Contact dynamics in a gently vibrated granular pile

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We use multi-speckle diffusive wave spectroscopy (MSDWS) to probe the micron-scale dynamics of a granular pile submitted to discrete gentle taps. The typical time-scale between plastic events is found to increase dramatically with the number of applied taps. Furthermore, this microscopic dynamics weakly depends on the solid fraction of the sample. This process is strongly analogous to the aging phenomenon observed in thermal glassy systems. We propose a heuristic model where this slowing down mechanism is associated with a slow evolution of the distribution of the contact forces between particles. This model accounts for the main features of the observed dynamics.

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Internal contact forces in dense granular systems are very inhomogeneous \cite{1, 2}, even for crystalline assemblies \cite{3}. The stress at a given contact can change macroscopically following a relative displacement of the two particles of order of microns. Hence, large modifications of the contact forces field can result from minute deformations of the pile. This phenomenon is crucial in understanding the catastrophic yielding occurring in granular systems submitted to a slowly varying stress (avalanches \cite{4}, shear-bands in triaxial tests \cite{5}). It also explains why the sound transmission through a granular sample can be strongly affected by very small deformations \cite{6}. To probe the evolution of the internal stress field, one needs to measure forces directly \cite{7, 8} which is difficult in 3D. In this letter, we propose a different approach: we use MSDWS to measure particle displacements on micronscales in a pile submitted to gentle discrete taps. These vibrations are too weak to induce large-scale rearrangements which would eventually lead to a compaction of the granular system \cite{9, 10}. We can therefore evaluate the micro-dynamics of the contacts without significantly perturbing the packing structure.

We use glass beads of diameter $45 \pm 2 \, \mu m$, contained in a glass cell ($30 \, \text{mm} \times 10 \, \text{mm} \times 2 \, \text{mm}$). To reduce electrostatic forces and the effects of moisture, the granular system is saturated with pure water. During the experiment, the mean packing fraction $\phi$ of is obtained by measuring the position of the upper surface of the pile with a CCD camera. Although a systematic error of 2% can not be avoided, this allows us to detect relative changes in $\phi$ as small as 0.01%. To produce motion in the pile, we use a piezoelectric actuator on which the cell is rigidly mounted. Vertical vibrations of precisely controlled amplitude, shape and durations can thus be applied to the granular column. In this experiment, we focus on a single type of mechanical excitation, later referred to as a "tap", which consists in a train of square wave vibrations of frequency 1 kHz and duration 100 ms. Different applied voltage are used, yielding various vertical amplitudes ranging from 50 to 300 nm.

In a standard experimental run, the pile is prepared by turning the cell upside down then allowing the particles to sediment for half an hour. This procedure yields reproducible structures of low volume fraction. The pile is then submitted to high amplitude taps (of vertical amplitude 300 nm) until it reaches a prescribed packing fraction $\phi_s$. During this compaction stage, the evolution of the packing fraction $\phi$ with the number of taps (Fig. 1) is highly reproducible, and consistent with previous experimental results on dry granular systems. We however note that the packing fraction we reach is well above the close packing limit expected for a fully disordered pile ($\approx 0.64$). This indicates that cristallisation does occur in our system under vibration. This first step is for us a mean to reproducibly prepare a granular sample of given packing fraction with essentially the same preparation history. We then start probing the dynamics of contacts by submitting the cell to very gentle taps (of amplitude 50 nm). As shown in Fig. 1, this second step does not induce significative evolution of the packing fraction except for initially very loose packs.

![FIG. 1: Four compaction curves : (+) is only excited by gentle vibrations; The other systems are prepared with (×) 50, (□) 150, (△) 8000 high impulsions pulses, before being excited by gentle vibrations. There is a systematic error of 2% on the measurements of the packing fraction.](image-url)
To probe the microscopic dynamics induced by these gentle taps, we use MSDWS \[13, 14\]. This technique, which allows one to resolve sub-micron displacements, has been successfully applied to granular dynamics by several groups \[13, 16\]. The sample is illuminated with a He-Ne laser beam at a depth of 2 cm below the pack upper surface (1 cm over the bottom). Photons are multiply scattered by the particles \[17\], and form a speckle pattern on the opposite cell wall that we record with a CCD camera. In the absence of vibrations, the speckle image does not change in time as temperature is insignificant for such large objects. By contrast, the taps induce some irreversible particles displacements which modify the speckle image. To quantify this internal dynamics, we measure the intensity correlation of speckle images, taken between taps, as a function of the number of taps \(t\) that separate them. This function generally depends on the total number of small amplitude taps \(t_w\) that have been performed. We therefore calculate the two-times correlation function \(g(t_w, t)\):

\[
 g(t_w, t) = \frac{\langle I(t_w + t) \cdot I(t_w) \rangle_{spkl} - \langle I(t_w)^2 \rangle_{spkl}}{\langle I(t_w)^2 \rangle_{spkl} - \langle I(t_w) \rangle_{spkl}^2} \tag{1}
\]

In this expression, \(\langle \rangle_{spkl}\) corresponds to averaging over different speckles. MSDWS thus allows one to rapidly access relaxation time by substituting time- with space-averaging and is therefore well suited to the study of non-stationary dynamical systems.

