SURGERY GROUPS OF THE FUNDAMENTAL GROUPS OF HYPERPLANE ARRANGEMENT COMPLEMENTS

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Abstract. Using a recent result of Bartels and Lück ([3]) we deduce that the Farrell-Jones Fibered Isomorphism conjecture in $L^{(-\infty)}$-theory is true for any group which contains a finite index strongly poly-free normal subgroup, in particular, for the Artin full braid groups. As a consequence we explicitly compute the surgery groups of the Artin pure braid groups. This is obtained as a corollary to a computation of the surgery groups of a more general class of groups, namely for the fundamental group of the complement of any fiber-type hyperplane arrangement in $\mathbb{C}^n$.

1. Introduction

The purpose of this short note is to compute explicitly the surgery ($L$-)groups of the Artin pure braid groups ($PB_n$). This computation requires the solutions of two other problems. Firstly, one has to compute the lower algebraic $K$-theory of the group and secondly, to show that the classical assembly map in $L$-theory is an isomorphism. This gives an interpretation of the surgery groups in terms of a generalized homology theory.

For $PB_n$ we had already computed the lower algebraic $K$-theory in [1]. In [7] we computed it for any subgroup of the full braid group ($B_n$), in particular for any subgroup of $PB_n$. Here we show that the classical assembly map in $L$-theory is an isomorphism for any subgroup of $B_n$. The main ingredients behind the proof is the $K$-theoretic vanishing result ([7], theorem 1.1] and a recent result of Bartels and Lück ([3], theorem B]). The later result is used to show that the $L^{(-\infty)}$-theory Fibered Isomorphism conjecture of Farrell and Jones ([6], §1.7)) is true for any subgroup of $B_n$. Finally, using the stable homotopy type
of the corresponding Eilenberg-Maclane space of $PB_n$ from [14] we do the computation of the surgery groups.

Let us first recall the definition of $PB_n$ (and $B_n$) before we state the computation of its surgery groups. Let $H_n$ be the set consisting of the hyperplanes $H_{ij} = \{(x_0, x_1, \ldots, x_n) \in \mathbb{C}^{n+1} \mid x_i = x_j\}$ for $i, j = 0, 1, \ldots, n$ and $i \neq j$ in the $(n + 1)$-dimensional complex space $\mathbb{C}^{n+1}$. The fundamental group of the complement $\mathbb{C}^{n+1} - \bigcup_{i \neq j} H_{ij}$ is denoted by $PB_n$ and is known as the Artin pure braid group on $n$ strings.

Note that the symmetric group $S_{n+1}$ on $(n + 1)$-symbols acts freely on $\mathbb{C}^{n+1} - \bigcup_{i \neq j} H_{ij}$ by permuting coordinates. The fundamental group of the quotient space $(\mathbb{C}^{n+1} - \bigcup_{i \neq j} H_{ij})/S_{n+1}$ is the Artin full braid group $B_n$. Therefore, there is an exact sequence of the following type.

$$1 \rightarrow PB_n \rightarrow B_n \rightarrow S_{n+1} \rightarrow 1.$$ 

**Corollary 1.1.** For all $n \geq 1$ the surgery groups of the Artin pure braid group $PB_n$ are computed as follows.

$$L_i(PB_n) = \begin{cases} \mathbb{Z} & \text{if } i \equiv 0 \mod 4 \\ \mathbb{Z}_{\frac{n(n+1)}{2}} & \text{if } i \equiv 1 \mod 4 \\ \mathbb{Z}_2 & \text{if } i \equiv 2 \mod 4 \\ \mathbb{Z}_{\frac{n(n+1)}{2}} & \text{if } i \equiv 3 \mod 4. \end{cases}$$

**Proof.** This is an immediate corollary of Theorem 2.2 since the arrangement $H_n$ is fiber-type and there are $\frac{n(n+1)}{2}$ hyperplanes in $H_n$. $\square$

We recall here that there are surgery groups for different kind of surgery problems and they appear in the literature with the symbols $L_i^*(\ast)$ where $\ast = h, s, \langle -\infty \rangle$ or $\langle i \rangle$ for $i \leq 0$. But all of them are naturally isomorphic for torsion free groups $G$ if the Whitehead group $Wh(G)$, reduced projective class group $\tilde{K}_0(ZG)$ and negative $K$-groups $K_{-i}(ZG)$ for $i \geq 1$ vanish. See [[11], remark 1.21 and proposition 1.23]. Therefore, we use the simplified notation $L_i(-)$ in this paper as the groups we consider have the required properties.

