Flatness-based embedded control in successive loops for spark-ignited engines

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Abstract. Embedded control units for transportation systems make use of advanced nonlinear control methods. In this research article a new nonlinear control method is applied to spark ignited (SI) engines. The proposed SI engine’s control scheme is based on differential flatness theory. The considered method succeeds the efficient control of the SI engine parameters such as intake pressure and turn speed. The method makes use of a state-space model of the SI-engine in the so-called triangular form. The controller design proceeds by showing that each row of the state-space model of the SI engine stands for a differentially flat system, where the flat output is chosen to be the associated state variable. Next, for each subsystem which is linked with a row of the state-space model, a virtual control input is computed, that can invert the subsystem’s dynamics and can eliminate the subsystem’s tracking error. From the last row of the state-space description, the control input that is actually applied to the SI engine is found. This control input contains recursively all virtual control inputs which were computed for the individual subsystems associated with the previous rows of the state-space equation. Thus, by tracing the rows of the state-space model backwards, at each iteration of the control algorithm, one can finally obtain the control input that should be applied to the SI-engine so as to assure that all its state vector elements will converge to the desirable setpoints.

1. Introduction
In this paper a new nonlinear control scheme is proposed for SI-engines making use of differential flatness theory [Rigatos, 2015], [Rigatos, 2011a]. The controller of the spark-ignited engines is structured in successive loops. The method makes use of a description of the engine’s state-space model in the so-called triangular form. The controller design proceeds by showing that each row of the state-space model of the SI engine stands for a differentially flat system, where the flat output is chosen to be the associated state variable. Next, for each subsystem which is linked with a row of the state-space model a virtual control input is computed, that can invert the subsystem’s dynamics and can eliminate the subsystem’s tracking error. From the last row of the state-space description, the control input that is actually applied to the spark-ignited engine is found. This control input contains recursively all virtual control inputs which were computed for the individual subsystems associated with the previous rows of the state-space equation. Thus, by tracing the rows of the state-space model backwards, at each iteration of the control algorithm, one can finally obtain the control input that should be applied to the SI engine so as to assure that all its state vector elements will converge to the desirable setpoints.

The paper’s results come to complement the significant research effort towards the development...
of embedded control systems for the automotive industry, aiming at improving the performance of vehicles’ engines in terms of produced power, at reducing fuel consumption and at eliminating the emission of exhaust gases. In particular, the problem of control of the rotation speed of SI-engines as well as the problem of control of the engine’s pressure manifolds has been approached with different methods [Sugihira, 2008],[Zhang, 2010]. In [Ali, 2006] a nonlinear state space controller for turn speed (and consequently for torque) of a spark-ignited engine is proposed. The controller’s design is based on feedback linearization in combination with pole placement. In [Zhang, 2009] time-varying internal model-based design is applied to compensate for the time-varying but angle dependent pressure pulsations in the fuel injection system of SI-engines. In [Leroy, 2009] a control method for the air-path system of SI-engines is presented. The first part considers generation of the motion-planning trajectory of the intake manifold pressure from a torque set point. Then, feedforward and feedback control laws are presented. In [Moulin, 2011] a feedback linearization approach based on differential flatness theory is proposed for the control of the air system of a turbocharged gasoline engine. Finally, in [Fl¨ ardh, 2014] a model-based approach is pursued to maximize an SI-engine’s torque through optimal control of the variable valve timing (VVT) and the variable gas turbine (VGT) model. Several other results on nonlinear control of spark ignited engines have been presented in [Sepuvelda, 2012], [Ballachi, 2013], [Blake Vance 2008], [Alipi, 2003],[Colin, 2007], [Nguyen, 2013]].

Moreover, the paper contributes a different perspective to the design and implementation of flatness-based controllers. Differential flatness theory is currently a main direction in nonlinear control systems analysis and synthesis [Rigatos, 2015],[Rigatos, 2011a]. A system is considered to be differentially flat if all its state variables and its control inputs can be expressed as functions of one single algebraic variable which is the so-called flat output, and also as functions of the flat-output’s derivatives [Rudolph, 2003],[Sira-Ramirez, 2004],[L´ evine, 2011],[Fliess, 1999],[Menhour, 2013]. The differential flatness property enables the transformation of the nonlinear system’s dynamics into the linear canonical form and the design of a state feedback controller through the application of pole placement techniques in the linearized equivalent model of the system [Rouchon, 2005], [Martin, 1999],[Bonouden, 2011], [Laroche, 2007],[Rigatos, 2014],[Rigatos, 2012]. In this paper, a different approach is developed for controller design in nonlinear dynamical systems which exhibit the differential flatness property. The method makes use of the initial nonlinear model of the system and of its decomposition into a set of nonlinear subsystems for which the differential flatness property holds. The proposed flatness-based controller consists finally of cascading interconnected loops.

