Fusion performance of spherical and conventional tokamaks: implications for compact pilot plants and reactors

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Abstract

Spherical tokamaks (STs) have features that make them a potentially attractive option for fusion power production compared to conventional tokamaks (CTs) including operation at high beta and high self-driven ‘bootstrap’ current. The thermal energy confinement time ($\tau_E$) also typically has a stronger dependence on toroidal magnetic field and a weaker dependence on plasma current, but so far it has not been established how this difference impacts performance under reactor conditions. This aspect is explored in this paper. Using empirical data from NSTX and MAST, and from multiple CTs, we investigate analytically and by using established fusion codes the potential fusion performance, characterised by the fusion triple product, $nT\tau_E$, and fusion power gain, $Q_{\text{fus}}$, where $n$ and $T$ are the density and temperature respectively. We find that for similar values of field and fusion power, but smaller volume, STs can have $nT\tau_E$ up to a factor of three higher and $Q_{\text{fus}}$ an order of magnitude higher than CTs. We identify the origin of this enhanced performance and outline a measurement to advance this finding. Potentially our results open an alternative and faster route to fusion power based on relatively small, low power STs.

Supplementary material for this article is available online

Keywords: magnetically confined plasmas, spherical tokamaks, fusion pilot plants and reactors

(Some figures may appear in colour only in the online journal)

1. Introduction

In magnetic confinement fusion, the highest performance has been obtained with tokamaks and thus far research has been concentrated on devices where the ratio of the plasma major radius, $R$, to plasma minor radius, $a$, known as the aspect ratio, $A$, is high, typically $\geq 3$. Notable examples are the JET device at the Culham Centre for Fusion Energy (CCFE), UK, and ITER which is currently under construction in Cadarache, France. Recently there has been increasing interest in the spherical tokamak (ST) where $A < 2$. STs have been found to have several advantages relative to conventional tokamaks (CTs): in particular, operation at relatively high values of the ratio of the plasma pressure to the confining magnetic field pressure, $\beta$, as well as operating in regimes where a significant fraction of the plasma current is produced by the self-driven ‘bootstrap’ current, therefore minimising the need for external current drive which is inefficient. When considering future fusion power plants these characteristics are important economic factors. Experiments on large STs, NSTX at the Princeton Plasma Physics Laboratory and MAST at CCFE,
have shown that in STs the scaling of the thermal energy confinement time \(\tau_E\) with device engineering parameters generally exhibits a stronger dependence on toroidal magnetic field, \(B\), and a weaker dependence on plasma current, \(I_p\), than that observed at large aspect ratio \([1–3]\). Also, direct measurements on MAST have shown that the dependence of \(\tau_E\) on the safety factor, \(q\), is much weaker in the case of the ST \([4]\). However, it has not yet been established how these differences impact performance under reactor conditions. This question is examined in this work and a significant advantage is found.

Using an established physics model and empirical confinement time scalings, we investigate the dependence of the fusion triple product, \(nT_{\text{E}}\), and the fusion power gain, \(Q_{\text{fus}}\), on device parameters for CTs and STs. For CTs, we use the established IPB98y2 scaling \([5]\) and for STs we use the device specific NSTX scaling \([1]\), which has been recently extended using dimensional arguments to include a size dependence \([6]\). Our investigation is carried out by analytical means using a simplified physics model and by calculations with a system code in which more accurate expressions for the key plasma parameters are used and some important phenomena, such as plasma radiation and the bootstrap current are included. These calculations are further validated using an established tokamak transport code. Our results show that STs can have \(nT_{\text{E}}\) up to a factor three higher, and \(Q_{\text{fus}}\) up to an order of magnitude higher, than CTs of similar magnetic field and fusion power but the ST plasma has a much smaller volume. Potentially, these results have major implications for the design of fusion pilot plants and reactors.

The paper has eight main sections. In section 2 we outline the physics model used and carry out an investigation by analytical means of the fusion performance of generic STs and CTs as characterised by \(nT_{\text{E}}\). Using a system code in section 3 we test a specific finding so as to check that the simplifications made in the analytical investigation are not substantially influencing the qualitative findings. In section 4 we generalise the analysis and thereby widen the basis of our results. In section 5 we examine the impact of our findings by comparing the fusion performance of three devices—a large size CT, a high field CT and a compact ST—and derive the values of the main device and operational parameters of a candidate relatively small ST fusion module. We use a transport code (ASTRA) in section 6 to derive the implied local values of the ion and electron thermal transport coefficients, and we compare these with expectations based on measurements on current STs. In section 7 we transform the generalised analysis into physics variables to reveal the key dependences and identify a specific measurement that would advance our findings. In section 8 we discuss the results and in section 9 we give a summary. Our paper is accompanied by online supplementary material (available online at: \(\text{https://stacks.iop.org/PPCF/63/035005/mmedia}\)) and therein we give the derivation steps of all equations along with additional explanatory figures and tables.

