Measurement of the $B^0\bar{B}^0$ Oscillation Frequency in Hadronic $B^0$ Decays

The BABAR Collaboration

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Abstract

$B^0\bar{B}^0$ flavor oscillations have been studied in 20.7 fb$^{-1}$ of $e^+e^-$ annihilation data collected in 1999 and 2000 with the BABAR detector at center-of-mass energies near the $\Upsilon(4S)$ resonance. The event sample consists of one $B^0$ meson fully reconstructed in a hadronic decay mode, while the flavor of the recoiling $B^0$ in the event is determined with a tagging algorithm that exploits the correlation between the flavor of the heavy quark and the charges of its decay products. By fitting the time development of the observed mixed and unmixed final states, the $B^0\bar{B}^0$ oscillation frequency, $\Delta m_d$, is determined to be $0.519 \pm 0.020 \pm 0.016 \, h \, ps^{-1}$.

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1 Introduction

In the Standard Model, $B^0\bar{B}^0$ mixing occurs through second-order weak diagrams involving the exchange of up-type quarks, with the top quark contributing the dominant amplitude. A measurement of $\Delta m_d$, the difference between the mass eigenstates of the $B^0-\bar{B}^0$ system, is therefore sensitive to the magnitude of the Cabibbo-Kobayashi-Maskawa matrix element $V_{td}$. At present the sensitivity to $V_{td}$ is not limited by experimental precision on $\Delta m_d$, but by other uncertainties in the calculation, in particular the quantity $f_{B^0}B_B$, where $f_{B^0}$ is the $B^0$ decay constant, and $B_B$ is the so-called bag factor, representing the strong interaction matrix elements.

The phenomenon of particle–anti-particle mixing in the neutral $B$ meson system was first observed almost fifteen years ago \[3\]. The oscillation frequency has been extensively studied with both time-integrated and time-dependent techniques \[4\].

In this paper we present a measurement of time-dependent mixing based on a sample of 20.7 fb\(^{-1}\) of data recorded at the $\Upsilon(4S)$ resonance with the BABAR detector at the Stanford Linear Accelerator Center. At the PEP-II asymmetric-energy $e^+e^-$ collider, resonant production of the $\Upsilon(4S)$ provides a copious source of $B^0\bar{B}^0$ pairs moving along the beam axis ($z$ direction) with a Lorentz boost of $\beta\gamma = 0.56$. The typical separation between the two neutral $B$ decay vertices is $\langle|\Delta z|\rangle \approx \beta\gamma c\tau_{B^0} = 260\,\mu$m, where $\tau_{B^0} = 1.548 \pm 0.032$ ps is the $B^0$ lifetime \[4\].

2 Analysis method

The $B^0\bar{B}^0$ mixing probability is a function of $\Delta m_d$ and the time difference between the $B$ decays, $\Delta t \approx \Delta z/\beta\gamma c$:

$$\text{Prob}(B^0 \bar{B}^0 \rightarrow B^0 B^0, \bar{B}^0 \bar{B}^0) \propto e^{-|\Delta t|/\tau_{B^0}}(1 - \cos \Delta m_d \Delta t)$$

$$\text{Prob}(B^0 \bar{B}^0 \rightarrow B^0 \bar{B}^0) \propto e^{-|\Delta t|/\tau_{B^0}}(1 + \cos \Delta m_d \Delta t)$$

resulting in a time-dependent probability to observe mixed, $B^0 B^0$ and $\bar{B}^0 \bar{B}^0$, or unmixed, $B^0 \bar{B}^0$, pairs produced in $\Upsilon(4S)$ decay. The effect can be measured by reconstructing one $B$ in a flavor eigenstate, referred to as $B_{\text{rec}}$, while using the remaining particles from the decay of the other $B$, referred to as $B_{\text{tag}}$, to identify, or “tag”, its flavor. The charges of identified leptons and kaons are the primary indicators, but other information in the event can also be used to identify the flavor of $B_{\text{tag}}$, resulting in a total of four non-overlapping tagging categories. The tagging algorithm used in this analysis is identical to that employed for $CP$ violation studies, in which one $B$ is fully reconstructed in a $CP$ eigenstate \[5\].

