Robust output regulation for voltage control in DC networks with time-varying loads

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In this paper, we propose novel control schemes for regulating the voltage in Direct Current (DC) power networks, where the loads are the superposition of time-varying signals and uncertain constants. More precisely, the proposed control schemes are based on the robust output regulation methodology and, differently from the results in the literature, where the loads are assumed to be constant, we consider time-varying loads whose dynamics are described by a class of differential equations with parametric uncertainty. The proposed control schemes achieve voltage regulation and guarantee the local robust stability of the overall network in case of impedance (Z), current (I), and power (P) load types and the global robust stability in case of ZI loads. The simulation results illustrate excellent performance of the proposed control schemes in different scenarios, where real load data are considered.

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1. Introduction

Power networks are categorized into Direct Current (DC) and Alternating Current (AC) networks. Recently, the design and operation of DC networks in presence of renewables and new types of load such as electric vehicles have attracted increasing research attention (Justo, Mwasilu, Lee, & Jung, 2013).

Voltage control is the main control purpose in DC networks ensuring the proper operation of the overall network, see for instance (Cucuzzella, Lazzari, Kawano, Kosaraju and Scherpen, 2019; Ferguson, Cucuzzella, & Scherpen, 2021; Iovine et al., 2018; Jeltsema & Scherpen, 2004; Kosaraju, Cucuzzella, Scherpen, & Pasumarthy, 2021; Machado, Arocas-Perez, He, Ortega, & Grino, 2018; Nahata, Soloperto, Tucci, Martinelli, & Ferrari-Trecate, 2020; Sadabadi, Shafiee, & Karimi, 2018; Strehle, Pfeifer, Malan, Krebs, & Hohmann, 2020). However, all these works and most of the results available in the literature ensure, to the best of our knowledge, voltage regulation and network stability in presence of constant load components only, while in practice loads are

time-varying (see for instance Aguirre, Rodrigues, Lima, & Martinez, 2008; Chiang, Wang, Huang, Chen, & Huang, 1997; Choi et al., 2006; Verdejo, Awerkin, Saavedra, Kliemann, & Vargas, 2016; Vignesh, Chakrabarti, & Srivastava, 2014 and the references therein), making it more difficult to guarantee the stability of the power grid by using existing control strategies (Aguirre et al., 2008; Verdejo et al., 2016). Consequently, new control schemes providing robust stability guarantees of the power network in presence of time-varying loads and uncertainty need to be developed in order to make the overall power system more reliable and resilient (Sira-Ramirez & Rosales-Diaz, 2014; Wilson, Neely, Cook, & Glover, 2014). More precisely, we consider the parallel combination of impedance (Z), current (I), and power (P) load types. Additionally and differently from Cucuzzella, Lazzari et al. (2019), Ferguson et al. (2021), Iovine et al. (2018), Jeltsema and Scherpen (2004), Kosaraju et al. (2021), Machado et al. (2018), Nahata et al. (2020), Sadabadi et al. (2018), Strehle et al. (2020) and other related papers, for each type of load we consider the superposition of time-varying and uncertain constant components, where the time-varying components are expressed as the outputs of suitable dynamical systems. However, achieving voltage regulation in DC power networks (or, similarly, frequency control in AC power grids) in presence of time-varying loads is attracting growing research interest (see Ferguson et al., 2021; Silani, Cucuzzella, Scherpen, & Yazdanpanah, 2021b; Sira-Ramirez & Rosales-Diaz, 2014; Wilson et al., 2014 for DC grids and Silani, Cucuzzella, Scherpen, & Yazdanpanah, 2021a; Silani & Yazdanpanah, 2019; Trip, Burger, & De Persis, 2016 for AC grids). However, the solutions proposed for DC grids in Sira-Ramirez and Rosales-Diaz
and Wilson et al. (2014) do not provide any closed-loop stability guarantee, while Ferguson et al. (2021) guarantee only (local) input-to-state stability, and the controller proposed in Silani et al. (2021b) considers only ZI loads and requires to solve Partial Differential Equations (PDEs). Differently, in this paper we propose robust control schemes based on the robust output regulation methodology (Huang, 2004), ensuring voltage regulation and robust stability also in presence of time-varying and uncertain constant components of ZI loads.

The contributions of the paper can then be listed as follows: (i) the voltage control problem in DC networks is formulated as a robust output regulation problem; (ii) for the loads we consider the superposition of time-varying and uncertain constant impedance (Z), current (I) and power (P) components, where the time-varying components of loads are modeled as outputs of dynamical ecosystems (see for instance Aguirre et al., 2008; Chiang et al., 1997; Choi et al., 2006; Sira-Ramirez & Rosales-Diaz, 2014; Trip et al., 2016; Vignesh et al., 2014), which is also of interest in agreement with output regulation theory (Huang, 2004); (iii) we propose two control schemes achieving voltage regulation and ensuring local robust stability in presence of ZI loads, where the local result is due to the use of linearization for control design; (iv) we also propose a control scheme achieving voltage regulation and ensuring global robust stability in presence of ZI loads.

1.1. Notation

The set of real and natural numbers are denoted by $\mathbb{R}$ and $\mathbb{N}$, respectively. The set of positive (nonnegative) real numbers is denoted by $\mathbb{R}_{>0}$ ($\mathbb{R}_{\geq 0}$). Let $\mathbb{B}$ be the vector of all zeros or the null matrix of suitable dimension(s) and let $I_{n} \in \mathbb{R}^{n}$ be the vector containing all ones. The $i$th element of vector $x$ is denoted by $x_{i}$. Given a vector $x \in \mathbb{R}^{n}$, $[x] \in \mathbb{R}^{n \times n}$ indicates the diagonal matrix whose diagonal entries are the components of $x$. Let $A \in \mathbb{R}^{n \times m}$ be a matrix. In case $A$ is a positive definite (positive semi-definite) matrix, we write $A > 0$ ($A \succeq 0$). Also, $\sigma(A)$ denotes the spectrum of matrix $A$. The $n \times n$ identity matrix is denoted by $I_{n}$. Let $x \in \mathbb{R}^{n}$, $y \in \mathbb{R}^{m}$ be vectors and $X \in \mathbb{R}^{n \times m}$ be row vectors, then we define $\text{col}(x, y) := (x^\top \ 1^\top)^\top \in \mathbb{R}^{n+m}$ and $\text{row}(x, y) := (x \ y) \in \mathbb{R}^{(n+m) \times 1}$. Consider the vector $x \in \mathbb{R}^{n}$ and functions $g : \mathbb{R}^{n} \to \mathbb{R}^{n+m}$, $h : \mathbb{R}^{n} \to \mathbb{R}^{n}$, then the Lie derivative of $h(x)$ along $g(x)$ is defined as $L_{g}h(x) := \frac{\partial}{\partial x}(h(g(x)))$ with $\frac{\partial}{\partial x} := \frac{\partial}{\partial x_{1}}, \ldots, \frac{\partial}{\partial x_{n}}$ for $i = 1, \ldots, n$. Let $\partial^{i}(v)$ denote a generic function of $v$ which is zero up to the $k$th order regardless of the dimension of its range space (see Huang, 2004, Definition 4.1). The bold symbols denote the solutions to regulator equations. A continuous function $\alpha : \mathbb{R}_{>0} \to \mathbb{R}_{>0}$ is said to be of class $\mathcal{K}$ if it is nondecreasing and $\alpha(0) = 0$, while it is said to be of class $\mathcal{K}_{\infty}$ if it also satisfies $\lim_{\tau \to \infty} \alpha(\tau) = \infty$.

2. Modeling and problem formulation

In this section, we introduce the considered DC power network model together with the dynamics of the load components. Then, the main control objective concerning the voltage regulation is introduced.

2.1. DC network model

The model of the considered DC network includes Distributed Generation Units (DGUs), loads and transmission lines Cucuzzella et al. (2019) and Tucci, Meng, Guerrero and Ferrari-Trecate (2018). Fig. 1 illustrates the structure of node $i$ and Table 1 reports the description of the used symbols. Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be a connected and undirected graph that describes the topology of the considered DC network. The nodes and the edges are denoted by $\mathcal{V} = \{1, \ldots, n\}$ and $\mathcal{E} = \{1, \ldots, m\}$, respectively. Then, the topology of the network is represented by the corresponding incidence matrix $A \in \mathbb{R}^{n \times m}$. Let the ends of each transmission line $k$ be arbitrarily labeled with a ‘−’ or a ‘+’, then the entries of $A$ are given by $A_{ik} = +1$, if $i$ is the positive end of $k, A_{ik} = -1$, if $i$ is the negative end of $k$, and $A_{ik} = 0$, otherwise. Before presenting the dynamics of the overall network, we first introduce and discuss the models of each component of the network.

\textbf{DGU model:} The dynamic model of DGU $i \in \mathcal{V}$ is described by

\begin{equation}
L_{g_{i}}\dot{I}_{g_{i}} = -V_{i} + u_{i},
\end{equation}

\begin{equation}
C_{v_{i}}\dot{V}_{i} = I_{g_{i}} - h_{i}(V_{i}) - \sum_{k \in \mathcal{E}_{i}} I_{k},
\end{equation}

where $I_{g_{i}}, V_{i}, u_{i} : \mathbb{R}_{>0} \to \mathbb{R}$, $L_{g_{i}}, C_{v_{i}} \in \mathbb{R}_{>0}$ and $\mathcal{E}_{i}$ is the set of the lines incident to node $i$. Moreover, $I_{k} : \mathbb{R} \to \mathbb{R}$ represents the current demand of load $i$ possibly depending on the voltage $V_{i}$.

