Matter Coupling
and Spontaneous Symmetry Breaking
in Topological Gravity

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Abstract

Matter is coupled to three-dimensional gravity such that the topological phase is allowed and the (anti-) de Sitter or Poincaré symmetry remains intact. Spontaneous symmetry breaking to the Lorentz group occurs if a scalar field is included. This Higgs field can then be used to couple matter so that the familiar form of the matter coupling is established in the broken phase. We also give the supersymmetrization of this construction.

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1. Introduction

Many attempts have been made to formulate a quantum theory of four-dimensional gravity (see [1]). In this approach it is hoped that if gravity can be formulated as a renormalizable theory, then this would improve the prospects of unifying gravity with the other known interactions. The recent developments in three-dimensional gravity provide an excellent testing ground for this program. There it was shown that by formulating three-dimensional gravity as a topological gauge theory of the groups $SO(1, 3), SO(2, 2)$ or $ISO(1, 2)$, the theory becomes finite [2]. The main difficulty in advancing this program is the coupling of matter.

A first difficulty of introducing matter lies in the nature of topological gravity. It allows for the unbroken phase of gravity where the dreibein is degenerate: $e_\mu^a = 0$. In this topological phase, the notion of geometry loses its meaning. Physics in the usual sense, where space-time is equipped with distances, arises only away from this phase. Actually, in [2] the partition function was seen to be dominated by geometrical universes, but the appearance of the topological phase was essential for its derivation.

Matter coupling, however, is usually formulated by using the inverse dreibein $e^\mu_a$ which would become singular in the topological phase. In topological gravity this coupling has to be introduced such that only $e_\mu^a$ is used. Moreover, we want to require the matter coupling to reproduce the familiar form if restricted to invertible dreibeins.

A second difficulty stems from the fact that $e_\mu^a$ is part of the gauge field $A$ and cannot be used by itself without breaking the gauge invariance. It is then suggestive to break the gauge symmetry to the Lorentz group $SO(1, 2)$ so that $e_\mu^a$ would correspond to the broken symmetry. To break the symmetry we employ some kind of Higgs mechanism. However, writing a usual Higgs potential in the action requires a metric for the volume element. Again, since no dreibein and therefore no metric $g_{\mu\nu} = e_\mu^a e^a_{\nu}$ can be used without breaking the tangent space symmetry ”by hand”, the Higgs field potential terms cannot be written in the usual way and an alternative construction will be applied.

It is surprising that despite of these difficulties matter interactions can be introduced. In the following we will discuss the case of a scalar field. The plan of this paper is as follows. In section 2 we give the coupling of three-dimensional topological gravity to matter. In section 3 the construction is generalized to the supersymmetric case. Some comments and the conclusion are in section 4.
2. Matter Coupling to three-dimensional Topological Gravity

From the work of Witten it is now established that three-dimensional quantum gravity becomes a finite theory when formulated as a gauge theory of $G = SO(1, 3)$, $SO(2, 2)$ or $ISO(1, 2)$ depending on the sign of the cosmological constant [2]. The gauge invariant action is of the Chern-Simons type

$$S_g = 4k_g \int < AdA + \frac{2}{3} A^3 >$$ (2.1)

where $A$ is an $SO(2, 2)$ gauge field (the other two cases are recovered by Wick rotation or an Inönu-Wigner group contraction)

$$A = \frac{1}{4} A^{AB} J_{AB}, \quad A = a, 3; \quad a = 0, 1, 2$$

and the quadratic form is defined by

$$< J_{AB} J_{CD} > = \epsilon_{ABCD}$$ (2.2)

The connection with gravity is made through the identification

$$A^{a3} = e^a, \quad A^{ab} = \omega^{ab}$$ (2.3)

In terms of $e$ and the spin connection $\omega$ the action (2.1) takes the form

$$S_g = k_g \int \epsilon_{abc} e^a (R^{bc} - \frac{1}{3} e^b e^c)$$ (2.4)

where $R^{bc} = d\omega^{bc} + \omega^{bd} \omega^d_c$. At the classical level, when $e_\mu^a$ is restricted to the subspace of invertible fields, the action (2.4) is equivalent to the Einstein-Hilbert action. However, this equivalence breaks down at the quantum level, where the quantum theory of (2.4) is finite.

