Kaluza–Klein reduction on a maximally non-Riemannian space is moduli-free

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We propose a novel Kaluza–Klein scheme where no graviscalar moduli can be generated. For this we set the internal space to be maximally non-Riemannian, meaning that no Riemannian metric can be defined for any subspace. Its description is only possible through Double Field Theory but not within supergravity. We spell out the corresponding Scherk–Schwarz twistable Kaluza-Klein ansatz, and show that the internal space prevents rigidly any graviscalar moduli. Plugging the ansatz into higher-dimensional pure Double Field Theory and also to a known doubled-yet-gauged string action, we recover heterotic supergravity as well as the heterotic worldsheet action. Our proposal can be applied to the uplift of a generic Yang–Mills theory, or as an alternative to string compactification.

Kaluza–Klein theory attempts to unify General Relativity and electromagnetism into higher-dimensional pure gravity. Yet, the (aesthetically unpleasing) cylindrical extra dimension brings along an unwanted additional massless scalar field, i.e. radion or graviscalar moduli, which is not observed in nature: it would spoil the Equivalence Principle by appearing on the right-hand side of the geodesic equation. This moduli stabilization problem is essentially rooted in the fact that there is no natural scale in pure gravity which would fix or stabilize the radius of the cylinder. The problem persists in modern string compactifications in view of the arbitrary size and shape of an internal space (mani/coni/orbifold, compact or not). Turning on fluxes or non-perturbative corrections might promise to solve the problem, e.g. 1, but such scenarios lack full calculational control, and are at the heart of current controversies, see e.g. 2 and references therein.

In this Letter we propose a novel Kaluza–Klein scheme to unify Stringy Gravity and Yang–Mills (including Maxwell), which postulates a non-Riemannian internal space and consequently does not suffer from any moduli problem. By Stringy Gravity, we mean the string theory effective action of the closed-string massless sector, conventionally represented by the three fields, \( (g_{\mu\nu}, B_{\mu\nu}, \phi) \). They transform into one another under O(D, D) T-duality 3, 4. Furthermore, within the framework of Double Field Theory (DFT) initiated in 5–9, O(D, D) T-duality becomes manifestly the principal symmetry and the effective action itself is identified as an integral of a stringy scalar curvature beyond Riemann. The whole closed-string massless sector may then be viewed as stringy graviton fields consisting of the DFT-dilaton, \( \Phi \), and the DFT-metric, \( \mathcal{H}_{AB} \) 10. The latter satisfies two defining properties:

\[
\mathcal{H}_{AB} = \mathcal{H}_{BA}, \quad \mathcal{H}_{A}^{C} \mathcal{H}_{B}^{D} \mathcal{J}_{CD} = \mathcal{J}_{AB},
\]

where \( \mathcal{J}_{AB} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \) is the O(D, D) invariant constant metric which can freely raise and lower the O(D, D) vector indices, \( A, B, \ldots \) (capital letters). A pair of symmetric projection matrices are then defined,

\[
P_{AB} = \frac{1}{2} ( \mathcal{J}_{AB} + \mathcal{H}_{AB} ), \quad \bar{P}_{AB} = \frac{1}{2} ( \mathcal{J}_{AB} - \mathcal{H}_{AB} ),
\]

while their ‘square roots’ give twofold DFT-vielbeins,

\[
P_{AB} = V_{A}^{P} V_{B}^{q} \eta_{pq}, \quad \bar{P}_{AB} = \bar{V}_{A}^{\bar{P}} \bar{V}_{B}^{\bar{q}} \bar{\eta}_{\bar{p}\bar{q}},
\]

which satisfy their own defining properties,

\[
V_{A} V^{A} = \eta_{pq}, \quad \bar{V}_{A} \bar{V}^{A} = \bar{\eta}_{\bar{p}\bar{q}}, \quad V_{A} \bar{V}^{A} = 0,
\]

or equivalently 11

\[
V_{A}^{P} V_{B}^{q} \eta_{pq} + \bar{V}_{A}^{\bar{P}} \bar{V}_{B}^{\bar{q}} \bar{\eta}_{\bar{p}\bar{q}} = \mathcal{J}_{AB}.
\]

Here \( \eta_{pq} \) and \( \bar{\eta}_{\bar{p}\bar{q}} \) are local Lorentz invariant metrics characterizing the twofold spin groups which have dimensions Tr\((P)\) and Tr\((\bar{P})\) separately. They are distinguished by unbarred and barred small letter indices.

