Constraints on the dark matter equation of state with redshift-space distortion

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In this paper, we study a model which is composed by the cosmological constant and dark matter with nonzero equation of state parameter, which could be called as \( \Lambda wDM \). In the synchronous gauge, we obtain the perturbation equations of dark matter, and deduce the evolution equations of growth factor about the dark matter and baryons. Based on the Markov Chain Monte Carlo method, we constrain this model by the recently available cosmic observations which include cosmic microwave background radiation, baryon acoustic oscillation, type Ia supernovae, and \( f\sigma_8(z) \) data points from redshift-space distortion. The results present a tighter constraint on the model than the case without \( f\sigma_8(z) \) data. In 3\( \sigma \) regions, we find the dark matter equation of state parameter \( w_{dm} = 0.000111^{+0.000068}_{-0.000136} + 0.00181 \). After an extra model parameter \( w_{dm} \) is considered, the difference between the minimum values of \( \chi^2 \) of our model and standard model is \( \Delta \chi^2_{min} = 0.598 \). Although the currently available cosmic observations mildly favor the nonzero dark matter equation of state parameter, no significant deviation from the \( \Lambda \)CDM model is found in 1\( \sigma \) region.

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I. INTRODUCTION

In March 2013, the European Space Agency and Planck Collaboration publicly released the new CMB data \cite{1,2}. The observational constraint on the standard model from alone Planck data shows us that the Universe is composed by 68.25\% dark energy (\( \Omega_\Lambda \)), 27.35\% dark matter (\( \Omega_{dm} \)), and 4.4\% baryons (\( \Omega_b \)) \cite{3}. The cosmological standard model is combined by the cosmological constant and cold dark matter, so it could be called as \( \Lambda \)CDM. The cosmological constant is a simple candidate of dark energy. The dark energy has been proposed to explain the observed accelerated expansion of the Universe. The cold dark matter is an usual candidate of dark matter, whose equation of state is zero. The cold dark matter can successfully explain the observed properties of large scale structures of the Universe, but there are the discrepancies on small scales \cite{4,8}. It leaves us the opportunity to probe an alternative to dark matter model, such as warm or hot dark matter. The hot dark matter has been ruled out due to the difficulty to form the observed large scale structure. The warm dark matter has been proposed as a potential solution to the small scales difficulties of cold dark matter \cite{6,9,13}. At the same time, on the large scales, warm dark matter yields the same results as cold dark matter and agrees with all the observations. To some extent, warm dark matter can successfully reproduce the cosmic observations from small to large scales. Lyman-\( \alpha \) forest data can be used to constrain on the warm dark matter with the free-streaming scale \cite{14,18}. Instead of measuring the warm dark matter particles, one could consider the dark matter equation of state as a nonzero parameter and measure it by the cosmic observations. \( w_{dm} \) might be a constant or time variable quantity \cite{19,21}, and could also be related to the tangential and radial pressure \cite{22,24}. In this paper, we assume \( w_{dm} \) to be a nonzero constant parameter and constrain it by the recent cosmic observations.

On the one hand, the nonzero equation of state parameter could bright about the deviation from the standard evolution of cold dark matter energy density, which will influences the matter density perturbations. This would alter the structure growth history of the matter. So, it is necessary to consider the effects of the cosmological constraints from the large scale structure information. On the other hand, the geometry information, which includes cosmic microwave background (CMB), baryon acoustic oscillation (BAO), and type Ia supernovae (SNIa), is not enough to break the possible degeneracy of cosmological models because the different models might undergo similar background evolution behavior, but the dynamical growth history would be different. Thus, the large scale structure information is an important tool to break the possible degeneracy of cosmological models. Via the redshift-space distortion (RSD), the measurement of the growth rate is related to the matter density contrast, that is, \( f = d\ln \delta / d\ln a \). However, the observed values of the growth rate \( f_{obs} = \beta b \), which are derived from the linear bias factor \( b \) and redshift-space distortion parameter \( \beta \) in the light of the \( \Lambda \)CDM model. It means that the recent \( f_{obs} \) data points could only be
where $p$ denotes the derivative with respect to the cosmic time,

\[
\sigma_8 = \frac{\rho_{\text{dm}}}{\rho_{\text{crit}}},
\]

