Generation of photon-number entangled soliton pairs through interactions

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(Dated: April 1, 2022)

Two new simple schemes for generating macroscopic (many-photon) continuous-variable entangled states by means of continuous interactions (rather than collisions) between solitons in optical fibers are proposed. First, quantum fluctuations around two time-separated single-component temporal solitons are considered. Almost perfect correlation between the photon-number fluctuations can be achieved after propagating a certain distance, with a suitable initial separation between the solitons. The photon-number correlation can also be achieved in a pair of vectorial solitons with two polarization components. In the latter case, the photon-number-entangled pulses can be easily separated by a polarization beam splitter. These results offer novel possibilities to produce entangled sources for quantum communication and computation.

PACS numbers: 03.67.Mn, 03.67.-a, 05.45.Yv, 42.65.Tg

Introduction Quantum-noise squeezing and correlations are two key quantum properties that can exhibit completely different characteristics when compared to the predictions of the classical theory. Almost all the proposed applications to quantum measurements and quantum information treatment utilize either one or both of these properties. In particular, solitons in optical fibers have been known to serve as a platform for demonstrating macroscopic quantum properties in optical fields, such as quadrature squeezing, amplitude squeezing, and both intra-pulse and inter-pulse correlations. The development of quantum theories of nonlinear optical pulse propagation in the past years has opened a way to analyze the quantum features of fiber-optic solitons. Experimental progress in demonstrating various quantum properties of these solitons has also been reported, see Refs. [1]-[4] and references therein.

In the field of quantum information processing and quantum computing, nonlocally entangled optical quantum states have been shown to be highly useful sources. The applications include quantum cryptography [5], teleportation [6], and algorithms [7, 8]. Following the Bohm’s gedanken experiment proposed by Einstein, Podolsky, and Rosen (EPR) utilized the continuous variables (the coordinate and momentum of a particle) to argue that the quantum mechanics is incomplete [9]. In 1992, Ou et al. used nondegenerate parametric amplification to demonstrate the EPR paradox with continuous variables [10]. Later, Vaidman proposed a generalized method for the teleportation of continuous-variable quantum states [11]. Braunstein and Kimble analyzed the entanglement fidelity of quantum teleportation with continuous variables [12]. Quantum teleportation of optical coherent states was experimentally realized by using the entanglement from squeezed states [13]. After that, quantum-information processing with continuous variables has attracted a lot of interest as an alternative to single-photon schemes.

In previous works, continuous-variable entangled beams have been generated by letting two squeezed fields (squeezed vacuum states [14], or amplitude-squeezed fields [15]) interfere through a beam splitter, which mathematically acts as the Hadamard transformation. By utilizing the continuous EPR-like correlations of optical beams, one can also realize quantum-key distributions [2] and entanglement swapping [3]. Thanks to these successful applications, squeezed states become essential for generating entangled continuous-variable quantum states, and play an important role in the study of the quantum-information processing.

It has been demonstrated that two independent squeezed pulse states can be simultaneously generated by using optical solitons in the Sagnac fiber loop configuration [1]. An EPR pulse source can be obtained by combining the two output pulse squeezed states by means of a 50:50 beam splitter. In contrast to this known method for achieving the entanglement, in this work we propose simple schemes for generating continuous-variable entangled states through soliton-soliton interactions, in single-mode and bimodal (two-component) systems, without using beam splitters. The quantum interaction of two time-separated solitons in the same polarization is described by the quantum Nonlinear Schrödinger Equation (NLSE), and in the bimodal system, including two polarizations, it is described by a system of coupled NLSEs.

