An Algorithm and Heuristic based on Normalized Mutual Information for Dimensionality Reduction and Classification of Hyperspectral images

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ABSTRACT

In the feature classification domain, the choice of data affects widely the results. The Hyperspectral image (HSI), is a set of more than a hundred bidirectional measures (called bands), of the same region (called ground truth map: GT). The HSI is modelized at a set of \( N \) vectors. So we have \( N \) features (or attributes) expressing \( N \) vectors of measures for \( C \) substances (called classes). The problematic is that it’s pratically impossible to investigate all possible subsets. So we must find \( K \) vectors among \( N \), such as relevant and no redundant ones; in order to classify substances. Here we introduce an algorithm based on Normalized Mutual Information to select relevant and no redundant bands, necessary to increase classification accuracy of HSI.

Keywords: Feature Selection, Normalized Mutual information, Hyperspectral images, Classification, Redundancy.

Mathematics Subject Classification: 68U10, 68R05.
Computing Classification System: \textsuperscript{I}.4.7, \textsuperscript{I}.4.8, \textsuperscript{I}.4.9.

1 Introduction

The Hyperspectral image AVIRIS 92AV3C, Airborne Visible Infrared Imaging Spectrometer, (Perdue, 97) is a substitution of 220 images for the region "Indiana Pine" at "north-western Indiana", USA (Landgrebe, 1997). The 220 called bands are taken between 0.4 \( \mu \)m and 2.5 \( \mu \)m. Each band has 145 lines and 145 columns. The ground truth map is also provided, but only 10366 pixels (49\%) are labeled fro 1 to 16. Each label indicates one from 16 classes. The zeros indicate pixels how are not classified yet, see Figure.1. The hyperspectral image AVIRIS 92AV3C contains numbers between 955 and 9406. Each pixel of the ground truth map has a set of 220 numbers (measures) along the hyperspectral image. This numbers (measures) represent the reflectance of the pixel in each band. So the pixel is shown as vector off 220 components,see Figure .2.

To classify pixels of HSI, we find some problems realed to their high dimentions, that needs
many cases to detect the relationship between the vectors and the classes, according to Hughes phenomenon (Hughes, 1968). Other problems are related to the redundant images (bands); they complicate the learning system and produce incorrect prediction (Lei and Huan, 2004). So we must pick up only the relevant bands. Now we can identify the problematic related to HSI as dimensionality reduction. It is commonly reencountered when we have \( N \) features (or attributes) that express \( N \) vectors of measures for \( C \) substances (called classes).

The problematic is to find \( K \) vectors among \( N \), such as relevant and no redundant ones; in order to classify substances. The number of selected vectors \( K \) must be lower than \( N \) regarding the problems above. So we must choose relevant vectors, that means there ability to predicate the classes. Indeed the bands don’t all contain the information; some bands are irrelevant like those affected by various atmospheric effects, see Figure.7, we can show the atmospheric effects on bands: 155, 220 and other bands; so the classification accuracy decreases. We can reduce the dimensionality of hyperspectral images by selecting only the relevant bands (feature selection or subset selection methodology), or extracting, from the original bands, new bands containing the maximal information about the classes, using any functions, logical or numerical (feature extraction methodology) (Kwak and Choi, 2007; Kwak and Kim, 2006; YANG, Yiming and PEDERSEN., 1997). Here we propose an algorithm based on mutual information, and normalised mutual information to reducing dimensionality. This will be as bellow: pick up the relevant bands first, and avoiding redundancy second. We illustrate the principal of this algorithm using synthetic bands for the scene of HIS AVIRIS 92AV3C(Landgrebe, 1997), see Figure.5. Then we validate its effect by applying it to real data of HSI AVIRIS 92AV3C. Here each pixel is shown as a vector of 220 components. Figure.2 shows the vector pixels notion (Kernéis, 2007). Reducing dimensionality means selecting only the dimensions caring a lot of
information regarding the ground truth map (GT).

In the literature, we can cite some methods related to the dimension reduction; but this action is accompanied by transformation of the multivariate data. Well-known methods include principal component analysis, factor analysis, projection pursuit, independent component analysis (ICA). Principal Component Analysis, or PCA (Oja, J., L. and Vigario, 95), is widely used in signal processing, statistics, and neural computing (Homayouni, 2005; Homayouni, 1998; Hyvarinen, 99). The PCA search another space with lower dimension, and when the data will be clearly separated. The independent component analysis ICA search also a space with lower dimension, and which the sources of data are separated (Hyvarinen, 99; Comon, 1994; Hyvarinen and Karhunen, 2001); so which desired action is minimizing the statistical dependence of the components and the components didn’t necessary orthogonal. Now due to the dimensionality of Hyperspectral Image, the question proposed is: can we find some principal components, or independant component among the 220 bands, without transformation of data? The response is to calculate the separation matrix using normalized mutual information as a cost function. We cite here that some works (Comon, 1994; Linsker, 1992; Barlow, 1961; Lee, M.Girolami, Bell and Sejnowski, 2000) had already use the mutual information (but not normalized form of MI) and neural network, to separate blinded sources.

