Constraining \( f(R) \) Gravity with Planck Sunyaev-Zel’dovich Clusters

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Clustering of galaxies have the potential of providing powerful constraints on possible deviations from General Relativity. We use the catalogue of Sunyaev-Zel’dovich sources detected by \textit{Planck} and consider a correction to the halo mass function for a \( f(R) \) class of modified gravity models, which has been recently found to reproduce well results from \( N \)-body simulations, to place constraints on the scalaron field amplitude at the present time, \( f_R^0 \). We find that applying this correction to different calibrations of the halo mass function produces upper bounds on \( f_R^0 \) tighter by more than an order of magnitude, ranging from \( \log_{10}(-f_R^0) < -5.81 \) to \( \log_{10}(-f_R^0) < -4.40 \) (95\% confidence level).

This sensitivity is due to the different shape of the halo mass function, which is degenerate with the parameters used to calibrate the scaling relations between SZ observables and cluster masses. Any claim of constraints more stringent than the weaker limit above, based on cluster number counts, appear to be premature and must be supported by a careful calibration of the halo mass function and by a robust calibration of the mass scaling relations.

Galaxy clusters are the most massive gravitationally bound structures in the universe \([1, 2]\). The dependence of their mass- and redshift-dependent number counts is usually described by the halo mass function (MF), \( n(M, z) \), the number density of halos in the mass range \([M, M + dM] \) at redshift \( z \). The MF is a sensitive cosmological probe of the late time universe, and can provide unique constraints on cosmological parameters and other fundamental physical quantities, like neutrino masses \([3, 4]\). Here we are interested in constraints on \( f(R) \) gravity \([5]\), which is characterized by a Lagrangian density of the form \( R + f(R) \), where \( f \) is a function of the Ricci scalar, \( R \). By a conformal transformation, the fourth order system of field equations can be reduced to Einstein gravity coupled to a scalar degree of freedom, \( f_R = df/(dR) \), the scalaron, of which \( f_R^0 \) represents the value at current epoch. Another important parameter of the theory is the present-day Compton wavelength squared of the scalaron, \( B_0 \), which is proportional to \( d^2 f/(dR)^2 \). Deviations from General Relativity (GR) are quantified by \( f \), and affect gravitational collapse and structure formation, resulting in a dependence of \( n(M, z) \) on \( f_R^0 \). In this letter we discuss viability of \( f(R) \) gravity \([6]\) by comparing redshift number counts predictions for galaxy clusters with the recently released \([7] \) all-sky, full-mission, \textit{Planck} catalogue of Sunyaev-Zel’dovich (SZ) sources (PSZ2), to constrain \( f_R^0 \).

We describe effects of modified gravity on the cluster MF following \([8, 9]\); in these studies \( N \)-body simulations are used to fit departures from GR predictions for the critical density contrast for the collapse of a top-hat spherical perturbation, \( \delta_c \), for models with \( f(R) \sim R^{-n} \), where \( n \) is a positive integer \([10, 11]\). In this context, the good agreement down to non-linear scales of recent numerical approaches, which compare theoretical models for the MF \([12, 16]\) with the results of different implementations of \( N \)-body simulations, motivates the use of an updated calibration of the MF to improve the robustness of existing constraints on modified gravity theories \([17, 18]\).

By taking into account Cosmic Microwave Background (CMB) lensing, constraints from primary CMB temperature anisotropies result in \( B_0 < 0.1 \) at 95\% confidence level (C.L.) \([19]\). Adding small scale information from redshift space distortions and weak lensing \([20, 21]\) further tightens this constraint to \( B_0 < 0.8 \times 10^{-4} \) (95\% C.L.). Similar results are obtained combining CMB and large scale structure (e.g., galaxy clustering) data \([22, 25]\). Here we are mainly interested in constraints coming from cluster number counts \([17, 18, 22]\), which have provided upper limits on \( f_R^0 \) in the range \([1.3 - 4.8] \times 10^{-4} \) by using different data sets and making somewhat different assumptions. More recently a quite stronger upper limit, \( |f_R^0| \lesssim 7 \times 10^{-5} \), was obtained from peak statistics in weak lensing maps \([29]\).

As for using clusters to derive constraints on cosmological models, a necessary ingredient is represented by a precise calibration of the halo MF. Significant progress in this direction has been made over the last decade in the context of GR, but only in a few cases deviations arising in modified gravity theories have been considered. In general, the MF can be written as \([27, 28]\)

\[
\frac{dn(M,z)}{dM} = F(\sigma_M) \rho_M \frac{d\log \sigma_M^{-1}}{d\log M},
\]

where \( \rho_M \) is the comoving density of matter, \( M \) the cluster mass, \( \sigma_M \) the variance of the linear matter power spectrum filtered on the mass-scale \( M \), and \( F \) the multiplicity function. Achitouv \textit{et al.} \([8]\) define a new func-
tional form for $F(\sigma)$ in $f(R)$ gravity, by fitting results obtained from $N$-body simulations. This is done by a re-parametrization of $\delta_c$ that, in contrast with the GR case, becomes scale dependent and a function of $f^0_R$.

