Inflationary Scenarios with Scale-invariant Spectral Tensorial Index

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Abstract

Next-to-leading order expressions related to Stewart-Lyth inverse problem are used to determine the inflationary models with a tensorial power spectrum described by a scale-invariant spectral index. Beyond power-law inflation, solutions are characterized by scale-dependent scalar indices. These models can be used as assumption on the generation of primordial perturbations to test for scale dependence of scalar index at large angular scales. If such a dependence is detected, a nonzero contribution of gravitational waves to the CMB spectrum must be expected.

PACS numbers: 98.80.Cq, 98.80.Es, 98.70.Vc
I. INTRODUCTION

With measurements carried out by experiments Boomerang and Maxima-1 [1] observations of cosmic microwave background (CMB) anisotropies entered a phase where different theories for structure formation at our Universe can be falsified. To date, inflation [2,3] is the cosmological model supported by analysis of these observations [4] as favored theory. Almost any model that produces an accelerated expansion of the early Universe (i.e., inflation) solves the well-known set of fundamental problems faced by the Standard Cosmological Model, provided that the inflationary period lasts long enough (see [5] for detailed explanations). The simplest inflationary scenario describes the classical and quantum dynamics of the early Universe dominated by a single scalar field (inflaton) evolving in a nearly flat potential.

Analyses such as those in Refs. [4] and [6–9] consist of maximizing a likelihood function over the space of model parameters. A given set of parameters yields a theoretical CMB spectrum to be compared with observations. The precise number of parameters depends on the version of the analysis and commonly is as small as 6 and as large as 11. They are classified as inflationary or cosmological parameters depending on whether they determine the initial power spectrum of fluctuations or its posterior evolution. To set the initial conditions what it is commonly assumed is a particular kind of single scalar field model namely, power-law inflation [10], characterized by scale-invariant scalar and tensorial spectral indices differing in unity each from the other. Amplitudes of tensorial perturbations are often neglected. The conclusion to be drawn is that, in the corresponding to observed Universe scales, the actual potential has a strong similarity with that of power-law inflation. No conclusions can be made about potential functional form in other scales, particularly at Planck scales.

When the relative contribution of kinetic energy to total scalar field energy (known as $\epsilon$) may be regarded as constant then, to leading order (LO) in an expansion in terms of $\epsilon$, the scalar and tensorial indices and $r$, the relative contribution of primordial gravitational waves to CMB anisotropies, are directly related and could be taken as constants too. In this situation, the model that better accomplishes the job of fitting the available data is still power-law inflation, with some distortion taking place for the relation between spectral indices [11]. Assumption $\epsilon = \text{const.}$ is not realistic since the equation of state must change near the end of inflation in order to return to the Standard Model expansion rate. Hence, several authors had pointed out that some scale dependence for the scalar index may be expected [12,13]. Furthermore, in Ref. [8] it was already found currently available data to be compatible with scale-dependent scalar index, though this dependence seems to be highly constrained [14]. If a scale-dependent scalar index is the actual case, then using a power-law index would affect the best-fit values of the entire set of cosmological parameters.

In this work we would like to address the question of which is the theoretical inflationary potential generating the primordial fluctuations characterized by a scale-dependent scalar index that could give the best fit to current as well as to next generation of observations.

Among other alternatives for deriving the inflaton potential from observations (see Ref. [15], and references therein), the Stewart-Lyth inverse problem (SLIP) was introduced [16] as a procedure consisting of solving a pair of non-linear differential equations derived in Refs. [17,18] from next-to-leading order (NLO) algebraic Stewart-Lyth equations for spec-
tral indices [19], and determining the corresponding inflaton potential. The full power of
this procedure can be used only when information on the functional forms of both spectral
indices is available. Unfortunately, this information is rather difficult to be directly obtained
from observations and, in addition, simple functional forms of the spectral indices involve
great difficulties while solving SLIP. However, there is an alternative way of using the SLIP
related expressions. Having independent information on the functional form of one of the
indices, the second one can be calculated and used as prior for maximizing the likelihood
function. The resulting maximized likelihood will approve or reject the used spectral indices
and the corresponding inflationary potential as a reliable scenario of the Early Universe.

Next generation of observations is expected to give information on CMB polarization
which, in turn, is linked to $r$ (see Ref. [20] for a review on this subject). Hence, future
CMB data, such as MAP and Planck [21] will provide, will determine a range of likely
values for tensor-modes parameters, in spite of the subdominance of the gravitational-waves
contribution to CMB anisotropies characteristic of most inflationary models. Nevertheless,
even if a central constant value of the tensorial parameters can be estimated from these
observations, it will be almost impossible to detect any scale- dependence for the tensorial
spectral index. With this regard, it makes sense to state the problem as finding out which
are the inflationary scenarios, other than power-law inflation, with scale-invariant spectral
tensorial index. A major theoretical advantage here is that this is one of the few cases where
the SLIP is analytically solvable.

In this paper we propose two models that could serve to fit the inflationary perturbations,
detecting, if there exist, scale dependence for the scalar spectral index at large angular scales
while increasing the scale range and resolution of CMB observations. These models are
obtained in a straightforward manner as SLIP solutions assuming a constant and almost
negligible tensorial index.

In next Sec. we briefly describe the theoretical frame for Stewart-Lyth calculations, the
main features of power-law model upon which these calculations are based and current ob-
servations are fitted, and present Stewart-Lyth algebraic NLO equations for the spectral
indices. In Sec. [II] SLIP is rewritten using $\epsilon$ as the basic parameter. Further, we introduce
a criterion that allows us to determine when a given SLIP solution is consistent with assump-
tions underlying calculations. Sec. [IV] is devoted to the main aspect of this manuscript, i.e.,
how to use NLO expressions related to SLIP to determine a theoretical potential yielding a
spectrum of perturbations able to fit observations likely to be done in the near future. We
summarize main results obtained in Sec.V.

