Transient Conjugate Heat Transfer Numerical Simulation in Superfluid Helium

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Abstract. Computational simulations of superfluid helium are needed in order to improve the cooling system design of superconducting magnets in particle accelerators and to achieve a better understanding of the transient phenomena during magnet quenches. A conjugate heat transfer numerical model based on the C++ toolbox OpenFOAM [1] is implemented to three-dimensional case studies involving superfluid helium and heating sources. The governing equations of the solver are modified according to the Kitamura’s model [2], a simplified version of the two-fluid model developed by Khalatnikov [3]. This simplified model is based on the assumption that the thermo-mechanical effect term and the Gorter-Mellink mutual friction term prevail on the others in the superfluid component momentum equation. Simulations are performed with the thermal conductivity function of superfluid helium both from theory [4] and the formulation used by Sato [5], who normalized the function according to a different conductive heat flux exponential coefficient determined from data analysis. An empirical calculation of the Kapitza conductance [6] is adopted in order to simulate the thermal resistance at the interface between helium and solids. Steady-state and transient simulations are compared to experimental data available in the literature. For such purpose, data are used from Van Sciver’s experiment in a helical coil [7] and a rectangular cross-section channel experiment conducted at CEA Paris-Saclay. The experiments comprised heaters and multiple temperature probes situated at different locations to track the temperature distribution and evolution of the superfluid helium state.

1. Introduction

Amongst the engineering applications of superfluid helium (He II), the cooling of low-temperature superconducting magnets plays an important role in the particle accelerators field. In order to maintain the superconducting state, low-temperature superconductors must be maintained at a lower temperature than a critical value that depends on the material. The equivalent thermal conductivity of He II, which is several orders of magnitude larger than any other liquid, ensures to keep the magnets at low temperature (below 2 K). One of the typical cooling processes involves a static He II bath in which the cold mass is submerged. The thermo-fluid dynamics simulations of helium in such applications are important in order to improve the cooling system.

In this paper, a new solver capable of simulating He II thermal and dynamic behaviour under unsteady conditions is presented. The solver, named heIIchtMultiRegionFoam, has been created by modifying the existing chtMultiRegionFoam, which is part of the toolbox OpenFOAM [1]. This solver simulates heat transfer and fluid flow transient phenomena involving both compressible fluids and solids. The governing equations adopted for the present study belong to the Kitamura’s one-fluid model [2]. Both the theoretical [4] and the Sato [5]
calculation of the He II heat conductivity function have been implemented in the new solver, allowing to perform comparisons. In order to take into account the Kapitza phenomenon \[6\] and the partial slip of He II at the walls, two new boundary conditions have been added to \textit{heIIchtMultiRegionFoam}.

The last part of this paper shows the validation of the model against experimental data.

2. Governing Equations and Boundary Conditions

Kitamura derived his model \[2\] under the assumption that all the terms in the superfluid component momentum equation of the classical two-fluid model are negligible apart from the thermo-mechanical effect and the mutual friction. This assumption allowed him to obtain equations in terms of a single fluid. The momentum equations contain additional terms with respect to the standard Navier-Stokes ones and the energy equation accounts for the divergence of the non-linear conductive heat flux.

2.1. He II Equations

The continuity equation remains unchanged with respect to the standard one since it is expressed in terms of the total density of the fluid:

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \tag{1}
\]

where \(\rho\) is the density, \(t\) the time and \(\mathbf{v}\) the velocity of the fluid.

The momentum equation takes the following form

\[
\frac{\partial (\rho \mathbf{v})}{\partial t} = -\nabla \cdot (\rho \mathbf{v} \otimes \mathbf{v}) - \nabla p - \nabla \cdot \left[ \frac{\rho_n \rho_s}{\rho} \left( \frac{s}{A_{GM} \rho_n |\nabla T|^2} \right)^{\frac{2}{3}} \nabla T \otimes \nabla T \right] + \nabla \cdot (2 \mu D(\mathbf{v})) - \nabla \left[ \frac{2}{3} \mu (\nabla \cdot \mathbf{v}) \right] - \mu \left( \frac{\rho_n^3 s}{A_{GM} \rho_n^3 \rho_s |\nabla T|^2} \right)^{\frac{1}{3}} \left[ \nabla^2 (\nabla T) + \frac{1}{3} \nabla (\nabla \cdot \nabla T) \right] + \rho g, \tag{2}
\]

where \(\rho_n\) and \(\rho_s\) are the densities of the normal and superfluid component respectively, \(s\) the entropy, \(A_{GM}\) the Gorter-Mellink mutual friction constant, \(\mu\) the dynamic viscosity, \(D(\mathbf{v})\) the deformation rate tensor, \(p\) the pressure, \(T\) the temperature.

