I. INTRODUCTION

Until 1994, two dimensional systems were widely believed to be ultimately insulators, as the presence of even a tiny disorder is enough to make the one-body electronic wavefunctions localized. In fact, until then, the experimental corpus was widely consistent with this paradigm: the electrical resistance was found to increase as one lowered the temperature, indicating an eventual divergence at zero temperature. The observation of metallic behaviors in low-density Silicon metal-oxide-semiconductor field-effect transistors (Si-MOSFETs) is controlled by a unique characteristic energy scale, the polarization energy. On one hand, we perform Quantum Monte Carlo calculations of the energy needed to polarize the two dimensional electron gas at zero temperature, taking into account Coulomb interactions, valley degeneracy and electronic mobility (disorder). On the other hand, we identify the characteristic energy scale controlling the physics in eight different sets of experiments. We find that our ab-initio polarization energies (obtained without any adjustable parameters) are in perfect agreement with the observed characteristic energies for all available data, both for the magnetic field and temperature dependence of the resistivities. Our results put strong constraints on possible mechanisms responsible for the metallic behavior. In particular, there are strong indications that the system would eventually become insulating at low enough temperature.

II. BASIC EXPERIMENTAL FACTS

A. Main observations

Let us quickly review the basic experimental facts that we want to understand. The first set of measurements that actually started the interest for these high mobility MOSFETs is the behavior of the resistivity $\rho(T)$ as a function of temperature (at temperatures lower than a
few Kelvin where phonon scattering no longer comes into play). The $\rho(T)$ behavior is well understood in two limiting cases. At high density, the resistivity is rather small and depends only weakly on temperature, except for a weak negative $\partial \rho/\partial T$ due to weak localization. At low density, the resistivity is much larger and one eventually reaches a clear insulating behavior where $\rho$ depends very strongly (exponentially) on temperature ($\partial \rho/\partial T < 0$). Before the report made in Ref. 4, the common behavior observed in Si-MOSFETs, as well as in other two dimensional electron gases was a simple crossover between the high and the low density limit. Hence, the observation of an intermediate density regime with $\partial \rho/\partial T > 0$ (hereafter referred as property P1) came as a large surprise. In the left panels of Fig. 1 we reproduced the data of three different experiments, including the original set of data from Kravchenko et al. One can observe a rather pronounced $\partial \rho/\partial T > 0$ as the resistivity increases by a factor 10 between 100 mK and 4 K. It was naturally surmised that in this regime, the resistivity could be extrapolated to a finite value at zero temperature, so that the system was in a metallic state, in contradiction with the prediction of scaling theory of localization.

The second important raw experimental feature is the dependence of the resistivity versus an in-plane magnetic field $B$: in presence of a magnetic field, the resistivity increases quickly before saturating (property P2). This is illustrated in the right panels of Fig. 1 for three different experiments. The increase of resistivity is comparable to the increase that one gets with increasing temperature, and $\partial \rho/\partial T$ quickly becomes negative (insulating like).

In our opinion, P1 and P2 are the two main experimental features that require an explanation. There have been an important effort to establish scaling laws and critical exponents close to the metal-insulator transition, but we will not discuss this aspect. Indeed, once the metallic behavior P1 is understood, there is no question that, as the system must be eventually insulating at low density, some sort of transition must occur. While this transition might be of interest, it is not fundamentally puzzling in itself.

### B. First analysis

One of the questions that many physicists had in mind when the first experiments came up was, what’s new in those devices? Why have we not seen this behavior before? In fact, there was nothing qualitatively new about those MOSFETs. The real novelty lied in the fact that the samples were of extremely high mobility (for Si-MOSFETs) which allowed the experiments to be carried out at much lower densities than previously possible. Working at lower densities has three consequences:

**Property P3.** The ratio $r_s$ of interaction over kinetic energy is large. This ratio $r_s = m^* e^2/(4\pi\epsilon_0\hbar^2\sqrt{\pi n_s})$ (where $n_s$ is the electronic density, $m^* = 0.19 m_e$ the effective mass, $e$ the electron charge, $\epsilon_0$ the dielectric constant and $\epsilon = 7.7$ the relative dielectric constant) takes values between $5 - 10$ for the metallic region, hence the system is intrinsically correlated.

