Electroweak Baryogenesis and New TeV Fermions

M. Carena \textsuperscript{a}, A. Megevand \textsuperscript{b}

M. Quirós \textsuperscript{b,c} and C.E.M. Wagner \textsuperscript{d,e}

\textsuperscript{a}Fermi National Accelerator Laboratory, P.O. Box 500, Batavia, IL 60510, USA
\textsuperscript{b}Theoretical Physics Group, IFAE/UAB, E-08193 Bellaterra (Barcelona) Spain
\textsuperscript{c}Institució Catalana de Recerca i Estudis Avançats (ICREA)
\textsuperscript{d}HEP Division, Argonne National Laboratory, 9700 S. Cass Ave., Argonne, IL 60439, USA
\textsuperscript{e}Enrico Fermi Institute, Univ. of Chicago, 5640 Ellis Ave., Chicago, IL 60637, USA

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Abstract

New fermions, strongly coupled to the Standard Model Higgs boson, provide a well motivated extension of the Standard Model (SM). In this work we show that, once new physics at heavier scales is added to stabilize the Higgs potential, such an extension of the SM can strengthen the first order electroweak phase transition and make the electroweak baryogenesis mechanism feasible. We propose a SM extension with TeV Higgsinos, Winos and Binos that satisfy the following properties: \textbf{a}) The electroweak phase transition is strong enough to avoid sphaleron erasure in the broken phase for values of the Higgs mass $m_H \lesssim 300$ GeV; \textbf{b}) It provides large CP-violating currents that lead to the observed baryon asymmetry of the Universe for natural values of the CP-violating phase; \textbf{c}) It also provides a natural Dark Matter candidate that can reproduce the observed dark matter density; \textbf{d}) It is consistent with electroweak precision measurements; \textbf{e}) It may arise from a softly broken supersymmetric theory with an extra (asymptotically free) gauge sector; \textbf{f}) It may be tested by electron electric dipole moment experiments in the near future.
1 Introduction

In spite of all our recent progress in the understanding of physics at the electroweak scale, the source of Dark Matter (DM) and the origin of the matter-antimatter asymmetry (BAU) still remain unclear. It is today well understood that the solution to either of these problems requires physics beyond the Standard Model (SM).

On the one hand if the Standard Model baryon number violating interactions cease to be in equilibrium in the bubbles of the broken phase, the matter-antimatter asymmetry may be generated at the electroweak phase transition via the mechanism of electroweak baryogenesis. For such a mechanism to be realized in nature a strongly first order electroweak phase transition is required. However the phase transition in the Standard Model for values of the Higgs mass consistent with the LEP bounds is a crossover and hence any baryon asymmetry generated at the weak scale would be erased. Moreover, the sources of CP-violation are insufficient to generate a baryon number consistent with the one observed in nature.

Electroweak Baryogenesis remains nevertheless as an attractive possibility in models of physics beyond the Standard Model at the weak scale. It has been shown that the minimal supersymmetric standard model (MSSM) is consistent with this mechanism provided one of the superpartners of the top quark is lighter than the top quark and the Higgs boson is lighter than $\sim 120$ GeV. This mechanism also demands the presence of charginos and neutralinos at the weak scale, which provide the necessary CP-violating sources and also a natural Dark Matter candidate. Electroweak Baryogenesis may also be realized in the next-to minimal supersymmetric extension of the Standard Model (NMSSM) where some of the MSSM constraints can be relaxed. In particular there are modifications to the tree-level effective potential that may ensure a strongly first order phase transition without a light stop.

On the other hand the Standard Model does not provide any natural source for the observed Dark Matter. Neutrinos are too light to give any sizeable contribution and there is no evidence of the possible existence of sufficient Jupiter-like, baryonic objects. Moreover, recent WMAP measurements exclude the presence of a significant baryonic contribution to the observed Dark Matter density. The natural candidates for the source of Dark Matter are new, neutral, stable, weakly interacting particles with masses of the order of the weak scale. These particles lead naturally to a relic density of the order of the critical density and appear in many models beyond the Standard Model. In particular they are present in models of softly broken supersymmetry at the TeV scale. The lightest supersymmetric particle in these models tends to be neutral and its stability is ensured by a Parity symmetry, $R_P$, which also ensures the proton stability.
Acceptable values of the Dark Matter density and a successful realization of the mechanism of electroweak baryogenesis may be simultaneously obtained in minimal supersymmetric models, in certain phenomenologically interesting regions of the parameter space \[41\], \[38\].

In the previous models, as well as in all successfully considered scenarios, the strengthening of the phase transition proceeds from the existence of new, extra scalars in the theory, while the CP-violating and Dark Matter sources proceed from new fermion fields. The common lore from all previous works was that the presence of extra bosons was a necessary requirement to induce a strong enough first order electroweak phase transition. In fact it is currently understood that bosons coupled to the Higgs field $\phi$ with coupling $h$ favor a first order phase transition: they create a cubic term in the Higgs effective potential $\sim (h\phi)^3$ either at the tree level, as in the NMSSM, or by its contribution to the one-loop thermal effective potential $\sim (h\phi)^3 T$. On the other hand fermions do not give rise to any cubic term in the high temperature expansion in powers of $h\phi/T$ of the thermal integrals and hence they were neither believed to give rise to a barrier between the symmetric $\phi = 0$ and broken $\phi \neq 0$ phases nor to trigger a first order phase transition. In this paper we will prove that while the latter statement remains true for weakly coupled fermions $h \ll 1$ it is not for strongly coupled (but still perturbative) ones $h \gtrsim 1$ that can indeed induce a strongly first order phase transition consistent with electroweak baryogenesis.

In this work we shall first show that in Standard Model extensions with extra fermions strongly coupled to the Higgs field the first order phase transition may be sufficiently strengthened in order to avoid erasure of the baryon asymmetry in the broken phase. We shall analyze in detail a simple model, which can be considered as a particular realization of split supersymmetry \[12\], where the standard supersymmetric relations between the Yukawa and gauge couplings are not fulfilled. We shall stress, however, that in such a model, the physical vacuum becomes unstable and therefore the strength of the electroweak phase transition may not be properly defined without an ultraviolet (UV) completion of this model, that includes the presence of heavier, stabilizing fields. An example of such fields may be provided by softly broken supersymmetry, although other extensions are possible.

We shall show that this low-energy effective theory, with Higgsinos and gauginos strongly coupled to the Higgs, may arise from a soft supersymmetry breaking model, based on a gauge extension of the Standard Model gauge group, with new (asymptotically free) gauge interactions that become strong at the TeV scale, and are responsible for the strong Yukawa couplings of Higgsinos and gauginos to the SM Higgs field. This gauge extension of the MSSM provides a UV completion of the model analyzed in this
paper and allows for large Higgs masses.

The article is organized as follows. In section 2 we present the general ideas leading to the strengthening of the phase transition in the presence of strongly coupled fermions, and the need for the presence of stabilizing fields. In section 3 the phase transition for the Standard Model extension containing Higgsinos, Winos and Binos strongly coupled to the Higgs field is worked out in detail. We show that a strong enough first order phase transition can be accommodated even with a heavy Higgs, $m_H \lesssim 300$ GeV. In section 4 the CP-violating currents induced by the charginos that lead to the observed baryon asymmetry of the universe for natural values of CP-violating phases are presented. We show that in order to reproduce the WMAP results on the BAU the CP-violating phases must be $\mathcal{O}(1)$ if all squarks are heavy enough to be decoupled from the thermal bath. Otherwise if some light squark (e.g. the right-handed stop) remains in the spectrum, the CP-violating phases can be as small as a few times $10^{-3}$. In section 5 the two-loop contributions to the electron electric dipole moment from the charginos and neutralinos in our model (in the absence of light squarks) are evaluated, assuming that all relevant CP-violating effects are associated with the new fermions. For the values of the parameters satisfying all other requirements the generated electric dipole moment is below the present experimental bound, although the model may be tested in the future if experimental bounds improve by a few orders of magnitude. In section 6 we discuss the Dark Matter constraints in our scenario, whereas compatibility of the strongly coupled fermions with electroweak precision measurements is considered in section 7. A natural region in the space of parameters is found where all requirements are fulfilled. A discussion on a possible UV completion of the model is presented in section 8. Finally we reserve section 9 for our conclusions.

2 Phase Transition and TeV Fermions

The finite temperature effective potential of the Higgs field $\phi$ is, by definition, the free-energy associated with $\phi$. The one-loop, finite temperature contribution to the free-energy density is given by

$$F_1(\phi, T) = \sum_i g_i T^4 \frac{m_i(\phi)}{2\pi^2} I_\mp \left( \frac{m_i(\phi)}{T} \right),$$  \hspace{1cm} (2.1)

where $g_i$ is the number of degrees of freedom (d.o.f.) of the particle, $m_i(\phi)$ is the Higgs-dependent particle mass, and

$$I_\mp(x) = \pm \int_0^\infty dy y^2 \log \left(1 \mp e^{-\sqrt{y^2+x^2}}\right),$$  \hspace{1cm} (2.2)
where $I_-(x) [I_+(x)]$ stands for the contribution from bosons [fermions]. In this section we will consider for simplicity masses of the form $m^2(\phi) = \mu^2 + h^2\phi^2$, where $h$ is the Yukawa coupling and $\mu$ is a constant mass parameter $^1$. For large masses ($m/T \gg 1$) the functions $I_\mp$ are exponentially small, which means that heavy species are decoupled from the thermal plasma. For small masses ($m/T \ll 1$) $I_\mp$ can be expanded in a power series of $m/T$. This is the case of the minimal standard model. In fact adding this expansion to the zero-temperature effective potential in the SM one obtains the well known expression for the free-energy density

$$F_{\text{SM}} (\phi, T) = -\frac{\pi^2}{90} g_* T^4 + V_{\text{SM}} (\phi, T),$$

where the first term comes from the entropy density contribution of relativistic particles, with $g_*$ the number of effectively light species in the plasma $^3$ ($g_* \simeq 107$ for the SM), and the second term is the field dependent effective potential

$$V_{\text{SM}} (\phi, T) = D \left(T^2 - T_0^2\right) \phi^2 - ET\phi^3 + \frac{\lambda_T}{4} \phi^4.$$  

In the SM the parameters of Eq. (2.4) are well known and given by $^4$

$$D = \frac{1}{8v^2} \left(2m^2_W + m^2_Z + 2m^2_t\right), \quad E = \frac{1}{6\pi v^3} \left(2m^3_W + m^3_Z\right),$$

$$T_0^2 = \frac{1}{4D} \left(m^2_H - 8Bv^2\right), \quad B = \frac{3}{64\pi^2 v^4} \left(2m^4_W + m^4_Z - 4m^4_t\right),$$

$$\lambda_T = \lambda - \frac{3}{16\pi^2 v^4} \left(2m^4_W \ln \frac{m^2_W}{a_B T^2} + m^4_Z \ln \frac{m^2_Z}{a_B T^2} - 4m^4_t \ln \frac{m^2_t}{a_F T^2}\right),$$

$$\lambda = \frac{m^2_H}{2v^2}, \quad \ln a_B = 3.91, \quad \ln a_F = 1.14.$$  

where the vacuum expectation value of the Higgs at zero temperature is normalized to $v \simeq 246$ GeV.