2 Principle of Feature Selection with Mutual Information

2.1 Definition of Mutual Information

The MI is a measure of information contained in both tow ensembles of random variables A and B:

$$I(A, B) = \sum p(A, B) \log_2 \frac{p(A, B)}{p(A).p(B)}$$

Let us consider the ground truth map (GT), and bands as ensembles of random variables, the MI between GT and each band calculates their interdependence. Fano (Lei and Huan, 2004) has demonstrated that as soon as mutual information of already selected features $X$ and the classe $C$ has high value, the error probability of classification decreases, according to the formula bellow:

$$\frac{H(C/X) - 1}{\log_2(N_c)} \leq P_e \leq \frac{H(C/X)}{\log_2}$$

with :

$$\frac{H(C/X) - 1}{\log_2(N_c)} = \frac{H(C) - I(C;X) - 1}{\log_2(N_c)}$$

and :

$$P_e \leq \frac{H(C) - I(C;X)}{\log_2} = \frac{H(C/X)}{\log_2}$$

Here the conditional entropy $H(C/X)$ is calculated between the ground truth map (i.e. the classes $C$) and the subset of bands candidates $X$. $N_c$ is the number of classes. So when the features $X$ have a higher value of mutual information with the ground truth map, (is more near to the ground truth map), the error probability will be lower. But it’s impractical to compute this conjoint mutual information $I(C;X)$, regarding the high dimensionality (Lei and Huan, 2004). Figure.6. shows the MI between the GT and synthetic bands. The figure.7 indicates the MI
between the GT and the real bands of HIS AVIRIS 92AV3C (Landgrebe, 1997).
Some studies use a thresholding to choice the more informative bands. Guo (Guo, Gunn, Senio and Nelson, 2006) uses the mutual information to select the top ranking band, and a filter based algorithm to decide if there neighbours are redundant or not. Sarhrouni (Sarhrouni, Hammouch and Aboutajdine, 2012a) use also a filter strategy based algorithm on MI to pick up relevant bands. A wrapper strategy based algorithm on MI, Sarhrouni (Sarhrouni, Hammouch and Aboutajdine, 2012b) is also introduced.
By a thresholding, for example with a threshold 0.4, see Figure.6, we eliminate the no informative bands: \( A_3, A_7 \) and \( A_9 \). With other threshold, we can retain fewer bands. We can visually show this effectiveness of MI to choice relevant features in Figure.6, and Figure.7.

2.2 Normalized Mutual information

In our study we use two forms of normalized mutual information, and we compare their results.

2.2.1 First Form

The first form of Mutual Information is defined as bellow:

\[
AS(A, B) = \frac{MI(A, B)}{H(A)};
\]

and

\[
AS(B, A) = \frac{MI(B, A)}{H(B)}
\]

\(H(X)\) is the Entropy of set random variable \(X\). This is an asymmetric measure of MI. Inspecting this formula We can make this observation: "When \(AS(A, B)\) is near to 1 i.e. 100%, it means that \(A\) exchange all its information with \(B\)."
So we can use this as a measure of redundancy. Numerous studies use Normalized Mutual Information for recalling images etc.(Denton, Holden, Chris, Jarosz, Russell-Jones, Goodey, Cox, and Hil, 2000; Holden, Denton, Jarosz, Cox, Studholme, Hawkes and Hilll, 1999; Otte, 2001).
Figure.3 illustrates the MI and Normalized MI.

2.2.2 Second Form

This is one of normalized form of Mutual Information introduced by (Witten, 2005). It’s defined as bellow:

\[
U(A, B) = \frac{MI(A, B)}{\sqrt[\frac{4}{3}]{H(A).H(B)}}
\]

\(H(X)\) is the Entropy of set random variable \(X\).
Figure.4 shows that normalized mutual information means how much information is partaged between \(A\) and \(B\) relatively at all information contained in both \(A\) and \(B\).
3 Principle of the Proposed Method and Algorithm

For this section we use 19 synthetic bands from the GT, Figure.1, by adding noise, cutting some substances etc. see Figure.5. Each band has 145 lines and 145 columns. Only 10366 pixels are labelled from 1 to 16. Each label indicates one from 16 classes. The zeros indicate pixels how are not classified yet, Figure.1. We can show the Mutual information of GT and the synthetic bands at Figure.6.

