Deterministic preparation of $W$ States via spin-photon interactions

Fatih Ozaydin, Can Yesilyurt, Sinan Bugu, and Masato Koashi

1Institute for International Strategy, Tokyo International University, 1-13-1 Matoba- kita, Kawasaki, Saitama, 350-1197, Japan
2Department of Physics, Faculty of Science, Istanbul University, 34416, Vezneciler, Istanbul, Turkey
3Department of Electrical and Electronic Engineering, Tokyo Institute of Technology, 2-12-1 Ookayama, Meguro-ku, Tokyo 152-8552, Japan
4Photon Science Center, Graduate School of Engineering, The University of Tokyo, 7-3-1 Bunkyo-ku, Tokyo 113-8656, Japan

Spin systems such as silicon or nitrogen vacancy centers in diamond, quantum dots and quantum dot molecules coupled to optical cavities appear as key elements for creating quantum networks as not only constituting the nodes of the network, but also assisting the creation of photonic networks. Here we study deterministic preparation of arbitrary size $W$ states with spin systems. We present an efficient operation on three qubits, two being the logical qubits and one being the ancillary qubit, where no interaction between the logical qubits are required. The proposed operation can create a $W$-type Einstein-Podolsky-Rosen (EPR) pair from two separable qubits, and expand that EPR pair or an arbitrary size $W$ state by one, creating a $W$-like state. Taking this operation as the fundamental building block, we show how to create a large scale $W$ state out of separable qubits, or double the size of a $W$ state. Based on this operation and focusing on nitrogen vacancy (NV) centers in diamond as an exemplary spin system, we propose a setup for preparing $W$ states of circularly polarized photons, assisted by a single spin qubit, where no photon-photon interactions are required. Next, we propose a setup for preparing $W$ states of spin qubits of spatially separated systems, assisted by a single photon. We also analyze the effects of possible imperfections in implementing the gates on the fidelity of the generated $W$ states. In our setups, neither post-measurement, nor post-processing on the states of spin or photonic qubit is required. Our setups can be implemented with current technology, and we anticipate that they contribute to quantum science and technologies.

I. INTRODUCTION

Preparing multipartite entangled systems belonging to basic classes such as GHZ \(^1\), cluster \(^2\), Dicke \(^3\) and $W$ \(^4\), is not only a critical step for enabling quantum technologies but also vital for understanding the quantum entanglement from a fundamental perspective. The preparation methods for cluster and GHZ states are usually straightforward \(^2,5\) with ongoing efforts on various physical systems such as quantum dot spins \(^6\). On the other hand, an intense theoretical and experimental effort has been devoted to develop efficient methods for preparing $W$ states \(^7,15\).

This is because an $n$-qubit cluster state in a generic form can be prepared via a series of controlled gates and single qubit rotation gates \(^16,17\), and a GHZ state in the form

\[
|\text{GHZ} \rangle = \left( |0 \rangle^{\otimes n} + |1 \rangle^{\otimes n} \right) / \sqrt{2}
\]

(1)
can be prepared via a series of controlled-NOT gates and Hadamard gates \(^5\) efficiently. However, efficient preparation of an $n$-qubit $W$ state requires more efforts due to its sophisticated form \(^18\)

\[
|W_n \rangle = \frac{1}{\sqrt{n}} (|0 \rangle^{\otimes(n-1)}|1\rangle + \sqrt{n-1}|W_{n-1}\rangle|0\rangle),
\]

(2)

which reads for $n = 4$ as follows

\[
|W_4 \rangle = \frac{1}{\sqrt{4}} (|0001\rangle + |0010\rangle + |0100\rangle + |1000\rangle).
\]

A probabilistic fusion setup for polarization based encoded photons was proposed \(^18\) and further improved for increasing the probability of success and simultaneous fusion of several $W$ states \(^19,20\). In a recent work, a quantum eraser model was proposed to enhanced the $W$ state fusion process \(^27\).

Fusion approach was applied to cavity-QED systems for creating $W$ states of atoms \(^28,32\) and quantum dot spins \(^33,34\). Recently, it was shown that a double quantum dot system can be used for fusing two $W$ states of spin qubits using Pauli spin blockade, requiring no cavities and ancillary photons \(^35\).

Deterministic expansion schemes were proposed to prepare $W$ states of 4-qubits \(^36\) and finally large-scale $W$ states of arbitrary number of polarization based encoded photonic qubits \(^37\). This strategy was then considered in preparing atomic qubits via cavities \(^31\), and in implementing efficient algorithms on IBM quantum computer \(^38\).

On the other hand, spin systems are of great importance in quantum science and technologies. However, a deterministic expansion strategy for spin systems based on spin-photon interactions is missing. Addressing this problem in the present paper, we propose an optimum three-qubit operation and present its circuit model decomposed into only two- and single-qubit gates. The operation applies on two logical qubits and one ancillary

*Corresponding author: can-yesilyurt@hotmail.com
qubit, such that through the ancillary qubit, the desired interaction between the logical qubits is realized without any direct interaction between them.

Then, we first show how this three-qubit operation can be implemented for a system where an ancillary spin qubit assists preparing a $W$ state of circularly polarized photons. Next, we present the essential contribution of this work, i.e., how to create large-scale $W$ states of spatially separated spins qubits, using an ancillary photon.

The proposed scheme can be realized in various spin systems that possess spin selective reflectivity. Quantum-dot-based technologies and NV centers are the most common platforms in which our proposed method can be applied. The required tasks to prepare $W$ states can be achieved by sequential detection of auxiliary photons in a system with quantum dots or quantum dot molecules coupled to plasmon or optical cavities. Similarly, instead of quantum dots, spin states in solid-state materials such as silicon-vacancy centers in diamond and silicon carbide can be considered as alternative platforms. Besides, various novel phenomena such as spin selective metasurfaces, chiral molecules, chiral metamirrors, and chiral coupling of valley excitons can be utilized to perform the operations required in the present scheme. Achieving high coherence times even in the room temperature, NV centers coupled to high quality optical cavities enable significant advances in quantum information science. Including the advantages and excluding the disadvantages of optical and solid state systems, NV centers are considered as strong candidates for distributed computing and quantum communication among large-scale quantum networks. Based on the interaction between an incident photon and the NV center, it has been shown that it is possible to create entanglement and even realize basic two- and three-qubit gates between the electronic spins of spatially separated NV centers. Very recently, the idea of fusing two existing $W$ states to create a larger one has been studied for NV centers, opening a new direction in this field. Hence, we choose NV centers to give an exemplary implementation of our general scheme based on spin-photon interactions in microcavities.

This paper is organized as follows: In Section II, we present the three-qubit operation in consideration. In Section III, we present the physics of the NV center briefly as the exemplary spin system, and the interaction between an incident photon and the spin which realizes a controlled-Z gate. In Section IV and V, we present the implementations of this operation to create $W$ states of circularly polarized photons and spins qubits, respectively. In Section VI, we analyze how possible imperfections in implementing the single- and two-qubit gates affect the fidelity of the generated $W$ state. In Section VII, we discuss some advantages and drawbacks of the present scheme and conclude.

