Applying the “snowplow” model for pulsar glitches to constrain nuclear symmetry energy

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Abstract. Glitches in pulsars are occasional, sudden increases in their rotation frequency as the pulsar otherwise steadily spins down. A broad class of glitch models suppose the sudden spin-ups are due to angular momentum transfer between some of the crustal superfluid neutrons and the rigid crust plus anything that couples to it on timescales shorter than the superfluid-crust coupling time. Using a set of neutron star equations of state (EOS) which span the experimentally constrained range of asymmetric nuclear matter properties, and estimates of the strength of the coupling between the crust and the core superfluid neutrons, we calculate the moment of inertia (MoI) of crustal superfluid neutrons involved in storing angular momentum in the glitch process. Following the recent, microscopically based “snowplow” model for glitches, we restrict the calculation to just those superfluid neutron vortices that are strongly pinned to the crustal lattice, a region which corresponds to an equatorial annulus of the inner crust. We compare calculations to observational estimates of the average rate of angular momentum transfer between superfluid neutrons and the rest of the star in the Vela pulsar, and also the relative acceleration of the crust post-glitch estimated from the 2002 Vela glitch. We found that to match the observations of the glitch sizes and recovery from Vela glitches, a value of $L \lesssim 50$ MeV for the density slope of the symmetry energy at saturation density is required and the coupling between core superfluid neutrons and crust is relatively weak.

1. Introduction
Neutron stars are the densest known observable objects in the universe, with interior densities reaching several times the nuclear saturation density, $\rho_0 \approx 2.7 \times 10^{14}$ g cm$^{-3}$. Matter in neutron stars are more isospin-asymmetric than any systems accessible to terrestrial experiments. They are thus a natural testing ground for theories of dense neutron-rich nucleonic matter [1]. Properties of neutron stars are still very mysterious because of our poor knowledge about matter under extreme conditions. One critical piece of information for understanding properties of neutron stars is the Equation of State (EOS) of nucleonic matter which can be written as

$$E(\rho, \delta) = E(\rho, \delta = 0) + E_{\text{sym}}(\rho)\delta^2 + O(\delta^4)$$  \hspace{1cm} (1)

where $\delta = (\rho_n - \rho_p) / (\rho_n + \rho_p)$ is the neutron-proton asymmetry and $E_{\text{sym}}(\rho)$ is the density-dependent nuclear symmetry energy. Around the saturation density, the nuclear symmetry energy can be expanded as

$$E_{\text{sym}}(\rho) = J + L\chi + \frac{1}{2}K_{\text{sym}}\chi^2 + \ldots$$  \hspace{1cm} (2)
where $\chi = \frac{E - E_0}{K_{sym}}$, $J$ is the symmetry energy, $L$ is the slope and $K_{sym}$ is the curvature all at saturation density. Neutron star properties such as the cooling mechanism, the core-crust transition density, the extent of exotic, deformed nuclei (the “pasta” phases) in the crust, the radius and moment of inertia are all sensitive to the density dependence of the $E_{sym}(\rho)$ [2]. Several properties of neutron stars, such as the mass-radius correlation [3, 4, 5], gravitational binding energy [6], quasi-periodic oscillations from soft gamma-ray repeaters [7, 8, 9], r-mode instability [10, 11] and gravitational waves from deformed pulsars[12], oscillations [13, 14] or tidal polarizability of neutron stars [15], have already been used to probe the $E_{sym}(\rho)$ at both low and high densities, see, e.g., ref. [16] for a recent review.

Pulsars are rotating neutron stars that emit a beam of electromagnetic radiation along their magnetic axis. They are among the most accurate clocks in the universe but long term study of pulsars has exposed irregularities in the frequencies [17]. The most conspicuous of these irregularities are glitches: sudden increases in frequency, often followed by a decay back towards the original pre-glitch frequency evolution. Of the 2000 known pulsars [18], about 100 pulsars have been known to glitch and 315 glitches have been recorded [19]. The glitches vary in size greatly from $\frac{\Delta \nu}{\nu_0} \sim 10^{-11}$ to $10^{-5}$, where $\nu_0$ is the pre-glitch frequency and $\Delta \nu$ is the size of the glitch [19]. Sixteen large glitches ($\frac{\Delta \nu}{\nu_0} \sim 10^{-6}$) have been observed in the Vela pulsar (PSR J0835-4510) [19], and have been shown to occur quasi-periodically [20]. Additionally, the 2002 Vela glitch yielded the first measurement of the post-glitch crust acceleration $\Delta \dot{\nu}_{gl}/\dot{\nu}_0$ [21].