3.1 Selection of relevant bands

With a threshold 0.4 of MI calculated in Figure.4 we obtain 16 relevant bands $A_i$ with :

\[ i = \{ 1, 2, 3, 4, 5, 6, 8, 10, 11, 12, 14, 15, 16, 17, 18, 19 \} \]

We can visually verify the resemblance of GT and the bands more informative, both in synthetic and the real data bands of AVIRIS 92AV3C. See Figure.6 and Figure.7.

3.2 Detection of no Redundant Bands

First: We order the remaining bands, in increasing order of there MI with the GT. So we have:

\[ \{ A_{12}, A_6, A_{15}, A_4, A_3, A_{14}, A_{16}, A_2, A_{10}, A_{17}, A_4, A_{19}, A_5, A_{11}, A_{18} \} \]
Second: We fixe a threshold to control redundancy, here 0.7. Then we compute the Normalised $AS(A_i, A_j)$ MI (Table 1), and the Symmetric Uncertainty $U(A_i, A_j)$ (Table 2): for all couple $(i, j)$ of the ensemble:

$$S = \{8, 15, 6, 1, 3, 14, 16, 2, 10, 17, 4, 19, 5, 11, 18\}.$$

Observation 1: Figure 5 shows that the band $A_{17}$ is practically the same at $A_4$. Table 1 shows that $AS(A_{17}, A_4)$ and $AS(A_4, A_{17})$ near to 100% (respectively 0.96 and 0.93). It means that both $A_{17}$ and $A_4$ share their information with each other. So this indicates a high redundancy. (Table 2 allows the same observation)
Table 1: THE NORMALIZED MI OF THE RELEVANT SYNTHETIC BANDS.

| Bands indices in ascending order of their MI with the GT | 12 | 8 | 15 | 6 | 1 | 3 | 16 | 14 | 2 | 10 | 17 | 4 | 19 | 5 | 11 | 18 |
|-------------------------------------------------------|----|----|----|---|---|---|----|----|---|----|----|---|----|---|----|----|
| 12                                                   | 0.12 | 0.13 | 0.11 | 0.14 | 0.12 | 0.14 | 0.13 | 0.14 | 0.12 | 0.13 | 0.14 | 0.15 | 0.16 |
| 8                                                    | 0.14 | 0.15 | 0.16 | 0.18 | 0.17 | 0.19 | 0.17 | 0.18 | 0.17 | 0.19 | 0.18 | 0.17 | 0.21 |
| 15                                                   | 0.16 | 0.15 | 0.16 | 0.17 | 0.17 | 0.19 | 0.16 | 0.17 | 0.17 | 0.18 | 0.18 | 0.17 | 0.21 |
| 6                                                    | 0.14 | 0.16 | 0.16 | 0.18 | 0.17 | 0.19 | 0.17 | 0.18 | 0.17 | 0.18 | 0.19 | 0.18 | 0.21 |
| 1                                                    | 0.19 | 0.19 | 0.19 | 0.19 | 0.18 | 0.21 | 0.20 | 0.32 | 0.20 | 0.19 | 0.19 | 0.18 | 0.23 |
| 3                                                    | 0.18 | 0.20 | 0.20 | 0.20 | 0.20 | 0.21 | 0.22 | 0.23 | 0.22 | 0.20 | 0.21 | 0.20 | 0.23 |
| 16                                                   | 0.19 | 0.21 | 0.21 | 0.21 | 0.22 | 0.20 | 0.23 | 0.24 | 0.22 | 0.23 | 0.27 | 0.31 | 0.32 |
| 14                                                   | 0.20 | 0.23 | 0.21 | 0.21 | 0.24 | 0.24 | 0.27 | 0.25 | 0.24 | 0.25 | 0.26 | 0.26 | 0.29 |
| 2                                                    | 0.21 | 0.22 | 0.22 | 0.22 | 0.38 | 0.24 | 0.23 | 0.24 | 0.24 | 0.22 | 0.23 | 0.23 | 0.32 |
| 10                                                   | 0.23 | 0.23 | 0.23 | 0.23 | 0.26 | 0.25 | 0.27 | 0.26 | 0.26 | 0.25 | 0.26 | 0.25 | 0.33 |
| 17                                                   | 0.21 | 0.24 | 0.23 | 0.22 | 0.24 | 0.21 | 0.28 | 0.26 | 0.24 | 0.25 | 0.03 | 0.37 | 0.37 |
| 4                                                    | 0.23 | 0.26 | 0.25 | 0.25 | 0.25 | 0.23 | 0.28 | 0.25 | 0.27 | 0.06 | 1.43 | 0.40 | 0.30 |
| 19                                                   | 0.24 | 0.25 | 0.25 | 0.24 | 0.26 | 0.39 | 0.29 | 0.26 | 0.27 | 0.39 | 0.64 | 1.07 | 0.51 |
| 5                                                    | 0.23 | 0.24 | 0.24 | 0.25 | 0.23 | 0.25 | 0.40 | 0.27 | 0.25 | 0.26 | 0.40 | 0.96 | 0.31 |
| 11                                                   | 0.29 | 0.30 | 0.31 | 0.31 | 0.35 | 0.33 | 0.40 | 0.34 | 0.36 | 0.33 | 0.35 | 0.36 | 0.36 |
| 18                                                   | 0.33 | 0.35 | 0.35 | 0.34 | 0.40 | 0.40 | 0.09 | 0.38 | 0.42 | 0.40 | 0.38 | 0.37 | 0.50 |

