Measuring thrust and predicting trajectory in model rocketry
Michael Courtney and Amy Courtney
Ballistics Testing Group, P.O. Box 24, West Point, NY 10996
Michael_Courtney@alum.mit.edu

Abstract: Methods are presented for measuring thrust using common force sensors and data acquisition to construct a dynamic force plate. A spreadsheet can be used to compute trajectory by integrating the equations of motion numerically. These techniques can be used in college physics courses, and have also been used with high school students concurrently enrolled in algebra 2.

I. Introduction
Model rocketry generates excitement and enthusiasm. However, many teachers aspire to impart quantitative understanding beyond the initial excitement of rocketry and qualitative understanding of Newton’s third law. This paper presents a relatively simple experimental method for measuring a rocket motor’s thrust curve and a theoretical approach for predicting the resulting trajectory that has been successfully implemented by high-school students concurrently enrolled in algebra 2. There are a number of excellent resources for hobbyists and teachers entering the field of rocketry.1,2,3,4,5

II. Experimental method
A number of educational and engineering instrument companies offer solutions for dynamic force measurements that can be successfully adapted for model rocketry. Here we use equipment that is readily available, can be configured with relative ease, and can be used with a software interface that is likely to be integrated with other science experiments so that the test system does not represent an entirely new learning curve. We also wanted a design that is easily calibrated and has sufficient dynamic range to yield accurate results with the smallest model rocket engines and handle thrusts up to 100 N.

The force plate employs three Vernier “dual-range force sensors” connected to a Vernier LabQuest. The three force sensors are attached to a bottom plate as shown in Figure 1, and a force plate rests on top of them and has a bolt to which the rocket engine is attached for static thrust testing. (The plate is held in place by gravity, there is no adhesive or connectors.) The total thrust is the sum of the three individual force readings. (The force plate is zeroed after the motor is attached.) Each force sensor has a selectable range of either 10N or 50N, so that if the plate was perfectly balanced, the full scale would be either 30N or 150N minus the static load, depending on the sensor setting. However, since the plate is not perfectly balanced, the three force readings are not equal, and the full scale ranges are closer to 20N and 100N for the 10N and 50N sensor settings, respectively. This system design is capable of measuring thrust curves for the full range (1/4A to E) of commonly available hobby rocket engines, as well as many experimental rocket motor designs.

The LabQuest can be configured for a variety of sample rates. The Logger Pro software both handles calibration and allows the three individual forces to be added and graphed as a total force, as well as reporting the total impulse (integrated area under the force curve). The time delay can also be subtracted from the time base to set the rocket ignition to t = 0 s.

Figure 2 shows the measured thrust curve for an Estes A10-PT model rocket engine. The shape of the thrust curve compares favorably with the published curve from the manufacturer.6 However, our experimental curve has a peak of 11.5 N and a total impulse of 1.991 Ns, compared with the...
manufacturer’s claims of a peak thrust of 13.0 N and a total impulse of 2.5 Ns. The total impulse (area under total thrust curve) is 20% lower than the manufacturer’s claim. The bulk of this discrepancy is likely due to the engine containing 19% less (3.08 g) than the specified quantity (3.78 g) of propellant, since our measured specific impulse (impulse divided by propellant weight) is 64.64 s, which is only 4% lower than the manufacturer’s specification of 67.49 s for specific impulse. (Measurements showing the A10-PT being below specification have been consistently repeated on different days using independent calibrations.)

\[ a = \frac{F_{\text{thrust}} - F_{\text{weight}} - F_{\text{drag}} - V \frac{dm}{dt}}{m} \]

where \( V \) is the instantaneous velocity, and \( m \) is the mass. Setting up the spreadsheet requires estimating the mass as a function of time. A linear interpolation between the initial and final masses can be used. Since the changing mass term and weight are much smaller than the thrust, and also smaller than the drag, there is only a small error from the linear interpolation.

Figure 3 shows the measured thrust curve for an experimental motor with a sugar-based propellant containing a 65/35/2 blend of KNO\(_3\), C\(_6\)H\(_{12}\)O\(_6\), and Fe\(_2\)O\(_3\). The theoretical specific impulse of this blend can be much higher (above 150 s)\(^7\) but this motor’s operating pressure is relatively low because of the propellant’s slow burn rate and the motor’s relatively large nozzle size. Filled to capacity (10 g) with a sucrose-based propellant, we believe this reloadable Maglite-based motor can have a specific impulse close to 100 Ns.

III. Predicting Trajectory
Since the mass of the rocket is changing during the burn phase of flight, the acceleration is given by

The drag force is \( F_{\text{drag}} = -\frac{1}{2} C_d \cdot A \cdot \rho \cdot V^2 \), where \( C_d \) is the drag coefficient (unitless), \( A \) is the frontal area of the rocket (m\(^2\)), \( \rho \) is the density of air (kg/m\(^3\)), and \( V \) is the rocket velocity (m/s). Care is required with the signs (adding or subtracting) in the spreadsheet. The weight is always subtracted, but the drag is always in the opposite direction of motion. The changing mass term enters the equation with a negative sign, but effectively increases the acceleration (a positive pseudo-force) because the change in mass is negative.

