Can late dark energy restore the Cosmic concordance?

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The tension between inferences of Hubble constant ($H_0$) is found in a large array of datasets combinations. Modification to the late expansion history is the most direct solution to this discrepancy. In this work we examine the viability of restoring the cosmological concordance within the scenarios of late dark energy. We explore two representative parameterizations: a novel version of transitional dark energy (TDE) and modified emergent dark energy (MEDE). We find that, the main anchors for the cosmic distance scale: cosmic microwave background (CMB), baryon acoustic oscillation (BAO), and SNe Ia calibrated by Cepheids form a “impossible trinity”, i.e., it’s plausible to reconcile with any of the two but unlikely to accommodate them all. Particularly, the tension between BAO and the calibrated SNe Ia can not be reconciled within the scenarios of late dark energy. Nevertheless, we still find a positive evidence for TDE model in analysis of all datasets combinations, while with the the exclusion of BOSS datasets, the tensions with SH0ES drops from 3.1σ to 1.1σ. For MEDE model, the tension with $H_0$ is much alleviated with the exclusion of SNe dataset. But unfortunately, in both TDE and MEDE scenarios, the $S_8$ tension is not relieved nor exacerbated.

I. INTRODUCTION

The precise determination of the Hubble constant $H_0$, which requires both accurate astrophysical and cosmological modeling, is crucial for modern cosmology. Historically, $H_0$ is measured directly via the classical distance ladder [1] in the local universe, while in the early universe, $H_0$ is extracted indirectly from angular size of the sound horizon $r_s$ and the shapes of the acoustic peaks in the cosmic microwave background (CMB) [2], [3]. These measurements disagree at 4 ~ 6σ significance e.g. $H_0 = 74.04 ± 1.42$ from SH0ES collaboration [4] and $H_0 = 67.4 ± 0.5$ from Planck 2018 data [5]. Several independent $H_0$ probes of the late-universe have reached competitive accuracy with the distance ladder, such as measurements of the strongly lensed quasar systems [6], obtaining $H_0 = 73.3_{-1.8}^{+1.7}$. In view of the fact that the $H_0$ tension between late and early universe is found in a large array of differing datasets combinations (see [7] for a recent review), it’s unnatural to attribute the disagreement solely to systematic errors. Hence, this tension may be a sign of physics beyond the standard ΛCDM cosmology.

There exist varied theoretical proposals to explain or ameliorate the $H_0$ discrepancy, ranging from new physics in early-time ($z > 1100$, pre-recombination) to late-time ($z \lesssim 2$) universe (see [8] for a review). Within the framework of General Relativity (GR), it is natural to consider the replacement of cosmological constant $\Lambda$ with dynamical dark energy [9–15] as a scalar field or a dark energy equation of state (EoS) parameterization (see [16] for a review). Other tentative solutions include interacting dark energy [17–19], modified gravity [20–24] as well as other possibilities [25–29]. These proposals must also stand up to the scrutiny of current Large-Scale Structure (LSS) surveys [30–33], which have delivered precise cosmological constraints. Interestingly, the inference of $H_0$ value in both CMB and LSS (BAO) surveys depends on the precise determination of sound horizon $r_s$, thus, a reduction of the sound horizon by increasing the expansion rate just prior to recombination (with additional energy density components e.g. exotic early dark energy (EDE) [34], acoustic dark energy (ADE) [35]) seems to be the “least unlikely to be successful” [36] approach to solve the $H_0$ tension. However, the addition of EDE or other component would suppress the growth of perturbations prior to recombination, which change the amplitudes and phases of the CMB acoustic peaks in complex ways and would bring in new tension with the density fluctuation amplitude, $\sigma_8$. Moreover, given a certain shift in $r_s$, the shift in $H_0$ needed in the CMB to match observations will be different with the one needed to match the LSS observations, introducing a currently non-existing tension [37].

In the late dark energy scenarios, the sound horizon at last scattering $r_s(z_*)$ is preserved from the modification of late expansion history. With the acoustic scale ($D_A(z_*)/r_s(z_*)$) fixed by CMB, an upward shift on $H_0$ is compensated by the increase of $D_A(z_*)$, which could be implemented by raising the “dilution rate” of dark energy density. This scenario can be realized by a wide class of “phantom-like” dark energy [14, 15, 38], including the recently proposed phenomenological emergent dark energy model (PEDE) [39]. In PEDE model, the dark energy is negligible at early times but dominates at late-time, providing an alternative solution to the coincidence problem [40]. If only the local $H_0$ measurements and CMB data are taken into consideration, this scenario seems ideal to solve the $H_0$ tension because the imprints of late-universe

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modification on the CMB spectra could be counteracted by a shift in $H_0$. However, the numerical analysis in [41, 42] suggests that, the PEDE model is not well compatible with SN Ia and LSS surveys, especially the $f\sigma_8(z)$ data from RSD observations [41]. Our research suggest that, this incompatibility is mainly caused by the sharp transition of EoS at present. In this work, our aim is to minimize the mismatches with SN Ia and LSS surveys while maintaining the ability of relieving the Hubble tension within the late dark energy scenarios. As a result of these concerns, we proposed a novel version of TDE model (based on the research in [43, 44]), which is captured by the transitional scale $a_c$. In this scenario, the dark energy behaves like the cosmological constant $\Lambda$ at late-time and goes through a rapid transition in the EoS at $a_c$. For comparison, we also included the discussion of an extended version of PEDE model (denoted as MEDE hereinbelow), which recovers the original PEDE model for $\alpha = 1$.

