\( \theta \)-instantons in SU(2) Higgs theory

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Abstract

We consider topology changing processes in SU(2)--Higgs theory. In the Standard Model of particle physics they are accompanied by baryon-- and lepton--number non--conservation. At fixed energy and multiplicity of initial state, these processes are described by classical \( \theta \)--instanton solutions. We describe these solutions and calculate the suppression exponents for the probabilities of the topology changing transitions at relatively low energies.

1 Introduction and Summary

Tunneling transitions between topologically distinct vacua in the electroweak theory are accompanied by baryon and lepton number violation \([1]\), which has important implications in particle physics and cosmology. At zero energy, such processes are described by instantons \([2]\), and their rate is suppressed by an exponentially small factor \(\exp(-2S(I)) \sim 10^{-170}\), where \(S(I) = 8\pi^2/g^2\) is the instanton action. It was found in Refs. \([3, 4]\) that at relatively low energies, the inclusive cross sections of the topology changing processes in particle collisions can be calculated by perturbative expansion about the instanton background, and that these cross sections grow rapidly with energy. In particular, the leading order instanton cross section becomes unsuppressed at energies comparable to the sphaleron energy \(E_{\text{sph}} \sim 10\text{TeV}\). However, further studies \([5, 6, 7, 8]\) (see Refs. \([9, 10]\) for reviews) revealed that the actual expansion parameter of the perturbation theory about the instanton is \((E/E_{\text{sph}})^{2/3}\), so the most interesting region \(E \gtrsim E_{\text{sph}}\) is unreachable for analytic methods. The results of the perturbative analysis at \(E \ll E_{\text{sph}}\) suggested the exponential form for the cross section,

\[
\sigma_0(E) \propto \exp \left\{ \frac{1}{g^2} F_0(E/E_{\text{sph}}) \right\},
\]

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where \( g \) is the weak coupling constant, while the suppression exponent \( F_0(E/E_{\text{sph}}) \) is negative and grows with energy. To the leading order in perturbation theory about the instanton
\[
F_0 = -16\pi^2 + 3 \left[ \frac{3g^4E^4}{8\pi^2v^4} \right]^{1/3} + O(g^2E^2/v^2),
\]
where \( v \) is the Higgs vacuum expectation value.

The exponential form (1) implies that there might exist some semiclassical–type procedure which would allow one to calculate the suppression exponent at all energies. The main difficulty with such a procedure resides in the fact that the initial state of the process contains two highly energetic particles and cannot be described semiclassically. A way out of this difficulty was suggested in Ref. [11]. The main idea is to study the instanton-like processes with parametrically large energy and initial-state particle number, \( E = \tilde{E}/g^2, N = \tilde{N}/g^2 \). One can then justify the use of the semiclassical methods for the semi–inclusive cross section \( \sigma(E,N) \): the multiparticle topology changing processes in the weak coupling limit \( g^2 \to 0 \) are described by \( \theta \)–instantons, solutions to a certain classical boundary value problem, and the multiparticle suppression exponent \( F(\tilde{E},\tilde{N}) \) is calculated as a value of an appropriately modified action functional on these solutions [11]. We review the boundary value problem for the \( \theta \)–instantons in Section 2.

It was argued in Refs. [12, 13] that the two-particle suppression exponent can be found as a limit of the multiparticle one,
\[
F_0(\tilde{E}) = \lim_{\tilde{N} \to 0} F(\tilde{E},\tilde{N}) .
\]
This conjecture was checked by explicit calculations in quantum mechanics in Ref. [14]. To summarize, the \( \theta \)–instanton method allows one to find, after performing the limiting procedure (2), the suppression exponent for the two-particle cross section at all energies.

