CLUMPY ACCRETION ONTO BLACK HOLES. I. CLUMPY-ADVECTION-DOMINATED ACCRETION FLOW STRUCTURE AND RADIATION

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ABSTRACT

We investigate the dynamics of clumps embedded in and confined by the advection-dominated accretion flows (ADAFs), in which collisions among the clumps are neglected. We start from the collisionless Boltzmann equation and assume that interaction between the clumps and the ADAF is responsible for transporting the angular momentum of clumps outward. The inner edge of the clumpy-ADAF is set to be the tidal radius of the clumps. We consider strong- and weak-coupling cases, in which the averaged properties of clumps follow the ADAF dynamics and are mainly determined by the black hole potential, respectively. We propose the analytical solution of the dynamics of clumps for the two cases. The velocity dispersion of clumps is one magnitude higher than the ADAF for the strong-coupling case. For the weak-coupling case, we find that the mean radial velocity of clumps is linearly proportional to the coefficient of the drag force. We show that the tidally disrupted clumps would lead to an accumulation of the debris to form a debris disk in the Shakura–Sunyaev regime. The entire hot ADAF will be efficiently cooled down by photons from the debris disk, giving rise to a collapse of the ADAF, and quench the clumpy accretion. Subsequently, evaporation of the collapsed ADAF drives resuscitate of a new clumpy-ADAF, resulting in an oscillation of the global clumpy-ADAF. Applications of the present model are briefly discussed to X-ray binaries, low ionization nuclear emission regions, and BL Lac objects.

Key words: accretion, accretion disks – black hole physics – hydrodynamics

1. INTRODUCTION

Accretion onto black holes is an energy source for various kinds of celestial high-energy objects. Radiation hydrodynamics of the accretion has been well established and is known as the standard accretion disk model (Shakura & Sunyaev 1973), the slim accretion disk (Abramowicz et al. 1988; Wang & Zhou 1999; Wang & Netzer 2003), and the advection-dominated accretion flows (ADAFs; Narayan & Yi 1994) in light of dimensionless accretion rates. These models are widely applied; however, it is not clear yet to what extent the known models represent a realistic description of the observed phenomena. Moreover, it should be noted that these models are based on continuous fluid with radiation fields, whereas the continuous disk is undergoing the thermal, viscosity, or photon bubble instabilities. Clearly, the popular treatment of accretion disks as continuous fluid only holds as a zeroth-order approximation.

We are motivated by both theoretical and observational facts that recently accretion onto black holes is clumpy rather than homogeneously continuous. Instabilities of the radiation-pressure-dominated regions driven by thermal (Krolik 1998), magnetorotational (Blaes & Socrates 2001, 2003), and photon bubble instabilities (Gammie 1998) create cold clumps in the disk, forming a multiphase medium around the black hole. As a simple and general case, the two-phase disk-corona model has been suggested for many years (e.g., Galeev et al. 1979; Haardt & Maraschi 1993; Mayer & Pringle 2007). More generally, a clumpy disk has been suggested for many years in light of the X-ray properties of X-ray binaries and active galactic nuclei (AGNs; Guilbert & Rees 1988; Celotti et al. 1992; Collin-Souffrin et al. 1996; Kuncic et al. 1997; Celotti & Rees 1999; Yuan 2003; Lawrence 2011). Recently, low-luminosity AGNs (LLAGNs), presumed to be powered by the ADAF, show components of big blue bumps like brighter AGNs and quasars (Barvainis 1993; Maoz 2007; but see Ho 2008 for a re-view), implying that there are cold matters in the hot flows. There are motivated arguments for the existence of clumpy disks in both AGNs (Kuncic et al. 1996; Kumar 1999), including LLAGNs (Celotti & Rees 1999) and X-ray binaries (Malzac & Celotti 2002; Merloni et al. 2006; Chiang et al. 2010). Similar to the LLAGNs, some X-ray binaries show broad Kα components in the low states (Miller et al. 2006a, 2006b; Tomskick et al. 2008; Reis et al. 2009, 2010). Most of the previous efforts focus on the internal state of clumps and their reprocessing properties (Guilbert & Rees 1988; Celotti et al. 1992; Kuncic et al. 1996, 1997; Malzac & Celotti 2002; Merloni et al. 2006; Barai et al. 2011); however, the dynamics of clumps in disks is not fully understood.

On the other hand, fates of the clumps embedded in accretion flows are poorly known when they are approaching the black hole. They would be tidally disrupted by the hole, of which the captured debris is eventually accreted onto the hole. Unlike the case of a black hole capturing stars, the capture rates of clumps are so fast that the debris of disrupted clumps are accumulating with time. In this paper, we show that the emission from the accretion of debris can efficiently cool the hot ADAF, leading it to collapse, and quench the clumpy accretion. Being triggered through α-viscosity or evaporation, the collapsed ADAF (cADAF) revives as a new clumpy-ADAF. This is a cycle between clumpy-ADAF and the cADAF, which is driven by the clumps. Radiation from the clumpy-ADAF shows interesting temporal properties.

In this paper, we presume that a clumpy structure in the ADAF has been formed through some mechanisms listed above in the ADAF, or perhaps it is produced in the transition regions between the ADAF and the Shakura–Sunyaev disk (SSD). Collisions among the clumps can be neglected in the present case. The goal of this paper is to derive the dynamical equations of the clumpy-ADAF, and we fortunately obtain the analytical solution of the clump dynamics in the ADAF. We find that
the scaling relation of ADAF, we have its bolometric luminosity above the disks, making the ionization parameter lower than that in SSD.

In the self-similar ADAF model, the presence of the debris disk is driving the global clumpy-ADAF to oscillate. The model is briefly applied to X-ray binaries and LLAGNs.

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2. ASSUMPTIONS AND DYNAMICAL EQUATIONS

2.1. Basic Assumptions

Figure 1 shows the regimes of accretion disk models. We simply note that slim disks have \( m_s > 1 \), the standard model of SSD works between \( 1 \sim m_s > m_{c1} \), and accretion flows become advection-dominated when \( m_{c2} < m_s \approx 0.1\alpha_1^{3/4} \), where \( \alpha = \alpha_k/0.3 \) is the viscosity parameter of the ADAF, \( m = M/M_{\text{Edd}} \), \( M_{\text{Edd}} = L_{\text{Edd}}/\eta c^2 = 1.39 \times 10^{18} \eta_{0.1} m_s g^{-1} \), \( c \) is the light speed, \( \eta_{0.1} = \eta/0.1 \) is the radiative efficiency, \( L_{\text{Edd}} = 1.25 \times 10^{38} m_s \) erg s\(^{-1} \) is the Eddington luminosity, and \( m_s = M/M_0 \) is the black hole mass. When the accretion rates are low enough \( (m < m_s) \), flows become a pure ADAF without clumps.

The critical accretion rate \( \dot{m}_1 \) can be roughly estimated from the ionization parameter, which is defined as \( \Xi = L/4\pi R^2 \times k_B T \), where \( k_B \) is the Boltzmann constant, \( L \) is the radiated luminosity, \( R \) is the distance to the ionizing source, \( n \) is the density, and \( T \) is the temperature of the cold clumps. The self-similar solution of the simple ADAF model gives the thermal pressure \( P_A = 1.7 \times 10^{10} \chi^{1/2} c_3^{1/2} m_s^{-1/2} \) g cm\(^{-2} \), where \( c_1 = (5 + 2\epsilon') g/3a^2, c_2 = [2(5 + 2\epsilon') g/9a^2]^{1/2}, c_3 = (5 + 2\epsilon') g/9a^2, g = \{1 + 18(5 + 2\epsilon')^2 \}^{1/2} - 1, \epsilon' = (5/3 - \gamma)/f(\gamma - 1), \gamma \) is the adiabatic index, and \( f \) is the advection-dominated factor (Narayan & Yi 1994). In this paper, we use \( c_1 = 0.46, c_2 = 0.48, \) and \( c_3 = 0.31 \) for \( \gamma = 1.4 \) and \( f = 0.9 \) unless we note their specific values. \( \epsilon' \) results in this paper are not very sensitive to the values of \( \gamma \) and \( f \). Using the scaling relation of ADAF, we have its bolometric luminosity \( L_{\text{ADAF}} = 0.2(m/\alpha)^2 L_{\text{Edd}} \) (Mahadevan 1997). Here we only use the single temperature of the ADAF model. The ionization parameter defined by \( \Xi = L/4\pi R^2 c P_\lambda \) is used to describe the two-phase medium, where \( P_\lambda \) is the internal pressure of the clumps. We assume that the clumps hold a pressure balance with the ADAF \( (P_\lambda = P_\lambda) \), yielding the critical accretion rate as

\[
\dot{m}_1 = 0.02\alpha_1^{3/2} \Xi_0^{1/2} 10^{-1/2}.
\]

where \( \Xi_0 = \Xi/0.1 \) and \( r_{\text{iso}} = R/1000R_\text{Sch} \). For cases with \( 10 > \Xi > 0.1 \) (Field 1965; Krolik 1998), the ionized gas holds a two-phase state with two different temperatures and a pressure balance between the hot and cold mediums. For gas with \( \Xi > 10 \) only a hot phase exists, whereas with \( \Xi < 0.1 \), only a cold phase exists. The timescale of the thermal instability is generally given by the line-cooling process, which determines the formation timescale of the clumps. With the cooling function (Börhringer & Hensler 1989), we have \( \Delta t_{\text{cl}} \sim n \theta_k T/2 L_{\text{Edd}} = 7 \times 10^5 n_{10}^{-1} T_2^{-3.3} \) s for a medium with one solar abundance. For an ADAF of \( 1 M_\odot \) black hole, clumps could be produced at the interacting regions between the ADAF and the SSD, namely the evaporation region, where \( \Delta t_{\text{cl}} \) will be much shorter than that of the Keplerian rotation. A detailed analysis is needed to show the production of clumps through thermal instability. Figure 2 shows a schematic of the clumpy-ADAF model.