We conclude the introduction by mentioning that in fact we prove the Fibered Isomorphism conjecture in $L^{(-\infty)}$-theory for a more general class of groups, namely for any finite extension $\Gamma$ of a strongly poly-free group (see [[1], definition 1.1] or Definition 2.1 below) and deduce the isomorphism of the classical assembly map in $L$-theory for any torsion free subgroup of $\Gamma$. As a consequence we compute the surgery groups of the fundamental group of any fiber-type hyperplane arrangement complement in the complex $n$-space $\mathbb{C}^n$ (see Theorem 2.2).
2. Statements of the Main Theorem and its consequences

Let us recall the definition of the strongly poly-free groups.

**Definition 2.1.** ([1]) A discrete group $\Gamma$ is called strongly poly-free if there exists a finite filtration of $\Gamma$ by subgroups: $1 = \Gamma_0 \subset \Gamma_1 \subset \cdots \subset \Gamma_n = \Gamma$ such that the following conditions are satisfied:
1. $\Gamma_i$ is normal in $\Gamma$ for each $i$
2. $\Gamma_{i+1}/\Gamma_i$ is a finitely generated free group
3. for each $\gamma \in \Gamma$ and $i$ there is a compact surface $F$ and a diffeomorphism $f : F \to F$ such that the induced homomorphism $f_\#$ on $\pi_1(F)$ is equal to $c_\gamma$ in $\text{Out}(\pi_1(F))$, where $c_\gamma$ is the action of $\gamma$ on $\Gamma_{i+1}/\Gamma_i$ by conjugation and $\pi_1(F)$ is identified with $\Gamma_{i+1}/\Gamma_i$ via a suitable isomorphism.

In such a situation we say that the group $\Gamma$ has rank $\leq n$.

We now state our main theorem.

**Theorem 2.1.** Let $\Gamma$ be a finite extension of a strongly poly-free group (the finite group is the quotient group). Then the Fibered Isomorphism conjecture of Farrell and Jones in $L^{(-\infty)}$-theory is true for any subgroup of $\Gamma$. In particular, it is true for any subgroup of $B_n$.

Although we prove Theorem 2.1 for the conjecture in $L^{(-\infty)}$-theory stated in [[6], §1.7], the proof goes through, under certain conditions (see [[13], 3(b) of theorem 2.2]), in a general setup of the conjecture in equivariant homology theory formulated in [2] and for a more general class of groups.

A corollary of the above theorem and [[7], theorem 1.1] is the following. This shows that for any torsion free subgroup $G$ of $\Gamma$ the surgery group $L_i(G)$ is isomorphic to the generalized homology group $H_i(BG, \mathbb{L}_0)$. Here $\mathbb{L}_0$ is the $\Omega$-spectrum whose homotopy groups compute the surgery groups of the trivial group.

**Corollary 2.1.** The classical assembly map in surgery theory is an isomorphism for any torsion free subgroup of $\Gamma$. That is, $H_i(BG, \mathbb{L}_0) \to L_i(G)$ is an isomorphism for all $i$ and for all torsion free subgroup $G$ of $\Gamma$. In particular, the assembly map is an isomorphism for any subgroup of $B_n$.

**Proof.** Let $H$ be a torsion free group so that the following are satisfied.
1. $Wh(H) = K_{-i}(\mathbb{Z}H) = K_0(\mathbb{Z}H) = 0$ for all $i \geq 1$.
2. The Isomorphism conjecture in $L^{(-\infty)}$-theory is true for $H$.

