Highlights

- Bézier extraction based IGA approach is successfully implemented for the nonlinear static and dynamic analyses of the FG plate reinforced by GPLs and integrated with piezoelectric layers.

- The combination of two porosity distribution types and three GPL dispersion patterns along the thickness direction of the FG plate is presented.

- The influences of the porosity coefficients, weight fractions of GPLs and the external electrical voltage are investigated.

- A constant displacement and velocity feedback control approaches are adopted to active control the responses of the plate structures with piezoelectric sensors and actuators.

- Numerical results demonstrate the efficiency and reliability of the proposed approach.
Analysis and control of geometrically nonlinear responses of piezoelectric FG porous plates with graphene platelets reinforcement using Bézier extraction

Nam V. Nguyen\textsuperscript{a}, Lieu B. Nguyen\textsuperscript{b}, Jaehong Lee\textsuperscript{c}, H. Nguyen-Xuan\textsuperscript{d,\textast}}

\textsuperscript{a}Faculty of Mechanical Technology, Industrial University of Ho Chi Minh City, Ho Chi Minh City, Vietnam
\textsuperscript{b}Faculty of Civil Engineering, University of Technology and Education Ho Chi Minh City, Vietnam
\textsuperscript{c}Department of Architectural Engineering, Sejong University, 98 Gunja-dong, Gwangjin-gu, Seoul 143-747, South Korea
\textsuperscript{d}CIRTech Institute, Ho Chi Minh City University of Technology (HUTECH), Ho Chi Minh City, Vietnam

Abstract

In this study, we propose an effective numerical approach to analyse and control geometrically nonlinear responses for the functionally graded (FG) porous plates reinforced by graphene platelets (GPLs) integrated with piezoelectric layers. The basis idea is to use isogeometric analysis (IGA) based on the Bézier extraction and the \(C^0\)-type higher-order shear deformation theory (\(C^0\)-HSDT). By applying the Bézier extraction, the original Non-Uniform Rational B-Spline (NURBS) control meshes can be transformed into the Bézier elements which allow us to inherit the standard numerical procedure like the finite element method (FEM). The mechanical displacement field is approximated based on the \(C^0\)-HSDT whilst the electric potential is assumed to be a linear function through the thickness of each piezoelectric sublayer. The FG plate contains the internal pores and GPLs dispersed in the metal matrix either uniformly or non-uniformly according to various different patterns along the thickness of plate. In addition, to control dynamic responses, two piezoelectric layers are perfectly bonded on the top and bottom surfaces of the FG plate. The geometrically nonlinear equations are solved by the Newton-Raphson iterative procedure and the Newmark’s time integration scheme. The influences of the porosity coefficients, weight fractions of GPLs as well as the external electrical voltage on the geometrically nonlinear behaviours of the plates with different porosity distributions and GPL dispersion patterns are evidently investigated through numerical examples. Then, a constant displacement and velocity feedback control approaches are adopted to active control the geometrically nonlinear static as well as the dynamic responses of the FG porous plates, where the effect of the structural damping is considered, based on a closed-loop control with piezoelectric sensors and actuators.

Keywords: Piezoelectric materials, FG porous plate, Graphene platelets reinforcement, Bézier extraction, Nonlinear dynamic, Active control

Corresponding author

Email address: ngx.hung@hutech.edu.vn (H. Nguyen-Xuan)

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1. Introduction

Nowadays, the demand for high performance structures with superior mechanical properties and chemical stability in engineering applications has been significantly increased. With these cellular structures, the porous materials whose the excellent properties such as lightweight, excellent energy absorption, heat resistance has been extensively employed in various fields of engineering including aerospace, automotive, biomedical and other areas [1–5]. However, the existence of internal pores leads to significant reduction in the structural stiffness [6]. In order to overcome this shortcoming, the reinforcement with carbonaceous nanofillers such as carbon nanotubes (CNTs) [7, 8] and graphene platelets (GPLs) [9, 10] into the porous materials is an excellent and practical choice to strengthen their mechanical properties. More importantly, this reinforcement aims also to maintain their potential for lightweight structures [11, 12]. In comparison with CNTs, GPLs have demonstrated great potentials to become a good candidate for reinforcement [13, 14] since GPLs have superior mechanical properties, a lower manufacturing cost, a larger specific surface area and two-dimensional geometry. In order to increase the performance of structure, the functionally graded (FG) porous structures reinforced by GPLs have been proposed in the literature to obtain the desired mechanical properties by modifying the sizes, the density of the internal pores in different directions as well as the dispersion patterns of GPLs [15–17]. In terms of numerical analysis, large number of investigations have been conducted to study the influences of the internal pores and GPLs on the behaviours of structures under various different conditions. Kitipornchai et al. [18] and Chen et al. [19] examined the free vibration, elastic buckling and the nonlinear free vibration, postbuckling behaviours of the FG porous beams reinforced with GPLs, respectively based on the Timoshenko’s beam theory and Ritz method. Yang et al. [20] utilized the first-order shear deformation plate theory (FSDT) and Chebyshev-Ritz method to study the uniaxial, biaxial, shear buckling and free vibration of the FG porous plates reinforced with GPLs uniformly or non-uniformly distributed in the metal matrix. Based on the isogeometric analysis (IGA), Li et al. [21] analysed the static, free vibration and buckling of the FG porous plates reinforced by GPLs using both first- and third-order shear deformation plate theories. Based on combination of the Galerkin method and the fourth-order Runge-Kutta approach, Li et al. [22] studied the nonlinear vibration and dynamic buckling of the sandwich FG porous plate reinforced by GPL resting on Winkler-Pasternak elastic foundation.

On the other hand, the piezoelectric materials have also been extensively applied to build up advanced smart structures for modern industrial products. One of the excellent and essential features of these materials is the ability of transformation between the electrical and mechanical energy which is known as the piezoelectric effect and the converse phenomenon [23]. Regarding the analysis for the plate structures integrated with piezoelectric layers, a lot of studies have been conducted to predict their behaviours in the literature [24–29]. In addition, the piezoelectric FG carbon nanotubes reinforced composite plates (FG-CNTRC) also attracted remarkable attention of researchers. Alibeigloo [30], [31] investigated the static and the free vibration analyses of FG-CNTRC plate as well as cylindrical panel embedded in thin piezoelectric layers using the three-dimensional theory of elasticity. Using FSDT and von Kármán strain assumptions, Raﬁee et al. [32] investigated the nonlinear parametric instability of initially imperfect the piezoelectric
FG-CNTRC plates under a combination of the electrical and thermal loadings. Then, Sharma et al. [33] investigated the active vibration control of FG-CNTRC rectangular plates with piezoelectric sensor and actuator layers using FEM based on FSDT. Selim et al. [34] studied the free vibration behaviour and active vibration control of FG-CNTRC plates with piezoelectric layers using element-free IMLS-Ritz model based on Reddy’s higher-order shear deformation theory. Nguyen-Quang et al. [35] studied the dynamic response of laminated CNTRC plates integrated with piezoelectric layers using IGA and HSDT. Recently, Malekzade et al. [36] employed the transformed differential quadrature method for the free vibration analysis of FG eccentric annular plates reinforced with GPLs and integrated piezoelectric layers.

It is known that the different basis functions are applied for approximation of the geometries and solutions in the framework of traditional FEM which leads to errors in the computational process. To improve the accuracy of solutions as well as reduce computational costs, the IGA [37] which employs non-uniform rational B-splines (NURBS) basis as shape functions had been discovered. The main idea of IGA is fulfilled by using the same basis functions to describe the geometry model and to approximate the solution field. The IGA has successfully been applied to various fields of engineering and science. In comparison with the standard FEM, the NURBS based IGA provides better accuracy and reliability for various engineering problems, especially for ones with complex geometries which are reported in the literature. The basic and review of IGA are presented in the established literature [38, 39]. Nevertheless, the implementation of NURBS based IGA approach is not often easy as their basis functions are not confined to a unique element but span over the entire domain instead. To overcome these obstacles, Borden et al. [40] proposed the Bézier extraction which represents the NURBS basis function in the form of Bernstein polynomials basis defined over \( C^0 \) continuous isogeometric Bézier element. By incorporating Bernstein polynomials which are similar to the Lagrangian basis functions as basis function in Bézier extraction, the implementation of IGA becomes analogous to the traditional FEM. As a result, the IGA approach can easily be embedded in most existing FEM codes while its advantages are still kept naturally.

