What is the Natural Size of Supersymmetric $CP$ Violation?

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It is well known that if phases and masses in the Minimal Supersymmetric Standard Model (MSSM) are allowed to have general values, the resulting neutron EDM ($d_n$) exceeds the experimental upper limit by about $10^3$. We assume that the needed suppression is not due to a fine-tuning of phases or masses, and ask what natural size of $CP$ violation (CPV) results. We show that (1) the phase of one of the superpotential parameters, $\mu$, does not contribute to any CPV in the MSSM and so is not constrained by $d_n$; (2) the MSSM contribution to $d_n$ is tiny, just coming from the CKM phase; (3) the phases in the MSSM cannot be used to generate a baryon asymmetry at the weak scale, given our assumptions; and (4) in non-minimal SUSY models, an effective phase can enter at one loop giving $d_n \sim 10^{-26}\text{e cm}$, $d_e \sim 10^{-27}\text{e cm}$, and allowing a baryon asymmetry to be generated at the weak scale, without fine-tunings. Our results could be evaded by a SUSY breaking mechanism which produced phases for the SUSY breaking parameters that somehow were naturally of order $10^{-3}$. 
I. INTRODUCTION

Predictions for $CP$ violating effects in supersymmetric (SUSY) theories have often been discussed with a certain ambiguity. On the one hand, it is well known that when the complex quantities in the theory are allowed to have phases of order unity, the predicted neutron electric dipole moment ($d_n$) is typically too large by perhaps $10^3$ \[1-3\]. In order to avoid this, the relevant quantities are often chosen to be real, in which case the theory predicts no non-Standard Model $CP$ violation or $d_n$ at all. On the other hand, it has often been assumed that the observation of $d_n$ around the current limit of $10^{-25} e \text{cm}$ \[4\] could easily be accommodated by a SUSY theory with the phases somehow reduced by just the right amount. These ideas are clearly in conflict: one cannot have a theory which avoids fine-tunings by having $d_n$ zero, and at the same time which gives $d_n$ near the current limit.

All of the $CP$ violation (CPV) induced by supersymmetry occurs because of phases in either the superpotential or the soft SUSY breaking Lagrangian \[3\]. The superpotential of the Minimal Supersymmetric Standard Model (MSSM) contains the Yukawa sector of the theory, $W_Y$, and a Higgs mixing term with coefficient $\mu$,

$$W = W_Y + \mu H_u \times H_d,$$

(1)

where $H_u$ and $H_d$ are Higgs doublet superfields, and where $\mu$ can be complex. The low energy supergravity (SUGRA) parametrization of the soft breaking Lagrangian can be written in terms of the superpotential and superpartner mass terms:

$$-\mathcal{L}_{soft} = |m_i|^2 |\varphi_i|^2 + \left(\frac{1}{2} \sum_\lambda \bar{m}_\lambda \lambda \lambda + A m_0^* [W_Y]_\varphi + B m_0^* [\mu H_u \times H_d]_\varphi + \text{h.c.} \right),$$

(2)

where $\varphi_i$ are the scalar superpartners, $\lambda$ are the gauginos, and $[ \ ]_\varphi$ means take the scalar part. Like $\mu$, the soft breaking parameters $A$, $B$, and $m_0$, and the gaugino masses $\bar{m}_\lambda$, can all be complex. These parameters contribute to $d_n$ at the order of $10^{-22} \tilde{\varphi}/\tilde{M}^2 e \text{cm}$, where $\tilde{\varphi}$ is a combination of the phases of the parameters, and $\tilde{M}^2$ is a combination of
superpartner masses, normalized to the weak scale. The only known ways to make such a large $d_n$ compatible with the experimental upper bound are to fine-tune the phase $\varphi$ to order $10^{-3}$; have superpartner masses of order a few TeV; or somehow require all the phases to naturally be zero $[6]$. Both the first and second approach eliminate much of the attractiveness of SUSY. For example, having large superpartner masses eliminates the possibility of radiative breaking of $SU_2 \times U_1$, which was one of the major successes of SUSY. Losing this is especially undesirable now that the top mass is large enough to make it work.

In this letter we consider what natural amount of CPV is expected in SUSY theories. By this we mean the size of CPV observables one expects in a SUSY theory constructed such that there are no fine-tunings of phases, parameters, or mass scales in order to satisfy the empirical upper bound on $d_n$. Since all the SUSY phases come into $\varphi$, these criteria force the parameters $A$, $B$, $m_0$ and $\tilde{m}_\lambda$ to be real $[6]$. However, we will show that $\mu$ does not have to be real in a large class of models.