\textbf{Load model:} In this work, we consider a general time-varying load model including the parallel combination of the following types of load: (i) impedance (Z): $G_{i} + \hat{G}_{i}$, (ii) current (I): $I_{k} + \hat{I}_{k}$, (iii) power (P): $P_{i} + \hat{P}_{i}$, where $G_{i}, I_{k}, P_{i} \in \mathbb{R}_{>0}$ are uncertain constant components and $\hat{G}_{i}, \hat{I}_{k}, \hat{P}_{i} : \mathbb{R}_{>0} \to \mathbb{R}$ are time-varying components, whose dynamics will be introduced in the next subsection. To refer to the load types above, the letters Z, I and P, respectively, are often used in the literature (see for instance De Persis, Weitenberg, & Dorfler, 2018). Thus, in presence of ZIP loads, $I_{k}(V_{i})$ in (1) is given by

\begin{equation}
I_{k}(V_{i}) = (G_{ik} + \hat{G}_{ik})V_{i} + I_{k}^{*} + \hat{I}_{k} + V_{i}^{-1}(P_{ik} + \hat{P}_{ik}).
\end{equation}

\textbf{Line model:} The dynamics of the current $I_{k}$ exchanged between nodes $i$ and $j$ are described by

\begin{equation}
L_{k}\dot{I}_{k} = (V_{i} - V_{j}) - R_{k}I_{k},
\end{equation}

where $R_{k}, I_{k} \in \mathbb{R}_{>0}$.

Now, the dynamics of the overall network can be written compactly as

\begin{equation}
\begin{align*}
L_{g_{i}}\dot{I}_{g_{i}} &= -V + u, \\
C_{v_{i}}\dot{V}_{i} &= I_{g_{i}} + AI - [G_{i}]V - l_{i} - [V^{-1}]P_{i}, \\
L_{k}\dot{I}_{k} &= -A^{T}V - RI,
\end{align*}
\end{equation}
where \( I_g, V, u : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}^n \) are the generated current, load voltage and control input vector, respectively, \( I : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}^m \) is the line current vector, \( I_g, C_g \in \mathbb{R}_{> 0}^{m \times m} \) and \( R, L \in \mathbb{R}_{> 0}^{m \times m} \) are positive definite diagonal matrices, e.g., \( I_g = \text{diag}(I_{g1}, \ldots, I_{gs}) \) (see Table 1 for description of the symbols). Moreover, \( G_i = \Gamma + \bar{G}_i, \ h = \bar{t}_i + \bar{h}_i \) and \( P_i = \bar{P}_i + \bar{p}_i \), where \( G_i, \ h, \ P_i \in \mathbb{R}^m \) represent the uncertain constant components and \( \bar{G}_i, \ \bar{h}, \ \bar{P}_i : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}^m \) the time-varying ones.

We note that the considered graph \( G \) is undirected, i.e., the edges are bidirectional, implying that the current through each edge can flow in both directions, and the information on the flow direction is enclosed in the incidence matrix \( A \). Finally, note also that for the sake of simplicity we consider load-connected topologies (see Fig. 1). This does not involve loss of generality in the case of linear loads (e.g., ZL), where such topologies are obtained by a Kron reduction of the original network (Zha & Dorfler, 2015).

2.2. Exosystems model

In this subsection, we introduce the dynamics of the time-varying components of the ZIP load, i.e., \( G_i, \ h, \ P_i \) in (2), which are modeled as outputs of dynamical exosystems (see for instance Aguirre et al., 2008; Chiang et al., 1997; Choi et al., 2006; Sirra-Ramirez & Rosales-Diaz, 2014; Trip et al., 2016; Vignesh et al., 2014). This is also conform with output regulation theory (Huang, 2004). Also, in Section 5 we use real data from the dataset (openEI, 0000), which provides load profile data for residential buildings in the United States to model the exosystems. Let \( y \) (or \( y \)) denote \( G, I, P \) (or \( G, I, P \)) in case of \( Z, I, P \) loads, respectively. Then, the exosystem dynamics can be expressed as (Silani et al., 2021b, Equation (2), Silani et al., 2021a, Equation (8), Trip et al., 2016, Equations (25), (26))

\[
\dot{y}_i = s_y d_y, \quad \dot{y}_i = \Gamma_i d_y, \quad (5)
\]

where \( d_y : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}^{2m} \) is the state of the exosystem describing the time-varying components of \( y \), which can be defined as \( d_y := \text{col}(d_y^1, d_y^2) \), with \( d_y^1, d_y^2 : \mathbb{R}_{\geq 0} 

(10a)

\[
\dot{d} = Sd \label{eq:10b}
\]

\[
h(x, d) = V - V^* \label{eq:10c}
\]

where \( h(x, d) \) is the output mapping representing the tracking error \( e(t) := h(x, d), S := \text{blockdiag} (S_1, S_2, S_3) \), \( g(x, d, w) := \text{col}(L_1, L_2, \omega_{mn}, \omega_{mn}) \) and \( i_{\text{ZIP}}(V, d, w) := \text{col} \left( \begin{array}{c} -L_1 V + \Delta t \bar{A} - i_{\text{ZIP}}(V, d, w) \end{array} \right), L^{-1}(-A'V - R), \),

\[
i_{\text{ZIP}}(V, d, w) := I_t^+ + \Gamma_i d_t + [(G_i^+ + [\Gamma_i^+ d_t])] V 
\]

\[
+ [V]^{-1}(P'_i + \Gamma i d_p) \label{eq:11}
\]

\[
\text{lim } V(t) = V^*, \label{eq:12}
\]

\[
V^* : \mathbb{R}_{\geq 0} \rightarrow \text{being the voltage reference (e.g. nominal value) at node } \label{eq:13}
\]

3. Local robust output regulation

In this section, we formulate the voltage control problem as a standard output regulation problem (Huang, 2004) in order to design control schemes achieving Objective 1. Let the network state \( x : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}^{m+2n} \) and the exosystems state \( d : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}^{mn} \) be defined as \( x := \text{col}(l_g, V, I) \) and \( d := \text{col}(d_c, d_v, d_p) \), respectively, and \( u : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}^m \) be the control input. Moreover, let \( w \in \mathbb{R}^3 \) be defined as \( w := \text{col}(\bar{G}_i, \bar{t}_i, \bar{P}_i) \), which represents the unknown constant components of the ZIP loads, i.e., parametric uncertainty. Then, we can rewrite (4) and (6) as the following system:

\[
\dot{x} = f(x, d, w) + g(x, d, w) u \label{eq:10a}
\]

\[
\dot{d} = Sd \label{eq:10b}
\]

\[
h(x, d) = V - V^* \label{eq:10c}
\]

where \( h(x, d) \) is the output mapping representing the tracking error \( e(t) := h(x, d), S := \text{blockdiag} (S_1, S_2, S_3) \), \( g(x, d, w) := \text{col}(L_1, L_2, \omega_{mn}, \omega_{mn}) \) and \( i_{\text{ZIP}}(V, d, w) := \text{col} \left( \begin{array}{c} -L_1 V + \Delta t \bar{A} - i_{\text{ZIP}}(V, d, w) \end{array} \right), L^{-1}(-A'V - R), \),

\[
i_{\text{ZIP}}(V, d, w) := I_t^+ + \Gamma_i d_t + [(G_i^+ + [\Gamma_i^+ d_t])] V 
\]

\[
+ [V]^{-1}(P'_i + \Gamma i d_p) \label{eq:11}
\]

\[
\text{lim } V(t) = V^*, \label{eq:12}
\]

\[
V^* : \mathbb{R}_{\geq 0} \rightarrow \text{being the voltage reference (e.g. nominal value) at node } \label{eq:13}
\]
Now, in the following lemma, we prove the stabilizability and detectability of the pairs \((A, B)\) and \((C, A)\), respectively.

**Lemma 2 (Stabilizability and Detectability of (13)).** The pair \((A, B)\) is stabilizable and the pair \((C, A)\) is detectable, with \(A, B, C\) given in (13).

**Proof.** To prove the stabilizability and detectability of \((A, B)\) and \((C, A)\), respectively, it is sufficient to show that \(A\) is Hurwitz, i.e., it has all the eigenvalues with negative real part. Therefore, consider the linear system
\[
\dot{x} = Ax - \bar{X},
\]
and the following Lyapunov function
\[
E(x) = \frac{1}{2} \left((I - \bar{I})^\top L_x(I - \bar{I}) + \frac{1}{2}(V - \bar{V})^\top C_g(V - \bar{V})
\right.
\]
\[
+ \frac{1}{2}(I - \bar{I})^\top R(I - \bar{I})
\]
(15).

Note that in the following subsections, we will show that \(V = V^*\). Then, the derivative of the Lyapunov function (15) along the solutions to (14) satisfies
\[
\dot{E}(x) = \frac{\partial E}{\partial x}(Ax - \bar{X})
\]
\[
= - (I - \bar{I})^\top R(I - \bar{I}).
\]
(16)

Then, we can conclude that \(A\) has no eigenvalue with positive real-part since \(R > 0\). Thus, in order to verify that \(A\) has no eigenvalues with real part equal to zero, we compute the determinant of \(A\) using the Schur complement of \(A\), i.e.,
\[
\det(A) = \det(-L^{-1}R) \det(C_g^{-1} - C_g^{-1}A R^{-1} A^\top)
\]
\[
= (-1)^n \det(-L^{-1}R) \det(-C_g^{-1}A R^{-1} A^\top) \neq 0.
\]
(17)

where we use the property that interchanging any pair of rows of a matrix multiplies its determinant by \(-1\). Consequently, we can conclude that all the eigenvalues of matrix \(A\) have negative real part. Then, the pair \((A, B)\) is stabilizable and the pair \((C, A)\) is detectable. ■

Before introducing the output regulation problem, we consider the following assumption in analogy with Huang (2004, Assumption 5.4), which guarantees that the Taylor series solution to the regulator equations exists (see Huang, 2004, Lemma 4.8 for more details). Indeed, it ensures that there exists an internal model of the \(k\)-fold exosystem (10b) (see Huang, 2004, Theorem 5.7 and Remark 5.8 (i)). Moreover, its physical meaning corresponds to the absence of resonance phenomena. Indeed, it does not allow that the linear combinations of the eigenvalues of \(S\) are equal to the eigenvalues of the linearized DC network.