The structure of the paper is as follows: in Section 2 the dynamic model of the SI-engine is analyzed and its state-space description is given. In Section 3 the implementation of flatness-based control in cascading loops is proposed for nonlinear dynamical systems having a state-space model in the triangular form. In Section 4 the dynamics of the closed loop of the spark-ignited engine is analyzed and asymptotic stability is proven. In Section 5 simulation experiments are carried out to assess the performance of the proposed nonlinear control scheme for the SI-engine. Finally, in Section 6 concluding remarks are stated.

2. Dynamic model of the Spark-Ignited Engine

It is possible to control the intake pressure \( p_m \) and the rotational speed of the engine’s shaft \( \omega \) by adjusting the angle of the air throttle. It is considered that the associated control loop is independent from the loops of the fuel injection control and spark timing control (Fig. 1).

The basic equations of the system are [Sugihira, 2008]:
\( \dot{\omega} = k_\omega p_m (t - \tau_d) + k_\omega_2 + k_\omega_3 T_{fm} \)
\( p_m = k_{p_1} \omega p_m + k_{p_2} \omega + k_{p_3} u \)
\( y_1 = \omega \) (1)

The above model is easy to be confirmed by considering the rotational motion dynamics of the engine’s shaft and the time delay in the measurement of the intake air pressure. The variable of the intake pressure appears with time delay in the equation of the turn speed in the second row of the model of the SI engine. Using that
\( p_{md} = p_m (t - \tau_d) \) and
\( p_m (t - \tau_d) = \frac{1}{\tau_s + 1} p_m \) (2)

while \( \tau = a_d / \omega \) and \( a_d \) is a parameter that is measured in radians. Denoting \( k_d = -1 / a_d \) one has about the dynamics of the delayed intake pressure variable
\( p_{md} = k_d \omega (p_{md} - p_m) \) (3)

Using the previous formulation, and defining the state variables \( x_1 = \omega, x_2 = p_{md} \) and \( x_3 = p_m \), the dynamics of the SI engine is written as [Rigatos, 2015], [Sugihira, 2008]
\[
\begin{align*}
\dot{x}_1 &= k_\omega x_2 + k_\omega_2 + k_\omega_3 T_{fm} \\
\dot{x}_2 &= K_d x_1 (x_2 - x_3) \\
\dot{x}_3 &= k_{p_1} x_1 x_3 + k_{p_2} x_1 + k_{p_3} u
\end{align*}
\] (4)

where \( T_{fm} \) are friction torques, which can be also perceived as disturbances. In the above equations coefficients \( k_{p_i}, i = 1, 2, 3, k_\omega, i = 1, 2, 3 \) and \( K_d \) are associated with the combustion cycle of the SI-engine and are defined in [Rigatos, 2015], [Sugihira, 2008]. The model also takes the matrix form \( \dot{x} = f(x) + g(x)u \) with
\[
\begin{align*}
f(x) &= \begin{pmatrix} k_\omega x_2 + k_\omega_2 + k_\omega_3 T_{fm} \\ k_d x_1 (x_2 - x_3) \\ k_{p_1} x_1 x_3 + k_{p_2} x_1 \end{pmatrix} \\
g(x) &= \begin{pmatrix} 0 \\ 0 \end{pmatrix}
\end{align*}
\] (5)
The considered model of the spark-ignited engine can be also written in the so-called triangular form. Indeed one has

\[
\begin{align*}
\dot{x}_1 &= f_1(x_1) + g_1(x_1)x_2 \\
\dot{x}_2 &= f_2(x_1, x_2) + g_2(x_1, x_2)x_3 \\
\dot{x}_3 &= f_3(x_1, x_2, x_3) + g_3(x_1, x_2, x_3)u
\end{align*}
\]  

(6)

where

\[
\begin{align*}
f_1(x_1) &= k_{\omega_1} + k_{\omega_1}T_m \\
f_2(x_1, x_2) &= k_d x_1 x_2 \\
f_3(x_1, x_2, x_3) &= k_{p_1} x_1 x_3 + k_{p_2} x_1 \quad g_1(x_1) = k_{\omega_1} \\
g_2(x_1, x_2) &= k_d x_1 \\
g_3(x_1, x_2, x_3) &= k_{p_1} 
\end{align*}
\]  

(7)

For the implementation of model-based control of the SI-engine it is important to use a reliable model of the engine’s dynamics and to have precise knowledge of the engine’s parameters. To this end, identification and model validation techniques are used, capable of providing estimates of the engine’s parameters through the processing of sensor data [Rigatos, 2007], [Basseville, 1993], [Rigatos, 2009].