If the flavor tagging were perfect, the asymmetry as a function of $\Delta t$

$$A_{\text{mixing}}(\Delta t) = \frac{N_{\text{unmixed}} - N_{\text{mixed}}}{N_{\text{unmixed}} + N_{\text{mixed}}}$$

would be a cosine with unit amplitude. However, the tagging algorithm incorrectly identifies the flavor of $B_{\text{tag}}$ with a probability $w_i$ for the $i^{th}$ tagging category. This mistag rate reduces the amplitude of the oscillation by a factor $(1 - 2w_i)$. A simultaneous fit to the mixing frequency and its amplitude allows the determination of both $\Delta m_d$ and the mistag rates, $w_i$.

Neglecting any background contributions, the probability density functions (PDFs) for the mixed (+) and unmixed (−) events, $H_{\pm,\text{sig}}$, can be expressed as the convolution of the decay
distribution for the $i^{th}$ tagging category

$$h_{\pm} (\Delta t; \Delta m_d, w_i) = \frac{e^{-|\Delta t|/\tau_{B^0}}}{4\tau_{B^0}} [1 \mp (1 - 2w_i) \cos \Delta m_d \Delta t],$$

with a time resolution function $R(\Delta t - \Delta t_{true}; \hat{a}_i)$,

$$H_{\pm, sig}(\Delta t; \Delta m_d, w_i, \hat{a}_i) = h_{\pm} (\Delta t_{true}; \Delta m_d, w_i) \otimes R(\Delta t - \Delta t_{true}; \hat{a}_i).$$

where $\hat{a}_i$ are the parameters of the resolution function. A log-likelihood function is then constructed from the sum of $\log H_{\pm, sig}$ over all mixed and unmixed events, and over the different tagging categories.

The log-likelihood is maximized to extract the mixing parameter $\Delta m_d$ and, simultaneously, the mistag rates, $w_i$. The correlation among these parameters is small, because the rate of mixed events at low values of $\Delta t$, where the $B^0 \bar{B}^0$ mixing probability is small, is principally governed by the mistag rate. Conversely, the sensitivity to $\Delta m_d$ increases at larger values of $\Delta t$; for $\Delta t$ near twice the lifetime, half of the $B^0$ mesons will have oscillated.

3 The BABAR detector

The BABAR detector is a 4$\pi$ charged and neutral spectrometer described in more detail elsewhere [6]. Charged particles are detected and their momenta measured by a combination of a 40-layer drift chamber (DCH) and a five-layer silicon vertex tracker (SVT) embedded in a 1.5-T solenoidal magnetic field. Decay vertices are typically reconstructed with a resolution along the boost direction of 65 $\mu$m for fully reconstructed $B$ mesons. A ring imaging Cherenkov detector, the DIRC, is used for charged hadron identification. A finely segmented CsI(Tl) electromagnetic calorimeter (EMC) is used to detect photons and neutral hadrons, and also for electron identification. The iron flux return (IFR) is segmented and instrumented with multiple layers of resistive plate chambers for the identification of muons and long-lived neutral hadrons.

4 Event selection and $B$ reconstruction

The analysis uses a sample of multihadron events, which are selected by demanding a minimum of three reconstructed charged tracks and a total charged and neutral energy greater than 4.5 GeV in the fiducial region of the detector, and a reconstructed event vertex within 0.5 cm of the measured interaction point [6] in the plane transverse to the beamline.

Electron candidates must satisfy a cut on the ratio of calorimeter energy to track momentum of $0.88 < E/p < 1.3$, a cluster shape consistent with an electromagnetic shower, and DCH d$E$/d$x$ and DIRC Cherenkov angle consistent with an electron.

