Manufacture of conical springs with elastic medium technology improvement

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Abstract. This article considers the manufacturing technology improvement by using an elastic medium in the stamping tool forming space to improve the conical springs performance characteristics and reduce the costs of their production. Estimation technique of disk spring operational properties is developed by mathematical modeling of the compression process during the operation of a spring. A technique for optimizing the design parameters of a conical spring is developed, which ensures a minimum voltage value when operated in the edge of the spring opening.

Keywords – conical springs, elastic medium, stamping, elastic stress-strain state, elastic deformation, solid mechanics.

1. Introduction
Conical springs are widely used in engineering for damping dynamic and shock loads arising during the operation of machines (shock absorbers, buffer devices, etc.).

However, according to numerous studies and practical tests, a large percentage of springs in the course of operation under cyclic loading does not withstand the number of cycles specified in the industry standard \((2 \cdot 10^6)\). Destruction occurs due to the formation of fatigue cracks in the edge of the hole and wear on their contact surfaces. Defects of the spring edges surface, appearing during the typical stamping, accelerate the fatigue cracks creation. In the case of standard springs, significant voltages arise in the operation therefore the resource decreases. The contact surface of the spring wears out quickly due to its small area. To increase the life of a conical spring, it is necessary to reduce stresses during operational deformation and to increase the contact surface area and to prevent the surface defects during its manufacturing.

Reduction of stresses is possible by optimizing the design parameters of the spring, based on the results of the analysis of stress strain behavior during compression in operation. Flanging of the spring edge will increase the area of its contact surface. To produce an improved spring with optimal design parameters, it is necessary to change the design of the die tool. Most FEM systems use the methods of calculating the stress-strain state on the basis of variational methods created by the works of many scientists, such as R. Hill [1], D. Banabic [2], F. Barlat [3–7], J.L. Bassani [8], et al. However, the use of FEM codes for complex shape parts requires considerable computer resources. One possible solution to this problem is a preliminary study in the 2D model, and then on the problem areas more accurate 3D modeling [14].
A large number of works have been devoted to the research of the conical springs sheet stamping process [7-8, 10]. The usual methods of sheet-metal stamping, widely used in large-scale and mass production, can be insufficient in the conditions of small-scale and fast-retooled production, since it takes a long time to produce structurally complex and expensive stamps. In some cases, the investment is not return. For the rapid development of new types of products, it is necessary to use new technological processes in the specified conditions using cheap universal or partially universal equipment. Improving the technology of manufacturing disc springs, eliminating the operation of the mechanical processing of the edges, will reduce production costs and improve the performance characteristics of the spring.

2. Optimization of the conical springs design parameters

Optimization of the conical springs design parameters consists of three stages. The first stage: determination and investigation of the spring surface areas, on which stresses state arises, causing fatigue failure, based on the finite element method. The second stage: improvement of the spring shape to reduce the tensile stresses in the area determined in the first stage. The third stage: the calculation of the optimal design parameters of the disk spring, based on the methods of linear programming.

The problem of calculating the elastic stress strain stage of a conical spring is solved on the basis of the potential energy minimum principle. The solution considered an axisymmetric stress strain stage in cylindrical coordinates, since the conical spring has axisymmetric shape.

Due to the symmetry of the body and the external forces symmetry, the directions of all radii \( r \) are equal. The stress components do not depend on the angle \( \phi \). The tangential stresses on the planes \( \phi - \tau_{r\phi} \) and \( \tau_{z\phi} \), as well as the \( \tau_{r\phi} \) and \( \tau_{z\phi} \) equal to them, will be zero. Neglecting infinitesimals of higher orders, taking into account the equilibrium equations, the potential energy functional takes the form:

\[
\Pi = \frac{1}{2} \int \left( \frac{1}{Z_1} \int \left[ \frac{1}{2} \mu (r) \varepsilon^T \cdot D \cdot \varepsilon \cdot rdrdz - k \cdot p \cdot \delta \right] R_z \cdot ud\phi \cdot \frac{2\pi}{r} \right) dz, \\
\]

where

\[
Z_1 = H_1 \frac{H_1}{R_1} r, \\
Z_2 = H_2 \frac{H_2}{R_2}, \\
H_1 = R_1 \cdot tg \alpha; \\
H_2 = H_1 + \frac{S}{cos \alpha}; \\
p_1 = \frac{F}{2\pi R_2 \cdot \delta}; \\
D = \frac{E}{1-\mu^2} \begin{bmatrix} 1 & \mu & 0 \\ \mu & 1 & 0 \\ 0 & 0 & (1-\mu)/2 \end{bmatrix}
\]

- \( k \) is the constant of friction;
- \( \mu \) is the Poisson ratio;
- \( \varepsilon \) is the deformation matrix,
- \( u \) is the displacement vector,
- \( \delta \) is the width of the contact surface

