Research Article

Estimation of Aperture and Stiffness of Fractures under High Water Pressure Using Hydrological Data and Slurry Consolidating Body

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The fractured limestone aquifers under coal seams bring great threat to coal production, and grouting engineering before mining is usually used to seal the fractures and prevent water inrushes. However, the grouting mechanism is still a difficult issue since the hidden fissures cannot be observed. In this paper, two characteristics of fractures filled with high-pressure water, deformation, and stiffness are studied to seek some good explanations for grouting. First, an ideal flow model composed of a borehole and a fracture is established with the fracture aperture as the main variable. The estimation methods for the aperture of fractures with high-pressure water before grouting and the stiffness of the fracture filled with slurry consolidating body during high-pressure grouting are proposed, and the millimeter-scale solution of deformation is obtained by using hydrological data and slurry consolidating body. Then, the methods are used for the field grouting practice, and its applicable conditions are given. The estimated apertures of the fractures with high-pressure water range from 2.08 to 2.56 mm. Under the grouting pressure of 15 MPa and the designed slurry diffusion radius of 30 m, the normal stiffness of the fracture filled with slurry consolidating body ranges from 0.5 to 14.4 GPa/m. The results meet the positive relationship between normal stiffness and fracture aperture got by the literature data and can be used for the study of grouting effect of the hidden fractures, including the evaluation of strength and water resistance of the slurry consolidating body. Our method of estimating the aperture and stiffness of fractures is applicable when grouting is used in the fractured limestone aquifers with high water pressure under coal seams before mining.

1. Introduction

Grouting reinforcement of the deep buried confined aquifers under coal seams is an important technology for the safety of coal production in northern China. However, in practice, although the aquifers in some mining areas are considered to be fully grouted, there are still water accidents occurring in the working face during mining. One main reason is that the fracture characteristics of hidden aquifers can not be seen, and grouting only depends on experience in practice, which has seriously troubled researchers. In order to study the fracture characteristics of grouted aquifers for the better implementation of grouting, this research on aperture and stiffness of fractures in deep buried limestone aquifers with high-pressure water is carried out. The related literatures on fracture apertures, fracture stiffnesses, fracture models, and estimating methods of apertures are first retrieved and analyzed, consistent with the main research content of this manuscript.

In discussion of fracture models, the equivalent model is usually used to simplify the study of the complexed fractured aquifers, such as discrete equivalent geometric model of nonlinear flow, fine pipe flow model [1], network model for fractures [2], and plane parallel plate model for a single fracture. According to the dip angles of fractures, the models for grouting [3, 4] and water movement [5] can be divided into horizontal fracture models and vertical fracture models in theoretical and similar experiment researches. The model
in this paper is an approximate vertical fracture model with an inclined borehole.

In terms of fracture apertures, according to the International Society for Rock Mechanics Commission on Standardization of Laboratory and Field Tests, an aperture between 0.5 and 10 mm is classified to have “gapped” features, which means that the aperture has a width that is open to moderately wide. Fractures with such an aperture are usually grouted to control groundwater inrush. In the latest study [6], aperture structures from 0.01 to 2 mm are selected for the study of fracture deformation and fluid flow. Specifically, there are many researches on fracture aperture relating to grouting, and the range of fracture aperture is large from 10 μm to 5 mm, including micron scale values [7–13] and millimeter scale values [14–18]. Obviously, the range of apertures is large from microns to millimeters, and the precise law has not been obtained. Moreover, the natural water pressure of the fractures and the grouting pressure are both smaller than that of this paper. Similar to the deformation study, the range of stiffness is also very wide. The estimating values of the normal stiffnesses of carbonate rock range from about 1.4 to 200 GPa/m [13]. For apertures ranging from 12 to 30 μm, the normal stiffnesses can vary from 10 to 1100 GPa/m [19, 20], which are smaller than the experimental values. For apertures ranging from 100 to 250 μm, the normal stiffnesses range from 2 to 10 GPa/m [21, 22]. The relationship between stiffness and deformation shows a positive correlation with fracture apertures rather than a nonlinear relationship, which provides space for research. In addition, the relationship between grouting and apertures is also used for the fracture dilation during grouting [23, 24], groutability prediction [25], and grouting effect [26–30]. However, it is difficult to determine the aperture of hidden fractures with high-pressure water in field application.

