Research on The Numerical Accuracy of Network Public Opinion Propagation Model Based on Runge-Kutta Method

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Abstract. In a SIsIoR public opinion transmission model with discussion mechanism and emotional tendency of public opinion, government intervention is added to list the corresponding dynamic differential equation. It is found that there is no exact solution to the equation. Theoretically, the equation satisfies Routh-Hurwitz stability and Lipschitz continuity, and finally runs to the local equilibrium point. Explicit Euler method and 4th order Runge-Kutta method are used for numerical simulation, and the results are real and effective. By comparing the two methods, it is found that the numerical results of the fourth-order Runge-Kutta method are more accurate, and the iteration step size is shorter, which is more consistent with the theoretical expectation. This method is more suitable for the simulation of network public opinion dissemination, and has the advantages of stronger verifiability and higher numerical accuracy of the model. The simulation of network public opinion transmission based on Runge-Kutta method can provide reliable numerical basis for the guidance of public opinion under the environment of netizens' emotional tendency, and provide scientific decision-making basis for the government to intervene in network public opinion transmission.

1. Introduction

With the rapid development of Internet technology in recent years, the spread speed of online public opinion is fast, the spread space is large, the potential level of harm is high and it is difficult to regulate the development of public opinion. Whenever a public crisis event breaks out, negative remarks, false information and one-sided views will bring adverse effects to the public institutions and the government, which have a wide audience. At this point, if effective methods are not taken in time for public opinion oriented intervention, the consequences of negative public opinion will be very serious. Therefore, the government attaches great importance to the intervention in the process of online public opinion transmission, and grasps the initiative of public opinion in the hands of the government to make it favor the government.

In recent years, the intervention of Internet public opinion dissemination and the guidance of netizens' emotional tendency have been a topic widely studied by scholars. At present, most scholars focus on the research of information acquisition algorithms, information extraction and mathematical models. Chen Bingxian\textsuperscript{[1]} studied an information sampling method with a higher sampling degree than the breadth-first algorithm for the sampling of social network data. Qian Chen et al.\textsuperscript{[2]} studied the emotional tendency of online public opinion and divided the public opinion groups into two mainstream groups, namely positive and negative, which further improved the classification efficiency of text information.
Xu Yejing [3] recognized that online public opinion would be largely affected by media, and analyzed and concluded the control strategy of online public opinion under big data. Zhang Lifan et al. [4] combined the similarity between public opinion and the spread of infectious diseases, established and improved the mathematical model of SIR, and obtained a relatively appropriate simulation diagram. Chong Dashuang et al. [5] added intervention and control factors and enumerated realistic cases of government intervention for explanation. Guo Dongwei et al. [6] studied the communication mechanism of public opinion information by using the game theory method, and simulated the interaction behavior of the two outputs in the game from a rational perspective. Zhang Yiwen et al. [7] proposed the trend of network public opinion and the mode discrimination method based on Bayesian network modeling by using Bayesian network modeling and network public opinion situation assessment. Liu Ji et al. [8] made an empirical analysis of the communication of microblog network public opinions with different themes, and proposed the research direction for the forwarding mechanism. Fang Wei et al. [9] studied the dissemination of public opinion information based on the cellular automata theory and designed the cellular automata model framework of public opinion dissemination. Gao Hang et al. [10] established the system dynamics model of public opinion and introduced the production function to study the possible risks caused by public opinion and the external relationship. All the above academic studies obtained data through the cases of online public opinion, brought the data into the model, carried out parameter verification and simulation intervention experiments, and then compared the results of real cases, which are helpful for the prediction of public opinion communication and case analysis and research.

Most researches are limited to the evolution process of public opinion communication [11] and the transformation of netizens' emotional trend state, but ignore the operational characteristics and research value of the network public opinion system itself. The system dynamics method can clearly describe the relationship between various groups of network public opinion and media intervention, but it lacks the description accuracy and result control of the method itself. Most research groups have developed a variety of simulation machines, but lack of simulation experiments of discrete models derived from differential models, so it is impossible to compare theory with simulation results.

