THE ROLE OF PEBBLE FRAGMENTATION IN PLANETESIMAL FORMATION. I. EXPERIMENTAL STUDY

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ABSTRACT

Previous work on protoplanetary dust growth shows a halt at centimeter sizes owing to the occurrence of bouncing at velocities of \( \geq 0.1 \text{ m s}^{-1} \) and fragmentation at velocities \( \geq 1 \text{ m s}^{-1} \). To overcome these barriers, spatial concentration of centimeter-sized dust pebbles and subsequent gravitational collapse have been proposed. However, numerical investigations have shown that dust aggregates may undergo fragmentation during the gravitational collapse phase. This fragmentation in turn changes the size distribution of the solids and thus must be taken into account in order to understand the properties of the planetesimals that form. To explore the fate of dust pebbles undergoing fragmenting collisions, we conducted laboratory experiments on dust-aggregate collisions with a focus on establishing a collision model for this stage of planetesimal formation. In our experiments, we analyzed collisions of dust aggregates with masses between 0.7 and 91 g mass ratios between target and projectile from 1 to 126 at a fixed porosity of 65\%, within the velocity range of 1.5–8.7 m s\(^{-1}\), at low atmospheric pressure of \( \sim 10^{-3} \) mbar, and in free-fall conditions. We derived the mass of the largest fragment, the fragment size/mass distribution, and the efficiency of mass transfer as a function of collision velocity and projectile/target aggregate size. Moreover, we give recipes for an easy-to-use fragmentation and mass-transfer model for further use in modeling work. In a companion paper, we use the experimental findings and the derived dust-aggregate collision model to investigate the fate of dust pebbles during gravitational collapse.

Key words: comets: general – methods: laboratory – planets and satellites: formation – protoplanetary disks – techniques: image processing

1. INTRODUCTION

Over the past decade, a significant amount of work on protoplanetary dust growth has been contributed by modelers and experimenters that has significantly advanced our understanding about the formation of planetesimals. In the field of planetesimal formation, broad consensus has been reached on the pre-gravitational dust-growth regime in which micrometer-sized dust grains grow to at least centimeter sizes by sticking collisions in protoplanetary disks. Based upon the first complete laboratory-based dust-aggregate collision model by Güttler et al. (2010), Zsom et al. (2010) showed that dust aggregates experience a bouncing barrier when they reach millimeter sizes, which limits growth and leads to relatively compact pebble-sized dust aggregates with volume filling factors of \( \phi \sim 0.4 \) (i.e., 60\% porosity).

The further growth from pebbles to planetesimals faces severe obstacles by the absence of direct hit-and-stick processes (Güttler et al. 2010), the onset of fragmentation in collisions between dust aggregates of similar size around \( \sim 1 \text{ m s}^{-1} \) (Güttler et al. 2010), and the strong influence of radial drift, which leads to the rapid depletion of boulders of around 1 m in size at 1 au (Weidenschilling 1977). This halt of growth at pebble sizes is in agreement with observations, which show the presence of millimeter- to centimeter-sized dust particles in protoplanetary disks (see Testi et al. 2014 for a review). However, Okuzumi et al. (2012) have shown that under very favorable conditions (submicrometer-sized water-ice particles), direct coagulation into planetesimals is feasible. As we are interested in a more generic formation scenario that is less restricted in terms of grain size, particle material, and location in the protoplanetary disk, we here assume that the growth pathway demonstrated by Okuzumi et al. (2012) is not feasible for micron-sized or warm dust particles.

Two competing models of planetesimal formation in the presence of the above-mentioned obstacles have been developed in the past years. Based upon an extensive body of laboratory work on mass transfer in high-velocity collisions between dust aggregates of dissimilar masses (Wurm et al. 2005a; Teiser & Wurm 2009a; Güttler et al. 2010; Kothe et al. 2010; Teiser et al. 2011; Deckers & Teiser 2014), Windmark et al. (2012a, 2012b) and Garaud et al. (2013) described the direct collisional formation of planetesimals, ignoring particle transport by radial drift. Although mass transfer in the process of fragmentation of the smaller projectile aggregate during an impact into the larger target aggregate has been clearly proven to exist, the formation of planetesimals of kilometer sizes or larger by this process faces severe problems, such as the rather long timescales required (Johansen et al. 2014), the role of counter-acting erosion (Schräpler & Blum 2011), and fragmentation in collisions between similar-sized planetesimals.

A planetesimal-formation model relying on particle concentration and self-gravity has been proposed by Johansen et al. (2007), who showed that the streaming instability, first described by Youdin & Goodman (2005), is capable of concentrating pebble-sized dust aggregates such that planetesimals can directly form by gravitational instability. Since then, this formation scenario has been refined and proven capable of forming planetesimals of up to several 100 km in size from dust aggregates with Stokes numbers in the range of \( St \sim 0.01 \) to \( St \sim 1 \) within the radii 1–10 au (Bai & Stone 2010; Johansen et al. 2012; Carrera et al. 2015). Here, the Stokes number is defined as the ratio between the gas-grain coupling time and the inverse Keplerian frequency (Cuzzi et al. 1993). At 1 au, this range in Stokes numbers corresponds to centimeter- to meter-sized dust aggregates in a minimum mass solar nebula model.
As the formation of dust aggregates at the upper end of the size range faces the above-mentioned drift and fragmentation problems, this planetesimal-formation scenario is likely to operate with pebble-sized rather than boulder-sized dust particles. One of the main issues with previous studies on planetesimal formation via gravitational collapse is the use of inert dust, i.e., dust agglomerates were indestructible. However, numerical simulations predict that they collide with rather high velocities, typically a few m s\(^{-1}\) according to Johansen et al. (2009). At these velocities, aggregates are supposed to fragment, as shown in the model of Güttler et al. (2010).

Additionally, during the gravitational collapse of the pebble clouds, speeds high enough for fragmentation can be reached for planetesimals above a few tens of kilometers in size (Wahlberg Jansson & Johansen 2014). This fragmentation changes the size distribution of the pebbles and thus influences the porosity and packing of the planetesimal that forms.

Nevertheless, Skorov & Blum (2012), Blum et al. (2014), and Blum et al. (2015) have shown that the dust activity of comets as they approach the Sun (as well as the low mass density and thermal conductivity) can only be explained by the gravitational instability scenario of planetesimal formation because of the resulting low tensile strengths of the accreted dust pebbles.

In this paper, we present new experimental work on the collision behavior of centimeter-sized dust aggregates in the velocity range up to 8.7 m s\(^{-1}\) for mass ratios between target and projectile agglomerates of 1–125. These results will be used in the companion paper (Wahlberg Jansson et al. 2016; hereafter Paper II) to simulate how fragmentation affects the gravitational collapse phase and the interior structure of planetesimals that form by gravitational instability.

In Section 2 we introduce our new experimental setup. Section 3 describes the sample preparation and sample properties. In Section 4 the results and analyses of our experiments are presented. Based on these results, in Section 5 we propose a simple empirical model to describe the general outcome in aggregate–aggregate collisions, which will be applied in Paper II. Section 6 briefly describes how we use the new data to better describe the collapse of a pebble cloud. In Section 7 we conclude our work and discuss its astrophysical implications.

2. EXPERIMENTAL SETUP

Regardless of whether the formation of planetesimals occurs by the process of mass transfer or through gravitational collapse of dust pebbles, centimeter-sized dust aggregates that collide with velocities in the range of 1–10 m s\(^{-1}\) play a crucial role. To analyze the collision outcome in this parameter range, we designed a new experimental setup, which is shown in Figure 1.

The heart of the experimental setup is a vacuum glass cylinder (labeled 1 in Figure 1), which has a length of 150 cm and a diameter of 50 cm and is mounted on top of a steel vacuum chamber (2). Inside this chamber, the projectile dust aggregate is placed on a sample holder, which is attached to a pneumatic accelerator (3). The pneumatic accelerator is connected to a pressurized gas bottle filled with nitrogen gas. The target dust aggregate is loaded on top of a double-winged trapdoor release mechanism (4), which is adjusted at the top of the glass cylinder. The trapdoor release system consists of solenoid magnets and eddy-current brakes, designed to release the target aggregate into a rotation-free free-fall. Bright-field illumination is accomplished by an LED panel, and the colliding dust aggregates are imaged by two megapixel high-speed cameras (6) operating at 7500 frames per second. The setup and its operation (with the exception of the pneumatic accelerator) are extensively described in Blum et al. (2014).

