Determining the CP Violation Angle $\gamma$ in $B_s$ Decays without Hadronic Uncertainty

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We study the rare decays $B_s^0 \rightarrow D^{\pm} \pi^{\mp}$ and $B_s^0 \rightarrow D^\mp \pi^\pm$, which can occur only via annihilation type $W$ exchange diagrams in the standard model. The time-dependent decay rates of the four channels can provide four CP parameters, which are experimentally measurable. We show that the CKM angle $\phi_3 = \gamma$ can be determined from these parameters without any theoretical model dependence. These channels can be measured in future LHCb experiments to provide a clean way for $\gamma$ measurement.

I. INTRODUCTION

The CP violation study is one of the hot topics in particle physics. After the measurement of CKM angle $\phi_1 = \beta$ in B factories, more attention has been drawn to the extraction of the other two angles, especially $\phi_3 = \gamma$, which is the most difficult one. Besides the method based on approximation of SU(3), a lot of other channels are discussed to measure this CKM angle, such as $B \rightarrow DK$ decays, $B \rightarrow K_S \pi^+ \pi^-$ and $B \rightarrow D^* K$ decays etc. Most of the methods require a number of measurements, some require measurements of rare decays with small branching ratio. Therefore the measurement of angle $\gamma$ is still difficult for experiments.

In this paper, we give another example to measure the CKM angle $\gamma$, which does not require any theoretical assumption, namely the rare decays $B_s^0 \rightarrow D^{\pm} \pi^{\mp}$ and $B_s^0 \rightarrow D^\mp \pi^\pm$. Similar to the $B \rightarrow DK$ decays, there are both contributions from $b \rightarrow c \bar{s}s$ and $b \rightarrow u \bar{c}s$ transitions, in these four modes. The interference between the two kinds of decay amplitudes will give out the information of CKM angle $\gamma$. Unlike the $B \rightarrow DK$ decays, there is no direct CP violation here, but mixing induced CP violation, since neutral $B_s$ meson decays are involved. The time dependent measurement of decay amplitudes can provide the ratio of two decay amplitudes and the CKM angle $\gamma$, without theoretical input.

Similar argument has been also proposed for $B_s (B_s^0 \rightarrow D^{+} K^{\mp})$ and $B_s^0 (B_s^0 \rightarrow D^{-} K^{+})$ decays some years ago, which is intensively discussed later in $\gamma$. These decays with emission diagram contributions will have a larger branching ratio than the channels discussed here. However the latter channels of $B_s (B_s^0 \rightarrow D^{\pm} K^{\mp})$ decays involve CKM matrix elements of $V_{ub} V_{cd}$ and $V_{ub} V_{ud}^*$. The large difference between these two matrix elements $|V_{ub} V_{cd}| \ll |V_{ub} V_{ud}^*|$ makes the two decay amplitudes differ too much, thus experimentally too difficult to measure. The $B_s (B_s^0 \rightarrow D^{\pm} K^{\mp})$ decays should be the best channels to measure CKM angle $\gamma$. Our newly proposed channels $B \rightarrow D^{\pm} \pi^{\mp}$ will be an alternative choice.

II. CP ASYMMETRY VARIABLES OF $B_s^0 (B_s^0 \rightarrow D^{\mp} \pi^{\pm})$

The non-leptonic $B_s^0$ decays $B_s^0 \rightarrow D^{+} \pi^{-}$ and $B_s^0 \rightarrow D^{-} \pi^{+}$ are rare decays, which can occur only at tree level operators. No penguin operators can contribute to avoid the penguin pollution. The perturbative diagrams for these decays are shown in Figs. $\gamma$. For decay $B_s^0 \rightarrow D^{+} \pi^{-}$ (Fig. $\gamma$), the decay amplitude is proportional to $V_{ub} V_{cs}$. And for decay $B_s^0 \rightarrow D^{-} \pi^{+}$ (Fig. $\gamma$), the decay amplitude is proportional to $V_{ub} V_{cd}^*$. They are pure annihilation type decays with $W$ exchange diagrams. Despite the perturbative picture, one may argue that they can get contribution from non-perturbative diagrams, such as soft-final state interaction diagrams shown in Fig. $\gamma$. $B_s^0 \rightarrow D^{\mp} K^{\pm}$ decays also have the same CKM matrix elements with the perturbative one in Fig. $\gamma$. Fortunately, these diagrams have the same CKM matrix elements with the perturbative one in Fig. $\gamma$ to make these channels clean for CP violation measurement. Therefore, the decay amplitudes for these decays can be parameterized

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FIG. 1: Perturbative Feynman diagrams contributing to decay $B_s^0 \to D^+ \pi^-$ (a) and $B_s^0 \to D^- \pi^+$ (b).

