Novel Multicriteria Decision Making Approach for Interactive Aggregation Operators of q-Rung Orthopair Fuzzy Soft Set

RANA MUHAMMAD ZULQARNAIN\textsuperscript{1}, IMRAN SIDDIQUE\textsuperscript{2}, AIYARED IAMPAN\textsuperscript{3}, JAN AWREJCEWICZ\textsuperscript{4}, MAKSYMILIAN BEDNAREK\textsuperscript{4}, RIFAQAT ALI\textsuperscript{5}, AND MUHAMMAD ASIF\textsuperscript{1}

\textsuperscript{1}Department of Mathematics, University of Management and Technology, Sialkot 51310, Pakistan
\textsuperscript{2}Department of Mathematics, University of Management and Technology, Lahore 54770, Pakistan
\textsuperscript{3}Department of Mathematics, School of Science, University of Phayao, Mueang, Phayao 56000, Thailand
\textsuperscript{4}Department of Automation, Biomechanics and Mechatronics, Lodz University of Technology, 90-924 Lodz, Poland
\textsuperscript{5}Department of Mathematics, College of Science and Arts, Muhayil, King Khalid University, Abha 61413, Saudi Arabia

Corresponding authors: Imran Siddique (imransiddique@umt.edu.pk) and Muhammad Asif (muhammad.asif@math.qau.edu.pk)

This work was supported by the Deanship of Scientific Research at King Khalid University through Large Groups Project under Grant R.G.P. 2/51/43.

\textbf{ABSTRACT} The q-rung orthopair fuzzy soft set is an advanced leeway of orthopair fuzzy sets, such as intuitionistic fuzzy soft set (IFSS) and Pythagorean fuzzy soft set (PFSS). The very lax requirement gives the evaluators excessive freedom to express their beliefs about membership degrees and non-membership degrees, making q-rung orthopair fuzzy soft set (q-ROFSS) have a broad scope of application in practical life. Considering the interaction, this paper will develop some novel operational laws for q-rung orthopair fuzzy soft numbers (q-ROFSNs). Some aggregation operators (AOs) such as q-rung orthopair fuzzy soft interactive weighted average (q-ROFSIWA) and q-rung orthopair fuzzy soft interactive weighted geometric (q-ROFSIWG) operators have been introduced. Also, we will prove some desirable properties such as idempotency, boundedness, and homogeneity for q-ROFSIWA and q-ROFSIWG operators. Multi-criteria decision-making (MCDM) plays a vital role to deals with the complications in manufacturing design for material selection. But, the prevailing MCDM methods habitually deliver incompatible outcomes. Based on the projected interactive aggregation operators, a robust MCDM method is planned for material selection in manufacturing design to accommodate this drawback. Furthermore, a comprehensive comparative analysis has been presented to confirm the pragmatism and validity of the proposed technique with some already available methods. The obtained outcomes through comparative studies show that our developed method is more efficient than existing approaches.

\textbf{INDEX TERMS} MCDM, q-rung orthopair fuzzy soft set, q-ROFSIWA operator, q-ROFSIWG operator.

\section{I. INTRODUCTION}

Problem-solving in our everyday lives is based on information assortment, data exploration, facts aggregation, etc. In decision analysis, the significant query is the absence of precise facts. This statistics gap can by taking a scientific model to fill suitable DM processes. The DM ideologies can support manufacturing enterprise, assemble, and rank several preferences from best to worst viable alternatives. Consequently, it is a source for choosing, categorizing, and establishing our prospects and helping the comprehensive evaluation of alternatives. MS is severe in the enterprise and growth of the product. The selected material influences the achievement and affordability of the manufacturer [1]. Manufacturing enterprise is obsessed with enactment, cost, and globally penetrating objectives, often inadequate by materials. The persistence of enhancing manufactured goods strategy is to select ingredients with the most advanced light design criteria while providing a supreme presentation at the lowest conceivable price [2]. However, in some contending circumstances, these objectives and constraints are common, so resolving which feature is more important than others is essential. The funding and resource aspects of the design method can be unexploited in the restructure or industrialized
MCDM is considered the most suitable method for finding the most acceptable alternative out of all possible, following criteria or attributes. Conventionally, it is supposed that all information that accesses the alternative in terms of attributes and their corresponding weights are articulated in the form of crisp numbers. On the other hand, most decisions are taken in situations where the objectives and limitations are usually indefinite or ambiguous in real-life circumstances. To overcome such ambiguities and anxieties, Zadeh offered the fuzzy set (FS) [5], a prevailing tool to handle the obscurities and uncertainties in DM. Such a set allocates to all objects a membership value ranging from 0 to 1. Experts mostly consider membership and a non-membership value in the DM process that FS cannot handle. Atanassov [6] introduced the idea of the intuitionistic fuzzy set (IFS) to overcome the limitation, as mentioned earlier. Wang and Liu [7] presented numerous operations on IFS, such as Einstein product, Einstein sum, etc., and constructed some novel AOs. They also discussed some essential properties of these operators and utilized their proposed operators to resolve multi-attribute decision making (MADM).

The presented FS and IFS models cannot handle the inappropriate and vague data because it is considered to envision the linear inequality between the membership (MG) and non-membership (NMG) grades. For example, if decision-makers choose MG and NMG values 0.9 and 0.6, respectively, the IFS mentioned above theory cannot deal with it because 0.9 + 0.6 ≥ 1. To resolve the limitation described above, Yager [8] presented the idea of the Pythagorean fuzzy set (PFS) by amending the basic condition \( \mu + \theta \leq 1 \) to \( \mu^2 + \theta^2 \leq 1 \) and developed some results associated with score function and accuracy function. Rahman et al. [9] developed the Pythagorean fuzzy Einstein weighted geometric operator and presented a MAGDM methodology utilizing their proposed operator. Zhang and Xu [10] developed some basic operational laws and prolonged the TOPSIS method to resolve multi-criteria decision-making (MCDM) complications for PFS information. Wei and Lu [11] offered the Pythagorean fuzzy power AOs and essential characteristics. They also established a decision-making (DM) technique to resolve MADM difficulties based on presented operators. Wang and Li [12] offered the interaction operational laws for Pythagorean fuzzy numbers (PFNs), and developed power Bonferroni mean operators. Ilbahr et al. [13] presented the Pythagorean fuzzy proportional risk assessment technique to assess the professional health risk. Zhang [14] proposed a novel DM approach based on similarity measures to resolve multi-criteria group decision making (MCGDM) difficulties for the PFS.

Over the past few decades, many intellectuals have resorted directly to MCDM techniques to identify some additional collective innovative mathematical tools for dealing with various real-world problems. The MCDM model explains how facts should be organized according to an appropriate purpose or article to assist in decision-making. In the literature, MCDM techniques have been used in various behavioral domains, along with manufacturing. There is no doubt that PFS theory plays a crucial part in resolving numerous real-world MCDM complications, but this model still has an inaccuracy. The PFS cannot cope with the situation when the square of the sum of MG and NMG exceeds one. To overcome these complications, Yager [15] proposed a generalization of intuitionistic fuzzy sets and Pythagorean fuzzy sets, referred to as q-rung orthopair fuzzy sets (q-ROFS). Which provides the systems modelers more freedom in expressing their beliefs about MG and NMG degrees. For example, \( (0.7, 0.8) \) is a q-rung orthopair membership degree \( q \geq 3 \) since \( 0.7^3 + 0.8^3 \leq 1 \), and so it is suitable to utilize the q-ROFS to treat such situations. Riazi et al. [16]–[18] described some aggregation operators such as geometric, hybrid, Einstein to fuse the q-rung orthopair fuzzy information. Liu and Wang [19] discussed some new q-rung orthopair fuzzy weighted averaging and geometric operators to deal with the decision-making, and their desirable properties are well proved. Liu et al. [20] introduced the complex q-rung orthopair fuzzy AOs to solve the MAGDM problem. Xu et al. [21] improved the existing q-rung orthopair fuzzy aggregation operators and applied them to multi-attribute group decision-making. Wang et al. [22] defined the similarity measures for q-ROFS utilizing cosine function. Peng and Liu [23] analyzed the relationships between distance measures and similarity measures of q-ROFS, improved the systematic transformation of information measures (distance measure and similarity measure) for q-ROFSs, and presented some new formulas of information measures of q-ROFS. Liu et al. [24] introduced the Heronian mean operators for q-ROFS and prolonged a MAGDM approach using their developed operators. Liu et al. [25] presented the power Maclaurin symmetric mean operators for q-ROFS and established a MAGDM technique.

All the methods mentioned above have too many applications in many fields. But due to their inefficiency, these methods have many limitations in terms of parameterization tools. Presenting the solution of this sort of obscurity and ambiguity in 1999, Molodtsov [26] proposed the soft set, a parameterized family of universe subsets. The soft set is an exciting and emerging concept that helps us deal with uncertainties in life and guide us to make effective decisions and choices. In recent years, many researchers have studied the fundamentals of the soft set. It is an efficient tool to deal with the problems of uncertainty and unpredictability compared to classical mathematical tools and helps us make a good decision. Maji et al. [27] extended the concept of SS and defined numerous basic operations with their features. While the exploration consequences of the SS leeway ideal
have been identical productive, the incompatible bipolarity is unescapable in the impartial domain. Just as a sample of a psychology disease-bipolar syndrome patient who has indications of obsession and melancholy, the sum of positive and negative symptoms value is more than one. Even both the two symptoms may reach extreme cases simultaneously. Shabir and Naz [28] presented the bipolar soft sets with some basic operations and associated properties. Recently, the incorporation of bipolar-valued soft sets had drained the devotion of several researchers, and worthy consequences attained.

Ali et al. [29] protracted the idea of q-rung orthopair fuzzy bipolar soft sets and established a MADM approach to choose the most competent student for the scholarship award. Maji et al. [30] proposed FSS by merging SS and FS. Maji et al. [31] presented the theory of IFSS with some fundamental operations and their properties. Garg and Arora [32] projected the general AOs for the IFSS. Arora and Garg [33] introduced AOs for IFSS with desirable properties. They also built the MCDM method utilizing their settled operators to resolve DM complications. Peng et al. [34] upgraded the extended the condition of $MG + NMG \leq 1$ to $MG^2 + NMG^2 \leq 1$ of IFSS and developed the PFSS. Athira et al. [35] protracted novel entropy measure for PFSS. Naeem et al. [36] developed linguistic PFSS and established TOPSIS and VIKOR methods under-considered setting. Riaz et al. [37] presented $m$ polar PFSS with some fundamental operations and constructed the TOPSIS method to solve multi-criteria group decision making (MCGDM) complications. Riaz et al. [38] established a DM approach for PFSS using their proposed similarity measures. Zulqarnain et al. [39] proposed the AOs for PFSS and constructed a DM approach utilizing their settled operators. Zulqarnain et al. [40] protracted the operational laws for PFSS and introduced interactive operational laws for PFSS. They also presented the interactive AOs for PFSS and developed an MCDM method to solve DM problems.

Hussain et al. [41] presented the concept of q-ROFSS by merging two well-known theories such as SS and q-ROFS. They also introduced the q-rung orthopair fuzzy soft weighted average operator for q-ROFSS and developed the MCDM approach to solving DM concerns. Chinram et al. [42] created the q-Rung orthopair fuzzy soft weighted geometric operator for q-ROFSS with desirable properties. They also utilized their developed operator to solve MCDM complications. However, the above methods are not appropriate for confirming the higher than q-rung orthopair fuzzy soft numbers (q-ROFSN). Neither can it consider the interaction among MG and NMG values. We can mainly vocalize the effect of MG and NMG at exceptional levels on the AOs.

