Double-layer quantum Hall antiferromagnetism at filling fraction $\nu = 2/(\text{odd integer})$

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A low energy action for double-layer quantum Hall systems at filling fractions $\nu = 2/m$ ($m$ an odd integer) is introduced. Interlayer antiferromagnetic exchange induces a phase with canting spin order, and also a spin-singlet phase. Universal properties of zero and finite temperature transitions are obtained. We compute the critical temperature at which the canting order vanishes in a Kosterlitz-Thouless transition. Implications for recent light scattering experiments at $\nu = 2$ are noted.

There has been much recent work on double-layer quantum Hall systems, the majority of which has focussed on the case where the electron tunneling rate between the two layers is small. Then, the electron layer index plays the role of a pseudospin, and for the case where the total filling factor $\nu = 1/m$ ($m$ an odd integer), very interesting new physics arises from long-range correlations in the pseudospin orientation. However, the tunneling term acts like a ‘magnetic field’ in pseudospin space, and so spontaneous long-range pseudospin order, and the associated finite temperature (T) phase transition, is only possible when the tunneling is vanishingly small.

Stimulated by recent light scattering experiments at $\nu = 2$, we present here a general low energy theory for double-layer systems at filling $\nu = 2/m$ in the presence of moderate interlayer tunneling. We find a rich phase diagram with interesting transitions both at $T = 0$ and $T > 0$. In contrast to the phases at $\nu = 1/m$, which are driven by ordering in pseudospin space, the phases at $\nu = 2/m$ are associated with ordering in the physical electronic spin space. Consequently, our order parameters are defined even in the presence of interlayer tunneling; indeed, moderate interlayer tunneling is required to stabilize some of our phases. We will use our results to interpret recent experiments, and argue that they show indirect evidence for our $T > 0$ phase transition.

It is useful to begin discussion of the physics at $\nu = 2/m$ by considering the case where the layer separation, $d$, is much larger than the magnetic length, $l_o$. Then the two layers (labeled 1, 2) are approximately decoupled, and each separately has filling fraction $\nu_1 = \nu_2 = 1/m$. Their ground states will be the familiar Laughlin states for $m > 1$, or a fully filled lowest Landau level at $m = 1$, both of which have a large energy gap to all charged excitations. These states are also fully spin polarized and there is significant intralayer ferromagnetic exchange. The low-lying excitations in each layer are spin waves which have a small excitation gap given precisely by the Zeeman energy $g\mu_B H$ (the gyromagnetic ratio $g$ and the Bohr magneton $\mu_B$ will henceforth be absorbed by a rescaling of the magnetic field $H$). For small $g$, a complete description of the low energy excitations of each layer can be given in terms of an action for unit vector fields $\vec{n}_{1,2}$ ($\vec{n}_{1,2}^2 = 1$) representing the orientation of the ferromagnetic orders. Spin waves are small fluctuations of $\vec{n}_{1,2}$ about an ordered state, while charged quasiparticles are skyrmion textures of $\vec{n}_{1,2}$.

Now reduce the value of $d$ and couple $\vec{n}_1$ and $\vec{n}_2$. The simplest allowed coupling between them is an antiferromagnetic exchange interaction. These considerations lead to the following imaginary-time ($\tau$) effective action (in units with $\hbar = k_B = 1$)

$$S_0 = \int d^2x \int_0^{1/T} d\tau \left( \mathcal{L}_F[\vec{n}_1] + \mathcal{L}_F[\vec{n}_2] + J\vec{n}_1 \cdot \vec{n}_2 \right)$$

$$\mathcal{L}_F[\vec{n}] = i M_0 \vec{A}(\vec{n}) \cdot \nabla \vec{n} - \frac{\rho_s^0}{2} (\nabla \times \vec{n})^2 - M_0 \vec{H} \cdot \vec{n}$$

