Baryon resonances as dynamically generated states in chiral dynamics

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Abstract. We discuss baryon resonances which are dynamically generated in hadron dynamics based on chiral coupled channels approach. With the dynamical description of the baryon resonance, we discuss the origin of the resonance pole, finding that for the description of $N(1535)$ some other components than meson and baryon are necessary. Since the chiral unitary model provides a microscopic description in terms of constituent hadrons, it is straightforward to calculate transition amplitudes and form factors of resonances without introducing further parameters. Finally we briefly discuss few-body nuclear kaonic systems as hadronic molecular states.

Keywords: dynamically generated resonances, chiral dynamics, coupled channel approach, form factor of $N(1535)$ resonance, hadronic molecular state

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DYNAMICAL DESCRIPTION OF HADRONIC RESONANCE

In principle, all the hadron states are dynamically generated by quark and gluon fields obeying QCD. Nevertheless, the current quarks and gluons appearing in QCD are not effective degrees of freedom for the understanding of the hadron structure. Thus, more efficient description of hadrons is favorable. There it is an important question what are the effective constituents in baryon resonances, or what are active ingredients in dynamical description of baryon resonances. Since baryon resonances are located where the strong decay channels are open, hadronic components are also important to understand the structure of the baryon resonances apart from the component originated from the quarks confined in the single potential. Therefore, for the investigation of the baryon resonance structure, the aspect of hadron dynamics should be unavoidably considered.

In dynamical description, resonance states are obtained as solutions of Schrödinger equation or Lippmann-Schwinger equation for scattering matrix with a given Hamiltonian $H = H_0 + V$, where $H_0$ is the Hamiltonian of free particles and $V$ represents the interaction of these particle. The Hamiltonian $H_0$ fixes the model space of dynamical elements, with which the resonance states are dynamically described, and the potential $V$ specifies dynamics of the elementary components. If states are obtained without introducing other components than those in $H_0$, we call these states dynamically generated state. If these ingredients are written in terms of hadrons, the resonance states are described by hadron dynamics.

In the coupled channel approach with chiral dynamics (chiral unitary model), the scattering amplitude is obtained as a solution of the Lippmann-Schwinger equation $T = V + VGT$ under the assumption that the model space spans the lowest lying octet mesons and baryons and their interaction is given by chiral perturbation theory [1]. The
off-shell behavior in the Lippmann-Schwinger equation is tamed with regularization of the integral by, for instance, introducing form factors or using the dimensional regularization. It is known that, based on the \(N/D\) method, one can simplify the scattering equation and obtain an algebraic solution \(T = (1 - GV)^{-1}V\) by using the on-shell values of the potential \(V\) and the loop function \(G\) with the dimensional regularization [2].

Explicit pole terms in the interaction kernel \(V\) represent states outside the model space. Thus, purely dynamically generated state should be obtained without the explicit pole terms. Even though the amplitudes are written in terms of hadron dynamics, resonances generated dynamically are not always genuine hadronic composite objects. According to Ref. [3], in chiral perturbation theory, the low energy constants in higher order terms are dominated by resonance contributions. This means that the contributions of the resonances which are not described dynamically in the present model space are hidden in the interaction kernel and this kind of resonances can be reconstructed after dynamical calculation [4, 5]. There is another source of contributions coming from outside the model space. In the regularization procedure, one fixes high-momentum behavior which is not controlled in the model space. This means that some contributions coming from the outside of the model space can be hidden in the regularization parameters [6].

**INTERPRETATION OF RESONANCE POLE**

Here we discuss the interpretation of the resonance pole within the chiral unitary model. In the previous section, we have discussed the possible sources of resonances from outside the model space. First of all, we discuss whether we exclude this source of the resonance from our description of the resonance theoretically. For the interaction kernel, since resonance contributions can be hidden in the higher order terms of chiral perturbation theory, we take only the leading Tomozawa-Weinberg term, which is understood as the \(t\)-channel vector meson exchange. For the regularization parameter of the loop function \(G\), it is possible to exclude the implicit source in a consistent way to the chiral expansion as the following way [6]. If there are no states other than the free scattering states in the loop function, the real part of the loop function should be negative below the threshold, since the spectral representation of the Green function reads \(G(W) = \sum_n \rho(W)/(W - E_n)\) with the positive definite spectral function \(\rho(W)\) and the total energy \(W\), and the lowest states is the threshold at \(W = E_0\), then \(\text{Re}\, G(W) \leq 0\) for \(W \leq E_0\). In addition, if the chiral expansion is applied, at some point in the low-energy region the scattering amplitude can be written in chiral perturbation theory, namely \(T = V\), which implies that \(G = 0\). Since the loop function is a decreasing function below the threshold, these conditions can be satisfied at \(W = M\) with the baryon mass \(M\) by

\[
G(M; a_{\text{natural}}) = 0, \tag{1}
\]

which we call natural renormalization scheme [6]. Equation (1) fixes the renormalization constant in a consistent way with chiral expansion and exclusion of the resonance source.