The above stringy graviton fields constitute the diffeomorphic DFT-Christoffel symbols 12, 13,

\[
\Gamma_{CAB} = 2 \left( P \partial_{C} P \right)_{(AB)} + 2 \left( P_{[A}^{\ D} \bar{P}_{B]}^{\ E} - P_{A}^{\ D} P^{\ E} \right) \partial_{D} \mathcal{P}_{EC} - 4 \left( \frac{P_{C}^{\ A} P_{B}^{\ D}}{P_{\rho}^{\ A}} + \frac{P_{C}^{\ A} P_{D}^{\ B}}{P_{\rho}^{\ B}} \right) \left( \partial_{D} (P \partial^{E} \mathcal{P}^{(\ E)}_{(\ E)D}) \right),
\]

and, consequently with \( \nabla_{A} := \partial_{A} + \Gamma_{A} \), twofold local Lorentz spin connections 14, 17,

\[
\Phi_{Apq} = V_{B}^{p} \nabla_{A} V_{Bq} = V_{B}^{p} (\partial_{A} V_{Bq} + \Gamma_{AB}^{C} V_{Cq}), \quad \Phi_{Ap\bar{q}} = \bar{V}_{B}^{\bar{p}} \nabla_{A} \bar{V}_{B\bar{q}} = \bar{V}_{B}^{\bar{p}} (\partial_{A} \bar{V}_{B\bar{q}} + \Gamma_{AB}^{C} \bar{V}_{C\bar{q}}).
\]

These connections form covariant curvatures, Ricci and scalar 13, 17, of which the latter can be constructed as

\[
S_{(0)} = (P^{AC} P^{BD} - P^{AC} P^{BD}) S_{ABCD}, \quad \mathcal{G}_{(0)} = \mathcal{F}_{ABpq} V^{A} V^{B} q + \frac{1}{2} \Phi_{Apq} \Phi_{Apq}, \quad \bar{\mathcal{G}}_{(0)} = \bar{\mathcal{F}}_{ABpq} \bar{V}^{A} \bar{V}^{B} \bar{q} + \frac{1}{2} \Phi_{Ap\bar{q}} \Phi_{Ap\bar{q}}.
\]
where, with $R_{CDAB} = \partial_A \Gamma_{BCD} + \Gamma_{AC} \bar{F}_{BDE} - (A \leftrightarrow B)$,

$$S_{ABCD} := \frac{1}{2} \left( R_{ABCD} + R_{CDAB} - \Gamma_{EAB} \Gamma_{ECD} \right),$$

$$\mathcal{F}_{ABpq} := \nabla_A \Phi_{Bpq} - \nabla_B \Phi_{A pq} + \Phi_A \nabla_p \Phi_{Bq} - \Phi_B \nabla_p \Phi_{Apq},$$

$$\mathcal{F}_{AB\hat{p}q} := \nabla_A \Phi_{B\hat{p}q} - \nabla_B \Phi_{A \hat{p}q} + \Phi_A \nabla_{\hat{p}} \Phi_{Bq} - \Phi_B \nabla_{\hat{p}} \Phi_{Apq}.$$ 

