Complementarity between ILC250 and ILC-GigaZ

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Abstract: In view of the very precise measurements on fermion couplings which will be performed at ILC250 with polarized beams, there is emerging evidence that the LEP1/SLC measurements on these couplings are an order of magnitude too imprecise to match the accuracies reachable at ILC250. This will therefore severely limit the indirect sensitivity to new resonances and require revisiting the possibility to run ILC at the Z pole with polarized electrons. This work was done as a contribution to the ESU 2018-2020.

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Introduction

An alternative source of positrons is under consideration by ILC which uses an auxiliary S-band accelerator operating at 3 GeV [1]. It would then be easy to run this machine at the Z pole with only polarized electrons. From a preliminary work performed by H. Yamamoto and collaborators, one can hope to achieve \(0.7 \times 10^{34} \text{ cm}^2 \text{ s}^{-1}\) luminosity at the Z pole, that is two orders of magnitude above LEP1 and comparable to the TESLA project, \(0.5 \times 10^{34}\text{cm}^2\text{s}^{-1}\). With e-driven positron source, the damping time limits the collision rate to about 6 Hz. A horizontal emittance smaller by 1/2 with respect to the TDR should be available at Z just like for 250 GeV. The number of bunches can be multiplied by two.
but the luminosity increase by disruption effect is reduced at Z compared to 250 GeV, so that the luminosity goes like $\sim E_c^{1.5}$.

Figure 1 provides the energy dependence of the ILC luminosity for its base-line and for its high current operation. The base-line assumes spills of 1312 bunches at 5 Hz. To collect 2000 fb-1, the ILC250 assumes a progressive increase to 2624 bunches and 10 Hz [2]. For the operation at the Z pole, one assumes 2624 bunches at 6 Hz.

In this paper, we intend to re-assess the need to go beyond the LEP1/SLC accuracies on Z couplings measurements. We will use as examples some predictions for new heavy $Z'$ resonances based on the Randall Sundrum model. This model offers a well-motivated approach to solve problems which plague the SM, in particular the hierarchy problem and the arbitrary appearance of the Higgs field. We will show how ILC250 and ILC-GigaZ, with polarized beams and large currents, can provide the necessary accuracies to test these models well beyond the sensitivity of LHC.

I. ILC-GigaZ: $\sin^2 \theta_{\text{eff}}$ measurement

This part is a reminder of the work done for TESLA in the so-called GigaZ scenario [3].

One can in principle repeat the same arguments with, however, an important difference: to reduce cost and complexity, one assumes that only $e^-$ are polarized and therefore the measurement of polarisation comes from the polarimeters and cannot be confirmed by the Blondel method, as when $e^+$ are polarized.

It is however true that ILC will be in a much better situation than SLC (which claimed 0.5% precision on $P_{e}$) since, at 250 GeV, it will be possible to confront the polarimeter results with the $WW$ measurements. This will no doubt reduce polarimetry uncertainties. To get a comparable precision
to TESLA where one uses the Blondel scheme with positrons polarized at 20%, one would need to measure $P_e$ to 0.05%, an order of magnitude more precise than at SLC. This gain still needs to be demonstrated but our confidence is based on the assumption, previously mentioned, that polarimetry can be calibrated at 250 GeV.

As for Tesla, one measures:

\[
ALR = \frac{NL-NR}{(NL+NR)P_e} = \frac{2veae}{(ae^2+ve^2)} = Ae
\]

$ae = -0.5$ and $ve = -0.5 + 2\sin^2 \theta_{eff}$ being the vector and axial vector couplings of the electron to the Z.

All hadronic decay modes are used and therefore GigaZ gives $\delta ALR = 3 \times 10^{-5}$, which requires $\delta P_e / P_e \sim 2 \times 10^{-4}$ to match the statistical accuracy. As discussed below, the monitoring of the beam energy spread gives a worse accuracy, $\delta ALR = 10^{-4}$. To reach this accuracy, the TESLA TDR takes an equivalent polarisation error of 0.05% which is achieved by using positrons with 20% polarisation [4].

In what follows one will assume for ILC-GigaZ:

\[
\delta P_e / P_e = 0.05\%
\]

which is a factor 10 better than achieved at SLC but, again, without the resources of calibrating the polarimeters using the WW channel.