We would like to point out that Kuncic et al. (1996) and Celotti & Rees (1999) present further arguments to support the general existence of cold clumps in the accretion disk. This lower limit (Equation (1)) for the clumpy-ADAF is only based on the thermal instability and is regarded as a characteristic critical value. The limit of the critical accretion rate could become lower if the magnetorotational instability (MRI) is included in the ADAF. Furthermore, we presume that the ADAF part of the global clumpy-ADAF can be described by the self-similar solution. We do not consider the effects of ADAF-driven outflow on the clumps known as advection-dominated inflow and outflows (ADIOs; Blandford & Begelman 1999). However, it would be very interesting to postulate a situation in which the clumps could dynamically follow the outflows, forming clumpy outflows. ADIOs with clumps as a potential scenario will be considered in the future (we acknowledge the referee for bringing this point to our attention).

2.1.1. Clumps in the Clumpy-ADAF

The existence of cold clumps can be simply justified by the thermal instability. Although a detailed analysis is beyond the scope of this paper, we can use simplified arguments to grasp the essentials here. The maximum size of the clumps is determined by the crossing distance of a sound wave within one Keplerian timescale; otherwise, the clumps are actually like a ring. This yields \( R_{\text{cl}} \sim c_s T_{\text{Kepler}} \approx 4.4 \times 10^{36} m_s T_4^{-1/2} r_{10}^{-1} \) cm at radius \( R \), where the sound speed \( c_s = 10^4 T_4^{-1/2} \) cm s\(^{-1} \), \( r_{10} = R/10R_\text{Sch} \), and \( T_4 = T_4/10^4 K \). On the other hand, the minimum size of the clumps is determined by the thermal conduction, below which the clumps will be evaporated.

In the self-similar ADAF model, \( \gamma = 4/3, 5/3 \) depends on the magnetic field density. Actually, the numerical solutions of the ADAF avoid the \( \gamma \neq 5/3 \) case (Mannote et al. 1997). For simplicity, we choose an intermediate value of \( \gamma = 1.4 \) in the calculations.

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\( ^4 \) The clumpy-SSD shows a different relation of the ionization parameter with the distance to the black hole. Since radiation pressure dominates in the inner regions, we have \( \Xi \propto n_{10}^{-1/2} \) where the radiation pressure \( P_{\text{rad}} \propto n_{10}^{-1} r_{10}^{-3/2} \). Details of the clumpy-SSD will be carried out in a forthcoming paper.

\( ^5 \) The clumps will have dynamical feedback to the ADAF, but the reprocessed emission could also significantly cool the hot ADAF. We neglect these effects in this paper. Full treatments of the problem should couple clump equations with the ADAF.
Therefore, the smaller eddies have an energy density like thermal conduction, the conduction rates are comparable in size to the clumps have smaller kinetic energies instead, we use the mass ratio defined as $\eta = M_\text{cl}/M_\text{ADAF}$ (see Equation (33)) as a free parameter, where $M_\text{cl}$ is the total mass of clumps and $M_\text{ADAF}$ is the total mass within the outer boundary of the ADAF in the present model. The mass of cold clumps in the ADAF is assumed to be comparable with the ADAF; otherwise, no significant effects can be created (see Section 4 for further discussion).

When a black hole has relatively low accretion rates, the inner part of the SSD becomes optically thin, forming the so-called hybrid disk (Shapiro et al. 1976; Wandel & Liang 1991). The ADAF, as the inner region of the SSD with relatively low accretion rates, then begins to develop from this radius. Actually, the cold disk will be evaporated by the hot corona to form the truncated disk and, namely, to form an ADAF that starts from the evaporation radius (Meyer & Meyer-Hofmeister 1994; Lu et al. 2004), where the evaporation rates are equal to the accretion rates. In this paper, we take the evaporation radius as the outer boundary radius of the clumpy-ADAF; $R_\text{out} = R_\text{evap} = 10^3 R_\text{Sch}$ (Liu & Taam 2009). The total mass of the ADAF can be simply estimated by $M_\text{ADAF} \approx 3.2 \times 10^{-2} M_\odot M_\text{Sch}/R_\text{ADAF}$, where $M_\text{Sch} = m/10^{-2}$ and $R_\text{ADAF} = R/1000 R_\text{Sch}$. Therefore, we have about $M_\text{tot} = 2 M_\odot M_\text{ADAF}/M_\text{cl} \sim 6 \times 10^9 M_\odot^{-1} m_{-2}^{-1}$ clumps, indicating that there are a plenty of small dense clumps in the ADAF. The mean distance of the clumps is given by $r = \mathcal{N}^{-1/3} \approx 1.0 R_\text{Sch}$, within $10^3 R_\text{Sch}$, the crossing timescale $\Delta t_\text{cross} = \langle l \rangle/(v_\text{K}^2)^{1/2} \approx 10^4 M_\odot M_\text{Sch}^{-3/2}$ s for a typical value $(v_\text{K}^2)^{1/2} \approx 0.1c$ (see Figure 4). Comparing with the Keplerian timescale $t_\text{K} \approx 10^8 M_\odot R_\text{Sch}^{-3/2}$ s, we find $\Delta t_\text{cross} > t_\text{K}$, indicating that collisions among the clumps can be neglected.

This guarantees the validity of the collisionless Boltzmann equation and its moment equations employed in this paper. This could only work for clumpy-ADAF, whereas collisions would be a key mechanism to transport angular momentum outward in a clumpy SSD.

2.1.2. Interaction Between Clumps and the ADAF

Clumps orbiting around the black hole deviate from the dynamics of the ADAF, which is a radial flow with a sub-Keplerian rotation. The motion of the clumps is controlled by two factors: (1) black hole potential, and (2) drag force arisen by the ADAF. Clumps gain or lose angular momentum through interaction with the ADAF, leading to moving outward or inward, respectively, and creating the velocity dispersion with

| Table 1 |
| --- |
| Values of Cold Cloud Parameters in the Clumpy Disk |

| Stellar Mass Black Hole | Supermassive Black Hole |
| --- | --- |
| $R_\text{cl}$ (cm) | $m_{\text{cl}}$ (g) | $n_{\text{cl}}$ (cm$^{-3}$) | $R_\text{cl}$ (cm) | $m_{\text{cl}}$ (g) | $n_{\text{cl}}$ (cm$^{-3}$) |
| $10^3$ | $4 \times 10^{10}$ | $10^{23}$ | $10^{11}$ | $4 \times 10^{23}$ | $10^{14}$ |

Considering that line cooling dominates in the cold clumps (with a temperature of $\sim 10^5 K$), the cooling rate $\dot{\Lambda}_\text{line} \approx 8.0 \times 10^2 n_{-2}^2 T_4^{-1.3} \text{ erg s}^{-1} \text{ cm}^{-3}$, where $n_{-2} = n_{\text{cl}}/10^{14} \text{ cm}^{-3}$ (see Figure 2 in Böhringer & Hensler 1989). For a Spitzer-like thermal conduction, the conduction rates are $\mathcal{H} = \nabla \cdot (k_\text{S} T^3 \nabla T) \approx k_\text{S} T^3.5/R_\text{cl}^2 \approx 5.4 \times 10^8 T_10^2/R_9^2 \text{ erg s}^{-1} \text{ cm}^{-3}$ (Spitzer 1962). The necessary condition of $\mathcal{H} \approx \dot{\Lambda}_\text{line}$ yields $R_\text{cl}^2 \geq 0.82 \times 10^2 T_10^{-1.7} n_{-2}^{-3} \approx 0.65 \text{ cm}$. According to the pressure balance, the density of clumps is roughly $n_{\text{cl}} = P_A/k_B T_9 = 2.1 \times 10^6 M_\odot^{-1} m_{-2}^{-3} R_9^{-5/2} \text{ cm}^{-3}$. Generally, the size and mass of the clumps could change with the distance to the black hole. For simplicity, we use the typical values of the cloud mass and radius of the clumps: $m_{\text{cl}} = 4\pi n_{\text{cl}} m_p R_\text{cl}^2/3 \approx 4 \times 10^2 g$ and $R_\text{cl} \sim 10^{11} \text{ cm}$ for a supermassive black hole with $10^8 M_\odot$. It should be noted that the Thompson scattering depth of an individual cloud is $\tau_\text{es} = n_{\text{cl}} R_\text{cl}^2/\sigma T_\odot = 6.6 n_{-2} R_9$. On the other hand, the temperature of the clumps could keep a constant value about $10^5 K$ in light of efficient line cooling in the range of temperature $\sim 10^5 K$ (Sutherland & Dopita 1993). Otherwise, the clumps will be evaporated by the surrounding medium. We take an approximately constant temperature of the clumps. Table 1 gives the values of typical clumps for stellar and supermassive black holes.

Turbulence excited by the MRI is responsible for the transfer of the angular momentum of the ADAF, and it interacts with the clumps. However, the MRI-turbulence cannot destroy the clumps in light of the energy argument. The energy density of the MRI-turbulence is about $\sim a P_A$, of which $P_A$ keeps balance with the thermal energy density of the clumps ($P_\text{cl}$); we use $a P_A < P_\text{cl}$ since $a < 1$. The MRI-turbulence is only a small disturbance to the clumps. On the other hand, the clump size is much smaller than the typical length of the turbulence ($a H_\text{Turb} \sim a R$). Furthermore, turbulent eddies that are comparable in size to the clumps have smaller kinetic energies according to the Kolmogorov’s law as $E_\text{Turb} \propto k^{-5/3}$ (Landau & Lifshitz 1959), where $E_\text{Turb}$ is the energy per unit wavelength number of turbulence, $k = 1/\lambda$, and $\lambda$ is the length of turbulence. Therefore, the smaller eddies have an energy density $\ll a P_A$, which is only a tiny fraction of clumps. It is, thus, a good approximation that clumps are simplified as particles, which can be described by the Boltzmann equation.

The total mass of cold clumps should be self-consistently determined by an analysis of global thermal instability, but, instead, we use the mass ratio defined as $\eta = M_\text{cl}/M_\text{ADAF}$ (see Equation (33)) as a free parameter, where $M_\text{cl}$ is the total mass of clumps and $M_\text{ADAF}$ is the total mass within the outer boundary of the ADAF in the present model. The mass of cold clumps in the ADAF is assumed to be comparable with the ADAF; otherwise, no significant effects can be created (see Section 4 for further discussion).

