Refactoring Metasurfaces without Spurious Diffraction

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Abstract—Refraction represents one of the most fundamental operations that may be performed by a metasurface. However, simple phase-gradient metasurface designs suffer from restricted angular deflection due to spurious diffraction orders. It has been recently shown, using a circuit-based approach, that refraction without spurious diffraction, or diffraction-free, can be achieved by a transverse (or in-plane polarizable) metasurface exhibiting either loss-gain, nonreciprocity or bianisotropy. Here, we rederive these conditions using a medium-based – and hence more insightful – approach based on Generalized Sheet Transition Conditions (GSTCs) and surface susceptibility tensors, and experimentally demonstrate two diffraction-free refractive metasurfaces that are essentially lossless, passive, bianisotropic and reciprocal.

I. INTRODUCTION

Metasurfaces represent a powerful electromagnetic technology that has experienced spectacular development over the past lustrum [1], [2]. They have already led to a diversity of applications, including single-layer perfect absorption [3], polarization twisting [4], power harvesting [5], orbital angular momentum multiplexing [6], spatial processing [7] and flat lensing [8], and there seems to be much more to be discovered and developed in this area.

One of the most fundamental operations that a metasurface may perform is refraction and reflection [11], as most metasurface field transformations involve these phenomena. Such operations have been achieved in blazed gratings [12], [13], and in planar phase-gradient metasurfaces [14], however with restriction to small angle differences between the incident and refracted or reflected beams and with the presence of spurious diffraction orders [8], [14], [15], [16].

Fortunately, it is possible to achieve refraction without spurious diffraction, that we shall hereafter refer to as diffraction-free refraction for short, by introducing more complexity in the metasurface design. This has been clearly demonstrated in [14], which shows that such operation may be accomplished by a transverse metasurface if that metasurface exhibits any one of the following three properties:

1) monoisotropy with loss and gain [18];
2) nonreciprocity [14], or
3) bianisotropy [19], [20], [21].

Among these properties, the practically most convenient is certainly the third one, since it allows to achieve diffraction-free generalized refraction with a metasurface that is purely lossless and passive, avoiding complex amplification, and at the same time reciprocal, avoiding non-integrable magnetic materials [22] or complex magnetostatic structures [23], [24], [25].

The work reported in [19], and related experimentation [20], represents the first synthesis of a diffraction-free generalized refractive metasurface. In that paper, the authors use a circuit-based approach with generalized scattering parameters to match the impedances of the oblique incident and transmitted waves across a layered metasurface structure. As a result, they obtain analytical expressions for the admittances of each of the layers constituting the metasurface. Here, as an extension of the short report [21], we present a fundamentally different and also more general approach of the same problem. This approach uses surface susceptibilities synthesized [26], [17], by Generalized Sheet Transition Conditions (GSTCs) [28], and is therefore a medium-based rather than a circuit-based approach, which inherently brings about greater insight into the physics of the problem. Moreover, it treats the metasurface as a global entity, without any restriction regarding its structure, and may therefore accommodate different implementations, via subsequent scattering parameter mapping [17].

II. REFRACTIVE TRANSVERSE METAFLUIDIC SYNTHESIS

A. Generalized Refraction and GSTC Synthesis

The problem of diffraction-free generalized refraction by a metasurface is represented in Fig. 1. The metasurface is placed at $z = 0$ in the $xy$-plane of a cartesian coordinate system. We denote $a$ and $b$ the media, possibly having different electromagnetic properties, bounding the metasurface at $z < 0$ and $z > 0$, respectively. A plane wave, with electric and magnetic fields $\mathbf{E}_{a1}$ and $\mathbf{H}_{a1}$, respectively, impinges from medium $a$ at angle $\theta_a$ onto the metasurface. The metasurface transforms, without any spurious reflection and scattering, this wave into a plane wave, with fields $\mathbf{E}_{b1}$ and $\mathbf{H}_{b1}$, propagating in medium $b$ at angle $\theta_b$.

