Practical Mathematical Model for the Evaluation of Main Parameters in Polymer Flooding: Rheology, Adsorption, Permeability Reduction, and Effective Salinity

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ABSTRACT: Polymer flooding is one of the most used chemical enhanced oil recovery (CEOR) technologies worldwide. Because of its commercial success at the field scale, there has been an increasing interest to expand its applicability to more unfavorable mobility ratio conditions, such as more viscous oil. Therefore, an important requirement of success is to find a set of design parameters that balance material requirements and petroleum recovery benefits in a cost-effective manner. Thus, for an effective enhanced oil recovery project management, a quick and feasible tool is needed to identify projects for polymer flooding applications, without giving up key physical and chemical phenomena related to the recovery process and avoiding activities or projects that have no hope of achieving adequate profitability. A detailed one-dimensional mathematical model for multiphase compositional polymer flooding is presented. The mathematical formulation is based on fractional flow theory, and as a function of fluid saturation and chemical compositions, it considers phenomena such as rheology behavior (shear thinning and shear thickening), salinity variations, permeability reduction, and polymer adsorption. Moreover, by setting proper boundary and initial conditions, the formulation can model different polymer injection strategies such as slug or continuous injection. A numerical model based on finite-difference formulation with a fully implicit scheme was derived to solve the system of nonlinear equations. The validation of the numerical algorithm is verified through analytical solutions, coreflood laboratory experiments, and a CMG-STARS numerical model for waterflooding and polymer flooding. In this work, key aspects to be considered for optimum strategies that would help increase polymer flooding effectiveness are also investigated. For that purpose, the simulation tool developed is used to analyze the effects of polymer and salinity concentrations, the dependence of apparent aqueous viscosity on the shear rate, permeability reduction, reversible−irreversible polymer adsorption, polymer injection strategies on petroleum recovery, and the flow dynamics along porous media. The practical tool and analysis help connect math with physics, facilitating the upscaling from laboratory observations to field application with a better-fitted numerical simulation model, that contributes to determine favorable scenarios, and thus, it could assist engineers to understand how key parameters affect oil recovery without performing time-consuming CEOR simulations.

1. INTRODUCTION
Taking into account that the world average of total production from an oil reservoir after primary and secondary recovery is about one-third of the original oil in place (OOIP) to 3 so that significant volume of petroleum is left behind underground due to capillary forces, bypassed oil as a result of viscous fingering (such as stratification, channeling, gravity segregation, etc.) and nearly 2 × 10^{12} barrels of conventional oil and 5 × 10^{12} barrels of heavy oil will remain in the reservoir. Therefore, along with the growing energy demand around the world, enhanced oil recovery (EOR) technologies need to contribute to increase the recovery from petroleum reservoirs. Now, because many oil fields have been under waterflooding, chemical EOR (CEOR) methods could be implemented with less additional facilities needed, compared with other EOR methods. Polymer flooding is probably the most practical CEOR technology applied successfully at the commercial scale for light or medium gravity oils and also in combination with horizontal wells for heavy high-viscosity oil (from 600 to 2000 cP), where thermal methods were not economically applicable.
The polymer flooding process improves waterflooding because a polymeric solution increases the viscosity of the displacing fluid and decreases the mobility ratio, thereby decreasing viscous fingering. As seen from experimental data, polymer adsorption onto rock surfaces causes reduction of effective permeability to water, contributing to increased volumetric sweep efficiency and water cut decrements.\(^{12-14}\)

For field experience, successful projects have been reported with considerable permeability reduction,\(^{15}\) such as the successful flood in the Shuanghe oilfield,\(^{16}\) which reported \(R_p\) values in the range from 20 to 60 and \(R_{RF}\) values in the range from 3 to 13.5, where \(R_p\) is the resistance factor, defined as the polymer mobility when the polymer flows divided by the water mobility when the polymer flows, and \(R_{RF}\) is the residual resistance factor, defined as the water mobility before the polymer flows divided by the water mobility after the polymer flows. Several pilot tests have been carried out for polymer flooding along with other EOR technologies so that it has combined the mobility control from polymer injection and the enhancement of microscopic displacement efficiency by reducing oil–water interfacial tension with surfactant injection or by wettability alteration through alkaline flooding,\(^{17-19}\) nanoparticle flooding,\(^{20}\) and low salinity injection.\(^{19-21}\) Besides, there are other endeavors to apply polymer flooding to more unfavorable environments such as high-temperature, high-salinity carbonate reservoirs,\(^{22}\) pH-sensitive polymer conditions,\(^{23}\) and formations with active clays,\(^{24}\) where geochemical modeling is recommended. In 2017, Seright\(^{25}\) examined and published a review of previous and current practices, where discussions with many operators and designers of current polymer floods revealed substantial differences of opinion for the appropriate way to design a polymer flood. As a concluding remark, polymer flooding is a mature and versatile technology that can be combined with other EOR methods, can be applied under a wide variety of reservoir conditions previously defined, and can be designed under different strategies to increase the recovery efficiency and profitability.

To find the optimum conditions of applying polymer flooding, a predictive model that represents physical and chemical behavior during displacement processes is a helpful tool. Polymer flooding entails phenomena such as rheology, salinity variation, permeability reduction, and adsorption. Some of the first multicomponent–multiphase displacement mathematical models for chemical flooding were presented by Pope and Nelson,\(^{26}\) Helfferich,\(^{27}\) and Hiraski,\(^{28}\) where models for polymer viscosity behavior did not consider the shear rate or salinity concentration dependence, and the adsorption behavior did not include salinity concentration, and permeability reduction was neglected. Hiraski and Pope\(^{29}\) discussed and proposed a model for adsorption and shear thickening behavior of a polymeric solution.

Afterward, with considerable simplifications, several authors introduced some analytical solutions to one-dimensional (1D) water and polymer flooding problems such as Buckley and Leverett (BL)\(^{30}\) and Welge\(^{31}\) described the classical frontal advance behavior for waterflooding, Pope\(^{32}\) presented the application of fractional flow theory to EOR, Sorbie\(^{33}\) and Lake\(^{34}\) documented the solution to the convection–dispersion equation for polymer transport, and more recent studies have presented analytical expressions\(^{33-37}\) for 1D polymer injection (Table 1). Although analytical methods cannot represent all the required physics of multiphase flow for EOR processes, they can draw a rough image of the flooding performance and can help build a decision framework for proceeding to support a decision.\(^{38}\)

Unlike analytical methods, reservoir simulation allows modeling more complex physical and chemical principles and reservoir conditions under specific development strategies. Then, at the beginning of the 70s were presented some of the numerical models,\(^{39,40}\) that, with basic relations, took into account the polymer content effect on polymer adsorption, aqueous viscosity, and polymer slug. Later, in 1978, for a chemical flooding compositional simulator, Pope and Nelson\(^{41}\) considered salinity as a component in the continuity equation but not in the polymer-rich phase viscosity or polymer adsorption. For economic analysis, by 1984, some researchers developed a prediction tool\(^{42}\) for polymer flooding which incorporated methods used in both, simplified and sophisticated prediction tools, by combining a two-dimensional (2D) cross-sectional model with area sweep correlations and injectivity functions. Based on the BL equation, Masuda et al.\(^{43}\) proposed a simple simulation model including the viscoelastic effect of polymer solution. In 1996, Delshad et al.\(^{44}\) described the formulation of a general chemical compositional simulator named UTCHEM. Zheng et al.\(^{44}\) introduced a new empirical model for relative permeability reduction as a function of polymer adsorption and a model for the shear rate based on mobility measurements. As the application of polymer flooding extended to viscous oils with polymers at high concentrations and with very high molecular weights, a mechanistic understanding of polymer rheology and accurate numerical modeling were essential; this led to improvements in the understanding of polymer rheology incorporating apparent viscosity models,\(^{45}\) that account for both shear-thinning and shear-thickening behavior of polymers in porous media. A critical review of the existing viscoelastic models was carried out, in 2019, by Azad and Trivedi,\(^{46}\) which presented the deficiencies of different methodologies used for quantifying the viscoelastic effects by polymer flooding. Nowadays, models and correlations developed for polymer flooding have been the basis for modeling specific problems, such as Manzoor in 2020\(^{47}\) studied the effect of injection pressure on flooding performance in a 1D mathematical model that considers several phenomena, neglecting the shear rate and salt effects on apparent viscosity.

Even for the coreflooding laboratory scale, commercial and academic reservoir simulators for CEOR applications such as CMG-STARS, ECLIPSE, and UTCHEM require more detailed information (related to fluid, component and reservoir properties, and operational conditions) and imply more economic resources than simplified numerical models. Additionally, for the

### Table 1. Analytical Model for 1D Flow Flooding

| reference | comments |
|-----------|----------|
| Pope\(^{32}\) | continuous polymer injection; constant viscosities; \(C_4\) is represented by constant retardation |
| Sorbie\(^{33}\) | continuous polymer injection; constant dispersion coefficient and \(Q\); \(C_4\) is a linear isotherm |
| Shapiro et al.\(^{34}\) | continuous chemical injection; aqueous multicomponent solution; constant viscosities; variable \(C_4\) |
| Vicente et al.\(^{35}\) | semi-analytical; slug polymer injection; aqueous viscosity is linear with \(C_4\); \(C_4\) is a Langmuir isotherm |
| De Paula and Pires\(^{36}\) | slug polymer injection; salinity variation; constant viscosities; \(C_4\) is a Langmuir isotherm with a salt effect |
| Borazjani et al.\(^{36}\) | slug polymer injection; salinity variation; non-Newtonian behavior; linear Henry’s sorption |
| Abdul-Hamid and Muggeridge\(^{37}\) | slug polymer injection; viscous fingering by Todd and Longstaff; absence of adsorption |
preliminary decision-making process, such as early stages of EOR process selection and laboratory investigations, a practical tool that allows us to capture the basic physical and chemical phenomena involved in the CEOR implementation is required. According to Hite et al., a successful EOR project depends on good planning and needs to avoid spending considerable time and effort on projects that have no hope of achieving adequate profitability. Screening studies should identify candidates, thus only focusing on worthy opportunities. Therefore, EOR predictive tools with different degrees of accuracy (from statistical techniques to reservoir numerical simulation) interact with economics, engineering planning, and data collection and should go hand-in-hand at each step for an effective EOR project management.

