Non-Gaussianity and direction-dependent systematics in HST Key Project data

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ABSTRACT

Two new statistics, namely $\Delta \chi^2$ and $\Delta \chi$, based on the extreme value theory, were derived by Gupta et al. We use these statistics to study the direction dependence in the HST Key Project data, which provides one of the most precise measurements of the Hubble constant. We also study the non-Gaussianity in this data set using these statistics. Our results for $\Delta \chi^2$ show that the significance of direction-dependent systematics is restricted to well below the $1\sigma$ confidence limit; however, the presence of non-Gaussian features is subtle. On the other hand, the $\Delta \chi$ statistic, which is more sensitive to direction dependence, shows direction dependence systematics to be at a slightly higher confidence level, and the presence of non-Gaussian features at a level similar to the $\Delta \chi^2$ statistic.

Key words: methods: statistical – cosmological parameters.

1 INTRODUCTION

Hubble’s observations (1929) can be approximated as $v \propto d$, where $v = cz$ is the velocity (towards or away from us) of the galaxy being observed. Apart from a very few nearby galaxies, redshifts are positive, indicating that the galaxies are receding from us. The fact that the recession velocity of a galaxy is proportional to its distance is explained by the expansion of the Universe. $H_0$, the constant of proportionality, is called the Hubble constant and it measures the rate of expansion at the present epoch. The Hubble constant is a part of various cosmological calculations and its importance can never be underestimated. It decides the value of the critical density $\rho_c$, the amount of matter and energy required to make the geometry of the Universe flat. By comparing $\rho_c$ to the observed density one can decide the geometry of the Universe. Most importantly it sets the age of the Universe ($t_0$) and hence the size of the observable Universe ($R_{ob} = c t_0$). Due to its importance, determining its accurate value is of paramount importance.

Determination of an accurate value of the Hubble constant can also lead to a test of the cosmological principle (CP hereafter). According to the CP the Universe is homogeneous and isotropic at any given cosmic epoch. If the cosmological principle is valid then one would expect the average value of the Hubble constant to be the same in different regions and in different directions. This issue has been addressed by various authors. We discuss some of the earlier results below.

Are we living in a bubble? If the matter distribution is not homogeneous, it causes variation in the value of the Hubble constant. Since gravity pulls, we expect that if a region of space has higher density then both the average density and the expansion rate will be negatively affected and thus the Hubble constant will be smaller in this region. In contrast to the local mass concentration, a region with low mass density will produce a larger value of $H_0$. Zehavi et al. (1998) provided the first evidence for a large local void. They determined $H_0$ both within and outside 70 $h^{-1}$ Mpc using type Ia supernovae (SNe Ia) and found that the value of $H_0$ was 6.5 per cent higher within 70 $h^{-1}$ Mpc than that outside. This indicates a low-density inner region compared to the outside one and is known as local bubble or Hubble bubble. The above authors had assumed a flat Friedmann–Robertson–Walker (FRW) universe with $\Omega_M = 1$ in their analysis. The variation in the inside and outside values of $H_0$ decreases to 4.5 per cent in the $\Lambda$ cold dark matter ($\Lambda$CDM) model ($\Omega_M = 0.3$, $\Omega_\Lambda = 0.7$); however, it does not disappear completely. Recently Jha, Riess & Kirshner (2007) revisited the problem using the latest SNe Ia data set and detected the local Hubble bubble at 3$\sigma$ confidence level. However, in a later publication Conley et al. (2007) claimed that it was a misinterpretation of the colour excess of supernovae. At this juncture it is difficult to say if the evidence for the local bubble is conclusive.

Variation in $H_0$ from HST key data: McClure & Dyer (2007) used HST Key Project data (see Section 2) to calculate the variation in $H_0$ value. They found that a statistically significant variation in $H_0$ of 9 $km s^{-1} Mpc^{-1}$ exists in HST Key Project data. The approximate directional uncertainty is 10' to 20'. Their results indicate two sets of extrema that dominate on different distance scales. They found differences as great as $\sim 35 km s^{-1} Mpc^{-1}$.

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within and $\sim20\text{ km s}^{-1}\text{ Mpc}^{-1}$ beyond our supercluster. Within 70 Mpc their results show a statistically significant difference of $\sim19\text{ km s}^{-1}\text{ Mpc}^{-1}$. This variation does not appear to be an artefact of Galactic dust since there is no consistent difference looking in or out of the plane of the Galaxy. In fact the overall structure in the map is inconsistent with the distribution of dust in the COBE dust maps (Schlegel, Finkbeiner & Davis 1998).

