X-Ray Quasi-periodic Oscillations in the Lense–Thirring Precession Model. II. Variability of the Relativistic Iron Kα Line

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Abstract

The reprocessing of primary X-ray emission in the accretion disk of black hole X-ray binaries (BHXRBs) produces a reflection spectrum with the characteristic Fe Kα fluorescent line. Strong low-frequency quasi-periodic oscillations (QPOs) are observed from BHXRBs, and the dependence of QPO properties (e.g., phase lag) on the inclination angle suggests that the observed QPO may be associated with a geometrical effect, e.g., the precession of the X-ray source due to frame dragging near the spinning black hole. Here, in the scenario of the Lense–Thirring precession of the X-ray source, we use a Monte Carlo simulation of radiative transfer to study the irradiation/reflection and the resultant spectral properties including the Fe Kα line as a function of precession phase (time). We found that the reflection fraction, i.e., the ratio of incident flux toward the disk and the direct flux toward the observer at infinity, is modulated by the precession phase, which depends on the truncation radius (i.e., the spectral state in the truncated disk model) and the inclination angle. The Fe Kα line profile also changes as the primary X-ray source precesses, with the line luminosity and the flux-weighted centroid energy varying with the precession phase. The periodically modulated 2–10 keV continuum flux could apparently lag the line luminosity in phase, if the truncation radius is small enough for Doppler effects due to disk orbital motion to significantly affect the observed radiation.

Unified Astronomy Thesaurus concepts: Accretion (14); Black hole physics (159)

1. Introduction

In black hole X-ray binaries (BHXRBs), soft seed photons from the accretion disk are inverse Comptonized by hot electrons near the black hole (BH), contributing to the observed Comptonization component in the spectrum at high energy (Galeev et al. 1979; Haardt & Maraschi 1991; Cao 2009; Qiao & Liu 2012; You et al. 2012). A fraction of the Comptonized photons will irradiate the accretion disk where they are absorbed and/or scattered to produce the reflection emission. The characteristic features of the reflection emission are the hump between ~20–30 keV and the Fe Kα emission line at around $E = 6.4$ keV in the rest frame (George & Fabian 1991; You et al. 2018; Zhang et al. 2019).

One simple reflection geometry is the lamp-post model, in which the accretion disk extends down to some radius close to the BH, whereas the illuminating X-ray source is assumed to be a point source located on the spin axis of the BH, at a certain height (Miniutti & Fabian 2004; Niedźwiecki et al. 2016; Vincent et al. 2016). This X-ray illumination source could also be vertically extended (Kara et al. 2019). Another possible reflection geometry is the truncated disk model, in which the accretion disk is truncated at some radius $R_\text{tr}$ which could be larger than the innermost stable circular orbit (ISCO). Interior to $R_\text{tr}$ is the spatially extended illuminating source, which is of high temperature, acting as the Comptonized component (Plant et al. 2015; Basak & Zdziarski 2016; Mahmoud et al. 2019). This truncated disk geometry has been mentioned in the literature to explain the spectral transition and the evolution of the disk reflection properties (Esin et al. 1997; Done et al. 2007; Gilfanov 2010). Recently, this truncation geometry coupled with the so-called Lense–Thirring precession was considered as the possible origin of the observed quasi-periodic oscillation (QPO) in BHXRBs (Stella & Vietri 1998; Motta et al. 2015; Stiele & Yu 2016; van den Eijnden et al. 2017; Huang et al. 2018; Xiao et al. 2019; Zhang et al. 2020; see Belloni & Motta 2016 and Ingram & Motta 2019 for reviews of alternative QPO models). It is the inner hot flow (hereafter the corona) that precesses around the BH axis due to the frame-dragging effect, periodically modulating the observed X-ray flux (Ingram et al. 2009; You et al. 2018). In this case, the illumination pattern of the precessing flow on the disk will also vary as a function of time. This will eventually result in the modulation of the reflection including the Fe Kα line. Ingram & Done (2012) investigated the effect of precession of the illumination pattern on the resultant Fe Kα line. Given the rotation of the accretion disk, when the approaching side of the disk is predominantly illuminated by the X-ray photons from the flow, the resultant iron line will be blueshifted and boosted; when the receding side of the disk is illuminated, the iron line will be predominantly redshifted. As the illumination pattern on the disk rotates due to the precession of the flow, the overall shape and peak flux of the iron line from the entire disk will periodically rock between blueshift and redshift. Observationally, the Fe Kα line has been reported to be variable in some BHXRBs, e.g., GRS 1915+105 (Miller & Homan 2005). Ingram et al. (2016, 2017, hereafter ID17) applied a phase-resolved analysis on XMM-Newton and NuSTAR observations of H1743–322 and found that both the Fe Kα line centroid energy and the reflection fraction vary systematically as a function of QPO phase, with high significance, suggesting that the observed QPO is produced by precession.
The phase-dependent behavior of the Fe Kα line in the Lense–Thirring precession model is a powerful diagnostic of not only the geometry but also the dynamics of the accretion flow very close to the BH. This means that the analysis of the reflection component including the Fe Kα line as a function of QPO phase during the outburst can be used to study the evolution of the accretion flow geometry and relativistic effect in BHXRBs. However, this potential cannot be realized without quantitatively modeling the phase-dependent reflection component including the Fe Kα line in the Lense–Thirring precession model. For the scenario of the truncated disk, You et al. (2018) developed a Monte Carlo code to deal with the radiative transfer process for photons in the precessing hot flow to simulate the QPO variability arising from the Lense–Thirring precession in the framework of full general relativity. In our simulation, the code is able to self-consistently simulate both Comptonization and reflection (including Fe Kα) processes. The contribution of the spectral components, i.e., the disk emission, the Comptonization, and the reflection emission, to the variability was studied, which allows us to study the overall QPO variability as a function of photon energy. More importantly, by using the Monte Carlo code, You et al. (2018) quantitatively studied the evolution of the QPO variability during the spectral transition from the hard to soft states, and the correlation between the QPO variability and QPO frequency. In this work, we study the QPO phase-dependent properties of disk reflection, including the Fe Kα line, as a function of time (precession phase), in the Lense–Thirring precession.

