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Spin-orbit splitting of $^9\Lambda$Be excited states studied with the SU$_6$ quark-model baryon-baryon interactions

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The previous Faddeev calculation of the two-alpha plus $\Lambda$ system for $^9\Lambda$Be is extended to incorporate the spin-orbit components of the SU$_6$ quark-model (QM) baryon-baryon interactions. We employ the Born kernel of the QM $\Lambda N$ LS interaction and generate the spin-orbit component of the $\Lambda$ potential by $\alpha$-cluster folding. The Faddeev calculation in the $jj$-coupling scheme implies that the direct use of the QM Born kernel for the $\Lambda N$ LS component is not good enough to reproduce the small experimental value $\Delta E_{\text{exp}}^{\text{LS}}=43\pm5$ keV for the $5/2^+\rightarrow 3/2^+$ splitting. This procedure predicts 3–5 times larger values in the models FSS and fss2. The spin-orbit contribution from the effective meson-exchange potentials in fss2 is argued to be unfavorable to the small $\ell s$ splitting, through the analysis of the Scheerbaum factors for the single-particle spin-orbit potentials calculated in the G-matrix formalism.

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The study of hypernuclei based on the fundamental baryon-baryon interactions is important, since the available scattering data for the hyperon-nucleon (YN) interaction are very scarce. We have recently proposed a comprehensive quark-model (QM) description of general baryon-octet baryon-octet ($B_B$) interactions, which is formulated in the $(3g)$-$(3g)$ resonating-group method (RGM) using the spin-flavor SU$_6$ QM wave functions, a colored version of the one-gluon exchange Fermi-Breit interaction, and effective meson-exchange potentials (EMEPs) acting between quarks [1–3]. The early version, the model FSS [1], includes only the scalar (S) and pseudoscalar (PS) meson-exchange potentials as the EMEPs, while the renovated one fss2 [2,3] introduces also the vector (V) meson exchange potentials and the momentum-dependent Bryan-Scott terms for the S and V mesons. Owing to these improvements, the model fss2 in the NN sector has attained an accuracy comparable to that of one-boson-exchange potentials (OBEPs).

These QM interactions can now be used for various types of many-body calculations. In the previous paper [4], we have carried out Faddeev calculations of the two-alpha plus $\Lambda\alpha\Lambda$ system, in which a two-range Gaussian $\Lambda N$ potential (called the SB potential), generated from the phase-shift behavior of the model fss2 [2,3] is employed. If we use the pure Serber-type $\Lambda N$ potential with the Majorana exchange mixture parameter $\varrho=1$, this Faddeev calculation with the proper treatment of the Pauli principle in the $\alpha\alpha$ RGM kernel can reproduce the ground-state and excitation energies of the $^9\Lambda$Be hypernucleus within 100–200 keV accuracy.

Another important piece of experimental information from $^9\Lambda$Be is the small spin-orbit splitting of the $5/2^+$ and $3/2^+$ excited states, $\Delta E_{\text{exp}}^{\ell s}=43\pm5$ keV [5,6], measured from the recent Hyperball $\gamma$-ray spectroscopy. It is widely known that the single-particle (s.p.) spin-orbit interaction of the $\Lambda$ hyperon seems to be extremely small, especially in light $\Lambda$ hypernuclei. In the nonrelativistic models of the YN interaction, this is a consequence of the strong cancellation of the ordinary LS component and the antisymmetric LS component ($LS^{-}$ force), the latter of which is a characteristic feature of baryon-baryon interactions between nonidentical baryons. For example, the SU$_6$ QM baryon-baryon interaction FSS [1] yields a strong $LS^{-}$ component [7], which is about one-half of the ordinary LS component, with the opposite sign. We performed the G-matrix calculation in symmetric nuclear matter, using this QM baryon-baryon interaction [8], and calculated the so-called Scheerbaum factor $S_{\text{p}}$, which indicates the strength of the s.p. spin-orbit interaction [9]. The ratio of $S_{\text{p}}$ to the nucleon strength $S_{\text{N}}\sim40$ MeV fm$^3$ is $S_{\text{p}}/S_{\text{N}}\sim1/5$ and $S_{\text{p}}/S_{\text{N}}\sim1/2$ in the Born approximation. The G-matrix calculation of the model FSS modifies $S_{\text{p}}$ to $S_{\text{p}}/S_{\text{N}}\sim1/12$. The significant reduction of $S_{\text{p}}$ in the G-matrix calculation of FSS is traced back to the enhancement of the antisymmetric LS component in the diagonal $\Lambda N$ channel, owing to the $P$-wave $\Lambda N$ coupling.