In terms of estimating methods, the equivalent hydraulic aperture is often used to evaluate the width of fractures and usually estimated by using the cubic law, according to the data including water inflow and hydraulic head [31, 32]. However, the natural water pressure of the fractures and the grouting pressure are both smaller than that of this paper (more than 6 MPa), while high pressure has an effect on the fracture apertures. When the fracture was just formed in the geological history, it had an original aperture. Then, under the action of high water pressure in the aquifer (more than 6 MPa), the fracture expands on the basis of the original aperture. The apertures studied in this paper are the expanded fracture apertures by high water pressure. In addition, there are many studies of fracture aperture and stiffness close to the tunnel during grouting, while the research on the aperture of the fractures with high-pressure water in the deep buried limestone aquifers under coal seams has not been studied.

This paper presents a borehole-fracture model of grouting for deep buried limestone aquifers under high water pressure. Based on the monitored hydrological parameters, a simple estimation method for aperture and normal stiffness of fractures with high-pressure water was put forward and then applied in the field grouting project. At last, the applicable conditions of the calculation results are discussed in detail.

2. Characteristics of Limestone Aquifers and Grouting Engineering

The mined coal seam located in Henan Province of China is threatened by hidden limestone aquifers with high-pressure water under coal seams, L9, L8, and L2. The spatial layout of aquifers and mined coal seams is shown in Figure 1. The water pressure of L2 is up to 6.4 MPa. During coal exploitation, the confined water may penetrate vertically upwards from L2 into working face through fractures, which increases the risk of water inrushes. For the safety of mine production, grouting is selected to seal the fractures and reconstruct the fractured limestone aquifers. In this paper, L2 was selected as the main studied aquifer.

Generally, when boreholes encounter wide fractures with high-pressure water during drilling, the water inflow from the borehole is large (e.g., Borehole 1#). The water inflow from the borehole is small when meeting narrow fractures (e.g., Borehole 4#), and there is no water flowing out when boreholes do not encounter fractures (e.g., Borehole 3#), which is in line with general cognition that the limestone aquifers contain a large number of inclined or vertical fractures and the distribution of fractures in these aquifers is uneven and irregular. The terminal of drilling depends on the water inflow, which put forward strict requirements for grouting technicians.
3. Analytic Model for Grouting in Fractures

3.1. Ideal Fracture-Borehole Flow Model. Usually, experimental research is used as an important means to study the fracture grouting. Two sketch maps of boreholes and inclined fractures are listed in Figure 2. The fractures are regarded as plane-parallel plates, and the fracture apertures are given in advance.

However, it is difficult to determine the hidden fracture aperture with high-pressure water. When the fractures in the deep buried aquifers are exposed by drilling holes, the confined groundwater can flow out along the borehole through fractures under the action of high water pressure. Based on grouting practice, the fracture-borehole systems with different apertures are drawn as shown in Figure 3. The ideal flow models under high water pressure contain a fracture and a borehole with diameter of 75 mm and length of \( L \). In both cases with different fracture widths (\( \delta_w \) and \( \delta_n \)), water inflow \( q \) and water pressure \( p \) can be measured at the borehole outlet.

In the model, the fracture aperture is regarded as the main variable to determine the flow patterns while other parameters remain unchanged. Different flow patterns may occur, such as free flow and fissure flow. Suppose \( \delta_b \) is the aperture of the critical state of free flow and fracture flow. When the aperture \( \delta_w \) is larger than \( \delta_b \), free flow occurs (Figure 3(a)); when the aperture \( \delta_n \) is less than \( \delta_b \), fracture flow occurs (Figure 3(b)). \( v_b \) is the critical boundary velocity corresponding to \( \delta_b \).

For a fracture-borehole flow system, if pure water is injected from the borehole outlet instead of slurry, the pure water can continuously flow into the fracture as long as the inject pressure is selected appropriately. The process of pure water injection is completely opposite to the fracture water inflow process. However, the grouting of clay-cement slurry is different from the injection of water due to its fluid characteristics, such as viscosity, deposition, and solidification, which is the focus of this paper.