The corresponding accuracy is $\delta \sin^2 \theta_{eff} = 1.3 \times 10^{-5}$, to be compared to the present world precision, $1.3 \times 10^{-4}$, and two times worse than $6 \times 10^{-6}$ claimed by FCCee with ~300 times more luminosity. Note in passing that the compensating gain due to polarisation was already observed with SLC that could overcome LEP1 results with ~30 times less integrated luminosity.

FCCee uses only the cleanest mode $e^+e^- \rightarrow \mu^+\mu^-$ to measure $AFB_\mu$, while ILC with polarisation can include all hadronic modes to measure the polarisation asymmetry $ALR$. Using $AFB_\mu$, FCCee has access to the combination $AeA_\mu$ and not to $Ae$ individually, as will be the case with ILC. It has therefore to assume lepton universality, $Ae = A_\mu$, to extract $Ae$ and $\sin^2 \theta_{eff}$. This assumption is by no means trivial and we will come back to this in the last section.

II. Systematics

Precision measurements at GigaZ are usually not limited by statistical errors but by various instrumental and theoretical effects.

II.1 Calibration in energy

A constraint to be remembered is the beam energy spread, which for certain EW measurements will be essential. Recall, as rightly reminded by the FCCee CDR, that the asymmetry measurements are very sensitive to the beam energy error through the influence of photon-Z interference effect.

For TESLA, it was assumed that there would be energy monitoring using the beam magnetic spectrometers. Quantitatively one should require a control on the energy which avoids degrading the accuracy on $ALR$. One predicts:
where this measurement uses all hadronic final states, u, d, s, c, b (see Appendix). Note in passing that this dependence is steeper if one selects lepton final states. Note also that this dependence will be an order of magnitude reduced for quantities like Rb or Rℓ.

Reaching a $10^{-4}$ accuracy on ALR is achievable with a spectrometer resolution of ~1 MeV as claimed by TESLA. The spectrometer can be calibrated using an energy scan around the well-known Z resonance. Beamstrahlung should also be kept under control since it produces a $9 \times 10^{-4}$ shift on ALR, again assumed to be feasible in the TESLA TDR [4]. The error quoted by TESLA is therefore achievable: $\Delta_{ALR}=10^{-4}$.

For what concerns the quantity $ALRFB$ which, as recalled in the Appendix, is used to extract $A_b$, $A_\mu$ and $A_\tau$, the energy dependence is similar. These results are summarized in the following table:

| Type of fermions | $\mu, \tau$ | b | c | All quarks |
|------------------|--------------|---|---|------------|
| $dALR/d\sqrt{s}$ | 10^{-5}/MeV  | 6 | 1.92 | 2.11 | 1.99 |
| $dALRFB/d\sqrt{s}$ | 0.9 | -0.34 | | | |

II.2 Polarisation measurement

At SLC, the positrons were produced from an energetic polarized electron beam (J. Seeman, private communication), which may have induced a remnant polarisation of the positrons. This effect was however unproven. At the ILC-GigaZ, the positron beam can be conventionally produced from an unpolarized 3 GeV electron beam. Therefore it is justified to suppose that the polarisation of the positrons is exactly 0 since no mechanism can be invoked to generate positron polarisation.

II.3 Theoretical errors

In [5], a review of theoretical errors has been presented and is recalled in the table below. For Rb, discussed in the next section, the expected error will be $1.5 \times 10^{-4}$, while the experimental error for ILC is of the same order. One also observes that the theoretical uncertainty on $\sin^2 \theta_{\text{eff}}$ is 3 times the expected accuracy at ILC-GigaZ, $1.3 \times 10^{-5}$. For Ab, the theoretical uncertainty $\delta A_b=3.3 \times 10^{-5}$ is smaller than our expected accuracy $5 \times 10^{-4}$.

This table recalls the accuracies expected at FCCee [5]. Extrapolations to better TH errors are expected in a foreseeable future.
III. Asymmetry measurements at ILC-GigaZ

TESLA assumes $\delta R_b = 0.21653 \pm 0.00014$. The gain in accuracy at FCCee very much depends on assumptions on systematics and can be as low as a factor two with respect to ILC.

Recall that for $R_b$, the luminosity error cancels on this ratio, while the b-tagging efficiency evaluation follows methods developed at LEP1 (ratio of double to single tag) but with improved b-tag efficiencies given the smaller radius of the ILD beam pipe, analogue to SLD.

