An inventory model for estimation of deterioration with time-dependent demand and storage cost

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Abstract. An inventory consists of items that are stocked temporarily before they are used or sold at a later time. An inventory model is required to enable the retailer to determine the optimal order quantity. The main goal is to determine how many items should be ordered and when to order to minimize the total cost. If retailers order too much, the holding cost will increase because the items that have to be stored increase and the items will deteriorate. If they order too little, then shortages may occur because the inventory could not meet the demand. In this paper, a mathematical model for inventories with deterioration, time-dependent demand, and time-dependent storage cost will be developed. A sensitivity analysis has been performed to examine how the changes in the model parameters affect the optimal solution. It can be concluded that if the deterioration rate decreases and the demand rate increases, then the total cost will also decrease.

1. Introduction
An inventory consists of goods that are stored for use or sale in a future period of time. If a company orders only a few items, the available inventory will quickly run out. This causes the ordering cost to be greater because replenishments will be made more often. But if the company orders a lot of goods, the higher the required storage/holding costs will be. Inventory models are needed to help the company determine the number of items to be ordered and when to order so that the costs incurred can be minimized and the profit maximized [1,7,8].

In fact, many things affect inventory. An example is the possibility of deterioration or decline in the quality of goods in the inventory held. Deterioration occurs over time, that is to say, the more time passes, the more the quality of goods decreases. This condition occurs when we are dealing with for example food products, fruit, vegetables or drugs. Other factors that are needed to consider are demand rate and storage cost function. In the early stage of the development of the mathematical model for inventory system considering the above factors, demand rate and storage cost are assumed to be constant, even for deterioration rate is also assumed to be constant. These assumptions are far from realistic and therefore people begin to develop other models by considering non-constant deterioration rate, demand rate and storage cost functions.

In this paper, we develop the model proposed by [6] by considering quadratic time-dependent demand and quadratic storage cost function. The time-dependent demand has been widely used in developing a mathematical model for the inventory control problem with its variation such as linear, quadratic or quadratic ramp-time demand. (see for example Kavithapriya R and Senbagam K (2018), Mishra and
Singh (2011) and Mishra et al (2013)). [6] develop a mathematical model with linear time-dependent demand and linear storage cost function. In developing the model, one cycle will be divided into two parts, namely the period of production and the period when production has ended. During the production period, inventory will increase due to the presence of the addition of goods. At the same time, supply will be reduced by demand and deterioration. When the production period ends, there will be no more items added, but there is still demand and deterioration. Because inventory continues to decrease, eventually the supply will run out and there can be a shortage of goods. When the latter occurs, the company can provide the option to place a backorder. Backorders are made so consumers will be willing to wait until the item is produced again. Needless to say, not all consumers will choose to make a back-order. There must be consumers who do not wish to wait until the goods are reproduced and decide to look for another company instead, which may lead to lost profit potential for the company. The model is made by taking all these factors into account.

By combining linear and quadratics for the demand rate and storage cost function, we propose four mathematical models, make comparison among these four models and perform sensitivity analysis for each model. The basic model was developed by [6] and we make modifications in the time-dependent demand and the storage cost function. The contribution of our paper lies in the more realistic assumptions on deterioration factor, time-dependent demand and quadratic storage cost function to our models. We also develop an algorithm to find the optimal solution. The decision variables in our models are the production length and the cycle length that give the minimum total cost. Sensitivity analysis is then performed in order to study the effect of changes in parameter value to the optimal solution.

The organization of the remainder of this paper is as follows. The model is introduced in section 2 along with the proposed algorithm to find the optimal solutions. Section 3 is devoted to results and discussion including the sensitivity analysis, and finally conclusions and further research direction are given in the last section.