**Property P4.** The effective disorder seen by the electrons, as parametrized by $1/k_F l$ ($k_F$ Fermi momentum and $l$ mean free path) is paradoxically not smaller: indeed the high mobility of the samples is compensated by the fact that the density is lower and therefore $k_F l \propto n_s$ remains of the order of a few units. Note that contrary to $n_s$ (hence $r_s$) which can be almost directly measured with Shubnikov-de-Haas measurements, there is no generic way to measure the disorder. At high density, $r_s \ll 1$ and the interaction effects can be neglected, so that $k_F l$ is simply related to the conductance $g$ of the system, $g = (2e^2/h) k_F l$. In this limit, we estimate the product $\eta = r_s \sqrt{k_F l}$ which should remain constant as one lowers

![FIG. 1: $\rho$ as a function of $T$ (left panels) and as a function of $B$ (right panels), for different $n_s$, in various Si-MOSFETs (a) from Ref. 4, (b) from Ref. 5, (c) from Ref. 6, (d) from Ref. 7, (e) from Ref. 8 and (f) from Ref. 9. The red circles are the corresponding polarization temperatures $T_p$ and polarization magnetic fields $B_p$ obtained from the numerical simulations.](image)
the density (as \( r_s \propto 1/\sqrt{n_s} \) and \( k_F \propto n_s \)). Hence, \( \eta \) is estimated at large density by the non-interacting formula
\[
\eta = \sqrt{\mu e^3/2m^*}/(4\pi \hbar^3/2e_0).
\]
For real samples where disorder have intrinsic characteristic lengths, the mobility \( \mu \) may have a density dependence. Typical values of \( \eta \) are of the order of 20. In Fig. 2 we have collected the trajectories in the \( r_s \) versus \( 1/\sqrt{k_Fl} \) phase diagram for ten different samples: as one lowers the density, the samples follow a straight line \( 1/\sqrt{k_Fl} = r_s/\eta \). The regions where the samples have a metallic behavior are plotted in blue while the insulating behaviors appear in dashed red. When comparing the samples where the metallic behavior is observed with the older samples that did not show such a behavior (the two on the left), one indeed observes that the metallic behavior is observed at larger \( r_s \) (P3) but in the same regime of disorder (P4).

**Property P5.** The Fermi energy \( E_F \) decreases so that the ratio \( T/T_F \) increases (where \( E_F = kT_F = \pi \hbar^2 n_s/2m^* \), \( T_F \) is the Fermi temperature). Typical experimental values are quite large, \( T/T_F \approx 20\% \) around the inflexion point of the \( \rho(T) \) curves as can be seen in Fig. 3(c). This last point, raised in Ref. \[22\] has been overlooked by many authors and is however crucial: the metallic behavior happens for temperatures which are of the same order of magnitude as the Fermi temperature. Hence, the usual "low energy" paradigm where one only looks at electrons close to the Fermi surface does not hold in those devices.

It is actually the purpose of this paper to identify the characteristic energy that controls the behavior of \( \rho(T) \) and \( \rho(B) \). As we shall see, both are controlled by a unique energy scale, the polarization energy \( E_p \) of the Si-MOSFET.

### III. A Minimum Model for Silicon MOSFETs

We now introduce the model that we will use for our calculations. It includes Coulomb repulsion, disorder, and a twofold valley degeneracy (present in Si-MOSFETs but not in their GaAs counterparts, Silicon being an indirect gap semi-conductor). This model is minimum in the sense that the first three ingredients are already known to be relevant experimentally, as discussed in the above section, while the fourth one is known to be present and we will show that it plays an important role for quantitative predictions. The system contains \( N \gg 1 \) particles in a lattice made of \( L_x \times L_y \) sites, the particles being equally split up into the two degenerate valleys, with \( N_\uparrow \) spin up and \( N_\downarrow \) spin down. The spin configuration defines the system polarization

\[
p = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow} \quad (1)
\]

