Product Acceptance Determination for Two Suppliers with Linear Profiles

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Abstract
In the management of suppliers, it is an important task to compare the performance of two suppliers using the linear profiles. In this paper, the product acceptance determination procedure is designed using a EWMA statistic based on the process-yield index applied to the linear profiles of two suppliers. The design parameters of the proposed plan are determined to satisfy both the producer’s and consumer’s risks. The efficiency of the proposed sampling plan is compared with the sampling plan developed based on the Wang’s test statistic in terms of the sample size required for the selection of a better supplier. A real example is given to explain the proposed sampling plan.

Key words:
Sampling plan; EWMA statistic; linear profile; process-yield index; difference statistic

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1 Introduction

Quality is the critical factor for supplier selection, evaluation of manufacturing firms, reducing rework and operation costs and to increase share of the companies in the market (Weber et al. [1], Olhager and Selldin [2]). The process yield which is based on the process capability index (PCI) is used to judge process performance of a supplier (Montgomery [3]). A yield index is used as one of the tools to judge the quality or performance of two or more suppliers. A higher index results in higher quality and a smaller fraction of nonconforming product. According to Wang [4] “if the index value of one supplier can be shown to be significantly greater than that of another supplier, then the supplier with higher index value will incur lower costs”. Some works to evaluate and to determine a better supplier including various PCIs under a normal distribution can be seen in [5-9]. Lin and Pearn [10] worked on comparing multiple suppliers using PCI. Lin and Kuo [11] presented a method for multiple comparisons based on PCI.

A functional relationship between the dependent variable and explanatory variables is called a profile. Profile monitoring has attracted the researchers in recent years because of its wide applications in quality engineering. A review on the linear and nonlinear profiles is provided by [12]. More details about this type of studies can be seen in [13-20].

An acceptance sampling plan is one of the tools for the inspection of the products at the final stage (Aslam et al. [21]). The sampling plan is also used to select a better supplier to provide a good quality product. Suppose that the null hypothesis is that the product from supplier 1 is better than the product from supplier 2. A random sample with a particular size is selected from the submitted lot of each supplier and the decision on the hypothesis can be made based on a suitable statistic. Wang [4] developed a difference test statistic for two suppliers using linear profile under a normal distribution. We may design a sampling plan based on this test statistic, however this test statistic has to be improved further by using exponentially weighted moving average (EWMA) statistic.

Usually a sampling plan provides the decision about the submitted product using the current state only. The use of EWMA statistic in a sampling plan increases the accuracy in decision about the acceptance of a lot of products. The EWMA statistic enables the engineer to use the current and the past information to make the final decision about the selection of supplier. According to Montgomery [3], this statistic weights sample in
exponentially decreasing order. Aslam et al. [22] designed a sampling plan using EWMA statistic when quality of interest follows the normal distribution. More details on the applications of the sampling plans can be seen in [23-32]. Also, some of the sampling plans have been developed based on EWMA statistic (see for example [33-43]).

In this paper, we improve the test statistic proposed by Wang [4] using a EWMA scheme when the profile data are available from two suppliers. The designing of a sampling plan is also given using the proposed EWMA statistic. The efficiency of the proposed sampling plan is compared with the one based on the Wang’s test statistic in terms of the sample size required for the selection of a better supplier among two. The application of the proposed sampling plan is discussed with the help of an industrial example.

2 Design of Proposed EWMA Plan

Let \( x_i \) denote the \( i \)-th level of the independent variable of interest and \( y_{ij} \) denote the \( j \)-th sample of the response variable at the fixed level \( x_i \). It is assumed that the following linear relationship of \( x_i \) and \( y_{ij} \) holds:

\[
y_{ij} = \beta_0 + \beta_1 x_i + \varepsilon_{ij} \quad i = 1, 2, \ldots, n; \quad j = 1, 2, \ldots, k
\]

where \( n \) is the number of levels of the independent variable and \( k \) is the number of observations (or sample size) while \( \beta_0 \) and \( \beta_1 \) are coefficients of the linear profile. Here, \( \varepsilon_{ij} \) is the error term and follows a normal distribution with mean 0 and variance \( \sigma^2 \). Note that \((y_{1j}, y_{2j}, \ldots, y_{nj})\) is called the \( j \)-th profile.

