The effect of problem based learning model and authentic assessment on mathematical problem solving ability by using numeric ability as the covariable

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Abstract. The research was intended to 1) recognize differences the ability in solving mathematic problems among students facilitated with problem-based learning model and conventional learning one, 2) determine the influence of authentic assessment towards mathematical problem-solving abilities, and 3) determine the effect of interaction between learning model with assessment model towards mathematical problem-solving abilities, once numerical ability is controlled. The research utilized experiment design using two factor measurements in factorial 2 X 2. The sample is 110 students. Data were collected using tests that had been validated both theoretically and empirically, then analyzed using Anacova statistics. The results of the analysis show that the group of students facilitated by problem-based learning model and authentic assessment get the highest average score of mathematical solving ability compared to other groups. Once the numerical ability is controlled, the results of hypothesis testing show significantly: 1) there is different problem solving ability of mathematic between student which facilitated model of problem based learning with conventional learning model, 2) There is a difference problem solving skills of mathematics between students who facilitated an authentic assessment model with conventional assessment model, 3) there is an interaction effect between the learning model and the assessment model on the ability mathematical problem solving. In an effort to improve the effectiveness of mathematics learning, the collaboration of problem-based learning models with authentic assessment models can be considered as one of the classroom learning models.

1. Introduction
The ability to solve problems is one of the important aspects in making the students literate in mathematics. This ability is needed very much by the society, and is the most important in mathematics, not only by those who will learn mathematics as their specialization, but also for those who will apply it in other disciplines and daily life. For Polytechnic students, especially in the field of engineering, the ability is a potential that is needed in their educational process. Since this ability is important, many efforts have been made intensively and continually by the Indonesian government to improve education. However, many research results show that the Indonesian students’ ability to understand and to solve mathematical problems in including Polytechnic students is still low. Among the results, Research and Development Agency in 2011 reported that the result of a survey conducted by Trends International Mathematics and Science Study (TIMSS) in 2011 shows that the learning achievement of the eighth grade students in Indonesia is in the 36th rank of 49 countries. The learning achievement of Indonesian students as reported by TIMSS 2015 is even worse, since Indonesia is in the 46th rank of 51 countries [1].
Similar to the result of TIMSS report, Programme for International Student Assessment (PISA) shows that the learning achievement of the Indonesian students aged about 13 is still low. In PISA in 2011, Indonesia only gets the 64th rank of 65 countries, with the mean score of 397.

The PISA result in the literacy assessment in reading, mathematics, and science in 2015 places Indonesia in the 67th rank of 76 countries. This data shows that the quality of education in Indonesia is still low [1].

The low quality of education in Indonesia as shown by TIMSS and PISA is obviously caused by many factors. One of them is the fact that Indonesian students are generally not well trained in solving mathematical problems, non-routine problems, which is the main characteristic of TIMSS and PISA problems. This can occur because the activity of problem solving in mathematics teaching has not been made the main activity. Another factor is the teaching model used by the teacher [2]. Teaching model plays an important role in creating success in the teaching and learning process.

Improvement in problem solving ability in the students can be done through an innovative and effective teaching. The teaching process is done systematically and structurally using an effective model and method according to the characteristics of the course. Problem-based Learning (PBL) is a teaching model which makes problems the basis of the students’ learning [3]. Barrows states same characteristics of PBL, i.e., 1) The teaching process is student-centered; 2) The teaching process occurs in small groups; 3) The teacher plays the role as facilitators or guide; 4) The problem presented becomes the stimulus for learning; 5) New information is obtained by self-directed learning and 6) problems become the means to develop the ability to solve problems [4]. Indirectly, the attainment of the ability to solve problems can be developed by the PBL model.

The implementation of PBL in teaching gives some benefits: (1) Preparing the students better for applying what they learn in the real world; (2) Enabling the students to become the producers of knowledge rather than only the consumers; and (3) Helping the students to develop communication, reasoning and critical thinking skill [5].

PBL compared to other teaching models, is a complex constructivist model of teaching and gives a great opportunity for the development of learning autonomy. PBL is very good for developing a high-order thinking skill, i.e., critical and creative thinking. Complex, contextual and ill-structured problems will give the opportunity to the students to develop analytic, evaluative, and reflective thinking and to develop their creativity in finding various kinds of information, in developing a possible solution, and creating various resources to solve the problem that has to be solved [4].

