Staggered Currents in the Vortex Core

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We study the electronic structure of the vortex core in the cuprates using the U(1) slave-boson mean-field wavefunctions and their Gutzwiller projection. We conclude that there exists local orbital antiferromagnetic order in the core near optimum doping. We compare the results with that of BCS theory and analyze the spatial dependence of the local tunneling density of states.

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The high-\(T_c\) superconductors are doped Mott insulators. Recently the possibility that the complex behavior of high-\(T_c\) systems is due to several competing orders came into focus. The contenders are the antiferromagnetic order, \(d\)-wave superconducting order, and stripe order. Most recently yet another item has been added to the above list - the staggered flux (SF) order. In the SF ground state there is a current circulating around each plaquette in the copper-oxide plane giving rise to orbital-magnetic moments. Like in a spin antiferromagnet, the orbital moment staggers from plaquette to plaquette.

Among different proposals of the SF order, Chakravarty \textit{et al.} suggest that such order actually exists in static form in the underdoped region of the high-\(T_c\) phase diagram. So far there is no direct experimental evidence for such a state. In another proposal, Ivanov, Lee, and Wen argue that the SF order appears in dynamic rather than static form. They substantiate their argument by showing that the Gutzwiller-projected \(d\)-wave superconducting state possesses an equal-time, power-law SF correlation. Such correlation is also seen in exact diagonalization of the t-J model. Based on SU(2) slave-boson mean-field theory, Lee and Wen further argued that the fluctuation of the SF order might slow down or even become frozen in the core of a superconducting vortex. They claim that the SF vortex core is more likely to be found in the underdoped systems, because the energy difference between the SF state and the \(d\)-wave superconducting state diminishes with underdoping.

In the following we present evidence for the existence of SF order in the vortex core using the U(1) slave-boson mean-field wavefunction and its Gutzwiller projection. Unlike Lee and Wen’s expectation, however, we find the strongest evidence for the SF vortex core near optimum doping, and a weakening evidence as the system becomes either more overdoped or underdoped.

The above discrepancy triggers the following cautionary remark. In an unbiased mean-field theory of the t-J model with the Coulomb interaction between the holes, the following orders compete with each other: antiferromagnetism, spin dimerization, staggered flux, and \(d\)-wave pairing. If no artificial constraint, such as preserving the translation symmetry or suppressing antiferromagnetism, is imposed the mean-field ground state exhibits antiferromagnetic order for small doping \((x<2\%)\), stripe order for intermediate doping \((2\%<x<14\%)\), and uniform \(d\)-wave superconducting order for high doping \((14\%<x<25\%)\). If the antiferromagnetism is omitted, the ground state for low doping shows spin-dimerization. Such dimerized insulating states also appear in the insulating region of the stripe phase in the intermediate doping. If one forbids antiferromagnetism, stripes and dimerization all together, then, aside from the \(d\)-wave superconducting phase, the SF phase is the closest contender for the ground state. In that case one expects that by frustrating the \(d\)-wave superconducting order the SF order will be revealed. This leads naturally to the prediction that the SF order appears in the vortex core or in the bulk when an external magnetic field suppresses the superconducting order. For sufficiently low doping this conclusion is likely to be invalid due to the presence of other competing orders.

We have previously carried out a numerical study of the vortex core within the U(1) slave-boson mean-field theory. The Hamiltonian we considered is the t-J + Coulomb model. In standard notation the Hamiltonian reads

\[
H = -t \sum_{\langle ij \rangle} (b_i^\dagger b_j + h.c.) + J \sum_{\langle ij \rangle} (\mathbf{S}_i \cdot \mathbf{S}_j - \frac{1}{4} n_i n_j) + \frac{V_c}{2} \sum_{i \neq j} \frac{1}{r_{ij}} (n_i - \bar{n})(n_j - \bar{n}) - \mu \sum_i n_i. \tag{1}
\]

