Effect of anisotropy on HBT radii using leptonpair interferometry

Payal Mohanty\textsuperscript{1,2}, Mahatsab Mandal\textsuperscript{1} and Pradip Roy\textsuperscript{2}
\textsuperscript{1}High Energy Nuclear and Particle Physics Division, Saha Institute of Nuclear Physics, 1/AF Bidhannagar, Kolkata-700 064, INDIA, \textsuperscript{2}Institute of Physics and Applied Physics, Yonsei University, Seoul 120-749, Korea

(Dated: November 11, 2014)

The effect of initial state momentum-space anisotropy on invariant mass dependence of HBT radii extracted from the leptonpair interferometry is presented here. We have studied the Bose-Einstein Correlation Function (BECF) for two identical virtual photons decaying to leptonpairs at most central collision of LHC energy having fixed transverse momentum of one of the virtual photons ($k_T=2$ GeV). The free streaming interpolating model with fixed initial condition has been used for the evolution in anisotropic Quark Gluon Plasma (aQGP) and the relativistic (1+2)d hydrodynamics model with cylindrical symmetry and longitudinal boost invariance has been used for both isotropic Quark Gluon Plasma (iQGP) and hadronic phases. We found a significant change in the spatial and temporal dimension of the evolving system in presence of initial state momentum-space anisotropy.

PACS numbers: 25.75.+t,25.75.-q,12.38.Mh

I. INTRODUCTION

Two-particle intensity interferometry, commonly known as HBT interferometry is one of the efficient ways to know the space-time description of particle emission zone created in high energy nucleus-nucleus collisions \cite{1,2}. This method was formulated and exploited by Hanbury, Brown and Twiss to measure the angular diameter of astronomical objects \cite{3}. It was first introduced in heavy ion collision (HIC) through pion interferometry which provided valuable information about the space-time description of the system at the freeze out surface only \cite{2}. It has been argued that in contrast to hadrons, two-particle intensity interferometry of photons and dileptons \cite{5-12} which are produced throughout the space-time evolution of the reaction zone and which suffer almost no interactions with the surrounding medium can provide information on the the history of the evolution of the hot matter created in HIC. However photons appear to be a more restrictive probe since they are characterized only by their transverse momentum ($p_T$) whereas the dileptons have two kinematic variables, $p_T$ and invariant mass(M) to play with. A soft photon (low $p_T$) in one frame of reference can be hard (high $p_T$) in another frame, whereas $p_T$ integrated invariant mass distribution of dileptons is independent of any frame. In addition to it, $p_T$ spectra is affected by the flow, however, $p_T$ integrated M spectra remain unaltered by the flow in the system \cite{13}. Also in the M spectra of dileptons, above $\phi$ peak, dilepton from QGP dominates over its hadronic counterpart \cite{14}. All these suggests that a judicious choice of $p_T$ and M windows will be very useful to characterize the QGP and hadronic phase separately. Moreover owing to rapid longitudinal expansion at the onset of QGP phase compared to the partonic interaction rate, the anisotropy arises in $p_T - p_L$ plane with $\langle p_L^2 \rangle \ll \langle p_T^2 \rangle$ in the local rest frame. With time, such asymmetry dies out with secondary partonic interactions. After which the system is considered to be isotropic and thermal at proper time $\tau_{iso}$ and beyond $\tau \geq \tau_{iso}$ the system can be treated hydrodynamically. To include such momentum anisotropy in pre-equilibrium stage of QGP, a simple phenomenological model is adopted from refs. \cite{15-17}. We assumed two time scales here; (i) the initial QGP formation time, $\tau_i$, and (ii) the isotropization time, $\tau_{iso}$, when the isotropy in momentum space is achieved and they should fulfill the criteria $\tau_i \leq \tau_{iso}$. In absence of anisotropy, we have $\tau_i = \tau_{iso}$.

In this letter, we present intensity interferometry with leptonpairs at most central LHC initial conditions at $\sqrt{s}=2.76$ TeV including momentum space anisotropy in pre-equilibrium QGP phase and attempt to study the effect of the anisotropy on the mass dependence of HBT radii extracted from the Bose Einstein Correlation Function, $C_2$ for two identical virtual photons which later decay into leptonpairs. We have discussed the definition and formalism of leptonpair interferometry in Sec. \textbf{III} the results are given in Sec. \textbf{III} and finally we have summarized in Sec. \textbf{IV}.

