Time reversal violation in $K^+ \to \mu^+ \nu \gamma$ decay and three Higgs model

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ABSTRACT

Transverse muon polarization in the $K^+ \to \mu^+ \nu \gamma$ decay is calculated in the model with scalar and pseudo scalar four-Fermi interactions. Combined with a similar calculation in the $K^+ \to \mu^+ \nu \pi^0$ decay, a possible constraint on parameters in the three-Higgs model is obtained assuming sensitivity of the up-coming KEK experiment. It is pointed out that the predictions for the two polarizations are strongly correlated in the three Higgs model.
1 Introduction

Although the present experimental knowledge on CP violation is consistently explained by a simple phase of the quark flavor mixing matrix, it may not be the only source of the CP violation. In fact, some types of physics beyond the standard model contain new physical phases which could induce different kinds of CP and T violating interactions. Therefore, it is important to search for such interactions in various processes.

Measurements of transverse muon polarization in the $K^+ \rightarrow \mu^+ \nu \pi^0$ process have received attentions as a process to probe a T violating interaction\cite{1}. Although the standard model prediction to the polarization is too small to be measured in the near future, possible extensions like multi Higgs models can induce measurable effects\cite{2}.

A new experiment is under preparation at KEK aiming at measuring the transverse polarization of muon ($P_T$) up to the level of $5 \times 10^{-4}$\cite{3}. This would be an improvement by a factor 10 from the present experimental bound which is $(−3.1 \pm 5.3) \times 10^{-3}$\cite{4}.

In the same experiment, the transverse muon polarization of the $K^+ \rightarrow \mu^+ \nu \gamma$ decay will be also measured. Since this is proportional to $(\vec{p}_\mu \times \vec{p}_\gamma) \cdot \vec{s}_\mu$ where $\vec{p}_\mu$ and $\vec{p}_\gamma$ are momenta of muon and photon and $\vec{s}_\mu$ is spin of muon, this quantity changes its sign under time reversal operation. Therefore, the measurement of the transverse muon polarization in this process will also give us useful information on possible new sources of T and CP violating interactions\cite{5}. For example, in Ref.\cite{6} this polarization was considered as a probe to possible tensor interactions in the kaon decay. Although the transverse muon polarization from the CP violation in the standard model is negligible, the electromagnetic final state interaction can mimic the T violation effects which are estimated to be as large as $10^{-3}$\cite{7}. This is in contract with the $K^+ \rightarrow \mu^+ \nu \pi^0$ process where the final state interaction can only produce the effect of $10^{-6}$\cite{8}. Since the sensitivity for the polarization measurement in this mode is expected to be similar to that of the $K^+ \rightarrow \pi^0 \mu^+ \nu$ mode at the coming experiment\cite{3}, we will be able to search for the T violating effects below $10^{-3}$ level if we can properly subtract the contributions from the final state interaction.

In this paper, we consider prediction of the transverse muon polarization in the $K^+ \rightarrow \mu^+ \nu \gamma$ process as well as the $K^+ \rightarrow \pi^0 \mu^+ \nu$ process in multi Higgs models. In these models, new scalar and pseudo scalar four Fermi interactions are induced from exchange of charged Higgs bosons and these interactions contain new physical phases. We will determine how the measurements of transverse muon polarization for these two processes put constraints on these new interactions. In particular, we will consider a three Higgs model and show that the predictions for the above two processes are strongly correlated.
after taking account of other phenomenological constraints. Therefore, it is very important to measure the transverse muon polarization in both processes to clarify the nature of possible CP violating effects in the three Higgs model.

This paper is organized as follows. In section 2, we will introduce scalar and pseudo scalar interaction and calculate the decay rate and transverse muon polarization for $K^+ \rightarrow \mu^+\nu\gamma$ and $K^+ \rightarrow \pi^0\mu^+\nu$ processes. In section 3, we consider a three Higgs doublet model and obtain constraints on Higgs coupling constants by these two processes. Discussions on the results are given in section 4. In the appendix A, the $K^+ \rightarrow \mu^+\nu\gamma$ amplitude due to the pseudo scalar interaction is derived. The appendix B contains several functions for the branching ratio and polarization calculations.

### 2 Muon Polarization in $K^+ \rightarrow \mu^+\nu\gamma$ and $K^+ \rightarrow \mu^+\nu\pi^0$ Decay

In this section we will present calculations of transverse muon polarization and decay rates for $K^+ \rightarrow \mu^+\nu\gamma$ process. For completeness, we also give results of a similar calculation for $K^+ \rightarrow \pi^0\mu^+\nu$ process.

