Novel DOA Estimation of Coherent Signal based on Virtual Array

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Abstract. Conventional direction-of-arrival (DOA) estimations perform poorly in coherent signals due to the rank loss of spatial covariance matrix, and most of the existing methods solve the problem at the expense of Degree of Freedom (DOF) or accuracy. In this paper, a novel DOA estimation technique for coherent signal is presented based on virtual array. The method comprises three steps: Firstly, the covariance matrix is generated according to the output signals of the array. Then, the first row and first column are selected from the matrix according to the data model of each element of the covariance matrix, which can be regarded as a uniform and continuous single snapshot signal of virtual array. Finally, the forward and backward spatial smoothing is used to achieve decorrelation of the single snapshot signal of the virtual array element. The proposed method has no loss of DOF and maintains high accuracy for low signal-to-noise ratio (SNR) conditions. The feasibility of the proposed method is proved by theoretical derivation, and its effectiveness is also verified by numerical results.

1. Introduction

Coherent signals exist widely in complex electromagnetic space due to the multipath effect in the signal propagation, resulting in the rank loss of covariance matrix, and the traditional MUSIC [1] or ESPRIT [2] algorithm fail to estimate the DOA. Therefore, some decorrelation algorithms are needed for relevant signals. At present, spatial smoothing [3] is a relatively mature technique in decoherence. Spatial smoothing method divides the receiving array into multiple subarrays with the same dimension and averages the covariance of these subarrays to obtain the non-singular matrix. Using the classical MUSIC algorithm on the newly obtained covariance matrix, the estimation of the direction of arrival for the relevant signals can be obtained. [4] proposes an improved ESPRIT-like algorithm to realize DOA estimation of coherent signals, and reconstruct a Toeplitz matrix by selecting specific elements in the covariance matrix. The rank of the reconstructed Toeplitz matrix is only related to the DOA of signals and cannot be affected by the coherency between them, and the signal subspace and noise subspace could be distinguished. Finally, the improved ESPRIT-like algorithm was used to estimate DOA. Vector Singular Value Method (SVD) [5] is also a classic decorrelation method. SVD method searches for a data vector that contains all the information about the desire signal, and then reconstruct a new matrix. Equally, DOA estimation is completed by SVD combined with MUSIC algorithm using the fresh attained matrix. Although the above algorithm has certain effect in solving the correlation problems, there are still many problems, such as the diminution of DOF.

Target at solving these issues, scholars begin to research the decorrelation method without the loss of DOF. In [6], a Toeplitz matrix was constructed by using the cross-correlation of the received signals.
The signal subspace and noise subspace are obtained by singular value decomposition of Toeplitz matrix, to realize DOA estimation of coherent signals. [7] leverages the Toeplitz characteristics of incoherent signals and noise autocorrelation matrix to separate the coherent signals and incoherent signals by the difference method, achieving the objective of estimating the DOA of coherent signals and incoherent signals. This algorithm expands the aperture of the spacing between two adjacent sensors and can estimate more source numbers. [8][9] improved the technique in [7], which improved the accuracy of DOA estimation of relevant signals to a certain extent. [10] raises an estimation approach to separate the independent signal and the coherent signal, which is based on the theorem that all the DOA information about a signal in a cluster is included in an eigenvector corresponding to a large eigenvalue. They proposed to use projection method to eliminate the interaction between related signals and make DOA more accurate. However, this algorithm requires a lot of eigenvalue decomposition in the process of calculation, which makes it difficult to realize in practical engineering. [11] adopts single snapshot number to realize DOA estimation of relevant signals, and uses single snapshot signal to construct Toeplitz matrix. This method has no loss of DOF, but precision falling with the reduction of SNR. [12] presents a DOA estimation method for relevant signals based on the principal feature vector and [13] modified the covariance matrix based on convex optimization to estimate DOA. The limitation of [13] lay in the high computational complexity, worse still, large deviations may sometimes emerge during the whole optimization progression, leading to the decline of estimation and weak stability.

Recently, the construction of virtual array provides a new method to obtain higher DOF in the estimation of signal parameters. Sparse array [14]-[17] based on the virtual array breaks through the restriction that the number of array elements must exceed signals amount in the traditional DOA parameter estimation and acquires a higher DOF. Typical sparse arrays include minimum-redundancy array [18], coprime array [19] and nested array [20].

Here, we apply the idea of virtual array to the DOA estimation of correlation signals to avoid the loss of DOF in the process of decorrelation. We establish the covariance matrix of the received signal, select the elements to get the single snapshot signal of a higher dimensional virtual array element, and then realize the single snapshot signal by spatial smoothing and other methods of uncorrelation. This procedure doesn't affect the degrees of freedom.