**Assumption 2 (Condition on the Eigenvalues of \(S\)).** The matrix
\[
\begin{pmatrix}
\lambda I_{2n + m} & C \\
B & 0
\end{pmatrix}
\]
has full rank for \(l \in \mathbb{N}\) and for all \(\lambda\) given by \(\lambda_l = \lambda_l^* + \ldots + \lambda_{l+m}^*\lambda_{l+m}^* + \ldots + \lambda_{l+n}^*\lambda_{l+n}^* = 1, 1, \ldots, l\), where \(\lambda^*_1, \ldots, \lambda^*_{n}\) are the eigenvalues of the matrix \(S\).

Now, given the parametric uncertainty \(w\), we define the robust output regulation problem for system (10) as follows (see Huang, 2004, Section 5.1):

**Problem 1 (Local Robust Output Regulation).** Design a state feedback controller
\[
u(t) = k(x(t), v(t), u(t))
\]
\[
n(t) = q(v(t), e(t)),
\]
(18)
such that the closed-loop system (10), (18), for every initial condition \((x(0), v(0), d(0))\) sufficiently close to the equilibrium point \((x, v, \overline{d} = 0, \overline{m})\) and sufficiently small elements of the uncertainty vector \(w\), has the following two properties:

**Property 1** The trajectories \(\{x(t), v(t), d(t)\}\) of the closed-loop system exist and are bounded for all \(t \geq 0\).

**Property 2** The trajectories \(\{|x(t), v(t), d(t)\}\) of the closed-loop system satisfy \(\lim_{t \to \infty} e(t) = 0\) achieving Objective 1.1

If a controller (18) exists such that the closed-loop system (10), (18) satisfies Properties 1, 2, Problem 1 is solvable. Also, the \(k\)-th-order robust output regulation problem is solvable if a controller exists such that the closed-loop system satisfies Properties 1, 2 with \(\lim_{t \to \infty} e(t) = 0\) achieving the steady-state tracking error is zero up to the \(k\)th order.

### 3.1. Linear robust controller

In this subsection, we propose a control scheme with a linear internal model achieving Objective 1 in presence of time-varying and uncertain constant ZIP loads.

First, we augment system (10) with a linear dynamic system incorporating an internal model of the \(k\)-fold exosystem (10b). Then, a controller stabilizing the linear approximation of the augmented system is proposed to solve the \(k\)-th order robust output regulation problem. Also, following Huang (2004, Theorem 3.8), the regulator equation associated with (10) can be expressed as
\[
\frac{\partial x(d, w)}{\partial d} = f(x(d, w), d, w) + g(x(d, w), d, w)u(d, w)
\]
(19a)
\[
0 = h(x(d, w), d),
\]
(19b)
whose solution we show to be polynomial in \(d\), implying, according to Huang (2004, Theorem 5.12), that the proposed controller also solves Problem 1.
Theorem 1 (Linear Robust Controller). Let Assumptions 1 and 2 hold.

(i) The linear state feedback controller
\[ u = K_1 x + K_2 v_a, \quad v_a = G_1 v_a + G_2 e, \]  
(22)
solves Problem 1, where \( v_a : \mathbb{R}_+ \to \mathbb{R}^{6m_0} \) is the state of the controller, \( K_1 \in \mathbb{R}^{1 \times (2n+2m)} \), \( K_2 \in \mathbb{R}^{n \times 6m_0} \) is chosen such that the matrix
\[ \begin{pmatrix} A + BK_1 & BK_2 \\ G_2 C & G_1 \end{pmatrix} \]  
(23)
is Hurwitz, with \( A, B, \) and \( C \) given in (13).

(ii) The linear output feedback controller
\[ u = K_1 v_b, \quad v_b = G_1 v_b + G_2 e, \]  
(24)
solves Problem 1, where \( v_b : \mathbb{R}_+ \to \mathbb{R}^{6m_0+2n+m} \) is the state of the controller, \( K := (K_1 \quad K_2) \in \mathbb{R}^{n \times (6m_0+2n+m)} \), with \( K_1, K_2 \) given in part (i), and \( G_1 \in \mathbb{R}^{(6m_0+2n+m) \times 2n+m}, \) \( G_2 \in \mathbb{R}^{(6m_0+2n+m) \times n} \) are given by
\[ G_1 := \begin{pmatrix} A + BK_1 - \bar{L}C & BK_2 \\ 0 & G_1 \end{pmatrix}, \]  
(25)
\[ G_2 := \text{col} (\bar{L}, \quad G_2), \]  
(26)
such that \( A - \bar{L}C \) is Hurwitz, with \( \bar{L} \in \mathbb{R}^{(2n+m) \times n} \).

Proof. The proof is provided in Appendix A due to lengthy calculations.

Remark 1 (Controller Properties in Theorem 1). Note that, even in presence of only 2I loads, the controllers (22) and (24) are incapable of ensuring global stability of the closed-loop system because of the linearity of the internal models in (22) and (24). Moreover, the controllers (22) and (24) do not guarantee the solvability of Problem 1 when the solutions to the regulator equation (19) are nonpolynomial in \( d \), (i.e., the solutions to the regulator equation (19) do not have polynomial expressions in \( d \)). Indeed, the solutions to the regulator equation (19) might be nonpolynomial in \( d \) if we consider for instance a nonlinear exosystem or an exosystem with a structure different from (6).

3.2. Stabilization technique for robust output regulation

In this subsection, we propose a control scheme solving Problem 1, but we use an internal model different from the ones in (22) and (24). Specifically, we follow the design framework introduced in Huang (2004, Chapter 6), which removes the assumption on the polynomial solutions to the regulator equation (19) and allows to incorporate global stabilization techniques which will be introduced in Section 4. This framework views the output regulation as a stabilization problem. However, for solving the “robust” output regulation problem (Problem 1), the solution to the regulator equation (19) cannot be used in the controller due to the parametric uncertainty \( u \). To address this issue, we augment system (10) with a generalized internal model based on the steady-state generator for system (10), which is a dynamic system that can reproduce a partial or whole solution to the regulator equation (19). In the following lemma, according to Huang (2004, Definitions 6.1, 6.2), a steady-state generator for system (10) is derived.

Lemma 3 (Steady-state Generator for System (10)). Let Assumption 2 hold. Then, system (10) has a steady-state generator \( \theta, \alpha, \beta \) with output \( g_0(x, u) = u \) with linear observability.

Proof. We first form the pairwise coprime polynomials based on the solution to the regulator equation (19), whose expression is given in (A.6) in Appendix. Then, we obtain the companion matrix associated with the minimal zeroing polynomials of such pairwise coprime polynomials. Finally, following Huang (2004, Lemma 6.17), we obtain a steady-state generator \( (\theta, \alpha, \beta) \) with output \( g_0(x, u) = u \) with linear observability.

Following Huang (2004, Lemma 6.17), we form the pairwise coprime polynomials \( \pi_1 : \mathbb{R}^{6m_0+3n} \to \mathbb{R}^m, \pi_2 : \mathbb{R}^{6m_0+3n} \to \mathbb{R}^m, \pi_3 : \mathbb{R}^{6m_0+3n \to \mathbb{R}_+} \) based on the solution to the regulator equation (19) for the output \( g_0(x, u) = u \) as:
\[ \pi_1(d, u) = \langle [o_0] d, \pi_2(d, u) = \langle [o_1] d^{[2]} \rangle, \pi_3(d, u) = \langle [o_2] d^{[2]} \rangle, \]  
(26)
with \( \bar{C}_0 \in \mathbb{R}^{6m_0} \) satisfying \( L_2 \gamma \bar{C}_0 \) - 1. Then, the minimal zeroing polynomials \( P_1 : \mathbb{R} \to \mathbb{R}^{6m_0+3m}, P_2 : \mathbb{R} \to \mathbb{R}^{6m_0+3m}, P_3 : \mathbb{R} \to \mathbb{R}^{6m_0+3m} \) of (26) are given by
\[ P_1(\lambda) = \lambda^2 - \langle [o_0] \rangle, \quad P_2(\lambda) = \lambda^2 - \langle [o_1] \rangle, \quad P_3(\lambda) = \lambda^2 - \langle [o_2] \rangle, \]  
(27)
Note that all the zeros of \( P_j(\lambda) \), with \( j = 1, 2, 3 \), are simple and pure imaginary. Now, we define the polynomials\( A_{1i} := \text{col}(\pi_{1i}, \pi_{2i}), A_{2i} := \text{col}(\pi_{2i}, \pi_{3i}), A_{3i} := \text{col}(\pi_{3i}, \pi_{3i}) \in \mathbb{R}^{2i}, \) where \( A_{1i}, A_{2i}, A_{3i} \) are the companion matrices of the polynomial \( \pi_j \), with \( i = 1, \ldots, n, j = 1, 2, 3, k = 1, \ldots, n_0 \). Then, we have \( g_0(x, u) = \Sigma(\Lambda), \) where \( \Sigma : \mathbb{R}^{6m_0+3n} \to \mathbb{R}^{6m_0+3n} \) represents \( u(d, u) \) and is defined as
\[ \Sigma(\Lambda) := L_2[V^*] \gamma, \]  
(28)
Note that (26) and (28) follow from the solution to the regulator equation (19), whose expression is given in (A.6) in Appendix. Now, the companion matrix associated with the minimal zeroing polynomials (27) can be expressed as \( \Phi_i = \text{blockdiag}(\Phi_{i1}, \ldots, \Phi_{in}) \), where \( \Phi_{ij} = \text{diag}(\Phi_{i1}, \ldots, \Phi_{ijn}) \).

