The Equivalence Principle in the Non-baryonic Regime

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March 14, 2018

Abstract

We consider the empirical validity of the equivalence principle for non-baryonic matter. Working in the context of the $TH\epsilon\mu$ formalism, we evaluate the constraints experiments place on parameters associated with violation of the equivalence principle (EVPs) over as wide a sector of the standard model as possible. Specific examples include new parameter constraints which arise from torsion balance experiments, gravitational red shift, variation of the fine structure constant, time-dilation measurements, and matter/antimatter experiments. We find several new bounds on EVPs in the leptonic and kaon sectors.

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I Introduction and Summary

The postulate that the equivalence between uniform acceleration and a uniform gravitational field apply to all physical phenomena allowed Einstein to construct a theory of gravitation, general relativity, which revolutionized our conceptual understanding of the universe. It allowed a description of physics in which the effects of gravitation are manifest as the dynamics of the geometry of a curved spacetime. That this geometry is unique for all forms of mass-energy is a consequence of Einstein’s equivalence postulate.

Only decades later was it realized that this postulate is the foundation for a rather broad class of theories of gravitation (which includes general relativity) known as metric theories. Any theory of gravity that describes spacetime via a symmetric, second-rank tensor field $g_{\mu\nu}$ that couples universally to all non-gravitational fields respects the aforementioned equivalence between uniform acceleration and uniform gravitational fields, and is by definition a member of this class. By definition, non-metric theories break gravitational universality by coupling additional gravitational fields to matter.

This understanding eventually resulted in a more precise formulation of Einstein’s equivalence postulate in terms of a number of physically distinct principles [1]. The most basic of these is the Weak Equivalence Principle, or WEP, which states that the trajectory of any test body (with a given initial velocity and spacetime position) in a given gravitational field is independent of its internal structure or composition. A natural extension of this to include all nongravitational phenomena states that, in addition to WEP, the outcomes of nongravitational test experiments performed within a local, freely falling frame are independent of the frame’s location (local position invariance, LPI) and velocity (local Lorentz invariance, LLI) in a background gravitational field. The combination of WEP, LLI and LPI embody what is now known as the Einstein Equivalence Principle, or EEP. The further extension of this principle to include self-gravitating systems is known as the Strong Equivalence Principle, or SEP.

Since a direct consequence of the EEP is that the outcome of local nongravitational experiments should be independent of the effects of an external (slowly varying) gravitational field, direct tests of EEP may be carried out as follows. Consider an Earth-based laboratory in which local nongravitational experiments are performed. External gravitational potentials generated by the Earth, the Sun, the planets, the Galaxy, etc. pervade this laboratory, and
any nonmetric couplings of these potentials to matter can cause the outcomes of experiments to depend on the laboratory’s position, orientation or velocity relative to these sources. This is a direct violation of (respectively) LPI and LLI. The character of a violation reflects the form of the specific nonmetric coupling responsible for it. It is only when LPI and LLI are valid that local nongravitational dynamics is indistinguishable from special relativistic dynamics as predicted by metric theories of gravity.

Tests of the validity of the various facets of the Equivalence Principle have been carried out to impressive levels of precision. The universality of free-fall (or UFF, a necessary consequence of WEP) has been empirically validated to within $10^{-12}$. Limits on violations of LLI to a precision of $\sim 10^{-22}$ have been imposed using laser experiments [2]. Recent experiments comparing the rates between H-maser and Hg clocks have imposed a stringent new limit on LPI violation via the bound $\dot{\alpha}/\alpha \leq 3.7 \times 10^{-14}$ where $\alpha$ is the fine-structure constant [3]. Alternate tests of LPI via gravitational redshift experiments could reach a precision up to $10^{-9}$ [4].

Why, then, ought one to resist the temptation to conclude that future experiments should ignore non-metric theories and focus only on winnowing out the correct metric theory of gravity? There are four basic reasons. One is the anticipated improvements in precision of upcoming experiments by as much as six orders of magnitude [5]. If such experiments yield improved limits on EEP-violation, this will afford us a much greater degree of confidence in our physical theories under the extreme conditions present in many astrophysical and cosmological situations. Another is historical: attempts to unify gravity with the other forces of nature have yielded a number of logically possible, physically well-motivated, alternatives to general relativity which do not naturally respect the EEP [6]. A third reason is that tests of the EEP can provide us with a unique way (perhaps the only way) of testing modern physical theories that unify gravity with the other forces of nature insofar as such theories typically generate new interactions which violate the equivalence principle [7]. Finally, EEP experiments to date have probed effects that are predominantly sensitive to nuclear electrostatic energy. Although violations of WEP/EEP due to other forms of energy (virtually all of which are associated with baryonic matter) have also been estimated [8], the bulk of our empirical knowledge about the validity of the equivalence principle is in the baryon/photon sector of the standard model.

Comparatively little is known about the empirical validity of the EEP for
systems dominated by other forms of mass-energy. Such systems include photons of differing polarization, antimatter systems, neutrinos, mesons, massive leptons, hypothesized dark matter, second and third generation matter, and quantum vacuum energies. There is no logically necessary reason why such systems should respect any or all of WEP, EEP or SEP.

In order to establish the universal behavior of gravity, we are therefore compelled to consider the validity of the EEP over as diverse a range of non-gravitational interactions as is possible. To this end, we consider in this paper an appraisal of the current experimental basis for the equivalence principle in the context of the $TH\epsilon\mu$ formalism. This formalism encompasses a broad class of non-metric theories of gravity and deals with the dynamics of charged particles and electromagnetic fields in a static, spherically symmetric gravitational field. Non-metric couplings are parameterized by a set of static spherically symmetric (SSS) functions which take on constant values for any physical systems whose spatio-temporal extent is such that the gravitational field(s) may be considered constant. The $TH\epsilon\mu$ formalism thereby provides an interpretative framework for EEP experiments, in that the results of any given EEP experiment set bounds on (or alternatively fix) these constant parameter values, henceforth referred to as EEP-violating parameters, or EVPs. Low-energy effective field theories that arise from more fundamental theories of quantum gravity, unification and spacetime structure will typically yield specific predictions for such EVPs. By providing bounds on these parameters, EEP tests give us an invaluable probe of the fundamental physical laws that govern our universe.

