Quantum Gravity Without Ghosts

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Abstract: An outline is given of a recently discovered technique for building a quantum effective action that is completely independent of gauge-fixing choices and ghost determinants. One makes maximum use of the geometry and fibre-bundle structure of the space of field histories and introduces a set of nonlocal composite fields: the geodesic normal fields based on Vilkovisky’s connection on the space of histories. The closed-time-path formalism of Schwinger, Bakshi, Mahantappa et al can be adapted for these fields, and a set of gauge-fixing-independent dynamical equations for their expectation values (starting from given initial conditions) can be computed. An obvious application for such equations is to the study of the formation and radiative decay of black holes, and to other back-reaction problems.

The back-reaction problem in quantum gravity is one of the most difficult in theoretical physics, firstly because one does not know how best to pose it, and secondly because quantum gravity is not a perturbatively renormalizable theory. One believes that the second difficulty can be overcome by means of string theory. But string theory is not yet a fully coherent discipline.

What one can do at present is treat standard quantum gravity as an effective field theory. Although this theory is not perturbatively renormalizable it will generate a unitary S-matrix if one performs dimensional regularization, and minimally subtracts the poles that arise in each perturbation order. No renormalization group is obtained, and each choice of auxiliary regularization mass yields a different theory at high energies. But by setting this mass equal to the Planck mass, one should obtain results that give a fair description of the physics even close to the Planck regime. Indeed, in view of the known utility of the WKB approximation in ordinary quantum mechanics, one may expect to get useful results already in 1-loop order. But what results?

Computing an S-matrix is of no help in tackling the backreaction problem. What is needed is the quantum effective action $\Gamma$, which generates the full vertex functions of the theory and which in principle describes all quantum effects, not merely scattering processes. But the functional form of $\Gamma$ is needed off shell.

It is not hard to construct a $\Gamma$ that is diffeomorphism invariant even off shell, but there are infinitely many ways of doing this, each corresponding to a different choice of covariant gauge-fixing term and associated ghost determinant in the functional integral that defines $\Gamma$. Each choice leads to a different set of nonlocal dynamical equations for the “mean value”, say, of the metric tensor $g_{\mu \nu}$. Therefore this “mean value” does not mean much and cannot be relied upon to give unambiguous insights.

One needs a quantum effective action that, like the classical action, yields dynamical equations that are ghost and gauge-fixing independent. The authors have recently shown how such an effective action can be built \cite{1}. One makes maximum use of the geometry of the space $\text{Riem}(M)$ of (pseudo) Riemannian 4-metrics on spacetime $M$. $\text{Riem}(M)$ may be viewed as a principal fibre bundle having for its typical fibre the diffeomorphism group $\text{Diff}(M)$. Real physics takes place in the base space $\text{Riem}(M)/\text{Diff}(M)$ of this bundle. $\text{Riem}(M)$, viewed as an infinite dimensional manifold, may itself be endowed with a diffeomorphism-invariant Riemannian metric. If this metric is required to be ultralocal (a requirement that is essential for the success of any renormalization program) it is not difficult to show that it must belong to the one-parameter family

$$\gamma^{\mu \nu, \tau} = \frac{1}{2} g^{1/2} (g^{\mu \sigma} g^{\nu \tau} + g^{\mu \tau} g^{\nu \sigma} + \lambda g^{\mu \nu} g^{\sigma \tau}) \delta(x, x'),$$

$$g = |\det(g_{\mu \nu})|.$$ (1)

A condensed notation is convenient here \cite{2}. The metric field, or any gauge field, will be denoted by $\varphi^i$, indices now being understood as ranging over a continuum of values that specify discrete components as well as spacetime points $x, x'$. . . . Summations over repeated indices then include integrations over spacetime. Infinitesimal gauge transformations (diffeomorphisms) may be expressed in the form

$$\delta \varphi^i = Q^i_\alpha \delta \xi^\alpha,$$ (2)

where the $\delta \xi^\alpha$ are infinitesimal parameters and the $Q^i_\alpha$ are components (in the “coordinates” $\varphi^i$) of an infinite set of vector fields $Q_\alpha$ that generate the fibres. The latter satisfy the Lie bracket relation

$$[Q_\alpha, Q_\beta] = -c^c_{\alpha \beta} Q_\gamma,$$ (3)

the $c^c_{\alpha \beta}$ being the structure constants of the gauge group.