The differential model of this paper is based on the classic SIR model, and the SIAIBR model of bi-affective tendency under the discussion mechanism is followed. Government intervention is added, intervention parameters are set according to the characteristics of government intervention, and the SIOISR model after government intervention is obtained. In the process of simulation, the Runge-Kutta method is compared with the explicit Euler method, and it is found that the numerical simulation results using Runge-Kutta method are more accurate, the iteration step size is shorter, and it is more consistent with the theoretical expectation. Based on the stability judgment methods provided by literature [12] and literature [13], the balance point of public opinion transmission, the reproduction number and the position of final balance under government intervention are calculated, and the balance of the equation is verified. On the basis of reference [14], in order to ensure the accuracy of explicit Euler's method, an improved scale rendering method is adopted to ensure that the scale relationship between various variables of the model remains unchanged. The two methods were used for simulation, and the experimental results were compared with the equilibrium point to obtain reliable results. Combined with the heat information of "hot search in Cainiao station", different parameter values were taken to simulate public opinion, with a high degree of agreement. Different from the influence setting of media in literature [4], government intervention does not affect the overall number of Internet users and does not change the refutation coefficient of both sides.

2. Related Work

2.1. The propagation dynamics model of the intermediate state and final evolution of public opinion in closed and open states

(1) SIR Model

SIR Model is a commonly used and the most classic infectious disease model, which divides the population within the epidemic scope of infectious diseases into three categories: S type, susceptible to
infection, I type, infected, R type, removed. In the process of transmission, the probability of easily infected people turning into infected people is set as $\alpha$, and the probability of infected people turning into removed people is set as $\beta$. The route of public opinion transmission is as follows:

![Figure 1. SIR Model](image)

Consider that in real life, when public opinion is spread, netizens' views will change accordingly. The process of public opinion transmission is roughly similar to the process of SIR infectious disease model, and the use of SIR model has certain reference value for the analysis of the development trend monitoring and intervention of public opinion.

(2) SIsIoR Model

According to the situation that the opinions of the media and netizens will be opposed in the process of network public opinion transmission, a network of network public opinion communication should be established first. According to the classification of sentiment of public opinion in reference [7], the susceptible population (I) of public opinion Is divided into two opposing discussion groups, supporters (Is) and opponents (Io). According to the trend of crowd transfer, a SIsIoR dynamic model was established.

In the SIsIoR model, definite that uninformed netizens belong to the S group, netizens who support public opinion dissemination belong to the Is group, netizens who oppose public opinion dissemination belong to the Io group, and netizens who are not interested in public opinion dissemination or not and public opinion information itself belong to the immune R group. However, several groups can influence each other and realize mutual transformation under certain probability conditions. According to literature [3], the SIsIoR dynamic model of network public opinion under government intervention is as follows:

$$\begin{align}
\frac{dS}{dt} &= \alpha_1 I_S + (\beta_2 - \beta_1) I_S I_O - \gamma_1 I_S \\
\frac{dI_S}{dt} &= \alpha_2 S I_S + (\beta_1 - \beta_2) I_S I_O - \gamma_2 I_S \\
\frac{dI_O}{dt} &= (\alpha_1 I_S + \alpha_2 I_O + \alpha_3) \cdot S \\
\frac{dR}{dt} &= \alpha_3 S + \gamma_1 I_S + \gamma_2 I_O
\end{align}$$

Parameter setting:
$\alpha_1 = \alpha_2 = \alpha \ , \ \alpha_3 = 1 - 2\alpha \ \gamma_1 = \gamma_2 = \beta$

![Figure 2. SIsIoR Dynamic Model](image)
\[ R \quad \text{Number of immunized persons} \]
\[ A \quad \text{Number of new Internet users} \]
\[ \alpha_1 \quad \text{The probability of uninformed people } S \text{ switching to disseminating supporters } I_s \]
\[ \alpha_2 \quad \text{The probability of uninformed people } S \text{ turning to disseminated opponents } I_o \]
\[ \alpha_3 \quad \text{The probability of switching from the uninformed } S \text{ to the immune } R \]
\[ \beta_1 \quad \text{Probability of switching from communication proponents } I_s \text{ to communication opponents } I_o \]
\[ \beta_2 \quad \text{Probability of switching from propagating opponents } I_o \text{ to propagating supporters } I_s \]
\[ \gamma_1 \quad \text{Probability of conversion of transmission supporters } I_s \text{ to immunizers } R \]
\[ \gamma_2 \quad \text{Probabilistic conversion of opponents } I_o \text{ to immunizers } R \]