II. DEFINITION AND FORMALISM

Leptonpair interferometry is based on computing the Bose-Einstein correlation (BEC) function for two
identical virtual photons which later decay in lepton-pairs and can be defined as \([11]\),

\[
C_2(k_1^*, k_2^*) = \frac{P_2(k_1^*, k_2^*)}{P_1(k_1^*) P_1(k_2^*)}
\]

(1)

where

\[
P_1(k_i) = \int d^4 x \int dM_i^2 \omega(x, k_i)
\]

(2)

and

\[
P_2(k_1^*, k_2^*) = P_1(k_1^*) P_1(k_2^*) + \frac{1}{3} \int d^4 x_1 d^4 x_2 dM_1^2 dM_2^2 \omega(x_1, K) \omega(x_2, K) \cos(\Delta x^\mu \Delta k_\mu)
\]

(3)

where \( k_i = (k_{iT} \cos \psi_i, k_{iT} \sin \psi_i, k_{iT} \sinh y_i) \) is the three momentum of the two identical virtual photons with \( i = 1, 2 \), \( K = (k_1 + k_2)/2 \) is the average transverse momentum, \( \Delta k_\mu = k_{1\mu} - k_{2\mu} = q_\mu \), \( x_i \) and \( k_i \) are the four co-ordinates for position and momentum variables respectively and \( \psi_i \)'s are the angles made by \( k_{iT} \) with the x-axis of each virtual photon. \( \omega(x, k) = dR/dM^2 d^3k dy \) is the source function related to the thermal emission rate of virtual photons per unit four volume. The possibility of dilution of the signal due to random pairs will not affect the HBT radii (discussed in ref \([11]\)), thus we have neglected the lepton pairs with different invariant masses. By ignoring the lepton pair with different invariant mass, i.e., \( M_1 = M_2 = M \), we can re write \( C_2 \) as:

\[
C_2(k_1^*, k_2^*) = 1 + \frac{\int d^4 x dM^2 \omega(x, K) \cos(\Delta \alpha)^2 + \int d^4 x dM^2 \omega(x, K) \sin(\Delta \alpha)\sin(\Delta \phi)^2}{P_1(k_1^*) P_1(k_2^*)}
\]

(4)

where \( \Delta \alpha = \alpha_1 - \alpha_2 \) and \( \alpha_k = \tau M_{iT} \cosh(y_i - \eta) - rk_{iT} \cos(\theta - \psi_i) \), \( M_{iT} = \sqrt{M^2 + k_{iT}^2} \). The inclusion of the spin of the virtual photon will reduce the value of \( C_2 - 1 \) by 1/3. The correlation functions can be evaluated by using Eqs. \([12, 21, 4]\) and \([4]\) for different average mass windows, \( \langle M \rangle \). We follow Ref. \([13, 16]\) for the dilepton production in aQGP. Beyond \( \tau \geq \tau_{iso} \), the leading order process through which lepton pairs are produced in QGP is \( qq \rightarrow l^+ l^- \). For low \( M \), dilepton production from the hadronic phase the decays of the light vector mesons \( \rho, \omega \) and \( \phi \) have been considered including the continuum \([19, 20]\). Since the continuum part of the vector meson spectral functions are included, the processes like four pions annihilation \([21]\) are excluded to avoid double counting.

In the present work the space time evolution is same as done in Ref. \([12, 22]\). For \( \tau \leq \tau_{iso} \), system evolves anisotropically and is described by free steaming interpolating model \([15, 17]\). The effect of radial flow is neglected in the anisotropic phase as it is not developed in the initial stage of the collision. For \( \tau \geq \tau_{iso} \), the system is described by (1+2)d ideal hydrodynamics model with cylindrical symmetry \([24]\) and boost invariance along the longitudinal direction \([25]\). The initial temperature

| TABLE I: Values of the various parameters used in the evolution dynamics. |
|--------------------------------------------|-----------------|
| \( \sqrt{s} \)          | 2.76 TeV        |
| \( T_\chi \)            | 646 MeV         |
| \( \tau_c \)            | 0.08 fm         |
| \( T_\phi \)            | 175 MeV         |
| \( T_{fo} \)            | 170 MeV         |
| \( \text{EoS} \)        | 2+1 Lattice QCD |

[23]
(\(T_1\)) and initial formation time (\(\tau_i\)) of the system is constrained by the hadronic multiplicity (\(dN/dy\)) as \(dN/dy \sim T_1^3 \tau_i\). The equation of state (EoS) which controls the rate of expansion/cooling has been taken from the lattice QCD calculations \(22\). The chemical (\(T_{ch}\)) and kinetic (\(T_{fo}\)) freeze-out temperatures are fixed by the particle ratios and the slope of \(p_T\) spectra of hadrons \(20\). The values of these parameters are displayed in Table \(\text{I}\).