The currently favored class of glitch models are the two-component model in which angular momentum is transferred from one component of the star to a second, presumed to be tightly coupled to the magnetic field and hence the pulse emission [22]. In most two component models, the component which acts as the angular momentum reservoir for the glitch is taken to be the superfluid neutrons in the crust. The superfluid neutrons throughout the star form an array of quantized vortices which must move radially outwards from the rotation axis in response to the spin-down of the star. Vortex-nucleus interactions in the inner crust can make it energetically favorable for vortices to pin to nuclei, or in-between nuclei, and hence decouple from the spin-down of the rest of the star. Only when the buoyant Magnus force pushing outward on the superfluid neutrons becomes great enough will they unpin and suddenly re-couple to the crust, transferring angular momentum and initiating the glitch.

The most recent incarnation of this model, used in this work, is the “snowplow” model [23, 24, 25]. Here the vortices are pinned only in the region of the crust in which they are totally immersed - the strong pinning region - which forms an annulus at the equatorial regions of the crust. These vortices accumulate in the density region of the crust in which the pinning force is at a maximum, before unpinning when the Magnus force becomes great enough. They then couple to, and spin-up, the crustal lattice of nuclei and that part of the star which couples to the crust on timescales less than the timescale for the glitch to occur, for which an observational upper limit is $\approx 40s$ [21].

The frequency evolution as a function of time through a glitch event may be modeled as a frequency jump plus a sum of exponentials modeling the recovery from the glitch towards the pre-glitch frequency evolution:

$$\nu(t) = \nu_0 + \Delta \nu_p + \sum_{n=1}^{4} \Delta \nu_n e^{-t/\tau_n}$$

where $\Delta \nu_p$ is the permanent change in frequency, $\Delta \nu_n$ and $\tau_n$ are the temporary changes in frequency and the recovery timescales respectively, and the secular spin-down part of the frequency evolution has been subtracted off [21]. It is thought that the four different timescales represent the different couplings between the components in the glitch process and the shortest timescale represents the relative acceleration of the rigid crust and everything coupled to it at
the time of glitch [23]. A representation of this frequency evolution is plotted in the left hand panel of Fig. 1: the glitch size and initial change in frequency derivative are indicated.

Only recently has the time resolution of observations been sufficient to measure the shortest recovery timescale $\tau_4$, and then only in one Vela glitch (the 2002 glitch). That one measurement can be translated into the relative change in the frequency derivative immediately after the glitch compared to the secular spin-down rate pre-glitch: [21].

$$\frac{\dot{\nu}_{gl}}{\dot{\nu}_0} = \frac{\Delta \nu_4/\tau_4}{\dot{\nu}_0} \equiv K \simeq 18 \pm 4$$  

(4)

where $\Delta \nu_4$ and $\tau_4$ are the short timescale parameters from equation (3) [21]. This quantity, $K$, will be calculated for our model pulsars later.

The glitch activity of a given pulsar is defined as follows. By summing the size of each glitch produced the pulsar, we can find dimensionless cumulative angular momentum transferred to the crust as

$$\frac{J}{2\pi \tilde{I} \tilde{\nu}} = \sum_i \frac{\Delta \nu_i}{\bar{\nu}},$$  

