Machine Learning Clustering Algorithms Based on the DEA Optimization Approach for Banking System in Developing Countries

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Abstract — The primary purpose of this paper is to combine Data Envelopment Analysis (DEA) optimization approach with machine learning clustering method in data mining in order to introduce the most efficient DEA Decision-Making Units (DMUs) and the best Clustering algorithm respectively. The main goal of this paper in optimization part is to evaluate bank efficiency with cross-efficiency over 2014-2019 with Data Envelopment Analysis (DEA) for 12 banks from two developing countries. The cross-efficiency evaluation is an extension of DEA that provides a ranking method and eliminates unrealistic DEA weighting schemes on weight restrictions, without requiring prior information. Applying cross-efficiency can be beneficial for managers to expand their comparison and evaluation. The ranking of decision-making units (DMUs) is one of the most critical topics in efficiency assessment. To find the superior model, we consider input-oriented BCC-CCR and CCR-BCC models. This study overcomes with some data and methodology issues in measuring the efficiency of developing country’s banks and highlights the importance of inspiring increased efficiency through the banking industry comparing new suggested models and the new results. After applying the optimization step, in the second part, in Machine learning step, clustering method has been applied. Clustering is the procedure of grouping similar items together. This group of the items is called the cluster. Different clustering algorithms can be used according to the behavior of data. Farthest First and Expectation Maximization algorithms have been applied. Finally, BCC-CCR and Farthest First algorithms have been proposed as a superior optimization model and machine learning algorithm, respectively.

Index Terms — Machine Learning, Optimization, Data Envelopment Analysis, Data Mining, Clustering, Cross-Efficiency, Banking System.

I. INTRODUCTION

Despite the unprecedented growth in the banking industry in developing countries, research on the performance and efficiency of this industry is almost challenging. Therefore, one of the aims of this study is measuring efficiency levels at the banks, which are an essential topic for administrators, stockholders, and customers.

Svitalkova [1] shows that non-parametric methods are more acceptable than parametric ones for ranking decision-making units (DMUs). Based on Wanke et al. [2], DEA is a critical non-parametric method presently applied for efficiency and productivity evaluation. This method, technologically advanced by Charnes et al. [3], is founded by a scientific way of measuring efficiency. DEA classifies the most efficient DMUs and specifies what inefficient units must do to become efficient. To clarify more, DEA shows the best observe to be recognized from an efficiency frontier [4].

Over time, it has correspondingly progressed to develop more diverse with highlighting on product diversity through novel blends and valuable formulation developments instead of cooperating quality to live only as a low-cost common substitute.

The primary purpose of this paper is based on the evaluation of efficiency with cross-efficiency. We have applied the two following models to find the superior one:

- \( \text{CCR}_{IO} - \text{BCC}_{IO} \)
- \( \text{BCC}_{IO} - \text{CCR}_{IO} \)

The rest of the paper clarifies as follows:

Part 2 distributes a careful evaluation of literature related to the calculation of productivity and stipulates the potential role of the current study. Part 3 discusses the four steps of research methodology, and evaluation of each step (step1: CCR-BCC, BCC-CCR evaluations ), step2: Inputs and outputs description, step3: Evaluation in cross-efficiency, step4: Process of efficiency with cross-efficiency, followed by a discussion and conclusion of the experimental consequences in Part 5 and 6 respectively. Finally, Figure 1 shows the whole process:
II. BACKGROUND AND LITERATURE REVIEW

Banks are crucial foundations in a country’s budget and economy. Based on Tsolas and Charles [5], the banking part plays an essential role in each country; consequently, difficulties in this area are the central part of the numerous papers. Based on the importance of economic institutes, many previous articles have pursued to assess the performance of banks in various countries [6-13]. Berger and Humphrey [14], in an outstanding study, surveyed 130 pieces of training that examined 21 multiple countries to evaluate bank efficiency based on parametric and non-parametric approaches, which shows the importance of education on efficiency evaluation in bank sectors.

Other related studies can be addressed through various computational and rating methods [15-24].