### III. RESULTS

With the initial conditions described in Table \(\text{I}\) we evaluate the correlation function, \(C_2\) for different invariant mass windows (for \(M\) = 0.3, 0.5, 0.77, 1.02, 1.6 and 2.5 GeV) for Pb+Pb collisions at \(\sqrt{s_{NN}} = 2.76\) TeV as a function of \(q_{\text{side}}\) and \(q_{\text{out}}\) which are related to the transverse momentum of individual pair as follows: \(2\):

\[
q_{\text{side}} = \frac{q_T - q_{\text{out}}}{k_T^* - K_T^*}
= \frac{2k_{1T}k_{2T}\sqrt{1 - \cos^2(\psi_1 - \psi_2)}}{\sqrt{k_{1T}^2 + k_{2T}^2 + 2k_{1T}k_{2T}\cos(\psi_1 - \psi_2)}}
\]

\[
q_{\text{out}} = \frac{q_T^2 - k_T^*}{|K_T^*|}
= \frac{(k_{1T}^2 - k_{2T}^2)}{\sqrt{k_{1T}^2 + k_{2T}^2 + 2k_{1T}k_{2T}\cos(\psi_1 - \psi_2)}}
\]

where \(k_{iT}\) is the individual transverse momentum and \(y_i\) is the rapidity. It may be mentioned that the BEC function has values \(1 \leq C_2(k_{1T}, k_{2T}) \leq 2\) for a chaotic source. These bounds are from quantum statistics.

These source dimensions can be obtained by parameterizing the calculated correlation function with the empirical Gaussian form \(1\):

\[
C_2(q, K) = 1 + \lambda \exp(-R_i^2 q_i^2)
\]

where \(i\) stands for side and out. Thus \(R_{\text{side}}\) and \(R_{\text{out}}\) appearing in Eq. \(5\) are commonly referred to as HBT radii, which are measures of Gaussian widths of the source size. The deviation of \(\lambda\) from 1/3 will indicate the presence of non-thermal sources. While the radius corresponding to \(q_{\text{side}}\) \(R_{\text{side}}\) is closely related to the transverse size of the system. The radius corresponding to \(q_{\text{out}}\) \(R_{\text{out}}\) measures both the transverse size and duration of particle emission \(2\ \ 3\ \ 27\ \ 29\).

In the present work, the corresponding HBT radii are extracted with the help of the parametrization expressed in Eq. \(6\) for three different values of \(\tau_{\text{iso}}\).

We choose \(\tau_{\text{iso}}\) in such a way that \(\tau_{\text{iso}} = \tau_i (0.08\ \text{fm})\) corresponds to the isotropic situation and \(\tau_{\text{iso}} > \tau_i\) \((2, 3\ \text{fm})\) corresponds to anisotropic scenario. So basically, we have attempted to examine the sensitivity of momentum anisotropy on spatial and temporal size of the evolving system by controlling the variable, \(\tau_{\text{iso}}\). Again we argue that large M region (beyond \(M \geq m_\rho\)) corresponds to the partonic phase as it is dominated by lepton pairs from the partonic interactions and \(M \sim m_\rho\) region corresponds to the hadronic region as leptonpairs are produced basically from the interaction of light vector mesons. So the study of invariant mass variations of \(R_{\text{side}}\) and \(R_{\text{out}}\) in these two M regions characterizes the two different phases of HIC \(11\). In Figure \(1\) and \(2\) we display the invariant mass dependence of \(R_{\text{side}}\) and \(R_{\text{out}}\) for three values of \(\tau_{\text{iso}} = 0.08, 2, 3\ \text{fm}\).

\(C_2\) as function of \(q_{\text{out}}\) is calculated by taking \(\psi_1 = \psi_2 = 0, y_1 = y_2 = 0\) and fixing transverse momentum of one photon \(k_{iT} = 2\ \text{GeV}\) and varying the other \(k_{3T}\) for different invariant masses and the values of corresponding \(R_{\text{out}}\) is extracted from it using the parametrization given in Eq. \(6\). \(R_{\text{out}}\) probes both the transverse size and the duration of emission. With the development of radial flow, the transverse dimension of the emission zone decreases. Although the effect of flow is small in the initial stage of collision which is dominant in large M region (corresponds to larger size) and the duration of emission is small - resulting in a small values of \(R_{\text{out}}\). Whereas \(M \sim m_\rho\) region suffers from larger flow effects which should have resulted in a minimum value in \(R_{\text{out}}\) in this M region. However, \(R_{\text{out}}\) probes the duration of emission too, which is large for hadronic phase because of the slower expansion due to softer EoS used in the present