2. The Model

2.1. Notations

We use the same notations as in [6] as follows:

- $A$: order cost per single order
- $c$: purchase price of goods per unit
- $T$: cycle length
- $\delta$: rate of back-order, $0 \leq \delta \leq 1$
- $s$: potential profit lost per unit (lost sales per unit)
- $I(t)$: inventory at time $t$
- $I_0$: total inventory during $[0, T]$
- $I_1(t)$: inventory during production time
- $I_2(t)$: inventory at the end of production
- $TC$: total cost
- $D(t)$: the rate of demand is time-dependent
- $k$: production rate
- $\theta(t)$: the rate of deterioration that follows the Weibull distribution, where $\theta(t) = \alpha \beta t^{\beta-1}, t \geq 0, 0 < \alpha \leq 1, \beta \geq 1$. 
- $Q$: inventory at the end of production
- $c_2$: shortage cost per unit
- $p(t)$: storage costs depending on time per unit of goods per time
2.2. Model Development

The model was developed by [6] where they divided the time interval \([0,T]\) into two periods of time. Production is carried out in the period \([0,t_1]\) and the number of inventory decreases due to demand and deterioration in that period. In the period \([t_1,T]\), goods cease to be produced. When the production period has passed, the remaining supply will be exhausted by demand and deterioration. Since there are no more items added, there will be a shortage of goods when the demand is greater than the actual supply. When there is a shortage of goods, the company can provide the option of placing a backorder. Backorders are made so that consumers continue to wait until the goods are available again and will not look for other companies. Some consumers will make a back-order, while others will not. Consumers who do not make backorders provide potential lost profits. After reaching \(t = T\), the production period will start again and this cycle will repeat itself.

In the model developed, [6] considered linear storage cost and linear time-dependent demand. In this paper we make some modifications of the model by considering four models regarding the time-dependent demand and storage costs. Model 1 has a quadratic demand rate \((D(t) = a + bt + rt^2)\), where \(t, a, b, r \geq 0\) and quadratic storage cost \((p(t) = h + yt^2)\), where \(t, y, h \geq 0\). (This type of demand and storage cost function can be found in Tripathi and Tomar (2018)). Model 2 has a quadratic demand rate \((D(t) = a + bt + rt^2)\), where \(t, a, b, r \geq 0\) and linear storage costs \((p(t) = h + jt)\), where \(t, j, h \geq 0\). Model 3 has a linear demand rate \((D(t) = a + bt)\), where \(t, a, b \geq 0\) and quadratic storage costs \((p(t) = h + yt^2)\), where \(t, y, h \geq 0\). Model 4 has a linear demand rate \((D(t) = a + bt)\), where \(t, a, b \geq 0\) and linear storage costs \((p(t) = h + jt)\), where \(t, j, h \geq 0\).

In this paper we will derive the amount of inventory and costs involved from the model by considering an inventory model with quadratic demand rate \((D(t) = a + bt + rt^2)\), where \(t, a, b, r \geq 0\) and the quadratic storage cost \((p(t) = h + yt^2)\), where \(t, y, h \geq 0\). The results are slightly different than [6]. The derivation of the other models can be found in Ichwanto [2].

The inventory during the production period \([0, t_1]\) can be described using the following first order ordinary differential equation as follows.

\[
\frac{dI_1(t)}{dt} = k - D(t) - \theta(t)I_1(t), 0 \leq t \leq t_1, I_1(0) = 0
\]

The rate at which the inventory during the production period decreases is affected by the demand \(D(t)\) and the deterioration rate \(\theta(t)\). The solution of the above differential equation is given by

\[
I_1(t) = (k - a) \left( t - a t^{1+\beta} + \frac{a t^{1+\beta}}{\beta + 1} \right) - b \left( \frac{t^2}{2} - \frac{a t^{2+\beta}}{2} + \frac{a t^{2+\beta}}{\beta + 2} \right) - r \left( \frac{t^3}{3} - \frac{a t^{3+\beta}}{3} + \frac{a t^{3+\beta}}{\beta + 3} \right)
\]

In the period \([t_1,T]\), the number of inventory at the end of production is described by the following differential equation. The rate at which the inventory declines is due to demand and deterioration rate.

\[
\frac{dI_2(t)}{dt} = -D(t) - \theta(t)I_1(t), 0 \leq t \leq t_1, I_1(t_1) = I_2(t_1) = Q
\]