The photon-number correlation between the two solitons can be numerically calculated by using the back-propagation method [14]. In addition to the transient multimode correlations induced by cross-phase modulation [4], we also find nearly maximum photon-number entanglement in the soliton pair. By controlling the initial separation of the two solitons, one can achieve a positive quantum correlation with the correlation parameter close

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The single-mode system
Neglecting loss and higher-order effects, which are immaterial for the experimentally relevant range of the propagation distance $z$, temporal solitons in optical fibers are described by the NLSE in the normalized form,

$$iU_z + \frac{1}{2}U_{tt} + |U|^2U = 0,$$

where $t$ is the retarded time. The input profile of the soliton pair is taken as

$$U(z,t) = \text{sech}(z, t + \rho) + \gamma \text{sech}(z, t - \rho)e^{i\theta},$$

with $\gamma$, $\theta$, and $2\rho$ being, respectively, the relative amplitude, phase and separation of the solitons. If $\theta = 0$ (the in-phase pair), the two solitons will collide periodically in the course of the propagation. Otherwise, they attract each other within a short propagating distance, and then move apart due to repulsion between them.

One can evaluate the multimode quantum fluctuations around the solitons by solving the linearized quantum NLSE. For the case of two-soliton collisions, König et al. used an exact classical solution for the in-phase soliton pair, and had found that the colliding solitons carry both intra-pulse and inter-pulse photon-number correlations. However, the photon-number correlation between the colliding solitons is transient, i.e., the inter-pulse correlation vanishes after the collision. Unlike the case of the collision, the photon-number correlations caused by the continuous interaction between the solitons belonging to the pair initiated by the configuration persist with the propagation.

In Fig. 1, we display the result of evaluation of the time-domain photon-number correlations for the out-of-phase ($\theta = \pi/2$) two-soliton pair. The correlation coefficients, which are defined through the normally ordered covariance,

$$C_{ij} \equiv \frac{\langle \Delta \hat{n}_i \Delta \hat{n}_j \rangle}{\Delta \hat{n}_i^2 \Delta \hat{n}_j^2},$$

were calculated by means of the above-mentioned back-propagation method. In Eq. 2, $\Delta \hat{n}_j$ is the photon-number fluctuation in the $i$-th slot $\Delta t_i$ in the time domain,

$$\Delta \hat{n}_i = \int_{\Delta t_i} dt [\hat{U}(z,t)\Delta \hat{U}^\dagger(z,t) + U^*(z,t)\Delta \hat{U}(z,t)],$$

where $\Delta \hat{U}(z,t)$ is the perturbation of the quantum-field operator, $U(z,t)$ is the classical unperturbed solution, and the integral is taken over the given time slot. As could be intuitively expected, nonzero correlation coefficients are found solely in the diagonal region of the spectra (intra-pulse correlations) if the interaction distance is short, as shown in Fig. 1(a). As the interaction distance increases, inter-pulse correlations between the two solitons emerge and grow, as shown in Figs. 1(b) and (c).

FIG. 1: The pattern of time-domain photon-number correlations, $C_{ij}$, of two interacting out-of-phase solitons, with $\theta = \pi/2$, $\rho = 3.5$, and $\gamma = 1.0$ in Eq. 1. The propagation distance is $z = 6$ (a), 30 (b), and 50 (c), in the normalized units. The width of the time slots is $\Delta t = 0.1$. Note the difference in the bar-code scales in the panels (a) and (b), (c).
of the relative phase (θ) of a pair (1) with the initial relative phase θ
\( \rho \). At the initial stage of the interaction, the photon-
number fluctuations are uncorrelated between the solitons, \( C_{12} \approx 0 \). After passing a certain distance, the photon-
number correlations between the two solitons gradually increase, and the pair may become a nearly maximum-positive-correlated one. The propagation distance needed to achieve the maximum positive photon-
number correlation depends on the initial separation of the two solitons; obviously, the interaction between them is stronger when the initial separation is smaller.

On the other hand, one can fix the initial separation but vary the initial relative phase. For this case, the results are shown in Fig. 2(b). Similar to the case of the soliton-soliton collision [4], the photon-number correlation coefficient oscillates with the period equal to that of the two-soliton breather, if the solitons are, initially, in-phase. Note that in the case which may be regarded as intermediate between the in-phase and out-of-phase ones, \( \theta = \pi/4 \), the correlation coefficient first becomes negative, and then positive.

Unless \( \theta = 0 \) (when the two solitons form a quasi-bound state in the form of a breather), the two solitons belonging to the initial configuration (1) will separate as a result of the propagation. Therefore, the interaction between them eventually vanishes, and thus the photon-
number correlation coefficient may saturate before it has a chance to reach the value corresponding to the total positive correlation, which is clearly seen in the inset to Fig. 2(a).

