Influence of flow topology on Lagrangian statistics in two-dimensional turbulence

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Abstract. The influence of flow topology on Lagrangian statistics in fluid turbulence is investigated. The Weiss criterion provides a tool to split the flow into topologically different regions: elliptic (vortex dominated), hyperbolic (deformation dominated), and intermediate (turbulent background) regions. The flow corresponds to forced two-dimensional Navier-Stokes turbulence in a double periodic or a circular domain, the latter with no-slip boundary conditions. A Lagrangian approach is adopted by tracking large ensembles of passively advected tracers. The pdfs (probability density functions) of the residence time in the topologically different regions are computed introducing the Lagrangian Weiss field, i.e., the Weiss field computed along the particles’ trajectories. In elliptic and hyperbolic regions, the pdfs of the residence time have self-similar algebraic decaying tails. In contrast, in the intermediate regions the pdf shows exponentially decaying tails. Furthermore, the conditional pdfs of the Lagrangian acceleration with respect to the flow topology confirm that both strong elliptic and hyperbolic regions contribute to the large values of Lagrangian acceleration.

1. Introduction

The purpose of this study is to understand the influence of flow topology on Lagrangian statistics. To distinguish the different topologies, we consider the Okubo-Weiss criterion (Weiss (1991); Elhmaïdi et al. (1993)) which enables us to split the flow into two contributions: elliptic (vortex dominated) and hyperbolic regions (deformation dominated). Indeed the emergence of coherent structures in two-dimensional turbulence has a strong influence on the flow dynamics and contributes to the different flow topologies. In this context, we are also interested in the influence of confinement on Lagrangian transport in two-dimensional turbulence, which changes the topology with respect to the unbounded case. A first study Kadoch et al. (2008) showed that the generation of vorticity at the walls, in freely decaying turbulence, has a strong influence on Lagrangian statistics, in particular we observed an increase of intermittency in the bounded case with respect to the unbounded case.
2. Numerical methods

In compressible turbulent flow is governed by the two-dimensional Navier-Stokes equations written in dimensionless form

\[ \frac{\partial \omega}{\partial t} + \mathbf{V} \cdot \nabla \omega - \nu \nabla^2 \omega - F_\omega = -\frac{1}{\eta} \nabla \times (\chi \mathbf{V}), \quad (1) \]

where, \( \mathbf{V} = (u, v) \) is the velocity, \( \omega = \nabla \times \mathbf{V} \) is the vorticity, \( \nu \) is the kinematic viscosity, and \( F_\omega \) an external force. Two different domains are considered: a double periodic domain (unbounded) and a circular domain with no-slip boundary conditions (bounded). The term on the right hand side of Eq. (1) is the volume penalization term which takes into account the no-slip boundary conditions using the mask function \( \chi \) Angot et al. (1999); Schneider (2005).

This term is zero for the periodic domain. To obtain a statistically stationary flow, a forcing term \( F_\omega \) is considered, where \( F_r \) denotes a random isotropic stirring at small wave-numbers. The friction term, \( -\beta \psi \), is added to avoid the accumulation of energy at large scales due to the inverse energy cascade. Thus we have \( F_\omega = F_r - \beta \psi \). For the periodic domain we choose \( \beta = 1 \). However, for the confined domain case we choose \( \beta = 0 \), since the wall plays the role of an energy sink at large scales. In the confined domain, we introduce a time correlation into the forcing term, by using a discrete Markov chain as in Lilly (1969).

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\[ a_L = -\nabla p + \nu \nabla^2 u + f, \quad (2) \]

where \( f \) is constructed by applying the Biot-Savart operator to \( F_\omega \). The Lagrangian statistics are computed by ensemble averaging over \( 10^4 \) trajectories. Each trajectory is integrated for \( 5 \times 10^6 \) time-steps, which correspond to \( 800 \tau \), where \( \tau = 1/\sqrt{2\langle \omega^2 \rangle} = 0.0622 \) is the eddy turnover time and \( Z = 1/2\langle \omega^2 \rangle \) the enstrophy.

To understand the influence of the topology onto the flow, we use the Okubo-Weiss criterion defined by: \( Q = 1/4(S^2 - \omega^2) \), where \( S \) is the deformation. We split then the flow in three contributions: strong elliptic regions \( Q < -\sigma \), with \( \sigma \) being the standard deviation of \( Q \), intermediate regions \( -\sigma < Q < \sigma \), and strong deformation regions \( \sigma < Q \).

