Visco thermo elastic waves in a nonhomogeneous piezoelectric resonator plate of polygonal shape

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Abstract. In this paper the propagation of non-homogeneous waves in visco thermo elastic piezoelectric plate is studied with the application of linear elasticity theory. The considered plate is resonator in nature. The stress strain equations are formulated using non homogeneous form of three dimensional linear elasticity theory. The solution of the problem is derived using the Bessel function of first and second kind. The irregular boundary conditions are evaluated using the Fourier expansion collocation method. The numerical computations are carried out for triangle, square, pentagon and hexagon shapes of resonator plates. The dispersion curves are presented for the physical variables.

1. Introduction
Resonators vibrate at high frequency. So that the resonator plates are used in the vehicles mustang of the engine to control the high frequency of sound. The sound frequency in the vehicles is nothing but the pressure wave emitted at a certain frequency. It increases the performance by decreases engine back pressure. The resonators involve in the design and operation of nuclear devices wherein the maximum growth and the gradient of temperature are emanating from centre of the nuclear. The only disadvantage of this is not adjustable. They are also used in biological sensing, motion sensing, signal filtering and micro electro mechanical system oscillator. Ponnusamy and Amuthalakshmi [1] analyzed the dispersion of non-homogeneous wave propagation in a plate of polygonal cross sections. Selvamani and Ponnusamy [2] discussed the wave propagation in piezoelectric bar which is immersed in fluid and also transversely isotropic. Later, Selvamani [3] considered a plate with inner and outer cross sections and discussed the behavior of stress waves in thermo elastic polygonal plate numerically and graphically. Wu Bin [4] concluded the behavior of the magneto electro elastic plate on the propagation of wave. Ponnusamy and Amuthalakshmi [5] derived the mathematical model for thermo electro elastic waves in a circular fiber. In ancient days, Biot [6] introduced the effect of thermo elasticity and the irreversibility on thermo dynamics. After that Turhurn [7] derived using the integration decay and growth of the wave in anistropic non homogenoeus thermo viscoelastic medium. Guo [8] analyzed the dissipation of thermo visco elasticity in circular micro plate resonators using the dual phase log model. For the non-homogeneous visco elastic medium, Shahin [9] discussed the transient waves. Othman [10] used the FEM to derive the fundamental solution of thermo visco elasticity. Rajneesh Kumar [11] referred [10] and derived the effect of viscosity on the medium considered by [7] using the three-phase-log model.
In this article, the visco thermo elasticity of the nonhomogeneous piezoelectric resonator plate of poly shape is discussed using the linear elasticity theory and the numerical computations are carried out for the polygonal shape plates.

2. Formulation of the problem
The considered visco thermo elastic waves in a piezoelectric resonator plate of dual-phase-lagging model with the stress strain model is having the following form in the cylindrical variables \((r, \theta, z)\) whose body force is ignored according to the linear elasticity theory,

\[
\begin{align*}
\sigma_{rr,r} + r^{-1}\sigma_{r\theta,\theta} + r^{-1}(\sigma_{rr} - \sigma_{\theta\theta}) &= \rho u_{rr,tt} \\
\sigma_{r\theta,r} + r^{-1}\sigma_{\theta\theta,\theta} + 2r^{-1}\sigma_{r\theta} &= \rho u_{r\theta,tt} \\
D_{rr,r} + r^{-1}D_{r\theta,\theta} &= 0
\end{align*}
\]

(1) \(2\) \(3\)

The generalized heat conduction equation in visco thermo elasticity by Guo[8] is,

\[
K \left[ T_{rr} + \frac{1}{r}T_{r\theta} + \frac{1}{r^2}T_{\theta\theta} \right] + \tau_r \frac{\partial}{\partial t} \left( T_{rr} + \frac{1}{r}T_{r\theta} + \frac{1}{r^2}T_{\theta\theta} \right) - \rho C_v \left( T_{rr} + \tau_q T_{r\theta} \right) +
\]

\[
\beta T \nabla^2 \left[ u_{rr,rr} + 1/r(u_{r\theta,rr} + u_{\theta,rr}) \right] + \tau_q \frac{\partial}{\partial t} \left[ u_{rr,rr} + 1/r(u_{r\theta,rr} + u_{\theta,rr}) \right] = 0
\]

