Confinement Admits Chiral Symmetry Breaking via Bag

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In this paper, it is pointed out for the first time that the linear effective potential between quarks is intrinsically relating to the bag model while concerning the asymptotically-free nature of colours. Based on the relationship we employ the symmetry method to analyze the quark-anti-quark system. By imposing the Poincare invariance on the quark-anti-quark bound-state, and translating the chiral transformation to its spatial manifestation, we can infer why the chiral symmetry breaking happens. Applying this knowledge to deep inelastic scattering we reach the conclusion that the measured proton spin in scattering experiments should be uncertain quantity.

keywords: confinement; linear potential; bag model; chiral symmetry breaking; conformal group; spin structure of hadrons

I. INTRODUCTION

The non-perturbative part of QCD can hardly be resolved analytically. Different methods have been proposed to simplify the process of searching solutions, all of which conclude with a linear potential between a quark and an anti-quark (It is also known as the flux tube.). Up to date it has been believed that the linear potential is compatible with colour confinement. In addition, Regge trajectories can be viewed as a compelling evidence for the rationale of linear potential. The linear potential has shown its essence in calculating the spectra of mesons with heavy quarks and has produced reliable results in agreement with experiments.

Chiral symmetry breaking (χSB) is the other side of non-perturbative QCD. Reconciling colour confinement with χSB becomes critical for understanding the non-perturbation of QCD. Since it agrees well with the hadron spectra even most of light hadrons, the linear potential turns out to be a common interpretation of colour confinement. Although Ref. tells that without χSB there is no confinement, the reverse reasoning, i.e. inferring χSB from the confinement, remained open for the past decades. There are efforts choosing different ways to infer χSB, and progresses have been made in using the method of Dyson-Schwinger equation. However, each effort has to rely on an extra input to circumvent a difficulty, i.e. also a fact, the finite-size effect hidden in the interaction vertex.

II. THE RELATIONSHIP BETWEEN LINEAR POTENTIAL AND BAG

Here we will associate confinement with χSB via bag. Physically, there exists a true bag in hadron, not just a model any longer, if only we combine the sea gluons with a quark. As for the linear potential, it provides a constant force, which implies in its own right that the lines of colour force (henceforth named "colour lines") are all absorbed by another (anti-)quark. In terms of gluon field, that means the gluons sent from one quark are all absorbed by another (anti-)quark, without any leaking. Moreover, knowing that protons always dress pion cloud, similarly we regard the gluons being dressed by the quark/anti-quark (to be the quark or the anti-quark depending on the reference frame we choose), subsequently we recognize that the dressing quark/anti-quark becomes spatially extended. Without losing generality, we propose that it is the quark that dresses all of the virtue gluons, forming a closed bag surrounding the anti-quark. And the anti-quark remains to be a point particle. But why should it be bag?

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TABLE I: We use this table to illustrate the almost equivalence between \([1/r] + \text{bag}\) and the linear potential model, where \(V_1(r) = -\alpha s/r\) while \(0 < r < R\) and \(V_1(r)\) becomes \(\infty\) while \(r > R\), \(V_2(r) = \sigma r\) as usual defined. The parameters are \(m_b = 4660\) MeV, \(\sigma = 3.3 \times 10^4\) MeV\(^2\), \(\alpha_s = 0.356, R = 1.925\)fm. Please note that the bag here is thin without thickness.

