Optimization of the Design Parameter for Standing Wave Thermoacoustic Refrigerator using Genetic Algorithm

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Abstract. Standing wave thermoacoustic refrigerator has a potential of replacing conventional vapor compression refrigeration system due to several factors. Firstly, this green technology does not contain any harmful chemicals or ozone depletion refrigerant. In fact, it uses the sound wave to generate the cooling effect. Thus, it is friendly to the environment. Besides, this technology is affordable, lightweight and easy to be constructed too. However, it has a low coefficient of performance compared to conventional refrigerator with the same cooling capacity. Recently, there is no noticeable breakthrough in the research of this technology. In this study, genetic algorithm is used to optimize the design parameters of the standing wave thermoacoustic refrigerator system to achieve the highest coefficient of performance. Multi-Objective Genetic Algorithm is used to provide the performance trend of a standing wave thermoacoustic refrigerator system. There are five normalized parameters which will undergo optimization using genetic algorithm, namely normalized stack length, normalized stack center position, blockage ratio, drive ratio and normalized gas thermal penetration depth. The results show that the highest COP that can be achieved by a thermoacoustic refrigerator system (working gas = air, temperature = 300 K and pressure = 1 atm) is 4.8904. The coefficient of performance is about 120.3 % higher than the previously published data. A standing wave thermoacoustic refrigerator prototype is built based on the optimized results obtained from genetic algorithm, whereby the system is operating under no load condition. It is found that the experimental results agree well with the optimization results.

1. Introduction
Refrigeration is a process of removing heat from a cold area and transferring it to a warm area. This heat transfer process requires the input of work. This work is normally driven by mechanical means. Typical refrigerator such as vapor compression refrigeration system uses mechanical work to provide the cooling effect. Other means can also drive this work, such as heat, magnetism, electricity and laser. Meanwhile, a standing wave thermoacoustic refrigerator (TAR) system uses acoustic work to perform the refrigeration process. A simple standing wave TAR system comprises an acoustic driver (loudspeaker), a resonator tube, a stack, a pair of heat exchangers and working gas. When the loudspeaker operates, it provides acoustic power for the working gas particles to vibrate near to the solid stack boundary. Therefore, there will be a thermal interaction between the fluid particles and solid boundaries. The gas particles undergo a reversible adiabatic expansion and compression, and isobaric heat transfer with the solid stack [1]. Thus, heat can be transferred from the cold side to the hot side of the heat exchanger.
Standing wave TAR system has the potential of replacing the conventional refrigerator due to several factors. TAR has a minimal moving component, this minimize the mechanical failure in the system. Besides, there is no issue on tight tolerance and sliding seal. So, it is more reliable compared to its counterpart. Standing wave TAR is more environment friendly because it does not contain any harmful refrigerant or chemical. The construction of the system is simple and it is cheap too due to its simplicity.

Some notable TAR models were built previously, such as TRITON TAR system [2] and Ben & Jerry’s ice-cream TAR system [3]. Although theoretical understanding of thermoacoustic effects is well developed, the quantitative knowledge on the TAR system is immature. TAR system has low coefficient of performance (COP) than that of the conventional refrigerator. Besides, progress on developing the standing wave TAR system is at a slow rate after the construction of the Ben & Jerry’s TAR system. Many experimental and numerical works were conducted by other researchers, but these techniques tend to work on one parameter and they were able to find the local optimum solution only.

The purpose of this research is to improve the COP of a standing wave TAR system using GA and Multi-Objective Genetic Algorithm (MOGA) optimisation schemes. Comparison of the optimization results with literature survey is conducted to benchmark the performance of a standing wave TAR system performance. Then, optimized geometry and operating parameters are used to validate the optimization results.

2. Design of the Standing Wave TAR System
Geometry parameters and operating parameters are the two main design factors that affect the performance of the standing wave TAR system. In this study, the parameters are extracted from the literature survey to construct the TAR system. Every actual parameter of a standing wave TAR system is decided in advance before the optimizations and experiment, except stack length ($L_s$), stack center position ($x_s$) and stack plate spacing ($2y_o$), sound pressure level ($L_p$), operating frequency ($f$) and resonator length ($L_{res}$). This is because these six parameters are to be optimized using GA optimization.

