WHICH THERMAL PHYSICS FOR GRAVITATIONALLY UNSTABLE MEDIA?

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ABSTRACT. We remind that the assumptions almost universally adopted among astronomers concerning the physics to use to describe rarefied cosmic gases remain often without justifications, mainly because the long range of gravitation invalidates the use of classical thermal physics. In turn, without sufficiently good local thermal equilibrium, macroscopic quantities, such as temperature and pressure, are not defined and the fundamental assumption that locally the medium is smoothed by “molecular chaos” to justify the use of differential equations is not granted. The highly inhomogeneous fractal state of the interstellar gas is probably a plain symptom of the large discrepancy between the available theoretical tools, predicting local homogeneity after a few sound crossing times, and reality. Such fundamental problems begin to occur in optically thin media such as stellar atmospheres, but become exacerbated in the interstellar medium, in cooling flows, and in the post-recombination gas, particularly when gravitation becomes energetically dominant, i.e., when the medium is Jeans unstable.

1. Introduction
The purpose of this paper is to remind that the over century old assumptions of classical thermodynamics and statistical physics have never been properly adapted to gravitating systems. This problem is central to astrophysics, because concepts such as temperature or pressure are constantly used throughout observational and theoretical astronomy, with often a poor perception of their limits.

Chemists, physicists or engineers use thermodynamics (i.e., thermostatics) and statistical mechanics everyday with often great success. They must be aware of the limits of these theoretical tools, when dealing with out-of-equilibrium systems such as convective fluid layers or biological systems. These systems often require new tools, to adapt from case to case. Indeed, the variety of possible systems out of thermal equilibrium is incomparably larger than the variety of systems in thermal equilibrium. Any system displaying long range correlations should be regarded as improper to be described with thermodynamics on the global scale. Much efforts have been and are currently made in statistical mechanics to describe and understand “phase transitions”, the meaning of which is often merely that such systems are in non-thermal states displaying long range correlations.

On the other hand astrophysicists were accustomed long ago (e.g., Kelvin in the late 19th century) to apply thermal physics with success in planetary and stellar conditions. The assumption of local thermodynamic equilibrium (LTE) may be very good inside solar type stars, thus the thermal physics applies well and leads to successful progresses. However, workers in stellar atmospheres realized early that when the photon mean free
path is larger than the density scale-length, physics becomes non-local and non-LTE corrections must be applied.

By lack of better tools, and by natural extension, the known thermal physics was applied by famous scientists such as Jeans, Eddington, Strömgren and Spitzer to interstellar gas. Often such pioneer attempts were considered by their authors as exploratory work, where LTE is assumed as a very simplified working hypothesis. However, since better theoretical tools continued to be lacking, it became admitted among astronomers to use these assumptions or variants of them (detailed balance), with decreasing awareness of their limits. Today these strong hypotheses are very often adopted without any cautionary remarks. A rereading of classical textbooks (e.g., Landau & Lifchitz 1966, “Statistical Physics”, in particular Chap. I) may be refreshing on this point. Much of the warnings reminded here are already stated there.

2. Thermostatics and Entropy Fluxes in Astrophysics

Basically, what is sometimes called the “fourth principle of thermodynamics” (Landsberg 1984), i.e., that macroscopic quantities (such as energy) are either intensive or extensive (additive), proportional to the 0th or 1st power of the volume \( V \), fails in systems with long range interactions. Thus the domains of astrophysics where the gravitational instability develops (i.e., when gravitational energy exceeds pressure) are particularly concerned, because the gravitational energy grows as \( G\rho_0^2 V^{5/3} \) in uniform media. In such situations long range correlations and matter motions induced by gravity propagate with the dynamical time-scale \( \sim 1/\sqrt{G\rho_0} \), comparable to the sound crossing time: the medium, not even in mechanical equilibrium, is then unable to thermalize, to relax faster than dynamics. There is then no justification to continue to apply thermostatics.

Perhaps not coincidental, such states are precisely related to the least understood situations in astrophysics: star formation, galaxy formation, and first structure formation after recombination. All involve some kind of gravitational collapse developing singularities, and various back reactions from the small scale physics stopping or slowing down the growth of singularities.

According to these remarks, the structures occurring in astrophysics could be classified in three categories.

