Macroscopic Parity Violation and Supernova Asymmetries

C. J. Horowitz ∗
Nuclear Theory Center and Department of Physics,
Indiana University, Bloomington, IN 47405, USA

J. Piekarewicz †
Supercomputer Computations Research Institute,
Florida State University, Tallahassee, FL 32306
(October 5, 2018)

Abstract

Core collapse supernovae are dominated by weakly interacting neutrinos. This provides a unique opportunity for macroscopic parity violation. We speculate that parity violation in a strong magnetic field can lead to an asymmetry in the explosion and a recoil of the newly formed neutron star. We estimate the size of this asymmetry from neutrino polarized neutron elastic scattering, polarized electron capture and neutrino-nucleus elastic scattering in a (partially) polarized electron gas.

I. INTRODUCTION

Core collapse supernovae emit approximately $10^{58}$ neutrinos. Apart from gravity, these neutrinos only have weak interactions—whose hallmark is parity violation. Can this lead to macroscopic parity violation? The important role played by weak interactions makes supernovae (or related phenomena) unique among all macroscopic systems in the present universe. No other systems have this potential for large scale parity violation.

One parity violating observable is a correlation $\mathbf{k}_\nu \cdot \mathbf{B}$ between the neutrino flux in the direction $\mathbf{k}_\nu$ and the direction of the magnetic field $\mathbf{B}$. As a result, the entire neutron star, formed in the explosion, could recoil. This can be at a significant velocity because of the tremendous momentum of the neutrinos (see below).

∗E-mail: Charlie@IUCF.indiana.edu
†E-mail: jorgep@scri.fsu.edu
Indeed, the observed velocities of neutron stars are large: with three dimensional galactic velocities of the order of 500 km/s [1]. Perhaps the simplest explanation for such large velocities is that neutron stars receive a significant kick at birth from an asymmetric supernova explosion. Note that this asymmetry need not be solely from parity violation. For example, one could have a preexisting asymmetry in the collapsing stellar core as speculated by Burrows and Hayes [2]. These authors also discuss how an asymmetry may impact a detectable gravitational radiation signal.

In any case, there is a simple relation between the final velocity of a neutron star and the asymmetry of the neutrinos $A_{\nu}$ and other matter $A_{m}$ ejected. Note, a supernova ejects the mantle and outer envelope comprising some 90 percent of the original star’s mass. The binding energy per nucleon of a neutron star is about 100 MeV/n. Thus, neutrinos carry off 100 MeV/c of momentum per nucleon of the neutron star (which then has a gravitational mass of the order of 839 MeV/n). In contrast, other matter only carries off about one percent of the binding energy (1 MeV/n). However, the mass of this material is about 10 times larger than that of the neutron star, so that its momentum $(p_{m} \approx 2 \cdot 1\text{MeV} \cdot 10 \cdot 939\text{MeV})^{1/2} \approx 140 \text{MeV}/c$ is comparable to that of the neutrinos.

The recoil velocity $v$ of a neutron star is approximately,

$$v/c \approx 0.1(A_{\nu} + A_{m}),$$  

with $c$ the speed of light and the asymmetry of the neutrinos is

$$A_{\nu} = \frac{|\sum p_{i}|}{\sum |p_{i}|},$$

(2)

where the sum runs over all neutrinos emitted. There is a similar expression for the asymmetry of the other matter $A_{m}$. To reproduce observed velocities of the order of $10^{-3}c$ one needs $A_{\nu}$—and/or $A_{m}$—to be about one percent.

Since matter and neutrinos are coupled, one expects both $A_{\nu}$ and $A_{m}$ to be nonzero, i.e., an original asymmetry in one will produce an asymmetry in the other. Thus, the interesting question is which came first (the chicken or the egg) $A_{\nu}$ or $A_{m}$?

In this paper we speculate that the original $A_{\nu}$ may stem from parity violation in a strong magnetic field. We assume conventional weak interactions for the neutrinos. Others have considered nonstandard neutrino magnetic moments [3] or matter enhanced neutrino oscillations [4]. These could enhance the effect. However, one does not require new interactions for a nonzero asymmetry. Thus, our speculation only depends on the existence of a strong magnetic field.

A number of Pulsars are thought to have magnetic fields around $10^{12}$ Gauss. However, the dipole field inside a supernova at early times could be stronger, perhaps $10^{14}$ Gauss [5]. Finally, it is possible that there are very strong non-dipole fields of the order of $10^{16}$ Gauss [6]. For example, differential rotation could wind up the B field into a very strong torus configuration. In section II we calculate the neutrino asymmetry that these fields induce because of

\textsuperscript{1}One can ask why do supernovae explode so that the momentum in matter is comparable to that in neutrinos?
several parity violating hadronic and electron reactions. We discuss possible enhancements of the asymmetry and conclude in section III.