We start from the following four-fermi interaction,

$$
\mathcal{L}_V = -\frac{G_F}{\sqrt{2}} \sin \theta_c \gamma \mu (1 - \gamma_5) \bar{u} \gamma^\mu (1 - \gamma_5) \mu \\
+ G_S \bar{s} \gamma \nu \frac{1 + \gamma_5}{2} \mu + G_P \bar{s} \gamma_5 \nu \frac{1 + \gamma_5}{2} \mu \\
+ h.c.,
$$

(2.1)

where $G_F$ is the Fermi constant and $\sin \theta_c = 0.22$. We have introduced two coupling constants $G_S$ and $G_P$. These constants are in general complex. In this section, we treat $G_S$ and $G_P$ as new coupling constants. Later, when we consider the multi Higgs models, these terms are supposed to be induced from the charged Higgs exchange and $G_S$ and $G_P$ are expressed as functions of charged Higgs masses and coupling constants in the multi Higgs model.

The $K^+ \rightarrow \mu^+\nu\gamma$ amplitude in the standard model can be divided into two parts, i.e., internal bremsstrahlung ($M_{IB}$) and structure dependent ($M_{SD}$) terms.$^2$

$$
M_{V-A} = M_{IB} + M_{SD},
$$

(2.2)

$^2$ Detailed account for the radiative semileptonic kaon decay within the standard model is found in Ref. [9].
\[ M_{IB} = -ie \frac{G_F}{\sqrt{2}} \sin \theta_c \sqrt{2} f_K m_\nu \epsilon^\nu_\nu(q) K^\nu, \quad (2.3) \]

\[ M_{SD} = i e \frac{G_F}{\sqrt{2}} \sin \theta_c L_\nu \epsilon^\nu_\mu(q) H^{\mu\nu}, \quad (2.4) \]

where

\[ L^\nu = \bar{u}(k) \gamma^\mu (1 - \gamma_5) U(\ell), \quad (2.5) \]

\[ H^{\mu\nu} = \frac{A}{m_K} p \cdot q (-g^{\mu\nu} + \frac{p^\mu q^\nu}{p \cdot q}) + i \frac{V}{m_K} \epsilon^{\mu\alpha\beta} q_\alpha p_\beta, \quad (2.6) \]

\[ K^\mu = \bar{u}(k)(1 + \gamma_5) \left( \frac{p^\mu}{p \cdot q} - \frac{q \cdot \gamma_5^\mu + 2 \ell^\mu}{2 \ell \cdot q} \right) U(\ell). \quad (2.7) \]

Here \( p^\mu, q^\mu, \ell^\nu, k^\nu \) are the \( K^+, \) photon, muon and neutrino four momenta, respectively and \( \bar{u}(k) \) and \( U(\ell) \) are neutrino and muon wave functions. \( \epsilon_\nu \) is the photon polarization vector. The kaon decay constant \( f_K \) is defined as,

\[ < 0|\bar{s}\gamma^\mu \gamma^5 u|K^+(p)> = -i \sqrt{2} f_K p^\mu, \quad (2.8) \]

and \( V \) and \( A \) are defined as follows,

\[ \int d^4 x e^{ixz} < 0|T(\bar{s}\gamma^\nu \gamma^5 u(0) J^\mu_{em}(x))|K^+(p)> = - \sqrt{2} f_K (g^{\mu\nu} + \frac{p^\mu (p - q)^\nu}{p \cdot q}) + \frac{A}{m_K} p \cdot q (g^{\mu\nu} - \frac{p^\mu q^\nu}{p \cdot q}), \quad (2.9) \]

\[ \int d^4 x e^{ixz} < 0|T(\bar{s}\gamma^\nu u(0) J^\mu_{em}(x))|K^+(p)> = i \frac{V}{m_K} \epsilon^{\mu\alpha\beta} q_\alpha p_\beta, \quad (2.10) \]

where \( J^\mu_{em}(x) \) is the electromagnetic current. Note that the form factors \( V \) and \( A \) are real since CP is conserved in the strong interaction.

