A Quantile-Based Approach for Transmission Expansion Planning

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ABSTRACT Transmission expansion planning is an integral part of power system planning and consists of generating and selecting transmission proposals for maintaining sufficient transmission capacity to satisfy the electric load. Specifically, the desire to increase the use of renewable energy has exposed the limitations of transmission networks and has elevated the importance of transmission expansion planning. However, considering the random nature of renewable sources in conjunction with the power outages makes the planning process very challenging. We present a new procedure for selecting the best transmission enhancement proposal from a set of finite proposals under uncertainty. The selection is based on the quantile value of the cost of each proposal. The procedure uses a combination of simulation and optimization and considers randomness of uncertain parameters of the network. Wind energy and network contingencies are among the considered random parameters. The procedure is suitable for evaluating investor-initiated enhancement proposals by the planner and statistically guarantees satisfaction of the planner’s prespecified probability of correct selection since simulation is involved. Two IEEE test networks are used for demonstrating the implementation of the new procedure. For these two test networks, solutions obtained using quantiles are compared with those when the expected value or a weighted combination of the expected value and the conditional-value-at-risk are used as selection criteria. The comparison shows that similar to the use of conditional-value-at-risk, the selection is sensitive to the choice of the quantile.

INDEX TERMS Decision making, quantile, simulation, stochastic optimization, transmission enhancement planning.

MODEL NOMENCLATURE

A. INDICES
- \( i, j \) Bus indices.
- \( l \) Transmission line index.
- \( w \) Load index.
- \( g \) Generator/wind farm index.
- \( k \) Proposal index.
- \( n \) The \( n \)th observation of the \( r \)th random sample index.

B. SETS
- \( B \) Set of buses.
- \( \Omega^i \) Set of loads at bus \( i \).
- \( \Omega^g \) Set of generators and wind farms located at bus \( i \).
- \( \Omega^G \) Set of generators at bus \( i \), \( \Omega^{G,i} \subset \Omega^G \).
- \( \Omega^W \) Set of wind farms at bus \( i \), \( \Omega^{W,i} \subset \Omega^G \).
- \( L \) Set of existing transmission lines.
- \( N \) Set of new transmission lines.
- \( N_k \) Set of new lines that form proposal \( k \), \( \Omega^N_k \subset \Omega^N \).
- \( ti \) Set of \( j \)-buses linked to bus \( i \) by line \( lij \).
- \( fi \) Set of \( j \)-buses linked to bus \( i \) by line \( lji \).

C. PARAMETERS
- \( IC_k \) Investment cost of the proposal \( k \) [$/h].
- \( c_k \) Hourly value of the discounted annual cost of proposal \( k \) [$/h].
- \( c^g_i \) Production cost of the generator/wind farm \( g \) located at bus \( i \) [$/MWh].
- \( c^wi \) The cost of load not supplied to load \( w \) located at bus \( i \) [$/MWh].
- \( P^lij \) Thermal limit capacity of the line \( l \) that connects buses \( i \) and \( j \) [MW].

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Proposals using the I-P process are also known as merchant proposals to the planner for consideration. The planner evaluates the ISO of New England implement the P-I process [3]. Enter into competitive bidding to execute the proposal. The mission proposals needed for implementation; and investors planner-investor (P-I) process and an investor-planner (I-P) process to a specific expansion problem. These processes are the effective way.

Customers are satisfied in an efficient, reliable, and cost-effective way. Mission enhancement needs to ensure the power needs of FERC; and its stakeholders include customers, utility companies, and investors. An RTO/ISO must address its transmission enhancement needs to ensure the power needs of customers are satisfied in an efficient, reliable, and cost-effective way.

In general, RTOs/ISOs follow one of two processes for a specific expansion problem. These processes are the planner-investor (P-I) process and an investor-planner (I-P) process [2]. Under the P-I process, planners identify transmission proposals needed for implementation; and investors enter into competitive bidding to execute the proposal. The Pennsylvania-New Jersey-Maryland (PJM) Interconnection and the ISO of New England implement the P-I process [3].

Under the I-P process, investors submit transmission proposals to the planner for consideration. The planner evaluates the proposals submitted and selects the best proposals. Proposals using the I-P process are also known as merchant transmission projects in which investors assume all market risks of a project and have no captive customers from which to recover the project’s cost [4]. The Midcontinent ISO and the Southwest Power Pool implement the I-P process [3]. The I-P process is the focus of this paper.

A planner operating under the I-P process may use a variety of methods to evaluate the operational and economic merits of the proposals. These methods range from ad hoc ranking methods to optimization models. The advantage of using an optimization model is that certain aspects of operating the network, such as the level of power generation at generation nodes, could be treated as decision variables. In this paper, we focus on the use of optimization model(s) to select the best proposal under the I-P process.

When using an optimization model to evaluate the merit of proposals and select the best proposal, the planner may use one combined optimization model incorporating all proposals or a set of specific optimization models, each representing one proposal. For a combined optimization model, a set of binary decision variables and constraints represents the selection or nonselection of each proposal. When using multiple optimization models, a specific model is created for a proposal to capture its characteristics. The advantage of using a specific optimization model for each proposal over a combined model is the ease of modeling and more accurate evaluation of the proposals. The planner evaluating these proposals generally wants to select the proposal with the lowest operational cost that meets certain operational constraints and also has the lowest or a reasonable construction bid cost [5]. In this paper, we are concerned with the I-P process where the planner is using a specific model for each proposal to evaluate the proposal’s operational merit.

The planner and operators of a transmission network face uncertainties related to load and generation levels as well as the availability of transmission lines [6]. Transmission network enhancement planning considers the impact of these uncertainties and explicitly accounts for them when evaluating the enhancement proposals. When uncertainty parameters are represented through probability distributions, the planning problem is formulated via stochastic optimization. The problem that we are addressing is one faced by a planner evaluating a set of transmission enhancement proposals in an uncertain environment by using stochastic optimization models to select the best proposal.