It was shown in Ref. [11] that at low energies, the \( \theta \)–instanton solution can be approximated by a chain of appropriately modified instantons and anti-instantons placed at certain positions along the Euclidean time axis. Although this approximation is justified only at \( E \ll E_{\text{sph}} \), the approximate solutions give an idea of the form of \( \theta \)–instantons in the whole region \( E < E_{\text{sph}} \). In Section 3 we investigate the properties of \( \theta \)–instantons at low energies in SU(2) gauge–Higgs theory, which is a close analog of the Standard Model. We show that the chain instanton approximation gives the multiparticle suppression exponent \( F(\tilde{E},\tilde{N}) \), up to corrections of order \( O(\tilde{E}^2) \). In the limit of small number of particles, our result for \( F(\tilde{E},\tilde{N}) \) coincides with the perturbative calculations of Refs. [15, 16, 5] for \( F_0(\tilde{E}) \), and thus eq. (2) is valid indeed. We find, however, that the low–\( \tilde{N} \) expansion of the suppression exponent contains a term \( \tilde{N} \log \tilde{N} \). This term does not depend on energy and therefore the derivative \( \partial F/\partial \tilde{E} \) is regular at \( \tilde{N} = 0 \).

During the last decade, sophisticated numerical techniques of finding the \( \theta \)–instanton solutions at high energies were developed [17, 18, 19, 20]. The approximate \( \theta \)–instanton solutions which we find in this paper may serve as a cross check of numerical methods. In Section 3 we perform this check in the SU(2)–Higgs model.
and find that after extrapolating to low energies, the numerical data obtained in Refs. [20, 21] agree with our analytical results.

2 \( T/\theta \) boundary value problem

The boundary value problem for the \( \theta \)-instanton involves two Lagrange multipliers, \( T \) and \( \theta \), which enable one to fix energy and initial particle number. The problem is naturally formulated on the contour ABCD in complex time plane (see fig. 1), with imaginary part of the initial time equal to \( T/2 \). In the internal points of the contour, the \( \theta \)-instanton fields, which we denote collectively by \( \varphi(x, t) \), satisfy the classical field equations,

\[
\frac{\delta S}{\delta \varphi} = 0 .
\]

(3a)

In the asymptotic future (region D of the contour) the fields are real and describe the evolution of the system after transition,

\[
\text{Im } \varphi(x, t) \bigg|_D = 0 .
\]

(3b)

In the asymptotic past (part A of the contour) the fields have to be in linear regime and satisfy the linearized field equations. Thus

\[
\varphi(x, t) \bigg|_A = \frac{1}{(2\pi)^{3/2}} \int \frac{dk}{\sqrt{2\omega_k}} \left[ f_k e^{-i\omega_k (t-iT/2)+ikx} + g_k^* e^{i\omega_k (t-iT/2)-ikx} \right] .
\]

The boundary conditions in the asymptotic past relate the positive and negative frequency components of the solution,

\[
f_k = e^{-\theta} g_k .
\]

(3c)

Note that at \( \theta \to +\infty \), the boundary conditions (3c) transform into the Feynman ones, and the corresponding \( \theta \)-instanton solution describes a transition from a state with vanishingly small number of particles. At finite \( \theta \), equation (3c) may be viewed as a deformation of the Feynman boundary conditions. Note that eq. (3c) implies
that the $\theta$–instanton solution is necessarily complex, except for the special periodic instanton [22] case $\theta = 0$.

By solving eqs. (3a)–(3c), one finds the $\theta$–instanton solution for given values of the Lagrange multipliers $T$ and $\theta$. The suppression exponent is the value of the following functional evaluated on this solution,

$$\frac{1}{g^2} F(E, N) = ET + N\theta - 2\text{Im} S_{ABCD} ,$$

(4)

where $S_{ABCD}$ is the action calculated along the contour ABCD. The extremization of the suppression exponent with respect to the Lagrange multipliers gives equations determining the values of $T$ and $\theta$:

$$E = 2\frac{\partial}{\partial T} \text{Im} S_{ABCD} ,$$

(5a)

$$N = 2\frac{\partial}{\partial \theta} \text{Im} S_{ABCD} .$$

(5b)