## II. A THREE-QUBIT OPERATION FOR DETERMINISTIC PREPARATION OF $W$ STATES

In the common scenario, a three-qubit operation can be considered for an ancillary photon and two spatially separated spin qubits to be entangled. We will introduce such a three-qubit operation as the basic building block, show how it can be realized and used for preparing arbitrary size $W$ states of spins qubits. We will also show that this operation can be used for preparing photonic $W$ states, such that instead of photon-photon interactions, each photon interacts only with the spin qubits separately. This operation can be considered as an optimization of three-qubit-extension of the two-qubit deterministic expansion circuit presented in Ref. which consists of a controlled-Hadamard and a controlled-NOT gate as shown in Fig. 1.

With a qubit in state $|0\rangle$ in input 1 and a qubit in state $|0\rangle$ in input 2, this circuit performs the transformation $|10\rangle \rightarrow (|01\rangle + |10\rangle)/\sqrt{2}$. However, as will be detailed later in this section, if the qubit in input 1 belongs to a $|W_n\rangle$ state, the transformation results in an $n+1$ qubit $W$-like state. Repeating the procedure for each qubit of the initial $|W_n\rangle$ state (with an additional qubit in $|0\rangle$ state in input 2), a genuine $|W_{2n}\rangle$ state is prepared.

Extending a general two-qubit operation -consisting of two controlled gates to a three qubit operation where the logical qubits do not interact directly but the interaction is realized through an ancillary qubit, requires two swap operations. Each swap operation can be realized by three two-qubit gates (between the ancillary qubit and the logical qubits), summing up to eight two-qubit gates. However, because the input qubits of the deterministic expansion operation in consideration have no general but some specific states, that is, not $\{00, 01, 10, 11\}$ but $\{00, 10\}$, it is possible to remove half of the two-qubit gates to realize this operation.

As illustrated Fig. 2, we consider two logical qubits in the inputs 1 and 2, and an ancillary qubit in input $Anc$. The circuit for the operation consists of four controlled-Z (CZ) gates and eight single qubit gates, i.e., six Hadamard gates and two $T'$ gates with the operation

$$T' = \begin{pmatrix} 
\cos \theta & \sin \theta \\
\sin \theta & -\cos \theta 
\end{pmatrix},$$

for $\theta = \frac{\pi}{8}$. It is straightforward to show that the overall operation on three qubits in the computational basis is

$$O = \begin{pmatrix} 
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\
0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} 
\end{pmatrix}.$$
Applying this operation and tracing out the ancillary qubit, a \( W \)-type Bell pair in the form \( \frac{1}{\sqrt{3}} (|01\rangle + |10\rangle) \) can be created from a three qubit system initially in the separable state \( |1\rangle|0\rangle|0\rangle \) in inputs 1, Anc and 2, respectively. In more detail, the transformation for the initial state \(|1\rangle|0\rangle|0\rangle \) is described as

\[
\begin{align*}
T'_1, CZ_1, T'_2 & \rightarrow |1\rangle \left( \frac{|0\rangle + |1\rangle}{\sqrt{2}} \right) |0\rangle, \\
H_1, CZ_2, H_2 & \rightarrow \left( \frac{|1\rangle|0\rangle + |0\rangle|1\rangle}{\sqrt{2}} \right) (|0\rangle), \\
H_3, CZ_3, H_4 & \rightarrow \left( \frac{|1\rangle|0\rangle|0\rangle + |0\rangle|1\rangle|1\rangle}{\sqrt{2}} \right), \\
H_5, CZ_4, H_6 & \rightarrow \left( \frac{|1\rangle|0\rangle|0\rangle + |0\rangle|0\rangle|1\rangle}{\sqrt{2}} \right), \\
\text{tr}_{\text{Anc}} & \rightarrow \left( \frac{|1\rangle|0\rangle + |0\rangle|1\rangle}{\sqrt{2}} \right),
\end{align*}
\]

that is, although the two qubits in input 1 and input 2 have never interacted directly, a two-qubit \( W \)-type Bell pair is created between them. Now, let us consider that the qubit in input 1 is the \( i \)'th qubit of an \( n \)-qubit \( W \) state in the form \( |W_n\rangle = \frac{1}{\sqrt{n}} \sum_{i=1}^{n} \otimes_{j=1}^{n} |\delta_{ij}\rangle \). Then, this operation expands the state with the qubit in input 2 (denoted as \(|0\rangle_2\)), creating an \( n+1 \) qubit \( W \)-like state as follows. The \( i \)'th term of the initial state with superposition coefficient \( 1/\sqrt{n} \) splits into two terms with superposition coefficients each \( 1/\sqrt{2n} \), as

\[
\begin{align*}
|0\rangle \otimes_{i=1}^{n-1} |1\rangle_i \otimes |0\rangle_2 \otimes |0\rangle \otimes_{i=n}^{\infty} & \rightarrow \left( \frac{|0\rangle \otimes_{i=1}^{n-1} |1\rangle_i \otimes |0\rangle_2 \otimes |0\rangle \otimes_{i=n}^{\infty}}{\sqrt{2n}} \right) \otimes_{i=1}^{n-1} |\delta_{ij}\rangle \\
& \rightarrow \left( \frac{|0\rangle \otimes_{i=1}^{n-1} |1\rangle_i \otimes |0\rangle_2 \otimes |0\rangle \otimes_{i=n}^{\infty}}{\sqrt{2n}} \right) \otimes_{i=1}^{n-1} |\delta_{ij}\rangle + \left( \frac{|0\rangle \otimes_{i=1}^{n-1} |0\rangle_i \otimes |1\rangle_2 \otimes |0\rangle \otimes_{i=n}^{\infty}}{\sqrt{2n}} \right) \otimes_{i=1}^{n-1} |\delta_{ij}\rangle,
\end{align*}
\]

Taking this expansion operation as the basic building block, it is easy to see that applying it to each qubit of the initial \( n \)-qubit \( W \) state with a new qubit in input 2, a \( 2n \)-qubit \( W \) state is created, having \( 2n \) terms all with coefficients \( 1/\sqrt{2n} \).

In Fig. 2, we illustrate the first three steps of our strategy for preparing a large scale \( W \) state, requiring no initial entanglement. We start with a qubit (red circle) in \(|1\rangle\) state. In the first row, applying the basic expansion operation (blue dashed rectangle) to the qubit with an additional qubit (green circle) in \(|0\rangle\) state (and the ancillary qubit in \(|0\rangle\) state, not shown in the figure) a \( W \)-type Einstein-Podolsky-Rosen (EPR) pair is created. In the middle row, the EPR pair is expanded with another qubit in \(|0\rangle\) state, creating a \( 3 \)-qubit \( W \)-like state in the form \( \frac{1}{\sqrt{4}} |100\rangle + \frac{1}{\sqrt{4}} |010\rangle + \frac{1}{\sqrt{4}} |001\rangle \). The third step is to expand the \( 3 \)-qubit \( W \)-like state with another qubit in \(|0\rangle\) state, obtaining a \( 4 \)-qubit \( W \) state in the form \( \frac{1}{\sqrt{8}} |1000\rangle + \frac{1}{\sqrt{8}} |1010\rangle + \frac{1}{\sqrt{8}} |0100\rangle + \frac{1}{\sqrt{8}} |0010\rangle + \frac{1}{\sqrt{8}} |0001\rangle \). It is straightforward to continue expanding the \(|W_4\rangle\) state in the same fashion.

