Load-displacement Relationship Modeling with Power Function on Cylindrical Underreamed Ground Anchor

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Abstract. An underreamed ground anchor technique is developed for high anchorage force demand based on shaft anchors worldwide. Engineers and scholars invented various types of underreamed ground anchors, such as those based on jet grouting, mechanical reaming, and blasting. To guarantee the quality of anchorage section and reduce cost, a new type of cylindrical underreamed ground anchor with geotextile bag was developed and applied throughout China. Through 33 Q–s curves in the field tests of underreamed anchor with geotextile bag, a power function model was established for describing the relationship between the load and displacement of a cylindrical underreamed anchor. Finally, the high precision of the function model was verified.

1. Introduction

Compared to other geotechnical reinforcement and supporting techniques, ground anchors are advantageous because they utilize the mechanical properties of rock and soil. They change the rock and soil from a total applied load to a partial carrier. Therefore, ground anchors not only guarantee the stability and safety of engineering projects, but also reduce engineering cost and shorten the construction period.

In ground anchor engineering, bearing capacity is an important indicator of an anchor. Improving the capacity of anchors yields more economic benefits. Function 1 can address the influence factors of an anchor directly [1~3].

\[ F = 2\pi \cdot R \cdot L \cdot \mu \] (1)

In function 1, the friction parameter \( \mu \) between the anchorage section and soil is determined when the soil properties and grouting process are known. Subsequently, the capacity of a ground anchor is determined by the anchor radius \( R \) and length \( L \). Meanwhile, the capacity is limited when lengthening the anchor length \( L \) [4]. Therefore, expanding the radius of the anchor is an efficient method to improve the anchor capacity.

The reaming technique of the ground anchor is from pile technology [5]. A conical anchor in series with high capacity was applied in hard clay in England in 1967. Since then, underreamed ground anchor has been accepted and applied worldwide. Some geotechnical engineering companies developed various types of underreamed anchors in succession. Liao [6,7] in Taiwan described a cone-shaped anchor with mechanical reaming technology; Hobst and Zajic [5] reported a blasting anchor method; Cheng [8] produced a type of deep mixing anchor; Massarsch [9] developed the Soilex
anchor system in which its capacity can reach 300 kN; Zeng [10] reported a bit expanded anchor by jet grouting.

The primary underreamed ground anchor technologies can be classified into three types: with jet grouting, with mechanical reaming, and with blasting. In the first two reaming techniques, it is difficult to guarantee the clarity in the borehole; therefore, the anchor section is mixed with soil chips and the strength becomes low. Regarding the blasting anchor, many counties forbid blasting in cities because of safety.

To present an underreamed ground anchor technology with low cost, high quality, and safety, a new type of underreamed anchor with geotextile bag (short for GB anchor) was developed in China at the beginning of the 21st century, as shown in Figure 1 and Figure 2. Through many years of application, the new anchor technology has been welcomed by Chinese construction companies because of its high capacity, low deformation, and quick installation.

The GB anchor exhibits the same mechanical properties as most other underreamed anchors. Underreamed anchors not only improve the anchor capacity, but also have a completely different mechanism from traditional shaft anchors. The expanded section of an underreamed anchor produces an end-bearing force to endure the uplift force similar to a reversed pile. Therefore, the total resistance of an underreamed anchor includes friction and end-bearing force, as shown in Figure 3. Because of the whole new bearing form, there is a vast difference between the bearing mechanism in underreamed anchors and normal shaft anchors. In [11~12], the bearing mechanism of underreamed anchors is described using model tests and numerical simulations. Articles [13~14] discussed different calculation methods and forecasting methods. However, different theories produced different results in confirming the capacity of an underreamed anchor. Overall, the theoretical research findings related to underreamed anchors lag the engineering practice. One of the most important issue in the research is that most of in-situ tests of underreamed anchor can’t be loaded to the bearing status which designers concerned.

In this study, the power function model for describing the load against displacement relationship of the GB anchor is established based on the Q~s~ curves from field tests across China. With the mathematical model, designers can forecast the bearing capacity value corresponding to a concerned displacement of an underreamed anchor based on a few experimental data.

2. Capacity forecasting method for underreamed anchor based on power function model
Compared to theoretical calculations, a field test can yield more accurate mechanical parameters of the ground anchors. The actual load against displacement curve was obtained for determining the ultimate
capacity and ultimate displacement of an anchor. In fact, the Q–s curve is a synthesis of the anchor and foundation soil, which contains friction, end-bearing capacity, anchor size, and soil mechanical parameters. It comprehensively states the operating characteristics of the anchor–soil system.