The main disadvantage in this formulation is the difficulty of introducing non-trivial matter. By "non-trivial" we mean couplings which, in the non-topological phase, reduce to the familiar interactions. The familiar form of the bosonic matter coupling requires the inverse dreibein $e_\mu^a$. This, however, is singular in the topological phase where $e_\mu^a = 0$. Moreover, $e_\mu^a$ is part of the gauge field and cannot be used by itself without breaking the symmetry. At the quantum level, the action (2.4) generates divergent 1-loop diagrams that are cancelled by ghost diagrams arising from Lorentz- and translation invariance. Since we do not want to lose these ghost diagrams, we should try to break the gauge spontaneously. Let us try to couple a scalar field $H$. 


Without using a metric, the only coupling that could be introduced would be to multiply the action \((2.1)\) by factors of \(H\). This, certainly, does not give interesting physics. Attempts have been made to introduce an additional antisymmetric tensor \([3]\) or fields living in representations of only the Lorentz-group \(\text{SO}(1,2)\) \([4]\). These, however, have a trivial physical content. Here, instead, we take a different strategy. We consider a field \(H^A\) and identify \(H^3 = H\) \([5]\). We will see that when expanding around a flat background and using a linear approximation, the \(H^A\) coupling will reproduce the familiar \(\partial^\mu H \partial_{\mu} H\) after eliminating the \(H^A\) by its equation of motion. Since the \(H^A\) will take a non-zero vacuum expectation value (vev) which breaks the symmetry to the Lorentz-group, we call it a Higgs field. This Higgs field can then be used to couple other matter fields.

Since no metric is at our disposal to write volume elements, the only Higgs terms that can be written (apart from possible factors \(H^A H_A\) multiplying them) are

\[
S_h = - \int \epsilon_{ABCD} H^A [\mu DH^B F^{CD} + \lambda DH^B DH^C DH^D] \tag{2.5}
\]

where

\[
D_\mu H^A = \partial_\mu H^A + A_{\mu}^{AB} H_B
\]

\[
F^{AB} = dA^{AB} + A^{AC} A_C^B
\]

With the Higgs terms given in (2.5) we may now ask for a possible vev in the translational direction:

\[
H^A = <H^A> + \bar{H}^A \tag{2.6}
\]

where

\[
<H^a> = 0 \quad , \quad <H^3> \equiv <H> \tag{2.7}
\]

Actually, the argument should have been reversed: It is the direction of the non-zero \(<H^A>\) that decides which part of the gauge fields in (2.3) separates to be identified with the dreibein. To look for non-zero \(<H>\) we have to consider the part of the action (2.5) given by

\[
S'_h = \int \epsilon_{abc} (\mu H^2 e^a R^{bc} + e^a e^b e^c [-\mu H^2 + \lambda H^4]) \tag{2.8}
\]

The \(\)' indicates that in (2.5) we set \(H^a = 0\) which is sufficient for obtaining a vev in the translation direction. With \(\frac{1}{3!} \epsilon_{abc} e^a e^b e^c = d^3 x \sqrt{g}\), the last two terms in (2.8) are seen to be the usual scalar potential. It is a well-known feature (and problem!) that the vev of a Higgs field changes the cosmological constant. For convenience, we may
assume that we tuned the coupling constants such that the effective cosmological constant vanishes. Then we may go to the flat background:

\[ < e^a_\mu > = \delta^a_\mu, \quad < \omega^{ab}_\mu > = 0 \] (2.9)

For such a background, the first term in (2.8) will not contribute. With \( \lambda > 0 \), the Higgs potential in (2.8) is then minimized by

\[ < H > = \sqrt{\frac{\mu}{2\lambda}} \] (2.10)

if \( \mu > 0 \), otherwise the vev will vanish. The vev (2.10) breaks the tangent space symmetry to the Lorentz-group \( SO(1,2) \) that leaves (2.7) invariant.

Plugging (2.10) back into (2.8), we find the total action to be

\[ S_g + S_h = \int \epsilon_{abc} [(k_g + \frac{\mu^2}{2\lambda}) e^a R^{bc} - \left( \frac{1}{3} k_g + \frac{\mu^2}{4\lambda} \right) e^a e^b e^c] + O(\bar{H}^4) \] (2.11)

Except for Higgs quantum fluctuations, this is of the same form as the gravity action (2.4) but with a different cosmological constant. We find this effective cosmological constant to be cancelled if

\[ \mu^2 = -\frac{4}{3} k_g \lambda \] (2.12)

This allows to use the flat background (2.9), and in the linear approximation the terms of (2.5) that are quadratic in \( H^a \) are

\[ 2\mu \int d^3x \left( 2H^a \delta^a_\mu \partial_\mu H - 3H^2 - H^a H_a \right) \] (2.13)

