Reconstructing the dark energy

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We put forward a reconstruction scheme for dark energy which employs a number of observationally motivated geometrical constraints expected to be satisfied by the luminosity-distance relation. This generalises a scheme recently put forward, with the important additional feature that it allows a wider range of possible equations of state to be explored.

Applying this scheme to the recent SNLS supernovae data we find a number of interesting results: (i) The $\Lambda$CDM model is compatible with the data within the 2$\sigma$ significance level, but not at the 1$\sigma$ level. (ii) Allowing the dark energy model to have an equation of state which is free to become phantom in the redshift interval $[0, 1]$, we find a sharp increase in the degeneracy in the reconstructed equation of state for redshifts greater than $\sim 0.6$, brought about by a sudden widening of the confidence contours. We trace the origin of this enhancement in degeneracy to the fact that in such models the equation of state can diverge. This is important in order to devise strategies to reduce this degeneracy, specially in the light of near future observations. There is, however, no such large degeneracies for dark energy models whose equations of state remain larger than $-1$ in the redshift interval $[0, 1]$. (iii) There exist reconstructed solutions which fit the data well and show transient acceleration, with a very recent phase of deceleration.

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I. INTRODUCTION

Recent observations have provided strong evidence to suggest that the universe has recently undergone a phase of accelerated expansion. These include evidence from supernovae data \cite{1, 2} as well as CMB measurements \cite{3, 4} and observations of large scale structure \cite{5, 6}.

The simplest way to explain this late acceleration is by invoking the presence of a positive cosmological constant. Although this is compatible with all observational constraints to date, it is not clear how to reconcile the presence of such a small parameter with fundamental theories. As a result a range of other scenarios have been considered to account for this acceleration. An important class of these scenarios - the so called ‘dark energy’ scenarios - models the underlying accelerating mechanism in terms of an effective perfect fluid with a negative equation of state (EOS). The attractive feature of these models is that they can, in addition to cosmological constant, mimic a wide range of scenarios with a variable EOS, including quintessence models, braneworld models \cite{7} as well as models based on loop quantum cosmology \cite{8}.

An urgent question at present is how to distinguish between these possibilities. Two approaches are possible in principle. One may choose a particular model and compare its predictions with observations. Alternatively, one can try to reconstruct the properties of the ‘dark energy’, including its equation of state, directly from observations.

A number of attempts have recently been made at such reconstructions, employing different data sets, including supernovae data as well as CMB and baryon acoustic oscillation (BAO) data (see \cite{10, 11} for a recent review and references therein). Among these schemes are the so called model-independent ones which attempt to reconstruct the cosmological parameters directly from the data without making particular assumptions regarding their forms \cite{13, 14, 15}. It is important to note that the interpretation of observational data always requires a theoretical framework. In that sense all reconstructions are framework dependent. The question is how wide a set of models such a framework allows.

The aim of this paper is to generalise the reconstruction scheme recently put forward in \cite{9} by allowing a wider set of equations of state to be explored.

For our reconstructions we shall employ supernova data recently released by the Supernova Legacy Survey (SNLS) \cite{2}, as well as the baryon acoustic oscillation data (BAO) \cite{16}. Using this scheme we shall reconstruct the luminosity-distance curves best fitting the data and hence the corresponding cosmological parameters including the Hubble parameter and the EOS. We find that the generalised scheme leads to a number of interesting results (see below).

The plan of the paper is as follows. In Section \text{I}I we give a brief description of the reconstruction scheme and the data sets used in the reconstructions. In Section \text{I}I\text{I} we summarise the results of our reconstructions. This includes a brief summary of our reconstruction of the mock data in subsection \text{I}I\text{I}A a summary of our re-

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construction of the luminosity-distance relation and its
derivatives as well as the Hubble parameter in subsec-
tion III B an estimate for the cold dark matter density
parameter \( \Omega_m \) using the BAO data in subsection III B 2
and finally a summary of our reconstructions of the EOS
and the deceleration parameter in subsection III B 3. In
Section IV we explain the origin of the degeneracy (i.e.
sudden widening of the confidence contours) appearing in
the reconstruction of the EOS and discuss how it can be
reduced. In Section V we consider two important subsets
of reconstructed curves representing respectively models
with quintessence dark energy and transient acceleration.
Finally, Section VI contains a summary of our results and
conclusions.

