EVALUATING METRICS FOR BIAS IN WORD EMBEDDINGS

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ABSTRACT

Over the last years, word and sentence embeddings have established as text preprocessing for all kinds of NLP tasks and improved the performances significantly. Unfortunately, it has also been shown that these embeddings inherit various kinds of biases from the training data and thereby pass on biases present in society to NLP solutions. Many papers attempted to quantify bias in word or sentence embeddings to evaluate debiasing methods or compare different embedding models, usually with cosine-based metrics. However, lately some works have raised doubts about these metrics showing that even though such metrics report low biases, other tests still show biases. In fact, there is a great variety of bias metrics or tests proposed in the literature without any consensus on the optimal solutions. Yet we lack works that evaluate bias metrics on a theoretical level or elaborate the advantages and disadvantages of different bias metrics. In this work, we will explore different cosine based bias metrics. We formalize a bias definition based on the ideas from previous works and derive conditions for bias metrics. Furthermore, we thoroughly investigate the existing cosine-based metrics and their limitations to show why these metrics can fail to report biases in some cases. Finally, we propose a new metric, SAME, to address the shortcomings of existing metrics and mathematically prove that SAME behaves appropriately.

There are two conference papers including new experiments to evaluate the properties of existing cosine scores (Schröder. et al. [2024]) and the SAME score (Schröder et al. [2024]).

1 Introduction

Word embeddings have been widely adopted in NLP tasks over the last decade, and have been further developed to embed the context of words, whole sentences or even longer texts into high dimensional vector representations. These are applied in a wide variety of downstream tasks, including translation and text generation as in question-answering systems (Radford et al. [2019]) or next sentence prediction (Devlin et al. [2019]), or sentiment and similarity analysis (Cer et al. [2018], Mikolov et al. [2013], Pennington et al. [2014], Reimers and Gurevych [2019]) of texts. However, many works have shown that these models capture various kinds of biases present in humans and society. Using biased word or text representations poses a great risk to produce unfair language models, which in turn can further amplify the biases in society.

The first works investigating bias in word embeddings focused on geometric relations between embeddings of stereotypically associated words. From there, several metrics based on cosine similarity have been proposed like the "Direct Bias" by Bolukbasi et al. [2016] and the Word Embedding Association Test (WEAT) by Caliskan et al. [2017]. Though initially introduced to prove the presence of bias in word embeddings, these tests have also been widely used to compare biases in different embedding models (e.g. May et al. [2019], Cer et al. [2018]) or validate debiasing method (e.g. Liang et al. [2020], Karve et al. [2019], Kaneko and Bollegala [2021]). Due to its popularity, there have been several variants introduced in the literature like the Sentence Encoder Association Test (SEAT) by May et al. [2019], the "Generalized WEAT" (Swinger et al. [2019]) or WEAT for gendered languages (Zhou et al. [2019]). Another cosine based metric is the Mean Average Cosine Similarity (MAC) score (Manzini et al. [2019]), though it has a different intuition than WEAT and the Direct Bias. However, there is literature criticizing WEAT and its variants. For instance May et al. [2019] found that SEAT produced rather inconsistent results on sentence embeddings and emphasized the positive predictability of the test. This prompted many other papers to doubt the meaningfulness or usability of cosine based metrics in general. Hence, methods to
quantify bias in downstream tasks have been proposed and increasingly replace cosine based metrics [Zhao et al., 2018, Gonen and Goldberg, 2019]. On the one hand, this is desirable since it bridges the gap to fairness evaluation in general and allows to apply other fairness definitions like statistical parity or counterfactual fairness. On the other hand, it poses the risk to confound biases from word representations with biases introduced by the downstream task itself. Hence, when investigating bias in word embedding models, apart from specific downstream tasks, metrics directly applicable to the embeddings are beneficial.

Nevertheless, the doubts regarding existing metrics show the need to evaluate these metrics on a theoretical level. To our knowledge, the only work doing this is Ethayarajh et al. [2019]. The authors showed that WEAT has theoretical flaws that can cause it to overestimate biases and demonstrated that one could make up attribute sets in order to make the test appear statistically insignificant. While this alone is highly worrying, we find even more theoretical flaws that, on one hand, cause WEAT to underestimate biases under certain conditions. On the other hand, these flaws make WEAT results incomparable between different embeddings. Moreover, these flaws extend to other versions like SEAT or the generalized WEAT, and we also find flaws for MAC and the Direct Bias.

Noteworthy, there exist also works criticising that cosine based bias scores do not capture biases entirely. However, they rely on different definitions of bias. As we discuss in Chapter 2.1, these definitions do not necessarily contradict the definition by Bolukbasi et al. [2016] or Caliskan et al. [2017]. Since there are applications like document clustering or text similarity estimations, where the cosine similarity is directly queried [Reimers and Gurevych, 2019, Cer et al., 2018, Gao et al., 2021, Subramanian et al., 2018, Liu et al., 2021], quantifying bias in terms of cosine similarity is highly relevant and thus further evaluation of cosine based metrics remains necessary.

The main contributions in this work are the following: (i) We define formal requirements for cosine based score functions which formalize a notion of meaningfulness in this context. (ii) We analytically demonstrate that the state of the art metrics WEAT, the Direct Bias and MAC are only partially meaningful, corresponding to the previous definitions. (iii) We propose a novel bias score function Scoring Association Means of Word Embeddings (SAME), that is based on some principles of WEAT, but reformulates potentially problematic parts. (iv) We experimentally investigate SAME and the state of the art score functions and substantiate the theoretic claims with real word embeddings.

The remainder of this paper is structured as follows: In Chapter 2 we give an overview over bias definitions and specifically cosine based bias metrics from the literature. The formal requirements for bias score functions are explained and applied to the existing score functions in Chapter 3.3. Next, in Chapter 4 we propose SAME with extensions for multi-attribute biases and skew and stereotype distinction. Chapter 5 describes our experiments, where we retrain BERT on biased data. Thereby we induce specific biases as a ground truth to evaluate the different bias score functions. Lastly, we conclude our findings and give an overview over the bias score functions in Chapter 6.

2 Related Work for Bias in Text Embeddings

In the following, we summarize the related work on bias in word embeddings. First, we focus on definitions and discuss our choice of cosine based scores. Then we revisit the different cosine based metrics from the literature.

2.1 Bias Definition

In this section, we describe several definitions of bias in word embeddings mentioned in the literature. The focus of this work lies on geometrical bias definitions, especially those regarding cosine similarity, and the skew and stereotype definitions. For the sake of clarity, we also discuss other bias definitions regarding downstream tasks. Lately, measuring bias in downstream tasks increasingly replaces geometrical bias scores in the literature. Yet, we argue that they cannot replace the geometrical definitions and scores entirely and thereby emphasize the usability of cosine based scores.

2.1.1 Geometrical Bias Definitions

Word embedding vectors are expected to reflect word relations by their geometrical relations, whereby distance in the vector space is said to correlate with the semantic similarity of words [Mikolov et al., 2013, Pennington et al., 2014]. Due to this intuition, biases in word embeddings are often considered as flawed vector representations, where words are embedded inappropriately close or far from other words. Since the cosine similarity is often used as a distance measure to estimate word or document similarity [Thongtan and Phienthrakul, 2019, Shahmirzadi et al., 2019, Reimers and Gurevych, 2019, Cer et al., 2018], it has also been used to measure bias in terms of geometric relations.
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Figure 1: Simplified (2D) visualization of gender stereotypes in occupation embedding vectors. The occupation vectors in dotted lines show a less biased representation of the occupations. Ideally, all gender neutral terms should be represented on the neutral axis, thus being equidistant to he and she. The red dotted line indicates the bias direction \( g = \pm (\text{she} - \text{he}) \)

Based on this idea, Bolukbasi et al. [2016] defined bias as the correlation between neutral words and a bias subspace or bias direction, that describes the relations between groups of words. For the simple case illustrated in Figure 1 this would be the gender direction \( g = \text{she} - \text{he} \). To get a more robust estimation of group relations (e.g. male and female terms), one would use several word pairs and determine a low dimensional subspace that captures most of the variance of individual bias directions. Caliskan et al. [2017] define bias as the difference in angular distance between a neutral word and two different groups of words. In the example depicted in Figure 1, they would consider secretary biased since it has a lower angular distance to she than he and engineer vice versa.

Both definitions follow a similar intuition and use the cosine similarity as correlation or distance measure. For the simple case illustrated in Figure 1, correlation with the bias direction matches up with the difference in angular distances. An important difference is that Caliskan et al. [2017] contrast over two groups of theoretically neutral words to determine a stereotype (see Section 2.2 for details) while Bolukbasi et al. [2016] take the mean bias correlation over all neutral words (see Section 2.2.3).

Another work by Ethayarajh et al. [2019] follows the definition from Bolukbasi et al. [2016]. However, they argue that the vector length holds important information and thus biases should rather be measured using the inner product of word vectors instead of cosine similarity. Yet, their definition of bias mostly concerns word co-occurrences in the training data and how these are represented in word vectors instead of the fairness implications in different use cases of word embeddings. Hence, it is not clear to what extent the information encoded in the vector length influences fairness in downstream tasks. On the other hand, there are downstream tasks such as document clustering that might rely directly on the cosine similarity or use it as a dissimilarity measure. Consequently, using the cosine similarity (as a modification of the inner product that removes the influence of vector lengths) is a valid option in cases where the vector length can be neglected. Moreover, they only consider single word biases. When computing the bias over a set of words, it seems preferable to normalize the word biases by word vector length to have all words contribute with the same weight to the overall bias, resulting in the same notion as if using cosine based scores.

### 2.1.2 Classification and Clustering Bias

Gonen and Goldberg [2019] found that stereotypical groups persisted after removing correlations of neutral words with a bias subspace or retraining on debiased text sources. This raised doubts on the usability of cosine based metrics. Instead they proposed to use clustering or classification tasks to investigate bias. In short, if a classifier or clustering algorithm can reconstruct stereotypical groups (according to previous correlation with the bias subspace), then there is still bias in the data, according to their definition. However, there are two possible explanations why they found this persisting bias: Either the debiasing methods were not performed thoroughly (leaving some geometrical bias associations in the embeddings or text sources) or as Ethayarajh et al. [2019] described it: "Because we define unbiasedness with respect to a set of word pairs, we cannot make any claims about word pairs outside that set". Another possibility is that these stereotypical groups simply reflect other relations independent of the bias attributes, but different enough to reconstruct stereotypical groups with a flawed classification task (e.g. different types of occupations that...
coincidentally relate to gender stereotypes). Since it is not clear whether one of these effects lead to their claims, we cannot take Gonen and Goldberg [2019]'s work as evidence against cosine based metrics.

### 2.1.3 Bias in other Downstream Tasks

Lately, more and more works started to measure bias in downstream tasks as opposed to geometrical metrics. An example are Zhao et al. [2018] and Zhao et al. [2019], who used a co-reference resolution task. For instance, the language model has to decide which word a gendered pronoun was associated with. Confronting the model with pro- and anti-stereotypical sentences can be used to determine whether the decisions are based on gender biases as opposed to semantic meaning.

Another work to be emphasized is Kurita et al. [2019], who proposed a bias test based on the masked language objective of BERT. Essentially, they query BERT for the probability to insert certain bias attributes in a masked sentence when associated with other neutral words. For instance they would compute the association between "programmer" and the male gender by probing BERT for the probability to replace "[MASK]" with "he" in "[MASK] is a programmer". They show that this is an accurate way to measure biases in BERT and can show a correlation with bias in the GPR task by Webster et al. [2018].

However, the bias measured by Kurita et al. [2019] is model-specific for BERT’s masked-language modeling (MLM) models and measuring bias in downstream tasks is always task specific. Both might be influenced by fine-tuning for MLM or the specific task. This is fine as long as one wants to quantify embedding models or probing embedding models without a specific use case in mind. On the other hand, cosine based metrics work for any kind of embedding models, from classical word embeddings to contextualized and sentence embeddings, and hence are worth further investigating.