The Hamiltonian for the 4-components plasma reads,

\[
H = -t \sum_{(\vec{r}\vec{r}')} \sum_{\sigma \sigma'} c^\dagger_{\vec{r}\sigma} c_{\vec{r}'\sigma'} + \frac{U}{2} \sum_{\vec{r} \neq \vec{r}' \uparrow \uparrow} V_{\vec{r} \vec{r}'} n_{\vec{r} \uparrow} n_{\vec{r}' \uparrow}
\]

where \( c^\dagger_{\vec{r}\sigma} \) and \( c_{\vec{r}\sigma} \) are the usual creation and annihilation operators of one electron on site \( \vec{r} \) with inner degree of freedom \( \sigma \), the sum \( \sum_{(\vec{r}\vec{r}')} \) is restricted to nearest neighbors and \( n_{\vec{r}\sigma} \) is the density operator. The internal degree of freedom \( \sigma = 1, \ldots, 4 \) corresponds to the spin and valley degeneracy degrees of freedom. The disorder potential \( v_\vec{r} \) is uniformly distributed inside \([-W/2, W/2]\), \( t \) is the hopping parameter and \( U \) is the effective strength of the two body interaction \( V_{\vec{r} \vec{r}'} \). To reduce finite size effects, \( V_{\vec{r} \vec{r}'} \) is obtained from the bare Coulomb interaction using the Ewald summation technique,

\[
V_{\vec{r} \vec{r}} = \sum_\vec{L} \frac{1}{|\vec{r} + \vec{L}|} \text{Erfc}(k_c |\vec{r} + \vec{L}|)
\]

\[
+ \frac{2\pi}{L_x L_y} \sum_{\vec{K} \neq \vec{0}} \frac{1}{|\vec{K}|} \text{Erfc}(\sqrt{3}|\vec{K}|/(2k_c)) \cos(\vec{K} \cdot \vec{r} \).
\]

In the previous equation, \( k_c \) is a (irrelevant) cut off. The vector \( \vec{L} \) takes discrete values \( \vec{L} = (n_x L_x, n_y L_y) \) with \( n_x \) and \( n_y \) integer numbers. The vector \( \vec{K} \) also takes discrete values, \( \vec{K} = (\frac{2\pi}{L_x} n_x, \frac{2\pi}{L_y} n_y) \) and \( (n_x, n_y) \neq \vec{0} \).
(0, 0). The complementary error function is defined as \( \text{Erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-t^2} \, dt \). We work at small filling factor \( \nu \equiv N/(L_x L_y) \ll 1 \), where we recover the continuum limit. The two dimensionless parameters that control the physics read,

\[
 r_s = U/(2t\sqrt{\pi \nu}), \quad k_F l = 48\pi \nu t^2/W^2. \tag{4}
\]

**IV. QUANTUM MONTE-CARLO CALCULATION OF THE POLARIZATION ENERGY.**

This section is devoted to the numerical calculation of the polarization energy of the 4-components plasma, as a function of \( r_s \) and \( k_F l \). We compute the ground state energy per particle \( E(p) \) of the system with the Green function Quantum Monte Carlo (QMC) technique in the Fixed Node approximation (see Ref.[32] for the numerical method). Particular care was given to the extrapolation to the thermodynamics limit \( (N \gg 1) \) as well as the continuum limit (the algorithm being implemented on a lattice). All the fits given below, as well as data points, are given with a precision better than \( \pm 0.02E_F \) for \( k_F l \geq 1.5 \) and \( \pm 0.04E_F \) for \( 0.3 < k_F l < 1.5 \). The data have been averaged on 50 to 200 samples for the strongest disorders.