Equation (1) is parametrized by two dimensionless ratios \(t/J\) and \(V_c/J\), where \(V_c\) is the strength of the Coulomb interaction at the nearest-neighbor distance. In Ref. 8 we have performed a partially-biased mean-field search where the spin-antiferromagnetic order is disallowed. The result shows that depending on doping there are two types of vortex core. The first type is insulating, and is favored at low doping. The second type is metallic, favored at high doping. In the insulating core, the frustration of the \(d\)-wave superconducting order gives rise to...
an insulating state known as the “box phase” \[13\]. (This state is known to be equivalent to the spin-dimer state at zero doping.) There is no orbital current flowing in this type of vortex core. Although it went unnoticed by the authors at the time, it turns out that in the metallic core SF order does exist \[8\]. The purpose of this paper is to present in detail the SF state in the metallic vortex core. We believe that the omission of antiferromagnetism is likely to have important effects on the structure of the insulating core \[14\], but much less so on the metallic one.

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Before showing the main results we first present the current pattern in the vortex core according to the d-wave BCS theory \[15\]. In Fig. 1 the direction and magnitude of the bond current in the core of a BCS d-wave vortex is shown. The result shown here is the self-consistent mean-field solution for the t-J model upon ignoring the occupation constraint. We use \( t/J = 2 \) and the average number of electrons per site equal to 0.88. It is clear that the current pattern is the diamagnetic pattern dictated by the vorticity. In the figure the numerals inside each plaquette is the lattice curl of the bond current, \( \phi(\mathbf{R}) \), defined in Eq. 2. The currents are normalized so that the maximum magnitude of the current is unity. The center of the vortex is shown in dark square, with one quadrant of the lattice in display.

FIG. 1. The current pattern near the core of a BCS vortex. The magnitude and direction of the bond current are shown. The numerals in the center of each plaquette is the lattice curl of the bond current, \( \phi(\mathbf{R}) \), defined in Eq. 2. The currents are normalized so that the maximum magnitude of the current is unity. The center of the vortex is shown in dark square, with one quadrant of the lattice in display.

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\[
J_{ij} = -i\epsilon_{ij}c_i^\dagger c_j - c_i^\dagger c_j^\dagger = -i(b_i^\dagger b_j^\dagger f_i f_j) + \text{h.c.}
\]

\[
\phi(\mathbf{R}) = (1/4) \sum_{(ij)\in \mathcal{P}_R} \eta_{ij} J_{ij}, \quad (2)
\]

Here \( \mathbf{R} \) labels the position of the plaquette \( \mathcal{P}_R \) and \( \epsilon_{ij} = \pm 1 \) is chosen so that the sign of \( \phi(\mathbf{R}) \) is the same for every plaquette attests to the absence of staggered current flow.

The results with the occupation constraint taken into account are shown in Fig. 2 for three doping concentrations \( x = 6\%, 12\%, 18\% \) with \( t/J = 2 \) and \( V_c/J = 1.5 \). The mean-field techniques are explained in Ref. \[8\]. Here we simply stress that in this calculation the slave-boson amplitude, fermion hopping and pairing amplitudes, and the Lagrange multipliers are all determined self-consistently without any restriction. Once the self-consistent mean-field wavefunction is obtained, we perform the Gutzwiller projection to remove double occupancy. The mean-field results shown here is for \( 16 \times 16 \) lattice with an open boundary condition. The central \( 10 \times 10 \) subset of the self-consistent mean-field parameters are used to construct the input wavefunction for the Gutzwiller projection. The evaluation of the current expectation value using the Gutzwiller projected wavefunction is achieved by a variational Monte Carlo technique. The results are obtained by running 5000 equilibration sweeps, and 10000 averaging sweeps. Finally we symmetrize the bond currents according to the point group symmetry of the square lattice.

Figures 2(a)-(c) illustrate the mean-field results for \( x = 6\%, 12\%, 18\% \) respectively. Since the point group symmetry is maintained in the solution, we only show one quadrant of the lattice with the center of the vortex sitting in the middle of the dark square located at the lower left corner of each figure. The arrows in the figure indicate the direction of the current flow. As in Fig. 1 we normalize the currents by their maximum value (See Table I for a list of maximal currents). It turns out that for all cases the maximal current occurs one plaquette away from the center. By inspection we conclude that the current pattern is the superposition of two components: the diamagnetic flow dictated by the vorticity and a staggered flow consistent with SF order. To study the degree of SF order we look at \( \Phi(\mathbf{R}) \) of each plaquette. It is striking that for all three cases, \( \Phi(\mathbf{R}) \) is almost perfectly staggered for all the plaquettes shown.