TABLE I: The data points of $f_\sigma(z)$ measured from RSD with the survey references. The former nine data points at $z \in [0.067, 0.78]$ were summarized in Ref. [34]. The data point at $z = 0.8$ was released by the VIPERS in Ref. [40]. Then, a lower growth rate from RSD than expected from Planck was also pointed out in Ref. [41].

| $z$     | $f_\sigma(z)$ | Survey and Refs |
|---------|----------------|-----------------|
| 0.067   | 0.42 ± 0.06    | 6dFGRS (2012)   |
| 0.17    | 0.51 ± 0.06    | 2dFGRS (2004)   |
| 0.22    | 0.42 ± 0.07    | WiggleZ (2011)  |
| 0.25    | 0.39 ± 0.05    | SDSS LRG (2011) |
| 0.37    | 0.43 ± 0.04    | SDSS LRG (2011) |
| 0.41    | 0.45 ± 0.04    | WiggleZ (2011)  |
| 0.57    | 0.43 ± 0.03    | BOSS CMASS (2012) |
| 0.60    | 0.43 ± 0.04    | WiggleZ (2011)  |
| 0.78    | 0.38 ± 0.04    | WiggleZ (2011)  |
| 0.80    | 0.47 ± 0.08    | VIPERS (2013)   |

adopted to test the consistency of the standard model. In order to avoid this disadvantage, the model-independent test $f_\sigma(z)$ have been proposed to constrain the cosmological models [33], where $\sigma_8$ is the root-mean-square mass fluctuation in spheres with radius $8h^{-1}$ Mpc. Motivated by this idea, testing on the cosmological models by use of RSD data have been studied in Refs. [26–31], and the constraint results presented tighter constraint on the model parameter space than the case without the $f_\sigma(z)$ measurement. For other cosmological constraints from the large scale structure information, Xu combined the WiggleZ Dark Energy Survey [32] with the geometry tests to constrain the AwDM model, and concluded that the cosmic observations favored a very small value of $w_{dm}$ [33]. Meanwhile, in this paper, we will try to combine the RSD measurement with the geometry tests to constrain the AwDM model. It is worthwhile to anticipate the $f_\sigma(z)$ data points will give a tight constraint on the parameter space. Up to now, the ten observed data points of $f_\sigma(z)$ are shown in Table 1.

The paper is organized as follows. In Sec. II, we present the perturbation equations of the dark matter, and obtain the evolution equations of growth factor about the dark matter and baryons. In Sec. III, we would probe the cosmological implications of the dark matter equation of state parameter. The varied $w_{dm}$ could alter the effective dark matter energy density, which would change the equality moment of matter and radiation at early time. This would affect the evolutions of the CMB temperature and matter power spectra. Moreover, different values of $w_{dm}$ might influence the matter density perturbations, which would alter the structure growth history of the matter. So, we also could look for the cosmological effects on the evolution of $f_\sigma(z)$. Then, via the Markov Chain Monte Carlo (MCMC) method, we would preform a global fitting for the AwDM model by using CMB, BAO, SNIa and RSD data sets, and present the observational constraint results. Sec. IV is the summary.

II. THE EQUATIONS OF BACKGROUND, PERTURBATION, AND STRUCTURE FORMATION

We consider the dark matter equation of state as a nonzero constant parameter, which can be written as

\[
w_{dm} = \frac{\rho_{dm}}{\rho_{c}}.
\]

where $\rho_{dm}$ and $\rho_{c}$ are the pressure and energy density of the dark matter.

In the AwDM model, the energy conservation equation for the dark matter is

\[
\dot{\rho}_{dm} + 3H(1 + w_{dm})\rho_{dm} = 0,
\]

where the dot denotes the a derivative with respect to the cosmic time, $H$ the Hubble expansion rate of the Universe.