\[ \Phi_{1i} := \begin{pmatrix} 0 & 1 \\ -\omega_i g_{ia} & 0 \end{pmatrix}, \quad \Phi_{2i} := \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad \Phi_{3i} := \begin{pmatrix} 0 & 1 \\ -\omega_i g_{ib} & 0 \end{pmatrix}, \]  
(29)
with \( i = 1, \ldots, n, j = 1, 2, 3, k = 1, \ldots, n_0 \). Then, we have \( \Phi = \text{blockdiag}(\Phi_1, \Phi_2, \Phi_3) \in \mathbb{R}^{7m_0+3m_0} \). Now, the Jacobian of \( \Sigma(\Lambda) \) at the origin can be given by \( \Psi = \text{row}(\Phi_1, \Phi_2, \Phi_3) \in \mathbb{R}^{7m_0+3m_0} \), where \( \Psi_i = \text{blockdiag}(\Psi_{i1}, \ldots, \Psi_{ijn}) \) with \( \Psi_{ij} = \text{row}(\Phi_{i1}, \ldots, \Phi_{ijn}) \).

\[ \theta(d, u) = T \Lambda, \quad \alpha(\theta) = T \Phi^T \theta, \quad \beta(\theta) = \Sigma(\Sigma \theta), \]  
(30)
where \( T \in \mathbb{R}^{7m_0+3m_0} \) is any nonsingular matrix. Since the pair \( (\Psi_{i1}, \Phi_{i1}) \) has a companion form, the pair \( (\Psi, \Phi) \) is observable; therefore, the steady-state generator \( (\theta, \alpha, \beta) \) is linearly observable. Also, it can be inferred from (30) that \( \theta = \alpha(\theta) \). Moreover, we note that the solution to the regulator equation (19) relates system (10) to the steady-state generator (30), while \( \beta(\theta) \) in (30) relates (28) to (30).

In the following, by virtue of Lemma 3, we use the steady-state generator (30) in order to find an internal model for system (10). Now, we define
\[ M := \text{blockdiag}(M_1, M_2, M_3) \in \mathbb{R}^{7m_0+3m_0}, \]  
(31)
\[ N := \text{col}(N_1, N_2, N_3) \in \mathbb{R}^{7m_0+3m_0}, \]  
(32)
\[ g_0(x, u) = u \]  
(33)
where $M_j := \text{blockdiag}(M_{j1}, \ldots, M_{jn})$, $N_j := \text{blockdiag}(N_{j1}, \ldots, N_{jn})$ with $M_j := \text{blockdiag}(M_{j1}, \ldots, M_{j\mu}), \ \ \ N_j := \text{col}(N_{j1}, \ldots, N_{j\mu})$.

$$
M_{1ik} := \begin{pmatrix} -a_{1ik} & -b_{1ik} \\ 0 & 1 \end{pmatrix}, \quad N_{1ik} := \text{col}(0, 1),
$$

$$
M_{2ik} := \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad N_{2ik} := \text{col}(0, 1, 1),
$$

$$
M_{3ik} := \begin{pmatrix} 0 & 1 \\ -a_{3ik} & -b_{3ik} \end{pmatrix}, \quad N_{3ik} := \text{col}(0, 1, 1),
$$

$\alpha_{1ik}, b_{1ik}, c_{ik} \in \mathbb{R}_{\ge 0}$, $i = 1, \ldots, n$, $j = 1, 2, 3, k = 1, \ldots, n_d$. Then, $T$ in (30) is defined as $T := \text{blockdiag}(T_{11}, T_{21}, T_{31})$. $T_{j1}, T_{j2}, T_{j3} \in \mathbb{R}_{\ge 0}^{2 \times 2}, T_{2k} \in \mathbb{R}_{\ge 0}^{2 \times 2}, i = 1, \ldots, n, j = 1, 2, 3, k = 1, \ldots, n_d$. Then, $T$ can be obtained by solving the following Sylvester equation

$$
T_{j1}\Phi_j = M_j T_{j1} + N_j \Psi_{ij}.
$$

(32)

Note that there exists a unique nonsingular matrix $T_{j1}$ satisfying the Sylvester equation (32) since the pair $(M_j, N_j)$ is controllable. $(\Psi_{ij}, \Phi_j)$ is observable and the spectra of $M_j$ and $\Phi_j$ are disjoint. Note that the controllability of the pair $(M_j, N_j)$ and the observability of the pair $(\Psi_{ij}, \Phi_j)$ follow from the controllable canonical form of the pair $(M_{jik}, N_{jik})$ and the companion form of the pair $(\Psi_{ij}, \Phi_j)$, respectively.

Now, according to Huang (2004, Proposition 6.21), the following proposition we introduce an internal model for system (10) different from the ones in (22) and (24).

**Proposition 1** (Internal Model for System (10)). Let Assumption 2 hold and consider the steady-state generator (30) with output $g_0(x, u) = u$. Then, the internal model for system (10) is given by

$$
\dot{\xi} = \mu(\xi, u) := M\xi + Nu,
$$

(33)

where $\mu : \mathbb{R}_{\ge 0} \to \mathbb{R}_{\ge 0}$ is the state of the internal model, $\mu : \mathbb{R}^{7m_d} \times \mathbb{R}^n \to \mathbb{R}^{7m_d}$, and $M, N$ are given in (31).

**Proof.** Since $\Sigma(A)$ given in (28) is a linear function, we can write $\beta(\xi_j) = \Psi_j T_{11}^{-1} \xi_j$, with $i = 1, \ldots, n, j = 1, 2, 3$. Thus, we can write $\mu_{ij}(\xi, u) = M_j \Psi_j T_{11}^{-1} \theta_j(d, w) + N_j \Psi_j T_{11}^{-1} \theta_j(d, w)$.

$$
\mu_{ij}(\theta_j, u) = M_{j1} \theta_j(d, w) + N_j \Psi_j T_{11}^{-1} \theta_j(d, w) = T_{j1} \Phi_j T_{11}^{-1} \theta_j(d, w),
$$

(34)

where $u$ is given in (A.6) in the Appendix and $i = 1, \ldots, n, j = 1, 2, 3$. Note that in (34) we apply $g_0(x, u) = \beta(\theta_j(d, w))$, where $\beta(\theta_j(d, w))$ is given in (30) and $M_j := N_j \Psi_j T_{11}^{-1} \theta_j(d, w)$, which is obtained from the Sylvester equation (32). Now, we rewrite (34) compactly as $\mu(\theta, u) = \alpha(\theta, u)$. Consequently, by Huang (2004, Definition 6.6), (33) is an internal model for system (10) with output $g_0(x, u) = u$. Also, we note that the Sylvester equation (32) relates $g_0(x, u) = u$. Also, we note that the Sylvester equation (32) relates $g_0(x, u)$ to the internal model (33).

In the following theorem, we use the internal model (33) to design a robust controller solving Problem 1.

**Theorem 2** (Linear Robust Controller with Internal Model (33)). Let Assumptions 1 and 2 hold. Consider system (10), the steady-state generator (30) and the internal model (33). Then, the output feedback controller

$$
\begin{aligned}
\hat{u} &= K\delta + \Psi T^{-1}\xi \\
\dot{\xi} &= \left( \begin{array}{c} A \\
B_c \end{array} \right) \left( \begin{array}{c} \Psi T^{-1} \delta + \left( \begin{array}{c} B \\
N \end{array} \right) K\delta \\
-\hat{L}(C) \end{array} \right) \delta + \hat{L} e
\end{aligned}
$$

where $\hat{L}(C) := \text{col}(\hat{L}_1, \hat{L}_2)$, $\hat{L}_1, \hat{L}_2 \in \mathbb{R}^{(m+2n+3n_d) \times m_d}, K \in \mathbb{R}^{\frac{n_x}{2} \times \frac{n_x}{2}}, L \in \mathbb{R}^{n \times \frac{n_x}{2}}$, solves Problem 1, where $\delta : \mathbb{R}^{n_x} \to \mathbb{R}^{7m_d+2n+3n_d}$ is the state of the controller, $A, B, C$ are given in (13) and $B_c := \text{col}(B, C)$. 