A. MOTIVATION AND DRAWBACK OF EXISTING APPROACHES

The q-rung orthopair fuzzy soft sets, a hybrid intellectual structure of soft sets, PFSS, and the q-rung orthopair fuzzy sets are influential mathematical tools for dealing with indefinite, asynchronous, and partial information. It has been detected that AOs are imperative in decision-making so that communally evaluated information from different sources can be composed in a special assessment. To the best of our understanding, interaction AOs with hybridization with a soft set of q-ROFS has no application in the literature. Still, not all of the above techniques are appropriate for summarizing q-rung orthopair fuzzy soft numbers (q-ROFSNs), and they cannot be deliberately connected with MG and NMG. In particular, we can say that the consequence of MG or other degrees of NMG on the related geometric or average AOs does not affect the overall process. In addition, the model states that the overall MG (NMG) function level is independent of its NMG (MG) function level. Therefore, according to these models, the outcomes are not constructive, so no proper preference is stated for alternatives. Thus, how to incorporate these q-ROFSNs over interactions is a captivating issue. We will progress some q-rung orthopair fuzzy soft interactive AOs for q-ROFSS such as q-ROFSIWA and q-ROFSIWG operators to address these issues. Since the prevailing FS, IFS, SS, IFSS, PFS, and PFSS are exceptional cases of q-ROFSS, the established interactive AOs are extra capable compared to AOs of existing hybrid structures of FS. Therefore, the results of the existing models are unfavorable, and no proper priority order for alternatives is given. So, adding these q-ROFSNs through interactive relationships is an exciting topic. The approaches designated in [41], [42] are insufficient to inspect the information in a deep intellect for superior conception and accurate conclusions. For example, $Q = \{Q^1, Q^2\}$ be a set of two experts and $e_1, e_2$ are two parameters. Where $X$ be an alternative, then preferences of experts can be summarized as $X = \left[\begin{array}{c}(0.7, 0) \\ (0.8, 0.7) \end{array}\right]$. Let $\Omega_i = (0.7, 0.3)^T$ and $\gamma_j = (0.4, 0.6)^T$ represents the weights of experts and parameters, respectively. Then, we obtained the aggregated value using the q-ROFSWA [41] operator is $(0.681981, 0)$. Similarly, we utilized the q-ROFSWG [42] operator and acquired an aggregated value $(0.666713, 0)$. This clearly shows that there is no effect on the collective result $\theta_c$. Because $\vartheta = \vartheta_{11} = 0, \vartheta_{12} = 0.7, \vartheta_{21} = 0.7, \vartheta_{22} = 0.2$, which is unreasonable. An enhanced sorting approach fascinates investigators to crack baffling and inadequate information. Rendering to the investigation outcomes, q-ROFSS plays a vital role in decision-making by collecting numerous sources into a single value.

B. CONTRIBUTION

The interaction AOs are a well-known charming gospelize AOs. It has been noticed that the prevailing AOs look unenthusiastic to mark the exact judgment through the DM procedure in some circumstances. To overwhelm these particular difficulties, these AOs need to be amended. Therefore, to inspire the current research and limitations mentioned above of q-ROFSS, we will state interaction AOs based on rough data, the fundamental objectives of the following study are given as follows:
1) We demonstrate advanced operational laws based on interaction for q-rung orthopair fuzzy soft numbers (q-ROFSNs).
2) The q-ROFSS competently deals with the complex issues considering the parameters in the DM process. To keep this advantage in mind, we establish the interaction AOs for q-ROFSS.
3) The q-ROFSIWA and q-ROFSIWG operators have been established using the interaction mentioned above operational laws with some fundamental properties.
4) A novel MCDM technique was established based on the proposed AOs to cope with DM issues under the q-ROFSS environment.
5) MS is a significant aspect of engineering as it sees the practical standards of all constituents. MS is a time-consuming but significant step in the enterprise procedure. The industrialist’s productivity, effectiveness, and character will suffer as an outcome deprived of material selection.
6) Comparative analysis of the developed MCDM technique and current approaches has been proposed to deliberate the practicability and supremacy.

This study is systematized as follows: Basic knowledge of some essential notions like SS, PFS, PFSS, q-ROFSS are deliberated in section 2. Section 3 demarcated some basic operational laws for q-ROFSS based on interaction and established the q-ROFSIWA and q-ROFSIWG operators. Also, some dynamic properties of the planned operators have been debated in the same section. In section 4, an MCDM approach is introduced using the q-ROFSIWA and q-ROFSIWG operators. In the same case, a study has been presented for material selection in the manufacturing industry. In Section 5, a comparison with some standing approaches is provided.

II. PRELIMINARIES

This section will assemble important definitions that assist us in constructing the subsequent article structure, like SS, PFS, and q-ROFSS.

A. DEFINITION 1 [26]

$\Omega$ and $E$ be the universe of discourse and set of attributes. $\mathcal{P}(\Omega)$ represents the power set of $\Omega$ and $A \subseteq E$. Then, a mapping $F$ from $A$ and $P(\Omega)$ is defined as follows:

$$F : A \rightarrow \mathcal{P}(\Omega)$$

It can be defined as follows:

$$(F, A) = \{ (F(e) \in P(\Omega) : e \in E, F(e) = \emptyset \text{ if } e \notin A) \}$$

B. DEFINITION 2 [8]

Let $\Omega$ be a universal set. A PFS over $\Omega$ is stated as $\varphi = \{(\mu, \varphi(\mu))(\mu \in \Omega), \mu(\varphi) and \varphi(\mu(\varphi)) represents the MD and NMD and satisfies the following conditions

$$\mu(\varphi) \leq [0, 1], (\mu(\varphi))^2 + (\varphi(\mu(\varphi)))^2 \leq 1$$

for all $\mu \in \Omega$. Where $\Sigma(\varphi(u)) = \sqrt{1 - (\mu(\varphi(u)))^2 + (\varphi(\mu(\varphi)))^2}$ represents the degree of hesitation.

C. DEFINITION 3 [34]

$\Omega$ and $E$ be the universe of discourse and set of attributes. The PFSS over $\Omega$ is defined as follows: $F(\omega, (\varepsilon)) = \{(\omega_i, (\mu_j(\omega_i)), \theta_j(\omega_i)) | \omega_i \in \Omega \}$, where $F$ is a mapping among $E$ and $PK^{\Omega}$ such as

$$F : E \rightarrow PK^{\Omega}$$

$PK^{\Omega}$ represents the collection of Pythagorean fuzzy subsets of $\Omega$ and satisfied the following condition $\mu^2 + \varphi^2 \leq 1$.

For the sake of readers’ convenience, the PFSN can be expressed as $\mathcal{N}(\omega) = \{(\mu_j, \varphi_j)\}$.

Assume three PFSNs such as $\mathcal{N}(\omega) = (\mu, \varphi, \omega) = (\mu(1), \omega(1))$, and $\mathcal{N}(\omega_1) = (\mu(1), \omega(1))$ and $\mathcal{N}(\omega_2) = (\mu(2), \omega(2))$ and $\alpha > 0$. Then, some fundamental operations are given as follows:

1) $\mathcal{N}(\omega_1) \oplus \mathcal{N}(\omega_2) = \{\sqrt{\mu(1)^2 + \mu(2)^2 - \mu(1)\mu(2)} + \sqrt{\omega(1)^2 - \omega(2)^2} \}$

2) $\mathcal{N}(\omega_1) \otimes \mathcal{N}(\omega_2) = \{\mu(1)\mu(2) + \sqrt{\omega(1)\omega(2)}\}$

3) $\alpha \mathcal{N}(\omega) = \{\mu^\alpha\varphi^\alpha, \omega\}$

4) $\mathcal{N}(\omega) = \{\mu^\alpha, \omega\}$

D. DEFINITION 4 [41]

Let $(\Omega, E)$ be a SS and $F \subseteq E$. Then, a pair $(\Omega, F)$ be a q-ROFSS over $\Omega$. Where, $\Omega : F \rightarrow q$ $ROFS^{(\Omega)}$, where represents the set of all $q$ $ROFS^{(\Omega)}$ sets and defined as:

$$\mathcal{N}(\omega_i) = \{(\omega_i, (\mu^q(\omega_i)), \theta^q(\omega_i)) | \omega_i \in \Omega, q \geq 3 \}$$

where $q$ $ROFS^{(\Omega)}$ be a collection of all subsets of q-ROFSS of $\Omega$ and $\mu^q(\omega_i)$, $\theta^q(\omega_i)$ represents the MD and NMD of an object of $\mathcal{N}(\omega)$ for each $\omega_i \in \Omega$ with the following condition $0 \leq (\mu^q(\omega_i)) + (\theta^q(\omega_i)) \leq 1$, $q \geq 3$. Simply, $\mathcal{N}(\omega_i) = \{(\omega_i, (\mu^q(\omega_i)), \theta^q(\omega_i)) \}$ can be written $\mathcal{N}(\omega) = (\mu, \varphi, \omega)$ that is called q-ROFS number (q-ROFSN). The degree of hesitancy for q-ROFSN can be expressed as $\sum_{\omega} = \sqrt{1 - (\mu^q(\omega))^2 + (\varphi^q(\omega))^2}$.

Assume three-q-ROFSNs such as $\mathcal{N}(\omega) = (\mu, \varphi, \omega) = (\mu(1), \omega(1))$, and $\mathcal{N}(\omega_1) = (\mu(1), \omega(1))$ and $\mathcal{N}(\omega_2) = (\mu(2), \omega(2))$ and $\alpha > 0$, then some operations are defined as follows [41]:

1) $\mathcal{N}(\omega_1) \oplus \mathcal{N}(\omega_2) = \{\sqrt{(\mu(1))^2 + (\mu(2))^2 - (\mu(1)\mu(2)) + (\varphi(1))^2 + (\varphi(2))^2 - (\varphi(1)\varphi(2))} \}$

2) $\mathcal{N}(\omega_1) \otimes \mathcal{N}(\omega_2) = \{\mu(1)\mu(2) + \varphi(1)\varphi(2)\}$

3) $\alpha \mathcal{N}(\omega) = \{\mu^\alpha, \omega\}$

4) $\mathcal{N}(\omega) = \{\mu^\alpha, \omega\}$

For the collection of q-ROFSNs $\mathcal{N}(\omega) = (\mu, \varphi, \omega)$, where $(i = 1, 2, \ldots, n, and j = 1, 2, \ldots, m)$. Where, $\Omega = (\Omega_1, \Omega_2, \ldots, \Omega_n)^T$ and $\gamma = (\gamma_1, \gamma_2, \ldots, \gamma_m)^T$ represents the weight vectors for experts and attributes such as $\Omega_i > 0$, $\sum_{i=1}^n \Omega_i = 1$, $\gamma_j > 0$, $\sum_{j=1}^m \gamma_j = 1$. 

VOLUME 10, 2022
Hussain et al. [41] offered the q-ROFSWA, and Chirn et al. [42] developed the q-ROFSWG operators such as follows:

\[
q - \text{ROFSWA} (\mathbf{N}_{e_{11}}, \mathbf{N}_{e_{12}}, \ldots, \mathbf{N}_{e_{e_{2u}}}) = \left( \sqrt[\gamma]{1 - \prod_{j=1}^{m} \left( \prod_{i=1}^{n} \left( 1 - \mu_{ij}^q \Omega^j \right) \right)^\gamma} \right),
\]

\[
q - \text{ROFSWG} (\mathbf{N}_{e_{11}}, \mathbf{N}_{e_{12}}, \ldots, \mathbf{N}_{e_{e_{2u}}}) = \left( \prod_{j=1}^{m} \left( \prod_{i=1}^{n} \left( \mu_{ij}^q \Omega^j \right) \right)^\gamma \right).
\]

**E. DEFINITION 5**

Let \( \mathbf{N}_{e_{ij}} = (\mu_{ij}, \vartheta_{ij}) \) be a q-ROFSN, then score function can be defined as follows:

\[
S(\mathbf{N}_{e_{ij}}) = \mu_{ij}^q - \vartheta_{ij}^q + \frac{e^{\mu_{ij}^q - \vartheta_{ij}^q} - 1}{2} - \frac{q}{\gamma} \mathbf{N}_{e_{ij}},
\]

for \( q \geq 3 \) and \( S(\mathbf{N}_{e_{ij}}) \in [-1, 1] \). (1)

Let \( \mathbf{N}_{e_{11}} = (\mu_{11}, \vartheta_{11}) \) and \( \mathbf{N}_{e_{12}} = (\mu_{12}, \vartheta_{12}) \) be two q-ROFSNs. Then

- If \( S(\mathbf{N}_{e_{11}}) > S(\mathbf{N}_{e_{12}}) \), then \( \mathbf{N}_{e_{11}} \geq \mathbf{N}_{e_{12}} \).
- If \( S(\mathbf{N}_{e_{11}}) < S(\mathbf{N}_{e_{12}}) \), then \( \mathbf{N}_{e_{11}} \leq \mathbf{N}_{e_{12}} \).
- If \( S(\mathbf{N}_{e_{11}}) = S(\mathbf{N}_{e_{12}}) \), then \( \mathbf{N}_{e_{11}} = \mathbf{N}_{e_{12}} \).

