Resummation and cancellation of the VIA source in electroweak baryogenesis

Marieke Postma, a Jorinde van de Visb and Graham Whitec

aNikhef, Science Park 105, 1098 XG Amsterdam, The Netherlands
bDeutsches Elektronen-Synchrotron DESY, Notkestr. 85, 22607 Hamburg, Germany
cKavli IPMU (WPI), UTIAS, The University of Tokyo, Kashiwa, Chiba 277-8583, Japan

E-mail: mpostma@nikhef.nl, jorinde.van.de.vis@desy.de, graham.white@ipmu.jp

ABSTRACT: We re-derive the vev-insertion approximation (VIA) source in electroweak baryogenesis. In contrast to the original derivation, we rely solely on 1-particle-irreducible self-energy diagrams. We solve the Green’s function equations both perturbatively and resummed over all vev-insertions. The VIA source corresponds to the leading order contribution in the gradient expansion of the Kadanoff-Baym (KB) equations. We find that it vanishes both for bosons and fermions, both in the perturbative and in the resummed approach. Interestingly, the non-existence of the source is a result of a cancellation between different terms in the KB equations, and not of a pathology in the vev-insertion approximation itself.
## Contents

1 Introduction 2

2 CTP formalism and Green’s functions 4
   2.1 Kadanoff-Baym equations 5
   2.2 Retarded/Advanced propagator and thermal corrections 6

3 Bosons 7
   3.1 Lagrangian 7
   3.2 KB equations 8
   3.3 Thermal corrections 9
   3.4 Vev-insertion approximation
      3.4.1 Constraint equation 10
      3.4.2 Kinetic equation 11
   3.5 Comparison with usual derivation of the VIA source 12
   3.6 Resummed Green’s functions and source term 13
   3.7 Discussion: flavor dynamics 14

4 Fermions 15
   4.1 Lagrangian 16
   4.2 KB equations 16
   4.3 Thermal corrections 17
   4.4 VIA approximation 19
   4.5 Resummed Green’s functions and source term 21

5 Conclusion 22

A Weyl basis 24

B Particle and hole modes 24
1 Introduction

In electroweak baryogenesis (EWB) the observed matter-antimatter asymmetry in the Universe is created during a first order electroweak phase transition [1–4] (for a review, see [5–10]). In order to satisfy the Sakharov conditions (violation of baryon number, charge (C) and charge-parity (CP) symmetry, and out-of-equilibrium dynamics), new physics at the electroweak scale is required [11]. Due to the relatively low energy scale, these new interactions can be probed by experiments. For example, the latest ACME results [12] on electric dipole moments put stringent constraints on CP violation in effective Yukawa interactions between the Higgs field and the Standard Model (SM) fermions, already ruling out a large class of models (see e.g. [13–18]).

Unfortunately, the theoretical uncertainties in the computation of the baryon asymmetry are large, as different approaches lead to predictions that may vary by orders of magnitude [16, 19, 20]. The main difference between methods is in the derivation of the source terms in the transport equations. In particular, using the vev-insertion approximation (VIA), models with CP-violating Yukawa couplings may still be marginally consistent with the electric dipole bounds [21, 22], whereas in a semi-classical computation the corresponding asymmetry is orders of magnitude too small [20]. In this paper we will take a careful look at the derivation of the VIA source term in the transport equations.

The transport equations describe the evolution of the phase space densities of the plasma particles as they interact with the expanding bubbles during the electroweak phase transition. The Higgs vacuum expectation value (vev) varies from zero in the false vacuum outside the bubble to some finite value inside.\(^1\) This leads to a spacetime-dependent mass for the SM fermions and other degrees of freedom with couplings to the Higgs field, and consequently to a semi-classical force [23–27]. In a multi-flavor set-up the mixing matrix between the flavor and mass eigenstates varies inside the bubble wall as well [28–30]. In the presence of CP violation, both effects can give a chiral asymmetry in front of the bubble wall, which is generated by appropriate source terms in the transport equations. Electroweak sphaleron transitions then transform the chiral asymmetry into a baryon asymmetry.

The vev-insertion approximation treats the space-time-dependent mass term as an interaction term, and calculates the source term in a perturbative expansion [31, 32]. Unlike the semi-classical and flavor source terms, there is no clear physical explanation for its origin. The VIA source crucially depends on thermal corrections, in particular on the thermal width, making it distinct from the other sources. Flavor dynamics is essential as well, as for example in a set-up with a left- and right-handed fermion, the left-handed source term is minus the right-handed source. Note that the use of the term ‘flavor’ is somewhat cavalier in this context, and used to distinguish different chiralities. We nevertheless stick to this terminology to highlight the similarities between a bosonic and fermionic source term; in both cases the source hinges on a misalignment between the propagating state and the state that interacts

---

\(^1\)In multi-field or multi-step transitions some combination of the scalar fields in the extended Higgs sector has a changing vev inside the bubble wall.
with the plasma. The VIA source already arises at leading order in the derivative expansion, which treats the bubble wall background as evolving slowly compared to all other relevant time scales. This explains why it generates larger asymmetries compared to other theoretical approaches [19, 20].

The authors of Ref. [33] tried to avoid the VIA expansion, but their approach is not based on a first principle derivation, but rather on a phenomenological method, including a phenomenological decay width, following [34, 35]. In Ref. [19] it was noted that for degenerate masses the source term diverges in the zero width limit. However, the approximations made to derive the VIA source already break down before this limit is reached [36]. Ref. [37] highlighted technical issues with the VIA source, such as omitted pole contributions and renormalization issues. While these indeed should be properly addressed, they do not directly invalidate the derivation, as the renormalization issue does not arise in a bosonic system, and the extra pole contributions can be included. More fundamentally compromising for the VIA approach, Ref. [37] also showed that VIA-like sources vanish at all orders in the derivative expansion if the interactions with the thermal bath are mediated by QCD. This result can be understood because the VIA source arises from the axial current equation, which vanishes in a vector-like theory. Indeed, all equations can be written in terms of the phase space densities of Dirac fermions, and thus no VIA source that is equal but opposite in sign for the left- and right-handed fermions is generated. In this paper we allow for chiral interactions with the bath which can yield a non-zero axial current, in line with the implicit assumption usually made in the VIA literature.

Although the VIA-method already appeared in [34, 38, 39], the source term was first derived in the closed-time-path formalism in [31, 35], and subsequently generalized to include a CP-conserving relaxation rate in [40]. Both the CP-violating and -conserving source terms are derived from the Schwinger-Dyson equations by expanding the self-energies to second order in the VIA expansion. This results in a self-energy diagram that is non-local [37] and not one-particle-irreducible (1PI) [36]. This approach makes it hard to count diagrams correctly, which is an obstacle to resumming all self-energy insertions. The source term is derived from the collision term in the transport equations, which is evaluated with a second order self-energy and zeroth order Green’s function. In [36] it was demonstrated that the same source term can be obtained from the mass commutator in the transport equations, if the self-energy is evaluated at first order – this is a 1PI diagram – and the Green’s function is computed to first order in VIA as well. This equivalence strongly suggest the technical issues noted in [37] do not arise from the non-local nature of the self-energy diagrams appearing in the standard derivation.

In this paper, we will follow the approach of Ref. [36] and only include 1PI, equilibrium, self-energy corrections, which is equivalent to treating the space-time dependent part of the mass matrix as a perturbation. The Green’s functions at any given order in VIA then follow from the Schwinger-Dyson equations (or in Wigner space, from the Kadanoff-Baym equations). The Green’s functions can be determined perturbatively, but remarkably, the full set of constraint equations can be solved exactly as well. The VIA source term is then derived
both perturbatively as well as fully resummed over all mass insertions. We find that the source term actually gets two contributions, from the mass-commutator and from the collision term in the transport equations. The cancellation happens order by order in VIA, and also for the fully resummed case. There is thus no VIA source term at leading order in the derivative expansion. This cancellation was missed before as the original non-1PI approach obscures this [31, 35, 40], and [36] only considered the mass-commutator but not the collision term.

When something cancels exactly, there usually is a deeper reason. Could we actually have foreseen our results? We think we could have. In VIA the Green’s functions are derived at leading order in the derivative expansion, and are thus indistinguishable from the Green’s functions in a constant background. The source term is also derived at leading order in the derivative expansion.\(^2\) In the limit of a constant background the source should vanish, as equilibrium is restored. But this is the same limit as taking the zeroth order in the derivative expansion, and one should thus have expected a zero source.

This paper is organized as follows. Section 2 introduces the Kadanoff-Baym equations and the Green’s functions that describe the out-of-equilibrium plasma. In Sections 3 and 4 we solve the Green’s functions with and without VIA expansion for bosons and fermions respectively. We also derive the corresponding source term and demonstrate that they are zero. We conclude in Section 5.

2 CTP formalism and Green’s functions

The closed-time path (CTP), or in-in formalism, provides a first-principle description in terms of Green’s functions\(^3\) of a system out-of-equilibrium. The Schwinger-Dyson equations can be split into hermitian and anti-hermitian parts, usually referred to as the constraint and kinetic equations. The constraint equations provide the spectral information of the system, whereas the kinetic equation determines the dynamical evolution. In this section we list the results and relations relevant for the discussion of the source terms for electroweak baryogenesis. This section also serves to set the notation. More details and derivations can be found in e.g. [26, 30, 35, 41]. We use the same conventions as [26].

The bosonic and fermionic CTP Green’s functions are, respectively,

\[
\begin{align*}
    i\Delta(u,v) &= \langle \Omega | T_C \left[ \phi(u)\phi^\dagger(v) \right] | \Omega \rangle, \\
    iS_{\alpha\beta}(u,v) &= \langle \Omega | T_C \left[ \psi_\alpha(u)\bar{\psi}_\beta(v) \right] | \Omega \rangle,
\end{align*}
\]

with \(T_C\) denoting time-ordering along the contour \(C\) which runs from initial time \(t_0 = -\infty\) to time \(t\) and back. The contour can thus be split into a forward- and backward-going branch, and fields are labeled by \(a = \pm\), depending on the branch they lie on. We define

\(^2\)To be precise, at leading order in the expansion of the diamond operator in the Kadanoff-Baym equations. In the standard VIA approach, in which this source does not cancel, the integrand is then subsequently expanded in something similar to a derivative expansion to extract the dominant CP-violating part – but this is not equivalent to the diamond expansion.

\(^3\)We will use the terms ‘Green’s function’ and ‘propagator’ interchangeably.
\[ G^{ab} = \{ \Delta^{ab}, S^{ab} \} \] for bosons and fermions respectively, and introduce the notation

\[ G^t = G^{++}, G^t = G^{-+}, \quad G^a = G^{+-}, G^a = G^{-+}. \]

The retarded and advanced propagators are

\[ G^r \equiv G^t - G^a = G^+ - G^-, \quad G^a \equiv G^t - G^r = G^- - G^+. \]  \hspace{1cm} (2.2)

from which it also follows that \[ G^t + G^\bar{t} = G^+ + G^- \]. Throughout we will label \( G^\lambda \) with \( \lambda = >, < \) for the so-called Wightman functions, and \( G^\alpha \) with \( \alpha = r, a \) for the retarded and advanced propagators. The hermiticity properties of the Wightman functions are\(^4\)

\[ (i \Delta^\lambda(u, v))^\dagger = i \Delta^\lambda(v, u) \] and \( (\gamma^0 i S^\lambda(u, v))^\dagger = \gamma^0 i S^\lambda(v, u) \) for bosons and fermions respectively. The retarded/advanced propagators can be split into hermitian and anti-hermitian parts via

\[ G^{\text{r,a}} = G^{\text{h}} \mp iG^{\text{A}} \]

with the upper (lower) sign for the retarded (advanced) propagator in the first expression.