While $S_{(0)}$ coincides exactly with the well-known expression of scalar curvature written in terms of $d, H_{AB}$ [18],

$$S_{(0)} = \frac{1}{8} H^{AB} \partial_A H_{CD} \partial_B H^{CD} + \frac{1}{2} H^{AB} \partial C H_{AD} \partial D H_{BC} - \partial_A \partial_B H^{AB} + 4 H^{AB} (\partial_A \partial B d - \partial_B \partial A d) + 4 \partial A \partial B H^{AB} d,$$

the other two accommodate the vielbeins and read [19]

$$+ \mathcal{G}_{(0)} = \frac{1}{2} S_{(0)} + 2 \partial_A \partial A d - 2 \partial_A \partial d A d + \frac{1}{2} \partial A V_{Bp} \partial A V^{BP},$$

$$- \mathcal{G}_{(0)} = \frac{1}{2} S_{(0)} - 2 \partial_A \partial A d + 2 \partial_A \partial d A d - \frac{1}{2} \partial_A V_{Bp} \partial A V^{BP}.$$ 

Clearly their differences would vanish upon the section condition, $\partial_A \partial A = 0$, but they provide precisely the ‘missing’ pieces in the Scherk–Schwarz reduction of DFT while relaxing the section condition on the internal space [20][22]. In particular, $\mathcal{G}_{(0)} - \mathcal{G}_{(0)}$ matches the action adopted in [22]. Below we focus on computing $\mathcal{G}_{(0)}$ and $-\mathcal{G}_{(0)}$ in higher dimensions with the Scherk–Schwarz twisted non-Riemannian Kaluza–Klein ansatz.

**Moduli-free non-Riemannian Kaluza–Klein ansatz**

With $\hat{D} = D + D$, the DFT Kaluza–Klein ansatz [14] breaks $O(\hat{D}, \hat{D})$ into $O(D', D') \times O(D, D)$ and takes the form:

$$\hat{\mathcal{H}} = \exp \left[ \hat{W} \right] \begin{pmatrix} \mathcal{H}' & 0 \\ 0 & \mathcal{H} \end{pmatrix} \exp \left[ W^T \right], \quad \mathcal{J} = \begin{pmatrix} J' & 0 \\ 0 & J \end{pmatrix}. \quad \text{(9)}$$

In our notation, hatted, primed, and unprimed symbols refer to the ambient, internal, and external spaces respectively: the ambient doubled coordinates split into the internal and external ones as

$$z^\mathcal{A} = (y^{\mathcal{A}'}, x^A), \quad \partial_A = (\partial_{A'}, \partial_A).$$

In [19], $\hat{W}$ is an off-block-diagonal $so(\hat{D}, \hat{D})$ element, $\hat{W} \mathcal{J} + \mathcal{J} \hat{W}^T = 0$, of the form,

$$\hat{W} = \begin{pmatrix} 0 & -W^c \\ W & 0 \end{pmatrix}, \quad W^c A' := W A' = \mathcal{J}^{AB} J_{A'B'} W_{B'B'},$$

in which the $2D \times 2D'$ block, $W A'$, should satisfy [24]

$$W A' W A' = 0, \quad W A' \partial_A = 0. \quad \text{(10)}$$

This condition sets half of the components to be trivial, truncates the exponential, $\exp \left[ \hat{W} \right] = 1 + \hat{W} + \frac{1}{2} \hat{W}^2$, and makes the above ansatz consistent with the ordinary Kaluza–Klein ansatz in supergravity. Moreover, crucially for the purpose of the present Letter, the ansatz [9] can accommodate non-Riemannian geometry in which the Riemannian metric cannot be defined, see [14] for classification and [25][27] for earlier examples. Henceforth, we focus on a specific internal space of which the DFT-metric is fully $O(D', D')$ symmetric and maximally non-Riemannian (namely $(D', 0)$ type as classified in [14]):

$$\mathcal{H}'_{A'B'} = J'_{A'B'}. \quad \text{(11)}$$

In general, from the defining relations [11], the infinitesimal variation of any DFT-metric should satisfy

$$\delta \mathcal{H}' = P' \delta \mathcal{H}' P' + \mathcal{J}' \delta \mathcal{H}' \mathcal{J}' P'. \quad \text{(12)}$$

