PBH Evaporation, Baryon Asymmetry, and Dark Matter

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Abstract—Sufficiently light primordial black holes (PBH) could evaporate in the very early universe and dilute the preexisting baryon asymmetry and/or the frozen density of stable relics. The effect is especially strong in the case that PBHs decayed if and when they dominated the cosmological energy density. The size of the reduction is first calculated analytically under the simplifying assumption of the delta-function mass spectrum of PBH and instant decay approximation. In the realistic case of exponential decay and for an extended mass spectrum of PBH the calculations are made numerically. Resulting reduction of the frozen number density of the supersymmetric relics opens for them a wider mass window to become viable dark matter candidate.

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1. INTRODUCTION

Primordial black holes might be abundant in the early universe and even dominate for a while the cosmological energy density. In the latter case they would have an essential impact on the baryon asymmetry of the universe, on the fraction of dark matter particles, and would lead to the rise of the density perturbations at relatively small scales.

Usually primordial black holes (PBH) are supposed to be created by the Zel’dovich–Novikov (ZN) mechanism \cite{1} (see also \cite{2}). According to ZN, a PBH could be created, if the density fluctuation, $\delta \rho / \rho$, at the horizon size happened to be larger than unity. In this case this higher density region would be inside its own gravitational radius and became a black hole. With the accepted Harrison–Zeldovich spectrum of primordial fluctuations \cite{3, 4} the process of PBH creation can result in a significant density of PBHs.

The mass inside horizon at the radiation dominated (RD) stage of the universe evolution is equal to

$$M_{\text{hor}} = m_{\text{pl}}^2 t,$$

where the Planck mass is $m_{\text{pl}} \approx 2.176 \times 10^{-5}$ g and $t$ is the cosmological time (universe age). So according to the pioneering estimates by ZN the initial moment of the creation of PBH with mass $M$ can be taken as

$$t_{\text{in}}(M) = M / m_{\text{pl}}^2.$$

More accurate calculations with an account of the equation of state of the primeval plasma lead to somewhat different result, but this difference is not of much importance for our approximate results.

It is mostly assumed that the mass spectrum of PBH created by ZN mechanism is very narrow. It is usually taken in a power law form or even as delta-function. There are, however, quite a few other scenarios of PBH formation. We can mention, in particular, the mechanism suggested in \cite{5, 6}, which leads to the log-normal mass distribution and may, in principle, create PBH with masses up to thousands and even millions solar masses due to production of the BH seeds during cosmological inflationary stage. Other mechanisms of PBH production initiated at inflation are considered in \cite{7, 8}. Some more work on PBH formation with extended mass spectrum include \cite{9–12}. The creation of PBH due to a phase transition in the primeval plasma is studied in \cite{13}. A recent review on massive PBH formation can be found in \cite{14}.

The log-normal mass spectrum became quite popular during last few years, being employed for the description of massive PBH observed in the present day universe. The analysis of chirp mass distribution of the LIGO events \cite{15} very well agrees with the log-normal mass spectrum. Other mass spectrum of PBH are of course possible but it is a formidable task to study all possible spectra, so we confine ourselves to the delta-function spectrum and to two examples of extended spectra, namely to a flat spectrum and to a power law spectrum close by shape to the log-normal one. Both types of extended spectra are assumed to be confined between some $M_{\text{min}}$ and $M_{\text{max}}$. All three types of spectra lead to similar results for the entropy suppression factors.

In what follows we consider rather small PBH masses such that the black holes evaporated early
enough, well before the Big Bang Nucleosynthesis (BBN). Though such short-lived PBH decayed long before our time, their impact on the present day universe may be well noticeable. Firstly, PBH decays could release a significant amount of entropy into the primeval plasma and diminish the magnitude of earlier created baryon asymmetry or diminish the relative (with respect to the relic photon background) density of dark matter particles \[16, 17\]. On the other hand, baryon asymmetry could be generated in PBH evaporation \[18, 19\], and dark matter could also be created in this process. We neglect however, the second kind of the processes and consider only dilution of baryons and dark matter particles by the PBH evaporation. Indeed it can be shown that with the chosen parameter values the stable supersymmetric relics produced in the process of PBH evaporation would make negligible contribution to the density of dark matter, see Appendix A.

An interesting well known effect, not touched in this work, is the rise of density perturbations during early matter dominated stage. If there existed an epoch of the early PBH domination, the rising density perturbations could create small scale clumps of matter in the present day universe such as globular clusters or even dwarf galaxies.

In the scenario, considered below, the universe is supposed to be initially in radiation dominated (RD) stage, when the cosmological matter mostly consisted of relativistic species. The cosmological energy density during this epoch was equal to

\[ \rho_{\text{rel}}^{(i)} = \frac{3m_{\text{pl}}^2}{32\pi^2} \]

and the scale factor at this epoch evolved as

\[ a_{\text{rel}}(t) = a^{(\text{in})} \left( \frac{t}{t_{\text{in}}} \right)^{1/2}. \]

If sufficiently large density of PBH were created during this period and if PBH were massive enough to survive up to the moment when they started to dominate in the universe, the cosmological expansion law turned into the non-relativistic one and the energy density started to tend asymptotically to:

\[ \rho_{\text{nr}} = \frac{m_{\text{pl}}^2}{6\pi^2} \frac{1}{(t + t_2)^3}, \]

where \( t_2 \) is determined from the condition of the equality of the energy densities \( \rho_{\text{nr}} \) (5) and \( \rho_{\text{rel}}^{(i)} \) (3) at the equilibrium time moment \( t_{\text{eq}} \). Comparing these two expressions we find that \( t_2 = t_{\text{eq}}/3 \), while \( t_{\text{eq}} \) can be found from the equations: \( a_{\text{nr}}/a_{\text{eq}} = (t_{\text{in}}/t_{\text{eq}})^{1/2} = \rho_{\text{nr}}^{(in)}/\rho_{\text{rel}}^{(in)} \), see below.

Ultimately all PBH evaporated producing relativistic matter and the expansion regime returned to the relativistic one at \( t_2 \) when all or a significant part of PBH evaporated:

\[ \rho_{\text{rel}}^{(2)} = \frac{3m_{\text{pl}}^2}{32\pi^2(t + t_2)^3}, \]

where \( t_2 \) is determined by the condition of the equality of \( \rho_{\text{nr}} \) (5) and \( \rho_{\text{rel}}^{(2)} \) (6) at the moment of PBH decay \( t = \tau_{\text{BH}} \).

Of course the above estimates are approximate and presented here to get an insight into the essence of the effect. Nevertheless they are applied below for the calculations in the instant decay approximation for the instant change of the expansion regime. Comparison with the exact calculations demonstrated that these “instant” results happened to be quite accurate.

In thermal equilibrium the energy density of relativistic particles is equal to

\[ \rho_{\text{rel}} = \frac{\pi^2 g_*(T)T^4}{30} \],

where \( T \) is the plasma temperature and \( g_*(T) \) is the number of relativistic species in the plasma at temperature \( T \).

It is known, see e.g. \[20, 21\], that in thermal equilibrium state of the cosmological plasma with zero chemical potentials the entropy in the comoving volume is conserved:

\[ s = \frac{\rho + \mathcal{P}}{T} a^3 = \text{const}, \]

where \( \rho \) is the energy density of the plasma and \( \mathcal{P} \) is its pressure.

In usual baryogenesis scenarios non-conservation of baryonic number took place at very high temperatures, while at low temperatures baryon non-conservation was switched off. So at late cosmological epochs baryonic number density, \( N_{\text{BH}} \), was also conserved in the comoving volume. Correspondingly the baryon asymmetry, i.e. the ratio

\[ \beta = N_{\text{B}}/s = \text{const} \]

remained constant in the course of the universe expansion if there was no entropy influx into the plasma.

