Q–phonon description of low lying $1^-$ two–phonon states in spherical nuclei

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Abstract

The properties of $1^-$ two-phonon states and the characteristics of E1 transition probabilities between low-lying collective states in spherical nuclei are analysed within the Q–phonon approach to the description of collective states. Several relations between observables are obtained. Microscopic calculations of the E1 $0^+_1 \rightarrow 1^-_1$ transition matrix elements are performed on the basis of the RPA. A satisfactory description of the experimental data is obtained.
I. INTRODUCTION

In systematic investigations of the E1 transitions \([1, 2, 3, 4, 5, 6, 7]\) the low–lying \(1^-\) have been observed in spherical nuclei states, which are characterized by strong \(B(E1;0^+_1 \rightarrow 1^-_1)\), of the order of several units\(\times 10^{-3} e^2 fm^2\). It was demonstrated that these low–lying \(1^-\) states arise as \(|2^+_1 \otimes 3^-_1; 1^- M\rangle\) due to coupling of the collective quadrupole \(2^+_1\) and the collective octupole \(3^-_1\) states. Similar \(1^-\) states have been observed in the Cd, Sn, Ba, Ce, Nd, and Sn isotopes. Their two–phonon character has been proved by the observed strong E2 and E3 transitions to the corresponding one–phonon states \([5, 6, 7]\) and by the fact that the energies of these \(1^-\) states are very close to the summed energies \(E(2^+_1) + E(3^-_1)\).

Strong correlations between the values of \(B(E1;0^+_1 \rightarrow 1^-_1)\) and the product of average squares of the quadrupole \(\langle \beta_2^2 \rangle\) and octupole \(\langle \beta_3^2 \rangle\) deformation parameters in nonmagic nuclei \([8]\) clearly show that away from the closed shells the large \(B(E1)\) values have a collective nature connected with the motion of the nuclear shape. The ratio \(B(E1)/((\langle \beta_2^2 \rangle(\beta_3^2)))\) is amazingly constant, although the \(B(E1)\) strength varies by an order of magnitude in the considered nuclei.

Accepting a two–phonon picture of the \(1^-\) states let us consider the other important experimental facts characterizing strong E1 transitions between low–lying states.

– Analyzing the experimental data on the ratio \(B(E1;1^-_1 \rightarrow 0^+_1)/B(E1;3^-_1 \rightarrow 2^+_1)\) it was shown in \([9]\) that this ratio is a constant equal to 1 within a factor of 2 although these E1 transition strengths can differ by about two orders of magnitude for different nuclei. Semimagic even–\(A\) nuclei and even–even nuclei with two or four nucleons outside a closed shell were considered in this analysis. These nuclei are rather vibrational and can be understood, at least qualitatively, in a harmonic phonon picture. In the last model \(B(E1;1^-_1 \rightarrow 0^+_1)/B(E1;3^-_1 \rightarrow 2^+_1) = \frac{7}{3}\). This correlation of the E1 strengths can be considered as an additional support for the quadrupole – octupole coupled character of the \(1^-\) states.

– The \(B(E1;1^-_1 \rightarrow 2^+_1)\) is small in Sn isotopes; however, it increase in Cd, Te isotopes and approach the Alaga value for the ratio \(B(E1;1^-_1 \rightarrow 2^+_1)/B(E1;1^-_1 \rightarrow 0^+_1)\) in deformed nuclei \([10]\).

– A minimum has been found in the \(A–dependence\) of \(B(E1;1^-_1 \rightarrow 0^+_1)\) in the Nd, Sm and Ba isotopes when the number of neutrons \(N\) is equal to 78 or \(86\) \([11, 12, 13]\). This characteristic feature of the behavior of \(B(E1;1^-_1 \rightarrow 0^+_1)\) as a function of \(N\) has been discovered earlier in
the RPA–based calculations in [14].

– Strong E1 transitions between the $3^-_1$ octupole–phonon states and the $2^+_{1,ms}$ mixed–symmetry quadrupole one–phonon states in the spherical nuclei $^{92}_{40}$Zr$_{52}$, $^{94}_{42}$Mo$_{52}$, $^{96}_{44}$Ru$_{52}$ and $^{142}_{58}$Ce$_{84}$ are observed [15]. In these four nuclei the E1 transitions to the $2^+_{1,ms}$ state are stronger than to the $2^+_1$ state. Probably, this is a consequence of the isovector nature of the E1 transition operator, which enhances E1 transitions between the mixed symmetry states and the isoscalar ones. In the last case, this is the isoscalar octupole vibrational state.

Since $B(E1;3^-_1 \rightarrow 2^+_{1,ms})$ are strong the question arises if there should appear strong E1 transitions from the ground state to the $(3^-_1 \otimes 2^+_{ms})_1$– state, which is expected at the excitation energy 4187 keV in $^{92}$Zr, 4601 keV in $^{94}$Mo, 4934 keV in $^{96}$Ru and 3658 keV in $^{142}$Ce. However, anharmonic effects, which can shift the energies of these two – phonon states, should not be excluded.

Summarizing the short review of the data we can see that the microscopical model describing strong E1 transitions at low energies basing on the two–phonon quadrupole–octupole model of the low lying $1^-_1$ states should explain:

– the fact that $B(E1;1^-_1 \rightarrow 0^+_1)/B(E1;3^-_1 \rightarrow 2^+_1) \approx 1$ at least in near magic nuclei;
– an increase of $B(E1;1^-_1 \rightarrow 2^+_1)$ when going away from the closed shell;
– the existence of the minimum in the $A$ – dependence of $B(E1;0^+_1 \rightarrow 1^-_1)$ when moving off the semimagic nuclei;

– the fact that $B(E1;3^-_1 \rightarrow 2^+_{ms}) \gg B(E1;3^-_1 \rightarrow 2^+_1)$.
– proportionality of $B(E1;0^+_1 \rightarrow 1^-_1)$ to $(\langle \beta^2_2 \rangle \langle \beta^3_3 \rangle)$ in collective nuclei;

II. MODEL

In [16, 17, 18] the Q–phonon approach to the description of the positive parity collective states was developed within the Interacting Boson Model. It was shown that the wave vectors of the yrast, the second $2^+$ and the second $0^+$ states could be described to a high accuracy over the whole parameter space of the consistent Q Hamiltonian, i.e., far outside of the region where a picture of harmonic vibrations was correct, by simple universal expressions containing only one or two multiple Q–phonon configurations. A simple structure of the wave vectors helps derive different relations between transition matrix elements.
In contrast to \[16, 17, 18\], where the \( Q \)-phonon approach was formulated for the bosonic configurational space of the Interacting Boson Model, in the present paper this approach is formulated for the fermionic configurational space. In addition, we consider both positive and negative parity states.