Among them, \( \alpha_1, \alpha_2, \alpha_3 \) are the propagation coefficients, the larger the value is, the faster the propagation speed of public opinion is; \( \beta_1, \beta_2 \) is the conversion coefficient, the higher the value is, the faster the conversion rate is. \( \gamma_1, \gamma_2 \) denote the drop-off coefficient, and the larger the value is, the faster the public opinion disappears.

3. Theoretical research and simulation with government intervention

3.1. Related theoretical research on SIsIoR model after government intervention is added

(1) Assumption of government intervention

It is assumed that the government's intervention in the transfer process of public opinion meets two limitations:

(a) The government should not guide the trend of public opinion more than the intensity of independent dissemination of public opinion, and intervene in the whole process by default rather than leading it.

(b) Compared with the platforms on which netizens have daily access to information, such as Circle of Friends, Microblog, etc., only a small amount can be deleted/increased influenced by the government, and spontaneous perceptual knowledge is not easy to change, i.e. \( \eta \in (0, 0.5) \).

The dynamic system composed of variables \( R \) unrelated to variables is removed, and the differential model SIsIoR that changes with time is shown as follows:

\[
\begin{align*}
\frac{dI_s}{dt} &= \alpha_1 S I_s + (\beta_2 - \beta_1) I_s I_o - \gamma_1 I_s \\
\frac{dI_o}{dt} &= \alpha_2 S I_o + (\beta_1 - \beta_2) I_s I_o - \gamma_2 I_o \\
\frac{dS}{dt} &= A - \alpha_1^* S I_s - \alpha_2^* S I_o - \alpha_3 S
\end{align*}
\]

\( S(0) = S_0, \quad I_s(0) = I_{s0}, \quad I_o(0) = I_{o0}, \quad R(0) = R_0 \)
Figure 3. The government interferes in the public opinion process

**Guide for the uninformed (S):** set the coefficient of government intervention and set the coefficient of transmission as $\lambda (\lambda > 0)$

$$\begin{align*}
\alpha_1^* &= \alpha_1 + \lambda \\
\alpha_2^* &= \alpha_2 - \lambda
\end{align*}$$

The government’s guidance on the positive and avoidance of negative public opinion makes the uninformed people more willing to become the positive supporters $I_S$ of public opinion in the process of expressing their opinions.

**Control of Disseminator (Is)(Io):** Set the government control coefficient, so that the new state transfer coefficient is $\eta (\eta > 0)$

$$\begin{align*}
\gamma_1^* &= \gamma_1 (1 - \eta) \\
\gamma_2^* &= \gamma_2 (1 + \eta)
\end{align*}$$

The government can appeal to the supporters to keep paying attention to the public opinion and maintain a positive attitude towards it, and constantly give positive related news to the supporters who support the public opinion of the government. Delay supporters’ time on the web, and speed up disconnection of opponents.

There are three basic judgments:
1. Government intervention will speed up the evolution of public opinion, that is, speed up the transformation of public opinion to the immune groups.
2. Government intervention will accelerate the upsurge of positive public opinion and reduce the upsurge of negative public opinion. Extend the upsurge of positive public opinion, and extend the overall discussion time of public opinion, and accelerate the participation of the uninformed in the discussion
3. Positive and negative public opinions will spontaneously open up the gap, and the role of government intervention is mainly reflected in further opening up the gap and accelerating the formation speed of the gap.