II. PERTURBATIONS PRODUCED BY INFLATIONARY MODELS

Many propositions for inflaton potential can be made fulfilling the conditions for suc-
cessful inflation (see Ref. [3] for a description of some inflationary models). Hence, the
criterion for choosing the potential must be the agreement between theoretical predictions
and measurements. Ultimately, testing this agreement involves the calculation of primor-
dial perturbations spectra. These spectra are used as initial conditions for the evolution
of perturbations which can be computed through the transfer functions (using for example
the CMBFAST package [3]), solutions being compared with current measurements of CMB
anisotropies. The simplest scenario where that comparison can be carried out is that of
a single and real scalar field rolling down a potential. In this scenario, a flat Friedmann-Robertson-Walker universe is assumed containing a single scalar field equivalent to a perfect fluid with equations of motions given by

$$H^2 = \frac{\kappa}{3} [T + V(\phi)], \quad (1)$$

$$\ddot{\phi} + 3H \dot{\phi} = -V'(\phi), \quad (2)$$

where \(\phi\) is the inflaton, \(T \equiv \dot{\phi}^2/2\), \(V(\phi)\) the inflationary potential, \(H = \dot{a}/a\) the Hubble parameter, \(a\) the scale factor, dot and prime stand for derivatives with respect to cosmic time and \(\phi\) respectively, \(\kappa = 8\pi/m_{Pl}^2\) is the Einstein constant and \(m_{Pl}\) the Planck mass.

In this framework, the first three slow-roll parameters were respectively defined in Ref. [22] and can be written as [15,16],

$$\epsilon(\phi) \equiv 3T [T + V(\phi)]^{-1} = \frac{2}{\kappa} \left(\frac{H'}{H}\right)^2 = \frac{\kappa T}{H^2}, \quad (3)$$

$$\eta(\phi) \equiv -\frac{\ddot{\phi}}{H \dot{\phi}} = -\frac{2H''}{\kappa H} = -\frac{\epsilon'}{\sqrt{2\kappa \epsilon}} = \frac{\kappa}{\epsilon} \frac{dT}{dH^2}, \quad (4)$$

$$\xi^2(\phi) \equiv \frac{4}{\kappa^2} \frac{H'H''}{H^2} = \epsilon \eta - \left(\frac{2}{\kappa}\right) \left(\frac{\epsilon'}{\epsilon}\right) \eta' = \kappa \epsilon \frac{dT}{dH^2} + 2\kappa \epsilon H^2 \frac{d^2T}{d(H^2)^2}. \quad (5)$$

Up to a constant, the first slow-roll parameter (3) is a measure of the relative contribution of kinetic energy to total field energy. By definition \(\epsilon \geq 0\) and, as it is well known and we shall see in details further in this manuscript, it has to be less than unity for inflation to proceed. This last assertion implies the potential being positive defined. We also note that while defining second and third slow-roll parameters it was assumed the potential to be a monotonically decreasing function. We shall return later to the point of assumptions behind the definitions and calculations to be used in this manuscript.

Few models of inflation allow exact determination of scalar and tensorial perturbations. One of them is power-law [10], a scenario of inflation where:

$$a(t) \propto t^p, \quad H(\phi) \propto \exp \left(-\sqrt{\frac{\kappa}{2p}} \phi\right), \quad V(\phi) \propto \exp \left(-\sqrt{\frac{2\kappa}{p}} \phi\right), \quad (6)$$

with \(p\) being a positive constant. It follows from Eqs. (3), (4), and (5) that in this case the slow-roll parameters are constant and equal each other, \(\epsilon = \eta = \xi = 1/p\). Note that condition \(\epsilon < 1\) implies \(p > 1\).

For perturbations produced during power-law inflation it can be shown [13] that \(A_T(k) = 10A_S(k)/\sqrt{p}\), where \(A_S(k)\) and \(A_T(k)\) stand for normalized scalar and tensorial spectral amplitudes, and \(k\) is the wavenumber corresponding to the scale matching the Hubble radius, \(k = aH\). Now, through definition of the spectral indices

$$n_S(k) - 1 \equiv \frac{d \ln A_S^2(k)}{d \ln k}, \quad n_T(k) \equiv \frac{d \ln A_T^2(k)}{d \ln k} \quad (7)$$

one can see that for power-law inflation
\[
\frac{n_T}{2} = \frac{n_S(k) - 1}{2} = \frac{1}{1 - p} = -\frac{1}{p} \left(1 + \frac{1}{p} + \frac{1}{p^2} + \cdots\right) \leq 0. \tag{8}
\]

where the inequality follows from above mentioned condition \(\epsilon < 1\). Obviously, relation (8) must be valid to any order in expansion of \(1/(1 - p)\). Since spectral indices are constant, for power-law inflation, definitions (7) can be rewritten as

\[
A_S^2(k) = A_S^2(k_0) \left(\frac{k}{k_0}\right)^{n_S - 1}, \quad A_T^2(k) = A_T^2(k_0) \left(\frac{k}{k_0}\right)^{n_T}, \tag{9}
\]

where \(k_0\) is a pivot scale usually taken to be the scale probed by COBE. From these expressions it is evident that \(n_S - 1\) and \(n_T\) measure the deviation of the spectra amplitudes from scale-invariance.

Up to the present, there are not general analytic expressions to calculate power spectra of inflationary models. Based on the power-law solution, Stewart and Lyth \[19\] derived approximated expressions for both spectra regarding as small the deviation of higher slow-roll parameters from \(\epsilon\) and also deviation of \(\epsilon\) with respect to zero. These approximations imply the slow-roll parameters to be slowly varying in time functions. In terms of spectral indices, NLO expressions are

\[
1 - n_S(k) \simeq 4\epsilon - 2\eta + 8(C + 1)\epsilon^2 - (10C + 6)\epsilon\eta + 2C\xi^2, \tag{10}
\]

\[
n_T(k) \simeq -2\epsilon \left[1 + (2C + 3)\epsilon - 2(C + 1)\eta\right], \tag{11}
\]

where notation is that of Ref. \[15\] and \(C \approx -0.73\) is a constant. Symbol \(\simeq\) is used to indicate that these equations were obtained using the slow-roll expansion. Hereafter we shall use equal sign in our calculations, but meaning of approximation should be added whenever it applies.

For a giving expression of scale factor, the Hubble parameter and potential are determined, and then substituting definitions (3), (4), and (5) in Stewart-Lyth equations (10) and (11), scale-dependent spectral indices are obtained. Imposing exact power-law relation \(\epsilon = \eta = \xi = 1/p\), expression (3) is recovered.