One can show that the values of $E$ and $N$ are equal to the classical energy and initial particle number calculated on the $\theta$–instanton solution,

$$E = \int dk \, \omega_k f_k g_k ,$$

$$N = \int dk \, f_k g_k .$$

3 $\theta$–instanton at low energy

We consider SU(2) gauge theory with a doublet Higgs field, which coincides with the bosonic sector of the standard electroweak theory with the weak mixing angle set equal to zero. The model possesses constrained instantons and anti–instantons of size $\rho$, which in a singular gauge have the following form [2, 23]:

$$A^a_{\mu(I)}(x, \tau) = \frac{2\rho^2}{g} \tilde{\eta}^a_{\mu\nu} x^\nu \frac{1}{x^2(x^2 + \rho^2)} ,$$

$$A^a_{\mu(A)}(x, \tau) = \frac{2\rho^2}{g} \eta^a_{\mu\nu} x^\nu \frac{1}{x^2(x^2 + \rho^2)} ,$$

where $x^0 \equiv \tau = it$ is the Euclidean time. We construct the $\theta$–instanton solution by placing instantons and anti–instantons along the Euclidean time axis, as shown in figure 2a, and suppressing them by factors $e^{-\theta|n|}$:

$$A^a_{\mu(\theta)}(x, \tau) = \sum_{n=-\infty}^{+\infty} e^{-\theta|n|} \left[ A^a_{\mu(I)}(x, \tau + T_1 + nT) + A^a_{\mu(A)}(x, \tau - T_1 + nT) \right] .$$

(6)

Hence, the (suppressed) instantons sit at $\text{Im } t = -\tau = T_1 + nT$, while anti–instantons are placed at $\text{Im } t = -\tau = -T_1 + nT$, $n = 0, \pm 1, \pm 2\ldots$. We will see that at low
energies the instanton size is small compared to the instanton separation, \(\rho \ll T\), \(\rho \ll T_1\), and \(T, T_1 \ll 1/(gv)\). This is the basic reason for approximating the solution as a sum of the instanton and anti–instanton fields.

Let us show that the solution (6) satisfies the field equations (3a) along the contour ABCD and boundary conditions (3c) and (3b) in the asymptotic regions A and D, respectively. It is convenient to work with the Fourier transform of the instanton field,

\[
A^a(I)(\k, \tau) = \int \frac{d\x}{(2\pi)^{3/2}} e^{i\k \cdot \x} A^a(I)(\x, \tau)
\]

with

\[
A^a_0(I)(\k, \tau) = -\frac{2i\rho^2}{g} \frac{\partial}{\partial k_a} \Phi(\k, \tau), \tag{7a}
\]

\[
A^a_1(I)(\k, \tau) = -\frac{2\rho^2}{g} \left(\delta_{ia} \frac{\partial}{\partial k_a} + i\epsilon_{ija} \frac{\partial}{\partial k_j}\right) \Phi(\k, \tau), \tag{7b}
\]

where \(k = |\k|\) and

\[
\Phi(\k, \tau) = \frac{\sqrt{2\pi}}{4|\tau|} e^{-k|\tau|} + O(\rho^2/\tau^3). \tag{7c}
\]

Along the contour ABCD of fig. 1 the approximate solution (6) has the form of the superposition of the “main instanton” at \(\text{Im } t = -\tau = T_1\), and small linearized asymptotics of other instantons and anti–instantons. Since outside the instanton core, the instanton field has low momenta, \(k \lesssim 1/T\) (see eq. (7c)), its interaction with the core of another instanton is suppressed by powers of \(\rho^2/T^2\). Therefore, up to corrections of order \(O(\rho^2/T^2)\) the linear combination (6) satisfies the field equations (3a).
In the asymptotic regions $A$ and $D$ one is able to sum up the contributions from all instantons and anti–instantons. After performing the transformation to the gauge $A_0^a = 0$, we obtain:

$$A_i^{(\theta)}(k, t)\bigg|_A = \frac{\rho^2 \sqrt{2\pi}}{g[1 - e^{-kT - \theta}]} \left[ e^{ikt} + e^{-ikt - kT - \theta} \right] \times \left\{ \sinh(kT_1)(\delta_{ia} - \frac{k_ik_a}{k^2}) + i\epsilon_{ija} \frac{k_j}{k} \cosh(kT_1) \right\}, \quad (8a)$$

$$A_i^{(\theta)}(k, t)\bigg|_D = -\frac{\rho^2 \sqrt{2\pi}}{g[1 - e^{-kT - \theta}]} \times \left\{ (\delta_{ia} - \frac{k_ik_a}{k^2}) \left[ e^{-kT_1} - e^{kT_1 - kT - \theta} \right] - i\epsilon_{ija} \frac{k_j}{k} \left[ e^{-kT_1} + e^{kT_1 - kT - \theta} \right] \right\}. \quad (8b)$$

We see that the boundary conditions (3b) and (3c) are satisfied indeed.

To calculate the suppression exponent (4) on the approximate solution (6), it is instructive to consider first the instanton—anti–instanton configuration and evaluate the imaginary part of its action along the contour ABCD (see fig. 2b). Since on this contour, anti–instanton is complex conjugate to instanton, the instanton—anti–instanton configuration is $C$–symmetric, and we have

$$2\text{Im} S^{(IA)}_{ABCD} = S^{(IA)}_{ABCD} - (S^{(IA)}_{ABCD})^* = S^{(IA)}_{ABCD} + S^{(IA)}_{D'C'B'A'} = S^{(IA)}_E, \quad (9)$$

where $S^{(IA)}_E$ denotes the Euclidean action of the instanton–anti–instanton pair. Note that the (anti)instanton singularities marked in fig. 2b by dashed lines do not permit one to move the contour ABCD to the regions $\tau \to \pm \infty$, where (anti)instanton field is equal to zero. The quantity $S^{(IA)}_E$ naturally divides into the sum of the instanton and anti–instanton actions,

$$S^{(I)} = S^{(A)} = \frac{8\pi^2}{g^2} + \pi^2 \rho^2 v^2, \quad (10)$$

where the second term represents the Higgs field contribution, and the interaction action, which was calculated in Refs. [24, 25, 26],

$$S^{(IA)}_{\text{int}} = -\frac{96\pi^2 \rho^4}{g^2 l^4}. \quad (11)$$

Here $l = 2T_1$ is the distance between the instanton and anti–instanton, and corrections involving powers of $\rho^2/l^2$ in (10) and (11) are omitted.

Finally, collecting formulas (10) and (11), we have

$$2\text{Im} S^{(IA)}_{ABCD} = \frac{16\pi^2}{g^2} + 2\pi^2 \rho^2 v^2 - \frac{96\pi^2 \rho^4}{l^4} + O(\rho^6/l^6). \quad (12)$$

The action of the $\theta$–instanton is the sum of the “main” instanton action (10) and the interaction terms, which, up to corrections of order $O(\rho^6/l^6)$, are quadratic with respect to the (anti)instanton fields. The instanton–instanton interaction is
of order $O(\rho^6/l^6)$ (see Refs. 24, 25, 26) and thus does not contribute into the action to the order we study. Thus, the interaction action of the $\theta$–instanton is the sum of interactions of different instanton—anti–instanton pairs. It is clear that if both instanton and anti–instanton are situated above (or below) the main instanton, the contour ABCD may be moved to the region $\tau \to \pm\infty$ without crossing the singularities, and therefore the interaction action of such pair is equal to zero. The above argument with C–conjugation shows that even if the instanton and anti–instanton are situated at different sides of the main instanton, their interaction action equals zero. Therefore, the only non-zero terms in the action emerge from the interaction of the “main” instanton with different anti–instantons. These terms were calculated above. Summing up all of them, we obtain