Wang [4] proposed the following process-yield index for the response variable at level \( x_i \):

\[
S_{pk_i} = \frac{1}{3} \Phi^{-1} \left[ \frac{1}{2} \Phi \left( \frac{USL_i - \mu_i}{\sigma_i} \right) + \frac{1}{2} \Phi \left( \frac{\mu_i - LSL_i}{\sigma_i} \right) \right] = \frac{1}{3} \Phi^{-1} \left[ \frac{1}{2} \Phi \left( \frac{1-c_{dr_i}}{c_{dp_i}} \right) + \frac{1}{2} \Phi \left( \frac{1+c_{dr_i}}{c_{dp_i}} \right) \right]
\]

where \( USL_i \) and \( LSL_i \) are the upper and the lower specification limits of the response variable at \( x_i \), \( \mu_i \) and \( \sigma_i \) are the mean and standard deviation, respectively of the response variable at \( x_i \), \( c_{dr_i} = (\mu_i - m_i) / d_i \), \( c_{dp_i} = \sigma_i / d_i \), \( m_i = (USL_i + LSL_i) / 2 \) and \( d_i = (USL_i - LSL_i) / 2 \). Here, \( \Phi(x) \) is the cumulative distribution function of a standard normal distribution.
Wang [4] considered the following estimator of the process-yield index for a simple linear profile model in Eq. (1) as

\[
S_{pk_i} = \frac{1}{3} \Phi^{-1} \left[ \frac{1}{2} \Phi \left( \frac{1 - \hat{c}_{dr_i}}{\hat{c}_{dp_i}} \right) + \frac{1}{2} \Phi \left( \frac{1 + \hat{c}_{dr_i}}{\hat{c}_{dp_i}} \right) \right]
\]

(3)

where \( \hat{c}_{dr_i} = (\hat{\mu}_i - m_i)/\hat{d}_i \) and \( \hat{c}_{dr_i} = \hat{d}_i/\hat{d}_i \).

Assume that there are two suppliers, supplier 1 and supplier 2, say. Our problem is whether to accept a lot from supplier 1 or 2 providing better quality. Suppose that the supplier 2 claims that their products are of better quality having a higher process-yield index than the products provided by supplier 1. Based on the claim by supplier 2, we set the following null and alternative hypotheses.

\[
\begin{align*}
H_0: & \ S_{pKA_2} - S_{pKA_1} \geq 0 \\
H_1: & \ S_{pKA_2} - S_{pKA_1} < 0
\end{align*}
\]

If sample information supports \( H_0 \), then it is concluded that supplier 2 is better than supplier 1 and therefore, a lot of products supplied by the supplier 2 should be accepted. Otherwise, a lot of products provided by the supplier 1 should be accepted. It is assumed that \( k_1 \) profiles are available at \( n_1 \) levels of the independent variable for supplier 1 and \( k_2 \) profiles are available at \( n_2 \) levels of the independent variable for supplier 2.

Wang [4] derived the approximate normal distribution of the difference statistic \( \hat{D} \) between supplier 1 and supplier 2 as

\[
\hat{D} = \hat{S}_{pKA_2} - \hat{S}_{pKA_1} \sim N \left( S_{pKA_2} - S_{pKA_1}, \sigma^2 + \sigma^2_{s_1} \right)
\]

