Spintronics and Quantum Computing
with Quantum Dots

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Abstract. The creation, coherent manipulation, and measurement of spins in nanostructures open up completely new possibilities for electronics and information processing, among them quantum computing and quantum communication. We review our theoretical proposal for using electron spins in quantum dots as quantum bits. We present single- and two qubit gate mechanisms in laterally as well as vertically coupled quantum dots and discuss the possibility to couple spins in quantum dots via superexchange. We further present the recently proposed schemes for using a single quantum dot as spin-filter and spin read-out/memory device.

Keywords: quantum computing, spin, spintronics, spin coherence, quantum dots

1. Introduction

Theoretical research on electronic properties in mesoscopic condensed matter systems has focussed primarily on the charge degrees of freedom of the electron, while its spin degrees of freedom have not yet received the same attention. But an increasing number of spin-related experiments\cite{1, 2, 3, 4, 5, 6} show that the spin of the electron offers unique possibilities for finding novel mechanisms for information processing and information transmission—most notably in quantum-confined nanostructures with unusually long spin dephasing times\cite{2, 3, 4} approaching microseconds, as well as long distances of up to 100 \( \mu \text{m} \) \cite{2} over which spins can be transported phase-coherently. Besides the intrinsic interest in spin-related phenomena, there are two main areas which hold promises for future applications: Spin-based devices in conventional\cite{1} as well as in quantum computer hardware\cite{7}. In conventional computers, the electron spin can be expected to enhance the performance of quantum electronic devices, examples being spin-transistors (based on spin-currents and spin injection), non-volatile memories, single spin as the ultimate limit of information storage etc.\cite{1}. On the one hand, none of these devices exist yet, and experimental progress as well as theoretical investigations are needed to
provide guidance and support in the search for realizable implementations. On the other hand, the emerging field of quantum computing[8, 9] and quantum communication[9, 10] requires a radically new approach to the design of the necessary hardware. As first pointed out in Ref.[7], the spin of the electron is a most natural candidate for the qubit—the fundamental unit of quantum information. We have shown[7] that these spin qubits, when located in quantum-confined structures such as semiconductor quantum dots, atoms or molecules, satisfy all requirements needed for a scalable quantum computer. Moreover, such spin-qubits—being attached to an electron with orbital degrees of freedom—can be transported along conducting wires between different subunits in a quantum network[9]. In particular, spin-entangled electrons can be created in coupled quantum dots and—as mobile Einstein-Podolsky-Rosen (EPR) pairs[9]—provide then the necessary resources for quantum communication.

It follows a short introduction of quantum computing and quantum communication and we will then present our current theoretical efforts towards a realization of quantum computing. We thereby focus on the implementation of the necessary gate and read-out operations schemes with quantum dots.

1.1. Quantum Computing and Quantum Communication

We give a brief description of the emerging research field of quantum computation. It has attracted much interest recently as it opens up the possibility of outperforming classical computation through new and more powerful quantum algorithms such as the ones discovered by Shor[11] and by Grover[12]. There is now a growing list of quantum tasks[9, 10] such as cryptography, error correcting schemes, quantum teleportation, etc. that have indicated even more the desirability of experimental implementations of quantum computing. In a quantum computer each quantum bit (qubit) is allowed to be in any state of a quantum two-level system. All quantum algorithms can be implemented by concatenating one- and two-qubit gates. There is a growing number of proposed physical implementations of qubits and quantum gates. A few examples are: Trapped ions[13], cavity QED[14], nuclear spins[15, 16], superconducting devices[17, 18, 19, 20], and our qubit proposal[7] based on the spin of the electron in quantum-confined nanostructures.