We do not propose any explanation for why the other parameters are real, but merely accept that to have a SUSY theory without fine-tunings, these conditions must somehow be satisfied, given the phenomenological and theoretical constraints. If fine-tunings such as large sparticle masses or phase cancellations turn out to be important, our arguments may or may not be relevant. For the remainder of this letter, we simply take $A$, $B$, $m_0$, and $\tilde{m}_\lambda$ to be real; see $[6]$ for a complete treatment of these criteria and a summary of previous discussions.

Imposing the above criteria means that the MSSM has no non-SM CPV, thus $d_n$ and $d_e$ are tiny, as predicted by the CKM mechanism. There are no CKM effects in the renormalization group equations of the SUSY parameters, and finite CKM effects are tiny $[6]$. In addition, there would not be enough CPV to explain the observed baryon asymmetry. With this in mind, we point out a mechanism by which a moderate amount of CPV can naturally arise in a non-minimal SUSY theory through loop corrections to the Higgs potential. The idea is that a phase which is unobservable at tree level can introduce an observable effective phase through loop effects. But the effective phase will always be smaller than a tree level
observable phase because of the usual factors of suppression associated with loops. Such a phase can make moderate contributions to $d_n$, $d_e$, and may be useful in explaining the baryon asymmetry. However it requires non-minimal extensions to the MSSM, and so its consideration will have important effects upon SUSY model building.

II. SOURCE OF $d_n$

All supersymmetric contributions to $d_n$ come from the mass matrices of squarks and gauginos—if the mass matrices can all be made real, the SUSY contribution to $d_n$ disappears. If they are complex, the gaugino-squark-quark couplings become complex and contribute to $d_n$ through loop diagrams [1]. Let us write the down squark mass matrix in a partially diagonalized basis [7]:

$$
\begin{pmatrix}
\mu_{dL}^2 1 + \hat{M}_D^2 & (A^* m_0 - \mu v_u/v_d) \hat{M}_D \\
(A^* m_0 - \mu v_u/v_d)^* \hat{M}_D & \mu_{dR}^2 1 + \hat{M}_D^2
\end{pmatrix},
$$

where $\hat{M}_D$ is the diagonal, real, $N_F \times N_F$ quark mass matrix (where $N_F$ is the number of families), and $\mu_{qL,R} \sim |m_{3/2}|^2$. As mentioned above, we take $A$, $B$, $m_0$ and $\tilde{m}_{\lambda}$ to be real to avoid fine-tuning. The only remaining possible phases in the squark mass matrices are those of $\mu$, and the vacuum expectation values (VEVs; see (3)).

We write the chargino mass matrix, $M_{\chi^+}$,

$$
\begin{pmatrix}
\tilde{m}_W & g_2 v_u^* \\
g_2 v_d & \mu
\end{pmatrix},
$$

in the basis of [8], where $\tilde{m}_W$ is the $SU_2$ soft breaking gaugino mass, and $g_2$ is the $SU_2$ coupling constant.

The neutralino mass matrix $M_{\chi^0}$ is (see [8]):

$$
\begin{pmatrix}
\tilde{m}_B & 0 & -g_1 v_d/\sqrt{2} & g_1 v_u^* / \sqrt{2} \\
0 & \tilde{m}_W & g_2 v_d/\sqrt{2} & -g_2 v_u^* / \sqrt{2} \\
-g_1 v_d/\sqrt{2} & g_2 v_d/\sqrt{2} & 0 & -\mu \\
g_1 v_u^* / \sqrt{2} & -g_2 v_u^* / \sqrt{2} & -\mu & 0
\end{pmatrix},
$$

(5)
where $g_1$ is the $U_1$ coupling constant, and $\tilde{m}_B$ is the $U_1$ gaugino mass. Most references do not keep track of the phases of the VEVs in (3)–(4). This is undoubtedly due to the fact that the tree level MSSM does not allow for spontaneous CP violation \[9,10\], so most authors have assumed $v_u$ and $v_d$ are real. As we will see, there can be a relative phase between the VEVs in some cases, so that it is important to use our form for $M_{\chi^+}$ and $M_{\chi^0}$.