Besides starting with qubits initially in a separable state, it is also possible to double the size of an existing \( n \)-qubit \( W \) state. Assuming that each intermediate step for expanding the state by one qubit is appropriately achieved -which can also be done in parallel, we now move to a more intuitive notation. We consider that the initial state is in the form \(|W_n\rangle_0 \otimes |0\rangle \otimes |\delta_{ij}\rangle_\text{Anc} \), which requires swapping qubits appropriately so that each triple of qubits shall consist of one qubit of the \( W \) state, one ancillary qubit, and one qubit to join the \( W \) state, respectively. For \( n = 3 \), the overall expansion process can be described as

\[
|W_3\rangle_0 \otimes |0\rangle_3 \otimes |\delta_{ij}\rangle_\text{Anc} \rightarrow \text{tr}_{2,5,8} (SW_{5,9} SW_{3,4} SW_{2,7}) (|W_3\rangle_0 \otimes |3\rangle_\text{Anc}) = |W_6\rangle,
\]

where \( \text{tr}_i \) denotes tracing out \( i \)'th qubit and \( SW_{i,j} \) denotes swapping \( i \)'th and \( j \)'th qubits. For \( n \) qubits, denoting the overall qubit swap operations as \( \text{SWAP} \), the expansion process reads...
rate. is the cavity decay rate and is the coupling strength of the cavity to the NV center, a photon pulse is found as [66–68].

Langevin equations can be solved and neglecting the vacuum input field in the rotating frame, assuming weak as-

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III. PHYSICAL MODEL

On the contrary to inherently probabilistic fusion or expansion operations presented in Refs. 812 [18–

24] 30, 32, 33, this operation is in principally deterministic, i.e. besides inevitable experimental imperfections -

which also exist in fusion operations, the resultant state is

predicted with certainty, requiring no post-measurement

that would possibly destroy some of the qubits and shrink

the size of the target state.

\[
W_n|0\rangle^{\otimes n}|0\rangle_{Anc} 
\rightarrow \text{tr}_{2, 5, ..., 2n-1}(O^{\otimes n}\text{SWAP}|W_n|0\rangle^{\otimes n}|0\rangle_{Anc})
\] (9)

FIG. 3. Large scale W-state preparation strategy, starting from initially separable qubits in |1⟩ and |0⟩ states in the first

step (first row), shown with red and green circles, respectively. Blue dashed rectangle represent the expansion process and

the ancillary qubit is not shown for clarity. A 3-qubit W-like

state is prepared in the second step (middle row) which is then expanded to a genuine |W_4⟩ state in the third step (last

row).

\[
W_n|0\rangle^{\otimes n}|0\rangle_{Anc} 
\rightarrow \text{tr}_{2, 5, ..., 2n-1}(O^{\otimes n}\text{SWAP}|W_n|0\rangle^{\otimes n}|0\rangle_{Anc})
\] (9)

\[
W_{2n}. \]

The reflection coefficients can be obtained for the resonant condition \(\omega_p = \omega_0 = \omega_C\) as

\[
r(\omega_p) = -\frac{\kappa \gamma + 4g^2}{\kappa \gamma + 4g^2}, \text{ and } r_0(\omega_p) = -1. \] (12)

FIG. 4. Λ type optical transitions possible in an NV center. The transitions \(|-\rangle \leftrightarrow |e\rangle\) and \(|+\rangle \leftrightarrow |e\rangle\) are associated with the left and right circular polarization of the photon, denoted as |L⟩ and |R⟩ respectively.

If the NV center is uncoupled from the cavity, the reflection coefficient for the input photon becomes

\[
r_0(\omega_p) = \frac{i(\omega_C - \omega_p) - \kappa}{i(\omega_C - \omega_p) + \frac{\kappa}{2}}. \] (11)

\[|R\rangle \text{ and } |L\rangle \text{ denoting the right and left circular polarization states, respectively; due to the spin-dependent optical transition rules [61] as simply illustrated in Fig. 4 and optical Faraday rotation, an } |R\rangle \text{ polarized incident photon receives a phase shift } e^{i\phi_{0}} \text{ because, due to large level splitting, the spin state of the NV center is decoupled from the incident pulse [62]. However, if the incident photon is } |L\rangle \text{ polarized, it will receive a phase shift } e^{i\phi} (e^{i\phi_{0}}) \text{ depending on the spin state of the NV center } |-\rangle \text{ } (|+\rangle), \text{ where } \phi \text{ and } \phi_{0} \text{ are the arguments of the complex numbers } r(\omega_p) \text{ and } r_0(\omega_p), \text{ respectively. For the resonant condition, and } g > 5\sqrt{\kappa\gamma}, \text{ one approximately finds } \phi = 0 \text{ and } \phi_{0} = \pi. \text{ Placing a } \pi \text{ phase shifter to the photon reflection path, a controlled-Z gate between the electronic spin of the NV center and the incident photon is realized as } |R\rangle|+\rangle \rightarrow |R\rangle|+\rangle, |R\rangle|-\rangle \rightarrow |R\rangle|-\rangle, |L\rangle|+\rangle \rightarrow |L\rangle|+\rangle, \text{ and } |L\rangle|-\rangle \rightarrow -|L\rangle|-\rangle. \] (6) (61) (63)

Note that the implementations of single qubit operations on NV center spins and incident photons are presented in the Appendix.

IV. CREATING OR EXPANDING A PHOTONIC W STATE USING AN NV CENTER

In this section, we first present how a W-type photonic Bell state in the form \(|W_2\rangle = 1/\sqrt{2}(|L\rangle|R\rangle + |R\rangle|L\rangle)\) can be created from an initially separable state of two photons in the circular polarization states |R⟩ and |L⟩. The strategy is as follows: A photon in input 1, spin in the NV center (as the ancillary qubit) and a photon in input 2 are prepared in the |L⟩ ⊗ |+⟩ ⊗ |R⟩ state,
We now present how to create a two-qubit $W$-type Bell pair, and how to expand an arbitrary size $W$ state of distant NV center spins in the form

$$\ket{W_n} = \frac{1}{\sqrt{n}} \left( |L_1 R_2 ... R_{n-1} R_n\rangle + |R_1 L_2 ... R_{n-1} R_n\rangle + \ldots + |R_1 R_2 ... R_{n-1} L_n\rangle \right)$$

