Flavour Physics in the Littlest Higgs Model with $T$-Parity: Effects in the $K$, $B_{d/s}$ and $D$ systems *

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The Littlest Higgs Model with $T$ parity (LHT) is an interesting alternative model for New Physics at the TeV scale. Although Flavour Physics was not the reason for creating the LHT model, significant effects (such as large $CP$ violation where not predicted by the SM) can be created without violating existing experimental bounds. We study the $B$-, $K$- and especially the $D$-sector.

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1. Introduction: Gauge hierarchy in the SM

A major problem in the Standard Model (SM) is the Gauge Hierarchy problem, Top-loop corrections make the Higgs mass unstable, $\Delta m_H^2 = -|\lambda_t|^2/8\pi^2 [\Lambda_{UV}^2 + ...]$. To prevent $m_H \rightarrow m_{\text{Planck}}$, we need incredible fine-tuning. One possible solution is SUSY, where the top-loop is cancelled with a stop-loop, $\Delta m_H^2 = 2|\lambda_s|^2/16\pi^2 [\Lambda_{UV}^2 + ...]$. It is also possible to lower the Planck mass with extra dimensions, another possible solution to the Gauge Hierarchy problem is the Little Higgs mechanism.

2. The Little(st) Higgs Model (with $T$ parity)

In the Little Higgs class of models [1], the Higgs Boson is a pseudo-Goldstone boson of a spontaneously broken global symmetry. Gauge and Yukawa couplings break the symmetry explicitely, but every single coupling conserves enough of the symmetry to keep the Higgs massless. This way, the radiative corrections to the Higgs mass are only logarithmically divergent at one loop (and not quadratically as in the SM).

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One popular implementation of the Little Higgs mechanism is the Littlest Higgs Model [2], where the Higgs boson is a pseudo-Goldstone boson from breaking a global $SU(5)$ symmetry to a global $SO(5)$ at the scale $f \sim \mathcal{O}(\text{TeV})$. The exact mechanism for symmetry breaking is unspecified, therefore the Littlest Higgs model is an effective theory valid up to $\Lambda \sim 4\pi f$.

There are 14 Nambu-Goldstone bosons from symmetry breaking: the SM Higgs, new heavy gauge bosons $W_H^\pm, Z_H, A_H$, a scalar triplet $\Phi$, and a heavy partner for the top quark, $T$. In the original Littlest Higgs, custodial $SU(2)$ is broken already at tree level, then electroweak precision (EWP) observables demand $f \gtrsim 2 - 3\text{ TeV}$, this leads to rather small ($10 - 20\%$) effects in Flavour Physics.

By introducing a new discrete symmetry ("T parity"), the Littlest Higgs Model with T parity (LHT) [3] avoids problems with the EWP observables: Under the new symmetry, all new particles (except $T_+$) are odd, all SM particles are even. There are therefore no contributions by T odd particles at the tree level, but the cancellation of divergences still works since it is a loop effect. This allows lowering the scale $f$ to $\sim 1\text{ TeV}$ (or even lower).

The LHT model contains three doublets of “mirror quarks” (T odd, heavy), three doublets of “mirror leptons” (T odd, heavy) and a T odd $T_-$ in addition to the T even $T_+$. (Just like R parity in SUSY, T parity can also produce a candidate for Dark Matter.)

The new parameters in the LHT model are $f$, the NP scale which also fixes $M_W, H$ etc. The mixing between $t$ and $T$ is described by $x_L$. There are three mirror quark masses: $m_{H1}, m_{H2}$ and $m_{H3}$ (the model is Minimal Flavour Violating (MFV) if these are degenerate) and a mirror quark mixing matrix $V_{Hd}$ containing three angles and three phases. The up-type mirror fermion mixing matrix is given by $V_{Hu}^\dagger V_{Hd} = V_{CKM}$. (There are also 9 mirror lepton parameters, but these are not of interest in the context of this study.)

3. Flavour effects from LHT

Although the LHT model does not introduce new operators in addition to the SM ones, it is not MFV because of the mirror quark mixing. New particles contribute to Flavour Changing Neutral Current (FCNC) processes as shown in the figure. A detailed discussion of Flavour Physics in the LHT model is given in [5].