The energy equation is enthalpy based in this solver. Since \textit{heIIchtMultiRegionFoam} has been applied to problems which do not involve forced flow, the enthalpy equation can be written like

\[
\frac{\partial (\rho h)}{\partial t} = -\nabla \cdot (\rho \mathbf{v} h) + \nabla \cdot \left( \frac{k_{eq}(T, p)}{c_p(T)} \nabla h \right), \tag{3}
\]

where \(h\) is the enthalpy and \(c_p\) is the specific heat capacity. The term \(k_{eq}(T, p)\) represents the equivalent thermal conductivity of He II:

\[
k_{eq} = \left( \frac{1}{f(T, p) |\nabla T|^n} \right)^{\frac{1}{n}}, \tag{4}
\]

where \(f(T, p)\) is the superfluid thermal conductivity function. The coefficient \(n\) may be either equal to 3 or 3.4, whether \(f(T, p)\) is computed according to theory or Sato’s formulation respectively.
2.2. Thermal Conductivity Function Calculation

The superfluid thermal conductivity function, which is part of the equivalent thermal conductivity of He II, can be computed in two different ways in this solver. One of these recalls the theoretical formulation:

\[ f(T, p) = \frac{A_G M \rho_n}{\rho_s^3 s^4 T^3}. \]  

(5)

The properties are temperature dependent and are extracted from the HEPAK package [8].

The other method is the empirical formulation of Sato [5]:

\[ f(T, p)^{-1} = h(t) g_{\text{peak}}(p), \]  

(6)

where \( h(t) \) and \( g_{\text{peak}}(p) \) are empirical functions. The reduced temperature \( t \) is equal to the ratio between the temperature and the lambda temperature \( T_\lambda \) at the pressure of interest.

2.3. Solid Equation

Only one equation is needed to describe the thermal behaviour of the solid part of the computational domain. The energy equation for solids takes the following form:

\[ \frac{\partial (\rho h)}{\partial t} = \nabla \cdot (\rho \alpha \nabla h) + S, \]  

(7)

where \( \alpha \) is the thermal diffusivity of the solid material and \( S \) a source term.

2.4. Kapitza Resistance

In the presence of heat flux across the interface between He II and a solid, a temperature discontinuity occurs due to thermal resistance. This thermal barrier, named Kapitza resistance [6], is caused by the mismatch in the phonon transport between the two media. This phenomenon is negligible at high temperatures or heat fluxes but in the range of temperatures below \( T_\lambda \) it plays an important role in the heat transfer between helium and a solid heater.

The heat flux across the interface can be described with

\[ q = h_K (T_s - T_{He}), \]  

(8)

where \( T_s \) and \( T_{He} \) are the temperatures of the solid and the helium respectively and \( h_K \) is the Kapitza conductance. The conductance \( h_K \) is approximately proportional to the third power of the bath temperature and can be empirically expressed as

\[ h_K = a T^b, \]  

(9)

where \( a \) and \( b \) are coefficients which depend on the solid material.

2.5. Partial Slip Boundary Condition

Since the superfluid component of He II has zero viscosity and supports no shear stress, at the walls the velocity of the fluid is not equal to zero and the condition of no-slip for Newtonian fluids cannot be applied. Therefore, a new boundary condition of partial slip must be implemented in the solver.

From the same assumption made by Kitamura, it is possible to write the partial slip condition as

\[ v_\parallel = \left( \frac{\rho_s^3 s A_G M \rho_n}{\rho_s^3 s^4 T^3} \right)^{\frac{2}{3}} (\nabla T)_\parallel, \]  

(10)

which is the component of the velocity vector tangential to the wall. This velocity is proportional to the temperature gradient in accordance with the thermo-mechanical effect.
3. Numerical Method

The governing equations are solved numerically using the open source toolbox OpenFOAM [1], which uses the finite volume method (FVM) to evaluate partial differential equations (PDEs) in three-dimensional geometries. In order to convert the PDEs into a set of algebraic equations, the differential terms are discretised with the Gaussian finite volume integration. This discretisation scheme sums the fluxes of a physical quantity over a control volume of the computational domain. The fluxes are evaluated by interpolation between cell centres values.