In a spatially flat FRW universe, the Friedmann equation reads

\[
H^2 = H_0^2 \left( \Omega_r a^{-4} + \Omega_b a^{-3} + \Omega_{dm} a^{-3(1+w_{dm})} + \Omega_{\Lambda} \right),
\]

where $\Omega_i = 8\pi G \rho_i/(3H^2)(i = r, b, dm, \Lambda)$ are, respectively, present dimensionless energy densities for radiation, baryons, dark matter and cosmological constant. They agree with $\Omega_r + \Omega_b + \Omega_{dm} + \Omega_{\Lambda} = 1$. 
Considering the scalar perturbations in a spatially flat universe, whose line element is
\[ ds^2 = -(1 + 2\phi)dt^2 + 2a\Lambda_i Bdx^i + a^2 [(1 - 2\psi)\delta_{ij} + 2\partial_i \partial_j E] dx^i dx^j. \] 
(4)

where \( \phi, B, \psi \) and \( E \) are the gauge-dependent scalar perturbations quantities.

In the general case of non-interacting fluids, the energy and momentum conservation equations for \( A \) fluid become
\[
\delta \rho_A + 3H (\delta \rho_A + \delta p_A) - 3 (\rho_A + p_A) \dot{\psi} + (\rho_A + p_A) \frac{\nabla^2}{a^2} (\theta_A + \sigma) = 0, 
\] 
(5)
\[
(\rho_A + p_A) \dot{\theta}_A - 3c_{s,Ad}^2 H (\rho_A + p_A) \theta_A + (\rho_A + p_A) \phi + \delta p_A + \frac{2}{3} \frac{\nabla^2}{a^2} \pi_A = 0.
\] 
(6)

where \( c_{s,Ad}^2 = p_A'/\rho_A' \) is the \( A \)-fluid adiabatic sound speed, the density contrast is \( \delta_A = \delta \rho_A/\rho_A \), and \( \theta_A \) is the volume expansion.

In the synchronous gauge, when the shear perturbation \( \sigma_{dm} = 0 \), the perturbation equations for the dark matter can be rewritten as
\[
\dot{\delta}_{dm} = -(1 + w_{dm}) \left( \frac{k^2}{a^2} \theta_{dm} + \frac{H}{2} \right) - 3H \left( \delta p_{dm}/\delta \rho_{dm} - w_{dm} \right) \delta_{dm},
\] 
(7)
\[
\dot{\theta}_{dm} = 3c_{s,Ad}^2 H \theta_{dm} - \frac{\delta p_{dm}/\delta \rho_{dm}}{1 + w_{dm}} \delta_{dm},
\] 
(8)

where the adiabatic sound speed of dark matter is \( c_{s,Ad}^2 = w_{dm} - w_{dm}/[3H(1 + w_{dm})] \). When \( w_{dm} \) is negative, the adiabatic sound speed is instable. So one should introduce an entropy perturbation and assume a positive or null effective sound speed. For a generalized dark matter \[46\], introducing the effective sound speed \( c_{s,eff}^2 = \delta p_{dm}/\delta \rho_{dm} \) in the rest frame, we can recast the perturbation equations into
\[
\dot{\delta}_{dm} = -(1 + w_{dm}) \left( \frac{k^2}{a^2} \theta_{dm} + \frac{H}{2} \right) - 3H (c_{s,eff}^2 - w_{dm}) \delta_{dm} + 9H^2 (c_{s,eff}^2 - c_{s,Ad}^2) (1 + w_{dm}) \theta_{dm},
\] 
(9)
\[
\dot{\theta}_{dm} = 3H c_{s,eff}^2 \theta_{dm} - \frac{c_{s,eff}^2}{1 + w_{dm}} \delta_{dm},
\] 
(10)

where \( c_{s,eff}^2 \) is another freedom to describe the micro scale property of dark matter in addition to \( w_{dm} \). Inspired by Refs. \[45\], we assume the effective sound speed \( c_{s,eff}^2 = 0 \) in this work.

Combining the above two equations and using \( H + 2H \dot{H} = -8\pi G(\delta \rho + 3\delta p) \), the second-order differential equation of density perturbation can be obtained
\[
\dot{\delta}_{dm} = (3w_{dm} - 2)H \delta_{dm} + 3w_{dm}(2H^2 + \dot{H}) \delta_{dm} + (1 + w_{dm})4\pi G(\delta \rho + 3\delta p)
+ 2(1 + w_{dm}) \frac{k^2}{a^2} H \theta_{dm} - 18w_{dm}(1 + w_{dm})(H^2 + \dot{H}) \theta_{dm}.
\] 
(11)

Following Refs. \[47\], numerical integration shows that the coupling to \( \theta_{dm} \) in Eq. \(11\) is subdominant for all scales that we are interested. If the dark matter is cold, we can return into the classic growth equation of the matter \( \delta_m + 2H \delta_m - 4\pi G(\delta \rho + 3\delta p) = 0 \) \[49\].

