Measuring the Indonesian provinces competitiveness by using PCA technique

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Abstract. Indonesia is a country which has vast territory. It has 34 provinces. Building local competitiveness is critical to enhance the long-term national competitiveness especially for a country as diverse as Indonesia. A competitive local government can attract and maintain successful firms and increase living standards for its inhabitants, because investment and skilled workers gravitate from uncompetitive regions to more competitive ones. Although there are other methods to measuring competitiveness, but here we have demonstrated a simple method using principal component analysis (PCA). It can directly be applied to correlated, multivariate data. The analysis on Indonesian provinces provides 3 clusters based on the competitiveness measurement and the clusters are Bad, Good and Best perform provinces.

1. Introduction
Indonesia is an archipelagic island country in South East Asia. There are many provinces under Indonesia’s auspices. In 2017, Indonesia has 34 provinces. Every province in Indonesia has a unique and distinctive charm. Moreover Indonesia has a vast territory, it’s area is 1.905 million km². Therefore it will be difficult for each province has similar development. To solve this problem, the central government made a regional autonomy (Otonomi Daerah) policy.

The new policy of decentralization and regional autonomy is outlined in Law No. 22, 1999 concerning “Local Government”¹ and Law No. 25, 1999 concerning “The Fiscal Balance Between the Central Government and the Regions.” Both these laws are based on five principles: 1) democracy, 2) community participation and empowerment, 3) equity and justice, 4) recognition of the potential and diversity within regions and 5) the need to strengthen local legislatures. These five principles support Indonesia’s push for reformasi, which continues to aim to eradicate the practices of corruption, collusion, and nepotism (known as KKN), within the government bureaucracy. One of the specific reasons behind the policy of decentralization and regional autonomy is that a centralized government system cannot possibly administer Indonesia’s large population of over 203 million (BPS, 2001) and its diverse socio-cultural and religious background. Strong, competent regional governments and greater autonomy are fundamental requirement for a country as diverse as Indonesia. The main aim of decentralization and regional autonomy is to bring the governments closer to their constituents so that government services can be delivered more effectively and efficiently. This is based on the assumption that district and municipal governments have a better understanding of the needs and aspirations of their communities than the central government [1].

We can rank each province by using PCA and try to find the cluster of each provinces. Therefore, it will build the local competitiveness. Building local competitiveness is critical to enhance the long-term national competitiveness. A competitive local government can attract and maintain successful firms and increase living standards for its inhabitants, because investment and skilled workers gravitate from uncompetitive regions to more competitive ones.

Asia Competitiveness Institute (ACI) defines competitiveness through four different environments, each with their three sub-components, namely [2]:

1) Macroeconomic stability
   a. Regional Economic Vibrancy
   b. Openness to Trade and Services
   c. Attractiveness to Foreign Investors
2) Government and institutional setting
   a. Government Policies and Fiscal Sustainability
   b. Institutions, Governance and Leadership
   c. Competition, Regulatory Standards and Rule of Law

3) Financial, business and manpower conditions
   a. Financial Deepening and Business Efficiency
   b. Labour Market Flexibility
   c. Productivity Performance

4) Quality of life and infrastructure development
   a. Physical Infrastructure
   b. Technological Infrastructure
   c. Standard of Living, Education and Social Stability

Those environment give us an idea about the condition of provinces. Two environments are specifically related to the economy and two other environment are more political, institutional and social in character. Macroeconomic stability encompassing aggregate economic conditions. It will be easy to see how strong the potential of a province in attracting foreign investors from this environment. Financial, business and manpower conditions represent condition related to the micro-economy. They include an analysis on the performance of firms as well as challenges that they face in running their companies, also from this environment we will be able to describe the state of the population in that area, the availability of labor in order to see the potential of human resources in the area. Government institutional setting will provide an overview of the rules, policies of local governments in addressing the problems in the region. And how responsive the local government in solving a case. Thus the stability of the government will be seen in the area. As for the quality of life and infrastructure development can be used as a reference in view of the security of the area, the power level of life in society, and the development of technology or infrastructure.