A general description of our simulation will be summarized in Section 2. The modeling of the illumination/reflection patterns on the disk and the resultant Fe Kα line, as a function of the precession phase, will be shown in Section 3. The phase lag between the spectral properties and the effect of the observer position at infinity on the QPO properties will be discussed in Section 4. The main conclusions of this work are listed in Section 5.

2. Model

2.1. Geometry Configuration

In this work, we use a Monte Carlo simulation of radiative transfer to simulate the phase-resolved emission, e.g., the Comptonization and the reflection including the characteristic Fe Kα line, in the Lense–Thirring precession model for the BHXRB. Figure 1 shows the assumed geometry of the accretion flow. The outer thin accretion disk is truncated at $R_\infty$, which is also assumed to be the outer radius of the precessing corona. The inner radius of the corona is $R_m$. The axis vector of the outer disk $\mathbf{J}_D$ is misaligned with that of the black hole $\mathbf{J}_BH$ by an angle $\alpha$. We define the beginning phase as the moment when the axis vector of the corona $\mathbf{J}_C$ is misaligned with $\mathbf{J}_BH$ by an angle $\beta$. As the corona undergoes Lense–Thirring precession, the vector $\mathbf{J}_C$ circularly rotates around the vector $\mathbf{J}_BH$, with the angle $\gamma$ with respect to the beginning phase, varying from 0 to $2\pi$. The Cartesian coordinate $(X, Y, Z)$ is defined in such a way that the Z-axis is directed toward $\mathbf{J}_D$, and the X-axis is on the plane confined by $(\mathbf{J}_BH, \mathbf{J}_D)$, pointing to the misalignment direction. The viewing angle of the observer $\theta$ is defined with respect to the Z-axis, ranging from 0 and $\pi/2$, and the azimuth of the observer $\varphi$ is defined with respect to the X-axis. The BH spin $a = 0.3$ is assumed.

Moreover, we define the Cartesian coordinates $(X, Y, Z)$ in such a way that the Z-axis is directed along $\mathbf{J}_D$, and the X-axis is on the plane confined by $(\mathbf{J}_BH, \mathbf{J}_D)$, pointing to the misalignment direction. The viewing angle of the observer $\theta$ is defined with respect to the Z-axis, ranging from 0 and $\pi/2$, and the azimuth of the observer $\varphi$ is defined with respect to the X-axis. At the beginning precession phase $\gamma = 0$, when $\varphi = 0$ or $\pi$, we observe the largest projected area of the corona. Note that the simulation results in Section 3 are for $\varphi = 0$. The effect of the observer azimuth on the phase-resolved emission will be discussed in Section 4.

Based on this geometry, we will simulate the phase-resolved spectrum from the system as a function of precession phase $\gamma/2\pi$, by using our relativistic radiative transfer code (You et al. 2018). The strategy of the simulation code is summarized as follows:

1. First, randomly initializing seed photons from the outer disk, which are required to obey the blackbody emission from the relativistic accretion disk (Novikov & Thorne 1973);
2. Second, determining the destination of each individual seed photon:
   (i) if going directly to infinity, then being saved as disk spectrum;
   (ii) if going to the corona, then being inverse-Compton scattered. The process of inverse Comptonization is implemented by following the prescriptions of Pozdnyakov et al. (1983) and Gorecki & Wilczewski (1984; see also Janiuk et al. 2000). If the Comptonized photons escaping from the corona travel to infinity, then it will be saved as the Comptonization spectrum (as the direct component).
3. For the Comptonized photons escaping from the corona, if they illuminate the disk (as the incident component), they will be reprocessed and reflected back to infinity, being saved as the contribution to the reflection spectrum. The reprocessing of the Comptonized photon in the disk is implemented by following the prescriptions of George & Fabian (1991) and Zycki & Czerny (1994). In this case, the predominant fluorescent photon at 6.4 keV, i.e., the Fe Kα line, is simulated as well.

More details on the simulation process can be found in You et al. (2018) in which the variability of the continuum due to the precession of the inner hot flow was studied. In this work, we will focus on the phase-resolved emission from the system, mainly concentrating on the reflection and the Fe Kα emission line, in the Lense–Thirring precession model.