1. The ones dominated by microscopic physics, such as small scale objects from atoms, molecules, grains up to planetary or stellar atmospheres which have little self-gravity. In such systems thermodynamics applies often well because the dominant interactions are short range.

2. The systems in which thermodynamics competes with gravity, such as stars, the cold interstellar medium, or the gas after radiation-matter decoupling. Here thermodynamics is sometimes applicable when LTE is a good approximation (when the medium is optically thick), but often not.

3. The systems dominated by gravity such as the planetary systems, stellar clusters, well formed galaxies, and large scale structures. Here thermodynamics is irrelevant at a global scale since dynamical relaxation is much longer than the system age.

The general growth of structures and correlations in the Universe is opposite to an approach to a thermal equilibrium. It requires a growth of phase space volume faster
than the entropy production by microscopic phenomena. For a definition of entropy and limits, see the Appendix. In other words, the general growth of structures in astrophysics requires states sufficiently out of thermal equilibrium, which is actually what makes fields like astrophysics or biology interesting. The steady production in time of ever increasingly complex structures, from large scale cosmological structures down to galaxies, stars and biological systems, follows from the negentropy (minus the entropy) generated by the universal expansion and cascading down the scales. In other words, as for biology, out of equilibrium systems characterize astrophysics.

If we take a close example of structural growth, complex biological structures are not produced by the Sun energy input, as often stated, but by the entropy flow: fortunately every erg received by the Earth from the Sun is re-radiated in the infrared, otherwise the Earth temperature would become rapidly uncomfortable! But since each UV photon is degraded into several IR photons, which increases the global entropy by increasing the number of degrees of freedom, works can be produced. In this non-equilibrium process living systems succeed to capture negentropy, allowing them to complexify.

3. Gravitationally Unstable Media

An infinite uniform medium raises fundamental difficulties that were apparent immediately after the discovery of the law of gravitation, as discussed in a famous letter exchange in 1692–1693 between Richard Bentley and Isaac Newton (Newton 1779). By symmetry an infinite uniform medium cancels exactly the attraction force at any point, but this is in fact a subtraction of infinities, the medium on the left of a point attracts it with an infinite force, as well as the medium on the right. In such situations the slightest perturbation to the medium at large distances may implies large residual forces. Therefore Bentley and Newton agreed that a God was required to constantly “stabilize” the Universe, seen then as necessarily eternal, infinite and uniform.

Such an intuitive notion of (in)stability was refined by Jeans (1902) with the mathematical description of the concept of stability (much developed earlier by Poincaré): Is a slight perturbation growing in time or not? If yes, how fast? This led to the condition of linear gravitational instability in uniform media with gas pressure, giving a scale, the Jeans length or mass, above which the medium is gravitationally unstable. Real physical systems are not infinite, therefore the Jeans criterion allows to distinguish between a box of gas in the laboratory, or in space, the size of which is practically infinite for thermodynamic applications, but perhaps not for the Jeans length. If the box size is much larger than the Jeans length the Bentley-Newton paradox applies, which in modern terms means that the medium is unstable, and cannot remain as such.

The linear stability analysis is rapidly unable to describe the non-linear phase of the gravitational instability due to the strongly chaotic nature of the problem. Some further description of the non-linear phase is necessary. The still most widespread mental model of the non-linear phase of a gravitational instability, alive since several decades, is the rapid and synchronous “crystallization” of the unstable uniform medium into a myriad of blobs of similar sizes given by the initial Jeans length. Subsequently the blobs are imagined to collapse synchronously nearly as if they were isolated spherical blobs.

In fact numerical simulations of the non-linear phases in cosmological and galac-
tic contexts show a very different typical behaviour. The unstable medium develops pancakes, filaments and clusters over time-scales much longer than the initial spherical collapse time. Filamentary structuration proceeds, and clumps are rarely isolated for long. Interactions, collisions, merging, but also disruptions and evaporation of clumps are frequent. Contrary to matter, voids do tend toward spherical shapes. The whole structure remains out of dynamical equilibrium as long as the large-scale ordered motion (the Hubble flow, the galaxy differential rotation) feeds the smaller scales. The smallest scales may dissipate energy and consume negentropy (i.e., they produce entropy). Eventually some scale-invariant order, scaling laws may emerge from such out of equilibrium non-linear processes.

