Thermodynamical features of Verlinde’s approach for a non-commutative Schwarzschild-anti-deSitter black hole in a broad range of scales

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Abstract

We try to study the thermodynamical features of a non-commutative inspired Schwarzschild-anti-deSitter black hole in the context of entropic gravity model, particularly for the model that is employed in a broad range of scales, from the short distances to the large distances. At small length scales, the Newtonian force is failed because one finds a linear relation between the entropic force and the distance. In addition, there are some deviations from the standard Newtonian gravity at large length scales.

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I. INTRODUCTION

The laws of black hole thermodynamics manifestly show that there is a profound connection between gravity and thermodynamics [1]. Two decades ago, Jacobson exhibited that the Einstein field equation can be derived from the first law of thermodynamics [2]. Recently, Padmanabhan applied the equipartition law of energy and the holographic principle to make a thermodynamical description of gravity [3] (see also [4]). Afterwards, Verlinde suggested a novel idea to interpret the gravity as an entropic force caused by alterations in the information connected to the position of massive particles [5]. He described the notions of inertia and Newton’s law of classical mechanics by joining the gravitational force with a special kind of force that is emerged from the thermodynamics on holographic screens. Verlinde’s conjecture has widely been studied in different theoretical frameworks [6–33]. There are also some criticisms on the entropic gravity scenario which show a number of deficiencies of the topic including some open challenges [34–40].

It is widely expected that underneath the emergent description of gravitational phenomena there is a deeper layer in which the fundamental microstructure of a quantum space-time plays an important role. Therefore, if the origin of gravity is an entropic force, it is essential within the emergent gravity to understand how the microscopic scale effects such as non-commutative geometry (NCG) could be emerged. One of the main references on the universal character of the quantum gravitational effects in the form of some non-commutative space-time can be found in Ref. [41]. In a physically inspired kind of NCG, established upon the coordinate coherent state technique caused by averaging non-commutative coordinate fluctuations, one can show that the short distance behavior of point-like structures is improved [42–49]. The NCG inspired model produces a class of solutions of Einstein equations which contain effects of quantum gravity at small scales [50]. Lately, some properties of the entropic picture of gravity in the present of some NCG inspired black holes have been investigated, e.g., the non-commutative Schwarzschild [51], Reissner-Nordström [52] and Schwarzschild-deSitter (SdS) [53] black holes. Here, it is worth mentioning that non-commutative point-like sources are also important in the non-commutative field theory in general, with potentially observable effects even in atomic physics. These effects are linear in the non-commutativity parameter [54], contrary to the typical situation in gravity, where such effects are quadratic.
In the NCG inspired model, it has been illustrated that the mean position of a point-like particle in a non-commutative manifold is no longer characterized by a Dirac-delta function distribution, but will be described by a Gaussian distribution of minimal width $\sqrt{\theta}$, where $\theta$ is the smallest fundamental unit of an observable area in the non-commutative coordinates, beyond which coordinate resolution is ambiguous. As a result, the curvature singularity at the origin of black holes is eliminated and a regular deSitter (dS) vacuum state is appeared instead. The appearance of a dS core in the centre of the black hole prohibits its collapse into a singular region. Indeed, a non-commutative black hole is a combination of the dS core around the origin with a standard metric of the black hole far away from the origin. In other words, the small scale behavior of point-like structures is improved such that the particle mass $M$, in lieu of being totally localized at a point, is distributed throughout a region of linear size $\sqrt{\theta}$ as a smeared-like particle. In fact, due to the appearance of extreme energies at short distances of a non-commutative manifold, the effects of manifold quantum fluctuations become considerable and prohibit any measurements to find a particle position with an accuracy greater than an inherent length scale. The smallness of the scale would reveal that non-commutativity effects can be visible just in extreme energy phenomena. In a general string theory framework one could presume that $\sqrt{\theta}$ would naturally not be far from the four-dimensional Planck scale, $L_{Pl}$. Most of the phenomenological investigations on non-commutative models have proposed that we live in a four-dimensional space-time and that the non-commutative energy scale is about $1 - 10$ TeV [53–58], accessible to colliders. However, the bounds coming from the non-commutative QCD are much stronger, at the level of $1/\sqrt{\theta} > 5 \times 10^{14}$ GeV [59]. Since, the minimal observable length is not exactly determined through deduction; therefore the scale is generally presumed as smaller than the typical scale of the standard model of particle physics, i.e. only less than $10^{-16}$ cm. In this paper, we include the non-commutativity correction in the Schwarzschild-anti-deSitter (SAdS) metric and find the entropic force for the small and large scales. Throughout the paper, we will use the definitions $\hbar = c = k_B = 1$. Also, Greek indices run from 0 to 3.
II. NON-COMMUTATIVE SADS BLACK HOLE

