Space-time analysis of reaction at RHIC

Fabrice Retière† §
† Lawrence Berkeley National Laboratory, One Cyclotron Road, Berkeley CA 94720-8169

Abstract.

Space-time information about the Au-Au collisions produced at RHIC are key tools to understand the evolution of the system and especially assess the presence of collective behaviors. Using a parameterization of the system’s final state relying on collective expansion, we show that pion source radii can be tied together with transverse mass spectra and elliptic flow within the same framework. The consistency between these different measures provide a solid ground to understand the characteristics of collective flow and especially the possible peculiar behavior of particles such as Ξ, Ω or φ. The validity of the short time scales that are extracted from fits to the pion source size is also addressed. The wealth of new data that will soon be available from Au-Au collisions at $\sqrt{s_{NN}} = 200$ GeV, will provide a stringent test of the space-time analysis framework developed in these proceedings.

1. Introduction

Ultra-relativistic heavy ions collisions are believed to produce initially hot systems that cool down by expanding until their temperature is low enough to release hadrons. Were the conditions within these systems adequate to free, for a time, quarks and gluons of their hadronic confinement? This question will remain pending at the end of the proceedings. Our aim will be to assess whether or not the particle space-time emission pattern is consistent with a scenario where initially hot systems have cooled down by expanding. In this scenario, only a small fraction of the particles measured in the detectors, are produced in the initial collisions between the nuclei. Most of the initially produced particles do not escape the system and reinteract inelastically with other particles. Only the high transverse momentum particles are dominantly produced in the initial nucleus-nucleus collisions as there is not enough energy available to produce them after the system has cooled down. Following this observation, we only consider particles with a transverse momentum smaller than 2 GeV/c.

Hydrodynamic models describe the evolution of systems from hot initial stages until freeze-out by assuming zero mean free path, which provides the limit of maximum transverse expansion [1]. However, this affirmation is not strictly true because the equation of state, and especially its main feature, the phase transition, regulates the

§ To whom correspondence should be addressed (fgretiere@lbl.gov)
pressure, i.e. the collective expansion strength. Hydrodynamic calculations may be used as a baseline to assess the presence of collective expansion, and ultimately to extract the equation of state of the system. Complementary to the hydrodynamic calculations, we will also base our discussion on parameterization of the system final state. Such parameterizations allow to quantify key points of the system final state as well as to investigate the consistency between various measures.

In these proceedings, we will first show that particle yield and transverse momentum distribution may be described assuming collective behaviors. Then, we will investigate how the spatial extent of the pion source may be related to collective expansion. Finally, we will discuss whether or not the time scales that are extracted assuming collective expansion make sense.

2. Collective flow and transverse momentum spectra

The relative yields of many different particle species are well described by parameterizations assuming that particles are statistically produced within the available phase space [2]. With the additional assumption that the system is in thermal equilibrium at freeze-out, the fit parameters may be understood as temperature and chemical potential. The extracted temperature is on the order of 170 MeV at RHIC energies, which is remarkably close to the phase transition temperature predicted by lattice QCD [3]. It suggests but does not prove that particle yields are frozen out at the boundary between partonic and hadronic matter.

Particle yields stop changing when the number of inelastic interactions become insignificant. However, the number of elastic or pseudo-inelastic interactions (such as $\pi^+\pi^- \rightarrow \rho \rightarrow \pi^+\pi^-$) may still remain significant. Thus, the particle momenta may still significantly change after the yields are frozen out (at chemical freeze-out) leading to a separate kinetic freeze-out stage. Furthermore, if the chemical freeze-out corresponds to the boundary between partonic and hadronic matter, the interactions that lead to kinetic freeze-out depend on hadronic cross-sections. Thus, the chemical and kinetic freeze-out of particles with low hadronic cross-sections, such as, presumably, Ξ, Ω or φ may coincide.