In the context of plate theories, there is a great deal of theories that have been introduced and developed to estimate the responses of plate structures under different conditions. While CPT or Kirchhoff-Love shows its drawbacks in the analysis of thick plates, FSDT which is capable of both thin and thick plates requires an appropriate shear correction factor. To overcome these shortcomings, several higher-order plate theories (HSDTs) have been proposed in the literature [41–45]. Nevertheless, these HSDTs require the \( C^1 \)-continuity of the generalized displacement field which leads to the second-order derivatives of deflection in the stiffness formulation. Therefore, several \( C^0 \)-continuous elements were proposed [46–48].

As previously mentioned, most of the studies mainly focused on studying the plates integrated with piezoelectric layers which address only the core layer composed of FGM or FG-CNTRC. Furthermore, the geometrically nonlinear static and dynamic analyses of the piezoelectric FG plates under various loading types are still somewhat limited. In this study, in order to fill the existing gap in the literature, the geometrically nonlinear static and transient responses of piezoelectric
FG plates which have the core layer composed of FG porous materials reinforced by GPLs using Bézier extraction via IGA and $C^0$-type HSDT. More importantly, the active control of the geometrically nonlinear static and dynamic responses of the FG porous plates with the effect of the structural damping based on a closed-loop control with piezoelectric sensors and actuators is investigated. By using the Bézier extraction, the IGA preserves the element structure, which allows the IGA approach to integrate conveniently into the existing FEM routine. For material distribution, the core layer is constituted by the combination of two porosity distributions and three dispersion patterns of GPLs along the thickness plate, while the piezoelectric layer is perfectly bonded on the both top and bottom surfaces of plate. The Newmark’s integration scheme incorporation with Newton-Raphson iterative procedure is utilized for the geometrically nonlinear static and dynamic analyses. Then, some verification investigations are also conducted to prove the accuracy and stability of the present method. The influence of some specific parameters such as different porosity distributions, porosity volume fractions and GPL dispersion patterns, input voltages on the nonlinear behaviours of plate is addressed and discussed in detail through various numerical examples.

The outline of this paper is as follows. Section 2 provides the material models, the variational and approximate formulations of the piezoelectric FG porous plates reinforced by GPLs based on $C^0$-type HSDT. Meanwhile, Section 3 describes the active control algorithm. Section 4 presents the numerical examples for the geometrically nonlinear static and transient analyses as well as the active control of the piezoelectric FG porous plates reinforced with GPLs before some concluding remarks are given in Section 5.

2. Theoretical formulations

2.1. Material models of the FG porous plate reinforced with GPLs

We consider a FG plate model whose core layer is made of metal foams reinforced by GPLs with piezoelectric layers as depicted in Fig. 1. The length, width and total the thickness of piezoelectric FG porous plate are defined as $a$, $b$ and $h = h_c + 2h_p$, respectively, in which $h_c$ and $h_p$ are the thicknesses of the porous core and the piezoelectric layers, respectively. The porous core layer of plate is constituted by combining of two different porosity distribution types and three GPL dispersion patterns along the thickness direction of plates which are depicted in Fig. 2, respectively. The material properties including the Young’s modulus, shear modulus and mass density through the thickness of the porous core layer corresponding to two porosity distribution types can be expressed as

$$
\begin{align*}
E(z) &= E_1 [1 - e_0 \lambda(z)], \\
G(z) &= E(z) / [2 (1 + \nu(z))], \\
\rho(z) &= \rho_1 [1 - e_m \lambda(z)],
\end{align*}
$$

where

$$
\lambda(z) = \begin{cases} 
\cos(\pi z/h_c), & \text{Porosity distribution 1} \\
\cos(\pi z/(2h_c + \pi/4)), & \text{Porosity distribution 2}
\end{cases}
$$

in which $E_1$ and $\rho_1$ denote the maximum values of Young’s modulus and mass density in the thickness direction of the porous core layer, respectively. Meanwhile, the coefficient of porosity

\[5\]
$e_0$ is determined by
\[ e_0 = 1 - E_2'/E_1'. \]  
(3)
where $E_1'$ and $E_2'$ stand for the maximum and minimum values of Young’s modulus for the porous core layer without GPLs, as shown in Fig. [2]. Based on Gaussian Random Field (GRF) scheme [49], the mechanical properties of closed-cell cellular solids can be given as
\[ \frac{E(z)}{E_1} = \left( \frac{\rho(z)/\rho_1 + 0.121}{1.121} \right)^{2.3} \text{ for } \left( 0.15 < \frac{\rho(z)}{\rho_1} < 1 \right). \]  
(4)
Then, the mass density coefficient $e_m$ in Eq. [1] can be determined as
\[ e_m = 1.121 \left( 1 - \frac{2^\sqrt{1 - e_0 \lambda(z)}}{1 - e_0 \lambda(z)} \right). \]  
(5)
Also according to the closed-cell GRF scheme [50], Poisson’s ratio $\nu(z)$ is determined by
\[ \nu(z) = 0.221 p' + \nu_1 \left( 0.342 p'^2 - 1.21 p' + 1 \right), \]  
(6)
where $\nu_1$ represents the Poisson’s ratio of metal without internal pores with $p'$ is given as
\[ p' = 1.121 \left( 1 - \frac{2^\sqrt{1 - e_0 \lambda(z)}}{1 - e_0 \lambda(z)} \right). \]  
(7)
The volume fraction of GPLs varies along the thickness direction of plate for three dispersion patterns which are illustrated in Fig. [2] can be given as
\[ V_{GPL} = \begin{cases} 
S_{i1} [1 - \cos (\pi z/h_c)], & \text{Pattern A} \\
S_{i2} [1 - \cos (\pi z/2h_c + \pi/4)], & \text{Pattern B} \\
S_{i3}, & \text{Pattern C} 
\end{cases} \]  
(8)
where $S_{i1}, S_{i2}$ and $S_{i3}$ are the maximum values of GPL volume fraction, in which $i = 1, 2$ correspond to two porosity distributions. The weight fraction of GPLs is related to its volume content which are given as follows
\[ \frac{\Lambda_{GPL} \rho_m}{\Lambda_{GPL} \rho_m + \rho_{GPL} - \Lambda_{GPL} \rho_{GPL}} \times \int_{-h_c/2}^{h_c/2} [1 - e_m \lambda(z)]dz = \int_{-h_c/2}^{h_c/2} V_{GPL} [1 - e_m \lambda(z)]dz. \]  
(9)
The effective Young’s modulus of porous core layer reinforced with GPLs without internal pores is determined by the Halpin-Tsai micromechanics model [51, 52] as
\[ E_1 = \frac{3}{8} \left( \frac{1 + \xi_L \eta_L V_{GPL}}{1 - \eta_L V_{GPL}} \right) E_m + \frac{5}{8} \left( \frac{1 + \xi_W \eta_W V_{GPL}}{1 - \eta_W V_{GPL}} \right) E_m, \]  
(10)
in which
\[ \xi_L = \frac{2 \xi_{GPL}}{\xi_{GPL}}, \quad \xi_W = \frac{2 \xi_{GPL}}{\xi_{GPL}}, \quad \eta_L = \frac{(E_{GPL}/E_m) - 1}{(E_{GPL}/E_m) + \xi_L}, \quad \eta_W = \frac{(E_{GPL}/E_m) - 1}{(E_{GPL}/E_m) + \xi_W}, \]  
(11)
where \( w_{GPL} \), \( l_{GPL} \) and \( t_{GPL} \) are dimensions of GPLs including the average width, length and thickness, respectively; Meanwhile, \( E_{GPL} \) and \( E_m \) are the Young’s modulus of GPLs and metal matrix, respectively. Finally, \( \rho_1 \) and \( \nu_1 \) denote the mass density and Poisson’s ratio of the GPLs reinforced porous metal matrix can be determined based on the rule of mixture [53]

\[
\rho_1 = \rho_{GPL} V_{GPL} + \rho_m V_m, \tag{12}
\]

\[
\nu_1 = \nu_{GPL} V_{GPL} + \nu_m V_m, \tag{13}
\]

where the mechanical properties for GPLs and metal matrix are denoted with subscript symbols \( GPL \) and \( m \), respectively. Meanwhile, \( V_{GPL} \) and \( V_m = 1 - V_{GPL} \) denote the volume fraction of GPLs and metal matrix, respectively.