2. Literature Review/Related Works

Malesky, in 2008 [3] with his research tim, make the research under the title The Vietnam Provincial Competitiveness Index: Measuring Economic Governance for Private Sector Development. 2008 Final Report indicated in last year's report, there is evidence that increasing inequality is beginning to appear across the country. Top performing provinces excel at all aspects of economic development, as shown in this report in terms of economic governance, infrastructure, and ICT capability. This group is pulling away from the rest of the country. At the same time, another group that must struggle with weak initial conditions and poor infrastructure has not been able to develop the good governance practices to compensate for their handicap. These are steadily falling behind the performance of their peers. The Provincial Competitiveness Index in Vietnam measured by 10 sub indices they are: Privat Sector Development services, Transparency, Labor training, Proactivity, Time cost of regulatory compliance, Legal institutions, SOE bias (competition environment), informal charges, Land access and security, and Entry costs. In This research, Malesky divided the provinces in Vietnam using the provincial competitiveness index into 6 classes. The classes are Excellent Province, High Province, Mid-High Province, Average Province, Mid-Low Province and las is Low Province.

In 2013, Tan and Amri [4] in their research under the title Subnational Competitiveness and National Performance: Analysis and Simulation for Indonesia said that the stable growth of Indonesia’s economy over the past eight years has provided momentum for investment in the country. One of the approaches taken by the central government is to allow healthy competition between its provinces. The Asia Competitiveness Institute (ACI) responds positively to that policy by ranking the competitiveness of Indonesia’s 33 provinces and providing simulation studies on how to improve each province’s competitiveness. ACI takes a comprehensive approach to the notion of provincial competitiveness, dissecting it from four major environments: macroeconomics, microeconomics, governance, and quality of life. Drawing on 91 indicators from formal sources as well as ACI’s own surveys and interviews, the study aggregates the indicators into 12 sub-environments, reaggregates them into four environments, and finally reaggregates them again into an overall competitiveness index. The conclusion highlights the high level of competitiveness in provinces where the country’s major urban regions are situated, as well as those closest to Singapore as the regional trading hub. Provinces endowed with natural resources also have the opportunity to be competitive, but not if they are wanting in
aspects such as governance and quality of life. The study’s findings invite further research on more specific topics such as labor market flexibility and regional cooperation.

3. Method

The data set that we use for this research is downloaded from Indonesian statistics website, www.bps.go.id, for the years 2014 and 2015. Non-Oil exports data is downloaded from www.kemendag.go.id. From both sources we have 29 variables which are belong to the 4 environments at ACI, namely:

1) Macroeconomic stability
   1. Realization Of Investment In Domestic Investment (Project) Year 2015
   2. Realization Of Investment In Domestic Investment By Province (Billion Rupiah) Year 2015
   3. Gdp Per Capita By Province Year 2014
   4. Non-Oil Export By Province Year 2015
   5. Number Of Foreign Tourist By Province Year 2015

2) Government And Institutional Setting
   6. Total Crime According To The Regional Police, 2000-2015
   7. Crime Rate Per 100,000 Population By Regional Police Year 2000-2015
   8. Percentage Of Completion Crime According To The Regional Police Year 2000-2015

3) Financial, Business And Manpower Conditions
   9. Unemployment Rate Year 2015
   10. Labor Force Participation Rate Year 2015
   11. Registered Job Seekers Years 2015
   12. Job Vacancies List Year 2015
   13. Placement/Fulfillment Of Labor Year 2015

4) Quality Of Life And Infrastructure Development
   14. Population By Province Year 2015
   15. Human Development Index (HDI) Year 2015
   16. Food Poverty Line Year 2015
   17. Gini Ratio Year 2015
   18. Number Of Poor Year 2015
   19. School Enrollment Rate (7-12 Years Old) Year 2015
   20. School Enrollment Rate (13-15 Years Old) Year 2015
   21. School Enrollment Rate (16-18 Years Old) Year 2015
   22. School Enrollment Rate (19-24 Years Old) Year 2015
   23. Number Of Universities Year 2014
   24. Number Of Students Year 2014
   25. Number Of Educational Personnel Year 2014
   26. Percentage Of Households By Source Of Improved Drinking Water Year 2015
   27. Percentage Of Households By Lighting Source Of Electricity Year 2015
   28. Percentage Of Households By Improved Sanitation Year 2015
   29. Percentage Of Population Having Health Complaint Year 2015