**III. INTERACTION AGGREGATION OPERATORS FOR Q-RUNG ORTHOPAIR FUZZY SOFT NUMBERS**

This section presents interaction operational laws for q-ROFSNs. Furthermore, some AOs for q-ROFSNs have been proposed, such as q-ROFSIWA and q-ROFSIWG operators with their desirable properties.

**A. INTERACTION OPERATIONAL LAWS FOR Q-ROFSNS**

1) **DEFINITION**

Let \( \mathbf{N}_e = (\mu, \vartheta) \), \( \mathbf{N}_{e_{11}} = (\mu_{11}, \vartheta_{11}) \), and \( \mathbf{N}_{e_{12}} = (\mu_{12}, \vartheta_{12}) \) be three q-ROFSNs and \( \alpha > 0 \). Then the interactive operational laws can be defined as:

1) \( \mathbf{N}_{e_{11}} \oplus \mathbf{N}_{e_{12}}, \) as shown at the top of the next page.
2) \( \mathbf{N}_{e_{11}} \otimes \mathbf{N}_{e_{12}}, \) as shown at the top of the next page.
3) \( \alpha \mathbf{N}_e = \left( \sqrt{1 - \left( 1 - \mu \right)^\alpha}, \sqrt{1 - \left( 1 - \vartheta \right)^\alpha} \right), \)

\[
\mathbf{N}_e = \left( \sqrt{1 - \left( 1 - \mu \right)^\alpha}, \sqrt{1 - \left( 1 - \vartheta \right)^\alpha} \right),
\]

4) \( \mathbf{N}_e = \left( \sqrt{1 - \left( 1 - \mu \right)^\alpha}, \sqrt{1 - \left( 1 - \vartheta \right)^\alpha} \right), \)

We will use the above presented operational laws to develop some AOs for q-ROFSSS by considering the interaction of q-ROFSIWA and q-ROFSIWG operators.

2) **DEFINITION**

Let \( \mathbf{N}_{e_{ij}} = (\mu_{ij}, \vartheta_{ij}) \) be a collection of q-ROFSNs, where \( i = 1, 2, \ldots, n \), and \( j = 1, 2, \ldots, m \). If q-ROFSIWA:

\[
q - \text{ROFSIWA} (\mathbf{N}_{e_{11}}, \mathbf{N}_{e_{12}}, \ldots, \mathbf{N}_{e_{e_{2u}}}) = \left( \prod_{j=1}^{m} \left( \prod_{i=1}^{n} \left( 1 - \mu_{ij}^q \Omega^j \right) \right)^\gamma \right).
\]

Then q-ROFSIW is called q-Rung orthopair fuzzy soft interactive weighted average operator. Where \( \Omega_i > 0, \sum_{i=1}^{n} \Omega_i = 1 \) and \( \gamma_j > 0, \sum_{j=1}^{m} \gamma_j = 1 \) denotes the weights of experts and attributes.

3) **THEOREM**

Let \( \mathbf{N}_{e_{ij}} = (\mu_{ij}, \vartheta_{ij}) \) be a collection of q-ROFSNs, where \( i = 1, 2, \ldots, n \), and \( j = 1, 2, \ldots, m \). Then, the obtained aggregated values is also a q-ROFSN and (3), as shown at the top of the next page, where \( \Omega_i \) and \( \gamma_j \) represents the weights of the experts and attributes respectively, such as \( \Omega_i > 0, \sum_{i=1}^{n} \Omega_i = 1, \gamma_j > 0, \sum_{j=1}^{m} \gamma_j = 1 \).

**Proof:** The proof of the q-ROFSIWA operator can be given by using mathematical induction such as follows:

We will prove for \( n = 2 \) and \( m = 2 \). Then, we have

\[
q - \text{ROFSIWA}(\mathbf{N}_{e_{11}}, \mathbf{N}_{e_{12}}, \mathbf{N}_{e_{21}}, \mathbf{N}_{e_{22}}).
\]

Now suppose that Equation (3) valid for \( n = \beta_1 \) and \( m = \beta_2 \). Then, we have \( \beta_2 \) q-ROFSIWA, as shown at the top of the page 7. For \( n = \beta_1 + 1 \) and \( m = \beta_2 + 1 \), we have \( \beta_2 + 1 \) q-ROFSIWA, as shown at the top of the page 7. This shows that Equation (3) is true for all \( n, m \geq 1 \).

The most interesting thing is that the obtained aggregated values using the q-ROFSIWA operator are also q-ROFSN. To prove this, consider \( \mathbf{N}_{e_{ij}} = (\mu_{ij}, \vartheta_{ij}) \), where \( i = 1, 2, \ldots, n, j = 1, 2, \ldots, m \), \( 0 \leq \mu_{ij}, \vartheta_{ij} \leq 1 \) and satisfying the 0 \( \leq \mu_{ij}^q + \vartheta_{ij}^q \leq 1 \). Where \( \Omega_i \) and \( \gamma_j \) are the weight vectors for experts and attributes respectively and having conditions that \( \Omega_i, \gamma_j \in [0, 1], \Omega_i > 0, \sum_{i=1}^{n} \Omega_i = 1, \gamma_j > 0, \sum_{j=1}^{m} \gamma_j = 1 \).

As we know that

\[
0 \leq \mu_{ij} \leq 1 \Rightarrow 0 \leq 1 - \mu_{ij} \leq 1 \Rightarrow 0 \leq \left( 1 - \mu_{ij}^q \right) \Omega_i^j \leq 1
\]

\[
0 \leq \prod_{i=1}^{n} \left( 1 - \mu_{ij}^q \right) \Omega_i^j \leq 1
\]

\[
0 \leq \prod_{i=1}^{n} \left( 1 - \mu_{ij}^q \right) \Omega_i^j \leq 1
\]

Similarly,

\[
0 \leq \mu_{ij}^q + \vartheta_{ij}^q \leq 1 \Rightarrow 0 \leq 1 - \left( \mu_{ij}^q + \vartheta_{ij}^q \right) \leq 1
\]
\[
\kappa_{c_{11}} \oplus \kappa_{c_{12}} = \left\{ \sqrt{\left( \mu_{11} \right)^{q} + \left( \mu_{12} \right)^{q} - \left( \mu_{11} \right)^{q} \left( \mu_{12} \right)^{q}}, \sqrt{\left( \mu_{11} \right)^{q} + \left( \mu_{12} \right)^{q} - \left( \mu_{11} \right)^{q} \left( \mu_{12} \right)^{q}} \right\}
\]

\[
\kappa_{c_{11}} \oplus \kappa_{c_{12}} = \left\{ \sqrt{\left( \mu_{11} \right)^{q} + \left( \mu_{12} \right)^{q} - \left( \mu_{11} \right)^{q} \left( \mu_{12} \right)^{q}}, \sqrt{\left( \mu_{11} \right)^{q} + \left( \mu_{12} \right)^{q} - \left( \mu_{11} \right)^{q} \left( \mu_{12} \right)^{q}} \right\}
\]

\[q \text{ - ROFSIWA } (\kappa_{c_{11}}, \kappa_{c_{12}}, \ldots, \kappa_{c_{c_{m}}}) = \Theta_{m}^{n} \bigotimes_{j=1}^{m} \Omega_{j} \kappa_{c_{j}}\]

\[= \left( \sqrt{1 - \prod_{j=1}^{m} \left( \prod_{i=1}^{n} \left( 1 - \mu_{ij}^{q} \right)^{\Omega_{j}} \right)^{\gamma_{j}}} \right)^{m} - \left( \prod_{j=1}^{m} \left( \prod_{i=1}^{n} \left[ 1 - \left( \mu_{ij}^{q} + \theta_{ij}^{q} \right) \right]^{\Omega_{j}} \right)^{\gamma_{j}} \right)^{m}\]

\[q \text{ - ROFSIWA } (\kappa_{c_{11}}, \kappa_{c_{12}}, \ldots, \kappa_{c_{c_{m}}}) = \Theta_{m}^{n} \bigotimes_{j=1}^{m} \Omega_{j} \kappa_{c_{j}}\]

\[= \left( \sqrt{1 - \prod_{j=1}^{m} \left( \prod_{i=1}^{n} \left( 1 - \mu_{ij}^{q} \right)^{\Omega_{j}} \right)^{\gamma_{j}}} \right)^{m} - \left( \prod_{j=1}^{m} \left( \prod_{i=1}^{n} \left[ 1 - \left( \mu_{ij}^{q} + \theta_{ij}^{q} \right) \right]^{\Omega_{j}} \right)^{\gamma_{j}} \right)^{m}\]

\[\Rightarrow 0 \leq \prod_{i=1}^{n} \left[ 1 - \left( \mu_{ij}^{q} + \theta_{ij}^{q} \right) \right]^{\Omega_{j}} \leq 1 \]
\[ q - \text{ROFSIWA} (\mathbf{N}_{e_1}, \mathbf{N}_{e_2}, \ldots, \mathbf{N}_{e_{p_1}}) \]
\[ = \sum_{j=1}^{p_2} \gamma_j \left( \prod_{i=1}^{n} (1 - \mu_{ij}^q) \right)^{\gamma_j} \]
\[ = \left( \prod_{j=1}^{p_2} \left( \prod_{i=1}^{n} (1 - \mu_{ij}^q) \right)^{\gamma_j} \right) - \left( \prod_{j=1}^{p_2} \left( [1 - (\mu_{ij}^q + \vartheta_{ij}^q)] \right)^{\gamma_j} \right) \]
\[ \Rightarrow \left( \prod_{j=1}^{p_2+1} \left( \prod_{i=1}^{n} (1 - \mu_{ij}^q) \right)^{\gamma_j} \right) - \left( \prod_{j=1}^{p_2+1} \left( [1 - (\mu_{ij}^q + \vartheta_{ij}^q)] \right)^{\gamma_j} \right) \]
\[ \Rightarrow \left( \prod_{j=1}^{p_2+1} \left( \prod_{i=1}^{n} (1 - \mu_{ij}^q) \right)^{\gamma_j} \right) - \left( \prod_{j=1}^{p_2+1} \left( [1 - (\mu_{ij}^q + \vartheta_{ij}^q)] \right)^{\gamma_j} \right) \]

As we know that
\[ 0 \leq \prod_{j=1}^{m} \left( \prod_{i=1}^{n} (1 - \mu_{ij}^q) \right)^{\gamma_j} \leq 1 \]

So, the equation as shown at the top of the next page.

Therefore, the equation as shown at the top of the next page.

So, it is proved that the obtained result through the q-ROFSIWA operator is also a q-ROFSN.

4) EXAMPLE

Let \( U = \{ u_1, u_2, u_3 \} \) be a set of experts with the given weights \( \Omega_u = (.143, .514, .343)^T \), who is going to describe the attractiveness of a vehicle under the defined set of attributes \( L' = \{ e_1 = \text{air conditioner}, e_2 = \text{airbag}, e_3 = \text{price} \} \) with weights \( \gamma_i = (0.4, 0.3, 0.3)^T \).