Some of the experimental results presented in Section 4 were conducted by using an electromagnetic accelerator, which is also described in Blum et al. (2014) and shown in Figure 2(a). It consists of a sledge (labeled 1 in Figure 2(a)), which is electromagnetically guided over a track of 1040 mm (2). The shaft (3), which operates partially in air and partially in vacuum, is fitted to the sledge, which remains outside the vacuum chamber. With the manufacturer-provided software, we can easily adjust the desired acceleration and reach a final velocity up to 5 m s\(^{-1}\). However, for some applications, the selected parameters (e.g., motor currents for the desired velocity) were not effective when the device was subjected to vacuum. High-vacuum conditions inside the vacuum chamber unintentionally accelerated the shaft, which required an unnecessarily high current for deceleration. This in turn induced undesirable jitter motion and resulted in pre-collision cracks within the projectile aggregate and complete fragmentation when the projectile was less massive. Thus, the success rate of launching an intact...
widely used in laboratory experiments before (The electromagnetic accelerator The Astrophysical Journal, Figure 2 we replaced the electromagnetic by a pneumatic accelerator. simple shaft housed in an aluminium casing pneumatic accelerator does not induce cracks or fragmentation (each holding a projectile (labeled S). The labeled components of the electromagnetic accelerator are (1) sledge, (2) track, (3) shaft, and of the pneumatic accelerator (4) shaft, (5) pressurized gas bottle, and (6) solenoid valve. projectile was too low for an efficient experimentation. Hence, we replaced the electromagnetic by a pneumatic accelerator. Figure 2(b) shows the pneumatic accelerator carrying a simulant aggregate (labeled S). The pneumatic accelerator consists of a simple shaft housed in an aluminium casing (4) and is connected to a pressurized gas bottle (5). Upon opening a solenoid valve (6), the gas bottle delivers a pressure of 4–5 bar to the shaft, sufficient to gently accelerate the projectile aggregate to a final velocity of up to ~5–6 m s⁻¹. Test experiments showed that the pneumatic accelerator does not induce cracks or fragmentation into the fragile dust aggregates.

3. SAMPLE PREPARATION AND SAMPLE PROPERTIES

3.1. Preparation of Centimeter-sized Dust Aggregates

The dust material used in this study is silicon dioxide (SiO₂). According to the manufacturer (Sigma-Aldrich), the dust is 99% pure, consists of irregular grains with a size of 0.5–10 μm (approximately 80% of the grains are between 1 and 5 μm in diameter), and possesses a material density of ρSiO₂ = 2.60 g cm⁻³. This dust-analog material has been widely used in laboratory experiments before (Blum 2006; Beitz et al. 2012; Schräpler et al. 2012; Deckers & Teiser 2013, 2014), so that the results published here can be related to earlier work.

However, as the dust provided by the manufacturer possesses a lumpy structure, it requires some processing before being used for centimeter-sized dust aggregates, because we require the aggregates to be as homogeneous as possible. To remove the lumps, the dust powder was first sifted using an electrically vibrated sieve with 500 μm mesh size. The mass of the sieved dust was carefully measured per desired fill factor and volume and then poured into a respective mold for further compression, which was done manually and hydraulically for aggregates of 5 cm. The resulting aggregates possess a cylindrical shape with lengths equalling diameters, both ranging between 1 and 5 cm. Details of the dust processing have been published in Blum et al. (2014).

3.2. Properties of the Dust Aggregates

The volume filling factor ϕ is defined as the ratio of the overall density of a dust aggregate and the material density of the dust particles. In this study, we fixed the volume filling factor to ϕ = 0.35, because previous work has shown that this is close to the expected value in the bouncing regime (Weidling et al. 2009; Zsom et al. 2010). Since the volume filling factor is one of the critical parameters in defining the collision outcome, it was carefully controlled throughout the dust processing. To investigate whether the compressed dust cylinders were homogeneous with a volume filling factor of ϕ = 0.35 throughout their volume, X-ray tomography (XRT) measurements were performed on selected dust aggregates.

Figure 3 shows reconstructed slices of the XRT analysis of a dust aggregate of 5 cm. The homogeneous gray matrix area possesses a standard deviation over the full length of 5 cm of 5%–6%, which translates into Δϕ = ±0.02 relative to its mean value ϕ = 0.35. However, occasionally also bright spots of dense regions are found, encircled in Figure 3(b), which occur sporadically throughout the cylindrical dust sample. The mean volume filling factor of these dense spots is ϕ ~ 0.57 and reaches up to ϕ = 0.68. However, the fractional volume occupied by the dense spots is <10⁻⁵, so that their influence on the collision behavior of the dust aggregates is negligible.

Figure 4 shows the global volume filling factor profile of an entire dust aggregate of 5 cm, which is very similar to the profile shown in Schräpler et al. (2012). The slightly higher volume filling factor in the first 15 slices (or 0.9 mm from the bottom of the cylinder) is an artifact due to the reflection of X-rays from the aluminum sample holder on which the aggregate was placed. The declining tail, starting from slice ~800 (or 46 mm from the bottom), is assumed to be the reflection from the air-material interface. Between slices 15 and 800, the slice-averaged volume filling factor slightly decreases from ϕ = 0.37 to ϕ = 0.33. This decline is caused by the unidirectional compression of the sample (Beitz et al. 2013).

4. DATA ANALYSIS AND EXPERIMENTAL RESULTS

We performed 142 individual aggregate–aggregate collisions in the following eight series (projectile diameter/height—target diameter/height): 1–1 cm, 1–2 cm, 1–2.6 cm, 1–5 cm, 2–2 cm, 2–5 cm, 3.5–5 cm, and 5–5 cm. Table 1 summarizes all collision parameters investigated here.

The velocity distributions of the individual collision experiments in the eight series listed in Table 1 are shown in Figure 5 in a cumulative way. We can see that the chosen velocities are distributed evenly in the respective ranges given in Table 1. The rather exceptionally high velocities of the 1–5 cm series are caused by the higher fragmentation threshold velocities that the targets possess in collisions with small projectiles. This is further discussed below.

Figure 6 shows two examples of pre-collision and post-collision images taken with one of the high-speed cameras. The
pre-collision images demonstrate the geometry of the collision, which has been set in such a way that the symmetry axis of the projectile was rotated by 90° with respect to the symmetry axis of the target. The target aggregate, which is dropped from the top, thus projects a rectangular shape onto the field of view of the cameras, while the projectile aggregate, shot from the bottom, appears as a circle (see Figure 6(c)). This geometry provides a minimum contact area between the aggregates at first contact and is representative of collisions between spherical aggregates, as shown by Beitz et al. (2011). However, in practice it has been challenging to guarantee these ideal geometrical conditions, first because of the slightly inherent nonalignment along the line joining the centers of release mechanism and accelerator, and second as a result of sometimes unavoidable rotation of the projectile aggregate. For instance, the projectile of 5 cm in Figure 6(a) is slightly tilted by the rotation. Figures 6(b) and (d) show examples of complete fragmentation of projectile and target (annotated CF in Table 1) and mass transfer from the fragmented projectile to the non-fragmented target (annotated FM in Table 1), respectively.

The imperfect alignment between projectile and target also results in not perfectly central collisions, which we describe using a one-dimensional impact parameter. Owing to the limitation of the experimental design, i.e., both cameras observing from the same direction, only one component of the two-dimensional impact parameter is accessible and taken into account. The velocities used in our data analysis have been corrected for impact parameter such that only the normal component of the relative velocity between projectile and target was taken into account. Formally the normal component of the collision velocity, \( v_n \), is derived from the relative collision speed, \( v_{rel} \), by

\[
v_n = v_{rel} \cdot \sin \left( \arctan \left( \frac{b}{r_p + R} \right) \right),
\]

with \( b \), \( r_p \), and \( R \) being the impact parameter, the radius of the projectile, and the radius of the target aggregate, respectively. For our cylindrical aggregates with length \( l \) and diameter \( d \), we obtain \( r_{p,t} = \frac{l_{p,t}}{2} = \frac{d_{p,t}}{2} \), with the indices \( p \) and \( t \) denoting the projectile and target aggregate, respectively.

4.1. Survival of the Target Aggregate and Mass Transfer

Survival of the target aggregate combined with mass transfer from the fragmenting projectile to the intact target was a non-negligible experimental outcome in the 1–2.6 cm, 1–5 cm, and
2–5 cm series, with single events present in the 1–1 cm and 2–2 cm series.

As both the fragmentation of the target and its survival were possible outcomes in the same velocity range, we analyzed the probability for the occurrence of mass transfer and thus for the intact survival of the target more closely by first determining the highest velocity \( v_{\text{sur}} \) for which mass transfer and intact survival of the target were observed. The sixth column of Table 1 shows this velocity. Then, we defined the probability for target survival, \( p_{\text{sur}} \), by the ratio between the number of mass-transfer events and the total number of experiments in the velocity range \( v < v_{\text{sur}} \). The seventh column of Table 1 lists the corresponding results. Plotting these probabilities in Figure 7(a) as a function of the size ratio \( f \) between target and projectile shows that \( p_{\text{sur}} \) steadily increases with increasing \( f \) values.