Figure 2 shows three correlation functions obtained with the same sandpile at different values \(t_w\). These functions are well fitted by stretched exponentials:

\[
 g(t_w, t) = \exp \left( - \frac{t}{\tau(t_w)} \right)^{\alpha(t_w)}
\]

\(\tau(t_w)\) and \(\alpha(t_w)\) are determined by a least-squares fit. The solid lines in figure 2 correspond to these fits for different values of \(t_w\). The quantity \(\tau(t_w)\) may correspond to catastrophic failures of the pack structure, which compete against a global reinforcement of the granular contacts.

For different packing fractions, we follow the evolution of the dynamics by monitoring the two parameters \(\tau(t_w)\) and \(\alpha(t_w)\) as a function of the total number of gentle pulses \(t_w\). We find that the exponent \(\alpha\) is roughly constant (\(\approx 0.8 \pm 0.2\)) and independent of the packing fraction \(\phi_s\). By contrast, the time \(\tau(t_w)\) increases by five decades over the range of \(t_w\) explored, as shown in figure 3. It should be noted that this dynamics can be immediately reset by submitting the system to a few taps of larger intensity (such as those used for compacting the sample). A careful examination of the \(\tau(t_w)\) curve also reveals large fluctuations in the internal dynamics, especially in looser packs for which the packing fraction slowly evolves. During certain periods of time, the dynamics is restarted as shown by a sudden decrease of \(\tau(t_w)\). This may correspond to catastrophic failures of the pack structure, which compete against a global reinforcement of the granular contacts.

This result demonstrates that the response of a granular system to small perturbations is strongly dependent on the history of its preparation (the number of applied taps \(t_w\)), and rather insensitive to the packing fraction. More quantitatively, the typical relaxation time associated with a given mechanical excitation evolves as a power law of the total number of taps \(t_w\):

\[
 \tau(t_w) \sim t_w^{1.2 \pm 0.2}
\]

This observation is strongly reminiscent of the aging behavior recently observed in many glassy colloidal systems \[13, 12, 20\]. In these materials, the longest \((\alpha-)\)relaxation time is found to grow as a power law of the time since the system was left to rest after a rapid shearing. In spite of this strongly analogous behavior, the microscopic processes leading to this dynamical arrest are qualitatively different. In colloidal systems, stress relaxation occurs by thermally activated rearrangements of the structure. In granular materials, temperature is effectively zero and relaxation only results from externally applied vibrations which induce the slippage of some contacts (the most fragile ones).

The frequency of these slipping events is directly

\[\tau(t_w)\]
probed by the MSDWS technique. Slipping events indeed result in some grain displacements (either translations or rotations) around the broken contact, until a new equilibrated configuration is found. In this gentle vibrations regime, these events are too small to contribute in a significant way to the compaction process. This means that the associated grain displacements are of order $\delta << D$, where $D = 45 \mu m$ is the particle diameter. Although tiny, these irreversible displacements control the speckle decorrelation. However, to get a quantitative estimate of their frequency, one needs to assume that they are uniformly distributed in space and have a unique characteristic amplitude $\delta$. Within this hypothesis, $\tau(t_w)$ is directly proportional to the inverse of the yielding frequency $\tau$.

We now turn to a tentative microscopic model to capture this slowing down process. As underlined before, the observed dynamical arrest is rather insensitive to the packing fraction of the sample. We thus need to introduce another internal variable that would control the instantaneous response of the pack to gentle vibrations. Here we propose to focus on contact stress distribution. It has been observed that the form of this distribution is almost independent of the volume fraction of the pack and the preparation history. However, standard measurements are not sensitive enough to detect small variations in these distributions especially in the low force limit, that may follow from very gentle mechanical vibrations. We will argue here that the observed evolution of the dynamics results precisely from a slow modification of the stress distribution inside the pack which effectively rigidify the granular pile.

To modelize such a dynamics, we picture the granular assembly as a set of independent contacts (which total number is supposed to be a constant.) Each contact is characterized at time $t$ by the normal and tangential component of the contact force which we denote $\sigma_n$ and $\sigma_t$ respectively. Mechanical equilibrium imposes that $\sigma_t < \mu \sigma_n$. For simplicity, we will make the friction coefficient $\mu$ equal to 1 in the rest of the letter. For a given packing structure, the state of the internal stress field is characterized by the two variable stress density $P_\sigma(\sigma_n, \sigma_t)$. As the cell is vibrated, mechanical waves travelling through the sample induce random stress fluctuations on each contact. Such perturbations can locally trigger the rupture of a contact, whenever the shear force $\sigma_t$ overcomes the normal force $\sigma_n$. We assume an exponential distribution $\chi(\delta \sigma)$ of the maximum force fluctuation induced by the tap on each contact:

$$\chi(\delta \sigma) = \frac{1}{\delta \sigma} \exp\left(-\frac{\delta \sigma}{\delta \sigma}\right)$$

