Shock Waves and Energy Dissipation in Magnetohydrodynamic Turbulence

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Abstract

Shock waves play an important role in turbulent astrophysical media by compressing the gas and dissipating the turbulent energy into the thermal energy. Here, we study shocks in magnetohydrodynamic turbulence using high-resolution simulations. Turbulent Mach numbers of $\mathcal{M}_{\text{turb}} = 0.5–7$ and initial magnetic fields of plasma beta $\beta_0 = 0.1–10$ are considered, targeting turbulences in interstellar and intracluster media. Specifically, we present the statistics of fast and slow shocks, such as the distribution of shock Mach numbers ($\mathcal{M}_s$) and the energy dissipation at shocks, based on refined methodologies for their quantifications. While most shocks form with low $\mathcal{M}_s$, strong shocks follow exponentially decreasing distributions of $\mathcal{M}_s$. More shocks appear for larger $\mathcal{M}_{\text{turb}}$ and larger $\beta_0$. Fast shock populations dominate over slow shocks if $\beta_0 \gg 1$, but substantial populations of slow shocks develop in the cases of $\beta \lesssim 1$, i.e., strong background fields. The shock dissipation of turbulent energy occurs preferentially at fast shocks with $\mathcal{M}_S \lesssim 1$ of a few to several, and the dissipation at strong shocks shows exponentially decreasing functions of $\mathcal{M}_s$. The energy dissipation at shocks, normalized to the energy injection, $\epsilon_{\text{shock}}/\epsilon_{\text{inj}}$, is estimated to be in the range of $\sim 0.1–0.5$, except for the case of $\mathcal{M}_{\text{turb}} = 0.5$ and $\beta_0 = 0.1$, where the shock dissipation is negligible. The fraction decreases with $\mathcal{M}_{\text{turb}}$; it is close to $\sim 0.4–0.6$ for $\mathcal{M}_{\text{turb}} = 0.5$, while it is $\sim 0.1–0.25$ for $\mathcal{M}_{\text{turb}} = 7$. The rest of the turbulent energy is expected to dissipate through the turbulent cascade. Our work will add insights into the interpretations of physical processes in turbulent interstellar and intracluster media.

Key words: galaxies: clusters: intracluster medium – ISM: general – magnetohydrodynamics (MHD) – methods: numerical – shock waves – turbulence

1. Introduction

Turbulence is present in a variety of astrophysical media, including the interstellar medium (ISM; see, e.g., Elmegreen & Scalo 2004; Mac Low & Klessen 2004; McKee & Ostriker 2007) and the intracluster medium (ICM; see, e.g., Schuecker et al. 2004; Churazov et al. 2012; Hitomi collaboration 2016; Vazza et al. 2017). Unlike terrestrial turbulence, astrophysical turbulence is mostly transonic/supersonic and in all the cases magnetized. The properties of flows change when the average flow speed is comparable to or exceeds the sound speed in the medium; high speeds usually mean that the flows are compressible. Magnetic fields permeated in turbulence are stretched and amplified by flows, and in turn, exert tension and pressure to flow motions altering flow properties; these processes lead to magnetohydrodynamics (MHDs). Therefore, astrophysical turbulence is by nature compressible MHD turbulence (see, e.g., Biskamp 2003).

Observations suggest that the turbulence in the ISM is transonic or supersonic with different turbulent Mach numbers, $\mathcal{M}_{\text{turb}}$, in different phases. In the warm ionized medium (WIM), for instance, the width of the H$\alpha$ line in the range of tens of km s$^{-1}$ (Tufte et al. 1999), suggesting that $\mathcal{M}_{\text{turb}}$ is of order unity. For the cold neutral medium, H I observations revealed internal gas motions corresponding to $\mathcal{M}_{\text{turb}} \sim$ a few (Heiles & Troland 2003). Molecular clouds (MCs) are characterized with nonthermal line widths due to highly supersonic motions of typically $\mathcal{M}_{\text{turb}} \gtrsim 10$ (see, e.g., Larson 1981). The strength of magnetic fields in the ISM also varies with location. The strength in the solar neighborhood, for instance, was estimated to be $B \sim 6 \mu$G, with a large-scale component of $B_{\text{regular}} \sim 2–3 \mu$G and a random component of $B_{\text{random}} \sim 3–4 \mu$G (Haverkorn 2015), and this corresponds to the plasma beta, the ratio of thermal to magnetic pressures, of $\beta \lesssim 1$ in the WIM. In MCs, magnetic fields are much stronger, typically of order $\sim$μG, corresponding to $\beta \sim 0.01–0.1$ (see, e.g., Crutcher 2012).

The turbulence in the ICM is driven by ongoing accretion and mergers, as well as galactic winds and feedback from active galactic nuclei (see, e.g., Brunetti & Jones 2014). Simulations of cosmic structure formation have shown that the turbulence is mildly transonic with $\mathcal{M}_{\text{turb}} \sim 1/2$ (see, e.g., Ryu et al. 2008; Vazza et al. 2017), and X-ray observations support it (Schuecker et al. 2004; Churazov et al. 2012). Observations of Faraday rotation measures and diffuse synchrotron emissions from radio halos and relics indicate the presence of μG-level magnetic fields over the whole volume of galaxy clusters (see, e.g., Carilli & Taylor 2002; Govoni & Feretti 2004). With these fields, the plasma beta of the ICM is estimated to be $\beta \sim 10–100$.

Shock waves commonly develop in astrophysical turbulences. In MCs, shocks are often responsible for the formation of filaments (see, e.g., Federrath 2016), and star formation appears to be linked to it, as high-density regions are the sites of prestellar cores (see, e.g., Mac Low & Klessen 2004). Shocks are also believed to be a mechanism for driving chemical evolution in the ISM; for instance, Pety & Falgarone (2000) invoked turbulent dissipation in shocks or vortices to explain chemical anomalies in diffuse MCs, and Pon et al. (2014) and Lehmann & Wardle (2016) used shocks driven by turbulence to model anomalous emission from high-J $^{13}$CO lines in giant MCs. In the ICM, shocks induced by turbulent flow motions are weak, but they play an important role in the evolution of vorticity (Porter et al. 2015) and significantly contribute to the gas heating (see, e.g., Ryu et al. 2003). Thus, describing the statistics of shocks in compressible MHD turbulence would be necessary to understand physical processes in astrophysical media.
The statistics of shocks were previously investigated, mostly through simulations of astrophysical turbulence. Smith et al. (2000, 2000b), for instance, presented the probability distribution function (PDF) of \( M_{\text{jump}} \equiv v_{\text{jump}}/c_s \), where \( v_{\text{jump}} \) is the velocity jump across flow-converging regions and \( c_s \) is the sound speed, in decaying and driven MHD turbulences. More recently, Porter et al. (2015) and Lehmann et al. (2016) identified “shocked” grid zones, i.e., the grid zones through which shocks pass, in simulations, and presented the PDF of the shock Mach number, \( M_{\text{sh}} \equiv v_s/c_s \), where \( v_s \) is the shock speed. While Porter et al. (2015) considered the turbulence of \( M_{\text{turb}} \sim 1/2 \) in the ICM plasma with \( \beta \gg 1 \), Lehmann et al. (2016) considered \( M_{\text{turb}} \sim 9 \) in \( \beta \lesssim 0.1 \), targeting the turbulence in MCs. Particularly, Lehmann et al. (2016) divided shocks into fast and slow populations, and separately presented their statistics. Both works showed that while weak shocks are common, strong shocks with \( M_{\text{sh}} \gg M_{\text{turb}} \) follow PDFs exponentially decreasing with \( M_{\text{sh}} \), Lesaffre et al. (2013), on the other hand, attempted to study shocks in observation, by applying PDFs of \( v_{\text{sh}} \) from different shock models to interpret molecular and atomic lines in Stephens Quintet and also in the diffuse ISM toward Chamaeleon. They argued that in both Stephens Quintet and Chamaeleon, shocks of low and moderate \( v_{\text{sh}} \) are important in shaping line emissions from interstellar gas.