Proof. According to Lemma 3, system (10) has the linearly observable steady-state generator (30) with output $g_0(x, u) = u$. Also, following Proposition 1 (33) is an internal model for system (10). Then, consider the following transformation based on the steady-state generator (30) and internal model (33) as:

$$
\begin{aligned}
\hat{\xi}_x := & I(g(d, u) - g_0(x, u)) \\
& = g_0(x, u) - AR^{-1}A^TV^* - I_n^* + I_3 d_1 - [(G_c^* + \xi)G_c^* + \epsilon_3] \\
& + [I_3^* d_1] V^* - [V^*]^{-1} (P_r + \Gamma_0 d_0) \\
\hat{\xi}_c := & I(\xi(d, w) = I - R^{-1}A^TV^* \\
\hat{\xi} := & \xi - \theta(d, w) \\
\hat{u} := & \dot{\xi} - \beta(\xi) = \hat{\xi} - \Psi T^{-1}\xi,
\end{aligned}
$$

(35)

where $\hat{I}_x(g(d, u), V(d, w), I(d, w))$ are given in (A.6) in Appendix. Then, the augmented system composed of the DC network dynamics (10a) and internal model (33) in the new coordinates is given by

$$
\begin{aligned}
\dot{\xi}_x & = I_x^* (-\hat{\xi}_b - V^* + \hat{\beta}(\hat{\xi}) + \theta) \\
& - I_3 S_{d_1} - [V^*] I_3 S_{d_1} - [V^*]^{-1} I_3 S_{d_0} \\
\dot{\xi}_b & = C^{-1} (\hat{\xi}_x + \hat{\beta}_c - \delta C) \\
& - [\delta c]^{-1} (P_r + \Gamma_0 d_0) \\
\dot{\xi}_c & = L^{-1} (-\hat{\xi}_b) \\
\hat{\xi} := & (M + \Psi T^{-1})\hat{\xi} + \hat{N}_u.
\end{aligned}
$$

(36)

According to Huang (2004, Corollary 6.9), a controller that locally stabilizes the equilibrium point of system (39) with $d = \text{col}(\hat{L}_1, \hat{L}_2)$, $u = \text{col}(\hat{L}_1, \hat{L}_2)$ solves Problem 1. Thus, we linearize system (39) around its equilibrium point and obtain

$$
\hat{x} = \hat{A} x + \hat{B}_x \hat{b} + \hat{B}_u \hat{u}. \\
\hat{\xi} = (M + \Psi T^{-1})\hat{\xi} + \hat{N}_u.
$$

(40)

Remark 2 (Local and Global Robust Stability). The solvability of Problem 1 guarantees the boundedness of the trajectories of the closed-loop system (10), (18) and the asymptotic regulation of
the error when the initial conditions are sufficiently close to the equilibrium point and the elements of the uncertainty vector $w$ sufficiently small. Even in the case of ZI loads, the linear controllers (22), (24) and (35) can ensure only local stability. In the next subsection, we design a controller that guarantees the global boundedness of the trajectories of the closed-loop system in presence of ZI loads and the asymptotic regulation of the error for any initial condition of the closed-loop system and uncertain parameter $w$, i.e., global robust output regulation.

4. Global robust output regulation

In this section, we first convert the system into a system in lower triangular form. To do so, we use a suitable coordinate transformation and dynamic extension. Then, we use the control design approach introduced in Section 3.2, i.e., transformation and dynamic extension. Then, we use the coordinate transformation.

Combining (45) with (46), we can write

$$
\dot{z} = Fz + G(d, w)v_x + D(d, w)
$$

(48a)

$$
v_x = \tilde{H}z + K(d, w)v_x + b\zeta + D(d, w)
$$

(48b)

$$
\zeta = -\zeta + u
$$

(48c)

$$
\dot{d} = Sd
$$

(48d)

$$
e = v_x - V^*
$$

(48e)

Combining (45) with (46), we can write

$$
\dot{z} = Fz + G(d, w)v_x + D(d, w)
$$

(48a)

$$
v_x = \tilde{H}z + K(d, w)v_x + b\zeta + D(d, w)
$$

(48b)

$$
\zeta = -\zeta + u
$$

(48c)

$$
\dot{d} = Sd
$$

(48d)

$$
e = v_x - V^*
$$

(48e)

Combining (45) with (46), we can write

$$
\dot{z} = Fz + G(d, w)v_x + D(d, w)
$$

(48a)

$$
v_x = \tilde{H}z + K(d, w)v_x + b\zeta + D(d, w)
$$

(48b)

$$
\zeta = -\zeta + u
$$

(48c)

$$
\dot{d} = Sd
$$

(48d)

$$
e = v_x - V^*
$$

(48e)

Combining (45) with (46), we can write

$$
\dot{z} = Fz + G(d, w)v_x + D(d, w)
$$

(48a)

$$
v_x = \tilde{H}z + K(d, w)v_x + b\zeta + D(d, w)
$$

(48b)

$$
\zeta = -\zeta + u
$$

(48c)

$$
\dot{d} = Sd
$$

(48d)

$$
e = v_x - V^*
$$

(48e)
then the solution to (49) can be expressed as
\[ z(d, w) = -[V^*]^T \gamma_C [(\omega_0 c_0^2 - 2 \pi w_m^2)]^{-1} d^c \]
\[ + [V^*]^T \gamma_C [(\omega_0 c_0^2 - 2 \pi w_m^2)]^{-1} \omega_0 d^c + \gamma((\omega_0 c_0^2 - 2 \pi w_m^2))^{-1} d^c \]
\[ - \gamma((\omega_0 c_0^2 - 2 \pi w_m^2))^{-1} \omega_0 d^c - L_m^{-1} V^* \]
\[ + [G^*_2] V^* - C_g V^* + I_g^* + \gamma R^{-1} A^{-1} V^* \]
\[ z_b(d, w) = -R^{-1} A^{-1} V^* \]
\[ u(d, w) = V^* \]
\[ \varsigma(d, w) = -L_g (z(d, w) + \lambda z_b(d, w) + C_g V^*) \]
\[ + ((G^*_2) + [I_g^*] d^c)) V^* + I_g^* + I_g^* d^c) \]
\[ (\Psi(d, w) = 2(\omega_0 c_0^2 - 2 \pi w_m^2))^{-1} \omega_0 d^c + L_m^{-1} V^* - 2 I_g^* \]
\[ \Pi(d, w) = (\omega_0 c_0^2 + \omega_0 c_0^2)^{-1} (\omega_0 c_0^2)^d \]
\[ - ((\omega_0 c_0^2 + \omega_0 c_0^2))^{-1} \omega_0 d^c + \tilde{C}_1, \]  
Then, in analogy with Huang (2004, Lemma 7.19), we form the following pairwise coprime polynomials \( \tilde{P}_1 : \mathbb{R}^{2m+2n} \to \mathbb{R}^{m}, \tilde{P}_2 : \mathbb{R}^{2m+2n} \to \mathbb{R}^{m}, \tilde{P}_3 : \mathbb{R} \to \mathbb{R}^{m+mn} \) for which \( \tilde{P}_1(z, \varsigma, \nu, \lambda) = \tilde{P}_1(\varsigma, \nu, \lambda) \) as:
\[ \tilde{P}_1(d, w) = \left( (\omega_0 c_0^2 - 2 \pi w_m^2) I + \omega_0 c_0^2 \right) d^c \]
\[ + (\omega_0 c_0^2 - 2 \pi w_m^2)^{-1} \omega_0 d^c + \gamma((\omega_0 c_0^2 - 2 \pi w_m^2))^{-1} d^c \]
\[ \tilde{P}_2(d, w) = (\omega_0 c_0^2 + \omega_0 c_0^2)^{-1} (\omega_0 c_0^2)^d \]
\[ - ((\omega_0 c_0^2 + \omega_0 c_0^2))^{-1} \omega_0 d^c + \tilde{C}_1, \]  
Note that the zeros of \( \tilde{P}_j(\varsigma, \lambda) \) with \( j = 1, 2 \) are simple and pure imaginary. Now, we define \( \Delta_{1j} := \text{col}(\tilde{P}_1, \tilde{P}_j), \Delta_{2j} := \text{col}(\tilde{P}_{2j}, \tilde{P}_{2j}, \tilde{P}_{3j}), \Delta_{ij} := \text{col}(\Delta_{1j}, \Delta_{2j}), \Delta_j := \text{col}(\Delta_{1j}, \ldots, \Delta_{mj}), \Delta := \text{col}(\Delta_{1j}, \ldots, \Delta_{nj}) \). Let \( \Lambda := \text{col}(\Lambda_1, \Lambda_2) \) be \( (d = 1, \ldots, n, \varsigma = 1, 2, k = 1, \ldots, n_0) \) such that \( \tilde{C}_1 \in \mathbb{R}^{m+mn+1} \) satisfies \( L_g \tilde{C}_1 = V^* - 2L_g J_{g}^* \). Then, the minimal zeroing polynomials \( \tilde{P}_1 : \mathbb{R} \to \mathbb{R}^{m+mn} \), \( \tilde{P}_2 : \mathbb{R}^{m+mn} \to \mathbb{R}^{m}, \tilde{P}_3 : \mathbb{R}^{m+mn} \to \mathbb{R}^{m+mn} \) of (51) are given by
\[ \tilde{P}_1(\varsigma, \nu, \lambda) = \lambda^2 I_m + \omega_0 c_0^2 \right], \quad \tilde{P}_2(\varsigma, \nu, \lambda) = \lambda I_m + \lambda^2 I_m. \]

In the following, we use the steady-state generator (56) in order to find an internal model for system (48). Thus, we define
\[ \hat{M} := \text{blockdiag}(\hat{M}_1, \hat{M}_2) \in \mathbb{R}^{m+mn}, \]
\[ \hat{N} := \text{col}(\hat{N}_1, \hat{N}_2) \in \mathbb{R}^{m+mn}, \]

where \( \hat{M}_i := \text{blockdiag}(\hat{M}_{1j}, \ldots, \hat{M}_{mj}), \hat{N}_i := \text{blockdiag}(\hat{N}_{1j}, \ldots, \hat{N}_{nj}) \) with \( \hat{M}_{ji} := \text{blockdiag}(\hat{M}_{ji}, \ldots, \hat{M}_{ji}), \hat{N}_{ji} := \text{col}(\hat{N}_{1j}, \ldots, \hat{N}_{nj}) \).

