Landau Gauge Fixing supported by Genetic Algorithm

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A class of algorithms for the Landau gauge fixing is proposed, which makes the steepest ascent (SA) method be more efficient by concepts of genetic algorithm. Main concern is how to incorporate random gauge transformation (RGT) to gain higher achievement of the minimal Landau gauge fixing, and to keep lower time consumption. One of these algorithms uses the block RGT, and another uses RGT controlled by local fitness density, and the last uses RGT determined by Ising Monte Carlo process. We tested these algorithms on SU(2) lattice gauge theory in 4 dimension with small $\beta$, 2.0, 1.75 and 1.5, and report improvements in hit rate and/or in time consumption, compared to other methods.

1. INTRODUCTION

Gauge fixing degeneracies (existence of Gribov’s copies) are generic phenomena\cite{1–3} in non-abelian continuum gauge theories, and thus there exist fundamental problems, e.g., what is the correct gauge fixed measure in the path integral formalism.

Although the foundation of lattice gauge theories does not necessitate gauge fixings, they are, however, often required for field theoretic transcriptions of their nonperturbative dynamics.

The principle of the Landau gauge fixing algorithm is given as an optimization problem of some functions along the gauge orbit of link variables\cite{3}. There are many extrema of the optimization function in general, and all these extrema points on the gauge orbit correspond to the Landau gauge, Gribov copies. The absolute maximum corresponds to the minimal Landau gauge\cite{4}. In order to fix the Landau gauge uniquely, the minimal Landau gauge is the most favourable goal.

Local search algorithms were developed by a chain of gauge transformations\cite{5,6}. Since there are many Gribov copies, the gauge orbit paths are easily captured by these extrema. We call these method as the steepest ascent method (SA). In such a situation that SA fails to attain an absolute extremum, one can try a simple random search\cite{7} on the gauge orbit with succeeding SA.

There is only unique method of Hetrick-de Forcrand\cite{8} (HdeF) aiming at the minimal Landau gauge fixing, which is systematic in the sense that random trial is not involved. However it works successfully only for large $\beta$ samples, rather smooth configurations. Thus finding efficient algorithm for the minimal Landau gauge fixing is still an open problem.

We report results of an attempt in some GA type methods in comparison with other methods as the simple RGT method. We work on SU(2) gauge theory of $8^4$ lattice, and define gauge fields as $A_{x,\mu} = \frac{1}{2i}(U_{x,\mu} - U_{x,\mu}^\dagger)$, then the optimization function, fitness, is given as

$$F_U(G) = \sum_{x,\mu} \frac{1}{2} Tr(U_{x,\mu}^G),$$  

(1)

where $U_{x,\mu}^G = G_x U_{x,\mu} G_{x+\mu}$. The fitness $F_U(G)$ can be viewed as negative of energy of SU(2) spin system $G$ sitting on sites, and thus the problem is equivalent to finding the lowest energy state under the randomized interaction $U$.

Straightforward GA strategy was applied to the minimal Landau gauge fixing\cite{9}. Their preliminary results of the Landau gauge fixing show that the straightforward application of GA is not so good as to become a practical method.

We tested three types of GA methods, which are different from each other in how RGT are incorporated in the algorithms. We compare them
Table 1
Search parameters and strategies on SU(2) with \( \beta = 2.0 \) performed on DEC \( \alpha \) 2100 4/275.

| Method   | Hitrate  | executing time [sec] |
|----------|----------|-----------------------|
| BRGT IS on \( N_{\text{div}} = 2 \) | 46/50 | 10.9 |
| IS off \( N_{\text{div}} = 2 \) | 42/50 | 11.6 |
| IRGT IS off | 32/50 | 11.2 |
| LFRGT \( R1 = 0.5,R2 = 0.85 \) | 47/50 | 11.6 |
| \( R1 = 0.35,R2 = 0.85 \) | 46/50 | 14.1 |
| \( R1 = 0.45,R2 = 0.9 \) | 45/50 | 10.3 |
| \( R1 = 0.5,R2 = 0.85 \) IS on | 44/50 | 12.7 |
| HdeF | 0/50 | 7.6 |
| TRGT | 47/50 | 14.7 |

with the simple RGT method in performance.