2.2. Spectral State Transitions

The electron temperature and the optical depth are the key parameters in shaping the Comptonization spectrum in terms of the spectral slope and the cutoff at high energy (Zdziarski et al. 1996; Qiao & Liu 2018; Yan et al. 2020). In our simulation, in order to implement the spectral transition, in the scenario of truncated disk geometry, we use the eqpair code to calculate the electron temperature and the optical depth for different truncation radii (You et al. 2018).

Given a truncation radius, the ratio of the hardness and softness compactness \( h_s/l_s \) (where \( h_s = L_{\text{eh}} \sigma T / R m_{\text{e}} c^3 \) and \( L_{\text{eh}} \) is the source luminosity) and the value of \( l_s \) are the key inputs in eqpair, as these two parameters dominantly determine the electron energy distribution, which is solved by the balance between the heating (including direct acceleration of particles) and cooling processes, including electron–positron pair balance, bremsstrahlung, and Compton cooling (Poutanen & Coppi 1998; Coppi 1999). We refer the reader to You et al. (2018) for the detailed computations of \( h_s/l_s \) and \( l_s \).

Under the geometric configuration of the disk/corona in Section 2.1, both \( h_s/l_s \) and \( l_s \) can be estimated, which have monotonous relationships with the truncation radius. Then, the parameters of the Comptonizing plasma (optical depth \( \tau \), phase-averaged electron temperature \( T_e \)) can be computed for a given \( R_{\text{in}} \). More specifically, when the truncation radius \( R_{\text{in}} = 90 \), then \( T_e \approx 110 \) keV, \( \tau \approx 2.1 \), and \( \Gamma \approx 1.3 \); when the truncation radius \( R_{\text{in}} = 10 \), then \( T_e \approx 85 \) keV, \( \tau \approx 1.0 \), and \( \Gamma \approx 1.7 \). It was suggested in previous studies that the density and the optical depth might be radially stratified (Axellson et al. 2013, 2014), which may affect the QPO properties. Studying the effect of the inhomogeneous corona on the QPO properties is beyond the scope of this work. Therefore, in this work, we simply assume that the corona is homogeneous with a constant density and take the optical depths above as those of the corona which are radially integrated from the inner radius \( R_{\text{in}} \) to the outer radius \( R_{\text{tr}} \). The optical depth \( \tau \) and electron temperature \( T_e \) are taken as input parameters in our Monte Carlo simulation to compute the X-ray energy spectrum and the associated variability in the following sections.

3. Results

In the truncated disk geometry, which is assumed in this work, seed photons from the outer disk are Comptonized in the corona. The Comptonized photons can escape out of the corona, some of which irradiate the outer disk, producing the reflection spectrum and the characteristic Fe Kα line as well. Therefore, in the scenario of Lense–Thirring precession, in order to study the variation of the disk reflection and Fe Kα line with the precession phase (Sections 3.2 and 3.3), it is essential to first study the variation of the irradiation (Section 3.1).

3.1. Variation of the Irradiation

3.1.1. The Case of the Lamp Post

In order to study the effect of the misalignment between the BH spin axis and the illuminating corona on the irradiation pattern over the disk, we first explore the simple case of the corona, i.e., the lamp-post geometry. In this case, the corona, as the X-ray source, is assumed to be a point-like illuminating source located on the disk symmetry axis above the disk at a height \( H \) (see the schematic in Figure 2). The illuminating photons from the corona are intercepted by the disk. When the BH spin is aligned with the disk axis, the irradiation pattern observed in the disk rest frame, i.e., the distribution of the illuminating photons over the disk, is perfectly symmetric, as expected. However, when the BH spin is misaligned with the disk axis, tilting toward the X-axis, as shown in the right panel of Figure 2, the axisymmetry of the irradiation pattern turns out to be broken due to the lens effect (light-bending effect) on the photon trajectory, which is prolonged along the X-axis. The color bar represents the scaled photon counts (in logarithm) intercepted by the disk. It can be seen that the difference in the counts of the intercepted photons could be two orders of magnitude. More importantly, we find that two bright patches are formed on the X-axis, with the left side being brighter than the right one. If this point-like X-ray source (corona) is vertically extended, e.g., a jet-like base (Wilkins et al. 2015;
precession angles. The difference in intercepted luminosity could be up to 1.8 orders of magnitude. Meanwhile, the corona is symmetric about the X-axis. As the corona precesses counterclockwise, the entire irradiation pattern will globally rotate counterclockwise, as shown in panels (b), (c), and (d). It should be noted that at the middle phase of the precession, i.e., $\gamma/2\pi = 0.5$, when the corona axis is aligned with the disk axis, the irradiation pattern turns out to be symmetric on the disk (panel c), which is expected from the geometrical point of view.

In Figure 4, we plot again the irradiation pattern of the precessing corona, but for the small truncation radius $R_t = 10$ where photons escaping from the corona are close to the BH. The relativistic effect in this case is significant, compared to the case of the larger truncation radius, so that the irradiation patterns are distorted at all precession phases and are shifted in the azimuthal direction due to the light-bending effect on the incident photons. As the corona precesses, the irradiation pattern on the disk rotates counterclockwise as well.