Meanwhile, the particular choice of the internal space [11] implies $P' = J'$ and $P' = 0$. Thus, the fluctuation should be trivial: no graviscalar modulus can be generated and the non-Riemannian internal space is rigid,

$$\delta \mathcal{H}_{A'B'} = 0. \quad \text{(13)}$$

In fact, [11] sets the “twofold” internal spin group to be $O(D', D') \times O(0, 0)$, such that the coset structures of the internal and the ambient DFT-metrics, $\mathcal{H}'$, $\mathcal{H}$, are ‘trivial’ and ‘heterotic’ respectively, if $\mathcal{H}$ is Riemannian [23],

$$\frac{O(D', D')}{O(D', D') \times O(0, 0)} = 1, \quad \frac{O(\mathcal{D}, \mathcal{D})}{O(D+1, D-1) \times O(D-1, 1)}.$$

The latter has dimension $D^2 + 2D'D'$, which matches the total degrees of the external DFT-metric, $\mathcal{H}_{AB}$ [23], and the gravivector, $W A'$. [10][28]. C.f. [22].

The corresponding DFT-vielbeins are [14]

$$\hat{V}_{\hat{B}\hat{p}} = \exp \left[ \hat{W} \right] \begin{pmatrix} V_{A'p} & 0 \\ 0 & V_{Ap} \end{pmatrix}, \quad \hat{V}_{\hat{B}\hat{p}} = \exp \left[ \hat{W} \right] \begin{pmatrix} 0 \\ V_{A'p} \end{pmatrix}, \quad \text{(15)}$$

where $V_{A'p}$ is a ‘square’ matrix as $V_{A'p} V_{B'p}' = J'_{A'B'}$.

**Reduction to heterotic DFT**

Before inserting the ansatz [9], [15] into the ambient scalar curvatures [8], or $\hat{G}_{(0)}, \hat{G}_{(0)}$ (all hatted), we perform a Scherk–Schwarz twist over the internal space. Following [19], we introduce a twisting matrix, $U_A^A(y)$, which depends on the internal coordinates only and is an $O(D', D')$ element satisfying

$$U_A^A U_B^B J_{A'B'} = \mathcal{J}_{A'B'}. \quad \text{(16)}$$

where $\mathcal{J}_{A'B'}$ coincides with $J'_{A'B'}$ numerically: both are $O(D', D')$ invariant constant metrics. Essentially the
In each transformation above, the first line with $\xi^A$ is the diffeomorphic DFT Lie derivative and the second with $A^A$ is the Yang–Mills gauge symmetry. In fact, every single term in (20) is diffeomorphism-invariant [36].

It is worth while to note the only difference between $2\tilde{g}_{(0)}$ and $-2\tilde{g}_{(0)}$: the former implies a DFT-cosmological constant term [13], $\frac{1}{2}e^{-2d}f_{ABC}f^{ABC}$, but the latter does not. As anticipated in [34, 37], our result (20) is manifestly symmetric for $O(D, D)$ as well as any subgroup of $O(D^\prime, D^\prime)$ which stabilizes the structure constant, $f_{ABC}$.

If we adopt the well-known Riemannian parametrization of the DFT-metric,

\[
\mathcal{H}_{AB} = \begin{pmatrix} g^{-1} & -g^{-1}B \\ Bg^{-1} & g^{-1} - Bg^{-1}B \end{pmatrix}, \quad e^{-2d} = \sqrt{-g}e^{-2\phi},
\]

and solve the external section condition, $\partial_\lambda A^A = 0$, and [10], by letting $\partial_\lambda = (\hat{\partial}^\mu, \nabla_\lambda)$ for $A^A = (0, W^A)$, our main result (20) gives precisely the heterotic supergravity action [38, 39], up to total derivatives,

\[
\int -2e^{-2d}\tilde{g}_{(0)} = \sqrt{-g}e^{-2\phi}(R + 4\partial_\mu\phi\partial^\mu\phi - \frac{1}{4}H_{\mu\nu}\tilde{H}^{\mu\nu} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}C),
\]

where $\tilde{H}_{\mu\nu} = 3\partial_{[\lambda}B_{\mu\nu]} - \omega_{\mu\nu}$ which is invariant under the Yang–Mills gauge symmetry [22], or specifically,

\[
\delta W_\mu^A = \partial_\mu A^A + f^{\hat{A}}_{\hat{B}C}W_\mu^\hat{B}A^\hat{C}, \quad \delta B_{\mu\nu} = W_{[\mu}^\hat{A}\partial_\nu\hat{C}}A^\hat{A}. \tag{24}
\]

As the starting scalar curvatures [8] are at most two-derivative, our final result (20) lacks the higher-derivative gravitational Chern–Simons term [40], c.f. [41, 43].