II. THE RECONSTRUCTION SCHEME

In our reconstruction scheme the luminosity-distance
\( d_l(z) \), expressed in terms of the Hubble parameter \( H \)
and redshift \( z \) as

\[
d_l(z) = c(1 + z) \int_0^z \frac{dz}{H},
\]

is reconstructed subject to two geometrical constraints
expected to be satisfied by the luminosity-distance relation.
These are:

[A] \( d'_l > 0 \)
[B] \( d''_l > 0 \),

where a prime denotes differentiation with respect to the
redshift \( z \).

The assumption A is simply a consequence of the ob-
served expansion of the universe. Assumption B can be
shown to imply that the deceleration parameter \( q < 1 \).
This would be satisfied by models whose current equa-
tion of state satisfy \( w_0 < 0.46 \) (see below). These include
non-accelerating models as well as the important class of
models that are currently accelerating (with \( q < 0 \)) and
which in the past tend to an Einstein de-Sitter model
(with \( q = 1/2 \)). We shall show that the assumption \( q < 1 \)
is preferred by the data at 3\( \sigma \) for \( z < 0.5 \), whereas at
larger redshift \( q \) is greater than 1 due to large
degeneracy if it was not constrained by condition [B].

Our reconstruction scheme generalises a scheme re-
cently considered by the authors [9], by dropping a third
geometrical constraint, namely \( d''_l \leq 0 \). The motivation
for this constraint was that it is satisfied by the Einstein-
de Sitter and \( \Lambda CDM \) solutions, which constitute im-
portant past and future phases of the ‘standard’ cosmolog-
ical model. Despite these nice asymptotic features, this
assumption, though compatible with observations, is not
directly imposed by them. One can in fact consider evol-
u tion histories for the universe which are compatible with
current observations and which do not satisfy this
assumption for all times. For example, this constraint
is not respected by some equations of state commonly
considered in the literature, such as \( w = w_0 + w_1 z \) or
\( w = w_0 + w_1 z/(1 + z) \), where \( w_0 \) and \( w_1 \) are real
constants. It is thus interesting to study the consequences
of dropping this assumption on the results of the recon-
structions.

To proceed we shall employ as our theoretical frame-
work General Relativity with a dark energy component
treated as a perfect fluid with a density \( \rho \) and an EOS\( w(z) \). The Hubble function \( H \) is in this case given by

\[
H^2 = H_0^2 \left[ \Omega_m (1 + z)^3 + \Omega_\Lambda \rho_0 / \rho_0 \right],
\]

where \( \Omega_m \) and \( \Omega \) are the dark matter and dark energy
density parameters respectively and the subscript "0" in-
dicates the values quantities take at the present time.
We shall employ the generalised scheme discussed above,
based on constraints [A] and [B], in order to reconstruct
the cosmological parameters. The data used in this study
are the 115 supernovae with redshifts \( z < 1.01 \) from
the Supernova Legacy Survey (SNLS) [2]. To constrain
\( \Omega_m \) we also employ the baryon acoustic oscillation peak
(BAO) recently detected in the correlation function of
luminous red galaxies (LRG) in the Sloan Digital Sky Survey [10].

To test the goodness of fit of our reconstructions we
shall, in line with many other works in this area [13, 17, 20], use
the standard \( \chi^2 \) minimization. For the supernovae data,
\( \chi^2 \) is defined by

\[
\chi^2 = \sum_{i=1}^n \frac{(m^\text{obs}_i - m^\text{th}_i)^2}{\sigma_i^2},
\]

where \( n \) is the number of data points, \( m^\text{obs}_i \) and \( m^\text{th}_i \)
are respectively the observed and the theoretical magnitudes
(obtained from the reconstructed luminosity-distance rel-
ations) and \( \sigma_i \) is the uncertainty in the individual \( m^\text{obs}_i \).