Since a set of parameters for the stochastic optimization model of an enhancement proposal are random variables, the objective function value for a feasible solution of the model is also a random variable. For the purpose of exposition, let Fig. 1 represent the distribution of the objective function, Z, for a feasible solution of the model. If this and other feasible solutions are to be evaluated based on their expected value, E[Z], then the objective function of the model is represented as minimizing the expected value of the operational and investment costs of the project given a set of random variables. The expected value is a summary statistic and is sensitive to the distribution’s outlier. Use of the expected

\[ p_{ij} \] Susceptance of the line \( l \) that connects buses \( i \) and \( j \) [S].

\[ \theta_k^{ref} \] Voltage angle at reference bus for the TEP\(_k\) model of proposal \( k \) [rad].

\[ N^s \] Number of identical wind turbines forming wind farm \( g \) located at bus \( i \).

\[ P_{g_{ref}} \] Power generation of a wind turbine given the \( r_{g}^{th} \) realization of wind speed at the location of the \( g^{th} \) wind farm [MW].

D. DECISION VARIABLES OF TEP\(_k\) MODEL FOR PROPOSAL \( k \)

\[ P_{g_{i}} \] Power generated by generator/wind farm \( g \) located at bus \( i \) [MW].

\[ P_{l_{i}} \] Load shed by load \( w \) located at bus \( i \) [MW].

\[ \theta_l \] Voltage angle at bus \( i \) [rad].

E. RANDOM VARIABLES (THEIR \( r_{g}^{th} \) REALIZATION)

\[ \xi_{w_{i}} \] Load level of load \( w \) located at bus \( i \) [MW].

\[ \xi_{g_{i}} \] Capacity level of generator \( g \) located at bus \( i \) [MW].

\[ \xi_{g_{i}}^{'} \] Wind speed at the location of wind farm \( g \) located at bus \( i \) [m/s].

\[ \eta_{g_{i}} \] Availability of generator \( g \) located at bus \( i \) [{0,1}].

\[ \eta_{l_{i}} \] Availability of line \( l \) that connects buses \( i \) and \( j \) [{0,1}].

I. INTRODUCTION

A Regional Transmission Organization (RTO) jointly with an Independent System Operator (ISO) are responsible for planning, operating, and maintaining the electric power transmission system within their geographical region of responsibility [1]. Operation of power transmission networks is regulated by the Federal Energy Regulatory Commission (FERC); and its stakeholders include customers, utility companies, and investors. An RTO/ISO must address its transmission enhancement needs to ensure the power needs of customers are satisfied in an efficient, reliable, and cost-effective way.

In general, RTOs/ISOs follow one of two processes for a specific expansion problem. These processes are the planner-investor (P-I) process and an investor-planner (I-P) process [2]. Under the P-I process, planners identify transmission proposals needed for implementation; and investors enter into competitive bidding to execute the proposal. The Pennsylvania-New Jersey-Maryland (PJM) Interconnection and the ISO of New England implement the P-I process [3].

Under the I-P process, investors submit transmission proposals to the planner for consideration. The planner evaluates the proposals submitted and selects the best proposals. Proposals using the I-P process are also known as merchant
value ignores the risk and is suitable for planners who are risk neutral [7].

Risk-conscious planners, however, are interested in bounding the risk of the selected proposal. For this purpose, a suitable risk measure is the quantile value that provides a cost target above which the cost values have a prespecified probability of occurrence. The quantile value for a risk of 1-q is defined by $z_q = \min \{z : P(Z \leq z) \geq q \}$ [8]. The $q^{th}$ quantile represents the cost that will not be exceeded with the probability of $q$. A planner using this risk measure is interested in the proposal that has the smallest $q^{th}$ quantile value.

Conditional value at risk (CVaR) is an alternative risk measure. For a given $0 < q < 1$, it is defined as $CVaR^q (Z) = E [Z \mid Z \geq z_q]$ [9]. Like the expected value measure, CVaR is a summary statistic representing the average of the cost exceeding the quantile value. Clearly, $z_q < CVaR^q$ and $z_q$ provide a selection criterion that indicates the planner is more concerned with the risk of any upside cost rather than the risk of the average upside cost. Therefore, we propose the use of $z_q$ as a risk-sensitive selection criterion in transmission planning. It should be noted that both quantile and CVaR originated from the field of finance as financial risk measures.

Quantile, unlike CVaR, is not a coherent risk measure. A coherent risk measure should satisfy four axioms: monotonicity, translation equivariance, positive homogeneity, and subadditivity [10]. The quantile value satisfies all axioms with the exception of the subadditivity, i.e., $z_q (X + Y) \leq z_q (X) + z_q (Y)$, where $X$ and $Y$ are random variables. Satisfying the subadditivity axiom in finance is important since it points to the advantage of diversification, i.e., diversifying a portfolio of investments reduces its risk. However, our proposed application is not concerned with diversification but rather with selecting the best proposal based on the behavior of one random variable, i.e., the risk of upside cost representing one random variable. Thus, we will compare the expansion proposals using the $z_q$ values of their objective functions.

The unique contributions of the paper are as follows:

- We propose, in the presence of uncertainty, the use of a quantile as the selection metric for selecting a transmission expansion proposal from among a set of competing proposals. The use of a quantile offers the planner a mechanism to control the level of cost risk important to the system operation. The planner specifies the desired risk level, $q$ quantile, for comparison of proposals. In general, the planners are interested in bounding the higher costs and thus the selected quantile resides at the upper end of the cumulative distribution of the cost.

- We offer a new procedure to select the best proposal from among transmission proposals submitted by investors. This procedure estimates quantile value, $z_q$, of the total cost function of each proposal and selects the proposal that has the smallest estimated quantile value. The estimated quantile values are obtained by random sampling from random parameters and solving a proposal centric optimization model using the random samples.

- The proposed procedure probabilistically guarantees the best proposal has been selected at the probability level $0 < P^* < 1$ specified by the planner. This feature is very important since random sampling is involved, and the selection decision should not be made until enough samples have been taken. This feature often is missing from simulation studies and its inclusion in the proposed procedure in novel.