In turbulent media, the turbulent energy cascades down to small scales and dissipates into heat at dissipation scales. However, a fraction of the turbulent energy can directly dissipate at shocks. To quantify it, often the dissipation timescale, \( t_{\text{diss}} \sim \tau_{\text{turb}}/E_{\text{turb}} \), is compared to the flow-crossing timescale, \( t_{\text{cross}} \sim L_{\text{sh}}/v_{\text{sh}} \), expecting that while \( t_{\text{diss}} \) counts for both the cascade and shock dissipations, \( t_{\text{cross}} \) only for the dissipation through a turbulent cascade. Here, \( E_{\text{turb}} \) and \( v_{\text{sh}} \) are the turbulent energy and flow speed, respectively, \( E_{\text{turb}} \) is the energy injection rate, and \( L_{\text{sh}} \) is the injection scale at which the turbulence is driven. Stone et al. (1998), for instance, estimated \( t_{\text{diss}} \) in MHD turbulence simulations, assuming that \( E_{\text{turb}} \) balances the energy injection rate in driven cases. They obtained \( t_{\text{diss}}/t_{\text{cross}} \sim 0.46-0.69 \) for supersonic turbulences with \( \beta = 0.01-1 \), when \( t_{\text{diss}} \) was defined with the kinetic energy. They got slightly smaller ratios for decaying turbulences. Also, there have been attempts to estimate turbulent dissipation in observation, for instance, by comparing the observations of CO rotational transitions to detailed shock models; Pon et al. (2014) estimated \( t_{\text{diss}}/t_{\text{cross}} \sim 1/3 \) for turbulent regions in the Perseus MC, and Larson et al. (2015) estimated \( t_{\text{diss}}/t_{\text{cross}} \sim 0.65 \) or 0.94, depending on the adopted shock model, for the Taurus MC.

There have been trials to directly estimate the amount of the energy dissipated at shocks. Lehmann et al. (2016), for instance, calculated the kinetic energy flux, \( (1/2) \rho v^2 \) (\( \rho \) is the preshock density), through shock surfaces, anticipating that it represents the shock dissipation. They estimated that for the turbulence of \( M_{\text{turb}} \sim 9 \) and \( \beta \lesssim 0.1 \), the timescale for the dissipation of kinetic energy at shocks would be comparable to \( t_{\text{cross}} \). Smith et al. (2000, 2000b) calculated the amount of the energy dissipated by “artificial viscosity” at shocks, and suggested that this would be \( \sim 1/2 \) and \( \sim 2/3 \) of the total energy dissipation for decaying and driven supersonic turbulences, respectively. All these numerical works and observations hint that the shock dissipation would account for a fraction of the turbulent energy dissipation, although it has not been previously defined.

In this paper, we study shocks in isothermal, compressible, driven MHD turbulence, focusing on their Mach number probability distribution and spatial frequency, as well as the energy dissipation at shocks. Wide ranges of \( M_{\text{turb}} \) and \( \beta \) are considered, intending to cover turbulences in the ISM and ICM. Specifically, we present an algorithm to identify shocked grid zones, separate them into fast and slow shocks, and calculate the Mach numbers. In particular, for the first time, we explicitly define the energy dissipation at shocks and introduce formulae to calculate it. We then apply those to a homogeneous set of simulation data with up to 1024\(^3 \) grid zones.

The paper is organized as follows. Section 2 describes the details of numerics, including the parameters for turbulence simulations. Section 3 presents the algorithm for shock identification and the formulae for the calculations of shock Mach number and energy dissipation at shocks. The main results, i.e., the shock statistics and the energy dissipation at shocks, are given in Section 4. A summary and discussion follow in Section 5.

2. Numerics

Turbulence simulations solved the following set of equations for isothermal, compressible MHDs:

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \tag{1}
\]

\[
\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} + \frac{1}{\rho} \nabla p - \frac{1}{\rho} (\nabla \times \mathbf{B}) \times \mathbf{B} = 0, \tag{2}
\]

\[
\frac{\partial \mathbf{B}}{\partial t} - \nabla \times (\mathbf{v} \times \mathbf{B}) = 0, \tag{3}
\]

where the unit of \( \mathbf{B} \) was chosen so that \( 4\pi \) does not appear in Equation (2). The gas pressure is given as \( p = \rho c_s^2 \) with a constant sound speed \( c_s \) (the isothermal condition). A multi-dimensional code described in Kim et al. (1999) was employed for simulations. It is based on the explicit, finite-difference “Total Variation Diminishing” scheme, which is a second-order accurate upwind, and enforces \( \nabla \cdot \mathbf{B} = 0 \) with a constrained transport scheme (Ryu et al. 1998). The code does not explicitly model viscous and resistive dissipations.

Simulations were performed in a three-dimensional (3D) periodic box of size \( L_0 = 1 \) using \( n_s^3 = 512^3 \) and 1024\(^3 \) grid zones. Initially, the medium was uniform with \( \rho_0 = 1, P_{\text{eq}} = 1 \) (so \( c_s = 1 \)), and \( B_0 = (B_0, 0, 0) \), and at rest with \( \mathbf{v} = 0 \). Turbulence was driven with traditional, “solenoidal” forcing (\( \nabla \cdot \mathbf{v} = 0 \)) (see, e.g., Stone et al. 1998; Mac Low 1999). We note that the properties of turbulence depend on the nature of forcing. Porter et al. (2015), for instance, demonstrated differences in ICM turbulence when solenoidal or compressive (\( \nabla \cdot \mathbf{v} = 0 \)) forcings were applied. And Federrath et al. (2010) argued that for modeling turbulence in MCs, a mixture of solenoidal and compressive drivings would be natural, based on degrees of freedom arguments. As the first trial to quantify the statistics and dissipation of shocks, we here consider turbulence driven by solenoidal forcing.