2.1. Geometry Parameters
A quarter-wavelength ($0.25 \lambda$) resonator is selected in the design of the TAR system. This is because it has smaller surface area for the heat dissipation and heat loss to the surrounding can be minimized [4]. A standing wave at the fundamental mode can be generated where pressure node is at the opened end while velocity node is at the closed end. Acrylic is a plastic material with low thermal conductivity is used as the resonator material. Then, spiral is selected as the stack geometry because it can provide thicker thermal boundary layer and larger stack wall’s surface area. However, this geometry was previously fabricated by rolling Mylar sheet or photographic film, where every rolled layer was divided using nylon or fishing line [5,6]. 3D printer is used to print out the spiral stacks. Next, Polyactide (PLA) is used for the stack material. This material has the lowest thermal conductivity ($K_s$) and highest specific heat capacity ($c_s$) among all the discovered stack materials, which is shown in Figure 1. Low $K_s$ allows lesser heat loss along the stack while high $c_s$ enables more heat to be retained for a small increase in temperature [1]. A thinner stack plate is used to reduce acoustic disturbance and gives larger stack surface area to allow more thermal interaction [7]. A minimum stack plate thickness of 8 times of the stack thermal penetration depth ($\delta_s$) is needed to ensure the stack plates have sufficient thickness to retain the heat during the thermoacoustic process [8]. The thickness of the stack plates ($2l$) of 0.5 mm is used for the TAR system design.

2.2. Operating Parameters
Air is chosen as the working gas as it does not bring any harmful effect to the occupant and environment. The TAR system is operating at the room condition, which are 300 K and 1 bar. The geometry parameters and operating parameters are given in Table 1. Meanwhile, the properties of air and PLA are given in Table 2 and Table 3, respectively. Then, figure 1 shows the comparison between the thermal properties of PLA and other stack materials used in other studies.
Table 1. Geometry and Operating Parameters.

| Parameters            | Values       |
|-----------------------|--------------|
| Resonator Length, $L_{res}$ | 0.25 λ m    |
| Resonator Material    | Acrylic      |
| Stack Geometry        | Spiral       |
| Stack Plate Thickness, 2t | 0.5 mm      |
| Stack Material        | PLA          |
| Working Gas           | Air          |
| Operating Temperature, $T_m$ | 300 K       |
| Operating Pressure, $p_m$ | 1 bar        |

Table 2. Properties of Air [9].

| Properties                        | Values         |
|-----------------------------------|----------------|
| Acoustic Velocity in Air, $a$     | 347.2 m s$^{-1}$|
| Density, $\rho_l$                | 1.1614 kg m$^{-3}$|
| Dynamic Viscosity, $\mu$         | $18.46 \times 10^6$ Pa-s |
| Heat Capacity Ratio, $\gamma$    | 1.4            |
| Isobaric Specific Heat Capacity, $c_p$ | 1 007 J (kg·K)$^{-1}$ |
| Thermal Conductivity, $K_s$      | 0.0263 W (m·K)$^{-1}$ |
| Prandtl Number, $\sigma$         | 0.707          |

Table 3. Properties of PLA [10].

| Properties                        | Values         |
|-----------------------------------|----------------|
| Thermal Conductivity, $K_s$      | 0.13 W (m·K)$^{-1}$ |
| Specific Heat Capacity, $c_s$    | 1 800 J (kg·K)$^{-1}$ |
| Density, $\rho_s$                | 1 300 kg m$^{-3}$ |

Figure 1. Properties of Stack Material [10,11].

3. GA and MOGA Optimizations

MATLAB is used to perform GA and MOGA optimizations on the standing wave TAR system. GA determines the highest COP (best fitness value) at certain cooling threshold by giving one set of global optimum solution. Meanwhile, MOGA focuses on maximizing the desired cooling power and minimizing the required acoustic power to obtain new COP. MOGA provides multiple optimum solutions and the trends of these objective functions. The analysis on a standing wave TAR is based on the system performance trends. Both of them use probabilistic transition rules to optimize on five design parameters simultaneously, which are normalized stack length ($L_{na}$), normalized stack center position ($x_{na}$), blockage ratio ($B$), drive ratio ($DR$) and normalized gas thermal penetration depth ($\delta_{na}$). Then, the optimum solution of these variables is used to determine the actual parameters, which are $L_{na}$, $x_{na}$, $2y_{na}$, $L_p$, $f$ and $L_{res}$. A best fitness function plot is plotted when the GA optimization is ended, while a Pareto Front plot is produced after the MOGA optimization is ended.

3.1. Design Parameters

Dimensionless parameters are introduced in the optimization by normalizing the actual parameters to reduce the dimensioned parameters in the objective functions and simplify the analysis work. The dimensionless parameters are normalized cooling power ($Q_{na}$) and normalized acoustic power ($W_{na}$).

