Turbulence Modelling and Stirring Mechanisms in the Cosmological Large-scale Structure

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Abstract. FEARLESS (Fluid mEchanics with Adaptively Refined Large Eddy SimulationS) is a numerical scheme for modelling subgrid-scale turbulence in cosmological adaptive mesh refinement simulations. In this contribution, the main features of this tool will be outlined. We discuss the application of this method to cosmological simulations of the large-scale structure. The simulations show that the production of turbulence has a different redshift dependence in the intra-cluster medium and the warm-hot intergalactic medium, caused by the distinct stirring mechanisms (mergers and shock interactions) acting in them. Some properties of the non-thermal pressure support in the two baryon phases are also described.

1. Introduction

In the framework of the physics of the cosmological large-scale structure, turbulent gas flows are an important link between the thermal and merger history of galaxy clusters, on the one side, and the non-thermal phenomena (cosmic ray acceleration, amplification of magnetic fields) in the intra-cluster medium (ICM) on the other. A central role in injecting turbulence in the cosmic flow is played by shocks, which contribute both to energy dissipation and gas stirring (Miniati et al. 2000; Vazza et al. 2009), and by hydrodynamical instabilities triggered by mergers (e.g., Heinz et al. 2003; Iapichino et al. 2008).

The evolution of turbulence in the ICM, as a result of the cluster merger history, has been explored in many works over the last decade using hydrodynamical simulations (Ricker & Sarazin 2001; Dolag et al. 2005; Iapichino & Niemeyer 2008; Vazza et al. 2011; Paul et al. 2011). In this contribution we want to highlight the results of a different approach to the study of turbulence, adopted by Iapichino et al. (2011). Instead of focusing on a single cluster, that work followed the evolution of turbulence in a large cosmological box (with side length of 100 Mpc h−1). This setup allows to study not only the injection of turbulence in the ICM, but also in the less dense warm-hot intergalactic medium (WHIM). Furthermore, the simulation code includes a subgrid scale (SGS) model for unresolved turbulence (Schmidt et al. 2006), coupled to the adaptive
mesh refinement (AMR). The resulting tool, called FEARLESS (Fluid mechanics with Adaptively Refined Large Eddy SimulationS), will be briefly outlined in the next section.

2. Numerical methods

FEARLESS consists of the combination of AMR with a SGS model for the unresolved kinetic energy. Details of this numerical tool have been presented elsewhere (Schmidt et al. 2006; Maier et al. 2009; Iapichino et al. 2011); in essence, the discretization onto a grid of the equations of fluid dynamics is equivalent to applying a filter formalism (Germano 1992) to them. Consequently, additional terms appear in the equations, which take into account the dynamics at unresolved length scales. For example, the filtered momentum equation of a viscous, compressible, self-gravitating fluid, becomes

\[
\frac{\partial}{\partial t} \langle \rho \rangle \hat{v}_i + \frac{\partial}{\partial r_j} \langle \rho \rangle v_j = -\frac{\partial}{\partial r_i} \langle p \rangle + \frac{\partial}{\partial r_j} \langle \sigma'_{i j} \rangle + \langle \rho \rangle \hat{g}_i - \frac{\partial}{\partial r_j} \hat{\tau}(v_i, v_j),
\]

where \( \rho \) is the baryon density, \( v_i \) are the velocity components, \( p \) is the pressure, \( g_i \) the gravitational acceleration and \( \sigma'_{i j} \) the viscous stress tensor. Given a variable \( f \), with \( \hat{f} \) we indicate the application of the filter operator to it (see Schmidt et al. 2006 for the a more rigorous treatment).

The last term on the right-hand side of equation (1) contains the turbulent stress tensor \( \hat{\tau}(v_i, v_j) \), which accounts for the interaction between the resolved flow and the SGS scales. This term can be expressed in analogy with the viscous stress tensor by means of the so-called eddy viscosity closure (cf. Pope 2000), although an improved closure has been recently adopted by Schmidt & Federrath (2011).

The turbulent stress tensor enters also the definition of the specific filtered kinetic energy, as a contribution from unresolved scales:

\[
\hat{e}_{\text{kin}} = \frac{1}{2} \hat{v}_i \hat{v}_i + \frac{1}{2} \hat{\tau}(v_i, v_j) / \langle \rho \rangle
\]

(2)

It is thus natural (Germano 1992) to interpret the trace of \( \hat{\tau}(v_i, v_j) / \langle \rho \rangle \) as the square of the SGS turbulence velocity \( q \). This leads to the definition of the SGS turbulence energy \( \epsilon_i \):

\[
\epsilon_i = \frac{1}{2} q^2 := \frac{1}{2} \hat{\tau}(v_i, v_j) / \langle \rho \rangle.
\]

(3)

The SGS turbulence energy is governed by an equation of the following form:

\[
\frac{\partial}{\partial t} \langle \rho \rangle \epsilon_i + \frac{\partial}{\partial r_j} \hat{\gamma}_j \langle \rho \rangle \epsilon_i = D + \Sigma + \lambda - \langle \rho \rangle (\lambda + \epsilon)
\]

(4)

The quantities on the right-hand side of equation (4) determine the evolution of \( \epsilon_i \) and are the turbulent diffusion term \( D \), the turbulent production term \( \Sigma \), the pressure dilatation term \( \lambda \) and the viscous dissipation term \( \epsilon \). Their closures represent the SGS model.

