Emergent Spin-Orbit Torques in Two-Dimensional Material/Ferromagnet Interfaces

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Heterostructures of two-dimensional (2D) materials provide a testbed for interface-induced magnetic phenomena owing to their hybridized electronic structure with a strong interplay between spin and lattice-pseudospin degrees of freedom. In this work, we present a general microscopic theory of spin-orbit torque (SOT) in heterostructures of 2D monolayers proximity coupled to ferromagnets. A number of unconventional and measurable interfacial effects are predicted, the most remarkable of which is a giant enhancement of antidamping SOT induced by a robust skew scattering mechanism, which is operative in realistic 2D materials. Our findings highlight the rich behavior of magnetized 2D Dirac fermions with multiple spin-like SU(2) degrees of freedom in atomically thin materials and suggest novel approaches to deterministic switching of magnetic memory devices.

When a current is driven through a surface with broken inversion symmetry, a nonequilibrium spin polarization is induced due the spin-orbital-entangled character of electronic wavefunctions. If coupled to a ferromagnetic system, the emergent spin polarization transfers angular momentum to local spin moments, changing their state by exerting a torque \( T \propto m \times S \) [1]. This allows for the efficient control of magnetization dynamics without external magnetic fields, which are cumbersome to employ at the nanoscale. For this reason, emergent spin-orbit-coupling (SOC) phenomena provides a fertile arena for the development of all-electrical spin memories [2–4].

Current-induced SOTs are conventionally classified into two broad categories depending on their behavior under time-reversal \( \mathcal{T} \): the \( m \)-odd or "field-like" SOT that affects the precession around the effective magnetic field and the \( m \)-even or "antidamping" torque that renormalizes the Gilbert damping and is responsible for the magnetization switching [5–7]. Thinning down heterointerfaces and devices by utilizing van der Waals (vdW) crystals opens up intriguing possibilities. Fueled by the discovery of ferromagnetism in 2D materials, recent works have reported strong out-of-plane antidamping torques, which are relevant for high-density magnetic memory applications.

Model.—The low-energy behavior of magnetized vdW heterostructures made of typical monolayer compounds, such as graphene and TMDs (see Figs. 1(a)-(b)), is governed by the following generalized Dirac-Rashba model, where \( \xi = \pm \) signs refer to valleys \( K(+\) and \( K'(-\),

\[
\mathcal{H}_\xi = \int dx \psi_\xi^\dagger \left[ v \Sigma \cdot (-i \nabla) + \mathcal{A}_\xi \right] \xi \Delta \Sigma_z + \mathcal{A}^\xi_0 - \epsilon \psi_\xi,
\]

(1)

\( (\psi_\xi, \psi_\xi^\dagger) \equiv (\psi_\xi(x), \psi_\xi^\dagger(x)) \) are 4-component spinor fields defined on the internal spaces of sublattices \( (\Sigma^\nu) \) and spin \( (``s^\nu``) \), \( v \approx 10^6 \) m/s is the bare Fermi velocity of 2D Dirac fermions and \( \epsilon \) is the Fermi energy [17–22]. The gauge-field components \( \mathcal{A}^\xi_\mu (\mu = 0, x, y, z) \) in the Hamiltonian (1) are 2 \times 2 matrices of the form \( \mathcal{A}^\xi_\mu = \sum_{a=x,y,z} \mathcal{A}^\xi_{\mathcal{A}^\xi a} s_a \), which account for all possible spin-dependent effects [23]. The Pauli matrices \( \Sigma^\nu \) and \( s^a (a = x, y, z) \) all anticommute with \( \mathcal{T} \), so that their products are invariant under time-reversal (which also interchanges valleys \( \xi \leftrightarrow -\xi \)). The staggered on-site potential \( (\Delta) \) describes orbital-gap opening due to broken sublattice symmetry [24, 25]. The ubiquitous interfacial Bychkov-Rashba (BR) effect, with coupling strength \( \lambda \), is captured by the gauge-field components \( \mathcal{A}^\xi_{xy} = -\mathcal{A}^\xi_{yx} = \lambda / v \) [26, 27]. Other spin-orbit effects include intrinsic spin-orbit-coupling (SOC) of McClure-Yafet-Kane-Mele type \( (\mathcal{A}^\xi = \lambda_0 s_z) \) and spin-
In Fig. 1(b) exists only in the anisotropic case ([33]). At high electronic density, the two spin-split bands with counterrotating spin textures are occupied. The spin texture of spin-majority states (colored blue in Fig. 1(b)), as well as the current-induced distortion of the Fermi surface, are illustrated in Fig. 1(d). The out-of-plane component of the spin texture can be triggered by an exchange field, spin-valley coupling or competition between BR and orbital effects.

