QCD Corrections to Charmless $b \rightarrow s$ Processes

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Abstract

QCD corrections to the penguin induced charmless inclusive $B$ decays are performed to an improved leading-log approximation. For process $b \rightarrow sq\bar{q}$ ($q=$uds) and $bq\rightarrow s\bar{q}$, four previous missed 4-quark operators are included to make a complete leading log QCD corrected result. Furthermore, part of the next-leading log effect was also given for the above processes and process $b \rightarrow sg$. In comparison to previous QCD corrected calculations by Grigjanis et al., the inclusive decay rates for process $b \rightarrow sq\bar{q}$ ($q=$ud), $b \rightarrow ss\bar{s}$ and $b\bar{q}\rightarrow s\bar{q}$ are suppressed 10%, 14% and 18% respectively. The branching ratio of process $b \rightarrow sg$ is enhanced by 2-3 orders, the result for this process is in agreement with the result of Ciuchini et al.

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1 Introduction

Recently the CLEO collaboration has observed the charmless decays of $B, B^0 \to \pi^+\pi^-, B^0 \to K^+\pi^-$ and $B^0 \to K^+K^-$\cite{1}. The upper limits on the branching fractions are $B_{\pi\pi} < 2.9 \times 10^{-5}$, $B_{K\pi} < 2.6 \times 10^{-5}$, $B_{KK} < 0.7 \times 10^{-5}$. The sum of $B_{\pi\pi}$ and $B_{K\pi}$ exceeds zero with a significance more than four standard deviations. One can expect, individuals of these exclusive channels and also the inclusive processes will be found at CESR or future B factories.

The $B$ meson system is expected to give rise to a very rich phenomenology, providing a wealth of information that will allow us to constrain the currently very successful standard model (SM), as well as many extensions that go beyond it. It also provides many channels for the search of CP violation.

The charmless $b$ quark decay has already been calculated in several papers\cite{2,3,4,5,6}. They are induced by both $b \to u$ transition and penguin diagrams. The $b \to u$ transition diagrams are easy to calculate, but penguin diagrams are more complicated. W.S. Hou showed the results of penguin diagrams without QCD corrections in ref.\cite{2}. Afterwards, results including some short distance QCD corrections were given by R. Grigjanis et al.\cite{3}. For different processes, the authors claimed that the branching ratios are either QCD-enhanced or suppressed by a notable factor. In other words, the strong interaction plays an important role in these decays. But this is not the end of the story, there are still some unsolved questions. The anomalous dimensions used in ref.\cite{3} were changed later by the authors themselves in ref.\cite{7}’s erratum. This change caused large effects in process $b \to sg$ which can be seen in the present paper. See also a recent work by Ciuchini et al.\cite{8}. Furthermore, only two four-quark operators were considered there, while the other four were neglected. Unfortunately, the missing four operators are responsible for contributing to the matrix elements of processes $b \to sq\overline{q}$ (q=uds) and $b\overline{q} \to s\overline{q}$. Although their coefficients of matching at $M_W$ can be transferred to the operator $O_7$ of ref.\cite{3} when running, these six four-quark operators will be mixed together, so the missing four will reappear with

\footnote{Note this operator $O_7$ is not the one that we defined in this paper}
nonzero coefficients. This is similar to the $b \rightarrow s\gamma$ case.\footnote{See discussions in ref.\cite{7,9}.}

Since charmless decays of $B$ meson caused a great interest in studying CP violation, and the present experimental limit falls in with the theoretical predictions of the standard model, a more accurate calculation is needed to reduce theoretical uncertainties. The aim of the present paper is to present an improved leading log QCD corrected result.

2 Matching at $M_W$

The basic effective field theory idea is by now quite well established\cite{10,11}. In our particular case of Minimal Standard Model, we integrate out the top quark and the weak W bosons at $\mu = M_W$ scale, generating an effective five-quark theory. By using the renormalization group equation, we run the effective field theory down to b-quark scale to calculate QCD corrections to charmless $b$ decay.