Muon candidates must satisfy requirements on the number of interaction lengths of IFR iron penetrated of $N_\lambda > 2.2$, on the difference in the measured and expected interaction lengths penetrated of $N^{exp}_\lambda - N_\lambda < 1$, on the position match between the extrapolated DCH track and the IFR hits, and on the average and spread of the number of IFR hits per layer.

Pairs of photons in the EMC with energy above 30 MeV are constrained to the known $\pi^0$ mass if they are within $\pm 20$ MeV/c$^2$ of the nominal invariant mass [4], and their summed energy is greater than 200 MeV.
$K^0_S \rightarrow \pi^+\pi^-$ candidates are required to have an invariant mass between 462 and 534 MeV/$c^2$, and a $\chi^2$ probability for the vertex fit of greater than 0.1%. The transverse flight distance of the $K^0_S$ candidate from the primary event vertex must be greater than 2 mm.

$\bar{D}^0$ candidates are identified in the decays channels $K^+\pi^-$, $K^+\pi^-\pi^0$, $K^+\pi^-\pi^-\pi^+$ and $K^0_S\pi^+\pi^-$. $D^-$ candidates are selected in the $K^+\pi^-\pi^-$ and $K^0_S\pi^-$ modes. Kaons from $D^-$ decays and charged daughters from $\bar{D}^0 \rightarrow K^+\pi^-$ are required to have a momentum greater than 200 MeV/$c$. All other charged $D$ daughters are required to have a momentum greater than 150 MeV/$c$. For $\bar{D}^0 \rightarrow K^+\pi^-\pi^0$, we only reconstruct the dominant resonant mode $\bar{D}^0 \rightarrow K^+\rho^-$, followed by $\rho^+ \rightarrow \pi^+\pi^-\pi^0$. The $\pi^-\pi^0$ mass is required to lie within $\pm 150$ MeV/$c^2$ of the nominal $\rho$ mass and the angle between the $\pi^-$ and $\bar{D}^0$ in the $\rho$ rest frame, $\theta_{\rho\pi}$, must satisfy $|\cos \theta_{\rho\pi}| > 0.4$. $D^0$ and $D^-$ candidates are required to have momentum greater than 1.3 GeV/$c$ in the $\Upsilon(4S)$ frame, an invariant mass within $\pm 3\sigma$ of the nominal value and a $\chi^2$ probability of the topological vertex fit greater than 0.1%. A mass constraint is applied to selected $\bar{D}^0$ candidates.

$D^{*-}$ candidates are formed by combining a $\bar{D}^0$ and a pion with momentum less than 450 MeV/$c$. The soft pion is constrained to originate from the beamspot when the $D^{*-}$ vertex is computed. Those $D^{*-}$ candidates with $m(D^0\pi^-) - m(\bar{D}^0)$ lying within $\pm 2.5\sigma$ of the nominal value are selected, where $\sigma = 1.1$ MeV/$c^2$.

$J/\psi \rightarrow e^+e^-$ or $\mu^+\mu^-$ candidates must have at least one decay product positively identified as an electron or muon. Electron candidates outside the calorimeter acceptance must have DCH dE/dz information consistent with that for an electron. The second muon candidate, if within the acceptance of the calorimeter, must be consistent with being a minimum ionizing particle. $J/\psi$ candidates are required to lie in the invariant mass interval 2.95 (3.06) to 3.14 GeV/$c^2$ for the $e^+e^-$ $(\mu^+\mu^-)$ channel.

$B^0$ candidates in the flavor eigenstate decay modes $D^{(*)-}\pi^+/\rho^+/a_1^+$ are formed by combining a $D^{*-}$ or $D^-$ candidate with a $\pi^+, \rho^+(\rho^+ \rightarrow \pi^+\pi^0)$ or $a_1^+(a_1^+ \rightarrow \pi^+\pi^-\pi^0)$; likewise $B^0 \rightarrow J/\psi K^{*0}$ candidates are reconstructed from combinations of $J/\psi$ candidates with a $K^{*0}$ ($K^{*0} \rightarrow K^+\pi^-)$.