In addition, since the received signals of virtual array elements come from the cross-correlation of the received signals of actual array elements, the received signals of actual array elements contain the same signal components, while the noise is independent of each other. The mutual correlation operation can reduce the noise to a certain extent; thus, the SNR of the constructed virtual array single snapshot data can be improved, and the algorithm has high precision. Meanwhile, the algorithm has no requirement on the type of incident signal and has strong applicability.

2. System model of mixed signals
Consider an array of \( M \) omnidirectional sensors linearly spaced that are receiving signals from multiple point sources at different angles, \( \lambda \) and \( d \) represents wavelength and array element spacing, respectively. Due to the multipath effect in the actual environment, the signals in the space include coherent signals and independent signals. It is assumed that there are \( N \) narrowband signals, including \( N_C \) coherent signals \( S_i(t) \). There is a total of \( K \) groups of coherent signals, and there are \( P_k \) coherent signals in the \( K \) group, \( k=1, 2, 3, \ldots, K \), which contains \( N_u \) independent signals \( S_j(t), j=N_C, N_C+1, \ldots, N \), \( \sigma_n^2 \) denotes signal power, \( \theta_j \) denotes the direction of signal arrival. \( N=1, 2, \ldots, N \). \( \beta_l \) denotes the correlation coefficient, \( l=1, 2, \ldots, N_C \).
Figure 1. $M \times 1$ uniform linear array.

\[
x(t) = \sum_{k=1}^{N_c} \sum_{p=1}^{P_k} a(\theta_{k,p}) \beta_{k,p} \ s_k(t) + \sum_{k=N_c+1}^{N} a(\theta_k) s_k(t) + n(t)
\]

(1)

\[
x(t) = A_c^T s_c(t) + A_s^T s_s(t) = \tilde{s} + n(t)
\]

(2)

\[
x(t) = [x_1(t), \ldots, x_M(t)]^T
\]

(3)

where $x_l(t)$ is the received signal of the $l$th array element, $s_c(t) = [s_1(t), \ldots, s_k(t)]^T$ is the coherent signal, $s_s(t) = [s_{k+1}(t), \ldots, s_N(t)]^T$ is the independent signal, $s(t) = [s_1^T(t), s_s^T(t)]^T$ is the mixed signal. $a(\theta) = [1, e^{-j2\pi \frac{d \sin \theta}{\lambda}}, \ldots, e^{-j2\pi \frac{M-1 \sin \theta}{\lambda}}]^T$ is a steering vector for an angle of arrival $\theta$, $A_c = [A_{c,1}, \ldots, A_{c,N_c}]$ denotes the steering vector of coherent signal, $A_{c,k} = [a(\theta_{k,1}), \ldots, a(\theta_{k,P_k})]$ represents the steering vector matrix of group $k$ coherent signals. $A_a = [a(\theta_{N_c+1}), a(\theta_{N_c+2}), \ldots, a(\theta_N)]$ is the steering vector matrix of independent signals, where $\beta_k = [\beta_{k,1}, \ldots, \beta_{k,P_k}]^T$ denotes the correlation coefficient of group $k$ correlation signals.

3. Proposed Algorithm

3.1. The covariance matrix of mixed signals

Since the covariance matrix $R_{xx} = E[x(t)x(t)^H]$ is unavailable even in signal-free applications, it is usually replaced by the sample covariance matrix $\hat{R}_{xx} = \frac{1}{T} \sum_{t=1}^{T} x(t)x(t)^H$, and $R_{xx}$ can be written as:

\[
R_{xx} = \begin{bmatrix}
  r(1,1) & r(1,2) & \cdots & r(1,M) \\
  r(2,1) & r(2,2) & \cdots & r(2,M) \\
  \vdots & \vdots & \ddots & \vdots \\
  r(M,1) & r(M,2) & \cdots & r(M,M)
\end{bmatrix}
\]

(4)

And any element $r(m,n)$ of the covariance matrix is given by:

\[
r(m,n) = \sum_{i=1}^{N} d_i e^{j2\pi (m-n) d \sin \theta_i / \lambda} + \sigma_a^2 \delta_{m,n}
\]

(5)

\[
\delta_{m,n} = \begin{cases}
  1 & m = n \\
  0 & m \neq n
\end{cases}
\]

(6)
The $d_i$ in the mathematics (5) is described as:

$$d_i = \begin{cases} 
\sigma_i^2 \beta_i^2 \sum_{p=1}^{N_i} \beta_p e^{-j 2 \pi m d (\sin \theta_i - \sin \theta_j) / \lambda} & i = 1, \cdots, P_1 \\
\sigma_i^2 \beta_i^2 \sum_{p=1}^{N_i} \beta_p e^{-j 2 \pi m d (\sin \theta_i - \sin \theta_j) / \lambda} & i = P_1 + 1, \cdots, P_1 + P_2 \\
\sigma_i^2 \beta_i^2 \sum_{p=1}^{N_i} \beta_p e^{-j 2 \pi m d (\sin \theta_i - \sin \theta_j) / \lambda} & i = \sum_{n=1}^{P_2} P_n + 1, \cdots, \sum_{n=1}^{P_2} P_n + P_n \n \end{cases}$$