Velocity perturbations, \( \delta \mathbf{v} \), were drawn from a Gaussian random field determined with a power spectrum, \( \delta \mathbf{v}(\mathbf{k})^2 \propto k^6 \exp(-8k/k_{\text{exp}}) \), where \( k_{\text{exp}} = 2k_0 \) (\( k_0 = 2\pi /L_0 \)), and added to \( \delta \mathbf{v} \) at each interval of \( \Delta t = 0.001L_0/c_s \). The amplitude of the perturbations was tuned in such a way that the desired
The plasma beta is defined as the ratio of the magnetic energy to the kinetic energy of the plasma. In this context, the plasma beta, $\beta$, is related to the magnetic field strength and the fluid velocity. The plasma beta at the saturated stage, $\beta_{sat}$, is calculated using the formula:

$$\beta_{sat} = \frac{B_{sat}^2}{\rho u_{rms}^2}$$

where $B_{sat}$ is the magnetic field strength at saturation, $\rho$ is the plasma density, and $u_{rms}$ is the root mean square velocity of the plasma.

The results presented in Section 4 were drawn from the data after the saturation, specifically, during $t/t_{cross} = 2–6$ and $2–4.8$ for $512^3$ and $1024^3$ simulations, respectively.

### 3. Methods

#### 3.1. Identification of Shocks

In simulation data, shocks (actually grid zones that are parts of shock surfaces) were identified with the following algorithm. Along each coordinate direction, grid zones were tagged as “shocked,” if $\nabla \cdot \mathbf{v} < 0$, i.e., the local flow is converging, and $\max(\rho_{i+1}/\rho_{i-1}, \rho_{i-1}/\rho_{i+1}) \geq 1.03^2$ around the zone $i$. The second condition, corresponding to the density jump of isothermal “hydrodynamic” shocks with sonic Mach numbers $\geq 1.03$ (see Section 3.2), excludes weak shocks; it was imposed to avoid confusions between very weak shocks and waves. In simulations, shocks are spread and captured over a few to several grid zones. The “shock center” was identified as the zone with $\nabla \cdot \mathbf{v}$ among attached shocked zones. The preshock and postshock zones were defined around the center zone, assuming the spread of shocks is typically over 2 to 4 zones. Specifically, for a shock center $i$, $i + 2$ and $i - 2$ were chosen as

$$v_{turb} \equiv v_{rms} = \langle v^2 \rangle^{1/2}$$

is achieved. Following previous works (e.g., Stone et al. 1998), the injection scale is set as that of $k_{exp}$, i.e., $L_{inj} = L_0/2$, although the forcing has a peak around $1.5k_0$. The problem is then defined by two parameters, the turbulent Mach number, $M_{turb} \equiv v_{rms}/c_s$, and the initial plasma beta, $\beta_0 = P_{B,0}/P_{rho,0} = \rho_0c_s^2/(B_0^2/2)$. To cover turbulences in different astrophysical environments, as mentioned in the introduction, the cases of $M_{turb} = 0.5$ (subsonic turbulence), 1 (transonic turbulence), and 2, 4, 7 (supersonic turbulence), and $\beta_0 = 0.1$ (strong field), 1 (moderate field), and 10 (weak field) were considered. Table 1 lists the simulations performed for this paper, along with their model names. High-resolution simulations using $1024^3$ grid zones were made only for $M_{turb} = 0.5$, 1, 7, while those of $512^3$ grid zones were done for all the cases. Figure 1 shows the kinetic and magnetic energy evolutions in those simulations; simulations run up to $t = 6$ and $4.8 \times t_{cross}$ for $512^3$ and $1024^3$ simulations, respectively, where $t_{cross} = L_0/v_{rms}$. The kinetic energy grows quickly within $\sim t_{cross}$ as shown in previous works (see, e.g., Federrath 2013; Porter et al. 2015). The growth of the magnetic energy follows and reaches saturation by $\sim 2t_{cross}$. The plasma beta at the saturated stage, $\beta_{sat}$, is listed in Table 1.

### Table 1

| Model Name | $N_{in}/n_g$ | $N_{sh}/n_g$ | $\epsilon_{m,0}$ | $\epsilon_{m,0}/\epsilon_{inj}$ | $\epsilon_{d}/\epsilon_{inj}$ | $\epsilon_{c}/\epsilon_{inj}$ | $\beta_{sat}$ |
|------------|--------------|--------------|------------------|-------------------------------|-------------------------------|-------------------------------|-------------|
| 512M0.5-b0.1 | 0.0018 | 0.103 | 1.20 | 0.0122 | 0.0004 | 0.807 | 0.099 |
| 512M0.5-b1 | 0.749 | 0.154 | 1.32 | 0.538 | 0.0691 | 0.715 | 0.932 |
| 512M0.5-b10 | 3.38 | 1.22 | 1.68 | 0.486 | 0.0192 | 0.567 | 5.580 |
| 512M1-b0.1 | 0.0520 | 0.303 | 1.30 | 0.0768 | 0.0144 | 0.728 | 0.097 |
| 512M1-b1 | 3.34 | 2.55 | 1.70 | 0.472 | 0.224 | 0.571 | 0.762 |
| 512M1-b10 | 7.77 | 4.52 | 2.17 | 0.390 | 0.0742 | 0.458 | 2.420 |
| 512M2-b0.1 | 0.480 | 1.06 | 1.50 | 0.135 | 0.0636 | 0.622 | 0.091 |
| 512M2-b1 | 6.16 | 4.09 | 1.88 | 0.285 | 0.0867 | 0.467 | 0.482 |
| 512M2-b10 | 11.8 | 4.42 | 1.94 | 0.328 | 0.0400 | 0.469 | 1.150 |
| 512M4-b0.1 | 1.58 | 1.57 | 1.58 | 0.111 | 0.0215 | 0.555 | 0.073 |
| 512M4-b1 | 7.32 | 2.54 | 1.81 | 0.192 | 0.0159 | 0.478 | 0.230 |
| 512M4-b10 | 13.7 | 2.59 | 1.55 | 0.275 | 0.0107 | 0.556 | 0.357 |
| 512M7-b0.1 | 9.15 | 1.34 | 1.72 | 0.193 | 0.0028 | 0.536 | 0.123 |
| 512M7-b1 | 512M7-b10 | 16.3 | 1.48 | 1.50 | 0.244 | 0.0020 | 0.618 | 0.332 |

Notes.

- The starting number is $n_g$, the number after $M$ is $M_{turb}$, and the number after $b$ is the initial plasma beta, $\beta_0$: $t_{cross} = 6$ and $4.8 \times t_{cross}$ for $512^3$ and $1024^3$ simulations, respectively.
- The number of grid zones identified as fast and slow shocks, normalized to $n_g^2$.
- The energy injection rate in units of $\rho_0L_0^2c_s^3M_{turb}$.
- The fraction of the energy dissipation at fast and slow shocks.
- The fraction of the energy dissipation through a turbulent cascade, estimated with the flow-crossing time. See Section 3.3 for the definition.
- The plasma beta calculated with $(B^2)_{sat}$ of the saturated stage.

