Neutron-Proton Pairing and its Impact on Gamow-Teller Transitions

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Abstract. We report a systematic study of Gamow-Teller (GT) transitions in the 2pf shell, using the nuclear shell model with a schematic Hamiltonian. The Hamiltonian used contains a quadrupole-quadrupole interaction, isoscalar ($T=0$) and isovector ($T=1$) pairing interactions as well as a single-particle energy term. The objective of the work is to observe the behavior of GT transitions in different isoscalar and isovector pairing scenarios. We also study their corresponding effect on the energy spectra of the parent and daughter nuclei involved in the GT decays. All results are discussed in relation to experimental data. A key conclusion reached is that with the schematic Hamiltonian we use it is essential to suppress the isoscalar pairing mode when treating nuclei with a neutron excess to avoid producing spurious results for the ground state spin and parity. Varying the strength parameters for the two pairing modes is found to exhibit different but systematic effects on GT transition properties and on the corresponding energy spectra.

1. Neutron-Proton Pairing and Gamow-Teller Transitions

Pairing in nuclei derives from the short-range attraction between nucleons. Pairs of nucleons can get close together to optimize the strong short-range attractive force by rotating opposite one another, achieving net orbital angular momentum zero.

Since nucleons occur in two types ($n$ and $p$), they can form four distinct types of correlated (Cooper) pairs ($nn$, $pp$, or $pn$ with either $T=0$ or $T=1$), each of which can in principle have net orbital angular momentum zero and thus exploit pairing correlations. In finite nuclei, however, the neutrons and protons must be in the same major shell if they are to exploit $pn$ pairing by coupling to zero orbital angular momentum. Otherwise, only $nn$ and $pp$ pairing is important, as in typical nuclei with a neutron excess.

It is generally accepted that $T=0$ $np$ pairing correlations are most important when $N \approx Z$ and when there are a reasonable number of active neutrons and protons [1], precisely the region in which quadrupole correlations are important and where deformation is often present. While mean-field methods are often used to treat the interplay of the various pairing modes in nuclei, it is known that they can lead to serious errors especially near $N = Z$ because of their violation of symmetries [2]. [Note: Recent efforts [3] to build a symmetry-restored mean field approach have proven promising.] For this reason it is especially useful to study how the various modes of pairing compete in nuclear systems near $N = Z$ in the context of the nuclear shell model,
whereby it is possible to treat all pairing modes on an equal footing, to preserve all symmetries, and also to naturally incorporate the effects of deformation.

Several years ago we initiated a program to study the interplay of the various pairing modes in nuclei near $N = Z$ using a simple shell model treatment [4]. We used a schematic Hamiltonian in which it was possible to systematically assess how the different modes impacted nuclear properties. In those earlier works we focused on their impact on energy spectra and to a lesser extent on electromagnetic transitions.

Our earlier work focused on the properties within a given nucleus. However, pairing correlations are often probed best by studying the relation between neighboring nuclei. In the case of $np$ pairing correlations, a particularly useful way to study them experimentally is through Gamow-Teller transitions between an even-even nucleus and its odd-odd neighbor.

Gamow-Teller transitions [5] are mediated by the $\sigma \tau$ operator. They are characterized as isovector-type spin-flip transitions with $\Delta T = 1$, $\Delta S = 1$, $\Delta L = 0$ and $\Delta J = 1$. The population of low-lying $1^+$ states in the odd-odd nucleus produced by such a transition is an especially important signal of isoscalar pairing correlations. These transitions are typically observed either directly through $\beta^-$ decay or through Charge Exchange (CE) Reactions such as $(p,n)$ or $(^3He,t)$. The use of CE Reactions has the advantage of not being limited by Q value considerations. Thus, most of the experimental information we will be discussing has come from Charge Exchange Reactions, and especially from $(^3He,t)$.