(4)

where

\[
\sigma^2_{s_1} = \frac{G_1^2 \cdot \var{3(3^2)}}{2n_1^2 k_1 \cdot \var{3(3^2)}}, \ G_1 = \frac{1}{3} \Phi^{-1} \left\{ \frac{n_1[2\Phi(3S_{pKA_1})-1]-(n_1-2)}{2} \right\}
\]

\[
\sigma^2_{s_2} = \frac{G_2^2 \cdot \var{3(3^2)}}{2n_2^2 k_2 \cdot \var{3(3^2)}}, \ G_2 = \frac{1}{3} \Phi^{-1} \left\{ \frac{n_2[2\Phi(3S_{pKA_2})-1]-(n_2-2)}{2} \right\}
\]

We propose the sampling plan using the EWMA scheme of the above difference test statistic. The proposed plan is stated as follows:

**Step-1:** At time \( t \), obtain \( k_1 \) random profiles at \( n_1 \) levels of the independent variable for supplier 1 and \( k_2 \) random profiles at \( n_2 \) levels of the independent variable for supplier 2.

**Step-2:** Calculate the difference statistic \( \hat{D}_t = \hat{S}_{pKA_2} - \hat{S}_{pKA_1} \). Then, compute the following EWMA statistic:
\[ \hat{D}_t^{EWMA} = \lambda \hat{D}_t + (1 - \lambda) \hat{D}_{t-1}^{EWMA} \]  

(5)

where \( \lambda \) is a smoothing constant ranging from 0 to 1. At \( t=1 \), we set \( \hat{D}_1^{EWMA} = \hat{D}_1 \).

**Step-3:** Accept the lot by supplier 1 if \( \hat{D}_t^{EWMA} \geq c \) or accept the lot by supplier 2, otherwise, where \( c \) is the acceptance constant to be determined.

The proposed sampling plan is based on the number of profiles (sample size) for each supplier (\( k_1 \) and \( k_2 \)) and the acceptance constant \( c \) when the number of levels is specified. We need to determine the acceptance constant \( c \) so as to minimize the sample size while satisfying the producer’s and the consumer’s risks.

The smoothing constant \( \lambda \) determines the rate at which “previous lots” enter into the calculation of the EWMA statistic. A value of \( \lambda=1 \) implies that only the most recent measurement influences the statistic. Thus, a large value of \( \lambda \) gives more weight to recent data and less weight to older data; a small value of \( \lambda \) gives more weight to older data. The value of \( \lambda \) ranging between 0.1 and 0.3 is recommended in practice [3].

First, the EWMA statistic for sufficiently large \( t \) follows the normal distribution given as

\[ \hat{D}_t^{EWMA} \sim N\left(S_{pkA_2} - S_{pkA_1}, (\lambda/2 - \lambda)(\sigma_{s_2}^2 + \sigma_{s_1}^2)\right) \]

Therefore, the operating characteristic (OC) function of the proposed plan is derived as follows

\[ P(\hat{D}_t^{EWMA} \geq c) = P\left(Z \geq \frac{c-(S_{pkA_2}-S_{pkA_1})}{\sqrt{(\lambda/2 - \lambda)\left[\sigma_{s_2}^2/2n_1^2k_1\phi(3\sigma_{s_2}^2) + \sigma_{s_1}^2/2n_2^2k_2\phi(3\sigma_{s_1}^2)\right]^2}}\right) \]

where \( Z \) is the standard normal random variable. Finally, the lot acceptance probability is given as
\[
P(D_{EWM}^c) = 1 - \Phi \left( \frac{c - (S_{pkA2} - S_{pkA1})}{\sqrt{\frac{\sigma_2^2[\phi(3\sigma_2^2)]^2}{2n_2^2k_2\phi(3S_{pkA2})} + \frac{\sigma_2^2[\phi(3\sigma_2^2)]^2}{2n_2^2k_2\phi(3S_{pkA1})}}\left(\frac{\lambda}{(2-\lambda)}\right)} \right)
\]

(6)

