Theory of the Anomalous Tunnel Hall Effect at Ferromagnet-Semiconductor Junctions.

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We report on theoretical investigations of carrier scattering asymmetry at ferromagnet-semiconductor junctions. By an analytical $2 \times 2$ spin model, we show that, when Dresselhaus interactions is included in the conduction band of III-V $T_d$ symmetry group semiconductors, the electrons may undergo a difference of transmission vs. the sign of their incident parallel wavevector normal to the in-plane magnetization. This asymmetry is universally scaled by a unique function independent of the spin-orbit strength. This particular feature is reproduced by a multiband $k \cdot p$ tunneling transport model. Astonishingly, the asymmetry of transmission persists in the valence band of semiconductors owing to the inner atomic spin-orbit strength and free of asymmetric potentials. We present multiband $14 \times 14$ and $30 \times 30$ $k \cdot p$ tunneling models together with tunneling transport perturbation calculations corroborating these results. Those demonstrate that a tunnel spin-current normal to the interface can generate a surface transverse charge current, the so-called Anomalous Tunnel Hall Effect.

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I - INTRODUCTION

Spinorbitronics is a science that uses the spin degree of freedom together with the spin-orbit interactions (SOI) to generate spin-currents without the need of a ferromagnetic material. Those spin-currents become essential in view to control transport asymmetry of carriers in semiconductor heterostructures, interfaces, tunnel barriers or quantum wells. Those are composed of ferromagnets and spin-orbit split electrodes made of semiconductors, e.g. III-V compounds, with magnetizations of opposite direction (AP). The symmetry of the structure allows a difference of transmission upon positive or negative incidence (that we will note $\xi = \pm k_\parallel$) with respect to the reflection plane defined by the magnetization and the surface normal. However, this quantum process departs from the effect of a beam deviation by the action of the Lorentz force and, unlike spin-filtering effects, the scattering asymmetry requires the simultaneous action of both in-plane and out-of-plane spin-orbit fields in the case of Dresselhaus interaction in the conduction band (CB). In a second part of the paper, we emphasize on the perturbation calculation techniques needed to understand this phenomena as well as the case of intrinsic SOI in the valence band (VB).

II - CASE OF THE CONDUCTION BAND OF SEMICONDUCTORS OF $T_d$ SYMMETRY GROUP.

We first consider the Dresselhaus interactions \( \hat{H}_D = \langle \hat{\gamma} \chi \rangle \cdot \hat{\sigma} \).

\[
\hat{H}_D = \begin{pmatrix}
-\gamma \xi^2 k & i\gamma \xi k^2 \\
i\gamma \xi k^2 & \gamma \xi^2 k
\end{pmatrix}
\]

in the conduction band of a semiconductor junction made of two magnetic materials in the AP state. We refer the structure to the $x, y, z$ cubic axes (unit vectors $\hat{x}, \hat{y}, \hat{z}$) and assume that electron transport occurs along the $z$ axis, whereas the magnetization lies along $x$. One then introduces $(0, \xi, k)$ the electron wavevector; $\hat{\sigma}$ the Pauli operator, and $\chi = [0, \xi k^2, -\xi^2 k]$ the D’yakonov-Perel’ (DP) internal field responsible for the spin splitting $\langle \hat{\gamma} \rangle$. One introduces the tensor $\hat{\gamma} = \langle \gamma_{i\delta_{ij}} \rangle$ which characterizes the DP-field strength, with $\gamma_x = \gamma_y = \gamma_z$. 
Asymmetry in the CB: 2D-map of the transmission coefficient is the longitudinal kinetic energy and wavevector component are more easily transmitted than those carried by $-\xi$. (Top right inset): Energy profile of the exchange step; $\tilde{\gamma}$ and $\gamma$ in the CB and VB, the exchange strength is $\gamma_1 = \tilde{\gamma}$ for $k_1$ and $\gamma_2 = \gamma$ for $k_2$ (pure imaginary) – is the $z$-component of the wavevector in the lower (upper) subband. The two eigenvectors write:

$$
\psi_{1,11} = [1 - 2i \nu \xi k_1 - \epsilon (1 - 2\mu \xi k_1)] / \sqrt{2}, \quad (3)
$$

$$
\psi_{1,12} = [1 + 2i \nu \xi k_2 + \epsilon (1 + 2\mu \xi k_2)] / \sqrt{2}, \quad (4)
$$

where $\mu = \gamma \xi/(2w)$ and $\tilde{\mu} = \gamma \xi/(2w)$. Note that the form of the eigenvectors does not foresee any tunneling transmission asymmetry in usual tunneling models [9] based on the density of states [28, 29]. The asymmetry arises from a full-quantum treatment discussed in terms of chirality. Because $k_{ij}$ is conserved, we are dealing with states with the same longitudinal kinetic energy $E$ along $z$ and a total kinetic energy $E = E + \gamma_2/\sqrt{2}$. The boundary conditions are the continuity of the wavefunction and of the current wave $J\psi_{1,11} = (1/h) \partial \hat{H}_{1,11} / \partial k \psi_{1,11}$, where $\hat{H}_{1,11}$ contains no more than quadratic $k$ terms [30, 34].

The transmission of a pure up-spin incident electron into a pure down-spin state is only possible under oblique incidence via SOI which introduces off-diagonal matrix elements. The spin-orbit field is also responsible for a discontinuity of the spin-current between incident (inc.) and transmitted (trans.) waves. Moreover, a non-vanishing diagonal part of SOI is necessary to obtain a non-zero asymmetry although the $z$ component of the DP field along $z$ does not depend on the sign of $k_{||}$ [9]. Then, from now on, we take $\tilde{\gamma} = \gamma$. The wavevector $k_1$ in the lower subband has to be real so that $k_{ij}$ is conserved, we are dealing with states with the same longitudinal kinetic energy $E$ along $z$ and a total kinetic energy $E = E + \gamma_2/\sqrt{2}$. The boundary conditions are the continuity of the wavefunction and of the current wave $J\psi_{1,11} = (1/h) \partial \hat{H}_{1,11} / \partial k \psi_{1,11}$, where $\hat{H}_{1,11}$ contains no more than quadratic $k$ terms [30, 34].

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(m, ξ, k) forms a direct frame and negative otherwise. Another striking feature is that an arbitrarily small perturbation is able to produce a 100% transport asymmetry i.e., a total quenching of transmission in the CB. Fig. 1b display the 2-dimensional map of the electron transmission at a given total energy in the reciprocal space calculated using both a 2 × 2 effective Hamiltonian and a full 14 × 14 band k · p treatment involving odd-potential coupling terms P' and Δ [35][37]. These calculations are based on the multiband transfer matrix technique detailed in Refs. [30][31].

Transverse Surface Currents.

The transmitted current summed, J[t, η] = Jx[ΨII(z)] + Jy[ΨII(z)], originates from incident waves of equal amplitude and opposite k\parallel. To the lowest order in γ, we find

\[ J_{y,z}[t, η] = \frac{4 (γ_c ω)^{1/2}}{ℏ} (1 + ν)^{1/2} T(t, η) [A(t, η) t\bar{y} + \bar{z}] \]

(7)

A non-zero A gives rise to a transverse carrier momentum and then to a tunneling surface current (per unit length) J\parallel = J\parallel × η, (η, f p is the electron mean free path), of the form J\parallel = m × J_{S,z} (C for current and S for spin-current), leading to a potentially large Anomalous Tunnel Hall Effect (ATHE). The ratio of the surface transverse to the longitudinal current density, J\parallel / Jz / Jz, leads to the ATHE length, or ATHE angles [9], in the spirit of the work dealing with IEE phenomenon [18][19].