The quark extended in space will be identical to either a point, or a bag topologically, according to the following arguments. Holding the picture that strong interaction works certainly by exchanging boson particles between points, then it is the familiar scenario whence the force is reversely proportional to an imagined spherical area \(4\pi r^2\), where \(r\) is the distance between the two points. If only the quark is not closed bag, this scenario holds, coinciding with asymptotic freedom \(30, 31\). Now assuming that the spatially-extended quark forms a closed bag and, the interaction between a point in quark and the anti-quark is still proportional to \(1/r\) [caution that the quark is sizable and the anti-quark remains a point particle], then the interaction term between the quark and anti-quark in Schrodinger equation should be an integral form like \(\int_{\text{bag}} \frac{1}{r} \psi(\vec{r}) d\vec{r} = \int_{\text{bag}} \frac{1}{r} \psi(\vec{r}) r^2 drd\Omega = 4\pi \int_{\text{bag}} r\psi(\vec{r}) dr = 4\pi R\psi(\vec{R})dR_0\). The last step assumes the ideal situation when the quark(bag) forms a spherical surface with fixed thickness \(dR_0\). So far one could conclude qualitatively with the linear potential.

We have used the potential \([1/r] + \text{bag}\) calculating the spectra for some states and comparing results with those using potential \([\sigma r]\). One notes that there does exist the set of parameters for the two methods to lead to the same tendency and similar separation between energy levels [Table I]. Now the bag looks indispensable in consistently bridging the asymptotic potential \([1/r]\) and the confining potential \([\sigma r]\), which are combined to be the very useful Cornell Potential \(12, 33\). Surprisingly the total effect of an asymptotic potential plus bag is almost equivalent to a linear potential \([1/r] + \text{bag} \sim [\sigma r]\). The bag plays a role in associating the perturbative dynamics with nonperturbative dynamics. The bag also manifests non-Abelian characteristic since it cannot be penetrated by other colour lines (gluons) which are all absorbed by the dressed gluons of the quark, i.e. the three-line-interaction-vertex of gluons works. It is totally a non-Abelian property not pertaining to the dressed photons by electron. \([1/r] + \text{bag}\) is in some sense better than \([\sigma r]\) since \([\sigma r]\) has its well-known Abelian origination \(1, 34\). Now based on the above arguments we accept the effectiveness that one of the quark and/or the anti-quark forms bag in the extreme state, i.e. meson ground state. We can picture a process for the above argument: with the decrease of system’s energy, one end of the flux tube (quark side) swells to be like an umbrella then finally forms the bag. In the process the gluon field evolves to be like many extra ribs to support the umbrella. In the final bag state the Regge trajectories remain effective in evaluating hadron spectra. If only the colour lines are not leaked, then the linear potential holds, as well as the effective flux tube.

There has been existing demonstration on how the bag induces \(\chi_{SB}\) from Lagrangian \(35\). But the explanation doesn’t comply physically with the massless situation while the chiral symmetry is restored, since it implies that the bag would disappear for pions or any goldstone bosons. And we are tempted to employ symmetry methods to reconcile the conflict.

III. THE REALIZATION OF CHIRAL TRANSFORMATION

In the non-perturbative regime, where the bag (with thickness \(dR_0\)) locates, the smallest symmetric group is assumed to be Poincare group which includes Lorentz group as sub-group. Of course, the quark-anti-quark system should be
Transformation and chiral transformation change the spinors of free particle. The chiral transformation varying the scale. It needs energy injection from or leakage to outside [42]. Now let’s have a glimpse at how scaling equivalent to that of the scaling operator $D$ represented by a Lagrangian as described above, scaling transformation $e^\gamma$ where $\gamma$’s are Dirac $\gamma$-matrices and we use $\rightarrow$ to represent the accurate mappings, and the same commutations as eq.(3) can be examined. We have recognized that the role of operator $\mu \frac{\partial}{\partial x_\mu}$ (or $x_\mu \frac{\partial}{\partial x_\mu}$) in the conformal group is equivalent to that of the scaling operator $D$, with its spinor representation being $\gamma_5$.