Even if it is already possible to include time dependence of \(\epsilon\) at lowest order, we shall show here that some interesting cases able to answer the question stated in this paper are only considered if NLO expressions are used. NLO expressions for the spectra have been tested and found to provide a high accuracy for theoretical perturbations calculations \[23\]. Even more, some authors have stressed that NLO expressions in terms of \(\epsilon\) will be compulsory in order to match analytic results with data to be obtained in the near future \[13,24\].

For Eq. (11) constrain \(n_T \leq 0\) remains valid, even for scale-dependent tensorial index. Assume \(n_T > 0\) then, from Eq. (11), we obtain inequality \(1 \leq 1 + \epsilon + 2(\eta - \epsilon)\). Evaluating \(C = -0.73\) yields \(\eta - \epsilon \geq 1.852\), in contradiction with the approximation \(|\eta - \epsilon| < 1\) used to derive Eq. (11). Thus, in general,

\[
n_T(k) \leq 0. \tag{12}
\]

For the kind of models we are considering here, even if tensorial perturbations are large enough to have a detectable imprint in CMB anisotropies, according with definition (7), their amplitudes will vanish in wave longitudes to be probed by most gravity waves interferometers.
A similar analysis for Eq. (10) yields that, although models with $n_s < 1$ are favored, not particular constrain exist upon values of $n_s$, thus allowing models with so called, blue spectra, i.e., $n_s > 1$.

III. STEWART-LYTH INVERSE PROBLEM

In Ref. [16] it was shown that using definitions (3), (4), (5) and defining $\tau \equiv \ln H^2$, $\delta(k) \equiv n_T(k)/2$ and $\Delta(k) \equiv [n_s(k) - 1]/2$, the indices equations in terms of the first slow-roll parameter $\epsilon$ and its derivatives with respect to $\tau$ ($\dot{\epsilon} \equiv d\epsilon/d\tau$ and $\ddot{\epsilon} \equiv d^2\epsilon/d\tau^2$), in a straightforward manner, can be written as [17,18]

$$2C\epsilon\ddot{\epsilon} - (2C + 3)\epsilon\dot{\epsilon} - \dot{\epsilon} + \epsilon^2 + \epsilon + \Delta = 0, \quad (13)$$

$$2(C + 1)\epsilon\ddot{\epsilon} - \epsilon^2 - \epsilon - \delta = 0. \quad (14)$$

Given expressions for scale-dependent spectral indices, corresponding inflaton potential can be found by solving Eqs. (13) and (14) for $\epsilon$, and using definitions of first slow-roll parameter (3). This procedure is what we called Stewart-Lyth inverse problem [16].

When SLIP was introduced in Ref. [16], the inflaton potential corresponding to a given solution of differential equations (13) and (14), was determined as a parametric function of $\tau$. Solutions of these equations are expressed with $\tau$ as an explicit function of $\epsilon$ and are difficult to convert to expressions for $\epsilon$ as explicit functions of $\tau$. Hence, it seems reasonable to look for a similar procedure, in terms of the first slow-roll parameter. Moreover, as we shall see later, using $\epsilon$ as a parameter allows us to analyze the solution restricted to the interval of $\phi$ where inflation is feasible, i.e., $0 \leq \epsilon < 1$.

The expression for the potential as a function of $\epsilon$ remains the same that in Ref. [16] and is obtained from definition (3),

$$V(\epsilon) = \frac{1}{\kappa} (3 - \epsilon) \exp [\tau(\epsilon)], \quad (15)$$

but here, instead substituting the first slow-roll parameter as function of $\tau$, we substitute $\exp[\tau(\epsilon)]$. On the other hand, using Eq. (3) [16],

$$\phi(\tau) = -\frac{1}{\sqrt{2\kappa}} \int \frac{d\tau}{\sqrt{\epsilon(\tau)}} + \phi_0, \quad (16)$$

where $\phi_0$ is an integration constant. Changing variables, and substituting $\dot{\epsilon}$ from the first order equation (14),

$$\phi(\epsilon) = -\frac{2(C + 1)}{\sqrt{2\kappa}} \int \frac{\sqrt{\epsilon} d\epsilon}{\epsilon^2 + \epsilon + \delta} + \phi_0. \quad (17)$$

This way, the inflaton potential is given by the parametric function,

$$V(\phi) = \begin{cases} \phi(\epsilon), \\ V(\epsilon). \end{cases} \quad (18)$$
Here is very important to recall that for SLIP solution (18) to be unique, \( \epsilon(\tau) \) must be solution of both equations (13) and (14). Need of information on the tensorial modes is also a conclusion stressed by Lidsey et al. [15] in their report about perturbative reconstruction of inflaton potential. With this regards, one can see that solution \( \epsilon = 1/p \) to Eqs. (13) and (14) automatically implies \( \Delta = \delta = 1/(1 - p) \) in full correspondence with relation (8).

A. Consistency criterion for SLIP solutions

Solving SLIP is not enough to state that solutions have any physical meaning. We recall that our calculations are based on several assumptions regarding the form of the potential (behavior of the inflaton as function of cosmic time) and range of slow-roll parameters. Hence, for SLIP to yield consistent results, conditions arising from this assumptions should be fulfilled by the obtained inflaton and its potential. Let us analyze these conditions in detail.

To derive Eqs. (10) and (11) and SLIP solution (18), no particular assumption was made about initial conditions for \( \phi \) nor for its expected value thus, solutions of SLIP are not constrained to chaotic or new inflation nor to a particular energy scale. Furthermore, no assumption was neither made about the potential convexity so, in principle, it could be in any of categories related to classification given in Ref. [7] and moreover, the same potential could have features characteristic of different categories. This is a consequence of dealing with NLO expressions, i.e., allowing a larger variation of slow-roll parameters during inflation. Particularly, SLIP solutions can be associated with a hybrid scenario of inflation [25]. Next, we shall analyze possible constrains upon solutions of SLIP. Deriving Eq. (1) with respect to cosmic time and inserting Eq. (2) it is obtained,

\[
T = -\frac{1}{\kappa} \dot{H}.
\]  

(19)

Now, taking into account that \( H' = \dot{H}/\dot{\phi} \), to determine a sign for \( H' \) it is necessary to fix the inflaton behavior as a function of cosmic time. In this paper, it was assumed that \( \dot{\phi} > 0 \), and, correspondingly, \( H' < 0 \). Further, comparing definitions in Eq. (3) for the first slow-roll parameter and eliminating \( \epsilon \) yields,

\[
T = \frac{2}{\kappa^2} H'^2,
\]  

(20)

and after substituting Eq. (20) in the equation of motion (1) and deriving with respect to \( \phi \) we obtain,

\[
V' = \frac{2}{\kappa} (3 - \eta) HH',
\]  

(21)

where the definition of the second slow-roll parameter (4) was used.