III. TREE LEVEL CANCELLATION

To see why the phase of $\mu$ does not actually contribute to $d_n$, we present two arguments. The first relies upon redefinition of the phases of Higgs superfields. If we redefine the phases of the Higgs and higgsino ($H_u$ and $\psi_{H_u}$), it turns out that the $F$ terms get rotated by the same amount, so that the procedure is equivalent to redefining the phase of the superfield $H_u$. If the soft terms come from the superpotential, as in (3), the phase of $\mu$ gets rotated away too. In the MSSM, the only new phase introduced by this rotation is in the Yukawa couplings in $W_Y$. Since those can absorb an arbitrary phase, we can define away the phase of $\mu$ without loss of generality, leaving no CPV in the MSSM \[13\], other than the CKM phase. This is true in the low energy SUGRA parametrization of the MSSM, and in many extensions of the MSSM.

We also give an alternate derivation of this result from minimizing the Higgs potential. In some ways it is more transparent, and we will need the results in the next section. We use two doublets of the same hypercharge,

$$\phi_1 = H_d \rightarrow \begin{pmatrix} 0 \\ v_d \end{pmatrix}, \quad \phi_2 = H_u \rightarrow \begin{pmatrix} 0 \\ v_u \end{pmatrix}, \quad (6)$$

which defines the two VEVs $v_u$ and $v_d$. The most general renormalizable $SU_2 \times U_1$ invariant scalar potential for two Higgs doublets $\phi_1$ and $\phi_2$ is \[9,11\]

$$V = m_1^2|\phi_1|^2 + m_2^2|\phi_2|^2 - (\mu_1^2\phi_1^\dagger\phi_2 + h.c.) + \lambda_1(\phi_1^\dagger\phi_1)^2$$
$$+ \lambda_2(\phi_2^\dagger\phi_2)^2 + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) +$$
$$[\lambda_5(\phi_1^\dagger\phi_2)^2 + \lambda_6|\phi_1|^2(\phi_1^\dagger\phi_2) + \lambda_7|\phi_2|^2(\phi_1^\dagger\phi_2) + h.c.] \quad (7)$$
We see that $m_{1-2}$ and $\lambda_{1-4}$ are real. Further, SUSY predicts that $\lambda_{5-7}$ are zero at tree level \cite{9,10}, so that only $\mu_{12}$ can be complex. Using (2), (3), and (4) we have, $\mu_{12}^2 = Bm_0^*\mu$, and we can define its phase in terms of that of $\mu$ \cite{13}: $\theta_\mu \equiv \text{Arg}\mu = \text{Arg}\mu_{12}^2$.

Minimizing the Higgs potential in (4), one condition we obtain is $\xi' \equiv \xi + \theta_\mu = 0$, which means that the relative phase between the VEVs, $\xi \equiv \text{Arg}\frac{\mu}{v_u/v_d}$, is non-zero. This phase can be rotated away everywhere in the gauge and matter sectors except in the squark and gaugino mass matrices. We consider the effects of $\xi' = 0$ on each.

The essential point is that the minimization condition $\xi' = 0$ implies that $\mu v_u/v_d$ is real, which means that the squark mass matrix in (3) is also real \cite{13}. Thus the phases of $\mu$ and the VEVs cancel, so there is no squark mass matrix contribution to $d_n$, even if $\mu$ is complex.

A necessary condition for this cancellation is that the Higgs scalar mixing term in $\mathcal{L}_{soft}$ has the same phase as the Higgs superfield mixing term in the superpotential; we must have $\mu_{12}^2 \sim \mu$ and the no fine-tuning criteria which implies $Bm_0^*$ is real. If there are contributions to $\mu_{12}^2$ which are not proportional to the phase of $\mu$, the cancellation goes away. Note that if $\mu$ is zero, the phase of $\mu_{12}^2$, whatever its source, can be rotated away.

The cancellation in the gaugino mass matrices is more subtle. Contributions to $d_n$ can come from phases in the gaugino-squark-quark vertices, which are introduced by the unitary matrices which diagonalize $M_{\chi^+}$ and $M_{\chi^0}$. But there is also a phase in the higgsino-squark-quark part of the couplings from the VEVs. In gaugino-squark-quark vertices, $\psi_{H_d}^- (\psi_{H_u}^+)$ higgsinos are always accompanied by $1/v_d^* (1/v_u)$. These phases can be rotated into the definition of $\psi_{H_d}^-$ and $\psi_{H_u}^+$, so that the weak basis gaugino-squark-quark couplings are real. This redefinition of the weak states also changes the gaugino mass matrices. It turns out that the only phase in the rotated $M_{\chi^+}$ and $M_{\chi^0}$ is $\xi'$, and thus the minimization condition implies that $M_{\chi^+}$ and $M_{\chi^0}$ are real! Remarkably, by having the $\mu_{12}^2 \sim \mu$, the VEVs are aligned so the mass matrices (3)–(5) are all real, and there is no SUSY contribution to $d_n$ at tree level even if $\mu$ is complex.
IV. A LOOP INDUCED OBSERVABLE PHASE

One might be worried that after inclusion of one loop effects that one could no longer rotate away the phase of \( \mu \), since there are new terms which involve \( H_u \). This cannot happen because at tree level all vertices are independent of the phase of \( \mu \), so there is no way for it to reappear at one loop. However, it is possible for a phase which makes negligible contribution to CPV in the tree level Lagrangian to appear more prominently at one loop.