By definition the scaling part of the conformal group will either stretch or press the whole bag homogenously while varying the scale. It needs energy injection from or leakage to outside [12]. Now let’s have a glimpse at how scaling transformation and chiral transformation change the spinors of free particle. The chiral transformation $e^{i\gamma_5}$ and the scaling transformation $e^{\pm \gamma_5}$ differ in an imaginary phase factor $i$. Now let the transformations perform on the simplest
spinor \( \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \), which are

\[
e^{\frac{u}{2} \gamma_5} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \cosh \frac{u}{2} \\ 0 \\ \sinh \frac{u}{2} \\ 0 \end{pmatrix},
\]

(5)

and

\[
e^{i \frac{u}{2} \gamma_5} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \cos \frac{u}{2} \\ 0 \\ i \sin \frac{u}{2} \\ 0 \end{pmatrix}.
\]

(6)

It is noted that they separately satisfy normalizations

\[
\psi^\dagger(x) \gamma_0 \psi(x) = u_1^* u_1 + u_2^* u_2 - u_3^* u_3 - u_4^* u_4 = 1, \quad \text{and} \quad \psi^\dagger(x) \psi(x) = u_1^* u_1 + u_2^* u_2 + u_3^* u_3 + u_4^* u_4 = 1,
\]

where \( \psi(x) \) is a complex spinor, and \( \psi^\dagger = (u_1^*, u_2^*, u_3^*, u_4^*) \), e.g. the eq. \( (6) \) is \( \psi^\dagger = (\cos \frac{u}{2}, 0, -i \sin \frac{u}{2}, 0) \). Though the former is Minkowski and the latter is Euclidean, the results keep the same

\[
cosh^2 \frac{u}{2} - \sinh^2 \frac{u}{2} = \cos^2 \frac{u}{2} + (-i \sin \frac{u}{2})(i \sin \frac{u}{2}) = 1.
\]

Now let’s examine the effect of chiral transformation on spinor in more detail. A regular Dirac spinor can be written as

\[
A \begin{pmatrix} s_1 \\ s_2 \end{pmatrix}_{4 \times 2} \varphi_\alpha
\]

where \( A \) is a normalization constant, and \( \varphi_\alpha, \alpha = 1, 2 \) are \( 2 \times 1 \) spin eigen states or helicity eigen states as \( \begin{pmatrix} 1 \\ 0 \end{pmatrix} \) or \( \begin{pmatrix} 0 \\ 1 \end{pmatrix} \). The part \( \begin{pmatrix} s_1 \\ s_2 \end{pmatrix} \) can be written as \( \begin{pmatrix} 1 \\ \vec{\varphi} \cdot \vec{p} \end{pmatrix} \) or \( \begin{pmatrix} \vec{p} \cdot \vec{E} + \mu m \\ 1 \end{pmatrix} \) corresponding to free particle or free anti-particle respectively. Now let’s check what if \( e^{i \frac{u}{2} \gamma_5} \) performs on \( \begin{pmatrix} s_1 \\ s_2 \end{pmatrix} \).

\[
e^{i \frac{u}{2} \gamma_5} \begin{pmatrix} s_1 \\ s_2 \end{pmatrix} = \cos \frac{u}{2} \begin{pmatrix} s_1 \\ s_2 \end{pmatrix} + i \sin \frac{u}{2} \begin{pmatrix} s_2 \\ s_1 \end{pmatrix}.
\]

We note this is another sort of spinors rotation, which mixes the states of both the particle and the anti-particle.

Then let’s consider here the spatial effect of chiral transformation on the quark-anti-quark system. According to the above analysis, in such case the chiral transformation is somehow equivalent to the partial exchange of quark state and anti-quark state, another sort of "rotation". With the aforementioned bag picture, if the quark and anti-quark masses are other than zero, one notes that the center of mass changes from anti-quark to quark while performing chiral transformation \( e^{i \frac{u}{2} \gamma_5} \) [Fig 1]. The chiral transformation becomes "observable" due to the need of injecting energy to displace the center of mass. Accordingly while the quark and anti-quark are massless, it is clear now such chiral transformation is not observable, i.e. the chiral symmetry holds. This is the picture of \( \chi \text{SB} \) in spatial manifestation via bag. Now \( \chi \text{SB} \) can be inferred from the confinement.