A free energy of the form of the SM one, Eq. (2.3), leads to a first order phase transition at a critical temperature given by

$$T_c = \frac{T_0}{\sqrt{1 - E^2/\lambda_T D}}.$$  

The value of the Higgs field at the minimum of the potential is

$$\phi_m (T) = \frac{3ET}{2\lambda_T} \left[1 + \sqrt{1 - \frac{8\lambda_T D}{9E^2} \left(1 - \frac{T_0^2}{T^2}\right)}\right],$$  

$^1$Mass eigenvalues are in general more complicated functions of $\phi$ as it will be the case in the model considered in section $^3$. $^2$
and the order parameter at the critical temperature is given by \( \phi_c/T_c \equiv \phi_m(T_c)/T_c = 2E/\lambda T_c \).

Notice that the parameter \( E \) is very small because only the (transverse) gauge bosons contribute to it and, as a consequence, \( \phi_c \) is much smaller than \( T_c \) (unless \( \lambda \lesssim 2E \)) and the phase transition is very weakly first-order \(^2\). Moreover, for physical values of the Higgs mass, the small value of \( \phi_c/T_c \) causes perturbation theory to break down and only non-perturbative calculations become reliable. To overcome this problem SM extensions containing extra bosons strongly coupled to the Higgs sector have been considered in the literature \(^3\). In general these bosons would contribute to the parameter \( E \) and would strengthen the first order phase transition. In this paper we will prove that a similar effect is produced by fermions strongly coupled to the Higgs sector, even if in the high temperature regime they do not contribute to the cubic term in the effective potential.

In general we will consider particle species that contribute to the one-loop effective potential as in Eq. (2.4), not light enough for the validity of the power expansion of \( I_\pm \) but not necessarily so heavy as to consider them to be decoupled in the typical range of temperatures of the electroweak phase transition. The \( \phi \)-dependent part of the free energy density would then be given by

\[
\mathcal{F}(\phi, T) = \mathcal{F}_{\text{SM}}(\phi, T) + \sum_i g_i V(m_i^2(\phi)) + T^4 \sum_i g_i I_\pm \left[ m_i(\phi)/T \right]/2\pi^2, \tag{2.8}
\]

where the first term is given by Eq. (2.3), the last one is the finite temperature contribution of the new, heavy particles, \( V(m_i^2) \) is the zero temperature contribution, and the plus and minus signs in front of \( V(m_i^2) \) correspond to bosons and fermions, respectively. The zero-temperature one-loop effective potential \( V(m^2(\phi)) \) is given by

\[
V(m^2(\phi)) = \frac{1}{64\pi^2} m^4(\phi) \log m^2(\phi) + P(\phi) \tag{2.9}
\]

where \( P(\phi) \) is a polynomial in \( \phi \) that contains quadratic and quartic terms with coefficients that depend on the renormalization conditions \(^4\). By imposing the renormalization conditions already used in the SM, Eq. (2.4), in particular that the tree-level values of the minimum and Higgs mass are not shifted by radiative corrections, i.e.

\[
\left. \frac{dV}{d\phi} \right|_{\phi=v} = 0, \quad \left. \frac{d^2V}{d\phi^2} \right|_{\phi=v} = m_H^2 \tag{2.10}
\]

we obtain

\[
P(\phi) = \frac{1}{2} \alpha \phi^2 + \frac{1}{4} \beta \phi^4 \tag{2.11}
\]

\(^2\)For \( E = 0 \) the transition becomes second order in the one-loop approximation.
with

\[
\alpha = \frac{1}{64\pi^2} \left\{ \left( -3 \frac{\omega'}{v} + \omega'^2 + \omega'' \right) \log \omega - \frac{3 \omega'}{2 v} + \frac{3 \omega'^2}{2} + \frac{1}{2} \omega'' \right\}
\]

\[
\beta = \frac{1}{128\pi^2 v^2} \left\{ 2 \left( \frac{\omega'}{v} - \omega'^2 - \omega'' \right) \log \omega + \frac{\omega'}{v} - 3 \omega'^2 - \omega'' \right\}
\]

(2.12)

where we are using the notation: \( \omega = m^2(v) \), \( \omega' = \left. \frac{dm^2(\phi)}{d\phi} \right|_{\phi=v} \), and so on. For the case where the \( \phi \) dependence of the mass eigenvalue is \( m^2(\phi) = \mu^2 + h^2\phi^2 \) the potential has the familiar expression

\[
V(m^2(\phi)) = \frac{1}{64\pi^2} \left[ m^4(\phi) \left( \log \left( \frac{m^2(\phi)}{m^2(v)} \right) - \frac{3}{2} \right) + 2m^2(\phi)m^2(v) \right].
\]

(2.13)

Let us first stress that, unless the Higgs is heavy, strongly coupled fermions may create a problem of vacuum stability at scales close to the electroweak scale. This can be easily understood from the fact that the tree-level quartic coupling, defined as the coefficient of the quartic term in the effective potential, is given by \( m_H^2 / 8v^2 \) and the radiative corrections are proportional to the fourth power of the Yukawa coupling, \( h \), and to the number of degrees of freedom. As we shall demonstrate, a relevant effect may only be obtained for a value of the number of degrees of freedom times \( h^4 \) larger than \( \mathcal{O}(10) \). For such values vacuum stability occurs at scales of the order of TeV and therefore the presence of new, stabilizing fields is necessary in order to define a consistent low-energy effective theory.

An efficient way of stabilizing the potential in the presence of strongly coupled fermions is to assume the presence of heavy bosonic degrees of freedom with similar couplings and number of degrees of freedom. For simplicity, let us here assume that the fermions have a dispersion relation \( m_f^2(\phi) = \mu_f^2 + h^2\phi^2 \), and a number of degrees of freedom \( g \), and there are bosonic, stabilizing fields with a dispersion relation \( m_S^2(\phi) = \mu_S^2 + h^2\phi^2 \) with equal number of degrees of freedom. Then, taking into account only the radiative corrections associated with these heavy fields, the maximum value of \( \mu_S \) consistent with vacuum stability may be obtained from the condition of a positive quartic coupling at scales much larger than \( v \), and it is given by

\[
\mu_S^2 \leq \exp \left( \frac{m_H^2 8\pi^2}{g h^4 v^2} \right) m_f^2(v) - h^2 v^2
\]

(2.14)

Observe that for heavy Higgs bosons and/or weakly coupled fermions \( \mu_S \) becomes much larger than the weak scale. If, however, \( h \) takes large values and \( m_H \) becomes light then \( \mu_S \) approaches \( \mu_f \) and the effect of stabilizing fields is to cancel the zero temperature contribution of fermions plus giving additional finite temperature contributions.
In order to get an understanding of the effects to be expected by the presence of these new particles, let us first consider a fermion particle with a mass, \( m(\phi) = h\phi \), much larger than \( T_c \) for \( \phi \simeq \phi_c \). If this effect is obtained for large values of the Yukawa coupling, then the sum of the effects of the fermions and the stabilizing fields at zero temperature is small and, in this extreme case that maximizes the contribution of fermions (and that of the stabilizing fields) to the phase transition, we can ignore the zero temperature contributions. Then, in the symmetric phase the species is light and contributes to \( g_* \), but in the broken-symmetry phase it is heavy and approximately decouples from the thermal plasma. Usually such a decoupling species would transfer its entropy to the thermal bath, causing a temperature rise. During a phase transition we would naively expect that the effect of such a reheating is to delay the appearance of the true vacuum, to decrease the critical temperature and subsequently to increase the value of \( \phi_c/T_c \).

More quantitatively, at constant \( g_* \) the critical temperature is given by the condition \( V(\phi_m(T'_c), T'_c) = 0 \), corresponding to degenerate minima of \( V \). As we will show below, this condition changes if the number of light degrees of freedom is different in the two phases, \( \Delta g_* \equiv g_* \text{symmetric} - g_* \text{broken} \neq 0 \). In our example, the decoupling particle has a mass \( m(\phi) = h\phi \), and its contribution to the free-energy density in the broken phase vanishes, while in the symmetric phase is equal to \(-\frac{\pi^2}{90}\Delta g_* T^4\), where \( \Delta g_* \) is its number of degrees of freedom. The condition of degenerate minima of \( F \), Eq. (2.8), then gives \( V_{SM}(\phi_m(T_c), T_c) = -\frac{\pi^2}{90}\Delta g_* T^4 \). (2.15)

This condition is attained at a lower critical temperature, \( T'_c > T_c \). Moreover we have that \( \phi_c = \phi_m(T_c) > \phi'_c \) and the phase transition is more strongly first-order. It was already noticed in Ref. \(^{48}\) that \( \Delta g_* \neq 0 \) in the context of the electroweak phase transition could be important for baryogenesis.

We can now estimate \( \Delta(\phi_c/T'_c) \equiv \phi_c/T'_c - \phi'_c/T'_c \) by noticing that the value of the effective potential \(^{24}\) at the minimum \( \phi_m \) can be written as

\[
\frac{V_{SM}(\phi_m(T), T)}{T^4} = \frac{\lambda}{4} \left( \frac{\phi_m(T)}{T} \right)^3 \left( \frac{\phi'_c}{T'_c} - \frac{\phi_m(T)}{T} \right),
\]

where we have made the approximation \( \lambda_T \simeq \lambda \). Eq. (2.16) implies that

\[
\left( \frac{\phi_c}{T_c} \right)^3 \Delta \left( \frac{\phi_c}{T_c} \right) = \frac{2\pi^2\Delta g_*}{45\lambda}.
\]

(2.17)

For the order parameter to increase from \( \phi'_c/T'_c = 2E/\lambda \ll 1 \) to the value \( \phi_c/T_c > 1 \), necessary to preserve the baryon asymmetry \(^{5}\), we need \( \Delta g_* \gtrsim 45\lambda/2\pi^2 \simeq 0.25 (m_H/115 \text{ GeV})^2 \).

\(^{3}\)We are normalizing the total effective potential as \( V(0, T) = 0 \).
So it seems that particles with few d.o.f. will produce an effect. Conversely given $\Delta g_*$, the bound on the Higgs mass is relaxed from $(m_H/v)^2 < 4E$ to

$$(m_H/v)^2 < 4E + \frac{4\pi^2\Delta g_*}{45}. \tag{2.18}$$

Notice that if the new value $\phi_c$ is $\sim T_c$, then the perturbative (one-loop) approach is fully justified even for small values of $E$.