(4)

where

\[
\sigma_{rr} = (\lambda + 2\mu)e_{rr} + \lambda e_{\theta\theta} - \beta T; \quad \sigma_{r\theta} = 2\mu e_{r\theta}; \quad \sigma_{\theta\theta} = \lambda e_{rr} + (\lambda + 2\mu)e_{\theta\theta} - \beta T
\]

(5)

Here \(\sigma_{rr}, \sigma_{r\theta}, \sigma_{\theta\theta}\) are the stress variables which includes the strains \(e_{rr}, e_{r\theta}, e_{\theta\theta}\) and \(T\) is the temperature, \(\rho\) is the mass density of the material, \(C_v\) is the specific heat capacity, \(\beta\) is the thermal capacity, \(K\) is the thermal conductivity of the material, \(\lambda, \mu\) are Lame’s constants. (3) represents the electric conduction of the considered plate with the components \(D_{rr}, D_{r\theta}\). In (4) \(\tau_r, \tau_q\) are the temperature phase lagging and the heat flux parameter.

The Strain components corresponding to the polar coordinates \((r, \theta)\) are given by,

\[
e_{rr} = u_{rr}; \quad e_{r\theta} = u_{r\theta} - 1/r(u_{\theta,rr} - u_{r,\theta\theta}); \quad e_{\theta\theta} = 1/r(u_{rr,\theta} + u_{r,\theta,\theta})
\]

(6)

The electric conduction equation (3) over the electric field \(\mathbf{E}_r, \mathbf{E}_\theta\) are related to the electric potential \(\mathbf{\mathcal{E}}\) as

\[
\mathbf{E}_r = -\mathbf{\mathcal{E}}_r; \quad \mathbf{E}_\theta = -r^{-1}\mathbf{\mathcal{E}}_\theta
\]

(7)

All the above constants and the mass density \(\rho\) can be expressed for the non-homogeneity of the material with the rational number \(m\) as the functions of radial coordinates are as follows

\[
\lambda = \lambda r^{2m}, \mu = \mu r^{2m}, e_{11} = e_{11} r^{2m}, \rho = \rho r^{2m}, \beta = \beta r^{2m}, K = K r^{2m}
\]

(8)

Using (6)-(8) and applying the non-homogeneity via (1)-(5), we reached the following displacement equations of motions
\[
(\lambda + 2\mu)[u_{rr,rr} + 1/r (u_{r,r} + u_{\theta,\theta}) - 1/r^2 u_r] + 1/r^2 \left[\frac{2\mu u_{\theta,\theta}}{(\lambda + 4\mu)u_{\theta,\theta}}\right] \\
+ 2m \left[\frac{(\lambda + 2\mu)u_{rr}}{\lambda + r(2u_r + u_{\theta,\theta})}\right] - \beta T_{rr} = \rho u_{rr,tt}
\]

\[
2\mu[u_{\theta,rr} + 1/r u_{\theta,r} - 1/r^2 (u_{\theta} - u_{\theta,\theta})] + 1/r^2 \left[\frac{2\mu u_{\theta,\theta}}{(\lambda + 4\mu)u_{\theta,\theta}}\right] \\
+ 4m/r \mu[u_{\theta,r} - 1/r (u_{\theta} - u_{\theta,\theta})] - \beta T_{\theta,\theta} = \rho u_{\theta,tt}
\]

\[
\varepsilon_{11}\left(\mathbf{3}_{rr} + 1/r \mathbf{3}_{r} + 1/r^2 \mathbf{3}_{\theta,\theta}\right) + 2m/r \varepsilon_{11} \mathbf{3}_{rr} = 0
\]

\[
K \left[ (T_{rr} + 1/r T_{r,r} + 1/r^2 T_{\theta,\theta} + \tau_{r} \frac{\partial}{\partial t} (T_{rr} + 1/r T_{r,r} + 1/r^2 T_{\theta,\theta}) \right] - \rho C_v (\dot{T}_{rr} + \tau_{r} \dot{T}_{rr}) \\
+ \beta T_0 \nabla^2 \left[ \frac{\tau_{r}}{\partial t^2} (u_{rr} + 1/r (u_{\theta,\theta} + u_{r})) + \right] = 0
\]