For $B^0 \rightarrow D^{*-}\rho^+$, the $\rho^0$ from the $\rho^+$ decay is required to have an energy greater than 300 MeV. For $B^0 \rightarrow D^{*-}a_1^+$, the $a_1^+$ is reconstructed by combining three charged pions, with invariant mass in the range 1.0 to 1.6 GeV/$c^2$ and a $\chi^2$ probability of the vertex fit of the $a_1^+$ candidate of greater than 0.1%. For most $B^0$ modes, no particle identification or only a loose requirement is enough to achieve reasonable signal purities.

Continuum background is rejected by requiring the normalized second Fox-Wolfram moment be less than 0.5. Further suppression is achieved by a mode-dependent restriction on the angle, $\theta_{th}$, between the thrust axes of decay products from $B_{rec}$ and $B_{tag}$ respectively in the $\Upsilon(4S)$ frame.

$B^0$ candidates are identified with the difference $\Delta E$ between the energy of the candidate and the beam energy $E^{cm}_{beam}$ in the center-of-mass frame and the beam-energy substituted mass $m_{ES} = \sqrt{(E^{cm}_{beam})^2 - (p^{cm}_B)^2}$. Those candidates with $m_{ES} > 5.2$ MeV/$c^2$ and $\Delta E = 0$ within $\pm 2.5$ standard deviations (typically $|\Delta E| < 40$ MeV) are selected. If there is more than one candidate satisfying these conditions only the one with the smallest $\Delta E$ is retained. Finally, a topological vertex fit of the candidate must converge.

## 5 Decay time difference determination

The decay time difference, $\Delta t$, between $B$ decays is determined from the measured separation along the $z$ axis between the reconstructed $B = B_{rec}$ and flavor-tagging decay $B = B_{tag}$ vertices $\Delta z = z_{rec} - z_{tag}$. This $\Delta z$ is then converted into $\Delta t$ using the known $\Upsilon(4S)$ boost and correcting
on an event-by-event basis for the direction of the $B$ mesons with respect to the $z$ direction in the $\Upsilon(4S)$ frame. The resolution of the $\Delta t$ measurement is dominated by the $z$ resolution of the tagging vertex. The $B_{\text{tag}}$ decay vertex uses all tracks in the event except those incorporated in $B_{\text{rec}}$. An additional constraint is provided by including a calculated $B_{\text{tag}}$ production point and three-momentum, determined from the three momentum of the $B_{\text{rec}}$ candidate, its decay vertex, and the average position of the interaction point and the $\Upsilon(4S)$ boost. Reconstructed $K^0_S$ or $\Lambda$ candidates are used as input to the fit in place of their daughters in order to reduce bias due to long-lived particles. Tracks with a large contribution to the $\chi^2$ are iteratively removed from the fit, until all remaining tracks have a reasonable fit probability or all tracks are removed. Only those candidates with $|\Delta z| < 3.0\,\text{mm}$ and $\sigma_{\Delta z} < 400\,\mu\text{m}$ are retained. The distribution of $m_{\text{ES}}$ for the surviving candidates is shown in Fig. 1 along with a fit with a Gaussian distribution for the signal and the ARGUS function [8] for the background. At this point the sample contains $6662 \pm 93$ signal events, with a purity, defined in the region $m_{\text{ES}} > 5.27\,\text{GeV}/c^2$, that varies between $80$–$95\%$ depending on the $B^0$ decay mode.

In the likelihood, the time resolution function can be approximated by a sum of three Gaussian distributions with different means and widths,

$$ R(\Delta t; \hat{a}_i) = \sum_{k=1}^{3} \frac{f_k}{\sigma_k \sqrt{2\pi}} e^{-(\Delta t - \Delta t_{\text{true}} - \delta_{k,i})^2 / 2\sigma_k^2}, $$

where, for the core and tail Gaussians, the widths $\sigma_{1,2} = S_{1,2} \times \sigma_{\Delta t}$ are the scaled event-by-event
measurement error, \( \sigma_{\Delta t} \), derived from the vertex fits. The third Gaussian, with a fixed width of \( \sigma_3 = 8 \) ps, accounts for less than 1% of outlier events with incorrectly reconstructed vertices. A separate core bias, \( \delta_{1,i} \), is allowed for each tagging category \( i \) to account for small shifts due to inclusion of charm decay products in the tag vertex, while a common bias \( \delta_2 \) is used for the tail component. The tail and outlier fractions and the scale factors are assumed to be the same for all decay modes, since the precision of the \( B_{tag} \) vertex dominates \( \Delta t \). This assumption is confirmed by Monte Carlo simulation studies.