There are several realistic mechanisms of entropy production in the early universe. For example, entropy rose in the course of the electroweak phase transition, even if it was second order (or mild crossover). The entropy rise could be at the level of 10% \[17\]. If in the course of the cosmological evolution a first order phase transition took place, e.g. the QCD one, the entropy rise can be gigantic. Some entropy rise could be created by the residual annihilation of out-of-equilibrium of nonrelativistic dark matter particle after they practically decoupled from the plasma (froze).

In this work we consider a hypothetical case of the universe which at some stage was dominated by PBHs
and calculate the dilution of the preexisting baryon asymmetry and a relative decrease of the number density of DM particles, called X-particles in what follows. We show that with a reasonable amount of PBHs the suppression of the number density of weakly interacting massive particles (WIMPs), may be sufficiently strong to allow them to be viable DM candidates, despite having interaction strength typical for superstrong to allow them to be viable DM candidates, (in particular having the annihilation cross-section equal to $\sigma_{\text{ann}}v = \alpha^2/m_X^2$ with $\alpha \sim 10^{-2}$. Here $m_X$ is the X-particle mass.

The parameter space of supersymmetry is known to be significantly restricted by LHC [22], but some types of the lightest supersymmetric particles (LSP) still remain viable candidates for dark matter [23, 24]. An excessive entropy release, discussed in this paper, can lead to a wider class of possible dark matter LSPs.

The paper is organized as follows. In the next section we present a simple estimate of the entropy release for the case of delta-function mass spectrum of PBHs, instant decay approximation for PBH, and instant change from the initial RD stage to MD stage and back. In Section 3 the exact solutions for the cosmological evolution and the entropy release for the mixture of relativistic matter and decaying PBHs with the delta-function mass spectrum are found numerically. Section 4 is devoted to the study of the evolution for two examples of the extended mass spectrum. In Section 5 we analyze the results and conclude. Appendix A is devoted to calculations of the number density of X-particles directly produced by PBH decays, the subject which is somewhat away from the main line of this paper. We show there that the density of DM particles produced by PBHs evaporation is not large enough to avoid the entropy suppression. In Appendix B the expressions of the analytically calculated integrals entering the evolution equations are presented.

2. INSTANT CHANGE OF EXPANSION REGIMES AND INSTANT EVAPORATION

We consider here the simplest model of PBHs with fixed mass $M_0$ with the number density at the moment of creation:

$$\frac{dN_{\text{BH}}}{dM} = \mu_i^3 \delta(M - M_0),$$  \hfill (10)

where $\mu_i$ is a constant parameter with dimension of mass.

All these PBHs were created at the same moment $t_0(M_0) = M_0/2m_{\text{pl}}^2$, see Eq. (2). Assume that the fraction of the PBH energy (mass) density at production was:

$$\frac{\rho_{\text{BH}}^{(\text{in})}}{\rho_{\text{rel}}^{(\text{in})}} = \epsilon \ll 1.$$  \hfill (11)

If we disregard the PBH evaporation and if the interaction between PBH and relativistic matter can be neglected, then both ingredients of the cosmic plasma evolve independently and so:

$$\rho_{\text{rel}}(t) = \left(\frac{a^{(\text{in})}}{a(t)}\right)^4 \rho_{\text{rel}}^{(\text{in})}, \quad \rho_{\text{BH}}(t) = \left(\frac{a^{(\text{in})}}{a(t)}\right)^3 \rho_{\text{BH}}^{(\text{in})}.$$  \hfill (12)

Let us consider the case when densities of relativistic and nonrelativistic (PBH) matters became equal at $t = t_{\text{eq}}$, before the PBH decay. According to Eqs. (11) and (12) it takes place when:

$$\frac{\rho_{\text{BH}}(t_{\text{eq}})}{\rho_{\text{rel}}(t_{\text{eq}})} = \epsilon \frac{a(t_{\text{eq}})}{a_{\text{in}}} = 1.$$  \hfill (13)

We assume in this section that at $t < t_{\text{eq}}$ the universe expansion is described by purely relativistic law, when the scale factor evolves according to Eq. (4). Correspondingly we find

$$t_{\text{eq}} = t_{\text{in}}/\epsilon^2.$$  \hfill (14)

PBHs would survive in the primeval plasma till equilibrium if $t_{\text{eq}} - t_{\text{in}} < \tau_{\text{BH}}$, where the life-time of PBH with respect to evaporation is given by the expression [25]:

$$\tau_{\text{BH}} = 3 \times 10^3 N_{\text{eff}}^{-1} M_{\text{BH}}^3 m_{\text{pl}}^{-3} \approx C M_{\text{BH}}^3 m_{\text{pl}}^{-4},$$  \hfill (15)

where $C \approx 30$, and $N_{\text{eff}}$ is the effective number of particle species with masses smaller than the black hole temperature. For the chosen values of the PBH masses $N_{\text{eff}} \approx 100$. (In reality $g_s$ is closer to 200, but this difference is not of much importance.) The black hole temperature is equal to:

$$T_{\text{BH}} = \frac{m_{\text{pl}}^2}{8\pi M_{\text{BH}}^3}.$$  \hfill (16)

Thus the condition that the RD/MD equality is reached prior to BH decay reads:

$$M_{\text{BH}} > \left[\frac{m_{\text{pl}}^2}{C(\frac{1}{\epsilon} - 1)}\right]^{1/2} = \frac{m_{\text{pl}}}{\sqrt{C\epsilon}}.$$  \hfill (17)

According to the assumption of the instant change of the expansion regime, the scale factor after the equilibrium moment is reached, i.e. for $t > t_{\text{eq}}$, started to evolve as

$$a_{\text{in}}(t) = a_{\text{eq}}(t_{\text{eq}}) \left(\frac{t + t_{\text{eq}}/2}{4t_{\text{eq}}/3}\right)^{2/3}$$  \hfill (18)

and the cosmological energy density drops according to the nonrelativistic expansion law:

$$\rho_{\text{BH}} = \frac{m_{\text{pl}}^2}{6\pi (t + t_{\text{eq}}/3)^3}.$$  \hfill (19)

Such forms of Eqs. (18) and (19) are dictated by the continuity of the Hubble parameter and of the energy
density (i.e. by equality of $\rho_{\text{rel}}$ and $\rho_{\text{BH}}$) at $t = t_{\text{eq}}$. The MD regime (19) lasted till $t = \tau_{\text{BH}}$, when instant explosion of PBHs created new relativistic plasma with the temperature:

$$T_{\text{rel}}^4 = \frac{5m_{\text{pl}}^2}{\pi^3 g^2(T_{\text{heat}})(\tau_{\text{BH}} + t_{\text{eq}}/3)^2}. \quad (20)$$

Here the instant thermalization is assumed.

The temperature of the relativistic plasma coexisting with the dominant PBH dropped down as the scale factor:

$$T_{\text{rel}} = T_{\text{eq}} \frac{a_{\text{eq}}}{a_{\text{in}}(\tau_{\text{BH}})} = T_{\text{eq}} \left( \frac{4t_{\text{eq}}}{3\tau_{\text{BH}} + t_{\text{eq}}} \right)^{2/3}. \quad (21)$$

Correspondingly the temperature of the newly created by the PBH decay relativistic plasma could be much higher than $T_{\text{rel}}$ given by Eq. (21). The entropy suppression factor, which is equal to the cube of the ratio of the temperatures of the new relativistic plasma created by the PBH instant evaporation to temperature of the “old” one, plus unity from the entropy of the old relativistic plasma is equal to:

$$S = 1 + \left( \frac{T_{\text{heat}}}{T_{\text{rel}}} \right)^3 = 1 + \left( \frac{a(\tau_{\text{BH}})}{a_{\text{eq}}} \right)^{3/4} = 1 + \frac{5\tau_{\text{BH}}}{4t_{\text{eq}}} \left( 1 + \frac{t_{\text{eq}}}{3\tau_{\text{BH}}} \right)^{1/2}. \quad (22)$$

Our approach is valid for $\tau_{\text{BH}} \geq t_{\text{eq}}$ and in the limiting case of $\tau_{\text{BH}} = t_{\text{eq}}$ the entropy suppression factor is $S = 2$ coming from the relativistic matter and from PBH in equal shares. Since the minimal value of the ratio

$$\frac{\tau_{\text{BH}}}{t_{\text{eq}}} = \frac{CM_{\text{BH}}^2\epsilon^2}{m_{\text{pl}}^2}$$

is equal to unity, the minimal mass of PBH for which we can trust the approximate calculations presented above is

$$M_{\text{BH}} > M_{\text{BH}}^{\text{min}} \equiv m_{\text{pl}} \frac{C}{\epsilon \sqrt{C}} = 4 \times 10^6 \left( \frac{10^{-12}}{\epsilon} \right), \quad (23)$$

where $C = 30$, according to Eq. (15).

