Riga – Plate flow of $\gamma \text{Al}_2\text{O}_3$-water/ethylene glycol with effective Prandtl number impacts

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1. Introduction

Magnetohydrodynamics is a branch of modern theory of fluid dynamics that characterizes the electromaginemagnetic processes arising in electric conducting flows under the influence of magnetic field. In classical MHD, the flow of highly electric conducting fluids could be dominated by an external magnetic field. But, the applied external magnetic field produces very small amount of current in weakly electric conducting fluids (e.g. sea water). The efficient flow control can be achieved only by applying the Lorentz force in wall parallel direction. Gailitis and Lielaukis [1] designed a device called Riga-plate to produce the Lorentz force in the direction which is parallel to the wall. Riga plate is an electromagnetic actuator which includes span wise aligned array of alternating electrodes and permanent magnets, mounted on a plane surface [2, 3]. It can be utilized to reduce the friction force and pressure drag of submarines by avoiding the boundary layer separation and decrease the production of turbulence. Tsinober and Shern [4] reported that the impacts of applying the Lorentz forces in wall-parallel direction are useful to increase the stability of Blasius flow over a Riga plate. The effects of this type of Lorentz force on the boundary layer flow of viscous fluid are investigate in recent years [5, 6].

In many of the industrial applications, the heat transfer enhancement methods are needed for high performance cooling or heating. But these methods are limited by the restriction of the low thermal conductivity of conventional heat transfer liquids like oil, water, ethylene glycol etc. Choi [7] introduced an advanced fluid called nanofluid and suggested to replace the conventional fluids with thses advanced fluids. The main idea is to combine the conventional fluids and nanosized solid particles of high thermal conductivity [8, 9, 10, 11, 12, 13, 14, 15, 16]. Nanofluids have better wetting, dispersion and separation properties on the surfaces such as stretching plate, Riga plate and surfaces with variable thickness. The suitable nanoparticle additives in the working fluid have much influence in the enhancement of thermal conductivity of base working fluid. Such investigations have significance in thermal treatment of cancer, aerospace, micro electronics and medical applications. The recent developments in the nanofluid theory, modelling and applications can be found in the articles of Mahian et al. 17, 18. Akbarzadeh [19] studied the MHD nanofluid flow between a porous layer in the presence of internal heat generation. Golafshan and Rahimi [20] investigated the effect of radiation on the third grade nanofluid over a stretching sheet with MHD effects. Freidoonimehr and Rahimi [21] examined the Brownian motion and slip effects on the three dimensional nanofluid flow. Khan et al. [22] addressed the impacts of nonlinear radiation on the flow of...
cross nano fluid. The development in the nano fluid flow with various physical aspects has been analysed in the papers [23, 24, 25, 26, 27]. Recently, the Al$_2$O$_3$ nano fluids are being studied by the many experimental and theoretical, researchers due to its variety of cooling applications. Maiga et al. [28, 29, 30] studied the heat transfer characteristics of Al$_2$O$_3$ nano fluids in heated tubes. Pop et al. [31] have made an analysis of laminar-to-turbulent threshold with Al$_2$O$_3$ nano fluids. Farajollahi et al. [32] reported the heat transfer characteristics of Al$_2$O$_3$/water and TiO$_2$/water in a shell and tube with turbulent flow condition. Sow et al. [33] have done an experimental study on the freezing point of Al$_2$O$_3$ water nano fluid. Beiki et al [34] considered the forced laminar flow of Al$_2$O$_3$/electrolyte nano fluid in a circular tube. Esmaeilzadeh et al. [35] studied the heat transfer and friction factor of Al$_2$O$_3$/water through circular tube with twisted tape inserts with different thicknesses. Abdul et al. [36] used Al$_2$O$_3$ nano fluid to analyse the effect of operating parameters on the gravity assisted heat pipe. Bayomy et al [37] have done a numerical and experimental work on the flow of Al$_2$O$_3$–water nano fluid through aluminum foam heat sink. Moghaieb et al [38] utilized Al$_2$O$_3$/Water nano fluids as an engine coolant in their study. Vishnu Ganesh and his co-authors [39, 40, 41, 42] studied the boundary layer flow of Al$_2$O$_3$ nano fluids with various physical effects. Ahmad et al. [43] studied the strong suction effects on Riga plate nano fluid flow region. Hayat et al. [44] investigated the characteristics of Riga plate flow of nano fluid in which the plate was convectively heated. The slip effects on Riga plate flow of nano fluid was analysed by Ayub et al [45]. Recently, Ahmad et al. [46] studied the vertical Riga plate flow of nano fluid.