As such, this derivation of the MF for $f(R)$ models should apply for halo masses computed at the virial radius. In order to calibrate the values of the parameters, Achitouv et al. [5] compared their predictions to the MF results from $f(R)$ $N$-body simulations in the redshift range $z \in [0,1.5]$ and for scalar values in the range $-f^0_R \in [10^{-5},10^{-6}]$ [14], with halos identified by applying a Friends-of-Friends (FoF) algorithm.

On the other hand, in the PSZ2 catalogue the cluster masses are given as $M_{500c}$, defined as the total mass within a radius, $R_{500c}$, chosen in such a way that the mean enclosed density is $500\rho_c$. In order to adapt this calibration of the $f(R)$ MF to our case, we decided to implement the Achitouv et al. [5] MF as a correction to the multiplicity function for the GR case, and apply it to GR multiplicity functions, computed at $R_{500c}$, that have been calibrated from large sets of $N$-body simulations of standard gravity:

$$F(\sigma) = F_{GR}(\sigma) \frac{F_A^{fR}(\sigma)}{F_A^{GR}(\sigma)},$$

(2)

Here $F_A^{fR}(\sigma)$ and $F_A^{GR}(\sigma)$ are the multiplicity functions defined in Ref. [8]. As for $F_{GR}(\sigma)$, the multiplicity function calibrated on $N$-body simulations in GR, we implement two alternative definitions: the Tinker et al. MF [29], and the Watson et al. MF [30]. In the following, we will refer to them as Tinker and Watson, respectively. We choose not to test the Achitouv et al. MF directly, but in the form of a correction to another MF, as in eq. (2), because its GR limit is markedly different from the Tinker and Watson results. These two MFs have been widely studied thus allowing us to compare our results to past literature.

Following this procedure, we are implicitly assuming that the $f(R)$ correction to the MF from Ref. [8] also applies at $R_{500c}$. This assumption clearly needs to be verified from an extensive calibration of the MF at different overdensities from large $f(R)$ $N$-body simulations.

Within the PSZ2 catalogue, we identify a sample of 429 clusters with a signal-to-noise ratio $q > 6$. These clusters have masses in the range $M_{500c} \in [1,10] \times 10^{14} M_\odot$, and redshift $z \in [0,1]$ and are hereafter denoted as the SZ data set. The characteristic mass scale of the cluster sample is a critical element in the number counts analysis. In the original analysis of the Planck collaboration [7], a calibration of a scaling relation between measured

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**FIG. 1:** Panel a: the joint marginalized posterior of $\log_{10}(-f^0_R)$ and $\sigma_8$. Different colors correspond to different data set combinations, as shown in legend. Constraints that do not include Planck have been obtained by using weak priors on $n_s$ and $\Omega_m$. The darker and lighter shades correspond to the 68% C.L. and the 95% C.L. regions, respectively. Panel b: comparison between the Planck measurements and the model predictions for the cluster number counts, as a function of redshift. Different colors correspond to different models and different mass functions, as shown in legend. The black data points are samples from the PSZ2 catalogue. The continuous lines represent the best fit prediction of the Planck and Planck cluster GR posterior. The dashed lines correspond to the same values of the parameters, but with $\log_{10}(-f^0_R) = -4$. 

The characteristic mass scale of the cluster sample is a critical element in the number counts analysis. In the original analysis of the Planck collaboration [7], a calibration of a scaling relation between measured...
cluster masses and integrated Compton-$y$ parameter is assumed. To parametrize the uncertain knowledge in the calibration of cluster masses, a mass bias parameter $b$ is introduced [7], which is defined as the ratio between the masses calibrated through XMM-Newton X-ray observations [31] and the true cluster masses. In the following, we assume true cluster masses to be given by the weak lensing results from the Weighting the Giants project [32]. This amounts to assume for the bias parameter $B_{SZ} = 1 - b$ a Gaussian prior with mean value 0.688 and variance 0.072. This choice of the bias parameter is motivated by the fact that it provides a better agreement with primary Planck CMB results. In this sense, this is a conservative choice since it leaves less freedom for deviations from the standard ΛCDM results. By choosing another prior, the tension between different data sets could result in artificially tighter constraints on $f_R$ when combining CMB and cluster number counts data. Compared to other X-ray selected cluster data sets, like CCCP [33] or REFLEX [34], the Planck sample is biased towards larger masses and higher redshift, and offers a unique opportunity to test the MF in a complementary regime. Another key parameter used in the likelihood analysis is $\alpha_{SZ}$, which sets the slope of the scaling relation between $Y_{500c}$, the strength of the SZ signal in terms of the Compton $y$-profile integrated within a sphere of radius $R_{500c}$, and $M_{500c}$.