We end this section with a speculative note regarding macroscopic parity violation and the origin of life. Cline suggests [7] that polarized leptons from a nearby supernova could influence homochiral organic molecule formation. This is an interesting but clearly very speculative attempt to explain why Terrestrial life uses almost exclusively L-amino-acid enantiomers (“left handed” mirror image molecules). Although not directly related, we speculate that supernovae produce another macroscopic parity violating effect. The hypothesis that parity violation leads to an asymmetry and recoil may be more directly observed and tested.

II. MICROSCOPIC PARITY VIOLATING REACTIONS

There are many possible sources of parity violation in a strong magnetic field. One would expect weak interactions involving electrons to be the most important because of the large electron magnetic moment. However, a large Fermi momentum can make it harder to polarize electrons. Furthermore, relativistic effects reduces the effective magnetic moment. Finally, interactions with nucleons often dominate the neutrino opacity rather than interactions with electrons. Therefore, it is possible for nucleon reactions to compete with electrons.

In this section we consider a number of reactions. Perhaps the simplest is neutrino elastic scattering from polarized neutrons. The longitudinal asymmetry for this process is very large,

$$|A_l| = \frac{2g_v g_a}{g_v^2 + 3g_a^2} \approx 0.46,$$

with $g_v = -1$ and $g_a = -1.26$. The polarization of neutrons in a magnetic field $B$ is,

$$P_e \approx eB/M_n kT,$$

where $M_n$ is the neutron’s mass, $T$ the temperature and we have neglected possible enhancements from the spin dependence of the strong interactions (more on this below). Elastic neutron scattering makes a significant contribution to the neutrino opacity. Therefore, this will produce an asymmetry in the neutrino flux of the order of $A_\nu \approx P_e A_l$, which at a temperature near 3 MeV is,

$$|A_\nu| \approx 1.3 \times 10^{-4} B_{14},$$

with $B_{14}$ the magnetic field in units of $10^{14}$ Gauss. This asymmetry, of the order of $eB/M_n kT$, is somewhat small compared to one percent. However, this simple mechanism is clearly present and provides a benchmark to compare other reactions.

Vilenkin [8] estimated the asymmetry of neutrino electron scattering (NES) while Bezchastnov and Haensel [9] calculated it in detail; it is of the order of $eB/k_F^2$, where $k_F \approx 20$ MeV is the electron Fermi momentum. Here, one power of $k_F^{-1}$ represents the relativistic electron magnetic moment and the second power describes the difficulty of polarizing
a degenerate Fermi gas. The NES asymmetry is about 10 times the neutron polarization. However, NES makes less then a 10 percent contribution to the total neutrino opacity (indeed it is probably less then one percent). Therefore the NES contribution to \( A_\nu \) is probably smaller than Eq. (3). Note, NES does change the neutrino energy and could introduce an asymmetry in the spectrum. One should investigate this effect; however, it is probably small. For example Haxton and Bruenn made large changes in the energy loss cross section and found only small effects on the dynamics.

We now consider neutrino-nucleus elastic scattering in a polarized electron gas. Nuclear scattering dominates the opacity, as long as many nuclei are present. Furthermore, this cross section is impacted by screening from the dense electron gas [11,12]. Polarization of the electron gas by the \( B \) field will introduce an asymmetry in the nuclear cross section even for spin zero nuclei. Furthermore, the asymmetry will depend on the large electron magnetic moment. Therefore this process could produce a significant neutrino asymmetry.

We are not aware of an exact calculation. Instead, we present a simple phase space estimate assuming the magnetic field slightly polarizes the electron gas. We are interested in the interference of the electron particle-hole excitation shown in Fig. 1B with the direct neutrino-nucleus scattering of Fig. 1A. Parity violation stems from the weak axial charge of the electron interfering with the weak vector charge of the nucleus. The electron particle-hole loop is described by (see ref. [11])

\[
\Pi_{\mu\nu}^n(q) = -i \int \frac{d^4p}{(2\pi)^4} \text{tr} [\gamma_5 \gamma_\mu G(p) \gamma_\nu G(p + q)],
\]

