1 INTRODUCTION

The missing matter problem in large scale physics is one that has been persisting through the turn of the century. With radio, x-ray and optical telescope strengths increasing, data driven observations of the rotation curves of spiral galaxies have shown that the missing matter problem is not due to inadequate technology. The most widely accepted explanation of the discrepancy in rotation velocities of galaxies has been attributed to Cold Dark Matter and can be modeled using formalisms such as the Navarro, Frenk, & White (1996). However, due to the lack of direct observational detection, more authors than ever have been exploring the possibility that the current Einstein Gravity may be in need of a modification or replacement to solve the missing matter problem. Although there are now many potential alternative gravitational theories in the literature, we will focus our efforts here to Conformal Gravity (CG).

A summary of the history and context of CG will be discussed in section 2. Any viable theory (standard gravity with dark matter or an alternative) must be able to address observational phenomena. Aside from the flattening rotation curves which has been ascribed to dark matter, there are other empirical relations that are of interest to the field. For example, the Tully-Fisher (TF) relation shows that for almost all observed rotation curves, there exists the relationship of $M_{disk} \propto v_{obs}^4$ in the outer regions of spiral galaxies. Recently, a new empirical phenomena was discovered by McGaugh, Lelli, & Schombert (2016) which shows a strong correlation between the predicted centripetal accelerations due to luminous Newtonian matter alone ($g_{bar}$) and the observed centripetal accelerations ($g_{obs}$). Here, the centripetal accelerations are the usual $g_{obs} = \frac{v_{obs}^2}{R}$ for a given observed velocity data point $v_{obs}$ at a distance $R$ from the center of the galaxy. Taking these values with a corresponding expectation value of the centripetal accelerations $g_{bar} = \frac{v_{bar}^2}{r}$, across every point in a rotation curve (and then across a survey of rotation curves), one can create a plot of the points ($g_{bar}, g_{obs}$). When viewed at an appropriate scale (see Lelli, McGaugh, Schombert, & Pawlowski (2017) ), the ($g_{bar}, g_{obs}$) plot highlights the correlation between observation and prediction of the luminous matter, and has become known as the Radial Acceleration Relation (RAR). Further, a fitting function is used,

$$g_{obs} = F(g_{bar}) = \frac{g_{bar}}{1 - e^{-\sqrt{g_{bar}/g_t}}},$$  \hspace{1cm} (1)  

where $g_t = 1.2 \times 10^{-10} m s^{-2}$ is the best fit free parameter found by McGaugh et al. from the chosen data set. The correlation described in Equation (1) is independent...
of any given theory, and should also be minimally coupled to any data set. McGaugh et al. used the Spitzer Photometry and Accurate Rotation Curves (SPARC) database (Lelli, McGaugh, & Schombert 2016) spanning a diverse population of 153 galaxies with a total of 2693 data points. One conclusion of McGaugh et al. (2016) is that the correlation seen in the plots of (cussed in McGaugh et al. (2016) is that the correlation seen in the plots of (g) is that the correlation seen in the plots of (g) may point towards new physics that could explain the phenomena and the apparent emergence of a fitting relation eq. (1). It is this latter conclusion that will be explored in this review, namely how conformal gravity could describe the new physics required in the RAR.

2 | REVIEW OF CONFORMAL GRAVITY ROTATION CURVE FITTING

Conformal Gravity, also known originally as Weyl Gravity, has enjoyed success in recent years in its ability to describe and predict rotation curve physics without the need for dark matter. The total number of galaxies fit by CG has now been expanded to over 230 with a diverse range of morphologies.

3 1 THE RAR PLOTS IN CONFORMAL GRAVITY

Since the RAR is independent of any galactic survey, we present a selection of 40 galaxies consisting of 1614 data points. These particular galaxies were chosen to span the full range of galactic rotation curves from large bulged spirals to small gas dominated dwarfs.

In eq. (3), \( v_{gr} \) is the standard formula of Freeman (1970). The constants in eq. (3) are \( \gamma^x = 5.42 \times 10^{-4} \) cm\(^{-1} \), \( \gamma_0 = 3.06 \times 10^{-30} \) cm\(^{-1} \) and \( \kappa = 9.54 \times 10^{-4} \) cm\(^{-2} \). M is the mass of the galaxy (measured in solar masses \( M_\odot \)) and \( R_0 \) is the disk scale length of the galaxy. The complete derivation of eq. (3), as well as the determination of the constants (independent of any particular galactic survey) can be found in Mannheim & O’Brien (2013). Other features of the fitting process, such as inclusion of bulges when applicable and galactic gas are assumed, and details can be found in Mannheim & O’Brien (2011).

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1 We note here that the value McGaugh et al. obtained for \( g \) is directly related to the data set chosen, as well as any assumptions that may have gone into the fitting. Specific details will follow in section 3.