If we solve the Lippmann-Schwinger equation with this renormalization parameter and the Tomozawa-Weinberg interaction \(V_{TW}\), we obtain a dynamical description of the
scattering amplitude in terms of hadrons as

\[ T_{\text{natural}}(W) = \left[ V_{TW}^{-1}(W) - G(W; a_{\text{natural}}) \right]^{-1}, \]  

(2)

and poles appearing in this amplitude correspond to a genuine dynamically generated states in terms of hadron dynamics. Nevertheless, it is not always the case that the scattering amplitude (2) and its poles agree with the experimental data. We discuss contribution from the outside of the model space by comparing the pole positions of the amplitude (2) and those of the scattering amplitude obtained phenomenologically so as to reproduce the experimental data. The phenomenological amplitude is obtained by using the Tomozawa-Weinberg interaction and the renormalization parameter determined by using experimental data:

\[ T_{\text{pheno}}(W) = \left[ V_{TW}^{-1}(W) - G(W; a_{\text{pheno}}) \right]^{-1}. \]  

(3)

In Fig. 1, we show the comparison of the pole positions for \( N(1535) \) and \( \Lambda(1405) \) obtained with the phenomenological and natural renormalization schemes. As one can see, for \( N(1535) \), the two solutions differ from each other. This implies that, to describe the \( N(1535) \) resonance, we need certain contributions coming from the components other than meson and baryon, which are possibly quark-originated components. In contrast, for \( \Lambda(1405) \), these two pole positions are almost the same. This shows that the \( \Lambda(1405) \) can be described dominantly by the meson-baryon component.

In the natural renormalization scheme with the Tomozawa-Weinberg interaction, \( \Lambda(1405) \) is successfully reproduced, while \( N(1535) \) is obtained not so satisfactorily. It is interesting to see which kind of interaction is necessary in the natural renormalization to reproduce a phenomenological description. First we note that, in the renormalization point of view, once the scattering amplitude \( T \) is fixed, the change of the renormalization parameter in \( G \) should be absorbed into the interaction \( V \). This implies that the equivalent scattering amplitude can be expressed by different sets of \( V \) and \( G \) depending on

FIGURE 1.  Comparison of the pole positions for \( N(1535) \) and \( \Lambda(1405) \) obtained with the phenomenological and natural renormalization schemes [6]. \( z \) denotes the pole positions of \( N(1535) \) and \( z_1 \) and \( z_2 \) are for the two states of \( \Lambda(1405) \). The triangle and cross mean the pole positions obtained by the phenomenological and natural renormalization schemes, respectively.
the renormalization scheme labeled by $a$:

$$T(W) = [V^{-1}(W;a) - G(W;a)]^{-1}. \quad (4)$$

Now if we obtain a good phenomenological description of the scattering amplitude with an appropriate $a_{\text{pheno}}$ as in Eq. (3) with the Tomozawa-Weinberg interaction, we can obtain $V(W;a_{\text{natural}})$ by equating Eqs. (3) and (4). After some algebra, we find in a single channel case

$$V(W;a_{\text{natural}}) = V_{WT}(W) + \frac{C}{2f^2} \frac{(W - M)^2}{W - M_{\text{eff}}} \quad (5)$$

with $V_{TW}(W) = -C(W - M)/(2f^2)$, $M_{\text{eff}} \equiv M - 2f^2/(C\Delta a)$ and $\Delta a \equiv G(W;a_{\text{natural}}) - G(W;a_{\text{pheno}}) = a_{\text{natural}} - a_{\text{pheno}}$. As seen in Eq. (5), the interaction kernel in the natural renormalization scheme is expressed by the WT term and a pole term with the mass $M_{\text{eff}}$ depending on the difference of two renormalization schemes $\Delta a$.

The relevance of the pole term depends on the value of $M_{\text{eff}}$. Using the values of $a_{\text{pheno}}$ obtained in the coupled channel [7, 8] and taking a natural renormalization scheme $G(MN;a_{\text{natural}}) = 0$ for all channels, we find the effective mass $M_{\text{eff}} \approx 1700 \pm 40i$ MeV for $N(1535)^2$. Since this pole appears in the relevant energy of the $N(1535)$ physics, it can be a source of the $N(1535)$ having some components other than dynamically generated state by meson and baryon. This pole may be interpreted as a genuine quark component and could be a chiral partner of nucleon discussed in Refs. [9, 10].

**APPLICATIONS OF DYNAMICAL DESCRIPTION**

Once we obtain a good and reliable description of hadron resonances, we can calculate further their dynamical properties. So far, there have been, for instance, the investigations of the magnetic moments of $\Lambda(1405)$ and $\Lambda(1670)$ [11], the radiative decay of $\Lambda(1405)$ [12], the helicity amplitudes of $N(1535)$, $\Lambda(1405)$ and $\Lambda(1670)$ [13, 14] and the electromagnetic form factors of $\Lambda(1405)$ [15, 16]. There are many works for reaction calculations with the dynamical description based on chiral dynamics.