A significant difference in the post test results in advance mathematics learning achievement between the students who were taught by using PBL and those who were taught by using traditional method, the advanced mathematical learning achievement of the students who were taught by using PBL was significantly better than that of those who were taught by traditional method [6]. Padmavathy, that PBL has an effect on mathematics teaching and improves the students’ ability, the ability to use concepts in the real life, PBL is more effective for learning mathematics [7].

Assessment is an integral part of teaching process. Assessment is often regarded as one of three very important pillars in teaching. The three pillars are planning, implementation, and assessment. If the three pillars are synergetic and continuous, then they will determine the quality of teaching. Assessment has to be designed and implemented according to the planning and implementation of teaching. The assessment system has to be developed in line with the development of the model and the strategy of teaching.

The implementation of the teaching model in the classroom to develop the students’ ability to solve mathematical problems will obviously become more optimal by the presence of an appropriate assessment. Wolf states that a good teaching quality has to be followed by a good assessment [8]. Assessment is an important part of teaching and a good teaching will not be successful without a good assessment. Good assessment is generally present with good teaching and learning activities. The teaching and learning process will occur effectively if it is supported by an effective assessment. An assessment activity has to be made a means for improving the effectiveness of the teaching and learning process [9].
Authentic assessment is a process of collecting, reporting and using information about the students’ learning achievement by applying assessment principles, continuous implementation, authentic, accurate, and consistent evidence as public accountability [9]. Authentic assessment is appropriate for assessing problem based learning process [10].

PBL which is integrated with authentic assessment, the students are expected to be able to solve the problems given as the process to master the existing mathematical concepts. For example, teaching the application of the concept of comparison of trigonometric functions through the problem of determining the width of a scout tent as follows.

**Problem**

![Image of a tent](image1)

Figure 1 is a prism-shaped tent that is shaped prism built by students during the camp activity. The height of the tent pole is 2 m and is visible from the front entrance of the same triangle foot tent. The student wants to measure the width of the tendon, but does not carry a length measurement tool. The student only carries the degree arc in his bag. When they measure the angle between the foot of the tent and the ground they get 45°. Can you help, how wide is the tent?

Illustration of the above learning, PBL with authentic assessment in the application of the concept of comparison of trigonometric functions. Students learn in small groups, lecturers play a role as facilitator to accompany students to discuss in their group to find problem solving which will then be presented. Assessment carried out during the learning process includes aspects of attitude and skills using checklist equipped rubric. While the knowledge aspect is implemented at the end of the learning using essay test which is completed with assessment rubric.

PBL situations encourage students to make connections between their knowledge and application in real life. The result of the discussion is then presented in the form of a simple report and is presented through a presentation activity which is one of the forms of authentic assessment. The authentic assessment which is integrated with PBL model can increase the students’ activities and motivation in learning and give the opportunities to the students to always assess themselves so that they can know their level of mastery. With the increase in their activities and motivation to learn and with the awareness of the relation between mathematics at school and their real life, it will be easier for them to understand the existing concepts. As the consequence, the students’ mathematics learning achievement will be better.

The teaching model and the technique of assessment used by the teacher in the classroom are actually determining factors for the students’ learning achievement, including for the ability to solve mathematical problems. In the attainment of this ability, every student has different characteristics and abilities. As an effort to improve mathematics learning achievement, in addition to considering the factor of teaching approach the teacher also has to consider the students’ internal factor which is a basic factor.

One of the basic abilities that the students have to possess in relation to the ability to solve mathematical problems is numeric ability. Numeric ability is the ability to operate numbers, in the form
of the ability to count which covers addition, substruction, multiplication, and division of numbers with speed and precision which facilitates the solution of mathematical problems.

Howard Gardner calls numeric ability numeric aptitude, i.e., logico mathematical intelligence in using numbers and reasoning. A person who has numeric intelligence, in general, has an organized way of thinking and solving problems. Those who are good at logical mathematics tend to learn mathematics quickly [11].

The students’ numeric ability needs to be considered in mathematics teaching. Mathematics involves arithmatical operations such as substruction, addition, multiplication, and division. The higher a student’s numeric ability the more chance he or she has to develop and achieve in mathematics. The students who have a high numeric ability tend to be active in learning, have an ability to solve problems, classify an categorize information, and do complex mathematical computation. Badru and Ademola also show a significant effect of numeric ability on mathematics learning achievement [12]. Singh and Kumar show a positive and significant correlation between numeric aptitude and academic achievement in mathematics among high school students [13].

