Two Flavour QCD Phase Transition

G. Boyd\textsuperscript{a},
with F. Karsch\textsuperscript{b}, E. Laermann\textsuperscript{b} and M. Oevers\textsuperscript{b}

(a) Dipartimento di Fisica dell’Università, I-56126 Pisa, Italy
(b) Fak. für Physik, Uni. Bielefeld, Postfach 100131, D-33501 Bielefeld, Germany

Abstract

Results on the phase transition in QCD with two flavours of light staggered fermions from an ongoing simulation are presented. We find the restoration of the chiral $SU(2) \times SU(2)$ symmetry, but not of the axial $U_A(1)$ symmetry.

1 Deconfinement and Chiral Symmetry

There are two symmetries in QCD, each giving rise to a phase transition. For infinite quark mass, i.e., pure $SU(3)$ gauge theory, there is the deconfinement transition. The order parameter is the Polyakov loop $\langle L \rangle$, zero in the confined $Z(3)$ symmetric phase, non-zero in the deconfined broken symmetry phase. For zero quark mass one has chiral symmetry, the chiral condensate $\langle \bar{\psi}\psi \rangle$ the order parameter. Chiral symmetry is spontaneously broken at $T = 0$, so $\langle \bar{\psi}\psi \rangle$ is non-zero at low temperature, and zero above the chiral transition.

In the real world with small quark masses it is not clear what role each symmetry assumes going from cold to hot QCD. There could be two different transitions, with two different transition temperatures. Confinement and chiral symmetry breaking may be related \cite{1}, with one causing the other. That they need not be coupled has been discussed in \cite{2}. In this project we saw deconfinement and chiral symmetry restoration at the same temperature, with in the susceptibilities corresponding to $\langle \bar{\psi}\psi \rangle$ and $\langle L \rangle$ peaking at the same temperature.

The order of the phase transition will depend on which symmetry dominates at the transition. Since the quark mass is close to zero, most likely the chiral symmetry will dominate. For the case of two light flavours there is no clear prediction of the order of the transition from effective theories \cite{3}. If it turns out to be of second order, then it should belong to the same universality class as the $O(4)$ spin model.

The lattice calculations of the order of the transition presented here extend earlier work \cite{4} (where details of the method may be found) to larger lattices. See also \cite{5}.

The bare quark mass in the QCD Langrangean behaves analogously to a magnetic field in a spin system, imposing an upper limit on correlation lengths. Of course, the physical size of the system must be larger than this limit. The order and universality

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Figure 1: The chiral susceptibility $\chi_{m,\text{peak}}$ at the deconfinement transition vs. $m_q$ for three different lattice volumes, $N^3 \times 4$. The points have been shifted slightly for ease of comparison. Also shown is the slope expected for a second order transition with O(4) exponents, and the slope expected for a first order transition.

class of the phase transition can then be determined by calculating the behaviour of various correlation lengths and corresponding susceptibilities as the quark mass is taken to zero near the deconfinement transition. For a second order chiral phase transition one expects the chiral susceptibility $\chi_m$ to grow without bound when the quark mass is taken to zero at the critical temperature, while for a first order transition it will grow to some maximum value.

$$
\chi_m(m_q) = \frac{T}{V} \frac{\partial}{\partial m_q} \langle \bar{\psi} \psi \rangle = \frac{T}{V} \frac{\partial^2}{\partial m_q^2} \ln(Z)
$$

(1)

$Z$ is the QCD partition function. Moving across the phase transition at a given quark mass $m_q$, the chiral susceptibility rises to a maximum, and then drops. The height of the peak depends on the quark mass via the critical exponent $\delta$

$$
\chi_{m,\text{peak}} = c_m m_q^{1/\delta - 1}.
$$

(2)

In figure 1 the peak height is plotted against quark mass for three different lattice sizes, $8^3$, $12^3$ and $16^3 \times 4$ and for three different quark masses, $m_q = 0.02, 0.0375$ and 0.075. Also plotted are two lines. One represents the slope expected if one has a second order transition with O(4) exponents, $1/\delta = 0.206$ [6], the other if there is a first order transition with $1/\delta = 0$. Both exponents fit the data more or less well, with larger lattices suggesting more that $1/\delta = 0$. However, the scaling arguments behind eq. (2) may well not hold for the heavy mass $m_q = 0.075$. If this mass is ignored then no conclusion can be drawn.
2 Axial Symmetry

There has been speculation about the possible restoration of the $U_A(1)$ axial symmetry at high temperature. Although the anomaly explicitly breaks the symmetry regardless of temperature, it is only present in one loop diagrams, and thus the effect may disappear as the temperature is increased. Examining the finite temperature masses of mesons with a mass splitting due to the axial anomaly, this effective restoration of the axial symmetry may be studied.

The particles to consider are the pion ($\pi$), the scalar isoscalar ($\sigma$) and the neutral component of the scalar isovector ($\delta$), the $\delta^0$. The $\sigma$ and $\delta^0$ differ only in diagrams with disconnected quark lines. In other words, the quark line connected diagrams are identical for both particles, and the disconnected diagrams give the $\sigma$ a new set of poles.

If chiral $SU(N) \times SU(N)$ symmetry is restored, the pion and sigma will be degenerate. If both $SU(N) \times SU(N)$ and $U_A(1)$ is restored, then all three will be degenerate. The following mass splittings

$$\Delta_{\pi\delta} = m_\pi - m_\sigma$$
$$\Delta_{\pi\delta} = m_\delta - m_\pi$$
$$\Delta_{\delta\sigma} = m_\delta - m_\sigma$$

(3)

can be used to determine which symmetry is restored, and at which temperature.

The short temporal direction at finite temperature prevents an easy determination of particle masses from a fit of an exponential to the correlator. However, one can also link the susceptibility, or correlator at zero four-momentum, to the mass:

$$1/m^2 \propto \chi = \int d^4x C(x).$$

(4)

This definition assumes that only one state contributes to the correlator. If the mass gap
is large, and the lowest state has a mass close to zero, then the bias introduced by using the full correlator will not be large.

This definition of the mass has been used in calculating the differences shown in figure 2. The figure shows quite clearly that although $\Delta_{\pi\sigma}$ goes to zero at $T_c$, neither $\Delta_{\pi\delta}$ nor $\Delta_{\delta\sigma}$ do\(^2\). So chiral symmetry has been restored, while $U_A(1)$ has not. Both $\Delta_{\pi\delta}$ and $\Delta_{\delta\sigma}$ do drop slowly to zero as the temperature increases, though, indicating an effective restoration of $U_A(1)$. See also [3].

So one concludes that at $T_c$ chiral $SU(N) \times SU(N)$ symmetry is restored and deconfinement takes place, while $U_A(1)$ symmetry is not restored. At some temperature above $T_c$, $U_A(1)$ becomes effectively restored. Finally, note that the order of the phase transition is still undecided.

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\(^2\)For technical reasons the \(\delta\) only corresponds to the physical \(\delta\) if the lattice is close to the continuum limit.