$v_{turb} \equiv v_{rms} = \langle v^2 \rangle^{1/2}$ is achieved. Following previous works (e.g., Stone et al. 1998), the injection scale is set as that of $k_{exp}$, i.e., $L_{inj} = L_0/2$, although the forcing has a peak around $\sim 1.5k_0$. The problem is then defined by two parameters, the turbulent Mach number, $M_{turb} \equiv v_{rms}/c_s$, and the initial plasma beta, $\beta_0 = P_{B,0}/P_{rho,0} = \rho_0c_s^2/(B_0^2/2)$. To cover turbulences in different astrophysical environments, as mentioned in the introduction, the cases of $M_{turb} = 0.5$ (subsonic turbulence), 1 (transonic turbulence), and 2, 4, 7 (supersonic turbulence), and $\beta_0 = 0.1$ (strong field), 1 (moderate field), and 10 (weak field) were considered. Table 1 lists the simulations performed for this paper, along with their model names. High-resolution simulations using $1024^3$ grid zones were made only for $M_{turb} = 0.5$, 1, 7, while those of $512^3$ grid zones were done for all the cases. Figure 1 shows the kinetic and magnetic energy evolutions in those simulations; simulations run up to $t = 6$ and $4.8 \times t_{cross}$ for $512^3$ and $1024^3$ simulations, respectively, where $t_{cross} = L_0/v_{rms}$. The kinetic energy grows quickly within $\sim t_{cross}$ as shown in previous works (see, e.g., Federrath 2013; Porter et al. 2015). The growth of the magnetic energy follows and reaches saturation by $\sim 2t_{cross}$. The plasma beta at the saturated stage, $\beta_{sat}$, is listed in Table 1.
either preshock or postshock zones, if \( i + 1 \) and \( i - 1 \) were shocked zones; otherwise, \( i + 1 \) or \( i - 1 \) were chosen.

Once shock centers were identified, the sonic Mach number, \( M_s \), was calculated using the flow quantities at the preshock and postshock zones with the formulae presented in the next subsection (Equations (9)). If a zone was identified as shock centers along more than one direction, the maximum value of \( M_s \) was taken as its Mach number; that is, \( M_s = \max(M_{sx}, M_{sy}, M_{sz}) \) if identified as shock centers in all coordinate directions.

The above algorithm is similar to the one used before with the data for cosmological, large-scale structure formation simulations (e.g., Ryu et al. 2003; Ha et al. 2018), but different from the one applied to MHD turbulence data in Porter et al. (2015). The current algorithm, which is based on the dimension-per-dimension identification of shocks, is simpler than the one in Porter et al. (2015), which uses boxes of \( 5 \times 5 \times 5 \) zones and identifies shocks in 3D by finding the principle direction of density variation. Yet, both algorithms produce almost identical results, as already noted in Porter et al. (2015).

### 3.2. Calculation of Shock Mach Number

Formulae to calculate \( M_s \) can be derived from the shock jump conditions. From the conservative form of Equations (1)–(3), the jump conditions for isothermal flows in the shock-rest frame are written as

\[
\begin{align*}
\rho_1 v_{||1} &= \rho_2 v_{||2}, \\
\rho_1 v_{\perp1}^2 + c_s^2 \rho_1 + \frac{1}{2} B_{||1}^2 &= \rho_2 v_{\perp2}^2 + c_s^2 \rho_2 + \frac{1}{2} B_{||2}^2, \\
\rho_1 v_{||1} v_{\perp1} - B_{||1} B_{\perp1} &= \rho_2 v_{||2} v_{\perp2} - B_{||2} B_{\perp2}, \\
v_{||1} B_{||1} - v_{\perp1} B_{\perp1} &= v_{||2} B_{||2} - v_{\perp2} B_{\perp2},
\end{align*}
\]

where the subscripts 1 and 2 indicate the preshock and postshock quantities, respectively, and || and \( \perp \) denote the components parallel and perpendicular to the shock normal, respectively. The parallel field is continuous across the shock, so \( B_{||1} = B_{||2} = B_{||} \).

The manipulation of the jump conditions becomes substantially simplified in the so-called preferred frame, in which \( \mathbf{v} \) and \( \mathbf{B} \) are parallel to one another on both sides of the shock, i.e., \( B_{||1}/B_{||} = v_{||1}/v_{||} \) and \( B_{\perp2}/B_{||} = v_{\perp2}/v_{||} \) (see, e.g., Shu 1992). In the frame, Equation (7) is automatically satisfied. By combining Equations (4)–(6), we can get, for instance,

\[
M_s^4 = \left[ \frac{1}{c_s^4} + \frac{v_{\perp1}^2}{c_s^4} + \frac{v_{\perp1}^2}{2c_s^2} \right] \chi + \frac{v_{\perp1}^2}{2c_s^2} \chi^2 M_s^2 - \frac{v_{\perp1}^2}{c_s^4} \chi^3 = 0,
\]

Figure 1. Time evolution of the kinetic energy, \( E_{\text{kin}} \) (thick lines), and the magnetic energy increase, \( \delta E_{\text{mag}} = (B - B_0)^2/2 \) (thin lines), for the simulations listed in Table 1.
where $M_s = v_{f1}/c_s$, $\chi = \rho_2/\rho_1 = v_{f1}/v_{f2}$, $v_{A,\perp} = B_{\perp}/\sqrt{\rho_1}$, and $v_{A,\parallel} = B_{\parallel}/\sqrt{\rho_1}$. The equation is written in such a way that the shock sonic Mach number, $M_s$, is calculated with the density compression and the preshock magnetic field. Note that the flow velocity in the shock-rest, preferred frame is different from that in the computational frame, so it can only be computed by $M_s$ with formulae derived in the shock-rest, preferred frame. The magnetic field, on the other hand, does not change by coordinate transformations, as long as $v \ll c$ (the speed of light), which is the case for the turbulence in the ISM and ICM.

Equation (8) is, however, a cubic equation for $M_s^2$, and solving it is rather complicated. Instead, a simpler formula, for instance,

$$M_s^2 = \chi + \frac{\chi}{\chi - 1} \frac{B_{\perp}^2 - B_{\parallel}^2}{2c_s^2 \rho_1}, \quad (9)$$

can be derived from Equations (4) and (5). This equation requires the preshock and postshock densities and magnetic fields, but those quantities can be extracted from simulation data. We employed Equation (9) for the calculation of $M_s$. For isothermal hydrodynamic shocks with $B = 0$, both Equations (8) and (9) reduce to $M_s^2 = \chi$, as expected.

In MHDs, shocks appear as fast and slow modes; the strength of the perpendicular magnetic field increases across the fast shock, i.e., $B_{\perp,2} > B_{\perp,1}$, while it decreases across the slow shock, i.e., $B_{\perp,2} < B_{\perp,1}$ (see, e.g., Shu 1992). In addition, the MHD equations accommodate shocks of the third mode, called “intermediate shocks,” which are, however, known to be non-evolutionary or unphysical, particularly in ideal MHDs (see, e.g., Landau et al. 1984). The intermediate mode is manifested as rotational discontinuity in simulations (see, e.g., Ryu & Jones 1995). Hence, we did not seek intermediate shocks.