Now, in the following proposition, we introduce an internal model for system (48) based on Huang (2004, Proposition 6.21).

**Proposition 2** *(Internal model for system (48)).* Let Assumption 2 hold and consider the steady-state generator (56) with output \( \tilde{g}(z, \varsigma, \nu, \lambda) = \text{col}(\varsigma, \nu) \). Then, the internal model for system (48) is given by
\[ \nu = \hat{N} \tilde{\varsigma} + \hat{N} \tilde{\nu} \]

where \( \hat{N} : \mathbb{R}^{m+mn} \to \mathbb{R}^{m+mn} \) is the state of the internal model, \( \tilde{\nu} : \mathbb{R}^{m+mn} \times \mathbb{R}^{m+mn} \to \mathbb{R}^{m+mn} \), and \( \hat{N} \) are given in (57).

**Proof.** The proof is similar to the one of Proposition 1. It is sufficient to replace \( z \) and \( u \) in (33) by \( \nu \) and \( \lambda \), respectively.

Before introducing the global robust controller with internal model (59), we define the following quantities, which are used to obtain a gain function for the lower triangular form of system (10) with (41) and represent the parameters of the proposed global robust controller:
\[ \rho := \left( \min_{\nu = 1, \ldots, n} \frac{1}{l_{c0}^2 \nu^2} + \frac{1}{\nu} \sum_{j=1}^{n} \gamma_j(\nu_j) \right)^2 \]
\[ + \frac{1}{\nu} \sum_{j=1}^{n} \gamma_j(\nu_j) \left( \left( \frac{C_j^* \alpha_j}{0} \right) + \left( \frac{C_j^* \beta_j}{0} \right) \right) \]
\[ + \left( \left( \frac{C_j^* \alpha_j}{0} \right) \right) + \left( \frac{C_j^* \beta_j}{0} \right) \right) \]

\[ + \left( \frac{C_j^* \alpha_j}{0} \right) + \left( \frac{C_j^* \beta_j}{0} \right) \right) \]
loads only and does not guarantee stability in presence of ZI loads. Also, the controller in Silani et al. (2021b) strongly depends on the solution to a Partial Differential Equation (PDE). On the other hand, the controllers (22), (24) and (35) achieve voltage regulation, guaranteeing local robust stability in presence of ZIP loads while the controller (64) additionally ensures global robust stability in presence of ZI loads. Furthermore, the controllers (22), (24), (35) and (64) guarantee robust stability with respect to the uncertain constants \( C_r, I_r \) and \( P_r \) while the controller presented in Silani et al. (2021b) only ensures local stability (not robust stability). Also, in this paper we provide the analytical solutions (A.6) and (50) to the regulator Eqs. (19) and (49), respectively, while the analytical solution for the PDE in Silani et al. (2021b) is not provided.

5. Simulation results

In this section, the performance of the control schemes proposed in Theorems 1–3 is evaluated. We consider a DC network composed of 4 nodes, whose electric parameters are equal to those reported in Cucuzzella, Trip et al. (2019, Tables II, III) and are identical or very similar to those used in Nahata et al. (2020), Strehle et al. (2020) and Tucci, Meng et al. (2018) for simulations and in Cucuzzella, Lazzari et al. (2019) for experimental validation. In the following, we consider a mismatch between the actual load profile and the one generated by the corresponding exosystem, showing that the controlled system is Robust Input-to-State Stable (RISS) with respect to such a mismatch and uncertain loads.

Let \( \mathcal{S}_i := 1.43 \sin(0.08t - 0.12) + 0.45 \sin(1.37t - 3.5) + 1 \), and \( \mathcal{S}_2 := 12.41 \sin(0.477t - 1.1) + 11.98 \sin(0.495t + 1.97) + 0.5 \). Then, consider the following load variations: \( \Delta I_i = \mathcal{S}_1 \ A, \Delta G_i = 0.005 \mathcal{S}_1, i = 1, 2, 3 \) and \( \Delta U_i = \mathcal{S}_2 \ A, \Delta G_i = 0.005 \mathcal{S}_2, i = 1, 2 \). Thus, the exosystem (6) can be expressed as

\[
\dot{y}_i = \begin{pmatrix} 0 & \omega_i \\ -\omega_i & 0 \end{pmatrix} y_i + \begin{pmatrix} d_{y1} \\ d_{y2} \end{pmatrix} ,
\]

where \( \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}^4 \) is the state of the exosystem and \( \omega_i \) is defined as \( \omega_i := \text{col}(\omega_{i1}, \omega_{i2}) \) with \( \omega_{i1}, \omega_{i2} \) equal to 0.08 and 1.37 rad/s for Nodes 1, 2 and 3, and 0.477 and 0.495 rad/s for Node 4, respectively. Moreover, the elements of the matrix \( \Gamma_{ij} \in \mathbb{R}^{4 \times 4} \), with \( y \) denoting \( G, I, P \) (or \( G, I, P \) in case of \( Z, I, P \) loads, respectively, can be obtained from the amplitude of the sinusoidal terms in \( \Delta I_i, \Delta G_i \) and \( \Delta P_i \). Consider the case in which the initial conditions of the voltages are not sufficiently close to the desired value, i.e., \( V(0) = \text{col}(340, 340, 340, 340) \). We can notice from Fig. 2 that only by applying the controller proposed in Theorem 3 the voltages converge to their desired values (i.e., 380 V). Differently, by applying the controllers proposed in Theorems 1 and 2, the voltages oscillate and do not converge to their desired values. Now, let the system initially be at the steady-state with \( \dot{h}(0) = \text{col}(30, 15, 30, 26) \), \( A_G(0) = \text{col}(0.07, 0.05, 0.06, 0.08) \) and \( P(0) = \text{col}(8, 4.5, 12) \). Then, at the time instant \( t = 1 \) s the loads vary according to the real data in openei (0000) while the controller uses the information of the exosystems, which differ from the real data. We can observe from Fig. 3 that by applying the controllers proposed in Theorems 1 and 2, the voltage at each node is kept very close to the corresponding desired value, showing that the controlled system is RISS with respect to the mismatch between the actual

\[\begin{align*}
\theta : & = \left\| C_k \right\| + \left\| L_{k}^{-1} \right\| + \left\| G \right\| \left\| L_{-1}^{-1} A \right\| \\
& + \left\| MNL_k C_k \right\| + \left\| NNl_k C_k \right\| + \left\| NNl_k G \right\|
\end{align*}\]

where \( G = \left[ G_i^{\text{max}} \right] + \left[ I_{Cd}^{\text{mix}} \right], \nu > 2, \varphi > 1 \) are constant parameters and \( \hat{P} \) is a symmetric positive-definite matrix of appropriate dimensions satisfying

\[
\hat{P} \sigma + \sigma^T \hat{P} = -\kappa \mathbf{1}_n + \kappa_n.
\]

where

\[
\sigma := \begin{pmatrix} -I_n & A & 0 \\ 0 & -L_{-1} R & 0 \\ -Nl_k & -Nl_k A & M \end{pmatrix}
\]

is Hurwitz. Moreover, \( \lambda_{\min}(\hat{P}), \lambda_{\max}(\hat{P}) \) are the minimum and maximum eigenvalues of \( \hat{P} \), respectively.

In the following theorem, we use the internal model (59) to design a robust controller solving Problem 2.

**Theorem 3 (Global Robust Controller).** Let Assumptions 1 and 2 hold. Consider system (10) with (41), dynamic extension (46), steady-state generator (56) and internal model (59). Then, the feedback controller

\[
u = -\rho \xi + \rho^2 e + (\rho + 1) \nu \tilde{t}^{-1} \nu + \nu \tilde{t}^{-1} \eta
\]

\[
\eta = \tilde{M} \eta + \tilde{N} \xi
\]

\[
\dot{\zeta} = -\zeta + u
\]

solves Problem 2, where \( \rho \) is given in (60).

**Proof.** The proof is provided in Appendix B due to lengthy calculations.

**Remark 3 (Controller Properties).** Note that the structures of the controllers (22), (24), (35) and (64) we propose in this section are more complex than other controllers proposed in the literature (see e.g., Cucuzzella, Lazzari et al., 2020; Ferguson et al., 2021; Lovine et al., 2018; Jeltsema & Schoth, 2004; Kosaraju et al., 2021; Machado et al., 2018; Nahata et al., 2020; Sadabadi et al., 2018; Strehle et al., 2020). More precisely, the proposed controllers require some information about the network parameters (e.g., \( L_p, C_p, L \) and \( R \)). However, in addition to the uncertain constant components of loads, the uncertainties of the network parameters can also be considered in the uncertainty vector \( w \), then the robustness of the proposed controllers can be guaranteed with respect to the uncertainties of the load components and network parameters as well. Note that the higher complexity is associated with the more challenging control objective we achieve. Indeed, differently from Cucuzzella, Lazzari et al., 2019, Ferguson et al. (2021), Lovine et al. (2018), Jeltsema and Schoth (2004), Kosaraju et al. (2021), Machado et al. (2018), Nahata et al. (2020), Sadabadi et al. (2018) and Strehle et al. (2020), the proposed controllers achieve voltage regulation in DC networks including time-varying and uncertain constant loads. Indeed, existing controllers in the literature cannot guarantee voltage regulation and stability in presence of load components that continuously vary with time. Finally, note that we do not consider P loads in the global output regulation problem because finding a gain function satisfying (8.7) in the Appendix is very challenging and left as a future work. Also, we do not consider the resistance of the buck converter in (4) because it is very small in practice and acts as an uncertain damping, thus not affecting the stability.