Let \( \alpha \) be the producer’s risk and \( \beta \) be the consumer’s risk. The producer is interested in guaranteeing that the lot acceptance probability for a good lot should be larger than his confidence level, \( 1-\alpha \), at acceptable quality level (AQL). The consumer requires that the lot acceptance for a bad lot should be smaller than his risk at lot tolerance percent defective (LTPD). Let \( C_{AQL1} \) be the AQL value of supplier 1 and \( C_{AQL2} \) be the AQL value of supplier 2, while \( C_{LTPD1} \) be the LTPD value of supplier 1 and \( C_{LTPD2} \) be the LTPD value of supplier 2. For the simplicity, it is assumed that \( n_1 = n_2 = n \) and \( k_1 = k_2 = k \). The plan parameters of the proposed plan using EWMA statistic will be determined by solving the following non-linear optimization problem:

Minimize \( k \) 

Subject to

\[
1 - \Phi \left( \frac{c - (C_{AQL2} - C_{AQL1})}{\sqrt{\frac{\sigma_2^2[\phi(3\sigma_2^2)]^2}{2n_2^2k_2\phi(3S_{CQL1})} + \frac{\sigma_2^2[\phi(3\sigma_2^2)]^2}{2n_2^2k_2\phi(3S_{CQL2})}}\left(\frac{\lambda}{(2-\lambda)}\right)} \right) \geq 1 - \alpha \tag{7b}
\]

\[
1 - \Phi \left( \frac{c - (C_{LTPD2} - C_{LTPD1})}{\sqrt{\frac{\sigma_2^2[\phi(3\sigma_2^2)]^2}{2n_2^2k_2\phi(3S_{CQL1})} + \frac{\sigma_2^2[\phi(3\sigma_2^2)]^2}{2n_2^2k_2\phi(3S_{CQL2})}}\left(\frac{\lambda}{(2-\lambda)}\right)} \right) \leq \beta \tag{7c}
\]

Here, we consider the cases where the quality level for supplier 2 is higher than that for supplier 1, that is, \( \Delta C_{AQL} = C_{AQL2} - C_{AQL1} > 0 \) and \( \Delta C_{LTPD} = C_{LTPD2} - C_{LTPD1} > 0 \). There are many combinations of \( (C_{AQL2}, C_{LTPD2}, C_{AQL1}, C_{LTPD1}) \) such as \( (1.5, 1.3, 1.0, 0.9), (1.6, 1.4, 1.1, 1.0) \). It should be noted that \( C_{AQL} > C_{LTPD} \).

Tables 1-2 present the plan parameters of the proposed sampling plan for various values of smoothing constant as well as the number of independent variable levels when
∆C_{AQL} = 0.5 and ∆C_{LTPD} = 0.4. Two combinations of (C_{AQL_2} = 1.5, C_{LTPD_2} = 1.3, C_{AQL_1} = 1.0, C_{LTPD_1} = 0.9) and (C_{AQL_2} = 1.6, C_{LTPD_2} = 1.4, C_{AQL_1} = 1.1, C_{LTPD_1} = 1.0) are considered. The producer’s and the consumer’s risks are chosen by \( \alpha = 0.05 \) and \( \beta = 0.10 \) for all tables.

From Tables 1-2, it is noted that for all other same values, as \( n \) changes from 2 to 30, there is a decreasing trend in \( k \). As expected, smaller sample size (or profile) is required as a larger number of variables is used. It is also seen that the sample size becomes smaller as a smaller smoothing constant is used.

Tables 3-4 are reported when ∆C_{AQL} = 0.6 and ∆C_{LTPD} = 0.5. Here again two combinations of (C_{AQL_2} = 1.6, C_{LTPD_2} = 1.4, C_{AQL_1} = 1.0, C_{LTPD_1} = 0.9) and (C_{AQL_2} = 1.7, C_{LTPD_2} = 1.5, C_{AQL_1} = 1.1, C_{LTPD_1} = 1.0) are considered.

It is observed that the values of \( k \) are increased for these cases as compared with the ones in Tables 1-2.