Our approach differs from the more traditional usage of the $TH\epsilon\mu$ formalism, which assumes a universal gravitational coupling for massive particles that is distinct from the gravitational/electromagnetic coupling. The only EVPs are the limiting speed of all massive bodies $c_0$ and the speed of electromagnetic radiation, $c_s$. For a theory respecting LLI and LPI, $c_s = c_0$, and so in this framework EEP experiments set bounds $|1 - c_s^2/c_0^2| \equiv \xi$. Since we wish to consider the possibility of EEP-violation for as many forms of mass-energy as possible, we generalize this approach by breaking universality for all species of particles. In this case the modified nongravitational action reads

$$S_{NG} = -\sum_a m_a \int dt \left( T_a - H_a v_a^2 \right)^{1/2} + \sum_{\nu} e_\nu \int dt v_\nu^\mu A_\mu(x_\nu)$$
\[ + \frac{1}{2} \int d^4x \left( \epsilon E^2 - B^2 / \mu \right), \tag{1} \]

where \( m_a, e_a, \) and \( x^\mu_a(t) \) are the rest mass, charge, and world line of particle \( a, \) \( x^0 \equiv t, \) \( v^\mu_a \equiv dx^\mu_a / dt, \) \( \vec{E} \equiv -\nabla A_0 - \partial \vec{A} / \partial t, \) \( \vec{B} \equiv \nabla \times \vec{A}. \) Violation of gravitational universality in this formalism implies that each given species of particle has its own distinct gravitational coupling (i.e. its own metric), described by \( g^{(a)}_{\mu\nu} = \text{diag}(T_a(r), -H_a(r), -H_a(r), -H_a(r)). \) These functions, along with \( \epsilon \) and \( \mu \) (which parameterize the metric for the electromagnetic field) are arbitrary functions of the static spherically symmetric (SSS) (background) Newtonian gravitational potential \( U = GM/r, \) which approaches unity as \( U \to 0. \) Expanding these functions about the origin \( X_0 \) of the local frame of reference, it is straightforward to see that, for systems in which spatio-temporal variations of the \( TH\epsilon\mu \) functions can be neglected, the limiting speed of the \( a \)-th species of massive particle is \( c_a = \left( \frac{T_a(X_0)}{H_a(X_0)} \right)^{1/2}, \) and that the speed of electromagnetic radiation (i.e. the speed of light) is \( c_* = 1/\sqrt{\epsilon(X_0)\mu(X_0)}. \)

Previous applications of the \( TH\epsilon\mu \) formalism assumed that all \( c_a \)'s were equal to \( c_0. \) In general this is not the case: the role of experiment in this context is to provide quantitative information on these EVPs or, more properly, on the ratios \( c_a/c_*, \) since the action can always be rescaled by an overall constant. Since we expect the symmetries implied by EEP to be at least approximate symmetries, it is in practice more useful to consider bounds on the quantities \( \xi_a \equiv |1 - c_*^2/c_a^2|. \)

We therefore consider in this paper the bounds experiment places on the various EVPs which follow from the action (\[1\]).

In the next section we review the present empirical evidence in support of EEP and find constrains on violations of EEP among the different species of massive particles. The main emphasis is to sort out the empirical knowledge related to the validity of EEP in the non-baryonic sector of the standard model. We start reviewing the empirical limits on WEP violations as found by torsion balance experiments. We then follow with violations of LPI in the context of gravitational red shift experiments and the variation of the atomic fine structure constant. Next we review the time-dilation experiment (Hughes-Drever type) as performed by Prestage et al. [\[15\]], in order to look for spatial anisotropy or LLI violations. We also consider limits on EVPs obtained from matter/antimatter experiments (CPT tests) as described by
Hughes [19]. In the last section we summarize and discuss the implications of our results.

II Empirical Review of the EEP

II.1 E"ötvos type Experiments

These experiments search for quantitative differences between the passive gravitational mass and the inertial mass of a given body. The former is a dynamical quantity that determines the gravitational force acting on a body (i.e. its weight), whereas the latter is a kinematical quantity that determines the response of a body to any applied force. There is no logically necessary reason why these quantities must be equal (in appropriate units), and so we therefore expect

\[ m_p = m_I + \sum_A \eta^A E_A^A/c^2 \]

where \( E_A^A \) is the internal energy generated by interaction \( A \), and \( \eta^A \) is a dimensionless parameter that measures the strength of the WEP violation for body \( A \).

For two different bodies we can write the acceleration as

\[ a_1 = (1 + \sum_A \eta^A E_1^A/c^2)g \quad a_2 = (1 + \sum_A \eta^A E_2^A/c^2)g \]

A measurement on the relative difference in acceleration yields the so called “E"ötvos ratio” given by

\[ \eta \equiv 2|a_1 - a_2|/a_1 + a_2| = \sum_A \eta^A \left( \frac{E_1^A}{m_1c^2} - \frac{E_2^A}{m_2c^2} \right) \]

Experiments carried out to date [3] imply that this ratio is constrained to be

\[ |\eta| < \begin{cases} 10^{-11} \\ 10^{-12} \end{cases} \]

This limit in turns constrains the violating parameter \( \eta^A \) related to each \( A \)-type interaction. This is possible provided the various interactions do not conspire towards special types of cancellations so that independent bounds
can be gathered in each case (see ref. [4] for quotations of those limits when referred to interactions stemming from the atomic nucleus: strong, electro-
static, magnetostatic, hyperfine, etc.).