(13)

via an ancillary photon, implementing the circuit model in Fig. 2. An NV center ($NV_2$) with the spin state $|+\rangle$ is in input 2, and the ancillary photon in the state $|R\rangle$ is in Anc input. In the $W$-type Bell pair-creation scenario, an NV center in the state $|-\rangle$, and in the $W$ state expansion scenario, an NV center spin as one of the qubits of the $W$ state is in input 1 ($NV_1$). As illustrated in Fig. 6, the photon interacts with $NV_1$ between two $T'$ gates. Before and after the second interaction, a Hadamard gate is applied to $NV_1$. Then the photon is sent to $NV_2$, which is subject to Hadamard gates before and after the first interaction. Finally, the photon interacts with $NV_2$ again, between two Hadamard gates. It is straightforward to show that this setup realizes the circuit model in Fig. 2, achieving the operation $O$ in Eq. 5. That is, a three-qubit system of $NV_1$, ancillary photon and $NV_2$ initially prepared in $|-\rangle |R\rangle |+\rangle$ state is transformed into $1/\sqrt{2}(-|R\rangle|+\rangle + |+\rangle |R\rangle |-)\rangle$ state, i.e. creating $W$-type Bell pair $1/\sqrt{2}(-|+\rangle |+\rangle + |+\rangle |+\rangle |-)\rangle$ between two distant NV center spins, leaving the ancillary photon in the $|R\rangle$ state.
FIG. 7. An ancillary photon (Anc) in $|R\rangle$ circular polarization state is sent to a pair of NV centers, first (red circle) belonging to the $|W_n\rangle$ state to be expanded, and second (green circle) being the one to join to the $W$ state. Each operation expands the state by one qubit, creating a $W$-like state, and with the last operation on $n$th pair, a $|W_{2n}\rangle$ state is obtained. Alternatively, $n$ ancillary photons can be used to realize the expansion process in parallel. Single qubit gates (in Fig. 6) and the $\pi$ phase shifter on the photon output paths are not shown.

ready for another operation.

The implementation of the strategy of Fig. 3 for expanding a $|W_n\rangle$ state to a $|W_{2n}\rangle$ state is as follows, and illustrated in Fig. 7. An ancillary photon in $|R\rangle$ state is sent to each pair of NV centers, first being a qubit of the $W$ state to be expanded, and second being the additional qubit to join the $W$ state. Single qubit operations are applied appropriately as shown in Fig. 6. Each of the $n - 1$ operations expands the $W$ state by one qubit, leading to a $W$-like state with weighted superposition coefficients, as explained in Section III. Finally, the last operation creates a genuine $|W_{2n}\rangle$ state.

Due to the flexibility of our model, the same ancillary photon can be used to apply the operations by traveling among each pair of NV centers consecutively in a serial manner, or $n$ distinct ancillary photons can be used in parallel. For the latter case, the transformation $|W_n\rangle \otimes |R\rangle \otimes (\ket{+}\rangle \otimes n) \rightarrow |W_{2n}\rangle \otimes |R\rangle \otimes n$ is achieved and a $W$ state of $2n$ NV center spins is prepared.

VI. FIDELITY ANALYSIS DUE TO IMPERFECTIONS

In this section, we analyze the effects of non-ideal gates of the circuit presented in Fig. 2 on the fidelity of the prepared $W$ state. We assume that the initial logical and ancillary qubits are prepared in the ideal state, and after the operations, the ancillary qubit is left in its ideal initial state (or that after each round, a fresh ancillary qubit can be prepared). Because no post-measurements are required, we focus on possible imperfections in implementing controlled-Z (CZ), Hadamard and $T'$ gates. We consider non-ideal Hadamard and $T'$ gates as follows:

$$H(\alpha) = \begin{pmatrix} \cos(\theta - \alpha) & \sin(\theta - \alpha) \\ \sin(\theta - \alpha) & -\cos(\theta - \alpha) \end{pmatrix},$$  

(15)

$$T'(\beta) = \begin{pmatrix} \cos(\theta - \beta) & \sin(\theta - \beta) \\ \sin(\theta - \beta) & -\cos(\theta - \beta) \end{pmatrix},$$  

(16)

with $\theta = \pi/4$ and $\theta = \pi/8$. They represent ideal Hadamard and $T'$ gates for $\alpha = 0$ and $\beta = 0$, respectively. Similarly, a non-ideal CZ gate is considered as a general controlled-phase gate

$$\text{CP}(\gamma) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \Exp[i(\pi - \gamma)] \end{pmatrix},$$  

(17)

representing an ideal CZ gate for $\gamma = 0$. Hence, for a non-zero $\alpha$, $\beta$ or $\theta$, not a genuine $W$ state but a $W$-like state is obtained. We calculate the fidelity of the $W$-like state obtained in the size doubling process, i.e. $|W_n\rangle \rightarrow |W_{2n}\rangle$ with respect to these imperfections. We first find that the fidelity does not depend on the number of qubits, $n$. This finding can be interpreted as follows, and be related to the robustness of $W$ states. Unlike a GHZ state for example, in each superposition term of a $W$ state, only one qubit is in $|1\rangle$ state.

Therefore, although we do not measure and learn which one, each controlled-gate in the circuit in Fig. 2 apply (ideal or non-ideal) $Z$ operation only when the logical qubit in input 1 is in $|1\rangle$, and apply an identity operator otherwise. In the latter case, applying two single qubit gates of the same type consecutively is equivalent to an identity operator, as well. Hence, for an arbitrary $n$, including the case ($n = 1$) where an EPR pair is created from two separable qubits, the fidelities with respect to each imperfection separately and the combined one are found as

$$F_H(\alpha) = \left| {1 \over 2} + {1 \over 2} \cos(2\alpha)^3 \right|^2,$$

$$F_T(\beta) = \left| {1 \over \sqrt{2}} \left[ \cos \left( {\pi + 8\beta \over 4} \right) + \sin \left( {\pi + 8\beta \over 4} \right) \right] \right|^2,$$

$$F_{\text{CP}}(\gamma) = \left| {1 \over 2} e^{-2i\gamma} \cos \left( {\pi \over 2} \right)^4 + {1 \over \sqrt{2}} \left[ \cos \left( {\pi \over 8} \right)^2 - e^{-i\gamma} \sin \left( {\pi \over 8} \right)^2 \right] \right|^2,$$

$$F_{\text{Combined}}(\alpha, \beta, \gamma) = \left| -e^{-4i\gamma}(1 + e^{i\gamma})^4 \cos(2\alpha)^3 \cos \left( {1 \over 2} (\pi + 8\beta) \right) \right|_{16\sqrt{2}}^2$$

$$+ \left| e^{-i\gamma} \left\{-1 + e^{i\gamma} + (1 + e^{i\gamma}) \sin \left( {1 \over 2} (\pi + 8\beta) \right) \right\} \right|^2.$$
We plot the fidelity results with respect to the same parameter, $0 \leq \Theta \leq \pi/60$ in Fig. 8, and find that for $\Theta = \pi/60$ with all the gate imperfections combined, fidelity is greater than 0.97.

VII. DISCUSSION

The analyses in Refs. [58, 65] suggest that the present setups are feasible with the current technology. Previous proposals and experimental demonstrations for preparing $W$ states based on fusion strategies are inherently probabilistic, requiring post-selection and possibly post-processing even in the ideal experimental conditions. This causes a significant decrease in the efficiency of the strategies, requiring a sub-exponential resource cost in the optimal case as shown in Monte Carlo simulations in Refs. [18, 21]. However, we call our schemes deterministic in principle because in an ideal experimental realization, no matter what technology and what kind of qubits are used, our circuit model illustrated in Fig. 2 operates in a deterministic way. Besides possible experimental imperfections - which are plausibly minimized using NV centers even in the room temperature - the spin-photon interaction is not ideal due to broad phonon sideband. However, being deterministic in principle, the present proposal can be significantly more efficient than the proposals which are probabilistic even without including the experimental imperfections.