1.2. Quantum Dots

Since quantum dots are the central objects of this work we shall make some general remarks about these systems here. Semiconductor quantum dots are structures where charge carriers are confined in all three
spatial dimensions, the dot size being of the order of the Fermi wavelength in the host material, typically between 10 nm and 1 µm [21]. The confinement is usually achieved by electrical gating of a two-dimensional electron gas (2DEG), possibly combined with etching techniques. Precise control of the number of electrons in the conduction band of a quantum dot (starting from zero) has been achieved in GaAs heterostructures [22]. The electronic spectrum of typical quantum dots can vary strongly when an external magnetic field is applied [21, 22], since the magnetic length corresponding to typical laboratory fields $B \approx 1$ T is comparable to typical dot sizes. In coupled quantum dots Coulomb blockade effects [23], tunneling between neighboring dots [21, 23], and magnetization [24] have been observed as well as the formation of a delocalized single-particle state [25].

2. Quantum Gate Operations with Coupled Quantum Dots

One and two qubit gates are known to be sufficient to carry out any quantum algorithm. For electron spins in nearby coupled quantum dots the desired two qubit coupling is provided by a combination of Coulomb interaction and the Pauli exclusion principle.

At zero magnetic field, the ground state of two coupled electrons is a spin singlet, whereas the first excited state in the presence of strong Coulomb repulsion is usually a triplet. The remaining spectrum is separated from these two states by a gap which is either given by the Coulomb repulsion or the single particle confinement. The low-energy physics of such a system can then be described by the Heisenberg spin Hamiltonian

$$H_s(t) = J(t) \mathbf{S}_1 \cdot \mathbf{S}_2,$$

where $J(t)$ is the exchange coupling between the two spins $\mathbf{S}_1$ and $\mathbf{S}_2$, and is given by the energy difference between the singlet and triplet states. If we pulse the exchange coupling such that $\int dt J(t)/\hbar = J_0 \tau_s/\hbar = \pi$ (mod 2π), the associated unitary time evolution $U(t) = T \exp(i \int_0^t H_s(\tau) d\tau/\hbar)$ corresponds to the “swap” operator $U_{\text{sw}}$ which exchanges the quantum states of qubit 1 and 2 [7]. Having an array of dots it is therefore possible to couple any two qubits. Furthermore, the quantum XOR gate can be constructed by applying the sequence [7]

$$U_{\text{XOR}} = e^{i(\pi/2)S_z^1} e^{-i(\pi/2)S_z^2} U_{\text{sw}}^{1/2} e^{i\pi S_z^2} U_{\text{sw}}^{1/2},$$

i.e. a combination of “square-root of swap” $U_{\text{sw}}^{1/2}$ and single-qubit rotations $\exp(i \pi S_z^i)$. Since $U_{\text{XOR}}$ (combined with single-qubit rotations) is proven to be a universal quantum gate [26], it can be used to assemble
any quantum algorithm. The study of universal quantum computation in coupled quantum dots is thus essentially reduced to the study of single qubit rotations and the exchange mechanism, in particular how the exchange coupling $J(t)$ can be controlled experimentally. Note that the switchable coupling mechanism described below need not be restricted to quantum dots: the same principle can be used in other systems, e.g. coupled atoms in a Bravais lattice, supramolecular structures, or overlapping shallow donors in semiconductors.

2.1. Laterally coupled quantum dots

We first discuss a system of two laterally coupled quantum dots defined by depleted regions in a 2DEG containing one (excess) electron each[27]. The electrons are allowed to tunnel between the dots (if the tunnel barrier is low) leading to spin correlations via their charge (orbital) degrees of freedom. We model the coupled system with the Hamiltonian $H = H_{\text{orb}} + H_Z$, where $H_{\text{orb}} = \sum_{i=1,2} h_i + C$ with

$$h_i = \frac{1}{2m} \left( p_i - \frac{e}{c} A(r_i) \right)^2 + V(r_i), \quad C = \frac{e^2}{\kappa |r_1 - r_2|}. \quad (3)$$