6 Flavor tagging

After the daughter tracks of the reconstructed \( B \) are removed, the remaining tracks are analyzed to determine the flavor of the \( B_{tag} \), and this ensemble is assigned a tag flavor, either \( B^0 \) or \( \bar{B}^0 \). For this purpose, we use the flavor tagging information carried by primary leptons from semileptonic \( B \) decays, charged kaons, soft pions from \( D^* \) decays, and more generally by high momentum charged particles to uniquely assign each event to a tagging category. The effective tagging efficiency \( Q_i = \varepsilon_i(1 - 2w_i)^2 \), where \( \varepsilon_i \) is the fraction of events assigned to category \( i \), is used as the basis for optimization of category selection criteria. The statistical error on \( \Delta m_d \) is proportional to \( 1/\sqrt{Q} \), where \( Q = \sum Q_i \).

Events are assigned to the Lepton category if they contain an identified lepton with a center-of-mass momentum greater than 1.0 or 1.1 GeV/c for electrons and muons, respectively. The momentum requirement selects mostly primary leptons by suppressing opposite-sign leptons from semileptonic charm decays.

Kaons are identified with a neural network based on the likelihood ratios in the SVT and DCH, derived from \( dE/dx \) measurements, and in the DIRC, calculated by comparing individual photomultiplier hits with the expected pattern of Cherenkov light for either kaons or pions. The charges of all identified kaons are summed, and if \( \sum Q_K \neq 0 \) the event is assigned to the Kaon category.

The final two categories involve a multivariable analysis using a neural network, which is trained to identify primary leptons, kaons, and soft pions, and the momentum and charge of the track with the maximum center-of-mass momentum. Depending on the value of the output variable from the neural net, events are given a \( B^0 \) or \( \bar{B}^0 \) tag and assigned to the mutually exclusive categories NT1 (more certain tags) or NT2 (less certain tags). About 30% of all events are assigned to no tagging category and are excluded from the analysis.

Tagging assignments for events are made mutually exclusive by the hierarchical use of the tagging categories. Events with a Lepton tag and no conflicting Kaon tag use the Lepton category. If no Lepton tag exists, then the Kaon category is used, if a tag exists. Otherwise the two neural network categories are used. The number of tagged events per category is given in Table 1. In total, there are 4538 ± 75 tagged signal events.

7 Background PDF

In the presence of backgrounds, the probability distribution functions \( H_{\pm, sig} \) must be extended to include a term for each significant background source, which are allowed to differ for each tagging
Table 1: Event yields for the different tagging categories used in this analysis, as obtained from fits to the $m_{ES}$ distributions after all selection requirements. The purity is quoted for $m_{ES} > 5.270 \text{MeV}/c^2$.

| Category | Tagged | Efficiency (%) | Purity (%) |
|----------|--------|----------------|------------|
| Lepton   | 754 ± 28 | 11.3 ± 0.4 | 97.1 ± 0.6 |
| Kaon     | 2317 ± 54 | 34.8 ± 0.6 | 85.2 ± 0.8 |
| NT1      | 556 ± 26  | 8.3 ± 0.3   | 88.7 ± 1.5 |
| NT2      | 910 ± 36  | 13.7 ± 0.4  | 83.0 ± 1.3 |
| Total    | 4538 ± 75 | 68.1 ± 0.9  | 86.7 ± 0.5 |

where the background PDFs, $B_{\pm,i,k}$, provide an empirical description for the $\Delta t$ distribution of the background events in the sample. The fraction of background events for each source and tagging category is given by $f_{i,k}$, while $b_{\pm,i,k}$ are parameters used to characterize each source of background by tagging category for mixed and unmixed events.