After applying the full QCD equations of motion\cite{12}, a complete set of operators relevant for charmless $B$ decays can be chosen to be:

\begin{align*}
O_1 &= (\bar{\tau}_L\gamma^\mu b_L\alpha)(\bar{\sigma}\gamma^\mu c_L\beta), & O_2 &= (\bar{\tau}_L\gamma^\mu b_L\alpha)(\bar{\sigma}\gamma^\mu c_L\beta), \\
O_3 &= (\bar{\tau}_L\gamma^\mu b_L\alpha)\sum_q(\bar{q}\gamma^\mu q_L\beta), & O_4 &= (\bar{\tau}_L\gamma^\mu b_L\alpha)\sum_q(\bar{q}\gamma^\mu q_L\alpha), \\
O_5 &= (\bar{\tau}_L\gamma^\mu b_L\alpha)\sum_q(\bar{q}\gamma^\mu q_R\beta), & O_6 &= (\bar{\tau}_L\gamma^\mu b_L\alpha)\sum_q(\bar{q}\gamma^\mu q_R\alpha), \\
O_7 &= (g_3/16\pi^2)m_b\bar{\sigma}\gamma^\mu T^a_{R\alpha}\bar{G}^a_{\mu\nu}.
\end{align*}

The covariant derivative is defined as

$$D_\mu = \partial_\mu - i\mu^\gamma g_3 X^a C^a_\mu,$$

with $g_3$ denoting the QCD coupling constant.

Then we can write down our effective Hamiltonian as

$$\mathcal{H}_{eff} = 2\sqrt{2}G_F V_{tb} V_{ts}^* \sum_i C_i(\mu)O_i(\mu).$$

\footnote{See discussions in ref.\cite{7,9}.}
One can find the coefficients of operators at $\mu = M_W$ scale by integrating out the weak gauge bosons and would-be Goldstone bosons at this scale. Since in standard model, there is no tree level flavor changing neutral current, all charmless $b \to s$ processes are through loop diagrams. They are QCD induced processes. Even if no radiative QCD corrections are performed, they are already order of $O(\alpha_s)$. The decay amplitudes of four-quark processes are proportional to $[f_1 \alpha_s \log(M_W/m_b)^2 + f_2 \alpha_s]$. The leading logarithmic QCD corrections to them are of $O([\alpha_s \log(M_W/m_b)^2]^n)$ with $n \geq 2$. They are summed by the one-loop renormalization group equation in the leading log approximation. So this approximation will include the leading log terms of all loops but miss the $f_2 \alpha_s$ terms of one-loop diagrams. Although this is usually considered as part of the next-to-leading log effect, it is actually part of the one-loop result. We will use the complete one-loop result (including $f_2 \alpha_s$) for four-quark operators in matching at $M_W$, so that we can get a result from both the complete one-loop contribution and summation of leading log contribution. This is an improvement in the perturbative calculation at least for the QCD-induced four-quark processes. The matching diagrams for four-quark operators are displayed in Fig.1 (There should also be three additional symmetric four quark diagrams at each side of the first equation in Fig.1.). Matching diagrams for operator $O_7$ are very similar to $b \to s\gamma$ calculations. Neglecting small terms proportional to $m_c^2$ or $m_u^2$ in these matching conditions, and using $V_{cb}V_{cs}^* = -V_{tb}V_{ts}^*$, one finds the following coefficients of the operators:

\begin{align*}
C_1(M_W) &= \frac{11 \alpha_s(M_W)}{8\pi}, \\
C_2(M_W) &= 1 - \frac{11 \alpha_s(M_W)}{24\pi}, \\
C_5(M_W) &= C_6(M_W) = \frac{\alpha_s(M_W)}{24\pi} F_1(\delta), \\
C_4(M_W) &= C_7(M_W) = -\frac{\alpha_s(M_W)}{8\pi} F_1(\delta) \\
C_7(M_W) &= \frac{-\frac{1}{8} + \frac{5}{8} \delta + \frac{1}{2} \delta^2}{(1 - \delta)^3} + \frac{3\delta^2}{(1 - \delta)^4} \log \delta
\end{align*}

with $\delta = M_W^2/m_t^2$. 

