THE CLUSTER $M_{\text{gas}}-T_X$ RELATION: EVIDENCE FOR A HIGH LEVEL OF PREHEATING

IAN G. MCCARTHY, ARIF BABUL, AND MICHAEL L. BALOGH

Received 2002 January 22; accepted 2002 March 13

ABSTRACT

Recent X-ray observations have been used to demonstrate that the cluster gas mass-temperature relation is steeper than theoretical self-similar predictions drawn from numerical simulations that consider the evolution of the cluster gas through the effects of gravity and shock heating alone. One possible explanation for this is that the gas mass fraction is not constant across clusters of different temperature, as is usually assumed. Observationally, however, there is no compelling evidence for gas mass fraction variation, especially in the case of hot clusters. Seeking an alternative physical explanation for the observed trends, we investigate the role in the cluster gas mass-temperature relation of the preheating of the intracluster medium by some arbitrary source for clusters with emission-weighted mean temperatures of $T_X \gtrsim 5$ keV. Making use of the physically motivated, analytic model developed in 2002 by Babul and coworkers, we find that preheating does, indeed, lead to a steeper relation. This is in agreement with previous theoretical studies on the relation. However, in apparent conflict with these studies, we argue that a "high" level of entropy injection is required to match observations. In particular, an entropy floor of $\gtrsim 300$ keV cm$^2$ is required. We also present a new test, namely, the study of the relation within different fixed radii. This allows one to indirectly probe the density profiles of clusters, since it samples different fractions of the virial radius for clusters of different temperature. This test also confirms that a high level of preheating is required to match observations.

Subject headings: cosmology: theory — galaxies: clusters: general — X-rays: galaxies: clusters

1. INTRODUCTION

Analytic models and numerical simulations of clusters of galaxies have been used to predict the existence of scaling relations between various observable quantities, such as the well-known luminosity ($L_X$)–temperature ($T_X$) and mass ($M$)–temperature relations, where $L_X \propto T_X^2$ and $M \propto T_X^{1.5}$, respectively. However, it is now fairly well established that X-ray properties of clusters do not scale in such a fashion. Most notable of these is the $L_X$–$T_X$ relationship, which is observed to be much steeper than predicted, $L_X \propto T_X^{5.3}$ (e.g., Markevitch 1998; Allen & Fabian 1998; Arnaud & Evrard 1999). Considerable effort has recently been directed toward explaining why the observed relations deviate from their predicted scalings (e.g., Tozzi & Norman 2001; Davé et al. 2001; Babul et al. 2002, hereafter BBLP02). In particular, it is the $L_X$–$T_X$ relation that has grabbed most of the spotlight because there is a wealth of published observational studies on the luminosities and temperatures of clusters with which to compare models and simulations. However, another important scaling relation is the cluster gas mass ($M_{\text{gas}}$)–$T_X$ relation. Neumann & Arnaud (2001) have suggested that a deviation from the self-similar scaling of $M_{\text{gas}} \propto T_X^{1.5}$ might "explain" the observed deviation in the $L_X$–$T_X$ relation. Indeed, a number of observational studies have indicated that the relation is much steeper, with $M_{\text{gas}} \propto T_X^{6.2}$ (Vikhlinin, Forman, & Jones 1999; Mohr et al. 1999, hereafter MME99; Neumann & Arnaud 2001). If the gas density profile is roughly self-similar, this does lead to consistency with the observed $L_X$–$T_X$ relation. However, we still need a physical explanation for why the relationship between a cluster's gas mass and its temperature deviates from its self-similar scaling.

Expressing the total gas mass within the cluster as $M_{\text{gas}} = f_{\text{gas}} M$, a steepening of the $M_{\text{gas}}$–$T_X$ relation can be interpreted as a dependence of $f_{\text{gas}}$ on cluster mass. That is, if $M \propto T_X^{5}$, as suggested by the self-similar model, then the observed $M_{\text{gas}}$–$T_X$ relation implies that $f_{\text{gas}} \propto T_X^{0.5}$. A varying gas mass fraction is expected if the efficiency of galaxy formation varies systematically across clusters of different mass. Observational support for this has been claimed recently by Bryan (2000). However, this is still controversial, and there is no compelling evidence for a variation of $f_{\text{gas}}$ with cluster temperature (but see Arnaud & Evrard 1999; MME99). This is especially true for the systems that we are specifically interested in: hot clusters with $T_X \gtrsim 3$ keV. This is apparent, for example, in Figure 1 (top) of Balogh et al. (2001), who carry out an accounting of stars and gas to estimate the fraction of cooling baryons in clusters. Moreover, Roussel, Sadat, & Blanchard (2000) have carried out a careful analysis of group and cluster X-ray data to estimate $f_{\text{gas}}$ directly and have found no trends. More recently, Grego et al. (2001) have analyzed Sunyaev-Zel'dovich effect observations of 18 hot clusters and have also found no correlations between a hot cluster's gas mass fraction and its temperature. Finally, observational studies of the total cluster mass ($M$)–temperature relation have indicated that $M \propto T_X^{6.2}$ (Horner, Mushotzky, & Scharf 1999; Ettori & Fabian 1999; Nevalainen, Markevitch, & Forman 2000; Finoguenov, Reiprich, & Böhringer 2001), which, given the observed $M_{\text{gas}}$–$T_X$ relation, is consistent with $f_{\text{gas}}$ being constant.

Theoretically, it is only now becoming possible to reliably investigate the dependence of $f_{\text{gas}}$ on temperature with the inclusion of radiative cooling, star formation, feedback, and other relevant processes in numerical simulations (e.g.,

---

1 Department of Physics and Astronomy, P.O. Box 3055, University of Victoria, Victoria, BC V8P 1A1, Canada; mccarthy@beluga.phys.uvic.ca.
2 Department of Physics, University of Durham, South Road, Durham, DH1 3LE, UK.
Lewis et al. 2000; Pearce et al. 2000; Muanwong et al. 2001; Davé et al. 2001). As of yet, however, there is little agreement in the approaches adopted to model these processes and prevent the so-called cooling crisis (compare, for example, the findings of Lewis et al. 2000 with those of Pearce et al. 2000). This is not surprising. As discussed in detail by Balogh et al. (2001), attempting to model the effects of cooling across the wide range of halo masses found in clusters is inherently very difficult. The addition of "subgrid" processes, such as star formation and feedback, further complicates matters. Thus, the effects that these additional physical processes have on the gas mass fraction of clusters will not be fully realized until such issues are resolved.

