Exclusive semileptonic decays of $\Lambda_b \to \Lambda l^+l^-$ in supersymmetric theories

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The weak decays of $\Lambda_b \to \Lambda l^+l^- (l = e, \mu)$ are investigated in Minimal Supersymmetric Standard Model (MSSM) and also in Supersymmetric (SUSY) SO(10) Grand Unified Models. In MSSM the special attention is paid to the Neutral Higgs Bosons (NHBs) as they make quite a large contribution in exclusive $B \to X_s l^+l^-$ decays at large $\tan \beta$ regions of parameter space of SUSY models, since part of SUSY contributions is proportional to $\tan^3 \beta$. The analysis of decay rate, forward-backward asymmetries, lepton polarization asymmetries and the polarization asymmetries of $\Lambda$ baryon in $\Lambda_b \to \Lambda l^+l^-$ show that the values of these physical observables are greatly modified by the effects of NHBs. In SUSY SO(10) GUT model, the new physics contribution comes from the operators which are induced by the NHBs penguins and also from the operators having chirality opposite to that of the corresponding SM operators. SUSY SO(10) effects show up only in the decay $\Lambda_b \to \Lambda + \tau^+\tau^-$ where the longitudinal and transverse lepton polarization asymmetries are deviate significantly from the SM value while the effects in the decay rate, forward-backward asymmetries and polarization asymmetries of final state $\Lambda$ baryon are very mild. The transverse lepton polarization asymmetry in $\Lambda_b \to \Lambda + \tau^+\tau^-$ is almost zero in SM and in MSSM model. However, it can reach to $-0.1$ in SUSY SO(10) GUT model and could be seen at the future colliders; hence this asymmetry observable will provide us useful information to probe new physics and discriminate between different models.

I. INTRODUCTION

From last decade, rare decays induced by flavor changing neutral currents (FCNCs) $b \to s l^+l^-$ have become the main focus of the studies due to the CLEO measurement of the radiative decay $b \to s\gamma$. In the standard model (SM) these decays are forbidden at tree level and can only be induced by Glashow-Iliopoulos-Maiani mechanism via loop diagrams. Hence, such decays will provide helpful information about the parameters of Cabbibo-Kobayashi-Maskawa (CKM) matrix elements as well as various hadronic form factors. In the literature there have been intensive studies on the exclusive decays $B \to P(V,A) l^+l^-$ both in the SM and beyond, where the notions $P,V$ and $A$ denote the pseudoscalar, vector and axial vector mesons respectively.

It is generally believed that supersymmetry (SUSY) is not only one of the strongest competitor of the SM but is also the most promising candidate of new physics. The reason is that it offers a unique scheme to embed the SM in a more fundamental theory where many theoretical problems such as gauge hierarchy, origin of mass, and Yukawa couplings can be resolved. One direct way to search for SUSY is to discover SUSY particles at high energy colliders, but unfortunately, so far no SUSY particles have been found. Another way is to search for its effects through indirect methods. The measurement of invariant mass spectrum, forward-backward asymmetry and polarization asymmetries are the suitable tools to probe new physics effects. For most of the SUSY models, the SUSY contributions to an observable appear at loop level due to the $R$-parity conservation. Therefore, it has been realized for a long time that rare processes can be used as a good probe for the searches of SUSY, since in these processes the contributions of
SUSY and SM arises at the same order in perturbation theory \[1 2\].

Motivated from the fact that in two Higgs doublet model and in other SUSY models, Neutral Higgs Bosons (NHBs) could contribute largely to the inclusive processes \( B \rightarrow X_s l^+ l^- \), as part of supersymmetric contributions is proportional to the \( \tan^3 \beta \) \[13\]. Subsequently, the physical observables, like branching ratio and forward-backward asymmetry, in the large \( \tan \beta \) region of parameter space in SUSY models can be quite different from that in the SM. In addition, similar effects in exclusive \( B \rightarrow K(K^*) \ l^+ l^- \) decay modes are also investigated \[12\], where the analysis of decay rates, forward-backward asymmetries and polarization asymmetries of final state lepton indicates the significant role of NHBs. It is believed that physics beyond the SM is essential to explain the problem of neutrino oscillation. To this purpose, a number of SUSY SO(10) models have been proposed in the literature \[14, 15, 16, 17\]. One such model is the SUSY SO(10) Grand Unified Models (GUT), in which there is a complex flavor non-diagonal down-type squark mass matrix element of 2nd and 3rd generations of order one at the GUT scale \[16\]. This can induce large flavor off-diagonal coupling such as the coupling of gluino to the quark and squark which belong to different generations. These couplings are in general complex and may contribute to the process of flavor changing neutral currents (FCNCs).

The above analysis of physical observables in \( B \rightarrow K(K^*) \ l^+ l^- \) decay is extended in SUSY SO(10) GUT model in Ref. \[18\]. It is believed that the effects of the counterparts of usual chromo-magnetic and electromagnetic dipole moment operators as well as semileptonic operators with opposite chirality are suppressed by \( m_s/m_b \) in the SM, but in SUSY SO(10) GUTs their effect can be significant, since \( \delta_{23}^{RR} \) can be as large as 0.5 \[14, 18\]. Apart from this, \( \delta_{23}^{RR} \) can induce new operators as the counterparts of usual scalar operators in SUSY models due to NHB penguins with gluino-down type squark propagator in the loop. It has been shown \[18\] that the forward-backward asymmetries as well as the longitudinal and transverse decay widths of \( B \rightarrow K(K^*) \ l^+ l^- \) decay, are sensitive to these NHBs effect in SUSY SO(10) GUT model which can be detected in the future \( B \) factories.

Compared to the \( B \) meson decays, the investigations of FCNC \( b \rightarrow s \) transition for bottom baryon decays \( \Lambda_b \rightarrow \Lambda l^+ l^- \) are much behind because more degrees of freedom are involved in the bound state of baryon system at the quark level. From the experimental point of view, the only drawback of bottom baryon decays is that the production rate of \( \Lambda_b \) baryon in \( b \) quark hadronization is about four times less than that of the \( B \) meson. Theoretically, the major interest in baryonic decays can be attributed to the fact that they can offer a unique ground to extract the helicity structure of the effective Hamiltonian for \( b \rightarrow s \) transition in the SM and beyond, which is lost in the hadronization of mesonic case. The key issue in the study of exclusive baryonic decays is to properly evaluate the hadronic matrix elements for \( \Lambda_b \rightarrow \Lambda \), namely the transition form factors which are obviously governed by non-perturbative QCD dynamics. Currently, there has been some studies in the literature on \( \Lambda_b \rightarrow \Lambda \) transition form factors in different models including pole model (PM) \[19\], covariant oscillator quark model (COQM) \[20\], MIT bag model (BM) \[21\] and non-relativistic quark model \[22\], QCD sum rule approach (QCDSR) \[23\], perturbative QCD (pQCD) approach \[24\] and also in the light-cone sum rules approach (LCSR) \[25\]. Using these form factors, the physical observables like decay rates, forward-backward asymmetries and polarization asymmetries of \( \Lambda \) baryon as well as of the final state leptons in \( \Lambda_b \rightarrow \Lambda l^+ l^- \) were studied in great details in the literature \[26, 27, 28, 29, 30, 31, 32, 33\]. It is pointed out that these observables are very sensitive to the new physics, for instance, the polarization asymmetries of \( \Lambda \) baryon in \( \Lambda_b \rightarrow \Lambda l^+ l^- \) decays heavily depend on the right handed current, which is much suppressed in the SM \[29\].

In this paper, we will investigate the exclusive decay \( \Lambda_b \rightarrow \Lambda l^+ l^- \ ( \ l = \mu, \tau \) both in the Minimal Supersymmetric Standard Model (MSSM) as well as in the SUSY SO(10) GUT model \[16\]. We evaluate the branching ratios,
forward-backward asymmetries, lepton polarization asymmetries and polarization asymmetries of Λ baryon with special emphasis on the effects of NHBs in MSSM. It is pointed out that different source of the vector current could manifest themselves in different regions of phase space. For low value of momentum transfer, the photonic penguin dominates, while the Z penguin and W box become important towards high value of momentum transfer [12]. In order to search the region of momentum transfer with large contributions from NHBs, the above decay in certain large tan β region of parameter space has been analyzed in SuperGravity (SUGRA) and M-theory inspired models [34]. We extend this analysis to the SUSY SO(10) GUT model [12], where there are some primed counterparts of the usual SM operators. For instance, the counterparts of usual operators in B → Xs γ decay are suppressed by ms/mb and consequently negligible in the SM because they have opposite chiralities. These operators are also suppressed in Minimal Flavor Violating (MFV) models [35, 36], however, in SUSY SO(10) GUT model their effects can be significant. The reason is that the flavor non-diagonal squark mass matrix elements are the free parameters and some of them have significant effects in rare decays of B mesons [37]. In our numerical analysis for Λb → Λl+ l− decays, we shall use the results of the form factors calculated by LCSR approach in Ref. [25], and the values of the relevant Wilson coefficient for MSSM and SUSY SO(10) GUT models are borrowed from Ref. [12, 18]. The effects of SUSY contributions to the decay rate and zero position of forward-backward asymmetry are also explored in this work. Our results show that not only the decay rates are sensitive to the NHBs contribution but the zero point of the forward-backward asymmetry also shifts remarkably. It is known that the hadronic uncertainties associated with the form factors and other input parameters have negligible effects on the lepton polarization asymmetries and polarization asymmetries of Λ baryon in Λb → Λl+ l− decays. We have also studied these asymmetries in the SUSY models mentioned above and found that the effects of NHBs are quite significant in some regions of parameter space of SUSY.

The paper is organized as follows. In Sec. II, we present the effective Hamiltonian for the dilepton decay Λb → Λl+ l−. Section III contains the definitions and numbers of the form factors for the said decay using the LCSR approach. In Sec. IV we present the basic formulas of physical observables like decays rate, forward-backward asymmetries (FBAs) and polarization asymmetries of lepton and that of the Λ baryon in Λb → Λl+ l−. Section V is devoted to the numerical analysis of these observables and the brief summary and concluding remarks are given in Sec. VI.