2.2. Weak form of the governing equations

The governing equations of motion for the piezoelectric FG plate can be obtained by applying the Hamilton’s variational principle [54] which can be expressed as follows

\[
\delta \int_{t_1}^{t_2} L \, dt = 0, \tag{14}
\]

in which \( t_1 \) and \( t_2 \) denote the starting and finish time values, respectively. Meanwhile, \( L \) is the general energy functional which contains the summation of the kinetic energy, the strain energy, the dielectric energy and the external work is expressed as follows

\[
L = \frac{1}{2} \int_{\Omega} \left( \rho \ddot{u}^T \dot{u} - \sigma^T \varepsilon + D^T E \right) \, d\Omega + \int_{\Gamma_s} \dot{u}^T f_s \, d\Gamma_s - \int_{\Gamma_\phi} \phi q_s \, d\Gamma_\phi + \sum \dot{u}^T F_p - \sum \phi Q_p, \tag{15}
\]

where \( \rho \) denotes the mass density; \( \dot{u} \) and \( \ddot{u} \) represent the mechanical displacement and velocity field vectors; \( \phi \) represents the electric potential; Meanwhile, \( f_s \) and \( F_p \) denote the external mechanical surface and concentrated load vectors; \( q_s \) and \( Q_p \) indicate the external surface and point charges, respectively; \( \Gamma_s \) and \( \Gamma_\phi \) denote the external mechanical and the electrical loading surface, respectively.

Then, the variational form for the equations of motion can be expressed as follows

\[
\int_{t_1}^{t_2} \int_{\Omega} \left( \rho \ddot{u}^T \dot{u} - \sigma^T \varepsilon + D^T E \right) \, d\Omega \, dt + \int_{t_1}^{t_2} \int_{\Gamma_s} \dot{u}^T f_s \, d\Gamma_s \, dt - \int_{t_1}^{t_2} \int_{\Gamma_\phi} \phi q_s \, d\Gamma_\phi \, dt + \sum \int_{t_1}^{t_2} \dot{u}^T F_p \, dt - \sum \phi Q_p \, dt = 0 \tag{16}
\]

In this study, the linear constitutive relationships of the FG porous plate reinforced by GPLs with the piezoelectric layers can be presented as follows [55]

\[
\begin{bmatrix} \sigma \\ D \end{bmatrix} = \begin{bmatrix} c & -e^T \\ e & g \end{bmatrix} \begin{bmatrix} \varepsilon \\ E \end{bmatrix}, \tag{17}
\]
in which \( \bar{\varepsilon} = [\varepsilon, \gamma]^T \) and \( \sigma \) represent the strain and stress vectors, respectively; \( \mathbf{D} \) denotes the dielectric displacement vector and \( \mathbf{e} \) represents the stress piezoelectric constant matrix; \( \mathbf{g} \) indicates the dielectric constant matrix; Meanwhile, the electric field vector \( \mathbf{E} \) which is calculated following the electric potential field \( \phi \) can be defined as \[56\]
\[
\mathbf{E} = -\text{grad}\phi = -\nabla\phi. \tag{18}
\]

And the material constant matrix \( c \) is defined as
\[
c = \begin{bmatrix}
A & B & L & 0 & 0 \\
B & G & F & 0 & 0 \\
L & F & H & 0 & 0 \\
0 & 0 & 0 & A^* & B^* \\
0 & 0 & 0 & B^* & D^*
\end{bmatrix}, \tag{19}
\]
in which \( (A, B, G, L, F, H) = \int_{-h_c}^{h_c} \frac{1}{2} (1, z, z^2, f(z), zf(z), f^2(z)) \mathbf{Q}^b_{ij} \, dz \), \( (A^*, B^*, D^*) = \int_{-h_c}^{h_c} (1, f'(z), f'^2(z)) \mathbf{Q}^s_{ij} \, dz \), \( \mathbf{Q}^b = \frac{E_e}{1 - \nu^2_e} \begin{bmatrix} 1 & \nu_e & 0 \\ \nu_e & 1 & 0 \\ 0 & 0 & 1 - \nu_e \end{bmatrix} \), \( \mathbf{Q}^s = \frac{E_e}{2(1 + \nu_e)} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \). \( \tag{21} \)

where \( E_e \) and \( \nu_e \) are the effective Young’s modulus and Poisson’s ratio, respectively.

2.3. Approximation of the mechanical displacement field

2.3.1. \( C^0 \)-type higher-order shear deformation theory

Considering a plate carrying a domain \( V = \Omega \times (-h_c, h_c) \), in which \( \Omega \in \mathbb{R}^2 \). Based on the higher-order shear deformation theory \([57]\), the displacement field at an arbitrary point in the plate can be presented as follows
\[
\mathbf{u}(x, y, z) = \mathbf{u}^0(x, y) + zu^1(x, y) + f(z)\mathbf{u}^2(x, y), \tag{22}
\]
where \( \mathbf{u} = \begin{bmatrix} u \\ v \\ w \end{bmatrix}, \mathbf{u}^0 = \begin{bmatrix} u_0 \\ v_0 \\ w_0 \end{bmatrix}, \mathbf{u}^1 = -\begin{bmatrix} w_{0,x} \\ w_{0,y} \\ 0 \end{bmatrix}, \mathbf{u}^2 = \begin{bmatrix} \theta_x \\ \theta_y \\ 0 \end{bmatrix}, \tag{23} \)
in which \( u_0, v_0, w_0, \theta_x \) and \( \theta_y \) are the displacement components in the \( x, y, z \) directions and the rotation components in the \( y- \) and the \( x- \) axes, respectively. Meanwhile, the subscript symbols \( x \) and \( y \) denote the derivatives of any function with respect to \( x \) and \( y \) directions, respectively; and \( f(z) \) is a function of the \( z \)-coordinate which is defined to describe the shear strains and stresses along the thickness of plate, as is listed in \([58]\). In this work, the famous third-order function proposed by Reddy is utilized as \( f(z) = z - \frac{z^3}{3h^2} \). \([59]\).
In order to avoid the high order derivations in approximate formulations and conveniently impose the boundary conditions, additional assumptions are formulated as follows

\[ w_{0,x} = \beta_x, \quad w_{0,y} = \beta_y. \tag{24} \]

Then, substituting Eq. 24 into Eq. 23 one obtains

\[
\begin{align*}
\mathbf{u}^0 &= \begin{cases} 
  u_0 \\
  v_0 \\
  w_0 
\end{cases}, \\
\mathbf{u}^1 &= \begin{cases} 
  \beta_x \\
  \beta_y \\
  0 
\end{cases}, \\
\mathbf{u}^2 &= \begin{cases} 
  \theta_x \\
  \theta_y \\
  0 
\end{cases}.
\end{align*}
\tag{25} \]

It can be seen that the compatible strain fields which are presented in Eq. 25 only require the \( C_0 \)-continuity of the generalized displacements. Therefore, this theory is called as the \( C_0 \)-type higher-order shear deformation theory (\( C_0 \)-HSDT).

The Green strain vector of a bending plate can be expressed in compact form as follows

\[
\varepsilon_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} + \frac{\partial u_k}{\partial x_i} \frac{\partial u_k}{\partial x_j} \right). \tag{26} \]

Employing the von Kármán assumptions, the strain-displacement relations can be rewritten as

\[
\begin{align*}
\varepsilon &= \begin{bmatrix} \varepsilon_{xx}, \varepsilon_{yy}, \gamma_{xy} \end{bmatrix}^T = \varepsilon_0 + z\kappa_1 + f(z)\kappa_2, \\
\gamma &= \begin{bmatrix} \gamma_{xz}, \gamma_{yz} \end{bmatrix}^T = \varepsilon_s + f'(z)\kappa_s,
\end{align*} \tag{27} \]

where

\[
\begin{align*}
\varepsilon_0 &= \begin{bmatrix} u_{0,x} \\
  v_{0,y} \\
  u_{0,y} + v_{0,x} 
\end{bmatrix}, \\
\kappa_1 &= \begin{bmatrix} \beta_x \\
  \beta_y \\
  \beta_x + \beta_y \n\end{bmatrix}, \\
\kappa_2 &= \begin{bmatrix} \theta_{x,x} \\
  \theta_{y,y} \\
  \theta_{x,y} + \theta_{y,x} \n\end{bmatrix}, \\
\varepsilon^0 &= \begin{bmatrix} w_{0,x} - \beta_x \\
  w_{0,y} - \beta_y \n\end{bmatrix}, \\
\kappa_s &= \begin{bmatrix} \theta_x \\
  \theta_y \n\end{bmatrix},
\end{align*} \tag{28} \]

where the nonlinear strain component is expressed as follows

\[
\varepsilon_{0}^{NL} = \frac{1}{2} \begin{bmatrix} w_x & 0 \\
  0 & w_y \\
  w_y & w_x \n\end{bmatrix} \begin{bmatrix} w_x \\
  w_y \n\end{bmatrix} = \frac{1}{2} \Theta \Lambda. \tag{29} \]