In the \( Q \)-phonon approach the \( 2^+ \) state is presented by the following expression

\[
|2^+_1, \mu\rangle = \mathcal{N}_{2^+_1} \hat{Q}_{2\mu} |0^+_1\rangle,
\]

(1)

where \( |0^+_1\rangle \) is the ground state vector, \( \mathcal{N}_{2^+_1} = \left( \frac{1}{\sqrt{5}} \langle 0^+_1 \vert (\hat{Q}_{2\mu} \hat{Q}_{2\mu}) \vert 0^+_1 \rangle \right)^{-1/2} \)

\[
= \sqrt{5}/ \langle 0^+_1 \vert Q_2 \vert 2^+_1 \rangle \]

and \( \hat{Q}_{2\mu} \) is in our case the standard shell model quadrupole moment operator

\[
\hat{Q}_{2\mu} = \sum_{jj'mm'} \langle jm \vert r^2 Y_{2\mu} \vert j'm' \rangle a_{jm}^+ a_{j'm'},
\]

(2)

expressed in terms of the nucleon creation \( a_{jm}^+ \) and annihilation \( a_{j'm'} \), operators.

Let us include into consideration the octupole mode

\[
|3^-_1, \nu\rangle = \mathcal{N}_{3^-_1} \hat{Q}_{3\nu} |0^+_1\rangle,
\]

(3)

where \( \mathcal{N}_{3^-_1} = \left( \frac{1}{\sqrt{7}} \langle 0^+_1 \vert (\hat{Q}_{3\nu} \hat{Q}_{3\nu}) \vert 0^+_1 \rangle \right)^{-1/2} = \sqrt{7}/ \langle 0^+_1 \vert Q_3 \vert 3^-_1 \rangle \)

and \( \hat{Q}_{3\nu} \) is the fermionic octupole moment operator

\[
\hat{Q}_{3\nu} = \sum_{jj'mm'} \langle jm \vert r^3 Y_{3\nu} \vert j'm' \rangle a_{jm}^+ a_{j'm'}.
\]

(4)

As we know from the RPA type calculations \[19\] both expressions (1) and (3) are very good approximations for the lowest collective \( 2^+ \) and \( 3^- \) states. In fact, this is a consequence of the well–known from the experiment result that E2 and E3 transitions from the ground state to the \( 2^+_1 \) and \( 3^-_1 \) states, correspondingly, are much stronger than transitions to higher lying \( 2^+ \) and \( 3^- \) states.

Continuing along this line and remembering that the \( 1^-_1 \) state is mainly a quadrupole – octupole two – phonon state we suggest for the \( 1^-_1 \) state vector that

\[
|1^-_1, M\rangle = \mathcal{N}_{1^-_1} \left( \hat{Q}_{2} \hat{Q}_{3} \right)_{1M} |0^+_1\rangle,
\]

(5)

where for \( \mathcal{N}_{1^-_1} \) we obtain the following relation:

\[
(\mathcal{N}_{1^-_1})^{-2} = (\mathcal{N}_{2^+_1})^{-2} (\mathcal{N}_{3^-_1})^{-2}
\]
\[
+ \frac{1}{\sqrt{5}} \sum_{n \neq 1} (0^+_1 | (Q_2 Q_2)_0 | 0^+_n) \langle 0^+_n | (Q_3 Q_3)_0 | 0^+_1 \rangle \\
+ \frac{2\sqrt{30}}{35} \langle 0^+_1 | ((Q_2 Q_2)_2 (Q_3 Q_3)_2)_0 | 0^+_1 \rangle \\
+ \frac{\sqrt{11}}{7\sqrt{5}} \langle 0^+_1 | ((Q_2 Q_2)_4 (Q_3 Q_3)_4)_0 | 0^+_1 \rangle 
\] (6)

The last three terms in the right-hand side of (6) can give a noticeable contribution only in the case of strong mixing of the two-phonon quadrupole-quadrupole and two-phonon octupole-octupole states. However, as it is known from the RPA type calculations [20], this mixing is insignificant: less than 1%. Neglecting these terms we obtain an approximate relation

\[
N_{1^-} \approx N_{2^-} N_{3^-}.
\] (7)

The wave vector (5) can be written in terms of the RPA collective phonons and the two-quasiparticle components which correspond to the noncollective RPA solutions. Written in this way the wave vector (5) has the two-phonon component with one quadrupole and one octupole collective RPA phonons as the main component, the three-phonon component with two collective quadrupole and one collective octupole RPA phonons, and the two-quasiparticle 1\(^-\) components. It means that the results of our \(Q\)–phonon approach should be compared not with the pure RPA calculations but with the RPA based calculations which include also anharmonic effects. A contribution of the last two components to the norm of the wave vector (5) is very small for nuclei considered in the present paper.

Of course, when we approach a deformed region the following \(Q\)–phonon component

\[
((Q_2 Q_2)_4 Q_3)_{1_M} | 0^+_1 \rangle
\] (8)

becomes more and more important since in this region there are not one but two 1\(^-\) states with \(K=0\) and \(K=1\). To construct the 1\(^-\) state with \(K\) as a good quantum number, it is necessary to take a mixture of the states (5) and (8). This mixing, which can be estimated as 70\% of one component plus 30\% of the other one can be considered as an upper limit for the error connected with the approximation (5). Of course, near spherical nuclei this mixing is much smaller. In transitional nuclei with soft \(\gamma\)–mode the \(Q\)–phonon state

\[
((Q_2 Q_2)_2 Q_3)_{1_M} | 0^+_1 \rangle
\] (9)

can also become important.
The strength of the $Q$–phonon scheme lies in a possibility to derive relations between electromagnetic transition matrix elements also outside the analytically solvable harmonic vibrator and rotor limits. This is possible because of the simple form of the wave vectors in this approach. This possibility is realized in this section below and in sections III and IV. However, the wave vectors (1) only look simple. The multipole moment operators in these expressions act on the exact ground state. Thus, for calculations we needed an expression for the ground state vector. In section V we take as an approximation to the ground state vector the vacuum state of the RPA phonons.