In this paper, however, we show that the observed variation of the $M_{\text{gas}}-T_{\text{X}}$ relation(s) arises quite naturally within the class of models that invoke preheating of the intracluster medium (ICM) during the early stages of cluster formation. In these models, $f_{\text{gas}}$ is constant on cluster scales ($T_{\text{X}} \gtrsim 3$ keV), and the self-similarity is instead broken by an entropy floor generated by early nongravitational heating events. Preheating has previously been shown to bring consistency between a number of other observed and predicted scaling relations for groups and clusters (e.g., BBLP02), and therefore one might expect that the $M_{\text{gas}}-T_{\text{X}}$ relation should also be modified.

The preheating model was originally put forward by Kaiser (1991) and has subsequently been investigated by a number of authors (e.g., Evrard & Henry 1991; Bower 1997; Cavaliere, Menci, & Tozzi 1997, 1998, 1999; Balogh, Babul, & Patton 1999; Wu, Fabian, & Nulsen 2000; Loewenstein 2000; Tozzi & Norman 2001; Borgani et al. 2001; Thomas et al. 2002; BBLP02). If the ICM is injected with enough thermal energy, the hot X-ray-emitting gas will become decoupled from the dark halo potential and break the self-similar scaling relations. The best estimates suggest that a substantial amount of energy ($\sim 1$ keV per particle) is required to reproduce the observed relations (mainly the $L_{\text{X}}-T_{\text{X}}$ relation). It is not yet known what source(s) could inject such a large amount of energy into the ICM. Both galactic winds (driven by supernovae) and ejecta from active galactic nuclei have been proposed, but because of the complexity of the physics, the exact details have yet to be worked out. For an in-depth discussion of potential sources of preheating and of alternative possibilities for reproducing the observed relations, we refer the reader to BBLP02.

In this paper, we adopt the physically motivated, analytic model developed in BBLP02 to explore the impact of cluster preheating on the $M_{\text{gas}}-T_{\text{X}}$ relation. In comparison with the $L_{\text{X}}-T_{\text{X}}$ and $M-T_{\text{X}}$ relations, it has drawn very little attention from theoretical studies. The only studies to have examined the effects of entropy injection on the $M_{\text{gas}}-T_{\text{X}}$ relation to date are Loewenstein (2000) and Bialek, Evrard, & Mohr (2001). To be specific, Loewenstein (2000) considered models in which the entropy injection occurs at the centers of groups and clusters after the latter have formed, whereas Bialek et al. (2001), like BBLP02, investigated preheated models. For reasons that are described below (in \S 5), we believe our work greatly improves upon both of these studies. The prevailing apathy by theorists is perhaps due in part to a near absence of published observational studies on the gas masses of clusters. However, in light of the recent important observations discussed above (e.g., Vikhlinin et al. 1999; MME99; Neumann & Arnaud 2001) and the new influx of high-resolution data from the Chandra and XMM-Newton X-ray satellites, which will likely provide even tighter constraints, we believe that a thorough examination of the $M_{\text{gas}}-T_{\text{X}}$ relation is timely.

The models we consider below were developed in a flat $\Lambda$CDM cosmology with $\Omega_m = 0.3$, $h = 0.75$, and a nucleosynthesis value $\Omega_b = 0.019$ $h^{-2}$ (Burles & Tytler 1998). They are computed for a number of different preheating levels, corresponding to entropy constants of $K_0 = 100, 200, 300$, and $427$ keV cm$^{-2}$. These span the range required to match the observed $L_{\text{X}}-T_{\text{X}}$ relations of groups and hot clusters (e.g., Ponman, Cannon, & Navarro 1999; Lloyd-Davies, Ponman, & Cannon 2000; Tozzi & Norman 2001; BBLP02). For the purposes of comparison, we also implement an "isothermal" model (see \S 2.3 in BBLP02), which mimics the self-similar result deduced from numerical simulations (e.g., Evrard, Metzler, & Navarro 1996).

2. CLUSTER MODELS

Since an in-depth discussion of the preheated cluster models can be found in BBLP02, we present only a brief description of the models here.

The preheated models can be summarized as follows: the dominant dark matter component, which is unaffected by the energy injection, collapses and virializes to form bound halos. The distribution of the dark matter in such halos is assumed to be the same as found in recent ultrahigh resolution numerical simulations (Moore et al. 1998; Klypin et al. 1999; Lewis et al. 2000) and is described by

$$\rho_{\text{dm}}(r) = \rho_{\text{dm},0} \left( \frac{r}{r_{c}} \right)^{-n} \left( 1 + \frac{r}{r_{s}} \right)^{-n-3},$$

where $n = 1.4$, $\rho_{\text{dm},0}$ is the profile normalization, and $r_{s}$ is the scale radius. While the dark component is unaffected by energy injection, the collapse of the baryonic component is hindered by the pressure forces induced by preheating. If the maximum infall velocity due purely to gravity of the dark halo is subsonic, the flow will be strongly affected by the pressure, and it will not undergo accretion shocks. It is assumed that the baryons will accumulate onto the halos isentropically at the adiabatic Bondi accretion rate (as described in Balogh et al. 1999). This treatment, however, is only appropriate for low-mass halos. If the gravity of the dark halos is strong enough (as it is expected to be in the hot clusters being considered here) that the maximum infall velocity is transonic or supersonic, the gas will experience an additional (generally dominant) entropy increase due to accretion shocks. In order to trace the shock history of the gas, a detailed knowledge of the merger history of the cluster/group is required but is not considered by BBLP02. Instead, it is assumed that at some earlier time, the most massive cluster progenitor will have had a mass low enough that shocks were negligible in its formation, similar to the low-mass halos discussed above. This progenitor forms an isentropic core of radius $r_{c}$ at the cluster center. The entropy of gas outside of the core, however, will be affected by shocks. Recent high-resolution numerical simulations suggest that the entropy profile for gas outside this core can be adequately represented by a simple analytic expression given by $\ln K(r) = \ln K_0 + \alpha \ln (r/r_{c})$ (Lewis et al. 2000), where $\alpha \sim 1.1$ for the massive, hot clusters ($T_{\text{X}} \gtrsim 3$ keV) of interest here (Tozzi & Norman 2001; BBLP02). It should be noted that in the case of these massive systems, the accretion...
of gas is limited by gravitational infall, and hence they accrete their full complement of baryons [i.e., \( M_{\text{gas}} = (\Omega_b/\Omega_m)M \)]. It is assumed that the mass of baryons locked up in stars is negligible (as suggested by, for example, Roussel et al. 2000; Balogh et al. 2001).

