Numerical Method for Approximate Solution of Fisher’s Equation

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ABSTRACT: In this paper, Fisher's reaction diffusion equation has been solved numerically by Strang splitting technique depending on collocation method with cubic B-spline. For our purpose, the initial and boundary value problem consisting of Fisher's equation is split into two sub-problems to be one linear and the other nonlinear such that each one contains the derivative in terms of time. Then, the whole problem is reduced to the algebraic equation system using finite element collocation method combined with the cubic B-spline for spatial discretization and the convenient classical finite difference approaches for time discretization. The effective and efficiency of the newly given method have been shown on the four examples. In addition, the newly obtained numerical results are shown in formats graphical profiles and tables to compare with studies available in the literature.

Keywords: Fisher’s equation, B-splines, Collocation method
INTRODUCTION

In this manuscript, we are going to consider one dimensional non-linear Fisher's equation

\[ U_t = \gamma U_{xx} + \mu U (1 - U), \quad x_L \leq x \leq x_R, \quad t \geq 0 \tag{1} \]

with conditions given at the boundaries and the initial time

\[
\begin{align*}
U(x,0) &= U_0(x), \quad x_L \leq x \leq x_R \\
U(x_L,t) &= h_0(t), \quad U(x_R,t) = h_1(t) \\
U_x(x_L,t) &= f_0(t), \quad U_x(x_R,t) = f_1(t)
\end{align*}
\]

(2)

Fisher's equation has influential implementations in many fields such as science and engineering. Firstly, Fisher's equation is investigated theoretically by (Kalmogoroff et al., 1937; Canosa, 1973). Outside of theoretical works, the approximate solution of Fisher's equation has been handled by lots of authors. (Gazdag and Canosa, 1974) used a pseudo-spectral approach for equation. The numerical work of Fisher's equation has been described by a moving mesh method by (Qiu and Sloan, 1998). (Zhao and Wei, 2003) solved equation by discrete singular convolution (DSC) algorithm. The wavelet-Galerkin approach using complex harmonic wavelets has been presented by (Cattani and Kudreyko, 2008). (Mittal and Wei, 2003) solved equation by discrete singular convolution (DSC) algorithm. The wavelet-Galerkin approach using complex harmonic wavelets has been presented by (Cattani and Kudreyko, 2008). (Mittal and Wei, 2003) solved equation by discrete singular convolution (DSC) algorithm. The wavelet-Galerkin approach using complex harmonic wavelets has been presented by (Cattani and Kudreyko, 2008). (Mittal and Wei, 2003) solved equation by discrete singular convolution (DSC) algorithm. The wavelet-Galerkin approach using complex harmonic wavelets has been presented by (Cattani and Kudreyko, 2008).

In this paper, we employ Strang splitting technique using collocation method with cubic B-spline for the numerical approach of given equation. For this purpose, firstly, in section 2, the finite element collocation method with cubic B-spline is explained and Fisher's equation split into two sub-equations and then the obtained sub-equations are applied Strang splitting technique with help of collocation method utilizing cubic B-spline with the proper conditions given at the boundaries and the initial time of problem. After that, the initial vector is formed using the condition at initial time and the conditions at the boundaries. In section 3, Fisher's equation is applied to four test problems and the error norms \(L_2\) and \(L_\infty\) are computed and then compared with existing studies in literature. In section 4, a brief conclusion is given.

MATERIALS AND METHODS

For the numerical behavior of Equation (1), we consider the solution domain \([x_L, x_R]\) and define \(x_L = x_0 < x_1 < \cdots < x_N = x_R\) as uniform partition of the solution range by the nodal points \(x_m\) with \(h = x_{m+1} - x_m = \frac{x_R - x_L}{N}, \ m = 0, 1, \ldots, N\). An approximate solution corresponding to the analytical solution \(U(x,t)\) can be given as
\[ U_N(x, t) = \sum_{m=j-1}^{j+1} \delta_m(t) \varphi_m \]  

where \( \delta_m(t) \) are unknown time-dependent parameters obtained using the boundary conditions and equation (1). It is presented cubic B-spline functions on the domain \([x_L, x_R]\) in terms of nodal points \(x_m\) by (Prenter, 1975) as follows