The interlayer ferromagnetic spin correlations are controlled by $\mathcal{L}_F$: $M_0 = 1/4\pi m l_o^2$ is the magnetization density per layer, $\rho_s^0$ is the spin stiffness of each layer when they are well separated (for $m = 1$, we have $\rho_s^0 = e^2/(16\sqrt{2} \pi \epsilon_0)$ and $\vec{A}$ accounts for the Berry phase accumulated under time evolution of the spins ($\epsilon_{ijk} \partial \vec{A}(\vec{n})/\partial n_j = n_k$). The interlayer antiferromagnetic correlations are induced by the positive coupling $J \sim M_0 \Delta_{\text{nas}}/U$ where $\Delta_{\text{nas}}$ is the tunneling matrix element between the layers, and $U \sim e^2/\epsilon_0$ is the Coulomb interaction energy.

Some potentially important terms have been omitted from $S_0$ and our analytic computations: the Hopf term which endows the skyrmions with fractional statistics, and the long-range Coulomb interaction between the skyrmions. We believe this is permissible because of the charge gap noted earlier. Further, as the layers are antiferromagnetically correlated, skyrmions in one layer will be correlated with anti-skyrmions in the other, and this neutralizes the leading contribution of both terms. This latter argument should continue to hold even if the charge gap were to vanish at a quantum critical point. Note that no new term is necessary to induce charge transfer between the layers: a hedgehog/anti-hedgehog pair in the two layers corresponds to an event transferring skyrmion number between them. Such space-time singu-
larities are absent in the strict continuum limit but appear when a short-distance regularization is introduced. Finally, for \( m \geq 1 \) and larger \( g \), the spin 0 Laughlin quasi-particles become the lowest energy charged excitations, but these can be neglected for similar reasons.

Now we parameterize

\[
\vec{n}_i = (-1)^i(1 - \vec{L}^2)^{1/2}\vec{n} + \vec{L} \tag{2}
\]

where the constraints \( \vec{n}_i^2 = 1 \) are now replaced by \( \vec{n}^2 = 1 \) and \( \vec{L} \cdot \vec{n} = 0 \). Because the layers are antiferromagnetically correlated we expect that \( \vec{L} \) will not be too large. We insert (3) into (1), expand to quadratic order in \( \vec{L} \), and then integrate out the \( \vec{L} \) degrees of freedom. This yields the following effective action for the antiferromagnetic order parameter \( \vec{n} \)

\[
S_1 = \frac{e}{2t} \int d^2 x \int_0^{1/T} d\tau \left[ \left( \nabla x \vec{n} \right)^2 + \frac{1}{c^2} \left( \frac{\partial \vec{n}}{\partial \tau} - i\vec{H} \times \vec{n} \right)^2 \right]
\]

where \( t = (J/2\rho_0 M_0^2)^{1/2} \) and \( c = (2\rho_0 J/M_0^2)^{1/2} \). This is precisely the action of the 2+1 dimensional quantum \( O(3) \) non-linear 

\[
sigma \text{ model in a field } \vec{H} \text{ coupling to the conserved global } O(3) \text{ charge. It is expected to apply to double-layer quantum Hall systems with } \nu = 2/m \text{ at length scales larger than } \Lambda^{-1} \sim t_o.
\]

For the special case \( m = 1 \), we will sharpen the quantitative theoretical predictions of \( S_1 \) by a Hartree-Fock (HF) analysis of a realistic, microscopic double-layer Hamiltonian \( [11] \). The HF theory will be used to compute renormalized \( T = 0 \) energy scales which completely specify the correlators of \( S_1 \) at low \( T \): in this manner we determine observables of the system with no free parameters (for \( m > 1 \) these energy scales remain as phenomenological parameters). We use the Hamiltonian

\[
\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_1 + \mathcal{H}_2
\]

where \( \mathcal{H}_0 \) annihilates an electron in the lowest Landau level in layer \( i \) (\( i = 1, 2 \)) with spin \( \sigma \) (\( \sigma = \pm 1 \)) in the \( z \) direction (we assume \( \vec{H} = (0, 0, H) \)) and with intra-Landau level index \( \alpha \). Interlayer tunneling induces the symmetric-antisymmetric energy separation \( \Delta_{\text{as}} \). The Coulomb interaction part of \( \mathcal{H} \) is