2. Algorithms

Our aim is to develop algorithms efficient for small \( \beta \)s. Basic building block for the algorithms is RGT. The simple RGT (TRGT) method which applies RGT on the whole lattice takes time, while it is difficult for SA paths to escape from a Gribov copy if the RGT is restricted in blocks of too small area on the lattice.

Thus main features of our algorithms consist in how to determine blocks to which RGT is applied. Given a Gribov copy link configuration \( U \), the following three types of definition of blocking for RGT are devised.

1. The whole lattice is partitioned into \( N_{\text{div}}^d \) chequered blocks, where the number of dimension, \( d = 4 \). Then RGT is applied on white blocks and a constant RGT on each black one, and vice versa. We call this method as blocked RGT (BRGT) method.

2. We set two parameters \( R1 \) and \( R2 \), and sites \( i \) where RGT is applied are chosen according to local fitness density, \( f(i) \), by \( f(i) < R1 \) or \( R2 < f(i) \). We call this method as local fitness density RGT (LFRGT).

3. Ising spin interaction on randomly chosen coarse lattice is defined from gauge spin interaction given by \( U \) such that at least one anti-ferro interaction should be involved. Through Monte Carlo simulation of this Ising system, one obtains up-spin blocks \( B_+ \) and down-spin blocks \( B_- \). Blocks for RGT is given by one of these blocks. \( \beta_{\text{Ising}} \) is so chosen that size of both blocks \( B_\pm \) becomes comparable. We call this method as Ising RGT (IRGT).

We use the local exact algorithm\[4,5\] for SA method. Given a Gribov copy \( U \), the SA method following RGT in use of one of three blockings, brings \( U \) to the extrema of fitness by steps of gauge transformations. This new copy in the Landau gauge is put as an initial copy for the next iteration. This sub-procedure is repeated \( M_{\text{itr}} \) times. The original Gribov copy and the maximum fitness copy among \( M_{\text{itr}} \) new gauge copies are compared in fitness value. If the fitness of the new one is higher, the sub-procedure is to be started again with this new one as an initial copy. Otherwise the process stops, and the initial copy is considered as the expected configuration with the maximum fitness value.

In addition to the above basic algorithm, two types of modification are devised as follows:

1. As the initial copy in the sub-process, the better fitness copy between the current copy and the preceding, is always chosen. We call this as the inter-selection-on (IS) scheme.

2. As the initial copy in the sub-process, the product of crossing is adopted, where chequered block crossing is done between the best fitness copy and the second best one among the obtained copies so far. We call this as the crossing-on (C) scheme.

We tested these algorithms on \( SU(2) \) \( 8^4 \) lattice with \( \beta = 2.0, 1.75, 1.5 \) and tuned some parameters, as \( N_{\text{div}} \), \( \beta_{\text{Ising}} \) and \( M_{\text{itr}} \), with or without IS and/or C scheme.
3. Results and Performance

Our three GA type methods were executed on the same set of randomly produced 50 copies from a suitably chosen copy from samples, $\beta = 2.0, 1.75, 1.5$. The TRGT method and the method of HdeF were also tested on the same set. Performance of these methods, hit rate of the minimal Landau gauge and average time consumption for the gauge fixing, are compared.

We fix $M_{itr} = 5$, and in tests of parameters search for $\beta = 2.0$, we found that BRGT with $N_{div} = 2$ shows a sufficient global search power, while BRGT with $N_{div} = 4$ shows a lower hit rate. The IS scheme could be viewed as a kind of elitism, and it is known that elitism is a suitable strategy when the global search power is available. BRGT with IS scheme, $N_{div} = 2$, shows a powerful and efficient search, while IRGT with IS, however, does not. For LFRGT, the cut parameters, $R_1$ and $R_2$, affect its search power. Without $R_2$ or with too large $R_2$, even high $R_1$ does not work well, nor low $R_2$ without $R_1$. Since LFRGT with both parameters suitably chosen, achieves the efficient search, IS scheme helps. From Table 1, our algorithms, BRGT with IS, $N_{div} = 2$, and LFRGT with IS, $R_1 = 0.5$ and $R_2 = 0.85$, have high hit rates and exhibit good performance comparable with TRGT. The HdeF method with small $\beta$ is known that it does not tend to the maximum fitness.

The hit rate performance for various methods is given in Figure 1, and the average time consumption is shown in Table 1.

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