Another essential aspect of the present proposal is that, in most of the proposals based on spin systems, after the interaction with the incident photon, an extra photon is required to interact with spin qubit to reveal the spin state or the entangled state created between the qubits. Our proposal on the contrary, does not require such a post-measurement or forward processing which requires additional interactions. Similarly to a recent work on implementing the Magic Square Game with quantum dots [69], the ancillary system herein is used just like a catalyst, left in the initial state after the operation and can be used again. Note that in any task based on distant spin or atomic qubits and traveling ancillary photons such as those in [54, 55, 58, 60, 64, 69], operations on three or more qubits are implicitly realized. Hence, considering a three-qubit operation as the basic building block enables a more systematic approach. The three-qubit operation $O$ presented in Eq. 5 and its applications to $W$ states of photonic or spin qubits can be used not only to expand a $|W_n\rangle$ with $n$ qubits to a $|W_{2n}\rangle$ in a deterministic way, but also to expand a $W$ state with arbitrary size of qubits in a probabilistic way with specific fidelities, following the approach presented in Ref. [37].

Note that we do not exclude the possibility of further decreasing the two-qubit operations (realized by the interaction of the incident photon with the spin). However, such a reduction would break the in-principle-deterministic nature of our proposal, requiring post-measurements and post-processing, achievable by introducing additional two-qubit operations, resulting in the increase of the number of overall two-qubit operations.

Regarding the feasibility of our setup, experimental demonstrations of polarization independent couplers with high efficiency were reported [70, 71]. Microtoroid resonators with whispering gallery modes (WGM) have been considered for realizing the photon-NV center spin interactions [55, 59, 60, 65]. Due to the non-transversality of WGM, the WGM resonators behaves differently from conventional ring or Fabry-Perot resonators [72]. That is, the orthogonal polarization states are correlated with the propagating directions of the photons, making the counter-propagating photons distinguishable. As an alternative to WGM microtoroid cavities, single-sided cavities can be utilized for realizing the interactions required in our setups. Single- and double-sided cavities have been recently attracted attention for realizing spin-photon interactions [73, 77]. On the other hand, due to their relatively low Q-factors, a major issue in using single-sided optical cavities is that photon losses and effective weak measurements can decrease the fidelity of the prepared state. However, considering the NV center in diamond in a photonic crystal (PC) modeled as a single-sided low-Q cavity, Young et al. showed that the NV center spin can be efficiently measured with high fidelity, and that with appropriate modifications, their system can be used for entangling spatially separated NV center spins [78]. Achieving Q-factors up to $10^7$ [79], PCs are promising as cavities for NV center spins, and experimental demonstrations have been reported [80, 83].

An interesting future problem is to design error correction mechanisms for systems such as the one presented herein based on spin-photon interactions.

In conclusion, we proposed a three-qubit operation for creating and expanding arbitrary size $W$ states and decomposed this operation into only two- and single-qubit gates. We showed that this operation can be implemented for, i) photonic systems, assisted by an ancillary spin qubit, and ii) spin qubits, assisted by an ancillary photonic qubit. We presented two exemplary setups for
NV center systems in microcavities. We analyzed the effects of experimental imperfections in implementing the gates on the fidelity of the prepared $W$ state.

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APPENDIX

Single qubit operations on photons considered in this paper can be realized simply by an half wave plate (HWP) with the operation [84]

$$
HWP(\theta/2) = \begin{pmatrix}
\cos \theta & \sin \theta \\
\sin \theta & -\cos \theta
\end{pmatrix},
$$

which realizes an Hadamard gate for $\theta = \pi/8$ and a $T'$ gate for $\theta = \pi/16$.

For the single qubit operations on NV center spins, EM pulses can be used as described in Refs. [85, 86].

Here, we follow holonomic quantum computation which attracted much attention recently [87, 88]. In Ref. [90] it was demonstrated that the highest excited state $|e\rangle$ can be connected to $|\rangle$ and $|\rangle$ states by a single tunable laser which can generate frequency side-bands and nanosecond pulses [91] by electro-optical modulation. In the rotating frame, the system is described by the Hamiltonian

$$
H(t) = \frac{\hbar \Omega(t)}{2} (u|e\rangle\langle-| + v|e\rangle\langle+| + h.c.) + \Delta|e\rangle\langle e|. \quad (19)
$$

Here, $\Omega(t)$ is the pulse envelope for both tones and the transitions associated with $|\rangle$ $\leftrightarrow |e\rangle$ and $|\rangle$ $\leftrightarrow |e\rangle$ are scaled by complex constants $u = \sin(\theta/2)$ and $v = -e^{-i\phi} \cos(\theta/2)$, respectively, controlled by the timing of the relative strength and phase between the carrier and sideband frequencies. $\Delta$ denotes the photon detuning. The dark state of the Hamiltonian decoupled from the dynamics is $|d\rangle = \cos(\theta/2)|\rangle - |\rangle$ and the bright state which undergoes an excitation to $|e\rangle$ is $|b\rangle = \sin(\theta/2)|\rangle - e^{i\phi} \cos(\theta/2)|\rangle$. For the pulse duration $\tau = 2\pi/\sqrt{\Omega^2 + \Delta^2}$, a purely geometric operator is realized via the cyclic, non-adiabatic evolution in $\{|b\rangle, |d\rangle\}$ subspace as $U(\theta, \phi, \Delta/\Omega) = |d\rangle\langle d| + e^{i\gamma} |b\rangle\langle b|$, with $\gamma = \pi(1 - \Delta/\sqrt{\Omega^2 + \Delta^2})$, which performs

$$
U(\theta) = \begin{pmatrix}
\cos \theta & \sin \theta \\
\sin \theta & -\cos \theta
\end{pmatrix},
$$

for $\gamma = \pi$ and $\phi = 0$ up to a global phase [88]. Hence, Hadamard and $T'$ gates can be realized for $\theta = \pi/4$ and $\theta = \pi/8$, respectively.

