Improved Detection Strategies for Nonlinear Frequency-Division Multiplexing

S. Civelli\textsuperscript{1}, E. Forestieri\textsuperscript{1} and M. Secondini\textsuperscript{1}

\textsuperscript{1}TeCIP Institute, Scuola Superiore Sant’Anna, Italy

Abstract—Two novel detection strategies for nonlinear Fourier transform-based transmission schemes are proposed. We show, through numerical simulations, that both strategies achieve a good performance improvement (up to 3 dB and 5 dB) with respect to conventional detection, respectively without or only moderately increasing the computational complexity of the receiver.

1. INTRODUCTION

Current optical fiber communication systems are limited by the Kerr nonlinearity, which is one of the main impairments hindering the increase of the transmission rate. To overcome this limitation and master nonlinearity, in the recent years, nonlinear spectrum modulation paradigms [1]–[8] have been investigated, devising the integrability of the nonlinear Schrödinger equation (NLSE)—which models the propagation of a signal in an optical fiber—with the nonlinear Fourier transform (NFT) [2],[3],[9]. The NFT is a sort of nonlinear analogue of the standard linear Fourier transform (FT), defining a nonlinear spectrum that undergoes just a phase rotation during propagation along the optical channel. The umbrella term nonlinear frequency-division multiplexing (NFDM) indicates NFT-based transmission schemes that encode the information directly on the nonlinear spectrum, such that deterministic propagation effects—dispersion and nonlinearity—can be exactly removed at the receiver (RX) with a single-tap operation. However, despite its theoretical robustness against nonlinearity, it is not yet clear whether NFDM can outperform conventional systems [4],[8]. Nevertheless, research about NFDM is still in progress, and NFDM schemes are far away from being fully optimized.

NFDM paradigms have been developed borrowing concepts from linear communication and, indeed, can be thought as a nonlinear version of the well known orthogonal frequency-division multiplexing (OFDM), using a backward NFT (BNFT) at the transmitter (TX) to encode information, and a forward NFT (FNFT) at the RX to recover it. The detection strategy commonly considered for NFDM, also borrowed from linear systems and optimal for an additive white gaussian noise (AWGN) channel, is not optimal for NFDM since it does not account for the actual statistics of noise in the nonlinear frequency domain. Therefore, the currently achieved NFDM performance can be much improved, and novel detection strategies tailored for NFDM might reveal its potential and allow to actually outperform conventional systems. A first attempt towards this direction is represented by the decision-feedback BNFT (DF-BNFT) detection strategy [10],[11], which provides a significant performance improvement with respect to conventional NFDM, at the expense of a significant computational complexity. In this paper, we introduce two novel detection strategies for NFDM by exploiting the same causality property of the NFT on which DF-BNFT detection is based. We compare this two novel strategies with standard detection and with DF-BNFT, both in terms of performance and computational complexity.

2. SYSTEM DESCRIPTION AND DETECTION STRATEGIES

The system setup is sketched in Fig. 1. The TX, similarly to the nonlinear inverse synthesis (NIS) technique [6], modulates a train of pulses \( g(t) \) with \( N_b \) quadrature phase-shift keying (QPSK) symbols, and its FT is mapped onto the continuous part of the nonlinear spectrum \( \rho(\lambda) \). Deterministic propagation effects are removed (full precompensation) multiplying the nonlinear spectrum by \( \exp(4j\lambda^2L) \), \( L \) being the normalized channel length, and a BNFT is performed to obtain the samples of \( q'(t) \). The optical signal is then obtained with a digital-to-analog converter (DAC) as \( q(t) = q'(-t) \) and launched into the fiber. Further details about the TX setup can be found in [11]. At the RX, the samples of the received noisy signal \( \tilde{r}(t) \) are obtained with an analog-to-digital converter (ADC) and used for detection. In this work, we consider the four different detection strategies schematically depicted in Fig. 2 and described later in this section. The incremental
FNFT (I-FNFT) and decision-feedback FNFT (DF-FNFT) strategies are introduced in this paper for the first time.

An important causality property of the NFT applies to the considered transmission scheme as follows. Let $T_s$ be the symbol time, and $t_k = (k - 1/2)T_s$, such that $(t_{k-1}, t_k)$ is the symbol time corresponding to the $k$-th symbol. Let $r(t)$ be the optical signal obtained propagating $q(t)$ in the ideal noiseless channel, i.e., the optical signal obtained at the TX if precompensation is not employed. If the pulsewidth is shorter than the symbol time in the ideal noiseless channel, i.e., the optical signal obtained at the TX if precompensation is not employed. If the pulsewidth is shorter than the symbol time $T_s$, then $r(t)$ for $t \leq t_k$ depends only on the first $k$ symbols $x_1, \ldots, x_k$, and not on the next ones $x_{k+1}, \ldots, x_{N_t}$ [11]. This causality property can be employed, as explained in the following, to detect symbols in an iterative way, and using decision feedback.

