Modeling and simulation of induction motor based on Time Scale Frame Transformation

Pengfei Tian¹, Qiumeng Qu², Yanmeng Cai³, Yu Cheng¹, Yizheng Xu¹, Shujun Yao²*, Yan Wang²

¹ State Key Laboratory of Power Grid Security and Energy Conservation (China Electric Power Research Institute), Beijing, 100192, China
² School of Electrical and Electronic Engineering, North China Electric Power University, Beijing, 102206, China
³ Jiangsu Power Transmission and Transformation Company, Nanjing, 211102, China

*yaoshujun@ncepu.edu.cn

Abstract. The time scale frame transformation is used to construct the analytic signal for single signal, and the large step simulation is carried out after the frequency reduction is rotated. It avoids the double frequency problem of Park transformation when the three-phase power system is asymmetrical, and also avoids the complexity of dynamic phasor method which needs to deal with the asymmetry problem in positive and negative zero order. Based on the principle of time scale frame transformation and the VBR model of induction motor, a time scale frame transformation model of induction motor is established. Then, the time scale frame transformation model of induction motor is programmed by Matlab software, and compared with the electromagnetic transient model of classical induction motor built by Simulink platform. The simulation example shows the accuracy and efficiency of the time scale frame transformation model of induction motor.

1. Introduction

As an important part of power system load and emergency standby power generation equipment, induction motor modeling has been studied since the early 20th century. The modeling methods of induction motor are mainly divided into finite element method¹, equivalent magnetic circuit method² and coupling circuit method. The first two methods are mainly used to analyze the detailed internal structure of the motor, and the coupling circuit method is used to simulate the dynamic characteristics of the motor in the power system. Due to the time-varying inductance matrix of the induction motor coupled circuit model seriously affects the simulation efficiency, the dq model of induction motor is established by Park transformation in [3]. The inductance matrix in the dq model is constant matrix, so the simulation efficiency is greatly improved. However, in the system simulation, there is a complex interface data conversion problem between the dq model and the external power network (abc static coordinate system), which will affect the numerical stability and simulation accuracy of the system simulation⁴. In order to solve the interface problem of induction motor dq model and give consideration to the simulation efficiency, VBR model of induction motor is proposed in references [5-7]. The stator side of the model is abc three-phase interface, and the rotor side still uses dq model of rotor system. The VBR model has the advantages of coupling circuit model and dq model, and it has higher numerical accuracy and smaller calculation load.
The principle of time scale frame transformation (TSFT) is proposed in [8], and a time scale frame transformation model of resistance, inductance and capacitance is established. The time scale transformation is to construct the analytic signal by using the Hilbert transformation (HT) for the original time domain signal, and then rotate it by using the rotation angle frequency $\omega_r$. When $\omega_r < 0$, the signal frequency is reduced and large step simulation can be carried out. The second section first briefly introduces the principle of TSFT. In the third section, according to the VBR model of motor, the motor model based on TSFT is established. In the fourth section, an example is given to illustrate the effectiveness of the model. Finally, the fifth section summarizes the thesis.

2. Time scale frame transformation
In reference [8], the principle and method of TSFT are described in detail by analyzing the rotation characteristics of analytical signals and the frequency reduction principle of Park transformation. It will not be described in detail here, but only the implementation process.

For a time domain signal $u(t)$, as shown in formula (1):

$$u(\omega,t) = A(t)\cos(\omega t + \theta)$$

(1)

The virtual orthogonal quantity can be constructed by HT, and the expression is:

$$v(t) = H[u(t)] = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{u(t-\tau)}{\tau} d\tau = u(t) * \frac{1}{\pi t}$$

(2)

Then the analytical signal $Z_{uv}(\omega,t)$ of the time domain signal $u(t)$ can be written as

$$Z_{uv}(\omega,t) = u(\omega,t) + jv(\omega,t)$$

$$= A(t)\cos(\omega t + \theta) + jA(t)\sin(\omega t + \theta)$$

(3)