A multiphase, compositional mathematical model, based on fractional flow theory that considers Non-Newtonian rheology (shear thinning and viscoelasticity), permeability reduction, and irreversible or reversible polymer adsorption, as a function of polymer and salinity concentrations, has been developed, including four components (water, oil, polymer, and salinity) and two phases (aqueous and oleic). Some limitations are temperature variations, mechanical degradation, and water hardness. The resulting non-linear mathematical system is solved numerically by using a finite-difference formulation with a fully implicit scheme in time and central differences in space. The numerical algorithm is supported by (1) the analytical solutions of BL for waterflooding and the extension of fractional flow theory for polymer flooding, (2) laboratory experiments reported by Koh et al. for waterflooding and polymer flooding where oil viscosities from 72 to 1050 cP were simulated and by Masuda et al. for viscoelastic rheology, and (3) a CMG-STARS model built as a reference for water and polymer flooding.

Additionally, this paper analyzes the interaction of main phenomena involved in polymer flooding processes, investigating the effects of the main controlling flow properties on saturation and composition profiles and on breakthrough time and recovery efficiency. The chemical injection strategy is deemed vital for the success of flooding in either continuous or slug modalities.

This paper is organized as follows: First, the governing equations are presented, and the mathematical flow model for polymer flooding is derived. Second, the resulting system of equations is discretized with the finite-difference method, and the numerical flow model is described. Third, polymer and salinity concentration effects on flow properties are explored. Fourth, the numerical model and algorithm are validated using well-known analytical solutions, laboratory experiments, and commercial simulator results. Fifth, the flow dynamics and recovery efficiency behavior of polymer flooding by saturation and concentration distribution under different flow parameter scenarios are investigated. Finally, the paper concludes with the interpretations of results.

2. MATHEMATICAL MODELING

2.1. Model Assumptions. The key assumptions made for developing the 1D model for multiphase multicomponent flow through porous media in a polymer flooding process are as follows:

1. The problem is reduced to 1D flow.
2. The medium is homogeneous and isotropic.
3. The system is isothermal.
4. All fluids and rocks are incompressible, and effects of pressure on the equilibrium and fractional flow are negligible.
5. There are two immiscible phases and four components.
6. Fluid properties are a function of composition only.
7. The phases are in local equilibrium.
8. There are no dispersive effects such as molecular diffusion, longitudinal and transverse dispersivities, convective dispersion, or capillary imbibition.
9. The adsorption isotherm has negative curvature, and it could be a reversible or irreversible process depending on the case analyzed.
10. The permeability is reduced as a function of the polymer concentration.
11. Gravity and capillary pressure are negligible.
12. Polymer and salt are transported only in the aqueous phase.
13. There are no changes in the inaccessible PV.
14. The rheological behavior can represent Newtonian and non-Newtonian (shear thinning and shear thickening) behavior.

2.2. General Flow Model. To understand any EOR method, the mass conservation equations for component $i$ that includes the interaction with the solid phase are described as

$$\frac{\partial W_i}{\partial t} + \nabla \cdot \vec{J}_i = R_i$$

where $W_i$ is the overall concentration (accumulation), $\vec{J}_i$ is the flux, and $R_i$ is the source–sink term. Upon incorporation of the definitions of each variable, eq 1 becomes

$$\frac{\partial}{\partial t} \left[ \phi \sum_{j=1}^{N_p} \rho_j S_{ij} \omega_j + (1 - \phi) R \omega_i \right]$$
$$+ \nabla \cdot \left[ \phi \sum_{j=1}^{N_p} \left( \rho_j S_{ij} \vec{R}_j - \phi \rho_j \vec{S}_{ij} \omega_j \right) \right]$$
$$= \phi \sum_{j=1}^{N_p} S_{ij} \omega_j + (1 - \phi) r_i, \quad i = 1, 2, ..., N_C$$

Considering incompressible fluids, constant porosity, ideal mixing, and other simplifications, eq 2 can be written in 1D form as

$$\frac{\partial}{\partial t} \left[ \phi \sum_{j=1}^{N_p} S_{ij} C_{ij} + (1 - \phi) C_i \right] + \frac{\partial}{\partial x} \left[ \phi \sum_{j=1}^{N_p} S_{ij} \frac{\partial C_{ij}}{\partial x} \right]$$
$$- \phi \frac{\partial}{\partial x} \left[ \sum_{j=1}^{N_p} \phi S_{ij} K_{ij} \frac{\partial C_{ij}}{\partial x} \right] = 0, \quad i = 1, 2, ..., N_C$$

This equation includes the main effects that control the flow of fluids and components through porous media in linear EOR processes (adsorption, gravity, capillarity, viscosity, and dispersion). The equation must be solved simultaneously with Darcy’s law, the definitions of relative mobility, capillary pressure, volume fractions, phase saturations, equations of states, phase equilibrium relations, and dispersion.

Authors have presented a special form of eq 3 to represent the physicochemical characteristics of oil and water
displacement under a chemical EOR process, which in a dimensionless form becomes
\[ \frac{\partial}{\partial t_D} [C_i + \dot{C}_i] + \frac{\partial^2}{\partial x_D^2} C_i = 0, \quad i = 1, 2, \ldots, N_C \] (4)

where
\[ x_D = \frac{x}{L} \] (5a)
\[ t_D = \frac{u}{\phi L} \] (5b)

\( C_i \) is the overall fluid phase concentration of component \( i \), given by
\[ C_i = \sum_{j=1}^{N_c} S_j C_{ij} \] (6a)
\( \dot{C}_i \) is the rock-phase concentration of \( i \) on a pore volume basis, and \( F_i \) is the overall flux of component \( i \), given by
\[ F_i = \sum_{j=1}^{N_c} f_j C_{ij} \] (6b)
in which
\[ f_j = \frac{\lambda_{ij}}{\sum_{j=1}^{N_c} \lambda_{ij}} \] (7)

2.3. Model for Polymer Flooding. The mathematical model based on fractional flow theory for linear polymer flood with salinity variation consists of four equations expressing conservation of mass for each component. The common notation \(^{6b} 14,26\) for chemical flooding is adopted so that the four components are water (1), oil (2), polymer (4), and salt (5).

Considering eqs 4–6b coupled with the assumptions listed, the resulting system of equations can be reduced to the following system of nonlinear equations:
\[ \frac{\partial}{\partial t_D} [S_1] + \frac{\partial}{\partial x_D} [f_1] = 0 \] (8)
\[ \frac{\partial}{\partial t_D} [C_{41} S_1 + \dot{C}_4] + \frac{\partial}{\partial x_D} [f_4 C_{41}] = 0 \] (9)
\[ \frac{\partial}{\partial t_D} [C_{51} S_1] + \frac{\partial}{\partial x_D} [f_{51} C_{51}] = 0 \] (10)

The fractional flow is a function of saturation and concentrations, as specified in eq 7:
\[ f_j = \frac{\lambda_{ij}}{\lambda_{i1} + \lambda_{i2}} \] (11)

where the relative mobilities for the two phases are
\[ \lambda_{i1} = \frac{k_{i1}}{\mu_i R_k} \] (12a)
\[ \lambda_{i2} = \frac{k_{i2}}{\mu_2} \] (12b)

Polymer flooding causes a degree of permeability reduction that decreases mobility, in addition to the increase in viscosity of the polymer solution.\(^ {14}\) To account for the reduction of permeability, the polymer solution viscosity is modeled using the permeability reduction factor, \( R_{\text{q}} \), which is the rock permeability when water flows divided by the rock permeability when aqueous polymer solution flows.\(^ {15}\)

Equation 12a considers the permeability reduction effect, \( R_{\text{q}} \). For relative permeabilities, Corey-type equations are used\(^ {14,26,52}\) as follows:
\[ k_{ij} = k_{ij}^0 [S_{ij}]^{\frac{5}{4}} \] (13)

where the normalized saturation is defined as
\[ S_{ij} = \frac{S_i - S_{ir}}{1 - S_{ir} - S_{2r}} \] (14)

Upon combining eqs 11, 13, and 14, the fractional flow of water is
\[ f_i = \frac{M^* S_{41}^{\frac{5}{4}}}{M^* S_{41}^{\frac{5}{4}} + (1 - S_{41})^{\frac{5}{4}}} \] (15)

where \( M^* \) is the end-point water–petroleum mobility ratio with permeability reduction, defined as
\[ M^* = \frac{k_{41}^0/(\mu_i R_k)}{k_{41}^0/\mu_2} = \frac{k_{41}^0}{k_{41}^0 (\mu_i R_k)} \] (16)

The advantages of polymers as water mobility control agents in porous media are indicated by large permeability reduction factor values and the increment of aqueous phase viscosity. This can be used in profile improvements to plug the more permeable streaks near injectors and to reduce permeability variation.\(^ {7}\)

Permeability reduction is caused by polymer adsorption. The most used in numerical simulation is the Langmuir-type mathematical expression,\(^ {13,52}\) even though it is not always the best choice. The flow model considers the option of irreversible adsorption, and we took into account the recommendation of Zhang and Seright\(^ {13}\) to reduce incorrect predictions: to reach the Langmuir plateau at very low polymer concentrations, the polymer front has to be sufficiently sharp, and the injected polymer concentration has to be relatively high. This model for adsorption could present inaccuracies in the dilute level for polymer solution according to Zhang. We use the following equation for \( \dot{C}_4 ^{i} \):
\[ \dot{C}_4 ^{i} = \min \left[ \dot{C}_4 ^{i}, \frac{a_4 (C_4 - \dot{C}_4 ^{i})}{1 + b_4 (\dot{C}_4 - \dot{C}_4 ^{i})} \right] \] (17)

where the dependence on the polymer and salinity concentrations is determined using the relation
\[ a_4 = (\alpha_1 + \alpha_2 C_{\text{SEP}}) \sqrt{(k_{\text{ref}}/k)} \] (18)
in which the effective salinity is expressed as
\[ C_{\text{SEP}} = C_{51}/C_{11} \] (19)

The Langmuir-type model is reversible in terms of the polymer concentration. When polymer adsorption needs to be irreversible, it requires tracking the adsorption history to detect decrements of the polymer concentration.\(^ {13}\)