[1] D. M. Greenberger, M. A. Horne, and A. Zeilinger, in Bell’s Theorem, Quantum Theory, and Conceptions of the Universe, edited by M. Kafatos (Kluwer, Dordrecht), p.69 (1989).
[2] H. J. Briegel and R. Raussendorf, Persistent Entanglement in Arrays of Interacting Particles, Phys. Rev. Lett. 86, 910 (2001).
[3] R. H. Dicke, Coherence in Spontaneous Radiation Processes, Phys. Rev. 93, 99 (1954).
[4] W. Dür, Multiparticle entanglement that is robust against disposal of particles, Phys. Rev. A 63, 3(R) (2001).
[5] M. A. Nielsen, I. L. Chuang, Quantum Computation and Quantum Information, Cambridge University Press (Cambridge) (2000).
[6] C. Miao, S-D. Fang, P. Dong, M. Yang and Z-L. Cao, Generation of large scale GHZ states with the interactions of photons and quantum-dot spins, Laser Phys. Lett. 15, 035204 (2018).
[7] T. Kobayashi, R. Ikuta, S. K. Ozdemir, M. Tame, T. Yamamoto, M. Koashi, and N. Imoto, Universal gates for transforming multipartite entangled Dicke states, New J. Phys. 16, 023005 (2014).
[8] T. Tashima, S. K. Ozdemir, T. Yamamoto, M. Koashi, and N. Imoto, Elementary optical gate for expanding an entanglement web, Phys. Rev. A 77, 030302 (2008).
[9] T. Tashima, S. K. Ozdemir, T. Yamamoto, M. Koashi, and N. Imoto, Local expansion of photonic $W$ state using a polarization-dependent beamsplitter, New J. Phys. 11, 023024 (2009).
[10] T. Tashima, T. Wakatsuki, S. K. Ozdemir, T. Yamamoto, M. Koashi, and N. Imoto, Local Transformation of Two Einstein-Podolsky-Rosen Photon Pairs into a Three-Photon $W$ State, Phys. Rev. Lett. 102, 130502 (2009).
[11] R. Ikuta, T. Tashima, T. Yamamoto, M. Koashi, and N. Imoto, Optimal local expansion of $W$ states using linear optics and Fock states, Phys. Rev. A 83, 012314 (2011).
[12] T. Tashima, T. Kitano, S. K. Ozdemir, T. Yamamoto, M. Koashi, and N. Imoto, Demonstration of Local Expansion Toward Large-Scale Entangled Webs, Phys. Rev. Lett. 105, 210503 (2010).
[13] R. Augusiak, M. Demianowicz, and J. Tura, Constructing genuinely entangled multipartite states with applications to local hidden variables and local hidden states models, Phys. Rev. A 98, 012321 (2018).
[14] A. Sharma, and A. A. Tulapurkar, Generation of n-qubit
W states using spin torque, Phys. Rev. A \textbf{101}, 062330 (2020).

[15] H.-R. Wei, W.-Q. Liu, and L.-C. Kwek, Efficient fusion of photonic W-states with nonunitary partial-swap gates, New J. Phys. \textbf{22}, 093051 (2020).

[16] R. Raussendorf, D. E. Browne, and H. J. Briegel, Measurement-based quantum computation on cluster states, Phys. Rev. Lett. \textbf{68}, 022312 (2003).

[17] M. S. Tame, Ş. K. Özdemir, M. Koashi, N. Imoto, and M. S. Kim, Compact Toffoli gate using weighted graph states, Phys. Rev. A \textbf{79}, 020302(R) (2009).

[18] S. K. Özdemir, E. Matsunaga, T. Tashima, T. Yamamoto, M. Koashi, and N. Imoto, An optical fusion gate for W-states, New J. Phys. \textbf{13}, 103003 (2011).

[19] S. Bugu, C. Yesilyurt, and F. Ozaydin, Enhancing the W-state quantum-network-fusion process with a single Fredkin gate, Phys. Rev. A \textbf{87}, 032331 (2013).

[20] C. Yesilyurt, S. Bugu, and F. Ozaydin, An optical gate for simultaneous fusion of four photonic W or Bell states, Quant. Inf. Process. \textbf{12}, 2965 (2013).

[21] F. Ozaydin, S. Bugu, C. Yesilyurt, A. A. Altintas, M. Tame, and S. K. Özdemir, Fusing multiple W states simultaneously with a Fredkin gate, Phys. Rev. A \textbf{89}, 042311 (2014).

[22] K. Li, F. Z. Kong, M. Yang, F. Ozaydin, Q. Yang, and Z.-L. Cao, Generating multi-photon W-like states for perfect quantum teleportation and superdense coding, Quant. Inf. Process. \textbf{15}, 3137 (2016).

[23] F. Diker, F. Ozaydin and M. Arik, Enhancing the W state fusion process with a Toffoli gate and a CNOT gate via one-way quantum computation and linear optics, Acta Phys. Pol. A \textbf{127}(4), 1189 (2015).

[24] K. Li, F.-Z. Kong, M. Yang, Q. Yang, and Z.-L. Cao, Qubit-loss-free fusion of W states, Phys. Rev. A \textbf{94}, 062315 (2016).

[25] X. Zang, M. Yang, W. Wu, and H.-Y. Fan, Generating multi-mode entangled coherent W and GHZ states via optical system based fusion mechanism, Quant. Inf. Process. \textbf{16}, 135 (2017).

[26] K. Li, T. Chen, H. Mao, and J. Wang, Preparing large-scale maximally entangled W states in optical system, Quant. Inf. Process. \textbf{17}, 307 (2018).

[27] Y.S. Kim, Y.W. Cho, H.T. Lim, S.W. Han, Efficient linear optical generation of a multipartite state via a quantum eraser, Phys. Rev. A \textbf{101}, 022337 (2020).

[28] M. Yang, Y. M. Yi, and Z. L. Cao, Scheme for preparation of W state via cavity QED, Int. J. Quantum Inform. \textbf{02}, 231 (2004).

[29] Z. J. Deng, M. Feng, and K. L. Gao, Simple scheme for generating an n-qubit W state in cavity QED, Phys. Rev. A \textbf{73}, 014302 (2006).

[30] X.-P. Zang, M. Yang, F. Ozaydin, W. Song, and Z.-L. Cao, Generating multi-atom entangled W states via light-matter interface based fusion mechanism, Sci. Rep. \textbf{5} 16245 (2015).

[31] X.-P. Zang, M. Yang, F. Ozaydin, W. Song, and Z.-L. Cao, Deterministic generation of large scale atomic W states, Opt. Exp. \textbf{24}(11) 12293 (2015).

[32] X. Zang, M. Yang, X. C. Wang, W. Song, and Z. L. Cao, Fusion of W states in a cavity quantum electrodynamic system, Can. J. Phys. \textbf{93}, 556 (2015).

[33] X. Han, S. Hu, Q. Guo, H. F. Wang, A.-D. Zhu, and S. Zhang, Effective W-state fusion strategies for electronic and photonic qubits via the quantum-dot-microcavity coupled system, Sci Rep. \textbf{5}, 12790 (2015).

[34] N. Li, J. Yang, and L. Ye, Realizing an efficient fusion gate for W states with cross-Kerr nonlinearities and QD-cavity coupled system, Quant. Inf. Process. \textbf{14}, 1933 (2015).

[35] S. Bugu, F. Ozaydin, T Ferrus, and T. Koder, Preparing multipartite entangled spin qubits via pauli spin blockade Sci. Rep. \textbf{10}, 1 (2020).

[36] C. Yesilyurt, S. Bugu, F. Diker, A. A. Altintas, and F. Ozaydin, An Optical Setup for Deterministic Creation of Four Partite W state, Acta Phys. Pol. A \textbf{127}(4), 1230 (2015).

[37] C. Yesilyurt, S. Bugu, F. Ozaydin, A. A. Altintas, M. Tame, L. Yang, and S. K. Özdemir, Deterministic local doubling of W states, J. Opt. Soc. Am. B \textbf{33}, 2313 (2016).