Let us apply the model formulated above to calculations of the matrix element of the $E1 \ 0^+ \rightarrow 1^-_{1^-}$ transition. We obtain

\[
\langle 1^-_{1^-}, M|\mathcal{M}_{1M}(E1)|0^+_1 \rangle = N_{1^-_{1^-}} \langle 0| (Q_2 Q_3)_{1M} \mathcal{M}_{1M}(E1)|0^+_1 \rangle,
\]

where $\mathcal{M}_{1M}(E1)$ is the operator of the electric dipole transition. Since multipole operators $Q_2, Q_3$ and $\mathcal{M}(E1)$ commute with each other we can rewrite the last expression as

\[
\langle 1^-_{1^-}, M|\mathcal{M}_{1M}(E1)|0^+_1 \rangle = N_{1^-_{1^-}} \sum_{\mu, \nu} \mathcal{C}^{1M}_{2\mu3\nu} (0^+_1)|(-1)^\nu Q_3 - \nu |M_{1M}(E1)(-1)^\mu Q_2 - \mu |0^+_1 \rangle.
\]

Using expressions (1), (3) and relation (7) we can rewrite (11) in the following way:

\[
\langle 1^-_{1^-}, M|\mathcal{M}_{1M}(E1)|0^+_1 \rangle = \sum_{\mu, \nu} \mathcal{C}^{1M}_{2\mu3\nu} (0^+_1)|(-1)^\nu Q_3 - \nu |M_{1M}(E1)(2^+_1, -\mu)\rangle.
\]

From the last expression using the Wigner – Eckart theorem we obtain

\[
\langle 1^-_{1^-} \parallel \mathcal{M}(E1) \parallel 0^+_1 \rangle = \langle 3^-_{1^-} \parallel \mathcal{M}(E1) \parallel 2^+_1 \rangle,
\]

and from (13) we get

\[
B(E1; 1^-_{1^-} \rightarrow 0^+_1)/B(E1; 3^-_{1^-} \rightarrow 2^+_1) = \frac{7}{3}.
\]

The last value coincides with that of the harmonic phonon picture. However, we did not use the assumption of harmonicity of quadrupole and octupole vibrations.

Thus, the $Q$–phonon model explains qualitatively one of the experimental results mentioned in Introduction. However, the value of the ratio $7/3$ deviates from the experimental observation $1 \div 2$. A possible reason for this deviation can be the presence of some admixtures in the $|1^-_{1^-} \rangle, |2^+_1 \rangle$ and $|3^-_{1^-} \rangle$ states which are not taken into account by the $Q$–phonon approach. The corrections to relation (11) discussed above will also influence the number in
the right-hand side of (14). Mention, however, a very small value of \( B(E1; 3^- \rightarrow 2^+) \) obtain in \([21]\). Taken together with the known value of \( B(E1; 1^- \rightarrow 0^+) \) it produces the value of the ratio \([14]\) which is much larger than 7/3.

III. \( E1 \) TRANSITION \( 1^- \rightarrow 2^+ \)

Let us consider the matrix element \( \langle 1^- M' \mid M_{1\mu}(E1) \mid 2^+ M \rangle \). This matrix element can be written in the following way:

\[
\langle 1^- M' \mid M_{1\mu}(E1) \mid 2^+ M \rangle = N_{2^+} \langle 1^- M' \mid M_{1\mu}(E1)Q_{2M} \mid 0^+ \rangle \\
= N_{2^+} \langle 1^- M' \mid Q_{2M}M_{1\mu}(E1) \mid 0^+ \rangle \\
(15)
\]

Having in mind that the \( 0^+ \rightarrow 1^- \) transition is the strongest one among the low-lying states we can rewrite approximately \((15)\) as

\[
\langle 1^- M' \mid M_{1\mu}(E1) \mid 2^+ M \rangle \approx N_{2^+} \langle 1^- M' \mid Q_{2M} \mid 1^- \mu \rangle \langle 1^- \mu \mid M_{1\mu}(E1) \mid 0^+ \rangle \\
= N_{2^+} \langle 1^- M' \mid Q_{2M} \mid 1^- \mu \rangle \frac{1}{\sqrt{3}} \langle 1^- \parallel M(E1) \parallel 0^+ \rangle \\
= N_{2^+} \frac{1}{3} C^{1M'}_{1\mu 2M} \langle 1^- \parallel \parallel M(E1) \parallel 0^+ \rangle \langle 1^- \parallel \parallel M(E1) \parallel 0^+ \rangle \\
(16)
\]

and as a consequence of \((16)\) we obtain

\[
\frac{\langle 1^- \parallel \parallel M(E1) \parallel 2^+ \rangle}{\langle 1^- \parallel \parallel M(E1) \parallel 0^+ \rangle} = \sqrt{\frac{5}{3}} \frac{\langle 1^- \parallel \parallel Q_2 \parallel 1^- \rangle}{\langle 1^- \parallel \parallel Q_2 \parallel 2^+ \rangle} \\
(17)
\]

The quadrupole moment of the \( 1^- \) state can be expressed approximately in terms of the quadrupole moments of the \( 2^+ \) and \( 3^- \) states. Finally, we have

\[
\frac{\langle 1^- \parallel \parallel M(E1) \parallel 2^+ \rangle}{\langle 1^- \parallel \parallel M(E1) \parallel 0^+ \rangle} \approx \frac{1}{\sqrt{35}} \frac{\langle 2^+ \parallel \parallel Q_2 \parallel 2^+ \rangle}{\langle 0^+ \parallel \parallel Q_2 \parallel 2^+ \rangle} + \frac{\sqrt{6}}{\sqrt{35}} \frac{\langle 3^- \parallel \parallel Q_2 \parallel 3^- \rangle}{\langle 0^+ \parallel \parallel Q_2 \parallel 2^+ \rangle} \\
(18)
\]