2.3.2. Isogeometric analysis based on Bézier extraction of NURBS

2.3.2.1. B-spline and NURBS basis functions. In one dimensional (1D) space, the B-spline basis functions can be expressed by a set of knot vector in the parametric space which is defined by \( \Xi = \{ \xi_1, \xi_2, ..., \xi_{n+p+1} \} \), where \( i = 1, \ldots, n+p \) denotes the knot index. Meanwhile \( n \) and \( p \) are
the number of basis functions and the polynomial order, respectively. For a given knot vector \( \Xi \), the B-spline basis functions are defined according to recursive form

\[
N_{i,0} (\xi) = \begin{cases} 
1, & \text{if } \xi_i \leq \xi \leq \xi_{i+1}, \text{ for } p = 0, \\
0, & \text{otherwise}
\end{cases},
\]

\[
N_{i,p} (\xi) = \frac{\xi - \xi_i}{\xi_{i+p} - \xi_i} N_{i,p-1} (\xi) + \frac{\xi_{i+p+1} - \xi}{\xi_{i+1} - \xi_{i+p}} N_{i+1,p-1} (\xi), \text{ for } p > 0.
\]

Then, B-spline curves can be determined by taking a linear association of B-spline basis functions and the control points \( P_i (i = 1, 2, ..., n) \) as

\[
T (\xi) = \sum_{i=1}^{n} P_i N_{i,p} (\xi).
\]

In two dimensional (2D) space, the B-splines basis functions can be also obtained by taking a tensor product of two basis functions in 1D space. Similarly, the B-splines surfaces are also expressed by

\[
S (\xi, \eta) = \sum_{i=1}^{n} \sum_{j=1}^{m} P_{i,j} N_{i,p} (\xi) M_{j,q} (\eta) = P^T N (\xi, \eta),
\]

in which \( N_{i,j} \) and \( M_{j,q} \) represent the basis functions with orders \( p \) and \( q \) in the \( \xi \) and \( \eta \) directions corresponding with the knot vectors \( \Xi = \{ \xi_1, \xi_2, ..., \xi_{n+p+1} \} \) and \( H = \{ \eta_1, \eta_2, ..., \eta_{m+q+1} \} \), respectively.

Due to B-splines basis functions are limited the ability to exactly description some conic shapes such as circles, cylinders, ellipsoids and spheres, the NURBS have been introduced based on the B-spline and a set of weights. Accordingly, the NURBS basis functions can be expressed as

\[
R_{i,j} (\xi, \eta) = \frac{N_{i,p} (\xi) M_{j,q} (\eta) w_{i,j}}{\sum_{i=1}^{n} \sum_{j=1}^{m} N_{i,p} (\xi) M_{j,q} (\eta) w_{i,j}},
\]

where \( w_{i,j} \) represents the weight values. Then, the NURBS surfaces can be determined as follows

\[
S (\xi, \eta) = \sum_{i=1}^{n} \sum_{j=1}^{m} R_{i,j} (\xi, \eta) P_{i,j}.
\]

2.3.2.2. Bézier extraction of NURBS. The major purpose of Bézier extraction is to instead the NURBS basis functions by the \( C^0 \)-continuous Bernstein polynomial basis functions defined over Bézier elements which have the similar element structure with standard FEM. By using the Bernstein polynomial as the basis function in Bézier extraction, the IGA approach is straightforwardly performed as well as can be integrated in most available FEM structures. It is well known that
the B-spline basis function of \( p \)th order has \( C^{n-k} \) continuity across each element, in which \( k \) represents the multiplicity of knots in the knot value. Therefore, the \( C^0 \)-continuity can be obtained by inserting the new knots into the B-spline basis function until \( k = p \). Accordingly, a new knot \( \bar{\xi} \in [\xi_k, \xi_{k+1}] \) with \( (k > p) \) is inserted into the original knot vector \( \Xi = \{\xi_1, \xi_2, ..., \xi_{n+p+1}\} \). As a result, a new set of control points are obtained and expressed as follows \[37\]

\[
P_i = \begin{cases} P_1, & i = 1, \\ \alpha_i P_i + (1-\alpha_i) P_{i-1} & 1 < i < n + 1, \\ P_n, & i = n + 1, \end{cases} \tag{36}
\]

where

\[
\alpha_i = \begin{cases} 1, & 1 \leq i \leq k - p, \\ \frac{\xi - \xi_i}{\xi_{i+p} - \xi}, & k - p + 1 \leq i \leq k, \\ 0, & i \geq k + 1, \end{cases} \tag{37}
\]

in which \( P_i \) and \( \bar{P}_i \) are the original and new control points, respectively.

Then, the Bézier extraction operator can be determined by using the new set of knots \( \{\bar{\xi}_1, \bar{\xi}_2, ..., \bar{\xi}_{n+1}\} \) as follows \[40, 60\]

\[
C^j = \begin{bmatrix} \alpha_1 & 1 - \alpha_2 & 0 & \cdots & 0 \\ 0 & \alpha_2 & 1 - \alpha_3 & 0 & \cdots & 0 \\ 0 & 0 & \alpha_3 & 1 - \alpha_4 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & 0 & \alpha_{n+j-1} & 1 - \alpha_{n+j} \end{bmatrix}. \tag{38}
\]

Applying the Bézier extraction operator \( C^j \), a new Bézier control points \( P^b \) associated with Bernstein polynomial basis can be determined as follows \[61\]

\[
P^b = C^T P, \tag{39}
\]

where the whole Bézier extraction operator \( C \) is defined as

\[
C = \prod_{j=1}^{n} C^j. \tag{40}
\]

It should be noted that the geometries will not change after inserting a new knot into the original knot vector. As a result, the B-spline surface is also obtained based on Bernstein polynomials and Bézier control points as

\[
S(\xi, \eta) = \sum_{i}^{n} \sum_{j}^{m} B_{i,j}(\xi, \eta) P^b_{i,j} = (P^b)^T B(\xi, \eta), \tag{41}
\]

in which the 2D Bernstein polynomials \( B(\xi, \eta) \) in terms of parametric coordinates \( \xi \) and \( \eta \) are defined recursively as

\[
B_{i,j,p}(\xi, \eta) = \frac{1}{4} (1 - \xi) (1 + \eta) B_{i,j-1,p-1}(\xi, \eta) + \frac{1}{4} (1 - \xi) (1 - \eta) B_{i,j,p-1}(\xi, \eta) + \frac{1}{4} (1 + \xi) (1 - \eta) B_{i-1,j,p-1}(\xi, \eta) + \frac{1}{4} (1 + \xi) (1 + \eta) B_{i-1,j-1,p-1}(\xi, \eta), \tag{42}
\]
where
\[ B_{1,1,0} (\xi, \eta) = 1, \quad B_{i,j,p} (\xi, \eta) = 0 \quad (i, j < 1 \text{ or } i, j > p + 1). \] (43)

From Eq. 33 and Eq. 41, yields the following relation
\[ (P^b)^T B (\xi, \eta) = P^T N (\xi, \eta). \] (44)

According to Eq. 39 for 2D, the B-spline basis functions in Eq. 44 can be rewritten based on Bernstein polynomials as follows
\[ N (\xi, \eta) = CB (\xi, \eta), \] (45)

Based on Eq. 45, the NURBS basis functions can be presented by Bernstein polynomials as follows
\[ R (\xi, \eta) = \frac{W}{W (\xi, \eta)} N (\xi, \eta) = \frac{W}{W (\xi, \eta)} CB (\xi, \eta), \] (46)

where \( W \) denotes the diagonal matrix of the local NURBS weights. Meanwhile, the weight functions \( W (\xi, \eta) \) are expressed with the Bernstein basis functions as follows
\[ W (\xi, \eta) = (C^T w)^T B (\xi, \eta) = (w^b)^T B (\xi, \eta), \] (47)

where \( w \) and \( w^b \) are the weights for the NURBS and Bézier, respectively. The relation of Bézier control points and NURBS ones is described by
\[ P^b = (W^b)^{-1} C^T WP. \] (48)

2.3.2.3. Bézier extraction of NURBS for FG porous plate formulations. Based on the Bézier extraction of NURBS, the mechanical displacement field \( u (\xi, \eta) \) of the FG porous plate can be approximated as follows
\[ u (\xi, \eta) = \sum_{A}^{m \times n} R_e^c (\xi, \eta) d_A, \] (49)

in which \( n \times m \) represents the number of basis functions. Meanwhile, \( R_e^c (\xi, \eta) \) denotes a NURBS basis function which is presented in Eq. 46. \( d_A = \{ u_{0A} v_{0A} w_{0A} \beta_{xA} \beta_{yA} \theta_{xA} \theta_{yA} \}^T \) is the vector of the nodal degrees of freedom associated with the control point A.