This result can be analyzed in more detail if the inertial mass of an atom of atomic number \(Z\) and mass number \(A\) is written as [9]:

\[
m_I(A, Z) = Z(m_e + m_p) + (A - Z)m_n + E(A, Z)/c^2
\]

and its gravitational mass as

\[
m_G(A, Z) = (1 + \delta_e)Zm_e + (1 + \delta_p)Zm_p + (1 + \delta_n)(A - Z)m_n + (1 + \delta_E)E(A, Z)/c^2
\]

where \(E(A, Z)\) is the sum of all the binding energies and \(\delta\) parameterizes possible violations of WEP by each of the constituents. Then the following limits are obtained [9]:

\[
|\delta_n| < 5 \times 10^{-9} \quad (8)
\]

\[
|\delta_e| < 4 \times 10^{-6} \quad (9)
\]

\[
|\delta_E| < 5 \times 10^{-9} \quad (10)
\]

where the possibility of fortuitous cancelation is ignored. The limit on \(\delta_E\) constrains the total atomic and nuclear binding energy. Note that the independent limit for electron quoted before is obtained provided \(\delta_n = \delta_p\).

Neutral atoms involved in these experiments contain equal number of protons and electrons, and so a more rigorous analysis actually yields [9]:

\[
|\delta_e \frac{m_e}{m_p} + \delta_p - \delta_n| < 2 \times 10^{-10} \quad (11)
\]

Since there is strong evidence supporting the validity of EEP in the baryonic sector and our aim is to extract as much information as possible to the less explored leptonic domain, we omit consideration of any gravitational anomaly stemming from baryonic matter.

In order to connect these results with the \(T\) formalism, we study first the case of a single particle falling in a SSS gravitational field \((U)\). Variation of the first term of the action [1] gives the particle’s equations of motion [1]:

\[
d\vec{v}_a \over dt = \frac{d}{dt} \left( \frac{H_a}{W_a} v_a \right) + \frac{1}{2} \frac{1}{W_a} \nabla (T_a - H_a v_a^2)
\]
where \( W_a \equiv (T_a - H_a v_a^2)^{1/2} \). If we expand the \( TH\epsilon\mu \) functions

\[
T(U) = T_0 + T_0 \vec{g}_0 \cdot \vec{X} + O(\vec{g}_0 \cdot \vec{X})^2
\]

about the origin \( X_0 = 0 \) of the system, then

\[
\frac{d\vec{v}}{dt} = -\frac{1}{2} T'_0 \vec{g}_0 + \frac{1}{2} H'_0 g_0 v_0 \left( \frac{T'_0}{T_0} - \frac{H'_0}{H_0} \right) (\vec{g}_0 \cdot \vec{v}) \vec{v} + \cdots
\]

\[
\equiv \vec{g}_a
\]

(14)

where the species-labeling index-\( a \) is implicit on \( T, H \) and \( v \).

To lowest order we obtain for the electron and proton the accelerations

\[
\vec{g}_e = -\frac{1}{2} \frac{T'_e}{H'_e} \vec{g}_0 + \cdots
\]

(15)

\[
\vec{g}_p = -\frac{1}{2} \frac{T'_p}{H'_p} \vec{g}_0 + \cdots
\]

(16)

The \( \delta_a \) parameters introduced before quantify differences in the acceleration of a given species of particle (electron, proton or neutron in the present case) with respect to a standard \( g \) (given by the choice of units), that is, \( \delta_a = |g_a - g|/g \).

The strongest constraint is on the neutron parameter \( \delta_n \), which is assumed to be equivalent to that of the proton. This suggests a choice of units in which \( g \equiv g_p \), to the order given in (16). In that case we can write (c.f. (13)):

\[
\delta_e = |1 - f(T) c_e^2 / c_p^2| < 4 \times 10^{-6}
\]

(17)

where \( f(T) \equiv T'_e T'_p / T_e T'_p \), and \( c_e \) and \( c_p \) the respective limiting speeds. Note that within the traditional usage of the \( TH\epsilon\mu \) formalism there is no EEP violation at this level, and so the constraint (17) is trivially satisfied.

Turning next to the binding energy \( E_A \), its dominant contribution arises from the atomic nucleus. In the case of electromagnetic interactions we can distinguish different internal energy contributions. For the electrostatic and magnetostatic nuclear energy, the violating EEP parameters are bounded by \( \delta_{ES} < 4 \times 10^{-10} \) and \( \delta_{MS} < 6 \times 10^{-6} \) respectively. A consideration
of these nuclear binding energies within the $\text{TH}e\mu$ formalism constrains the LPI violating parameters as [4]:

$$|\Gamma_B| < 2 \times 10^{-10} \quad |\Lambda_B| < 3 \times 10^{-6}$$ \hspace{1cm} (18)

where

$$\Gamma_B \equiv \frac{T_B}{T'_B} \ln \left[ \frac{T_B \mu^2}{H_B} \right]|_{\vec{X}_0} \quad \Lambda_B \equiv \frac{T_B}{T'_B} \ln \left[ \frac{T_B \mu^2}{H_B} \right]|_{\vec{X}_0},$$ \hspace{1cm} (19)

where the sub-index $B$ labels the metric related to baryonic matter (which is assumed to be the same among baryons). Note that the former parameters are both equal to zero if the EEP is valid. The above result sets an upper limit on violations of WEP due to a different gravitational coupling between baryonic matter and electromagnetism.