The outline of this paper is as follows: in section II we take a brief review of the key observations associated with the $H_0$ tension. In section III, we briefly introduce the MEDE and TDE model as well as their cosmological features. In section IV we describe our numerical implementation of the EDE model and the datasets used in our analysis. The numerical results of MCMC and analysis are presented in section VI. The discussion and conclusions are presented in section VI.

II. OBSERVATIONS

Measurements of the CMB spectra precisely determine the angular acoustic scale $\theta_s$ (transverse direction) and the “shift parameter” $R$ (line-of-sight direction), defined as [45]

$$\theta_s = r_s(z_s)/D_A(z_s), \quad R(z_s) \propto \omega_m D_A(z_s),$$

(1)

where $z_s$ is the redshift at the photon decoupling epoch, $D_A(z) = \int_0^z \frac{dz'}{H(z')} r_s$ is the angular diameter distance and $r_s$ is the comoving sound horizon, defined by

$$r_s(z_s) = \int_{z_s}^\infty \frac{dz}{H(z)c_s(z)}.$$  

(2)

In matter-radiation dominated era

$$H(z) \propto \left[\omega_r(1+z)^4 + \omega_m(1+z)^3\right]^{\frac{1}{2}},$$  

(3)

Noting that, if we fix physical density of matter $\omega_m \equiv \Omega_{m,0} h^2$ and radiation $\omega_r \equiv \Omega_{r,0} h^2$ (hereinbelow), then, a shift of the local expansion rate $H_0$ as well as modifications to the expansion history after last scattering ($z < z_s$) preserves the sound horizon $r_s(z_s)$.

In LSS surveys, the angular acoustic scale $\theta_s$ could also be extracted from galaxy power spectrum at low-redshifts ($z \approx z_{\text{LSS}} \sim 0.3$). The galaxy BAO measurement constrains the angular scale parallel and perpendicular to the line of sight [46, 47]

$$\theta_{\parallel} \simeq \frac{r_s(z_s)}{D_A(z_{\text{LSS}})}, \quad \theta_{\perp} \propto \frac{r_s(z_s)}{D_H(z_{\text{LSS}})},$$

(4)

or the combination of the angles

$$\theta_{\text{LSS}} \simeq \frac{r_s(z_s)}{(D_A(z_{\text{LSS}})^2 \cdot cz_{\text{LSS}}/H(z_{\text{LSS}}))^\frac{1}{2}},$$

(5)

with the approximation $r_s(z_s) \sim r_s(z_d)$, where $z_d$ is redshift at drag epoch, and $D_H(z) \equiv 1/H(z)$.

To relief the $H_0$ tension, it is natural to think of an reduction of the sound horizon $r_s$ so that the inferred value of $H_0$ increases. Usually, this is achieved by the addition of light degrees of freedom prior to recombination [34, 37]. However, even if we allow for the reduction of $r_s$, it’s still not likely to fully resolve the tension among SH0ES, LSS and CMB. As can be seen straightforward from Eq(4) that $\theta_{\parallel} \propto r_s H_0$ and $\theta_{\perp} \propto r_s H_0^{0.7}$, while Taylor expand Eq(2) around the Planck best fit indicates $\theta_s(z_s) \propto r_s H_0^{0.5}$. Consequently, given a certain shift in $r_s$, the shift in $H_0$ needed in the CMB to match observations will be different than the one needed to match the LSS observations.

The modification of the late universe provides an alternative solution to this dilemma. In view of the fact that, the LSS survey anchors the distance scale at $z \sim 0.3$, one may consider the modification of the expansion rate within small redshifts i.e., $z \lesssim 0.3$. This could be realized by a sharp transition of the EoS at present e.g. the MEDE scenario (see Fig.1). However, such a modification inevitably leads to the tension with SNe surveys, owing to the fact that the SNe Ia distances calibrated by the local $H_0$ measurement do not agree with the distances inferred from LSS surveys [43] (see Fig.3). Apart from anchoring the cosmic distance scale, the “large-scale structure data” also constrain the growth history. More specifically, the LSS data constrain the matter power spectrum $P(k)$ across a decade in $k$-space, while the overall amplitude is primarily captured by $S_8 \equiv \sigma_8(\Omega_{m,0}/0.3)^{0.5}$ [33, 48, 49]. It is suggested that the DES-Y1 [48] measurements are the most statistically powerful LSS data with publicly available likelihoods and could be well approximated by a simple Gaussian prior on $S_8$ [37]. Also, the growth rate $f \equiv d\ln(D)/d\ln(a)$ can be obtained from redshift space distortion (RSD) measurements via the peculiar velocities of galaxies [31, 50, 51]. To marginalize the galaxy bias [52], most of growth rate measurements are reported as the combination $f(z) \sigma_8(z) = f \sigma_R(z)$. We show in Fig.4 the compilation of 63 observational $f \sigma_R(z)$ RSD data points collected by [50].

In this work, we focus on the scenarios of late Dark energy which have no effective presence in the past. Unlike EDE changes the power spectrum in a complex way [53], the presence of late Dark Energy changes only the traveling distance of the free streaming photons in the late universe, resulting in a linear shift in power spectrum for $\ell \gtrsim 10$. Note that, such a shift in power spec-
trum is highly degenerate with an adjustment of the local $H_0$ value. As is indicated in Eq.(1), the predictions on CMB observables ($\theta_s$, “shift parameter” R, etc) would be the same for late dark energy scenarios and ΛCDM, provided that the inferences on $D_A(z_*)$ is equalized. So it is convenient to give a fast check on the ability of late dark energy scenario to solve $H_0$ tension by “shooting” the Planck best-fit $D_A(z_*)$ (ΛCDM) with varying $H_0$. We show in Fig.2 the “shooting” results of their selected late dark energy model, i.e., WCDM, phenomenological emergent dark energy (MEDE) and transitional dark energy(TDE), and the details of these models will be discussed in the next section. With the best fit $H_0$ value, we can calculate other key observables ($S_8, f\sigma_8(z)$) to see whether the solution of $H_0$ tension is at the cost of bringing in new tensions with other observations.