Fig. 1. Problem of diffraction-free generalized refraction by a metasurface.
We shall next perform the synthesis of the diffraction-free general-
ized refractive metasurface in Fig. 1 using the GSTC-susceptibility
theory described in [7]. This technique consists in specifying
the incident, reflected and transmitted fields and computing the
respective metasurface susceptibility tensors via GSTCs. According
to the definition of diffraction-free generalized refraction, the reflected
field will be specified to be zero and the transmitted field will be
specified as a pure plane wave without any spurious diffraction orders.

In the case of a general transverse bianisotropic metasurface, the
GSTCs, defined at z = 0, are given as
\[ \hat{\Delta} \times \mathbf{H} = j \omega \mu_0 \frac{\chi}{\varepsilon} \mathbf{E} + j \omega \varepsilon_0 \mathbf{H}, \]
\[ \Delta \mathbf{E} \times \hat{z} = j \omega \mu_0 \chi_{\text{mee}} \mathbf{H}, \]
where the \( \Delta \) symbol and 'av' subscript represent the differences and
averages of the tangential electric or magnetic fields on both sides of
the metasurface, and \( \chi_{\text{ee}}, \chi_{\text{em}}, \chi_{\text{mm}} \) are the bianisotropic
susceptibility tensors describing the metasurface. In order to realize
the simplest and most fundamental general refraction operation, we
require the metasurface to be non-gyrotropic so as to avoid polariz-
ation alteration. As a result, the s-polarization and p-polarization
problems are independent from each other and can therefore be
treated separately. We shall therefore, without loss of generally, only
treat the p-polarization problem.

\[ \text{B. Monoanisotropic Metasurface} \]

We heuristically start with a metasurface having the simplest
constitutive parameters, a monoanisotropic metasurface. In the
p-polarization case, for which the metasurface appears monoanisotropic,
the fields corresponding to the scenario of Fig. 4 are

\[ \mathbf{E}_{\text{a1}} = (\cos \theta_x \hat{x} + \sin \theta_x \hat{z}) e^{-j(k_{ax} x + k_{az} z)}, \mathbf{H}_{\text{a1}} = \frac{e^{-j k_{ax} x}}{\eta_a} \hat{y}, \]
\[ \mathbf{E}_{\text{a1}} = T_p(\cos \theta_y \hat{x} + \sin \theta_y \hat{z}) e^{-j(k_{bx} x + k_{bz} z)}, \mathbf{H}_{\text{a1}} = T_p \frac{e^{-j k_{bx} x}}{\eta_p} \hat{y}, \]
where \( T_p \) is the (parallel-polarization) transmission coefficient,
\( \eta_{\text{a,b}} = \sqrt{\mu_{\text{a,b}}/\varepsilon_{\text{a,b}}}, \)
\[ k_{\text{a,b}} = k_{\text{ax, bx}}, \]
\[ k_{\text{a,b}} = k_{\text{ax, bx}} \cos \theta_{\text{a,b}}, \]
where \( k_{\text{a,b}} = \frac{\sqrt{\mu_{\text{a,b}}/\varepsilon_{\text{a,b}}}}{k_0} \) with \( k_0 = \omega/c_0. \)

Based on the above assumption of a transverse metasurface, the
GSTCs [1] involve only the \( x \) and \( y \) components of these fields,
evaluated at \( z = 0 \), which, also accounting for the p-polarization
monoanisotropy, results into

\[ \chi_{\text{ee}}^{\text{xx}} = \frac{-\Delta H_{y1}}{j \omega \mu_0 \mathbf{E}_{\text{av},y1}}, \]
\[ \chi_{\text{yy}}^{\text{yy}} = \frac{-\Delta E_{x1}}{j \omega \mu_0 \mathbf{H}_{\text{y},y1}}, \]
with