Global picture.—We first determine the nonzero SOTs and underlying mechanisms using the semiclassical transport equations, then we derive a general microscopic picture and discuss its consequences. Here we restrict the derivation to $C_{6v}$-invariant models, which already display the essential phenomenology. A general symmetry-based analysis of the spin-charge response function is given in the Supplemental Material (SM). The first step is to determine the spin texture at the Fermi energy. Perturbation theory in the anisotropy parameter yields, after a long but straightforward calculation, $\delta S_\text{K} = (\gamma|| + \mu||) \mathbf{m} \cdot \hat{k} + 2\mu\parallel \hat{k} \times (\hat{k} \times \mathbf{m}) + m_{\mu\perp} \hat{k} \times \mathbf{m}$ the correction induced by $m_{\mu\perp}$, or, in a more intuitive form conduction band (CB)

$$\delta S_\text{K} = m_x (\gamma|| \hat{x} + \mu|| (\cos 2\theta \hat{x} + \sin 2\theta \hat{y}) - m_{\mu\perp} \sin \theta \hat{z})$$

with $\theta$ the wavevector angle. In these expressions, all the coefficients $\{\gamma||, \gamma\perp, \mu||, \mu\perp\}$ are functions of $k = |\mathbf{k}|$, $\lambda$ and $m_x^2$ (see SM). The spin-helical component (in $s^{||}_k$) yields the well-known inverse spin-galvanic effect ($S \propto$...
self-consistently, we extend the controlled diagrammatic semiclassical effects and disorder corrections (impurity scattering) mechanisms are summarized in Fig. 1(d). This generates a field-like SOT that is sensitive to the direction of the applied current, $T_{\alpha2} \propto m \times (m \times \hat{z}) (m \cdot J)$. This newly unveiled effect, which can be traced back to the unique Dirac-Rashba character of electronic states, still occurs when the two spin-slit bands (with opposite-in-sign $\hat{z}$-polarizations) are populated. This avoided cancellation of nonequilibrium out-of-plane spin polarization stems from the interplay between pseudospin and spin angular momentum (see SM), which renders contributions from spin-split bands inequivalent. This differs from 2D electron gases, for which the only robust SOT is $T_{\alpha1}$.

To explain the emergence of robust antidamping SOTs, we add the effect of a nonzero transverse scattering to the picture. Semiclassically, the nonequilibrium spin polarization is obtained as $S = \sum_{k\nu} \delta f_{k\nu} \propto \tau_{||}^* \hat{k}$. $E + \tau_{||}^* (\hat{k} \times E)$, is the deviation of the distribution function away from equilibrium [39]. Consider an electric field along $\hat{x}$. The Fermi surface is shifted perpendicular to the applied current by an amount $\delta f_{k\nu} \propto (\tau_{||}^* \sin \theta) E_x$. This results in an extrinsic anomalous Hall effect [33], but it also provides an efficient mechanism for current-induced collinear spin polarization $S_x$. Skew scattering plays an essential role as there must be an imbalance between scattering cross sections at angles $\pm \theta$ relative to $E$, otherwise all the states in the Fermi surface will have their $S_x$ component canceled by states with opposite angle. This mechanism is operative under rather general conditions because the spin-orbit-coupled carriers experience an average out-of-plane Zeeman field $\hat{z}$ ($s_{k\nu})_{FS} \neq 0$ that breaks the left/right symmetry of scattering events. After performing the angular integration accounting for a finite $\tau_{||}^*$, we easily find the magneto-electric effect: $S \propto m_z \varphi_z \cdot E$. The generation of collinear nonequilibrium spin polarization can be extremely efficient due to its inherent semiclassical scaling $\tau_{||} \propto \tau_{||} \propto n^{-1}$ (where $n$ is the impurity density) [40]. This phenomenon, which we term \textit{collinear Edelstein effect}, contributes with an antidamping SOT $T_{\alpha2} \propto m \times (m \times \hat{z} \times J)$. From Eq. (2), one can easily conclude that the skewness also activates an out-of-plane spin response, $S^z \propto \tau_{||} m_x m_y E_x \propto m_x m_y^2 E_x$. This yields an anisotropic antidamping torque $T_{\alpha2} \propto m \times \hat{z} (m \cdot J)$. These novel SOTs, which scale favorably with the conductivity $\sigma \propto \epsilon \tau_{||} / \hbar \gg 1$, are our central result. The semiclassical mechanisms are summarized in Fig. 1(d).