A summary of the presentation is as follows. In Section II, we briefly describe our model and discuss how we choose the optimal parameters that provide the starting point of our analysis. In Section III, we discuss the results of that analysis, first for Gamow-Teller transitions and then for energy spectra. Finally in Section IV we summarize the principal conclusions that emerged.

2. Our Model

Our model consists of neutrons and protons restricted to the orbitals of the $2p1f$ shell outside a doubly-magic $^{40}Ca$ core and interacting via a schematic Hamiltonian

$$\hat{H}_2 = \sum_i \varepsilon_i \hat{n}_i + \chi \left( \hat{Q} \cdot \hat{Q} + a \hat{P}^\dagger \cdot \hat{P} + b \hat{S}^\dagger \cdot \hat{S} \right).$$ (1)

Here $\hat{Q} = \hat{Q}_n + \hat{Q}_p$ is the quadrupole mass operator and $\hat{Q} \cdot \hat{Q}$ is the two-body part of the quadrupole-quadrupole operator. Also $\hat{P}^\dagger$ is the operator that creates a correlated pair with $L = 0$, $S = 1$, $J = 1$, $T = 0$, whereas $\hat{S}^\dagger$ is the operator that creates a correlated pair with $L = 0$, $S = 0$, $J = 0$, $T = 1$. Finally, the first term is the contribution of single-particle energies to the Hamiltonian.

2.1. Optimal Parameters of the Hamiltonian

Our approach is to first identify an optimal set of parameters for our model Hamiltonian and then to systematically vary those associated with the two pairing terms to see how they impact Gamow-Teller transitions in this region. We will also study their systematic effect on energy properties for the nuclei involved in those transitions.

For the single-particle energies of our model, we have chosen to use those from the KB3 shell model Hamiltonian [6], namely $\varepsilon_{7/2} = 0.0MeV$, $\varepsilon_{3/2} = 2.0MeV$, $\varepsilon_{1/2} = 4.0MeV$ and $\varepsilon_{5/2} = 6.5MeV$. In the earlier work in which we focussed primarily on energy spectra we used a different set of single-particle energies. There our single-particle energies had two contributions, one coming from the one-body part of the quadrupole-quadrupole interaction and the other coming from the one-body part of a spin-orbit interaction.

As to the strength of the quadrupole-quadrupole interaction we likewise used a slightly different value than in our earlier work. The value we use here, $\chi = -0.065 \ MeV$, is better able
to produce a meaningful Gamow-Teller fragmentation pattern, as we will see in the subsequent section.

Next we turn to the optimal values of the isoscalar and isovector pairing strengths. Here we have found it necessary to make two different choices, one for $N = Z$ nuclei and one for $N \neq Z$ nuclei, for reasons we now describe.

In the case of $N = Z$ nuclei, we find that a reasonable choice for these parameters is $a = b = 6$. The rationale for this choice can be seen in Fig. 1, where we show the calculated energy spectrum for $^{42}\text{Sc}$ as a function of the two pairing strengths $a$ and $b$ in comparison with the experimental spectrum. As is readily evident from the figure, the choice of $a = b = 6$ provides almost perfect reproduction of the energy of the lowest $1^+$ state in this nucleus, while also providing a good description of most of the other low-lying states. The fact that it cannot reproduce the energy of the lowest $7^+$ state is related to the simplicity of the schematic Hamiltonian we use, as was discussed in ref. [4]. Note that these shell-model calculations and all others reported herein have been carried out using the ANTOINE shell-model program [7, 8, 9]. Likewise all experimental spectra shown are from ref. [10].

Next we consider $N \neq Z$ nuclei. In Figs. 2 and 3 we show the calculated energy spectra for two such nuclei, $^{44}\text{Sc}$ and $^{48}\text{V}$, again in comparison with the experimental data. What we see is that with the same choice of $a = b = 6$ as above we obtain a $1^+$ ground state for these two nuclei, whereas in the first case the correct ground state spin and parity is $2^+$ and in the second $4^+$. It is only by turning off isoscalar pairing, i.e. setting $a = 0$, that we obtain the correct ground state spin and parity.

