \Delta \varepsilon \) is thermally reemitted and heats up the bath.

The reason we can extract energy from the heat bath in this process is that we made use of information about which states are excited. Without that information, we would be more likely to lose energy into absorption as to stimulate emission, thus not getting a net gain.
Let us define the energy conversion factor $h = W / (W_1 + W_2)$ as the ratio of the work extracted by the machine over the energy obtained by the machine from the demon. In the case $n = 1$, we have then $h = 2 / (1 + \epsilon)$, where $\epsilon = \Delta \varepsilon / \Delta E$. That is provided that the probability of the signal to cause stimulated emission is equal to one. In the general case, it will be $h = (1 + \sigma_{10}) / (1 + \epsilon)$, where $\sigma_{10}$ denotes the probability of the demon’s signal to stimulate the emission from $E_1$ to $E_0$.

### 2.2 General Case

In the general case the demon, $D$, and the machine, $M$, operate in series of four steps illustrated in figure 2. In the figure the demon is composed of two parts, but this is merely to make the process better to illustrate. The demon has access to an energy source and is coupled to the heat bath at temperature $T_D$. The boxes with numbers in the machine are the cells of the machine. We label them with 1 if all elements of the cell are excited and with 0 otherwise.

![Figure 2: Flow of work and energy for the general case of the remote Maxwell demon. Description see text.](image)

For $n > 1$ we can imagine the elements of the cells being lined up in the direction perpendicular to the image. The part of the demon that is left in the figure would then be located on the perpendicular axis too. This 3-dimensional spatial arrangement is not shown in the figure for graphical reasons.

**Step 1 (measure):** The demon at $T_D$ is in a low entropy state, shown as series of 0s in Figure 2. The machine is in thermal equilibrium with its heat bath and has a thermal distribution of excited states, shown as 1s and 0s. The right part of
the demon emits the measuring signal of energy $\Delta \varepsilon_1$, the resonance energy of the excited states of $M$. To emit this signal, the demon needs an energy $W_1 \geq Nn\Delta \varepsilon$, which is partly absorbed and reemitted by $M$ into the heat bath as $Q_1$.

**Step 2 (copy and compute):** The signal not absorbed or scattered is detected at the left side of the demon. Since the resonant state is short-lived compared to the excited state $E_1$, this measurement is unlikely to kick the excited state into the ground state. This means that the measurement effectively copies the information from $M$ to $D$.

The demon now sends a signal of energy $\Delta E$ to the cells with all excited elements, using energy $W_2 \geq \Delta E$ each. For this the demon has to first condense the information about $n$ elements per cell. It will in general not succeed in stimulating emission for all cells in state 1, both because the probability for inducing emission is smaller than 1 and because the demon, if not at zero temperature, will make errors due to thermal fluctuations. This is indicated in the figure as a remaining 1 in the machine.

**Step 3 (cool):** Since the demon targets only cells that are at energies above the statistical average, it has an above average probability of inducing emission, for estimates see next section. In that process the machine extracts energy $Q$ from the heat bath by use of the information the demon has gathered and produces work $W$.

**Step 4 (erase):** The demon erases the information it had copied from the machine by using work $W_4$, and returns to its initial state and. The machine returns to equilibrium with the heat bath and the demon can resume with Step 1.

### 3 Estimated Efficiency

In thermal equilibrium, each level ($i \in \{1, 2, 3, 4\}$) of the machine’s cells will be occupied with probability

$$p_i^M = \exp \left( -\frac{E_i}{T} \right) / Z^M ,$$

(1)

with the normalization

$$Z^M = \sum_i \exp \left( -\frac{E_i}{T} \right) .$$

(2)

The average number of machine elements in state $i$ is $nNp_i^M$ and the average number of cells with $n$ elements that contain only excited elements is $N(p_1^M)^n$. Let us first assume that $T_D = 0$. 

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If the demon targets all excited cells correctly, it will send an average energy of \( W_2 = \Delta E N(p^M_1)^n \) and the average energy gained by the machine will be \( W = \Delta E (n + 1)N(p^M_1)^n \). The energy down-conversion factor is thus

\[
h_n = 1 + \frac{n}{1 + \epsilon_n} , \quad \epsilon_n = (p^M_1)^{-n}\frac{\Delta \varepsilon}{\Delta E} .
\]

The larger we make \( h_n \) by increasing \( n \), the less work the machine will in total generate because the probability of finding a cell with all excited elements decreases exponentially with \( n \). Or, to put it differently, we would have to make the machine exponentially larger to keep the total work the same with a larger conversion factor.