One may note that pancakes, filaments and clusters do occur in the computer without including any radiative cooling. This is important because it shows that gravity alone is able to develop dense states without requiring any atomic cooling phenomena. This is related to the “negative specific heat” of gravitational systems. In general, systems with negative specific heat are thermally unstable and tend to develop spontaneously large fluctuations in temperature or density.

A nice illustration of the development and persistence of large scale fractal-like structures is provided by the shearing sheet experiments aimed at representing a small patch of a differentially rotating self-gravitating disk (Wisdom & Tremaine 1988; Toomre & Kalnajs 1991; Salo 1992, 1995). The anti-thermodynamic persistence of long range correlations must be understood by the continuous flow of order from large-scale time-dependent and correlated motion down to small-scale dissipative and chaotic dynamics. In this case gravitational instability does not lead to “collapse” as long as the system, driven by the large scale time-dependence of the boundary conditions, absorbs at small scale the negentropy by chaos and dissipative terms.

This aspect is very important to understand because usually the simple minded picture of gravitational instability considers collapsing blobs in isolation developing almost everywhere as the main outcome of gravitational instability (thus, it is often believed that stars should form in large amount in the early Universe). Here we argue that a better understanding of the non-linear regime of gravitational instability leads to the possibility that point-like collapses are not necessarily widespread, but that long range fractal order can persist as long as the largest scale boundary conditions are time-dependent. The main examples supporting this view are:

1. The fractal-like structure of galaxy and galaxy cluster distribution (e.g., Coleman & Pietronero 1992) over some range in scale, fed in our view by the universal expansion.
2. The fractal structure of the interstellar medium (Scalo 1985) even when not perturbed by star formation, is fed by galaxy differential rotation (Fleck 1981). This large-scale ordered motion cascades down the cold interstellar clouds, which tend to follow scaling laws (e.g., the size–velocity-dispersion relation, Larson 1981).

In these situations, scaling laws may persist as long as the larger scale is driving the gravitational instability. The boundary conditions being time-dependent, a steady flow of entropy can self-organize the system with a statistical quasi-steady state with scaling laws. The Kolmogorov picture of turbulence in incompressible fluids is similar, except that cosmic gases are necessarily highly compressible (since motion is trans- or
supersonic), and the force responsible to degrade the large-scale order by a cascade of “eddies” is gravity instead of molecular forces.

4. The Isothermal Gravitating Gas Sphere and Beyond

The limits of classical thermodynamics are particularly clear when using a simple model of gravitating gas as close as possible to the perfect gas box at a temperature fixed by a heat bath, used extensively in thermodynamics. To simplify the treatment a spherical box is adopted (Lynden-Bell & Wood 1968). Such a configuration does allow an equilibrium when the temperature is sufficiently high (Fig. 1). The high temperature limit tends toward the usual perfect gas without self-gravity, where the gas confinement is due to the solid walls of the box. When the temperature drops, the total energy of the gas decreases, and reaches negative values: the gas is then confined mostly by self-gravity, and secondly by the box. This is the regime analogous to stars. Yet, in this simple model nothing forbids to drop the box temperature even more. But then a last equilibrium model is reached at $T_c \approx 0.397$, which corresponds to the Jeans critical temperature for infinite uniform gas. Below this critical temperature no thermal or dynamical equilibrium exists.

Real cosmic gases are not perfect. At sub-critical temperature one expects wild fluctuations and the growth of singularities, “revealing” at some point new physical ingredients. By necessity an eventual asymptotic statistical state, if it exists, must then depend directly on the properties of the new ingredients revealed by the growth of singularities, and on gravity. The triggering of nuclear reactions in stars is an example of possible new physical ingredients, leading to long-term out-of-equilibrium situations, but other outcomes could be considered, such as planets or black-holes.

Among the most necessary modifications of the isolated isothermal perfect gas to
represent cosmic gas, one may include energy dissipation and time-dependent boundary conditions, since interstellar clouds are far from being energetically isolated. Indeed, cosmic gas cools rapidly, is often optically thin, so energy conservation is bad in practice. And molecular clouds (whose long-term persistence is still mysterious) are known to collide with substantial strength with a time-scale similar to their internal crossing time. So eventually models including these aspects might explain better the statistical properties of cosmic clouds. One can show that if a gravitational system finds a hierarchical order with fractal dimension \( D \), clumps tend to increase/decrease their kinetic “temperature” with size for \( D > 1 \), which might thus be a criterion for star formation. Further, a steady energy flow across the scales occurs for \( D = 5/3 \approx 1.67 \) (Pfenniger & Combes 1994).

