Constraints on supersymmetry from FCNC and CP violation

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Abstract

We consider FCNC and CP violating processes mediated by gluino exchange in generalized supersymmetric theories. We present the constraints on flavour (F) changing squark mass terms at the electroweak energy scale, focusing our analysis on $\Delta F = 2$ transitions. Results are also given for $\Delta F = 1$ CP conserving processes both in the hadronic and leptonic sectors, while we limit ourselves to some remarks on the relevance of box diagrams in the evaluation of $\epsilon'/\epsilon$.

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Flavour changing neutral currents (FCNC) and CP violating processes provide us with a powerful tool to constrain the mass spectrum of supersymmetric particles [1]-[3]. Lately, there has been a renewed interest in the analysis of these processes, in connection with the study of supersymmetric grand-unified theories (SUSY-GUTs) [4] and of supersymmetry (SUSY) breaking in effective supergravities which emerge as the low energy limit of superstring theories [5]. Indeed, from the FCNC tests one can obtain relevant constraints on the mechanism of SUSY breaking and on the interactions of fermions up to the supergravity breaking scale. One very efficient way to obtain such constraints is to analyse gluino-mediated FCNC processes, which can be sizeable due to the presence of the strong coupling [2, 3]. These flavour changing (FC) transitions arise if the squark mass matrices are not diagonalizable with the same transformations which diagonalize quark mass matrices. In this case a convenient choice is to keep the gluino-quark-squark (\(\tilde{g} - q - \tilde{q}\)) vertices diagonal in flavour space. This can be obtained by applying to the squark fields exactly the same transformations that diagonalize the quark mass matrices. In this basis, usually called the super-KM one, all flavour changing effects are due to off-diagonal mass insertions in squark propagators [3]. As long as the ratio of the off-diagonal entries over an average squark mass remains a small parameter, the first term in the expansion obtained by an off-diagonal mass insertion represents a suitable approximation. The advantage of this method is that it avoids the specific knowledge of the sfermion mass matrices, and simplifies the phenomenological analysis.

The presence of off-diagonal mass terms at the electroweak scale can be due to the initial conditions: SUSY breaking terms may yield contributions which are not universal, i.e. they are not proportional to the unit matrix in flavour space. Otherwise, even starting with universal mass contribution to sfermions in the SUSY soft breaking sector, renormalization effects from the starting point, i.e. the scale of supergravity breaking, down to the Fermi scale can bring about a misalignment between \(q\) and \(\tilde{q}\) mass matrices [4-5]. This latter situation is what we encounter in the minimal SUSY standard model. For instance, consider the mass matrix squared of the scalar partner of the left-handed down-quarks \(d_L\). At the scale of supergravity breaking this matrix consists of the SUSY conserving contribution \(m_d m_d^\dagger\) (where \(m_d\) denotes the down quark mass matrix) and the SUSY breaking universal contribution \(\tilde{m}^2\). However, the term \(h_u Q H u^c\) of the superpotential generates a logarithmically divergent contribution which is proportional to \(h_u h_u^\dagger\) and, hence, to \(m_u m_u^\dagger\) (\(m_u\) being the up-quark mass matrix). Hence the resulting \(\tilde{d}_L\) mass matrix squared at the Fermi scale is:

\[
m_{\tilde{d}_L \tilde{d}_L}^2 = m_d m_d^\dagger + \tilde{m}^2 + c m_u m_u^\dagger
\]  