The solution of the differential equation above is given by:

\[
I_2(t) = Q \left[ 1 + a \left( t_1^{\beta} - t^{\beta} \right) \right] + a \left( t_1 - t \right) - \frac{a}{\beta + 1} \left( t_1^{\beta+1} - t^{\beta+1} \right) + a \left( t_1^{1+\beta} - t_1 t^{\beta} \right) - b \left( \frac{t_1^2}{2} - \frac{t^2}{2} + \frac{a}{\beta + 2} \left( t_1^{2+\beta} - t^{2+\beta} \right) - \frac{a}{2} \left( t_1^{2+\beta} - t^{2+\beta} \right) \right) - r \left( \frac{t_1^3}{3} - \frac{t^3}{3} + \frac{a}{\beta + 3} \left( t_1^{3+\beta} - t^{3+\beta} \right) - \frac{a}{3} \left( t_1^{3+\beta} - t^{3+\beta} \right) \right)
\]

The total inventory during the production period is given by:

\[
I_0 = \int_0^{t_1} I_1(t) \, dt
\]
The total order cost is given by $OC = A$, while the storage cost ($HC$) can be calculated as follows.

$$HC = \int_0^{t_1} p(t) I_1(t) \, dt$$

$$= h(k - a) \left[ \frac{t_1^2}{2} - \frac{\alpha \beta t_1^{2+\beta}}{(1+\beta)(2+\beta)} \right] - hb \left[ \frac{t_1^3}{6} - \frac{\alpha \beta t_1^{3+\beta}}{2(\beta + 2)(\beta + 3)} \right] - r \left[ \frac{t_1^4}{12} - \frac{\alpha \beta t_1^{4+\beta}}{3(\beta + 3)(\beta + 4)} \right]$$

Shortage can only occur when the production process is over or in the period of $[t_1, T]$ and the total shortage cost can be calculated as follows.

$$SC = c_2 \int_{t_1}^{T} I_2(t) \, dt$$

$$= QC_2 + ac_2 \left[ \frac{2Tt_1 - t_1^2 - T^2}{2} + \frac{\alpha}{\beta + 1} \left( t_1^{1+\beta} (T - t_1) - \frac{T^{2+\beta} - t_1^{2+\beta}}{2 + \beta} \right) + \alpha \left( \frac{T^{2+\beta} - t_1^{2+\beta}}{2 + \beta} - \frac{t_1^{2+\beta} - t_1^{1+\beta}}{2+\beta+1} \right) \right]$$

$$+ bc_2 \left[ \frac{\alpha}{3 + \beta} \left( \frac{T^{3+\beta} - t_1^{3+\beta}}{3 + \beta} - \frac{t_1^{3+\beta} - t_1^{2+\beta}}{2 + \beta} \right) + \frac{\alpha}{4 + \beta} \left( \frac{T^{4+\beta} - t_1^{4+\beta}}{4 + \beta} - \frac{t_1^{4+\beta} - t_1^{3+\beta}}{3 + \beta + 1} \right) \right]$$

Deterioration occurs in the production process due to the unsold goods. Therefore, the deterioration cost can be formulated as:

$$DC = c \int_0^{t_1} k - D(t) \, dt = c \left[ t_1(k - a) - \frac{bt_1^2}{2} - \frac{rt_1^3}{3} \right]$$

In the case where the consumers do not wish to take backorders, there is a potential of profit lost (lost sales cost). The lost sales cost is given by:

$$LSC = s \int_{t_1}^{T} (1 - \delta) D(t) \, dt = s(1 - \delta) \left[ a(T - t_1) + \frac{b}{2} (T^2 - t_1^2) + \frac{r}{3} (T^3 - t_1^3) \right]$$

The total purchase cost is calculated from all the inventory including the number of backorders, and calculated as:
\[ PC = c \left[ l_0 + \int_{t_1}^{T} \delta D(t)dt \right] \]
\[ = c(k - a) \left[ \frac{t_1^2}{2} - \frac{\alpha \beta t_1^{2+\beta}}{(\beta + 1)(\beta + 2)} \right] - bc \left[ \frac{t_1^3}{6} - \frac{\alpha \beta t_1^{3+\beta}}{(2(\beta + 2)(\beta + 3))} \right] \]
\[ - rc \left[ \frac{t_1^4}{12} - \frac{\alpha \beta t_1^{4+\beta}}{3(\beta + 3)(\beta + 4)} \right] + \delta c a(T - t_1) + \frac{\delta cb}{2} (T^2 - t_1^2) + \frac{\delta cr}{3} (T^3 - t_1^3) \]