II. EFFECTIVE HAMILTONIAN

After integrating out the heavy degrees of freedom in the full theory, the general effective Hamiltonian for b → sl+ l− in SUSY SO(10) GUT model, can be written as [18]

\[
H_{eff} = -\frac{G_F}{2\sqrt{2}} V_{tb} V_{ts}^{*} \sum_{i=1}^{10} C_i(\mu) O_i(\mu) + \sum_{i=3}^{10} (C_i(\mu) O_i(\mu) + C'_i(\mu) O'_i(\mu)) + \sum_{i=1}^{8} (C_{Q_i}(\mu) Q_i(\mu) + C'_{Q_i}(\mu) Q'_i(\mu)),
\]

where O_i(\mu) (i = 1, ..., 10) are the four-quark operators and C_i(\mu) are the corresponding Wilson coefficients at the energy scale \(\mu\) [38]. Using renormalization group equations to resume the QCD corrections, Wilson coefficients are evaluated at the energy scale \(\mu = m_b\). The theoretical uncertainties associated with the renormalization scale can
be substantially reduced when the next-to-leading-logarithm corrections are included \[30\]. The new operators \(Q_i(\mu)\) \((i = 1, \ldots, 8)\) come from the NHBs exchange diagrams, whose manifest forms and corresponding Wilson coefficients can be found in \[40\], \[41\]. The primed operators are the counterparts of the unprimed operators, which can be obtained by flipping the chiralities in the corresponding unprimed operators. It needs to point out that these primed operators will appear only in SUSY SO(10) GUT model and are absent in SM and MSSM \[12\].

The explicit expressions of the operators responsible for \(\Lambda_b \to \Lambda c\) transition are given by

\[
\begin{align*}
O_7 & = \frac{e^2}{16\pi^2} m_b (\bar{s}\sigma_{\mu\nu} P_L b) F^{\mu\nu}, & \quad O_7' & = \frac{e^2}{16\pi^2} m_b (\bar{s}\sigma_{\mu\nu} P_L b) F^{\mu\nu} \\
O_9 & = \frac{e^2}{16\pi^2} (\bar{s}\gamma_\mu P_L b)(\bar{l}\gamma_\mu l), & \quad O_9' & = \frac{e^2}{16\pi^2} (\bar{s}\gamma_\mu P_R b)(\bar{l}\gamma_\mu l) \\
O_{10} & = \frac{e^2}{16\pi^2} (\bar{s}\gamma_\mu P_L b)(\bar{l}\gamma_\mu \gamma_5 l), & \quad O_{10}' & = \frac{e^2}{16\pi^2} (\bar{s}\gamma_\mu P_R b)(\bar{l}\gamma_\mu \gamma_5 l) \\
Q_1 & = \frac{e^2}{16\pi^2} (\bar{s}P_R b)(\bar{l}l), & \quad Q_1' & = \frac{e^2}{16\pi^2} (\bar{s}P_L b)(\bar{l}l) \\
Q_2 & = \frac{e^2}{16\pi^2} (\bar{s}P_R b)(\bar{l}\gamma_5 l), & \quad Q_2' & = \frac{e^2}{16\pi^2} (\bar{s}P_L b)(\bar{l}\gamma_5 l)
\end{align*}
\]
with \(P_{L,R} = (1 \pm \gamma_5)/2\). In terms of the above Hamiltonian, the free quark decay amplitude for \(b \to s l^+l^-\) can be derived as \[13\]:

\[
\mathcal{M}(b \to s l^+l^-) = -\frac{G_F a}{\sqrt{2}} V_{tb} V_{ts}^* \left\{ C_9^{\tau f}(\bar{s}\gamma_\mu P_L b)(\bar{l}\gamma_\mu l) + C_{10}(\bar{s}\gamma_\mu P_L b)(\bar{l}\gamma_\mu \gamma_5 l) \right. \\
\left. - 2m_b C_{7}^{\tau f}(\bar{s}\sigma_{\mu\nu} \frac{q^\nu}{s} P_R b)(\bar{l}\gamma_\mu l) + C_{Q_1}(\bar{s}P_R b)(\bar{l}l) + C_{Q_2}(\bar{s}P_R b)(\bar{l}\gamma_5 l) + (C_1(m_b) \leftrightarrow C_1'(m_b)) \right\}
\]

(3)

where \(s = q^2\) and \(q = p_{\Lambda_b} - p_{\Lambda}\) is the momentum transfer. Due to the absence of \(Z\) boson in the effective theory, the operator \(O_{10}\) can not be induced by the insertion of four-quark operators. Therefore, the Wilson coefficient \(C_{10}\) does not renormalize under QCD corrections and hence it is independent on the energy scale. Moreover, the above quark level decay amplitude can receive additional contributions from the matrix element of four-quark operators, \(\sum_{i=1}^{6}|l^+l^-s|O_i|b\rangle\), which are usually absorbed into the effective Wilson coefficient \(C_9^{\tau f}(\mu)\). To be more specific, we can decompose \(C_9^{\tau f}(\mu)\) into the following three parts \[42\], \[43\], \[44\], \[45\], \[46\], \[47\], \[48\].

\[
C_9^{\tau f}(\mu) = C_9(\mu) + Y_{SD}(z, s') + Y_{LD}(z, s'),
\]

where the parameters \(z\) and \(s'\) are defined as \(z = m_c/m_b\), \(s' = q^2/m_b^2\). \(Y_{SD}(z, s')\) describes the short-distance contributions from four-quark operators far away from the \(c\bar{c}\) resonance regions, which can be calculated reliably in the perturbative theory. The long-distance contributions \(Y_{LD}(z, s')\) from four-quark operators near the \(c\bar{c}\) resonance cannot be calculated from first principles of QCD and are usually parameterized in the form of a phenomenological Breit-Wigner formula making use of the vacuum saturation approximation and quark-hadron duality. The manifest expressions for \(Y_{SD}(z, s')\) and \(Y_{LD}(z, s')\) can be written as \[25\], \[26\], \[27\], \[28\], \[29\].

\[
Y_{SD}(z, s') = h(z, s')(3C_1(\mu) + C_2(\mu) + 3C_3(\mu) + C_4(\mu) + 3C_5(\mu) + C_6(\mu)) \\
- \frac{1}{2} h(1, s')(4C_1(\mu) + 4C_2(\mu) + 3C_3(\mu) + C_6(\mu)) \\
- \frac{1}{2} h(0, s')(3C_3(\mu) + 3C_4(\mu)) + \frac{2}{9} (3C_3(\mu) + C_4(\mu) + 3C_5(\mu) + C_6(\mu)),
\]

(4)
\[ Y_{LD}(z, s') = \frac{3}{\alpha^2_{em}}(3C_1(\mu) + C_2(\mu) + 3C_3(\mu) + C_4(\mu) + 3C_5(\mu) + C_6(\mu)) \sum_{j=\psi, \psi'} \omega_j(q^2)k_j \frac{\pi \Gamma(j \rightarrow l^+l^-)M_j}{q^2 - M_j^2 + i\Gamma_{j\text{tot}}}, \]

with

\[ h(z, s') = -\frac{8}{9} \ln z + \frac{4}{27}x - \frac{2}{9}(2 + x)|1 - x|^{1/2} \left\{ \ln \frac{\sqrt{1 - x} + 1}{\sqrt{1 - x} - 1} - i\pi \right\} \quad \text{for } x \equiv 4s^2/s' < 1, \]

\[ h(0, s') = \frac{8}{27} - \frac{8}{9} \ln \frac{m_b}{\mu} - \frac{4}{9} \ln s' - \frac{4}{9}i\pi. \]

The non-factorizable effects \([49, 50, 51, 56]\) from the charm loop can bring about further corrections to the radiative \(b \rightarrow s\gamma\) transition, which can be absorbed into the effective Wilson coefficient \(C_7^{\text{eff}}\). Specifically, the Wilson coefficient \(C_7^{\text{eff}}(\mu)\) is given by \([29]\)

\[ C_7^{\text{eff}}(\mu) = C_7(\mu) + C_{b \rightarrow s\gamma}(\mu), \]

with

\[ C_{b \rightarrow s\gamma}(\mu) = i\alpha_s \left[ \frac{2}{9} \eta^{14/23}(G_1(x_t) - 0.1687) - 0.03C_2(\mu) \right], \]

\[ G_1(x) = \frac{x(x^2 - 5x - 2)}{8(x - 1)^3} + \frac{3x^2\ln^2 x}{4(x - 1)^2}, \]

where \(\eta = \alpha_s(m_W)/\alpha_s(\mu)\), \(x_t = m_t^2/m_W^2\), \(C_{b \rightarrow s\gamma}\) is the absorptive part for the \(b \rightarrow s\bar{c}c \rightarrow s\gamma\) rescattering and we have dropped out the tiny contributions proportional to CKM sector \(V_{ub}V_{us}^\ast\). In addition, \(C_7^{\text{eff}}(\mu)\) and \(C_9^{\text{eff}}(\mu)\) can be obtained by replacing the unprimed Wilson coefficients with the corresponding prime ones in the above formula.