III - CASE OF THE CB: PERTURBATION CALCULATIONS INVOLVING SOI.

Using advanced perturbation procedures, one may give a general expression for the change of the transmission amplitude δΨ\σ'(z) of a propagative spin↑ wave from the left transmitted into a propagative spin↓ wave to the right, after having experienced a SOI potential Vσσ′ (spin-flip) in a confined region of space. The calculation is based on Ref. [41] and we will demonstrate in fine that

\[ δΨ_{in}^σ(z) = \int_0^a G_{in}^σ(z, z') Vσσ′(z') Ψ_{in}^σ(z') dz', \]

(8)

and from the expression of G\σ, that t\bar{z} may be written:

\[ δΨ_{out}^σ(z) = \frac{-im*}{ℏ^2k} \int_0^a Ψ_{out}^σ(z') Vσσ′(z') Ψ_{in}^σ(z') dz', \]

(9)

where (in) and (out) refer respectively to the unperturbed incoming wave at left and outgoing wave at right [41]. G\σ is the (spin-diagonal) Green’s function (GF) to consider and we are searching for. Such perturbative scattering approach has hardly been employed to investigate the role of the evanescent waves in transport like investigated here. The method is particularly suitable for the case of non-degenerate orbital systems but however could be applied, in a future work, to the case of the valence band (VB). We consider the Green’s function (GF) G\sigma of an Hamiltonian system \( H_0 = \left( \frac{κ^2}{2} \right) \frac{∂^2}{∂z^2} - U(z) \) in a homogenous potential U1 for z < 0, and U2 for z > 0 satisfying:

\[ (E - H_0) G\sigma(z, z') = δ(z - z'), \]

(10)

Green’s function without orbital degeneracy.

The strategy to find the GF is then i) to find two different ground states \{ ΨL, ΨR \} of the homogenous Schrödinger equation \( (E - H_0) Ψ = 0 \) (L for left and R for right with characteristic wavevectors k1 and k11), ii) to find the relevant linear combinations of \ΨL and \ΨR that make y1 and y2 solution of the equation \( (E - H_0) y = 0 \) finite at z = -∞ \[ y1(z) \] and z = +∞ \[ y2(z) \] depending on the use of the retarded or advanced quantities, and iii) to define the correct GF by making use of:

\[ G\sigma(z, z') = \begin{cases} \left( \frac{ΨL(z)ΨR(z')}{W(y1, y2)} \right) -∞ < z < z' < +∞ , & \text{if } y1(y2), \text{where } W(y1, y2) = \frac{ℏ^2}{2m} \left[ y1(z) \frac{∂y2(z')}{∂z} - \frac{∂y1(z')}{∂z} y2(z') \right] \text{ is the Wronskian.} \\ \text{the Wronskian. The homogenous Schrödinger equation,} \end{cases} \]

(11)

where \( W(y1, y2) = \frac{ℏ^2}{2m} \left[ y1(z) \frac{∂y2(z')}{∂z} - \frac{∂y1(z')}{∂z} y2(z') \right] \) is the Wronskian. The homogenous Schrödinger equation, \( (E - H_0) Ψ = 0 \), admits the solutions:

\[ \Psi_{L,R}^0 = \left( e^{ikz^<} + e^{-ikz^>} \right) |σ > \]

and

\[ \Psi_{R,L}^0 = \left( e^{-ikz^<} + e^{ikz^>} \right) |σ > \]

(12)

where \( z^< \) and \( z^> \) stand for z < 0 and z > 0. If we chose \( y1 = Ψ_{L,R}^0 \) and \( y2 = Ψ_{L,R}^0 \), Eq. [10] admits a particular solution:

\[ G\sigma(z, z') = \frac{Ψ_{L,R}^0(z')Ψ_{L,R}^0(z)}{W(Ψ_{L,R}^0, Ψ_{L,R}^0)} , \]

(13)