Now that we have chosen our optimal Hamiltonian parameters, we are in a position to proceed with the primary goal of the project: to systematically vary the two pairing strengths so as to assess how they impact GT transition rates and (to a lesser extent) the associated energy spectra.

3. Results

3.1. Impact of different pairing modes on GT transition strengths

3.1.1. $A=42$ The GT transition strengths for $^{42}\text{Ca} \rightarrow ^{42}\text{Sc}$ are shown in Fig. 4, as functions of the isovector pairing strength $b$, which applies to both the parent and daughter nucleus, and the isoscalar pairing strength $a$, which only contributes to the properties of the $^{42}\text{Sc}$ daughter nucleus. The calculated results are compared with the experimental data from ref. [11].

As we can see from the figure, increasing the isoscalar pairing strength $a$ leads to an increase in the strength that is focussed on the lowest state, albeit slowly. At the same time, the energy of that peak goes down, eventually becoming the ground state. As the isovector strength $b$ is increased (for a given value of $a$), the main peak moves up in energy with no noticeable change in its strength.

3.1.2. $A=44$ Fig. 5 depicts the GT intensities for $^{44}\text{Ca} \rightarrow ^{44}\text{Sc}$. As both nuclei have a neutron excess, we set $a = 0$ for both and vary the isovector pairing strength $b$ only, once again comparing the calculated results with those from experiment [12].

Unlike for the $A = 42$ decay, the experimental data exhibits satellite peaks at low energy, and this is likewise produced by the calculations. For $b = 6$, the calculation produces greater enhancement of lowest peak than seen in the data. The lowest peak moves up in energy as $b$ is increased, and becomes progressively more dominant.

3.1.3. $A=46$ Next, we analyze the GT results obtained for the nuclei with mass $A = 46$. While the parent nucleus involved in the GT decay $^{46}\text{Ti} \rightarrow ^{46}\text{V}$ has a neutron excess, the daughter nucleus does not. Thus we assume $a = 0$ for the parent and present the results as a function of the $a$ value used in describing the daughter only. In addition, the results are shown as a function
Figure 1. Energy spectra of $^{42}$Sc as a function of the isovector $b$ and isoscalar $a$ pairing strengths in comparison with the experimental data. Results are shown for $a, b = 0, 6, 12$.

of the isovector pairing strength $b$ used for both nuclei. These results are shown in Fig. 6, with the experimental data taken from [13].

The key points that can be seen in the figure are: (1) Strong fragmentation is produced in the calculation for the optimal $a = b = 6$ values; (2) the effect of increasing $a$ is - as for lighter decays - to focus strength in the lowest state while lowering its energy, and (3) the effect of increasing $b$ is to enhance the population of the lowest state while moving it up in energy.

3.1.4. $A=48$ Finally, we treat the GT transitions in $A = 48$. In this case the relevant decay is $^{48}$Ti $\rightarrow$ $^{48}$V, for which the parent and daughter nuclei both have a neutron excess. Thus, in Fig. 7, where we compare the experimental [14] and calculated transition rates, the theoretical analysis is only shown as a function of the isovector pairing strength $b$.

The first point to note is that the model with $b = 6$ produces its lowest excitation in good
Figure 2. Energy spectra of $^{44}$Sc as a function of the isovector $b$ and isoscalar $a$ pairing strengths in comparison with the experimental data [10]. Results are shown for $a, b = 0, 6,$ and 12.

agreement with experiment, but the level of fragmentation is not well reproduced. The strength is concentrated in a single state near 2 MeV, but with substantially more strength than in experiment. As $b$ is increased, the overall effect is to increase the level of fragmentation across an increasingly larger range of excitations.