For the polymer-rich phase, viscosity considers the influence of the polymer concentration and effective salinity using the modified Flory–Huggins equation.\(^ {13,52}\)
\[ \mu_i ^{2} = \mu_i [1 + (\alpha_i C_{41} + \alpha_2 C_{41}^2 + \alpha_3 C_{41}^3) C_{\text{SEP}} S_1] \] (20)
The rheology of polymeric solutions for polymer flooding is generally pseudoplastic, and the viscosity related to this type of flow is shear thinning viscosity ($\mu_{1sh}$); the apparent viscosity appears to be less at increasing shear rates.\textsuperscript{12} The shear thinning behavior of the polymer solution is caused by the uncoiling and unsnagging of the polymer chains when they are elongated in shear flow.\textsuperscript{11} To capture the polymer solution viscosity and shear rate relationship, the Carreau equation (eq 21a)\textsuperscript{12,13,54} was used:

$$\mu_{1sh} = \mu_0 + (\mu_f - \mu_0)(1 + (\lambda \dot{\gamma}_eq)\alpha)(n-1)/n$$  \hspace{1cm} (21a)

where $\mu_0$ is the viscosity at a high shear limit that generally is taken as water viscosity; $\alpha$ affects the shape transition region between the zero shear rate plateau and the rapidly decreasing portion and is generally taken\textsuperscript{13} to be 2; $\lambda$ is a parameter for material relaxation time correlated with the molecular structure,\textsuperscript{54} which determines the shear rate at which the transition occurs from the zero shear rate plateau to the power law portion; and $\dot{\gamma}_eq$ is the power-law-like exponent. Another type of flow is elongational, or extensional, when the fluid flows through a series of pore bodies and throats in a porous medium. In this type of flow, the apparent viscosity is increased as the shear rate increases, and the viscosity related to this flow is shear thickening viscosity or elongational viscosity, $\mu_{1el}$\textsuperscript{13} which can be represented with the model developed by Masuda\textsuperscript{42} (eq 21b):

$$\mu_{id} = \mu_{1sh} C^\kappa m_c \gamma_{eq}$$  \hspace{1cm} (21b)

in which the parameter $C^\kappa = C_M t_i^{mc}$ depends on the empirical constant $C_M$ and the relaxation time of the fluid $t_i$ (i.e., considered constant); then, $C^\kappa$ and $m_c$ are empirical constants determined using the flow experiment of the fluid through porous media.\textsuperscript{55} To describe the viscosity in the entire shear rate range, we consider two parts: the shear-viscosity-dominant part ($\mu_{1sh}$) and elongational-viscosity-dominant part $\mu_{1el}$ (eq 21c):

$$\mu_{app} = \mu_{1sh} + \mu_{1el}$$  \hspace{1cm} (21c)

The shear rate ($\dot{\gamma}$) is an equivalent shear rate for flow in permeable medium applications ($\dot{\gamma}_{eq}$), and it could be modeled as Sorbie\textsuperscript{12} and UTCHEM,\textsuperscript{52} but to consider the applicability of the model to low-permeability porous formations,\textsuperscript{13} an expression that includes permeability reduction was chosen:\textsuperscript{55,56}

$$\dot{\gamma}_{eq} = \dot{\gamma}/\sqrt{k_c_S f_i} / R_k$$  \hspace{1cm} (22)

The permeability reduction factor is defined\textsuperscript{52} as

$$R_k = 1 + (R_{k_{max}} - 1) \frac{b_t C_i}{1 + b_t C_i}$$  \hspace{1cm} (23)

where

$$R_{k_{max}} = \left[1 - \left(\frac{\epsilon(r_p C_{SEP} S_i^{3/2})}{\sqrt{k}/\phi}\right)^{1/3}\right]^{-1}$$  \hspace{1cm} (24)

For the previous equation, it is common to consider an empirical value for a maximum permeability reduction factor\textsuperscript{13} of 10.

The nonlinear system of differential equations must be solved simultaneously with Darcy’s law, definitions for relative mobility, mass fractions, saturations, and other auxiliary relations for polymer flooding.

For the initial and boundary conditions, it was considered that porous medium was initially saturated with oily and irreducible water at some initial salinity. At the injection boundary, water saturation is always at its maximum values. For continuous polymer injection, the polymer concentration is fixed to a specified value. For slug injection, the polymer concentration is maintained at the desired value during the slug size, after which chase water injection follows.

### 2.4. Numerical Model of the Polymer Flooding Equation

The system of advection–reaction equations, developed for describing the polymer flooding process, has no analytical solution for the general case, but it is suitable for numerical solution. Although the flow model developed could work under any initial and boundary conditions, Dirichlet-type boundary conditions were considered as it was described before.

The nonlinear polymer flooding model, eqs 8–10, could adopt the following general form:

$$\frac{\partial}{\partial t_D} [C_i S_i + \dot{C}_i] + \frac{\partial}{\partial x_D} [\dot{C}_i S_i] = 0$$  \hspace{1cm} (25)

This equation was discretized using finite differences, resulting in an unconditionally stable numerical model for the flow problem, using an upwind fully implicit difference scheme and Newton–Raphson (NR) method.\textsuperscript{57,58} The mesh assumes that the block size is constant. The discretized nonlinear equations at node $i$ and time step $n + 1$ take the residual ($R_e$) form

$$R_{ei, i}^{n+1} \approx \left(\frac{(C_i S_i)_{t}^{n+1} - (C_i S_i)_i^n}{\Delta t_D} + \frac{\dot{C}_i_{t}^{n+1} - \dot{C}_i^n}{\Delta x_D}\right)$$  \hspace{1cm} (26)

For either continuous injection or slug injection schemes, the boundary conditions are Dirichlet-type, and thus, $i = 2, 3, ..., I - 1$, and for time discretization, $n = 0, 1, 2, ...$

For linearizing the system of non-linear equations, the iteration level $v + 1$ of the $n + 1$ time step solution is required, and a generic form of the linearized equation for component $i$ and node $i$ is as follows:

$$-R_{ei, i}^{n+1} (\xi_i, \xi_{i+1}, ..., \xi_{i+\alpha}) = \sum_{m=1}^{M} \left(\frac{\partial R_{ei, i}^v}{\partial \xi_m^v}\right) \partial \xi_m^{v+1}$$  \hspace{1cm} (27)

where $m$ is the index for the unknown variables ($\xi$) around the node $i$ and $\partial \xi_m^{v+1}$ are the iteration changes of the unknowns:

$$\partial \xi_m^{v+1} = \xi_m^{v+1} - \xi_m^v$$  \hspace{1cm} (28)

The generic form of the non-zero coefficients that linearized the system of equations for component $i$ and unknown variable $\xi_m$ at node $i$ is the following:

$$\left(\frac{\partial R_{ei, i}^v}{\partial \xi_m^v}\right)_{i,v} = \frac{1}{\Delta x_D} \left(\frac{\partial (C_i S_i)}{\partial \xi_m^v}\right)_{i,v}$$  \hspace{1cm} (29a)

$$\left(\frac{\partial R_{ei, i}^v}{\partial \xi_m^v}\right)_{i,v} = \frac{1}{\Delta x_D} \left(\frac{\partial (C_i S_i)}{\partial \xi_m^v}\right)_{i,v} + \frac{1}{\Delta x_D} \left(\frac{\partial (C_i S_i)}{\partial \xi_m^v}\right)_{i,v}$$  \hspace{1cm} (29b)

The Jacobian matrix takes into account all components and unknown variables for each node.
To summarize, Figure 1 presents the flowchart that the numerical simulator applies for the numerical algorithm. In each time step, the simulator sets values for saturation and concentrations and begins the iterative process to solve implicitly the saturation and concentration variables. All the newly updated variables and properties are provided for the initial values of the next time step. This continues until it reaches the final time.

The validation for the numerical algorithm by using the analytical solutions of BL\(^{14,30}\) for waterflooding and the extension of fractional flow theory for polymer flooding\(^{12,32}\) and by using experimental data carried out in 1D corefloods is presented later.

3. RESULTS AND DISCUSSION

3.1. Effects of Polymer and Salinity Concentrations on Flow Properties. Applying fractional flow theory concepts along with the most important phenomena and flow property models related to polymer flooding, this section presents a qualitative analysis to get a quick idea of a favorable scenario for oil recovery with less computing effort and time. As a first qualitative analysis to get a quick idea of a favorable scenario for oil recovery with less computing effort and time. As a first qualitative analysis to get a quick idea of a favorable scenario for oil recovery with less computing effort and time.

Based on eq 15, fractional flow depends on the relative permeability, fluid viscosity, and permeability reduction factor. Several experimental studies reported\(^{39}\) that the effect of viscosity on relative permeabilities is very small and many times insignificant. Nevertheless, many tests conducted\(^{13}\) show that the relative permeability curve for polymer solution is significantly lower than the corresponding relative permeability curve for water before polymer transport. This is caused by permeability reduction, which is a consequence of polymer adsorption. Therefore, to account for the mobility decrease due to permeability reduction, the viscosity of the aqueous phase is multiplied by the value of the permeability reduction factor as eq 16 illustrates.

To analyze the influence of the flow properties on each other and on the fractional flow according to eqs 11–24, the polymer and reservoir data described by Sorbie\(^{12}\) are used as a starting point (see Table 2).

### Table 2. Polymer and Reservoir Data for the 1D Flow Problem

| property                          | input data          |
|-----------------------------------|---------------------|
| phase viscosities                 | \(\mu_1 = 0.5 \text{ cP}\) \(\mu_2 = 3.0 \text{ cP}\) |
| water relative permeability       | \(k_W = 0.3\) \(k_W = 0.9\) |
| oil relative permeability         | \(k_o = 0.25\) \(k_o = 0.22\) |
| residual saturations              | \(S_r = 0.25\) \(S_r = 0.22\) |
| polymer solution viscosity        | \(\mu_p = 7.8 \text{ cP}\) \(C_{41} = 500 \text{ ppm}\) |
| polymer adsorption                | \(\hat{C}_4 = 3.12 \times 10^{-3} \text{ lb/ft}^3\) \(R_0 = 2\) |
| reservoir geometry                | \(L = 2000 \text{ ft}\) \(A = 2500 \text{ ft}^2\) |
| reservoir properties              | \(\phi = 0.25\) \(k = 1000 \text{ mD}\) |
| fluid injection rate              | \(Q = 685 \text{ lb/ft}^3/\text{D}\) |

#### 3.1.1. Polymer Adsorption. A Langmuir-type isotherm is used to describe polymer adsorption (\(\hat{C}_4\) or \(\text{Cad}_4\)), eq 17. Figure 2 illustrates a typical behavior of adsorption for the reference case (\(a_4\) is constant and \(a_4/b_4\) is fixed to 0.0163). As the polymer concentration increases, the polymer adsorption also increases until it reaches asymptotically an adsorption level of \(\hat{C}_4 = 0.0163\) wt %. For the case where no salinity effect is included (reference case) and for a Langmuir isotherm, \(b_4\) controls the curvature of the isotherm, and the ratio \(a_4/b_4\) determines the plateau value for adsorption.\(^{14}\)

To capture the influence of salinity on polymer adsorption, eq 18 is used, where \(a_4\) is a function of the salinity and polymer concentration. For comparison with the reference case, a salinity of \(C_{41} = 0.1\) wt % and a set of values for \(a_4\) and \(\alpha_4\) are selected so that \(a_4/b_4\) approaches 0.0163. Figure 2 shows that at low polymer concentrations, the curves for \(C_{41} = 0.1\) wt % and no salinity effect seem identical, but after a certain value of \(C_{41}\), the curves start to separate as it is illustrated. Additionally, this figure also includes a case where salinity is lower (0.01 wt %), so it can be observed that as salinity decreases, the adsorption level diminishes and tends to flatten.

