Study of thermal and hydraulic efficiency of supersonic tube of temperature stratification

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Abstract. Efficiency of supersonic pipe for temperature stratification with finned subsonic surface of heat transfer is the major of this paper. Thermal and hydraulic analyses of this pipe were conducted to assess effects from installation of longitudinal rectangular and parabolic fins as well as studs of cylindrical, rectangular and parabolic profiles. The analysis was performed based on refined empirical equations of similarity, dedicated to heat transfer of high-speed gas flow with plain wall, and Kármán equation with Nikuradze constants. Results revealed cylindrical studs (with height-to-diameter ratio of 5:1) to be 1.5 times more efficient than rectangular fins of the same height. At the same time, rectangular fins (with height-to-thickness ratio of 5:1) were tend to enhance heat transfer rate up to 2.67 times compared to bare walls from subsonic side of the pipe. Longitudinal parabolic fins have minuscule effect on combined efficiency of considered pipe since extra head losses void any gain of heat transfer. Obtained results provide perspective of increasing efficiency of supersonic tube for temperature stratification. This significantly broadens device applicability in thermostating systems for equipment, cooling systems for energy converting machinery, turbine blades and aerotechnics.

1. Introduction
Temperature stratification devices without moving parts have a broad practical application. This class includes Ranque-Hilsch vortex tube, resonance tubes, Leontiev supersonic tube etc. The major downside of such devices is low efficiency. Efficiency of Leontiev tube [1] is determined by intensity of heat transfer between subsonic and supersonic flows, which are separated with wall. Experimental study [2] showed inverse correlation between temperature drop and flow rate in subsonic channel.

Efficiency of energy separation may be increased by enhancement of heat transfer with subsonic flow. The most straightforward ways of heat transfer intensification are surface intensifiers [3]: fins, studs etc. However, these intensifier provide extra head losses when installed in subsonic channel of Leontiev tube for temperature stratification.

The study reveals thermohydraulic parameters of supersonic device for temperature stratification with surface intensifiers in subsonic channel.

2. Methods
On figure 1 one can see schematic of gas-dynamic temperature stratification in Leontiev supersonic tube. Gas had entered the device via chamber 1 and proceeded to subsonic channel 2. Supersonic flow was fed in corresponding channel 3 from Laval nozzle 4. Heat transfer was driven
by temperature drop between gas temperature in near-wall region and stagnation temperature of the flow. Amount of heat transferred from subsonic to supersonic flow was deduced as follows:

\[
Q = k \cdot F (T^* - T_{r2}),
\]

(1)

\[
k = \frac{1}{\alpha_1 + \sum \delta_i \lambda_i + \frac{1}{\alpha_2}},
\]

(2)

where \(\alpha_1, \alpha_2\) – heat transfer coefficients for subsonic and supersonic flows respectively; \(\delta_i, \lambda_i\) – thickness and thermal conductivity of separating wall respectively.

To increase supersonic device efficiency at given temperature drop one can intensify heat transfer or increase area of heat transfer surface on subsonic side. This is valid at pressure below 80 bar when efficiency of heat transfer with subsonic flow is much lower than it’s counterpart. To increase efficiency one can use fins or studs, mounted on subsonic side of heat transfer surface.

Considered configurations of fins and studs are presented on figure 2.

(a) Rectangular fin  (b) Parabolic fin  (c) Parabolic stud  (d) Rectangular stud  (e) Cylindrical stud

Figure 2: Fins and studs configurations: a, b – longitudinal fins; c, d, e – studs.

Heat transfer coefficients and head losses were evaluated to analyze thermal and hydraulic efficiency of Leontiev device for temperature stratification. Heat transfer coefficients were considered for both subsonic and supersonic channels. Subsonic heat transfer coefficient was calculated with known expression [4]:

\[
St_{w1} = 0.029 \cdot Re_{w1}^{-0.2} \cdot Pr_{w1}^{-0.6}.
\]

(3)

Supersonic flow is known to be strongly affected by compressibility of the medium. Thus corresponding heat transfer coefficient was calculated with modified equation (3) according to previous studies [4, 5]:

\[
St_{w2} = 0.029 \cdot Re_{w2}^{-0.2} \cdot Pr_{w2}^{-0.6} \cdot \left(1 + r \cdot \frac{\gamma - 1}{2} \cdot M_s^2\right)^{0.11},
\]

