Two atomic quantum dots interacting via coupling to BECs

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Abstract – We consider a system of three weakly coupled Bose-Einstein condensates and two atomic quantum dots embedded in the barriers between the condensates. Each dot is coupled to two neighboring condensates by optical transitions and can be described as a two-state system, or a pseudospin 1/2. Although there is no direct coupling between the dots, an effective interaction between the pseudospins is induced due to their coupling to the condensate reservoirs. We investigate this effective interaction, depending on the strengths of the dot-condensate coupling $T$ and the direct coupling $J$ between the condensates. In particular, we show that an initially ferromagnetic arrangement of the two pseudospins stays intact even for large $T/J$. However, antiferromagnetically aligned spins undergo peculiar “breathing” modes for weak coupling $T/J<1$, while for strong coupling the behaviour of the spins becomes uncorrelated.

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Introduction. – Atomic quantum dots (AQDs) are a relatively new topic, referred to as nanobosonics in the context of cold atoms. An AQD constitutes a lowest atomic level of a tight trapping potential which can be maximally singly occupied due to a large repulsive interaction energy between the atoms. In analogy to nanoelectronics the question arises whether one can couple an atomic dot to large bosonic reservoirs, i.e. “leads” and investigate particle transport properties in such a combined system. Another interesting problem is the control and manipulation of the single spin behaviour, since such a quantum dot can be described in terms of a two-state system (the occupied state is equivalent to a “spin-up”, and the empty state corresponds to a “spin-down”).

An interesting variant of the optical coupling between an AQD and a uniform superfluid reservoir was considered by Recati et al \cite{1}. Provided the dot atoms and the atoms of the superfluid reservoir are of different hyperfine species, one can (by an external laser) induce Raman transitions between dot atoms and those of the condensate. At low energies the spin degrees of freedom couple to the lowest excitation modes of a superfluid bath —phonons, and therefore the combined system can be mapped onto a spin-boson model, and the dissipative behaviour of the spin can be explored \cite{1}. From a quantum optical point of view this kind of Raman coupling and its consequences were numerically investigated by Zippilli and Morigi \cite{2}. The generalization of such a spin-boson model to the case of many spins has been considered by Orth \textit{et al.} \cite{3}. They have demonstrated that the coupling to the reservoir sound modes induces a ferromagnetic coupling as well as dissipation \cite{3}.

An atomic quantum dot can be considered as an impurity in a Bose-Einstein condensate. If two impurity atoms are embedded in a condensate, the effective interaction between them can arise due to, for example, Bogoliubov phonon exchange with the condensate (Klein and Fleischhauer \cite{4}). This phonon-induced interaction leads to the renormalization of impurity levels. Apart from that there is a direct phonon exchange between the impurities, which can be employed for a transfer of quantum information between the atoms \cite{4}. Another example of an effective interaction between two localized impurities in one-dimensional quantum liquid was considered by Recati \textit{et al.} \cite{5}. The authors show that for fermionic impurities the effective interaction is long-ranged, while for bosons it turns out to be short-ranged, exponentially decaying on the scale of the healing length \cite{5}.

In the present work we consider the correlations induced between two atomic quantum dots, or two impurities localized in between the condensates (fig. 1) and coupled...
to them by optical Raman transitions. The correlation dynamics which we observe here cannot be found, when two dots are coupled to only a single condensate between them. The three-well case considered here is motivated by the experimental possibility of a coupling between two optical lattices. A condensate in a three-well potential was previously studied in detail for example by Buonsante et al. Another motivation is that the condensate dynamics under certain conditions reflects the properties of an induced correlation between the impurities (especially, in the strong-coupling limit, as is explained below).

In previous works by Bausmerth et al. and Fischer et al., a single AQD coherently coupled by optical transitions to two BEC-reservoirs with finite number of particles has been considered. A direct Josephson tunneling between the two BECs has also been taken into account. One should note here, that a Bose Josephson junction exhibits an unusually rich dynamical behaviour, which was also confirmed experimentally.