The rest of this paper is organized as follows. In Section II, we present a literature review that concisely summarizes the selection criteria used in stochastic optimization for solving the transmission planning problem. In Section III, we describe the proposed procedure that is suitable for implementing any optimization model and its random parameters. In Section IV, numerical examples are presented. The section uses a generic optimization model for testing the proposed procedure. Use of this model is for demonstration purposes; a planner may use a more or less complicated model. Section V presents some concluding remarks.

II. SELECTION CRITERION REVIEW

The random variables incorporated into the planning models in most published papers are the load, renewable generation, and operation and maintenance costs. Other variables include system line’s and/or generator’s contingencies, e.g., in [11]–[19]. The variables of the models are represented by using robust sets, resulting in solving robust optimization models, e.g., in [20]–[24], and by using probabilities distributions, resulting in solving stochastic optimization models that is the case of this paper.

For a majority of stochastic models, optimizing the expected value of the objective function is the selection criterion, e.g., [25]–[27]. However, some models consider minimizing only the CVaR, e.g., [17], [28], the sum of the $E [Z]$ and $\beta$-CVaR, e.g., [29], [30], or $E [Z]$ subject to $\beta$-CVaR, e.g., [31], [32], where $\beta$ is the planner’s specified trade-off between the expected value and the risk, as represented by CVaR. Other selection criteria include the standard deviation (std). The std is the square root of the variance of the objective
function and measures the dispersion of its values with respect to the expected value of the objective function [33]. Some models’ objective function include both the $E[Z] + \theta \cdot std$, e.g., [34], [35], and others are of the form $stpE[Z]/std$, e.g., [36], and $E[Z] + \theta \cdot std$, e.g., [37], where $\theta \geq 0$ is the planner’s specified trade-off between the expected value and the risk as represented by the deviation.

Solution approaches to stochastic optimization problems incorporate simulation [38]. In simulation-optimization approaches, random variables are sampled from their respective distributions; and the optimization model is solved deterministically for each realization of the random variable, while a stopping rule is used to end the simulation. The rules deal with the computational efficiency and solution accuracy of the solution and include measures of variance, e.g., in [16], [26]. Our proposed procedure uses a simulation-optimization approach to estimate the selection criterion value, i.e., the $q$-quantile value of the cost distribution function ($z^q$). The procedures statistically guarantee to estimate the $q$-value with the specified level of confidence $P^* \times 100$%.

III. THE PROPOSED PROCEDURE

A generic optimization model for evaluating a transmission enhancement proposal has the structure given by (1).

\[
\begin{align*}
    \text{TEP}(\mathcal{Z}) : & \{ Z = \min f(X), & (1.a) \\
                      & g(X) = 0 , & (1.b) \\
                      & h(X) \leq 0 , & (1.c) \\
                      & X^{\min} \leq X \leq X^{\max} \} , & (1.d)
\end{align*}
\]

where $X$ represents a vector of decision variables, $Z$ represents the value of the objective function, and $\mathcal{Z}$ is the parameter set of the model. Equation (1.a) is the objective function. Equations (1.b) and (1.c) represent equality and inequality constraints of the problem. Equation (1.d) represents the lower and upper limits of the decision variables.

The elements of set $\mathcal{Z}$ may be divided into two subsets, one containing the random parameters ($\Phi \subseteq \mathcal{Z}$) and the other containing the deterministic parameters ($\Psi \subseteq \mathcal{Z}$), where $\Phi \cup \Psi = \mathcal{Z}$. When the set $\Phi$ is not empty and contains parameters associated with the constraints and, possibly, the objective function, the feasible space of the model is a random set; and the objective function is a random variable. The proposed procedure assumes the set $\Phi$ is not empty and the planner is using $z^q$ of the objective function as the selection criterion. Under the simulation-optimization procedure, we can customize a specific optimization model, TEP$_k(\mathcal{Z})$, for the $k^{th}$ enhancement proposal.

To determine the $z^q$ for each enhancement proposal, the probability distribution of its models’ objective functions needs to be obtained. To this end, we repeatedly generate random samples from $\Phi$; and for each sample set, we solve a proposal’s optimization model to obtain its objective function value. By using the objective function values obtained, its probability distribution is determined.

The procedure is flexible to accommodate the random treatment of the load, generation capacity, wind speed, and transmission line, and/or generator availability. The random variable probability distributions could be empirical or known distributions. Historical data could be used to create the empirical distributions, or the data could be fitted into a known distribution. Before we describe the proposed procedure in detail, we describe the procedures for random sample generation and estimation of $z^q$.

A. GENERATION OF RANDOM SAMPLES

We generate a realization $x$ from the cumulative probability distribution of the random variables $X$ using the inverse transform method [39]. The sampling method first generates a uniformly distributed random number $u \in (0,1)$ and then uses it for computing a realization from the inverse of the cumulative distribution function ($F^{-1}$) of the variable. The inverse of the cumulative distribution $F(x)$ is $F^{-1}(u) = \min \{ X : F(x) \geq u \}, u \in (0,1)$.

Using this method, we generate $R$ random sets, each containing $N$ samples for each element of $\Phi$. We refer to the $r^{th}$ random sample from the set $\Phi$ by $\Phi_r, r = 1, \ldots, R$, and refer to its $n^{th}$ observation by $\Phi_{rn}, n = 1, \ldots, N$. The $r^{th}$ observation is a realization of random variables.

B. QUANTILE ESTIMATION

By solving the model TEP$_k(\mathcal{Z})$ for each sample set $\Phi_{rn}$, we obtain $N$ observations for the objective function of proposal $k$. By using these $N$ observations of the objective function for the proposal $k$, we estimate $z^q$.

We use an $L$-estimator for estimating $z^q$ of the objective function. An $L$-estimator is a weighted function of the sample’s order statistics. In the proposed procedure, we implement the Harrell and Davis (HD) [40] $L$-estimator described below.