2 Of the 40 galaxies chosen, 15 galaxies are high surface brightness (HSB) consisting of 909 points and 25 galaxies are low surface brightness (LSB) consisting of 705 points to create a fairly robust and balanced overall set.
and depend only on constants and the relative mass of the system. There are no other free parameters that can be changed or modified when modeling. Using eq. (1), one can generate $g_{CG} = g_R^{1/2}$. We show the results across the 1614 data points in two ways. Figure 2 shows the same data as in Fig. 1 in blue, but with the conformal gravity prediction $g_{CG}$ overlaid in purple. This plot shows that the CG prediction almost completely covers the data. Figure 3 shows an alternative method for viewing the CG prediction. In Fig. 3, we replace the x axis of the standard plot with $g_{CG}$ instead of $g_{NEW}$ to plot the points $(g_{CG} \cdot g_{OBS})$. This plot is illustrative since CG makes different predictions than standard gravity. Hence, the RAR plot for CG should be the one constructed in Fig. 3.

4 | DISCUSSION AND EXTENSIONS

The plots shown in Fig. 3 highlight how CG can capture the essence of the Radial Acceleration Relation without the need for dark matter. Further, since CG both covers the data in Fig. 1 and makes a prediction in Fig. 3 that shows the correlation to be a one to one relation between luminous matter alone $g_{CG}$ and observation $g_{OBS}$, there is no need for an equivalent to eq. (1) in CG. Instead, with the absence of dark matter, CG predicts that $g_{dm} = g_{OBS} - g_{CG} = 0$. This can be seen in Fig. 3, where 97% of the 1614 points lie along the line of unity. This fact is not only supportive of the claim that CG can describe the RAR, but without a fitting function as in eq. (1), one does not have to worry about a specific value of a fitting parameter $g_z$. This is important since the phenomena should be independent of a chosen data set. It was shown in O’Brien, Chiarelli, & Mannheim (2018) that depending on the masses and distances used, the value for $g_z$ can vary by up to a factor of four, thus adding to the universal power of conformal gravity. To illustrate how the individual rotation curve of each galaxy is used in constructing the RAR plots, we present in Figure 5 the fits of eight of the chosen galaxies. The rotation curve of each galaxy in Figure 5 is followed (below) by a plot showing the respective data points contributions to the $(g_{NEW} \cdot g_{OBS})$ as well as its contribution to the plot of $(g_{CG} \cdot g_{OBS})$. Since McGaugh et al. (2016) used eq. (1) to show the universal trend in the overall data, we present in each fit how eq. (1) applies to each individual galaxy for this data set (using the NED distances and derived masses from fitting as described above).

Since the RAR is an empirical finding, a theory which explains the RAR should also be able to explain other empirical trends in the data. To this end, we present in Fig. 4, a plot of the Tully Fisher relationship for the last data point of the 40 galaxy sample. The authors chose to express this plot as $v \propto M^{4/3}$ so that the velocities natural error can be retained. The last

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3It should be noted that we are using the Baryonic Tully Fisher relationship (BTF), which uses the full mass of the galaxy (stars, gas and bulge). See McGaugh (2012) for more details.
The Radial Acceleration Relation has added a new observation about rotation curve fitting. It is an empirical feature of the data which can be used to help constrain the dark matter parameter space, as well as challenge an alternative theory which would explain the current dark matter paradigm with new physics. In this work, we have selected a random, diverse population of galaxies and applied conformal gravity to the set, and have shown that CG can capture the Radial Acceleration Relation. Further, if we posit that CG is the new physics responsible for the RAR, then the predictions shown in Fig. 5 highlights that CG explains the RAR without the need for dark matter or a fitting function as in eq. (1). Application of CG to the Baryonic Tully Fisher relation, further establishes how the theory can accommodate all of the current empirical findings in spiral galaxy physics. Conformal gravity fitting has recently been extended to include the entire SPARC set so that the data can be tested on a one to one basis and will be published in a future paper. The authors would like to thank IW ARA for a warm welcome, and to thank the audience at IWARA 2016 who helped inspire this work. The authors hope that this work highlights the importance of these international meetings and collaborations.

5 CONCLUSIONS

The Radial Acceleration Relation has added a new observation about rotation curve fitting. It is an empirical feature of the data which can be used to help constrain the dark matter parameter space, as well as challenge an alternative theory which would explain the current dark matter paradigm with new physics. In this work, we have selected a random, diverse population of galaxies and applied conformal gravity to the set, and have shown that CG can capture the Radial Acceleration Relation. Further, if we posit that CG is the new physics responsible for the RAR, then the predictions shown in Fig. 5 highlights that CG explains the RAR without the need for dark matter or a fitting function as in eq. (1). Application of CG to the Baryonic Tully Fisher relation, further establishes how the theory can accommodate all of the current empirical findings in spiral galaxy physics. Conformal gravity fitting has recently been extended to include the entire SPARC set so that the data can be tested on a one to one basis and will be published in a future paper. The authors would like to thank IW ARA for a warm welcome, and to thank the audience at IWARA 2016 who helped inspire this work. The authors hope that this work highlights the importance of these international meetings and collaborations.

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FIGURE 5 Fitting to the rotational velocities (in km sec$^{-1}$) of the selected eight galaxy sample with their reported errors as plotted as a function of radial distance (in kpc) is given in rows one and four. The dashed line shows the luminous Newtonian contribution, while the full curve is the conformal gravity fit. Rows two and five show the respective contributions of the eight galaxies to Fig. 1 and rows three and six show their respective contributions to Fig. 3.
\[ \log[g(NEW)] [m s^{-2}] \]

\[ \log[g(OBS)] [m s^{-2}] \]