The more conventional detection strategy for NFDM—referred to as FNFT detection in the following—consists in computing the FNFT of the received signal to obtain the nonlinear spectrum, and then detecting symbols in the nonlinear frequency domain after matched filtering and symbol-time sampling [2], [3], [6], [11]–[13]. This simple detection strategy would be optimal if the noise in the nonlinear spectrum were AWGN, which, however, is true only at low power. Indeed, at higher power, signal and noise interact during propagation (much like in conventional systems) and, more importantly, the FNFT operation—a nonlinear operation—significantly affects noise statistics. More details about this can be found in [11]. The DF-BNFT detection strategy, already introduced and investigated in previous papers [10], [11], avoids this problem by detecting symbols in time-domain (before the FNFT), using decision feedback and BNFT. This detection strategy, though not optimal, provides a significant performance improvement (up to 7dB [11]), but requires to compute $M$ BNFTs, $M$ being the constellation order. A detailed investigation about DF-BNFT is available in [11].

The I-FNFT strategy is based on deciding symbols in an iterative way in the nonlinear frequency domain, analyzing only a portion of the received signal to reduce noise in the nonlinear spectrum, which increases with signal energy [12]. Specifically, the $k$-th symbol is detected operating as in conventional FNFT detection (i.e., FNFT, matched filtering, and sampling), but on the signal

$$\hat{r}_k(t) = \begin{cases} \hat{r}(t) & t \leq t_k \\ 0 & \text{else} \end{cases}$$

(1)

This detection strategy can be implemented with the same computational complexity of FNFT detection. Indeed, the nonlinear spectrum is computed (e.g., with the Boffetta Osborne method [2], [3]) recursively adding a small portion of the optical signal and multiplying for the transfer matrix of this contribution; in our case, this means that at the $k$-th step, one has already computed the contribution of the optical signal for $t \leq t_{k-1}$ and needs to add only the contribution of the signal in $(t_{k-1}, t_k]$, resulting overall in a single FNFT.

The DF-FNFT strategy adds a further step to the I-FNFT one: besides considering only a portion of the signal in detection, it also takes advantage of the feedback given by already decided
symbols to clean the received signal. Specifically, given the symbols \( \hat{x}_1, \ldots, \hat{x}_{k-1} \) already decided, the \( k \)-th symbol is decided with two steps: (i) digitally perform a BNFT to obtain for \( t < t_{k-1} \) the noiseless signal \( r_{k-1}(t) \) which corresponds to the symbol sequence \( \hat{x}_1, \ldots, \hat{x}_{k-1} \) (this is obtained performing the same operation of the TX, but for precompensation), and (ii) perform standard detection (i.e., FNFT, matched filter, and sampling) on the signal

\[
\hat{r}_k(t) = \begin{cases} 
    r_{k-1}(t) & t \leq t_{k-1} \\
    \tilde{r}(t) & t_{k-1} < t \leq t_k \\
    0 & \text{else}
\end{cases}
\]  

(2)

to detect \( \hat{x}_k \). Importantly, DF-FNFT requires to perform at the RX a total of one BNFT and two FNFT. Indeed, as far as it concerns (i), at the \( k \)-th step one needs to evaluate \( r_{k-1}(t) \) performing a BNFT only for \( t \in (t_{k-2}, t_{k-1}] \), since the values for \( t \leq t_{k-2} \) have already been evaluated at the previous step, resulting overall in a single BNFT. Regarding (ii), similarly to the I-FNFT case, one needs to add the contribution of the signal in two symbol times for \( t \in (t_{k-2}, t_k] \), therefore resulting in two FNFT.

Remarkably, both detection strategies, as well as DF-BNFT, choose the \( k \)-th symbol \( x_k \) accounting only for its contribution in the time window \((t_{k-1}, t_k] \). While \( x_k \) does not contribute to the signal before \( t_{k-1} \), it does for \( t > t_k \), with this contribution increasing at higher energies. Therefore, these detection strategies do not consider all the available information, thus reducing the effective signal to noise ratio (SNR). However, removing part of the signal also improves performance, as shown in the next section. Moreover, for what it concerns DF-FNFT, considering \( r_k(t) = \tilde{r}(t) \) for \( t > t_{k-1} \) would drive to a much more computationally complex detection: at the \( k \)-th step, one should perform FNFT adding the contribution of the signal to \( N_b - k + 2 \) symbols (rather than 1).