On the assumption of a same effective mass, the Wronskian \( W = \frac{ℏ^2}{2m} \left( e^{-ikz^<} + e^{ikz^>} \right) \) is a constant \( (∂W/∂z') = 0 \) and we recover the retarded GF introduced in Ref. [42] according to:

\[ G_0(z_<, z^>) = \frac{2m^*}{ℏ^2k_f} \left( e^{-ikz^<} + e^{ikz^>} \right), \]

(14)
The two solutions of the homogeneous Schrödinger equation, \( \Psi_{\uparrow \uparrow}^{0} \), and \( \Psi_{\uparrow \downarrow}^{0} \) are given by Eq. [12] with reflection, \( r_{L\uparrow} = r_{R\uparrow} = k_{L\uparrow}k_{R\uparrow} \) and transmission, \( t_{L\uparrow} = t_{R\uparrow} = e^{i k_{L\uparrow} z} \) (\( t_{R\uparrow} = t_{L\uparrow} = 0 \)), amplitudes found via the matching conditions at \( z = 0 \). This allows possible transmission from propagative to evanescent states (\( t_{L\downarrow} \) and \( t_{L\uparrow} \)) and vice-versa (\( t_{L\uparrow} \) and \( t_{L\downarrow} \)).

The SOI, \( \hat{H}_{\text{SO}}' \), is then introduced as a perturbation potential according to:

\[
\hat{H}_{\text{SO}}' = -\frac{\xi^{2} (\gamma k + k^{2}) (\gamma k + (k')^{2})}{2} - \frac{\xi^{2} (\gamma k + (k')^{2})}{2} \left( \frac{\partial^{2}}{\partial z^{2}} - \frac{\gamma}{\partial z} \right) - \frac{\xi^{2} (\gamma k + (k')^{2})}{2} \left( \frac{\partial^{2}}{\partial z^{2}} + \frac{\gamma}{\partial z} \right) ,
\]

(15)

with \( \gamma = \gamma (z) \). \( \hat{H}_{\text{SO}}' \) acquires a pure non-diagonal form like:

\[
V_{\uparrow \downarrow} = \left( \begin{array}{c}
\frac{\xi^{2} \gamma}{2} & \frac{\xi^{2} \gamma}{2}
\end{array} \right)
\]

and

\[
V_{\uparrow \uparrow} = \left( \begin{array}{c}
\frac{\xi^{2} \gamma}{2} & \frac{\xi^{2} \gamma}{2}
\end{array} \right) - \left( \begin{array}{c}
\frac{\xi^{2} \gamma}{2} & \frac{\xi^{2} \gamma}{2}
\end{array} \right) \left( \frac{\partial^{2}}{\partial z^{2}} + \frac{\gamma}{\partial z} \right).
\]

(16)

From Eq. [8] and \( W = e^{i k_{L\uparrow} z} t_{R\uparrow} \), the correction to the amplitude of transmission is:

\[
\delta t_{\uparrow \downarrow} = -\frac{m^{*}}{i \hbar^{2} k_{L\uparrow}^{2}} \int_{-\infty}^{+\infty} \psi_{\uparrow \uparrow}^{0} (z') \left( i \frac{\xi^{2}}{2} \frac{\partial^{2} \psi_{\uparrow \downarrow}^{0} (z')}{\partial z^{2}} - i \frac{\xi^{2}}{2} \frac{\partial^{2} \psi_{\uparrow \downarrow}^{0} (z')}{\partial z^{2}} \right) d z'.
\]

(17)

We are now going to calculate the properties of the carrier transmission \( A \) for the different SOI configurations: at left, at right, and SOI in both contacts for an incoming left electron.