The results for \( \Phi_{oa fm} \) is shown in table I.

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\[
\Phi_{oa fm} = \sum_{\mathbf{R}} (-1)^{\mathbf{R}} \phi(\mathbf{R}). \quad (3)
\]

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Figures 2(d)-(f) illustrate the Gutzwiller projected results for \( x = 6\%, 12\%, 18\% \) respectively. The projection enhances the maximal current in the first two cases. It is clear that there is still substantial amount of staggered order near the center of the vortex. The values of the SF order parameter for these figures as well as their maximal currents are shown in table I.

By studying the value of \( \Phi_{oa fm} \) as shown in Table I we conclude that the \( x = 12\% \) system shows the strongest evidence for the SF order in the vortex core. We should also stress that at still lower doping the vortex core becomes insulating, and the bond current should vanish.
By comparing Fig. 1 and Fig. 2 it is clear that the emergence of staggered currents is a strong correlation effect. An experimental detection of the SF core would strongly indicate that our understanding of the local correlation in the cuprates based on the t-J model is qualitatively correct.

How to detect the SF order in the vortex core is an important question. Lee and Wen suggested using the NMR as a probe of the local magnetic field generated by the orbital currents. From an STM experimental point of view it is important to ask whether the signature of the SF core can be decoded from the tunneling spectroscopy. For example one might think that the doubling of unit cell associated with the SF order can give rise to real space patterns of the tunneling spectra suggestive of the presence of two sublattices. Motivated by such a question we computed the real space maps of the tunneling density of state at a fixed voltage. The result shows a strong sublattice dependence for a bias close to $\pm 0.1 J$. This is shown in Figs. 3(a),(b) for $x = 12\%$. In order to determine whether such a sublattice dependence is indeed a consequence of the SF order we compute the same map for the BCS vortex shown in Fig. 1. As is clear in Figs. 3(c),(d), the sublattice dependence is also present in the BCS vortex. We are then forced to conclude that the sublattice dependence is not correlated with the SF order. Instead we believe it is due to the nature of the quasiparticle wavefunctions near the gap nodes. We have also studied the tunneling spectra as a function of bias voltage in a range of doping where the SF core is found. Despite the common presence of SF order, the detailed shape of the spectra depends sensitively on the doping concentration. We therefore conclude that it is difficult to draw an unequivocal prediction for the STM spectroscopy based on SF order.

In summary, we have studied in detail the current distribution near the vortex core both in the BCS and strongly-correlated systems. We find in the latter case an evidence of staggered flux order near the optimum doping. We take this as an indication that, next to the $d$-wave superconducting state, the staggered flux state is a close contender for the ground state in the studied doping range. The strength of SF order weakens when the system is doped away from optimal one. Combining the
result of Ref. [8] and this work we have identified three distinctive signatures of strong correlation in the core of a superconducting vortex. 1) There are two ways to suppress the superfluid density inside the vortex core, one by de-pairing and the other by locally depleting the holes. They respectively give rise to the metallic and insulating vortex cores. 2) At energies below the charge gap, the single-particle spectral weight is not conserved. 3) The suppression of the pairing in the metallic core results in the appearance of staggered flux order.

FIG. 3. A conductance profile at a fixed bias near ±V, V ≈ 0.1J, from slave-boson (SB) mean-field theory ((a) and (b)) and BCS-like (BCS) mean-field theory ((c) and (d)) of the vortex core. The interior 8 × 8 region is shown. The average hole density in both cases is 12%.

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TABLE I. Maximal Currents |Jij| and the staggered orbital magnetization Φoafm for various doping (See text for exact definition). The superscripts MF/GP indicate mean-field and Gutzwiller projection results, respectively.

| x     | |Jij|MF | |Jij|GP | Φoafm^MF | Φoafm^GP |
|-------|----------------|----------------|----------------|----------------|
| 6%    | 0.012          | 0.023          | 0.096          | 0.135          |
| 12%   | 0.039          | 0.059          | 0.445          | 0.553          |
| 18%   | 0.040          | 0.036          | 0.267          | 0.305          |