Because our simulation code can record the information of both the incident flux on the disk and the observed direct flux at infinity, we can study the reflection fraction, i.e., the ratio of the number of photons incident on the disk to those going to the observer, which is widely used to infer the relative geometry of the X-ray source with respect to the disk in many codes, e.g., pexray (Magdziarz & Zdziarski 1995) and relxill (García et al. 2013); also see the discussion in Ingram et al. (2019). In Figure 5, the incident flux (red line) and the observed Comptonization flux (or direct; blue line) as a function of precession angle/phase are plotted in the upper panels. The resultant reflection fraction (green lines) as a function of precession angle/phase is plotted in the bottom panels. The solid lines are for the large truncation radius $R_t = 90$, while the dashed lines are for the small truncation radius $R_t = 10$. From the left to right panels, the viewing angles of the observer decreases from $\cos i \sim 0.1$ to $\cos i \sim 0.9$. It can be seen that the incident flux shows variation due to the precession, as expected. The minimum of the incident flux occurs at the middle phase when the corona is aligned with the disk, while
the maximum occurs at the the beginning phase when the corona highly tilts up with the maximum misalignment. The variation of the incident flux with the precession phase is independent of the truncation radius. However, as for the observed direct Comptonization flux, that is not the case. It depends on not only the truncation radius, but also the viewing angle. For the truncation radius \( R_t = 10 \), the direct Comptonization flux viewed at a small viewing angle \( \cos \theta \approx 0.9 \); face-on) reaches minimum at the middle phase and maximum at the beginning/ending phase. However, if the viewing angle is large with \( \cos i \approx 0.1 \) (edge on), the direct Comptonization flux shows the variation in wave-like form, with the maximum and minimum fluxes roughly occurring at the first half \( \gamma/2\pi \approx 0.3 \) and the second half \( \gamma/2\pi \approx 0.7 \), respectively. In our simulation, the Keplerian rotation of the corona is assumed, which will affect the observed Comptonization flux. At the first half of the precession cycle \( \gamma/2\pi < 0.5 \), the approaching side of the corona is facing the observer, so that the Comptonization flux is boosted. At the second half of the precession cycle \( \gamma/2\pi > 0.5 \), the receding side of the torus is facing the observer, so that the Comptonization flux is reduced.

Consequently, the resultant reflection fraction shows variation due to the precession and depends on not only the truncation radius (spectral state) but also on the viewing angle. For large truncation radius, i.e., \( R_t = 90 \), the fraction shows subtle variation with the precession phase, even being constant for a large viewing angle. The phase-averaged value roughly ranges from 0.2 to \( \gamma/2\pi \approx 0.4 \), which is in the observed range of the reflection fraction of BHXRBs in the low hard state (Zdziarski et al. 1999). For a small truncation radius, i.e., \( R_t = 10 \) (in this case the spectrum being softened), the fraction also shows variation with the precession phase, but the pattern depends on the viewing angle. At low viewing angle, because the direct Comptonization flux is roughly constant, the reflection fraction almost follows the incident flux, i.e., the minimum occurring at the middle phase when the corona aligns with the disk, while the maximum occurs at the the beginning phase when the corona highly tilts up with the maximum misalignment. However, at high viewing angle, because the direct Comptonization flux shows the waveform variation, the resultant reflection fraction displays the waveform variation as well, but with opposite sign. This means that there will be a significant phase difference between the direct Comptonization flux and the reflection fraction when the truncation radius is small. We also note that the phase-averaged reflection fraction for the small truncation radius, ranging from 0.8 to 1.0, is larger than that for the large truncation radius, which agrees with the observed anticorrelation between the hardness and the reflection fraction (Zdziarski et al. 1999).

### 3.2. Variation of the Reflection

The corona photons illuminate the disk and will be reprocessed to produce the reflection spectrum including the characteristic Fe Kα line. Figure 6 plots the reflection pattern taken at four different values of precession angles, \( \gamma/2\pi = 0, 1/4, 1/2, 3/4 \), in terms of luminosity \( L = \int F_{\gamma} dE \), for the truncation radius \( R_t = 90 \), as seen by an observer with low inclination angle \( \cos \theta \approx 0.9 \). It can be seen that the reflection pattern follows the incident patterns. At the beginning of the precession cycle, \( \gamma/2\pi = 0 \), i.e., the maximum misalignment between the corona and disk (leading to the maximum illumination of the disk), the reflection pattern appears to be bright on the left side of the disk (\( \sim 180° \)), but faint on the right region (\( \sim 0° \)). The difference in the incident luminosity can be up to about 1.8 orders of magnitude. As the corona precesses counterclockwise, as a consequence, the entire irradiation pattern will globally counterclockwise rotates, as shown in panels (b), (c), and (d). At the middle phase of the precession, i.e., \( \gamma/2\pi = 0.5 \), when the corona axis is aligned with the disk axis, the reflection pattern turns out to be symmetric on the disk (panel c).
In our simulation, the Keplerian (counterclockwise) rotation of the accretion disk is assumed. When a reprocessed photon leaves the disk, disk rotation will affect not only the observed energy, but also the direction of motion. Therefore, the observed reflection pattern should depend on the inclination angle as well. In order to demonstrate the effect of the disk rotation on the observed reflection, the reflection pattern at the same truncation radius, but for the high viewing angle, is plotted in Figure 7. Compared to the face-on case, as for the observed reflection pattern in the edge-on case, the observed reflection from the receding region of the disk is reduced, while the observed reflection from the approaching region of the disk is boosted.