**Remark 4 (Comparison with Silani et al., 2021b).** Note that the controller proposed in Silani et al. (2021b) is designed for ZI
load profile and the one generated by the exosystem, achieving in practice voltage regulation (Objective 1). We also observe that the controller proposed in Theorem 2 performs better than the one proposed in Theorem 1 in terms of voltage undershoots and deviations. Also, Fig. 4 shows that in presence ZI loads and by applying the controllers proposed in Theorems 2 and 3, the voltage at each node converges to the desired value, achieving voltage regulation (see Objective 1) and the controller system is RISS with respect to the actual load profile. We also observe that the controller proposed in Theorem 3 performs better than the one proposed in Theorem 2 in terms of voltage undershoots, deviations and time response. Finally, for the sake of comparison, we apply (to the same network under the same conditions) the controller in Tucci, Riverso and Ferrari-Trecate (2018), which is designed to deal with constant loads only. Fig. 5 depicts that in presence of time-varying ZIP loads the controller in Tucci, Riverso et al. (2018) is not able to achieve voltage regulation.

Remark 5 (Region of Attraction). Note that for the controllers proposed in Theorems 1 and 2 when at the initial time instant, the initial conditions are outside of the region of attraction, then, only during the initialization phase, a conventional controller can be used to steer the system trajectories within such region.

However, if the region of attraction is not exactly known, the conventional controller can use a sufficiently small region around the equilibrium point instead of the region of attraction.

6. Conclusions and future work

In this paper, we have considered time-varying dynamics with constant uncertainties for the load components of a DC network. Then, we have proposed control schemes achieving voltage regulation and guaranteeing robust stability of the overall network. Finally, we have proposed a control scheme achieving voltage regulation and guaranteeing global robust stability. Future research directions include the use of (global) robust output regulation theory to tackle the problem of average voltage regulation and current sharing in DC networks with time-varying loads. Additionally, we would like to extend the global robust stability also to the case of P loads via finding a suitable gain function for the lower triangular form of the system.

Appendix

The proofs of Theorems 1 and 3 are presented in the following.
Appendix A. Proof of Theorem 1

Proof. We first form the regulator equation associated with (10) and show that its solution is a degree $k$ polynomial in $d$. Then, based on this solution we obtain the minimal polynomial of the $k$-fold exosystem (10b) to show that the pair $(G_1, G_2)$ given in (20) and (21) incorporates an internal model of the $k$-fold exosystem (10b). Finally, following Huang (2004, Theorems 5.7, 5.12), we show that the controllers (22) and (24) solve Problem 1.

In analogy with Huang (2004, Theorem 3.26), we first compute $H_0(x, d, w) = \begin{pmatrix} h(x, d) \\ I_d h(x, d) \end{pmatrix}$. Then, we notice that the solution to $H_0(x, d, w) = 0_{2n}$ for system (10) can be expressed as follows:

$$V = V^* + L_g + i_{2P}(V^*, d, w),$$

(A.1)

with $i_{2P}$ given by (11). Thus, the partition $x^d := \text{col}(I_g, V, x^d_0 := I$ and a smooth function $\tau(x^d, d, w) := \text{col}(-AL + I_g^T + I_g^T(d_1 + (G^T_1 + G^T_2)d_2)\dot{V}^* + \dot{V}^* - (P^* + I_g^T d_2), \dot{V}^*)$ exist such that $H_0(x, d, w) = 0_{2n}$. Recalling the relative degree of system (10) (see Lemma 1), the equivalent control input $u_0(x, d, w)$ can be computed by posing the second-time derivative of the output mapping (10c) equal to zero, i.e.,

$$I_d^2 h(x, d) + L_g I_d h(x, d) u_0(x, d, w) = 0,$$

(A.2)

Now, let $u_0^T(x, d, w) := u_0(x, d, w)\|_{s\in\tau(x^d, d, w)}$. By replacing $V$ and $L_g$ in the solution to (A.2) with the right-hand side of (A.1), we obtain

$$u_0^T(x, d, w) = V^* + L_g AL^{-1}A^T V^* + L_g AL^{-1} R I$$

$$= V^* + L_g (IV_1C_0S_2d_2 + I_g SI_1s_0)$$

$$+ L_g (IV_1C_1S_2d_2 + I_g SI_1s_0),$$

(A.3)

According to Lemma 1, the zero dynamics of (10) can be expressed as

$$\dot{L} = -A^T V^* - R I, \quad \dot{d} = Sa.$$ 

Then, $I(d, w)$ in (19) can be obtained by solving

$$-A I(d, w) = Sd = -L^{-1}(A^T V^* + R I(d)).$$

Since the exosystem model (6) is linear, we can find the analytical expression for the solutions to (A.5). Thus, by virtue of Assumption 1, Lemma 2 and following Huang (2004, Theorem 3.26), the solutions to (19) can be given by

$$x(d, w) = \left(\begin{array}{c} AR^{-1}A^T V^* + i_{2P}(V^*, d, w) \\ V^* \\ -A^T V^* \end{array}\right)$$

$$u(d, w) = u_0^T(x(d, w), d, w) = V^* + L_g (IV_1C_0S_2d_2 + I_g SI_1s_0)$$

$$+ [V^* - (P^* + I_g^T d_2)],$$

(A.6)

with $i_{2P}$ given by (11). Then, we can infer from (A.6) that the solutions $x(d, w)$ and $u(d, w)$ to the regulator equation (19) are linear in $d$, i.e., degree $k = 1$ polynomials in $d$. Consequently, according to Huang (2004, Theorem 5.16), the minimal polynomial of the $k$-fold exosystem (10b) with $k = 1$ can be expressed as:

$$\alpha^k(\lambda) = \prod_{i=1}^{n} \prod_{j=1}^{n_d} (\lambda^2 + a_{ij}^0, \lambda^2 + a_{ij}^1, \lambda^2 + a_{ij}^2, \lambda^2 + a_{ij}^3).$$

(A.7)

Thus, it follows that the pair $(G_1, G_2)$ given by (20) and (21) is the minimal internal model of the $k$-fold exosystem (10b) with $k = 1$. Moreover, by virtue of Assumption 2, it can be inferred from (A.7) that $G_1$ satisfies (Huang, 2004, Property 1.5), i.e., for all $\lambda \in \sigma(G_1)$ the matrix 

$$\begin{pmatrix} A - \lambda I_{2n+m} & B \\ C & 0 \end{pmatrix}$$

has full rank. Then, by virtue of Assumptions 1, 2 and Lemma 2, we have

(i) In analogy with Huang (2004, Theorem 5.7 (i)), the controller (22) solves the first order ($k = 1$) robust output regulation in the sense as described in Huang (2004, Section 5.1). Furthermore, since the solutions $x(d, w)$ and $u(d, w)$ to the regulator equation (19) are degree $k = 1$ polynomials in $d$, following Huang (2004, Theorem 5.12 (i)), the controller (22) solves Problem 1.

(ii) In analogy with Huang (2004, Theorem 5.7 (ii)), the controller (24) solves the first order ($k = 1$) robust output regulation in the sense as described in Huang (2004, Section 5.1). Furthermore, since the solution $u(d, w)$ to the regulator equation (19) is a degree $k = 1$ polynomial in $d$, following Huang (2004, Theorem 5.12 (ii)), the controller (24) solves Problem 1.

Appendix B. Proof of Theorem 3

Proof. We first apply a coordinate transformation to system (48), (59) to find a lower triangular form for system (10) with (41). Then, following Huang (2004, Remark 7.23), we investigate the Robust Input-to-State Stability (RISS) (Huang, 2004, Definition 2.22) of the internal model in the new coordinate to show that the controller (64) satisfies the condition given in Huang (2004, Theorem 7.21 and Remark 7.22). Finally, we show that $\rho$ given in (60) fulfills the conditions given in Huang (2004, Remark 7.10).

Now, we apply the following coordinate transformation to system (48), (59) in order to find a lower triangular form for system (10) with (41)

$$\tilde{z} := z - z(d, w), e := v_e - V^*, \tilde{\zeta} := \zeta - \hat{b}_1(n),$$

$$\dot{\zeta} = \zeta - \bar{\Psi}\dot{T}^{-1}\eta, \dot{\eta} = \eta - \bar{c}(d, w), \dot{u} := u - \tilde{\beta}(\eta),$$

(B.1)

where $z(d, w)$ is given in (50), $\tilde{\beta}, \bar{\Psi}$ are given in (54), (55), respectively, and $\tilde{T}$ is the solution to the Sylvester equation (58), respectively. Then, the augmented system $(48a)$, $(48b)$, $(48c)$, (59) in the new coordinates is given by

$$\dot{\tilde{z}} = F\tilde{z} + G(d, w)e$$

$$\dot{\tilde{e}} = \bar{H}\tilde{z} + K(d, w)e + b\bar{\Psi}\dot{T}^{-1}\eta + b\tilde{\zeta}$$

$$\dot{\tilde{\zeta}} = -(1a + \bar{\Psi}\dot{T}^{-1}\tilde{N})\tilde{\zeta} + \tilde{u}$$

$$\dot{\tilde{\eta}} = (\bar{M} + \bar{\Psi}\dot{T}^{-1})\tilde{\eta} + \bar{\Gamma}\tilde{\zeta},$$

(B.2)

with $F, \bar{H}, b, G(d, w), K(d, w)$ as in (48) and $\bar{M}, \bar{N}$ in (57). Now, we consider the coordinate transformation $\tilde{\eta} := \eta - \bar{N}\dot{T}^{-1}e$. Then, we have

$$\dot{\tilde{\eta}} = \bar{N}\tilde{\eta} + \bar{M}\bar{N}b\dot{T}^{-1}e - \bar{N}\bar{b}\dot{T}^{-1}k(d, w)e - \bar{N}\bar{b}\bar{H}\tilde{z}.$$ 