III. MATRIX ELEMENTS AND FORM FACTORS IN LIGHT CONE SUM RULES

With the free quark decay amplitude available, we can proceed to calculate the decay amplitudes for \(\Lambda_b \rightarrow \Lambda\gamma\) and \(\Lambda_b \rightarrow \Lambda^{+}l^-\) at hadron level, which can be obtained by sandwiching the free quark amplitudes between the initial and final baryon states. Consequently, the following four hadronic matrix elements

\[ \langle \Lambda(P)|\bar{s}\gamma_\mu b|\Lambda_b(P + q) \rangle, \quad \langle \Lambda(P)|\bar{s}\gamma_\mu \gamma_5 b|\Lambda_b(P + q) \rangle, \]

\[ \langle \Lambda(P)|\bar{s}\sigma_{\mu\nu} b|\Lambda_b(P + q) \rangle, \quad \langle \Lambda(P)|\bar{s}\sigma_{\mu\nu} \gamma_5 b|\Lambda_b(P + q) \rangle, \]

need to be computed. Generally, the above matrix elements can be parameterized in terms of the form factors as \([29, 30, 31, 32, 33]\):

\[ \langle \Lambda(P)|\bar{s}\gamma_\mu b|\Lambda_b(P + q) \rangle = \overline{\mathcal{X}}(P)\left[ g_{1b}^\gamma \gamma_\mu + g_{2b}^\sigma \sigma_{\mu\nu} q^\nu + g_{3b}^q q_\mu \right] \Lambda_b(P + q), \]

\[ \langle \Lambda(P)|\bar{s}\gamma_\mu \gamma_5 b|\Lambda_b(P + q) \rangle = \overline{\mathcal{X}}(P)\left[ G_1^\gamma \gamma_\mu + G_{2b}^\sigma \sigma_{\mu\nu} q^\nu + G_{3b}^q q_\mu \right] \gamma_5 \Lambda_b(P + q), \]

\[ \langle \Lambda(P)|\bar{s}\sigma_{\mu\nu} b|\Lambda_b(P + q) \rangle = \overline{\mathcal{X}}(P)\left[ h_{1b}^\sigma \sigma_{\mu\nu} - i h^2_{2b} \gamma_\mu q_\nu - \gamma_\nu q_\mu \right] \Lambda_b(P + q), \]

\[ \langle \Lambda(P)|\bar{s}\sigma_{\mu\nu} \gamma_5 b|\Lambda_b(P + q) \rangle = \overline{\mathcal{X}}(P)\left[ H_{1b}^\sigma \sigma_{\mu\nu} - i H^2_{2b} \gamma_\mu q_\nu - \gamma_\nu q_\mu \right] \gamma_5 \Lambda_b(P + q), \]
where all the form factors $g_i, G_i, h_i$ and $H_i$ are functions of the square of momentum transfer $q^2$. Contracting Eqs. (12, 13) with the four momentum $q^\mu$ on both side and making use of the equations of motion

$$q^\mu (\bar{\psi}_1 \gamma_\mu \psi_2) = (m_1 - m_2) \bar{\psi}_1 \psi_2$$  \hspace{1cm} (14)
$$q^\mu (\bar{\psi}_1 \gamma_\mu \gamma_5 \psi_2) = -(m_1 + m_2) \bar{\psi}_1 \gamma_5 \psi_2$$  \hspace{1cm} (15)

we have

$$\langle \Lambda(P) | \bar{s} i \sigma_{\mu \nu} q^\nu b | \Lambda_b(P + q) \rangle = \overline{\Lambda}(P)(f_1 \gamma_\mu + f_2 i \sigma_{\mu \nu} q^\nu + f_3 q_\mu) \Lambda_b(P + q),$$  \hspace{1cm} (16)
$$\langle \Lambda(P) | \bar{s} i \sigma_{\mu \nu} \gamma_5 q^\nu b | \Lambda_b(P + q) \rangle = \overline{\Lambda}(P)(F_1 \gamma_\mu + F_2 i \sigma_{\mu \nu} q^\nu + F_3 q_\mu) \gamma_5 \Lambda_b(P + q),$$  \hspace{1cm} (17)

with

$$f_1 = \frac{2h_2 - h_3 + h_4 (m_{\Lambda_b} + m_\Lambda)}{2} q^2,$$  \hspace{1cm} (18)
$$f_2 = \frac{2h_1 + h_3 (m_\Lambda - m_{\Lambda_b}) + h_4 q^2}{f_1},$$  \hspace{1cm} (19)
$$f_3 = \frac{m_\Lambda - m_{\Lambda_b}}{f_1},$$  \hspace{1cm} (20)
$$F_1 = \frac{2H_2 - H_3 + H_4 (m_{\Lambda_b} - m_\Lambda)}{2} q^2,$$  \hspace{1cm} (21)
$$F_2 = \frac{2H_1 + H_3 (m_\Lambda + m_{\Lambda_b}) + H_4 q^2}{2},$$  \hspace{1cm} (22)
$$F_3 = \frac{m_\Lambda + m_{\Lambda_b}}{q^2} F_1.$$  \hspace{1cm} (23)

Due to the conservation of vector current, the form factors $f_4$ and $g_4$ do not contribute to the decay amplitude of $\Lambda_b \to \Lambda l^+ l^-$. To incorporate the NHBs effect one need to calculate the matrix elements involving the scalar $\bar{s} b$ and the pseudoscalar $\bar{s} \gamma_5 b$ currents, which can be parameterized as

$$\langle \Lambda(P) | \bar{s} b | \Lambda_b(P + q) \rangle = \frac{1}{m_b + m_s} \overline{\Lambda}(P)[g_1 (m_{\Lambda_b} - m_\Lambda) + g_3 q^2] \Lambda_b(P + q),$$  \hspace{1cm} (24)
$$\langle \Lambda(P) | \bar{s} \gamma_5 b | \Lambda_b(P + q) \rangle = \frac{1}{m_b - m_s} \overline{\Lambda}(P)[G_1 (m_{\Lambda_b} + m_\Lambda) - G_3 q^2] \gamma_5 \Lambda_b(P + q).$$  \hspace{1cm} (25)

The various form factors $f_i$ and $g_i$ appearing in the above equations are not independent in the heavy quark limit and one can express them in terms of two independent form factors $\xi_1$ and $\xi_2$ in HQET defined by

$$\langle \Lambda(P) | \bar{s} \Gamma s | \Lambda_b(P + q) \rangle = \overline{\Lambda}(P)[\xi_1 (q^2) + \xi_2 (q^2)] \Gamma \Lambda_b(P + q).$$  \hspace{1cm} (26)

with $\Gamma$ being an arbitrary Lorentz structure and $v_\mu$ being the four-velocity of $\Lambda_b$ baryon. Comparing Eqs. (10, 11), (16, 17) and the Eq. (26), one can arrive at

$$f_1 = F_1 = \frac{q^2}{m_{\Lambda_b}} \xi_2,$$  \hspace{1cm} (27)
$$f_2 = F_2 = g_1 = G_1 = \xi_1 + \frac{m_\Lambda}{m_{\Lambda_b}} \xi_2,$$  \hspace{1cm} (28)
$$f_3 = \frac{m_\Lambda - m_{\Lambda_b}}{m_{\Lambda_b}} \xi_2,$$  \hspace{1cm} (29)
$$F_3 = \frac{m_\Lambda + m_{\Lambda_b}}{m_{\Lambda_b}} \xi_2,$$  \hspace{1cm} (30)
$$g_2 = G_2 = g_3 = G_3 = \frac{\xi_2}{m_{\Lambda_b}}.$$  \hspace{1cm} (31)
Due to our poor understanding towards non-perturbative QCD dynamics, one has to rely on some approaches to calculate the form factors answering for $\Lambda_b \to \Lambda$ transition. It is suggested that the soft non-perturbative contribution to the transition form factor can be calculated quantitatively in the framework of LCSR approach, which is a fully relativistic approach and well rooted in quantum field theory, in a systematic and almost model-independent way. As a marriage of standard QCDSR technique and theory of hard exclusive process, LCSR cure the problem of QCDSR applying to the large momentum transfer by performing the operator product expansion (OPE) in terms of twist of the relevant operators rather than their dimension. Therefore, the principal discrepancy between QCDSR and LCSR consists in that non-perturbative vacuum condensates representing the long-distance quark and gluon interactions in the short-distance expansion are substituted by the light cone distribution amplitudes (LCDAs) describing the distribution of longitudinal momentum carried by the valence quarks of hadronic bound system in the expansion of transverse-distance between partons in the infinite momentum frame.

Considering the distribution amplitude up to twist-6, the form factors for $\Lambda_b \to \Lambda l^+ l^−$ have been calculated to the accuracy of leading conformal spin, where the pole model was also employed to extend the results to the whole kinematical region. Specifically, the dependence of form factors on transfer momentum are parameterized as

$$\xi_i(q^2) = \frac{\xi_i(0)}{1 - a_1 q^2/m_{\Lambda_b}^2 + a_2 q^4/m_{\Lambda_b}^4},$$

where $\xi_i$ denotes the form factors $f_2$ and $g_2$. The numbers of parameters $\xi_i(0)$, $a_1$, $a_2$ have been collected in Table I. To the leading order and leading power, the other form factors can be related to these two as

$$\begin{align*}
\xi_2(q^2) &= \xi_2(0) \\
\xi_4(q^2) &= \xi_4(0) \\
\xi_5(q^2) &= \xi_5(0) \\
\xi_6(q^2) &= \xi_6(0)
\end{align*}$$

TABLE I: Numerical results for the form factors $f_2(0)$, $g_2(0)$ and parameters $a_1$ and $a_2$ involved in the double-pole fit of eq. (32) for both twist-3 and twist-6 sum rules with $M_{\Lambda_b}^2 \in [3.0, 6.0] \text{ GeV}^2$, $s_0 = 39 \pm 1 \text{ GeV}^2$.

| parameter | twist-3 | up to twist-6 |
|-----------|---------|--------------|
| $f_2(0)$  | $0.14^{+0.02}_{-0.01}$ | $0.15^{+0.02}_{-0.02}$ |
| $a_1$     | $2.91^{+0.10}_{-0.07}$  | $2.94^{+0.11}_{-0.06}$  |
| $a_2$     | $2.26^{+0.13}_{-0.08}$  | $2.31^{+0.14}_{-0.10}$  |
| $g_2(0)(10^{-2}\text{GeV}^{-1})$ | $-0.47^{+0.06}_{-0.06}$ | $1.21^{+0.06}_{-0.4}$ |
| $a_1$     | $3.40^{+0.06}_{-0.05}$  | $2.91^{+0.12}_{-0.09}$  |
| $a_2$     | $2.98^{+0.09}_{-0.08}$  | $2.24^{+0.17}_{-0.13}$  |

$$\begin{align*}
F_1(q^2) &= f_1(q^2) = q^2 g_2(q^2) = q^2 G_2(q^2), \\
F_2(q^2) &= f_2(q^2) = g_1(q^2) = G_1(q^2),
\end{align*}$$

where the form factors $F_3(q^2)$ and $G_3(q^2)$ are dropped out here due to their tiny contributions.