In the case of the 1–5 cm series, i.e., for \( f = 5 \), the formal mass-transfer probability is \( p_{\text{sur}} = 0.8 \), as shown by the black square in Figure 7(a). However, the collision velocities in this series had been chosen systematically higher than for all the other series (see Figure 5) to achieve fragmentation of the target at all. Thus, \( p_{\text{sur}} = 0.8 \) is most likely a lower limit to the true mass-transfer probability. As an upper limit, we chose \( p_{\text{sur}} = 1.0 \) for the size ratio \( f \geq 5.83 \) (see red square in Figure 7(a)). For \( f = 1 \), the mass-transfer probability is rather low and slightly decreases from \( p_{\text{sur}} = 0.2 \) to \( p_{\text{sur}} = 0 \) when the aggregate size increases from 1 to 5 cm. As the mass-transfer probabilities for the 1–2.6 cm and the 2–5 cm series are very similar, we conclude that these probabilities are merely dependent on the size (or mass) ratio between target and projectile and not on their absolute values. Thus, we approximated the mass-transfer probability by

\[
p_{\text{sur}} = \begin{cases} 
0.19f - 0.13 & \text{for } 1 \leq f \leq 5.83 \text{ and } p_{\text{sur}}(f = 5) = 0.8 \\
1 & \text{for } f > 5.83 \text{ and } p_{\text{sur}}(f = 5) = 0.8 \\
0.24f - 0.20 & \text{for } 1 \leq f \leq 5 \text{ and } p_{\text{sur}}(f = 5) = 1 \\
1 & \text{for } f > 5 \text{ and } p_{\text{sur}}(f = 5) = 1
\end{cases}
\]

(2)

Figure 7. Normalized cumulative velocity distributions of the eight experimental series listed in Table 1. The mean collision velocity in this study is \( v_{\text{coll}} = 4.51 \text{ m s}^{-1} \).

For the three experiment series for which \( f \geq 2.5 \) and mass transfer was a common outcome (1–2.6 cm, 1–5 cm, and 2–5 cm, see Figure 9 below), we can also state that mass transfer always occurs down to the lowest investigated collision velocities, but it possesses an upper velocity limit in the cases of 1–2.6 cm and 2–5 cm. Thus, for \( v > v_{\text{sur}} \), fragmentation is the only outcome. However, for 1–5 cm, there is no such upper limit, although in this series we extended the investigated velocity range up to 8.7 m s\(^{-1}\). Moreover, for velocities \( v < 5.6 \text{ m s}^{-1} \), mass transfer is the only outcome. The sixth column in Table 1 summarizes our findings for \( v_{\text{sur}} \) and Figure 7(b) shows the data as a function of \( f \). We can recognize that \( v_{\text{sur}} \) increases with increasing target-to-projectile size ratio so that dust-evolution models need to take mass transfer into account.

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**Table 1**

| No. | Projectile Size–Target Size | No. of Collisions | Velocity Range (m s\(^{-1}\)) | Outcome | \( v_{\text{sur}} \) (m s\(^{-1}\)) | \( p_{\text{sur}} \) |
|-----|---------------------------|------------------|-------------------------------|---------|-------------------------------|--------------|
| 1   | 1.0–1.0 cm                | 13               | 2.1–3.0                        | CF+FM*  | 3.1                           | 0.2 ± 0.14   |
| 2   | 2.0–2.0 cm                | 18               | 2.0–6.0                        | CF+FM*  | 2.6                           | 0.14 ± 0.14  |
| 3   | 5.0–5.0 cm                | 21               | 2.0–6.2                        | CF       | <2                            | 0            |
| 4   | 1.0–2.0 cm                | 10               | 2.2–7.7                        | CF       | <2.1                          | 0            |
| 5   | 1.0–2.6 cm                | 19               | 2.5–5.7                        | CF+FM    | 4.5                           | 0.53 ± 0.19  |
| 6   | 1.0–5.0 cm                | 21               | 3.6–8.7                        | CF+FM    | >8.4                          | 0.80 ± 0.20  |
| 7   | 2.0–5.0 cm                | 24               | 1.6–7.1                        | CF+FM    | 4.6                           | 0.52 ± 0.17  |
| 8   | 3.5–5.0 cm                | 16               | 1.5–4.4                        | CF       | <1.5                          | 0            |

**Note.** Projectile and target aggregate possess a cylindrical shape of equal height and diameter and a fixed volume filling factor of \( \phi = 0.35 \). CF stands for catastrophic fragmentation, in which the target and projectile aggregates both fragment. FM stands for fragmentation with mass transfer, in which only the projectile fragments and transfers part of its mass to the intact target. \( p_{\text{sur}} \) is the probability with which the target survives the impact for collision velocities \( v < v_{\text{sur}} \) and \( v_{\text{sur}} \) is the highest velocity for which the target survives the impact velocity. The asterisks indicate that mass transfer was observed in only a few events, see Figure 9 below.
account, particularly in cases when small projectiles hit large targets. These latter cases have been extensively studied in previous works (Wurm et al. 2005a; Teiser & Wurm 2009a; Güttler et al. 2010; Kothe et al. 2010; Teiser et al. 2011; Deckers & Teiser 2014).

The target survival velocity $v_{\text{sur}}$ can then be compared with the critical fragmentation velocity $v_{0.5}$ (which we derived in Section 4.4 below), for which the mass of the largest fragment of the target equals half the initial target mass. In Figure 7(b) we also show $v_{0.5}$ for comparison. As can be seen, both velocities are very similar in those series in which mass transfer is a regular outcome (1–2.6 cm, 1–5 cm, and 2–5 cm), so that we can conclude that mass transfer is only possible (and will occur with a probability $p_{\text{sur}}$ as shown above) as long as the projectile is unable to destroy the target (for more details, we refer to Sections 4.3 and 4.4).

The typical mass-transfer efficiencies, here defined by $\Delta m/m_p$, with $\Delta m$ being the mass transferred from projectile to target, varied from $\Delta m/m_p \approx 0.08$ to $\Delta m/m_p \approx 0.30$ and are in the same range as was previously found in collisions among centimeter-sized spherical dust agglomerates (Beitz et al. 2011, their Figure 8) or in collisions with varying mass ratios (Wurm et al. 2005b, their Figure 6; Kothe et al. 2010, their Figure 5; Deckers & Teiser 2014, their Figure 8).

One novelty of our experimental data over previous work is that we are now able to derive the dependency of the mass-transfer efficiency on velocity, projectile size, and target size. Earlier work either used equal-sized aggregates or studied impacts of dust aggregates on semi-infinite targets. The events of mass transfer have been observed in five series, providing 37 data points on the relative mass transfer $\Delta m/m_p$ (shown in Figure 8(a)), which we analyzed according to their (assumed power-law) dependency on velocity $v_a$ and projectile/target sizes $P$ and $T$, respectively, i.e.,

$$\log \left( \frac{\Delta m}{m_p} \right) = C_{\text{int}} + \sigma \log \left( \frac{v_a}{1 \text{ m s}^{-1}} \right) + \zeta \log \left( \frac{P}{1 \text{ cm}} \right) + \Gamma \log \left( \frac{T}{1 \text{ cm}} \right).$$

In order to derive the coefficients $C_{\text{int}}$, $\sigma$, $\zeta$, and $\Gamma$ in the above equation, we minimized the reduced chi-squared value $\chi^2_{\text{red}}$ for two cases. In the first case, all 37 events of mass-transfer were taken into account. In the second case, the three

Figure 6. Examples of pre-collision and post-collision images. (a) and (b): Projectile and target aggregates of 5 cm each, colliding at 6.2 m s$^{-1}$ and resulting in catastrophic fragmentation of both aggregates. (c) and (d): A projectile aggregate of 2 cm, colliding with a target aggregate of 5 cm at 2.8 m s$^{-1}$, resulting in the fragmentation of the projectile alone. Part of the projectile mass has been visibly transferred to the target, which typically has a cone shape (highlighted by the circle in (d)).
Figure 7. (a) Probability \( P_{\text{sur}} \) for the target aggregate to survive the impact of a projectile aggregate in the velocity range \( v < v_{\text{sur}} \), with \( P_{\text{sur}} \) and \( v_{\text{sur}} \) shown in Table 1, as a function of the size ratio of target and projectile, \( f \). The red solid line is a linear fit to the data, including the black square at \( f = 5 \); the red dashed line is the same for the red square (see text). (b) Highest collision velocity for which the target stayed intact, \( v_{\text{sur}} \), as a function of the target-to-projectile size ratio, \( f \) (black filled squares). In addition, we also show the critical fragmentation velocity of the target, \( v_{0.5} \) (blue open squares). In both figures the aggregates of similar \( f \) tend to have similar values.

Events from the series with \( f = 1 \) were dropped because of their low mass-transfer efficiency (see Figure 7(a)), so that only 34 events from the series with \( f > 1 \) were considered. The respective values of the coefficients and \( \chi_{\text{red}}^2 \) in both cases are given in Table 2, where we can see that the restriction to \( f > 1 \) significantly reduces the \( \chi_{\text{red}}^2 \) value. The resulting correlations and fits in both cases are shown in Figure 8.