III. MODELS

In this work, we introduce a novel version of transitional dark energy (TDE), and consider two extra forms of parameterizations within the scenarios of dark energy, summarized as:

- WCDM: a natural extension to ΛCDM model, with dark energy EoS state being a free constant ($w_0$).
- MEDE: the dark energy emergent at late-time...
ble parameter within this framework can be expressed for MEDE and EDE model (when BAO data point are corrected with the factor distance ladder i.e., $D_L/\theta_{BAO}$, so the absolute scale of the SNe data points in this figure is calibrated by the local distance ladder i.e., $H_0 = 74.03$ , while the same value of $H_0$ is set for both TDE and MEDE model. The BAO measurements constrain $D_L \times (r_d_{\text{fid}}/r_d)$ and $D_H \times (r_d_{\text{fid}}/r_d)$, so the BAO data point are corrected with the factor $r_d/r_d$, where $r_d$ is corrected by AP factor \(^5\). The 63 observational $f(z)\sigma_8(z)$ RSD data points collected by \(^5\), which have corrected by AP factor \(^5\) calculated based on TDE model.

where $\Omega_{\text{DE}}$ is ratio of DE density to the critical density ($\rho_{\text{crit}}/\rho_{\text{crit},0}$). The exponential part of Eq.7 indicates that, a smaller value of $w$ could translate into a faster dilution rate of dark energy, which suppresses dilation of the late universe. This effect is well compensated by an upward shift in $H_0$, thus, the $H_0$ tension is relieved.

\section{MEDE}

We consider a modified version of PEDE (MEDE) captured by a single parameter $\alpha$, which recovers $\Lambda$CDM model for $\alpha = 0$ and the original PEDE model for $\alpha = 1$ \(^4\). The expression of MEDE energy density could be written as follows:

$$\Omega_{\text{DE}}(a) = \Omega_{\text{DE},0} \left(1 - \tanh \left(\alpha \log_{10} \left(\frac{1}{a}\right)\right)\right)$$ \quad (8)

The EoS of this scenario could be derived as:

$$w_{\text{DE}}(a) = -1 - \frac{\alpha}{3 \ln(10)} \left(1 + \tanh \left(\alpha \log_{10} \left(\frac{1}{a}\right)\right)\right)$$ \quad (9)
In this model, the EoS of dark energy evolves asymptotically to \( w(z) = -1 \) in far future from \( w(z) < -1 \). As can be seen from Fig.1), the dark energy density undergo a fast drop in the near past \( (z \lesssim 1) \), which relieves the suppression of the growth of the structure. A faster rate of the structure growth increases the the growth function, \( f(z) \), hence exacerbating the tension with \( f\sigma_8(z) \) data (see from Fig.4). Recent works [42] show that The MEDE model is disfavored in MCMC analysis with the inclusion of \( f\sigma_8(z) \) data. However, measurement of \( f\sigma_8(z) \) in RSD surveys requires the presumption of fiducial model, hence, it can hardly be viewed as an conclusive evidence against MEDE model. Meanwhile, a rapid change of EoS in the near past leads to a fast transformation of the distance scale (compared with \( \Lambda \)CDM), while such a transformation is disfavored by the current SNe data points, which constrain the ratio of distances, \( D_L/D_H \) (see from Fig.3).

B. TDE

We formulate a novel version of transitional dark energy (TDE) model (see [43, 44, 54] for discussion of similar models). The dark energy EoS remains constant at late-time, then endures a rapidly transition at the critical scale \( a_c \), written as:

\[
w(z) = w_0 - \frac{1}{2} \left[ \tanh \left( 3 \left( \frac{1}{a} - \frac{1}{a_c} \right) \right) + 1 \right].
\]

which recovers \( \Lambda \)CDM model for \( w_0 = -1 \) and \( a_c \sim 0 \). In this work, we only consider the case with \( w_0 = -1 \), then the critical scale \( a_c \) is the only considered free parameter in this model. In such case we have the dark energy EoS evolving from \( w = -2 \) at high redshift to \( w = -1 \) at low redshift, which recovers the case in Covariant Galileon cosmology under the right initial conditions [55]. As is shown in Fig.2, the inferred values (from Planck distance prior) of several observable approaches to that of \( \Lambda \)CDM model with the transitional scale \( a_c \) approaching to zero. With the increase of \( a_c \), the inferred value of \( H_0 \) increases, while the shifts on \( S_8 \) is the reverse. Due to the resemblance to \( \Lambda \)CDM model at late universe, TDE could largely avoid the mismatch with SNe and \( f\sigma_8(z) \) datasets brought by the sudden change of the EoS of dark energy. However, the shift of the local expansion rate \( H_0 \) creates the tension between the acoustic scale measured in local LSS surveys and CMB measurement. Such a dilemma could not be fully resolved even with the reduction of sound horizon at drag epoch, \( r_s(z_d) \) (see section II).

