Spin polarization observables of the deuteron photodisintegration at low energies in pionless effective field theory

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Abstract

Spin polarization observables of the deuteron photodisintegration at low energies are studied in a pionless effective field theory up to next-to-next-to-leading order (NNLO). The total and differential cross sections, induced neutron polarization $P_{y'}$, and tensor analyzing powers $T_{20}$ and $T_{22}$ of the process are calculated at photon energies from the breakup threshold to 20 MeV. We find that the NNLO corrections in the cross sections and $P_{y'}$ converge well whereas they turn out to be important contributions in $T_{20}$ and $T_{22}$. We discuss the discrepancy between theory and experiment in $P_{y'}$ still persisting as well as an implication of our result to the first measurement of $T_{20}$ at low energies in the HIGS facility.

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I. INTRODUCTION

Observables related with polarization provide more detailed information on nuclear reactions than the unpolarized ones [1]. There still remain couples of problems which show discrepancy between experiment and theory even in few-nucleon systems at low energies [2–4]. Induced neutron polarization \( P_y' \) of the photodisintegration of deuteron is an example and one can find more information in the HIGS2 proposal [5]. Proposal for the photodisintegration of deuteron at HIGS is focusing on the role of the final state. Tensor analyzing power \( T_{20} \) is chosen for the investigation, which depends on the \( d \)-state of the deuteron wavefunction as well as the polarization states. Main objective of the experiment is to figure out or resolve the discrepancy between experiment and theory at photon energies around 10 MeV, and obtain precise data for \( T_{20} \) which has not been measured yet in the low-energy regime.

Pionless effective field theory (EFT) for low energy phenomena, in which the pion can be treated as a heavy degree of freedom and integrated out of the effective Lagrangian, provides us a model-independent and perturbative calculation method [6–8]. An effective Lagrangian of the pionless EFT can be constructed by using only the symmetry property of the system and momentum expansion in the low energy. The pionless EFT is able to successfully explain many low energy properties of nuclear two-body systems with and without external electromagnetic probes [7–10]. Because the measured induced neutron polarization \( P_y' \) in \( \gamma + d \rightarrow n + p \) and theoretical calculation using phenomenological potential model show a discrepancy [2, 4], it is desirable to have a model independent calculation.

In this work, we compute the photodisintegration cross section of deuteron by using a pionless EFT in dibaryon formalism [8, 11, 12] up to next-to-next-to-leading order (NNLO) which includes \( sd \) wave mixing effects of the deuteron wavefunction. Dibaryon fields which have the quantum numbers of two-nucleon systems in either scattering or bound states are introduced to facilitate the resummation of effective range effects to infinite order. The details of formalism up to next-to-leading order (NLO) for the photodisintegration of deuteron have been reported in [4, 8, 13]. The result of pionless EFT up to NLO in [4] shows discrepancy with the experimental data of \( P_y' \) [14], similar to the phenomenological nuclear force model calculations of [1, 2], but the result is different from the potential model calculation with increasing photon energy. Even though the unpolarized cross sections could be well
described up to NLO, the spin-dependent observables can be sensitive to the higher order corrections. The $d$ state in the deuteron wavefunction could be properly accounted when we increase the expansion up to NNLO. An NNLO calculation in the pionless EFT with dibaryon formalism for the electrodisintegration of deuteron has been reported by Christlmeier and Grießhammer [15]. Because the photodisintegration of deuteron has the same electromagnetic hadronic currents of the electrodisintegration of deuteron up to NNLO, we employ the expression of the hadronic currents reported in [15]. Thus, we calculate the total cross section, the differential cross section, $P_y$, $T_{20}$ and $T_{22}$ from threshold to 19.8 MeV in photon energy in the center of mass frame. We compare the results up to NNLO with those of NLO, and other theoretical results. Experimental data are compared when available.

The paper is organized as follows. In Sect. II we present basic formalism of the pionless EFT and the analytic forms of the transition amplitudes and probabilities. In Sect. III numerical results and related discussions are given. In Sect. IV we summarize the present work.