2. Physics model and initial analysis

Our employed model of the tokamak plasma has been documented previously \([7]\). According to the model, the plasma operates in steady-state where the fusion (alpha) power and the externally supplied power balance the power lost \((P_L)\) by thermal diffusion from the plasma core. Key parameters are \(I_p, B, R, \alpha, \beta, \kappa\), plasma vertical dimension, \(b\), plasma elongation, \(\kappa = b/a, \beta = \kappa nT/\rho_0 a\), and the safety factor \(q = 5R B/\kappa A^2\). Key relationships are \(V \propto R \alpha^2 \propto R^2 b/\kappa A^2, P_{\text{fus}} \propto n^2 T^2 V \propto n^2 T^2 R \kappa^2 /A^2, P_L \propto W/\tau_E\) where \(W = nTV\) is the stored energy. In steady-state the plasma operates at a fixed fraction of the density limit, \(n_{\text{lim}} \propto I_p/\alpha^2\). Experimentally it has been found that to avoid disruptions \(q\) must be \(\geq 2\) and devices typically operate with \(q > 3\). For a deuterium-tritium plasma operating in steady state there is a simple relationship between \(nT_{\text{E}}\) and \(Q_{\text{fus}}\) and \(P_{\text{fus}}/P_{\text{in}}\) where \(P_{\text{in}}\) is the input power: \(Q_{\text{fus}} = 5cnT_{\text{E}}/\left(5 - cnT_{\text{E}}\right)\), where \(c\) is a constant \([8]\).

Empirical confinement times derived from parameter scans of multiple plasmas operating under the same conditions, for example ELM by mode, are typically of the form \((\tau_E)_{\text{scaling}} \propto f(I_p, B, R, n, \kappa, A, P_L)\). By a straightforward forward analysis that has been used previously \([8]\) we derive an expression for \(nT_{\text{E}}\). In brief, in the expression for \((\tau_E)_{\text{scaling}}\) we write \(P_L \propto W/\tau_E\) and solve for \(\tau_E\). We use the density limit to eliminate \(n\) and the safety factor expression to eliminate \(I_p\) and thus obtain \(nT_{\text{E}}\) in terms of the device dimensional parameters, \(R, A, \kappa, B\) and \(q\). We include \(H = \tau_E/(\tau_E)_{\text{scaling}}\) a simple multiplier that accounts for the fact that the confinement time of individual plasmas may differ from that predicted by empirical confinement time scalings.

Applying this analysis to CTs, which typically have \((\tau_E)_{\text{scaling}} \propto I_p R^2 n_{\text{ave}}^{1/2} q_{\text{asym}}^{3/4}/A^{1/2} P_L^{1/2}\), we find:

\[
(nT_{\text{E}})_{\text{CT}} \propto \frac{H_{\text{CT}}^2}{q^2} R^2 B^4 \left(\frac{\kappa^{7/2}}{A^3}\right)
\]

where the subscript ‘CT’ signifies that this applies to a CT plasma. Here we see the origins of the two main routes to fusion energy that have been pursued thus far: the large device approach pursued by ITER and the big DEMO reactors in which size (major radius and volume) are emphasised, and the high field approach notably with tokamaks at MIT and ENEA, Italy. Note the strong inverse dependence of \(nT_{\text{E}}\) on \(q\).