Polymer adsorption is an important phenomenon that must be evaluated for polymer flooding projects due to loss of the polymer from solution. This results in a reduction of polymer performance during the displacement. Hence, in polymer...
flooding modeling, the impact of polymer adsorption should be included.

3.1.2. Permeability Reduction Factor. When a polymer solution flows throughout a porous medium, some of the polymer molecules are adsorbed on the rock surface, causing permeability reduction or pore throat blocking. Hence, the reduction of permeability is accounted for using eqs 23 and 24.

Equation 23 shows that the permeability reduction factor depends on $R_{k_{\text{max}}}$ and $C_{41}$. The behavior of the maximum value for the permeability reduction factor, eq 24, is illustrated in Figure 3, and it implies that as effective salinity ($C_{\text{SEP}}$) increases, $R_{k_{\text{max}}}$ initially decays rapidly, and then, it is asymptotic. $S_p$ is a power parameter that also contributes to the salinity effect, as Figure 3 also depicts; when $S_p$ increases, the maximum value of the permeability reduction factor decreases.

As seen from eq 23 and a fixed value of $R_{k_{\text{max}}}$, the relationship between the permeability reduction factor and polymer solution concentration is similar to the relationship between the polymer adsorption and polymer solution concentration with no salinity effect considered. For $1 \leq R_k \leq R_{k_{\text{max}}}$, $b_k$ controls the curvature of the permeability reduction.

It is important to notice that the effect of increasing salinity on the permeability reduction factor is different from that observed on polymer adsorption. This means that permeability reduction from eqs 23 and 24 does not correlate with the increase in polymer adsorption. Nevertheless, the prediction of $R_{k_{\text{max}}}$ agrees with the reported values in the literature by Sheng and Martin.

3.1.3. Viscosity at the Zero Shear Rate. Equation 20 describes the polymer solution viscosity at the zero shear rate ($\mu_1$°) as a function of the effective salinity and polymer concentration. Figure 4 depicts the behavior of $\mu_1$° for four specific salinity effects ($C_{\text{SEP}}$), where the increment in the polymer concentration raises the viscosity at zero shear rate, with a cubic dependency. As seen in this figure, when $C_{\text{SEP}}$ and $S_p$ increase, the polymer viscosity decreases at zero shear rate. Higher salinities decrease the benefits of polymer solution viscosity.

3.1.4. Equivalent Shear Rate. To estimate the effects of the shear rate, eq 22 is used by considering the water flow rate, permeability reduction factor, and other multiphase flow properties. We assumed viscosity at the zero shear rate (eq 20) as an initial approximation to calculate the water fractional flow (eq 15) and then the equivalent shear rate.

Three cases for the water fractional flow and equivalent shear rate were computed and are illustrated in Figure 5 as a function of water saturation. For cases where $R_k = 1$, as $\mu_1$° increases, the water saturation behind the front increases, and $\chi_{eq}$ decreases. For cases where $\mu_1$° = 7.8 cP, as $R_k$ increases, the water saturation behind the front increases, and $\chi_{eq}$ reaches higher values. The cases in this figure show that the fractional flow curve is more affected by variations in permeability reduction than by variations in viscosity at the zero shear rate, so this is something to consider in cases where permeability reduction has an important role.

3.1.5. Aqueous Phase Viscosity. To estimate the apparent polymer solution viscosity ($\mu_{1\text{app}}$), eq 21a is used. Figure 6 displays two cases with $\alpha = 2$ as a typical value, $n_c = 0.45$, $1/\lambda = 40$ s$^{-1}$, and varying total fluid velocity. When velocity is lower, $\chi_{eq} > 40$ s$^{-1}$, $\mu_{1\text{app}}$ presents slight variations. On the other hand, as total fluid velocity increases and provides $\chi_{eq} > 40$ s$^{-1}$ (power law region), apparent polymer viscosity decreases considerably (this decrease in apparent viscosity could be different for shear

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**Figure 3.** Maximum value for the permeability reduction factor vs effective salinity.

**Figure 4.** Viscosity at the zero shear rate vs polymer concentration.

**Figure 5.** Water fractional flow and the equivalent shear rate vs water saturation.

**Figure 6.** Aqueous phase viscosity vs water velocity.
thickening fluids as the onset of dilatancy and dilatant behavior dictate the increment of viscosity). Depending on the combination of total fluid velocity and $\lambda$, it is important to consider the impacts of the equivalent shear rate in modeling.

3.1.6. Aqueous Phase Fractional Flow. We analyzed the cases in previous figures, focusing on the relation between fractional flow and apparent viscosity. Figure 7 shows that as the total fluid velocity increases, the apparent polymer viscosity is reduced by shear rate effects, and it reduces the water saturation behind the front, and so it provides a lower oil recovery performance.

For the case of shear thickening fluids, as the shear rate increases, the start of the shear thickening and the increment of apparent viscosity will depend on the onset of dilatancy and dilatant behavior. In this rheological regime, the increment of apparent viscosity leads to increase in water saturation behind the displacement front and thus to a better oil recovery.

3.2. Numerical Solution Validation. 3.2.1. Comparison of the Numerical Model and Analytical Solution. The validation of the model versus the analytical solution to the fractional flow of Buckley–Leverett $^{14,30}$ and its extension for polymer flooding $^{14,32}$ were done using the case described by Sorbie.$^{12}$

Figures 8 and 9 compare the analytical and numerical solutions of the aqueous phase saturation profile for waterflooding and polymer flooding, respectively. The saturation profiles obtained numerically give a good approximation to the analytical solutions. As the mesh is refined, the numerical approximation improves.

To analyze the performance of the numerical approach, the approximation error ($\varepsilon_A$) is used to estimate the saturation profile error of the numerical solution versus the analytical solutions. $\varepsilon_A$ represents the area bordered by analytical and numerical results,$^{15}$ and it is derived using the trapezoidal rule:

\[
\varepsilon_A = \frac{\Delta x_D}{2} \left\{ \sum_{i=1}^{n} \left| S_i(x_D, t_f) - S(x_D, t_f) \right| - \sum_{i=1}^{n} \left| S_i(x_D, t_f) - S(x_D, t_f) \right| \right\} + 2 \sum_{i=2}^{n} \left| S_i(x_D, t_f) - S(x_D, t_f) \right|
\]

Figures 10 and 11 illustrate the sensitivity of $\varepsilon_A$ to the number of nodes for a saturation profile at a specific time, for water
flooding and polymer flooding, respectively. These figures show a similar behavior for the numerical performance; $\varepsilon_A$ tends to decrease when increasing the number of nodes or the number of time steps. Then, the computational effort is worthless from time steps above 400. As seen from these cases, 200 nodes and 200 time steps yield acceptable results.

Although applying the analytical approach from the extension of fractional flow theory to polymer flooding is very useful to provide practical results, it has limitations in cases where salinity variations, shear rate effects on polymer rheology, and permeability reduction are important for estimating fluid distribution and petroleum recovery. The modeling presented in this work applies to a wide range of flow cases and provides a more realistic physical representation of the polymeric solution flow performance by describing phases and composition distribution along porous media and oil recovery behavior.

### 3.2.2. Comparison of the Numerical Model and Coreflood Experiments

The numerical model developed in this work and CMG-STARS software were used to reproduce experimental data reported by Koh and Masuda of water and polymer flooding in sandpacks and reservoir cores. From Koh, the first five experiments (1–5) of waterflooding (WF) and polymer flooding (PF) were used to evaluate our proposed numerical model since data of relative permeability curves, rheological and adsorption behavior dependence of salinity and polymer concentrations were available. We fitted eqs 13–24 of our model to experimental data to find the parameter required by the new numerical model. As a brief reference, Table 3 indicates some properties used for experimental corefloods, where the properties of polymer solution are indicated to get the desired apparent viscosity. For the five experiments shown in Table 3, water velocities are in the shear thinning region.

For experiments 1 through 3, the same brine was used, and the authors determined by a tracer test that the sands were homogeneous. The oil viscosities under reservoir conditions were 80 cP, 120 cP, and 250 cP, while polymer viscosities were 16 cP, 28 cP, and 108 cP. These polymer viscosities correspond to the shear rates specified in Table 3 for each experiment. In the publication, the polymer solution viscosity for each experiment was chosen to give an endpoint mobility ratio close to 1.

#### Table 3. Brief Description of Coreflood Properties for Laboratory Polymer Solution Flooding

| property                     | experiment #1 | experiment #2 | experiment #3 | experiment #4 | experiment #5 |
|------------------------------|---------------|---------------|---------------|---------------|---------------|
| oil viscosity, cP            | 80            | 120           | 250           | 1050          | 72            |
| oil–water viscosity ratio    | 0.167         | 0.250         | 0.521         | 0.1591        | 0.144         |
| porosity                     | 0.35          | 0.36          | 0.37          | 0.39          | 0.28          |
| water velocity, R/D          | 13            | 14            | 14            | 5             | 3.3           |
| polymer concentration, ppm   | 1200          | 1300          | 2450          | 3500          | 2000          |
| equivalent shear rate, 1/s   | 10.1          | 10.7          | 9.8           | 8             | 54            |
| polymer viscosity, cP         | 16            | 28            | 108           | 46            | 12            |
| initial oil saturation       | 0.87          | 0.88          | 0.88          | 0.89          | 0.68          |
| initial water saturation     | 0.12          | 0.12          | 0.12          | 0.11          | 0.32          |

**Figure 10.** Behavior of $\varepsilon_A$ as a function of the number of nodes in the grid for waterflooding.

**Figure 11.** Behavior of $\varepsilon_A$ as a function of the number of nodes in the grid for polymer flooding.
We start with experiment #3 since it was the most discussed in the publication. Figure 12 illustrates the recovery efficiency against PV injected of the experimental and simulated data of water and polymer flooding. Simulated data were obtained using the new model and CMG-STARS software. From the figure it is observed that numerical results yield a good approximation to experimental data.