According to Refs. \[50\], the growth factor \( g(a) \) is proportional to the linear density perturbation \( \delta = \delta \rho/\rho \), \( g(a) = \delta(a)/a \), one can obtain the evolution equations of the growth factor for the dark matter and baryons
\[
g_{dm}'' + \frac{1}{2} \left[ 5 + 3\Omega_A(a) - 6w_{dm} - 3w_{dm}\Omega_{dm}(a) \right] g_{dm}' + \frac{3}{2} (1 - 3w_{dm})(1 + \Omega_A(a) - w_{dm}\Omega_{dm}(a)) g_{dm}
= \frac{3}{2} \left[ 1 + w_{dm} \right] \Omega_{dm}(a) g_{dm} + \Omega_b(a) g_{b},
\] 
(12)
\[ g''_b + \frac{1}{2} [5 + 3\Omega_{\Lambda}(a) - 3w_{dm}\Omega_{dm}(a)] g'_b + \frac{3}{2} [1 + \Omega_{\Lambda}(a) - w_{dm}\Omega_{dm}(a)] g_b = \frac{3}{2} [\Omega_{dm}(a)g_{dm} + \Omega_b(a)g_b], \tag{13} \]

where the prime denotes the derivative with respect to \( \ln a \) and

\[ \Omega_{dm}(a) = \frac{H^2}{H^2_0} \Omega_{dm}a^{-3(1+w_{dm})}, \quad \Omega_b(a) = \frac{H^2}{H^2_0} \Omega_b a^{-3}, \quad \Omega_{\Lambda}(a) = \frac{H^2}{H^2_0} \Omega_{\Lambda}. \tag{14} \]

The varied dark matter equation of state parameter leads to a bias between the density perturbations of standard dark matter and baryons. According to \( g_m = (\rho_{dm}g_{dm} + \rho_b g_b)/(\rho_{dm} + \rho_b) \), we can obtain the growth factor of the matter. Then, the growth factor \( g_m \) can connect to the growth function \( f_m \) with the relation \( f_m = (\ln \delta_m)' = 1 + (\ln g_m)' \).

In order to adopt the RSD measurement, we modify the \textit{CAMB} code \cite{60} and \textit{CosmoMC} package \cite{61}, and add a new module to calculate the theoretical values of \( f\sigma_8(z) \). For the details, please see Refs. \cite{26–31}.

### III. COSMOLOGICAL IMPLICATIONS AND CONSTRAINT RESULTS

#### A. Cosmological implications

We illustrate how the CMB temperature and matter power spectra are characterized by different values of the parameter \( w_{dm} \). The effects on the CMB temperature and matter power spectra are shown in Figs. 1 and 2 respectively. At the same time, in order to clearly explain the change of power spectra, we also plot the evolution curves for the ratio of dark matter and radiation \( \Omega_{dm}/\Omega_r \). From Figs. 1 and 3 increasing the values of \( w_{dm} \), which is equivalent to increasing the value of the effective dimensionless energy density of dark matter \( \Omega_{dm} \), will make the equality of matter and radiation earlier; therefore, the sound horizon is decreased. As a result, the first peak of CMB temperature power spectra is depressed. At large scales \( l < 100 \), the integrated Sachs-Wolfe (ISW) effect is dominant, the changed parameter \( w_{dm} \) affects the CMB power spectra via ISW effect due to the evolution of gravitational potential. The opposite rule can be seen from Figs. 2 and 3, with the increasing the values of \( w_{dm} \), the matter power spectra \( P(k) \) is enhanced due to the earlier matter-radiation equality.

