Dust settling in magnetorotationally-driven turbulent discs – II.
The pervasiveness of the streaming instability and its consequences

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ABSTRACT
We present a series of simulations of turbulent stratified protostellar discs with the goal of characterizing the settling of dust throughout a minimum-mass solar nebula. We compare the evolution of both compact spherical grains, as well as highly fractal grains. Our simulations use a shearing-box formulation to study the evolution of dust grains locally within the disc, and collectively our simulations span the entire extent of a typical accretion disc. The dust is stirred by gas that undergoes magnetorotational instability-driven turbulence. The turbulence tends to lift the dust above the disc’s mid-plane, while gravity tends to draw it back to the mid-plane. This establishes a steady state scaleheight for the dust that is different for dust of different sizes. This sedimentation of dust is an important first step in planet formation and we predict that the Atacama Large Millimeter Array (ALMA) should be able to observationally verify its existence.

When significant sedimentation occurs, the dust will participate in a streaming instability that significantly enhances the dust density. We show that the streaming instability is pervasive at many of the outer radial stations in a disc. We characterize the scaleheights of dust whose size ranges from a few micrometres all the way up to a few centimetres. We find that for spherical grains, a power-law relationship develops for the scaleheight with grain size, with a slope that is slightly steeper than $-1/2$. The sedimentation is strongest in the outer disc and increases for large grains. The results presented here show that direct measurements of grain settling can be made by ALMA and we present favourable conditions for observability. The streaming instability should also be directly observable and we provide conditions for directly observing it. We calculate collision rates and growth rates for the dust grains in our simulations of various sizes colliding with other grains, and find that these rates are significantly enhanced through the density enhancement arising from the streaming instability.

Key words: instabilities – MHD – turbulence – methods: numerical – planetary systems: protoplanetary discs.

1 INTRODUCTION
Dust has many important roles in evolving protostellar discs. Dust regulates the heating and cooling of the gas in the disc through photoabsorption and shielding, and thus determines the temperature structure. As the disc evolves, its dust and gas distributions change, with concomitant evolution in the spectral energy distribution (SED) (Wood et al. 2002; Whitney et al. 2003). The evaluation of SEDs of discs becomes an important diagnostic of the distribution and nature of the dust in these discs (Wolf & Hillenbrand 2003), as well as the overall structure of young protostellar discs like HH 30 (Burrows et al. 1996). However, the dust distributions for such SED modelling has always been derived from analytical models, such as Dubrulle, Morfill & Sterzik (1995) or Miyake & Nakagawa (1993). These models are based on a certain distribution of turbulence in the disc, and it has now become possible to test these distributions through the use of dynamical models of dust and gas (Balsara et al. 2009, hereafter Paper I). Dust also plays an important role in building planets. The settling of dust to the mid-plane of an accretion disc and its resultant coagulation and growth plays an important role in planet formation (Safronov 1969;...
Goldreich & Ward 1973). A derivation of dust distributions that is obtained from a dynamical model is presented here and it is one of the goals of this paper to catalogue such information at several representative radii in a minimum-mass solar nebula (MMSN).

Dust is a key component in the process of planet formation. Observations of young stellar discs show that the removal of gas from the disc can occur in \( \sim 10 \) Myr, setting a limit on the time frame available for growing Jovian planets (Sicilia-Aguilar et al. 2005). The core accretion model for the formation of gas giant planets postulates the initial formation of a rocky core on to which the gas can accrete. Mizuno (1980) and Pollack et al. (1996) show that radiative effects limit the rate of accretion in the core accretion model. Their calculations show that forming a gas giant planet requires more than a few million years. If coagulation and sedimentation occur, however, the opacity from dust could be significantly reduced and the time-scale for planetary core formation can be reduced significantly (Hubickyj, Bodenheimer & Lissauer 2005). Planet migration within the disc could also accelerate the growth of planetary cores (Rice et al. 2003). Gravitational instability models for gas giant planet formation, which rely on collective processes in a massive disc acting on global scales, suggest that these planets can form on much smaller time-scales, as low as a few thousand years (Boss 1997, 2008). Furthermore, driving gravitational instabilities might require very massive discs, with masses that exceed several per cent of the mass of the central star. Reconciling the core accretion and gravitational instability models with the observed disc masses and time-scales is thus a necessary step for progress in understanding planet formation. We explore here the evolution of dust within gaseous turbulent discs, with the goal of characterizing the time-scales for possible grain growth in such discs.

Coagulation can occur when dust grains collide, and this process can grow dust into pebble- and boulder-sized objects. These more massive particles are not as efficiently stirred by the turbulent motions in the gas, and can sediment into a thinner, more gravitationally unstable layer. Grain growth needs to be fast through this regime, as particles of these intermediate sizes are subject to a headwind-induced drag force that can lead them to spiral in to the protostar in a few hundred orbital time-scales (Weidenschilling 1977). Therefore, it is important to seek out rapid mechanisms for coagulation and grain growth. As coagulation is essentially proportional to the square of the density, the problem becomes one of raising the density sufficiently high to ensure that coagulation is fast. Whatever the mechanism for coagulation, the dust density in discs is low enough that it forms an essentially pressureless fluid. It will sediment to the mid-plane unless it can be stirred by turbulent motions transmitted to it by the gaseous component of the disc. Observations by Rettig et al. (2006), Bouwman et al. (2008) and Fedele et al. (2008) show evidence that dust does at least partially decouple from the gas and settle towards the mid-plane in accretion discs. Sedimentation will lead to greater dust densities in the mid-plane, and hence more rapid grain growth.

Dust densities can be further enhanced by significant amounts if it is subject to growing instabilities. Even in the absence of self-gravity, such instabilities do exist. Johansen, Henning & Klahr (2006) examined the turbulence produced by the Kelvin–Helmholtz instability in 2D models of stratified discs. The streaming instability of Youdin & Goodman (2005) can occur on fast time-scales if the density of the dust is comparable to the gas density in the mid-plane. The streaming instability develops when grains get entrained in the wake of nearby grains as they plough through the gas. This entrainment reduces the headwind that a grain experiences relative to the grains around it, leading it to move with a similar velocity. Thus, dust grains in close proximity to each other will start to move collectively. Initial studies of this phenomenon in unstratified discs were performed by Youdin & Johansen (2007) and Johansen & Youdin (2007). Further studies by Johansen et al. (2007) extended these results to the case where dust self-gravity was incorporated, allowing the large dust grains to sediment into a thin layer within an unstratified gas disc, and then grow due to the combined effects of the streaming and gravitational instabilities. Paper I examined the case with grains having a distribution of sizes in a stratified gaseous disc and found that the largest grain sizes can sediment to the mid-plane and undergo a streaming instability while smaller grains remain well mixed with the gas. They studied shearing sheet models that were stationed at two radii, 0.3 and 10 au, in an MMSN. However, the streaming instability in this work only prevailed at one radius, 10 au. It is interesting to examine whether the instability persists at several radii in the disc, and if it becomes more vigorous at larger radii. In particular, the reduced gas densities in the outer regions of a protostellar disc should lead to stronger sedimentation in smaller grains. If the streaming instability can develop in these grains as well, it could serve as an even better precursor to planetesimal formation. Furthermore, it should be observable in the outer parts of these discs. Sedimentation of smaller grains would also be easier to observe and in this paper we show that ALMA could be well positioned to observationally verify sedimentation in the outer parts of protostellar discs. A further goal of this paper is to show that the streaming instability is indeed quite pervasive in protostellar discs.