The simulations above are for large truncation radius, which means the reflecting photons are far away from the BH. If the disk is truncated at small radius, close to the BH, e.g., $R_c = 10$, then the reflecting photons will inevitably suffer from relativistic effects (e.g., light bending), which will significantly affect the photon trajectory to infinity. In Figure 8, we plot the reflection pattern for small truncation radius $R_c = 10$ and high viewing angle $\cos \theta \sim 0.1$. Differing from the case of large truncation radius $R_c = 90$, as the corona precesses, the brightest reflection keeps coming from the approaching side of the disk, while the dimmest reflection keeps coming from the receding side of the disk. In other words, the receding side of the disk never dominates over the approaching side of the disk in the observed reflection flux.

### 3.3. Variation of the Fe Kα Line

The reflection patterns shown above could also represent the contribution to the overall luminosity of the Fe Kα line from different regions of the disk. Because the shift in photon energy due to both gravitational and Doppler effects depends on the emitting location on the disk, the overall spectroscopic profile of the Fe Kα line is determined by the reflection pattern. Because the reflection pattern varies as the corona precesses and the variation depends on not only the truncation radius but also the inclination angle, the overall spectroscopic shape of the Fe Kα line should depend on the truncation radius and the inclination angle as well.

In Figure 9, we plot the reflection pattern for large truncation radius $R_c = 90$ as seen by an observer at the middle viewing angle $\cos \theta \sim 0.5$. The corresponding spectroscopic profile of the Fe Kα line is plotted in Figure 10. At the beginning of the precession cycle, the brightest reflection primarily occurs at the region hidden from the observer (at the azimuth of $\sim 180^\circ$ on the disk), where the Doppler effect is not strong (from the point of view of the observer). Besides, the brightest reflection also partially occupies the approaching side of the disk. Consequently, the overall peak of the Fe Kα line is subtly shifted to
respectively. The BH spin \( \alpha = 0.3 \) is assumed. The color bar represents the scaled luminosity (in logarithm) intercepted by the disk. The four panels (a), (b), (c), and (d) correspond to the four precession angles \( \gamma/2\pi = 0, 1/4, 1/2, \) and \( 3/4 \), respectively. In each panel, the blue arrow at the center represents the projection of the corona axis \( \vec{A} \) on the disk plane. The cartoon eye indicates the azimuthal position of the observer, \( \varphi = 0 \), at infinity.

Figure 9. Reflection pattern in the observer rest frame, in terms of luminosity \( L = \int E dE \), for the truncation radius \( R_t = 90 \), being observed at middle inclination angle \( \alpha \approx 0.5 \). The BH spin \( \alpha = 0.3 \) is assumed. The color bar represents the scaled luminosity (in logarithm) intercepted by the disk. The four panels (a), (b), (c), and (d) correspond to the four precession angles \( \gamma/2\pi = 0, 1/4, 1/2, \) and \( 3/4 \), respectively. In each panel, the blue arrow at the center represents the projection of the corona axis \( \vec{A} \) on the disk plane. The cartoon eye indicates the azimuthal position of the observer, \( \varphi = 0 \), at infinity.

\[ \cos \theta = 0.1 \quad \cos \theta = 0.5 \quad \cos \theta = 0.9 \]

Figure 11. Spectral properties of the observed 2–10 keV radiation, as a function of precession angle \( \gamma/2\pi \), for large truncation radius \( R_t = 90 \). From top to bottom, each panel corresponds to the central energy of the line \( E_c \), the iron line luminosity \( L_L \), the spectral slope \( \Gamma \), and continuum flux \( L_C \). Correspondingly, the phase-averaged values are labeled as \( E_{c,\text{av}}, L_{L,\text{av}}, \Gamma_{\text{av}}, \) and \( L_{C,\text{av}} \). The blue, green, and red points are for the viewing angles \( \cos \theta \approx 0.1, 0.5, \) and 0.9, respectively. The BH spin \( \alpha = 0.3 \) is assumed.

In order to quantitatively demonstrate the variation of the Fe K\( \alpha \) line, we define the flux-weighted central energy of the line \( E_{\text{ch}} \) as follows:

\[ E_i = \frac{\int E F(E) dE}{\int F(E) dE}. \quad (1) \]

We plot the flux-weighted centroid energy of the Fe K\( \alpha \) line \( E_i \) as a function of precession angle \( \gamma/2\pi \), for large truncation radius \( R_t = 90 \), in Figure 11. Also plotted are the line luminosity \( L_L \), the spectral slope \( \Gamma \), and the continuum flux \( L_C \) between 2 and 10 keV. For clarity, these values are plotted as the ratio between the phase-dependent values and the phase-averaged values. It can be seen that the line flux \( L_L \) and the continuum flux \( L_C \) between 2 and 10 keV, with respect to the phase-averaged values, show large variation by up to 50\% and 20\%, respectively. The variation of these two spectral parameters with the precession phase is insensitive to the inclination angle. The spectral slope \( \Gamma \) also varies with precession phase by about \( \sim 5\% \). The spectrum becomes softest at the precession angle \( \gamma/2\pi \approx 0.5 \), when the corona is aligned with the disk axis. We note that the centroid energy \( E_i \) varies with precession phase by up to \( \sim 2\% \) for large inclination angles (blue and green lines), being dominated by the blueshift effect at the first half of the precession period while being dominated by the redshift effect at the second half of the precession period. The centroid energy was observed to systematically vary in H1743–322 (ID17), although the observed variation pattern is complex in comparison to the prediction here. For low inclination angle (red line), the predicted variation pattern is almost constant. Given the dependence of these spectral properties on the inclination in Figure 11, the variation of the centroid energy \( E_c \) with phase could be a good diagnostic for whether the inclination angle is low or large.