Here, $h_i$ describes the single-electron dynamics in the 2DEG confined to the $xy$-plane, with $m$ being the effective electron mass. We allow for a magnetic field $B = (0, 0, B)$ applied along the $z$-axis that couples to the electron charge via the vector potential $A(r) = \frac{B}{2}(-y, x, 0)$, and to the spin via a Zeeman coupling term $H_Z$. The single dot confinement as well as the tunnel-coupling is modeled by a quartic potential, $V(x, y) = \frac{m\omega_0^2}{2} \left( \frac{1}{4\omega_0} (x^2 - a^2)^2 + y^2 \right)$, which, in the limit $a \gg a_B$, separates into two harmonic wells of frequency $\omega_0$ where $2a$ is the interdot distance and $a_B = \sqrt{\hbar/m\omega_0}$ is the effective Bohr radius of a dot. This choice for the potential is motivated by the experimental observation[22] that the low-energy spectrum of single dots is well described by a parabolic confinement potential. The (bare) Coulomb interaction between the two electrons is described by $C$ where $\kappa$ denotes the dielectric constant of the semiconductor. The screening length $\lambda$ in almost depleted regions like few-electron quantum dots can be expected to be much larger than the bulk 2DEG screening length (about 40 nm for GaAs). Therefore, $\lambda$ is large compared to the size of the coupled system, $\lambda \gg 2a \approx 40 \text{nm}$ for small dots, and we will consider the limit of unscreened Coulomb interaction ($\lambda/a \gg 1$). At low temperatures $kT_B \ll \hbar\omega_0$ we are allowed to restrict our analysis to the two lowest orbital eigenstates of $H_{\text{orb}}$, leaving us with a symmetric (spin-singlet) and an antisymmetric (three triplets $T_0$, $T_\pm$) orbital state. In
Figure 1. The exchange coupling $J$ (full line) for GaAs quantum dots with confinement energy $\hbar \omega = 3$ meV and $c \approx 2.42$. For comparison we plot the usual short-range Hubbard result $J = 4t^2/U$ (dashed-dotted line) and the extended Hubbard result\cite{27} $J = 4t^2/U + V$. In (a), $J$ is plotted as a function of magnetic field $B$ at fixed inter-dot distance ($d = a/a_B = 0.7$), and in (b) as a function of the inter-dot distance $d = a/a_B$ at $B = 0$.

this reduced (four-dimensional) Hilbert space, $H_{\text{orb}}$, can be replaced by the effective Heisenberg spin Hamiltonian Eq. (1) where the exchange coupling $J = \epsilon_t - \epsilon_s$ is given by the difference between the triplet and singlet energy. We make use of the analogy between atoms and quantum dots (artificial atoms) and calculate $\epsilon_t$ and $\epsilon_s$ with variational methods similar to the ones used in molecular physics. With the Heitler-London approximation using single-dot groundstate orbitals we find\cite{27},

$$J = \frac{\hbar \omega_0}{\sinh \left( \frac{2d^2}{2b-1} \right)} \left\{ \frac{3}{4b} (1 + bd^2) + c \sqrt{\pi/(2a^2)} \left[ e^{-bd^2} I_0 (bd^2) - e^{d^2(b-1)/b} I_0 \left( d^2 \frac{b-1}{b} \right) \right] \right\},$$

(4)

where we introduce the dimensionless distance $d = a/a_B$ and the magnetic compression factor $b = B/B_0 = \sqrt{1 + \omega_L^2/\omega_0^2}$, where $\omega_L = eB/(2mc)$ is the Larmor frequency. $I_0$ denotes the zeroth Bessel function. The first term in Eq. (4) comes from the confinement potential. The terms proportional to $c = \sqrt{\pi/2}(e^2/k\alpha_B)/\hbar \omega_0$ are due to the Coulomb interaction $C$, where the exchange term appears with a minus sign. Note that typically $|J/\hbar \omega_0| \ll 1$ which makes the exclusive use of ground-state single-dot orbitals in the Heitler-London ansatz a self-consistent approach. The exchange $J$ is given as a function of $B$ and $d$ in Fig. 1. We observe that $J > 0$ for $B = 0$, which is generally true for a two-particle system with time reversal invariance. The most remarkable feature of $J(B)$, however, is the change of sign from positive (ferromagnetic) to negative (antiferromagnetic), which occurs at
some finite $B$ over a wide range of parameters $c$ and $a$. This singlet-triplet crossing is caused by the long-range Coulomb interaction and is therefore absent in the standard Hubbard model that takes only into account short range interaction and, in the limit $t/U \ll 1$, is given by