### 3 Comparative Study

We may design a sampling plan for the comparison purpose using the Wang’s difference statistic given in Eq. (4). In fact, the plan based on the Wang’s test statistic is a special case of the proposed sampling plan with \( \lambda = 1 \). In this section, we compare the proposed sampling plan with the plan developed based on Wang’s testing procedure in terms of the sample size required. To compare the efficiency of both sampling plans, we select same values of all specified parameters. The plan parameters of the sampling plan by Wang [4] are placed in the last columns of Table 2 (when ∆C_{AQL} = 0.5 and ∆C_{LTPD} = 0.4) and Table 4 (when ∆C_{AQL} = 0.6 and ∆C_{LTPD} = 0.5).

By comparing these with the ones having \( \lambda \)’s smaller than 1, we note that the proposed EWMA plan provides smaller values of \( k \) as compared to the Wang’s sampling plan. For
example, when $C_{AQL_2} = 1.5$, $C_{LTPD_2} = 1.3$, $C_{AQL_1} = 1.0$, $C_{LTPD_1} = 0.9$ and $n = 5$, the value of $k$ for the proposed plan with $\lambda = 0.1$ is 32 and it is 66 with $\lambda = 0.2$, while it is 589 for the sampling plan based on Wang’s test statistic. So, the proposed EWMA sampling plan looks more efficient than the sampling plan without using EWMA scheme. Practically, the values of $\lambda$ lie between 0.1 and 0.5 are preferable in the industries. So, the proposed plan is more efficient in this range.

4 Application of the Proposed Plan

In this section, we discuss an application of the proposed sampling plan in the leather industry [44]. Wang [4] used the leather industry data to discuss the application of his difference statistic. According to Wang [4] “the quality performance of leather dyeing process is characterized by a relationship between the leather color effluent and temperature. The corresponding color effluent was examined in 150 ml water at five different temperatures, including 25, 32, 39, 46, and 53 °C”.

Suppose the industry is using the proposed sampling plan with $\lambda = 0.29$ and $n=5$ by specifying the values of $\Delta C_{AQL} = 0.5$ and $\Delta C_{LTPD} = 0.4$. Then, from Table 1, we find $k=100$ and $c = 0.4300$. So, we need to collect data for 100 profiles from two suppliers. Suppose now that the specification limits, means and standard deviations (from both supplier 1 and supplier 2) at the five levels of the independent variable are given in Table 5.

| Table 5 is around here |

Based on the data given in Table 5, the EWMA statistic based on Eq. (5) can be obtained, as $D_t^{EWMA} = 0.4896541$. According to the proposed plan, since $D_t^{EWMA} > c = 0.4300$, we conclude that the supplier 2 has significantly better capability than supplier 1. So, we accept the leather product by supplier 2 and reject the lot of leather products supplied by supplier 1.

5 Concluding Remarks
In this paper, the designing methodology of a sampling plan based on EWMA statistic has been proposed for the inspection of products supplied by two suppliers. The tables for various profiles have been provided for practical use. The performance of the proposed sampling plan has been compared with the plan based on the Wang’s test statistic in terms of the sample size required. By comparing the both sampling plans, it is concluded that the proposed plan using EWMA statistic requires a small sample size when deciding about the lot of products by two suppliers. The application of the proposed plan has also been given with the help of industrial data. By applying the proposed plan in the industry for product acceptance, the product by two suppliers can be inspected at the same time. The proposed sampling plan considering multiple suppliers can be considered as future research.