(5)

where $J$ is the angular momentum, $\tilde{I}$ is the MoI of the rigid crust plus the component of the core coupled to it at the time of the glitch, $\bar{\nu}$ is the average frequency of the pulsar and $\Delta \nu_i$ is the size of the $i$th glitch in terms of frequency [26]. The dimensionless cumulative angular momentum for the Vela pulsar is plotted in the right panel of Fig. 1. The slope of the plot is the rate at which angular momentum is transferred to the rigid crust on long timescales, and is what we define as the activity parameter $A$. For the Vela pulsar, $A = 1.95311 \pm 0.03367 \times 10^{-9}$ day$^{-1}$. The activity parameter is related to the fraction of the pulsar’s MoI which acts as the angular momentum reservoir in between glitches via [26]:

$$\frac{I_{\text{res}}}{I} \geq \frac{\bar{\nu}}{|\dot{\nu}|} A \equiv G$$  

(6)

where $G$ is the minimum fraction of the star’s MoI that stores angular momentum, $|\dot{\nu}|$ is the average spin-down rate over the total observation time and $I_{\text{res}}$ is the MoI of the reservoir that
stores the angular momentum which in the “snowplow” model is that of the superfluid neutrons in the strong pinning region in the inner crust [24, 26]. As of 2011, \(G = 0.01615 \pm 0.0002\).

We now outline the calculation of \(G\) and \(K\) given an input EOS and crust composition.

### 2. Modeling the average glitch properties.

Using the EOSs, we can build the stellar profiles of pressure, energy density and mass from the TOV equations:

\[
\frac{dP(r)}{dr} = -G\frac{\mathcal{E}(r) + P(r)}{r^2 (1 - 2GM(r)/r)} \left( M(r) + 4\pi r^3 P(r) \right),
\]

\[
\frac{dM(r)}{dr} = 4\pi r^2 \mathcal{E}(r),
\]

where \(\mathcal{E}(r)\) is energy density as a function of radius \(r\), \(P(r)\) is the pressure as a function of radius and \(M(r)\) is the mass contained in the star within radius \(r\). The stellar radius is given by \(R\), defined by \(P(R) = 0\). For an individual NS of mass \(M(R)\), we can calculate the total MoI for a slowly rotating neutron star via

\[
I_{\text{tot}} = \frac{J}{\nu} = \frac{8\pi}{3} \int_0^R r^4 e^{-\nu(r)} \bar{\omega}(r) \left( \mathcal{E}(r) + P(r) \right) \frac{\mathcal{E}(r) + P(r)}{\sqrt{1 - 2GM(r)/r}} dr,
\]

where \(J\) is the angular momentum, \(\nu(r)\) is a radially-dependent metric function given by

\[
\nu(r) = \frac{1}{2} \ln \left( 1 - \frac{2GM}{R} \right) - G \int_r^R \frac{M(x) + 4\pi x^3 P(x)}{x^2 (1 - 2GM(x)/x)} dx,
\]

and \(\bar{\omega}\) is the frame dragging angular velocity relative to the inertial frame rotating at \(\nu\) [27]

\[
\frac{1}{r^3} \frac{d}{dr} \left( r^4 j(r) \frac{d\bar{\omega}(r)}{dr} \right) + 4 \frac{dj(r)}{dr} \bar{\omega}(r) = 0,
\]

and inside the star \(r \leq R\) [28]

\[
j(r) = e^{-\nu(r) - \bar{\lambda}(r)} = \sqrt{1 - 2GM(r)/r} e^{-\nu(r)}.
\]

On timescales on order of the glitch rise time \(<40s\), only a fraction \(Y_{gl}\) of the core superfluid neutrons will be coupled to the crust and therefore take part in the initial spin-up of the star. The core protons couple to the crust on very short timescales, and therefore, assuming no more exotic components to the neutron star core, the moment of inertia of that part of the star that is spun up at the time of glitch is given by

\[
I_c = \frac{8\pi}{3} \int_0^R r^4 \left[ 1 - Q(r)(1 - Y_{gl}) \right] e^{-\nu(r) - \bar{\lambda}(r)} \frac{\mathcal{E}(r) + P(r)}{\sqrt{1 - 2GM(r)/r}} dr,
\]