3.2. Objective Functions

The equations for $Q_{na}$, $W_{na}$, and COP are given by [1]. COP without modulus is the only objective function used in the GA optimization. GA attempts to find a global minimum for this function during the optimization. After the optimization, the maximum COP can be obtained by substituting this global minimum into Equation (3). The threshold for $Q_{na}$ is $5.07 \times 10^6$, which is decided by referring to the result obtained in a previous study [6]. Then, $W_{na}$ must be always negative due to the input of $W$ into TAR system. Meanwhile, negative $Q_{na}$, $W_{na}$ and COP without modulus are the objective functions for MOGA. Inequalities are set for $Q_{na}$, $W_{na}$ and COP in MOGA, where COP and $Q_{na}$ are positive, while $W_{na}$ is negative. $Q_{na}$ is positive and it denotes the removal of heat by the TAR system.
\[ Q_{cn} = -\frac{\delta_{ln} DR^2 \sin 2x_m}{8\gamma(1 + \sigma)\Lambda} \times \left\{ \left[ \Gamma(1 + \sqrt{\sigma + \sigma}) \right] - \left( 1 + \sqrt{\sigma - \delta_{ln}\sqrt{\sigma}} \right) \right\} \]  

\[ W_n = \frac{\delta_{ln} L_m DR^2 B(\gamma - 1)\cos^2 x_m}{4\gamma} \left( \frac{\Gamma}{1 + \sqrt{\sigma}\Lambda} - 1 \right) - \left( \frac{\delta_{ln} L_m DR^2}{4\gamma} \right) \times \frac{\sqrt{\sigma}\sin^2 x_m}{B\Lambda} \]  

\[ COP = \frac{Q_{cn}}{W_n} \]  

3.3. Imposed Constrained Bounds

The boundaries imposed on the optimization to reduce the computational effort and refine the search. Firstly, a typical stack occupies less than 10% of \( L_{res} \) [12]. The short stack approximation theory suggested a shorter stack has lesser interruption of acoustic waves in the resonator [13]. Secondly, the stack has to be inserted and located within the resonator. Therefore, the \( x_m \) must be always larger than zero and not longer than \( L_{res} \). On the other hand, the \( 2\gamma \) has to be limited between \( 2\delta_k \) and \( 4\delta_k \) [13]. Next, the \( f \) should lies between 300 Hz and 500 Hz [1]. But, the lower and upper bounds for \( f \) are set at 50 Hz and 1 000 Hz, respectively. Wider parameter range and search interval are chosen to allow more explorations on these parameters. By normalizing the boundaries of these actual parameters, the constrained bounds applied to the design parameters can be obtained. \( DR \), which is not inter-related with other design parameters and it is set between 0.015 and 0.030 based on the literature survey [1,6].

3.4. Operators

Proper options allow the algorithms to obtain desirable and reasonable optimum solutions during the optimization. A large population size of 200 allows higher chance of approaching to a global optimum due to better exploration and exploitation of the search space. Rank fitness scaling can remove the spread of the raw fitness scores, which in turn gives minimum complexity in the optimization. Tournament selection uses lesser computational time because it has a better convergence characteristic than other options. Next, heuristic crossover with a ratio of 2.0 and a fraction of 0.95 is used to achieve maximum improvement to the “offspring” after every iteration. This crossover setting gives a higher rate of convergence and avoids the chance of settling for a false optimum due to early convergence. Adaptive feasible mutation enables the algorithms to exploit better result while maintaining its feasibility with respect to the imposed constrained bounds.

| Table 4. Imposed Constraints. | Table 5. Operators and Options for GA and MOGA. |
|--------------------------------|-----------------------------------------------|
| Constraints of Design parameters | Operators | Options |
| a) \( 0.0000 < L_{in} < 0.05\pi \) | Population Size | 200 |
| b) \( 0.0000 < x_m < 0.5\pi \) | Scaling Function | Rank |
| c) \( 0.0150 < DR < 0.0300 \) | Selection Function | Tournament |
| d) \( 0.5000 < \delta_{ln} < 1.0000 \) | Crossover Function | Heuristic |
| e) \( 0.4036 < B < 0.6021 \) | Crossover Fraction, \( p_c \) | 0.95 |
| | Crossover Ratio, \( r \) | 2.0 |
| | Mutation Function | Adaptive Feasible |

3.5. Results

The optimized design parameters and best fitness values can be obtained after GA optimization. The values of five optimized design parameters are \( L_{in} = 0.0769 \), \( x_m = 0.1040 \), \( B = 0.6021 \), \( DR = 0.0300 \) and \( \delta_{ln} = 0.5695 \). The best fitness value obtained in the GA optimization is 4.8904. The actual parameters for getting highest COP can be obtained by de-normalizing the optimized design parameters. 