The assumed rating values for each attribute in the form of q-ROFSNs (\( q = 3 \)) \( (\mathbf{N}, L') = (\mu_{ij}, \vartheta_{ij})_{3 \times 3} \) given as:

\[
(\mathbf{N}, L') = \begin{bmatrix}
(7, .8) & (4, .6) & (3, .6) \\
(8, .3) & (7, .4) & (7, .3) \\
(3, .6) & (5, .7) & (9, .4)
\end{bmatrix}
\]

For \( q = 3 \) using Equation 3, \( q - \text{ROFSIWA} (\mathbf{N}_{e_1}, \mathbf{N}_{e_2}, \ldots, \mathbf{N}_{e_{p_1}}) \), as shown the top of the next page. Utilizing the above developed q-ROFSIWA operator, some fundamental properties for the collection of q-ROFSNs based presented as follows:
\[
0 \leq \prod_{j=1}^{m} \left( \prod_{i=1}^{n} \left( 1 - \mu_{ij}^q \right)^{\Omega_i} \right)^{\gamma_j} - \prod_{j=1}^{m} \left( \prod_{i=1}^{n} \left[ 1 - \left( \mu_{ij}^q + \theta_{ij}^q \right) \right]^{\Omega_i} \right)^{\gamma_j} \leq 1
\]
\[
\Rightarrow 0 \leq \sqrt[m]{\prod_{j=1}^{m} \left( \prod_{i=1}^{n} \left( 1 - \mu_{ij}^q \right)^{\Omega_i} \right)^{\gamma_j}} - \sqrt[m]{\prod_{j=1}^{m} \left( \prod_{i=1}^{n} \left[ 1 - \left( \mu_{ij}^q + \theta_{ij}^q \right) \right]^{\Omega_i} \right)^{\gamma_j}} \leq 1
\]
\[
0 \leq \sqrt[m]{1 - \prod_{j=1}^{m} \left( \prod_{i=1}^{n} \left( 1 - \mu_{ij}^q \right)^{\Omega_i} \right)^{\gamma_j}} + \sqrt[m]{\prod_{j=1}^{m} \left( \prod_{i=1}^{n} \left[ 1 - \left( \mu_{ij}^q + \theta_{ij}^q \right) \right]^{\Omega_i} \right)^{\gamma_j}} - \prod_{j=1}^{m} \left( \prod_{i=1}^{n} \left[ 1 - \left( \mu_{ij}^q + \theta_{ij}^q \right) \right]^{\Omega_i} \right)^{\gamma_j} \leq 1.
\]

\[q - ROFSWA (\mathbf{K}_{x_{11}}, \mathbf{K}_{x_{12}}, \ldots, \mathbf{K}_{x_{x_{35}}})\]
\[
= \sqrt{1 - \prod_{j=1}^{m} \left( \prod_{i=1}^{n} \left( 1 - \mu_{ij}^q \right)^{\Omega_i} \right)^{\gamma_j}} - \prod_{j=1}^{m} \left( \prod_{i=1}^{n} \left[ 1 - \left( \mu_{ij}^q + \theta_{ij}^q \right) \right]^{\Omega_i} \right)^{\gamma_j}
\]
\[
= \sqrt{1 - \prod_{j=1}^{m} \left( \prod_{i=1}^{n} \left( 1 - \mu_{ij}^q \right)^{\Omega_i} \right)^{\gamma_j}} + \prod_{j=1}^{m} \left( \prod_{i=1}^{n} \left[ 1 - \left( \mu_{ij}^q + \theta_{ij}^q \right) \right]^{\Omega_i} \right)^{\gamma_j} - \prod_{j=1}^{m} \left( \prod_{i=1}^{n} \left[ 1 - \left( \mu_{ij}^q + \theta_{ij}^q \right) \right]^{\Omega_i} \right)^{\gamma_j}
\]
\[
\sqrt{1 - \prod_{j=1}^{m} \left( \prod_{i=1}^{n} \left( 1 - \mu_{ij}^q \right)^{\Omega_i} \right)^{\gamma_j}} + \prod_{j=1}^{m} \left( \prod_{i=1}^{n} \left[ 1 - \left( \mu_{ij}^q + \theta_{ij}^q \right) \right]^{\Omega_i} \right)^{\gamma_j} - \prod_{j=1}^{m} \left( \prod_{i=1}^{n} \left[ 1 - \left( \mu_{ij}^q + \theta_{ij}^q \right) \right]^{\Omega_i} \right)^{\gamma_j} = \mathbb{K}_{x_{e}}
\]
\( \leq \left(1 - \min_{j} \min_{i} \{ \mu_{ij}^{q} \} \right) \sum_{j=1}^{m} \gamma_{j} \)
\( \quad \Rightarrow 1 - \max_{j} \max_{i} \{ \mu_{ij}^{q} \} \)
\( \leq \prod_{j=1}^{m} \left( \prod_{i=1}^{n} \left(1 - \mu_{ij}^{q} \right) \right) \gamma^{q} \)
\( \leq 1 - \min_{j} \min_{i} \{ \mu_{ij}^{q} \} \)
\( \quad \Leftrightarrow \min_{i} \min_{j} \{ \mu_{ij}^{q} \} \)
\( \leq 1 - \prod_{j=1}^{m} \left( \prod_{i=1}^{n} \left(1 - \mu_{ij}^{q} \right) \right) \gamma^{q} \)
\( \leq \max_{i} \max_{j} \{ \mu_{ij}^{q} \} \)
\( \quad \Rightarrow \min_{i} \min_{j} \{ \mu_{ij}^{q} \} \)
\[ \leq \max_{j} \max_{i} \{ \mu_{ij}^{q} \} \]  
\[ \text{(4)} \]

Similarly, (5), as shown at the bottom of the next page. Let \( \delta = q - ROFSIWA (\mathbf{S}_{e_{11}}, \mathbf{S}_{e_{12}}, \ldots, \mathbf{S}_{e_{mn}}) = (\mu_{\delta}, \nu_{\delta}) = \mathbf{S}_{\delta} \), then utilizing Equation 1.

\[ S(\delta) = \mu_{\delta}^{q} - \nu_{\delta}^{q} + \left( \frac{e^{\mu_{\delta}^{q} - \nu_{\delta}^{q}} - 1}{e^{\mu_{\delta}^{q} - \nu_{\delta}^{q}} + 1} - \frac{1}{2} \right) \pi_{\delta}^{q} \]
\[ \leq \left( \max_{i} \max_{j} \{ \mu_{ij}^{q} \} \right)^{q} - \left( \min_{i} \min_{j} \{ \nu_{ij}^{q} \} \right)^{q} \]
\[ + \left( \frac{e^{\max_{i} \max_{j} \{ \mu_{ij}^{q} \}} - \left( \min_{i} \min_{j} \{ \nu_{ij}^{q} \} \right)^{q}}{e^{\max_{i} \max_{j} \{ \mu_{ij}^{q} \}} - \left( \min_{i} \min_{j} \{ \nu_{ij}^{q} \} \right)^{q}} - \frac{1}{2} \right) \pi_{e_{ij}}^{q} \]
\[ = S(\mathbf{S}_{e_{ij}}^{+}) \Rightarrow S(\delta) \leq S(\mathbf{S}_{e_{ij}}^{+}) \]

and

\[ S(\delta) = \mu_{\delta}^{q} - \nu_{\delta}^{q} + \left( \frac{e^{\mu_{\delta}^{q} - \nu_{\delta}^{q}} - 1}{e^{\mu_{\delta}^{q} - \nu_{\delta}^{q}} + 1} - \frac{1}{2} \right) \pi_{\delta}^{q} \]
\[ \geq \left( \min_{i} \min_{j} \{ \mu_{ij}^{q} \} \right)^{q} - \left( \max_{i} \max_{j} \{ \nu_{ij}^{q} \} \right)^{q} \]
\[ + \left( \frac{e^{\min_{i} \min_{j} \{ \mu_{ij}^{q} \}} - \left( \max_{i} \max_{j} \{ \nu_{ij}^{q} \} \right)^{q}}{e^{\min_{i} \min_{j} \{ \mu_{ij}^{q} \}} - \left( \max_{i} \max_{j} \{ \nu_{ij}^{q} \} \right)^{q}} - \frac{1}{2} \right) \pi_{e_{ij}}^{q} \]
\[ = S(\mathbf{S}_{e_{ij}}^{-}) \Rightarrow S(\delta) \geq S(\mathbf{S}_{e_{ij}}^{-}) \]

Considering the above procedure, we have the following cases:

If \( S(\delta) < S(\mathbf{S}_{e_{ij}}^{+}) \) and \( S(\delta) > S(\mathbf{S}_{e_{ij}}^{-}) \), then by comparing two q-ROFNS, we obtain \( \mathbf{S}_{e_{ij}} < q - ROFSIWA (\mathbf{S}_{e_{11}}, \mathbf{S}_{e_{12}}, \ldots, \mathbf{S}_{e_{mn}}) < \mathbf{S}_{e_{ij}}^{+} \).
∑_{i=1}^n Ω_i = 1 and γ_j > 0. ∑_{j=1}^m γ_j = 1 denotes the weights of experts and attributes.

5) THEOREM
Let \( \mathbf{N}_{ij} = (\mu_{ij}, \theta_j) \) be a collection of q-ROFSNs, where \( i = 1, 2, \ldots, n \), and \( j = 1, 2, \ldots, m \). Then, the attained aggregated values is also a q-ROFSN and (7), as shown at the top of the next page, where \( \Omega_i \) and \( γ_j \) represents the weights of the experts and attributes respectively, such as \( \Omega_i > 0 \), \( ∑_{i=1}^n Ω_i = 1, \gamma_j > 0 \), \( ∑_{j=1}^m γ_j = 1 \).

Proof: The proof of the q-ROFSIWG operator can be given by using mathematical induction such as follows:

We will prove for \( n = 2 \) and \( m = 2 \). Then, we have \( q - ROFSIWG (\mathbf{N}_{e11}, \mathbf{N}_{e12}, \ldots, \mathbf{N}_{e1m}) \), as shown at the top of the next page. For \( m = 1 \), we get \( γ_1 = 1 \). Then, we have \( q - ROFSIWG (\mathbf{N}_{e11}, \mathbf{N}_{e12}, \ldots, \mathbf{N}_{em}) \), as shown at the top of the next page. So, Equation 7 satisfies for \( n = 1 \) and \( m = 1 \).

Consider Equation 7 holds for \( m = β_1 + 1, n = β_2 \) and \( m = β_1, n = β_2 + 1 \), such as \( ∏_{j=1}^{β_1+1} γ_j (∏_{i=1}^{β_2+1} Ω_i N_{ej}) \), as shown at the top of the next page. For \( m = β_1 + 1 \) and \( n = β_2 + 1 \), we have \( ∏_{i=1}^{β_2+1} γ_i (∏_{j=1}^{β_1+1} Ω_j N_{ji}) \), as shown at the top of the page 12. Hence, it is true for \( m = β_1 + 1 \) and \( n = β_2 + 1 \).

6) REMARK
1) If \( µ_{ij} + \theta_j^q \leq 1 \) and \( q = 2 \). Then, the q-ROFS was reduced to the PFSS [30].
2) If \( µ_{ij}^q + \theta_j^q \leq 1 \) and \( q = 1 \). Then, the q-ROFS was reduced to the IFSS [27].
3) If the set of attributes contains only one attribute, then the q-ROFSIWA operator is reduced to the q-ROFIWA operator [43] \( q - ROFIWA (\mathbf{N}_{e11}, \mathbf{N}_{e21}, \ldots, \mathbf{N}_{em}) \), as shown at the top of the page 12.
4) If the set of attributes contains only one attribute, then the q-ROFSIWG operator is reduced to the q-ROFIWG operator [43] \( q - ROFIWG (\mathbf{N}_{e11}, \mathbf{N}_{e21}, \ldots, \mathbf{N}_{em}) \), as shown at the top of the page 12.

7) EXAMPLE
Let \( \mathcal{U} = \{ \mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3 \} \) be a set of experts with the given weights \( \Omega_i = (1.43, .514, .343)^T \), who is going to describe the attractiveness of a vehicle under the defined set of attributes \( \mathcal{L}' = \{ e_1 = \text{air conditioner}, e_2 = \text{airbag}, e_3 = \text{price} \} \) with weights \( γ_j = (0.4, 0.3, 0.3)^T \). The assumed rating values for each attribute in the form of q-ROFSNs (\( q = 3 \)) \( \mathbf{N}, \mathcal{L}' = (\mu_{ij}, \theta_j)_{3 \times 3} \) given as:

\[
\begin{bmatrix}
(0.7, 0.8) & (4.6, 4.6) & (3.6, 3.6) \\
(0.8, 0.3) & (7.4, 7.4) & (7.3, 7.3) \\
(3.6, 3.6) & (5.7, 5.7) & (9.3, 9.4)
\end{bmatrix}
\]

For \( q = 3 \) using Equation 7, \( q - ROFSIWG (\mathbf{N}_{e11}, \mathbf{N}_{e12}, \ldots, \mathbf{N}_{em}) \), as shown at the bottom of the page 12. Some desirable properties have been discussed for q-ROFSNs using the q-ROFSIWG operator given as follows:

C. PROPERTIES OF Q-ROFSIWG OPERATOR
1) IDEMPOTENCY
If \( \mathbf{N}_{ej} = \mathbf{N} = (\mu_{ij}, \theta_j) \) ∀i, j, then,
\( q - ROFSIWG (\mathbf{N}_{e11}, \mathbf{N}_{e12}, \ldots, \mathbf{N}_{em}) = \mathbf{N} \).
Proof: As we know that \( \mathbf{N}_{ej} = \mathbf{N} = (\mu_{ij}, \theta_j) \) ∀i, j (\( i = 1, 2, \ldots, n \), \( j = 1, 2, \ldots, m \)). Then using Equation 7 \( q - ROFSIWG (\mathbf{N}_{e11}, \mathbf{N}_{e12}, \ldots, \mathbf{N}_{em}) \), as shown at the top of the page 13.