(6)

here \( q \) is the four-momentum transferred to the nucleus and the electron propagator \( G = G_F + G_D \) has the usual Feynman piece \( G_F(p) = (\not{p} - m + i\epsilon)^{-1} \) and a density dependent part \( G_D \) which describes the occupied states in the (slightly polarized) Fermi sea,

\[
G_D(p) = (\not{p} + m) \frac{i\pi}{\sqrt{p^2 + m^2}} \delta(p_0 - \sqrt{p^2 + m^2}) \left\{ n_+ \left( \frac{1 + \gamma_5 \hat{s}}{2} \right) + n_- \left( \frac{1 - \gamma_5 \hat{s}}{2} \right) \right\}.
\]

(7)

Here \( n_+ \) and \( n_- \) are the occupations of the Fermi gas states polarized along or against \( \hat{s} \); for an electron at rest, the relativistic spin vector \( s \) is chosen in the direction of \( B \). We assume a dispersion relation,

\[
\epsilon_\pm^2 \approx p^2 + m^2 \pm eB,
\]

(8)

which describes the interaction of the spin magnetic moment with \( B \). We assume that the orbital contributions are small since in the weak field limit a very large number of Landau levels will be filled. In any event, it is hoped that this simple approximation provides an order of magnitude estimate. We evaluate Eq. (6) in the extreme relativistic limit \( m \to 0 \), for zero energy transfer \( q_0 = 0 \), and assume weak fields \( eB \ll k_F^2 \). Moreover, at low temperatures one has \( n_\pm(p) = \theta(k_F - \epsilon_\pm) \), with \( k_F \) the Fermi energy. In a coordinate system with \( q \) along \( \hat{3} \) and \( \hat{s} \) in the 1-3 plane one has,

\[2\]Not 1000 times as would be expected from the ratio of magnetic moments in nonrelativistic non-degenerate gases
\[ \Pi_{10}^{va}(q) = \frac{\sin \alpha}{4\pi^2} eB \left\{ \frac{k_F}{q} \left( 1 - \frac{q^2}{4k_F^2} \right) \ln \left| \frac{2k_F + q}{2k_F - q} \right| + 1 \right\}, \]  

with \( \alpha \) the angle between \( \mathbf{B} \) and \( \mathbf{q} \). In the limit of \( q \ll k_F \) one has simply,

\[ \Pi_{10}^{va} \approx \sin \alpha \frac{eB}{2\pi^2}. \]  

The density independence of this result arises from a cancellation. The polarization of a relativistic electron gas decreases with density as \( eB/k_F^2 \) while the density of states for particle-hole excitations goes like \( k_F^2 \). The asymmetry in the neutrino-nucleus elastic cross section is,

\[ |A| \approx \frac{2Ze^2 c_a}{Cq^2} \Pi_{01}^{va}. \]  

Here \( c_a = \pm 1/2 \) is the weak axial charge of the electron (+ for \( \nu_\mu, \nu_\tau \) and − for \( \nu_e \)), \( C = Z(1/2 - 2\sin^2 \theta_W) - N/2 \) is the weak vector charge of a nucleus with charge \( Z \) and neutron number \( N \) and the momentum transfer is \( q \approx E_\nu \approx 10MeV \). As long as the opacity is dominated by neutrino nucleus scattering this will produce a neutrino asymmetry of about,

\[ |A_\nu| \approx 5 \times 10^{-5} B_{14}, \]  

for nuclei near \(^{56}\text{Fe} \). This somewhat small number arises from the factor of \( e^2 = 4\pi\alpha \) in Eq. (11). Nevertheless, because of the large electron magnetic moment, \( A_\nu \) is comparable to that from polarized neutrons in Eq. (5).

Finally, we make a simple phase space estimate for the asymmetry from polarized electron capture on protons. For simplicity, we neglect the electron mass and the neutron-proton mass difference. The rate for left handed electrons moving in the direction \( \theta_e \) to be captured and produce a neutrino moving in the direction \( \theta_\nu, \phi_\nu \) into \( d\Omega \) is,

\[ \frac{dR}{d\Omega} \propto \int \left( 3g_a^2 + g_v^2 + (g_v^2 - g_a^2)\cos \theta_{\nu e}\right)E_\nu^2 n_{k\downarrow}(\theta_e)d^3k. \]  

Here \( \mathbf{k} \) is the momentum of the initial electron and \( n_{k\downarrow} \) is the occupation of left handed spin states (for an electron moving at an angle \( \theta_e \) with respect to \( \mathbf{B} \)). Finally, \( \theta_{\nu e} \) is the scattering angle of the neutrino with respect to the initial electron’s direction.