Let us consider the effects of the scalar and pseudo scalar couplings. In the appendix A, we show that only the pseudo scalar coupling can contribute to this process and that the amplitude induced by the \( G_P \) coupling constant is proportional to \( M_{IB} \), so that no new form factor is necessary. This is quite different from the case of the tensor interaction where a new form factor should be introduced\[6\]. The amplitude is given by,

\[ M_P = -ie G_P \frac{\sqrt{2} f_K m_K^2}{2} \epsilon^\nu_\mu(q) K^\mu, \quad (2.11) \]
and $m_s$ and $m_u$ are the strange and up quark masses. Combining two expression, the total amplitude becomes

$$M = M_{IB} + M_{SD} + M_P$$

$$= -ie\frac{G_F}{\sqrt{2}}(1 + \Delta_P)\sin \theta_c \sqrt{2} f_K m_\mu \epsilon^*_\mu(q) K^\mu$$

$$+ ie\frac{G_F}{\sqrt{2}} \sin \theta_c L_\nu \epsilon^*_\mu(q) H^{\mu
u}, \quad (2.12)$$

where

$$\Delta_P = \frac{G_P}{\sqrt{2}G_F \sin \theta_c} \frac{m_K^2}{(m_s + m_u)m_\mu}. \quad (2.13)$$

From this amplitude a partial decay width and transverse polarization of the muon are calculated. Since the effect of the $G_P$ coupling is just to replace the coupling constant in the $M_{IB}$ term by a complex one, the calculation of the transverse polarization essentially reduces to the old calculation of T-odd asymmetry in this process where the structure-dependent term were assumed to be complex numbers\[10\]. The partial decay width is given by,

$$\frac{d^2\Gamma}{dxdy} = \rho(x, y), \quad (2.14)$$

$$\rho(x, y) = A_{IB}[1 + \Delta_P^2]f_{IB} + A_{INT}(1 + Re\Delta_P)((V + A)f^*_\mu + (V - A)f^-\mu)$$

$$+ A_{SD}\frac{1}{2}((V + A)^2 f^*_\mu + (V - A)^2 f^-\mu), \quad (2.15)$$

where $x$ and $y$ are normalized energies of the photon and muon, ie. $x = \frac{2E_\gamma}{m_K}$, $y = \frac{2E_\mu}{m_K}$, and $A_{SD}$ etc. are defined by,

$$A_{SD} = \frac{m_K^5}{32\pi^2} \alpha G_F^2 \sin^2 \theta_c, \quad (2.16)$$

$$A_{IB} = 2r_\mu (\sqrt{2} f_K m_K)^2 A_{SD}, \quad (2.17)$$

$$A_{INT} = 2r_\mu (\sqrt{2} f_K m_K) A_{SD}, \quad (2.18)$$

and $r_\mu = (\frac{m_\mu}{m_K})^2$. Functions $f_{IB}(x, y)$ etc. are defined in the appendix B. Using a unit vector $\vec{n}_T = \vec{p}_\gamma \times \vec{p}_\mu / |\vec{p}_\gamma \times \vec{p}_\mu|$, the muon transverse polarization is defined as

$$P_\perp = \frac{d\Gamma(\vec{n}_T) - d\Gamma(-\vec{n}_T)}{d\Gamma(\vec{n}_T) + d\Gamma(-\vec{n}_T)}, \quad (2.19)$$
where $\vec{p}_\gamma$ and $\vec{p}_\mu$ are the photon and muon momenta in the $K^+$ rest frame and $d\Gamma(\pm \vec{n}_T)$ is the partial decay width with the muon polarization $\pm \vec{n}_T$. $P_\perp$ is given by

$$P_\perp = \sigma(x, y) \cdot Im \Delta_S,$$

(2.20)

where

$$\sigma(x, y) = -A_p \cdot \frac{2\sqrt{(1 - y + r_\mu)((1 - x)(x + y - 1) - r_\mu)}}{\rho(x, y)} \cdot \{(V + A)f_\mu^+ + (V - A)f_\mu^-\}.$$  

(2.21)

$A_p$ is defined by,

$$A_p = \sqrt{r_\mu} \frac{\sqrt{2 f_K}}{m_K} A_{SD},$$

(2.22)

and $f_\mu^\pm$ are given in the appendix B.

Next, for completeness, we present the partial width and the transverse muon polarization in $K^+ \rightarrow \mu^+ \nu \pi^0$ decay [2]. Contrary to the $K^+ \rightarrow \mu^+ \nu \gamma$ process, this process is sensitive to the scalar coupling $G_S$. The form factors $f_\mu^\pm$ are defined as,

$$< \pi^0 | \bar{s} \gamma^\mu u | K^+ > = f_\mu^+(p + q)^\mu + f_\mu^-(p - q)^\mu,$$