Identified shocks were classified into either fast or slow populations according to the criterion of $B_{\perp,2} > B_{\perp,1}$ or $B_{\perp,2} < B_{\perp,1}$, and the statistics of the two populations were calculated separately. The speeds of fast and slow waves were calculated as

$$c_{s,\text{fl}} = \frac{1}{2} (c_s^2 + v_{A,\parallel} + v_{A,\perp}^2) \pm \frac{1}{2} \sqrt{(c_s^2 + v_{A,\parallel}^2 + v_{A,\perp}^2)^2 - 4v_{A,\parallel}^2c_s^2}, \quad (10)$$

with the + and − signs referring to the fast slow modes, respectively. Here and below, we used the quantities along the direction of shock identification for those with ||, and the quantities perpendicular to the direction for those with ⊥. After the sonic Mach numbers of shocks, $M_s$, were obtained with Equation (9), their fast and slow Mach numbers were calculated as $M_{\text{fl}} = M_s c_s/c_{s,\text{fl}}$ and $M_{\text{sl}} = M_s c_s/c_{s,\text{sl}}$, where $c_{s,\text{fl}}$ and $c_{s,\text{sl}}$ are the wave speeds in the preshock zones.

To avoid confusions from complex flow patterns and shock surface topologies, only shocks with $\max(\rho_2/\rho_1, \rho_{\perp,2}/\rho_{\perp,1}) > 1.03^2$ and $M_{\text{fl}} > 1.06$ (for fast shocks) and $M_{\text{sl}} > 1.06$ (for slow shocks) were used for the statistics presented in Section 4. Weak shocks with $M_{\text{fl,sl}} < 1.06$ are expected to dissipate little energy.

In general, the identification of fast shocks is relatively straightforward. Slow shocks, on the other hand, are harder to reliably identify, mainly due to the following two reasons. First, the surfaces of slow shocks could be subject to a corrugation instability (see, e.g., Stone & Edelman 1995), and hence their surfaces are distorted and fragmented (see Section 4.1). Then, they could be easily confused with waves, especially when the shock normal is not aligned with coordinate axes. Second, some slow shocks can have small shock speeds, $v_{\|} \ll c_s$, when the preshock flows have small slow wave speeds, $c_{s,\text{fl}} \ll c_s$. In these cases, the distinction between shocks and fluctuations is often not very clear. So for slow shocks, an additional constraint, $c_{s,\text{fl}}/c_s > 0.3$, was imposed. In the cases of low $M_{\text{corr}}$ and small $\beta_{\text{r}}$, slow shocks with $c_{s,\text{fl}}/c_s < 0.3$ mostly propagate perpendicular to the background magnetic field and have small $M_s$ (see Section 4.2). In the cases of high $M_{\text{corr}}$, however, the constraint might exclude some shocks with substantial $M_s$. On the other hand, some waves or fluctuations may have been counted as slow shocks. Hence, the population of slow shocks, presented in the next section, might have been estimated less accurately than that of fast shocks.

Table 2 lists some of the average quantities of identified fast and slow shocks. See the next section for discussions.

![Table 2 Properties of Identified Shocks](image-url)
In Appendix A, the dependence of the statistics on the shock identification parameters, i.e., the constraints on $\max(\rho_{i+1}/\rho_{i-1}, \rho_{i-1}/\rho_{i+1})$, $M_{\text{flux}}$, and $c_{d,1}$, is presented.

3.3. Calculation of Energy Dissipation

As noted in the introduction, some of the turbulent energy directly dissipates at shocks, while the rest cascades down and dissipates into heat. For adiabatic flows, the conservation equation for the total energy, composed of the kinetic, magnetic, and thermal energies, follows actions in the energy. In the case of isothermal flows, Mouschovias (1974) showed that the “heat energy” or the “effective internal energy,” $P \ln P$, which obeys

$$\frac{\partial P \ln P}{\partial t} + \nabla \cdot (P \ln P \nu) + P \nabla \cdot \nu = 0,$$

(11)
can be introduced, and the equation for the “effective total energy” can be written down as

$$\frac{\partial Q}{\partial t} \left( \frac{1}{2} \rho v^2 + P \ln P + \frac{1}{2} B^2 \right) + \nabla \cdot \left( \left( \frac{1}{2} \rho v^2 + P \ln P + P \right) \nu + (B \times \nu) \times B \right) = 0.$$

(12)

The above equation holds in smooth parts of flows, but does not at shocks because the isothermal equation of state assumes an instantaneous loss of the thermal energy converted from the kinetic and magnetic energies at shocks. The jump of the energy flux in the shock-rest frame,

$$\left[ \left( \frac{1}{2} \rho v^2 + P \ln P + P + B^2 \right) v_{\parallel} - B \cdot \left( B v_{\parallel} + B_{\perp} v_{\perp} \right) \right]_2 \equiv Q,$$

(13)
hence, should estimate the energy lost at isothermal shocks. Here, $|f|_2 = f_1 - f_2$ denotes the jump between the preshock and postshock quantities. So we define $Q$ as the energy dissipation rate per unit area at shock surfaces.

In the preferred frame, $Q$ is simplified to

$$Q = \left( \frac{1}{2} \rho_1 v_1^2 + P_1 \ln P_1 \right) v_{1 \parallel} - \left( \frac{1}{2} \rho_2 v_2^2 + P_2 \ln P_2 \right) v_{2 \parallel},$$

(14)

and further written as

$$\frac{Q}{\rho_1 M_0 c_s^4} = \frac{1}{2} M_0^2 \left[ 1 - \frac{1}{\chi^2} + \frac{v_{\perp}^2}{\chi^2} (\chi - 1) \left( \frac{v_{\perp}^2}{\chi^2} \chi + 1 \right) - 2 M_0^2 c_s^2 \right] - \ln \chi.$$

(15)

Here, $Q$ is expressed in terms of the shock Mach number and the flow quantities independent of coordinate transformations, and hence can be calculated with simulation data. For hydrodynamic shocks, Equation (15) reduces to

$$Q = \frac{1}{2} \rho_1 M_0^3 c_s^4 \left( \frac{M_0^4 - 1}{M_0^4} - 4 \ln M_0 \right).$$

(16)

The term involving $\ln M_0$ inside the parentheses (also the $-\ln \chi$ term in Equation (15)) originates from the heat energy term in Equation (12), and so can be attributed to the assumption of isothermal flows. If flows are adiabatic and the gas cools instantaneously behind the shock, the energy lost at the shock is

$$Q = \frac{1}{2} \rho_1 v_{1 \parallel}^3 - \frac{1}{2} \rho_2 v_{2 \parallel}^3,$$

(17)

assuming the adiabatic index $\gamma = 1$ (see, e.g., Ryu et al. 2003); then, the first term inside the parenthesis in Equation (16) is recovered.