The shapes of dust grains in discs are not well constrained by the data. Models and experiments of dust growth suggest that micrometre-sized dust grains might form a variety of fractal shapes (Ossenkopf 1993; Henning & Stognienko 1996; Blum et al. 2000) that reduce the mass as \( \sigma/m \sim \sigma/m^{0.5} \). One of the goals of this study is to understand the differences in the way spherical and fractal dust sediment in a protostellar disc. Due to the lack of direct observations of the shapes of grains, we simulate the settling of spherical grains and BCCA grains, which represent limits for the most compact and least compact grains we would expect to find in protostellar discs. By characterizing the behaviour of spherical and BCCA grains, we will be able to understand the role of fractal dust in sedimentation.

We present simulations of dust sedimentation in an MMSN through shearing-box simulations performed at several radial stations around a 0.5 M\(_\odot\) star. The disc is dynamically active and undergoes turbulence generated from the magnetorotational instability (MRI), with levels of...
turbulence that are consistent with observed accretion rates seen in T Tauri-type protostars (Hartmann 1998). We incorporate an ensemble of dust grains with sizes ranging from 2.5 \( \mu \)m to 2.5 cm, with mass ratios derived from the number distribution from Mathis, Rumpl & Nordsieck (1977; MRN distribution henceforth) and a total dust-to-gas mass ratio of 0.01. We consider the role of both spherical and fractal dust grains. The structure of the turbulence has been described in Paper I for standard dust-to-gas ratios at 10 au. In this paper, we extend our study to examine the structure of the dust throughout the disc from 0.3 to 300 au, and to study the dynamics of MRI turbulence in gas-depleted discs.

In Section 2, we describe our simulation techniques, initial conditions and the parameter space we explore. Section 3 describes the results of our simulations and the consequences for developing the streaming instability throughout a disc. We summarize our conclusions in Section 4.

2 DESCRIPTION OF INITIAL CONDITIONS, GOVERNING EQUATIONS AND SET-UP OF THE SIMULATIONS

We perform a suite of simulations in a shearing-box geometry to model the evolution of the gas and dust at several stations throughout a protostellar disc, using the Riemann magnetohydrodynamics (MHD) code (Balsara 1998a,b; Balsara & Spicer 1999a,b; Balsara 2004). We evolve the gas as a continuous fluid using the equations of ideal MHD. The particles are treated using a superparticle approximation, so that one computational superparticle represents an ensemble of dust grains. The forces arising from the interaction of these superparticles with the mesh-based fluid are interpolated to and from the underlying grid using triangular-shaped cell (TSC) interpolation. The particles do not interact with each other directly, but only through their interactions with the gas.

We detail the equations for dust–gas friction and our numerical methods for solving them in Paper I. The friction force per unit volume is

\[ f_d = \alpha \rho_d \rho_g (v_g - v_d), \]

where \( \rho_g \) and \( v_g \) are the density and velocity of the gas, and \( \rho_d \) and \( v_d \) are the density and velocity of the dust grains. The friction coefficient \( \alpha \) described the strength of the momentum transfer between dust and the gas. We parameterize this friction coefficient through an Epstein drag law when the dust grain size \( r_j < \frac{1}{4} \lambda_{\text{MFP}} \) (Cuzzi, Dobrovolskis & Champney 1993), and a Stokes draw law when the grain size is larger than this. The form of the coefficient under these two drag laws, \( \alpha_{\text{Epstein}} \) and \( \alpha_{\text{Stokes}} \), are

\[ \alpha_{\text{Epstein}} = \frac{4\sigma}{3m} \sqrt{c_s^2 + |v_d - v_g|} \]

\[ \alpha_{\text{Stokes}} = \frac{3D}{8 \rho_d \sigma_{\text{grain}}} |v_d - v_g|, \]

where \( \sigma_d \) is the cross-section of the grain, \( m_d \) is the mass of the grain, \( c_s \) is the isothermal sound speed, \( \rho_{\text{grain}} \) is the mass of the solid material from which the grain is made and \( D \) is a coefficient that depends on a Reynolds number \( \text{Re}_{\text{grain}} = 2r_j |v_j - v_g| / \nu \) of the gas moving past the dust grain (\( \nu = 0.5 c_s \lambda_{\text{MFP}} \) is kinematic viscosity of the gas; Lang 1999):

\[ D = \begin{cases} 
24 / \text{Re}_{\text{grain}} & \text{if } \text{Re}_{\text{grain}} \leq 1 \\
24 / \text{Re}_{\text{grain}}^{0.6} & \text{if } 1 < \text{Re}_{\text{grain}} \leq 800 \\
0.44 & \text{if } \text{Re}_{\text{grain}} > 800 
\end{cases} \]

The stopping time for a particle \( \tau_s = (\alpha \rho_g)^{-1} \).

The mass and cross-section of the dust grains depend on whether the dust is spherical or fractal. For spherical grains, the cross-section and mass are \( \sigma_d = \pi r_d^2 \) and \( m_d = \frac{4}{3} \pi \rho_{\text{grain}} r_d^3 \), respectively. For fractal grains, we use the BCCA models of Ossenkopf (1993), a particularly loose and fluffy fractal model that likely represents an extreme of grain non-compactness. In this model, if \( r_d \) represents the extremal radius of the grain (i.e. the smallest radius that encloses the entire grain), then in the fractal limit the filling fraction of grain material within the extremal radius is \( f_d = 0.279 (r_d/0.01 \mu \text{m})^{-1.04} [1 + 4.01 (r_d/0.01 \mu \text{m})^{-1.54}] \) (Henning & Stognienko 1996), where we have followed Henning & Stognienko (1996) and Suttner & Yorke (2001) in assuming that the constituent particles that make up the fractal grain are 0.01 \( \mu \)m in size. The mass of the grain is thus \( m_d = f_d m_{\text{q bluff}} \) and the cross-section is \( \sigma_d = 0.692 f_d \rho_{\text{q bluff}} (r_d/0.01 \mu \text{m})^{0.95} f_d^{0.95} [1 + 0.301 / \ln f_d + 3 \ln (r_d/0.01 \mu \text{m})] \), where \( m_{\text{q bluff}} \) and \( \sigma_{\text{q bluff}} \) are the mass and cross-section of a spherical grain with the same extremal radius.