\textit{Nonperturbative microscopic approach}.—To derive an accurate microscopic theory of SOT that includes intrinsic effects and disorder corrections (impurity scattering) self-consistently, we extend the controlled diagrammatic technique developed in Refs. [19–22] to arbitrary multiband models. Our approach has two essential features. First, it is fully nonperturbative in the energy scales of the bare Hamiltonian, which includes orbital mass $\Delta$, exchange field vector, BR interaction $\lambda$ and other couplings. This technique allows us to explore rich scenarios, including the experimentally relevant regime of proximized materials with competing energy scales e.g., $\lambda \approx \Delta_{xc} \approx \epsilon$. Simple analytical expressions can be obtained to leading order in $m_x$ by developing the Green’s functions in Dyson series [7]. Second, the three-leg spin-charge correlation or vertex function $\Gamma_{\sigma \alpha \sigma}(x, y, z) = (T J_\ell (x) \Psi_\sigma (y) \Psi_\sigma^\dagger (z))$ is evaluated by re-summation of all single-impurity Feynman diagrams, which provides the dominant contribution to the spin-charge response functions in the dilute impurity regime. This is accomplished by writing a Bethe-Salpeter equation with $T$-matrix insertions [19], which is more general and accurate than the standard approach based on ladder diagrams (Figs. 2(a)-(b)). This allows us to obtain virtually exact results and explore the crossover between the standard weak Gaussian limit and the important unitary scattering regime, which physically corresponds to resonant scattering from vacancies or adatoms [41, 42].

We are interested in SOTs generated by weakly disordered 2D electron gases and thus focus our subsequent analysis on Fermi surface processes. The latter are captured by the spin density–charge current response function $\langle G^+ J_i G^- \rangle$.

\[
K_{ai} = \frac{1}{2\pi} \text{Tr} \left[ s_\alpha \langle G^+ J_i G^- \rangle \right],
\]

with $G^\pm$ the retarded(+)advanced(-) Green’s function, $J_i = -e \partial_i H_p = -e v_{\sigma i} J_i$ the charge current operator and $\text{Tr}$ the trace over all degrees of freedom. Here the angular brackets denote disorder averaging and $H_p$ is the extension of the single-particle Hamiltonian to the space of two valleys. Since our aim is to develop a generic SOT theory, which does not rely on the existence of spinful scattering centers of a particular nature, such as spin-orbit-active impurities [44–48], we assume a standard scalar short-range potential $V(x) = u_0 \sum_i \delta(x - x_i)$, where $x_i$ are random impurity locations and $u_0$ parametrizes the potential scattering strength. Leading terms $K_{ai} \propto 1/n$ in
the dilute impurity regime are obtained by replacing in Eq. (3) \((G^+J,G^-) → G^{+}_p J G^{-}_p\), where \(G^{±}_p\) is the disorder-averaged Green’s function and \(\tilde{J}_i\) is the renormalized vertex (Figs. 2(a)-(b)). The final trace in Eq. (3) is carried out using an exact SO(5) decomposition of the response function; technical details are given in the SM.

Armed with this formalism, we evaluate the SOTs and determine their efficiency. Within linear response theory, we write \(\mathbf{T} = d^{-1} \mathbf{m} \times \mathbf{H}_\tau\), where \(d\) is the FM thin film thickness and \(\mathbf{H}_\tau = -\Delta_{xc} K^J \mathbf{J}\) is the current-induced spin-orbit field and \(K^J \equiv K \cdot \hat{\sigma}^{-1}\), with \(\hat{\sigma}\) is the conductivity tensor, is a 3 × 2 matrix that quantifies the underlying SOC transport effects. The earlier semiclassical picture suggests the decomposition (to leading order in \(m_z\))

\[
K^J = \begin{pmatrix}
  m_z \kappa_{zz}^{ss} & \kappa_E \\
  -\kappa_E & m_z \kappa_{zz}^{ss}
\end{pmatrix},
\]

where the superscript \(ss\) marks the responses activated by skew scattering. The Fermi energy dependence of \(K^J\) for a graphene heterostructure is shown in Fig. 1(e). The highly efficient Edelstein-type response \((\kappa_E \sim 0.4\) for \(\epsilon \sim 0.1\) eV) is reminiscent of topological surface states and nonmagnetic graphene/TMD bilayers [22, 38]. This process is accompanied by the generation of robust out-of-plane spin polarization (Fig. 1(e)). This is at variance with 2D electron gases in Rashba ferromagnets, for which \(K^J_{zy} \rightarrow 0\) in the weak scattering limit [7]. Concurrently, the newly unveiled skew scattering mechanism, which is operative in all systems with \(m_z \neq 0\), enriches the class of SOTs to include \(T\)-odd (\(m\)-even) terms. Despite the moderate scattering potential strength, a collinear Edelstein response is induced \(K^J_{ii}(i=x,y)\); see Fig. 1(e). The total spin-orbit field comprises Rashbatype \(\mathbf{H}_\tau^0 \propto \mathbf{m} \times \mathbf{J} \times \hat{z}\) and out-of-plane \(\mathbf{H}_\tau^2 \propto \hat{z} \cdot \mathbf{J}\) antidamping contributions. Owing to its skew-scattering origin, \(\mathbf{H}_\tau = \mathbf{H}_\tau^0 + \mathbf{H}_\tau^2\) scales linearly with the Drude conductivity with an efficiency \(K^J_{ii} \sim (\sigma_0)^0 \sim n^0\). This behavior is notoriously different from predicted \(T\)-odd torques for topological insulators, whose quantum-side-jump origin [20] yields \(K^J_{ii} \sim 1/\sigma_0\) in the clean limit [38].