With $Q$ in Equation (15), we estimated the dissipation rate of turbulent energy at shocks, inside the whole computational box, as

$$\epsilon_{\text{fa or sl}} = \sum_{\text{fast or slow shocks } j} Q_{\text{fa or sl}} (\Delta x)^2,$$

(18)

separately for fast and slow shock populations. Here, $\Delta x = L_0/n_x$ is the size of grid zones, and the summation is over all the identified shock zones.

We also estimated the energy dissipation rate through turbulence cascade as

$$\epsilon_{\text{cas}} = \frac{1}{L_{\text{cross}}} \int \frac{1}{2} \rho v^2 dV,$$

(19)

where the integration is over the whole computational box (see, e.g., Stone et al. 1998). Note that if the total energy, i.e., the kinetic energy plus the magnetic energy increase, is used instead of the integral, $\epsilon_{\text{cas}}$ would be $\sim 10\%$–$60\%$ larger (see Figure 1). The estimate would be “independent” of the scale and hence a fair value, only for incompressible Kolmogorov turbulence with spectral slope $5/3$. Hence, we here quote it only as a supplementary quantity.

The above energy dissipation rates were compared to the “energy injection rate,” $\epsilon_{\text{inj}}$. Turbulence was driven by adding velocity perturbations, $\delta v$, at intervals $\Delta t$, as described in Section 2. Hence,

$$\epsilon_{\text{inj}} = \frac{1}{\Delta t} \int \frac{1}{2} \rho [(\nu + \delta v)^2 - v^2] dV.$$

(20)

Below the energy dissipations, normalized to the energy injection, $\epsilon_{\text{sh}}/\epsilon_{\text{inj}}$ and $\epsilon_{\text{cas}}/\epsilon_{\text{inj}}$, are presented (also in Table 1).

4. Results

4.1. Spatial Distribution of Shocks

Figure 2 shows the spatial distributions of $M_0$ for fast and slow shocks, along with the distributions of the density and the flow convergence ($\nabla \cdot \nu$), in a two-dimensional (2D) slice, for $1024M_{\odot}$-b0.1 with $\mathcal{M}_{\text{turb}} = 7$ and $\beta_0 = 0.1$, which intends to reproduce the turbulence in MCs, and for $1024M_{\odot}$-b10 with $\mathcal{M}_{\text{turb}} = 0.5$ and $\beta_0 = 10$, which targets the turbulence in the ICM, at $t_{\text{end}}$. The density images display the characteristic morphologies in turbulence with different $\mathcal{M}_{\text{turb}}$. The supersonic case ($1024M_{\odot}$-b0.1, top panels) exhibits density concentrations of dots and strings, which are filaments and sheets in 3D, as was previously discussed for hydrodynamic turbulence (see, e.g., Kim & Ryu 2005) and MHD turbulence (see, e.g., Lehmann et al. 2016). The subsonic case ($1024M_{\odot}$-b10, bottom panels), on the
other hand, includes curves of discontinuities, which are surfaces of shocks with density jumps of mostly $\ll 2$.

Figure 3 shows the 3D distributions of $M_s$ for fast and slow shocks, from the same data as in Figure 2. The images of $M_s$ reveal a few points. Note that while $M_s > 1$ for fast shocks, $M_s$ could be smaller than unity for slow shocks. First, the distribution of $M_s$ shows a clear correlation to that of $\nabla \cdot v$, as expected. Second, fast shock populations dominate over slow shocks for $1024M_7-b0.1$, a weak background magnetic field case. On the other hand, slow shocks are as frequent as fast shocks for $1024M0.5-b10$, a strong magnetic field case. This trend is true, regardless of $\mathcal{M}_{\text{turb}}$, as further discussed in Section 4.2. Third, shock surfaces are neither smooth nor homogeneous, but composed of shocks with different Mach numbers. Yet, fast shocks show organized structures of strings in 2D cuts and surfaces in 3D distributions. Slow shocks, on the other hand, display fragmented structures. This could be partly because the surfaces of some slow shocks may be subject to a corrugation instability (see, e.g., Stone & Edelman 1995), as noted above.

4.2. Statistics of Shock Mach Numbers

The PDFs of $M_s$ for fast shocks (upper panels) and slow shocks (lower panels) in the cases of $\mathcal{M}_{\text{turb}} = 0.5, 1$, and 7, for which both $1024^3$ and $512^3$ simulations are available, are shown in Figures 4. Only fast shocks with $M_{fa} \geq 1.06$ and slow shocks with $M_{sl} \geq 1.06$ and $c_{s,1}/c_s \geq 0.3$ were included, as described in Section 3.2. The PDFs were obtained by averaging those from 16 and 26 data dumps for $1024^3$ and $512^3$ simulations, respectively, over the saturated state. They were normalized with $n_g^2$ to compensate the resolution effect. The average values of $M_s$ for all models are given in Table 2. Simulations of different resolutions produced reasonably converged results.

A noticeable feature is the “flat” parts with $M_s < 1$ for slow shocks in the lower panels of Figure 4. With $c_{s,1} < c_s$, those shocks still have slow Mach numbers $M_{sl} > 1$. They substantially contribute to the population of slow shocks. Yet, these are the slow shocks affected by the difficulty in their identification (see Section 3.2); different values in the constraint of $c_{s,1}/c_s$ result in differences in this part (see...
Appendix A). There is almost no such population in the case of $M_{\text{turb}} = 0.5$ and $\beta_0 = 0.1$, since weak perturbations propagating mostly perpendicular to strong background fields do not easily develop into shocks.

Another noticeable feature is the lack of fast shocks with $M_s \lesssim 2-3$ for $M_{\text{turb}} \lesssim 1$ in the case of $\beta_0 = 0.1$, in the upper panels of Figures 4; with subsonic/transonic flows, the strong background magnetic field is only mildly perturbed, and hence $c_{\text{fs},1} > a \times c_s$. In the cases of weak background magnetic fields or highly supersonic flows, magnetic field lines are easily tangled and turbulence becomes isotropic, hence the PDFs follow the usual shape, which is described below.

Except for the “anomalies” described above, the PDFs are characterized as follows. Most shocks are “weak” with low sonic Mach numbers, as previously shown (see, e.g., Smith et al. 2000; Porter et al. 2015; Lehmann et al. 2016). Shocks with high Mach numbers are rare and mostly fast shocks. They show exponentially decreasing probability distributions. When the exponential part of high $M_s$ is fitted to

\[
\frac{dN_{\text{fa}}}{dM_s} \propto \exp\left( -\frac{M_s}{M_{\text{cha},N}} \right),
\]

the characteristic Mach number, $M_{\text{cha},N}$ increases with $M_{\text{turb}}$, but decreases with $\beta_0$ although the dependence on $\beta_0$ is not strong. For instance, $M_{\text{cha},N} \approx 0.1$ for $1024M0.5-b10$ with $M_{\text{turb}} = 0.5$ and $\beta_0 = 10$; it was estimated to be $\sim 0.08$ for $M_{\text{turb}} = 0.5$ and $\beta_0 = 10^6$ in Porter et al. (2015). For $1024M7-b0.1$ with $M_{\text{turb}} = 7$ and $\beta_0 = 0.1$, on the other hand, $M_{\text{cha},N} \approx 3$, indicating that shocks with high $M_s$ are quite common. For the cases listed in Table 1, the ratio $M_{\text{cha},N}/M_{\text{turb}}$ is in a rather narrow range of $\sim 0.2-0.4$, except for $1024M0.5-b0.1$ with $M_{\text{turb}} = 0.5$ and $\beta_0 = 0.1$, where fast shocks with high $M_s$ are rare, so their PDF is not very well defined.