When we remove the 3 events from the series where \( f = 1 \) and fit the remaining 34 data points to Equation (3), we obtain the coefficients shown in the second row of Table 2. It can be seen that \( \chi_{\text{red}}^2 \) is reduced by a factor \( \sim5 \), while the velocity dependence becomes significant. Surprisingly, the previous strong dependence on \( T \) is now negligible, suggesting no role of the target in the case of \( f > 1 \). As the error of the exponent of \( P \) is almost twice as large as the exponent itself, we also argue that the \( P \) dependence of the mass-transfer is negligible. Therefore we neglect the coefficients of \( P \) and \( T \) by setting \( \zeta = 0 \) and \( \Gamma = 0 \), which leads to Equation (4). The resulting values are shown in the third row of Table 2. We can see that the omission of \( P \) and \( T \) further strengthens the dependence on velocity and even slightly reduces the value of \( \chi_{\text{red}}^2 \). Figure 8(b) is the graphical representation of mass-transfer for size ratios \( f > 1 \) as a function of velocity alone. Thus, we rewrite Equation (3), such that

\[
\log \left( \frac{\Delta m}{m_p} \right) = C_{\text{int}} + \sigma \log \left( \frac{v_t}{1 \text{ m s}^{-1}} \right)
\]

and obtain \( C_{\text{int}} = -1.42 \pm 0.07 \) and \( \sigma = 0.91 \pm 0.11 \), respectively, with \( \chi_{\text{red}}^2 = 0.021 \). Obviously, if the aggregates are intrinsically different in size, i.e., \( f > 1 \), the dependence of mass-transfer on the size of individual aggregates becomes negligible and only the impact velocity \( v_t \) is the primary factor on which mass-transfer depends.

Formally, Equation (4) breaks down when \( \Delta m/m_p > 1 \), i.e., for \( v > 10^{-C_{\text{int}}/\sigma} = 36.34 \text{ m s}^{-1} \). However, at these high impact velocities, other processes like cratering are important, but these are not the subject of this study. We should therefore be careful to use extrapolations of Equation (4) to too high velocities.

4.2. Fragmentation Strength \( \mu \)

For each collision, we measured the fragmentation strength \( \mu \), which we define as the mass ratio of the largest fragment \( m_1 \) observed after the collision to the initial target mass \( m_t \), i.e.,

\[
\mu = \frac{m_1}{m_t}.
\]

With this definition, we can use \( \mu \) to distinguish between different collisional outcomes, i.e.,

\[
\begin{cases} 
\mu > 1 & \text{mass transfer from projectile to target} \\
\mu = 1 & \text{bouncing} \\
\mu < 1 & \text{fragmentation of projectile and target}
\end{cases}
\]

Contrary to many previous studies in which \( \mu \) has been investigated as a function of impact velocity, here we analyze it as a function of the kinetic energy \( E_{\text{cm}} \) in the center-of-mass system of projectile and target, i.e.,

\[
E_{\text{cm}} = \frac{1}{2} m_1 v_1^2,
\]

where \( m \) is the reduced mass of the projectile-target system, given by \( m^{-1} = m_p^{-1} + m_t^{-1} \), with \( m_p \) and \( m_t \) being the projectile and target mass, respectively. We use the kinetic energy, because we later intend to derive the collision strength \( Q^* \) (see Section 4.4), and the reduced mass, because only this value has a contribution to the mass loss (the remainder of the kinetic energy refers to the motion of the center of mass). Figure 9 compiles the results for the fragmentation strength of all eight collision series listed in Table 1.

4.3. The Catastrophic Threshold Energy \( E_{0.5} \)

In previous studies, power-law dependencies between the fragmentation strength and the collision energy have frequently...
The catastrophic threshold energy varies systematically between $E_{0.5} \sim 10$ mJ for the smaller projectiles/targets and $E_{0.5} \sim 117$ mJ for the larger projectiles/targets. This is analyzed in more detail in Section 4.4.

### 4.4. The Collision Strength $Q^*$

If the catastrophic threshold energy $E_{0.5}$ is known, the collision strength $Q^*$, which we define through

$$Q^* = \frac{E_{0.5}}{m_t},$$

can be calculated. Here, we use the target mass for normalization rather than the total mass of the system, $m_t + m_p$, as used by Stewart & Leinhardt (2009) and Beitz et al. (2011).

The reasons for doing this are (1) that in the previous studies the variation in mass ratio between projectile and target was not so extreme, but here it varies by more than two orders of magnitude, and (2) that the largest fragment always stems from the target (in the case of equal-mass dust aggregates, the target is the one that delivers the largest fragment). As we are interested in systematically following the fate of the more massive of the colliding dust aggregates, we normalize the

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**Table 2**

| Data Points | $C_{cm}$  | $\sigma$ (exponent of $v_p$) | $\zeta$ (exponent of $P$) | $\Gamma$ (exponent of $T$) | $\chi^2_{red}$ |
|-------------|-----------|-----------------------------|---------------------------|---------------------------|---------------|
| 37          | $-2.30 \pm 0.24$ | 0.52 $\pm 0.44$ | $-0.72 \pm 0.69$ | $1.82 \pm 0.40$ | 0.111 |
| 34          | $-1.37 \pm 0.14$ | 0.81 $\pm 0.23$ | $-0.22 \pm 0.39$ | $0.05 \pm 0.32$ | 0.022 |
| 34          | $-1.42 \pm 0.07$ | 0.91 $\pm 0.11$ | 0 | 0 | 0.021 |

**Note.** The first row shows the results when all 37 data points are taken into account. The second row gives the results for 34 events of mass-transfer in the series with $f > 1$ (see text for details). The third row shows the fit values when projectile and target size are neglected for the same data set.

been used to determine the catastrophic threshold energy $E_{0.5}$ for which $\mu = 0.5$ (see, e.g., Ryan et al. 1991). Here, we rather use a Hill function (Hill 1910), given by

$$\mu(E_{cm}) = 1 - \frac{E_{cm}^n}{E_{0.5}^n + E_{cm}^n} = \frac{E_{0.5}^n}{E_{0.5}^n + E_{cm}^n},$$

with the exponent $n$ as a free parameter to describe the dependency of the fragmentation strength (i.e., the relative mass of the largest fragment; see Section 4.2) on impact energy. The functional form of Equation (8) has the advantage of approaching the natural limit of $\mu \rightarrow 1$ (i.e., bouncing) for low impact energies, $E_{cm} \ll E_{0.5}$, and a power law $\mu \propto E_{cm}^{-n}$ for high energies, $E_{cm} \gg E_{0.5}$, and $n > 0$.

Table 3 lists the results for $E_{0.5}$ and $n$; the data on the fragmentation strength and corresponding fit curves are shown in Figure 9. Owing to the small number of data points in the 1–5 cm series, $E_{0.5}$ was estimated to be $E_{0.5} \approx 19$ mJ.

However, from Table 3, it can be seen that the exponent $n$ is constrained to values between $n \sim 0.2$ and $n \sim 0.8$ without an obvious dependence on the projectile/target size. Thus, we determined the weighted average of the exponents shown in Table 3 and obtained $\bar{n} = 0.55 \pm 0.11$. We note that all values for $n$ in Table 3 are within their individual errors consistent with $n = 0.55$. For this fixed exponent, the resulting energy values $E_{0.5}(n = 0.55)$ are also shown in Table 3 and are used below.

![Figure 8.](image-url) (a) Dependence of the mass-transfer efficiency $\Delta m/m_p$ in all 37 events as a function of a power-law combination of the collision parameters, i.e., $v_p^n P^{-T^\zeta}$, with the respective coefficients given in first row of Table 2. Since the mass gain was determined by measuring the projected area of the transferred mass (as encircled in Figure 6), we estimate the mean error as being a factor 2 in mass, which is shown by the red error bar at top left. (b) Removing the three data points with $f = 1$ reduces $\chi^2_{red}$ considerably and shows that the dependence on $P$ and $T$ vanishes (see second row of Table 2). Thus, we refitted the 34 remaining data points with a power-law dependence on velocity alone (see third row of Table 2).
catastrophic fragmentation energy to the target mass. Another advantage of doing this is the comparison with different projectile sizes for a constant target size, which can be interpreted as the fragmentation efficiency of the projectile as a function of target mass. The higher $Q^*$, the lower the fragmentation efficiency of the projectile. The absolute values of $Q^*$ are given in Table 3.