![FIG. 1: The variation of \(R_{\text{out}}\) with \(\langle M \rangle\) for different values of \(\tau_{\text{iso}}\). The solid line corresponds to isotropic scenario (\(\tau_{\text{iso}} = \tau_i = 0.08 \text{ fm}\)) and the dotted and dashed line corresponds to \(\tau_{\text{iso}} = 2.3 \text{ fm}\) respectively which is related to anisotropic scenario.](attachment:image.png)
work for the hadronic phase. The larger duration compensates the reduction of $R_{\text{out}}$ due to flow resulting in a bump in $R_{\text{out}}$ for $M \sim m_\rho$ (see Fig. 1). Again by increasing $\tau_{\text{iso}}$, the duration of particle emission shortens and results in smaller value of $R_{\text{out}}$.

\[ C_2 \text{ as function of } q_{\text{side}} \text{ is calculated by taking } k_{1T} = k_{2T} = 2 \text{ GeV}, y_1 = y_2 = 0 \text{ and fixing } \psi_1 = 0 \text{ for different invariant masses and the values of corresponding } R_{\text{side}} \text{ is extracted from it using the parametrization given in Eq. (6). In Fig. 2 we display the variation of } R_{\text{side}} \text{ with } M \text{ for three different values of } \tau_{\text{iso}} \text{ and observe quantitative as well as qualitative changes in magnitude. It can be shown that } R_{\text{side}} \text{ is related to the collective motion of the system through the relation: } R_{\text{side}} \sim 1/(1 + E_{\text{collective}}/E_{\text{thermal}}). \text{ In large M region, as the flow is not developed fully so the values of } R_{\text{side}} \text{ is affected only due to initial thermal energy. As } \tau_{\text{iso}} \text{ is inversely related to the temperature, thus increasing } \tau_{\text{iso}} \text{ results in decrease in the values of } R_{\text{side}} \text{ in large M region. In the mass region corresponds to the hadronic phase, } M \sim m_\rho, \text{ the flow is fully developed and the thermal energy is reduced. The ratio of collective to thermal energy is large in this M region and hence shows smaller } R_{\text{side}} \text{ in isotropic scenario. However, the situation is complex in presence of initial momentum space anisotropy. By increasing } \tau_{\text{iso}}, \text{ the flow is reduced resulting in larger values of } R_{\text{side}} \text{ in anisotropic scenario.}

The HBT radii are proportional to the average size of the system [27]. The average size of the system is related to the HBT radii (here $R_{\text{out}}$ and $R_{\text{side}}$) extracted from correlation function, $C_2$ using Eq. (6). However, some of the model dependence gets canceled out by taking the ratio of $R_{\text{out}}$ to $R_{\text{side}}$. Thus the quantity, $R_{\text{out}}/R_{\text{side}}$ gives the duration of particle emission for $\sqrt{s_{NN}} = 2.76$ TeV for different values of $\tau_{\text{iso}}$. Both show a non-monotonic dependence on $M$. The smaller values of both the quantities, particularly at high mass region, reflect the contributions from the early partonic phase of the system. The peak around $\rho$-meson mass reflects dominance of the contribution from the late hadronic phase in isotropic scenario. However, increasing values of $\tau_{\text{iso}}$ result in shorter duration of particle emission hence both these quantities have smaller value in anisotropic scenario compared to the isotropic one. However, by increasing the values of $\tau_{\text{iso}}$ we observe qualitative change in the magnitude of both of these quantities. This is because the duration of particle emission reduces as the system takes more time to become thermalized (by increasing $\tau_{\text{iso}}$). Hence both these quantities have smaller value in the anisotropic scenario compared to the isotropic one.

\[ \text{FIG. 2: The variation of } R_{\text{side}} \text{ with } \langle M \rangle \text{ for different values of } \tau_{\text{iso}}. \text{ The solid line corresponds to isotropic scenario (} \tau_{\text{iso}} = \tau = 0.08 \text{ fm} \text{) and the dotted and dashed line corresponds to } \tau_{\text{iso}} = 2.3 \text{ fm respectively which is related to anisotropic scenario.} \]

\[ \text{FIG. 3: The ratio } R_{\text{out}}/R_{\text{side}} \text{ as a function of } \langle M \rangle. \]

\[ \text{FIG. 4: The ratio of collective to thermal energy. As } \tau_{\text{iso}} \text{ increases, the duration of particle emission shortens and results in smaller value of } R_{\text{out}}. \]