\[
\varphi_m(x) = \begin{cases} 
\frac{(x - x_{m-2})^3}{h^3} + 3h(x - x_{m-1})^2 - 3(x - x_{m-1})^3, & [x_{m-2}, x_{m-1}] \\
\frac{h^3 + 3h^2(x - x_{m-1}) + 3h(x - x_{m-1})^2 - 3(x - x_{m-1})^3}{h^3 + 3h^2(x_{m+1} - x) + 3h(x_{m+1} - x)^2 - 3(x_{m+1} - x)^3}, & [x_{m-2}, x_{m-1}] \\
\frac{(x_{m+2} - x)^3}{h^2}, & otherwise 
\end{cases}
\]

where \( \{\varphi_1, \varphi_0, ..., \varphi_N, \varphi_{N+1}\} \) is a base on the domain \([x_L, x_R]\). Equation (1) contains the term \( U_m \), the first and second derivatives of \( U_m \). So, we need the values of the 1st \( U_m' \), the 2nd \( U_m'' \) with respect to space variable \( x \) and the values \( U_m \) in terms of cubic B-spline functions using the approximations (3), (4) and time-dependent parameters \( \delta(t) \). These values are obtained as follows

\[
U_m = U(x_m) = \delta_{m-1} + 4\delta_m + \delta_{m+1}
\]

\[
U_m' = U'(x_m) = \frac{3}{h}(\delta_{m-1} + \delta_{m+1})
\]

\[
U_m'' = U''(x_m) = \frac{6}{h}(\delta_{m-1} - 2\delta_m + \delta_{m+1}).
\]

The time split form of Equation (1) is as follows

\[
U_t - \gamma U_{xx} - \mu U = 0,
\]

\[
U_t + \mu U U = 0
\]

By substituting the values \( U_m, U_m' \) and \( U_m'' \) in system (5) in equations (6) and (7), we obtain the 1st order system of ODE as follows:

\[
\delta_{m-1} + 4\delta_m + \delta_{m+1} - \frac{6}{h^2} \mu (\delta_{m-1} - 2\delta_m + \delta_{m+1}) = 0
\]

\[
\delta_{m-1} + 4\delta_m + \delta_{m+1} + \mu \varepsilon_m (\delta_{m-1} + 4\delta_m + \delta_{m+1}) = 0.
\]

Here ‘‘.’’ denotes derivative in terms of time variable \( t \) and for linearization form, the value of \( z_m \) is taken as

\[
z_m = (\delta_{m-1} + 4\delta_m + \delta_{m+1}).
\]

When it is written \( \frac{\delta_{m+1} + \delta_m}{2} \) instead of the parameter \( \delta_m \) and \( \frac{\delta_{m+1} - \delta_m}{\Delta t} \) instead of the \( \delta_m \), we have the following equations

\[
v_1 \delta_{m-1}^{n+1} + v_2 \delta_{m}^{n+1} + v_3 \delta_{m+1}^{n+1} = v_4 \delta_{m-1}^{n} + v_5 \delta_{m}^{n} + v_6 \delta_{m+1}^{n}
\]

\[
z_1 \delta_{m-1}^{n+1} + z_2 \delta_{m}^{n+1} + z_3 \delta_{m+1}^{n+1} = z_4 \delta_{m-1}^{n} + z_5 \delta_{m}^{n} + z_7 \delta_{m+1}^{n}
\]

respectively and here

\[
v_1 = 1 - \frac{3\gamma \Delta t}{h^2} - \frac{\mu \Delta t}{2}, \quad v_2 = 4 + \frac{6\gamma \Delta t}{h^2} - 2\mu \Delta t, \quad v_3 = 1 - \frac{3\gamma \Delta t}{h^2} - \frac{\mu \Delta t}{2},
\]