After one loop corrections to \( V \), there will be non-zero contributions to \( \lambda_{5-7} \) and \( \mu_{12}^2 \). In the natural MSSM (satisfying our criteria), these contributions (call them \( \delta \lambda \)) will be real. Suppose that our theory contains some new complex parameter which does not lead to large CPV through tree level vertices. Suppose further that the complex parameter appears in vertices which involve Higgs fields. Then the parameter could introduce complex \( \delta \lambda \)'s through loops. The VEVs would get a relative phase, and that could in turn introduce observable CPV effects.

To see how this works, let us consider the one loop Higgs potential with arbitrary coefficients \( \delta \lambda \) added to the tree level \( V \). Let us also use our results from the last section to set \( \xi = \xi' = \theta_\mu = 0 \) at tree level. At one loop, the minimization point will shift to

\[
\sin \xi \simeq \frac{v^2}{|\mu_{12}^2|} \text{Im} \left[ \delta \lambda_5 \sin 2\beta \pm \right. \\
\left. \left(-\delta \mu_{12}^2/v^2 + \delta \lambda_6 \cos^2 \beta + \delta \lambda_7 \sin^2 \beta \right) \right]
\]  

(8)

where the + (−) corresponds to \( \xi \) near 0 (\( \pi \)), and \( \tan \beta \equiv |v_u/v_d| \). We have assumed \( \mu_{12}^2 >> \delta \lambda v^2 \) for simplicity.

This new induced phase \( \sin \xi \) contributes to both \( d_n \) and \( d_e \) through the sfermion and gaugino mass matrices. Let us define the phase of the quantity in brackets in (8) to be \( \theta_{\delta \lambda} \), and take the magnitude of these loop corrections to be of order \( 10^{-3} \). If we take \( B \sim .5 \), colored superpartners \( \sim 300 \text{GeV} \), sleptons \( \sim 175 \text{GeV} \), and all other superpartners \( \sim 100 \text{GeV} \), we estimate that

\[
d_n \sim 10^{-26} \tan \beta \sin \theta_{\delta \lambda} \text{e cm},
\]  

(9)
\[ d_e \sim .6 \times 10^{-27} \tan \beta \sin \theta_{3\lambda} \text{ e cm}. \]  

(10)

These estimates do depend upon the parameters and the mass scales in the theory, but the point is that the contributions entering at one loop are naturally much smaller than those from SUSY phases which contribute through tree level vertices. Note that this effect will not work in the MSSM \[ ] because the CKM contribution to \( \theta_{3\lambda} \) is negligible. We must look beyond the MSSM.

As an example, let us add a gauge singlet superfield \( N \) to the minimal model \[ ] . Suppose we replace the superpotential in (1) with

\[ W = W_Y + \mu H_u \times H_d + h N H_u \times H_d + \frac{1}{3} k N^3, \]

(11)

where \( h \) and \( k \) are complex, and \( N \) does not get a VEV. By redefinition of the phase of \( N \) we can make \( h \) real, leaving \( k \) arbitrarily complex. With \( \langle N \rangle = 0 \) the phase of \( k, \theta_k \), does not appear in any vertices outside the Higgs sector at tree level. Some scalar-pseudoscalar mixing is induced, but one loop contributions to \( d_n \) and \( d_e \) from this are negligible \[ ], though two loop diagrams may be larger \[ ]. However, one loop corrections to \( V \) can depend upon \( \theta_k \), and thus perhaps \( \sin \theta_{3\lambda} \sim \sin \theta_k \), which could be of order unity. Thus the phase \( \theta_k \), which at present is probably unobservable at tree level, can give \( d_n \) and \( d_e \) near their current limits through loop effects.

**V. REMARKS**

We have shown that it is not necessary for \( \mu \) to be real for a SUSY theory to satisfy the bound on \( d_n \) without fine-tunings. The soft breaking parameters \( A, B, m_0 \) and \( \tilde{m}_\lambda \) must be real or a new mechanism must be found to give them phases at the \( 10^{-3} \) level. A MSSM which satisfies the no fine-tuning criteria gives no new CPV beyond the SM CKM phase, and thus \( d_n \) and \( d_e \) are negligible. There may still be observable CPV effects due to SUSY CKM contributions to various processes, notably in B decays \[ ]. We showed that
non-minimal SUSY models can potentially give a loop induced observable phase, which can give \( d_n \) and \( d_e \) near their current bounds.