(2.23)

where $p$, $q$ are the $K^+$ and $\pi^0$ momenta. Using the fact $|f_\mu^+| > |f_\mu^-|$, we get for the partial width

$$\frac{d^2 \Gamma}{dx dy} = \rho_\pi(x, y),$$

(2.24)

$$\rho_\pi(x, y) = \frac{m_K^5}{128\pi^3} G_F^2 \sin^2 \theta_c f_\pi^2 \rho(x, y)$$

$$\{(3 + r_\mu - r_\pi - x - 2y)(x + 2y - 1 - r_\mu - r_\pi)$$

$$-(1 + r_\pi + x)(1 + r_\pi - r_\mu - x)\},$$

(2.25)

where $x$ and $y$ are normalized energies for the pion and muon in the $K^+$ rest frame, i.e. $x = \frac{2E_\pi}{m_K}$ and $y = \frac{2E_\mu}{m_K}$. The muon transverse polarization defined by the Eq. (2.19) with $\vec{n}_T = \vec{p}_\pi \times \vec{p}_\mu / |\vec{p}_\pi \times \vec{p}_\mu|$ where $\vec{p}_\pi$ is the pion momentum is given by

$$P_\perp = \sigma_\pi(x, y) \cdot Im \Delta_S,$$

(2.26)

$$\sigma_\pi(x, y) = \frac{2\sqrt{r_\mu}}{\rho(x, y)}$$

$$\cdot \frac{\sqrt{(x^2 - 4r_\pi)(y^2 - 4r_\mu) - 4(1 + r_\mu + r_\pi + \frac{r_\mu - y}{2}) - (x - y)^2}}{(3 + r_\mu - r_\pi - x - 2y)(x + 2y - 1 - r_\mu - r_\pi) - (1 + r_\pi + x)(1 + r_\pi - r_\mu - x)},$$

(2.27)

where

$$Im \Delta_S = \frac{(m_K^2 - m_\pi^2)Im G^*_S}{(m_s - m_u)m_\mu \sqrt{2} G_F \sin \theta_c},$$

(2.28)
and \( r_\pi = \frac{m_\pi^2}{m_K^2} \). In the calculation of the partial width in Eq. (2.24), we have assumed that the V-A contribution is dominant and have kept only the \( G_F \) term. Note that the polarization does not depend on the form factor \( f_+ \) in the limit of \( (f_+) \gg (f_-) \).

The Dalitz plots and transverse muon polarizations for \( K^+ \to \mu^+ \nu \gamma \) and \( K^+ \to \mu^+ \nu \pi^0 \) are shown in figure 1, and 2. In the calculations for \( K^+ \to \mu^+ \nu \gamma \) the values of \( V_+A \) and \( V_-A \) have to be specified. These values can be obtained in the analysis of radiative semileptonic kaon decay. However, the present experimental knowledge is not enough to extract finite numbers to these quantities, so that we use the values obtained from calculation in the one loop chiral perturbation theory which are \( V_+A = -0.137, V_-A = -0.052 \) [3]. In the \( K^+ \to \mu^+ \nu \gamma \) process, the polarization effect is large in the region 0.3 ≤ \( x \) ≤ 0.8. Although the differential decay width is large in the limit of soft photon \( (x \to 0) \) the polarization vanishes in this limit. This is because the transverse polarization is caused by interference of \( M_P \) and \( M_{SD} \) term, whereas the large branching in the limit of \( x \to 0 \) is caused by the \( |M_{IB} + M_P|^2 \) term. Therefore, the sensitivity to the polarization is determined by the intermediate \( x \) region and average polarization in this region is given by \( P_\perp = (0.1 \sim 0.2) \times Im \Delta_P \) depending on the experimental cut. This is compared to the corresponding average polarization for the \( K^+ \to \pi^0 \mu \nu \) process where we get \( P_\perp \sim 0.3 \times Im \Delta_S \). This shows that \( K^+ \to \mu^+ \nu \gamma \) process has a comparable sensitivity to new physics and the both processes give valuable informations. In general multi-Higgs models, \( ImG_S \) and \( ImG_P \) are not related, therefore two process gives independent informations. On the other hand, if we restrict ourself to the three Higgs model they are expressed by the same parameters of the model, so that we are able to obtain specific predictions.

3 Transverse Muon Polarization in Three Higgs Model

In this section we consider prediction of muon transverse polarization in \( K^+ \to \mu^+ \nu \gamma \) and \( K^+ \to \mu^+ \nu \pi^0 \) decay in the context of the three Higgs model. We show that taking account of present phenomenological constraints the predictions for the two processes are strongly correlated, therefore it is important to search for T-odd polarization in both processes.