It should be noticed that Eqs. (2.15)–(2.17) give only an estimate of $T_c$ and $\phi_c$. In fact, the effective number of light d.o.f. varies continuously from $g_*$ to $g_* - \Delta g_*$, as $\phi$ goes from 0 to $\phi_c$. Therefore we have a function $g_*(\phi)$ that contributes to $F(\phi, T)$, and the correct value of the vacuum expectation value (VEV) is obtained by minimizing the complete free energy, Eq. (2.8). Once the dependence of $g_*$ on $\phi$ is taken into account, we cannot use analytical approximations and we need to resort to numerical calculations. Observe that in the limit studied above fermions and bosons gave equally important contributions to the phase transition strength and the number of degrees of freedom is the sum of the one associated to the fermions and that of stabilizing fields. In the rest of this work we will consider bosons that are heavier than the fermions, and therefore lead to a smaller finite temperature contribution than in the example above.

We will now consider adding a fermion particle with mass $m^2(\phi) = \mu^2 + h^2 \phi^2$ to the SM with a Higgs mass $m_H = 120$ GeV, but we will retain the effects of the heavy particles in the broken phase. We consider only fermion species and their (heavier) stabilizing fields since, as explained above, the effect of bosons on the phase transition has been extensively studied in the literature \[5\], \[17\]–\[31\]. In Fig. 1 we plot the number of degrees of freedom $g$ that give $\phi_c/T_c = 1$ as a function of the mass parameter $\mu$, for different values of the Yukawa coupling $h$. The invariant mass of the stabilizing fields has been set to their maximum value consistent with vacuum stability, Eq. (2.14). As anticipated, for $\mu \approx 0$, and for large values of the Yukawa couplings, only a small number of degrees of freedom are necessary in order to obtain a strongly first order phase transition.

A minimal Standard Model extension (i.e. the introduction of a single species) is possible with bosons but not with fermions, since the SM Higgs is an $SU(2)$ doublet. In order to construct an invariant Yukawa Lagrangian the simplest possibilities are, either doublet and singlet fermions (as e.g. a generation of mirror leptons and/or quarks) or doublet and triplet fermions (as e.g. light Higgsinos and gauginos) remnant from (split) supersymmetry \[42\]. In the next section we will consider the latter possibility.
Figure 1: Curves of constant $\phi_c/T_c = 1$ and $m_H = 120$ GeV, for a fermion with mass $m^2 = \mu^2 + h^2 \phi^2$ and $g$ degrees of freedom. From top to bottom the curves correspond to $h = 1.5, 2, 2.5,$ and $3$.

3 HIGGSINOS AND GAUGINOS: THE PHASE TRANSITION

In this section we will consider the particular case of Higgsinos ($\tilde{H}_{1,2}$), Winos and Binos ($\tilde{W}^a$, $\tilde{B}$) coupled to the SM Higgs doublet $H$ with the Lagrangian

$$
\mathcal{L} = H^\dagger \left(h_2 \sigma_a \tilde{W}^a + h'_2 \tilde{B}\right) \tilde{H}_2 + H^T \epsilon \left(-h_1 \sigma_a \tilde{W}^a + h'_1 \tilde{B}\right) \tilde{H}_1
+ \frac{M_2}{2} \tilde{W}^a \tilde{W}^a + \frac{M_1}{2} \tilde{B} \tilde{B} + \mu \tilde{H}_2^T \epsilon \tilde{H}_1 + h.c. \tag{3.1}
$$

where $\epsilon = i \sigma_2$ and the Yukawa couplings $h_{1,2}$ and $h'_{1,2}$ are arbitrary $^4$.

The chargino mass matrix is

$$
\begin{pmatrix}
M_2 & h_1 \phi \\
h_2 \phi & \mu
\end{pmatrix}, \tag{3.2}
$$

and the squared mass matrix has eigenvalues

$$
\lambda_{c\pm} = \left(\sqrt{M_+^2 + h_2^2 \phi^2} \pm \sqrt{M_-^2 + h'_2 \phi^2}\right)^2, \tag{3.3}
$$

where $M_{\pm} = \frac{1}{2} (M_2 \pm \mu)$, $h_{\pm} = \frac{1}{2} (h_1 \pm h_2)$. The mass matrix for neutralinos is

$$
\begin{pmatrix}
M_2 & 0 & -h_2 \phi/\sqrt{2} & h_1 \phi/\sqrt{2} \\
0 & M_1 & h'_2 \phi/\sqrt{2} & -h'_1 \phi/\sqrt{2} \\
-h_2 \phi/\sqrt{2} & h'_2 \phi/\sqrt{2} & 0 & -\mu \\
h_1 \phi/\sqrt{2} & -h'_1 \phi/\sqrt{2} & -\mu & 0
\end{pmatrix}. \tag{3.4}
$$

$^4$The matching at high scale with the MSSM couplings would be $h_2 = g \sin \beta/\sqrt{2}$, $h_1 = g \cos \beta/\sqrt{2}$, $h'_2 = g' \sin \beta/\sqrt{2}$, $h'_1 = g' \cos \beta/\sqrt{2}$, where $g$ and $g'$ are the SU(2) and U(1) gauge couplings respectively, and the Higgs doublet is related to the MSSM Higgses by $H = \sin \beta H_2 - \cos \beta \epsilon H_1^\dagger$. The matching with the couplings of a possible UV completion of the model will be done in section 8. For the moment we will just assume that there is such a UV completion and that it provides the necessary stabilizing fields as it was discussed in section 2.
The eigenvalues of this matrix are cumbersome, so we consider the particular case \( M_1 = M_2 \equiv M, \ h_1 = h_2 \equiv h, \) and \( h'_1 = h'_2 \equiv h'. \) The eigenvalues of the squared mass matrix are thus
\[
\lambda_{n1} = \mu^2, \ \lambda_{n2} = M^2, \tag{3.5}
\]
and
\[
\lambda_{n\pm} = \left( M_+ \pm \sqrt{M_+^2 + h^2\phi^2 + 2h'^2\phi^2} \right)^2, \tag{3.6}
\]
In this case the chargino eigenvalues become very similar to those in Eq. \( (3.6) \): in particular we have \( h_- = 0, \ h_+ = h, \) and \( M_2 = M \) in Eq. \( (3.3) \). We can further simplify the problem by also setting \( h' = 0 \), since in this case Eqs. \( (3.3) \) and \( (3.6) \) become
\[
\lambda_{\pm} = \left( M_+ \pm \sqrt{M^2 + h^2\phi^2} \right)^2. \tag{3.7}
\]
that corresponds to 6 degrees of freedom (a Dirac spinor and a Majorana spinor) with squared mass \( \lambda_+ \), 6 with squared mass \( \lambda_- \), and 2 (light) Majorana particles with masses \( \mu \) and \( M \). A total of 16 fermionic degrees of freedom out of which only 12 are coupled to the SM Higgs. Clearly, for \( h' = 0 \) the Majorana particle with mass \( M \) is just a pure Bino state of mass \( M_1 = M \), which decouples from the other low-energy states and therefore plays no role in determining the strength of the electroweak phase transition. In the following, we shall concentrate on this particularly simple case.

From Eq. \( (3.7) \) it is clear that all the results in this case will be symmetric under \( \mu \leftrightarrow M \). The simplest limiting case is when \( M_+ = 0 \) and \( M_- = M = -\mu \). In this case the eigenvalues are degenerate, \( \lambda_{\pm} = M^2 + h^2\phi^2 \), with 12 degrees of freedom corresponding to 3 Dirac spinors, and the situation is identical to the simple example illustrated in the previous section. One expects that other limits will be less favorable for the phase transition. For instance if we take \( M_- = 0 \) (i.e. \( M_+ = M = \mu \)), the eigenvalues are \( (M \pm h\phi)^2 \). This means that, unless \( h\phi \geq 2M \), in the broken-symmetry phase half of the particles become heavier than in the symmetric phase but the other half become lighter. Therefore, unless \( M_+ \) is also small, less degrees of freedom than in the case \( M_+ = 0 \) will contribute to this effect.

In Fig. 2 we plot the free energy at different temperatures for a Yukawa coupling \( h = 2 \), a Higgs mass of 120 GeV and \( \mu = -M \simeq 200 \) GeV. We can explicitly see from the figure shape that there is a first order phase transition with an order parameter \( \phi_c/T_c \approx 1.75 \).

In Fig. 3 we plot the ratio of the Higgs VEV to the temperature, evaluated at the critical temperature, as a function of the mass \( M \) for the case \( \mu = -M \) and \( m_H = 120 \) GeV. As expected, the strength of the phase transition decreases with \( M \). This illustrates the fact that for large \( M \) the particle is decoupled already in the symmetric
phase, hence the VEV $\phi_c$ has a smaller value, which corresponds in the $M \to \infty$ limit, to that of the electroweak phase transition in the Standard Model.

In Fig. 4 we plot the values of the Yukawa coupling $h$ necessary to induce a strongly first order phase transition for the case $M = -\mu$ and $m_H = 120$ GeV. It is clear from the plot that for such a small value of the Higgs mass, a strengthening of the phase transition may only be achieved for $h \gtrsim 1.5$. Let us stress that this lower bound may be weakened by assuming slightly smaller values of $\mu_S$. For instance, taking $\mu_S^2$ to be 0.9 times the maximum value allowed in Eq. (2.14) is enough to ensure that values of $h = 1.5$ and masses $\mu$ of order of 100 GeV are allowed.

We are interested in a strong enough phase transition for baryogenesis, which means that the Higgs VEV must be large enough for sphaleron processes to be suppressed in the broken-symmetry phase (in order to avoid the washout of the BAU after the phase transition). In the case of Fig. 3 this happens up to a value $M = M_{\text{max}}$ determined by
the condition $\phi_c/T_c \sim 1$. Observe that the top-quark, with 12 degrees of freedom (similar to the chargino-neutralino system) and a Yukawa coupling $h_t \simeq 0.7$ in our normalization, is not able to generate such a strong first-order phase transition in the SM.