(2) Explanation of model assumptions and parameters
It is assumed that the propagation parameters of the SIR model with anarchic intervention are $\alpha$、$\beta$、$\gamma$

$$\begin{align*}
\alpha_1^* &= \alpha + \lambda, & \alpha_2^* &= \alpha - \lambda, & \alpha_3^* &= 1 - 2\alpha, & \gamma &= 1 - \alpha, \\
\varepsilon &= \alpha_1^* + \alpha_2^*, & \gamma_1^* &= \beta (1 - \eta), & \gamma_2^* &= \beta (1 + \eta), & \eta &\in (0, 0.5), & \lambda &\in (0, \alpha)
\end{align*}$$

It can be seen from the above that there is the following relationship between the parameters

$$\begin{align*}
\alpha_1^* &= \alpha + \lambda, & \alpha_2^* &= \alpha - \lambda, & \alpha_3^* &= \gamma - \varepsilon
\end{align*}$$

$\alpha$、$\beta$、$\gamma$.
And $\gamma = 1 - \alpha$, because of $\lambda \in (0, a), \eta \in (0, 0.5)$, so with the government intervention, there is
\[ \alpha^*_1 > \alpha > \alpha^*_2, \gamma^*_1 < \beta < \gamma^*_2 \]

Considering the practical significance, the range of parameters is as follows:
\[ 0 \leq \gamma^*_1 = \gamma_1 (1 + \gamma) \leq 1, \quad 0 \leq \gamma^*_2 = \gamma_1 (1 - \gamma) \leq 1, \]
\[ \alpha_1, \alpha_2 \in [0, 1], \quad 0 \leq \alpha_1 + \lambda \leq 1, \quad 0 \leq \alpha_2 - \lambda \leq 1 \]

(3) The phase orbit line of $I_S - S$ and $I_O - S$

The simplified model is taken $\beta_1 = \beta_2$ in this section. The simplified $S - I_S, S - I_O$ relationship is as follows:
\[
\begin{align*}
\frac{dI_S}{ds} &= \frac{\alpha^*_1 S - \gamma^*_1 I_S}{A - \alpha_1 I_S - \alpha_2 I_O S - \alpha_3 S} \\
\frac{dI_O}{ds} &= \frac{\alpha^*_2 S - \gamma^*_2 I_O}{A - \alpha_1 I_S - \alpha_2 I_O S - \alpha_3 S}
\end{align*}
\]

(4) Analysis of propagation equilibrium point and stability

According to the dynamic propagation theory, there is a propagation threshold $R_0$, also known as the basic regenerative number, in the transmission dynamic system of network public opinion, which determines whether the network public opinion is spread or not. When $R_0 \leq 1$, transmission will gradually disappear; When $R_0 > 1$, the network public opinion will be spread in a certain area.

Let $X = (I_S, I_O, S)^T$, then Model (2) can be expressed as $dx = F(x) - V(x)$. Here, the method of reference [12] is used to calculate the basic regenerative number [4]:
\[
F(x) = \begin{pmatrix}
\alpha^*_1 S I_S \\
\alpha^*_2 S I_O \\
0
\end{pmatrix}
\]
\[
V(x) = \begin{pmatrix}
(\beta_1 - \beta_2) I_S I_O + \gamma^*_1 I_S \\
(\beta_1 - \beta_2) I_S I_O + \gamma^*_2 I_O \\
-A + (\alpha_1 I_S + \alpha_2 I_O + \alpha_3 S)
\end{pmatrix}
\]

There are
\[
F = \begin{pmatrix}
\alpha_1 S_0 & 0 \\
0 & \alpha_2 S_0
\end{pmatrix}
V = \begin{pmatrix}
\gamma^*_1 & 0 \\
0 & \gamma^*_2
\end{pmatrix}
\]

From $A = FV^{-1}$, the basic regenerative number $R_0$, namely the spectral radius, can be obtained
\[
\rho(A) = \max \left\{ \frac{\alpha^*_1 A}{\gamma^*_1 \alpha_3}, \frac{\alpha^*_2 A}{\gamma^*_2 \alpha_3} \right\}
\]
Using Mathematica 11.0's Solve function to solve Model (2), and make the left end of the model equal to 0. The equilibrium point is as follows:

\[ P_0: \left( S = \frac{A}{\alpha_3}, I_s^* = 0, I_o^* = 0 \right) \]

\[ P_1: \left( S = \frac{\gamma_1^*}{\alpha_1^*}, I_s^* = -\frac{\alpha_3 \gamma_1^* + \alpha_1^* A}{\alpha_1^* \gamma_1^*}, I_o^* = 0 \right) \]