### 2.1.4 Skew and Stereotype

Recently, de Vassimon Manela et al. [2021] introduced another definition of bias. They also used the GPR task by Webster et al. [2018] to obtain word biases, but distinguish between two types of bias: Skew and stereotype. While stereotype shows the presence of groups stronger associated with different bias attributes than other groups, which is also the intuition behind Caliskan et al. [2017]'s work, the skew describes the effect that all words from a set are (on average) biased towards the same attribute. For instance, occupations could be stereotypically associated with male/female terms, but it’s also possible that (on average) all occupations are rather biased towards one gender. They further suggest that there is a certain trade-off between these two forms of bias. Distinguishing these kinds of biases seems to be beneficial for investigating bias in embeddings more thoroughly. Hence, we apply their definition to construct cosine based skew and stereotype metrics (see Section 4.4) and show their benefit over general bias metrics in the experiments.

### 2.2 Cosine-based measure

Cosine-based proximity measures are the most used measures in terms of geometric relations. While it is possible to refer to other similarity/dissimilarity measures, mostly the cosine similarity

\[
\cos(u, v) = \frac{u \cdot v}{||u|| \cdot ||v||},
\]

is used to determine the association between two vector representations of words \(u\) and \(v\).

Usually the similarity of words \(w\) towards groups of words (e.g. terms specific for one gender/ religion/ race...), so called attribute sets, is measured. The similarity of \(w\) and one set of attributes \(A\) is then

\[
s(w, A) = \text{mean}_{a \in A} \cos(w, a).
\]

Based on this, different bias metrics are defined in literature. We detail the most prominent ones in the following sections, using the notations from the literature.

#### 2.2.1 WEAT

The Word Embedding Association Test (WEAT), as defined by Caliskan et al. [2017], is based on the Implicit Association Test (Greenwald et al., 1998), which is used in psychological studies to measure reaction times regarding pro and anti stereotypical associations.

The test compares two sets of target words \(X\) and \(Y\) with two sets of bias attributes \(A\) and \(B\) of equal size \(n\). When observing gender stereotypes in occupations, which might be \(A = \{he, man, male, ...\}, B = \{she, woman, female, ...\}\), the WEAT score is calculated as:

\[
\text{WEAT}(X, Y, A, B) = \frac{\text{mean}_{a \in A} \cos(w_a, X) - \text{mean}_{a \in B} \cos(w_a, Y)}{\text{std}(\cos(w_a, X)) + \text{std}(\cos(w_a, Y))},
\]

where \(w_a\) is the vector representation of the word associated with the attribute \(a\).
\{she, woman, female, ...\}, \(X = \{engineer, doctor, policeman, ...\}\) and \(Y = \{secretary, nurse, teacher, ...\}\), anticipating that the occupations in \(X\) would be closer to words in \(A\) than \(B\), and vice versa for occupations in \(Y\).

The association of a single word \(w\) with the bias attribute sets \(A\) and \(B\) including \(n\) attributes each, is given by

\[
s(w, A, B) = \frac{1}{n} \sum_{a \in A} \cos(w, a) - \frac{1}{n} \sum_{b \in B} \cos(w, b).
\]

To quantify bias in the sets \(X\) and \(Y\), the effect size is used, which is a normalized measure for the association difference between the target sets

\[
d(X, Y, A, B) = \frac{\text{mean}_{x \in X} s(x, A, B) - \text{mean}_{y \in Y} s(y, A, B)}{\text{stddev}_{x \in X} s(x, A, B)},
\]

where \(\text{mean}_{x \in X} s(x, A, B)\) refers to the mean of \(s(x, A, B)\) with \(x\) in \(X\) and \(\text{stddev}_{x \in X} s(x, A, B)\) to the standard deviation over all word biases of \(x\) in \(X\).

A positive effect size confirms the hypothesis that words in \(X\) are rather stereotypical for the attributes in \(A\) and words in \(Y\) stereotypical for words in \(B\), while a negative effect size indicates that the stereotypes would be counter-wise.

Based on the Implicit Association Tests, the WEATs include several tests probing for associations between pleasant/unpleasant words and race as well as tests probing for gender stereotypes comparing career and family, math and arts as well as science and arts.

To determine the statistical significance of biases measured in those tests, the authors use the test statistic

\[
s(X, Y, A, B) = \sum_{x \in X} s(x, A, B) - \sum_{y \in Y} s(y, A, B)
\]

for a permutation test with partitions \((X_i, Y_i)\) of \(X \cup Y\):

\[
p = P[s(X_i, Y_i, A, B) > s(X, Y, A, B)].
\]

There are several extensions to WEAT: For instance WEAT for gendered languages by Zhou et al. [2019] and the "Generalized WEAT" by Swinger et al. [2019], which allows to apply WEAT to more than two different attribute and target groups.

May et al. [2019] propose the Sentence Encoder Association Test (SEAT), which basically applies WEAT to sentence representations. The sentences are obtained by inserting WEAT terms into simple templates such as "this is a <term>".

\[2.2.2\] MAC

The Mean Average Cosine Similarity (MAC) score was introduced by Manzini et al. [2019] to provide a metric for multi-class fairness problems. They compute the bias of a word \(t\) towards an attribute set \(A_j\) using the cosine distance as the reciprocal of the cosine similarity:

\[
S(t, A_j) = \frac{1}{N} \sum_{a \in A_j} 1 - \cos(t, a).
\]

In contrast to WEAT, only one set of target words \(T\) is utilized. The bias of words in \(T\) towards several bias attribute sets \(A = \{A_1, \ldots, A_n\}\) is then given by the MAC score as

\[
MAC(T, A) = \frac{1}{|T||A|} \sum_{t \in T} \sum_{A_j \in A} S(t, A_j).
\]

Equation \[(8)\] applies a different intuition to WEAT and the Direct Bias (see Figure [1] for the bias definition). However, MAC uses the direct associations instead of contrasting terms like Caliskan et al. [2017] or measuring the correlation with a contrastive bias direction (or subspace) like Bolukbasi et al. [2016]. Most works use the MAC score only for multi-class problems that (the original) WEAT cannot be applied to. For instance, Liang et al. [2020] use it in addition to WEAT to evaluate their debiasing method.

\[2.2.3\] Direct Bias (Bolukbasi)

Bolukbasi et al. [2016] define the Direct Bias as the correlation of neutral words \(w \in N\) with a bias direction (in their example gender direction \(g\)):

\[
\text{DirectBias} = \frac{1}{|N|} \sum_{w \in N} |\cos(w, g)|^c
\]
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Figure 2: Bias of stereotypical male and female jobs as indicated by WEAT’s $s(w, A, B)$ calculated on BERT embeddings. Differences along the y-axis are arbitrary shift for better visibility.

with $c$ determining the strictness of bias measurement. The gender direction is either obtained by a gender word-pair e.g. $g = \text{he} - \text{she}$ or - to get a more robust estimate - it is obtained by computing the first principal component over a set of individual gender directions from different word-pairs.

The authors used it as a preliminary experiment to show bias present in embeddings, but not to evaluate their debiasing algorithm (which follows the same intuition). The Direct Bias is applied e.g. in Chen et al. [2021] and Costa-juss and Casas [2019].

In terms of their debiasing algorithm Bolukbasi et al. [2016] describe how to obtain a bias subspace given defining sets $D_1, \ldots, D_n$. A defining set $D_i$ includes words $w$ that only differ by the bias relevant topic e.g. for gender bias $\{\text{man}, \text{woman}\}$ could be used as a defining set. Given these sets, the authors construct individual bias directions $w - \mu_i \forall w \in D_i, i \in \{1, \ldots, n\}$ and $\mu_i = \sum_{w \in D_i} w / |D_i|$. To obtain a k-dimensional bias subspace $B$ they compute the $k$ first principal components over these samples. Manzini et al. [2019] applied this debiasing algorithm to mitigate multi-class bias. For this purpose they determined the bias subspace using defining sets $D_i$ with more than two elements. This could be also applied to the Direct Bias to measure multi-class bias.

3 Requirements for Bias Metrics

3.1 Motivation

While WEAT is frequently used in the literature (Zhou et al. [2019], Swinger et al. [2019], Kaneko and Bollegala [2021], Liang et al. [2020], Karve et al. [2019], Ravfogel et al. [2020]), there does exist literature documenting its behaviour to be not always as expected, e.g. May et al. [2019], who emphasize that WEAT cannot prove the absence of bias, or Ethayarajh et al. [2019], who already showed theoretical flaws of WEAT. Apart from this, we show further examples where WEAT, the Direct Bias and MAC fail to report biases.

To emphasize the need of formal requirements for bias scores, we look into how WEAT as the most prominent cosine based bias score is used in the literature. The authors (Caliskan et al. [2017]) introduce WEAT to prove the presence of bias in word embeddings. However, there are other papers that use WEAT as a quantification of bias, either to compare biases in different embeddings (May et al. [2019]) or to evaluate debiasing algorithms (Liang et al. [2020], Ravfogel et al. [2020], Karve et al. [2019], Kaneko and Bollegala [2021]). Those use cases require a bias score whose magnitude can be interpreted analogously to more or less bias. However, we present an example that shows that the WEAT effect size cannot be interpreted in that way. Therefor, its meaningfulness for bias quantification is severely limited.

Consider the example of gender bias with $A$ containing embeddings of male terms and $B$ containing those of female terms, and 6 occupations, half of those stereotypically male/female. All words were embedded with BERT. We computed the word-wise biases of 6 occupations according to WEAT’s $s(w, A, B)$. The results are shown in Figure 2 (the y-axis is added for better visibility and does not imply any difference).
We then used groups $X = \{\text{footballer, bishop}\}$ and $Y = \{\text{hairdresser, sociologist}\}$ as target words and measured the effect size $d(X, Y, A, B) = 1.7299$, which indicates a high stereotype regarding the groups $X$ and $Y$. Furthermore, we constructed groups $X' = \{\text{footballer, lawmaker}\}$ and $Y' = \{\text{hairdresser, nurse}\}$, using the more extremely stereotyped occupations according to the word-wise bias scores. Now, when computing the effect size using $X'$ and/or $Y'$ instead of their counterparts, one would expect a bias metric to report an even larger bias with respect to these groups.

However, we found $d(X, Y', A, B) = -0.0865$ and $d(X', Y', A, B) = 1.6448e-05$, which would be interpreted as significantly lower biases, although looking at individual words we know that this is not the case.

This is just one example to illustrate how WEAT as the most prominent cosine based bias score can be misleading when quantifying biases. Hence, in the following sections, we derive formal requirements for bias scores to ensure their results are reliable and comparable.

### 3.2 Formal Bias Definition and Notations

In Section 2.1 we described the geometrical definition of bias that is commonly used in the literature (e.g. Caliskan et al. [2017] and Bolukbasi et al. [2016]). However, this definition considers only the simple case of comparing one neutral word with two other words representing two protected groups. Thus, in this section, we will provide a formal definition of bias covering arbitrary numbers of words and groups to compare against.

Given a fairness critical concept like gender, religion or ethnicity, we select $n$ protected groups that might be subject to biases. Each protected group is defined by a set of attribute words $a_{ik} \in A_i$ with $i \in \{1, ..., n\}$ the group’s index. We summarize these attribute sets as $A = \{A_1, ..., A_n\}$. The intuition is that the attributes define the relation of protected groups by contrasting specifically over the membership to the different groups. Therefore, it is important that any attribute $a_{ik} \in A_i$ has a counterpart $a_{jk} \in A_j$ and $j \in A, j \neq i$ that only differs from $a_{ik}$ by the group membership.

For instance, if we used $A_1 = \{\text{she, female, woman}\}$ and $A_2 = \{\text{he, male, man}\}$ would be the proper choice of male terms.

Given these attribute sets the association of a words $w$ with a protected group is given by $s(w, A_i)$ (Equation 2). We are particularly interested in the difference of associations towards the different groups, i.e. is $w$ more similar to one protected group than the others. Whether such associations are harmful depends on whether $w$ is theoretically neutral to the protected groups. For example, terms like “aunt” or “uncle” are associated with one or the other gender per definition, while a term like “transsexual” is associated with the overall concept of gender but should not be stronger associated with female terms than male terms.

Noteworthy, by words and attributes we refer to vectorial representations of words in a $d$-dimensional embedding space, i.e. let $V = \{v_1, ..., v_m\}$ be a vocabulary, then an embedding model $E : V \rightarrow \mathbb{R}^d$ transforms words $v$ into $d$-dimensional word vectors $w$.

As baseline for our bias score requirements and the following analysis of bias scores from the literature, we suggest two intuitive definitions of bias for single words $w$ and sets of words $W$. For words $w$ we apply the intuition of WEAT extended to $n$ protected groups instead of only two.