The measure of $A_b$ at SLC uses the following combination of measurements (see Appendix):

$$A_{LRFB} = \frac{1}{P_e} \frac{(\sigma_F - \sigma_B)L - (\sigma_F - \sigma_B)R}{(\sigma_F + \sigma_B)L + (\sigma_F + \sigma_B)R} = \frac{3}{4} A_f$$

This quantity directly provides $A_b$ when selecting the $b\bar{b}$ channel, noting that for this channel one can define forward and backward events by selecting charged B mesons or charged kaons, as currently performed with ILD [6].

![Figure 2: Estimated measurement errors at ILC-GigaZ compared to LEP1/SLC and FCCee](image)

The accuracy on $A_b$ can be understood easily by extrapolating from the result of SLD which gives an error on $A_b$ of 2% with $0.5 \times 10^9$ hadronic events. ILC-GigaZ will collect $0.7 \times 10^9$ hadronic Z decays, hence an error of 0.05%. With better tracking and kaon identification in ILD, one expects a statistical error $\sim 0.015\%$, meaning that the polarisation error will become dominant.

Using the ratio $A_{LRFBb}/A_{LR}$, one can also extract $A_b/A_e$ independently of the beam polarisation, which offers an additional tool.

The same is true for $\mu$ and $\tau$, giving the ratios $A_\mu/A_e$ and $A_\tau/A_e$. These numbers provide a test on lepton universality, as will be discussed later.
As for ALR, one has checked how this various methods are affected by energy spread.

Figure 2 summarizes what can be expected from ILC-GigaZ, showing an impressive progress with respect to LEP1/SLC.

IV. BSM physics from ILC250 with ILC-GigaZ measurements

The reaction $ee \rightarrow bb$ will be studied at ILC250 [7]. Present results are based on a new study, under preparation, using 2000 fb$^{-1}$ at ILC250 and increased efficiency. Polarized beams allow to separate the 4 different chirality combinations $LeLb$, $LeRb$, $ReRb$, $ReLb$. As will be shown on examples, these four modes can be differently influenced by BSM physics and their separation allows to identify the underlying mechanism and extract $MZ'$. These four amplitudes can be decomposed as follows:

$$LeLb = QeQb + \frac{LeZLbZ}{s^2wc^2w}BWZ + \sum_{z'} \frac{LeZ'LbZ'}{s^2wc^2w}BWZ'$$

where the charges are $Qe=-1$, $Qb=-1/3$, the SM Z couplings are $LeZ=Ie3-Qes^2w$, $LbZ=Ib3-Qbs^2w$ and $BWZ=s/(s-MZ^2)$ is the Breit Wigner of the Z. One can write similar formulae for the three other combinations $LeRb$, $ReRb$ and $ReLb$.

$LeZ'$ and $LbZ'$ are provided by BSM models, as illustrated by figure 4 given below.

While the first term, the photon exchange, should remain unaltered by virtue of the U(1)em symmetry, the second term can be affected by Z-Z' mixing or b-b' mixing. As is well known, this channel has provided the most significant deviations at LEP1 and in [8] Z-Z' mixing has been invoked to explain the deviation on $AFB_b$.

If one considers, figure 3, the two most precise measurements at the Z pole providing $\sin^2\theta_{eff}$, ALR from SLC and $AFB_b$ from LEP1, one observes that they differ by 3.5 s.d. This alone shows the interest to re-measure these two quantities with much higher accuracy. Moreover, it would be far preferable to measure $A_b$ directly, as this quantity is unaffected by the $A_e$ behaviour, in contrast to $AFB_b$ measured at circular colliders w/o beam polarisation.

Generally speaking, the third term of the equation above contains an ensemble of vectorial heavy resonances which, in the case of Randall Sundrum (RS) models are Kaluza-Klein recurrences of the photon, the Z and new resonances of the type $Z'$, due to additional symmetry groups. The first and third term are negligible on the Z resonance given that there is almost no interference with the Z imaginary amplitude, while this interference can become manifest at ILC250 where all three terms are real. To unambiguously isolate this new contribution, it is therefore essential to measure accurately the second term at the Z resonance. In the absence of a run at the Z pole, one would be left with the unsatisfactory LEP/SLC measurements, with insufficient accuracy to match in precision the measurements performed at ILC250 and would therefore limit the sensitivity of ILC to new physics, as explained by a numerical example in the next section.
One would still be able to witness the presence of BSM physics but unable to isolate the contribution from new resonances (in red in above equation).