3. A new approach to flatness-based control for nonlinear dynamical system

The control method to be applied to the SI-engine’s model is also suitable for a wide class of nonlinear dynamical systems of the form:

\[
\begin{align*}
\dot{x} &= f(x) + g(x)u \quad x \in \mathbb{R}^n \\
y &= h(x) \quad u \in \mathbb{R}
\end{align*}
\]  

(8)

It is considered that such systems can be written in the so-called triangular form:

\[
\begin{align*}
\dot{x}_1 &= f_1(x_1) + g_1(x_1)x_2 \\
\dot{x}_2 &= f_2(x_1, x_2) + g_2(x_1, x_2)x_3 \\
\dot{x}_3 &= f_3(x_1, x_2, x_3) + g_3(x_1, x_2, x_3)x_4 \\
&\vdots \\
\dot{x}_i &= f_i(x_1, x_2, \cdots, x_i) + g_i(x_1, x_2, \cdots, x_i)x_{i+1} \\
&\vdots \\
\dot{x}_{n-1} &= f_{n-1}(x_1, x_2, \cdots, x_{n-1}) + g_{n-1}(x_1, x_2, \cdots, x_{n-1})x_n \\
\dot{x}_n &= f_n(x_1, x_2, \cdots, x_n) + g_n(x_1, x_2, \cdots, x_n)u
\end{align*}
\]  

(9)

The following virtual control inputs \(\alpha_i = x_{i+1}\) are defined per row of the state-space model of Eq. (9)

\[
\begin{align*}
\dot{x}_1 &= f_1(x_1) + g_1(x_1)\alpha_1 \\
\dot{x}_2 &= f_2(x_1, x_2) + g_2(x_1, x_2)\alpha_2 \\
\dot{x}_3 &= f_3(x_1, x_2, x_3) + g_3(x_1, x_2, x_3)\alpha_3 \\
&\vdots \\
\dot{x}_i &= f_i(x_1, x_2, \cdots, x_i) + g_i(x_1, x_2, \cdots, x_i)\alpha_i \\
&\vdots \\
\dot{x}_{n-1} &= f_{n-1}(x_1, x_2, \cdots, x_{n-1}) + g_{n-1}(x_1, x_2, \cdots, x_{n-1})\alpha_{n-1} \\
\dot{x}_n &= f_n(x_1, x_2, \cdots, x_n) + g_n(x_1, x_2, \cdots, x_n)u
\end{align*}
\]  

(10)

The system of Eq. (10) is a differentially flat one. It is considered that \(y = x_1\) is the flat output of system. It can be easily shown that each virtual control input \(\alpha_i = x_{i+1}\), \(i = 1, 2, \cdots\) can be expressed as a function of the flat output and its derivatives, since it holds
\[ \alpha_i = \frac{1}{g_i(x_1, x_2, \ldots, x_i)}(\dot{x}_i - f(x_1, x_2, \ldots, x_i)) \]  

For \( i = 1 \) one has

\[ \alpha_1 = \frac{1}{g_1(x_1)}(\dot{x}_1 - f(x_1)) \]  

which means that \( \alpha_1 \) is a function of the flat output and its derivative. For \( i = 2 \) one has

\[ \alpha_2 = \frac{1}{g_2(x_1, x_2)}(\dot{x}_2 - f_2(x_1, x_2)) \]  

which means that \( \alpha_2 = x_3 \) is a function of the flat output \( y = x_1 \) and its derivatives. Continuing in a similar manner one has that \( \alpha_{n-1} = x_n \) and consequently \( \alpha_n = u \) is also a function of the flat output \( y = x_1 \) and its derivatives. According to the above, one has a nonlinear dynamical system in which all its state variables and the control input can be written as functions of the flat output and its derivatives. Therefore, such a system is differentially flat.