The non-commutative SAdS metric is given by

\[ ds^2 = -\left(1 - \frac{2GM_\theta}{r} - \frac{\Lambda}{3} r^2 \right)dt^2 + \left(1 - \frac{2GM_\theta}{r} - \frac{\Lambda}{3} r^2 \right)^{-1}dr^2 + r^2d\Omega^2, \]  

(1)

where the smeared mass distribution \( M_\theta \) is found to have the form

\[ M_\theta = M \left[ \mathcal{E} \left( \frac{r}{2\sqrt{\theta}} \right) - \frac{r}{\sqrt{\pi\theta}} e^{-\frac{r^2}{4\theta}} \right], \]  

(2)

where the Gaussian error function is determined by \( \mathcal{E}(x) \equiv 2/\sqrt{\pi} \int_0^x e^{-t^2} dt \). The non-commutative SAdS metric is comprised of the non-commutative black hole solution with a negative cosmological term \( \Lambda = -3/l^2 \), where \( l \) is the cosmological length associated with the \( \Lambda \). In the limit \( r/\sqrt{\theta} \to \infty \), we have the standard (commutative) SAdS metric. In the commutative limit, the Gaussian error function tends to one and the second term in Eq. (2) will exponentially be reduced to zero and finally one recovers the standard mass totally localized at a point, i.e. \( M_\theta/M \to 1 \). However, in the regime that non-commutative fluctuations are important, i.e. \( r \sim \sqrt{\theta} \), the non-commutative SAdS metric deviates significantly from the standard one and provides a new physics at short distances.

In Fig. 1 we present the behavior of \(-g_{00}\) versus the radius, \( r/\sqrt{\theta} \) for the metric (1). This figure shows two situations. The situation one displays an AdS background for two values of \( \Lambda \theta \) and the situation two displays an asymptotically flat space. For both situations, there is no curvature singularity and the metric is regular at the origin. The possibility of having two distinct horizons for \( M > M_0 \) is shown (an inner \( r_i \) and an outer black hole horizon \( r_o \)), where \( M_0 \) is the minimal mass corresponding to an extremal black hole with one degenerate horizon in \( r_o \). For different values of \( \Lambda \theta \), the minus peak of the curves corresponding to \( r_0 \) is almost fixed. It is evident from the figure that when the cosmological constant deviates from the zero, the outer black hole horizon diminishes however the inner horizon and the minimal non-zero radius become almost unchanged. In the limit \( \theta \to 0 \), the inner horizon disappears and the outer horizon is the Schwarzschild value, \( r_o = 2M \). However at small scales or high energies, due to the effect of strong quantum fluctuations at short distances, quantum gravity corrections commence to be most significant wherein there is a considerable deviation from the standard SAdS metric. By expanding Eq. (1) for \( r \gtrsim r_0 \), one can find...
the asymptotic form of the metric as follows:

$$-g_{00} \approx 1 - \frac{\Lambda_{\text{eff}}}{3} r^2,$$

(3)

with

$$\Lambda_{\text{eff}} = \Lambda + MG/\sqrt{\pi \theta^3},$$

(4)

where $r_0$ is a cut-off in the radial direction at small scales and $\Lambda_{\text{eff}}$ is the effective cosmological constant at short distances. The first expression of Eq. (1) is the negative background AdS term, while the second expression is the positive non-commutative fluctuations of the geometry. It should be emphasized that Eq. (3) is simply obtained from a Taylor-series-expansion around $r = 0$ just to second order in $r$. We carry out the calculations just under the circumstance that $r \gtrsim r_0$. The physical description of $r_0$ is the radius of the smallest holographic surface which can not be probed by a test particle that is located within some distance from the source. If one considers the screen radius to be less than the radius of
The mass, $M/\sqrt{\theta}$, versus the radius, $r/\sqrt{\theta}$. On the right-hand side of the figure, from top to bottom, the solid lines correspond to the non-commutative SAdS black hole for $\Lambda = -10^{-2}/\theta$, and $\Lambda = -3 \times 10^{-3}/\theta$, respectively. The dashed line refers to the case of $\Lambda = 0$.