(4)
where $\gamma$ – adiabatic index; $M$ – Mach number; $r$ – temperature recovery factor, evaluated as follows [5, 6]:

$$r = \frac{T_{r2} - T_{f2}}{T^* - T_{f2}} = \frac{r_0}{1 - 28.67 \cdot G^{0.3}} = \frac{\sqrt{Pr}}{1 - 28.67 \cdot G^{0.3}}. \quad (5)$$

Relative heat flux between working mediums in Leontiev tube was evaluated as follows:

$$q = \frac{q_{max}}{k_{max}} \cdot \frac{T_{r1} - T_{r2}}{T^* - T_{f2}}. \quad (6)$$

In equation (6) thermodynamic temperature $T_{f2} = 0$ at $q = q_{max}$. Equation (6) was modified according to previous studies [5, 6] to estimate efficiency of Leontiev pipe for temperature stratification:

$$q = (1 - r) \cdot \epsilon \cdot \left( \frac{T_{f1}}{T^*} - \frac{T_{f2}}{T^*} \right) / \left( \frac{F_2}{(F_{in} + \eta_r \cdot F_r)} + \frac{\alpha_1}{\alpha_2} \right), \quad (7)$$

where $\epsilon$ – characteristic of working medium properties [5].

For air and dispersed flow $\epsilon$ was calculated as follows:

$$\epsilon = \left( M^2 \cdot Pr \right)^{0.4} \left[ \left( \frac{2}{\gamma + 1} \right) \cdot \left( 1 + \frac{\gamma - 1}{2} \cdot M^2 \right) \right]^J, \quad (8)$$

where $J = \frac{0.4(1 + \gamma)}{\gamma - 1}$.

Finning efficiency was assessed with coefficient $\eta_r$, which is determined by geometry and thermophysical properties of fins and studs [3]. For rectangular longitudinal fins (figure 2 (a)) efficiency coefficient was calculated as follows:

$$\eta_r = \frac{th \left( \frac{m \cdot l}{m \cdot l} \right)}{th \left( \frac{l \cdot \sqrt{2} \cdot Bi}{l \cdot \sqrt{2} \cdot Bi} \right)} = \frac{2}{1 + \sqrt{1 + (2m \cdot l)^2}}. \quad (9)$$

Efficiency of longitudinal parabolic fins (figure 2 (b)), which has minimal specific quantity of metal, was calculated as follows:

$$\eta_r = \frac{2}{1 + \sqrt{1 + \frac{8}{9} \cdot m^2 \cdot l^2}} = \frac{2}{1 + \sqrt{1 + 8 \cdot \left( \frac{1}{\delta} \right)^2 \cdot Bi}}. \quad (10)$$

Efficiency of parabolic stud (figure 2 (c)) was calculated as follows:

$$\eta_r = \frac{2}{1 + \sqrt{1 + \frac{8}{9} \cdot m^2 \cdot l^2}} = \frac{2}{1 + \sqrt{1 + \frac{16}{9} \cdot \left( \frac{1}{\delta} \right)^2 \cdot Bi}}. \quad (11)$$

Efficiency of rectangular stud (figure 2 (d)) was calculated as follows:

$$\eta_r = \frac{th \left( \frac{m \cdot l}{m \cdot l} \right)}{th \left( \frac{4 \cdot \alpha_1 \cdot (l_1 + l_2)}{\lambda_{ef} \cdot l_1 \cdot l_2} \right)} / \sqrt{\frac{4 \cdot \alpha_1 \cdot (l_1 + l_2)}{\lambda_{ef} \cdot l_1 \cdot l_2}}. \quad (12)$$

In special case when $l_1 = l_2 = \delta$ equation (12) was being transformed into:

$$\eta_r = \frac{th \left( \sqrt{8 \cdot Bi} \cdot \frac{l}{\delta} \right)}{\sqrt{8 \cdot Bi} \cdot \frac{l}{\delta}}. \quad (13)$$

Efficiency of cylindrical stud (figure 2 (e)) with diameter $d = \delta$ was calculated as follows:
\[ \eta_r = \frac{th(m_3 \cdot l)}{(m_3 \cdot l)} = th \left( \frac{2 \cdot \sqrt{Bi} \cdot l}{\delta} \right) / \left( \frac{2 \cdot \sqrt{Bi} \cdot l}{\delta} \right). \] (14)

Equations (9)-(14) were obtained with trivial transformations of known expressions for finning efficiency [3]. Herewith coefficients \( m, m_1, m_2 \) and \( m_3 \) were calculated as in mentioned work [3].