\[ \Delta E_{x1} = E_{bx1} - E_{ax1} = T_p \cos \theta_x e^{-j k_{bx} x}/\eta_a - \cos \theta_x e^{-j k_{ax} x}/\eta_a, \]
\[ \Delta H_{y1} = H_{by1} - H_{ay1} = T_p e^{-j k_{bx} x}/\eta_b - e^{-j k_{ax} x}/\eta_p, \]
\[ E_{x1} = \frac{E_{ax1} + E_{bx1}}{2} = \frac{\cos \theta_x e^{-j k_{ax} x} + T_p \cos \theta_x e^{-j k_{bx} x}}{2}, \]
\[ H_{y1} = \frac{H_{ay1} + H_{by1}}{2} = \frac{e^{-j k_{ax} x}/\eta_a + T_p e^{-j k_{bx} x}/\eta_p}{2}, \]
where the subscript ‘1’ has been introduced for later convenience.

The only unknown in these relations is the transmission coefficient,
\( T_p \). This coefficient may be obtained by enforcing power conservation
across the metasurface,

\[ \frac{1}{2} \text{Re} ((E_{ax1} \hat{x}) \times (H_{ay1} \hat{y})) = \frac{1}{2} \text{Re} ((E_{bx1} \hat{x}) \times (H_{by1} \hat{y})), \]
whose resolution for \( T_p \) with (7) yields

\[ T_p = \sqrt{T_p \cos \theta_x/\eta_a \cos \theta_b}, \]
which is thus a fundamental condition for power conserving
diffraction-free refraction.

Inserting (5) into (4) yields the periodic complex susceptibility functions

\[ \text{Re}(\chi_{\text{ee}}^{\text{xx}}) = \frac{-2 k_0 k_p T_p (\eta_a k_a k_{ax} + \eta_p k_b k_{bx}) \sin(\alpha x)}{\epsilon_0 \omega \mu_0 \eta_a (k_a^2 k_{ax}^2 + k_b^2 k_{bx}^2 + 2 k_a k_b k_{ax} k_{bx} T_p^2 \cos(\alpha x))}, \]
\[ \text{Im}(\chi_{\text{ee}}^{\text{xx}}) = \frac{2 k_0 k_p (\eta_a k_a k_{ax} T_p^2 - \eta_p k_b k_{bx} + T_p (\eta_a k_b k_{ax} - \eta_p k_a k_{bx}) \cos(\alpha x))}{\epsilon_0 \omega \mu_0 \eta_a (k_a^2 k_{ax}^2 + k_b^2 k_{bx}^2 + 2 k_a k_b k_{ax} k_{bx} T_p^2 \cos(\alpha x))}, \]
\[ \text{Re}(\chi_{\text{yy}}^{\text{yy}}) = \frac{-2 \eta_a \eta_p (\eta_a k_b k_{bx} + \eta_p k_a k_{ax}) T_p \sin(\alpha x)}{k_a k_b \mu_0 \omega (\eta_a^2 \eta_b^2 T_p^2 + 2 \eta_a \eta_p T_p) \cos(\alpha x)}, \]
\[ \text{Im}(\chi_{\text{yy}}^{\text{yy}}) = \frac{2 \eta_a \eta_p (\eta_a k_a k_{ax} T_p^2 - \eta_p k_b k_{bx} + (\eta_a k_b k_{bx} - k_a k_b k_{bx} T_p^2) \cos(\alpha x))}{k_a k_b \mu_0 \omega (\eta_a^2 \eta_b^2 T_p^2 + 2 \eta_a \eta_p T_p) \cos(\alpha x)}, \]
with \( \alpha = k_{ax} - k_{bx} \). Plots of these functions may be found in [8].