\[
v_4 = 1 + \frac{3\gamma \Delta t}{h^2} + \frac{\mu \Delta t}{2}, \quad v_5 = 4 - \frac{6\gamma \Delta t}{h^2} + 2\mu \Delta t, \quad v_6 = 1 + \frac{3\gamma \Delta t}{h^2} + \frac{\mu \Delta t}{2},
\]
\[ z_1 = 1 + \frac{\mu z_m \Delta t}{2}, \quad z_2 = 4 + 2 \mu z_m \Delta t, \quad z_3 = 1 + \frac{\mu z_m \Delta t}{2}, \]
\[ z_4 = 1 - \frac{\mu z_m \Delta t}{2}, \quad z_5 = 4 - 2 \mu z_m \Delta t, \quad z_6 = 1 - \frac{\mu z_m \Delta t}{2}. \]

Systems (10) and (11) contain \((N + 1)\) equations and \((N + 3)\) unknowns \(\delta_m\) parameters, \(m = 0, 1, \ldots, N + 1\). We have to exterminate \(\delta_{-1}\) and \(\delta_{N+1}\) to obtain a solvable system using the boundary conditions \(U_x(x_R, t) = f_0(t)\), \(U_x(x_L, t) = f_1(t)\) for the systems (10) and (11). Thus, we obtain the following equations

\[ \delta_{-1} = \delta_1 - \frac{h}{3} U_x(1), \quad \delta_{N+1} = \delta_{N-1} + \frac{h}{3} U_x(N + 1) \]  \hspace{1cm} (12)

The parameters \(\delta_{-1}\) and \(\delta_{N+1}\) are eliminated in systems (10) and (11) using the equation (16) and eventually obtained a three diagonal \((N + 1)\times(N + 1)\) band matrix. A unique solution of these systems is obtained using Thomas algorithm. We need the initial parameters \(\delta^0_m\) to solve ones. For this, the initial parameters \(U(x, 0) = U_0(x)\) are firstly obtained. The initial vector \(\delta^0_m\) can be found out using the following IC and BCs

\[ U(x, 0) = U_0(x), \]
\[ U_x(x_L, t) = f_0(t), \quad U_x(x_R, t) = f_1(t) \]

Thus, the initial vector \(\delta^0_m\) is obtained as follows

\[ A^* = \begin{bmatrix} 4 & 2 & 0 & \cdots & \cdots & 0 \\ 1 & 4 & 1 & \cdots & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots \\ 1 & 4 & 1 & \cdots & \cdots & 0 \\ 0 & \cdots & \cdots & \cdots & \cdots & 4 \\ 2 & 0 & \cdots & \cdots & \cdots & 0 \end{bmatrix}, \quad \delta^0 = \begin{bmatrix} \delta^0_0 \\ \delta^0_1 \\ \vdots \\ \delta^0_{N-1} \\ \delta^0_N \end{bmatrix}, \quad B^* = \begin{bmatrix} U(x_0) + \frac{h}{3} f_0 \\ U(x_0) \\ \vdots \\ U(x_{N-1}) \\ U(x_N) - \frac{h}{3} f_1 \end{bmatrix} \]

Namely, it is written in the form of \(A^* \delta^0 = B^*\). From here, the dimensional band matrix \((N + 1)\times(N + 1)\) for parameters \(\delta^0_m\) is found.

**RESULTS AND DISCUSSION**

In the present section, we will consider four numerical examples to determine the effectiveness of the proposed approach using Strang splitting technique via collocation method with cubic B-spline for Fisher’s equation. For our goal, we compute the error norms \(L_2\) and \(L_{\infty}\) determined as

\[ L_2 = \| U - U_N \|_2 = \sqrt{\sum_{j=0}^{N} (U - U_N)^2}, \]
\[ L_{\infty} = \| U - U_N \|_{\infty} = \max_j |U - U_N| \]

Relative error (Madzvamuse, 2006)\(= \frac{\sqrt{\sum_{j=0}^{N} |U_{n+1}^j - U_n^j|^2}}{\sqrt{\sum_{j=0}^{N} |U_{n+1}^j|^2}}\).
Example 1

In the present example, we are going to deal with Equation (1) with BCs $U(x_L, t) = U(x_R, t) = 0$ and IC given by

$U_0(x) = \text{sech}^2(10x)$.