Based on the explanation above, naturally, mathematical problems solving needs much mathematics computation. To find out the effect of PBL model and authentic assessment on the ability to solve mathematical problems, there is a need to control numeric ability scores so that the effect that occurs can be believed to be purely caused by PBL model and assessment.

The key problems investigated in this study, i.e., after numeric ability is controlled: 1) is there any difference in the ability to solve mathematical problems between the students who are facilitated by PBL and those who are facilitated by conventional teaching model?; 2) is there any difference in the ability to solve mathematical problems between the students who are facilitated by authentic assessment model and those who are facilitated by conventional assessment model?; and 3) is there any effect of interaction between teaching model and assessment on the ability to solve mathematical problems?

2. Research Method

This study was conducted at Mechanical Engineering Department in Politeknik Negeri Bali, by using quasi-experiment design by measuring two factors in 2x2 factorial version. The sample taken consisted of 110 students, distributed in four classes.

The data were collected by using mathematical problem solving ability test and numeric ability test. Each instrument has been validated both theoretically and empirically. Theoretically, content validity was tested by using Aiken’s statistical index approach, while empirically, it was tested by using classical test approach.

The data were analyzed descriptively and by using 2x2 factorial ANACOVA. Before it was analyzed by anacova, prerequisite analysis testing was done, i.e; data distribution normality test, intergroup variance homogeniety, linearity test and multicolinearity test. The results showed that the assumptions were met.

3. Results And Discussion

After treatment, PBL-AA group showed the highest mean in mathematical problem solving ability. The group of students who were facilitated by conventional teaching model and authentic assessment were better than those who were facilitated by PBL and conventional assessment. While the group of students facilitated by conventional model and conventional assessment were lower than other groups.

The testing of the hypotheses was done by using ANACOVA. The testing of all hypotheses was done consecutively, the recap of the testing can be seen in table 3.1 as follows.
Table 3.1 Results of 2x2 Factorial ANACOVA of Mathematics Problem Solving Ability Data

| Source          | Type III Sum of Squares | Df | Mean Square | F    | Sig.  |
|-----------------|-------------------------|----|-------------|------|-------|
| Corrected Model | 2758.335                | 4  | 689.584     | 35.996 | .000  |
| Intercept       | 20371.015               | 1  | 20371.015   | 1063.373 | .000  |
| X               | 1013.613                | 1  | 1013.613    | 52.911 | .000  |
| A               | 1266.843                | 1  | 1266.843    | 66.130 | .000  |
| N               | 300.847                 | 1  | 300.847     | 15.704 | .000  |
| X * A           | 303.062                 | 1  | 303.062     | 15.820 | .000  |
| Error           | 2011.483                | 105| 19.157      |       |       |
| Total           | 611578.000              | 110|             |       |       |
| Corrected Total | 4769.818                | 109|             |       |       |

Notes:
Nu = Covariate (Numeric Ability)
X = Teaching Model
A = Assessment

Before the effect of the variable of numeric ability was controlled, the analysis shows that the effect of the interaction between teaching model and assessment on mathematical problem solving ability was significant. This was evidenced by the value of $F = 39.846$ ($p < 0.05$). It means there is an effect of interaction between teaching model and assessment on the ability to solve mathematical problems. The effect of teaching model on the ability to solve mathematical problems depends on the assessment used.

The Anacova analysis with numeric ability covariable shows that the interaction between teaching model and assessment on ability to solve mathematical problems is still significant as shown by the value of harga $F = 15.820$ ($p < 0.05$). Also significantly, numeric ability has an effect on the ability to solve mathematical problems ($F = 15.704$; $p < 0.05$).

The ability to solve mathematical problems is one of the form of the result of learning mathematics, and is the highest capability in thinking skill and intellectual skill. As one of the forms of the result of learning it is of course influenced by various abilities that are basic for the students, i.e. numeric ability. Numeric ability is the ability to operate numbers, in the form of arithmetic which covers addition, subtraction, multiplication, and division of numbers with precision so that it facilitates the solving of mathematical problems. The students who have a high numeric ability tend to be active in learning, show the ability to solve problems, classify and categorize information and do a complex mathematical computation.