Switching now to the super-KM basis, rotating the \(\tilde{q}\) fields together with the \(q\) ones, we obtain off-diagonal mass terms due to the presence of the last term in eq. [5]. From eq. [5] it is easy to realize that the mass insertion needed to accomplish the transition from \(\tilde{d}_{iL}\) to \(\tilde{d}_{jL}\) (\(i, j\) flavour indices) is given by:

\[
(\Delta_{LL}^q)_{ij} = c \left[ K (m_u^\text{diag})^2 K^\dagger \right]_{ij}
\]  

where \(K\) is the Cabibbo-Kobayashi-Maskawa matrix and \(m_u^\text{diag}\) denotes the diagonalized up-quark mass matrix. In the following, with the notation \((\Delta_{AB}^q)_{ij}\), we mean the mass insertion...
needed for a transition from a squark $\tilde{q}_{iA}$ to $\tilde{q}_{jB}$ with $A = (L, R)$ and $B = (L, R)$. Actually, as particularly emphasized in ref. [8], the above expression for $(\Delta^d_{LL})_{ij}$ may be somewhat misleading since one might think that the term $c$ in the r.h.s. of eq. (2) is a constant (i.e. independent of the SUSY breaking scale) or at most depends logarithmically on it. On the contrary, the one-loop RGE’s show that $c$ depends quadratically on that scale. Since also the average squark mass is proportional to this scale, the meaningful parameter for our mass insertion approximation is the dimensionless quantity $\delta = \Delta / \tilde{m}^2$, where $\tilde{m}$ denotes at the same time the average squark mass and the typical SUSY breaking scale. This observation is of utmost relevance if one wants to understand the scaling of the SUSY contribution to FCNC with increasing squark masses. The powers of $\tilde{m}$ in the denominator which are present to compensate for $\Delta$ mass insertions in the numerator do not have to be considered if also $\Delta$ is proportional to $\tilde{m}^2$. This justifies why gluino-induced FCNC SUSY contributions remain still sizeable even for $\tilde{q}$ masses above 1 TeV [8], as we will see in what follows. Two peculiar features of the MSSM should be noted. The first is that in this model there is a sharp hierarchy among the $\Delta_{AB}$: $(\Delta_{LL})_{ij} \gg (\Delta_{LR})_{ij} \gg (\Delta_{RR})_{ij}$ with $i \neq j$, due to the different number of mass insertions needed to accomplish the various transitions. The second peculiar feature is the smallness of these terms due to the super- GIM mechanism. We stress that there is no reason in the general case to expect such a pattern of FC effects.

There exist three main analysis of the constraints on $\delta$’s in the literature: ref. [7, 8, 9]. In particular, in [9] some previous discrepancies are discussed and more emphasis is provided on the constraints on the imaginary parts of the $\delta$’s given by CP violation.

We now come to the results of our analysis concerning the terms $(\Delta_{LL})_{ij}$, $(\Delta_{LR})_{ij}$ and $(\Delta_{RR})_{ij}$ in the u- and d-sectors. In the following we consider the case in which $(\Delta_{LR})_{ij} \simeq (\Delta_{RL})_{ij}$. We will comment later on the analogous contributions in the charged lepton sector.
First we consider $\Delta F = 2$ FCNC processes. In the down sector the $\Delta_{ij}$ mass insertions are bounded by the $K - \bar{K}$ mass difference and by the CP violating parameter $\epsilon$ ($\delta_{12}$) and the $B_d - \bar{B}_d$ mixing ($\delta_{13}$), while the only available bound in the up-sector concerns $\delta_{12}$ from $D - \bar{D}$ mixing.

The effective hamiltonian for $\Delta S = 2$ processes can be obtained from the calculation of the diagrams in fig. 1. We give below the mass difference $\Delta m_K$, obtained by considering the matrix element of the effective hamiltonian between Kaon states:

$$\Delta m_K = \frac{\alpha_f^2 f_m^2 m_K}{81 m^2} \left\{ \left[ \left( \delta_{12}^d \right)_{LL}^2 + \left( \delta_{12}^d \right)_{RR}^2 \right] (-6 x M(x) + 11 G(x)) + \left( \delta_{12}^d \right)_{LR}^2 \left( \delta_{12}^d \right)_{RL}^2 \left[-24 Z - 14\right] G(x) \right\} \right\} \tag{3}$$

where we have denoted by $Z$ the quantity

$$Z = \frac{m^2_K}{(m_s + m_d)^2} \tag{4}$$

and the functions $M(x)$ and $G(x)$ can be found in ref. [7]. Concerning this calculation we find some discrepancies with ref. [7, 8]. More details on the subject will be provided in ref. [10].