An unprecedented sensitivity of the SOT efficiency to the potential scattering strength is borne out by our theory. In contrast to the Edelstein efficiency \((\kappa_E)\), which receives slow (logarithmic) disorder corrections [22, 49], all damping-like efficiencies exhibit a monotonic increase with \(u_0\). This important feature is illustrated in Fig. 3(a), where a ten-fold increase in both \(K^J_{zz}\) and \(K^J_{xy}\), approaching the unitary regime of a resonant scatterer \((u_0 \rightarrow \infty)\) can be observed. In the weak scattering regime \((u_0 \rho \ll 1)\) with \(\epsilon \gg \{\lambda, \Delta_{xc}\}\), where \(\rho\) is the clean density of states, the coefficients in the \(m\)-expansion of the current-induced torque \((t_{e(o)l} \equiv \Delta_{xc} \tau_{e(o)l})\) admit a compact analytic form (to leading order in \(m_z\))

\[
t_{e1} \approx 2\lambda^3/\epsilon f_c, \quad t_{o2} \approx 2\Delta_{xc}^2/\epsilon f_c, \quad t_{e2} \approx -u_0 \Delta_{xc} \lambda^5/(\epsilon f_c^2), \quad t_{o1} \approx -t_{e1},
\]

where \(f_c = v/(\lambda^2 + \Delta_{xc}^2 m_z^2)\). For interpreting these results, it is important to note that the BR coupling should be not too small compared to \(k_B T\) so that the torques are appreciable in realistic conditions. Notably, the slow algebraic decay \(\propto \epsilon^{-1}\) in Eqs. (5)-(6) effectively quenches the effect of thermal fluctuations [22], which in principle allows room-temperature SOT operation even in samples with weak BR effect \(\lambda \approx 1\) meV. Recent observations of gate-tunable and reversible spin galvanic effect in graphene/TMD bilayers at room temperature [50-52] provide extra confidence that the interfacial SOTs unveiled here can be demonstrated experimentally.

**TMDs and related heterostructures**—Next, we consider models with broken sublattice symmetry \((C_{6v} \rightarrow C_{3v})\); examples are shown in Fig. 1(a). As a case study, we focus here on semiconducting TMD/FM, for which interfacial magnetic exchange coupling can be up to 100 times greater than in graphene [53-56]. The presence of an orbital-gap in TMDs \((E_g = 2\Delta)\) modifies the \(k\)-space spin texture dramatically. We find that the “orbital mass” \((\Delta > \lambda)\) stabilizes a giant equilibrium out-of-plane spin polarization around \(K\) points even in the absence of spin-valley coupling [57]. To determine the antidamping spin-orbit field \(\mathbf{H}_\tau^0\), we evaluate the spin-charge correlation vertex of the full model [Eq. (1)]. Figure 3b shows the SOT evolution with the orbital gap. Its most salient feature is a strong enhancement of out-of-plane antidamping efficiency i.e. \(K_{zz}\). This phenomenon
is accompanied by anisotropic collinear Edelstein effect, \( \partial_\lambda [K_{xx} - K_{yy}] > 0 \), as indicated by the colored arrows. The total antidamping SOT is highly anisotropic, \( \delta \tau_{12} \equiv (|\tau e_2| - |\tau e_1|)/|\tau e_1| \approx 0.5 \), in contrast to the graphene \((C_{Bi})\) case [Eq. (6)]. For increasing in-plane magnetization \( m_{e} \), the SOT efficiency tensor \( K_i \) acquires multiple higher-order harmonics, which invalidates a simplistic analysis in terms of low-order torques \( t_{e1(2)} \) and \( t_{e1(2)} \); a full nonperturbative treatment as employed here becomes indispensable. The onset of the strong nonperturbative regime \((\phi_{in} \gtrsim \pi/8)\) is shown in the inset.

Several extensions to our work are possible. The novel noncoplanar inverse spin galvanic effect and resulting antidamping SOTs unveiled here result from a skew scattering mechanism that is operative in high-symmetry environments. This suggests that breaking the hexagonal symmetry (e.g. with straining) could unlock a yet richer picture, including other types of unconventional SOT as recently observed in noncentrosymmetric TMDs [12–14].

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Here, it is fundamental to keep the full matrix structure of $\Sigma^\pm$ in order to obtain physically sensible results that comply with exact symmetry relations of the four-point vertex function, known as Ward identities.