It is well known that the value of the Higgs mass plays a prominent role in the strength of the phase transition in the Standard Model and extensions thereof. Up to now we have fixed it to a “minimal” value, $m_H = 120$ GeV. However our mechanism of strengthening the phase transition by using strongly coupled fermions, although certainly sensitive to the value of the Higgs mass, permits to go to higher values. In Fig. 5 we plot, for fixed values of the Yukawa coupling, the values of $M$ that give $\phi_c/T_c = 1$ as a function of the Higgs mass. We can see that, as expected, there is an upper bound on the Higgs mass that depends on the Yukawa coupling $m_H^{\max} = m_H^{\max}(h)$. Moreover the upper value of the Higgs mass has an approximate linear behaviour as a function of the Yukawa coupling $h$ as can be readily deduced from Fig. 5. Imposing perturbativity of the theory at the low scale sets a generic upper limit on the Yukawa coupling $h \lesssim \sqrt{4\pi} \sim 3.5$ which from Fig. 5 yields a corresponding upper limit on the Higgs mass of $m_H \lesssim 400$ GeV. In general, for large values of $h$, the requirement of perturbative consistency of the theory up to high energies may only be fulfilled by embedding this model into a more complete theory where couplings become asymptotically free (see section 8). For the particularly interesting UV completion proposed in section 8 the upper limit on the Yukawa coupling is, as we will see, more restricted, $h \lesssim 2$, which translates into the stronger Higgs mass upper bound $m_H \lesssim 175$ GeV for $\mu = -M$ of order 100 GeV.

The model we have proposed contains Higgsinos $\tilde{H}_i$ and electroweak gauginos $\tilde{W}_a$, $\tilde{B}$, coupled to the SM Higgs through the Lagrangian (3.1) with Yukawa couplings $h_i$, $h'_i$.
and Majorana masses $M_i$. We have concentrated on the case $h_i' = 0$, in which the Bino state decouples from the other low energy states. Similar results for the phase transition would be obtained in any model in which the Bino would decouple from the low energy theory, for instance if $h_i' \ll h_i$ and $M_1$ is much larger than the weak scale. Although the Bino is absent at low energies, there is still a state with mass approximately equal to $|\mu|$ that is a candidate for Dark Matter. This will be analyzed in section 6 where we will show that a modest splitting between the Yukawa couplings $h_1$ and $h_2$ will be necessary to accommodate the observed DM energy density, although a large separation is not permitted by electroweak precision measurements. In this way introducing a small splitting between $h_2$ and $h_1$ is not expected to modify substantially the previous results in this section.

In fact in the approximation of neglecting terms of order $h_2^2$ in the diagonalization of the mass matrices one can prove that the mass eigenvalues still correspond to one Majorana spinor with mass $\mu$, two Dirac spinors with squared masses given in Eq. (3.3) and two Majorana spinors with squared masses,

$$\lambda_{\pm} = \left( M_+ \pm \sqrt{M_+^2 + h_+^2 \phi^2} \right)^2,$$

which of course coincides with the degenerate result of Eq. (3.7) in the limit of $h_1 = h_2$.

The result of Eq. (3.8) also proves that the modification of the strength of the phase transition due to the non-degeneracy will be small at least in the validity region where Eq. (3.8) holds. More explicitly, in Fig. 6 we show numerically the variation of the order parameter $\phi_c/T_c$ away of the degenerate point as a function of $\delta h = h_- - h_+$ for $h_+ = 2$ and $M = -\mu = 50$ GeV. We see that the phase transition is weakened by $\lesssim 20\%$ for $h_-/h_+ \lesssim 0.3$.
Figure 6: Values of $\phi_c/T_c$ as a function of $\delta h$, for a fixed value of the average value $h_+ = 2$, $m_H = 120$ GeV and $M = -\mu = 50$ GeV.

Up to this point we have discussed the electroweak phase transition in the absence of CP-violating phases. However for the baryogenesis mechanism to work we need a non-vanishing CP-violating phase in the parameters of the theory that will trigger baryon number generation. As in the MSSM studies, we shall consider real Yukawa couplings (related to gauge couplings in the MSSM) and Majorana gaugino masses, and a complex mass parameter $\mu = |\mu|e^{i\varphi}$. To conclude this section we would like to consider how the phase transition and in particular the order parameter $\phi_c/T_c$ vary with the phase $\varphi$. The result is presented in Fig. 7 where we plot $\phi_c/T_c$ as a function of $\varphi$ for $M_2 = M_1 = |\mu| = 50$ GeV and Yukawa couplings $h_+ = 2$, $h_- = 0$. The plot at $\varphi = \pi$ is consistent with the corresponding results that can be read off from Fig. 3 while the strength of the first order phase transition decreases by only a tiny amount for $\mathcal{O}(1)$ CP violating phases, i.e. for
\[ \varphi = \pi/2. \]

4 CP-VIOLATING SOURCES AND BARYOGENESIS

The chargino sector in the model presented in section 3 has a similar structure to the chargino sector in the minimal supersymmetric Standard Model. The only difference is that the couplings \( g \sin \beta / \sqrt{2} \) and \( g \cos \beta / \sqrt{2} \) are replaced by arbitrary couplings \( h_2 \) and \( h_1 \), respectively, as can be seen in the corresponding mass matrix (3.2). As in the MSSM the CP-violating phase can have its origin, after field redefinitions, in the phase \( \varphi \) of the (complex) \( \mu \)-parameter. A general method for computing the effects of CP-violating mass terms on particle distributions was introduced in Ref. [17] leading to an efficient transport of CP-violating quantum numbers into the symmetric phase where weak sphalerons are active and can trigger electroweak baryogenesis for all bubble wall widths. The method was adapted to the MSSM by a number of papers [18]-[31] where a set of coupled differential equations, that include the effect of diffusion, particle number changing reactions and CP violating terms, were solved to find various particle number densities diffused from the bubble wall, where CP-violation takes place, to the symmetric phase where sphalerons are active. These methods can be adapted to the present model by just considering the particular structure of the chargino mass matrix given by Eq. (3.2).

We will further make the simplifying assumption that all CP violation resides in the fermionic sector. Otherwise there should be extra contributions to the CP violating currents from the bosonic (stabilizing) fields, although these contributions are expected to be suppressed with respect to the fermionic ones because the stabilizing fields are heavier than the fermions. So from this point of view our results will be mostly conservative.

In particular we will follow the formalism of Refs. [27, 31] where a method was developed to compute the CP-violating sources induced by the passage of a bubble wall in a system of fermions that interact in a way similar to the one described above, in an expansion of derivatives of the Higgs fields. The method allows for the computation of the currents in a resummation to all orders of the Higgs vacuum expectation value effects. It was found that there are two different CP-violating sources from the chargino sector which the total baryon asymmetry depends upon. The leading contribution is provided by

\[ \epsilon_{ij} H_i \partial_\mu H_j = v^2(T) \partial_\mu \beta \]

that is proportional to the variation of the angle \( \beta = \arctan [v_2(T)/v_1(T)] \) at the wall of the expanding bubble. The source (4.1) has a resonant behaviour for \( M_2 = |\mu| \) and it is the leading contribution in the MSSM. However, the Higgs sector of our model (which
contains just the SM Higgs doublet) can be considered as the \( m_A \to \infty \) limit of the MSSM Higgs sector (where \( m_A \) is the pseudoscalar mass) in which case \( \partial_\mu \beta \to 0 \) and the source (4.1) does not contribute to the diffusion equations.

The second contribution to the baryon asymmetry is proportional to

\[
H_1 \partial_\mu H_2 + H_2 \partial_\mu H_1 = v^2 \cos(2\beta) \partial_\mu \beta + v \partial_\mu v \sin(2\beta). \tag{4.2}
\]

In the limit \( m_A \to \infty \) only the second term in (4.2) survives. Moreover we will consider in this section \( h_1 \simeq h_2 \equiv h \) (i.e. \( \tan \beta \simeq 1 \)) \(^5\) in which case the only remaining source is proportional to \( \partial_\mu v^2 \). This region is, as it was proven in Ref. [31], very insensitive to the resonance region relating \( M_2 \) and \( |\mu| \) and it provides a very natural region of parameters where electroweak baryogenesis can hold. Although in the MSSM such a region provided a very tiny amount of baryon asymmetry, in the present model all effects are enhanced by the strong Yukawa couplings of the Higgs to charginos as we will see in this section.

In the MSSM there is an additional suppression of the source, Eq. (4.2), due to the large values of \( \tan \beta \) necessary to fulfill the LEP bounds on the lightest CP-even Higgs boson mass \(^49\), \( \tan \beta \) of order 10. As stressed above in this model, instead, \( \tan \beta \simeq 1 \).

Following the formalism of Refs. [27, 31] the solution of the diffusion equations, in the limit where the strong sphaleron (\( \Gamma_{ss} \)) and Yukawa processes (\( \Gamma_Y \)) are fast enough, provide quark number density for third generation doublets \( n_Q \) and singlets \( n_T \) as functions of the number density for the Higgs doublet coupled to the top quark \( n_H \), as

\[
\begin{align*}
n_Q(z) &= \frac{k_Q(9k_T - k_B)}{k_H(k_B + 9k_Q + 9k_T)} n_H(z) + \mathcal{O}\left(\frac{1}{\Gamma_{ss}}, \frac{1}{\Gamma_Y}\right) \\
n_T(z) &= -\frac{k_T(9k_Q + 2k_B)}{k_H(k_B + 9k_Q + 9k_T)} n_H(z) + \mathcal{O}\left(\frac{1}{\Gamma_{ss}}, \frac{1}{\Gamma_Y}\right) \tag{4.3}
\end{align*}
\]

where \( k_i \) are statistical factors \(^50\)

\[
\begin{align*}
k_B(m^2) &= \frac{3}{2\pi^2} \int_0^\infty dp \frac{p^2}{\sinh^2\left(\sqrt{(p^2 + m^2)/T^2}/2\right)} \tag{4.4} \\
k_F(m^2) &= \frac{3}{2\pi^2} \int_0^\infty dp \frac{p^2}{\cosh^2\left(\sqrt{(p^2 + m^2)/T^2}/2\right)} \tag{4.5}
\end{align*}
\]

that satisfy the condition \( k_F(0) = 1 \) (\( k_B(0) = 2 \)) for Weyl fermions (complex bosons). In turn the density \( n_H(z) \) \(^6\) is obtained from the diffusion equations as a function of the

\(^5\)This choice will be motivated by the contribution of charginos and neutralinos to the electroweak parameter \( T \) as we will see in section 7.

\(^6\)The spatial coordinate \( z \) is transverse to the bubble wall and we are neglecting the bubble curvature.
particle number changing rates, CP-violating sources and diffusion constants, as explained in Ref. 31, yielding from Eq. (4.3) the quark number densities \( n_{Q,T} \).