**The essential mission of the ILC is however to discover new physics and provide information on the masses of new resonances for future colliders.**

To get a feeling of what can be expected from BSM, one can use above table from [9], a BSM model with three heavy resonances with masses from 7 to 9 TeV.

From figure 4, one observes that the $bR$ couplings to the heavier resonances can be up to four times the SM coupling while the $eR$ coupling reaches 7 times the SM coupling. This explains how ILC250 can reach such a sensitivity on the ReRb combination as shown by figure 5. Moreover these effects will also be marked for the leptonic couplings, including the $ee\rightarrow ee$ reaction which requires a more subtle analysis [11].

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Using three explicit RS models based on extra dimensions, figure 5 shows the expected number of standard deviations from a combination of an ILC-GigaZ measurement giving RbZ and LbZ and from the 4 helicity combinations measured at 250 GeV. The Djouadi model [5] was designed to reproduce the LEP1 effect and corresponds to a $Z'$ with a $\sim 3$ TeV mass which mixes with $Z$. This $Z'$ does not couple to $e^+e^-$, and the KK excitations of the photon and the $Z$ are very weakly coupled. The model of
Hosotani et al. [9], already discussed in the previous section, assumes KK resonances at 7-9 TeV and no mixing at the Z resonance. Peskin and Yoon provide two hypotheses referred as Peskin 4 (bL in 5 and bR in 4 of SO(5)) and Peskin 5 (bL and bR in 5 of SO(5)).

These results suggest that ILC250+GigaZ can discover RS signals and, more importantly, pick up the right underlying model. Then these precision measurements would allow to predict indirectly the mass of the new resonance. How far in mass can ILC reach is discussed in the next section.

V.1 Sensitivity to very high Z’ masses

At a modest energy of 250 GeV, ILC has a reach which extends well above LHC direct searches. For what concerns the Hosotani model, the large deviation which would be seen at ILC for a 8 TeV resonance indicates that ILC250 can extend it sensitivity even far beyond this mass. Since these resonances are very wide, LHC, is already able to exclude them up to ~9 TeV [9]. In the absence of a signal at ILC250, one would conclude that such a Z’ is heavier than 20 TeV at 95% C.L. This limit could reach 40 TeV with ILC500. Furthermore, as discussed in [11], leptonic channels, with beam polarisation and with ~100% reconstruction efficiency and negligible background can also reach mass limits above 20 TeV at ILC250.

These figures illustrate how ILC could pave the way to the design of a pp collider which would be able to observe directly such heavy resonances.

V.2 The need for ILC-GigaZ

Recall again that effects from Z’ propagators can only be established if we re-measure the lepton and b couplings at ILC-GigaZ. To see this quantitatively, let’s compare the measurement error on the amplitude ReRb which has, according to above plot, the highest occurrence of deviations, to the uncertainty due to the b couplings measurement at LEP1. At ILC250 one has:

\[ \text{ReRb} = QeQb + \text{ReZRbZBWZ}(s^2w/c^2w) = 1/3 + 0.115 \]

where the second term, ~0.115, is the Z contribution. To match the expected measurement accuracy with 2000 fb⁻¹ collected at 250 GeV, \( \delta \text{ReRb} = 0.0007 \), one needs a relative accuracy on \( R_{bZ} \) of 0.0005/0.115=0.7%, while the present LEP1 measurement gives ~10%. In the absence of a GigaZ baseline, one would be able to observe a significant deviation from the SM but be totally unable to decide if it comes from an alteration of SM Z couplings or from the presence of Z’ propagators. This would result in a major waste of information.

At ILC-GigaZ, one expects: \( \delta R_{bZ} /R_{bZ}=0.55\% \) (and \( \delta L_{bZ}/L_{bZ}=0.04\%) \) which perfectly matches the accuracy achievable with ILC250.