When skew scattering diagrams are considered, the one-to-one mapping between $K_{ia}$ and $\gamma_a$ breaks down because the $\Upsilon$ matrix mixes different renormalized vertex components.
SUPPLEMENTARY INFORMATION

I. ELECTRONIC STRUCTURE: ENERGY REGIMES AND PSEUDOSPIN-SPIN TEXTURE

The low-energy Hamiltonian of $C_{3v}$-invariant vdW monolayers reads as

$$H\xi = \sigma\mu (v p + A_{\text{SO}} + A_{\text{xc}} + A_{\text{orb}})^\mu,$$

where $p^\mu \equiv (-\epsilon/v, p_x, p_y)$ is the 3-momentum, $\sigma^\mu = (\sigma_0, \vec{\sigma})$ and

$$A_{\text{SO}} = \lambda (s_y \hat{x} - s_x \hat{y}) + \lambda_{\text{KM}} s_z \hat{z} + \xi s_\nu s_z \hat{t},$$

$$A_{\text{xc}} + A_{\text{orb}} = (m \cdot s) \hat{t} + \xi \Delta \hat{z},$$

are non-Abelian gauge fields capturing all symmetry-allowed SOCs [Eq. (8)], on-site staggered potential and interfacial exchange coupling [Eq. (9)]. Here, $m = -\Delta_{\text{xc}} \equiv m(\sin \theta_m \hat{x} + \cos \theta_m \hat{z})$ with $\pi \geq \theta_m \geq 0$ parametrizes the exchange field. For brevity, in this supplementary information, the analytical expressions are provided for the strong SOC regime with $\lambda > m_z \gg m_x$ and $\lambda_{\text{KM}} = 0$, where $m_z \equiv m_z + \xi s_\nu$.

Figure 4 shows the low-energy spectrum for two representative systems: (a) TMD/graphene/FM and (b) TMD/FM. In panel (a) only low-energy states (within the TMD gap) are shown. In-plane magnetization ($m_x$) breaks the $C_{\text{voc}}$ symmetry of the continuum model, rending the Fermi surface anisotropic.

The electronic structure comprises four distinct spectral regions:

- **Regime Ia**: Low energy regime where the Fermi level crosses an electron/hole pocket for
  $$\epsilon_{\text{Ia}} < |\epsilon| < \epsilon_{\text{Ib}};$$

- **Regime Ib**: Very narrow energy range where the Fermi level crosses two different Fermi rings both belonging to the spin majority band. This happens for
  $$\epsilon_{\text{Ib}} < |\epsilon| < \epsilon_{\text{Ic}};$$

Figure 4. Electronic structure of a TMD|graphene|FM (a) and TMD|FM (b) heterostructure plotted along $p = (0, p_y)$ (full line) and $p = (p_x, 0)$ (dashed lines) directions. Shaded areas highlight the spectral region II. Parameters: for (a-b) $\lambda = 40$ meV, $m_x = 20$ meV, $m_z = 30$ meV and for (b) $\Delta = 0.4$ eV, $\lambda_{\text{ov}}(\epsilon < 0) = 150$ meV and $\lambda_{\text{ov}}(\epsilon > 0) = 5$ meV.
\[ \Delta = 0 \quad \sqrt{m_z^2 + 4\lambda^2} \]
\[ \Delta \neq 0 \quad \tilde{m}_z + \Delta \quad \sqrt{(\Delta - \tilde{m}_z)^2 + 4\lambda^2} \]

\[ \lambda \tilde{m}_z \sqrt{m_z^2 + \lambda^2} + \frac{\lambda m_x \tilde{m}_z \sqrt{2\lambda^2 + m_z^2}}{(\lambda^2 + m_z^2)^{3/2}} \]
\[ \lambda \tilde{m}_z \sqrt{m_z^2 + \lambda^2} - \frac{\lambda m_x \tilde{m}_z \sqrt{2\lambda^2 + m_z^2}}{(\lambda^2 + m_z^2)^{3/2}} \]

Table I. Spectral regimes of the \( C_{3v} \) model. To ease the notation, all couplings are taken to be positive.

- **Regime Ic**: Intermediate regime where the Fermi level crosses only the spin majority band, hinting at stronger spin density responses for
  \[ \epsilon_{lc} < |\epsilon| < \epsilon_{II}; \]

- **Regime II**: Typical high-electronic density regime in the experiments. Here, we have \( \epsilon > \epsilon_{II} \) and the Fermi level crosses two Fermi rings with opposite spin textures;

The expressions for the different limits are given in Table I.

In regime II, the equilibrium spin operators have the following average values at the Fermi energy, in the asymptotic limit \( \epsilon \gg m_z \equiv \tilde{m}_z > m_x \) (with \( \theta \) the wavevector angle with respect to \( \hat{x} \) axis):

\[ \langle s_x \rangle \approx \frac{\lambda}{\sqrt{\lambda^2 + m_z^2}} \left( 1 - \frac{(\Delta m_z + \lambda^2)^2}{2\epsilon^2 (\lambda^2 + m_z^2)} \right) \sin \theta + \frac{m_x}{2\sqrt{\lambda^2 + m_z^2}} \left( 1 + \frac{m_x^2 + \lambda^2 \cos 2\theta}{\lambda^2 + m_z^2} \right), \]

\[ \langle s_y \rangle \approx -\frac{\lambda}{\sqrt{\lambda^2 + m_z^2}} \left( 1 - \frac{(\Delta m_z + \lambda^2)^2}{2\epsilon^2 (\lambda^2 + m_z^2)} \right) \cos \theta + \frac{m_x}{2\sqrt{\lambda^2 + m_z^2}} \frac{\lambda^2 \sin 2\theta}{\lambda^2 + m_z^2}, \]