3. Results

Visualisations of the spatial distribution of the three-level, normalized Eulerian Weiss field, \( \hat{Q} = Q/Q_0 \), in double periodic and bounded domains, are shown in Fig. 1(top). The values of \( Q_0 \) are 170 and 300 for the periodic and confined domain, respectively. The negative regions are concentrated inside cyclones, \( \omega > 0 \), and anti-cyclones, \( \omega < 0 \), i.e., the coherent vortices while the hyperbolic regions corresponding to strong deformation regions are dominated by circulation cells which surround the vortices. The background turbulent field is characterized by small intermediate positive and negative values of the Weiss field. In both the double periodic and the bounded domain, we can observe in Fig. 1 (bottom) that particles tend to spend relatively short times in strongly hyperbolic regions. This is probably due to the fact that these regions are dynamically unstable. However, the relatively long stay of particles in strongly elliptic regions results from the trapping of particles by vortices. These latter ideas are quantified in Fig. 2, where the pdfs of the residence time, \( \tau \), in the strongly elliptic, \( P^e(\tau) \), strongly hyperbolic,
Figure 1. Snapshots of three levels Eulerian Weiss field $\hat{Q} = Q/Q_0$ (top) and typical passive tracer orbits in two-dimensional forced turbulence color-coded by the instantaneous value of the Lagrangian Weiss field (bottom), in a double periodic domain (left panel) and in a bounded circular domain (right panel). Blue denotes strongly elliptic regions with $\hat{Q} \leq -1$; Red denotes strongly hyperbolic regions with $\hat{Q} \geq 1$; and Pink denotes intermediate regions with $-1 < \hat{Q} < 1$.

$P^h(\tau)$, and intermediate, $P^i(\tau)$, regions are shown. These pdfs determine the probability that a given Lagrangian tracer stays in a region with the same value of the three-level normalized Weiss field for a given time $\tau$. The exponential decay of $P^i(\tau)$ is due to the Gaussian fluctuations, characteristic of the turbulent background, which is consistent with the results in Moisy & Jiménez (2006) for three-dimensional turbulence. A Poisson process which is characterized by a small time-correlation and an exponential pdf could be a possible explanation. On the other hand, $P^e(\tau)$ and $P^h(\tau)$ exhibit a self-similar behavior, corresponding to algebraic tails in the pdf. This is probably due to the weak chaos and strong correlations characteristic of the vortex cores and the circulating cells surrounding the vortices. The strong dynamical instability of the circulating cells is responsible for the considerably larger decay exponent of $P^h(\tau)$. Since vortex trapping is a local phenomenon which is not dependent on the boundaries, $P^e(\tau)$ shows the same decay in the periodic and the bounded domain. However, since particles in the tur-
bulent background evolve in the entire domain $P^i(\tau)$ exhibits some dependence on the boundary.

\[
\begin{align*}
10^{-4} & \quad 10^{-3} & \quad 10^{-2} & \quad 10^{-1} & \quad 10^0 & \quad 10^1 \\
0 & \quad 5 & \quad 10 & \quad 15 & \quad 20 & \quad 25 & \quad 30
\end{align*}
\]

PDF

Residence time

exp(-0.24*t)

\[
\begin{align*}
10^{-4} & \quad 10^{-3} & \quad 10^{-2} & \quad 10^{-1} & \quad 10^0 & \quad 10^1 \\
0 & \quad 5 & \quad 10 & \quad 15 & \quad 20 & \quad 25 & \quad 30 & \quad 35 & \quad 40
\end{align*}
\]

PDF

Residence time

exp(-0.12*t)

\textbf{Figure 2.} Pdfs of residence time for periodic (left) and circular domain (right).

The conditional pdfs of the Lagrangian acceleration are plotted in Fig. 3. We can observe that the level of intermittency, which is characterized by the heavy tails, is qualitatively comparable in the double periodic and the bounded domain. Indeed, the tails decay exponentially for both geometries. This is different to freely decaying two-dimensional turbulence Kadoch et al. (2008) in which the circular domain shows a stronger level of intermittency. The conditional pdf for the intermediate values of the Weiss field exhibits a sharp peak around zero indicating a stronger probability of weak acceleration values in the intermediate region.

\[
\begin{align*}
10^{-9} & \quad 10^{-8} & \quad 10^{-7} & \quad 10^{-6} & \quad 10^{-5} & \quad 10^{-4} & \quad 10^{-3} & \quad 10^{-2} & \quad 10^{-1} & \quad 10^0 \\
-600 & \quad -400 & \quad -200 & \quad 0 & \quad 200 & \quad 400 & \quad 600
\end{align*}
\]

PDF

Conditional Lagrangian acceleration

Q < -σ

-σ < Q < σ

σ < Q

\textbf{Figure 3.} Conditional pdf of Lagrangian acceleration in the x-direction with respect to the three-level Weiss field in the double periodic domain (left) and in the circular domain (right).

4. Conclusions

We have presented a study of the influence of the flow topology onto Lagrangian statistics with the help of the Weiss criterion to characterize the topologies. The Weiss criterion is a tool which enables us to split the flow field into topologically different regions. The pdfs of the residence
time, which characterize statistically the time that particles stay in the different topologies of the flow domain: strongly elliptic, hyperbolic regions and intermediate regions. The pdfs $P^e(\tau)$ and $P^h(\tau)$ exhibit algebraic decaying tails, and the pdf $P^i(\tau)$ exhibits exponentially decaying tails. This motivates the use of this pdf to model waiting time pdfs in continuous time random walk descriptions of anomalous diffusion in turbulent systems with coherent trapping structures. To study the dependence of the Lagrangian statistics on the flow topology, we also investigated the conditional pdfs of the Lagrangian acceleration with respect to the Lagrangian Weiss field. It is shown that strong elliptic and hyperbolic regions contribute most to the Lagrangian acceleration. Further results are reported in Kadoch et al. (2011).

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