FIG. 2: The mass, $M/\sqrt{\theta}$, versus the radius, $r/\sqrt{\theta}$. On the right-hand side of the figure, from top to bottom, the solid lines correspond to the non-commutative SAdS black hole for $\Lambda = -10^{-2}/\theta$, and $\Lambda = -3 \times 10^{-3}/\theta$, respectively. The dashed line refers to the case of $\Lambda = 0$.

the smallest holographic surface at the Planckian regime, i.e. $r < r_0$, then one encounters some unusual dynamical features, leading to negative entropic force and negative energy [51]. As a consequence, we make the requirement that the screen radius is bigger than the radius of the smallest holographic surface but is smaller than the radius corresponding to the maximum extremal temperature. According to the original work proposed by Nicolini et al. [61-70], for $r \sim \sqrt{\theta}$, the temperature of the black hole grows during its evaporation until it reaches to a maximum extremal value and then falls down to a zero temperature black hole remnant configuration, entirely governed by microscopic fluctuations of the manifold, encoded in the parameter $\theta$. In other words, instead of the ordinary divergent treatment for the ultimate phase of the Hawking evaporation at small radii, there exists a value at which the temperature vanishes. In addition, for $M < M_0$ there is no solution for $g_{00}(r_0) = 0$ and no horizon occurs. This means that, if $r < r_0$ there cannot be a black hole and we cannot speak of an event horizon and then no temperature can be defined, so the final zero temperature configuration can be considered a black hole remnant. To emphasize this point, we consider the internal energy of the black hole which is nothing but the mass of the black
hole as a function of the event horizon (for more details, see [71]). Following an approach analogous to Ref. [71], the mass parameter $M$ is a function of the horizon by requiring $g_{00}(r_0) = 0$. Thus, one can show that there is a minimum $M_0$ in the following form

$$M_0 \equiv M(r_0) = \frac{\sqrt{\pi \theta} r_0}{2G} \left[ \sqrt{\pi \theta} \mathcal{E} \left( \frac{r_0}{2\sqrt{\theta}} \right) - r_0 e^{-\frac{r_0^2}{\theta}} \right]^{-1} \left( 1 - \frac{\Lambda}{3} r_0^2 \right).$$

(5)

The numerical results of the mass versus the horizon radius are depicted in Fig. 2. As can be seen from the figure, the existence of a minimal non-zero radius ($r = r_0$), corresponding to the case of an extremal black hole configuration ($M = M_0$), is clear. For different values of $\Lambda \theta$, the minimal non-zero radius and the minimal non-zero mass are nearly fixed, i.e. $r_0 \approx 3\sqrt{\theta}$ and $M_0 \approx 1.9\sqrt{\theta}$. As expected, the non-commutativity discloses a minimal non-zero mass, namely the black hole remnant, in order to have an event horizon. So, the black hole in the non-commutative case does not allow to decay lower than the remnant, and for $M < M_0$ there is no event horizon.

III. ENTROPIC FORCE AT SMALL LENGTH SCALES

In order to define the temperature, we first need to introduce the generalized form of the Newtonian potential $\phi$ via the timelike Killing vector $\xi^\alpha$:

$$\phi = \frac{1}{2} \log \left( -g^{\alpha\beta} \xi_\alpha \xi_\beta \right),$$

(6)

where $\xi_\alpha$ satisfies the Killing equation

$$\partial_\alpha \xi_\beta + \partial_\beta \xi_\alpha = 2\Gamma^\gamma_{\alpha\beta} \xi_\gamma.$$

(7)

The redshift factor is denoted by $e^\phi$ and it relates the local time coordinate to that at a reference point with $\phi = 0$. In accord with Ref. [72], there is a problem to normalize a timelike Killing vector in a curved space-time. To study the entropic force for the non-commutative SAdS metric, which is not an asymptotically flat space-time, we may consider the normalization of a timelike Killing vector $\xi_\alpha$, as follows

$$\xi_\alpha = \sigma (\partial_0)_\alpha,$$

(8)

where $\sigma$ is a normalization constant. To find Eq. (8) we have used the Killing equation and the condition of static spherical symmetry $\partial_0 \xi_\alpha = \partial_3 \xi_\alpha = 0$, and also the infinity condition.
\( \xi^a \xi^a = -1 \). It is clear that for the infinity condition we should have \( \sigma = 1 \). The gravitational potential for the non-commutative SAdS metric is then found to be

\[
\phi = \frac{1}{2} \log \left( -\sigma^2 g_{00} \right). 
\]  
(9)