The probability that a $B^0$ candidate is a signal or a background event is determined from a separate fit to the observed $m_{ES}$ distributions of $B_{rec}$ candidates in each of the four tagging categories. We describe the $m_{ES}$ shape with a single Gaussian distribution $S(m_{ES})$ for the signal and an ARGUS parameterization $A(m_{ES})$ for the background. Based on this fit, the event-by-event signal probabilities $f_{i,sig}$ are given by

$$f_{i,sig}(m_{ES}) = \frac{S(m_{ES})}{S(m_{ES}) + A(m_{ES})}$$

The sum of signal and background fractions is forced to unity.

The $\Delta t$ distributions of the combinatorial background are assumed to be described with a zero lifetime component and a non-oscillatory component with non-zero lifetime. We fit for separate resolution function parameters for the signal and the background in order to minimize correlations of the time structure between background and signal. Candidates with low signal probability, i.e., in the $m_{ES}$ sideband region below 5.27 GeV/c$^2$, dominate the determination of these background parameters.

8 Extraction of $\Delta m_d$

The value of $\Delta m_d$ is extracted from the tagged flavor-eigenstate $B^0$ sample with an unbinned maximum likelihood fit involving a total of 34 parameters, including $\Delta m_d$. The value of $\Delta m_d$ was kept hidden throughout the analysis until the systematic errors were finalized, in order to eliminate possible experimenter’s bias.

The results from the likelihood fit to the tagged $B^0$ sample are summarized in Table 2. The probability of obtaining a likelihood smaller than the one observed is determined to be 28% from a
Table 2: Results from the likelihood fit to the $\Delta t$ distributions of the tagged hadronic $B^0$ decays. $\Delta m_d$ and the mistag rates include small corrections corresponding to the difference between the generated and reconstructed values in simulated signal events.

| Parameter        | Value     | Parameter        | Value     |
|------------------|-----------|------------------|-----------|
| $\Delta m_d$ (ps) | $0.519 \pm 0.020$ | $\delta_{1,\text{Lepton}}$ (ps) | $+0.11 \pm 0.07$ |
| $w_{\text{Lepton}}$ | $0.085 \pm 0.018$ | $\delta_{1,\text{Kaon}}$ (ps) | $-0.20 \pm 0.05$ |
| $w_{\text{Kaon}}$ | $0.167 \pm 0.014$ | $\delta_{1,\text{NT1}}$ (ps) | $+0.01 \pm 0.09$ |
| $w_{\text{NT1}}$ | $0.195 \pm 0.026$ | $\delta_{1,\text{NT2}}$ (ps) | $-0.20 \pm 0.08$ |
| $w_{\text{NT2}}$ | $0.326 \pm 0.024$ | $\delta_2$ (ps) | $-1.2^{+0.9}_{-1.6}$ |
| $S_1$            | $1.42^{+0.08}_{-0.09}$ | $f_2$            | $0.032^{+0.03}_{-0.02}$ |
| $S_2$            | $5.5^{+2.0}_{-1.6}$ | $f_3$            | $0.001 \pm 0.004$ |

Figure 2: $\Delta t$ distributions in data for the selected mixed and unmixed tagged hadronic $B^0$ decays ($m_{ES} > 5.27$ GeV/c$^2$), with overlaid the projection of the likelihood fit. The background contribution is indicated by the dashed curve.
large number of simulated experiments, each generated according to the parameters obtained from the fit. The measured value for $\Delta m_d$ is:

$$\Delta m_d = 0.519 \pm 0.020(stat) \pm 0.016(syst) \, h \, ps^{-1}.$$ 

where the sources of systematic error are discussed below. The observed distribution of mixed and unmixed events and the asymmetry, $A_{\text{mix}}$, are shown in Fig. 2 and 3 as a function of $\Delta t$ along with projections of the likelihood fit result.