It is perceived that various DEA models are commonly utilized in different studies to compare, rank, and evaluate energy efficiency. Thus, a comprehensive comparison of several efficiencies delivers insight into the bank’s efficiency. This comparison is of considerable significance to bank practitioners who desire to assess efficiency at a proper step of its progression. A unique comparing exclusive four models in cross-efficiency is applied, which
eventually results in comparing several efficient and inefficient DMUs. Finally, finding the superior model provide valuable information for bank managers to select the best model. Meanwhile, comparing various bank’s companies from different developing countries is one of the novelties of our research, which considers large laboratories at the same time. Thus, it can be beneficial for managers to have superior evaluating, remove unrelated data, and more effective processes.

III. RESEARCH METHODOLOGY

The objective of this study is to compare companies’ efficiency effectively. Using a comparative DEA with cross efficiency is established to determine the features of banks in terms of some DMUs with two suggested models. Finally, the entire progression can be divided into four steps, as follows.

A. CCR-BCC and BCC-CCR models:

1) CCR-BCC Model:
Consider manufacturing technology where if it produces \( X_0 \) and \( Y_0 \) then \( \lambda X_0 \) can produce \( \lambda Y_0 \). Only when we have \( \lambda \leq 1 \). We make a set of production possibilities that include observations and apply the principles of convexity and feasibility. This series will be introduced as follows.

Suppose the purpose of evaluating the DMU with input \( X \) and output \( Y \) concerning the abovementioned technology will be the following definition: T is defined as the set of possible production.

The main goal in the input-oriented method is to find a virtual unit in which the input \( X_0 \) is not more than \( X_0 \), and the minimum production should be \( Y_0 \). In fact:

\[
M_{\text{in}}\theta
\]
\[
\text{St.}
\]
\[
(\theta X_0, Y_0) \in T_{ND}
\]

\[
T_{\text{CCR-BBC}} = T_{ND} = \{(X,Y) | X \geq \sum_{j=1}^{n} \lambda_j X_j \& Y \leq \sum_{j=1}^{n} \lambda_j Y_j \& \sum_{j=1}^{n} \lambda_j \leq 1 \& \lambda \geq 0 \}
\]

(1)

2) BCC-CCR Model:
Consider manufacturing technology where if it produces \( X_0 \) and \( Y_0 \) then \( \lambda X_0 \) can produce \( \lambda Y_0 \). Only when we have \( \lambda \geq 1 \). We make a set of production possibilities that include observations and apply the principles of convexity and feasibility. This series will be introduced as follows.

Suppose the purpose of evaluating the DMU with input \( X \) and output \( Y \) in relation to the abovementioned technology will be the following definition: T is defined as the set of possible production.

The goal in the input-oriented method is to find a virtual unit in which the input \( X_0 \) is not more than \( X_0 \) and at least produce \( Y_0 \). In fact:

\[
M_{\text{in}}\theta
\]
\[
\text{St.}
\]
\[
(\theta X_0, Y_0) \in T_{ND}
\]

\[
T_{\text{CCR}} = T_{ND} = \{(X,Y) | X \geq \sum_{j=1}^{n} \lambda_j X_j \& Y \leq \sum_{j=1}^{n} \lambda_j Y_j \& \sum_{j=1}^{n} \lambda_j \geq 1 \& \lambda \geq 0 \}
\]

(2)

Based on the \( T_{KD} \) structure for BCC\(_{10} \) – CCR\(_{10} \):

\[
M_{\text{in}}\theta
\]
\[
\text{St.}
\]
\[
(\theta X_0, Y_0) \in T_{ND}
\]

\[
T_{\text{BCC-CRC}} = T_{ND} = \{(X,Y) | X \geq \sum_{j=1}^{n} \lambda_j X_j \& Y \leq \sum_{j=1}^{n} \lambda_j Y_j \& \sum_{j=1}^{n} \lambda_j \geq 1 \& \lambda \geq 0 \}
\]

(3)

B. Inputs and outputs Description:

As soon as the six suggested models, the orientation, and the DMUs were established, the next step was to create which variables would be involved in the model. This involvement is the most important step in applying DEA. After applying the DMU \( j \) \((j=1 ... n)\) which are banking facilities in eight selected developing countries or decision-making units, we propose the following three inputs and two outputs in our study:

- \( \text{Xij} \text{ (i= 1 ... m)}\): Fixed assets
- \( \text{Ncj} \text{ (C= 1 ... c): Personnel expenses} \)
- \( \text{Qoj} \text{ (O= 1 ... o): Total deposits} \)
- \( \text{Yrj} \text{ (r = 1 ... s): Total loans} \)
- \( \text{Mhj} \text{ (H = 1 ... h): Total profits} \)

Drake et al. [25] divide the selecting variables for financial institutes, into two following parts:

- Production: Based on Benston [26], banks are mainly measured to be service providers for customers. The inputs involve physical variables such as staff, capital, and materials. The outputs are generally related to the services available to customers, which may include deposits and loans
- Intermediation: Based on Sealey and Lindley [27], the critical role of banks is to gather assets and change them into investments and other profitable assets. The bank is chiefly playing an essential intermediary among extra managers and a lack of managers.

The production approach is more appropriate for assessing agencies, while the intermediation method is more suggested for bank evaluation. Several papers such as Svitalkova [1], Liu et al. [10] Zimkova [28], and Assaf et al.
[29] have used the same inputs in our study. However, these papers measured the number of employees instead of personnel expenses.

Concerning the outputs, many papers used the total loans as the output such as Drake et al. [25], Liu et al. [10], Assaf et al [26] and Yılmaz and Güneş [30]. Numerous papers used the intermediation method also used total loan output, like in our study, based on the primary duty of banks, is to take deposits and to lend money. We consider the total profits in our research as a second output too.

The data analysis was directed applying R language and the Benchmarking package, which makes many DEA models available.

The linear and dual evaluation for all the above-mentioned models are presented below:

1) Linear model in $CCR_{IO} - BCC_{IO}$:

Max $= \sum_{r=1}^{s} u_r x_{rp} + \sum_{b=1}^{n} e_b m_{bp}$

St.

\[ \begin{align*}
    \sum_{i=1}^{m} v_i x_{ip} + \sum_{i=1}^{c} f_i n_{ip} + \sum_{o=1}^{n} k_o q_{op} & \leq 1 \\
    \sum_{i=1}^{s} u_r y_{ij} + \sum_{i=1}^{n} e_b m_{bij} + \sum_{j=1}^{j} g_j d_{ij} - \sum_{i=1}^{m} v_i x_{ij} & - \sum_{i=1}^{c} f_i n_{ij} - \sum_{o=1}^{n} k_o q_{oj} \leq 0 \\
    u_r, e_b, v_i, f_i, k_j & \geq 0 , j = 1, \ldots, n
\end{align*} \]

2) Dual model in $CCR_{IO} - BCC_{IO}$:

Min $\theta$

\[ \begin{align*}
    \sum_{j=1}^{n} \lambda_j x_{ij} & \leq \theta_p x_{ip} \\
    \sum_{j=1}^{n} \lambda_j n_{ij} & \leq \theta_p n_{cp} \\
    \sum_{j=1}^{n} \lambda_j q_{ij} & \leq \theta_p q_{op} \\
    \sum_{j=1}^{n} \lambda_j y_{rij} & \geq y_{rp} \\
    \sum_{j=1}^{n} \lambda_j m_{hj} & \geq m_{hp} \\
    \sum_{j=1}^{n} \lambda_j & \leq 1 \\
    \lambda_j & \geq 0 \quad \theta_p \: free
\end{align*} \]

3) Linear model in $BCC_{IO} - CCR_{IO}$:

Max $= \sum_{r=1}^{s} u_r y_{rp} + \sum_{b=1}^{n} e_b m_{bp}$

St.

\[ \begin{align*}
    \sum_{i=1}^{m} v_i x_{ip} + \sum_{i=1}^{c} f_i n_{ip} + \sum_{o=1}^{n} k_o q_{op} & \geq 1 \\
    \sum_{i=1}^{s} u_r y_{ij} + \sum_{i=1}^{n} e_b m_{bij} + \sum_{j=1}^{j} g_j d_{ij} - \sum_{i=1}^{m} v_i x_{ij} & - \sum_{i=1}^{c} f_i n_{ij} - \sum_{o=1}^{n} k_o q_{oj} + w \leq 0 \\
    u_r, e_b, v_i, f_i, k_j & \geq 0 , j = 1, \ldots, n
\end{align*} \]