**Definition 1** (word bias). Given $n$ protected groups represented by attribute sets $A_1, ..., A_n$ and a word $w$ that is theoretically neutral to these groups, we consider $w$ biased if

$$\exists A_i, A_j \in A : s(w, A_i) > s(w, A_j)$$  \hspace{1cm} (10)

**Definition 2** (set bias). Given $n$ protected groups represented by attribute sets $A_1, ..., A_n$ and a set of word $W$ containing only words that are theoretically neutral to these groups, we consider $W$ biased if at least one word $w \in W$ is biased:

$$\exists A_i, A_j \in A, w \in W : s(w, A_i) > s(w, A_j)$$  \hspace{1cm} (11)

The idea behind Definition 2 is that even when looking at sets of words, each individual word’s bias is important, i.e. as long as there is one biased word in the set, we cannot call the set of words unbiased, even if word biases cancel out on average or the majority of words is unbiased.

Apart from this general notion of bias, we further introduce the definition of skew and stereotype bias from de Vassignon Manela et al. [2021] to geometrical biases. These two types of bias describe how a set of words $W$ is biased. The skew describes whether all words are (on average) biased towards on protected group, while the stereotype describes the occurrence of stereotypical groups associated with different protected groups.
Definition 3 (skew). Given \( n \) protected groups represented by attribute sets \( A_1, \ldots, A_n \) and a set of word \( W \), we consider \( W \) skewed towards a group \( i \) compared to group \( j \) if
\[
\text{mean}_{w \in W} s(w, A_i) > \text{mean}_{w \in W} s(w, A_j)
\] (12)

Definition 4 (stereotype). Given \( n \) protected groups represented by attribute sets \( A_1, \ldots, A_n \) and a set of word \( W \), we consider \( W \) containing stereotypes if:
\[
\exists A_i, A_j \in A, w_1, w_2 \in W : s(w_1, A_i) - s(w_1, A_j) \neq s(w_2, A_i) - s(w_2, A_j)
\] (13)

Note that our definition of stereotype does not require the words in \( W \) to form distinct stereotypical groups but also considers a continuous distribution of word biases stereotypical as long as there are words closer to a certain group and other words closer to another groups. Therefore, our stereotype definition is not limited to extreme cases with fundamentally different groups but is also sensitive to individual words being stereotypical for certain protected groups. Yet, the greater the differences or the more distinct groups appear, the higher is the stereotype considered.

In the following we will use a notation for bias score functions in general: \( b(w, A) \) measuring the bias of one word. For sets of words, there are two notations reflecting two different strategies presented in the literature: Bias scores measuring bias over all neutral words jointly (MAC and Direct Bias) will be written as \( b(W, A) \) and bias scores measuring the bias over two groups of neutral words \( X, Y \subseteq W \) (WEAT), will be written as \( b(X, Y, A) \). Since the bias scores from the literature have different extreme values and different values indicating no bias, we use the following notations: \( b_{min} \) and \( b_{max} \) are the extreme values of \( b(\cdot) \) and \( b_0 \) is the value of \( b(\cdot) \) that means \( w \) or \( W \) is unbiased. Note that \( b_{min} \) and \( b_0 \) are not necessarily equal. This results in the following notations for WEAT, MAC and the Direct Bias:

1. For WEAT \( b_{WEAT}(w, A) = s(w, A_1, A_2) \) and \( b_{WEAT}(W, A) = d(X, Y, A_1, A_2) \) with \( A = \{A_1, A_2\} \) limited to two attribute sets and \( X \cup Y = W, X \cap Y = \emptyset \) two predefined groups of the neutral words. \( b_{min} = -1, b_{max} = 1 \) and \( b_0 = 0 \).
2. For MAC \( b_{MAC}(w, A) = \frac{1}{|A|} \sum_{A_j \in A} s(w, A_j) \) (MAC score for \( W = \{w\} \) and \( b_{MAC}(W, A) = MAC(W, A) \) (in the MAC notation, \( T \) is corresponding to \( W \)). \( b_{min} = 0, b_{max} = 2 \) and \( b_0 = 0 \).
3. For the Direct Bias \( b_{DirectBias}(w, A) = |\cos(w, g)| \) and \( b_{DirectBias}(W, A) = \frac{1}{|W|} \sum_{w \in W} |\cos(w, g)| \) (in the Direct Bias notation (Equation 9) \( N \) is used instead of \( W \)). Here \( g \) refers to the bias direction, which is obtained using two attribute sets \( A_1 \) and \( A_2 \) as described in Section 2.2.3. Hence, \( A = \{A_1, A_2\} \) is limited to two attribute sets. \( b_{min} = 0, b_{max} = 1 \) and \( b_0 = 0 \).

3.3 Requirements for Bias Metrics

Based on the definitions of bias explained in Section 3.2, we introduce two novel properties which a bias score function should fulfill to quantify bias in a meaningful way: *trustworthiness* and *magnitude-comparability*. Both properties will be explained and formally defined in the following. The goal of both properties is to ensure that biases can be quantified in a way such that bias scores can be safely compared between different embedding models and debiasing methods can be evaluated without risking to overlook bias. We acknowledge that there are also use cases where these properties can be neglected, for example the tests of Caliskan et al. [2017]. Nevertheless, there plenty of statements in the literature (e.g. May et al. [2019]) that suggest biases being overlooked by state-of-the-art bias scores, so it is highly relevant to determine which bias scores suffice these properties and which do not. Furthermore, in Section 4.4 we propose sensitivity to skew and stereotype as two additional criteria that allow a closer look into what kinds of biases certain bias scores can detect.

3.3.1 Comparability

The goal of our first property, *magnitude-comparability*, is to ensure that bias scores are comparable between different embeddings. This is necessary to make statements about embedding models being more or less biased than others, which includes comparing debiased embeddings with their original counterparts. A necessary condition for such comparability is the possibility to reach the extreme values of \( b_{min} \) and \( b_{max} \) of \( b(\cdot) \) in different embedding spaces depending only on the neutral words.

More formally, assume a word embedding is fixed with embedding space \( E \). Considering \( W \), a set of words in \( E \) and \( A_1 \subseteq A \) sets of attributes in \( E \), a bias score function \( b(W, A) \) maps \( W \) and the set of attribute sets \( A \) to a real number. The set of words \( W \) could also be divided into subsets \( X \) and \( Y \), corresponding to two attribute sets \( A \) and \( B \) (as done in WEAT). We propose the following requirements:

Definition 5 (magnitude-comparable). We call the bias score function \( b(W, A) \) magnitude-comparable if, for a fixed number of target words in set \( W \) (including the case \( W = \{w\} \), the maximum bias score \( b_{max} \) and the minimum bias
We call the bias score function \( \hat{b}(W, A) \) analogously for scores that use a set of words \( W \). We formulate two additional criteria in alignment to the skew and stereotype definitions. Since the requirements are independent of the attribute sets in \( A \):

\[
\max_{W, |W| = \text{const}} \hat{b}(W, A) = \hat{b}_{\text{max}}, \quad \min_{W, |W| = \text{const}} \hat{b}(W, A) = \hat{b}_{\text{min}} \quad \forall A.
\]  

(14)

### 3.3.2 Trustworthiness

The second novel property of trustworthiness defines whether we can trust a bias score to report any bias in accordance to definitions 1 and 2 i.e. the bias score can only reach \( b_0 \), which indicates fairness, if the observed word is equidistant to all protected groups or all words in the observed set of words are unbiased. This is important, because even if a set of words is mostly unbiased or word biases cancel out on average, individual biases can still be harmful and should thus be detected. The requirement for the consistency of the minimal bias score \( b_0 \) can be formulated in a straightforward way using the similarities to the attribute sets \( A \).

**Definition 6** (unbiased-trustworthy). Let \( b_0 \) be the bias score of a bias score function, that is equivalent to no bias being measured. We call the bias score function \( b(W, A) \) unbiased-trustworthy if

\[
b(W, A) = b_0 \iff s(w, A_i) = s(w, A_j) \forall A_i, A_j \in A.
\]

Analogously for scores that use at set of words \( W = \{w_1, ..., w_m\} \), we say \( b(W, A) \) is unbiased-trustworthy if

\[
b(W, A) = b_0 \iff s(w_k, A_i) = s(w_k, A_j) \forall A_i, A_j \in A, k \in \{1, ..., m\}.
\]

(15)

### 3.3.3 Sensitivity to Skew and Stereotype

We formulate two additional criteria in alignment to the skew and stereotype definitions. Since the requirements introduced in [3.3] are critical to achieve comparable and trustworthy measurements in terms of a bias magnitude, the following definitions are simply meant to distinguish what type of biases a bias score function is sensitive towards.

**Definition 7** (skew-sensitive). Let \( b_0 \) be the value of a bias score function, that is equivalent to no bias being measured. We call the bias score function \( b(W, A) \) skew-sensitive if

\[
\exists A_i, A_j \in A : mean_{w \in W} s(w, A_i) > mean_{w \in W} s(w, A_j) \Rightarrow b(W, A) \neq b_0.
\]

(17)

In lay terms, skew-sensitivity reflects the fact that a bias score should increase if neutral words are (on average) more similar to one protected group than another.

**Definition 8** (stereotype-sensitive). Let \( b_0 \) be the value of a bias score function, that is equivalent to no bias being measured. We call the bias score function \( b(W, A) \) stereotype-sensitive if

\[
\exists w_1, w_2 \in W, A_i, A_j \in A : s(w_1, A_i) - s(w_1, A_j) \neq s(w_2, A_i) - s(w_2, A_j) \Rightarrow b(W, A) \neq b_0.
\]

(18)

In lay terms, this definition refers to the fact that an embedding, which places words differently with respect to its relative distance as regards to (opposing) protected groups is biased as regards the axis which is described by these two groups.

As stated in the following theorem, bias score functions that are unbiased-trustworthy are both skew-sensitive and stereotype-sensitive. From there follows that a metric that is not skew-sensitive or not stereotype-sensitive cannot be unbiased-trustworthy.

**Theorem 1.** A bias score function \( b(W, A) \) that is unbiased-trustworthy is also skew-sensitive and stereotype-sensitive.

**Proof.** Let \( W \) be skewed with regard to attributes in \( A \), then, according to Definition 3

\[
\exists A_i, A_j \in A : mean_{w \in W} s(w, A_i) > mean_{w \in W} s(w, A_j).
\]

In that case

\[
\exists w \in W, A_i, A_j \in A : s(w, A_i) \neq s(w, A_j).
\]

Further, let \( b(W, A) \) be a unbiased-trustworthy bias score function. From Definition 6 directly follows that \( b(W, A) \neq b_0 \) given a skewed set of words \( W \).

Analogously, for \( W = W_1 \cup W_2 \) with \( W_1 \) and \( W_2 \) reflecting different stereotypes with regard to (at least) two attributes \( A_i, A_j \in A \) and \( b(W, A) \) an unbiased-trustworthy bias score function:

\[
\exists w_1, w_2 \in W_1 : s(w_1, A_i) - s(w_1, A_j) \neq s(w_2, A_i) - s(w_2, A_j) \Rightarrow \exists w \in \{w_1, w_2\} : s(w, A_i) \neq s(w, A_j)
\]

(21)

\[
\Rightarrow b(W, A) \neq b_0
\]

(22)

(23)
3.4 Properties of bias scores from the literature

As a major contribution of this work, we apply the novel properties for bias score functions to all cosine based bias scores from the literature. Table 1 gives an overview over our findings. The detailed analyses follow in Section 3.4.1 for WEAT, Section 3.4.2 for MAC and Section 3.4.3 for the Direct Bias.

3.4.1 Analysis of the WEAT Score

In the following, we detail properties of the WEAT score in light of the definitions stated above. First, we focus on the word-wise biases as reported by $s(w, A, B)$.

**Theorem 2.** The bias score function $s(w, A, B)$ of WEAT is not magnitude-comparable.

*Proof.* For the proof see Section A in the supplementary material.

**Theorem 3.** The bias score function $s(w, A, B)$ of WEAT is unbiased-trustworthy.

*Proof.* For the proof see Appendix A.

Next, we focus on the properties of the effect size $d(X, Y, A, B)$. Note that the effect size is not specified for cases, where $s(w, A, B) = s(w', A, B)$ or $w, w' \in X \cup Y$ due to its denominator. This is highly problematic considering Definition 6 which states that a bias score should be 0 in that specific case (which implies perfect fairness). Furthermore, the following theorem shows that WEAT can report no bias even if the embeddings contain associations with the bias attributes.