Here we assume that three different Higgs doublets can couple to up-type, down-type quarks and leptons, respectively. Details on this model can be found in Refs.[2, 3]. There are two physical charged Higgs and one Goldstone mode and the mixing matrix among them can contain a new physical phase. The original Yukawa coupling of this model is given by,
\[
\mathcal{L} = \bar{q}_L y_d d_R H_d + \bar{q}_L y_u u_R \tilde{H}_u + \tilde{e}_L e_R H_\ell + \text{h.c.}
\]  
(3.1)

We assume that the vacuum expectation values are in general complex. If we define

\[
H_d = e^{i\theta_1} \begin{pmatrix} H_d^+ \\ (v_1 + \rho_1 + i\chi_1) \end{pmatrix},
\]

(3.2)

\[
H_u = e^{i\theta_2} \begin{pmatrix} H_u^+ \\ (v_2 + \rho_2 + i\chi_2) \end{pmatrix},
\]

(3.3)

\[
H_\ell = e^{i\theta_3} \begin{pmatrix} H_\ell^+ \\ (v_3 + \rho_3 + i\chi_3) \end{pmatrix},
\]

(3.4)

where \( H_{ui} \equiv \epsilon_{ij} \tilde{H}_{uj}^* \), the above three charged Higgs fields \( H_d^+, H_u^+, H_\ell^+ \) are related to mass-diagonalized states by the following \( 3 \times 3 \) matrix.

\[
\begin{pmatrix} H_d^+ \\ H_u^+ \\ H_\ell^+ \\ v_1 \\ v_2 \\ v_3 \end{pmatrix} = \frac{1}{v} \begin{pmatrix} 1 & \alpha_1 & \alpha_2 \\ 1 & -\beta_1 & -\beta_2 \\ 1 & \gamma_1 & \gamma_2 \end{pmatrix} \begin{pmatrix} G^+ \\ H_1^+ \\ H_2^+ \end{pmatrix},
\]

(3.5)

where \( v = \sqrt{v_1^2 + v_2^2 + v_3^2} \), \( G^+ \) is the Goldstone mode and \( H_i \ (i = 1, 2) \) are physical charged Higgses. The couplings between fermions and the charged Higgses are determined as follows.

\[
\mathcal{L} = (2\sqrt{2}G_F)^{\frac{1}{2}} \sum_{i=1}^{2} \{ \alpha_i \bar{\nu}_L K M_D d_R H_i^+ + \beta_i \bar{u}_R M_U K d_L H_i^+ + \gamma_i \bar{\nu}_L M_E e_R H_i^+ \} + \text{h.c.},
\]

(3.6)

where \( K \) is the ordinary flavor mixing matrix for the quark sector. \( M_D, M_U \) and \( M_E \) are diagonal down-type quark, up-type quark and lepton mass matrix respectively. For the complex coupling constants \( \alpha_i, \beta_i \) and \( \gamma_i \), the following relations exist from the requirement of unitarity of the mixing matrix.

\[
\frac{I_m(\alpha_2 \beta_2^*)}{I_m(\alpha_2 \beta_1^*)} = \frac{I_m(\alpha_2 \gamma_2^*)}{I_m(\alpha_1 \gamma_1^*)} = \frac{I_m(\beta_2 \gamma_2^*)}{I_m(\beta_1 \gamma_1^*)} = -1,
\]

(3.7)

From this Lagrangian we can derive four-Fermi interaction constants \( G_S \) and \( G_P \).

\[
G_P = \sum_{i=1}^{2} 2\sqrt{2}G_F \sin \theta_{c_i} m_\mu \frac{\gamma_i}{m_i^2} (m_u \beta_i^* - m_s \alpha_i^*),
\]

(3.8)

\[
G_S = \sum_{i=1}^{2} 2\sqrt{2}G_F \sin \theta_{c_i} m_\mu \frac{\gamma_i}{m_i^2} (m_s \alpha_i^* + m_u \beta_i^*).
\]

(3.9)
In the formulas of the previous section, the above expressions should be substituted to the coupling constants $G_S$ and $G_p$.