Fatoke, Ogunlade, and Ibidirin state that one of their findings shows that the problem solving learning strategy, numeric ability of the students and their performance improve in chemistry [14]. Thus, after numeric ability was controlled, the mean of the ability to solve mathematical problems was caused by the effect of interaction between teaching model and assessment used, rather than solely caused by numeric ability.

PBL pedagogy helps to show and clarify the way of thinking and the richness of cognitive structure and process involved in it. In addition, in PBL the students can become self directed learners who have the desire to understand and learn, formulate learning needs, the ability to select and used the best learning resources. A self directed learner is an individual who directs himself or herself in the learning process. Sadia firmly states that in PBL situation, the students obtain learning experience to develop a high order thinking skill, analytic skill and problem solving skill as well as to develop autonomy and competitiveness. In such a situation the students learn how to use an interactive process and evaluate what they know, identify what they know, collect information, and collaborate in evaluating a hypothesis based on the data that have been collected [4]. This is similar to what is stated by Ibrahim and Nur, in PBL situation the students can be involved in a high order thinking and problem solving oriented to the real world problems, including learning how to learn. The students are
expected to have a wholistic understanding of a material that is formulated in the problem, a positive attitude and gradual and continuous skill. The students’ mental activity is required to understand a concept, a principle, and skill through a situation or problems presented in the beginning of the lesson. The students understand concepts and principles of a material from working and learning in a situation or problem given through investigation, inquiry, and problem solving. The students develop concepts and principles through their own ability which integrates the skills and knowledge that they have understood before [14].

While conventional teaching model is teacher centered. Coleman states that conventional teaching is an assimilation of information or classroom teaching, with the following characteristics: 1) acquisition of information through symbolic sources such as through teachers and reading; 2) assimilation or organization of information so that general information is understood, 3) use of a general principle in cases of specific nature; and 4) application of a general principle in a new condition [15]. Thus, in the conventional teaching model, the teacher’s authority tends to dominate learning. The teacher has to demonstrate knowledge or skill to be trained to the students step by step. The teacher is required to play the role as an attractive model for the students, the material developed should meet students’ interest. The students are passively receptive, their reasoning and knowledge develop only as far as the limit of the teacher’s. The students’ activities become limited and this causes them unable to learn optimally. It is clear that it is better to implement PBL than conventional teaching model for the students since in PBL all of the student’s senses of perception are involved in the teaching process. Therefore, the teaching outcome in the form of ability to solve mathematical problems of the students who learned through PBL is better than that of those who learn through conventional teaching model.

Assessment is a process of collecting various data which can give a description of the students’ learning development [9]. This assessment stresses the importance of the ability of the students to demonstrate the knowledge that they have meaningfully and realistically. The activity of assessment does not only consist of asking or tapping the knowledge that the students have known, but of asking them to show the real performance about knowledge that has been understood and mastered. In addition, authentic assessment requires the students to demonstrate knowledge, skills and strategies by creating answers or products. The students are not only required to respond like in the conventional test, but are asked to create and produce answers based on theoretical knowledge.

The applications of authentic assessment in teaching that happens in the classroom can give a good psychological impact on the students. The involvement of the students in the assessment process can increase social interactions and mutual trust among themselves and with the teacher. The perceptions of the students that are involved in assessment activity generally improve their self-confidence, responsibility and motivation. Social interactions can make the students and the teacher control the result of assessment made by each party, so that the assessment becomes valid, objective, and can be accepted by both students and teacher.

The conventional assessment is often called paper and pencil assessment. Such an assessment is separated from teaching. Paper and pencil assessment cannot be regarded as an integral part of the teaching process. The giving of feedback for the students will be delayed and even almost cannot be done, so that the objectives of correcting the teaching as the result of assessment cannot be done immediately and even it is not if it will be done or not. This assessment model doesn’t provide information to the students during the learning process. Based on discussion, it seems that authentic assessment model tends to be better than the conventional assessment model in the attainment of the ability to understand concepts in mathematical problem solving.

