State transition for de-escalation in the graph model for conflict resolution framework

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Abstract

This study discusses the analysis method for obtaining solutions that allow de-escalation in international conflicts within the framework of the Graph Model for Conflict Resolution. CD games, a conflict type in which two decision-makers can choose Cooperate (C) or Defect (D) strategies, are analyzed by examining the state transition and stability of decision makers with a focus on reachability. Based on the analysis, a new conceptual set is proposed, called ‘de-escalation reachability,’ which enables focused analysis on the conflict states while avoiding any influence of arbitrators’ attributes.

Keywords conflict resolution, graph model for conflict resolution, de-escalation

Research Activity Group Mathematical Politics

1. Introduction

The Graph Model for Conflict Resolution (GMCR) is a mathematical model derived from game theory, which describes conflicts in terms of transitions according to the preferences of the decision-maker (DM). Thus, an intuitive manner of modeling of conflicts is possible. This framework, developed by Kilgour, Hipel, and Fang [1], treats preferences in a ‘relative’ manner and describes the sequential responses of DMs. It can explicitly incorporates the ‘irreversibility of choice’ and ‘infeasibility of state.’ The framework of GMCR is flexible and suitable for analyzing issues in which irreversibility is a critical point of contention, such as international relations involving national interests. Extended concepts that incorporate GMCR are being studied in addition to the standard framework, including coalitions [2] and time of state transitions [3].

In international conflicts where national interests conflict, decision-makers try to achieve new stability by exercising rational choices that maximize their interests while threatening others. If both decision-makers do not want the conflict to escalate beyond what they initially envisioned, deterrent stability will be established in which they check each other. Even in that case, the decision-maker’s gain that uses the threat more strongly, up to the outburst limit, will be more significant.

Previous research on conflict escalation and deterrence have been conducted by Brams, Zagare, Fraser and Hipel [4–6] using game theory and metagame approaches. This paper focuses on the irreversibility of decision-makers’ actions, proposes a new concept of adding and deleting state transitions in GMCR, and describes de-escalation by applying this concept.

GMCR and the CD game are reviewed first, followed by a discussion of conflict states in which de-escalation is possible within the newly proposed framework. Finally, the new framework is used to analyze the Cuban missile analysis.

2. Methods and Analysis

2.1 The Graph Model for Conflict Resolution

The definition of GMCR is given as follows.

**Definition 1** Basic concepts of GMCR.

- **Graph Model of Conflict:** \((N, S, (A_i)_{i \in N}, (\succ_i)_{i \in N})\).
- **Set of all DMs:** \(N : |N| \geq 2\).
- **Set of all states:** \(S : |S| \geq 2\).
- **Directed graphs of DMi:** \(G_i : (S, A_i)\).
- **Preference of DMi satisfies completeness and transitivity:**
  \(s \succ_i s' : s\) is equally or more preferred to \(s'\) by DMi;
  \(s \succ_i s' : s\) is strictly preferred to \(s'\) by DMi;
  \(s \sim_i s' : s\) is equally preferred to \(s'\) by DMi.
- **DMi’s reachable list from \(s\) to \(s'\) by unilateral moves:**
  \(R_i(s) = \{s' \in S \mid (s, s') \in A_i\}; \ (s, s') \in A_i\) denotes the reachability; DMi can reach from \(s\) to \(s'\).
- **DMi’s reachable list from \(s\) to \(s'\) by unilateral improvements:**
  \(R_i^+(s) = \{s' \in R_i(s) \mid s' \succ_i s\}\).
- **DMi’s reachable list from \(s\) to \(s'\) by equally or less preferred moves:**
  \(R_i(s) = \{s' \in S \mid s \succ_i s'\}\).
- **DMi’s reachable list from \(s\) to \(s'\) by equally or less preferred moves:**
  \(R_i^+(s) = \{s' \in S \mid s \succ_i s'\}\).
- **DMi’s reachable list from \(s\) to \(s'\) by equally or less preferred moves:**
  \(R_i(s) = \{s' \in S \mid s \succ_i s'\}\).
- **DMi’s reachable list from \(s\) to \(s'\) by equally or less preferred moves:**
  \(R_i(s) = \{s' \in S \mid s \succ_i s'\}\).
Stability definitions in GMCR in terms of graphs and preferences described in Definition 1 are given as follows.