By substituting Eq. 49 into Eq. 28, the in-plane and shear strains can be expressed as
\[ [\varepsilon, \gamma]^T = \sum_{A=1}^{m \times n} \left( B_A^{L} + \frac{1}{2} B_A^{NL} \right) d_A, \] (50)
in which \( \mathbf{B}^L_A = [ \mathbf{B}^1_A \mathbf{B}^2_A \mathbf{B}^3_A \mathbf{B}^{s1}_A \mathbf{B}^{s2}_A]^T \), where

\[
\mathbf{B}^1 = \begin{bmatrix}
R_{A,x} & 0 & 0 & 0 & 0 & 0 \\
0 & R_{A,y} & 0 & 0 & 0 & 0 \\
R_{A,y} & R_{A,x} & 0 & 0 & 0 & 0
\end{bmatrix}, \quad \mathbf{B}^2 = -\begin{bmatrix}
0 & 0 & 0 & R_{A,x} & 0 & 0 \\
0 & 0 & 0 & 0 & R_{A,y} & 0 \\
0 & 0 & 0 & R_{A,y} & R_{A,x} & 0
\end{bmatrix},
\]

\[
\mathbf{B}^{s1} = \begin{bmatrix}
0 & 0 & -R_A & 0 & 0 & 0 \\
0 & 0 & R_{A,y} & 0 & -R_A & 0
\end{bmatrix}, \quad \mathbf{B}^{s2} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & R_A \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix},
\]

and \( \mathbf{B}^{NL}_A = \Theta \Lambda \), in which

\[
\Theta = \begin{bmatrix}
w_{A,x} & 0 \\
0 & w_{A,y} \\
w_{A,y} & w_{A,x}
\end{bmatrix}, \quad \Lambda = \begin{bmatrix}
0 & 0 & R_{A,x} & 0 & 0 & 0 \\
0 & 0 & R_{A,y} & 0 & 0 & 0
\end{bmatrix}.
\]

\[(52)\]

2.4. Approximation of the electric potential field

By discretizing the piezoelectric layer into finite sublayers along the thickness, the electric potential field on each layer is then approximated. Accordingly, in each sublayer, the electric potential variation is considered to be linear and is approximated through the thickness as follows

\[(53)\]

where \( \mathbf{R}^i_\phi \) is the shape function of the electric potential function which is determined in Eq. 46 with \( p = 1 \). Meanwhile, \( \phi^i = [ \phi^{i-1}, \phi^i] \) with \( i = 1, 2, \ldots, n_{sub} \) denotes the electric potentials at the top and bottom surfaces of the sublayer, where \( n_{sub} \) represents the number of piezoelectric sublayers.

In each sublayer element, the values of the electric potentials are estimated to equal at the same height according to \( z \) – direction [55]. Therefore, the electric potential field \( \mathbf{E} \) for each sublayer element which is presented in Eq. 18 can be expressed as follows

\[(54)\]

where

\[
\mathbf{B}_\phi = \left\{ 0 \ 0 \ \frac{1}{n_p} \right\}^T.
\]

13
Finally, the stress piezoelectric constant matrix $e$, the strain piezoelectric constant matrix $k$ and the dielectric constant matrix $g$ can be determined by \[62\]

$$e = \begin{bmatrix}
e_{11} & e_{12} & e_{13} & 0 \\
e_{21} & e_{22} & e_{23} & 0 \\
e_{31} & e_{32} & e_{33} & 0 \\
0 & 0 & 0 & 0
\end{bmatrix},
\quad k = \begin{bmatrix}
k_{11} & k_{12} & k_{13} & 0 \\
k_{21} & k_{22} & k_{23} & 0 \\
k_{31} & k_{32} & k_{33} & 0 \\
0 & 0 & 0 & 0
\end{bmatrix},
\quad g = \begin{bmatrix}
p_{11} & p_{12} & 0 \\
p_{21} & p_{22} & 0 \\
p_{31} & p_{32} & p_{33}
\end{bmatrix},$$

(56)

2.5. Governing equation of motion

By substituting Eqs. [49] and [54] into Eq. [16], the final form of the elementary governing equation can be obtained and expressed as follows \[62\]

$$\begin{bmatrix}
M_{uu} & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}\ddot{d} + \begin{bmatrix}
K_{uu} & K_{u\phi} \\
K_{\phi u} & -K_{\phi\phi}
\end{bmatrix}\begin{bmatrix}
d \\
\phi
\end{bmatrix} = \begin{bmatrix}
f \\
Q
\end{bmatrix},$$

(57)

where

$$K_{uu} = \int_{\Omega} (B^L + B^{NL})^T c(B^L + \frac{1}{2}B^{NL}) d\Omega, \quad K_{\phi\phi} = \int_{\Omega} B^T g B d\Omega,$$

(58)

$$K_{u\phi} = \int_{\Omega} (B^L)^T e B \phi d\Omega, \quad M_{uu} = \int_{\Omega} \bar{N}^T m \bar{N} d\Omega, \quad f = \int_{\Omega} \bar{q} \bar{N} d\Omega,$$

in which

$$\bar{e} = [e_m^T \ z e_m^T \ f(z) \ e_s^T \ f'(z) \ e_s^T],$$

$$\bar{N} = \begin{bmatrix} N^0 & N^1 & N^2 \end{bmatrix}^T,$$

(59)

where

$$e_m = \begin{bmatrix} 0 & 0 & 0 \\
0 & 0 & 0 \\
e_{31} & e_{32} & e_{33}
\end{bmatrix},
\quad e_s = \begin{bmatrix} 0 & e_{15} & 0 \\
e_{15} & 0 & 0 \\
0 & 0 & 0
\end{bmatrix},
\quad N^0 = \begin{bmatrix} R_A & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & R_A & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & R_A & 0 & 0 & 0 & 0 & 0
\end{bmatrix},
\quad N^1 = -\begin{bmatrix} 0 & 0 & 0 & R_A & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & R_A & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix},
\quad N^2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & R_A & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & R_A & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix},$$

(60)

and

$$\mathbf{m} = \begin{bmatrix} I_1 & I_2 & I_3 & I_4 \\
I_2 & I_3 & I_4 & I_5 \\
I_4 & I_5 & I_6 \end{bmatrix},$$

(61)

in which the mass inertia terms $I_i$ with $(i = 1 : 6)$ are given as

$$(I_1, I_2, I_3, I_4, I_5, I_6) = \int_{-h/2}^{h/2} \rho(z) \begin{bmatrix} 1, z, z^2, f(z), zf(z), f^2(z) \end{bmatrix} dz.$$

(62)
Since the electric field \( E \) exists only according to the \( z \) direction, \( K_{u\phi} \) in Eq. 58 can be rewritten as
\[
K_{u\phi} = \int_{\Omega} \left( (B^1)^{T} e_{m}^{T} B_{\phi} + z (B^2)^{T} e_{m}^{T} B_{\phi} + f(z) (B^3)^{T} e_{m}^{T} B_{\phi} \right) d\Omega.
\] (63)

Now, substituting the second equation into the first one of Eq. 57, one obtains
\[
M_{uu} \ddot{d} + \left( K_{uu} + K_{u\phi} K_{\phi u}^{-1} K_{\phi u} \right) d = F + K_{u\phi} K_{\phi u}^{-1} Q.
\] (64)

3. Active control analysis

In this section, a piezoelectric FG porous plate, as depicted in Fig. 3, is considered for the active control the static and dynamic responses of the FG plates. Whereas the bottom layer is a piezoelectric sensor labeled with the subscript \( s \), the top layer represents a piezoelectric actuator denoted with the subscript \( a \). The combination between the displacement feedback control [55], which helps the piezoelectric actuator to generate the charge, and the velocity feedback control [54] [63] [64], which can provide a velocity component based on an appropriate electronic circuit, is utilized in this study. Furthermore, a consistent method [64] [65] which can predict the dynamic responses of piezoelectric FG plate is also applied. Two constant gains \( G_d \) and \( G_v \) of the displacement and velocity feedback control, respectively, are adopted in order to couple the input actuator voltage vector \( \phi_a \) and the output sensor voltage vector \( \phi_s \) as follows [64]
\[
\phi_a = G_d \phi_s + G_v \dot{\phi}_s.
\] (65)

Assuming without any the external charge \( Q \), the generated potential from the piezoelectric sensor layer can be obtained from the second equation of Eq. 57
\[
\phi_s = [K_{\phi u}]_s [K_{u\phi}]_s \dot{d}_s,
\] (66)
and the sensor charge resulted due to the deformation is determined by
\[
Q_s = [K_{\phi u}]_s \dot{d}_s.
\] (67)
which can be understood that when the FG plate structures deform, the electric charges are generated and gathered in the sensor layer because of the piezoelectric effect. After that, these electric charges are amplified based on a closed loop control in order to convert into the voltage signal before being sent and applied to the actuator layer. Due to the converse piezoelectric effect, the strains and stresses of structures are formed which can be applied to actively control the dynamic response of the FG porous plate.