At very high density, i.e. no disorder \( (1/k_F l = 0) \) and interaction \( (r_s = 0) \), it is straightforward to show that the energy depends on the polarization \( p \) in a quadratic manner: \( E(p) = E_F (1 + p^2)/2 \). In fact, we find that such a quadratic dependence is verified with good precision for a wide range of parameters \( 0 \leq r_s \leq 10 \) and \( k_F l > 1 \), corresponding to the disorder regime of the experiments, as shown in the upper left panel of Fig. [3]. We found (small) deviations from the quadratic behavior for rather large disorder in the localized regime (see triangles at \( 1/k_F l = 2.65 \) and \( r_s = 6 \)). Hence we find with good precision that,

\[
 E(p) = E(0) + E_{p} p^2, \tag{5}
\]

where the coefficient \( E_{p}(r_s, k_F l) \) is the polarization energy of the system and is proportional to the Fermi energy. Without electron-electron interaction and disorder \( E_p = E_F/2 \), and as interaction is turned on, \( E_p \) decreases (indeed the ferromagnetic state tends to minimize Coulomb energy as antisymmetric orbital wave-functions have a lower probability for two electrons to be close to each other than symmetric ones).

In presence of an in-plane (i.e. purely Zeeman) magnetic field \( B \), Eq.(5) turns into \( E(p) = E(0) + E_{p} p^2 - g\mu_B B p^2 /2 \) \((g = 2 \text{ Lande factor and } \mu_B \text{ Bohr magneton})\). Introducing \( B_p = 4E_p/(g\mu_B) \), the minimum of energy is found for a polarization \( p^* \) given by,

\[
 p^* = B/B_p \quad B < B_p, \tag{6}
\]

\[
 p^* = 1 \quad B > B_p. \tag{7}
\]

**TABLE I: Parameters \( A_i \) of Eq. (5) and (9).**

| \( A_i \) | 0 | 1.0 | 2 | 3 | 4 | 5 |
|-------|---|----|---|---|---|---|
| \( A_0 \) | 27.93 | 0.83 | 56.5 | 46 | 1.77 | 0.019 |

Hence, the spin susceptibility \( \chi = n_s/2(dE/dB)_{B=0} \) is directly related to the field \( B_p \) at which the polarization saturates. Defining the non-interacting susceptibility \( \chi_0 = g\mu_B m^*/(2\pi\hbar^2) \), one finds \( E_F/E_p = 2\chi/\chi_0 \). In addition to the characteristic magnetic field \( B_p \), \( E_p \) is also associated with the characteristic temperature \( kT_p = E_p \) at which the polarized excited states get significantly populated.

In the lower panels of Fig. [4] we plotted our \( E_p \) data as a function of disorder \( 1/\sqrt{k_F l} \) (left) and interaction \( r_s \) (right). Without disorder (squares in the lower right panel), our results are in close agreement with previous Diffusion Monte Carlo calculations performed in Ref.[34] (dashed line). As expected, the polarization energy \( E_p \) decreases with interaction \( r_s \). However, contrary to the single-valley case where \( E_p \) becomes extremely small and could even become negative (ferromagnetic instability), we find that \( E_p \) is always positive up to \( r_s = 20 \) where \( E_p = 0.13E_F \).

We find that disorder tends to increase \( E_p \) for very small interactions \( r_s \leq 0.25 \). At \( r_s = 0 \), the disorder correction to \( E_p \) is very well described by the second order perturbative correction \( E_p/E_F = 1/2 + \log 2/(\pi k_F l) \) (dashed line in lower left panel of Fig. [3]). However, as soon as small interactions are switched on \( (r_s > 0.3) \), disorder tends to decrease \( E_p \). We also find that the second order perturbative correction \( \propto 1/k_F l \) is valid only for very tiny disorder \( 1/k_F l < 0.04 \) above which the correction becomes proportional to \( 1/\sqrt{k_F l} \). In order to make our set of data easily retrievable, we use the following fit, valid for our entire data set for \( 0.25 < r_s < 10 \),

\[
 E_p(k_F l, r_s) = E_p^{cl}(r_s) + \frac{\alpha(r_s)}{\sqrt{k_F l}} E_F + A_5 E_F, \tag{8}
\]

where \( E_p^{cl} \) is the polarization energy of the clean system fitted (for \( 0 < r_s < 20 \)) with the Pade formula:

\[
 E_p^{cl}(r_s) = \frac{A_0 + A_1 r_s}{A_2 + A_3 r_s + A_4 r_s^2} E_F. \tag{9}
\]