II. FORMALISM

In this work, we employ the standard counting rules of the pionless EFT, whose expansion parameter, typical momentum divided by the pion mass, satisfies $Q \sim 1/3$ [6, 7]. In this section, we display the Lagrangian up to NNLO and the expression of the cross sections, and obtain the spin polarization observables of the photodisintegration of deuteron.

A. Lagrangian

Effective Lagrangian in the pionless EFT up to NNLO can be written as [8]

$$\mathcal{L} = \mathcal{L}_N + \mathcal{L}_s + \mathcal{L}_t + \mathcal{L}_{st},$$

(1)

where $\mathcal{L}_N$ is a standard one-nucleon Lagrangian, $\mathcal{L}_{s,t}$ is a Lagrangian for two-nucleon part in $s$-wave spin singlet and triplet channel, respectively, and $\mathcal{L}_{st}$ is a Lagrangian for spin mixing
channel. Thus, one has

\[ \mathcal{L}_N = N^\dagger \left[ iD_0 + \frac{\vec{D}^2}{2M} + \frac{e}{2M}(\kappa_0 + \kappa_1 \tau_3) \vec{\sigma} \cdot \vec{B} \right] N, \]  

(2)

\[ \mathcal{L}_s = -s_a^\dagger \left[ iD_0 + \frac{\vec{D}^2}{4M} - \Delta_s \right] s_a - y_s \left[ s_a^\dagger N^T P_a^{(1S_0)} N + h.c. \right], \]  

(3)

\[ \mathcal{L}_t = -t_i^\dagger \left[ iD_0 + \frac{\vec{D}^2}{4M} - \Delta_t \right] t_i - y_t \left[ t_i^\dagger N^T P_i^{(2S_1)} N + h.c. \right], \]  

(4)

\[ \mathcal{L}_{st} = \frac{eL_1}{M \sqrt{\tau_0 \rho_d}} \left[ t_i^\dagger s_3 B_i + h.c. \right], \]  

(5)

where \( N \) is the nonrelativistic nucleon field, and \( s_a \) and \( t_i \) are the dibaryon fields in the \( 1S_0 \) and \( 3S_1 \) states, respectively. The dibaryon fields couple with two nucleons in each partial waves or with other dibaryon fields. Spin projection operators, \( P_a^{(1S_0)} \) and \( P_i^{(2S_1)} \), for the \( s \)-wave spin singlet and triplet channels are given as

\[ P_a^{(1S_0)} = \frac{1}{\sqrt{8}} \sigma_2 \tau_2 \tau_a, \quad P_i^{(2S_1)} = \frac{1}{\sqrt{8}} \sigma_2 \sigma_i \tau_2, \]  

(6)

where \( \sigma_i \) and \( \tau_a \) are the Pauli matrices for spin and isospin spaces, respectively. The projection operators satisfy the normalization condition

\[ \text{Tr} \left( P_j^\dagger P_k \right) = \frac{1}{2} \delta_{jk}. \]  

(7)

In addition, a spin projection operator for \( sd \) wave mixing channel is given as

\[ O_{xy,j} = \frac{1}{4} \left( \vec{D}_x \vec{D}_y P_j^{(3S_1)} + P_j^{(3S_1)} \vec{D}_x \vec{D}_y - \vec{D}_x P_j^{(3S_1)} \vec{D}_y - \vec{D}_y P_j^{(3S_1)} \vec{D}_x \right), \]  

(8)

where \( D_\mu \) is a covariant derivative, \( D_\mu = \partial_\mu + ieQ_{em}A_\mu \) with a charge operator \( Q_{em} \), the electric charge \( e \), and a photon field \( A_\mu \) (and \( \vec{B} = \vec{\nabla} \times \vec{A} \)). Three parameters, \( M, \kappa_0, \) and \( \kappa_1, \) appear in the one-nucleon part of the Lagrangian: \( M \) is the nucleon mass, and \( \kappa_0 \) and \( \kappa_1 \) are magnetic momenta of the nucleon for isosinglet and isovector channels, respectively, \( \kappa_0 = 0.44 \) and \( \kappa_1 = 2.35 \). Six parameters, \( \Delta_s, \Delta_t, y_s, y_t, C_{sd}, \) and \( L_1, \) appear in the two-nucleon part. First four parameters are fixed by the effective range parameters: scattering length and effective range for each \( NN \) scattering channel. Thus, one has \[ x \]

\[ \Delta_s = \frac{2}{Mr_0} \left( \frac{1}{\alpha_0} - \mu \right), \quad \Delta_t = \frac{2}{Mr_d} \left( \gamma - \frac{\rho_d}{2} \gamma^2 - \mu \right), \]

\[ y_s = \frac{\sqrt{8\pi}}{M\sqrt{r_0}}, \quad y_t = \frac{\sqrt{8\pi}}{M\sqrt{\rho_d}}, \]  