Additionally, by considering each row of the model of Eq. (10), one has a set of \( n \) subsystems of the form

\[ \dot{x}_i = f_i(x_1, x_2, \ldots, x_i) + g_i(x_1, x_2, \ldots, x_i)\alpha_i \]  

where each subsystem describes the dynamics of the single state variable \( x_i \). For each one of these subsystems one can consider the state variable \( x_i \) as the flat output. Obviously, the virtual control input \( \alpha_i \) is a function of this flat output and its derivatives. Therefore, each local subsystem is also differentially flat.

Next, one can compute the virtual inputs which are applied to each subsystem. For the first subsystem, which is associated with the first row of Eq. (9), and by defining \( z_i = x_i - x_i^* = x_1 - \alpha_{i-1} \), the virtual control input is given by

\[ \alpha_1 = x_2^* = \frac{1}{g_1(x_1)}\cdot(\dot{x}_2^* - f(x_1)) - K_1^1(x_1 - x_1^*) \]  

\[ \alpha_1 = x_2^* = \frac{1}{g_1(x_1)}\cdot(\dot{x}_2^* - f(x_1)) - K_1^1z_1 \]  

From the second row of Eq. (9), and using that \( z_2 = x_2 - x_2^* = x_2 - \alpha_1 \) one has

\[ \alpha_2 = x_3^* = \frac{1}{g_2(x_1, x_2)}\cdot(\dot{x}_3^* - f_2(x_1, x_2)) - K_2^1(x_2 - x_2^*) \]  

\[ \alpha_2 = x_3^* = \frac{1}{g_2(x_1, x_2)}\cdot(\dot{x}_3^* - f_2(x_1, x_2)) - K_2^1z_2 \]  

From the third row of Eq. (9), and using that \( z_3 = x_3 - x_3^* = x_3 - \alpha_2 \) one has

\[ \alpha_3 = x_4^* = \frac{1}{g_3(x_1, x_2, x_3)}\cdot(\dot{x}_4^* - f_3(x_1, x_2, x_3)) - K_3^1(x_3 - x_3^*) \]  

\[ \alpha_3 = x_4^* = \frac{1}{g_3(x_1, x_2, x_3)}\cdot(\dot{x}_4^* - f_3(x_1, x_2, x_3)) - K_3^1z_3 \]
Continuing in a similar manner and from the \(i\)-th row of the state-space description of the system given in Eq. (9), and also using that \(z_i = x_i - x_i^* = x_i - \alpha_{i-1}\) one obtains

\[
\alpha_i = x_{i+1}^* = \frac{1}{g_i(x_1, x_2, \ldots, x_i)} \cdot (\dot{x}_i^* - f_i(x_1, x_2, \ldots, x_i) - K_i(x_i - x_i^*)) \Rightarrow
\]

\[
\alpha_i = x_{i+1}^* = \frac{1}{g_i(x_1, x_2, \ldots, x_i)} \cdot (\dot{\alpha}_{i-1} - f_i(x_1, x_2, \ldots, x_3) - K_1(z_i))
\]

Equivalently, from the \(n-1\)-th row of the state-space model of Eq. (9) and using that \(z_{n-1} = x_{n-1} - x_{n-1}^* = x_{n-1} - \alpha_{n-2}\) one has

\[
\alpha_{n-1} = x_{n}^* = \frac{1}{g_{n-1}(x_1, x_2, \ldots, x_{n-1})} \cdot (\dot{x}_{n-1}^* - f_{n-1}(x_1, x_2, \ldots, x_{n-1}) - K_{n-1}^n(x_{n-1} - x_{n-1}^*)) \Rightarrow
\]

\[
\alpha_{n-1} = x_{n}^* = \frac{1}{g_{n-1}(x_1, x_2, \ldots, x_{n-1})} \cdot (\dot{\alpha}_{n-2} - f_{n-1}(x_1, x_2, \ldots, x_{n-1}) - K_{n-1}^n z_{n-1})
\]

Finally, from the \(n\)-th row of the state-space model of Eq. (9) and using that \(z_n = x_n - x_n^* = x_n - \alpha_{n-1}\) one has

\[
\alpha_n = u = \frac{1}{g_n(x_1, x_2, \ldots, x_n)} \cdot (\dot{x}_n^* - f_n(x_1, x_2, \ldots, x_n) - K^n_1(x_n - x_n^*)) \Rightarrow
\]

\[
\alpha_n = x_n^* = \frac{1}{g_n(x_1, x_2, \ldots, x_n)} \cdot (\dot{\alpha}_{n-1} - f_n(x_1, x_2, \ldots, x_n) - K^n_1 z_n)
\]

The computation of the control input \(u\) that should be actually applied to the nonlinear system is performed in a recursive manner by processing backwards the virtual control inputs described in Eq. (15) to Eq. (20).