For large $\tau \gg t_{\text{eq}}$, when $S$ is large, it is approximately equal to

$$S \approx \frac{3\tau_{\text{BH}}}{4t_{\text{eq}}} = \frac{\sqrt{3C}M_{\text{BH}}}{2m_{\text{pl}}} = 2.14 \times 10^{-7} (\epsilon/10^{-12})(M_{\text{BH}}/g). \quad (25)$$

The PBH mass is bounded from above by the condition that the heating temperature after evaporation should be higher than the BBN temperature, ~1 MeV. From Eq. (20) it follows that

$$T_{\text{heat}} = 0.06m_{\text{pl}} \left( \frac{m_{\text{pl}}}{M_{\text{BH}}} \right)^{3/2}. \quad (26)$$

Hence the PBH masses should be below $10^9$ g.

The entropy suppression factors for $\epsilon = 10^{-12}$ as functions of $M_{\text{BH}}$ are presented in Fig. 1 for small and large masses respectively.

## 3. EXACT SOLUTION FOR DELTA-FUNCTION PBH MASS SPECTRUM

Here we relax the instant decay approximation and solve numerically equations describing evolution of the cosmological energy densities of non-relativistic PBHs and relativistic matter. It is convenient to work in terms of dimensionless time variable $\eta = t/\tau_{\text{BH}}$, when the equations can be written as:

$$\frac{d\rho_{\text{BH}}}{d\eta} = -(3H\tau_{\text{BH}} + 1)\rho_{\text{BH}}, \quad (27)$$

$$\frac{d\rho_{\text{rel}}}{d\eta} = -4H\tau_{\text{BH}}\rho_{\text{rel}} + \rho_{\text{BH}}. \quad (28)$$
We present the energy densities of PBH and relativistic matter respectively in the forms:
\[ \rho_{BH} = \rho_{BH}^{(in)} \exp(-\eta + \eta_{in}) y_{BH}(\eta)/z(\eta)^3, \]  
(29)
\[ \rho_{rel} = \rho_{rel}^{(in)} y_{rel}(\eta)/z(\eta)^4, \]  
(30)
where \( y_{rel}^{(in)} = y_{BH}^{(in)} = 1 \) and 
\[ \eta_{in} = \frac{m_{Pl}^2}{CM_{BH}^2} \ll 1. \]  
(31)

The constant \( C \) is determined in Eq. (15).

The redshift factor \( z(\eta) = a(\eta)/a_{in} \) satisfies the equation:
\[ \frac{dz}{d\eta} = H \tau_{BH} z, \]  
(32)
where the Hubble parameter \( H \) is determined by the usual expression for the spatially flat universe:
\[ \frac{3H^2m_{Pl}^2}{8\pi} = \rho_{rel} + \rho_{BH}. \]  
(33)

Using Eqs. (30) and (29) with \( \rho_{rel}^{(in)} \) given by Eq. (3) at \( t = t_{in} \) and bearing in mind that \( \rho_{BH}^{(in)} = \epsilon \rho_{rel}^{(in)} \) we find
\[ H \tau_{BH} = \frac{C M_{BH}^2}{2 m_{Pl}^2} \left( \frac{y_{rel}}{z^4} + \frac{\epsilon}{z^3} e^{-\eta_{BH}} \right)^{1/2}. \]  
(34)

Evidently Eq. (27) with \( \rho_{BH} \) given by (29) is solved as
\[ y_{BH}(\eta) = y_{BH}^{(in)} = 1, \]  
(35)
while \( \rho_{rel}(\eta) \) satisfies the equation:
\[ \frac{dy_{rel}}{d\eta} = \epsilon z(\eta) e^{-\eta + \eta_{BH}}. \]  
(36)

Equations (32) and (36) can be solved numerically with the initial conditions at \( \eta = \eta_{in} \)
\[ y_{BH} = y_{rel} = z = 1. \]  
(37)

However, a huge value of the coefficient \( H\tau \) makes the numerical procedure quite slow. To avoid that we introduce the new function \( W \) according to:
\[ z = \sqrt{W}/\epsilon \]  
(38)
and arrive to the equations:
\[ \frac{dW}{d\eta} = C e^{2} \left( \frac{M}{m_{Pl}} \right)^2 (y_{rel} + \sqrt{W} e^{-\eta_{BH}})^{3/2}, \]  
(39)
\[ \frac{dy_{rel}}{d\eta} = \sqrt{W} e^{-\eta + \eta_{BH}}, \]  
(40)
where \( W(\eta_{in}) = \epsilon^2 \). Entropy release from PBH evaporation can be calculated as follows. In the absence of PBHs the quantities conserved in the comoving volume evolved as \( 1/z^3 \). With extra radiation coming from the PBH evaporation the entropy evolves as \( y_{rel}^{3/4}/z^3 \), see Eq. (30). Hence the suppression factor of the relative number density of frozen dark matter particles or earlier generated baryon asymmetry tends to:
\[ S = [y_{rel}(\eta)]^{3/4} \]  
(41)
when time goes to infinity. The temporal evolution of \( S \) is depicted in Fig. 2 for different values of \( M_{BH} = 10^7, 10^8, 10^9 \) g and \( \epsilon = 10^{-12} \).

For large \( \eta \) (in fact for \( \eta > 15 \)) the suppression factor \( S \) tends, as expected, to a constant value. The results presented in Fig. 2, very well agree with the approximates ones demonstrated in Fig. 1.

In Fig. 3 the asymptotic values of the entropy suppression factor are presented for different PBH masses
and $\epsilon = 10^{-12}$. They are also very close to the limiting values which can be extracted from Fig. 1.

The ratio of the entropy suppression factor of the exact fixed mass calculations to that performed in the approximation of the instant decay and of the instant change of the expansion regime as a function of mass for $\epsilon = 10^{-12}$ is presented in Fig. 4. A rise of this ratio at small $M$ can be understood by underestimation of entropy release in the instant approximation. Indeed for $M$ smaller than the boundary value given by Eq. (24) the entropy release would be zero while the exact calculations lead to nonzero result, so their ratio would tend to infinity.

4. EXTENDED MASS SPECTRUM

Let us now consider, instead of delta-function, an extended mass distribution:

$$
\frac{dN_{BH}}{dM} = f(M,t),
$$

(42)

where $N$ is the number density of PBH. Since PBHs are nonrelativistic, their differential energy density is

$$
\frac{d\sigma_{BH}}{dM} \equiv \sigma(M,t) = Mf(M,t),
$$

(43)

PBH created by the old conventional mechanism [1, 2] are supposed to have sharp, even delta function mass spectrum. However, in several later works the mechanisms leading to extended mass spectrum have been explored, for the early papers see e.g. [5–8].

We assume that the number and energy densities of PBHs are strictly confined between $M_{\text{min}}$ and $M_{\text{max}}$. The value of $M_{\text{max}}$ should be below the upper limit $M = 10^9$ g, which is imposed by the condition that PBH evaporation would not distort successful results of BBN-theory. However, a small fraction of PBHs may have masses higher than $10^9$ g and their impact on BBN can be interesting, though not yet explored in full.