Hydrodynamic calculations are successful at reproducing particle yields and transverse momentum spectra that have been measured at RHIC [4]. As shown in Figure 1, the blast wave parameterization [5] is also successful at reproducing π, K, p, Λ transverse momentum spectra measured in Au-Au collisions at $\sqrt{s_{NN}} = 130$ GeV published by the PHENIX [6] and STAR [7] collaborations. This parameterization is designed to mock up the final state of the hydrodynamic evolution. Thus, it relies on transverse expansion to reproduce the spectra of different particle species with a single set of parameters: a freeze-out temperature and a mean flow rapidity (or equivalently velocity). The freeze-out temperature is independent of the particle species the freeze-out time or position. The system is confined within an infinitely long cylinder along the beam line. Longitudinal boost invariance is assumed. The transverse flow rapidity
Figure 1. Comparison of measured spectra in Au-Au collisions at $\sqrt{s_{NN}} = 130$ GeV with the blast wave calculations performed with the best fit parameters in three centrality bins. Closed circle are central data, open circle are mid-central data and crosses are peripheral data.

|                  | Central     | Mid-central | Peripheral |
|------------------|-------------|-------------|------------|
| $\pi$, K, p spectra | 0-5%       | 15-30%      | 60-92%     |
| $\Lambda$ spectra | 0-5%       | 20-35%      | 35-75%     |
| $\chi^2$/dof     | 55.1/58     | 105.6/58    | 64.0/37    |
| T (MeV)          | 108 ± 3     | 106 ± 3     | 95 ± 4     |
| $\langle \beta_T \rangle (c)$ | 0.53 ± 0.01 | 0.52 ± 0.02 | 0.47 ± 0.02 |

Table 1. Upper section: data used in the fit with their different centrality range. Bottom section: bet's fit parameters and $\chi^2$/dof.

linearly increases from zero at the center to a maximum value ($\rho_0 = 3/2\langle \rho \rangle$) at the edge of the system. In addition to the temperature and flow rapidity, the particle yields are also free parameters. The best fit parameters and $\chi^2$/dof are summarized in table 1.

The kinetic freeze-out temperature that is obtained from blast wave fits to transverse...
momentum spectra (≈ 100 MeV) is significantly lower than the chemical freeze out temperature (≈170 MeV). However, the study reported in Ref [8] shows that the separation between chemical and kinetic freeze-out vanishes if resonance feed-down is properly accounted for. The authors of this paper are able to reproduce the π, K and p transverse momentum spectra published by the PHENIX collaboration [6] with a temperature of 165 MeV by decaying all the resonances that feed-down into π, K and p (such as ρ, ω, K∗, Δ,...). However, this result is contradicted by a study reported in Ref. [9] where it is shown that forcing the chemical and kinetic freeze-out temperature to coincide significantly increases the $\chi^2$/dof. Thus, the effect of resonances on transverse momentum spectra remains to be clarified in order to conclude whether freeze-out occurs in one or two stages. In these proceedings, we will further investigate the later hypothesis where chemical freeze-out precedes kinetic freeze-out.

One important consequence of a freeze-out in two stages is the possibility that the Ξ, Ω and φ behave differently than π, K, p and Λ because of their presumably lower hadronic cross-sections. In that case their kinetic freeze-out temperature is expected to be close to the chemical freeze-out temperature. Hence, if the chemical freeze-out stage coincides with the transition from partonic to hadronic matter, those particles allow to measure the amount of collective expansion that build up at the partonic stage. Such speculations can be investigated using the data reported at this conference on Ξ, Ω and φ spectra in Au-Au collisions at $\sqrt{s_{NN}} = 200$ GeV. From Ξ, Ω and φ spectra measured in Au-Au collisions at $\sqrt{s_{NN}} = 130$ GeV, blast wave fits allow to conclude only that Ξ reach kinetic freeze-out at a higher temperature and lower transverse flow velocity than π, K, p and Λ [7]. Due to the lack of statistics such claim cannot be made from Ω or φ spectra. Interestingly, data reported by the NA49 [10] and NA57 [11] collaborations show that Ξ and Ω spectra are well described by a blast wave calculation using the same parameters that reproduce π, K, p and Λ spectra. However, the statistical significance of these results remain to be addressed.

We have shown that particle yields and transverse momentum spectra fit in a scenario where the initial hot systems expand and cool down. On the other hand, a few important issues have not been unambiguously established: does the chemical freeze-out coincide with the partonic to hadronic matter transition? Do the hadronic cross-sections influence the temperature and flow velocity at which particle freeze-out? Are the chemical and kinetic freeze-out stages decoupled? We will revisit this last question when discussing the time scale issues. Before that, we will investigate the consequences of the collective expansion on the position where particles are emitted. This study will allow us to discuss a fundamental questions that we have ignored so far; is collective expansion the only scenario that is consistent with RHIC data?