4) Dual model in $BCC_{IO} - CCR_{IO}$

Min $\theta$

\[ \begin{align*}
    \sum_{j=1}^{n} \lambda_j x_{ij} & \leq \theta_p x_{ip} \\
    \sum_{j=1}^{n} \lambda_j n_{ij} & \leq \theta_p n_{cp} \\
    \sum_{j=1}^{n} \lambda_j q_{ij} & \leq \theta_p q_{op} \\
    \sum_{j=1}^{n} \lambda_j y_{rij} & \geq y_{rp} \\
    \sum_{j=1}^{n} \lambda_j m_{hj} & \geq m_{hp} \\
    \sum_{j=1}^{n} \lambda_j & \leq 1 \\
    \lambda_j & \geq 0 \quad \theta_p \: free
\end{align*} \]

C. Definition of cross-efficiency:

Suppose there are N DMUs to be evaluated, indexed by $j = 1, \ldots, N$, and each DMU is assumed to produce s different outputs from m different inputs, denoted by two following relations:

\[ x_j = (x_{1j}, \ldots, x_{mj}) \]
\[ y_j = (y_{1j}, \ldots, y_{sj}) \]

All the components of the vectors $x_j$ and $y_j$ for all the DMUs are assumed to be non-negative. Each DMU is assumed to have at least one strictly positive input and one strictly positive output as follows:

\[ x_{io} = (i = 1, 2, \ldots, m) \]
\[ y_{ro} = (r = 1, 2, \ldots, s) \]

Based on the inputs as mentioned above and outputs the relative efficiency score is defined as bellow:

\[ \theta_o(u_r, v_i) = \max \frac{\sum_{i=1}^{m} u_r x_{ip}}{\sum_{i=1}^{m} v_i x_{ip}} \]

where $u_r$ and $v_i$ are the non-negative vectors that represent the output and input weights, respectively. Additionally, we require that the same weights, when applied to all the DMUs, do not provide any unit with an efficiency greater than one. This condition appears in the following set of constraints:

\[ \sum_{i=1}^{m} u_r x_{ij} \leq 1 \quad j = 1, \ldots, N \]

Cross-efficiency matrix for N DMUs and the average cross-efficiency is available in Table I:
Therefore, the fractional model for obtaining the relative efficiency of \( DMU_0 \) is as follows:

\[
\theta_0 = \max \frac{\sum_{r=1}^{m} u_r y_{r0}}{\sum_{j=1}^{n} v_j x_{0j}}
\]

\[
\text{St.} \quad \frac{\sum_{r=1}^{m} u_r y_{rj}}{\sum_{j=1}^{n} v_j x_{0j}} \leq 1 \quad j = 1, 2, \ldots, N
\]

\[
v_{i} \geq \varepsilon \quad u_{i} \geq \varepsilon \quad i = 1, 2, \ldots, m; \quad r = 1, 2, \ldots, s
\]

where \(\varepsilon > 0\) is a non-Archemidean construct to ensure strongly efficient solutions. In order to ensure the above model, have finite optimal values, Ali, and Seiford [31] and Mehrabian et al. [32] argued that \(\varepsilon\) should satisfy the following inequality:

\[
\varepsilon < 1 / \max \{\sum_{i=1}^{m} v_{i} x_{0i}\}
\]

In the above model, each unit is evaluated by its best weights. DMU0 is efficient if it lies on the Pareto frontier. Model (3) is a fractional programming problem and can be converted to the following linear programming:

\[
\text{Max } \sum_{r=1}^{m} u_r y_{r0}
\]

\[
\text{St.} \quad \sum_{j=1}^{n} v_j x_{0j} = 1
\]

\[
\sum_{r=1}^{m} u_r y_{rj} - \sum_{j=1}^{n} v_j x_{0j} \leq 0 \quad j = 1, 2, \ldots, N
\]

\[
\lambda_{j} \geq 0 \quad j = 1, \ldots, n
\]

\[
v_{i} \geq \varepsilon \quad u_{i} \geq \varepsilon \quad i = 1, 2, \ldots, m; \quad r = 1, 2, \ldots, s
\]

where \(\varepsilon > 0\) is a non-Archemidean construct.