![FIG. 1: The effects on CMB temperature power spectra for the different values of model parameter \( w_{dm} \). The black solid, red thick dashed, green dotted-dashed, and blue dotted lines are for \( w_{dm} = 0, 0.000111, 0.03, \) and \( -0.03 \), respectively; the other relevant parameters are fixed with the mean values as shown in the fourth column of Table II.](image)

Then, in order to test the effects on the evolution of \( f\sigma_8(z) \) for the model parameter \( w_{dm} \), we fix the relevant cosmological parameters according to the fourth column of Table II but consider \( w_{dm} \) to be varied in a range. The
FIG. 2: The effects on matter power spectra for the different values of model parameter $w_{dm}$. The black solid, red thick dashed, green dotted-dashed, and blue dotted lines are for $w_{dm} = 0, 0.000111, 0.03,$ and $-0.03$, respectively; the other relevant parameters are fixed with the mean values as shown in the fourth column of Table II.

FIG. 3: The evolution curves for the ratio of dark fluid and radiation $\Omega_{dm}/\Omega_r$ when the parameter $w_{dm}$ is varied. The different lines correspond to the cases of the Fig. 1; the horizontal gray thick line responds to the case of $\Omega_{dm} = \Omega_r$, and the other relevant parameters are fixed with the mean values as shown in the fourth column of Table II.

evolution curves of $f\sigma_8(z)$ with respect to the redshift $z$ are shown in Fig. 4. With the increasing the values of $w_{dm}$, the curves of $f\sigma_8(z)$ are enhanced at both lower and higher redshifts.

Importantly, one can mildly see that the cases of $w_{dm} =$ mean value ($w_{dm} = 0.000111$) and that of $w_{dm} = 0$ (correspond to the ΛCDM model) are slightly distinguishing from the evolution curves of $f\sigma_8(z)$, which is different from the evolutions of CMB temperature and matter power spectra. It means that, to some extent, the $f\sigma_8(z)$ data
set could mildly break the degeneracy between the $\Lambda w_{\text{DM}}$ model and the $\Lambda \text{CDM}$ model.

![Graph](image)

**FIG. 4:** The fitting evolution curves of $f\sigma_8(z)$ about the redshift $z$ for the varied model parameter $w_{\text{dm}}$. The black solid, red dashed, green dotted-dashed, and blue dotted lines are for $w_{\text{dm}} = 0, 0.000111, 0.003$, and $-0.003$, respectively; The gray error bars denote the observations of $f\sigma_8$ are listed in Table I; the other relevant parameters are fixed with the mean values as shown in the fourth column of Table II.

### B. Data sets and constraint results

We use the cosmic observational data sets which include $f\sigma_8(z)$ data points from RSD in Table I. For the SNIa data set, we adopt the SNLS3 which consists of 472 SN calibrated by SiFTO and ASLT2 $^{52-54}$. For the BAO data set, we use the measured ratio of $r_s/D_v$ as a ‘standard ruler’ at three different redshifts: $r_s/D_v(z = 0.106) = 0.336 \pm 0.015$ $^{56}$; $r_s/D_v(z = 0.35) = 0.1126 \pm 0.0022$ $^{57}$; $r_s/D_v(z = 0.57) = 0.0732 \pm 0.0012$ $^{58}$. After Planck data, the CMB data which includes two main parts: one is the high-$l$ TT likelihood ($\text{CAMSpec}$) up to a maximum multipole number of $l_{\text{max}} = 2500$ from $l = 50$ $^{3}$; the other is the low-$l$ TT likelihood up to $l = 49$ $^{3}$ and the low-$l$ TE, EE, BB likelihood up to $l = 32$ from nine-year WMAP data $^{59}$. In our numerical calculations, the total likelihood is calculated by $L \propto e^{-\chi^2/2}$, where $\chi^2$ can be constructed as

$$
\chi^2 = \chi^2_{\text{CMB}} + \chi^2_{\text{BAO}} + \chi^2_{\text{SNIa}} + \chi^2_{\text{RSD}},
$$

(15)

where the four terms in the right hand side, respectively, denote the likelihood contribution from CMB, BAO, SNIa, and RSD data sets. The constraint methodology of observed data sets have been shown in Appendix of this paper.

