Empowered Neural Cellular Automata

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ABSTRACT
Information-theoretic fitness functions are becoming increasingly popular to produce generally useful, task-independent behaviors. One such universal function, dubbed empowerment, measures the amount of control an agent exerts on its environment via its sensorimotor system. Specifically, empowerment attempts to maximize the mutual information between an agent’s actions and its received sensor states at a later point in time. Traditionally, empowerment has been applied to a conventional sensorimotor apparatus, such as a robot. Here, we expand the approach to a distributed, multi-agent sensorimotor system embodied by a neural cellular automaton (NCA). We show that the addition of empowerment as a secondary objective in the evolution of NCA to perform the task of morphogenesis, growing and maintaining a pre-specified shape, results in higher fitness compared to evolving for morphogenesis alone. Results suggest there may be a synergistic relationship between morphogenesis and empowerment. That is, indirectly selecting for coordination between neighboring cells over the duration of development is beneficial to the developmental process itself. Such a finding may have applications in developmental biology by providing potential mechanisms of communication between cells during growth from a single cell to a multicellular, target morphology. Source code for the experiments in this paper can be found at: https://github.com/caitlingrasso/empowered-nca.

CCS CONCEPTS
• Computing methodologies → Artificial life.

KEYWORDS
neural cellular automata, empowerment, morphogenesis

1 INTRODUCTION
Biological development, arguably one of the most complex processes in nature, still admits a host of open questions [25, 32]. Many of these relate to the as yet unknown processes by which cells

Figure 1: (a): Neural cellular automata (NCA) are evolved to grow from an initial seed into a desired target shape (green). The CA’s signaling channel (greyscale) can be used to share information between cells but remains constant during simulation. (b) NCA are evolved for empowerment maximize mutual information between cells’ past actions (cell’s signal) and cells’ future sensors (neighboring signals) resulting in a greater diversity of action, sensor pairs and coordination of the CA’s signaling throughout space and time. (c) We here report that NCAs evolved for both shape matching and empowerment are better able to match the target shape than those evolved for shape matching alone.

and tissues coordinate their actions in space and time. Although many computational models of morphogenesis have been created
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section below, we apply a multiobjective optimization approach
later during development. As described in the methods
CA as an agent, and evolve for maximimal mutual information
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genesis likely requires large-scale and long-term control and/or
shape, a simplification of biological morphogenesis, as morpho-
may guide them toward the desired behavior of matching a target
selecting for empowerment within an evolving population of CAs
increasingly long time horizons. Given this, we hypothesize that
outcomes among its near or possibly distant neighbors, over
is empowered if it can act in such a way as to influence particu-
for agent dynamics seemingly unrelated to the desired behavior
often result in the evolution of the desired behavior anyway. Such
approaches range from diversity [12, 35] and novelty [24] metrics
to information theoretic measures [13, 22, 23, 35]. In one such study
[16], an information theoretic-based fitness function was employed
to increase diversity. Information theoretic fitness functions have
proved particularly useful in the domain of evolutionary robotics
[3], where information flows between the agent’s sensory system,
motor system and environment often result in coordinated, inter-
esting and/or useful robot behaviors [1, 11, 15].

This interest in information flows within robots mirrors an in-
terest in information flows within organisms. Neuroscience has a
long history of using information theory to study functional con-
nectivity in the brain [5] and abstract cognitive phenomena [38],
but there is also some work on using information theory to study
the more grounded aspects of intelligent behavior: sensorimotor
coordination [21, 30]. Robots and cellular automata are simplified
models of biological organisms and processes respectively, and are
thus ideal substrates within which to study growth and behavior
from an information theoretic point of view.

One particular information theoretic metric of interest for evolu-
tionary robotics and cellular automata alike is that of empowerment:
maximizing mutual information between actions in the past and
sensor states in the future [20]. Empowerment is of interest be-
cause it describes, in information theoretic terms, a coordination
and control that can extend over space and time: an agent
is empowered if it can act in such a way as to influence particu-
lar outcomes among its near or possibly distant neighbors, over
increasingly long time horizons. Given this, we hypothesize that
selecting for empowerment within an evolving population of CAs
may guide them toward the desired behavior of matching a target
shape, a simplification of biological morphogenesis, as morpho-
genesis likely requires large-scale and long-term control and/or
coordination.