\( \text{From (18)} \) we can see that going from the magic or semimagic nuclei to nuclei with open shell the ratio \( B(E1; 1^- \rightarrow 2^+) / B(E1; 1^- \rightarrow 0^+) \) increases with the quadrupole moments of the first \( 2^+ \) and \( 3^- \) states. In order to obtain the Alaga rule for the ratio of the matrix elements \( \langle 1^- \parallel \parallel M(E1) \parallel 2^+ \rangle / \langle 1^- \parallel \parallel M(E1) \parallel 0^+ \rangle \), we must take into account an admixture of \( (|4^\uparrow \rangle \otimes |3^- \rangle)_1 \) to the \( |1^- \rangle \) state.
IV. IBM ANALYSIS OF THE NEUTRON NUMBER DEPENDENCE OF THE B(E1; 0^{+}\rightarrow 1^{-})

For completeness of the consideration the results of the very schematic IBM–based analysis of the E1 transitions between the low–lying collective states are given in this section below. A detailed analysis is presented in [15, 22]. Let us analyse qualitatively the quantity B(E1;0^{+}\rightarrow 1^{-}) in order to understand a possible reason for the appearance of the minimum in the neutron number dependence of this quantity when going away from the closed neutron shell N=82. As it follows from relation (13), instead of \langle 1^{-}_{1}\parallel Q(E1)\parallel 0^{+}_{1}\rangle we can consider the matrix element \langle 3^{-}_{1}\parallel Q(E1)\parallel 2^{+}_{1}\rangle. In the IBM |2^{+}_{1}\rangle and |3^{-}_{1}\rangle are the states with the maximum value of the F–spin: F = F_{max}. The dipole transition operator \mathcal{M}(E1) is mainly the F–spin vector. To simplify the discussion, we assume below that the E1 transition operator is exactly the F–spin vector. Then

\begin{equation}
\langle 3^{-}_{1}\parallel Q(E1)\parallel 2^{+}_{1}\rangle \sim (N_{\pi} - N_{\nu}),
\end{equation}

where 2N_{\pi} and 2N_{\nu} are the numbers of the valence protons and neutrons, respectively. If we consider a nucleus with the closed neutron shell, then as it is seen from (15), only protons will contribute to the matrix element \langle 3^{-}_{1}\parallel Q(E1)\parallel 2^{+}_{1}\rangle. If we start to increase the number of valence neutrons, then a neutron contribution to \langle 3^{-}_{1}\parallel Q(E1)\parallel 2^{+}_{1}\rangle will increase compensating, partly, a proton contribution. This matrix element takes the minimum value when N_{\pi} = N_{\nu}. Of course, this phenomenological analysis cannot give us the value of N_{\pi} at which \langle 3^{-}_{1}\parallel Q(E1)\parallel 2^{+}_{1}\rangle has a minimum because the physical E1 transition operator does not reduce to the F – spin vector. In the IBM–based consideration it is assumed, in fact, that a microscopic structure of the proton and the neutron bosons does not change from nucleus to nucleus. However, it is not evident that this assumption will be supported by the microscopic calculations. The results of the RPA based microscopic analysis are presented below.

In a similar way, transitions between the 3^{-}_{1} and 2^{+}_{ms} states [15] can be considered. In contrast to the preceding case the |2^{+}_{ms}\rangle state is a state with F = F_{max} – 1. Therefore,

\begin{equation}
\langle 3^{-}_{1}\parallel Q(E1)\parallel 2^{+}_{ms}\rangle \sim \frac{F_{max}M_{F}}{F_{max} - M_{F}} 10 \sqrt{\frac{(F_{max} + M_{F})(F_{max} - M_{F})}{(2F_{max} - 1)F_{max}}},
\end{equation}

where
where $F_{\text{max}} = \frac{1}{2}(N_\pi + N_\nu)$ and $M_F = \frac{1}{2}(N_\pi - N_\nu)$. Therefore,

$$
\langle 3^-_1 \parallel Q(E1) \parallel 2^+_m \rangle \sim \sqrt{\frac{N_\pi N_\nu}{(2N_\pi + 2N_\nu - 1)(N_\pi + N_\nu)}}. 
$$

(21)

It is seen from (20) and (21) that in contrast to (19) there is no cancellation of the proton and neutron contributions to the matrix element. Thus, this analysis indicates a possible reason for a large value of the $\langle 3^-_1 \parallel Q(E1) \parallel 2^+_m \rangle$ matrix element [15].

Let us assume, by analogy with section II, that the $|2^+_m\rangle$ state is created by an action of the operator

$$
\left(\frac{Q_0^\pi}{\langle 0^+_1|(Q_0^\pi Q_0^\pi)_0|0^+_1\rangle^{1/2}} - \frac{Q_0^\nu}{\langle 0^+_1|(Q_0^\nu Q_0^\nu)_0|0^+_1\rangle^{1/2}}\right) 
$$

(22)
on the ground state. Here $Q_0^\nu$ is the neutron (proton) quadrupole operator. This state is constructed so as to be orthogonal to the $2^+_1$ state. Then using as above a commutativity of the operators (22) and $\mathcal{M}(E1)$ we obtain from the inequality

$$
\langle 3^-_1 \parallel \mathcal{M}(E1) \parallel 2^+_m \rangle \gg \langle 3^-_1 \parallel \mathcal{M}(E1) \parallel 2^+_1 \rangle \quad \text{and} \quad \langle (3^-_1 \otimes 2^+_m)_1 \parallel \mathcal{M}(E1) \parallel 0^+_1 \rangle \gg \langle (3^-_1 \otimes 2^+_1)_1 \parallel \mathcal{M}(E1) \parallel 0^+_1 \rangle.
$$

The last statement seems to be in disagreement with the analytical result presented in [22] (Table 3). A possible reason is connected with a different proton–neutron structure of the shell model E1 transition operator and the two–body term in the sdf IBM–2 E1 transition operator. Roughly speaking, the proton and neutron parts of the shell model E1 transition operator has opposite signs. Only a small part of this operator proportional to $(N - Z)/A$, which appears because of substraction of the center of mass motion, contains proton and neutron contributions with the same signs. However, the two–body term in the sdf IBM–2 E1 transition operator used in [22] contains the proton and neutron parts with the same sign. As a consequence, the value of $B(E1;1^-_{ms} \rightarrow 0^+_1)$, to which only a two–body term contributes, becomes small and if $\eta \equiv e_\nu/e_\pi = 1$, i.e., a two–body term becomes the $F$–spin vector, $B(E1;1^-_{ms} \rightarrow 0^+_1) = 0$.

