The Bayesian Estimation of Customer Satisfaction Based on MCMC Method

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Abstract. In this paper, using MCMC method, numerical computing problems of the Bayes model of customer satisfaction are solved. Firstly, we introduced MCMC method. Afterwards, We have carried on the instance analysis using the customer satisfaction of the Bayes model, and the Bayes model of simulation data is given under the prior distribution for the Dirichlet distribution using OpenBUGS software.

Introduction

Since the 1970s, western countries, such as Europe and American countries, began to study Customer Satisfaction. At the end of last century, China also began to design the national satisfaction index model. At present, Measure and study customer satisfaction has become an important part of the overall quality plan of the enterprise, And widely used in the country, industry and other macroscopic, mesoscopic analysis. Many universities and market research companies have already participated in the research of customer satisfaction model. Among them, Miao[1] proposed the Bayes model of customer satisfaction. In this paper, markov chain Monte Carlo method (MCMC) is used to solve the numerical calculation problems in Bayes model of customer satisfaction.

Bayes Model of Customer Satisfaction

The Basic Idea of Bayesian Statistic

The difference between Bayesian statistics and classical statistics is that the former uses prior information(experience or historical data). Suppose \( \theta \) is an unknown quantity that we’re interested in(The set of all its possible values is called the parameter space, which is denoted as \( \Theta \)), according to Bayes’ view, we view it as a random variable, so, it can be described by a probability distribution, this distribution is called a prior distribution[2]. Bayesian formula combines the prior of \( \theta \) and sample information \( (x) \) to obtain posterior distribution, the posterior distribution is denoted as \( \pi(\theta | x) \). All decisions and extrapolations are based on a posterior distribution, the determination method of posterior distribution is given below.

Suppose \( \pi(\theta) \) is the prior distribution of \( \theta \), \( x \) is sample and it depend on \( \theta \), \( p(x | \theta) \) is conditional density function of \( X \) and \( \theta \). The joint density function of \( X \) and \( \theta \) is

\[
h(x, \theta) = p(x | \theta) \pi(\theta)
\]

The marginal probability density function of \( X \) as follows

\[
m(x) = \int_{\Theta} h(x, \theta) d\theta = \int_{\Theta} p(x | \theta) \pi(\theta) d\theta
\]

So

\[
\pi(\theta | x) = \frac{h(x, \theta)}{m(x)} = \frac{p(x | \theta) \pi(\theta)}{\int_{\Theta} p(x | \theta) \pi(\theta) d\theta}
\]
This equation is called Bayes’ rule of density function form. The posterior distribution combines prior \( \theta \) and sample information \( x \), so the posterior distribution is closer to the actual situation than the prior distribution. Generally, a digital feature of the posterior distribution is chosen as the estimate of \( \theta \), for example mathematical expectation or quantile. This paper adopts mathematical expectation \( \theta = E[\pi(\theta| x)] \).

**Bayesian Estimation of Customer Satisfaction**

Bayes estimation of customer satisfaction is essentially a Bayes estimation of multi-grade scoring. Bayes estimation of multilevel scoring has been proved in detail in literature \([3]\), this paper quotes its main conclusions. In the first-level indicator system, the possible score of each indicator is \( K, \ldots, 1, 0 \).

Suppose the index system consists of \( n \) indicators, there are \( K \) indicators for evaluation of \( K \), ..., there are \( 1 \) indicators for evaluation of \( 0 \). For each reviewer, his score is recorded as:

\[
x = (x_0, x_1, \ldots, x_K), \quad \sum_{i=0}^{K} x_i = n.
\]

Suppose that the score of a certain reviewer is \( T = (T_0, T_1, \ldots, T_K) \), where \( T_0, \ldots, T_K \) are the scores given by the reviewer. Then \( \theta \) can be used as a probability of scoring. When \( \theta \) is known, \( x \) obeys multinomial distribution:

\[
p(x|\theta) = n! \prod_{i=0}^{K} \frac{\theta_i^{x_i}}{x_i!}
\]

(4)