In Figure 12, we plot the reflection patterns taken at four different precession angles \( \gamma/2\pi = 0, 1/4, 1/2, \) and \( 3/4 \) for small truncation radius \( R_t = 10 \), as seen by an observer at the middle viewing angle \( \cos \theta \approx 0.5 \). The corresponding
spectroscopic shape of the Fe Kα line is plotted in Figure 13. Differing from the case of the larger truncation radius, in the case of the smaller truncation radius, the brightest reflection pattern constantly comes from the approaching side of the disk, while the dimmest reflection constantly comes from the receding side of the disk. The entire reflection is dominated by the approaching side of the disk. Therefore, over the full precession cycle, the corresponding Fe Kα line always peaks at the higher energy $E \sim 6.8$ keV due to Doppler shift and is skewed to low energy $E \sim 4$ keV. The main difference between the four Fe Kα line profiles in Figure 13 is the peak luminosity.

We plot the flux-weighted centroid energy of the Fe Kα line $E_l$, line luminosity $L_l$, the spectral slope $\Gamma$, and the continuum flux $L_C$ between 2 and 10 keV, as a function of precession angle $\gamma/2\pi$, for small truncation radius $R_{tr} = 10$, in Figure 14. Again, these values are plotted as the ratio of the phase-dependent values to the phase-averaged values. Although the truncation radius decreases from $R_{tr} = 90$ to $R_{tr} = 10$, the variation pattern of the Fe Kα line $E_l$, line luminosity $L_l$ and the spectral slope $\Gamma$ with the precession phase in Figures 11 and 14, are not changed. We note that in the case of $R_{tr} = 10$, the variation pattern of the continuum flux $L_C$ with precession phase depends on the inclination angle, i.e., showing modulation by up to $\sim 20\%$ for large inclination angles $\cos \theta \sim 0.1$ (blue points), but is less variable for low inclination angle $\cos \theta \sim 0.9$ (red points). From this perspective, the phase-dependent continuum flux $L_C$ could be a good diagnostic for the evolution of the truncation geometry, if the source is viewed with low inclination angle.

4. Discussion

In You et al. (2018), the effect of the parameters, e.g., the accretion geometry (i.e., the truncation radius of the disk), the observer position (in terms of inclination angle and azimuthal angle), etc., on the observed QPO properties (e.g., energy-dependent fractional variability amplitude) of the continuum emission was studied in the scenario of Lense–Thirring precession. In this work, we mainly studied the phase-resolved irradiation/reflection (including the Fe Kα line) and the resultant spectral properties, in the same scenario of precession. This will allow us to further study the phase lag between these spectral properties.

4.1. Phase Lag

In Figures 11 and 14, the luminosity of the Fe Kα line $L_l$ is modulated by the precession phase. But, the variation pattern of the line luminosity relative to the phase-averaged value is independent of the inclination angle and the truncation radius (i.e., corresponding to the spectral state in the truncation model). This means that the Fe Kα line luminosity $L_l$ could be taken as a good reference to study the phase lag of other spectral properties with respect to it. As the truncation radius decreases from $R_{tr} = 90$ to 10, there is no obvious phase lag for the spectral slope $\Gamma$. For a large inclination angle, the minimum of
the centroid energy of the Fe Kα line $E_l$ is roughly at the same phase $\gamma/2\pi \sim 0.65$ in the cases of $R_{tr} = 90$ and 10, although the line profiles and their variation with the precession phase are absolutely different for different truncation radii. At the truncation radius $R_{tr} = 90$, we found that there is no significant phase lag between the continuum flux $L_C$ and the Fe Kα line luminosity $L_l$ when the inclination angle is small with $\cos \theta \sim 0.9$. However, when the inclination angle is large, e.g., $\cos \theta \sim 0.1$, the continuum flux $L_C$ slightly lags the Fe Kα line luminosity $L_l$. This phase lag is caused by the Doppler effect due to the rotation of the corona in the case of high inclination angle. This effect could be more significant for the faster rotation of the corona when the truncation radius (i.e., the outer radius of the corona) is small, $R_{tr} = 10$. Therefore, the continuum flux $L_C$ apparently lags the luminosity of the Fe Kα line $L_l$.

### 4.2. Effect of the Observer Azimuth

The simulation results above are for the case of the observer with the azimuthal position of $\varphi = 0$, i.e., the precessing corona is directed toward the observer. However, the azimuth $\varphi$ of the observer is unknown. It has been shown that the QPO properties (variability amplitude and polarization) can be very different for different azimuthal positions of the observer (Ingram et al. 2015; You et al. 2018). In order to study the effect of the observer azimuth on the phase-resolved emission, in this section we consider the case of the observer being behind the precessing torus, i.e., the azimuthal position of $\varphi = \pi$.