IV. STATISTICAL METHODOLOGY AND DATASETS

We implement the MEDE and TDE scenarios as modifications to the publicly available Einstein-Boltzmann code CLASS [56, 57] package. The non-linear matter power spectrum required by redshift-space distortion (RSD) likelihoods are computed using the “Halofit” prescription implemented in CLASS. The Markov chain Monte Carlo (MCMC) analyses are preformed using the publicly available code Cobaya [58] package with a Gelman-Rubin [59] convergence criterion \( R - 1 < 0.05 \). The plots have been obtained using the GetDist [60] package.

The following datasets are considered in our MCMC analyses:

A. CMB

CMB distance prior from final Planck 2018 release. Noted that, in late dark energy scenario, the shape of the power spectrum (with the same matter and radiation density) is identical to that of \( \Lambda \)CDM cosmology at \( \ell \gtrsim 10 \) (for small \( \ell s \) the covariance is largely dominated by cosmic variance). Consequently, the employment of distance prior is equivalent to the full shape CMB power spectrum for late dark energy scenarios, while this is not true for EDE scenarios.

B. Hubble constant

The most recent SH0ES measurement \( H_0 = 74.03 \pm 1.42 \) [4], which shows a tension at 4.4\( \sigma \) with the Planck [5] value of \( H_0 \) assuming a minimal \( \Lambda \)CDM model.

C. Supernovae Type Ia

The 1048 Supernova Type Ia data points distributed in the redshift interval \( z \in [0.01, 2.3] \), known as the Pantheon sample [61], which provide accurate relative luminosity distances. We utilize the likelihood as implemented in Cobaya.

| Parameter          | Prior     |
|-------------------|-----------|
| \( H_0 \)        | [50, 90]  |
| \( \Omega M_h^2 \) | [0.01, 0.1] |
| \( \Omega b^2 \)  | [0.05, 0.3] |
| \( \alpha \) (MEDE) | [-4, 4]  |
| \( a_c \) (TDE)  | [0.01, 1.0] |

Table I. Flat priors assumed on the cosmological parameters associated with the TDE model and MEDE model.
Table II. The mean and 1-σ constraints on $H_0$ and $S_8$ in the TDE model (top panel); MEDE model (middle panel); and ΛCDM model (bottom panel), as well as the the constraints on late dark energy parameters, i.e., $a_c$ for TDE model and $\alpha$ for MEDE model. We also shown in the third to the last column the effective number of $\sigma$’s that the Hubble measurement is away from SH0ES. The $\chi^2$ statistics are shown in the last two columns, where $\Delta \chi^2$ is the inferred $\chi^2_{\text{min}}$ compared with ΛCDM model.
The mean ±1σ constraints on the cosmological parameters in ΛCDM and in the EDE scenario with $n = 3$, as inferred from the combination of Planck 2018 distance prior and the latest SH0ES $H_0$ constraint.

### D. LSS

The LSS data set which probes the low-redshift universe are considered, which include:

- **BAO**: measurements of the BAO signal with their full covariance matrix from BOSS DR12 [31]. Radial and transverse BAO measurements from Lyα Forests SDSS DR12 [62]. Angle averaged BAO measurement from 6DF Galaxy [63], from quasar sample from BOSS DR14 [64].

- **RSD**: SDSS BOSS DR12 [31] measurements of $f\sigma_8(z)$, at $z = 0.38, 0.51$ and 0.61. We include the full covariance of the joint BOSS DR12 BAO and RSD data (denote as FS).

- **DES**: shear-shear, galaxy-galaxy, and galaxy-shear two-point correlation functions (“3x2pt”), measured from 26 million source galaxies in four redshift bins and 650,000 luminous red lens galaxies in five redshift bins, for the shear and galaxy correlation functions, respectively. We utilize a Gaussian prior on $S_8$ derived by the DES-Y1 likelihood as is suggested in [37].

Noted that, TDE and MEDE are extended form ΛCDM model and all these models share with the six standard ΛCDM parameters. We fix three of the six parameter ($n_s, A_s, \tau_{reio}$) associated with the overall shape of the CMB spectrum to the best fit value given by Planck 2018 [5]. Next, we consider as baseline a 4-dimensional parameter space described by the following parameters: the Hubble constant $H_0$, the baryon energy density $\Omega_b h^2$, cold dark matter energy density $\Omega_c h^2$, the parameter $\alpha$ for MEDE and parameter $a_c$ for TDE. We assume flat uniform priors on all these parameters, as shown in Table 1.

### V. RESULTS AND DISCUSSION

For the cosmological analysis of the TDE model, we fix the EoS at present to be identical with that of ΛCDM model, i.e., $w_0 = -1$, then $a_c$ is the only considered free parameters in this scenario. As a comparison, we also fit the ΛCDM model and MEDE model to the above datasets. For brevity, we denote the combination of likelihoods without LSS data as PHS (Planck+SH0ES+SNe). In Table II we report the constraints at 68% CL on the $H_0$ of the key derived quantities for several dataset combinations considered in this work. Detailed constrains on cosmological parameters and $\chi^2$ statistics could be found in Tables III~V. The triangular plot with the 1D posterior distributions and the 2D contour plots for these parameters are shown in Fig.5~7.