III. RECONSTRUCTIONS

Our reconstruction procedure begins by generating a
large set of luminosity-distance curves which satisfy the
constraints [A] and [B] while fitting the SNLS supernovae
data (see next Section for details). Each luminosity-
distance curve \( d_l \) is constructed using a discrete set of
equally spaced points, and satisfying the constraints [A]
and [B], which when discretised respectively take the forms

\[
d_{i+1} \geq d_i,
\]

\[
d_{i+2} - d_{i+1} \geq d_{i+1} - d_i.
\]

In practice we subdivided the interval \( z \in [0, 1] \) into eight
sub-intervals of different length allowing us to recover the
\( \Lambda CDM \) model at 1\( \sigma \) level with the mock data (see section III A). Taking a larger (smaller) number of sub-intervals
leads to over-fitting (under-fitting) the mock data, re-
resulting in the \( \Lambda CDM \) model not to be recovered at the
1σ level. We have checked that using different choices of sub-intervals \( z_i \) (with the same number of sub-intervals) do not qualitatively change our results. In the figures presented here, second order interpolation was used to obtain the reconstructed values between the steps. We have checked that using third order interpolation does not qualitatively change our results.

Using the set of luminosity-distance curves thus determined, we calculate for each curve

\[
H = c \left[ \left( \frac{d_i'}{1 + z} - \frac{d_i}{1 + z} \right) \frac{1}{1 + z} \right]^{-1} \tag{2}
\]

and

\[
H' = -c(1 + z) \frac{2d_i' + (1 + z)[-2d_i'' + (1 + z)d_i''']}{(dl - (1 + z)d_i')^2} \tag{3}
\]

which can then be used to reconstruct the cosmological parameters, including the equation of state, without assuming specific ansatzes (see [9] for further details.)

A. Reconstruction using mock data

To test our generalised reconstruction scheme we first employed mock data, consisting of supernovae data as in [9] (see also [18]). The data was generated by assuming the supernovae have the same distribution in redshift and error bars as the real (SNLS) data, but with the luminosity-distances replaced by those corresponding to the \( \Lambda \)CDM model plus a gaussian noise. The corresponding \( d_i \) curves were then obtained through interpolation. The reconstructed curves were chosen such that their \( \chi^2 \) per degree of freedom \( (\chi^2_{DOF}) \) was less than 1.20. The best fit \( \chi^2 \) was found to be 113.15 which is to be compared with \( \chi^2 = 114 \) for the \( \Lambda \)CDM model. Figures 1 show the reconstructions of \( d_i' \) and \( d_i'' \) together with their 1σ, 2σ and 3σ confidence contours respectively. As can be seen, the \( \Lambda \)CDM model lies within the 1σ confidence contours, thus demonstrating that the reconstruction scheme is reliable.

B. Reconstruction using real data

1. Reconstruction of \( d_i \), \( d_i' \), \( d_i'' \) and the Hubble function

In this subsection we briefly summarise the results of our reconstructions, using the real data, for the quantities \( d_i \), \( d_i' \), \( d_i'' \) (depicted in Fig. 2) as well as the Hubble function (shown in Fig. 3). As can be seen, the \( \Lambda \)CDM model is compatible with the data at 2σ significance level, but not at 1σ level since the best fit \( \chi^2 = 112.11 \). Although this is not significant enough to rule out the \( \Lambda \)CDM model, it does highlight a consequence of dropping the constraint on the third derivative of the luminosity-distance (i.e. \( d_i''' < 0 \)) used in [9]. Keeping this constraint has the consequence that \( \Lambda \)CDM model becomes compatible with the data at 1σ level.

2. Using BAO data to constrain \( \Omega_{m0} \)

The supernovae data - and hence the reconstructed luminosity-distance curves obtained here - do not constrain tightly the value of \( \Omega_{m0} \) (see [6] for an explanation about degeneracy on this parameter). To further constrain \( \Omega_{m0} \) we require additional input from observations. Here we use supernovae together with baryon oscillation data [16] to recalculate the \( \chi^2 \) values corresponding to each luminosity-distance curve. The results are depicted in Fig. 4. As can be seen, the best fit value for \( \Omega_{m0} \) is 0.28, with the 1σ confidence interval given by \( 0.26 < \Omega_{m0} < 0.3 \). We shall use this best fit value in the rest of the paper.

3. Reconstruction of the EOS and the deceleration parameter

Our reconstructions of the EOS are depicted in Figs. 5. A number of comments are in order.