IV. FORMULA FOR OBSERVABLES

In this section, we proceed to perform the calculations of some interesting observables in phenomenology including decay rates, forward-backward asymmetry, polarization asymmetries of final state lepton and of \( \Lambda \) baryon. From Eq.
it is straightforward to obtain the decay amplitude for $\Lambda_b \to \Lambda l^-$ as

$$M_{\Lambda_b \to \Lambda l^-} = \frac{G_F}{2\sqrt{2}\pi} V_{tb} V_{ts}^* \left[ T_1^1 (i\gamma^\mu l) + T_2^2 (i\gamma^\mu \gamma_5 l) + T_3^3 (i\bar{l}) \right],$$

(34)

where the auxiliary functions $T_1^1$, $T_2^2$ and $T_3^3$ are given by

$$T_1^1 = \bar{\Xi}(P) \left[ \{ \gamma_\mu (g_1 - G_1\gamma_5) + i\sigma_{\mu\nu} q^\nu (g_2 - G_2\gamma_5) \} C_9^{eff} + \{ \gamma_\mu (g_1 + G_1\gamma_5) + i\sigma_{\mu\nu} q^\nu (g_2 + G_2\gamma_5) \} C_9^{eff} - 2m_b/s \{ \gamma_\mu (f_1 + F_1\gamma_5) + i\sigma_{\mu\nu} q^\nu (f_2 + F_2\gamma_5) \} C_9^{eff} - 2m_b/s \{ \gamma_\mu (f_1 - F_1\gamma_5) + i\sigma_{\mu\nu} q^\nu (f_2 - F_2\gamma_5) \} C_9^{eff} \right] \Lambda_b (P + q),$$

(35)

$$T_2^2 = \bar{\Xi}(P) \left[ \{ \gamma_\mu (g_1 - G_1\gamma_5) + i\sigma_{\mu\nu} q^\nu (g_2 - G_2\gamma_5) + (g_3 - G_3\gamma_5) q_\mu \} C_{10} + \{ \gamma_\mu (g_1 + G_1\gamma_5) + i\sigma_{\mu\nu} q^\nu (g_2 + G_2\gamma_5) + (g_3 + G_3\gamma_5) q_\mu \} C_{10} - \frac{q_\mu}{2m_l (m_\Lambda + m_s)} \{ (g_1 (m_\Lambda - m_\Lambda) + g_3 q^2 + G_1 (m_\Lambda + m_\Lambda) - G_3 q^2) \} C_{Q_2} - \frac{q_\mu}{2m_l (m_\Lambda + m_s)} \{ (g_1 (m_\Lambda - m_\Lambda) + g_3 q^2 - G_1 (m_\Lambda + m_\Lambda) + G_3 q^2) \} C_{Q_2} \right] \Lambda_b (P + q),$$

(36)

and

$$T_3^3 = \bar{\Xi}(P) \left[ \{ (g_1 (m_\Lambda - m_\Lambda) + g_3 q^2 + G_1 (m_\Lambda + m_\Lambda) - G_3 q^2) \} C_{Q_1} + \{ (g_1 (m_\Lambda - m_\Lambda) + g_3 q^2 - G_1 (m_\Lambda + m_\Lambda) + G_3 q^2) \} C_{Q_1} \right] \Lambda_b (P + q).$$

(37)

It needs to point out that the terms proportional to $q_\mu$ in $T_1^1$ do not contribute to the decay amplitude with the help of the equation of motion for lepton fields. Besides, one can also find that the above results can indeed reproduce that obtained in the SM with $C_i^L = 0$ and $T_3^3 = 0$.

### A. The differential decay rates of $\Lambda_b \to \Lambda l^-$

The differential decay width of $\Lambda_b \to \Lambda l^-$ in the rest frame of $\Lambda_b$ baryon can be written as

$$\frac{d\Gamma(\Lambda_b \to \Lambda l^-)}{ds} = \frac{1}{(2\pi)^3} \frac{1}{32m_{\Lambda_b}^3} \int_{u_{min}}^{u_{max}} |\bar{M}_{\Lambda_b \to \Lambda l^-}|^2 du,$$

(38)

where $u = (p_\Lambda + p_{l^-})^2$ and $s = (p_{l^+} + p_{l^-})^2$; $p_\Lambda$, $p_{l^+}$ and $p_{l^-}$ are the four-momenta vectors of $\Lambda$, $l^+$ and $l^-$ respectively. $\bar{M}_{\Lambda_b \to \Lambda l^-}$ denotes the decay amplitude after performing the integration over the angle between the $l^-$ and $\Lambda$ baryon. The upper and lower limits of $u$ are given by

$$u_{max} = (E_{\Lambda}^* + E_{l^+}^*)^2 - (\sqrt{E_{\Lambda}^{*2} - m_\Lambda^2} - \sqrt{E_{l^+}^{*2} - m_{l^+}^2})^2,$$

$$u_{min} = (E_{\Lambda}^* + E_{l^+}^*)^2 - (\sqrt{E_{\Lambda}^{*2} - m_\Lambda^2} + \sqrt{E_{l^+}^{*2} - m_{l^+}^2})^2,$$

(39)

where $E_{\Lambda}^*$ and $E_{l^+}^*$ are the energies of $\Lambda$ and $l^+$ in the rest frame of lepton pair

$$E_{\Lambda}^* = \frac{m_{\Lambda}^2 - m_\Lambda^2 - s}{2\sqrt{s}}, \quad E_{l^+}^* = \frac{\sqrt{s}}{2}.$$

(40)
Putting everything together, we can achieve the decay rates and invariant mass distributions of $\Lambda_b \to \Lambda l^+ l^-$ with and without long distance contributions as
\[
\frac{d\Gamma}{ds} = \frac{\alpha^2 G_F^2 |V_{tb} V_{tb}^*|^2 (u_{\text{max}} - u_{\text{min}})}{128 m_{\Lambda_b}^3 \pi^3} \times 
\left\{ \frac{m_l}{2m_{\Lambda_b}} \left( f_2 - g_2((m_\Lambda + m_{\Lambda_b})) C_{\alpha_{eff}Q (C_{\alpha_{eff}Q}^* + C_{\alpha_{eff}Q}^*\tilde{C}_{\alpha_{eff}Q}) + \frac{m_l}{2m_{\Lambda_b}} (f_2(m_\Lambda + m_{\Lambda_b}) - g_2 s)(C_{\alpha_{eff}Q}^* C_{\alpha_{eff}Q} + C_{\alpha_{eff}Q}^* C_{\alpha_{eff}Q}^*) 
\right. 
\right.
\left. + \frac{s}{2m_{\Lambda_b}} \left( (f_2 - g_2 m_{\Lambda_b})^2 - g_2^2 m_{\Lambda_b}^2 \right) (C_{\alpha_{eff}Q}^* C_{\alpha_{eff}Q}^* + C_{\alpha_{eff}Q}^* C_{\alpha_{eff}Q}^*) 
\right.
\left. + m_b (f_2^2 - g_2^2 s)(C_{\alpha_{eff}Q}^* C_{\alpha_{eff}Q}^* + C_{\alpha_{eff}Q}^* C_{\alpha_{eff}Q}^*) \right) (2m_\Lambda^2 + m_\Lambda^2 + m_{\Lambda_b}^2 - s - u_{\text{max}} - u_{\text{min}}) 
\left. + \frac{m_l}{2m_{\Lambda_b}} f_2^2 (m_\Lambda - m_{\Lambda_b})((m_\Lambda + m_{\Lambda_b})^2 - s)(C_{\alpha_{eff}Q}^* C_{\alpha_{eff}Q}^* + C_{\alpha_{eff}Q}^* C_{\alpha_{eff}Q}^*) 
\right. 
\left. - \frac{m_b}{s} (2m_\Lambda^2 + s)((m_\Lambda + m_{\Lambda_b})^2 - s)(f_2^2 - 4s m_\Lambda f_2 g_2 + g_2^2 s(m_\Lambda^2 - m_{\Lambda_b}^2 + s))(C_{\alpha_{eff}Q}^* C_{\alpha_{eff}Q}^* + C_{\alpha_{eff}Q}^* C_{\alpha_{eff}Q}^*) 
\right. 
\left. + \frac{s m_{\Lambda_b}}{2m_{\Lambda_b}} f_2^2 (m_{\Lambda} + m_{\Lambda_b})((m_{\Lambda} + m_{\Lambda_b})^2 - s)(C_{\alpha_{eff}Q}^* C_{\alpha_{eff}Q}^* + C_{\alpha_{eff}Q}^* C_{\alpha_{eff}Q}^*) \right) (6m_\Lambda^2 (sm_\Lambda^2 + (m_\Lambda^2 - m_{\Lambda_b}^2)^2 - s(s + u_{\text{max}} + u_{\text{min}})) 
\left. + s (3m_\Lambda^4 + 3m_{\Lambda_b}^4 - 3(m_\Lambda^2 + m_{\Lambda_b}^2)(s + u_{\text{max}} + u_{\text{min}}) + 2u_{\text{max}}^2) 
\right. 
\left. + 3s (u_{\text{max}} + u_{\text{min}} + 2u_{\text{max}} u_{\text{min}}) f_2^2 + 6f_2 g_2 m_{\Lambda} (m_\Lambda^2 - m_{\Lambda_b}^2 - s)(2m_\Lambda^2 + s) 
\right. 
\left. + g_2^2 s^2 (-6m_\Lambda^4 + 6(s + u_{\text{max}} + u_{\text{min}}) m_\Lambda^2 - 3s^2 - 2(u_{\text{max}}^2 - u_{\text{min}}^2) 
\right. 
\left. - 3s (u_{\text{max}} + u_{\text{min}} + 3m_{\Lambda_b}^2 (s + u_{\text{max}} + u_{\text{min}}) - 2u_{\text{max}} u_{\text{min}} 
\right. 
\left. + 3m_{\Lambda_b}^2 (s + u_{\text{max}} + u_{\text{min}})) \left| C_{\alpha_{eff}Q}^* \right|^2 
\left. + \frac{1}{4m_{\Lambda_b}} \left( -6m_\Lambda^4 + (6m_\Lambda^2 - 3s)(u_{\text{max}} + u_{\text{min}}) - 3s^2 - 2(u_{\text{max}}^2 + u_{\text{min}}^2 + u_{\text{max}} u_{\text{min}}) + 3m_{\Lambda_b}^2 (s + u_{\text{max}} + u_{\text{min}}) 
\right. 
\left. + 3m_{\Lambda_b}^2 (-2m_{\Lambda_b}^2 + s + u_{\text{max}} + u_{\text{min}}) f_2^2 - 6f_2 g_2 m_{\Lambda} (m_\Lambda^2 - m_{\Lambda_b}^2 - s)(2m_\Lambda^2 + s) 
\right. 
\left. + g_2^2 (6s m_\Lambda^4 + 6m_\Lambda^2 ((m_\Lambda^2 - m_{\Lambda_b}^2)^2 - s(s + u_{\text{max}} + u_{\text{min}})) + s (3m_\Lambda^4 + 3m_{\Lambda_b}^4 
\right. 
\left. - 3(m_\Lambda^2 + m_{\Lambda_b}^2)(s + u_{\text{max}} + u_{\text{min}}) + 2(u_{\text{max}}^2 + u_{\text{min}}^2 + u_{\text{max}} u_{\text{min}}) + 3t (u_{\text{max}} + u_{\text{min}})) \right) \left| C_{\alpha_{eff}Q}^* \right|^2 
\right. 
\left. + \frac{1}{4m_{\Lambda_b}} \left( (6m_\Lambda^4 + 6(2m_\Lambda^2 + m_{\Lambda_b}^2 - 2s - u_{\text{max}} - u_{\text{min}}) m_\Lambda^2 + 2(u_{\text{max}}^2 + u_{\text{min}}^2 + u_{\text{max}} u_{\text{min}}) 
\right. 
\left. + 3(s - m_{\Lambda_b}^2 - m_{\Lambda_b}^2)(s + u_{\text{max}} + u_{\text{min}}) f_2^2 + 6g_2 f_2 (4m_\Lambda^2 - s)(m_{\Lambda_b}^2 - m_{\Lambda_b}^2 + s) 
\right. 
\left. + g_2^2 (-6s m_\Lambda^4 + 6(m_\Lambda^4 - 2m_{\Lambda_b}^2 + s)m_\Lambda^2 + m_{\Lambda_b}^4 - m_{\Lambda_b}^2 s + s (s + u_{\text{max}} + u_{\text{min}}) m_\Lambda^2 
\right. 
\left. + s (-3m_\Lambda^4 + 3(m_\Lambda^2 + m_{\Lambda_b}^2)(s + u_{\text{max}} + u_{\text{min}}) - 3m_{\Lambda_b}^4 
\right. 
\left. - 2(u_{\text{max}}^2 + u_{\text{min}}^2 + u_{\text{max}} u_{\text{min}}) - 3t (u_{\text{max}} + u_{\text{min}})) \right) \left| C_{\alpha_{eff}Q}^* \right|^2 
\right\} \right\}
\end{align}
\right)}
\end{equation}