Numerical experiments with dissipative \( N \)-body systems are underway and will be reported elsewhere. It suffices to say here that, as the shearing sheet experiments, 3D \( N \)-body systems can also find some kind of statistical state with fractal dimension \( D \sim 1 - 2 \) when ordered energy is injected at large-scale (representing cloud collisions), and dissipated at small scale (by particle inelastic collisions). This is not fundamentally different from the situation where a box in an external gravity field and half-filled with sand, is periodically shaken. The sand may quickly adopt spontaneously a well-defined vertical distribution unrelated to thermostatics, but with balancing the macroscopic energy and entropy fluxes from the box scale down to the grain scale.

5. Conclusions
The frequent assumption that cosmic gas is near a thermal equilibrium is particularly questionable in the case of gravitationally unstable media in which the basic requirements for applying thermodynamics are not met (e.g., additivity of energy). Indeed, as long as the medium is gravitationally unstable, the speed of matter disturbances is comparable to the sound speed, and thermalization cannot be established.

Instead of classical thermodynamics one should tend to use what has been learned in non-equilibrium statistical mechanics. Scaling laws are ubiquitous in open systems in which entropy flows. We suggest that in many situations gravitationally unstable media can reach a dynamical equilibrium which transforms large scale ordered motion into small scale “turbulence”. A fractal state can be maintained by interaction with higher scales as long as the largest scale is not in equilibrium, and energy and negentropy are dissipated at small scale.

The main astrophysical situations where one might expect the formation of fractal states are: 1. At the formation of the first structures in the Hubble flow just after recombination. 2. In cooling flows in galaxies and galaxy clusters. 3. In spiral galaxies as long as the amount of gas is sufficient to allow efficient cooling. 4. Similarly, in other smaller self-gravitating gaseous disks.

The conventional isolation hypothesis in scenarios of star and galaxy formation is probably inappropriate. Much can be changed in the way of understanding these processes if the interactions between upper and lower scales are taken into account. In particular, we see no ground to necessarily expect an intense phase of star formation just after recombination just because the medium becomes gravitationally unstable.
Appendix: Entropy in Gravitating Systems

A huge literature exists discussing “entropy” in widely different contexts. However much confusion occurs in astrophysics because some forms of entropy are used, in particular Boltzmann’s form, without consideration whether the original assumptions (e.g., additivity of energy) are applicable.

So here is a limited attempt to clarify the situation, largely based on the clear exposition in Landau & Lifchitz (1966), with some additional remarks, inspired by David Ruelle (oral communication), including the modern notion of chaos.

Entropy has a clear interpretation for \( \text{bounded} \) dynamical systems,

\[
\dot{z} = F(z),
\]

where \( z \) is the phase space vector, and \( F \) the right-hand side of the “equations of motion”. Hamiltonian systems are a particular case of dynamical system. Entropy is defined as,

\[
S = k \log \frac{\Omega}{\Omega_0},
\]

where \( \Omega \) is the “a priori available phase space volume” during the time \( \Delta t \), \( \Omega_0 \) is a constant volume of phase space (e.g., an incompressible elementary volume \((2\pi\hbar)^N\), where \( N \) is the number of degrees of freedom), and \( k \) a constant used to relate \( S \) to the temperature unit. Actually, entropy is best viewed as a pure number, the logarithm of a phase-space volume ratio, so often \( k = 1 \) is chosen.

The meaning of “a priori available phase space” becomes clearer when considering the chaos of most dynamical systems. In strictly deterministic systems, or when \( \Delta t \) is much smaller than the relaxation time, the potentially available phase space is restricted to just one elementary cell of phase space, fixed by the initial conditions, so entropy is zero. But real physical systems cannot be modeled as deterministic, not necessarily because of quantum effects as often argued, but because they are often very chaotic, i.e., sensitive to perturbations.