In order to evaluate the baryon asymmetry generated in the broken phase \( n_B \) we first need to compute the density of left-handed quarks and leptons, \( n_L \), in front of the bubble wall in the symmetric phase. These chiral densities bias weak sphalerons to produce a net baryon number. Considering particle species that participate in fast particle number changing transitions, and neglecting all Yukawa couplings except those corresponding to the top quark, only quark doublets do contribute to \( n_L \). Then assuming that all quarks have nearly the same diffusion constant it turns out that \[ n_{Q_1} = n_{Q_2} = 2(n_Q + n_T) \] and therefore from Eq. (4.3)

\[
n_L(z) = 5n_Q(z) + 4n_T(z) = An_H(z) + \mathcal{O}\left(\frac{1}{\Gamma_{ss}}, \frac{1}{\Gamma_Y}\right)
\]

\[
A = \frac{5k_Qk_B + 8k_Tk_B - 9k_Qk_T}{k_H(k_B + 9k_Q + 9k_T)}.
\]

(4.6)

It turns out that the baryon asymmetry can be written as 31

\[
n_B = -\frac{n_F\Gamma_{ws}}{v_\omega} \int_{-\infty}^{0} dz \ n_L(z) \exp\left(-\frac{5n_F\Gamma_{ws}z}{4v_\omega}\right)
\]

(4.7)

where \( n_F = 3 \) is the number of families, \( v_\omega \) the bubble wall velocity and \( \Gamma_{ws} = 6\kappa\alpha_5^5 T \), where \( \kappa \simeq 20 \) 4, is the weak sphaleron rate.

For the model presented in this paper, where squarks and the non-SM Higgs bosons are superheavy, \( m_Q, m_T, m_B \gg T_c \approx 100 \text{ GeV} \) \((k_B = k_T \approx 3, k_Q \approx 6 \text{ and } k_H \approx 8)\), it turns out that the coefficient in Eq. (4.6) is \( A \simeq 0 \). This SM suppression was already pointed out by Giudice and Shaposhnikov 50 and consequently the baryon asymmetry \( n_B \) in our model is produced by sub-leading effects. Assuming \( \Gamma_Y \gg \Gamma_{ss} \) we can go beyond the approximation of Eq. (4.3) and work out corrections of \( \mathcal{O}(1/\Gamma_{ss}) \). This was done in Ref. 17 leading to an \( \mathcal{O}(1/\Gamma_{ss}) \) correction to \( n_L(z) \), \( \Delta_{ss} n_L(z) \), in our model as

\[
\Delta_{ss} n_L(z) = -\frac{3}{112} \frac{D_q n'_H(z) - v_\omega n''_H(z)}{\Gamma_{ss}}
\]

(4.8)

where \( D_q \approx 6/T \) is the quark diffusion constant and the strong sphaleron rate is given by \( \Gamma_{ss} = 6\kappa'\frac{8}{3}\alpha_4^4 T \), where \( \kappa' \) is an order one parameter 17. When (4.8) is inserted into (4.7) it produces the baryon asymmetry generated by the sub-leading \( \mathcal{O}(1/\Gamma_{ss}) \) effects. We have numerically checked that this correction is insufficient to generate the observed baryon asymmetry of the universe.

Another, more important, correction that can lead to a non-zero value of the baryon asymmetry to leading order in \( \Gamma_{ss} \) are Yukawa and gauge radiative corrections to statistical
coefficients $k_i$ (or equivalently thermal masses) \[50\]. Expanding Eq. (4.5) in a power series of $m^2/T^2$ we can write

$$k_F(m^2) = 1 - \frac{3}{2\pi^2} \frac{m^2}{T^2} + \mathcal{O}(m^4/T^4) \ .$$

(4.9)

Keeping only the strong gauge ($g_s$) and top Yukawa ($h_t$) couplings one obtains in our model the statistical coefficients,

\begin{align*}
k_T & = 3(1 + \Delta_s + \Delta_Y) \\
k_B & = 3(1 + \Delta_s) \\
k_Q & = 6(1 + \Delta_s + \frac{1}{2}\Delta_Y)
\end{align*}

(4.10)

with

$$\Delta_s = -\frac{g_s^2}{2\pi^2}, \quad \Delta_Y = -\frac{3h_t^2}{8\pi^2} \quad (4.11)$$

where $h_t \simeq 1/\sqrt{2}$ is the top quark Yukawa coupling and $g_s \simeq 1.2$ the strong gauge coupling. Numerically, $|\Delta_s| \sim 7 \times 10^{-2}$ is more important than $|\Delta_Y| \sim 2 \times 10^{-2}$ but since the strong correction to all quarks is universal it cancels in the contribution to the baryon asymmetry. In fact to linear order in $\Delta_i$ one can write the density $n_L(z)$ in Eq. (4.6) as

$$n_L(z) = -\frac{3}{16} \Delta_Y n_H(z) \quad (4.12)$$

and the proportionality coefficient turns out to be $A \sim 4 \times 10^{-3}$. This is the reduction factor we get from such a sub-leading effect. The numerical calculation of the baryon-to-entropy ratio $\eta$ is presented in Fig. 8 (lower solid line) where we plot its ratio to the experimentally determined value $\eta_{\text{BBN}} = (8.7 \pm 0.3) \times 10^{-11}$ \[59\] as a function of the Yukawa coupling $h$ and we have fixed the CP-violating phase $\sin \varphi = 1$. In fact the phase $\sin \varphi$ that would be required for fixing $\eta = \eta_{\text{BBN}}$ is given by the inverse value plotted in Fig. 8. We have chosen the case $\mu = -M_2 \exp(i\varphi)$, and small values of $M_2 = |\mu| \simeq 50$ GeV, where the phase transition is favored, and typical values of the bubble width and velocity \[7\]. Since the computation of the baryon asymmetry has been performed by ignoring corrections of order one, the main conclusion one can extract from the results of Fig. 8 is that CP-violating phases such that $\sin \varphi$ is of order one are necessary to obtain a value of the baryon asymmetry consistent with the experimentally determined values, for any value of $h \gtrsim 1.5$.

\[7\] A general feature of first-order phase transitions is that the release of latent heat causes a slow-down of bubble expansion \[48\]. The electroweak bubble-wall velocity thus decreases during the phase transition \[51\] from its initial value $v_w \sim 10^{-1}$ \[52\] given by the friction of the plasma. Calculating the exact value of $v_w$ is out of the scope of this paper. However, as noticed in Ref. \[53\], the effect of the velocity variation on the BAU is likely to be an $\mathcal{O}(1)$ one effect and should not modify the main conclusions of this paper.
Figure 8: The ratio $\eta/\eta_{\text{BBN}}$ as a function of the Yukawa coupling $h$ for $\mu = -M_2 \exp(i\varphi)$, $M_2 = 50$ GeV, maximal CP-violating phase, $\sin \varphi = 1$, and bubble parameters $L_\omega = 10/T_c$, $v_\omega = 0.1$. Left-handed squarks and right-handed sbottoms are heavy (in the few TeV range). The lower (upper) solid line corresponds to heavy (light) right-handed stops, $m_T \gtrsim 1$ TeV ($m_T \simeq 100$ GeV). Dashed line corresponds to right-handed stops with $m_T \simeq 500$ GeV.

The amount of generated baryon number density can be increased if some squark is light enough to be in equilibrium with the thermal bath during the phase transition, in which case the SM suppression is avoided. The typical case that was considered in previous studies is that of a light right-handed stop [9] that corresponds to values of the supersymmetry breaking soft masses $m_Q, m_B \gg T_c$ and $m_T \lesssim T_c$. In that case the statistical coefficients are given by $k_Q \simeq 6$, $k_T \simeq 9$, $k_B \simeq 3$ and $k_H \simeq 8$ and the coefficient $A$ in Eq. (4.6) does not vanish to leading order in $\mathcal{O}(1/\Gamma_{ss})$. In fact it is given by $A \simeq 1/6$ and this produces an enhancement factor of $\mathcal{O}(100)$ with respect to the case where all squarks are superheavy. This enhancement factor produces larger values of $\eta$ (and so smaller phases are allowed) as can be seen in Fig. 8 upper solid line. Now fixing $\eta = \eta_{\text{BBN}}$ can be consistent with phases $\sin \varphi \simeq 10^{-2}$. Notice finally that a similar enhancement would also appear if other squark species (i.e. right-handed sbottom or left-handed doublet) are light; this effect is not particularly linked to the lightness of right-handed stops.

As mentioned above, the upper solid line in Fig. 8 corresponds to the extreme case where there are no extra bosons in the low energy spectrum, or equivalently stop masses in
the TeV range or larger. Of course there can be intermediate situations where the stop is heavy but still does not fully decouple from the thermal plasma. In this case it contributes to the statistical factor $k_T$ with some small value that can significantly contribute to the $A$-parameter and departure its value from zero. For instance if $m_T = 5 T_c \simeq 500$ GeV, its contribution to the statistical factor $k_T$ as given from (4.4) is $\sim 0.24$ which produces in (4.6) a value $A \sim 1.2 \times 10^{-2}$ and enhances the value of $\eta$ from its value with fully decoupled right-handed stops. The corresponding value of $\eta$ is plotted in Fig. 8 (dashed line). We can see that the CP-violating phase for $\eta = \eta_{BBN}$ is now $\sin \varphi \simeq 0.1$. A similar effect would be produced by other not-so-heavy third generation squarks.

5 Electron Electric Dipole Moment

In the absence of light squarks, baryon number generation demands the presence of large phases in the chargino and neutralino sectors. In the previous section, we assumed gaugino masses and Yukawa couplings to be real, and therefore the relevant phase is the one of the $\mu$ parameter. Although one-loop corrections to the electron electric dipole moments are suppressed in the limit of heavy fermions, as has been stressed in Ref. [54], two-loop contributions become relevant. In this section we will evaluate the two-loop contribution to the electron electric dipole moment from the fermion and Higgs sector.

In Fig. 9 we plot the chargino contribution to the electron electric dipole moment [55], for $\mu = -M_2 \exp(i\varphi)$, with $M_2$ real and positive ($M_2 = |\mu|$), maximal CP-violating phase, $\sin \varphi = 1$, $h_- = 0$, and for $h_+ = 1.5$, $m_H = 120$ GeV (solid line); $h_+ = 2$, $m_H = 120$ GeV (dashed line); and $h_+ = 2$, $m_H = 200$ GeV (dot-dashed line). We have verified that the results vary only slightly for non-vanishing values of $|h_-/h_+| < 0.1$, which, as we will show in sections 6 and 7, are preferred by dark matter and precision electroweak constraints.

As discussed above, for values of $h_+ < 1.5$, the electroweak phase transition is weakly first order and the generated baryon asymmetry is not preserved. For slightly larger values, $h_+ \lesssim 1.6$, small values of $|\mu| < 100$ GeV and Higgs mass values smaller than 125 GeV are demanded in order to make the phase transition strongly first order. The present electric dipole moment bound, $|d_e|/(ecm) < 1.7 \times 10^{-27}$ [56] does not put a bound on this model for these values of $h_+$ and $|\mu|$. For larger values of $h_+$ and similar values of $|\mu|$, Fig. 9 shows that the electron electric dipole moment contributions become smaller and

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8For instance for $m_T = 10 T_c \simeq 1$ TeV the stop contribution to $k_T$ is $\sim 3 \times 10^{-3}$ and $A \sim 1.8 \times 10^{-4}$, much smaller than the previously considered thermal effects.