For STs, the device database is not sufficiently large to enable an empirical \(\tau_E\) scaling to be derived from multiple tokamaks; nevertheless, the dependences on \(I_p, B\) and \(n\) can be obtained from experiments on individual devices, and in principle the size dependence can be inferred by dimensional analysis. By this technique, a ST scaling based on NSTX data has been derived \([6]\): \((\tau_E)_{\text{scaling}} = 0.21 R_p^{1.5} A^{1.9} n_{\text{ave}}^{-0.05} P_L^{-0.38}\) which we refer to as ‘NSTX-ext’. Here \(n_{\text{ave}}\) is the line-averaged electron density. The \(I_p\) and \(B\) dependences observed on MAST are similar \((I_p^{0.53} B^{1.6})\) \([3]\). For the purpose of this analysis, we simplify to \((\tau_E)_{\text{scaling}} \propto I_p^{1/2} B^2 P_L^{-2.5}\) and, applying the same analysis as above yields \((nT_{\text{E}})_{\text{ST}} \propto \frac{H_{\text{ST}}^2}{q^2} n_{\text{ave}}^{17/6} R^{11/6} \kappa^{1/2} T^{1/3} q^{7/6} A^{1/3}\). In reactor studies, it is usual to assume operation in a limited temperature range around the maximum of the fusion
reactivity ($T \sim 14–20$ keV), and thus, assuming constant temperature, we find approximately:

$$nT_M \propto \frac{P_{\text{INS}}^{5/3}}{q} B_1^R A^{1/3}$$

(2)

which is almost identical to (1) except that the inverse $q$ dependence is now weaker by a factor $\sim q^3$. Given that $q$ is usually 2–3 this is a significant difference potentially approaching an order of magnitude and is in the direction to give higher $nT_M$. We note the shape factor, $\kappa^{1/2}/A^{1/3}$, is relatively weak compared to that for the CT. However, it is probably not complete because there are no dependences on $\kappa$ and $A$ in $(\tau_M)$ scaling for the ST. These could not be determined empirically because the scaling is derived from experiments on one device with a limited range of $\kappa$ and $A$. By adding dependences of the form of $\kappa^{\alpha\kappa} A^{\alpha A}$, the expression for $nT_M$ with these maintained can be derived and is given in the supplementary material (section 2.3).

For a tokamak plasma, $P_{\text{INS}} \propto \beta^2 B^2 V \propto \beta^2 B^4 R^3 \kappa^3 / q^2 A^4$. Using this, we eliminate $B$ (or $R$) and reveal the relationship between $nT_M$ and $P_{\text{INS}}$. For the CT, we find:

$$nT_M \propto \frac{H_{\text{CT}} P_{\text{INS}}^{5/3} A^{5/4}}{\beta_N^{3/2} R^{1/4} q^{4/3}}$$

(3)

and for the ST:

$$nT_M \propto \frac{H_{\text{ST}} P_{\text{INS}}^{5/3} A^{5/4}}{\beta_N^{3/2} R^{1/4} q^{4/3}}$$

(4)

Thus, we see that for the CT, the $q$ dependence serves to decrease $nT_M$ for a given operating $P_{\text{INS}}$ whereas for an ST it serves to increase it, and the difference is again $\sim q^3$.

The relationship $P_{\text{INS}} \propto \beta^2 B^4 R^3 \kappa^3 / q^2 A^4$ can alternatively be used to eliminate $q$ to give:

$$nT_E \propto \frac{H_{\text{CT}} A^3}{\kappa \beta_N^2} P_{\text{INS}}^{3/2} B^3 R^{5/2}$$

(5)

and

$$nT_E \propto \frac{H_{\text{ST}} A^3}{\kappa \beta_N^2} P_{\text{INS}}^{3/2} B^3 R^{5/2}$$

(6)

The full and more accurate expressions are given in the supplementary material (equations (S6) and (S8)).

The concept of the $BR$ combination has been suggested as an effective size metric [9]. Adopting that, we see that for a CT at constant fusion power if the effective size is increased then $nT_M$ and hence $Q_{\text{INS}}$ are reduced. In order to maintain $nT_M$, $P_{\text{INS}}$ must also be increased and this could limit design options. On the other hand, for a ST if the effective size is increased then $P_{\text{INS}}$ can be reduced and that could make finding feasible engineering solutions easier, especially at small physical size. This is a significant qualitative difference.

3. Tests with the system code

In order to make our equations tractable, we have made simplifications in our analysis and some phenomena, for example, the bootstrap current and plasma radiation are not included. Potentially these could influence our qualitative findings. To check whether this is the case we have evaluated the $q$ dependences in the CT and ST expressions for $nT_M$ using a system code. We use the Tokamak Energy system code, which has been documented previously [7].

