On the superconductivity in the system with preformed pairs

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We discuss the phenomenology of the superconductivity resulting from the bose condensation of the preformed pairs coexisting with unpaired fermions. We show that this transition is more mean field like than usual bose condensation, i.e. it is characterized by a relatively small value of the Ginzburg parameter. We consider the Hall effect in the vortex flow regime and in the fluctuational regime above \( T_c \) and show that in this situation it is much less than in the transition driven entirely by bose condensation but much larger than in a usual superconductivity. We analyse the available Hall data and conclude that this phenomenology describes reasonably well the data in the underdoped materials of \( YBaCuO \) family but is not an appropriate description of optimally doped materials or underdoped \( LaSrCuO \).

I. INTRODUCTION

It is known for a long time that excitation spectrum in underdoped high \( T_c \) cuprates shows formation of the pseudogap at temperature \( T_s \) far above \( T_c \); this phenomena was observed in the NMR responses \( [1] \) and in optics \( [2] \). Recently the photoemission experiments showed that this phenomena can be attributed to the electrons in the corners of the Fermi surface which acquire a gap in these materials at about the same temperature at which pseudogap is observed in optics and NMR \( [3] \). Below \( T_c \) the value of the gap does not change significantly with temperature, instead the electron spectral function develops coherence peaks at the gap edges. These data invite the interpretation that this gap formation is due to the pairing of electrons in the corners of the Fermi surface into the bosons which later bose condense at \( T_c \). The description of the superconductivity in the cuprates as a bose condensation of preformed pairs was proposed also in different physical contexts \( [4][5] \). Unfortunately, all these scenarios would lead to the conclusion that superconducting transition is similar to the bose condensation and has a wide fluctuation region near \( T_c \). This conclusion does not agree with the data which show that the transition is more mean field like and that it is characterized by a small value of Ginzburg parameter. In this paper we show that the bose condensation description and mean-field nature of the superconducting transition can be reconciled if bose condensation happens against the background of the Fermi liquid and processes that convert bosons into the fermions on the Fermi surface are allowed. We formulate the model which describes this physics in Section II and derive its physical properties in Section III. Another problem of the descriptions based on bose condensation is that it leads to a large value of the Hall effect in the superconducting state. Our analysis of the data shows that usual bose condensation is not consistent with the data whereas bose condensation which happens against the background of the Fermi liquid might be consistent with the available Hall data in the underdoped \( YBaCuO \) materials but is not consistent with the data on optimally doped \( YBaCuO \) or underdoped \( LaSrCuO \). We emphasize however that the data presently available are not sufficient to make the definite conclusion, especially for the underdoped materials; we discuss the data in more detail below in the Introduction and in Section IV. It is not important for the foregoing discussion what is the microscopic mechanism resulting in the formation of the preformed pairs but for the sake of concreteness we shall discuss the model where these pairs are formed from the electrons in the corners of the Fermi surface.

Qualitatively, the relative weakness of superconducting fluctuations in high \( T_c \) is clear from the following arguments. In these highly anisotropic materials the coherence length in \( c \)-direction, \( \xi_c(T = 0) \), is much smaller than the interlayer distance, \( d \) making them almost two dimensional superconductors. In a purely two dimensional superconductor bose condensation would show up as Berezinskii-Kosterlitz-Thouless transition in which the superfluid density jumps from \( \rho_S(T_c) \approx \rho_S(0) \) to \( \rho_S = 0 \). Weak three dimensional effects would only smear this transition a little. Such \( \rho_S(T) \)
dependence was not observed in any cuprates; instead the observed temperature dependence of $\rho_S(T)$ is mean field like in the broad temperature range even for underdoped cuprates, for instance in $\mathrm{YBa}_2\mathrm{Cu}_3\mathrm{O}_8$ $\rho_S(T) \propto (T_c - T)$ for $T_c - T \gtrsim 0.05 T_c$. Note here that critical three dimensional behavior of optimally doped $\mathrm{YBa}_2\mathrm{Cu}_3\mathrm{O}_{7}$ reported in [1] does not contradict the conclusion that superconducting fluctuations are relatively weak. In this material the $\rho_S(T)$ dependence remains linear in $T$ in a wide temperature range [1] and the mere fact that these critical fluctuations are three dimensional implies that they occur only in the vicinity of $T_c$ where the correlation length in $c$ direction becomes large, $\xi_c \gg d$.