\[
\mathcal{H}_1 = \frac{1}{2} \sum_{\sigma_1 \sigma_2} \sum_{ij} \sum_{\alpha_1 \alpha_2} \frac{1}{\Omega} \sum_q V_{ij}(q) e^{-q^2l_z^2/2\epsilon_c q} e^{(\alpha_1 - \alpha_2)l_z^2} C_{i\alpha_1 \sigma_1}^\dagger C_{j\alpha_2 \sigma_2} C_{j\alpha_2 \sigma_2} C_{i\alpha_1 \sigma_1}, \tag{3}
\]

where \( q \) is a wavevector, \( \Omega \) is the area of the sample, and the interaction potentials are \( V_{ij} = 2\pi e^2/\epsilon q \) for \( i = j \) and \( V_{ij} = (2\pi e^2/\epsilon q) e^{-|q|^2d} \) for \( i \neq j \). The \( T = 0 \) phase diagram is obtained in the HF computation \( [11] \), and is shown as an inset. The quantum phase transitions between these phases are continuous and are accompanied by the softening of the intersubband spin density excitations. The phases are described below:

(I) Fully Polarized Ferromagnet (FPF): In \( S_1 \) this is present for \( H > \sim ct\Lambda^2 \). This phase has \( \langle n_{1z} \rangle = \langle n_{2z} \rangle = 1 \). It is continuously connected to the large \( d \) limit discussed earlier.

(II) Canted (C): We now have \( \langle n_{1z} \rangle = \langle n_{2z} \rangle \neq 0 \), and, for example, \( \langle n_{1x} \rangle = -\langle n_{2x} \rangle \neq 0 \). This phase has a broken spin rotational \( O(2) \) symmetry in the \( x - y \) plane. For \( m = 1 \), the HF phase boundary between the FPF and \( C \) phases is \( V_c/\Delta_{\text{as}} = (\Delta_{\text{as}}/H)[1 - (H/\Delta_{\text{as}})^2] \), and that between the \( C \) and \( SS \) phases is \( V_c/\Delta_{\text{as}} = 1 - (H/\Delta_{\text{as}})^2 \), where \( V_c = \frac{1}{\Omega} \sum_q e^{-q^2l_z^2/2}\left[V_{11}(q) + V_{22}(q)\right] \); a wavefunction for the \( C \) phase is obtained by the standard HF methods. For \( m > 1 \) a caricature of the wavefunction is two separate Laughlin states at \( \nu_1 = \nu_2 = 1/m \) but polarized in the orientations shown.

(III) Neel (N): This is the limiting case of \( C \) with \( \langle n_{1z} \rangle = \langle n_{2z} \rangle = 0 \) achieved at \( H = 0 \). Now an \( O(3) \) spin rotation symmetry is broken.

(IV) Spin Singlet (SS): This corresponds to the quantum disordered phase of the \( O(3) \) sigma model. The ground state is a spin-singlet and is therefore unaffected by \( H \): its wavefunction is the same as that at \( H = 0 \). For \( m = 1 \), in the independent electron HF picture, the
electrons fill the layer-symmetric subband, with spin-up and spin-down levels equally populated. However, it is well-known that HF theory overestimates the energy of a nonmagnetic phase like SS because correlations between opposite spin electrons, important for reducing the Coulomb energy, are now absent. It is likely, therefore, that the real SS phase is stable over a larger parameter region than that in our HF approximation, but the topology of the HF phase diagram in the inset of Fig. 6 (which is identical to that for $S_1$) should be correct. To build in charge correlations, one can use an approach similar to the Heitler-London picture of the hydrogen molecule, and consider pairs of electrons with their charge localized in opposite layers, while their spins form singlet bonds. Indeed, such a charge-localized picture was behind our introduction of the actions $S_0, 1$. In such an approach, an alternative wavefunction for the SS phase (valid for $m = 1$ and $m > 1$) can be obtained in the $J \to \infty$ limit: pairs of electrons in opposite layers bind to form spin singlet, charge $2e$ bosons, which then condense into a boson Laughlin state at filling fraction $1/2m$, as demanded by the strength of the magnetic flux.