2) BOUNDEDNESS
Let \( \mathbf{N}_{ej} \) be a collection of q-ROFSNs and \( \mathbf{N}_{ej}^- = \left\{ \min \min_i (\mu_{ij}), \max \max_j (\theta_j) \right\} \) and 
\( \mathbf{N}_{ej}^+ = \left\{ \max \max_i (\mu_{ij}), \min \min_j (\theta_j) \right\} \), then \( \mathbf{N}_{ej}^- \leq q - ROFSIWG (\mathbf{N}_{e11}, \mathbf{N}_{e12}, \ldots, \mathbf{N}_{em}) \leq \mathbf{N}_{ej}^+ \).

\[
\min \min_i (\theta_j) \leq \sqrt[γ_j]{\prod_{j=1}^{m} \left( \prod_{i=1}^{n} \left( 1 - \mu_{ij}^q Ω_i \right) \right)^{γ_j} - \prod_{j=1}^{m} \left( \prod_{i=1}^{n} \left( 1 - (\mu_{ij}^q + \theta_j^q) Ω_i \right) \right)^{γ_j} \leq \max \max_i (\theta_j).} \tag{5}
\]
\[ q - ROFSIWG (\mathbf{x}_{e1}, \mathbf{x}_{e12}, \ldots, \mathbf{x}_{e_un}) = \sum_{j=1}^{m} (\sum_{i=1}^{n} \Omega_{i} \mathbf{x}_{e_i})^{\gamma_j} \]

\[ = \left( \sqrt[\gamma_j]{\prod_{j=1}^{m} \left( \prod_{i=1}^{n} \left( 1 - \theta_{ij}^{q} \right)^{\Omega_{i}} \right)^{\gamma_j}} - \prod_{j=1}^{m} \left( \prod_{i=1}^{n} \left[ 1 - (\mu_{ij}^{q} + \theta_{ij}^{q}) \right]^{\Omega_{i}} \right)^{\gamma_j} \right) \sqrt[\gamma_j]{1 - \prod_{j=1}^{m} \left( \prod_{i=1}^{n} \left( 1 - \theta_{ij}^{q} \right)^{\Omega_{i}} \right)^{\gamma_j}} \]

\[ = \left( \sqrt[\gamma_j]{\prod_{j=1}^{m} \left( \prod_{i=1}^{n} \left( 1 - \theta_{ij}^{q} \right)^{\Omega_{i}} \right)^{\gamma_j}} - \prod_{j=1}^{m} \left( \prod_{i=1}^{n} \left[ 1 - (\mu_{ij}^{q} + \theta_{ij}^{q}) \right]^{\Omega_{i}} \right)^{\gamma_j} \right) \sqrt[\gamma_j]{1 - \prod_{j=1}^{m} \left( \prod_{i=1}^{n} \left( 1 - \theta_{ij}^{q} \right)^{\Omega_{i}} \right)^{\gamma_j}} \]

Proof: For each \( i = 1, 2, \ldots, n, j = 1, 2, \ldots, m \), we have

\[ \min_{j} \min_{i} \left[ \theta_{ij}^{q} \right] \leq \theta_{ij}^{q} \leq \max_{j} \max_{i} \left[ \theta_{ij}^{q} \right] \]

\[ \Rightarrow 1 - \max_{j} \max_{i} \left[ \theta_{ij}^{q} \right] \leq 1 - \theta_{ij}^{q} \leq 1 - \min_{j} \min_{i} \left[ \theta_{ij}^{q} \right] \]

\[ \Leftrightarrow \left( 1 - \max_{j} \min_{i} \left[ \theta_{ij}^{q} \right] \right)^{\Omega_{i}} \leq \left( 1 - \theta_{ij}^{q} \right)^{\Omega_{i}} \leq \left( 1 - \min_{j} \max_{i} \left[ \theta_{ij}^{q} \right] \right)^{\Omega_{i}} \]

\[ \Leftrightarrow \left( 1 - \min_{j} \max_{i} \left[ \theta_{ij}^{q} \right] \right)^{\sum_{i=1}^{n} \Omega_{i}} \leq \prod_{i=1}^{n} \left( 1 - \theta_{ij}^{q} \right)^{\Omega_{i}} \leq \left( 1 - \max_{j} \min_{i} \left[ \theta_{ij}^{q} \right] \right)^{\sum_{j=1}^{m} \gamma_{j}} \]

\[ \Leftrightarrow \left( 1 - \min_{j} \max_{i} \left[ \theta_{ij}^{q} \right] \right)^{\sum_{i=1}^{n} \Omega_{i}} \leq \prod_{j=1}^{m} \left( \prod_{i=1}^{n} \left( 1 - \theta_{ij}^{q} \right)^{\Omega_{i}} \right)^{\gamma_j} \leq \left( 1 - \max_{j} \min_{i} \left[ \theta_{ij}^{q} \right] \right)^{\sum_{j=1}^{m} \gamma_{j}} \]
R. M. Zulqarnain et al.: Novel MCDM Approach for Interactive AOs of q-ROFSS

\[ \otimes_{j=1}^{\beta_1+1} \gamma_j \left( \otimes_{i=1}^{\beta_1+1} \Omega_i \mathcal{N}_{e_{ij}} \right) \]

\[ = \otimes_{j=1}^{\beta_1+1} \gamma_j \left( \otimes_{i=1}^{\beta_1} \Omega_i \mathcal{N}_{e_{ij}} \otimes \Omega_{\beta_1+1} \mathcal{N}_{e_{i(\beta_1+1)}} \right) \]

\[ = \otimes_{j=1}^{\beta_1+1} \gamma_j \Omega_i \mathcal{N}_{e_{ij}} \otimes_{j=1}^{\beta_1+1} \gamma_j \Omega_{\beta_1+1} \mathcal{N}_{e_{i(\beta_1+1)}} \]

\[ = \left\{ \prod_{j=1}^{\beta_1+1} \left( \prod_{i=1}^{\beta_1} \left( 1 - \theta_{ij}^q \right)^{\Omega_i} \right)^{\gamma_j} - \prod_{j=1}^{\beta_1} \left( \prod_{i=1}^{\beta_1} \left[ 1 - \left( \mu_{ij}^q + \theta_{ij}^q \right) \right]^{\Omega_i} \right)^{\gamma_j} \right\} \]

\[ q - \text{ROFIWA} \left( \mathcal{N}_{e_{i1}}, \mathcal{N}_{e_{i2}}, \ldots, \mathcal{N}_{e_{im}} \right) \]

\[ = \left\{ \sqrt{1 - \prod_{j=1}^{m} \left( 1 - \mu_{ij}^q \right)^{\gamma_j}} \right\} - \prod_{j=1}^{m} \left( 1 - \left( \mu_{ij}^q + \theta_{ij}^q \right) \right)^{\gamma_j} \]

\[ q - \text{ROFIWG} \left( \mathcal{N}_{e_{i1}}, \mathcal{N}_{e_{i2}}, \ldots, \mathcal{N}_{e_{im}} \right) \]

\[ = \left\{ \sqrt{\prod_{j=1}^{m} \left( 1 - \theta_{ij}^q \right)^{\gamma_j}} - \prod_{j=1}^{m} \left[ 1 - \left( \mu_{ij}^q + \theta_{ij}^q \right) \right]\right\} \]

\[ q - \text{ROFSIWG} \left( \mathcal{N}_{e_{i1}}, \mathcal{N}_{e_{i2}}, \ldots, \mathcal{N}_{e_{i5}} \right) \]

\[ = \left\{ \prod_{j=1}^{5} \left( \prod_{i=1}^{3} \left( 1 - \theta_{ij}^q \right)^{\Omega_i} \right)^{\gamma_j} - \prod_{j=1}^{5} \left( \prod_{i=1}^{3} \left[ 1 - \left( \mu_{ij}^q + \theta_{ij}^q \right) \right]^{\Omega_i} \right)^{\gamma_j} \right\} \]

\[ = \left\{ \sqrt{\left( \{0.488\}^{143} \cdot \{0.973\}^{514} \cdot \{0.784\}^{343} \right)^4} \right\} - \left\{ \sqrt{\left( \{0.145\}^{143} \cdot \{0.461\}^{514} \cdot \{0.757\}^{343} \right)^4} \right\} \]

\[ \Rightarrow 1 - \max \min_j \left\{ \theta_{ij}^q \right\} \]

\[ \leq \prod_{j=1}^{m} \left( \prod_{i=1}^{n} \left( 1 - \theta_{ij}^q \right)^{\Omega_i} \right)^{\gamma_j} \]

\[ \leq 1 - \min \min_i \left\{ \theta_{ij}^q \right\} \]

\[ \leq 1 - \prod_{j=1}^{m} \left( \prod_{i=1}^{n} \left( 1 - \theta_{ij}^q \right)^{\Omega_i} \right)^{\gamma_j} \]

\[ \leq \max \max_i \left\{ \theta_{ij}^q \right\} \]

\[ \Rightarrow \min \min_i \left\{ \theta_{ij}^q \right\} \]

\[ \leq 1 - \prod_{j=1}^{m} \left( \prod_{i=1}^{n} \left( 1 - \theta_{ij}^q \right)^{\Omega_i} \right)^{\gamma_j} \]

\[ \leq \max \max_i \left\{ \theta_{ij}^q \right\} \]
Similarly (9), as shown at the bottom of the next page. Let \( \delta = q - \text{ROFSIWG}(\mathcal{N}_{e_{11}}, \mathcal{N}_{e_{12}}, \ldots, \mathcal{N}_{e_{mn}}) \) = \( (\mu_\delta, \vartheta_\delta) = \mathcal{N}_\delta \), then utilizing Equation 1.

\[
\delta = \sqrt[q]{\prod_{j=1}^{m} \left( \prod_{i=1}^{n} (1 - \vartheta_\delta^{q}) \right) - \prod_{j=1}^{m} \left( \prod_{i=1}^{n} \left( 1 - \left( \mu_\delta^{q} + \vartheta_\delta^{q} \right) \right) \right) ^{\frac{1}{q}}}.
\]

If \( S(\delta) = S(\mathcal{N}_{e_{j}^+}) \), i.e.,

\[
\mu_\delta^{q} - \nu_\delta^{q} + \left( \frac{e^{\mu_\delta^{q}-\nu_\delta^{q}}}{e^{\mu_\delta^{q}-\nu_\delta^{q}} + 1} \right) \pi_\delta^{q} = \left( \max \max \{ \mu_j \} \right)^q - \left( \min \min \{ \vartheta_j \} \right)^q
\]

utilizing the above inequalities, we obtain \( \mu_\delta = \max \max \{ \mu_j \} \), and \( \vartheta_\delta = \min \min \{ \vartheta_j \} \). Hence, \( \pi_\delta^{q} = \pi_{\mathcal{N}_{e_{j}^+}}^{q} \). Then by comparing two q-ROFSNs, we obtain \( q - \text{ROFSIWG}(\mathcal{N}_{e_{11}}, \mathcal{N}_{e_{12}}, \ldots, \mathcal{N}_{e_{mn}}) = \mathcal{N}_{e_{j}^+} \).

If \( S(\delta) = S(\mathcal{N}_{e_{j}^-}) \), i.e.,

\[
\mu_\delta^{q} - \nu_\delta^{q} + \left( \frac{e^{\mu_\delta^{q}-\nu_\delta^{q}}}{e^{\mu_\delta^{q}-\nu_\delta^{q}} + 1} \right) \pi_\delta^{q} = \left( \min \min \{ \mu_j \} \right)^q - \left( \max \max \{ \vartheta_j \} \right)^q
\]

utilizing the above inequalities, we obtain \( \mu_\delta = \min \min \{ \mu_j \} \), and \( \vartheta_\delta = \max \max \{ \vartheta_j \} \). Hence, \( \pi_\delta^{q} = \pi_{\mathcal{N}_{e_{j}^-}}^{q} \), then by comparing two q-ROFSNs, we obtain \( q - \text{ROFSIWG}(\mathcal{N}_{e_{11}}, \mathcal{N}_{e_{12}}, \ldots, \mathcal{N}_{e_{mn}}) = \mathcal{N}_{e_{j}^-} \).