The reconstructed EOS seems to have a maximum at \( z = 0 \). This is indeed expected and is a consequence of the fact that we have assumed \( d_i''' > 0 \). This maximum value can be readily calculated in the following way. Recall that at present \( (z = 0) \) the equation of state is given by (see e.g. [19]):

\[
w_0 = \frac{\left( 2 \frac{H_0}{\Omega_{m0}} - 1 \right)}{(1 - \Omega_{m0})}
\]
Fig. 2: Shown are the reconstructions of $d_l$ (with SNLS data), $d'_l$ and $d''_l$. The $\Lambda$CDM model, with $\chi^2 = 114$, is compatible with the data at 2σ confidence level, but not at 1σ level.

We also have

$$d''_l(z = 0) = \frac{c}{H_0} \left( 2 - \frac{H'_0}{H_0} \right).$$

Now since $d''_l(z = 0) > 0$, this implies that $H'_0 < 2H_0$ and thus $w_0 < \frac{1}{3(1-\Omega_m)} \approx 0.46$.

The reconstructions of EOS also show a sudden large degeneracy, brought about by a sudden widening of the confidence contours, at around a redshift of $z \sim 0.6$. This makes it extremely difficult to deduce the nature of EOS beyond this redshift. We shall discuss this issue in the next subsection.

An alternative way of testing the closeness of EOS to $-1$ is to use the weighted average of the equation of state, the so called w-probe ($\bar{w}$) [14]. The advantages of this probe are that it is independent of the value of $\Omega_m$ and does not depend upon the second derivative of the luminosity-distance $d_l$. The calculated values of $\bar{w}$ are depicted in Fig. 6. As can be seen the constraints on the $\bar{w}$ are better than those on $w$, even for larger values of $z$ for which $\bar{w}$ remains finite while $w$ diverges. The w-probe provides an alternative confirmation of our results concerning the reconstruction of the EOS, namely that the $\Lambda$CDM model is in agreement with the data at 2σ confidence level.

We also reconstructed the deceleration parameter, which is depicted in Fig. 7. An interesting feature of these reconstructions is that they show some solutions can possess a transient acceleration. For example at 1σ significance level, decelerated expansion can occur for $0 < z < 0.13$ and for $z > 0.38$, while for $0.13 < z < 0.38$ the results show an accelerating expansion. We shall return to this issue in section V B below.

IV. NATURE OF EOS DEGENERACY AND STRATEGIES TO REDUCE IT

Given the important consequences of such sudden widening of the confidence contours for the determin-
nation of EOS, it is important to understand their origin in order to find strategies to reduce them.

To do this, recall that the dark energy EOS can be written as \[ w = \frac{2(1+z)H' - 1}{1 - (2H^*)^2\Omega_m(1+z)^3} \]

This shows that the EOS diverges when \( H/H_0 = \sqrt{\Omega_m(1+z)^3} \). To see the role of this divergence for the above degeneracies we have plotted this curve (referred to as \( C \)) in Fig. 3. As can be seen the curve \( C \) intersects the reconstructed curves \( H/H_0 \), fitting the data, at around \( z > 0.6 \), which leads to the sudden widening of the confidence contours and the resulting degeneracies observed in the reconstruction of the EOS for \( z > 0.6 \) (see the top panel of Fig. 5). Thus as the reconstructed Hubble function approaches the curve \( C \), the confidence contours widen and degeneracies increase. In particular, the EOS blows up when the reconstructed Hubble function intersects the curve \( C \). Consequently some very large values of the EOS can fit the data beyond the redshift \( z > 0.6 \), leading to the observed degeneracy. In fact as can be seen from the plot of curve \( C \), some reconstructed solutions within the 1\( \sigma \) confidence contour lines correspond to diverging EOS. It is worth noting here that a diverging effective EOS cannot necessarily be ruled out as unphysical, since it could be an effective mathematical representation of some physical models such as generalised gravity models \[12\] as well as braneworld models.

Having seen the cause of the sudden widening of the confidence contours and the resulting large degeneracies, it is important to determine how they could be reduced. Clearly to decrease the observed degeneracy in the EOS, one needs to prevent \( C \) from intersecting the reconstructed \( H/H_0 \) curves, which implies that the data will have to be made more precise for redshifts beyond \( z = 0.6 \).