System codes use the full and precise expressions for the thermal energy scalings and so we first derive by our analytical method the expressions for $nT_M$ using the full scalings. For the CT we take the IPB98y2 scaling [5]:

$$nT_M \propto \frac{I_p^{0.93} B_T^{0.15} P_0^{0.97} \kappa^{0.41} A^{-0.58} P_L^{-0.69}}{q^{1.38}}$$

(7)

and for the ST we use equation (13) [6]:

$$nT_M \propto \frac{I_p^{0.54} B_T^{0.01} P_0^{2.14} \kappa^{0.05} A^{-0.38}}{q^{1.17}}$$

(8)

and find:

$$nT_M \propto \frac{H_{\text{CT}}^{1.22} B_T^{0.58} P_L^{-0.39}}{q^{1.18} A^{0.52}}$$

(9)

and

$$nT_M \propto \frac{H_{\text{ST}}^{1.61} B_T^{0.65} P_L^{0.39}}{q^{1.18} A^{0.52}}$$

(10)

These are the more complete and accurate versions of equations (1) and (2). We notice that here again the strong difference in the dependences on $q$ with the $q$ dependence being less punitive for the ST.

Ideally, we would carry out a scan in fusion performance, i.e. $nT_M$, for a given device, keeping all parameters on the right hand side of these equations, including plasma temperature, constant, only allowing $q$ to vary. However, for operation at a fixed fraction of the density limit as assumed in our analysis, this would be over constrained. In order to carry out the scan we have to allow variation in one other parameter. We use temperature and take that variation into account in order to determine the dependence on $q$.

We choose a device and fix $R, B, A, \kappa$ and $H$ factor. We specify required $Q_{\text{INS}}$ and adjust $P_{\text{INS}}$ and $T$ to find the parameters corresponding to operation at 0.8 of the density limit. We record $nT_M$, $T$ and $q$. We repeat for different values of $Q_{\text{INS}}$ in the range 0.5–5.0. The values of the fixed parameters and the ranges of the variable parameters that we used are given in the supplementary material (table S1).

For the CT scaling, we plot $nT_M(T)^{1.22}$ versus $q$ and for the ST scaling we plot $nT_M(T)^{0.39}$ versus $q$. We assume a non-linear power law dependence $\propto q^n$ and determine $n$ by a simple fit. The results are shown in figures 1(a) and (b).

For the CT scaling, we find the exponent of the $q$ dependence is $-3.00$ and for the ST scaling we find $-1.17$; these values are in excellent agreement with the dependences in equations (9) and (10). We conclude that despite the
simplicity of our analysis it is accurate in determining the $q$ dependence of $nT_{\text{fus}}$, and that the approximate difference $\sim q^2$ between the CT and ST expressions found with the simple analysis, and the derived consequential qualitative findings, are confirmed.

4. Generalisation of the analysis and application to other scaling data

In addition to the IPB98y2 confinement time scaling, other empirical scalings for CTs have been derived. Similarly, but to a lesser extent, empirical scalings in addition to the NSTX scaling have been derived for the ST; for example the MAST scaling $(\tau_E)_{\text{scaling}} \propto R^{0.5}B^{1.6}$. By generalising the analysis we can find the $q$ dependence of $nT_{\text{fus}}$ for other sets of derived scalings for both configurations and thereby widen the basis of our findings.

We generalise the empirical scaling $(\tau_E)_{\text{scaling}} \propto I_p^{\alpha_p}B^{\alpha_B}n^{\alpha_n}T^{\alpha_T}P_{\text{L}}^{\alpha_L}P_{\text{mag}}^{\alpha_{\text{mag}}}A^{\alpha_A}$ and proceed through the same steps as before keeping the dependence on the parameters general at all stages. We find:

$$
nT_{\text{fus}} \propto H^{(\alpha_p+\alpha_B+\alpha_n+\alpha_T)/\alpha_p'} \times (\alpha_p+\alpha_B+\alpha_n+\alpha_T)/\alpha_p' \times q^{-\alpha_p'/\alpha_p} \propto q^{-\alpha_p'/\alpha_p} \times q^{-\alpha_p'/\alpha_p} \times A^{(\alpha_A-2\alpha_B)/\alpha_p'} \times q^{-\alpha_p'/\alpha_p}$$

where $\alpha_p' = 1 + \alpha_p$. We note that the exponent of $q$ depends on the exponents of $I_p$, $n$ and $P_L$, in the scaling expression, and, in particular, does not depend on the exponents of $R$ or $B$. Therefore, we can derive the $q$ dependence even when those exponents are not known. The results are collated in table 1.