\[ \langle s_z \rangle \approx \frac{m_z}{\sqrt{\lambda^2 + m_z^2}} \left( 1 + \frac{\lambda^2 \Delta^2 - m_z^2}{2\epsilon^2 \lambda^2 + m_z^2} - \frac{m_x \lambda}{\lambda^2 + m_z^2} \sin \theta \right) + \frac{\lambda^4}{\epsilon^2} \left( 1 - \frac{\Delta - m_z}{\lambda^2 + m_z^2} \right)^{3/2}, \]

for the spin majority band, the other band has opposite polarity. The pseudospin texture, on the other hand, is:

\[ \langle \sigma_x \rangle \approx \left( 1 - \frac{\lambda^2 + \Delta^2}{2\epsilon^2} \right) \cos \theta \pm \frac{\lambda m_x}{2\epsilon} \frac{\sin 2\theta}{\sqrt{\lambda^2 + m_z^2}}, \]

\[ \langle \sigma_y \rangle \approx \left( 1 - \frac{\lambda^2 + \Delta^2}{2\epsilon^2} \right) \sin \theta \pm \frac{m_x}{2\epsilon} \frac{1 + \cos 2\theta}{\sqrt{\lambda^2 + m_z^2}}, \]

\[ \langle \sigma_z \rangle \approx \frac{\Delta}{\epsilon} \mp \frac{m_z}{\sqrt{\lambda^2 + m_z^2}} \frac{\Delta m_z + \lambda^2}{\epsilon^2} \left( 1 - \frac{\lambda m_x}{\lambda^2 + m_z^2} \sin \theta \right), \]

where the ± refers to the spin majority and minority bands, respectively. Figure (5) shows the spin and pseudospin profiles along \( de \hat{x} \) and \( \hat{y} \) directions in momentum space. The orbital mass \( (\Delta) \) broadens up the \( p \)-space spin texture dramatically, which boosts the generation of out-of-plane spin polarization in applied current.

II. FERMI-SURFACE \( \kappa \)-RESPONSE: SYMMETRIES

The Fermi surface response function are determined from the zero temperature limit of the term I in the Kubo-Streda formula, namely

\[ K_{ai} = \frac{1}{2\pi} \int (dp) \text{Tr} \left\{ s_a G_p^+ \hat{j}_p G_p^- \right\}. \]
Figure 5. Emergent pseudospin-spin texture of spin-majority band for a 2D material|FM interface in the presence (thick lines) and absence (thin lines) of orbital gap. Spin-pseudospin textures along \( \hat{k}_x \)-direction for spin. (c-d) Textures along \( \hat{k}_y \)-direction.

Band structure parameters: \( \lambda = 20 \) meV, \( \Delta = 400 \) meV, \( \Delta_{xc} = 15 \) meV, \( \lambda_{sv} = 0 \) with \( \theta_m = \pi/8 \).

where \( \tilde{J}_i \) is the renormalized current density vertex and \( (d\mathbf{p}) \equiv d\mathbf{p}/(2\pi)^2 \).

To determine the parity of \( K_{ai} \) with respect to the field reversal \( m \to -m \), it suffices to consider the “empty” bubble (first bubble in Fig. 6). The disorder-averaged Green’s functions satisfy the following symmetry relations

\[
\begin{align*}
    s_x \sigma_y G^a(-p_x, p_y) \sigma_y s_x &= G^a(p_x, p_y) \big|_{m_z \to -m_z, \lambda_{sv} \to -\lambda_{sv}, \Delta \to -\Delta} \equiv S_1, \\
    s_y \sigma_x G^a(p_x, -p_y) \sigma_x &= G^a(p_x, p_y) \big|_{m_z \to -m_z, m_x \to -m_x, \lambda_{sv} \to -\lambda_{sv}, \Delta \to -\Delta} \equiv S_2, \\
    s_z \sigma_z G^a(-p_x, -p_y) s_z &= G^a(p_x, p_y) \big|_{m_x \to -m_x} \equiv S_3,
\end{align*}
\]

Using these symmetries in (19), we find

\[
\begin{align*}
    K_{xx(yy)} &= -K_{xx(yy)} \big|_{S_1}, \\
    K_{xy(yx)} &= +K_{xy(yx)} \big|_{S_2}, \\
    K_{zx} &= +K_{zx} \big|_{S_3}, \\
    K_{zx} &= -K_{zx} \big|_{S_3}, \\
    K_{xy} &= -K_{xy} \big|_{S_1}, \\
    K_{xy} &= +K_{xy} \big|_{S_2}, \\
    K_{xy} &= -K_{xy} \big|_{S_3}.
\end{align*}
\]