$$J = 4t^2/U > 0$$

(see Fig. 1). Large magnetic fields ($b \gg 1$) and/or large interdot distances ($d \gg 1$) reduce the overlap between the dot orbitals leading to an exponential decay of $J$ contained in the $1/\sinh$ prefactor in Eq. (4). This exponential suppression is partly compensated by the exponentially growing exchange term $\propto \exp(2d^2(b - 1/b))$. As a consequence, $J$ decays exponentially as $\exp(-2d^2b)$ for large $b$ or $d$. Thus, $J$ can be tuned through zero and then exponentially suppressed to zero by a magnetic field in a very efficient way (exponential switching is highly desirable to minimize gate errors). Further, working around the singlet-triplet crossing provides a smooth exchange switching, requiring only small local magnetic fields. Qualitatively similar results are obtained[27] when we extend the Heitler-London result by taking into account higher levels and double occupancy of the dots (using a Hund-Mullikan approach). In the absence of tunneling ($J = 0$) direct Coulomb interaction between the electron charges can still be present. However the spins (qubit) remain unaffected provided the spin-orbit coupling is sufficiently small, which is the case for s-wave electrons in GaAs structures with unbroken inversion symmetry. Finally, we note that a spin coupling can also be achieved on a long distance scale by using a cavity-QED scheme[28] or superconducting leads to which the quantum dots are attached[29].

2.2. Vertically coupled quantum dots

We also investigated vertically coupled Quantum dots[30]. This kind of coupling can be implemented with multilayer self-assembled quantum dots[31] as well as with etched mesa heterostructures[32].

We model the vertical coupled dot system by a potential $V = V_l + V_v$ where $V_l$ describes the parabolic lateral confinement and $V_v$ models the vertical dot coupling assumed to be a quartic potential similar to the one introduced above for the lateral coupling. We allow for different dot sizes $a_{B,\pm} = \sqrt{\hbar/m\alpha_0\omega_z}$ with $\omega_z$ being the vertical confinement (see Fig. 2), implying an effective Bohr radius $a_B = \sqrt{\hbar/m\omega_z}$ and a dimensionless interdot distance $2d = 2a/a_B$. By applying an in-plane electric field $E_{\parallel}$ (see Fig. 2) an interesting new switching mechanism arises. The dots are shifted parallel to the field by $\Delta x_{\pm} = E_{\parallel}/E_0\alpha_0^{\pm}$, where $E_0 = \hbar\omega_z/ea_B$. Thus, the larger dot is shifted a greater distance $\Delta x_+ > \Delta x_-$ and so the mean distance between the electrons grows as $d' = \sqrt{d^2 + A^2(E_{\parallel}/E_0)^2} > d$, taking $A = (\alpha_0^2 - \alpha_0^-)/2\alpha_0^2 + \alpha_0^-$. 
Figure 2. (a) Sketch of the vertically coupled double quantum-dot system. The two dots may have different lateral diameters, $a_B^+$ and $a_B^-$. We consider an in-plane electric field $E_\parallel$. (b) The model potential for the vertical confinement is a double well, which is obtained by combining two harmonic wells with frequency $\omega_z$ at $z = \pm a$. (c) Switching of the spin-spin coupling between dots of different size by means of an in-plane electric field $E_\parallel$ ($B = 0$). The exchange coupling is switched “on” at $E = 0$ (see text). We have chosen $\hbar \omega_z = 7 \text{ meV}$, $d = 1$, $\alpha_{0+} = 1/2$ and $\alpha_{0-} = 1/4$. For these parameters, $E_0 = \hbar \omega_z/ea_B = 0.56 \text{ mV/\mu m}$ and $A = (\alpha_{0+}^2 - \alpha_{0-}^2)/2\alpha_{0+}^2\alpha_{0-}^2 = 6$. The exchange coupling $J$ decreases exponentially on the scale $E_0/2A = 47 \text{ mV/\mu m}$ for the electric field.