We perform three sets of eight simulations. Table 1 provides a list of simulation parameters for the MMSN model. The first set of simulations evolves spherical dust grains in a MMSN, with a dust-to-gas mass ratio of 0.01. The second set considers BCCA fractal grains in a MMSN, also with a dust-to-gas mass ratio of 0.01. The final set considers spherical grains in a gas-depleted disc, such that the gas density has decreased by a factor of 10 while the dust mass remains the same as in the first set. In other words, \( \Sigma \) is one-tenth of its values in Table 1 for this gas-depleted set of simulations. Thus, the dust-to-gas ratio in this suite of simulations is 0.1, and might represent a later transitional stage in the evolution of the disc.

As the shearing-box approximation requires that we simulate only a small portion of a disc, in each set of simulations we perform eight individual simulations at different radial locations spanning the entire likely range of a MMSN, from 0.3 to 300 au. We initialize the simulations as described in Paper I, using a resolution of \( 96 \times 96 \times 192 \) zones in the \( x, y, \) and \( z \) directions, respectively. To increase our resolution in the vertical direction, we use a ratioed grid spacing in \( z \) above and below the mid-plane with a ratio of 1.00732. In the first two sets of simulations, we assume a disc surface mass density of \( \Sigma(R) = (1700 \text{ g cm}^{-2})(R/1 \text{ au})^{-1.5} \) and temperature distribution from D’Alessio...
Table 1. List of simulation parameters.

| Radius (au) | T (K)  | $c_s$ (km s$^{-1}$) | $H_{gas}$ (au) | $\Sigma$ (g cm$^{-2}$) | $\rho$ (g cm$^{-3}$) | $t_{ caut}$ (yr) |
|------------|--------|---------------------|----------------|--------------------------|------------------------|----------------|
| 0.3        | 365.0  | 1.33                | 0.0146         | $1.04 \times 10^4$      | $2.67 \times 10^{-8}$  | 0.232          |
| 1.0        | 232.6  | 1.06                | 0.0711         | $1.70 \times 10^3$      | $9.02 \times 10^{-10}$ | 1.41           |
| 3.0        | 148.1  | 0.845               | 0.295          | $3.28 \times 10^2$      | $4.17 \times 10^{-11}$ | 7.35           |
| 10.0       | 95.85  | 0.680               | 1.44           | 53.8                     | $1.41 \times 10^{-12}$ | 44.7           |
| 30.0       | 69.12  | 0.577               | 6.37           | 10.4                     | $6.13 \times 10^{-14}$ | 2.32 $\times 10^2$ |
| 100.0      | 50.24  | 0.492               | 33.0           | 1.70                     | $1.94 \times 10^{-15}$ | 1.41 $\times 10^3$ |
| 300.0      | 42.72  | 0.454               | 56.0           | 0.925                    | $6.24 \times 10^{-16}$ | 2.60 $\times 10^3$ |

et al. (1998). In the third set of simulations, we use $\Sigma(R) = (170 \text{ g cm}^{-2})(R/1 \text{ au})^{-1.5}$, i.e. 10 times lower. From the corresponding sound speed, assuming an isothermal equation of state, and a rotation rate $\Omega^2 = GM/R^3$, we obtain a scaleheight $H_{gas} = \sqrt{2 c_s}/\Omega$. We use this to derive a mid-plane gas density, $\rho_0 = \sqrt{\pi \Sigma(R)}/H_{gas}$. Our initial conditions for the gas disc as a function of height above the mid-plane are Gaussian profiles $\rho_g = \rho_0 e^{-z^2/H_{gas}^2}$. We initialize our dust particles within 2.5 gas scaleheights of the mid-plane so that they have the same vertical profile as the gas density. We set the masses of the superparticles such that the total mass of the dust of all families is either 1 per cent of the gas mass (Sets 1 and 2) or 10 per cent (Set 3). The dust grains are divided into five dust families of different sizes from 2.5 $\mu m$ to 2.5 cm (where ’size’ refers to either the radius of the spherical dust grain in Sets 1 and 3, or the extremal radius of the fractal dust grains in Set 2), and the relative fraction of the total dust mass in each family is given by an MRN distribution (Mathis et al. 1977).

In our simulations we do not include the effects of coagulation and growth, which take place over time-scales that are substantially longer than the ones simulated here. While Bouwman et al. (2003) have measured the grain distribution in HD 100546 and found a flatter distribution $N(r) \sim r^{-2}$, such a distribution represents the specific history of coagulation and growth in one system. As a result, the use of an MRN distribution in this paper represents but one possible conservative choice. Most of the qualitative results in this paper associated with the pervasiveness of the streaming instability would remain unchanged when different grain distributions are tried. Some of the quantitative results associated with collision times and mean free paths would depend sensitively on the dust distribution. However, the consequence of having a flatter distribution would be to collect a greater percentage of the dust mass in a few large particles. With more large dust grains, the collision times between large grains would be reduced. Consequently, flatter distributions of grains would only enhance our conclusions about vigorous grain growth in the presence of instabilities.

3 RESULTS

The dust layers in our simulations experience sedimentation, as found in Paper I. As the density of these layers increases due to sedimentation, the dust layer begins to break up into clumps due to collective effects from the streaming instability. We show in Section 3.1 the role played by these collective instabilities in maintaining a scaleheight for the dust. In Section 3.2, we further examine the nature of this collective instability.

3.1 Dust scaleheights

The thickness of the dust layer in a disc is related to the interplay between the turbulence in the dust and gravity. If the turbulence affects the dust as a diffusion process, and the dust is well coupled to the gas, then balancing the diffusion rate with the sedimentation rate leads to an estimate for the ratio of dust and gas scaleheights that varies as $H_d/H_{gas} \propto \tau_{\epsilon}^{-1/2}$ (Dubrulle et al. 1995), where $\tau_{\epsilon} = (\alpha \rho_g)^{-1}$ is the friction time-scale between dust and gas. In the Epstein regime, the friction time-scale and grain size are related as $\tau_{\epsilon} \propto r_{\epsilon}$ for spherical grains and hence $H_d/H_{gas} \propto r_{\epsilon}^{-1/2}$. As a result, the thickness of the dust layers for grains of different sizes will vary with grain size. The estimate of Dubrulle et al. (1995) required several assumptions in order to make the problem analytically tractable. First, they assumed that the gaseous component of the disc was in hydrostatic equilibrium, with no vertical motions, so that the role of gravity was to induce sedimentation at the terminal velocity of the grains. If the gaseous disc is turbulent as well, this assumption breaks down. Secondly, they assumed a particular $\alpha$-disc model for the turbulence in their calculation of the dust diffusion rate. Our goal here is to determine how the dust scaleheights are set when we use 3D self-consistent simulations of turbulence driven via the MRI in the gaseous component of the disc, which in turn generates turbulence in the dust via frictional forces.