There are many ways one might attempt to empower a CA. Here,
we start with the most obvious approach: we treat each cell in the
CA as an agent, and evolve for maximal mutual information
between cells’ actions early in development and cells’ sensor states
later during development. As described in the methods
section below, we apply a multiobjective optimization approach
which allows us to evolve for diversity within the CA population,
morphogenesis, empowerment, or any subset of these three.

As we report in the result section, we find here that CAs evolved
for morphogenesis and empowerment exhibit better morphogene-
sis than CAs evolved for morphogenesis alone. This supports the
findings from the evolutionary robotics literature that evolving for
empowerment can indirectly exert selection pressure for desired
behaviors that require sensor/motor coordination. In the discus-
sion section we investigate the patterns of growth, signaling and
empowerment in several random and evolved CAs: the lack of any
obvious relationships there indicate that there remain many sub-
tle interdependencies between information flows within growing
cellular automata and developing organisms yet to be investigated.

2 RELATED WORK

This work builds off of seminal work on empowerment by applying
the concept to a cellular automaton system. Empowerment was
first introduced by Klyubin et al. [19, 20] as a local, universal, task-
dependent intrinsic motivation that can be used as the sole fitness
function in the evolution of an agent’s sensor/actuator placements
to achieve a task. For an in-depth review of empowerment, we refer
the reader to [34].

Most empowerment work to date focuses on an individual agent,
however, of particular interest are the few studies that consider
a multi-agent scenario. Capdepuy et al. [6, 7] investigated the in-
teractions between agents that individually attempt to maximize
empowerment in a shared environment and found that interesting
structures can emerge. Similarly, Huckelberger and Polani
[17] evolved agents for empowerment maximization in a resource-
centric environment which resulted in agents exhibiting biolog-
ically plausible behaviors, such as greed and parsimony. Lastly,
Clements and Polani [9] introduce the idea of team empowerment.
They showed that maximizing empowerment of a sports team of
agents in an Ultimate frisbee simulation can produce recognizable
team sport behaviors. In all current work on multi-agent empower-
ment, agents are considered in the traditional robotic sense with
a conventional sensorimotor apparatus. Additionally, with the ex-
ception of team empowerment, agents work to maximize their
individual empowerment as opposed to that of a collective. We
expand the approach to a non-traditional sensorimotor apparatus
embodied by a neural cellular automata where each cell is an agent
with sensors/actuators defined by the input/output of the neural
network rules. Cells in the CA not only share the same environment
but also are working together to achieve a global task.

The NCA model presented here is largely inspired by two cel-
lar automaton models of morphogenesis. First, Miller [26] used
a boolean feed-forward circuit to represent the rules of a cellular
automata and evolved these circuits to produce CA capable of grow-
ing from a single cell into a French flag. The second is Mordvintsev
et al. [29] who introduced neural cellular automata and successfully
trained a CNN to learn the update rules for a CA capable of growing
from a single cell into complex shapes.

Recently, there have been many works that use NCA models to
achieve morphogenesis in various platforms. Most rely on objective
functions that aim to minimize the distance between the current
shape of the cellular automaton and a pre-specified target shape [8,
We evolve a population of NCAs that attempt to grow from a single initial state. The NCA model is a discrete cellular automaton consisting of two channels: a binary live/dead channel and a signaling channel. The output of the network produces the updated CA grid at time step \( n + 1 \) (e). In computing empowerment, each cell is considered an agent. The sensor state of cell \( i, j \) at time step \( n \), \( s_{ijn} \), is the average signal of the cell’s neighbors (f). Cell \( i, j \) produces an action at time step \( n \), \( a_{ijn} \), which updates its own signal value for neighboring cells to sense (g).

Others are purely open-ended with no particular target shape, aiming only to generate interesting structures and/or behaviors [39]. Here, we employ a simple neural network to achieve the task of shape matching. Rather than adding complexity to the network to better achieve the task, we introduce empowerment as an additional objective and hypothesize that NCA intrinsically motivated by empowerment are better capable of morphogenesis than those that are not.

3 METHODS

We evolve a population of NCAs that attempt to grow from a single cell into a target shape, a simplified version of biological morphogenesis. NCAs are evaluated based on their ability to match the target shape and/or their empowerment during development. Those NCAs scoring poorly on the specified objective(s) are removed from the population while those scoring well are randomly modified to produce the next generation. Evolution thus produces NCAs that are capable of shape matching, highly empowered, or both.