For the $\Lambda w_{\text{DM}}$ model, we consider the seven-dimensional parameter space which reads

$$
P \equiv \{\Omega_b h^2, \Omega_{dm} h^2, \Theta_S, \tau, w_{\text{dm}}, n_s, log[10^{10} A_S]\},
$$

(16)

where $\Omega_b h^2$ and $\Omega_{dm} h^2$, respectively, stand for the density of the baryons and dark matter, $\Theta_S = 100\theta_M$ denotes the ratio of sound horizon and angular diameter distance, $\tau$ indicates the optical depth, $w_{\text{dm}}$ is the dark matter equation of state parameter, $n_s$ is the scalar spectral index, and $A_s$ represents the amplitude of the initial power spectrum. Here, the pivot scale of the initial scalar power spectrum $k_{\text{pivot}} = 0.05 \text{Mpc}^{-1}$ is used. The following priors to model parameters are adopted: $\Omega_b h^2 \in [0.005, 0.1], \Omega_{dm} h^2 \in [0.01, 0.99], \Theta_S \in [0.5, 10], \tau \in [0.01, 0.8], w_{\text{dm}} \in [-0.2, 0.2], n_s \in [0.5, 1.5], log[10^{10} A_S] \in [2.7, 4]$. Based on the MCMC method, we modify the publicly available $\text{CAMB}$ code $^{60}$ according to the perturbation equations for dark matter, and add a new module in $\text{COSMOMC}$ package $^{61}$ to calculate the theoretical value of $f\sigma_8(z)$, one can also see Refs. $^{26-31}$ for the details.
After running eight chains in parallel on the computer, the constraint results for the AwDM model are presented in the fourth and fifth columns of Table II. We show the one-dimensional (1D) marginalized distributions of parameters and two-dimensional (2D) contours with 68% confidence levels (C.L.), 95% C.L., and 99.7% C.L. in Fig. 5. From this figure, we could analyze the degeneracy between the model parameters \( \Omega_m \) and \( w_{dm} \). The amount of \( \Omega_m \) at the decoupling epoch is determined by the CMB power spectra; with the decreasing present values of \( \Omega_m \), increasing values of \( w_{dm} \) could maintain the amount of \( \Omega_m \) at the same early time. The similar case have occurred in Ref. [33]. In order to clearly see the significance of large scale structure information, we also list the constraint results without \( f_\sigma(z) \) test in the second and third columns of Table II. As is expected, RSD measurement give a tighter constraint on the parameter space. Moreover, in order to compare with the ACDM model, we also constrain the ACDM model with \( f_\sigma(z) \) data set, the results are shown in the sixth and seventh columns of Table II. Based on the same observational data sets (CMB from Planck + WMAP9, BAO, SN1a, and RSD), the AwDM model has another parameter \( w_{dm} \) which gives rise to the difference of the minimum \( \chi^2 \) with the ACDM model, \( \Delta \chi^2_{\text{min}} = 0.598 \). Besides, the constraint results for the AwDM model is compatible with the previous results of Ref. [33]. However, because of using different measurements of the large scale structure, the results have a little difference, such as the mean values of \( w_{dm} \) and \( \sigma_8 \) are smaller than those in the Table I of Ref. [33]. It means that the recent cosmic observations mildly favor the nonzero dark matter equation of state parameter, but no significant deviation from the ACDM model is found in 1\sigma region.

### IV. SUMMARY

In this paper, the AwDM model, which is combined by the cosmological constant and dark matter with a nonzero equation of state parameter, has been studied. In the synchronous gauge, we obtained the perturbation equations, and firstly presented the equation of structure formation for this model. Based on the density perturbations of dark matter and baryons, we added a new module to calculate the theoretical values of \( f_\sigma(z) \) which could be adopted to constrain the AwDM model. For the cosmological implications of varied parameter \( w_{dm} \), we have plotted the evolutions of the CMB power spectra, matter power spectra and \( f_\sigma(z) \). When \( w_{dm} \) was taken as mean value or zero, from the power spectra, we were difficult to distinguish the AwDM model from the ACDM model. However, due to using the \( f_\sigma(z) \) data set, the evolution curves of \( f_\sigma(z) \) could mildly break the degeneracy of the two models. Then, based on the MCMC method, a global fitting was performed on this model by adopting the CMB from Planck + WMAP9, BAO, SNIa, and RSD data sets. We obtained tighter constraint for the parameter space than the case without \( f_\sigma(z) \) data points. The constraint results for the AwDM model is compatible with the results of Ref. [33]. However, because of using different measurements of the large scale structure, the constraint results have a little difference. With the \( f_\sigma(z) \) data set, the cosmic observational data sets favor a very small value of \( w_{dm} \) which is up to the order of 10\(^{-4}\). The AwDM model has another parameter \( w_{dm} \) which gives rise to the difference of the minimum \( \chi^2 \) with the ACDM model, \( \Delta \chi^2_{\text{min}} = 0.598 \). Although the currently available cosmic observations mildly