The LHT amplitudes can be written as (e.g. $K$ sector)

\[
\sum_{i=u,c,t} \lambda_i^K F_i(m_i, m_{T_+}, \ldots) + \xi_i^K G_i(m_i^H, M_{WH}, \ldots),
\]

where the first term is the T even contribution and the second term is the T odd contribution. This way the Inami-Lim functions become $X_K = X_{SM} + X_{\text{even}} + \xi_i^K / \lambda_i^K X_{\text{odd}}$, with the CKM factors $\lambda_i^K = V_{ts}^* V_{td}$ and the mirror...
quark mixing $\xi^K_i = V^*_{iHd}V_{idHd}$. Because of the CKM hierarchy $1/\lambda^K_i \gg 1/\lambda^B_d \gg 1/\lambda^B_s$, we expect the largest effects in $K$ physics, but suitable $\xi^j_i$ can produce large effects also in $B_d, B_s$.

It has to be checked very carefully whether the LHT effects do not violate existing experimental FCNC constraints. We studied [6] the constraints on $\Delta M_K$ and $\epsilon_K$ from the $K$ system, the mass differences in the $B$ system $\Delta M_{B_d}$ and $\Delta M_{B_s}$, as well as the CP asymmetry in $B_d$ decays $S_{J/\psi K_S}$. (Constraints from $b \to s\gamma$ are not a problem, the effects from LHT in this channel are very moderate.)

We generated random points in the LHT parameter space, checked these constraints and kept only points that fulfill all constraints. The input parameters were evenly distributed over their respective $1\sigma$ ranges. Although a lot of points in parameter space have to be tried to find one that does not violate any of the experimental constraints, fine tuning is not really a problem: Typically, $\epsilon_K$ as generated by arbitrary model parameters is one or two orders of magnitude too large, but there are also many points that generate correct $\epsilon_K$ without large fine tuning $\Delta_{BG}(O) = \max_j \left| \frac{\partial O}{\partial p_j} \right| [7]$. Some of the most spectacular points need no fine tuning at all.

4. General results from LHT flavour study

The decays $K^+ \to \pi^+\nu\bar{\nu}$ and especially $K_L \to \pi^0\nu\bar{\nu}$ are excellent probes of new physics because they can be calculated very cleanly. In the LHT model, $K_L \to \pi^0\nu\bar{\nu}$ can be enhanced significantly over the SM value (black dot) up to a factor of 3-5, and also $K^+ \to \pi^+\nu\bar{\nu}$ can easily be enhanced to the central value (dashed line) of the current experimental range. Most data points lie on two axes: One of constant $K_L \to \pi^0\nu\bar{\nu}$ and one parallel to the Grossmann-Nir bound, this is due to the specific operator structure of the LHT model and distinguishes the experimental signature from other models.
The CP-asymmetry $S_{\psi\phi}$ of the decay $B_s \rightarrow \psi\phi$ is much smaller in the SM than $S_{J/\psi K_S}$ because the corresponding CKM angle $\beta_s$ is only about $-1$ deg. In the LHT model, large effects between -0.3 and +0.4 are observed, but simultaneous large effects in $K_L \rightarrow \pi^0\nu\bar{\nu}$ and $S_{\psi\phi}$, though possible, seem unlikely. This is very different from the situation between $Br(B_s \rightarrow \mu^+\mu^-)$ and $S_{\psi\phi}$, here simultaneous significant effects are rather likely because both observables profit from a modified $b \rightarrow s$ penguin. The enhancement of $Br(B_s \rightarrow \mu^+\mu^-)$ of up to 30% over the SM result is, however, rather moderate compared to e.g. SUSY.

Another interesting signature of the LHT model is the correlation between the $Br$'s of $K_L \rightarrow \mu^+\mu^-_{SD}$ and $K^+ \rightarrow \pi^+\nu\bar{\nu}$, which is very different from e.g. the RS model with custodial protection (c.f. contribution by Börn Duling in this volume). Correlations like these might prove instrumental in distinguishing different models of NP in the experiment.

5. $D\bar{D}$ Oscillations (in the LHT model)

(This section is based on [8, 9].) $D\bar{D}$ is more complicated than $K\bar{K}$ and $B\bar{B}$ mixing: $K\bar{K}$ and $B\bar{B}$ mixing is dominated by short-distance physics, i.e. charm/top loops (c.g. figure). $D\bar{D}$ has almost no short-distance contribution: The corresponding CKM factors are small and the down-type quarks in the loops too light. Therefore the SM contribution to $D\bar{D}$ mixing is long-distance and therefore difficult to estimate. In our analysis, we vary the SM contribution in a reasonable range and use theoretical estimates only to bound the values.