We also use Planck measurements of CMB fluctuations in both temperature and polarization [35] in the multipoles range $\ell \leq 29$. We account for CMB anisotropies at smaller angular scales by using Plik likelihood [36] for CMB measurements of the TT, TE and EE power spectra. Finally, we include the Planck 2015 full-sky lensing potential power spectrum [37] in the multipoles range $40 \leq \ell \leq 400$.

We then complement CMB measurements with the Joint Light-curve Analysis “JLA” Supernovae sample, as introduced in [38], and with BAO measurements of: the SDSS Main Galaxy Sample at $z_{\text{eff}} = 0.15$ [39]; the BOSS DR11 “LOWZ” sample at $z_{\text{eff}} = 0.32$ [40]; the BOSS DR11 CMASS at $z_{\text{eff}} = 0.57$ [41]; and the 6dFGS survey at $z_{\text{eff}} = 0.106$ [42]. We refer to this data combination CMB+BAO+JLA as Planck.

We use EFTCAMB and EFTCosmoMC [43, 44], modifications of the CAMB/CosmoMC codes [45, 46], to compute cosmological predictions, and compare them with observations. The EFTCosmoMC code is modified to account for the $f(R)$ cluster likelihood, which has been obtained from a suitable modification of the original likelihood described in [7].

**Results.** In Table I we show the marginalized constraints, at 95% confidence level, obtained from the Planck+SZ data set, with the $f(R)$ correction applied to, both, Tinker and Watson MFs. In the case of the Tinker MF we obtain the tightest constraints on $f(R)$ to date. In particular we improve the bounds of [15] on $\log_{10}(-f_R^0)$ by one order of magnitude and the ones in [20] by almost an order of magnitude. In addition these constraints improve substantially on the bounds coming from large scale cosmological observations [21], confirming the leading role of galaxy clusters in constraining modified gravity theories.

We notice, however, that the upper bound on $\log_{10}(-f_R^0)$ strongly depends on the choice of the MF, which can affect observational constraints by more than one order of magnitude. This strong dependence is clear also in Fig. 1: the Tinker MF produces the tightest bounds, while the Watson MF is less constraining. Noticeably, SZ cluster measurements break the degeneracy between $\sigma_8$ and $\log_{10}(-f_R^0)$ that Planck CMB measurements clearly display. Furthermore we consider a run with SZ clusters without Planck data, adding the BAO constraints that we described previously. We also include a prior on $n_s$, $n_s = 0.9624 \pm 0.014$, taken from [45] and we adopt Big Bang nucleosynthesis constraints from [46], $\Omega_k = 0.022 \pm 0.002$ (SZ+BAO data set). The results obtained are shown in Fig. 1, where we report both the Tinker (in yellow) and Watson (in orange) contour plots. We can notice that, at least for the Tinker run, the addition of CMB data significantly improves the constraints on $f(R)$ by more than two orders of magnitude. We also stress that, in the case of SZ+BAO, we do not get the strong dependence on the GR calibration of the MF that we obtain for the SZ+Planck runs. In the latter case the constraints obtained for the choice of Watson MF are weaker because the shape of this MF is different from Tinker MF in the range of mass and redshift probed by SZ Planck clusters. More precisely, as shown in Fig. 1, $N(z)$ falls off at high redshift for the Watson MF more slowly compared to Tinker case: when combined with CMB Planck data, in order to fit the tail at high redshift in GR, a lower $B_{SZ}$ is required; a lower $\alpha_{SZ}$ is instead preferred in order to fit the low-redshift trend for $N(z)$. When we, instead, consider $f(R)$ models for Watson MF, there is a more effective way to change the slope of $N(z)$ with this parameter (Fig. 1) than by using $\alpha_{SZ}$, which is now fairly unconstrained and degenerate with $f_R^0$. The same is not true for the Tinker case: this degeneracy is

| Parameter | Tinker (95 % C.L.) | Watson (95 % C.L.) |
|-----------|-------------------|-------------------|
| $\log_{10}(-f_R^0)$ | $<-5.81$ | $<-4.40$ |
| $\log_{10}B_0$ | $<-5.60$ | $<-4.06$ |
| $\sigma_8$ | (0.79, 0.83) | (0.80, 0.83) |
| $\alpha_{SZ}$ | (1.68, 1.91) | (1.57, 1.89) |
| $B_{SZ}$ | (0.55, 0.67) | (0.50, 0.63) |
In Figure 2 we show the contour plots for $f(R)$ and for the SZ parameters $\alpha_{SZ}$ and $B_{SZ}$. This figure summarizes the interplay between cosmology and astrophysical parameters of the cluster scaling relations. In the first two panels we can see the degeneracy between $f(R)$ and the other two parameters: this is clear in the Watson case, but absent in the Tinker one. The wider range of $\alpha_{SZ}$ probed by Watson MF when compared to Tinker MF in $f(R)$ models is evident and explains the weaker constraints obtained in the former case.