Table 2: THE SYMMETRIC UNCERTAINTY OF THE RELEVANT SYNTHETIC BANDS.

| Bands indices in ascending order of their MI with the GT | 12 | 8 | 15 | 6 | 1 | 3 | 16 | 14 | 2 | 10 | 17 | 4 | 19 | 5 | 11 | 18 |
|-------------------------------------------------------|----|----|----|---|---|---|----|----|---|----|----|---|----|---|----|----|
| 12                                                   | 0.13 | 0.14 | 0.12 | 0.16 | 0.15 | 0.17 | 0.16 | 0.17 | 0.18 | 0.16 | 0.17 | 0.18 | 0.23 |
| 8                                                    | 0.21 | 0.16 | 0.16 | 0.18 | 0.18 | 0.20 | 0.20 | 0.20 | 0.20 | 0.22 | 0.21 | 0.20 | 0.27 |
| 15                                                   | 0.14 | 0.16 | 0.16 | 0.18 | 0.18 | 0.19 | 0.19 | 0.20 | 0.19 | 0.20 | 0.19 | 0.21 | 0.24 |
| 6                                                    | 0.12 | 0.16 | 0.16 | 0.18 | 0.19 | 0.21 | 0.19 | 0.20 | 0.20 | 0.19 | 0.21 | 0.22 | 0.24 |
| 1                                                    | 0.21 | 0.16 | 0.18 | 0.18 | 0.19 | 0.22 | 0.22 | 0.23 | 0.21 | 0.22 | 0.21 | 0.20 | 0.29 |
| 10                                                   | 0.23 | 0.19 | 0.19 | 0.19 | 0.19 | 0.23 | 0.23 | 0.23 | 0.21 | 0.22 | 0.23 | 0.20 | 0.34 |
| 17                                                   | 0.23 | 0.24 | 0.24 | 0.25 | 0.25 | 0.25 | 0.40 | 0.27 | 0.25 | 0.26 | 0.40 | 0.96 | 0.31 |
| 4                                                    | 0.23 | 0.25 | 0.25 | 0.25 | 0.25 | 0.23 | 0.35 | 0.25 | 0.27 | 0.38 | 0.30 | 0.39 | 0.33 |
| 19                                                   | 0.24 | 0.25 | 0.25 | 0.24 | 0.26 | 0.39 | 0.29 | 0.26 | 0.27 | 0.39 | 0.64 | 1.07 | 0.51 |
| 5                                                    | 0.23 | 0.24 | 0.24 | 0.25 | 0.23 | 0.25 | 0.40 | 0.27 | 0.25 | 0.26 | 0.40 | 0.96 | 0.31 |
| 11                                                   | 0.29 | 0.30 | 0.31 | 0.31 | 0.35 | 0.33 | 0.40 | 0.34 | 0.36 | 0.33 | 0.35 | 0.36 | 0.36 |
| 18                                                   | 0.33 | 0.35 | 0.35 | 0.34 | 0.40 | 0.40 | 0.09 | 0.38 | 0.42 | 0.40 | 0.38 | 0.37 | 0.50 |

Observation 2: Figure.5 shows that the bands $A_{16}$ and $A_{18}$ are practically disjoint, i.e. they are not redundant. Table 1. shows $AS(A_{16}, A_{18}) = 0.06$ and $AS(A_{18}, A_{16}) = 0.09$. It means that no information is shared between $A_{16}$ and $A_{18}$. So this indicates no redundancy (Table 2 allows the same observation).

This makes an interest result : the ensemble of selected bands became $SS = \{16, 18\}$. $A_{16}, A_{18}$ will be discarded from the Table .1. (The same for Table 2 )

Now we can emit this rule:

"Each band candidate will be added to the ensemble of already selected ones, SS, if and only if their Normalized Mutual Information values with all elements of SS, are less than the
thresholds (here 0.7)."