3.2. Theoretical Solution of Fracture-Borehole Flow Model

3.2.1. General Equations for Free Flow. The free flow pattern with critical aperture of \( \delta_b \) is selected to be studied. It is easy to get the hydrodynamic equations for free flow in a wide fracture-borehole system, as shown in Figure 4. In the system, confined water can flow out freely at a certain velocity when the borehole outlet is open. When the borehole outlet is closed, the water pressure of aquifers can be monitored.
Applying Bernoulli equation in sections $M$ and $O$ of Figure 4, the basic equations in the fracture-borehole flow system with critical aperture at cross section $M$ and $O$ can be easily written down as follows:

$$Z_M + \frac{P_M}{\gamma} + \frac{\alpha_1 v_M^2}{2g} = Z_O + \frac{P_O}{\gamma} + \frac{\alpha_2 v_O^2}{2g} + h_{f, O} + h_{j, O}. \quad (1)$$

Point $M$ is in the aquifer, and point $O$ is at the borehole outlet. $z_M$, $P_M$, and $v_M$ are, respectively, water head height, water pressure, and fluid velocity at cross section $M$; $z_O$, $P_O$, and $v_O$ are, respectively, water head height, water pressure, and fluid velocity at cross section $O$; $h_{f, O}$ is the friction resistance of drilling hole; $h_{j, O}$ is the local resistance at the intersection of fracture and borehole. $\alpha_1$ and $\alpha_2$ are the kinetic energy correction factors, usually taken as 1.

Another two different forms of Bernoulli equation are easily got through transformation as follows:

$$v_M^2 = \frac{1}{\alpha_1} \left[ 2g(Z_O - Z_M) + \frac{2(P_O - P_M)}{\rho} + \alpha_2 v_O^2 + 2g h_{f, O} + h_{j, O} \right],$$

$$v_O^2 = \frac{\left[ ((2\rho M - P_O)\rho) - 2g(Z_O - Z_M) + \alpha_1 v_M^2 \right]}{(\alpha_2 + \lambda(L/d) + \zeta)}.$$

The friction resistance $h_{f, O}$ and local resistance $h_{j, O}$ can be, respectively, calculated by equations $h_{f, O} = \lambda(L/d)(v^2/2g)$ and $h_{j, O} = \zeta v^2/2g$, where $\lambda$ is the friction factor, $L$ is the borehole length, $d$ is the diameter of the borehole, and $v$ is the fluid velocity in the borehole. $\zeta$ is the local resistance coefficient.

### 3.2.2. Flow Model of a Wide Fracture with High-Pressure Water

For wide fracture, free flow occurs, and the flow at the junction from a wide fracture not penetrated by a borehole is shown in Figure 5. The model of this paper shown in Figure 5 is similar to orifice outflow in the hydrodynamic textbooks. The water in the fracture is sufficient and far greater than the water output from the borehole. Cross section $C-C$ is set as a reference plane. Cross section $S-S$ is another virtual cross section not far away from $C-C$, and the continuity equation is applicable to these two cross sections.

Thus, the energy conservation equation at cross sections $C-C$ and $S-S$ is written as follows:

$$z_s + \frac{P_s}{\rho g} + \frac{v_s^2}{2g} = z_C + \frac{P_C}{\rho g} + \frac{v_C^2}{2g} + \zeta v_s^2,$$

where $z_O$, $P_O$, and $v_O$ are, respectively, water head height, water pressure, and fluid velocity at section $S-S$ and $z_C$, $P_C$, and $v_C$ are, respectively, head height, water pressure, and fluid velocity at cross section $C-C$. $h_{j, O} = \zeta v_s^2/2g$ is the local resistance at the intersection of the fracture and the borehole. The kinetic energy correction coefficient is temporarily taken as 1.
The borehole diameter is very small compared with the aquifer thickness; the friction resistance can be ignored but the local resistance needs to be considered.

Similarly, the energy equation is also applied at both ends of the borehole, written as follows:

$$Z_C + \frac{p_C}{\rho g} + \frac{\alpha_C v_C^2}{2g} = Z_O + \frac{p_O}{\gamma} + \frac{\alpha_S v_O^2}{2g} + h_{f,O},$$  \hspace{1cm} (4)

where \(Z_O, p_O,\) and \(v_O\) are, respectively, water head height, water pressure, and flow velocity at the cross section of borehole outlet; \(h_{f,O} = \lambda (L/d) (v_O^2/2g)\) is the friction resistance of drilling hole; \(\alpha_C\) is the kinetic energy correction factors, taken as 1.

