An optimal combination prediction method of turnover spare parts consumption based on certain weight

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Abstract. Aimed at the situation that the first, second and third exponential smoothing method and the three and four term moving average method are used to forecast the consumption of spare parts separately, the combined prediction model is established by optimizing the combination of these single prediction methods, and the result with the minimum prediction error is taken as the predicted value of consumption. A concrete example shows that the combined prediction model is more accurate and has higher practicability and generalization.

1. Introduction

Turnover spare parts refers to the spare parts for daily supply support. There are many types and quantities of turnover spare parts, which takes up a lot of aviation spare parts support funds. How to accurately predict the consumption of turnover spare parts has always been the focus and difficulty of naval aviation spare parts support work. At present, the single prediction method we use has some shortcomings in the prediction accuracy of turnover spare parts consumption, which will produce large prediction error. However, the combined prediction method is not sensitive to single poor prediction method, which can improve the accuracy and reliability of prediction.

There are many research achievements on the combination prediction method of aviation spare parts consumption at home and abroad, but there is no systematic study on the combination prediction method of turnover spare parts consumption. WAN Yucheng and others put forward a new combination prediction model, generalized weighted function proportion average combination prediction model based on the grey system and neural network methods, gave the parameter estimation method of the weighting coefficients by using quadratic programming algorithm, and illustrated its prediction effect by an example [1]. MAO Kaiyi founded a combination prediction model, comprehensively compared and analyzed the weight of the combination prediction model through several weighting methods, and discussed the application of different combination prediction weight determination methods in practice through an example. The results show that the prediction accuracy of the proposed combined prediction model is higher than that of the single prediction models [2]. GENG Jianjun and others used quadratic programming to determine the combination weight of each period of samples in order to make the weight of combination prediction change with different sample data, so as to minimize the sum of squares of errors [3].

Therefore, this paper studies the consumption law of repairable parts and consumable parts of turnover spare parts respectively, establish appropriate consumption combination prediction...
models by several single prediction models for different turnover spare parts, and seek the optimal combination prediction model, which can better solve the consumption prediction problem of turnover equipment.

2. Single prediction model

2.1. Moving average method

Moving average method is a method that is carried out item by item according to the data of time series, calculating the time series average including a certain number of items in turn, in order to reflect the long-term trend. When the value of time series fluctuates greatly and is not easy to show the development trend due to the influence of periodic and irregular changes, moving average method can be used to eliminate the influence of these factors and analyze and predict the future trend of the series. Moving average method has many types, this paper uses three term moving average and four term moving average methods.

2.1.1. Three term moving average method

Suppose the order of annual consumption of turnover spare parts is \( y_1, y_2, y_3, \cdots \), three moving average formula is

\[
M_t = \frac{y_t + y_{t-1} + y_{t-2}}{3}, \ t \geq 3
\]

Where, \( M_t \) is \( t \) period of moving average, then the forecast value is \( \hat{y}_{t+1} = M_t \).

2.1.2. Four term moving average method

Suppose the order of annual consumption of turnover spare parts is \( y_1, y_2, y_3, y_4, \cdots \), four moving average formula is

\[
M_t = \frac{y_t + y_{t-1} + y_{t-2} + y_{t-4}}{4}, \ t \geq 4
\]

Where, \( M_t \) is \( t \) period of moving average, then the forecast value is \( \hat{y}_{t+1} = M_t \).

2.2. Exponential smoothing method

Exponential smoothing method eliminates the random fluctuation in the historical statistical series and finds out the main development trend by some average way. According to the different smoothing times, exponential smoothing method can be divided into primary exponential smoothing, secondary exponential smoothing and cubic exponential smoothing methods \([4-6]\).

2.2.1. Primary exponential smoothing method

Suppose the order of annual consumption of turnover spare parts is \( y_1, y_2, \cdots \), the weighting coefficient is \( \alpha \), \( 0 < \alpha < 1 \), the primary exponential smoothing value is

\[
S_t^{(1)} = \alpha y_t + (1-\alpha)S_{t-1}^{(1)}.
\]

Then, the forecast value of the primary exponential smoothing method is

\[
\hat{y}_{t+1}^{(1)} = S_t^{(1)} \text{ or } \hat{y}_{t+1}^{(1)} = \alpha y_t + (1-\alpha)\hat{y}_t.
\]

Determinate initial value \( S_0^{(1)} \): if the number of samples is large, for example, \( n \geq 20 \), the initial value has little influence on the future forecast value, then the first period of data can be selected as the initial value, that is \( S_0^{(1)} = y_1 \); if the number of samples is small, the initial value
has great influence on the future forecast value, then the average value of the previous two 
periods of actual data can be selected as the initial value, that is \( S_y^{(1)} = \frac{y_1 + y_2}{2} \).