As a result of the displacement approximation on each of the elements by linear functions and using the rules of matrix differentiation, we calculated the partial derivatives of elemental energies in terms of the nodal displacements vectors and obtained an elementary matrix equation from the equation of the functional (1):
\[
\frac{\partial z^e}{\partial u^e} = 2\pi \left( B^T \cdot D \cdot B \cdot U^e \cdot I \cdot D^e \cdot k \cdot p \cdot I \right).
\]

where \(D^e\) is the area of the triangular element;

\(L^e\) - length of the contact side, loaded boundary elements;

\(U^e\) - approximation of the each finite elements displacements by linear functions

\[
I = \int \frac{rdD^e}{D^e};
\]

\[
L^e = \int \frac{rdL^e}{L^e};
\]

\[
B = \begin{bmatrix}
\frac{\partial}{\partial r} & 0 \\
0 & \frac{\partial}{\partial \theta} \\
0 & \frac{\partial}{\partial \phi}
\end{bmatrix} \cdot N^e;
\]

\(N^e\) are the basis functions.

Further, after using the extremum condition for the function (1) and combining the elemental matrix equations (2), we obtain a system of linear equations [15]:

\[
\sum_{e=1}^{E} K^e \cdot U^e = \frac{F \cdot S \cdot (R_e - R_s)}{288 \cdot \pi \cdot R_s \cdot \cos \alpha} = 0,
\]

where \(K^e = B^T \cdot D \cdot B \cdot I \cdot D^e\) is conical springs stiffness matrix.

To numerical implementation of this model, a software product in the C# was developed [16]. To check the obtained results, simulation in the CAE "ANSYS" system was conducted.

3. Experiment results

As a result of the study of the stress strain stage of standard springs, it is determined that the region with the greatest tensile stresses is the edge of the 2 holes (Fig. 1). These stresses create favorable conditions for the appearance and development of fatigue cracks.

Figure1. Stresses \(\sigma_{\phi}\) in a loaded conical spring
With the purpose of reducing the value of stress in the edge 2, a technique for calculating the optimal design parameters of a disk spring was developed [17]. The technique is based on the calculation formulas given in the national standard 3057-90 "Conical Springs".

As a target criterion, the minimum stresses at the edge 2 for compression are chosen. As a limitation, the geometric dimensions of the conical spring are adopted, which must correspond to the location of its installation and its rigidity. To implement the proposed methodology for calculating the optimal design parameters of the disk spring, application software was developed.

The disk spring was also improved by a change in shape, forming a generatrix curve of the cone in the longitudinal section (Fig. 2). This led to a reduction of the stress value in the edge 2.

The design of the conical springs using this technique of calculating and improving the shape leads to a decrease of the stress value in the edge of hole 2 to 30%.

4. Discussion

Using the developed software product, a series of test calculations of design parameters of standard and improved conical springs were done. In Table 1 the results of calculations of the stress in edge 2 at 0.6 f3 spring deformation shown (Fig. 2) for standard springs taken from standard tables and optimized springs.

The analysis of test calculations results showed that for a given diameter of the spring D and the stiffness range, a decrease in stress $\sigma_2$ (stress in edge 2 (Fig. 1)) was recorded by 14%. For other design parameters of the conical springs, a reduction in the stress $\sigma_2$ to 30% was recorded. The results were discussed and approved at conferences [18, 19].

5. Conclusions

A technique for calculating the optimal design parameters and the shape of the plate springs is developed, consisting of three stages. The first stage: with the help of the developed mathematical model of the elastic compression process during the operation of the spring, a surface area of the spring is defined, where the greatest stresses occur that promote fatigue failure.

The second stage: improvement of the shape of the Belleville spring in order to reduce the stresses on the found area. The most loaded of them is the lower edge of the hole. The third stage: optimizing the design parameters of the spring, based on linear programming methods, providing a minimum value of the voltage at the lower edge of the hole during operation.

The use of the proposed calculation procedure led to decrease in the value of the stresses in the lower edge of the hole to 30% in comparison with the standard spring.

| Table 1. Results of test calculations |
|--------------------------------------|
| Spring type | $D$, mm | $d$, mm | $S$, mm | $f_\alpha$, mm | $C$, H/ mm | $\sigma_2$, MPa | $r_1$, mm | $r_2$, mm |
|-----------------|--------|--------|--------|----------------|----------|------------|--------|--------|
| standard spring | 31.5   | 12.5   | 1.05   | 1              | 1094     | 532.07     | -      | -      |
| optimized spring| 31.5   | 15.75  | 0.97   | 0.83           | 1200     | 459.20     | 1.22   | 0.5    |

Figure 2. Improved conical spring cross section
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