**Theorem 4.** The effect size $d(X, Y, A, B)$ of WEAT is not unbiased-trustworthy.

*Proof.* For the WEAT score $b_0 = 0$. 

Figure 3: Simplified visualization of gender stereotypes in embeddings. The occupation vectors are biased if $0 < \alpha < \theta^2$ or $0 < \beta < \theta^2$. 

| bias score | comparable | trustworthy | bias score | comparable | trustworthy | skew | stereotype |
|------------|-------------|-------------|------------|-------------|-------------|------|------------|
| WEAT_word  | x           | ✓           | WEAT       | ✓           | x           | x    | (✓)        |
| MAC_word   | x           | x           | MAC        | x           | x           | x    | x          |
| DirectBias_word | ✓           | x           | DirectBias | ✓           | x           | x    | x          |
For four words \( w_1, w_2, w_3, w_4 \) and \( s(w_1, A, B) = s(w_3, A, B) \) and \( s(w_2, A, B) = s(w_4, A, B) \), the effect size

\[
d(\{w_1, w_2\}, \{w_3, w_4\}, A, B) = \frac{(s(w_1, A, B) + s(w_2, A, B)) - (s(w_3, A, B) + s(w_4, A, B))}{2 \cdot \text{stddev}_{w \in \{w_1, w_2, w_3, w_4\}} s(w, A, B)}
\]

(24)

(25)

is 0, if \( s(w_1, A, B) \neq s(w_2, A, B) \) (otherwise \( d \) is not defined). Now, for the simple case \( A = \{a\}, B = \{b\} \) and assuming all vectors having length 1, we see

\[
s(w_1, A, B) = s(w_3, A, B) \\
\iff a \cdot w_1 - b \cdot w_1 = a \cdot w_3 - b \cdot w_3 \\
\iff a \cdot (w_1 - w_3) - b \cdot (w_1 - w_3) = 0 \\
\iff (a - b) \cdot (w_1 - w_3) = 0.
\]

(26)

This implies that, if the two vectors \( a - b \) and \( w_1 - w_3 \) are orthogonal (and e.g. \( s(w_2, A, B) = 0 \)), the WEAT score returns 0. In this case, there exist \( a, b, w_1, w_3 \) with \( s(w_1, A, B) = s(w_3, A, B) \neq 0 \) and accordingly \( s(w_1, A) \neq s(w_1, B) \). See Figure 4 for an example, with \( a = \text{he}, b = \text{she}, w_1 = \text{engineer}, w_3 = \text{secretary} \).

In order to show that the effect size of WEAT is magnitude-comparable, we need the following lemma.

**Lemma.** Let \( x_1, \ldots, x_n \in \mathbb{R} \) be real numbers. Let \( \hat{\mu}, \hat{\sigma} \) denote the empirical estimate of mean and standard deviation of the \( x_i \). Then, for any selection of indices \( i_1, \ldots, i_m \), with \( i_j \neq i_k \) for \( j \neq k \), the following bound holds

\[
\left| \sum_{j=1}^{m} \frac{x_{i_j} - \hat{\mu}}{\hat{\sigma}} \right| \leq \sqrt{m \cdot (n-m)}.
\]

Furthermore, for \( 0 < m < n \) the bound is obtained if and only if all selected resp. non-selected \( x_i \) have the same value, i.e. \( x_{i_j} = \hat{\mu} + s \sqrt{\frac{n-m}{m}} \hat{\sigma} \) and all other \( x_{i_k} = \hat{\mu} - s \sqrt{\frac{m}{n-m}} \hat{\sigma} \) with \( s \in \{-1, 1\} \).

**Proof.** For the proof see Appendix A.

**Theorem.** The effect size \( d(X, Y, A, B) \) of WEAT with \( X = \{x_1, \ldots, x_m\}, Y = \{y_1, \ldots, y_m\} \) is magnitude-comparable.

**Proof.** For the proof see Appendix A.

The proof of Theorem shows that the effect size reaches its extreme values only if all \( x \in X \) achieve the same similarity score \( s(x, A, B) \) and \( s(y, A, B) = -s(x, A, B) \) \( \forall y \in Y \), i.e. the smaller the variance of \( s(x, A, B) \) and \( s(y, A, B) \) the higher the effect size. For one thing, this limits WEAT’s predictability to the partitioning of neutral words into \( X \) and \( Y \). On the other hand, the differences \( \text{mean}_{x \in X} s(x, A, B) - \text{mean}_{y \in Y} s(y, A, B) \) does not affect
the effect size values. The dependence on the variance explains the behavior shown in Section 3.1. This shows that we cannot take low effect sizes as a guarantee for low biases in the embeddings.

Finally, we probe the effect size for skew and stereotype sensitivity:

**Theorem 6.** The effect size $d(X, Y, A, B)$ of WEAT is stereotype-sensitive assuming that the groups $X$ and $Y$ correctly classify for the direction of stereotype, i.e. all words in $X$ are closer to the attribute set $A$ (compared to $B$) than words in $Y$:

$$s(x, A) - s(x, B) > s(y, A) - s(y, B) \quad \forall x \in X, y \in Y,$$

or vice-versa:

$$s(x, A) - s(x, B) < s(y, A) - s(y, B) \quad \forall x \in X, y \in Y$$

*Proof.* For the proof see Appendix A.

**Theorem 7.** The bias score function $d(X, Y, A, B)$ of WEAT is not skew-sensitive.

*Proof.* For the proof see Appendix A.

To conclude this theorems, we found that the word-wise bias reported by WEAT is in fact unbiased-trustworthy, but not magnitude-comparable. The effect size, which is applied to measure bias of two sets of target words, is magnitude-comparable, but not unbiased-trustworthy. This is due to the fact that the effect size is sensitive to stereotypical differences between the sets $X$ and $Y$ (which is exactly the intuition behind WEAT), but it is not skew-sensitive. Furthermore the magnitude of the effect size depends on the inner-group variance of $s(w, A, B)$ for groups $X$ and $Y$, but not one the difference between those groups, which should be taken into account when interpreting it.

This shows that WEAT can only be used to quantify stereotypical biases as anticipated by the groups $X$ and $Y$, while skew and stereotypes that contradict the partitioning will not be detected. And most importantly, while high effect sizes are a certain indicator of bias, low effect sizes are not that meaningful. Hence, when it comes to quantitatively comparing biases between different embeddings or evaluating debiasing methods, we argue that WEAT is not sufficient. At most, it could be used as an additional measure for specific stereotypes using, for instance, the tests from Caliskan et al. [2017]. Of course, if ones goal is to only prove the presence of bias instead of quantifying it, WEAT is still useful.

### 3.4.2 Analysis of the MAC Score

On contrary to the bias definitions summarized in Section 2.1, the MAC score follows a slightly different intuition. Instead of contrasting over the association of words towards different attributes (as WEAT does) or measuring the correlation with a bias direction (Direct Bias), it depicts the average correlation of neutral words and bias attributes. In the following we show several situations where the metric thereby contradicts our bias definition. This includes detecting bias on fair embeddings and measuring no bias in biased embeddings.

**Theorem 8.** The bias score function $MAC(T, A)$ of MAC is not unbiased-trustworthy.

*Proof.* For the proof see Appendix A.

**Theorem 9.** The bias score functions $MAC$ and $MAC_{word}$ are not magnitude-comparable.

*Proof.* For the proof see Section A in the supplementary material.

**Theorem 10.** The bias score function $MAC(T, A)$ of MAC is neither stereotype-sensitive nor skew-sensitive.

*Proof.* For the proof see Appendix A.

Considering these theorems, we strongly argue against using the MAC score at all, since it is neither reliable to measure skew nor stereotype. Furthermore, it might falsely detect a bias simply because the embedding vectors are represented closely in general.
3.4.3 Analysis of the Direct Bias

Theorem 11. The Direct Bias is magnitude-comparable for $c \geq 0$.

Proof. For the proof see Appendix A.

Theorem 12. The Direct Bias is not unbiased-trustworthy.

Proof. For the Direct Bias $b_0 = 0$ indicates no bias. Consider a setup with two attribute sets $A = \{a_1, a_2\}$ and $C = \{c_1, c_2\}$ (we cannot call our attribute set $B$ as usual because this could be confused with the bias subspace $B$ or bias direction $b$ from the Direct Bias).

Using the notation from Section 2.2.3, this gives us two defining sets $D_1 = \{a_1, c_1\}$ and $D_2 = \{a_2, c_2\}$. Let $a_1 = (-x, rx)^T, a_2 = (-x, -rx)^T, c_1 = -c_2$ and $r > 1$.

The bias direction is obtained by computing the first principal component over all $(a_i - \mu_i)$ and $(c_i - \mu_i)$ with

$$\mu_i = \frac{a_i + c_i}{2} = 0.$$ Due to $r > 1$, $b = (0, 1)^T$ is a valid solution for the 1st principal component as it maximizes the variance

$$b = \text{argmax}_{\|v\|=1} \sum_i (v \cdot a_i)^2 + (v \cdot c_i)^2. \quad (29)$$

According to the definition in Section 2.1, any word $w = (0, w_y)^T$ would be considered neutral to groups $A$ and $C$ with $s(w, A) = s(w, C)$ and being equidistant to each word pair $\{a_i, c_i\}$.

But with the bias direction $b = (0, 1)^T$ the Direct Bias would report a maximal bias of 1 instead of $b_0 = 0$, which contradicts Definition 6.

On the other hand, we would consider a word $w = (w_x, 0)^T$ maximally biased, but the Direct Bias would report no bias.

Note that the example shown in Theorem 12 is an extreme case. Yet, it shows that a bias direction obtained by the PCA does not necessarily represent individual bias directions appropriately. This can lead to both over- and underestimation bias. If the variance inside the attribute groups is higher than the differences between the attributes, the bias direction (or subspace) is likely to be misleading. (Note that this critique of the Direct Bias does not affect the debiasing algorithm of Bolukbasi et al. [2016], due to the equalize step.)

4 Proposed Metric

To address the shortcomings of the existing bias scores, we propose a new score based on the before mentioned bias definition: Scoring Association Means of Word Embeddings (SAME). It has a similar intuition to WEAT, in terms of measuring polarity between attribute sets, but fulfills the criteria set up in the last chapter. In a first step we detail SAME for a setting with binary attributes and then extend it to the multiple attributes case.

4.1 The binary case

We use one set of target words $W$ and measure the association with two attribute sets $A_i$ and $A_j$. Furthermore, we assume that each attribute vector $a_i \in A_i$ is normalized to unit length, so that $\cos(w, a_i) = \frac{w}{\|w\|} \cdot a_i$. Then, we can write the similarity of a word $w$ towards attribute set $A_i$ as

$$s(w, A_i) = \frac{1}{|A_i|} \sum_{a_i \in A_i} \frac{w}{\|w\|} \cdot a_i \quad (30)$$

$$= \frac{w}{\|w\|} \cdot \left( \frac{1}{|A_i|} \sum_{a_i \in A_i} a_i \right) \quad (31)$$

$$= \frac{w}{\|w\|} \cdot \hat{a}_i \quad (32)$$

where $\hat{a}_i = \left( \frac{1}{|A_i|} \sum_{a_i \in A_i} a_i \right)$.  

![Image](https://via.placeholder.com/150)
Similar to WEAT, we define a pairwise bias comparing the similarity of a word $w$ towards two attribute sets $A_i$ and $A_j$ with $\hat{a}_i \neq \hat{a}_j$:

$$b(w, A_i, A_j) = \frac{s(w, A_i) - s(w, A_j)}{\|\hat{a}_i - \hat{a}_j\|}$$  \hspace{1cm} (33)

Contrary to equation (3) of WEAT we normalize the term, resulting in bias scores in $[-1, 1]$ independent of the attributes. By transforming the equation, we can show that it has a similar notion to the Direct Bias from Bolukbasi et al. [2016], measuring the correlation of words with a bias direction between two attributes. The only difference is that we obtain the pairwise bias direction by averaging over individual directions instead of using the PCA.