#### A. Constraints from CMB and SH0ES

As is discussed in section II, there is a strong degeneracy between $H_0$ and late dark energy parameter $a_c$ ($\alpha$) which quantifies the deviation of these scenarios from ΛCDM model (see Fig.2). Consequently, the Planck data alone is insufficient to give a tight constrain to these parameters so that a prior on $H_0$ should be included. To test the above argument, we first consider the fit to Planck 2018 distance prior alone. The results tabulated in Table II indicates a lower bound of $a_c > 0.562$ at 68% CL, and $\alpha$ is unconstrained within the range $-4 < \alpha < 4$, while $H_0$ is loosely constrained in both scenarios. Next, we consider the fit to a combination of Planck and SH0ES datasets. In Table III, we find the Hubble constant to be $H_0 = 73.4 \pm 1.4$ and $H_0 = 72.8 \pm 1.7$ for TDE model and MEDE model, respectively. The $H_0$ tension in both TDE and MEDE scenarios are largely removed when a prior on $H_0$ (SH0ES) is added. These results suggest that, the resolve of the tension between local distance ladder and CMB could be achieved by a wide class of “phantom-like” dark energy. The contours in Fig.5 shows a clear degeneracy between $a_c(\alpha)$ and $H_0$ in TDE (MEDE) model. The $\chi^2$ statistic in III shows that, the goodness of the fit is significantly improved. Noted that, this improvement is mainly due to the reduction of the disaccordance with SH0ES data.

#### B. Constraints from accumulative DataSets

We can see from Table II, different combinations of datasets yield different values of best fit parameters. A clear trend should be noticed that, with the inclusion of more datasets, the best fit cosmological parameters as well as the $\chi^2_{min}$, value approach to that of ΛCDM model for both TDE and MEDE scenarios. For example, with SH0ES and Planck combination we find $H_0 = 73.4 \pm 1.4$ and $S_8 = 0.828 \pm 0.013$, in $1\sigma$ to $3\sigma$ tension with the fit of the same datasets in ΛCDM scenario. This tension drops...
Figure 6. In right panels show the cosmological parameters constraints from the combination of Planck 2018 distance prior; BAO data from 6dF, SDSS DR14, and BOSS DR12; Pantheon SNIa data; the latest SH0ES $H_0$ constraint; BOSS DR12 RSD data; and the $S_8$ prior derived from DES-Y1 3x2pt data. The red (blue) contours show $1\sigma$ and $2\sigma$ posteriors for the TDE($\Lambda$CDM) model (top right panel) and for MEDE($\Lambda$CDM) model (bottom right panel). In the top left panel, we show the show $1\sigma$ and $2\sigma$ posteriors for the TDE($\Lambda$CDM) model by constraints from the above combination of without the full covariance of the joint BOSS DR12 BAO and RSD data (denote as without BOSS); In the bottom left panel, we show the show $1\sigma$ and $2\sigma$ posteriors for the MEDE($\Lambda$CDM) model by constraints from the above combination of without the Pantheon SNIa data (denote as without SNe).

to less than $1\sigma$ with the inclusion of all considered likelihoods, while the differences in $\chi^2_{\text{min}}$ value reduce from 21.3 to 3.7. With the inclusion of SNe dataset, the tension with SH0ES increased $1.6\sigma$ for TDE model. Noted that, the SNe dataset which constrains the expansion history of the late universe, i.e., $D_L(z)/H_0$, is well compatible with the $\Lambda$CDM model, thus, modification of the dark energy EoS at $\alpha \gtrsim 1/2$ is disfavored. Consequently, the $\pm 1\sigma$ constrains on transitional scale $a_c$ decrease from $0.623^{+0.044}_{-0.039}$ to $0.536 \pm 0.028$, which results in a decline of the best fit value of $H_0$. While for MEDE model, the EoS of dark energy deviates from $\Lambda$CDM at $z \sim 0$, hence it is strongly disfavored by the SNe dataset, i.e $\alpha$ declines from $1.20 \pm 0.37$ to $0.64^{+0.23}_{-0.21}$ and in agreement with zero within $2\sigma$. Interestingly, the tension with SH0ES reduces with the addition of a prior on $S_8 = 0.773^{+0.026}_{-0.020}$ (
Figure 7. Constraints on TDE scenario (left panel) and MEDE scenario (right panel) from Planck 2018 distance prior; BAO data from 6dF, SDSS DR14, and BOSS DR12; Pantheon SNIa data; the latest SH0ES $H_0$ constraint; BOSS DR12 RSD data; and the $S_8$ prior derived from DES-Y1 3x2pt data. The contours show $1\sigma$ and $2\sigma$ posterior envelopes for various data set combinations. The green contours show results for the combination Planck distance prior, SNIa and SH0ES; the dark contours additionally include a prior on $S_8$; and the red contours further include BAO data. The blue contours add the RSD data (constrains on $f\sigma_8$). For TDE model, the inclusion of a prior on $S_8$ slightly alleviate the tension with SH0ES and shift $\alpha_c$ toward 1. The inclusion of BOSS DR12 BAO and RSD data significantly weakens the evidence for TDE, i.e., $\alpha_c$ shift significantly towards zero, and the tension with SH0ES is no longer reconciled. The evidence for MEDE model is also weakened with the inclusion of BOSS DR12 BAO and RSD data.