Among them \( x_i = 0, 1, \ldots, n(i = 0, 1, \ldots, K), \sum_{i=0}^{K} x_i = n, \sum_{i=0}^{K} \theta_i = 1 \); When the survey was conducted on \( m \) evaluator, denote sample as \( X = (x^{(1)}, \ldots, x^{(m)}), x^{(j)} = (x_{0,j}, \ldots, x_{K,j}), j = 1, \ldots, m \). Where \( x^{(j)} \) represents the score of the \( j \)th evaluator, So there is always: \( \sum_{i=0}^{K} x_{ij} = n, j = 1, \ldots, m \). Suppose \( \pi(\theta) \) is prior distribution of \( \theta \), when \( \theta \) is known, \( X \) obeys joint distribution as follows:

\[
f(x|\theta) = (nt)^n \prod_{i=0}^{K} \frac{\sum_{i=0}^{K} \theta_i^{x_i}}{\prod_{i=0}^{K} x_i!}
\]

(5)

We will study the Bayes estimation of \( \theta \).

**MCMC Method**

Many of the problems in Bayesian statistics can be reduced to integration calculations of posterior distributions, but when the posterior distribution is complex, such calculations are difficult. Therefore, it is necessary to explore some new methods of calculation. Markov chain Monte Carlo (MCMC) method is an effective way to solve such problems. The basic idea of the MCMC method is to get samples of \( \pi(x) \) by establishing a Markov chain with a stable distribution of \( \pi(x) \), these samples are then used to make statistical inferences. In other words, Markov chain monte carlo method is essentially a monte carlo comprehensive program. The generation of random sample is associated with a markov chain. Gibbs\(^4\) sampling is the simplest and most common MCMC method, it is an iterative sampling method based on conditional distribution.

Let \( \pi(x_1, \ldots, x_m) \) denotes \( m \)-dimensional joint distribution, constructor the transition nuclear in Gibbs sampling as follows:
\[ P_{x,y} \triangleq P(x,y) = \prod_{k=1}^{m} \pi(y_k \mid y_1, \ldots, y_{k-1}, x_{k+1}, \ldots, x_m) \]  

(6)

among them \( x = (x_1, \ldots, x_m), y = (y_1, \ldots, y_m), x_i \in D, y_j \in D \) (\( D \) denotes a \( m \)-dimensional region), \( \pi(y_1 \mid y_2, \ldots, y_m), x_i \in D \) is the conditional distribution. Gibbs sampling specific steps are as follows,

By Markov chain \( X_i(\omega) \) sample, a sample of \( X_{n+1}(\omega) \) can be obtained according to the following procedure:

first, \( y_1 \) is obtained by a random variable \( X_{n+1}(\omega) \), \( X_{n+1}(\omega) \) obey the distribution of \( \{\pi(y_1 \mid y_2, \ldots, y_m), y_i \in D\} \) from \( X_i(\omega) \).

secondly, \( y_2 \) is obtained by a random variable \( X_{n+1}(\omega) \), \( X_{n+1}(\omega) \) obey the distribution of \( \{\pi(y_2 \mid y_1, y_3, \ldots, y_m), y_2 \in D_2\} \). In this down, \( y_k(k = 1, \ldots, m-1) \) is obtained by a random variable \( X_{n+1}(\omega) \), \( X_{n+1}(\omega) \) obey the distribution of \( \{\pi(y_k \mid y_1, \ldots, y_{k-1}, x_{k+1}, \ldots, x_n, y_k \in D_k\} \).

In the end, \( y_m \) is obtained by a random variable \( X_{n+1}(\omega) \), \( X_{n+1}(\omega) \) obey the distribution of \( \{\pi(y_m \mid y_1, \ldots, y_{m-1}), y_m \in D\} \).

Definition \( y \triangleq (y_1, \ldots, y_m) \) is a sample of \( X_{n+1}(\omega) \).