3. Solution of the problem

The above coupled solutions can be uncoupled by considering the following form

\[
u_r(r, \theta) = \sum_{n=0}^{\infty} \varepsilon_n \left[ (\psi_{n,0} - \phi_{n,r}) + \left( r^{-1} \psi_{n,\theta} - \phi_{n,\theta} \right) \right] e^{i\omega t}
\]

\[
u_\theta(r, \theta) = \sum_{n=0}^{\infty} \varepsilon_n \left[ (r^{-1} \phi_{n,\theta} - \psi_{n,r}) + \left( r^{-1} \phi_{n,\theta} - \psi_{n,\theta} \right) \right] e^{i\omega t}
\]

\[
\mathbf{3}(r, \theta) = \sum_{n=0}^{\infty} \varepsilon_n \left( \mathbf{3}_n + \overline{\mathbf{3}}_n \right) e^{i\omega t}
\]

\[
T(r, \theta) = (\lambda + 2\mu / \alpha^2 \beta) \sum_{n=0}^{\infty} \varepsilon_n \left( T_n + \overline{T}_n \right) e^{i\omega t}
\]

where \( \varepsilon_n = 1/2 \) for \( n = 0 \) and \( \varepsilon_n = 1 \) for \( n \geq 1 \) and \( \omega \) is the angular frequency.

\( \phi_n(r, \theta), \psi_n(r, \theta), E_n(r, \theta), T_n(r, \theta) \) are the displacement potentials. The bar symbol of the displacement potentials denote the anti-symmetric modes of vibrations.

To get the solution substituting (10) in (9), we get

\[
(\lambda + 2\mu)\nabla^2 \phi_n - \alpha^2 T_n + 2m(r^{-1}(\lambda + 2\mu)\phi_{n,r} - r^{-2} \lambda \phi_n) + \rho \phi_{n,tt} = 0
\]

\[
\mu(\nabla^2 \psi_n + \alpha^2 T_n) + 2\mu m(r^{-1}(\psi_{n,r} - \psi_n)) = 0
\]

\[
\varepsilon_{11}\nabla^2 \mathbf{3}_n + 2mr^{-1} \varepsilon_{11} \mathbf{3}_n = 0
\]

\[
K \left[ (1 + \tau_{r}) \nabla^2 T_n - i\omega C_v (1 + i\tau_{q}) T_n + \beta T_0 \omega (1 + i\tau_{q}) \nabla^2 \phi_n \right] = 0
\]

We consider the following solution which is suitable for the non-homogeneous resonator plate of polygonal shape.
\[
\phi_n(r, \theta, t) = r^{-m} \phi_n(r) \cos n \theta e^{i \omega t}
\]
\[
\psi_n(r, \theta, t) = r^{-m} \psi_n(r) \cos n \theta e^{i \omega t}
\]
\[
E_n(r, \theta, t) = r^{-m} E_n(r) \cos n \theta e^{i \omega t}
\]
\[
T_n(r, \theta, t) = r^{-m} T_n(r) \cos n \theta e^{i \omega t}
\]

We can obtain the solution by substituting (12) in (11) and eliminating \( T_n \) and we get,

\[
\phi_n''(r) + \frac{1}{r} \phi_n'(r) + \left( \rho \omega^2 \left( \lambda + 2\mu \right) - \frac{1}{r^2} \left( \lambda + 2\mu \right) \right) \phi_n(r) = 0
\]

Eq. (13) is a Bessel equation of order \( \zeta \)

\[
\phi_n(r) = [P_{1n} J_\zeta(\kappa r) + P_{1n}' Y_\zeta(\kappa r)] \cos n \theta
\]

\[
\psi_n''(r) + \frac{1}{r} \psi_n'(r) + \left( \mu \rho \omega^2 - \frac{1}{r^2} \left( \lambda + 2\mu \right) \right) \psi_n(r) = 0
\]

\[
\psi_n''(r) + \frac{1}{r} \psi_n'(r) + \left( \kappa^2 r^2 - \zeta^2 \right) \psi_n(r) = 0
\]

Eq. (15) is a Bessel equation of order \( \zeta \)

\[
\psi_n(r) = [P_{2n} J_\zeta(\gamma r) + P_{2n}' Y_\zeta(\gamma r)] \cos n \theta
\]

\[
\mathcal{I}_n''(r) + \frac{1}{r} \mathcal{I}_n'(r) - \left( \frac{1}{r^2} \lambda + 2 \mu \right) \mathcal{I}_n(r) = 0
\]

The solution of (17) is,

\[
\mathcal{I}_n(r) = (P_{3n} r^\rho + P_{3n}' r^{-\rho}) \cos n \theta
\]