For this problem, discretization parameters are chosen as $h = 0.025, \Delta t = 0.05$ on the domain $[-50, 50]$ for $\gamma = 0.1$, $\mu = 1$ as in the studies (Dağ et al., 2010) and (Dağ and Ersoy, 2016). Physical behaviour of equation (1) has been drawn in graphical profiles. In Figure 1, for different time level $t = 0$ to $t = 0.5$, we have seen that near $x = 0$, $U(x, t)$ reaches maximum value $U = 1$. However, the peak rapidly comes down since diffusion term $U(1-U)$ dominates over reaction. Because of the reaction influence, Figure 2 indicates that the peak value is gradually increasing in time from $t = 0$ to 5. Also it is seen that the peak value reaches until the top $U = 1$ in at the time levels $0, 5, 10, 15, 20, 25, 30, 40$ in Figure 3. Tables 1 presents a comparison of the relative error at various times and shows that our results are much better.

![Figure 1. The numerical approaches of Example 1 for $t = 0(0.1)5$](image1)

![Figure 2. The numerical approaches of Example 1 for $t = 0(1)5$](image2)

![Figure 3. The numerical approaches of Example 1 for $t = 0(5)40$](image3)
Table 1. Comparison of relative errors for Example 1 at various times.

| Relative Error | t=5        | t=10       | t=15       | t=20       | t=40       |
|----------------|------------|------------|------------|------------|------------|
| Present        | 1.383E−2   | 7.835E−3   | 6.029E−3   | 5.067E−3   | 3.417E-3   |
| (Dağ et al., 2010) | 1.386E−2   | 7.860E−3   | 6.054E−3   | 5.090E−3   | 3.434E-3   |

Example 2

In this example, Fisher's equation is taken with local boundary condition and initial condition as follows:

\[
\begin{cases}
  e^{10(x+1)}, & x < 1 \\
  1, & -1 \leq x \leq 1 \\
  e^{-10(x+1)}, & x > 1
\end{cases}
\]

or this problem, we use coefficients \( \alpha = 0.1 \), \( \beta = 1 \) and parameters \( h = 0.025 \), \( \Delta t = 0.05 \) as in the first problem over domain \([-50, 50]\) until time 40 considering to studies (Dağ et al., 2010) and (Dağ and Ersoy, 2016). In Figure 4 and 5, it is graphically performed the solutions at early times. In these figures, the reaction-diffusion effective is quite minor. Because the reaction effect is more effective than the diffusion effect. Thus, they become smooth from having sharp. Also, figure 6 shows that the top of the wave have risen and shown that it is getting more and more flat. Table 2 submits a comparison of the relative error at various times and indicates that our results are much better.

Figure 4. The numerical approaches of Example 2 for \( t = 0(0.1)0.5 \)

Figure 5. The numerical approaches of Example 2 for \( t = 0(1)5 \)

Figure 6. The numerical approaches of Example 2 for \( t = 0(5)40 \)
Example 3

In the present example, we handle Eq. (1) with BCs $U(x_L, t) = 1, U(x_R, t) = 0, t \geq 0$ and the analytical solution given as follows:

$$U(x, t) = \left[1 + \exp \left(\frac{\mu}{\sqrt{6}}x - \frac{5\mu}{6}t\right)\right]^{-2}$$