To put the above analysis alternatively, one notes the Lagrangian eq. \( (1) \) is invariant under Poincare group, which includes translations of energy and momentum. According to the eq. \( (4) \) the chiral transformation happens not to
Anti-quark

The quark

Mass center locates at Anti-quark.

The anti-quark

Quark

Mass center locates at quark.

FIG. 1: The two-quark system before and after the chiral transformation.

...commutate with these four translations, \([\gamma_5, \gamma_\mu(1+\gamma_5)] \neq 0\), especially the energy part. Thus while the energy (mass) of the system changing (displacement of mass center), then the chiral symmetry becomes varying, i.e. breaking or restoring.

IV. DISCUSSIONS

The model in this paper offers a new insight into the spin structure of hadrons \([43–54]\). In contrast to confinement, while a large amount of external energy is injected into hadrons, we know the asymptotic freedom would be reached by scattered fragments (including partons, quarks etc.) and chiral symmetry will be restored by these fragments. The fragments take almost zero masses. In such ultra-high-energy scattering case, the aforementioned pictured process evolves backward with the increase of energy, i.e. from the bag back to flux tube. Actually there are many such flux tubes in the fragment jets \([55, 56]\). Now these flux tubes are surrounded almost isotropically by identical particles (fermions like partons, quarks or bosons like meson). The flux tubes together with identical particles form dual composite particles, whose angular momenta exclusively depend on environment \([57]\).

We call these composite particles anyons which live in 2-dimension environment. How can this 2-dimension condition be satisfied in our model? Without losing generality, we know the composite particles must locate in one of the final-state-planes, and in fact only one plane dominates there \([58]\). As for how flux tubes are perpendicular to the plane and identical particles are parallel to the plane, please refer to \([59]\). The interactions among composite particles are complicated according to \([57]\, as well as the statistics and the total angular momenta of these particles. So in this hot state of mixed particles, the angular momentum is consequently not certain due to the anyons. Accordingly, the sum of angular momenta we measured from the jets cannot be used to infer reversely the original spin of hadrons. Alternatively, maybe from the factor \(\frac{\delta m}{m}\) we can also conclude that the total spin of quarks are uncertain, which causes...
the total angular momenta uncertain. This uncertainty stems from the 2-d condition and gauge transformation (the transformation of gauge potential $\vec{A}$) [57], plus the varying chiral parameter (the quark mass). The result is consistent with the reference [60], telling that only while quarks are in light-cone gauge, could the spin and angular momentum be physical observables.

Another interesting aspect of this research is the topology shown by transformation of the chiral symmetry, i.e. while interchanging the spatial position of quark and anti-quark. It is very mimicking the Mobius band in four dimension, the Klein bottle [61]. We may call it the inner-outer transform, which sounds a bit strange since one cannot imagine how to drag the anti-quark from the inner of the closed bag to its exterior, unless you think of things like Klein bottle realized in 4-dimension space. Here we have attributed the topology effect to single quark. The alternative way of expressing such topology is to use the $\chi$SB result directly, leading to such as PCAC or the linear sigma model, from which one can derive topology effects for emerging quasi-particles [62]. At the present stage we conjecture that these two ways of deriving topology may be equivalent. To speak alternatively, to view the quark being extended in space, is somehow equivalent to understand hadrons as something emergent formed by a lot of point particles. Realistically, while the energy of quark is truly low and thus its wavelength becomes very large, the quark becomes spatially extended completely, with dressed gluons as its wings [63]. In such cases the quark can be viewed as nonlocal entity without bias. It could be a constituent quark. This view coincides with the straton model proposed in the mid 1960s [64]. The term straton suggests the layers' structure of constituents in hadron, like an onion. It has the similar dynamics to chromo-dynamics. It gets some points of the truth with nowadays hindsight.

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