Quantitatively, the strength of superconducting fluctuations in quasi two dimensional systems is determined by the superfluid density, $\rho_S$. The measured absolute values of $\rho_S$ in cuprates turn out to be too large for the Bose condensation scenario; in $\mathrm{YBa}_2\mathrm{Cu}_4\mathrm{O}_8$ the in-plane penetration lengths are $\lambda_a = 800$ Å and $\lambda_b = 2000$ Å [2].

This unrealistic value indicates that $\rho_S(T)$ must decrease by a factor of 10 before the thermal fluctuations become important in agreement with linear $\rho_S(T)$ dependence observed in [3]. A related evidence of the weakness of superconducting fluctuations is provided by a small value of the Ginzburg parameter which is $G_i \sim 0.02$ in this material (see Eq. [4]).

Another important argument against Bose condensation is provided by the Hall effect data near $T_c$. Bose condensation of charged particles would lead to a huge Hall effect in the superconducting state: $\rho_{xy}$ becomes important in agreement with linear $\rho_{xy}$ dependence observed in [7]. This sign change is preempted by the negative fluctuational contribution to the positive Hall effect in the normal state near $T_c$. The mere fact that these critical fluctuations are three dimensional implies that they occur only in the vicinity of $T_c$ where the correlation length in $c$ direction becomes large, $\xi_c \gg d$.

A similar explanation of the pseudogap phenomena is based on the spin charge separation model [8]. In this model the gap formation is due to the pairing of spinons which carry no charge, such pairing does not lead to superconductivity; it happens only at lower temperature and is due to Bose condensation of holons. This model has the same difficulty as the condensation of the preformed pairs discussed above; there seems to be no reason to expect a narrow fluctuation region if the transition is driven by the Bose condensation of holons.

A somewhat different view point on this problem is provided by the models which interpolate between BCS like transition in Fermi liquid and ordinary Bose condensation of preformed pairs as the interaction strength is varied [4,5]. In this framework the data discussed above would make one to conclude that high $T_c$ cuprates are well inside the Fermi liquid regime and very far from the preformed pairs in contradiction to the observed gap formation above $T_c$ in underdoped cuprates.

Another puzzling property of the superconducting transition is the change of the Hall effect sign occurring below $T_c$. This sign change is preempted by the negative fluctuational contribution to the positive Hall effect in the normal state [4,7], qualitatively both the sign change in the superconducting phase and the fluctuational Hall effect can be explained if Cooper pairs are responsible for superconductivity and are in fact negatively charged. In this case these pairs give a large negative contribution to the Hall conductivity in the vortex state below $T_c$ which is proportional to $1/\mu$; and produce negative fluctuational Hall conductivity observed in [6]. Both Hall conductivity in the vortex state near $T_c$ and the fluctuational conductivity above it can be described in the framework of the time dependent Ginzburg Landau (TDGL) equation; for this equation the negative sign of the Cooper pair implies that the imaginary part of the relaxation rate $\text{Im} \gamma < 0$,

\begin{equation}
\gamma \frac{\partial \Delta}{\partial t} = - \frac{\partial F}{\partial \Delta^*} \tag{1}
\end{equation}

Here $F$ is the usual Ginzburg-Landau free energy [18].