Figure 2. Schematics of clumpy-ADAF. Left panel: a tiny debris disk forms through an accumulation of the tidally disrupted clumps by the black hole within the tidal radius ($R_\text{tidal}$). The disk can be accumulated up to the Shakura–Sunyaev regime. We set the tidal radius as the inner edge of the clumpy-ADAF, which is the outer radius of the debris disk. The radiation from the debris disk has strong feedback to the hot ADAF through Compton cooling, giving rise to a collapse of the ADAF that quenches the clump accretion. Right panel: collapsed clumpy ADAF (cADAF). States of clumpy-ADAF transit to a cADAF through the debris disk, and resume ADAF through disk evaporation. This leads to a kind of quasiperiodical oscillation of the global accretion flows. The fates of the clumps remain open: they either totally disappear, are orbiting around the black hole, or collide with the cADAF. See the text for details.
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The angular momentum of the clumps is then carried away by the ADAF, in which α-viscosity is responsible for the transfer of the ADAF’s outward momentum. In this paper, we use the drag force as $F_R = f_R(v_R - v_R^*)^2$ and $F_\phi = f_\phi(v_\phi - V_\phi)^2$, where $V_R$ and $V_\phi$ are the radial and $\phi$-velocity of the ADAF, respectively, and $f_R$ and $f_\phi$ are two coefficients (Mathews 1990; Cinzano et al. 1999). In principle, we should use velocities of the MRI-turbulence to estimate the drag force.

The drag force employed here is an approximation of the drag force in laminar flows. The drag force can actually be expressed by $F_{R,\phi} = f_{R,\phi}(v_{R,\phi} - (V_{R,\phi}^* + \sigma_{R,\phi,cl}^2)^{1/2}) = f_{R,\phi}(v_{R,\phi} - \beta_{R,\phi}VR^2)$, where $\beta_{R,\phi} = [1 + (\sigma_{R,\phi,cl}^2 / VR^2)]^{1/2}$, $\sigma_{R,\phi}$ are the turbulent velocities, and the subscripts represent the $R$- and $\phi$-directions, in turbulent flows. Since $V_{R,\phi}^* \gtrsim \alpha^2 \sigma_{R,\phi}^2$, we have $\beta_{R,\phi} \approx (1 + \alpha)^{1/2}$. Considering $\alpha < 1$, it would be a good approximation for us to use the laminar drag force ($\beta_{R,\phi} = 1$ in this paper).

The two coefficients $f_R$ and $f_\phi$ can be approximately taken as constants, which are independent of the ADAF density and the distance to the black hole. Clumps are undergoing contraction along with spiraling-in. For a simple estimation, we have $n_{cl}R_{cl}^3 = n_{cl,0}R_{cl,0}^3$, where the subscript “0” indicates the initial value (i.e., at the outer boundary) if the individual clumps keep their mass. With the help of the pressure balance with the ADAF, we have $R_{cl} = (P_{\Lambda} / P_{\Lambda,0})^{1/3} R_{cl,0}$. Since the coefficients $f_{R,\phi} = n_{cl} / (n_{cl,0}R_{cl,0}) = (T_{cl} / T_{\Lambda,0})^{1/3} R_{cl,0}$, we have $F_{R,\phi} = (P_{\Lambda} / P_{\Lambda,0})^{1/3} R_{cl,0}^{1/3} T_{cl,0}^{-1}$. Considering that the contraction makes the clumps a little bit hotter (efficiently being cooled by radiation), we assume $T_{cl} \propto R^{-1}$ and $\zeta \ll 1$.

We obtain $f_{R,\phi} \propto R^{1-(1/6)} R_{cl,0}^{1/3} \propto R^{1/3} R_{cl,0}^{-1}$ if $\zeta = 0.2$. Without more details of $\zeta$, we find that the coefficients are not sensitive to the distance to the black holes. Therefore, we use the coefficients as two constants in this paper.

2.1.3. Inner Edge of the Clumpy-ADAF: Tidal Disruption

The orbiting clumps are suffering from the tidal disruption governed by the black hole. Self-gravitation of clumps is negligible; however, clumps still survive by keeping a pressure balance with the surroundings if the tidal distortion can be overcome by the thermal pressure of the ADAF. The tidal force reads $F_{\rm tidal} \approx GM_m R_{cl} / R^2$ for a cloud, where $G$ is the gravitational constant and $R$ is the distance to the hole. This tidal force is balanced by the thermal pressure of the ADAF, namely $F_{\rm tidal} = 4\pi P_{\Lambda} R_{cl}^2$. Since the clumps keep a pressure balance with the ADAF, we have $P_{\Lambda} = n_{cl}k_BT_{cl}$, where $n_{cl} = m_{cl} / (4\pi/3)R_{cl}^3$ is the mass density of clumps and $k_B$ is the Boltzmann constant. Tidal disruption happens when $F_{\rm tidal} \gg 4\pi n_{cl}k_BT_{cl}R_{cl}^2$, yielding a natural limit of the inner boundary radius

$$R_{\rm in} = \left( \frac{GM_m m_{cl} R_{cl}^2}{4\pi k_B T_{cl}} \right)^{1/3} \approx 8.0 M_8^{2/3} R_{11}^{2/3} T_{4}^{-1/3} R_{sch},$$

(2)

where $M_8 = M_\ast / 10^8 M_\odot$, $M_1 = M_\ast / 1 M_\odot$, $R_{11} = R_{cl,0} / 10^{11}$ cm, $R_3 = R_{cl} / 10^3$ cm, and $T_{4} = T_{cl,0} / 10^4$ K. Here we set the temperature of clumps $T_{cl} = 10^4$ K. Within the tidal radius, clumps are destroyed to form a disk of debris or are mixed with the ADAF. Some papers have studied the formation of the accretion disk after the tidal disruption of stars by supermassive black holes (e.g., Cannizzo et al. 1990; Strubbe & Quataert 2009). Investigating the detailed processes of destroyed clumps to form an accretion disk is not the main goal of this paper; however, we focus on its influence on the ADAF. Following popular treatment, we assume that the tidal radius is the location of the disk after the captured debris’ orbit has been circularized.

We should stress that we use the parameters of the clumps at the tidal radius throughout the entire ADAF even though the clumps are undergoing contraction. Collisions are neglected for the clumpy-ADAF; thus, the contraction is not important in the present situation. We use the values of the parameters listed in Table 1 throughout the ADAF. Clumps could vary in size and density in the outer parts of the ADAF, in particular, in a standard disk with clumps. In such a case, depending on the size of the clumps, they merge and fragment through collisions. This is, however, beyond the scope of this paper.

2.2. Collisionless Boltzmann Equation

By defining the distribution function as $f = \Delta N / R \Delta R \Delta x \Delta \phi \Delta v$, the dynamics of the clumps can be generally described using the Boltzmann equation. Unlike the normal stellar system, the clumps are moving in the supermassive black hole potential, but they are also being dragged by the ADAF. We start from the origin of collisionless Boltzmann equation (4-11) in Binney & Tremaine (1987)

$$\frac{\partial f}{\partial t} + \sum_{a=1}^{6} \frac{\partial (f v_a)}{\partial v_a} = 0,$$

(3)

where $(x, v) \equiv w$ and $w \equiv (x, v)$ are the coordinates in the phase space. Generally, for a stellar system, $\sum_{a=1}^{6} \partial w_a / \partial v_a = 0$ holds. The Boltzmann equation reduces to $\partial f / \partial t + \sum_{a=1}^{6} \partial f / \partial v_a \partial v_a = 0$. The drag force on the clumps depends on the velocity, making the Boltzmann equation complicated. Considering the dependence of acceleration of the clumps on their velocity, we recast the Boltzmann equation, so that

$$\frac{\partial f}{\partial t} + \sum_{i=1}^{3} \left( \frac{v_i}{\partial x_i} \frac{\partial f}{\partial v_i} + \frac{\partial f}{\partial v_i} \frac{\partial v_i}{\partial v_j} \right) = 0.$$  

(4)

The third term in the bracket arises from the drag force, which disappears in a conservative system as described by Equation (4-13a) in Binney & Tremaine (1987). In a cylindrical coordinate, we have

$$\frac{\partial f}{\partial t} + R \frac{\partial f}{\partial R} + \phi \frac{\partial f}{\partial \phi} + \frac{\partial f}{\partial z} + v_R \frac{\partial f}{\partial v_R} + v_\phi \frac{\partial f}{\partial v_\phi} + v_z \frac{\partial f}{\partial v_z} + R \frac{\partial f}{\partial v_R} \frac{\partial v_R}{\partial v_R} + \frac{\partial f}{\partial v_\phi} + \frac{\partial f}{\partial v_z} = 0,$$

(5)

where $R = v_R$, $\phi = v_\phi / R$, and $z = v_z$. Motion equations of an individual cloud read

$$\dot{v}_R = -\frac{\partial \Phi}{\partial R} + \frac{v_R^2}{R} + F_s; \dot{v}_\phi = -\frac{1}{R} \frac{\partial \Phi}{\partial \phi} - \frac{v_R v_\phi}{R} + F_\phi;$$

$$\dot{v}_z = -\frac{\partial \Phi}{\partial z},$$

(6)

where $\Phi = GM_m / (R^2 + z^2)^{1/2}$, $F_s = f_R(v_R - V_s)^2$, and $F_\phi = f_\phi(v_\phi - V_\phi)^2$ are the drag forces per unit mass in the
$R$- and $\phi$-directions, respectively, and $V_R$ and $V_\phi$ are the radial and rotational velocities of the ADAF. It should be noted that $f_R$ and $f_\phi$ might be functions of the cloud’s parameter, such as radius and density. Here we neglect the drag forces in the $z$-direction. We have \( \sum_{\alpha=1}^{6} \partial \tilde{w}_\alpha / \partial w_\alpha = 2 f_\phi(V_\phi - V_\phi) + 2 f_R(v_R - V_R) \) in the cylindrical coordinate frame. Considering the $\phi$-symmetry of the clumpy disk, we have

$$
\frac{\partial \mathcal{F}}{\partial t} + v_R \frac{\partial \mathcal{F}}{\partial R} + v_\phi \frac{\partial \mathcal{F}}{\partial z} + \left( \frac{v_\phi^2}{R} - \frac{\partial \Phi}{\partial R} + F_\phi \right) \frac{\partial \mathcal{F}}{\partial v_R} + \left( \frac{v_R v_\phi}{R} \right) \frac{\partial \mathcal{F}}{\partial v_\phi} - \frac{\partial \Phi}{\partial z} + 2 \mathcal{F} [f_\phi(v_\phi - V_\phi) + f_R(v_R - V_R)] = 0.
$$

(7)

This is the final version of the Boltzmann equation used for the dynamics of the clumps in the following sections.