The term $b(w, A_i, A_j)$ could be contrasted over pairs $w_1 \in W_1, w_2 \in W_2$, similarly as done in the WEAT score, but this would result in the potential problems stated in Theorem 4 / Figure 4. Hence, to achieve a trustworthy and comparable metric for binary bias attributes, we propose to take the mean absolute values of word-wise biases. This results in the following bias score for a set of target words $W$:

$$b(W, A_i, A_j) = \frac{1}{|W|} \sum_{w \in W} |b(w, A_i, A_j)|.$$  \hspace{1cm} (37)

### 4.2 The multiple attributes case

Now we extend SAME to cases with multiple attributes. Let $A = \{A_0, ..., A_n\}$ with $n \geq 1$ contain at least 2 attribute sets. To measure the bias with respect to all attributes in $A$, we construct a $n$-dimensional bias subspace from binary bias directions $\hat{a}_i - \hat{a}_0$ with $i \in \{1...n\}$ and $\hat{a}_i$ the mean of attributes in $A_i$. Thereby, we assume that $\hat{a}_i \neq \hat{a}_j \forall i, j \in \{1...n\}, i \neq j$.

Let $B$ be the bias subspace, defined by an orthonormal basis $\{b_1, ..., b_n\}$. The first basis vector $b_1$ is obtained from the first binary bias direction, i.e.

$$b_1 = \frac{\hat{a}_1 - \hat{a}_0}{\|\hat{a}_1 - \hat{a}_0\|}. \hspace{1cm} (38)$$

The other basis vectors are obtained from the successive binary bias directions, after removing linear correlations with previous basis vectors, which ensures orthogonality

$$b'_i = (\hat{a}_i - \hat{a}_0) - (\hat{a}_i - \hat{a}_0, b_{i-1})b_{i-1} - ... - (\hat{a}_i - \hat{a}_0, b_0)b_0$$

$$b_i = \frac{b'_i}{\|b'_i\|}. \hspace{1cm} (39)$$

Thereby, we assume that $\|b'_i\| \geq 0$. In the case of linear dependency of the binary bias directions, i.e. one calculated $b'_i$ is a zero vector, this vector can be left out, resulting in a smaller bias space $B$.

Given this subspace, the bias of a word vector $w$ is described by the cosine similarities with the basis vectors of $B$

$$w_B = (\cos(w, b_1), ..., \cos(w, b_n))^T$$  \hspace{1cm} (41)

and the bias magnitude is

$$SAME_{word}(w) := b(w, A) = \|w_B\|$$  \hspace{1cm} (42)

Consequently the overall bias of all target words in $W$ is

$$SAME(W) := b(W, A) = \frac{1}{|W|} \sum_{w \in W} b(w, A).$$  \hspace{1cm} (43)
A major benefit of using a vector \( \mathbf{w}_B \) as an intermediate step to obtain a bias magnitude over multiple attributes is that we achieve an interpretable representation of \( \mathbf{w} \) in terms of the protected groups, i.e. each element in \( \mathbf{w}_B \) represents the association of \( \mathbf{w} \) with the two groups, whose attributes were used to obtain the respective basis vector. Noteworthy, all elements represent bias in comparison to the same default group that is represented by \( A_0 \) and users must be aware that if bias directions correlate with each other, their share will be only accounted for by the first basis vector. Therefore, analysing how the different bias directions correlate with each other is highly recommended to allow more sophisticated insights into how biases manifest in embedding spaces.

4.3 Analysis of SAME

The following theorems and their proofs detail properties of SAME in light of the above stated definitions and show that \( \text{SAME} \) and \( \text{SAME}_{\text{word}} \) are unbiased-trustworthy and magnitude-comparable. Hence, we can state that SAME is a reliable bias score to quantify bias in embeddings and it can be compared between different embedding models.

**Theorem 13.** The bias score function \( b(\mathbf{w}, A) \) and \( b(\mathbf{W}, A) \) are unbiased-trustworthy.

**Proof.** For the proof see Section B in the supplementary material.

According to Theorem 1, this shows that SAME is sensitive to both skew and stereotype.

**Theorem 14.** The bias score functions \( b(\mathbf{w}, A) \) and \( b(\mathbf{W}, A) \) are magnitude-comparable, assuming the number of protected groups \( n \) is not larger than the dimensionality of the embedding space.

**Proof.** For the proof see Section B in the supplementary material.

4.4 Measuring Skew and Stereotype

Using SAME as defined in the last subsections, we can reliably quantify and compare biases in different embeddings or domains. However, we only get one measurement of the average or word-wise bias magnitude. To get a closer look on how a set of target words is biased, we propose two additional metrics to determine the skew and stereotype. The basic intuition is that the skew is given by the mean of the word-wise bias distribution and the stereotype by the standard deviation. In the following we detail how to calculate this in the case of binary and multiple attributes.

4.4.1 The binary case

The Skew of words in \( W \) given the two attribute sets \( A_i, A_j \) is simply the mean of word-wise biases. Contrary to Equation 37 we obtain a sign indicating the direction of bias.

\[
b_{\text{skew}}(W, A_i, A_j) = \frac{1}{|W|} \sum_{w \in W} b(w, A_i, A_j). \tag{44}
\]

Respectively, the Stereotype is given by the standard deviation:

\[
b_{\text{stereo}}(W, A_i, A_j) = \frac{1}{|W|} \sqrt{\sum_{w \in W} (b(w, A_i, A_j) - b_{\text{skew}}(W, A_i, A_j))^2}. \tag{45}
\]

4.4.2 The multiple attributes case

To obtain a meaningful and interpretable Skew and Stereotype in terms of \( n > 2 \) attribute sets, we suggest looking at all pairwise bias directions, i.e. all pairs \( (A_i, A_j) \) \( A_i, A_j \in A, i \neq j \) or contrasting over each group \( A_i \) compared to the union of all other sets \( \bigcup_{A_j \in A, j \neq i} A_j \). This allows to identify the major directions of skew or stereotype, indicating attribute groups that are particularly prone to biases.

4.4.3 Analysis of Skew and Stereotype Extensions

**Theorem 15.** The Skew extension \( b_{\text{skew}}(W, A_i, A_j) \) is skew-sensitive, but not stereotype-sensitive.

**Proof.** For the proof see Appendix B.

**Theorem 16.** The Stereotype extension \( b_{\text{stereo}}(W, A_i, A_j) \) is stereotype-sensitive, but not skew-sensitive.
Proof. For the proof see Appendix B.

Since both extensions are either not skew-sensitive or not stereotype-sensitive, they are not unbiased-trustworthy (see Theorem 1). In this regard, one of these score functions cannot replace SAME in its general form as explained in Sections 4.2 and 4.1. However, one could use the Skew and Stereotype score functions jointly.

Theorem 17. The Skew $b_{\text{skew}}(W, A_i, A_j)$ is magnitude-comparable.

Proof. For the proof see Appendix B.

Theorem 18. The Stereotype $b_{\text{stereo}}(W, A_i, A_j)$ is magnitude-comparable.

Proof. For the proof see Appendix B.

5 Experiments

To emphasize the drawbacks and benefits of the different score functions, we conducted several experiments. The most important point is to highlight the sensitivity of the score functions to different kinds of biases in the training data (skew, stereotype, absolute bias as explained later). Furthermore, we emphasize the important differences between our and the state of the art score functions, in relation to the theoretical flaws discussed in Chapter 3.

In order to achieve a ground truth for biases to measure, we constructed biased datasets and fine-tuned BERT on these using the masked language objective. The datasets consisted of a variety of sentences, including a gender-neutral occupation and either male or female pronoun referring to that occupation. Only the pronouns were masked out during training, forcing the model to explicitly learn the gender probabilities associated with each occupation. After training all bias score functions were applied to measure gender bias with these occupations. We ran this procedure for a large number of datasets, varying with regard to each occupation’s gender probability and the overall gender distribution. This allowed us to produce a large number of models with different expressions of bias. To confirm that the biases were actually learned, we probed the resulting masked language models for probabilities of inserting male/female pronouns in the training sentences. This approach is similar to the one from Kurita et al. [2019], who also measured bias using the masked language objective. In the following, we explain the procedure of generating datasets and training in detail. Then we evaluate the performance of bias scores, first regarding individual words, then regarding sets of words. In both cases, we conduct additional experiments, highlighting the robustness of SAME in comparison to WEAT and the drawbacks of computing the Bias Directions by PCA instead of our approach.

5.1 Training Procedure

Our goal was to test for both word-specific biases as well as biases in terms of a set/domain of words. Therefor, we decided to observe gender bias in occupations, similarly to the experiments of Bolukbasi et al. [2016]. We used the gender attributes and occupations from their implementation, although we removed some occupations that were not gender neutral, resulting in 258 occupations. The final lists can be found in Table 5 in the appendix.

We further constructed 30 template sentences (see Table 6 in the appendix), e.g. ‘[MASK] is a OCCUPATION’ or ‘the OCCUPATION enjoyed [MASK] lunch’, where [MASK] could either be substituted by he/she or his/her. Given a gender probability for each occupation (e.g. ‘nurse’ is 60% male, 40% female), we inserted the occupation in each of the template sentences and assigned a pronoun, which was randomly selected based on the gender probability, as solution for the masked language task. This results in 7770 sentences for fine-tuning BERT, holding 30% back as validation set. We used a pre-trained model (‘bert-base-uncased’) and trained for 5 epochs.

To verify that BERT learned the gender associations implied by the training data, we ran an unmasking task on the fine-tuned model using a separate test set constructed with another set of 20 template sentences (see Table 7 in the appendix), where we inserted the occupations. For each sentence, we queried the probability of inserting the male/female pronoun (later called unmasking bias), then took the average over all sentences with the same occupation and measured the $R^2$ correlation of the results with the gender probabilities in the training data (later called training bias). If the correlation was above a threshold of 0.7, the model was assumed to have learned the bias well enough to be considered for further experiments.

We repeated this to generate a variety of biased models. For each model, we defined a normal distribution of gender probabilities (for inserting the male pronoun) by $\mu$ and $\sigma$, then randomly selected the occupation’s gender probability

\[1\] https://github.com/tolga-b/debiaswe
Figure 5: Example for correlation of word-wise biases measured with the different bias scores compared to training biases (probability for male pronoun) $p$. Since MAC and the Direct Bias do not measure the direction of bias, we display $0.5 - p$ on the x-axis.

(later referred as predefined bias) from this distribution, then constructing the training data as described above. For $\mu$ we used values in \{0.25, 0.3, 0.35, ..., 0.75\} and $\sigma$ in \{0.1, 0.15, ..., 0.35\} to elaborate how well the bias scores react to either shifted distributions (skew) or variance of distributions (stereotype). For each combinations of $\mu$ and $\sigma$, we trained 5 models using the same predefined biases, but generating the sentences and selecting the training set each time to produce randomness. This resulted in 330 models in total.

In the following sections, we usually refer to the training bias (the gender probabilities in the actual training data) as opposed to the predefined biases. We first experiment with word-wise biases. Later, we focus on the bias distribution parameters $\mu$, $\sigma$ and the absolute amount of bias comparing them with the bias scores over all occupation, where each model makes up one data point.

5.2 Performance of word-wise bias scores

To evaluate the performance of bias scores on a word-level, we measured the $R^2$ correlations of word-wise biases with the respective training biases as illustrated in Figure 5. As explained in the last section, we consider the unmasking task as a sanity check and baseline for the best possible outcome. The word-wise bias scores reported here are Eq. 36 for SAME, Eq. 9 with $N = \{w\}$ and $c = 1$ for the Direct Bias and Eq. 8 with $T = \{w\}$ for MAC. Since Eq. 3 of WEAT only differs from our pairwise bias (Eq. 36) by a constant factor (given fixed attribute sets), it will result in the same correlation. Hence, both bias scores are reported in one graph. As can be seen in the figure, WEAT and SAME outperform MAC and the Direct Bias significantly.

We further calculated these correlations for every fine-tuned model and display the mean and standard deviation of correlations in Figure 6. The cosine based metrics cannot match the unmasking tasks, but with the mean correlations of around 0.4 for WEAT/SAME, they still outperform MAC with mean correlation around 0 and Direct Bias around 0.1. However, we also see a high standard deviation for WEAT/SAME, which indicates that for some models, the score functions word considerably well (similar to Figure 5), while for other models, they perform quite bad. This could be an indication that those models represent the biases in a non-linear way that impacts the unmasking task (and possibly other downstream tasks), but does not show with cosine based scores.