At this point a pertinent question is, ‘are these variations in the measurement of $H_0$ due to a real departure from the cosmological principle?’ On the contrary it is also possible that the data themselves have some systematic errors due to some uncorrected physical processes in the Universe, or there could be some real issues with the data reduction/calibration process. In order to comment on ‘what is the real cause of the variations?’, a critical review of the measurement methods is required. Determining an accurate value of the Hubble constant is a challenging task. One requires an accurate measurement of redshift $z$ and distance $d$. Although redshift can be measured with good accuracy from the spectrum of the light emitted by the object, distance measurement is difficult. Various methods are employed to measure distances, namely the Tully–Fisher relation (TFR), surface brightness (SB) fluctuations, the Fundamental Plane (FP) relation, type II SNe, type Ia SNe, the Sunyaev–Zeldovich effect (SZE), gravitational lensing, etc. Unfortunately, most of these methods suffer from systematic effects arising from many different causes. For instance the SZE requires the 3D distribution/shape of the plasma (hot gas) in the galaxy clusters. Radio and X-ray images of the clusters provide only the projected X-ray surface brightness and cosmic microwave background (CMB) decrement. Hence simplified assumptions about the shape of the cluster are made. Again, the assumptions about the phase and temperature of the plasma are ad hoc (Sulkanen 1999). For another example, the uncertainty in the physical basis of the TFR can cause subtle systematic variances in the TFR with environment. A critical review of the subject of $H_0$ measurement methods can be found in Jacoby et al. (1992).

Since there are many sources of systematics, special attention was needed to determine the accurate value of $H_0$. The most accurate experiment to achieve this was the Hubble Space Telescope (HST) Key Project (Freedman et al. 2001). We will discuss the HST Key Project in the next section. However, the issues with these data have been mentioned in the previous paragraphs. In the present paper we intend to put constraints on the CP using the $H_0$ data. This can be achieved by looking for direction-dependent signatures in the data. As has been pointed out earlier, detecting the direction-dependent signatures does not guarantee departure from isotropy and hence the CP. In that case one can constrain the reliability of the data. We use a technique (Gupta, Saini & Laskar 2008; Gupta & Saini 2010) based on the extreme value theory in order to accomplish this. Another important issue with any data set is the presence of non-Gaussian errors. Since the central limit theorem (CLT) predicts that the errors in the data should be Gaussian, non-Gaussian errors are undesirable and may indicate some unresolved issues. As a by-product of our method we are able to detect the presence of non-Gaussian features in the errors, which makes our methods useful not only in the case of $H_0$ data but for any data set in general.

The plan of this paper is as follows. We discuss the HST Key Project in Section 2, which is the main source of our data. We discuss our methods in Section 4. Since our techniques are based on the extreme value theory (EVT hereafter), we discuss it briefly in Section 3. Results and conclusions are presented in Sections 5 and 6, respectively.

2 HST KEY PROJECT AND THE DATA SET

Determining an accurate value of $H_0$ was one of the motivating reasons for building the NASA/ESA Hubble Space Telescope (HST). Measurement of $H_0$ with the goal of 10 per cent accuracy was designated as one of the three ‘Key Projects’ of the HST (Aaronson & Mould 1986; Kennicutt, Freedman & Mould 1995). The overall goal of the HST Key Project was to measure $H_0$ based on a Cepheid calibration of a number of independent, secondary distance determination methods. Many times the systematic errors dominate the accuracy of distance measurement. To overcome this the HST team averaged over the systematics and used a number of different methods to measure distances instead of relying on a single method.

Determining $H_0$ accurately requires the measurement of distances far enough away so that both the small- and large-scale motions of galaxies become small compared with the overall Hubble expansion. To extend the distance scale beyond the range of the Cepheids, a number of methods that provide relative distances were chosen. The HST Cepheid distances were used to provide an absolute distance scale for these otherwise independent methods, including type Ia supernovae, the TFR, the FP for elliptical galaxies, surface brightness fluctuations and type II supernovae. The final result of the $H_0$ Key Project (Freedman et al. 2001) was $H_0 = 72 \pm 8 \text{ km s}^{-1}\text{ Mpc}^{-1}$.