Considering the above procedure, we have the following cases:

If \( S(\delta) < S(\mathcal{N}_{e_{j}^+}) \) and \( S(\delta) > S(\mathcal{N}_{e_{j}^-}) \), then by comparing two q-ROFSNs, we obtain \( \mathcal{N}_{e_{j}^-} < q - \text{ROFSIWG}(\mathcal{N}_{e_{11}}, \mathcal{N}_{e_{12}}, \ldots, \mathcal{N}_{e_{mn}}) < \mathcal{N}_{e_{j}^+} \).

3) HOMOGENEITY

Prove that \( q - \text{ROFSIWG}(\alpha \mathcal{N}_{e_{11}}, \alpha \mathcal{N}_{e_{12}}, \ldots, \alpha \mathcal{N}_{e_{mn}}) = \alpha q - \text{ROFSIWG}(\mathcal{N}_{e_{11}}, \mathcal{N}_{e_{12}}, \ldots, \mathcal{N}_{e_{mn}}) \) for any positive real number \( \alpha \).
**Proof**: Let \( \mathbf{x}_{ij} \) be a q-ROFSN and \( \alpha > 0 \), then by using Definition 7 (3), we have \( \alpha \mathbf{x}_{ij} = \left( 1 - (1 - \mu_{ij}^q)^\alpha, (1 - \mu_{ij}^q)^\alpha \right) \). Let \( q - \text{ROFSIWG}(\alpha \mathbf{x}_{e1}, \alpha \mathbf{x}_{e2}, \ldots, \alpha \mathbf{x}_{eum}) \), as shown at the bottom of the page.

### IV. NOVEL MCDM APPROACH FOR INTERACTIVE AGGREGATION OPERATORS OF q-ROFSNs

The following section will construct an MCDM technique utilizing our settled interaction AOs. Also, a numerical example has been presented to prove the validity of the planned operators.

#### A. PROPOSED MCDM APPROACH

Consider \( \mathcal{X} = \{X_1, X_2, X_3, \ldots, X_s\} \) be a set of \( s \) alternatives \( \mathcal{Q} = \{Q^1, Q^2, Q^3, \ldots, Q^n\} \) be a set \( n \) experts. Let \( \Omega = (\Omega_1, \Omega_2, \ldots, \Omega_m)^T \) and \((\gamma_1, \gamma_2, \gamma_3, \ldots, \gamma_m)^T\) represents the weight vectors for experts and attributes such as \( \Omega_i > 0, \sum_{i=1}^{m} \Omega_i = 1 \) and \( \gamma_j > 0, \sum_{j=1}^{m} \gamma_j = 1 \). Experts \( Q^i: \{i = 1, 2, \ldots, n\} \) evaluate the considered alternatives \( \{X_1, z = 1, 2, \ldots, s\} \) in the form of q-ROFSNs such as \( D^{(i)} = \langle \mathbf{x}_{eij} \rangle_{n \times m, n} = (\mu_{ij}, \theta_{ij})_{n \times m} \) where \( 0 \leq \mu_{ij}, \theta_{ij} \leq 1 \) and \( 0 \leq \mu_{ij}^q + \theta_{ij}^q \leq 1 \) for all \( i, j \) given in Table 1. To use the valuations of senior specialists, the aggregated q-ROFSNs for alternatives \( X_i (s = 1, 2, \ldots, l) \) given by employing the planned interactive AOs. Finally, it utilized the score function to analyze the ranking of the alternatives. The step-wise procedure of the above-presented approach is given as follows:

1) Assemble the specialist’s valuation data for each alternative to their conforming parameters and then construct a decision matrix such as follows (10), as shown at the bottom of the next page.

\[
\begin{align*}
\min_j \min_i \{\mu_{ij}\} & \leq \sqrt{\prod_{j=1}^{m} \left( \prod_{i=1}^{n} (1 - \theta_{ij}^q)_{\alpha \Omega_i} \right)_{\gamma_j} - \prod_{j=1}^{m} \left[ 1 - (\mu_{ij}^q + \theta_{ij}^q)_{\alpha \Omega_i} \right]_{\gamma_j}} \\
& \leq \max_j \max_i \{\mu_{ij}\},
\end{align*}
\]

(9)

\[
q - \text{ROFSIWG}(\alpha \mathbf{x}_{e1}, \alpha \mathbf{x}_{e2}, \ldots, \alpha \mathbf{x}_{eum})
= \left\langle \sqrt{\prod_{j=1}^{m} \left( \prod_{i=1}^{n} (1 - \theta_{ij}^q)_{\alpha \Omega_i} \right)_{\gamma_j} - \prod_{j=1}^{m} \left[ 1 - (\mu_{ij}^q + \theta_{ij}^q)_{\alpha \Omega_i} \right]_{\gamma_j}} \right\rangle_{\alpha}
= \left\langle \left( \prod_{j=1}^{m} \left( \prod_{i=1}^{n} (1 - \theta_{ij}^q)_{\alpha \Omega_i} \right)_{\gamma_j} \right)^{\alpha} - \left[ 1 - (\mu_{ij}^q + \theta_{ij}^q)_{\alpha \Omega_i} \right]_{\gamma_j} \right\rangle_{\alpha}
= \alpha q - \text{ROFSIWG}(\mathbf{x}_{eij}, \mathbf{x}_{e1}, \mathbf{x}_{e2}, \ldots, \mathbf{x}_{eum}).
\]
Greenhouse gas radiations means less usage of fossil fuels. But, this is not an informal assignment since the invention. What is formed from hydrocarbons is an energy transporter and the critical energy cause. To have a substantial impression on decarbonization, it would be included in a globally friendly way. In 2017, fossil fuels accounted for extra than 85% of global energy production [55].

Consequently, energy scarcities resolve instantaneously if the world completely alters to a hydrogen budget that eradicates fossil fuel feasting. This component delivers significant tasks in verdict an appropriate power source [56]. Though, this investigation will not insurance this issue. As mortality is impending, the "end of low-priced oil" era," with complete compromise in science and power engineering that essential discover new energy exporters. Severe reduction procedure across nations exposed hydrogen will be the eventual optimal. Hydrogen conceivable as complementary energy in cars influences industrial innovations such as hydrogen fuel cells to deliver manufacturing deprived of producing any CO₂ involuntary transmission authority and straight fuel for internal burning engines. One of its compelling features is the propensity for hydrogen fabrications in the variation of feedstocks it produces. Since there is virtually no abundant hydrogen in wildlife. The single choice is to proclaim it from the organic bond of other grains. There are two conducts to produce hydrogen, amongst other belongings: furious hydrocarbons or cracking water. Condensation fermentation is used to disrupt depressed hydrocarbons. Water ex卤cuting can be completely straight in compelling circumstances, temperature, or energy use. a new way to produce hydrogen from water is to burn coal in the attendance of water suspension [57].

It kinds intelligence to renovate fossil discarded energies such as natural gas is earliest transformed into hydrogen. In conclusion, while fossil fuels develop excessively and prospective unlawfully for worldwide warming, renewable most important energy will originate into the depiction for financial or environmental causes. In expressions of power, the recently formed hydrogen fuel is dissimilar to the frequently used ones in gratified weight and volume. This hydrogen is frivolously associated with its energy capability is the top prominent feature. The energy content of hydrogen per kilogram is 120 MJ. The benefit of methanol is six times as extraordinary [58]. Hydrogen has a little volumetric energy compactness related to its exceptional gravimetric density. The compactness of hydrogen is committed by its accumulation state.

UnFluctuating densities up to 700 bar are not massive sufficient belongings of hydrocarbons similar to gasoline and diesel. Only fluid hydrogen can influence a reasonable amount, still less than a quarter of the amount of gasoline. So, hydrogen containers for motorized solicitations will conquer more than fluid hydrocarbons formerly used containers [59]. Cryogenic storing containers are also recognized as cryogenic holding vessels. Dewar flask is, in fact, a double-walled super-insulator container. It vehicles fluid oxygen, nitrogen, hydrogen, helium, and argon, temperatures <110 K/163 °C. Fluid hydrogen has been familiar as a more significant energy cause. Since water is impartial a surplus gas, it’s unbelievably non-toxic ecological security when rehabilitated to power. Constituents used in cryogenic container enterprises are contingent on protection and budget [60], [61]. Essentially the exertion of cryogenic vessels is security apprehensions and enterprise conditions in perspective short temperature embrittlement can be designated as follows:

Fracture toughness: The steaming point of melted nitrogen is around –196 °C, whereas the steaming point of liquefied nitrogen is around. The temperature of hydrogen gas is approximate - 253°C. The substantial cannot find ductility and converts hard. So, the considerable requirement is robust sufficient to endure Inelastic crack. Face-centered cube metals webs are suitable since they are impervious to low temperatures. All nickel-copper compounds, aluminum, its compounds, and austenitic stainless steels contain an extra 7% nickel to construct a storing cryogenic vessel [62]. Heat transfer: heat transient over low-temperature container barriers are principally conductive. Constituents with low, warm air conductivity are chosen. Thermal stress: due to slight temperature, interior the barriers contract, instigating thermal straining. So, constituents with slight thermal conductivity are suitable. Thermal diffusivity: in practice, the collective thermal isolation is ridiculous. The material must be selected in such a tactic that it can disperse heat as rapidly as conceivable.

Material assortment in any manufacturing arena is a very significant enterprise phase. Manufacturing enterprise is prepared by enactment, budget, ecological compassion objectives, And commonly inadequate by the material. The most acceptable product strategy selects the best appropriate material design criteria by providing an extreme presentation
at the lowermost probable budget. Material selection is By seeing numerous contradictory DM procedures. The existing AOs have originated as a DM procedure in this circumstance. These AOs must be modernized to talk about these definite concerns. We intend some novel operations and escorting AOs for aggregating innumerable q-ROFSNs. Conferring to the clarification stated above and DM perception, all structures can be categorized. The case study was shown in a motorized portions engineering corporation in Malaysia, and a motorized constituent, cryogenic storing, accompanied the study. As part of applying the concept of sustainability, companies must choose suitable materials for produced parts. It focuses first on cryogenic storage containers and then on other factors input of weights for gathering parameters and materials from DM. q-ROFSS theory and proposed interaction AOs used to overcome complexity and indecision human judgment. MS with three remember the essential pillar of sustainability: materials must be reasonable, ecologically pleasant, and beneficial to humanity. The most imperative aspects (parameters) to consider when selecting a substantial dashboard DM. Choose the procedure starts with an initial screening of material used for dashboards, captivating into justification structures intrinsic to the application. In the screening process, identify the fabrics that may be appropriate. It is serious about deciding the material that can be used initial MS for the instrument board process. Four materials are selected subsequently examining the abilities: \( x_1 = \text{Ti-6Al-4V}, \ x_2 = \text{SS301-FH}, \ x_3 = \text{70Cu-30Zn}, \) and \( x_4 = \text{Inconel 718}. \) The attribute of material selection is given as follows: \( e_1 = \text{Specific gravity}, \ e_2 = \text{Toughness index}, \ e_3 = \text{Yield stress}, \ e_4 = \text{Easily accessible}, \ e_5 = \text{Sustainability requirements with weights (0.26, 0.22, 0.1, 0.27, 0.15)}^T \). Let \( Q = \{Q^1, \ Q^2, \ Q^3, \ Q^4, \ Q^5\} \) be a set of five experts with weights \((0.18, 0.24, 0.21, 0.15, 0.22)^T\). To judge the optimum alternative, specialists provide their preferences in q-ROFSNs. The method to find the most acceptable alternative is presented in 4.1.

C. q-ROFSIWA OPERATOR

Step 1: The experts scrutinize the environments and give their preferences in q-ROFSN. The summary of their score values is shown in Table 1-Table 5.

Step 2: There is no need to normalize because all parameters are similar.