(9)
where $\mu$ is a dimensional parameter from the power divergence subtraction (PDS) regularization scheme of the loop diagrams $^{1}$ $a_0$ and $r_0$ are scattering length and effective range in $^1S_0$ channel, $a_0 = -23.71$ fm and $r_0 = 2.73$ fm. $\gamma$ is the deuteron binding momentum, $\gamma = \sqrt{M B} = 45.70$ MeV with the deuteron binding energy $B$, $B = 2.225$ MeV, and $\rho_d$ is effective range in the deuteron channel, $\rho_d = 1.764$ fm. The remaining two parameters, $C_{sd}$ and $L_1$, are fixed by using asymptotic ratio $\eta_{sd} = 0.0254$ of $s/d$ wave of deuteron wavefunction and thermal neutron capture cross section of proton, respectively. We obtain $C_{sd} = \frac{6\sqrt{\pi} \eta_{sd}}{\sqrt{M \gamma^2}}$ and $L_1 = -4.41$ fm.

The amplitude $\mathcal{A}$ of photodisintegration, $\gamma + d \to n + p$, can be written as

$$\mathcal{A} = \epsilon^{(\gamma)}_\mu(q) J^\mu_{\text{hadr}}$$

where $\epsilon^{(\gamma)}_\mu(q)$ is a polarization vector of the photon with four-momentum $q$, and $J^\mu_{\text{hadr}}$ is a hadronic current. Hadronic current $J^\mu_{\text{hadr}}$ is divided into electric and magnetic currents, $J^{(E)\mu}_{\text{hadr}}$ and $J^{(M)\mu}_{\text{hadr}}$, which are given as

$$J^{(E)\mu}_{\text{hadr}} = i \frac{\sqrt{Z}}{\sqrt{8}} (N^i_p \sigma^i \sigma^2 N^*_n) \epsilon^j_{(d)} J^\mu_{E,ij},$$

$$J^{(M)k}_{\text{hadr}} = \frac{\sqrt{Z}}{\sqrt{8}} \epsilon^{ijk} \epsilon^i_{(d)} (N^i_p \sigma^2 N^*_n) J^j_{M},$$

where $N_p$ and $N_n$ are the Pauli spinors of the proton and neutron, and $Z$ is a wavefunction normalization factor of the deuteron, $Z = \sqrt{\frac{\rho_d}{\rho_d}}$. Since the isoscalar magnetic moment is smaller than the isovector magnetic moment ($\kappa_0/\kappa_1 \lesssim Q$), isoscalar magnetic currents are treated as a numerically higher order in this work and thus not included in the currents. The hadronic magnetic currents consist of contributions at leading order (LO) and NLO (See Fig. 1), while the electric currents have LO (see Fig. 2) and NNLO (see Fig. 3) contributions which come from the mixing of $s$ wave and $d$ wave. The explicit expression of the hadronic currents in Eq. (11) can be found in the appendix of Ref. $^{15}$. $^{2}$

$^{1}$ In the calculation of loop integrals for hadronic currents in Ref. $^{15}$, the power divergence subtraction scheme is combined with dimensional integration in spatial dimensions after using contour integration for the energy part. For more details, one may refer to Appendix in Ref. $^{15}$.

$^{2}$ Since no explicit multipole expansion is done and plane wave is used for final nucleons, the currents in Eq. (11) contains various multipole amplitudes including dominant $M1$ and $E1$ amplitudes.
FIG. 1: Diagrams for LO (a-c) and NLO (d) of isovector magnetic currents (denoted as MV). Double line represents dibaryon field of spin triplet (t) and singlet (s). Square of (d) represent the NLO \( L_1 \) term. Only \( ^1S_0 \) final states contribute to the magnetic currents in (c-d) while (a-b) includes spin singlet partial waves with \( L = 0, 1, \ldots \).