Gauge invariance of the metric (1) is the statement

$$\mathcal{L} Q_\alpha \gamma = 0.$$ (4)

The $Q_\alpha$ are Killing vector fields for the metric $\gamma$ and vertical vector fields on the space of histories. The $Q_\alpha$ and $\gamma$ together define a unique connection 1-form on this space:

$$\omega^\alpha = \Theta^{\alpha \beta} Q_\beta \cdot \gamma,$$ (5)
where $\mathcal{G}^{\alpha\beta}$ is the Greens’s function, appropriate to the boundary conditions at hand (typically “in-out” or “in-in”), of the operator
\[\mathcal{S}_{\alpha\beta} = -Q_\alpha \cdot \gamma \cdot Q_\beta.\] (6)

It is easy to see that $\omega^\alpha \cdot Q_\beta = \delta^\alpha_\beta$ and that horizontal vectors are those that are perpendicular to the fibres. A horizontal vector may be obtained from any vector by applying the horizontal projection operator:
\[\Pi^i_j = \delta^i_j - Q^i_\alpha \omega^\alpha_j.\] (7)

The trick for obtaining an effective action yielding ghost and gauge-fixing-independent effective field equations is to make use of the following connection on the space of histories, first proposed by Vilkovisky [2]:
\[\Gamma^i_{jk} = \Gamma^i_{jk} - Q^i_\alpha \omega^\alpha_k - Q^i_{\alpha k} \omega^\alpha_j + \frac{1}{2} \omega^\alpha_j Q^i_{\alpha l} Q^\alpha_j \omega^\alpha_k + \frac{1}{2} \omega^\alpha_j Q^i_\alpha l Q^\alpha_j \omega^\alpha_k.\] (8)

$\Gamma$ is the Riemannian connection associated with $\gamma$ and the dots denote covariant functional differentiation based on it. Vilkovisky’s connection has the following properties:

1. If a geodesic based on it is horizontal (vertical) at one point it is horizontal (vertical) throughout its length.
2. If $A$ is a gauge invariant functional on the space of histories then, for all $n$,
\[A_{ij...in} = A_{ij...in} \Pi^j_i ... \Pi^k_n,\] (9)

where the semicolon denotes covariant functional differentiation based on Vilkovisky’s connection and parentheses indicate that a symmetrization of the indices they embrace is to be performed.

Introduce a convenient base point $\varphi_*$ in the space of histories, e.g., Minkowski, Friedmann-Robertson-Walker, black hole. Let $\varphi$ be an arbitrary point and $\lambda$ a Vilkovisky geodesic connecting it to the base point, and let $s$ and $s_*$ be the values, at $\varphi$ and $\varphi_*$ respectively, of an affine parameter along $\lambda$. Define
\[\phi = (s - s_*) \left( \frac{\partial}{\partial s} \right)_{\lambda(s_*)}.\] (10)

$\phi$ is a vector at $\varphi_*$, invariant under rescaling of $s$. Its components $\phi^\alpha$ in an arbitrary frame at $\varphi_*$ may be called \textit{geodesic normal fields}.

The $\hat{\phi}^\alpha$ can be used [1, 2] to carry out covariant functional Taylor expansions about $\varphi_*$, and in reference [1] it is shown that they lead in a simple and straightforward way to the desired effective action:
\[\Gamma[\varphi_*, \phi] = S[\varphi_*, \phi] + \Sigma[\varphi_*, \phi].\] (11)

Here $S$ is the classical action, $\phi$ is the mean field, and $\Sigma$ is the logarithm of a functional integral:
\[\Sigma[\varphi_*, \phi] = -i \left( \ln \int e^{i \left( \frac{1}{2} F[\varphi_*, \phi] \chi \chi + \frac{1}{4} S_3[\varphi_*, \phi] \chi \chi + \ldots \right) } |d\chi| \right)_{1PI}.\] (12)