To illustrate this point, we take as example the classical deterministic description of molecular motion in a gas, perturbed by the minute gravitational perturbation of a single electron at 1 Gpc distance. The additional acceleration may look negligible \((\Delta a \approx 6 \cdot 10^{-90} \text{cm s}^{-2})\). The collision-time of atmospheric molecules is of the order of \( \tau = 10^{-12} \text{s} \), during which the molecule paths are deviated by the electron gravitational force by \( 3 \cdot 10^{-114} \text{cm} \). Each collision, because of uncorrelated directions, doubles the previous squared deviation, such that \(|\Delta x|^2(n\tau) \approx 2^{n-1}|\Delta x|^2(\tau)\), so after only \( 10^{-9} \text{s} \), \( n = 10^3 \), and \(|\Delta x|^2(1000\tau)^{1/2} \approx 10^{37} \text{cm}\), very large indeed! This is sufficiently huge to be obliged to drop very quickly the assumption of strict isolation. Thus for such chaotic systems a probabilistic attitude must be taken. The “molecular chaos” invoked by Boltzmann is fully justified when taking into account the chaotic nature, in the modern sense of chaos, of most systems with many degrees of freedom.

Thus the exponential growth of perturbations compels to take into account in realistic models of microscopic dynamics the uncontrolled minute terms, for example in the form of a stochastic component \( \epsilon \zeta(z) \),

\[
\dot{z} = F(z) + \epsilon \zeta(z),
\]
which represents all the minute forces that by necessity cannot be explicitly included in
the system, but which leads to non-negligible deviations of $z$ over $\Delta t$.

In a dynamical system subject to perturbations by stochastic forces the potentially
accessible phase space is in fact rapidly the whole chaotic region of the unperturbed
system. The notion of entropy therefore is particularly useful in systems in which nearly
all the phase space is chaotic. The measure of entropy is then well approximated by the
whole phase space, and the ergodic hypothesis is a good approximation.

In integrable or nearly integrable dynamical systems the available phase-space due to
stochastic perturbations may grow much slower in regular than in chaotic regions. If the
time to have a sizable probability to visit all parts of phase-space due to the stochastic
forces is much longer than the observation time $\Delta t$, the system looks predictable and no
statistical physics is required. A good case is the Solar System which in the present state
is well described by deterministic equations over less than a few Myr (Laskar 1994), but
for longer time-scales the most chaotic variables (associated with the largest positive
Liapunov exponents) must be considered in a statistical sense.

The general definition of entropy in Equ. (2) can be applied to non-additive bounded
dynamical systems, while Boltzmann’s entropy, $S_B = -k \sum_i p_i \log p_i$, where $p_i$ is the
probability of state $i$, requires a system with additive energy (see Landau & Lifchitz,
1966, §7). So Boltzmann entropy should not be used in gravitating systems.

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References

Binney, J., Tremaine, S.: 1987, Galactic Dynamics, Princeton Univ. Press.
Coleman, P.H., Pietronero, L.: 1992, Phys. Rep. 231, 311.
Fleck, R.C.: 1981, Astrophys. J. 246, L151.
Jeans, J.H.: 1902, Phil. Trans. Roy. Soc. London 199, 1.
Landau, L.D., Lifchitz, E.M.: 1966, Statistical Physics, Pergamon Press, Oxford; 1967,
Physique Statistique, Mir, Moscow.
Landsberg, P.T.: 1984, J. Stat. Phys. 35, 159.
Larson R.B., 1981, Mon. Not. R. S. Astr. Soc. 194, 809.
Laskar J.: 1994, Astron. Astrophys. 287, L9.
Lynden-Bell, D., Wood, R.: 1968, Mon. Not. R. S. Astr. Soc. 138, 495.
Newton I.: 1779-85, Opera, IV
Pfenniger, D., Combes, F.: 1994, Astron. Astrophys. 285, 94.
Salo, H.: 1992, Icarus 96, 85.
Salo, H.: 1995, Icarus 117, 287.
Scalo, J.M.: 1985, in Protostars and Planets II, D.C. Black & M.S. Matthews eds., Univ.
of Arizona Press, Tucson, p. 201.
Toomre, A., Kalnajs, A.J.: 1991, in Dynamics of Disc Galaxies, B. Sundelius ed.,
Göteborg, 341.
Wisdom, J., Tremaine, S.: 1988, Astron. J. 95, 925.