The values of fitting parameters \( A_i \) are listed in Table I. The parameter \( \alpha \) is plotted in the upper right part of Fig. [3] and is equal with good precision for \( r_s \geq 2 \) \( \alpha = -0.1 \). The small parameter \( A_5 \) roughly accounts for the fact that at very low disorder, the disorder-correction to \( E_p/E_F \) is not linear but quadratic in \( 1/\sqrt{k_F l} \). We note that Eq.(8) and (9) are merely a compact way of describing our numerical data.
Disorder corrections to the polarization energy $E$ right panel: $\alpha \rho$

FIG. 3: Polarization energy. Upper left panel: $E(p)$ as a function of $p^2$, for $1/k_F l = 0$, $r_s = 0$ (diamonds), $1/k_F l = 0$, $r_s = 10$ (circles), $1/k_F l = 0.66$, $r_s = 2$ (squares) and $1/k_F l = 2.65$, $r_s = 6$ (triangles). Straight lines are linear fits. Upper right panel: $\alpha$ as defined in Eq. (5) as a function of $r_s$. Lower left panel: $E_p$ as a function of disorder $1/\sqrt{k_F l}$, for $r_s = 0$ (circles), $r_s = 0.25$ (left triangles), $r_s = 0.5$ (down triangles), $r_s = 1$ (diamonds), $r_s = 2$ (up triangles), $r_s = 5$ (squares) and $r_s = 8$ (right triangles). Dashed line stands for $E_p/E_F = 1/2 + \log 2/(\pi k_F l)$. Lower right panel: $E_p$ as a function of $r_s$ for $1/k_F l = 0$ (squares), $1/k_F l = 0.66$ (circles), $1/k_F l = 1.49$ (diamonds) and $1/k_F l = 3.51$ (triangles). Dashed line is the result of Ref. 22 without disorder. In both lower panels, the lines are fits given by Eq. (5) for $r_s \geq 0.25$. All energies are in unit of $E_F$.

V. COMPARISON OF THE POLARIZATION ENERGY $E_p$ WITH THE EXPERIMENTAL CHARACTERISTIC ENERGY SCALE.

Fig. 4 shows raw experimental data $\rho(B)$ and $\rho(T)$ on low density Si-MOSFETs from six different experiments, including the original set of data from Kravchenko et al. For each sample, we estimate the parameter $\eta$ from the peak mobility $\mu^{\text{peak}}$ obtained at high density (hence probably overestimating $\eta$) which sets how $\sqrt{k_F l} = \eta/r_s$ evolves with density. Then, for each density, we calculate the polarization energy $E_p$ according to Eq. (5) and (9). Disorder corrections to $E_p$ are rather small for those high mobility samples, so that our results are dominated by $E_p^{\text{sat}}(r_s)$ and not very sensitive to our estimate of $\eta$. For each curve $\rho(B)$ ($\rho(T)$), we draw a circle at the calculated saturation field $B_p$ (characteristic temperature $T_p$). The calculations are performed without any adjustable parameters and we find an extremely good match between $B_p$ ($T_p$) and the field (temperature) at which the resistivity saturates (has its inflexion point).

A. Magneto-resistance experiments

Describing the full $\rho(B)$-dependence is a complicated task. However, as the only effect of an in-plane magnetic field is to polarize the sample through Zeemann coupling (the orbital effect of an in-plane field can be neglected with good approximation), one expects that when the polarization saturates, the resistivity also saturates, so that $\rho(B)$ provides a direct measurement of $B_p$.