3.1 NCA Model

The NCA model is a discrete cellular automaton consisting of two channels (Figure 2a): a binary live/dead channel (Figure 2b), and a signaling channel (Figure 2c). At each time step, a neural network (d) is applied sequentially to each cell in the CA. The rules of the CA are embodied by a feed-forward neural network with no hidden layers (Figure 2d). The inputs for a given cell include neighboring cells’ signals and the live/dead values for neighboring cells and the cell itself. The network outputs an updated live/dead value and signal for the cell itself and four binary values that determine whether the cell should replicate its state into each of its four neighbors. These binary replication outputs allow for growth or death of contiguous patches of cells. Outputs of the network pass through a sigmoid activation function and are then binarized or scaled depending on the output type. For the binary outputs, if the activation is greater than \( \epsilon \), the output is 1, otherwise it is set to 0. For the signal concentration node, if the output is greater than \( \epsilon \), the value is scaled between \([127, 255]\) and binned to an integer, otherwise it is set to 0. In this way, the NCA is heavily biased towards growth and signaling. As in biological morphogenesis, proliferation and signaling are more common than cell death or quiescence making controlled growth a challenging task.

At each time step, the neural network is executed sequentially for each cell in the grid to determine the global state of the CA at the following time step (Figure 2e). In a similar manner to biological signaling gradients, signal diffuses through the CA at each time step at a decaying rate. Each CA simulation begins with a seed cell at the center of an \( M \times M \) grid with a signal value of zero. The CA develops for \( N \) time steps at which point the resulting morphology can be compared to a pre-determined target shape.

3.2 NCA Evaluation

It is currently unknown what cellular communication strategies are required to ensure faithful morphogenesis in nature, but it is likely that large-scale coordination is necessary. This concept of coordination can be efficiently captured by the information theoretic concept of empowerment [20]: the actions of an agent in the present yields high mutual information with the states of other agents at a future time. To that end, two functions are used to evaluate NCAs: loss, which evaluates the CA’s morphogenetic capability of the distance between a target shape and the final shape of the CA after \( N \) time steps, and empowerment, computed from a CA’s signaling dynamics during development.

3.2.1 Loss. To determine how well the CA matches a static, pre-defined target shape, \( T \), the L2 loss, \( \mathcal{L} \), is computed between the binary state of the \( M \)-dimensional CA at a given time step, denoted by \( C_n \), and the target averaged over the number of CA time steps starting at time step \( n_0 \) and terminating at step \( n_1 \). Loss, as defined in Equation 1, is to be minimized.

\[
\mathcal{L}(n_0, n_1) = \frac{1}{n_1 - n_0} \sum_{n \in [n_0, n_1]} \sum_{i,j \in M} |C_{ijn} - T_{ij}|^2 \quad (1)
\]

3.2.2 Empowerment. We expand empowerment, as defined in [20], to NCA by defining an agent as a single cell. The environment of the cell is defined by the rest of the CA grid. Cells’ only have direct influence over their neighbors via CA neural network ruleset. Empowerment is applied only to the signaling dynamics of the CA. We define the sensor state of cell \( i, j \) at time step \( n \), \( s_{ijn} \), as the average signal of its Von Neumann neighbors (Figure 2f). The action state of cell \( i, j \) at time step \( n \), \( a_{ijn} \), is the cell’s own signal value as output from the neural network (Figure 2g).

Parallel to a traditional robotic sensorimotor system, the cell can be thought of as an agent with four sensors (one on each side) facing outward to sense the signals of its neighbors and one actuator which acts on its environment by updating the signal the

Figure 2: A single update of the NCA model. The NCA at an arbitrary time step \( n \) (a) is comprised of two channels: a binary live/dead channel (b) and a signaling channel (c). At each time step, a neural network (d) is applied sequentially to each cell in the CA. For cell \( i, j \), the network takes as input the neighboring cells’ live/dead values and signals. The output of the network produces the updated CA grid at time step \( n + 1 \) (e). In computing empowerment, each cell is considered an agent. The sensor state of cell \( i, j \) at time step \( n \), \( s_{ijn} \), is the average signal of the cell’s neighbors (f). Cell \( i, j \) produces an action at time step \( n \), \( a_{ijn} \), which updates its own signal value for neighboring cells to sense (g).
cell itself emits for neighboring cells to sense. We define empowerment as maximizing the mutual information between the set of all cells’ action states during one time period in CA development and the corresponding set of all cells’ sensor states during an equal-duration time period later in development. These time periods were arbitrarily chosen to be the first $N/2$ time steps and the last $N/2$ time steps where $N$ is the total number of developmental time steps. Action ($A$) and sensor ($S$) sets are defined as follows where $M$ is the dimension of the CA grid.