$$S_{\text{int}}^{(\theta)} = -\frac{96\pi^2 \rho^4}{g^2} \sum_{n=-\infty}^{+\infty} \frac{e^{-\theta|n|}}{(2T_1 + nT)^4} =$$

$$-\frac{16\pi^2 \rho^4}{g^2} \int_{0}^{\infty} \frac{dk}{1 - e^{-kT-\theta}} \left[ e^{-2kT_1} + e^{2kT_1-kT-\theta} \right], \quad \text{(13)}$$

where we have used the integral representation for the sum in last equality. Finally, eq. (14) gives the expression for the suppression exponent,

$$\frac{1}{g^2} F = E T + N \theta - \frac{16\pi^2}{g^2} \left[ \frac{1}{2} \rho^2 \right] + \frac{16\pi^2 \rho^4}{g^2} \int_{0}^{\infty} \frac{dk}{1 - e^{-kT-\theta}} \left[ e^{-2kT_1} + e^{2kT_1-kT-\theta} \right] \quad \text{(14)}$$

Note that, apart from the Lagrange multipliers $T$ and $\theta$, which are determined by equations (5a) and (5b), the solution (6) has two free parameters: the instanton size $\rho$ and the position of the “main” instanton $T_1$. The values of these parameters are to be chosen to give the extremum of the suppression exponent $F$. The extremization of (14) with respect to $T_1$ determines the ratio $t_1 \equiv T_1/T$ as a function of $\theta$. This ratio satisfies the equation

$$\int_{0}^{\infty} \frac{dq}{1 - e^{-q-\theta}} \left[ e^{2qt_1-q-\theta} - e^{-2qt_1} \right] = 0. \quad \text{(15)}$$

When $\theta = 0$ (periodic instanton case), one finds $t_1 = 1/4$, so the anti–instantons are situated exactly in the middle between the instantons. In the limiting case $\theta \to +\infty$ (corresponding to the limit $N \to 0$), one has $t_1 \to 1/2$, and instantons approach the neighbouring anti–instantons. As $\theta$ changes from 0 to $+\infty$, $t_1$ smoothly interpolates between the two limiting values, see fig. 3.

It is convenient to express the saddle point values of the quantities $\rho$, $T$ and $\theta$
in terms of two integrals:

\[ I_E(\theta) = \frac{1}{2} \int_0^\infty dq \frac{q^4}{(1 - e^{-q - \theta})^2} e^{-q - \theta} \cosh[2qt_1(\theta)], \quad (16a) \]

\[ I_N(\theta) = \frac{1}{2} \int_0^\infty dq \frac{q^3}{(1 - e^{-q - \theta})^2} e^{-q - \theta} \cosh[2qt_1(\theta)]. \quad (16b) \]

The extremization of (14) with respect to \( \rho \) gives

\[ \rho^2 = \frac{g^2 v^2 T^4}{16 I_E(\theta)}, \quad (17) \]

while equations (5a) and (5b) imply,

\[ T = \left[ \frac{4EI_E(\theta)}{\pi^2 g^2 v^4} \right]^{1/3}, \quad (18) \]

\[ \frac{N}{E^{4/3}} = I_N(\theta) \left[ \frac{2}{\pi g v^2 I_E(\theta)} \right]^{2/3}. \quad (19) \]

Equation (19) determines \( \theta \) explicitly as function of \( N/E^{4/3} \). In terms of the integrals \( I_E(\theta) \) and \( I_N(\theta) \), the suppression exponent takes the form

\[ \frac{1}{g^2} F(\theta, E) = -\frac{16\pi^2}{g^2} + \left[ \frac{E^4}{16\pi^2 g^2 v^4} \right]^{1/3} \left( 3I_E(\theta)^{1/3} + 4\theta \frac{I_N(\theta)}{I_E(\theta)^{2/3}} \right) + O(\rho^6/g^2 T^6) \quad (20) \]