FIG. 2: Diagrams for LO electric currents. Double line represents triplet dibaryon field. While only \( ^3S_1 \) final state contributes in (b-c), other partial wave contributions are included in (a) which is dominated by the isovector E1 amplitude to final spin triplet P-wave states.

FIG. 3: Diagrams for NNLO electric currents (denoted as E). Double line represents triplet dibaryon field. Black blob represents the \( sd \) mixing \( C_{sd} \) contribution. While only \( ^3S_1 \) final state contributes to (d,e,g,h) and only \( ^3D_1 \) final state contribute to (b,c), (a,f) include other spin triplet partial waves contributions too.
B. Cross section and observables

An unpolarized differential cross section for the $\gamma + d \rightarrow n + p$ process is given in terms of an amplitude $A_{m_n, m_p, \lambda, m_d}$ as

$$\frac{d\sigma_0}{d\Omega_p} = \frac{\alpha}{4\pi} \frac{p_p E_p}{E_\gamma} \sum_{m_n, m_p, \lambda, m_d} |A_{m_n, m_p, \lambda, m_d}|^2,$$

(12)

where $\lambda$ denotes the polarization of photon and, $m_d, m_n, m_p$ are deuteron, neutron, and proton spin projections. The unpolarized differential cross section in the center of mass frame with non-relativistic approximation is obtained by summing the spin states. $\alpha$ is the fine structure constant, $E_\gamma$ is the photon energy in the center of mass frame. The magnitude of three-momentum $p_p$ and the energy $E_p$ of the proton read

$$p_p = \frac{1}{2} \sqrt{(E_\gamma + \sqrt{M_d^2 + E_\gamma^2})^2 - 4M^2}, \quad E_p = \sqrt{M^2 + p_p^2},$$

(13)

where $M_d$ is the deuteron mass. From now on, the spin indices of the amplitude will be implied for the amplitude $A$ for convenience. Any spin dependent observables can be written in appropriate combination of differential cross section $\frac{d\sigma}{d\Omega_p}(m_n, m_p, \lambda, m_d)$.

In this work, we will focus on the spin polarization observables: the induced polarization $P_{y'}$ of the neutron, and the tensor analyzing powers $T_{20}$ and $T_{22}$ in the deuteron photodisintegration. Let us choose the incoming photon momentum as $z$-direction $\hat{q} = \hat{z}$ which is also a spin quantization axis and then introduce a second reference frame $\hat{x}'$, $\hat{y}'$, and $\hat{z}'$ such that the direction of the outgoing neutron momentum is along the $z'$ axis, $\hat{z}' = \hat{p}_n$, and $\hat{y}' \propto \hat{q} \times \hat{p}_n$ for convenience. In the form of components, $\hat{x}'$, $\hat{y}'$ and $\hat{z}'$ are represented as $(\cos \theta_n \cos \phi_n, \cos \theta_n \sin \phi_n, -\sin \theta_n)$, $(-\sin \phi_n, \cos \phi_n, 0)$ and $(\sin \theta_n \cos \phi_n, \sin \theta_n \sin \phi_n, \cos \theta_n)$, respectively. Thus, $P_{y'}$ is defined in terms of polarized differential cross sections,

$$P_{y'}(\theta_n) \equiv \frac{\sigma_{+y'}(\theta_n) - \sigma_{-y'}(\theta_n)}{\sigma_{+y'}(\theta_n) + \sigma_{-y'}(\theta_n)},$$

(14)

where $\sigma_{+y'}(\sigma_{-y'})$ is a differential cross section in which the spin of outgoing neutron is parallel (anti-parallel) to the $\hat{y}'$ direction.