2.1 Data set

We have chosen data from the HST Key Project (Freedman et al. 2001) as our primary data set. This set contains 74 data points and provides a reasonably full sky coverage. The different methods used in order to derive this data set are the TFR, the FP relation, SB fluctuations, type Ia SNe and type II SNe. In addition we have chosen two data points from Sakai et al. (2000). In all the cases recessional velocities have been corrected to the CMB frame and thus all the $H_0$ values belong to the CMB frame. The full data set is published in table 1 of McClure & Dyer (2007).

3 EXTREME VALUE THEORY

In the present paper we investigate the direction-dependent systematic effects which exhibit anisotropy in the cosmological data. We identify the direction where the effect of the systematics is maximum. To estimate its statistical significance we need to know the distribution of this maximum, which can be computed using the EVT. Since it is not a common tool in the arsenal of astronomers, we begin with a brief introduction to the EVT. It was developed in parallel with the CLT. While the CLT describes the limiting distribution of partial sums, the EVT describes what the distribution of extremes (maxima/minima) looks like. Below we discuss the theory of maxima; however, the results obtained can be easily reformulated to obtain the distribution of minima.

We will outline the basic ingredients to obtain the theoretical distribution. Let $F$ be a distribution with its right-hand end point $x^*$ which may be infinite, i.e.

$$x^* = \sup\{x : F(x) \leq 1\}.$$  

We randomly choose a sample $(X_1, X_2, \ldots, X_n)$ of size $n$ from this distribution. The maximum of this sample will approach $x^*$ for a large $n$, i.e. $\max(X_1, X_2, \ldots, X_n) \to x^*$ as $n \to \infty$. The distribution

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of maxima has the following probability:

\[
P(\max(X_1, X_2, \ldots, X_n) \leq x) = P(X_1 \leq x, X_2 \leq x, \ldots, X_n \leq x)
\]

\[
= P(X_1 \leq x)P(X_2 \leq x) \cdots P(X_n \leq x)
\]

\[
= F^n(x),
\]

where \(X_1, X_2, \ldots, X_n\) are assumed to be independent. This converges to zero for \(x < x'\) and to unity for \(x \geq x'\), which means that \(F(x)\) is a degenerate distribution. A normalization of variable max(\(X_1, X_2, \ldots, X_n\)) is required in order to get a non-degenerate distribution. We choose linear normalization (Haan et al. 2006). Let us assume that there exists a sequence \(a_n > 0\), and \(b_n\) real such that

\[
t = \max(X_1, X_2, \ldots, X_n) - b_n
\]

has a non-degenerate limiting distribution as \(n \to \infty\), i.e.

\[
\lim_{n \to \infty} F^n(a_n x + b_n) = G(x),
\]

where \(0 \leq G(x) \leq 1\), \(G(x)\) is the required distribution. It is very difficult to derive the general form of \(G(x)\); we mention some of the results available in the literature. We state a theorem given by Fisher and Tippett (Fisher & Tippett 1928) which describes the required distribution.

**Theorem 1.** The distribution of maxima \(G(x; \varepsilon)\) has the following form:

\[
G(x; \varepsilon) = \exp \left( -[1 + \varepsilon x]^{-1/\varepsilon} \right),
\]

(2)

with \(1 + \varepsilon x > 0\), where \(\varepsilon \in R\) is called the shape parameter or the extreme value index.

Presenting the proof of this theorem (Fisher & Tippett 1928) is beyond the scope of this paper. We only outline a few interesting facts below.

(i) When \(\varepsilon = 0\), Taylor expansion of \([1 + \varepsilon x]^{-1/\varepsilon}\) gives \(e^{-x}\). Thus equation (2) gives

\[
G(x) = \exp(-e^{-x}).
\]

This is known as the Gumbel distribution or extreme value distribution of type I. The right-hand end point of this distribution is infinity. Also, \(1 - G(x) \sim e^{-x}\) as \(x \to \infty\), indicating that the distribution has a thin tail. It is clear from the form of Gumbel that it is unbounded on either side, that is, there is no \(x\) for which \(G(x) = 0\). A sketch of the Gumbel distribution is shown in Fig. 1.