![Diagram of the force distribution of an underreamed anchor](image)

Figure 3. Diagram of the force distribution of an underreamed anchor with (d) diameter of the normal anchor section; (H) embedment depth; (L) length of underreamed section; (D) diameter of underreamed section; (1) friction of normal section; (2) end bearing force; (3) friction of underreamed section.

Because the GB anchor has a much higher capacity than the shaft anchor, the steel-bar strength, and the measuring capacity and strength of the test equipment can hardly satisfy the high capacity of the GB anchor in practice. When added to the effects of test objective and budget, few field tests acquired the ultimate capacities of GB anchors. Accordingly, the method to determine the ultimate capacity and ultimate displacement based on the data obtained is important for engineering practice and theoretical research. An efficient method to solve this problem is to establish a mathematical model describing the Q–s relationship. Typical mathematical models for describing the Q–s curves of geotechnical components contain hyperbolic models [15], trilinear models [16], and exponential curve models [1]. To describe the relationship between the load and displacement of the GB anchor, this study summarized 33 Q–s curves from eight field tests throughout China and established a power function model. The mathematical model is shown in Formula 2.

$$y = \frac{x^c}{A + Bx^c}$$  \hspace{1cm} (2)

When the numerator and denominator are divided by $x^c$ in the right hand of Formula 2, we obtain

$$y = \frac{1}{\frac{A}{x^c} + B}$$  \hspace{1cm} (3)

Because $C$ is greater than zero in the model, Formula 4 can be acquired from Formula 3.

$$\lim_{x \to \infty} y = \frac{1}{B}$$  \hspace{1cm} (4)

Therefore, the theoretical ultimate capacity $Q'$ of the GB anchor is $1/B$. Substituting $x = 1/B$ into Formula 2, the ultimate displacement can be obtained. Figure 4 to Figure 7 display fourteen of the 33 Q–s curves from eight field tests separately and the corresponding fitting curves by Formula 2.
Figure 4. The Q–s curves and fitting curves of GB anchors tested in Beijing.

To explain the reasonability and accuracy of the power function model, Table 1 lists the fitting parameters, the correlation coefficients and theoretical ultimate bearing capacity Q’ of the anchors.

Table 1. Fitting factors and correlation coefficients of the Q–s curves of GB anchors.

| No. | Name            | A      | B      | C   | R²  | Q’ (kN) |
|-----|-----------------|--------|--------|-----|-----|---------|
| 1   | Beijing No.A1   | 9.52E-03 | 1.79E-05 | 0.61 | 0.994 | 55892   |
| 2   | Beijing No.A2   | 8.34E-03 | 1.36E-05 | 0.55 | 0.984 | 73534   |
| 3   | Beijing No.A3   | 7.34E-03 | 5.95E-06 | 0.53 | 0.995 | 167935  |
| 4   | Fujian No.E1    | 8.10E-02 | 7.61E-04 | 1.08 | 0.993 | 1314    |
| 5   | Fujian No.E2    | 3.41E-02 | 7.48E-04 | 0.88 | 0.999 | 1337    |
| 6   | Fujian No.E3    | 4.26E-01 | 9.20E-04 | 1.63 | 0.994 | 1086    |
| 7   | Hebei No.A1     | 2.93E+00 | 1.26E-03 | 2.63 | 0.986 | 794     |
| 8   | Hebei No.A2     | 2.40E-02 | 1.21E-03 | 1.18 | 0.983 | 827     |
| 9   | Hebei No.A3     | 1.88E-02 | 1.37E-03 | 1.03 | 0.982 | 729     |
| 10  | Hebei No.A4     | 1.42E-02 | 1.67E-03 | 1.20 | 0.986 | 600     |
| 11  | Hebei No.A5     | 1.64E-01 | 7.42E-04 | 1.61 | 0.995 | 1349    |
| 12  | Hebei No.A6     | 4.14E-01 | 9.03E-04 | 1.71 | 0.988 | 1108    |
| 13  | Hebei No.A7     | 1.13E-01 | 8.43E-04 | 1.44 | 0.98  | 1187    |
| 14  | Hebei No.A8     | 2.21E-01 | 7.78E-04 | 1.64 | 0.995 | 1285    |
| 15  | Henan No.A1     | 4.87E-02 | 5.87E-04 | 0.96 | 0.996 | 1703    |
| 16  | Henan No.A2     | 1.54E-02 | 3.01E-05 | 0.53 | 0.983 | 33195   |
| 17  | Henan No.A3     | 3.20E-02 | 7.64E-04 | 0.85 | 0.994 | 1310    |
| 18  | Henan No.A4     | 1.73E-02 | 6.84E-04 | 0.60 | 0.97  | 1461    |
| 19  | Henan No.E1     | 3.83E-02 | 1.23E-03 | 0.89 | 0.958 | 811     |
Analyzing the factor C in Formula 2 referring to the Q–s curves, it is found that C controls the opening size between the Q–s curve and the X-axis. Referring to the C-related ultimate bearing capacity Q’ when the C value is smaller, the asymptote of the Q–s curve becomes higher. When the C value is higher than 1.0, the Q’ value becomes approximately the real ultimate bearing capacity of the GB anchor. However, when the C value is less than 1.0, the Q’ value becomes larger than the real value. In this case, the Q’ value cannot be used as the ultimate capacity, such as the Beijing No.A1 to No.A3 anchors in Table 1; however, the designer can forecast the bearing capacity of a pointed displacement, such as that shown in Table 2.