\begin{align*}
F_1(\delta) &= \frac{2}{3} + \frac{-\frac{1}{12} - \frac{11}{12} \delta + \frac{3}{2} \delta^2}{(1 - \delta)^3} + \frac{-\frac{3}{2} \delta^2 + \frac{5}{3} \delta^3 - \frac{2}{3} \delta^4}{(1 - \delta)^4} \log \delta
\end{align*}
Table 1: Numerical results for coefficients of operators $C_i(m_b)$ with $\alpha_s(m_Z) = 0.117$.

| $m_{top}$ (GeV) | $C_3(m_b)$ | $C_4(m_b)$ | $C_5(m_b)$ | $C_6(m_b)$ | $C_7(m_b)$ |
|----------------|------------|------------|------------|------------|------------|
| 100            | 0.012      | -0.027     | 0.008      | -0.034     | -0.158     |
| 140            | 0.012      | -0.028     | 0.008      | -0.035     | -0.170     |
| 180            | 0.013      | -0.028     | 0.008      | -0.035     | -0.177     |
| 220            | 0.013      | -0.028     | 0.008      | -0.036     | -0.182     |
| 260            | 0.013      | -0.029     | 0.008      | -0.036     | -0.185     |
| 300            | 0.013      | -0.029     | 0.008      | -0.036     | -0.187     |

3 Renormalization group running from $M_W$ to $m_b$

We then use the renormalization group equation satisfied by the coefficient functions $C_i(\mu)$, to continue running the coefficients of operators from $\mu = M_W$ to $\mu = m_b$.

$$\mu \frac{d}{d\mu} C_i(\mu) = \sum_j (\gamma^T)_{ij} C_j(\mu),$$

(5)

Where $\gamma_{ij}$s are anomalous dimensions of operators. These anomalous dimensions have been calculated by many authors for the process $b \rightarrow s\gamma$. Only recently it is completely solved by Ciuchini et al.\[15\], and their calculation is confirmed by Cella et al.\[16\].

$$\gamma = \begin{pmatrix}
-1 & 3 & 0 & 0 & 0 & 0 & 3/2 \\
3 & -1 & -1/9 & 1/3 & -1/9 & 1/3 & 38/27 \\
0 & 0 & -11/9 & 11/3 & -2/9 & 2/3 & 557/54 \\
0 & 0 & 22/9 & 2/3 & -5/9 & 5/3 & 271/27 \\
0 & 0 & 0 & 0 & 1 & -3 & -37/6 \\
0 & 0 & -5/9 & 5/3 & -5/9 & -19/3 & -673/54 \\
0 & 0 & 0 & 0 & 0 & 0 & 14/3
\end{pmatrix} \frac{g_3^2}{8\pi^2}$$

(6)

In the leading order of $g_3$, the solution to eqn.\(6\) in matrix notation is given by

$$C(\mu_2) = \left[ \exp \int_{g_3(\mu_1)}^{g_3(\mu_2)} dg \frac{\gamma^T(g)}{\beta(g)} \right] C(\mu_1).$$

(7)
After insertion of anomalous dimension matrix (6), we get the coefficients of operators at \( \mu = m_b \) scale. Here we made use of \( M_W = 80.22 \text{GeV} \), \( m_b = 4.9 \text{GeV} \), \( \alpha_s(m_Z) = 0.117 \) [17]. Table 1 shows the numerical values of coefficients of operators \( O_i \), with different input of top quark mass. In this table, coefficients of four-quark operators \( O_{3,4,5,6} \) have very little dependence on the top quark mass, while \( C_7(m_b) \) is getting bigger when top mass increases. \( C_2(m_b) = 1.077 \). Its coefficient does not vary with top mass at all, because operator \( O_2 \) is generated from \( W \) exchange diagram and does not mix with top quark loop induced operators.