Let \(u_{i}(r = 1, \ldots, s)\) and \(v_{i}(i = 1, \ldots, m)\) be the optimal output and input weights to the above model, respectively.

\[
\theta_0 = \sum_{r=1}^{m} u_r y_{r0}
\]

is referred to as the CCR efficiency of \( DMU_0 \), which is the best relative efficiency that \( DMU_0 \) can achieve and reflects the self-evaluated efficiency of the \( DMU_0 \). \( DMU_0 \) is CCR efficient if \(\theta_0 = 1\): otherwise, \( DMU_0 \) is CCR-inefficient.

The cross-efficiency value of \( DMU_j \) is defined as \( \theta_0 = \max \frac{\sum_{r=1}^{m} u_r y_{r0}}{\sum_{j=1}^{n} v_j x_{0j}} \), which reflects the peer evaluation of \( DMU_0 \) to \( DMU_j \) (\( j = 1, \ldots, N; \ j \neq 0 \)). Consequently, we obtain an \( N \times N \) matrix in which the diagonal members show the CCR-efficiency scores. We can compute the average of the cross-efficiency scores (the common method) in each row to proceed with a cross-evaluation. See Table 1 for details.

Because alternative input and output weights exist, Sexton et al. [33] suggested introducing a secondary goal to compute the unique efficiency score for DMUs. Doyle and Green [34] proposed aggressive and benevolent models, which minimize or maximize, respectively, the efficiency of the composite DMU constructed for other DMUs as compared to \( DMU_0 \). The most commonly used secondary goal is the aggressive model, which minimizes the efficiency of the composite DMU constructed for \( n \) DMUs. The aggressive model is given as:

\[
M_{\text{in}} \sum_{r=1}^{m} u_r \{ \sum_{j=1}^{n} v_j (y_{rj}) \}
\]

\[
\text{St.}
\]

\[
\sum_{r=1}^{m} u_r y_{r0} - \theta_0 \sum_{j=1}^{n} v_j x_{0j} = 0
\]

\[
\lambda_{j} \geq 0 \quad j = 1, \ldots, n
\]

\[
v_{i} u_{i} \geq \varepsilon \quad i = 1, 2, \ldots, m; \quad r = 1, 2, \ldots, s
\]

where \(\varepsilon > 0\) is a non-Archemidean construct, and \(\theta_0\) is the CCR efficiency of \( DMU_0 \) derived from the CCR model. The benevolent formulation for cross-efficiency evaluation is achieved by putting max in the objective function of Model (19) in place of min. See Model (20) for details.

\[
\text{Max } \sum_{r=1}^{m} u_r \{ \sum_{j=1}^{n} v_j (y_{rj}) \}
\]

\[
\text{St.}
\]

\[
\sum_{r=1}^{m} u_r y_{r0} - \theta_0 \sum_{j=1}^{n} v_j x_{0j} = 0
\]

\[
\sum_{r=1}^{m} u_r y_{rj} - \sum_{j=1}^{n} v_j x_{ij} \leq 0
\]

\[
\lambda_{j} \geq 0 \quad j = 1, \ldots, n
\]

\[
v_{i} u_{i} \geq \varepsilon \quad i = 1, 2, \ldots, m; \quad r = 1, 2, \ldots, s
\]

If a cross-efficiency matrix is given, it must be decided how to aggregate cross-efficiencies to one score for each DMU in the next step. Other associated studies can be addressed through several similar methods [35-45].