Since the exchange coupling $J$ is exponentially sensitive to the inter-dot distance $d'$ (see Eq. (4)) we have another exponential switching mechanism for quantum gate operations at hand.

2.3. COUPLING TWO SPINS BY SUPEREXCHANGE

There is a principal problem if one wants to couple two “extended” dots whose energy levels are closely spaced (i.e. smaller than $k_B T$), as would be the case if there is a sizable distance between the two confined qubits before the barrier is lowered. In this case, the singlet-triplet splitting becomes vanishingly small, and it would not take much excitation energy to get states which are not entangled at all. In other words, the adiabatic switching time[27] which is proportional to the inverse level spacing becomes arbitrarily large. A better scenario for coupling the two spin-qubits is to make use of a superexchange mechanism to obtain a Heisenberg interaction[7]. Consider three aligned quantum dots where the middle dot is empty and so small that only its lowest levels will be occupied by 1 or 2 electrons in a virtual hopping process (see Fig. 3). The left and right dots can be much larger but still small enough such that the Coulomb charging energies $U_L \approx U_R$ are high enough (compared to $k_B T$) to suppress any double occupancy. Let us assume now that the middle dot has energy levels higher than the ground states of right and left dots, assumed to be approximately the same. These levels include single particle energy (set to zero) and Coulomb charging energy $N^2e^2/2C$, with $N$ the number of electrons and $C$ the
capacitance of the middle dot, and thus the ground state energy of the middle dot is 0 when empty, $\epsilon = e^2/2C$ for one electron, and $4\epsilon$ for 2 electrons. The tunnel coupling between the dots is denoted by $t_0$.

Now, there are two types of virtual processes possible which couple the spins but only one is dominant. First, the electron of the left (right) dot hops on the middle dot, and then the electron from the right (left) dot hops on the same level on the middle dot, and thus, due to the Pauli principle, the two electrons on the middle dot form a singlet, giving the desired entanglement. And then they hop off again into the left and right dots, respectively. (Note that $U$ must be larger than $k_B T$, otherwise real processes involving 2 electrons in the left or right dot will be allowed). It is not difficult to see that this virtual process leads to an effective Heisenberg exchange interaction with exchange constant $J = 4t_0^4/4\epsilon^3$, where the virtual energy denominators follow the sequence $1/\epsilon \rightarrow 1/4\epsilon \rightarrow 1/\epsilon$.

In the second type of virtual process the left (right) electron hops via the middle dot into the right (left) dot and forms there a singlet, giving $J = 4t_0^4/U_{RL}\epsilon^2$. However, this process has vanishing weight because there are also many nearby states available in the outer dots for which there is no spin correlation required by the Pauli principle. Thus, most of the virtual processes, for which we have 2 electrons in the left (right) dot, do not produce spin correlations, and thus we can neglect these virtual processes of the second type altogether. It should be possible to create ferroelectrically defined nanostructures for which superexchange is the dominant mechanism for coupling neighboring electrons. The geometry will resemble closely that of Fig. 3, except that the central barrier becomes a narrow well.
3. Single-Spin Rotations