Despite some scatter in the results, there is a clear difference in the $q$ dependence of $nT_{\text{fus}}$ between the CT scalings, where the dependence is $\sim q^{(2.2-3.5)}$, and the $q$ dependence for the ST scalings where the dependence is $\sim q^{(0.6-1.6)}$. The results indicate that the difference is generic for these two types of tokamaks and thus the benefits of that difference will also be generic; that is, they apply relative to both large size and high field CTs. To illustrate the impact of these findings, we examine the fusion performance of specific devices. We use the system code for these investigations.

5. Impact on fusion performance

We take three devices: an ST with $R = 1.5$ m, an ITER size device with $R = 6.36$ m and an ARC size device [13] with $R = 3.30$ m. The fixed parameters of the devices are given in the supplementary material (table S2). We change the operating point by changing the density, temperature, current and field, maintaining operation at fixed $\beta_N$, and fixed fraction of the density limit (0.7). For the ST we use the CT scaling NSTX-ext and for the ITER size and ARC size devices we use the IPB98y2 scaling (equation (30) in [5]). We keep $H = 1$ throughout the scans and note that $q$ is typically $\sim 2-3$. The results are shown in figure 2. We see that $Q_{\text{fus}}$ for the ST is approximately an order of magnitude higher than for the CT for $P_{\text{fus}}$ of a few 100 MWs, confirming the result of the simplified analysis. During the scan key device parameters such as $I_p$ and $B$ are allowed to vary and these variations are given in the supplementary material (figure S1).

When modelling expected pilot plant and reactor performance, frequently $H > 1$ is used to allow for enhanced confinement which can sometimes be achieved in experiments. For example, for the modelling of steady-state performance of ITER $H = 1.4$ has been assumed [14], while for ARC $H = 1.8$ [13]. These enhanced $H$ factors lead to higher $Q_{\text{fus}}$ for a given $P_{\text{fus}}$. We include in the figure the published operating points of ITER and ARC. For comparison we select an operating point on the ST power scan for a candidate ST pilot plant operating at $H = 1$, $Q_{\text{fus}} = 10$ and $P_{\text{fus}} = 189$ MW. The details of the ST operating point are given in table 2. The plasma volumes and fields are $58 \text{ m}^3$, $3.7 \text{ T}$ (ST150); $141 \text{ m}^3$, $9.2 \text{ T}$ (ARC); $730 \text{ m}^3$, $5.2 \text{ T}$ (ITER). It is notable that although the ST is considerably smaller and operates at lower values of the magnetic field, $Q_{\text{fus}}$ is similar. Potentially the ST operating point could form a design point for a compact pilot plant.

6. Transport code modelling

The analysis and system code calculations thus far have been carried out using global device and plasma performance parameters. To check if the implied values of the local ion and electron thermal diffusivities, $\chi_i$ and $\chi_e$, are reasonable in the light of what is known from direct measurements on STs, albeit in different parameter ranges, we use the ASTRA transport code [15]. The ASTRA model includes more accurate calculations.
Table 1. Dependence of \( nT \tau_E \) on safety factor, \( q \), derived using equation (11) for different CT and ST scaling data.

| Confinement time scaling | \( \alpha_I \) | \( \alpha_n \) | \( \alpha_P \) | Analytically derived \( q \) dependence (equation (11)) |
|--------------------------|---------|---------|---------|-------------------------------|
| Generic CT               | 1       | 1/2     | -1/2   | -3.0                          |
| IPB98y2 (analytic approx.) | 1       | 1/2     | -2/3   | -3.5                          |
| IPB98y2 (exact) (equation (30) [5]) | 0.93   | 0.41    | -0.69  | -3.1                          |
| Petty (2008) (equation (36) [10]) | 0.75   | 0.32    | -0.47  | -2.1                          |
| McDonald (2004) (equation (9) [11]) | 0.96   | 0.40    | -0.59  | -2.9                          |
| Cordey (2005) (equation (9) [12]) | 0.85   | 0.26    | -0.45  | -2.2                          |
| NSTX-ext (analytic)       | 1/2     | 0       | -2/5   | -1.2                          |
| NSTX-ext (equation (13) [6]) | 0.54   | 0.05    | -0.38  | -1.2                          |
| NSTX (2006) (equation (3) [1]) | 0.52   | 0.27    | -0.50  | -1.6                          |
| NSTX (2006) (equation (4) [1]) | 0.56   | 0       | -0.40  | -1.3                          |
| MAST (2009) (equation (2) [3]) | 0.51   | -0.06   | -0.61  | -0.6                          |