In this paper, we did not calculate the matrix element $\langle 3^-_1 \parallel \mathcal{M}(E1) \parallel 2^+_m \rangle$. However, if we take expression (22) as an approximation for the operator, which produces the $2^+_m$ state acting on the ground state, then the proton and the neutron contributions to this matrix element will have the same sign and for this reason $|\langle 3^-_1 \parallel \mathcal{M}(E1) \parallel 2^+_m \rangle|$ will be larger than $|\langle 3^-_1 \parallel \mathcal{M}(E1) \parallel 2^+_1 \rangle|$.
V. RPA BASED MICROSCOPIC CONSIDERATION

The aim of this section is to calculate the dipole transitional matrix element \( \langle 1^-_1 | M(E1) | 0^+_0 \rangle \), basing on expressions (1), (3) and (5) for the state vectors \(|2^+_1\rangle\), \(|3^-_1\rangle\) and \(|1^-_1\rangle\) and on the RPA approximation for the ground state. Thus, in the calculations below it is assumed that the wave vectors of the \(2^+_1\), \(3^-_1\) and \(1^-_1\) states are

\[
|2^+_1, \mu\rangle = \tilde{N}_{2^+_1} \hat{Q}_{2 \mu} |0^+_1, RPA\rangle, \quad (23)
\]

\[
|3^-_1, \nu\rangle = \tilde{N}_{3^-_1} \hat{Q}_{3 \nu} |0^+_1, RPA\rangle, \quad (24)
\]

and

\[
|1^-_1, M\rangle = \tilde{N}_{1^-_1} (\hat{Q}_{2} \hat{Q}_{3})_{1M} |0^+_1, RPA\rangle, \quad (25)
\]

Since the derivation of relations (23) and (24) is independent of a concrete structure of the \(0^+_1\) state and is based only on the \(Q\)–phonon form of the wave vectors, the results obtained below are consistent with relations (14) and (18).

We assume below that the ground state contains only those correlations which are produced by the quadrupole–quadrupole and the octupole–octupole interactions. The quadrupole and octupole interaction constants are fixed so as to reproduce the experimental values of the \(B(E2;0^+_1 \rightarrow 2^+_1)\) and the \(B(E3;0^+_1 \rightarrow 3^-_1)\), respectively. The ground state correlations related to the dipole–dipole interaction are not included. Therefore, calculating the strength of the E1 transitions we must introduce the core polarization factor \(\chi\) which takes into account a shift of a part of the E1 strength to the giant dipole resonance \cite{23}. Mention that the RPA–type calculations of the E1 transitions were performed in \cite{14, 24, 25, 26}.

In our approach based on the \(Q\)–phonon description the following expressions describe a microscopic structure of the \(|2^+_1\rangle\) and \(|3^-_1\rangle\) state vectors

\[
|2^+_1, \mu\rangle = \tilde{N}_{2^+_1} \left( b^+_2 - \frac{2\sqrt{21}}{P(2)} \sum_{ss'} W_{ss'}(3)(\alpha_s^+ \alpha_{s'}^+) b^+_3 \right) |0\rangle, \quad (26)
\]

\[
|3^-_1, \mu\rangle = \tilde{N}_{3^-_1} \left( b^+_3 + \frac{2\sqrt{15}}{P(3)} \sum_{ss'} W_{ss'}(2)(\alpha_s^+ \alpha_{s'}^+) b^+_2 \right) |0\rangle, \quad (27)
\]

where

\[
W_{ss'}(3) = \sum_t \langle t \parallel r^2 Y_2 \parallel s \rangle (-1)^{j_s+j_{s'}} (u_t u_s - v_t v_s) \left\{ \begin{array}{ccc} 3 & 2 & 1 \\ j_s & j_{s'} & j_t \end{array} \right\} \varphi_{is'}, \quad (28)
\]
In the above expressions summation is performed over both proton and neutron single particle states. In (26) and (27) \( b_{2\mu}^+ \) and \( b_{3\mu}^+ \) are the creation operators of the most collective quadrupole and octupole RPA phonons corresponding to the first roots of the RPA secular equation; \( \alpha^+_s \) is the quasiparticle creation operator; \( \psi^{(\lambda)}_{ss'} \) and \( \varphi^{(\lambda)}_{ss'} \) are the RPA amplitudes describing a microscopic structure of the most collective phonons \( b_{2\mu}^+ \) and \( b_{3\mu}^+ \); \( u \) and \( v \) are the coefficients of the Bogoliubov transformation; \( \tilde{N}_\lambda \) is the normalization factor. In (26) and (27) dipole excitations are presented by the two–quasiparticle and not by the phonon operators because dipole–dipole correlations in the ground state are not taken into account. Of course, summation is performed over all \( I^\pi = 1^- \) two–quasiparticle states. Contributions of the second terms in (26) and (27) to the total norm are of the order of 1% or less.

The wave vector of the \( 1_1^- \) state contains the following components: \( (b_{2\mu}^+ b_{3\mu}^+)_1|0\rangle \), \( \sum_{ss'} W_{ss'}(3)(\alpha^+_s\alpha^+_s)_1|0\rangle \) and \( \sum_{ss'} W_{ss'}(2)(\alpha^+_s\alpha^+_s)_1|0\rangle \). Among them the first one gives the main contribution to the norm, which is close to 100%. There are also other components in the wave vectors of the \( 2_1^+, 3_1^- \) and \( 1_1^- \) states. However, they are small and unimportant for calculations of the E1 transitions. A similar conclusion about a microscopic structure of the \( 1_1^- \) state was obtained in [24].