Figure 10 shows $Q^*$ as a function of the projectile size $P$ for a fixed target size of $T = 5$ cm (Figure 10(a)) as well as a function of the target size $T$ for a fixed projectile size of $P = 1$ cm (Figure 10(b)). The $Q^*$ value of the 2–2 cm series is not shown, as it belongs neither to the fixed projectile ($P = 1$ cm) nor to the fixed target ($T = 5$ cm) parameter space. However, this series is included in the collective analysis (Figure 11). As can be seen in Figure 10, the collision strength roughly follows a power law of the projectile size with an exponent of $\sim 1.12 \pm 0.25$ and a power law of the target size with an exponent of $\sim 2.92 \pm 0.57$. Varying the projectile size by a factor of 5 changes the value of $Q^*$ by a factor of $\sim 6$, whereas a variation in the target size by the same factor 5 changes $Q^*$ by a factor of $\sim 100$. The collision strength of a target for a given projectile size thus becomes considerably weaker for increasing target sizes.

Although the data analysis as shown in Figure 10 is very intuitive, we favor the simultaneous correlation of $Q^*$ with $P$ and $T$, which also has the advantage that all eight available $Q^*$ data points can be used. Thus, we used the ansatz

$$\log \left( \frac{Q^*(P, T)}{1 \text{ J kg}^{-1}} \right) = C_Q + \kappa \log \left( \frac{P}{1 \text{ cm}} \right) + \lambda \log \left( \frac{T}{1 \text{ cm}} \right).$$

(10)

Least-squares fitting of all eight data points of $Q^*$ to the respective $(P, T)$ pairs delivers the coefficients $C_Q = 1.24 \pm 0.16$, $\kappa = 1.12 \pm 0.35$, and $\lambda = -2.70 \pm 0.37$, which slightly differs from the previously independently determined values, but is within the standard errors. The data and fit are shown in Figure 11. We here recall that the 2–2 cm series is included. However, the values of $\kappa$ and $\lambda$ did not vary significantly when $Q^*$ was separately analyzed as a function of $P$ and $T$. Figure 11 shows the goodness of the fit, with $\log \left( \frac{Q^*(P, T)}{1 \text{ J kg}^{-1}} \right)$ as y-values and $\log \left( \left( \frac{P}{1 \text{ cm}} \right)^{\kappa} \cdot \left( \frac{T}{1 \text{ cm}} \right)^{\lambda} \right)$ as x-values.

Our results can be applied to the calculation of the catastrophic fragmentation velocity (for which $\mu = 0.5$) when we recall (see Equation (9)) that $Q^* = E_{0.5}/m_1 = \frac{1}{2} m_p m_2 v_{0.5}^2/m_1 \approx \frac{1}{2} m_p v_{0.5}^2/m_1 \propto v_{0.5} P^2/T^3$, so that we obtain, using Equation (10),

$$v_{0.5} \propto \left( \frac{Q^* \cdot T^3}{P^3} \right)^{1/2} \propto P_{-0.5} T_{0.15}^2 \propto P_{-0.94} T_{0.15}^{0.15},$$

(11)

For equal-sized dust aggregates, i.e., $P = T$, we obtain

$$v_{0.5} \propto T_{-0.79} T_{-0.26}^2 \propto P_{-0.26}.$$ 

(12)

This is a rather weak dependence of the catastrophic fragmentation velocity on aggregate mass for equal-sized collision partners.

As an alternative data analysis of the dependence of the fragmentation strength on impact velocity and projectile/target size, we assume a power-law relationship among these values (and thus circumvent the collision strength) of the following form

$$\log \mu = C_\mu + \epsilon \log \left( \frac{v_n}{1 \text{ m s}^{-1}} \right) + \omega \log \left( \frac{P}{1 \text{ cm}} \right) + \Omega \log \left( \frac{T}{1 \text{ cm}} \right).$$

(13)

Minimizing the squared deviations between measured $\mu$ values and those calculated, we derive a reasonable fit to Equation (13) (shown in Figure 12) with $C_\mu = 0.18 \pm 0.07$, $\epsilon = -0.66 \pm 0.12$, $\omega = -0.58 \pm 0.10$, and $\Omega = 0.13 \pm 0.11$, respectively, which is only valid as long as $\log \mu \leq 0$. This can be applied to derive the general expression for the onset-velocity for fragmentation, for which $\mu = 1$, i.e.,

$$v_1 \propto (P^\omega \cdot T^{0.7})^{-1/\omega} = P^{-0.88} \cdot T^{0.20},$$

(14)

and for $T = P$ we obtain

$$v_1 \propto T^{-0.68} \propto m_1^{-0.23},$$

(15)

As these results also hold for $\mu = 0.5$ and thus for $v_{0.5}$, this is comparable with the results shown in Equation (12). It should, however, be mentioned that a power-law description of $\mu$ as shown in Equation (13) does not adequately describe the asymptotic behavior of the largest fragment mass for low velocities, $\mu \rightarrow 1$. As can be seen in Figure 12, the high-velocity behavior of $\mu$ is not well represented by Equation (13).

This can also be seen by comparing the velocity dependence $\mu \propto v_{0.5}^{-0.66}$ in Equation (13) with that of the Hill function (Equation (8)). Using $E_{cm} \propto v_n^2$, we obtain for the asymptotic velocity behavior of the Hill function (Equation (8)) $\mu \propto v_n^{2.1} = v_{0.5}^{-1.1}$. Thus, the velocity dependence of the largest fragment is probably better described by Equation (8).

It should be mentioned that Equation (10) can be rewritten for $P = T$ as $\left( \frac{Q^*}{1 \text{ J kg}^{-1}} \right) = 10^{C_Q} \left( \frac{T}{1 \text{ cm}} \right)^{-\kappa} = 17.38 \left( \frac{T}{1 \text{ cm}} \right)^{-1.58}$. This is about one order of magnitude higher than the data given by Beitz et al. (2011), but with a comparable slope, which Beitz et al. (2011) give as $-0.95 \pm 0.38$. However, our new results (see Figure 11 and Equation (10)) indicate that $Q^*$ independently depends on both the projectile and the target size. As far as we know, this aspect has not been described before and has thus not been considered so far in collisional evolution models. As shown in Equation (11), the catastrophic threshold velocity scales with projectile and target size as $v_{0.5} \propto P_{-0.26} \cdot T_{0.15}^{0.15} \propto P_{-0.94} \cdot T_{0.15}^{0.15}$. This means that smaller projectiles require higher impact velocities to achieve the same collisional result with the same target. On the other hand, for a given projectile size, larger targets require higher impact speeds according to $v_{0.5} \propto T_{0.15}^{0.15}$, but with a much shallower size dependence. As a result, in Figure 7(b) we see that the series where $f = 1$ tends to have a lower catastrophic threshold velocity than that of the series where $f > 1$. However, as far as the relative strength $Q^*$ is concerned, the larger aggregates (of the same filling factor) are intrinsically weaker.

4.5. The Fragment Size Distribution

A physical model for the fragmentation in aggregate–aggregate collisions requires more than the knowledge of the mass of the largest fragment or the catastrophic fragmentation
Figure 9. The fragmentation strength $\mu$ as a function of center-of-mass kinetic energy. The dotted horizontal line at $\mu = 1$ is the line of bouncing that separates the region of mass transfer ($\mu > 1$, represented by the filled data points) from the region of fragmentation ($\mu < 1$, represented by the open data points). We note the break in scale between $\mu < 1$ and $\mu > 1$. The curves in the $\mu < 1$ region follow Equation (8) and were fitted to the $\mu < 1$ data points alone. Owing to the small number of data points with $\mu < 1$ in the 1–5 cm series, Equation (8) could not be fitted to this data set.
energy. Thus, we also measured the fragment size distribution in all collisions.

Fragment size distributions of colliding dust aggregates have previously been observed to be composed of two parts, with a high count of smaller fragments following a power law in size-frequency distribution and fewer counts of the largest ones (see, e.g., Blum & Münch 1993; Deckers & Teiser 2014). Technically, the largest fragment in such collisions is the remnant of the original target aggregate. Deckers & Teiser (2014) showed that the mass fraction of the largest fragment decreases with increasing impact energy, in agreement with our findings (see Figure 9) and discussions in Section 4.2.

In previous experiments, the factors influencing the fragment size distribution have not been fully revealed owing to technical limitations. Thanks to the high frame rate of 7500 frames per second of our high-speed cameras (i.e., a temporal resolution of $\sim 130$ ms), however, we were able to trace back the trajectories of the distinguishable fragments and analyze the time evolution of the fragmentation process. In order to count the fragments, a time series of frames was generated with the trajectories of the distinguishable fragments and analyze the cumulative number of fragments with area $x$, $N_{\text{cum}}(x)$, and the cross-sectional area $x$ of the form

$$N_{\text{cum}}(x) = \sum_{x' = 1}^{x_{\text{max}}} N(x') = C_N x^{-\alpha}. \quad (16)$$

Here, $C_N$ is a normalization constant, the bin width of the summation is $\Delta x = 1$ pixel, and the summation starts with the largest discernible fragment of the continuous area-frequency distribution function, $x_{\text{max}}$, in each image sequence. The latter is not necessarily equal to the projected area of the largest fragment (see Section 4.2), particularly in the cases where $\mu \approx 1$.

