On the $0^+$ excited states in even-even deformed nuclei

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Abstract. The low-lying $0^+$ excited states remain an object of particular interest in the nuclear structure physics. Recently long sets of $0^+$ excited states were experimentally observed. Our analysis of the experimental data have shown that in even-even nuclei of the rare-earth and actinide regions the energies of all low lying $0^+$ excited states with great accuracy can be distributed on parabolic functions of the number of monopole excitations building these states. Along with the classification of the energies of the $0^+$ excited states in respect to the number of bosons that build the band heads we analyze the role of their collectivity in the structure and evolution of the yrast bands and the B(E2) transition probabilities within these bands. The experimental determination in $^{160}$Dy of the predicted in this way $0^+$ excited state with energy 0.6813 MeV is presented.

1. Introduction

The complexity in the structure of excited $0^+$ states springs up a variety of approaches ranging from microscopic studies of anharmonic effects [1], consideration of quadrupole and pairing vibrational modes in conversional electrons and internal pair decay [2], or exact diagonalization in the restricted space of collective phonons of different types [3]. On the other hand, the description of the energies and electromagnetic decay properties of the excited states are important tests in the evaluation of the applicability of the different models, like the shell model, cluster-vibrational model, quasi-particle - phonon model, a deformed configuration mixing shell model, the interacting boson approximation and in particular its O(6) limit.

In this work, we suggest an approach for the evaluation of the yrast bands energies and B(E2) probabilities between yrast band states. The basic assumption is, that some of the states with $K^\pi = 0^+$ are the band heads of crossing rotational bands, parts of which form the yrast band and we show the importance of the structure of band head, which determines the value of the moment of inertia of the nucleus in this state. The different behavior of such rotational bands gives us the possibility to consider the yrast band as composed from several crossing rotational bands which start from different band heads with $K^\pi = 0^+$. Hence, our first step is to review in short the energy distribution of the $0^+$ excited states in terms of their collectivity (number of boson excitations) within the same nucleus.
2. Energy distribution of the $0^+$ excited states

Usually the classifications of experimental data is mainly done from a "horizontal perspective" in sequences of nuclei where the investigated nuclear characteristics are empirically studied as functions of the numbers of their valence nucleons. In contrast we analyze the experimental data on the low-lying spectra in terms of the order of collectivity (number of monopole bosons constructing the corresponding $0^+$ excited state) in the same nucleus. Such a classification of large amount of experimental data in terms of integer classification parameter has been done in [4], based on phenomenological collective Hamiltonian for single level approach, written in terms of boson creation and annihilation operators $R_+$, $R_-$ and $R_0$.

$$H = \alpha R_+ R_- + \beta R_0 R_0 + \beta \Omega R_0.$$ (1)

The operators in (1) are constructed from pairs of fermion creation and annihilation operators $a^\dagger$ and $a$ on a single $j-$ level.

$$R_+ = \frac{1}{2} \sum_m (-1)^{j-m} a_m^\dagger a_{j-m}^\dagger,$$

$$R_- = \frac{1}{2} \sum_m (-1)^{j-m} a_{j-m} a_m,$$

$$R_0 = \frac{1}{4} \sum_m (a_m^\dagger a_{j-m} - a_{j-m} a_m^\dagger).$$

$$[R_0, R_\pm] = \pm R_\pm, \quad [R_+, R_-] = 2R_0.$$ (2)

Applying the Holstein-Primakoff transformation on the operators (2) (where $\Omega = \frac{2j+1}{2}$)

$$R_- = \sqrt{2\Omega - b\dagger b} b, \quad R_+ = b^\dagger \sqrt{2\Omega - b\dagger b} R_0 = b^+ b - \Omega$$ (3)

we express them in terms of ideal bosons, defined by $[b, b^\dagger] = 1$ $[b, b^\dagger] = [b^\dagger, b^\dagger] = 0$, where $b|0\rangle = 0$ $|n\rangle = \frac{1}{\sqrt{n!}} (b^\dagger)^n |0\rangle$ and the initial Hamiltonian (1) becomes:

$$H = a^\dagger b^\dagger b - \mathbf{b}^\dagger \mathbf{b} b^\dagger b.$$ (4)