By substituting Eqs. 65 and 66 into the second equation in Eq. 57 one obtains
\[
Q_a = [K_{\phi u}]_a \dot{d}_a - G_d [K_{\phi \phi}]_a [K_{\phi u}]_s \dot{d}_s - G_v [K_{\phi \phi}]_a [K_{\phi u}]_s \dot{d}_s.
\] (68)
Then, substituting Eq. 68 into Eq. 64 yields

$$M\ddot{d} + C\dot{d} + K^*d = F,$$

(69)
in which

$$K^* = K_{uu} + G_d[K_{u\phi}]_a[K_{\phi\phi}^{-1}]_s[K_{\phi u}]_s,$$

(70)
and $C$ is the active damping matrix which is expressed as

$$C = G_v[K_{u\phi}]_a[K_{\phi\phi}^{-1}]_s[K_{\phi u}]_s.$$

(71)

Considering the effect of the structural damping, Eq. 69 can be rewritten as

$$M\ddot{d} + (C + C_R)\dot{d} + K^*d = F,$$

(72)
in which $C_R$ denotes the Rayleigh damping matrix which is defined based on a linear association between $M$ and $K_{uu}$ as follows

$$C_R = \alpha_RM + \beta_RK_{uu},$$

(73)
where $\alpha_R$ and $\beta_R$ are Rayleigh damping coefficients that can be defined from experiments. In this study, the procedure in order to define the Rayleigh damping coefficients was reported in [66].

4. Numerical examples

In this study, the Newton-Rapshon iterative procedure [67] is employed to obtain the solutions of the nonlinear problems. Accordingly, the iterations, where the solutions of current time step can be obtained based on the solutions of previous time step, are repeated until the solutions converge. For the geometrically nonlinear dynamic analysis of the FG plate under various dynamic loadings, which the equations of dynamic problem depend on both the time domain and unknown displacement vector, the Newmark’s integration scheme [68] is selected. In all numerical examples, the PZT-G1195N piezoelectric is employed and perfectly bonded on the top and bottom surfaces of the FG plate structure as well as ignored the adhesive layers.

4.1. Validation analysis

In this section, various numerical studies regarding the geometrically nonlinear static and dynamic analyses of the isotropic as well as the piezoelectric FG square plates are carried out in order to demonstrate the accuracy and stability of the present approach. Firstly, a fully clamped (CCCC) isotropic square plate is considered to show the validity of the present formulation for the geometrically nonlinear analysis. The plate is subjected to uniformly distributed load while the width-to-thickness ratio $(a/h)$ is taken equal to 100. The material properties of plate are $E = 3 \times 10^7$ psi and $\nu = 0.316$. In this example, the normalized central deflection and load parameter can be defined as $\overline{w} = w/h$ and $P = q_0a^4/(Eh^2)$, respectively. Table 1 presents the normalized central deflections of the isotropic square plate which are compared with those of the
Levy’s analytical solution [69], Urthaler and Reddy’s mixed FEM using FSDT [70] and Nguyen et al. based on IGA and refined plate theory (RPT) [71]. As can be observed that the proposed results are in good agreement with the existing analytical solution as well as other approximate results.

Next, in order to verify the accuracy of the proposed approach for the geometrically nonlinear transient analysis, a fully simply supported (SSSS) orthotropic square plate subjected to uniform load with $q_0 = 1.0$ MPa is conducted in this example. The material properties and the geometry of plate are considered as follows: Young’s modulus $E_1 = 525$ GPa, $E_2 = 21$ GPa, shear modulus $G_{12} = G_{23} = G_{13} = 10.5$ GPa, Poisson’s ratio $\nu = 0.25$, mass density $\rho = 800 \text{ kg/m}^3$, length of the plate $L = 0.25$ m and thickness $h = 5$ mm. Fig. 4 depicts the geometrically nonlinear transient response of the square plate subjected to uniform load. It can be seen that the present results are in an excellent agreement with those obtained from the finite strip method, which was reported by Chen et al. [72].

Last but not least, a cantilever piezoelectric FG square plate is exhaustively presented to demonstrate the accuracy and validity of the present method for the static analysis of the FG plates integrated with piezoelectric layers. The FG plate which is bonded by two piezoelectric layers on both the upper and the lower surfaces is made of aluminum oxide and Ti-6Al-4V materials whose material properties are given in Table 2. In this study, the rule of mixture [53] is utilized to describe the distribution of the ceramic and metal phases in core layer. The plate has a side length $a = b = 0.4$ m while the thickness of the FG core layer and each piezoelectric layer are $h_c = 5$ mm and $h_p = 0.1$ mm, respectively. The cantilever piezoelectric FG plate is subjected to simultaneously a uniformly distributed load with $q_0=100 \text{ N/m}^2$ and various input voltage values. The centerline linear deflections of the piezoelectric FG square plate are plotted in Fig. 5 while the tip node deflections are also listed in Table 3 with various material index $n$. The results which are generated from the proposed method are compared with those reported in [73] using a cell-based smoothed discrete shear gap method (CS-DSG3) based on FSDT. It can be observed that the results obtained by the present formulation generally agree well with the reference solutions.

In the next part, investigations into the geometrically nonlinear static and dynamic responses of the piezoelectric FG porous square plate reinforced by GPLs will be presented.

4.2. Geometrically nonlinear static analysis

Firstly, the geometrically nonlinear static analysis of a piezoelectric FG plate subjected to uniform load with parameter load $\bar{q} = q_0 \times 10^3$ is addressed. A SSSS piezoelectric FG square plate which is made of aluminum oxide and Ti-6Al-4V has a side length $a = b = 0.2$ m, thickness of FG core layer $h_c = 2$ mm and thickness of each piezoelectric layer $h_p = 0.1$ mm. Fig. 6 illustrates the influence of the material index $n$ on the normalized linear and nonlinear central deflections of the piezoelectric FG plates under mechanical load. As can be seen that, by increasing the material index $n$, the deflection of the piezoelectric FG plate decreases gradually. The largest deflection is obtained when material index $n = 0$, where the plate consists only of Ti-6Al-4V leads to the
decrease in the bending stiffness. Furthermore, the values of the central deflection of the geometrically nonlinear analysis are always smaller than that of the linear one and this difference reduces with the increase of the material index.

Next, a SSSS piezoelectric FG plate with porous core layer which is constituted by combining of two porosity distribution types and three GPL dispersion patterns, respectively, is considered in this example. The piezoelectric FG plate is subjected to sinusoidally distributed load which is defined as \( q = q_0 \sin(\pi x/a) \sin(\pi y/b) \) in which \( q_0 = 1.0 \) MPa. The plate has a side length \( a = b = 0.4 \) m, thickness of the FG porous core layer \( h_c = 20 \) mm and thickness of each piezoelectric layer \( h_p = 1 \) mm. In this study, the copper is chosen as the metal matrix whose material properties are given in Table 2 while the dimensions of GPLs are \( l_{GPL} = 2.5 \) µm, \( w_{GPL} = 1.5 \) µm, \( t_{GPL} = 1.5 \) nm. Fig. 7 examines the influence of the porosity coefficients on the nonlinear deflection of the piezoelectric FG porous plate with GPL dispersion pattern \( A \) (\( \Lambda_{GPL} = 1.0 \) wt. %) for two porosity distribution types, respectively. It can be observed that an increase of the porosity coefficients leads to the increase of the nonlinear deflection of the FG porous plate since the higher density of internal pores in material yields the reduction stiffness of plate structures. In addition, Fig. 8 depicts the effect of the weight fraction and the GPL dispersion patterns on the nonlinear deflection of the piezoelectric FG porous plate with \( e_0 = 0.2 \) and two porosity distribution types, respectively. It can be observed that the effective stiffness of the FG porous core layer is greatly strengthened when adding a small amount of GPLs (\( \Lambda_{GPL} = 1.0 \) wt. %) into metal matrix as evidenced by decreasing the nonlinear deflection of the FG plate. More importantly, the reinforcing effect of GPLs also depends significantly on the dispersion of GPLs in material matrix. Accordingly, with the same weight fraction of GPLs, the dispersion pattern \( A \), where GPLs are dispersed symmetric through the midplane of porous core layer, achieves the smallest nonlinear deflection while the asymmetric dispersion pattern \( B \) provides the largest one. For further illustration, Fig. 9 depicts the variation of the nonlinear deflection of the piezoelectric FG porous plate reinforced by GPLs which is constituted by porosity distribution 1 and three different GPL dispersion patterns corresponding to parameter load 10, respectively.

