PROBABILISTIC INVENTORY MODEL WITH LEAD TIME EQUALS SINGLE SCHEDULING PERIOD AND VARYING DETERIORATING COST UNDER CONSTRAINT

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ABSTRACT

In this paper inventory model of declining goods with ambiguous and imprecise details about available storage has been established. Here, our targets are: The optimal scheduling period. The optimal order-level. Minimize the wastage cost due to the deterioration. Minimize the expected average total cost under a restriction on the expected average varying deteriorating cost by using the Lagrange method. This model, is developed for continuously deterioration rate is constant or follows a two-parameter Weibull distribution with varying and constrained expected deteriorating cost, Where the lead time is only one period of time, without shortage and when demand is a random variable during any scheduling time. These probabilistic models are studied in two cases: crisp numbers and trapezoidal fuzzy numbers. Some special cases are deduced. There is a numerical illustration to illustrate the proposed model in the crisp case and the fuzzy case and the sensitivity analysis is performed.

Keywords: Deterioration, Probabilistic demand rate, Scheduling period, Varying cost, Weibull distribution.

1 INTRODUCTION

In many inventory systems the impact of the deterioration is so critical that it cannot be disregarded. By deterioration we mean decay, degradation or spoilage in such a way that the object cannot be used for its original purpose, i.e. the object undergoes a shift in storage such that it loses its value partly or completely over time. In some models the item may be lost over time and therefore its price may decline based on its age, although in some other systems the item will become obsolete due to changes in design or technical advances. In any such cases, the loss of inventory due to deterioration cannot be ignored when evaluating the system, because it provides an incomplete model of inventory systems operations. Photographic videos, medical products and pharmaceuticals, other chemicals, electronic components are some of the examples of items where there may be significant degradation during their usual inventory storage period. Therefore, the failure must be taken into account when deciding on their storage policies. Many probabilistic models have been created for goods that are continually deteriorating in time for example, [13] produced periodic review inventory model for gumbel deteriorating items when demand follows pareto distribution. [12] studied a probabilistic inventory model with two-parameter exponential deteriorating rate and pareto demand distribution. [15] introduced an inventory model for deteriorating items with weibull deterioration with time dependent demand and shortages. [4] presented an inventory model for deteriorating items with quadratic demand and partial backlogging. [3] studied optimal control of production inventory model with exponential deterioration. [6] produced a probabilistic scheduling period inventory system for deteriorating items with lead time equal to one scheduling period. [5] studied m-scheduling-period inventory model for deteriorating items with instantaneous demand. [2] explained a note on an order-level inventory model for a system with constant rate of deterioration. Many probabilistic inventory models assume that the cost units are constant or that one of these units differs. Hundreds of articles and books present models for this under a wide range of conditions and assumptions. [10] studied probabilistic multi-item inventory model with varying mixture shortage cost
under restrictions. [9] deduced probabilistic periodic review \(<Q_{m_r}, N_r>\) inventory model using Lagrange technique and fuzzy adapting particle swarm optimization. [14] produced multi-item EOQ model with varying holding cost a geometric programming approach. [11] examined periodic review probabilistic multi-item inventory system with zero lead time under constraints and varying ordering cost. [8] introduce probabilistic single-item inventory problem with varying order cost under two linear constints. [1] studied probabilistic multi-item inventory model with varying order cost under two restrictions. [7] explained procurement and inventory system: theory and analysis.

In this paper, we use a different approach, in the absence of shortages we find a stochastic stock management mechanism that deals with goods that deteriorate. Radioactive materials used in medical diagnostics, coal or any other form of fuel used for heating boilers—particularly in glass, cement or any other similar industry where heating a furnace up to the certain temperature takes a considerable amount of time and also some medicines used during the treatment of deadly diseases are examples of failing products that could lead to disaster and should therefore not be permitted.

This problem of finding the optimum scheduling period for an inventory model which is subject to continuous decline and stochastic demand is considered here, varying deteriorating costs, a restriction on the expected deteriorating costs, multi-items \((r = 1,2,3,...,n)\), non-zero lead time and with lead time equivalent to one scheduling period and then each scheduling period \(N_r\) is the lead time for the next scheduling period, that might be the case with certain inventory systems in where the ordered lot comes at the time next order is placed. Under the conditions of no shortage and continuous variable time, the model is built. The mathematical model is analyzed for a deterioration function in a general form, the deterioration rate is constant or follows a two-parameter Weibull distribution and then its particular case is presented. The minimum expected total cost of the system is obtained. We evaluated the optimal policy variables in two sub models: the first is, as usual, when the cost components are considered as crisp values, and the second one is when the cost components are fuzzified as a trapezoidal fuzzy numbers, which is called the fuzzy model. Finally, a numerical example is solved and the sensitivity analysis is conducted to demonstrate the effects of increasing parameter values on the optimal solution for the system.

2 Notations and Assumptions

To develop the inventory model of deteriorating items with varying deteriorating cost, the following notations and assumptions will be used in this paper:

2.1 Notations

\(n\) \hspace{1cm} Number of items.

\(MISS\) \hspace{1cm} The multi-item, single source.

\(t\) \hspace{1cm} An element of a random variable represents the time to the deterioration.

\(X_r\) \hspace{1cm} The demand for the \(r^{th}\) item.

\(N_r\) \hspace{1cm} The decision variable representing the length of the any scheduling period for the \(r^{th}\) item.

\(N_r^+\) \hspace{1cm} The optimum scheduling period for the \(r^{th}\) item.

\(X_r/N_r\) \hspace{1cm} The demand \(X_r\) during any scheduling period \(N_r\) for the \(r^{th}\) item.

\(E(X_r/N_r)\) \hspace{1cm} The mean demand \(X_r\) during any scheduling period \(N_r\) for the \(r^{th}\) item.

\(\bar{D}_r\) \hspace{1cm} The average demand rate for the \(r^{th}\) item.

\(\theta_r(t)\) \hspace{1cm} The deteriorating rate of the on hand inventory at time \(t\) for the \(r^{th}\) item.

\(\theta_r\) \hspace{1cm} Constant rate of deterioration for the \(r^{th}\) item.

\(Q_{mr}\) \hspace{1cm} The order-level of the system for the \(r^{th}\) item.

\(Q_{mr} - I_r\) \hspace{1cm} The on hand stock at the time an order is received i.e. at the start of the lead
time period for \( r^{th} \) item, where \( I_r \) is an order place units for the \( r^{th} \) item.