The total cost is a function of \( t_1 \) and \( T \) consists of the order, storage, purchase, deterioration, shortage and lost sales costs.

\[ TC(T,t_1) = \frac{1}{T}(OC + HC + PC + DC + SC + LSC) \]

2.3. Algorithm
In this section, we provide the algorithm to a solution that minimizes the total cost by taking the following steps:

1. Finding the value of \( t_1^* \) and \( T^* \) derived from the equation \( \frac{\partial TC}{\partial t_1} = 0, \frac{\partial TC}{\partial T} = 0 \)

2. Check whether the obtained values of \( t_1^* \) and \( T^* \) found from step 1 satisfy the condition of the determinant of Hessian matrix (Vanberg et al (2007)),

\[ H = \begin{vmatrix}
\frac{\partial^2 TC}{\partial t_1^2} & \frac{\partial^2 TC}{\partial t_1 \partial T} \\
\frac{\partial^2 TC}{\partial t_1 \partial T} & \frac{\partial^2 TC}{\partial T^2}
\end{vmatrix} = \left( \frac{\partial^2 TC}{\partial t_1^2} \right)^2 - \left( \frac{\partial^2 TC}{\partial T^2} \right)^2 > 0, \left( \frac{\partial^2 TC}{\partial t_1} \right) > 0, \left( \frac{\partial^2 TC}{\partial T} \right) > 0 \]

3. Once all conditions have been met, then \( TC \) can be calculated with the \( t_1^* \) and \( T^* \) obtained.

4. The optimal solution is \((T^*, t_1^*, TC^*)\) where \( T^* \) is the optimal value of \( T \), \( t_1^* \) the optimal value of \( t_1 \) and \( TC^* \) the optimal value of \( TC \).

3. Results and Discussion

3.1 Numerical Examples
Numerical examples are provided as an illustration of the mathematical model used. In this example, Model 1 is used, which is a model with a quadratic demand rate and a quadratic storage cost. Most of the parameters’ values in this numerical examples come from \([6]\) except for order cost (A) and the parameters for quadratic demand rate and storage cost.

A company has a production rate of \( k \) 35 units per unit time, with a purchase price of goods per unit \( c \) of $20. The price for a one-time order \( A \) is $20. The total inventory at the end of the production period \( Q \) amounts to 25 units. When the production period is over, there can be a shortage of goods with the cost of shortages of goods per unit \( c_s \) is $6. When there is a shortage of goods, there are consumers who will make a back-order and some will not. The rate of consumer back-orders \( \delta \) is 0.7. Consumers who do not carry out back-orders give potential lost profit \( s \) of $8 per unit item. The request is made at the rate of \( D(t) = a + b(t - t^2) \), \( a \geq 0 \), where \( a = 10, b = 20 \) and \( r = 1 \). There is a deterioration rate that follows the Weibull distribution \( \theta(t) = \alpha \beta t^{\beta - 1} \), where \( \alpha = 0.5 \) and \( \beta = 1.5 \). In addition, there are time-dependent storage costs \( p(t) = h + y t^2 \), where \( h = 0.8 \) and \( y = 0.95 \). Following the solution search procedure, the optimal results are cycle length \( T = 0.77247 \) units of time, time of production \( t_1 = 0.56961 \) units of time, and the total cost amounts to \( TC = 531.22 \).
3.2 Sensitivity Analysis

In this section, we will show how the change of order parameters per one-time order \((A)\), purchase price of goods per unit \((c)\), rate of back-orders \((\delta)\), potential profit lost per unit \((s)\), cost of goods shortage per time units \((c_2)\), time-dependent demand rates \((a, b, r)\), the deterioration rate that follows the Weibull distribution \((\alpha, \beta)\), and time-dependent storage costs \((h, j, \gamma)\) affect the total cost \((TC)\).