Eqs. (10) and (11) were obtained using the slow-roll expansion, i.e, the absolute value of \( \eta \) should be close to the value of \( \epsilon \) which, in turn, should be near zero. Therefore, \( (3 - \eta) > 0 \) and the sign of \( V' \) is determined by that of \( H' \). Hence, according to the previous assumptions, the potential must be a monotonically decreasing function of the inflaton. Summarizing, any feasible solution of SLIP should fulfill the following conditions:
These correspond to conditions for the inflaton to roll down the potential from lower to higher values of the scalar field. At least in the case we shall analyze here, it seems to be useful to write conditions (22) in an equivalent manner. Note that \( \frac{d\tau}{dt} = 2\dot{H}/H < 0 \), then conditions (22) imply

\[
\begin{aligned}
\hat{\phi} &< 0, \\
\hat{V} &> 0,
\end{aligned}
\quad \Leftrightarrow \quad
\begin{aligned}
\hat{\epsilon} \frac{d\phi}{d\epsilon} &< 0, \\
\hat{\epsilon} \frac{dV}{d\epsilon} &> 0.
\end{aligned}
\] (23)

Similar conditions could be derived for the case of an inflaton rolling down from higher to lower values by properly choosing the behavior of the inflaton as a function of cosmic time. For that case conditions (23) will read,

\[
\begin{aligned}
\hat{\epsilon} \frac{d\phi}{d\epsilon} &> 0, \\
\hat{\epsilon} \frac{dV}{d\epsilon} &> 0.
\end{aligned}
\] (24)

In fact, of all the expressions used here, the sign of \( \dot{\phi} \) only affects Eq. (17), the modification being \( \phi(\epsilon) \rightarrow -\phi(\epsilon) \). That changes the SLIP solutions by the mirror equivalent solutions.

On the other hand, inflation is defined as a period where scale factor grows accelerately, i.e., \( \ddot{a} > 0 \). This is equivalent to say that \( d(H^{-1}/a)/dt < 0 \), definition remarking that, during inflation, the comoving Hubble radius decrease with time. Deriving, using expression (19) and definition (3) we obtain already mentioned upper value for first slow-roll parameter, i.e., \( \epsilon < 1 \). By definition \( \epsilon \geq 0 \), then criteria (23) and (24) must be tested in the interval \( \epsilon \in [0, 1) \). Looking at graphs of inflaton and its potential as functions of \( \epsilon \), and plots of \( \epsilon(\tau) \) in the corresponding range, one is able to find out whether SLIP solutions will be consistent with underlying assumptions. Note that these conditions are sufficient for given potential to be inflationary but they do not ensure the inflationary epoch to be long enough.

**IV. MODELS WITH CONSTANT TENSORIAL SPECTRAL INDEX**

To settle down the cosmological parameters, the set containing the parameters that determine the realization of the cosmological model together with the parameters that determine the initial conditions is tuned in order to maximize a likelihood function \([4,6–9]\).

Commonly, the initial power spectra are set to the form corresponding to power-law inflation with negligible amplitudes for primordial gravitational waves. The assumption behind is that during the inflationary lapse where quantum fluctuations were imprinted in scales currently reentering our causal Universe, slow-roll parameters behave roughly as constants.

An important parameter is the tensor-scalar ratio of contribution to the CMB spectrum which can be defined as (see Ref. \([8]\) for alternative definitions),

\[
r \equiv 12.4 \frac{A_T^2}{A_S^2}.
\] (25)

In principle, the value of \( r \) can be estimated from observations of CMB polarization \([20]\). To NLO, \( r \) is related with the tensorial index by \([15]\)
\[ n_T \simeq -2 \frac{A^2_T}{A_s^2} (1 + 3\epsilon - 2\eta). \] 

From Eqs. (10) and (11), LO expressions are recovered by neglecting second order terms of slow-roll parameters, and from Eq. (26) by setting the expression within parenthesis equal to unity. This way, if in the desired scale range \( \epsilon \) and \( \eta \) can be approximated by constant values, to LO, \( n_S, n_T \) and \( r \) must be regarded as constants too. Hence, if some information on \( r \) is available, power-law inflation can still be the model providing the best fit to data given some distortion of the relation between indices. If some degree of scale dependence is hidden in the CMB, the error of assuming power-law will be reflected in the best-fit values of remaining parameters. Hence, to consider \( \epsilon \simeq \text{const.} \) is a big restriction regarded as fair if the fit of data using a constant scalar index give a good result. Nevertheless, even now, such a good overall fit can also be achieved for some potentials yielding scale dependent scalar index, for example, running mass models \([8]\) though the recent results in Ref. \([14]\) constrain this dependence to be very weak. Several authors have pointed out that observations with higher resolution and wider scale-range to be provided by satellites Planck and Map, and galaxies surveys should be able to discern a time dependent \( \epsilon \) \([23]\), making of power-law a poor assumption for the inflationary period. Therefore, question arises of which model could provide the best fit to upcoming data. To answer this question, an option is to find out if there exist models with slowly-varying tensorial and scalar spectral indices which, using current data, can be accurately fitted by a power-law and smoothly departs from power-law while making broader the range of scales or increasing the resolution of measurements. With this aim, one can make assumptions on the functional form for \( n_T \) and look for solutions of SLIP and corresponding functional forms of the scalar spectral index which, in turn, can be compared with observations of CMB anisotropies. In general, the tensorial spectral index should be a scale-dependent function slowly varying close to zero thus, a well-based assumption for \( \delta \) is

\[ \delta(\ln k) = \delta(\ln k_0) + a_1 \ln \frac{k}{k_0} + a_2 \ln^2 \frac{k}{k_0} + \cdots. \] 

The first reliable observation of \( r \) to be obtained is hardly expected to detect any dependence on the scale \([20]\). Hence, from this information only a constant value for \( n_T \) would be estimated. With this regard, in this paper, the analysis will be restricted to zero order in expansion (27). This approach has also the advantage of dealing with one of the few cases when SLIP is analytically solvable. We remark that to LO it is a nonsense to consider a constant tensorial index while regarding a time dependent \( \epsilon \). Solutions presented here can be obtained only by using NLO expressions related to SLIP.

