Effect of Thermal Radiation on MHD Stagnation Point Flow over a Shrinking Sheet

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Abstract. In this present paper, we investigate a two dimensional magnetohydrodynamic (MHD) stagnation flow over shrinking sheet in the presence of thermal radiation. In this study, the effects of magnetic, heat generation or absorption, radiation and Prandtl number parameter are taken into consideration. An appropriate similarity transformation is introduced to transform the governing partial differential equations into a system of ordinary differential equations then solved using bvp4c in MATLAB. The effects of the involved parameters such as magnetic field, velocity, heat generation/absorption, Prandtl number as well as local Nusselt number are calculated numerically. Result on skin friction and temperature increased as the magnetic field increased. While, the temperature decreased as the radiation parameter increased as well as the Prandtl number.

1. Introduction

Over the years, the study on stagnation point flow has grown drastically among researchers due to its various applications such as in cooling industries, submarines and aircraft flows, hydrodynamics processes and etc. Hiemenz [1] was the first person who studied the steady two dimensional (2D) point flow and reduced the governing Navier-Stokes equations using similarity transformation. Later, Eckert [2] include the energy equation, and obtain an exact solution for thermal field. Chiam [3] tried to extend the works of Hiemenz [1] by considered equally stretching and straining velocities but unfortunately he was unable to obtain any boundary layer near the sheet. Mahapatra and Gupta [4] then reinvestigate the stagnation point flow towards a stretching sheet by considering different stretching and straining velocities. Many researchers have been working on stagnation point flow by taking consideration on its types of fluid, physical conditions and the effects towards the flow.

Literature study shows that researchers have studied the flow caused by stretching sheet because of its distinctive solution. Sahar[5] investigated the impact of magnetic field flow over a permeable stretching wall in porous medium with heat radiation and suction / injection.Meanwhile, in the presence of radiation and buoyancy effects, Rashidi et al.[6] conducted a study on free convective heat and mass transfer for MHD flow over a permeable vertical stretch sheet. Later, the research was proceeded over a stretching porous sheet by Yahaya et al.[7] and the issue was solved using the technique of homotopy analysis method. The findings of their study found that when the parameter of buoyancy rises, the velocity of the fluid rises and the heat boundary layer reduces where it was in fact a good agreement with prior studies. As in case of thermal radiation, increasing the thermal radiation parameter produces significant increases in the thermal conditions of the fluid temperature. Besides, Bhukta et al. [8] study
the dissipation effect with non-uniform heat source/sink. Recently, Ibrahim et al. [9] has conducted a study on laminar flow over a vertical stretching sheet with the existence of heat sink/source and magnetic field. Problem with induced magnetic field has been carried out by Ali et al. [10]. Ishak[11] and Khedr et al.[12] solved the problem with induced magnetic field, followed by micro-polar liquid in stretch sheets with distinct consequences. Some scientists [13]–[16] also addressed issues with various physical circumstances and impacts on stretching sheet.

Wang [17], on the other hand, was the first to observe the flow around the shrinking sheet while studying liquid film behavior over unstable shrinking sheet. Goldstein [18] pointed out that the shrinking sheet flow is essentially a backward flow that differs from the stretching flow in its physical events. Wang [19] again performed two studies, first on a two-dimensional shrinking sheet stagnation stream and two axisymmetric stagnation flows on an axisymmetric shrinking sheet. He discovered that there are no alternatives for a higher declining rate and may be non-unique in two-dimensional cases. Later Wang’s work was extended by Bhattacharyya and Layek [20] and Ishak et al. [21] under difference circumstances. Kandasamy et al. [22] showed the effects of heat and mass transfer on MHD boundary layer flow over a shrinking sheet with suction. Bhattacharyya[23] then added to his research the impacts of heat source / sink on the flow and heat transfer of magnetohydrodynamics (MHD) over a shrinking sheet with mass suction. Nazar et al. [24] stressed on nanofluid shrinking sheet stagnation point flow. While Fazlul et al. [25] extended the work from Rana and Bhargava [26] by incorporating the impact of radiation on a nanofluid's boundary layer flow over a non-linearly permeable stretch sheet. In their research, they explore the flow, concentration and heat transfer parameter of the parameter of thermal radiation, thermophoresis parameter, Brownian movement parameter and suction parameter. The nonlinear shrinking sheet with slip effects on stagnation point flow has been studied by Fauzi et al. [27]. In addition, with Newtonian heating and radiation impact, Mat Yasin et al. [28] addressed the issue of MHD stagnation point flow in ferrofluid. Their research clearly shows that the temperature flow will be increased by adding the radiation parameter. On the other hand, the skin friction and Nusselt number will increase as the magnetic parameter increase. Later, Waini et al. [29] proceed the study on heat transfer in unsteady hybrid nanofluid. The objective of this present study is to extend the work from stretching sheet in Chenna et al. [30] to shrinking sheet with the existence of thermal radiation, which has not been considered before.