The zero non-imaginary parts of \( \chi_{\text{ee}}^{\text{xx}} \) and \( \chi_{\text{yy}}^{\text{yy}} \), tensorially corresponding
to the loss and gain relations \( \chi_{\text{em}}^{\text{xx}} \neq \chi_{\text{em}}^{\text{yy}} \), \( \chi_{\text{em}}^{\text{xx}} \neq \chi_{\text{em}}^{\text{yy}} \), where the superscripts \( T \) and \( * \) denote the transpose and conjugate operation
respectively [29], indicate the presence of loss (negative imaginary
part) and gain (positive imaginary part) alternating along the
metasurface. This synthesis corresponds to the first way of obtaining a
diffraction-free refractive metasurface, as shown in [14].

\[ \text{C. Biaxial Metasurface} \]

Since specifying a monoanisotropic (or monoisotropic) metasur-
face leads only to the loss and gain option for diffraction-free refraction,
as just found, complexity must be added to the metasurface to
obtain the nonreciprocity and bianisotropy options. The
non-gyrotropy assumption requires \( \chi_{\text{em}}^{\text{xx}} = \chi_{\text{em}}^{\text{yy}} = \chi_{\text{em}}^{\text{xx}} = \chi_{\text{em}}^{\text{yy}} = 0 \),
and hence eliminates 8 of the 16 terms of a transverse metasurface,
and, among the remaining 8 terms, 4 are for p-polarization and 4
are for s-polarization. Therefore, still assuming p-polarization,
only the bianisotropic two terms \( \chi_{\text{em}}^{\text{xx}} \) and \( \chi_{\text{em}}^{\text{yy}} \) can be added to \( \chi_{\text{ee}}^{\text{xx}} \) and
\( \chi_{\text{yy}}^{\text{yy}} \). This 4-element susceptibility set allows for two fundamentally
new possibilities: a) \( \chi_{\text{em}}^{\text{xx}} \neq -\chi_{\text{em}}^{\text{yy}} \) and b) \( \chi_{\text{em}}^{\text{xx}} \neq -\chi_{\text{em}}^{\text{yy}} \). The latter
tensorially generalizes to \( \chi_{\text{em}}^{\text{xx}} = -\chi_{\text{em}}^{\text{yy}} \), where the superscript \( T \)
represents the transpose operation, which is the only condition for
reciprocity in the prevailing non-gyrotropic situation [29],
and the former corresponds thus to a nonreciprocal metasurface.
These two possibilities correspond to options 2) and 3), respectively, in [14].

In each of the two cases, one has to describe the phenomenon
(reciprocity or nonreciprocity) by also specifying the transformation
in the reverse direction, namely the direction from medium \( b \)
to medium \( a \), which brings about two additional equations, leading to
a full-rank matrix system of order 4.

In the nonreciprocal case, one may specify any reverse trans-
formation, such as for instance refraction in different directions or
with \( \alpha = k_{ax} - k_{bz} \). These relations are plotted in Fig. 3 for \( \theta_a = 0^\circ \) and \( \theta_b = 70^\circ \), and considering air on both sides of the metasurface.

**D. Properties of the Synthesized Metasurface**

The metasurface characterized by the susceptibilities in (12) possesses the following properties:

- It is *bianisotropic*, as already noted in Sec. II-C since \( \chi_{xy}^{mm} \neq 0 \) and \( \chi_{yx}^{mm} \neq 0 \).
- As a result of bianisotropy, it is *asymmetric*, as will be shown in the corresponding scattering parameters to be given in Sec. III.
- It is *reciprocal*, as noted in Sec. II-C since \( \chi_{yy}^{mm} = -\chi_{yy}^{mm} \), which is equivalent to \( T_p = \sqrt{\eta_p \cos \theta_x / \eta_p \cos \theta_y} \) (power conservation), tensorially corresponding to the relation \( \chi_{yy}^{mm} = -\chi_{yy}^{mm} \).
- It is *passive* and *lossless*, since \( \chi_{xx}^{em}, \chi_{yy}^{em} \in \mathbb{R} \) and \( \chi_{xy}^{em}, \chi_{yx}^{em} \in \mathbb{I} \), tensorially corresponding to the relations \( \chi_{xx}^{em} = \chi_{xx}^{em} \) and \( \chi_{yy}^{em} = \chi_{yy}^{em} \) (29).
- It is *periodic* in \( x \) with period \( k_{ax} - k_{bx} \), as seen in Fig. 3 corresponding to the periodic field momentum transformation (39) operated by the metasurface.