Let $z_{k,r} = \{ z_{k,r1}, z_{k,r2}, \ldots, z_{k,rN} \}$ be the $r^{th}$ set of values of the objective function, and let $z_{k,r1}, z_{k,r2}, \ldots, z_{k,rN}$ be its corresponding order statistics. The $z^q$ estimator for the objective function of proposal $k$ is determined by (2).

\[
z^q_{k,r} = \sum_{n=1}^{N} w_n \cdot z_{k,r(n)}
\]

where $w_n = I_{n}[N \cdot a, b] - I_{(n-1)/N}[a, b]$; $I_c[a, b]$ is the incomplete beta function defined by the interval $[0, c]$, parameterized by $a = q(N + 1)$ and $b = (1-q)(N + 1)$, and determined by solving (3).

\[
I_c[a, b] = \int_0^c \int_0^1 (1-t)^{b-1} t^{a-1} \, dt /
\]

\[
\int_0^1 t^{a-1} (1-t)^{b-1} \, dt
\]

The efficiency of the HD estimator depends on the sample size $N$, the quantile $q$, and the shape of the random variable probability distribution. Simulation studies using the Kolmogorov-Smirnov nonparametric test have shown that for the uniform and normal distributions, the HD estimator is adequate when the sample size is larger than 20, when
FIGURE 2. Pseudocode for estimating an HD quantile value of the objective function of each proposal.

```
EST (q, K, \Phi_r, TEP_r)
//Input: q: quantile; K: proposals; \Phi_r = \{\Phi_{r1}, \Phi_{r2}, ..., \Phi_{rn}\}; random sample r of size N; TEP_r: TEP model for each proposal.
//Output: The value of a quantile estimate for the objective function of each proposal.
1 for k ← 1 to K do
2 for n ← 1 to N do
3 solve TEP_r(\Psi \cup \Phi_{rn}) to determine the optimal value of the objective function, z^q_{k,r}.
4 estimate the quantile value z^q_{k,r} using Equation (2)
5 Return z^q_{k}, k = \{1, ..., K\}
```

q = 0.5, or around 50, when q = 0.95. For asymmetric distributions (e.g., exponential), sample sizes as large as 100 may be required for q = 0.9 or above [40]. The subroutine EST renders an HD estimate for the \(z^q\) of the objective function for all K transmission proposals, and its pseudocode is shown in Fig. 2.

Line 1 to Line 3 of the pseudocode (L1-L3) obtain N optimal values for each proposal’s objective function using the random sets of sample r. To obtain the \(n^{th}\) value, L3 solves the TEP\(_k\) model using the samples’ \(n^{th}\) random set; L4 estimates the value of the \(q\)-quantile of the objective function for each proposal using its N optimal values; and L5 returns the K estimates.

C. DESCRIPTION OF THE PROCEDURE

Since distributions of the objective functions are sampled distributions, the correctness of any decision using these distributions cannot be guaranteed with certainty [41]. Therefore, we want to be at least \(P^*\times 100\%\) confident that we have selected the best proposal. Thus, \(P^*\times 100\%\) represents the lower bound on the probability of correct selection. The value of \(P \in (0,1)\) is user specified, and a larger \(P^*\) requires a larger number of samples. Therefore, the proposed procedure’s selection criterion is shown by (4).

\[
k^* = \arg \max_{k \in K} \{ k \mid \text{Pr} \left( z^q_{k,r} \leq z^q_v \right) \geq P^* \}
\]  

(4)

To ensure this level of confidence in our decision, we propose a three-stage procedure. The first stage of the procedure obtains a set of quantile value estimates and their respective means and variances for all proposals. The second stage determines whether additional quantile values are needed in order to achieve the desired level of confidence that the selected proposal is indeed optimal. If additional samples are needed, they are obtained. The third stage compares the quantile values of the proposals and selects the best proposal. A schematic of the procedure is shown in Fig. 3.

The input of the selection procedure is the TEP\(_k\) model and the random parameter set (\(\Phi\)) and its distributions for each proposal, an initial number of samples (\(R \geq 2\)), the sample size (N), a desired confidence level (\(P^*\)), the indifference zone (\(\delta\)), and the quantile (\(q\)).

In Stage 1, the proposed procedure obtains \(R\) quantile value estimates (\(z^q_{k,1}, z^q_{k,2}, ..., z^q_{k,R}\)) for each proposal. To obtain those estimates, we generate \(R\) random samples of size N from the parameter set \(\Phi\). For each random sample r, we then execute subroutine EST to estimate the quantile value for the proposals’ objective functions. These steps are repeated \(R\) times to produce the \(R\) estimates for each proposal.

In Stage 2, the proposed procedure determines whether additional quantile values are required to statistically validate the final selection with at least \(P^*\) confidence. The subroutine REQ returns that required number (\(R_{\text{max}}\)) of samples. If the required number is larger than \(R\), we then obtain \(R_{\text{max}} - R\) additional quantile estimates for each proposal. The pseudocode for subroutine REQ is shown in Fig. 4.

In our quantile-based selection procedure, a correct selection (CS) is defined as the event in which the procedure selects the best proposal \(k\) when its quantile value \(z^q_{k,r} \leq z^q_v - \delta\), \(\forall v \neq k\); the parameter \(\delta\) is the indifference-zone
REQ (P*, δ, R, K, $\mathbf{\hat{z}_k}$)

//Input: P: confidence level; δ: indifference-zone; R: number of samples; K: proposals; $\mathbf{\hat{z}_k}$: R quantile value estimates ($\mathbf{\hat{z}_{k1}}, \mathbf{\hat{z}_{k2}}, \ldots, \mathbf{\hat{z}_{kR}}$) for each proposal.

//Output: required number of samples.
1 for k ← 1 to K do
2 $\overline{z}_k^q = \frac{1}{R} \sum_{r=1}^{R} \mathbf{\hat{z}}_{kr}$ to determine
3 $S_k^q = \sqrt{\frac{1}{R-1} \sum_{r=1}^{R} (\mathbf{\hat{z}}_{kr} - \overline{z}_k^q)^2}$ and then calculate
4 $R_k = \max \left\{ R \left( \frac{h \overline{z}_k^q}{S_k^q} \right)^2 \right\}$
5 $R_{max} = \max (R_1, \ldots, R_K)$
6 Return $R_{max}$

FIGURE 4. Pseudocode for determining the required number of quantile samples. In Line 4, $[\lfloor \cdot \rfloor]$ represents the smallest integer greater than or equal to the value within the ceiling bracket.