Assuming the rotation transformation $e^{j\omega t}$, where $\omega_r$ is the rotation speed. Multiply both sides of the analytical signal $Z_{uv}(\omega,t)$ of formula (3) by $e^{j\omega t}$ to obtain:

$$Z_{uv}(\omega,t)e^{j\omega t} = A(t)e^{j\omega t}e^{j\omega t} = A(t)e^{j\omega t}e^{j(\omega t+\omega_r)t} = A(t)e^{j\omega t}e^{j\omega t} = X_{dq}(\Delta \omega, t)$$

(4)

Where, $\Delta \omega = \omega + \omega_r$

$$X_{dq}(\Delta \omega, t) = A(t)e^{j\omega t}e^{j\omega t}$$

(5)

$$= x_q(\Delta \omega, t) + jx_d(\Delta \omega, t)$$

(6)

The formula (4) can be expanded according to the real part and the imaginary part as:

$$\begin{bmatrix} x_d(\Delta \omega, t) \\ x_q(\Delta \omega, t) \end{bmatrix} = \begin{bmatrix} \cos(\omega t) & \sin(\omega t) \\ -\sin(\omega t) & \cos(\omega t) \end{bmatrix} \begin{bmatrix} u(\omega, t) \\ v(\omega, t) \end{bmatrix}$$

(7)

From equation (7), it can be seen that the coordinate system $u-v$ where $Z_{uv}$ is located is the static coordinate system, and the rotation speed of $Z_{uv}$ in $u-v$ is $\omega$. The coordinate system $d-q$ is the rotation coordinate system, the rotation speed is $\omega_r$, and the relative rotation speed of $Z_{uv}$ projecting $X_{dq}$ in the coordinate system $d-q$ is $\Delta \omega = \omega + \omega_r$, as shown in Figure 1. In fact, $\omega_r$ can be any real number. When $\omega_r < 0$, the frequency of the xdq signal decreases. When $\omega_r > 0$, the frequency of $X_{dq}$ signal increases. If $\Delta \omega$ is a small number, that is, the relative rotation speed $\Delta \omega$ of $X_{dq}$ after transformation is small, the corresponding analytical signal can be simulated in large step.
3. Modeling of Induction motor

3.1. VBR model of induction motor

The VBR model is based on the dq0 coordinate system, and the stator voltage equation in the dq0 coordinate system is returned to the abc coordinate system’s stator voltage equation through the inverse Park transformation. Assuming that the induction motor operates under the normal operation condition of three-phase symmetry, the stator and rotor state equations of the induction motor in dq0 coordinate system can be expressed as follows:

\begin{align}
    v_q &= \left( R_s + \frac{L_m^d}{L_{lt}} \right) i_{q} + \omega (L_{ls} + L_m^q) i_{d} + p(L_{ls} + L_M^q) i_q + e_q^q \\
    v_s &= \left( R_s + \frac{L_m^d}{L_{lt}} \right) i_{d} - \omega (L_{ls} + L_M^q) i_q + p(L_{ls} + L_M^q) i_d + e_d^s \\
    v_{0s} &= R_{ls} i_{0s} + p \lambda_{0s}
\end{align}

Where,

\begin{align}
    p \lambda_{qs} &= -\frac{R_s}{L_{ls}} (\lambda_{qs} - \lambda_{ms}) - (\omega - \omega_r) \lambda_{qs} + v_q \\
    p \lambda_{qs} &= -\frac{R_s}{L_{ls}} (\lambda_{qs} - \lambda_{ms}) - (\omega - \omega_r) \lambda_{qs} + v_q
\end{align}

By inverse Park transformation of equation (8-1), we can get

\begin{align}
    v_{ABC} = R_{ABC} i_{ABC} + L_M^q i_{ABC} + e_{ABC}
\end{align}

Where,
Then the state equation of induction motor VBR model can be expressed by formula (8-2), formula (9) and formula (10).