The angular distribution is important. Electron capture on a proton involves both Fermi transitions with a forward peaked angular distribution \( g_v^2(1 + \cos \theta_{\nu e}) \) and backward peaked Gamow Teller transitions involving \( g_a^2(3 - \cos \theta_{\nu e}) \). The sum is only weakly backward peaked \( (g_v^2 > g_a^2) \). A flat angular distribution will not produce a neutrino asymmetry because the neutrino “forgets” the initial electron direction, i.e., neutrinos are emitted isotropically, independent of any asymmetry in the initial electron direction. Therefore only the \( \cos \theta_{\nu e} \) term will contribute and it involves the small factor \( g_v^2 - g_a^2 \).

The occupation of left handed electrons along \( \mathbf{k} \) can be decomposed into spin up \( n_+ \) and down \( n_- \) occupations along \( \mathbf{B} \): 

\[ n_{k\downarrow}(\theta_e) = \left( \frac{1 + \cos \theta_e}{2} \right)n_- + \left( \frac{1 - \cos \theta_e}{2} \right)n_+ \]  

(14)
As above we use \( E_2^2 = E_e^2 \approx k^2 \pm eB \), \( n_\pm = \theta[k_F^2 - (k^2 \pm eB)] \) so that,

\[
\frac{dR}{d\Omega} \propto \int d^3k \left[ 1 + \frac{g_a^2 - g_v^2}{3g_a^2 + g_v^2} \cos \theta_{\nu e} \right] \left\{ \left( \frac{1 + \cos \theta_e}{2} \right) (k^2 - eB)n_- + \left( \frac{1 - \cos \theta_e}{2} \right) (k^2 + eB)n_+ \right\}.
\] (15)

This gives,

\[
\frac{dR}{d\Omega} \propto 1 + \left( \frac{g_a^2 - g_v^2}{3g_a^2 + g_v^2} \right) \left( \frac{5eB}{12k_F^2} \right) \cos \theta_{\nu e},
\] (16)

and produces a neutrino asymmetry of,

\[
A_\nu \approx \left( \frac{g_a^2 - g_v^2}{3g_a^2 + g_v^2} \right) \frac{5eB}{12k_F^2}.
\] (17)

We estimate the contribution from the emission of neutrinos near the neutrinosphere at a density of the order of \( 10^{11} \text{ g/cm}^3 \) and at an electron fraction (number of electrons per baryon) of the order of 0.1. Here \( k_F \approx 10 \text{ MeV} \) and

\[
|A_\nu| \approx 2.4 \times 10^{-4} B_{14}.
\] (18)

This asymmetry is somewhat larger than the others that we have calculated. Yet, it has been reduced by a factor of about 30 because of the nearly flat angular distribution. If it were not for this reduction, this reaction could produce a one percent asymmetry for \( B_{14} \) near unity. Note that one should check our phase space estimate with an exact calculation. If the \( B \) field modifies the angular distribution it could substantially increase the asymmetry.

We illustrate the sign of the electron capture asymmetry in Fig. (2). A vertical \( B \) field polarizes electrons with spin down. As a result, only electrons moving up are left handed and can be captured. However these electrons produce neutrinos moving down because the angular distribution is (weakly) backward peaked. Thus the star recoils in the direction of the \( B \) field (up).

**III. DISCUSSION AND CONCLUSIONS**

In this section we discuss possible enhancements of the asymmetry and conclude. Spin dependent strong interactions can enhance the magnetic response of neutron rich matter. This could increase the asymmetry from polarized neutrons. For example, Kutschera and Wojcik [14] speculate that a ferromagnetic state could form because of spin-dependent neutron-proton interactions. Indeed the asymmetry could still be increased by a significant amount, even if a ferromagnetic state did not form. However, this enhancement is unlikely to be an order of magnitude or more unless one is very close to a ferromagnetic transition—which we think unlikely. Therefore, we do not expect very large enhancements from the strong interactions.

Alternatively, the nonlinear dynamics in a supernova could enhance small microscopic asymmetries. Indeed, the explosion energy may be very sensitive to the neutrino heating
rate. Consider the extreme limit of a heating rate that just fails to produce an explosion. A small asymmetry in the neutrino flux will produce a large asymmetry in the ejected matter by restarting the shock wave on only one side of the supernova. Perhaps a $10^{-3}$ asymmetry in the neutrino flux can lead to a $10^{-2}$ asymmetry in the ejected matter. This should be explored with simulations incorporating a small microscopic asymmetry.