COMP (K, $\mathbf{\hat{z}_k}$)

//Input: K: proposals; $\mathbf{\hat{z}_k}$: $R_{max}$ quantile value estimates for each proposal ($\mathbf{\hat{z}_{k1}}, \mathbf{\hat{z}_{k2}}, \ldots, \mathbf{\hat{z}_{kR_{max}}}$).

//Output: the best proposal $Y$
1 for k ← 1 to K do
2 determine $\mathbf{\hat{z}_k}$
3 $Y \leftarrow K$
4 for v ← 1 to K do
5 for k ← 1 to K do
6 if v ≠ k
7 if k ∉ Φ & $\mathbf{\hat{z}_v} < \mathbf{\hat{z}_k}$
8 $Y \leftarrow Y \setminus k$
9 Return Y

FIGURE 5. Pseudocode for determining average quantile value of proposals and selecting the proposal with the smallest average quantile value.

IV. NUMERICAL EXAMPLES

We used the Institute of Electrical and Electronics Engineers (IEEE) 24-bus and 118-bus test systems to demonstrate the implementation of the proposed procedure. The optimization model and random and nonrandom parameters chosen for the experiments are for demonstration purposes only. For other cases, different models and parameters could be used. We assumed the planner was interested in selecting the proposal that had the smallest 90 percentile quantile of the total cost, i.e., $min_{k} \delta_{k}^{0.9}$, with the probability of correct selection of $P^* = 0.99$. We set the initial quantile sample size to R=10 and used N=100 to determine a quantile estimate. The IZ parameter was set to $\delta = $500.

We added wind-based generation to the IEEE systems and used the historical wind speed data from July 2007 to July 2017 available at [44] to determine the scale ($\lambda$) and shape ($\delta$) parameters of the Weibull distribution $W(\lambda, \delta)$ representing the probability of wind speed, i.e., $\xi^{wi} \sim W(6.02\, m/s, 2.1)$. The distribution obtained and the power curve in [45] were considered for modeling the power generation of the wind farms located at specified buses. For the other generators in the systems, we considered a possible 20% fluctuation in the level of generation from their nominal values at any given time; therefore, we used a uniform distribution $\xi^{gi} \sim U(a,b)$ for the nonrenewable generators.

We considered the historical data from July 2014 to July 2017 of the hourly load of the ISO of New England [46] to be representative of the network’s total load. We used this data to estimate the parameters of a normal distribution and scaled it to obtain the parameters of the load distribution at each bus. In Section A of the Appendix, we present the equations used for determining the normal distribution of the random load at bus $i$, i.e., $\xi^{wi} \sim N(\mu^{wi}, \sigma^{wi})$. Moreover, the availability of each line follows a Bernoulli distribution $\eta^{li} \sim Brn(p^{li})$, and each generator is represented by a Bernoulli distribution $\eta^{gi} \sim Brn(p^{gi})$. The distribution’s parameter is defined by $p = 1 - FOR$, where FOR is the forced outage rate of lines/generators and indicates the probability of an unexpected failure putting the network under contingency [47]. For each test system, we used either its published FORs for its lines and generators or made a reasonable assumption for their values.
For each proposal $k$, we formulated a customized deterministic TEP$_k$ optimization model whose objective function was to minimize the sum of the production cost of generators and wind farms and the cost of load not supplied plus the investment cost of the proposal. The generic TEP$_k$ model for proposal $k$ given the $n_k$ random set of the $r$th sample, $\Phi_{n_k} = \{\xi_{wi}^r, \xi_{wi}^l, \zeta_{ri}^r, \zeta_{ri}^l, \eta_{ri}^r, \eta_{ri}^l\}$, is shown by Equation (6). It should be noted that the proposed procedure is not tied to a specific type of optimization model and is flexible to accommodate different problem-specific optimization models.

$$\begin{align*}
\text{TEP}_k (\Psi \cup \Phi_{n_k}) : & \{ z_{k,n_k} = \min \chi_k \sum_{i \in \Omega^B} \left( \sum_{g \in \Omega^{Gi}} c_{gi} \cdot P_{gi}^k + \sum_{w \in \Omega^{wi}} c_{wi} \cdot P_{wi}^k \right) + c_k \} \\
& \text{subject to} \sum_{g \in \Omega^{Gi}} P_{gi}^k + \sum_{j \in \Omega^B} P_{lj}^k - \sum_{j \in \Omega^B} P_{lj}^k = \sum_{w \in \Omega^{wi}}\xi_{wi}^k - P_{wi}^k \} \\
& P_{lj}^k = n_{lj}^k \cdot \left( \theta_{lj}^k - \theta_{lj}^k \right) \} \\
& -p_{lj}^k \leq P_{lj}^k \leq p_{lj}^k \} \\
& 0 \leq P_{wi}^k \leq \xi_{wi}^k \} \\
& 0 \leq P_{gi}^k \leq \eta_{gi}^k \} \forall g \in \Omega^{Gi} \\
& 0 \leq P_{gi}^k \leq N^{Gi} \cdot \xi_{gi}^k \} \forall g \in \Omega^{Wi} \\
& -\pi \leq \theta_{lj}^k \leq \pi \} \forall i = \text{ref} \\
& \theta_{lj}^k = 0 \} \\
& \forall i, j \in \Omega^B; \forall g \in \Omega^{Gi}; \forall w \in \Omega^{wi}; \forall l \in \Omega^L \cup \Omega^N_k \\
\end{align*}$$

\[
\mathbb{I} \{ \theta_{kj}^k, P_{kj}^k, \bar{P}_{kj}^k \} \]

where $X_k (\{\theta_{kj}^k, P_{kj}^k, \bar{P}_{kj}^k\})$ is the set of continuous decision variables. Equation (6.a) is the objective function, and $z_{k,n_k}$ represents the optimal value given random set $\Phi_{k,n_k}$. To determine the equivalent hourly value of the investment cost of the $k$th proposal, $c_k$, we first calculated the discounted annual value of its investment cost (IC) by using the capital recovering formula shown in (7) and then divided that discounted value by 8,760 hr. We assumed an annual interest rate (ir) of 5% and an economic life (el) of 40 years for the proposals.