- \( \text{Q}_{mr}^* \) The optimum order-level for the \( r^{th} \) item.
- \( \text{Q}_{1r} \) Inventory level initial for the \( r^{th} \) item for the current period.
- \( \text{q}_{1r}(t) \) The inventory level of the system at various points of time during the lead time for \( r^{th} \) item.
- \( \text{q}_{2r}(t) \) The inventory location of \( r^{th} \) item at various points of time during the current period.
- \( \text{q}_{dr}(N_r) \) The number of units required to deteriorate for the \( r^{th} \) item.
- \( \text{C}_{or} \) The order cost per unit for the \( r^{th} \) item.
- \( \text{C}_{hr} \) The inventory holding cost per unit for the \( r^{th} \) item.
- \( \text{C}_{dr} \) The deteriorating cost per unit for the \( r^{th} \) item.
- \( \text{C}_{dr}(N_r) \) The varying deteriorating cost per unit for the \( r^{th} \) item,

\[
\text{C}_{dr}(N_r) = N_r^\beta \text{C}_{dr} \tag{1}
\]

- \( \beta \) A constant real number chose to provide the best fit for estimated expected cost function, \( 0 < \beta < 1 \).
- \( \text{K}_{dr} \) The limitation on the expected average deteriorating cost for the \( r^{th} \) item.
- \( \lambda_{dr} \) Lagrange multiplier for the \( r^{th} \) item.
- \( \bar{H}_r(N_r) \) The average inventory level per time unit for the \( r^{th} \) item.
- \( \text{E}(\text{OC}_r) \) The expected ordering cost for the \( r^{th} \) item.
- \( \text{E}(\text{HC}_r) \) The expected holding cost for the \( r^{th} \) item.
- \( \text{E}(\text{DC}_r(N_r)) \) The expected varying deteriorating cost for the \( r^{th} \) item.
- \( \text{E}(\text{TC}_r(N_r)) \) The expected total cost function for the \( r^{th} \) item.
- \( \text{E}(\text{TC}) \) The expected total cost function of the system.
- \( \text{min}\text{E}(\text{TC}) \) The minimum expected total cost function.

2.2 Assumptions

1. The demand \( X_r \) is a random variable at any scheduling period \( N_r \), for the \( r^{th} \) item with probability density function (p.d.f.) \( f(x_r/N_r) \) and \( (X_r \text{min} \leq x_r \leq X_r \text{max}) \).
2. The average demand rate is \( D_r = \mu_r(N_r)/N_r \) where

\[
\text{E}(X_r/N_r) = \mu_r(N_r) = \int_{x_r \text{min}}^{x_r \text{max}} x_r f(x_r/N_r)dx
\]

is the mean demand during \( N_r \).
3. The replenishment rate is infinite.
4. Not permitted shortages.
5. Non-zero lead time and equal to one scheduling period.
6. The rate of deterioration is constant or follows a Weibull two-parameter distribution,

\[
\theta_r(t) = \eta_r \pi_r t^{\pi_r-1}
\]

where \( 0 < \eta_r < 1 \) is the scale parameter, \( \pi_r > 1 \) is the shape parameter. It is assumed that the deterioration of units increases with time \( t > 0 \).
7. The deteriorated inventory is not being repaired or replaced during the time under consideration.
3 The Mathematical Model for Crisp Environmental

Consider the total cost of the system composed of three components. Since the number of replenishments per unit for \( r^{th} \) item is \( 1/N_r \), the expected average total cost per unit time of the system during the period is composed of the expected average order cost per unit time of \( r^{th} \) item, the expected average holding cost per unit time of \( r^{th} \) item and the expected average varying deteriorating cost of \( r^{th} \) item as follows:

\[
E(\text{TC}) = \sum_{r=1}^{n} \left[ E(\text{OC}_r) + E(\text{HC}_r) + E(\text{DC}_r(N_r)) \right]
\]

\[
E(\text{TC}) = \sum_{r=1}^{n} \left[ \frac{C_{or}}{N_r} + C_{hr} \bar{H}_r(N_r) + C_{dr}(N_r) \frac{q_{dr}(N_r)}{N_r} \right]
\]

(3)

Notice that every scheduling period \( N_r \) is the lead time for the next scheduling period. Then we considered a pair of consequential scheduling periods and we called them respectively “the lead time” and “the current period”. Figure 1 shows the inventory level of this system.

![Figure 1: Graphical representation of the inventory level during the lead time period and the current period](image)

Then, note that the degradation in inventory is caused by demands and the deterioration of units, the differential equation representing the inventory level \( q_{lr}(t) \) of the system at time \( t \) \((0 \leq t \leq N_r)\) during the lead time is demonstrated by the following:

\[
\frac{dq_{lr}(t)}{dt} + \theta_r(t)q_{lr}(t) = -\frac{x_r}{N_r}, \quad 0 \leq t \leq N_r
\]

(4)

Using the \( q_{lr}(0) = Q_{mr} - I_r \) limit condition, the solution of Eq. (4) is:

\[
q_{lr}(t) = \left[ Q_{mr} - I_r - \frac{x_r}{N_r} \int_0^t A(t)dt \right] / A(t), \quad (0 \leq t \leq N_r)
\]

(5)

Where

\[
A(t) = e^{\int_0^t \theta_r(t)dt}
\]

(6)

After the order of \( I_r \) unit has been realized then, the final inventory is:

\[
q_{lr}(N_r) + I_r' = \left[ Q_{mr} - I_r - \frac{x_r}{N_r} \int_0^{N_r} A(t)dt \right] / A(N_r) + I_r'
\]

Which will be independent of \( I_r' \), iff \( I_r' = I_r/A(N_r) \). For this, an order must be put for \( I_r/A(N_r) \) units. So that the final amount of inventory for the lead period after completion of the order is:

\[
Q_{lr} = \left[ Q_{mr} - \frac{x_r}{N_r} \int_0^{N_r} A(t)dt \right] / A(N_r)
\]

(7)
PROBABILISTIC INVENTORY MODEL WITH LEAD TIME EQUALS...

Where \( Q_{1r} \) will be the inventory level initial for the current period, the inventory would then deplete due to the demand and deteriorate with the preceding consideration. The inventory location \( q_{2r}(t) \) of the \( r^{th} \) item in the interval, \( 0 \leq t \leq N_r \) follows the differential equation in the current period with demands \( x_r \) is given by:

\[
\frac{dq_{2r}(t)}{dt} + \theta_r(t)q_{2r}(t) = -\frac{x_r}{N_r}, \quad (0 \leq t \leq N_r)
\]

Using the \( q_{2r}(0) = Q_{1r} \) limit condition, the solution of Eq. (8) is:

\[
q_{2r}(t) = \left[ Q_{1r} - \frac{x_r}{N_r} \int_0^t A(t) dt \right] / A(t), \quad (0 \leq t \leq N_r)
\]

Some drugs and medicines used in the remedy of deadly diseases are examples of a number of the deteriorating items whose shortage may lead to disastrous results and should not be allowed afterwards. So even the \( x_{r_{\text{max}}}(N_r) \) maximum demand and the degradation of the units during the lead time and also the current period should not allow shortage. That means the \( Q_{mr} \) order level has to be from Eqs. (7) and (9):

\[
Q_{mr} = \frac{(1 + A(N_r)) x_{r_{\text{max}}}(N_r)}{N_r} \int_0^{N_r} \frac{A(t) dt}{t}
\]