Step 3: Expert’s evaluation for each alternative can be measured using Equation 3 such as follows q – ROFSIWA(\( N_{e_1}, N_{e_2}, \ldots, N_{e_5}\)), as shown at the bottom of the next page.

For \( q = 3 \), \( L_1 = (0.7226, 0.384), \ L_2 = (0.7453, 0.4629), \ L_3 = (0.7746, 0.237), \) and \( L_4 = (0.7539, 0.17758) \).

Step 4: Compute the score values using Equation 1. \( S(L_1) = 0.297879, S(L_2) = 0.476531, S(L_3) = 0.396731, S(L_4) = 0.453902. \)

Step 5: \( x_2 \) has most outstanding score value, so \( x_2 \) is the most acceptable option.

D. FOR Q-ROFSIWG OPERATOR

Step 1: Same as above.

Step 2: Same as above.

Step 3: Expert’s evaluation for each alternative can be measured using Equation 7 such as follows q-ROFSIWG, as shown at the bottom of the next page. For \( q = 3 \), \( L_1 = (0.7279, 0.2250), \ L_2 = (0.7365, 0.2147), \ L_3 = (0.7781, 0.19487), \) and \( L_4 = (0.7535, 0.1847) \).

Step 4: Compute the score values using Equation 1. \( S(L_1) = 0.37859, S(L_2) = 0.53417, S(L_3) = 0.41386, S(L_4) = 0.48721. \)

Step 5: \( x_2 \) has most incredible score value, so \( x_2 \) is the most acceptable option.

Step 6: Alternatives ranking using q-ROFSIWA operator given as follows: \( S(L_2) > S(L_4) > S(L_3) > S(L_1) \). So, \( x_2 > x_4 > x_3 > x_1 \). Hence, the above calculation shows that \( x_2 \) be a most suitable material.

V. COMPARATIVE ANALYSIS AND DISCUSSION

The consequent section will examine the planned method’s effectiveness, flexibility, and benefits. We also assessed the succeeding: a planned technique with existing methods.

A. BENEFITS AND SUPERIORITY OF THE PROPOSED METHOD

The anticipated method is competent and practicable; we extant an innovative procedure under the q-ROFSS setting via q-ROFSIWA or q-ROFSIWG operators. Our predictable model is extra proficient than prevailing systems and can carry the most delicate consequences in MCDM problems. The incorporated model is multipurpose and informal to adjust to sprouting fluctuations, involvement, and productivity. Different models have specific ranking procedures, so there are direct variances among the proposed technique’s rankings to be viable based on their contemplations. From this scientific study and evaluation, we now conclude that the results obtained from the existing approach are unpredictable.
TABLE 1. q-ROFS decision matrix for $X_1$.

|   | $e_1$       | $e_2$       | $e_3$       | $e_4$       | $e_5$       |
|---|-------------|-------------|-------------|-------------|-------------|
| $q^1$ | (0.7, 0.25) | (0.7, 0.22) | (0.88, 0.1) | (0.9, 0.1)  | (0.73, 0.2) |
| $q^2$ | (0.6, 0.1)  | (0.6, 0.13) | (0.85, 0.12)| (0.65, 0.25)| (0.81, 0.16)|
| $q^3$ | (0.54, 0.15)| (0.7, 0.2)  | (0.75, 0.24)| (0.68, 0.25)| (0.6, 0.26) |
| $q^4$ | (0.65, 0.2) | (0.8, 0.18) | (0.85, 0.13)| (0.8, 0.18) | (0.7, 0.28) |
| $q^5$ | (0.6, 0.3)  | (0.75, 0.18)| (0.67, 0.25)| (0.6, 0.3)  | (0.45, 0.15)|

TABLE 2. q-ROFS decision matrix for $X_2$.

|   | $e_1$       | $e_2$       | $e_3$       | $e_4$       | $e_5$       |
|---|-------------|-------------|-------------|-------------|-------------|
| $q^1$ | (0.71, 0.25)| (0.78, 0.1) | (0.88, 0.11)| (0.81, 0.18)| (0.78, 0.2) |
| $q^2$ | (0.8, 0.15) | (0.85, 0.12)| (0.9, 0.1)  | (0.65, 0.25)| (0.74, 0.23)|
| $q^3$ | (0.76, 0.1) | (0.88, 0.11)| (0.84, 0.12)| (0.86, 0.1) | (0.79, 0.2) |
| $q^4$ | (0.78, 0.22)| (0.75, 0.25)| (0.74, 0.2) | (0.75, 0.25)| (0.65, 0.16)|
| $q^5$ | (0.6, 0.25) | (0.8, 0.19) | (0.75, 0.16)| (0.6, 0.2)  | (0.5, 0.1)  |

TABLE 3. q-ROFS decision matrix for $X_3$.

|   | $e_1$       | $e_2$       | $e_3$       | $e_4$       | $e_5$       |
|---|-------------|-------------|-------------|-------------|-------------|
| $q^1$ | (0.8, 0.15) | (0.75, 0.22)| (0.76, 0.1) | (0.8, 0.19) | (0.7, 0.25) |
| $q^2$ | (0.75, 0.18)| (0.8, 0.15) | (0.8, 0.18) | (0.5, 0.25) | (0.8, 0.16) |
| $q^3$ | (0.78, 0.13)| (0.7, 0.2)  | (0.7, 0.25) | (0.76, 0.21)| (0.76, 0.23)|
| $q^4$ | (0.9, 0.1)  | (0.65, 0.33)| (0.76, 0.15)| (0.87, 0.12)| (0.65, 0.18)|
| $q^5$ | (0.65, 0.3) | (0.55, 0.2) | (0.6, 0.3)  | (0.7, 0.23) | (0.55, 0.15)|

TABLE 4. q-ROFS decision matrix for $X_4$.

|   | $e_1$       | $e_2$       | $e_3$       | $e_4$       | $e_5$       |
|---|-------------|-------------|-------------|-------------|-------------|
| $q^1$ | (0.76, 0.22)| (0.75, 0.22)| (0.85, 0.14)| (0.78, 0.2) | (0.65, 0.26)|
| $q^2$ | (0.72, 0.12)| (0.79, 0.18)| (0.6, 0.12) | (0.73, 0.15)| (0.8, 0.14) |
| $q^3$ | (0.82, 0.16)| (0.83, 0.1) | (0.84, 0.13)| (0.82, 0.12)| (0.77, 0.2) |
| $q^4$ | (0.6, 0.27)| (0.6, 0.3)  | (0.7, 0.2)  | (0.83, 0.13)| (0.6, 0.25) |
| $q^5$ | (0.55, 0.1) | (0.81, 0.12)| (0.8, 0.15) | (0.72, 0.17)| (0.5, 0.15) |

$q - ROFSIWA (\kappa_{e_{11}}, \kappa_{e_{12}}, \ldots, \kappa_{e_{33}})$

$$= \left\langle \sqrt[r]{1 - \prod_{j=1}^{5} \left( \prod_{i=1}^{5} (1 - \mu_{ij}^q)^{ \Omega_i} \right)^{ \Omega_j}} \right\rangle - \left\langle \prod_{j=1}^{5} \left( \prod_{i=1}^{5} (1 - \mu_{ij}^q + \vartheta_{ij}^q)^{ \Omega_i} \right)^{ \Omega_j} \right\rangle$$

$q - ROFSIWG$

$$= \left\langle \prod_{j=1}^{5} \left( \prod_{i=1}^{5} (1 - \vartheta_{ij}^q)^{ \Omega_i} \right)^{ \Omega_j} \right\rangle - \left\langle \prod_{j=1}^{5} \left( \prod_{i=1}^{5} [1 - (\mu_{ij}^q + \vartheta_{ij}^q)]^{ \Omega_i} \right)^{ \Omega_j} \right\rangle$$

compared to the hybrid structure. Furthermore, due to some fortunate circumstances, many hybrid structures of FS, IFSS, and PFSS have become uncommon for q-ROFSS. It is a simple method of syndicating incomplete and uncertain information in the DM technique. Data about the object can be made more exact and logically expressed. It is a moderately
simple tool to mix inaccurate and insecure data in the DM procedure. So, our planned method will be extra competent, significant, superior, and healthier than numerous mixed structures of FS. The characteristic analysis of the proposed approach with some existing models is presented in the following Table 6.

**B. COMPARATIVE ANALYSIS**

To demonstrate the dominance and impact of the planned approach, a comparative study has been presented among the developed method and some prevailing works, based on some AOs. If we consider the MG = 0.9 and NMG = 0.6, then $MG + NMG \leq 1$ and $MG^2 + NMG^2 \leq 1$. So, the existing IFWA [46], IFSWA [33], IFSWG [33], PFSWA [39], and PFWSG [39] cannot handle the circumstances. Similarly, q-ROFWA [21], q-ROFWG [21], q-ROFIWA [47], and q-ROFIWG [47] cannot handle scenarios due to a lack of parametrization. If $X$ be an alternative, $Q = \{Q^1, Q^2\}$ be two experts and $e_1, e_2$ are two parameters. Then preferences of experts $X' = \begin{bmatrix} (0.7, 0) \\ (0.8, 0.7) \end{bmatrix}$ in the form of q-ROFSS. Where $\Omega_i = (0.7, 0.3)^T$ and $\gamma_j = (0.4, 0.6)^T$

---

**TABLE 5. Alternatives score values using developed operators.**

| Method      | $X_1$    | $X_2$    | $X_3$    | $X_4$    | Alternatives ranking    |
|-------------|----------|----------|----------|----------|-------------------------|
| q-ROFSIWA   | 0.297879 | 0.476531 | 0.396731 | 0.453902 | $X_2 > X_4 > X_3 > X_1$ |
| q-ROFSIWG   | 0.37859  | 0.53417  | 0.41386  | 0.48721  | $X_2 > X_4 > X_3 > X_1$ |

**TABLE 6. Characteristic analysis of different approaches.**

|       | Fuzzy information | Aggregated parameters information | Aggregated parameters information considering the interaction |
|-------|-------------------|----------------------------------|----------------------------------------------------------|
| IFWA  [46] | ✓                  | ✗                                | ✗                                                        |
| IFSWA [33] | ✓                  | ✓                                | ✗                                                        |
| PFSWA [39] | ✓                  | ✓                                | ✗                                                        |
| PFSWG [39] | ✓                  | ✓                                | ✗                                                        |
| PFSIWA [40] | ✓                  | ✓                                | ✓                                                        |
| PFSIWG [40] | ✓                  | ✓                                | ✓                                                        |
| q-ROFWA [21] | ✓                  | ✗                                | ✗                                                        |
| q-ROFWG [21] | ✓                  | ✗                                | ✗                                                        |
| q-ROFIWA [47] | ✓                  | ✗                                | ✓                                                        |
| q-ROFIWG [47] | ✓                  | ✗                                | ✓                                                        |
| q-ROFSWA [41] | ✓                  | ✓                                | ✗                                                        |
| q-ROFSWG [42] | ✓                  | ✓                                | ✗                                                        |
| Proposed operators | ✓                  | ✓                                | ✓                                                        |

**TABLE 7. Comparative analysis with existing operators.**

| Method      | $X_1$    | $X_2$    | $X_3$    | $X_4$    | Ranking order    |
|-------------|----------|----------|----------|----------|------------------|
| IFWA [46]   | 0.220169 | 0.332146 | 0.211734 | 0.270078 | $X_2 > X_4 > X_3 > X_1$ |
| IFWG [46]   | 0.46119  | 0.5572   | 0.49917  | 0.55418  | $X_2 > X_4 > X_3 > X_1$ |
| IFWA [46]   | 0.232465 | 0.32471  | 0.225013 | 0.240629 | $X_2 > X_4 > X_3 > X_1$ |
| IFSWA [33]  | 0.516859 | 0.604673 | 0.548324 | 0.590214 | $X_2 > X_4 > X_3 > X_1$ |
| IFSWG [33]  | 0.51160  | 0.56806  | 0.47536  | 0.55816  | $X_2 > X_4 > X_3 > X_1$ |
| PFSWA [39]  | 0.522097 | 0.621904 | 0.565965 | 0.593809 | $X_2 > X_4 > X_3 > X_1$ |
| PFSWG [39]  | 0.46427  | 0.57343  | 0.55374  | 0.51474  | $X_2 > X_4 > X_3 > X_1$ |
| q-ROFWA [21] | 0.099586 | 0.147617 | 0.079845 | 0.081579 | $X_2 > X_4 > X_3 > X_1$ |
| q-ROFWG [21] | 0.29865  | 0.40678  | 0.35021  | 0.38045  | $X_2 > X_4 > X_3 > X_1$ |
| q-ROFSWA [41] | 0.414877 | 0.522354 | 0.46537  | 0.484856 | $X_2 > X_4 > X_3 > X_1$ |
| q-ROFSWG [42] | 0.4337  | 0.464826 | 0.404847 | 0.349273 | $X_2 > X_4 > X_3 > X_1$ |
| q-ROFSIWA   | 0.297879 | 0.476531 | 0.396731 | 0.453902 | $X_2 > X_4 > X_3 > X_1$ |
| q-ROFSIWG   | 0.37859  | 0.53417  | 0.41386  | 0.48721  | $X_2 > X_4 > X_3 > X_1$ |
represents the weights of experts and parameters, respectively. Then \( q - ROFSWA = (0.681981, 0) \), which shows that there is no effect on the collective result \( \theta_e \). So, q-ROFSWA [41] and q-ROFSWG [42] are failed to cope with this issue. However, the core benefit of a deliberate method is that it consents extra data to contact the DM issues. It is also a valuable tool for resolving imprecise and unexplainable information considering the interaction in the DM procedure. The obtained ranking using different models has been presented in TABLE 7.