(ii) When \(\varepsilon > 0\), \(G(x; \varepsilon) < 1\) for all \(x\), i.e. the right-hand end point of the distribution is infinity. Also, as \(x \to \infty\), \(1 - G(x, \varepsilon) \sim (\varepsilon x)^{-1/\varepsilon}\), which indicates that the distribution has a heavy tail. This is called the Frechet distribution, or type II extreme value distribution. One can clearly see that this distribution has a lower bound.

(iii) When \(\varepsilon < 0\), the right-hand end point of the distribution is \(-1/\varepsilon\). This is known as the Weibull distribution, or extreme value distribution of type III.

In the above discussion we have not mentioned the location and scale parameters for simplicity. When we introduce these parameters, the form of \(G(x)\) becomes complicated as shown below:

\[
G(x; m, s, \varepsilon) = \exp \left( -\left[1 + \varepsilon \left(\frac{x - m}{s}\right)\right]^{-1/\varepsilon} \right),
\]

(3)

where \(m = b_n \in R\) is the location parameter and \(s = a_n > 0\) is the scale parameter.

For our analysis we will use the Gumbel distribution since the variable of interest there does not have bound on either side. Thus our emphasis will be on the Gumbel distribution in the rest of this paper, which with scale and location parameters takes the form

\[
G(x; m, s) = \exp \left( -\left[1 + \varepsilon \left(\frac{x - m}{s}\right)\right]^{-1/\varepsilon} \right).
\]

(4)

The probability distribution (pdf) can be derived by differentiating equation (4).

### 4 METHODOLOGY

Two new techniques, namely the \(\Delta_x^2\) and \(\Delta_y\) statistics based on the EVT, were derived in Gupta et al. (2008) and Gupta & Saini (2010). We apply the same method here; however, we briefly mention these techniques for completeness.

#### 4.1 \(\Delta_x^2\) statistic

First we calculate the best-fitting value of \(H_0\) for the complete data set by minimizing \(\chi^2\), defined below:

\[
\chi^2 = \sum_i \left( \frac{H_{0i} - H_i}{\sigma_i} \right)^2,
\]

(5)

where \(H_{0i}\) is the \(i\)th point in the data set and \(\sigma_i\) is the observed standard error. This minimization gives us the best-fitting value \(H_{0}\).

Using this best-fitting value we now define \(\chi_i\) for the \(i\)th data point as

\[
\chi_i = \frac{H_{0i} - H_{ib}}{\sigma_i},
\]

(6)

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where $H_{0b}$ is the best-fitting value of the Hubble constant. We will consider subsets of the data set to construct our statistic. We define the reduced $\chi^2$ in terms of $\chi$, as follows:

$$\chi^2 = \frac{1}{N_{\text{subset}}} \sum_i \chi^2_i,$$

(7)

where it should be noted that by ‘reduced’ we do not mean ‘per degree of freedom’, since we do not fit the model separately to the subsets of the data. Here $\chi^2$ is an indicator of the statistical scatter of the subset from the best-fitting value $H_{0b}$.

If the CP holds then the value of the Hubble constant should not depend upon the direction in which it is measured. We use this fact to choose specific subsets of data. We divide the complete data into two hemispheres, labelled by the direction vector $n_i$, and take the difference of the reduced $\chi^2$ computed for the two hemispheres separately to obtain $\Delta \chi^2 = \chi^2_{\text{north}} - \chi^2_{\text{south}}$, where we have defined ‘north’ as that hemisphere towards which the direction vector $n_i$ points. We are only interested in the magnitude of this difference, therefore we take the absolute value of $\Delta \chi^2$, and then vary the direction $n$ across the sky to obtain the maximum absolute difference:

$$\Delta \chi^2 = \max(|\Delta \chi^2|).$$

(8)

We note that since the same data point appears in several subsets, the maximization is not done over statistically independent measures of our statistic. Another noteworthy fact is that if the direction dependence has a forward–backward symmetry then this statistic will not be able to detect it. However, due to the ease of its construction and use, we consider this simplest of possible statistics.