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**Chi-Hyuck Jun** was born in Seoul, Korea in 1954. He received a B.S. (1977) in mineral and petroleum engineering from Seoul National University, an M.S. (1979) in industrial engineering from KAIST, and a Ph.D. (1986) in operations research from University of California, Berkeley. Since 1987, he has been with the department of industrial and management engineering, POSTECH; and he is now a professor, and the department head. He is interested in reliability and quality analysis, and data mining techniques. He is a member of IEEE, INFORMS, and ASQ.

| Table 1: The plan parameters for the proposed plan when $\Delta C_{AQL} = 0.5$ and $\Delta C_{LTPD} = 0.4$ |
|---------------------------------------------------------------|

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| $n$ | $\lambda = 0.10$ | $\lambda = 0.15$ | $\lambda = 0.20$ | $\lambda = 0.29$ |
|-----|----------------|----------------|----------------|----------------|
|     | $k$ $c$ $k$ $c$ $k$ $c$ $k$ $c$ |
| 2   | 34 0.4243 53 0.4246 73 0.4249 109 0.4228 |
| 4   | 32 0.4288 50 0.4287 70 0.4298 105 0.4286 |
| 5   | 32 0.4306 48 0.4299 66 0.4298 100 0.4300 |
| 10  | 26 0.4306 40 0.4306 57 0.4316 85 0.4311 |
| 15  | 24 0.4299 34 0.4291 48 0.43 71 0.4293 |
| 20  | 20 0.4277 30 0.427 41 0.4278 61 0.4273 |
| 25  | 18 0.4254 27 0.4262 36 0.4255 55 0.4253 |
| 30  | 17 0.425 24 0.4244 34 0.4255 49 0.4241 |

$(C_{AQ_2} = 1.5, C_{LTPD_2} = 1.3, C_{AQ_1} = 1.0, C_{LTPD_1} = 0.9)$

| $n$ | $\lambda = 0.50$ | $\lambda = 0.75$ | Wang’s test statistic |
|-----|----------------|----------------|----------------|
|     | $k$ $c$ $k$ $c$ $k$ $c$ |
| 2   | 30 0.4105 46 0.4105 62 0.4107 96 0.4108 |
| 4   | 30 0.419 47 0.4187 62 0.4174 95 0.4183 |
| 5   | 30 0.4211 47 0.4192 64 0.4214 95 0.4197 |
| 10  | 29 0.4254 44 0.4253 61 0.4253 93 0.4254 |
| 15  | 27 0.4278 42 0.428 56 0.4272 86 0.4271 |
| 20  | 26 0.4271 39 0.4279 54 0.4277 84 0.4281 |
| 25  | 25 0.4292 37 0.4279 50 0.4281 77 0.4282 |
| 30  | 23 0.4272 35 0.4282 47 0.4274 72 0.4276 |

$(C_{AQ_2} = 1.6, C_{LTPD_2} = 1.4, C_{AQ_1} = 1.1, C_{LTPD_1} = 1.0)$

Table 2: The plan parameters for the proposed plan when $\Delta C_{AQ} = 0.5$ and $\Delta C_{LTPD} = 0.4$
Table 3: The plan parameters for the proposed plan when $\Delta C_{AQL} = 0.6$ and $\Delta C_{LTPD} = 0.5$

| $n$ | $\lambda = 0.10$ | $\lambda = 0.15$ | $\lambda = 0.20$ | $\lambda = 0.29$ |
|-----|-----------------|------------------|------------------|------------------|
|     | $k$ | $c$ | $k$ | $c$ | $k$ | $c$ | $k$ | $c$ |
| 10  | 100 | 0.4243 | 117 | 0.4237 | 289 | 0.4239 |
| 15  | 139 | 0.4294 | 252 | 0.4296 | 417 | 0.4292 |
| 20  | 121 | 0.4273 | 216 | 0.4274 | 368 | 0.4276 |
| 25  | 109 | 0.4262 | 195 | 0.4263 | 322 | 0.4254 |
| 30  | 100 | 0.4243 | 173 | 0.4237 | 289 | 0.4239 |