(B.3)

Now, let define $\tilde{x} := \text{col}(z_1^T, z_2^T) := \text{col}(z_1^T, z_2^T)$ then, the lower triangular form of the system (10) with (41) can be expressed as

$$\tilde{x}_a = F\tilde{x}_a + G\tilde{x}_b$$

$$\tilde{x}_b = M\tilde{x}_b + \bar{M}\bar{N}b\dot{T}^{-1}\tilde{x}_b - \bar{N}\bar{b}\dot{T}^{-1}\bar{H}\tilde{z}$$

(B.4a)

$$\tilde{x}_a = H\tilde{x}_a + K\tilde{x}_b + b\bar{\Psi}\dot{T}^{-1}\tilde{x}_b + b\tilde{\zeta}$$

(B.4b)

$$\tilde{x}_b = -(\bar{M} + \bar{\Psi}\dot{T}^{-1})\tilde{\eta} + \tilde{u}.$$ 

(B.4d)

Now, let (B.3) view $\tilde{\eta}$ as state and col($\tilde{z}, e$) as input. Then, in analogy with Huang (2004, Remark 7.23), we investigate the
RISS (Huang, 2004, Definition 2.22) of (B.3) with respect to the uncertainty \( w \). Suppose that there exists a constant value \( 0 < r_0 < 1 \), satisfying \( 0 \leq (1 - r_0) \tilde{\eta} \). Moreover, since \( M \) is Hurwitz, there exists a symmetric positive-definite matrix \( P \in R^{n_{\mathrm{Lg}} \times n_{\mathrm{Lg}}} \) satisfying \( PM + M^TP = -s_{\mu}^{\mathrm{Lg}} \). Let \( S_1(\tilde{\eta}) = \frac{2}{r_0} \tilde{\eta}^T P \tilde{\eta} \), then \( \frac{2}{r_0} \lambda_{\min}(P) \| \tilde{\eta} \|^2 \leq S_1(\tilde{\eta}) \leq \frac{2}{r_0} \lambda_{\max}(P) \| \tilde{\eta} \|^2 \). The derivative of \( S_1(\tilde{\eta}) \) satisfies
\[
S_1(\tilde{\eta}) = \frac{2}{r_0} \left( 2 \tilde{\eta}^T P \tilde{\eta} - \tilde{\eta}^T (\tilde{\eta}^T P \tilde{\eta}) - \tilde{\eta}^T \tilde{\eta} \right) \leq -\frac{\| \tilde{\eta} \|^2}{r_0} + \frac{2}{r_0} \left( 2 \tilde{\eta}^T P \tilde{\eta} - \tilde{\eta}^T (\tilde{\eta}^T P \tilde{\eta}) - \tilde{\eta}^T \tilde{\eta} \right) \leq -\left( \left( \frac{1}{r_0} \right)^2 - \frac{2}{r_0} \right) \| \tilde{\eta} \|^2 + \left( \frac{2}{r_0} \right)^2 \left( \frac{2}{r_0} \right) \| \tilde{\eta} \|^2,
\]
along the solutions to (B.3). Then, we have
\[
\| \frac{2}{r_0} \left( 2 \tilde{\eta}^T P \tilde{\eta} - \tilde{\eta}^T (\tilde{\eta}^T P \tilde{\eta}) - \tilde{\eta}^T \tilde{\eta} \right) \| \leq a \| \tilde{\eta} \|^2,
\]
where \( a = 1 + \| P \| \| \tilde{\eta} \| + \| P \| \| \tilde{\eta} \| - \frac{1}{r_0} \left( \| \tilde{\eta} \| - \frac{1}{2} \| \tilde{\eta} \|^2 \right) \). Consequently, according to Huang (2004, Theorem 7.21 and Remark 7.22), the controller (64) solving Problem 2 is obtainable by replacing (B.5) in (B.8).

References

Aguirre, L. A., Rodrigues, D. D., Lima, S. T., & Martinez, C. B. (2008). Dynamical prediction and pattern mapping in short-term load forecasting. Electrical Power and Energy Systems, 13, 73–82.

Chiang, H. D., Wang, J. C., Huang, T. C., Chen, Y. T., & Huang, Chang-Hong (1997). Development of a dynamic ZIP-motor load model from on-line field measurements. International Journal of Electrical Power & Energy Systems, 19(7), 459–468.

Choi, B. K., Chang, H. D., Li, Y., Chen, Y. T., Huang, D. L., & Lauby, M. G. (2006). Development of composite load models of power systems using online measurement data. In IEEE power engineering society general meeting, Toronto, QC, Canada.

Cucuzzella, M., Lazzari, R., Kawano, Y., Kosaraju, K. C., & Scherpen, J. M. A. (2019). Robust passivity-based control of boost converters in DC microgrids. In 58th IEEE conference on decision and control (CDC 2019).

Cucuzzella, M., Trip, S., De Persis, C., Cheng, X., Ferrara, A., & van der Schaft, A. J. (2019). A robust consensus algorithm for current sharing and voltage regulation in DC microgrids. IEEE Transactions on Control Systems Technology, 27(4), 1583–1595.

De Persis, C., Weitenberg, E. R., & Dorfler, F. (2018). A power consensus algorithm for DC microgrids. Automatica, 89, 364–375.

Ferguson, J., Cucuzzella, M., & Scherpen, J. M. A. (2021). Exponential stability and local ISS for DC networks. IEEE Control Systems Letters, 5(3), 893–898.

Huang, J. (2004). Nonlinear output regulation theory and applications. Siam.

Jeltsma, D., & Scherpen, J. M. A. (2004). Tuning of passivity-preserving controllers for switched-mode power converters. IEEE Transactions on Automatic Control, 49(8), 1333–1344.

Justo, J. J., Mwasilu, F., Lee, J., & Jung, J. W. (2013). AC-microgrids versus Sustainable Energy Reviews. 135 (2022) 109997.

Kosaraju, K. C., Cucuzzella, M., Scherpen, J. M. A., & Pasumarthy, R. (2021). Differentiation and passivity for control of Brayton–Moser systems. IEEE Transactions on Automatic Control, 66(3), 1087–1101.

Machado, J. E., Aracil-Perez, J., He, W., Ortega, R., & Grino, R. (2018). Active damping of a DC network with a constant power load: An adaptive passivity-based control approach. arXiv preprint:1809.04920.

Nahata, P., Soloperto, R., Tucci, M., Martinelli, A., & Ferrari-Trecate, G. (2020). A passivity-based approach to voltage stabilization in DC microgrids with ZIP loads. Automatica, 13.
Commercial and Residential Hourly Load Profiles for all TMY3 Locations in the United States, https://openEI.org/dataset/commercial-and-residential-hourly-load-profiles-for-all-tmy3-locations-in-the-united-states.

Sadabadi, M. S., Shafiee, Q., & Karimi, A. (2018). Plug-and-play robust voltage control of DC microgrids. *IEEE Transactions on Smart Grid*, 9(6), 6886–6896.

Silani, A., Cucuzzella, M., Scherpen, J. M. A., & Yazdanpanah, M. J. (2021a). Output regulation for load frequency control. *IEEE Transactions on Control Systems Technology*, Early Access.

Silani, A., Cucuzzella, M., Scherpen, J. M. A., & Yazdanpanah, M. J. (2021b). Output regulation for voltage control in DC networks with time-varying loads. *IEEE Control Systems Letters*, 5(3), 797–802.

Silani, A., & Yazdanpanah, M. J. (2019). Distributed optimal microgrid energy management with considering stochastic load. *IEEE Transactions on Sustainable Energy*, 10(2), 729–737.

Sira-Ramírez, H., & Rosales-Díaz, D. (2014). Decentralized active disturbance rejection control of power converters serving a time varying load. In *Proceedings of the 33rd Chinese control conference* (pp. 1934–1768).

Strehle, F., Pfeifer, M., Malan, A. J., Krebs, S., & Hohmann, S. (2020). A scalable port-Hamiltonian approach to plug-and-play voltage stabilization in DC microgrids. *arXiv preprint:2002.05050*.

Trip, S., Burger, M., & De Persis, C. (2016). An internal model approach to (optimal) frequency regulation in power grids with time-varying voltages. *Automatica*, 64, 240–253.

Tucci, M., Meng, L., Guerrero, J. M., & Ferrari-Trecate, G. (2018). Stable current sharing and voltage balancing in DC microgrids: A consensus based secondary control layer. *Automatica*, 95, 1–13.

Tucci, M., Riverso, S., & Ferrari-Trecate, G. (2018). Line-independent plug-and-play controllers for voltage stabilization in DC microgrids. *IEEE Transactions on Control Systems Technology*, 26(3), 1115–1123.

Verdejo, H., Awerkin, A., Saavedra, E., Kliemann, W., & Vargas, L. (2016). Stochastic modeling to represent wind power generation and demand in electric power system based on real data. *Applied Energy*, 173, 283–295.

Vignesh, V., Chakrabarti, S., & Srivastava, S. C. (2014). An experimental study on the load modelling using PMU measurements. In *IEEE PES T & D conference and exposition*. Chicago, IL, USA.

Wilson, D. G., Neely, J. C., Cook, M. A., & Glover, S. F. (2014). Hamiltonian control design for DC microgrids with stochastic sources and loads with applications. In *International symposium on power electronics, electrical drives, automation and motion*.

Zhao, J., & Dorfler, F. (2015). Distributed control and optimization in DC microgrids. *Automatica*, 61, 18–26.

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