$$c_k IC^k \cdot \frac{1 + ir}{(1 + ir)^{el} - 1} \cdot \frac{ir}{(1 + ir)^{el} - 1} \cdot (7)$$

Equation (6.b) balances the inflows of power to outflows of power at each bus. Equation (6.c) defines the DC power flow of existing lines and new lines that form proposal $k$; the lines’ power flow is a function of the realization of their availability. Equation (6.d) keeps the power flowing through the lines within their thermal capacity. Equations (6.e)-(6.h) maintain the decision variables between their lower and upper limits.

In (6.f), the model assumes the generator’s capacity level is conditioned by its availability. In (6.g), the model assumes that a wind farm has $N^{gi}$ identical wind turbines. Section B of the Appendix presents the equations for determining the power generation of a turbine, $P_{gi}^{\text{el}}$, given its power curve and a wind speed’s realization, $\zeta_{gi}^r$. For the IEEE systems, we assumed a $0/MWh wind-based generation cost and $N^{gi} = 50$.

Equation (6.i) fixes the voltage angle to zero at the reference bus of the network. Equation (6.j) defines the decision variables, $\mathbb{I}^{[wi]}$ represents the coordinate space of the continuous variables, and $|\cdot|$ is the set cardinality.

If the planner opts for not considering network contingency, either the Bernoulli distribution’s parameter should be set to $p = 1$, or the procedure should not include random variables associated with the availability of lines and generators and the model needs to be modified. The modified model replaces Equation (6.c) by $P_{lij}^k = b_{lij} \cdot (\theta_{lj}^k - \theta_{lj}^k)$ and Equation (6.f) by $0 \leq P_{gi}^{\text{el}} \leq \xi_{gi}^k$.

MATLAB®’s commands, icdf, fitdist, and betainc, were used to implement the sampling method, to fit historical data to the distributions, and to determine the value of the incomplete beta function (3), respectively. The values of $h(t, p_e, R)$ used in subroutine REQ are found in [43]. The TEP models were coded using the General Algebraic Modeling System (GAMS) [48] and solved on a Dell Inspiron TM 3847 computer with Intel® Core™ i7-4790 at 3.6 GHz and 16 GB of RAM.

A. THE IEEE 24-BUS RELIABILITY TEST SYSTEM (RTS)

The IEEE 24-bus RTS, shown in Fig. 6, has 17 loads, 18 generators distributed in 10 buses, and 34 transmission lines. All network data can be found in [22] and includes the data of
seven transmission proposals ($K = 7$) and their discounted annual cost. Since the lines’ and generators’ FOR are not included in the data, we used FOR data available in [49]. The FOR of the 20MW-thermal capacity lines and transmission proposals was set to $FOR = 0.0031575$; and for generators, the FORs were set to 0.04. Both values represent the median value of all lines and generators, respectively.

For this IEEE system, we first present the selection of the proposed procedure by describing each of its stages. In Stage 1, we first generated $R$ random sets of size $N$, i.e., $(10 \times 100)$ random vectors representing the uncertain parameters of the system. For each 100-random set of observations, we solved the respective TEP model of the proposal $k$ and obtained 100 optimal values for its objective function. Those values were used to estimate the $\hat{z}_{k,r}^{0.9}$ of the objective function. These steps were repeated ten times to estimate ten values ($\hat{z}_{k,1}^{0.9}$, $\hat{z}_{k,2}^{0.9}$, …, $\hat{z}_{k,10}^{0.9}$) for each proposal.

In Stage 2, we executed subroutine $REQ$ to determine the required number of quantile samples to ensure the probability of correct selection. Table 1 shows the mean and variance of the 10 quantile estimates and the required number of samples for each proposal. The value of constant $h$ used in the subroutine’s $L_4$ is ($K = 7, P^* = 0.99, R = 10$) = 5.213.

The required number of samples for all proposals was set at $max_k R_k = 23$ to guarantee, with at least $99\%$ confidence, the selection of the proposal with the smallest quantile value. The additional samples ($\hat{z}_{k,1}^{0.9}$, $\hat{z}_{k,2}^{0.9}$, …, $\hat{z}_{k,10}^{0.9}$) for each proposal were generated.

In Stage 3, we executed the subroutine $COMP$ to calculate the average of 23 quantile estimates for each proposal. The procedure selected proposal $k_4$ with the $\overline{VaR}_{k_4} = 14.396 \times 10^3$. The procedure’s solution time per proposal, including the CPU time and time spent by GAMS performing input/output operations (i.e., reading files) was 11.53 [min:ss].

Using the variance of the quantile estimates reported in Table 1, we assessed the sensitivity of the $R_{max}$ (Line 5 of Fig. 4) to the values of the probability of correct selection ($P^*$) and the IZ parameter ($\delta$). Table 2 presents the number of samples required for three $P^*$ and five $\delta$ values. As anticipated, we observed that the number of required samples increased as the probability of correct selection $P^*$ and/or the size of the IZ decreased.

We replicated the implementation of the procedure four times, and the results are reported in Table 3. The Table shows the procedure’s recommendation, the value of its quantile, and the number of samples needed for meeting the correct selection probability. For all replications, proposal $k_4$ was the selection.

We compared the recommendation of the proposed procedure using $c^f$ selection criterion with that using $\varphi (\beta) = (1 - \beta) \cdot E[Z] + \beta \cdot CVaR^\beta (Z)$ selection criterion, where $\beta (0 < \beta < 1)$ is the planner’s specified trade-off between the expected value and the risk, as represented by $CVaR$. It should be noted that when $\beta = 0$, the selection criterion is the expected value. Table 4 presents the results for three different quantiles, as represented by the columns, and five levels of mean-risk trade-off, as represented by the rows. The top three rows represent the quantile, the number of samples needed for meeting the probability of correct selection of at least 0.99, and the recommendation of the proposed procedure based on the quantile values. The bottom five rows represent the recommendations when a weighted combination of the mean and $CVaR$ is used as the selection criterion. The values of the mean and $CVaR$ were estimated from the simulated data.