\[
\begin{align*}
\frac{d\theta}{dt} &= \omega_r \\
J \left( \frac{2P}{P_m} \right) P_s &= T_m - T_e
\end{align*}
\]

Where, \( P \) is the pole number of the induction motor, \( J \) is the moment of inertia of the rotor, and \( T_e = \frac{3P}{4} \left[ \lambda_{m_d} i_{qs} - \lambda_{m_q} i_{ds} \right] \cdot T_m = k \left[ \alpha + (1 - \alpha)(1-s) \right] \)

Where, \( \alpha \) is the proportion of the part of the mechanical torque unrelated to the speed of the induction motor; \( P_m \) is the index related to the mechanical load characteristics; \( k \) is the load factor.

In addition, when the induction motor is a squirrel cage structure and the star connected stator winding neutral point is not grounded, the VBR model of the induction motor will be greatly simplified. At this time, the rotor voltage is zero and there is no zero sequence component in the induction motor. The simplified VBR model is shown in Figure 2. The RL circuit branches on the stator side are naturally decoupled and constant.

\[
\begin{align*}
R'_{ABC} &= \begin{bmatrix} r_s^r & r_m^r & r_m^r \\ r_m^r & r_s^r & r_m^r \\ r_m^r & r_m^r & r_s^r \end{bmatrix}, \\
L'_{ABC} &= \begin{bmatrix} L_s & L_m & L_m \\ L_m & L_s & L_m \\ L_m & L_m & L_s \end{bmatrix}, \\
\begin{cases} r_s = R_s + \frac{2}{3} \frac{L_M^e}{L_n^r} R_t \\ r_m = -\frac{1}{3} \frac{L_M^e}{L_n^r} R_t \end{cases}, \\
\begin{cases} L_s = L_{ts} + \frac{2}{3} L_M^s \\ L_m = -\frac{1}{3} L_M^s \end{cases}
\end{align*}
\]

Figure 2. Simplified VBR model structure

In figure 2, the state equation of the rotor side is unchanged, and the voltage equation of the stator side can be expressed as

\[
v_{ABC} = \text{diag}(r_D, r_D, r_D)i_{ABC} + \text{diag}(L_D, L_D, L_D)pi_{ABC} + e''_{ABC}
\]

Where,

\[
\begin{align*}
r_D &= R_s + \frac{L_M^e}{L_n^r} R_t \\
L_D &= L_{ts} + L_M^s
\end{align*}
\]
3.2. Induction motor model based on TSFT

3.2.1. Stator system of induction motor model based on TSFT. Note that the capital letters in English are the phasor forms of each physical quantity after TSFT. According to the principle of TSFT, the stator voltage equation (equation (11)) of VBR model can be rewritten as

\[
\text{Re}\{\mathbf{V}_{ABC}^T\} = \text{Re}\{\mathbf{R}_D \mathbf{I}_{ABC}^T + \mathbf{L}_D p\mathbf{I}_{ABC}^T + \mathbf{E}_{ABC}^s\}
\]

(12)

Where,

\[
\begin{align*}
\mathbf{R}_D &= \text{diag}(r_{D_1}, r_{D_2}, r_{D_3}) \\
\mathbf{L}_D &= \text{diag}(L_{D_1}, L_{D_2}, L_{D_3}) \\
T &= e^{j\omega t}
\end{align*}
\]

Simplify equation (12) to

\[
\mathbf{V}_{ABC} = (\mathbf{R}_D + j\omega_0 \mathbf{L}_D) \mathbf{I}_{ABC}^d + \mathbf{L}_D p\mathbf{I}_{ABC}^q + \mathbf{E}_{ABC}^s
\]

(13)