In the framework of the usual BCS theory the sign and the magnitude of $\text{Im} \gamma$ (and therefore the effective charge of the Cooper pair) is determined by the derivative of the density of states at the Fermi surface, $\partial \nu / \partial \epsilon$, namely $\text{Im} \gamma \sim - \partial \nu / \partial \epsilon$. This conclusion remains valid for any weak coupling BCS theory of superconductivity regardless of the nature of the interaction [19]. For the high $T_c$ cuprates $\partial \nu / \partial \epsilon$ is controlled by the proximity to a van Hove singularity and photoemission data show that Fermi surface is always a hole like, so that $\partial \nu / \partial \epsilon < 0$ and BCS theory would predict the hole sign of $\text{Im} \gamma$ in contrast to the data. One can also relate $\partial \nu / \partial \epsilon$ to $\partial T_c / \partial \mu$ and avoid the use of photoemission data, this would lead to the prediction $\text{Im} \gamma \sim - \partial T_c / \partial \mu$ which implies that the hydrodynamic contribution to the Hall effect is hole-like for the underdoped cuprates and electron-like for the overdoped cuprates in a striking contrast to the study of $\mathrm{La}_{2-x}\mathrm{Sr}_x\mathrm{CuO}_4$ [20] which reported the opposite correlation. We emphasize here
that the sign change of the Hall effect in the superconducting state does not itself contradicts the BCS theory, it is only the disagreement between the sign of $\partial \nu / \partial \epsilon$ (or $\partial T_c / \partial \mu$) and the sign of the hydrodynamic contribution to the Hall effect which indicates that the weak coupling BCS theory is not valid. In conventional, BCS-like superconductors, the Hall effect might change sign if $-\partial \nu / \partial \epsilon$ has the sign opposite to the sign of the charge carriers which is measured by the normal state Hall effect. The sign of the charge carriers in the normal state is determined by the topology of the Fermi surface. The sign change might occur if $\partial \nu / \partial \epsilon < 0$ on the electron-like Fermi surface or if $\partial \nu / \partial \epsilon > 0$ on the hole-like Fermi surface.

Qualitatively, the notion of electron-like preformed pairs agrees with the non-BCS behavior of the Hall effect of the superconductive pairs, but it is difficult to reconcile both of them with the small value of the Ginzburg parameter and with a moderate Hall effect in the superconducting state. In this paper we resolve this dichotomy suggesting the model where preformed pairs coexist with usual fermions and show that in such systems the Hall effect might still be unusual but the Ginzburg parameter is small. One can justify this model using the following qualitative arguments.

II. MODEL

It is well established that cuprate Fermi surface lies in the vicinity of the van Hove points. Moreover, it is remarkable how small is dispersion of fermions near $(\pi, 0)$ points in the underdoped cuprates according to the photoemission data [5]. It is natural to assume that interaction between these fermions can easily exceed their kinetic energy and that the interaction with momentum transfer $q \sim (\pi, \pi)$ is less repulsive than interaction with small momentum transfer. Such interaction gives fermions a gap which is due to the pairing in the antiferromagnetic or superconducting channels. In the weak coupling approximation the d-wave superconductive pairing dominates if the Fermi surface is not nested.

In the cuprates both photoemission data [6] and band structure calculations [21] show that the Fermi surface is not nested; a simplest Fermi surface which agrees with photoemission data shown in Fig. 1. Therefore, it is reasonable to assume that fermions near the corners of the Fermi surface (which lie inside the discs shown in Fig. 1) are paired into bosons, $b^\dagger$, with charge $2e$ and no dispersion; this is the key assumption of our model. So one-particle fermionic excitations acquire a gap; the soft modes appearing instead of these fermionic excitations are spinless bosons

$$H = \varepsilon b^\dagger_q b_q$$  \hspace{1cm} (2)
where $\varepsilon$ is phenomenological parameter of the model. Note that in this model the bose condensation does not occur because bosons have no dispersion (i.e., are infinitely heavy). Another assumption of the model is that interaction, $V$, transferring electrons from the 'discs', where they are paired to the other parts of the Fermi surface (where Fermi velocity is large) is weak. This assumption can be justified in the spinon-holon model of charge separation \[8\] where this interaction is suppressed by gauge field fluctuations. If $V$ is small we may neglect the effects of these transfer processes on the gap formation in the corners of the Fermi surface, clearly in this case the gap formation in the corners does not necessarily result in the superconductivity and it does not give a gap to the electrons away from the 'discs'. At higher temperatures the effects of the remaining fermions can be neglected and bosons form a normal liquid without long range order. Only at sufficiently low temperatures the boson-mediated Cooper pairing between remaining unpaired electrons results in the superconductivity.

Hamiltonian describing this physics is

$$H = \sum_q \varepsilon b_q^\dagger b_q + \sum_{p,q} V_{p,q}(b_q^\dagger c_{p}\gamma_{p,q} + h.c.) + \sum_p \xi_p c_p^\dagger c_p$$

(3)

here $\sum'$ denotes the sum over Brillouin zone excluding the 'disc' area.