In order to perform one qubit gates single-spin rotations are required. This is done by exposing a single spin to a time-varying Zeeman coupling \( (g \mu_B \mathbf{S} \cdot \mathbf{B})(t) \) [27], which can be controlled through both the magnetic field \( \mathbf{B} \) and/or the g-factor \( g \). We have proposed a number of possible implementations[7, 27, 9, 33] for spin-rotations: Since only relative phases between qubits are relevant we can apply a homogeneous \( \mathbf{B} \)-field rotating all spins at once. A local change of the Zeeman coupling is then possible by changing the Larmor frequency \( \omega_L = g \mu_B B / \hbar \). The equilibrium position of an electron can be changed through electrical gating, therefore if the electron wavefunction is pushed into a region with different magnetic field strength or different (effective) g-factor, the relative phase of such an electron then becomes \( \phi = (g' B' - g B) \mu_B \tau / 2 \hbar \). Regions with an increased magnetic field can be provided by a magnetic (dot) material while an effective magnetic field can be produced e.g. with dynamically polarized nuclear spins (Overhauser effect)[27]. We shall now explain a concept for using g-factor-modulated materials[9, 33]. In bulk semiconductors the free-electron value of the Landé g-factor \( g_0 = 2.0023 \) is modified by spin-orbit coupling. Similarly, the g-factor can be drastically enhanced by doping the semiconductor with magnetic impurities[4, 3]. In confined structures such as quantum wells, wires, and dots, the g-factor is further modified and becomes sensitive to an external bias voltage[34]. We have numerically analyzed a system with a layered structure (AlGaAs-GaAs-InAlGaAs-AlGaAs), in which the effective g-factor of electrons is varied by shifting their equilibrium position from one layer to another by electrical gating[35]. We have found that in this structure the effective g-factor can be changed by about \( \Delta g_{\text{eff}} \approx 1 \) [33].

Alternatively one can use electron-spin-resonance (ESR) techniques [27] to perform single-spin rotations, e.g. if we want to flip a certain qubit (say from \( | \uparrow \rangle \) to \( | \downarrow \rangle \)) we apply an ac-magnetic field perpendicular to the \( \uparrow \)-axis that matches the Larmor frequency of that particular electron. Due to paramagnetic resonance[36] the spin can flip.

Furthermore, localized magnetic fields can be generated with the magnetic tip of a scanning force microscope, a magnetic disk writing head, by placing the dots above a grid of current-carrying wires, or by placing a small wire coil above the dot etc.
4. Read-Out of a Single Spin

The final step of each (quantum) computation, consists in reading out the state of each qubit, i.e. if the electron spin is in the $|\uparrow\rangle$ or $|\downarrow\rangle$ state. It is very hard to detect an electron spin over its tiny (of the order of $\mu_B$) magnetic moment directly. We proposed several devices for read out like tunneling of the electron into a supercooled paramagnetic dot\cite{7, 9}, thereby inducing a magnetization nucleation from the metastable phase into a ferromagnetic domain. The domain’s magnetization direction is along the measured spin and can be detected by conventional methods and provides a 75%-reliable result for the read out of the electron spin. Another possibility is to use a spin selective tunnel barrier (conventional spin filter), that let pass only one spin direction. If an electron passes the barrier to enter another dot an electrometer can detect the charge\cite{7}.

4.1. Quantum Dot As Spin Filter and Read-Out/Memory Device

We recently proposed\cite{37} another setup using a quantum dot attached to in and outgoing current leads $l = 1, 2$—that can work either as a spin filter or as a read-out device, or as a spin memory where the spin stores the information. A new feature of this proposal is that we lift the spin-degeneracy with different Zeeman splittings for the dot and the leads, e.g. by using materials with different effective g-factors for leads and dot\cite{37}. This results in Coulomb blockade oscillation peaks and spin-polarized currents which are uniquely associated with the spin state on the dot.