Expand equation (13) according to the d-axis and q-axis of the rotating coordinate system to obtain

\[
\begin{bmatrix}
\mathbf{V}_{ABC}^d \\
\mathbf{V}_{ABC}^q
\end{bmatrix} =
\begin{bmatrix}
\mathbf{L}_D & 0 \\
0 & \mathbf{L}_D
\end{bmatrix}
\begin{bmatrix}
\mathbf{I}_{ABC}^d \\
\mathbf{I}_{ABC}^q
\end{bmatrix} +
\begin{bmatrix}
\mathbf{R}_D & -j\omega_0 \mathbf{L}_D \\
\omega_0 \mathbf{L}_D & \mathbf{R}_D
\end{bmatrix}
\begin{bmatrix}
\mathbf{I}_{ABC}^d \\
\mathbf{I}_{ABC}^q
\end{bmatrix} +
\begin{bmatrix}
\mathbf{E}_{ABC}^d \\
\mathbf{E}_{ABC}^q
\end{bmatrix}
\]

(14)

Where the superscripts "d" and "q" respectively represent the projection of single-phase electrical quantities in dq rotation coordinate system.

3.2.2. Rotor system of induction motor model based on TSFT. Compared with the electromagnetic transient process, the mechanical rotation process of the rotor system changes slowly, so it is not meaningful to use the time scale transformation to increase the step size of the rotor system. Therefore, the rotor system is still expressed in dq0 coordinate system. However, the data exchange between the stator system and the rotor system of VBR model needs to transform the sub-transient potential \(e_q^s\) and \(e_d^s\) in dq0 coordinate system of the rotor side into the sub-transient potential in abc coordinate system of the stator side. And the stator current and voltage need to be transferred to the rotor side through park transformation. So it is necessary to restudy the conversion process of stator and rotor variables which is suitable for time scale transformation.

The time-domain form and phasor form of the single-phase sub-transient potential in abc coordinate system are shown in formula (15):

\[
\begin{align*}
\mathbf{e}_A^s &= e_q^s \cos \theta + e_d^s \sin \theta \\
\mathbf{E}_A^s &= e_q^s e^{j(0 - \omega_0 t)} + e_d^s e^{j(\frac{2\pi}{3} - \omega_0 t)}
\end{align*}
\]

(15)

The time-domain form and phasor form of the sub-transient potential of B and C phases have similar transformation characteristics, so the form of the sub-transient potential phasor in the stator side ABC coordinate system can be expressed as

\[
\mathbf{E}_{ABC}^s = P_s \begin{bmatrix}
e_q^s \\
e_d^s \\
0
\end{bmatrix}^T
\]

(16)

Where,

\[
P_s =
\begin{bmatrix}
e^{j(0 - \omega_0 t)} & e^{j(\frac{2\pi}{3} - \omega_0 t)} & 1 \\
e^{j(\frac{2\pi}{3} - \omega_0 t)} & e^{j(\frac{2\pi}{3} - \omega_0 t)} & 1 \\
e^{j(\frac{2\pi}{3} - \omega_0 t)} & e^{j(\frac{2\pi}{3} - \omega_0 t)} & 1
\end{bmatrix}
\]
Secondly, stator current and voltage in phasor form need to be transmitted to time domain current and voltage signals in dq0 coordinate system of rotor side. This process can be realized by formula (17).

\[
\begin{align*}
    i_{q0s} &= K_i \cdot \Re \{ I_{ABe}e^{j\omega t} \} \\
    v_{q0s} &= K_v \cdot \Re \{ V_{ABe}e^{j\omega t} \}
\end{align*}
\]

Where,

\[
K_i = \frac{2}{3} \begin{bmatrix}
    \cos \theta & \cos\left(\theta - \frac{2\pi}{3}\right) & \cos\left(\theta + \frac{2\pi}{3}\right) \\
    \sin \theta & \sin\left(\theta - \frac{2\pi}{3}\right) & \sin\left(\theta + \frac{2\pi}{3}\right) \\
    \frac{1}{2} & \frac{1}{2} & \frac{1}{2}
\end{bmatrix}
\]

4. Example verification

In order to verify the accuracy and efficiency of the proposed induction motor TSFT model, this section compares the simulation results and CPU time consumption of the classic induction motor electromagnetic transient model (Matlab/Simulink platform) and the induction motor TSFT model (m file preparation) through the test of the step response of the induction motor load torque. 3-hp induction motor parameters are: 220V, 1725r/min, 60Hz, \( R_s = 0.435 \, \Omega \), \( L_s = 2.0e-3mH \), \( R_r = 0.816 \, \Omega \), \( L_r = 2.0e-3mH \), \( J = 0.089kg\cdot m^2 \).