In thermal equilibrium, the Wightman functions can be written in terms of the spectral function

\[ G^\lambda = \rho g^\lambda, \]

with \( g^> = (1 + s n_s) \) & \( g^< = s n_s \) and \( \rho \equiv (G^r - G^a) \). \hspace{1cm} (2.4)

The temperature-dependent self-energies arise from loop corrections in the plasma background. To emphasize the similarity between the bosonic and fermionic system, we will denote the self-energies for both by \( \Pi^{ab} \), rather than using the more conventional \( \Sigma^{ab} \) notation for the fermionic self-energy. When approximated by their thermal equilibrium expressions, the self-energies \( \Pi^{ab} \) satisfy the same relations as the Green functions \( G^{ab} \) listed above. In particular, the advanced/retarded self-energy can be split into real and imaginary parts \( \Pi^{r,a} = \Pi^{h} \mp i\Pi^{A} \), with \( \Pi^{h} \) containing the thermal mass corrections and \( \Pi^{A} \) the thermal widths.

\[ K(u)G^{ab}(u, v) = a\delta_{ab}\delta^4(u - v) + \sum_c c \int d^4 w \Pi^{ac}(u, w)G^{cb}(w, v), \]  \hspace{1cm} (2.5)

with \( a, b, c = \pm \). For bosons \( K(u) = -(-\partial^2_u + M_0^2(u))^2 \) is the Klein-Gordon operator with \( M_0^2 \) the tree-level mass matrix, and for fermions \( K(u) = (i\gamma_u - M_0(u))^2 \) is the Dirac operator with \( M_0 \) the tree-level Dirac mass.

For thick bubble walls, one can perform a gradient expansion in the slowly varying bubble background. It is then useful to define the relative and center-of-mass coordinates \( r = u - v \)

\[^4\]The fermionic Wightman functions can be projected onto a helicity basis, and eq. (2.4) then applies to the projected Wightman functions, see eqs. (4.17) and (4.18) for the exact expressions for fermions.
and $x = \frac{1}{2}(u + v)$, and to make a Wigner transform, which is defined as the Fourier transform with respect to the relative coordinate:

$$G(k, x) = \int d^4(u - v) e^{i k \cdot (u - v)} G(u, v) = \int d^4 r e^{i k \cdot r} G(x + \frac{1}{2} r, x - \frac{1}{2} r).$$  

(2.6)

The gradient expansion in the slowly varying background is then equivalent to an expansion in the diamond operator, which is defined as

$$\diamond (A(k, x) B(k, x)) = \frac{1}{2} (\partial_x A(k, x) \cdot \partial_k B(k, x) - \partial_k A(k, x) \cdot \partial_x B(k, x)).$$  

(2.7)

Wigner transforming the hermitian and anti-hermitian part of the Schwinger-Dyson equation (2.5) gives the constraint and kinetic equations, known as the Kadanoff-Baym (KB) equations. For bosons this gives

$$(k^2 - \frac{1}{4} \partial^2_x) G^{ab} = a \delta_{ab} \otimes 1 + \frac{1}{2} e^{-i \phi} \left( \{ M_0^2, G^{ab} \} + \sum_c c(\Pi^{ac} G^{cb} + G^{ac} \Pi^{cb}) \right),$$

$$2i k \cdot \partial_x G^{ab} = e^{-i \phi} \left( [ M_0^2, G^{ab} ] + \sum_c c(\Pi^{ac} G^{cb} - G^{ac} \Pi^{cb}) \right).$$  

(2.8)

Note that $G^{ab}, \Pi^{ab}, M_0^2$ and 1 are $n \times n$-matrices in flavor space with $n$ complex bosonic degrees of freedom. For fermions we get

$$\frac{1}{2} \{ \gamma, G^{ab} \} = a \delta_{ab} \otimes 1 + \frac{1}{2} e^{-i \phi} \left( \{ M_0, G^{ab} \} + \sum_c c(\Pi^{ac} G^{cb} + G^{ac} \Pi^{cb}) \right),$$

$$\frac{i}{2} \{ \gamma, G^{ab} \} = e^{-i \phi} \left( [ M_0, G^{ab} ] + \sum_c c(\Pi^{ac} G^{cb} - G^{ac} \Pi^{cb}) \right).$$  

(2.9)

For a single Dirac fermion $G^{ab}, \Pi^{ab}, \delta M$ and 1 are matrices in Dirac space; projecting onto left and right chiralities, this can be viewed as $2 \times 2$ in ‘flavor’ space.

### 2.2 Retarded/Advanced propagator and thermal corrections

Introducing the matrices

$$\tilde{G} = \begin{pmatrix} G^t & -G^< \\ G^\ge & -G^\dagger \end{pmatrix}, \quad \tilde{\Pi} = \begin{pmatrix} \Pi^t & -\Pi^< \\ \Pi^> & -\Pi^\dagger \end{pmatrix},$$  

(2.10)

the Schwinger-Dyson eq. (2.5) takes the simple form [35]

$$K(u) \tilde{G}(u, v) = \delta^4(u - v) + \int d^4 w \tilde{\Pi}(u, w) \tilde{G}(w, v).$$  

(2.11)

For a single flavor system in thermal equilibrium the self-energies and Wightman functions satisfy the KMS relations

$$(1 + sn_s) G^{<} = sn_s G^\ge, \quad (1 + sn_s) \Pi^{<} = sn_s \Pi^>.$$  

(2.12)
We note that in the presence of flavor mixing the equilibrium conditions are no longer of the simple KMS form eq. (2.12). We will return to this point in section 3.4.1.

We can diagonalize the propagator and self-energy matrices by an orthogonal matrix \( V \) (with \( VV^{-1} = 1 \))

\[
V \tilde{G} V^{-1} = \begin{pmatrix} G_r & 0 \\ 0 & G_a \end{pmatrix}, \quad V \tilde{\Pi} V^{-1} = \begin{pmatrix} \Pi_r & 0 \\ 0 & \Pi_a \end{pmatrix}, \quad \text{with} \quad V = \begin{pmatrix} (1 + sn_s) & -sn_s \\ 1 & -1 \end{pmatrix}.
\] (2.13)

The diagonal entries \( G_r,a \) and \( \Pi_r,a \) are the retarded/advanced propagators and self-energies respectively. Eqs. (2.10) and (2.13) contain the same information as the relation between the Green’s functions in eqs. (2.2) and (2.4). Using this rotation, the Schwinger-Dyson equations, and thus also the Wigner space KB equations, can be diagonalized.

3 Bosons

In this section we will derive the VIA source for a set-up with two flavors and an off-diagonal spacetime-dependent mass. We assume that the interactions with the bath are flavor-diagonal and are different for the two flavors. The results can be generalized to other bosonic systems as well. After discussing the set-up and the thermal corrections, we will first solve the constraint equation and derive the VIA source perturbatively, expanding in the number of mass insertions. However, the constraint equations can also be solved exactly, without such an expansion, and we also present the corresponding resummed results.

3.1 Lagrangian

For definiteness, consider a toy model with two bosonic flavors, labeled by \( L, R \), with a CP-violating interaction with the Higgs field. In terms of the flavor doublet \( \phi = 1/\sqrt{2}(\phi_L, \phi_R)^T \), the quadratic Lagrangian in the bubble wall background is

\[
\mathcal{L}^{(2)} = (\partial_{\mu} \phi)^{\dagger} (\partial^{\mu} \phi) - \phi^{\dagger} M_{0}^{2} \phi, \quad M_{0}^{2}(v) = \begin{pmatrix} m_{LL,0}^{2} & m_{LR}^{2}(v) \\ m_{RL}^{2}(v) & m_{RR,0}^{2} \end{pmatrix}.
\] (3.1)

The subscript 0 indicates that this is the zero temperature mass matrix. The field-independent diagonal mass terms will receive finite temperature corrections, which are diagonal in flavor space for a flavor-diagonal coupling to the heat bath. The off-diagonal masses depend on the background Higgs vev \( v \), which is spacetime dependent, and violates CP in the bubble wall background. The vev insertion approximation consists of treating \( m_{LR}^{2} = (m_{RL}^{2})^* \) as a perturbation, and expanding the KB-equations in this small quantity.
3.2 KB equations

The constraint equations for the Wightman functions $G^\lambda$ with $\lambda = >, <$ and time-ordered propagator $G^t$ follow from eq. (2.8)

\[
\begin{align*}
(k^2 - \frac{1}{4} \partial_x^2) G^\lambda &= \frac{1}{2} e^{-i\omega} \left( \{M^2, G^\lambda\} + \{\Pi^\lambda, G^h\} + \frac{1}{2} (\Pi^> G^< - [\Pi^>, G^<]) \right), \\
(k^2 - \frac{1}{4} \partial_x^2) G^t &= 1 + \frac{1}{2} e^{-i\omega} \left( \{M^2 + \Pi^t - \Pi^h, G^t\} - \Pi^< G^> - G^< \Pi^> \right). \tag{3.2}
\end{align*}
\]

Here we rewrote the right-hand side of the first equation using the following relations (and the equivalent ones with $\Pi$ and $G$ flipped)

\[
\begin{align*}
\Pi^> G^t - \Pi^t G^> &= \Pi^h G^> + \Pi^> G^h + \frac{1}{2} (\Pi^> G^< - \Pi^< G^>) \\
\Pi^t G^< - \Pi^< G^t &= \Pi^h G^< + \Pi^< G^h + \frac{1}{2} (\Pi^> G^< - \Pi^< G^>). \tag{3.3}
\end{align*}
\]

We have further defined $M^2 = M^2_0 + \Pi^h$, where $M^2_0$ is the tree-level mass matrix, and $\Pi^h$ the thermal mass, as shown in the next subsection. Note that since the system of equations only depends on $M^2$, it is fully equivalent to include the space-time dependent mass in $M^2_0$ and expand the mass matrix, or to include it in the interaction Lagrangian and add the 1PI diagram with one mass insertion to $\Pi^h$ [36]. Here we follow the former approach.

The kinetic equation for the Wightman functions is

\[
2ik \cdot \partial_x G^\lambda = e^{-i\omega} \left( \{M^2, G^\lambda\} + \{\Pi^\lambda, G^h\} + \frac{1}{2} \left( \{\Pi^>, G^<\} - \{\Pi^<, G^>\} \right) \right). \tag{3.4}
\]

Integrating the left-hand side over four-momenta and adding up the two different Wightman functions gives [35]

\[
\frac{1}{2} \partial_\mu \int \frac{d^4k}{(2\pi)^4} i k^\mu \left( G^>(k, x) + G^<(k, x) \right) = -i \langle \phi^\dagger(x) \phi(x) \rangle = -\partial_\mu \langle J^\mu(x) \rangle. \tag{3.5}
\]

We can then identify the source term for the left- and right-handed flavors from the right-hand side of the kinetic equation. The integration over 4-momenta only picks out the leading order term in the diamond expansion, which amounts to setting $e^{-i\omega} = 1$. Explicitly, the left-handed source is

\[
S_{LL} = -\int \frac{d^4k}{(2\pi)^4} \left( [M^2, G^> + G^<] + [\Pi^> + \Pi^<, G^h] + \{\Pi^>, G^<\} - \{\Pi^<, G^>\} \right)_{LL}
\]

\[
\equiv -\int \frac{d^4k}{(2\pi)^4} \tilde{S}_{LL}, \tag{3.6}
\]

where the label $LL$ refers to the $(1,1)$-component of the resulting source matrix. We introduced the notation $\tilde{S}_{LL}$ for the integrand for future convenience.
3.3 Thermal corrections

Let’s first consider the solutions of the constraint equation for a flavor-diagonal mass matrix and self-energy, and see how the thermal self-energies affect the solution. This would be the starting point in the vev-insertion approximation, where at leading order the off-diagonal mass terms are set to zero. The constraint equations for different flavors decouple, and we can focus on a single flavor. We work at leading order in the derivative expansion and set $e^{-i\phi} = 1$.