We can characterize the vertical distribution of dust by interpolating the dust density to the mesh and averaging in each $x$–$y$ plane to create a vertical density profile. The Dubrulle et al. (1995) model predicts that the vertical distribution of dust remains Gaussian as sedimentation takes place. Numerical studies suggest that above 2–3 scaleheights, the dust is strongly depleted relative to a Gaussian profile, which Fromang & Nelson (2009) attribute to increased velocity fluctuations in the upper layers of the disc. However, the dust does maintain a Gaussian profile within the inner two scaleheights, especially for larger grains that experience sedimentation (Fromang & Nelson 2009; Paper I). Thus, the use of a Gaussian profile to estimate the vertical distribution of sedimented dust is valid. We can thus fit the Gaussian profile $\rho(z) = \rho_0 e^{-z^2/H_{gas}^2}$ to the vertical density profile in order to extract a scaleheight for the dust of each size. We note that both the form $e^{-z^2/H_{gas}^2}$ and $e^{-z^2/2H_{gas}^2}$ commonly appear in the literature; we have chosen the former. The scaleheights calculated according to the former are larger by a factor of $\sqrt{2}$ than the latter. Generally, however, we will be making comparisons of the ratio of dust-to-gas scaleheights, for which this factor of $\sqrt{2}$ cancels out.
We can use the scaleheight information to test whether our simulations have reached a steady-state balance between turbulent mixing and sedimentation. We plot the scaleheight $H_d$ for the spherical grain simulations as a function of time in Fig. 1. Sedimentation begins immediately in all of the simulations as the dust responds to the initial conditions, with the sedimentation occurring slowly for small (i.e. well-coupled) grains and quickly for larger grains. At the smaller radii, the smallest dust undergoes no sedimentation at all, i.e. $H_{\text{dust}} \sim H_{\text{gas}}$. At distances greater than 30 au, we also see that the largest dust grains are so poorly coupled to the gas that they undergo underdamped oscillations through the mid-plane. This behaviour is possible because our superparticle treatment of the dust allows streams of particles to interpenetrate; a continuum (mesh-based) representation of the dust would not show it, and indeed may not be valid when $\Omega r_s \sim 1$ (Garaud & Lin 2004). The oscillations of the 2.5 cm grains continue even at the end of our simulations at 100, 150 and 300 au, as do the 2.5 mm grains at 300 au. The stopping times for these dust grains are several tens of orbits, and as a result we do not expect them to reach a steady state over the duration of these simulations. As our goal in this work is to characterize the dust once it has settled, we exclude those grain families that have not reached a steady state from our analysis. The dust scaleheights of the other dust families do, however, become practically constant when averaged over one orbital time, and we consider these to have reached a steady state suitable for further analysis.

The scaleheights for the BCCA fractal dust are quite different, as shown in Fig. 2 for our simulations of fractal grains at 0.3 and 10 au. The simulations at 1, 3, 30, 100, 150 and 300 au show very similar trends to the simulation at 10 au, and are not displayed here. The fractal grains of
different sizes have nearly identical scaleheights everywhere throughout the disc, as the ratio of cross-sectional area to mass is nearly constant with grain size and hence the grains experience nearly identical accelerations. The low volume filling fractions for the fractal grains result in significantly smaller grain masses than for spherical grains at centimetre sizes. Consequently, the gas–dust friction forces are reduced by a factor of $\sim3 \times 10^{-2}$ for fractal grains compared to spherical grains of the same size, and are more than sufficient to keep the dust well mixed with the gas. The one exception to the concordance between BCCA fractal grains of different sizes is for the 2.5 cm grains at 0.3 au, which have experienced some degree of sedimentation as a consequence of being in the Stokes rather than the Epstein regime. Our choices of spherical and BCCA grains are essentially the most extreme grain shapes we could have chosen, in terms of compactness versus non-compactness. Other fractal grain models tend to be somewhere within this range. Ballistic particle–cluster aggregation (BPCA) fractal models (Ossenkopf 1993; Henning & Stognienko 1996), for instance, are significantly more centrally condensed than BCCA grains and have masses that scale with radius similarly to spherical grains, but with a filling fraction of $\sim4.6$ per cent. Such grains would thus be expected to behave intermediate to the spherical and BCCA grains we discuss here. Our simulations thus bound the dust scaleheights that are likely achieved in protostellar discs.

In Fig. 3(a), we plot the scaleheight of the dust in physical units as a function of radius in the disc after it has reached the steady state seen in Figs 1 and 2. The scaleheight of the gas is shown by the solid black line and provides an upper bound to the scaleheight of the dust.

![Figure 3. Dust and gas scaleheights as a function of radius in the disc. Top: MMSN. Bottom: Disc with the gas density depleted by a factor of 10.](image-url)
The spherical grain scaleheights (Fig. 3a) are well mixed with the gas and closely follow the gas scaleheights in the inner disc, a trend that can also be seen in Fig. 1. At a distance of a few au, the largest (2.5 cm) grains begin to sediment out, developing into an almost flat profile with increasing radius. Successively smaller families of dust do show strong sedimentation at larger radii. The greater sedimentation in the outer disc arises from two factors; the increased dust–gas friction time-scale due to the reduced gas density in the outer disc, leading to poor coupling; and the weaker turbulence available to stir the dust layer away from the mid-plane, due to the decreased sound speed from the lower temperature in the disc.

In order to fit scaleheights to the dust, we have interpolated the particle densities back to the mesh. The Gaussian fits to the scaleheight that we calculate will be at least as large as the resolution of the grid, \( \sim 2\Delta z_{\text{mid-plane}} \), which increases with the radius in the disc. This resolution limit is shown with a dashed line in Fig. 3, and we can see that the 2.5 cm grains at 30 au and the 2.5 mm grains at 150 au fall very close to this line. The 2.5 cm grains at 100 au and beyond, and the 2.5 mm grains at 300 au, are oscillating rather than settling and are not shown. Even though these sedimented grains only interact with a few zones in the vertical extent, they still sample the time evolution of the gas turbulence (Carballido, Fromang & Papaloizou 2006). It is an open question as to whether the time sampling of the gas turbulence by the dust is sufficient to allow us to measure the density structure of the dust on scales smaller than the grid scale. The resolution limit marked by the dashed line in Fig. 3 shows that the dust families that do settle over the course of our simulations are all at least marginally resolved on our original mesh.

We also show in Fig. 3(b) the scaleheights for a disc with the same initial conditions as the MMSN, but in which the gas density has been decreased by a factor of 10. The scaleheights have been substantially decreased for all grains as the momentum available to stir the dust has been reduced. The mass of dust in these simulations is the same as those for the MMSN discussed previously, and so the smaller scaleheights in turn lead to an increased dust density in the mid-plane. We will show later that this has important consequences for the possibility of grain growth in later stages of disc evolution.