Since the transverse muon polarizations in $K^+ \rightarrow \mu^+\nu\gamma$ and $K^+ \rightarrow \mu^+\nu\pi^0$ are sensitive to the $ImG_P$ and $ImG_S$ respectively, $Im\Delta_P$ and $Im\Delta_S$ defined from Eqs. (2.13) and (2.28) are given by,

\[
Im\Delta_P = \frac{m^2_K}{m_s + m_u} \sum_{i=1}^{2} Im\{\frac{\gamma_i}{m_i^2}(m_u\beta_i^* - m_s\alpha_i^*)\}
\]

\[
\simeq -m^2_K(\frac{1}{m_1^2} - \frac{1}{m_2^2})(Im\gamma_1\beta_1^* - \frac{m_u}{m_s}Im\gamma_1\beta_1^*),
\]  

(3.10)

\[
Im\Delta_S = \frac{-m^2_K - m^2_\pi}{m_s - m_u} \sum_{i=1}^{2} \frac{1}{m_i^2}Im\{\gamma_i(m_s\alpha_i^* + m_u\beta_i^*)\}
\]

\[
\simeq -m^2_K(\frac{1}{m_1^2} - \frac{1}{m_2^2})(Im\gamma_1\alpha_1^* + \frac{m_u}{m_s}Im\gamma_1\beta_1^*),
\]  

(3.11)

where we have neglected $m_u$ term in the denominator in Eqs. (3.10) and (3.11) and $m^2_\pi$ term compared to $m^2_K$ term in Eq. (3.11) and used the unitarity relation to rewrite $Im\gamma_2\alpha_2^*$ and $Im\gamma_2\beta_2^*$ in terms of $Im\gamma_1\alpha_1^*$ and $Im\gamma_1\beta_1^*$. If we assume $m_1^2 \leq m_2^2$ then the polarization effect is maximal when $m_2 \rightarrow \infty$, on the other hand, it vanishes when $m_1 = m_2$. The two measurements of the muon polarization can put constraints on the coupling constraints $Im\gamma_1\alpha_1^*$ and $Im\gamma_1\beta_1^*$ for a given set of charged Higgs masses.

In order to determine sensitivity, we first discuss current bounds on these parameters from other processes. From now on we are concentrating on the case $m_1 \ll m_2$ and see what kinds of constraints are obtained for the coupling constants of the lighter charged Higgs. We denote the lighter charged Higgs mass as $m_H$. Among $\alpha_i, \beta_i, \gamma_i$, the coupling constant $\beta_i$ is most severely constrained since it is related to the top Yukawa coupling constant. We use the result of the analysis in Ref. [11]. For the $Im\gamma_1\beta_1$, the present bound is given by a product of the bounds of $|\gamma_1|$ and $|\beta_1|$. The bound of $|\beta_1|$ is given by $B^0 - \bar{B}^0$ mixing, the parameter of CP violating amplitude of $K$ decay ($\epsilon$) and the $Z \rightarrow b\bar{b}$ vertex. For $m_t > 140 GeV$, $|\beta_1| < 1.3 \sim 2.0$ corresponding to the charged Higgs mass 45 GeV $\sim$ 200 GeV. The bound on $|\gamma_1|$ is given by $e - \mu$ universality in $\tau$ decay and by the perturbative bound:

\[
|\gamma_1| < \min(1.93m_H GeV^{-1}, 340).
\]  

(3.12)

Then, $Im\gamma^* \beta_1$ is bounded by,

\[
|Im\gamma_1^*| \beta_1| < 110 \sim 650,
\]  

(3.13)
depending on \( m_H = 45 \text{GeV} \sim 200 \text{GeV} \). The strongest bound on \( \text{Im}\gamma \alpha^* \) is obtained from \( B \to X\tau\nu_\tau \) decay in the range of \( m_H < 400 \text{ GeV} \).

\[
|\text{Im}\gamma_1^* \alpha_1| < 0.23 \left( \frac{m_H}{\text{GeV}} \right)^2,
\]

which varies from 465 to 9200 for the range of \( m_H \) from 45 GeV to 200 GeV. Note that in the two expressions, \( \text{Im}\Delta_P \) and \( \text{Im}\Delta_S \), the second term \( \frac{m_u}{m_s} \text{Im}\gamma_1 \beta_1^* \) is more strongly constrained than the first term \( \text{Im}\gamma_1 \alpha_1^* \).

In figure 3, we show constraints on \( \text{Im}\gamma_1 \beta_1^* \) and \( \text{Im}\gamma_1 \alpha_1^* \) space expected from future muon polarization measurements for different values of \( m_H \). We have assumed \( P_\perp = 0.2 \times \text{Im}\Delta_P \) and \( P_\perp = 0.3 \times \text{Im}\Delta_S \) for the \( K^+ \to \mu^+\nu\gamma \) and \( K^+ \to \mu^+\nu\pi^0 \) processes respectively. In the analysis we have used \( V + A = -0.137, V - A = -0.052 \) as before and \( \frac{m_u}{m_s} = \frac{1}{69} \). The bounds from these two processes are presented. Also the present experimental constraints from other processes are shown.