The setup is described by a standard tunneling Hamiltonian $H_0 + H_T$ \cite{38}, where $H_0 = H_L + H_D$ describes the leads and the dot. $H_D$ includes the charging and interaction energies of the electrons in the dot as well as their Zeeman energy $\pm g\mu_B B/2$ in an external magnetic field $\mathbf{B}$. Tunneling between leads and the dot is described by $H_T = \sum_{l, k, p, \sigma} t_{lp} c_{lk\sigma}^\dagger d_{p\sigma} + \text{h.c.}$, where $c_{lk\sigma}$ annihilates electrons with spin $\sigma$ and momentum $k$ in lead $l$ and $d_{p\sigma}$ annihilates electrons in the dot. We work in the Coulomb blockade regime\cite{21} where the charge on the dot is quantized. We use a stationary master equation approach\cite{21, 37} for the reduced density matrix of the dot and calculate the transition rates in a “golden-rule” approach up to 2nd order in $H_T$. The first-order contribution to the current is the sequential tunneling (ST) current $I_s$\cite{21}, where the number of electrons on the dot fluctuates and thus the processes of an electron tunneling from the lead onto the dot and vice versa are allowed by energy conservation. The second-order
contribution is the cotunneling (CT) current $I_c$[39], where charge is transported over intermediate virtual states of the dot.

We now consider a system, where the Zeeman splitting in the leads is negligible (i.e. much smaller than the Fermi energy) while on the dot it is given as $\Delta_s = \mu_B |gB|$. We assume a small bias $\Delta \mu = \mu_1 - \mu_2 > 0$ between the leads at chemical potential $\mu_{1,2}$ and low temperatures so that $\Delta \mu, k_B T < \delta$, where $\delta$ is the characteristic energy-level distance on the dot. First we tune the system to the ST resonance $\mu_1 > \Delta E > \mu_2$ where the number of electrons can fluctuate between $N$ and $N+1$. $\Delta E$ is the energy difference between the $N+1$ and $N$-particle groundstates (GS) of the dot. We first consider a quantum dot with $N$ odd and total spin $s = 1/2$ with the $N$-particle GS to be $|\uparrow\rangle$ and to have energy $E_{\uparrow} = 0$. In this state the dot can receive an electron from the leads and, depending on the spin of the incoming electron form a singlet $|S\rangle$ with energy $E_S$ (for spin down) or a triplet $|T_{\pm}\rangle$ with energy $E_{T_{\pm}}$ (for spin up). The singlet is (usually) the GS for $N$ even, whereas the three triplets $|T_{\pm}\rangle$ and $|T_0\rangle$ are excited states. In the regime $E_{T_{\pm}} - E_S, \Delta_s > \Delta \mu, k_B T$, energy conservation only allows ground state transitions. Thus, spin-up electrons are not allowed to tunnel from lead 1 via the dot into lead 2, since this would involve virtual states $|T_{\pm}\rangle$ and $|\downarrow\rangle$, and so we have $I_s(\uparrow) = 0$ for ST. However, spin down electrons may pass through the dot in the process $\downarrow\uparrow_i \rightarrow \downarrow\downarrow_f$, followed by $\uparrow\downarrow_i \rightarrow \uparrow\downarrow_f$. Here the state of the quantum dot is drawn inside the circle, while the states in the leads are drawn to the left and right, resp., of the circle. This leads to a spin-polarized ST current $I_s = I_s(\downarrow)$, which we have calculated as[37]

$$I_s(\downarrow)/I_0 = \frac{\Delta \mu}{4k_B T} \cosh^{-2} \left( \frac{E_S - \mu}{2k_B T} \right), \quad k_B T > \Delta \mu, \tag{6}$$

$$I_s(\downarrow)/I_0 = \theta(\mu_1 - E_S) - \theta(\mu_2 - E_S), \quad k_B T < \Delta \mu, \tag{5}$$