It is also a convenient tool for cracking imprecise and inaccurate information in the DM process. The advantage of the intentional method and related measures over current approaches is to avoid inferences based on uncomplimentary reasons. So, it is an appropriate tool for combining imprecise and indeterminate data in the DM procedure.

VI. CONCLUSION

Decision-making is a pre-planned procedure to classify and select logical options from multiple alternatives. DM is a complicated process because it can change from one scene to another. Thus, it is imperative to judge the features and boundaries of the alternatives. In addition, DM is a healthier technique to raise the chances of indicating the most suitable alternative. It is essential to distinguish how much real background data the decision-makers require. The most operative approach in DM is to pay close attention and focus on your goals. Molodtsov investigates the pioneering paradigm of SS by attaching parametric tools to the common set. SS theory does not inherit complexity and is an excellent mathematical tool that can handle uncertainty relationships parametrically. The core objective of this study is to introduce some novel operational laws for q-ROFSS considering the interaction. Based on the presented concept, interactive AOs have been planned: q-ROFSWA and q-ROFSWG operators. Moreover, some fundamental properties have been discussed of the proposed operators. Based on the projected model, a medical DM problem is presented in the q-ROFSS setting. Then, we use some existing methods to show the rationality of the developed method and through characteristic analysis to establish the impact and dominance of the established technique on the general studies. The benefit of the settled notion is that they can resolve realistic complications using their parameterized properties. Thus, the established concept can solve the DM problem instead of other existing operators in the q-ROFSS environment. In the future, several other hybrids AOs for q-ROFSS have been introduced with their decision-making techniques. Furthermore, the developed AOs can be extended to T-spherical fuzzy soft settings with decision-making approaches.

ACKNOWLEDGMENT

This work was supported by the Deanship of Scientific Research at King Khalid University through Large Groups Project under Grant R.G.P. 2/51/43.

REFERENCES

[1] P. Chatterjee and S. Chakraborty, “Material selection using preferential ranking methods,” Mater. Des., vol. 35, pp. 384–393, Mar. 2012.
[2] A. Thakker, J. Jarvis, M. Buggy, and A. Sahed, “A novel approach to materials selection strategy case study: Wave energy extraction impulse turbine blade,” Mater. Des., vol. 29, no. 10, pp. 1973–1980, Dec. 2008.
[3] K. L. Edwards, “Materials influence on design: A decade of development,” Mater. Des., vol. 32, no. 3, pp. 1073–1080, Mar. 2011.
[4] G. P. Reddy and N. Gupta, “Material selection for microelectronic heat sinks: An application of the Ashby approach,” Mater. Des., vol. 31, no. 1, pp. 113–117, Jan. 2010.
[5] L. A. Zadeh, “Fuzzy sets,” Inf. Control, vol. 8, pp. 338–353, Jun. 1965.
[6] K. T. Atanassov, “Intuitionistic fuzzy sets,” in Fuzzy Sets and Systems, vol. 20, no. 1, 1986, pp. 87–96, doi: 10.1016/0165-0114(86)80034-3.
[7] W. Wang and X. Liu, “Intuitionistic fuzzy geometric aggregation operators based on Einstein operations,” Int. J. Intell. Syst., vol. 26, no. 11, pp. 1049–1075, Nov. 2011.
[8] R. R. Yager, “Pythagorean fuzzy subsets,” in Proc. Joint IFSA World Congr. NAFIPS Annu. Meeting (IFSA/NAFIPS), Jun. 2013, pp. 57–61.
[9] K. Rahman, S. Abdullah, R. Ahmed, and M. Ullah, “Pythagorean fuzzy Einstein weighted geometric aggregation operator and their application to multiple attribute group decision making,” J. Intell. Fuzzy Syst., vol. 33, no. 1, pp. 635–647, 2017.
[10] X. Zhang and Z. Xu, “Extension of TOPSIs to multiple criteria decision making with Pythagorean fuzzy sets,” Int. J. Intell. Syst., vol. 29, no. 12, pp. 1061–1078, 2014.
[11] G. Wei and M. Lu, “Pythagorean fuzzy power aggregation operators in multiple attribute decision making,” Int. J. Intell. Syst., vol. 33, no. 1, pp. 169–186, 2018.
[12] L. Wang and N. Li, “Pythagorean fuzzy interaction power Bonferroni mean aggregation operators in multiple attribute decision making,” Int. J. Intell. Syst., vol. 35, no. 1, pp. 150–183, Jan. 2020.
[13] E. Ilbahar, A. Karas ˛an, S. Cebi, and C. Kahraman, “A novel approach to risk assessment for occupational health and safety using Pythagorean fuzzy AHP & fuzzy inference system,” Saf. Sci., vol. 103, pp. 124–136, Mar. 2018.
[14] X. Zhang, “A novel approach based on similarity measure for Pythagorean fuzzy multiple criteria group decision making,” Int. J. Intell. Syst., vol. 31, no. 6, pp. 593–611, 2016.
[15] R. R. Yager, “Generalized orthopair fuzzy sets,” IEEE Trans. Fuzzy Syst., vol. 25, no. 5, pp. 1222–1230, Oct. 2017.
[16] M. Riaz, H. M. A. Farid, F. Karaaslan, and M. R. Hashmi, “Some q-rung orthopair fuzzy hybrid aggregation operators and TOPSIS method for multi-attribute decision-making,” J. Intell. Fuzzy Syst., vol. 39, no. 1, pp. 1227–1241, Jul. 2020.
[17] M. Riaz, A. Razzaq, H. Kalsoom, D. Pamuˇcar, H. M. A. Farid, and Y.-M. Chu, “q-Rung orthopair fuzzy geometric aggregation operators based on generalized and group-generalized parameters with application to water loss management,” Symmetry, vol. 12, no. 8, p. 1236, Jul. 2020.
[18] M. Riaz, W. Salabun, H. M. A. Farid, N. Ali, and J. Wątróbski, “A robust q-runq orthopair fuzzy information aggregation using Einstein operators with application to sustainable energy planning decision management,” Energies, vol. 13, no. 9, p. 2155, May 2020.
[19] P. Liu and P. Wang, “Some q-rung orthopair fuzzy aggregation operators and their applications to multi-attribute decision making,” Int. J. Intell. Syst., vol. 33, no. 2, pp. 259–280, Feb. 2018.
[20] P. Liu, T. Mahmood, and Z. Ali, “Complex q-rung orthopair fuzzy aggregation operators and their applications in multi-attribute group decision making,” Information, vol. 11, no. 1, p. 5, Dec. 2019.
[21] L. Xu, Y. Liu, and H. Liu, “Some improved q-rung orthopair fuzzy aggregation operators and their applications to multiattribute group decision-making,” Math. Problems Eng., vol. 2019, pp. 1–18, Dec. 2019.
[22] F. Wang, J. Wang, G. Wei, and C. Wei, “Similarity measures of q-rung orthopair fuzzy sets based on cosine function and their applications,” Mathematics, vol. 7, no. 4, p. 340, Apr. 2019.
[23] X. Peng and L. Liu, “Information measures for q-rung orthopair fuzzy sets,” Int. J. Intell. Syst., vol. 34, no. 8, pp. 1795–1834, Aug. 2019.
[24] Z. Liu, S. Wang, and P. Liu, “Multiple attribute group decision making based on q-rung orthopair fuzzy Heronian mean operators,” Int. J. Intell. Syst., vol. 33, no. 12, pp. 2341–2363, Dec. 2018.
Z. Xu, “Intuitionistic fuzzy aggregation operators,” IEEE Trans. Fuzzy Syst., vol. 22, no. 4, pp. 797–807, Aug. 2014.

[56] Major General. (2001). Well-to-Wheel Energy Use and Greenhouse Gas Emissions of Advanced Fuel/Vehicle Systems—North American Analysis, Part 1. [Online]. Available: http://www.transportation.gov/pdfs/TA/163.pdf

[57] R. Arora and H. Garg, “A robust aggregation operators for multi-criteria decision-making with intuitionistic fuzzy soft set environment,” Int. J. Intell. Syst., vol. 34, no. 2, pp. 215–246, Feb. 2019.

[58] R. Zhang, Z. Yuan, and S. Wang, “Fuzzy soft weighted aggregation operators and their application in multi-criteria decision-making,” IEEE Access, vol. 6, pp. 7657–7670, 2018.

[59] B. Petroleum, “BP statistical review of world energy 2017,” Stat. Rev. World Energy, vol. 65, pp. 1–56, Jun. 2017.

[60] A. Godula-Jopek, W. Jehle, and J. Wellnitz, “Near-Horizontal Two-Phase Flow Patterns of Helium in a Vertical Tube,” Cryogenic Engineering, vol. 33, no. 2, pp. 99–107, 2012.

[61] N. Van Dresar and J. Siegwarth, “Near-Horizontal Two-Phase Flow Patterns of Helium in a Vertical Tube,” Cryogenic Engineering, vol. 33, no. 2, pp. 99–107, 2012.

[62] A. Godula-Jopek, W. Jehle, and J. Wellnitz, “Near-Horizontal Two-Phase Flow Patterns of Helium in a Vertical Tube,” Cryogenic Engineering, vol. 33, no. 2, pp. 99–107, 2012.
AIYARED IAMPAN was born in Nakhon Sawan, Thailand, in 1979. He received the B.S., M.S., and Ph.D. degrees in mathematics from Naresuan University, Phitsanulok, Thailand. He was the Founder of the Group for Young Algebraists, University of Phayao, in 2012, and one of the Co-Founder of the Fuzzy Algebras and Decision-Making Problems Research Unit, University of Phayao, in 2021. He is currently an Associate Professor at the Department of Mathematics, School of Science, University of Phayao, Phayao, Thailand. His research interests include algebraic theory of semigroups, ternary semigroups, and Gamma-semigroups, lattices and ordered algebraic structures, fuzzy algebraic structures, and logical algebras.

JAN AWREJCEWICZ is currently a Full Professor and the Head of the Department of Automation, Biomechanics and Mechatronics, Lodz University of Technology. His research interests include mechanics, material science, biomechanics, applied mathematics, automation, physics, and computer oriented sciences, with main focus on nonlinear processes.

MAKSYMILIAN BEDNAREK graduated in computer science from the University of Lodz. He is currently pursuing the Ph.D. degree with the Department of Automation, Biomechanics and Mechatronics, Lodz University of Technology. His research interests include artificial intelligence, control, electromagnetic springs, and tailoring stiffness characteristic.

MUHAMMAD ASIF received the Ph.D. degree in mathematics from Quaid-i-Azam University, Islamabad, Pakistan, in 2020. He is currently an Assistant Professor with the Department of Mathematics, University of Management and Technology, Sialkot, Pakistan. His research interests include large scale of coding theory, cryptography, information theory, and fuzzy algebra.

RIFAQAT ALI is currently working at the Department of Mathematics, College of Science, King Khalid University. His current research interests include linear approximation, growth of the polynomial, and differential geometry.