The minimal value of PBH mass $M_{\text{min}}$ should be

$$
M_{\text{min}} \geq \frac{M_{\text{BH}0}}{x} M_{\text{min}},
$$

(44)

where $M_{\text{BH}0}$ is the mean value of the mass density distribution or the value where $\sigma(M, t)$ reaches maximum, and $x$ is a nonzero in the limits:

$$
x_{\text{min}} \equiv \frac{M_{\text{min}}}{M_{\text{BH}0}} \leq x \leq x_{\text{max}} \equiv \frac{M_{\text{max}}}{M_{\text{BH}0}}.
$$

(45)

We define now the dimensionless “time” $\eta$ as $\eta = t/\tau(M_{\text{BH}0})$ where $\tau(M_{\text{BH}0}) = \tau_{\text{eq}}$ is the life time of PBH with mass $M_{\text{BH}0}$. All the PBHs have different masses and hence their lifetimes (15) and the moments of formation (2) are different.

The time evolution of the differential energy density of PBHs, is governed by the equation:

$$
\frac{d\sigma}{d\eta} = \left[3H + \frac{1}{\tau(M)}\right] \sigma(M,t),
$$

(46)

where $\Gamma(M) = 1/\tau(M) = m_{\text{Pl}}^2/(CM^2)$, see Eq. (15).

In terms of dimensionless time $\eta$, the above expression takes the form:

$$
\frac{d\sigma}{d\eta} \equiv \sigma' \equiv \sigma' = \left[3H \tau_0 + \left(M_{\text{BH}0}/M\right)^{3/2}\right] \sigma.
$$

(47)

The initial value of $\eta$ is the moment of BH formation. It depends upon $M$ and, according to Eq. (31), is equal to

$$
\eta_{\text{form}}(M) = \frac{m_{\text{Pl}}^2 M}{CM_{\text{BH}0}^2}.
$$

Evidently $\sigma(M) = 0$ when $\eta(M) < \eta_{\text{form}}$. 

The equation describing the evolution of the energy density of relativistic matter now takes the form:

\[
\frac{d\rho_{\text{rel}}}{d\eta} = -4H\tau_0\rho_{\text{rel}} + \int dM(M_0/M)^3\sigma(M). \tag{48}
\]

In analogy with the previous section we introduce the red-shift function normalized to the value of the scale factor when the least massive PBH was formed:

\[
z(\eta) = a(\eta)/a_{\text{form}}(M_{\text{min}}). \tag{49}
\]

The evolution of \(z(\eta)\) is determined by the equation, analogous to Eq. (32):

\[
\frac{dz}{d\eta} = H\tau_0z \tag{50}
\]

with the Hubble parameter now given by

\[
\frac{3H^2m_{\text{pl}}^2}{8\pi} = \rho_{\text{rel}} + \rho_{\text{PBH}} = \rho_{\text{rel}} + \int dM\sigma(M). \tag{51}
\]

Equation (46) has the following solution

\[
\sigma(M, \eta) = \theta(\eta - \eta_f)\sigma(M, \eta_f) \times \exp\left[\left(\eta_f - \eta\right)\left(M_0/M\right)^3\right] \left(z(\eta_f)/z(\eta)\right)^3, \tag{52}
\]

where for brevity we have introduced the new notation \(\eta_f \equiv a_{\text{form}}(M)\) and the theta-function ensures vanishing of the solution for \(\eta < \eta_f\). The initial value of the PBH density at the moment of formation \(\sigma(\eta_f/(M))\) (47) is determined by the fraction \(\epsilon(M)\) of the energy density of PBH with mass \(M\) with respect to the energy density of the relativistic matter at the moment of PBH formation:

\[
\sigma(M, \eta_f(M)) = \epsilon(M)\rho_{\text{rel}}(\eta_f(M))/M, \tag{53}
\]

where \(\epsilon(M)\) depends upon the scenario of PBH formation and will be taken below according to some reasonable assumptions. In any case we assume that \(\epsilon(M)\) vanishes if \(M < M_{\text{min}}\) and \(M > M_{\text{max}}\).

Let us assume that in the time interval \(\eta_f/(M_{\text{min}}) < \eta < \eta_f/(M_{\text{max}})\), the total fraction of PBH mass density is negligibly small in comparison with the energy density of relativistic matter, and so the expansion regime is the nondisturbed relativistic one, see Eqs. (3), (4). Accordingly using Eq. (2), we find that the energy density of relativistic matter at the moment of the creation of the “first” lightest black holes is

\[
\rho_{\text{rel}}(\tau_{\text{in}}) = \frac{3}{32\pi M_{\text{min}}^2}m_{\text{pl}}^6. \tag{54}
\]

If the energy density of PBH remains small in comparison with that of relativistic matter till formation of the heaviest PBHs, then the last term in the r.h.s. of Eq. (48) can be neglected and thus in the time interval \(\eta(M_{\text{min}}) < \eta < \eta(M_{\text{max}})\), the energy density \(\rho_{\text{rel}}\) is equal to:

\[
\rho_{\text{rel}} = \frac{3}{32\pi M_{\text{min}}^2}m_{\text{pl}}^6 \frac{1}{\zeta(\eta)^3}. \tag{55}
\]

Hence the differential PBH energy density evolves as

\[
\sigma(M, \eta) = \frac{3m_{\text{pl}}^6}{32\pi M_{\text{min}}^2} \epsilon(M) \times \left(\frac{\eta_f(\eta_f)}{\eta - \eta_f(M)}\right)\exp\left[\left(\eta_f - \eta\right)\left(M_0/M\right)^3\right]\left(z(\eta_f)/z(\eta)\right)^3. \tag{56}
\]

In this equation \(\eta\) runs in the limits \(\eta(M_{\text{min}}) < \eta < \eta(M_{\text{max}})\) or \(\eta_f/(M) < \eta < \eta_f(M_{\text{max}})\), depending upon which lower limit is larger.

Since \((M_0/M)^3\eta_f/(M) = m_{\text{pl}}^2/(CM^2) \ll 1\), for any \(\eta\), we may expand the exponent as

\[
\exp[-(M_0/M)^3(\eta - \eta_f(M))] = \exp[-(M_0/M)^3\eta(1 + m_{\text{pl}}^2/(CM^2))]. \tag{57}
\]

Due to the necessity to integrate over \(M\) the relevant evolutionary equations are integro-differential and the numerical calculations generally become quite cumbersome. However, we can consider some simplified forms of the initial mass distribution of the PBH for which the integrals over \(M\) can be taken analytically and after that the differential equations can be quickly and simply solved. Using such toy models we can understand essential features of the entropy production by PBH with extended mass spectrum. Unfortunately we could not find a workable toy model for a realistic log-normal mass spectrum, see [12]. Nevertheless the spectra which allows for analytic integration can be quite close numerically to the log-normal one.

In what follows we consider a couple of illustrative examples, assuming that the function

\[
F(x) = \epsilon(M)/(\eta_f(M)) \tag{58}
\]

is confined between \(x_{\text{min}} = (M_{\text{min}}/M_0)\) and \(x_{\text{max}} = (M_{\text{max}}/M_0)\). Here according to Eq. (53) \(\epsilon(M)\) is the fraction of the energy density of PBH with mass \(M\) at the moment of PBH creation. For simplicity we assume that \(F(x)\) is a polynomial function of integer powers of \(x\), though the latter is not necessary.

We take two examples for \(F\):

\[
F_1(x) = \epsilon_0/(x_{\text{max}} - x_{\text{min}}) \tag{59}
\]

for \(x_{\text{min}} < x < x_{\text{max}}\) and \(F_1 = 0\) for \(x\) outside of this interval. Evidently the point \(x = 1\) should be inside this interval.