A. Constant scalar and tensorial indices

Let us start by proving that using \( \delta = C_1 \) and \( \Delta = C_2 \) as SLIP input (with \( C_1 \) and \( C_2 \) being some constants) just yields power-law solution. Recall that power-law inflation has a number of characteristic features: slow-roll parameters are constant and equal each other, spectral indices are constant, and spectra given by these indices are red tilted in the same magnitude from Harrison-Zeldovich spectra, i.e., \( C_1 = C_2 \neq 0 \). To proceed with, we note
that in Eqs. (13) and (14) $\epsilon$ and its derivatives depend on $\tau$ while $\Delta$ and $\delta$ explicitly depend on $k$. So, we shall rewrite these equations in terms of $k$ in the same fashion it was done in Ref. [26]. The conversion between the inflaton values and wavenumbers while crossing the Hubble radius can be done using expression [15]

$$d\ln k/d\phi = \kappa H/2H'(\epsilon - 1).$$

(28)

Using Eq. (28) it is obtained,

$$d\ln k/d\tau = \frac{1}{2} \frac{\epsilon - 1}{\epsilon}.$$  

(29)

After conversion to derivatives in term of $\ln k$, Eqs. (13) and (14) become [26],

$$C(\epsilon - 1)^2 \dddot{\epsilon} + \frac{C(\epsilon - 1)}{2\epsilon} \ddot{\epsilon}^2 - [(2C + 3)\epsilon + 1] \frac{\epsilon - 1}{2\epsilon} \dot{\epsilon} + \epsilon^2 + \epsilon + \Delta = 0, \quad \text{or} \quad (C + 1)(\epsilon - 1)\dddot{\epsilon} - \epsilon^2 - \epsilon - \delta = 0, \quad \text{(30)}$$

where $\dddot{\epsilon} \equiv d^3\epsilon/d(\ln k)^2$.

Differentiating Eq. (31) with respect to $\ln k$ we can replace expressions for $\dddot{\epsilon}$ and $\ddot{\epsilon}$ obtained from this equation into Eq. (30) and the following algebraic expression for $\epsilon$ is obtained:

$$\epsilon(k)^4 + P\epsilon(k)^3 + Q(\tilde{\delta}, \delta, \Delta)\epsilon(k)^2 + R(\tilde{\delta}, \delta)\epsilon(k) + S(\delta) = 0, \quad \text{(32)}$$

where

$$P = C + 2, \quad Q(\tilde{\delta}, \delta, \Delta) = -(C + 1) \left[ C\tilde{\delta} - (2C + 3)\delta + 2(C + 1)\Delta - 1 \right],$$

$$R(\tilde{\delta}, \delta) = (C + 1)C\tilde{\delta} + (2C + 1)\delta,$$

$$S(\delta) = C\delta^2.$$

Roots of Eq. (32) can be calculated but they are very complicated expressions not necessary for further analysis. They can be simply written as

$$\epsilon_i = \epsilon_i(\Delta, \delta, \tilde{\delta}), \quad \text{(33)}$$

where $i = 1, \ldots, 4$. Thus, for $\delta = C_1$ and $\Delta = C_2$ the solution is $\epsilon = \text{const.}$. Substituting back this solution into Eqs. (30) and (31) we finally obtain $\delta = \Delta$. Thus, as it was expected, power-law inflation is a trivial solution of our problem. Notice that, according with Eq. (7) any inflationary potential that behaves like a decaying exponential for a range of $\phi$ corresponding to scales currently probed can be, in this range, approximated by a power-law. While this will offer good enough information on the inflationary period corresponding to currently observed Universe, it perhaps will hide clues about physics on higher energy scales.
Previously it has been stressed the relevance of Eq. (14) for constraining solutions of Eq. (13). As was already proved, for constant $\delta$, a trivial SLIP solution is power-law inflation, corresponding to $\delta = \Delta$. We shall analyze here remaining solutions involving scale-dependent $\Delta$. The methodology used in this research to solve the problem of finding models with scale-invariant tensorial index but scale-dependent scalar index consists of solving Eq. (14) for $\epsilon$ using as input a constant $\delta$, substituting the solutions on Eq. (13) and deriving the corresponding functional forms for the inflaton potential and $\Delta$.

With the aim of obtaining an expression for the scalar spectral index as function of scale, let us first to express $\Delta$ as a function of $\epsilon$. It can be derived from algebraic Eq. (32) considering a constant $\delta$,

$$\Delta(\epsilon) = \frac{1}{2 (C + 1)^2} \left\{ \epsilon^2 + (C + 2) \epsilon + (C + 1) [(2C + 3) \delta + 1] + (2C + 1) \frac{\delta}{\epsilon} + C \frac{\delta^2}{\epsilon^2} \right\} . \quad (34)$$

Now, we shall look for an expression of the comoving scale as a function of $\epsilon$. This relation is obtained by integrating Eq. (31):

$$\ln k(\epsilon) = \frac{(C + 1)}{2} \left( \ln |\epsilon^2 + \epsilon + \delta| - 3 \int \frac{d\epsilon}{\epsilon^2 + \epsilon + \delta} \right) . \quad (35)$$

This way, the scalar spectral index can be expressed as a parametric function given by

$$n_S(k) = \begin{cases} k(\epsilon), \\ 2\Delta(\epsilon) + 1 . \end{cases} \quad (36)$$

The relevant inflationary parameters are the scalar and tensorial spectra amplitudes evolved through the transfer functions into the CMB anisotropies spectrum. Having Eq. (36), a parametric expression for the scalar amplitudes $A_S(k)$ can be readily derived using definition (7) and Eq. (31):

$$A_S(k) = \begin{cases} k(\epsilon), \\ A_S(\epsilon) , \end{cases} \quad (37)$$

with $A_S(\epsilon)$ given by

$$A_S(\epsilon) = A_0 \exp \left[ (C + 1) \int \Delta(\epsilon) \frac{\epsilon - 1}{\epsilon^2 + \epsilon + \delta} d\epsilon \right] , \quad (38)$$

where $A_0$ is an integration constant, $\Delta(\epsilon)$ is given by expression (34) and the scale as function of $\epsilon$ by Eq. (35).