\hspace{1cm} i = N_i + 1, \cdots, N 

\hspace{1cm} (7)

3.2. Establishment the model of virtual array

We can construct the $(2 \times M - 1) \times 1$ received single snapshot signal of the virtual array through the elements in the covariance matrix. $Y(t)$ represents the received signal of the virtual array, where $y_{n-m}(t) = r(n,m)$ is the received single snapshot signal of the $(n-m)$th virtual array element.

$$Y(t) = [r(1,M), r(1,M-1), \cdots, r(1,1), \cdots, r(M-1,1), r(M,1)]^T = [y_{1-M}(t), y_{2-M}(t), \cdots, y_0(t), y_{M-1}(t)]^T$$

\hspace{1cm} (8)

For the building virtual array of single Snapshot, the output signal snapshot number is reduced compared to the original signal snapshot number. Since $r(n,m)$ comes from the cross-correlation of the received signals of each array element and the received signal of each sensor contain the same signal component. However, noise components are independent of each other, hence the intensity of noise will be weakened when cross-correlation is carried out. Therefore, the SNR$_V$ of the virtual array is improved compared with the SNR$_R$ of the actual signal, and the improved SNR and the decreased snapshot number satisfy the following equation (except for the intermediate array element):

$$\frac{\text{SNR}_V}{\text{SNR}_R} = \frac{\text{SNR}_{\text{Snapshot}_V}}{\text{SNR}_{\text{Snapshot}_R}}$$

\hspace{1cm} (9)
3.3. **DOA estimation**

The single snapshot signal of the virtual array also has the problem of rank deficit when generating the covariance matrix and needs to be decoherent. In this paper, the forward and backward spatial smoothing is used to decoherent the single snapshot signal of the virtual array.

![Diagram of forward and backward spatial smoothing](image)

**Figure 3.** Forward and backward spatial smoothing.

\[
Y_f(t) = [y_{-M}(t), y_{-M+1}(t), y_{-M+2}(t) \ldots y_{-1}(t)]
\]

\[
Y_b(t) = [y_{M}(t), y_{M-1}(t), y_{M-2}(t) \ldots y_{1}(t)]
\]

\[
R_f = \frac{1}{M} \sum_{l=1}^{M} R_f^l
\]

\[
R_b^l = E[Y_f^l(t)Y_f^l(t)^H]
\]

\[
R_b = \frac{1}{M} \sum_{l=1}^{M} R_b^l
\]

where \( Y_f^l(t) \) denotes the \( l \) th forward subarray, \( Y_b^l(t) \) is the \( l \) th backword subarray, \( R_f \) is the forward spatial smooth covariance matrix. \( R_b \) is the backward spatial smooth covariance matrix. \( R_b \) is actually the conjugate transpose matrix of \( R_f \), and the covariance matrix for backward and forward space smoothing can be defined as

\[
\hat{R} = \frac{1}{2} (R_f + R_b)
\]

Finally, we utilize the MUSIC algorithm to realize DOA estimation. \( U_N \) is the noise subspace of \( \hat{R} \), and DOA estimation is obtained by spectral peak search.
4. Simulation Results

In this section, we present simulation results that illustrate the performance of the algorithm we proposed in this paper. Our approach has no loss of freedom. The performance of the algorithm is mainly analysed in three aspects: resolution, root-mean-square error (RMSE) and accuracy (the setting error of the DOA within ±2° is considered as success). RMSE and accuracy are mathematically expressed by

\[
RMSE = \sqrt{\frac{1}{NL} \sum_{k=1}^{N} \sum_{l=1}^{L} (\theta_{\hat{\theta},k,l} - \theta_{\theta,k,l})^2}
\]

where \( \theta_{\hat{\theta},k,l} \) is the estimated angle, \( \theta_{\theta,k,l} \) is the real angle, \( N \) is the number of Examples while \( F_d \) is the successful trials. We assume that there are \( N \) signals incident from the far field of space in the form of plane waves on the uniform linear array (ULA) with \( M \) omnidirectional sensors spaced a half wavelength apart is used. For each scenario, \( L \) Monte-Carlo runs are performed.

In Example 1, we considered has five (\( N = 5 \)) planar wavefronts including two group of coherent signals and one independent signal (each group of coherent signals included two coherent signals) imping on a ULA of eleven (\( M = 11 \)) sensors with half-wavelength spacing. The correlation coefficients of the first group are 1, 0.9550+0.0995j respectively, and the correlation coefficients of the second group are 1, 1+0.01j respectively. The signal directions are -35°, -25.4°, 3°, 10°, 40°, respectively. The SNR in each sensor is set to be fixed at 20 dB and the number of samples (snapshots) taken from the array is 200. Compared with forward spatial smoothing method, [4] method (because [4] used ESPRIT algorithm to estimate DOA, it could not prove the observation resolution from spatial spectrum) and vector singular value method, the Angle grid used in simulation is 0.1°, and the simulation results are shown in Figure 4 below.