3. Spatial issues, assessing collective flow

Transverse momentum spectra do not show unambiguously that the systems produced in ultra-relativistic heavy ion collisions undergo collective transverse expansions. Indeed,
transverse mass scaling if it is established may arise from models such as the Color Glass Condensate [13]. Furthermore, the NA49 collaboration has shown [14] that the transverse momentum distribution of protons produced by the projectile in p-Pb collisions is similar to the distribution of protons from Pb-Pb collisions. Thus, initial state effects such as Color Glass Condensate or random walk of partons may mock up the effect of transverse flow. However, the collective expansion also affects the position of particle emission. Hence, measures which are sensitive to the position may be used to assess the presence of collective flow.

![Figure 2](image.png)

**Figure 2.** Comparison of the pion source data measured by the STAR (circles and crosses) [15] and the PHENIX (boxes) [16] collaborations in Au-Au collisions at \( \sqrt{s_{NN}} = 130 \) GeV with the blast wave calculations. Only the STAR data were used to constrain the blast wave parameters. The closed circles are the central data, the open circles are the mid-central data and the crosses are peripheral data.

Pion source sizes are measured by two-pion interferometry techniques. One very important feature of this technique is that it probes the separation between pions at low relative momentum. Thus when space-momentum correlations are present, the measured source size does not reflect the whole source size. Space-momentum correlations are at the core of any collective expansion. Indeed, in such scenario, particles push each other away from the hot center toward the vacuum. In other words, particles are emitted outward, their spatial and momentum azimuthal angles are close to each other. Furthermore, the particles that pick up a large momentum kick from the collective expansion tend to be emitted close to the edge of the system. These features, which are characteristic of the collective expansion, are obtained in hydrodynamic calculations and parameterized in the blast wave framework.
Pion source sizes have been measured in Au-Au collisions at $\sqrt{s_{NN}} = 130$ GeV by the STAR and PHENIX collaboration [15, 16]. The measured source radii shown in figure 2 are decomposed in the three directions, $r_{\text{out}}$, $r_{\text{side}}$, and $r_{\text{long}}$. They are calculated in the Longitudinally Comoving System where the pair velocity is zero. $r_{\text{long}}$ is parallel to the beam axis, $r_{\text{out}}$ is parallel to the pair transverse momentum and $r_{\text{side}}$ is perpendicular to both beam axis and pair transverse momentum. It can be shown that $r_{\text{side}}$ only probes the spatial extent of the source while $r_{\text{out}}$ and $r_{\text{long}}$ are sensitive to the system lifetime and the duration of particle emission [17].

Hydrodynamic calculations fail to reproduce the measured radii [18]. In most cases, these calculations underestimate $r_{\text{side}}$ and overestimate $r_{\text{out}}$ and $r_{\text{long}}$. In other words, they underestimate the system size and over estimate its lifetime and emission duration. Does it mean that the collective expansion scenario is ruled out? Not necessarily, since the transverse momentum dependence of the radii is, in general, well described and it is only the magnitude of the radii that cannot be reproduced. This observation is confirmed when using the blast wave parameterization as shown in figure 2. In this case, the radii are well reproduced because in the blast wave parameterization the system size, life time and emission duration are tunable parameters. The values of the best fit parameters are shown in table 2. The exact same parameterization that was used to the fit the transverse momentum spectra is applied [19]. The temperature and the flow velocity have been fixed to the values obtained from fits to transverse momentum spectra. The remaining free parameters are the radius (R) of the cylinder that confines the system, the system proper time ($\tau = \sqrt{t^2 - z^2}$) and the emission duration ($\Delta t$). The longitudinal boost invariance assumption motivates the use of the parameter $\tau$ rather than the time, $t$, in the laboratory frame.