Thus the energy spectrum raised by the Hamiltonian (4) is a parabolic function of the number $n$ of the ideal monopole bosons

$$E_n = a^\dagger a n - b^\dagger b n^2$$ (5)

where $a = (2\Omega + 1)\alpha - \Omega \beta$ and $\mathbf{b} = \alpha - \beta$. There are more realistic models that reproduce similar dependence of the energies of the collective states on the number of collective excitations, for instance, the Interacting vector boson model (IVBM) [5] based on the introduction of two kinds of vector bosons (called $p$- and $n$-bosons), that "built up" the collective excitations in the nuclear system. The Hamiltonian, corresponding to one of the dynamical symmetries of the IVBM [5], expressed in terms of the first and second order invariant operators of the different subgroups of its corresponding chain, is obviously diagonal in the basis labelled by their irreducible representations with eigenvalues:

$$E((N, T); KLM; T_0) = d' N + b' N^2 + \alpha_3 T(T + 1) + \beta_3 L(L + 1) + \alpha_1 T_0^2$$ (6)

where $N$ is the total number of vector bosons, $T, T_0$, are the T-spin, which defines the parity of the states and its third projection and $L$ is the angular momentum of each state. For fixed parity and angular momentum we have obviously an expression similar to (5).

Thus we choose the parabolic function to be a rule for the distribution of the $K^\pi = 0^+$ state energies in the space of monopole bosons. On Figure 1. we show the respective new representation of the sets of recent experimental data [6]. Now we can label every $K^\pi = 0^+$
state by an additional characteristic \( n \), the number of monopole bosons determining it’s collective structure. The parameters \( a \) and \( b \) are evaluated by fitting the experimental energies of the different \( 0^+ \) states of a given nucleus to the theoretical ones applying all possible permutations of the classification numbers \( n \) and evaluating the distribution corresponding to the minimal value of \( \chi^2 \)-square. Often the first excited \( 0^+ \) state is considered as less collective than the next one in increasing energy. In some even-even nuclei the first in energy excited state is not necessarily the state with the smallest \( n \) (degree of collectivity). For instance the \( 0^+ \) state with excitation energy 0.2548 MeV (\( n = 20 \)) observed in \(^{158}\text{Gd} \) [7] is much more collective than the \( 0^+ \) state with energy 0.5811 (\( n = 1 \)) MeV. Similar behavior of the distribution of the energies of \( 0^+ \) states, can be seen on Figure 1 for \(^{180}\text{W} \) and \(^{152}\text{Gd} \), as well as the high accuracy of the distribution of all the experimental states on the second order curves. In comparison, the calculations of the \( 0^+ \) excited states’ energies in \(^{158}\text{Gd} \) within the semi-microscopic approach of the ”quasi-particle - phonon model” [8] are far from the accuracy obtained in our simple approach.

![Figure 1](image.png)

**Figure 1.** Experimental data [6] distributed on parabolic curves, defined by the parameters \( a \) and \( b \) given for each nucleus, in the space of monopole bosons. \( \Delta \) gives the mean square deviation of the experimental data from the theoretical curve (5) \( E_n = a n^2 - b n \).

3. Density distribution and moment of inertia
As a first example of the application of the above results we present the description of the yrast bands of non-magic even-even nuclei. For this purpose we take the nuclear surface oscillation model [9] giving the density distribution of the nucleus placed in \( n \)-boson excited state \( |n\rangle \):

\[
\rho_{nn}(r, R_0, S) = \frac{3\rho_{00}(r, R_0, S)}{4\pi R_0(3S^2 + R_0^2)} + \sum_{k=1}^{n} \binom{n}{k} \left( \frac{\pi R_0^2}{4k^2} \right) \frac{\frac{d^2}{dR_0^2} \rho_{00}(r, R_0, S)}{k!} \frac{n_k R_0^2}{\pi} + \frac{4}{3} \pi (3S^2 + R_0^2) R_0
\]  

(7)
Then, having the density distribution as a function of the number of monopole bosons $n$ and choosing any nuclear form, defined by the nuclear size $R_0$ and the diffuseness parameter $S$, the moment of inertia can be calculated. As a result it also becomes function of number of bosons, through the density distribution (7). We suppose that the shape of the nucleus is an ellipsoid with a spherical core inside and half-axis $c(n)$ and $d(n)$. The moment of inertia corresponding to this nuclear shape (see Figure 2) is $\mathcal{I}(c(n), d(n)) = \mathcal{I}_{\text{ellip}}(c(n), d(n)) - 3_{\text{sphere}}(c(n), c(n))$. In the case of spherical nucleus $d = c$ the moment of inertia $\mathcal{I}(c(n), c(n)) = 0$.