where $u_{\text{max}}$ and $u_{\text{min}}$ are defined in Eq. (39). In Eq. (41) we have given the result in MSSM with NHBS and ignored the contribution from the primed operators which appear SUSY SO(10) GUT model as the results are very tiny.

**B. FBAs of $\Lambda_b \to \Lambda l^+ l^-$**

Now we are in a position to explore the FBAs of $\Lambda_b \to \Lambda l^+ l^-$, which is an essential observable sensitive to the new physics effects. To calculate the forward-backward asymmetry, we consider the following double differential decay
rate formula for the process $\Lambda_b \to \Lambda^+ l^-$

\[
\frac{d^2\Gamma(s, \cos \theta)}{dsd \cos \theta} = \frac{1}{(2\pi)^3} \frac{1}{64m^3_{\Lambda_b}} \lambda^{1/2}(m^2_{\Lambda_b}, m^2_{\Lambda}, s) \sqrt{1 - \frac{4m^2}{s} |\vec{M}_{\Lambda_b \to \Lambda^+ l^-}|^2},
\]

where $\theta$ is the angle between the momentum of $\Lambda_b$ baryon and $l^-$ in the dilepton rest frame; $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$. Following Refs. [29, 48], the differential and normalized FBAs for the semi-leptonic decay $\Lambda_b \to \Lambda^+ l^-$ are defined as

\[
\frac{dA_{FB}(q^2)}{ds} = \int_0^1 d\cos \theta \frac{d^2\Gamma(s, \cos \theta)}{dsd \cos \theta} - \int_{-1}^0 d\cos \theta \frac{d^2\Gamma(s, \cos \theta)}{dsd \cos \theta}
\]

and

\[
A_{FB}(q^2) = \frac{\int_0^1 d\cos \theta \frac{d^2\Gamma(s, \cos \theta)}{dsd \cos \theta} - \int_{-1}^0 d\cos \theta \frac{d^2\Gamma(s, \cos \theta)}{dsd \cos \theta}}{\int_0^1 d\cos \theta \frac{d^2\Gamma(s, \cos \theta)}{dsd \cos \theta} + \int_{-1}^0 d\cos \theta \frac{d^2\Gamma(s, \cos \theta)}{dsd \cos \theta}}
\]

Following the same procedure as we did for the differential decay rate, one can easily get the expression for the forward-backward asymmetry.

### C. Lepton Polarization asymmetries of $\Lambda_b \to \Lambda^+ l^-$

In the rest frame of the lepton $l^-$, the unit vectors along longitudinal, normal and transversal component of the $l^-$ can be defined as [73]:

\[
\begin{align*}
\vec{s}_L^- &= (0, \vec{e}_L) = \left(0, \frac{\vec{p}_-}{|\vec{p}_-|}\right), \\
\vec{s}_N^- &= (0, \vec{e}_N) = \left(0, \frac{\vec{p}_A \times \vec{p}_-}{|\vec{p}_A \times \vec{p}_-|}\right), \\
\vec{s}_T^- &= (0, \vec{e}_T) = \left(0, \vec{e}_N \times \vec{e}_L\right),
\end{align*}
\]

where $\vec{p}_-$ and $\vec{p}_A$ are the three-momenta of the lepton $l^-$ and $\Lambda$ baryon respectively in the center mass (CM) frame of $l^+ l^-$ system. Lorentz transformation is used to boost the longitudinal component of the lepton polarization to the CM frame of the lepton pair as

\[
(s_L^-)_{CM} = \left(\frac{|\vec{p}_-|}{m_l}, \frac{E_l \vec{p}_-}{m_l |\vec{p}_-|}\right)
\]

where $E_l$ and $m_l$ are the energy and mass of the lepton in the CM frame. The normal and transverse components remain unchanged under the Lorentz boost.

The longitudinal ($P_L$), normal ($P_N$) and the transverse ($P_T$) polarizations of lepton can be defined as:

\[
P_i^{(\mp)}(s) = \frac{d\Gamma_i^{(\mp)}}{ds}(\xi^\mp = \vec{e}^\mp) - \frac{d\Gamma_i^{(\pm)}}{ds}(\xi^\mp = -\vec{e}^\mp) + \frac{d\Gamma_i^{(\mp)}}{ds}(\xi^\mp = -\vec{e}^\mp)
\]

where $i = L, N, T$ and $\xi^\mp$ is the spin direction along the leptons $l^\mp$. The differential decay rate for polarized lepton $l^\mp$ in $\Lambda_b \to \Lambda^+ l^-$ decay along any spin direction $\xi^\mp$ is related to the unpolarized decay rate with the following relation

\[
\frac{d\Gamma(\xi^\mp)}{ds} = \frac{1}{2} \left(\frac{d\Gamma}{ds}\right) [1 + (P_L^\mp \vec{e}_L^\mp + P_N^\mp \vec{e}_N^\mp + P_T^\mp \vec{e}_T^\mp) \cdot \xi^\mp].
\]
We can achieve the expressions of longitudinal, normal and transverse polarizations for $\Lambda \rightarrow \Lambda t^+t^-$ decays as collected below, where only the results in MSSM models with NHB are given for the conciseness of paper. Thus the longitudinal lepton polarization can be written as

$$P_L(s) = (1/ds) \frac{\alpha^2 G_F^2 |V_{tb}V_{ts}^*|^2 u_{\max}}{768(m_b^2 - m_s^2) m_{\Lambda s}^3} \sqrt{1 - \frac{4m_t^2}{s}} \times \left\{ -6m_b(m_b - m_s)f_2(m_\Lambda - m_{\Lambda s})(s - (m_\Lambda + m_{\Lambda s})^2)(C_{Q_1} C_{10}^* + C_{Q_1}^* C_{10}) \\
-6m_b^2 f_2(s - (m_\Lambda + m_{\Lambda s})^2)(C_{Q_1}^* C_{Q_2} + C_{Q_1} C_{Q_2}) \\
+(m_b^2 - m_s^2)((3m_\Lambda^2 + 3m_{\Lambda s}^2 - 3s^2)(f_2^2 + g_2^2 s) - 6m_b^2 m_\Lambda^2 (f_2^2 + g_2^2 s) \\
-\frac{\lambda(m_\Lambda^2, m_{\Lambda s}^2, s)}{1 - 4m_b^2/s}(f_2^2 - g_2^2 s))(C_{9}^{eff} C_{10} + C_{9}^{c10}) \\
-12(m_b^2 - m_s^2)(m_b^2 - m_{\Lambda s}^2 - s - m_{\Lambda} (m_\Lambda^2 - m_{\Lambda s}^2 - s)) \\
(f_2^2 + g_2^2 s) + f_2g_2(m_s((m_\Lambda^2 - m_{\Lambda s}^2)^2 - s) - 4m_b m_{\Lambda s} m_\Lambda s)(C_{10}^{eff} C_7 + C_{10}^{eff} C_7 f)ight\}. \quad (49)$$