### 4 The charmless b decay rate

There are four types of nonleptonic charmless decays considered here. Their inclusive decay widths are given by the sum of operators \( O_i \) in our effective field theory.

(1) Operator \( O_5 \) and \( O_7 \) in our operator basis contribute to process \( b \to s q \) [15]. The decay width is,

\[
\Gamma_{b \to s q} = \frac{8\alpha_s}{\pi} \left| C_7^{\text{eff}}(m_b) \right|^2 \Gamma_0, \tag{8}
\]

where

\[
\Gamma_0 = \frac{G_F^2 m_b^5}{192\pi^3} |V_{ts}V_{tb}|^2, \quad C_7^{\text{eff}}(m_b) = C_7(m_b) + C_5(m_b).
\]

(2) For process \( b \to s q\bar{q}(q \text{ denotes } u \text{ or } d \text{ quark}) \), the Feynman diagrams are displayed in Fig.4. Operators \( O_{2,3,4,5,6,7} \) contribute to this process in the effective field theory. The decay rate is,

\[
\Gamma_{b \to s q\bar{q}} = \left[ 3 \sum_{i=3}^6 \left| C_i^{\text{eff}}(m_b) \right|^2 + 2 \left\{ C_3^{\text{eff}}(m_b)C_4^{\text{eff}}(m_b) + C_5^{\text{eff}}(m_b)C_6^{\text{eff}}(m_b) \right\} + \frac{8\alpha_s}{3\pi} \left\{ C_4^{\text{eff}}(m_b) + C_6^{\text{eff}}(m_b) \right\}C_7(m_b) \right] \Gamma_0, \tag{9}
\]

where

\[
C_i^{\text{eff}}(m_b) = C_i(m_b) - \frac{\alpha_s}{36\pi} \log \frac{m_c^2}{m_b^2} C_2(m_b), \quad i = 3, 5, 
\]
\[ C_{i}^{\text{eff}}(m_b) = C_i(m_b) + \frac{\alpha_s}{12\pi} \log \frac{m_c^2}{m_b^2} C_2(m_b), \quad i = 4, 6, \]

with \( m_c = 1.5 GeV \). Here we included all terms proportional to \( \alpha_s \), to make a complete one-loop \( O(\alpha_s) \) result.

(3) For process \( b \to ss\bar{s} \), there are additional diagrams concerning momentum exchange of two \( s \) quarks. The decay width is,

\[
\Gamma_{b \to s s \bar{s}} = \left[ 4 \left| C_3^{\text{eff}}(m_b) + C_4^{\text{eff}}(m_b) \right|^2 + 3 \left| C_5^{\text{eff}}(m_b) \right|^2 + 3 \left| C_6^{\text{eff}}(m_b) \right|^2 \right. \\
\left. + 2C_5^{\text{eff}}(m_b)C_6^{\text{eff}}(m_b) + \frac{8\alpha_s}{3\pi} \left\{ C_3^{\text{eff}}(m_b) + C_4^{\text{eff}}(m_b) + C_6^{\text{eff}}(m_b) \right\} C_7(m_b) \right] \Gamma_0. \tag{10}
\]

(4) The Feynman diagrams contributing to the nonspectator process \( b\bar{q} \to s\bar{q} \) are also seen in Fig. 2. The decay width is,

\[
\Gamma_{b\bar{q} \to s\bar{q}} = 32\pi^2 \left| C_5^{\text{eff}}(m_b) + 3C_6^{\text{eff}}(m_b) \right|^2 \left( \frac{f_B}{m_b} \right)^2 \Gamma_0. \tag{11}
\]

Where the B decay constant is \( f_B = 200 MeV \).