3. SYSTEM PERFORMANCE

System performance was evaluated through simulations. The channel is a standard single mode fiber of length \( L = 4000 \) km (group velocity dispersion parameter \( \beta_2 = 20.39 \) ps\(^2\)/km, nonlinear coefficient \( \gamma = 1.22 \) W\(^{-1}\)km\(^{-1}\), and attenuation \( \alpha = 0.2 \) dB/km) with ideal distributed amplification (spontaneous emission factor \( \eta_{sp} = 4 \)). The DAC and ADC bandwidth is 100 GHz. The symbol rate is \( R_s = 1/T_s = 10 \) GBAud and the basic pulse \( g(t) \) is Gaussian with 99% of the energy contained into a symbol time \( T_s \). To avoid overlapping of different bursts during propagation and ensure the vanishing boundary conditions of the NFT type considered here, \( N_s = 160 \) guard symbols are inserted between different bursts. To account for the loss in spectral efficiency due to guard symbols insertion between bursts we consider the rate efficiency term \( \eta = N_b/(N_b + N_s) \).

Numerical NFT operations are performed with an oversampling factor of 8 samples per symbol. The FNFT is numerically performed using the Boffetta-Osborne method \[2,11\], while the BNFT is computed with an enhanced version of the Nystrom method \[11,14\]. System performance is measured in terms of Q-factor as \( Q_{dB} = 20 \log_{10}[\sqrt{2}\text{erfc}^{-1}(2P_b)] \), where the bit error rate \( P_b \) is estimated by direct error counting.

The performance obtained through simulations are shown in Fig. 3(a), 3(b), and 4(a) for \( N_b = 128, 256, 512 \), respectively. Firstly, the figures show that FNFT standard detection for NFDM performs worse, as a consequence of being a detection strategy not optimal in the nonlinear frequency domain. Secondly, I-FNFT performs better than FNFT detection allowing for an improvement of up to 3 dB without increasing the computational complexity at all. Next, the figures show that a further performance improvement can be achieved with DF-FNFT detection, at the expense of increasing the computational complexity by additionally performing one FNFT and one BNFT. Finally, DF-BNFT detection provides the best performance, with a gain of up to about 3 dB with respect to DF-FNFT, and about 7 dB with respect to the conventional FNFT.

Figure 4(b) reports the optimal performance as a function of the rate efficiency \( \eta \). The figure shows that increasing the rate efficiency, i.e., the number of information symbols per burst, performance decreases \[11,12\]. Moreover, Fig. 4(a) emphasizes the relative behavior of the considered detection strategies: FNFT performs worse than all others, I-FNFT achieves better results than FNFT, but worse than DF-FNFT, DF-BNFT performs better than all others.

4. CONCLUSION

In this paper, motivated by the fact that, performance-wise, the currently considered detection strategy is one of the main critical aspects of NFDM, we proposed two novel detection strategies.
Figure 3: NFDM performance for different detection strategies vs power for: (a) $N_b = 128$ ($\eta = 44\%$), and (b) $N_b = 256$ ($\eta = 62\%$).

Figure 4: NFDM performance for different detection strategies: (a) vs power for $N_b = 512$ ($\eta = 76\%$), and (b) optimal performance as a function of the rate efficiency $\eta$.

specifically designed for this transmission technique. We showed that, by relying on an NFT causality property, the received signal can be partly “cleaned” before computing its nonlinear spectrum, thus reducing the detrimental “noise amplification” effect taking place on the latter and badly affecting conventional FNFT detection. In the I-FNFT detection technique, the noise cleaning effect is limited to the signal portion following the symbol to be detected; in the DF-FNFT technique, it is extended also to the previous part of the signal by using decision feedback. In both cases, decisions are made in the nonlinear frequency domain, after matched filtering and symbol-time sampling, exactly as in conventional FNFT detection and in contrast to DF-BNFT detection, which completely avoids the noise amplification effect by making decisions in the time domain [1]. We compared the two novel detection strategies with the conventional FNFT and the DF-BNFT ones. Both the I-FNFT and DF-FNFT strategies perform better than the conventional FNFT one, with gains of up to 3 and 5 dB, respectively, proving that currently considered NFDM schemes are not optimized and can be enhanced. Even if both detection strategies turn out to be inferior to the DF-BNFT strategy, their computational complexity is considerably lower and comparable to that of the conventional FNFT one.

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