Study on PSD of track irregularities of Beijing Metro Line

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Abstract. Track irregularities are the significant factor of vehicle vibration. In this paper, track spectrum is combined with the theory of finite element method. Dynamic responses of several track spectrums such as spectrums of the USA and Europe are compared with the field investigation result. The comparison prove that the spectrums which are mentioned above are not applied well to the construction and maintenance of the metro line. Using the method of wavelet theory and error removing algorithm, process the raw data which is measured by the track inspection car. And by using nonlinear least square method, get the track spectrum of the metro line, and constructed a FEM model for verification. The results say the spectrum we put forward has significant advantages compare with other track spectrum. Through this way, this work fills the blanks on field of metro line track spectrum.

1. Introduction
Rail irregularity has a crucial impact on the wheel-rail system\cite{1}. Firstly, it is not only the source of disturbance in the wheel-rail system, but also is the main reason of the vibration and the dynamic force of the wheel-rail movement. It has a significant impact on the traffic safety, stability, comfort, environmental noise and the life-cycle of the track parts and vehicle\cite{2}. Secondly, the track smoothness is also an important manifestation of the performance and carrying capacity of the track structure, which is an important indicator for the operation and maintenance department to formulate the maintenance plan. Track spectrum which use the method of FFT (Fast Fourier Transform) to evaluate the spectrum of the whole condition is the most effective way to investigate the rail irregularity. Although metropolitans like Beijing, Shanghai are developing its metro line rapidly but there is no track irregularities spectrum for metro line.

Therefore, this paper based on the data of Beijing Metro Line 6 measured by the track inspection car, proposed a track spectrum for urban rail transit to fill the blanks in current research.

2. Analysis of Typical Track Irregularity Spectrum
Since the mid-1960s, many foreign countries have begun to research a series of track irregularity studies. Naeimi\cite{3} et al, analyzed the dynamic response of sleeper by means of finite element simulation. Li\cite{4} et al, studied and put forward China Wuhan-Guangzhou high-speed rail irregularity spectrum, and evaluated its status of irregularity. Chen\cite{5} et al, put forward the general track spectrum for Chinese main railway lines. As for the US spectrum and German spectrum, it has been widely used as the most authoritative track spectrum and widely used in related design work and research.
In this paper, the US track spectrum and the German track spectrum are taken as an example to inspect whether the existing irregularity spectrum adapts to the urban rail transit condition.

2.1. Finite element model

Figure 1 is schematic of vehicle-track coupling system. The type B vehicle which is widely used in China metro line is used in construction of the vehicle system, and LM wear-type tread is used in wheel tread modeling.

2.2. Import of the irregularity

To get the amplitude of the typical spectrum we use the IFFT (Inverse Fourier transform) method to derive the corresponding irregular amplitude. Then the value we calculated is added to the vehicle-track model to simulate the actual track irregularities.

The specific method of the import process is achieved by offsetting the vertical and horizontal coordinates of the model with the amplitude of the results of IFFT.

Irregularities of spectrum of the US, German low interference and German high interference data are added to the coupling model as the different working conditions to facilitate the comparison of the next kinetic responses.

2.3. Analysis of calculation results

In order to analyze the influence of the irregular condition on the vehicle-track system, spectrums like the United States spectrum, the German high interference spectrum and the German low interference spectrum are considered, and the irregularities of the gauge and longitude are added. And take the rail force and acceleration for comparison of the various conditions.

For a vehicle-track coupled system, the static and dynamic loads of the vehicle are transferred by the contact between the rail and the wheel to the substructure. Therefore, analyzing the vertical and lateral forces of the rail are necessary in researching the dynamic responses of the track system.

Figure 2 shows the comparison of vertical and horizontal wheel-rail forces under different conditions. The lateral force is much smaller than the vertical force, and both of the lateral and vertical force increase as the irregularity deteriorates. The amplitude order of the rail force from large to small is the US spectrum, the German high interference spectrum, the German low interference spectrum.