Note that the cancellation of the phase of \( \mu \) depends crucially on the assumption that \( \mu_{12}^2 \) is proportional to the phase of \( \mu \). This will not be true if \( \mu_{12}^2 \) has sources other than the superpotential, e.g. \( \mu_{12}^2 \) is put in by hand. Knowing that the phase of \( \mu \) can be rotated away is important because SUGRA models do not usually generate superpotential parameters, so explaining why the phase of \( \mu \) was zero might have proven much harder than for the other parameters.

In the MSSM a non-zero \( d_n \) or \( d_e \) near the present limits would probably have pointed away from SUSY theories, since there would have to have been some fine-tuning in the theory. For SUSY believers a non-zero observation of \( d_n \) or \( d_e \) would point toward non-minimal SUSY models which allow CPV to enter at one loop as we have described above, or require a new mechanism to give the soft breaking parameters a tiny phase.

There are also SUSY contributions to \( d_n \) from a three gluon operator [18], which are probably smaller than the quark EDM contribution [3]. If the no fine-tuning criteria we have used is satisfied, this operator will give only a small numerical correction to \( d_n \), proportional to the phase \( \theta_{\delta \lambda} \). Thus (9)–(10) still reflect the natural level of CPV possible in a non-minimal SUSY model.

It may be possible to have spontaneous \( CP \) violation in supersymmetric theories at one loop [11], though that may conflict with Higgs mass limits [11]. That could be circumvented by extending the MSSM with a singlet \( N \) and allowing it a complex VEV, giving spontaneous \( CP \) violation at tree level [12]. In either case a fine-tuning is probably required of the VEV phase \( \xi \) to keep \( d_n \) below the experimental bound; in fact we argue that spontaneous \( CP \) violation generally requires more fine tuning than hard \( CP \) violation. One of the minimization conditions necessary for non-zero spontaneous CPV is

\[
\cos \hat{\xi} = \frac{X}{Y} \simeq 1 - \frac{1}{2} \hat{\xi}^2, \tag{12}
\]

where \( \hat{\xi} \) is \(|\xi|\) or \(|\xi - \pi|\), and \( X \) and \( Y \) are some combination of loop integrals, parameters,
and VEVs. We need $\hat{\xi}$ to be small to satisfy the bound on $d_n$, which we can achieve only if $\Delta \equiv (Y - X)/Y$ is of order $\hat{\xi}^2$. For example, if we need $\hat{\xi} \sim 10^{-3}$, then $\Delta$ must be fine-tuned to be of order $10^{-6}$, which is completely unacceptable. Thus SUSY theories of spontaneous CPV may have even more trouble in justifying the small experimental bound on $d_n$ than hard CPV. However, it should be fairly easy in most models to require there be no spontaneous CPV, without any fine-tunings.

Finally we note that the loop mechanism of section IV could be very important to the CPV aspects of the problem of the baryon asymmetry. A recent interesting model of electroweak baryogenesis used CPV from the Higgs scalar mixing coefficient $\mu_{12}^2$ [19]. It was pointed out [20] that $\mu_{12}^2$ can be rotated out of the Higgs potential, but the resulting phase in the gaugino mass matrices was then used by [21]. They found that with the small phase allowed by the limit imposed by $d_n$, there is probably sufficient CPV for the baryon asymmetry. Our results change these conclusions in two ways: At tree level, there is no phase in the gaugino mass matrices [13], and no way for $\theta_{\mu}$ to cause CPV. At one loop in non-minimal SUSY models, there can be an effective phase $\xi$ introduced into the gaugino mass matrix of order $\xi \sim 10^{-3}\theta_{\delta\lambda}$. Although this is a large suppression, $\theta_{\delta\lambda}$ can be of order unity and so $\xi$ should generate the same level of CPV as the phase used in [21], which was bounded by $d_n$ anyway.

From the standpoint of explaining the baryon asymmetry or a non-zero $d_n$ or $d_e$, a loop induced observable phase provides an attractive alternative to the fine-tuning needed in the MSSM. If $d_n$ or $d_e$ is observed in the near future, or the baryon asymmetry is generated at the weak scale, it will either force believers in SUSY toward non-minimal models, or require a way of somehow naturally generating phases of order $10^{-3}$.

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