### 2.3. Moment Equations

Following the popular treatment, we solve the moment equations of the Boltzmann equation. We define the averaged parameter in velocity-space as \( \langle X \rangle = \mathcal{N}^{-1} \int X \mathcal{F} d\mathbf{v} \), where $X$ is a parameter. Here we use $\mathcal{N} = \int N d\mathbf{v}$. The zeroth-order moment equation can be obtained by integrating Equation (7):

$$
\frac{\partial \mathcal{N}}{\partial t} + \frac{1}{R} \frac{\partial}{\partial R} (R \mathcal{N}(v_R)) + \frac{\partial}{\partial z} (\mathcal{N}(v_z)) = 0.
$$

(8)

The first-order moment equations can be gained through multiplying Equation (7) by $v_R$, $v_\phi$, and $v_z$, respectively, and integrating the equations in velocity-space. The first moment equation is given by

$$
\frac{\partial}{\partial t} \mathcal{N}(v_R) + \frac{1}{R} \frac{\partial}{\partial R} (R \mathcal{N}(v_R)) + \frac{\partial}{\partial z} (\mathcal{N}(v_z)) + \mathcal{N} \left( \frac{v_\phi^2}{R} - \frac{\partial \Phi}{\partial R} \right) - \mathcal{N} f_R \langle v_R \rangle - 2 \langle v_R \rangle V_R + V_R^2 = 0,
$$

(9)

the second as

$$
\frac{\partial}{\partial t} \mathcal{N}(v_\phi) + \frac{1}{R^2} \frac{\partial}{\partial R} (R^2 \mathcal{N}(v_R v_\phi)) + \frac{\partial}{\partial z} (\mathcal{N}(v_\phi v_z)) + \mathcal{N} \langle v_R^2 \rangle - 2 \langle v_\phi \rangle V_\phi + V_\phi^2 = 0,
$$

(10)

and the third as

$$
\frac{\partial}{\partial t} \mathcal{N}(v_z) + \frac{1}{R} \frac{\partial}{\partial R} (R \mathcal{N}(v_R v_z)) + \frac{\partial}{\partial z} (\mathcal{N}(v_z^2)) + \mathcal{N} \langle v_\phi^2 \rangle + \frac{\partial \Phi}{\partial z} \mathcal{N} = 0.
$$

(11)

We will use these moment equations to discuss the dynamics of the clumpy disk.

For the $\phi$- and $z$-direction symmetric clumpy-ADAF, we have $\langle v_z \rangle = 0$, $\langle v_R v_\phi \rangle = 0$, and $\langle v_\phi v_z \rangle = 0$, and we recast the continuity equation as

$$
\frac{1}{R} \frac{\partial}{\partial R} (R \mathcal{N}(v_R)) = 0,
$$

(12)

yielding the accretion rates of the clumps as $\dot{M}_{cl} = -2 \pi R H \mathcal{N} m_{cl}(v_k)$. The first moment equation is reduced to

$$
\frac{\partial}{\partial R} (\mathcal{N} \langle v_R^2 \rangle) + \mathcal{N} \left( \langle v_\phi^2 \rangle - \langle v_\phi \rangle \right) \frac{\partial \Phi}{\partial R} - \mathcal{N} f_R \langle v_R \rangle - 2 \langle v_R \rangle V_R + V_R^2 = 0,
$$

(13)

the second to

$$
\frac{1}{R^2} \frac{\partial}{\partial R} (R^2 \mathcal{N} \langle v_R v_\phi \rangle) - \mathcal{N} f_\phi \left( \langle v_\phi^2 \rangle - 2 \langle v_\phi \rangle V_\phi + V_\phi^2 \right) = 0,
$$

(14)

and the third to

$$
\frac{\partial}{\partial z} (\mathcal{N} \langle v_\phi^2 \rangle) + \frac{\partial \Phi}{\partial z} \mathcal{N} = 0.
$$

(15)

Table 2 gives a summary of input and output parameters used in the present model. The above moment equations describe the dynamics of clumps; however, these are not close. We must supplement additional physical considerations to proceed. We distinguish two classes of the dynamics in light of the strength of coupling between the clumps and the ADAF. When $f_R$ and $f_\phi$ are large enough, the clumps are strongly coupled with the ADAF so that the average dynamics of the clumps follows the ADAF. For small $f_R$ and $f_\phi$, the clumps are weakly coupled with the ADAF, showing weak dependence on the ADAF.

### 3. STRUCTURE OF THE CLUMPY DISK

#### 3.1. Strong-coupling Case

In the strong-coupling case, the drag force is so strong that the average dynamics of the clumps follow the ADAF, namely $\langle v_z \rangle = V_z$ and $\langle v_\phi \rangle = R \Omega_\alpha$, where $\Omega_\alpha$ is the rotational velocity of the ADAF. So, the factor $f_\phi$ should be larger than a critical one. This can be understood by the fact that both the specific angular momentum and the kinetic energy of the clumps are...
about the same as the gas of the ADAF since they are born in the ADAF. Therefore, we have
\[
\langle v_x \rangle = -2.12 \times 10^{10} \alpha c_1 r^{-1/2} \text{ cm s}^{-1},
\]
\[
\langle v_\phi \rangle = 2.12 \times 10^{10} c_2 r^{-1/2} \text{ cm s}^{-1},
\]
\[
(v_x)/(v_\phi) = -\alpha c_1/c_2,
\]
and the height of the clumpy disk
\[
H_\Lambda = 2.95 \times 10^5 c_1^{1/2} c_2^{-1} m r \text{ cm}.
\]
From the continuity equation, we have
\[
\mathcal{N} = -\frac{\dot{M}_d}{4\pi R H_\Lambda (v_x) m_c},
\]
where \(\dot{M}_d\) is the averaged accretion rate of clumps. From \(\phi\)-motion equation, we have
\[
\langle v_\phi^2 \rangle = \frac{1}{2} \frac{d}{dR} \mathcal{N} f_\phi - 2 \langle v_\phi \rangle V_\phi - V_\phi^2
\]
\[
= -\frac{1}{2} \frac{\langle v_x v_\phi \rangle}{f_\phi R} + 2 \langle v_\phi \rangle V_\phi - V_\phi^2
\]
(20)
and \(\langle v_\phi^2 \rangle/\langle v_\phi \rangle^2 = 1 + \alpha c_1/2 c_2 f_\phi R\). The radial motion is rewritten by
\[
d\langle v_R^2 \rangle/R + \frac{1}{R} \left( 1 + \frac{d \ln \mathcal{N}}{d \ln R} - f_R R \right) \langle v_R^2 \rangle
\]
\[
+ \left( \frac{\partial \Phi}{\partial R} - \frac{1}{R} \langle v_R^2 \rangle + f_R V_\phi^2 \right) = 0,
\]
(21)
where
\[
\frac{\partial \Phi}{\partial R} = \frac{1}{H_\Lambda} \int_0^{H_\Lambda} \frac{\partial \Phi}{\partial R} d\zeta \approx \frac{GM_\bullet}{R^2}.
\]
Considering \(d \ln \mathcal{N}/d \ln R = -3/2\) from Equation (19), we have
\[
d\langle v_R^2 \rangle/R - \frac{1}{R} \left( f_R R + \frac{1}{2} \right) \langle v_R^2 \rangle + R \left( \Omega_R^2 - \langle v_R^2 \rangle \frac{\partial \Phi}{\partial R} \right)
\]
\[
+ f_R V_\phi^2 = 0.
\]
Clearly, the \(\phi\)-motion has a strong influence on the radial motion. Inserting \(\langle v_R^2 \rangle\) and \(\langle v_\phi \rangle\), we have
\[
d\langle v_R^2 \rangle/R - \frac{1}{R} \left( f_R R + \frac{1}{2} \right) \langle v_R^2 \rangle + \left[ (1 - c_2^2) \right] R
\]
\[
- \frac{1}{2} \alpha c_1 c_2 f_\phi^{-1} \Omega_R + f_R V_\phi^2 = 0.
\]
(23)
With the outer boundary condition of \(\langle v_R^2 \rangle = V_{\text{out}}^2\) at \(R = R_{\text{out}}\), we have the solution as
\[
\langle v_R^2 \rangle = e^c \left[ \frac{\Omega_R^2 \Lambda_2^2 (\Gamma_R, r)}{2 \Gamma_\phi} - (1 - c_2^2) \Lambda_2 (\Gamma_R, r) \right]
\]
\[
- \frac{\alpha c_1 c_2}{2 \Gamma_\phi} \lambda_2 (\Gamma_R, r) + \frac{V_{\text{out}}^2 e^{-\Gamma_{\text{out}}} r^{1/2} e^{\Gamma_{\text{out}}}}{c_2^2 R_{\text{out}}^{1/2} e^{\Gamma_{\text{out}}}} \right] r^{1/2} e^{\Gamma_R},
\]
(24)
where \(\Gamma_R = f_R R_{\text{Sch}}, \Gamma_\phi = f_\phi R_{\text{Sch}},\) and the function
\[
\Lambda_\phi (\Gamma_R, r) = \int_r^{r_{\text{out}}} x^{-q} e^{-\Gamma_r} dx,
\]
and \(q = 3/2, 5/2, 7/2\). Figure 3 shows the properties of the function \(\Lambda_\phi\). Equation (24) gives the solution of the clumps, which deviates from the ADAF.