This also leads to a preliminary conclusion: To control the amplitude of wheel-rail force, the control of irregularity state is indispensable.
From the analysis above, it can be concluded that the track structure dynamic response is significantly larger with the deterioration of the rail irregularity condition. Therefore, it is necessary to select the appropriate track spectrum to guide the urban rail transit control and design.

3. Field test

In order to investigate whether the existing spectrum is applicable to the urban subway conditions, a dynamic field test for rail and track slab was carried out.

3.1. Testing device

The vertical force and lateral force of rail are more important dynamic responses, which have great influence on passenger comfort and driving safety. In order to ensure the reliability of the modeling data, the field test was carried out on Beijing metro line 6. The test mainly included lateral and vertical forces of the rail and vertical acceleration of track bed. The test devices layout is as shown in Figure 3.

3.2. Result comparison

Table 1 is the comparison between the field test and the calculation results of the chapter 1. It can be seen that the irregularity condition of the Beijing metro line 6 is between the German high interference spectrum and the US spectrum, but the differences between them are significant.

| Type     | Vertical force (kN) | Lateral force (kN) | Rail acceleration (g) | Slab acceleration (g) |
|----------|---------------------|--------------------|-----------------------|------------------------|
| FRA6     | 125.1               | 22.6               | 90.44                 | 1.72                   |
| GRSLI    | 49.61               | 3.96               | 62.52                 | 0.87                   |
| GRSHI    | 87.76               | 11.07              | 70.4                  | 0.95                   |
| Field test | 104.43             | 15.09              | 76.65                 | 1.28                   |
| MRD      | 15.96%              | 26.64%             | 10.85%                | 25.78%                 |

In the table, the MRD (minimum relative difference) is defined as follows:

\[ D_j = \min \left\{ \frac{|a_i - b_j|}{b_j} \right\} \]

Where \( D_j \) means MRD, \( a_i \) is the results from the simulation, \( b_j \) is results from the field test, \( i \) means the different spectrum, \( j \) means different dynamic responses.

Table 1 shows that the MRD between the indicators are greater than 10%, and some even more than 25%, indicating that the existing spectrum and urban subway line irregularities conditions are not consistent, if the actual construction using the existing spectrum to construction work and maintenance of the guidance, it will inevitably lead to the corresponding engineering error, affecting the long-term operation of the track line.

4. Estimation and fitting of the track spectrum

In order to remove the abnormal value of track inspection car, set K which is the rate of overrun value as 3‰. As shown in Figure 4, we can conclude that the algorithm is effectively removed the outlier data and the effect is obvious.

Due to the structural characteristics of the track structure itself, there are trend term in the rail inspection data which means the mean value is not zero. As can be seen from Figure 4, the axis of the graph is not zero but tends to a negative value. The need to eliminate the trend of the items. At present, there are two kinds of trend term elimination methods which are applied to track irregularity, EMD and wavelet analysis.

The bior 4.4 is selected as the wavelet base of the wavelet analysis, the results of the elimination of trend term is as shown in Figure 5, which uses the bior 4.4 as the wavelet base and make the raw data
reconstructed 6 times. We can see the effect of elimination of trend term, the irregularity data
symmetry in the zero axis, the data close to the stationary random process data.

Figure 4. Results of outlier elimination

Figure 5. Result of elimination of trend term

4.1. Track spectrum estimation

By previous studies[6], the track irregularity data can be approximated as a stationary random process, and because the number of measured points involved is large, the PSD of the track irregularity can be acquired by using the average periodic method.

For a stationary stochastic process, take a function in the range. Then the sample function of the
process satisfies the finite energy condition[7], which means PSD of track irregularity can be obtained
by the method of fast Fourier transform (FFT). The expression of PSD of track irregularity $G_x(f)$ is
as follows:

$$G_x(f) = \begin{cases} 2S_x(f) & 0 \leq f < \infty \\ 0 & \text{Others} \end{cases}$$  \hspace{1cm} (1)

In which, $S_x(f)$ denotes a bilateral power spectrum. The calculation expression is as follows:

$$S_x(f) = \lim_{T \to +\infty} S_{X_T}(f) = \lim_{T \to +\infty} \frac{1}{T} \int_{-T}^{T} R_{X_T}(e^{-2\pi f \tau}) d\tau = \lim_{T \to +\infty} \frac{1}{2T} \left| F_{X_T}(f) \right|^2$$  \hspace{1cm} (2)

In which, $F_{X_T}(f)$ is the results of the sample function $X_T(t)$ thorough FFT; $S_{X_T}(f)$ is estimation
of the track spectrum; $R_{X_T}$ means the correlation function.