The components of the wave vectors of the \( 2_1^+, 3_1^- \) and \( 1_1^- \) states, containing operators creating two–quasiparticle \( 1^- \) states, appear because the quadrupole and the octupole multipole operators representing the corresponding state vectors according to (1), (3) and (5), are not exhausted by the one–boson term expressed in collective bosons only. In the calculations based on the Quasiparticle–Phonon model [14, 19, 20, 24, 25, 26] this admixture is generated by the quasiparticle–phonon coupling term which is produced by the terms in \( Q_2 \) and \( Q_3 \) additional to the one–boson term. However, the weight of this contribution in the norm of the eigenstate is similar in both the approaches. For the reduced matrix element \( \langle 1_1^- \| \mathcal{M}(E1) \| 0_1^+ \rangle \) we obtain the following expression:

\[
\langle 1_1^- \| \mathcal{M}(E1) \| 0_1^+ \rangle = \frac{e}{2}(1 + \chi)\sqrt{\frac{4\pi}{3}}\left(1 + \frac{N - Z}{A}\right)B^{(\pi)} - \left(1 - \frac{N - Z}{A}\right)B^{(\nu)}\right),
\]  

(31)
where

\[ B^{(\pi)} = -\sqrt{\frac{35}{(1 + P^{(\nu)}(2)/P^{(\pi)}(2))(1 + P^{(\nu)}(3)/P^{(\pi)}(3))}} \left( Z^{(\pi)} + 2 \frac{T^{(\pi)}(3)}{P^{(\pi)}(3)} + 2 \frac{T^{(\pi)}(2)}{P^{(\pi)}(2)} \right) \]

\[ B^{(\nu)} = -\sqrt{\frac{35}{(1 + P^{(\pi)}(2)/P^{(\nu)}(2))(1 + P^{(\pi)}(3)/P^{(\nu)}(3))}} \left( Z^{(\nu)} + 2 \frac{T^{(\nu)}(3)}{P^{(\nu)}(3)} + 2 \frac{T^{(\nu)}(2)}{P^{(\nu)}(2)} \right) \]

\[ Z^{(\tau)} = \sum_{s,s',t \in \tau} \langle s \| irY_1 \| s' \rangle (-1)^{j_s+j_t} (u_s u_{s'} - v_s v_{s'}) \left\{ \begin{array}{ccc} 2 & 3 & 1 \\ j_s & j_{s'} & j_t \end{array} \right\} \left( \psi_{st}^{(3)} \psi_{st'}^{(2)} + \varphi_{st}^{(3)} \varphi_{st'}^{(2)} \right), \]

\[ T^{(\tau)}(3) = -\sum_{ss' \in \tau} \langle s \| irY_1 \| s' \rangle (u_s v_{s'} + u_{s'} v_s) W_{ss'}^{(\tau)}(2), \]

\[ T^{(\tau)}(2) = -\sum_{ss' \in \tau} \langle s \| irY_1 \| s' \rangle (u_s v_{s'} + u_{s'} v_s) W_{ss'}^{(\tau)}(3). \]

In \( \tau = \pi \) or \( \nu \). The quantities \( P^{(\tau)}(\lambda) \) and the matrices \( W_{ss'}^{(\tau)}(\lambda) \) are determined by the expressions analogous to (30) and (32 – 29), respectively, though with summation over proton or neutron single particle states only.

The results of the calculations of the electric dipole transitional matrix element are presented in Figures 1 and 2 and in Tables 1–9. Here besides the total calculated dipole transitional matrix element \( |\langle 1^{-} \| M(E1) \| 0^{+} \rangle|_{total} \) including all contributions the results obtained without inclusion of the contribution coming from the \( I^\pi = 1^{-} \) two–quasiparticle admixture denoted by \( |\langle 1^{-} \| M(E1) \| 0^{+} \rangle|_{T=0} \) are also shown.

The results presented in Tables 1 and 2 and in Figs. 1 and 2 show that in Cd, Sn, Ba, Ce and partly in Nd and Sm isotopes the experimental data are between the results of calculations obtained with and without a contribution of an admixture of a dipole two–quasiparticle component to the \( 1^{-} \) state. However, in many cases the matrix element \( |\langle 0^{+} \| M \| 1^{-} \rangle|_{T=0} \), which includes only a collective contribution, is closer to the experimental data than the total matrix element, which includes both collective and two–quasiparticle contributions. Nevertheless, we can see that the two–quasiparticle admixture to the collective quadrupole–octupole two–phonon component of the \( 1^{-} \) state should be taken into account to improve agreement with the experimental data. In the present calculations, this contribution of the two–quasiparticle component is overestimated. However, since the weight of this component is smaller than 1% it is difficult to expect such an accuracy from the microscopic
calculations. We can conclude that the two-phonon component \((|2_1^+ \otimes 3_1^-\rangle)\), which gives the main contribution to the norm of the \(1_1^-\) state, determines an excitation energy and the E2 and E3 decay properties of the \(1_1^-\) state. However, for description of the electric dipole transitions \(0_1^+ \to 1_1^-\) it is necessary to take into account also an admixture of the dipole two-quasiparticle component.

As it is seen from expressions (26) and (27), the components containing two-quasiparticles coupled to angular momentum \(I=1\) contribute also to the structure of the \(2_1^+\) and \(3_1^-\) states. Since the microscopic E1 transitional operator contains in addition to the quadrupole–octupole phonon term a term changing the number of quasiparticles by two units, these components contribute to the \(\langle 3_1^- \parallel \mathcal{M}(E1) \parallel 2_1^+ \rangle\) transition matrix element.

Both sets of the results obtained with and without inclusion of the contribution of the two-quasiparticle component of the \(1_1^-\) state reproduce also an experimental \(A\)–dependence of the reduced matrix element. Only in \(^{146}\)Nd and \(^{148}\)Sm a depth of the minimum in the \(A\)–dependence of \(\langle 0_1^+ \parallel \mathcal{M} \parallel 1_1^- \rangle\) is significantly larger than in the calculated results. As it was noted in Introduction this matrix element has a minimum when the number of neutrons is equal to the magic number plus(minus) four neutrons. In Tables 3–8 the calculated ratios of the matrix elements \(\langle 0_1^+ \parallel \mathcal{M} \parallel 1_1^- \rangle|_A/\langle 0_1^+ \parallel \mathcal{M} \parallel 1_1^- \rangle|_{\text{semimagic}}\) are compared with the experimental ones. It is seen that in the case of Cd, Sn, Ba and Ce isotopes our calculations reproduce an \(A\)–dependence of the reduced dipole matrix element. However, in Nd and Sm isotopes experimental data have a deeper minimum than the calculated values. Calculations rather show a delay in an increase of the absolute value of the matrix element with increasing a number of the valence particles or holes.