The combination influences of two porosity distribution types and three GPL dispersion patterns on the nonlinear deflection of the piezoelectric FG porous plate with \( \Lambda_{GPL} = 1.0 \) wt. % and \( e_0 = 0.4 \) is also investigated. As evidently depicted in Fig. 10, for all the considered associations, the combination between the porosity distribution 1 and the GPL dispersion pattern \( A \) obtains the best reinforcing performance in the geometrically nonlinear static analysis of the piezoelectric FG porous plate. This indicated that the plate structures, where the internal pores are distributed on the midplane and GPLs are dispersed around the top and bottom surfaces, can provide the optimum reinforcement.

4.3. Geometrically nonlinear dynamic analysis

In this part, the geometrically nonlinear dynamic responses of a CCCC piezoelectric FG porous plate reinforced by GPLs are studied. The dimensions and the material properties of the FG plate
are the same previous example. The plate is assumed to be subjected to time-dependent sinu-
soidally distributed transverse loads which are expressed as follows $q = q_0 \sin(\pi x/a) \sin(\pi y/b) F(t)$, where $F(t)$ is defined as

$$F(t) = \begin{cases} 
1 & 0 \leq t \leq t_1, \\
0 & t > t_1, \\
1 - \frac{t}{t_1} & 0 \leq t \leq t_1, \\
0 & t > t_1, \\
\sin\left(\frac{\pi t}{t_1}\right) & 0 \leq t \leq t_1, \\
e^{-\gamma t} & t > t_1,
\end{cases}$$

in which $q_0 = 100 \text{ MPa}$, $\gamma = 330 \text{ s}^{-1}$ and the time history $F(t)$ is plotted in Fig. [11]

Fig. [12] illustrates the influence of the porosity coefficient on the nonlinear transient re-
sponse of the piezoelectric FG porous plate with porosity distribution 1 and dispersion pattern $A (\Lambda_{GPL} = 1.0 \text{ wt. } \%)$ under step and sinusoidal loads, respectively. It can be seen that by increasing the porosity coefficients, the amplitude of the transverse deflection of the FG porous plate can be increased while the period of motion does not seem to affect. It can be concluded that the presence of porosities in core layer of the FG plate reduces the capacity of itself against external excitation. Furthermore, Fig. [13] demonstrates the influence of the weight fraction and the dispersion pattern of GPLs on the nonlinear transient response of the piezoelectric FG porous plate with $e_0 = 0.2$ and porosity distribution 2 corresponding to triangular and explosive blast loads, respectively. As expected, smaller magnitude of the deflection can be obtained when the weight fraction of GPLs in metal matrix increase. Again, the dispersion of GPLs into the metal matrix also affects the reinforcing performance of structure that dispersion pattern $A$ provides the smallest magnitude of the deflection.

Next, the combination influences of various porosity distribution types and the GPL disper-
sion patterns on the nonlinear dynamic response of the piezoelectric FG plate is also examined and indicated in Fig. [14]. For this specific example, the porous core layer of the piezoelectric plate has the porosity coefficient $e_0 = 0.4$ and the GPL weight fraction $\Lambda_{GPL} = 1.0 \text{ wt. } \%$. As clearly demonstrated in Fig. [14] the combination between the porosity distribution 1 and the GPL dispersion pattern $A$ always provides the best reinforcement as evidenced by obtaining the smallest amplitude of the deflection. Moreover, the dynamic responses of the linear and nonlinear of the FG porous plate with porosity distribution 2 ($e_0 = 0.3$) and the GPL dispersion pattern $C (\Lambda_{GPL} = 1.0 \text{ wt. } \%)$ under triangular and sinusoidal loads are also considered and depicted in Fig. [15]. As can be observed, the geometrically nonlinear responses generally obtain smaller magnitudes of the deflection and periods of motion.

4.4. Static and dynamic responses active control

In this section, the active control for the static and dynamic responses of the FG porous plate reinforced by GPLs using integrated sensors and actuators is investigated. Firstly, the active con-
control for the linear static responses of a SSSS FG plate which is subjected to a uniformly distributed load with \( q_0 = 100 \text{ N/m}^2 \) is investigated to perform the accuracy of the proposed approach. The FG plate composed of Ti-6Al-4V and aluminum oxide materials with material index \( n = 2 \) has the side length \( a = b = 0.2 \text{ m} \) while thickness of core FG layer and each piezoelectric layer are taken to be 1 mm and 0.1 mm, respectively. Fig. 16 illustrates the linear static deflections of the FG plate with various the displacement feedback control gains \( G_d \). As can be observed that the present results agree well with the reference solution which is reported in [73] who employed the CS-DSG3 based on FSDT. As expected, when the displacement feedback control gain \( G_d \) increases, the linear static deflection of the FG plate decreases. Furthermore, the active control for the linear dynamic responses of the FG plate is also investigated based on a constant velocity feedback control algorithm \( G_v \) and a closed loop control. In this specific example, the FG plate is initially subjected to a uniform load \( q_0 = 100 \text{ N/m}^2 \) and then the load is suddenly removed. In this study, the modal superposition is adopted in order to reduce the computational cost and the first six modes are considered in the modal space analysis, while the initial modal damping ratio for each mode is assumed to be 0.8\%. Fig. 17 shows the linear dynamic responses of the central deflection of the FG plate. The results which are generated from present method agree well with the reference solution [73].

Next, the active control for the nonlinear static responses of the SSSS FG porous plate reinforced with GPLs is further investigated in this part. The FG plate consisting of combined the porosity distribution 1 and GPL dispersion pattern A, which provides the best structural performance, is selected to study. The material properties of the FG porous plate are the same in Section 4.2. The plate has a side length \( a = b = 0.4 \text{ m} \), thickness of the FG porous core layer \( h_c = 20 \text{ mm} \) and thickness of each piezoelectric layer \( h_p = 1 \text{ mm} \) under sinusoidally distributed load which is defined as \( q = q_0 \sin(\frac{\pi x}{a})\sin(\frac{\pi y}{b}) \) with \( q_0 = 1.0 \text{ MPa} \). Fig. 18 depicts the nonlinear static deflection of the FG porous reinforced by GPLs with the porosity coefficient \( e_0 = 0.4 \) and the GPL weight fraction \( \Lambda_{GPL} = 1.0 \text{ wt. \%} \) corresponding to various displacement feedback control gains. As can be observed that the deflection of the FG porous plate decreases significantly when the displacement feedback control gain increase.

In the last example, the active control for the geometrically nonlinear dynamic responses of the CCCC FG porous plate reinforced by GPLs is conducted. The plate has the both length and width set the same at 0.2 m with the thickness of core layer \( h_c = 10 \text{ mm} \) and each piezoelectric layer \( h_p = 0.1 \text{ mm} \). The FG plate with the porosity distribution 1 \( (e_0 = 0.4) \) and dispersion pattern \( A (\Lambda_{GPL} = 1.0 \text{ wt. \%}) \) is subjected to sinusoidally distributed transverse loads which are the same as those in Section 4.3. Fig. 19 illustrates the nonlinear dynamic responses of the central deflection of the FG plate corresponding to various the velocity feedback control gains \( G_v \). It can be observed that when the control gain \( G_v \) is equal to zero corresponding to without control case, the nonlinear dynamic response of the FG porous plate still attenuates with respect to time since the effect of the structural damping is considered in this study. More importantly, the geometrically nonlinear dynamic response can be suppressed more faster in the case controlled by higher velocity feedback control gain values. As a result, depending on the specific cases, the responses of the FG porous plate structures including deflection, oscillation time or even both can be controlled to
satisfy an expectation by designing an appropriate value for the velocity feedback control gain. It should be noted that the feedback control gain values could not be increased without limit since piezoelectric materials have their own breakdown voltage values. In addition, Fig. 20 depicts the influence of the velocity feedback control gain $G_v$ on the linear and nonlinear responses of the CCCC FG porous square plate subjected to step load. As expected, the geometrically nonlinear dynamic responses provide smaller magnitudes of the deflection and periods of motion.

5. Conclusions

In this study, the IGA based on the Bézier extraction and the $C^0$-HSDT was successfully presented for the geometrically nonlinear static and dynamic responses for FG porous plates with GPLs reinforcement and integration with piezoelectric layers. The equations of motion were derived based on the $C^0$-HSDT in conjunction with von Kármán strain assumptions. Whereas the mechanical displacement field is approximated using the $C^0$-HSDT based on the Bézier extraction of NURBS, the electric potential field was considered as a linear function through the thickness of each piezoelectric sublayer. Two porosity distributions and three dispersion patterns of GPLs with various related parameters were exhaustively carried out through numerical examples. The control algorithms based on the constant displacement and velocity feedbacks were utilized to control the geometrically nonlinear static and dynamic responses of the FG porous plate reinforced with GPLs. Through the present numerical results, several major remarks can be drawn:

- By applying Bernstein polynomials as basis functions in the Bézier extraction, the IGA approach can easily be integrated into most existing FEM structures while its advantages are maintained effectively.