As it was discussed in Sec. I while analyzing constrain given by Eq. (12), we have that possible values of the tensorial index are restricted to the interval $\delta \leq 0$. With this regard, in Secs. IV B 1 and IV B 2 we shall solve the problem of finding models with scale-invariant tensorial index and scale-dependent scalar index for $\delta = 0$ and $\delta < 0$ respectively.
1. Null tensorial index, $\delta = 0$

We begin by analyzing the case $\delta = 0$ which, being easy to explicitly solve, is a good example of using SLIP to test the reliability of given functional form for tensorial index and of corresponding analysis of solutions. However, we shall see that the results fail to match current observations.

Explicit integration of Eq. (14) with $\delta = 0$ yields,

$$\exp(\tau - \tau_0) = (\epsilon + 1)^{2(C+1)},$$  \hspace{1cm} (39)

where $\tau_0$ is the integration constant. This solution, first reported in Ref. [16], is plotted in Fig. 1. Corresponding to $\delta = 0$ expression for $\phi(\epsilon)$ is obtained by straightforward integration of Eq. (17),

$$\phi(\epsilon) = -\frac{4(C+1)}{\sqrt{2}\kappa} \arctan\left(\sqrt{\epsilon}\right) + \phi_0.$$  \hspace{1cm} (40)

Expression for $V(\epsilon)$ is obtained by substituting solution (39) in Eq. (15),

$$V(\epsilon) = V_0(3 - \epsilon)(\epsilon + 1)^{2(C+1)},$$  \hspace{1cm} (41)

where $V_0 = \exp(\tau_0)/\kappa$. Separating $\epsilon$ in the expression for $\phi$ and substituting in Eq. (41) finally yields for the inflaton potential,

$$V(\phi) = V_0 \frac{3 - \tan^2 \left[\frac{\sqrt{2}\kappa}{4(C+1)}(\phi - \phi_0)\right]}{\cos^4(\phi)}.$$  \hspace{1cm} (42)

Plot of this solution for range corresponding to $0 \leq \epsilon < 1$ is presented in Fig. 2.

Looking at expression (42) and Fig. 2, one can conclude that exist a n interval of $\phi$ where the potential is not consistent with the assumption $\dot{\phi} > 0$, i.e., where the potential increases
with the inflaton value. One can wonder whether there is another sector of this potential where inflation can take place. Indeed, such a sector can exist but the primordial fluctuations generated during the corresponding inflaton rolling down could be different to those assumed as SLIP input. Note that the range of $\phi$ used to plot solution (42) was possible to determine using Eq. (40) which describe the inflaton as a function of the first slow-roll parameter. The valid range of values for $\epsilon$ determine the corresponding inflaton values.

Let us look at plots of $\phi$ and $V$ as functions of $\epsilon$, Fig. 3. Regarding criterion (23) and Figs. 1, and 3, it can be concluded that solution is consistent only for $\epsilon \in [0, \epsilon_0^*]$ with $\epsilon_0^* \equiv (6C + 5)/(2C + 3) \simeq 0.4$. This way, the correct functional form for the potential with $\delta = 0$ is that in Fig. 4. Observe that for this case the potential curvature undergoes changes. Such a behavior for the potential is obtained as a SLIP solution due to the use of NLO expressions.
After integrating Eq. (31), the comoving number $k$ as a function of $\epsilon$ is given by

$$k = k_0 \left( \frac{(\epsilon + 1)^2}{\epsilon} \right)^{C+1},$$

(43)

with $k_0$ being the integration constant. From here, $\epsilon(k)$ is obtained,

$$\epsilon(k) = \frac{1}{2} \left\{ \left( \frac{k}{k_0} \right)^{1/(C+1)} - 2 - \sqrt{\left( \frac{k}{k_0} \right)^{1/(C+1)} \left[ \left( \frac{k}{k_0} \right)^{1/(C+1)} - 4 \right]} \right\},$$

(44)

where the root was chosen corresponding to $0 \leq \epsilon < 1$.

Substituting expression (44) in Eq. (34) with $\delta = 0$, the corresponding expression for $\Delta(k)$ is,

$$\Delta(k) = \frac{1}{8(C+1)^2} \left\{ \left( \frac{k}{k_0} \right)^{1/(C+1)} - 2 - \sqrt{\left( \frac{k}{k_0} \right)^{1/(C+1)} \left[ \left( \frac{k}{k_0} \right)^{1/(C+1)} - 4 \right]} \right\}$$

(45)

$$\times \left\{ \left( \frac{k}{k_0} \right)^{1/(C+1)} + 2C - \sqrt{\left( \frac{k}{k_0} \right)^{1/(C+1)} \left[ \left( \frac{k}{k_0} \right)^{1/(C+1)} - 4 \right]} \right\}.$$
FIG. 5. $\Delta$ as a function of the comoving number $k$ and $\ln A_S^2$ as a function of the $\ln k$ for $\delta = 0$.

\[
+ \frac{1}{8(C+1)} \left\{ \left( \frac{k}{k_0} \right)^{1/(C+1)} - 2 - \sqrt{\left[ \left( \frac{k}{k_0} \right)^{1/(C+1)} - 4 \right]} \right\} \\
\times \left\{ \left( \frac{k}{k_0} \right)^{1/(C+1)} - 2 + 4C - \sqrt{\left[ \left( \frac{k}{k_0} \right)^{1/(C+1)} - 4 \right]} \right\}. \tag{46}
\]

The value for $A_0$ must be chosen taking into account the observational constrain $A_S^2 \sim 10^{-5}$ given by COBE measurements.

From these figures it is observable that the scalar index could be regarded as scale independent for relevant to measurements scales. The corresponding constant value is $n_S = (C + 2)/(C + 1) \approx 4.7$, which is too far from values allowed by theory and experiments. It seems to be not possible an inflationary model to exist such that the tensorial spectrum generated in its framework will be of Harrison-Zeldovich type and the corresponding scalar index will be scale-dependent. Therefore, to this order and considering Eqs. (26) and (11) with $\delta = 0$, it can be concluded that any model with a scale-dependent scalar index matching observations will give a nonzero tensorial contribution to the CMB spectrum.