We repeat the simulation results above but for $\varphi = \pi$. In Figure 15, we plot the reflection patterns taken at four different precession angles $\gamma/2\pi = 0, 1/4, 1/2, \text{ and } 3/4$ for small truncation radius $R_{tr} = 10$ as seen by an observer at the middle viewing angle $\cos \theta \sim 0.5$. The corresponding spectroscopic profiles of the Fe Kα line are plotted in Figure 16. At the first half of the precession period ($\gamma/2\pi < 0.5$), the incident luminosity concentrates on the approaching side of the disk, from the point of view of the observer. Due to the Keplerian rotation of the disk, however, the flux from the approaching side will be boosted and the flux from the receding side will be suppressed. Therefore, given the combination of these two effects, the reflection pattern shows marginal difference over the disk (see panels (a) and (b)). As a result, the corresponding Fe Kα line is roughly flat between 4 and 8 keV (the black and blue lines in Figure 16). However, at the second half of the precession period ($\gamma/2\pi > 0.5$), the incident flux concentrates on the approaching side of the accretion disk where the reflection pattern is much brighter than the receding side (see panels (c) and (d)). Consequently, the blue wing of the emission line is enhanced (red and green lines in Figure 16).

The spectral analysis between 2 and 10 keV including the Fe Kα line is replotted in Figure 17, but for the observer azimuth $\varphi = \pi$. The BH spin $a = 0.3$ is assumed.

Figure 15. Reflection pattern in the observer rest frame, in terms of luminosity $L = \int L_{\gamma} dE$, for the truncation radius $R_{tr} = 10$, being observed at middle inclination angle $\cos \theta \sim 0.5$. The color bar represents the scaled luminosity (in logarithm) intercepted by the disk. The four panels (a), (b), (c), and (d) correspond to the four precession angles $\gamma/2\pi = 0, 1/4, 1/2, \text{ and } 3/4$, respectively. The azimuthal angle of the observer $\varphi = \pi$. The BH spin $a = 0.3$ is assumed.

Figure 16. Observed Fe Kα line for large truncation radius $R_{tr} = 10$ and middle inclination angle $\cos \theta \sim 0.5$. The black, blue, green, and red profiles correspond to the precession angles, $\gamma/2\pi = 0, 1/4, 1/2, \text{ and } 3/4$, respectively. The azimuthal angle of the observer $\varphi = \pi$. The BH spin $a = 0.3$ is assumed.

Figure 17. Spectral properties of the observed 2–10 keV radiation, as a function of precession angle $\gamma/2\pi$, for large truncation radius $R_{tr} = 10$. From top to bottom, each panel corresponds to the central energy of the line $E_0$, the iron line flux $L_0$, the spectral slope $\Gamma$, and continuum flux $L_C$. Correspondingly, the phase-averaged values are labeled as $E_0$, $L_0$, $\Gamma$, and $L_C$. The blue, green, and red points are for the viewing angles $\cos \theta \sim 0.1, 0.5, \text{ and } 0.9$, respectively. The azimuthal angle of the observer $\varphi = \pi$. The BH spin $a = 0.3$ is assumed.
to $\sim 5\%$. More importantly, when the source is viewed at large inclination angle, in the case of $\varphi = 0$, the phase difference between the centroid energy $E_c$ and the line luminosity $L_4$ is small with $\Delta \gamma / 2\pi < 0.1$; in the case of $\varphi = \pi$, the phase difference between the centroid energy $E_c$ and the line luminosity $L_4$ is large with $\Delta \gamma / 2\pi > 0.3$. Moreover, for the latter case, the phase difference between the continuum luminosity $L_4$ and the line luminosity $L_4$ is large with $\Delta \gamma / 2\pi \sim 0.4$. Therefore, given this effect, measuring the phase difference between the line flux $F_{\text{line}}$ and $E_c/F_{2-10}$ could be a good diagnostic for the azimuthal position of the observer.

### 4.3. Effect of the Optical Depth

In this work, the corona is assumed to be torus-like with $H/R = \tan 15^\circ \sim 0.3$. For the truncation radius $R_\text{tr} = 90$, the corona is optically thick with the maximum optical depth $\tau \simeq 2.1$ (see Section 2.2) when the line of sight is perpendicular to the corona axis. As the viewing angle decreases, the effective optical depth (i.e., being integrated over all lines of sight leading to different points within the corona) along the line of sight will then decrease. Therefore, in Figure 5, the observed direct flux for the truncation radius $R_\text{tr} = 90$ depends on the inclination angle. In contrast, for the truncation radius $R_\text{tr} = 10$, the corona is optically thin with maximum optical depth $\tau \approx 1.0$, and then the phase-averaged direct flux is fairly independent of the inclination angle.

In order to study the effect of the optical depth on the variability properties, we run the simulation for $R_\text{tr} = 90$ again, but assuming the maximum optical depth $\tau \sim 0.5$. In Figure 18, we plot the direct/incident flux and the resultant reflection fraction, as a function of precession phase, for the optically thin and thick cases. The difference in the phase-averaged direct flux between high and low inclinations turns out to be reduced. However, the variation pattern of the incident flux is still identical to the one in the case of optically thick corona, and there is no significant change in the patterns of the reflection fraction. Moreover, we check that there is no qualitative difference in the corresponding variation patterns of the disk reflection including Fe K$\alpha$, due to changes in the optical depth.