Hiyama et al. [10] calculated the $\Lambda N$ spin-orbit splitting in $^9\Lambda$Be and $^{13}\Lambda C$ in their cluster model, by using simple approximations of the Nijmegen one-boson-exchange $\Lambda N$ interactions. They employed several two-range Gaussian LS potentials for the $\Lambda N$ interaction, which simulate the $LS$ and $LS^{-}$ parts of the $G$-matrix interactions derived from Nijmegen model-D (ND), model-F (NF), and NSC97a-f interactions. For example, they obtained $\Delta E_{\text{LS}}^{\ell s}=0.16$ MeV for NSC97f. When the $LS^{-}$ force is switched off, they obtained 0.23 MeV. Since these values are too large to compare with the experiment, they adjusted the strength of the $LS^{-}$ potentials, guided by the relative strength of the QM $LS^{-}$ force. Such a procedure, however, does not prove the adequacy of the QM spin-orbit interaction for the experimental data.

The purpose of this Brief Report is to show that, if we carry out more serious calculations starting from the the QM...
baryon-baryon interactions, the situation is not so simple as stated in Ref. [10]. Here we concentrate only on the spin-orbit interaction and use the QM exchange kernel directly, following our basic idea in other applications of our SU$_6$ QM baryon-baryon interactions [11–13]. The Λα spin-orbit interaction is generated from the Born kernel of the ΛN LS QM interaction, and the Faddeev equation is solved in the jj-coupling scheme, by using the central plus spin-orbit Λα interactions. We find that our model FSS yields spin-orbit splittings of almost 2/3 of the Nijmegen NSC97f result. We find a large difference between FSS and fss2 for the effect of the short-range correlations, especially in the way of the P-wave ΛN-ΣN coupling.

We assume that the ΛN LS interaction is given by the Born kernel of the ΛN QM interaction [9]:

$$u^L_{\Lambda N}(q_f,q_i) = \sum_{\Omega} \sum_{\tau} \Omega^L \cdot \Omega^{S^+} \cdot \Omega^{S^-} \cdot \Omega^T \cdot \Omega^S \cdot \Omega_D$$

where $\Omega=LS$, $L^-(\cdot)$ and $S^-(\cdot)$ are the three different types of spin-orbit operators $\Omega^L = in \cdot S$, $\Omega^{S^+} = in \cdot S^+$, and $\Omega^{S^-} = in \cdot S^-$, and $\Omega^T$, $\Omega^S$, $\Omega_D$, and $\Omega$ stands for various interaction types originating from the quark antisymmetrization. Here we use the standard notation $n=(q_f \times q_i)$, $S=(\sigma_f + \sigma_i)/2$, $S^-=(-\sigma_f - \sigma_i)/2$, $S^+=(\sigma_f + \sigma_i)/2$, $P=(1+\sigma_f \cdot \sigma_i)/2$, etc. The up-down and strange spin-flavor factors are explicitly given in Refs. [7] and [9]. If we take the matrix element of Eq. (1) with respect to the spin-flavor functions of the Λα system, the nucleon spin operator part disappears due to the spin saturated property of the α cluster and we obtain the spin-flavor part as $X^L_{\Lambda N}$ and $X^S_{\Lambda N}$ with $X^L_{\Lambda N} = (\sigma_f + \sigma_i)/2$. $X^S = \left[ (X^L_{\Lambda N})^{us} + (X^L_{\Lambda N})^{ds} \right] / 2$, and $X^S = \left[ (X^L_{\Lambda N})^{us} + (X^L_{\Lambda N})^{ds} \right] / 2$. We therefore only need to calculate the spatial integrals of $f^L_{\Lambda N}(-\theta)$ and $f^S_{\Lambda N}$ with $\cos \theta = (q_f \cdot q_i)$. For this calculation, we can use a convenient formula Eq. (B6) given in Appendix B of Ref. [4]. The calculation is carried out analytically, since it only involves Gaussian integration. We finally obtain

$$V^L_{\Lambda N}(q_f,q_i) = \sum_{\tau} \left[ T^L \cdot T^S \cdot d(q_f,q_i) + T^L \cdot T^S \cdot e(q_f,q_i) \right] \{ n \cdot S \}.$$  