**SOI at left for electrons incoming from left.**

We first note that the zero-order transmission coefficient, \( t_{0}^{\uparrow \downarrow} = 0 \) is zero without spin-mixing interactions. Then, from Eq. [18], the transmission amplitude, \( t_{L\uparrow \downarrow} \), with SOI at left is:

\[
\delta t_{R\uparrow} = -\frac{1}{2w (1 + i \lambda \lambda)} \left\{ \frac{\xi^{2} K^{2}}{K (3 \lambda^{2} - 1) + 2 \lambda (\lambda^{2} - 1)} \right\}
\]

(20)

**SOI at right for electrons incoming from left.**

The transmission changes from the previous case by changing the integral from \( f_{-\infty}^{0} \) into \( f_{0}^{+\infty} \) giving \( t_{R\uparrow}^{\uparrow \downarrow} = t_{L\uparrow \downarrow}^{\uparrow \downarrow} \).
exchange potential,  

...generally different from the previous treatment. One then ... 

...electrons may scatter, now, at the two ... 

...with a single incident propagative wave of a pure spin \( \uparrow \)  

...height is \(-0.55\) eV.  

...the transmission, \( \delta t \)  

...the GF is given in Ref. [43]. To the first order of perturbation, ... 

...at the two ... 

...to prevent any back and forth scattering. The calculation of the most general shape of the GF is given in Ref. [43]. To the first order of perturbation, the transmission, \( \delta t^{\uparrow \downarrow} \), now equals:  

\[
\delta t^{\uparrow \downarrow} = 2\gamma |k_{2}\rangle_{\uparrow} \langle k_{1}\downarrow| \gamma c (k_{1} + ik_{2})^{2} (\xi + k_{2}).
\]  

...&\langle k_{1}\downarrow| \gamma c (k_{1} + ik_{2})^{2} (\xi + k_{2}).
\]  

...where we remind that \( a \) is the barrier thickness.  

...SOI perturbation, the transmission coefficient is also zero in the situation of pure spin states, and consequently, \( T^{\uparrow \downarrow} = |\delta t^{\uparrow \downarrow}|^{2} \). If one defines again the incidence parameter \( t = \tan \theta = \xi/K \) for and \( \eta = \frac{1}{k_{1}} \xi = \frac{\xi}{k_{1}} \) the reduced incident kinetic energy, we find the asymmetry of transmission for the tunnel barrier like:  

\[
A = \frac{\xi + k_{2}}{\xi - k_{2}} \left| \frac{\xi + k_{2}}{\xi - k_{2}} \right|^{2} = 2 \sqrt{|1 - \eta|/|1 + \eta|} t^{2}(1 + \eta) + (1 - \eta).
\]  

...one obtains a perfect agreement between the perturbative scattering method and our multiband calculations for \( |t^{\uparrow \downarrow}|^{2} \) and \( A \) (Fig. 2b). The transmission coefficient for an incoming propagative spin-\( \uparrow \) electron into an outgoing propagative spin-\( \downarrow \) electron is non-zero after SOI is branched on. The transmission vs. incident kinetic energy and incident angle is different from the case of a simple exchange-step. The maximum of transmission depends also on the incidence angle or \( t \) parameter. The \( k \cdot p \) theory gives a maximum of asymmetry when the evanescent wavevector equals in magnitude the parallel incoming wavevectors in the CB.  