Since it is periodic, it inherently supports an *infinite number of space harmonics* (31). From the fact that the metasurface is synthesized so as to scatter only in one direction, all the space harmonics (or diffraction orders) *other than that corresponding to the specified refraction angle* must be evanescent (i.e. transformed to leaky or surface waves).

**III. Scattering Parameter Mapping**

The task now is to establish a proper link between the mathematical transverse susceptibility functions (12) and the corresponding real metasurface, composed of an array of scattering particles. Specifically, this includes discretizing these susceptibility functions in subwavelength cells and determining the appropriate particle geometries for all the cells.

To build a metasurface with the assumed purely transverse susceptibility, \( \chi_{xx}^{em} \), we shall proceed as follows:
1) Map the synthesized susceptibility parameters onto normal-incidence scattering parameters. The reason to use normal incidence in the design procedure is twofold. First, this is a necessary condition to ensure that only the transverse terms of the particle susceptibilities or, more precisely, polarizabilities, get excited. Second, this will lead to the simplest possible simulation set-up for each specific particle (step 3 below), obliqueness being produced by the phase gradient between cells. For p-polarization, and assuming that the metasurface is reciprocal and surrounded by air, the corresponding relations are found, following the procedure in (12), as

\[
S_{11}^{xx} = \frac{-2j}{D^{xx}} \left(2k_0 \eta_1 \chi^{xx}_{\text{inc}} + \mu_0 \omega \chi^{yy}_{\text{lin}} - \eta_0^2 \varepsilon_0 \omega \chi^{ee}_{\text{lin}} \right),
\]

\[
S_{22}^{xx} = \frac{2j}{D^{xx}} \left(2k_0 \eta_1 \chi^{xx}_{\text{inc}} - \mu_0 \omega \chi^{yy}_{\text{lin}} + \eta_0^2 \varepsilon_0 \omega \chi^{ee}_{\text{lin}} \right),
\]

\[
S_{21}^{xx} = S_{21}^{yy} = \frac{-j}{D^{xx}} \left(4 + k_0^2 \chi^{xx}_{\text{lin}} \right)^2 + \mu_0 \omega \chi^{ee}_{\text{lin}} \left(-2j \eta_1 + \mu_0 \omega \chi^{yy}_{\text{inc}} \right),
\]

with

\[
D^{xx} = -2j \mu_0 \omega \chi^{yy}_{\text{lin}} + \eta_1 \left(4 + k_0^2 \chi^{xx}_{\text{lin}} \right)^2 + \omega \chi^{ee}_{\text{lin}} \left(-2j \eta_1 + \mu_0 \omega \chi^{yy}_{\text{inc}} \right),
\]

where the x's in the superscript x correspond to the transverse component of the p-polarized fields [Eqs. (3) and (10)], assuming also non-tyropeutropy. We have thus obtained the scattering matrix periodic functions \( S^{xx} \), \( S^{yy} \) and \( S^{xy} \), or \( S^{yx} \) (x, y) in sub-wavelength cells in order to ensure their safe sampling in terms of Nyquist theorem. This leads to the discrete function \( S\hat{xx} \) or \( S\hat{yy} \), or \( S\hat{xy} \) or \( S\hat{yx} \) (x, y), for \( i, ..., N_x \) and \( j, ..., N_y \), where \( N_x \) and \( N_y \) represent the number of cells along the x and y directions, respectively.

2) Select a generic particle structure and geometry that may be adjusted to cover the phase and amplitude range of \( S^{xx} \) or \( S^{yy} \) across the entire metasurface. For simplicity and computational efficiency, compute the scattering parameters (under normal incidence) of each cell separately and within periodic boundary conditions. Even though the final metasurface will be locally aperiodic, i.e., made of different adjacent cells, periodic boundary conditions will reasonably approximate the coupling to slightly different neighbours.