where $\mu = (\mu_1 + \mu_2)/2$ and $I_0 = e\gamma_1 \gamma_2/((\gamma_1 + \gamma_2))$. Here $\gamma_l = 2\pi \nu |A_{nn'}|^2$ is the tunneling rate between lead $l$ and the dot. $n$ and $n'$ denote the $N$ and $N+1$ particle eigenstates of $H_D$ involved in the tunnel process. The dependence of $A_{nn'} = \sum_{\rho \sigma} t_{lp}(n'|\rho d_{\rho n}|n)$ on $n$ and $n'$ is weak compared to the resonant character of the tunneling current considered here[37]. Similarly, for $N$ even we find $I_s(\downarrow) = 0$ while for $I_s(\uparrow)$ a similar result holds[37] as in Eqs. (5), (6).

Even though $I_s$ is completely spin-polarized, a leakage of current with opposite polarization arises through cotunneling processes[37]; still the leakage is small, and the efficiency for $\Delta_s < |E_{T_{\pm}} - E_S|$ for spin filtering in the sequential regime becomes[37]

$$I_s(\downarrow)/I_c(\uparrow) \sim \frac{\Delta^2}{(\gamma_1 + \gamma_2) \max\{k_B T, \Delta \mu\}}, \tag{7}$$

where $\Delta^2 = \Delta \mu^2$. The cotunneling contribution $I_c$ is given by the cotunneling rate $I_c = \frac{\Delta^2}{\gamma_1 + \gamma_2}$.
and equivalently for $I_s(\uparrow)/I_s(\downarrow)$ at the even-to-odd transition. In the ST regime we have $\gamma_i < k_B T, \Delta \mu$, thus, for $k_B T, \Delta \mu < \Delta_z$, we see that the spin-filtering is very efficient. Above or below a ST-resonance the system is in the CT regime where the current is solely due to CT-processes. Again, in the regime $E_{T\pm} - E_S, \Delta_z > \Delta \mu, k_B T$ the current is spin-polarized and the spin filter also works in the CT regime[37].

We discuss now the opposite case where the leads are fully spin polarized with a much smaller Zeeman splitting on the dot[37]. Such a situation can be realized with magnetic semiconductors (with effective g-factors reaching 100 [3]) where spin-injection into GaAs has recently been demonstrated for the first time[3, 4]. Another possibility would be to work in the quantum Hall regime where spin-polarized edge states are coupled to a quantum dot[40]. In this setup the device can be used as read-out for the spin state on the dot. Assume now that the spin polarization in both leads is up, and the ground state of the dot contains an odd number of electrons with total spin $1/2$. Now the leads can provide and take up only spin-up electrons. As a consequence, a ST current will only be possible if the dot state is $|\downarrow\rangle$ (to form a singlet with the incoming electron, whereas the triplet is excluded by energy conservation). Hence, the current is much larger for the spin on the dot being in $|\downarrow\rangle$ than it is for $|\uparrow\rangle$. Again, there is a small CT leakage current for the dot-state $|\uparrow\rangle$, with a ratio of the two currents given by Eq. (7) (assuming $E_S > \Delta_z$). Thus, we can probe (read out) the spin-state on the quantum dot by measuring the current which passes through the dot. Given that the ST current is typically on the order of $0.1 - 1$ nA [21], we can estimate the read-out frequency $I/2\pi e$ to be on the order of $0.1 - 1$ GHz. Combining this with the initialization and read-in techniques, i.e. ESR pulses to switch the spin state, we have a spin memory at the ultimate single-spin limit, whose relaxation time is just the spin relaxation time. This relaxation time can be expected to be on the order of 100’s of nanoseconds[2], and can be directly measured via the currents when they switch from high to low due to a spin flip on the dot[37].

5. Conclusions

We have described a scalable scenario for the implementation of a solid state quantum computer based on the electron spin in quantum dots as the qubit. We have shown how electron spins can be manipulated through their charge (orbital) degrees of freedom to implement single and two-qubit gates as well as the possibility of read in/out a single qubit (spin).
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