If the thermal plasma is close to equilibrium, to first approximation the thermal self-energies and propagators satisfy the KMS relations eq. (2.12). We can then rotate the KB equations to the basis of retarded and advanced propagators using eq. (2.13):

$$k^2G^\alpha - \frac{1}{2} \{ M^2_0 + \Pi^\alpha, G^\alpha \} = (k^2 - M^2_0 - \Pi^\alpha)G^\alpha = 1,$$

(3.7)

with $\alpha = r, a$. The self-energies for each flavor can be rewritten in terms of the hermitian and anti-hermitian parts eq. (2.3), which are related to the thermal mass $m_T$ and width $\Gamma_T$ respectively, via [35, 42, 43]

$$\Pi^\alpha = \Pi^h \mp i\Pi^A,$$

(3.8)

with the upper (lower) sign for the retarded (advanced) propagator. The sign (and factor 2) in the bosonic case are taken to obtain the same result as in [31, 40] for the spectral function. The constraint equation for a single flavor is solved by

$$(G^\alpha)^{-1} = k^2 - m^2_0 - \Pi^h_T \pm i\Pi^A_T.$$  (3.9)

The Wightman functions are given by eq. (2.4) in terms of the spectral function

$$\rho = G^r - G^a = \frac{-2i\Pi^A}{(k^2 - m^2_0 - \Pi^h)^2 + (\Pi^A)^2} = \frac{\gamma}{(k^2 - m^2)^2 - \gamma^2/4},$$

(3.10)

with $m^2 = m^2_0 + \Pi^h$ the tree-level plus thermal mass, and $\gamma = -4ik_0\Gamma$.

3.4 Vev-insertion approximation

Just as in the standard VIA approach, we work at leading order in the derivative expansion and set $e^{-i\phi} = 1$ in the KB equations. In VIA the off-diagonal mass term is treated as a perturbation, and one can solve the KB equations perturbatively as a series in the number of ‘mass insertions’, i.e. vev insertions [31, 32, 35, 40]. The solution of the constraint equations can be substituted into the kinetic equation to get the dynamics of the system and derive the source term eq. (3.6). The source term is calculated at 2nd order in VIA, which requires solving the constraint equation to 2nd order as well.
3.4.1 Constraint equation

The mass matrix can be expanded as \( M^2 = M_d^2 + \delta M^2 \), with \( M_d \) containing the diagonal tree-level plus thermal masses, and \( \delta M^2 \) the off-diagonal and field-dependent masses. The constraint equations at leading order in VIA – i.e. setting \( \delta M^2 = 0 \) – are

\[
\begin{align*}
    k^2 G^{\lambda}_{(0)} &= \frac{1}{2} \left( \{ M_d^\ast, G^\lambda_{(0)} \} + \{ \Pi^\lambda, G^\mu_{(0)} \} + \frac{1}{2} \left( [\Pi^>, G^\lambda_{(0)}] - [\Pi^<, G^\lambda_{(0)}] \right) \right), \\
    k^2 G^\mu_{(0)} &= 1 + \frac{1}{2} \left( \{ M_d^\ast + \Pi^I - \Pi^\lambda, G^\mu_{(0)} \} - \Pi^< G^\mu_{(0)} - G^\mu_{(0)} \Pi^> \right),
\end{align*}
\]

(3.11)

where we have set \( e^{-i\omega} = 1 \). We have also dropped the \( \frac{1}{4} \partial_t^2 G^\lambda_{(0)} \) term, as this term likewise leads to derivatives of the background dependent mass, which is assumed to be sub-leading in the derivative expansion. This is not different from the standard assumption in the VIA computation. The thermal corrections arise from (approximately) thermal equilibrium physics, and we assume that the self-energies satisfy KMS relations, and are flavor diagonal:

\[
\Pi^\lambda_{IJ} = g^\lambda_{IJ} \gamma_I \delta_{IJ}, \quad \gamma_I = -4i k_0 \Gamma_I.
\]

(3.12)

with \( I, J = L, R \) and \( g^\lambda \) given in eq. (2.4). The off-diagonal self-energies vanish.

The thermal self-energies are given in eq. (3.12). The 0th order KB equations are solved by Wightman functions that are flavor diagonal and that satisfy the KMS relations

\[
G^\lambda_{(0),IJ} = g^\lambda_{IJ} \rho_{(0),I} \delta_{IJ}.
\]

(3.13)

Substituting this and eq. (3.12) into the leading order constraint equations (3.11), one can solve for the spectral function\(^5\)

\[
\rho_{(0),I} = \frac{\gamma_I}{(k^2 - m_I^2)^2 - \gamma_I^2/4} = \frac{\gamma_I}{D_I D_I^*},
\]

(3.14)

with \( D_I = k^2 - m_I^2 - \gamma_I/2 \) and \( D_I^* = k^2 - m_I^2 + \gamma_I/2 \). The spectral function agrees with the result derived in the basis of retarded and advanced propagators in thermal equilibrium eq. (3.10). The solution for the time-ordered Green’s function is

\[
G^d_{(0),IJ} = \left[ n_I (G^r - G^a) + G^r \right] \delta_{IJ} = \rho_{(0),I} \left[ n_I + \frac{D_I^*}{\gamma_I} \right] \delta_{IJ},
\]

(3.15)

where the first expression can also be derived from eq. (2.13), and we used that \( G_I^r = D_I^{-1} \) and \( G_I^a = (D_I^*)^{-1} \) in the right-most expression. The anti-time-ordered Green’s function is

\[
G^d_{(0),IJ} = \left[ n_I (G^r - G^a) - G^a \right] \delta_{IJ} = \rho_{(0),I} \left[ n_I - \frac{D_I}{\gamma_I} \right] \delta_{IJ}.
\]

(3.16)

\(^5\)At the pole \( k_0 \approx \omega_I = \sqrt{|k|^2 + m_I^2} \) and we can write the spectral function in the form

\[
\rho_{(0),I} = \frac{1}{2 \omega_I} \left[ \frac{1}{k^0 - \omega_I + i \Gamma_I} - \frac{1}{k^0 + \omega_I + i \Gamma_I} \right] - \left( \frac{1}{k^0 - \omega_I - i \Gamma_I} - \frac{1}{k^0 + \omega_I - i \Gamma_I} \right).
\]

This is the standard form of the zeroth order propagator used in the VIA approach [40].
The propagators at 1st and 2nd order in VIA can be expressed in terms of the zeroth order propagators by means of the Schwinger-Dyson equation

\[
G_{(1),II}^{ab} = \sum_c e G_{(0),II}^{ac}(\delta M^2)_{IJ} G_{(0),IJ}^{cb}, \\
G_{(2),II}^{ab} = \sum_{cd} cd G_{(0),II}^{ac}(\delta M^2)_{IJ} G_{(0),IJ}^{cd}(\delta M^2)_{IJ} G_{(0),IJ}^{db},
\]

with \( J \neq I \). The first and second order propagators are flavor-off-diagonal and -diagonal respectively.

### 3.4.2 Kinetic equation

For notational convenience we will drop the (0) subscript on the zeroth order Green’s functions and spectral function in this subsection. The higher order Green’s functions are labeled by a subscript, and there should thus be no source of confusion.

At zeroth order the source term vanishes as the zeroth order propagators are diagonal and therefore the commutator with the mass vanishes. As the zeroth order propagators satisfy the KMS relation the collision term also vanishes. At first order the diagonal entries still vanish by virtue of the KMS relations for the zeroth order propagators. To show that the off-diagonal source vanishes as well is a bit more non-trivial, but straightforward. Here we focus on the diagonal source terms at 2nd order, which is the usual order at which the VIA source is derived, given by eq. (3.6)

\[
\tilde{S}^{(2)} = [\delta M^2, (G_{(1)}^> + G_{(1)}^<)] + [M^2, (G_{(2)}^> + G_{(2)}^<)] + [\Pi^>, \Pi^<, G_{(2)}^h] \\
+ \left( \{\Pi^>, G_{(2)}^<\} - \{\Pi^<, G_{(2)}^>\} \right).
\]

(3.18)

Only the first and last terms on the right-hand-side contribute to the diagonal source term \((S^{(2)})_{II}\). The higher order Green’s functions can be expressed in terms of the zeroth order Green’s functions using eq. (3.17).

The contribution of the first term in eq. (3.18) to the LL-component of the source is

\[
\tilde{S}^{(2)}_{M,LL} \equiv [\delta M^2, (G_{(1)}^> + G_{(1)}^<)] = m^2_{LR}(G_{(1),RL}^> + G_{(1),RL}^<) - (G_{(1),LR}^> + G_{(1),LR}^<)m^2_{RL} \\
= |m_{LR}|^4 \left( (G^> - G^<)_{RR}(G^t + G^t)_{LL} - (G^> - G^<)_{LL}(G^t + G^t)_{RR} \right) \\
= |m_{LR}|^4 \rho_L \rho_R [(2n_L - 1) - (2n_R - 1)] = 2|m_{LR}|^4 \rho_L \rho_R (n_L - n_R),
\]

(3.19)

where we used that \((G^t + G^t)_{II} = (2n_I - 1)\rho_I\) and \((G^> - G^<)_{II} = \rho_I\). The collision term contributes

\[
\tilde{S}^{(2)}_{C,LL} \equiv \left( \{\Pi^>, G_{(2)}^<\} - \{\Pi^<, G_{(2)}^>\} \right)_{LL} \\
= 2|m_{LR}|^4 \left[ \{\Pi^>_{LL} G_{LL}^< - G_{LL}^< \Pi^<_{LL}\}G_{RR}^t G_{RR}^t + G_{LL}^t G_{RR}^t \right] \\
+ \left( \Pi^<_{LL} G_{LL}^< G_{RR}^< G_{LL}^< - \Pi^>_{LL} G_{LL}^< G_{RR}^> G_{LL}^< \right) + (\Pi^<_{LL} G_{RR}^> - \Pi^>_{LL} G_{RR}^<) G_{LL}^t G_{LL}^t.
\]

(3.20)
The term on the 2nd line vanishes by virtue of the KMS relation for the zeroth order propagator and self-energy. The first term on the last line becomes

\[
\text{term}_1 \equiv (\Pi^\gamma_{LL} G^\gamma_{LL} G^\gamma_{RR} G^\gamma_{LL} - \Pi^\gamma_{LL} G^\gamma_{LL} G^\gamma_{RR} G^\gamma_{LL}) \\
= \gamma_L \rho^2_{LR} \rho_R (n_L + n_L) (n_R(1 + n_L) - n_L(1 + n_R)) = -\gamma_L \rho^2_{LR} \rho_R (n_L(n_L - n_R)).
\] (3.21)

The second term on the last line equals

\[
\text{term}_2 \equiv (\Pi^\gamma_{LL} G^\gamma_{RR} - \Pi^\gamma_{LL} G^\gamma_{RR}) G^\gamma_{LL} G^\gamma_{LL} \\
= \gamma_L \rho^2_{LR} \rho_R (n_L - n_R) \left(n_L \frac{D_L^*}{\gamma_L} + \frac{D_L}{\gamma_L} \right).
\] (3.22)

Adding them up we find

\[
(\text{term}_1 + \text{term}_2) = -\gamma_L (\rho_L^2)^2 \rho_R (n_L - n_R) \left[n_L(1 - n_L) + \left(n_L + \frac{D_L^*}{\gamma_L} \right) \left(n_L - \frac{D_L}{\gamma_L} \right) \right] = -\gamma_L \rho^2_{LR} \rho_R (n_L - n_R) \frac{D_L^* D_L}{\gamma_L^2} = -\rho_L \rho_R (n_L - n_R),
\] (3.23)

where we used that \(D_L - D_L^* + \gamma_L = 0\). The final result is then

\[
S^{(2)}_{LL} = \tilde{S}_{M,LL}^{(2)} + \tilde{S}_{C,LL}^{(2)} = 2|m_{LR}|^2 \rho_L \rho_R (n_L - n_R) - 2|m_{LR}|^2 \rho_L \rho_R (n_L - n_R) = 0.
\]

The VIA source thus cancels to 2nd order in the VIA expansion. Likewise, the right-handed source term and all off-diagonal contributions to the source term vanish.