D. Evaluation in clustering:

Clustering is a foremost duty of explorative data mining, and a public procedure for numerical data analysis utilized in several areas, containing machine learning, pattern recognition, and bioinformatics. In this study, 70 percent of the data were designated as training data sets, and 30 percent of the data were selected as experimental data sets. To randomly select the experimental data, the Excel software has been used. Finally, to compare and to find the superior algorithms, eight designated clustering algorithms in WEKA software are widely discussed below [46]:

1) Farthest First

This algorithm is a Modified of K-means that seats each cluster center in sequence at the point farthest from the exiting cluster center lying inside the data range. It is appropriate for large-scale data sets. Farthest first is a heuristic created process of clustering. It also selects centroid and allocates the items in clusters. This algorithm
provides fast clustering in most of the cases since less relocation and modification is required.

2) Expectation Maximization (EM)

This algorithm is an iterative technique for the detection of maximum possibility in statistical models, and undetected hidden variables determine it. It provides a valuable result for the actual world data set. The EM iteration substitutes among acting an expectation (E) step, which produces a purpose intended for the expectation of the log-possibility assessed utilizing the existing evaluation for the factors, and a maximization (M) step, which calculates factors maximizing the expected log-possibility establish on the E step. These factor-estimates are then utilized to conclude the delivery of the hidden variables in the next E step.

IV. DISCUSSION

A. Discussion in cross-efficiency in CCR-BCC model for 2014, CCR-BCC OVER 2014-2019 and BCC-CCR over 2014-2019:

Just as an example, we consider the first period of the CCR-BCC model as a cross-efficiency evaluation. As we discussed above, the diagonal members show the CCR-BCC efficiency scores. Table II represents a cross-efficiency ranking in the first period (2014) for 12 banks.

It can be concluded from Table II:
- DMU_{10} has the first and the highest average efficiency score in the CCR-BCC model for the first period
- DMU_{6} has the second average efficiency score in the CCR-BCC model for the first period
- DMU_{3} has the third average efficiency score in the CCR-BCC model for the first period

The data covers in this study are six years from 2014 to 2019 for 12 banks in the two developing countries. The number of DMUs is N or 12, and the period is T or 6.

It can be concluded from Table II:
- DMU_{10} has the first and the highest average efficiency score in the CCR-BCC model for the first period
- DMU_{6} has the second average efficiency score in the CCR-BCC model for the first period
- DMU_{3} has the third average efficiency score in the CCR-BCC model for the first period

The average cross-efficiency-CCR-BCC for all banks over 2014-2019 is given in Table III.

| Banks | Efficiency Score (2014) | Ranking (2014) | Efficiency Score (2015) | Ranking (2015) | Efficiency Score (2016) | Ranking (2016) | Efficiency Score (2017) | Ranking (2017) | Efficiency Score (2018) | Ranking (2018) | Efficiency Score (2019) | Ranking (2019) | Avg. |
|-------|------------------------|----------------|------------------------|----------------|------------------------|----------------|------------------------|----------------|------------------------|----------------|------------------------|----------------|------|
| 1     | 91.96                  | 5              | 61.63                  | 10             | 27.24                  | 12             | 64.58                  | 8              | 44.89                  | 10             | 87.96                  | 7              | 62.93%. |
| 2     | 45.88%                 | 11             | 50.47%                 | 12             | 35.55%                 | 11             | 45.11%                 | 11             | 23.31%                 | 12             | 39.48%                 | 10             | 39.96% |
| 3     | 99.96%                 | 3              | 50.69%                 | 11             | 45.81%                 | 9              | 99.96%                 | 3              | 55.09%                 | 8              | 99.95%                 | 4              | 76.74% |
| 4     | 57.62%                 | 10             | 99.94%                 | 4              | 99.94%                 | 5              | 59.92%                 | 10             | 64.98%                 | 7              | 32.47%                 | 11             | 69.14% |
| 5     | 34.13%                 | 12             | 85.95%                 | 7              | 99.95%                 | 4              | 33.76%                 | 12             | 50.90%                 | 9              | 49.76%                 | 9              | 59.08% |
| 6     | 99.97%                 | 2              | 81.19%                 | 9              | 99.96%                 | 3              | 99.97%                 | 2              | 99.94%                 | 9              | 99.94%                 | 5              | 96.82% |
| 7     | 62.31%                 | 9              | 99.97%                 | 2              | 71.80%                 | 6              | 62.39%                 | 9              | 99.96%                 | 3              | 99.96%                 | 3              | 82.73% |
| 8     | 73.33%                 | 8              | 99.96%                 | 3              | 49.22%                 | 7              | 72.65%                 | 7              | 40.62%                 | 11             | 99.97%                 | 2              | 72.62% |
| 9     | 87.08%                 | 7              | 85.34%                 | 8              | 49.16%                 | 8              | 85.42%                 | 6              | 99.95%                 | 4              | 27.57%                 | 12             | 72.41% |
| 10    | 100%                   | 1              | 99.94%                 | 5              | 45.30%                 | 10             | 100%                   | 1              | 99.94%                 | 5              | 100%                   | 1              | 90.84% |
| 11    | 99.95%                 | 4              | 100%                   | 1              | 100%                   | 1              | 99.95%                 | 4              | 100%                   | 1              | 75.72%                 | 8              | 95.92% |
| 12    | 99.17%                 | 6              | 96.47%                 | 6              | 99.97%                 | 2              | 99.94%                 | 5              | 78.14%                 | 6              | 99.94%                 | 6              | 95.60% |