In the case of the Nd and Sm isotopes the results of the microscopic calculations deviate from the picture which follows from the analysis of an \(A\)–dependence of \(|\langle 0_1^+ \parallel \mathcal{M} \parallel 1_1^- \rangle|\) in the Interacting Boson Model, where it is clearly seen that the reduced matrix element \(|\langle 0_1^+ \parallel \mathcal{M} \parallel 1_1^- \rangle|\) decreases if the number of valence protons is fixed, but the number of valence neutrons increases from zero. In the microscopic calculations, a proton contribution to the E1 transition matrix element changes when the number of valence neutrons varies although the number of valence protons continue to be fixed. This fact is illustrated by the results presented in Table 9 where the quantities \(B^{(\pi)}\) and \(B^{(\nu)}\) determined by Eqs. (32–33) are shown for the Nd isotopes.

The nature of the electric dipole two-quasiparticle component admixed to the wave func-
tion of the $1^{-}_1$ state is an interesting question. Since this component is a mixture of many spherical two-quasiparticle states, we can consider it formally as the $p$–boson introduced in \cite{27}. If we separate this component from the wave function of the $1^{-}_1$ state, change its normalization coefficient so as to obtain a wave function with the norm equal to one and calculate the E1 reduced matrix element between this artificially constructed state and the ground state, we obtain the value of the order of 0.6–0.7 \text{e}·\text{fm}. For comparison, for a pure alpha–cluster state we obtain approximately 6 \text{e}·\text{fm}. For the $1p–1h$ $(1h_{11/2} 1g_{9/2})_{1^{-}}$ proton state this matrix element is equal to 1.7 \text{e}·\text{fm}. So, it is difficult to do a definite conclusion about the nature of this two–quasiparticle admixture.

Concluding this section we should like to stress that any microscopic consideration of the enhanced E1 transitions cannot be performed in a harmonic approximation, i.e., cannot be carried out in the framework of the pure RPA and requires from the beginning an inclusion of the anharmonic effects. Indeed, in the pure RPA calculations only those parts of the one–body fermion operators are taken into account which change the number of quasiparticles by two units. The terms of the form $\alpha^+_s\alpha_i$ are neglected. The only exception is the part of the nuclear Hamiltonian which describes noninteracting quasiparticles because of its leading role. However, just the term of the form $\alpha^+_s\alpha_i$ describes $3^{-}_1 \rightarrow 2^{+}_1$ E1 transition. Therefore, the terms of this kind in the $Q_2$ and the $Q_3$ operators should be included into consideration. However, this inclusion leads to appearance of the anharmonic terms in the Hamiltonian since the latter contains quadrupole – quadrupole and octupole – octupole interactions. An inclusion into the operators $Q_2$, $Q_3$ and $\mathcal{M}(E1)$ of the terms proportional to $\alpha^+\alpha$ is necessary to conserve commutativity of these operators. This conclusive remark shows that a comparison of the present microscopic consideration, which from the beginning contains anharmonic effects, with the U(5) limit of IBM is difficult.

VI. SUMMARY

In conclusion, based on the $Q$–phonon representation of the wave vectors of the low–lying collective states we have derived the relations between different reduced matrix elements of the E1 transition operator. These relations explain qualitatively the experimentally observed correlations among data on E1 transitions.

Using the RPA approximation for the ground state vector and the $Q$ – phonon form of
the $1^-_1$ state we performed microscopic calculations of the reduced matrix element of the E1 transition operator between $|0^+_1\rangle$ and $|1^-_1\rangle$ states. It is shown that the two–quasiparticle component of the wave vector of the $|1^-_1\rangle$ state should be taken into account to achieve an agreement with the experimental data in spite of a small contribution of this component to the norm of the $|1^-_1\rangle$ state.

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FIG. 1: Absolute values of the reduced matrix elements for the E1 $0^+_1 \rightarrow 1^-_1$ transitions in Cd, Sn and Ba isotopes calculated with an inclusion of all contributions (dashed line), without inclusion of the contribution coming from the two–quasiparticle admixture (dot line) and the experimental data (solid line).

FIG. 2: The same as in Fig. 1, but for Ce, Nd and Sm isotopes.

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TABLE I: The experimental (exp) and calculated electric dipole transition matrix elements for Cd, Sn and Ba isotopes obtained including all contributions (total) and without contribution of the two–quasiparticle admixture to the \(1^-\) state \((T=0)\) (in units \(e \cdot fm\)).

| Nucleus | \(|\langle 1^- | M(E1) | 0^+_1 \rangle|_{\text{total}}\) | \(|\langle 1^- | M(E1) | 0^+_1 \rangle|_{T=0}\) | \(|\langle 1^- | M(E1) | 0^+_1 \rangle|_{\text{exp}}\) |
|---------|---------------------------------|-----------------|-----------------|
| \(^{108}\text{Cd}\) | 0.140                           | 0.018           | 0.050           |
| \(^{110}\text{Cd}\) | 0.119                           | 0.024           | 0.048           |
| \(^{112}\text{Cd}\) | 0.115                           | 0.026           | 0.041           |
| \(^{114}\text{Cd}\) | 0.114                           | 0.031           | 0.044           |
| \(^{116}\text{Cd}\) | 0.109                           | 0.036           | 0.034           |
| \(^{116}\text{Sn}\) | 0.170                           | 0.066           | 0.081           |
| \(^{118}\text{Sn}\) | 0.173                           | 0.073           | 0.085           |
| \(^{120}\text{Sn}\) | 0.169                           | 0.076           | 0.087           |
| \(^{122}\text{Sn}\) | 0.156                           | 0.073           | 0.085           |
| \(^{124}\text{Sn}\) | 0.134                           | 0.063           | 0.078           |
| \(^{134}\text{Ba}\) | 0.086                           | 0.019           | 0.048           |
| \(^{136}\text{Ba}\) | 0.113                           | 0.040           | 0.071           |
| \(^{138}\text{Ba}\) | 0.160                           | 0.082           | 0.114           |
| \(^{140}\text{Ba}\) | 0.152                           | 0.074           | –               |
| \(^{142}\text{Ba}\) | 0.153                           | 0.078           | –               |
| \(^{144}\text{Ba}\) | 0.158                           | 0.079           | –               |
TABLE II: The same as in Table 1 but for Ce, Nd and Sm isotopes.