### 3.5 Comparison with usual derivation of the VIA source

Let’s compare our results with the standard derivation of the VIA source found in the literature [31, 40]. In this approach the self-energy is expanded to 2nd order

\[
\Pi^\lambda_{II,(2)} = - (\delta M^2)_{II,JJ} G^\lambda_{II,JJ}(\delta M^2)_{II},
\] (3.24)

and Wightman functions are taken at leading order. The diagonal source originates from the collision term as all other terms in the kinetic equation vanish

\[
\tilde{S}_{LL}^{(2)}_{\text{usual}} = \{\Pi^\gamma_{LL,(2)}, G^\gamma_{LL}\} - \{\Pi^\gamma_{(LL),(2)}, G^\gamma_{LL}\} \\
= -|m_{LR}|^2 \{G^\gamma_{RR}, G^\gamma_{LL}\} = -|m_{LR}|^2 \rho_L \rho_R (n_L - n_R). \] (3.25)

This is the same expression for the collision term as in our approach derived above eq. (3.23). To bring this into more standard VIA notation we perform an inverse Wigner transform on the first line in eq. (3.25) above to evaluate the source eq. (3.6) in position space:

\[
S^{(2)}_{LL}(x) = -2 \int d^4 y \Re \left[ m_{LR}^2(x) G^\gamma_{RR}(x, y) m_{RL}^2(y) G^\gamma_{LL}(y, x) \right] \\
- m_{RL}^2(x) G^\gamma_{RR}(x, y) m_{LL}^2(y) G^\gamma_{LL}(y, x) \\
= -2 \int d^4 y \left( \Re \left[ g(x, y) + g(y, x) \right] \Re \left[ G^\gamma_{RR}(x, y) G^\gamma_{LL}(y, x) - G^\gamma_{RR}(x, y) G^\gamma_{LL}(y, x) \right] \right) \\
+ \Re \left[ g(x, y) + g(y, x) \right] \Re \left[ G^\gamma_{RR}(x, y) G^\gamma_{LL}(y, x) - G^\gamma_{RR}(x, y) G^\gamma_{LL}(y, x) \right].
\] (3.26)
with \( g(x, y) = m_{LR}^2(x)m_{RL}^2(y) \), and where for notational convenience we have dropped the ‘usual’ subscript. Now expand the masses in the integrand

\[
m_{IJ}^2(y) = m_{IJ}^2(x) + (x - y) \mu \partial_\mu m_{IJ}^2(x) + \mathcal{O}(\partial^2).
\] (3.27)

Although this expansion also assumes a slowly varying bubble wall background, it is different from the derivative expansion, i.e. the expansion in terms of the diamond operator, of the KB equations. The leading order term of this mass expansion gives a CP-conserving source term, whereas from the next-to-leading term we recover the usual CP-violating (CPV) source term found in the literature:

\[
S_{\text{CPV}}^{(2)} = 2 \text{Im} \left[ m_{LR}^2 \partial_\mu m_{RL}^2 \right] \int d^4y \ (y - x)^\mu \left( G^>_R(x, y)G^>_L(y, x) - G^>_R(x, y)G^>_L(y, x) \right).
\] (3.28)

Hence, it was concluded that the VIA source term is non-zero. We believe that this erroneous result is a consequence of the non-1PI approach, which is not a systematic perturbative expansion, and obscures the correct counting of diagrams. Indeed, our more systematic approach of the previous subsections shows that if all contributions are carefully taken into account the source term cancels.

In [44] the self-energy was calculated to next-to-leading order, that is, to 4th order in the VIA expansion. This calculation thus likewise works with non 1PI-diagrams, and thus suffers from the same issues as the 2nd order calculation. As we will see in the next subsection, the VIA source cancels to all orders in the vev-insertion expansion.

### 3.6 Resummed Green’s functions and source term

With the thermal equilibrium approximation for the thermal energies in eq. (3.12), the KB equations at leading order in the derivative expansion can be solved exactly for the off-diagonal mass matrix, and there is no need to actually perform a vev-insertion expansion. The equations to solve are

\[
k^2 G^\lambda = \frac{1}{2} \left( \{M^2, G^\lambda\} + \{\Pi^\lambda, G^h\} + \frac{1}{2} ([\Pi^>, G^<] - [\Pi^<, G^>]) \right),
\]

\[
k^2 G^t = 1 + \frac{1}{2} \left( \{M^2 + \Pi^t - \Pi^h, G^t\} - \Pi^< G^> - G^< \Pi^> \right),
\] (3.29)

with thermal self-energies given in eq. (3.12). The solutions for the Wightman functions are

\[
G^\lambda_{LL} = \frac{1}{D_+D_-} \frac{\gamma_R\gamma_L}{\rho_R} \left( g^\lambda_L + g^\lambda_R\rho_R |m_{LR}|^2 \right),
\]

\[
G^\lambda_{LR} = \frac{m_{LR}^2}{D_+D_-} \left( \gamma_R g^\lambda_R(k^2 - m^2_R) + \gamma_L g^\lambda_L(k^2 - m^2_L) + \frac{1}{2} \gamma_R \gamma_L (g^\lambda_R - g^\lambda_L) \right),
\] (3.30)

with \( g^\lambda_I \) defined in eq. (2.4), \( \rho_L, \rho_R \) the zeroth order spectral function eq. (3.14), and

\[
D_\pm = (k^2 - m^2_L \pm \gamma_L/2)(k^2 - m^2_R \pm \gamma_R/2) - |m_{LR}|^2.
\] (3.31)
The other flavor components can be found through $G^\lambda_{RR} = G^\lambda_{LL}|_{L\leftrightarrow R}$ and $G^\lambda_{RL} = G^\lambda_{LR}|_{L\leftrightarrow R}$. Note that

$$
\lim_{m^2_{LR}, m^2_{RL} \to 0} \left\{ \frac{1}{D^+D^-} \frac{\gamma_R \gamma_L}{\rho_R}, \frac{1}{D^+D^-} \frac{\gamma_R \gamma_L}{\rho_L} \right\} = \{\rho_L, \rho_R\},
$$

and we retrieve the perturbative zeroth order Green’s function in this limit.

Note that the denominators of the resummed propagators are shifted by the off-diagonal mass squared $|m_{LR}|^2$. Consequently, the separate contributions from the collision term and from the mass commutator term do not suffer from a pinch singularity, as foreseen in [37], as the divergence in the limit of equal hermitian self-energies $\Pi^h_{LL} = \Pi^h_{RR}$ and vanishing decay widths $\gamma_I \to 0$ is cut off.

To evaluate the off-diagonal source term one also needs the (anti) time-ordered Green’s function

$$
G^t_{LR} = \frac{m^2_{LR}}{D^+D^-} \left( (k^2 - m^2_R) \gamma_R (n_R + \frac{1}{2}) + (k^2 - m^2_L) \gamma_L (n_L + \frac{1}{2}) - |m_{LR}|^2 \right.
$$

$$
+ \left. \frac{1}{4} \gamma_L \gamma_R (2n_R - 2n_L + 1) + (k^2 - m^2_R)(k^2 - m^2_L) \right),
$$

and $G^t_{RL} = -G^t_{LR}|_{\gamma_I \to -\gamma_I}$.

Using the explicit solutions, the source term eq. (3.6)

$$
\tilde{S} = [M^2, (G^> + G^<)] + [\Pi^> + \Pi^<, G^h] + (\{\Pi^>, G^<\} - \{\Pi^<, G^>\}) = 0,
$$

vanishes identically. For the flavor-diagonal source terms, this is due to a cancellation between the commutator with the off-diagonal mass $\delta M^2$ and the collision term. Explicitly, using eq. (3.30), we find

$$
[\delta M^2, (G^> + G^<)]_{LL} = 2|m_{LR}| \frac{\gamma_L \gamma_R (n_L - n_R)}{D^+D^-},
$$

$$
\{\Pi^>, G^<\}_{LL} - \{\Pi^<, G^>\}_{LL} = -2|m_{LR}| \frac{\gamma_L \gamma_R (n_L - n_R)}{D^+D^-}.
$$

And thus $\tilde{S}_{LL} = 0$ exactly, to all orders in the vev-insertion expansion.

### 3.7 Discussion: flavor dynamics

The thermal corrections arise from model-dependent interactions with the bath that are (approximately) described by thermal equilibrium physics. The flavor physics is only non-trivial when the interaction and mass bases are not aligned. The standard VIA approach generates a source term for the two flavors that is equal but opposite in sign, and thus depends crucially on non-trivial flavor physics. Our choice of the self-energy eq. (3.12) yields at leading order in the VIA expansion a Wightman function $G^\lambda_{II} \propto \Pi^\lambda_{II} \propto g^\lambda_{II}$ as in eq. (3.13), which is of the usual form used in the VIA literature [31, 35]. In this work it is our aim to show that, following the assumptions made in the literature, the VIA source cancels at leading order in the derivative expansion. We note, though, that the cancellation of the source
is independent of the specific choice of self-energy. This can be checked explicitly by solving the KB equations for generic (flavor-diagonal) \( \Pi^{\lambda}_{II} \).

At leading order in the gradient expansion the individual terms in the kinetic equation are proportional to \((n_L - n_R)\). Although the source term cancels at this order in the derivative expansion, a non-zero source may still arise at higher order. This would require non-trivial flavor dynamics, different interactions for the flavor states, to allow for a symmetry \( n_L \neq n_R \) to build up. This is consistent with Ref. [37], who did not find a VIA source at any order of the derivative expansion for fermions with vector-like bath interactions (i.e. no flavour dynamics). The fermionic case is discussed in the next section.

Lastly, we note that the propagators at higher order in the VIA expansion eq. (3.17) and the fully resummed propagators eq. (3.30) no longer satisfy the KMS relations as in eq. (3.13) with the spectral function derived at the same order in VIA. The reason is that at lowest order in VIA the mass and flavor eigenstates still coincide, but at higher order this is no longer the case. Whereas the spectral function gives information about the mass eigenstates, the KMS relations for the self-energies are associated to the number densities of the flavor eigenstates. This is motivated by the rapid interactions with the bath, and to a good approximation the bath can be regarded as a collection of particles in flavor eigenstates. Had \( \delta M^2 \) instead been flavor-diagonal, and thus flavor and mass eigenstates aligned at all orders in VIA, then one would have found a KMS relation eq. (3.13) for the Wightman functions at all orders. The collision term in the kinetic equations vanishes for Wightman functions and self-energies that satisfy the KMS relations. Since this is not the case at higher order in VIA, the collision term will contribute to the source.