Experiments #1 and #2 describe flow cases with oil/water viscosity ratios of 167 and 250, flooded with a polymer concentration of 0.12 wt % and 0.13 wt %, respectively. Figures 13 and 14 show the recovery efficiency against injected PV for polymer flooding experimental data, simulated using the new model and CMG-STARS software. From the figures, we can observe that although the numerical representation in both flow cases provides good approximation to experimental data, the numerical results in the case of Figure 14 are more accurate to experimental data after 0.8PV of flooding than in the case of Figure 13. Additionally, it is noticed that simulated data (New model and CMG-STARS) behave a bit differently from experimental data in the transition from water breakthrough to polymer breakthrough (from 0.4PV to 0.8PV); this behavior is not observed in the results for experiment #3 (Figure 12).

For experiments #4 and #5, the injection strategy is different; both started with a waterflooding followed by polymer flooding. In experiment #4, the oil and polymer viscosities were 1050 cP and 46 cP, respectively, with an end-point mobility ratio of 10. Figure 15 displays the WF + PF behavior, and it is observed that numerical results have good approximation to experimental data even though in the polymer flooding stage, numerical results behave a bit differently from experimental data near 2.5PV of injected time. This behavior is not observed in the results for experiment #5 (Figure 16).

Experiment # 5 considers a polymer injection (polymer viscosity of 12 cP) after a stage of waterflooding to displace an oil with a viscosity of 72 cP. In this experiment, the end-point
mobility ratio was set to 1. The results are shown in Figure 16, where the recovery efficiency against injected PV for the experimental data and simulated data during water and polymer flooding is illustrated. A good match between experimental and numerical data is observed. By using the properties and parameters extracted from the publication of Koh, from Figures 12 to 16, we can identify that numerical results provide a good approximation to experimental data for experiments #1 to #5. In some experiments for polymer flooding, a slight difference was detected between numerical and experimental results in the transition period from water breakthrough time to polymer breakthrough time; this difference could be reduced by adjusting the polymer retardation factor for these experiments. Additionally, for all experiments, the performance of the new numerical model developed in this research and the results from CMG-STARS models are in close agreement. Therefore, despite the assumptions of the new model, the implemented numerical algorithm provides a good representation.

An additional set of experimental data used in the validation was the one reported by Masuda, in which a polymer injection process was carried out to displace a mineral oil with a viscosity of 25 cP. To reproduce this experiment, we used the specified core properties (relative permeability curves, rheological behavior, and fractional flow curves) to fit eqs 13–16, 20, and 21 to find the parameters and properties required by the new numerical model. For oil relative permeability, we noticed that Corey-type behavior was not the best fit (this could yield some inaccuracies to reproduce the efficiency recovery obtained from the publication of Masuda), while water relative permeability was very well fitted with Corey behavior.

The rheological behavior of polymer solution for shear thinning and viscoelastic behavior was obtained from the experimental data of Masuda, which are depicted in Figure 17, together with the fitting of the models for shear thinning (eq 21a), shear thickening (eq 21b), and viscoelastic (eq 21c) behaviors.

Beginning with the shear thinning rheological behavior, we reproduce the Ellis model results taken from Masuda and presented in Figure 18; it is observed that results of the new model for shear thinning display good approximation to Ellis model solution even though near the polymer breakthrough time, numerical results are different; this could be explained from the Masuda publication, where it is identified that the flow model was built for an aqueous phase with a polymer that is mixed with a mobile water instantaneously at the displacement front, but its concentration is kept constant; therefore, the aqueous phase behaves as only a one-component phase. On the other hand, the new model proposed in this research considers more than one component in the aqueous phase.

With the same data set of the new model (shear thinning) but changing the rheological model to viscoelastic (Figure 17), we obtained the new model (viscoelastic) results to reproduce the experimental data of Masuda. From Figure 18, a close agreement between experimental data and new model (viscoelastic) results is identified; then, the experiment from Masuda is represented in the shear thickening or dilatant behavior regime. Additionally, the new model presents better results than the results of the Masuda model (viscoelastic).

We conclude from this section that although a commercial numerical simulator can be accommodated to represent simple
models, considering the simplifications of the mathematical model built in this work, we identify that the CMG-STARS model needed more information such as the black oil properties for different pressures and temperatures, molecular weight for each component, mass density, thermal expansion coefficient, reference depth and pressure, well geometry parameters, and temperature of injected fluid, among other information. The model developed provides a practical tool that allows us to represent crucial phenomena involved in polymer flooding in CEOR implementations at the laboratory scale, with less effort.

3.3. Flow Dynamic Analysis. By using the numerical tool developed, the behavior of flow properties is obtained, like Figure 19 displays, such as the aqueous saturation ($S_1$) and polymer concentration ($C_{41}$) distributions and the viscosity at the zero shear rate ($\mu^0_1$) and apparent viscosity ($\mu_{1,app}$) for 0.5PV injected. This figure shows that the displacement begins with the formation of a water front followed by a polymer front; this last one is due to chemical retention as the normalized polymer concentration front illustrates. Additionally, the difference between viscosity at the zero shear rate and apparent viscosity of the polymer solution is observed, which is a consequence of shear rate effects and degradation of polymer solution.

Although in this example, salinity concentrations remain constant during the entire injection process, the simulation tool can describe the changes in salinity concentrations along the porous media and during the evolution of the petroleum displacement by polymer flooding, as it will be discussed later.

To understand better flow dynamics and main controlling aspects of polymer flooding, we applied the numerical model developed and investigated the effects of the flow properties and injection strategy on phase saturation, chemical composition distribution, and oil recovery. This increases the chances to achieve an optimum design that balances material requirements and petroleum recovery benefits.

3.3.1. Effect of the Shear Rate and Polymer Concentration. As mentioned in Figure 19 and eqs 21a, 21b, 21c, and 22, the flow rate and shear rate impact the apparent viscosity of the polymeric solution. Getting into more details on the effects of the shear rate on polymer flooding, Figure 20 shows the relationship between the injection flow rate and recovery efficiency ($E_R$) for several polymer injection concentrations. As the injection flow rate increases, the $E_R$ decreases according to the curve for $C_{41} = 0.1$ wt %, and as the polymer concentration decreases, the influence of the injection flow rate diminishes, up to the point of having no influence at all, as the curve for $C_{41} = 0.001$ wt % exhibits.

Figure 20 also shows that for high $C_{41}$ curves, as the injection flow rate increases, curves tend to converge to a value of recovery efficiency. At lower values of $C_{41}$ and for high flow rates, the minimum asymptotic value of recovery efficiency decreases up to a certain value where changes in the flow rate do not matter at all. To present the effect of the shear rate on flow properties, the curve for $C_{41} = 0.1$ wt % from Figure 20 is taken and compared with two cases for flow rates 0.274 and 27.4 ft/D, where $E_R$ drops from 90 to 83%. Then, Figure 21 illustrates the difference in phase saturation and viscosity distributions along the porous medium, and it is identified that for a slower displacement, the water front saturation tends to decrease, while the polymer front saturation increases. In this way, fluids are distributed more favorably with a longer breakthrough time. With respect to aqueous viscosity, for a slower displacement, the differences
between the viscosity at the zero shear rate and apparent viscosity are less than for a faster displacement.

As indicated, Figure 20 shows the influence of the changing polymer concentration upon recovery efficiency. Therefore, Figure 22 exhibits, for a fixed value of the injection rate (0.274 ft/D) and for three cases ($C_41 = 0.1, 0.01, \text{ and } 0.0 \text{ wt }\%$), the effects of polymer concentration variations on phase saturation and aqueous viscosity distributions, and it is observed that as the amount of the polymer in the injected fluid increases, the apparent viscosity of the aqueous phase increases, and as a consequence, the water front saturation decreases, whereas the polymer front saturation increases. This results in a more efficient displacement; nevertheless, for practical applications, economical aspects must be considered to optimize the quantity of the polymer to be used.

The pore volume injected and recovery efficiency relationship could work in a complementary way to visualize the cases in Figure 22. Figure 23 shows that for a fixed injection rate (0.274 ft/D) and the highest polymer concentration ($C_41 = 0.1 \text{ wt }\%$), the recovery efficiency is nearby 0.8 when the aqueous front reaches the outlet; for the same pore volume injection time, the recovery efficiency for the intermediate polymer concentration ($C_41 = 0.01 \text{ wt }\%$) case is about 0.7 and close to 0.6 for the water injection case. This clearly shows in a quantitative way the benefits of injecting the largest amount of the polymer at the lowest possible injection rate to obtain the highest petroleum recovery.

3.3.2. Effect of the Salinity Concentration. The salinity concentration has an important effect on apparent viscosity and as a consequence on petroleum recovery during a chemical flooding. Equations 20 and 21 relate the salinity concentration to the apparent viscosity.

Figure 24 displays the relationship between the salinity concentration and petroleum recovery efficiency for four polymer injection concentration scenarios. In general, for a polymer concentration case, when the salinity concentration increases, the recovery efficiency decreases up to a minimum asymptotic value. It can also be observed that when the polymer concentration decreases to the minimum value ($C_41 = 0.001 \text{ wt }\%$), the recovery efficiency does not depend on the salinity concentration. Additionally, the figure shows that regardless of the polymer concentration level, as salinity increases, $E_R$ converges to the value of the flow case with the minimum polymer content.

A chart with polymer volume injected and recovery efficiency is a complementary way to describe the salinity and polymer concentration effect on petroleum recovery. Figure 25 takes four cases included in Figure 24, and it can be identified that for the case of the highest polymer concentration ($C_41 = 0.05 \text{ wt }\%$) and the lowest salinity ($C_51 = 0.01 \text{ wt }\%$), the water front arrives later, while the polymer-rich front arrives earlier to the production well than the other cases. Additionally, from two cases with almost the same recovery efficiency at 1.5PV injected, it is observed that after breakthrough and before 1.5PV, the case with $C_41 = 0.02$ and $C_51 = 0.01 \text{ wt }\%$ has better recovery efficiency than the case with $C_41 = 0.05$ and $C_51 = 0.2 \text{ wt }\%$; thus, at a specific time, there could be a value of recovery efficiency that can be obtained with different combinations of salinity and polymer concentrations.