[38] D. Cruz, R. Fournier, F. Gremion, A. Jeannerot, K. Komagata, T. Toshi, J. Thiesbrummel, C. L. Chan, N. Macris, M.-A. Dupertuis, and C. Javerzac-Galy, Efficient quantum algorithms for GHZ and W states, and implementation on the IBM quantum computer, Adv. Quantum Technol. \textbf{2}, 1900015 (2019).

[39] D. E. Westmoreland, K. P. McClelland, K. A. Perez, J. C. Schwabacher, Z. Zhang, and E. A. Weiss, Properties of quantum dots coupled to plasmons and optical cavities, J. Chem. Phys. \textbf{151}(21), 210901 (2019).

[40] J. Cui, Y. E. Panfil, S. Koley, D. Shamalia, N. Waiskopf, S. Remennik, I. Popov, M. Oded, and U. Banin, Colloidal quantum dot molecules manifesting quantum coupling at room temperature, Nat. Commun. \textbf{10}(1), 5401 (2019).

[41] D. D. Sukachev et al., Silicon-Vacancy Spin Qubit in Diamond: A Quantum Memory Exceeding 10 ms with Single-Shot State Readout, Phys. Rev. Lett. \textbf{119}, 223602, (2017).

[42] Z.-H. Zhang, P. Stevenson, G. Thiering, B. C. Rose, D. Huang, A. M. Edmonds, M. L. Markham, S. A. Lyon, A. Gali, and N. P. de Leon, Optically Detected Magnetic Resonance in Neutral Silicon Vacancy Centers in Diamond via Bound Exciton States, Phys. Rev. Lett. \textbf{125}, 093602, (2020).

[43] M. Widmann, M. Niethammer, D. Y. Fedyanin, I. A. Khramtsov, T. Rendler, I. D. Booker, J. Ul Hassan, N. Morioka, Y.-C. Chen, I. G. Ivanov, N. T. Son, T. Ohshima, M. Bockstedte, A. Gali, C. Bonato, S.-Y. Lee, and J. Wrachtrup, Electrical Charge State Manipulation of Single Silicon Vacancies in a Silicon Carbide Quantum Optoelectronic Device, Nano Lett., \textbf{19}(10), 7173 (2019).

[44] K. C. Miao, A. Bourassa, C. P. Anderson, S. J. Whiteley, A. L. Crook, S. L. Bayliss, G. Wolowicz, G. Thiering, P. Udvarhelyi, V. Ivády, H. Abe, T. Oshshima, A. Gali, and D. D. Awschalom, Electrically driven optical interferometry with spins in silicon carbide, Sci. Adv., \textbf{5}(11), eaay0527 (2019).

[45] Y. Huang, T. Xiao, Z. Xie, J. Zheng, Y. Su, W. Chen, K. Liu, M. Tang, P. Müller-Buschbaum, and L. Li, Single-Layered Reflective Metasurface Achieving Simultaneous Spin-Selective Perfect Absorption and Efficient Wavefront Manipulation, Adv. Opt. Mater., \textbf{9}(5), 2001663 (2021).

[46] R. Naaman, Y. Paltiel, and D. H. Waldeck, Chiral Molecules and the Spin Selectivity Effect, J. Phys. Chem. Lett. \textbf{11}(9), 3660 (2020).

[47] L. Jing, Z. Wang, Y. Yang, B. Zheng, Y. Liu, and H. Chen, Chiral metamirrors for broadband spin-selective...
absorption, Appl. Phys. Lett., 110(23), 231103 (2017).

[48] P. Chen, T. W. Lo, Y. Fan, S. Wang, H. Huang, and D. Lei, Chiral Coupling of Valley Excitons and Light through Photonic Spin–Orbit Interactions, Adv. Opt. Mater., 8(5), 1901233 (2020).

[49] F. Jelezko, T. Gaebel, I. Popa, M. Domhan, A. Gruber, and J. Wrachtrup, Observation of coherent oscillation of a single nuclear spin and realization of a two-qubit conditional quantum gate, Phys. Rev. Lett. 93, 130501 (2004).

[50] L. Childress, M. G. Dutt, J. M. Taylor, A. S. Zibrov, F. Jelezko, J. Wrachtrup, P. R. Hemmer and M. D. Lukin, Coherent dynamics of coupled electron and nuclear spin qubits in diamond, Science, 314, 281 (2006).

[51] P. Neumann, N. Mizuochi, F. Rempp, P. Hemme, H. Watanabe, S. Yamasaki, V. Jacques, T. Gaebel, F. Jelezko and J. Wrachtrup, Multiparticle entanglement among single spins in diamond, Science, 320, 1326 (2008).

[52] S. C. Benjamin, B. W. Lovett, and J. M. Smith, Prospects for measurement-based quantum computing with solid state spins, Laser Photo. Rev. 3, 556 (2009).

[53] L. Childress and R. Hanson, Diamond NV centers for quantum computing and quantum networks, MRS Bull. 38, 134 (2013).

[54] E. Togan, Y. Chu, A. S. Trifonov, L. Jiang, J. Maze, L. Childress, M. V. G. Dutt, A. S. Sørensen, P. R. Hemmer, A. S. Zibrov, and M. D. Lukin, Quantum entanglement between an optical photon and a solid-state spin qubit, Nature 466, 730 (2010).

[55] H. R. Wei and G. L. Long, Universal photonic quantum gates assisted by ancilla diamond nitrogen-vacancy centers coupled to resonators, Phys. Rev. A 91, 032324 (2015).

[56] H.-R. Wei, G. L. Long, Hybrid quantum gates between flying photon and diamond nitrogen-vacancy centers assisted by optical microwavities, Sci. Rep. 5, 12918 (2015).

[57] C. Cao, T. J. Wang, R. Zhang, and C. Wang, Concentration on partially entangled W-class states on nitrogen-vacancy centers assisted by microresonator, J. Opt. Soc. Am. B 32, 1524 (2015).

[58] B. C. Ren, G. Y. Wang, and F. G. Deng, Universal hyperparallel hybrid photonic quantum gates with dipole-induced transparency in the weak-coupling regime, Phys. Rev. A 91, 032328 (2015).

[59] Q. Chen, W. Yang, M. Feng, and J. F. Du, Entangling separate nitrogen-vacancy centers in a scalable fashion via coupling to microtoroidal resonators, Phys. Rev. A 83, 054305 (2011).

[60] K. Xia, G. K. Brennen, D. Ellinas, and J. Twamley, Deterministic generation of an on-demand Fock state, Opt. Express 20, 27198 (2012).

[61] H. R. Wei and F. G. Deng, Compact quantum gates on electron-spin qubits assisted by diamond nitrogen-vacancy centers inside cavities, Phys. Rev. A 88, 042323 (2013).

[62] L. Y. Cheng, H. F. Wang, and S. Zhang, Simple schemes for universal quantum gates with nitrogen-vacancy centers in diamond, J. Opt. Soc. Am. B 30, 1821 (2013).

[63] S. Liu, J. Li, R. Yu, and Y. Wu, Achieving maximum entanglement between two nitrogen-vacancy centers coupling to a whispering-gallery-mode microresonator. Opt. Express 21, 3501 (2013).

[64] L. Y. Cheng, H. F. Wang, S. Zhang, and K. H. Yeon, Quantum state engineering with nitrogen-vacancy centers coupled to low-Q microresonator, Opt. Express 21, 5988 (2013).