Because $b$ describes fermions paired into the state with $d$-wave symmetry, $V_{p,q}$ also has this symmetry and we may approximate it by

$$V_{p,q} = V a^2 (p_x^2 - p_y^2)$$

(4)

and neglect its $q$-dependence at small $q$. Superconducting transition in this model occurs at $T_c$ given by

$$\epsilon = g \ln \Lambda / T_c, \quad g = \frac{1}{(2\pi)^2} \int v_F^2 \frac{dp}{v_F(p)}$$

(5)

where integral $\int dp$ is taken over the Fermi line and $\Lambda \sim \epsilon_F$ is upper cut off.

Depending on the parameter $\epsilon$ model \[3\] describes somewhat different physical situations. At $\epsilon >> T_c$ even at low temperatures bosons exist only as virtual states, in this case the superconducting transition is almost conventional. At $\epsilon \sim T_c$ the density of bosons at $T \sim T_c$ is significant so the superconducting transition acquires some features of the bose condensation. We anticipate that the former case is relevant for optimally doped cuprates whereas the latter is more appropriate for the underdoped ones. Model \[3\] is somewhat similar to the model of disordered quasilocalized pairs coexisting with Fermi liquid introduced in \[22\]. In the latter model the quasilocalized pairs are assumed to form resonances with energies $E$ that are randomly distributed around the Fermi level. We do not know any experimental justification for this assumption and we believe that the phase transition in the presence of such large disorder in the energy levels will become quite broad.

The superconducting transition at $T_c$ can be described as a bose condensation which occurs only because bosons become coherent due to the exchange of fermions. Alternatively, one might integrate out the bosons and get the fermion model with retarded short-range interaction. Both approaches lead to the same physical results. Here we shall adopt the bose formalism because it is shorter and more physical in the regime when $\epsilon \sim T$ so the density of bosons is significant; we shall argue below that this regime is relevant for the underdoped cuprates. At $T_c$ the gap begins to open on the remaining part of the Fermi surface

$$\Delta(p) = V(p)\langle b \rangle$$

(6)

Clearly $\phi = \langle b \rangle$ plays the role of the order parameter in this model; its thermal fluctuations are governed by the action $S(\phi)$ which is obtained after integrating out the fermion degrees of freedom:

$$S(\phi) = \sum_{\omega} \phi_\omega^2 \left\{ -i\gamma''\omega - g \left[ T - \frac{T_c}{\Lambda} + \frac{\pi|\omega|}{8T_c} + \xi_0^2 \left[ (\nabla - \frac{2ie}{c}A) \right]^2 \right] \right\} \phi_\omega - \frac{1}{2}\beta \int |\phi_t|^4 dt.$$  

(7)

Here we use imaginary time representation; we introduce coefficients

$$\xi_0^2 = \frac{7\zeta(3)}{2(8\pi^2T)^2} \int V_F^2 v_F(p) dp, \quad \beta = \frac{7\zeta(3)}{2(4\pi^2T)^2} \int V_F^4 \frac{dp}{v_F(p)}$$

(8)

and $\gamma'' = -1$, the latter we introduced to facilitate the comparison with the usual time dependent Ginzburg-Landau equation where this coefficient is determined by a particle-hole asymmetry near the Fermi surface and is usually small.
Generally, the coefficient $\gamma''$ has contributions from the bare action of the bosons, $S_b = -b^\dagger \partial_t b - b^* b$, described by Hamiltonian (2) and from the fermions that we integrated out but the latter is always small in parameter $T_c/\epsilon_F$ leading to a simple result, $\gamma'' = -1$.

Action of a generic form (7) but with different parameter values describes also usual BCS type superconductivity in the Fermi liquid, bose condensation and interpolation between these two regimes [14]. The crucial difference between the interpolation scheme [14] and model considered here is that the latter leads to such parameters that the condensate amplitude, $|\phi|^2 = g^2/\beta$, always remains small even far from $T_c$ and so the fluctuation region is narrow and the Hall effect never becomes too large.