The values of the parameters \(A, a, b, \alpha, \beta, \gamma, h, s, \delta, c,\) and \(c_2\) are changed one by one by + 50%, + 25%, + 10%, -10%, -25%, and -50% while other parameter values remain. The model sensitivity analysis is carried out based on numerical examples. The same parameters are used for Model 2, Model 3 and Model 4 with the difference among those models lie in the parameters of the demand rate and storage cost function. For the quadratic demand rate, we have \(D(t) = a + bt + rt^2, t \geq 0\), while for the quadratic storage costs \(p(t) = h + jt + \gamma t^2\), where \(h = 0.8, j = 0.38\) and \(\gamma = 0.95\). The values of \(a, b, r, h, j, \gamma\) are the same depending on the demand rate function and storage cost function used for Model 2, Model 3 and Model 4.

Table 1. Sensitivity Analysis based on the four models

| Parameter | % Change | TC Change (%) |
|-----------|----------|---------------|
|           | MODEL 1  | MODEL 2  | MODEL 3  | MODEL 4  |
| \(A\)     | + 50     | 2.33    | 2.32    | 2.21    | 2.20    |
|           | + 25     | 1.19    | 1.19    | 1.13    | 1.13    |
|           | + 10     | 0.48    | 0.48    | 0.46    | 0.46    |
|           | - 10     | -0.49   | -0.49   | -0.47   | -0.47   |
|           | - 25     | -1.25   | -1.25   | -1.19   | -1.19   |
|           | - 50     | -2.57   | -2.57   | -2.45   | -2.45   |
| \(h\)     | + 50     | 0.31    | 0.31    | 0.32    | 0.32    |
|           | + 25     | 0.15    | 0.15    | 0.16    | 0.16    |
|           | + 10     | 0.06    | 0.06    | 0.06    | 0.06    |
|           | - 10     | -0.06   | -0.06   | -0.06   | -0.06   |
|           | - 25     | -0.15   | -0.15   | -0.16   | -0.16   |
|           | - 50     | -0.31   | -0.31   | -0.33   | -0.32   |
| \(s\)     | + 50     | 1.33    | 1.34    | 1.34    | 1.35    |
|           | + 25     | 0.68    | 0.69    | 0.69    | 0.70    |
|           | + 10     | 0.28    | 0.28    | 0.28    | 0.29    |
|           | - 10     | -0.29   | -0.29   | -0.29   | -0.30   |
|           | - 25     | -0.74   | -0.74   | -0.75   | -0.76   |
|           | - 50     | -1.53   | -1.55   | -1.57   | -1.59   |
| \(c\)     | + 50     | 42.86   | 42.84   | 42.93   | 42.91   |
|           | + 25     | 21.39   | 21.38   | 21.43   | 21.42   |
|           | + 10     | 8.54    | 8.54    | 8.56    | 8.56    |
|           | - 10     | -8.57   | -8.50   | -8.53   | -8.52   |
|           | - 25     | -21.13  | -21.11  | -21.19  | -21.18  |
|           | - 50     | -41.09  | -41.07  | -41.28  | -41.26  |
| \(c_2\)   | + 50     | 3.33    | 3.35    | 3.29    | 3.31    |
|           | + 25     | 1.61    | 1.62    | 1.60    | 1.61    |
|           | + 10     | 0.64    | 0.64    | 0.63    | 0.63    |
|           | - 10     | -0.62   | -0.63   | -0.62   | -0.63   |
|    | -25 | -1.55 | -1.56 | -1.54 | -1.55 |
|----|-----|-------|-------|-------|-------|
| -50| -3.04| -3.06 | -3.02 | -3.02 |
| $\delta$ | $+$50 | 4.20 | 4.22 | 4.17 | 4.20 |
|     | $+$25 | 2.24 | 2.25 | 2.24 | 2.26 |
|     | $+$10 | 0.95 | 0.95 | 0.95 | 0.96 |
|     | $-$10 | -1.05| -1.06| -1.07| -1.08|
|     | $-$25 | -2.89| -2.92| -3.01| -3.04|
| $\gamma$ | $+$50 | 0.06 | 0.05 | 0.05 | 0.00 |
|      | $+$25 | 0.03 | 0.03 | 0.03 | 0.00 |
|      | $+$10 | 0.01 | 0.01 | 0.01 | 0.00 |
|      | $-$10 | -0.01| 0.00 | -0.01| -0.01|