5.2.1 Approaches to compute the Bias Direction

As seen in the previous section, though similar to SAME, the Direct Bias performs significantly worse. Comparing the Equations 9 and 36, we can see that the word-wise biases both measure the cosine between a word $w$ and a bias direction. In case of the Direct Bias this direction is obtained by PCA, in case of the proposed metric simply by the mean bias direction. Hence, the difference in performance must be linked to the way the bias direction is determined. To confirm this, we conducted the following experiment:

We selected a number of models where the word-wise bias scores reported by the Direct Bias correlated similar well to those reported by SAME (difference in correlation < 0.25) and a number of models, where the Direct Bias correlated worse (difference in correlation > 0.4). In both cases we considered only such models, where SAME correlated with at least 0.6. For both sets of models, we reported the mean angle between bias directions computed by PCA and by averaging over individual bias directions.
Evaluating Metrics for Bias in Word Embeddings

Figure 6: The average correlation of word-wise bias scores with the training and unmasking bias. WEAT and SAME are reported jointly since they only differ by a constant factor and thus share the same correlation.

Figure 7: In this example, the Direct Bias shows similar results to WEAT/own metric regarding the word-wise correlation, while both bias directions are very similar.

| similar correlation | 10.859 |
|---------------------|--------|
| worse correlation   | 17.551 |

According to these results, the bias direction computed by PCA differs from the average bias direction in an actual use case and this can be linked with lower performance of the Direct Bias. The following Figures 7 and 8 show exemplary cases with similar bias directions and performance and one with a greater difference between bias directions and performance.

5.2.2 Comparability of word-wise biases

As stated in Definition 5, an important property of a bias score is the comparability of its results. We showed in Theorem 2 that the extrema of word-wise biases calculated by WEAT depend on the average distance between bias attributes in the embedding space. Hence those biases are not comparable between different embedding models. Opposed to this, our bias score, SAME, mitigates this effect by normalizing over said distance and is thus comparable. To highlight this effect in a practical setting and show that our proposed metric is more robust in this regard, we conducted the following experiment:

As mentioned before, we generated 66 different distributions (combinations of $\mu$ and $\sigma$) of predefined biases, where each occupation was assigned a certain gender probability. Based on each distribution (with fix gender probability per occupation) we generated 5 sets of training and testing data and fine-tuned BERT on each of these. Since the predefined biases were identical for each of the 5 models (yet with random noise due to train/test split and sentence generation),
Evaluating Metrics for Bias in Word Embeddings

Table 2: Robustness of word-wise bias scores. We compute the standard deviations of word-wise biases for each model and then the percentage difference between each set of 5 models trained with the same training bias. Lower values indicate more robust scores.

| Direct Bias | WEAT | SAME | unmask |
|-------------|------|------|--------|
| 0.306       | 0.150| 0.121| 0.096  |

Figure 8: In this example, the Direct Bias performs far worse than WEAT/own metric regarding the word-wise correlation, since the bias direction computed by the PCA differs from the ideal bias direction.

Table 2 shows that SAME is indeed more robust than WEAT and achieves nearly the same robustness as the unmasking bias. However, the Direct Bias performs even worse than WEAT. Considering the similarity of the Direct Bias and SAME, a possible explanation is that the different fine-tuned models also represent the bias attributes in a distinctive way leading to greater variations in the bias direction obtained by PCA compared to our approach. This suggest that using the mean bias direction is preferable to the bias direction by PCA.

5.3 Performance of bias scores for sets of words

In the next step, we evaluated the bias scores applicable to a set of words $W$ or sets of words (e.g. $X$, $Y$ for WEAT). This is critical to distinguish SAME from WEAT and evaluate the skew and stereotype adaptions. In our case $W$ is the list of occupations and the groups $X$, $Y$ for WEAT are selected using the stereotype ratings provided with the occupations from Bolukbasi et al. [2016], reflecting actual stereotypes in society.

In Figure 9, we illustrate the correlations of all bias scores discussed in the paper. Here we use the unmasking task as reference to measure correlation. We expect this to be more accurate than comparison to the training bias, since the unmasking bias undoubtedly reflects biases learned by the model. We consider the mean and standard deviation of word-wise unmasking biases, as well as the absolute bias ($|0.5 - p|$ with $p$ the probability of male pronouns as predicted by the unmasking task).

As expected after investigating the word-wise bias correlations, MAC and the Direct Bias perform rather bad. WEAT produces better results compared to the unmasking standard deviation, but SAME (standard version) also outperforms WEAT with regard to the unmasking standard deviation and absolute biases. Other than that, our proposed stereotype metric outperforms all metrics from the literature in terms of standard deviation and absolute bias by a large margin. Regarding the mean of word biases, our skew implementation is the only score that achieves a significant correlation with 0.21. Hence, our proposed score function and its versions are clearly preferable over the metrics from the literature and this experiment further emphasizes the benefits of using the skew and stereotype scores. Yet, keep in mind that these two should be used jointly to prevent overlooking the other kind of bias. Nevertheless, we conduct another experiment to highlight the robustness of our score function.

5.3.1 Robustness to Permutation and Subset Selection

When observing bias in certain domains or groups of target words, we might not always have the resources or knowledge to measure bias over all relevant words (or sentences). An example are the different WEATs (and their
Figure 9: $R^2$ correlation of bias scores (measured over all occupations) with unmasking bias. Bold values indicate a p-value below 0.05. Regarding the mean of biases only our skew score produces a significant correlation. Regarding the standard deviation SAME, our stereotype and WEAT correlate significantly, though the stereotype clearly outperforms the others. Regarding the absolute bias, SAME also surpasses WEAT, though the stereotype version works even better.

Table 3: Robustness to permutation and subset selection of neutral words. We report the mean difference in biases reported over subsets $W_i$ compared to biases reported over all neutral words in $W$. Lower values indicate more robust models.

| MAC   | WEAT  | Direct Bias | SAME  | SAME_{stereo} | SAME_{skew} |
|-------|-------|-------------|-------|---------------|-------------|
| 0.0010| 0.0397| 0.0018      | 0.0016| 0.0016        | 0.0022      |

versions/ implementations in the literature), where only a limited number of words is used to measure bias in terms of a concept (e.g. the career/family test). Using only a small subset poses the risk to misjudge bias due to noise or individual words’ associations rather than associations of the whole concept. In this experiment, we investigate how robust the different bias scores are to this problem. For each model trained, we determine the bias over all occupations in $W$, then run a permutation test ($n = 100$ iterations) and determine the bias over subsets $W_i$ with half the number of occupations, randomly selected from $W$. We report $\frac{1}{n} \sum_{i=1}^{n} |b(W_i) - b(W)|$ for each model, i.e. the mean differences in biases over $W_i$ compared to $W$ with $b(W')$ to be replaced by concrete bias scores. We further take the mean over all models and normalize by the size of bias score intervals. Thereby, we obtain the results shown in Table 3. We observe similar results for all score functions beside WEAT, for which we report differences larger by one order of magnitude. This is most likely due to the splitting of target words into groups $X, Y$, which is unique for WEAT. Hence, WEAT is far less robust to subset selection than the other metrics.

6 Conclusions

In conclusion, we proved in Chapter 3.3 that all existing cosine based bias scores have one or several drawbacks that make them unreliable to quantify bias. The baseline definition for this claim is linked in the literature, e.g. Bolukbasi et al. [2016], Caliskan et al. [2017]. Nevertheless, we also showed in our experiments that the weaknesses of the bias scores manifest in practice, which further confirms our statement. Furthermore, we proposed three new bias scores:
Table 4: Overview over the properties of bias scores.

| bias score     | comparable | trustworthy | bias score     | comparable | trustworthy | skew       | stereotype |
|----------------|------------|-------------|----------------|------------|-------------|------------|------------|
| WEAT\textsuperscript{word} | x          | ✓           | WEAT           | ✓          |             | x          |            |
| MAC\textsuperscript{word}   | x          | x           | MAC            | x          | x          | x          |            |
| DirectBias\textsuperscript{word} | ✓          | x           | DirectBias     | ✓          | x          | x          | x          |
| SAME\textsuperscript{word}   | ✓          | ✓           | SAME           | ✓          | ✓          | ✓          | ✓          |
| SAME\textsuperscript{stereo} | ✓          | ✓           | ✓              | ✓          |             |            |            |
| SAME\textsuperscript{skew}   | ✓          | ✓           | ✓              | ✓          |             |            |            |

SAME for bias quantification that suffices all requirements stated in Chapter 3.3 and two versions to distinguish between skew and stereotype.

An overview over the capabilities and drawbacks of the bias scores can be found in Table 4.

Based on both the theoretical evaluation and our experiments we argue against using the Direct Bias and MAC, as we don’t see advantages in them. We acknowledge that WEAT can be useful when people are interest in a specific kind of stereotype (aligning with groups X and Y), but caution to treat the results carefully in light of our findings. While high effect sizes with a low p-value prove the presence of stereotypes, low effect sizes (and high p-values) are not particularly meaningful. Hence WEAT cannot be used for quantitative bias measurements, but only as an indicator. SAME on the other hand, can be used for quantification as it will report any bias in terms of the geometrical definition and provide comparable results. The skew and stereotype complement this to allow better insights and proved particularly useful in the experiments regarding both the correlation with our baseline and robustness.

Yet, our experiments also show that none of the cosine based scores can entirely grasp the biases manifesting in the models (both in relation to the training data as well as the unmasking task). This shows the need to evaluate biases also in terms of a downstream task if deploying the model in that context or possibly considering other tests in addition. In that context, we want to point out two works that used a multitude of tests to enrich their bias evaluation: [Ravfogel et al. 2020] and [Costa-juss and Casas 2019]. While we cannot advocate their choice of cosine based metric, having multiple tests to allow a more thorough evaluation of biases seems like a wise choice as long as there is no consensus about the ideal bias metrics for word embeddings.

Of course, this again shows the importance of research in the field of bias evaluation methods. We would highly recommend looking further into when cosine based metrics fall short or showing whether other tests from the literature (e.g. clustering test) can fill this gap appropriately.

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A Supplementary Materials for Section 3 (Requirements for Bias Metrics)

Theorem 2. The bias score function $s(w, A, B)$ of WEAT is not magnitude-comparable.

Proof. With $\hat{a} = \frac{1}{|A|} \sum_{a \in A} \frac{a}{|a|}$ and $\hat{b}$ analogously defined, we can rewrite

$$s(w, A, B) = \frac{w \cdot \hat{a}}{|w|} - \frac{w \cdot \hat{b}}{|w|}$$

(46)

Thus, applying $f$ does not change the bound and therefore we may reduce to case of $\hat{a}$ and $\hat{b}$.

$$= \frac{w}{|w|} \cdot (\hat{a} - \hat{b})$$

(47)

$$= \cos(w, \hat{a} - \hat{b})||\hat{a} - \hat{b}||.$$  

(48)

Hence we can show that the extrema depend on the attribute sets $A$ and $B$:

$$\max_{w} s(w, A, B) = ||\hat{a} - \hat{b}||,$$  

(49)

$$\min_{w} s(w, A, B) = -||\hat{a} - \hat{b}||$$  

(50)

The statement follows.

Theorem 3. The bias score function $s(w, A, B)$ of WEAT is unbiased-trustworthy.

Proof. This follows directly from the definition of $s(w, A, B)$ (equation 3):

$$s(w, A, B) = s(w, A) - s(w, B) = 0$$

(51)

$$\iff s(w, A) = s(w, B)$$

(52)

Lemma 1. Let $x_1, \ldots, x_n \in \mathbb{R}$ be real numbers. Let $\hat{\mu}, \hat{\sigma}$ denote the empirical estimate of mean and standard deviation of the $x_i$. Then, for any selection of indices $i_1, \ldots, i_m$, with $i_j \neq i_k$ for $j \neq k$, the following bound holds

$$\left| \sum_{j=1}^{m} x_{i_j} - \hat{\mu} \right| \leq \sqrt{m \cdot (n-m)}.$$  

Furthermore, for $0 < m < n$ the bound is obtained if and only if all selected resp. non-selected $x_i$ have the same value, i.e. $x_i = \hat{\mu} + s \sqrt{\frac{n-m}{n-m}} \hat{\sigma}$ and all other $x_k = \hat{\mu} - s \sqrt{\frac{m}{n-m}} \hat{\sigma}$ with $s \in \{-1, 1\}$.