These relations imply the following general structure of the response function (sum over repeated indices is implied)

\[
\{K_{ai}\} = \begin{pmatrix}
    m_x \kappa_{xx} + m_\sigma^2 \tilde{\alpha}_x \cdot \tilde{f}_{zx} + \tilde{\alpha}_x \cdot \tilde{g} & \kappa_{xy} + m_x \tilde{\alpha}_y \cdot \tilde{f}_{yx} + m_\sigma^2 h_{xy}^a \\
    -\kappa_{xy} - m_\sigma \tilde{\alpha}_x \cdot \tilde{f}_{zy} + m_\sigma^2 h_{yx}^a & m_x \kappa_{xx} + \tilde{\alpha}_x \cdot \tilde{g} + m_\sigma^2 \tilde{\alpha}_y \cdot \tilde{f}_{yy}^a
\end{pmatrix}.
\]
where $\vec{\alpha} = \xi(\lambda_{sv}, \Delta)$. The coefficients $\{\kappa_{ia}, f_{ij}, f_{ij}^a, g, h_{ij}^a, z_i\}$ are even functions of $m_x$ and $m_z$. The terms linear in $\vec{\alpha}$ are activated by the breaking of sublattice symmetry, vanishing upon the summation over the two valleys.

III. $T$-MATRIX LADDER RESUMMATION

Local operators admit the SO(5) Clifford decomposition $O = \sum_\alpha O_\alpha \gamma_\alpha$ where $\gamma_\alpha \equiv \sigma_i \otimes s_j$ are Dirac matrices defined on pseudospin-spin space and we have agglutinated the $i$ and $j$ indices into $\alpha = 0, \ldots, 16$. Thus, the renormalized currents $\tilde{J}_x = -ev\tilde{\gamma}_\beta$ and $\tilde{J}_y = -ev\tilde{\gamma}_\beta$ (Fig. 6(a)) can be obtained from the generalised Bethe-Salpeter equation

$$\tilde{\gamma}_\beta = \gamma_\beta + n \int (dp) \{ T^+ G_p^+ \tilde{\gamma}_\beta G_p^- T^- \}$$

with

$$T^a = (u_0^{-1} - g_0^a)^{-1}, \quad G_p^a = (\epsilon - \mathcal{H}_p - nT^a)^{-1},$$

and where

$$g_0^\pm = \int (dp)(\epsilon - \mathcal{H}_p \pm \imath \eta)^{-1}$$

is the integrated clean Green’s function (LDOS). The $T$-matrix insertions in the ladder [Eq. (28)] generate the entire set of noncrossing two-particle diagrams (Fig. 6; see also Fig. 2 main text). To compute the full response function

$$\mathcal{X}_{\alpha \beta} \equiv \int (dk) \text{Tr} \left\{ \gamma_\alpha G_p^+ \tilde{\gamma}_\beta G_p^- \right\},$$

we use the Clifford decomposition of the T-matrix ladder to write

$$\mathcal{X}_{\alpha \beta} = \frac{2}{\pi} \left[ \Upsilon^{-1} \left( (1 - \Upsilon \mathcal{N})^{-1} - 1 \right) \right]_{\alpha \beta}$$

with the $16 \times 16$ matrices

$$\mathcal{N}_{\alpha \beta} = \frac{1}{4} \int (dp) \text{Tr} \left\{ \gamma_\alpha G_p^+ \gamma_\beta G_p^- \right\}, \quad \Upsilon_{\alpha \beta} = \frac{n}{4} \text{Tr} \left\{ \gamma_\alpha T^R \gamma_\beta T^A \right\}.$$ (33)

The full response function is determined once the simple two-dimensional integrals in Eq. (33) have been performed. This bypasses the computation of the momentum integral in Eq.(31) and allow us to compute the response function nonperturbatively in all couplings at the $T$ matrix level.

IV. WEAK SCATTERING REGIME: ANALYTICAL RESULTS

A. Response functions

For 2D materials with $C_{6v}$ symmetry ($\Delta = \lambda_{sv} = 0$), the self energy at high carrier density reads as

$$\Sigma^\pm (\epsilon) \simeq \pm i\eta \left( 1 + \frac{m_x}{\epsilon} s_x + \frac{m_z}{\epsilon} s_z \right),$$

where $\eta = n\lambda_0^2(4\eta^2)$ is the disorder-induced quasiparticle broadening. There is already a significant difference at this stage between magnetized Dirac fermions and 2DEGs since, in the latter, the self-energy is a scalar [58].

In the main text, we presented full nonperturbative results obtained with a numerical inversion of the Bethe-Salpeter equation (see Sec. III). Analytical expressions for the weak scattering regime can be obtained by evaluating special subsets of diagrams. Three responses, $K_{xy}$, $K_{yx}$ and $K_{zy}$, are activated at the “Gaussian level”. They are obtained as

$$\mathcal{X}_{\alpha \beta} \simeq \frac{2}{\pi n\lambda_0^2} \left( \frac{1}{1 - n\lambda_0^2 \mathcal{N}} \right)_{\alpha \beta},$$

(35)
which is the familiar “vertex correction” obtained by resumming the ladder series, i.e. impurity scattering events with only two potential insertions generated by the Gaussian correlator \( \langle V(x)V(x') \rangle = nu_0^3 \delta(x - x') \).