|      | $-$25 | -0.03| 0.00 | -0.03| -0.03|
|      | $-$50 | -0.06| 0.00 | -0.07| -0.06|
| $j$  | $+$50 | 0.00 | 0.05 | 0.05 | 0.06 |
|      | $+$25 | 0.00 | 0.03 | 0.03 | 0.03 |
|      | $+$10 | 0.00 | 0.01 | 0.01 | 0.01 |
|      | $-$10 | 0.00 | 0.01 | 0.01 | 0.01 |
|      | $-$25 | 0.00 | 0.03 | 0.03 | 0.03 |
|      | $-$50 | 0.00 | 0.05 | 0.05 | 0.06 |
| $a$  | $+$50 | -7.55| -7.54| -7.73| -7.71|
|      | $+$25 | -5.94| -5.92| -6.03| -6.01|
|      | $+$10 | -2.65| -2.63| -2.68| -2.66|
|      | $-$10 | 2.84 | 2.82 | 2.85 | 2.82 |
|      | $-$25 | 7.26 | 7.19 | 7.24 | 7.15 |
|      | $-$50 | 14.67| 14.46| 14.35| 14.10|
| $b$  | $+$50 | -1.45| -1.43| -1.53| -1.51|
|      | $+$25 | -1.27| -1.25| -1.34| -1.32|
|      | $+$10 | -0.68| -0.67| -0.72| -0.71|
|      | $-$10 | 0.98 | 0.96 | 1.04 | 1.00 |
|      | $-$25 | 3.09 | 2.97 | 3.14 | 2.96 |
|      | $-$50 | 6.39 | 5.85 | 2.36 | 1.76 |
| $\alpha$ | $+$50 | -0.90| -0.90| -1.00| -1.00|
|      | $+$25 | -0.43| -0.42| -0.47| -0.47|
|      | $+$10 | -0.18| -0.18| -0.20| -0.20|
|      | $-$10 | 0.17 | 0.17 | 0.19 | 0.19 |
|      | $-$25 | 0.44 | 0.43 | 0.48 | 0.48 |
|      | $-$50 | 0.87 | 0.87 | 0.96 | 0.96 |
| $\beta$ | $+$50 | 0.49 | 0.49 | 0.50 | 0.49 |
|      | $+$25 | 0.26 | 0.26 | 0.26 | 0.26 |
|      | $+$10 | 0.11 | 0.11 | 0.11 | 0.11 |
|      | $-$10 | -0.11| -0.11| -0.11| -0.11|
|      | $-$25 | -0.27| -0.26| -0.26| -0.26|
|      | $-$50 | -0.45| -0.44| -0.43| -0.42|
Based on the sensitivity analysis, it can be concluded that the total cost increases and the deterioration rate increases and demand rate decreases. Therefore, the minimum total cost is obtained for lower deterioration rate and higher demand rate. We also found that for all four models, changes in deterioration rates (α and β) and time-dependent storage costs (h, j, and γ) have no significant effect (below 1%) on total costs. Also, changes to the ordering cost per message (A), the potential profit lost per unit (s), the cost of shortages of goods per unit (c2), and the rate of back-orders (δ) have a considerable effect (ranging from 1% to 10%) on total costs, for all four models. Changes in the rate of demand and the purchase price of goods per unit (c) have a large influence (more than 10%) on total costs, for all four models.

4. Conclusions and Further Research
In this paper, four inventory models have been developed that consider the following as time dependent demand, time-dependent storage costs, and the presence of deterioration factors that are not constant. Model 1 has a demand rate and a quadratic storage cost, while Model 2 has a linear demand rate and linear storage costs, Model 3 has a linear demand rate and quadratic storage cost, whereas Model 4 has a linear demand and storage cost rate. Model 1 gives the highest total cost results while Model 4 gives the smallest total cost results. Based on the sensitivity analysis it can be concluded that the total cost will increase along with the increasing deterioration rate and reduced demand rate. So the minimum total cost is obtained when the rate of deterioration is small and the rate of demand is large.

For further development, the model can be developed into a model with a probabilistic rate of demand with more than one type of item.

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