General form of polarized deuteron cross section with unpolarized photons is given as [17]

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma_0}{d\Omega} \left[ 1 + \sum_{l=1,2} \sum_{M \geq 0} P^l M_0(\theta_d) \cos(M(\phi_d - \phi) - \delta_{l1} \frac{\pi}{2}) \delta_{lM_0(\theta_d)} \right],$$

(15)

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where $P_I^d$ and $T_{IM}(\theta)$ are orientation parameters and analyzing powers, respectively, for $I = 1, 2$. $(\theta, \phi)$ represents a direction of the outgoing proton, the deuteron is oriented in a direction $(\theta_d, \phi_d)$, and $d_{IM}^d$ is a rotation matrix. When a density matrix of the deuteron is diagonal in a quantization axis $\rho_{m'm}^d = p_m \delta_{m'm}$ where $p_m$ is a probability of finding a deuteron with a spin projection $m$, orientation parameters $P_I^d$ are related to $p_m$ as

$$P_1^d = \sqrt{\frac{3}{2}}(p_1 - p_{-1}), \quad P_2^d = \sqrt{\frac{1}{2}}(1 - 3p_0).$$  \hspace{1cm} (16)$$

From the rotation matrix, let us define polarized cross sections $d\sigma^\circ$ for the deuteron polarization axis orienting to $(\theta_d = \pi/2, \phi_d = 0)$, $d\sigma^\uparrow$ to $(\theta_d = \pi/2, \phi_d = \pi/2)$ and $d\sigma^\downarrow$ to $(\theta_d = \pi/2, \phi_d = -\pi/2)$. Thus, we have

$$\frac{d\sigma^z}{d\Omega} \equiv \frac{d\sigma_0}{d\Omega} \left[ 1 + P_2^d T_{20}(\theta) \right],$$

$$\frac{d\sigma^\circ}{d\Omega} \equiv \frac{d\sigma_0}{d\Omega} \left[ 1 + P_2^d T_{22}(\theta) \sqrt{\frac{3}{8}} + P_2^d T_{20}(\theta)(-\frac{1}{2}) \right],$$

$$\frac{d\sigma^{\uparrow,\downarrow}}{d\Omega} \equiv \frac{d\sigma_0}{d\Omega} \left[ 1 + P_1^d T_{11}(\theta)(\mp \frac{1}{\sqrt{2}}) + P_2^d T_{22}(\theta)(-\sqrt{\frac{3}{8}}) + P_2^d T_{20}(\theta)(-\frac{1}{2}) \right].$$  \hspace{1cm} (17)$$

The tensor analyzing powers, thus, can be obtained from the polarized cross sections as

$$T_{11} = \frac{1}{\sqrt{2}P_1^d} \frac{d\sigma^{\uparrow,\downarrow}}{d\sigma^0},$$

$$T_{20} = \frac{1}{P_2^d} \left( 2 - \frac{d\sigma^\circ}{d\sigma^0} + \frac{1}{2} \frac{d\sigma^{\uparrow,\downarrow}}{d\sigma^0} \right) = \frac{1}{P_2^d} \left( \frac{d\sigma^z}{d\sigma^0} - 1 \right),$$

$$T_{22} = \frac{\sqrt{2}}{\sqrt{3}P_2^d} \frac{d\sigma^\circ - \frac{1}{2} d\sigma^{\uparrow,\downarrow}}{d\sigma^0}.  \hspace{1cm} (18)$$

where $d\sigma^{\pm\pm} = d\sigma^\uparrow \pm d\sigma^\downarrow$. By choosing $p_1 = 1$, we have $P_1^d = \sqrt{\frac{3}{2}}$, $P_2^d = 1/\sqrt{2}$. This corresponds to the choice of $\vec{e}(d)$ in Eq. (11) such that $\vec{e}(d) = -\frac{1}{\sqrt{2}}(0, +i, -1)$ for $d\sigma^\circ$, $\vec{e}(d) = -\frac{1}{\sqrt{2}}(-i, 0, -1)$ for $d\sigma^\uparrow$, $\vec{e}(d) = -\frac{1}{\sqrt{2}}(+i, 0, -1)$ for $d\sigma^\downarrow$, and $\vec{e}(d) = -\frac{1}{\sqrt{2}}(1, +i, 0)$ for $d\sigma^z$.