Algorithm 1 implements this rule.

**Algorithm 1**: Band is the HSI. Let $\text{Th}_{\text{relevance}}$ the threshold for selecting bands more informative, $\text{Th}_{\text{redundancy}}$ the threshold for redundancy control.

1) Compute the Mutual Information ($MI$) of the bands and the Ground Truth map.
2) Make bands in ascending order by their $MI$ value.
3) Cut the bands that have a lower value than the threshold $\text{Th}_{\text{relevance}}$, the subset remaining is $S$.
4) Initialization: $n = \text{length}(S), i = 1$, $D$ is a bidirectional array $\text{values} = 1$;
   //any value greater than 1 can be used, it’s useful in step 6)
5) Computation of bidirectional Data $D(n,n)$:
   for $1 := 1$ to $n$ step 1 do
   for $j := 1$ to $n$ step 1 do
   $D(i,j) = AS(\text{Band}_S(i), \text{Band}_S(j));$ // with $AS(A,B) = \frac{MI(A,B)}{H(A)}$
   //Or $D(i,j) = U(\text{Band}_S(i), \text{Band}_S(j));$ with $U(A,B) = \sqrt{\frac{MI(A,B)}{H(A) H(B)}}$
   end for
   end for
6) $SS = \{\}$ ; // Initialization of the Output of the algorithm
   while $\text{min}(D) < \text{Th}_{\text{redundancy}}$ do
   // Pick up the argument of the minimum of $D$
   $(x, y) = \text{argmin}(D(:, .))$;
   if $\forall l \in SS \ D(x, l) < \text{Th}_{\text{redundancy}}$ then
   // check $x$ is not redundant with the already selected bands
   $SS = SS \cup \{x\}$
   end if
   $D(x, y) = 1$; // The cells $D(x, y)$ will not be checked as minimum again
   end while
7) Finish: The final subset $SS$ contains bands according to the the couple of thresholds ($\text{Th}_{\text{relevance}}, \text{Th}_{\text{redundancy}}$).

4 **Application On HIS AVIRIS 92AV3C**

The Algorithm 1 implement the proposed method. Here we use real data, hyperspectral image AVIRIS 92AV3C (Landgrebe, 1997), as input of the algorithm. 50% of the labelled pixels are randomly chosen and used in training; and the other 50% are used for testing classification (Guo et al., 2006; Sarhrouni et al., 2012a; Sarhrouni et al., 2012b). The classifier used is the SVM (Chang and Lin, 2011; Hsu and Lin, 2002; Guo, Gunn, Senio and Nelson., 2008). The Algorithm 1 implement the proposed method.
4.1 Mutual Information Curve of Bands

We must eliminate no informative bands from the remaining subset bay thresholding, see the proposed algorithm. Figure 7 gives the MI of the HSI AVIRIS 92AV3C with the ground truth GT.

![Figure 7: Mutual information of GT and AVIRIS bands.](image)

4.2 Results

From the remaining subset bands, we must eliminate redundant ones using the proposed algorithm. Table 3 and Table 4 give the accuracy of classification for a number of bands with several thresholds, for the two forms of normalized MI.

4.3 Discussion

Results in Table 3 and Table 4 allow us to distinguish six zones of couple values of thresholds \((TH, IM)\):

First case: lower values of TU and higher values of TH, this is practically no control of relevance and no control of redundancy. So there is no action of the algorithm (Zone1).

Second case: Higher values TU and lower values of TH, this is a hard selection: a few more relevant and no redundant bands are selected (Zone2).

Third case (Zone3): This is an interesting zone. We can have easily 80% of classification accuracy with about 41 bands.

Fourth case (Zone4): This is the very important zone; we have the very useful behaviours of the algorithm. For example with a few numbers of bands 17 we have classification accuracy
Table 3: Classification Accuracy for several couples of thresholds (TH,IM) and their corresponding number of bands retained, (First Form of Normalized MI).