Following this prescription and specifying the parameters \( r_c \), \( \rho_{\text{gas}}(r_c) \), and \( \alpha \) (as discussed in BBLP02) completely determines the models. Under all conditions, the gas is assumed to be in pressure-supported hydrostatic equilibrium within the dark halo potential. The effects of radiative cooling are neglected by these models.

3. HOW PREHEATING AFFECTS THE \( M_{\text{gas}}-T_X \) RELATION

Preheating affects the \( M_{\text{gas}}-T_X \) relation in two ways: (1) by altering the temperature profile and increasing the emission-weighted gas temperature of a cluster and (2) by altering the gas density profile and reducing the gas mass in the cluster core. We are interested in the strength of these effects and whether or not they can be distinguished by current or future observational data. First, we consider the effect of preheating on the temperature of a cluster.

Figure 1 is a plot of \( T_X \) as a function of entropy floor level (i.e., \( K_0 \)) for three clusters of different total masses. The thin line represents a cluster with \( M \approx 5.6 \times 10^{14} M_\odot \), the next thickest line represents a cluster with \( M \approx 10^{15} M_\odot \), and the thickest line represents a cluster with \( M \approx 1.8 \times 10^{15} M_\odot \). As expected, the gas temperature of a cluster increases as the level of preheating is increased. On average, an increase in \( T_X \) of about 1 keV (10%–25%) occurs when a cluster is preheated to the level of \( K_0 \approx 300 \text{ keV cm}^2 \) (over the range 3 keV \( \leq T_X \leq 10 \text{ keV} \)). This effect will primarily manifest itself as a normalization shift in the \( M_{\text{gas}}-T_X \) relation.

Figure 2 presents the dimensionless gas density profile of a cluster with \( T_X = 4 \text{ keV} \) (left panel) and a cluster with \( T_X = 8 \text{ keV} \) (right panel) as a function of the level of preheating. The dot–short-dashed line is the self-similar result (i.e., isothermal model of BBLP02). The long-dashed, short-dashed, dotted, and solid lines represent the preheated models of BBLP02 with \( K_0 = 100, 200, 300 \), and 427 keV cm\(^2\), respectively. Preheating reduces the gas density, and therefore the gas mass, in the central regions of a cluster. In \( \frac{3}{8} \) we investigate the \( M_{\text{gas}}-T_X \) relation within three different radii: \( r = 0.25 \) and 0.50 \( h^{-1} \text{ Mpc} \) and \( r_{500} \) (the radius within which the mean dark matter mass density is 500 times the mean critical density \( \rho_{\text{crit}} \) at \( z = 0 \)). These radii are indicated in Figure 2 by the open squares, pentagons, and triangles, respectively. Clearly, the effect on the \( M_{\text{gas}}-T_X \) relation will be strongest when \( M_{\text{gas}} \) is evaluated within \( r = 0.25 \ h^{-1} \text{ Mpc} \). Furthermore, because \( r = 0.25 \ h^{-1} \text{ Mpc} \) is a fixed radius that samples different fractions of the virial radius (\( R_{\text{halo}} \)) for clusters of different temperature, the (fractional) reduction in gas mass within that radius will be largest for the lowest temperature systems. This will lead to both a normalization shift and a steepening of the \( M_{\text{gas}}-T_X \) relation. To illustrate how strong the effect is, we examine the reduction of gas mass within \( r = 0.25 \) and 0.50 \( h^{-1} \text{ Mpc} \) and \( r_{500} \) for a low-mass cluster with a total mass of \( M \approx 5.6 \times 10^{14} M_\odot \) and for a high-mass cluster with \( M \approx 1.8 \times 10^{15} M_\odot \). We find that when the low-mass cluster has undergone preheating at the level of \( K_0 = 100 \text{ keV cm}^2 \), it has \( \approx 32\% \) more gas in mass within \( r = 0.25 \ h^{-1} \text{ Mpc} \) than when the same cluster has undergone preheating at the level of \( K_0 = 427 \text{ keV cm}^2 \). Using the same test on the \( M \approx 1.8 \times 10^{15} M_\odot \) cluster, however, yields a difference of only 22%. When we

![Figure 1](image1.png)

**Fig. 1.—** Effect of preheating on a cluster’s temperature. The thin line, the next thickest line, and the thickest line represent clusters with \( M \approx 5.6 \times 10^{14}, 10^{15}, \) and \( 1.8 \times 10^{15} M_\odot \), respectively. The squares indicate the discrete points at which the model was actually evaluated.

![Figure 2](image2.png)

**Fig. 2.—** Effect of preheating on a cluster’s gas density profile. Left: Cluster with \( T_X = 4 \text{ keV} \). Right: Cluster with \( T_X = 8 \text{ keV} \). The dot–short-dashed line is the self-similar result. The long-dashed, short-dashed, dotted, and solid lines represent the preheated models of BBLP02 with \( K_0 = 100, 200, 300 \), and 427 keV cm\(^2\), respectively. The squares, pentagons, and triangles indicate the radii \( r = 0.25 \) and 0.50 \( h^{-1} \text{ Mpc} \) and \( r_{500} \), respectively, for each of the models.
probe the larger radius $r = 0.50 \, h^{-1} \, \text{Mpc}$, we find the effect is less pronounced (as expected). The difference in $M_{\text{gas}}$ between the $K_0 = 100 \, \text{keV cm}^2$ model and $K_0 = 427 \, \text{keV cm}^2$ model is 8% for the low-mass cluster and 7% for the high-mass cluster. Finally, when the gas mass is evaluated within $r_{500}$, the difference is 4% for the low-mass cluster as opposed to 2% for the high-mass cluster.