Figure 5 plots the total number of shock zones for fast and slow shocks and all together, $N_{\text{fa}}, N_{\text{sl}},$ and $N_{\text{fa}+\text{sl}}$, normalized to $n_{\infty}^2$, in all the cases considered (see also Table 1 for the values). The trend is summarized as follows. First, $N_{\text{fa}+\text{sl}}$ increases with $M_{\text{turb}}$ and $\beta$; that is, shocks are more common or shock surface area is larger if the turbulent flow velocity is larger and the background magnetic field is weaker. Second, fast shocks are more common than slow shocks for $\beta_0 = 10$. For $\beta_0 = 0.1$, on the other hand, slow shocks are as frequent as fast shocks, or even more frequent if $M_{\text{turb}}$ is small.
The spatial frequency of shocks may be presented as the mean distance between shock surfaces, as was done in Ryu et al. (2003). For fast and slow shocks altogether, that is, with $N_{fa+sl}$ in the bottom panel of Figure 5, the mean distance is estimated to be, for instance, $\sim (1/2) L_{inj}$ for $1024M0.5$-$b10$ and $1024M7$-$b0.1$, while it is $\sim (1/10) L_{inj}$ for $1024M7$-$b10$ with $\beta_0 = 7$ and $\kappa_0 = 10$.

4.3. Energy Dissipation at Shocks

Figures 6 shows the distributions of the energy dissipation rate in Equation (18), normalized to the energy injection rate in Equation (20), as functions of $M_s$ for fast shocks (upper panels) and slow shocks (lower panels) in the cases of $M_{turb} = 0.5$, 1, and 7. They were obtained by averaging the data over the saturated state. Again, simulations of different resolutions produced reasonably converged results.

As for the PDFs of $M_s$, the energy dissipation is contributed mostly by shocks with low Mach numbers, although the distributions of the energy dissipation have peaks at higher $M_s$ than the PDFs; the peak locates close to $M_s \sim 1$ for $1024M0.5$-$b10$ with $M_{turb} = 0.5$ and $\beta_0 = 10$, while it occurs at $M_s \sim$ a few to several in the cases of $M_{turb} = 7$. At shocks with high $M_s$, the energy dissipation per unit area, on average, scales as $\propto M_s^3$ (see Equation (15)), but the area decreases exponentially with $M_s$. Hence, the energy dissipation rate also shows exponentially decreasing distributions of $M_s$. When the...
The exponential part of high $M_s$ is fitted to

$$\frac{d\varepsilon_{fa}}{dM_s} \propto \exp \left( - \frac{M_s}{M_{cha,r}} \right),$$

(22)

the characteristic Mach number, $M_{cha,r}$, is larger for larger $M_{turb}$ and smaller $\beta_0$. For instance, $M_{cha,r}$ is estimated to be $\sim 0.14$ and $\sim 3.5$ for 1024M0.5-b10 and 1024M7-b0.1, respectively. For the cases considered here, the ratio $M_{cha,r}/M_{turb}$ is in the range of $\sim 0.3$–0.5, except for 1024M0.5-b10.

In general, the amount of energy dissipated at fast shocks is larger than that at slow shocks (see also Figure 7). The bottom panels of Figure 6, however, indicate that the energy dissipation at slow shocks with $M_s < 1$ is substantial, especially in the cases of low $M_{turb}$. This is mostly because the MHD part inside the parentheses in Equation (15), i.e., the term involving $v_A$, is positive for slow shocks, while it is negative for fast shocks; the term accounts for the decrease of the magnetic energy across the slow shock. On the top of that, as listed in Table 2, also previously shown by Lehmann et al. (2016), slow shocks, on average, form in preshock conditions with stronger magnetic fields than fast shocks. However, for slow shocks in the range of $M_s < 1$, the population was estimated with large uncertainties, as described in Section 4.2, and so should have been the energy dissipation.

Bearing the uncertainties in mind, Figure 7 plots the fraction of the total energy dissipated at fast and slow shocks and all together, normalized to the injected energy, in all the cases considered (see also Table 1 for the values). The fraction decreases with increasing $M_{turb}$, except for low $M_{turb}$ and low $\beta_0$, where the number of shocks is very small, so is the energy dissipation. The fraction of the total shock energy dissipation, $\varepsilon_{fa+sl}/\varepsilon_{inj}$, is in the range of $\sim 0.1$–0.6. It is close to $\sim 0.4$–0.6 in the cases of subsonic turbulences, $M_{turb} = 0.5$, with $\beta_0 = 1$–10. The fraction is smaller with $\sim 0.1$–0.25 for highly supersonic turbulences, $M_{turb} = 7$. The dissipation at fast shocks is larger by at least a few times, and sometimes a few tens of times, than that at slow shocks, partly because fast shocks have higher Mach numbers, $M_s$, but also because fast shocks are more abundant. An exception is the case of 1024M1-b1 with $M_{turb} = 1$ and $\beta_0 = 1$, for which the energy dissipations at both modes of shocks are comparable. However, this is perhaps the case where the uncertainties could have seriously affected the statistics, as noted above.

The rest of the turbulent energy should dissipate through the turbulent cascade. The fractions of the cascade dissipation (Equation (19)), normalized to the energy injection, $\varepsilon_{cas}/\varepsilon_{inj}$, are listed in Table 1. Again, they were obtained by averaging the data over the saturated stage. The values are in the range of
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5. Summary and Discussion

In this paper, we studied shock waves in compressible, MHD turblences with Mach numbers $M_{\text{turb}} = 0.5–7$ and initial magnetic fields of plasma beta $\beta_0 = 0.1–10$, using a set of “isothermal” simulations with up to 1024$^3$ grid zones. The ranges of the parameters were chosen to cover turbulence in the ISM and ICM. Turbulence was driven by “solenoidal” forcing, leaving the examination of the dependence on the nature of forcing as a follow-up work. We separately identified fast and slow shock populations; slow shocks are harder to reliably identify, partly because their surfaces are subject to a corrugation instability (e.g., Stone & Edelman 1995) and also because of their Mach numbers $M_s < 1$. We then obtained the PDFs of $M_s$ and calculated the dissipation of turbulent energy at shocks. In order to minimize confusion between weak shocks and waves, only fast shocks with fast Mach numbers $M_{\text{fl}} \geq 1.06$ and slow shocks with slow Mach numbers $M_{\text{sl}} \geq 1.06$ and slow wave speeds $c_{\text{s,sl}} \geq 0.3c_s$ were considered for the statistics presented. Our main findings are summarized as follows.