Since the resonance is described dynamically in terms of the constituent meson and baryon microscopically, it is straightforward to calculate the helicity amplitude and form factors of the resonance by implementing the photon coupling to the constituent hadrons. Once we fix the elementary couplings of the meson and hadrons to the photon, there are no additional parameters. The helicity amplitudes of $N(1535)$ have been discussed in the chiral unitary model [13], in which the $N(1535)$ is described with the phenomenological renormalization scheme, and the electromagnetic transitions $\gamma^*N \rightarrow N(1535)$ are calculated. It is very interesting that this model reproduces the observed helicity amplitudes, especially the neutron-proton ratio $A_{1/2}^n/A_{1/2}^p$ in good agreement with experiment.

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1 Generalization to the coupled channel is straightforward and discussed in Ref. [6]
2 This value quantitatively has moderate dependence on the values of $a_{\text{pheno}}$ and choice of the natural renormalization condition in the coupled channel.
although this model implicitly has the quark-originated pole for $N(1535)$ as discussed above and the direct photon couplings to the quark components were not considered in this calculation. Therefore, the success of this model implies that the meson-baryon components of $N(1535)$ are essential for the structure of $N(1535)$ probed by a low-energy virtual photon. The effect of the quark core and its relation to the meson cloud are discussed in Refs. [17, 18].

The electromagnetic form factors of $\Lambda(1405)$ were also calculated within the chiral unitary model using the chiral effective theory for the couplings of the external currents with the hadronic constituents [11, 15, 16]. The first moment of the form factor, which corresponds to the mean-squared radius in the limit that the resonance width goes to zero, was also calculated. The electric first moment of $\Lambda(1405)$ at the pole position was obtained as a complex number $-0.13 + 0.30i \text{ fm}^2$, whose modules, 0.33 fm$^2$, is much larger than the neutron charge radius. This may imply that the $\Lambda(1405)$ has a spatially larger size than the typical hadronic size.

The dynamical description of $\Lambda(1405)$ shows that one of the $\Lambda(1405)$ states is almost a bound state of $\bar{K}N$. The scalar mesons, $f_0(980)$ and $a_0(980)$, have been also considered to have large components of $\bar{K}K$. A state which is essentially described by hadronic components is called hadronic molecular state. In the hadronic molecular states, constituent hadrons keep their identities as they are in isolated systems. Estimating the strength of the inter-hadron interactions between $\bar{K}N$ and $\bar{K}K$ by chiral effective theory, in which the Weinberg-Tomozawa interaction is responsible for the low-energy s-wave $\bar{K}N$ and $\bar{K}K$ attractions, we see that this interaction is strong enough to produce $\bar{K}N$ and $\bar{K}K$ bound states with a few tens MeV binding energies. If one compares this chiral effective interaction with the $NN$ interaction, the $\bar{K}N$ and $\bar{K}K$ attractions are very strong, because the $NN$ bound state, that is deuteron, has as small as 2 MeV binding energy. But if one compares the binding energies to the typical hadronic scale of several hundred MeV, one should say that these inter-hadron interactions are weak.

It is also interesting to mention the fact that the $\bar{K}N$ and $\bar{K}K$ attractions obtained from the Tomozawa-Weinberg interaction have very similar strengths, because the strength of the Tomozawa-Weinberg interaction is given by the SU(3) flavor symmetry and $K$ and $N$ are classified into the same state vector in the octet representation. This similarity between $K$ and $N$ leads to systematics of three-body kaonic systems [19], $\bar{K}NN$, $\bar{K}KN$ and $\bar{KK}K$. The $\bar{KK}N$ quasibound state is an important resonance for the $N^*$ physics, since it has the same quantum number as $N^*(P_{11})$. This state was studied first with a simple single-channel non-relativistic potential model [20] and later was investigated in a more sophisticated calculation [21, 22] based on a coupled-channels Faddeev method and a simple fixed center approximation of three-body calculation [23]. These approaches lead to a very similar $N^*$ resonance state appearing around 1910 MeV. The potential model calculation shows that the root mean-squared radius of the $\bar{K}NN$ state is as large as 1.7 fm, which are similar with the radius of $^4\text{He}$. The inter-hadron distances are comparable with an average nucleon-nucleon distance in nuclei. Thus, this $N^*$ resonance has an much larger spatial size than typical $N^*$ resonances which are made of constituent quarks confined in 1 fm. The hadronic molecular states could be identified by production rates in heavy ion collisions, since coalescence of hadrons to produce loosely bound hadronic molecular systems is more probable than quark coalescence for compact multi-quark systems [24].
SUMMARY

Coupled-channel approaches, for instance the chiral unitary model, provide us with a dynamical description of meson-baryon scattering, describing simultaneously both resonance and nonresonant scattering applicable to reaction calculations. They also give us hadronic description in which all contents of the models are hadrons. Nevertheless, obtained hadron resonances are not necessarily genuine hadronic composite objects and sources of quark dynamics can be hidden everywhere. Thus, detailed theoretical analyses are necessary to interpret the structure of resonances. In the chiral unitary model, the resonance can be described microscopically in terms of the constituent hadrons based on fundamental interactions given in the chiral effective theory. This is well suited to the calculation of form factors of resonances. For the effective constituents in baryons structure hadrons themselves can be effective constituents in some hadron resonances, for instance few-body systems with nucleons and kaons.

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