Another interesting form of \(F\) is

\[
F_2(x) = \frac{\epsilon_0}{N} a^2b^2(1/a - 1/x)^2(1/x - 1/b)^2. \tag{60}
\]

Here \(N\) is the normalization factor, chosen such that the maximum value of \(F_2/\epsilon = 1\). This function vanishes at \(x = x_{\text{min}} = a\) and \(x = x_{\text{max}} = b\), with vanishing derivatives at these points, and \(F_2\) being identically
zero outside of this interval. \( F_2 \) reaches maximum at 
\[ x_0 = \frac{2ab}{(a + b)} \]  
with 
\[ F_2^{(\text{max})} = \frac{\varepsilon_0}{16} N a^2 b^2 \left( \frac{1}{a} - \frac{1}{b} \right)^4 = 1. \]  
(61)
\( F_2 \) can be quite close numerically to the log-normal distribution with a proper choice of parameters. As a working example we take \( a = 1, b = 30 \) and compare \( F_2 \) with the log-normal function:
\[ F_{\text{LN}} = \varepsilon \exp[-1.5(\log^2(15x))]. \]  
(62)
There are two following integrals, which enter the evolution Eqs. (51) and (48):
\[ I_0 = \int dM \sigma(M, \eta) \]  
(63)
and
\[ I_3 = \int dM \left( \frac{M_0}{M} \right)^3 \sigma(M, \eta). \]  
(64)

We can calculate them explicitly making some simplifying assumptions about the form of \( F \) (58), which are discussed in the following subsections.

4.1. Calculations for the Flat Spectrum

Here we find the entropy suppression factor for the “flat” \( F(x) \):
\[ F_1(x) = \frac{\varepsilon_0 (M)}{z(\eta_f(M))} = \frac{\varepsilon_0}{b - a} = \text{const} \]  
(65)
if \( x \) is in the limits \( a \equiv x_{\text{min}} < x < b \equiv x_{\text{max}} \) and \( F_1(x) = 0 \) outside this region. Parameters \( a \) and \( b \) here and in what follows, Eq. (75), define the width of the mass spectrum. There is some but rather mild dependence of \( S \) on them. Since there is no essential difference between the entropy suppression for extended and delta function mass spectra, the variation of \( a \) and \( b \) is not of much importance.

Using Eq. (56) we find:
\[ f_0^{(i)} = \int_{M_{\text{min}}}^{M_{\text{max}}} dM \sigma(M, \eta) = \frac{3m_0^6 \varepsilon_0}{32\pi \varepsilon^2 (\eta) M_{\text{min}}^2 (b - a)} \]
\[ \times \left[ \int_{M_{\text{min}}}^{M_{\text{max}}} \frac{\theta(\eta - \eta_f(M))}{M} \exp\left[\left(M_0/M\right)^3 (\eta - \eta_f(M))\right] \right] \]
\[ = K(\eta) \frac{b}{b - a} \int_{x_{\text{min}}}^{x_{\text{max}}} \frac{dx}{x^3} \exp\left[ x^3 (\eta - \eta_f(M)) \right] \]
\[ \equiv K(\eta) j_{10}(a, b, \eta, \eta_f), \]
where \( x = M_0/M \) and
\[ K(\eta) = \frac{3m_0^6 \varepsilon_0}{32\pi \varepsilon^2 (\eta) M_{\text{min}}^2}. \]  
(67)

\[ I_3^{(i)} = \int_{M_{\text{min}}}^{M_{\text{max}}} dM \left( \frac{M_0}{M} \right)^3 \sigma(M, \eta) \]
\[ = K(\eta) \frac{b}{b - a} \int_{x_{\text{min}}}^{x_{\text{max}}} \frac{dx}{x^4} \exp\left[ x^3 (\eta - \eta_f(M)) \right] \]
\[ \equiv K(\eta) j_{13}(x_{\text{min}}, x_{\text{max}}, \eta, \eta_f). \]  
(68)

We take integrals \( j_{10} \) and \( j_{13} \) analytically, using Mathematica, and substitute them into Eqs. (47) and (48), and (49), which solve numerically. Since \( \eta_f(M) \ll \eta \) in almost all integration interval we neglect \( \eta_f \), see also Eq. (57). The results are presented in Appendix B.

We will search for the solution as it is done in Section 4 taking \( \rho_{\text{rel}} \) in the form:
\[ \rho_{\text{rel}} = y_{\text{rel}}^{(\text{in})} / z^4, \]  
(69)
where \( \rho_{\text{rel}}^{(\text{in})} = 3m_0^6 / (32\pi M_{\text{min}}^2) \) and so \( y_{\text{rel}} \) and \( z \) satisfy the equations:
\[ y_{\text{rel}} = \varepsilon_0 z(\eta) j_{13}, \]  
(70)
\[ z(\eta) = \frac{CM_0^3}{2m_0^2 M_{\text{min}}} \left( \frac{y_{\text{rel}} + \varepsilon_0}{z^4} \right)^{1/2} j_{10}. \]  
(71)

In analogy with Eq. (38) we introduce new function \( W_e \) according to
\[ z = \sqrt{W_e / \varepsilon_0}. \]  
(72)
and obtain:

\[
\frac{dW_e}{d\eta} = \frac{C\varepsilon_0^2M_0^2}{m_{\text{pl}}M_{\text{min}}} (y_{\text{rel}} + \sqrt{W_e\bar{f}(\eta)})^{1/2}
\]

\[
eq \frac{C\varepsilon_0^2M_0^2}{m_{\text{pl}}a} (y_{\text{rel}} + \sqrt{W_e\bar{f}(\eta)})^{1/2}, \tag{73}
\]

\[
\frac{dy_{\text{rel}}}{d\eta} = \sqrt{W_e\bar{f}(\eta)} \tag{74}
\]

with the initial conditions \(W_e^{(\text{in})} = \varepsilon^2\) and \(y_{\text{rel}}^{(\text{in})} = 1\).

These equations can be integrated numerically. The asymptotic value of \(y_{\text{rel}}^{3/4}\) at large \(\eta\), which is the entropy suppression factor according to Eq. (41) is presented in Figs. 6, 7 all for \(\varepsilon = 10^{-12}\) and \(x_{\text{min}} = 1/3\) and \(x_{\text{max}} = 5/3\). The result is proportional to \(M_{\text{BH}}\) and reasonably well agrees with the approximate results calculated in the instant decay and in the instant change of regime approximations (25).
Here we assume that

\[ F_2(x) = \epsilon(M)/z(\eta_j(M)) = \frac{\epsilon_0 a^2 b^2 (1/a - 1/x)^2 (1/x - 1/b)^2}{16a^2 b^2 (1/a - 1/b)^4}. \] (75)

Correspondingly Eqs. (66) and (68) are modified by insertion of the factor \( F_2(x) \) into the integrands. The expressions for \( j_{(20)} \) and \( j_{(23)} \) are presented in Appendix B.

Evolution equations coincides with those in the previous subsection after the change \( j_{(10)} \to j_{(20)} \) and \( j_{(13)} \to j_{(23)} \). The entropy suppression factor for the continuous mass spectrum and different values of the parameters, indicated in the figure captions, are presented in Figs. 8, 9.

We see that the entropy suppression factor for both forms of extended mass spectra, the rectangular and more realistic log-normal one, behave as a function of the central value of the PBH mass and are essentially similar to that calculated for the delta-function mass spectrum in Sections 2 and 3 and changes from the factor 2–3 for \( M = 10^7 \) g up to 100–300 for \( M = 10^9 \) g.

However, the comparison is ambiguous because it depends upon the normalization of the spectra, e.g. if...
we compare them at the equal mass densities of PBHs or at their equal number densities. It also depends upon the widths of the extended spectra. Anyhow the outcome is the same within an order of magnitude. The dependence on $e$ is very accurately the same as it was found in the analytical calculations of Section 2.

5. CONCLUSIONS

As it is shown in this work, the suppression of thermal relic density or of the cosmological baryon asymmetry may be significant if they were generated prior to PBH evaporation. In the simplified approximation of the delta-function mass spectrum of PBH, instant decay of PBH, and instant change of the expansion regimes from the initial dominance of relativistic matter to non-relativistic BH dominance and back the entropy suppression factor, $S$, can be calculated analytically, Eq. (25). Exact calculations but still with delta-function mass spectrum are in very good agreement with the approximate one.