Now, Take an initial value \( X_0(\omega) = y^{(0)}, y^{(0)} \) is obtained by the random variable \( X_i(\omega) \). According to the above methods. To get the sample \( y^{(1)}, y^{(n)} \) from \( X_0(\omega), \ldots, X_i(\omega) \). When \( n \) is large, the distribution of \( X_i(\omega) \) approximation of \( \pi(x_1, \ldots, x_m) \). Can approximate thought, \( y^{(n)} \) is a sample obey the distribution of \( \pi(x_1, \ldots, x_m) \).

Many software and applications for MCMC methods have been developed, for example OpenBUGS[5][6] software. Using OpenBUGS, you can easily sample Gibbs for many commonly used models and distributions. When using OpenBUGS, the user does not need to know the prior density of the parameter or the exact expression of the likelihood function, As long as the prior distribution of variables is set and the model under study is generally described, You can easily implement the bayesian analysis of the model, it doesn't require complex programming.

Data Simulation

We take the example in literature [1] as an example to illustrate the application process of MCMC method.

This example is an enterprise human resource about the company employee satisfaction survey. Sample size 79, involved 33 indicators, the index system adopts the international general scale. The scoring rules are as follows: 1-be sure; 2-once in a while is; 3-uncertainty; 4-once in a while not; 5-certainly not. Table of survey statistics is shown in literature [1], it will not be repeated here.

Suppose the prior distribution of \( \theta \) is Dirichlet distribution, The OpenBUGS program is shown below:

```r
model
{for(i in 1:79){
   x[i,1:5]~dmulti(theta[],33)
}
theta[1:5]~ddirich(prior[]);
}
```

We are in the process of running the model, at first, proceed 1000 pre-iterations, to ensure the convergence of the parameters. Then discard the initial pre-iteration and do another 10,000 iterations. The following is the partial information of the parameters of \( \theta \) obtained.
Table 1. OpenBUGS computational results.

| Parameters | mean    | std      | MC error | 2.5% | median | 97.5% | start | sample |
|------------|---------|----------|----------|------|--------|-------|-------|--------|
| theta1     | 0.08496 | 0.005438 | 5.019E-5 | 0.07454 | 0.08484 | 0.09583 | 1001   | 10000  |
| theta2     | 0.1143  | 0.006172 | 6.405E-5 | 0.1027 | 0.1142 | 0.1267 | 1001   | 10000  |
| theta3     | 0.171   | 0.007322 | 7.127E-5 | 0.1568 | 0.1709 | 0.1855 | 1001   | 10000  |
| theta4     | 0.2484  | 0.008456 | 8.678E-5 | 0.232  | 0.2484 | 0.2652 | 1001   | 10000  |
| theta5     | 0.3813  | 0.009522 | 9.143E-5 | 0.3626 | 0.3813 | 0.3998 | 1001   | 10000  |

We can see that in the table, Bayes of employee satisfaction is estimated to be: \( \theta = (\theta_1, \ldots, \theta_5) = (0.08496, 0.1143, 0.171, 0.2484, 0.3813) \). \( \theta \) is graded as 1...5 estimate of the probability of 5 points. About 38% of employees were very satisfied with the company, about 25% were more satisfied, about 17 percent had no special opinion, about 11% were dissatisfaction, about 9 per cent were very dissatisfied. The results are consistent with literature[1]. In addition to getting the mean of the parameter, the standard deviation of the posterior distribution of the parameter can also be obtained from the table, information such as 95% confidence interval and median. The kernel density estimates of the posterior distribution of the parameters can also be obtained from OpenBUGS software, dynamic locus, A series of information such as iteration history and convergence statistical diagnostic map. In addition to writing code programs directly, we can also use the directed graph model approach in OpenBUGS(Doodle model). This does not repeat itself here.

**Conclusion**

The birth of MCMC method led to the rapid development of bayesian statistics, its application dramatically reshaped the way statisticians work. A statistical research model based on stochastic simulation is constructed. Models that have long been considered intractable can now be studied quantitively. We apply the MCMC approach to the customer satisfaction model, it shows the wide application of this method. The study in this paper provides some implications for applying this method to more bayesian models.

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