9 Systematic studies and cross checks

The conversion of $\Delta z$ to $\Delta t$ introduces a systematic uncertainty ($\pm 0.007 \, h \, ps^{-1}$) due to the limited knowledge of the PEP-II boost, the $z$ length scale of $\text{B} \overline{\text{A} \text{B} \text{A} \text{R}}$ (determined from secondary interactions in a beam pipe section of known length) and the $B_{\text{tec}}$ momentum vector in the $\Upsilon(4S)$ frame.

The signal $\Delta t$ resolution parameters are determined directly from data by the fit, contributing $\pm 0.006$ in quadrature to the statistical error. Residual uncertainties ($\pm 0.005 \, h \, ps^{-1}$), are attributed to the choice of the parameterization, its description of the outliers, and the capability of the resolution model to deal with various plausible misalignment scenarios applied to the Monte Carlo simulation.

The parameters of the background $\Delta t$ distribution are left free in the likelihood fit, but systematic errors ($\pm 0.005 \, h \, ps^{-1}$), are introduced by the residual uncertainty from the $m_{\text{ES}}$ fit used
to determine the signal probability, the assumed parameterization of the background $\Delta t$ distributions and resolution function, and the small amount of correlated $B^+$ background remaining in the sample.

Finally, statistical limitations of Monte Carlo validation tests ($\pm 0.004 \, \text{h ps}^{-1}$), the full size of a (negative) correction obtained from Monte Carlo ($\pm 0.009 \, \text{h ps}^{-1}$), and the variation of the fixed $B^0$ lifetime (due to its negative correlation with $\Delta m_d$) within known errors ($\pm 0.006 \, \text{h ps}^{-1}$) contribute. A summary of these sources of systematic error for the hadronic $B^0$ sample is shown in Tables \ref{tab:sys}.

Various checks on the consistency of the result were performed by splitting the data in sub-samples according to several key variables, including (but not limited to) the decay modes of $B_{\text{rec}}$, the tagging category of $B_{\text{tag}}$, and the flavor of either $B_{\text{rec}}$ or $B_{\text{tag}}$. The value of $\Delta m_d$ was found to be consistent for all sub-samples.

| Source                        | $\delta \Delta m_d [\text{h ps}^{-1}]$ |
|-------------------------------|----------------------------------------|
| Beamspot                      |                                        |
| position and size             | 0.001                                  |
| $\Delta z$ to $\Delta t$ conversion |                                        |
| PEP-II boost                  | 0.003                                  |
| $z$ scale                     | $< 0.005$                              |
| method                        | 0.004                                  |
| $\Delta t$ resolution        |                                        |
| outliers                      | 0.002                                  |
| parameterization              | 0.003                                  |
| SVT alignment                 | 0.004                                  |
| Backgrounds                   |                                        |
| $\Delta t$ model              | 0.001                                  |
| Resolution parameterization   | 0.003                                  |
| Fractions                     | 0.003                                  |
| Correlated                    | 0.002                                  |
| Monte Carlo                   |                                        |
| statistics                    | 0.004                                  |
| correction                    | 0.009                                  |
| $B^0$ lifetime                | 0.006                                  |
| Total Systematic Error        | 0.016                                  |

**10 Summary**

We have measured the value for $\Delta m_d$ to be:

$$\Delta m_d = 0.519 \pm 0.020 (\text{stat}) \pm 0.016 (\text{syst}) \, \text{h ps}^{-1}.$$  

This result is one of the best single measurements available and is consistent with the current world average \cite{4}. Moreover, the error on $\Delta m_d$ is still dominated by the statistical size of the reconstructed


$B^0$ sample, leaving substantial room for further improvement as more data is accumulated at BABAR. The measurement shares the same flavor-eigenstate sample as used for the determination of $\sin 2\beta$. Thus, it provides an essential validation for the reported $\sin 2\beta$ result and, in particular, the mistag rates that appear as coefficients of the mixing asymmetry.

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