Eliminating the \( H^a \) by its equation of motion from (2.13), this turns into

\[ 2\mu \int d^3x \left( \partial^a H \partial_\mu H - 3H^2 \right) \] (2.14)

In (2.14) we recognize the usual kinetic term for the Higgs field around the flat background (2.9). For a general gravitational background and including also higher than quadratic terms in (2.13), the elimination of \( H^a \) by equations of motion becomes a formidable task and will not be attacked here. We take (2.14) as sufficient in determining the structure of the \( H^A \) sector. Alternatively, for analyzing the system (2.5), the gauge condition \( H^a = 0 \) could be imposed and a kinetic term for the Higgs field \( H \) would be generated by a Weyl scaling that absorbs the \( H^2 \) in the first term of (2.8).
Having included the Higgs field $H^A$, we are now able to couple other matter fields. The simplest matter interaction to construct is that of a scalar multiplet. Let $X^A$ be a scalar multiplet in the fundamental representation of $SO(2,2)$ with the identifications $X^a = \pi^a, X^3 = \phi$. One possible action that reproduces the familiar form at the classical level is

$$S_m = k_m \int \epsilon_{ABCD} H^A D H^B D H^C (X^D D X^E H_E) \quad (2.15)$$

If we expand the Higgs field around the broken phase (2.7), (2.10) the matter action (2.15) takes the form

$$S_m = -k'_m \int d^3 x \epsilon^{\mu \nu \rho} \epsilon_{abc} e^a_{\mu} e^b_{\nu} \pi^c (\partial_\rho \phi - e^d_\rho \pi_d) + O(\bar{H}^A) \quad (2.16)$$

where $k'_m = k_m \mu^2 / 4 \lambda^2$. The action (2.16) is just the first-order formulation of a scalar field action. To see this, assume the non-topological phase where $e^a_\mu$ is invertible, and substitute the equation of motion of $\pi_a$,

$$\pi_a = \frac{1}{2} e^\mu_a \partial_\mu \phi + O(\bar{H}^A) \quad (2.17)$$

into the action (2.16) to get

$$S_m = - \frac{k'_m}{2} \int d^3 x \ e^a e^{\nu a} \partial_\mu \phi \partial_\nu \phi + O(\bar{H}^A) \quad (2.18)$$

Thus (2.15) reproduces the canonical form at the classical level. In the spontaneously broken phase, the total action, which is the sum of (2.11) and (2.16), has only the $SO(1,2)$ Lorentz symmetry.

3. Topological Supergravity and Matter Coupling

Since $SO(2,2) \cong SO(1,2) \times SO(1,2)$ and $OSP(2|1)$ is the graded version of $SO(1,2)$, the supersymmetric analogue of the construction given in the previous section is achieved by gauging $OSP(2 | 1) \times OSP(2 | 1)$ [2,6].

We shall adopt the notation of [7] for the matrix representation of $OSP(2 | 1)$. Let $\Phi_1$ and $\Phi_2$ be the gauge fields of the two $OSP(2 | 1)$ gauge groups transforming as

$$\Phi_1 \rightarrow \Omega_1 \Phi_1 \Omega_1^{-1} + \Omega_1 d \Omega_1^{-1}$$
$$\Phi_2 \rightarrow \Omega_2 \Phi_2 \Omega_2^{-1} + \Omega_2 d \Omega_2^{-1} \quad (3.1)$$
where $\Omega_1$ and $\Omega_2$ are two elements of the two respective groups. These can be represented in the matrix form

$$\Phi = \begin{pmatrix} A_\alpha^\beta & \psi_\alpha \\ \bar{\psi}_\beta & 0 \end{pmatrix}$$

(3.2)

where

$$A_{\alpha\beta} = A_{\beta\alpha}, \quad \psi_\alpha = \epsilon_{\alpha\beta}\bar{\psi}_\beta$$

(3.3)

It is also convenient to write

$$A_\alpha^\beta = A^a_{\alpha} (\tau_a)_\beta^\alpha$$

where the $\tau_a$ are the $SO(2,1)$-generators

$$\tau_0 = \frac{1}{2} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \tau_1 = \frac{1}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \tau_2 = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Introduce now the Higgs field $G$ transforming as

$$G \rightarrow \Omega_1 G \Omega_2^{-1}$$

(3.4)

and the covariant derivative of $G$, transforming as $G$, is defined by

$$DG = dG + \Phi_1 G - G\Phi_2$$

(3.5)

In order to distinguish the group indices of the second $OSP(2|1)$ let us denote them by $\dot{\alpha}, \dot{\beta}, ...$. Then the matrix representation of $G$ is

$$G = \begin{pmatrix} H_{\dot{\alpha}}^\beta & \eta_\alpha \\ \bar{\xi}_\beta & \phi \end{pmatrix}$$

(3.6)

where both $\eta_\alpha$ and $\xi_\dot{\alpha}$ are Majorana spinors, and $H_{\dot{\alpha}}^\beta$ and $\phi$ are real.