From the figure 3, we can see that the both processes are quite useful to put constraints on the value of \( \text{Im}\gamma_1 \alpha_1^* \). Also a strong correlation between the prediction of the two polarizations can be seen. This is because \( \text{Im}\gamma_1 \beta_1^* \) is already strongly constrained from other processes. Therefore, if the coming experiment gives non-null result for the polarization measurements, the \( \text{Im}\gamma_1 \alpha_1^* \) term will be dominant and the prediction of two polarizations are strongly correlated. This is important for the experiment because if the T-odd polarization is observed in one process then the polarization in the other process is also expected to be within reach. Notice that this strong correlation is a unique feature of this three Higgs model where the parameter \( \text{Im}\gamma_1 \beta_1 \) is strongly bounded because the coupling constant \( \beta_i \) is related to the processes involving top Yukawa coupling. If we allow more Higgses, the predictions for two polarizations are not necessarily correlated.

4 Discussions

We have calculated the partial decay width and muon transverse polarization in the processes \( K^+ \to \mu^+\nu\pi^0 \) and \( K^+ \to \mu^+\nu\gamma \) in the model with complex scalar and pseudo scalar couplings. For the calculation for the \( K^+ \to \mu^+\nu\gamma \) process we do not need to introduce any new form factor, therefore, no new theoretical ambiguity exists to extract short distance effects of new physics. Improvements on the polarization measurements expected at the new experiment will give remarkable impacts on the search for a new source of CP violation in the Higgs sector. Especially in the three Higgs model we have shown that the predictions of two polarization are strongly correlated, therefore it is important to search for T-violation in both processes.
In the actual experiment, the final state interaction due to the electromagnetic interaction induces a T-odd effect which mimics the T-violation. For the $K^+ \rightarrow \mu^+ \nu \pi^0$ process, this effect is evaluated to be $10^{-6}$ and therefore negligible. On the other hand for the $K^+ \rightarrow \mu^+ \nu \gamma$ process the final state interaction can induce the muon transverse polarization at the level of $10^{-3}$. At first sight this looks a problem for measuring the T-violating effects in this process. This is, however, not the case since the effect is induced by the electromagnetic interaction and can be estimated without much ambiguity. Moreover, in a very good approximation, the total transverse polarization is expected to be a simple sum of a term due to the final state interaction and that from the pseudo scalar coupling since each term comes from the interference with the standard model tree amplitude. Therefore, the subtraction procedure is straightforward. Detailed evaluation of the final state interaction for this process is called for.

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A Appendix A

In this appendix we derive the formula (2.11) for the amplitude of \( K^+ \rightarrow \mu^+ \nu \gamma \) from the pseudo scalar and scalar couplings. There are two types of diagrams for this process from the scalar and pseudo scalar couplings defined in Eq. (2.1). The first diagram is the one in which the photon is originated from the external muon line. We get the following amplitude for this diagram,

\[
M_1 = e \frac{G_P}{2} \langle 0 | \bar{s} \gamma^5 u | K^+(p) > \| \epsilon_\mu(q) \bar{u}(k) (1 + \gamma_5) (\frac{q \cdot \gamma \gamma^\mu + 2 \ell \mu}{2 \ell \cdot q}) \nu(\ell). \tag{A.1}
\]

In the above expression the contribution from \( G_S \) has dropped because of parity conservation in the matrix element.

The second diagram corresponds to the situation in which the photon comes from the hadronic system. These contribution can be written as,

\[
M_2 = -ie \frac{G_P}{2} \epsilon_\mu(q) \bar{u}(k) (1 + \gamma_5) \nu(\ell) (I_\mu^S + I_\mu^P), \tag{A.2}
\]

where

\[
I_\mu^S = \int d^4 x e^{ixp} < 0 | T(\bar{s}u(0)J_{em}(x)) | K^+(p) > , \tag{A.3}
\]

\[
I_\mu^P = \int d^4 x e^{ixp} < 0 | T(\bar{s} \gamma^5 u(0)J_{em}(x)) | K^+(p) > . \tag{A.4}
\]

Here, \( I_\mu^S \) and \( I_\mu^P \) are functions of two momenta \( p^\mu, q^\mu \). Since we cannot construct axial vector quantity from two independent momenta we can set \( I_\mu^S = 0 \). On the other hand, \( I_\mu^P \) can be expanded as

\[
I_\mu^P = I_1 \cdot p^\mu + I_2 \cdot q^\mu. \tag{A.5}
\]

The \( I_2 \) part does not contribute to the on-shell photon amplitude. The \( I_1 \) is determined using the Ward-Takahashi identity. In fact we can show

\[
q_\mu I_\mu^P = -i < 0 | \bar{s} \gamma^5 u | K^+(p) > . \tag{A.6}
\]