This discussion illustrates the specific role of b quark measurements with respect to top quark measurements: they allow a perfect separation between Z coupling anomalies and propagator contribution. To achieve the same separation for top quarks one needs to run at two energies above the top threshold.
V.3 Summary

These examples clearly show:

- The importance of running at the Z pole to measure the Zbb couplings, eventually detecting an anomaly which needs to be taken into account for the measurement at 250 GeV
- The sensitivity of ILC to detect the presence of Z' propagators at 250 GeV, provided the Zbb couplings are re-measured with ILC-GigaZ.
- The importance of the measurements with polarized beams to disentangle the four amplitudes LeLb, LeRb, ReRb, ReLb, allowing to discriminate between the various models

VI. The lepton universality and BSM physics

Recalling the various anomalies observed at B factories where, e.g. b->sµµ differs from b->see, it will be important to verify lepton universality at the Z pole and at ILC250.

At the Z pole, it will be straightforward to measure Rµ and Aµ using the same observables as for b quarks. The rate is ~5 times smaller but the efficiency is close to 100% for both measurements which gives an almost complete compensation for the efficiency for b quarks. One has δRµ/Rµ=0.017% and δAµ=0.01%. As shown in the Appendix, one can even improve on this by getting rid of the beam polarisation uncertainty and provide a test on universality for Aµ/Ae and Aτ/Ae at the 3x10⁻⁴ level.

At FCCee, one can measure the ratio Rτ/Rµ with a relative error of 5x10⁻⁵ which allows a stringent test of universality for the combination g²Lℓ+g²Rℓ, while ILC-GigaZ allows to measure (g²Lℓ-g²Rℓ)/(g²Lℓ+g²Rℓ) which may show different deviations.

At ILC250, one will be able to measure separately ReRµ, ReLµ, LeLµ and LeRµ and the same is true for τ leptons which, together with the b coupling measurements, will also allow to identify the origin of BSM contributions (see e.g. [9]). For ee->µµ and ee->ττ one expects ~30 s.d. on ReRµ and ReRτ in this model. The uncertainty due to Z couplings is negligible given the accuracies expected from ILC-GigaZ but this would not be the case if one were left with LEP/SLC accuracies ten times worse.

Conclusion

From this brief survey, one can conclude that ILC-GigaZ is a necessity which should not be disregarded since it offers a powerful access to EW precision measurements, comparable to FCCee. For the emblematic measurement of sin²θeff, it allows to improve the present accuracy by a factor 10, beyond present theoretical uncertainties.

This achievement requires either polarized positrons, at the expense of a more complicated scheme, or, as for SLC, an electron beam with a polarisation measured 10 times better than at SLC. While very challenging, the latter goal does not seem out of reach given that polarimetry can be cross-checked at ILC250 where one can use the presence of the WW channel and, eventually, the Blondel method if there are polarized positrons at 250 GeV.
As shown in a concrete example, these measurements are complementary to those performed by ILC250. This example also demonstrates that beam polarisation plays an essential role in distinguishing between various RS models.

In the absence of a run at the Z pole, one would be left with the LEP/SLC measurements accuracies which would not match in precision the measurements performed at ILC250 which would therefore decrease the sensitivity of ILC to new physics and, more importantly, limit the capability of ILC250 to predict the masses of the new resonances.

Running ILC-GigaZ is therefore required to benefit from the high accuracies provided by ILC250 on fermion coupling.

Not to be forgotten is the issue of HO theoretical corrections, in particular EW corrections which for the top channel have revealed very significant effects [13]. Given that experimental errors will reach the 0.1% level, it is essential to launch an important effort on this topic.

Note in passing that ILC-GigaZ with polarisation can provide a rigorous and unique test of lepton universality by measuring separately $A_e$, $A_\mu$ and $A_\tau$. This test could be crucial given some departures of lepton universality observed in B factories.

This ensemble of results would constitute a major step forward in our comprehension of fermion properties with a major potential for BSM discoveries.

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APPENDIX

Asymmetries

1/ Recall here the methods used by SLD to extract \( A_b \), see [12].

The Born level differential cross-section, with a polarized electron beam, is given by:

\[
\frac{d\sigma_{b}}{d\cos\theta} = \frac{3}{8} \sigma_b [(1 - P_e A_e) (1 + \cos^2 \theta) + 2 (A_e - P_e) A_b \cos \theta]
\]

Where the electron beam polarisation \( P_e \) is positive for right-handed beam. A forward-backward-left-right asymmetry can be formed as:

\[
A_{LRFB} = \frac{1}{P_e} (\sigma_F - \sigma_B) L - (\sigma_F - \sigma_B) R
\]

\[
A_{b} = \frac{3}{4} A_f
\]

This measurement can be applied to all fermionic final states where one can measure the fermion charges: \( \mu, \tau, b \) and \( c \). In particular it gives \( A_b \).