| TH: Threshold for control the relevance (MI of bands with GT) |
|-------------------------------------------------------------|
| 0 N.B. [ac(%)] | MI > 0.4 N.B. [ac(%)] | MI > 0.45 N.B. [ac(%)] | MI > 0.57 N.B. [ac(%)] | MI > 0.6 N.B. [ac(%)] | MI > 0.9 N.B. [ac(%)] | MI > 0.91 N.B. [ac(%)] | MI > 0.94 N.B. [ac(%)] |
| 0.10 25 36.06 3 46.31 | 2 46.62 | 1 37.05 | - | - | - | - | - |
| 0.15 20 43.93 7 59.28 | 7 59.21 | 3 46.74 | 2 43.25 | - | - | - | - |
| 0.20 35 46.02 9 61.64 | 9 67.06 | 5 54.70 | 3 58.06 | - | - | - | - |
| 0.25 29 46.64 13 68.75 | 13 68.67 | 7 60.32 | 7 62.42 | - | - | - | - |
| 0.30 30 47.48 21 76.19 | 20 75.34 | 15 69.96 | 14 66.71 | - | - | - | - |
| 0.35 57 47.05 29 77.77 | 29 77.67 | 23 71.87 | 21 71.30 | 2 60.28 | 2 55.48 | - | - |
| 0.40 73 46.56 42 81.77 | 41 81.75 | 30 76.74 | 29 75.86 | 5 71.73 | 4 63.80 | - | - |
| 0.43 78 46.43 47 83.15 | 46 82.51 | 35 77.25 | 33 76.97 | 6 72.96 | 4 61.56 | - | - |
| 0.45 88 45.96 56 84.45 | 54 83.54 | 42 80.13 | 41 80.01 | 11 76.80 | 7 65.69 | 2 52.09 | - |
| 0.46 94 45.57 61 84.88 | 58 84.53 | 48 81.67 | 44 80.19 | 12 78.08 | 8 66.78 | 2 52.09 | - |
| 0.47 100 45.16 68 86.21 | 63 85.17 | 49 80.81 | 48 80.64 | 17 79.05 | 12 72.70 | 2 52.09 | - |
| 0.48 108 44.91 70 86.34 | 65 85.26 | 53 82.02 | 52 81.73 | 20 80.42 | 16 72.82 | 3 55.19 | - |
| 0.49 115 44.69 78 86.85 | 73 86.44 | 60 82.64 | 57 82.88 | 22 81.47 | 18 74.38 | 3 55.19 | - |
| 0.50 122 44.54 80 87.14 | 75 86.40 | 63 84.28 | 60 83.58 | 25 81.86 | 20 74.83 | 5 57.77 | - |
| 0.51 129 44.05 86 87.51 | 81 87.01 | 68 84.84 | 67 84.73 | 29 82.54 | 24 76.54 | 6 57.82 | - |
| 0.52 135 43.76 88 87.51 | 83 87.03 | 71 84.84 | 70 84.90 | 30 80.58 | 25 76.78 | 8 58.29 | - |
| 0.53 141 43.48 94 87.78 | 89 87.36 | 75 85.23 | 74 84.88 | 33 83.34 | 29 77.48 | 10 59.75 | - |
| 0.54 147 43.19 104 87.78 | 99 87.53 | 81 85.95 | 79 85.02 | 38 83.89 | 33 77.91 | 14 61.52 | - |
| 0.55 154 42.57 107 87.90 | 102 87.63 | 83 86.07 | 81 85.66 | 40 84.01 | 35 78.67 | 15 61.84 | - |
| 0.56 159 42.29 112 88.10 | 107 87.70 | 85 86.03 | 83 86.05 | 42 84.24 | 36 78.49 | 16 61.89 | - |
| 0.70 220 38.96 173 88.72 | 163 88.41 | 128 87.88 | 126 87.55 | 67 86.71 | 54 81.77 | 22 63.72 | - |
| 0.90 220 38.96 176 88.72 | 163 88.41 | 128 87.88 | 126 87.55 | 67 86.71 | 54 81.77 | 22 63.72 | - |

N.B.: Number of Bands retained for the couple of threshold (MI,TH)
ac(%): The accuracy of classification calculated for the couple of threshold (MI,TH)

80%.

Fifth case: Here we make a hard control of redundancy, but the bands candidates are more near to the GT, and they may be more redundant. So we can’t have interesting results. (Zone5)

Sixth case (Zone6): When we do not control properly the relevance, some bands affected by transfer affects may be non redundant, and can be selected, so the accuracy of classification is decreasing.

We conclude that this algorithm is very effectiveness for redundancy and relevance control, in feature selection area.

The most difference of this heuristic regarding previous works is the separation of the operations: avoiding redundancy and selecting more informative bands. Sarhrouri (Sarhrouri et al., 2012a) use also a filter strategy an MI based algorithm to select bands, and another wrapper strategy algorithm also based on MI (Sarhrouri et al., 2012b). Guo (Guo et al., 2006) used a filter strategy with threshold control redundancy, but in those works, the tow process, i.e avoiding redundancy and avoiding no informative bands, are made at the same time by the
same threshold. Figure 8 illustrates the reconstruction of the ground truth map GT, for the first form of normalised MI. The redundancy threshold 0.56 and relevance threshold IM=0.9. The accuracy classification is 84.24% for 42 bands selected. The generalisation of classification to the entire scene Indiana Pin (Landgrebe, 1997) illustrates the power of the proposed method: the pixels not labelled in GT, are here classified.