Each row of the spreadsheet corresponds an instant of time. Each column corresponds to a
The forces, velocity, and height curves are shown in Figures 4, 5, and 6 for the Estes A10-PT motor. The trajectory has three distinct phases: the thrust phase from 0.0 s to 0.8 s. This phase ends when the thrust falls to zero and the net force suddenly becomes negative. No longer dominated by the thrust, the net force is then dominated by air drag and rocket weight, which are both negative and rapidly robbing the rocket of velocity ($a = -25.88 \text{ m/s}^2$ just after burnout). After burnout, the rocket continues ascending during the coast phase until approximately $t = 5.0$ s when the rocket reaches its peak because drag and weight have reduced the velocity to zero. After reaching the peak, the rocket falls in the descent phase. Care is required when entering the drag formula so that it will be upward (change directions) after the peak. Rather than squaring the velocity, the $V^2$ term is expressed as $V|V|$ to ensure that the drag force properly changes sign when the rocket begins to descend. The rocket returns to the ground at approximately $t = 10.5$s.
IV. Discussion
A number of simplifying assumptions have been made to make the problem solvable and tractable with algebra 2 level skills. First, we assume a straight upward launch with no deviation from vertical. Second, we model the changing mass with a linear interpolation. Third, typical values are used for the coefficient of drag and the density of air. Knowing these values with confidence requires additional considerations.

However, even with these simplifying assumptions, the rocket trajectory computation here goes significantly beyond the typical introductory projectile problem where the force is constant, mass is constant, and drag is neglected. The approach presented here allows for modeling these effects with reasonable accuracy and is a significant step above the qualitative description that a rocket’s motion is merely the equal and opposite reaction to the expanding gases expelled from the nozzle.

The force plate demonstrated here can also be useful for evaluating experimental rocket motor designs including propellant grain shape, nozzle design, propellant chemistry, and grain production techniques. Each propellant has a theoretical specific impulse which can be computed from the theory of rocketry or using widely available software such as PROPEP and GUIPEP. Comparing the theoretical specific impulse with the experimentally determined value indicates the combustion pressure and the efficiency of the rocket motor design (grain, nozzle, etc.)

V. Appendix
The first ten rows of the spreadsheet predicting the trajectory of a 30 g rocket using an Estes A10-PT engine are shown in the table below. Using a step size of 0.02 s from t=0 s to t=1.0 s and a step size of 0.1 s from t=1.0 s to t = 11.0 s requires 151 lines total and yields the predictions shown in Figures 4, 5, and 6.

| Time (s) | Thrust (N) | Mass (g) | -V(dm/dt) (N) | Weight (N) | Drag (N) | Fnet (N) | a (m/s²) | V (m/s) | height (m) | Impulse (Ns) |
|----------|------------|----------|---------------|------------|----------|----------|----------|---------|------------|--------------|
| 0.0000   | 0.0000     | 30.0000  | 0.0000        | 0.2940     | 0.0000   | 0.0000   | 0.0000   | 0.0000  | 0.0000     | 0.0000        |
| 0.0200   | 0.0305     | 29.9249  | 0.0000        | 0.2933     | 0.0000   | 0.0000   | 0.0000   | 0.0000  | 0.0000     | 0.0000        |
| 0.0400   | 0.3964     | 29.8498  | 0.0000        | 0.2925     | 0.0000   | 0.0000   | 0.1039   | 3.4805  | 0.0696     | 0.0014        |
| 0.0600   | 1.0633     | 29.7746  | 0.0003        | 0.2918     | 0.0000   | 0.7717   | 25.9194  | 0.5880  | 0.0132     | 0.0213        |
| 0.0800   | 2.0076     | 29.6995  | 0.0022        | 0.2911     | 0.0000   | 1.7187   | 57.8706  | 1.7454  | 0.0481     | 0.0402        |
| 0.1000   | 3.8322     | 29.6244  | 0.0066        | 0.2903     | -0.0004 | 3.5480   | 119.7676 | 4.1408  | 0.1309     | 0.0766        |
| 0.1200   | 7.7592     | 29.5493  | 0.0156        | 0.2896     | -0.0022 | 7.4830   | 253.2370 | 9.2055  | 0.3150     | 0.1552        |
| 0.1400   | 9.3685     | 29.4742  | 0.0346        | 0.2888     | -0.0111 | 9.1032   | 308.8531 | 15.3826 | 0.6226     | 0.1874        |
| 0.1600   | 10.7396    | 29.3990  | 0.0578        | 0.2881     | -0.0309 | 10.4783  | 356.4158 | 22.5109 | 1.0729     | 0.2148        |
| 0.1800   | 11.5009    | 29.3239  | 0.0846        | 0.2874     | -0.0662 | 11.2318  | 383.0250 | 30.1714 | 1.6763     | 0.2300        |
| 0.2000   | 10.2216    | 29.2488  | 0.1133        | 0.2866     | -0.1190 | 9.9293   | 339.4776 | 36.9609 | 2.4155     | 0.2044        |

VI. References

1. http://www.jacobsrocketry.com/
2. http://www.nakka-rocketry.net/
3. http://www.jamesyawn.net/
4. Conkling JA, Chemistry of Pyrotechnics, Basic Principles and Theory, CRC Press (1985).
5. Jacobs G, Home Built Model Rocket Engines, Product Engineering and Development Company (1979).
6. 2007 Estes Catalog, Estes Industries, Penrose, CA. p 39-40, (2007).
7. http://www.nakka-rocketry.net/dexchem.html
8. http://www.nakka-rocketry.net/articles/nakka_theory_pages.pdf
9. http://www.nakka-rocketry.net/th_prope.html