\[ P_2: \left( S = \frac{\gamma_2^*}{\alpha_2^*}, I_s^* = 0, I_o^* = -\frac{\alpha_3 \gamma_2^* + \alpha_2^* A}{\alpha_3^* \gamma_2^*} \right) \]

\[ P_3: \begin{cases} I_s = \frac{\alpha_3 \beta_1 \gamma_2^* - \alpha_3^* \beta_2 \gamma_2^* - \alpha_2 \gamma_1^* \gamma_2^* + \alpha_1 \gamma_2^* \gamma_2^* - \alpha_2 \beta_1 A + \alpha_2 \beta_2 A + \alpha_2^* \beta_1 A + \alpha_2^* \beta_2 A}{(\beta_1 - \beta_2) (\alpha_3 \beta_1 - \alpha_3^* \beta_2 - \alpha_2 \gamma_1^* + \alpha_1 \gamma_2^*)} \\
I_o = -\frac{-\alpha_3 \beta_1 \gamma_1^* + \alpha_3 \beta_2 \gamma_1^* + \alpha_1 \gamma_2^* \gamma_1^* - \alpha_1 \gamma_2^* \gamma_2^* + \alpha_3 \beta_1 A - \alpha_3 \beta_2 A}{(\beta_1 - \beta_2) (\alpha_3 \beta_1 - \alpha_3^* \beta_2 - \alpha_2 \gamma_1^* + \alpha_1 \gamma_2^*)} \end{cases} \]

The above equilibrium point \( P_0, P_1, P_2 \) and \( P_3 \), is the equilibrium point, which means that the variables of the dynamic system no longer change with respect to time \( t \), and the system reaches the equilibrium state. Where, \( P_0 \) is the disease-free equilibrium point, \( P_1, P_2 \) and \( P_3 \), is the endemic disease equilibrium point, \( P_1 \) and \( P_2 \) is the equilibrium point symmetric to \( I_s \) and \( I_o \) population. In \( I_s \) and \( I_o \), one of which is 0 in \( P_1, P_2 \), while in the other parameter in \( P_1 \) and \( P_2 \), \( \gamma_1^*, \gamma_2^* \) and \( \gamma_1, \gamma_2 \) etc. are all replaced. When \( I_s \) and \( I_o \) is 0, netizens expressing support or opposition to online public opinion completely disappear, and the trend of public opinion is eliminated. In \( P_3, I_o, I_s \) and \( S \) all exist at the same time and do not change with time \( t \), indicating that the communication public opinions exist at the same time and are in a state of equilibrium.

In a closed environment, when \( A = 0 \), \( R_0 = 0 \), at this time, the spread of online public opinion will gradually disappear and eventually become stable, and everyone in the system will become immune. It's an equilibrium point \( P_0 \) because

\[ P_0: \left( S = \frac{A}{\alpha_3}, I_s = 0, I_o = 0 \right) \]

\[ S = 0, \quad I_s = 0, \quad I_o = 0 \]

Public opinion tends to stabilize, everyone becomes immune, and public opinion no longer spreads.

When \( A = 0, S = 0 \), there are \( I_s = \frac{\gamma_2^*}{(\beta_1 - \beta_2)^2}, I_o = -\frac{\gamma_1^*}{(\beta_1 - \beta_2)^2} \)

Since there must be a number less than zero in \( I_s, I_o \), when \( A = 0 \), the equilibrium point \( P_3 \) does not exist.

When \( \rho(A) = \max \left( \frac{\alpha_1^* A}{\gamma_1^* \alpha_3^* \gamma_2^* \alpha_3} \right) > 1 \), that is \( A > \min \left( \frac{\gamma_1^* \alpha_3}{\alpha_1^*} \frac{\gamma_2^* \alpha_3}{\alpha_2^*} \right) \), public opinion will be spread within a range. And because of the parameter setting, when \( R_0 \geq 1 \),

\[ \gamma_1^* = \beta(1-\eta), \gamma_2^* = \beta(1+\eta), \gamma_1^* < \gamma_2^* \]

\[ \alpha_i^* = \alpha + \lambda, \alpha_1^* = \alpha - \lambda, \alpha_i^* > \alpha_2^* \]