To demonstrate our analysis, we randomly selected an experiment, named 3Jun-6, from the 3.5–5 cm series. Out of the images of this experiment, six non-overlapping equal time intervals of 50 frames were selected, covering a total time of 40 ms after the collision. We applied a power law of the form presented in Equation (16) to the area range depicted in Figure 13(a) with the rectangular box, in order to avoid small-number effects with the largest fragments. Here the curve of black squares T1 and the curve of green triangles T6 represent the first and the last time interval, respectively. For reference, the complete cross-section data of the 3Jun-6 experiment (comprising all data from the six sequences T1-T6) are also plotted, here represented by red vertical dashes. As the time after the collision elapses from T1 through T6, the cumulative count of each fragment bin increases, which shifts each subsequent curve upward. At the same time, the larger fragments become separated from the power-law tail of small fragments and become countable, so that the curves shift rightward to higher cross sections. However, the slope $\alpha$ (see

| No. | Projectile Size–Target Size | $E_{0.8}$ (mJ) | $n$ in Equation (8) | $E_{0.8}$ (n = 0.55) (mJ) | $Q^*$ (J kg$^{-1}$) |
|-----|-----------------------------|----------------|---------------------|---------------------------|------------------|
| 1   | 1.0–1.0 cm                  | 9.20 ± 7.70    | 0.69 ± 0.36         | 13.58 ± 5.57              | 18.61 ± 7.63     |
| 2   | 2.0–2.0 cm                  | 22.52 ± 7.84   | 0.70 ± 0.35         | 24.82 ± 9.10              | 4.28 ± 1.57      |
| 3   | 0.5–0.5 cm                  | 109.43 ± 50.21 | 0.60 ± 0.34         | 104.10 ± 34.06            | 1.14 ± 0.37      |
| 4   | 1.0–0.5 cm                  | 58.24 ± 109.63 | 0.38 ± 0.31         | 30.42 ± 11.08             | 5.24 ± 1.91      |
| 5   | 1.0–0.26 cm                 | 15.27 ± 37.32  | 0.21 ± 0.47         | 8.43 ± 3.02               | 0.67 ± 0.24      |
| 6   | 1.0–5.0 cm                  | ~19.0          | ~19.0               | ~19.0                     | ~0.21            |
| 7   | 2.0–5.0 cm                  | 53.42 ± 14.97  | 0.80 ± 0.30         | 59.40 ± 21.65             | 0.65 ± 0.29      |
| 8   | 3.5–5.0 cm                  | 117.29 ± 43.88 | 0.52 ± 0.23         | 114.24 ± 33.20            | 1.26 ± 0.36      |

Note. The data shown in Figure 9 were fitted with the function shown in Equation (8) and deliver the catastrophic threshold energy $E_{0.8}$ and the exponent $n$. We note that owing to the small number of data points, the catastrophic threshold energy for the 1.0–5.0 cm series was estimated. $E_{0.8}(n = 0.55)$ is the catastrophic threshold energy for a fixed exponent of $n = 0.55$. The collision strength $Q^*$ is the catastrophic threshold energy (for $n = 0.55$) per target mass, which inherits its error from the error in $E_{0.8}(n = 0.55)$.
Equation (16) does not change much over time, as Figure 13(b) demonstrates. With the exception of T1, the slopes remain within a narrow interval of $\hat{\alpha} = \pm 0.897 \pm 0.0086$, determined by averaging the five $\alpha$ values for the intervals T2-T6. This mean slope corresponds very well to the reference slope $\alpha = 0.907$ of 3Jun-6 for the full time interval. From this analysis, we conclude that the slope of the power-law part of the cumulative area-frequency distribution of fragments can be derived from the full interval of images. In other words, the slope of the fragment area distribution, observed at any instance after the collision, remains almost the same.

In Figure 14 we show all 16 size-frequency distributions in the 3.5-5 cm series derived for full time intervals of $\sim 50$ ms and the assumptions mentioned on page 28. The numbers next to the symbols in the legend indicate the collision energy in units of mJ.

We have seen above that the cumulative area-frequency distribution of the fragments is well represented by a power law for small fragments (see Equation (16)), with a rather sharp cutoff at the high-mass end. In order to mathematically describe the cumulative area-frequency distribution for the full range of fragment sizes, we used an exponential cutoff of the form

$$N_{\text{cum}}(x) = \sum_{x=x_{\text{min}}}^{x=x_{\text{max}}} N(x') = C_N x^{-\alpha} e^{-\left(\frac{x}{x_i}\right)^\nu}.$$  

As the area of the fragments increases, $N_{\text{cum}}(x)$ declines with a slope $-\alpha$, up to about the critical fragment area $x \approx x_i$, the knee of the distribution, which indicates the end of the continuous regime of the power law. Above $x \approx x_i$, the cumulative count drops exponentially with an exponent $-\left(\frac{x}{x_i}\right)^\nu$. Fitting the experimental data to Equation (17) using the four fit parameters $C_N$, $\alpha$, $x_i$, and $\nu$, respectively, shows that $\nu \approx 2$ for all data sets. The slope varies between $\alpha \approx 0.2$ and $\alpha \approx 2$, and the critical fragment area ranges between $x_i \approx 50$ pixels and $x_i \approx 10,000$ pixels. In Appendix A we show all 142
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respectively, with $\chi^2_{\text{red}} = 0.066$. Obviously, the dependence on $P$ is very weak and statistically not significant. A variation of $P$ by a factor 5 results in a maximum deviation of $\sim0.01$ in $\alpha$, which is very small compared to the range of slopes. Here we also like to show that if only the collision velocity is considered, then it increases the $\chi^2_{\text{red}}$. For this we set $\epsilon$ and $\psi$ equal to zero in Equation (18), and we obtain

$$\alpha = C_\alpha + \delta \log \left( \frac{v_n}{1 \text{ m s}^{-1}} \right),$$  

(19)

which yields $C_\alpha = 0.30 \pm 0.07$ and $\delta = 1.06 \pm 0.12$, with $\chi^2_{\text{red}} = 0.071$, slightly higher than when using Equation (18).

In Figure 15(a) we show the best-fit $\alpha$ values of all 141 impact experiments as a function of collision velocity and target size according to Equation (19), whereas the dependence of $\alpha$ on velocity according to Equation (19), is shown Figure 15(b) (one outlier data point from the 1–1 cm series was not used in either fit; this data point is circled).

In addition to $\alpha$, we also analyzed the critical fragment size, that is, the cutoff size of the area-frequency distribution function $x_i$. This is the size of $x_i$ that determines the boundary between the continuous and the discrete size distribution. Therefore we are interested in seeing its dependence on collision parameters. We realize that in the case of mass transfer (37 cases in which the target stayed intact and only the projectile had fragmented), $x_i$ exclusively belongs to the projectile, therefore we eliminate 37 events of mass-transfer and consider the remaining 105 events of complete fragmentation of projectile and target. We fitted the $x_i$ data to a combined power law

$$\log x_i = C_x + \theta \log \left( \frac{v_n}{1 \text{ m s}^{-1}} \right)$$

$$+ \eta \log \left( \frac{P}{1 \text{ cm}} \right) + \tau \log \left( \frac{T}{1 \text{ cm}} \right),$$

(20)

and obtained $C_x = 3.03 \pm 0.11$, $\theta = -0.83 \pm 0.19$, $\eta = 1.44 \pm 0.17$, and $\tau = -0.05 \pm 0.17$, respectively, with $\chi^2_{\text{red}} = 0.087$. As the dependence on target size turned out to
We obtained new fit values $C_x = 3.02 \pm 0.11$, $\theta = -0.85 \pm 0.17$, and $\eta = 1.40 \pm 0.11$, with $\chi^2_{\text{red}} = 0.085$, which are slightly better than before. Figure 16 shows the dependence of $x_i$ on $v_n$ and $P$. Contrary to the results for the slope of the area-frequency distribution function, the largest fragment area of the continuous distribution is strongly dependent on the projectile size, but not at all on the target size. We recall that $x_i$ is the cross section of the cutoff fragment and $P$ is the diameter of the projectile. Equation (20) shows that the size of the cutoff fragment almost linearly scales with the projectile size. Thus, the cutoff size of the continuous area-frequency distribution function is dominated by the contribution of the fragmenting projectile, which is also seen by the vanishing dependence on the target size. Figure 16 demonstrates the excellent correlation of $x_i$ with $v_n^2 P^\beta$. As the collision velocity increases, the size of the cutoff area decreases, as expected.