as

$$g = \langle D^+ \pi^- | H | B_s^0 \rangle = V_{ub} V_{cs}^* A_1, \quad h = \langle D^+ \pi^- | H | \bar{B}_s^0 \rangle = V_{cb} V_{us}^* A_2,$$

$$\bar{g} = \langle D^- \pi^+ | H | B_s^0 \rangle = V_{ub} V_{cs}^* A_1, \quad \bar{h} = \langle D^- \pi^+ | H | \bar{B}_s^0 \rangle = V_{cb} V_{us}^* A_2,$$ (1)

which determine the decay matrix elements of $B_s^0 \to D^+ \pi^-$ and $D^- \pi^+$, and of $\bar{B}_s^0 \to D^- \pi^+$ and $D^+ \pi^-$. There is only one kind of contribution for each of the decay modes, thus there is no direct CP violation for them. However there is still a CP violation induced by mixing, although they are decays with charged final states (non CP eigenstates).

The neutral $B_s - \bar{B}_s$ mixing is usually described as

$$B_H = |p|^2 |B_s^0 > + |q| |\bar{B}_s^0 >,$$ (2)

$$B_L = |p|^2 |B_s^0 > - |q| |\bar{B}_s^0 >,$$ (3)

with $|p|^2 + |q|^2 = 1$. $|p|$ and $|q|$ come from the information of $B_s^0$ and $\bar{B}_s^0$ transition from each other. Thus,

$$\frac{q}{p} = \frac{V_{tb} V_{ts}^*}{V_{tb} V_{ts}^*}$$ (4)

for $B_s - \bar{B}_s$ mixing. In Wolfenstein parameterization $[9]$, $\text{arg}(q/p) = 2\lambda^2 \eta < 2^\circ$, which is negligible. Since we are only interested in the CKM angle measurement, the normalized time-dependent decay rates for $B_s \to D^\pm \pi^\mp$ are given by $[10]$

$$\Gamma^{D^\pm \pi^\mp}(t) = (1 \pm A_{CP}) \frac{e^{-t/\tau_{B_s}}}{8\tau_{B_s}} \left\{ 1 + (S_{D\pi} \pm \Delta S_{D\pi}) \sin \Delta m t \right. + \left. (C_{D\pi} \pm \Delta C_{D\pi}) \cos \Delta m t \right\},$$ (5)

and $\bar{B}_s \to D^\pm \pi^\mp$ by

$$\tilde{\Gamma}^{D^\pm \pi^\mp}(t) = (1 \pm A_{CP}) \frac{e^{-t/\tau_{B_s}}}{8\tau_{B_s}} \left\{ 1 - (S_{D\pi} \pm \Delta S_{D\pi}) \sin \Delta m t \right. - \left. (C_{D\pi} \pm \Delta C_{D\pi}) \cos \Delta m t \right\},$$ (6)
where $\Delta m$ is the mass difference of the two mass eigenstates $B_H$ and $B_L$, and

$$
C_{D\pi} = \frac{1}{2}(a_e + a_\bar{e}), \quad \Delta C_{D\pi} = \frac{1}{2}(a_e - a_\bar{e}),
$$

$$
S_{D\pi} = \frac{1}{2}(a_e + a_\bar{e} + a_e^* + a_\bar{e}^*), \quad \Delta S_{D\pi} = \frac{1}{2}(a_e + a_\bar{e} - a_e^* - a_\bar{e}^*).$$

They can be expressed by another set of parameters as

$$
a_e = \frac{|g|^2 - |h|^2}{|g|^2 + |h|^2}, \quad a_e + a_\bar{e} = \frac{-2Im(h/g)}{1 + |h/g|^2},
$$

$$
a_e = \frac{|g|^2 - |h|^2}{|h|^2 + |\bar{g}|^2}, \quad a_e + a_\bar{e} = \frac{-2Im(\bar{g}/\bar{h})}{1 + |\bar{g}/\bar{h}|^2}.
$$

Utilizing eq. (11), we can get

$$
C_{D\pi} = A_{CP} = 0, \quad \Delta C_{D\pi} = \frac{1 - R^2}{1 + R^2}, \quad \Delta S_{D\pi} = \frac{-2R \sin \delta \cos \gamma}{1 + R^2},
$$