\[ Q_{cn} = -\frac{\delta_{ln} DR^2 \sin 2x_m}{8\gamma(1 + \sigma)\Lambda} \times \left\{ \left[ \Gamma(1 + \sqrt{\sigma + \sigma}) \right] - \left( 1 + \sqrt{\sigma - \delta_{ln}\sqrt{\sigma}} \right) \right\} \]

\[ W_n = \frac{\delta_{ln} L_m DR^2 B(\gamma - 1)\cos^2 x_m}{4\gamma} \left( \frac{\Gamma}{1 + \sqrt{\sigma}\Lambda} - 1 \right) - \left( \frac{\delta_{ln} L_m DR^2}{4\gamma} \right) \times \frac{\sqrt{\sigma}\sin^2 x_m}{B\Lambda} \]

\[ COP = \frac{Q_{cn}}{W_n} \]
parameters. The optimum actual parameters are \( L_0 = 2.76 \text{ cm}, x_2 = 3.7 \text{ cm}, y_0 = 0.38 \text{ mm}, L_P = 160.5 \text{ dB}, f = 154.\text{Hz} \) and \( L_{Res} = 56.3 \text{ cm} \). The \( Q_{cn} \), \( W_n \) and COP can be calculated using equations (1), (2) and (3) respectively. The \( Q_{cn} \) is \( 5.070 \times 10^6 \), \( W_n = -1.037 \times 10^6 \) and COP = 4.8904. On the other hand, a set of non-dominating solutions can be obtained from the MOGA optimization.

| Number of Variables | Design parameters | Optimization Methods | \( Q_{cn} \times10^6 \) | \( W_n \times10^6 \) | COP |
|---------------------|--------------------|----------------------|-------------------------|-----------------|-----|
| Current Study       | 5                  | \( L_{init}, x_{init}, B, DR, \delta_{init} \) | GA                      | 5.07           | -1.037 | 4.8904 |
| Other Study [6]     | 3                  | \( L_{init}, x_{init}, B \)              | MOGA                    | 5.07           | -2.29  | 2.22   |

![Figure 2. Best Fitness Plot.](image1.png)  
![Figure 3. Pareto Front Plot.](image2.png)

The Pareto Front plot obtained in the MOGA shows that the magnitude of \( Q_{cn} \) increases with the magnitude of \( W_n \). This implies that higher cooling power requires more acoustic work. As both magnitudes further increase, this will lead to large increase in the magnitude of \( W_n \) than the \( Q_{cn} \). By considering the equation (3), it shows that the COP decreases when the magnitudes of both \( Q_{cn} \) and \( W_n \) increase, provided the results are the non-dominating solutions. On the other hand, this study’s result is used to compare with the result from [6] because both have many similarities in the geometry and operating parameters. The highest COP achieved in this study is 4.8904, which is 120.3\% higher than the previous literature by Zolpakar et al. [6]. Both studies have same \( Q_{cn} \) because GA maintained the \( Q_{cn} \) at its minimum threshold \( (5.07 \times10^6) \) while attempting to get the highest COP. More design parameters in the GA optimization allows it to focus on the discovery of solution with higher fitness values from a wider search space compared to MOGA with lesser design parameters. The optimized \( DR \) is 0.030, which is the same with the literature survey. This implies that the optimized \( \delta_{init} \) influences the most in the improvement in the COP of a standing wave TAR system. Thus, this leads to the huge improvement in the current TAR system’s COP. With this outcome, the TAR system is able to compete with the conventional refrigerator with the same cooling capacity if the suitable operating pressure, working gas and resonator size are used for it in the real-life situation.

4. Experimental Study

4.1. Setup and Prototype

The experiment setup is shown in figure 4. There is no heat exchanger installed in the prototype. The TAR prototype is operated under no load condition as it is meant to verify the reliability of GA’s result. Then, small Styrofoam balls are poured into the resonator until half of it is being filled before the experiment. This is to prove that a quarter wavelength standing sound wave can be produced in the system. It is found that the balls excited the most at the speaker side while the balls did not vibrate at the closed end. Meanwhile, the sound pressure level conducted in the experiment was 139 dB only, which is different from the optimized sound pressure level \( (160.5 \text{ dB}) \). This is due to the limitation of the sound meter available for this experiment, whereby it has an operating range of up to 140 dB.
4.2. Results

The result shows that there is a temperature difference between the stack ends where the hot and cold ends achieved 34.7 °C and 19.3 °C respectively. The temperature difference recorded was 15.4 °C. This shows that the prototype using the optimized parameters is able to provide the expected heat transfer from cold stack end to hot stack end.

![Figure 4. TAR Prototype.](image)

![Figure 5. Experiment Result.](image)

5. Conclusion

The optimization on five design parameters of a standing wave TAR system using GA was successfully conducted, where the highest COP achieved during the optimization is about 120.3% higher than the previous study by other researchers. δkn influences the most in the improvement of COP. Then, the trend of the TAR system performance is analyzed from the result obtained from MOGA. Experiment was carried out according to the optimized parameters. The cold end temperature and temperature difference achieved were 19.3 °C and 15.4 °C, respectively.

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