3) Since \( S^{xx} \) or \( S^{yy} \) is periodic, the period includes the complete set of all the different cells, and the overall structure will consist in the periodic repetition of the corresponding super-cell. Now, simulate this supercell within periodic boundary conditions with the specified incidence angle, and optimize the geometry of the particles so as to maximize energy refraction in the specified direction, i.e., specifically, in the proper defraction order corresponding to the supercell.

Note that it is practically difficult to realize scattering particles with purely transverse polarizability and hence purely transverse susceptibility. Practical metasurfaces typically always include small non-zero longitudinal susceptibility terms, \( \chi_{xx}(x, y) \). Such terms are not excited in 3 above, due to normal incidence, but would play a role in 4 above, given the oblique angle. We shall select a generic particle without longitudinal metallizations, to avoid strong perpendicular electric moments, and without transverse loops, to avoid strong perpendicular magnetic moments. We may therefore expect negligible \( \chi_{xx}(x, y) \) and a design essentially correspond to the assumed purely transverse one.

IV. DESIGN OF SCATTERING PARTICLES

We shall design here two diffraction-free refractive transverse metasurfaces to illustrate the theory of the previous sections: the first metasurface with \( (\theta_a, \theta_b) = (20^\circ, -28^\circ) \) at 10 GHz and the second with \( (\theta_a, \theta_b) = (0^\circ, -70^\circ) \) at 10.5 GHz. For this purpose, we shall follow the procedure described in Sec. II. For experimental simplicity, we assume that the metasurface is entirely surrounded by free space \( (k_a = k_b = \infty, \eta_a = \eta_b = \eta_0) \).

As the step 1), we insert the susceptibilities given by (12), with \( k_a = k_b = k_0 \) into (13). This yields the scattering parameter functions

\[
S_{11}^{xx} = \left(-k_0^2 + k_a k_b\right) \sin(k_a x - k_b y) + j k_0 \left(k_a - k_b\right) \cos(k_a x - k_b y),
\]

\[
S_{22}^{xx} = \left(-k_0^2 + k_a k_b\right) \sin(k_a x - k_b y) - j k_0 \left(k_a - k_b\right) \cos(k_a x - k_b y),
\]

\[
S_{21}^{xx} = S_{21}^{yy} = -2j k_0 \sqrt{k_a k_b}\left(k_a^2 + k_b^2\right) \sin(k_a x - k_b y) + j k_0 \left(k_a + k_b\right) \cos(k_a x - k_b y),
\]

It may a priori seem contradictory with the initial assumption of reflection-less refractive to obtain \( S_{11}^{xx} \neq 0 \) and \( S_{22}^{xx} \neq 0 \). However, there is no contradiction if one recalls that Eqs. (13) are associated in the design procedure with normal incidence, both to isolate out transverse susceptibility components and to simulate the cells one by one, whereas the specified incidence angle is generally nonzero. When excited under the specified oblique incidence angle, the metasurface realized by this design methodology will naturally be reflection-less. Note that the metasurface asymmetry predicted in Sec. III-C is still clearly apparent from the fact that \( S^{yy} \neq S^{xx} \), since asymmetry for normal incidence implies asymmetry.

As step 2), we discretize each of the two metasurfaces in 6 different unit cells of size \( 6 \times 6 \text{ mm} \) \((\sim 0.5\text{ mm})\) for the metasurface with \( (\theta_a, \theta_b) = (20^\circ, -28^\circ) \) and \( 5.1 \times 5.1 \text{ mm} \) \((\sim 0.5\text{ mm})\) for the metasurface with \( (\theta_a, \theta_b) = (0^\circ, -70^\circ) \).