Similarly, the normal lepton polarization is

$$P_N(s) = (1/ds) \frac{\alpha^2 G_F^2 |V_{tb}V_{ts}^*|^2 u_{\max}}{1024(m_b + m_s)m_{\Lambda s}^3 \pi^2} \sqrt{1 - \frac{4m_t^2}{s}} \times \left\{ (s - 4m_b^2)(g_2s - f_2(m_\Lambda + m_{\Lambda s}))m_{\Lambda s}(C_{Q_1}, C_{10}^* + C_{Q_1}^* C_{10}) \\
-2m_b^2 (m_b^2 - m_s^2)(g_2(m_\Lambda + m_{\Lambda s}) - f_2)(C_{Q_2}^{eff} C_7 + C_{Q_2}^{c10}) \\
-2m_b^2 m_{\Lambda s}(f_2(m_\Lambda + m_{\Lambda s}) - g_2s)(C_{Q_2}^{eff} C_9 + C_{Q_2}^{c10}) \\
-2f_2 m_s(m_\Lambda + m_{\Lambda s})(f_2^2(m_\Lambda^2 - m_{\Lambda s}^2) + g_2 m_{\Lambda s} s)(C_{9}^{eff} C_{10}^* + C_{9}^{c10} C_{10}) \\
+8m_b(m_b + m_s)(m_b m_{\Lambda s}(f_2^2 + g_2^2 s) \\
+m_b m_{\Lambda s}(f_2^2 - g_2^2 s) - f_2 g_2 m_b m_s(m_\Lambda^2 - m_{\Lambda s}^2 + s))(C_{9}^{eff} C_7 + C_{9}^{c10} C_7 e_7) \\
+4m_b(m_b + m_s)(f_2 g_2 m_b m_s - f_2 m_b m_{\Lambda s} + m_\Lambda m_s(f_2 - g_2 m_\Lambda))(C_{7}^{eff} C_{10} + C_{7}^{eff} C_{10}^* ) \\
-\frac{16(m_b + m_s)^2(m_b - m_s)m_\Lambda}{s}(f_2(m_\Lambda - m_\Lambda) + g_2 s)(g_2 s - f_2(m_\Lambda - m_\Lambda)) |C_{9}^{eff}|^2 \\
+4m_b(m_b + m_s)(f_2 - g_2 m_\Lambda)(|C_{9}^{eff}|^2) \right\}. \quad (50)$$
and the transverse one is given by

\[ P_T(s) = \left\{ 2m_{\Lambda_b}f_2(g_2(m_\Lambda + m_{\Lambda_b}) - f_2)(C_{Q_1}C_{7}^{* eff} - C_{Q_1}^{* eff}) - \frac{m_{\Lambda_b}}{m_b + m_s}f_2(g_2s - f_2(m_\Lambda + m_{\Lambda_b}))(C_{Q_1}C_{9}^{* eff} - C_{9}^{* eff}C_{Q_1}) - \frac{m_{\Lambda_b}}{m_b - m_s}f_2(g_2s - f_2(m_\Lambda + m_{\Lambda_b}))(C_{Q_2}C_{10}^{*} - C_{10}^{*}C_{Q_1}) - 2m_l((f_2 - g_2m_\Lambda)^2 - g_2^2m_{\Lambda_b}^2)(C_{9}^{* eff}C_{10} - C_{10}^{* eff}C_{10}) + \frac{4m_l}{s}(m_s m_\Lambda + m_b m_{\Lambda_b})f_2^2 - f_2g_2 m_s (m_\Lambda^2 - m_{\Lambda_b}^2 + s) + g_2^2(m_s m_\Lambda - m_b m_{\Lambda_b})s)(C_{10}^{*}C_{7}^{* eff} - C_{7}^{* eff}C_{10}) \right\} , \]

\[ \frac{d\Gamma}{ds} \]

appearing in the above equation is the one given in Eq. (41).

D. \textbf{A polarization in } \Lambda_b \rightarrow \Lambda^+ l^-

To study the \textbf{A} spin polarization, one needs to express the \textbf{A} four spin vector in terms of a unit vector \( \hat{\xi} \) along the \textbf{A} spin in its rest frame as

\[ s_0 = \frac{\vec{p}_\Lambda \cdot \vec{\xi}}{m_\Lambda}, \quad \bar{s} = \vec{\xi} + \frac{s_0}{E_\Lambda + m_\Lambda}\vec{p}_\Lambda, \]

where the unit vectors along the longitudinal, normal and transverse components of the \textbf{A} polarization are chosen to be

\[ \hat{e}_L = \frac{\vec{p}_\Lambda}{|\vec{p}_\Lambda|}, \quad \hat{e}_N = \frac{\vec{p}_\Lambda \times (\vec{p}_- \times \vec{p}_\Lambda)}{|\vec{p}_\Lambda \times (\vec{p}_- \times \vec{p}_\Lambda)|}, \quad \hat{e}_T = \frac{\vec{p}_- \times \vec{p}_\Lambda}{|\vec{p}_- \times \vec{p}_\Lambda|}. \]

Similar to the lepton polarization, the polarization asymmetries for \textbf{A} baryon in \( \Lambda_b \rightarrow \Lambda^+ l^- \) can be defined as

\[ P_i^{(\mp)}(s) = \frac{\frac{d\Gamma}{ds}(\xi = \hat{e}) - \frac{d\Gamma}{ds}(\xi = -\hat{e})}{\frac{d\Gamma}{ds}(\xi = \hat{e}) + \frac{d\Gamma}{ds}(\xi = -\hat{e})} \]

where \( i = L, N, T \) and \( \hat{\xi} \) is the spin direction along the \textbf{A} baryon. The differential decay rate for polarized \textbf{A} baryon in \( \Lambda_b \rightarrow \Lambda^+ l^- \) decay along any spin direction \( \hat{\xi} \) is related to the unpolarized decay rate \( 3\bar{s} \) through the following relation

\[ \frac{d\Gamma(\hat{\xi})}{ds} = \frac{1}{2} \left[ \frac{d\Gamma}{ds} \right] [1 + (P_L \vec{e}_L + P_N \vec{e}_N + P_T \vec{e}_T) \cdot \hat{\xi}]. \]
Following the same procedure as we did for the lepton polarizations, we can derive the formulae for the longitudinal, normal and transverse polarizations of \( \Lambda \) baryon in the MSSM as

\[
P_L(s) = \left( \frac{d\Gamma}{ds} \right)^{\alpha^2 G_F^2 |V_{tb}V_{ts}^*|^2 u_{max}^2} \frac{\alpha^2 G_F^2 |V_{tb}V_{ts}^*|^2 u_{max}^2}{64 \alpha m^2_{\Lambda}} \frac{\pi^2}{s^{3/2}} \times \left\{ \begin{array}{l}
m_{\Lambda} m_{\Lambda b} m_{\Lambda b} \left( m_{\Lambda} + m_{\Lambda b} \right) s^{3/2} \lambda_p f_2^2 \left( C_{Q_7} C_{Q_9}^* C_{Q_7}^* + C_{Q_9} C_{Q_7}^* \right) \\
+ m_{\Lambda} m_{\Lambda b} m(2m_{\Lambda b}^2 + s) \left( g_2 s - f_2 \right) \sqrt{\lambda_p} \sqrt{s} \left( C_{Q_7}^* C_{Q_9}^* + C_{Q_9} C_{Q_7}^* \right) \\
+ \frac{m_{\Lambda b}^2}{3} \left( 12m_{\Lambda}^2(m_{\Lambda} - m_{\Lambda b}) \lambda_p + s \sqrt{1 - \frac{4m_{\Lambda b}^2}{s}} u_{max}(m_{\Lambda}^2 - m_{\Lambda b}^2 + s) - 3 \lambda_p s(m_{\Lambda}^2 - m_{\Lambda b}^2 + s) \right) f_2^2 \\
- g_2 f_2 \left( 24m_{\Lambda}^2 \lambda_p m_{\Lambda b}^2 - \lambda_p \lambda_{max} \lambda_p + \sqrt{1 - \frac{4m_{\Lambda b}^2}{s}} \lambda_p \right) \right\} |C_{Q_7}^*|^2 \\
+ \frac{s}{12} \left[ -m_{\Lambda}(12m_{\Lambda}^2 \lambda_p - \lambda_{max} \lambda_p) - g_2 m_{\Lambda} \sqrt{s} (12m_{\Lambda}^2 \lambda_p s m_{\Lambda b}^2 + s) \\
+ \lambda_{max} \sqrt{s} \left( 1 - \frac{4m_{\Lambda b}^2}{s} \lambda_p \right) - 3 \lambda_p \lambda_{max} \lambda_p \right] |C_{Q_7}|^2 \\
+ \frac{s}{12} \left[ -m_{\Lambda}(12m_{\Lambda}^2 \lambda_p - \lambda_{max} \lambda_p) - g_2 m_{\Lambda} \sqrt{s} (12m_{\Lambda}^2 \lambda_p s m_{\Lambda b}^2 + s) \\
+ \lambda_{max} \sqrt{s} \left( 1 - \frac{4m_{\Lambda b}^2}{s} \lambda_p \right) - 3 \lambda_p \lambda_{max} \lambda_p \right] |C_{Q_9}|^2 \\
\right\} 
\]

\[
P_N(s) = \left( \frac{d\Gamma}{ds} \right)^{\alpha^2 G_F^2 |V_{tb}V_{ts}^*|^2 u_{max}^2} \frac{\alpha^2 G_F^2 |V_{tb}V_{ts}^*|^2 u_{max}^2}{512 \alpha m_{\Lambda}} \frac{\pi^2}{s^{3/2}} \times \left\{ \begin{array}{l}
2m_{\Lambda} m_{\Lambda b} (s - (m_{\Lambda} + m_{\Lambda b})^2) (f_2(m_{\Lambda b} - m_{\Lambda}) + g_2 s)(C_{Q_7} C_{Q_9}^* + C_{Q_9} C_{Q_7}^*) \\
+ m_{\Lambda} m_{\Lambda b} (s - (m_{\Lambda} + m_{\Lambda b})^2)(f_2 + g_2 s)(m_{\Lambda} - m_{\Lambda b}) \left( C_{Q_7} C_{Q_9}^* + C_{Q_9} C_{Q_7}^* \right) \\
- m_{\Lambda} m_{\Lambda b} (s - (m_{\Lambda} + m_{\Lambda b})^2)(f_2 + g_2 s)(m_{\Lambda} - m_{\Lambda b}) \left( C_{Q_7} C_{Q_9}^* + C_{Q_9} C_{Q_7}^* \right) \\
+ m_{\Lambda} m_{\Lambda b} (s - (m_{\Lambda} + m_{\Lambda b})^2)(f_2 + g_2 s)(m_{\Lambda} - m_{\Lambda b}) \left( C_{Q_7} C_{Q_9}^* + C_{Q_9} C_{Q_7}^* \right) \\
\right\} 
\]

\[
P_T(s) = \left( \frac{d\Gamma}{ds} \right)^{\alpha^2 G_F^2 |V_{tb}V_{ts}^*|^2 u_{max}^2} \frac{\alpha^2 G_F^2 |V_{tb}V_{ts}^*|^2 u_{max}^2}{512 \alpha m_{\Lambda}} \frac{\pi^2}{s^{3/2}} \times \left\{ \begin{array}{l}
2m_{\Lambda} m_{\Lambda b} (s - (m_{\Lambda} + m_{\Lambda b})^2) (f_2(m_{\Lambda b} - m_{\Lambda}) + g_2 s)(C_{Q_7} C_{Q_9}^* - C_{Q_9} C_{Q_7}^*) \\
+ s(g_2 m_{\Lambda} - m_{\Lambda b}) f_2 \left( C_{Q_7} C_{Q_9}^* - C_{Q_9} C_{Q_7}^* \right) \right\} 
\right\} 
\]

where \( \lambda_p = \lambda(m_{\Lambda b}^2, m_{\Lambda b}^2, s) \) and the mass of strange quark is neglected to make the expressions more compact.
V. NUMERICAL ANALYSIS