![Figure 2](image)

**Figure 2.** Nuclear shape. For the nucleus in the ground state $n = 0$ the half-axis $c(0)$ and $d(0)$ do not depend on the number of bosons $n$

After introducing the oscillations of the nuclear surface and using the density distribution (7) the moment of inertia of the nucleus with this form becomes:

$$\mathcal{I}(c(n), d(n), x, n) = \frac{M(d(0)^2 - c(0)^2)}{6400\pi^4} (27n^2x^2 - 360n^2x^2\pi + 48n\pi^2(25nx - 2\sqrt{15})x + 6402\sqrt{15}nx^3 + 1280\pi^4).$$

(8)

where $x = \frac{E_0}{C_0}$, $E_0$ - the energy of the one-boson state and $C_0$ - the nuclear surface compressibility parameter in respect to the monopole excitations. In all our calculations we impose for the ratio of the ellipsoid’s half-axis (of any nuclear excited state $|n >$), the condition $\frac{c(n)}{d(n)} = \frac{c(0)}{d(0)} = Const$, where $c(n) = c(0)\left(1 + \sqrt{nx\left(\frac{3}{2\pi} + \frac{9}{40\pi^2}\right)}\right)$ and $d(n) = d(0)\left(1 + \sqrt{nx\left(\frac{3}{2\pi} + \frac{9}{40\pi^2}\right)}\right)$. For the

![Figure 3](image)

**Figure 3.** Yrast line of $^{236}U$ energies described as three crossing rotational bands. $x = 0.054$; red line - ground band, blue line - $n = 3$, green line - $n = 8$.

![Figure 4](image)

**Figure 4.** Comparison of B(E2) values of $^{236}U$ calculated within rigid rotor (dash-dotted lines) and IVBM (full lines) approaches with experiment.
ground band \( n = 0 \) and the moment of inertia for the rest of the \( 0^+ \) excited bands depends only on the number of bosons that build their heads structures. This leads to the possibility of separation of the yrast band into two or more parts belonging to different rotational bands having different values of the moment of inertia because of the difference in the collective structure of the rotational bands heads (see for example Figure 3). In the next Figure 4, we present the comparison of the calculated and experimental \( B(E2) \) values for \( ^{236}U \) nucleus, when the yrast line separation is taken into account.

4. Analysis of the experimental situation in \( ^{160}Dy \)

Calculated energy distributions of the \( 0^+ \) states in the \( ^{160}Dy \) nucleus plotted on Figure 5 for the best fit number of monopole bosons show that apart from the \( 0^+ \) states with the number of bosons \( n = 2, 5, 6, \) and \( 7 \) (stars on Figure 5) currently known from the experiments [10], states with the number of bosons \( n = 1, 3, 4, \) and \( 8 \) (circles in Figure 5) should be observed. In the light of the above-mentioned theoretical predictions of possible existence of new excited \( 0^+ \) states in the \( ^{160}Dy \) nucleus new experimental investigations were carried out with the aim to look for new possible E0 transitions between known and predicted \( 0^+ \) states in the spectrum of internal conversion electrons (ICE) measured at the \( \beta \) decays \( ^{160}Ho \rightarrow ^{160}Dy \) and \( ^{160}Er \rightarrow ^{160}Ho \rightarrow ^{160}Dy \). In order to obtain the ICE spectrum, several photo-emulsion plates (irradiated at JINR DLNP, Dubna, in different years) were measured, using permanent magnet beta-spectrographs [11] at the \( \beta \) decay of Ho and Er fractions. These fractions were separated by means of the chromatographic method [12] from the tantalum target irradiated by a 660 MeV proton beam with an intensity of 2 \( \mu A \) from the DLNP phasotron. The plates were measured by a new technique developed at ITEP (Moscow) and described in detail in the paper [13]. An example of the results of such measurement are illustrated on Figure 6.

