Financial Rogue Waves*

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Abstract We analytically give the financial rogue waves in the nonlinear option pricing model due to Ivancevic, which is nonlinear wave alternative of the Black–Scholes model. These rogue wave solutions may be used to describe the possible physical mechanisms for rogue wave phenomenon in financial markets and related fields.

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1 Introduction

Rogue waves have generated many marine misfortunes in the oceans.[1] The New Year’s wave or Draupner wave was regarded as the first rogue wave recorded by scientific measurement in North Sea. Recently, they were paid much attention in order to understand better their physical mechanisms.[1–8] Rogue waves are also known as freak waves, monster waves, killer waves, giant waves, or extreme waves. The rogue wave phenomenon remain poorly understood. It was not until 2007 that Solli et al.[9] first observed the optical rogue waves in an optical fibre and found that they could be used to stimulate supercontinuum generation.[10] The basic solution (rogon) was first presented by Peregrine[11] to describe the rogue wave phenomenon, which was known as by Peregrine soliton (or Peregrine breather). Recently, the multi-rogon solutions were also presented by using the deformed Darboux transformation in Refs. [12–13]. The matter rogue waves were realized by using the numerical simulation[14] and the rogon-like solutions were also found.[15] In addition, the atmospheric rogue waves were also presented.[16]

To the best of our knowledge, there is no theoretical research for the financial rogue waves (or financial crisis/storm) that have been occurred (e.g. 1997 Asian financial crisis/storm) and are taking place (e.g. the current global financial crisis/storm).

Based on the the geometric Brownian motion (i.e. the stochastic differential equation) $dS = \mu S dt + \sigma dW(t)$ satisfied by the stock (asset) price $S$ and the Itô lemma,[17] the celebrated Black–Scholes linear partial differential equation

$$\frac{\partial C}{\partial t} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} + rS \frac{\partial C}{\partial S} - rC = 0,$$

was deduced,[18–19] where $C \equiv C(S,t)$ is the values of European call option on the asset price $S$ at time $t$, $\mu$ is the instantaneous mean return, $\sigma$ is the stock volatility, $W$ is a Wiener process, and $r$ is the risk-free interest rate. In 1997, Merton and Scholes received the Nobel Prize in Economy for their method to determine the price of a European call option. But the model can not describe long-observed features of the implied volatility surface.

2 Ivancevic Option Pricing Model

Recently, Ivancevic, based on the modern adaptive markets hypothesis due to Lo[20–21] and Elliott wave market theory,[22–23] and quantum neural computation approach,[24] proposed a novel nonlinear option pricing model (called the Ivancevic option pricing model)

$$i \frac{\partial \psi(S,t)}{\partial t} = -\frac{1}{2} \sigma^2 \frac{\partial^2 \psi(S,t)}{\partial S^2} - \beta |\psi(S,t)|^2 \psi(S,t),$$

in order to satisfy efficient and behavioral markets, and their essential nonlinear complexity, where $\psi = \psi(S,t)$ denotes the option-price wave function, the dispersion frequency coefficient $\sigma$ is the volatility (which can be either a constant or stochastic process itself), the Landau coefficient $\beta = \beta(r,w)$ represents the adaptive market potential. Some periodic wave solutions of Eq. (2) have been obtained.[25]

3 Financial Rogue Waves

Here, based on the approach developed in Refs. [12–13], we show that the Ivancevic option pricing model (2) also possesses the financial multi-rogon (rogue wave) solutions, which may be used to describe the possible formation mechanisms for rogue wave phenomenon in financial markets. Here we give the first two representative financial rogon solutions of the Ivancevic option pricing model (2).