If we take into account the finite probability \( \sigma_{10} \) of the demon’s signal to stimulate emission, we find by counting the possibilities of failure to induce emission in any combinations of the cell’s elements that

\[
W = \Delta E N(p^M_1)^n q_n , \quad h_n = \frac{1}{1 + \epsilon_n} q_n ,
\]

where

\[
q_n := \sum_{k=0}^{n} \binom{n}{k} (1 + k) \sigma_{10}^k (1 - \sigma_{10})^{n-k} .
\]

The possible distance between \( D \) and \( M \) is limited by the lifetime of \( E_1 \). We can take it into account by replacing \( \sigma_{10} \) with \( \sigma_{10}(t) = \sigma_{10} \exp(-\Delta t/\tau_{10}) \). If the demon needs much longer to send its signal than the lifetime of the state, then the possible energy gain will deteriorate.

We define the efficiency of the machine as

\[
\eta_M = \frac{W - W_2 - Q_1 - Q_4}{Q} .
\]

That is, we subtract from the work \( W \) that the machine generates the work that the demon sent as input, and take into account that \( Q_1 \) and \( Q_4 \) are deposited into the heat bath to erase the information of the measurement.

The 4-step procedure does not pin down exactly which part of the heat the demon deposits into the bath at each step. We will assume that the machine absorbs part of the measuring signal \( W_1 \) that came from the demon and returns it as heat to the bath. The remainder of the signal, which is not absorbed and serves to copy the machine’s state, is removed by the demon in step 4. There is thus no work done with \( Q_1 + Q_4 \). The energy \( Q \) that is extracted from the heat bath is the energy necessary to refill the depleted energy levels of the machine, i.e. \( Q = W - W_2 \).

At \( T_D > 0 \) the demon has a non-vanishing probability to falsely send a signal to a cell that is not excited, or to fail to send a signal to an excited cell. This can be estimated.
considering the demon’s information-processing part (left side in figure 2) to be composed of $N$ two-level system – one per each element of the machine – with an energy difference $\Delta E$, like the cells of the machine.

At a temperature of $T_D$ the probability for an accidental emission is typically $p^D_1 := v/(1 + v)$ with $v = \exp(-\Delta E/T_D)$, i.e. if the temperature of the demon is considerably higher than the energy of the signal it sends, it will be error-prone.

With the noise added, the demon sends a signal

$$W_2 = N\Delta E(p^M_1)^n p^D_0 + N\Delta E p^D_1 (1 - (p^M_1)^n), \quad (7)$$

where $p^D_0 = 1 - p^D_1 = 1/(1 + v)$. Here, the first term is the desired signal reduced by the demon’s failure to respond because there was nothing in the lower level, and the second term is faulty emission due to thermal excitation. The first term thus decreases the work that the machine can do, while the latter increases its heat because the signal is absorbed.

The machine can extract work only from the error-free part of the signal that comes from the demon

$$W = q_n N\Delta E(p^M_1)^n p^D_1, \quad (8)$$

and so

$$W - W_2 = (q_n - 1)N\Delta E(p^M_1)^n p^D_0 - N\Delta E p^D_1 (1 - (p^M_1)^n). \quad (9)$$

Taken together we get

$$\eta_M = 1 - \frac{(p^M_1)^n - 1)p^D_1 + n\epsilon_n}{(q_n - 1)p^D_0}. \quad (10)$$

The machine’s efficiency is not bound by the Carnot limit because it is not a heat engine; it receives a signal from the demon to do its work. The efficiency is shown for some parameter values in figure 3.

Since we do not want to construct a machine that does the demon’s work, we just show that any machine which does this work brings the total efficiency of the combined system of demon and machine down below the Carnot limit. We do this by noting that the demon must be able to empty $N$ energy levels, the result of its computation, to emit the signal to the machine. This amounts to reducing its entropy and, since the process is cyclic, that it must have extracted from its heat bath at least the energy necessary for this reduction.

To derive an upper bound we use the simplified case in which $\sigma_{10} = 1$ and $\Delta \epsilon = 0$ and the signal is noise-free, because these additional corrections would only decrease the efficiency. To further simplify the estimate we assume that the machine returns to the demon the energy of the signal that triggered the stimulated emission, $W_2$, and the work output is thus only the surplus (again, if this return is imperfect it would only decrease
Figure 3: Estimated efficiency of machine, \( \eta_M \), not taking into account efficiency of demon, for \( \sigma_{10} = 1 \) and \( \Delta \varepsilon = 0 \). The efficiency of the machine goes to one if the signal is error free and the heat produced by the measurement is negligible. The efficiency can become negative since there is a parameter regime in which the machine produces less work than it obtains energy from the demon.
the efficiency). Since the process is cyclic and reversible, \( \Delta S = Q_2/T_D \), where \( \Delta S \) is the change in entropy in the demon’s two-level system

\[
\Delta S = - (1 - (p_M^n)^n) \log (1 - (p_M^n)^n) + (p_M^n)^n \log (p_M^n)^n
\]