2. Formulation of the problem
In this present paper, let us consider the two dimensional magnetohydrodynamic (MHD) stagnation flow over a permeable shrinking horizontal sheet in the presence of heat generation/absorption and radiation effects. Assume that the velocity of the shrinking sheet is in the form \( \lambda u_w(x) \), with \( \lambda < 0 \) for a shrinking surface where \( u_w(x) = ax \), \( a \) is a positive constant and \( x \) is the distance from the slit where the sheet is investigate. While, the free stream velocity is \( u_e(x) = bx \). The simplified two-dimensional equations, which models the flow is written as:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]  

\[
u \frac{\partial u}{\partial x} + u \frac{\partial u}{\partial y} = \frac{d u_e}{dx} + \nu \frac{\partial^2 u}{\partial y^2} + \frac{\sigma B^2}{\rho}(u_e - u)
\]  

\[
u \frac{\partial T}{\partial x} + \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho C_p} \frac{\partial q_v}{\partial y} + \frac{Q_0}{\rho C_p}(T - T_{\infty})
\]
where \( u \) and \( v \) are the velocity components along \( x \) and \( y \) axes. \( T \) denotes as the fluid temperature, \( T_w \) is the constant temperature at the surface of the sheet while \( T_\infty \) is the ambient fluid temperature. \( \nu \) is the kinematic viscosity of the fluid, \( \sigma \) is the electric conductivity of the fluid, \( B \) is magnetic induction, \( \alpha \) is thermal diffusivity of the fluid, \( \rho \) is the fluid density, \( C_\rho \) is specific heat capacity at constant pressure, \( Q_o \) is the temperature dependent heat generation/absorption coefficient and \( q_r \) is radiative heat flux. The boundary conditions that satisfied Eqs. (1) - (3) can be written as:

\[
y = 0: \quad v = 0, \quad u = \lambda u_w(x), \quad T = T_w \\
y \to \infty: \quad u \to u_e(x), \quad T \to T_\infty
\] (4)

By using Rosseland approximation Brewster [31] and El-Arabawy [32] following papers by Ishak [33] and Babu et al. [34], Eq. (3) can be written as

\[
u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \left( 1 + N_R \right) \frac{\partial^2 T}{\partial y^2} + \frac{Q_0}{\rho C_\rho T_\infty} \left( T - T_\infty \right)
\] (5)

where \( N_R = \frac{16 \sigma T^3_\infty}{3k \kappa^\nu} \) is the radiation parameter.

The governing Eqs (1), (2) and (3) subject to the boundary conditions (4) can be expressed in a simpler form by introducing the following similarity transformation:

\[
\eta = y \sqrt{\frac{\nu}{a}}, \quad \psi = \sqrt{a v x f} \left( \eta \right), \quad \theta(\eta) = \frac{T - T_w}{T_w - T_\infty}
\] (6)

Where \( \eta \) is the similarity variable and \( \psi(x, y) \) is the dimensionless stream function which is usually define as \( u = \frac{\partial \psi}{\partial y} \) and \( v = -\frac{\partial \psi}{\partial x} \). Therefore, we have

\[
u = a x f' \left( \eta \right), \quad v = -\sqrt{a v f} \left( \eta \right)
\] (7)

Substitute Eqs. (6) and (7) into Eqs. (1), (2) and (5). Therefore, Eq. (1) is approved. While Equations. (2) and (5) turns into

\[
ff'' + f'f''' - \left( f' \right)^2 + M \left( 1 - f' \right) + 1 = 0
\] (8)

\[
\frac{1}{Pr} \left( 1 + N_R \right) \theta'' + f \theta' + Q \theta = 0
\] (9)

subject to boundary conditions

\[
f(0) = 0, \quad f'(0) = \lambda, \quad \theta(0) = 1.
\]

\[
f'(\eta) \to 1, \quad \theta(\eta) \to 0 \quad \text{as} \quad \eta \to \infty.
\] (10)

The dimensionless constants such as \( Pr, M, \lambda, N_R \) and \( Q \) denote the Prandtl number, magnetic parameter, velocity ratio parameter where \( \lambda < 0 \) for shrinking while \( \lambda > 0 \) for stretching,
radiation parameter and heat source \((Q > 0)\) or sink \((Q < 0)\) parameter, respectively, which are written as:

\[
\Pr = \frac{\nu}{\alpha}, \quad M = \frac{\sigma B^2}{\alpha \rho}, \quad Q = \frac{Q_0}{\alpha \rho C_p}.
\] (11)

The interest of physical quantities in this problem are the skin friction coefficient, \(C_f\) and the local Nusselt number, \(N\theta_u\) which are defined as

\[
C_f = \frac{\tau_w}{\rho u^*_w(x)} \quad \text{and} \quad N\theta_u = \frac{xq_w}{k(T_w - T_\infty)}.
\] (12)

where \(\tau_w\) and \(q_w\) are the skin friction/shear stress and heat flux from the sheet which are given by

\[
\tau_w = \mu \left( \frac{\partial u}{\partial y} \right)_{y=0} \quad \text{and} \quad q_w = -\left( \frac{\partial T}{\partial y} \right)_{y=0} + (q_r)_{y=0} \quad \text{(13)}
\]