Where

\[
p^2 = m^2 + n^2
\]

\[
C_1 \nabla^2 \phi_n(r) + C_2 \nabla^2 T_n(r) - C_3 T_n(r) = 0
\]

Where

\[
C_1 = \frac{\beta T_n(1 + i \tau_q)}{K}, C_2 = \frac{1 + \tau_q}{K}, C_3 = \frac{i \omega C_n(1 + i \tau_q)}{K}
\]

The solution of the thermal effect is

\[
T_n(r) = [P_{4n} J_n(\alpha r) + P_{4n}' Y_n(\alpha r)] \cos n \theta
\]

where

\[
\alpha = \frac{\beta T_n(1 + i \tau_q)}{\rho C_n}
\]

The solution of the non-homogeneous solid plate of polygonal cross sections can be considered as

\[
\phi_n(r, \theta, t) = P_{1n} J_\zeta(\kappa r) \cos n \theta
\]

\[
\psi_n(r, \theta, t) = P_{2n} J_\zeta(\gamma r) \sin n \theta
\]

\[
\mathcal{I}_n(r, \theta, t) = P_{3n} r^\rho \cos n \theta
\]

\[
T_n(r, \theta, t) = P_{4n} J_n(\alpha r) \cos n \theta
\]

The solution is derived for the anti-symmetric mode by taking the sine function via cosine function.
4. Boundary conditions and frequency equations  
The considered material is having the irregular boundary condition. Hence we are using the method of 
Nagaya [13] to obtain the boundary conditions as,  
\( (\sigma_{xx})_j = (\sigma_{yy})_j = (D_e)_j = (T_r)_j = 0 \)  

(25)  

Transforming the vibration displacements into the Cartesian coordinates \( x_1, a_n, y_j \), the relation between 
the displacements for the \( i \)-th segment of straight line boundaries are (Fig(1))  
\[ u_x = u_r \cos(\theta - \gamma_j) - u_\theta \sin(\theta - \gamma_j) \]  
\[ u_\theta = u_\theta \cos(\theta - \gamma_j) + u_r \sin(\theta - \gamma_j) \]  

(26)  

(27)  

And  
\[ \frac{\partial r}{\partial x_i} = \cos(\theta - \gamma_j) \frac{\partial \theta}{\partial x_i} = -r^{-1} \sin(\theta - \gamma_j) \]  
\[ \frac{\partial r}{\partial y_j} = \sin(\theta - \gamma_j) \frac{\partial \theta}{\partial y_j} = r^{-1} \cos(\theta - \gamma_j) \]  

(28)  

(29)  

Reference [1] and (26)-(29) help to derive the stress equations for the non-homogeneity as,  
\[ \sigma_{xx} = (i(\lambda + 2\mu)\cos^2(\theta - \gamma_j) + 2\lambda \sin^2(\theta - \gamma_j))u_{r,r} \]  
\[ + 1/r \{(\lambda + 2\mu)\sin^2(\theta - \gamma_j) + \lambda \cos^2(\theta - \gamma_j)\}(u_r + u_{\theta,\theta}) + \frac{\mu}{2} \left( \frac{1}{r} (u_\theta - u_{r,\theta}) - u_{\theta,\theta}\right) \sin 2(\theta - \gamma_j) - T\beta = 0 \]  
\[ \sigma_{xy} = \mu \{(u_{r,r} - r^{-1}u_{\theta,\theta} - r^{-1}u_r)\sin 2(\theta - \gamma_j) + (r^{-1}u_{r,\theta} + u_{\theta,\theta} - r^{-1}u_\theta)\cos 2(\theta - \gamma_j)\} = 0 \]  
\[ \sigma_{xx} = -\epsilon_{11} \mathcal{S}_{xx} = 0 \]  

The transformed form of the solution by applying the boundary conditions (25)  
\[ \left[ (S_{xx})_j + (S_{xy})_j \right] e^{i\omega} = 0 \]  
\[ \left[ (S_{xy})_j + (S_{yy})_j \right] e^{i\omega} = 0 \]  
\[ \left[ (\mathcal{S}_{xx})_j + (\mathcal{S}_{xy})_j \right] e^{i\omega} = 0 \]  
\[ \left[ (T_x)_j + (T_y)_j \right] e^{i\omega} = 0 \]  