Thus, from the last row of the state-space description, the control input that is actually applied to the nonlinear system is found. This control input contains recursively all virtual control inputs which were computed for the individual subsystems associated with the previous rows of the state-space equation. Consequently, by tracing the rows of the state-space model backwards, at each iteration of the control algorithm, one can finally obtain the control input that should be applied to the nonlinear system so as to assure that all its state vector elements will converge to the desirable setpoints.

4. Closed-loop dynamics

By substituting Eq. (20) into the last row of the state space model of Eq. (9), and using the definition \(\dot{x}_n - \dot{\alpha}_{n-1} = z_n\), one obtains:

\[
\dot{x}_n = \dot{\alpha}_{n-1} - K^n_1(x_n - \alpha_{n-1}) \Rightarrow
\]

\[
(\dot{x}_n - \dot{\alpha}_{n-1}) + K^n_1(x_n - \alpha_{n-1}) = 0 \Rightarrow \\
\dot{x}_n + K^n_1 z_n = 0
\]

(21)
By substituting Eq. (19) into the last row of the state space model of Eq. (9), and using the definition \( \dot{x}_{n-1} - \dot{a}_{n-2} = z_{n-1} \), one obtains:

\[
\begin{align*}
\dot{x}_{n-1} &= \dot{a}_{n-2} - K_1^{n-1}(x_{n-1} - \alpha_{n-2}) \\
(\dot{x}_{n-1} - \dot{a}_{n-2}) + K_1^{n-1}(x_{n-1} - \alpha_{n-2}) &= 0 \Rightarrow \\
\dot{z}_{n-1} + K_1^{n-1}z_{n-1} &= 0
\end{align*}
\]  

(22)

By substituting Eq. (18) into the last row of the state space model of Eq. (9), and using the definition \( \dot{x}_i - \dot{a}_{i-1} = z_i \), one obtains:

\[
\begin{align*}
\dot{x}_i &= \dot{a}_{i-1} - K_1^{n-1}(x_i - \alpha_{i-1}) \\
(\dot{x}_i - \dot{a}_{i-1}) + K_1^{n-1}(x_i - \alpha_{i-1}) &= 0 \Rightarrow \\
\dot{z}_i + K_1^{n-1}z_i &= 0
\end{align*}
\]  

(23)

while continuing backwards and by substituting Eq. (16) into the second row of the state space model of Eq. (9), and using the definition \( \dot{x}_2 - \dot{a}_1 = z_2 \), one gets:

\[
\begin{align*}
\dot{x}_2 &= \dot{a}_1 - K_2^{n-2}(x_2 - \alpha_1) \\
(\dot{x}_2 - \dot{a}_1) + K_2^{n-2}(x_2 - \alpha_1) &= 0 \Rightarrow \\
\dot{z}_2 + K_2^{n-2}z_2 &= 0
\end{align*}
\]  

(24)

Finally, by substituting Eq. (15) into the first row of the state space model of Eq. (9), one has:

\[
\begin{align*}
\dot{x}_1 &= \dot{x}_1 - K_1^{n-1}(x_1 - x_1^d) \\
(\dot{x}_1 - \dot{x}_1^d) + K_1^{n-1}(x_1 - x_1^d) &= 0 \Rightarrow \\
\dot{z}_1 + K_1^{n-1}z_1 &= 0
\end{align*}
\]  

(25)

Therefore, after the application of the feedback control law, the closed-loop dynamics becomes

\[
\begin{align*}
\dot{z}_1 + K_1^{n}z_1 &= 0, \quad \dot{z}_2 + K_2^{n}z_2 &= 0, \quad \ldots, \quad \dot{z}_i + K_i^{n}z_i &= 0, \quad \ldots, \quad \dot{z}_{n-1} + K_{n-1}^{n-1}z_{n-1} &= 0, \quad \dot{z}_n + K_n^{n}z_n &= 0.
\end{align*}
\]