It is directly dependent on the electron polarisation.

The dominant systematical error comes from \( P_e \) (luminosity and efficiency go away on this ratio): \( \delta A_b / A_b = \delta P_e / P_e \). Taking 0.05% on \( P_e \), we find:

\[
\delta A_b = 0.0005
\]

For \( A_\mu \) and \( A_\tau \) the efficiency is \( \sim 100\% \) (instead of \( \sim 30\% \) for \( A_b \)) which allows to get almost the same statistical accuracy, \( \sim 0.02\% \), also almost negligible, while the error due to \( P_e \), \( \delta A_\mu / A_\mu = \delta P_e / P_e = 0.05\% \), gives: \( \delta A_\mu = \delta A_\tau = 0.000075 \)

Above errors do not include energy smearing effects which explain why they appear better than \( \delta A_e \).

When included, they will read:

\[
\delta A_\mu = \delta A_\tau \sim 0.0001
\]

2/ Another method allows eliminating the error on \( P \)

Take the difference \( \Delta \) between \( P \& -P' \) (we do not need to assume a perfect flip, \( P=P' \)), then:

\[
\frac{d\Delta \sigma_{b}}{d\cos\theta} = \frac{3}{8} \sigma_b (P + P') [A_e (1 + \cos^2 \theta) + 2 A_b \cos \theta]
\]

After angular integration, one has: \( F_\Delta - B_\Delta = 2 \sigma_b (P + P') A_b \) \( F_\Delta + B_\Delta = 8 \sigma_b (P + P') A_e / 3 \) and the ratio:

\[
R_\Delta = F_\Delta - B_\Delta / (F_\Delta + B_\Delta)
\]

does not depend on \( P, P', L, \sigma_b \) nor efficiency:

\[
R_\Delta = 3 A_b / 4 A_e
\]
It is therefore possible to define a combination free of the knowledge of polarisation. One can proceed in an equivalent way by simply taking the ratio:

\[ R_{Fe} = \frac{ALRF_B}{ALR} = \frac{3A_f}{4A_e} \]

This measurement provides a precise test of \( e/\mu \) and \( \tau/e \) universality.

The statistical error is very small, dominated by the statistics on \( ee \rightarrow \mu\mu \), at the level of:

\[ \frac{\delta R_{\mu e}}{R_{\mu e}} = \frac{\delta R_{\tau e}}{R_{\tau e}} = 0.03\% \]

One should add the energy smearing effect where there is partial cancellation between the \( ALRF_B \) and \( ALR \) contribution (see below).

Universality \( \mu/e \) and \( \tau/e \) can thus be tested at the \( \approx 3 \times 10^{-4} \) level, independently of the accuracy on \( Pe^- \).

Recall that at LEP1 this test using leptonic branching ratios is at the \( 3 \times 10^{-3} \) level.

Energy dependence

For \( ALR \) and \( ALRF_B \) one needs to worry about the energy dependence induced by the photon-Z interference. Due to beamstrahlung and beam energy resolution, this effect cannot be eliminated by sitting on the resonance. Detailed computations will not be reproduced here. To evaluate analytically the differential dependence of these quantities, the principle is rather simple. One can easily show that:

\[
\frac{dALR}{d\sqrt{s}} = \frac{1.99 \times 10^{-5}}{\text{MeV}}
\]

in very good agreement with the TESLA TDR. It also appears that in the expression of \( ALR = (L-R)/(L+R) \), the main energy dependence comes from the numerator while the denominator shows a variation more than ten times slower. This means that the \( R_b \) measurement will show a negligible effect.

One could measure \( ALR \) using lepton final states but in this case the derivative is larger:

\[
\frac{dALR_{\mu}}{d\sqrt{s}} = 6.2 \times 10^{-5}/\text{MeV}
\]

This method also allows to compute the energy variation of \( ALRFB \):

\[
\frac{dALRFB_B}{d\sqrt{s}} = -0.34 \times 10^{-5}/\text{MeV} \quad \text{and} \quad \frac{dALRFB_{\mu}}{d\sqrt{s}} = 0.9 \times 10^{-5}/\text{MeV}
\]

showing a similar dependence as \( ALR \).