Therefore, according to Equations (3) and (4), the following Equation (5) with \(v_O\) as the dependent variable can be obtained. \(v_O\) is the flow velocity at the cross section of borehole outlet and can be monitored by instruments.

$$v_O = \left[ \frac{(2(p_S - p_O)/\rho) - 2g(Z_O - Z_S) + \alpha_S v_S^2}{(\alpha_S + \lambda (L/d) + \zeta)} \right].$$  \hspace{1cm} (5)

The term, \(p_S - p_O,\) in Equation (5) has a great influence on \(v_O,\) especially when the water pressure is very high. The term of \(v_O,\) in Equation (5) has a positive correlation effect on \(v_O.\) The greater the aquifer velocity is, the greater the borehole outlet velocity will be. However, in practice, the water pressure is much larger than the square of the velocity of the aquifer. So the influence of aquifer velocity on the flow velocity at the borehole outlet is small, and the theoretical verification is completed as follows.

3.2.3. Estimation of the Free Flow Velocity at the Borehole Outlet. Take borehole D23-8 as an example, the case when the water flow in the fracture is static is first considered with the water pressure of 5.8 MPa.

According to the basic theory of fluid mechanics [33], when the inlet section of boreholes is mounted into the end-face wall at an angle, the inlet local resistance increases. The local resistance coefficient \(\zeta\) can be calculated as follows:

$$\zeta = 0.5 + 0.3 \sin \theta + 0.2 \sin^2 \theta.$$  \hspace{1cm} (6)

The inclination angle of borehole D23-8 is 35°, so the local resistance coefficient \(\zeta\) is 0.74. Use the Moody diagram to determine the friction factor. Referring to the roughness of the cement pipe, the relative roughness of the borehole wall is obtained by \(\Delta/d = 0.33/75 = 0.005333.\) From the Moody diagram, the friction factor is preliminarily determined as \(\lambda = 0.032.\)

Therefore, when the fracture is filled with static high pressure water, the velocity of borehole outlet \(v_O\) is calculated as 12.69 m/s by substituting parameters into Equation (5).

The flow velocity of aquifers sometimes may be greater than or equal to zero \((v_M \geq 0).\) As the aquifer velocity \(v_M\) gradually increases from 0 to 11.3 m/s, the velocity of borehole outlet \(v_O\) changes very slowly from 12.69 m/s to 12.77 m/s. The detailed relationship between \(v_S\) and \(v_O\) for free flow is obtained by theoretical calculation, as shown in Figure 6. \(v_O\) is the velocity at the borehole outlet, and \(v_S\) is the aquifer velocity. The aquifer velocity has little effect on the outlet velocity under high water pressure.

It is worth noting that the velocity difference, \(\Delta v_O,\) is smaller than the aquifer velocity difference \(\Delta v_S,\) and the approximate derivative function of the \(v_O - v_S\) curve is \(12E\) -06x. Compared with high water pressure (the order of magnitude of the item containing water pressure is \(10^3\)), the flow velocity of the aquifer (the order of magnitude of the item containing flow velocity is \(10^2\)) has little effect on the velocity at the borehole outlet.

For free flow, when the flow velocity of the aquifer changes from \(v_S\) to \(v_S + \Delta v_S,\) the flow velocity at borehole outlet \(v_O\) changes very little, and the difference of flow velocity at borehole outlet, \(\Delta v_O,\) can be ignored especially from engineering scale. That is, when the flow velocity of the aquifer changes from \(v_S\) to \(v_S + \Delta v_S,\) the minimum free flow velocity \(v_{\text{min}}\) for free flow at the borehole outlet is got by

$$v_{\text{min}} = \lim_{\Delta v_O \to 0} (v_O + \Delta v_O) = v_O.$$  \hspace{1cm} (7)

This formula for the minimum free flow velocity \(v_{\text{min}}\) is important and can be used in engineering scale. In many cases, the flow velocity does not reach this value, which is mainly attributed to the small crack width in this paper.

3.2.4. Velocity Coefficient of Narrow Fractures. In practice, the flow velocity at the borehole outlet does not reach \(v_{\text{min}},\) far less than \(v_{\text{min}},\) although the water pressure is very high. There is a big gap between the actual velocity and the theoretical velocity. This paper mainly studies the decrease of fracture water inflow due to the fracture width. High water pressure and low water inflow conform to the jet flow characteristics. A jet model of a fracture penetrated by a borehole is shown in Figure 7. The junction between the fracture and the borehole is tentatively determined as regular circle.