By examining the values in the Table, the following observations can be made. As anticipated, the required number of samples increased as $q$’s value increased. The recommendation using the quantile criterion was sensitive to the level of risk $q$. The selection based on the expected value criterion

| Table 1. Results of executing lines L1-L4 of subroutine REQ. |
|-----------------|-----------------|-----------------|-----------------|
| Proposal | From | To | $\hat{z}_{k,r}^{0.9}$ [\$10^3] | $\overline{VaR}_{k}$ [\$10^3] | $R_k$ |
| k_1 | Bus 1 | Bus 18 | 14.526 | 0.420 | 20 |
| k_2 | Bus 1 | Bus 12 | 14.673 | 0.428 | 20 |
| k_3 | Bus 2 | Bus 23 | 14.780 | 0.410 | 19 |
| k_4 | Bus 3 | Bus 14 | 14.441 | 0.459 | 23 |
| k_5 | Bus 6 | Bus 19 | 14.700 | 0.429 | 20 |
| k_6 | Bus 9 | Bus 15 | 14.489 | 0.419 | 20 |
| k_7 | Bus 9 | Bus 20 | 14.758 | 0.458 | 23 |

| Table 2. The number of samples required to PrCS $\geq P^*$ for the q=0.9. |
|-----------------|-----------------|-----------------|
| $P^*$ (K=7, P^*=0.9) | $k$ | $R_{max}$ |
| 0.90 | 3.445 | 10 |
| 0.95 | 3.993 | 10 |
| 0.99 | 5.213 | 12 |

| Table 3. Results of replicating the proposed procedure. |
|-----------------|-----------------|-----------------|
| Test | Proposal | $\hat{z}_{k,r}^{0.9}$ [\$10^3] | $R_{max}$ |
| T_1 | k_4 | 14.327 | 21 |
| T_2 | k_4 | 14.385 | 20 |
| T_3 | k_4 | 14.431 | 27 |
| T_4 | k_4 | 14.665 | 26 |

| Table 4. The selected proposal values in parentheses are in [\$10^3]. |
|-----------------|-----------------|-----------------|
| q | 0.900 | 0.925 | 0.975 |
| $R_{max}$ | 23 | 31 | 42 |
| $\varphi (\beta)$ | $\beta = 0$ | $\beta = 0.3$ | $\beta = 0.5$ |
| k_4(12.002) | k_4(13.095) | k_4(13.825) |
| k_4(11.984) | k_4(13.216) | k_4(14.029) |
| k_4(12.003) | k_4(13.694) | k_4(14.800) |
| $\beta = 0.7$ | $\beta = 0.7$ | $\beta = 0.7$ |
| k_4(15.544) | k_4(14.827) | k_4(15.899) |
| k_4(16.018) | k_4(15.647) | k_4(17.534) |
TABLE 5. The transmission proposals for IEEE 118-bus system.

| Proposal | From     | To        | $b$ [S] | $P$ [MW] | $IC_k$ [$\text{SM}$] |
|----------|----------|-----------|---------|----------|---------------------|
| $k_1$    | Bus 2    | Bus 12    | 16.2    | 400      | 30                  |
| $k_2$    | Bus 3    | Bus 12    | 6.3     | 400      | 32                  |
| $k_3$    | Bus 6    | Bus 7     | 48.1    | 400      | 25                  |
| $k_4$    | Bus 7    | Bus 12    | 29.4    | 400      | 28                  |
| $k_5$    | Bus 11   | Bus 13    | 13.7    | 400      | 18                  |

TABLE 6. The selected proposal under normal operation ($\delta = 5000$).

| $q$     | 0.900    | 0.925    | 0.975    |
|---------|----------|----------|----------|
| $R_{max}$ |         |          |          |
| $z^q$   |          |          |          |
| $\beta = 0$ | $k_2$ (58.064) | $k_2$ (58.064) | $k_2$ (58.064) |
| $\beta = 0.3$ | $k_3$ (59.768) | $k_3$ (59.768) | $k_3$ (60.395) |
| $\beta = 0.5$ | $k_4$ (60.904) | $k_4$ (61.208) | $k_4$ (61.941) |
| $\beta = 0.7$ | $k_5$ (62.025) | $k_5$ (62.454) | $k_5$ (63.470) |
| $\beta = 1.0$ | $k_5$ (63.699) | $k_5$ (64.311) | $k_5$ (65.763) |

TABLE 7. The Selected Proposals Under Contingency ($\delta = 20000$).

| $q$     | 0.900    | 0.925    | 0.975    |
|---------|----------|----------|----------|
| $R_{max}$ |         |          |          |
| $z^q$   |          |          |          |
| $\beta = 0$ | $k_2$ (60.657) | $k_2$ (60.757) | $k_2$ (60.737) |
| $\beta = 0.3$ | $k_3$ (64.345) | $k_3$ (65.012) | $k_3$ (66.210) |
| $\beta = 0.5$ | $k_4$ (66.789) | $k_4$ (67.826) | $k_4$ (69.838) |
| $\beta = 0.7$ | $k_5$ (69.836) | $k_5$ (71.343) | $k_5$ (74.373) |
| $\beta = 1.0$ | $k_5$ (72.883) | $k_5$ (74.860) | $k_5$ (78.908) |

($\beta = 0$) was insensitive to the level of risk $q$. Selection using the $\varphi(\beta)$ criterion was sensitive to the user-specified trade-off value $\beta$ and the risk level.

B. THE IEEE 118-BUS SYSTEM

We used the IEEE 118-bus system data available in [50]. The network has 91 loads, 54 generators, and 186 transmission lines. We assumed the network has three wind farms located at Buses 7, 12, and 13; and five enhancement proposals ($K = 5$) are available for consideration [51]. Table 5 shows the technical data of the proposals and their investment cost; all lines’ FOR is set to 0.0021986, the median value of lines’ FOR data.