At small length scales, we use the condition of \( r \sim \sqrt{\theta} \) just under the circumstance that \( M > \frac{3\sqrt{\frac{\pi}{G}}}{\Lambda_{\text{eff}}} \) (or \( \Lambda_{\text{eff}} > 0 \)). For \( \Lambda_{\text{eff}} > 0 \) there are a dS core at the origin and a local gravitational repulsion. Using the Bousso-Hawking reference point \([73]\), one can observe the temperature on the holographic screens in short distances. Bousso and Hawking set up a reference point in the radial direction, wherein the force vanishes. They have indicated that this reference point can play a role of a point at infinity in an asymptotically flat space-time. They selected a normalization in which the norm of the Killing vector is unity at the region where the force vanishes, the gravitational attraction becomes precisely balanced out by the cosmological repulsion. Adopting this normalization is associated with the choosing a special observer who follows geodesics. To find the correct value for the temperature, one must normalize the Killing vector in the right way. In the Schwarzschild case (\( \Lambda = 0 \)) the natural choice is to have \( \xi^2 = -1 \) at infinity; this corresponds to \( \sigma = 1 \) for the standard Schwarzschild metric. However, in our case there is no infinity, and it would be a mistake to set \( \sigma = 1 \). Instead one may choose the radius \( r_0 \) as a Bousso-Hawking reference point due to the fact that the temperature becomes zero at that point. An observer at \( r_0 \) will need no acceleration to stay there, just like an observer at infinity in the Schwarzschild case. One must normalize the Killing vector on this geodesic orbit. We assume that the region between the inner and outer black hole horizon is separated by a boundary at the reference point \( r = r_0 \). Then the two regions divided by this boundary cannot have thermal exchange between them because the temperature at the reference point vanishes wherein a thermally insulating wall is existed. The notion of thermally insulating wall in our consideration is similar to that of wholly reflecting wall in the Gibbons-Hawking’s work \([74]\). The two regions separated by the surface at \( r = r_0 \) can be thought as independent systems: the total system becomes the sum of two independent systems, the inner \( (r < r_0) \) and the outer \( (r > r_0) \) regions. But, as mentioned before, we should emphasize that the existence of a dS core in the centre of the black hole yields a outward push to prevent its collapse into a singular one. For that reason, it is impossible to set up a measurement to find more precise particle position than \( r_0 \) and for the pattern of the metric for \( r < r_0 \) no temperature can be defined.
To characterize the foliation of space, and for recognizing the holographic surfaces $\Omega$ at screens of the constant redshift, we should consider the acceleration $a^\alpha$ on the spherical holographic screen with radius $r$ in a general relativistic form as

$$a^\alpha = -g^{\alpha\beta} \nabla_\beta \phi. \quad (10)$$

The temperature on the holographic screen seen by an observer located at the Bousso-Hawking reference point is given by the Unruh-Verlinde temperature associated with the proper acceleration of a particle near the screen which is written as

$$T = \frac{1}{2\pi} e^\phi n^\alpha a_\alpha = \frac{e^\phi}{2\pi} \sqrt{g^{\alpha\beta} \nabla_\alpha \phi \nabla_\beta \phi}, \quad (11)$$

where $n^\alpha$ is a unit vector in which it is normal to the holographic screen and to $\xi_\alpha$. The unit vector is given by

$$n^\alpha = \frac{\nabla^\alpha \phi}{\sqrt{g^{\alpha\beta} \nabla_\alpha \phi \nabla_\beta \phi}}. \quad (12)$$

The redshift factor $e^\phi$ is identical to unity at the Bousso-Hawking reference point. The temperature for the non-commutative SAdS metric in short distances becomes

$$T = \frac{\sigma}{4\pi} \left| \frac{dg_{00}}{dr} \right| \approx \frac{\sigma}{2\pi} \left( \frac{MG}{3\sqrt{\pi \theta^3}} - \frac{1}{l^2} \right) r. \quad (13)$$

It is straightforward to show that

$$\sigma = \left[ -g_{00}(r_0) \right]^{-\frac{1}{2}}. \quad (14)$$

The energy on the holographic screen according to the equipartition law of energy can be written as

$$E = \frac{1}{4\pi} \int_\Omega e^\phi \nabla_\phi dA = 2\pi r^2 T, \quad (15)$$

where $A$ is the area of the surface. The energy on the non-commutative SAdS screen is given by

$$E \approx \sigma \left( \frac{MG}{3\sqrt{\pi \theta^3}} - \frac{1}{l^2} \right) r^3. \quad (16)$$