The electron played no role in the derivation of (18). In order to compare (gravitationally) electrons and photons we should look at atomic binding energies. The electrostatic interaction amongst the electrons themselves and between them and the protons in the nucleus is the dominant form of energy in this case. In general these energies are within the range of $E_{\text{atom}} \sim 10eV$. Although the experimental limit (5) is related via (4) to the specific interaction under consideration, thereby involving the difference between the atomic electrostatic energies of the two bodies being tested (aluminum and platinum in the case of the strongest limit quoted in (5)), we can make a crude estimate by assuming that this difference contributes as $E_{\text{atom}}/m_p \sim 10^{-8}$. This allows us to constrain the EVP related to this interaction (the corresponding $\eta^A$), as $\delta_{\text{atom}} < 10^{-3}$. In the case of hydrogen atoms the $\text{TH}e\mu$ formalism implies $\delta_{\text{atom}} = 2\Gamma_e$, which is given by (19) with the appropriate electron label. This result comes after solving the Schrödinger equation (within the $\text{TH}e\mu$ context) for the principal atomic energy levels (it can be inferred from eq.(2.113) in ref.[4]). From here we can extract the approximate limit $|\Gamma_e| < 10^{-3}$ for the EVP associated with the relative gravitational coupling between electrons and electromagnetism. This limit is well below the analogous constraint for baryonic matter (18).

\section*{II.2 Gravitational Red Shift Experiments}

In a redshift experiment the local energies at emission $w_{em}$ and at reception $w_{rec}$ of a photon transmitted between observers at different points in an
external gravitational field are compared in terms of

\[ Z = \frac{w_{em} - w_{rec}}{w_{em}} \equiv \Delta U \left(1 - \Xi\right) \tag{20} \]

The anomalous redshift parameter (\(\Xi\)) measures the degree of LPI violation. It signals the breakdown of the universality of gravity, and so depends on the nature of the transition involved in the experiment (e.g., fine, hyperfine, etc.).

The most accurate test for the gravitational red shift corresponds to the gravity probe A experiment [20], which was able to constrain \(|\Xi^H| < 2 \times 10^{-4}\). This experiment employed hydrogen maser clocks, where the governing energy transition is given by the hyperfine splitting due to the interaction between the magnetic moment (spin) of the nucleus (proton) and electron.

In the following we proceed to review this experiment assuming different gravitational couplings between electrons and protons (or baryons in general). In solving the Hydrogen atom, the relevant metric is the electron metric, the proton (or more generally, the nucleus in Hydrogenic atoms) playing only the role of a static charge at rest with magnetic moment \(\vec{\mu}_p\).

The electromagnetic field produced by that source is [4]

\[ A_0 = -\frac{e}{\epsilon_0 r} \quad \vec{A} = \frac{1}{2} \mu_0 \vec{\mu}_p \times \frac{\vec{r}}{r^3}. \tag{21} \]

where the magnetic moment of the proton is given by (in a manner analogous to that for the electron):

\[ \vec{\mu}_p = \frac{T_B^{1/2}}{H_B} g_p \left( \frac{e \sigma}{2m_p} \right) \tag{22} \]

where the various parameters have their usual meaning. Note that the proton metric affects only the hyperfine splitting (due to \(\vec{\mu}_p\)), since it arises from the interaction between the magnetic moments of the electron and proton (nucleus). The principal and fine structure atomic energy levels depend only upon the electron metric. It is simple to check from ref.[4] that the hyperfine splitting scales as

\[ \Delta E_{hf} = \mathcal{E}_{hf} \frac{T_B^{1/2} H_B^2 \mu_0}{H_B T_e \epsilon_0^3} \tag{23} \]

where \(\mathcal{E}_{hf}\) depends on atomic parameters only.
If we expand (23) according to (13), then we can identify

\[ \Delta E_{hf} = \mathcal{E}_{hf}(1 - U) + \mathcal{E}_{hf}U \Xi_{hf} \]  

(24)

with

\[ \Xi_{hf} = 3\Gamma_B - \Lambda_B + \Delta \]  

(25)

where we chose units such that the gravitational potential is given by

\[ U = -\frac{1}{2} T'_B \vec{g}_0 \cdot \vec{X}, \]  

and \( \Gamma_B \) and \( \Lambda_B \) are given by (19). In (24) we rescaled the atomic parameters to absorb the \( TH\epsilon\mu \) functions and chose units such that \( c_B = 1 \).

The quantity \( \Delta \) is given by

\[ \Delta = 2 \frac{T_B}{T'_B} \left[ 2 \left( \frac{H'_B}{H_B} - \frac{H'_e}{H_e} \right) - \frac{T'_B}{T_B} + \frac{T'_e}{T_e} \right] \]  

(26)

and would vanish under the assumption that the leptonic and baryonic \( TH\epsilon\mu \) parameters were the same.

The gravity probe A experiment constrains the corresponding LPI violating parameter related to hyperfine transitions:

\[ |\Xi_{hf}| = |3\Gamma_B - \Lambda_B + \Delta| < 2 \times 10^{-4} \]  

(27)

Note that if protons and electrons couple identically to gravity then \( \Delta = 0 \). On the other hand we can use the Eötvös result (18) to assign the above limit to the \( \Delta \) function only and so have just a constraint on the relative baryonic and leptonic metrics.

II.3 Variations of the Fine Structure Constant

Experiments searching for a temporal variation of the fine structure constant \( \alpha \) can be divided into two categories: cosmological and laboratory measurements. The first ones look for variations within cosmological time scales and the others are based on clock comparisons over time durations of months or years.

Laboratory measurements make use of clocks with ultrastable oscillators of differing physical composition, such as the superconducting cavity oscillator vs. cesium hyperfine clock transition. They rely on the ultrahigh stability of the atomic standard clocks and set limits a few orders of magnitude
less stringent than the cosmological measurements. One of the most sensitive tests for $\alpha$–variation comes from the clock comparison between Hg$^+$ and H hyperfine transitions [3]. This experiment set an upper bound of $\dot{\alpha}/\alpha \leq 3.7 \times 10^{-14}/\text{yr}$ after a 140 day observation period. Note that any variation of $\alpha$, whether a cosmological time variation or a spatial variation via a dependence of $\alpha$ on the gravitational potential, will force a variation in the relative clock rates between any such pair of clocks.