Physical meanings of each term in Equation (24) can be examined under some extreme cases. Generally, clumps are controlled by two factors: (1) black hole potential, and (2) \(\phi\)- and \(R\)-direction drag forces. A drag-free cloud’s, orbit is determined purely by the black hole. When \(f_R\) tends to zero, namely \(\Gamma_R = 0\), clumps are orbiting around the black hole with \(\phi\)-drag. The angular momentum of the clumps is transferred by the ADAF, in which the popular \(\alpha\)-prescription works toward an outward transportation of the angular momentum of the ADAF, giving rise to a fast spiral down to the black hole. In such a case, we have
\[
\langle v_R^2 \rangle = \frac{1}{3} \left[ 1 - c_2^2 \right] \frac{r}{r_{\text{out}}}^{3/2} - \frac{\alpha c_1 c_2}{20 \Gamma_\phi} \left[ 1 - \left( \frac{r}{r_{\text{out}}} \right)^{5/2} \right] \left( \frac{r}{r_{\text{out}}} \right)^{1/2} V_{\text{out}}^2.
\]
(25)
The first term is the orbital motion around the black hole. The second term results in the accordance of cloud motion with the ADAF. The \(\phi\)-drag decreases the velocity dispersion between the clumps and the ADAF. When the \(\phi\)-drag is strong enough, we have the critical value
\[
\Gamma_\phi^* \approx \frac{3 \alpha c_1 c_2}{10 (2 - 2 c_2^2 - 3 \alpha c_1^2) r_{\text{in}}} \approx 8.7 \times 10^{-4} \alpha_{0.2} r_{10}^{-1}.
\]
(26)
Here we neglect the terms of \((r_{\text{in}}/r_{\text{out}})\). When \(\Gamma_\phi = \Gamma_\phi^c\), coupling with the ADAF is so strong that the velocity dispersion of the clumps is zero at the tidal capture radius \(R_{\text{in}}\), namely \((v_{R}^2)^{1/2} = V_{\text{in}}\) at \(R = R_{\text{in}}\), where \(V_{\text{in}}\) is the radial velocity of the ADAF. So, the strong coupling is referred to the case with \(\Gamma_\phi > \Gamma_\phi^c\). Models with \(\Gamma_\phi < \Gamma_\phi^c\) are the weak coupling, so the strong coupling approximation does not work. We will discuss the case below.

For an extremely strong coupling, namely \(\Gamma_\phi \to \infty\), clumps tend to have
\[
\langle v_R^2 \rangle = c^2 \left[ \frac{1}{2} \left( a^2 c_t^2 \Gamma R \Lambda_2 (\Gamma R, r) + (1 - c_t^2) \Lambda_2 (\Gamma R, r) \right) \right] \\
\quad + \frac{V_{\text{out}}^2}{c_t^2 \Gamma R_{\text{out}}^2} \alpha_{t} \Gamma R_{\text{out}} \approx \frac{1}{3} (1 - c_t^2) c_t^2 \\
\quad \times \left[ 1 - \left( \frac{r}{r_{\text{out}}} \right)^{3/2} \right] + \left( \frac{r}{r_{\text{out}}} \right) \frac{V_{\text{out}}^2}{c_t^2} \\
\text{reaching their maximum. Here, the first term with } \Lambda_2 \text{ is always smaller than the others.}
\]

Figure 4 shows the solutions of the clumpy disk for the different parameters of the drag forces. For a fixed \(\Gamma_\phi\) case, the radial drag strongly influences the velocity dispersion of the clumps at large radii. On the other hand, for fixed \(\Gamma R\) cases, \((v_R^2)^{1/2}\) is mainly determined by the \(\Gamma_\phi\) and reaches its maximum as shown in Figure 4. We find that the term involving \(\Lambda_{3/2}\) is always smaller than the other two in Equation (24). This is due to the radial velocity of the ADAF, which is smaller than the rotational, and it is represented by multiplying the factor \(\alpha\). Although the influence of the radial drag can be neglected, the present treatments are complete. The most important point is that \((v_R^2)^{1/2} \sim 10\langle v_R \rangle\) for the extremely strong coupling case from Figure 4. This means that the accretion rates of the clumps are actually enhanced.

### 3.2. Weak-coupling Case

When the birth of the clumps is not very tightly linked with the ADAF in dynamics, the strong coupling between the clumps and the ADAF is relaxed. For cases of weak coupling with the ADAF, the assumptions of \((v_R) = V_{\phi}\) and \((v_\phi) = V_\phi\) do not work, and cloud dynamics resembles the stellar system. We introduce the velocity dispersions \(\sigma_z^2 = \langle v_z^2 \rangle - \langle v_\phi \rangle^2\) and \(\sigma_\phi^2 = \langle v_\phi^2 \rangle - \langle v_\phi \rangle^2\) to solve the moment equations. The moment equations should be closed up by physical considerations. Similar to stellar dynamics, we assume \(\sigma_z^2 = k_R \sigma_z^2\) and \(\sigma_\phi^2 = k_\phi \sigma_\phi^2\), where \(k_R\) and \(k_\phi\) are two constants. Since the clumps are produced by the ADAF in light of thermal instability, the velocity dispersion of the clumps should be less than the sound speed of the ADAF, and the vertical height of the clumpy disk does not exceed the ADAF height. We assume \(H = H_\Lambda\). Employing \(\langle v_\phi \rangle = 0\) and \(\sigma_\phi^2 = \langle v_\phi^2 \rangle\), we have
\[
\sigma_z = H_\Lambda \Omega_K. \tag{28}
\]

After some algebraic manipulations, we recast Equations (17) and (18) as
\[
\left( \langle v_\phi \rangle^2 - \sigma_\phi^2 \right) \frac{d}{dR} \langle v_\phi \rangle - \langle v_\phi \rangle^2 + \frac{\sigma_\phi^2}{R} + \frac{\sigma_z^2}{R} + \frac{\partial \Phi}{\partial R} = 0 \tag{29}
\]
and
\[
\langle v_\phi \rangle \frac{d}{dR} \langle v_\phi \rangle + \langle v_\phi \rangle - f_\phi \sigma_\phi^2 + \langle \langle v_\phi \rangle - V_\phi \rangle^2 = 0. \tag{30}
\]

where Equation (19) is used.

For a weak-coupling case, the clumps are mainly orbiting around the black hole. The first term with \(d\langle v_\phi \rangle/dR, \sigma_\phi^2/R\) and \(d\sigma_z^2/R\) are of the same order, but the term is much smaller than \(\langle v_\phi \rangle^2/R\) and \(\partial \Phi/\partial R\). So, we have
\[
\langle v_\phi \rangle \approx - \left( \frac{\partial \Phi}{\partial R} \right) = V_K. \tag{31}
\]
where \( V_K \) is the Keplerian velocity. Since \( \langle v_{\phi} \rangle \approx V_K \), we have \( \sigma_\phi \approx 0 \) and \( dv_{\phi}/dR \approx -v_{\phi}/2R \), and Equation (30) yields
\[
\langle v_\phi \rangle = \frac{2f_0 R}{V_K} (V_K - V_{\phi})^2 = 7.0 \times 10^4 \Gamma_{-5}^{-1/2} \text{ cm s}^{-1},
\] (32)
where \( \Gamma_{-5} = \Gamma/10^{-5} \). The mean velocity of the clumps is linearly proportional to the factor \( f_0 \). The velocity dispersions \( \langle v_{\phi}^2 \rangle = \sigma_\phi^2 + \langle v_{\phi} \rangle^2 \) and \( \langle v_R^2 \rangle = \sigma_R^2 + \langle v_R \rangle^2 \) are known only if \( k_R \) and \( k_\phi \) are also known; however, the present model cannot determine \( k_R \) and \( k_\phi \) in a self-consistent way. The two parameters should be constrained by observations. The weak-coupling case does not have significant observable effects; therefore, we do not use it further.

As a brief summary, the dynamics of the clumps are very different from the ADAF both for strong- and weak-coupling cases between the clumps and the ADAF, but the dynamical properties of the clumps depend on the ADAF. The prominent properties are the velocity dispersion of clumps is one order higher or much smaller than the radial velocity of the ADAF in strong- and weak-coupling cases, respectively. For weak-coupling cases, solutions with the clumps weakly coupled with the ADAF show that their properties are similar to stars in the black hole potential field. These properties of the strong-coupled clumpy-ADAF determine the variabilities of the accretion disk of black holes.

Finally, we would like to point out the roles of magnetic fields discussed by Kuncic et al. (1996). For magnetized clumps, magnetic fields might be in equipartition with thermal pressure. The magnetic field plays a role in resisting the MRI-turbulence, making the existence of the clumps more persistent. Furthermore, magnetic fields lower the thermal conductivity (Spitzer 1962); thus, they create a more robust clump. In reality, this may be much more complicated. Future numerical simulations may be able to uncover more details into the roles of magnetic fields in the clumpy-ADAF.

4. RADIATION: OBSERVATIONAL APPEARANCES

Radiative properties of the clumps in AGNs and X-ray binaries have been extensively studied by several authors (Kuncic et al. 1997; Celotti & Rees 1999; Malzac & Celotti 2002; Merloni et al. 2006). Much attention is given to the reprocessed emission from the clumps; emission lines are the main features. In such a case, profiles of emission lines could be broadened by the motions of the clumps, and they can be calculated through the method used by Whittle & Saslaw (1986). This paper investigates a case in which the debris of the tidally disrupted clumps accumulates with time until a transient disk of the debris forms around the black hole. We show that the debris disk plays a key role in the radiation of the global accretion flows, and, in particular, in the feedback to the ADAF driven by the debris disk, leading to a kind of quasiperiodical oscillation of the flows.