Principal Component Analysis (PCA) is one of famous techniques for dimension reduction, feature extraction, and data visualization. In general, PCA is defined by a transformation of a high dimensional vector space into a low dimensional space [5]. Manage and Scariano [6] describe PCA as a nonparametric variable reduction technique well suited for correlated data and one objective of principal component analysis is to reduce a set of correlated variables into fewer uncorrelated variables as linear combinations of the original ones.

In general, the PCA procedures can be explained [7-9] explain as follows.

Suppose we have a random vector X

\[ X = \begin{pmatrix} X_1 \\ X_2 \\ \vdots \\ X_p \end{pmatrix} \]
with population variance-covariance matrix

\[
\text{var}(X) = \sum = \begin{pmatrix} \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1p} \\ \sigma_{21} & \sigma_2^2 & \cdots & \sigma_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{p1} & \sigma_{p2} & \cdots & \sigma_p^2 \end{pmatrix}
\]

Consider the linear combinations:

\[
Y_1 = e_{11}X_1 + e_{12}X_2 + \cdots + e_{1p}X_p \\
Y_2 = e_{21}X_1 + e_{22}X_2 + \cdots + e_{2p}X_p \\
\vdots \\
Y_n = e_{p1}X_1 + e_{p2}X_2 + \cdots + e_{pp}X_p
\]

Each of these can be thought of as a linear regression, predicting \(Y_i\) from \(X_1, X_2, \ldots, X_p\). There is no intercept, but \(e_{1i}, e_{2i}, \ldots, e_{pi}\) can be viewed as regression coefficients.

The \(Y_i\) is a function of random data and it has a population variance

\[
\text{var}(Y_i) = \sum_{k=1}^{p} \sum_{l=1}^{p} e_{ik} e_{il} \sigma_{kl} = e_i' \sum e_i
\]

\(e_i' \sum e_j\) is population covariance of \(Y_i\) and \(Y_j\)

\[
\text{cov}(Y_i, Y_j) = \sum_{k=1}^{p} \sum_{l=1}^{p} e_{ik} e_{jl} \sigma_{kl} = e_i' \sum e_j
\]

The coefficients of \(e_i\) are

\[
e_i = \begin{pmatrix} e_{i1} \\ e_{i2} \\ \vdots \\ e_{ip} \end{pmatrix}
\]

The \(p^{th}\) principal component is built by selecting \(e_{i1}, e_{i2}, \ldots, e_{ip}\) such that maximizes the variance of the new component.

\[
\text{var}(Y_i) = \sum_{k=1}^{p} \sum_{l=1}^{p} e_{ik} e_{il} \sigma_{kl} = e_i' \sum e_i
\]

The maximization considers two things: the sums of squared coefficients add up to one and the new component will be uncorrelated with all the previously defined components.

\[
e_i' e_i = \sum_{j=1}^{p} e_{ij}^2 = 1
\]

\[
\text{cov}(Y_1, Y_i) = \sum_{k=1}^{p} \sum_{l=1}^{p} e_{1k} e_{il} \sigma_{kl} = e_i' \sum e_i = 0
\]

\[
\text{cov}(Y_2, Y_i) = \sum_{k=1}^{p} \sum_{l=1}^{p} e_{2k} e_{il} \sigma_{kl} = e_i' \sum e_i = 0
\]

\[
\vdots
\]

\[
\text{cov}(Y_{i-1}, Y_i) = \sum_{k=1}^{p} \sum_{l=1}^{p} e_{(i-1)k} e_{il} \sigma_{kl} = e_{i-1}' \sum e_i = 0
\]
4. Results and Discussion
The calculations for the average of each environment are shown as following:

| Infromations | Macroeconomic Stability (MS) | Government and Institutional Setting (GIS) | Financial, Business and Manpower Conditions (FBI) | Quality of Life and Infrastructure Development (QL) |
|--------------|------------------------------|---------------------------------------------|--------------------------------------------------|--------------------------------------------------|
| Averages     | 8.48994E-17                  | -6.75522E-17                               | 1.18206E-15                                     | 2.35106E-16                                     |
| Conclusions  | Lower Is The Best            | Higher Is Best                              | Lower Is The Best                               | Lower Is The Best                               |