In `heIIchtMultiRegionFoam`, the convection terms have been discretised using the upwind differencing scheme to ensure the stability of the simulation. Whereas the central differencing scheme has been adopted for the diffusion terms. The initial value problem has been solved through the implicit Euler method, which is first order accurate in time. The additional terms in the momentum equation (Eq. 2) due to the superfluid component of He II have been obviously solved explicitly, being derivatives of the temperature. The diffusion term of the energy equation (Eq. 3) has been treated implicitly even though the equivalent thermal conductivity (Eq. 4), which is temperature gradient dependent, is calculated using the temperature field of the previous time step in order to overcome the non-linearity of the problem.

OpenFOAM heat transfer solvers adopt a segregated solution strategy. Therefore, the flow equations of the three components of velocity and pressure are solved sequentially in an uncoupled manner. The same principle applies to the coupling between fluid and solid. The coupling between momentum and mass conservation equations is realized through the pressure-implicit split-operator (PISO) [9] iterative algorithm for transient problems, with a double pressure correction loop. The resulting systems of linear algebraic equations have been solved using the preconditioned conjugate gradient method [9] with a residual tolerance of $10^{-7}$.

4. Model Validation

This section is dedicated to the comparison between simulations carried out with `heIIchtMultiRegionFoam` and data sets of two different experiments designed to measure the temperature evolution of He II following an applied heat flux. The first one was developed by Van Sciver [7]. The second experiment was conceived at the cryogenics laboratory at CEA Paris-Saclay.

4.1. Van Sciver’s Helical Coil Experiment

The experimental set-up is constituted by a stainless steel helical coil filled with He II and situated in a vacuum environment. The tubing is 9.01 m long and has an internal diameter of 9.02 mm. The top extreme of the tube is exposed to a He II bath at 1.802 K and saturation conditions. Whereas the lower end is soldered to a copper rod which is heated up by a Manganin wire wrapped around it. The rod, in turn, releases a heat flux equivalent to 22.2 kW/m² to the helium contained in the coil. The temperature of the He II is measured four times by four sensors located at different distances from the heater.

The applied heat flux is high enough to make the helium undergo nucleate boiling and eventually film boiling, instant at which the last set of measurements is recorded. As mentioned by the author of the experiment, the highest temperature reached by the helium next to the heater is 0.1 K greater than the saturation one at a pressure equal to the saturation pressure of the bath plus the hydrostatic head. Therefore, the saturation temperature at the bottom of the tubing, which is not explicitly provided, has to be less than or equal to 1.92 K. Taking into account the data showed in the paper, it is easy to notice that before the second set of data (2.5 s) the helium is already in the nucleate boiling regime.

Since the present model is not meant to simulate phase transition, two assumptions have been considered in order to account for the coexistence of liquid and vapour. Firstly, in the zone close to the heater where the fluid reaches saturation temperature, the thermal conductivity is
constructed as an averaged value:

\[ k = (1 - \alpha)k_{\text{HeII}} + \alpha k_{\text{vap}}, \]  

(11)

where \( k_{\text{vap}} \) and \( k_{\text{HeII}} \) are the thermal conductivity at saturation conditions of the vapour phase and He II respectively, whereas \( \alpha \) is the quality of the fluid and it varies according to the enthalpy of the mixture. In particular, since the value of \( f(T, p)^{-1} \) in the proximity of the \( \lambda \) point tends to zero, \( k_{\text{HeII}} \) is set to the value (Eq. 4) that He II has at the onset of nucleate boiling at a distance of a helium bubble size from the heater. This is justified by considering that bulk He II replenishes the zone where the bubbles collapse because of condensation. The average bubble size of helium was found to be 0.4 mm and independent of the fluid mass fraction with satisfying approximation [10]. The other assumption concerns the density of the fluid, which has been kept constant at the He II value, to not affect the fluid dynamics of the liquid.

Since the space between coil turns is small and the mean diameter of the coil (0.12 m) is much larger than the tubing internal one, the geometry of the computational domain is assumed to be a horizontal straight cylinder. The mesh, generated using the open source software SALOME [11], is of type structured and accounts for more than \( 6 \times 10^5 \) elements. The geometry, which does not take into account the stainless steel tubing, is split into the helium part and the copper part. The heat flux (Eq. 8) rules the coupling at the interface. The coefficients of Eq. 9 for copper are taken from [12]. At the other end of the helium domain, a Dirichlet boundary condition is applied with a temperature fixed at the bath value. Since the rod is in a vacuum environment, the opposite end of the copper domain is considered adiabatic. The adiabatic boundary condition is also used for the lateral patch of the helium domain, because of the negligible thermal conductivity of the stainless steel. The external heat flux is applied onto the lateral patch of the copper domain, in order to simulate the joule dissipation of the wiring.