In Table 3, we have firstly presented a comparison of the error norms $L_2$ and $L_{\infty}$ of Equation (1) with discretization parameters $h=1, \Delta t=0.01$ on range $-10 \leq x \leq 10$ for reaction-diffusion coefficients $\gamma=1$ and $\mu=2$ by considering the study (Mittal and Jain, 2012) and also, we have calculated the error norms $L_2$ and $L_{\infty}$ for values $h=0.5, \Delta t=0.01$ at times $t=5,10,15,20$ in Table 4. For values $h=0.25, \Delta t=0.01, t \leq 2$, in Figure 7, it is shown graphically together a comparison of analytical and numerical scheme of Example 3. Secondly, in Table 5, we have presented a comparison of the error norms $L_2$ and $L_{\infty}$ for the numerical approach of Example 3 by taking $N = 64, 150$ and $\Delta t = 0.000005$ at times $t = 0.0005, 0.0015, 0.0025, 0.0035$ over region $[-0.2,0.8]$ with reaction-diffusion coefficient $\gamma=1, \mu =10000$ taking into account some studies in literature. Table 3 shows that our results are very good and Table 4 indicates that we have achieved very low results. Figure 7 exhibits that the numerical scheme of the problem show fairly a good physical behaviours for $h=0.25, \Delta t=0.01$ at times $t \leq 2$. Table 5 displays that results of the error norms $L_2$ and $L_{\infty}$ computed by Strang splitting technique utilizing collocation method combined with cubic B-spline are better than in (Dağ and Ersoy, 2016), CN (Qiu and Sloan, 1998) and close to results in (Dağ et al., 2010), ASD (Qiu and Sloan, 1998) and it shows that the results of DSC (Qiu and Sloan, 1998) are better than the presented method. Also, it is seen that solution profiles and absolute error distributions in Figure 8 exhibit fairly accurate physical behaviors for parameters $N = 200$ and $\Delta t = 0.000005$ at times $t = 0.0005, 0.001, 0.0015, 0.002, \ldots, 0.0035$ as in (Dağ et al., 2010). So, the method presented can be recommended as alternative solution to other non-linear equations such as the Fisher's equation. Additionally, to indicate the effectiveness and performance of the suggested method, it is presented together the numerical and analytical solution graphically at different times in Figure 9 taking $\mu = 2000$ and $5000$ for $N = 200$ with $\Delta t = 0.00001$ on the solution region $[-0.2,0.8]$ as in studies (Mittal and Jain, 2013) and (Kappoor and Joshi, 2020).

| t  | $L_2$ Present | $L_{\infty}$ Present | $L_2$ Mittal and Jain (2012) | $L_{\infty}$ Mittal and Jain (2012) |
|----|--------------|----------------------|-----------------------------|----------------------------------|
| 0.5| 1.77E-03     | 1.10E-03             | 1.76E-03                    | 1.10E-03                        |
| 1  | 2.93E-03     | 1.75E-03             | 2.92E-03                    | 1.75E-03                        |
| 1.5| 3.65E-03     | 1.85E-03             | 3.65E-03                    | 1.86E-03                        |
| 2  | 4.28E-03     | 2.93E-03             | 4.50E-03                    | 3.00E-03                        |
**Table 4.** The error norms $L_2$ and $L_\infty$, for $\Delta t = 0.01$, $h = 1$ of Example 3 at some various times.

| Errors | t=5   | t=10  | t=15  | t=20  |
|--------|-------|-------|-------|-------|
| $L_2$  | 1.66E-03 | 1.30E-03 | 1.76E-03 | 1.10E-03 |
| $L_\infty$ | 0.33E-03 | 0.08E-03 | 2.92E-03 | 1.75E-03 |

**Figure 7.** The numerical solutions of Example 3 for $t \leq 2$ ($\Delta t = 0.01, h = 0.25$)

**Table 5.** Comparison of the error norms $L_2$ and $L_\infty$ at various times $t$ of Example 3 for $\alpha = 1$, $\beta = 10000$