9We will assume here that squarks and stabilizing bosons are heavy enough not to contribute appreciably to electric dipole moments. However when particular UV completions of this model will be considered this assumption should be re-checked and if the new contributions are relevant they should be added to the fermionic ones.
in addition, as shown in Fig. 8, smaller phases are demanded for the baryon number generation. Therefore, the electric dipole moment bounds become even weaker in this case.

However the anticipated improvement of a few orders of magnitude in this quantity \cite{57} will be sufficient to test this model even for values of $h_+ = 2$, provided the values of $|\mu|$ are in the range necessary to obtain a good dark matter source and avoid the LEP invisible constraints: For large values of $M_1$, (or for $M_2 = M_1$) and small values of $h_-$, the lightest neutralino mass is mainly a Higgsino with mass approximately equal to $|\mu|$. For $h_+ \geq 1.5$, the chargino and the second lightest neutralino are heavier than 200 GeV and therefore, the stronger experimental bound comes from the $Z$ invisible width. Surprisingly, for $h_- = 0$, the tree-level coupling of the lightest neutralino to the $Z$ vanishes and therefore the invisible width bounds become very weak. However as we shall analyze in section 6 assuming thermal production of the lightest neutralino, an acceptable relic density may only be obtained for a small but non-vanishing coupling to the $Z$. Quite generally, if the dark matter density is determined by the s-channel $Z$-annihilation cross section of a neutralino, an acceptable relic density may only be obtained for values of the neutralino mass larger than 35 GeV, what in this case implies $|\mu| \gtrsim 35 \text{ GeV}$ \cite{38}. For such a range of values of $|\mu|$, a bound on the electric dipole moment of about $10^{-29}$ e cm will be enough to test this model for $h_+ \lesssim 2$ and $\sin \varphi \simeq \mathcal{O}(1)$, as required for baryogenesis in the absence of light squarks.
6 Dark Matter

One of the most attractive features of the model presented above is that the particles that lead to a strengthening of the electroweak phase transition are the same as the ones leading to a generation of the baryon number at the weak scale. It would be most important if the same particles would also provide a good Dark Matter candidate. As stated in the introduction stable, neutral, weakly interacting particles lead naturally to a Dark Matter relic density of the order of the one present in nature. As we will see, under the assumption of an $R$-Parity symmetry, the lightest neutralino of the model presented above becomes a good Dark Matter candidate.

Let us work in the simplest case, in which the Bino mass $M_1$ takes large values ($M_1 \gg M_2$) and mixes only weakly with the Higgsino ($h'_{1,2} \ll h_{1,2}$). In such a case, due to the strong Yukawa couplings $h_1$ and $h_2$, the charginos and two of the neutralinos acquire masses of about $h_+ v$. The mass of the lightest neutralino is close to $|\mu|$ and the lightest neutralino is therefore an almost pure Higgsino state.

Assuming that all squarks, stabilizing bosons and heavy Higgses, if present, are considerably heavier than the lightest Higgsino, the states which determine the neutralino annihilation cross section are the light SM-like Higgs boson and the $Z$-gauge boson. Due to the small coupling of the SM Higgs boson to quarks and leptons, the annihilation cross section via s-channel Higgs boson production is very small, unless the neutralino mass is very close to $m_h/2$, a quite unnatural possibility that we shall discard for the aim of this work. Hence the annihilation cross section is governed by the coupling of the lightest neutralino to the $Z$-gauge boson.

The coupling of a neutralino state to the $Z$-gauge boson is proportional to the difference of the square of the components $N_{\tilde{\chi}_i}$ of the neutralino into the two weak Higgsino states $\tilde{H}_i$, 

$$g_{\tilde{\chi}Z} \propto (|N_{\tilde{\chi}_1}|^2 - |N_{\tilde{\chi}_2}|^2) \quad (6.1)$$

This difference vanishes for values of $h_1 = h_2$, and increases for increasing values of $h_-$. Considering small differences between the values of $h_1$ and $h_2$, the lightest neutralino, with mass approximately equal to $|\mu|$, is given by

$$\tilde{\chi} \simeq \frac{h_1}{\sqrt{h_1^2 + h_2^2}} \tilde{H}_2 + \frac{h_2}{\sqrt{h_1^2 + h_2^2}} \tilde{H}_1 \quad (6.2)$$

and hence

$$g_{\tilde{\chi}Z} \propto \frac{h_2^2 - h_1^2}{h_1^2 + h_2^2}. \quad (6.3)$$

The annihilation cross section is proportional to the square of $g_{\tilde{\chi}Z}$ and inversely proportional to the square of the difference between the lightest neutralino mass and the resonant
mass value, $M_Z/2$. Therefore, the smaller the coupling $g_{\tilde{\chi}Z}$, the closer the modulus of the parameter $\mu$ should be to the resonant mass value. Hence, in order to get a value of the relic density consistent with the one determined by WMAP, there must be a correlation between the departure of $|\mu|$ from $M_Z/2$ and the difference of the Yukawa couplings of the two Higgsinos to the Wino and Higgs field.

The numerical estimates of the values of $|\mu|$ for a given value of $h_+$ have been obtained by computing the relic density, which is inversely proportional to the thermal average annihilation cross section,

$$\Omega h^2 = \left(\frac{1.07 \times 10^9 \text{ GeV}^{-1}}{M_{Pl}}\right)^{-1} \left(\int_{x_f}^{\infty} dx \frac{\langle \sigma v \rangle (x)}{x^2} g^{1/2}_{*f} \right)^{-1},$$  \hspace{1cm} (6.4)

where $x = m_{\tilde{\chi}}/T$, $m_{\tilde{\chi}}$ is the mass of the lightest neutralino particle and $T$ the temperature of the Universe \cite{58}. The value of the variable $x$ at the freeze-out temperature, $x_f = m_{\tilde{\chi}}/T_f$, is given by the solution to the Eq. \cite{43}

$$x_f = \ln \left[\frac{0.038 (g/g^{1/2}_{*f}) m_{\tilde{\chi}} M_{Pl} \langle \sigma v \rangle (x_f)}{x_f^{1/2}}\right],$$  \hspace{1cm} (6.5)

where $g = 2$ is the number of degrees of freedom of the neutralino, $g_{*f}$ is the total number of relativistic degrees of freedom at temperature $T_f$, and $M_{Pl}$ is the Planck mass. The thermal average of the annihilation cross section may be computed by standard methods and, for a particle of mass $m_{\tilde{\chi}}$, is given by \cite{58}

$$\langle \sigma v \rangle = \int_{4m_{\tilde{\chi}}^2}^{\infty} ds \sqrt{s - 4m_{\tilde{\chi}}^2} W K_1(\sqrt{s}/T) / 16 m_{\tilde{\chi}}^4 T K_2(m_{\tilde{\chi}}/T),$$  \hspace{1cm} (6.6)

where $s$ is the usual Mandelstam parameter, $K_1$ and $K_2$ are modified Bessel functions, and the quantity $W$ is defined to be

$$W = \left[\prod_{f} \frac{d^3 p_f}{(2\pi)^3 2E_f}\right] (2\pi)^4 \delta^{(4)}(p_1 + p_2 - \sum_f p_f) |\mathcal{M}|^2,$$  \hspace{1cm} (6.7)

where $|\mathcal{M}|^2$ is the squared matrix element averaged over initial states, and summed over final states.

In Fig. \cite{10} we plot the required value of $|\mu|$ in order to obtain the central value of the relic density consistent with the recent experimental results $\Omega h^2 \simeq \Omega_{\text{WMAP}} h^2 = 0.11 \pm 0.01$ \cite{59} as a function of $h_+$. The average values of the Yukawa couplings have been fixed to $h_+ = 1.5$ (dashed lines) and $h_+ = 2$ (solid lines). Lower and upper lines
Figure 10: Values of the $\mu$ parameter leading to a value of the neutralino relic density consistent with the central value of the WMAP observation, as a function of $h_-$, for $\mu = -M_2 \exp(i\varphi)$ and $M_2 = |\mu|$, assuming that the average value of the Yukawa couplings $h_+$ is fixed to the value $h_+ = 2$ (solid lines) and $h_+ = 1.5$ (dashed lines). Lower and upper lines for each set of $h_+$ values correspond to $\varphi = 0$ and $\varphi = \pi/2$, respectively.

for each set of $h_+$ values correspond to two different values of the CP-violating phase, $\varphi = 0$ and $\varphi = \pi/2$ respectively. Larger values of $|h_-|$ lead to larger values of $|\mu|$. We show the results only for small values of $|h_-|$, since as we will discuss in the next section, large values of $|h_-|$ lead to unacceptably large corrections to the precision electroweak observables. A non-vanishing phase has only a significant impact on the relic density determination for values of $h_+ \simeq 1.5$ and $|\mu| > 80$ GeV ($|h_-| \gtrsim 0.2$), for which the phase transition is no longer strongly first order and/or unacceptably large corrections to the precision electroweak data are generated. Otherwise, the variation induced by the CP-violating phases is of the order of (or smaller than) the present experimental uncertainty in the relic density. Indeed, a one sigma variation of the relic density results for values of $|\mu| \simeq 80$ GeV (60 GeV) would be obtained by varying $|\mu|$ by about 3 GeV (1.5 GeV).

It is important to compare the results shown in Fig. 10 with those in Fig. 5. For $h_+ < 1.6$ it follows from Fig. 5 that in order to preserve a strongly first order phase transition one needs $|\mu| < 100$ GeV and a Higgs mass smaller than 125 GeV. As seen in Fig. 10 such small values of $|\mu|$ are also consistent with Dark Matter constraints, provided that $|h_-| \lesssim 0.3$. As emphasized before, and as we will show in detail in section 7, the requirement of consistency with the precision electroweak data further constraints the allowed parameter space.

For $h_+ = 2$, the constraints coming from the requirement of a sufficiently strong phase
transition are much weaker. Values of $|\mu|$ and of the Higgs mass as large as 200 GeV are consistent with this constraint. This is a much larger region of values of the parameter $|\mu|$ than the one consistent with Dark Matter constraints, for the small values of $|h_-|$ that are required by precision electroweak data (see section 7). Therefore, also in this case the allowed region of parameters may only be determined once the analysis of the constraints coming from the consistency with precision electroweak data are evaluated.

Finally notice that, for a fixed value of $h_+$ in Fig. 10 the region above (below) the line $\Omega = \Omega_{\text{WMAP}}$ corresponds to $\Omega > \Omega_{\text{WMAP}}$ ($\Omega < \Omega_{\text{WMAP}}$). Therefore while the region above the corresponding curve is excluded since it would predict too much DM density, the region below it is not excluded provided there is another candidate for Dark Matter in the theory.