| Dataset | Without BOSS | Without SNe |
|---------|-------------|-------------|
| Model   | TDE | MEDE | $\Lambda$CDM | TDE | MEDE | $\Lambda$CDM |
| $H_0$   | $72.20^{+0.82}_{-0.71}$ | $70.5 \pm 1.0$ | $68.26 \pm 0.37$ | $69.31 \pm 0.70$ | $70.46 \pm 0.91$ | $68.20 \pm 0.39$ |
| $\Omega_m h^2$ | $0.0224 \pm 0.00015$ | $0.0225 \pm 0.00014$ | $0.0226 \pm 0.00013$ | $0.0226 \pm 0.00015$ | $0.0225 \pm 0.00012$ | $0.0226 \pm 0.00014$ |
| $\Omega_c h^2$ | $0.119 \pm 0.0011$ | $0.119 \pm 0.0011$ | $0.117 \pm 0.00074$ | $0.1181^{+0.0012}_{-0.0013}$ | $0.1186 \pm 0.0011$ | $0.1171 \pm 0.0008$ |
| $\alpha_c$ | $0.560^{+0.034}_{-0.040}$ | $-$ | $-$ | $0.385^{+0.11}_{-0.038}$ | $-$ | $-$ |
| $\alpha$ | $-$ | $0.63^{+0.18}_{-0.22}$ | $-$ | $-$ | $0.64 \pm 0.23$ | $-$ |
| $S_8$   | $0.819^{+0.012}_{-0.010}$ | $0.812 \pm 0.013$ | $0.7996 \pm 0.0087$ | $0.813 \pm 0.012$ | $0.813 \pm 0.011$ | $0.801 \pm 0.0093$ |
| $\chi^2_{\text{SH0ES}}$ | $2.0 \pm 1.7$ | $6.8 \pm 3.4$ | $16.6 \pm 2.1$ | $11.8 \pm 3.5$ | $6.7 \pm 3.4$ | $16.9 \pm 2.3$ |
| $\chi^2_{\text{Planck}}$ | $3.5 \pm 2.8$ | $4.7 \pm 3.5$ | $9.6 \pm 3.7$ | $6.0 \pm 3.8$ | $4.4 \pm 2.7$ | $9.6 \pm 3.7$ |
| $\chi^2_{\text{tot}}$ | $1050.2 \pm 3.2$ | $1055.4 \pm 4.5$ | $1062.7 \pm 2.3$ | $32.5 \pm 3.1$ | $32.0 \pm 2.9$ | $36.3 \pm 2.4$ |

Table IV. The the mean $\pm 1\sigma$ constraints on the cosmological parameters in TDE, MEDE and $\Lambda$CDM models, as inferred from the combination of Planck 2018 distance prior; BAO data from 6dF, SDSS DR14; Pantheon SNIa data; the latest SH0ES $H_0$ constraint; and the $S_8$ prior derived from DES-Y1 3x2pt data (left panel), and Planck 2018 distance prior; BAO data from 6dF, SDSS DR14; SH0ES; and the $S_8$ prior (right panel). $\Delta\text{AIC}$ and $\Delta\chi^2$ are the value of AIC criteria and $\chi^2_{\text{min}}$ compared with $\Lambda$CDM, respectively.
Table V. The the mean ±1σ constraints on the cosmological parameters in TDE, MEDE and ΛCDM models, as inferred from the combination of Planck 2018 distance prior; BAO data from 6dF, SDSS DR14, and BOSS DR12; Pantheon SNIa data; BOSS DR12 RSD data; and the $S_8$ prior derived from DES-Y1 3x2pt data [left panel], and Planck 2018 distance prior; BAO data from 6dF, SDSS DR14; SH0ES; SNe dataset and the $S_8$ prior (right panel). $\Delta AIC$ and $\Delta \chi^2$ are the value of AIC criteria and $\chi^2_{\text{min}}$, compared with ΛCDM, respectively.

| Dataset          | Without SH0ES | All combined |
|------------------|---------------|--------------|
| Model            | TDE           | MEDE         | ΛCDM         | TDE           | MEDE         | ΛCDM         |
| **H_0**          | 67.82^{+0.45}_{-0.57} | 68.7^{+1.0}_{-1.2} | 67.67 ± 0.43 | 69.13 ± 0.73 | 69.40 ± 0.53 | 68.06 ± 0.41 |
| **Ω_c h^2**      | 0.0225 ± 0.00014 | 0.0225 ± 0.00012 | 0.0226 ± 0.00013 | 0.0225 ± 0.00013 | 0.0226^{+0.00012}_{-0.00010} | 0.0226^{+0.00013}_{-0.00011} |
| **Ω_M h^2**      | 0.1180 ± 0.0009 | 0.1188 ± 0.0010 | 0.1181 ± 0.0009 | 0.1181 ± 0.0010 | 0.1181 ± 0.0009 | 0.1174 ± 0.00086 |
| **α_c**          | < 0.252       | < 0.252      | < 0.252      | < 0.252       | < 0.252       | < 0.252      |
| **S_8**          | 0.818 ± 0.011 | 0.818^{+0.011}_{-0.010} | 0.814 ± 0.010 | 0.813 ± 0.012 | 0.809 ± 0.0094 | 0.805 ± 0.010 |
| $\chi^2_{\text{Planck}}$ | 4.9 ± 3.0    | 4.2 ± 3.2    | 5.5 ± 3.1    | 5.2 ± 3.4    | 5.5 ± 2.9    | 8.1 ± 4.0    |
| $\chi^2_{\text{SN}}$ | 1034.99 ± 0.24 | 1040.6 ± 5.1 | 1034.99 ± 0.23 | 1035.0 ± 0.3 | 1036.3 ± 1.2 | 1034.84 ± 0.12 |
| $\chi^2_{\text{tot}}$ | 1051.8 ± 2.7 | 1052.5 ± 5.4 | 1051.9 ± 2.2 | 1067.2 ± 2.8 | 1068.9 ± 2.5 | 1070.8 ± 2.8 |