4 Fermions

Although the constraint and kinetic equations for bosons eq. (2.8) and fermions eq. (2.9) have a very similar structure, the spinor structure of the latter seems to complicate the analysis considerably. For example, the structure of the fermionic thermal self-energy is such that it leads to the appearance of collective plasma modes, see Appendix B. Fortunately, we do not have to delve into these complications for the derivation of the VIA source. In fact, as we will see, we can expand both the self-energies and Green’s functions in helicity eigenstates. The spinor structure then drops out of the KB equations, and the derivation and cancellation of the VIA source is then in almost one-to-one correspondence to the bosonic case.

With a slight abuse of terminology, we will now refer to the two chiralities as different ‘flavors’, because the chiralities have different interactions with the thermal plasma in a chiral theory such as the SM. Once thermal corrections are included, the propagating and interaction states are not aligned, which is essential for the VIA source.
4.1 Lagrangian

For definiteness, consider two Weyl fermions with left and right chirality coupled via a background-dependent Dirac mass \( M_0 = M_0(v) \)

\[
\mathcal{L} = i \bar{\psi} \gamma^\mu \partial_\mu \psi - \bar{\psi} M_0 \psi = i \bar{\psi} \gamma^\mu \partial_\mu \psi_L m_{LR} \psi_R - \bar{\psi}_R m_{RL} \psi_L,
\]

with \( m_{LR} = (m_{RL})^* \), \( P_R M_0 P_R = m_{LR} \) and \( P_L M_0 P_L = m_{RL} \). The vev insertion approximation consists of treating \( m_{LR} = (m_{RL})^* \) as a perturbation, and expanding the KB-equations in this small quantity.

4.2 KB equations

The KB equations eq. (2.9) for the Wightman functions \( G^\lambda \) and time-ordered propagator \( G^t \) are

\[
\{ \bar{k}, G^\lambda \} = e^{-i\phi} \left( \{ M, G^\lambda \} + \{ \Pi^\lambda, G^h \} + \frac{1}{2} \left( \{ \Pi^>, G^< \} - \{ \Pi^<, G^> \} \right) \right),
\]

\[
\frac{1}{2} \{ \bar{k}, G^t \} = 1 + \frac{1}{2} e^{-i\phi} \left( \{ M + \Pi^t - \Pi^h, G^t \} - \{ \Pi^>, G^< - G^<\Pi^> \} \right),
\]

where we defined \( M = M_0 + \Pi^h \) with \( M_0 \) the tree-level Dirac mass matrix, and \( \Pi^h \) the thermal mass. The source terms can be derived from the kinetic equation for the Wightman functions

\[
\frac{i}{2} \{ \bar{\phi}_x, G^\lambda \} = e^{-i\phi} \left( [M, G^\lambda] + [\Pi^\lambda, G^h] + \frac{1}{2} \left( \{ \Pi^>, G^< \} - \{ \Pi^<, G^> \} \right) \right).
\]

Integrating the left-hand side over 4-momentum gives [32]

\[
\text{Tr} \left[ \int \frac{d^4 k}{(2\pi)^4} i \{ \bar{\phi}_x, G^\lambda(k, x) \} \right] = \lim_{r \to 0} \text{Tr} \left[ i(\bar{\phi}_x G^\lambda(r, x) + G^\lambda(r, x) \bar{\phi}_x) \right]
\]

\[
= \partial_\mu \langle \bar{\psi}(x) \gamma^\mu \psi(x) \rangle = \partial_\mu \langle J^\mu \rangle,
\]

with Tr the trace over Dirac spinor indices. The position-space limit \( r \to 0 \) corresponds to taking the leading order derivative expansion in Wigner space, i.e. setting \( e^{-i\phi} = 1 \), which is picked out by the integration over 4-momenta. The evolution of the left-handed current is then defined as

\[
\partial_\mu \langle J^\mu_L \rangle \equiv \partial_\mu \langle \bar{\psi}_L(x) \gamma^\mu \psi_L(x) \rangle = \text{Tr} \left[ \int \frac{d^4 k}{(2\pi)^4} i \left( P_R \bar{\phi}_x P_L G^\lambda(k, x) P_R \right. \right.
\]

\[
+ \left. \left. P_L G^\lambda(k, x) P_R \bar{\phi}_x P_L \right) \right].
\]

Hence, the source term for the left-handed fields can be written as

\[
S_{LL} = \text{Tr} \left[ \int \frac{d^4 k}{(2\pi)^4} \left( [M, G^> + G^<] + [\Pi^> + \Pi^<, G^h] + \left( \{ \Pi^>, G^< \} - \{ \Pi^<, G^> \} \right) \right) \right]_{LL}
\]

\[
\equiv \int \frac{d^4 k}{(2\pi)^4} S_{LL},
\]

with \( (AB)_{LL} = \sum_{I=L,R} A_{LI} B_{IL} \). Note in this respect that for the propagator \( G_{LL} = P_L G P_R \) while for the mass/self-energy \( M_{LL} = P_R M P_L, \Pi_{LL} = P_R \Pi P_L \). In the last line we defined the integrand \( S_{LL} \) for future reference.
4.3 Thermal corrections

Let’s first consider massless fermions and set the Dirac mass to zero. This would be the starting point in the vev-insertion approximation, where at leading order the off-diagonal vev-dependent mass terms are set to zero. The self-energies are flavor-diagonal, and we approximate them with the thermal self-energies that satisfy the KMS relations eq. (2.12). We can then assess the effects of the self-energies in the diagonal basis of retarded and advanced propagators using eq. (2.13), where we work at leading order in the derivative expansion.

The self-energies for the left and right chiralities may differ in a chiral theory. For a massless fermion the hermitian and anti-hermitian self-energy can be written as

\[ \Pi^h = P_R (a_L \hat{k} + b_L \sigma) P_L + P_L (a_R \hat{k} + b_R \sigma) P_R, \]
\[ \Pi^A = P_R (2 \Gamma_L \gamma_0) P_L + P_L (2 \Gamma_R \gamma_0) P_R, \]

(4.7)

with \( a_I, b_I \) Lorentz invariants and functions of \( k = |k| \). In vacuum \( b_I = 0 \) by Lorentz invariance, but at finite temperature the center of mass frame of the plasma introduces a special Lorentz frame. In the plasma rest frame the plasma four velocity is \( u^\mu = (1, 0, 0, 0) \) and \( \hat{\sigma} = \gamma^0 \). The anti-hermitian self-energy is calculated in the zero-momentum limit [45], and can then be related to the decay rate. Away from this limit, the expression is likely of the same general form as \( \Pi^h \), but for simplicity we will stick to the simpler expression for \( \Pi^A \) in eq. (4.7).

It will be useful to work in the basis of 2-dimensional Weyl spinors, and we define

\[ M_0 = \begin{pmatrix} m_{RL} & 0 \\ 0 & m_{LR} \end{pmatrix}, \quad \Pi^{ab} = \begin{pmatrix} 0 & \Pi_{RR}^{ab} \\ \Pi_{LL}^{ab} & 0 \end{pmatrix}, \quad G^{ab} = \begin{pmatrix} G_{LR}^{ab} & G_{LL}^{ab} \\ G_{RR}^{ab} & G_{RL}^{ab} \end{pmatrix}, \]

(4.8)

with the full matrices \( M_0, \Pi, G \) all 4-by-4 in Dirac spinor space, and the sub-blocks \( \Pi_{II}, G_{II}, M_{II} \) 2-by-2 matrices in Weyl spinor space. Note that this is a change of notation, as below eq. (4.6) the same notation was used for the projection of propagators and self-energies in Dirac space. From now on, all propagators, self-energies and masses will be given in Weyl space, and we do not expect this to lead to confusion.

We introduce the helicity projection operators

\[ P^\pm = \frac{1}{2} (1 \pm \sigma \cdot \hat{k}), \]

(4.9)

which satisfy \( P^+ P^- = 0 \), \( (P^\pm)^2 = P^\pm \) and \( P^+ + P^- = 1 \). The hermitian self-energy can then be written as follows

\[ \Pi_{LL}^h = \Pi_{LL,+}^h P^+ + \Pi_{LL,-}^h P^-, \quad \Pi_{RR}^h = \Pi_{RR,+}^h P^+ + \Pi_{RR,-}^h P^-, \]

(4.10)

with \( \Pi_{II,\pm}^h = a_I K_\pm + b_I \) and \( K_\pm = k^0 \pm k \). To leading order in the derivative expansion the equation for the retarded/advanced Green’s function is \( \frac{1}{2} \{ \hat{k}, G^\alpha \} - \frac{1}{2} \{ \Pi^\alpha, G^\alpha \} = 1 \). We rewrite \( \hat{k} \) in the Weyl basis

\[ \hat{k} = \begin{pmatrix} 0 & k^0 - k \cdot \sigma \\ k^0 + k \cdot \sigma & 0 \end{pmatrix} = \begin{pmatrix} 0 & K_+ P^+ + K_- P^- \\ K_- P^- + K_+ P^+ & 0 \end{pmatrix}. \]

(4.11)
The solution for the inverse retarded propagator becomes
\[
(G_{rLL}^{-1}) = (K_+ - \Pi_{LL,+}^h + 2i\bar{\Gamma}_L)P^+ + (K_- - \Pi_{LL,-}^h + 2i\bar{\Gamma}_L)P^-,
\]
\[
(G_{rRR}^{-1}) = (K_+ - \Pi_{RR,+}^h + 2i\bar{\Gamma}_R)P^+ + (K_- - \Pi_{RR,-}^h + 2i\bar{\Gamma}_R)P^-.
\] (4.12)

Inverting this relation gives
\[
G_r = G_{rLL} + P^+ + G_{rRR}P^-,
\]
\[
G_r = G_{rRR}P^- + G_{rLL}P^+.
\] (4.13)

The advanced propagator is obtained by taking $i\bar{\Gamma}_I \to -i\bar{\Gamma}_I$. To re-express the results in the basis of Wightman functions, we first derive the spectral function. Define
\[
\rho_{I,\pm} = \left( G_{rI,\pm}^r - G_{II,\pm}^a \right) = \frac{1}{D_{I,\pm}} - \frac{1}{D_{I,\pm}^*} = \frac{\bar{\gamma}_I}{D_{I,\pm}D_{I,\pm}^*} = \frac{\bar{\gamma}_I}{(K_+ - \Pi_{II,\pm}^h)^2 - \bar{\gamma}_I^2/4},
\] (4.15)

with $\bar{\gamma}_I = -4i\bar{\Gamma}_I$, where we use the barred notation to avoid possible confusion with a Dirac matrix. Then $\rho_L = \rho_{L,+}P^+ + \rho_{L,-}P^-$ and $\rho_R = \rho_{R,+}P^+ + \rho_{R,+}P^-$. With this notation we write the self-energies as
\[
\Pi_{II}^\lambda = \bar{\gamma}_I g^{\lambda I},
\] (4.16)

with $g^{\lambda}$ defined in eq. (2.4). This is analogous to our choice for the bosonic case eq. (3.12), and follows the assumptions in the VIA literature. It is at this point that our treatment diverges from the analysis in Ref. [37], which is based on vector-like interactions with the bath, whereas eq. (4.16) is expected in a chiral theory. The discussion on flavor dynamics in section 3.7 carries over to the fermionic case.