| Parameters | AwDM without RSD | Best fit | AwDM with RSD | Best fit | ACDM with RSD | Best fit |
|------------|------------------|----------|---------------|----------|---------------|----------|
| \( \Omega_b^2 \) | 0.0220\( ^{+0.000362}_{-0.000357} \) | 0.0220 | 0.0223\( ^{+0.000261}_{-0.000257} \) | 0.0223 | 0.0223\( ^{+0.000245}_{-0.000246} \) | 0.0225 |
| \( \Omega_c^2 \) | 0.117\( ^{+0.00169}_{-0.00168} \) | 0.117 | 0.116\( ^{+0.00171}_{-0.00170} \) | 0.117 | 0.116\( ^{+0.00144}_{-0.00146} \) | 0.115 |
| 1000MC | 1.04\( ^{+0.00566}_{-0.00572} \) | 1.04 | 1.04\( ^{+0.00554}_{-0.00553} \) | 1.04 | 1.04\( ^{+0.00543}_{-0.00545} \) | 1.04 |
| \( \tau \) | 0.0890\( ^{+0.0124}_{-0.0121} \) | 0.0895 | 0.0857\( ^{+0.0119}_{-0.0116} \) | 0.0931 | 0.0937\( ^{+0.0117}_{-0.0116} \) | 0.0788 |
| \( w_{dm} \) | 0.00102\( ^{+0.000253}_{-0.000252} \) | 0.000568 | 0.000111\( ^{+0.000848}_{-0.000836} \) | 0.000195 | -- | -- |
| \( n_s \) | 0.962\( ^{+0.00255}_{-0.00254} \) | 0.968 | 0.968\( ^{+0.00255}_{-0.00254} \) | 0.970 | 0.969\( ^{+0.00384}_{-0.00390} \) | 0.972 |
| \( \Delta m \) | 3.99\( ^{+0.0375}_{-0.0375} \) | 3.11 | 3.07\( ^{+0.0370}_{-0.0366} \) | 3.09 | 3.07\( ^{+0.0322}_{-0.0313} \) | 3.06 |
| \( \Delta \Omega \) | 0.709\( ^{+0.0185}_{-0.0184} \) | 0.707 | 0.706\( ^{+0.0180}_{-0.0179} \) | 0.699 | 0.710\( ^{+0.0191}_{-0.0190} \) | 0.713 |
| \( \Omega_m \) | 0.291\( ^{+0.00237}_{-0.00235} \) | 0.293 | 0.290\( ^{+0.00238}_{-0.00236} \) | 0.301 | 0.290\( ^{+0.00185}_{-0.00184} \) | 0.287 |
| \( \sigma_8 \) | 0.851\( ^{+0.0253}_{-0.0252} \) | 0.854 | 0.818\( ^{+0.0353}_{-0.0353} \) | 0.807 | 0.810\( ^{+0.0395}_{-0.0394} \) | 0.802 |
| \( z_{re} \) | 11.0\( ^{+0.11}_{-0.11} \) | 11.7 | 10.5\( ^{+0.10}_{-0.11} \) | 11.2 | 10.6\( ^{+0.10}_{-0.11} \) | 9.90 |
| \( H_0 \) | 69.3\( ^{+0.13}_{-0.13} \) | 69.1 | 69.3\( ^{+0.13}_{-0.13} \) | 69.2 | 69.3\( ^{+0.13}_{-0.13} \) | 69.5 |
| \( \Delta \text{Age/Gyr} \) | 13.7\( ^{+0.053}_{-0.052} \) | 13.7 | 13.8\( ^{+0.0537}_{-0.0536} \) | 13.8 | 13.8\( ^{+0.0536}_{-0.0535} \) | 13.7 |

Table II: The mean values with 1, 2, 3\sigma errors and the best fit values of model parameters for the AwDM and ACDM models, where CMB from Planck + WMAP9, BAO, SNIa, with or without RSD data sets have been used.
FIG. 5: The 1D marginalized distributions on individual parameters and 2D contours with 68\%C.L., 95\%C.L., and 99.7\%C.L. between each other using the combination of the observational data points from the CMB from Planck + WMAP9, BAO, SNIa, and RSD data sets.

favor the nonzero dark matter equation of state parameter, no significant deviation from the standard model is found in 1\% region.