We calculate the spin-flavor factors and spatial integrals for each of the interaction types, $T=D_-, D_+$, and $S(S')$. From our previous paper [7], we find the spin-flavor factors given in Table I. Note that the most important knock-on term of the $D_-$ type turns out to be zero in the Λα direct potential, because of the exact cancellation between the LS and L$^-$ factors in the up-down sector. As a result, the main contribution to the Λα spin-orbit potential in the present formalism comes from the strangeness exchange $D_-$ term, which is non-local and involves a very strong momentum dependence. If the quark mass ratio $\lambda = (m_f/m_{ud})$ goes to infinity, all of these spin-flavor factors vanish, which is a well-known property of the spin-flavor SU$_6$ wave function of the Λ particle. Only the strange quark of Λ contributes to the spin-related quantities like the magnetic moment, since the up-down diquark is coupled in the spin-isospin zero for Λ. The explicit expressions of the spatial integrals $V^L_{\Lambda N}(q_f,q_i)$ and $V^S_{\Lambda N}(q_f,q_i)$ will be given elsewhere, since they are rather lengthy. The partial-wave components of Eq. (2) are calculated from the formula in Appendix C of Ref. [14] by using the Gaussian-Legendre 20-point quadrature formula. Since the model fss2 contains the LS components from the EMEPs, we should also include these contributions to the Λα spin-orbit interaction. A detailed derivation of the EMEP Born kernel for the Λα system is deferred to a separate paper.

For the Faddeev calculation, we use the same conditions as used in Ref. [4], except for the exchange mixture parameter $u$ of the SB ΛN potential. We here use a repulsive ΛN odd interaction with $u=0.82$ in order to reproduce the ground-state energy of $^9$Be. This is because the 5/2$^+$–3/2$^+$ $\ell s$ splitting is rather sensitive to the energy positions of these states, measured from the $^9$He-α threshold. We also use the Nijmegen-type ΛN potentials from Ref. [15]. The α RGM kernel is generated from the three-range Minnesota force with $u=0.946$ 87. The harmonic oscillator width parameter of the α cluster is assumed to be $\nu=0.257$ fm$^{-2}$. The partial waves up to $\lambda_{max}^\alpha = 10$ are included both in the $\alpha$ and Λα channels. The momentum discretization points are selected by $n_1-n_2-n_3=10-10-5$ with the midpoints $p$, $q=1$, 3, and 6 fm$^{-1}$. The Coulomb force is incorporated in the cutoff Coulomb prescription with $R_c=10$ fm.

Table II shows the results of Faddeev calculations in the jj-coupling scheme. First we note that the ground-state energies do not change much from the LS-coupling calculation, which implies the dominant S-wave coupling of the Λ hyperon. The final values for the $\ell s$ splitting of the 5/2$^+$–3/2$^+$ excited states are $\Delta E_{\ell s}=137$ keV for FSS and 198 keV for fss2, when the SB force with $u=0.82$ is used for the ΛN central force. If we compare these results with the experimental value $43\pm5$ keV, we find that our QM predictions are 3–5 times too large. If we use the G-matrix-simulated NSC97f LS potential in Ref. [10], we obtain 209 keV for the same SB force with $u=0.82$. The difference from 0.16 MeV in Ref. [10] is due to the model dependence to the $\alpha\alpha$ and $\Lambda\alpha$ central interactions. We find that our QSS prediction for $\Delta E_{\ell s}$ is less than 2/3 of the NSC97f prediction, while fss2
TABLE II. The ground-state energy \( E_g (1/2^+), 5/2^+ , 3/2^+ \) excitation energies \( E_x (5/2^+), E_x (3/2^+) \), and spin-orbit splitting \( \Delta E_{\ell x} \) calculated by solving the Faddeev equations for the \( \alpha \Lambda N \) system in the \( jl \)-coupling scheme. The exchange interaction parameter of the SB \( \Lambda N \) force is assumed to be \( u=0.82 \). The \( \Lambda \alpha \) spin-orbit force is generated from the Born kernel of the FSS and fss2 \( \Lambda N LS \) interactions. For the fss2 \( LS \) interaction, the \( LS \) component from the EMEPs is also included.