In Example 2, \( M \) and \( N \) is set to be nine and eight respectively (\( M = 9 \), \( N = 8 \)), and the signal arrival direction are evenly distributed between -60° and 60°. The signals include two groups of coherent signals and two independent signals, each group of coherent signals contains three coherent signals, the correlation coefficients of the first group of coherent signals are respectively, and the correlation coefficients of the second group of coherent signals are respectively, the noise is white Gaussian noise, and the number of snapshots is the same as Example 1. The Angle grid used for simulation is 0.1°, and the simulation results are shown in Figure 5 below.

![Figure 4. Resolution comparison of different methods](image1)

![Figure 5. Space spectrum without loss of freedom](image2)
Figure 4 shows that our approach has the highest resolution in the four kinds of methods mentioned, because the third signal close to the fourth signal DOA, singular value vector when signals to wave direction is close already could not tell the difference between the two signals. Although forward spatial smoothing can distinguish it, its spectral peak is too flat. Compared with smooth spectral peak, our algorithm has a sharp peak in all signal DOA, and all signals can be clearly distinguished. As can be seen from figure 5, when the number of signals increases, the method in this paper can also effectively estimate all signals. By projecting into the virtual domain, the array aperture is enlarged, the resolution is improved, and the number of signals can be estimated is increased compared with the traditional method.

In Example 3, we further demonstrate the performance of our new method, the root square error (RMSE) and accuracy of $\theta_0$ estimated with our new algorithm versus SNR are plotted in figure 6 and figure 7 respectively, as well as that with spatial forward smoothing method and the vector singular value method. We set $M=11$, $N=5$, the correlation coefficients of the first group are 1, 0.9550+0.0995j respectively, and the correlation coefficients of the second group are 1, 1+0.01j respectively. The signal directions are $-35^\circ$, $-25.4^\circ$, $3^\circ$, $10^\circ$, $40^\circ$, respectively. The noise is Gaussian white noise, and the snapshot number is 200, the number of Monte Carlo Examples is 500 ($L=500$).

![Figure 6. RMSE versus SNR](image6.png)

![Figure 7. Probability of success versus SNR](image7.png)

Figure 6 and figure 7 indicate that the spatial smoothing, the algorithm proposed in [4] and the vector singular value method all have higher accuracy when $SNR>5dB$, but the RMS error is larger and the success rate is lower when the SNR is low. On the contrary, our algorithm still maintains a higher success probability and a lower RMS error when SNR is low. The proposed algorithm is transformed into a virtual array, which reduces the influence of noise on the estimation accuracy, hence the algorithm can still maintain the intersection precision under the condition of low SNR.

In Example 4, we test the performance of our algorithm versus the number of snapshots in figure 8 and figure 9 with three other DOA methods: the spatial forward smoothing method, the algorithm proposed in [4] and the vector singular value method. In this Example, we set the $SNB=0dB$, and test RMSE and accuracy of four methods with the number of snapshots ranging from 200 to 2000 (stride length=200). Other Experimental configurations are the same as in Example 3.
In figure 8 and figure 9, RMSE and accuracy for the tested methods are illustrated against the number of snapshots respectively. It can be seen from these figures that the proposed DOA algorithm always performs better than other three methods. Furthermore, Spatial smoothing, [4] proposed algorithm, and singular value vector method under the condition of the high number of snapshots have high precision. But in the case of low fast, RMSE is bigger, estimated accuracy gradually declined and cannot even complete the DOA estimate. The proposed algorithm, by contrast, converges quickly and maintains high accuracy and low RMSE under the low number of snapshots, which is suitable for the actual engineering needs.

5. Conclusion
In this correspondence, we propose a DOA estimation method for correlated independent mixed signals which is suitable for low SNR, low number of snapshots. The received signal of the array is transformed into a single snapshot signal of the virtual array. This algorithm extends the array aperture, improves the SNR, estimates the related signal without loss of freedom at the same time, and saves the hardware overhead. The simulation results demonstrate that the proposed method is effective.

Appendices
In this paper, we use uppercase and lowercase boldfaces to denote matrices and vectors, respectively; $(\cdot)'$, $(\cdot)\hat{}$, and $(\cdot)^H$ denote conjugate, transpose, and conjugate transpose respectively; $E(\cdot)$ denotes statistical expectation; $\text{diag}(\cdot)$ denotes diagonalization and $I$ denote the identity matrix.

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