Table 2. Corresponding bearing capacity values of pointed displacement values on the fitting curves of GB anchors.

| No. | Name          | A   | B   | C   | R²   | s_u/mm | Q_u/kN | Q_c/kN | Error |
|-----|---------------|-----|-----|-----|------|---------|--------|--------|-------|
| 1   | Beijing No.A1 | 9.77E-03 | 9.54E-06 | 0.61 | 0.993 | 73      | 1360   | 1386   | 1.96% |
| 2   | Beijing No.A2 | 8.46E-03 | 8.97E-06 | 0.56 | 0.978 | 84      | 1360   | 1381   | 1.59% |
| 3   | Beijing No.A3 | 6.94E-03 | 7.92E-06 | 0.52 | 0.977 | 83      | 1458   | 1397   | -4.18%|
| 4   | Fujian No.E1  | 4.77E-02 | 3.19E-04 | 0.85 | 0.997 | 134     | 840    | 952    | 13.36%|
| 5   | Fujian No.E2  | 2.94E-02 | 5.75E-04 | 0.80 | 1.000 | 124     | 800    | 838    | 4.79% |
| 6   | Fujian No.E3  | 1.04E+00 | 1.06E-03 | 1.93 | 1.000 | 132     | 950    | 875    | -7.93%|
| 7   | Hebei No.A1   | 1.66E+00 | 1.41E-03 | 4.00 | 0.980 | 84      | 781    | 708    | -9.40%|
| 8   | Hebei No.A2   | 3.68E-02 | 1.49E-03 | 1.50 | 0.995 | 99      | 781    | 656    | -15.98%|
| 9   | Hebei No.A3   | 1.42E-02 | 8.59E-04 | 0.77 | 0.994 | 98      | 639    | 789    | 23.41%|
| 10  | Hebei No.A4   | 1.15E-02 | 1.10E-03 | 0.87 | 0.999 | 79      | 568    | 737    | 29.76%|
| 11  | Hebei No.A5   | 1.73E-01 | 7.55E-04 | 1.63 | 0.992 | 61      | 1045   | 1038   | -0.69%|
| 12  | Hebei No.A6   | 4.69E-01 | 9.19E-04 | 1.75 | 0.981 | 179     | 1045   | 1030   | -1.44%|
| 13  | Hebei No.A7   | 9.04E-02 | 7.99E-04 | 1.36 | 0.973 | 170     | 1092   | 1131   | 3.61% |
| 14  | Hebei No.A8   | 2.78E-01 | 8.06E-04 | 1.72 | 0.994 | 132     | 1183   | 1152   | -2.65%|
| 15  | Henan No.A1   | 5.76E-02 | 5.41E-04 | 0.98 | 0.987 | 143     | 1000   | 1007   | 0.75% |
| 16  | Henan No.A2   | 2.73E-02 | 9.79E-06 | 0.67 | 0.995 | 82      | 650    | 712    | 9.57% |
| 17  | Henan No.A3   | 2.91E-02 | 6.17E-04 | 0.77 | 0.991 | 213     | 900    | 930    | 3.35% |
| 18  | Henan No.A4   | 3.86E-02 | 1.18E-03 | 0.90 | 0.982 | 281     | 800    | 703    | -12.16%|
| 19  | Henan No.E1   | 2.85E-02 | 5.26E-06 | 0.63 | 0.956 | 116     | 550    | 711    | 29.33%|
| 20  | Henan No.E2   | 1.70E-02 | 4.77E-05 | 0.51 | 0.965 | 164     | 550    | 770    | 40.00%|
| 21  | Henan No.E3   | 1.73E-02 | 3.44E-05 | 0.50 | 0.994 | 133     | 550    | 658    | 19.62%|
| 22  | Jiangsu No.A1 | 7.90E-02 | 1.92E-03 | 1.33 | 0.979 | 96      | 450    | 474    | 5.39% |
| 23  | Jiangsu No.A2 | 6.78E-01 | 3.33E-03 | 2.15 | 0.979 | 103     | 375    | 297    | -20.77%|
It is supposed the last Q–s tested points of all the 33 GB anchors are unknown. Table 2 lists the fitting parameters A, B, C, and the correlation coefficients of the 33 GB anchors without the supposed unknown points. Besides, the displacements of ‘unknown’ points are arranged to the pointed displacements \( s_0 \), they are also listed in Table 2 together with the tested real capacities \( Q_0 \) corresponding to the pointed displacements \( s_0 \). Subsequently, the calculated capacities \( Q \) from the pointed displacements and the calculation errors are all shown. From the data in Table 2, the calculation errors range from -22.07%–40.00%. Therefore, when considering the load corresponding to a pointed displacement on the Q–s curve as the ultimate bearing capacity of a GB anchor, it is safe setting the safety factor of working load to 1.5. It can be concluded that the capacity calculation results of the pointed displacements out of the tested range are reasonable.

