1-Dimension magnetotelluric data inversion using MOEA/D algorithm

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Abstract. Magnetotelluric (MT) data is used to derive resistivity imaging of subsurface. The subsurface resistivity is obtained by inversion of MT data. Generally, MT data contains two parts, namely: apparent resistivity and phase or real and imaginary parts. Inversion of MT data for reconstructing resistivity value of each layer is to minimize single objective (combination two parameters MT data) which used global or local optimization method. Nevertheless, single objective optimization method has several disadvantages, such as; (1) weight value to combine two parameters of MT data is needed, where this weigh value depend on the amplitude of both MT data; (2) there is no validation of the inversion results. In this research, Inversion MT data to estimate 1D resistivity of subsurface uses multi-objective evolutionary algorithm based on decomposition (MOEA/D) to minimize root mean square error (RMSE) of calculated and observed data for apparent resistivity and phase data simultaneously. The algorithm has applied to synthetic and field data. This result shows that MOEA/D algorithm is robust and accurate to determine subsurface resistivity and lithology.

1. Introduction
Magnetotelluric is one of passive geophysical methods which utilizes natural electromagnetic field to understand physical parameters distribution within a subsurface. The magnetotelluric method uses Maxwell law as the basic concept as defined in [1], [2]. The observation of this technique is magnetic and electric fields, further analyzed to estimate apparent resistivity and phase or complex number. Forward modeling of one dimension magnetotelluric (MT 1D) can be solved by the analytic method or using recursion equation [1]. Forward modeling of MT 1D is quite simply, a correlation between data and parameter model is not linear. However, non-linear inversion using linearization is needed initial value which is close to the solution value. Besides, the solution of linearization method is often trapped in local minimum, where the optimum solution of non-linear inversion always associates with global minimum.

On the other hand, a global minimum can be found by stochastic method. Meanwhile, there are two kinds of the stochastic methods, namely: finding an optimum solution using single objective and multi-objective approaches. Considering that MT 1D has two sorts data, i.e. apparent resistivity and phase or complex number. Thus, optimum solution of MT 1D can be found by minimizing both of them. Minimizing two variable using single objective is probably trapped in local minimum
Whereas, the weight factor is needed. Furthermore, weight factor in every data is different each other. To get the best of weight factor for each data should be done trial and error. In addition, using single objective causes difficult to validate the model with the true geological condition [4], [5]. Consequently, Dal Moro and Colleagues suggest that inversion process using more than two variables should use a multi-objective method. One of the best multi-objective methods is multi-objective evolution algorithm based on decomposition (MOEA/D) [6].

In the last decades, MOEA/D is more efficiency and effective than other multi-objective algorithms which base on non-dominating sorting. In this research, MT 1D is inverted using MOEA/D to estimate subsurface resistivity in a way minimizing both of objective functions, namely; the error of observed and calculated data for phase and apparent resistivity. This algorithm has been tested on synthetic and observed data.

2. Methods
MOEA/D purposes to re-decompose multi-objective problem becoming several sub-problem. Generally, this method is started by input parameter and defining search space. Furthermore, the number of parameters has the closest relation. Wider search is needed more individual to evaluate and also time-consuming. The next step is generating initial population using one of the methods which has been chosen, like Weighed sum or Tchebycheff approaches. Then, evaluating objective function, defining the non-dominant individual, reproduction, and cross over process are calculated [6]. Reproduction and cross over aim to avoid solution trapped in the local minimum (exploration process). In this paper, to invert synthetic data 250 populations, 250 maximum non-dominant archives, and 500 times iterations are needed. Meanwhile, 150 populations, 150 maximum non-dominant archives, and 500 iterations are needed for field data inversion.

2.1. Inversion of data synthetic
Data synthetic inversion proposes to know how performance (robust) MOEA/D algorithm for simultaneously MT 1D data inversion. This process is also to determine the reliability of MOEA/D. Conductive layer between the resistive layers of structure is applied as synthetic model in this research. Furthermore, two sorts of approaching layer which are implemented in this study are inversion using same and difference layers.