It will also be necessary to define the equivalent representation $\tilde{G}$ transforming as

$$\tilde{G} \rightarrow \Omega_2 \tilde{G} \Omega_1^{-1}$$

(3.7)

and whose matrix form is

$$\tilde{G} = \begin{pmatrix} [\epsilon H^T \epsilon^{-1}]_{\dot{\alpha}}^\beta & -\xi_\alpha \\ -\bar{\eta}_\beta & \phi \end{pmatrix}$$

(3.8)

We first write the pure supergravity action [6]

$$S_{sg} = -\frac{k_{sg}}{2} \int [\text{Str}(\Phi_1 d\Phi_1 + \frac{2}{3}\Phi_3^3) - 1 \rightarrow 2]$$

(3.9)
whose component form is

\[ S_{sg} = \frac{k_{sg}}{4} \int \left[ (A_{1a}dA_{1}^{a} - \frac{1}{3} \epsilon_{abc}A_{1}^{a}A_{1}^{b}A_{1}^{c}) + 4\bar{\psi}_{1}D_{1}\psi_{1} - 1 \rightarrow 2 \right] \tag{3.10} \]

where \( D_{i} = d_{i} + A_{i} \). The action in (3.10) can be put into a more familiar form by reexpressing it in terms of [6]

\[
\begin{align*}
\omega^{a} &= \frac{1}{2} (A_{1}^{a} + A_{2}^{a}) \\
e^{a} &= \frac{1}{2} (A_{1}^{a} - A_{2}^{a}) \\
\psi_{\pm} &= \frac{1}{2} (\psi_{1} \pm \psi_{2})
\end{align*}
\tag{3.11}
\]

Then

\[ S_{sg} = k_{sg} \int \left[ e^{a}(R_{a} - \frac{1}{6} \epsilon_{abc}e^{b}e^{c}) \\
+ 4\bar{\psi}_{-}(d + \omega)\psi_{+} + 2\bar{\psi}_{+}e\psi_{+} + 2\bar{\psi}_{-}e\psi_{-} \right] \tag{3.12} \]

where \( R_{a} = d\omega_{a} - \frac{1}{2} \epsilon_{abc}\omega^{b}\omega^{c} \). Using \( \omega^{a} = \frac{1}{2} \epsilon^{abc}\omega_{bc} \) the bosonic part agrees with (2.4).

Apart from trace factors \( Str(G\tilde{G}) \), the most general expression for the Higgs interactions compatible with (3.9) and the diagonalization in (3.11) is

\[ S_{sh} = \int \left\{ \frac{\mu}{2} \left[ Str(G\tilde{D}G(d\Phi_{1} + \Phi_{1}^{2})) - Str(\tilde{G}DG(d\Phi_{2} + \Phi_{2}^{2})) \right] \\
+ \frac{\lambda}{4} Str(G\tilde{D}G \tilde{G}D\tilde{G}) \right\} \tag{3.13} \]

Analogously to the bosonic case, we may look for a non-zero vev of

\[ G = \langle G \rangle + \tilde{G} \tag{3.14} \]

where

\[ \langle G \rangle = \begin{pmatrix} \langle h \rangle & 0 \\ 0 & \langle \varphi \rangle \end{pmatrix} \tag{3.15} \]

and \( h \) is in the unit direction of \( H = h + H^{a}\tau_{a} \). The supergroup \( OSP(2 | 1) \times OSP(2 | 1) \) has ten degrees of freedom. Out of these, seven may be used to rotate \( \langle G \rangle \) into the direction given by (3.15). Therefore, a non-zero vev (3.15) would leave only three degrees of freedom and we will see that these correspond to the Lorentz-group.
Since we have to vary the action in the direction given by (3.15) we only need to look at the terms