In matrix form, the closed-loop dynamics is written as

\[
\begin{pmatrix}
\dot{z}_1 \\
\dot{z}_2 \\
\dot{z}_3 \\
\vdots \\
\dot{z}_i \\
\vdots \\
\dot{z}_{n-1} \\
\dot{z}_n
\end{pmatrix} = 
\begin{pmatrix}
-K_1^1 & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 \\
0 & -K_2^1 & 0 & \cdots & 0 & \cdots & 0 & 0 \\
0 & 0 & -K_3^1 & \cdots & 0 & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & \cdots & -K_i^1 & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & \cdots & 0 & \cdots & -K_{n-1}^{n-1} & 0 \\
0 & 0 & 0 & \cdots & 0 & \cdots & 0 & -K_n^{n}
\end{pmatrix}
\begin{pmatrix}
z_1 \\
z_2 \\
z_3 \\
\vdots \\
z_i \\
\vdots \\
z_{n-1} \\
z_n
\end{pmatrix}
\]  

(26)

or equivalently

\[
\dot{Z} = KZ
\]  

(27)

By selecting the eigenvalues of matrix \( K \) to be in the left complex semiplane, one has that

\[
\lim_{t \to \infty} Z = 0_{n \times 1}
\]  

(28)

which also implies that \( \lim_{t \to \infty} x_1 = x_1^* \), \( \lim_{t \to \infty} x_2 = x_2^* \), \( \lim_{t \to \infty} x_3 = x_3^* \), \ldots, \( \lim_{t \to \infty} x_i = x_i^* \), \ldots, \( \lim_{t \to \infty} x_{n-1} = x_{n-1}^* \), and \( \lim_{t \to \infty} x_n = x_n^* \).

To prove asymptotic stability for the proposed control scheme the following Lyapunov function can be defined
\[ V = \sum_{i=1}^{N} \frac{1}{2} z_i^2 \]  

The time derivative of the aforementioned Lyapunov function is

\[ \dot{V} = \sum_{i=1}^{N} z_i \dot{z}_i \Rightarrow \dot{V} = -\sum_{i=1}^{N} K_i z_i^2 \Rightarrow \dot{V} < 0 \]  

By selecting the feedback control gains \( K_i, i = 1, \cdots, n \) to be \( K_i > 0 \), the asymptotic stability of the control loop is assured.

5. Simulation tests
Simulation tests have been performed for the evaluation of the proposed nonlinear control scheme, that is based on implementation of a flatness-based controller in successive loops. Indicative results where obtained for the tracking of three different setpoints. It can be observed that the SI engine’s state vector elements (that is engine’s turn speed \( \omega \), input pressure subjected to time delay \( P_{m, d} \), and input pressure \( P_m \) ) converged fast and smoothly to the associated setpoints. The associated results in the case of tracking of setpoint 1 are depicted in Fig. 2 to Fig. 3. Additional results for the case of tracking of setpoint 2 are shown in Fig. 4 to Fig. 5. Finally, the case of tracking of setpoint 3 which exhibits more rapid variations is depicted in Fig. 6 to Fig. 7.

![Figure 2](image1.png)

(a) (b)

Figure 2. Tracking of setpoint 1 from (a) state variables \( x_1 \), (b) state variables \( x_2 \)

It can be also noted that the so-called backstepping control which is based on the recursive computation of the control signal of the system after applying virtual control inputs to the individual rows of the state-space model, can be completely substituted by the proposed flatness-based control method [Khalil, 1996], [Rigatos, 2011a],[Rigatos, 2011b],[Rigatos, 2006]. Moreover, it is noted that the proposed control method stands for a different implementation of flatness-based control in successive loops, in which there is no need for prior transformation of the system’s dynamics in the canonical (trivial form) through the application of a change of variables (diffeomorphisms). Therefore, the control input is computed directly on the initial nonlinear state-space description of the system and not on an equivalent linearized form of it.
6. Conclusions
A nonlinear embedded control scheme for spark-ignited (SI) engines has been proposed in this article. The method is based on differential flatness theory. The method makes use of a description of the state-space model of the SI-engine in the so-called triangular form. In the proposed controller design method each row of the state-space model of the SI-engine is shown to be a differentially flat system, while the associated state variable is taken to be the flat output. Next, a virtual control input is computed for each subsystem which is obtained from a row of the state-space model. The virtual control input can invert the subsystem’s dynamics while also eliminating the subsystem’s tracking error. The control input that is actually applied to the SI engine is computed from the last row of the state-space description. This control input contains recursively all virtual control inputs which were found for the individual subsystems associated with the previous rows of the state-space equation. Thus, at each iteration of the
control algorithm and by tracing the rows of the state-space model backwards, one can finally obtain the control input that should be applied to the SI-engine so as to assure that all its state vector elements will converge to the desirable setpoints.

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Figure 7. (a) Tracking of setpoint 3 from state variables $x_3$, (b) control input $u$. 

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