4.1. Capture Rates of Clumps

Considering the mass of clumps \( \Delta M_{\text{cl}} = 4\pi R H_A \rho m_{\text{cl}} dR \) within \( R \) to \( R + \Delta R \), whereas the gas mass of the ADAF is \( \Delta M_A = 4\pi R H_A \rho A dR \), we find that the mass fraction of the clumps to the ADAF is
\[
\mathfrak{m} = \frac{\Delta M_{\text{cl}}}{\Delta M_A} = \frac{M_{\text{cl}}}{M_A},
\] (33)
where \( M_A \) is the accretion rate of the ADAF. In principle, the parameter \( \mathfrak{m} \) should be self-consistently determined using the cloud formation model, but this is beyond the scope of this paper. Instead, we treat it as a free parameter in the model. The capture rates are given by the number of clumps that enter the tidal radius per unit time. Considering a time interval of \( \Delta t \), we include the number of captured clumps through the surface of the tidal radius as \( \Delta M_{\text{cl}} = 2\pi R \dot{m} m_{\text{cl}} \Delta \langle v_R \rangle^{1/2} \Delta t \), where \( \dot{m} \) and \( \langle v_R \rangle \) are the height of the clumpy disk and the clump number density at the tidal radius, respectively. Therefore, the capture rates are \( \dot{R} = \lim_{\Delta t \rightarrow 0} (\Delta M/\Delta t) \)
\[
\dot{R} = \frac{\delta m_{\text{cl}} \dot{M}_{\text{edd}}}{m_{\text{cl}}} \approx \begin{cases} 1.4 \times 10^7 n_{0,1}^{-1} \delta_1 \mathfrak{m} m_{-2} m_{10}^{-1} M_1 \text{ s}^{-1}, \\ 3.6 \times 10^9 n_{0,1}^{-1} \delta_1 \mathfrak{m} m_{-2} m_{22}^{-1} M_8 \text{ month}^{-1}, \end{cases}
\] (34)
where \( \delta = \langle v_R^2 \rangle / \langle v_{\phi} \rangle \), \( \delta_1 = \delta/10 \), and \( m_{-2} = M_A/10^{-2} \dot{M}_{\text{edd}} \). We note that the capture rates are very high since \( \langle v_R^2 \rangle / \langle v_{\phi} \rangle \sim 10 \).

Since the capture timescale \( \dot{R}^{-1} \) is much smaller than the accretion of the debris onto black holes, the captured clumps monotonically accumulate with time. For a single capture event, the tidally disrupted debris has complicated fates, which may resemble the captured stars (e.g., Rees 1988). However, the difference of the present case from the captured star should be noted: the debris of disrupted clumps will interact with the ADAF. The circularization timescale could be a multiple Keplerian timescale orbiting the black hole, possibly mixing with the local ADAF. On the other hand, after a series of captured clumps, a tiny disk of debris will be formed within the timescale of \( \Delta t_{\text{cir}} \) if the clumps take enough kinetic energy. The interaction among the debris of the captured clumps is very complicated; however, it is reasonable to assume that the timescale used to form the debris disk is of the order of \( \Delta t_{\text{cir}} \).

4.2. Fates of the Captured Clumps

Fates of the disrupted clumps depend on the cloud’s properties as well as the density of the ADAF. Similar to the case of the captured star, the debris of a disrupted cloud has a specific kinetic energy \( \varepsilon \sim 3(G M_A/R_p)(R_{\text{cl}}/R_p) \) for clumps approaching the black hole with a parabolic orbit, where \( R_p \) is the pericenter distance of the parabolic orbit (Lacy et al. 1982; Evans & Kochanek 1989). After multiple cycles of the parabolic orbits, the gas flow will be circularized and form a disk. The fallback timescale is given by
\[
\Delta t_{\text{cir}} \approx \begin{cases} 0.7 M_1^{5/2} R_1^{3/5} R_{\text{cl},3}^{-3/2} \text{ s}, \\ 2.2 M_8^{5/2} R_8^{3/5} R_{\text{cl},11}^{-3/2} \text{ yr}, \end{cases}
\] (35)
where \( R_{\text{cl}} = R_{\text{cl}}/3 R_{\text{sch}} \) (e.g., Strubbe & Quataert 2009). Since the debris of the clumps is embedded in the ADAF, the interaction timescale is shorter than that of the circularization; otherwise, the debris will form a disk of its own, called a debris disk. This timescale is actually for the accumulation of the debris disk (\( \Delta t_{\text{acc}} \sim \Delta t_{\text{cir}} \)).
4.2.1. Mixed with the ADAF

The swept mass rates of the debris are \( \dot{M} \sim \pi R^2 c_i \langle v_{\|}^2 \rangle^{1/2} m_p \) and the lost energy rates are \( \dot{E}_K \sim \dot{M} c_i^2 \), where \( c_i \) is the sound speed of the ADAF. The timescale of dissipating the kinetic energy of the debris is given by

\[
\Delta t_{\text{diss}} = \frac{E_K}{\dot{E}_K} \sim \frac{m_p \langle v_{\|}^2 \rangle R_{\text{cl}}}{k_B T c_i \langle v_{\|} \rangle} \approx \left\{ \begin{array}{ll}
1.2 \delta_i \langle v_{\|}^2 \rangle^{1/2} R_3 T_4^{-1} \text{s}, \\
3.8 \delta_i \langle v_{\|}^2 \rangle^{1/2} R_1 T_4^{-1} \text{yr},
\end{array} \right.
\]

where \( \langle v_{\|}^2 \rangle^2 = \langle v_{\|}^2 \rangle / 10^8 \text{ cm s}^{-1} \), and the approximation of the pressure balance between clumps and the ADAF is used. We find that \( \Delta t_{\text{diss}} \sim \Delta t_{\text{min}} \) generally holds for small \( \langle v_{\|}^2 \rangle \) cases, but \( \Delta t_{\text{diss}} \) is significantly longer than \( \Delta t_{\text{min}} \) for large \( \langle v_{\|}^2 \rangle \) cases. This indicates the formation of a debris disk, especially for the very strong coupling case.

When the debris is mixed with the ADAF, the density of the ADAF within the tidal radius will be generally enhanced, so that the cooling of the ADAF increases. Cooling enhancement could be moderate by mixing the ADAF with the captured clumps. We do not focus on this case; instead, we focus on the formation of a debris disk.

4.2.2. Formation of a Debris Disk

As we have shown above, capturing the clumps is much faster than the accretion onto black holes, yielding an accumulation of debris around the hole. Since the orbits of the clumps are in the ADAF, the new disk of the accumulated debris will be inside the ADAF. Detailed formation of the debris disk could be very complicated (more so than the case of the tidally disrupted stars; see Rees 1988; Strubbe & Quataert 2009). Here we assume that the formed disk holds the approximation of radiation-pressure-dominated regions of the SSD since most of the gravitational radiation will be released in this region.

The accretion timescale driven by the viscosity is given by

\[
\Delta t_{\text{debris}} \approx \frac{R_{\text{debris}}}{v} = \alpha^{-1} (H_{\text{debris}}/R)^2 t_\kappa, \quad \text{where} \quad t_\kappa = 1/\Omega_K
\]

is the Keplerian rotation timescale, \( R_{\text{debris}} \) and \( H_{\text{debris}} \) are the typical radius and scale height of the debris disk, respectively, and \( v \) is the kinetic viscosity. Using the SSD solution, we have the accretion timescale

\[
\Delta t_{\text{debris}} = \left\{ \begin{array}{ll}
1.56 \omega_0^{-1} M_1 m_{0.1}^{2.7/2} \text{s}, \\
4.96 \omega_0^{-2} M_1 m_{0.1}^{2.7/10} \text{yr},
\end{array} \right.
\]

where \( \omega_0^{-1} \ll \Delta t_{\text{debris}} \). The accumulation of the debris around the black hole follows the capture of the clumps.

For an interval \( \Delta t \), the accumulated mass of the captured clumps is \( M_\Delta = m_{\text{cl}} \Delta t \), where we neglect the swallowed clumps by the black hole during accumulation. The accretion rate of the debris disk is \( \dot{M}_{\text{debris}} = \Delta M_{\text{cl}} / \Delta t_{\text{debris}} \), and the dimensionless rate is

\[
\dot{m}_{\text{debris}} = \delta \Delta t_{\text{debris}} \frac{\Delta M}{\Delta t_{\text{debris}}} = 0.1 \delta \dot{M} \dot{m}_{0.1} \Delta t_1
\]

where \( \Delta t_1 = \Delta t / 1.0 \Delta t_{\text{debris}} \). We stress that the accretion rates of the debris disk are one order higher than the underlying ADAF, which arises from the fast radial velocity of clumps (\( \delta \sim 10 \) for the strong-coupling case. The storage of the debris is undergoing through capturing the clumps since it carries too much kinetic energy to be directly accreted or mixed with the local ADAF.

It should be pointed out that the above estimation is based on the radiation-pressure-dominated solution of the Shakura–Sunyaev model. The transition radius from gas to radiation-pressure-dominated regions is \( R_{\text{c}} / R_{\text{Sch}} \sim 104.1 (\alpha m_p)^{1/2} (m_{0.1})^{1/16} 21 \). For \( m_{0.1} = 0.1 \), we find that \( R_{\text{c}} \sim 18.0 (\alpha m_p)^{1/2} R_{\text{Sch}} > R_{\text{in}} \) generally holds. This means that the debris disk should be generally radiation-pressure-dominated. Debris disks dominated by gas pressure could still be in the ADAF regime until they reach the SSD regimes. The present estimations are valid.