The differences in the degree of sedimentation between the dust of different sizes will have important consequences for the prospects of observing dust in protostellar systems. We can use the scaleheights in Fig. 3 to interpolate the scaleheight of dust of a particular size at any point in the disc, using a linear interpolation method in \( \log(r) \) and \( \log(H_d) \). Similarly, we can calculate the density of dust in the mid-plane as a function of radius in each of our simulations, and interpolate those mid-plane densities to any point in the disc. With these interpolated scaleheights and mid-plane densities for dust of different sizes as a function of radius, we can construct the global density structure of the disc in the \( \log r \)–\( \log z \) plane. This is shown in Fig. 4, along with the minimum resolution of our data. Note that the axes are logarithmically scaled. Neither the 2.5 cm dust at 100, 150 and 300 au nor the 2.5 mm dust at 300 au have reached a steady state by the end of the simulation; as a result, we do not display the density of 2.5 cm dust beyond 30 au, or 2.5 mm dust beyond 150 au, in Fig. 4. However, all of the scientific goals of this paper can be achieved by using the dust families that have indeed reached steady state.

The density structure of the small, 2.5 \( \mu \)m dust closely follows that of the gas. Similarly, the 25 \( \mu \)m grains closely follow the gas density in the inner parts of the disc, but start to sediment out beyond 100 au. Sedimentation begins to appear in the 250 \( \mu \)m grains at \( \sim 30 \) au, and drops precipitously at 150 au. The 25 and 250 \( \mu \)m grains have become clearly separated from the 2.5 \( \mu \)m grains in the outer disc, and we predict that the distinction in their scales’ heights could be observable with high-resolution instruments viewing edge-on discs. The ALMA 350 \( \mu \)m and 1 mm bands (Wootten & Thompson 2009), for instance, would be able to resolve structures larger than \( \sim 0.7 \) and 2.1 au, respectively, at a distance of 140 pc, the distance of the Taurus molecular cloud. The 250 \( \mu \)m grains beyond 30 au and the 2.5 mm grains beyond 3 au have sedimented into a flat subdisc, with virtually no dust of this size to be found beyond 5 and 1 au, respectively, above the mid-plane. Similarly, in the regime in which we can resolve the 2.5 cm dust grains, we see that all of the 2.5 cm dust is contained within \( \sim 0.1 \) au of the mid-plane, and the flatness of this dusty subdisc in the region of the disc that our simulations can adequately resolve suggest that little 2.5 cm dust would be found higher above the mid-plane than this further out in the disc. Since this sedimentation of dust is an important first step in planet formation, we believe that an observational demonstration of sedimentation would be a very important discovery. The simulations also show that one is most likely to make that discovery in the outer parts of discs. By identifying the dust sizes that sediment at different radii, the simulations even help observers in identifying the wavebands in which the observations of dust settling need to be made.

Spitzer observations of the spectral indices of protostellar discs have indicated that small dust grains are often depleted in the upper atmospheres of the inner discs (e.g. Furlan et al. 2009). As our simulations only span up to three scaleheights, we are not sensitive to the depletion of small grains in the upper layers of the disc, as seen by Fromang & Nelson (2009). We note, however, that if a dead zone is present in the inner disc (Gammie 1996), we would expect reduced levels of turbulence and hence depleted levels of small dust grains.

We are now in a position to test the dependence of the scaleheight with grain size. In Fig. 5(a), we plot the ratio of the dust scaleheight of spherical grains to the gas scaleheight as a function of grain size for our simulations at each radial station. We see that the large grains sediment significantly, forming an approximate power-law relationship between scaleheight and grain size. This is most pronounced in the outer disc, where the slope is the steepest. In the inner disc, the degree of sedimentation is quite small, and we see a much shallower slope. The dust scaleheight is limited by the gas scaleheight (Dubrulle et al. 1995), and the flattening we see in the slope of the dust scaleheights when it becomes large is consistent with this expectation. These trends generalize consistently with the specific results at 10 au that we presented previously in Paper I. Furthermore, Fromang & Nelson (2009) have reported that \( \frac{H_{\text{dust}}}{H_{\text{gas}}} \propto r^{-0.3} \) for dust grains in the size range of 1–100 \( \mu \)m at orbital radii of 1–8 au, consistent with the flattening of the slopes for smaller grain sizes that our results here produce.

The spherical grains in Fig. 5(a) at different radii are in very different environments, due to the overall structure of the disc. In Fig. 5(b), we plot the spherical grain scaleheights as a function of the dimensionless stopping time \( \xi \tau_r \). The grains at all radii now fall on the same curve, and we can clearly identify a power-law slope for large stopping times (and hence, large grain sizes). We fit the data to the semi-analytic
Figure 4. Gas and dust density interpolated from the final mid-plane density and scaleheights from our simulations, over the entire $R-z$ structure of the disc with logarithmic scaling of the axes. The solid line marks the resolution of our simulation, and effectively places a lower limit on the scaleheights we can achieve. The top left panel shows the gas density; the top right panel shows the density of the 2.5 μm grains; the middle left panel shows the 25 μm grain density; the middle right panel shows the 250 μm density; the lower left panel shows the 2.5 mm grain density and the lower right panel shows the 2.5 cm grain density.

curves of Dubrulle et al. (1995):

$$\frac{H_{\text{dust}}}{H_{\text{gas}}} = h(1 + h^2)^{-1/2}$$

where $h^2 = (3/8)^{1/2} \alpha / \Omega_\tau$. Here, we determine $\alpha$ from a fit to our data, and find $\alpha \sim 1.79 \times 10^{-3}$. Our simulations produce a slightly steeper power-law relation between scaleheight and stopping time than the predicted slope of $(\Omega_\tau)^{-1/2}$ from several semi-analytical models that consider the balance between sedimentation and turbulent diffusion (Dubrulle et al. 1995; Fromang & Papaloizou 2006), although the fit produces a value for $\alpha$ that is a good match to the turbulent stresses that we measure in our code.

3.2 Existence of the streaming instability

Youdin & Goodman (2005) demonstrated that when a dust layer couples marginally to the gas and is sufficiently dense, it can become unstable in the presence of a steady drift with respect to the gas due to a collective instability they dubbed a streaming instability. Specifically, the growth rates of this instability in the linear regime are strongly dependent on both the ratio of the density of dust to the density of gas and the...
Figure 5. Ratio of the scaleheight of dust to the scaleheight of gas in the disc when the dust layer reaches a steady state. (a) Scaleheights for spherical grains as a function of grain size. The largest grains at 100 au and beyond oscillate through the mid-plane and do not reach a steady state, and so are not shown here. The horizontal dashed line is the resolution limit for our simulations. (b) Scaleheights for spherical grains, as a function of dimensionless stopping times.

Figure 6. (a) Maximum dust-to-gas ratio in the disc for spherical grains of different sizes at each radial station. The dust that oscillates rather than settles is not shown. (b) Mean dust-to-gas ratio in the mid-plane of the disc for grains of different sizes at each radial station.

ratio of the dust–gas friction time-scale to the orbital time-scale. The growth rate becomes largest when the dust-to-gas density ratio becomes unity. Furthermore, it is known to increase with $\Omega \tau_s$. The Youdin & Goodman (2005) linear analysis relied on the sub-Keplerian rotational velocity of the gas to generate a constant initial relative velocity between the dust and gas in order to drive the instability. However, similar collective effects have been observed in simulations where the dust–gas relative velocities arise from turbulence generated through the MRI (Johansen et al. 2007; Paper I). Furthermore, in Paper I we presented evidence that while the dust in our simulation at 10 au become unstable due to this effect, no corresponding instability appeared at 0.3 au. We know from Fig. 3 that sedimentation is the strongest in the outer disc, and hence the dust-to-gas ratio in the mid-plane will be larger further away from the protostar. It is therefore of interest to ask where one would expect the streaming instability to operate in the disc.