When the load applied to the GB anchor does not reach its ultimate capacity value in the field test, the ultimate bearing capacity value or the capacity value of a pointed displacement can be forecasted with the acquired test data based on the power function model of Formula 2. When the C value is higher than 1.0, the value of parameter \( 1/B \) will be the ultimate capacity value. However, when the C value is less than 1.0, one can point to a concerned displacement to calculate the corresponding capacity value with Formula 2.

Actually, because ground anchors are flexible structure elements, the ultimate working condition of ground anchors are usually decided by displacements. Therefore, it has significance of engineering practice when the designers calculate the ultimate capacities of GB anchors by deciding their ultimate displacements with test data and Formula 2.

It is noteworthy that it is better to obtain the test data of a large displacement in field test when adopting Formula 2. Hence, the C value will be higher than 1.0. Therefore, the theoretical ultimate bearing capacity will be approximately the real value. Besides, though the field tests are based on GB anchors, the theoretical results are appropriate for other types of underreamed anchor with the same geometry.

### 3. Conclusions

An innovative anchor system of an underreamed ground anchor with GB was introduced herein. A reasonable power function model for describing the relationship between load and displacement was established based on 33 Q–s curves from the field test results of the GB anchor.

With regard to an innovative geotechnical technology such as the GB anchor, it was important to establish a simple and efficient mathematical model of the Q–s curve for determining its ultimate capacity and researching its mechanical property.

Based on 33 Q–s curves of the GB anchors, a power function model for describing the load against displacement the relationship of cylindrical anchor was established. The model exhibited a high fitting precision: the correlation coefficients were more than 0.929, and the average value reached 0.986. Using the model, the ultimate bearing capacity and the capacity value of the displacement that designers concerned could be acquired with a small number of test data.

| 24 | Jiangsu No.A3 | 2.41E-01 | 1.93E-03 | 1.74 | 0.984 | 99 | 525 | 496 | -5.55% |
| 25 | Shenzhen No.E1 | 2.79E-02 | 9.35E-04 | 0.98 | 0.992 | 53 | 699 | 664 | -4.96% |
| 26 | Shenzhen No.E2 | 4.33E-02 | 7.36E-05 | 0.79 | 0.996 | 105 | 827 | 852 | 2.99% |
| 27 | Shenzhen No.E3 | 1.18E-02 | 3.72E-04 | 0.58 | 0.962 | 75 | 827 | 760 | -8.13% |
| 28 | Guangdong No.1 | 9.21E-02 | 6.02E-04 | 1.30 | 0.997 | 48 | 827 | 838 | 1.25% |
| 29 | Guangdong No.2 | 3.76E-02 | 6.38E-06 | 0.78 | 0.871 | 76 | 827 | 775 | -6.29% |
| 30 | Guangdong No.3 | 4.48E-02 | 2.60E-05 | 0.86 | 0.994 | 68 | 827 | 819 | -0.93% |
| 31 | Guangdong No.4 | 4.12E-02 | 1.58E-04 | 0.90 | 1.000 | 117 | 1166 | 1375 | 18.01% |
| 32 | Guangdong No.5 | 3.54E-02 | 3.20E-05 | 0.85 | 0.998 | 71 | 1034 | 1014 | -1.95% |
| 33 | Guangdong No.6 | 2.39E-02 | 1.38E-04 | 0.78 | 1.000 | 98 | 1166 | 1252 | 7.42% |

It is noteworthy that it is better to obtain the test data of a large displacement in field test when adopting Formula 2. Hence, the C value will be higher than 1.0. Therefore, the theoretical ultimate bearing capacity will be approximately the real value. Besides, though the field tests are based on GB anchors, the theoretical results are appropriate for other types of underreamed anchor with the same geometry.
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