For a narrow fracture, cross section A-A and cross section B-B are selected. Section B-B is close to the hole wall. Section A-A is an ideal section not far away from section B-B, and the mass conservation satisfies on sections A-A and B-B.
Similarly, according to the conservation of energy, write the equations of cross section A-A and cross section B-B

\[
\frac{p_A}{\rho g} + \frac{v_j^2}{2g} = \frac{p_B}{\rho g} + \frac{v_j^2}{2g} + \frac{v_j^2}{2g} + \zeta_n \frac{v_j^2}{2g},
\]

(8)

where \( v_j \) is the flow velocity of section B-B very close to the hole wall.

Ignore the frictional resistance between section A-A and section B-B. \( \zeta_n \) is the local resistance coefficient to be solved when the fracture water jets into the borehole in case of narrow fractures.

According to the continuity equation, the formula,

\[
v_A = v_j A_B = v_O A_O,
\]

can be got, where \( A \) is the section area.

So,

\[
v_A = \frac{(d/D)^2}{\pi} v_j
\]

\( D \) is the diameter of circle marked on section A-A.

The velocity \( v_j \) can be obtained by transforming Equation (8) as follows:

\[
v_j = \frac{1}{\sqrt{1 + \zeta_n - (d/D)^4}} \sqrt{2 \Delta p/\rho}.
\]

(9)

For \( d \ll D \), \( (d/D)^4 = 0 \), then

\[
v_j = \frac{1}{\sqrt{1 + \zeta_n}} \sqrt{2 \Delta p/\rho} = C_r \sqrt{2 \Delta p/\rho},
\]

(10)

where \( C_r = 1/\sqrt{1 + \zeta_n} \) is the velocity coefficient.

The velocity coefficient \( C_r \) is the ratio of actual velocity to theoretical velocity of free flow,

\[
C_r = \frac{v_O}{v_{\text{min}}}
\]

(11)

The actual velocity is got by drilling boreholes, and the theoretical velocity has been calculated above. The velocity coefficient is obtained, and then, \( v_j \) can be obtained. Finally, the fracture aperture can be calculated.

3.2.5. Estimation of Fracture Aperture. The area \( A \) of the annular junction of borehole and aquifer is calculated by

\[
A = \pi d \delta_n
\]

where \( d \) is the diameter of the borehole and \( \delta_n \) is the narrow fracture aperture. According to the continuity equation, the fracture aperture \( \delta_n \) can be deduced as follows:

\[
\delta_n = \frac{(q/3600)}{\rho d} / (C_r \sqrt{2 \Delta p/\rho} / \pi d)
\]

(12)

where \( q \) is the water inflow monitored at the borehole outlet and \( d \) is the borehole diameter.

So far, the fracture aperture has been determined, and its application is carried out in the following sections.

3.2.6. Estimation of Stiffness of the Fractures Filled with Slurry Consolidating Body. Under the action of high grouting pressure, fracture expansion occurs. In order to explain the problem easily, the average deformation of fractures is used in the next calculation. After grouting is completed, the fractures affected by high injection pressure include consolidation zone and water separating zone, and the fracture filled with consolidating body of slurry is the effective area to prevent water inrush, which is regarded in this paper as an important parameter for calculating the stiffness.

\( \delta_0 \) is the fracture aperture under high water pressure before grouting, and \( \delta_f \) is the expanded average aperture of the fracture filled with consolidating body after grouting. \( \Delta u_n \) is the difference of the deformation before and after grouting and can be obtained as follows:

\[
\Delta u_n = \delta_f - \delta_0,
\]

(13)
where $\delta_j$ is determined by the quantity of the consolidating body (effective grouting slurry stone) in the fracture $Q_s$. The final grouting slurry stone is the key part that can block water. For the convenience of calculation, it is assumed to be ideally and evenly distributed in the fracture. Here, $\delta_0 = \delta_n$ and

$$
\delta_j = \frac{Q_s}{L_d H}, \quad (14)
$$

where $L_d$ is the distance of fracture effectively filled with slurry consolidating body.

The difference between grouting pressure and water pressure, $\Delta \sigma$, can be got by

$$
\Delta \sigma = p_j - p_w, \quad (15)
$$

where $p_j$ is the grouting pressure and $p_w$ is the water pressure in the fracture.