Then, from Eq. (7), (9) and (10), the average inventory level per unit time of the \( r^{th} \) item is given by:

\[
\bar{H}_r(N_r) = \frac{1}{N_r} \int_0^{N_r} E(q_{2r}(t)) dt
\]

\[
= \left\{ \left( 1 + A(N_r) \right) x_{r_{\text{max}}}(N_r) - N_r \bar{D}_r \right\} \int_0^{N_r} \frac{A(t) dt}{N_r} - \frac{\bar{D}_r}{N_r} \int_0^{N_r} \frac{A(t) dt}{A(t)}
\]

and the number of units expected to decline for the \( r^{th} \) item is given by:

\[
q_{dr}(N_r) = E(Q_{1r}) - \mu(N_r) - E(q_{2r}(N_r))
\]

\[
= \left\{ x_{r_{\text{max}}} - \frac{(x_{r_{\text{max}}} - N_r \bar{D}_r)}{(A(N_r))^2} \right\} \int_0^{N_r} \frac{A(t) dt}{N_r} - N_r \bar{D}_r
\]

Then substituting from Eq. (11) and (12) in (3) we shall obtain the expected average total cost per unit time of the system for the period as follows:

\[
E(TC) = \sum_{r=1}^{n} \frac{C_{or}}{N_r} \int_0^{N_r} \frac{A(t) dt}{A(t)} - C_{dr} N_r \beta_\bar{D}_r
\]

\[+ C_{hr} \left\{ \left( 1 + A(N_r) \right) x_{r_{\text{max}}}(N_r) - N_r \bar{D}_r \right\} \int_0^{N_r} \frac{A(t) dt}{N_r} - \frac{1}{A(t)} \int_0^{N_r} dt
\]

\[+ C_{dr} N_r \beta_2 \left\{ x_{r_{\text{max}}} - \frac{(x_{r_{\text{max}}} - N_r \bar{D}_r)}{(A(N_r))^2} \right\} \int_0^{N_r} \frac{A(t) dt}{A(t)}
\]

(13)
In order to find the optimal \( N_r \) scheduling period, it is important to know the implied structure of the \( x_{r, \text{max}} \) and \( \theta_r(t) \) function. 

Consider the following relationship

\[
x_{r, \text{max}}(N_r) = a_r \mu_r(N_r) = a_r N_r \bar{D}_r, \quad a_r \geq 1
\]

hence

\[
x_{r, \text{min}}(N_r) = b_r \mu_r(N_r) = b_r N_r \bar{D}_r, \quad b_r \leq 1
\]

where \( a_r \) and \( b_r \) are known positive constants.

### 3.1 Constant Deterioration Rate

The model for:

\[
\theta_r(t) = \theta_r \rightarrow \lambda(t) = e^{\lambda(t)} = e^{\theta_r t}, (0 \leq t \leq N_r)
\]

Therefore substituting from Eq. (14) and (15) in Eq. (13) we get:

\[
E(\text{TC}) = \sum_{r=1}^{n} \left[ \frac{C_{or}}{N_r} + \left( \frac{C_{hr}}{\theta_r} + C_{dr} N_r^\beta \right) \bar{D}_r \left( \frac{[a_r - (a_r - 1)e^{-2\theta_r N_r}]\left(e^{\theta_r N_r} - 1\right)}{\theta_r N_r} - 1 \right) \right]
\]

The objective is to minimize the expected total cost \( E(\text{TC}(N_r)) \) under the constraint:

\[
C_{dr} N_r^\beta \bar{D}_r \left( \frac{[a_r - (a_r - 1)e^{-2\theta_r N_r}]\left(e^{\theta_r N_r} - 1\right)}{\theta_r N_r} - 1 \right) - K_{dr} \leq 0
\]

To solve this primary function which is a problem of convex programming, let us write the equations of the previews in the form below:

\[
E(\text{TC}) = \sum_{r=1}^{n} \left[ \frac{C_{or}}{N_r} + \left( \frac{C_{hr}}{\theta_r} + C_{dr} N_r^\beta \right) \bar{D}_r \left( \frac{[a_r - (a_r - 1)e^{-2\theta_r N_r}]\left(e^{\theta_r N_r} - 1\right)}{\theta_r N_r} - 1 \right) \right]
\]

Subject to:

\[
C_{dr} N_r^\beta \bar{D}_r \left( \frac{[a_r - (a_r - 1)e^{-2\theta_r N_r}]\left(e^{\theta_r N_r} - 1\right)}{\theta_r N_r} - 1 \right) - K_{dr} \leq 0
\]

The Lagrange multipliers technique is used as follows to find the optimal values \( N_r^* \) for a given \( Q_{mr}^* \) which minimize (16) under the constraint (17):

\[
L(N_r, \lambda_{dr}) = \sum_{r=1}^{n} \left[ \frac{C_{or}}{N_r} + \left( \frac{C_{hr}}{\theta_r} + C_{dr} N_r^\beta \right) \bar{D}_r \left( \frac{[a_r - (a_r - 1)e^{-2\theta_r N_r}]\left(e^{\theta_r N_r} - 1\right)}{\theta_r N_r} - 1 \right) \right]
\]

\[
+ \lambda_{dr} \left[ C_{dr} N_r^\beta \bar{D}_r \left( \frac{[a_r - (a_r - 1)e^{-2\theta_r N_r}]\left(e^{\theta_r N_r} - 1\right)}{\theta_r N_r} - 1 \right) - K_{dr} \right]
\]

The optimal value \( N_r^* \) may be determined by setting each of the corresponding first partial derivatives Eq. (18) equal to zero, which is minimizing \( E(\text{TC}) \) the expected total cost. 

i.e.

\[
\frac{\partial L}{\partial N_r} \bigg|_{N_r=N_r^*}, \lambda_{dr}=\lambda_{dr}^* = 0, \quad \frac{\partial L}{\partial \lambda_{dr}} \bigg|_{N_r=N_r^*}, \lambda_{dr}=\lambda_{dr}^* = 0
\]
then we obtain:

\[
(1 + \lambda_{dr}^*)N_r^* \beta \mathcal{C}_{dr}\left[\lambda_{dr}^* g_r - \beta N_r^* \left(2N_r^* + g_r\right)e^{-2N_r^* \theta_r} + \frac{1}{\theta_r} \left(2N_r^* + g_r\right)e^{-N_r^* \theta_r} \right]
\]

and

\[
\mathcal{C}_{dr}N_r^* \beta \mathcal{D}_r \left[\left(\alpha_r - J_r \right) e^{-2\theta_r N_r^*} \left(\frac{e^{\theta_r N_r^* - 1}}{\theta_r N_r^*} - 1\right)\right] = K_{dr}
\]

where \( g_r = (1 - \beta)/\theta_r \), \( J_r = a_r - 1 \)