Therefore, \( \frac{\gamma_1^* \alpha_3}{\alpha_1} < \frac{\gamma_2^* \alpha_3}{\alpha_2} \), \( P_1 \) stability must come before \( P_2 \). So in the case of government intervention, only the \( P_1 \) situation will appear, but \( P_2 \) not appear.
Then stability analysis is performed. When $A > \min\left(\frac{\gamma_1 \alpha_3}{\alpha_1}, \frac{\gamma_2 \alpha_3}{\alpha_2}\right)$, is $R_0 > 1$, according to the above analysis, there is only an equilibrium point $P_1$, and the asymptotic stability is proved $P_1$ as follows$^{[4][15]}$.

There are

$$
X = A - \alpha_1^s SI_s - \alpha_2^s SI_o - \alpha_3 S
$$
$$
Y = \alpha_1^s SI_s + \beta_2^i I_o - \beta_1^i I_o - \gamma_1^i I_s
$$
$$
Z = \alpha_2^s SI_o + \beta_1^i I_o - \beta_2^i I_o - \gamma_2^i I_o
$$

Let $X$, $Y$ and $Z$ are respectively applied to $S$, $I_s$ and $I_o$, to get the matrix:

$$
\begin{pmatrix}
\frac{dX}{dS} = -\alpha_1^s I_s - \alpha_2^s I_o - \alpha_3, & \frac{dX}{dI_s} = -\alpha_1^s S, & \frac{dX}{dI_o} = -\alpha_2^s S \\
\frac{dY}{dI_s} = \alpha_1^s I_s, & \frac{dY}{dI_o} = \alpha_1^s S + \beta_2^i I_o - \beta_1^i I_o - \gamma_1^i, & \frac{dY}{dI_0} = \beta_2^i I_o - \beta_1^i I_o \\
\frac{dZ}{dS} = \alpha_2^s I_s, & \frac{dZ}{dI_o} = \beta_1^i I_o - \beta_2^i I_o, & \frac{dZ}{dI_0} = \alpha_2^s S + \beta_1^i I_o - \gamma_2^i I_o
\end{pmatrix}
$$

Substitute corresponding values of, and $S$, $I_s$, $I_o$ in the equilibrium point $P_1$: $S = \frac{\gamma_1^*}{\alpha_1}, I_s = \frac{-\alpha_3 \gamma_1^* + \alpha_1^A}{\alpha_1 \gamma_1^*}, I_o = 0$ into the matrix, and the $P_1$ Jacobian matrix can be obtained as

$$
\begin{pmatrix}
-\alpha_1^* I_s & -\gamma_1^* & \frac{-\alpha_2^* \gamma_1^*}{\alpha_1} \\
\alpha_1^* I_s & 0 & (\beta_2^i - \beta_1^i) I_o \\
0 & 0 & \frac{-\alpha_2^* \gamma_1^*}{\alpha_1} - \gamma_2^i - (\beta_2^i - \beta_1^i) I_0
\end{pmatrix}
$$

Its characteristic equation is as follows:

$$\lambda^3 + M_1 \lambda^2 + M_2 \lambda + M_3 = 0$$

$$
M_1 = (\beta_2^i - \beta_1^i) I_s + \frac{\alpha_1^* A}{\gamma_1^*} + \gamma_2^i - \frac{\alpha_2^* \gamma_1^i}{\alpha_1}
$$

$$
M_2 = \frac{\alpha_1^* A}{\gamma_1^*} \left[(\beta_2^i - \beta_1^i) I_s + \gamma_2^i - \frac{\alpha_2^* \gamma_1^i}{\alpha_1}\right] + \alpha_3 \gamma_1^i (R_0 - 1)
$$

$$
M_3 = \alpha_3 \gamma_1^i (R_0 - 1) \left[(\beta_2^i - \beta_1^i) I_s + \gamma_2^i - \frac{\alpha_2^* \gamma_1^i}{\alpha_1}\right]
$$

$$
I_s = \frac{-\alpha_3 \gamma_1^i + \alpha_1^* A}{\alpha_1 \gamma_1^*}, \alpha_1^* > \alpha_2^* \text{ and } \gamma_2^i > \gamma_1^i, \beta_2^i > \beta_1^i, \text{ except } R_0 > 1, \text{ all the other parameters are greater than 0. Thus, the coefficient of the characteristic equation } M_1 > 0, M_2 > 0, M_3 > 0, \text{ and } M_1 M_2 - M_3 > 0. \text{ According to Routh-Hurwitz criterion}^{[13]} \text{ so all the real parts of the } \lambda \text{ are negative, and it can be proved that the equation at the point } P_1 \text{ is locally asymptotically stable.}
$$