III. RESULTS

The gradient term in the effective action (7) is determined by the fermion properties. As a result the superconducting transition is mean field like and thermal fluctuations become large only in the narrow vicinity, $G_i$, of the transition temperature; it is convenient to express it in terms of the screening length, $\lambda_0$, $G_i$ is given by

$$G_i = \frac{(4\pi\lambda_0)^2 T_c}{\sqrt{2} \delta \Phi_0^2}$$

Here we define $\lambda_0$ as the value of the physical screening length interpolated from the vicinity of the transition temperature to low temperatures; it is expressed through the coefficients $g$, $\xi_0^2$ and $\beta$ of the effective action (7) by

$$\lambda_0^2 = \frac{c^2 \beta d}{32\pi e^2 g^2 \xi_0^2}$$

We computed the coefficient $g$, $\xi_0^2$ and $\beta$ for the fermions with the spectrum $\xi_p = -2t(\cos p_x + \cos p_y) - 4t' \cos p_x \cos p_y - \mu$. Because fermions in the discs of size $p_0$ around van Hove points are paired and do not contribute to the effective action we excluded these regions from the integrals over the Fermi surface (8). We get

$$\lambda_0^2 = \frac{c^2 d \Upsilon(\delta)}{e^2 t}, \quad G_i = \frac{16 T_c}{\sqrt{2} t} \Upsilon(\delta).$$

Here $\Upsilon(\delta)$ is dimensionless function of the doping density $\delta$ which we plot in Fig. 2 for $t' = -0.3t$ and different sizes of the excluded regions, $p_0$. We observe that once the regions near the corners of the Fermi surface are excluded the doping dependence of the penetration length becomes relatively weak and the dependance on the size of the excluded regions become far more important. Qualitatively we expect that the size of the excluded region becomes large in the underdoped bilayered cuprates where large pseudogap was observed in spin responses and photoemission so that their penetration length is larger than the one in the optimally doped cuprates in agreement with the data. However we can not make a quantitative comparison because we do not know the value of $p_0$. 

![Graph](image-url)
The results above do not depend on the dynamical part of the action (9), but it becomes important for the fluctuational conductivity [28]:

\[ \delta \sigma_{xx} = \frac{1}{16d^2} \frac{e^2}{\hbar} \frac{T_c}{T - T_c} \]

where \( d \) is the distance between planes. This result depends only weakly on the properties of the electrons if they form a Fermi liquid even with a large relaxation rate. Note that in the conventional bose condensation scenario real part of the relaxation time is absent [14] leading to a much larger fluctuational correction to the longitudinal conductivity. Thus, it would be important to understand whether this universal behavior (11) is indeed observed in high \( T_c \) cuprates. Fluctuational Hall conductivity in low field is also controlled by the coefficients of the effective action (7) [19,24]:

\[ \delta \sigma_{xy} = \frac{e^2}{3\pi d^2} \gamma'' \frac{T_c}{g} \frac{\xi_0^2}{\hbar} \left( \frac{T_c}{T - T_c} \right)^2 \]

Here \( \gamma'' \) is the coefficient of the non-dissipative term in the action (7), in this model \( \gamma'' = -1 \). This contribution should be added to the normal state Hall conductivity. As a result a sign change of the Hall effect would occur above \( T_c \) at

\[ \frac{T - T_c}{T} = \sqrt{\frac{2}{3}} \left| \frac{\gamma'' T_c}{g} \right|^{1/2} \frac{\xi_0}{l}, \]

where \( l \) is the mean free path; here we used usual Drude formula, \( \sigma_{xx}^n = \frac{ee'c}{B}(\omega_c \tau)^2 \), for the conductivity in the normal state. If \( |\gamma'' T_c/g| > (T_c/\mu)^2 \) the correction to \( \sigma_{xy} \) is small and the Hall effect changes sign in the region where the longitudinal conductivity is still close to the normal state value [11].

In the vortex state the hydrodynamic contribution to the Hall effect is [28]

\[ \sigma_{xy}^V = \frac{2ee'c}{B} \gamma'' |\phi|^2 = \frac{H^2(T)}{2\pi(T_c - T)} \left( \frac{\gamma'' T_c}{g} \right) \frac{ee'c}{B} \]

In the generic time dependent Ginzburg Landau theory [28] this contribution might be different by a numerical factor \( \beta_V \approx 1 \); the physical effect taken into account by this numerical factor is electric field generated by the moving vortex. This effect is small and \( \beta_V \approx 1 \) if the length, \( \xi_E = 4\xi_0\sqrt{2\sigma_n e'^2 N} \approx \xi_0 \sqrt{\frac{2}{\pi} \tau_{\nu} T_c} \), which sets the scale for the electric field variations is long, \( \xi_E \gg \xi_0 \), which seems to be an appropriate limit for cuprates. In conventional notations [28] TDGL dimensionless parameter \( u = (\xi_E/\xi_0)^2 \ll 1 \) for these materials.