There was an effect of interaction between teaching model and assessment on the ability to solve mathematical problems. The students facilitated by PBL and authentic assessment had the highest mean score for the ability to solve mathematical problems compared to other groups. The students facilitated by PBL and authentic assessment discussed in groups intensively, so that they would ask each others, answer, criticize, correct, and clarify every concept or mathematical argument that occurred in the discussion. In such a discussion will develop the students’ ability to make, refine, and
explore conjectures, so that it will strengthen their understanding of the mathematical concept that they are learning, or mathematical problem that they are solving. In the authentic assessment situation they have very broad opportunity to construct learning outcome. Since the teacher does not only ask the students to repeat what they have learned. The students are asked to construct what they have got when they face a concrete situation. In this way the students select and organize answers based on the knowledge that they have and analyze the situation in order their answers are relevant and meaningful. Such mathematical teaching will be able to improve the students’ ability to understand concepts and solve mathematical problems.

The students facilitated by conventional teaching and authentic assessment also have a higher mean score for the ability to solve mathematical problems than that of the group of the students facilitated by PBL model and conventional assessment and the group of students facilitated by conventional teaching and conventional assessment.

The students in the class facilitated by conventional teaching model and authentic assessment model experienced teaching process that tend to be teacher centered. In the design of the teaching program, the teacher does not consider prior knowledge that the students have. The teaching process occurs one way, the role of the teacher is no longer as motivator, facilitator and mediator, but as a teacher with an authority in teaching. In the teaching activities, the teacher tends to used lecturing method with a few questions and answers. The teacher tries to transfer or transmit knowledge that he or she has to the students. This condition tends to make the students passive in receiving the lesson. Such teaching activities do not create active learning and even the students become very passive. But authentic assessment requires the students to demonstrate all skills, knowledge and competences needed to solve problems given, passivity of the students can be lessened and even they can become active. The students are given the opportunity to construct their learning outcome since the assessment does not only ask the students to repeat what they have learned. The students are asked to construct what they have got when they are facing a concrete situation. In this way the students will select and organize answers based on the knowledge that they have and the result of analyzing the situation in order the answers are relevant and meaningful. Such a mathematical teaching situation will facilitate the students in improving their ability in understanding concepts and solving mathematical problems.

From the above discussion each teaching model and assessment has the same direction, i.e., the attainment of the learning objectives. The learning objectives will be achieved when the teachers and students feel the learning process to be meaningful. This occurs if the students follow correct and appropriate steps of learning according to the characteristics /syntax of the teaching model implemented. Thus the optimization of the attainment of the learning objectives can be achieved. Based on the above discussion, we can see the strengths and the weaknesses of each teaching model, depending on the assessment model used. Thus in the teaching process the teacher should consider the assessment model used. The teaching process will occur effectively if it is supported by and effective assessment, assessment activities should be viewed as the means to improve the teaching process.

The implications that can be drawn from the results of this study is that the collaboration of teaching model and assessment needs to be considered in teaching mathematical problem solving. The application of PBL is very effective if it is collaborated with authentic assessment. Conventional teaching becomes effective if it is collaborated with authentic assessment. PBL is less effective if collaborated with conventional assessment in improving concept understanding mathematical problem solving ability. The implementation of each model obviously needs to consider the students, numeric ability.

4. Conclusion, Implication and Suggestion
In conclusion, once the numerical ability is controlled: there is different problem solving ability of mathematic between student which facilitated model of problem based learning with conventional learning model; there is a difference problem solving skills of mathematics between students who facilitated an authentic assessment model with conventional assessment model; there is an interaction
effect between the learning model and the assessment model on the ability mathematical problem solving. Authentic assessment strongly supports the effectiveness of problem-based learning in the classroom compared with conventional assessment. Authentic assessment can also improve the effectiveness of conventional learning.

The implication is that improving students’ mathematical problem-solving abilities can be pursued through the collaboration of problem-based learning models with authentic assessment. Implementation of learning models and assessments in the classroom is necessary to consider the numerical ability of each student.

Suggestion. Before the problem-based learning model and authentic assessment are implemented in the classroom, the lecturers desperately need to make well-prepared preparations. Lecturers really need to provide a detailed explanation so that students understand the problems faced well. The motivational plants learn the students so that they have the confidence to succeed, so they become more active, creative and interested in solving the problems given. More time is maximized to lead students in discussion. Facilitate student discussions only when absolutely necessary. In a state of discussion of deadlock, lecturers can provoke students’ ideas with challenging questions, or give key clues without turning off creativity.

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