The diagonal Green's functions are expanded in projection operators
\[
G_{ab}^{ab} = G_{LL}^{ab} + G_{RR}^{ab}P^+ + G_{LL}^{ab}P^-,
\]
\[
G_{ab}^{ab} = G_{RR}^{ab}P^- + G_{LL}^{ab}P^+.
\] (4.17)

The Wightman functions are then
\[
G_{II}^\lambda = g^{\lambda I} \rho_I^\pm.
\] (4.18)

The (anti) time-ordered propagators $G_{II}^{\pm} = -(G_r - G_{II})_{II} n_I + G_{II}^{\pm}$ and $G_{II}^{\mp} = -(G_r - G_{II})_{II} n_I - G_{II}^{\mp}$ can be written as
\[
G_{II}^\pm = \left( -n_I + D_{I,\pm}^* \bar{\gamma}_I \right) \rho_I^\pm,
\]
\[
G_{II}^{\pm} = \left( -n_I - D_{I,\pm} \bar{\gamma}_I \right) \rho_I^\pm,
\] (4.19)

where we used eq. (4.15).

The constraint equations at leading order in both the derivative expansion and the VIA expansion can be solved to directly find the Wightman functions and time-ordered propagators. This is what we have shown explicitly for bosons in section 3.4.1. We will not repeat the
exercise here for fermions, but it is straightforward to check that indeed the solutions given above solve the leading order constraint equations.

To calculate the source term and show that also in the fermionic case there is a cancellation, we can work with the spectral function given in eq. (4.15), which is a solution of the constraint equation at leading order in VIA. Although there is no need to determine the pole structure and identify the particle and collective hole modes, this is further discussed in appendix B, to show that this is indeed the same spectral function as used in the usual VIA approach.

4.4 VIA approximation

The propagators found in the previous subsection are the leading order propagators in the VIA expansion. The KB equations have a similar form as those for bosons, and it is no surprise that the (leading order) Green’s functions found in the previous subsection have the same structure as those for bosons as well. Likewise, as we will show, the cancellation of the source term derived from the kinetic equation goes along the same lines as what we have already seen for bosons. In this subsection we will discuss the VIA approximation, and in the next subsection we derive the source from the resummed propagators.

The propagators at 1st and 2nd order in VIA can be expressed in terms of these zeroth order propagators by means of the Schwinger-Dyson eq. (3.17), which becomes in Weyl space

\[ G_{(1),LR}^{ab} = m_{LR} \sum_c \left( G_{LL}^{ac} G_{RR}^{db} P_+ + G_{LL}^{ac} G_{RR}^{db} P_- \right), \]

\[ G_{(2),LL}^{ab} = |m_{LR}|^2 \sum_{cd} \left( G_{LL}^{ac} G_{RR}^{cd} P_+ + G_{LL}^{ac} G_{RR}^{cd} P_- \right). \]  

In this subsection, all propagators without a (..) subscript are again to be understood as the zeroth order propagator.

We can then evaluate the source term in the VIA expansion. Let’s focus on the left-handed source eq. (4.6) at 2nd order in VIA. We insert \( 1 = (P^- + P^+) \) in the trace, and denote the projections of the source onto \( P^\pm \) by \( \bar{S}^\pm \). We concentrate here on \( \bar{S}_{LL}^+ \), but it is straightforward to check that all other diagonal source contributions \( S_{II}^\pm \) vanish as well. At 2nd order in VIA,

\[ \bar{S}_{LL}^{(2)+} = \text{Tr} \left[ P^+ \left( [M_0, G_{(1)}^+] + \{\Pi^+, G_{(2)}^+\} \right) \right] \]

with now Tr denoting the trace in Weyl space, and \( (AB)_{LL} = \sum_{L,R} A_{LI} B_{IL} \) in Weyl space notation. \( M_0 \) is the tree-level (Dirac) mass.

Consider first the commutator term

\[ \bar{S}_{M,LL}^{(2)+} \equiv \text{Tr} \left[ P^+ [M_0, G_{(1)}^+] \right] = \text{Tr} \left[ m_{LR} P^+ (G_{(1),RL} + G_{(1),RL}) - P^+ (G_{(1),LR} + G_{(1),LR}) \right] \]

\[ = \text{Tr} [P^+] m_{LR}^2 \left( (G^t + G^t)_{LL} + (G^t - G^t)_{RR} - (G^t)_{RR} - (G^t - G^t)_{LL} \right) \]

\[ = -2|m_{LR}|^2 \rho_L^+ \rho_R (n_L - n_R), \]  

(4.22)
where we used that $(G^l_{\pm} + G^r_{\pm})_{II} = (1 - 2n_I)\rho^\pm_I$ and $(G^> - G^<)_{\pm,I} = \rho^\pm_I$. Further $\text{Tr}(P^+) = 1$. This has exactly the same structure as in the bosonic case eq. (3.19), except for an overall sign difference because of the difference between bose and fermi statistics.

The collision term contributes
\[
\tilde{S}^{(2)+}_{C,LL} \equiv 2 \text{Tr} \left[ P^+ \left( G_{(2),LL}^< \Pi^<_{LL} - G_{(2),LL}^> \Pi^>_{LL} \right) \right] = 2|m_{LR}|^2 (\text{term}_1 + \text{term}_2),
\] (4.23)
where we have used the KMS relations to already cancel some contributions, fully analogous to what happens in the bosonic expression eq. (3.20). The two remaining terms are then
\[
\text{term}_1 \equiv \text{Tr} \left[ P^+ \left( \Pi^<_{LL} G^G_{LL} G^<_{RR} G^>_{LL} - \Pi^<_{LL} G^G_{LL} G^>_{RR} G^<_{LL} \right) \right] = \text{Tr} \left[ P^+ \left( \Pi^<_{LL} G^<_{+,LL} G^<_{-,RR} G^>_{+,LL} - \Pi^<_{LL} G^<_{+,LL} G^<_{-,RR} G^<_{+,LL} \right) \right] = -\tilde{\gamma}_L (\rho^+_L)^2 \rho^-_R (1 - n_L)(n_L - n_R),
\] (4.24)
and
\[
\text{term}_2 \equiv \text{Tr} \left[ P^+ (\Pi^<_{LL} G^G_{RR} - \Pi^<_{LL} G^<_{RR}) G^d_{LL} G^d_{LL} \right] = (\Pi^<_{LL} G^>_{-,RR} - \Pi^>_{LL} G^<_{-,RR}) G^d_{+,LL} G^d_{+,LL} = -\tilde{\gamma}_L (\rho^+_L)^2 \rho^-_R \left( n_L - \frac{D^*_{L,+}}{\tilde{\gamma}_L} \right) \left( n_L + \frac{D_{L,+}}{\tilde{\gamma}_L} \right) (n_L - n_R).
\] (4.25)
Just as for bosons, the result simplifies when we add the two terms
\[
(\text{term}_1 + \text{term}_1) = \rho^+_L \rho^-_R (n_L - n_R)
\] (4.26)
where we used that $D_{L,+} - D^*_{L,+} + \tilde{\gamma}_L = 0$. The final result is then
\[
\tilde{S}^{(2)+} = \tilde{S}^{(2)+}_M,LL + \tilde{S}^{(2)+}_C,LL = -2|m_{LR}|^2 \rho^+_L \rho^-_R (n_L - n_R) + 2|m_{LR}|^2 \rho^+_L \rho^-_R (n_L - n_R) = 0.
\] (4.27)
We can likewise calculate the source term projected onto negative helicity using the $P^-$ operator. This amounts to interchanging the plus and minus labels, and thus also gives a vanishing source term. We conclude that the 2nd order VIA source cancels also for fermions. In the next subsection we will show that this cancellation occurs at all orders in VIA.

We can again compare with the usual derivation of the fermionic source term in the literature \[31, 40\], which is derived expanding the self-energy to 2nd order in VIA. In this approach only the collision term contributes, and the contribution is equal to our result for the collision term. To write that contribution to the source in a more familiar form, perform an inverse Wigner transform on eq. (4.23) and expand the masses in the integrand to linear order, just as we did in section 3.4.2 for the bosonic case. However, as noted before, this standard non-IPI approach misses the cancellation of the source term.
4.5 Resummed Green’s functions and source term

We will now derive the fully resummed source term. The KB constraint equations at leading derivative order are

\[ \{ \bar{\gamma}, G^\lambda \} = \{ M, G^\lambda \} \} = \{ \Pi^\lambda, G^h \} + \frac{1}{2} \{ [\Pi^\lambda, G^\lambda] - [\Pi^\lambda, G^\lambda] \}, \]

\[ \frac{1}{2} \{ \bar{\gamma}, G^d \} = 1 + \frac{1}{2} \{ \{ M + \Pi^d - \Pi^h, G^d \} - \Pi^d \} \} = 1 + \frac{1}{2} \{ \{ \Pi^\lambda, G^\lambda \} - \Pi^\lambda \} \} = 1 + \frac{1}{2} \{ \{ \Pi^\lambda, G^\lambda \} - \Pi^\lambda \} \} = 1 \]

We express all propagators and self-energies in the Weyl basis as before, see Appendix A for explicit expressions.

We then project all four components in Weyl space onto the \( P^+ \) and \( P^- \) propagators, which gives two decoupled systems of equations. The solutions for \( G_{LL,+}, G_{LR,+} \) and \( G_{RL,-} \) are

\[ G_{LL,+}^\lambda = \frac{1}{D^+D^-} \frac{\bar{\gamma}_R \bar{\gamma}_L}{\rho_R^+} \left( g^\lambda_L + g^\lambda_R \rho_R^+ |m_{LR}|^2 \right), \]

\[ G_{LR,+}^\lambda = \frac{m_{LR}}{D^+D^-} \left( \bar{\gamma}_R g^\lambda_L (K_+ - \Pi^h_{LL,+}) + \bar{\gamma}_L g^\lambda_R (K_- - \Pi^h_{RR,-}) + \frac{1}{2} \bar{\gamma}_L \bar{\gamma}_R (g^\lambda_L - g^\lambda_R) \right), \]

\[ G_{RL,-}^\lambda = \frac{m_{RL}}{D^+D^-} \left( \bar{\gamma}_R g^\lambda_L (K_+ - \Pi^h_{LL,+}) + \bar{\gamma}_L g^\lambda_R (K_- - \Pi^h_{RR,-}) + \frac{1}{2} \bar{\gamma}_L \bar{\gamma}_R (g^\lambda_L - g^\lambda_R) \right), \]

with \( \rho^+_L \) the zeroth order spectral functions eq. (4.15), and

\[ D^\pm = (K_+ - \Pi^h_{LL,+} \pm \bar{\gamma}_L/2)(K_- - \Pi^h_{RR,-} \pm \bar{\gamma}_R/2) - |m_{LR}|^2. \]

In the limit of vanishing Dirac masses

\[ \lim_{m_{LR}, m_{RL} \to 0} \left\{ \frac{1}{D^+D^-} \frac{\bar{\gamma}_R \bar{\gamma}_L}{\rho_R^+}, \frac{1}{D^+D^-} \frac{\bar{\gamma}_R \bar{\gamma}_L}{\rho_L^+} \right\} = \{ \rho_L^+, \rho_R^- \}, \]

and the 0th order Wightman functions are retrieved. The fermionic solutions have a similar structure as their bosonic counterpart eq. (3.30). Pinch singularities in the separate mass commutator and collision term are again avoided, as the denominators in the propagator are shifted by the non-zero Dirac mass eq. (4.30).