Proof. For cases $m = 0$ or $m = n$ the statement is trivial. So assume $0 < m < n$. Let $f(x) = ax + b$ be an affine function. Then the images of $x_i$ under $f$ have mean $a\hat{\mu} + b$ and standard deviation $|a|\hat{\sigma}$. On the other hand, we have

$$\frac{f(x_i) - (a\hat{\mu} + b)}{|a|\hat{\sigma}} = \frac{ax_i + b - (a\hat{\mu} + b)}{|a|\hat{\sigma}} = \text{sgn}(a) \frac{x_i - \hat{\mu}}{\hat{\sigma}}.$$  

Thus, applying $f$ does not change the bound and therefore we may reduce to case of $\hat{\mu} = 0$ and $\hat{\sigma} = 1$. This allows us to rephrase the problem of finding the maximal bound as an quadratic optimization problem:

$$\min\ s^\top x$$

s.t. $x^\top x = n$  

$1^\top x = 0,$

where $s = (1, \ldots, 1, 0, \ldots, 0)^\top$, $x = (x_1, \ldots, x_n)^\top$ and $1$ denotes the vector consisting of ones only. Notice, that we assumed w.l.o.g. that $i_1, \ldots, i_m = 1, \ldots, m$. Furthermore, we made use of the symmetry properties to replace $\max_{x} |s^\top x|$ by the minimizing statement above, $\hat{\mu} = 0$ is expressed by the last and $\hat{\sigma} = 1$ by the first constrained (recall that $\hat{\sigma} = \sqrt{1/nx^\top x - \hat{\mu}^2}$). Notice, that $\nabla_x x^\top x - n = 2x$ and $\nabla_x 1^\top x = 1$ are linear dependent if and only if $x = a 1$ for some $a \in \mathbb{R}$, thus, as $0 = a 1^\top 1 = an$ if and only if $a = 0$ and $(0 1)^\top (0 1) = 0$, there is no feasible $x$ for which the KKT-conditions do not hold and we may therefore use them to determine all the optimal points.
The Lagrangian of the problem above and its first two derivatives are given by

\[ L(x, \lambda_1, \lambda_2) = s^\top x - \lambda_1(x^\top Ix - n) - \lambda_2 1^\top x \]
\[ \nabla_x L(x, \lambda_1, \lambda_2) = s - 2\lambda_1 x - \lambda_2 1 \]
\[ \nabla^2_{xx} L(x, \lambda_1, \lambda_2) = -2\lambda_1 I. \]

We can write \( \nabla_x L(x, \lambda_1, \lambda_2) = 0 \) as the following linear equation system:

\[
\begin{bmatrix}
2x_1 \\
2x_2 \\
\vdots \\
2x_n
\end{bmatrix}
\begin{bmatrix}
1 \\
1 \\
\vdots \\
1
\end{bmatrix}
= \begin{bmatrix}
\lambda_1 \\
\lambda_2
\end{bmatrix}
\]
\[
= s
\]

Subtracting the first row from row 2, …, m and row m + 1 from row m + 2, …, n we see that \( 2(x_k - x_1)x_1 = 0 \) for \( k = 1, ..., m \) and \( 2(x_k - x_{m+1})x_1 = 0 \) for \( k = m + 2, ..., n \), which either implies \( x_1 = 0 \) or \( x_1 = x_2 = ... = x_m \) and \( x_{m+1} = x_{m+2} = ... = x_n \). However, assuming \( x_1 = 0 \) would imply that \( x_2 = 1 \) from the first row and \( x_2 = 0 \) from the \( m + 1 \)th row, which is a contradiction. Thus, we have \( x_1 = x_2 = ... = x_m \) and \( x_{m+1} = x_{m+2} = ... = x_n \). But the second constraint from the optimization problem can then only be fulfilled if \( mx_1 + (n - m)x_{m+1} = 0 \) and this implies \( x_{m+1} = \frac{m}{n-m}x_1 \). In this case the first constraint is equal to \( n = mx_1^2 + (n - m)\left(\frac{m}{n-m}x_1\right)^2 \), which has the solution \( x_1 = \pm \sqrt{\frac{n-m}{m}} \).

Set \( x^* = (-\sqrt{\frac{n-m}{m}}, ..., -\sqrt{\frac{n-m}{m}}, \sqrt{\frac{n-m}{m}}, ..., \sqrt{\frac{n-m}{m}}) \). Then \( x^* \) and \( -x^* \) are the only possible KKT points as we have just seen. Plugging \( x^* \) into the equation system above and solving for \( \lambda_{1/2} \) we obtain

\[ \lambda_1^* = -\frac{1}{2\left(\sqrt{\frac{m}{n-m}} + \sqrt{\frac{n-m}{m}}\right)} \]
\[ \lambda_2^* = \frac{\sqrt{\frac{m}{n-m}}}{\sqrt{\frac{m}{n-m}} + \sqrt{\frac{n-m}{m}}} \]

Now, as \( \nabla^2_{xx} L(x^*, \lambda_1^*, \lambda_2^*) = \left(\sqrt{\frac{m}{n-m}} + \sqrt{\frac{n-m}{m}}\right)^{-1} I \) is positive definite, we see that \( x^* \) is a global optimum, indeed. The statement follows.

**Theorem 5**  The effect size \( d(X, Y, A, B) \) of WEAT with \( X = \{x_1, \ldots, x_m\}, Y = \{y_1, \ldots, y_m\} \) is magnitude-comparable.

**Proof.** With \( c_i = s(x_i, A, B), c_{i+m} = s(y_i, A, B), n = 2m, \hat{\mu} = 1/n \sum_{i=1}^{n} c_i \) and \( \hat{\sigma} = \sqrt{1/n \sum_{i=1}^{m} (c_i - \mu)^2} \), we have

\[
d = \frac{1}{m} \sum_{i=1}^{m} c_i - \frac{1}{m} \sum_{i=m+1}^{2m} c_i \]
\[
= \frac{\sum_{i=1}^{m} c_i - \sum_{i=m+1}^{2m} c_i + \sum_{i=1}^{m} c_i - \sum_{i=1}^{m} c_i}{m\hat{\sigma}}
\]
\[
= \frac{2\sum_{i=1}^{m} c_i - 2m\hat{\mu}}{m\hat{\sigma}}
\]
\[
= \frac{2}{m} \sum_{i=1}^{m} \frac{c_i - \hat{\mu}}{\hat{\sigma}} \in [-2, 2]
\]

where the last statement follows from Lemma \( \text{[1]} \) with \( \sum_{i=1}^{m} \frac{c_i - \hat{\mu}}{\hat{\sigma}} \in [-m, m] \). The extreme value \( \pm 2 \) is reached if \( c_1 = \ldots = c_m = -c_{m+1} = \ldots = -c_{2m} \), which can be obtained by setting \( x_1 = \ldots = x_m = -y_1 = \ldots = -y_m \), independently of \( A \) and \( B \) as long as \( A \neq B \) and \( \sum_{a_i \in A} a_i / ||a_i|| \neq 0 \neq \sum_{b_i \in B} b_i / ||b_i|| \).
Theorem 6. The effect size $d(X, Y, A, B)$ of WEAT is stereotype-sensitive assuming that the groups $X$ and $Y$ correctly classify for the direction of stereotype, i.e. all words in $X$ are closer to the attribute set $A$ (compared to $B$) than words in $Y$:

$$s(x, A) - s(x, B) > s(y, A) - s(y, B) \forall x \in X, y \in Y,$$

(27)

or vice-versa:

$$s(x, A) - s(x, B) < s(y, A) - s(y, B) \forall x \in X, y \in Y$$

(28)

Proof. We transform the effect size (Equation (4)) into

$$d(X, Y, A, B) = \frac{\text{mean}_{(x,y) \in (X,Y)} s(x, A, B) - s(y, A, B)}{\text{stddev}_{w \in X \cup Y} s(w, A, B)}.$$  (55)

From our assumption follows that $|d(X, Y, A, B)| \neq 0$. □

Theorem 7. The bias score function $d(X, Y, A, B)$ of WEAT is not skew-sensitive.

Proof. Let $X \cup Y = \{x, y\}$ be a skewed set with $s(w, A) > s(w, B) \forall w \in X \cup Y$. Assuming that $s(x, A, B) = s(y, A, B) \neq 0$, which depicts a skew but no stereotype, from the definition of the effect size follows:

$$d(X, Y, A, B) = 0,$$

(56)

which contradicts equation (17) with $b_0 = 0$ for WEAT. □

Theorem 8. The bias score function $MAC(T, A)$ of $MAC$ is not unbiased-trustworthy.

Proof. An ideal MAC score, indicating fairness, would be $b_0 = \pm 1$. We again consider the example case depicted in Figure 3 with $w = \text{secretary}, A_1 = \{\text{she}\}, A_2 = \{\text{he}\}$.

First, consider $\theta = \pi$ and $0 < \alpha < \frac{\theta}{2}$. In that case, secretary would be closer to she than he, which reflects a biased representation and does not satisfy Eq. (15). Yet, if we apply the MAC score $MAC(T, A)$ with $T = \{w\}, A = \{A_1, A_2\}$, due to the symmetry of the cosine function we get

$$MAC(T, A) = \frac{1}{2}((1 - \cos(\alpha)) + (1 - \cos(\pi - \alpha)))$$

(57)

$$= 1,$$

(58)

which implies the word secretary to be perfectly fair embedded.

As a second example, if we considered $\alpha = \frac{\theta}{2}, \theta \neq \pi$, and thus secretary being equidistant to $A_1$ and $A_2$. Yet, we would get

$$MAC(T, A) = \frac{1}{2}((1 - \cos(\alpha)) + (1 - \cos(\theta - \alpha)))$$

(59)

$$= 1 - \cos \left( \frac{\theta}{2} \right).$$

(60)

Now, due to $\theta \neq \pi$, we still measure a bias, although based on eq. (15) the occupation is unbiased towards both gender words. Also, the larger $|\theta - \pi|$ the more extreme is the bias according to the MAC score. □

Theorem 9. The bias score functions $MAC$ and $MAC_{\text{word}}$ are not magnitude-comparable.

Proof. With $\hat{a}_i = \frac{1}{|A_i|} \sum_{a_i \in A_i} \frac{a_i}{||a_i||}$, $|A_i| = |A_j| \forall A_i, A_j \in A$ and $W = \{w\}$ we can rewrite MAC:

$$MAC(W, A) = \frac{1}{|A|} \sum_{A_j \in A} \frac{w \cdot \hat{a}_i}{||w||}$$

$$= \frac{w}{||w||} \cdot \frac{1}{|A|} \sum_{A_j \in A} \hat{a}_i$$

$$= \cos(w, \hat{a}) ||\hat{a}||,$$

(61)

(62)

(63)
with $\hat{a} = \frac{1}{|A|} \sum_{A_i \in A} \hat{a}_i$. Hence we can show that the extrema depend on the attribute sets $A$ and $B$:

\begin{align}
\max_W MAC(W, A) &= ||\hat{a}||, \\
\min_W MAC(W, A) &= -||\hat{a}||
\end{align}

Choosing $|W| > 1$ does not change the extrema. The statement follows.

**Theorem 10** The bias score function $MAC(T, A)$ of MAC is neither stereotype-sensitive nor skew-sensitive.

**Proof.** We showed in the proof of Theorem 8 that if the angle between both attributes is $\pi$, the average bias of an individual word will be $b_0 = 1$. Now consider a purely stereotypical scenario with secretary and engineer as depicted in Figure 3 with $\theta = \pi$, and $\alpha = \beta$. With both individual words’ MAC scores being 1, the MAC of both words will also be 1. On the other hand, a purely skewed representation with engineer = secretary would also result in a MAC score of 1.

**Theorem 11** The Direct Bias is magnitude-comparable for $c \geq 0$.

**Proof.** For $c \geq 0$ the bias of one word $|\cos(w, g)|^c$ is in $[0, 1]$. Calculating the mean over all words in $W$ does not change this bound.

## B Supplementary Materials for Section 4 (Proposed Metric)

**Theorem 13** The bias score function $b(w, A)$ and $b(W, A)$ are unbiased-trustworthy.

**Proof.** The bias score indicating no bias is $b_0 = 0$. First, we can state that

\begin{equation}
b(w, A_i, A_j) = 0 \iff s(w, A_i) = s(w, A_j), \tag{66}
\end{equation}

which directly follows from the definition. Hence

\begin{align}
b(w, A) &= ||w_B|| = ||(b(w, A_0, A_1), ..., b(w, A_0, A_n))^T|| = 0, \\
&\iff s(w, A_i) = s(w, A_0) \forall A_i \in A, \tag{67}
\end{align}

and

\begin{equation}
b(W, A) = 0 \iff b(w, A) = 0 \forall w \in W. \tag{70}
\end{equation}

**Theorem 14** The bias score functions $b(w, A)$ and $b(W, A)$ are magnitude-comparable, assuming the number of protected groups $n$ is not larger than the dimensionality of the embedding space.