The experimental data of resistivity as a function of in-plane magnetic field $B$ (at low temperature) shown in panels (d), (e) and (f) of Fig. 4 correspond to three samples with mobility $\mu^{\text{peak}} = 25000 \text{cm}^2/\text{Vs}$, $41000 \text{cm}^2/\text{Vs}$ and $20000 \text{cm}^2/\text{Vs}$. We find that the magnetic field $B_{\text{sat}}$ at which the resistivity saturates is in extremely good agreement with our calculated $B_p$. This agreement between the experimental $B_{\text{sat}}$ and the calculated $B_p$ can be considered as a validation of the minimum model that we have used as well as of the accuracy of the QMC method.

Let us now discuss a publication which claimed to observe a divergence of the spin susceptibility $\chi$. Such a finding would contradict our calculation as $\chi \propto 1/B_p$, and although $B_p$ decreases as one lowers the density, we did not observe any divergence in the numerical calculations (one should however keep in mind that in the lower right panel of Fig. 3 the energies are in unit of $E_F$ which itself is proportional to the electronic density $n_s$). The corresponding data are shown in the right panel of Fig. 4 (our $B_p$: full line, experimental $B_{\text{sat}}$: dashed line). The experimental $B_{\text{sat}}$ were obtained in Ref. 22 from rescaling the in-plane magneto-resistivity data ($\mu^{\text{peak}} = 30000 \text{cm}^2/\text{Vs}$) at small field. Here, we find a good, but not very good, agreement between $B_p$ and $B_{\text{sat}}$. To understand the discrepancy, let us come back to the raw $\rho(B)$ experimental data for two values of the density corresponding to the blue squares (experiments) and red circles (our numerics). Those raw data are shown on the left panel of Fig. 4 where we also reproduce our predictions as well as the value of $B_{\text{sat}}$ obtained from the rescaling procedure. Clearly, the rescaling procedure underestimates $B_{\text{sat}}$ as the resistivity has not saturated yet at this value while our calculated $B_p$ matches precisely the end of the resistivity rise. We find that the agreement between the experiments and the numerics is actually extremely good and that the rescaling procedure only give approximate estimates of $B_{\text{sat}}$. As the divergence of $\chi$ claimed in Ref. 22 is extremely sensitive to this procedure, we conclude that there is probably no divergence of the susceptibility as defended in Refs. 27,28 but in contradiction with Ref. 22. New thermodynamic measurements are in qualitative and quantitative agreement with our numerics (hence we disagree with the conclusions of Ref. 22 which claim that there is a divergence of
the spin susceptibility).

Finally, we add a last data set that corresponds to the measures made in Ref.55 (green squares in right panel of Fig. 4) from the analysis of Shubnikov de Haas oscillations, at a similar mobility ($\mu_{\text{peak}} = 24000 \text{cm}^2/\text{Vs}$). We find that they agree perfectly with our calculations. Overall, we have a quantitative agreement between QMC $B_p$ and blue squares to $B_{\text{sat}}$ from Ref.55, at two fixed densities.

VI. A MINIMUM SCENARIO FOR THE 2D METAL-INSULATOR TRANSITION

We now have a good understanding of what happens on the horizontal of the $\rho(T)$ and $\rho(B)$ curves. To gain a full understanding of properties P1 and P2, we need to understand what happens on the vertical axis, i.e. why the resistivity increases and not decrease upon increasing temperature or magnetic field. In other words, in order to account for both P1 and P2, it is necessary and sufficient to show that polarized states have a higher resistivity than non polarized one (hereafter referred as property P6). Indeed, it is necessary, since the $\rho(B)$ curves show that the polarized system (high field) has a higher resistivity than the non polarized one (zero field). It is also sufficient as, once P6 is established, it naturally follows that when one will increase temperature in a range around $T_p$, the highly resistive excited states will be significantly populated and the overall resistivity will increase.