7 ELECTROWEAK PRECISION MEASUREMENTS

Heavy fermions, with large couplings to the Higgs field, may induce large corrections to the electroweak precision measurement parameters. Extra contributions may come from the stabilizing fields, but in this section we shall assume that the UV completion of the model is such that they are small. A very trivial (ad hoc) way of achieving this is if the stabilizing fields are a set of scalar singlet fields strongly coupled to the Higgs boson. For instance we can consider the case of $N$ scalar (complex) singlets giving a contribution to the scalar effective potential given by

$$V_S = h^2|\vec{S}|^2|H|^2 + \mu_S^2|\vec{S}|^2 + \lambda_S|\vec{S}|^4,$$

(7.1)

with $\mu_S^2, \lambda_S > 0$. This set of singlet fields contributes to the one-loop effective potential of the Higgs field as $g_b = 2N$ bosonic degrees of freedom with a mass squared $m^2(\phi) = \mu_S^2 + h^2\phi^2$ that corresponds to the typical case of stabilizing fields introduced in section 2. As it is obvious this system of stabilizing (singlet) fields would not contribute to the electroweak precision observables or to the CP-violating observables analysed in the previous sections. In the following, we will just concentrate on the contribution to the electroweak precision measurement parameters from the fermionic sector of the theory.

It is well known that if the fermion masses proceed from the usual contraction of a left-handed fermion doublet and a right-handed fermion singlet with the Higgs doublet, and if the mass difference of the fermion components of the doublet field is small, the

---

\footnote{Strictly speaking, for the stabilization of the effective potential, it is not necessary that the bosonic and fermionic number of degrees of freedom, $g_b$ and $g_f$, are equal and/or that the corresponding Yukawa couplings $h_b = h_f$, as we have been assuming in section 2. It is easy to see that a necessary stabilization condition is provided by $g_b h_b^4 \geq g_f h_f^4$.}
contribution of heavy fermions to the $S$-parameter is about $1/6\pi$. On the other hand the contribution to the $T$-parameter would depend on the size of the mass difference between the up and down fermions. Cancellation of anomalies requires the presence of at least two such new heavy fermion doublets and therefore the contribution to the $S$-parameter tends to be large, $S > 0.1$.

In the case under analysis, however, the symmetry breaking masses proceed from the coupling of the Higgsino doublets to the Higgs field and an $SU(2)_L$-triplet of Winos. Contrary to the standard case of heavy fermions, the contribution to the $S$-parameter becomes small in this case. For a given value of the average Yukawa coupling $h_+$, the contribution to the $T$ parameter, instead, becomes sizeable for large values of $h_-$, while the contribution to the parameter $U$ is an order of magnitude smaller than the contribution to $T$.

We shall work in the limit in which $M_1$ is large. Thus the mixing of Binos with the Winos and Higgsinos becomes small and therefore the Binos decouple from the precision measurement analysis. For large values of the Yukawa couplings, the lightest neutralino has a mass close to $|\mu|$ and a coupling to the $Z$-gauge boson given by Eq. (6.3). As explained in section 3 for $h_- = 0$ there are two Dirac charginos degenerate in mass with two Majorana neutralinos and the $T$ parameter contribution vanishes. The mass difference between the neutralinos and charginos grows linearly with $h_-$, as does the coupling of the lightest neutralino to the $Z$-gauge boson, and the $T$ parameter grows quadratically with $h_-$, as shown in Fig. 11, for values of $h_+ = 2$ (solid line) and $h_+ = 1.5$ (dashed line).

Moderate contributions to the $T$-parameter are not in conflict with electroweak precision measurements. Even in the absence of any other physics at the weak scale the corrections to the $T$-parameter coming from the neutralino and chargino sector may be largely compensated by the negative contribution induced by the presence of a heavy Higgs, which contributes to the $S$ and the $T$ parameters in a way proportional to the logarithm of its mass,

$$\Delta S = \frac{1}{12\pi} \log \left( \frac{m_h^2}{m_{h_{\text{ref}}}^2} \right),$$

$$\Delta T = -\frac{3}{16\pi c_W^2} \log \left( \frac{m_h^2}{m_{h_{\text{ref}}}^2} \right),$$

(7.2)

where $m_{h_{\text{ref}}}$ is a reference Higgs mass value.

The model under analysis falls therefore under the class of models which give a small contribution to the $U$ parameter and sizeable contributions to the $T$ parameter. Although the new physics gives only negligible contribution to $S$, a sizeable contribution to the $S$ parameter may also be induced by a heavy Higgs boson. A fit to the precision electroweak
Chargino and neutralino contributions to the $T$-parameter as a function of $h_-$, assuming that the average values of the Yukawa couplings has been fixed to $h_+ = 2$ (solid line) and $h_+ = 1.5$ (dashed line) and considering $M = -\mu$ determined as a function of $h_-$ from Figure 10.

data in this class of models has been done by the LEP electroweak working group [60].

For a reference Higgs mass value of 150 GeV, they find

$$
S = 0.04 \pm 0.10
$$

$$
T = 0.12 \pm 0.10
$$

(7.3)

with an 85% correlation between the two parameters. Taking this into account we show in Fig. 12 the 68% C.L. (solid lines, hatched ellipses) and 95% C.L. (dashed lines) region of allowed values of the new physics contribution to the $S$ and $T$ parameters for values of the Higgs boson mass equal to 115 GeV, 200 GeV and 300 GeV, respectively. Observe that, since we are presenting a fit to the allowed new physics contribution to the $S$ and $T$ parameters, for each value of the Higgs mass the origin of coordinates represents the SM value.

From the results of Figs. 10 11 and 12 we see that for a fixed value of the average Yukawa coupling, $h_+$, the requirement of an acceptable relic density and a good fit to the precision electroweak data implies an interesting correlation between values of $h_-$, the parameter $|\mu|$ and the Higgs mass. The value of the Higgs mass is also bounded
Figure 12: 68 % (solid lines) and 95 % (dashed lines) C.L. allowed values of the $S$ and $T$ parameter based on the fit to the precision electroweak data in Ref. [60], for three different values of the Higgs mass: 115 GeV, 200 GeV and 300 GeV.

from above from the requirement of a successful generation of the baryon asymmetry. For instance, for $h_+ = 2$, Fig. 5 shows an upper bound on the Higgs mass of about 200 GeV and therefore from Fig. 12 it follows that a good fit to the electroweak data may only be obtained for values of the new physics contribution to the parameter $T \lesssim 0.27$ (if the new physics contributions to the parameter $S$ are small, as in the case under analysis). The upper bound on $T$ translates into an upper bound on $h_-$. From Fig. 11 we get that $|h_-| \lesssim 0.18$, and from Fig. 10 this leads to an acceptable relic density only for $|\mu| \lesssim 65$ GeV.

For $h_+ \lesssim 1.6$ the corrections to the precision electroweak parameters are slightly smaller than for $h_+ = 2$. However, as emphasized before, from Fig. 5 we see that consistency with a strong electroweak phase transition can only be obtained for a Higgs mass value close to its present lower bound, $m_h < 125$ GeV, and small values of $|\mu| < 100$ GeV. From Fig. 12 this implies that the new physics contribution to the parameter $T \lesssim 0.15$, and therefore from Fig. 11 one obtains that $|h_-| \lesssim 0.14$. Interestingly enough, from Fig. 10 we
find that the same region of parameters is consistent with the Dark Matter relic density provided $|\mu| < 70$ GeV. Observe that the allowed values of $|\mu|$ approximately coincide with the ones obtained for $h_+ = 2$.

8 LARGE YUKAWA COUPLINGS IN LOW ENERGY SUPER-SYMMETRIC THEORIES

In this work we have analyzed the properties of a model with light charginos and neutralinos which are strongly coupled to the SM Higgs. Such large values of the Yukawa couplings do not arise in minimal supersymmetric extensions of the Standard Model, since these couplings are determined by supersymmetric relations to the weak couplings. It is therefore an important question to understand under which conditions such a low energy effective theory may be obtained.

One possibility is to assume that, although the quantum numbers of the light particles are those of Higgsinos and gauginos of supersymmetric theories, the theory is not related to any supersymmetric theory at high energies. A more interesting possibility would be to consider this theory as a particular realization of split supersymmetry [42], but where the particular relation between the Yukawa couplings and the gauge couplings has been broken by supersymmetry breaking interactions. One of the problems with this alternative is the one of vacuum stability. In addition, due to the strong Yukawa couplings, perturbative consistency is lost at scales much lower than the GUT scale and therefore perturbative unification of the gauge couplings cannot be achieved.

In this section we shall show that this low energy effective theory may also arise in low energy supersymmetric models with extra, strongly coupled gauge sectors, if the scale of supersymmetry breaking is larger than the scale of spontaneous symmetry breakdown of the extended gauge sector to the Standard Model one. A supersymmetric extension provides the necessary stabilizing fields in a natural way. On the other hand in this extension the strong Yukawa couplings are proportional to the strong gauge couplings, which become asymptotically free at high energies. This ultraviolet completion of the theory allows the preservation of perturbative consistency up to high energies and therefore the possibility of perturbative unification of gauge couplings.

In order to illustrate this property let us consider the model analyzed in Ref. [61]. The model is based on the low energy gauge group

$$SU(3)_C \otimes SU(2)_1 \otimes SU(2)_2 \otimes U(1)_Y.$$  \hspace{1cm} (8.1)

First and second generation left-handed quark and leptons transform in the fundamental
representation of $SU(2)_1$, i.e. $(2, 1)$ of $SU(2)_1 \otimes SU(2)_2$, while the third generation left-handed quark and lepton and the two MSSM Higgs bosons transform in the fundamental representation of $SU(2)_2$, $(1, 2)$. The model also includes a bifundamental $(\bar{2}, 2)$ Higgs field $\Sigma$, as well as a singlet field $S$. The scalar component of the bifundamental takes a vacuum expectation value $\langle \Sigma \rangle = u \cdot I$, breaking $SU(2)_1 \otimes SU(2)_2 \rightarrow SU(2)_L$. The gauge bosons

$$W^\mu = \frac{g_2 W_1^\mu - g_1 W_2^\mu}{\sqrt{g_1^2 + g_2^2}}$$

(8.2)

remain massless after this symmetry breakdown and interact with an effective gauge coupling $g_W = g_1 g_2 / \sqrt{g_1^2 + g_2^2}$. This model can be made consistent with gauge coupling unification by embedding the group $SU(3)_c \otimes SU(2)_1 \otimes SU(2)_2 \otimes U(1)_Y$ into the grand unified product group $SU(5) \otimes SU(5)$ broken by bi-fundamental field $(\bar{5}, 5)$ diagonal VEV’s [61, 62].