\[
S'_{sh} = \int \left\{ \mu h^2 e^a R_a - \frac{1}{2} \epsilon_{abc} e^a e^b e^c (\mu h^2 - \lambda h^4) \right. \\
+ 2\mu(h^2 + \varphi^2)\bar{\psi}_-(d + \omega)\psi_+ \\
+ [\mu(3h^2 + \varphi^2 - 2h\varphi) - \frac{\lambda}{2}(2h^4 - 5h^3\varphi + 4h^2\varphi^2 - h\varphi^3)]\bar{\psi}_+ e\psi_+ \\
+ [\mu(3h^2 + \varphi^2 + 2h\varphi) - \frac{\lambda}{2}(2h^4 + 5h^3\varphi + 4h^2\varphi^2 + h\varphi^3)]\bar{\psi}_- e\psi_- \\
+ \frac{\lambda}{2}(hd\varphi - \varphi dh)(h^2 - \varphi^2)\bar{\psi}_- \psi_+ \left\} \\
\] (3.16)

where ' indicates that we set Higgs components orthogonal to (3.15) to zero. Like the bosonic case, we may for convenience assume that the coefficients \(k_{sg}, \mu\) and \(\lambda\) are tuned such that the effective cosmological constant vanishes. Then we may go to a flat background:

\[
< e^a_\mu > = \delta^a_\mu \quad , \quad < \omega^a_\mu > = 0 \quad , \quad < \psi_\pm > = 0 \quad (3.17)
\]

With \(\lambda > 0\), the potential in (3.16) will then be minimized by

\[
< h > = \sqrt{\frac{\mu}{2\lambda}} \\
\] (3.18)

if \(\mu > 0\). For \(\mu < 0\) the vev of \(h\) would be zero. The field \(\varphi\) is not driven to a certain value; in the background (3.17) any value for the \(\varphi\) is allowed. The total action takes a particularly interesting form if we shift

\[
\varphi \rightarrow h + \varphi \\
\] (3.19)

Then the sum of (3.9) and (3.13) becomes

\[
S_{sg} + S_{sh} = \int \left\{ (k_{sg} + \mu h^2)[e^a R_a + 4\bar{\psi}_-(d + \omega)\psi_+ + 2\bar{\psi}_+ e\psi_+] \\
+ (k_{sg} + 3\mu h^2 - 3\lambda h^4)[-\frac{1}{6} \epsilon_{abc} e^a e^b e^c + 2\bar{\psi}_- e\psi_-] \right. \\
+ O(\varphi, \bar{G}) \left\} \\
\] (3.20)

Except for Higgs quantum fluctuations and with zero \(\varphi\), the terms appearing in the action (3.20) are of the same form as the original supergravity action (3.12), but with a different cosmological constant. We find the cosmological constant to be cancelled if

\[
\mu^2 = -\frac{4}{3} k_{sg} \lambda \\
\] (3.21)
This will then allow for the flat background (3.17). Notice, that in this background the quadratic Higgs terms in (3.13) are

$$\mu \int d^3 x \left[ H^a \delta^a \partial_\mu h - 3h^2 - \frac{1}{4} H^a H_a + \frac{1}{2} \bar{\eta} \tau^\mu \partial_\mu \eta - \frac{1}{2} \bar{\xi} \tau^\mu \partial_\mu \xi - \frac{3}{8} (\bar{\eta} \eta + \bar{\xi} \xi) \right] (3.22)$$

After rescaling $H^a \rightarrow 2H^a$, the $h$ and $H^a$ terms are of the same form as in (2.13), and the $\eta, \xi$ terms are of the Dirac type.

The presence of the Higgs field $G$ does now allow to couple another matter field $X$ which is a multiplet transforming like $G$. The matrix representation of $X$ is given by

$$X = \left( \frac{1}{2} (\phi + s) \tilde{\delta}^\alpha + \pi^a (\tau_a)_{\dot{\alpha} \alpha}, \lambda_\alpha - \chi_\alpha \right) (3.23)$$

A matter interaction which reproduce the bosonic matter interactions (2.16) is

$$S_{sm} = k_{sm} \int Str \left( \bar{D} G \bar{G} X \right) Str(\bar{G} D X) (3.24)$$

This can be seen by using the vev (3.15), (3.18). Then (3.24) reduces to

$$S_{sm} = -k'_{sm} \int \left( \epsilon_{abc} e^a e^b \pi^c + 4 \bar{\psi}_- \tau_a \psi_+ \pi^a - 4 \bar{\psi}_- e \lambda \right) (d\phi - e^d \pi_d - 2 \bar{\psi}_- \lambda) + 0(\varphi, \bar{G})$$

where we used the shift (3.19) and did not write the $\varphi$-contributions. They always appear with gravitinos and will not influence the bosonic part. Although this action has the correct bosonic interactions for $\pi^a$ and $\phi$, however, $s$ and $\chi$ decouples, and $\lambda$ does not acquire a propagator.