In summary, preheating will significantly affect the $M_{\text{gas}}-T_X$ relation by increasing the emission-weighted gas temperature of clusters. Whether or not the relation is also affected by the reduction of gas mass in the cores of clusters depends on within which radius $M_{\text{gas}}$ is evaluated and what temperature regime is being probed. The effect will be strongest for low-temperature systems and when $M_{\text{gas}}$ is probed within small radii (e.g., $r = 0.25 \, h^{-1} \, \text{Mpc}$). An evaluation of the $M_{\text{gas}}-T_X$ relation within large radii, such as $r_{500}$, however, probes the integrated properties of a cluster and will be sensitive only to the temperature shift.

In the next section, we compare the results of the BBLP02 preheated models with genuine observational data. As we show below, only models with $K_0 \gtrsim 300 \, \text{keV cm}^2$ are consistent with the data.

4. RESULTS

In Figure 3 we present the $M_{\text{gas}}-T_X$ relation as predicted by the BBLP02 preheated models within $r_{500}$. The radius $r_{500}$ is typically comparable in size to the observed radius of a cluster and represents the boundary between the inner, virialized region and the recently accreted, still settling outer region of a cluster (Evrard et al. 1996). Thus, as already mentioned, the $M_{\text{gas}}(r_{500})-T_X$ relation can be regarded as a probe of the integrated properties of a cluster and can be directly compared with the self-similar result of $M_{\text{gas}} \propto T_X^{5/3}$.

In Figures 4 and 5 we present the $M_{\text{gas}}(r_{500})-T_X$ relation as predicted by the BBLP02 preheated models within the fixed radii $r = 0.25$ and $0.50 \, h^{-1} \, \text{Mpc}$, respectively. As mentioned above, the determination of the $M_{\text{gas}}-T_X$ relation within some fixed radius, such as $r = 0.25$ or $0.50 \, h^{-1} \, \text{Mpc}$, can be used as an indirect probe of the gas density profiles of clusters because it samples different fractions of the virial radius for clusters of different temperature. For the purposes of clarity, we discuss the $M_{\text{gas}}-T_X$ relation at these three radii separately.

4.1. Test 1: $M_{\text{gas}}(r_{500})-T_X$

The solid squares in Figure 3 represent the gas mass determinations of MME99 within $r_{500}$ using surface brightness profile fitting (with isothermal $\beta$ models) of ROSAT Position Sensitive Proportional Counter data and mean emission-weighted temperatures from the literature, for clusters with $T_X \gtrsim 3 \, \text{keV}$ and whose error bars are 1 keV or smaller. We compare this with the self-similar result represented by the “isothermal” model of BBLP02 (dot–short-dashed line). Finally, the long-dashed, short-dashed, dotted, and solid lines represent the preheated models of BBLP02 with $K_0 = 100$, 200, 300, and $427 \, \text{keV cm}^2$, respectively. The thick dot–long-dashed line represents the predictions of the best-fit heated model of Loewenstein (2000). This model is discussed further in § 5.1.

It is readily apparent that only the preheated models of BBLP02 with $K_0 \gtrsim 200 \, \text{keV cm}^2$ have a reasonable chance of being consistent with the data of MME99. The normalization clearly indicates that the observed gas tem-
Fig. 5.—Comparison of $M_{\text{gas}}(r = 0.50 \, h^{-1} \, \text{Mpc})-T_X$ relations. The solid pentagons represent the gas mass determinations of White et al. (1997) within $r = 0.50 \, h^{-1} \, \text{Mpc}$. The dot–short-dashed line is the semi-similar result (i.e., isothermal model of BBLP02). The long-dashed, short-dashed, and solid lines represent the preheated models of BBLP02 with $K_0 = 100$, 200, 300, and 427 keV cm$^2$, respectively.

The temperature of clusters with a given gas mass is hotter than predicted by models with entropy floors of $K_0 \lesssim 100$ keV cm$^2$. We note that this discrepancy can be remedied by assuming a smaller value of $\Omega_b/\Omega_m$. However, a similar offset, in the same sense, is seen in the correlation with total dark matter mass and gas temperature (Horner et al. 1999; Nevalainen et al. 2000; Finoguenov et al. 2001). This will not be reconciled by lowering $\Omega_b/\Omega_m$. The reason why the preheated models with $K_0 \gtrsim 200$ keV cm$^2$ are better able to match the normalization of the observational data than models with $K_0 \lesssim 100$ keV cm$^2$ is, as mentioned above, because an increase in the amount of preheating directly leads to an increase in the emission-weighted gas temperature.

We have attempted to quantify how well (or poorly) the preheated and self-similar models match the observational data. We have fit both the theoretical results and the observational data with simple linear models of the form $\log M_{\text{gas}} = m \log T_X + b$ over the range $3 \, \text{keV} \lesssim T_X \lesssim 10$ keV. For the theoretical results, we calculate the best-fit slope and intercept using the ordinary least squares test. We stress that the results of these fits, which are presented in Table 1, are only valid for clusters with $T_X \gtrsim 3$ keV. At lower temperatures, the role of preheating becomes much more important [as $M_{\text{gas}}$ becomes less than $(\Omega_b/\Omega_m)M$], and as a result, the relations steepen dramatically. For example, the preheated model with $K_0 = 427$ keV cm$^2$ is well approximated by a power law with $M_{\text{gas}} \propto T_X^{1.58}$ over the range $3 \, \text{keV} \lesssim T_X \lesssim 10$ keV but is significantly steeper over the range $1 \, \text{keV} \lesssim T_X \lesssim 3$ keV with $M_{\text{gas}} \propto T_X^{2.94}$. Thus, it is absolutely essential that comparisons between theoretical models and observations are done over the same range in temperatures.

To fit the observational data of MME99, we have used a linear-regression technique that takes into account measurement errors in both coordinates as well as intrinsic scatter (the BCES test of Akritas & Bershady 1996). As a consistency check, we have also employed 10,000 Monte Carlo bootstrap simulations. No significant deviations between the two tests were found. The results of the linear-regression fits to the observational data are also presented in Table 1.