$$A_{0}^{N/2} = \{ a_{00}, \cdots, a_{MM} \}$$

$$S_{N/2}^{N} = \{ s_{00}, \cdots, s_{MMN} \}$$

Thus, empowerment, $E$, measured in bits, is the mutual information between these sets of actions and sensor states and can be maximized via evolutionary search.

$$E = I(A_{0}^{N/2}, S_{N/2}^{N}) = -\sum_{a_{ij},s_{ij}} p(a_{ij},s_{ij}) \log_2 \frac{p(a_{ij},s_{ij})}{p(a_{ij})p(s_{ij})}$$ (2)

Empowerment, as stated in Equation 2, was implemented using PyInform, a Python package for information-theoretic computation [27]. Empowerment is to be maximized, however, for ease of implementation it was converted into a minimization function by multiplying by negative one. In the following figures, empowerment is displayed as a maximization function by undoing this negation.

4 EXPERIMENTAL DESIGN

NCAs were evolved using Age-Fitness Pareto Optimization (AFPO) [35], a multi-objective evolutionary algorithm which uses age of genetic material as an additional objective to maintain diversity in a population. Four variations of AFPO outlined in Table 1 were tested with either one or two additional objectives (age is always the first objective). To rule out the possibility that adding more objectives makes evolutionary search for this task easier, we include an tri-objective loss-only control ensuring a fair comparison with the tri-objective loss-empowerment treatment. This is achieved by splitting loss (Equation 1) into the first half of CA development and the second half of CA development.

| Treatment          | Obj. 1  | Obj. 2  | Obj. 3       |
|--------------------|---------|---------|--------------|
| Bi-error           | Age     | $\Psi(0,N)$ [1] | -            |
| Tri-error-empowerment | Age     | $\Psi(0,N)$ [1] | $\Psi$ [2]   |
| Tri-error          | Age     | $\Psi(0,N/2)$ [1] | $\Psi(N/2,N)$ [1] |
| Bi-empowerment     | Age     | $\Psi$ [2] | -            |

Table 1: Objective functions for four different treatments of Age-Fitness Pareto Optimization (AFPO).

For each treatment, 25 evolutionary runs were performed for 2,000 generations each and with a population size of 400 NCAs. A CA grid dimension of $M = 25$ is used and the target shape for all runs is a square centered around the initial seed. During evaluation, the NCA is run for $N = 50$ time steps and evaluated on the objectives described in Table 1. Wall time for a single evolutionary run was about 10 hours. Experiments were performed in parallel using CPU resources provided by the Vermont Advanced Computing Core.

5 RESULTS

Results of the evolutionary runs for each treatment are shown in Figure 3. The tri-loss-empowerment treatment achieves the lowest loss. That is, NCAs evolved for both loss minimization and empowerment maximization on average come closer to matching the target shape than those evolved for matching the target alone. The bi-loss and tri-loss curves achieve similar performance indicating that the tri-loss treatment is an adequate tri-objective implementation of the bi-loss treatment. Additionally, empowerment of the tri-loss-empowerment runs is higher than that of the loss-only
Figure 4: The most lowest loss NCAs resulting from each of the 25 evolutionary runs for each AFPO variation in the loss-empowerment space. Arrows corresponding to each treatment indicate the direction of selection pressure for that treatment. Shown in the top-left in red are the lowest loss NCAs from 25 different populations at generation zero. No evolution occurs on these individuals, thus, there is no corresponding arrow.

controls suggesting evolution is occurring on the empowerment objective. As expected, the bi-empowerment treatment achieved the highest loss as well as the highest empowerment as NCAs were only trained to maximize empowerment and there was no direct selection pressure for them to grow the target shape.

Interestingly, empowerment of the two loss-only treatments (blue and orange curves in Figure 3 bottom panel), though not directly selected for, increases during evolution. Likewise, the loss of the bi-empowerment treatment (purple curve in Figure 3 top panel) decreases at the beginning of evolution before flattening. These trends suggest that the tasks of shape matching and empowerment have a synergistic relationship. In other words, evolving for morphogenesis alone naturally produces NCAs with a higher empowerment and vice versa. Thus, introducing direct selection pressure for empowerment pushes the NCAs into part of the fitness landscape that is also beneficial for the shape matching task and evolution speeds up.