Estimation of head losses produced by finning or studding of subsonic channel of Leontiev tube was conducted with Kármán equation introduced with Nikuradze constants. This equation is applicable at Reynolds range \( 3 \cdot 10^4 \div 10^6 \) and was approximated as follows:

\[ f = 0.046 \cdot Re^{-0.25}. \] (15)

Relative friction coefficient in subsonic channel was calculated as Reynolds ratio:

\[ f_r/f_{bare} = \left( \frac{Re_r}{Re_{bare}} \right)^{-0.25}. \] (16)

Hydraulic diameter of annular channel was used as characteristic linear dimension for calculation of \( Re_{bare} \) for bare walls. For finned (studded) channel corresponding \( Re_r \) was calculated with equivalent diameter as characteristic length.

Analysis of thermal and hydraulic efficiency of supersonic tube with finned surface of heat transfer was conducted with the following expression:

\[ \frac{\bar{q}}{F_r/F_{bare}} = \left( 1 - \frac{3 \sqrt{Pr}}{1 - 28.67 \cdot G^{0.3}} \right) \cdot \epsilon \cdot \left( \frac{Re_r}{Re_{bare}} \right)^{0.25} \cdot \frac{T_f}{T^*} - \frac{T_f}{T^*} \bigg/ \left( \frac{F_2}{(F_{in} + \eta_r \cdot F_r)} + \frac{\alpha_1}{\alpha_2} \right), \] (17)

where \( G \) – indicator of condensed particles in flow of working medium [5, 6, 7]; \( F_2 \) – area of heat transfer surface on supersonic side; \( F_r \) – area of heat transfer surface on subsonic side; \( T_f, T_{f1}, T_{f2} \) – thermodynamic temperature of subsonic and supersonic flow respectively; \( \alpha \) – heat transfer coefficients; index ”r” corresponds to finned (studded) surface; index ”bare” corresponds to bare wall; indices ”1” and ”2” correspond to subsonic and supersonic flow.

Analysis of thermal and hydraulic efficiency of the device for gas-dynamic temperature stratification was conducted for air and dispersed flow (carrier medium was air) provided the inertial deposition of particles on the wall of the supersonic channel.

### 3. Results and discussion

Analysis of thermal and hydraulic efficiency of temperature stratification tube was conducted under following assumptions: working medium – air or dispersed flow with air as carrier medium at 80 bar or more, initial Mach number \( M_0 = 0.1 \), Reynolds number near bare walls \( Re_{bare} = 6 \cdot 10^5 \), relative fins (studs) length \( l/\delta = 5 \), Bio criterion \( Bi = 0.1 \), indicator of condensed particles \( G = 5 \cdot 10^{-7} \) and device length \( L = 1 \) m.

Results of conducted analysis (figure 3) showed strong effect of working medium properties and fins (studs) geometry on efficiency of temperature stratification in Leontiev supersonic tube. Here with finning (studding) was considered from subsonic side of heat transfer surface. Presence of dispersed phase provides a significant increase of process efficiency (more than 3 times), considering inertial deposition of dispersed particles.

The most efficient among considered were cylindrical and rectangular studs. These studs provide increase of thermal and hydraulic efficiency in a range of 1.1 ÷ 2.67 times depending...
on supersonic flow intensity. Longitudinal parabolic fins appeared to be ineffective, because increased heat transfer efficiency was compensated by excessive head losses. Thus finned surface provides no apparent benefits compared to bare walls.

![Graphs showing thermal and hydraulic efficiency of Leontiev tube.](image)

**Figure 3:** Thermal and hydraulic efficiency of Leontiev tube.

**Conclusions**

The study of Leontiev tube showed significant efficiency increase of temperature stratification in presence of dispersed particles regardless of finning or studding of heat transfer surface. However, this applies to the case when dispersed particles depositing on heat transfer surface.

Fins and studs geometry affects efficiency of temperature stratification in Leontiev tube as well. Highest thermal and hydraulic efficiency was achieved with cylindrical and rectangular studs installed on subsonic side of heat transfer surface. These studs provide increase of thermal and hydraulic efficiency in a range of $1.1 \div 2.67$ times depending on supersonic flow intensity.

**References**

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