To interpret our results we need to know the range of $\Delta \chi^2$ that we can expect if there were no direction dependence in the data, and the noise in the measurements were Gaussian. The spatial distribution of measurements is not uniform in the sky, therefore the number of measurements in the two hemispheres, for a given direction, varies with the direction $n$ in a complicated manner. Therefore one might expect the probability distribution function $P(\Delta \chi^2)$ to be extremely complicated; however, the EVT in Section 3 shows that the distribution is, in fact, a simple, two-parameter Gumbel distribution, characteristic of extreme value distribution type I:

$$P(\Delta \chi^2) = \frac{1}{s} \exp \left[ -\frac{\Delta \chi^2 - m}{s} \right] \exp \left[ -\exp \left( -\frac{\Delta \chi^2 - m}{s} \right) \right].$$

(9)

where the position parameter $m$ and the scale parameter $s$ completely determine the distribution.

To quantify departures from isotropy we need to know the theoretical distribution $P_{\text{theory}}(\Delta \chi^2)$. Even though we know what to expect in a general manner, it is difficult to obtain the parameters $s$ and $m$ analytically; therefore we calculate this distribution numerically by simulating several sets of Gaussian-distributed $\chi_i$ on the measurement positions and obtaining $\Delta \chi^2$ from each realization.

If the noise in the data is Gaussian then the above distribution adequately quantifies the directional dependence in the data. But if the data have non-Gaussian noise then the theoretical distribution cannot be used to quantify the level of significance of our possible discovery of anisotropy. We construct an independent test for directional dependence by obtaining the bootstrap distribution $P_{BS}(\Delta \chi^2)$, which is constructed in the following manner. The observed $\chi_i$ values are assumed to be drawn from some unknown, direction-dependent probability distribution. We shuffle the data values $\chi_i$, $H_{0b}$ and $\sigma(\chi_i)$ over the measurement positions, thus destroying any directional alignment they might have had due to noise. Thus we are able to generate several realizations of data and estimate the distribution $P_{BS}(\Delta \chi^2)$.

In order to know what to expect from this statistic we simulate data and calculate the above-mentioned bootstrap and numerical distributions. These are shown in Fig. 2. As is discussed in Gupta & Saini (2010), there exists a specific bias between the two distributions. Since the numerical distribution is obtained by assuming $\chi_i$ values to be Gaussian random variables with a zero mean and unit variance, therefore it does not have any bounds. However, the bootstrap distribution is obtained by shuffling through a specific realization of $\chi_i$, where the $\chi_i$ values are obviously bounded. It is clear that on an average this should produce slightly smaller values of $\Delta \chi^2$ in comparison to what one expects of Gaussian distributed $\chi_i$ values.

Our results for the $\Delta \chi^2$ statistic in this paper should be interpreted with respect to Fig. 2. Concerns regarding the small number of data points in the data set and its effect on the efficacy of our method can be addressed (as in Gupta & Saini 2010) by noting that this figure is produced with only 76 points and that the theoretical and the bootstrap distributions look similar.

4.2 $\Delta \chi$ statistic

As mentioned above, $\chi^2_i$ does not contain information about whether the measurement is above or below the fit, i.e. greater or smaller than $H_{0b}$. An obvious generalization that does contain this information can be obtained by considering a statistic based on $\chi_i$ values. We consider two subsets of data defined by two hemispheres labelled by the direction vector $n$, containing $N_{\text{north}}$ and $N_{\text{south}}$ data points, where the total number of data points is $N = N_{\text{north}} + N_{\text{south}}$ and we define the quantity

$$\Delta \chi_n = \frac{1}{\sqrt{N}} \left( \sum_{i=1}^{N_{\text{north}}} \frac{\chi_i}{\sigma_i} - \sum_{j=1}^{N_{\text{south}}} \frac{\chi_j}{\sigma_j} \right).$$

(10)

Clearly, $\langle \Delta \chi_n \rangle = 0$ and $\langle (\Delta \chi_n)^2 \rangle = 1$. From the CLT (Kendall & Stuart 1977) it follows that for $N \gg 1$, the quantity $\Delta \chi$ follows a...
Gaussian distribution with a zero mean and unit variance. As in the previous case we maximize this quantity by varying the direction \( n \) across the sky to obtain the maximum absolute difference

\[
\Delta_x = \max(\lvert \Delta x_n \rvert).
\]  