4.3. Feedback: Collapse of ADAF?

4.3.1. Compton Cooling as Feedback to the ADAF

We show that a debris disk forms within \( \Delta t \sim \Delta t_{\text{debris}} \) in the regime of the Shakura–Sunyaev model. The disk is radiating at a quite large luminosity, \( L_{\text{debris}} = \eta \dot{M}_{\text{debris}} c^2 \sim 10^{45} m_{0.1} M_8 \text{ erg s}^{-1} \), where \( m_{0.1} = m_{\text{debris}}/0.1 \). Photons from the debris disk spanning from optics to UV for supermassive black holes and from UV to \( \lesssim 1.0 \) keV for a few solar mass black holes provide extra sources to cool the hot electrons in the ADAF through Compton cooling with a timescale

\[
\Delta t_{\text{Comp}} = n_e k_B T \left( \frac{\lambda_{\text{Comp}}}{10^9} \right)^{1/2} \text{ms},
\]

where \( \lambda_{\text{Comp}} = 2 \pi k_B T \sigma_T F_{\text{debris}} / m_p c^2 \) is the Compton cooling rate, the energy flux from the debris disk is \( F_{\text{debris}} = L_{\text{debris}} / 4 \pi R^2 \), and \( L_{\text{debris}} = \dot{m}_{\text{debris}} L_{\text{Edd}} \). Using \( \Delta t_{\text{Comp}} = \Delta t_{\text{debris}} \), we have the Compton radius, within which the ADAF is driven by the emergent photons from the debris disk to collapse through Compton cooling

\[
R_{\text{Comp}} = 5447.0 n_{0.2}^{1/2} M_{1000}^{1/4} m_{0.1}^{-1/2} R_{\text{Sch}}.
\]

We find that \( R_{\text{Comp}} > (R_{\text{exp}}, R_{\text{out}}) \), suggesting that the global ADAF will be cooled through Compton cooling. With such a strong feedback, the ADAF collapses into a geometrically thin disk since it has angular momentum. The collapse timescale is mainly controlled by the vertical gravity of the black hole. The collapsing velocity is given by \( v_{\text{th}} = H_{\text{Comp}} \Omega_K \) and the timescale is \( \Delta t_{\text{collapse}} = \Omega_K^{-1} = 0.44 n_{1000}^{1/2} M_1 s = 7.3 n_{1000}^{1/2} M_8 \text{ yr} \). This timescale is much shorter than the dynamical; thus, shocks could be formed during the collapse and could heat the cADAF. In this paper, we neglect this heating, which could be balanced by the cooling of the condensed gas. The collapse stops until the cADAF reaches a new dynamical equilibrium of the SSD with a scale height of \( H_{\text{ADAF}} / R = 4 \times 10^{-3} m_{0.1}^{3/2} M_8 \). In such a case, clumps are then orbiting around the black holes without the ADAF-driven drag if they are bound by the magnetic field. It is also plausible for clumps to collide with the cold disk and then captured by the disk. They could also undergo a fast expansion.

\( \Delta \) The steady ADAF is formed by the balance between gravity heating and cooling (free–free, synchrotron, and inverse Compton scattering). Since the photon fluxes from the debris disk are much larger than the ADAF-generated energy flux (\( L_{\text{debris}} \sim 0.1 L_{\text{Edd}} \) whereas gravity heating \( L_G \lesssim 10^{-2} L_{\text{Edd}} \) in the ADAF itself), the ADAF is rapidly cooled through the Compton cooling without sufficient heating of the released gravitational energy. Esin (1997) discussed the influence of nonlocal radiation on the ADAF, but the present case is different.
and totally disappear since the pressure balance is broken. This feedback gives rise to a quenching of the clumpy accretions. The debris disk is acting as a switch during these processes.

4.3.2. cADAF and Revived Clumpy-ADAF

The cADAF is undergoing two processes: (1) it proceeds the accretion onto black holes at a viscosity timescale, and (2) it may be evaporated by the hot corona from the outer to inner regions. Although the formation of the hot corona on the SSD remains open, we presume here that the two competing processes determine the post-appearance of the cADAF. For the cADAF as an SSD, it is still radiation-pressure-dominated and has a timescale of

\[
\Delta t_{\text{cADAF}} = \begin{cases} 
0.5\alpha_0^{-1} M_{\text{cADAF}}^{-1/2} t_{1000}^{3/2} \text{yr}, \\
5 \times 10^n \alpha_0^{-1} M_{\text{cADAF}}^{-1/2} t_{1000}^{3/2} \text{yr}, 
\end{cases}
\]

(41)

which is much longer than \( \Delta t_{\text{collapse}} \). This indicates that the cADAF has the same accretion rates as the previous ADAF, and it radiates at \( L_{\text{cADAF}} = \dot{m}_{\text{cADAF}} L_{\text{Edd}} \approx 1.25 \times 10^{40} \dot{m}_{0.1}^{-2} M_{8} \text{ erg s}^{-1} \). The disk enters a relatively brighter state than the ADAF. The rising timescale is about \( \Delta t_{\text{debris}} + \Delta t_{\text{comp}} \) from the ADAF state. However, the fate of the cADAF is determined by the competition between the fueling black hole and the evaporating cADAF.

According to numerical calculations (Liu & Taam 2009), the evaporation timescale is given by

\[
\Delta t_{\text{evap}} = \frac{M_{\text{cADAF}}}{\dot{M}_{\text{cADAF}}} \approx 1.45 \alpha_0^{-1} r_{1000}^{3/2} m_{\text{cADAF}}^{-1/2} \text{yr},
\]

(42)

where \( M_{\text{cADAF}} \) is the mass of the cADAF, which is roughly equal to the total mass of the ADAF within \( R_{\text{out}} \), and \( \dot{M}_{\text{cADAF}} \) is the evaporation rate. The estimation simply follows from \( \dot{M}_{\text{cADAF}} \sim 10^{-2} \dot{M}_{\text{Edd}} \), which somehow depends on viscosity. We should stress here that the evaporation timescale does not depend on the mass of the black hole. Comparing \( \Delta t_{\text{cADAF}} \) with \( \Delta t_{\text{evap}} \), we have \( \Delta t_{\text{evap}} = \min(\Delta t_{\text{cADAF}}, \Delta t_{\text{comp}}) \), indicating that evaporation could govern the postappearance of the cADAF in the AGNs, whereas the viscosity of the accretion governs it in X-ray binaries. We point out that accretion onto black holes in the AGNs and X-ray binaries are different in this way. After the interval of \( \Delta t_{\text{tran}} \), the clumpy-ADAF is revived and a new cycle starts.

Figure 5 shows a schematic of the state transition cycle in a black hole clumpy-ADAF. After the time \( \Delta t_{\text{tran}} \), a new ADAF develops, and the object enters a low/hard state, switching on the clumpy accretion. Clumpy-ADAF is at low/hard state, but the cADAF corresponds to the high/soft state since it is an SSD. State transition happens through the feedback of the transient disk of the debris. The cADAF that powers the high state will be brighter than the ADAF at the low/hard state. These processes correspond to the transition of states in black hole X-ray binaries. For LLAGNs, there could be a component seen as a big blue bump in some low ionization nuclear emission regions (LINERs), especially in some LINERs with broad components of emission lines (Ho 2008; Yones et al. 2011, 2012). Furthermore, BL Lac objects have ADAF and some of them show light curves with quasiperiodical modulations, which could be explained by the present model. We stress that the transient disk plays a key role in feedback to the ADAF. The present model predicts a transition of accretion flows. There are two processes of shorter bursts: (1) emission from the debris disk, and (2) cooling of the ADAF through the inverse Compton scattering. The first outburst proceeds to the second, especially it is in the soft band, and the second is a burst in the hard X-ray band. A detailed comparison with observations of the X-ray binaries and AGNs would determine the size of the cADAF and the disk of the debris cloud.

4.4. Discussions

When an accretion flow has low enough rates, it turns into an ADAF through evaporation (Meyer & Meyer-Hofmeister 1994) and will become a clumpy-ADAF with a critical accretion rate of \( \dot{m} \geq 0.02 \alpha_{0.1} \) as shown in Figure 1. Clumps are finally disrupted by the tidal force of the black hole, forming a transient tiny disk. Liu et al. (2007) and Meyer-Hofmeister & Meyer (2011) suggest that an inner disk may be formed through condensations of the ADAF if the accretion rates are in a reasonable range. In principle, this condensation model results from thermal instability, and it is physically equivalent to the clumpy-ADAF model suggested in this paper. However, their model is different from the one presented in this paper for three reasons: (1) the tiny disk is persistent in Liu et al. (2007), but it is transient in the present model; (2) there are only two components (the cold disk and ADAF) in Liu et al. (2007) and many clumps in the present model; and (3) the dynamics are different. As we have shown above, the present model can conveniently explain more observations, besides the observed iron K\( \alpha \) lines in the AGNs (Meyer-Hofmeister & Meyer 2011), such as variabilities of X-ray binaries and radio-loud AGNs.

In the present model, we assume that clumps in the ADAF keep a constant mass and radius before being tidally disrupted. Despite these assumptions, it holds the main features of the clumpy accretion. We may relax some of these assumptions in future studies. Here, we do not discuss the weak-coupling case of the clumpy-ADAF because it may give results without any significant differences from the pure ADAF. On the other hand, if the accretion rates are in the SSD regime, the dynamics of the clumps could be changed into a phase driven by collisions...
among the clumps as well as the drag force in the \( R \)- and \( \phi \)-directions. Observations show that the X-ray regions are partially covered by clumps (Gallo et al. 2004; Ballantyne et al. 2004; Ricci et al. 2010; see a review of Turner & Miller 2009), implying that the clumps in the clumpy-SSD will be much larger than those predicted by the present Clumpy-ADAF model. These contents will be discussed as the main goals of second paper. We treat the outer boundary as the evaporation radius, where the production of clumps is likely happening due to the thermal instability there. This should be issued in more detail in a future paper.

Finally, numerical simulations of the dynamics and radiation of the clumpy-ADAF are worth performing to provide more details like in a dusty torus (Stalevski et al. 2011). We briefly discuss the radiation from the clumpy-ADAF in light of timescales of the undergoing processes. The collapse of the ADAF deals with the release of gravitational energy in the vertical direction, although this energy is smaller than that of the debris disk. The radiated cADAF is simplified as an SSD. This is valid when the collapsed timescale is much shorter than the accretion timescale. Future numerical simulations will uncover the full processes and radiation from the cADAF.

5. OBSERVATIONAL TESTS AND APPLICATIONS

The present clumpy-ADAF model only applies to those black holes that have a steady accretion rate in the ADAF regime with \( m \gtrsim 10^{-2} \) as shown by Equation (1). These accreting black holes could be used to test the predictions of the present model. We briefly illustrate how the present model can be applied to X-ray binaries and LLAGNs to potentially explain the related phenomena, but detailed applications will be given in a separate paper.