The result of ref. [3] is based on the increasing importance of relativistic contributions to the hyperfine splitting as $Z$ increases in the group I alkali elements and alkali ions. Using the theoretical expressions for the hyperfine splitting in hydrogen ($a_h$) and the alkali atom or ion ($A_{Hg^+}$ in this case), along with the experimental result for the drift between the two clocks, namely $d\ln(A_{Hg^+}/a_h)/dt < 2.1(0.8) \times 10^{-16}/\text{day}$, they constrained:

$$\frac{1}{\alpha} \frac{d\alpha}{dU} dt \leq 3.7 \times 10^{-14}/\text{yr}$$  \hspace{1cm} (28)

where we have explicitly shown the position dependence of $\alpha$ via the gravitational potential $U$. Note that within the $TH\epsilon\mu$ formalism $\alpha$ rescales according to:

$$\alpha \rightarrow \alpha \frac{H_e}{\epsilon_0 \sqrt{T_e}}$$

where $T_e$ and $H_e$ are the electron metric locally evaluated at the center mass position of the atom (for example $\vec{X} = 0$). If these functions along with $\epsilon$ are expanded according to (13), then

$$\alpha \rightarrow \alpha (1 + \Gamma_e U)$$  \hspace{1cm} (29)

where $\Gamma_e$ is defined by (19), and adequate units are used to define the gravitational potential. It is clear that we can evaluate:

$$\frac{1}{\alpha} \frac{d\alpha}{dU} = \Gamma_e + O(U)$$  \hspace{1cm} (30)

A crude estimation for the time variation of the gravitational potential, which can be considered as the solar gravitational potential at the laboratory, is obtained by taking the extreme variation of $U$ over the 140 day period, that is the seasonal change, $\Delta U \sim 3 \times 10^{-10}$ [9]. This altogether bounds:

$$|\Gamma_e| < 5 \times 10^{-5}$$  \hspace{1cm} (31)
Baryonic matter contributes to the ratio $A_{Hg^+}/a_h$ only via the nuclear $g$ factors related to each atom. These parameters are determined mainly by the strength of the strong interactions. These interactions were assumed to obey EEP, when deriving eq. (28) from the empirical variation of the hyperfine ratio. Hence the purely leptonic contribution to the change on $\alpha$ allows us to improve the upper bound for the LPI violating parameter related to electrons (relative to photons), as stated in eq. (31).

II.4 Time Dilation Experiments

LLI requires that the local, nongravitational physics of a bound system of particles be independent of its velocity and orientation relative to any preferred frame. If LLI were violated the energy levels of a bound system such as a nucleus could be shifted in a way that correlates the motion of the bound particles in each state with the preferred direction, leading to an orientation-dependent binding energy (anisotropy of inertial mass). The most precise experiments of this sort \cite{2} search for a time dependent quadrupole splitting of Zeeman levels. They compare the nuclear-spin-precession frequencies between two gases with nuclear spin $I = 3/2$ and $I = 1/2$, the latter being insensitive to a quadrupole splitting. These results place the constraint

$$\xi_B = (1 - c^2/c_B^2) < 6 \times 10^{-21}$$

on the relative gravitational coupling between electromagnetism and baryonic matter. This result stems from nuclear interactions only and so the leptonic metric plays no role in its derivation.

A weaker bound was obtained by Prestage al. \cite{18} by comparing the frequency of a nuclear spin-flip transition in $^9Be^+$ to the frequency of a hydrogen maser transition. In the context of the standard $TH\epsilon\mu$ formalism the EEP violating anomalies coming from the hyperfine transitions of both clocks are negligible in comparison to those originating from the electric quadrupole moment of the $^9Be^+$ nucleus. The derived limit of $\xi_B < 10^{-18}$ was obtained under the assumption that the relative gravitational interaction between electromagnetism and electrons is the same as that between electromagnetism and baryons.

In the following we drop this assumption and proceed to interpret this experiment accordingly. Experiments which search for the time dependent
quadrupole Zeeman splitting suggest that $\xi_B \approx 0$. We shall assume this for simplicity, so that we have $c_e \neq c_B \approx c_\ast$. In other words, we analyze the effects from EEP violations in the electron/photon sector of the standard model.

Let us review the result of ref. [18]. They studied the ratio of the frequency of clocks defined by the hyperfine transition of the $^9$Be$^+$ (nuclear spin-flip) and $H$ (electron spin-flip) atoms. They searched for variation in the clock frequencies of the form:

$$\nu = \nu_0 + A_2 P_2(\cos \Theta) \quad (32)$$

where $P_2(\cos \Theta) = 3(\cos^2 \Theta - 1)/2$, and $\Theta$ is the angle between the quantization axis and the direction of matter anisotropy in the nearby universe.