Our numerical results on the limits obtained from $\Delta F = 2$ processes are shown in Tables 1 and 2, and in figures 2, 3, 4.

Let us now turn to $\Delta F = 1$ FCNC processes. We obtain limits from $b \to s \gamma$ decays ($\delta_{23}^d$) in the squark sector. Numerical results are given in table 3.

Table 3 shows that the decay ($b \to s + \gamma$) does not limit the $\delta_{LL}$ insertion for a SUSY breaking of O(500 GeV). Indeed, even taking $m_{\tilde{q}} = 100$ GeV, the term $(\delta_{23})_{LL}$ is only marginally limited ( $(\Delta_{LL})_{23} < 0.3$ for $x = 1$). Obviously, $(\delta_{23})_{LR}$ is much more constrained since with a $\Delta_{LR}$ FC mass insertion the helicity flip needed for $(b \to s + \gamma)$ is realized in the gluino internal line and so this contribution has an amplitude enhancement of a factor $m_{\tilde{g}}/m_b$ over the previous case with $\Delta_{LL}$.

A similar analysis can be performed in the leptonic sector where the masses $\tilde{m}$ and $m_{\tilde{q}}$ are replaced by the average slepton mass and the photino mass $m_{\tilde{\gamma}}$ respectively. A clear but important point to be stressed is that the severe bounds that we provide on the $\delta_{LL}$ and $\delta_{LR}$ mass insertions in the leptonic sector and the consequent need for high degeneracy of charged sleptons, only apply if separate lepton numbers are violated. It is well known that in the MSSM the lepton numbers $L_e$, $L_\mu$ and $L_\tau$ are separately conserved because of the diagonality of the soft breaking terms and the masslessness of neutrinos. If at least one of this two properties is not present one can have partial lepton number violation. A particularly interesting example is the case where neutrinos acquire a mass through a see-saw mechanism (for its SUSY version and the implications for FCNC see [11]). In table 4 we exhibit the bounds on $\delta_{LL}$ and $\delta_{LR}$.
| $x$ | $\sqrt{\text{Re} (d_{12}^d)_{LL}^2}$ | $\sqrt{\text{Re} (d_{12}^d)_{LR}^2}$ | $\sqrt{\text{Re} (d_{12}^d)_{LL} (d_{12}^d)_{RR}^2}$ |
|-----|------------------|------------------|------------------|
| 0.3 | $1.9 \times 10^{-2}$ | $7.9 \times 10^{-3}$ | $2.5 \times 10^{-3}$ |
| 1.0 | $4.0 \times 10^{-2}$ | $4.4 \times 10^{-3}$ | $2.8 \times 10^{-3}$ |
| 4.0 | $9.3 \times 10^{-2}$ | $5.3 \times 10^{-3}$ | $4.0 \times 10^{-3}$ |

| $x$ | $\sqrt{\text{Re} (d_{13}^d)_{LL}^2}$ | $\sqrt{\text{Re} (d_{13}^d)_{LR}^2}$ | $\sqrt{\text{Re} (d_{13}^d)_{LL} (d_{13}^d)_{RR}^2}$ |
|-----|------------------|------------------|------------------|
| 0.3 | $4.6 \times 10^{-2}$ | $5.6 \times 10^{-2}$ | $1.6 \times 10^{-2}$ |
| 1.0 | $9.8 \times 10^{-2}$ | $3.3 \times 10^{-2}$ | $1.8 \times 10^{-2}$ |
| 4.0 | $2.3 \times 10^{-1}$ | $3.6 \times 10^{-2}$ | $2.5 \times 10^{-2}$ |

| $x$ | $\sqrt{\text{Re} (d_{12}^u)_{LL}^2}$ | $\sqrt{\text{Re} (d_{12}^u)_{LR}^2}$ | $\sqrt{\text{Re} (d_{12}^u)_{LL} (d_{12}^u)_{RR}^2}$ |
|-----|------------------|------------------|------------------|
| 0.3 | $4.7 \times 10^{-2}$ | $6.3 \times 10^{-2}$ | $1.6 \times 10^{-2}$ |
| 1.0 | $1.0 \times 10^{-1}$ | $3.1 \times 10^{-2}$ | $1.7 \times 10^{-2}$ |
| 4.0 | $2.4 \times 10^{-1}$ | $3.5 \times 10^{-2}$ | $2.5 \times 10^{-2}$ |