The average cross-efficiency-BCC-CCR for all banks over 2014-2019 is given in Table IV.
It can be concluded from Table III, and IV:

- BCC-CCR and CCR-BCC models have the same ranking for all DMUs.
- BCC-CCR model has the first average efficiency score over 5-years period for 12 DMUs.
- CCR-BCC model has the second average efficiency score over 5-years period for 12 DMUs.

According to the evaluation of cross-efficiency in two suggested models in Table III and IV for 12 banks with quantified input and output principles:

Based on the CCR-BCC model in Table III:

- The 6th bank has the 1st or the highest average efficiency score of 96.82.
- The 11th and 12th banks are in the 2nd and 3rd places with efficiency scores of 95.92 and 95.60, respectively.
- The 2nd bank has the 12th and the lowest efficiency score of 39.96.
- The 7th and 9th banks are in the 11th and 12th places with efficiency scores of 62.93 and 59.08, respectively.

Based on the BCC-CCR model in Table IV:

- The 6th bank has the 1st or the highest average efficiency score of 96.83.
- The 11th and 12th banks are in the 2nd and 3rd places with efficiency scores of 95.93 and 95.61, respectively.
- The 2nd bank has the 12th and the lowest efficiency score of 39.97.
- The 22nd and 30th banks are in the 28th and 29th places with productivity scores of 62.94 and 59.09, respectively.

BCC-CCR and CCR-BCC models are in the 1st, and 2nd places, respectively. Finally, the following relation is applicable for all DMUs in all cross-efficiency and all years:

\[
\text{BCC-CCR} > \text{CCR-BCC} \quad (21)
\]

B. Discussion in the clustering

After applying DEA models in the DEA-SOLVER at the first step, we apply all the efficient and inefficient DMUs in WEKA software. Finally, the accuracy and average accuracy are presented in Table V.

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**TABLE IV: COMPARING CROSS-EFFICIENCY-BCC-CCR MODEL FOR ALL BANKS OVER 2014-2019**