As step 3), we choose scattering particles made of three dog-bone shaped metallic layers separated by 1.52 mm-thick \((\sim 0.5\text{ mm})\) Rogers 3003 \((\varepsilon_r = 3, \tan\delta = 0.0013)\) dielectric slabs. The generic dog-bone metallization is shown in Fig. 4(a), while Fig. 4(b) shows the corresponding three-layer unit cell. Each unit
cell is then optimized with periodic conditions using a commercial software (CST Studio 2014), which provides a reasonable initial guess for the geometry of the dog-bone patterns.

As step 4), we combine the six different unit cells into a supercell, which is periodically repeated to form the whole metasurface. Figure 4 (c) and (d) show the generic structure of the supercell. Finally, the supercell, automatically taking into account the exact guess for the geometry of the dog-bone patterns.

Figure 4 (c) and (d) show the generic structure of the supercell. The full-wave simulated fields of the two diffraction-free bianisotropic reciprocal refractive metasurfaces are plotted in Fig. 5. Being perfectly periodic, the metasurface supports in principle an infinite number of space harmonics, as mentioned in the last item of Sec. II-D. In both designs, only the space harmonics \( m = 0 \) and \( m = -1 \) are propagating, while the others are evanescent. The horizontal extent of each figure corresponds to two supercells. The propagating field includes the space harmonics \( m = 0 \) and \( m = -1 \), whose directions are indicated by the arrows in the center, while the other space harmonics are evanescent. (a) Metasurface with \((\theta_a, \theta_b) = (20^\circ, -28^\circ)\) at 10 GHz. (b) Metasurface with \((\theta_a, \theta_b) = (0^\circ, -70^\circ)\) at 10.5 GHz. (c) Superell composed of 6 unit cells, front view. (d) Supercell perspective view.

**Table I**

| Layer | W   | L   | G   | S   |
|-------|-----|-----|-----|-----|
| Cell 1| 4   | 0.25| 0.5 | 0.5 |
| Cell 2| 3.75| 0.5 | 0.75| 0.5 |
| Cell 3| 4.25| 0.5 | 1   | 0.5 |
| Cell 4| 3.75| 0.5 | 1   | 0.5 |
| Cell 5| 4.25| 0.5 | 1   | 0.5 |
| Cell 6| 4.25| 0.5 | 1   | 0.5 |

**Table II**

| Layer | W   | L   | G   | S   |
|-------|-----|-----|-----|-----|
| Cell 1| 3   | 0.5 | 0.375| 0.5 |
| Cell 2| 2.25| 0.5 | 0.5  | 0.5 |
| Cell 3| 3.75| 0.5 | 0.75 | 0.5 |
| Cell 4| 4.25| 0.5 | 1   | 0.5 |
| Cell 5| 4.25| 0.5 | 1   | 0.5 |
| Cell 6| 4.25| 0.5 | 1   | 0.5 |

**V. SIMULATION AND EXPERIMENT**

The full-wave simulated fields of the two diffraction-free bianisotropic reciprocal refractive metasurfaces are plotted in Fig. 5. Being perfectly periodic, the metasurface supports in principle an infinite number of space harmonics, as mentioned in the last item of Sec. II-D. In both designs, only the space harmonics \( m = 0 \), \( m = -1 \) and \( m = +1 \) are propagating, while the others are evanescent, and the incident and refracted waves correspond to the space harmonics \( m = 0 \) and \( m = -1 \), respectively. Ideally, from synthesis, 100% of the scattered power should reside in the \( m = -1 \) space harmonic. Practically, the harmonics \( R_0, R_{-1}, R_{+1}, T_0 \) and \( T_{+1} \) are also weakly excited, due to the imperfections of the metasurface associated with discretization and fabrication restrictions (essentially limited resolution of the metallic particles), already taken into account at this simulation stage.

The corresponding scattering parameter simulations are shown in Fig. 6 and Fig. 7 for the \((\theta_a, \theta_b) = (20^\circ, -28^\circ)\) metasurface and the \((\theta_a, \theta_b) = (0^\circ, -70^\circ)\) metasurface, respectively. As expected from
synthesis, most of the incident power, except for small conducting and dielectric dissipation loss and negligible coupling to undesired space harmonics, is refracted to the specified direction ($\sim -0.6$ dB for the $(\theta_a, \theta_b) = (20^\circ, -28^\circ)$ metasurface and $\sim -0.9$ dB for the $(\theta_a, \theta_b) = (0^\circ, -70^\circ)$ metasurface) with low reflection ($< -15$ dB).