In order to find the branching ratios of these kinds of charmless B decays, the semileptonic decay of B is used.

\[
BR(\bar{B} \to X_s \text{ no charm}) = \frac{\Gamma(b \to s \text{ no charm})}{\Gamma(b \to ce\bar{\nu})} BR(\bar{B} \to X_c e\bar{\nu}), \tag{12}
\]

where

\[
\Gamma(b \to ce\bar{\nu}) \simeq g(m_c/m_b) \left( 1 - \frac{2\alpha_s(m_b)}{3\pi} f(m_c/m_b) \right) \Gamma_0, \tag{13}
\]

with \( g(m_c/m_b) \simeq 0.447 \) and \( f(m_c/m_b) \simeq 2.4 \) correspond to the phase space factor and the one-loop QCD correction to the semileptonic decay, respectively [18]. Here we use experimental result \( Br(\bar{B} \to X_c e\bar{\nu}) = 11\%[17] \).

The branching ratios for different processes are given in Fig. 3. In comparison to the previous QCD-corrected results of ref. [3], the decay rate of process \( b \to sg \) is strongly enhanced by QCD corrections rather than severely suppressed. The differences are of 2-3 orders. The reason for such a large difference, is obvious, for the authors of ref. [3] used the wrong anomalous dimensions of
ref.\[7\], which has already been corrected in its erratum. If right values are taken, the differences are not so large. Our result for \(b \to sg\) is consistent with the recent work by Ciuchini et al.\[8\] in leading log approximation. The part of next-to-leading log contributions we considered in this paper is not essential to this process. For process \(b \to sq\), \(b \to ss\) and \(b \to s\), after we include all the dimension-6 four-quark operators, the complete leading log QCD corrections make the branching ratios suppressed by 15\%, 19\%, and 25\% respectively (comparing to the result of ref.\[3\]), when \(m_{top} = 175\text{GeV}\). Furthermore, we also include \(O(\alpha_s)\) terms in the matching condition, which is the first term of next-to-leading log effects. The branching ratios for these processes are slightly enhanced. Comparing with ref.\[3\], the total effects are 10\%, 14\%, and 18\% suppression of the branching ratios, respectively.

The branching ratios obtained by W.S. Hou\[2\] without QCD running from \(M_W\) to \(m_b\) are also given at Fig. 4. Comparison with this result shows that our decay rate of process \(b \to sg\) is strongly enhanced, e.g. a factor of 3.4 at \(m_{top} = 175\text{GeV}\), this is almost the same as \(b \to s\) decay\[13, 15\]. While for process \(b \to sq\), \(b \to ss\) and \(b \to s\), the branching ratios are only increased by 49\%, 34\% and 17\%, respectively.

5 Conclusion

In conclusion, we have given the full leading log QCD corrections to penguin induced charmless \(b \to s\) processes together with some next-leading log corrections. The branching ratios of processes \(b \to sq\) (\(q=uds\)), and \(b \to s\) are found to change slowly with the top quark mass. For process \(b \to sg\), the decay rate is getting larger when top mass increases. The result also shows that for \(b \to sg\) process (gluon on shell), QCD corrections are as important as in the \(b \to s\) case. The QCD corrected results for four-quark processes are all enhanced, the amount of the increase vary notably for different processes.

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Figure Captions

Fig. 1: Matching conditions at $\mu = M_W$ for Green functions in the full standard model (left hand side) and effective field theory below W scale (right hand side) with the heavy dots denoting high dimension operators.

Fig. 2: Feynman diagrams contributing to $b \to sq\bar{q}$, $b\bar{q} \to s\bar{q}$ in effective field theory.

Fig. 3: QCD corrected branching ratios as function of top quark mass. From top to bottom are lines for process $b \to sg$, $b \to sq\bar{q}$, $b \to ss\bar{s}$ and $b\bar{q} \to s\bar{q}$.

Fig. 4: Branching ratios without QCD radiative corrections, obtained by W.S. Hou as function of top quark mass. From top to bottom are lines for process $b \to sq\bar{q}$, $b \to ss\bar{s}$, $b\bar{q} \to s\bar{q}$ and $b \to sg$. 

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