For all 38 clusters taken from MME99, we derive a best fit that is inconsistent with the results of all the theoretical models considered at greater than the 90% confidence level. However, as is apparent from Figure 3, the slope and intercept of the best-fit line are sure to be heavily dependent on the two low-temperature clusters with the lowest measured gas masses (and gas mass fractions): the Hya I cluster (Abell 1060) and the Cen cluster (Abell 3526). A number of other studies (both optical and X-ray) have also identified very unusual properties in both clusters. For example, Fitchett & Merritt (1988) were unable to fit a spherical equilibrium model to the kinematics of galaxies in the core of Hya I. They suggest that substructure is present and is likely why Hya I does not lie along the $L_X-\sigma$ relation for galaxy clusters. More recently, Furusho et al. (2001) have found that the metal-abundance distribution implies that the gas in Hya I is well mixed (i.e., it does not contain an obvious metallicity gradient), suggesting that a major merger event may have occurred sometime after the enrichment of the ICM. Measurements of the bulk motions of the intracluster gas in the Cen cluster (through Doppler shifting of X-ray spectral lines) reveal strange gas velocity gradients indicative of a large merger event in the not too distant past (Dupke & Bregman 2001). This picture has also been supported by Furusho et al. (2001), who found large temperature variations across the cluster’s surface. Thus, neither Hya I nor Cen can be regarded as typical “relaxed” clusters and are probably not representative of the majority of low-temperature systems.

One way to ameliorate the impact of the two clusters would be to increase the number of systems of this temperature. However, there are very few published gas mass

---

Our best fit differs slightly from MME99’s best fit to their own data because we implemented a different selection criteria. Namely, we have used only clusters with $T_X \geq 3$ keV and whose error bars are 1 keV or smaller.
estimates of cool clusters and groups within \(r_{500}\). X-ray emission from groups is usually only detected out to a small fraction of this radius. The one study that does present group gas masses for a radius at fixed overdensity, Roussel et al. (2000), does so for \(r_{200}\) and is not directly comparable to the results presented in Figure 3. Also, in that study, gas masses were determined by extrapolating the surface brightnesses far outside the limiting radius for which X-ray emission was actually detected. This can lead to biases in determining group/cluster properties (see Mulchaey 2000; Balogh et al. 2001).

In recognition of the above, we have tried removing Hya I and Cen from the sample and fitting the remaining 36 clusters using the same procedure. We find that the preheated model with \(K_0 = 427\) keV cm\(^{-2}\) is then consistent with the data at the 90% level. The \(K_0 = 200\) and 300 keV cm\(^{-2}\) models are marginally inconsistent with the MME99 data. The isothermal model is ruled out at \(\gtrsim 99\%\) confidence irrespective of whether these clusters are dropped or not.

The other two observational studies that have investigated the \(M_{\text{gas}}-T_X\) relation, Neumann & Arnaud (2001) and Vikhlinin et al. (1999), unfortunately did not present gas mass determinations within \(r_{500}\) for individual clusters in a table or graphically. They did, however, present their best-fit values for the slope of the relation. These were deduced from samples of clusters that have temperatures spanning roughly the same range as that considered in Figure 3. The best-fit slopes of the preheated models are shallower than the best fit claimed by Neumann & Arnaud (2001) of \(\log M_{\text{gas}} \propto T_X^{-0.94}\) for a sample of 15 hot clusters. However, an estimate of the uncertainty on this result was not reported; thus, we are unable to say whether this result is inconsistent with the predictions of the preheated models. The predicted slopes of all four preheated models studied here are in excellent agreement with the findings of Vikhlinin et al. (1999), who report \(M_{\text{gas}} \propto T_X^{0.71 \pm 0.13}\) for their sample of 39 clusters. We also note that the results of Neumann & Arnaud (2001) and Vikhlinin et al. (1999) differ significantly from the predictions of the self-similar model.

In summary, we find that the class of models that invoke preheating are much better able to match the observed \(M_{\text{gas}}(r_{500})-T_X\) of hot clusters than that of the isothermal self-similar model, which is ruled out with a high level of confidence. A careful analysis of the MME99 data also suggests that only those models that invoke a “high” level of energy injection (i.e., \(K_0 > 300\) keV cm\(^2\)) are able to match observations.

**4.2. Test 2: \(M_{\text{gas}}(r = 0.25\ h^{-1}\ \text{Mpc})-T_X\)**

The solid triangles and pentagons in Figure 4 represent the gas mass determinations within \(r = 0.25\ h^{-1}\ \text{Mpc}\) of Peres et al. (1998) and White, Jones, & Forman (1997), respectively. These data were obtained using surface brightness profile fitting of ROSAT data (Peres et al. 1998) and Einstein data (White et al. 1997) and emission-weighted temperatures from the literature, for clusters with \(T_X \geq 3\) keV and whose error bars are 1 keV or smaller. The predictions of the isothermal self-similar model are represented by the dot-short-dashed line. Once again, the long-dashed, short-dashed, dotted, and solid lines represent the preheated models of BBLP02 with \(K_0 = 100, 200, 300,\) and 427 keV cm\(^2\), respectively.

**Table 2**

| Model                  | Entropy Floor (keV cm\(^2\)) | \(m^b\) | \(b^b\) |
|------------------------|-----------------------------|---------|---------|
| Isothermal model....... | 0                           | 0.84    | 12.61   |
| Preheated models....... | 100                         | 0.91    | 12.43   |
|                        | 200                         | 0.95    | 12.34   |
|                        | 300                         | 1.06    | 12.21   |
|                        | 427                         | 1.19    | 12.03   |
| Peres et al. and White et al. data ... | ... | 1.11 ± 0.22 | 12.06 ± 0.17 |

* Uncertainties correspond to the 90% confidence level.