Forward modeling in this inversion process aims to obtain apparent resistivity and phase from model parameter. As a result, objective function can be calculated. Meanwhile, objective functions conducted in this inversion are root mean square error (RMSE) between true data and calculation data as described below

\[
E = \frac{1}{N} \sum_{f} \left( \log_{10}(d_{obs}^{a} / d_{pred}^{a}) \right)^{2}
\]

\[
E = \frac{1}{N} \sum_{f} \left( d_{obs}^{\phi} - d_{pred}^{\phi} \right)^{2}
\]

Where \(d_{obs}^{a}\) and \(d_{pred}^{a}\) denote observed and calculated apparent resistivity data, respectively, \(d_{obs}^{\phi}\) and \(d_{pred}^{\phi}\) are observed and predicted phase data, respectively, \(N\) is number of frequency or period.

On the other hand, inversion result on the basis of the multi-objective algorithm is Pareto front model which shows both RMSE of MT data. It can be called Pareto optimum. All of individual non-dominant in Pareto front model (Pareto optimum) probably becomes a solution in this inversion. Thus selection optimum solution is needed. A method to determine a optimum solution is relative distance
considering both minimum and maximum values in for each objective function through normalization approach [3]. Mathematically, it is given by eqs. (3) and (4):

\[ f_{xi}(j) = \frac{f_i(j) - \min(f_i)}{\max(f_i) - \min(f_i)} \quad i=1,2; j=1,1…Npop \quad (3) \]

\[ D = \sqrt{f_{x1}^2 - f_{x2}^2} \quad (4) \]

Where \( f_{xi} \) and \( D \) denote normalization of each objective function and distances between utopia point and Pareto front. Optimum model is identified by the lowest of \( D \).

2.2. Field data inversion

Field data inversion is done to test the reliability of MOEA/D in field MT 1D data. Sasaki field data [13;14] at first station is used. Inversion process which is tackled in this field data is similar to invert process in the synthetic data described before.

3. Results and Discussion

3.1. Inversion of Synthetic Data

First of all, the first step is inversion using the same of true layer. Previously, forward modeling is done to obtain synthetic. Skin depth is calculated to know search space which is applied in the inversion process. In this synthetic data, inversion uses the search space: 1000 and 5000 meter for maximum thickness, 100 and 500 meter for minimum thickness; 500, 1500, and 100 Ωm for maximum resistivity; and also 10, 500, and 1 Ωm for minimum resistivity. Second inversion, inversion synthetic data using difference layer approaching (5 layers) is also done. It proposes to know the influence of defining layer in the inversion results. Search spaces is used in this inversion are 1000, 500, 5000, and 100 meter for maximum thicknesses; 10, 100, 500, 50 meter for minimum thicknesses; 500, 1000, 2000, 1000, and 100 Ωm for maximum resistivity; and also 5, 50, 100, 10, and 1 Ωm for minimum resistivity.

Fig. 1 is the result of synthetic data inversion using three layers which is done by MOEA/D algorithm simultaneously. Fig 1a and 1b are comparing between calculated data (blue) and observed data (red) for apparent resistivity and phase, respectively. Therefore, Fig 1c demonstrates Pareto front model with all of the probable solution of individual non-dominant. Based on this Pareto front model, it can be seen that resistivity and phase error about \( 0 - 3\times10^{-3} \) and \( 0-2.5\times10^{-3} \) particularly. Fig 1d is comparing between subsurface modeling for the true and inverted model. The figure shows that both of subsurface models are close.
Figure 1 Inversion result of synthetic data using three layers. Inversion result using five layers approaching is shown by Fig 2. Figs. 2a and 2b illustrate the comparison between observed or field and calculated for apparent resistivity and phase, respectively. The figures show that both the data are very close. Fig.2c illustrates Pareto front model (RMSE for both data) which shows that apparent resistivity has RMSE about 0–0.15x10^{-3}, while phase has RMSE around 0-0.08. Fig 2d is subsurface modeling from true model and inverted model using 5 layers. This figure shows both of data looked similar or close. Even though the second layer shows slightly difference and it does not give significantly influence. Consequently, it can be seen that MT data inversion which is conducted by the higher number of layer than the true layer can obtain subsurface modeling which is quite similar to the true model.