Table 1 shows the average of each environments from ACI and the the scores for each provinces. It is based on the standardized of original values of each variable for every environments. If the value of variable tends as higher is better then we multiply the standardization with minus, and otherwise.

Figure 1 Provinces performance according to ACI environments

Before applying PCA, the standarization is needed to be applied cause the variables have different scales. The processes of finding the principal components by using the standardized variables are equivalent to finding the principal components by using the correlation matrix instead of the covariance matrix. Furthermore, it is used to compute the eigen values and vectors.
Figure 2 PCA’s Output

The output of PCA, Figure 2, shows that the PC1 gives the highest proportion of variance, 31.08%, then it is used to rank the provinces. Furthermore, its eigen vectors are in the Table 2.

| No | Variables                               | Eigen Vectors | No | Variables                               | Eigen Vectors |
|----|-----------------------------------------|---------------|----|-----------------------------------------|---------------|
| 1  | HDI                                     | -0.166        | 16 | Number Of Universities                 | -0.306        |
| 2  | Food Poverty Line                       | 0.105         | 17 | Number Of Students                     | -0.290        |
| 3  | Gini Ratio                              | -0.135        | 18 | Number Of Educational Personnel       | -0.309        |
| 4  | Number Of Poor                          | 0.099         | 19 | Household by Drinking Water Sources    | -0.150        |
| 5  | School Enrollment Rate (7-12 yo)        | -0.113        | 20 | Households by Electric Power Sources  | -0.170        |
| 6  | School Enrollment Rate (13-15 yo)       | -0.111        | 21 | Households by Improved Sanitation     | -0.157        |
| 7  | School Enrollment Rate (16-18 yo)       | 0.037         | 22 | Total Crime                            | 0.018         |
| 8  | School Enrollment Rate (19-24 yo)       | 0.041         | 23 | Crime Rate                             | 0.046         |
| 9  | Unemployment Rate                       | -0.051        | 24 | Percentage of Completion Crime         | 0.129         |
| 10 | Labor Force Participant Rate            | 0.104         | 25 | Health Complaint                       | -0.121        |
| 11 | Registered Job Seeker                   | -0.273        | 26 | Population (Thousands)                | -0.288        |
| 12 | Job Vacancies List                      | -0.269        | 27 | Non Oil Exprots                       | -0.169        |
| 13 | Labor Fulfillment                       | -0.271        | 28 | GDP                                    | -0.011        |
| 14 | Investment Realization (Project)        | -0.295        | 29 | Foreign Tourist                        | -0.072        |
| 15 | Investment Realization (billion)        | -0.291        |     |                                         |               |

Table 2 gives the first principal component as follows:

\[ Y_1 = e_{1,1}X_1 + e_{1,2}X_2 + \ldots + e_{1,29}X_{29} \]
\[ Y_1 = -0.166(HDI) + 0.105(Poverty \ Line) - 0.135(Gini \ Ratio) + 0.099 \ (Number \ of \ Poor) \\
- 0.113 \ (School \ Enrollment \ Rate \ (7 - 12 \ yo)) \\
- 0.111 \ (School \ Enrollment \ Rate \ (13 - 15 \ yo)) \\
+ 0.037 \ (School \ Enrollment \ Rate \ (16 - 18 \ yo)) \\
+ 0.041 \ (School \ Enrollment \ Rate \ (19 - 24 \ yo)) \\
- 0.051 \ (Open \ Unemployment \ Rate) + 0.104 \ (Labor \ Force \ Participant \ Rate) \\
- 0.273 \ (Registered \ Job \ Seeker) - 0.269 \ (Job \ Vacancies \ List) \\
- 0.271 \ (Labor \ Fulfillment) - 0.295 \ (Realization \ Investment \ (Project)) \\
- 0.291 \ (Investment \ Realization \ (billion)) - 0.306 \ (Number \ Of \ Universities) \\
- 0.290 \ (Number \ Of \ Students) - 0.309 \ (Number \ Of \ Educational \ Personnel) \\
- 0.150 \ (Household \ by \ Drinking \ Water \ Sources) \\
- 0.170 \ (Households \ by \ Improved \ Sanitation) + 0.018 \ (Total \ Crime) \\
+ 0.046 \ (Crime \ rate) + 0.129 \ (Percentage \ of \ Completion \ Crime) \\
+ 0.121 \ (Health \ Complaint) - 0.288 \ (Population \ (Thousands)) \\
- 0.169 \ (Non \ Oil \ Exports) - 0.011 \ (GDP) - 0.072 \ (Foreign \ Tourist) \]

Therefore, the scores for each province are as in Table 3:

| Provinces            | PC 1 score | Standardized PC 1 score | Ranking |
|----------------------|------------|-------------------------|---------|
| Aceh                 | 0.437172   | 0.143654                | 15      |
| Sumatera Utara       | -0.88412   | -0.29052                | 8       |
| Sumatera Barat       | 0.875282   | 0.287616                | 18      |
| Riau                 | 0.02172    | 0.007137                | 11      |
| Jambi                | 0.96015    | 0.315504                | 19      |
| Sumatera Selatan     | 0.245784   | 0.080764                | 14      |
| Bengkulu             | 2.051826   | 0.674226                | 28      |
| Lampung              | 1.125308   | 0.369774                | 21      |
| Kep. Bangka Belitung | 1.407645   | 0.462549                | 23      |
| Kep.Riau             | 0.184928   | 0.060767                | 13      |
| DKI Jakarta          | -5.78365   | -1.9005                 | 4       |
| Jawa Barat           | -7.62536   | -2.50568                | 2       |
| Jawa Tengah          | -5.86413   | -1.92694                | 3       |
| DI Yogyakarta        | -1.08801   | -0.35752                | 7       |
| Jawa Timur           | -9.47617   | -3.11385                | 1       |
| Banten               | -2.07725   | -0.68258                | 5       |
| Bali                 | -0.5356    | -0.176                  | 9       |
| Nusa Tenggara Barat | 0.082932   | 0.027251                | 12      |
| Nusa Tenggara Timur  | 2.425648   | 0.797063                | 29      |
| Kalimantan Barat     | 1.430153   | 0.469946                | 24      |
| Kalimantan Tengah    | 1.893612   | 0.622237                | 25      |
| Kalimantan Selatan   | 0.645778   | 0.212202                | 16      |
| Kalimantan Timur     | -0.26374   | -0.08666                | 10      |
| Kalimantan Utara     | 2.540385   | 0.834766                | 32      |
Table 3 shows that Jawa Timur has the first ranking and Papua is the last one. In 2012, Tan and Amri [4] has ranked the competitiveness provinces in Indonesia by using 2010 data. The results are the first ranking is DKI Jakarta and the second is Jawa Timur. It means since 2010, Jawa Timur has improved itself and becomes the best province. The same condition is applied to other provinces. For examples, in 2010 Nusa Tenggara Barat has a ranking 29 and in 2015 Nusa Tenggara Barat has a ranking 12. In Tan and Amri [4] results, DKI Jakarta have become the first ranking in 2000 and 2010. But in 2015 competitiveness, DKI Jakarta has dropped to number 4.

The ranking of provinces based on PCA is determined by the following procedures. The scores that we got from PCA needs to be normalized. The PCA scores are divided into three areas, as in Figure 3.