In the bose condensation scenario \( \gamma'' = 1 \), at low temperatures \( |\phi|^2 \) coincides with the boson density and the Hall conductivity \( \sigma_{xy} = n_0 ee'c/B \) is huge. In the present model \( |\phi|^2 \sim \frac{T}{\beta'} n_c \) is small leading to a smaller value of the Hall conductivity. In the conventional BCS theory the coefficient \( \gamma'' \) is determined by the dependence of the density of states, \( \nu(\mu) \) on the chemical potential \( \gamma''_{BCS} = -\frac{2}{\pi} \frac{\partial \ln \nu}{\partial \mu} \); here it is controlled by the bosons mediating the interaction between fermions. So, in the conventional BCS theory \( \gamma''_{BCS} \) is small, \( \gamma''_{BCS} \sim \frac{T_c}{\mu} \), whereas here it is large. Formally we get a large \( \frac{\sigma''_{BCS}}{g} \) in the model (3) because we assumed that bosons are coupled to the pairs of electrons, not holes which introduced a large particle-hole asymmetry. This assumption can be justified if the fermion dispersion near van Hove points is small so that properties of the bosons are determined by the relative number of electrons and holes in the 'disc' area. Further, if the number of electrons is small, the bosons are entirely electron-like and we get the phenomenological model (3); if the numbers of electrons and the holes in the 'disc' area are close we would need to introduce two types of bosons (electron-like and hole-like). This would lead to the effective action (4) with \( \gamma'' \ll 1 \) and the resulting Hall effect would be much smaller.
These results show that $\gamma''$ is not necessarily related to $\frac{\partial \ln T_c}{\partial \mu}$ as was conjectured in [19]. The arguments of [19] were based on the gauge invariance and on the assumption that $T_c$ dependence on $\mu$ implies a dependence on the gauge invariant object $T_c(\mu + i\frac{\gamma''}{\mu})$. This is true in the BCS model with weak interaction where this dependence is due to the density of states dependence on the chemical potential. However, in a general case one should distinguish two sources of $T_c(\mu)$ dependence: the dependence via the energy of pairing electrons, $\epsilon_F$, and the dependence via the total density of particles, $n$. The gauge invariance indeed requires that $T_c(\epsilon_F)$ is converted into the $T_c(\epsilon_F + i\omega)$ in the dynamical action but the dependence via the total density is not modified by the frequency so generally the quadratic term in the action is

$$S^{(2)} = -\sum_\omega g \ln \left( \frac{T}{T_c(\epsilon_F + i\omega, n)} \right) b_\omega^* b_\omega$$  \hspace{1cm} (15)

In other words, $n(\mu)$ dependence does not imply a non-gauge invariant action, it can be reformulated in an explicitly gauge invariant manner as a dependence on $\varphi = \nabla^{-2}(\nabla E)$.

In the phenomenological model [3] the $T_c$ dependence on the doping, $\delta$, is due to the interaction term, $V(\delta)$ so that $T_c$ grows with doping. One possible microscopic mechanism of this dependence is suppression of the interaction $V(\delta)$ by the gauge field fluctuations discussed in [3] which becomes less in more doped systems.

IV. ANALYSIS OF THE DATA

Equations (12,14) can be directly compared with the data. Note here that fluctuational Hall conductivity and Hall conductivity in the flux flow regime are controlled by the same dimensionless parameter $(\frac{\gamma''}{\sigma})$; this is a general feature of any hydrodynamic description based on time-dependent Ginzburg-Landau equation. Experimental verification that one gets the same parameter if it is extracted from the Hall data in the fluctuational regime above $T_c$ and if it is extracted from the data in the vortex flow regime would be a very important proof of the validity of the hydrodynamic approach. The comparison of these parameters becomes more complicated in weakly anisotropic materials such as $YBa_2Cu_3O_7$ where the fluctuational data are further complicated by the crossover between two and three dimensional behaviors; to avoid these problems it is better to compare the data obtained on more anisotropic materials.