**Proof.** For SAME we now show that $b_{\min} = 0$, $b_{\max} = 1$ and both can be reached independent of $A$. Since $A$ defines the bias space $B$ used by SAME, we need to show that the extreme values can be reached independent of $B$. First, we can state that

\begin{align}
\max_W b(W, A) &= \max_w b(w, A), \\
\min_W b(W, A) &= \min_w b(w, A)
\end{align}

which is derived directly from the definition of $b(W, A)$. With an embedding space in $\{d_1, ..., d_n\}$ the orthonormal basis for the bias space $B$, we can write any vector $w \in \mathbb{R}^d$ as a linear combination of its parts $w_{||B} \in B$ and $w_{\perp B} \notin B$ and the former one as the sum of projections onto the basis vectors

\begin{equation}
w = w_{||B} + w_{\perp B} = w_{||B} + \sum_i (w, d_i) d_i. \tag{73}
\end{equation}

With $||b_i|| = 1$ and $b_i \perp b_j \forall i, j \in \{1, ..., n\}, i \neq j$ and

\begin{equation}
w_B = \left(\cos(w, b_1), ..., \cos(w, b_n)\right)^T = \frac{1}{||w||} \left(\langle w, d_1 \rangle, ..., \langle w, d_n \rangle\right)^T \tag{74}
\end{equation}
follows

\[ b(w, A) = \frac{1}{||w||} ||w_B||. \] (76)

With \( ||w_B|| \leq ||w|| \) we can determine the upper bound of the bias magnitude and the lower bound follows directly from the definition:

\[ 0 \leq b(w, A) \leq 1 \] (77)

To show that both extreme cases can be reached independent of \( A \), we consider the following extreme cases: First, let \( w \) be orthogonal to the bias space \( B \), i.e. \( w = w_{\perp B} \), which is possible as long as the bias space \( B \) is lower dimensional than the embedding space. Then follows \( ||w_B|| = 0 \implies b(w, A) = 0 \). Since \( B \) has \( n-1 \) dimensions for \( n \) attribute groups, this requires that an embedding space with \( d \geq n \) dimensions as provided in the statement. Secondly, let \( w \) be entirely defined in the bias space, i.e. \( w = w_B \). Then follows \( ||w_B|| = ||w|| \implies b(w, A) = 1 \). Hence the statement follows.

\[ \square \]

**Theorem 15** The Skew extension \( b_{skew}(W, A_i, A_j) \) is skew-sensitive, but not stereotype-sensitive.

**Proof.** The bias score indicating no bias is \( b_0 = 0 \). Let \( A_i, A_j \) be attribute sets with

\[ mean_{w \in W} s(w, A_i) > mean_{w \in W} s(w, A_j), \] (78)

i.e. the set of words \( W \) is skewed towards \( A_i \). We can reshape Eq. 44 to

\[ b_{skew}(W, A_i, A_j) = \frac{1}{|W|} \sum_{w \in W} b(w, A_i, A_j) \] (79)

\[ = \frac{mean_{w \in W} s(w, A_i) - s(w, A_j)}{|\hat{a}_i - \hat{a}_j|} \] (80)

\[ = \frac{mean_{w \in W} s(w, A_i)}{|\hat{a}_i - \hat{a}_j|} - \frac{mean_{w \in W} s(w, A_j)}{|\hat{a}_i - \hat{a}_j|} > 0 \] (81)

Hence, Equation 44 meets the condition of Definition 7.

Regarding the stereotype, let \( W = \{w_1, w_2\} \) be a word pair with \( s(w_1, A_i) - s(w_1, A_j) = -(s(w_2, A_i) - s(w_2, A_j)) \), i.e. one word \( w_i \) is stereotypical for \( A_i \) and the other for \( A_j \), both with the same magnitude of bias. However, according to Equation 44 \( b_{skew}(W, A_i, A_j) = 0 \), which contradicts Definition 8.

\[ \square \]

**Theorem 16** The Stereotype extension \( b_{stereo}(W, A_i, A_j) \) is stereotype-sensitive, but not skew-sensitive.

**Proof.** The bias score indicating no bias is \( b_0 = 0 \). Let \( w_1, w_2 \in W \) and \( A_i, A_j \) attribute sets with \( s(w_1, A_i) - s(w_1, A_j) \neq (s(w_2, A_i) - s(w_2, A_j)) \), i.e. \( w_1, w_2 \) have different biases w.r.t. \( A_i \) and \( A_j \). Hence we consider one stereotypical for \( A_i \) and the other stereotypical for \( A_j \). From there we can follow that

\[ b(w, A_i, A_j) \neq b_{skew}(W, A_i, A_j) \forall w \in W \] (82)

and hence

\[ b_{stereo}(W, A_i, A_j) = \frac{1}{|W|} \sqrt{\sum_{w \in W} (b(w, A_i, A_j) - b_{skew}(W, A_i, A_j))^2} > 0, \] (83)

which meets the condition of Definition 8.

For the Skew let \( s(w, A_i) - s(w, A_j) = s(w', A_i) - s(w', A_j) > 0 \forall w, w' \in W \). From there follows

\[ mean_{w \in W} s(w, A_i) > mean_{w \in W} s(w, A_j), \] (84)

i.e. the set of words \( W \) is skewed towards \( A_i \). However, it also follows that

\[ b_{stereo}(W, A_i, A_j) = \frac{1}{|W|} \sqrt{\sum_{w \in W} (b(w, A_i, A_j) - b_{skew}(W, A_i, A_j))^2} = 0 \] (85)

since \( b(w, A_i, A_j) = b_{skew}(W, A_i, A_j) \forall w \in W \), which contradicts Definition 7.

\[ \square \]
Theorem 17. The Skew \( b_{skew}(W, A_i, A_j) \) is magnitude-comparable.

Proof. The minimal score is \(-1\), the maximum 1. As shown in Theorem 14 \( b(w, A_i, A_j) \in [-1, 1] \), independent of \( w \). Taking the mean over \( w \in W \) does not change the extrema, hence Equation 44 meets the condition of Definition 5.

Lemma 2. Given values \( x \in [-1, 1] \) \( \forall x \in X \) (or more general a random variable \( X \) taking on values in \([-1, 1]\)), the standard deviation \( \sigma(X) \) is in bounds \([0, 1]\).

Proof. The standard deviation is defined as
\[
\sigma(X) := \sqrt{\text{var}(X)} = \sqrt{\mathbb{E}(X^2) - \mathbb{E}(X)^2}.
\]
Thus, by Jensens inequality and the fact that the bounds of \( X \) also bound the expectation, it follows
\[
0 \leq \mathbb{E}(X)^2 \leq \mathbb{E}(X^2) \leq 1 \quad | - \mathbb{E}(X)^2 \Rightarrow \mathbb{E}(X^2) - \mathbb{E}(X)^2 \leq \mathbb{E}(X^2) - \mathbb{E}(X)^2 \leq 1 \quad \mathbb{E}(X)^2 \leq \mathbb{E}(X)^2 \leq 1.
\]
Hence, the statement follows.

Theorem 18. The Stereotype \( b_{stereo}(W, A_i, A_j) \) is magnitude-comparable.

Proof. The minimal score is 0, the maximum 1. As shown in Theorem 14 \( b(w, A_i, A_j) \in [-1, 1] \), independent of \( w \). Taking the standard deviation over all \( w \in W \) results in values in \([0, 1]\) as explained in Lemma 2. Hence Equation 45 meets the condition of Definition 5.

C Supplementary Materials for Section 5 (Experiments)
### List of occupations

| List of occupations |
|---------------------|
| caretaker | dancer | homemaker | librarian | nurse | hairdresser |
| housekeeper | secretary | teacher | librarian | receptionist | stylist |
| interior designer | clerk | educator | instructor | environmentalist | fashion designer |
| paralegal | therapist | dermatologist | soloist | organist | planner |
| radiologist | singer | songwriter | socialite | artist | tutor |
| violinist | vocalist | pediatrician | performer | photographer | lyricist |
| mediator | naturalist | sociologist | baker | substitute | publicist |
| realtor | singer | poet | flight attendant | nurse | counselor |
| photographer | pianist | employee | entertainer | epidemiologist | cellist |
| correspondent | war | secretary | teacher | musician | gardener |
| guidance counselor | student | dancer | homemaker | receptionist | psychologist |
| painter | writer | librarian | nurse | fashion designer | author |
| artist | dancer | librarian | nurse | fashion designer | author |
| musician | painter | student | dancer | fashion designer | author |
| dancer | painter | student | dancer | fashion designer | author |

Table 5: List of occupations used in the experiments.
### Template sentences for training

| pronouns | sentences |
|----------|-----------|
| he/she   | [MASK]’s a OCCUPATION  
 [MASK] is a OCCUPATION  
 [MASK] just started working as a OCCUPATION  
 [MASK] is working as a OCCUPATION  
 [MASK] was a OCCUPATION  
 the OCCUPATION was in a hurry because [MASK] was late for work  
 [MASK] is interested in becoming a OCCUPATION  
 [MASK] does not like being a OCCUPATION  
 [MASK] likes being a OCCUPATION  
 [MASK] always wanted to be a OCCUPATION  
 [MASK] never wanted to be a OCCUPATION  
 [MASK] had an interview for a position as a OCCUPATION  
 is [MASK] a OCCUPATION  
 [MASK] is a OCCUPATION, right?  
 [MASK] is proud to be a OCCUPATION  
 the OCCUPATION took a week off, because [MASK] wanted to go on vacation  
 "i want to be a OCCUPATION when i grow up", [MASK] said  
 "being a OCCUPATION sounds boring", [MASK] said  
 the OCCUPATION is very skilled. [MASK] has a lot of experience |
| him/her  | the OCCUPATION enjoyed [MASK] lunch  
 the OCCUPATION missed [MASK] bus  
 the OCCUPATION arrived in [MASK] car  
 the OCCUPATION asked [MASK] boss for a promotion  
 the OCCUPATION collected [MASK] check  
 the OCCUPATION was chatting with [MASK] colleagues  
 the OCCUPATION loves [MASK] job  
 the OCCUPATION does not like [MASK] job  
 the OCCUPATION went on vacation with [MASK] family  
 the OCCUPATION went on vacation with [MASK] friends  
 it was [MASK] choice to become a OCCUPATION  
 the OCCUPATION thanked [MASK] colleagues  
 the OCCUPATION likes to spend [MASK] free time with friends  
 the OCCUPATION likes to spend [MASK] free time with the family  
 the OCCUPATION gets along really well with [MASK] colleagues |

Table 6: Template sentences used for training BERT in the experiments. The [MASK] token was to be replaced by the respective pronouns and OCCUPATION by one of the occupations mentioned in Table 5.
### Template sentences for testing

| pronouns | sentences |
|----------|-----------|
| **he/she** | [MASK] is one of the OCCUPATIONS  
[MASK] is well paid for a OCCUPATION  
[MASK] is paid less than the other OCCUPATIONs  
[MASK] thinks about becoming a OCCUPATION  
as a child [MASK] wanted to become a OCCUPATION  
the OCCUPATION was early because [MASK] overslept  
I thought [MASK] was a OCCUPATION  
the OCCUPATION was early because [MASK] could not sleep anyways  
the OCCUPATION is nice but [MASK] is not very competent  
the OCCUPATION is very knowlegeable but [MASK] is bad with people |
| **him/her** | the OCCUPATION started [MASK] new position on monday  
the OCCUPATION lost [MASK] job  
the OCCUPATION left early to pick up [MASK] children from school  
the OCCUPATION told everyone about [MASK] weekend  
the OCCUPATION likes to keep [MASK] workplace clean  
the OCCUPATION does not like [MASK] colleagues  
the OCCUPATION likes [MASK] colleagues very much  
the OCCUPATION smiled and showed [MASK] teeth  
the OCCUPATION wore [MASK] favorite jacket  
i asked the OCCUPATION about [MASK] new book |

Table 7: Template sentences used for testing the unmasking bias in the experiments. The [MASK] token was to be replaced by the respective pronouns and OCCUPATION by one of the occupations mentioned in Table 5.