Then it is a known fact that for all $i$, $H_i(BH, \mathbb{L}_0) \to L_i(H)$ is an isomorphism. For a details proof see [[11], theorem 1.28] or [[6], 1.6.3].
Now the proof of the Corollary is immediate since (1) is satisfied for $G$ by [[7], theorem 1.1] and (2) is satisfied by Theorem 2.1.

The particular case follows since $PB_n$ is strongly poly-free and $B_n$ is torsion free (see the discussion after the following Remark). □

**Remark 2.1.** Here we recall that the isomorphism of the above assembly map is expected when the group is torsion free. The integral Novikov conjecture in $L$-theory states that this assembly map should be split injective.

Before we state our main computation of the surgery groups we recall the definition of a fiber-type hyperplane arrangement from [[12], p. 162]. An arrangement $A_n \subset \mathbb{C}^n$ is called *strictly linearly fibered* if after a suitable linear change of coordinates, the restriction of the projection of $\mathbb{C}^n - A_n$ to the first $(n - 1)$ coordinates is a fiber bundle projection whose base space is the complement of an arrangement $A_{n-1}$ in $\mathbb{C}^{n-1}$ and whose fiber is the complex plane minus finitely many points. By definition the arrangement 0 in $\mathbb{C}$ is fiber-type and $A_n$ is defined to be *fiber-type* if $A_n$ is strictly linearly fibered and $A_{n-1}$ is of fiber type. It follows by repeated application of homotopy exact sequence of a fibration that the complement $\mathbb{C}^n - A_n$ is aspherical. And hence $\pi_1(\mathbb{C}^n - A_n)$ is torsion free.

The hyperplane arrangement $H_n$ for $PB_n$ as described in the Introduction is an example of a fiber-type arrangement.

Now recall from [[7], theorem 5.3] that if $A$ is a fiber-type hyperplane arrangement in $\mathbb{C}^n$, then the fundamental group $\pi_1(\mathbb{C}^n - \cup A)$ is strongly poly-free. In particular $PB_n$ is also strongly poly-free. This was proved in [[1], theorem 2.1].

As a consequence of Theorem 2.1 we prove the following.

**Theorem 2.2.** Let $A = \{A_1, A_2, \ldots, A_N\}$ be a fiber-type hyperplane arrangement in $\mathbb{C}^n$, then the surgery groups of $\Gamma = \pi_1(\mathbb{C}^n - \cup_{j=1}^N A_j)$ are given by the following.

$$L_i(\Gamma) = \begin{cases} \mathbb{Z} & \text{if } i \equiv 0 \mod 4 \\ \mathbb{Z}^N & \text{if } i \equiv 1 \mod 4 \\ \mathbb{Z}_2 & \text{if } i \equiv 2 \mod 4 \\ \mathbb{Z}_2^N & \text{if } i \equiv 3 \mod 4. \end{cases}$$

### 3. The Isomorphism Conjecture and Related Results

The Isomorphism conjecture of Farrell and Jones ([[6], §1.6, §1.7]) is a fundamental conjecture and implies many well-known conjectures in algebra and topology (see [10] for a quick introduction to the conjecture...
or see [11]). The statement of the conjecture has been stated in a very general setup of equivariant homology theory in [2]. We recall the statement below.

Let $\mathcal{H}_*^G$ be an equivariant homology theory with values in $R$-modules for $R$ a commutative associative ring with unit.

A family of subgroups of a group $G$ is defined as a set of subgroups of $G$ which is closed under taking subgroups and conjugations. If $C$ is a class of groups which is closed under isomorphisms and taking subgroups then we denote by $C(G)$ the set of all subgroups of $G$ which belong to $C$. Then $C(G)$ is a family of subgroups of $G$. For example $\mathcal{VC}$, the class of virtually cyclic groups, is closed under isomorphisms and taking subgroups. By definition a virtually cyclic group has a cyclic subgroup of finite index.