The good fit of the data obtained with the blast wave parameterization shows that transverse momentum spectra and pion source size can be interpreted consistently in terms of collective expansion. Furthermore, preliminary data from the STAR collaboration shows that the blast wave parameterization predicts an average space-time separation between pion, kaon and proton sources that is consistent with the data [21]. The list of measures that can be used to test the blast wave parameterization also

|                  | Central | Mid-central | Peripheral |
|------------------|---------|-------------|------------|
| $\chi^2$/dof    | 10.9/15 | 0.7/3       | 0.6/3      |
| T (MeV) (fixed) | 108     | 106         | 95         |
| $\langle \beta_T \rangle$ (c) (fixed) | 0.53    | 0.52        | 0.47       |
| $R$(fm)         | 12.9 ± 0.2 | 11.0 ± 0.4 | 9.1 ± 0.3  |
| $\tau$(fm/c)   | 8.9 ± 0.3 | 7.4 ± 1.2   | 6.5 ± 0.8  |
| $\Delta t$(fm/c)| 0.002 ± 1.4 | 0.8 ± 3.2  | 0.8 ± 1.9  |

Table 2. Blast wave parameters that best fit the pion source radii published by the STAR collaboration. The temperature and flow velocity parameters were obtained by fit to transverse mass spectra (see table 1).
includes pion source radii measured with respect to the reaction plane \cite{22}, and kaon source radii. Thus, collective transverse expansion as parameterized in the blast wave framework provide a consistent, well constrained description of the final state of the systems produced in Au-Au collisions at RHIC. Bringing in two-particle correlation analyses has allowed us to assess the presence of collective expansion.

However, this definite conclusion may not hold when new and more precise data are available from Au-Au collisions at 200 GeV. For example, the invariant radius extracted from two $K^0_S$ interferometry reported by the STAR collaboration \cite{23} is too large to be reproduced in the blast wave framework. On the other hand, the blast wave parameterization may prove to be a very valuable tool to assess whether $\Xi$, $\Omega$ and $\phi$ undergo the same collective expansion than $\pi$, $K$, $p$ and $\Lambda$, since it can be used to interpret transverse momentum spectra as well as $v_2$ and results from two-particle correlation analysis such as $\pi^\pm - \Xi^-$ or $\pi^\pm - \Omega^-$. 

4. A problem with the time scales?

The agreement of the blast wave parameterization with data is reached because the system lifetime and emission duration are small. Hydrodynamic calculations are unable to produce such a short life-time. But, do such short time scales make sense? Only extreme models achieve short time scales (\cite{24,25} for example). Thus, we will investigate whether or not these short time-scales are supported by other measurements.

![Figure 3. Comparison of $v_2$ for pions and protons measured by the STAR collaboration \cite{20} in Au-Au collisions at $\sqrt{s_{NN}} = 130$ GeV with the blast wave calculation obtained with the parameters that best fit the data. The crosses are the most peripheral events (45-85% of centrality), the open circle at the mid-peripheral events (11-45%) and the closed circles are the central data (0-11%).](image)

The spatial source shape carries qualitative information about the system lifetime. When the colliding ions do not fully overlap, the initial energy density is distributed
over an ellipse. The spatial energy density gradients are larger along the ellipse short radius (in-plane radius) than along the long radius (out-of-plane radius), which gives rise to pressure gradient stronger in-plane than out-of-plane. This phenomenon is experimentally quantified by the parameter $v_2$ as an azimuthal anisotropy of the particle emission, where more particles are emitted in-plane than out-of-plane \cite{20}. Furthermore, the collective expansion fights against the initial anisotropy of the system. Along its lifetime the system evolves from an out-of-plane extended shape towards a cylindrical shape and even towards an in-plane extended shape depending on how long it lives \cite{26}. Thus, a short life time would be associated with an out-of-plane extended source while a long life time would be associated with an in-plane source. The blast wave parameterization may be used to extract the source shape from the data by adding two additional parameters allowing the final state of the system to be an ellipse instead of a disk and by allowing the flow velocity to vary with the azimuthal angle. The ellipse shape is parameterized by splitting the system radius in two: a radius in-plane and a radius out-of-plane. Figure \ref{fig:3} shows a fit to the $v_2$ parameters performed with the blast wave parameterization by fixing the temperature and overall flow velocity ($\langle \beta_T \rangle$) to the values obtained by fits to transverse momentum spectra. From these fits, we extract the source aspect ratio (in-plane radius divided by the out-of-plane radius).