To capture the antidamping responses \( K_{xx}, K_{yy} \) and \( K_{zz} \), one supplements the ladder series with the “Y” diagrams generated by the high-order correlator \( \langle V(x)V(x')V(x'') \rangle = nu_0^3 \delta(x - x')\delta(x' - x'') \) (see Fig. 6).

We find for the Gaussian Fermi-surface responses:

\[
K_{xy} = -K_{yx} = \frac{2}{v\eta} \frac{\lambda \epsilon^2 (\epsilon^2 + m_y^2)}{\epsilon^4 (\lambda^2 + m_z^2) - m_x^2 (\epsilon^2 - 3\lambda^2)},
\]

\[
K_{zy} = \frac{2}{v\eta} \frac{\lambda m_z m_y}{\epsilon^2 + m_z^2} + O(\epsilon^{-3}),
\]

\[
\sigma_{xx} = \sigma_{yy} = \frac{\epsilon}{\eta} \left( 1 - \frac{4\lambda^2 m_y^2 (\epsilon^2 - 2\lambda^2)}{\epsilon^4 (\lambda^2 + m_z^2) - m_x^2 (\epsilon^2 - 3\lambda^2)} \right),
\]

and

\[
K_{xx}^Y = K_{yy}^Y = \frac{u_0 m_z \lambda^5 \epsilon^2 (\epsilon^2 - m_z^2) (\epsilon^2 + m_y^2)^2}{v^3 \eta (\epsilon^4 (m_y^2 + \lambda^2) - m_x^2 (\epsilon^2 - 3\lambda^2))^2},
\]

\[
K_{zz}^Y = -\frac{u_0 \lambda^5 m_z m_y}{2v^3 \eta (\lambda^2 + m_z^2)^3} + O(\epsilon^{-3}),
\]

\[
\sigma_{xy}^Y = -\sigma_{yx}^Y = -\frac{2u_0 \lambda^3 \epsilon^2 (\epsilon^2 + m_z^2)^3}{v^2 \eta (\epsilon^4 (\lambda^2 + m_z^2) - m_x^2 (\epsilon^2 - 3\lambda^2))^2},
\]

for the \( Y \) diagrams. Eq. (39) corresponds to the “collinear” inverse spin galvanic effect predicted in this work.

### B. Structure of the renormalized vertex

The linear response function at the Gaussian level is determined by a single component of the renormalized vertex[59], which provides a transparent scheme to identify candidate nonzero responses \( K_{ia} \) based on a symmetry analysis (see table II).

The current vertex transverse to the anisotropy direction (\( \tilde{J}_z \)) displays a rich matrix structure with a term proportional to \( s_z \), which shows that \( K_{zy} \) is finite already at the Gaussian level, in agreement with the analysis of Boltzmann transport equations outlined in the main text.

The nonzero matrix coefficients \( \tilde{\gamma}_{ij} = \sum_{kl} c_{ijkl} \tilde{\gamma}_{kl} \) at the Gaussian level read as

\[
c_{1010} = c_{2020} = 2 - \frac{4\lambda^2 m_z^2 (\epsilon^2 - 2\lambda^2)}{\epsilon^4 (\lambda^2 + m_z^2) - m_x^2 (\epsilon^2 - 3\lambda^2)},
\]
$$J_x \sigma_{y} s_0 (\gamma_0), \sigma_{y} s_0 (\gamma_2), \sigma_{y} s_0 (\gamma_{10}), \sigma_{y} s_0 (\gamma_{10})$$

$$J_y \sigma_{y} s_0 (\gamma_0), \sigma_{y} s_0 (\gamma_2), \sigma_{y} s_0 (\gamma_{10}), \sigma_{y} s_0 (\gamma_{10}), \sigma_{y} s_0 (\gamma_{11})$$

Table II. Gaussian matrix structures of the renormalized current vertex. In-plane magnetic anisotropy generates additional orbital-spin mixings, which are fundamental for the accurate description of SOTs.

$$c_{1002} = - c_{2001} = - \frac{2 \lambda^2 \epsilon (\epsilon^2 + m_z^2)}{\epsilon^4 (\lambda^2 + m_z^2) - m_z^4 (\epsilon^2 - 3 \lambda^2)}. \quad (43)$$

$$c_{2003} = - \frac{2 \lambda m_z m_z}{\epsilon (\lambda^2 + m_z^2)} + O \left( \epsilon^{-2} \right). \quad (44)$$

The longitudinal conductivity is determined by Eq.(42), the Rashba-Edelstein Effect is encoded in Eq.(43), and Eq.(44) determines the generation of out-of-plane nonequilibrium spin polarization. Apart from a prefactor these expressions are the Gaussian response functions presented earlier.