| Method          | N   | Error | t   |
|-----------------|-----|-------|-----|
|                 |     |       | 0.0005 | 0.0015 | 0.0025 | 0.0035 |
| Present         | 64  | $L_2$ | 1.50E−3 | 0.21E−1 | 4.99E−2 | 0.79E−1 |
|                 |     | $L_\infty$ | 6.41E−3 | 0.89E−1 | 2.12E−1 | 3.25E−1 |
| Present         | 150 | $L_2$ | 4.45E−4 | 0.36E−2 | 0.86E−2 | 1.40E−2 |
|                 |     | $L_\infty$ | 3.27E−2 | 1.52E−2 | 3.64E−2 | 5.90E−2 |
| Present         | 200 | $L_2$ | 0.35E−3 | 0.02E−1 | 0.48E−2 | 0.78E−2 |
|                 |     | $L_\infty$ | 2.84E−3 | 0.83E−2 | 2.00E−2 | 3.27E−2 |
| Dağ et.al., 2010| 150 | $L_2$ | 6.89E−5 | 1.30E−2 | 1.55E−2 | 8.82E−3 |
|                 |     | $L_\infty$ | 2.57E−4 | 5.65E−2 | 6.63E−2 | 3.93E−2 |
| Dağ and Ersoy, 2016 (p = 1) | 64  | $L_\infty$ | 1.10E−2 | 1.49E−1 | 3.44E−1 | 5.08E−1 |
| CN(Zhao and Wei, 2003) | 64  | $L_2$ | 1.92E−3 | 2.65E−2 | 6.18E−2 | 9.91E−1 |
|                 |     | $L_\infty$ | 1.03E−2 | 1.25E−1 | 2.80E−1 | 4.48E−1 |
| ASD(Zhao and Wei, 2003) | 64  | $L_2$ | 2.09E−3 | 1.06E−2 | 2.02E−2 | 2.35E−2 |
|                 |     | $L_\infty$ | 1.07E−2 | 4.93E−2 | 9.37E−2 | 9.44E−1 |
| DSC(Zhao and Wei, 2003) | 64  | $L_2$ | 1.24E−6 | 5.92E−7 | 1.16E−6 | 1.64E−6 |
|                 |     | $L_\infty$ | 6.28E−6 | 1.98E−6 | 4.46E−6 | 6.22E−6 |

**Figure 8.** Solution profiles and absolute errors and of Example 3 for $N = 200$
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Example 4

In the last example, we get non-linear Fisher's equation given as

\[ U_t - \alpha U_{xx} = -\alpha_1 U^2 + \beta_1 U; \quad -\infty \leq x \leq \infty, \ t \geq 0, \]

having the following initial and boundary conditions

\[ U(x, t) = \begin{cases} \frac{\beta_1}{4\alpha_1} \left[ \text{sech}^2 \left( -\frac{\sqrt{\beta_1}}{24c} x \right) - 2\tanh \left( -\frac{\sqrt{\beta_1}}{24c} x \right) - 2 \right], & \text{for } x_L, t = 0.5, U(x_R, t) = 0. \]

The analytical solution for the present problem is taken as

\[ U(x, t) = \begin{cases} \frac{\beta_1}{4\alpha_1} \left[ \text{sech}^2 \left( \pm \frac{\sqrt{\beta_1}}{24c} x + \frac{5\beta_1}{12} t \right) - 2\tanh \left( \pm \frac{\sqrt{\beta_1}}{24c} x + \frac{5\beta_1}{12} t \right) - 2 \right]. & \end{cases} \]

The coefficients in this problem are chosen as \( \alpha=1, \alpha_1=1, \beta_1=0.5, c=1 \) for h=0.25, \( \Delta t=0.01 \) at times \( t=2 \) and \( t=4 \) on solution domain \([-30,30]\) as in studies (Cattani and Kudreyko, 2008), (Mittal and Arora, 2010) and (Mittal and Jain, 2013). Table 6 and Table 8 report a comparison of the presented method solutions with those obtained in (Cattani and Kudreyko, 2008) and (Mittal and Arora, 2010). Also, Tables 7 and 9 give a comparison of the absolute error results found out by the presented method. From these tables, it is seen that our results are better than those of the previous studies. Figure 10 clearly illustrates a comparison between numerical and analytical solutions at times \( t = 1,2,3,4,5 \) and this figure displays that it can be found a good conformity with those given the earlier studies.