Then, the fracture normal stiffness under the action of water pressure, $k_n$, is expressed [13] as follows:

$$
k_n = \frac{\Delta \sigma}{\Delta u_w} = \frac{p_j - p_w}{Q_s/L_d H - \delta_0}. \quad (16)
$$

4. Application in Grouting Engineering for Fractured Aquifers

4.1. Grouting Engineering. The drilling boreholes for grouting in a drilling site of a working face are shown in Figure 8. The total length of boreholes is up to 20,000 m, and the injection consumptions of cement and clay are more than 3000 t. The designed slurry diffusion radius is less than 30 m, according to the grouting engineering practice. At the termination stage of grouting, minimum flow criterion is used. Maintain a minimum flow rate of 58 L/min for 30 minutes with the grouting pressure of 15 MPa and then stop grouting.

4.2. Aperture Estimation of the Fracture under High Water Pressure by Using Hydrological Data of Typical Boreholes. First, take borehole D23-8 as an example. The key fluid parameters of borehole D23-8 are listed in Table 1.

The water inflow of borehole D23-8 is 1 m$^3$/h, and the flow velocity in the borehole is 6.29 $\times$ 10$^{-2}$ m/s. The water pressure $p_w$ monitored at the borehole is 5.0 MPa. By substituting the monitored data of borehole D23-8 into Equations (5), (6), (7), (10), (11), and (12), the fracture aperture penetrated by borehole D23-8 is calculated as 2.38 $\times$ 10$^{-3}$ m (2.38 mm).

Literature retrieval is completed for this theoretical research on the grouting of fractures, and the research results are listed together in Table 2, which is conducive to quickly understand the research of fractures and make a rapid judgment or comparison. The research methods include laboratory research and field test. Grouting pressure varies from 0.2 MPa to 15 MPa. The aperture scale ranges from microns to millimeters. The apertures of fractures with high-pressure water of this paper belong to millimeter scale.

Then, substitute the monitored parameters of 9 typical boreholes into Equations (11) and (12), the corresponding theoretical flow velocity $v_{min}$ of free flow at the borehole outlet, the velocity coefficient of the fracture-borehole system $C_v$, and the fracture aperture $\delta_n$ in the limestone aquifers are got and listed in Table 3. In practice, compared the

![Figure 8: Sketch map of boreholes during grouting engineering.](image-url)

Table 1: Key fluid parameters of borehole D23-8.

| Parameters               | Value   |
|--------------------------|---------|
| Borehole diameter        | 0.075 m |
| Kinematic viscosity (20°C)| $1.0067 \times 10^{-6}$ m$^2$/s |
| Water density            | 1000 kg/m$^3$ |
| Borehole length          | 141.5 m |
| Flow velocity at the outlet of borehole | 0.0629079 m/s |
| Drilling angle           | 35°     |

Table 2: Fracture apertures and its application conditions.

| Aperture | Reference         | Grouting pressure | Methods   |
|----------|-------------------|-------------------|-----------|
| 1-3 mm   | This paper        | 15.0 MPa          | Field     |
| 1-5 mm   | Lee (2020)        | <1 MPa            | Lab       |
| 1-4 mm   | Sui (2015, 2019)  | 1.5 MPa           | Lab       |
| 0.1-0.025 mm | Funehag (2018) | 0.2 MPa           | Lab       |
| 10-230 μm | Ghafar (2017)    | 1.5 MPa           | Lab       |
| 0.3 mm   | Tani (2017)       | 0.6 MPa           | Lab       |
| 5 mm     | Li (2016)         | 350 KPa           | Lab and field |
| 100 μm   | Place (2016)      | 1.5 MPa           | Lab       |
| 100-500 μm | Mohammed (2015) | 1.5 MPa           | Lab       |
| 0.1-2 mm | Hakami (1995)     |                  | Lab       |
| 0.2-2 mm | Jung (1989)       | 1.9 MPa           | Lab       |
theoretical water inflow with the actual water inflow, the water inflow monitored by boreholes does not reach the theoretical calculation value, and the fracture aperture is less than 3 mm and far less than the borehole diameter 75 mm.