Once the network public opinion reaches the equilibrium point $P_2$, it will eventually stabilize at the equilibrium point. At the equilibrium point, there are only ignorant people $S$ and supporters $I_s$ in the whole public opinion communication system, but there are no opponents $I_o$. So the whole model will degenerate into a steady state model $SI_s R$.

3.2. Based on the 4th order Runge-Kutta simulation

1. Explicit Euler method
The changing trend of each group in public opinion is related to the size of other groups in the current public opinion, as shown in Formula (1). According to the study of Liao Shu and Yang Weiming on cholera transmission [5], the discretization of the differential equation with global asymptotic convergence is stable, which naturally proves the Lipschitz continuity of the equation and satisfies the basic conditions of the numerical solution method.

The study in Section 3.1 can prove the global asymptotic convergence of differential equations in this paper and the model modified by explicit Euler method. The recursion formula is as follows:

Changes in the Uninformed:
\[ S(t_{n+1}) - S(t_n) = Ah - \alpha_1 I_S(t_n) S(t_n)h - \alpha_2 I_O(t_n) S(t_n)h - \alpha_3 S(t_n)h \] (8)

Changes in supporters:
\[ I_S(t_{n+1}) - I_S(t_n) = \alpha_1 S(t_n) I_S(t_n)h + (\beta_2 - \beta_1) I_O(t_n) I_S(t_n)h - \gamma_1 I_S(t_n)h \] (9)

Changes in opposition:
\[ I_O(t_{n+1}) - I_O(t_n) = \alpha_2 S(t_n) I_O(t_n)h + (\beta_1 - \beta_2) I_O(t_n) I_S(t_n)h - \gamma_2 I_O(t_n)h \] (10)

Changes in immunized persons:
\[ R(t_{n+1}) - R(t_n) = \alpha_3 S(t_n)h + \gamma_1 I_S(t_n)h + \gamma_2 I_O(t_n)h \] (11)

Divide both sides \( h \) of the above discrete models (8), (9), (10) and (11), then let the above discrete model be reduced to a dynamic differential model that changes with time, as shown in Formula (1) above.

When \( t_0 = 0 \), the initial state is as follows: \( S(t_0) = S_0, I_S(t_0) = I_{S_0}, I_O(t_0) = I_{O_0}, R(t_0) = R_0 \).

In order to improve the problem that the deviation of the explicit Euler method in the numerical value will be transmitted and become larger and larger, the step size \( h \) value should be obtained as small as possible. After comparison, the step size is taken as \( h = 0.001 \), the unit of time in the experimental environment, namely, one thousandth of a day.

(2) Fourth-order Runge-Kutta method

In the numerical solution method, Runge-Kutta provided the solution method for different time nodes \( t_n \) of the function. Starting from the initial node, according to the conditions of the differential equation, the slope was divided according to a certain step size and progressed slowly, so as to realize the calculation of numerical nodes and the drawing of curves.

The basic iteration process of the fourth-order Runge-Kutta method is as follows:

\[
\begin{align*}
Y_{n+1} &= Y_n + \frac{h}{6}(K_1 + 2K_2 + 2K_3 + K_4) \\
K_1 &= F(t_n, Y_n) \\
K_2 &= F(t_n + h/2, Y_n + K_1*h/2) \\
K_3 &= F(t_n + h/2, Y_n + K_2*h/2) \\
K_4 &= F(t_n + h, Y_n + K_3*h) \\
\end{align*}
\]

Where, \( y_i = f_i(t, y_1, y_2, ..., y_n) \), \( Y_n = (y_1, y_2, ..., y_n)^T \), \( F_n = (f_1, f_2, ..., f_n)^T \) meets Lipschitz's condition.