It was observed in [7] and [8], that one should distinguish between the two important limiting cases: i) the weak-coupling regime, in which the coupling between dot and condensate \( T \) is smaller or of the same order as the direct Josephson coupling \( J \) between two condensates, and ii) the strong-coupling regime, in which the coupling between dot and condensate is dominating, so that the tunneling between the two condensates occurs predominantly through the dot channel.

Since in the weak-coupling regime the effect of the AQD on the tunneling between the condensates is very small, it is the favorable regime in order to investigate the behaviour of the pseudospin under the influence of an effective time-dependent field, induced by the condensates dynamics. It turns out, that the pseudospin behaves in this case as a quantum top and undergoes multi-frequency rotations depending on the initial conditions and the mean-field interaction parameter.

In the strong-coupling limit (\( T \gg J \)), it was shown that an atomic quantum dot, surprisingly, can cause large particle imbalance oscillations between the wells similar to the Josephson effect, which are characterized by the two frequencies. For non-interacting condensates these frequencies could be derived analytically.

In this work we show that in the case of two impurities, although the AQDs do not directly interact with each other, their coupling to the condensates induces an effective coupling between the two pseudospins, associated with the two dots, which can be studied numerically. We show below that symmetric initial conditions favour the ferromagnetic (FM) coupling between the dots, while antiferromagnetic (AFM) spin alignment does not survive the increasing coupling \( T/J \). These features are especially clearly observed in the strong-coupling limit.

Model. — We investigate the setup depicted in fig. 1. The condensate in a three-well potential \( V_{\text{BEC}} \) is described within a three-mode approximation, which is a generalization of the two-mode case [9,10], so that the wave function is \( \Psi(r,t) = \sum_{i=1}^{3} \phi_i(r) \bar{\Psi}_i(t) \), where the \( \phi_i(r) \)'s are the normalized local mode solutions for well \( i \), and

\[
\Psi_i(t) = \sqrt{N_i(t)} e^{i \theta_i(t)}.
\]

Here \( N_i \) is the number of particles in the \( i \)-th condensate, and \( \theta_i \) is its phase.

The Hamiltonian of our system reads

\[
H = \sum_{i=1}^{3} E_i |\Psi_i|^2 + \frac{U}{2} \sum_{i=1}^{3} |\Psi_i|^4 - J(\Psi_i^\dagger \Psi_2 + \Psi_2^\dagger \Psi_1)
\]

\[
- J(\Psi_2^\dagger \Psi_3 + \Psi_3^\dagger \Psi_2) - \hbar \omega \sum_{i=1}^{2} \left( \frac{1 + \sigma_z^{(i)}}{2} + \hbar c \right),
\]

where \( \sigma_z^{(j)} = (\sigma_x^{(j)} + i \sigma_y^{(j)})/2 \), where \( \sigma_x^{(j)} \), \( \sigma_y^{(j)} \), and \( \sigma_z^{(j)} \) are the Pauli matrices describing the pseudospin degrees of freedom of the first \( j=1 \) or the second \( j=2 \) atomic quantum dot. Each quantum dot is described by a state-vector \( \bar{\Psi}(j) |\Psi_d^j \rangle \) with \( j=1,2 \) and \( \Psi_d^j \) being a two-state wave function of the \( j \)-th dot. This formalism is applicable in the limit of large interaction on the atomic dots.

For simplicity we consider a symmetric system: the Josephson couplings between the condensates are the equal: \( J_{12} = J_{23} \equiv J \), defined through

\[
J = - \int dr \left[ \frac{\hbar^2}{2m} (\nabla \phi_i(r) \cdot \nabla \phi_{i+1}(r)) + \phi_i(r) V_{\text{BEC}}(r) \phi_{i+1}(r) \right], \quad i = 1, 2, 3.
\]