Given a group homomorphism $\phi : G \to H$ and a family $C$ of subgroups of $H$ define $\phi^*C$ to be the family of subgroups $\{K < G \mid \phi(K) \in C\}$ of $G$. Given a family $C$ of subgroups of a group $G$ there is a $G$-CW complex $E_C(G)$ which is unique up to $G$-equivalence satisfying the property that for $H \in C$ the fixpoint set $E_C(G)^H$ is contractible and $E_C(G)^H = \emptyset$ for $H$ not in $C$.

Let $G$ be a group and $C$ be a family of subgroups of $G$. Then the Isomorphism conjecture for the pair $(G, C)$ states that the projection $p : E_C(G) \to pt$ to the point $pt$ induces an isomorphism

$$\mathcal{H}_n^G(p) : \mathcal{H}_n^G(E_C(G)) \simeq \mathcal{H}_n^G(pt)$$

for $n \in \mathbb{Z}$.

And the Fibered Isomorphism conjecture for the pair $(G, C)$ states that for any group homomorphism $\phi : K \to G$ the Isomorphism conjecture is true for the pair $(K, \phi^*C)$.

In this article we are concerned with the equivariant homology theory arising in $L^{(-\infty)}$-theory and when $C = \mathcal{VC}$ and $R = \mathbb{Z}$. This (Fibered) Isomorphism conjecture is equivalent to the Farrell-Jones conjectures stated in ([6], §1.7) ([6], §1.6). (For details see [2], §5 and §6).

We say that the FICwF$^L$ (FIC$^L$) is true for a group $G$ if the Fibered Isomorphism conjecture in $L^{(-\infty)}$-theory is true for $G \wr H$ ($G$) for any finite group $H$. Here $G \wr H$ denotes the semidirect product $G^H \wr H$ with respect to the regular action of $H$ on $G^H = G \times G \times \cdots \times G$ ($|H|$ number of factors).

Next, we recall some standard results and some recent development in this area which we need for the proof of Theorem 2.1. Also we prove some basic results. Let us start by recalling that the Fibered Isomorphism conjecture has hereditary property, that is if it is true for a group then it is true for any of its subgroups.
Lemma 3.1. Let $G$ be a group acting properly discontinuously and cocompactly by isometries on a metric space $X$. Then for any finite group $H$ the group $G \wr H$ acts properly discontinuously and cocompactly by isometries on the product metric space $X^H = X \times X \times \cdots \times X$ (number of factors).

Proof. This follows from the proof of Serre’s theorem in [[5], theorem 3.1, p.190-191].

An immediate corollary to the above Lemma is the following. Recall that a CAT(0)-space is a connected simply connected metric space which is nonpositively curved in the sense of distance comparison. For example the universal cover $\tilde{M}$ of a closed nonpositively curved Riemannian manifold $M$ with respect to the lifted metric is CAT(0). For some more information on this subject see [4].

Corollary 3.1. If $G$ acts properly discontinuously and cocompactly on a CAT(0)-space, then for any finite group $H$, $G \wr H$ also acts properly discontinuously and cocompactly on a CAT(0)-space.

Proof. The proof is immediate as the product of two CAT(0)-spaces is again CAT(0).

A group $G$ is called CAT(0) if it acts properly and cocompactly by isometries on a CAT(0)-space. Hence if $M$ is as above then $\pi_1(M)$ is a CAT(0)-group. Therefore, by Corollary 3.1 for any finite group $H$, $\pi_1(M) \wr H$ is also a CAT(0)-group.

Lemma 3.2. The FIC$^L$ is true for $V_1 \times V_2$ for any two virtually cyclic groups $V_1$ and $V_2$.

Proof. Since the FIC$^L$ is true for any virtually cyclic group we can assume that both $V_1$ and $V_2$ are infinite. Hence $V_1 \times V_2$ contains $\mathbb{Z} \times \mathbb{Z}$ (=A, say) as a finite index normal subgroup. By the algebraic lemma in [7] $V_1 \times V_2$ is a subgroup of $A \wr H$ where $H = (V_1 \times V_2)/A$. Let $T$ be a flat 2-dimensional torus. Then by Corollary 3.1 $A \wr H$ is a CAT(0)-group. Therefore by [[3], theorem B] the FIC$^L$ is true for $A \wr H$ since the CAT(0)-space $T^H$ is finite dimensional. Here $\tilde{T}$ denotes the universal cover of $T$ with the lifted metric. Hence FIC$^L$ is true for $V_1 \times V_2$ by the hereditary property.