In this expression, the mean force fluctuation $\langle \delta \sigma \rangle$ is an increasing function of the applied vibration amplitude. Thus the probability for a given contact to yield following a single tap writes:

$$\omega_y(\sigma_n, \sigma_t) = \exp\left(-\frac{\sigma_n - \sigma_t}{\delta \sigma}\right)$$

This expression is a consequence of the peculiar form (Eq. 2) taken for the tap induced force fluctuations $\chi(\delta \sigma)$. However, the main results of the present model remain valid for any fast decaying distribution (faster than a power law).

After a yielding event, the force at the renewed contact is chosen from a given "rejuvenated" distribution which we consider intrinsic to the system. The distribution of normal forces in a granular pile under moderate load is known to exhibit an exponential tail at high forces and a plateau below the mean force. Numerical measurements have also shown that, for a given value of the normal force, the tangential forces are uniformly distributed between 0 and the sliding limit $\sigma_t = \mu \sigma_n$ (in a 2D case). We use these different observations to infer the form of the "rejuvenated" distribution $P_{rej}(\sigma_n, \sigma_t)$, which is thus written:

$$P_{rej}(\sigma_n, \sigma_t) = \frac{1}{\sigma_0 \cdot \sigma_n} \cdot \exp\left(-\frac{\sigma_n}{\sigma_0}\right)$$

where $\sigma_0$ is the mean stress inside the pile. For simplicity, we have omitted the plateau saturation of the distribution at low forces. We can now derive the dynamical equation of evolution of the stress distribution $P_\sigma$:

$$\frac{\partial P_\sigma(\sigma_n, \sigma_t)}{\partial t} = -P_\sigma(\sigma_n, \sigma_t) \cdot \omega_y(\sigma_n, \sigma_t) \left[+ P_{rej}(\sigma_n, \sigma_t) \cdot F(P_\sigma) \right]$$

where $F(P_\sigma)$ is the total frequency of sliding events which is self-consistently defined as:

$$F(P_\sigma) = \int_{\sigma_0}^{\sigma_{n}} \cdot P_\sigma(\sigma_n', \sigma_t') \cdot \omega_y(\sigma_n', \sigma_t') \cdot d\sigma_n' d\sigma_t'$$

The present description exhibits many common features with Bouchaud’s trap model of glass transition. In the latter, the internal dynamics of a glassy liquid is pictured as a succession of thermal escapes from energy wells of various depths. In an analogous way, each contact here can be considered as frozen in a mechanical trap (the local solid friction cone), the depth of which depends on the relative amplitude of the normal and shear components of the contact force. Moreover, in the absence of temperature, mechanical vibrations play the role of the energy source by allowing individual contacts to hop out of their trap.

As in the trap model, we thus observe two limiting regimes depending on the relative values of the intensity
of the applied stress $\delta \sigma$ and the width $\sigma_0$ of the rejuvenated distribution. For large vibrations, i.e. $\delta \sigma > \sigma_0$, the rejuvenated stress distribution is a stationary solution of Eq. (6), and the yielding frequency is constant with time. By contrast, for $\delta \sigma < \sigma_0$, the stress distribution keeps evolving endlessly. Figure 4 shows the time evolution of $P_\sigma$ obtained by numerically solving Eq. (6) for $\delta \sigma = \sigma_0/20$, starting with $P_\sigma = P^*$. It shows that fragile contacts - contacts of low normal force or close to the sliding limit (inset) - are slowly depleted. As a result, the number of sliding events per unit time unit decays. More quantitatively, we find that the characteristic time between events grows linearly with the elapsed time. This is to be compared with the $\tau \sim t^{1.2}$ scaling behavior observed in the experiment (Figure 3).

We have evidenced, through MSDWS measurements, the existence of a slowing down of the micro-scale dynamics over more than five decades in gently vibrated granular piles. This behavior is reminiscent of the aging process observed in glassy systems. This dynamics appears to be weakly connected to the overall grain-scale structure, which suggests a two-level description of granular systems. At high enough vibration, a granular pile evolves through the restructuration of the piling geometry, leading to a slow irreversible compaction. In this regime, forces networks are rapidly renewed and show no history-dependent behavior. At very low vibrations however, the geometry of the pile is essentially frozen, but the forces network can still evolve by slowly depleting the most fragile contacts. This leads to an effective reinforcement of the pack structure as is evidenced in the present study by the decrease of vibration induced plastic events.

The precise nature of the yielding events remains however unclear. In particular, one might expect large spatial and temporal correlations between them, which we cannot probe with DWS. Another important question concerns the relevance of such modifications to the onset of macroscopic flow. For instance, does the observed reinforcement of the force networks play a role in changing the threshold of avalanche triggering or shear-banding appearance?

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