- After adding a small amount of GPLs into the metal matrix, the stiffness of the structures is significantly improved while an increase of the porosity coefficients leads to the decrease of the reinforcing effect. Furthermore, the distribution of porosities and GPLs in metal matrix also affect significantly the reinforcing performance of the structures. For all the combinations, the association between the porosity distribution type 1 with internal pores distributed on the midplane and the GPL dispersion pattern $A$, where GPLs are dispersed around the top and bottom surfaces, obtained the best reinforcing performance.

- For geometrically nonlinear static responses control of the FG porous plates, two effective algorithms are considered including the input voltage control with opposite signs applied across the thickness of two piezoelectric layers and the displacement feedback control algorithm. In addition, the dynamic response of the FG porous plate can be expectantly suppressed based on the effectiveness of the velocity feedback control algorithm.

- Finally, the combination advantages of both the porous architecture and GPL reinforcement into material matrices is a good choice to provide the advanced ultra-light high-strength structures in engineering.
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Figure 1: Configuration of a piezoelectric FG porous plate reinforced by GPLs.
Figure 2: Porosity distribution types and dispersion patterns of GPLs [18].

Figure 3: A schematic diagram of a FG porous plate with integrated piezoelectric sensors and actuators.
Figure 4: Normalized nonlinear transient central deflection of a square orthotropic plate under the uniform load.
Figure 5: Centerline linear deflections of the cantilever piezoelectric FG plate under the uniform loading and various actuator input voltages with $n = 0$ and $n = 0.5$.

Figure 6: Effect of the material index $n$ on the linear and nonlinear central deflections of the piezoelectric FG plate under the mechanical load.
Figure 7: Effect of the porosity coefficients on the nonlinear deflection of the piezoelectric FG porous square plate with GPL dispersion pattern A and $A_{GPL} = 1.0 \text{ wt. } \%$. 

(a) Porosity distribution 1  

(b) Porosity distribution 2
Load parameter
-0.45
-0.4
-0.35
-0.3
-0.25
-0.2
-0.15
-0.1
-0.05
0

(a) Porosity distribution 1
(b) Porosity distribution 2

Figure 8: Effect of the weight fractions and dispersion patterns of GPLs on the nonlinear deflection of the piezoelectric FG porous square plate with $e_0 = 0.2$. 
Figure 9: Effect of the porosity coefficients and weight fractions of GPLs on the nonlinear deflection of piezoelectric FG porous square plate for porosity distribution 1 and different GPL dispersion patterns.

(a) Pattern A

(b) Pattern B

(c) Pattern C
Figure 10: Effect of the porosity distributions and GPL dispersion patterns on the nonlinear deflection of the piezoelectric FG porous square plate with $\epsilon_0 = 0.4$ and $\Lambda_{GPL} = 1.0$ wt. %.

Figure 11: Time history of load factor.
Figure 12: Effect of the porosity coefficients on the nonlinear dynamic responses of the CCCC piezoelectric FG porous plate with GPL dispersion pattern A and $\Lambda_{GPL} = 1.0$ wt. %.

Figure 13: Effect of the weight fractions and dispersion patterns of GPLs on the nonlinear dynamic responses of the CCCC piezoelectric FG porous square plate with porosity distribution 2 and $\epsilon_0 = 0.2$. 
Figure 14: Effect of the porosity distributions and GPL dispersion patterns on the nonlinear dynamic responses of the CCCC piezoelectric FG porous square plate with $\epsilon_0 = 0.4$ and $\Lambda_{GPL} = 1.0$ wt. %.

![Graph](a) Step load![Graph](b) Sinusoidal load

Figure 15: Linear and nonlinear dynamic responses of the CCCC piezoelectric FG porous square plate with porosity distribution 2 ($\epsilon_0 = 0.3$) and dispersion pattern C ($\Lambda_{GPL} = 1.0$ wt. %).

![Graph](a) Triangular load![Graph](b) Sinusoidal load
Figure 16: Effect of the displacement feedback control gain $G_d$ on the linear static responses of the SSSS plate subjected to uniformly distributed load.

Figure 17: Effect of the velocity feedback control gain $G_v$ on the linear dynamic response of the SSSS FG square plate.
Figure 18: Effect of the displacement feedback control gain $G_d$ on the nonlinear static responses of the SSSS FG porous plate with porosity distribution 1 ($e_0 = 0.2$) and GPL dispersion pattern A ($\Lambda_{GPL} = 1.0$ wt. %).
Figure 19: Effect of the velocity feedback control gain $G_v$ on the nonlinear dynamic responses of the CCCC FG porous square plate subjected to dynamic loadings.
Figure 20: Effect of the velocity feedback control gain $G_v$ on the linear and nonlinear dynamic responses of the CCCC FG porous square plate subjected to step load.

Table 1: Normalized central deflection $\bar{w}$ of CCCC isotropic square plate under the uniform load with $a/h = 100$.

| $P$  | Present | Analytical [69] | MXFEM [70] | IGA-RPT [71] |
|------|---------|-----------------|------------|--------------|
| 17.79| 0.2348  | 0.237           | 0.2328     | 0.2365       |
| 38.3 | 0.4663  | 0.471           | 0.4738     | 0.4692       |
| 63.4 | 0.6873  | 0.695           | 0.6965     | 0.6908       |
| 95.0 | 0.8983  | 0.912           | 0.9087     | 0.9024       |
| 134.9| 1.1016  | 1.121           | 1.1130     | 1.1060       |
| 184.0| 1.2960  | 1.323           | 1.3080     | 1.3008       |
| 245.0| 1.4875  | 1.521           | 1.5010     | 1.4926       |
| 318.0| 1.6728  | 1.714           | 1.6880     | 1.6784       |
| 402.0| 1.8492  | 1.902           | 1.8660     | 1.8552       |
Table 2: Material properties of the core and piezoelectric layers.

| Properties          | Core layer | Piezoelectric layer |
|---------------------|------------|---------------------|
|                     | Ti-6Al-4V  | Aluminum oxide      | Copper | GPLs      | PZT-G1195N |
| Elastic properties  |            |                     |        |           |            |
| $E_{11}$ (GPa)      | 105.70     | 320.24              | 130    | 1010      | 63.0       |
| $E_{22}$ (GPa)      | 105.70     | 320.24              | 130    | 1010      | 63.0       |
| $E_{33}$ (GPa)      | 105.70     | 320.24              | 130    | 1010      | 63.0       |
| $G_{12}$ (GPa)      | –          | –                   | –      | –         | 24.2       |
| $G_{13}$ (GPa)      | –          | –                   | –      | –         | 24.2       |
| $G_{23}$ (GPa)      | –          | –                   | –      | –         | 24.2       |
| $\nu_{12}$          | 0.2981     | 0.26                | 0.34   | 0.186     | 0.30       |
| $\nu_{13}$          | 0.2981     | 0.26                | 0.34   | 0.186     | 0.30       |
| $\nu_{23}$          | 0.2981     | 0.26                | 0.34   | 0.186     | 0.30       |
| Mass density $\rho$ (kg/m$^3$) | 4429       | 3750                | 8960   | 1062.5    | 7600       |
| Piezoelectric coefficients |            |                     |        |           |            |
| $k_{31}$ (m/V)      | –          | –                   | –      | –         | $254 \times 10^{-12}$ |
| $k_{32}$ (m/V)      | –          | –                   | –      | –         | $254 \times 10^{-12}$ |
| Electric permittivity |            |                     |        |           |            |
| $p_{11}$ (F/m)      | –          | –                   | –      | –         | $15.3 \times 10^{-9}$ |
| $p_{22}$ (F/m)      | –          | –                   | –      | –         | $15.3 \times 10^{-9}$ |
| $p_{33}$ (F/m)      | –          | –                   | –      | –         | $15.3 \times 10^{-9}$ |

Table 3: Tip node deflection of the cantilever piezoelectric FGM plate subjected to the uniform load and various input voltages ($\times 10^{-4}$ m).

| Method     | Input voltages (V) |
|------------|---------------------|
|            | 0       | 20      | 40      |
| Present    | -2.5437 | -1.3328 | -0.1229 |
| CS-DSG3    | -2.5460 | -1.3346 | -0.1232 |
| Present    | -1.6169 | -0.8418 | -0.0667 |
| CS-DSG3    | -1.6199 | -0.8440 | -0.0681 |
| Present    | -1.1233 | -0.5808 | -0.0382 |
| CS-DSG3    | -1.1266 | -0.5820 | -0.0375 |
| Present    | -0.8946 | -0.4608 | -0.0271 |
| CS-DSG3    | -0.8947 | -0.4609 | -0.0271 |