The modified Newtonian force law as the entropic force is found to be

$$F_\alpha = T \nabla_\alpha S, \quad (17)$$

where, the change in entropy for the test mass at a fixed position near the screen can be written as $\nabla_\alpha S = -2\pi mn_\alpha$. Note that, the entropy is defined on the freely falling
holographic screen located outside the horizon and the minus sign for the change in entropy comes from the fact that the entropy increases when we cross from the outside to the inside. In other words, we assume that the entire mass distribution is contained inside the volume enclosed by the holographic screen, and all test particles are located in the emerged space outside the screen. This yields the other reason that compels us to consider the holographic screen located at the distance greater that \( r_0 \).

We then obtain the entropic force in the presence of the non-commutative SAdS black hole at small scale,

\[
F = \sqrt{g^{\alpha\beta} F_\alpha F_\beta} \approx \frac{\sigma m \Lambda_{\text{eff}}}{3} r, \tag{18}
\]

which is not a Newtonian force. The entropic force vanishes at the origin and there exists a linear relation between the force and the distance. The non-Newtonian kind of an entropic force might potentially be of interest for the domain of validity at small length scales.

Note that, if we had chosen a different type of the smeared mass distribution, the overall qualities would be led to completely similar outcomes to those above. Therefore the fundamental characteristics of the non-commutativity method are not specifically sensitive to the Gaussian nature of the smearing. To prove that we choose a Lorentzian distribution of the smeared mass as follows \[51\]

\[
M_{\theta'} = \frac{2M}{\pi} \left[ \tan^{-1} \left( \frac{r}{\sqrt{\theta'}} \right) - \frac{r \sqrt{\theta'}}{r^2 + \theta'} \right]. \tag{19}
\]

The new non-commutativity parameter \( \theta' \), is not accurately similar to \( \theta \). In the commutative limit, i.e. \( \theta' \to 0 \), one obtains \( M_{\theta'} \to M \). In the limit \( r \gtrsim r_0' \) \( (r \sim \sqrt{\theta'}) \), by expanding the following Lorentzian profile of the metric

\[
ds^2 = - \left( 1 - \frac{2GM_{\theta}'}{r} - \frac{\Lambda}{3} r^2 \right) dt^2 + \left( 1 - \frac{2GM_{\theta}'}{r} - \frac{\Lambda}{3} r^2 \right)^{-1} dr^2 + r^2 d\Omega^2, \tag{20}
\]

where \( r_0' \) is a Lorentzian cut-off in the radial direction at small scales, we can immediately write the asymptotic form of \( g_{00} \) as

\[-g_{00} \approx 1 - \frac{\Lambda_{\text{eff}}'}{3} r^2, \tag{21}\]

where \( \Lambda_{\text{eff}}' = \Lambda + 8MG/\pi \sqrt{\theta'^3} \) is the effective cosmological constant for a Lorentzian distribution in short distances. For the entropic force, we have finally

\[
F \approx \frac{\sigma m \Lambda_{\text{eff}}'}{3} r, \tag{22}\]

[72x710]
which is similar to that found in the Gaussian profile. Hence, most of the consequences obtained in the Gaussian profile, at least for asymptotic values of \( r \), remain intact if we pick out other profiles of probability distributions.

### IV. ENTROPIC FORCE AT LARGE LENGTH SCALES

The time-like Killing vector \( \xi_\alpha \) for the non-commutative SAdS metric which is an asymptotically AdS space-time, can be achieved by a normalization constant \( \sigma \) as \( \xi_\alpha = \sigma (\partial_\theta)_\alpha \). In an asymptotically flat space-time, the standard Killing vector normalization, i.e. \( \sigma = 1 \), is retrieved. Similarly, at large length scales or \( r \gg \sqrt{\theta} \), we use the explicit form of the metric (1) and obtain the Unruh-Verlinde temperature for the non-commutative SAdS black hole

\[
T = \frac{\sigma}{2\pi} \left( \frac{GM_\theta}{r^2} - rf(r) \right),
\]

with

\[
f(r) = \frac{GM}{2\sqrt{\pi\theta^3}} e^{-\frac{r^2}{4\theta}} - \frac{1}{l^2}.
\]

Here, it should be noted that the second term of Eq. (24) is extremely small. This means that in the limit \( r \gg \sqrt{\theta} \), the cosmological length scale is extremely large and an asymptotically flat space-time may approximately be retrieved at that regime.