4. Conclusions and Comments

We have constructed matter interactions coupled to gravity in a topological way. The dreibein separates from the other Poincare or (anti-) de Sitter gauge fields only by spontaneous symmetry breaking. The matter coupling was introduced without using the inverse dreibein, thereby allowing for the unbroken phase of gravity. This became possible by including a Higgs field and using a first order formalism. Restricting to the invertible dreibeins, the matter coupling takes the familiar form if the equations of motion are used. We worked out the three-dimensional case, but the generalization
to topological gravity in higher dimensions [8] is straightforward. We also presented
the supersymmetric analogue of this construction.

Future work should examine the quantum theory of the proposed matter in-
teraction in a perturbative setting. Since we included matter only by spontaneous
symmetry breaking, we can immediately deduce that the pure gravity sector remains
finite. Since three-dimensional gravity is a specific example of a Chern-Simons theory,
the perturbative analysis may be performed along the lines of [9]. For the case of pure
gravity perturbation could be performed in the unbroken background $e_\mu^a = 0$. Having
included matter interactions, the fields used will obtain propagators only if one ex-

pands around some non-zero background. For the case of pure gravity this expansion
and the perturbative analysis was performed in [10]. With matter many new vertices
and diagrams arise. Apart from questions about non-zero backgrounds any quantum
analysis of a topological theory requires the introduction of a background metric to
fix the gauge and derive propagators. For pure gravity, the resulting quantum theory
remains independent of this background metric [2,11]. In general, however, it is not
guaranteed that a theory which is metric independent at the classical level would
remain so at the quantum level [12]. It is then of interest to study whether the prop-
erty of metric independence is lost in the presence of matter. We want to emphasize
that for proving a possible metric dependence it is not enough to find divergences
that can only be cancelled by using the background metric. The situation may be
compared to Yang-Mills theory in the axial gauge $n^\mu A_\mu = 0$ where $n^\mu$ plays the
role of the background metric. There, at the one-loop level counterterms have been
found that were dependent on $n^\mu$ [13]. Later, the situation was re-investigated by
using BRST methods and it became possible to control this gauge dependence [14].
A BRST analysis along the lines of [14] should also be applied to study a possible
background dependence of topological gravity in the presence of matter. This will
then decide whether topological gravity keeps all of its nice features after matter is
coupled.

References

[1] P. van Nieuwenhuizen, Phys. Rep. 68 (1981) 191, and references therein.
[2] E. Witten, Nucl.Phys. B311 (1988) 96; B323 (1989) 113.
[3] J. Gegenberg, G. Kunstatter and H.P. Leivo, Phys.Lett. B252 (1990) 381.
[4] S. Carlip and J. Gegenberg, Phys. Rev. D44 (1991) 424.
[5] A.H. Chamseddine, *Anal.Phys.* **113** (1978) 219; *Nucl.Phys.* **B131** (1977) 494; H.G. Pagels, *Phys.Rev.* **D27** (1983) 2299.

[6] A. Achúcarro and P.K. Townsend, *Phys.Lett.* **B180** (1986) 89.

[7] A.H. Chamseddine, A. Salam and T. Strathdee, *Nucl.Phys.* **B136** (1978) 248.

[8] A.H. Chamseddine, *Nucl.Phys.* **B346** (1990) 213.

[9] L. Alvarez Gaumé, J. Labastida and A.V. Ramallo, *Nucl.Phys.* **B334** (1990) 103; E. Guadagnini, M. Martellini and M. Mintchev, *Phys.Lett.* **B334** (1989) 111; C.P. Martin, *Phys.Lett.* **B241** (1990) 513.

[10] S. Deser, J. McCarthy and Z. Yang, *Phys.Lett.* **B222** (1989) 61.

[11] D. Ray and I. Singer, *Adv.Math.* **7** (1971) 145; *Ann.Math.* **98** (1973) 154.

[12] M. Blau and G. Thompson, *Phys.Lett.* **B255** (1991) 535.

[13] D. M. Capper and G. Leibbrandt, *Phys. Rev.* **D 25** (1982) 1002; *Phys. Rev.* **D 25** (1982) 1009.

[14] P. Gaigg, O. Piguet, A. Rebhahn and M. Schweda, *Phys. Lett.* **175** B (1986) 53.