To show this more clearly we proceeded as follows. We removed all reconstructed \( H \) curves which intersected the \( C \) curve, as can be seen in Fig. 8. The corresponding \( d_l \) curves are shown in Fig. 9. This shows the extent to which the accuracy of observations needs to be enhanced in order to prevent the intersections of reconstructed \( d_l \) curves with the curve \( C \), up to redshifts \( z = 1 \). The corresponding reconstructed EOS is shown in Fig. 10. As can be seen the removal of \( H \) curves intersecting the \( C \) curve has reduced the degeneracy for larger redshifts. The result is rather modest, however, and a further reduction of degeneracies, specially at higher redshifts would require more severe narrowing of the reconstructed \( d_l \) cone which in turn implies more accurate observations (smaller error bars) as well as less dispersion.

Another important point to note in this regard is the relation between the \( C \) curve and the \( \Lambda CDM \) curve (see Fig. 3). It is easy to see that these curves intersect at \( z = \infty \). More importantly the distance between the two curves decreases as \( z \) increases. This implies that to avoid the intersections (and the resulting degeneracies) the data has to become more and more accurate as \( z \).
increases, which is of course a very difficult constraint to meet observationally.

Figure 8: Reconstruction of the Hubble parameter $H$ with the reconstructed curves intersecting the $C$ curve removed. The upper solid curve represents the $\Lambda CDM$ curve and the lower one the $C$ curve.

Figure 9: Reconstruction of the $d_1$ curves corresponding to the $H$ curves depicted in Fig. 8.

Figure 10: Reconstruction of the EOS corresponding to the reconstructed curves in Figs. 8 and 9.

V. IMPORTANT SUBSETS OF RECONSTRUCTED CURVES

In this section we consider different subsets of the reconstructed luminosity-distance curves corresponding to quintessence dark energy and transient acceleration.

A. The quintessence subset

In this subsection we identify the dark energy with a minimally coupled scalar field and reconstruct the so-called quintessence solutions defined by $w > -1$ (see [9] for details). The reconstructed kinetic and potential energies of the scalar field, with their corresponding confidence contours, are depicted in Figs. 11 and 12. As can be seen from these reconstructions, the kinetic and potential energies are well constrained over the intermediate times in the range $z \in [0, 1]$ while they are poorly constrained at present ($z \sim 0$) and at early times ($z \sim 1$). The former is due to the fact that we have no data in future and the latter is the result of the fact that we have not used any data beyond the redshift of $z > 1$ and that the error bars are larger for earlier data.

The best $\chi^2$ value found is 114, i.e. very close to that for the $\Lambda CDM$ model, indicating that we do not find models that fit the data qualitatively better than the $\Lambda CDM$ model. Thus $\Lambda CDM$ model belongs to the $1\sigma$ confidence contour.

We note that the quintessence dark energy does not suffer from the degeneracy discussed in connection with the dark energy EOS in section [9]. To see why, consider two functions of $z$, $f_1$ and $f_2$, say. If $f'_1 > f'_2$ for all $z$ and initially $f_1(0) \geq f_2(0)$, then $f_1 > f_2$. Now, the dark energy density $\rho$ in general be written as $\rho/\rho_0 = e^{\int_{0}^{z} \frac{3}{w(z) + 1}/(1 + z)dz}$, while for the $\Lambda CDM$ model $w = -1$. Hence, if we define $f'_1 = 3(w + 1)/(1 + z)$ and $f'_2 = 3(w_{\Lambda CDM} + 1)/(1 + z) = 0$ (since $w_{\Lambda CDM} = -1$), we see that for the quintessence dark energy models (with $w > -1$) $f'_1 \geq f'_2$ and hence $f_1 > f_2$ (since initially $\rho/\rho_0 = 1$ implying that $f_1 = f_2 = 0$). Thus we have $\rho/\rho_0 \geq \rho_{\Lambda CDM}/\rho_{\Lambda CDM0} = 1$.

Now since the Hubble function is given by $H = H_0 \sqrt{\Omega_{m0}(1 + z)^3 + \Omega_{r0}\rho/\rho_0}$, we see that for quintessence dark energy models $H$ should be such that $H \geq H_{\Lambda CDM} > C$. Hence, quintessence dark energy models do not undergo the large degeneracies described in section [9]. for the range of redshifts covered by the supernovae observations. A similar reasoning, on the other hand, shows that ghost dark energy models can suffer from similar large degeneracies, since in that case $H$ can intersect the $C$ curve.