Table 1: Limits on $\text{Re} (d_{ij})_{AB} (d_{ij})_{CD}$, with $A, B, C, D = (L, R)$, for a squark mass $\tilde{m} = 500\text{GeV}$ and for different values of $x = m_{\tilde{g}}^2/\tilde{m}^2$.

| $x$ | $\sqrt{\text{Im}(d_{12}^d)_{LL}^2}$ | $\sqrt{\text{Im}(d_{12}^d)_{LR}^2}$ | $\sqrt{\text{Im}(d_{12}^d)_{LL} (d_{12}^d)_{RR}^2}$ |
|-----|------------------|------------------|------------------|
| 0.3 | $1.5 \times 10^{-3}$ | $6.3 \times 10^{-4}$ | $2.0 \times 10^{-4}$ |
| 1.0 | $3.2 \times 10^{-3}$ | $3.5 \times 10^{-4}$ | $2.2 \times 10^{-4}$ |
| 4.0 | $7.5 \times 10^{-3}$ | $4.2 \times 10^{-4}$ | $3.2 \times 10^{-4}$ |

Table 2: Limits on $\text{Im} (d_{12}^d)_{AB} (d_{12}^d)_{CD}$, with $A, B, C, D = (L, R)$, for a squark mass $\tilde{m} = 500\text{GeV}$ and for different values of $x = m_{\tilde{g}}^2/\tilde{m}^2$.

| $x$ | $|d_{23}^d|_{LL}$ | $|d_{23}^d|_{LR}$ |
|-----|------------------|------------------|
| 0.3 | 4.4 | $1.3 \times 10^{-2}$ |
| 1.0 | 8.2 | $1.6 \times 10^{-2}$ |
| 4.0 | 26 | $3.0 \times 10^{-2}$ |

Table 3: Limits on the $|d_{23}^d|$ from $b \to s\gamma$ decay for a squark mass $\tilde{m} = 500\text{GeV}$ and for different values of $x = m_{\tilde{g}}^2/\tilde{m}^2$. 
Table 4: Limits on the $|\delta^d_{ij}|$ from $l_j \rightarrow l_i \gamma$ lepton decay for a slepton mass $\tilde{m} = 100\text{GeV}$ and for different values of $x = m_{\tilde{\gamma}}^2/\tilde{m}^2$. 

| $x$  | $|\left(\delta^l_{12}\right)_{LL}|$ | $|\left(\delta^l_{12}\right)_{LR}|$ |
|------|---------------------|---------------------|
| 0.3  | $4.1 \times 10^{-3}$ | $1.4 \times 10^{-6}$ |
| 1.0  | $7.7 \times 10^{-3}$ | $1.7 \times 10^{-6}$ |
| 5.0  | $3.2 \times 10^{-2}$ | $3.8 \times 10^{-6}$ |

| $x$  | $|\left(\delta^l_{13}\right)_{LL}|$ | $|\left(\delta^l_{13}\right)_{LR}|$ |
|------|---------------------|---------------------|
| 0.3  | 15 | $8.9 \times 10^{-2}$ |
| 1.0  | 29 | $1.1 \times 10^{-1}$ |
| 5.0  | $1.2 \times 10^2$ | $2.4 \times 10^{-1}$ |

| $x$  | $|\left(\delta^l_{23}\right)_{LL}|$ | $|\left(\delta^l_{23}\right)_{LR}|$ |
|------|---------------------|---------------------|
| 0.3  | 2.8 | $1.7 \times 10^{-2}$ |
| 1.0  | 5.3 | $2.0 \times 10^{-2}$ |
| 5.0  | 22 | $4.4 \times 10^{-2}$ |
coming from the limits on $\mu \to e\gamma$, $\tau \to e\gamma$ and $\tau \to \mu\gamma$, for a slepton mass of $O(100 \text{ GeV})$ and for different values of $x = m_{\tilde{g}}^2/\tilde{m}^2$. Our results confirm those obtained in refs [7, 8].