Clearly, from (19) and (20) we calculate the optimal scheduling period \( N_r^* \) which are used to determine the minimum \( \mathbb{E}(TC) \) expected total cost and substituting from (14), (15) in (10), the optimal order-level is given by:

\[
Q_{mr}^* = \left(\frac{e^{2\theta_r N_r^*} - 1}{\alpha_r \theta_r^*}\right)
\]

### 3.2 Deterioration Rate Follows Two-Parameter Weibull Distribution

Using \( \theta_r(t) = \eta_r t^\tau \) \( \rightarrow A(t) = a^{\eta_r t^\tau} = \sum_{i=0}^{\infty} \frac{(n_0 t_{\tau})^i}{i!} \), \( 0 \leq t \leq N_r \)

As \( 0 < \eta_r << 1 \), neglecting the \( \eta_r^2 \) and higher powers terms, Then substituting from Eq. (14) and (22) in Eq. (13) we get:

\[
\mathbb{E}(TC) = \sum_{r=1}^{n} \left[\frac{C_{dr}^*}{N_r} + \mathcal{C}_{dr} \mathcal{D}_r \left[\frac{a_r + (a_r - 1)(2\pi_r + 1)\eta_r N_r^{\beta + \pi_r}}{\pi_r + 1}\right] + \frac{\{\pi_r - (a_r - 1)(\pi_r + 1)(\pi_r + 2)\eta_r N_r^{\pi_r + 1}\}}{(\pi_r + 1)(\pi_r + 2)}\right]
\]

To find an optimal values \( N_r^* \) and \( Q_{mr}^* \) which minimize \( \mathbb{E}(TC) \) under the limitations, the Lagrange multiplier technique with the Kuhn-Tucker conditions is used as follows:

\[
L(N_r, \lambda_{dr}) = \sum_{r=1}^{n} \left[\frac{C_{dr}^*}{N_r} + \mathcal{C}_{dr} \mathcal{D}_r \left[\frac{\{\pi_r - (a_r - 1)(\pi_r + 1)(\pi_r + 2)\eta_r N_r^{\pi_r + 1}\}}{(\pi_r + 1)(\pi_r + 2)}\right] + \frac{\frac{a_r + (a_r - 1)(2\pi_r + 1)\eta_r N_r^{\beta + \pi_r}}{\pi_r + 1}} + \lambda_{dr} \left(\left(\frac{C_{dr} \mathcal{D}_r \{a_r + (a_r - 1)(2\pi_r + 1)\eta_r N_r^{\beta + \pi_r}}{\pi_r + 1}\right) - K_{dr}\right)\right]
\]
The optimal value $N_r^*$ may be determined by setting each of the corresponding first partial derivatives Eq. (24) equal to zero, which is minimizing the expected total cost. Then the following equations are obtained:

$$\frac{C_{or}}{N_r^*} = C_{dr} \bar{D}_r \left\{ 2a_r - \frac{3}{2} + \frac{(\mu_r - (a_r - 1)(\mu_r + 1)(\mu_r + 2))\eta_r N_r^*}{(\mu_r + 2)} \right\}$$

$$+ (1 + \lambda_{dr}^*) (\beta + \mu_r) \eta_r C_{dr} \bar{D}_r \frac{a_r + (a_r - 1)(2\mu_r + 1)N_r^* (\beta + \mu_r - 1)}{\mu_r + 1}$$

and

$$C_{dr} \eta_r (a_r + (a_r - 1)(2\mu_r + 1))N_r^* (\beta + \mu_r) = k_{dr}$$

Clearly, from equations (25) and (26) we calculate the optimal scheduling period $N_r^*$, which are used to determine the minimum $E(TC)$ expected total cost and the optimal order-level is given by:

$$Q_{opt}^* = \alpha_r \bar{D}_r \left( 2N_r^* + \frac{(\mu_r + 3)\eta_r N_r^* (\mu_r + 1)}{\mu_r + 1} \right)$$

4 Fuzzy Model and Solution Procedure

In actual inventory systems, the cost parameters and various parameters that include price, marketing, and demand elasticity of providers are in nature imprecise and uncertain. This confusion introduced the Fuzziness notion. Since the model proposed is in a fuzzy environment, a fuzzy decision should be taken to meet the requirements for the decision, and the results should be fuzzy. So that we consider the model in fuzzy environment. Because of uncertainty it is not easy to precisely define all parameters.

Let

$$\tilde{\theta}_r = (\theta_r - \delta_{1r}, \theta_r - \delta_{2r}, \theta_r + \delta_{3r}, \theta_r + \delta_{4r}),$$

$$\tilde{\nu}_r = (\nu_r - \delta_{5r}, \nu_r - \delta_{6r}, \nu_r + \delta_{7r}, \nu_r + \delta_{8r}),$$

$$\tilde{\sigma}_{or} = (\sigma_{or} - \delta_{9r}, \sigma_{or} - \delta_{10r}, \sigma_{or} + \delta_{11r}, \sigma_{or} + \delta_{12r}),$$

$$\tilde{\sigma}_{hr} = (\sigma_{hr} - \delta_{13r}, \sigma_{hr} - \delta_{14r}, \sigma_{hr} + \delta_{15r}, \sigma_{hr} + \delta_{16r})$$

and

$$\tilde{C}_{dr} = (C_{dr} - \delta_{17r}, C_{dr} - \delta_{18r}, C_{dr} + \delta_{19r}, C_{dr} + \delta_{20r})$$

be trapezoidal fuzzy numbers, where $\delta_{ir}, i = 1, 2, ..., 20, r = 1, 2, ..., n$, are arbitrary positive numbers under the following restrictions:

$$\theta_r > \delta_{1r} > \delta_{2r}, \delta_{3r} < \delta_{4r}$$

$$\nu_r > \delta_{5r} > \delta_{6r}, \delta_{7r} < \delta_{8r}$$

$$\sigma_{or} > \delta_{9r} > \delta_{10r}, \delta_{11r} < \delta_{12r}$$

$$\sigma_{hr} > \delta_{13r} > \delta_{14r}, \delta_{15r} < \delta_{16r}$$

and

$$C_{dr} > \delta_{17r} > \delta_{18r}, \delta_{19r} < \delta_{20r}$$

Hence, the left and right limits $\alpha$ -cuts of $\theta_r, \nu_r, \sigma_{or}, \sigma_{hr}$ and $C_{dr}$ are given as follows:

$$\tilde{\theta}_{r\gamma}(\alpha) = \theta_r - \delta_{1r} + (\delta_{1r} - \delta_{2r})\alpha$$

$$\tilde{\nu}_{r\gamma}(\alpha) = \nu_r - \delta_{5r} + (\delta_{5r} - \delta_{6r})\alpha$$

$$\tilde{\sigma}_{or\gamma}(\alpha) = \sigma_{or} - \delta_{9r} + (\delta_{9r} - \delta_{10r})\alpha$$

$$\tilde{\sigma}_{hr\gamma}(\alpha) = \sigma_{hr} - \delta_{13r} + (\delta_{13r} - \delta_{14r})\alpha$$

and
By using signed distance method, the defuzzified value of fuzzy number is given by:
\[ \hat{\theta}_r = \frac{1}{4} [4\hat{\theta}_r - \delta_1 + \delta_2 + \delta_3] \]
\[ \hat{\eta}_r = \frac{1}{4} [4\hat{\eta}_r - \delta_5 + \delta_6 + \delta_7 + \delta_8] \]
\[ \hat{C}_{or} = \frac{1}{4} [4C_{or} - \delta_9 + \delta_{10} + \delta_{11} + \delta_{12}] \]
\[ \hat{C}_{hr} = \frac{1}{4} [4C_{hr} - \delta_{13} + \delta_{14} + \delta_{15} + \delta_{16}] \]
and
\[ \hat{C}_{dr} = \frac{1}{4} [4C_{dr} - \delta_{17} + \delta_{18} + \delta_{19} + \delta_{20}] \]