Figure 11: Reconstruction of the scalar field kinetic energy for a quintessence dark energy model with the corresponding 1, 2 and 3$\sigma$ confidence contours.
Figure 12: Reconstruction of the scalar field potential for a quintessence dark energy model with the corresponding 1, 2 and 3σ confidence contours.

B. The transient subset

We also find a subset of reconstructed luminosity-distance curves which correspond to models with transient accelerating phases - with the universe presently decelerating. Figures 13 and 14 show examples of such reconstructions for $d_l''$ and the EOS. We have plotted the curves with $\chi^2 < 118$ in order to show that such curves fit the data quite well.

The results also show that to be compatible with the observational data, the existence of a transient acceleration phase, with the universe currently decelerating, requires that $d_l''$ should be very small today.

Interestingly such transient solutions were not found in our recent work [9], where an additional assumption, namely $d_i'' < 0$, was made. The reason is easy to see. Given the smallness of $d_i''$ at present (which follows from the present reconstructions), then the constraint $d_i''' < 0$ would imply that $d_i''$ would soon become negative and stay negative throughout the past, which is excluded by our constraint [B] above. In the absence of the constraint $d_i''' < 0$, on the other hand, $d_i''$ could have increased, thus allowing it to be small today and yet positive in the past, resulting in a decelerated phase of expansion for small $z$.

Figure 13: Reconstruction of $d_l''$ when Universe undergoes a transient acceleration with the corresponding 1, 2 and 3σ confidence contours. The solid line represents $d_l''$ for the $\Lambda CDM$ model.

Figure 14: Reconstruction of the EOS with the Universe undergoing a transient acceleration with the 1σ confidence contour. The upper solid line represents the $w = -1/3$ which demarcates between an accelerated and a decelerated expansion.

VI. CONCLUSIONS

We have studied the effects of generalizing a recently proposed model-independent scheme by dropping the assumption constraining the third derivative of the luminosity-distance. We have seen that this has the consequence of allowing a wider class of EOS to be explored.

Using the SNLS supernovae data, we have found a number of interesting results reconstructing the luminosity-distance curves $d_l$, subject to constraints [A] and [B] above.

In particular we have found that although the $\Lambda CDM$ model is compatible with the data within the 2σ significance level, it is not at the 1σ level.

For models whose EOS becomes ghost somewhere in the redshift interval $[0, 1]$, large degeneracies can appear in the reconstructions of the EOS due to sudden widening of the significance contours, for redshifts larger than $\sim 0.6$. We have shown that these degeneracies are a consequence of the fact that the EOS can in these cases take very large values or even diverge, due to the fact that the Hubble function can approach or intersect the curve $C$. Our results demonstrate that to decrease these degeneracies, the error bars need to be reduced dramatically, particularly for data at redshifts larger than $\sim 0.6$. We have made a concrete study of how such degeneracies can be reduced and found that constraints on the accuracy of data increase with increasing redshifts. Studies of this type could allow limits to be put on the extent to which the degeneracies on EOS may be reduced in near future. Interestingly, we find that this divergence is absent for quintessence dark energy models (with $w > -1$), with the consequence that quintessence dark energy models are not affected by such degeneracies. In this case $\Lambda CDM$ model appears to be among the best fitting models.

We should add that the interpretation of such possible blows ups in the EOS will depend on the theoretical framework chosen to interpret the observations. Clearly such blow ups can occur if the EOS is treated as an effective mathematical representation of a theory such as the $F(R)$ theories [12]. On the other hand if the EOS is treated as the physical representation of dark energy, then according to standard physics the blow up needs
to be avoided if it corresponds to a blow up in pressure. Things are less clear if it corresponds to the vanishing of the density at a finite redshift.

We have also found examples of reconstructed solutions showing transient acceleration while fitting the data very well. All such curves, however, require a very small $d_l''$ today, corresponding to a very recent phase of deceleration, which thus makes them rather artificial.

These new results highlight the range of new possibilities that the generalisation to the reconstruction scheme considered in this paper allows.

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