![Figure 2](image1.png)

**Figure 2.** \( \dot{Q}_{\text{fus}} \) versus \( \dot{P}_{\text{fus}} \) for operation of a representative ST (ST150) according to the ST scaling, and two CTs, an ITER-size (large size) device and an ARC-size (high field) device, according to IPB98y2 scaling. Operation at 0.7 of the density limit and \( H = 1 \) is assumed in all cases. For comparison, we show the published operating points of ITER (red filled circle, \( H = 1.4 \) [14]) and ARC (yellow filled circle, \( H = 1.8 \) [13]), and a candidate operating point for ST150 (blue filled circle, \( H = 1 \)).

In our application, the plasma geometry, magnetic field, plasma current, and external heating and current drive parameters are prescribed using the system code values. The external heating and current drive power is assumed to be provided by electron cyclotron waves, which directly heat the electrons, and a simple Gaussian deposition profile centred on axis is assumed. The alpha heating power to the ions and electrons is calculated assuming all fast alphas are confined and that they thermalise on the flux surface on which they are born. The plasma density profiles and central values are the same as those used in the system code and the temperature profile shape is the same with the central value adjusted to match the system code fusion power. The effective ion and electron thermal conductivities are calculated based on the temperature, density and heating profiles. The ion thermal conductivity is compared against the neoclassical level calculated using the NCLASS package [16], which calculates the neoclassical transport properties of a multi-species axisymmetric plasma of arbitrary aspect ratio, geometry and collisionality.

We find very good agreement between the values of the plasma parameters obtained with the system code and ASTRA (supplementary material, table S3). Thus, we have consistency of results from our three methods of investigation: analytical, system code and transport code. For the local transport coefficients, results for the representative operating point of the ST, \( \dot{Q}_{\text{fus}} = 10 \), and \( \dot{P}_{\text{fus}} = 189 \) MW are shown in figure 3.

In the example shown, the ion transport is found to be approximately at the neo-classical level. Measurements on
Table 2. Values of the main plasma and device parameters for ST150.

| Parameter | Value from TESC |
|-----------|-----------------|
| R (m) | 1.51/0.84/1.8/0.05 |
| V (m/s) | 58.0/101.0 |
| Q_{ fus } | 10.0 |
| P_{ fus } (MW) / P_{ aux } (MW) | 189/18.9 |
| B (T) / I_P (MA) | 3.73/7.1 |
| T_0 (keV) | 18.9/0.7 |
| β_{ Nu,Trim } | 9.1/4.0/0.82 |
| n_w (MW m^{-2}) / P_{ aux } R^{-1} (MW m^{-1}) | 1.5/22.8 |
| H(ST150-ext) / H(IPB98S2) | 1.0/2.27 |

\( g \) = plasma-wall gap, \( δ \) = plasma triangularity, \( S \) = area of first wall, \( P_{ aux } \) = auxiliary current drive power, \( P_{ dis } \) = first wall heat load, \( P_{ dis } / R \) is a metric of the power load on the divertor exhaust system.

NSTX and MAST have shown that ion transport, \( χ_i \), can be close to the neo-classical level [2, 4]. As a further check the relative ion temperature gradient \( R / L_T \), where \( L_T = |T_i / \nabla T_i| \), is compared to the typical ion-temperature-gradient mode, ITG, instability threshold [17] of \( R / L_T \leq 4 \), and is found to be below this threshold over much of the minor radius. The ITG mode is a drift wave instability that is commonly found to be the dominant instability in CTs. As the relative ion temperature gradient approaches the ITG threshold the core kinetic profiles are often observed to become ‘stiff’, having roughly the same gradient regardless of the applied heating power.

While the ion transport can be close to neo-classical level, the electron transport is always anomalous, and for NSTX it is typically around 1–10 m^2 s^{-1} for electron temperatures in the keV range [18, 19]. Whilst it is not possible to make direct comparisons to the experimental electron thermal diffusivities, due to differences in the operating points, measurements and modelling on NSTX have shown that \( χ_e \) reduces with reducing collisionality [19, 20]. Collusionality reduces as \( \sim 1 / T_i \) and so at the operating temperature of the fusion plasma, \( \sim 20 \) keV, as compared to NSTX at \( \sim 1 \) keV, the values of \( χ_e \) found here appear reasonable.