The spectrum was calibrated in energy and intensity against the well studied intense single lines of K electrons and three of them are shown on Figure 6. Our more thorough investigations of particular ICE spectrum regions (Figure 7) on the left of the K682.3 keV, corresponding to the known 682.3 keV \( \gamma \) transition in the \( ^{160}Dy \) nucleus [10], revealed an earlier unknown line. Analysis of the available experimental data on \( \gamma \) transitions in the \( ^{160}Dy \) nucleus, showed that in the \( \gamma \) ray spectrum, \( \gamma \) transitions "responsible" for internal conversion electrons which could give rise to the observed unknown line, do not exist.

Therefore, we assumed that this line may correspond to the E0 transition between a new excited \( 0^+ \) state with the energy 681.3 keV and the ground \( 0^+ \) state in \( ^{160}Dy \). We were further motivated by the observation that this energy (681.3 keV) is close in value to the predicted by the theoretical model energy for \( 0^+ \) state built by \( n = 8 \) bosons (see Figure 5). Since population of this \( 0^+ \) state directly from the \( \beta \) decay of \( ^{160}g,mHo \) is practically impossible because of the

![Figure 5.](image-url)
quantum characteristics of the ground and isomeric states with $I^\pi = 5^+$ and $I^\pi = 2^-$ of the parent nucleus, it may only be populated by transitions from higher lying excited states of $^{160}\text{Dy}$. We found at least two such transitions. One of them was the E0 transition with the energy K1271.0 keV observed in the ICE spectrum on the left of the known K1271.89 keV line. It’s energy exactly corresponds to the energy of the transition between the 1952.3 keV and 681.3 keV $0^+$ states. As the analysis showed, in the $\gamma$ spectrum [10] there were no $\gamma$ rays of appropriate energy whose conversion electrons could be responsible for this line. Further analysis made it possible to reveal one more transition which could be treated as a transition populating the

![Figure 6. ICE spectrum in the energy region 578 - 1078 keV](image)

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![Figure 7. Two fragments of the $^{160}\text{Dy}$ ICE spectrum in the electron energy ranges from 625 to 632 keV and from 1215 to 1223 keV.](image)

Figure 7. Two fragments of the $^{160}\text{Dy}$ ICE spectrum in the electron energy ranges from 625 to 632 keV and from 1215 to 1223 keV.
Figure 8. Fragment of the scheme of $^{160}$Dy excited states.

681.3 keV 0$^+$ state. It was the $\gamma$ transition reliably identified in the experiment [10], but not placed in the $^{160m,g}$Ho $\rightarrow$ $^{160}$Dy decay scheme. This transition with energy 1822.4(3) keV fits best between the quite reliably established 2503.8 keV 2$^+$ state and the predicted 681.3 keV 0$^+$ state. As is evident from Figure 8, de-excitation of the 681.3 keV 0$^+$ state is possible and should only be due to the E0 transition to the ground state and its competing E2 transition to the 86.8 keV 2$^+$ level of the ground state band in $^{160}$Dy. The sum of their intensities should completely compensate the sum of the intensities of the transitions populating the 681.3 keV state. In other words, the balance of total intensities of all transitions to and from this level should be zero. This situation is observed for all known 0$^+$ states in the $^{160}$Dy nucleus [10]. The major part of the intensity in the de-excitation of the 0$^+$ states in question is attributed to the E2 transitions to the 86.8 keV 2$^+$ level. Fulfillment of the zero balance condition for the 681.3 keV 0$^+$ state also requires the E2 transition to the 86.8 keV 2$^+$ level. The energy of this transition should be 594.5 keV. Based on the aforesaid, we analyzed the energy region involving the energy 594.5 keV in the $\gamma$ ray spectrum obtained in our measurements. This region is very difficult for analysis because of the insufficient energy resolution, and therefore it is impossible to separate explicitly the 594.5 keV line against the background of two lines with similar energies 593.5 and 595.3 keV. Nor was it done in [10], where only two lines with energies 593.5 and 595.3 keV appear in the list of $\gamma$ rays for the region in question. However, it seems possible to determine the upper limit for the intensity of the 594.5 keV E2 transition. According to our evaluation, the intensity of this transition should not be higher than 0.3 relative units. This evaluation allowed us to calculate the limit for the dimensionless Rasmussen parameter for the 681.3 keV 0$^+$ state $X_{calc.}$ $\approx$ 0.6 and to compare it with the known $X_{exp.}$ for this nucleus. The values of $X_{exp.}$ are presented in Table 1, where $I_{K}(E0)$ is the relative intensity of the E0 transition K line, $\Omega_{K}$ is the electron factor, $E_{\gamma}(E2)$ and $I_{\gamma}(E2)$ are the energy and relative intensity of the E2 transition. It is evident that
Table 1.