4.6. Fragment Mass Distribution

In the next step, we analyzed the mass-frequency distribution of the fragments. This is required for the implementation of the model, which is discussed in Paper II. Here, we derive a normalized distribution, required for the modeling (details in Section 5). In a first step, we derive the mass $m_i(x)$ of an individual fragment with a measured cross-sectional area $x$, assuming spherical aggregate-fragments with a mass density $\rho_{\text{agg}}$ (see Section 3). Using this approach, we obtain $m_i(x) = 5.46 \times 10^{-3}$ $\text{cm}^3$, $\rho_{\text{agg}} = 0.91$ $\text{g cm}^{-3}$, and $x$ in pixels and $m_i$ in grams. Assuming a simple power law for the cumulative area-frequency distribution, as shown in Equation (16), we can derive for the cumulative mass distribution function

$$M_{\text{cum}}(m_i) = \begin{cases} M_{\text{tot}} \left(1 - \frac{m_i}{m_{\text{max}}} \right)^\beta & \text{for } \beta > 0, \\ M_{\text{tot}} \left( \frac{m_{\text{max}}}{m_0} \right)^\beta \left( \frac{m_i}{m_{\text{max}}} \right)^\beta & \text{for } \beta < 0, \end{cases}$$

with $\beta = 1 - \frac{2\alpha}{3}$ and $m_{\text{max}}$ being the maximum fragment mass of the continuous mass-frequency distribution function. The functional conversion of cross section into mass for the truncated power law as shown in Equation (17) will be presented in Paper II.

As $\alpha < 3/2$ for the majority of the cases considered here (see Figure 15), and thus $\beta > 0$, $M_{\text{cum}}(m)$ asymptotically approaches a constant total mass $M_{\text{tot}}$ for $m \to 0$ with no need to insert a smallest fragment mass. Thus, Equation (22) is a good fit to the fragment mass distribution function as long as the deviations from the power law (compare

Figure 15. (a) Slope of the area-frequency distribution function, $\alpha_i$, introduced in Equation (17), as a function of collision parameters. The blue solid line shows the fit to the data following Equation (18) with $\delta = 1.02$, $\epsilon = -0.02$ and $\psi = 0.36$. (b) Same as (a), but assuming that $\alpha$ depends only on the collision velocity. The blue line shows the fit to Equation (19) with $\delta = 1.06$. In both cases, one data point (encircled) from the series 1–1 cm has been ignored during the fitting process.

Figure 16. Correlation between the cutoff area $x_i$ in the cumulative area-frequency distribution function as introduced in to Equation (17) and $v_n^2 P^\beta$, with $\theta = -0.85$ and $\eta = 1.40$. 

\[ \log x_i = C_x + \theta \log \left( \frac{v_n}{1 \text{ m s}^{-1}} \right) + \eta \log \left( \frac{P}{1 \text{ cm}} \right). \] (21)
Equations (16) and (17)) are small. In those cases for which 
\( \alpha > 3/2 \), particularly for the highest experimental velocities and for reasons of extrapolating to even higher impact 
energies, Equation (22) is still valid, but \( M_{\text{cum}}(m_t) \to \infty \) for \( m_t \to 0 \), so that a reasonable smallest fragment mass, \( m_0 \), 
needs to be inserted to keep the total mass finite. Thus, for \( \beta < 0 \), the fragment mass must be constrained to 
\( m_0 \leq m_t \leq m_{\text{max}} \).

To determine \( m_{\text{max}} \) in Equation (22) and correlate it with \( x_i \) 
in Equation (17), we fitted Equation (22) to the cumulative 
mass-frequency distribution of all experiments, which we 
derived from the corresponding cumulative area-frequency 
distributions, with only \( m_{\text{max}} \) and \( M_{\text{tot}} \) as fit parameters and \( \beta \) 
derived from the respective \( \alpha \) through \( \beta = 1 - \frac{2\alpha}{3} \). In 
Figure 17(a) we show an example of the fit of Equation (22) 
to the data, and in Appendix B we present all 142 experiments 
with their corresponding fit functions. Figure 18 shows the 
relation between the fit value of \( m_{\text{max}} \) and the cutoff area \( x_i \) 
from Equation (17), with both values normalized to their 
respective target values \( m_t \) and \( x_t \), respectively. As expected, 
these values correlate such that we can state that

\[
\log \left( \frac{m_{\text{max}}}{m_t} \right) = C_m + 1.5 \log \left( \frac{x_t}{x_i} \right), \tag{23}
\]

\[
\frac{m_{\text{max}}}{m_t} = 10^{C_m} \cdot \left( \frac{x_t}{x_i} \right)^{1.5} \tag{24}
\]

where \( C_m = 0.32 \pm 0.02 \).

Finally, we investigated the correlation between the normalized 
cutoff value of the cumulative fragment mass distribution, 
\( m_{\text{max}}/m_0 \), derived from Equation (24) and the mass of the 
largest fragment \( \mu \) from Equation (5) and found that there is no 
such correlation (see Figure 19). Thus, the cutoff mass and the 
mass of the largest fragment are independent and should be 
treated separately. As generally \( m_{\text{max}}/m_0 \leq \mu \), this also means 
that the continuum part of the fragment mass-frequency 
distribution, whose upper mass is \( m_{\text{max}} \), and the largest 
fragment, whose mass is \( \mu \cdot m_0 \) are distinct for the projectile/target sizes and impact velocities studied in this 
study. From their scaling behavior, i.e., \( m_{\text{max}} \propto (x_t)^{3/2} \propto v_n^{1.23} \) 
(Equation (20)) and \( \mu \propto v_n^{-1.1} \) (Equation (8) with \( n = 0.55 \)) 
or \( \mu \propto v_n^{-0.66} \) (Equation (13)), respectively, we can argue that 
this is also the case for higher impact speeds.

Figure 17(b) shows the log-log representation of the cumulative 
area-frequency distribution function, now displayed in log-log 
form with the cutoff area included at the low-mass end. It is obvious 
that for the smallest fragment masses the cumulative mass-frequency 
distribution function follows a power law, but flattens for higher masses. This is generally the case for all 
collisions. In Figure 20 we show the slope of the initial power 
law as determined in the example shown in Figure 17(b) as a 
function of the normal collision velocity. Except for a few 
outliers, these slopes fall within a narrow range around the 
average of 0.72 with a standard deviation of 0.13 (see 
Figure 20), independent of impact velocity.

5. A UTILITARIAN FRAGMENTATION MODEL FOR AGGREGATE–AGGREGATE COLLISIONS

Following the results of our extensive study on aggregate–aggregate collisions, which include the determination of the 
mass of the largest fragment as a function of the normal component of the impact velocity and the projectile/target size 
as well as a full description of the mass distribution function of the 
escaping fragments, we suggest the following recipe for a complete description of the fragmentation event:

1. Determination of whether mass transfer or fragmentation 
of the target occurs. The outcome in aggregate–aggregate 
collisions is not unique. In the velocity range above the fragmentation threshold of \( \sim 1 \text{ m s}^{-1} \), mass transfer or complete fragmentation can coexist. In the former case, 
the target agglomerate stays intact and acquires part of the 
mass of the projectile. In the latter case, both colliding aggregates lose mass. Based on our experimental results, 
we propose the following approach:

(a) Determine whether mass transfer can occur. For given 
sizes (i.e., diameters) of projectile and target aggregate, 
\( P \) and \( T \), and their masses \( m_p \) and \( m_t \) respectively, the 
necessary condition for the occurrence of mass transfer is

\[ m_p \times \frac{T}{P} > \frac{m_t}{m_p} \times \frac{m_t}{m_p} \]
Determination of the relative mass of the largest fragment

that the collision velocity must not exceed the critical fragmentation velocity \( v_{0.5} \). This can be determined through the strength of the target agglomerate \( Q^* \) as a function of \( P \) and \( T \) using Equation (10),

\[
\log \left( \frac{Q^*(P,T)}{1 \text{ J kg}^{-1}} \right) = C_Q + \kappa \log \left( \frac{P}{1 \text{ cm}} \right) + \lambda \log \left( \frac{T}{1 \text{ cm}} \right),
\]

with \( C_Q = 1.24, \kappa = 1.12 \) and \( \lambda = -2.70 \). Then, the critical fragmentation velocity is given by \( v_{0.5} = \sqrt{2Q^* \left( 1 + \frac{m}{m_\eta} \right)} \) (see discussion before Equation (11)).

For \( v_n > v_{0.5} \), mass transfer is not possible, because the target aggregate necessarily fragments.

(b) If mass transfer can occur, determine the probability for mass transfer against the probability for fragmentation of the target. According to our analysis in Section 4.1, the probability for mass transfer (and the intact survival of the target aggregate) is given by Equation (2), \( p_{\text{sur}} = 0.194 \) for \( f - 0.13 \) for \( 1 \lesssim f \lesssim 5.83 \) and \( p_{\text{sur}} = 1 \) for \( f > 5.83 \). Thus, the probability for the fragmentation of the target is \( p_{\text{frag}} = 1 - p_{\text{sur}} \).

(c) In case of mass transfer, determine the mass gain of the target. According to Equation (4), the mass-transfer efficiency \( \Delta m/m_\eta \) is given by

\[
\sigma \log \left( \frac{\Delta m}{m_\eta} \right) = C_{\text{mt}} + \sigma \log \left( \frac{v_n}{1 \text{ m} s^{-1}} \right),
\]

with \( C_{\text{mt}} = -1.50 \) and \( \sigma = 0.99 \).