The financial one-rogon solution of Eq. (2) for the option-price wave function $\psi(S,t)$ by means of the complex rational functions of the stock price $S$ and time $t$ in the form

$$\psi_1(S,t) = \alpha \sqrt{\beta} \left[ 1 - \frac{4(1 + i \sigma \alpha^2 t)}{1 + 2 \alpha^2 (S - \sigma k t)^2 + \sigma^2 \alpha^4 t^2} \right],$$

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\begin{equation}
x \exp \left\{ i \left[ kS + \sigma/(2(\alpha^2 - k^2)t) \right] \right\}, \quad \sigma \beta > 0, \quad (3)
\end{equation}

which involves four free parameters \( \sigma, \beta, \alpha, \) and \( k \) to manage the different types of financial rogue wave propagations whose intensity \(|\psi_1(S, t)|^2\) is displayed in Fig. 1 for the chosen volatility \( \sigma = 0.3 \), adaptive market potential \( \beta = 0.03 \), the scaling \( \alpha = 2 \), and the gauge \( k = 0, -1.5 \).

Notice that time \( t \) in Fig. 1 can be chosen to be negative since the solution is invariant under the translation transformation \( t \rightarrow t + t_0 \).

Moreover, the financial two-rogon solutions of Eq. (2) can be written as
\begin{equation}
\psi_2(S, t) = \alpha \sqrt{\frac{\sigma}{2\beta}} \left[ 1 + \frac{P_2(S, t) + iQ_2(S, t)}{R_2(S, t)} \right] \exp \left\{ i[kS + \sigma/(2(\alpha^2 - k^2)t)] \right\}, \quad \sigma \beta > 0, \quad (4)
\end{equation}

with these functions \( P_2(x, t), Q_2(x, t), \) and \( R_2(x, t) \) being of polynomial forms of the stock price \( S \) and time \( t \)
\begin{align*}
P_2(S, t) &= \frac{3}{8} - \frac{1}{2} \alpha^4(S - \sigma kt)^4 - \frac{3}{2} \sigma^2 \alpha^6 t^2 (S - \sigma kt)^2 - \frac{5}{8} \sigma^4 \alpha^8 t^4 - \frac{3}{2} \alpha^2 (S - \sigma kt)^2 - \frac{9}{4} \sigma^2 \alpha^4 t^2, \\
Q_2(S, t) &= -\frac{1}{2} \sigma \alpha^2 t \left[ \alpha^4(S - \sigma kt)^4 + \sigma^2 \alpha^6 t^2 (S - \sigma kt)^2 + \frac{1}{4} \sigma^4 \alpha^8 t^4 - 3 \alpha^2 (S - \sigma kt)^2 + \frac{1}{2} \sigma^2 \alpha^4 t^2 - \frac{15}{4} \right], \\
R_2(S, t) &= \frac{3}{32} + \frac{1}{12} \alpha^6(S - \sigma kt)^6 + \frac{1}{8} \sigma^2 \alpha^8 t^2 (S - \sigma kt)^2 + \frac{1}{16} \sigma^4 \alpha^{10} t^4 (S - \sigma kt)^2 + \frac{1}{96} \sigma^6 \alpha^{12} t^6 \\
&\quad + \frac{1}{8} \sigma^4(S - \sigma kt)^4 - \frac{3}{8} \sigma^2 \alpha^6 t^2 (S - \sigma kt)^2 + \frac{9}{32} \sigma^4 \alpha^8 t^4 + \frac{9}{16} \alpha^2 (S - \sigma kt)^2 + \frac{33}{32} \sigma^2 \alpha^4 t^2,
\end{align*}

which contains four free parameters \( \sigma, \beta, \alpha, \) and \( k \) to manage the different types of financial rogue wave propagations whose intensity \(|\psi_2(S, t)|^2\) is depicted in Fig. 2 for the chosen volatility \( \sigma = 0.3 \), adaptive market potential \( \beta = 0.03 \), the scaling \( \alpha = 0.8 \), and the gauge \( k = 0, -1.5 \).
Fig. 2  Rogue wave propagations (left) and contour plots (right) for the intensity $|\psi|^2$ of the one-rogon solution (3) for $\sigma = 0.3$, $\beta = 0.03$, $\alpha = 0.8$. (a)–(b) $k = 0$; (c)–(d) $k = -1.5$.

4 Conclusion

In conclusion, we have shown that the nonlinear option pricing model (2) also possesses the analytical financial one- and two-rogon solutions. This may further excite the possibility of relative researches and potential applications for the financial rogue wave phenomenon in the financial markets and related fields.

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