In a previous publication48, we have performed a systematic study of the interplay between Anderson localization and electron-electron interactions in Si-MOSFETs. We found that upon increasing interactions, the localization length of the non polarized ground state strongly increases (in absolute value, but in particular with respect to the polarized excited states), so that one naturally accounts for P6. This result is also shown in the right panel of Fig. 5 where we compare as a function of $r_s$ the localization lengths $\xi$ (in unit of the average distance between electrons a) computed at zero temperature for the non-polarized electron gas and for the fully polarized one. At very weak interaction ($r_s \lesssim 0.5$), the polarized system is less localized than the non-polarized one. Indeed, polarizing the system raises the Fermi level $E_F$ (by a factor 2), hence also $k_Fl$, hence $\xi$ which depends exponentially on the latter. On the contrary, at stronger interaction ($r_s \gtrsim 0.5$), the situation is reversed and the polarized system is more localized than the non-polarized one. We checked that we also get such an inversion of the localization lengths between the non-polarized ground state and partially polarized states, so that we indeed recover P6. In Fig. 4 of Ref.55 we presented the effective phase diagram constructed out of this mechanism which should be compared with the experimental one presented in Fig. 2. The agreement is semi-quantitative, meaning that our scenario also captures properties P3 and P4 (as well as P5 discussed above). Hence, we find that the minimum model discussed in this paper is enough to account for all the relevant experimental facts discussed in the beginning of this paper.
We end this section with a discussion of the existence of "true" metal at zero temperature, i.e. does the localization length diverge in the region where the metallic behavior is observed. We emphasize that in light of the above discussion, this is a rather academic question as above discussion. In Ref. 48, we have claimed that there probably does not exist a "true" metal at zero temperature. Our main argument was the absence of any visible deviation to the one-parameter scaling theory in presence of interactions, despite a huge delocalization effect. Here we give another argument. In the left panel of Fig. 3 we plot $a/\xi$ as a function of $r_s$ for the non-polarized (NP, full lines) and polarized (P, dashed lines) Si-MOSFET, at $k_Fl = 1.51$ (two lower green curves) and $k_Fl = 0.89$ (two upper blue curves). The arrows indicate the interaction strength $r_s$ at which the polarized excited state becomes more localized than the non-polarized ground state. In both panels, the lines are given by Eq. (10), (11) and (12).

Below we provide the fitting parameters that account for our localization length data. We have

$$\frac{a}{\xi}(k_Fl, r_s) = \frac{B(r_s)}{k_Fl} \exp(-C(r_s)k_Fl) , \tag{10}$$

where two different sets of parameters are needed for the non-polarized ($B^{NP}, C^{NP}$) and the polarized ($B^{P}, C^{P}$) case. In turn, the parameters $B$ and $C$ are well fitted by:

$$B(r_s) = b_0 + b_1 \sqrt{r_s} + b_2 r_s + b_3 r_s^3 \sqrt{r_s} + b_4 r_s^2 + b_5 r_s^2 \sqrt{r_s} + b_6 r_s^3$$

and:

$$C(r_s) = c_0 + c_1 \sqrt{r_s} + c_2 r_s + c_3 r_s^3 \sqrt{r_s} + c_4 r_s^2 + c_5 r_s^2 \sqrt{r_s} + c_6 r_s^3 , \tag{12}$$

where the values of the $b_i$ and $c_i$ fitting parameters are given in Table III for the non-polarized system ($b_i^{NP}$ and $c_i^{NP}$) and for the polarized one ($b_i^{P}$ and $c_i^{P}$), with the convention that $k_Fl$ always refers to the Fermi level of the non-polarized gas. Eq. (10) supports the idea that the two-dimensional electron gas behaves as a Fermi liquid (at least for moderate interactions $r_s \leq 10$) and that the effect of electron-electron interactions is to renormalize its effective characteristics.