The best (lowest loss) NCAs resulting from each of the 25 runs for every treatment were visualized in the loss-empowerment space and compared to 25 random NCAs, shown in Figure 4. Random NCAs were produced by selecting the lowest-loss NCAs in 25 generation zero populations (of size 400). Arrows indicate direction of selection pressure for each treatment. Random NCAs are clustered in the top right and exhibit the highest loss and lowest empowerment. Clusters for the other treatments are positioned in the direction of their selection pressures relative to the random cluster. Notably, loss of the bi-empowerment cluster is lower than that of the random cluster. Also interesting to note is that the loss-only controls are not as clustered near low empowerment compared to random. Again, suggesting that the two objectives are not independent.
6 DISCUSSION

The above results provide evidence that adding empowerment as an objective in evolutionary search produces NCAs that are better able to perform the task of morphogenesis than evolving for morphogenesis alone. That is, pushing NCA towards a signaling mechanism that maximizes the amount of information shared between the actions of cells in the past and the senses of cells at a future time point is useful for the process of morphogenesis. Understanding this signaling mechanism that results from maximizing empowerment is a difficult task due to the number of cells in the CA and the long time horizon between a cell’s action and its corresponding sensor value. However, visualizing signaling dynamics over time and local empowerment heatmaps may provide some clues regarding its characteristics. Local empowerment measures the contribution of a single cell’s sensor, action pairs to the overall empowerment score of the CA.

Figure 5 depicts the development of the lowest-loss NCAs from each treatment. In accordance with the loss curves in Figure 3, the bi-loss, tri-loss, and tri-loss-empowerment treatments are best at matching the target shape, though none achieve the task perfectly. These treatments also appear to show similar signaling dynamics with the majority of cells having high signal concentrations. They also yield similar local mutual information heatmaps: corner or edge cells have the highest local mutual information and also seemingly high mutual information in the center of the shape that fades outwards. In contrast, the bi-empowerment NCA appears to make use of a broader range of signaling values which produce a striped pattern. Whether these patterns are indicative of some underlying useful dynamic or are simply artefacts of the CAs chosen for representation remains to be determined.

Striped signaling patterns and those that use a wider range of signal concentrations appear to be more common in NCAs with higher empowerment (Figure 6). For the tri-loss-empowerment and bi-empowerment treatments, local empowerment appears highest in cells that appear earlier in development. Growth then proceeds outwards from these regions. Notably, in both Figures 5 and 6, evolved NCAs exhibit more cohesive shapes with fewer holes and more controlled growth than the random NCAs. This is particularly interesting for the bi-empowerment NCAs where shape of the NCA is not considered in the evaluation or selection of NCAs during evolution.

To explore whether results are specific to the square target shape and 25 × 25 grid resolution, experiments were repeated for four different target shapes (triangle, circle, biped, and circular biped) and at double the resolution (50 × 50 grid). Figure 7 displays loss curves for four different target shapes while Figure 8 displays those for the square target on a grid of double the original resolution. Experiments were conducted as described in Section 3 with the exception of the number of generations which was decreased from 2,000 to 1,000 for computational reasons. Loss curves in all scenarios display similar trends to those in Figure 3 suggesting that empowerment is beneficial for and generalizes to various morphogenetic processes and increasingly complex tasks.

7 CONCLUSIONS AND FUTURE WORK

In this paper, we have expanded the application of empowerment, a universal, information-theoretic objective function, to a neural cellular automaton system whereby a cell collective attempts to coordinate its behavior to achieve a global task. We show that including empowerment as an objective in the evolutionary search of NCAs with the primary task of shape matching results in more
When cells are selected to exert control over their environment (neighboring cells) via their signaling dynamics they are better able to coordinate their actions through space and time to achieve a collective task.

This work suggests that there may be certain information-theoretic signatures, such as empowerment, of CA dynamics that produce generally useful, task-independent behaviors. Such a notion has clear parallels to biology in which complex networks of cells share information by means of chemical, bioelectric, and mechanical signals in order to achieve specific tasks, such as morphogenesis. Uncovering these information signatures by means of simulation suggests potential mechanisms by which biological cells communicate and coordinate behavior. Much future work remains to better understand the relationship between empowerment and NCA signaling dynamics including decomposing highly-empowered NCAs and examining their dynamics. Additionally, it will be interesting to investigate different variations of empowerment (altering the time horizon, expanding the cells’ neighborhood, etc.) as well as examine the impact of empowerment on evolutionary search for more diverse CA tasks.

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