Analyzing the effects of the salinity and polymer concentration on the distribution of saturation, effective salinity concentration, and apparent viscosity, Figure 26 takes two
cases ($C_{41} = 0.2$ and $C_{51} = 0.4$ wt %) included in the curve for $C_{41} = 0.05$ wt % from Figure 24 and illustrates a snapshot for 0.5 pore volume injected time. From Figure 26, it is shown that as salinity increases, the apparent viscosity of the polymeric solution deceases. Additionally, taking into account that adsorption increases with salinity (see Figure 2) and is a retardation term in the flow equation, it is identified that the combined effect (polymer viscosity and adsorption) tends to anticipate the water shock front and delay the polymer-rich shock front, so the benefits of the polymer injection arrive later. Depending on the salinity degree, this could affect the economics of the recovery process.

Figure 26 also allows us to examine the relationship between the salinity concentration and effective salinity. Considering any case from the figure and keeping in mind that the salinity concentration is constant along the porous media and during the flooding, the effective salinity concentration responds to the polymer solution shock front location due to polymer concentration increments from zero to the injection concentration. The magnitude of the abrupt change in the effective salinity depends on the degree of the salinity concentration.

3.3.3. Effect of Polymer Adsorption. As we mentioned before, the flow equation that describes the displacement of petroleum by a polymer solution considers a term for the representation of chemical adsorption, and as we indicated previously, this could be modeled using a Langmuir-type isotherm.

To investigate the influence of the adsorption phenomenon on fluid distribution and recovery efficiency, the second case of Figure 2 is chosen, where salinity of water formation is $C_{41} = 0.1$ wt %. This case is considered as 100% of the adsorption level (reference case) in Figures 27 and 28, so by reducing the percentage of the adsorption level of the flow case at $t_{D} = 0.5$ and $C_{41} = 0.1$ wt %, the polymer adsorption effects on fluid distribution and petroleum recovery are studied.

Figure 27 shows the aqueous phase and polymer concentration adsorbed profiles for the three cases, and it can be observed that as the adsorption level decreases, the water and polymer-rich fronts get closer; the water front is delayed, and the polymer front goes faster. In addition, with the less adsorption level, the water saturation front reduces, and the polymeric solution front tends to be less sharp and spreads out toward the water front. This results in differences for recovery efficiency
behavior; the flow case with the less adsorption level reaches faster the maximum recovery efficiency value, even though the three cases get to the maximum recovery at different times, as Figure 28 illustrates. Figures 27 and 28 expose the adsorption phenomena as a retardation term that defines the ratio between front velocities and recovery times.12,13,62

3.3.4. Effect of Permeability Reduction. Polymer flooding causes permeability reduction due to the adsorption phenomenon. This has an important effect on mobility control for the increment of recovery efficiency. Therefore, in this section, we research the effect of permeability reduction on fluid distribution and petroleum recovery.

As a measure of mobility control for the displacement, the mobility ratio concept is used. For fractional flow theory with Corey-type equations, one of the easiest expressions used is the end-point water—oil mobility ratio, \( M^* \).14,63,64

For waterflooding, one displacement front is obtained because viscosities and relative permeabilities are not affected during the process, so \( M^* \) is constant in time and distance. Nevertheless, during polymer flooding, a water front and a polymer-rich front are presented because aqueous phase viscosity is a function of time and distance. Additionally, the end-point water—oil mobility ratio should consider the effect of permeability reduction as eq 16 indicates.

\[ M^* \] needs \( \mu_{\text{app}} \) and \( R_k \), so a value for the water front and another for the polymer-rich front could be estimated. Thus, eq 16, as a profile, is used to analyze the behavior of mobility control. Therefore, Figure 29 presents four flow cases that compare the benefits of the growing polymer concentration from \( C_{41} = 0.0 \text{ wt }\% \) to \( C_{41} = 0.1 \text{ wt }\% \) and the benefits of the increasing permeability reduction factor from \( R_k = 1 \) to \( R_k = 3.36 \).

The waterflooding case is for reference, and it can be observed that the four cases have the same \( M^* \) value in the water displacement zone but different values after the polymer-rich displacement front.

First, the cases with the unit permeability reduction factor are considered, and it is identified that as the polymer concentration increases, the displacement is more favorable, up to the point of \( M^* = 0.07 \) for the largest \( C_{41} \). This value of \( M^* \) can also be achieved when we keep \( C_{41} = 0.05 \text{ wt }\% \) and increment permeability retention to \( R_k = 3.36 \). Then, with a different combination of \( C_{41} \) and \( R_k \), the same degree of \( M^* \) is obtained but at different polymer-rich front locations, as Figure 29 depicts.

Figure 30 contributes to complement the flow cases in Figure 29, and it shows the fluid distribution and aqueous apparent viscosity behavior. It is identified that for a \( R_k = 1 \), as the polymer concentration grows, the water front velocity and water saturation front decrease, while the polymer front velocity and polymer-rich saturation front increase; this makes the flow cases more favorable. Considering the two cases with the same \( M^* = 0.07 \), for the one with \( R_k = 3.36 \), its aqueous phase saturation behind the polymer-rich front tends to the same trend as the case with \( R_k = 1 \); also, as the permeability reduction factor increases, the water velocity and water saturation at the front increase, while the polymer-rich front velocity decreases.

From Figure 31, the recovery efficiency behavior for the flow cases included in Figures 29 and 30 is shown, and it can be seen that for cases with \( R_k = 1 \), after water breakthrough and as the

![Figure 29](https://example.com/figure29)

**Figure 29.** Profiles for the end-point water—oil mobility ratio for different polymer concentrations (\( C_{41} \)) and permeability reduction factors (\( R_k \)).

![Figure 30](https://example.com/figure30)

**Figure 30.** Saturation profiles and apparent viscosity distributions for different polymer concentrations (\( C_{41} \)) and permeability reduction factors (\( R_k \)).

![Figure 31](https://example.com/figure31)

**Figure 31.** Recovery efficiency vs pore volume injected for different polymer concentrations (\( C_{41} \)) and permeability reduction factors (\( R_k \)).
polymer concentration increases, the efficiency recovery increases. Comparing the two cases where $M^* = 0.07$ (as Figure 29 illustrates), both eventually reach the same maximum recovery efficiency ($E_R = 0.86$), with the difference that the case of the unit permeability reduction factor gets the maximum recovery efficiency before the cases for $R_k = 3.36$, so the former presents more favorable conditions for polymer flooding.

3.3.5. Effect of Slug Size. The flow cases discussed are related to a continuous injection scheme at a fixed polymer concentration. The chemical injection strategy has an important role in the success of a CEOR project. In this section, two types of scenarios are considered: (1) a fixed chemical slug size with varying polymer concentrations and (2) a fixed polymer concentration with changing slug size.

The numerical tool built allows us to track flow properties, as the example in Figure 32 presents a snapshot of 0.5PV injected with a polymer slug size of 0.25PV and chemical concentrations for the polymer and salinity of $C_{41} = 0.1$ wt % and $C_{51} = 0.1$ wt %, respectively. After slug injection, chase water follows with the same water salinity. This figure illustrates the aqueous saturation, polymer, effective salinity, and adsorption concentration profiles and viscosity at the zero shear rate and apparent viscosity of polymeric solution.

Figure 32 illustrates that the aqueous saturation profile shows two shock fronts, and between them, all properties are practically constant. At the rich polymer front, chemical concentrations abruptly increase from zero to the near-injection concentration, and at the end of the chemical slug, the polymer and effective salinity content gradually decreases to initial concentrations. Polymer viscosities (at the zero shear rate and apparent viscosities) and adsorption behave similarly to the chemical concentration.

The evolution of the saturation profiles and polymer concentration for a slug injected size of 0.25PV with chemical concentrations of $C_{41} = 0.1$ wt % and $C_{51} = 0.1$ wt %, respectively. After slug injection, chase water follows with the same water salinity. This figure illustrates the aqueous saturation, polymer, effective salinity, and adsorption concentration profiles and viscosity at the zero shear rate and apparent viscosity of polymeric solution.

Figure 33 illustrates that the aqueous saturation profile shows two shock fronts, and between them, all properties are practically constant. At the rich polymer front, chemical concentrations abruptly increase from zero to the near-injection concentration, and at the end of the chemical slug, the polymer and effective salinity content gradually decreases to initial concentrations. Polymer viscosities (at the zero shear rate and apparent viscosities) and adsorption behave similarly to the chemical concentration.

The evolution of the saturation profiles and polymer concentration for a slug injected size of 0.25PV with chemical concentrations of $C_{41} = 0.1$ wt % and $C_{51} = 0.1$ wt % at three times is illustrated in Figure 33. $t_0 = 0.25$ is the time when the polymer slug injection finishes, $t_0 = 0.63$ is the water breakthrough time, and $t_0 = 0.44$ is the intermediate time between the two previously mentioned. From this figure, it is observed that as time goes by, the separation between the water front and the polymer-rich front increases because the water front moves faster than the other front, and the peak level and width around the peak of the polymer concentration curve (bell-shaped curve) decrease. In addition, it can be deduced that due to the level of the polymer concentration applied to this case, the effect of polymer dilution does not have a relevant role in changing the polymer shock front and in reducing the benefits of polymer injection. Therefore, this indicates that the content of the polymer to be used could be optimized by considering an appropriate combination of the polymer concentration and polymer slug size to be injected.

To analyze the consequences of reducing the polymer content in the displacement process of Figure 33, the polymer concentration is reduced about 70% (from 0.1 to 0.032 wt %), and the results are presented in Figure 34. Comparing both cases, it can be seen that in the case with the less polymer concentration, water breakthrough appears earlier with a higher water saturation, and the polymer-rich front moves slower with a lower water saturation at the front. As a result, the case in Figure 34 takes less benefits from the polymer flooding than the case in Figure 33.