\section*{VII. CONCLUSION}

Let us discuss a few theoretical scenarios that have been proposed by other authors. In light of the findings of this paper, we believe that models whose characteristic energies do not involve the polarization energy should not be applied to those experiments. This includes for example the percolation scenario proposed in Refs. 31, 32 (although percolation probably plays an important role close to the insulating region in GaAs heterostructures). This also includes the proposal related to Wigner crystal Ref. 24 (which we dismissed in Ref. 53 on other grounds). Ref. 25 has some overlap with our scenario (on the role of polarization energy in particular) but relies on the existence of some form of local order (which we did not observe in our simulations) and does not include the presence of disorder. Extrinsic models as the temperature dependent disorder scenario proposed in Refs. 31, 32 can also be excluded on the same ground (but these authors were the first to recognize the presence of a high energy scale in the experiments). An important theoretical advance was made in Ref. 24 where the authors studied the interplay between interaction and disorder in the diffusive limit, in the limit of an infinitely large number of valleys. We believe that those authors correctly pointed out the important role of valley degeneracy in the experiments, at the origin of the large differences between the behaviors observed in Silicon MOSFETs and in other heterostructures. However, the limit of an infinite number of valleys artificially increases the role of electron-electron interactions. In particular those authors find that even a very weak interaction will drive the system toward a non Fermi-liquid
fixed point. Those results, which imply a complete breakdown of one parameter scaling theory are in contradiction with our numerical results. More importantly, the corresponding analysis of the experiments is in contradiction with the present analysis of the characteristic energy scales involved in these systems.

In summary, we have considered a minimum model of Si-MOSFETs taking into account Coulomb repulsion, spin, valley degeneracy, and disorder in a non perturbative way. Our chief result is that the polarization energy $E_p$ calculated for this model is in quantitative agreement with the characteristic energy scale that controls the metallic behavior of those high mobility Si-MOSFETs. Beside spin, Coulomb repulsion plays a crucial role here as it decreases $E_p$ by a factor 2–3 (see Fig.3). The presence of valley degeneracy is also very important as it also decreases $E_p$ by a factor 2. On the other hand, disorder only gives corrections of 10–30% to $E_p$ for the samples considered in this survey. We note that the polarization temperature is the “crossover temperature” that was foreseen in Ref.22. However, we find that this “crossover temperature” is an intrinsic property of the electron gas, and does not involve any additional extrinsic ingredient. A simple corollary of our chief result is that the non-polarized system has to be a much better conductor than the polarized one (to explain the sign of $dp/dT$ and $dp/dB$ in the metallic region). We have argued before that our minimum model accounts for this counter-intuitive point (P6), as the interplay between Coulomb repulsion and Anderson localization depends strongly on the polarization. These findings imply that our “necessary” model (in the sense that all its ingredients are known to be present and a priori relevant) is also “sufficient” to capture the essential features of the metallic behaviors. Another consequence of our model is the apparent failure of one parameter scaling. For a given polarization, we could not find any deviation to the (one parameter) scaling theory of localization. However, at finite temperature excited states of different localization lengths come into play so that the physics is no longer controlled by a single parameter. In this work, we have focused on Si-MOSFETs, but at the qualitative level, many statements also apply to other materials like GaAs heterostructures. In our view, the situation in the latter is made a bit different by the conjunction of three elements. First the absence of valley degeneracy makes the delocalization effect of the ground state much less effective. Second, the present of the doping layer close to the gas induces some long range disorder which can mask the local physics. Last, extremely high mobility are available for these systems, making it possible to study almost ballistic samples.

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|  | 1  | 2  | 3  | 4  | 5  | 6  |
|---|---|---|---|---|---|---|
| $b_i^{NP}$ | 0.191 | -0.325 | 0.24 | -0.0526 | 0.0035 | 0  |
| $b_i' P$ | 0.136 | -0.09006 | -0.03197 | 0.18232 | -0.17749 | 0.07549 | 0.010827 |
| $c_i^{NP}$ | 0.84 | -0.177 | 1.094 | 2.0616 | -1.6784 | 0.3027 | 0  |
| $c_i'$ | 1.68 | -0.712 | 2.418 | -1.1874 | 0.1594 | 0  |

TABLE II: Parameters $b_i$ and $c_i$ of Eq. (11) and (12) for the non-polarized Si-MOSFET ($b_i^{NP}$ and $c_i^{NP}$) and for the polarized one ($b_i'$ and $c_i'$).

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