Determinate weighting coefficient \( \alpha \): if the basic trend of the known data is relatively stable 
and the fluctuation is small, the weighting coefficient should be smaller, for example, \( \alpha = 0.2 \); if 
the known data fluctuates greatly, the weighting coefficient should be larger, for example, 
\( \alpha = 0.8 \), in order to make the prediction model more sensitive. If there are not many known data, 
several more values can be taken for calculation respectively, and the value with small mean 
square error can be selected as the weighting coefficient in actual prediction.

The determination method of the initial value and weighting coefficient of secondary 
exponential smoothing and cubic exponential smoothing method is the same as the above method.

2.2.2. Secondary exponential smoothing
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exponential smoothing method is that an exponential smoothing is performed again on 
the basis of the primary exponential smoothing. The second exponential smoothing value is 
\( S_y^{(2)} = \alpha S_y^{(1)} + (1-\alpha)S_{t-1}^{(2)} \) \hspace{1cm} (5)

Then, the forecast value of the second exponential smoothing method is 
\( \hat{y}_{t+T} = a_t + b_T \), \hspace{1cm} (6)

where, \( T = 1,2,\cdots \), \( a_t = 2S_y^{(1)} - S_y^{(2)} \), \( b_T = \frac{\alpha}{1-\alpha}(S_y^{(1)} - S_y^{(2)}) \).

2.2.3. Cubic exponential smoothing method
Cubic exponential smoothing method is that an exponential smoothing is performed again on the 
basis of the second exponential smoothing. The second exponential smoothing value is  
\( S_y^{(3)} = \alpha S_y^{(2)} + (1-\alpha)S_{t-1}^{(3)} \) \hspace{1cm} (7)

Then, the forecast value of the cubic exponential smoothing method is 
\( \hat{y}_{t+T} = a_t + b_T + c_T^2 \), \( T = 1,2,\cdots \) \hspace{1cm} (8)

where, 
\( a_t = 3S_y^{(1)} - 3S_y^{(2)} + S_y^{(3)} \), 
\( b_T = \frac{\alpha}{2(1-\alpha)} \left[ (6-5\alpha)S_y^{(1)} - 2(5-4\alpha)S_y^{(2)} + (4-3\alpha)S_y^{(3)} \right] \), 
\( c_T = \frac{\alpha^2}{2(1-\alpha)^2} \left( S_y^{(1)} - 2S_y^{(2)} + S_y^{(3)} \right) \).

3. Combination prediction model
Different single prediction models have their own advantages and disadvantages. They are not 
mutually exclusive, but interrelated and complementary. Different prediction methods use 
different data and get different information. If we take it for granted that a prediction method has 
a large error and discard it, some useful information may be lost. At this time, if we can optimize 
the combination of different single prediction methods, we can use the different useful 
information provided by different single prediction methods to greatly improve the accuracy of 
prediction, so the new model after the optimization combination is the combination prediction model.
3.1. Found combination prediction model
Assume that \( x_i \) is the weighting coefficient of the \( i \) method and \( \hat{Y}_it \) is the predicted value of the \( t \) year of the \( i \) method, then the combined prediction model is:

\[
Y_t = \sum_{i=1}^{N} x_i \hat{Y}_{it}
\]

(9)

Where \( N \) represents the number of selected single prediction methods, \( N=2,3,4,5 \).

3.2. Determine weighting coefficient
The core problem of combination forecasting is to calculate the weighting coefficients of single prediction models, so that the combination forecasting model can more effectively improve the prediction accuracy. In the process of solving the weighted coefficients, we must ensure that the sum of all the weight coefficients is 1, that is \( \sum_{i=1}^{N} x_i = 1 \). At the same time, the condition \( x_i \geq 0 \) should be met. In this paper, the following methods are used.

3.2.1. Arithmetic average method
Arithmetic average method is also called equal weight average method. Its characteristic is that the weighting coefficient of each single forecasting method are completely equal, that is, each single forecasting model is treated equally. Generally, it is applicable when the prediction accuracy of each single prediction method is similar or the prediction accuracy of each single prediction method is unknown. Its mathematical formula is

\[
x_i = \frac{1}{N}, \quad i = 1,2,\ldots, N
\]

(10)

3.2.2. Simple weighted average method
Simple weighted average method is a non equal weight average method. First, the sum of the squares of the prediction errors of each single prediction model is sorted; then, according to the basic principle that the square sum of prediction error of each single prediction model, namely \( E_i (i = 1,2,\ldots,N) \), is inversely proportional to its weight coefficient, the higher the order of single forecasting model, the smaller the weighting coefficient in combination forecasting. Suppose \( E_1 > E_2 > \cdots > E_N \), the corresponding coefficients are

\[
x_i = \frac{i}{\sum_{i=1}^{N} i} = \frac{2i}{N(N+1)}, \quad i = 1,2,\ldots, N
\]

(11)

where, \( E_i \) refers to the mean square error of single prediction method \( i \)th.

3.3. Model evaluation
In order to comprehensively evaluate the accuracy of the proposed combined forecasting model, according to the evaluation principles and conventions of combined forecasting effect, generally, at least three error performance indexes are used to evaluate the prediction error comprehensively, and the combination with the smallest prediction error is selected as the optimal combined forecasting model\[5,7,8\]. This paper uses the following three error performance indicators to evaluate.