| \( u_{\Lambda N}^{LS} \) | \( u_{\Lambda N}^{C} \) | \( E_{g}^{(1/2^+)} \) (MeV) | \( E_{x}^{(5/2^+)} \) (MeV) | \( E_{x}^{(3/2^+)} \) (MeV) | \( \Delta E_{\ell x} \) (keV) |
|---|---|---|---|---|---|
| SB | -6.623 | 2.854 | 2.991 | 137 |
| NS | -6.744 | 2.857 | 2.997 | 139 |
| FSS | ND | -7.485 | 2.872 | 3.024 | 152 |
| NF | -6.908 | 2.877 | 3.002 | 125 |
| JA | -6.678 | 2.866 | 2.991 | 124 |
| JB | -6.476 | 2.858 | 2.980 | 122 |
| SB | -6.623 | 2.828 | 3.026 | 198 |
| NS | -6.745 | 2.831 | 3.033 | 202 |
| fss2 | ND | -7.487 | 2.844 | 3.064 | 220 |
| NF | -6.908 | 2.853 | 3.035 | 182 |
| JA | -6.678 | 2.843 | 3.024 | 181 |
| JB | -6.477 | 2.834 | 3.012 | 178 |
| Expt. [6] | -6.62(4) | 3.024(3) | 3.067(3) | 43(5) |

Figure 1 shows the comparison of the \( \Lambda \alpha \) spin-orbit potentials predicted by the Wigner transform of FSS with \( q=0, 1, 2, 3 \) fm\(^{-1}\) and \( \bar{R}q=0 \) (solid curves), the Scheerbaum potential with \( S^\ell_{\Lambda}=-10.12 \) MeV fm\(^5\) (dotted curve), and the G-matrix-simulated NSC97f-type potential [10] (dashed curve). An appropriate \( S^\ell_{\Lambda} \) is shown in Figure 1. The \( \Lambda N \) potential for this \( \Lambda N \) potential is calculated to be \( S_{\Lambda}=-10.34 \) MeV fm\(^5\) for \( \bar{q}=0.7 \) fm\(^{-1}\). If we use the Scheerbaum potential with \( S^\ell_{\Lambda}=-13.41 \) MeV fm\(^5\), we obtain \( \Delta E_{\ell x} = 194 \) keV, which is close to 209 keV.

Table III lists the results of G-matrix calculations for the Scheerbaum factors \( S_{\Lambda} \) in symmetric nuclear matter. The Fermi momentum \( k_F = 1.07 \) fm\(^{-1}\), corresponding to half of the normal density \( \rho_0 = 0.17 \) fm\(^{-3}\), is assumed. For solving G-matrix equations, the continuous prescription is used for intermediate spectra. Table III also shows the decompositions into various contributions and the results when the \( \Lambda N \)-\( \Sigma N \) coupling through the \( LS^{(-)} \) and \( LS^{(+)} \) \( \sigma \) forces is switched off (coupling off) in the G-matrix calculations. For FSS, we find a large reduction of \( S_{\Lambda} \) value from the Born value \(-7.8 \) MeV fm\(^5\), especially when this (dominantly) \( P \)-wave \( \Lambda N \Sigma N \) coupling is properly taken into account. When all