In this section, we would like to present the numerical analysis of decay rates, FBAs and polarization asymmetries of the lepton and Λ baryon. The numerical values of Wilson coefficients and other input parameters used in our analysis are borrowed from Ref. [12, 18, 25] and collected in Tables II, III and IV. In the subsequent analysis, we will focus on the parameter space of large \( \tan \beta \), where the NHBs effects are significant owing to the fact that the Wilson coefficients corresponding to NHBs are proportional to \( (m_\mu m_t/m_h) \tan^3 \beta \) \((h = h^0, A^0)\). Here, one \( \tan \beta \) comes from the chargino-up-type squark loop and \( \tan^2 \beta \) comes from the exchange of the NHBS. At large value of \( \tan \beta \) the \( C_{Q_i}^{f} \)

**TABLE II:** Values of input parameters used in our numerical analysis

| Parameter                      | Value                  |
|--------------------------------|------------------------|
| \( G_F \) \( \times 10^{-2} \) GeV\(^{-2} \)   | 1.166 ± 0.10      |
| \( |V_{ts}| \) \( \times 10^{-3} \)          | 0.4161 ± 0.080       |
| \( \mu_b \) \( \pm 0.03 \) GeV              | 4.68 ± 0.03          |
| \( m_e \) (GeV)                         | 1.275 ± 0.015        |
| \( m_s \) (1 GeV)                        | (142 ± 28) MeV       |
| \( m_{\tilde{A}_b} \) \( \pm 0.4 \) GeV   | 5.62 ± 0.04          |
| \( f_{\tilde{A}_b} \) \( \times 10^{-3} \) GeV\(^2 \) | 3.9 ± 0.4 | 6.0 ± 0.4 \( \times 10^{-3} \) GeV\(^2 \)

**TABLE III:** Wilson Coefficients in SM and different SUSY models but without Neutral Higgs boson contributions. The primed Wilson coefficients corresponds to the operators which are opposite in helicities from those of the SM operators and these comes only in SUSY SO(10) GUT model.

| Wilson Coefficients | \( C_7^{eff} \) | \( C_7^{eff} \prime \) | \( C_9 \) | \( C'_9 \) | \( C_{10} \) | \( C'_{10} \) |
|---------------------|----------------|----------------------|---------|---------|----------|----------|
| SM                  | -0.313         | 0                    | 4.334   | 0       | -4.669   | 0        |
| SUSYI               | +0.3756        | 0                    | 4.7674  | 0       | -3.7354  | 0        |
| SUSYII              | +0.3756        | 0                    | 4.7674  | 0       | -3.7354  | 0        |
| SUSYIII             | -0.3756        | 0                    | 4.7674  | 0       | -3.7354  | 0        |

**TABLE IV:** Wilson coefficient corresponding to NHBS contributions. SUSYI corresponds to the regions where SUSY can destructively contribute and can change the sign of \( C_7 \), but contribution of NHBS are neglected, SUSYII refers to the region where \( \tan \beta \) is large and the masses of the superpartners are relatively small. SUSY III corresponds to the regions where \( \tan \beta \) is large and the masses of superpartners are relatively large. The primed Wilson coefficients are the contribution of NHBS in SUSY SO(10) GUT model. As the neutral Higgs bosons are proportional to the lepton mass, and the values shown in the table are for \( \mu \) and \( \tau \) case. The values in the bracket are for \( \tau \).

| Wilson Coefficients | \( C_{Q_1} \) | \( C_{Q_1} \prime \) | \( C_{Q_2} \) | \( C_{Q_2} \prime \) |
|---------------------|---------------|---------------------|---------------|---------------------|
| SM                  | 0             | 0                   | 0             | 0                   |
| SUSYI               | 0             | 0                   | 0             | 0                   |
| SUSYII              | 6.5 (16.5)    | 0                   | -6.5 (16.5)   | 0                   |
| SUSYIII             | 1.2 (4.5)     | 0                   | -1.2 (4.5)    | 0                   |

**TABLE IV** (cont.)

| Wilson Coefficients | \( C_{Q_1} \) | \( C_{Q_1} \prime \) | \( C_{Q_2} \) | \( C_{Q_2} \prime \) |
|---------------------|---------------|---------------------|---------------|---------------------|
| SUSY SO(10) \( A_0 = -1000 \) | 0.106 + 0i    | -0.247 + 0.242i    | -0.107 + 0i   | -0.250 + 0.246i     |
|                     | (1.775 ± 0.002i) | (-4.148 + 4.074i)  | (-1.797 ± 0.002i) | (-4.202 + 4.128i)   |
compete with $C_i^{(i)}$ and can overwhelm $C_i^{(i)}$ in some region as can be seen from the Tables III and IV \cite{13}. Apart from the large tan $\beta$ limit, the other two conditions responsible for the large contributions from NHBs are: (i) the mass values of the lighter chargino and lighter stop should not be too large; (ii) the mass splitting of charginos and stops should be large, which also indicate large mixing between stop sector and chargino sector \cite{12}. Once these conditions are satisfied, the process $B \rightarrow X_s \gamma$ will not only impose constraints on $C_7$ but it also puts very stringent constraint on the possible new physics. It is well known that the SUSY contribution is sensitive to the sign of the Higgs mass term $\mu$ and SUSY contributes destructively when the sign of this term becomes minus. It is pointed out in literature \cite{12} that there exist considerable regions of SUSY parameter space in which NHBs can largely contribute to the process $b \rightarrow s l^+ l^-$ due to change of the sign of $C_7$ from positive to negative, while the constraint on $b \rightarrow s \gamma$ is respected.

Also, when the masses of SUSY particles are relatively large, say about 450 GeV, there exist significant regions in the parameter space of SUSY models in which NHBs could contribute largely. However, in these cases $C_7$ does not change its sign, because contributions of charged Higgs and charginos cancel each other. Hopefully, we can distinguish between these two regions of SUSY by observing $\Lambda_b \rightarrow \Lambda l^+ l^-$ with $(l = \mu, \tau)$.

The numerical results for the decay rates, FBAs and polarization asymmetries of the lepton and $\Lambda$ baryon are presented in Figs. 1-8. Fig. 1 describes the differential decay rate of $\Lambda_b \rightarrow \Lambda l^+ l^-$, from which one can see that the supersymmetric effects are quite significant for SUSY I and SUSY II model in the high momentum transfer regions for the final states being muon pair, whereas these effects are extremely small for SUSY III and SUSY SO(10) GUT models in this case. The reason for the increase of differential decay width in SUSY I model is the relative change in the sign of $C_7^{eff}$; while the large change in SUSY II model is due to the contribution of the NHBs. As for the SUSY III and SUSY SO (10) models, the value of the Wilson coefficients corresponding to NHBs is small and hence one expects small deviations from SM. For the tauon case, the values of Wilson coefficients corresponding to NHBs in SUSY III are larger than that for the muon case and therefore their effects are quite significant as shown in Fig. 1(c, d). The numerical values of the branching fractions for $\Lambda_b \rightarrow \Lambda l^+ l^-$ $(l = \mu, \tau)$ with and without long-distance contribution in SM and different SUSY models are given in Table V.

| Branching Ratio $\Lambda_b \rightarrow \Lambda l^+ l^-$ $(l = \mu, \tau)$ in units of $10^{-6}$ in SM and different SUSY models. |
|---------------------------------------------------------------|
| Branching Ratio $\Lambda_b \rightarrow \Lambda l^+ l^-$ $(l = \mu, \tau)$ in units of $10^{-6}$ |
| SM | 5.9 | 39 | 2.1 | 4 |
| SUSYI | 7.9 | 47 | 3.5 | 5.7 |
| SUSYII | 25 | 65 | 31 | 33 |
| SUSYIII | 5.6 | 45 | 3.2 | 5.6 |
| SUSY SO(10) ($A_0 = -1000$) | 5.92 | 23 | 2 | 2.6 |

In Fig. 2, the FBAs for $\Lambda_b \rightarrow \Lambda l^+ l^-$ are presented. Fig. 2(a,b) describe the FBAs for $\Lambda_b \rightarrow \Lambda \mu^+ \mu^-$ with and without long-distance contributions, from which one can easily distinguish different SUSY models. It is known that in the SM the zero position of FBAs is due to the opposite sign of $C_7^{eff}$ and $C_9^{eff}$. In SUSY I and SUSY II models, the sign of $C_7^{eff}$ and $C_9^{eff}$ are the same and hence the zero point of the FBAs disappears. Whereas, in SUSY III model due to the opposite sign of $C_7^{eff}$ and $C_9^{eff}$ forward-backward asymmetry passes from the zero but this zero position
FIG. 1: The differential width for the $\Lambda_b \rightarrow \Lambda l^+ l^-$ ($l = \mu, \tau$) decays as functions of $q^2$ without long-distance contributions (a, c) and with long-distance contributions (b, d). The solid, dashed, dashed-dot, dashed-double dot and dashed-triple dot line represents, SM, SUSY I, SUSY II, SUSY III and SUSY SO(10) GUT model.

shifts to the right from that of the SM value due to the contribution from the NHBs. Similar behavior is expected in SUSY SO(10) GUT model but in this case the shifting is very mild as the contribution from the NHBs is very small. For $\Lambda_b \rightarrow \Lambda \tau^+ \tau^-$ the FBAs with and without long-distance contributions are represented in Fig. 2(c,d). Again, one can easily distinguish between the contributions from different SUSY models. Here, the most interesting point is that the FBAs pass through zero point in SUSY SO(10) GUT model. This is due to the same sign of the $C'_Q$, which suppress the large contribution coming from the $C_{eff}^7$ and $C_{eff}^9$ in this model. Though SUSY effects are more distinguishable in FBAs in this case, however, it is too difficult to measure it experimentally due to its small value.