The $D$ mass eigenstates are $|D_{1/2}| = 1/\sqrt{|p|^2 + |q|^2} (p|D^0\rangle \pm q|\bar{D}^0\rangle)$, the observables are the normalised mass and width differences, $x_D \equiv$
\[ \Delta M_D/\Gamma_D, y_D \equiv \Delta \Gamma_D/2\Gamma_D, \] as well as \( q/p \equiv \sqrt{(M_{12}^{D*} - \frac{i}{2}\Gamma_{12}^{D*})/(M_{12}^D - \frac{i}{2}\Gamma_{12}^D)}. \]

Obviously CP is violated when \( |q/p| \neq 1. \)

Rather recently, \( DD \) oscillations have been observed \([10]\), a measurement received with great interest by the community: \( x_D = 0.0100_{+0.0024}^{-0.0026}, y_D = 0.0076_{+0.0017}^{-0.0018}, |q/p| = 0.86_{+0.17}^{-0.15}. \) Although this establishes oscillation, CP violation has not (yet) been observed, \( |q/p| \) is consistent with 1. In the SM, no significant CP violation is expected.

To establish whether the LHT model can produce a significant CP violation in the \( D \) system, we determine \( (M_{12}^D)_{\text{SM}} \) and \( (\Gamma_{12}^D)_{\text{SM}} \) so that together with the LHT contribution, \( x_D \) and \( y_D \) coincide with experiment. This approach is reasonable, because even the expected relative sign of \( (M_{12}^D)_{\text{SM}} \) and \( (\Gamma_{12}^D)_{\text{SM}} \) \([11]\) does not match the values necessary to reproduce the measured values of \( x_D \) and \( y_D \) with the SM contributions, i.e. very little is known about these quantities from the theoretical side. We obtain two solutions for each LHT parameter point as shown in the figure.

Essentially all LHT parameter points are consistent with expectations for the magnitude of SM contributions. In some cases, \( (M_{12}^D)_{\text{SM}} / (\Gamma_{12}^D)_{\text{SM}} \) can be rather large, but these are not our most spectacular/interesting data points.

Obviously, requiring \( x_D \) and \( y_D \) to coincide with experiment restricts the allowed points to a rather narrow region in the \( \text{Abs}/\text{Arg} M_{12}^D \) plane. Since \( V_{H_u}^\dagger V_{H_d} = V_{\text{CKM}} \) and the CKM-matrix is rather close to the unity matrix, the experimental constraints on \( \epsilon_K \) exclude points with large \( \text{Arg} M_{12}^D \) (light blue/grey triangles).

Even without these points, i.e. observing all experimental constraints, very large (for the \( D \) system) CP asymmetries of several percent are possible. The LHT model could even generate asymmetries of \( \pm 5\% \) for \( D \to K\phi \), but this would correspond to semileptonic asymmetries \( a_{SS}^D \) close to unity. Such large values
of $a^D_{SL}$ are already excluded by the measurements of $|q/p|_{\text{exp}} = 0.86^{+0.17}_{-0.15}$ because $a^D_{SL} = (|q|^4 - |p|^4)/(|q|^4 + |p|^4)$. We can therefore conclude that the LHT model can easily saturate the CP violation in the $D$ system that is still allowed by current measurements.

Let us last look at the correlation between the $D$ system and the $B_s$ system: We find that simultaneous large NP effects in both systems are possible, but unlikely, just as we found that simultaneous large effects in the $K$ and the $B$ system are unlikely in the LHT model. Again, it is easier to produce large NP effects that do not violate existing experimental constraints in one sector than in two.

6. Conclusions

The LHT model is an interesting, economical alternative to SUSY etc. in solving the Little Hierarchy problem. There are rather few parameters, the model passes the EW precision tests and (surprisingly, because this is not what the model was created for) there are interesting, sometimes spectacular effects on Flavour observables. For example, large $CP$ violation in $D\bar{D}$ oscillations is possible. We hope that in the near future, experimental results will show us whether nature has chosen anything like the LHT model for physics at the TeV scale.

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