Finally we make a short comment on the gluino-induced FC contribution to $\epsilon'/\epsilon$ (the interested reader should consult ref. [9] for a more thorough discussion). A common feature of the past literature on this point was the emphasis on the role of superpenguins, while superbox contributions were thought to be negligible. In our recent work [9] we show that box contributions yield a sizeable interference effect with the superpenguins ones. The results concerning the bounds on the imaginary parts of $(\delta_{12}^d)_{LL}$ and $(\delta_{12}^d)_{LR}$ from $\epsilon'/\epsilon$ for an average squark mass of 500 GeV are provided in table 5. It is apparent from a comparison of table 2 and table 5 that SUSY models with predominantly $\delta_{LL}$ or $\delta_{RR}$ contributions to CP violation (like the MSSM) tend to be superweak, while models with sizeable $\delta_{LR}$ are likely to be milliweak. Complete expressions for the separate box and penguin contributions to the $\Delta S = 1$ effective hamiltonian will be provided in a forthcoming paper [10]. The implications of the constraints obtained in this work on SUSY-GUTs and on theories with non-universal soft breaking terms are presently under study [10].

References

[1] J. Ellis and D.V. Nanopoulos, *Phys. Lett.* B 110 (1982) 44; R. Barbieri and R. Gatto, *Phys. Lett.* B 110 (1982) 211.

[2] M.J. Duncan, *Nucl. Phys.* B 221 (1983) 285; J.F. Donoghue, H.P. Nilles and D. Wyler, *Phys. Lett.* B 128 (1983) 55; A. Bouquet, J. Kaplan and C.A. Savoy, *Phys. Lett.* B 148 (1984) 69; M.J. Duncan and J. Trampetic, *Phys. Lett.* B 134 (1984) 439; T. Inami and C.S. Lim, *Nucl. Phys.* B 207 (1982) 533; J.-M. Frère and M.B. Gavela, *Phys. Lett.* B 132 (1983) 107; E. Franco and M. Mangano, *Phys. Lett.* B 135 (1984) 445; S. Bertolini, F. Borzumati and A. Masiero, *Phys. Lett.* B 192 (1987) 437; G. Altarelli and P.J. Franzini, *Zeit. f"ur Physik* C 37 (1988) 271; S. Bertolini, F. Borzumati and A. Masiero, *Phys. Lett.* B 194 (1987) 545, (Erratum, *Phys. Lett.* B 198 (1987) 590); T.M. Aliev and M.I. Dobroliubov, *Phys. Lett.* B 237 (1990) 573;
S. Bertolini, F. Borzumati, A. Masiero and G. Ridolfi, *Nucl. Phys. B* **353** (1991) 591; J. Hisano, T. Moroi, K. Tobe, M. Yamaguchi and T. Yanagida, TU-476, hep-ph/9501407.