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Appendix A: The constraint methodology of observed data sets

For the BAO data set, we use the measured ratio of $r_s/D_v$ as a ’standard ruler’; $r_s$ is the comoving sound horizon at the baryon drag epoch, $D_v$ is the effective distance which is determined by the angular diameter distance $D_A$ and Hubble parameter $H$ \[ D_v(z) = \left[ (1 + z)^2 D_A(a)^2 \frac{z}{H(z)} \right]^{1/3}. \] (A1)
At three different redshifts, \( r_s(z) / D_V(z) = 0.106 \) = 0.336 ± 0.015 is from 6-degree Field Galaxy Redshift Survey (6dFGRS) data 56, \( r_s(z) / D_V(z) = 0.35 \) = 0.1126 ± 0.0022 comes from Sloan Digital Sky Survey Data Release 7 (SDSS DR7) data 57, and \( r_s(z) / D_V(z) = 0.57 \) = 0.0732 ± 0.0012 is from SDSS DR9 58. So, the likelihood for BAO reads

\[
\chi^2_{BAO} = \chi^2_{6dF} + \chi^2_{DR7} + \chi^2_{DR9} = \frac{[(r_s(zd)/D_V(0.106))_{th} - 0.336]^2}{0.015^2} + \frac{[(r_s(zd)/D_V(0.35))_{th} - 0.1126]^2}{0.0022^2} + \frac{[(r_s(zd)/D_V(0.57))_{th} - 0.0732]^2}{0.0012^2}.
\]

For the SNIa data set, we use the SNLS3 data, which consists of 472 SN calibrated by SiFTO and SALT2 52–54. The expected apparent magnitudes of cosmological model are given by 53–54.

\[
\chi^2_{SN1a} = (\vec{m}_B - \vec{m}_B^{model})^T C^{-1}_{SN}(\vec{m}_B - \vec{m}_B^{model}).
\]

where \( \vec{m}_B \) is the vector of effective absolute magnitudes and \( C^{-1}_{SN} \) is the sum of non-sparse covariance matrices of quantifying statistical and systematic errors 53. The expected apparent magnitudes of cosmological model are given by 53–54.

\[
m_B^{model} = 5\log_{10} D_L(z_{hel}, z_{cmb}, w_\Lambda, \Omega_m, \Omega_\Lambda) - \alpha(s - 1) + \beta C + M_B,
\]

where \( D_L \) is the Hubble-constant free luminosity distance, \( z_{cmb} \) and \( z_{hel} \) are the CMB frame and heliocentric redshifts of the SN, \( s \) is the stretch (a measure of the shape of the SN light curve), and \( C \) is color measure for the SN. \( \alpha \) and \( \beta \) are nuisance parameters. \( M_B \) is another nuisance parameter which absorbs the Hubble constant. As in Ref. 54, one could express values of the parameter \( M_B \) in term of an effective absolute magnitude, \( m_B = M_B - 5\log_{10}(c/H_0) - 25 \).

After Planck data, the CMB data which includes two main parts: one is the high-l temperature likelihood (CAM-Spec) up to a maximum multipole number of \( l_{max} = 2500 \) from \( l = 50 \); the other is the low-l temperature likelihood up to \( l = 49 \) and the low-l polarization likelihood up to \( l = 32 \) from nine-year WMAP data 59.

The likelihood of RSD measurement is given by

\[
\chi^2_{RSD} = \sum \frac{[f\sigma(z_i)_{th} - f\sigma(z_i)_{obs}]^2}{\sigma_i^2}.
\]

Therefore, the total likelihood is calculated by \( \mathcal{L} \propto e^{-\chi^2/2} \), where \( \chi^2 \) can be constructed as

\[
\chi^2_{total} = \chi^2_{BAO} + \chi^2_{SN1a} + \chi^2_{CMB} + \chi^2_{RSD}.
\]
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