The limit $A_2 < 10^{-5}$ Hz was obtained from looking for variations with respect to motion through the mean rest frame of the universe. On the other hand the $TH\epsilon\mu$ formalism predicts a change of the form:

$$\delta\nu = \nu - \nu_0 = \xi_B \delta\nu_{ES}^{nuclear} P_2(\cos \Theta) \quad (33)$$

where $\delta\nu_{ES}^{nuclear}$ corresponds to the nuclear electrostatic energy coming from the electric quadrupole moment of the $^9$Be$^+$ nucleus. This is reoriented by the clock transition which flips the nuclear spin and therefore contributes to the energy transition. Comparing the former expression with the experimental result for $A_2$ yields the previously quoted limit on $\xi_B$. Note that the nonmetric anomalies stemming from the hyperfine transition of the $^9$Be$^+$ and maser clocks were ignored in deriving (33). By including them and neglecting the baryonic contributions, we obtain an expression of the form

$$\delta\nu = \xi_e \delta\nu_{Hf}^{atom} P_2(\cos \Theta) \quad (34)$$

where $\xi_e = 1 - c_B/c_e$ and $\delta\nu_{Hf}^{atom}$ accounts for the hyperfine anomalies mentioned above. These can be expressed as

$$\delta\nu_{Hf}^{atom} = (A_{Be} - A_H) \nu_0 \quad (35)$$

where the $A$-coefficients account for LLI violations stemming from the hyperfine structure related to each atom (similar to the time dilation coefficients introduced by Gabriel and Haugan [21]), and $\nu_0$ for the clock transition being measured.
In the $TH\mu\mu$ framework we can find an expression for $A$, which to lowest order gives: $A_{Be} - A_H = -V^2/2$ (see appendix for details), where $V$ is the Earth velocity with respect to the preferred frame. Assuming $\xi_B = 0$, the Prestage experiment implies the limit

$$\xi_e(A_{Be} - A_H)\nu_0 \sim A_2 < 10^{-5}\,[\text{Hz}]$$  \hspace{1cm} (36)

when looking for variations with respect to motion through the mean rest frame of the universe ($V \sim 10^{-3}$). This, along with the $^9Be^+$ clock frequency value $\nu_0 = 303\,\text{MHz}$, imposes the constraint

$$\xi_e < \frac{2}{3} \times 10^{-7}$$  \hspace{1cm} (37)

on the leptonic EEP violating parameter.

\section{II.5 Antimatter/matter experiments}

The universality of gravity embodied by EEP makes no distinction between particles and antiparticles. This assumption goes beyond the CPT theorem, because the gravitational acceleration of a particle is not an intrinsic property but involves its interaction with the external gravitational field.

Hughes made use of experiments testing CPT-violation in the Kaon system to deduce an improved test of the WEP for the antiproton \cite{Hughes}, via potentially differing gravitational couplings of quarks and antiquarks. We wish here to review the influence of a possibly anomalous gravitational interaction on neutral kaons within the context of the $TH\mu\mu$ formalism.

In the presence of an external gravitational field the non gravitational action for the kaon system can be expressed as \cite{Hughes}

$$S = -\int_0^t dt' \frac{M}{\gamma}$$  \hspace{1cm} (38)

with

$$\gamma^{-1} = \sqrt{g_a^{\mu\nu} \frac{dx_\mu}{dt} \frac{dx_\nu}{dt}}$$  \hspace{1cm} (39)

where the gravitational interaction of particle $a$ ($K_0$ or $\bar{K}_0$) is mediated by

$$g_a^{\mu\nu} = \text{diag}(T_a, -H_a, -H_a, -H_a),$$  \hspace{1cm} (40)
and \( x_\mu \) are the space-time coordinates along the kaon’s world line. \( \mathbf{M} \) is the mass matrix, whose diagonal terms correspond to the \( K_0 \) and \( \bar{K}_0 \) masses.\[1\]

We assume that the gravitational field changes slowly along the dimensions of the laboratory system, so that we can expand the \( TH \) functions according to (13). In that case

\[
g^{\mu\nu} = \eta_*^{\mu\nu} + U h_*^{\mu\nu} + O(U^2)
\]

with

\[
\eta_*^{\mu\nu} = \text{diag}(T_0, -H_0, -H_0, -H_0) \quad h_*^{\mu\nu} = \text{diag}(T'_0, -H'_0, -H'_0, -H'_0),
\]

and \( U = \vec{g}_0 \cdot \vec{X} \) as the gravitational potential yielding the gravitational acceleration \( \vec{g}_0 \). The \( a \)-label related to each particle is implicit on each \( TH \) parameter. The former expansion reduces to Hughes’ work provided \( T_0 = H_0 = 1 \) and \( T'_0 = -H'_0 = 2\alpha \), where \( \alpha \) is dependent on the species of particle.

By using (41) we can approximate

\[
\gamma^{-1} = \gamma_*^{-1} + \frac{U}{2} \gamma_* t_* + O(U^2)
\]

where

\[
\gamma_*^{-1} = \sqrt{\eta_*^{\mu\nu} \frac{dx_\mu}{dt} \frac{dx_\nu}{dt}} \quad t_* = h_*^{\mu\nu} \frac{dx_\mu}{dt} \frac{dx_\nu}{dt},
\]

which in turns reduces the Lagrangian to (see footnote)

\[
L = -\mathbf{M} \left[ \gamma_*^{-1} + \frac{U}{2} \gamma_* t_* + O(U^2) \right]
\]

Hughes argued that for the experiment under consideration the potential \( U \) can be regarded as constant, and so the gravitational terms can be absorbed into redefinitions of the mass matrix elements. The apparent potential and velocity dependence of those elements can be partially removed by introducing physical (or local) time and length units (which are also influenced by gravity), that are gathered from instruments that are assumed to

\[\footnotemark\]

\[\footnotetext{Actually eq. (38) is a short notation to express the dependence of the kaon and antikaon on the gravitational field, since \( \gamma \) is also dependent on particle \( a \), and so factors each diagonal term of \( \mathbf{M} \) in a different way. The off-diagonal terms of \( \mathbf{M} \) are irrelevant since the experimental data to be used later on relates the diagonal elements alone.} \]
obey EEP. Actually this is irrelevant when considering the difference between the kaon and antikaon masses, since if the instruments obey EEP they are going to affect the kaon and anti-kaon system in the same way and therefore it will give no contribution to the mass difference.