Table 3 shows a comparison of the initial aspect ratio obtained by a simple Wood-Saxon nuclear overlap calculation \cite{27} with the final aspect ratio obtained by blast wave fits to $v_2$. The important point to notice is that the source remains out-of-plane extended, which is consistent with short lived systems. The same qualitative conclusion has been independently obtained by analysis of the preliminary STAR data of the pion source size with respect to the reaction plane angle \cite{22}. The short system life time obtained from analysis of the pion radii and the out-of-plane extended source of the system final state are consistent.

Is the very short emission duration extracted from pion source radii also consistent with other measures? Before discussing this issue, it is important to recall that the emission duration is mostly determined by the difference (or the ratio) between the $r_{side}$ and $r_{out}$. As pointed out by the CERES collaboration \cite{28}, the ratio $r_{out}/r_{side}$ and

|                  | Central | Mid-central | Peripheral |
|------------------|---------|-------------|------------|
| $\chi^2$/dof    | 14.6/13 | 47.4/16     | 9.7/13     |
| $T$ (MeV) (fixed)| 108     | 106         | 95         |
| $\langle \beta_T \rangle$ (c) (fixed)| 0.53 | 0.52 | 0.47 |
| $\langle \beta_{T_{in-plane}}/\beta_{T_{out-of-plane}} \rangle$ | $1.067 \pm 0.009$ | $1.060 \pm 0.007$ | $1.05 \pm 0.01$ |
| $R_{in-plane}/R_{out-of-plane}$ | $1.01 \pm 0.03$ | $0.86 \pm 0.06$ | $0.79 \pm 0.05$ |
| Initial state   |         |             |            |
| $R_{in-plane}/R_{out-of-plane}$ | 0.80 | 0.59 | 0.42 |

Table 3. Blast wave parameters that best fit the pion and proton $v_2$. The last row is a calculation of the source aspect ratio from a nuclear overlap approximation \cite{27}.
hence the emission duration may be artificially increased by an inappropriate Coulomb correction applied to the two-pion correlation functions. With this point in mind, we argue that emission duration cannot be very small (i.e. less than 1 fm/c) because it wouldn’t leave enough time for the system to cool down from chemical to kinetic freeze-out and because it is not consistent with the measured yield of $K^*$ \cite{29} and $\Lambda^*(1520)$ \cite{30} resonances. The first point is valid if the chemical freeze-out temperature is truly higher than the kinetic freeze-out temperature. In this case, the requirement that the entropy cannot decrease imposes, in a bounded system such as the one described by the blast wave parameterization, that the time between chemical and kinetic freeze-out is larger than 4-5 fm/c \cite{31}. The second point has to do with the fact that the measured $K^*$ and $\Lambda^*(1520)$ yields are lower than calculated at chemical freeze-out using the same parameters (temperature and chemical potential) that reproduce the relative yield of all the other hadrons. This suppression is understood by arguing that the $K^*$ and $\Lambda^*$ decay products lose the invariant mass correlation when reinteracting (pseudo-)elastically before freezing-out. These data challenge the scenario where chemical and kinetic freeze-out coincide, as this interpretation is only valid if (pseudo)elastic interactions occur after chemical freeze-out. The lifetime of the $K^*$ and $\Lambda^*$ provide a gauge of the time between chemical and kinetic freeze-out which is on the order of 4-5 fm/c \cite{30}. Thus, the short emission duration extracted from fit to pion radii is inconsistent with other measures. However, this conclusion will need to be revisited when pion source radii are extracted using the proper Coulomb correction technique.

5. Conclusions

Assessing the presence and characteristics of collective behaviors in ultra-relativistic heavy ion collisions is a key step towards the discovery of partonic matter. This step is well underway; we have shown that the data from Au-Au collisions at $\sqrt{s_{NN}} = 130$ GeV are consistent with systems undergoing a collective expansion. However, definite conclusions await the resolution of the following pending issues:

- Carefully assess the effect of resonance feed-down on transverse momentum spectra.
- Improve the two-pion interferometry data by performing a proper Coulomb correction and extending the data range to higher transverse momentum. This may clarify the time scale issues.
- Establish in a statistically significant manner whether or not $\Xi$, $\Omega$ and $\phi$ freeze-out at the same temperature and transverse flow velocity as $\pi$, $K$, and protons.
- Investigate the no flow hypothesis by studying p-p and d-Au collisions. Indeed, space-momentum correlations such as the one that arise from jet fragmentation may mock up the effect of flow.