**Definition 2 Stability in GMCR.**

1. **Nash stability (Nash)** [7]: \( R_i^1(s) = \emptyset \).
2. **General metarationality (GMR)** [8]: \( \forall s' \in R_i^1(s), R_{i\setminus\{i\}}^1(s') \cap \phi_i^R(s) \neq \emptyset \).
3. **Symmetric metarationality (SMR)** [8]: \( \forall s' \in R_{i\setminus\{i\}}^1(s'), \exists s'' \in R_i^1(s') \cap \phi_i^R(s), R_i(s'') \subseteq \phi_i^R(s) \).
4. **Sequential stability (SEQ)** [9, 10]: \( \forall s_1 \approx s_2 \subseteq s_3 \subseteq s_4 \subseteq s_5 \cdots \).

Let \( S_i^{Nash}, S_i^{GMR}, S_i^{SMR}, S_i^{SEQ} \) denote the sets of all states which satisfies the above stability definitions, respectively, then:

- \( S_i^{Nash} \subset S_i^{SEQ} \subset S_i^{GMR} \).
- \( S_i^{Nash} \subset S_i^{SMR} \subset S_i^{GMR} \).

### 2.2 Inter-state conflicts

Based on Definitions 1 and 2, we conduct further analysis of reachability in the CD game. The object of analysis is the case of \((D, D)\), where Nash equilibrium is established in a state of all-out confrontation, and the possibility of escaping from the stalemate in \((D, D)\) is examined by the effects of depriving or granting reachability for DMs. The CD game can be defined in terms of the set of DMs and conflict states in Def. 1 of the GMCR as follows; \( N = \{D1, D2\}, S = \{s_1, s_2, s_3, s_4\}\), where \( s_1 = (C, C), s_2 = (C, D), s_3 = (D, C), s_4 = (D, D) \).

If we assume that the most reasonable conditions of preference order for cases that are limited to inter-state conflicts as \( s_3 \succ s_4 \) and \( s_1 \succ s_2 \), i.e., that a country seeks its own national interests but does not wish to have catastrophic conflicts, and would rather make concessions than defects, then we obtain only 6 applicable sets of preferences for analysis as follows [11]:

- \( a.s_3 \succ s_1 \succ s_4 \succ s_2 \).
- \( b.s_3 \succ s_1 \succ s_2 \succ s_4 \).
- \( c.s_1 \succ s_3 \succ s_4 \succ s_2 \).
- \( d.s_1 \succ s_3 \succ s_2 \succ s_4 \).
- \( e.s_3 \succ s_4 \succ s_1 \succ s_2 \).
- \( f.s_1 \succ s_2 \succ s_3 \succ s_4 \).

In the 6 types of preference indicated by \( a \) to \( f \), \( a \) and \( b \) correspond to the Prisoners’ dilemma (PD) and Chicken game CD, respectively. The graph models of CG are given in Fig. 1; the left is for DM1 and the right is for DM2. The numbers 1–4 denote each state, and the directed arcs represent the DMs’ reachability from a state to another state. Within 36 games, in which 6 pairs of DMs with the preferences from \( a \) to \( f \) conflict with each other, the Nash in \( s_4 \) holds in 9 games. In these 9 games, we analyzed the reachability of DMs avoiding \( s_4 \) and reaching another Nash equilibrium state.