The method is coupled with AMR in a way that consistently accounts for cut-off length scales varying in time and space (see Maier et al. 2009 for a detailed description). The resulting tool is particularly suitable in the study of turbulence in strongly
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3. Turbulence and non-thermal pressure support

The definition of the baryon phases in Iapichino et al. (2011) is based on a threshold in temperature $T$ and baryon overdensity $\delta$. The gas is defined as belonging to the WHIM if $T > 10^5$ K and $\delta < 10^3$; if $\delta > 10^3$, it belongs to the ICM. The former phase is found in filaments and cluster outskirts, and the latter in the denser, collapsed structures. In Fig. 1 the time evolution of the average internal and SGS energy for these two phases is shown. We notice a different redshift evolution for $e_i$: in the ICM, it shows a peak at $z$ between 1.0 and 0.65, followed by a mild decrease. In the WHIM phase, there is a steady increase to $z = 0$.

In Iapichino et al. (2011) we provide an interpretation of these features in terms of different stirring mechanisms acting in the two baryon phases: the evolution of turbulence in the ICM follows closely the cluster merging history, therefore $e_i$ peaks approximately during the major merger epoch (cf. Giocoli et al. 2007) and its decline is halted by the subsequent minor mergers. Recently, Hallman & Jeltema (2011) studied the evolution of turbulence in clusters, and found that the fraction of clusters with large turbulence in the core evolves in time with a trend very similar to the ICM in Fig. 1.
evolution of turbulence in the WHIM is governed by the gas accreted on filaments and cluster outskirts, and closely resembles the evolution of the kinetic energy flux through external shocks (Miniati et al. 2000; Skillman et al. 2008).

Another interesting problem which was explored in Iapichino et al. (2011) concerns the dynamical pressure support of the cosmic gas. We notice that, starting from the definition of $e_t$, one can identify the non-thermal pressure caused by unresolved, SGS velocity fluctuations with $p_t = 2/3 \rho e_t$. This term has been included in an analysis of the dynamical support against gravitational contraction of the gas (Zhu et al. 2010). We refer to Iapichino et al. (2011) for a more thorough derivation of the support equations; here it is sufficient to say that the analysis is based on the Laplacians of the thermal and dynamical pressure (the latter one including both resolved and SGS terms).

It is found that the turbulent support is stronger in the WHIM gas at baryon overdensities $1 < \delta < 100$, and less relevant for the ICM. A fairly large fraction of the WHIM and ICM gas has a large vorticity (28.7 and 52.3 per cent in mass at $z = 0$, respectively), but this is usually associated with an equally large thermal pressure support, and only in a small volume fraction (of the order of 10 percent) the non-thermal pressure is dynamically significant.

This result is apparently in contradiction with the idea that the cluster outskirts, consisting mostly of gas which is newly accreted in the potential well, could have significant departures from hydrostatic equilibrium. Our work shows that this is the case only in localized regions, but not globally (see also Valdarnini 2011). More resolved cosmological simulations, focused on single clusters, will be used to investigate further this point.

References

Dolag, K., Vazza, F., Brunetti, G., & Tormen, G. 2005, MNRAS, 364, 753
Germano, M. 1992, Journal of Fluid Mechanics, 238, 325
Giocoli, C., Moreno, J., Sheth, R. K., & Tormen, G. 2007, MNRAS, 376, 977
Hallman, E. J., & Jeltema, T. E. 2011, ArXiv e-prints. [1108.0934]
Heinz, S., Churazov, E., Forman, W., Jones, C., & Briel, U. G. 2003, MNRAS, 346, 13
Iapichino, L., Adamek, J., Schmidt, W., & Niemeyer, J. C. 2008, MNRAS, 388, 1079
Iapichino, L., & Niemeyer, J. C. 2008, MNRAS, 388, 1089
Iapichino, L., Schmidt, W., Niemeyer, J. C., & Merklein, J. 2011, MNRAS, 414, 2297
Maier, A., Iapichino, L., Schmidt, W., & Niemeyer, J. C. 2009, ApJ, 707, 40
Miniati, F., Ryu, D., Kang, H., Jones, T. W., Cen, R., & Ostriker, J. P. 2000, ApJ, 542, 608
O’Shea, B. W., Bryan, G., Bordner, J., Norman, M. L., Abel, T., Harkness, R., & Kritsuk, A. 2005, in Adaptive Mesh Refinement – Theory and Applications, ed. T. Plewa, T. Linde, V.G. Weirs (Berlin; New York: Springer), vol. 41 of Lecture Notes in Computational Science and Engineering, 341
Paul, S., Iapichino, L., Miniati, F., Bagchi, J., & Mannheim, K. 2011, ApJ, 726, 17
Pope, S. B. 2000. Turbulent Flows (Cambridge University Press)
Ricker, P. M., & Sarazin, C. L. 2001, ApJ, 561, 621
Schmidt, W., & Federrath, C. 2011, A&A, 528, A106
Schmidt, W., Niemeyer, J. C., & Hillebrandt, W. 2006, A&A, 450, 265
Skillman, S. W., O’Shea, B. W., Hallman, E. J., Burns, J. O., & Norman, M. L. 2008, ApJ, 689, 1063
Valdarnini, R. 2011, A&A, 526, A158
Vazza, F., Brunetti, G., & Gheller, C. 2009, MNRAS, 395, 1333
Vazza, F., Brunetti, G., Gheller, C., Brunino, R., & Brüggen, M. 2011, A&A, 529, A17
Zhu, W., Feng, L., & Fang, L. 2010, ApJ, 712, 1