where \(Q(r)\) is the neutron fraction at radius \(r\). The protons in the core account for \(\sim 10\%\) the total MoI while the fraction of neutrons that are coupled to the protons in the core is rather more uncertain \(\sim 1 - 50\%\) [23], depending as it does on the relatively uncertain physics of the coupling mechanisms between crustal superfluid neutrons and the crustal lattice, and between core superfluid neutrons and core protons. We shall assume the former is a result of excitation of lattice phonons by vortex Kelvin waves (the strength of which is given by the drag parameter \(B_k\) [24]) and the latter by entrainment of neutrons by protons and subsequent scattering of electrons off neutron vortices (strength given by \(B_c\)). We can approximate the respective coupling times
by \( \tau \approx 1/20B \) \[24\]. Using \( B_k \approx 5 \cdot 10^{-2} - 5 \cdot 10^{-3} \) and \( B_c \approx 10^{-3} - 5 \cdot 10^{-5} \) \[24\] we get \( \tau_c \) on the order of \( 10^0 - 10^1 \)s while the value of \( \tau_k \) is on the order of \( 10^2 \)s. We can approximate the coupled core neutron superfluid fraction by \( Y_{gl} \approx \frac{B_c}{B_k} \approx \frac{\tau_c}{\tau_k} \). Taking the values cited above, in this paper we test the range of \( Y_{gl} \) from 0.5% to 50%.

The MoI of the superfluid neutrons in the crust, that are our presumed angular momentum reservoir for the glitches, is given by

\[
\Delta I_{ic} = \frac{8\pi}{3} \int_{R_{\text{inner}}}^{R_{\text{outer}}} r^4 e^{-\nu(r)} \bar{\omega}(r) \frac{\left( \varepsilon_n(r) + P_n(r) \right)}{\nu} \sqrt{1 - 2GM(r)/r} \, dr,
\]

where \( \varepsilon_n(r) \) is the energy density of crustal superfluid neutrons, \( P_n(r) \) is the pressure of the crustal superfluid neutrons and \( R_{\text{inner}} \) and \( R_{\text{outer}} \) are the radius boundaries for the inner crust. If we let

\[
r^2 \mathcal{I} := \frac{8\pi}{3} r^4 e^{-\nu(r)} \bar{\omega}(r) \frac{\left( \varepsilon_n(r) + P_n(r) \right)}{\nu} \sqrt{1 - 2GM(r)/r}
\]

we can write the MoI of the superfluid neutrons in the strong pinning region of the inner crust as:

\[
\Delta I_{sp} = \int_{\theta_{\text{outer}}}^{\pi/2} \left[ \int_{R_{\text{inner}}}^{R(\theta)} r^2 \mathcal{I} dr \right] \sin \theta d\theta
\]

where \( R(\theta) \) is the radius from the core of the star to the point where the strong pinning region occurs at an angle \( \theta \).

Finally we can write the observationally related quantities \( G \) and \( K \) as \[23\]

\[
G = \frac{\Delta I_{sp}}{I_c}, \quad K = \frac{I_{\text{tot}} - I_c}{I_c}
\]

In order to calculate the above quantities, we need to supply the EOS: \( \varepsilon(r), P(r), \varepsilon_n(r), P_n(r) \) throughout crust and core. In this paper, we use the phenomenological modified Skyrme-Like (MSL) model \[29, 30, 31\] for nuclear matter in crust and core. With this model, we can smoothly vary the symmetry energy and slope at saturation density, \( J \) and \( L \), while the symmetric nuclear matter EOS is held constant \[29\]. The sequence of EOSs that we will be using fits the results of microscopic pure neutron matter (PNM) at low densities, which imposes a relation between \( J \) and \( L \) given by \( J = 20.53 + 0.207L \) \[29\]. For the inner crust properties such as the phase transitions between the core, bubble phase, pasta phase, we use a compressible liquid drop model (CLDM) \[29\].