| Nucleus | $\langle 1^- || \mathcal{M}(E1) || 0^+_1 \rangle_{\text{total}}$ | $\langle 1^- || \mathcal{M}(E1) || 0^+_1 \rangle_{\tau=0}$ | $\langle 1^- || \mathcal{M}(E1) || 0^+_1 \rangle_{\text{exp}}$ |
|---------|------------------|------------------|------------------|
| $^{140}\text{Ce}$ | 0.179 | 0.099 | 0.129 |
| $^{142}\text{Ce}$ | 0.170 | 0.092 | 0.108 |
| $^{144}\text{Ce}$ | 0.190 | 0.114 | – |
| $^{146}\text{Ce}$ | 0.209 | 0.134 | – |
| $^{142}\text{Nd}$ | 0.189 | 0.108 | 0.128 |
| $^{144}\text{Nd}$ | 0.181 | 0.101 | 0.098 |
| $^{146}\text{Nd}$ | 0.187 | 0.111 | 0.071 |
| $^{148}\text{Nd}$ | 0.223 | 0.141 | 0.119 |
| $^{144}\text{Sm}$ | 0.193 | 0.115 | 0.140 |
| $^{146}\text{Sm}$ | 0.177 | 0.103 | – |
| $^{148}\text{Sm}$ | 0.190 | 0.116 | 0.052 |
| $^{150}\text{Sm}$ | 0.212 | 0.132 | 0.099 |

TABLE III: The experimental and calculated absolute values of the reduced matrix element $\langle 0^+_1 || \mathcal{M}(E1) || 1^- \rangle$ in Cd isotopes given in units of $\langle 0^+_1 || \mathcal{M}(E1) || 1^- \rangle$.

| Nucleus | $^{108}\text{Cd}$ | $^{110}\text{Cd}$ | $^{112}\text{Cd}$ | $^{114}\text{Cd}$ | $^{116}\text{Cd}$ |
|---------|------------------|------------------|------------------|------------------|------------------|
| Exp.    | 1                | 1.0              | 0.80             | 0.90             | 0.70             |
| Calc.   | 1                | 0.85             | 0.82             | 0.82             | 0.78             |

TABLE IV: The experimental and calculated absolute values of the reduced matrix element $\langle 0^+_1 || \mathcal{M}(E1) || 1^- \rangle$ in Sn isotopes given in units of $\langle 0^+_1 || \mathcal{M}(E1) || 1^- \rangle$ for $^{116}\text{Sn}$.

| Nucleus | $^{116}\text{Sn}$ | $^{118}\text{Sn}$ | $^{120}\text{Sn}$ | $^{122}\text{Sn}$ | $^{124}\text{Sn}$ |
|---------|------------------|------------------|------------------|------------------|------------------|
| Exp.    | 1                | 1.0              | 1.06             | 1.0              | 0.94             |
| Calc.   | 1                | 1.02             | 0.99             | 0.91             | 0.78             |
TABLE V: The experimental and calculated absolute values of the reduced matrix element $\langle 0_1^+ \parallel M(E1) \parallel 1_1^- \rangle$ in Ba isotopes given in units of $|\langle 0_1^+ \parallel M(E1) \parallel 1_1^- \rangle|$ for $^{138}$Ba.

| Nucleus | $^{134}$Ba | $^{136}$Ba | $^{138}$Ba | $^{140}$Ba | $^{142}$Ba |
|---------|------------|------------|------------|------------|------------|
| Exp.    | 0.43       | 0.61       | 1          | —          | —          |
| Calc.   | 0.53       | 0.70       | 1          | 0.94       | 0.95       |

TABLE VI: The experimental and calculated absolute values of the reduced matrix element $\langle 0_1^+ \parallel M(E1) \parallel 1_1^- \rangle$ in Ce isotopes given in units of $|\langle 0_1^+ \parallel M(E1) \parallel 1_1^- \rangle|$ for $^{140}$Ce.

| Nucleus | $^{140}$Ce | $^{142}$Ce | $^{144}$Ce | $^{146}$Ce |
|---------|------------|------------|------------|------------|
| Exp.    | 1          | 0.85       | —          | —          |
| Calc.   | 1          | 0.94       | 1.06       | 1.16       |

TABLE VII: The experimental and calculated absolute values of the reduced matrix element $\langle 0_1^+ \parallel M(E1) \parallel 1_1^- \rangle$ in Nd isotopes given in units of $|\langle 0_1^+ \parallel M(E1) \parallel 1_1^- \rangle|$ for $^{142}$Nd.

| Nucleus | $^{142}$Nd | $^{144}$Nd | $^{146}$Nd | $^{148}$Nd |
|---------|------------|------------|------------|------------|
| Exp.    | 1          | 0.77       | 0.56       | 0.92       |
| Calc.   | 1          | 0.96       | 0.99       | 1.18       |

TABLE VIII: The experimental and calculated absolute values of the reduced matrix element $\langle 0_1^+ \parallel M(E1) \parallel 1_1^- \rangle$ in Sm isotopes given in units of $|\langle 0_1^+ \parallel M(E1) \parallel 1_1^- \rangle|$ for $^{144}$Sm.

| Nucleus | $^{144}$Sm | $^{146}$Sm | $^{148}$Sm | $^{150}$Sm |
|---------|------------|------------|------------|------------|
| Exp.    | 1          | —          | 0.38       | 0.69       |
| Calc.   | 1          | 0.91       | 0.98       | 1.10       |
TABLE IX: Dependence of the calculated proton $B^{(\pi)}$ and neutron $B^{(\nu)}$ contributions to the reduced matrix element $\langle 0^+_1 \parallel M(E1) \parallel 1^-_1 \rangle$ on the number of neutrons in Nd isotopes. For definition of the quantities $B^{(\pi)}$ and $B^{(\nu)}$ see (31–36).

| Nucleus  | $B^{(\pi)}$ (in fm) | $B^{(\nu)}$ (in fm) |
|----------|---------------------|---------------------|
| $^{142}\text{Nd}_{82}$ | 0.78                | 0.17                |
| $^{144}\text{Nd}_{84}$ | 0.80                | 0.21                |
| $^{146}\text{Nd}_{86}$ | 0.87                | 0.26                |
| $^{148}\text{Nd}_{88}$ | 1.05                | 0.32                |
$$<1^-_1 \| M(E1) \| 0^+ > \text{ (e fm)}$$

![Graph showing $<1^-_1 \| M(E1) \| 0^+ >$ for Ce, Nd, and Sm isotopes](image)