It is worth noting explicitly here that the HF computations at $m = 1$ allow us to assert that all the different phases of $S_1$ are the ground states in realistic parameter regimes. For $m > 1$, it remains an open question as to whether the phases of $S_1$ other than FPF are accessible, although we consider it a likely possibility that at least C will exist.

We now turn to the physics at $T > 0$. Only the $N$ and $C$ ground states have a broken spin rotation symmetry; the $O(3)$ symmetry of the former implies that the symmetry is restored at any $T > 0$, while the $O(2)$ symmetry of the latter implies a Kosterlitz-Thouless phase transition at $T = T_c > 0$. We may characterize the order parameter fluctuations in both phases by a $T = 0$ spin stiffness $\rho_s(H)$ such that the energy cost of rotations of the order parameter by a slowly varying angle $\phi(r)$ is $E_\phi = (\rho_s(H)/2) \int d^2 r |\nabla \phi(r)|^2$. A crude estimate of $T_c$ is $T_c \approx \rho_s(H)$, although this must fail as $H \to 0$. In the latter limit it is possible to obtain an exact leading asymptotic result $T_c = 2\pi \rho_s(0)/\ln(\rho_s(0)/H)$ for $\ln(\rho_s(0)/H) \gg 1$. For $m = 1$ we computed $\rho_s(H)$ in the HF calculation and the results are shown in Fig. 6. We see that the $T_c$ estimates are well in the experimentally accessible regimes for typical GaAs-based semiconductor samples. We emphasize that the Kosterlitz-Thouless transition at $T_c$ is present even in the presence of interlayer tunneling, unlike the case for the pseudospin transition at $\nu = 1/m$.

A more precise approach to the $T > 0$ properties is to expand in the deviation from the $H = 0$ quantum critical point between the $N$ and SS phases at $t = t_c \sim \Lambda^{-1}$. This quantum critical point is described by a renormalizable quantum field theory (with upper-critical spatial dimension $d = 3$), and so all thermodynamic properties are universal functions of energy scales characterizing ‘relevant’ perturbations from this critical point; corrections due to irrelevant operators require additional energy scales and will be neglected here. Two of the relevant energy scales are the ‘bare’ couplings $T$ and $H$ (there is no renormalization of the scale of $H$ because it couples to a conserved charge) and a third (the last) measures deviation of $t$ from $t_c$. For $t > t_c$ we choose [4] this energy scale to be $\Delta$, the energy gap of the SS state at $T = H = 0$, while for $t < t_c$ we choose the renormalized spin stiffness $\rho_s(0)$ of the $N$ state also at $T = H = 0$. As $t$ approaches $t_c$ we have $\Delta \sim (t - t_c)^\nu$, while $\rho_s(0) \sim (t_c - t)^{(d-1)/\nu}$, where $\nu$ is the correlation length exponent of the classical three-dimensional $O(3)$ ferromagnet. For $m = 1$, the microscopic HF calculation gives $\Delta = \Delta_\text{ssas} \sqrt{1 - V_- / \Delta_\text{ssas}}$, and $\rho_s(0) = ((1 - (\Delta_\text{ssas} / V_-)^2)/8\pi\Omega) \sum_{q} q^2 e^{-q^2 \Lambda^2 / 2V_{11}(q)}$. Notice that these are consistent with the mean field exponent $\nu = 1/2$ in the upper-critical dimension $d = 3$.