Table 3

| Provinces          | PC 1 score | Standardized PC 1 score | Ranking |
|--------------------|------------|-------------------------|---------|
| Sulawesi Utara     | 0.743866   | 0.244433                | 17      |
| Sulawesi Tengah    | 1.93902    | 0.637158                | 26      |
| Sulawesi Selatan   | -1.91624   | -0.62967                | 6       |
| Sulawesi Tenggara  | 1.064083   | 0.349656                | 20      |
| Gorontalo          | 1.261665   | 0.414581                | 22      |
| Sulawesi Barat     | 2.428965   | 0.798153                | 30      |
| Maluku             | 2.033066   | 0.668061                | 27      |
| Maluku Utara       | 2.542381   | 0.835422                | 33      |
| Papua Barat        | 2.480462   | 0.815075                | 31      |
| Papua              | 4.692438   | 1.541926                | 34      |

In Figure 3, the first area represents the best perform provinces, the second one represents the good perform provinces and the third one represents the bad perform provinces. The scores for best perform provinces are less than and or equal to -0.43, for good perform provinces are between -0.43 until 0.43 half right inclusive, and for bad perform provinces are greater than 0.43. The best, good and bad perform provinces based on their scores are shown at Table 4.
Table 4 Members of each PCA clusters

| BEST PERFORM PROVINCES | GOOD PERFORM PROVINCES | BAD PERFORM PROVINCES |
|-------------------------|-------------------------|-----------------------|
| Jawa Timur              | DI Yogyakarta           | Aceh                  |
| Jawa Barat             | Sumatera Utara          | Kalimantan Selatan    |
| Jawa Tengah            | Bali                    | Sulawesi Utara        |
| DKI Jakarta            | Kalimantan Timur        | Sumatera Barat        |
| Banten                 | Riau                    | Jambi                 |
| Sulawesi Selatan       | Nusa Tenggara Barat     | Sulawesi Tenggara     |
|                        |                        |                       |

Table 4 shows the members of every cluster using PCA. The best perform provinces has 7 members, good perform provinces contains 16 members and for bad perform provinces has 12 members. If we compare the average of each ACI environment in Table 2 and the scores of each provinces, then

a. from the best perform provinces, there are some provinces that worst than the average score. For example, in Financial, Bussiness and Manpower environment, there are two provinces have lower scores than its average, DKI Jakarta and Banten. For Macroeconomy stability environment Sulawesi Selatan has lower score than its average. In two others environments, each member of the Best perform provinces have better scores than the average.

b. from the good perform provinces, with 16 members. In Macroeconomic stability environment there are 5 provinces have scores better than the average, namely DI Yogyakarta, Sumatera Utara, Kep. Riau, Sulawesi Utara and Jambi. In Government and Institutional Setting environment, there are two provinces have scores lower than the average, namely Nusa Tenggara Barat and Lampung. In Financial, Bussiness and Manpower Condition environment, there are 4 provinces have better scores than its average, namely DI Yogyakarta, Bali, Nusa Tenggara Barat and Kalimantan Selatan. For Quality of Life and Infrastructure development environment, there are six provinces have lower scores than its average. They are Sumatera Selatan, Kalimantan Selatan, Jambi, Sulawesi Tenggara, Lampung and Gorontalo.

c. from the last cluster is Bad perform Provinces which has 12 members, in Macroeconomic stability environment, it is only Kalimantan Barat has a core better than its average. In Government and institutional setting environment, there are two Provinces which have lower scores than its average. They are Maluku Utara and Maluku. In Financial, bussiness and manpower condition environment, four Provinces, Kep. Bangka Belitung, Papua Barat, Maluku Utara and Maluku, have lower scores than its average. And for the last environment, Quality of life and infrastructure development, only Maluku Utara has a better score than its average.
5. Conclusion and Remarks

Measuring the competitiveness of every province in Indonesia is a challenging task. The result from competitiveness can be used to make some government’s rules to make the provinces in Indonesia better. This result can also be used by some investors to decide where they can invest their money according to its competitiveness. In this paper, we show that the principal component analysis can be used to rank the provinces competitiveness based on 29 variables, where Jawa Timur is the best perform provinces (the most competitive) and Papua is the least competitive. Further investigation could be applied to measure the provinces competitiveness simultaneously and thoroughly by adding more information through some additional variables as in the ACI ([2] and [4]).

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