Changing polymer slug size could be another way to adjust the amount of the polymer needed for a flooding. As seen from Figures 33 and 35, the slug size is reduced from 0.25PV to 0.08PV, and it is observed that the latter case takes less
advantage of the polymer flooding benefits than the case in Figure 33, although water breakthrough occurs almost at the same time. This displacement with polymer injection during 0.08PV lets us appreciate a condition where the polymer shock front and its benefits are considerably affected by polymer dilution to the extent that even though the maximum of the polymer concentration at the end of chemical injection is 0.1 wt %, as in Figure 33, it is reduced about 70% at water breakthrough. Besides, it is noticed that in the less-efficient process, water saturation does not remain constant in the region between water front and polymer-rich front saturations, thus reducing the performance of the polymer displacement.

Figure 36 complements the analysis of the flow cases from Figures 33 to 35. Then, it can be seen that the case with a higher polymer concentration and longest polymer injection time is the case with the better benefits of polymer flooding because it has the longest water breakthrough time and the highest recovery efficiency. For the other two cases, considering that both contain the same mass of the polymer inside the reservoir, the case in Figure 35 yields a better scenario than the case in Figure 34 because the former case reaches higher cumulative petroleum production and the longest water breakthrough time. It can also be identified that as time increases, the case in Figure 34 tends to achieve the recovery efficiency of the case in Figure 35.

3.3.6. Effect of Reversible and Irreversible Adsorption. So far, the polymer flooding cases have assumed reversible adsorption, in terms of the polymer concentration. However, depending on rock-fluid chemical composition, some cases need to consider adsorption as an irreversible process. Therefore, the effect of the adsorption irreversibility is explored, by comparing the reversible adsorption case, explained in Figure 35, and the flow case with the same conditions of such figures but including irreversible adsorption.

Applying the simulation tool developed, Figure 37 represents the adsorption behavior during the polymer flooding case of Figure 35, and we can observe that as time goes by, the area under the curve of polymer adsorption tends to increase according to Langmuir eq 17, and by considering the mass of the polymer adsorbed onto formation rock, the polymer slug is degraded.

For an irreversible adsorption process, the Langmuir model cannot be used directly when the polymer concentration is declining. An additional parameter must be used to track the adsorption history, so the maximum polymer adsorption concentration without exceeding the adsorption capacity is selected. Taking into account this artifact for the numerical simulation tool, Figure 38 shows the theoretical approximation of polymer flooding performance in Figure 37 but including irreversible adsorption. Also, the evolution of polymer adsorption is observed, such that the polymer concentration and polymer adsorption concentration near the polymer front decrease to almost zero at water breakthrough time. Besides, the polymer adsorbed onto the rock is larger than the case in Figure 37; in fact, it is so large that the polymer slug is almost completely consumed when water reaches the production well, and so at this time, almost all polymer mass injected is adsorbed onto the rock surface.

Figure 39 presents the recovery behavior comparison of the cases for reversible and irreversible adsorption, and we identify that although both cases yield almost the same water breakthrough time, the case with a reversible process gets more benefits by polymer flooding than the one with an irreversible process. This is because consumption of the polymer...
is larger in the case of irreversible adsorption than in the reversible one, as Figures 37 and 38 display.

**4. CONCLUSIONS**

A practical mathematical model for multiphase compositional polymer flooding was presented. Then, a numerical technique was applied to solve it, and a validation of the numerical algorithm was carried out. Finally, the effects of several phenomena and parameters on polymer flooding were studied. The following conclusions can be drawn from the investigation:

1. Based on fractional flow theory, a practical reservoir simulator to evaluate the viability of polymer flooding is developed. It considers the non-Newtonian rheology (shear thinning and shear thickening), permeability reduction, reversible—irreversible adsorption, and salinity effect. The validation of the numerical algorithm was carried out; numerical solutions for water flooding and polymer flooding here developed are in close agreement with analytical solutions, coreflood laboratory experiments, and a CMG-STARS numerical model. Therefore, despite the assumptions or simplifications of the model, it provides a good representation to laboratory data, without losing reliability.

2. This investigation presented an analysis procedure that considers fractional flow theory concepts and the most important phenomena related to polymer flooding. The procedure could be applied prior to building a more elaborate predictive model to get a quick idea of a favorable scenario for oil recovery with less computing effort and time, thus identifying key phenomena worth considering without losing the deserved representativeness.

3. Based on the effect of the shear rate and polymer content on polymer flooding behavior, it can be concluded that as polymer concentrations decrease and the injection flow rate increases, the $E_R$ decreases to a minimum asymptotic value. Now, for a fixed $C_{41}$ and as the injection rate lessens, water front saturation and velocity tend to decrease, while the polymer front saturation increases; also, the differences between the viscosity at the zero shear rate and apparent viscosity profiles are reduced. In this way, fluids are distributed more favorably with a longer breakthrough time.

4. During polymer flooding, to diminish the polymer adsorption level tends to get closer to the water front and polymer-rich front; the water front is delayed, and the polymer front goes faster. The water saturation front reduces, and the polymeric solution front tends to be less sharp and spreads out toward the water front. Therefore, a flow case with a less adsorption level reaches faster to the maximum recovery value than a flow case with more adsorption degrees, even though both eventually get to the maximum recovery efficiency at different times.

5. Looking into water salinity concentration variations, it was established that bigger salinity effects tend to decrease benefits of the polymer concentration on polymeric solution viscosity and increase the retardation effect due to polymer adsorption. The combination of rheology and adsorption phenomena contributes to anticipate water breakthrough and delay the polymer-rich shock front so that the benefits of the polymer injection arrive later, and in consequence, the economics of the recovery process could be affected depending on the salinity degree.

6. At a specific time after water breakthrough, because of rheology and adsorption effects, there could be a value of recovery efficiency that can be obtained with different combinations of salinity and polymer concentrations. In general, for a fixed $C_{41}$ and as $C_{51}$ increases, the recovery efficiency decreases to a minimum asymptotic value regardless of salinity increments.

7. End-point mobility ratio ($M^*$) is controlled through the product of the apparent viscosity and permeability reduction factor. From the flow cases presented with the same $M^*$, it was identified that increasing $\mu_{app}$ has better recovery efficiency benefits than increasing $R_k$.

8. The amount of the polymer needed for the slug injected should be reduced as much as possible. This depends on the polymer concentration and slug size, so it was presented that different combinations of these two parameters can yield the same mass of the polymer injected, but the better recovery was obtained with the highest polymer concentration and the smallest slug size. This should be considered as much as the technical and economic restrictions allow it.

**Figure 38.** Behavior of the water saturation and polymer concentration profiles for a slug size of 0.08PV, $C_{41} = 0.1$ wt % and irreversible adsorption.

**Figure 39.** Recovery efficiency vs pore volume injected for reversible and irreversible adsorption.
9. As excepted, reversible or irreversible polymer adsorption has different effects on the degree of polymer consumption; irreversible adsorption has a larger detrimental effect than the reversible one, and therefore, maximum recovery efficiency is less. Then, defining the reversible or irreversible adsorption degree is another key element to consider for polymer flooding predictions.

10. In general, from a technical point of view, conditions under which the polymer concentration increases, injection rate decreases, polymer adsorption reduces, and water salinity lessens provide a more favorable displacement process. For practical applications, it is also mandatory to consider economical aspects to optimize the quantity of the polymer and salinity for a polymer flooding project.

11. The numerical simulator for polymer flooding developed in this research has been applied as a practical tool which allowed us to capture key aspects for representing physical—chemical phenomena and petroleum recovery behavior with a less amount of detailed information, effort, and time than commercial or academic field reservoir simulators for CEOR applications. Therefore, as an EOR project management practice, this predictive tool could work in screening studies to identify problems and avoid unnecessary engineering work with low chances to get profitability.

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■ NOMENCLATURE

Normal text

- $A$, cross-sectional area, $L^2$
- $a_{p1}$, $a_{p2}$, $a_{p3}$, $S_{np}$, polymer viscosity at zero shear rate model parameters
- $a$, $b$, adsorption model parameters
- $b_{sr}$, $c_{sr}$, permeability reduction model parameters
- $C_p$, overall concentration of species $i$ in the mobile phases
- $C_{op}$, overall concentration of species $i$ in the mobile and stationary phases
- $C_{ij}$, rock-phase concentration of species $i$
- $C_{ijp}$, volume fraction of component $i$ in phase $j$
- $C_{SEP}$, effective salinity for a polymer
- $C^a$, $C_M$ empirical constants in the polymer viscosity model
- $E_{fr}$, recovery efficiency
- $e_p$, relative permeability exponent of phase $j$
- $F_{i}$, fractional flux of component $i$
- $f_{ij}$, fractional flow of phase $j$
- $f_{ijp}$, mass flux of species $i$, $M L^{-2} T^{-1}$
- $g_{ij}$, dispersion tensor for species $i$ in phase $j$, $L^2 T^{-1}$
- $k_r$, relative permeability of phase $j$
- $k_{r1}^*$, end-point relative permeability of phase $j$
- $L$, length of the porous medium, $L$
- $M^e$, end-point water-oil mobility ratio
- $N_{c1}$, total number of components
- $N_j$, total number of phases
- $n_{ex}$, shear rate dependence of polymer viscosity model parameter
- $Re_{ij}$, residual function of component $i$ and node $j$ for the NR method
- $R_p$, mass rate of production of species $i$, $M L^{-3} T^{-1}$
- $R_F$, resistance factor
- $R_{per}$, permeability reduction factor
- $R_{max}$, maximum permeability reduction factor
- $R_{res}$, residual resistance factor
- $r_{ij}$, homogeneous reaction rate of species $i$ in phase $j$, $M L^{-3} T^{-1}$
- $S_{ij}$, saturation of phase $j$
- $S_{ijp}$, residual saturation of phase $j$
- $S_{np}$, normalized saturation of phase $j$
- $t$, time, $T$
- $t_d$, dimensionless time
- $t_r$, relaxation time of the fluid
- $Q$, fluid injection rate, $L^3 T^{-1}$
- $u$, superficial velocity of phase $j$, $L T^{-1}$
- $u_s$, magnitude of superficial velocity, $L T^{-1}$
- $W_{ij}$, overall concentration of species $i$, $M L^{-3}$
- $x$, position, $L$
- $x_{dr}$, dimensionless position

Greek symbols

- $\alpha$, $\lambda$, shear rate dependence of polymer viscosity model parameters
- $\alpha_1$, $\alpha_2$, adsorption model parameters
- $\epsilon_A$, approximation error
- $\gamma_{ij}$, equivalent shear rate model parameter, $T^{-1}$
- $\gamma_{ij}^{eq}$, equivalent shear rate, $T^{-1}$
- $r_{ij}^{rel}$, relative mobility of phase $j$, $L M^{-1} T^{-1}$
- $\Pi_1$, polymer solution viscosity at the zero shear rate, $M L^{-1} T^{-1}$
- $\Pi_{app}$, apparent polymer solution viscosity, $M L^{-1} T^{-1}$
- $\Pi_{thick}$, shear thickening viscosity or elongational viscosity, $M L^{-1} T^{-1}$
- $\mu$, viscosity of phase $j$, $M L^{-1} T^{-1}$
- $\mu_{thick}$, shear thinning viscosity, $M L^{-1} T^{-1}$
- $\mu_{h}$, viscosity at a high shear limit (generally taken as water viscosity), $M L^{-1} T^{-1}$
- $\xi$, unknown variable for the generic numerical expression
- $\rho$, density of phase $j$, $M L^{-3}$
\( \phi \), porosity, fraction
\( \omega_{ij} \), mass fraction of species \( i \) in phase \( j \)

**Superscripts**

- \( n \), time level of the numerical simulation
- \( \iota \), iteration level for a time step

**Subscripts**

- \( i \), species index (1 = water, 2 = oil, 4 = polymer, 5 = salt)
- \( j \), phase index (1 = aqueous, 2 = oleic)
- \( m \), unknown variable index for the generic numerical expression
- \( \text{ref} \), reference parameter for permeability in the adsorption model

## REFERENCES

(1) Donaldson, E. C.; Yen, T. F.; Chilingarian, G. V. *Enhanced Oil Recovery - 1 Fundamentals and Analyses*; Elsevier: New York, 1985; pp 1–10.