In our case, in order to make the redefinition of masses possible, we need to introduce the standard of units. We have chosen units such that the speed of light is equal to one, and so a kaon system obeying the EEP can be reduced to the form:

$$L = -\frac{M}{\gamma_0}$$  \hspace{1cm} (46)$$

with $$\gamma_0 = (1 - v^2)^{-1/2}$$. Note that $$\gamma_\ast = T_0^{-1/2}(1 - v^2/c_a^2)^{-1/2}$$, where $$c_a = (T_0/H_0)^{1/2}$$ corresponds to the local limiting speed of the massive $$a$$–type particle. We can then expand

$$\gamma_\ast = T_0^{-1/2}\gamma_0(1 - \xi_a^2 v^2 \gamma_0^2) + O(\xi_a^2)$$  \hspace{1cm} (47)$$

with $$\xi_a = (c_a^2 - 1)/c_a^2$$.

Hence, by comparing (45) to (46), we can introduce an effective mass, up to $$O(\xi_a)O(U)$$ of the form:

$$m_a^{(\text{eff})} = m_a T_0^{1/2} \left( 1 + \frac{\xi_a}{2} v^2 \gamma_0^2 + \frac{U^2}{2 \gamma_0^2 T_0} t_\ast \cdots \right)$$  \hspace{1cm} (48)$$

The CERN experiment with kaons at energies $$\sim 100$$ GeV constrained (see [19] and references therein):

$$|(m(K^0) - m(\bar{K}^0))/m(K^0)| < 5 \times 10^{-18}$$  \hspace{1cm} (49)$$

which along with (48) bounds the difference between the limiting speed of the kaon and the anti-kaon as:

$$|\xi_K - \xi_{\bar{K}}| \simeq |c_K^2 - c_{\bar{K}}^2| < 1.5 \times 10^{-22}$$  \hspace{1cm} (50)$$

Moreover if we assume that there is no fortuitous cancelation between the two EEP violating terms in (48), we can constrain the $$O(U)$$ contribution as:

$$\frac{U}{2} \gamma_0^2 \left| \frac{t_\ast}{T_0} - \frac{\bar{t}_\ast}{\bar{T}_0} \right| < 5 \times 10^{-18}$$  \hspace{1cm} (51)$$
By taking the supercluster potential at the surface of the Earth to be $U \sim 3 \times 10^{-5}$, we can bound the relative non-universal gravity coupling in the neutral kaon system as:

$$\left| \frac{T_0'}{T_0} - \frac{\dot{T}_0'}{T_0} + \frac{\ddot{H}_0'}{T_0} - \frac{H_0'}{T_0} \right| < 10^{-17}$$

(52)

Note that eqs. (50) and (52) bound the relative metric between the neutral kaon and antikaon in terms of the $TH$ functions and their derivatives, evaluated locally. In Hughes’ analysis there is no EEP violation at the level described by eq. (50).

In the absence of gravity the $TH$ functions approach unity, restoring CPT symmetry. We note that violation of EEP in the kaon-antikaon sector need not imply violation of CPT, as has recently been demonstrated [23].

III Concluding Remarks

Up to now, the strongest constraints on the EEP have involved the baryonic sector of the standard model relative to the electromagnetic one. This is not surprising given the relative strength of $10^6$ between nuclear and atomic interactions. We have demonstrated in this paper that many of the experiments which test EEP actually provide us with a broader degree of empirical information over the different sectors of the standard model. However the constraints on EEP-violation in these other sectors are typically much weaker than those in the baryon sector. This difference is manifest in the limits imposed on LPI-violating parameters related to baryons and electrons, which are extracted from torsion balance experiments. These tests look for anomalous gravitational accelerations for an entire atom, and so the effects of possible EEP violations coming from electron-nucleus interactions will be suppressed by the total atomic mass.

In this respect, gravitational redshift experiments provide cleaner tests for leptonic matter insofar as its relative gravitational coupling to electromagnetism is concerned. Such experiments probe atomic energy transitions, where the nucleus plays the role of a static electromagnetic source. In the case of experiments involving the principal, fine or Lamb shift transitions, the effect of any anomalous redshift will be to constrain only the relative gravitational coupling between electrons and photons [22]. In those energy
transitions the effect of the nuclear magnetic moment (and so the nuclear metric) can be neglected. However the most precise gravitational redshift experiment employed hydrogen maser clocks, where the energy shift is produced by the magnetic interaction between the nucleus and the electron, and so a hybrid bound is obtained (27).

By reviewing laboratory tests for variations of the fine structure constant, we were able to obtain the strongest bound on the leptonic LPI violating parameter. Although it involved hyperfine transitions, the empirical output was the hyperfine ratio between two atoms and so cancelled out effects of the nuclear metric (assumed to be the same among nucleons).

The experiment of Prestage (et al.) has the distinct feature amongst Hughes-Drever type experiments of employing hyperfine transitions stemming from the electron-nucleus interaction, as opposed to nucleon-nucleon ones. A re-analysis of the results of this experiment yielded a bound on the leptonic LLI violating parameter to a level comparable to those coming from Lamb shift or $g-2$ experiments [14].

Finally, we employed the $TH\epsilon\mu$ formalism within the context of matter/antimatter experiments. We set new constraints on EEP violations coming from CPT tests in the neutral kaon-antikaon system. The remarkable limit of $10^{-22}$ was obtained for the difference between the limiting speed of kaons/antikaons.

The empirical validity of the EEP must be checked separately for each sector of the standard model. We have shown in this paper how existing experiments can provide us with some empirical information to this end in several non-baryonic sectors. However it is the proposal and design of new experiments that probe the validity of the EEP in the non-baryonic sector that will ultimately provide us with the best empirical foundation for a metric description of gravity. It is our hope that this paper will further motivate such work.