($C_{AQL2} = 1.6$, $C_{LTPD2} = 1.4$, $C_{AQL1} = 1.1$, $C_{LTPD1} = 1.0$)

| $n$ | $\lambda = 0.10$ | $\lambda = 0.15$ | $\lambda = 0.20$ | $\lambda = 0.29$ |
|-----|-----------------|------------------|------------------|------------------|
| 2   | 186 | 0.4106 | 333 | 0.4110 | 551 | 0.4108 |
| 4   | 191 | 0.4178 | 331 | 0.4176 | 562 | 0.4175 |
| 5   | 184 | 0.4199 | 338 | 0.4199 | 553 | 0.4196 |
| 10  | 179 | 0.4255 | 327 | 0.4255 | 543 | 0.4255 |
| 15  | 170 | 0.4275 | 305 | 0.4274 | 504 | 0.4272 |
| 20  | 159 | 0.4280 | 291 | 0.4276 | 480 | 0.4278 |
| 25  | 149 | 0.4278 | 268 | 0.4279 | 446 | 0.4278 |
| 30  | 141 | 0.4276 | 251 | 0.4274 | 426 | 0.4276 |

($C_{AQL2} = 1.6$, $C_{LTPD2} = 1.4$, $C_{AQL1} = 1.0$, $C_{LTPD1} = 0.9$)
Table 4: The plan parameters for the proposed plan when $\Delta C_{AQL} = 0.6$ and $\Delta C_{LTPD} = 0.5$

| $n$ | $\lambda = 0.50$ | $\lambda = 0.75$ | Wang’s test statistic |
|-----|------------------|------------------|----------------------|
|     | $k$ | $c$     | $k$ | $c$ | $k$ | $c$     |
|     |     |         |     |   |     |         |
| 2   | 234 | 0.5223  | 408 | 0.5222 | 682 | 0.5222  |
| 4   | 223 | 0.5274  | 399 | 0.5276 | 667 | 0.5276  |
| 5   | 216 | 0.5285  | 396 | 0.5286 | 647 | 0.5284  |
| 10  | 185 | 0.5289  | 337 | 0.5298 | 551 | 0.5290  |
| 15  | 159 | 0.5281  | 286 | 0.5275 | 473 | 0.5276  |
| 20  | 139 | 0.5256  | 249 | 0.5255 | 417 | 0.5258  |
| 25  | 124 | 0.5238  | 226 | 0.5246 | 373 | 0.5243  |

$(C_{AQL2} = 1.7, C_{LTPD2} = 1.5, C_{AQL1} = 1.1, C_{LTPD1} = 1.0)$
\[
\begin{array}{cccccc}
30 & 114 & 0.5220 & 203 & 0.5221 & 340 & 0.5221 \\
\end{array}
\]

\(C_{AQL_2} = 1.7, C_{LTPD_2} = 1.5, C_{AQL_1} = 1.1, C_{LTPD_1} = 1.0\)

| Level | \(X_i\) | \(LSL_i\) | \(USL_i\) | \(Target_i\) | Supplier 1 Mean | Supplier 1 S.D. | \(\hat{\delta}_{pki}\) | Supplier 2 Mean | Supplier 2 S.D. | \(\hat{\delta}_{pki}\) |
|-------|--------|----------|----------|-------------|----------------|----------------|--------------|----------------|----------------|--------------|
| 1     | 25     | 0.00400  | 0.06600  | 0.03500     | 0.03498        | 0.01249        | 0.8274       | 0.03453        | 0.01020        | 1.0120       |
| 2     | 32     | 0.00600  | 0.10600  | 0.05600     | 0.05570        | 0.02675        | 0.6230       | 0.05508        | 0.01134        | 1.4650       |
| 3     | 39     | 0.00800  | 0.16600  | 0.08700     | 0.08657        | 0.03405        | 0.7734       | 0.08543        | 0.01120        | 2.3312       |
| 4     | 46     | 0.01600  | 0.20000  | 0.11000     | 0.11002        | 0.01806        | 1.6881       | 0.10998        | 0.01645        | 1.8517       |
| 5     | 53     | 0.02000  | 0.24000  | 0.13000     | 0.12808        | 0.01853        | 1.9687       | 0.12934        | 0.01598        | 2.2926       |

**Table 5: Comparison table**