Now we calculate the left-handed source eq. (4.6) projected onto \( P^+ \):

\[ \bar{\Sigma}_{LL}^+ = \frac{1}{2} \text{Tr}(P^+) \left( [M, G^\lambda] + [M, G^\lambda] + [\Pi^\lambda, G^h] + [\Pi^\lambda, G^h] + ([\Pi^\lambda, G^\lambda] - [\Pi^\lambda, G^\lambda]) \right)_{LL} \]

\[ = \frac{1}{2} \text{Tr}(P^+) \left( m_{LR}(G^\lambda_{RL,-} + G^\lambda_{RL,-}) - (G^\lambda_{LR,+} + G^\lambda_{LR,+})m_{RL} + 2(\Pi^\lambda_{LL} G^\lambda_{LL} - \Pi^\lambda_{LL} G^\lambda_{LL}) \right) \]

\[ = 0. \]

Only the commutator with the tree-level Dirac mass \( M_0 \) and the collision term contribute, and they add up to zero. There is thus no fermionic VIA source at all orders in the vev-insertion expansion.
5 Conclusion

In this work we have shown that the resonantly enhanced source term traditionally derived in the vev insertion approximation gets cancelled exactly when all terms in the Kadanoff-Baym equations at leading order in the derivative expansion are included. The derivation of the original VIA source relied crucially on the existence of non-trivial flavor dynamics, i.e. a non-alignment between the propagating and flavor eigenstates. In the case of fermions, the term ‘flavor’ is (ab)used to refer to the two different chiralities, which obtain distinct thermal corrections from the plasma. In our analysis we have followed the original VIA literature, and allowed for flavor-dependent interactions (chiral interactions for fermions) with the thermal bath. This generalizes the results of [37], which considered vector -like interactions.

We suspect that the cancellation of the contributions to the source went unnoticed in the original VIA literature, because of the use of 1-particle-reducible (non-1PI) self-energies. Instead, our analysis relies solely on 1PI self-energies, and uses a systematic perturbative expansion. In hindsight our result may have been foreseen. Indeed, the VIA expansion source arises at leading order in the derivative expansion, at which order no distinction between a constant and an evolving background is made – and certainly no source term is expected in the former case.

Interestingly, the problem with the original derivation of the VIA source term does not stem from a pathology in the vev insertion expansion itself, or from the existence of so-called pinch singularities (which are absent when the fully resummed propagator solutions are used), but instead from the approach based on non-1PI diagrams that obscures a proper counting of all contributions.

To show the cancellation of the source one must include the collision term in the kinetic equation, which is non-zero because of the non-trivial flavor dynamics. The assumption is that the interactions with the thermal bath are fast, and to a good approximation the bath can be regarded as a collection of particles in flavor eigenstates. The thermal self-energies then satisfy KMS relations eq. (2.12), formulated in terms of the number densities of flavor eigenstates. The spectral function, on the other hand, contains information on the mass spectrum of the system. If the mass and flavor states are not aligned, the Wightman functions no longer satisfy KMS relations in terms of the number densities of flavor eigenstates, and thus the collision term does not necessarily vanish. Our explicit expressions for the collision term, see e.g. eq. (3.35), indeed vanish in the limit that the flavor off-diagonal mass term \( m_{LR} \) is set to zero, and flavor mixing is absent.

Computations of the baryon asymmetry using the VIA source suggested a rosier picture of the feasibility of EWB in a number of BSM models. The larger yield was mainly due to the CP-violating VIA source containing only one spacetime derivative, and consequently there is no partial cancellation in particle asymmetries when integrated over the bubble wall, in sharp contrast to the semi-classical and flavor oscillation sources [20]. We do expect that the VIA approximation gives a non-zero source term at the next (first) order in the derivative expansion, and that the inclusion of thermal effects may lead to resonant enhancement. But
it remains to be seen whether this will be enough to save large classes of models from the ever tightening constraints imposed by the non-observation of EDMs.

Acknowledgements

MP is supported by the Netherlands Organization for Scientific Research (NWO). JvdV is supported by the Deutsche Forschungsgemeinschaft under Germany’s Excellence Strategy – EXC 2121 “Quantum Universe” – 390833306. GW is supported by World Premier International Research Center Initiative (WPI Initiative), MEXT, Japan. We thank Jordy de Vries and Thomas Konstandin for helpful discussions. This work benefitted from discussions at the 2021 Lorentz Center workshop ‘Computations that Matter’.
A Weyl basis

In this appendix we list all propagators and self-energies expressed in the Weyl basis, defined as 2-by-2 subblocks of the matrices in Dirac space.

\[
\begin{align*}
\mathbf{k} &= \begin{pmatrix}
0 & k^0 - \mathbf{k} \cdot \sigma \\
\mathbf{k}^0 + \mathbf{k} \cdot \sigma & 0
\end{pmatrix} = \begin{pmatrix}
0 & K^-P^+ + K^+P^- \\
K^+P^- + K^-P^+ & 0
\end{pmatrix}, \\
\Pi^h &= \begin{pmatrix}
0 & a_R(k^0 - \mathbf{k} \cdot \sigma) + b_R \\
a_L(k^0 + \mathbf{k} \cdot \sigma) + b_L & 0
\end{pmatrix} = \begin{pmatrix}
\Pi^h_{LL,+}P^+ + \Pi^h_{LL,-}P^- & 0 \\
0 & \Pi^h_{RR,-}P^+ + \Pi^h_{RR,+}P^-
\end{pmatrix}, \\
M_0 &= \begin{pmatrix}
m_{LR} & 0 \\
0 & m_{RL}
\end{pmatrix}, \\
\Pi^\lambda &= \begin{pmatrix}
0 & \tilde{\gamma}R \gamma^\lambda_R \\
\tilde{\gamma}L \gamma^\lambda_L & 0
\end{pmatrix}, \\
G^{ab} &= \begin{pmatrix}
G_{LR}^{ab} & G_{LL}^{ab} \\
G_{RR}^{ab} & G_{RL}^{ab}
\end{pmatrix} = \begin{pmatrix}
G_{LR,+}^{ab}P^+ + G_{LR,-}^{ab}P^- & G_{LL,+}^{ab}P^+ + G_{LL,-}^{ab}P^- \\
G_{RR,+}^{ab}P^+ + G_{RR,-}^{ab}P^- & G_{RL,+}^{ab}P^+ + G_{RL,-}^{ab}P^-
\end{pmatrix}, (A.1)
\end{align*}
\]

where we defined \( K_\pm = k^0 \pm k \).

B Particle and hole modes

In this appendix we give the connection between our propagator solutions to the usual form of the Dirac propagators – with particle and hole modes – used in the derivation of the VIA source in the literature, see in particular Ref. [40]. In contrast to the analysis in Section 4, but in correspondence with [40, 46], we will work with 4-dimensional Dirac spinors in this appendix.

The self-energies for the left and right chiralities may differ in a chiral theory and we split

\[
\Pi^\alpha = \Pi^\alpha_{LL} + \Pi^\alpha_{RR} = P_R \Pi^\alpha P_L + P_L \Pi^\alpha P_R, (B.1)
\]

with \( \Pi^\alpha = \Pi^h \mp i \Pi^A \). For a massless fermion the self-energy can be written as

\[
\Pi^h_J = P_J (a_J \mathbf{\hat{k}} + b_J \mathbf{\hat{\gamma}}) P_I, \quad \Pi^A_J = P_J (2 \mathbf{\hat{\gamma}}_I \gamma_0) P_I, (B.2)
\]

with \( J \neq I \). The solution to the leading order constraint equation for the retarded/advanced Green’s function, \((G^r)^{-1} = (G_{LL}^r)^{-1} + (G_{RR}^r)^{-1} = P_R (G^r)^{-1} P_L + P_L (G^r)^{-1} P_R\), is

\[
(G^r_{II})^{-1} = P_J \left( (A_{I,0} + 2 \mathbf{\hat{\gamma}}_I) \gamma^0 - A_{I,s} \mathbf{\gamma} \cdot \mathbf{\hat{k}} \right) P_I = D_{I,+} A^{J+}_I + D_{I,-} A^{J-}_I, (B.3)
\]

with \( I = L, R \). In the middle expression the coefficients are

\[
A_{I,0} = k_0 (1 - a_I) - b_I, \quad A_{I,s} = k (1 - a_I), (B.4)
\]

– 24 –
and on the RHS we rewrote this defining

\[
D_{I,\pm} = A_{I,0} + 2i\Gamma_I = A_{I,s}, \quad \Lambda_\pm = \frac{1}{2}(\gamma^0 \pm \gamma \cdot \hat{k}), \quad \Lambda^{IJ}_\pm = P_I \Lambda_\pm P_J \text{ for } I \neq J. \tag{B.5}
\]

The advanced propagator is of the same form with \(i\Gamma \to -i\Gamma\).

To invert the propagator we use that \(\Lambda^\pm \Lambda^\mp = 0\) and \(\Lambda^+ \Lambda^- + \Lambda^- \Lambda^+ = 1\). Then \(G^r = G^r_{LL} + G^r_{RR} = P_L G^r P_R + P_R G^r P_L\) with

\[
G^r_{II} = \frac{\Lambda^{IJ}_+}{D_{I,+}} + \frac{\Lambda^{IJ}_-}{D_{I,-}}. \tag{B.6}
\]

The poles of the propagators are located at \(k_0 = \mathcal{E}_{I,p}\) (and \(k_0 = -\mathcal{E}_{I,p}^*\)), which is the solution of \(D_{I,+} = 0\), and at \(k_0 = \mathcal{E}_{I,h}\) (and \(k_0 = -\mathcal{E}_{I,h}^*\)), which is the solution of \(D_{I,-} = 0\). The real parts of the poles \(E_I = \text{Re}(\mathcal{E}_I)\) correspond to the particle and hole energies (antiholes and antiparticles) respectively, as indicated by the subscript. For the retarded propagator the poles of the propagator have negative imaginary parts, and in the narrow width approximation

\[
\mathcal{E}_{I,p} = E_{I,p} - i\Gamma_{I,p}, \quad \mathcal{E}_{I,h} = E_{I,h} - i\Gamma_{I,h}, \tag{B.7}
\]

with \(\Gamma_I = Z_{I,\rho,\Gamma_I}\) and

\[
Z_{p,h} = (\partial_{k_0} D_{II})^{-1} |_{k_0 = \mathcal{E}_{p,h}} = \frac{E^2_{p,h} - k^2}{2m^2_I}, \tag{B.8}
\]

the residues of the poles [40, 46]. The retarded propagator can then be written as

\[
G^r_{II} = \left(\frac{Z_{I,p}}{k_0 - \mathcal{E}_{I,p}} + \frac{Z_{I,h}^*}{k_0 + \mathcal{E}_{I,h}} - f_I(k_0, k)\right)\Lambda^{IJ}_+ + \left(\frac{Z_{I,h}}{k_0 - \mathcal{E}_{I,h}} + \frac{Z_{I,p}^*}{k_0 + \mathcal{E}_{I,p}} + f_I^*(-k_0, k)\right)\Lambda^{IJ}_-. \tag{B.9}
\]

The function \(f_I\) gives the non-pole part of the propagator, which is subdominant near a resonance. The poles at \(k_0 = -\mathcal{E}_{I,p}^*, -\mathcal{E}_{I,h}^*\) correspond to excitations of anti-particles and anti-holes. The spectral function is \(\rho = (G^r - G^a) = \rho_L + \rho_R = P_L \rho P_R + P_R \rho P_L\) with,

\[
\rho_I = P_I (G^r - G^a) P_J = \Lambda^{IJ}_+ \rho_{I,+} + \Lambda^{IJ}_- \rho_{I,-}, \tag{B.10}
\]

and

\[
\rho_{I,+} = \frac{Z_{I,p}}{k_0 - \mathcal{E}_{I,p}} - \frac{Z_{I,p}^*}{k_0 - \mathcal{E}_{I,p}^*} + \frac{Z_{I,h}}{k_0 + \mathcal{E}_{I,h}} - \frac{Z_{I,h}^*}{k_0 + \mathcal{E}_{I,h}^*} + \ldots \tag{B.11}
\]

and \(\rho_{I,-}(k_0, k) = (\rho_{I,+}(-k_0, k))^*\), and the elipses denoting the sub-dominant non-pole contributions (see [47] for their explicit form). The Wightman functions, generalized for non-zero chemical potential in Ref. [40] \(^6\), are

\[
G^\lambda_{II} = g_I^\lambda \left(\Lambda^{IJ}_- \rho_{I,+} + \Lambda^{IJ}_+ \rho_{I,-}\right), \tag{B.12}
\]

\(^6\)Note that Ref. [40] uses \(\tilde{m}_T^2 = 2m_T^2\) and there is no factor 1/2 in the residue. Their dispersion relation (A.18) differs a factor 2 compared to [47, 48] given their definition of the thermal mass \(\tilde{m}_T^2 = g^2 C_2(r)T^2/2\).
with $n_I(k^0 - \mu_I) = (e^{(k_0 - \mu_I)/T} + 1)^{-1}$ the Fermi-Dirac distribution.