In spite of the scatter, it is apparent that only those preheated models with entropy floors of \(K_0 \gtrsim 300\) keV cm\(^2\) are consistent with the 57 clusters plotted in Figure 4. As with the \(M_{\text{gas}}(r_{500})-T_X\) relation, the normalization of the self-similar model and the preheated model with \(K_0 = 100\) keV cm\(^2\) suggests that the ICM is observed to be much hotter than predicted by either of these models. Fitting both the theoretical predictions and observational data in a manner identical to that presented in the previous subsection, we find that only the preheated models with \(K_0 \gtrsim 300\) keV cm\(^2\) have both slopes and intercepts that are consistent with the observational data (see Table 2). On the basis of normalization (intercept), the self-similar model is ruled out with greater than 99% confidence.

In § 3 we briefly discussed the potential of the gas density profile to affect the \(M_{\text{gas}}(r = 0.25\ h^{-1}\ \text{Mpc})-T_X\) relation. This effect is obvious in Figure 4, with mild breaks at \(T_X \approx 10\) keV for the \(K_0 = 427\) keV cm\(^2\) model and at \(T_X \approx 5\) keV for the \(K_0 = 300\) keV cm\(^2\) model. However, with the large scatter obscuring any potential breaks in the \(M_{\text{gas}}-T_X\) relation, all we can conclude is that the data are consistent with predicted profiles of the BBLP02 preheated models with \(K_0 \gtrsim 300\) keV cm\(^2\).

The exact nature of the scatter in Figure 4 is unclear. While some of the scatter is likely attributable to the large uncertainties in the temperature measurements made using Einstein, Ginga, and EXOSAT data, some of it may also be due to unresolved substructure (e.g., cooling flows) and point sources. Such issues become particularly important when investigating the central regions of clusters as opposed to their integrated properties. Indeed, new high-resolution data obtained by Chandra support this idea (see, e.g., Stanford et al. 2001). We anticipate that future data obtained by both Chandra and XMM-Newton will place much tighter constraints on the \(M_{\text{gas}}(r = 0.25\ h^{-1}\ \text{Mpc})-T_X\) relation and possibly even allow one to probe the mild break in the relationship predicted by the preheated models.

**4.3. Test 3: \(M_{\text{gas}}(r = 0.50\ h^{-1}\ \text{Mpc})-T_X\)**

The solid pentagons in Figure 5 represent the gas mass determinations of White et al. (1997) within \(r = 0.50\ h^{-1}\ \text{Mpc}\) using surface profile fitting of Einstein data and emission-weighted gas temperatures from the literature, for clusters with \(T_X \geq 3\) keV and whose error bars are 1 keV or smaller. Again, the predictions of the isothermal self-similar...
5. COMPARISON WITH PREVIOUS THEORETICAL STUDIES

Only two other theoretical studies have examined the effects on the \( M_{\text{gas}}-T_X \) relation of entropy injection into the ICM: Loewenstein (2000) and Bialek et al. (2001). Both studies investigated the \( M_{\text{gas}}(r_{500})-T_X \) relation and demonstrated that entropy injection does, indeed, steepen the relation, in agreement with the present work (however, neither implemented the \( M_{\text{gas}}-T_X \) relation at fixed radii test). These studies suggest that models that produce an entropy floor with a level that is consistent with measurements of groups (\( K_0 \approx 100 \text{ keV cm}^2 \); Ponman et al. 1999; Lloyd-Davies et al. 2000) are capable of matching the observations of even hot clusters (up to 10 keV). This is in apparent conflict with the results presented in \( \frac{3}{4} \) that suggest that a high entropy floor of \( K_0 \gtrsim 300 \text{ keV cm}^2 \) is required to match the observations of hot clusters. A low value of the entropy floor is also in apparent conflict with a number of other studies that have focused mainly on the \( L_X-T_X \) relation of hot clusters. For example, da Silva et al. (2001), Tozzi & Norman (2001), and BBLP02 have all concluded that such low levels of entropy injection do not bring consistency between observations and theoretical models of hot clusters. As such, a closer analysis of Loewenstein (2000) and Bialek et al. (2001) studies is warranted.

5.1. The Loewenstein (2000) Models

To model the observed deviations of the cluster X-ray scaling relations, Loewenstein (2000) has constructed a suite of hydrostatic polytropic models (which are normalized to observations of high-temperature clusters and numerical simulations) and then modified them by adding various amounts of heat per particle at the cluster center. Strictly speaking, the Loewenstein (2000) models cannot be characterized as preheated models, since the injection of entropy into the ICM occurs after the cluster has formed. Thus, a straightforward comparison between the Loewenstein (2000) and BBLP02 models is not trivial. However, success in matching the \( M_{\text{gas}}(r_{500})-T_X \) relation (the data of MME99) is claimed by Loewenstein (2000) for a model that “produces an entropy-temperature relation with the observed entropy floor at \( \approx 100 \text{ keV cm}^2 \).” Regardless of how the entropy floor actually arose, this contradicts the results presented in \( \frac{4}{4} \), which suggest that an entropy floor of \( \gtrsim 300 \text{ keV cm}^2 \) is required to match the observations. Can the analysis of Loewenstein (2000) and that of the present work be reconciled?

A closer investigation of Figure 4 of Loewenstein (2000) reveals that first of all, his heated models were not compared to the actual data, but rather to points that represent MME99’s best-fit power-law match to their data. Second, this power-law relationship was assumed to hold true, and hence was extrapolated to span a wider range in temperatures than considered by MME99. Of the 45 clusters studied by MME999, only one had a temperature below 3 keV (it was 2.41 keV), yet Loewenstein (2000) compared his heated models to the best-fit relation of MME99 over the range \( 1 \text{ keV} < T_X < 10 \text{ keV} \). As previously mentioned, however, entropy injection preferentially affects low-temperature systems, and therefore, extrapolating scaling relations derived from high-temperature systems down to the low-temperature regime is not safe.

In Figure 3 we compare the best-fit heated model of Loewenstein (2000) (his \( \epsilon = 0.35 \) model; thick dot–long-dashed line) with the predictions of BBLP02 models and the data of MME99. The plot clearly demonstrates that his best-fit model does not match the data of MME99 nearly as well as the BBLP02 preheated models with \( K_0 \gtrsim 300 \text{ keV cm}^2 \), especially at the high-temperature end. The difference in temperature ranges examined by Loewenstein (2000) and the present study (whose range of temperatures was intentionally chosen to match the observational data) has likely led to an underestimation of the entropy floor in these clusters by Loewenstein (2000). We once again reiterate that it is extremely important that comparisons between theoretical models and observations are done over the same range in temperatures.