3. Preliminary Results and Discussions

Figure 2 shows the fractional MoI \( G \) and the initial crustal acceleration \( K \) versus the neutron star mass for three different values of the slope of the symmetry energy, \( L \), and \( B_c \) while \( B_k \) is fixed. As the strength of the coupling of the core neutrons to the crust, \( B_c \), increases, \( G \) and \( K \) both decrease; the former because more of the core is coupled to the crust at glitch time, and therefore there is more for the superfluid neutrons in the crust to spin up, and the latter because the remaining portion of the core yet to couple to the crust is smaller, so the torque it can exert on the crust is correspondingly smaller. One can see that the constraint on \( K \) from the 2002 Vela glitch is satisfied only when the core superfluid-crust coupling is relatively weak \( B_c = 5 \cdot 10^{-5} \). As \( L \) increases from 40 MeV to 60 MeV, we see that for constant \( B_c \), the maximum mass that satisfies the observational constraint on \( G \) increases. This gives a broader range of possible masses for the Vela pulsar as \( L \) increases. Conversely, the \( K \) constraint favors lower \( K \), being satisfied only when \( L < 50 \) MeV for the EOSs shown. At \( L = 40 \) MeV, the
The fractional moment of inertia (MoI) of the crustal superfluid neutrons in the strong pinning region $G$, top, and the relative acceleration of the crust immediately post-glitch $K$, bottom, plotted for the crust superfluid neutron-crust lattice coupling strength $B_k = 5 \cdot 10^{-3}$, core superfluid neutron - crust coupling strength $B_c = 10^{-3}$ (red), $10^{-4}$ (green), and $5 \cdot 10^{-5}$ (blue) and symmetry energy slopes $L = 40, 50$ and 60 MeV. The observational constraints are shown for $G$ as a black horizontal line designating the lower bound that should be satisfied, and for $K$ by two black horizontal lines designating the upper and lower bounds from the 2002 Vela glitch.

acceptable mass range that simultaneously satisfies both the MoI and the initial rate of recovery is $\sim 1.4 - 1.8 M_\odot$. This limits the value of the slope of the symmetry to less than 50 MeV, and the core superfluid neutron - crust coupling to $B_c = 5 \cdot 10^{-5}$.

We have demonstrated that by comparing to observations from the Vela pulsar using a consistently constructed sequence of crust and core EOSs that conform to, among other things, the pure neutron matter EOS extracted from state-of-the-art microscopic calculations, it is possible to constrain the slope of the symmetry energy $L$ as well as other physical properties of neutron star matter such as the strength of the drag the core superfluid neutrons experience which couples them to the crust, $B_c$. A more rigorous exploration of the available model parameter space is currently under way. As reported by [25, 32], there are two calculated densities in the neutron star that maximum to the pinning force for superfluid neutrons in the crust occurs ($\rho_{\text{max}} = 0.14 \rho_0$ and 0.325$\rho_0$) and the pinning force outward from that point quickly drops off. This will make the strong pinning region smaller and thus the MoI of the strong pinning region will decrease and provide more stringent constraints. Also neglected in this initial study is the effect of entrainment of superfluid neutrons by the crustal lattice by Bragg scattering, which might greatly reduce the proportion of crustal superfluid neutrons which are able to pin, and hence would decrease the MoI of the strong pinning region significantly, possible even
prohibiting the superfluid neutrons from causing glitches big enough to explain those observed in Vela [33, 34, 35, 36, 37]. Another possibility is that the superfluid neutron vortices in the outer core are pinned to the type II superconducting protons [38]. Such pinning would decouple the vortices from the rest of the star over timescales of weeks or longer and timescales would increase based on the age of the star [38]. This, in inclusion with the maximum pinning region, would provide tough conditions for both the MoI and the initial rate of recovery to simultaneously be satisfied, and will be reported upon in an upcoming paper.

4. Summary
In summary, it is shown that the nuclear symmetry energy can be probed by comparing model calculations and observations for pulsar glitches. Using a recent glitch model that assumes the angular momentum reservoir for the spin-up is due to the superfluid neutrons in the inner crust, we calculated the fractional moment of inertia and the initial acceleration of the crust after the glitch. Based on a preliminary study neglecting entrainment, we found that to match the observations of the glitch sizes and recovery from Vela glitches, a value of $L \lesssim 50$ MeV is required and the coupling between core superfluid neutrons and crust is relatively weak.

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