For this IEEE system, we compared the recommendations of the proposed procedure when the selection criterion was $z^q_k$ and $\varphi(\beta)$ for three different quantiles and five different values of $\beta$. Tables 6 and 7 show the recommendations under normal operation (no contingencies) and under lines’ and generators’ contingency, respectively. The table values in parentheses are in $10^3$.

For all quantile $q$, the proposed procedure recommended the proposed $k_5$. Using only the expected value criterion, the proposal $k_2$ was recommended. When the criterion $\varphi(\beta)$ was used, the proposal $k_5$ was recommended with the exception of when $\beta = 0.3$ and $q = 0.9$ recommending proposal $k_2$. This demonstrates the sensitivity of $\varphi(\beta)$ (a combination of the expected value and CVaR) to two parameters as opposed to one for the quantile-based recommendation. Furthermore, the values in the table show the sensitivity of the selected proposal cost to both $q$ and $\beta$.

V. CONCLUSION

Cautious planners are cognizant of uncertainty related to the parameters of the transmission expansion and the risks they impose on the selection of adequate transmission proposals for their networks. Thus, we proposed the use of quantile as a selection criterion for evaluating a set of proposals submitted by investors and selecting the proposal that has the least cost for a desired risk level $q$. The use of quantile avoids the implicit assumption of risk neutrality that is embedded in the use of expected value and the additional interpretation of the trade-off $\beta$ between the expected value and surrogate for the risk as represented by CVaR.

We proposed a three-stage procedure using quantile as the selection criterion for identifying the best proposal from among a set of transmission enhancement proposals under uncertainty. The procedure is flexible to fully accommodate consideration of random variables leading to contingencies, combines simulation and optimization, and guarantees the quantile value estimation with a user-specified level of confidence that is representative of the procedure selection confidence. Numerical examples demonstrated the implementation of the procedure and the sensitivity of the recommendation to user-specified parameters. The selection criterion was highlighted.

APPENDIX

A. NORMAL DISTRIBUTION OF THE RANDOM LOAD AT BUS I

Since the historical load data of RTO/ISOs is, e.g., on the order of thousands of MW and the IEEE system’s generation and load data, e.g., is on the order of ten/hundreds MW, the hourly load data needs to be scaled for the systems. To determine the normal distribution $N$ for the system load of the IEEE systems, we scaled the hourly load values of the historical data by a factor $\rho$ and estimate $N(\mu, \sigma)$ using the scaled load; mean ($\mu$) and variance ($\sigma$) were the distribution’s parameters.

The normal distribution’s parameters for the random load $w$ located at bus $i$, $\xi^w_i \sim N(\mu^w_i, \sigma^w_i)$ can be obtained by (A.1) and (A.2), where $L F^{w_i} \in [0, 1]$ is the fraction of the system load $P^{sys}$ that the $w^{th}$ load withdraws from the network.

$$\mu^w_i = \mu \cdot LF^{w_i}, \forall i \in \Omega^B \& \forall w \in \Omega^w$$  \hspace{1cm} (A.1)

$$\sigma^w_i = \sigma \cdot LF^{w_i}, \forall i \in \Omega^B \& \forall w \in \Omega^w$$  \hspace{1cm} (A.2)

$$\mu = \sum_{i \in \Omega^B} \sum_{w \in \Omega^w} \mu^w_i, \sigma = \sum_{i \in \Omega^B} \sum_{w \in \Omega^w} \sigma^w_i$$ and the load data of the IEEE system were used to determine $LF^{w_i}$ for all loads.
For the numerical examples in Section 4, we set $\rho = 0.05$ for the 24-bus system, obtaining the $\mathcal{N}(718.8, 129.9)$ [MW], and $\rho = 0.25$ for the 118-bus system, obtaining the $\mathcal{N}(3594.1, 649.6)$ [MW].

**B. POWER OUTPUT OF A WIND TURBINE**

The power curve of the turbines of wind farm $g$ located at bus $i$ is shown in (B.1). The turbine’s power curve is a function of its nominal power ($P_{gi}^{\text{nom}}$), rated wind speed ($v_{gi}^{\text{r}}$), and cut-in and cut-out wind speeds ($v_{gi}^{\text{in}}$ and $v_{gi}^{\text{out}}$). Given the $v_{gi}^{\text{in}}$ wind speed’s realization, $v_{gi}^{\text{in}}$, (B.1) determines the power generation $P_{gi}^{\text{out}}$ of the turbine for (6.g).

\[
P_{gi}^{\text{out}} = \begin{cases} 
0 & \text{if } v_{gi}^{\text{in}} < \frac{\varepsilon_{gi}^{\text{in}}}{\nu_{gi}^{\text{in}}} \cdot \frac{v_{gi}^{\text{r}}}{v_{gi}^{\text{r}} - v_{gi}^{\text{in}}} < v_{gi}^{\text{out}} \\
\frac{\varepsilon_{gi}^{\text{out}}}{\nu_{gi}^{\text{out}}} \cdot \frac{v_{gi}^{\text{r}}}{v_{gi}^{\text{r}} - v_{gi}^{\text{out}}} & \text{if } v_{gi}^{\text{in}} \leq \varepsilon_{gi}^{\text{in}} < v_{gi}^{\text{r}} \\
\frac{\varepsilon_{gi}^{\text{out}}}{\nu_{gi}^{\text{out}}} & \text{if } v_{gi}^{\text{in}} < \varepsilon_{gi}^{\text{in}} < v_{gi}^{\text{out}} \\
\end{cases} 
\]

(B.1)

For the numerical examples in Section 4, we considered the Vestas V110 turbine that has $P_{gi}^{\text{nom}} = 2000$ kW, $v_{gi}^{\text{r}} = 13$ m/s, $v_{gi}^{\text{in}} = 3$ m/s, and $v_{gi}^{\text{out}} = 20$ m/s.

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