In the hard thermal loop approximation

$$A_{I,0} = k_0 - \frac{m_I^2}{k_0} x Q_0(x), \quad A_{I,s} = k + \frac{m_I^2}{k} (1 - x Q_0(x)),$$

with $Q_0 = \frac{1}{2} \ln \frac{x+1}{x-1}$ the Legendre function of the 2nd kind, $x = k_0/k$, and $m_I^2 \equiv g^2 C^I_x T^2 / 8$, with $C^I_x$ denoting the quadratic Casimir operator of the gauge group under which particle $I$ is charged. In the limit $k \to 0$ of zero momentum, $x Q_0 \to 1$, which gives $E_{I,p} = E_{I,h} = m_I$ and $Z_{I,p} = Z_{I,h} = 1/2$. This motivates the definition of the thermal mass $m_I$.

In the limit $k \to \infty$ the hole contribution becomes subdominant $Z_{I,h} \ll Z_{I,p}$. Already for $k \gtrsim m_I$ this becomes a good approximation and this limit was often used in the literature on the VIA source, to avoid the complication of hole modes. The particle pole is located at $E_{I,p} = k + m_I^2 / k$, which is the high momentum limit of a particle with energy $E_{I,p} = \sqrt{k^2 + 2 m_I^2}$, and $Z_p = 1$. In this case it is more useful to define the thermal mass as $\bar{m}_I^2 = 2 m_I^2$. 


References

[1] V.A. Kuzmin, V.A. Rubakov and M.E. Shaposhnikov, On the Anomalous Electroweak Baryon Number Nonconservation in the Early Universe, Phys. Lett. B 155 (1985) 36.

[2] M.E. Shaposhnikov, Baryon Asymmetry of the Universe in Standard Electroweak Theory, Nucl. Phys. B 287 (1987) 757.

[3] L.D. McLerran, M.E. Shaposhnikov, N. Turok and M.B. Voloshin, Why the baryon asymmetry of the universe is approximately 10^{-10}, Phys. Lett. B 256 (1991) 451.

[4] A.G. Cohen, D.B. Kaplan and A.E. Nelson, Baryogenesis at the weak phase transition, Nucl. Phys. B 349 (1991) 727.

[5] A.G. Cohen, D.B. Kaplan and A.E. Nelson, Progress in electroweak baryogenesis, Ann. Rev. Nucl. Part. Sci. 43 (1993) 27 [hep-ph/9302210].

[6] M. Trodden, Electroweak baryogenesis, Rev. Mod. Phys. 71 (1999) 1463 [hep-ph/9803479].

[7] J.M. Cline, Baryogenesis, in Les Houches Summer School - Session 86: Particle Physics and Cosmology: The Fabric of Spacetime, 9, 2006 [hep-ph/0609145].

[8] D.E. Morrissey and M.J. Ramsey-Musolf, Electroweak baryogenesis, New J. Phys. 14 (2012) 125003 [1206.2942].

[9] G.A. White, A Pedagogical Introduction to Electroweak Baryogenesis, IOP concise series (2016).

[10] B. Garbrecht, Why is there more matter than antimatter? Calculational methods for leptogenesis and electroweak baryogenesis, Prog. Part. Nucl. Phys. 110 (2020) 103727 [1812.02651].

[11] A.D. Sakharov, Violation of CP invariance, c asymmetry, and baryon asymmetry of the universe, Pis'ma Zh. Eksp. Teor. Fiz. 5 (1967) 32.

[12] ACME collaboration, Improved limit on the electric dipole moment of the electron, Nature 562 (2018) 355.

[13] J.M. Cline, K. Kainulainen and M. Trott, Electroweak Baryogenesis in Two Higgs Doublet Models and B meson anomalies, JHEP 11 (2011) 089 [1107.3559].

[14] S. Liebler, S. Profumo and T. Stefaniak, Light Stop Mass Limits from Higgs Rate Measurements in the MSSM: Is MSSM Electroweak Baryogenesis Still Alive After All?, JHEP 04 (2016) 143 [1512.09172].

[15] G.C. Dorsch, S.J. Huber, T. Konstandin and J.M. No, A Second Higgs Doublet in the Early Universe: Baryogenesis and Gravitational Waves, JCAP 05 (2017) 052 [1611.05874].

[16] J.M. Cline, Is electroweak baryogenesis dead?, 1704.08911.

[17] J. de Vries, M. Postma, J. van de Vis and G. White, Electroweak Baryogenesis and the Standard Model Effective Field Theory, JHEP 01 (2018) 089 [1710.04061].

[18] D. Bodeker and W. Buchmuller, Baryogenesis from the weak scale to the grand unification scale, Rev. Mod. Phys. 93 (2021) 035004 [2009.07294].

[19] J.M. Cline and K. Kainulainen, Electroweak baryogenesis at high bubble wall velocities, Phys. Rev. D 101 (2020) 063525 [2001.00568].
[20] J.M. Cline and B. Laurent, *Electroweak baryogenesis from light fermion sources: A critical study*, Phys. Rev. D 104 (2021) 083507 [2108.04249].

[21] J. De Vries, M. Postma and J. van de Vis, *The role of leptons in electroweak baryogenesis*, JHEP 04 (2019) 024 [1811.11104].

[22] E. Fuchs, M. Losada, Y. Nir and Y. Viernik, *CP violation from τ, t and b dimension-6 Yukawa couplings - interplay of baryogenesis, EDM and Higgs physics*, JHEP 05 (2020) 056 [2003.00099].

[23] M. Joyce, T. Prokopec and N. Turok, *Nonlocal electroweak baryogenesis. Part 2: The Classical regime*, Phys. Rev. D 53 (1996) 2958 [hep-ph/9410282].

[24] J.M. Cline, M. Joyce and K. Kainulainen, *Supersymmetric electroweak baryogenesis*, JHEP 07 (2000) 018 [hep-ph/0006119].

[25] K. Kainulainen, T. Prokopec and M.G. Schmidt and S. Weinstock, *First principle derivation of semiclassical force for electroweak baryogenesis*, JHEP 06 (2001) 031 [hep-ph/0105295].

[26] T. Prokopec, M.G. Schmidt and S. Weinstock, *Transport equations for chiral fermions to order h bar and electroweak baryogenesis. Part 1*, Annals Phys. 314 (2004) 208 [hep-ph/0312110].

[27] T. Prokopec, M.G. Schmidt and S. Weinstock, *Transport equations for chiral fermions to order h-bar and electroweak baryogenesis. Part II*, Annals Phys. 314 (2004) 267 [hep-ph/0406140].

[28] T. Konstandin, T. Prokopec and M.G. Schmidt, *Kinetic description of fermion flavor mixing and CP-violating sources for baryogenesis*, Nucl. Phys. B716 (2005) 373 [hep-ph/0410135].

[29] T. Konstandin, T. Prokopec, M.G. Schmidt and M. Seco, *MSSM electroweak baryogenesis and flavor mixing in transport equations*, Nucl. Phys. B 738 (2006) 1 [hep-ph/0505103].

[30] V. Cirigliano, C. Lee and S. Tulin, *Resonant Flavor Oscillations in Electroweak Baryogenesis*, Phys. Rev. D 84 (2011) 056006 [1106.0747].

[31] A. Riotto, *Towards a nonequilibrium quantum field theory approach to electroweak baryogenesis*, Phys. Rev. D 53 (1996) 5834 [hep-ph/9510271].

[32] A. Riotto, *Supersymmetric electroweak baryogenesis, nonequilibrium field theory and quantum Boltzmann equations*, Nucl. Phys. B 518 (1998) 339 [hep-ph/9712221].

[33] M. Carena, J.M. Moreno, M. Quiros, M. Seco and C. Wagner, *Supersymmetric CP violating currents and electroweak baryogenesis*, Nucl. Phys. B 599 (2001) 158 [hep-ph/0011055].

[34] P. Huet and A.E. Nelson, *Electroweak baryogenesis in supersymmetric models*, Phys. Rev. D 53 (1996) 4578 [hep-ph/9506477].

[35] A. Riotto, *The More relaxed supersymmetric electroweak baryogenesis*, Phys. Rev. D58 (1998) 095009 [hep-ph/9803357].

[36] M. Postma, *A different perspective on the vev insertion approximation for electroweak baryogenesis*, JHEP 09 (2021) 055 [2107.05971].

[37] K. Kainulainen, *CP-violating transport theory for electroweak baryogenesis with thermal corrections*, JCAP 11 (2021) 042 [2108.08336].
[38] P. Huet and E. Sather, *Electroweak baryogenesis and standard model CP violation*, Phys. Rev. D 51 (1995) 379 [hep-ph/9404302].

[39] P. Huet and A.E. Nelson, *CP violation and electroweak baryogenesis in extensions of the standard model*, Phys. Lett. B 355 (1995) 229 [hep-ph/9504427].

[40] C. Lee, V. Cirigliano and M.J. Ramsey-Musolf, *Resonant relaxation in electroweak baryogenesis*, Phys. Rev. D 71 (2005) 075010 [hep-ph/0412354].

[41] V. Cirigliano, C. Lee, M.J. Ramsey-Musolf and S. Tulin, *Flavored Quantum Boltzmann Equations*, Phys. Rev. D81 (2010) 103503 [0912.3523].

[42] C. Greiner and S. Leupold, *Stochastic interpretation of Kadanoff-Baym equations and their relation to Langevin processes*, Annals Phys. 270 (1998) 328 [hep-ph/9802312].

[43] P.A. Henning, M. Blasone, R. Fauser and P. Zhuang, *Thermalization of a quark - gluon plasma*, hep-ph/9612492.

[44] M. Postma and J. van De Vis, *Source terms for electroweak baryogenesis in the vev-insertion approximation beyond leading order*, JHEP 02 (2020) 090 [1910.11794].

[45] E. Braaten and R.D. Pisarski, *Calculation of the quark damping rate in hot QCD*, Phys. Rev. D 46 (1992) 1829.

[46] H.A. Weldon, *Structure of the quark propagator at high temperature*, Phys. Rev. D 61 (2000) 036003 [hep-ph/9908204].

[47] M.L. Bellac, *Thermal Field Theory*, Cambridge Monographs on Mathematical Physics, Cambridge University Press (3, 2011), 10.1017/CBO9780511721700.

[48] H.A. Weldon, *Effective Fermion Masses of Order gT in High Temperature Gauge Theories with Exact Chiral Invariance*, Phys. Rev. D 26 (1982) 2789.