2. Determination of the relative mass of the largest fragment \( \mu \). As the sizes of projectile and target, \( P \) and \( T \), their masses \( m_\eta \) and \( m_n \), the reduced mass \( m = (m_\eta^{-1} + m_n^{-1})^{-1} \), and the normal component of the collision velocity, \( v_n \), are known, we can

(a) first determine the strength of the target agglomerate \( Q^* \) as a function of \( P \) and \( T \) using Equation (10),

\[
\log \left( \frac{Q^*(P,T)}{1 \text{ J kg}^{-1}} \right) = C_Q + \kappa \log \left( \frac{P}{1 \text{ cm}} \right) + \lambda \log \left( \frac{T}{1 \text{ cm}} \right),
\]

with \( C_Q = 1.24, \kappa = 1.12 \) and \( \lambda = -2.70 \),

(b) then calculate \( E_{0.5} \) using Equation (9), \( Q^* = \frac{E_{0.5}}{m_\eta} \),

(c) and finally derive the relative mass of the largest fragment using Equation (8), \( \mu (E_{\text{cm}}) = 1 - \frac{E_{\text{cm}}}{E_{0.5} + E_{\text{cm}}} \),

with \( n = 0.55 \). Here, \( E_{\text{cm}} \) is the center-of-mass kinetic energy given by \( E_{\text{cm}} = \frac{1}{2} m_n^2 \).

(d) In the case of mass transfer, the mass of the largest fragment is \( \mu = 1 + \frac{\Delta m}{m_n} \) (determination of \( \mu \), see above).

3. Determination of the exponent \( \beta \) in the fragment mass-frequency distribution function. The exponent in the continuous part of the fragment mass-frequency distribution function Equation (22), \( M_{\text{cum}}(m_t) = M_{\text{tot}} \left( 1 - \left( \frac{m_t}{m_{\text{max}}} \right)^{\beta} \right) \),

can be calculated using its relation to the slope of the area-frequency distribution function, \( \beta = 1 - \frac{2\eta}{\tau} \). The latter is solely a function of the collision velocity, as expressed in Equation (19), \( \alpha = C_\alpha + \delta \log \left( \frac{v_n}{1 \text{ m} s^{-1}} \right) + \psi \log \left( \frac{T}{1 \text{ cm}} \right) \),

with \( C_\alpha = 0.14, \delta = 1.02 \) and \( \psi = 0.34 \).

4. Determination of the largest relative fragment mass of the continuous distribution \( m_{\text{max}} \). The continuous fragment mass distribution function, Equation (19), requires an upper mass limit \( m_{\text{max}} \), which we propose to equate to the cutoff mass of the area-frequency distribution function Equation (17). Following Equation (24),

\[
\log \left( \frac{m_{\text{max}}}{m_\eta} \right) = 2.1m_1 \cdot \left( \frac{x_i}{x_1} \right)^{\eta / 2}.
\]

with (see Equation (20))

\[
\alpha = C_\alpha + \theta \log \left( \frac{v_n}{1 \text{ m} s^{-1}} \right) + \eta \log \left( \frac{T}{1 \text{ cm}} \right) + \tau \log \left( \frac{T}{1 \text{ cm}} \right),
\]

and coefficients \( C_\alpha = 3.01, \theta = -0.82, \eta = 1.44 \) and \( \tau = -0.04 \) in the case of complete fragmentation of projectile and target, and \( C_\alpha = 2.60, \theta = -0.28, \eta = 3.00 \) and \( \tau = -1.15 \), respectively, in the case of mass transfer.

5. Total-mass scaling. With this information, the functional behavior of the cumulative mass distribution of the fragments, \( M_{\text{cum}}(m_t) \), is fully determined, following Equation (22), except for the scaling parameter \( M_{\text{tot}} \).

However, the latter can easily be derived by acknowledging the fact that the full mass distribution function consists of two parts, (i) a continuous fragment mass distribution function, and (ii) isolated from this, the largest fragment mass. That the latter is really distinct from the former can be seen by the fact that \( m_{\text{max}}/m_\eta \) is practically always lower than \( \mu \) (see Figure 19 and scaling behavior \( m_{\text{max}} \propto v_n^{-1} \).
6. SIMULATING THE COLLAPSE

The results of our experiments can be used to study the formation of planetesimals. In a protoplanetary disk, gravitationally bound pebble clouds can form through the interaction between pebbles and the gas in the disk, e.g., by the streaming instability (see Section 1). Such a cloud will collapse into a solid planetesimal thanks to the negative heat capacity property of gravitationally bound systems and energy dissipation in pebble-pebble collisions. Wahlberg Jansson & Johansen (2014) studied the collapse process of such a cloud to find the internal structure of the resulting planetesimal. They find that the density of a planetesimal formed increases with planetesimal mass. More massive clouds have more fragmenting collisions and a wide range of particle sizes in the resulting planetesimal, leading to better packing capabilities. In their numerical simulations, however, the authors use a simplified model of fragmenting collisions, treating fragmentation as erosion. In Paper II, the collapse process is investigated with an updated model. The new model includes the results of our experiments (critical speeds, fragment size distribution, and mass transfer probability), to achieve more physically realistic simulations. For low-mass planetesimals ($R_{\text{solid}} \lesssim 40$ km) the results are similar (they end up as porous pebble-piles). For more massive planetesimals, however, the internal structure shows a strong dependence on both fragmentation model and pebble composition (silicates versus ice).

7. CONCLUSION AND DISCUSSION

We developed a new experimental setup dedicated to the study of the low-velocity fragmentation behavior of porous dust aggregates. Aggregates consisted of micrometer-sized SiO$_2$ grains and possessed volume filling factors of $\phi = 0.35$, i.e., porosities of 65%. The sizes of the dust aggregates ranged between 1 and 5 cm, with collision velocities in the range from 1.5 to 8.7 m s$^{-1}$ (see Figure 5).

In all cases we studied, the smaller (or equal-sized) projectile aggregate fragmented. The larger (or equal-sized) target aggregate survived impact when the target-to-projectile size ratio was high and the impact velocity rather low (see Figure 7b). However, we found that the outcome in these cases is probabilistic between target survival and target fragmentation, with a probability for target survival given by Equation (2) (see Figure 7a).

We described the fragmentation of the colliding dust aggregates by the mass of the largest fragment and a continuous area-frequency distribution function of the smaller fragments. When we express the mass of the largest fragments in units of the target-aggregate mass, we can describe its dependence on impact energy with a Hill function (see Equation (8)) with two free parameters, the energy $E_0$, for which the largest fragment is $\mu = 0.5$, and an exponent $n$, for which we find that $n = 0.55$. Following our recipe that we summarized in Section 5, a full description of the fragmentation process in collisions between arbitrary dust aggregates is possible.

In addition to the application of our high-velocity dust-aggregation collision model in the description of the fate of dust aggregates in collapsing pebble clouds (see Section 6 and Paper II), the model will also be useful for mass-transfer based formation models of planetesimals (Windmark et al. 2012a, 2012b; Garaud et al. 2013). The successive growth of dust aggregates beyond the bouncing barrier by mass transfer in catastrophic collisions between dissimilar-sized dust aggregates is an essential part of these models. With the data and formal descriptions of the collision outcomes presented in this paper, the validity of models for the formation of planetesimals by direct sticking via mass transfer can be assessed with more realistic collision outcomes.

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APPENDIX A
ELECTRONIC APPENDIX 1

Figures 21–28.
Figure 21. Area-frequency distribution of all individual experiments in the 1–1 cm series.
Figure 22. Area-frequency distribution of all individual experiments in the 1–2 cm series.
Figure 23. Area-frequency distribution of all individual experiments in the 1–2.6 cm series.
Figure 24. Area-frequency distribution of all individual experiments in the 1–5 cm series.
Figure 25. Area-frequency distribution of all individual experiments in the 2–2 cm series.
Figure 26. Area-frequency distribution of all individual experiments in the 2–5 cm series.
Figure 27. Area-frequency distribution of all individual experiments in the 3.5–5 cm series.
Figure 28. Area-frequency distribution of all individual experiments in the 5–5 cm series.
Figure 29. Mass-frequency distribution of all individual experiments in the 1–1 cm series.
Figure 30. Mass-frequency distribution of all individual experiments in the 1–2 cm series.
Figure 31. Mass-frequency distribution of all individual experiments in the 1–2.6 cm series.
Figure 32. Mass-frequency distribution of all individual experiments in the 1–5 cm series.
Figure 33. Mass-frequency distribution of all individual experiments in the 2–2 cm series.
Figure 34. Mass-frequency distribution of all individual experiments in the 2–5 cm series.
Figure 35. Mass-frequency distribution of all individual experiments in the 3.5–5 cm series.
Figure 36. Mass-frequency distribution of all individual experiments in the 5–5 cm series.
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