Fig. 6. Scattering parameters of the $(\theta_a, \theta_b) = (20^\circ, -28^\circ)$ 10 GHz metasurface. (a) Transmitted propagating space harmonics. (b) Reflected propagating space harmonics. The $T_{\pm 1}$ and $R_{\pm 1}$ harmonics are also propagating but not visible in these graphs as their magnitudes are lower than -80 dB.

Fig. 7. Scattering parameters of the $(\theta_a, \theta_b) = (0^\circ, -70^\circ)$ 10.5 GHz metasurface. (a) Transmitted propagating space harmonics. (b) Reflected propagating space harmonics. (Note that the optimum has been up-shifted by around 0.1 GHz in the optimization procedure.)

The two metasurfaces were fabricated and measured. Figure 8 shows a photograph of them. In the measurement, we used a horn antenna placed $\sim 400$ mm from the metasurface and a near-field probe scanning over a plane parallel to the metasurface in the transmission region. We then applied a near-field to far-field transformation [32] to evaluate the transmission response of the metasurface. The measurement results are shown, superimposed with the simulations, in Figs. 9 and 10 for the $(\theta_a, \theta_b) = (20^\circ, -28^\circ)$ metasurface and the $(\theta_a, \theta_b) = (0^\circ, -70^\circ)$ metasurface, respectively. The observed discrepancies between simulation and measurement may be attributed to different factors, including fabrication tolerance, horn antenna excitation (instead of ideal plane wave) and probe antenna imperfection (spurious edge diffraction).

The performance of our metasurfaces is limited only by dissipation loss in the metal scatterers and in the dielectric. In [33], the authors established a theoretical limit in efficiency for a lossless monoanisotropic metasurface, which is found to be $\sim 76\%$ for $(\theta_a, \theta_b) = (0^\circ, -70^\circ)$; it is interesting to note that our bianisotropic metasurface exceeds this lossless monoanisotropic limit by around 5% despite the natural presence of loss. The experimental work in [20], for a similar wide-angle refraction, had lower efficiency due to higher scattering into other diffraction orders and higher absorption compared to our metasurface.

Fig. 8. Photographs of the two fabricated metasurfaces. (a) $(\theta_a, \theta_b) = (20^\circ, -28^\circ)$ metasurface. (b) $(\theta_a, \theta_b) = (0^\circ, -70^\circ)$ metasurface.

Fig. 9. Measured (thick lines) and simulated (thin lines) scattering parameters in transmission of the $(\theta_a, \theta_b) = (20^\circ, -28^\circ)$ metasurface.

VI. CONCLUSION

We have derived the conditions for diffraction-free refraction in a metasurface using a medium-based approach based on Generalized Sheet Transition Conditions (GSTCs) and surface susceptibility tensors, and experimentally demonstrated two diffraction-free metasurfaces that are essentially lossless, passive, bianisotropic and reciprocal.

Following [14], we have considered refractive metasurfaces possessing only transverse susceptibility components. However, diffraction-less refraction might also be achieved by metasurfaces including normal polarizabilities, which would lead to other possibilities than the three reported in [14]. However, solving the synthesis problem for metasurfaces with nonzero normal susceptibility components is not trivial since the corresponding GSTCs relations form a set of differential equations instead of just algebraic equations. At this stage, the design of such structures remains an open avenue for further investigation.
Fig. 10. Measured (thick lines) and simulated (thin lines) scattering parameters in transmission of the $(\theta_a, \theta_b) = (0^\circ, -70^\circ)$ metasurface. Note that the optimal frequency after fabrication was up-shifted by about 0.2 GHz compared to the simulation.

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