The energy on the non-commutative SAdS screen is immediately written as

\[
E = \sigma \left( GM_\theta - r^3 f(r) \right).
\]

Finally, the entropic force in the presence of the non-commutative SAdS black hole in large distances becomes

\[
F = \sigma \left( \frac{GM_\theta m}{r^2} - mrf(r) \right).
\]

The numerical computation of the entropic force and the energy as a function of the radius for two cases, an AdS background and an asymptotically flat space-time, are depicted in Figs. 3 and 4 respectively. As can be seen from Fig. 3, as the cosmological constant deviates from the zero, the entropic force increases but the peak in the entropic force in the vicinity of the minimal non-zero radius \( r_0 \) remains nearly intact. Similarly, the energy in Fig. 4 increases with deviating the cosmological constant from the zero. As we have already mentioned, the case of \( r < r_0 \) leads to some out of the standard dynamical features like negative entropic force, i.e. gravitational repulsive force, and negative energy; as a result, one should make
FIG. 3: The entropic force $F$ versus the radius, $r/\sqrt{\theta}$. We have set $M = 10.0\sqrt{\theta}/G$. On the right-hand side of the figure, from top to bottom, the solid lines correspond to the non-commutative SAdS black hole for $\Lambda = -10^{-2}/\theta$, and $\Lambda = -3 \times 10^{-3}/\theta$, respectively. The dashed line refers to the non-commutative Schwarzschild black hole so that it corresponds to $\Lambda = 0$.

FIG. 4: The energy, $E/\sqrt{\theta}$, versus the radius, $r/\sqrt{\theta}$. We have set $M = 3.0\sqrt{\theta}/G$. On the right-hand side of the figure, from top to bottom, the solid lines correspond to the non-commutative SAdS black hole for $\Lambda = -10^{-2}/\theta$, and $\Lambda = -3 \times 10^{-3}/\theta$, respectively. The dashed line refers to the non-commutative Schwarzschild black hole so that it corresponds to $\Lambda = 0$. 
the requirement that $E \geq 0$. Accordingly, the appearance of a lower finite cut-off at the short-scale gravity compels a bound on any measurements to determine a particle position in a non-commutative gravity theory.

Notice that one can define the following effective gravitational constant:

$$G_{\text{eff}} = G \left[ \mathcal{E} \left( \frac{r}{2 \sqrt{\theta}} \right) - \frac{r}{\sqrt{\pi \theta}} e^{-\frac{r^2}{4\theta}} \left( 1 + \frac{r^2}{2 \theta} \right) \right], \quad (27)$$

and rewrite the entropic force of the metric (1) in terms of the effective gravitational constant at large length scales as follows

$$F = \sigma \left( \frac{G_{\text{eff}} M m}{r^2} + \frac{m}{\ell^2} r \right). \quad (28)$$

One can easily observe that the NCG inspired model can anticipate an effective gravitational constant as well. The effective gravitational constant includes effects of the non-commutativity of coordinates such that in the limit $r/\sqrt{\theta} \rightarrow \infty$, we have the standard gravitational constant, i.e. $G_{\text{eff}} \rightarrow G$.

Finally, the last point we should denote here is related to a recent paper [75] that analyzed the question of possible quantum corrections in the entropic scenario of emergent gravity. In our present work we presume that it is possible to analyze the effects of the underlying non-commutativity on the entropic gravity via concept of a smooth commutative holographic screen at small scales. However, the authors of Ref. [75] claim that the holographic screen in short distances should also be considered as a non-commutative one. They used a fuzzy sphere as a natural quasiclassical approximation for the spherical holographic screen to analyze whether it is possible to observe such corrections to Newton’s law in principle. The main outcome of their analysis is that it is difficult to draw any conclusive prediction unless there is a complete control over the dynamics of the microscopic degrees of freedom leading to the entropic picture.

V. SUMMARY

In summary, we have studied the thermodynamical aspects of a non-commutative SAdS black hole in the framework of Verlinde’s conjecture. We have obtained the energy and the entropic force at small and large scales. The entropic force is linear in $r$ at small length scales; as a consequence, the Newtonian force is broken down. Our calculations do not show
any severe differences between Gaussian and Lorentzian profiles. At large length scales, we have found some deviations from the standard Newtonian gravity. The NCG inspired model anticipates the presence of an effective gravitational constant in addition to a lower finite cut-off at the short-scale gravity.

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