where $R = |h/g| = |V_{cb}V_{ub}^*/A_2|/|V_{cb}^*V_{ub}A_1|$, is the relative size of the two kinds of decay amplitudes. And $\delta = \arg(A_2/A_1)$ is the relative strong phase between them. From eq. (9), one can see that the ratio $R$ can be determined from $\Delta C_{D\pi}$, and strong phase $\delta$ and the CKM angle $\phi_3 = \gamma$ can be solved from $S_{D\pi}$ and $\Delta S_{D\pi}$, without uncertainty, if $\Delta C_{D\pi}, S_{D\pi}$ and $\Delta S_{D\pi}$ have been gotten from experiments. From eq. (11), one can also see that, $\Delta C_{D\pi}, S_{D\pi}$ and $\Delta S_{D\pi}$ are measurable by experiments through the time-dependent decay rate. In the standard model (SM), the strong phase of $A_1$ and $A_2$ should be the same, since CP is conserved in strong interaction. Therefore, $\delta = 0$, and $\Delta S_{D\pi} = 0$. Hence only $\Delta C_{D\pi}$ and $S_{D\pi}$ are used to determine $R$ and $\sin \gamma$. In a word, the CKM angle $\phi_3 = \gamma$ can be determined cleanly without any theoretical model dependence, provided the experimental measurements of time-dependent decay rates.

The parameter $C_{D\pi} = A_{CP} = \Delta S_{D\pi} = 0$ is a consequence of the fact that there is only one kind of contribution for each of the decays. If there is any new physics contribution, which usually provides a different weak phase, these two parameters will not be zero any longer. The non-zero measurement of these parameters experimentally will be a signal of new physics.

Since these decays are rare, one may worry about the decay branching ratios are too small to be measured. A perturbative QCD approach (PQCD) based on $k_T$ factorization shows that they are at least at the order of $10^{-6}$ [11]. This is consistent with naive argument that the annihilation topology is power suppressed as $1/m_b$, which is order of 10%. Translating to branching ratios, the $B_s \to D\pi$ branching ratio $(10^{-6})$ should be at 1% level of the emission type decay $B_s \to D_s K$ $(10^{-4})$. The CP parameters $S_{D\pi}$ and $\Delta S_{D\pi}$ are also sensitive to the relative amplitude $R$ through eq. (9). If $R$ is too small or too big, $S_{D\pi}$ and $\Delta S_{D\pi}$ will be too small to be measured. The same study in PQCD shows $R \simeq 1.8$ [11], to make the extraction of CKM angle $\phi_3 = \gamma$ realistic. In fact, the ratio $R$ can not deviate from 1 too much, since the CKM parameter $|V_{cb}V_{ub}^*|/|V_{cb}^*V_{ub}|$ for these two kinds of decays are at the same level $O(\lambda^3)$ in Wolfenstein parameterization [3]. More precisely, the value of $|V_{cb}^*V_{ub}|$ is about half of $|V_{cb}^*V_{ub}|$.

The same argument shown above is applicable to the $B_s(B_s) \to D^\pm K^{\mp}$ decays. Since there are emission diagram contributions for these decays, their decay branching ratios are much higher at order of $10^{-4}$ [12]. It is easier for experiments to measure. The branching ratio of the proposed channel $B_s(B_s) \to D^\pm \pi^{\mp}$ is two order magnitude smaller, but it will provide a test for SM to measure the same quantity using different channels. The situation is similar in the $\beta$ measurement, where people try to measure CP asymmetry of $B \to K_\alpha \phi$, after $B \to J/\psi K_s$. Recently, many new physics discussions have been made on this issue due to the different results for the two channels [13].

Since the current B factories do not produce $B_s$ mesons, there are no data for these decays in experimental side up to now. But the designed LHCb experiment will produce $10^{12}$ $b\bar{b}$ pairs per year where 10% of them will be $B_s(B_s)$ [14]. The $B_s - B_s$ mixing parameter $\Delta m_{B_s}$, which is predicted to be $25$ ps$^{-1}$ in SM, can be measured in LHCb in one month. With $10^{11}$ $B_s$ mesons produced, the LHCb experiment can measure decays with branching ratio as small as $10^{-7}$ even if the detection efficiency is only several percent. Therefore the $B_s(B_s) \to D^\pm \pi^{\mp}$ decay is measurable in the near future, although the time-dependent CP asymmetry measurement could be challenging.

The $B^0(B^0) \to D^\pm \pi^{\mp}$ decays, can be easily measured in the current B factories, however, the same argument does not apply to them, since its ratio $R \simeq |V_{ub}V_{cd}^*|/|V_{cb}V_{ud}| \simeq 0.02$ is too small, making the measurement of $S_{D\pi}$ and $\Delta S_{D\pi}$ nonrealistic.
III. SUMMARY

In this paper, we show that the four time-dependent decay rates of $B_s^0 \to D^\pm \pi^\mp$ and $\bar{B}_s^0 \to D^{\mp} \pi^{\pm}$, can provide four CP parameters which are experimentally measurable. These parameters are functions of CKM angle $\phi_3 = \gamma$. The measurement of these parameters at the future LHCb experiment can provide a method to extract CKM angle $\gamma$ without any hadronic uncertainty. These channels occur purely via annihilation type $W$ exchange diagrams, and have a branching ratio of $10^{-6}$ which is measurable in future LHCb experiment.

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