All four dot-condensate couplings are expressed by a single parameter \( T \)

\[
T = \hbar \Omega_R \int dr \phi_j(r) \phi_j^*(r) = \hbar \Omega_R \int dr \phi_j(r) \phi_j^*(r).
\]
where \( j = 1, 2 \), and the spatial wave function of each dot \( \phi_j(r) \) is normalized to unity. \( \Omega_R \) is the Rabi frequency of the Raman transition. The Rabi frequency is an externally controllable parameter, that can be tuned to any desirable value. Spontaneous emission is suppressed by a large detuning from the excited electronic states, which is absorbed into the effective dot energy \( \hbar \delta \) [1]. Another important parameter in the problem is the standard mean-field two-particle interaction between the condensate atoms

\[
U = g \int \mathrm{d}r |\phi_1(r)|^4,
\]

where \( g = 4\pi a_s/m \) with \( a_s \) being the s-wave scattering rate. The zero-point energies are taken equal to each other

\[
E_{\text{z}} = \int \left[ -\frac{\hbar^2}{2m} \nabla^2 \phi_j(r) + |\phi_j(r)|^2 V_{\text{BEC}}(r) \right] \mathrm{d}r \equiv E,
\]

and in the actual calculation we assume \( E = 0 \).

We can now derive equations of motion for the condensate wave functions, which take the following form:

\[
i\partial_t \Psi_1 = (E + U|\Psi_1|^2) \Psi_1 - J \Psi_2 + T s_3^{(1)},
\]

\[
i\partial_t \Psi_2 = (E + U|\Psi_2|^2) \Psi_2 - J (\Psi_1 + \Psi_3) + T (s_1^{(1)} + s_3^{(2)}),
\]

\[
i\partial_t \Psi_3 = (E + U|\Psi_3|^2) \Psi_3 - J (\Psi_1 + \Psi_2) + T s_3^{(2)}.
\]

The equations of motion for the pseudospin components of the two quantum dots are expressed as two Bloch equations

\[
\hbar \partial_t s^{(j)} = \vec{\omega}^{(j)} \times \vec{s}^{(j)}.
\]

The Bloch equations for the two spins are coupled through the rotation frequencies

\[
\vec{\omega}^{(1)} = \begin{pmatrix} 2T(\Psi_1 + \Psi_2) \\ -2T(3\Psi_1 + 3\Psi_2) \\ -\hbar\delta \end{pmatrix},
\]

\[
\vec{\omega}^{(2)} = \begin{pmatrix} 2T(\Psi_2 + \Psi_3) \\ -2T(3\Psi_2 + 3\Psi_3) \\ -\hbar\delta \end{pmatrix},
\]

which are time-dependent due to the coupling to the condensate dynamics eqs. (6)–(8).

We solve this set of equations numerically for different initial conditions, and analyze the behaviour of the condensates and the dots in both regimes of weak and strong coupling. In the case of the condensate we are interested in the behaviour of normalised particle imbalances

\[
n_{12}(t) = \frac{N_1(t) - N_2(t)}{N_0}, \quad n_{23} = \frac{N_2(t) - N_3(t)}{N_0},
\]

where \( N_0 = N_1(0) + N_2(0) + N_3(0) \). Note, that conserving quantities are \( N_{\text{tot}} = N_0 = (s_z^{(1)} + 1)/2 + (s_z^{(2)} + 1)/2 \) and \( (s^{(j)})^2 = (s_x^{(j)})^2 + (s_y^{(j)})^2 + (s_z^{(j)})^2 \) for \( j = 1, 2 \).

The time dependence of \( \cos \gamma \) for different coupling constants \( T/J \) is shown in fig. 3. The angle \( \gamma \) oscillates...
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Fig. 3: (Colour on-line) Relative orientation $\vec{s}_1 \cdot \vec{s}_2 = \cos(\gamma)$ vs. time for different coupling constant $T/J$. The initial conditions are the same as in fig. 2 and initial alignment of the two pseudospins is AFM. One can observe the “breathing” modes, whose period decreases for growing $T/J$.

Fig. 4: (Colour on-line) Behaviour of the pseudospins for asymmetric initial condensate sizes in the case of weak coupling: $N_1(0) = 10000$, $N_2(0) = 1000$, $N_3(0) = 500$, interaction is fixed to $UN_0/2J = 1$, and the relative coupling of the dot to the condensate is $T/J = 0.1$.

One can also consider asymmetric initial conditions: $N_1(0) \neq N_2(0) \neq N_3(0)$ as, for instance, in fig. 4. In this case the dynamics of the two AQDs becomes very complicated, although not chaotic, as the Fourier transform of $(\cos \gamma)$ does not produce a typical white noise signal.