3.2. Impact of different pairing modes on energy spectra

Next we ask: What is the impact of isoscalar and isovector pairing on energy spectra? We will focus our analysis on the $N = Z$ nucleus $^{46}$V, where both are relevant. The results are shown in Fig. 8 for this nucleus, but the same general conclusions apply for all other $N = Z$ nuclei.

The first point to note is that when both pairing strengths are reduced simultaneously, the spectrum energies are reduced. When only the isoscalar strength $a$ is reduced, no appreciable effect is seen on the even-$J$
Figure 3. Energy spectra of $^{48}$V as a function of the isovector $b$ and isoscalar $a$ pairing strengths in comparison with experimental data [10]. Results are shown for $a, b = 0, 6, \text{ and } 12$.

states, but the the odd-J states are compressed and the ground state gradually changes from $0^+$ to $1^+$.

In contrast, when the isovector strength $b$ is reduced, there is no appreciable effect seen on the odd-J states but the even-J states are gradually compressed,

4. Concluding Remarks
In this work, we have carried out a shell-model study of GT transitions in the $2p1f$ shell from $A = 42 - 48$ using a schematic shell-model Hamiltonian that includes a quadrupole-quadrupole interaction, isovector and isoscalar pairing interactions and single-particle energies.

We first obtained the optimal parameters of our schematic Hamiltonian by focusing on the lowest $1^+$ state and the other low-lying states in these nuclei and also on obtaining a reasonable GT fragmentation pattern. We found that to be able to meaningfully reproduce these features with this model it was necessary to choose a different set of optimal parameters for $N = Z$ nuclei.
Figure 4. Comparison of the experimental [11] and theoretical results for B(GT) transition strengths for $^{42}\text{Ca} \rightarrow ^{42}\text{Sc}$ as a function of the isovector pairing strength $b$ and the isoscalar pairing strength $a$, which only acts in the daughter nucleus.

Figure 5. Comparison of the experimental [12] and theoretical results for B(GT) transition strengths for $^{44}\text{Ca} \rightarrow ^{44}\text{Sc}$ as a function of the isovector pairing strength $b$. Since both nuclei involved have neutron excesses, the isoscalar pairing strength is set to $a = 0$.

Figure 6. Comparison of the experimental [13] and theoretical results for B(GT) transition strengths for $^{46}\text{Ti} \rightarrow ^{46}\text{V}$ as a function of the isovector pairing strength $b$ and the isoscalar pairing strength $a$, which only acts in the daughter nucleus.
Figure 7. Comparison of the experimental [14] and theoretical results for B(GT) transition strengths for $^{48}$Ti $\rightarrow$ $^{48}$V as a function of the isovector pairing strength $b$. Since both nuclei involved have neutron excesses, the isoscalar pairing strength is set to $a = 0$ for both.

Figure 8. Energy spectra of $^{46}$V as a function of the isovector $b$ and isoscalar $a$ pairing strengths in comparison with the experimental data [10]. Results are shown for $a$, $b = 0$, 6, and 12.
and $N \neq Z$ nuclei. Only by turning off the isoscalar pairing term for $N \neq Z$ nuclei were we able to obtain the correct ground state spins for such nuclei.

With our optimal Hamiltonian in hand, we then systematically varied the strengths of the active pairing terms to see how this impacted both the GT fragmentation pattern and the associated spectral properties.

For the GT fragmentation pattern the key effects seen were: (1) that the lowest peak moved up in energy as the isovector pairing strength was increased and became progressively more dominant, and (2) that the lowest peak moved down in energy as the isoscalar pairing strength was increased (when it was active) and it became progressively more dominant.

When studying the spectral properties as a function of the two pairing strengths, we found (1) that when the isovector pairing strength was reduced, no appreciable effect on odd-J states was observed, but that the even-J states were compressed, and (2) when the isoscalar pairing strength was reduced (where active), no appreciable effect on the even-J states was observed, but the odd-J states were compressed.

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