1. Most shocks form with low $M_s$. Strong shocks with high $M_s$, which are mostly fast shocks, are rare and follow exponentially decreasing probability distributions, $\propto \exp(-M_s/M_{\text{cha},N})$. The characteristic Mach number, $M_{\text{cha},N}$, is larger for larger $M_{\text{turb}}$, indicating that shocks with high $M_s$ are more common if turbulent flows have higher speeds, as expected. In addition, $M_{\text{cha},N}$ is larger for smaller $\beta_0$, although the dependence on $\beta_0$ is not strong. The ratio $M_{\text{cha},N}/M_{\text{turb}}$ is in the range of $\sim 0.2–0.6$ for the cases studied in this paper.

2. More shocks are induced, if turbulent flows have higher speeds and the background magnetic field is weaker. Fast shocks are more common than slow shocks, if $\beta_0 \gg 1$, i.e., in weak field cases. Slow shocks, on the other hand, are less frequent, or even more common than fast shocks, if $\beta_0 \lesssim 1$, i.e., in strong field cases. The mean distance between shock surfaces is estimated to be $\sim 1/2$ of the injection scale, $L_{\text{inj}}$, for typical turbulence expected in the ICM and MCs. It is smaller in the cases with highly supersonic turbulences in weak background magnetic fields.

3. The energy dissipation is mostly due to shocks with low $M_s$. The peak of the dissipation is close to $M_s \sim 1$ for subsonic turbulences expected in the ICM, while it is at $M_s \sim 0.5$ for several for highly supersonic turbulences in MCs. The energy dissipation at shocks with high $M_s$ follows exponentially decreasing distributions, $\propto \exp(-M_s/M_{\text{cha},N})$: this is because while the dissipation per unit area of shock surfaces, on average, increases as $\propto M_s^3$, the shock population exponentially decreases with $M_s$. The ratio $M_{\text{cha},N}/M_{\text{turb}}$ is in the range of $\sim 0.3–0.5$ for the cases studied in this paper.

4. The energy dissipation is attributed mostly to fast shocks, partly because fast shocks are stronger than slow shocks, but also because fast shocks are more common in most cases. The fraction of the turbulent energy dissipated at shocks, $\epsilon_{\text{turb}}/\epsilon_{\text{inj}}$ (the energy dissipation at both fast and slow shocks, normalized to the energy injection), decreases with increasing $M_{\text{turb}}$, ranging from $\sim 0.1$ to 0.25 for highly supersonic turbulences with $M_{\text{turb}} = 7$, to $\sim 0.4–0.6$ for subsonic turbulences with $M_{\text{turb}} = 0.5$. Note that both $\epsilon_{\text{turb}}/\epsilon_{\text{inj}}$ and $\epsilon_{\text{inj}}$ increase with $M_{\text{turb}}$; however, $\epsilon_{\text{inj}}$ increases faster than $\epsilon_{\text{turb}}$. The rest of the turbulent energy should dissipate through the turbulent cascade.

The statistics presented in this paper should be applicable within the context of the models considered here. For turbulences with different characteristics (different forcings, different equations of state, etc.), quantitative estimates would be different, although we still expect that more shocks form if turbulent flows have higher speeds, fast shock populations dominate over slow shocks if the background magnetic field is weak, etc.

Finally, our work could have implications for physical processes in turbulent ISM and ICM. For instance, the dissipation of turbulent energy at shocks may have consequences for observed shock structures and emission line spectra. We leave the investigation of those to future works.
Appendix A

Dependence on Shock Identification Parameters

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The statistics present in Section 4 inevitably depend on the parameters employed in the identification of shocks. We chose \( \max(\rho_{i+1}/\rho_{i-1}, \rho_{i-1}/\rho_{i+1}) \geq 1.03^2 \), \( M_{fa} \geq 1.06 \), \( M_{sl} \geq 1.06 \), and \( c_{sl,1} \geq 0.3 \times c_s \), after a number of experiments with different values. Smaller lower bounds result in more confusion with waves, while larger values exclude some shocks. Here, we present the statistics for a couple of different sets of parameter values to demonstrate the dependence of our results on them.

Figure 8 shows the total number of shock zones normalized to \( n_g^2 \) (left panels) and the turbulent energy dissipation at shocks normalized to the energy injection (right panels), obtained with \( M_{fa} \geq 1.1 \) and \( M_{sl} \geq 1.1 \), keeping the minimum value of \( c_{sl,1} \) the same; \( \max(\rho_{i+1}/\rho_{i-1}, \rho_{i-1}/\rho_{i+1}) \geq 1.05^2 \) was used, but once the minimum value of the square-root of it is smaller than those of \( M_{fa} \) and \( M_{sl} \), the results are insensitive to it. The trends still hold; the number of shock zones, \( N_{fa+sl}/n_g^2 \), is larger if turbulence has larger \( \mathcal{M}_{turb} \) and the background magnetic field is weaker, and the fraction of the energy dissipation at shocks, \( \epsilon_{fa+sl}/\epsilon_{inj} \), decreases with increasing \( \mathcal{M}_{turb} \), except for the cases of low \( \mathcal{M}_{turb} \) and low \( \beta_0 \). However, \( N_{fa+sl}/n_g^2 \) and \( \epsilon_{fa+sl}/\epsilon_{inj} \) are smaller, as expected. The decrements are larger for smaller \( \mathcal{M}_{turb} \). For instance, \( \epsilon_{fa+sl}/\epsilon_{inj} \) is close to \( \sim 0.2-0.25 \) for \( \mathcal{M}_{turb} = 0.5 \) and \( \beta_0 = 1-10 \), while it is in the range of \( \sim 0.05-0.2 \) for \( \mathcal{M}_{turb} = 7 \).

Figure 9 shows the total number of shock zones and the energy dissipation at shocks, obtained with \( c_{sl,1} \geq 0.4 \times c_s \), keeping the lower bounds of \( M_{fa} \) and \( M_{sl} \) the same. Obviously, the increase in the minimum value of \( c_{sl,1} \) results in the decrease in slow shock populations, without affecting fast shock populations. But \( N_{fa+sl}/n_g^2 \) and \( \epsilon_{fa+sl}/\epsilon_{inj} \) are not much affected, since they are contributed mostly from fast shock populations.
Figure 9. Statistics for shocks with $c_{sl,1}/c_s > 0.4$. Left panels: number of the grid zones identified as fast (top) and slow (middle) shocks and their sum (bottom), normalized to $n_g^2$, as a function of $M_{turb}$. Right panels: fraction of the energy dissipation at fast (top) and slow (middle) shocks and their sum (bottom) as a function of $M_{turb}$.

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