To estimate the degree to which the streaming instability could operate in our disc models, we plot in Fig. 6 the maximum dust-to-gas ratio in the mid-plane as well as the mean dust-to-gas ratio in the mid-plane for spherical dust grains. We see that while the mean dust-to-gas ratio for the largest dust can increase significantly over its starting value from the initial MRN distribution, it remains below 0.1 at all radii for all of the dust families with resolved scaleheights. The dust layers that collapse to an unresolved thickness would likely have greater dust-to-gas ratios than are observed here, but this data are not shown in Fig. 6. The maximum dust-to-gas ratios in the mid-plane at each radial station is $\sim 1$ for $R \geq 30$ au, so local enhancements in the dust density can bring the dust-to-gas ratio up high enough that collective effects could begin.

The other important quantity Youdin & Goodman (2005) found that controls the growth rate of the streaming instability is the dimensionless stopping time $\Sigma \tau_s$. We plot this quantity for the spherical grains in Fig. 7. In making Fig. 7, in order to determine whether the grain is in the Epstein or Stokes regime, we assumed that the relative dust–gas velocity was 0.1 times the sound speed, which is the typical velocity dispersion in our simulations. We clearly set the transition from the Epstein to Stokes drag laws in the dust at 0.3 and 1 au.

As we would expect, the dimensionless stopping time is maximized for large dust and in the outer disc. The 2.5 cm grains begin to decouple ($\Omega \tau_s > 0.1$) at 10 au, and fully decouple ($\Sigma \tau_s > 1$), indicating that the particle motions are becoming rather ballistic over the course of an orbit) at 100 au. Similarly, the 2.5 mm grains begin to decouple at 100 au, as do the 250 $\mu$m grains at 300 au. The streaming instability is strongest for $\Omega \tau_s \sim 1$, at least in the case when the instability is being driven by pressure gradients in the gas and hence the time-scales for coupling and driving are balanced, and our results indicate that a similar results hold for MRI-driven turbulence.
In Fig. 8(a), we show the total dust density of large grains in a MMSN (i.e. the sum of the 250 μm, 2.5 mm and 2.5 cm grain densities), in order to ascertain where streaming instabilities might be developing. We see very little density variation in the mid-plane in the inner disc, from 0.3 to 3 au, suggesting that no streaming instability is occurring within this region and hence the only density enhancement is from sedimentation. At 10 au and beyond, we see that the dust layer in the mid-plane break up into clumps with large dust densities. In Paper I, we showed that at 10 au, this was the result of the streaming instability operating in the mid-plane for the 2.5 cm dust grains. Further out in the disc, the condensations we see arise from clumpings of progressively smaller dust grains. Since the ALMA instrument favours observation of smaller grains and can resolve features of ~5 mas in its 350 μm band (or ~0.7 au at a distance of 140 pc) (Wootten & Thompson 2009), it suggests that the au-scale condensations of 250 μm dust that we see develop at 300 au are resolvable by ALMA at the distance of Taurus. In Fig. 8(b), we show only the density of 250 μm dust at 10 (top) and 100 au (bottom) for the MMSN models (on the left) and gas-depleted disc models (on the right). We see that at 10 au, the 250 μm grains in both the MMSN and the gas-depleted disc show little signs of clumping. At 100 au, however, we see that submillimetre-sized grains will settle and exhibit strong clumping due to the streaming instability in the gas-depleted disc, while no such clumping is seen in the MMSN. Thus, as discs age we predict that the streaming instability lends itself more easily to observational confirmation.

3.3 Possibilities for grain growth

Grain growth for large numbers of colliding particles is determined by the Smoluchowski equation (Smoluchowski 1916), and the rate of growth is proportional to the product of the densities of the colliding particles. The streaming instability discussed above causes the dust density to increase significantly over the enhancement due solely to sedimentation. Furthermore, the streaming instability is also known to reduce the relative velocity between particles (Paper I) which is also known to increase the probability that two particles will stick during a collision, rather than destroy each other (Blum & Wurm 2008). Consequently, it is worthwhile to examine the implications this density enhancement has for the growth of dust grains. This is the first time such a calculation has been attempted based on simulated data. The sticking probabilities used here are the best we could cull from the literature.

The growth of dust in protostellar discs has been studied extensively through the use of semi-analytic models (Suttner & Yorke 2001; Dullemond & Dominik 2005, etc). In these models, grain–grain collisions arise through parametrizations of grain turbulence and sedimentation. In the absence of fragmentation, grain growth is extremely rapid, with centimetre- and metre-sized grains forming on timescales of a few thousand years and micrometre-sized grains depleting on a similar time-scale. Such a rapid depletion of the small grains is inconsistent with the infrared emission that is seen in discs a million years old. Dullemond & Dominik (2005) posited that significant levels of grain fragmentation were required in order to maintain a population of small grains to explain the infrared emission. They found that even with high levels of fragmentation, they could get large grains to form.

A common feature in these simulations is that the collision velocities between dust grains arise from semi-analytic models of disc turbulence, such as those studied by Völk et al. (1980). In these models, the average collision velocity between grains arises from perturbations to laminar motions arising from turbulent eddies of different sizes and the differences in the strength of the coupling of the grains to gas motions within these eddies. The dust–gas interaction in these models is usually considered only in the limits where the friction time-scale is much less or much larger than the eddy turnover time, which is comparable to the orbital time-scale, although Carballido, Stone & Turner (2008) demonstrate some degree of concordance between these models and numerical simulations when the friction and eddy time-scales are comparable. As we discussed in Section 3.2, the streaming instability is most sensitive to the case where the friction time-scale is comparable to the orbital time-scale, so further clarity of the dust–dust collision velocities in this regime is desirable. Furthermore, it is not clear whether a semi-analytic model will capture the intermittency of the gas turbulence, or treat the interaction of eddies. Our simulations do include these physics, and thus we can estimate a growth rate for dust and compare it to the results that arise from semi-analytic models.

The collision rate per unit volume between dust of family $i$ with dust of family $j$ is given by $R_{ij}(x) = n_i(x)n_j(x)\sigma_{ij}(x)v_i(x)v_j(x)S(v_i(x))$, where $n_i(x)$ and $n_j(x)$ are the particle densities at grid zone located at $x$, $\sigma_{ij}(x)$ is the interaction cross-section for particle $i$ with particle $j$,
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\[ v_i(x) \] is the collision speed and \( S(v_i(x)) \) is a sticking coefficient (see e.g. equation 32 of Suttner & Yorke 2001). The time-scale for a dust grain of family \( i \) to collide with a number of dust grains of family \( j \) with total mass equal to the mass of grain \( i \) is thus \( \tau_i(x) = n_i(x) m_i/[m_j R_{ij}(x)] \). In principle, if every collision resulted in the particles sticking together (i.e. if \( S = 1 \) for all collision velocities), then this would also be the time-scale for grains of family \( i \) to grow by accreting grains from family \( j \). However, only those collisions that take place with a small enough relative velocity result in the sticking of the two grains.