Table 6. Comparison of approximate solutions of Example 4 at various values of \( x \) for \( t = 2 \).

| x    | Cattani and Kudreyko,2008 | Mittal and Arora, 2010 | Present | Exact  |
|------|---------------------------|------------------------|---------|--------|
| -20  | 0.498681                  | 0.498653               | 0.498650| 0.498652|
| -16  | 0.495130                  | 0.495745               | 0.495739| 0.495740|
| -12  | 0.486758                  | 0.486679               | 0.486668| 0.486669|
| -8   | 0.459576                  | 0.459478               | 0.459476| 0.459478|
| -4   | 0.386681                  | 0.386742               | 0.386787| 0.386791|
| 2    | 0.158878                  | 0.159011               | 0.158859| 0.158850|
| 6    | 0.041822                  | 0.041877               | 0.041852| 0.041851|
| 10   | 0.006455                  | 0.006426               | 0.006465| 0.006465|
| 14   | 0.000750                  | 0.000746               | 0.000754| 0.000755|
| 18   | 7.617E-05                 | 7.79E-05               | 7.91E-05| 7.92E-05|
Tablo 7. Comparison of absolute error at various values of $x$ for $t = 2$ of Example 4.

| $x$  | Mittal and Arora, 2010 | Present |
|------|------------------------|---------|
| -20  | 1.52E−0 6              | 1.37E−0 6 |
| -16  | 4.56E−06               | 1.15E−0 6 |
| -12  | 9.42E−06               | 7.87E−0 7 |
| -8   | 2.39E−07               | 1.24E−0 7 |
| -4   | 4.91E−05               | 4.37E−0 6 |
| 2    | 1.61E−04               | 8.77E−0 6 |
| 6    | 2.54E−05               | 8.28E−0 6 |
| 10   | 3.92E−05               | 2.65E−0 6 |
| 14   | 9.46E−06               | 6.30E−0 7 |
| 18   | 1.23E−06               | 8.24E−0 8 |

Tablo 8. Comparison of numerical approach at various values of $x$ for $t = 4$ of Example 4.

| $x$  | Cattani and Kudreyko, 2008 | Mittal and Arora, 2010 | Present | Exact |
|------|-----------------------------|------------------------|---------|-------|
| -20  | 0.498678                    | 0.499412               | 0.499411 | 0.499413 |
| -16  | 0.498525                    | 0.498146               | 0.498140 | 0.498142 |
| -12  | 0.494757                    | 0.494149               | 0.494139 | 0.494140 |
| -8   | 0.481776                    | 0.481763               | 0.481754 | 0.481756 |
| -4   | 0.445508                    | 0.445372               | 0.445394 | 0.445398 |
| 2    | 0.279025                    | 0.280082               | 0.279947 | 0.279941 |
| 6    | 0.116980                    | 0.117196               | 0.116975 | 0.116963 |
| 10   | 0.025927                    | 0.025881               | 0.025967 | 0.025974 |
| 14   | 0.003695                    | 0.003559               | 0.003618 | 0.003622 |
| 18   | 0.000409                    | 0.000395               | 0.000405 | 0.000406 |

Tablo 9. Comparison of absolute error at various values of $x$ for $t = 4$ of Example 4.

| $x$  | Mittal and Arora, 2010 | Present |
|------|------------------------|---------|
| -20  | 1.35E−0 6              | 1.93E−0 6 |
| -16  | 4.01E−06               | 1.86E−0 6 |
| -12  | 8.86E−06               | 1.40E−0 6 |
| -8   | 7.28E−06               | 1.40E−0 6 |
| -4   | 2.53E−05               | 3.51E−0 6 |
| 2    | 1.41E−04               | 5.96E−0 6 |
| 6    | 2.33E−04               | 1.20E−0 5 |
| 10   | 9.30E−05               | 7.02E−0 6 |
| 14   | 6.29E−05               | 4.20E−0 6 |
| 18   | 1.12E−05               | 7.45E−0 7 |

Figure 10. Approximate and exact solutions of Example 4 for $\Delta t = 0.01$, $h = 0.25$. at $t = 1$ to $t = 5$
CONCLUSION

In the current study, the approximate results of nonlinear Fisher's equation have been obtained via Strang splitting technique using finite element collocation method combined with cubic B-spline. To display the correctness and validity of the presented method, the four examples given with suitable the initial-boundary condition available in literature have been considered and computed the error norms $L_2$ and $L_\infty$. It has been seen that numerical results acquired with the presented method are very good. Consequently, we can say that the solutions of this study gotten Strang splitting technique can be both effectively implemented and considered as an alternative to obtain numerical results of these type of problems.

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