In addition to the measure described in these proceedings, new analyses will available soon. The large statistics available at RHIC will allow to study the behavior of $\Xi$, $\Omega$ and $\phi$, with never used before tools such as $v_2$ or two-particle correlations. Balance
function analyses may also bring crucial information. Furthermore the development of parameterizations such as the blast wave provides new opportunities to assess the consistency of the data within a single framework. However, with the quality and the variety of the data increasing significantly such parameterizations will be very significantly challenged in the coming future. In any cases, combining space-time analysis with yield and spectra analysis will remain the key to reach a global understanding of the ultra-relativistic heavy ion collisions.

I wish to thank the conference organizers for inviting me and hence providing me with the opportunity of developing these ideas. The blast wave fits were developed together with Mike Lisa following ten years of development within the community. These proceedings is the results of fruitfull discussions with Ulrich Heinz, Mike Lisa, Dan Magestro, Thomas Peitzmann, Kai Schweda, Nu Xu, and Zanghu Xu, Eugene Yamamoto. I also wish to thank the whole STAR collaboration for the stimulating discussions that have gone along the release of the preliminary data shown at this conference.

[1] For example, D. Teaney, J. Lauret, E.V. Shuryak, nucl-th/0110037 or P. Huovinen, nucl-th/0305064
[2] P. Braun-Munzinger at al. Phys. Lett. B 518 (2001) 41
[3] F. Karsch, Lect. Notes Phys. 583 (2002) 209 and hep-lat/0106019
[4] For example P. Huovinen, nucl-th/0210024
[5] E. Schnedermann, J. Sollfrank, and U. Heinz, Phys. Rev. C48 (1993) 2462
[6] K. Adcox et al., Phys. Rev. Lett. 88 (2002) 242301
[7] C. Adler et al. Phys. Rev. Lett. 89 (2002) 092301
[8] W. Broniowski, A. Baran, W.Florkowski, nucl-th/0212053 and nucl-th/0212052
[9] T. Peitzmann, Eur.Phys.J. C26 (2003) 539-549
[10] V. Friese (NA49 collaboration) these proceedings, nucl-ex/0305017
[11] L. Sandor (NA57 collaboration) these proceedings
[12] J. Castillo (STAR collaboration) these proceedings
[13] J. Schaffner-Bielich, D. Kharzeev, L. McLerran, R. Venugopalan, Nucl.Phys. A705 (2002) 494
[14] H.G. Fischer (NA49 collaboration), hep-ex/0209043
[15] C. Adler et al., Phys. Rev. Lett. 87 (2001) 082301
[16] K. Adcox et al., Phys. Rev. Lett. 88 (2002) 192302
[17] S. Pratt, Phys. Rev. Lett. 53 (1984) 53
[18] For example, S. Soff, hep-ph/0202240
[19] B. Tomasik, U. Achim Wiedemann, U.Heinz, Heavy Ion Phys. 17 (2003) 105-143
[20] C. Adler et al. Phys. Rev. Lett. 87 (2001)182301
[21] F. Retière (STAR collaboration) , nucl-ex/0212026
[22] M. Lisa (STAR collaboration), Beckenbridge proceedings
[23] S. Bekele (STAR collaboration), these proceedings
[24] T. J. Humanie, nucl-th/0205053
[25] D. Molnar and M. Gyulassy, nucl-th/0211017
[26] U. Heinz, these proceedings, and P. Kolb and U. Heinz, nucl-th/0208047
[27] J. Eskola, Nucl. Phys. B523 (1989) 37
[28] D. Adamova et al., Nucl.Phys. A714 (2003) 124-14
[29] H. Zhang (STAR collaboration), these proceedings
[30] L. Gaudichet (STAR collaboration), these proceedings
[31] O. Baranikova and F. Wang, private communication. Arguments based on C.Y. Wong in "Introduction to High Energy Heavy-Ion Collisions", World Scientific, Singapore (1994)