IV. SUMMARY

In summary, the correlation functions for dilepton pairs have been evaluated for different values of $\tau_{\text{iso}}$ and the HBT radii have been extracted from it for Pb+Pb collision at $2.76$ TeV LHC energies for different $\langle M \rangle$ windows. We observe both qualitative as well as quantitative change in the variation of HBT radii with M for dilepton pairs in presence of the initial momentum anisotropy. We argue that the variation of HBT radii with M for dilepton pairs can be used as an efficient tool to follow...
The change of the spatial and temporal dimensions of the evolving system with time in presence of momentum space anisotropy in the initial stage of collision. The invariant mass dependence of two experimentally challenging quantities, $R_{out}/R_{side}$ and $R_{diff}$, also show a quantitative change due to the incorporation of anisotropy in the momentum space in the initial stage of collision. The experimental challenges for the study of dilepton interferometry is addressed in ref. [11], where it has been argued that increasing luminosity of the experiment will be a motivating factor for such measurements.

V. ACKNOWLEDGEMENTS

This work is supported by Department of Atomic Energy, India. PM is partially supported by Korean National Research Foundation.

[1] S. Pratt, Phys. Rev. D 33, 1314 (1986).
[2] U. A. Weidemann and U. Heinz, Phys. Rep. 319, 145 (1999).
[3] U. Heinz and B. V. Jacak, Ann. Rev. Nucl. Part. Sci. 49, 529 (1999); T. Csörgő and B. Lőrstad, Phys. Rev. C 54, 1390 (1996); B. R. Schlei and N. Xu, Phys. Rev. C 54, R2155 (1996); D. H. Rischke and M. Gyulassy, Nucl. Phys. A 608, 479 (1996).
[4] R. Hanbury Brown and R. Q. Twiss, Nature 178, 1046 (1956).
[5] D. Peressounko, Phys. Rev. C 67, 014905 (2003).
[6] E. Frodermann, U. Heinz, Phys. Rev. C 80 044903 (2009).
[7] J. Alam, B. Mohanty, P. Roy, S. Sarkar and B. Sinha Phys. Rev. C 67, 054902 (2003).
[8] S. A. Bass, B. Mueller and D. K. Srivastava, Phys. Rev. Lett. 93, 162301 (2004).
[9] D. K. Srivastava and J. I. Kapusta, Phys. Lett. B 319, 407 (1993); D. K. Srivastava, Phys. Rev. D 49, 4523 (1994).
[10] D. K. Srivastava and R. Chatterjee, Phys. Rev. C 80, 054914 (2009).
[11] P. Mohanty, J. Alam and B. Mohanty, Phys. Rev. C 84, 024903 (2011); P. Mohanty, J. Alam and B. Mohanty, Nucl. Phys. A 862 301-303, (2011). P. Mohanty, J. Alam, PoS(WPCF2011)040, (2012), arXiv : 1202.2189[Nucl-th].
[12] P. Mohanty, M. Mandal and P. Roy, Phys. Rev. C 80, 054915 (2014).
[13] P. Mohanty, J. K. Nayak, J. Alam and S. K. Das, Phys. Rev. C 82 034901 (2010); J. K. Nayak and J. Alam, Phys. Rev. C 80 (2009) 064906.
[14] P. Mohanty, V. Roy, S. Ghosh, S. K. Das, B. Mohanty, S. Sarkar, J. e. Alam and A. K. Chaudhuri, Phys. Rev. C 85 (2012) 031903.
[15] M. Martinez and M. Strickland, Phys. Rev. Lett. 100, 102301 (2008).
[16] M. Martinez and M. Strickland, Phys. Rev. C 78, 034917 (2008).
[17] B. Schenke and M. Strickland Phys. Rev. D 76, 025023 (2007).
[18] J. Cleymans, J. Fingberg and K. Redlich, Phys. Rev. D 35, 2153 (1987).
[19] J. Alam, S. Raha and B. Sinha, Phys. Rep. 273, 243 (1996); J. Alam, S. Sarkar, P. Roy, T. Hatsuda and B. Sinha, Ann. Phys. 286, 159 (2001); R. Rapp and J. Wambach, Adv. Nucl. Phys. 25, 1 (2000).
[20] E. V. Shuryak, Rev. Mod. Phys. 65, 1 (1993).
[21] P. Lichard and J. Juran, Phys. Rev. D 76, 094030 (2007).
[22] L. Bhattacharya and P. Roy, Phys. Rev. C 78, 064904 (2008). L. Bhattacharya and P. Roy, Phys. Rev. C 79, 054910 (2009).