Lemma 3.3. Let $p : G \to Q$ be a surjective group homomorphism and assume that the FIC$^{wF^L}$ is true for $Q$, for $\ker(p)$ and for $p^{-1}(C)$ for any infinite cyclic subgroup $C$ of $Q$. Then $G$ satisfies the FIC$^{wF^L}$.

Proof. The proof is immediate using Lemma 3.2 and [[13], lemma 3.4].
Lemma 3.4. Let $G$ be isomorphic to one of the following groups.

- Fundamental group of a closed nonpositively curved Riemannian manifold.
- A finitely generated virtually free group.
- Fundamental group of a compact 3-manifold $M$ with nonempty boundary so that there is a fiber bundle projection $M \to S^1$.

Then the FICwF$^L$ is true for $G$.

Proof. Since FIC$^L$ is true for all finite groups we can assume that $G$ is infinite.

Let $M$ be a closed nonpositively curved Riemannian manifold so that $\pi_1(M) \simeq G$. Then by Corollary 3.1 for any finite group $H$, $G \wr H$ is a CAT(0)-group and hence the FIC$^L$ is true for $G \wr H$ by [3], theorem B since the CAT(0)-space $\tilde{M}^H$ is finite dimensional. This completes the proof of the first item.

Now we give the proof for the third item, then the second one will follow using the hereditary property of the Fibered Isomorphism conjecture.

Let $S$ be a fiber of the fiber bundle $M \to S^1$ with monodromy diffeomorphism $f : S \to S$. Then $M$ is diffeomorphic to the mapping torus of $f$. Therefore $M$ is a compact Haken 3-manifold (that is an irreducible 3-manifold which has a $\pi_1$-injective embedded surface, see [8]) with boundary components of zero Euler characteristic. We now apply [9, theorem 3.2 and 3.3] to get a complete nonpositively curved Riemannian metric in the interior of $N$ so that near the boundary the metric is the product flat metric, that is each end is isometric to $X \times [0, \infty)$ for some flat 2-manifold $X$. Therefore if we take the double $D$ of $M$, we get a closed nonpositively curved Riemannian manifold. Hence by the first item FICwF$^L$ is true for $\pi_1(D)$ and consequently for $\pi_1(M)$ also by the hereditary property.

This completes the proof of the Lemma. \(\square\)

4. Isomorphism of the Assembly map and computation of the surgery groups

In this section we give the proofs of Theorems 2.1 and 2.2.

Proof of Theorem 2.1. Let $G$ be a finite index strongly poly-free normal subgroup of $\Gamma$. Then by the algebraic lemma in [7] $\Gamma$ can be embedded as a subgroup in $G \wr (\Gamma/G)$. Therefore using the hereditary property it is enough to proof the FICwF$^L$ for any strongly poly free group. Hence we can assume that $\Gamma$ is strongly poly-free.
The proof is by induction on the rank of $\Gamma$ and the framework of the proof is same as that of the proof of [[13], 3(b) of theorem 2.2].

If the rank is $\leq 1$ then $\Gamma$ is a finitely generated free group and hence the theorem follows from the second item in Lemma 3.4.

Therefore assume that the rank of $\Gamma \leq k$ and that the FICwF is true for all strongly poly-free group $\Gamma$ of rank $\leq k - 1$.

Let $1 = \Gamma_0 < \Gamma_1 < \cdots < \Gamma_k = \Gamma$ be a filtration of $\Gamma$.

Consider the following exact sequence.

$$1 \rightarrow \Gamma_1 \rightarrow \Gamma \rightarrow \Gamma/\Gamma_1 \rightarrow 1.$$  

Let $q : \Gamma \rightarrow \Gamma/\Gamma_1$ be the above projection. The following assertions are easy to verify.