Note that, while applying criterion (24) to the mirror image with respect to the ordinate axis of solution (42), the same result is obtained.

2. SLIP solution for negative tensorial index, $\delta < 0$

We shall find the inflationary potentials producing perturbations that, to next-to-leading order, are characterized by constant and negative tensorial index and scale-dependent scalar index. Since for $\delta < 0$, SLIP equations are not explicitly solvable in terms of $\phi$, we must look for a parametric expression for the inflaton potential.

The solution of Eq. (14) for $\delta < 0$ is [12]...
exp(\(\tau - \tau_0\)) = | \epsilon^2 + \epsilon + \delta |^{C+1} \left( \frac{2\epsilon + 1 + \sqrt{1 - 4\delta}}{2\epsilon + 1 - \sqrt{1 - 4\delta}} \right)^{\frac{C+1}{2}}. \tag{47}

The three branches corresponding to this expression are plotted in Fig. 6. It is observed that, along with stationary solution

\[ \epsilon_+ = -\frac{1}{2} + \frac{1}{2} \sqrt{1 - 4\delta}, \tag{48} \]

solutions can increase unbounded or decrease bounded by value \(\epsilon = 0\). Regarding that \(\tau\) and cosmic time have opposite signs, there is some time interval where \(\epsilon \simeq \epsilon_+\), i.e., the solution asymptotically behaves like that of power-law. After integrating Eq. (17) for \(\delta < 0\) and inserting solution (47) in expression (13), the parametric potential is given by

\[
\begin{align*}
V(\phi) &= \left\{ \begin{array}{l}
\phi(\epsilon) = \frac{2(C+1)}{2\epsilon + 1} \left[ -\sqrt{1 + \sqrt{1 - 4\delta}} \arctan \left( \frac{\sqrt{2\epsilon + \sqrt{-1 + \sqrt{1 - 4\delta}}}}{\sqrt{2\epsilon - \sqrt{-1 + \sqrt{1 - 4\delta}}} \epsilon} \right) \right] + \phi_0, \\
V(\epsilon) &= V_0(3 - \epsilon) | \epsilon^2 + \epsilon + \delta |^{C+1} \left( \frac{2\epsilon + 1 + \sqrt{1 - 4\delta}}{2\epsilon + 1 - \sqrt{1 - 4\delta}} \right)^{\frac{C+1}{2}}.
\end{array} \right.
\tag{49}
\]

The inflaton and corresponding potential as functions of \(\epsilon\) are respectively plotted in Fig. 7. Note that the graph of the potential has also a maximum around

\[ \epsilon^* \equiv \left( \frac{(6C + 5) + \sqrt{(6C + 5)^2 - 4\delta(2C + 3)}}{4C + 6} \right) \simeq 0.4 \]

for \(\delta = -0.01\), not shown in the figure in order to observe the details for small values of \(\epsilon\). Hence, the analysis for \(\delta < 0\) should be done for three intervals of \(\epsilon\), namely, \(I_1 = [0, \epsilon_+]\), \(I_2 = (\epsilon_+, \epsilon^*]\), and \(I_3 = (\epsilon^*, 1]\).
FIG. 7. Inflaton and its potential as functions of the first slow-roll parameter for $\delta = -0.01$.

Making use of criterion (23), consistent SLIP solutions are determined to exist in the intervals $I_1$ and $I_2$. Corresponding plots are presented in Fig. 8. Here, the value of the constant $V_0$ could be chosen to make the potential flat enough for conditions of successful inflation to be satisfied. For $\epsilon \in I_3$, the solution fails to fulfill criterion (23).

The main difference between both solutions is the curvature of the inflaton potential near the origin, but is not a trivial one. For the same parametric expression of the potential we have two rather different realization of inflation depending not only on initial conditions for $\phi$ but also for $\dot{\phi}$ (i.e., depending on the initial conditions for $\epsilon$). In the case corresponding to the left graph on Fig. 8, the term in Eq. (2) given by the first derivative of the potential will generically dominate during the inflationary epoch meanwhile, in the remaining case, the evolution of $\phi$ will be dominated by the friction term due to Universe expansion, $3H\dot{\phi}$.

For $\delta < 0$, after integration of Eq. (38) for $\delta < 0$, the logarithm of squared scalar amplitudes is given by

$$\ln \frac{A_s(\epsilon)}{A_0^2} = \frac{\epsilon^2 + \epsilon + \delta}{2} \ln \left( \frac{2(C+1) \ln \left( \frac{2(\epsilon + 1 + \sqrt{1 - 4\delta})}{2(\epsilon + 1 - \sqrt{1 - 4\delta})} \right)}{\sqrt{1 - 4\delta}} \right)^{\frac{3(C+1)}{2}} + \frac{3\delta(C+1)}{\epsilon^2 + 2C\epsilon^2 + 2C\delta} \ln \left( \frac{2\epsilon + 1 + \sqrt{1 - 4\delta}}{2\epsilon + 1 - \sqrt{1 - 4\delta}} \right),$$

where $k_0$ is the integration constant. With $\Delta(\epsilon)$ given by Eq. (34), parametric plots for $\Delta(k)$ corresponding to $\epsilon \in I_1$ and $\epsilon \in I_2$ are presented in Fig. 9. Again, the scale range was chosen to allow details observation at lowest scales. Observe that in both cases the present error for $n_S$ (e.g., $n_S = 0.99 \pm 0.09$ in Refs. [4]) can mask the scale dependence at large $k^{-1}$.

After integrating Eq. (38) for $\delta < 0$, the logarithm of squared scalar amplitudes is given by

$$\ln \frac{A_s(\epsilon)}{A_0^2} = \frac{\delta C - 1 - C}{C + 1} \ln \epsilon + \delta (C + 1) \ln \left( \frac{\epsilon^2 + \epsilon + \delta}{2} \right) + \frac{3\delta(C+1)}{\epsilon^2 + 2C\epsilon^2 + 2C\delta} \ln \left( \frac{2\epsilon + 1 + \sqrt{1 - 4\delta}}{2\epsilon + 1 - \sqrt{1 - 4\delta}} \right) + \frac{\epsilon^3 + 2C\epsilon^2 + 2C\delta}{2(C+1)\epsilon}.$$
FIG. 8. Consistent SLIP solutions for $\epsilon \in I_1$ and $\epsilon \in I_2$, respectively, with $\delta = -0.01$.