We have considered a number of electron reactions which yield somewhat small asymmetries for perhaps accidental reasons. For example, polarized electron capture produces a reduced asymmetry because of a broad angular distribution. The natural size of an asymmetry is of the order of $eB/k_F^2$, for degenerate conditions or $eB/E_{\nu}^2 \approx 0.6 \times 10^{-2} B_{14}$ for nondegenerate conditions (with $E_{\nu} \approx 10$ MeV the neutrino energy). This is near one percent for $B_{14}$ near unity. Thus, one should search for other electron reactions with large asymmetries.

One possibility is $\nu\bar{\nu} \rightarrow e^+e^-$ which will be examined in future work. Note that Kuznetsov and Mikheev [15] considered the related $\nu \rightarrow \nu e^+e^-$ in a strong field. They find a relatively small asymmetry in $A_m$ of the order of $10^{-5} B_{14}$. Perhaps this is because $B$ is necessary to produce, both, the asymmetry and to make the process kinematically allowed. Thus we expect $\nu\bar{\nu} \rightarrow e^+e^-$ to be more important because it is allowed even in the absence of $B$.

Supernovae, because they are dominated by weakly interacting neutrinos, provide a unique opportunity for macroscopic parity violation. One parity violating observable is a correlation between the neutrino flux and the magnetic field directions. We have examined possible asymmetries in a supernova from known parity violating weak interactions in strong magnetic fields. To explain the large velocities of neutron stars one needs an asymmetry in the radiated neutrinos and or the ejected matter of the order of one percent. We have looked at a number of hadronic and electron reactions that give asymmetries of the order of a few times $10^{-4} B_{14}$. These are somewhat small to directly produce the recoil velocity from a dipole magnetic field. However, they only involve known weak interactions and, thus, are clearly present and provide a benchmark to compare with more speculative possibilities. Furthermore, these asymmetries are important if supernovae involve very strong non-dipole magnetic fields (possibly as high as $10^{16}$ Gauss). One should continue to search for new reactions and or enhancements since the natural size of electron reaction asymmetries $eB/E_{\nu}^2 \approx 0.6 \times 10^{-2} B_{14}$ could produce a large enough effect.

ACKNOWLEDGMENTS

This work was supported by the DOE under grant numbers DE-FG02-87ER40365, DE-FC05-85ER250000 and DE-FG05-92ER40750.

---

The sensitivity may be somewhat reduced by requiring the shock to reproduce observed (relatively large) explosion energies.
REFERENCES

[1] Lyne A.G. and Lorimer D.R. Nature 369(1994)127.
[2] Burrows A. and Hayes J., PRL76(1996)352.
[3] Chugai N.N., Sov. Astron. Lett. 10(1984)87.
[4] Kusenko A. and Segre G., PRL77(1996)4872.
[5] Tompson C. and Duncan R. C., Mon.Not.R.Astron.Soc., 275 (1995) 757.
[6] Mueller E. and Hillebrandt, Astron. Astrophys. 80(1979)147.
[7] Cline D.B. ed., “Physical Origin of Homochirality in Life”, AIP Conference Proceedings 379 (1995)231; see also Chyba C.F., Nature 348 (1990) 113; Hegstrom R.A., Nature 297 (1982)643.
[8] Vilenkin A., ApJ451(1995)700.
[9] Bezchastnov V.G. and Haensel P., PRD54(1996)3706.
[10] Bruenn S. W. and Haxton W. C., ApJ376(1991)678.
[11] Horowitz C. J. and Wehrberger K., PRL66(1991)272.
[12] Leinson L.B., Oraevsky V.N. and Semikoz V.B., PhysLB209(1988) 80.
[13] Serot B.D. and Walecka J.D., Adv. Nucl. Phys. 16(1986)1.
[14] Kutschera M. and Wojcik W., Phys. Lett. B221(1989) 11.
[15] Kuznetsov A.V. and Mikheev N.V., hep-ph/9612312 submitted to Phys. Lett. B.

FIG. 1. Neutrino-nucleus elastic scattering in a polarized electron gas. The direct neutrino-nucleus diagram (a) interferes with the electron screening diagram (b) where the neutrino first excites an electron particle-hole state. This introduces a parity violating asymmetry in the angular distribution because the electron gas is (slightly) polarized by a magnetic field as indicated by the X.
FIG. 2. Recoil of a neutron star from polarized electron capture. A vertical $B$ field polarizes electrons with spin down (indicated by the arrow $S_e$). These electrons when moving up are left handed and can be captured. However, the electron capture angular distribution is backward peaked so that neutrinos will tend to be emitted down. As a result the neutron star will recoil up, in the direction of the magnetic field.