**Reduction to heterotic string**

The completely covariant doubled-yet-gauged string action [44, 23, 27],

\[
S_{\text{string}} = \frac{1}{4\kappa_s} \int d^2\sigma \mathcal{L}_{\text{string}}, \tag{25}
\]

given by

\[
\mathcal{L}_{\text{string}} = \frac{1}{4\kappa_s} \sqrt{-h}h^{\alpha\beta}D_\alpha \dot{z}^\beta D_\beta \dot{z}^\alpha \mathcal{H}_{AB} - e^{\alpha\beta}D_\alpha \dot{z}^\beta A_{\beta A},
\]

where $D_\alpha \dot{z}^\beta = \partial_\alpha \dot{z}^\beta - A_\alpha \dot{z}^\beta$ and $A_\alpha \dot{z}^\beta$ is an auxiliary gauge potential satisfying $A_{\alpha \dot{z}^\beta} \dot{A} = 0$, $A_\alpha \dot{A} \dot{A} = 0$.

This worldsheet action realizes concretely the assertion that doubled coordinates in DFT are actually (half) gauged and a gauge orbit in the doubled coordinate system corresponds to a single physical point [45].

As before, we put $y^A = (y^\mu, y^\nu)$, $x^\alpha = (\bar{x}_\mu, x^\nu)$, and further set $W_\mu^A = (W^\mu_\nu, \bar{W}_\mu^\nu)$, $A_\alpha \dot{A} = (A_\alpha \dot{A}, A_\alpha \dot{A})$, $A_\alpha \dot{A} = (A_{\alpha \dot{A}}, 0)$, $A_\alpha \dot{A} = (A_{\alpha \dot{A}}, 0)$. With [9] and [23]...
assumed, the above action (25) becomes quadratic in the combination $A_{\mu} - \tilde{W}_{\mu} A_{\nu}$, and linear in $A_{\nu}$. Hence, while the former leads to a Gaussian integral, the latter plays the role of a Lagrange multiplier [14]. Integrating out the auxiliary gauge potential, the string action (25) plays the role of a Lagrange multiplier [14]. Integrating does not admit any linear fluctuation, to be completely

\[ \mathcal{L}_{\text{Het}} = -\frac{1}{2\kappa^2} \sqrt{-h} \partial_\sigma x^\mu \partial_\sigma x^\nu g_{\mu\nu} + \frac{1}{2} \epsilon^{\alpha\beta} \partial_\sigma x^\nu \partial_\sigma x^\nu B_{\mu\nu} + \frac{1}{2} \epsilon^{\alpha\beta} \partial_\sigma y^\nu \partial_\sigma y^\nu W_{\mu\nu} + \partial_\sigma x^\nu \partial_\sigma (B_{\mu\nu} W_{\alpha\beta}^\nu) + \frac{1}{2} \epsilon^{\alpha\beta} [\partial_\sigma x^\nu - W_{\mu A'}(\partial_\sigma y^{A'} + \frac{1}{2} \partial_\sigma x^\nu W_{\nu A'})] \partial_\sigma x^\mu. \]

(26)

Here $\partial_\sigma y^{\mu'} + \partial_\sigma x^\nu \hat{W}_{\mu}^{\nu'}$ should be chiral, as

\[
\left( h^{\alpha\beta} + \frac{1}{\kappa^2} \alpha^{\alpha\beta} \right) \left( \partial_\beta y^{\mu'} + \partial_\beta x^\nu \hat{W}_{\mu}^{\nu'} \right) = 0.
\]