One of our main results, which follows from the considerations above, is that the critical temperature $T_c$ at which the ordering of the $C$ phase disappears obeys, for $t > t_c$,

$$T_c = H \Psi_>(\Delta/H).$$

Here $\Psi_>(u)$ is a universal function of $u$ with no arbitrary scale factors, and obeys the exact relation $\Psi_>(u \geq 1) = 0$ (because [1][13] the $T = 0$ boundary of the SS phase...
is given precisely by the condition $\Delta = H$). A similar scaling form holds for $t < t_c$ with $T_c = H \Psi_<(\rho_s(0)/H)$. We computed the functions $\Psi_>, \Psi_<$ in an expansion in $\varepsilon = 3 - d$ using recently developed methods and found to leading order

$$\Psi_>(u) = [33(1 - u^2)/(10\pi^2\varepsilon)]^{1/2};$$

the structure of the subleading terms is quite complicated and is similar to that discussed elsewhere. This result is valid for all $u$, except for $u$ very close to 1; in that case we find, by a mapping to the dilute Bose gas problem, the exact asymptotic result $\Psi_>(u \to 1) = y\ln(1/y)/(4\ln(1/y))$ with $y = 1 - u$, which holds for $\ln(1/y) \gg 1$. For $t > t_c$, the $\varepsilon$ expansion holds for $\Psi_<(u/\sqrt{\varepsilon})$ and we obtained

$$\Psi_<(u/\sqrt{\varepsilon}) = [(33 + 3u^2)/(10\pi^2\varepsilon)]^{1/2}$$

Again this result is valid for all $u$, but now fails for $u \to \infty$ (which is $H\to 0$). While $T_c(H = 0) > 0$ for all $\varepsilon < 1$, we noted earlier that $T_c(H = 0) = 0$ for $\varepsilon = 1$; the latter property will not appear at any order in the $\varepsilon$ expansion. Using results special to $d = 2$ for $H \to 0$ discussed earlier, we have instead the exact asymptotic form $\Psi_<(u \to \infty) = 2\pi u/\ln u$. We draw attention to a particularly simple and striking limit of the above results. At $t = t_c$, we have $T_c = KH$ where $K = \Psi_>(0) = \Psi_<(0)$ is a universal number. Further, we do not expect any large or singular variation in $T_c$ if $t$ is close to but not exactly $t_c$. As both $H$ and $T_c$ are directly measurable energies, this relationship is amenable to a direct experimental test. On the theoretical side, while at present there is only the leading term in an $\varepsilon$ expansion for the value of $K$, it should be possible to obtain a reasonably precise result using quantum Monte Carlo simulations of double-layer lattice spin systems, which have been limited to $H = 0$ so far. Universality implies that these lattice models will have the same value of $K$ as the quantum Hall system, and it appears to us that the simulation for $H \neq 0$ should also be free of the fermion sign problems.

We have also obtained results in the $\varepsilon$ expansion for the crossovers of the dynamic spin susceptibility at frequency $\omega$ as universal functions of the energy ratios $\omega/T, H/T$ and $\Delta/T (\rho_s(0)/T)$: the methods are similar to those of ref. and results will be presented elsewhere.

We turn now to a comparison with recent light scattering experiments. The high density sample (the right sample in the inset of Fig. shows 'mode-softening' consistent with a $T > 0$ phase transition which we identify with that above our $C$ phase. Using input parameters from the HF calculation in, we obtain the prediction of $T_c \sim 0.5K$, to be compared with the experimental value $T_c \sim 0.52K$: the good agreement must be considered fortuitous until the accuracy of the $\varepsilon$ expansion is better understood. The same sample also shows marked $T$ dependence for $T > T_c$ in the light scattering spectrum at $\omega$ of order or greater than $T$: a natural explanation for this could be a crossover into the 'high-$T$' region above the quantum-critical point. The lower density sample (the left sample in the inset of Fig. shows no mode softening and little $T$ dependence in the light scattering spectrum: we suggest that this sample is in the $SS$ phase. The HF computation puts this sample in the $C$ phase, but as we discussed earlier, this could be in error because the HF theory overestimates the stability of the $C$ phase.

Finally we note that we expect similar considerations to apply to all double-layer quantum Hall systems with $\nu = 2\nu_1$ where a single layer at filling $\nu_1$ forms a fully polarized quantum Hall state with a charge gap. This work was supported by the U.S.-ONR (S.D.S. and L.Z.) and by NSF Grant No DMR 96-23181 (S.S.)

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