III. NUMERICAL RESULT AND DISCUSSION

A. Total and differential cross sections

In Fig. 4 we plot curves of the total cross section calculated up to NLO and NNLO and include experimental data [18–27] as well. With the parameters fixed to low-energy data,
FIG. 4: (Color Online) Unpolarized total cross section $\sigma_{tot}$ of deuteron photodisintegration up to NLO and NNLO. Experimental data [18–27] are also displayed in the figure.

The curves up to NLO and NNLO give the results consistent with the data to $E_{\gamma}^{lab} = 30$ MeV. The correction from NNLO is about 10% of the contribution up to NLO at $E_{\gamma}^{lab} \sim 20$ MeV, and about 20% at $E_{\gamma}^{lab} \sim 30$ MeV. Although the total cross section turns out to be in good agreement with the experimental data up to relatively high energy, in principle, the pionless theory must be applied to the photon energies below 10 MeV which corresponds to nucleon momentum close to the pion mass. Thus, in this work, we limit the photon energy to the range $E_{\gamma}^{lab} \leq 20$ MeV in the study of the spin observables.

In Fig. 5, curves of the unpolarized differential cross section divided by the total cross section at $E_{\gamma} = 19.8$ MeV are plotted by using our results up to NLO and NNLO, and the experimental data are also included [28]. Similar to the total cross section, the difference between NLO and NNLO is negligible up to 10 MeV, and the NNLO correction converges well even at $E_{\gamma} = 19.8$ MeV.

B. Induced neutron polarization $P_{y'}$

In Fig. 6 our results of $P_{y'}$ up to NLO and NNLO at $E_{\gamma} = 2.75, 5, 10, 19.8$ MeV are plotted as functions of $\cos \theta_n$, where $\theta_n$ is the angle for the outgoing neutron in the center of mass frame. Experimental data labeled by “John” [29] and “Jewell” [14] at $E_{\gamma} = 2.75$ MeV
FIG. 5: (Color Online) Ratio of unpolarized differential cross section and total cross section for the photon energy 19.8 MeV up to NLO (blue dashed) and NNLO (red solid). Experimental data \cite{28} are also displayed.

are also included in the figure. We find that the NNLO corrections converge well at all the photon energies and are small compared to those up to NLO. Discrepancy between theory and experiment of $P_y'$ at $E_\gamma = 2.75$ MeV, which has been reported in our previous work in the pionless EFT up to NLO \cite{4}, cannot be resolved by including the NNLO corrections.

Figure 7 shows the results of $P_y'$ in three different angles with increasing photon energy. The pionless EFT results up to NLO and NNLO agree well with phenomenological potential model calculation of Av18 in impulse approximation and Av18 calculation with exchange currents \cite{2} at low energies, respectively. The agreement implies that model-independent calculation of pionless EFT supports the Av18 results rather than the measurement by Nath \cite{30}. The results of Av18 and pionless EFT converge up to $E_\gamma \sim 10$ MeV, and start to deviate for $\theta_n = 45^\circ$ and $135^\circ$ while $\theta_n = 90^\circ$ results show good agreement even at $E_\gamma \geq 10$ MeV. This could be related with higher order corrections which are neglected in the present work.

C. Tensor analyzing power $T_{20}$ and $T_{22}$

By computing differential cross sections, $d\sigma^\circ$, $d\sigma^\uparrow$, and $d\sigma^\downarrow$, we can obtain the tensor analyzing powers $T_{11}$, $T_{20}$ and $T_{22}$. However, because $d\sigma^\uparrow \simeq d\sigma^\downarrow$ up to NNLO, we obtain
FIG. 6: (Color Online) \( P_{y'} \) for the photon energies 2.75 MeV (top left), 5 MeV (top right), 10 MeV (bottom left), and 19.8 MeV (bottom right) up to NLO (blue dashed) and NNLO (red solid). Experimental data labeled by “John” [29] and “Jewell” [14] are included in the figure at \( E_\gamma = 2.75 \) MeV.

\( T_{11} \approx 0 \). We would need to include higher order corrections to have a sizable contribution to \( T_{11} \). Also, \( T_{22} \) is dominated by the NNLO contribution since \( T_{22} \approx 0 \) at NLO, which implies a contribution to \( T_{22} \) mostly comes from the \( sd \) mixing effects.