Two different methods are used to simulate the same set of data with Python programming software. The results are shown in Figure 4.

After running for a period of time, the two methods have both entered the theoretical equilibrium point, but there are great differences in the process before the theoretical equilibrium point. Runge-Kutta curve is smooth and public opinion fluctuates; The curve of Euler method has jitter and instability in the iteration part, and the curve fluctuation is not obvious, and the simulation process of public opinion is not significant. Through comparison, it is found that Euler's method can only obtain final results with high accuracy from the simulation, but cannot guarantee the accuracy in the process. Therefore, it is
more appropriate to choose the fourth-order Runge Kutta method in the process state estimation such as the peak of public opinion.

Figure 4. Comparison of propagation and phase orbit diagrams between Runge Kutta method (solid line) and Euler method (dashed line)

Through crawling, we get a set of search volume data about the "hot searches in Cainiao Station" in 2019, as follows:

Table 2. Cainiao station hot search "event search volume data

| Time t | S     | Is    | Io    | R     |
|--------|-------|-------|-------|-------|
| 1      | 138600| 700   | 700   | 0     |
| 2      | 71139 | 32141 | 27386 | 9334  |
| 3      | 3268  | 54213 | 30089 | 52430 |
| 4      | 287   | 31856 | 21373 | 86484 |
| 5      | 64    | 18056 | 14391 | 107489|
| 6      | 30    | 10089 | 9619  | 120262|
| 7      | 15    | 6703  | 5257  | 128025|
| 8      | 11    | 4126  | 3132  | 132731|
| 9      | 8     | 1265  | 896   | 137831|

Using \( A = 0, \alpha_1 = \alpha_2 = 0.45, \gamma_1 = \gamma_2 = 0.05, \beta_2 = 0.15, \beta_1 = 0.1, \lambda = 0.15, \eta = 0.5, \alpha_1^* = 0.6, \alpha_2^* = 0.3, \gamma_1^* = 0.025, \gamma_2^* = 0.075 \) perform simulation.
Figure 5. Comparison of actual data of hot search and simulation results

On the right, the solid line shows no government intervention, and the dotted line shows government intervention. As can be seen from the simulation without government intervention, the process data is in good agreement with the real data of the event, and the peak value, the rise range of the curve and the inflection point are also very similar, showing a steep rise and decline trend at first and then slowly. The intervention of the government could predict that the number of people in favour of it would quickly outnumber the number against it. And the time of the dissenting crowd will be half as fast. The effect of government intervention and control on public opinion is consistent with the theoretical expectation.

4. Conclusion

Based on the numerical solution method of the fourth-order Runge-Kutta method, the curve of propagation dynamics drawn has strong continuity and smooth curve, which is conducive to explaining the development trend of public opinion in any section of the curve. The experimental part of this paper is aimed at "hot search events in Cainiao station", and has a high degree of agreement. It is a better solution to other simulation that relationship mechanism is complex, the simulation results are difficult to interpret and solutions can not correspond to any scale of the problem. It provides a controllable method for the study of the propagation direction of public opinion in the communication dynamics system, so that the scale control of public opinion and the results of public opinion distribution can be predicted, and effectively match the real data, theoretical results and simulation results.

Compared to the Eulerian step methods the scale drawing, Runge-Kutta method in the simulation run, after a period of time the results are based on the closed-form solution of can explain or balance solution, good way to solve the Euler iterative error in numerical computation is constantly increasing, at the same time, the improved method can effectively improve the authenticity of simulation results, is conducive to the analysis of public opinion transmission mechanism.

In addition to the curve drawing of each variable, the phase trajectory method calculated in this paper can more clearly obtain the scale change relationship between any two variables, and can also more intuitively show the difference in the operating scale of public opinion before and after the government intervention. The node where the specific phase orbit curve intersects is set as the scale node, which is another public opinion transmission node besides the time node, and provides a new idea for the control of the change trend of the public opinion transmission in the scale. It is helpful for the government to better analyze and process public opinion data and provide theoretical support for the decision of the optimal timing of intervention.
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