Similarly, in the crisp case, the same steps can be applied here, except that the crisp values of \( \theta_r, C_{or}, C_{hr}, C_{dr} \) and \( \eta_r \) will be replaced by the fuzzy values of \( \hat{\theta}_r, \hat{C}_{or}, \hat{C}_{hr}, \hat{C}_{dr} \) and \( \hat{\eta}_r \). Then optimal values \( N_r^*, Q_{mr}^* \) can be calculated using the same previous equations to minimize expected annual total cost \( \mathbb{E}(TC(N_r)) \) for fuzzy case i.e. equations (16): (21) for constant deterioration and equations (23): (27) for Weibull deterioration.

5 Special Cases

5.1 Put \( \beta = 0 \) and \( K_{dr} = -\infty \) \( \Rightarrow C_{dr}(N_r) = C_{dr} \) and \( \lambda_{dr} = 0 \). Thus Eq. (16) and (19) for the single item is given by:
\[ \mathbb{E}(TC) = \frac{C_o}{N} + \frac{D}{\theta} \left( \frac{C_h + C_d}{2} \right) \left[ \frac{a - (a - 1)e^{-2\theta N}}{\theta N} - 1 \right] \]  (28)
and
\[ -\frac{C_o}{D} + \left( \frac{C_h}{\theta} + C_d \right) \left[ a \left( N^* - \frac{1}{\theta} \right) e^{N^* \theta} + (a - 1) \left( N^* + \frac{1}{\theta} \right) e^{-N^* \theta} - (a - 1) \left( 2N^* + \frac{1}{\theta} \right) e^{-2N^* \theta} + \frac{a}{\theta} \right] = 0 \]  (29)

Eqs. (28) and (29) are the same as those obtained by [7] for the similar model for deteriorating items without any varying or constraint of deteriorating cost for a single item if the lead time is equal to \( N_r \).

5.2 When there is no deterioration, i.e. \( \theta_r = 0 \) then Eq. (16), (19) and (21) for the single item is given by:
\[ \mathbb{E}(TC) = \left[ \frac{C_o}{N} + C_h (2a - 3) \right] \]  (30)
and
\[ N^* = \sqrt{2C_o/C_h (4a - 3)D} \]  (31)

Eqs. (30) and (31) are the same as those obtained by [7] for the similar model for nondeteriorating items.

6 A Numerical Example

To explain the inventory model described above, take the following parameter values for a hypothetical inventory system as shown in Table 1, Table 2 and Table 3. During the scheduling period \( N_r \) the demand \( X_r \) follows the uniform distribution defined by:
\[ f(x_r/N_r) = \begin{cases} 
1/50N_1 & x_1 \in [0,50N_1] \\
1/50N_2 & x_2 \in [0,50N_2] \\
1/50N_3 & x_3 \in [50N_3,100N_3] \\
0 & \text{otherwise}
\end{cases} \]
Table 1: The crisp parameters for multi-item

| Parameters          | Item 1       | Item 2       | Item 3       |
|---------------------|--------------|--------------|--------------|
| $C_{o,r}$ (per order) | 200 $      | 250 $      | 300 $      |
| $C_{p,r}$ (perunit/day) | 0.005 $    | 0.0055 $    | 0.0060 $    |
| $C_{d,r}$ (perunit)   | 5 $       | 6 $       | 7 $       |
| $\theta_r$            | 0.01       | 0.014      | 0.018      |
| $\eta_r$              | 0.001      | 0.005      | 0.003      |
| $K_{d,r}$ (Constant)   | 25         | 36         | 58         |
| $K_{d,r}$ (Weibull)    | 14         | 27         | 31         |

Table 2: The fuzzy parameters for multi-item

| Parameters          | Item 1              | Item 2              | Item 3              |
|---------------------|---------------------|---------------------|---------------------|
| $\bar{C}_{o,r}$ (per order) | (30,70,210,250) | (40,80,260,300) | (50,90,310,340) |
| $\bar{C}_{p,r}$ (perunit/day) | (0.0045,0.0048,0.0053,0.0055) | (0.0050,0.0051,0.0058,0.0061) | (0.0052,0.0054,0.0062,0.0065) |
| $\bar{C}_{d,r}$ (perunit) | (3.4,6,8) | (3.5,8,10) | (5.6,10,12) |
| $\bar{\theta}_r$      | (0.004,0.006,0.010,0.014) | (0.006,0.010,0.014,0.018) | (0.010,0.014,0.018,0.022) |
| $\bar{\eta}_r$        | (0.0005,0.0010,0.0011,0.0015) | (0.003,0.005,0.006,0.007) | (0.002,0.0025,0.003,0.004) |
| $\bar{K}_{d,r}$ (Constant) | 19       | 28       | 47       |
| $\bar{K}_{d,r}$ (Weibull) | 11       | 22       | 25       |

SOLUTION:

We are solve that $(\mu_1, \mu_2, \mu_3) = (25N_1, 25N_2, 75N_3)$, i.e. $(\overline{D}_1, \overline{D}_2, \overline{D}_3) = (25, 25, 75)$ and if $X_{T_{max}}(N_r) = \bar{a}_1N_r\overline{D}_1$, then $(\alpha_1, \alpha_2, \alpha_3) = (2, 2, 4/3)$. The optimal value $N_{r^*}$ can be obtained by solving Eqs. (19) and (20) of the constant deterioration and Eqs. (25) and (26) of the Weibull deterioration for different values of $\beta$ and consequently the corresponding optimum order-level $Q_{mr^*}$ and minimum expected total cost of both the crisp and fuzzy environmental for three items are illustrated in Table 3 and Table 4.