Strong-coupling limit. – The peculiarities, which we found out in the dynamics of the initially AFM aligned spins in the weak-coupling limit, are expected to become even more pronounced in the strong-coupling regime, corresponding to $T \gg J$. In this case, the relevant time scale is set by $T$. It was previously shown that the ratio $UN_0/\delta$ plays an important role in the physical behaviour of the system [8]. Thus, the most interesting regime, when the AQD serves as a quantum “shuttle” of the atoms between the condensates, is achieved for $\delta \gg UN_0$. Naively one expects, that the oscillations induced solely due to the coupling of the condensates to the dot, would be always very small, however, surprisingly large amplitude Josephson-type oscillations between the wells were obtained [8].

In our case we also observe this property of AQDs (fig. 5). For initially parallel spins we observe symmetric oscillations of the particle imbalances $n_{12}$ and $n_{23}$ (see figs. 5(a) and (c)). These are similar to the oscillations in a triple-well potential which are always symmetric for symmetric initial conditions. Note, that these oscillations are induced purely by the dots.

The pseudospins remain parallel, independent of the coupling strength between them, so that $\cos \gamma(t) = \cos \gamma(0) = 1$. The frequency $\omega$ (which is the same for left and right spin) exhibits precession (fig. 6(a)), which means that each of the pseudospins nutates.

The situation is very different for initially antiparallel pseudospins. First of all, the dynamics of interwell tunneling is of different character and seems to be not very sensitive to initial conditions for $N_1(0)$ (figs. 5(b) and (d)). Second of all, one clearly observes asymmetric features, which we associate with the uncorrelated spin behaviour. This can be also seen from the behaviour of $\omega^{(1)}$ and $\omega^{(2)}$. The black curve denotes the left pseudospin, and red curves correspond to the right pseudospin.
Conclusions and discussion. – We have discussed the problem of an induced interaction between two atomic quantum dots which are embedded between three trapped Bose-Einstein condensates. As it is a multi-parameter problem, it helps to distinguish two different regimes: a weak-coupling limit (when the direct Josephson coupling between the condensates is dominating) and a strong-coupling case, in which the tunneling between the condensates occurs predominantly through the additional channels mediated through the dots.

Our main conclusion is that indeed we clearly observe the signs of an effective interaction, induced between the two dots due to their coupling to the same condensate system. For this interaction to appear, $T$ should be at least of the order of $J$. Interestingly, this induced interaction produces very different effects on the pseudospins, depending on whether they were initially FM or AFM aligned.

It turns out, that initially parallel spins are stable with respect to coupling between the dots and the condensates. They remain parallel even in the strong-coupling limit, while undergoing two-frequency behaviour, as we can judge from the precessions of $\vec{\omega}$.

Initially antiparallel spins perform what we call “breathing” motion, which is characterized by small periodic deviations of the relative spin angle $\gamma$ from $\pi$. The breathing modes, however, do not survive the strong-coupling limit, when the effective interaction between the two spins becomes large. Instead, spins become rather uncorrelated.

A possible experimental setup for the realization of an atomic quantum dot coupled to Bose-Einstein condensates on a microchip is proposed in [8]. The dot is trapped in a tight magnetic potential created by two crossed conductors on a chip. We also note, that in our case the induced correlation between the impurities can be indirectly probed by the condensate dynamics in the strong-coupling limit, as can be seen from fig. 5, where (a)
and (c) differ from (b) and (d) only by initial conditions for the dots.

Our analysis has important consequences for the periodic model of the system under consideration, which is the subject of future research. First of all, in a periodic system one shall be able to locally address and modify the interaction between the neighboring spins just by changing the occupation of each well. Second of all, by changing the coupling between AQDs and condensates (or optical lattice sites), one can control the different phase transitions in such a coupled system. For example, if an optical lattice was initially in the Mott state, a finite $T$ should drive the system to a superfluid state once some critical value is exceeded. At the same time spins of the AQDs will become correlated, and the question will be if this correlation is associated with a phase transition.

In the future it will be definitely worth studying quantum effects in such a model, as well as temperature effects, which would lead to decoherence phenomena and temperature-dependent phase transitions.

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