Where  
\[ S_{xx} = 0.5 \{ P_{10} e_0^4 + P_{40} e_0^4 + P_{30} e_0^4 \} + \sum_{n=1}^{\infty} \{ P_{1n} e_n^4 + P_{2n} e_n^4 + P_{3n} e_n^4 + P_{4n} e_n^4 \} \]  
\[ S_{yy} = 0.5 \{ P_{10} f_0^2 + P_{20} f_0^2 + P_{30} f_0^2 \} + \sum_{n=1}^{\infty} \{ P_{1n} f_n^2 + P_{2n} f_n^2 + P_{3n} f_n^2 + P_{4n} f_n^2 \} \]  
\[ \mathcal{S}_{xx} = 0.5 \{ P_{10} h_0^4 + P_{20} f_0^4 + P_{30} f_0^4 \} + \sum_{n=1}^{\infty} \{ P_{1n} h_n^4 + P_{2n} h_n^4 + P_{3n} h_n^4 + P_{4n} h_n^4 \} \]  
\[ T_x = 0.5 \{ P_{10} h_0^4 + P_{20} h_0^4 + P_{30} h_0^4 \} + \sum_{n=1}^{\infty} \{ P_{1n} h_n^4 + P_{2n} h_n^4 + P_{3n} h_n^4 + P_{4n} h_n^4 \} \]  

For anti-symmetric mode  
\[ \widetilde{S}_{xx} = 0.5 \{ \overline{P}_{40} e_0^4 \} + \sum_{n=1}^{\infty} \{ \overline{P}_{1n} \overline{e}_n^4 + \overline{P}_{2n} \overline{e}_n^4 + \overline{P}_{3n} \overline{e}_n^4 + \overline{P}_{4n} \overline{e}_n^4 \} \]  
\[ \widetilde{S}_{xy} = 0.5 \{ \overline{P}_{40} \overline{f}_0^4 \} + \sum_{n=1}^{\infty} \{ \overline{P}_{1n} \overline{f}_n^4 + \overline{P}_{2n} \overline{f}_n^4 + \overline{P}_{3n} \overline{f}_n^4 + \overline{P}_{4n} \overline{f}_n^4 \} \]
\[ \overline{3}_x = 0.5 \left( \overline{P}_{40} \tilde{I}_0 \right) + \sum_{n=1}^{\infty} \left( \overline{P}_{1n} \tilde{I}_1 + \overline{P}_{2n} \tilde{I}_2 + \overline{P}_{3n} \tilde{I}_3 + \overline{P}_{4n} \tilde{I}_4 \right) \]

\[ \overline{T}_x = 0.5 \left( \overline{P}_{40} \overline{P}_0 \right) + \sum_{n=1}^{\infty} \left( \overline{P}_{1n} \overline{P}_1 + \overline{P}_{2n} \overline{P}_2 + \overline{P}_{3n} \overline{P}_3 + \overline{P}_{4n} \overline{P}_4 \right) \]

Where

\[ e_n^1 = 2 \left[ \zeta (\zeta - 1) J_\zeta (kr) + (\kappa) J_{\zeta + 1}(kr) \right] \cos 2(\theta - \gamma_i) \cos n\theta - r^2 \left[ \kappa^2 (\lambda + 2 \cos^2 (\theta - \gamma_i)) \right] J_\zeta (kr) \cos n\theta \]

\[ e_n^2 = 2 \left[ \zeta (\zeta - 1) J_\zeta (kr) + (\kappa) J_{\zeta + 1}(kr) \right] \sin 2(\theta - \gamma_i) \sin n\theta - r^2 \left[ \kappa^2 (\lambda + 2 \cos^2 (\theta - \gamma_i)) \right] J_\zeta (kr) \sin n\theta \]

\[ e_n^3 = 0, e_n^4 = 0 \]

\[ f_n^1 = \left[ \frac{2 \left[ J_\zeta (kr) - (\kappa) J_{\zeta + 1}(kr) \right]} {\left[ (\kappa)^2 - \zeta^2 - n^2 \right] J_\zeta (kr)} \right] \cos n\theta \sin 2(\theta - \gamma_i) + 2n \left[ \zeta - 1 \right] J_\zeta (kr) \sin n\theta \cos 2(\theta - \gamma_i) \]

\[ f_n^2 = 2n \left[ \zeta J_\zeta (kr) - (\kappa) J_{\zeta + 1}(kr) \right] \cos n\theta \sin 2(\theta - \gamma_i) + \left[ (\kappa)^2 - \zeta^2 - n^2 \right] J_\zeta (kr) \sin n\theta \cos 2(\theta - \gamma_i) \]

\[ f_n^3 = 0, f_n^4 = 0, h_n^1 = 0 = h_n^4 \]

\[ h_n^1 = n J_n (kr) - (\kappa) J_{n+1}(kr) \cos n\theta \]

\[ h_n^2 = n J_n (kr) \cos n\theta \]

\[ h_n^3 = n J_n (kr) \sin n\theta \]