Motivated by the above works, an attempt has been taken to study the Al$_2$O$_3$–Water/Ethylene glycol nano fluid flow over a stretchable Riga plate with the impacts of effective Prandtl number and electromagnetohydrodynamics. The related mathematical formulation has been done with an effective Prandtl number. The Grinberg term [3] has been used to model the electro MHD flow of nano fluids. Numerical solutions are carried out by fourth order RK method and special case analytical solutions are presented.

2. Methodology

2.1. Problem formulation

Two dimensional, steady, electro MHD flow of Al$_2$O$_3$–Water/Ethylene glycol nano fluid over a stretching Riga plate with stretching velocity $u_w = a x$ is considered (See Figs. 1 and 2). The prescribed surface temperature at the Riga plate is $T_w = T_w + b x$. Where $T_w$ is the ambient temperature and $a$ and $b$ are constants. It is assumed that no-slip condition and thermally equilibrium state between Al$_2$O$_3$ nanoparticles and base fluids. With the above assumptions, the governing equations of the problem are as follows
Eqs. (6), (7), (8), and (9) are the dynamic viscosity and the effective density, respectively,
where \( \mu \) is the dynamic viscosity, \( \rho \) is the density, \( \phi \) is the solid volume fraction of nano fluid, and \( \gamma \) is the solid volume fraction of nano fluid.

The effective dynamic density \((\rho_{ef})\) and the heat capacitance \((\rho C_v)_{ef}\) are given by

\[
\rho_{ef} = (1 - \phi) \rho_0 + \phi \rho_f, \quad (\rho C_v)_{ef} = (1 - \phi) (\rho C_v)_0 + \phi (\rho C_v)_f,
\]

where \( \phi \) is the solid volume fraction of nano fluid.

The dynamic viscosity of the nano fluid is defined as

\[
\mu_{ef} = 123\phi^2 + 7.3\phi + 1, \quad \text{(for } \gamma \text{ Al}_2\text{O}_3 \text{ Water)},
\]

\[
\mu_{ef} = 306\phi^2 - 0.19\phi + 1, \quad \text{(for } \gamma \text{ Al}_2\text{O}_3 \text{ Ethylene glycol)}.\]

The effective thermal conductivity of the nano fluid is given by

\[
k_{ef} = 4.97\phi^2 + 2.72\phi + 1, \quad \text{(for } \gamma \text{ Al}_2\text{O}_3 \text{ Water)},
\]

\[
k_{ef} = 28.905\phi^2 + 2.8273\phi + 1, \quad \text{(for } \gamma \text{ Al}_2\text{O}_3 \text{ Ethylene glycol)}.\]

The effective Prandtl number of the nano fluid is given by

\[
Pr_{ef} = 82.1\phi^2 + 3.9\phi + 1, \quad \text{(for } \gamma \text{ Al}_2\text{O}_3 \text{ Water)},
\]

\[
Pr_{ef} = 254.3\phi^2 - 3\phi + 1, \quad \text{(for } \gamma \text{ Al}_2\text{O}_3 \text{ Ethylene glycol)}.\]

Eq. (5) is the common correlation used to calculate the \( \rho_{ef} \) and \((\rho C_v)_{ef}\). Eqs. (6), (7), (8), and (9) are the dynamic viscosity and the effective thermal conductivity of \( \gamma \text{ Al}_2\text{O}_3 \text{ nano fluid} \) that have been obtained by performing a curve fitting (least square) of some experimental data [28, 29, 30, 48, 49, 50]. Eqs. (8) and (9) are obtained from Hamilton and Crosser model [51]. Eqs. (10) and (11) are the effective Prandtl number models which are obtained by a curve fitting using regression laws [31, 40].

By using the following relations

\[
\eta = \frac{u}{v} = 0, \quad u = \alpha f'(\eta) = 0, \quad v + (a\eta)^{1/2} f(\eta) = 0 \quad \text{and} \quad \theta = \frac{T - T_w}{T_m - T_w},
\]

Eqs. (2) and (3) are transformed to non-dimensional form as follow:

\[
f'' = -\frac{\left(1 - \phi + \phi \left(\frac{\mu_f}{\mu_0}\right)\right) (f'' - f^*)}{(123\phi^2 + 7.3\phi + 1)} - \frac{Z_{ef}^{hn}}{123\phi^2 + 7.3\phi + 1} (f'' - f^*) \quad \text{(for } \gamma \text{ Al}_2\text{O}_3 \text{ Water}),
\]

\[
f'' = -\frac{\left(1 - \phi + \phi \left(\frac{\mu_f}{\mu_0}\right)\right) (f'' - f^*)}{(306\phi^2 - 0.19\phi + 1)} - \frac{Z_{ef}^{hn}}{306\phi^2 - 0.19\phi + 1} (f'' - f^*) \quad \text{(for } \gamma \text{ Al}_2\text{O}_3 \text{ Ethylene glycol)}
\]

\[
\theta'' = -\frac{Pr_f \left(1 - \phi + \phi \left(\frac{\mu_f}{\mu_0}\right)\right) (82.1\phi^2 + 3.9\phi + 1)}{123\phi^2 + 7.3\phi + 1} (f'' - f^*) \quad \text{(for } \gamma \text{ Al}_2\text{O}_3 \text{ Water}),
\]

\[
\theta'' = -\frac{Pr_f \left(1 - \phi + \phi \left(\frac{\mu_f}{\mu_0}\right)\right) (254.3\phi^2 - 3\phi + 1)}{306\phi^2 - 0.19\phi + 1} (f'' - f^*) \quad \text{(for } \gamma \text{ Al}_2\text{O}_3 \text{ Ethylene glycol)}.
\]