[3] B.A. Campbell, *Phys. Rev. D* **28** (1983) 209; F. del Aguila, J.A. Grifols, A. Mendez, D.V. Nanopoulos and M. Srednicki, *Phys. Lett. B* **129** (1983) 77; J.–M. Gerard, W. Grimus, A. Raychaudhuri and G. Zoupanos, *Phys. Lett. B* **140** (1984) 349; J.–M. Gerard, W. Grimus, A. Masiero, D.V. Nanopoulos and A. Raychaudhuri, *Phys. Lett. B* **141** (1984) 79; P. Langacher and R. Sathiapalan, *Phys. Lett. B* **144** (1984) 401; J.–M. Gerard, W. Grimus and A. Raychaudhuri, *Phys. Lett. B* **145** (1984) 400; M. Dugan, B. Grinstein and L. Hall, *Nucl. Phys. B* **255** (1985) 413; L.J. Hall, V.A. Kostelecky and S. Raby, *Nucl. Phys. B* **267** (1986) 415; J.S. Hagelin and L.S. Littenberg, *Prog. Part. Nucl. Phys.* **23** (1989) 1; E. Gabrielli and G. Giudice, *Nucl. Phys. B* **433** (1995) 3.

[4] R. Barbieri and L.J. Hall, *Phys. Lett. B* **338** (1994) 212; R. Barbieri, L.J. Hall and A. Strumia, *Nucl. Phys. B* **445** (1995) 219; R. Barbieri, L.J. Hall and A. Strumia, *Nucl. Phys. B* **449** (1995) 437.

[5] M. Dine, A. Kagan and S. Samuel, *Phys. Lett. B* **243** (1990) 250; L. Ibanez and D. Lüst, *Nucl. Phys. B* **382** (1992) 305; V. Kaplunovsky and J. Louis, *Phys. Lett. B* **306** (1993) 269; R. Barbieri, J. Louis and M. Moretti, *Phys. Lett. B* **312** (1993) 451, (Erratum, *Phys. Lett. B* **316** (1993) 632); B. de Carlos, J.A. Casas and C. Muñoz, *Phys. Lett. B* **299** (1993) 234; *Nucl. Phys. B* **399** (1993) 623; A. Brignole, L.E. Ibanez and C. Muñoz, *Nucl. Phys. B* **422** (1994) 125, (Erratum, *Nucl. Phys. B* **436** (1995) 747); Y. Nir and N. Seiberg, *Phys. Lett. B* **309** (1993) 337; M. Dine, R. Leigh and A. Kagan, *Phys. Rev. D* **48** (1993) 4269; A. Lleyda and C. Muñoz, *Phys. Lett. B* **317** (1993) 82; N. Polonski and A. Pomarol, *Phys. Rev. Lett. 73* (1994) 2292; D. Matalliotakis and H.P. Nilles, *Nucl. Phys. B* **435** (1995) 115; M. Olechowski and S. Pokorski, *Phys. Lett. B* **344** (1995) 201; D. Choudhury, F. Eberlein, A. König, J. Louis and S. Pokorski, *Phys. Lett. B* **342** (1995) 180.

[6] L.J. Hall, V.A. Kostelecky and S. Raby, in ref. 3.

[7] F. Gabbiani and A. Masiero, *Nucl. Phys. B* **322** (1989) 235.

[8] J.S. Hagelin, S. Kelley and T. Tanaka, *Nucl. Phys. B* **415** (1994) 293.

[9] E. Gabrielli, A. Masiero and L. Silvestrini, Rome1 – 1109/95, ROM2F/95/20, hep-ph/9509379.

[10] F. Gabbiani, E. Gabrielli, A. Masiero and L. Silvestrini, in preparation.

[11] F. Borzumati and A. Masiero, *Phys. Rev. Lett. 57* (1986) 961.
Figure 2: The $\sqrt{\text{Re} \left( \delta_{12}^d \right)_{LL}^2}$ as a function of $x = m_g^2/\tilde{m}^2$, for a squark mass $\tilde{m} = 100\text{GeV}$.

Figure 3: The $\sqrt{\text{Re} \left( \delta_{12}^d \right)_{LR}^2}$ as a function of $x = m_g^2/\tilde{m}^2$, for a squark mass $\tilde{m} = 100\text{GeV}$.
Figure 4: The $\sqrt{\Re (\delta_{12}^d)_{LL} (\delta_{12}^d)_{RR}}$ as a function of $x = m_{3/2}^2 / \tilde{m}^2$, for a squark mass $\tilde{m} = 100\text{GeV}$.