### 4.4. Comparison to Previous Work

The phase-dependent QPO properties including Fe K$\alpha$, which are simulated in this work, qualitatively agree with the far simpler earlier treatment of Ingram & Done (2012, hereafter ID12). ID12 also found the behavior displayed in Figures 10 and 13, i.e., the line varies in such a way that the red wing can dominate over the blue wing for a large $R_\text{tr}$, but not for a small $R_\text{tr}$. There are a few important differences though. ID12 concluded that the bluest line always happened during the rising phase of the continuum flux, and the reddest line always happened during the falling phase of the continuum flux. This is no longer the case with the new analysis for a number of reasons. First, in the new analysis, the bottom of the corona provides the strongest illumination, meaning that the brightest patch of the disk is always in the opposite direction to where the projection of the corona axis on the disk plane points. This means that for the viewer azimuth whereby the corona maximally faces the observer at maximum misalignment (i.e., observer at $\varphi = 0$), the bluest line is the falling phase and the reddest line is the rising phase (i.e., the opposite of ID12). Second, we include the effect of variation of seed photons as the misalignment angle between disk and corona varies and also light bending, which was ignored by ID12. This leads to the precession phase of the peak continuum flux depending not only on the angle between the corona axis and the line of sight (as in ID12), but also on the angle between the corona and disk axes. We also include light bending here, which is far more important for $R_\text{tr} = 10$ than for $R_\text{tr} = 90$. 

![Figure 18](image-url)
4.5. Comparison to Observations

ID17 applied a phase-resolved analysis to XMM-Newton and NuSTAR observations of H1743–322 to study its spectral properties, including the Comptonization continuum flux, the centroid energy of Fe Kα line, and the reflection fraction as a function of QPO phase. It was found that the continuum normalization by the spectral fit varies in wave-like form with high significance. In this work, we show that the simulated Comptonization flux is modulated due to Keplerian rotation of the corona, with the maximum and minimum fluxes roughly occurring at the first half, \( \gamma/2 \pi \sim 0.3 \), and the second half, \( \gamma/2 \pi \sim 0.7 \) (see Figure 5 and the discussion in Section 3.1.2), which qualitatively agrees with the results of ID17.

In ID17, the centroid energy of the Fe Kα line of H1743–322 shows a characteristic variation with QPO phase, with two maxima at \( \sim0.2 \) and \( \sim0.7 \) of the QPO cycles. However, in this work, we only see one maximum of the iron line energy within one precession cycle. Given that the iron line intrinsically arises from the illumination of the disk in the reflection model, the two maxima of the iron line centroid energy require two bright patches on the disk surface. But we are still only seeing one bright patch in the current simulation (e.g., see Figure 3). This is because, in our current simulation, we assume the corona to be torus-like radially extended. We note that in the case of lamp-post (point-like) geometry, two bright patches on the disk are seen (see Figure 2). If this point-like corona is vertically extended, e.g., a jet-like base (Wilkins et al. 2015; Kara et al. 2019; B. You et al. 2020, in preparation) and precesses around the BH spin axis with some period (Liska et al. 2018), then we would expect these two bright patches on the disk to vary, which may explain the two maxima of the iron line in H1743–322. The simulation for such a geometry of the corona, i.e., a jet base coupling with an extended corona, will be done in detail in our next paper.

The reflection fraction of H1743–322 in ID17 also varies with QPO phase. We note that H1743–322 was in the hard state according to XMM-Newton and NuSTAR observations in ID17. In our simulation results, we found that the predicted reflection fraction indeed is highly modulated as well, when the truncation radius \( R_t \) is small and the inclination to the source is large. However, the modulation of the measured reflection fraction in ID17 is much more complex, in comparison to our results. Given that the reflection fraction intrinsically depends on the relative geometry of the X-ray source with respect to the disk, the complex variation pattern of the reflection fraction indicates the geometry of the X-ray source should be more advanced, as we discuss above, e.g., a jet base coupling with an extended corona.

5. Summary

In this work, we studied the phase-resolved irradiation/reflection (including the Fe Kα line) and the resultant spectral properties if the X-ray primary source undergoes Lense-Thirring precession around the spin axis of the BH. In the truncation geometry of the primary X-ray source, the main conclusions are summarized as follows:

1. The incident pattern over the outer disk rotates with the precession of the inner corona. Because the direct radiation from the corona varies as a function of precession phase, the derived reflection fraction will correspondingly vary with the precession phase, which depends on the truncation radius (i.e., the spectral state) and the inclination angle.

2. The reflection off the disk also rotates with the precession, the observed pattern of which depends on the truncation radius and the inclination angle.

3. The Fe Kα line profile will correspondingly change with the precession phase. More specifically, the line luminosity and the flux-weighted centroid energy vary with the precession phase.

4. The continuum luminosity from the precessing corona could apparently lag the line luminosity in phase, if the truncation radius is small when the Doppler effect of the observed radiation becomes significant.

However, the geometry of the primary X-ray source is still uncertain, which may alternatively be described by the lamp-post geometry or as a vertically extended structure, like a jet base. Then, two bright patches are formed on the disk, the flux of which will be modulated by the precession of the X-ray primary source, but likely with half of the precession period. This effect may be responsible for the observed harmonic in the power spectrum of BHXRBs, which will be further studied in the next paper.

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