The subscript “1PI” means “retain only 1–particle–irreducible graphs”. $S_{\alpha \beta}[\varphi_*, \phi]$ denotes the nth functional derivative of $S[\varphi_*, \phi]$ with respect to the $\phi$’s, and $F$ has components given by
\[F_{ab}[\varphi_*, \phi] = S_{ab}[\varphi_*, \phi] + \kappa_{a\beta} P^a_\beta \] (13)

where $\kappa_{a\beta}$ is any $\phi$-independent ultralocal invertible continuous matrix and $(P^a_\beta)$ is any $\phi$-independent continuous matrix for which the operator
\[\hat{\mathcal{S}}_{\alpha\beta} = P^a_\alpha Q^a_\beta[\varphi_*, \phi].\] (14)

is nonsingular. The $Q^a_\alpha$ are the components of $Q_\beta$ in the geodesic normal coordinate system.

The vertex functions $S_{n\beta}[\varphi_*, \phi]$ have the structure
\[S_{n\beta}[\varphi_*, \phi] = \sum_{m=0}^{\infty} \frac{1}{m!} S^{n\beta}_{\{\varphi_* \ldots \, \phi^{b_m}\} \ldots \phi^{b_m}},\] (15)

and if one is computing amplitudes for physical processes taking place in the background $\varphi_*$, these functions reduce to
\[S_{n\beta}[\varphi_*, 0] = S^{n\beta}_{\{\varphi_* \ldots \phi^{b_m}\} \ldots \phi^{b_m}}[\varphi_*],\] (16)

which are easily computed. The secret of ghost-free gauge theory is simple: Replace all traditional vertex functions by (16) and eliminate the ghost graphs. It is easy to show that every graph in the loop-expansion of $\Sigma$ is individually invariant under both gauge transformations and changes in the $P$’s and $\kappa$’s. This result may seem surprising in view of the presence of the $P$’s and $\kappa$’s in expression (13). However, do the following: Proceed in the usual way, including the determinant of the ghost operator (14) in the functional integral. Using dimensional regularization discover that, because of the special forms taken by the ghost vertices generated by the $Q^a_\alpha$, every graph containing a ghost line vanishes!

The authors have applied this formalism to the Yang-Mills field, with standard “in-out” boundary conditions. The renormalization program proceeds as in the conventional approach, with these differences: The $\phi^\alpha$ are non-local composite operators and cannot serve directly as interpolating fields for the S-matrix. To get from them to the local fields $\varphi^\alpha$ further renormalizations are required. The latter do not spoil the asymptotic freedom of Yang-Mills theory but they lead to slightly different $\beta$-functions for $\phi^\alpha$ and $\varphi^\alpha$.

If one is interested in expectation values rather than S-matrix elements then the $\phi^\alpha$ will serve as well as the $\varphi^\alpha$. With use of the “closed-time-path” formalism [1], which
replaces the $\phi^a$ by “forward-time” and “backward-time” fields $\phi^a_+ \text{ and } \phi^a_-$ respectively, one can construct an “in-in” effective action that generates the time evolution of expectation values $\langle \phi^a \rangle$. The pole subtractions needed for the “in-out” effective action suffice to renormalize the “in-in” effective action as well. This is true for both gravity and Yang-Mills theory.

Although the computations for quantum gravity are algebraically far more complicated than for Yang-Mills theory, Mirzabekian and Vilkovisky [5] have been able to show that 1-loop contributions to the gravitational effective action already account for the Hawking radiation from black holes, and they obtain general expression for the relevant form factors describing back-reaction and decay. With a ghost-and-gauge-fixing-independent effective action one can hope to pin these form factors down so as to yield believable results in the near Planckian regime. The ultimate aim is to solve the effective field equations for $\langle \phi^a \rangle$, starting from an imploding scalar-field shell and following the dynamics through black hole formation and decay. Short of a full blown string-theoretical analysis this is the only way of resolving such issues as information loss or tunneling to other universes.

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