Table 3: Crisp and fuzzy values for constant deterioration

| Item 1 | Crisp Case | Fuzzy Case |
|--------|------------|------------|
| $\beta$ | $N_r$ | $\lambda_{dr}$ | $Q_{mr}$ | $E(T_C^r(N_r))$ | $\bar{N}_r$ | $\lambda_{dr}$ | $\bar{Q}_{mr}$ | $E(T_C^f(\bar{N}_r))$ |
| 0.1    | 6.75257   | 0.022467   | 722.979   | 56.6837   | 5.80520 | 0.08448 | 610.131   | 44.9108   |
| 0.2    | 5.74426   | 0.119338   | 608.723   | 61.5797   | 5.00491 | 0.17283 | 522.400   | 48.5229   |
| 0.3    | 5.01275   | 0.195265   | 527.264   | 66.4397   | 4.41641 | 0.23991 | 458.643   | 52.0701   |
| 0.4    | 4.46209   | 0.254428   | 466.725   | 71.1965   | 3.96843 | 0.29030 | 410.535   | 55.5112   |
| 0.5    | 4.03503   | 0.300007   | 420.232   | 75.8104   | 3.61773 | 0.32745 | 373.129   | 58.8232   |
| 0.6    | 3.69565   | 0.334506   | 383.566   | 80.2588   | 3.33677 | 0.35400 | 343.322   | 61.9950   |
| 0.7    | 3.42040   | 0.359911   | 354.010   | 84.5300   | 3.10729 | 0.37205 | 319.083   | 65.0228   |
| 0.8    | 3.19322   | 0.377909   | 329.739   | 88.6202   | 2.91377 | 0.38324 | 299.029   | 67.9069   |
| 0.9    | 3.00297   | 0.389765   | 309.499   | 92.5299   | 2.75634 | 0.38889 | 282.194   | 70.6510   |
| 1.0    | 5.89359   | 0.045779   | 640.776   | 80.3930   | 4.98563 | 0.07163 | 529.616   | 63.7702   |
## Table 4: Crisp and fuzzy values for Weibull deterioration

| Item | Crisp Case | Fuzzy Case |
|------|------------|------------|
|      | \( \beta \) | \( N_p \) | \( \lambda_{dr} \) | \( Q_{mr} \) | \( E(TC_F(N_p)) \) | \( \bar{N}_p \) | \( \bar{\lambda}_{dr} \) | \( \bar{Q}_{mr} \) | \( E(TC_F(N_p)) \) |
| Item 1 | 0.1 | 6.31817 | 0.01222 | 652.835 | 47.6029 | 5.503348 | 0.02870 | 564.225 | 38.150 |
|      | 0.2 | 5.81033 | 0.06062 | 597.379 | 50.2168 | 5.09286 | 0.07155 | 520.294 | 40.0751 |
|      | 0.3 | 5.38238 | 0.10326 | 551.232 | 52.8240 | 4.74487 | 0.10865 | 483.389 | 41.9845 |
|      | 0.4 | 5.01784 | 0.14075 | 512.313 | 55.4127 | 4.44681 | 0.14069 | 452.009 | 43.8706 |
|      | 0.5 | 4.70432 | 0.17362 | 479.107 | 57.9734 | 4.18916 | 0.16826 | 425.042 | 45.7276 |
|      | 0.6 | 4.43232 | 0.20234 | 450.488 | 60.4991 | 3.96459 | 0.19185 | 401.652 | 47.5512 |
|      | 0.7 | 4.19452 | 0.22734 | 425.602 | 62.9844 | 3.76741 | 0.21193 | 381.197 | 49.3384 |
|      | 0.8 | 3.98514 | 0.24900 | 403.788 | 65.4252 | 3.59311 | 0.22888 | 363.176 | 51.0871 |
|      | 0.9 | 3.79960 | 0.26766 | 384.532 | 67.8187 | 3.43808 | 0.24305 | 347.195 | 52.7959 |
| Item 2 | 0.1 | 5.43322 | 0.02416 | 574.286 | 74.8417 | 4.41062 | 0.05387 | 460.366 | 62.0269 |
|      | 0.2 | 4.91834 | 0.07224 | 515.975 | 79.4903 | 4.04192 | 0.08894 | 419.711 | 65.4215 |
|      | 0.3 | 4.50178 | 0.11209 | 469.527 | 84.0566 | 3.74014 | 0.11653 | 386.797 | 68.7154 |
|      | 0.4 | 4.15906 | 0.14484 | 431.781 | 88.5193 | 3.48928 | 0.13784 | 359.674 | 71.9000 |
|      | 0.5 | 3.87298 | 0.17149 | 400.582 | 92.8642 | 3.27793 | 0.15387 | 336.985 | 74.9711 |
|      | 0.6 | 3.63114 | 0.19288 | 374.421 | 97.0827 | 3.09775 | 0.16547 | 317.756 | 77.9275 |
|      | 0.7 | 3.42442 | 0.20975 | 352.207 | 101.170 | 2.94257 | 0.17332 | 301.275 | 80.7698 |
|      | 0.8 | 3.24597 | 0.22270 | 333.139 | 105.124 | 2.80768 | 0.17802 | 287.009 | 83.5003 |
|      | 0.9 | 3.09057 | 0.23230 | 316.613 | 108.944 | 2.68947 | 0.18007 | 274.552 | 86.1220 |
| Item 3 | 0.1 | 4.73933 | 0.03328 | 987.972 | 96.7710 | 3.96935 | 0.00548 | 817.267 | 76.7695 |
|      | 0.2 | 4.38461 | 0.06707 | 909.178 | 101.709 | 3.70496 | 0.02882 | 760.282 | 80.1872 |
|      | 0.3 | 4.08661 | 0.09519 | 843.808 | 106.545 | 3.48096 | 0.04728 | 712.390 | 83.5047 |
|      | 0.4 | 3.83331 | 0.11841 | 788.803 | 111.264 | 3.28909 | 0.06158 | 671.640 | 86.7176 |
|      | 0.5 | 3.61577 | 0.13737 | 741.954 | 115.860 | 3.12316 | 0.07234 | 636.589 | 89.824 |
|      | 0.6 | 3.42722 | 0.15262 | 701.627 | 120.327 | 2.97842 | 0.08007 | 606.153 | 92.8239 |
|      | 0.7 | 3.26246 | 0.16464 | 666.588 | 124.662 | 2.85120 | 0.08523 | 579.503 | 95.7185 |
|      | 0.8 | 3.11740 | 0.17385 | 635.892 | 128.865 | 2.73858 | 0.08818 | 555.992 | 98.5099 |
|      | 0.9 | 2.98885 | 0.18061 | 608.801 | 132.937 | 2.63828 | 0.08923 | 535.111 | 101.201 |
7 Sensitivity Analysis

A sensitivity analysis is performed to verify the stability of the model according to different values of parameters $\theta_r$ and $\eta_r$. We trade one parameter at a time retaining the opposite unchanged parameters, find $N_r^*, Q_{mr}^*, \min E_r(TC(N_r))'$, then calculate the following sensitivity measure 
$$S(N_r) = \left[\left(\frac{N_r^*}{N_r^*} - 1\right) \times 100\right]$$
similarly for $Q_{mr}$ and $E_r(TC(N_r))$. The results are summarized in Table 4 and Table 5.