5.1. LLAGNs: LINERs and BL Lac Objects

The AGNs well known in the ADAF regime include LINERs (e.g., Ho 2008) and some BL Lac objects (e.g., Wang et al. 2002, 2003). Since the timescale of the transient disk is much longer than that seen in X-ray binaries, the component of the transient disk can be easily observed, but it varies at a timescale of years.

**LINERs.** Evidence has been found for the presence of the significant component of the big blue bump observed in normal AGNs and quasars (Maoz 2007). Pure reprocessing emission from clumps (Figure 1 in Celotti & Rees 1999) is not enough to explain the component since the reprocessing emission achieves a peak at \( 10^{14.5} \) Hz. The transient disk originated from the captured clumps definitely contributes to its emission to \( 10^{14}–10^{16} \) Hz in light of the SSD model. It is trivial to test this model in LINERs since the component of the debris disk has a variability with a timescale of years depending on the black hole masses and accretion rates of the ADAF. It should be noted that the selected LINERs to test the model should have relatively higher accretion rates in the clumpy-ADAF regime rather than in the pure ADAF mode. These LINERs have broad H\( \alpha \) components that are 25% LINERs (Ho 2008). They could have relatively higher accretion rates (Elitzur & Ho 2009) and thus contain clumps in the ADAF. These LINERs are expected to be monitored for variabilities to test the accretion processes.

**BL Lac objects.** They have lower accretion rates, likely in the ADAF mode (Wang et al. 2002; 2003; Barth et al. 2003). It is expected that some of them could have clumpy-ADAF. The observed emission is overwhelmed by the boosted emission of a relativistic jet, and the emission from the debris disk or cADAF is not directly visible. It is generally postulated that the ADAF will somehow produce a relativistic jet (e.g., Meier 2001), which is evidenced by X-ray binaries (e.g., Fender et al. 2010), e.g., the radio galaxy 3C 120 (Marscher et al. 2002) and 3C 111 (Tombesi et al. 2011). As we have shown, the presence of the debris disk drives the disappearance of the ADAF through efficient Compton cooling, resulting in quenching jet formation. In such a case, thermal components \( L_{\text{debris}} + L_{\text{Comp}} + L_{\text{cADAF}} \) could be observed if the relativistic boosting jet emission underwhelms this component. More interestingly, the model predicts a periodical presence of the clumpy-ADAF, which could lead to light curves with quasiperiodical modulation in light of the intermittent production of the jet in the clumpy-ADAF mode. Jet production quenches in the period of \( \Delta L_{\text{debris}} + \Delta L_{\text{Comp}} + \Delta L_{\text{cADAF}} \). Radio light curves from the Web site (http://www.astro.lsa.umich.edu/obs/radiotel/umrao.php) with quasiperiodical modulations of a few years observed in some BL Lac objects and radio galaxies could be explained by the present model, e.g., 0235+164, 3C 120, 0607–157, 0727–115, 1127–145, 1156+295, 1308+326, 1335–127, OT 129, BL Lac, 3C 446, and 3C 345. These objects have supermassive black holes with masses of \( 10^6–10^9 M_\odot \) (Ghisellini & Celotti 2001; Barth et al. 2003; Wang et al. 2002, 2003) and Eddington ratios of \( 10^{-3}–10^{-2} \) (Wang et al. 2003).

We note that some sources from this Web site, such as 3C 273, are different from the sources mentioned above. These sources may have higher accretion rates than the ADAF in BL Lac type sources. This implies the necessity of the standard clumpy accretion disk discussed in previous sections.

5.2. Black Hole X-Ray Binaries: State Transition

Extensive reviews of observed properties of black hole X-ray binaries and related theoretical explanations have been given by Remillard & McClintock (2006), McClintock & Remillard (2006), Done et al. (2007), and Belloni et al. (2011). Some black hole X-ray binaries show a repeat transition from low to high states and vice versa. The present model predicts repeat transitions. From the long-term light curves of 4U 1630-47, XTE J1650-500, XTE J1720-318, H 1743-322, SLX 1746-331, XTE J1859, and Cyg X-1, they usually have an Eddington ratio of \( \log (L/L_{\text{Edd}}) \approx -1 \), but they show variabilities with two orders (Done et al. 2007). They show bursts with very steep rising and slow declining luminosity. For a simple estimation at the hard state, the hard X-ray luminosity (at \( \sim 100 \) keV) from the ADAF is \( L_{\text{HX}} \approx 2.5 \times 10^{38}(M_*/M_\odot)m_{-2} \) erg s\(^{-1} \) (Mahadevan 1997). Once the debris disk is formed, it radiates at a luminosity of \( L_{\text{SX}} \approx 1.3 \times 10^{37}(M_*/M_\odot)(m_{\text{debris}}/0.1) \), and the accreting black hole transits to a high state with around a two-order change (\( L_{\text{HX}}/L_{\text{SX}} \approx 10^{-2} \)). Detailed calculations of the state transition and variability will be carried out in a forthcoming paper.

On the other hand, the presence of clumps in the ADAF will increase the radiative efficiency of the ADAF, driving the states of the ADAF into hard states with high luminosity (e.g., Figure 5 in the most recent review of Belloni et al. 2011). After the ADAF collapses, jet production quenches and the accretion flows then transit to the high state. We note that the collapse of the ADAF will squeeze itself. The squeezed gas could be blown away by the radiation, yielding outflows. This happens during the transition of states, especially from low to high states. Detailed comparisons with observations will be given in a future paper.
6. CONCLUSIONS

We show that an ADAF becomes a clumpy-ADAF composed of cold clumps arising from thermal instability when \( \dot{m} \gtrsim 0.02 \dot{m}_{\text{Edd}}^{2/3} \). We set up the dynamics of the clumpy accretion onto black holes and focus on the clumpy-ADAF in this paper. This model fills the regimes of the accretion mode from standard disks to pure ADAF. The angular momentum of the clumps is transported by the ADAF. We separately discuss the strong- and weak-coupling cases. The inner edge of the clumpy disk is set at the radius of the tidal disruption. Analytic solutions of the clumpy-ADAF are obtained for the two cases. For the strong-coupling case, the root of the averaged radial velocity square can be one order higher than the ADAF, resulting in a fast capture of the clumps through tidal force. For the weak-coupling case, the clumps are mainly orbiting around the black hole. The tidally disrupted clumps are accumulating with time until an efficiently transported by the ADAF. We separately discuss the strong- and weak-coupling cases. The inner edge of the clumpy disk is set at the radius of the tidal disruption. Analytic solutions of the clumps. We use the strong-coupling condition to close up the moment equation (9), obtaining the energy equation

\[
\frac{\partial}{\partial R} \left( N \frac{v_{\phi}^2}{2} \right) + \frac{\partial}{\partial z} (N v_{\phi} v_z) - \frac{N v_{\phi}^3}{2} + N v_{\phi} \frac{\partial \Phi}{\partial R} = 0,
\]

where

\[
\sigma_{\phi}^2 = \langle v_{\phi}^2 \rangle - \langle v_{\phi} \rangle^2.
\]

When \( f_{\phi} = 0 \), Equation (A6) reduces to Equation (4-51) in Binney & Tremaine (1987). The energy equation introduces a new unknown parameter \( \langle v_{\phi} v_z \rangle \). To close up the moment equations, observational relations should be employed, such as ratios of velocity dispersion. This can be done in the same way as in galactic dynamics, but it is very difficult for the present case. Thus, we use the strong coupling approximations.

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APPENDIX

ENERGY EQUATION OF CLUMPS: THE SECOND MOMENT EQUATION

Section 2.2 gives the zeroth- and first-order moment equations of the clumps. We use the strong-coupling condition to close up the moment equations. The energy equation for the clumps can be obtained from the second-order moment equation, which can be obtained by multiplying and integrating \( v_{\phi} v_{\phi} dv \):

\[
\frac{\partial}{\partial R} \left( N \frac{v_{\phi}^2}{2} \right) + \frac{\partial}{\partial z} (N v_{\phi} v_z) - \frac{N v_{\phi}^3}{2} + N v_{\phi} \frac{\partial \Phi}{\partial R} = 0,
\]

\[
- f_{\phi} N \langle v_{\phi}^3 \rangle + f_{\phi} N \langle v_{\phi} v_{\phi} v_z \rangle - f_{\phi} N \langle v_{\phi}^2 \rangle \frac{\partial \Phi}{\partial R} = 0.
\]

For the \( \phi \) - and \( z \)-direction symmetric clumpy-ADAF, we have the following relations:

\[
\langle v_{\phi}^2 (v_{\phi} - \langle v_{\phi} \rangle) \rangle = \langle v_{\phi}^2 \rangle - \langle v_{\phi}^2 \rangle \langle v_{\phi} \rangle = 0,
\]

\[
\langle v_{\phi} v_z (v_{\phi} - \langle v_{\phi} \rangle) \rangle = \langle v_{\phi} v_z v_{\phi} \rangle - \langle v_{\phi} v_z \rangle \langle v_{\phi} \rangle = 0,
\]

\[
\langle (v_{\phi} - \langle v_{\phi} \rangle)^3 \rangle = \langle v_{\phi}^3 \rangle - 3 \langle v_{\phi}^2 \rangle \langle v_{\phi} \rangle + 2 \langle v_{\phi} \rangle^3 = 0,
\]

\[
\langle v_{\phi} v_z \rangle = 0,
\]

and subtract \( \langle v_{\phi} \rangle \) times the moment equation (9), obtaining the energy equation

\[
\frac{\partial}{\partial R} \left( N \frac{v_{\phi}^2}{2} \right) + f_{\phi} N \langle v_{\phi} v_{\phi} v_z \rangle - f_{\phi} N \langle v_{\phi}^2 \rangle \frac{\partial \Phi}{\partial R} = 0.
\]

where

\[
\sigma_{\phi}^2 = \langle v_{\phi}^2 \rangle - \langle v_{\phi} \rangle^2.
\]