The first error performance index is Squared Sum Error (SSE), namely

\[
E_{SSE} = \sum_{t=1}^{n} (y_t - \hat{y}_t)^2
\]

(12)
The second error performance index is Mean Squared Error (MSE), namely

$$E_{\text{MSE}} = \frac{1}{n} \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$  \hfill (13)

The third error performance index is Theil Inequality Coefficient (Theil IC), namely

$$\text{Theil IC} = \frac{\sqrt{\frac{1}{n} \sum_{i=1}^{n} (y_i - \hat{y}_i)^2}}{\sqrt{\frac{1}{n} \sum_{i=1}^{n} y_i^2} + \sqrt{\frac{1}{n} \sum_{i=1}^{n} \hat{y}_i^2}}$$  \hfill (14)

where, $\hat{y}_i$ is the predicted consumption of year $i$th, $y_i$ is the actual consumption of year $i$th, $n$ is the number of samples. The value of Theil IC is usually between 0 and 1, and the smaller the value is, the higher the fitting degree is.

4. Example analysis

The consumption data of a certain type of tire from 2005 to 2017 are shown in Table 1. Try to predict the consumption number of the tire in the next year.

| Year   | 2005 | 2006 | 2007 | 2008 | 2009 | 2010 | 2011 | 2012 | 2013 | 2014 | 2015 | 2016 | 2017 |
|--------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| Number of consumption | 50   | 113  | 102  | 130  | 161  | 121  | 125  | 177  | 220  | 166  | 167  | 150  | 200  |

4.1. Calculate the forecast errors of single prediction methods

The forecast values and three kind of forecast errors of seven single prediction methods are shown in Table 2.

| Single prediction method | The forecast value in the next year | MSE      |
|--------------------------|------------------------------------|----------|
| The three term moving average method | 173 | 1524.46 |
| The four term moving average method | 171 | 1435.33 |
| The primary exponential smoothing method | 191 | 1172.09 |
| The secondary exponential smoothing method | 225 | 2007.63 |
| The cubic exponential smoothing method | 274 | 3893.79 |

By comparing the mean square error of five single forecasting methods, it can be seen that the first exponential smoothing method has the best prediction effect, and the cubic exponential smoothing method has the worst prediction effect.

4.2. Determinate weighting coefficient

Optimize the combination of five single prediction methods, select different methods to determine the weighting coefficient for comparison, and select the best one. The forecast values and two kind of forecast errors based on different weight determination methods are shown in Table 3.
Table 3. The forecast values and errors based on different weight determination methods

| Weight determination method                  | The forecast value in the next year | MSE     |
|---------------------------------------------|------------------------------------|---------|
| Arithmetic average method                   | 207                                | 1338.18 |
| The simple weighted average method          | 192                                | 1231.32 |

Through the comparison of the above two methods for solving the weighted coefficient, it can be seen that the simple weighted average method has higher prediction accuracy, so this method should be used for prediction.

4.3. Seek optimization combination model

The different quantity of single forecasting methods are selected for combination, and the quadratic programming method is used to solve the problem. The forecast values and three kind of forecast errors based on different combination modes are shown in Table 4.

Table 4. The forecast values and errors based on different combination modes

| The combination mode                  | The adopted single prediction models                                      | The quantity of single prediction models | The forecast value in the next year | SSE      | MSE      | Theil IC |
|---------------------------------------|---------------------------------------------------------------------------|----------------------------------------|------------------------------------|----------|----------|----------|
| The first combination mode            | The primary exponential smoothing method, the three term moving average method | 2                                      | 185                                | 13026.11 | 1302.61  | 0.1141   |
| The second combination mode           | The primary exponential smoothing method, the three term moving average method, the four term moving average method | 3                                      | 182                                | 11566.46 | 1285.16  | 0.1113   |
| The third combination mode            | The primary exponential smoothing method, the secondary exponential smoothing method, the three term moving average method, the four term moving average method | 4                                      | 185                                | 11381.26 | 1264.58  | 0.1101   |

By comparing the results in Table 2, Table 3 and Table 4, it can be seen that the precision of combined prediction is higher than that of single prediction. The prediction accuracy is the highest when the five single prediction methods are selected for combination. At the same time, giving different weighting coefficients to different single prediction methods can effectively reduce the importance of single prediction methods with large prediction errors in the combined prediction model, so as to improve the prediction accuracy.

5. Conclusion

Based on the first, second and third exponential smoothing method and the three and four term moving average method, this paper uses different number of single term forecasting methods to establish the combined forecasting model through the optimization combination. An example shows that the optimal combination prediction model obtained in this paper has higher prediction accuracy. The model has the advantages of simple operation, good prediction effect and easy popularization and application. It can not only analyze and provide reference for the prediction of spare parts consumption quantity, but also have certain reference value for the consumption prediction of other equipment spare parts.
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