As observed in Fig. 10, where the parametric plots of scalar amplitudes for $\epsilon \in I_1$ and $\epsilon \in I_2$ is presented in the same figure, differences are almost impossible to note when the full range of scales is considered. Differences arise at large angular scales which could be out of reach for measurements. It means that, from the observational point of view, there could be not differences between these two realizations of inflation if the scales where differences arise are not probed or the resolution is not high enough to detect the scale dependence. These scales are precisely those where higher energies physics could leave an imprint.

Recalling the behavior of solution (39) for small $\tau$, i.e., large $t$ (see Fig. 6) and taking the limit $\epsilon \to \epsilon_+$ of Eqs. (34) and (50) it is found out that for small $k^{-1}$, $\Delta \simeq \delta$ (see also Fig. 9). That the scale interval where this behavior is observed corresponds to sufficient large number of e-folds to solve the Standard Model problems is provided by the asymptotic behavior of $\epsilon$ near the value $\epsilon_+$. These models have the desired feature of an almost negligible $\delta = \text{const.}$ and the scalar index being nearly constant in a wide range of scales, with the possibility of choosing values that can accurately match current observations. In fact, the value $\delta = -0.01$ chosen in the figures of this section corresponds (in the limit $\epsilon \to \epsilon_+$) to $n_S = 0.98$ compatible with values given in Refs. [4]. Any other constant value for $n_S$ arising from analysis similar to that of Refs. [4,6–9], except for blue spectra, can also be fitted with models given by Eq. (49). The weak scale-dependence of $n_S$ obtained is in good agreement with results in Ref. [14]. Furthermore, the above mentioned value of $\delta$ approximately corresponds to $r \simeq 0.12$ which is greater than $r^* = 0.1$, the value given in Ref. [3] as the lower limit for $r$ to be detectable with a 95-percent confidence regarding the error reported in Ref. [6] as the estimate for Planck measurements. Lower values for $r$ can be appropriately taken into account.

Because models given by Eq. (49) do not have a graceful exit to the Standard Model stage of Universe evolution ($\epsilon$ converges to $\epsilon_+$ not to 1), $\phi$ must be regarded here as the dominant scalar field in a hybrid scenario with $\epsilon_+$ being the value corresponding to the critical value of $\phi$ near which the false vacuum becomes unstable and the multiple scalar fields roll to the true potential minimum.
FIG. 9. $\Delta$ as a function of the comoving number for $\epsilon \in I_1$ and $\epsilon \in I_2$, respectively, and $\delta = -0.01$.

Finally, all of the statements done in this section with regards to solution (49) are also valid after changing $\phi(\epsilon)$ by $-\phi(\epsilon)$ and using criterion (24).

V. CONCLUSIONS

We presented a version of Stewart-Lyth inverse problem using the first slow-roll parameter as the basic variable in the procedure of finding the inflaton potential. That allows us to analyze the solutions in the range of this parameter where inflation is feasible. A criterion was introduced to check for solutions consistent with the assumptions underlying the derivation of the Stewart-Lyth inverse problem equations.

It was shown that expressions related to Stewart-Lyth inverse problem can be used to determine inflationary models corresponding to given observations. We proved that power-law inflation is a trivial solution of this problem when constant spectral indices are used as input in related equations. Next-to-leading-order in the slow-roll expansion makes possible to consider more general scenarios where slow-roll parameters can slowly vary with time. In a near future, these scenarios could be more realistic than common assumption regarding slow-roll parameters as constants during inflation.

Looking for a potential generating the primordial perturbations able to grow into CMB anisotropies and matching current and future observations, we solved the Stewart-Lyth inverse problem with constant tensorial index as input. Inflationary models were found which, unlike power-law inflation, yield scalar modes characterized by a scale-dependent index. For negative tensorial index, solutions were given as an expression depending on the first slow-roll parameter.

The special case of a Harrison-Zeldovich spectrum of tensorial perturbations, i.e., constant amplitudes of gravitational waves, is ruled out by comparison of our results with current observations. It means that it seems to not exist inflationary models with scale dependent scalar index and null tensorial index. Hence, for any model exhibiting some degree
of scale dependence of the index of curvature perturbations it must be expected a nonzero contribution of primordial gravitational waves to the amplitudes of the CMB spectrum.

Potentials obtained for strictly negative tensorial index fulfill the conditions for successful inflation. Evolution of the scalar field given by these potentials can be considered as the dynamical element in a hybrid inflation scenario, the value of the inflaton corresponding to power-law solution acting like the instability value for the false vacuum.

These models can be used as assumption on the origin of primordial perturbations to test for scale dependence of the scalar index. Using them to fit inflationary perturbations is almost as easy as using power-law inflation. Only one parameter, the constant tensorial index, need to be fitted. If CMB polarization fails to give a value of the tensor-scalar ratio greater than the threshold value $r^* = 0.1$, then the tensorial index must be fitted in the interval $(-0.02, 0)$. Otherwise, if some value for the tensor-scalar ratio is measured, then an approximated value for the tensorial index can be estimated to serve like pivot value for the fitting procedure.

We would like to stress that if any of the potentials here presented makes possible to reach an overall good fit for CMB anisotropies and to detect scale dependence of the scalar index from the next generation of observations, the conclusion to be drawn is that the actual inflaton potential is similar in the probed scale to the used one. In turn, if the quality of the overall fit is not improved compared to the result obtained using power-law inflation, either the scale dependence of the scalar index is practically negligible, or the information on the scale dependence of the tensorial index is fundamental in order to account for the features in the CMB anisotropies.

The spectra of scalar perturbations produced by these models differ from those of power-law inflation at scales corresponding to earlier times in the Universe evolution. Thus, if while increasing the quality of observations a good fit using any of these potentials is achieved, it could give some hints about physics taking place at very high energies.
ACKNOWLEDGMENTS

This research is supported in part by the CONACyT grant 32138-E. The work of CATE was also partially founded by the Sistema Nacional de Investigadores (SNI) and CINVESTAV. We want to thank Andrew Liddle and Dominik Schwarz for helpful discussions.
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