(2) Chierici, G. L. *Principles of Petroleum Reservoir Engineering*, 1st ed.; Springer: New York, 1995; Vol. 2, pp 255–375.

(3) Green, D. W.; Willhite, P. G. *Enhanced Oil Recovery*; SPE Textbook Series, Vol. 6; Henry L. Doherty Memorial Fund of AIME, Society of Petroleum Engineers, 1998; pp 1–11.

(4) Thomas, S. *Enhanced Oil Recovery – An Overview*. *Oil Gas Sci. Technol.* **2008**, *63*, 9–19.

(5) Sheng, J. J. *A Comprehensive review of Alkaline-Surfactant-Polymer (ASP) flooding*. *Asia-Pac. J. Chem. Eng.* **2014**, *9*, 471–489.

(6) Taber, J. J.; Martin, F. D.; Seright, R. S. EOR Screening Criteria Revisited- Part 1: Introduction to Screening Criteria and Enhanced Recovery Field Projects. *SPE Reservoir Eng.* **1997**, *12*, 189–198.

(7) Caroçano, A. *Applied Enhanced Oil Recovery*. Prentice-Hall, 1992, pp 135–158.

(8) Dickson, J. L.; Leahy-Dios, A.; Wylie, P. L. Development of Improved Hydrocarbon Recovery Screening Methodologies. Presented at the SPE Improved Oil Recovery Symposium, 2010.

(9) Wassmuth, F. R.; Green, K.; Arnold, W.; Cameron, N. Polymer Flood Application to Improve Heavy Oil Recovery at East Bodo. *J. Can. Pet. Technol.* **2009**, *48*, 55–61.

(10) Delaplace, P.; Renard, G.; Delamaide, E.; Euzen, T.; Roggero, F.; Kopcency, P. Reservoir Simulation of a Polymer Flood Pilot in the Pelican Lake Heavy Oil Field (Canada): A Step Forward. Presented at the SPE Reservoir Characterization and Simulation Conference and Exhibition, 2013.

(11) Delamaide, E.; Zaitoun, A.; Renard, G.; Tabary, R. Pelican Lake Field: First Successful Application of Polymer Flooding In a Heavy-Oil Reservoir. *SPE Reservoir Eval. Eng.* **2014**, *17*, 340–345.

(12) Sorbie, K. S. *Polymer-Improved Oil Recovery*. Springer: Dordrecht, The Netherlands, 1991, pp 126–164.

(13) Sheng, J. J. *Modern Chemical Enhanced Oil Recovery: Theory and Practice*, 1st ed.; Gulf Professional Publishing, 2011; pp 101-206.

(14) Lake, L. W. *Enhanced Oil Recovery*, 1st ed; Prentice-Hall, 1989; pp 17–344.

(15) Standnes, D. C.; Skjervak, I. *Literature Review of Implemented Polymer Field Projects*. *SPE J.* **2014**, *122*, 761–775.

(16) He, J.; Song, Z.; Qiu, L.; Xie, F.; Tan, Z.; Yue, Q.; Li, X.; Li, X. N. High Temperature Polymer Flooding in Thick Reservoir in ShuangHe Oilfield. Presented at the International Oil and Gas Conference and Exhibition in China, 1998.

(17) Olajire, A. A. *Review of ASP EOR (alkaline surfactant polymer enhanced oil recovery) Technology in the Petroleum Industry - Prospects and Challenges*. *Energy* **2014**, *77*, 963–982.

(18) Kamal, M. S.; Adevunmi, A. A.; Sultan, A. S.; Al-Hamad, M. F.; Mehmood, U. Recent Advances in Nanoparticles Enhanced Oil Recovery - Rheology, Interfacial Tension, Oil Recovery, and Wettability Alteration. *J. Nanomater.* **2017**, *2017*, 2473175.
Polymer Solution. SPE Reservoir Eng. 1992, 7, 247–252. Society of Petroleum Engineers SPE-19499.

(43) Delshad, M.; Pope, G. A.; Sepehrnoori, K. A Compositional Simulator for Modeling Surfactant Enhanced Aquifer Remediation, 1 Formulation. J. Contam. Hydrol. 1996, 23, 303–327.

(44) Zheng, C. G.; Gall, B. L.; Gao, H. W.; Miller, A. E.; Bryant, R. S. Effects of Polymer Adsorption and Flow Behavior on Two-Phase Flow in Porous Media. SPE Reservoir Eval. Eng. 2000, 3, 216–223.

(45) Delshad, M.; Kim, D. H.; Magbagbeola, O. A.; Huh, C.; Pope, G. A.; Tarahhom, F. Mechanistic Interpretation and Utilization of Viscoelastic Behavior of Polymer Solutions for Improved Polymer-Flood Efficiency. Mechanistic Interpretation and Utilization of Viscoelastic Behavior of Polymer Solution for Improved Polymer-Flooding Efficiency; Society of Petroleum Engineers SPE-113620, 2008.

(46) Azad, M. S.; Trivedi, J. J. Quantification of the Viscoelastic Effects During Polymer Flooding: A Critical Review. SPE J. 2019, 24, 2731–2757.

(47) Manzoor, A. A. Modeling and Simulation of Polymer Flooding with Time-Varying Injection Pressure. ACS Omega 2020, 5, 5258–5269.

(48) Hite, J. R.; Bondor, P. L. Planning EOR Projects; Society of Petroleum Engineers SPE-92006, 2004. DOI: 10.2118/92006-MS.

(49) Rai, K.; Johns, R. T.; Lake, L. W.; Delshad, M.; Goudarzi, A. Oil-Recovery Predictions for Surfactant Polymer Flooding. Oil-Recovery Predictions for Surfactant Polymer Flooding; Society of Petroleum Engineers SPE-124001-MS, 2009.

(50) Koh, H.; Lee, V. B.; Pope, G. A. Experimental Investigation of the Effect of Polymers on Residual Oil Saturation. SPE J. 2018, 23, 1–17.

(51) Druetta, P.; Picchioni, F. Surfactant-Polymer Flooding: Influence of the Injection Scheme. Energy Fuels 2018, 32, 12231–12246.

(52) Reservoir Engineering Research Program; Center for Petroleum and Geosystems Engineering. Volume II: Technical Documentation for UTCHEM-9.0. A Three Dimensional Chemical Flood Simulator: The University of Texas at Austin: Austin, TX, 2000; pp 2.1–2.31.

(53) Zhang, G.; Seright, R. S. S. Effect of Concentration on HPAM Retention in Porous Media. SPE J. 2014, 19, 373–380.

(54) Morrison, F. A. Understanding Rheology; Oxford University Press: New York, 2001, pp 225–256.

(55) Wang, J.; Liu, H. A Novel Model and Sensitivity Analysis for Viscoelastic Polymer Flooding in Offshore Oilfield. J. Ind. Eng. Chem. 2014, 20, 656–667.

(56) Sharafi, M. S.; Jamialahmadi, M. Mathematical model for prediction of oil recovery of core-flood tests in process of viscoelastic polymer flooding. Pet. Sci. Technol. 2016, 34, 1098–1105.

(57) Ertekin, T.; Abou-Kassem, J. H.; King, G. R. Basic Applied Reservoir Simulation: Textbook series; Society of Petroleum Engineers, 2001; Vol. 7, pp 256–258.

(58) Honarpour, M.; Koederitz, L.; Harvey, A. H. Relative Permeability of Petroleum Reservoirs; CRC Press: Boca Raton, FL, 1986, pp 45–97.

(59) Martin, F. D.; Hatch, M. J.; Shepitka, J. S.; Ward, J. S. Improved Water-Soluble Polymers for Enhanced Recovery of Oil. SPE Oilfield and Geothermal Chemistry Symposium; Society of Petroleum Engineers SPE 11786, 1983.

(60) Rosado-Vazquez, F. J.; Rangel-German, E.; Rodriguez-De La Garza, F. Analysis of Capillary, Gravity and Viscous Forces Effects in Oil/Water Displacement; Society of Petroleum Engineers SPE-108722, 2007.

(61) Herrera, I.; Pinder, G. F. Mathematical Modeling in Science and Engineering: An Axiomatic Approach, 1st ed.; John Wiley & Sons, Inc., 2012; pp 63–83.

(62) Dake, L. P. Developments in Petroleum Science, 8. Fundamentals of Reservoir Engineering, 1st ed.; Elsevier: Amsterdam, 1978; pp 100–125.

(63) Dake, L. P. Developments in Petroleum Science, 8. Fundamentals of Reservoir Engineering, 1st ed.; Elsevier: Amsterdam, 1978; pp 100–125.

(64) Rosado-Vazquez, F. J.; Rangel-German, E. R.; Rodriguez-De La Garza, F. Analytical Model for Vertical Oil-Water Displacement Under Combined Viscous, Capillary and Gravity Effects; Society of Petroleum Engineers SPE-100659, 2006. DOI: 10.2118/100659-MS.