In the model of Ref. [61] there are extra fields transforming under $SU(2)_1$ but not under $SU(2)_2$. With this particle content the coupling $g_2$ of $SU(2)_2$ is asymptotically free, but $g_1$ of $SU(2)_1$ is not. We shall work under the assumption that the coupling $g_2$ becomes strong at the scale $u$ of spontaneous symmetry breaking of the symmetry $SU(2)_1 \otimes SU(2)_2$, and therefore the Winos of $SU(2)_2$ interact strongly with the Higgs and Higgsinos. For our analysis here, the relevant term in the superpotential is $W = M_{\Sigma} \Sigma \Sigma$, and the relevant supersymmetry breaking terms in the gaugino-Higgsino sector are the masses of the gauginos, $M_i$, corresponding to the two groups $SU(2)_i$. In the absence of supersymmetry breaking terms the superpartners of $W^\mu$ would become the low energy Winos, interacting with Higgsinos and Higgs bosons with the weak coupling $g_W$. In order to get a strongly interacting Wino-Higgsino-Higgs sector at low energies we need to decouple the bifundamental Higgsinos $\tilde{\Sigma}$, as well as the weakly interacting Wino $\tilde{W}_1$ from the weak scale theory. This may be achieved by choosing the supersymmetry breaking parameters $M_i$ and the Higgsino mass parameter $M_{\Sigma}$ to verify

$$M_1, M_{\Sigma} \gg g_2 u, \quad M_2 \ll g_2 u$$

(8.3)

Supersymmetry is therefore broken before the scale of breakdown of $SU(2)_1 \otimes SU(2)_2$ group to $SU(2)_L$. The Wino of $SU(2)_1$ is only weakly coupled with the bidoublet Higgsinos and its large supersymmetry breaking mass ensures its decoupling from the low energy theory.

For the parameters given above the low energy Wino has a mass

$$M_2 \simeq M_2 - \frac{g_2^2 u^2}{M_{\Sigma}}$$

(8.4)
and has a component on the strongly coupled Wino of $SU(2)$ of order $\cos \theta_{\Sigma}$, with

$$\sin \theta_{\Sigma} \simeq g_2 u / M_{\Sigma}.$$  

(8.5)

The effective Yukawa couplings between the low energy Winos and the two Higgsinos are therefore given by

$$h_1 \simeq g_2 \cos \theta_{\Sigma} \cos \beta / \sqrt{2}$$
$$h_2 \simeq g_2 \cos \theta_{\Sigma} \sin \beta / \sqrt{2}$$  

(8.6)

where we have assumed that the CP-odd Higgs mass is larger than the weak scale, and therefore a single, SM-like, CP-even Higgs boson remains in the low-energy theory. Assuming $\alpha_2 = g_2^2 / 4\pi < \sim 1$ we get that, in this realization of the low energy theory, the Yukawa couplings $h_i \lesssim 2$.

On the other hand, a very large supersymmetric mass $M_{\Sigma}$ would also demand to be compensated in a precise way by a similarly large supersymmetry breaking mass in order to get the proper $SU(2)_1 \otimes SU(2)_2$ breaking scale. Lower values of $M_{\Sigma}$ would reduce the strongly coupled gaugino component of the low-energy Wino state and would require a fine-tuning between the two terms in Eq. (8.4) in order to obtain a small value of the effective low-energy Wino mass. Therefore in this model, a moderate amount of fine-tuning is required to get consistency with phenomenological constraints and a strong electroweak phase transition.

We can now ask if the strongly coupled gauge bosons may serve as the stabilizing bosonic fields defined in section 2. Let us first stress that the particular extension of the MSSM presented above leads to extra contributions to the precision electroweak observables. Small values of these extra contributions may only be obtained for values of $g_2 u$ larger than a few TeV [61, 63]. Since $g_2 u$ acts as the bosonic mass $\mu_S$ defined in Eq. (2.14), these particles can only act as stabilizing fields if the Higgs is heavier than the range consistent with a strongly first order phase transition. It would be interesting to investigate possible regions of parameter space in which cancelations between different contributions take place and consistency with data may be achieved for lighter gauge boson masses. Otherwise, additional fields would be necessary to stabilize the Higgs potential.

A very simple possibility, that has already been pointed out in section 7 is that the stabilizing fields are gauge singlets, harmless from the point of view of electroweak precision measurements. These singlets should couple strongly to the Higgs sector, in order to stabilize the fermionic part of the zero-temperature effective potential, but should not contribute strongly to the Higgs quartic coupling since in that case the phase transition

\footnote{We thank D. Morrissey for helpful discussions on this point.}
would become weakly first order. A simple example of a model producing the required effect is given by a singlet superfield $P$ coupled to the Higgs field doublets as well as to a set of $N$ singlet fields $S_i$, with superpotential

$$ W = \lambda_1 \vec{S}^2 P + \lambda_2 P H_1 H_2 + \frac{1}{2} M_S \vec{S}^2 - \frac{1}{2} M_P P^2 + \mu H_1 H_2 $$

(8.7)

where all couplings and the masses $M_S$ and $M_P$ are positive. The absence of a coupling of the singlet fields $S_i$ to the Higgs doublets as well as the appearance of only terms proportional to $\vec{S}^2$ in the superpotential may be understood as a result of the invariance of the theory under a global $O(N)$ symmetry.

The supersymmetric masses of the singlet fields, $M_S$ and $M_P$, are assumed to be much larger than the weak scale, suppressing the mixing of light singlet fermions with the standard gauginos and Higgsinos. We shall also assume that there are supersymmetry breaking effects in the bosonic $\vec{S}$-sector that prevent the possibility of integrating out the superfields $\vec{S}$ from the weak-scale theory and allow the bosonic $\vec{S}$-sector to be the stabilizing fields with mass given by (2.14). Such supersymmetry breaking effects should also ensure the preservation of the modifications to the low-energy Higgs quartic coupling induced by the presence of the superfields $\vec{S}$ in the theory.

Furthermore we will assume that the superfield $P$ can be integrated out supersymmetrically so that for scales below $M_P$ it gives rise to an effective superpotential as

$$ W_{\text{eff}} = \frac{1}{M_P} \left( \lambda_1 \vec{S}^2 + \lambda_2 H_1 H_2 \right)^2 + \frac{1}{2} M_S \vec{S}^2 + \mu H_1 H_2 $$

(8.8)

The above superpotential gives rise to a coupling in the tree-level potential between the $S$ sector and the Higgs sector, $h_b^2 |\vec{S}|^2 |H|^2$, where

$$ h_b^2 \sim \frac{M_S}{M_P} \lambda_2 \lambda_1 \sin 2\beta, $$

(8.9)

to an $H$-quartic coupling as $\frac{1}{4} \Delta \lambda |H|^4$, where

$$ \Delta \lambda \sim \frac{\mu}{M_P} \lambda_2^2 \sin 2\beta, $$

(8.10)

and to a self-interacting quartic coupling $\lambda_S |\vec{S}|^4$ fields

$$ \lambda_S \sim \frac{M_S}{M_P} \lambda_1^2. $$

(8.11)

Since we have concentrated in this work on the results obtained in the model with strongly interacting gauginos and Higgsinos for small values of $h_\gamma$, the value of $\tan \beta \approx 1$, and therefore $\sin 2\beta \approx 1$. For $M_S \sim M_P$ and $\lambda_2 \lambda_1 \gtrsim 1$, one obtains values $h_b^2 \gtrsim 1$,
necessary for the singlets \( S_i \) to stabilize the one-loop Higgs potential against the fermion contribution. On the other hand, for \( \mu \ll M_P \), as required to obtain a good description of the dark matter relic density and precision electroweak observables, the contribution to the quartic coupling is not too large. This has implications for the Higgs boson mass. In fact, for values of \( \tan \beta \simeq 1 \), the D-term contribution to the Higgs mass is small and the Higgs mass may be raised above (or around) the MSSM values, due to the tree-level contribution

\[
m_H^2 \simeq 2\Delta \lambda v^2 + \text{(loop - effects)} + \text{(D - term)}. \tag{8.12}
\]

Values of the Higgs mass of order 115–200 GeV may be naturally obtained for \( \lambda_2 \mu / M_P \simeq 0.1–0.3 \), that appear in this model for values of \( M_P \) of the order of the TeV scale and values of \( \lambda_2 \) somewhat larger than one. By choosing the appropriate value of \( N \) and the values of the couplings \( \lambda_i \), one can reach a situation similar to the one described in the previous sections. A detailed analysis of the parameter space in this particular model is outside the scope of the present paper.

### 9 Conclusions

In this article we have shown that heavy fermions with strong couplings to the Higgs fields may induce a strengthening of the electroweak phase transition and can also provide the proper CP-violating sources for the generation of baryogenesis. These heavy fermions, however, also induce for light Higgs bosons an instability of the Higgs potential at zero temperature and therefore require an ultraviolet completion of the theory to recover the consistency of the low-energy theory. In this work, we have assumed that the heavier, stabilizing fields have opposite statistics but similar couplings and number of degrees of freedom as the fermion fields. The above properties are then associated with the low energy theory consisting both of the heavy fermions and the heavier, stabilizing fields.

We have illustrated this possibility by considering a model with TeV scale Higgsinos and gauginos that may lead to a sufficiently strong first order electroweak phase transition for values of the Higgs mass as large as 300 GeV. This is quite different from the results of the MSSM, in which a light stop is necessary, and the Higgs mass should be lower than \( \sim 120 \) GeV to enhance the strength of the electroweak phase transition. Also at variance with the case of the MSSM is the fact that in this scenario the particles that induce a strong first order phase transition are the same ones responsible for the generation of the baryon asymmetry at the weak scale.

This model preserves most of the properties of low energy supersymmetry, including a good Dark Matter candidate. Beyond the problem of vacuum stability, however, the
low-energy Yukawa couplings deviate from the ones obtained in minimal supersymmetric extensions of the SM, and their strength spoils the perturbative consistency of the theory at scales of about a few TeV. We have shown, however, that the model may be considered as the low energy effective description of a gauge extended supersymmetric standard model, in which the strength of the Yukawa couplings is related to the gauge couplings of an extended asymptotically free gauge sector, that becomes strongly interacting at TeV scales. This ultraviolet completion of the model solves the strong coupling problem and introduces new heavy particles that, due to their supersymmetric relations to the Higgsinos and gauginos, tend to stabilize the Higgs potential in a natural way. The stability of the potential, however, may only be achieved for values of the heavy particle masses smaller than about 1 TeV. These relatively small values of the heavy gauge boson masses lead to large corrections to the precision electroweak observables, unless cancellations between different contributions occur. Alternatively, we also have shown that the model can be completed with singlets that may stabilize the zero-temperature effective potential while providing Higgs masses consistent with present experimental bounds and with the required strongly first order phase transition. Finally, we have shown that this model may be tested by electron electric dipole moment experiments in the near future.

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