A Proof Procedure for Hybrid Logic with Binders, Transitivity and Relation Hierarchies

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Abstract. A tableau calculus constituting a decision procedure for hybrid logic with the converse modalities, the global ones and a restricted use of the binder has been defined in a previous paper. This work shows how to extend such a calculus to multi-modal logic equipped with two features largely used in description logics, i.e. transitivity and relation inclusion assertions. An implementation of the proof procedure is also briefly presented, along with the results of some preliminary experiments.

1 Introduction

This work considers multi-modal hybrid languages (see, for instance, [3]) that, beyond the standard modalities, nominals, the satisfaction operator and the binder, include the converse modalities ($\langle - \rangle$ and $\langle - \rangle^\circ$), the global ones ($E$ and $A$) and a feature largely used in description logics, i.e. the possibility of declaring an accessibility relation to be transitive and/or included in another one. Basic hybrid logic (with nominals only, beyond the modal operators $\langle - \rangle$ and $\langle - \rangle^\circ$) will be denoted by $HL$, and basic multi-modal hybrid logic by $HL_m$. Logics extending $HL$ or $HL_m$ with operators $O_1, \ldots, O_n$ (and their duals) are denoted by $HL(O_1, \ldots, O_n)$ and $HL_m(O_1, \ldots, O_n)$, respectively. Multi-modal languages including transitivity assertions and/or relation hierarchies are denoted in the same way, just including Trans (for transitivity) and/or $\sqsubseteq$ (for relation inclusion) among $O_1, \ldots, O_n$.

The satisfiability problem for formulae of any hybrid logic $HL(O_1, \ldots, O_n)$ or $HL_m(O_1, \ldots, O_n)$ – where $O_i \in \{@, \diamond -, E\}$ is decidable [4]. Unfortunately, due to the high expressive power of the binder, $HL(\downarrow)$ is undecidable [1, 4].

There are both semantic and syntactic restrictions allowing for regaining decidability of hybrid logic with the binder. Restricting the frame class is a way of restoring decidability, but the interplay with multi-modalities (or the addition of other operators) is not always harmless. For instance, $HL(\downarrow)$ over transitive frames is decidable [18], but $HL(@, \downarrow)$ and $HL_m(\downarrow)$ are not [18, 17].

In [20] it is proved that the satisfiability problem for formulae in $HL(@, \downarrow, E, \diamond -)$ is decidable, provided that their negation normal form contains no universal operator (i.e. either $\square$ or $\square^\circ$ or $A$) scoping over a binder, that in turn has scope over a universal operator. Such a fragment of hybrid logic is denoted by $HL(@, \downarrow, E, \diamond -) \backslash \square \downarrow \square$. The result is proved by showing that there exists a satisfiability preserving translation of $HL(@, \downarrow, E, \diamond -) \backslash \square \downarrow \square$ into $HL(@, \downarrow, E, \diamond -) \backslash \downarrow \square$, i.e.
the set of formulae in negation normal form where no universal operator occurs in the scope of a binder. The standard translation of hybrid logic into first order classical logic [1, 20] maps, in turn, formulae in $\text{HL}(\oplus, \downarrow, E, \Diamond^-) \ \downarrow \Box$ into universally guarded formulae, that have a decidable satisfiability problem [12].

Decidability of $\text{HL}_m(\oplus, \downarrow, E, \Diamond^-) \ \Box \downarrow \Box$ can be proved by the same reasoning, and the separate addition of either relation hierarchies or transitive relations can easily be shown to stay decidable, by reduction to the first order guarded fragment and by resorting to results already proved in the literature [19]. However, such results do not directly allow for concluding whether the logic including both features is still decidable.

This work is a continuation of previous works, where terminating tableau calculi for decidable fragments of Hybrid Logic with the binder have been defined [8, 9]. In particular, [9] presents a tableau calculus constituting a satisfiability decision procedure for $\text{HL}(\oplus, \downarrow, E, \Diamond^-) \ \Box \downarrow \Box$. Such a procedure is here extended to multi-modal hybrid logic $\text{HL}_m(\oplus, \downarrow, E, \Diamond^-, \text{Trans}, \sqsubseteq) \ \Box \downarrow \Box$: a tableau calculus is presented, which terminates and is sound and complete for formulae in the fragment $\text{HL}_m(\oplus, \downarrow, E, \Diamond^-, \text{Trans}, \sqsubseteq) \ \downarrow \Box$, i.e. formulae in negation normal form where no occurrence of a universal operator is in the scope of a binder, with the addition of transitivity assertions and relation hierarchies. A preprocessing step along the lines of [20] turns the calculus into a satisfiability decision procedure for the fragment $\text{HL}_m(\oplus, \downarrow, E, \Diamond^-, \text{Trans}, \sqsubseteq) \ \downarrow \Box$. Soundness, completeness and termination of the tableaux calculus thus imply that the satisfiability problem for the fragment of multi-modal hybrid logic $\text{HL}_m(\oplus, \downarrow, E, \Diamond^-, \text{Trans}, \sqsubseteq) \ \Box \downarrow \Box$ is decidable. The proof procedure has been implemented in a prover called Sibyl, which will be briefly presented along with the results of some preliminary experiments.

The language of $\text{HL}_m(\oplus, \downarrow, E, \Diamond^-, \text{Trans}, \sqsubseteq) \ \Box \downarrow \Box$ subsumes the description logic $\text{SHOIL}$ enriched with restricted occurrences of the binder, and allows for representing some interesting frame properties, such as, for instance, symmetry ($R^- \sqsubseteq R$), reflexivity ($A \downarrow x. \Diamond R x$), “at most” restrictions on the number of states ($E \downarrow x_1, \ldots, E \downarrow x_n. A(x_1 \lor \cdots \lor x_n)$), and “at least” restrictions on the number of $R$-successors of each state ($A \downarrow x. \Diamond R \downarrow y_1.(x : \Diamond R (\neg y_1 \land \downarrow y_2 \land \downarrow y_3 \land \ldots))$).

This section concludes with a brief introduction to the syntax and semantics of multi-modal hybrid logic with transitive relations and inclusion assertion. Well-formed expressions of $\text{HL}_m(\oplus, \downarrow, E, \Diamond^-, \text{Trans}, \sqsubseteq)$ are partitioned into two categories: formulae (for which the metasymbols $F, G$ are used) and assertions.

Formulae are built out of a set $\text{PROP}$ of propositional letters, a set $\text{NOM}$ of nominals, an infinite set $\text{VAR}$ of state variables, and a set $\text{REL}$ of relation symbols (all such sets being mutually disjoint), and defined by the following grammar:

$$F ::= p \mid a \mid x \mid \neg F \mid F \land F \mid F \lor F \mid \Diamond RF \mid \Box RF$$
$$\mid \Diamond RF \mid \Box RF \mid EF \mid AF \mid a:F \mid x:F \mid \downarrow x.F$$

where $p \in \text{PROP}$, $a \in \text{NOM}$, $x \in \text{VAR}$ and $R \in \text{REL}$. In this work, the notation $t:F$ is used (for $t \in \text{NOM} \cup \text{VAR}$) rather than $@_t F$. We use metavariables $a, b, c$ for nominals, $x, y, z$ for state variables and $R, S, P$ for relation symbols.