(11)

The total efficiency is then always smaller than

\[
\eta \leq \frac{n(p_M^n)^n - T_D \Delta S/\Delta E}{n(p_M^n)^n}.
\]

(12)

Now we note that the 2-level distribution with probability \( (p_M^n)^n \) in the upper level that enters the entropy change (11) is just that of a thermal distribution with temperature \( T/n \) and total energy \( \Delta E (p_M^n)^n \), thus

\[
\Delta S \geq \frac{n(p_M^n)\Delta E}{T}.
\]

(13)

This also leads us to suspect that the system will not work efficiently if the temperature of the demon \( T_D \geq T/n \), because it will become very costly to empty the energy levels. Combining (12) and (13) gives

\[
\eta \leq 1 - \frac{T_D}{T}.
\]

(14)

The total efficiency is plotted for some representative values in figure 4.

4 Discussion

In the example for the remote demon discussed here the work is produced by stimulated emission in multiples of the incoming energy from demon. This feature is specific to the amplification procedure used and likely not a general property of the remote demon. As mentioned earlier, if one uses a small potential wall separating two potential minima of different energies, then the amplification does not necessarily have to occur in multiples of the incident energy. Alas, it remains to be seen if a concrete example can be constructed for this case.

Acknowledgements

I thank Ralf Eichhorn, Holger Müller and Stefan Scherer for helpful discussion.

References

[1] Maxwell, J. C. Theory of Heat. Longmans, London (1871).
[2] Szilárd, L. On the decrease of entropy in a thermodynamic system by the intervention of intelligent beings. *Z. Phys.* **53**, 840856 (1929).

[3] Landauer, R. Irreversibility and heat generation in the computation process. *IBM J Res. Dev.* **5**, 183-191 (1961).

[4] Bennett, C. H. The thermodynamics of computation. *Int. J Theor. Phys.* **21**, 905-940 (1982).

[5] Sagawa, T., Ueda, M. Minimal energy cost for thermodynamic information processing: Measurement and information erasure. *Phys. Rev. Lett.* **102**, 250602 (2009) [arXiv:0809.4098 [quant-ph]].

[6] Maruyama, K., Nori, F., Vedral, V. The physics of Maxwell’s demon and information. *Rev. Mod. Phys.* **81**, 123 (2009).

[7] Dillenschneider, R., Lutz, E. Memory erasure in small systems. *Phys. Rev. Lett.* **102**, 210601 (2009) [arXiv:0811.4351 [cond-mat.stat-mech]].

[8] Mandal, D., Jarzynski, C., Work and information processing in a solvable model of Maxwell’s demon. *PNAS* **109**, 11641 (2012) [arXiv:1206.5553 [cond-mat.stat-mech]].

[9] Toyabe, S., Sagawa, T., Ueda, M., Muneyuki, E. & Sano, M. Experimental demonstration of information-to-energy conversion and validation of the generalized Jarzynski equality. *Nature Physics* **6**, 988992 (2010).

[10] Serreli, V., Lee, C., Kay, E. R. & Leigh, D. A. A molecular information ratchet. *Nature*, **445**, 523 - 527 (2007).

[11] Kim, S. W., Sagawa, T., De Liberato, S. & Masahito Ueda, M. Quantum Szilard Engine. *Phys. Rev. Lett.* **106**, 070401 (2011) [arXiv:1006.1471 [quant-ph]]

[12] Del Rio, L., Aberg, J., Renner, R., Dahlsten, O., Vedral, V. The thermodynamic meaning of negative entropy. *Nature Physics* **74**, 61–63 (2011) [arXiv:1009.1630 [quant-ph]].

[13] Hotta, M. Quantum measurement information as a key to energy extraction from local vacuums. *Phys. Rev. D* **78**, 045006 (2008) [arXiv:0803.2272 [physics.gen-ph]].

[14] Hotta, M., Matsumoto, J., & Yusa, G. Quantum Energy Teleportation without Limit of Distance. *Phys. Rev. A* **89**, 012311 (2014) [arXiv:1305.3955 [quant-ph]].

[15] You, J. Q. & Nori, F. Atomic physics and quantum optics using superconducting circuits. *Nature* **474**, 589-597 (2011) [arXiv:1202.1923 [quant-ph]].

[16] Hoppenau, J. & Engel, A. On the energetics of information exchange. *EPL* **105**, 50002 (2014) [arXiv:1401.2270 [cond-mat.stat-mech]].
Figure 4: Upper limit for efficiency of total system for $\sigma_{10} = 1$ and $\Delta \varepsilon = 0$. The solid (red) horizontal lines indicate the Carnot limit.