Several remarks are in order:

(i) Note that the corrections to the suppression exponent calculated within the instanton chain approximation are of order \( O(\rho^6/T^6) \), where

\[ \frac{\rho^6}{T^6} = \frac{E^2 g^2}{256\pi^4 v^2 I_E(\theta)}. \quad (21) \]
As $I_E(\theta)$ is a bounded function, one has $(\rho/T)^6 \sim O(E^2 g^2/v^2)$. Thus, our approximation is indeed valid at $E^2 \ll v^2/g^2$. We note also that corrections due to the Higgs field interactions are also of order $(\rho/T)^6$.

(ii) Although we cannot solve eq. (19) analytically in the entire region $\theta \in [0; +\infty)$, we are able to find the asymptotics of large $\theta$ (small $N$). In this way we obtain at small $N$,

$$
\frac{1}{g^2} F = -\frac{16\pi^2}{g^2} + f \left\{ 3 - 5x \log x + 5x + \frac{5}{3} x^2 - \frac{35}{27} x^3 + O(x^4) \right\},
$$

where

$$
f \equiv \left[ \frac{3E^4}{8\pi^2 g^2 v^4} \right]^{1/3},
$$

and

$$
x \equiv N/f.
$$

Note that although the limit $N \to 0$ is singular, $N \log N$ term does not depend on energy (at least, to the first order of the perturbation theory). Thus, the “period” $T(E, N)$ is a regular function at $N = 0$:

$$
T = \frac{4f}{E} \left\{ 1 + \frac{5}{3} x - \frac{5}{9} x^2 + \frac{70}{81} x^3 + O(x^4) \right\}.
$$

This validates the method of Ref. [21], where the period is extrapolated to the region of small $N$ by polynomials. Rescaled period $T$ as function of $x = N/f$ at fixed energy, together with its linear asymptotics at small $N$ is shown in fig. 4.

(iii) In the limiting case $N = 0$ (two–particle collisions) and $\theta = 0$ (periodic instanton) the leading terms of order $(g^2E)^{4/3}$ in the suppression exponent were found in Refs. [5] and [22], respectively. From eqs. (20), (22) we obtain

$$
\frac{1}{g^2} F \bigg|_{\theta=0} = -\frac{16\pi^2}{g^2} + 3 \left[ \frac{\pi^2 E^4}{g^2 v^4} \right]^{1/3} + O(E^2/v^2),
$$

$$
\frac{1}{g^2} F \bigg|_{N=0} = -\frac{16\pi^2}{g^2} + 3 \left[ \frac{3E^4}{8\pi^2 g^2 v^4} \right]^{1/3} + O(E^2/v^2).
$$
These coincide with the results of Refs. [5, 22].

(iv) The $T/\theta$ boundary value problem summarized in Section 2 may be solved numerically. In Refs. [20, 21], the numerical study has been performed in the SU(2) Higgs model, so we can directly check the numerical data of Refs. [20, 21] against our analytical results. Since the latter apply to relatively low energies only, this check involves extrapolation of the numerical results to $E \rightarrow 0$. To this end, let us notice that according to eqs. (19), (20) the quantity

$$W \equiv \frac{(F + 16\pi^2/g^2)}{f}$$

depends only on the combination $x \propto N/E^{4/3}$, up to corrections $O(g^2E^2/v^2)$. In figure 5 we have plotted functions $W(x)$ extracted from numerical data of Refs. [20, 21] for different values of energy, and our analytical low energy prediction. We see that though for energies close to the sphaleron energy ($Eg\sqrt{2}/(4\pi v) = 2.4$) the deviation between numerical and analytical results is fairly large, the extrapolation of the numerical results to zero energy agrees with our analytical result.

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