### 7. Analysis using physics variables and proposed next step

Rather than using engineering variables it is possible to express the empirical confinement time in dimensionless physics variables, and this can provide additional insight; in particular, it can indicate which additional experiments are needed to test these findings. The variables usually employed are the ion cyclotron frequency, \( ω_i \), the normalised ion Larmor radius, \( r_s \), the normalised collisionality, \( ν_s \), the mass of the ion species, \( M \), and the inverse aspect ratio, \( ε = 1 / A \). The values of the dimensionless variables in terms of the engineering variables are well documented [21]. Expressed in these variables \((τ_E)_{ scaling } \sim χ_i^{−1} ω_i^{−1} r_s^{−1} ν_s^{−1} B^{−3} M^q / q^8 A^4 \).

There are established relationships between the exponents of the variables in the engineering expression of \((τ_E)_{ scaling }\) and those in the equivalent dimensionless expression; for example, those in Table 2 [6]. Using those relationships, we transform equation (11) into physics variables. In terms of the three key variables, \( R, B \) and \( q \), we find:

\[
 nT(τ_E)_{ scaling } \propto \frac{R^{2(q + τ_s)} B^{(−1)q} (ε + q)}{q^{(q + τ_s)} (ε + q)}
\]  

(12)

Direct experiments have shown that usually \( x_{ P_{ aux } } \sim 3 \) (gyro-Bohm like transport) and \( x_{ ν_s } \) is usually \( \ll 1 \) [10] and so approximately:

\[
 nT(τ_E)_{ scaling } \propto \frac{R^{2B^3}}{q^{(q + τ_s)} (ε + q)}
\]  

(13)

and thus we see that the fusion performance is dominated by the \( q \) and \( β \) dependences of \( τ_E \). For several years, the concern has been whether the apparent confinement advantages of the ST will remain at lower values of collisionality [22] but this analysis shows that it is the \( q \) and \( β \) dependences that are the most important. Experiments on both CTs and STs have shown that the \( β \) dependence is low, \( x_β \sim 0 \), especially at the low collisionalities expected for reactor grade plasmas. Thus we see that it is the \( q \) dependence that is the critical aspect and so far this has not been well examined. Measurements of this dependence, especially at low collisionalities, are required.

Some measurements at lower collisionalities have been made on NSTX [18]. These have shown that the dependence of \( τ_E \) on the engineering parameters \( I_p \) and \( B \) changes to \( I_p^{0.76} B^{-0.15} \) and thus becomes more like that seen in CTs. On the other hand, the dependence on collisionality appears constant at \( \sim 0.8 \) and it is suggested that this unifies the scaling at lower and higher collisionalities [18]. However, from the relationships between the engineering and dimensionless parameters it is readily shown that:

\[
 x_q = x_{ P_{ aux } } - \frac{α_q}{(1 + α_P)}
\]  

(14)

and so a change in the \( I_p \) dependence at constant \( ν_s \) reflects a change in the \( q \) dependence. The change in the \( q \) dependence is \( Δ x_q = −Δα_q / (1 + α_P) \) and since \( α_P \) is typically \( \sim 0.5 \) and \( Δα_q \sim 0.4 \) the change in the \( q \) dependence is \( Δ x_q \sim 0.8 \), that is the \( q \) dependence is becoming more CT like. On the other hand, the \( ν_s \) dependence is stronger than typically observed in CTs, which is \( \sim −0.2 \) to \( −0.5 \). Thus at low collisionality in NSTX the ST scaling appears to be entering a different regime. Further experiments at low collisionality are clearly required and specifically the determination of the \( q \) dependence of \( τ_E \). Essentially what is required is a repeat of the measurement made on MAST [4] but at lower collisionality. NSTX and MAST are currently being upgraded for operation at fields \( \sim 1 \) T, and Tokamak Energy Ltd is currently upgrading the spherical tokamak, ST-40, which has a unique capability to operate at 3 T. Thus it should be possible to carry out this measurement in the near future.
Figure 3. ASTRA code calculations. (a) Electron and ion thermal diffusivities calculated with the ASTRA transport code and the neo-classical ion transport, $\chi_{i,NC}$, calculated using a dedicated module (NCLASS [16]), which calculates the neoclassical transport properties of a multi-species axisymmetric plasma of arbitrary aspect ratio, geometry and collisionality. (b) Relative ion temperature gradient, $R/L_T$, compared to the typical ion-temperature-gradient mode, ITG, instability threshold.