5.2. The Bialek et al. (2001) Simulations

In similarity to the present work, Bialek et al. (2001) investigated the impact of preheating on the \( M_{\text{gas}}-T_X \) relation for a number of different levels of entropy injection, spanning the range \( 0 \text{ keV cm}^2 \leq K_0 \leq 335 \text{ keV cm}^2 \). Fitting their \( M_{\text{gas}}(r_{500})-T_X \) simulation data over the range \( 2 \text{ keV} \leq T_X \leq 9 \text{ keV} \), which is similar (but not identical) to the MME99 sample, they claim success in matching the observations of MME99 for models with entropy injection at the level of \( 55 \text{ keV cm}^2 \leq K_0 \leq 140 \text{ keV cm}^2 \), at least on the basis of slope. Their models with higher levels of entropy

| Model                  | Entropy Floor (keV cm²) | \( m^a \) | \( b^a \) |
|------------------------|-------------------------|---------|---------|
| Isothermal model......  | 0                       | 0.97    | 12.87   |
| Preheated models......  | 100                     | 0.98    | 12.78   |
|                        | 200                     | 0.98    | 12.75   |
|                        | 300                     | 0.99    | 12.72   |
|                        | 427                     | 1.01    | 12.68   |
| White et al. data...... | ...                     | 1.12 ± 0.25 | 12.53 ± 0.20 |

Note.—We have fitted models of the form \( \log M_{\text{gas}} = m \log T_X + b \) over the range \( 3 \text{ keV} \leq T_X \leq 10 \text{ keV} \). * Uncertainties correspond to the 90% confidence level.
injection, apparently, predict relations much too steep to be consistent with the data of MME99. These predictions are inconsistent with the results of the BBLP02 analytic models with similar levels of entropy injection (e.g., for $K_0 \approx 300$ keV cm$^{-2}$, BBLP02 predict $M_{\text{gas}} \propto T_X^{1.9}$, while Bialek et al. find $M_{\text{gas}} \propto T_X^{0.7}$). However, we believe the difference in the predictions (and conclusions) of Bialek et al. (2001) and the present work can be reconciled.

As noted by Neumann & Arnaud (2001), Bialek et al. (2001) have simulated very few hot clusters, and although they fit their $M_{\text{gas}}(r_{500})-T_X$ simulation data over a range similar to MME99, the results are too heavily weighted by the cool clusters ($T_X \lesssim 3$ keV) to be properly compared with the data of MME99. As an example, we consider their “S6” sample of 12 clusters that have $K_0 = 335$ keV cm$^{-2}$. According to the present study, this model should give a reasonably good fit to the MME99 observational data, much better than that of a model with $K_0 \approx 100$ keV cm$^{-2}$. Although the normalization of the S6 model is in excellent agreement with the MME99 data (as is apparent in Table 3 of Bialek et al. and in the general trends in their Fig. 1), they rule this model out based on the fact that the predicted slope is 2.67, much steeper than the 1.98 found by MME99. However, a closer analysis reveals that the fraction of cool clusters in the simulation data set is much higher than the fraction of cool clusters in the MME99 sample. For example, in the MME99 sample of 45 clusters, only one cluster has a temperature below 3 keV. In the Bialek et al. (2001) S6 set, however, five of the 12 clusters have temperatures below 3 keV. In addition, the mean temperature of clusters in the MME99 sample is $\approx 5.5$ keV, while it is only about 3.8 keV in the Bialek et al. (2001) S6 data set. As previously mentioned, preheating preferentially affects low-temperature systems, and therefore comparisons between theory and observations should be done over the same range in temperatures. To illustrate the problems of comparing theoretical models and observations that span different temperature ranges, we tried to reproduce the fit of Bialek et al. (2001) to their S6 data set. We used data presented in their Table 2 for clusters with $T_X > 2$ keV (we used their preferred “processed” temperatures) and fit it with a linear model and found $M_{\text{gas}} \propto T_X^{2.0 \pm 0.17}$. This is slightly different from the value listed in their Table 3, presumably because Table 2 is based on data within $r_{200}$, while Table 3 is based on data within $r_{500}$ (they note that a change of up to 6% in the predicted slope can occur when switching between the two). To match the conditions of the present work, we then discarded all simulated cluster data below 3 keV (the mean temperature for the remaining seven clusters was then 5.1 keV, similar to the MME99 data) and found a best fit of $M_{\text{gas}} \propto T_X^{3.03 \pm 0.30}$. This is in excellent agreement with the results of MME99 and only marginally inconsistent with the BBLP02 models of similar entropy injection.

What about their favored models? We have tried the same type of test on their S3 data set ($K_0 \approx 100$ keV cm$^{-2}$). Fitting all simulated clusters with $T_X > 2$ keV (mean temperature of 3.8 keV), we find $M_{\text{gas}} \propto T_X^{1.86 \pm 0.12}$, which is in good agreement with the results of MME99. When we remove all clusters below 3 keV (mean temperature of 4.9 keV), however, the best fit is $M_{\text{gas}} \propto T_X^{0.77 \pm 0.38}$. In this case, the best-fit relation is not very constraining. It is even indistinguishable from the self-similar result. It is apparent from their Figure 1, however, that the predicted normalization for this model (and all other low-entropy models) does not match the observations of MME99. This is not by the authors themselves. They claim the difference in the zero point can be resolved by reducing the baryon fraction by $\approx 20\%$. As we noted earlier, however, a similar normalization offset is also seen in the total cluster mass-temperature ($M-T_X$) relation, and this cannot be resolved by reducing the baryon fraction. This suggests that the problem lies with the temperature, rather than the gas mass. Alternatively, Bialek et al. (2001) also suggest that rescaling their simulations for $H_0 = 70$ km s$^{-1}$ Mpc$^{-1}$ (instead of 80 km s$^{-1}$ Mpc$^{-1}$) would bring consistency between the normalization of this model and the observations. This would be true only if the baryon fraction was held fixed at 0.1 and not rescaled for the new cosmology. Given that they assume $\Omega_m = 0.3$, this would imply $\Omega_b = 0.015$ h$^{-2}$, which is roughly 30% lower than observed in quasar absorption spectra (Burles & Tytler 1998). Thus, while the normalization offset between their theoretical model and the observations of MME99 is directly reduced by decreasing the value of $h$, it is indirectly increased by roughly the same proportion through the increased value of $\Omega_b/\Omega_m$.