### IV - CASE OF INTRINSIC CORE SOI IN THE VALENCE BAND: CHIRALITY  

We now turn on the case of the VB of a tunnel junction composed of two \( p \)-type ferromagnets separated by a thin tunnel barrier (3 nm in the present case). The barrier height have been chosen so as to match with the exchange strength (0.3 eV). The structure is free of any odd-potential \( k \)-terms (\( \tilde{H}_{D} = 0 \)) and only includes core SOI (\( p \)-orbitals). Results are displayed in Fig. 1c for the transmission maps and Fig. 2c for the corresponding asymmetry resulting from a multiband \( k \cdot p \) treatment. In the 2D-map calculation procedures obtained for a hole kinetic energy of \( \epsilon = 0.23 \) eV, we have checked (Fig. 1c) that the whole numerical approaches (6, 14, 18 and 30-bands models) provide about exact similar data. The transmission scales within the range \( 15 - 45 \times 10^{-3} \) with \( |P| = 0 \).
and $\Delta' = 0$. Those results demonstrate that the absence of inversion symmetry ($T_d$) is not mandatory to observe an asymmetry $A$. Fig. 2c displays the asymmetry $A$ vs. hole energy $E$ for $k_z = 0.05 \text{ nm}^{-1}$. The energy range covers the spin-$\uparrow$, $\downarrow$ heavy ($HH$) and light ($LH$)-hole subbands whereas the respective spin $\uparrow$ and $\downarrow$ split-off bands are not represented here. We refer to points (1) to (4) marked by vertical arrows in the following discussion. Here, the energy of the $HH \uparrow$ ($HH \downarrow$) corresponds to $0.15 \text{ eV} [-0.15 \text{ eV}]$ as indicated by point (1) [(4)], the energy zero being taken at the top of the VB of the non-magnetic material. Correspondingly, one observes a large negative transmission asymmetry ($-60\%$) in this energy range for predominant majority spin $\uparrow$ injection as far as $HH \downarrow$ does not contribute to the current. At more negative energy [$E < -0.15 \text{ eV}$: point (4)], a sign change of $A$ occurs at the onset of $HH \downarrow$ to reach about $+20\%$. From Ref. [9] $A$ changes sign two times at characteristic energy points corresponding to a sign change of the injected particle spin. Also, we have performed similar calculation for a simple contact [9]. Remarkably, $A$, although smaller, keeps the same trends as for the tunnel junction, $A$ abruptly disappears as soon as $SO \downarrow$ contributes to tunneling $i.e.$, when evanescent states disappear. In the case of tunnel junction, $A$, although small, subsists in this energy range and this should be related to the evanescent character of the wavefunction in the barrier.

V - CONCLUSIONS

We have presented theoretical evidence for large interfacial tunneling asymmetry of carriers (scattering), electrons or holes, vs. their incidence in exchange-split semiconductor structures. The effect of transmission asymmetry occurs in the CB via the SOI Dresselhaus interactions whereas intrinsic SOI of the p-type VB is sufficient. This transmission asymmetry have been revealed by taking into account boundary wavefunctions matching, advanced multiband $k \cdot p$ calculations as well as scattering perturbation theory. After averaging over incoming states, a large surface current parallel to the barrier is results in an Anomalous Tunnel Hall effect.