First we compare the values of $(\frac{\gamma''}{\sigma})$ obtained on similar optimally doped materials. The extensive study [14] of the fluctuation regime in $Bi_2Sr_2CaCu_2O_x$ shows that in the regime of 2D fluctuations $\delta \sigma_{xy} \approx 0.08 \frac{1}{\Omega_{cm}}$ at $B = 0.7 \ T$. Using the value $\frac{dH_c}{dT} \approx 2 \ T/K$ and $\delta = 18.5 \ \AA$ we obtain $(\frac{\gamma''}{\sigma}) \approx -0.003$. Unfortunately we are not aware of the Hall effect data in the vortex flow regime on this material, so we compare this value with the other optimally doped cuprates. It is convenient to characterize Hall conductivity data in the vortex flow regime by the value of $\sigma_{xy}(0)$ obtained by a linear extrapolation to low temperatures. For $YBa_2Cu_3O_7$ we use extrapolated value $\sigma_{xy}(0) = \frac{3.10^3 \Omega_{cm}}{B[2]}$ and $dH_c/dT = 0.02 \ T/K$; we get $(\frac{\gamma''}{\sigma}) \approx -0.03$. For $Tl_2Ba_2Cu_2O_8$ we use $\sigma_{xy}(0) = \frac{3.10^3 \Omega_{cm}}{B[2]}$, $dH_c/dT = 0.01 \ K/T$ and we get $\gamma''/\sigma \approx -0.0016.$ The fit of the fluctuational Hall conductivity data obtained on the same sample agree with theoretical predictions if one chooses $dH_c/dT = 1 \ T/K$ and $(\frac{\gamma''}{\sigma}) \approx -0.002$. These data indicate that hydrodynamic approach is likely to be valid but do not allow to make a definite conclusion. They also show that in optimally doped materials $\epsilon \sim g \gg T_c$, so the bosons may exist only as virtual states of electron pair.

The situation is different for underdoped bilayered cuprates. We take extrapolated value $\sigma_{xy} = \frac{4 \times 10^3 \Omega_{cm}}{B[2]}$ and $dH_c/dT = 0.006 \ T/K$ appropriate for $60 \ K \ YBa_2Cu_3O_{7-x}$; we get $(\frac{\gamma''}{\sigma}) \approx -0.9$ in agreement with our initial expectations that bosons exist as real electron pairs in these materials. However the data on the underdoped $La_{2-x}Sr_xCuO_4$ lead to a different conclusion. Here we take $\sigma_{xy} = \frac{300 \Omega_{cm}}{B[2]}$ and $dH_c/dT = 0.006 \ T/K$ for the material with $x = 0.1$, we get $(\frac{\gamma''}{\sigma}) \approx -0.001$. This estimate implies that bosons are unlikely exist as real pairs in underdoped $La_{2-x}Sr_xCuO_4$. We emphasize that we do not know of any data which would allow us to check that hydrodynamic approach remains valid for underdoped materials.

The independent check of the validity of the hydrodynamic (time dependent Ginzburg-Landau) description in the flux flow regime is provided by the Hall angle data in weak field region $B \ll H_c$. In the framework of the effective
The data [13] for Hall angle tangent in 60 $K\ YBa_2Cu_3O_{7-x}$ show that its value extrapolated to $T = 0$ is $\tan(\theta_H) \approx 1$, for 90 $K\ YBa_2Cu_3O_7$ it is much smaller, $\tan(\theta_H) \approx 10^{-2}$, finally for $x = 0.1 \ Lao_{2-x}Sr_xCuO_4 \ tan(\theta_H) \approx 10^{-3}$ [20]. All these values are in a resonable agreement with the above estimates for the parameter $\left(\frac{\epsilon''T}{g}\right)$ and usual expectation that $\ln(\xi_E/\xi_0) \sim 1$.

Another physical property of the phenomenological model (3) is anomalous thermopower in the normal state. The magnitude of this effect is very sensitive to the value of $\epsilon_R/T$ where $\epsilon_R = \epsilon - g \ln(\frac{T}{T_c})$ is the effective chemical potential of the pairs. We have only a rough estimate of this parameter based on the following arguments. The boson density in the phase space is $n_0 = n_B(\epsilon_R/T) \leq 1$ (here $n_B$ is Bose factor), so $\epsilon_R/T = \ln 1/n_0 \gtrsim 1$; such $\epsilon_R$ makes possible the scattering of electrons with energies larger than $\epsilon_R$ resulting in a large relaxation rate for these fermions. Because this relaxation mechanism is effective only for fermions above the Fermi energy it results in a large particle-hole asymmetry and leads to a large thermopower. Assuming that this contribution to the relaxation rate $1/T_R$ is much larger than the typical relaxation rate for the fermions with energies less than $\epsilon_R$ we get Seebeck coefficient

$$S_0 = \ln\left(\frac{1}{n_0(T)}\right)n_0(T) = \frac{g \ln(T)}{T} \exp\left(-\frac{g \ln(T)}{T}\right)$$