Table 4: Classification Accuracy for several couples of thresholds (TH,IM) and their corresponding number of bands retained, (Second Form of Normalized MI).

| MI  | N.B | TH | N.B | TH | N.B | TH | N.B | TH | N.B | TH | N.B | TH | N.B | TH | N.B | TH | N.B | TH | N.B | TH | N.B | TH | N.B | TH | N.B | TH |
|-----|-----|----|-----|----|-----|----|-----|----|-----|----|-----|----|-----|----|-----|----|-----|----|-----|----|-----|----|-----|----|
| 0.00| 26  | 34 | 36  | 37 | 38  | 39 | 40  | 41 | 42  | 43 | 44  | 45 | 46  | 47 | 48  | 49 | 50  | 51 | 52  | 53 | 54  | 55 | 56  | 57 |
| 0.10| 26  | 34 | 36  | 37 | 38  | 39 | 40  | 41 | 42  | 43 | 44  | 45 | 46  | 47 | 48  | 49 | 50  | 51 | 52  | 53 | 54  | 55 | 56  | 57 |
| 0.15| 30  | 37 | 38  | 39 | 40  | 41 | 42  | 43 | 44  | 45 | 46  | 47 | 48  | 49 | 50  | 51 | 52  | 53 | 54  | 55 | 56  | 57 | 58  | 59 |
| 0.20| 34  | 40 | 41  | 42 | 43  | 44 | 45  | 46 | 47  | 48 | 49  | 50 | 51  | 52 | 53  | 54 | 55  | 56 | 57  | 58 | 59  | 60 | 61  | 62 |
| 0.25| 40  | 46 | 47  | 48 | 49  | 50 | 51  | 52 | 53  | 54 | 55  | 56 | 57  | 58 | 59  | 60 | 61  | 62 | 63  | 64 | 65  | 66 | 67  | 68 |

Figure 8: In the middle the GT of AVIRIS 92AV3C. In the left: Reconstructed Truth map (GT) with the proposed algorithm for TH=0.56 and IM=0.9; the accuracy = 84.24% for only 42 bands. In right the generalization of classification for all Indiana Pine regions.
Figure 9 also, illustrates the reconstruction of the ground truth map GT, for the second form of normalised MI. The redundancy threshold 0.56 and relevance threshold IM=0.9. The accuracy classification is 84.16% for 42 bands selected. The generalisation of classification to the entire scene Indiana Pin (Landgrebe, 1997) illustrates the power of the proposed method: the pixels not labelled in GT, are here classified.

Now, we can view that the two forms of normalized MI give practically the same performances. But Table 2 is a symmetric matrix; and affects largely the complexity of the algorithm.

5 Conclusion

The features selection in high dimensionality is problematic that is always open. The combinatorial search and test for all possible subset, is practically impossible. We can use some heuristic methods and algorithms to select no optimal subset features, but useful in practical field. The subset retained must be relevant and no redundant. In this paper we introduce an heuristic in order to process separately the relevance and the redundancy. We apply our method to classify the region Indiana Pin with the Hyperspectral Image AVIRIS 92AV3C. This algorithm is a Filter strategy (i.e. with no call to classifier during the selection). In the first step, by thresholding we use mutual information to pick up relevant bands (like most method already used). The second step uses Normalized Mutual information to measure redundancy. We conclude the effectiveness of our method and algorithm to select the relevant and no redundant bands. This algorithm allows us a wide area of possible fasted applications. But no guaranties that the chosen bands are the optimal ones; and some times redundancy can be important to reinforcement of learning classification system. So the thresholds controlling relevance redundancy is a very useful tool to calibre the selection, in real time applications, regarding that in commercial applications, the inexpensive filtering algorithms are urgently preferred.

References

Barlow, H. 1961. A unifying information-theoretic framework for independent component analysis, *International journal of computers and mathematics with applications*. 
Chang, C.-C. and Lin, C.-J. 2011. LIBSVM: A library for support vector machines, *ACM Transactions on Intelligent Systems and Technology* **2**: 27:1–27:27. Software available at [http://www.csie.ntu.edu.tw/~cjlin/libsvm](http://www.csie.ntu.edu.tw/~cjlin/libsvm).

Comon 1994. Independent component analysis a new concept, *Signal Processing* **36**: 287–314.

Denton, E. R. E., Holden, M., Chris, E., Jarosz, J. M., Russell-Jones, D., Goodey, J., Cox, T. C. S., and Hill, D. L. G. 2000. The identification of cerebral volume changes in treated growth hormonedeficient adults using serial 3d mr image processing, *Journal of Computer Assisted Tomography* **24**(1): 139–145.