In this paper, we use MATLAB programming, estimated the PSD of the irregularities on the metro
line by means of periodic method.

4.2. Track spectrum fitting

Because the US spectrum was widely used in previous research, which means it has a high reliability. Therefore, fitting the spectrum using the US formula. According to Levenberg-Marquardt optimization nonlinear least squares method, the process of gauge irregularity spectrum fitting is as follows:

$$f(x) = \frac{4kAB^2}{(x^2 + B^2)(x^2 + C^2)}$$  \hspace{1cm} (3)

In which, $f$ means the spatial frequency, in units of 1/m; A, B, C for the relevant fitting parameters; $k$ generally take for 0.25.

4.2.1. Parameter fitting

A series of least squares algorithms were applied to the PSD of track irregularity fitting work. The results are shown in Figure 6 and Figure 7. From Table 2, the fitting correlation coefficient of the
The gauge irregularity is 0.8091. The data fit well with the fitting formula, the PSD of longitude irregularity perfectly describes the distribution and characteristics of the track irregularity in the 1~30m wavelength segment. As can be seen from the figure, it fits better in the 10~20m wavelength segment, and the measured data is lower compared with the fitting analytic value in the wavelength of 2~5m.

Table 2. Evaluation of longitudinal irregularity

| Type        | Value  |
|-------------|--------|
| A           | 0.0185 |
| B           | -0.2010|
| Correlation coefficient | 0.8091 |

The results show that the track spectrum which is proposed by us can describe the characteristic of the urban rail irregularity well, and better than other existing track spectrum.

4.3. Verification of the fitting spectrum

In order to verify the correctness of the fitting spectrum, get the amplitude of the track spectrum we proposed by the method mentioned in Chapter 1 and import it to the FEM model to calculate the dynamic responses in the condition of the metro line spectrum.

Table 3. Dynamic response of fitting spectrum and field test data

| Type       | Vertical force (kN) | Lateral force (kN) | Rail acceleration (g) | Slab acceleration (g) |
|------------|--------------------|--------------------|------------------------|-----------------------|
| Fitting result | 98.72              | 13.37              | 73.48                  | 1.16                  |
| Field test  | 104.43             | 15.09              | 76.65                  | 1.28                  |
| Relative difference | 5.46%             | 11.40%             | 4.14%                  | 9.38%                 |

The results of the comparison is shown in Table 3. It can be seen that the simulation results are similar with the field test data for the rail force and acceleration of the both rail and slab. The degree of the simulation has a great improvement compare with the existing track spectrum, the minimum is 5.46% and the maximum is 11.4% of the relative difference. The difference between the measured value and the simulation value of the fitting spectrum is mainly related to the assumption and limited calculation condition.

The results show that the spectrum we proposed can describe the characteristics of subway irregularity condition well. It also has great advantages on evaluating the urban track irregularities compared with other existing spectrum.

5. Conclusion

(1) In order to confirm whether existing spectrum is fitted the condition of the metro line conditions, the field tests were carried out. The comparison results shows that the existing irregularity spectrum is
not suitable for construction and irregular control work on urban rail transit.

(2) During the preprocessing of track data, the outlier elimination algorithm and the wavelet analysis method are used to process the track data, and the final filtering result is good, which can be used for the later excavation of relevant research.

(3) Using the programming method of MATLAB, the PSD of irregularity of Beijing urban rail transit is obtained, and the fitting formula is used to fit the least squares method. Finally, the fitting spectrum of Beijing subway irregularity is obtained. The results of the dynamic simulation show that the PSD of urban track irregularity which we proposed, has great advantages compared with other existing spectrum.

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