Using (6), (12) and (13), we get

\[
\text{Re}_x^{1/2} C_f = f''(0) \quad \text{and} \quad \text{Re}_x^{1/2} N\theta_u = -\left(1 + N\theta_u\right)\theta'(0).
\] (14)

3. Results and Discussions

Using the similarity transformation, a system of ordinary equations is obtained from the governing partial differential Eqs. (1)-(3). Thus, numerical solutions to the system of ordinary equations (8) and (9) with transformed boundary conditions (10) are obtained using bvp4c solver in Matlab. The present results are given to show the influence of non-dimensionless parameters which is the Prandtl number parameter, \(\Pr\), magnetic field parameter, \(M\), shrinking/stretching \(\lambda\) parameter and heat source/sink, \(Q\) parameter.
Fig. 1 Variation of the skin friction coefficient, $f''$ with various $\lambda$ for different $M$.

![Variation of skin friction coefficient](image1)

Fig. 2 Variation of temperature number coefficient, $\theta'$ with various $\lambda$ for different $M$.

Figs. 1-2 show the variation of skin friction coefficient and temperature for various value of lambda in shrinking sheet case $\lambda < 0$ and different values of magnetic parameter, M. These graphs show that there is a distinctive solution exist for Eqs. (8) and (9) with boundary conditions (10) when $\lambda \geq 0$ and dual solution (first and second solution) exists for $\lambda_c < \lambda < 0$ where $\lambda_c < 0$ is the critical values of $\lambda$. It is clearly shows that in Fig 1, as the magnetic field parameter increase ($M=0, 0.2, 0.4$) the skin friction coefficient will increase. The presence of magnetic field in fluid is observed to have a resistive force which known as Lorentz force. This force creates a retardation flow effect which will enlarges the velocity gradient. Thus, according to Pal and Mandal [35] the ability of retarding force in controlling the velocity can be applied in electromagnetic coating of wires and MHD power generation. Figure 2 depicts the increment of the temperature as the magnetic field increase.

![Variation of temperature number coefficient](image2)

**Figure 3** Effect of various thermal radiation, Nr on temperature.

Figure 3 displays the various values of thermal radiation, Nr ($Nr=0, 1, 2$) on temperature. As the Nr increases, the temperature gradient decreases at the surface. It is due to the effect of radiation is to lower
the rate of energy transport to the fluid, hence reducing the temperature of the fluid. As a result, the transfer rate at the surface decreases in the present of radiation.

Figure 4 shows the effect of Prandtl number, Pr on temperature. We considered three types of Prandtl number, Pr=0.03, 0.7 and 2 which represents gaseous, water and liquid, respectively. From the graph, we can see that the second solution is actually invertible with the first solution. As the value of Pr increases, the temperature decreases. Moreover, the thermal boundary layer thickness is seen to reduce as the Prandtl number increases. According to Chenna et al. [30], this is due to the thermal diffusivity decrement for the larger values of Prandtl value number.

Figure 5 Temperature profile for various Q where Q = 0.05, 0.1, 0.2.
Figure 5 illustrates the influence of heat generation on temperature. The temperature increase with the increment of the heat generation values. This is because the energy received from the heat source in the boundary layer which cause the temperature of the fluid to increase.

![Figure 5](image)

**Figure 6** Velocity profiles \( f(\eta) \) for various \( M \).

![Figure 6](image)

**Figure 7** Temperature profile, \( \theta(\eta) \) for various \( M \).

![Figure 7](image)
The validity of these numerical solutions and dual nature solutions is supported by the velocity and temperature profile presented in Figures 6-9. As we can see, these profiles produce two different profile for a particular value parameter which is the solid line and dash line represent the first solution and second solution, respectively. From these graphs, it clearly shows that the boundary layer thickness for the first solution is thinner compared to the second solution. All profiles satisfy the above boundary conditions (10).
4. Conclusion
This paper considers two dimensional MHD stagnation point flow of a shrinking sheet with thermal radiation effect. The effects of thermal radiation, Prandtl number and heat generation as well as magnetic field parameter on skin friction coefficient and temperature were investigated and discussed. The results obtained are as follow:

- Dual solution exists for $\lambda < 0$ for shrinking sheet case for a particular value of parameter and unique solution for $\lambda \geq 0$ for stretching sheet case.
- The skin friction and temperature increase as the magnetic field increase.
- The temperature decrease as the radiation number increase.
- The value of Pr increases, the temperature decreases.
- The thermal boundary layer thickness is reducing as the Prandtl number increases.
- The temperature increase with the increment of the heat generation values.
- The boundary layer thickness for the first solution is thinner compared to the second solution.

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