In practice, we calculate this time-scale from our distribution of superparticles. To do this, we use the same triangular-shaped cloud interpolation strategy that we use to treat the dust–gas interaction, as detailed in Paper I. If \( W_{TSC}(x - x_i) \) is the interpolation kernel for a

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**Figure 8.** First row: The total mid-plane density for 250 \( \mu \)m–2.5 cm grains at 0.3, 1, 3 and 10 au in a MMSN. Second row: The total mid-plane density for 250 \( \mu \)m–2.5 cm grains at 30, 100, 150 and 300 au in a MMSN. Third row: A comparison of the 250 \( \mu \)m dust density at 10 au, in the MMSN (left) and gas-depleted disc (right). Fourth row: A comparison of the 250 \( \mu \)m dust density at 100 au, in the MMSN (left) and gas-depleted disc (right).
particle at location \( x_k \) to the grid zone at \( x \), \( dV(x) \) is the volume of the zone at \( x \) and \( N_k \) is the number of dust grains that are contained in the superparticle \( k \), then the density of dust family \( i \) at any point in the simulation is given by

\[
n_i(x) = \sum_{k \in i} W_{TSC}(x - x_k) N_k dV(x)
\]

and the collision time is given by

\[
\tau_j(x) = \frac{m_j}{m_j \sum_{k \in p j} \sum_{p \in j} W_{TSC}(x - x_k) W_{TSC}(x - x_p) \sigma_{ij} |v_k - v_p| S(v_k - v_p) N_k N_p dV(x)^2}
\]

(5)

Note that this formulation also allows us to consider the case where \( i = j \), i.e. we can consider collisions with grains of the same family. If \( \tau_j \) is much smaller than the orbital period for any particular families \( i \) and \( j \), then we would expect significant amounts of momentum to be exchanged between those families and our collisionless approximation for the dust breaks down. If the grains also stick together during these collisions, we would further expect extremely rapid grain growth. If \( \tau_j \) is much larger than the orbital time, then grain collisions are very infrequent and grain growth is slow, even if the grains always stick together during a collision. The intermediate regime, where \( \tau_j \) is comparable to the orbital time, is of great interest as the rate of momentum exchange between grains is small enough to justify our collisionless approximation, but the potential for rapid grain growth exists.

We use equation (5) to calculate the time-scale for dust particles to interact with their own mass with other dust families of different sizes, assuming perfect sticking (\( S = 1 \)). We plot the minimum and mean of these interaction time-scales in Fig. 9. In each of the panels in Fig. 9, the black stars denote the time-scale for 2.5 cm grains to interact with an equal mass of grains of different sizes from the x-axis. Similarly, the orange diamonds are the interaction time-scale for 2.5 mm grains with grains of 2.5 mm or smaller, the blue crosses are the interaction time-scales for 250 \( \mu \)m grains with grains of equal size or smaller, the green triangles are the interaction time-scales for 25 \( \mu \)m grains with an equal mass of grains of equal size or smaller and the red square is the time-scale for 2.5 \( \mu \)m grains to interact with other 2.5 \( \mu \)m grains. The left column of Fig. 9 shows the minimum interaction time-scales in the simulations at 1, 3, 10, 30 and 100 au, from top to bottom respectively. The right column similarly shows the mean interaction time-scale.

A general trend that can be seen in Fig. 9 is that large dust grains tend to collide with grains of the same size on a time-scale as short as a few orbits beyond 3 au, and on time-scales of tens of orbits at radii of 3 au or less. Smaller grains collide with themselves on time-scales less than an orbit. The enhanced mid-plane density from sedimentation has reduced the collision time-scale for the large dust significantly. The streaming instability in the mid-plane has further enhanced the density of sedimented particles and led to another reduction of about one order of magnitude in the collision time-scale.

In Fig. 9, we see that the sedimentation of large grains leads to an increased predominance of large grains colliding with other large grains, rather than large grains colliding with smaller grains. For instance, the minimum time-scale for 2.5 mm dust at 100 au to interact with its own mass of 250 \( \mu \)m dust is 18.4 orbits. The same 2.5 mm dust will interact with itself on time-scales as low as 2.2 orbits. Similarly, the minimum time-scale for 2.5 cm grains at 10 au to interact with 2.5 mm grains is 307 orbits, while the minimum time-scale for 2.5 cm grains to interact with themselves is 10.5 orbits. Conversely, interior to 3 au grains primarily collide with smaller grains, as seen in Figs 9e and 10e. As a result, we would expect from this argument that if we had perfect sticking in every collision, then grains would grow predominantly through collisions with similar-sized grains in the outer disc – the scenario that forms the BCCA structure for fractal grains – while growing primarily through accreting smaller grains in the inner disc, a scenario predicted to form much more compact grain structures.

We also see from Fig. 9 that the smaller grains – which do not experience significant amounts of sedimentation – will primarily collide with smaller families of dust, and would thus likely grow as compact grains.

In Fig. 10, we show the interaction time-scales at 30 au in the mid-plane for four typical grain–grain collisions. In Fig. 10(a), we show the collisions of 2.5 mm grains with an equal mass of 2.5 \( \mu \)m grains. We see that this typically occurs on time-scales of a few tens of orbits, and that there are no strong concentrations where the collision rate is significantly enhanced. In Fig. 10(b), we show the collisions of 2.5 cm grains with an equal mass of 2.5 mm grains, and in Fig. 10(c) we show the collisions of 2.5 mm grains with 2.5 mm grains. In both of these panels, we see that the lowest collision times are found only in a few highly concentrated regions. In Fig. 10(d), we show the collision time-scales for 2.5 cm dust with itself. Here again, we see that the collision rate is maximal in only a very small volume in the mid-plane. It is only in these regions where the density is enhanced by the streaming instability that significant enhancements in the growth rate can be found.