Moreover, we know that the optimal values of $N_r$ would have been $N_{r0}^*$ for three items from Eq. (31) was disregarded degradation. Then find $\min E_r(TC(N_{r0}^*))'$ for the three items that would have been in this case with each change in $\theta_r$ and $\eta_r$. Finally, measure the potential cost reductions using the next formula as a compare model for nondeteriorating products:
$$PCR_r = \left[1 - \left(\frac{\min E_r(TC(N_r))'}{\min E_r(TC(N_{r0}^*))'}\right)\right] \times 100$$

The results of the sensitivity analysis mentioned above and the potential savings of both the crisp and fuzzy environmental for the three items are illustrated in Table 5: Table 8.

| Item 1 | $\theta_r$ | $N_r^*$ | $Q_{mr}^*$ | $\min E_r(TC_r(N_r))'$ | $S(N_r)$ | $S(Q_{mr})$ | $S(E_r(TC_r))$ | $\min E_r(TC_r(N_{r0}^*))'$ | PCR_r |
|--------|------------|---------|------------|-------------------------|----------|-------------|-----------------|----------------------------|------|
| 0.006  | 10.73      | 1145.6  | 46.9181    | 58.967                  | -17.228  | 78.0760     | 39.907          |                            |      |
| 0.008  | 8.268      | 883.99  | 51.7194    | 22.443                  | 22.271   | 8.7579      | 47.109          |                            |      |
| 0.010  | 6.753      | 722.98  | 56.6837    | 0.0000                  | 0.0000   | 0.0000      | 51.593          |                            |      |
| 0.012  | 5.723      | 613.47  | 61.6965    | -15.247                 | -15.147  | 8.8436      | 54.665          |                            |      |
| 0.014  | 4.976      | 533.93  | 66.7139    | -26.310                 | -26.148  | 17.695      | 56.910          |                            |      |
| Item 2 | $\theta_r$ | $N_r^*$ | $Q_{mr}^*$ | $\min E_r(TC_r(N_r))'$ | $S(N_r)$ | $S(Q_{mr})$ | $S(E_r(TC_r))$ | $\min E_r(TC_r(N_{r0}^*))'$ | PCR_r |
| 0.010  | 7.997      | 867.20  | 69.9423    | 35.690                  | 35.336   | -12.999     | 52.899          |                            |      |
| 0.012  | 6.778      | 736.05  | 75.1549    | 15.007                  | 14.868   | -6.5156     | 56.512          |                            |      |
| 0.014  | 5.894      | 640.78  | 80.3930    | 0.0000                  | 0.0000   | 0.0000      | 59.155          |                            |      |
| 0.016  | 5.221      | 568.29  | 85.6289    | -11.407                 | -11.313  | 6.5129      | 61.184          |                            |      |
| 0.018  | 4.692      | 511.19  | 90.8495    | -20.383                 | -20.233  | 13.007      | 62.803          |                            |      |
| Item 3 | $\theta_r$ | $N_r^*$ | $Q_{mr}^*$ | $\min E_r(TC_r(N_r))'$ | $S(N_r)$ | $S(Q_{mr})$ | $S(E_r(TC_r))$ | $\min E_r(TC_r(N_{r0}^*))'$ | PCR_r |
| 0.014  | 5.737      | 1244.8  | 113.271    | 25.644                  | 25.403   | -10.153     | 62.171          |                            |      |
| 0.016  | 5.082      | 1103.7  | 119.673    | 11.292                  | 11.191   | -5.0748     | 64.661          |                            |      |
| 0.018  | 4.566      | 992.62  | 126.071    | 0.0000                  | 0.0000   | 0.0000      | 66.661          |                            |      |
| 0.020  | 4.150      | 902.76  | 132.453    | -9.1267                 | -9.0522  | 5.0619      | 68.315          |                            |      |
| 0.022  | 3.805      | 828.51  | 138.812    | -16.663                 | -16.532  | 10.106      | 69.717          |                            |      |

Table 6: Sensitivity analysis and potential of the fuzzy values savings for the three items with respect to $\tilde{\theta}_r$

| Fuzzy | $\tilde{\theta}_r$ | $\tilde{N}_r^*$ | $\tilde{Q}_{mr}^*$ | $\min E_r(TC_r(N_r))'$ | $S(\tilde{N}_r)$ | $S(\tilde{Q}_{mr})$ | $S(E_r(TC_r))$ | $\min E_r(TC_r(N_{r0}^*))'$ | $\tilde{PCR}_r$ |
|--------|-------------------|-----------------|-------------------|------------------------|-----------------|-----------------|----------------|----------------------------|---------------|
| Item 1 | (0.000,0.002)     | 10.341          | 1083.8            | 35.7375                | 78.136          | 77.628          | -0.24225       | 54.1261                    | 33.974        |
| (0.002,0.0048) | 7.4060          | 777.42          | 40.1937           | 27.575                 | 27.419          | -10.5031        | 71.5984        | 43.862                     |               |
| (0.004,0.006)   | 5.8052          | 610.13          | 44.9108           | 0.0000                 | 0.0000          | 0.0000          | 88.7164        | 49.377                     |               |
| (0.006,0.008)   | 4.7919          | 504.13          | 49.6967           | -17.455                | -17.373         | 10.6565         | 105.5260       | 52.906                     |               |
| (0.008,0.010)   | 4.9905          | 430.69          | 54.4897           | -29.5381               | -29.410         | 21.3289         | 122.072        | 55.363                     |               |
| Item 2 | (0.002,0.006)     | 7.2034          | 763.49            | 54.0777                | 44.4829         | 44.1591         | -15.2922       | 108.677        | 50.295                     |               |
| (0.004,0.008)   | 5.8828          | 624.28          | 58.8716           | 17.9946                | 17.8743         | -7.68158        | 130.784        | 58.985                     |               |
| (0.006,0.010)   | 4.9856          | 529.62          | 63.7702           | 0.0000                 | 0.0000          | 0.0000          | 152.526        | 58.191                     |               |
| (0.008,0.012)   | 4.3347          | 460.88          | 68.6715           | -13.0553               | -12.9791        | 7.68596         | 173.963        | 60.525                     |               |
| (0.010,0.014)   | 3.8400          | 408.59          | 73.5579           | -22.9781                | -22.8513        | 15.3484        | 195.153        | 62.308                     |               |
Table 7: Sensitivity analysis and potential savings for the three items with respect to $\eta_r$