| Banks | Efficiency Score (2014) | Ranking (2014) | Efficiency Score (2015) | Ranking (2015) | Efficiency Score (2016) | Ranking (2016) | Efficiency Score (2017) | Ranking (2017) | Efficiency Score (2018) | Ranking (2018) | Efficiency Score (2019) | Ranking (2019) | Avg. |
|-------|------------------------|---------------|------------------------|---------------|------------------------|---------------|------------------------|---------------|------------------------|---------------|------------------------|---------------|-----|
| 1     | 91.91%                 | 5             | 61.64%                 | 10            | 27.25%                 | 12            | 64.59%                 | 8             | 44.9%                 | 10            | 87.97%                 | 7             | 62.94% |
| 2     | 45.89%                 | 11            | 50.48%                 | 12            | 35.56%                 | 11            | 45.12%                 | 11            | 23.32%                 | 12            | 39.49%                 | 10            | 39.97% |
| 3     | 99.97%                 | 3             | 59.70%                 | 11            | 45.82%                 | 9             | 99.97%                 | 3             | 55.10%                 | 8             | 99.96%                 | 4             | 76.75% |
| 4     | 57.63%                 | 10            | 99.96%                 | 4             | 99.95%                 | 5             | 99.93%                 | 10            | 64.94%                 | 7             | 32.48%                 | 11            | 69.15% |
| 5     | 34.14%                 | 12            | 85.96%                 | 7             | 99.96%                 | 4             | 33.77%                 | 12            | 50.91%                 | 9             | 49.77%                 | 2             | 59.09% |
| 6     | 99.98%                 | 2             | 81.20%                 | 9             | 99.97%                 | 3             | 99.98%                 | 2             | 99.98%                 | 2             | 99.95%                 | 5             | 96.83% |
| 7     | 62.32%                 | 9             | 99.98%                 | 2             | 71.81%                 | 6             | 62.40%                 | 9             | 99.97%                 | 3             | 99.97%                 | 3             | 82.74% |
| 8     | 73.34%                 | 8             | 99.97%                 | 3             | 49.23%                 | 7             | 72.66%                 | 7             | 40.63%                 | 11            | 99.98%                 | 2             | 72.63% |
| 9     | 87.09%                 | 7             | 85.53%                 | 8             | 49.17%                 | 8             | 85.43%                 | 6             | 99.96%                 | 4             | 27.58%                 | 12            | 72.42% |
| 10    | 100%                   | 1             | 99.95%                 | 5             | 45.31%                 | 10            | 100%                   | 1             | 99.95%                 | 5             | 100%                   | 1             | 90.85% |
| 11    | 99.96%                 | 4             | 100%                   | 1             | 100%                   | 1             | 99.96%                 | 4             | 100%                   | 1             | 75.73%                 | 8             | 95.93% |
| 12    | 99.18%                 | 6             | 96.48%                 | 6             | 99.98%                 | 2             | 99.95%                 | 5             | 78.15%                 | 6             | 99.94%                 | 6             | 95.61% |
| Avg.  | 79.28%                 | 8             | 85.06%                 | 6             | 68.67%                 | 7             | 76.98%                 | 7             | 71.44%                 | 6             | 76.05%                 |               |     |

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**TABLE V: ACCURACY COMPARISON CONTAINED BY CLUSTERING ALGORITHMS (ALL NUMBERS ARE IN PERCENT)**

| Algorithms            | CCR-BCC | BCC-CCR |
|-----------------------|---------|---------|
| Farther First         | 74.8182 | 72.5455 |
| Expectation Maximization | 65.4831 | 69.6431 |
| Average               | 70.1506 | 73.5943 |

It can be concluded from Table V, from CCR-BCC to BCC-CCR model:

- The maximum of accuracy within two assessment approaches is improved.
- The average accuracy within two algorithms is augmented.
- The accuracy of two algorithms is increased.

And finally, after applying BCC-CCR and CCR-BCC models at the first step, in the extraction of hidden rules of the second clustering step, BCC-CCR was the superior model.

V. CONCLUSION

In this study, we describe how banks operate in the presence of similar banks. Therefore, those banks which have a higher score can improve their efficiency. The more taking available information, the higher accurate and accessible data will be available. Each bank needs a productivity measurement to know its current status. So, efficient banks are the best reference for increasing the efficiency of inefficient banks. The BCC-CCR model has a more positive impacts on efficiency and accuracy scores in DEA optimization approach and machine learning clustering method compare with CCR-BCC model. The proposed approach, geometric average, results, and predictions derived from the period and efficiencies in cross-efficiencies can help the practitioner to compare the efficacy of uncertain cases and instruct accordingly. In the future, applying window analysis and Malmquist Productivity Index (MPI) and comparing final productivities and efficiencies result with cross-efficiency will be valuable. Since the window analysis method is based on a moving average, it is useful for finding per efficiency trends over time. Meanwhile, using fuzzy and random data for cross-efficiency will be interesting as a final comparison. So, the results and predictions can be helpful for managers of these banks and other managers who benefit from this approach to achieve a higher relative efficiency score. Besides, managers can compare the efficiency of the current year with other similar companies over the past years.
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