$\Delta \chi^2$ = −0.1, 0.6, −3.6, −1.9, $\Delta AIC$ = 1.9, 2.6, −1.6, 0.1

Table VI. The the mean ±1σ constraints on the cosmological parameters in TDE model parameterized by $a_c$ (top panel) and in TDE model parameterized by $z_c$ (bottom panel), as inferred from the combination of Planck 2018 distance prior; SH0ES $H_0$ constraints; BAO data from 6dF, SDSS DR14, and BOSS DR12; Pantheon SNIa data; BOSS DR12 RSD data; and the $S_8$ prior derived from DES-Y1 3x2pt data.
discordance with the other. Noted that the BAO measurements relies on the presumption of fiducial cosmology, which means the measured distance scale should be calibrated by the factor $r_{a,68}/r_a$. While in the scenarios of late dark energy, the inferred value of $r_a$ is close to that of fiducial ΛCDM model (see Section II), so in this case the discordance between BAO and SNe($H_0$) datasets could not be reconciled. To account for the possibility that the tension is originated by some unknown systematics of single dataset, we selected several subclasses of whole datasets as follows:

- Without BOSS: Comparing the result tabulated in Table IV and Table V, we find that for TDE model, the $\pm 1\sigma$ constrains on $H_0$ reduce from $71.66 \pm 0.83$ to $69.09^{+0.60}_{-0.61}$ while the tension with SH0ES increase from $1.4\sigma$ to $3.1\sigma$ (see Fig.6 for a view), which indicate that the BOSS data set is not consistent with SH0ES in this scenario. For MEDE model, the tension with SH0ES increased $1.1\sigma$ with the inclusion of BOSS, indicating a relatively milder tension between these two datasets. For ΛCDM model the constraint results on cosmological parameters are almost identical for both the case.

- Without SNe: See Table IV and Table V for a contrast between the constraint results with and without SNe dataset. For MEDE model, the $\pm 1\sigma$ constrains on $H_0$ reduce from $70.46 \pm 0.91$ to $69.40 \pm 0.53$. While for TDE and ΛCDM model, there is only a minor shift on the best fit cosmological parameters. Noted that the $\pm 1\sigma$ constrains on $z_c$ is $4.0^{+1.2}_{-3.4}$ with the absence of SNe dataset, i.e., the transition of EoS happens at the scale where dark energy plays a negligible role on the cosmic expansion, suggesting that this scenario is largely in agreement with the ΛCDM model. Such an agreement is also indicated by the best fit parameters e.g. $H_0 = 68.49$ (TDE) and $H_0 = 68.20$ (ΛCDM).

- Without SH0ES: From Table V one can see that, the difference on the $\pm 1\sigma$ constrains on $H_0$ for the three model is marginal as is the $\chi^2$ statistics, i.e, compared with ΛCDM model $\delta \chi^2 = -3.3(0.6)$ for TDE(MEDE) model, indicating that there is little evidence to support the extension from ΛCDM model. The constrains on the late dark energy parameters also support the above argument, e.g. $\alpha = 0.29 \pm 0.24$, in agreement with zero within $1\sigma$.

C. Constraints from Selected DataSets

In the context of TDE model, the tension with SH0ES increases significantly (from $1.4\sigma$ to $2.8\sigma$) with the inclusion of BAO data. While in MEDE scenario, the tension is mainly brought by the incorporation of SNe dataset (see Table II). In another word, the TDE model appeals to the SNe dataset, while the MEDE model appeals to BAO dataset. As can be seen from Fig.3, both BAO and SNe measurements (calibrated by the local distance ladder) anchor the distance scale at $z \lesssim 1.0$, yet in good agreement with each other. Such a tension seemed to be inherited in the data set itself. Therefore, the conformity with one of anchor almost certain leads to the

Figure 8. Constraints on TDE scenario with a uniform prior on $z_c$ (denote as TDE2) from Planck 2018 distance prior; BAO data from 6dF, SDSS DR14, and BOSS DR12; Pantheon SNIa data; the latest SH0ES $H_0$ constraint; BOSS DR12 RSD data; and the $S_8$ prior derived from DES-Y1 3x2pt data. The contours show $1\sigma$ and $2\sigma$ posteriors for various data set combinations. The green contours show results for the combination Planck distance prior, SNe and SH0ES; the dark contours additionally include a prior on $S_8$; and the red contours further include BAO data. The blue contours add the RSD data (denote as FS); The constraints results on cosmological parameters closely matches the results shown in the left panel of Fig.7.

$S_8 = 0.833 \pm 0.016$ (for the Planck best fit ΛCDM). This is in accord with the shooting result shown in Fig.2, i.e., applying such a prior on $S_8$ would result in an upward shift on the transitional scale factor $a_c$ (higher $\alpha$ values), which in turn increase the inferred value of $H_0$. For a brief summary, the TDE scenario is disfavored by the BAO dataset, while the MEDE is disfavored by SNe dataset calibrated by $H_0$. This discordance could not be fully removed within the context of late dark energy. Neither TDE model nor MEDE model is favored without the inclusion of SH0ES dataset.
D. Prior Dependence

We have assumed uniform prior probability distributions for effective TDE parameter $a_e$, which correspond to non-uniform priors on the transitional redshifts $z_c = 1/a_e - 1$. The EoS in parameterized by $z_c$ (denote as TDE2) could be rewritten as:

$$w(z) = w_0 - \frac{1}{2} \tanh(3(z - z_c)) + 1$$  \hspace{1cm} (11)$$

An obvious concern is the dependence of the posterior distributions on the choice of priors. Accounting for that, we recompute the TDE parameter constraints with a uniform prior imposed on $z_c$, i.e., $z_c \in [0, 100]$, which corresponds to $a_e \in [0.01, 1]$. The posterior distributions are shown in Fig.8 and the parameter constraints are tabulated in Table VI. It’s obvious to see that, the difference in priors have negligible impact on the $\chi^2$ statistics, e.g $\chi^2_{\text{min}} = 1067.2$ for the TDE fit to the all combined datasets and $\chi^2_{\text{min}} = 1067.1$ for that of TDE2 (quoted Table VI) fit of all combined datasets. It is notable that, there is a slight decline on the inferred value of $H_0$ in TDE2, which could be understood as the effect of a shift in prior probability distribution toward smaller value of $a_e$ when a uniform prior on $z_c$ is assumed. The difference between the fit of TDE and TDE2 becomes insignificant with the inclusion of growing number of datasets (see Table VI).