In summary, as with the Loewenstein (2000) models, we find that the difference in the results and conclusions of Bialek et al. (2001) and the present work can be explained on the basis that different temperature ranges were examined. In particular, we have shown that the fraction of cool clusters in Bialek et al.’s simulated data set is much larger than that found in the MME99 sample, and this has likely led to an underestimation of the entropy floor in these clusters. In order to safely and accurately compare the preheated models of BBLP02 with observations, we have paid special attention to only those hot clusters with $T_X \gtrsim 3$ keV. As such, we believe our comparison is more appropriate.

6. DISCUSSION AND CONCLUSIONS

Motivated by a number of observational studies that have suggested that the $M_{\text{gas}}-T_X$ relation of clusters of galaxies is inconsistent with the self-similar result of numerical simulations and by the launch of the Chandra and XMM-Newton satellites, which will greatly improve the quality of the observed $M_{\text{gas}}-T_X$ relation, we have implemented the analytic model of BBLP02 to study the impact of preheating on the $M_{\text{gas}}-T_X$ relation. The predictions of the model have previously been shown to be in very good agreement with observations (e.g., the $L_X-T_X$ relation and the $L_X-\sigma$ relation).

In agreement with the previous theoretical studies of Loewenstein (2000) and Bialek et al. (2001), our analysis indicates that injecting the ICM with entropy leads to a steeper relationship than predicted by the self-similar result of numerical simulations of clusters that evolve through the effects of gravity alone. Loewenstein (2000) and Bialek et al. (2001) have found that models that produce an entropy floor of $K_0 \sim 100$ keV cm$^{-2}$, which is consistent with measurements of galaxy groups, are capable of reproducing the $M_{\text{gas}}-T_X$ relation of hot clusters. This is inconsistent with our analysis, which indicates that a “high” level of entropy injection ($K_0 \gtrsim 300$ keV cm$^{-2}$) is required to match the observational data of hot clusters of White et al. (1997), Peres et al. (1998), and MME99. It is also inconsistent with BBLP02’s best-fit value of $K_0 \approx 330$ keV cm$^{-2}$ found via an investigation of the $L_X-T_X$ relation of both groups and hot clusters. They note that the strongest constraints for a high
entropy floor come from hot clusters. Moreover, a high value of $K_0$, one that is inconsistent with the predictions of the best-fit models of Loewenstein (2000) and Bialek et al. (2001), has also been reported by Tozzi & Norman (2001). Finally, da Silva et al. (2001) used numerical simulations with a "low" value of $K_0 \sim 80$ keV cm$^2$ (which is similar to predictions of the best-fit models of Loewenstein 2000 and Bialek et al. 2001) and found that they could not reproduce the observed X-ray scaling relations. Our result, on the other hand, is consistent with the results of BBLP02, Tozzi & Norman (2001), and da Silva et al. (2001). As discussed in § 5, we believe the difference between the studies of Loewenstein (2000) and Bialek et al. (2001) and the present work can be explained by considering the difference in temperature ranges studied. In particular, we have focused only on hot clusters in an attempt to match the majority of the observational data as closely as possible. The results and conclusions of the other two studies, however, are strongly influenced by their low temperature model data.

We have proposed that the $M_{\text{gas}}-T_X$ relation can be used as a probe of the gas density profiles of clusters if it is evaluated at different fixed radii. This is a new test. The preheated models of BBLP02 predict a mild break in the scaling relations when small fixed radii (such as $r = 0.250$ $h^{-1}$ Mpc) are used. The scatter in the current observational data is consistent with the predictions of the BBLP02 models with $K_0 \gtrsim 300$ keV cm$^2$; however, the exact shape of the gas density profiles is not tightly constrained. We anticipate that large samples of clusters observed by Chandra and XMM-Newton will place much stronger constraints on the gas density profiles of clusters and allow for further testing of the preheating scenario.

Finally, the high level of energy injection inferred from our analysis has important implications for the possible sources of this excess entropy. Valageas & Silk (1999), Balogh et al. (1999), and Wu et al. (2000) have all shown that galactic winds driven by supernovae can only heat the intracluster/intergalactic medium at the level of $\lesssim 0.3$–0.4 keV per particle. This is lower than the 1–2 keV per particle result found here. Thus, if the BBLP02 preheated models provide an accurate description of the ICM, supernovae winds alone cannot be responsible for the excess entropy. It has also been speculated that quasar outflows may be responsible (e.g., Valageas & Silk 1999; Nath & Roychowdhury 2002). This remains an open possibility.

The role of radiative cooling also remains an open issue. Recently, it has been suggested that both radiative cooling and preheating together could be actively involved in shaping the X-ray scaling relations (e.g., Voit & Bryan 2001; Voit et al. 2002). Radiative cooling (and subsequent star formation) would serve to remove the lowest entropy gas, which in turn would help to compress the highest entropy gas, thus increasing the emission-weighted gas temperature and steepening the $M_{\text{gas}}-T_X$ relation (e.g., the discussion of entropy in the “Cool+SF” simulation of Lewis et al. 2000). In this way, the combination of cooling and preheating may reduce the best-fit entropy level, perhaps even to a level that can be provided by supernovae winds (Voit et al. 2002). Further study is required to determine the relative roles that both preheating and cooling have on cluster evolution.

We would like to thank Mike Loewenstein and the anonymous referee for many useful comments and suggestions. I. G. M. is supported by a postgraduate fellowship from the Natural Sciences and Engineering Research Council of Canada (NSERC) and by the Petrie Fellowship at the University of Victoria. He also acknowledges additional assistance in the form of a John Criswick Travel Bursary. A. B. is supported by an NSERC operating grant, and M. L. B. is supported by a Particle Physics and Astronomy Research Council rolling grant for extragalactic astronomy and cosmology at the University of Durham.