Figure 1: Comparison between simulations (lines) and Van Sciver’s experiment data (markers with black edge): temperature difference evolution with respect to the bath temperature.
Figure 1 shows the comparison between experimental data (black edge markers) and simulations carried out both with (Eq. 6) (solid line) and (Eq. 5) (dotted line). The last set of data (5 s) has been omitted because of the presence of film boiling, which impedes to use the assumption (Eq. 11). As can be seen, the method by Sato provides accurate results despite the assumptions, whereas the theoretical formulation becomes less precise over time.

4.2. Rectangular Cross Section Channel Experiment

The experimental mock-up is constituted by a 14 cm long, 5 cm wide and 1 mm thick channel with rectangular cross section. The channel is enclosed laterally by four stainless steel parts, the bottom of which supports eight temperature sensors (bare chip Cernox) spaced 1.5 cm from each other along the channel centerline. One aperture of the channel is exposed to a He II bath at atmospheric pressure (Fig. 2a), whereas the opposite side faces a NiChrome (80Ni 20Cr) strip which provides a constant heat flux to the helium contained in the channel. The heater is installed in a G10 support that insulates the experiment from the external environment (Fig. 2b). A structured mesh of orthogonal hexahedrons has been used to take advantage of the simplicity of the geometry. The computational domain is split into the support, heater and helium parts. The Neumann boundary condition is applied in every external patch apart from the side of the channel facing the bath, which is at constant temperature. The heat flux flowing from the strip to the helium is calculated through (Eq. 8) and the coefficients of the conductance (Eq. 9) are assumed to be the ones for nickel [13]. The support and the heater are considered thermally coupled at the interface since the Kapitza resistance between solids is negligible [14]. The volumetric heat generation of the NiChrome strip, electrically supplied externally, is taken into account through a source term in the energy equation of the heater.

4.2.1. Convergence Analysis and Validation

In order to ensure the convergence of the solution, space and time convergence studies have been performed with non-adaptive time-steps and grids. Multiple simulations with constant time-step and a different number of grid elements have been run for the spatial convergence study. Figure 3a shows the relative error (calculated using the 2-norm of the array of values) of the steady state solution of each simulation with respect to the experimental data. It is clear how the numerical error decreases for finer meshes until convergence is reached. The time-step independence study has been checked with the grid indicated in Fig. 3a. The highest utilized time-step showed an inaccurate solution due to high Courant numbers. No relevant difference in accuracy has been found at lower time-steps (Fig. 3b).
(a) Grid convergence  
(b) Time convergence

Figure 3: Convergence Study

In Fig. 4 the simulation results are compared to temperature measurements of an experiment performed at 1.81 K with an equivalent heat flux of 22 kW/m$^2$, which does not cause phase change in the He II. The steady state temperature distribution is well represented by Sato’s formulation. The transient results show discrepancies with respect to the data probably because

Figure 4: Comparison between simulations (lines) and the channel experiment data (markers with black edge): temperature difference evolution with respect to the bath temperature.
the coefficients in the empirical functions of Eq. 6 are fitted experimentally at steady state conditions. Apart from the beginning of the transient, the theoretical method underestimates the temperature increase in the helium.

5. Conclusions

A transient conjugate heat transfer numerical model capable of simulating superfluid helium thermo-fluid dynamics in 3D geometries has been presented. An existing OpenFOAM solver has been modified taking the equations of Kitamura [2] as a reference and modifying the energy equation into an enthalpy based version. The Kapitza resistance and the partial slip of He II have been implemented in the code through two novel OpenFOAM boundary condition classes. The code is able to perform simulations with both the theoretical and Sato’s formulation of the conductive heat flux and thermal conductivity function. Convergence studies have been demonstrated for the steady state case. In case of presence of nucleate boiling, the averaged thermal conductivity allowed to simulate an approximate phase change, which will be further developed into a complete phase transition module in future works. The validation of the model against experimental data showed good agreement using Sato’s empirical equation and a considerable underestimation using the theoretical equation.

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Acknowledgments

EASITrain – European Advanced Superconductivity Innovation and Training. This Marie Sklodowska-Curie Action (MSCA) Innovative Training Networks (ITN) receives funding from the European Union’s H2020 Framework Programme under grant agreement no. 764879.