Figure 2 Inversion result of synthetic data using five layers. Pareto front models for inversion synthetic data MT as described above demonstrate the low number (Figs. 1c and 2c). The low of RMSE for both data is owing to the fact that MT data inverting is free
noise data (synthetic data). Thus, Pareto front resulted by MOEA/D can also estimate noise contaminated in the MT data [7], [8], where high RMSE is correlated to the high noise and vice versa. Further, Dal Moro and colleagues [4], [5] point out that Pareto front can be used to validity resulted model which affected by wrong assumption in the inversion process. Generally, the correct inversion process is illustrated by the location of minimum distances around the center of Pareto front. Figs. 1c and 2c can be known that the inversion assumption is correct. Thus, Fig. 1d and Fig. 2d show that the true and inverted model are similar.

### 3.2. Inversion Field Data

This inversion aims to recognize MOEA/D Algorithm in field data. This research uses field data which applied to test performances algorithm by previous researchers [9], [10] in the station 1. This inversion utilizes four rock layers approaching, and also search spaces are given by: 15000, 10000, and 10000 for maximum thicknesses; 100, 1000, 1000 for minimum thicknesses; 50, 300, 100, and 100 for maximum resistivity; 2,10,2, and 1 for minimum resistivity. Inversion result of the field data is presented in the Fig. 3. Fig. 3a and 3b demonstrate calculated and observed data for apparent resistivity and phase, respectively. Therefore, Fig. 3c is Pareto front the field data which gives information about noise contained in the field data. RMSE value for phase and resistivity are about 2-2.08 and 0.05-0.065 particularly. Similar to invert problem in synthetic data, error value in phase is higher than resistivity error value owing to the fact that phase data is worse than resistivity data [9].

![Figure 3](image)

**Figure 3** Inversion result using MOEA/D algorithm for field data [7].

Fig 3d shows subsurface modeling for the result inversion. Based on the figure, it can be seen that data in station 1 [10] has about four rock layers. The top layer has resistivity value about 10 Ωm in the shallowest depth. Then, second layer has the highest resistivity value about 95 Ωm. Meanwhile, the following layer has the lowest resistivity value. Therefore, fourth layer has higher value than previous layer just about 10 Ωm. Further, inversion MT data also invers using MT2DinvMatlab software [10] to determine 2D resistivity and to provide comparison resistivity model. Fig. 5 is resistivity 2D resulted by inversion process using MT2DinvMatlab. The figure shows that the first station is located in zero point. The figure demonstrates that the site contains three layers materials within subsurface. The first layer has the highest resistivity value. Meanwhile, the second layer has lower value than the last layer.
This result is similar trend to the inversion using MOEAD result which shows four layers with very thick layer in the first layer, which can be ignored.

Figure 4. Joint inversion results of field data by MT2DinvMatlab software [10]

Conclusion
MT 1D data inversion to minimize RMSE between measured and calculated data for apparent resistivity and phase simultaneously can be solved by MOEA/D. Using the algorithm, MT 1D inversion do not need weight value in the objective function. Consequently, MOEA/D can be used to invert MT 1D data accurately, easily, and also fast for estimating subsurface resistivity which has been tested on synthetic and field data. Besides, inversion results also provide information about validity model of inversion result which can be described by Pareto front properties and also give information about uncertainty of model (owing to MOEA/D solution as much as populations which are used).

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