| Model | \( LS \) | \( LS^{(-)} \) | \( LS^{(+)} \) |
|---|---|---|---|
| FSS | -17.36 | -18.43 | -18.37 |
| | -0.38 | 0.22 | 0.26 |
| | -19.70 | 8.37 | 0.30 |
| total | -1.93 | -10.77 |
| fss2 | -19.97 | -8.64 | -11.26 |
| | -0.14 | 0.21 | -14.89 |
the \(\Lambda N-\Sigma N\) couplings, including those by the pion tensor force, are switched off, the \(LS^{-1}\) contribution is just a half of the \(LS\) contribution with the opposite sign (in the dominant odd partial waves), which is the same result as in the Born approximation. The \(1^{P_{2}}+1^{F_{2}}\) \(\Lambda N-\Sigma N\) coupling enhances the attractive \(LS\) contribution slightly, while the \(1^{P_{1}}+1^{F_{1}}\) \(\Lambda N-\Sigma N\) coupling enhances the repulsive \(LS^{-1}\) contribution largely. If we use this reduction of the \(S_{\Lambda}\) factor from \(-7.8\) to \(-1.9\) MeV fm\(^{5}\) in the realistic \(G\)-matrix calculation, we find that the present \(\Delta E_{\text{ls}}\) value \(-137\) MeV is reduced to an almost correct value \(-33\) keV. However, such a reduction of the Scheerbaum factor due to the \(\Lambda N-\Sigma N\) coupling is supposed to be hindered in the \(\Lambda\alpha\) system in the lowest-order approximation from the isospin consideration. On the other hand, the situation of \(\text{fss}2\) in Table III is rather different, although the cancellation mechanism between the \(LS\) and \(LS^{-1}\) components and the reduction effect of \(S_{\Lambda}\) factor in the full calculation are equally observed. When all the \(\Lambda N-\Sigma N\) coupling is neglected, the ratio of the \(LS^{-1}\) and \(LS\) contributions in the quark sector is still one-half. Since the EMEP contribution is mainly for the \(LS\) type, it amounts to about \(-6\) MeV fm\(^{3}\), which is very large and remains with the same magnitude even after the \(P\)-wave \(\Lambda N-\Sigma N\) coupling is included. Furthermore, the increase of the \(LS^{-1}\) component is rather moderate, in comparison with the FSS case. This is because the model \(\text{fss}2\) contains an appreciable EMEP contribution (~40\%) which has very few \(LS^{-1}\) contributions. As a result, the total \(S_{\Lambda}\) value in \(\text{fss}2\) \(G\)-matrix calculations is 3–6 times larger than the FSS value, depending on the Fermi momentum \(k_{F}=1.35\)–\(1.07\) fm\(^{-1}\). Such an appreciable EMEP contribution to the \(LS\) component of the \(YN\) interaction is not favorable to reproduce the negligibly small \(\ell s\) splitting of \(^{8}\)Be.

Summarizing this work, we have performed the \(jj\)-coupling Faddeev calculations for \(^{8}\)Be by incorporating \(\Lambda\alpha LS\) interactions generated from the Born kernel of the QM baryon-baryon interactions. This calculation corresponds to an evaluation of the Scheerbaum factors in the Born approximation. Since the \(P\)-wave \(\Lambda N-\Sigma N\) coupling is not properly taken into account, the present calculation using the FSS Born kernel yields too large spin-orbit splitting of the \(5/2^{+}\) and \(3/2^{+}\) excited states of \(^{8}\)Be by a factor of 3. In the model FSS, a reduction by a factor of \(1/2–1/4\) is expected in the \(G\)-matrix calculation of the Scheerbaum factor \(S_{\Lambda}\) [9], depending on the Fermi momentum \(k_{F}=1.35\)–\(1.07\) fm\(^{-1}\). In \(\text{fss}2\), the \(G\)-matrix calculation for the Scheerbaum factor yields a rather large value \(S_{\Lambda}\sim11\) MeV fm\(^{2}\), with very weak \(k_{F}\) dependence, due to the appreciable EMEP contributions. The QM baryon-baryon interaction with a large spin-orbit contribution from the meson-exchange potentials is, in general, unfavorable to reproduce the very small \(\ell s\) splitting observed in \(^{8}\)Be. It is a future problem how to incorporate the \(P\)-wave \(\Lambda N-\Sigma N\) coupling in cluster model calculations like the present one.

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