In Figure 8, we summarize our results of \( T_{20} \) at NLO and NNLO. There are interesting behaviors absent or appearing weakly in other observables. First of all, in the observables considered so far, the results of NNLO is almost identical to those of NLO up to \( E_\gamma = 10 \) MeV, but for \( T_{20} \) non-negligible difference appears already at \( E_\gamma = 5 \) MeV. As the energy increases, the NLO result becomes the shape of flat and wide plateau. The value of \( T_{20} \) in the flat region is close to zero and independent of the energy. Similarly the values at \( \theta = 0^\circ \)
FIG. 7: (Color Online) $P_y'$ at three different angles with increasing photon energy up to NLO (blue dashed) and NNLO (red solid). Top left is for $\theta_n = 45^\circ$, top right is for $\theta_n = 90^\circ$ and the bottom is for $\theta_n = 135^\circ$. Experimental results from Nath [30] and Holt [31] are also displayed in the figures at $\theta_n = 45^\circ$ and $90^\circ$. and $180^\circ$ remain the same regardless of the energy.

The result at $E_\gamma = 19.8$ MeV is similar to the results with those obtained from the Bonn potential model calculations, lying between “N+MEC+IC” and “N+MEC+IC+RC” in Fig. 7.4.13 in [1]. The effect of NNLO can be distinguished from that of NLO even at $E_\gamma = 10$ MeV, which is the energy of interest in the HIGS proposal [3]. Therefore measurement of $T_{20}$ will provide a unique opportunity to test the role of higher orders in the pionless theory. At the same time, it might help understand the origin of discrepancy between experiment and theory in the polarization observables in the few-body systems.

In Fig. [3] curves of $T_{22}$ up to NLO and NNLO at $E_\gamma = 2.75, 5, 10, 19.8$ MeV are plotted...
FIG. 8: (Color Online) $T_{20}$ for the photon energies 2.75 MeV (top left), 5 MeV (top right), 10 MeV (bottom left), and 19.8 MeV (bottom right) up to NLO (blue dashed) and NNLO (red solid). Bonn potential result is from Ref. [1] (black dotted).

as functions of $\theta$. As mentioned before, there is no contribution from the NLO corrections, and the NNLO corrections, mainly the $sd$ wave mixing term, are a leading contribution to $T_{22}$ at the low energies. Our result at $E_\gamma = 19.8$ MeV is compared to a result of the Bonn potential model [1]. One can see quantitative agreement between the NNLO result and the Bonn potential one.

IV. SUMMARY

Motivated by the proposal of measurement of the tensor analyzing power $T_{20}$ at HIGS facility, we studied the photodisintegration of the deuteron at low energies. Pionless EFT
For the quantities that have non-vanishing contributions from LO such as the cross sections and $P_y'$, NNLO terms give perturbative corrections to the NLO results, thus the theory shows good convergent behavior. For $P_y'$, including NNLO contributions, our result becomes closer to a sophisticated calculation with Av18 model. The discrepancy between measurement and NLO result remains unsolved even if we include the NNLO corrections.

For $T_{20}$, NNLO gives negligible change to the NLO result at $E_\gamma = 2.75$ MeV, but the correction becomes more significant as the energy increases. No data are available below

FIG. 9: (Color Online) $T_{22}$ for the photon energies 2.75 MeV (top left), 5 MeV (top right), 10 MeV (bottom left), and 19.8 MeV (bottom right) up to NLO (blue dashed) and NNLO (red solid). Bonn potential result is from Ref. [1] (black dotted).
19.8 MeV, and we can make comparison to the result with the Bonn potential model at $E_\gamma = 19.8$ MeV. The agreement to the Bonn model result depends on the angle, but as a whole the NNLO result agrees well with that of the Bonn model quantitatively.

For $T_{22}$, contributions up to NLO are null, and non-vanishing values appear at NNLO. NNLO result agrees to the Bonn model result fairly well. The agreement in the tensor analyzing power proves that increase of the order in both wavefunctions and operators for the external probe in the pionless EFT can give results as accurate as those of the most elaborate calculation with modern phenomenological potential models. Since our work includes electric and magnetic hadronic currents up to NNLO, it would be interesting to check the effects of other multipole operators.

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