VI. DISCUSSION AND CONCLUSIONS

The $\Lambda$CDM model calibrated by Planck data is well compatible with a large bunch of independent observational datasets (BAO, SN Ia, etc), but in severe tension with the local distance ladder ($H_0$). Consequently, the ideal solution to the discordance lies in the scenarios which disagree with $\Lambda$CDM solely on the inferences of $H_0$, while recover the predictions of $\Lambda$CDM in most cases. However, the shift in $H_0$ inevitably leads to the modification in late expansion history. A practical way to capture this modification is to add an extra parameter to the six standard $\Lambda$CDM parameters named as “late dark energy parameter”, which degenerates only with $H_0$ while is uncorrelated with the other five parameters.

In this work, we explore two representative parameterizations within the scenario of late dark energy: the Modified Emergent Dark energy (MEDE) and a novel version of Transitional dark energy (TDE). For TDE model, the “late dark energy parameter” refers to the scale ($a_e$) at which EoS of dark energy endures a rapid transition, which recovers $\Lambda$CDM model when $a_e$ approach to zero; For MEDE model, the deviation from $\Lambda$CDM is quantified by the extra parameter $\alpha$, which recovers $\Lambda$CDM model for $\alpha = 0$ and PEDE model for $\alpha = 1$.

We analyze these scenarios accounting for Planck 2018 distance prior, BAO+FS (DF6, BOSS DR12 and DR14), $S_8$ prior derived from DES-Y1 3x2pt, as well as SH0ES $H_0$ measurement and SNeIa dataset from the Pantheon compilation. The results are shown in Table III~V and Fig.5~7. To give a clear view on the influence of different datasets, we conduct the MCMC analysis in an accumulative way. Due to the degeneracy between $H_0$ and the extra late dark energy parameter, $H_0$ is loosely constrained with CMB alone, i.e., $H_0 = 76.7^{+7.1}_{-9.9}$ and $H_0 = 75.7 \pm 8.1$ for TDE and MEDE model, respectively. When added with the $H_0$ prior by SH0ES ($H_0 = 74.03 \pm 1.42$), the constraint results for TDE ($H_0 = 73.9 \pm 1.5$) closely matches the SH0ES $H_0$ prior, which further confirms the degeneracy. With the growing number of included datasets, one can see clear trend of the late dark energy scenarios (TDE,MEDE) degenerate into the standard $\Lambda$CDM model while the tension with SH0ES grows continuously (see Table II). However, there is still a positive evidence for the EoS transition in TDE scenario with the combination of all considered datasets, i.e $a_e = 0.380^{+0.11}_{-0.039}$ at 68% CL, which is inconsistent with zero, also, the goodness of the fit is slightly improved with $\Delta \chi^2 = -3.6$ ($\Delta \text{AIC} = -1.6$) compared with $\Lambda$CDM.

Within TDE scenario, the tension on $H_0$ is not solved, but slightly alleviated, i.e., $\sigma_{\text{SH0ES}} \sim 3.1\sigma$. The results also show that, the $S_8$ tension is neither relieved nor exacerbate within the scenarios of late dark energy, due to the fact that the structure growth is more rapid in the case of “phantom-like” dark energy than that of cosmological constant. The upward shift on $S_8$ is largely compensated by a lower inferred value of $\Omega_m,0$, resulting in a minor influence on the $S_8$ constraints.

The three main anchors of the cosmic distance scale e.g. CMB ($z \sim 1100$), BAO ($z \sim 0.3$), and SNe Ia calibrated by the local $H_0$ measurements ($z \lesssim 1.5$) forms a “Impossible trinity”, which means it’s plausible to reconcile with any of the two but hard to accommodate all of them. Within the scenario of late dark energy, it’s nearly impossible to reconcile the tension between BAO($z \sim 0.3$) and SNe Ia calibrated by $H_0$ (see Fig.3). Even if we allow for an reduction of the sound horizon $r_s$ it is not likely to simultaneously match the distance anchored by CMB and BAO (see section II). However, the late universe measurements require the modeling of complex astrophysical systems which may bring about some systematic error either in the measurements or in the astrophysical modeling [46]. To account for the possibility of unknown systematics, we selected several subclasses of the datasets, the constraint results can be seen in Table IV, V and Fig.6. In the analysis without BOSS measurement, TDE model have shown its potential to solve the $H_0$ tension, i.e., $H_0 = 72.20^{+0.62}_{-0.70}$ in 1.1$\sigma$ tension with SH0ES. For MEDE model, the tension with SH0ES is reduced to 2.1$\sigma$ in analysis without SNeIa dataset. While the analyses without the SH0ES measurement have $a_e$ (TDE) and $\alpha$ (MEDE) peaked toward zero, suggesting no evidence for both TDE and PEDE model.

In summary, our results suggest that, owing to the irreconcilable tension between BAO and SNeIa calibrated...
by $H_0$ within the scenarios of late dark energy, the cosmological concordance can not be restored. However, with the presumption of unknown systematics, the late dark energy could still be viewed as viable candidate. What is the “true” origin of these discordances? We hope that more investigators will be motivated to explore this sector.

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