4.3. Stiffness of the Fracture Filled with Slurry Consolidating Body. High grouting pressure can affect the normal deformation of fractures, and the normal deformation is related to the damage degree of rock masses, which can be used to evaluate the quality of rock mass to a certain extent. When the grouting amount \( Q_f \) is given, the effective filling volume, \( Q_s \) (slurry stone), can be obtained, and the grout-induced fracture aperture, \( \delta_j \), can be calculated. Next, the normal deformation difference due to grouting, \( \Delta \delta \), is calculated by subtracting \( \delta_n \) from \( \delta_j \). At last, the normal stiffness of the fracture can be determined. \( Q_s \) is the volume of the slurry stone, \( p_g \) is the grout pressure, and \( Q_f \) is the volume of slurry only in the fracture.

The volume of the slurry consolidating body depends on the water separating proportion. The water separating proportion of clay-cement slurry is approximately linear with the cement content. For the slurry with the same specific gravity, the water separating proportion decreases with the increase of cement content. In this paper, the mass fraction \( C/S \) of cement in the total solid material is 19.2%, and the specific gravity of slurry is 1.16 g/cm\(^3\). According to the experimental results of the relationship between cement content and water separating proportion for the clay cement slurry with different specific gravity [34], the water separating proportion \( W_{sep} \) is taken as 70%.

Substitute the monitored data of 9 boreholes into Equation (16), the corresponding normal stiffness of fractures in the limestone aquifers is got and listed in Table 4.

The apertures of fractures filled with high-pressure water before grouting range from 2.08 to 2.56 mm. Under the grouting pressure of 15 MPa with the designed slurry diffusion radius of 30 m, the normal stiffnesses of the fractures filled with slurry consolidating body range from 0.75 to 14.4 GPa/m. The minimum normal stiffness of the fracture exposed by borehole D23-8 is 0.75 GPa/m, and the maximum normal stiffness calculated by using the data of borehole D23-8 is 14.4 GPa/m. In practice, the grouting of borehole D23-8 is more difficult than other boreholes, and pure cement slurry has to be used for better plugging effect during grouting.

4.4. Discussion

4.4.1. Fracture Stiffness and Weathering Degree. The normal stiffness of fracture in the aquifers can be affected by weathering. According to the relative weathered degree of limestones, the fractures can be divided into several levels [35]: no weathered to slightly weathered, moderately weathered, and completely weathered. The corresponding lower limit

| Table 3: Monitored data, velocity coefficient, and apertures of typical boreholes. |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Borehole        | Length/m        | \( q \) (m\(^3\)/h) | \( p_g \) (MPa) | \( v_{max} \) (m/s) | \( C_v \) | \( v_j \) (m/s) | Aperture \( \delta_j \) (mm) |
| D17-13          | 187             | 1               | 5               | 11.08            | 5.68E-03 | 0.57            | 2.08            |
| D18-3           | 147             | 2.5             | 5.5             | 13.06            | 1.20E-02 | 1.26            | 2.34            |
| D18-8           | 162             | 1               | 4.8             | 11.64            | 5.40E-03 | 0.53            | 2.23            |
| D19-9n          | 122             | 1               | 4.8             | 13.35            | 4.71E-03 | 0.46            | 2.56            |
| D20-10n         | 142             | 1.2             | 4.5             | 12.02            | 6.28E-03 | 0.60            | 2.38            |
| D21-3           | 147             | 1.2             | 3.8             | 10.86            | 6.95E-03 | 0.61            | 2.34            |
| D23-10          | 122             | 1.2             | 5               | 13.63            | 5.54E-03 | 0.55            | 2.56            |
| D23-3           | 147             | 4               | 5.2             | 12.70            | 1.98E-02 | 2.02            | 2.34            |
| D23-8           | 141.5           | 1               | 5               | 12.69            | 4.96E-03 | 0.50            | 2.38            |