The result is proportional to the product $\epsilon M_{\text{BH}}$, and e.g. for $M_{\text{BH}} = 10^9 g$ and $\epsilon = 10^{-12}$ the suppression factor is $S \approx 400$. The black hole mass equal to $10^9 g$ is the maximum allowed value of the early evaporated PBH mass permitted by BBN, see conclusion below Eq. (26). This statement is true if PBH dominated in the early universe before the onset of BBN. This could take place if the minimal PBH mass is given by Eq. (24).

The calculations with more realistic extended mass spectra of PBHs show similar features of the suppression factor $S$, which is also proportional to $e$ and to the central value of the mass distribution. There is some dependence on the form of the spectrum and on the values of $M_{\text{max}}$ and $M_{\text{min}}$, but they do not change our results essentially.

The significant restriction of the parameter space of the minimal supersymmetric model by LHC created some doubts about dark matter made of the lightest supersymmetric particles. Moreover, the usual WIMPs with masses below teraelectron-volts seem to be excluded. The mechanism considered here allows to save relatively light WIMPs and open more options for SUSY dark matter. Since according to famous Zeldovich result the ratio of the cosmological energy density of massive relic species with respect to the entropy density is proportional to the square of these particle mass, the possible rise of the entropy density by factor 100 permits to increase the allowed particle mass 10-fold thus opening the option for SUSY dark matter above the limits imposed by LHC.

Similar dilution of cosmological baryon asymmetry by an excessive entropy release may look not so essential, because theoretical estimates of the asymmetry are rather uncertain since they strongly depends upon the unknown parameters of the theory at high energies. However, there are a couple of exceptions for which the dilution may be of interest. Firstly, there is the Affleck–Dine [26] scenario of baryogenesis, which naturally leads to the magnitude of the asymmetry much higher than the observed one, $\beta \sim 10^{-9}$. The suppression by 1–2 orders of magnitude might be helpful, though not always sufficient.

Another example is baryo-thru-lepto genesis [27], for a review see [28]. According to this model cosmological baryon asymmetry arose from initially generated lepton asymmetry, which was produced in the decays of heavy Majorana neutrinos. In some models the parameters of CP-violating decays of this heavy neutrino can be related to the CP-odd phases in light neutrino oscillations. Hence one can predict the magnitude and sign of the lepton asymmetry. With an unknown dilution of the asymmetry the magnitude cannot be predicted but the sign probably can.

APPENDIX A

We estimate here the density of stable supersymmetric relics produced in PBH evaporation and show that their contribution to the cosmological dark matter is insignificant, due to very low density of the PBHs and because of their quick cooling by the background relativistic particles. To this end we will present here a few simple estimates and numerical values.

The moment of PBH production with mass $M$ is (2):

$$ t_{\text{in}} = \frac{M}{m_{\text{Pl}}^2} = 2.5 \times 10^{31} M_8 \text{ s}, $$

(76)

where $M_8 = M/(10^8 \text{ g}).$

By assumption at the moment of production PBHs make a small fraction $e \ll 1$ of the energy density of relativistic matter. So the energy and number densities of PBH at $t = t_{\text{in}}$ are respectively:

$$ \rho_{\text{BH}}^{(\text{in})} = \frac{3n_{\text{Pl}}^6}{32 \pi M^2}, \quad n_{\text{BH}}^{(\text{in})} = \frac{3n_{\text{Pl}}^6}{32 \pi M^2}. $$

(77)

The energy density of the relativistic matter at $t = t_{\text{in}}$ is:

$$ \rho_{\text{rel}}^{(\text{in})} = \frac{3m_{\text{Pl}}^6}{32 \pi M^2} = \frac{\pi^2 g_*^{(\text{in})}}{30} T_{\text{in}}^4, $$

(78)

where $g_*^{(\text{in})} \approx 100$ is the number of relativistic species at $T = T_{\text{in}}$. Correspondingly the temperature of the relativistic cosmological plasma at the moment of PBH production is equal to

$$ T_{\text{in}} = 1.72 \times 10^{12} \text{ GeV}/\sqrt{M_8}. $$

(79)

The ratio on PBH number density to that of relativistic particles at the moment of creation can be estimated as:

$$ r_{\text{in}} = \frac{n_{\text{BH}}^{(\text{in})}}{n_{\text{rel}}^{(\text{in})}} = \rho_{\text{BH}}^{(\text{in})}/\rho_{\text{rel}}^{(\text{in})} \frac{T_{\text{in}}}{0.3M} \approx 0.9 \times 10^{-31} \epsilon_3 M_8^{3/2}, $$

(80)
where $\epsilon_{12} = 10^{12} \epsilon$ and $n_{\text{rel}} \approx 0.3 \rho_{\text{rel}} / T$.

This ratio remains approximately constant till the PBH decay because both densities are almost conserved in the comoving volume up to the entropy release created by massive particle annihilation. As we see in what follows, the temperature of the relativistic matter at the moment of PBH decay is about 20–30 MeV and so at that moment $g_* \sim 10$. Hence the ratio $r$ drops down by factor 10.

The average distance between PBHs at the moment of their creation is

$$d_{\text{in}}^{(\text{BH})} = (n_{\text{BH}}^{(\text{in})})^{1/3} = 2.4 \times 10^{-16} M_8 \epsilon_{12}^{-1/3} \text{ cm.} \quad (81)$$

At the moment of equilibrium, when densities of BH and relativistic matter became equal, the average distance of BH separation was

$$d_{\text{eq}}^{(\text{BH})} = d_{\text{in}}^{(\text{BH})} / \epsilon = 2.4 \times 10^{-4} M_8 \epsilon^{-4/3} \text{ cm.} \quad (82)$$

The temperature of the relativistic matter at the equilibrium moment was

$$T_{\text{eq}} = \epsilon T_{\text{in}}^{1/3} = 3.7 \epsilon_{12} M_8^{-1/2} \text{ GeV}, \quad (83)$$

where $S_{\text{eq}}$ is the ratio of the number of particle species at $T_{\text{in}}$ to that at $T_{\text{eq}}$:

$$S_{\text{eq}} = g_*(10^5 \text{ GeV}) / g_*(3 \text{ GeV}) \approx 10. \quad (84)$$

Since before the equilibrium the universe expanded in relativistic regime, when the scale factor rose as $a(t) \sim t^{1/2}$, the equilibrium is reached at the moment of time:

$$t_{\text{eq}} = t_{\text{in}} / \epsilon^2 = 2.5 \times 10^{-7} M_8 \epsilon_{12}^2 \text{ s.} \quad (85)$$

After that and till the moment of BH decay at

$$t = \tau = 30 M_{\text{BH}}^3 / m_{\text{Pl}}^4 = 1.6 \times 10^{-4} M_8^3 \text{ s} \quad (86)$$

the universe expanded in matter dominated regime, $a(t) \sim \rho^{1/3}$. So during this MD stage the scale factor rose as:

$$z(t) = \left( \frac{t}{t_{\text{eq}}} \right)^{2/3} = 74 \epsilon_{12} M_8^{4/3}. \quad (87)$$

Correspondingly the energy density of PBHs just before the moment of their decay would be larger than the energy density of the relativistic background by this redshift factor, $z(t)$:

$$\rho_{\text{BH}}(t) / \rho_{\text{rel}}(t) = 74 \epsilon_{12} M_8^{4/3}. \quad (88)$$

The temperature of the relativistic background just before the BH decay was

$$T_{\text{cool}} = T_{\text{rel}}(t) = T_{\text{eq}} / z(t) = 50 \epsilon_{12}^{-1/3} M_8^{-11/6} \text{ MeV.} \quad (89)$$

The temperature of the particles produced in the BH decay is equal to:

$$T_{\text{BH}} = \frac{m_{\text{Pl}}^2}{8\pi M_8} = 10^5 M_8^{-1} \text{ GeV.} \quad (90)$$

So the lightest supersymmetric particles (LSP) of the minimal SUSY model with the mass $m_X \sim 10^3 \text{ GeV}$ should be abundantly produced in the process of the PBH evaporation with $T_{\text{BH}} \gg m_X$, contributing about 0.01–0.1 to the total number of the produced particles.