### 2.3 Reachability analysis

Assume that graphs of DM1 and DM2 are provided as in the following \( A_1 \) and \( A_2 \) in the standard GMCR method as follows:

\[
A_1 = \{(s_1, s_3), (s_3, s_1), (s_2, s_4), (s_4, s_2)\},
A_2 = \{(s_1, s_2), (s_2, s_1), (s_3, s_4), (s_4, s_3)\}.
\]

Then, for each \( s_1 \rightarrow s_4 \) state, we perform the analysis in three ways: 1) normal stability analysis, 2) deleting one arc from \( A_2 \) and 3) adding the \( s_4 \rightarrow s_1 \) reachability arc to each of \( A_1 \) and \( A_2 \). Table 1 shows the results of whether Nash in \( s_4 \) disappeared given the changes in 2) and 3) in conflicts between DMs with \( a \) to \( f \) preference types indicated in 2.2. \( D \) represents the deletion defined in 2) from the initial reachability, \( A-DM1 \) is the addition 3) to \( DM1 \), and \( A-DM2 \) is that to \( DM2 \). These changes can be summarized as follows.

- **Deletion of 1 arc from DM2’s graph \( A_2 \):**
  - Nash holds on \( s_1 \) in 2 games by deleting \((s_1, s_2)\).
  - Nash holds on \( s_3 \) in 6 games by deleting \((s_3, s_4)\).

- **Addition of 1 arc \( s_4 \) to \( s_1 \) to each DM:**
  - Nash disappears on \( s_4 \) in 6 games by adding \((s_4, s_1)\) to \( A_1 \).
  - Nash disappears on \( s_4 \) in 6 games by adding \((s_4, s_1)\) to \( A_2 \).

As results of the changes in reachability, Nash disappears in \( s_4 \), which is an all-out conflict state, or a new Nash can be found in other states. In other words, for DMs, de-escalation is possible if they can control the conflict by effecting such changes.

### 3. Transitions for de-escalation

#### 3.1 State transition for de-escalation

Based on the previous section’s analysis results, we study the new reachability that DMs can obtain by adding and deleting the original arcs. In the standard method of analysis of GMCR, the introduction of new requirements to conflicts is performed with information on DMs’ preferences and reachability, while changes of reachability are not treated as separate objects of analysis. When the purpose of the analysis is clear as to whether escalation can be avoided, it is more efficient to...
focus on the states and analyze only the effects of escalation. Thus, we define a new type of state transition to avoid the worst-case conflict stalemate, as described in the previous section.

**Definition 3** Preference in inter-state conflicts.
- DMi’s preference in inter-state conflicts $\succ_{i}^{\psi}$:
  - $s_{3} \succ_{i}^{\psi} s_{4}, s_{1} \succ_{i}^{\psi} s_{2}$.

**Definition 4** De-escalation reachability (DESR) in inter-state conflicts.
- $A_{i}^{desD}$ and $A_{i}^{desA}$ denote de-escalational reachability achieved by deleting or adding arcs in $A_{i}$, respectively.
- Reachable list:
  - $R_{i}^{desD}(s) = \{ s' \in S | (s, s') \in A_{i}^{desD} \}$,
  - $R_{i}^{desA}(s) = \{ s' \in S | (s, s') \in A_{i}^{desA} \}$.

$A_{i}^{desA}$ and $A_{i}^{desD}$ represent the transitions with the directed graphs newly acquired by DMi as a result of some change by adding or deleting arcs that lead to de-escalation in conflicts. Such changes occur through influences brought by other than $N$, which may be caused by some environmental factors or intervention by third parties. With $A_{i}^{des}$, we can independently analyze state transition from the DM attributes that brought the change into the conflicts. This framework, de-escalation reachability (DESR), enables us to focus on the state of conflicts rather than the DMs, which is especially useful when considering interventions such as arbitration in actual international disputes. It also allows conflict resolution mechanisms to be analyzed more flexibly and usefully.

4. Application to Cuban missile crisis

This section analyzes a real-life conflicts using the concept of state transitions for de-escalation, as defined in the previous section. As an object of analysis, this paper discusses the Cuban missile crisis as a representative case of contemporary interstate conflict.