| Table 4: Normal stiffness of fractures using the data of typical grouting boreholes. |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Bh              | \( p_g \) (MPa) | \( \Delta p \) (MPa) | \( C/S \) (%) | \( W/S \) (%) | \( W_{sep} \) (%) | \( Q_s \) (m\(^3\)) | \( \Delta \delta \) (m) | \( \delta_j \) (m) | \( Q_f \) (m\(^2\)) | \( k_f \) (GPa/m) |
| D17-13          | 15              | 10.0            | 23.8            | 3.39            | 65             | 3.61            | 0.002351        | 0.004429        | 10.30            | 4.25            |
| D18-3           | 15              | 9.5             | 19.2            | 3.72            | 70             | 5.21            | 0.004061        | 0.006396        | 17.36            | 2.34            |
| D18-8           | 15              | 10.2            | 19.2            | 3.72            | 70             | 6.46            | 0.005705        | 0.007933        | 21.53            | 1.79            |
| D19-9n          | 15              | 10.2            | 30.6            | 3.54            | 50             | 3.56            | 0.00182         | 0.004375        | 7.12             | 5.61            |
| D20-10n         | 15              | 10.5            | 19.2            | 3.72            | 70             | 8.71            | 0.008323        | 0.010698        | 29.03            | 1.26            |
| D21-3           | 15              | 11.2            | 19.2            | 3.72            | 70             | 11.88           | 0.012258        | 0.014593        | 39.61            | 0.91            |
| D23-10          | 15              | 10.0            | 19.2            | 3.72            | 70             | 3.65            | 0.00193         | 0.004485        | 12.17            | 5.18            |
| D23-3           | 15              | 9.8             | 19.2            | 3.72            | 70             | 12.52           | 0.013038        | 0.015374        | 41.72            | 0.75            |
| D23-8           | 15              | 10.0            | 100             | 3.62            | 70             | 2.50            | 0.000694        | 0.003073        | 8.34             | 14.4            |
values of stiffnesses are, respectively, 7.9, 4.9, and 3.8 GPa/m. The fractured limestone aquifers in this paper are weathered to a certain extent in some areas.

4.4.2. Fracture Stiffness and Other Influencing Factors. Fracture stiffness is also affected by many other factors, including weathering during deposition in geological history, adjacent fractures, hydraulic erosion, and cyclic loading times. (1) In geological history, after the fracture was formed, it would first suffer from natural weathering during the deposition process, which will reduce the stiffness of the fracture. (2) Then, under the action of in situ stress field and high-pressure water, the local rock adjacent to the fracture may be eroded, and local large deformation will occur during high-pressure grouting, which will also reduce the normal stiffness of the crack. (3) The distance between fractures will also control the normal stiffness. When two fractures are far away, they have no influence on each other. When the spacing is not too large, the adjacent fracture may make the grouting relatively easy.

4.4.3. Engineering Applicable Conditions. High-pressure water and drainage conditions are two important conditions for the estimating of fracture apertures in this paper. The apertures under high water pressure range from 2.08 to 2.56 mm, and the normal stiffnesses of the fractures filled with slurry consolidating body range from 0.75 to 14.4 GPa/m. The results meet the positive relationship between normal stiffness and fracture aperture got by the literature data. Our method of estimating the aperture and stiffness of fractures is applicable when grouting is used in the fractured limestone aquifers with high water pressure under coal seams before mining. It also lays a foundation for the theoretical evaluation of grouting effect of the hidden fractures, including the evaluation of strength and water resistance of the slurry stone.

5. Conclusions

(1) An ideal flow model composed of a borehole and a fracture is established with the fracture aperture as the main variable. For critical free flow, when the flow velocity of the aquifer changes from \( v_s \) to \( v_s + \Delta v_s \), the minimum free flow velocity \( v_{\min} \) for free flow at the borehole outlet is got by \( v_{\min} = \lim_{\Delta v_s \to 0} (v_O + \Delta v_O) = v_O \). The velocity coefficient \( C_v \) is the ratio of actual velocity to theoretical velocity \( v_O/v_{\min} \). According to the continuity equation, the fracture aperture \( \delta_n \) can be deduced as \( \delta_n = (q/3600)/v/\pi d = (q/3600)/(C_v \sqrt{2 \Delta p/\rho})/\pi d \). Then, the fracture normal stiffness, \( k_n \), is got as \( k_n = (p_f - p_w)/(Q_s/(L_d H - \delta_n)) \).

(2) The estimated fracture apertures with high-pressure of typical boreholes range from 2.08 to 2.56 mm, and the fracture normal stiffness ranges from 0.75 to 14.4 GPa/m. The results meet the positive relationship between normal stiffness and fracture aperture got by the literature data. Compared with the method of determining apertures by cubic law, the method in this paper pays more attention to engineering application, considering the hydrogeological characteristics of fractured limestone aquifers. The influencing factors of fracture stiffness, including sedimentary weathering in geological history, adjacent fractures, and hydraulic erosion, are comprehensively discussed. The research results provide a foundation for the next theoretical evaluation of grouting effect in fractured limestone aquifers, including the evaluation of strength and water resistance of the slurry stone.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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