| $E(0^+)$ level [keV] | $I_K(E0)$ rel.un. | $\Omega_K$ | $E_\gamma(E2)$ [keV] | $I_\gamma(E2)$ rel.un. | $X_{exp.}$ |
|----------------------|-------------------|-----------|----------------------|------------------------|------------|
| 681.3                | 0.024             | 4.089 · 10^{10} | 594.5                | < 0.3                  | > 0.3      |
| 1280.0               | 0.053             | 7.778 · 10^{10} | 1193.2               | 13.8                   | 0.3        |
| 1456.7               | -                 | 9.068 · 10^{10} | 1369.9               | 40.0                   | -          |
| 1708.2               | 0.033             | 1.103 · 10^{11} | 1621.4               | 12.7                   | 0.6        |
| 1952.3               | 0.015             | 1.310 · 10^{11} | 1865.6               | 6.1                    | 0.9        |

our evaluation of $X_{calc.}$ $\approx$ 0.6 for the predicted 681.3 keV $0^+$ state is in good agreement with $X_{exp.}$ $> 0.3$ for the other $0^+$ states in $^{160}Dy$.

Summing up, we think it possible to deduce that the 681.3 keV $0^+$ state exist in the spectrum of the $^{160}Dy$ nucleus. Naturally, this conclusion would be more convincing if one or more other E0 transitions from upper $0^+$ states to the 681.3 keV level were found. However, if these transitions do exist, their negligibly low intensities make them very difficult to observe. The really low values of the expected intensities follows from the fact that the balance of the intensities of the 681.3 keV level is already almost zero, and any significant addition of intensity from above will result in braking it. Nevertheless, we do not lose interest in searching for E0 transitions and even plan more thorough measurements of ICE spectra.

In conclusion, we hope that our theoretical predictions of vacant places in the distributions of the energies of the $0^+$ states (for example, see Figure 1 and Figure 5), the evaluated energies positions of bands heads and the estimations of E0 transition probabilities between different $0^+$ states in the same nucleus, may be used by experimentalists in identifying the spins, parities and energies of new excited collective states like in the presented case of the $^{160}Dy$ nucleus.

Acknowledgments

The Bulgarian authors wish to acknowledge financial support from Contract Φ – 1501 with the National Science Foundation of the Bulgarian Ministry of Education and Science and a grant for scientific collaboration with the Bogolyubov Laboratory for Theoretical Physics of JINR, Dubna.

References

[1] Silvestre-Brac B and Piepenbring R 1977 Phys. Rev. C 16, N4 1638 ; 1978 Phys. Rev. C 17 N1
[2] Passoja A et al. 1983 Physics Letters 124B, N3 4 157
[3] Kantele Jet al. 1986 Physics Letters 171B N2 3 151
[4] Vladimir P Garistov e-Print Archive:nucl-th/0008067(August 2000);nucl-th/0309058(September 2003) ;
[5] Georgieva A, Ganev H G, Draayer J P and Garistov V P 2009 PEPAN, 40, issue 4, 461
[6] Meyer D A, et.al. 2006 Phys. Rev. C 74, 044309
[7] Lesher S R et al. 2002 Phys. Rev. C66 051305 (R); Aprahamian A et al. 2002 Phys. Rev. C 65 031301(R)
[8] Lo Iudice N et al. 2004 Phys. Rev. C 70, 064316
[9] Garistov V P 1995 IJMP-E 4 N2 371
[10] Adam J et al. 2002 Izv. RAN, Ser. Fiz. 66 1384; Reich C W 2005 Nuclear Data Sheets 105, 557
[11] Abdurazakov A A et al. 1972 Permanent Magnet Beta-Spectrographs, Tashkent, FAN
[12] Molnar F et al. 1973 Fiz. Elem. Chastits At. Yadra 4 (4) 1077-1155
[13] Egorov O K et al., JTF 2003 vol. 46 no. 3 p.870