| $\eta_r$ | $N_r$ | $Q_{mr}$ | $\min E(\text{TC}_r(N_r))$ | $S(N_r)$ | $Q(N_r)$ | $\min E(\text{TC}_r(N_r))$ | $P_{CR}$ |
|----------|------|---------|-------------------------------|--------|--------|-------------------------------|--------|
| 0.0006   | 8.0581 | 831.97  | 41.3052                      | 27.5388| 27.4401 | -13.2297                      | 169.513| 75.633 |
| 0.0008   | 7.0265 | 725.78  | 44.6306                      | 11.2110| 11.1731 | -6.24403                      | 220.746| 79.782 |
| 0.0010   | 6.3182 | 652.84  | 47.6029                      | 0.00000| 0.00000 | 0.00000                       | 271.980| 82.498 |
| 0.0012   | 5.7928 | 598.71  | 50.3118                      | -8.31577| -8.29003| 5.69057                       | 323.214| 84.434 |
| 0.0014   | 5.3828 | 556.47  | 52.8149                      | -14.8048| -14.7605| 10.9489                       | 374.447| 85.895 |
| 0.0030   | 7.4767 | 788.95  | 62.9549                      | 37.61177| 37.3786 | -15.8826                      | 227.522| 72.330 |
| 0.0040   | 6.2463 | 659.74  | 69.1262                      | 14.9658 | 14.8800 | -7.63687                      | 297.182| 76.740 |
| 0.0050   | 5.4332 | 574.29  | 74.8417                      | 0.00000| 0.00000 | 0.00000                       | 366.843| 79.598 |
| 0.0060   | 4.8481 | 512.75  | 80.1981                      | -10.7698| -10.7147| 7.1569                        | 436.503| 81.627 |
| 0.0070   | 4.4028 | 465.90  | 85.2633                      | -18.9655| -18.8726| 13.9248                       | 506.163| 83.155 |
| 0.0010   | 8.4495 | 1757.4  | 70.9118                      | 78.2857 | 77.8791 | -26.7220                      | 248.176| 71.427 |
| 0.0020   | 5.8667 | 1222.0  | 85.1949                      | 23.7883 | 23.6822 | -11.9624                      | 471.251| 81.922 |
| 0.0030   | 4.7393 | 987.97  | 96.7710                      | 0.00000| 0.00000 | 0.00000                       | 694.327| 86.063 |
| 0.0040   | 4.0734 | 849.68  | 106.772                      | -14.0506| -13.9974| 10.3343                       | 917.403| 88.362 |
| 0.0050   | 3.6221 | 755.90  | 115.714                      | -23.5746| -23.4901| 19.5752                       | 1140.48| 89.854 |

Table 8: Sensitivity analysis and potential of fuzzy values savings for the three items with respect to $\tilde{\eta}_r$

| $\tilde{\eta}_r$ | $\tilde{N}_r$ | $\tilde{Q}_{mr}$ | $\min E(\tilde{\text{TC}}_r(N_r))$ | $S(\tilde{N}_r)$ | $\tilde{Q}(\tilde{N}_r)$ | $\min E(\tilde{\text{TC}}_r(N_r))$ | $\tilde{P}_{CR}$ |
|-------------------|-------------|------------------|----------------------------------|-----------------|-----------------|----------------------------------|--------|
| (0.0001, 0.0001)  | 7.0189      | 719.18           | 33.1288                          | 27.5388         | 27.4633         | -13.1618                         | 123.791| 73.238 |
| (0.0003, 0.0003)  | 6.1203      | 627.32           | 35.7776                          | 11.2110         | 11.1820         | -6.21873                         | 160.634| 77.727 |
| (0.0005, 0.0005)  | 5.0333      | 564.23           | 38.1500                          | 0.00000         | 0.00000         | 0.00000                          | 197.478| 80.681 |
| (0.0007, 0.0007)  | 4.0457      | 517.42           | 40.3149                          | -8.31577        | -8.29003        | 5.69057                          | 234.321| 82.795 |
| (0.0009, 0.0009)  | 3.6886      | 480.88           | 42.3171                          | -14.8048        | -14.7605        | 10.9489                          | 374.447| 85.895 |
| (0.001, 0.001)    | 5.9521      | 620.49           | 52.5643                          | 34.9499         | 34.7828         | -15.2557                         | 196.442| 73.242 |
| (0.002, 0.002)    | 5.0333      | 525.07           | 57.4682                          | 14.1186         | 14.0558         | -7.34957                         | 252.194| 77.213 |
| (0.003, 0.003)    | 4.1046      | 460.37           | 62.0269                          | 0.00000         | 0.00000         | 0.00000                          | 307.946| 79.858 |
| (0.004, 0.004)    | 3.9522      | 413.03           | 66.3110                          | -10.3243        | -10.2831        | 6.96702                         | 363.699| 81.798 |
| (0.005, 0.005)    | 3.6049      | 376.58           | 70.3708                          | -18.2687        | -18.1989        | 13.4520                         | 419.451| 83.223 |
| (0.000, 0.000)    | 7.4239      | 1525.9           | 55.3700                          | 87.0296         | 86.7047         | -27.8502                        | 179.241| 69.109 |
| (0.001, 0.001)    | 4.9708      | 1022.8           | 67.2539                          | 25.2286         | 25.1488         | -12.3951                        | 361.154| 81.378 |
| (0.002, 0.002)    | 3.9694      | 817.27           | 76.7695                          | 0.00000         | 0.00000         | 0.00000                          | 543.068| 85.864 |
| (0.003, 0.003)    | 3.3923      | 698.77           | 84.9410                          | -14.5383        | -14.4995        | 10.6441                         | 724.982| 88.284 |
| (0.004, 0.004)    | 3.0062      | 619.46           | 92.2242                          | -24.2650        | -24.2039        | 20.1289                         | 906.895| 89.831 |
8 CONCLUSION

From the above described example and sensitivity analysis we found:

- The optimal values of $N_r, Q_{mr}$ decreases when $\beta$ is increase of the constant deterioration and the Weibull deterioration for the three items as shown in Table 3 and Table 4 respectively of both the crisp and fuzzy environmental.
- The minimum expected total cost decrease when $\beta$ is decrease of the constant deterioration and the Weibull deterioration for the three items as shown in Table 3 and Table 4 respectively of both the crisp and fuzzy environmental.
- The optimal values of $N_r$ and $Q_{mr}$ increase when $\theta_r$ and $\eta_r$ are increases for the three items in both the crisp and fuzzy cases as shown in Table 5: Table 8, respectively.
- The minimum expected total cost and the potential savings decrease when $\beta$ decrease for the three items in both the crisp and fuzzy cases as shown in Table 5: Table 8, respectively.

In other words, if $\beta = 0.1$, it gives the best value for the minimum expected total cost, we can conclude that the minimum expected total cost in fuzzy case is less than in the crisp case, which indicates that the fuzziness is very close to the actuality of life and gets minimum expected total cost less than the crisp case. The optimal values of $N_r, E(TC_r)$ and $Q_{mr}$ are all sensitive with respect to $\theta_r$ and $\eta_r$ and significant potential cost reductions are often present as compared to an analogous nondeteriorating model. This means that the parameter $\theta_r$ plays an important role in the assumed inventory system in the sense that a small change in it can cause significant disruptions in optimal system decisions and should therefore be precisely controlled.

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