[65] Z. Jin, Y. Q. Ji, A. D. Zhu, H. F. Wang, and S. Zhang, Deterministic implementation of optimal symmetric quantum cloning with nitrogen-vacancy centers coupled to a whispering-gallery microresonator, J. Opt. Soc. Am. B, 31, 2516 (2014).

[66] X. Han, Q. Guo, A.-D. Zhu, S. Zhang and H.-F. Wang, Effective W-state fusion strategies in nitrogen-vacancy centers via coupling to microtoroidal resonators, Opt. Exp. 25, 17701 (2017).

[67] C. Y. Hu, A. Young, J. L. O’Brien, W. J. Munro, and J. G. Rarity, Giant optical Faraday rotation induced by a single-electron spin in a quantum dot: Applications to entangling remote spins via a single photon, Phys. Rev. B 78, 085307 (2008).

[68] D. F. Walls and G. J. Milburn, Quantum Optics (Springer-Verlag 1994).

[69] C. Y. Hu, W. J. Munro, and J. G. Rarity, Deterministic photon entangler using a charged quantum dot inside a microcavity, Phys. Rev. B 78, 125318 (2008).

[70] S. Bugu, F. Ozaydin, and T. Kodera, Surpassing the classical limit in magic square game with distant quantum dots coupled to optical cavities, Sci. Rep. 10, 22202 (2020).

[71] B. Ben Bakir, A. Vazquez de Goyes, R. Orobtechouk, P. Ryan, C. Porzier, A. Roman, and J.-M. Fedeli, Low-loss and polarization-insensitive edge fiber couplers fabricated on 200-mm silicon-on-insulator wafers, IEEE Photonics Technol. Lett. 22(11), 739 (2010).

[72] P. Cheben, J. H. Schmid, S. Wang, D.-X. Xu, M. Vachon, S. Janz, J. Lapointe, Y. Painchaud, and M.-J. Picard, Broadband polarization independent nanophotonic coupler for silicon waveguides with ultra-high efficiency, Opt. Exp. 23(17), 22553 (2015).

[73] C. Junge, D. O’Shea, J. Volz, and A. Rauschenbeutel, Strong Coupling between Single Atoms and Nontransversal Photons, Phys. Rev. Lett. 110, 213604 (2013).

[74] H.R. Wei, F.G. Deng, Universal quantum gates on electron-spin qubits with quantum dots inside single-side optical microcavities, Opt. Express 22, 593 (2014).

[75] C. Y. Hu, Photonic transistor and router using a single quantum-dot-confined spin in a single-sided optical microcavity, Sci. Rep. 7, 45582 (2017).

[76] J. Heo, Implementation of controlled quantum teleportation with an arbitrator for secure quantum channels via quantum dots inside optical cavities, Sci. Rep. 7, 14905 (2017).

[77] B.-C. Ren, A. H. Wang, A. Alsaeedi, T. Hayat, and F.-G. Deng, Three-Photon Polarization-Spatial Hyperparallel Quantum Fredkin Gate Assisted by Diamond Nitrogen Vacancy Center in Optical Cavity, Ann. Phys. 530, 1800043 (2018).

[78] M. Li, J.-Y. Lin and M. Zhang, High-Fidelity Hybrid Quantum Gates between a Flying Photon and Diamond Nitrogen-Vacancy Centers Assisted by Low-Q Single-Sided Cavities, Ann. Phys. 531, 1800312 (2018).

[79] A. Young, C. Y. Hu, L. Marseglia, J. P. Harrison, J. L. O’Brien and J. G. Rarity, Cavity enhanced spin measurement of the ground state spin of an NV center in diamond New J. Phys. 11, 013007 (2009).

[80] B.-S. Song, S. Noda, T. Asano and Y. Akahane, Ultra-high-Q photonic double-heterostructure nanocavity, Nat. Mater. 4, 207 (2005).
[80] D. Englund, B. Shields, K. Rivoire, F. Hatami, J. Vuckovic, H. Park, and M. D. Lukin, Deterministic Coupling of a Single Nitrogen Vacancy Center to a Photonic Crystal Cavity, Nano Lett. 10(10), 3922 (2010).

[81] A. Faraon et al., Coupling of Nitrogen-Vacancy Centers to Photonic Crystal Cavities in Monocrystalline Diamond, Phys. Rev. Lett. 109, 033604 (2012).

[82] M. Schukraft, J. Zheng, T. Schröder, S. L. Mouradian, M. Walsh, M. E. Trusheim, H. Bakhru, and D. R. Englund, Invited Article: Precision nanoimplantation of nitrogen vacancy centers into diamond photonic crystal cavities and waveguides, APL Photonics 1, 020801 (2016).

[83] K. G. Fehler, A. P. Ovvyan, N. Gruhler, W. H. P. Pernice and A. Kubanek, Efficient Coupling of an Ensemble of Nitrogen Vacancy Center to the Mode of a High-Q, $\text{Si}_3\text{N}_4$ Photonic Crystal Cavity, ACS Nano 13(6), 6891 (2019).

[84] M. Bartkowiak and A. Miranowicz, Linear-optical implementations of the iSWAP and controlled NOT gates based on conventional detectors, J. Opt. Soc. Am. B 27, 2369 (2010).

[85] K. Nemoto, M. Trupke, S. J. Devitt, A. M. Stephens, B. Scharfenberger, K. Buczak, T. Nöbauer, M. S. Everitt, J. Schmiedmayer, and W. J. Munro, Photonic Architecture for Scalable Quantum Information Processing in Diamond, Phys. Rev. X 4, 031022 (2014).

[86] K. Nemoto, M. Trupke, S. J. Devitt, B. Scharfenberger, K. Buczak, J. Schmiedmayer, and W. J. Munro, Photonic Quantum Networks formed from NV centers, Sci. Rep. 6, 26284 (2016).

[87] S. A. Camejo, A. Lazarijev, S. W. Hell and G. Balasubramanian, Room temperature high-fidelity holonomic single-qubit gate on a solid-state spin, Nat. Comm. 5, 4870 (2014).

[88] J. Zhou, B. J. Liu, Z. P. Hong and Z. Y. Xue, Fast holonomic quantum computation on solid-state spins with all-optical control, arXiv:1705.08852 (2017).

[89] Y. Sekiguchi, N. Nikura, R. Kuroiwa, H. Kano and H. Kosaka, Optical holonomic single quantum gates with a geometric spin under a zero field, Nature Phot. 11, 309 (2017).

[90] B. B. Zhou, P. C. Jerger, V. O. Shkolnikov, F. J. Heremans, G. Burkard, and D. D. Awschalom, Holonomic Quantum Control by Coherent Optical Excitation in Diamond, Phys. Rev. Lett. 119, 140503 (2017).

[91] B. B. Zhou, A. Baksic, H. Ribeiro, C. G. Yale, F. J. Heremans, P. C. Jerger, A. Auer, G. Burkard, A. A. Clerk, and D. D. Awschalom, Accelerated quantum control using superadiabatic dynamics in a solid-state lambda system, Nature Phys. 13, 330 (2017).