Guo, B., Gunn, S. R., Senio, R. I. D. and Nelson, J. D. B. 2006. Band selection for hyperspectral image classification using mutual information, *IEE GEOSCIENCE AND REMOTE SENSING LETTERS* **Vol 3**(4).

Guo, B., Gunn, S. R., Senio, R. I. D. and Nelson, J. D. B. 2008. Customizing kernel functions for svm-based hyperspectral image classification, *IEEE TRANSACTIONS ON IMAGE PROCESSING* **Vol 17**(4).

Holden, M., Denton, E. R. E., Jarosz, J. M., Cox, T. C. S., Studholme, C., Hawkes, D. J. and Hill, D. L. G. 1999. Detecting small anatomical changes with 3d serial mr subtraction images, in *Medical Imaging: Image Processing, Proc. SPIE* **366**(1): 44–55.

Homayouni, S. 1998. Assessment of multi source data fusion methods for improvement of accuracy in urban area classification from remotely sensed data, *Mémoire de D.E.A.*.

Homayouni, S. 2005. *Caractérisation des Scènes Urbaines par Analyse des Images Hype-spectrales*, Thesis, Ecole Nationale Supérieure des Télécommunications de Paris.

Hsu, C.-W. and Lin, C.-J. 2002. A comparison of methods for multiclass support vector machines, *Neural Networks, IEEE Transactions on* **13**: 415–425. Sponsored by: IEEE Computational Intelligence Society.

Huges, G. 1968. On the mean accuracy of statistical pattern recognizers, *Information Thaory, IEEE Transactionon* **Vol 14**(1): 55–63.

Hyvarinen 99. Survey on independent component analysis, *Neural Computing Surveys* **2**: 94–128.

Hyvarinen and Karhunen, E. 2001. *Independent Component Analysis*.

Kernéis, D. 2007. *Amélioration de la classification automatique des fonds marins par la fusion multicapteurs acoustiques*, PhD thesis, ENST BRETAGNE, université de Rennes. Chapitre3, Rédution de dimensionalité et classification, Page.48.

Kwak, N. and Choi, C. 2007. Feature extraction based on direct calculation of mutual information, *IJPRAI* **Vol 21**(7): 1213–1231.
Kwak, N. and Kim, C. 2006. Dimensionality reduction based on ica regression problem, *Lecture Notes in Computer Science* **1431**: 1–10.

Landgrebe, D. 1997. *On information extraction principles for hyperspectral data: A white paper*, Purdue University, West Lafayette, IN, Technical Report, School of Electrical and Computer Engineering, Téléchargeable ici : http://dynamo.ecn.purdue.edu/landgreb/whitepaper.pdf.

Lee, T.-W., M. Girolami, Bell, A. and Sejnowski, T. 2000. A unifying information-theroretic framework for independent component analysis, *International Journal of Computers and Mathematics with Applications* **39**: 1–21.

Lei, Y. and Huan, L. 2004. Efficient feature selection via analysis of relevance and redundancy, *Journal of Machine Learning Research* **5**: 1205–1224.

Linsker, R. 1992. Local sunaptic learning rules suffice to maximize mutual information in a linear network, *Neural Computation* **4**: 691–702.

Oja, J., K., L., W. and Vigario, R. 95. Principal and independent components in neural networks - recent developments, *In Proc. VII Italian Workshop Neural Networks WIRN'95*, Vietrisul Mare, Italy pp. 16–35.

Otte, M. 2001. Elastic registration of fmri data using b´ezier-spline transformations, *IEEE Transactions on Medical Imaging* **20**(3): 193–206.

Perdue 97. document avaible at: ftp://ftp.ecn.purdue.edu/biehl/MultiSpec/.

Sarhrouni, E., Hammouch, A. and Aboutajdine, D. 2012a. Dimensionality reduction and classification feature using mutual information applied to hyperspectral images: a filter strategy based algorithm, *Appl. Math. Sci* **6**(101-104): 5085–5095.

Sarhrouni, E., Hammouch, A. and Aboutajdine, D. 2012b. Dimensionality reduction and classification feature using mutual information applied to hyperspectral images: a wrapper strategy algorithm based on minimizing the error probability using the inequality of fano, *Appl. Math. Sci* **6**(101-104): 5073–5084.

Witten, E. F. 2005. *Data Mining: Practical Machine Learning Tools and Techniques*, Morgan Kaufmann,2 edition, San Francisco.

YANG, Yiming and PEDERSEN., J. O. 1997. A comparative study of feature selection in text categorization, *Proceedings of the Fourteenth International Conference on Machine Learning(ICML'97)* **5**: 412–420.