- $\Gamma/\Gamma_1$ is strongly poly-free and has rank $\leq k - 1$.
- $q^{-1}(C)$ is a finitely generated free group or isomorphic to the fundamental group of a compact Haken 3-manifold $M$ with nonempty boundary so that there is a fiber bundle projection $M \rightarrow S^1$, where $C$ is either the trivial group or an infinite cyclic subgroup of $\Gamma/\Gamma_1$ respectively.

Now we can apply the induction hypothesis, Lemma 3.3, and Lemma 3.4 to complete the proof of the Theorem.

The particular case for $B_n$ follows as $PB_n$ is strongly poly-free (([[1], theorem 2.1])) and is an index $(n + 1)!$ normal subgroup of $B_n$.

To prove Theorem 2.2 we need the following lemma regarding the topology of an arbitrary hyperplane arrangement complement in the complex $n$-space $\mathbb{C}^n$.

**Lemma 4.1.** The first suspension $\Sigma(\mathbb{C}^n - \cup_{j=1}^N A_j)$ of the complement of a hyperplane arrangement $A = \{A_1, A_2, \ldots, A_N\}$ in $\mathbb{C}^n$ is homotopically equivalent to the wedge of spheres $\lor_{j=1}^N S_j$ where $S_j$ is homeomorphic to the 2-sphere $S^2$ for $j = 1, 2, \ldots, N$.

**Proof.** Let $A = \{A_1, A_2, \ldots, A_N\}$ be an arrangement by linear subspaces of $\mathbb{C}^n$. Then in [[14], (2) of proposition 8] it is proved that $\Sigma(\mathbb{C}^n - \cup_{j=1}^N A_j)$ is homotopically equivalent to the following space.

$$\Sigma(\lor_{p \in P}(S^{2n-\text{d}(p)}-1) - \Delta P_{<p})$$

We recall the notations in the above display from [[14], p. 464]. $P$ is in bijection with $\mathcal{A}$ under a map, say $f$. Also $P$ is partially ordered by the rule that $p < q$ for $p, q \in P$ if $f(q)$ is a subspace of $f(p)$. $\Delta P$ is a simplicial complex whose vertex set is $P$ and chains in $P$ define simplices. $\Delta P_{<p}$ is the subcomplex of $\Delta P$ consisting of all $q \in P$ so...
that \( q < p \). \( d(p) \) denotes the dimension of \( f(p) \). The construction of the embedding of \( \Delta P_{<p} \) in \( S^{2n-d(p)-1} \) is also a part of [[14], proposition 8].

Now in the situation of the Lemma obviously \( d(p) = 2n - 2 \) and \( \Delta P_{<p} = \emptyset \) for all \( p \in P \). Finally note that the suspension of a wedge of spaces is homotopically equivalent to the wedge of the suspensions of the spaces. This completes the proof of the lemma. \( \Box \)

**Proof of Theorem 2.2.** Let \( X = \mathbb{C}^n - \bigcup_{j=1}^{N} A_j \). Then by Lemma 4.1 \( \Sigma X \) is homotopically equivalent to \( \vee_{j=1}^{N} S_j \) where \( S_j \) is homeomorphic to the 2-sphere \( S^2 \) for \( j = 1, 2, \ldots, N \). Let \( h_i \) be a generalized homology theory. Then \( h_i(X) \) is computed as follows.

\[
h_i(X) = h_i(*) \oplus h_{i-2}(*)^N.
\]

Where \( h_{i-2}(*)^N \) denotes the direct sum of \( N \) copies of \( h_i(*) \) and \( '*' \) is a single point space.

Now replacing \( h_i(*) \) by \( H_i(\ast, \mathbb{L}_0) \) and using the following known computation we complete the proof of the theorem.

\[
H_i(\ast, \mathbb{L}_0) = \begin{cases} 
\mathbb{Z} & \text{if } i \equiv 0 \mod 4 \\
0 & \text{if } i \equiv 1 \mod 4 \\
\mathbb{Z}_2 & \text{if } i \equiv 2 \mod 4 \\
0 & \text{if } i \equiv 3 \mod 4.
\end{cases}
\]

\( \Box \)
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