This statistic differs from the previous one in that the \( \Delta_x \) statistic has a theoretical limit where the position and the shape parameters can be determined analytically. Given \( N_d \) independent directions in the sky we are essentially determining the maximum of a sample of size \( N_d \) where the individual numbers are drawn from a Gaussian distribution with a zero mean and unit variance. In the limit \( N_d \gg 1 \) the parameters are given by Haan et al. (2006):

\[
m = \sqrt{2 \log N_d - \log \log N_d - \log 4\pi},
\]

\[
s = \frac{1}{m},
\]  

where we have to additionally assume that the number of measurements \( N \gg 1 \) since the distribution for \( \chi \) becomes Gaussian only in this limit. This is convenient since at least for large data sets, which will be available in the future, a comparison with theory becomes simpler. However, for a smaller number of measurements (data points) there is a possibility that not all directions are independent; in fact it is quite possible that two directions contain exactly the same subsets in the two hemispheres. In this situation it is clear that the total number of independent directions is a smaller number than \( N_d \) and thus the theoretical distribution would be rightward shifted and also more sharply peaked. For this reason we also calculate the bootstrap distribution and the theoretical distribution in the same manner as for the previous statistic.

5 RESULTS

First we obtain the best-fitting value, \( H_{0\beta} \), of the Hubble constant for the data, which is shown in Table 1. The large value of \( \chi^2 \) suggests that the error bars may have been underestimated.

\( \Delta_x \) statistic. We have applied the \( \Delta_x \) statistic to these data and calculated the bootstrap and theoretical distributions for the HST Key Project data, as discussed earlier. The theoretical and bootstrap distributions for these data are shown in Fig. 3. To interpret our results we compare with Fig. 2. We see that the theoretical distribution has been shifted far away on the left-hand side of the bootstrap distribution. This contradicts the fact that the bootstrap distribution should lie slightly to the left of the theoretical distribution, as explained above, indicating the non-Gaussian nature of the errors in the data. HST key data lie outside the theoretical distribution, but are close to the peak of the bootstrap distribution. This suggests that either the data are free from direction-dependent systematics or a more sensitive technique is required to put constraints on these systematics.

\( \Delta_x \) statistic. We apply the \( \Delta_x \) statistic to the data and calculate the bootstrap and numerical distributions. These are shown in Fig. 4. Here also the numerical distribution lies to the left of the bootstrap distribution, which is in contradiction to the expected relative positions of the two. Interestingly, in this case we have an advantage, since we can calculate the theoretical distribution analytically. This analytic distribution is shown by the dotted line in Fig. 4. It was

\begin{tabular}{lll}
Table 1. Best-fitting value for \( H_0 \). & \\
Best fit & \( \chi^2 \) & \( \chi^2/\text{d.o.f.} \) \\
\hline
72.0 & 194.1 & 2.6
\end{tabular}

Figure 3. Here we plot the probability density of \( \Delta_{x^2} \). The solid curve represents the bootstrap distribution while the broken curve represents the theoretical distribution assuming Gaussian errors. The two curves do not match.

Figure 4. Here we plot the probability density of \( \Delta_x \). The solid curve represents the bootstrap distribution while the broken curve represents the numerically calculated distribution assuming Gaussian errors. The two curves do not match very well. The analytically calculated distribution is shown by the dotted lines. \( \Delta_x \) for the HST key data lies outside the 1σ region around the peak of the bootstrap distribution.
discussed in Section 4.2 that if all the directions are not independent then the analytic distribution should lie on the right-hand side of the numerical distribution and should be peaked sharply. This is what we observe in Fig. 4. The, bootstrap distribution lies on the right of the analytic distribution; however, we find that it is wider than what we would expect from a true Gumbel distribution. This indicates that the bootstrap distribution is not truly Gumbel, therefore it should be compared with the numerical distribution, which is calculated in a manner that is identical to the bootstrap one. Comparison shows that the two distributions differ in a manner identical to the difference seen in Fig. 3, based on the $\Delta_{\chi^2}$ statistic; indicating a similar level of non-Gaussianity. We also find that the position of the data lies outside the 1σ region from the peak of the bootstrap distribution.

6 CONCLUSIONS

We have applied the $\Delta_{\chi^2}$ and $\Delta_{\chi}$ statistics to the HST Key Project data. We find that in both cases the bootstrap and the theoretical distributions are very different from each other. Thus we conclude that the nature of the errors in the data is non-Gaussian. The $\Delta_{\chi^2}$ statistic does not show direction dependence in the data; however, the $\Delta_{\chi}$ statistic, which is more sensitive to the direction dependence, shows the presence of direction-dependent systematics at around the 1σ level.

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