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[1] M. I. Dyakonov and V.I. Perel, Pis’ma Zh. Eksp. Teor. Fiz. 13, 657 (1971) [JETP Lett. 13, 467 (1971)].
[2] M. I. Dyakonov and V. I. Perel, Phys. Lett. A35, 459 (1971).
[3] J. Sinova, D. Culcer, Q. Niu, N. A. Sinitsyn, T. Jungwirth, and A. H. MacDonald, Phys. Rev. Lett. 92, 126603 (2004).
[4] Y. K. Kato, R. C. Myers, A. C. Gossard, and D. D. Awschalom, Science 306, 1910 (2004).
[5] J. Wunderlich, B. Kaestner, J. Sinova, and T. Jungwirth, Phys. Rev. Lett. 94, 047204 (2005).
[6] S. O. Valenzuela and M. Tinkham, Nature 442, 176 (2006).
[7] V. V. Rylkov, S. N. Nikolaev, K. Yu. Chernoglavoz et al., Phys. Rev. B 95, 144202 (2017).
[8] M. Jamet, A. Barski, T. Devillers, V. Poydenot et al., Nat. Mat. 5, 653-659 (2006).
[9] T. Huong Dang, H. Jaffrès, T. L. Hoai Nguyen, and H.-J. Drouhin, Phys. Rev. B 92, 060403(R) (2015).
[10] A. Matos-Abiague and J. Fabian, Phys. Rev. Lett. 115, 056602 (2015).
[11] S. D. Ganichev, M. Trushin, and J. Schliemann, Spin-polarization by current, “Handbook of spin-transport & magnetism”, (Chapman and Hall), 2016.
[12] L. Liu, O. J. Lee, T. J. Gudmundsen, D. C. Ralph, and R. A. Buhrman, Phys. Rev. Lett. 109, 096602 (2012).
[13] I. M. Miron, G. Gaudin, S. Aufrêt, B. Rodmacq, A. Schuhl et al., Nat. Mat. 9, 230-234 (2010).
[14] K. Garello, I. M. Miron, C. O. Avci, F. Freimuth, et al., Nat. Nano. 8, 587-593 (2013).
[15] J. C. Rojas, N. Reyren, P. Laczowski, W. Savero et al., Phys. Rev. Lett. 112, 106602 (2014).
[16] Y. A. Bychkov and E. I. Rashba, Sov. Phys. JETP Lett. 39, 78 (1984).
[17] P. Gambardella and I. M. Miron, Phil. Trans. R. Soc. A, 369, 3175 (2011).
[18] J. C. Rojas Sánchez, L. Vila, G. Desfonds, S. Gambarelli et al., Nat. Comm. 4, 2944 (2013).
[19] E. Lesne, Y. Fu, S. Oyarzun, J. C. Rojas-Sanchez, et al., Nat. Mat. 15, 1261-1266 (2016).
[20] C. O. Avci, K. Garello, A. Ghosh, M. Gabureac, S. F. Alvarado, and P. Gambardella, Nat. Phys. 11, 570-575 (2015).
[21] K. Olejnuk, V. Novak, J. Wunderlich, and T. Jungwirth, Phys. Rev. B 91, 180402(R) (2015).
[22] K. Yasuda, A. Tsukazaki, R. Yoshimi, K. S. Takahashi, M. Kawasaki, and Y. Tokura, Phys. Rev. Lett. 117, 127202 (2016).
[23] P. S. Alekseev, JETP Lett. 92, 788-792, (2010).
[24] V. I. Perel, S. A. Tarasenko, I. N. Yassievich, S. D. Ganichev et al., Phys. Rev. B 67, 201304(R) (2003).
[25] S. A. Tarasenko, V. I. Perel; and I. N. Yassievich, Phys. Rev. Lett. 93, 056601 (2004).
[26] G. Dresselhaus, Phys. Rev. 100, 580 (1955).
[27] M. D’yakonov and V. I. Perel’, Zh. Eksp. Teor. Fiz. 60, 1954 (1971); Sov. Phys. JETP 33, 1053 (1971).
[28] M. Jullière, Phys. Lett. 54A, 225 (1975).
[29] J. C. Slonczewski, Phys. Rev. 117, 155306 (2014).
[30] J.-M. Jancu, R. Scholz, E. A. de Andrada e Silva, and G. C. La Rocca, Phys. Rev. B 85, 235313 (2012).
[31] M. Cardona, N. E. Christensen, and G. Fasol, Phys. Rev. B 38, 1806 (1988).
[32] P. Pfeffer and W. Zawadzki, Phys. Rev. B 41, 1561 (1990).
[33] S. Richard, Frédéric Aniel, and Guy Fishman, Phys. Rev. B 70, 235204 (2004).
[34] J. M. Luttinger, Phys. Rev. 102, 1030 (1956).
[35] E. O. Kane, J. Phys. Chem. Solids, 1, 249 (1957).
[36] P. S. Alekseev, M. M. Glazov, and S. A. Tarasenko, Phys. Rev. B89, 155306 (2014).
[42] D. A. Stewart, W. H. Butler, X.-G. Zhang, and V. F. Los, Phys. Rev. B 68, 014433 (2003).

[43] M. A. M. de Aguiar, Phys. Rev. A 48, 2567 (1993).