**Acknowledgments**

This work was supported by the Natural Sciences and Engineering Research Council of Canada.
Appendix

The spacetime scale of atomic systems allows one to ignore the spatial variations of $T, H, \epsilon, \mu$, and evaluate them at the center of mass position of the system, $\vec{X} = 0$. After rescaling coordinates, charges, and electromagnetic potentials, the field theoretic extension of the action (1) can be written in the form

$$S = \int d^4x \bar{\psi}(i \gamma^0 \partial_0 - \xi_\star \gamma^0 Q_0)\psi + \frac{1}{2} \int d^4x (E^2 - B^2),$$

where local natural units are used, $Q_\mu \equiv p_\mu - eA_\mu$, $Q = \gamma_\mu Q^\mu$, and $\xi_\star = (1 - c_\star/c_e)$, with $c_\star$ and $c_e$ as the local speed of light and limiting speed of electrons respectively. Note that local natural units were used in action (53), by taking $\hbar = 1$, and $c_\star = 1$ (which is taken to be identical to the baryonic limiting speed $c_B$).

It is clear that under a Lorentz transformation, the electromagnetic part of (53) is invariant but not the fermion–photon interaction sector, which adds to the total density Lagrangian a non metric term of the form:

$$\mathcal{L}_\xi = -\xi_\star \gamma^2 \bar{\psi} \gamma \beta \cdot Q \psi$$

where $\gamma^2 \equiv 1/(1 - \vec{V}^2)$ and $\beta^\mu \equiv (1, \vec{V})$; henceforth $\beta^2 \equiv 1 - \vec{V}^2$, with $V$ the relative velocity between the preferred frame defined by (53) and the laboratory system (Earth).

We are interested in the non relativistic terms of (54). These are

$$\mathcal{L}_\xi \simeq -\xi_\star \gamma^2 \left[ \psi^\dagger \frac{\vec{\sigma}}{2m} \cdot (\vec{\nabla} \times (\beta \cdot Q)\vec{\beta})\phi + \cdots \right]$$

where $\phi$ represents the large component of the Dirac spinor and the ellipsis denotes terms that do not depend on the electron spin.

From (55) we can read the interaction term between the electron spin and an external magnetic field, which added to the metric contributions gives up to $O(\xi_\star O(V^2))$:

$$H_{hf} = -\vec{\mu}_e \cdot \left[ \vec{\nabla} \times A + \xi_\star \vec{\nabla} \times (\vec{V} \cdot A)\vec{V} \right]$$

where in units of $\mu_B = e/2m_e$, $\vec{\mu}_e = g_e\mu_B\vec{J}$, $\vec{J}$ being the electron spin ($J = 1/2$). Recall that the magnetic field produced by the magnetic moment $\vec{\mu}_N = \vec{\mu}_e$.
$g_N\mu_B\vec{I}$ of the nucleus is $\vec{A} = -\frac{1}{4\pi}\vec{\mu}_N \times \nabla I$, where $\vec{I}$ represents the nuclear spin ($I = 1/2$ for hydrogen). We have also introduced the corresponding gyromagnetic ratios for the electron and nucleus.

The $^9\text{Be}^+$ atom can be treated as an alkali atom, where the main interaction governing the hyperfine transitions comes from the magnetic moment interaction between the valence electron and the nucleus only. Hence it can be considered as a hydrogenic atom with $I = 3/2$ and $J = 1/2$. The effects of the electrons in the closed shell are accounted for in the values for the principal quantum number and effective charge of the atom which are empirically determined [24].

Let us start reviewing the Maser clock. Here the relevant atomic transition takes place under a weak magnetic field, which does not break the coupling between the electron and nuclear spin. That is the atomic states can be labeled by the total angular momentum $\vec{F} = \vec{J} + \vec{I}$ and the corresponding quantum number $M_F$. The hyperfine transition in this case is described by $\Delta F = 1$, $\Delta M_F = 0$.

It is straightforward within this $TH\psi\mu$ formalism to obtain from (56) the corresponding hyperfine transition

$$\nu^{(H)}_{hf} = \nu_0 \left[ 1 + \xi \frac{V^2}{6} (1 + P_2(\cos \Theta)) \right]$$

(57)

where $\nu_0 = -\langle \vec{\mu}_e \cdot \nabla \times \vec{A} \rangle |_{\Delta F = 1, \Delta M_F = 0}$. Since experiments search for frequency changes of the form

$$\delta \nu = \xi A \nu_0 P_2(\cos \Theta)$$

(58)

the parameter $A$ in (55), for the maser clock is identified as $A_H = \frac{V^2}{6}$.

The situation is different for the $^9\text{Be}^+$ clock. Here the relevant atomic transition occurs in the presence of a strong magnetic field that breaks the nuclear electron spin coupling. The main energy contribution to the electron comes from the electron spin interaction with that external field and not with the field of the nucleus. Hence the atomic states are described by the quantum numbers $M_I$ and $M_J$. The $^9\text{Be}^+$ hyperfine transition corresponds to a nuclear resonance with $\Delta M_J = 0, \Delta M_I = 1$.

After computing (56) for the relevant states we obtain

$$\nu^{(Be)}_{hf} = \nu_0 \left[ 1 + \xi \frac{V^2}{6} (4 - 2P_2(\cos \Theta)) \right]$$

(59)
for the modified hyperfine transition for the $^9\text{Be}^+$ clock, where $\nu_0 = -\langle \vec{\mu}_e \cdot \vec{V} \times \vec{A} \rangle|_{\Delta M_J=0,\Delta M_I=1}$. We neglected the nuclear spin interaction with the external magnetic field, since it contributes as $O(m_e/m_p)$ when $B \sim 1[T]$. By identifying the former result with (58) we finally obtain: $A_{\text{Be}} = -\frac{V^2}{3}$.

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