The interaction time-scales above are the time-scales for the collisions of particles with other grains of their own size, or for collisions of the grains of a certain size with an equal mass of grains of a smaller size. There are many possible outcomes for each collision, though. Particles can stick together, particularly if the collision velocity is low. The grains can also scatter off each other, or they can fragment and break apart. Blum & Wurm (2008) exhaustively catalogue the current status of dust collision experiments that test the collision behaviour of dust in laboratory settings. However, the collision velocities from their disc model that they use to guide their experimental setups tend to be smaller than the typical collision velocities that we find in our simulations, particularly for the smallest dust. In their disc models, subcentimetre-sized grains have collision velocities less than 1 cm s\(^{-1}\); the vigorous MRI-driven turbulence in our simulations, however, leads to collision velocities between dust grains that are typically a few m s\(^{-1}\). Thus, there is some uncertainty as to what the appropriate velocity threshold for fragmentation should be for our simulations. However, as a general rule these experiments show that fragmentation predominates over coagulation when the collision velocity exceeds 1–2 m s\(^{-1}\). To assess the impact this has in the context of our simulations, we repeat the calculation of equation (5), but now use a sticking coefficient that is more consistent with experiments. Blum & Wurm (2000)
Figure 9. Left: Minimum collision time-scales in the mid-plane for grains of family $i$ to collide with an equal mass of grains from family $j$, where $i$ is labelled next to the line in question and $j$ is the $x$ axis. Right column: Mean time for grains of family $i$ to collide with an equal mass of grains from family $j$. For both columns, from top to bottom are the data for spherical grains at 1, 3, 10, 30 and 100 au, respectively. At 100 au, the 2.5 cm grains oscillate about the mid-plane instead of settling, and so are not shown.
found that the rate of grain destruction exceeded the rate of coagulation when the collision velocities were greater than \( \sim 1.9 \text{m s}^{-1} \). For the purposes of this calculation, we will simplify this slightly and use \( S(v_k - v_p) = 1 \) for \( |v_k - v_p| < 1.9 \text{m s}^{-1} \), and \( S(v_k - v_p) = 0 \) otherwise. In other words, we only add a contribution to the collision rate if the collision velocity is less than the critical velocity of \( 1.9 \text{m s}^{-1} \).

This restrictive condition has a large impact on the growth rates that we estimate from our simulations. We plot in Fig. 11 the growth rates for grains of different sizes and different radii, analogous to Fig. 9. Similarly, in Fig. 12 we plot the growth rates in the mid-plane at 30 au for 2.5 mm grains growing through collisions with 2.5 \( \mu \text{m} \) grains, for 2.5 cm grains to grow via collisions with 2.5 mm grains, for 2.5 mm grains to grow through collisions with 2.5 mm grains and for 2.5 cm grains to grow through collisions with other 2.5 cm grains. The minimum growth time-scales have been reduced by a factor of \( \sim 100–1000 \) to the collision time-scales we found previously. At 10 au, for instance, the minimum growth time-scale for 2.5 cm grains through collisions with other 2.5 cm grains is \( 3.10^5 \) orbits; the minimum growth time-scale for 2.5 cm grains with 2.5 mm grains is \( 2.71 \times 10^5 \) orbits. The minimum growth time-scale for 2.5 \( \mu \text{m} \) grains is a little faster, at \( 2.93 \times 10^3 \) orbits. The other panels in Fig. 11 show that these trends persist for other dust sizes and at other radii in the disc; the growth rate for grains is substantially depressed when we consider just the grain collisions with collision velocities less than \( 1.9 \text{m s}^{-1} \).

The large collision velocities that we find will not necessarily prevent any grain growth from occurring. First of all, one of the models considered by Dullemond & Dominik (2005) considered fragmentation in addition to grain growth. In this model, those authors found that while the majority of grains were fragmented in collisions before they could grow to centimetre sizes, there was a fraction of the grains that escaped undergoing a fragmentation event that grew to metre sizes over the course of a few hundred thousand years. Secondly, highly porous grains may undergo compactification in collisions, rather than fragmentation or cratering. Furthermore, the smaller masses of fractal grains for a given size reduces the collision energy, allowing collisions to occur at higher velocities before the grains begin to fragment. Finally, the collision time-scales that we find are at best comparable to the lifetime of the clumps they are in, and in many cases are longer. As a result, dust grains may be swept up in a clump, undergo some degree of coagulation over the course of a few orbits, and then disperse back into the ambient disc where fragmenting collisions are more rare.
Figure 11. The time-scales for grains of various sizes to grow through collisions with grains of other sizes. Grains will only grow if their mutual collision velocities are less than 1.9 m s$^{-1}$ (see the main text). Left column: Minimum time-scales in the mid-plane for grains of family $i$ to grow via collisions with an equal mass of grains from family $j$, where $i$ is labelled next to the line in question and $j$ is the $x$-axis. Right column: Mean time for grains of family $i$ to grow via collisions with an equal mass of grains from family $j$. For both columns, from top to bottom are the data for spherical grains at 1, 3, 10, 30 and 100 au, respectively. At 100 au, the 2.5 cm grains oscillate about the mid-plane instead of settling, and so are not shown.
The time-scale for momentum transfer between dust particles in the same family that we extract from Figs 9 and 10 generally correspond to at minimum tens to hundreds of time-steps per collision. Thus, our approximation that the gas is collisionless is verified. The only exception comes for the smallest dust at 0.3 and 1 au. The collision time-scale for the 2.5 μm dust at 0.3 au is approximately one-third of a time-step, and the time-scale for 25 μm dust at 0.3 au is approximately one collision every three time-steps. The 2.5 μm dust at 1 au has approximately one collision every two time-steps. We would thus expect dust self-collisions to have some small role for these dust families in the inner disc. However, the momentum exchange rate between 2.5 μm dust and the gas is of the order of $10^4$ larger, and thus should dominate any dust pressure.

4 SUMMARY

The streaming instability is pervasive throughout sedimented dusty discs, and can lead to significant enhancements in the densities of large grains. We have catalogued the sedimentation and development of streaming instabilities throughout a prototypical protostellar disc. We find the following.

1. Dust of different grain sizes sediment to different degrees, with larger dust sedimenting to a greater extent than smaller grains and more sedimentation occurring in the outer disc. This effect can become quite pronounced, producing dusty sublayers with thicknesses of 1–2 au. Such layers should be distinguishable from thicker discs via observations by ALMA at submillimetre wavelengths for discs at the distance of Taurus.

2. We show that the streaming instability produces clumps in 250 μm grains, 2.5 mm grains and 2.5 cm grains that are of the order of a few au across in the outer disc, especially in gas-depleted systems like transitional discs. ALMA at 350 μm should be able to resolve features as small as 0.7 au at a distance of 140 pc, and can thus potentially detect these dust clumps when viewing face-on discs.
(3) The scaleheight of the sedimented dust approaches a power-law relationship with dust size for compact grains, with a slope slightly steeper than $-1/2$. Highly porous fractal grains have a scaleheight that is largely independent of grain radius.

(4) The combination of sedimentation and the streaming instability leads to dust-to-gas ratios of $\sim 1$ over the course of a few orbits, and increase of $\sim 100$ over our initial conditions.

(5) The enhanced grain densities that are produced by sedimentation and the streaming instability in turn lead to a reduced collision time-scale for both large grains with other large grains, as well as for large grains with small grains, leading to the possibility of enhanced dust growth rates.

(6) The collision velocities of the dust are of the order of a several metres per second to a few tens of metres per second, significantly higher than that assumed by many semi-analytic models of dust growth in protostellar discs. As a result, while the collision rates between dust particles can be significantly enhanced, the most likely result of the majority of such collisions is likely to be grain destruction, rather than grain growth.

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