The corresponding non-dimensional Riga plate flow boundary conditions are
Where $B$ is a non-dimensional number, $f(\eta, 0) = f'(0), \theta(0) = 0 = \theta(\infty)$. 

(17)

Where $B$ is a non-dimensional number, $Z = \left( \frac{\sigma B M \mu}{\pi \rho V^2} \right)$ is the modified Hartmann number, $Pr$ is the Prandtl number and $\phi$ is volume fraction of nanoparticles.

One can observe that if modified Hartmann number $Z = 0$, the present problem reduces to the stretching sheet problem of nanoparticles.

The hypergeometric function solution for the energy Eqs. (8) and (9) along with Eq. (10) with $Z = 0$ is obtained as [40],

$$f(\eta) = \frac{1 - e^{-\alpha \phi}}{\alpha} \theta(\eta)$$

Where

$$\alpha = \frac{1}{\sqrt{1 - \phi \left( \frac{\rho_s}{\rho_f} \right)}} \left( 123 \phi^2 + 7.3 \phi + 1 \right)^{-1} (\text{for } \gamma Al_2O_3 \text{ Water}),$$

$$\alpha = \frac{1}{\sqrt{1 - \phi \left( \frac{\rho_s}{\rho_f} \right)}} \left( 306 \phi^2 - 0.19 \phi + 1 \right)^{-1} (\text{for } \gamma Al_2O_3 \text{ Ethylene glycol}).$$

The hypergeometric function solution for the energy Eqs. (8) and (9) along with Eq. (10) with $Z = 0$ is obtained as [40] (see Table 1).

$$\theta(\eta) = e^{-\left(1 + \alpha \phi \right) \eta} \left[ \begin{array}{c} -M(E \alpha^2 - 1, 1 + \alpha \phi) + \alpha \eta E \phi \alpha^2 \end{array} \right]$$

where $E = \frac{Pr \left( 1 - \phi \left( \frac{\rho_s}{\rho_f} \right) \right)}{123 \phi^2 + 7.3 \phi + 1}$ (for $\gamma Al_2O_3$ Water) and

$$E = \frac{Pr \left( 1 - \phi \left( \frac{\rho_s}{\rho_f} \right) \right)}{254 \phi^2 - 3 \phi + 1}$$

(18)

The local skin friction coefficient $Re_{\infty}^{1/2} C_f$ and the reduced Nusselt number $Re_{\infty}^{1/2} N_u$ are derived and given in Table 2 for $\gamma Al_2O_3$ Water/Ethylene Glycol nanofluids.

2.2. Numerical procedure

The transformed Eqs. (13), (14), (15), and (16) and the Riga plate flow BCs in (17) can be written in the following IVP form

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \\ y_8 \end{bmatrix} = C \begin{bmatrix} y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \\ y_8 \end{bmatrix}$$

The impact of modified Hartmann number on velocity profile with $\phi = 0.1$.

The impact of nanoparticle volume fraction ($\phi$) on temperature profile with $Z = 2.0, Pr = 6.96 (\gamma Al_2O_3$ Water) and $Pr = 204 (\gamma Al_2O_3$ Ethylene glycol).
and on local skin friction coefficient. Computations, a convergence criterion of \(10^{-6}\) has been used.

\[
\begin{bmatrix}
 y_1 \\
 y_2 \\
 y_3 \\
 y_4 \\
\end{bmatrix} =
\begin{bmatrix}
 0 \\
 1 \\
 R_1 \\
 R_2 \\
\end{bmatrix}
\]

(19)

Where \(C = \frac{1 - \phi + \phi \left( \frac{\gamma}{\gamma} \right)}{(123\phi^2 + 7.3\phi + 1)}\) and \(D = \frac{1}{(123\phi^2 + 7.3\phi + 1)}\) (for \(\gamma \text{Al}_2\text{O}_3-\text{Water}\)).

And

\(C = \frac{1 - \phi + \phi \left( \frac{\gamma}{\gamma} \right)}{(306\phi^2 - 0.19\phi + 1)}\) and \(D = \frac{1}{(306\phi^2 - 0.19\phi + 1)}\) (for \(\gamma \text{Al}_2\text{O}_3-\text{Ethylene glycol}\)).