Then,

\[
I_\mu^P = -i \frac{p_\mu}{p \cdot q} < 0 | \bar{s} \gamma^5 u | K^+(p) > . \tag{A.7}
\]

Therefore, the second amplitude is written as follows;

\[
M_2 = -e \frac{G_P}{2} < 0 | \bar{s} \gamma^5 u | K^+(p) > \left( \frac{p_\mu}{p \cdot q} \right) \epsilon_\mu(q) \bar{u}(k) (1 + \gamma_5) \nu(\ell). \tag{A.8}
\]

Combining Eqs. (A.1) and (A.8), and expressing the scalar matrix element by the decay constant as

\[
< 0 | \bar{s} \gamma^5 u | K^+(p) > = i \frac{\sqrt{2} f_K m_K^2}{m_s + m_u}, \tag{A.9}
\]

we obtain Eq. (2.11).
B Appendix B

In this appendix the functions for evaluation of the partial width and transverse muon polarization for the $K^+ \to \mu^+ \nu \gamma$ process are listed.

\begin{align*}
    f_{IB}(x, y) & = \frac{1 - y + r_\mu}{x^2(x + y - 1 - r_\mu)} (x^2 + 2(1 - x)(1 - r_\mu) - \frac{2x r_\mu(1 - r_\mu)}{x + y - 1 - r_\mu}), & (B.1) \\
    f_{SD}^+(x, y) & = (x + y - 1 - r_\mu)((x + y - 1)(1 - x) - r_\mu), & (B.2) \\
    f_{SD}^-(x, y) & = (1 - y + r_\mu)((1 - x)(1 - y) + r_\mu), & (B.3) \\
    f_{INT}^+(x, y) & = \frac{1 - y + r_\mu}{x(x + y - 1 - r_\mu)}((1 - x)(1 - x - y) + r_\mu), & (B.4) \\
    f_{INT}^-(x, y) & = \frac{1 - y + r_\mu}{x(x + y - 1 - r_\mu)}(x^2 - (1 - x)(1 - x - y) - r_\mu), & (B.5) \\
    f_p^+(x, y) & = \frac{(1 - x)(x + y - 1) - r_\mu}{x(x + y - 1 - r_\mu)}, & (B.6) \\
    f_p^-(x, y) & = \frac{1 + r_\mu - y}{x(x + y - 1 - r_\mu)}. & (B.7)
\end{align*}

In the above, $x$ and $y$ are defined as $x = \frac{2E_\gamma}{m_K}$ and $y = \frac{2E_\mu}{m_K}$ using the photon and muon energies in the $K^+$ rest frame.
Figure Captions

Figure 1  Partial decay width (a) and transverse muon polarization (b) for the $K^+ \rightarrow \mu^+ \nu\gamma$ process as a function of $x = \frac{2E_\gamma}{m_K}$ and $y = \frac{2E_\mu}{m_K}$. (a) represents the partial decay width $\rho(x, y)$ normalized by a constant $A_{SD}$ and (b) represents the function $\sigma(x, y)$.

Figure 2  Partial decay width (a) and transverse muon polarization (b) for the $K^+ \rightarrow \mu^+ \nu\pi^0$ process as a function of $x = \frac{2E_\pi}{m_K}$ and $y = \frac{2E_\mu}{m_K}$. (a) represents the partial decay width $\rho_\pi(x, y)$ normalized by $\frac{m_\pi^5}{128\pi^3}G^2_F \sin^2 \theta_c f^2_\pi$ and (b) represents the function $\sigma_\pi(x, y)$.

Figure 3  Constraints on the parameters of the three Higgs model obtained from transverse muon polarization measurements for the charged Higgs mass 45 GeV (a) and 200 GeV (b). The solid (dotted) lines correspond to the $K^+ \rightarrow \mu^+ \nu\gamma$ ($K^+ \rightarrow \mu^+ \nu\pi^0$) process. From left to right the lines represent $P_\perp = 5 \times 10^{-3}, 1 \times 10^{-3}, 5 \times 10^{-4}, -5 \times 10^{-4}, -1 \times 10^{-3}, -5 \times 10^{-3}$ for both cases. The shaded parameter regions are already excluded by other phenomenological constraints.
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Figure 1 (a)
Figure 1 (b)

$y = 2E \mu / MK$

$x = 2E \gamma / MK$
Figure 3 (a)

$m_H = 45$ GeV
Figure 3 (b)

$m_H = 200 \text{ GeV}$