The load torque step response is mainly to simulate the slow dynamic response process of induction motor when the load torque step changes. In the initial state, the 3-hp induction motor runs without load. When \( t=1.5s \), apply 11.9N\*m load torque to the induction motor. When \( t=2.0s \), the load torque is reversed to -23.8 N\*m. The total simulation time is 3s. Figures 3 and 4 show the dynamic response process of stator current \( i_{A} \) and slip \( s \) of induction motor during load torque step response, respectively.

![Figure 3. Dynamic response diagram of stator A phase current](image)

![Figure 4. Slip rate of induction motor](image)

It can be seen from figure 3 and 4 that the slow dynamic response curves of the two induction motor models are almost the same in the process of load torque step. After calculation, the maximum relative error of the simulation results of the two induction motor models is about 5%. The simulation time of TSFT model of induction motor is 0.266s, and the simulation time of classical induction motor model is 3.974s. Therefore, without sacrificing the necessary numerical accuracy, the calculation efficiency of the TSFT model of induction motor is far higher than that of the classical model.
5. Conclusions
In this paper, based on the VBR model of induction motor and the principle of time scale transformation, the mathematical model of induction motor based on time scale transformation is established. Finally, the validity of the model is verified by an example.

1) Firstly, this paper introduces the principle of TSFT. The TSFT is to construct the analytical signal of the original signal through HT, and to rotate the analytical signal with \( \omega_r \). Different steps can be used for simulation according to the different value of \( \omega_r \).

2) Based on the principle of TSFT and the VBR model of induction motor, a fast simulation model of induction motor is established in this paper. Finally, an example of load torque step response shows that the TSFT model can accelerate the simulation speed while maintaining a certain accuracy.

Acknowledgment
This work was supported in part by open fund of the state key laboratory of power grid security and energy conservation (China Electric Power Research Limited Company).

References
[1] Salon, S. J. (1995) Finite Element Analysis of Electrical Machines. Norwell, MA: Kluwer.
[2] Xiao, Y., Slemon, G. R., Iravani, M. R. (1994) Implementation of an equivalent circuit approach to the analysis of synchronous machines. IEEE Transactions on Energy Conversion, 9(4): 717-723.
[3] Krause, P. C., Wasyczuk, O., Sudhoff, S. D. (2002) Analysis of Electric Machinery And Drive Systems. 2nd. Piscataway, NJ: Wiley-IEEE Press.
[4] Wang, L., Jatskevich, J., Dommel, H. W. (2007) Re-examination of Synchronous Machine Modeling Techniques for Electromagnetic Transient Simulations. IEEE Transactions on Power Systems, 22(3): 1221-1230.
[5] Wang, L., Jatskevich, J., Pekarek, S. D. (2008) Pekarek. Modeling of Induction Machines Using a Voltage-Behind-Reactance Formulation. IEEE Transactions on Energy Conversion, 23(2): 382-392.
[6] Wang, L., Jatskevich, J. (2008) A Voltage-Behind-Reactance Induction Machine Model for the EMTP-Type Solution. IEEE Transactions on Power Systems, 23(3): 1226-1238.
[7] Therrien, F., Chapariha, M., Jatskevich, J. (2015) Constant-Parameter Voltage-Behind-Reactance Induction Machine Model Including Main Flux Saturation. IEEE Transactions on Energy Conversion, 30(1): 90-102.
[8] Yao, S. J., Han, M. X, Huang, W. E. D. (2019) A Large Time Step Electromagnetic Transients Simulation Based on Time-scale-frame Transformation. Proceedings of the CSEE, 39(02):436-446+641.