8. Discussion

The impact of the different safety factor dependences in the derived expressions for $nT\tau_E$, and hence $Q_{ fus }$, is clear and substantial and naturally raises the question why there should be a difference in confinement between ST and CT devices. The reason is not fully understood but it is likely due to differences in the dominant transport mechanisms as aspect ratio and beta are varied. At large aspect ratio both the ion and electron transport are typically turbulent and dominated by electrostatic modes such as the ion and electron temperature gradient modes (ITG and ETG respectively) and trapped electron modes (TEM). In high-performance STs, ion transport is typically observed to be neoclassical as the high-beta low-aspect-ratio equilibrium is effective at stabilising ITG and TEM instabilities. Relatively high perpendicular $ExB$ flow shear can also provide stabilisation in STs that use NBI heating (and torque) due to the low moment of inertia. However, the electron transport remains anomalously high, with the primary candidates for transport being ETG, as well as electromagnetic mechanisms such as micro-tearing modes (MTM) and kinetic ballooning modes (KBM), which are not typically observed in CTs, only becoming unstable at sufficiently high beta.

While the performance of a relatively small ST fusion module looks promising from a physics perspective, the construction of a device requires the solution of multiple engineering challenges. Thermal loads on the first wall and the divertor are two important areas. For the operating point of ST150, the wall load is $\sim 1.5$ MW $m^{-2}$, which is in the range of those expected for much larger devices and so the solutions that are being developed there could potentially be used. Similarly, for the divertor the figure of merit, $P_{div}/R$ is $\sim 23$ MW $m^{-1}$, which is in the range of that expected on much larger and more powerful devices. Probably the most critical area is the magnet and here novel solutions are needed. The field on the conductor in the central column is $\sim 25$ T, which is too high for low-temperature superconductors which typically have a limit $\sim 12$ T. However, high temperature superconductors (HTS) can withstand 20 T or higher. Good progress is being made with the development and supply of HTS tape of sufficient quality and quantity, and with the design and construction of the magnets suitable for a tokamak [23, 24]. Unavoidably, even with substantial shielding the HTS will be subject to high energy neutrons and
gamma radiation. Progress is also being made with effective radiation shields [25] and on the determination of the impact of this radiation on the performance of the conductors [26, 27]. There are several groups and commercial companies working on these challenges and rapid progress is being made.

Our analysis has assumed steady state operation but, of course, the plasma will have to be initiated, ramped up and down, and in a ST the space for a central solenoid is limited. Several non-inductive plasma initiation and ramp up techniques are under development, for example Transient Co-axial Helicity Injection [28], Electron Bernstein Waves [29], and Merging Compression and Double Null Merging [30]. In some cases, full plasma initiation and ramp-up have been achieved with these methods.

9. Summary
In summary, using empirical data from NSTX and MAST, and from multiple CTs, the potential fusion performance of STs and CTs has been investigated analytically and by using established fusion codes. The simplifications made in the analytical investigation have been checked and found not to significantly influence the qualitative findings. By generalising the analysis the basis of the findings has been widened. The fusion performances of three devices—a large size CT, a high field CT and a compact ST—have been compared and it is found that, for similar values of field and fusion power but smaller volume, the ST has a fusion triple product up to a factor of three higher and fusion power gain an order of magnitude higher. The implied local values of the ion and electron thermal transport coefficients for a candidate operating point of the ST have been derived using a transport code (ASTRA). The derived values are reasonable compared with expectations based on measurements on current STs, although a significant extrapolation, especially in collisionality, is involved. By transforming the generalised analysis into physics variables the key dependences of the scaling of the confinement time are revealed and these are the beta and safety factor dependences; the safety factor dependence dominates since the beta dependence is expected to be close to zero. Thus it is found that the measurement to advance this finding is the safety factor dependence of the confinement time at low collisionality. With the upgrades currently on going of NSTX and MAST, and ST40 at Tokamak Energy, UK, there should be the opportunity to make this measurement, at least at lower collisionalities than presently examined, in the near future. The possible origin of the enhanced performance has been discussed and this is an area that would benefit from more investigation. In order to realise compact devices based on the ST, novel technologies will be needed especially for the magnets. Substantial progress is being made with magnets employing high temperature superconductors and could make the realisation of such devices feasible in the near term.

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