**Stability of the results.** To test the dependence of our results on other effects, we first add the contribution of baryons, and implement the baryonic correction to the MF discussed in [47]. In particular, we consider the correction to the MF obtained when including the effect of feedback from active galactic nuclei (AGN) in hydrodynamic simulations. We obtain the constraint $\log_{10}(-f_R^0) < -5.84$ at 95% C.L. when considering the Tinker MF and the SZ+Planck data set. We thus conclude that the presence of baryons does not have a substantial influence on our results unlike the larger effects found in Ref. [3], where, however, cluster data probed smaller masses, which are more affected by feedback effects than those probed by SZ clusters.

We then investigate the dependence from the signal-to-noise ratio of Planck data, by using the most conservative choice $q > 8.5$, that reduces the sample to 40% of the original one. In this case we obtain $\log_{10}(-f_R^0) < -5.54$, at 95% C.L., using Tinker MF, now with this reduced SZ+Planck data set. Again, we can then conclude that our constraints are stable, in the sense that a change in $q$ affects them much less than a change in the MF would.

**Discussion.** We compare our results with a recent work [18], where galaxy clusters have been used in order to get constraints on $f(R)$ gravity theory. In that case the authors got $\log_{10}(-f_R^0) < -4.79$ by considering the case of Tinker MF. In this sense, with the same choice of the MF, our work improves the constraint by one order of magnitude and gives $\log_{10}(-f_R^0) < -5.81$. We wish to stress that this result should be compared with the one in [18], since both come from the same choice of the MF, i.e. Tinker. However, the main result of this letter goes far beyond the mere exposition of a tighter constraint. Indeed, we also show that the implementation of a $f(R)$ correction to the MF strongly depends on the calibration of the MF in GR. In this context we prove that, by keeping the $f(R)$ correction constant and changing the MF for GR, e.g. by switching from Tinker to Watson, we obtain a change of more than one order of magnitude in the $f_R^0$ constraint. In the case of Tinker we get $\log_{10}(-f_R^0) < -5.81$ (at 95% C.L.), while for Watson $\log_{10}(-f_R^0) < -4.40$ (at 95% C.L.). As we already pointed out, this strong dependence on the MF arises from the degeneracy between $f_R^0$ and the SZ parameters, $\alpha_{SZ}$ and $B_{SZ}$. In order to reduce this dependence, it would be effective to further constrain the clusters mass bias; by reducing the distribution of this parameter, and thus of $B_{SZ}$, one would minimize the region of the parameter space in which the degeneracy occurs. Thus, we expect that a better determination of the variables describing SZ clusters would directly translate into a more robust estimation of modified gravity parameters.

In our analysis we also considered stability of the fi-
nal results. First of all we implemented the corrections on the MF induced by considering the effect of baryons and, more specifically, the effect arising when including star formations and AGN feedback in hydrodynamic simulations, as described in [47]. In particular we speculated that the baryonic processes would not depend on the model of gravity, i.e. on the value of $f_R$. In principle, since these effects strongly influence the shape of the MF and, consequently, of the cluster number counts, we would expect some change in the final constraints on the scalaron amplitude. However, as described before, taking into accounts these effects did not influence appreciably the result.

We also investigated the effects of the signal-to-noise ratio $q$ for the identification of the clusters in the Planck catalogue. Setting this threshold to the most conservative one, $q > 8.5$, we obtain $\log_{10}(-f_R^0) < -5.54$, at 95% C. L., which implies a correction of about 5% on the original result for $\log_{10}(-f_R^0)$. Also in this case, we can then support a remarkable stability of the result.

In conclusion, we quantitatively investigated the important role that SZ clusters have in constraining theories of modified gravity once cluster physics is properly understood and modeled, by using a state-of-the-art data set and recent results in terms of cluster MF. While studies in GR are already at an advanced stage, modified gravity theories can benefit from additional insight on cluster physics that can be directly translated in tighter constraints on gravitational physics. The work presented in this letter is thus relevant to present and future cosmological surveys, like Euclid and CMB-S4, that are expected to deliver unprecedented quality cluster measurements. A deep understanding of the physics of clusters will then be the essential to fully exploit the constraining power of these observations [48, 49].

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