4.1 The conflict

The Cuban missile crisis was a confrontation between the United States and the Soviet Union that erupted in 1962 when the Soviet Union deployed a missile base in Cuba. This crisis threatened to escalate into a nuclear war on a scale that could have been a third World War. Ultimately, such a war was averted following this crisis through talks between the leaders of the two countries. The conflict between the two countries is often analyzed as a CG, as shown in Table 2. However, analysis of the CG alone does not provide a solution to how the two Nash in (US: No attack, USSR: Attack) and (US: Attack, USSR: No-attack) can be resolved. Historical testimony suggests that both countries did not want the conflict to escalate and contained the conflict locally [12,13]. Several measures to avoid escalation were considered, including informal channels of communication. Those channels of arbitration have been known to include a recommendation of ‘moratorium’ by the United Nations [14] and the secret contact between US President Kennedy’s brother, Attorney General Robert Kennedy, and the Soviet ambassador to the US [15]. A modified normal form game, taking these circumstances into account, is given in Table 3. The preferences of both countries can be considered to be $s_{1} \succ s_{3} \succ s_{4} \succ s_{2}$ for the United States , and $s_{1} \succ s_{2} \succ s_{4} \succ s_{3}$ for the Soviet Union in this game, assuming that both countries had a strategy of fighting for hegemony while having a deterrent preference rather than a disastrous CG preference. The preference order corresponds to the type $c$ defined in 2.2. However, because the deterrence game has two Nash in $s_{1}$ and $s_{4}$, it is not possible to analyze further, as in CG. Thus, we apply the new analysis concept DESR to the crisis in the next section.

4.2 Analysis by GMCR

Four possible states for the Cuban crisis in the simplest structure can be defined as follows:
- $s_{1}$: Both the United States and the Soviet Union make concessions and stop the conflict.
- $s_{2}$: The Soviet Union makes no concessions, and the United States stops attacking.
- $s_{3}$: The United States makes no concessions, and the Soviet Union makes concessions.
- $s_{4}$: Neither country makes any concessions and escalate to a full-scale conflict.

| Table 2. Cuban crisis-CG. |
|---------------------------|
|                           |
| **USSR**                  |
| No attack 3.3 2.4         |
| Attack 4.2 1.1            |

| Table 3. Cuban crisis-Deterrence game. |
|----------------------------------------|
| **USSR**                               |
| No attack 4.4 1.3                      |
| Attack 3.1 2.2                         |

| Table 4. Stability analysis-Cuban crisis. |
|-------------------------------------------|
| **s_{1} s_{2} s_{3} s_{4}**              |
| **Nash**                                 |
| US ✓ ✓ ✓                               |
| USSR ✓ ✓                               |
| Eq ✓ ✓                                 |
| **GMR**                                 |
| US ✓ ✓ ✓                               |
| USSR ✓ ✓                               |
| Eq ✓ ✓                                 |
| **SMR**                                 |
| US ✓ ✓ ✓                               |
| USSR ✓ ✓                               |
| Eq ✓ ✓                                 |
| **SEQ**                                 |
| US ✓ ✓                                 |
| USSR ✓ ✓                               |
| Eq ✓ ✓                                 |
5. Conclusion

As shown in the previous sections, this paper proposes an extended framework in GMCR, called DESR, which describes solutions for de-escalation in international conflicts between DMs that have representative preferences. To analyze the de-escalation of conflicts, we may not need as much detailed information about the numerous parties involved in a given conflict outside of the main DMs. It is sufficient if we can analyze the impact on the movements onto and between the main DMs. In the case of the Cuban missile crisis, the priority issues are how escalation can be avoided and the states of conflict. Analysis of other DMs that might bring about such de-escalation should be made separately, and only when the necessary amount of information can be collected. From this perspective, DESR can be evaluated as simple and easy to apply to real-world problems, because it can only focus on changes in the state of the conflict being analyzed.

This paper focuses on Nash in CD games, especially the deterrence game, which is frequently found in international conflicts. In the future, we would like to conduct analysis in a framework that includes other stability concepts such as GMR, SMR, and SEQ. We would also like to incorporate the framework of deletion and addition of reachability into the analysis of other extended concepts in GMCR, such as coalition analysis or transition time analysis to develop a broader range of conflict analysis methods.

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