5. Numerical computation

To illustrate the analytical result presented earlier, now we consider the numerical example for which computational results are presented. The physical constants for the numerical computation is taken
from copper at 42$^\circ$K. Poisson ratio $\nu=0.3$, density $\rho=8.96\times10^3$ kg/m$^3$ Young’s modulus $E=2.139\times10^4$ N/m$^2$, $\lambda=8.20\times10^4$ kg/m$^2$, $\mu=4.20\times10^3$ kg/m$^2$, $c_v=9.1\times10^{-2}$ m$^2$/ks$^2$ and $K=113\times10^{-2}$ kgm/ks$^2$. The material properties of the magneto- electro elastic material based on Peng-Fei Hou et al.[13] are given by $\varepsilon_{ii}=8.26\times10^{-11}$ C$^2$ N$^{-1}$ m$^{-2}$, $\mu_{ii}=-5\times10^{-6}$ Ns$^2$/C$^2$, $m_{ij}=-3612\times10^{-11}$ Ns/VC. Nagaya [12] has given the geometric relations for the polygonal cross sections as $R/b=[\cos(\theta-\gamma)]^{-1}$. The thermo-elastic damping factor is defined as $Q^{-1}=2|\frac{\text{Im}(\omega)}{\text{Re}(\omega)}|$

**Figure 1.** Line Segment
\begin{align*}
\theta_0 &= 0^\circ & \gamma_1 &= 0^\circ \\
\theta_1 &= 36^\circ & \gamma_2 &= 72^\circ \\
\theta_2 &= 108^\circ & \gamma_3 &= 144^\circ \\
\theta_4 &= 180^\circ & I &= 3
\end{align*}

\begin{align*}
\theta_0 &= 0^\circ & \gamma_1 &= 30^\circ \\
\theta_1 &= 60^\circ & \gamma_2 &= 72^\circ \\
\theta_2 &= 120^\circ & \gamma_3 &= 150^\circ \\
\theta_4 &= 180^\circ & I &= 3
\end{align*}

**Figure 2.** Polygonal resonator plates (a). Triangular (b). Square (c) Pentagon (d) Hexagon

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{figure2}
\caption{Thermo elastic damping with thickness of triangular resonator plate.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{figure3}
\caption{Thermo elastic damping with thickness of square resonator plate.}
\end{figure}
The dispersion curves are plotted in Figs. 3 and 4 for thermo elastic damping with the thickness for different temperature values of triangle and square cross sectional plates. Fig.3 shows that the increase in thickness will lead to the reduction in thermo elastic damping for the different values of temperature values. In Fig.4, the square cross section experiences high values of damping in the wave propagation compared with triangular in Fig.3. It can be noted that the effect of temperature and noncircular cross section shows increasing effect on the damping magnitude.

![Figure 5](image)

**Figure 5.** 3D curve of thermo elastic damping of pentagonal resonator plate.

![Figure 6](image)

**Figure 6.** 3D curve of thermo elastic damping of hexagonal resonator plate.

A comparative illustration is made among the thermo elastic damping, radial and circumferential distance of the piezoelectric pentagon and hexagon resonator plates in Fig.5 and 6, respectively. From the Figs.5 and 6, it is clear that, the lower range of radial and circumferential distances the damping attain a wave propagation behaviour in both type of plates, after that, it became linear with the increase of distances. These curves explain the dependence of distances on the damping of thermo elasticity.
6. Conclusion
The visco thermo elasticity of the nonhomogeneous piezoelectric resonator plate of poly shape is discussed using the linear elasticity theory and dual phase lagging model. The irregular boundaries are derived via FECM. The numerical computations are carried out for thermo elastic damping of the polygonal shape plates. We concluded that

- The piezoelectricity and non-homogeneity of the poly plate have important role on the distribution of damping values.
- As the thickness value increases the damping decreases and the square cross section plate attain higher values of damping compared with triangular cross section.
- The variation of radial and circumferential distance also influences the damping values in a oscillating nature in pentagon and hexagon cross sections.
- The FECM gives faster convergence in irregular boundaries.

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