Since PBHs were produced at temperatures much higher than the temperature of the surrounding relativistic plasma, initially they did not lose mass through evaporation but instead gained it by the accretion of matter. However, this effect is quite weak. Indeed, the PBH surface is $4\pi r^2$ and the flux of the external radiation is $\sim T^4$. So the rate of mass increase would be

$$\dot{M} = 4\pi r^2 T^4. \quad (91)$$

According to Eq. (3) $T \approx m_{\text{Pl}}/30$ and thus $dt = -m_{\text{Pl}} dT/(15 T^3)$. Hence the mass increase is equal to

$$\Delta M = \frac{4\pi}{30} m_{\text{Pl}} r^2 T_{\text{in}}^2, \quad (92)$$

where $T_{\text{in}}$ is given by Eq. (79). So finally

$$\frac{\Delta M}{M} = 0.1. \quad (93)$$

It is interesting that the relative mass increase does not depend upon the PBH mass value.

This result contradicts to that advocated in [2] as indicated to us by the referee. Indeed based on the estimate of the accretion rate, presented in [1], the calculations of [2] led to the huge increase of the PBH mass up to billions of solar masses or even much beyond that. This is surely an overestimate. The arguments of [1, 2] are based on the application of the Bondi accretion mechanism to the early universe. However, the physical situations in the matter dominated contemporary universe and in the radiation dominated early universe are very much different. PBHs are supposed to be at rest in the comoving frame and the flux of the relativistic particles could be induced only by the gravitational attraction of the surrounding relativistic particles to a PBH, which has very low efficiency, especially due to the Hubble expansion. The calculation of the accretion efficiency at the different epochs of the cosmological evolution is reviewed and performed in [33]. Quoting this work: “For PBHs much smaller than $M_{\text{cr}}$ accretion is completely unimportant,” where $M_{\text{cr}}$ is about $5 \times 10^{14} \text{ g}$. PBH with mass larger than $M_{\text{cr}}$ could survive to the present time, while lighter ones would evaporate before they reach favorable to accretion times. Since we deal with BHP lighter than $10^9 \text{ g}$, we neglect their possible mass rise after formation.

The average distance between PBH just before their decay was:
\[ d^{BH}(\tau) = d_{\text{BH}}^{(BH)} z(\tau) = 1.75 \times 10^{-2} M_8^{7/3} \text{ cm}. \quad (94) \]

The total number of energetic particles produced by the decay of a single BH is:

\[ N_{\text{hot}} = \frac{M_{\text{BH}}}{3 \tau_{\text{BH}}} = \frac{8\pi}{3} \left( \frac{M}{m_p} \right)^2 = 1.8 \times 10^{26} M_8^2. \quad (95) \]

We assume the following model: as a result of BH instant evaporation each black hole turns into a cloud of energetic particles with temperature \( T_{\text{BH}} = 10^5 M_8 \) GeV, with radius \( \tau_{\text{BH}} \), see e.g. Eq. (86):

\[ \tau_{\text{BH}} = 4.8 \times 10^{10} M_8^3 \text{ cm}. \quad (96) \]

This radius is much larger than the average distance between the BHs (94) and the number of PBHs in this common cloud is

\[ N_{\text{cloud}} = (\tau_{\text{BH}}/d_{\text{BH}}(\tau))^3 = 2 \times 10^{25} M_8^7, \quad (97) \]

so their number density just before the decay was

\[ n_{\text{BH}}(\tau) = d(\tau)^{-3} = 1.9 \times 10^5 M_8^{-7} \text{ cm}^{-3}. \quad (98) \]

The density of the hot particles with temperature \( T_{\text{BH}} \), created by the evaporation of this set of black holes is:

\[ n_{\text{hot}} = n_{\text{BH}}(\tau) H_{\text{hot}} = 3.4 \times 10^{31} M_8^{-5} \text{ cm}^{-3}. \quad (99) \]

The density of cool background particles with temperature \( T_{\text{cool}} \) is

\[ n_{\text{cool}} = 0.1 g* T_{\text{cool}}^3 = 1.6 \times 10^{37} e_1^{-1} M_8^{-11/2} \text{ cm}^{-3}, \quad (100) \]

where we took \( g* = 10 \) at \( T < 100 \text{ MeV} \). Note that 

\[ n_{\text{cool}} \gg n_{\text{hot}}. \]

The particles produced by PBH evaporation consist predominantly of some light or quickly decaying species and a little of stable lightest supersymmetric particles (or any other stable particles, would-be dark matter), denote them as \( X \). Since by assumption \( T_{\text{BH}} \) is higher than the SUSY mass scale, the total number of all supersymmetric partners created through evaporation should be equal to the number of all other particles. Each SUSY partner produces one LSP (\( X \)-particle) in the process of its decay and a few other particle species. So the number of \( X \)-s became roughly about one per cent of the number of other particle number. More precise value is not of much importance here. This ratio further significantly dropped down in the process of thermalization, see below.

The ejected energetic particles propagate in the background of much colder plasma and cool down simultaneously heating the background. The cooling proceeds, in particular, through the Coulomb-like scattering, so the momentum of hot particles decreases according to the equation (the term related to the universe expansion is neglected there because the characteristic time scale of cooling is much shorter than the Hubble time at \( T \sim 100 \text{ MeV} \)): \[ E_{\text{hot}} = -\sigma n_{\text{cool}} \delta E, \quad (101) \]

where \( \delta E \) is the momentum transfer from the hot particles to the cold ones. The scattering cross section can be approximated as \( a = \alpha^2 g*/(\rho_1 - \rho_2)^2 \). For massless particles

\[ q^2 = (p_1 - p_2)^2 = -2(E_1 E_2 - \rho_1 \rho_2). \quad (102) \]

Here \( E_1 \) and \( E_2 \) are the initial and final energies of cold particles, \( E_1 \sim T_{\text{cool}} \) and \( \delta E \equiv (E_2 - E_1) \sim E_2 \). For a noticeable energy transfer large angle scattering is necessary, so \( q^2 \sim E_1 E_2 \). Finally

\[ \dot{E} = 0.1 g* T_{\text{cool}}^3 \alpha^2 / E_1 = 10^{-4} T_{\text{cool}}^2 \]

\[ = 6 \times 10^{18} \text{ MeV/s}. \]

Correspondingly the energy loss of hot particles equal by the order of magnitude to their temperature (90) would be achieved during the very short time:

\[ t_{\text{cool}} = 10^{-10} \text{ s}. \quad (104) \]

Such a quick cooling is ensured by a huge number density of cool particles: there are about a million of cool particles over each hot one, see Eqs. (99), (100).

As a result of mixing and thermalization of the two components, hot and cool, the temperature of the resulting plasma would become:

\[ T_{\text{fin}} = T_{\text{cool}} (\rho_{\text{hot}} / \rho_{\text{cool}})^{3/4} = 147 M_8^{3/2} \text{ MeV}. \quad (105) \]

Correspondingly the total number density of relativistic particles would be equal to:

\[ n_{\text{rel}} = 0.1 g* T_{\text{fin}}^3 = 4 \times 10^{38} M_8^{-9/2} / \text{cm}^3. \quad (106) \]

According to Eq. (99) the number density of \( X \)-particles immediately after evaporation should be about \( 10^{30} M_8^{5} \text{ cm}^{-3} \). After fast thermalization the ratio of number densities of \( X \)-s to that of all relativistic particles becomes:

\[ n_X / n_{\text{rel}} = 3 \times 10^{-9}. \quad (107) \]

The evolution of the number density of \( X \)-particles is governed by the equation:

\[ \dot{n}_X + 3H n_X = -\sigma(\text{ann}) v n_X^2, \quad (108) \]

where the inverse annihilation term is neglected because hot particles from the PBH evaporation cool down very quickly with characteristic time (104) and hence the plasma temperature became much smaller than \( M_8 \). Evidently since \( m_X \gg T_{\text{fin}} \) (105), the distribution of \( X \)-particles would be very much different from the equilibrium Bose–Einstein or Fermi–Dirac distributions but the kinetic equilibrium should be quickly established leading to the distribution over energy close to the equilibrium one with nonzero and
equal chemical potentials of $X$ and anti-$X$, assuming zero $X/X$-asymmetry. If total kinetic and chemical equilibrium would be established, the number densities of $X$ (and $\bar{X}$) would be extremely small and the problem of their over-abundance would not appear. The key point here is the fast cooling of the plasma of the produced hot particles, much faster than the cosmological expansion rate, see Eq. (104).