Figs. (3-5) describe the lepton polarization asymmetries for $\Lambda_b \rightarrow \Lambda l^+ l^-$. Before, we try to explain the behavior of different polarization asymmetries with the help the formulas (49, 50, 51) given above. Eq. (14) shows the dependence of the longitudinal lepton polarization on different Wilson coefficients, from which one can expect that the value of
FIG. 2: Forward-backward asymmetry for the $Λ_b \rightarrow Λ l^+ l^-$ ($l = \mu, \tau$) decays as functions of $q^2$ without long-distance contributions (a, c) and with long-distance contributions (b, d). The line conventions are same as given in the legend of Fig. 1.

lepton polarization asymmetries in the SUSY I model should be greatly modified from that of the SM due to the change of sign for the term proportional to $C_{7}^{\text{eff}}/C_{10}$. Due to this change in sign, the large positive contribution comes and the magnitude of the longitudinal polarization asymmetry decreases from that of the SM value. However, this value is expected to increase in SUSY II model because of the NHBs contribution which lies in the first and second term of Eq. (49). In SUSY III model, this asymmetry lies close to that of the SM value due to the same sign of $C_{7}^{\text{eff}}/C_{10}$ and small contribution from the NHBs. As we have considered all the primed Wilson coefficients to be zero therefore the effect of SUSY SO(10) on longitudinal lepton polarization asymmetry will be explained by plotting it with the square of the momentum transfer.

Now, Fig. 3(a,b) shows the dependence of longitudinal polarization asymmetry for the $Λ_b \rightarrow Λ \mu^+ \mu^-$ on the square of momentum transfer. The value in SUSY I model is significantly different from that of the SM, however, this
FIG. 3: Longitudinal lepton polarization asymmetries for the \( \Lambda_b \to \Lambda l^+l^- \) (\( l = \mu, \tau \)) decays as functions of \( q^2 \) without long-distance contributions (a, c) and with long-distance contributions (b, d). The line conventions are same as given in the legend of Fig. 1

value is close to that in the SM for SUSY II and SUSY III models. Furthermore, the absolute value of longitudinal polarization asymmetry in the SUSY SO(10) is small compared to the SM model due to the complex part of the Wilson coefficients and also due to small contributions of the NHBs in this model.

Fig. 3(c,d) are for the longitudinal lepton polarization asymmetries of \( \Lambda_b \to \Lambda \tau^+\tau^- \) with and without long-distance contributions, where the different SUSY models are easily distinguishable. Contrary to the muon case, the values of this asymmetry in SUSY II and SUSY III models are even larger in magnitude than that obtained in the SM owing to the large contributions form NHBs which is attributed to the first and second term of Eq. (49). Though the large contributions come from the first term which is proportional to \( m_l \) in Eq. (49), but this is overshadowed by the much larger term proportional to \( m_{\Lambda_b}(C^*_{Q_1}C_{Q_2} + C^*_{Q_2}C_{Q_1}) \).

The dependence of lepton normal polarization asymmetries for \( \Lambda_b \to \Lambda l^+l^- \) on the momentum transfer square are
presented in Fig. 4. In terms of Eq. (50), one can observe that this asymmetry is sensitive to the contribution of NHBs in SUSY II and SUSY III models, while it is insensitive to the contributions from SUSY I and SUSY SO(10) model. It can be seen that $P_N$ changes its sign in the case of large contributions from NHBs as indicated in Fig. 4 and this is also clear from the first three terms of Eq. (50). As expected, the contribution of NHBs from the $\tau^+\tau^-$ channel is much more significant than that from the $\mu^+\mu^-$ channel. Now, the normal polarization is proportional to the $\lambda$ which approaches to zero at large momentum transfer region and hence the normal polarization is suppressed by $\lambda$ in this region.

Fig. 5 shows the dependence of transverse polarization asymmetries for $\Lambda_b \to \Lambda l^+ l^-$ on the square of momentum transfer. From Eq. (51) we can see that it is proportional to the imaginary part of the Wilson coefficient which are negligibly small in SM as well as in SUSY I, SUSY II and SUSY III models. However, complex flavor non-diagonal down-type squark mass matrix elements of 2nd and 3rd generations are of order one at GUT scale in SUSY SO(10) model, which induce complex couplings and Wilson coefficients. As a result, non zero transverse polarization asymmetries for $\Lambda_b \to \Lambda l^+ l^-$ exist in this model. Even though in this case, the asymmetry effects are quite small in
FIG. 5: Transverse lepton polarization asymmetries for the $\Lambda_b \rightarrow \Lambda' l^+ l^-$ ($l = \mu, \tau$) decays as functions of $q^2$. 

(a) 

(b)
FIG. 6: Longitudinal Λ polarization asymmetries for the $\Lambda_b \to \Lambda l^+l^-(l = \mu, \tau)$ decays as functions of $q^2$.

$\tau^+\tau^-$ channel, but the value of transverse polarization asymmetry can reach to $-0.1$ when the momentum transfer is around $15\text{GeV}^2$. Experimentally, to measure $\langle P_T \rangle$ of a particular decay branching ratio $B$ at the $n\sigma$ level, the required number of events are $N = n^2/(B \langle P_T \rangle^2)$ and if $\langle P_T \rangle \sim 0.1$, then the required number of events are almost $10^8$ for $\Lambda_b$ decays. Since at LHC and BTeV machines, the expected number of $b\bar{b}$ production events is around $10^{12}$ per year, so the measurement of transverse polarization asymmetries in the $\Lambda_b \to \Lambda l^+l^-$ decays could discriminate the SUSY SO(10) model from the SM and other SUSY models.

Fig. 6 shows the dependence of longitudinal polarization of Λ baryon on the square of momentum transfer. One can see that the effects of NHBs are quite distinguishable in SUSY II and SUSY III models both for the $\mu^+\mu^-$ and $\tau^+\tau^-$ channels; but the values for SUSY I and SUSY II are almost close to that of the SM. As observed from Eq. (55), the effects of NHBs are proportional to the mass of leptons, therefore the large deviations from the SM are expected for the tauon as presented in Fig. 6. In the SUSY II model, the value of the longitudinal polarization even changes its sign for $\tau^+\tau^-$ channel.

With the help of Eq. (55), one can see that the normal polarization asymmetry of Λ baryon is sensitive to the $C_{Q_1}$ and the sign of the $C_{eff}^{l\bar{l}f}$. It is shown that the sign of $C_{eff}^{l\bar{l}f}$ is negative in SUSY I and II models. In particular, this asymmetry in the SUSY II model differs from that in the SM remarkably due to the large value of $C_{Q_1}$. Moreover, the contributions from SUSY III and SUSY SO(10) models are also quite distinguishable from the SM as shown in Fig. 7. Therefore, the measurements of normal polarization asymmetries for $\Lambda_b \to \Lambda l^+l^-$ in future experiments will help to distinguish different scenarios beyond the SM.

Similar to the transverse polarization asymmetry of lepton, the transverse polarization asymmetry of Λ baryon is also proportional to the imaginary part of $C_{Q_1}C_{eff}^{l\bar{l}f}$ and $C_{Q_1}C_{eff}^{l\bar{l}f}$ (c.f. Eq. (55)). It is known that these imaginary parts are quite small in SM, SUSY I, SUSY II and SUSY III model and hence the values of the transverse polarization asymmetries of Λ baryon are almost zero in these models. However, the imaginary part of Wilson coefficient in SUSY SO(10) model is large and hence its effects are quite different from the other models discussed above as collected in Fig. 8. For the muon case, the transverse polarization asymmetry can reach the number $-0.1$ in SUSY SO(10) model;
whereas the value is too small to measure experimentally for the tauon case.

VI. CONCLUSION

We have carried out the study of invariant mass spectrum, FBAs, polarization asymmetries of lepton and Λ baryon in $\Lambda_b \to \Lambda l^+ l^- \ (l = \mu, \tau)$ decays in SUSY theories including SUSY SO(10) GUT model. Particularly, we analyze the effects of NHBs to this process and our main outcomes can be summarized as follows:

- The differential decay rates deviate sizably from that of the SM especially in the large momentum transfer region. These effects are significant in SUSY II model where the value of the Wilson coefficients corresponding to the NHBs is large. However, the SUSY SO(10) effects in differential decay rate of $\Lambda_b \to \Lambda l^+ l^-$ are negligibly small.

- The SUSY effects show up for the FBA of the $\Lambda_b \to \Lambda l^+ l^- \ (l = \mu, \tau)$ decays and the deviations from the SM are very large especially in SUSY I and SUSY II model where the FBAs did not pass from zero. The reason is the same sign of $C_{\tau}^{eff}$ and $C_{\phi}^{eff}$ in these two models, but in SUSY III scenario it passes from the zero-point different from that of the SM. The effects of SUSY SO(10) are quite distinguishable when the final state leptons are the tauon pair but these are too small to be measured experimentally.
FIG. 8: Transverse $\Lambda$ polarization asymmetries for the $\Lambda_b \rightarrow \Lambda l^+ l^- \ (l = \mu, \tau)$ decays as functions of $q^2$.

The longitudinal, normal and transverse polarizations of leptons are calculated in different SUSY models. It is found that the SUSY effects are very promising which could be measured at future experiments and shed light on the new physics beyond the SM.

Following the same line, the longitudinal, normal and transverse polarizations of $\Lambda$ baryon in $\Lambda_b \rightarrow \Lambda l^+ l^- \ (l = \mu, \tau)$ decays are calculated at length. The different SUSY effects are clearly distinguishable from each other and also from that of the SM. The transverse polarization asymmetries are proportional to the imaginary part of the Wilson coefficients. Hence it is almost zero in SM as well as in MSSM model; however, the Wilson coefficients in SUSY SO(10) GUT model have large imaginary part and hence the value of the transverse polarization is expected to be non-zero. The maximum value of transverse polarization of $\Lambda_b \rightarrow \Lambda \mu^+ \mu^-$ decay reaches to $-0.1$ for the square of momentum transfer around $10\text{GeV}^2$ and hence can be measurable at future experiments like LHC and BTeV machines where a large number of $b\bar{b}$ pairs are expected to be produced.

In short, the experimental investigation of observables, like decay rates, FBAs, lepton polarization asymmetries and the polarization asymmetries of $\Lambda$ baryon in $\Lambda_b \rightarrow \Lambda l^+ l^- \ (l = \mu, \tau)$ decay will be used to search for the SUSY effects, in particular the NHBs effect, encoded in the MSSM as well as SUSY SO(10) models.
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