(27)

In particular, when $W_{\mu A'}$ vanishes, the second and the third lines in (26) merge into a known topological term, $\frac{1}{2} \epsilon^{\alpha\beta} \partial_\sigma \tilde{z}^\alpha \partial_\sigma \tilde{z}^\beta$, while (27) gets simplified to make the internal coordinates chiral: $y^{\mu'}(\tau, \sigma) = y^{\mu'}(0, \tau + \sigma)$. This implies – at least classically [14, 20] – that a closed string subject to a periodic boundary condition cannot vibrate in the internal space. Thus, we recover the usual heterotic string action which agrees with the rigidity [13].

**Conclusion**

The maximally non-Riemannian DFT background specified by the DFT-metric, $H_{AB'} = J_{AB'}$, is singled out to be completely $O(D', D)$ symmetric and rigid: it does not admit any linear fluctuation, $\delta H_{AB'} = 0$ [13], nor graviscalar moduli, and the coset structure is trivial [14].

For the DFT Kaluza–Klein ansatz, [6] and [15], we set the internal space to be maximally non-Riemannian, performed (optionally) a Scherk–Schwarz twist [17], [19], and computed the ambient higher $(D'+ D)$-dimensional scalar curvatures, $\hat{G}_{(0)} + -\hat{G}_{(0)}$ [5], which lead to the $O(D, D)$-manifest formulation of the non-Abelian heterotic supergravity [20]. Only the former, $\hat{G}_{(0)}$, contains a DFT-cosmological constant.

Plugging the same non-Riemannian Kaluza–Klein ansatz [9] into the doubled-yet-gauged string action (24) may reproduce the usual heterotic string action,

\[ \frac{1}{2} \int_{\Sigma} \left( -\sqrt{-h} \epsilon^{\alpha\beta} g_{\mu\nu} + \epsilon^{\alpha\beta} B_{\mu\nu} \partial_\alpha x^\nu \partial_\beta x^\nu + \epsilon^{\alpha\beta} \partial_\alpha \tilde{z}^\alpha \partial_\beta \tilde{z}^\beta \right), \]

with chiral internal coordinates, $(h^{\alpha\beta} + \frac{1}{\kappa^2} \alpha^{\alpha\beta}) \partial_\beta y^{\mu'} = 0$. This is the case when the Yang–Mills sector is trivial. We leave the study of the nontrivial case with $W_{\mu A'} \neq 0$ for future work, including the worldsheet aspect of the relaxed section condition designed for the Scherk–Schwarz twist.

Uplift of the Standard Model of particle physics coupled to DFT [24] to higher dimensions would be of interest, as well as applications to string compactifications, possibly on other types of non-Riemannian internal space [14, 17].

We wish to thank Stephen Angus, Kanghoon Lee for useful comments, and Wonyoung Cho for valuable help at the early stage of the project. This work was supported by the National Research Foundation of Korea through the Grants, NRF-2016R1D1A1B01015196 and NRF-2018H1D3A1A01030137 (Korea Research Fellowship Program).

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With (10), we can rewrite the Yang–Mills kinetic term,
\[ -\frac{1}{4} H^{AC} H^{BD} F_{AB} F_{CD} = \frac{1}{4} P_{AC} P_{BD} F_{CD}, \]
where
\[ P_{AC} := \nabla_A W_C - \nabla_C W_A + f_{AB} W_A W_B. \]
Irrespective of (10), we have
\[ \text{is diffeomorphism-covariant, but } F_{AB} \text{ is so if } (10) \text{ holds} \]

With the numerical model term,
\[ -\frac{1}{4} H^{AC} H^{BD} F_{AB} F_{CD} = \frac{1}{4} P_{AC} P_{BD} F_{CD}. \]
Irrespective of (10), we have
\[ P_{AC} := \nabla_A W_C - \nabla_C W_A + f_{AB} W_A W_B. \]
Irrespective of (10), we have
\[ F_{AB} \text{ is so if } (10) \text{ holds} \]

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