The Hubble parameter $H$ which enters Eq. (108) is given by the expression:

$$H = \left( \frac{8\pi^3}{90} g_* \right)^{1/2} \frac{T^2}{m_{pl}^2} = \frac{0.4 T_{in}^2}{z m_{pl}},$$

(109)

where $z = a_{in}/a$ is the ratio of the initial scale factor to the running one and for the initial value of the temperature, $T_{in}$ we take $T_{in}$ given by Eq. (105). Hopefully it will not lead to confusion.

Introducing $r = n_X z^2$ and changing the time variable to $z$, we arrive to the equation:

$$\frac{dr}{dz} = -\sigma_{ann} v r^2 H z = -\sigma_{ann} v m_{pl} r^2 0.4 T_{in}^2 z^2,$$

(110)

which is easily solved leading to

$$n_X = \frac{n_m}{z^2 (1 - 1/z)} \to \frac{1}{Q z^2},$$

(111)

where $Q = (\sigma v m_{pl})/(0.4 T_{in}^2)$.

The total annihilation cross-section can be fixed by the condition that $X$-particles are the dominant carriers of the cosmological dark matter. According to the numerous observational data:

$$\Omega_{DM} = 0.26 \quad \text{and} \quad \Omega_{CMB} = 5.5 \times 10^{-5}$$

(112)

or $(\rho_{X}/\rho_{CMB})_{obs} = 5 \times 10^3$.

As calculated e.g. in [32], the frozen cosmological mass density of $X$-particles is determined by the equation:

$$\Omega_{X} h^2 = 10^9 x_f m_{pl} / \text{GeV} (\sigma_{ann} v) \approx 0.12,$$

(113)

where $h \approx 0.67$ is the dimensionless Hubble parameter and $x_f = T_f/m_X = 20 - 30$ is the ratio of the freezing temperature to the $X$ mass. The last term in the equation above is the observed value. Hence

$$\sigma_{ann} v m_{pl} \sim 3 \times 10^{11} \text{ GeV}^{-1}$$

(114)

and

$$n_X \sim 10^{-12} z^{-3} T_{in}^2 \text{ GeV}.$$  

(115)

So for the ratio of the density of $X$ to the relativistic particle density we find:

$$\frac{n_X}{n_{rel}} \to 10^{-12} \text{ GeV}/T_{in} = 7 \times 10^{-12},$$

(116)

so the ratio of the corresponding energy densities at the present time is

$$\frac{\rho_X}{\rho_{CMB}} = \frac{m_X}{3 T_{CMB} n_{rel} g_*(150 \text{ MeV})} < 10^{3} \frac{m_X}{\text{TeV}},$$

(117)

which is safely below the observer ratio $\rho_X/\rho_{CMB} = 5 \times 10^3$, especially if $m_X < 1 \text{ TeV}$. Here we took $g_* = 50$ at $T = 150 \text{ MeV}$ and $g_* < 10$ at $T = 0.1 \text{ MeV}$.

One can see that the values presented in this Appendix disagree with the published works [30] and [31] on production of possible dark matter particles by PBH evaporation. But the disagreement is natural, since in these papers some essential physical effects are disregarded. Firstly, it is assumed that the evaporation goes into an empty space, while in our case the universe was filled by cooler relativistic plasma. Secondly, the residual annihilation of the created DM particles is disregarded, while as it is shown above it is very much essential. On the other hand, the cooling of DM particles is so fast that their inverse annihilation does not take place, so the are not created by this process.

**APPENDIX B**

We present here analytic expressions for the integrals of $I_0$ (63) and $I_3$ (64) for two forms of PBH mass spectrum: flat one and (the first index of $j$ is (1) and the continuous smooth spectrum, which is numerically close to the log-normal one (the first index of $j$ is (2), see Eq. (60) and above. The second indices 1 or 3 correspond $I_0$ and $I_3$, respectively. For brevity we use notations $t$ instead of $\eta$.

$$j_{10}(t, a, b) = \frac{1}{3} \left[ -\Gamma \left[0, \frac{t}{a^3} \right] + \Gamma \left[0, \frac{t}{b^3} \right] \right].$$

(118)

This is the analytic result of the integral $j_{10}$ defined in Eq. (66).

$$j_{13}(t, a, b) = -\exp[-t/a^3] + \exp[-t/b^3] \frac{3t}{a^3}.$$  

(119)

The analytic result for the integral $j_{13}$ defined in Eq. (68).

$$j_{20}(t, a, b) = -\frac{1}{9(a-b)^3} 8a^4 b^2 \left[ 27 \exp \left[ -\frac{t}{a^3} \right] - 8a \Gamma \left[ \frac{2}{3} \right] \right. + 24\sqrt{3} \pi \frac{8b^3 [1 - 1/3, t/a^3]}{t^{1/3} [1/3, t/a^3]} - 2b(4a + b) \Gamma [1/3, t/a^3] \left. + 6\Gamma [0, t/a^3] + \frac{2a^2 b^2 \Gamma [1/3, t/a^3]}{t^{4/3}} \right].$$

(120)
\[ \frac{1}{9(a-b)^2} \left( \frac{27 \exp \left[ -\frac{t}{b^2} \right] - 8 \Gamma \left[ -\frac{2}{3} \right] \frac{24 \sqrt{3} \pi}{t^{1/3}} \right) + \frac{8 \Gamma - 2/3, t/b^3}{t^{1/3}} \right) + \frac{2a^2 b^2}{t^{4/3}} \left( \frac{36 \Gamma / 4, t/b^3}{t^{1/3}} + \frac{9 b^2 \Gamma / 5, t/b^3}{t^{2/3}} \right) + \frac{2a^2 b^2 \Gamma / 1, t/b^3}{t^{2/3}} \right) \]

The analytic result for the integral \( j_{23} \) as explained in Subsection 4.2.

\[ j_{23}(t, a, b) = -\frac{1}{27(a-b)^2} \left( 16a^2 b^2 (6a t \Gamma / 1, t/a^3) + 6a^2 t^{2/3} \Gamma [2, t/a^3] + bl - 18a \exp [-t/a^3] \right] \]

\[ -18ab \exp [-t/a^3] \frac{\sqrt{3} \pi a}{\Gamma [1/3]} - 9 a^2 b \Gamma / 7, t/a^3] \]

\[ -18 \Gamma / 4, t/a^3] + 9(4a + b) \exp [t/a^3] \Gamma / 5, t/a^3] + 9 a^2 b \Gamma / 7, t/a^3] \]

\[ + 9 a \exp [t/a^3] \]

\[ - 8 \sqrt{3} \pi a \frac{b^2 \Gamma [1/3]}{\Gamma [1/3]} - 9 a^2 b \Gamma / 7, t/a^3] \]

\[ - 18 \Gamma / 4, t/a^3] + 9(4a + b) \exp [t/a^3] \Gamma / 5, t/a^3] + 9 a \exp [t/a^3] \]

\[ - 8 \sqrt{3} \pi a \frac{b^2 \Gamma [1/3]}{\Gamma [1/3]} - 9 a^2 b \Gamma / 7, t/a^3] \]

The analytic result for the integral \( j_{23} \) as explained in Subsection 4.2.

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