Eq. (18) and the initial conditions in (19) are solved using R-K integration technique along with shooting method. For the numerical computations, a convergence criterion of \(10^{-6}\) has been used.

3. Results and discussion

Numerical results for velocity and temperature profiles are obtained by fourth order RK method with shooting techniques. To analyse the impacts of various pertinent parameters which involved in the problem are discussed via graphical illustrations. The verification of current numerical code has been done by comparing the reduced Nusselt number values with Isak [52] in the absence of modified Hartmann number and nanoparticle volume fraction. The comparisons of these values are in good agreement (Table 3).

The impacts of nanoparticle volume fraction \(\phi\) of \(\gamma \text{Al}_2\text{O}_3\) nanoparticles on the velocity profile with water and ethylene glycol as base fluids is described in Fig. 3. The velocity profile enhances with nanoparticle volume fraction of \(\gamma \text{Al}_2\text{O}_3\) nanoparticles. The \(\gamma \text{Al}_2\text{O}_3\) nanoparticles with same nanoparticle volume fraction show variations in velocity profile with different base fluids. On comparing the velocity profile of \(\gamma \text{Al}_2\text{O}_3-\text{Water}\) and \(\gamma \text{Al}_2\text{O}_3-\text{ethylene glycol}\), it can be observe that the \(\gamma \text{Al}_2\text{O}_3\)-ethylene glycol has larger velocity.

Characteristics of the modified Hartmann number \(Z\) on the velocity profile of the nanofluid are exposed in Fig. 4. An increasing behaviour of velocity profile due to the increase of modified Hartmann number has been seen in this figure. In fact, the larger values of this parameter lead to enhance the external electric field. This enhancement in external electric field leads to the production of wall parallel Lorentz force which slowing down the growth of the momentum boundary layer. Closely examining the figure, it is noted that the velocity suddenly rises near the plate with larger modified Hartmann number.

Fig. 5 is prepared to show the influences of \(\phi\) on the temperature profile of \(\gamma \text{Al}_2\text{O}_3\) nanofluids. It is clear that the temperature profile is a decreasing function of nanoparticle volume fraction. Experimental studies have shown that the \(\gamma \text{Al}_2\text{O}_3\) nanofluids are used for the cooling purposes [33, 34, 35, 36, 37, 38]. Thus the present theoretical result revealed that the same behaviour of \(\gamma \text{Al}_2\text{O}_3\) nanofluids. On comparing the thermal boundary layers of \(\gamma \text{Al}_2\text{O}_3\)-Water and \(\gamma \text{Al}_2\text{O}_3\)-Ethylene glycol, it is observed that the thermal boundary layer of \(\gamma \text{Al}_2\text{O}_3\)-Ethylene glycol is thinner than \(\gamma \text{Al}_2\text{O}_3\)-Water. This is due to the fact that the value of \(k\) is higher for water than ethylene glycol. Fig. 6 portrays the influences \(Z\) on the temperature profile of \(\gamma \text{Al}_2\text{O}_3\) nanofluids. Larger values of modified Hartmann number lead to decay the temperature of \(\gamma \text{Al}_2\text{O}_3\) nanofluids.

The impacts of \(Z\) and the \(\phi\) on the skin friction coefficient and reduced Nusselt number are displayed in Figs. 7 and 8. It is seen that both skin friction and reduced Nusselt number are the increasing function of \(\phi\) of \(\gamma \text{Al}_2\text{O}_3\) nanofluids. The larger values of modified Hartmann number
reduce the skin friction and increase the reduced Nusselt number.

4. Conclusion

Thermal transfer characteristics of steady state electro-MHD flow of γ-Al2O3-Water/Ethylene glycol nano fluid over a stretchable Riga plate in two dimensional case are studied numerically. An effective Prandtl number model is used to analyse velocity and thermal boundary layers. The main results are summarized as follow:

- Higher nanoparticle volume fraction increases the velocity profile and decreases the temperature profile in γ-Al2O3 nanofluids.
- The γ-Al2O3-Ethylene glycol has larger velocity and lower temperature than γ-Al2O3-Water.
- The velocity distribution increases with higher modified Hartmann number due to external electric field. The higher values of modified Hartmann number reduce the temperature profile.
- The local skin friction and reduced Nusselt number are the decreasing function of modified Hartmann number.

Declarations

Author contribution statement

N. Vishnu Ganesh, Qasem M. Al-Mdallal: Conceived and designed the analysis; Analyzed and interpreted the data; Contributed analysis tools or data; Wrote the paper.
Sara Al Fahel, Shymaa Dadoa: Analyzed and interpreted the data; Wrote the paper.

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Competing interest statement

The authors declare no conflict of interest.

Additional information

No additional information is available for this paper.

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