(17)

which is much larger than the usual value, $T/\epsilon_F$, for the normal metal. The sign of the thermopower is positive. Its temperature dependence is non-monotonic, at $G_T, T_c \ll T \ll T_c$ the thermopower decreases with temperature due to the temperature dependence of $\epsilon_R = g \ln(\frac{T}{T_c})$, at higher temperatures, $T \gg T_c$ the temperature dependence of $\epsilon_R$ becomes negligible and thermopower becomes small and it increases with temperature. The sign and the value of the thermopower are in agreement with the experiment [21], but its temperature dependence at high temperatures is not. This is not very surprising because this model does not describe the transport properties at high temperatures which are due to a new physics associated with the appearance of low energy modes. The disagreement between the predictions of the model (3) and data implies that these low energy modes are responsible for the temperature dependence of the thermopower at high temperatures.

V. CONCLUSION

The model (3) applies to the superconductivity in the underdoped cuprates where gap opens above $T_c$, we expect a more usual transition in the overdoped cuprates. The crossover from underdoped to overdoped occurs in the framework of the model (3) if $\epsilon$ and $V$ is increased with doping; at large $\epsilon$ the transition can be described in terms of the virtual pair formation and becomes very similar to a usual BCS picture. However, even in this regime the contribution of these virtual pairs to the $\gamma''$ coefficient in time dependent Ginzburg Landau equations can be much larger than the contribution coming from the density of states dependence near the Fermi surface and may result in a sign change of the Hall effect. In the optimally doped cuprates $n_0$ is still non-zero and we expect a large hydrodynamic contribution to the Hall effect and large positive thermopower.

In the optimally doped cuprates and in the underdoped ones above the temperature of the pseudogap formation one expects new physical effects due to the appearance of new low energy modes. These soft modes are responsible for the anomalous transport relaxation rates. Another probe of the effect of these modes in the optimally doped cuprates (where they are expected to exist down to the transition temperature) is the fluctuational conductivity which should no longer be given by universal form (11). It is important to determine experimentally whether fluctuational conductivity agrees with a phenomenological Fermi liquid picture with large relaxation rate which gives universal form (11), if the data do not fit the universal form (11) it means that this phenomenological Fermi liquid picture is not applicable at all even for the in-plane properties.

A model similar to (3) but in real space also describes a phase transition of the system of superconducting grains embedded in the normal matrix. In this case the mixed $b'cc$ term corresponds to the Andreev reflection at the NS boundary. In this system the Hall effect in the superconducting state is governed by the particle hole assymetry of the grains and may change sign close to $T_c$. 
In conclusion we have shown that the phenomenological description of the superconductivity which follows from the concept of preformed pairs coexisting with electrons on some patches of the Fermi surface agrees semi-quantitatively with available data on Hall conductivity in the fluctuation and flux flow regime and with the small value of the Ginzburg parameter for underdoped bilayered cuprates. However in order to describe the data on optimally doped bilayered cuprates or underdoped $LaSrCuO$ one needs to assume that the value of the chemical potential for these pairs is large so that preformed pairs exist only as virtual states. The important necessary ingredients of this model are (1) the assumption that the pairs have very little dispersion of their own and (2) their coupling to the electrons on the Fermi surface is weak. The hydrodynamic contribution to the Hall effect in this model is controlled by the pairs and has electron-like sign; it explains the Hall sign change observed experimentally.

It is not possible to test thoroughly the predictions of the model because experiments which give data on the Hall and longitudinal conductivity in the fluctuational and vortex flow regime obtained on the same sample are scarce. Such data on underdoped (spin gapped) materials do not exist at all. It would be very important to verify experimentally that hydrodynamic approach is still valid for the underdoped cuprates (i.e. that the parameters extracted from the fluctuational regime are the same as those extracted from vortex flow regime) and that the parameters needed for the hydrodynamic description are indeed in agreement with the picture of preformed pairs coexisting with fermions as we conclude here using a limited number of data.

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