**The bimodal system** The time-division entangled soli-
ton pair considered above can be separated with an optical switch. Since the time separation between the two solitons is, typically, on the order of a few picoseconds, a lossless ultrafast optical switch will be required for the actual implementation of the scheme. The experimental difficulties can be greatly reduced if another scheme is used, which utilizes vectorial solitons in two polarizations. The model is based on the well-known system of coupled NLSEs [17],

\[
\frac{\partial U}{\partial z} + \frac{1}{2} \frac{\partial^2 U}{\partial t^2} + A|U|^2U + B|V|^2U = 0, \quad (3)
\]

\[
\frac{\partial V}{\partial z} + \frac{1}{2} \frac{\partial^2 V}{\partial t^2} + A|V|^2V + B|U|^2V = 0. \quad (4)
\]

Here \( U \) and \( V \) are the fields in orthogonal circular polar-
izations, \( A \) and \( B \) being the self-phase- and cross-phase-
modulation coefficients, respectively, with the relation \( A : B \) as 1 : 2 in the ordinary optical fibers [17]. We take the following initial configuration for the soliton pair [cf. Eq. (1)],

\[
U = \text{sech}(t+t_1) + \text{sech}(t-t_1), \quad V = \text{sech}(t+t_1) - \text{sech}(t-t_1).
\]

Using the methods for the analysis of the classical vectorial solitons developed in Refs. [18, 20, 21], we calculated the respective quantum fluctuations and the photon-number correlators numerically. It should be noted the total intensity of the vectorial solitons, defined

\[
\rho, \quad \frac{\partial}{\partial \rho} \frac{\partial}{\partial \rho} + A|U|^2 &= 0, \quad (5)
\]

\[
\frac{\partial}{\partial \gamma} \frac{\partial}{\partial \gamma} + B|V|^2 &= 0. \quad (6)
\]

Here \( \Delta N_{1,2} \) are the perturbations of the photon-number operator of the two soliton, the solitons being numbered (first and second) according to their position in the time domain.

In Fig. 2(a), we show the coefficient \( C_{12} \) for the soliton pair (1) with the initial relative phase \( \theta = \pi/2 \), equal amplitudes (\( \gamma = 1.0 \)), and different values of the separation distance needed to achieve the maximum positive photon-
number correlation in (b). The inset in (a) shows the evolution of the interaction solitons by means of contour plots.

FIG. 2: The photon-number correlation parameter \( C_{12} \) for the soliton pair with different values of the separation (\( \rho = 3.0, 3.5, 4.0 \), while \( \theta = \pi/2 \) and \( \gamma = 1.0 \)) in (a), and different values of the relative phase (\( \theta = 0, \pi/4, \pi/2 \), while \( \rho = 3.5, \gamma = 1.0 \)) in (b). The inset in (a) shows the evolution of the interaction solitons by means of contour plots.
FIG. 3: The photon-number correlation coefficient of interacting vectorial solitons. The inset displays the evolution of the $x$-component of the classical field.

in terms of the circular polarizations, remain unchanged during the propagation, but the intensities of the linearly-polarized ($x$- and $y$-) components, $E_x = (U + V)/\sqrt{2}$ and $E_y = (U - V)/i\sqrt{2}$, evolve periodically, as shown in the insert of Fig. 3. In this figure, we display the evolution of the photon-number correlation between the $x$- and $y$- components of the vectorial solitons, which are originally uncorrelated, and then become negatively correlated. Recently, Lantz et al. have showed that vectorial solitons in the spatial domain can also develop an almost perfect negative correlation between quantum fluctuations around an incoherently-coupled soliton pair [22].

**Conclusion** We have studied the quantum photon-number correlations induced by interactions between two solitons in the time-division and polarization-division pairs. In the former case, using the pair with suitable initial separation and relative phase, one can generate positive or negative photon-number-correlated soliton pairs. An ultrafast optical switch will be needed to separate the two entangled solitons into different channels. On the other hand, by using the vectorial solitons with two polarization components, pairs with negative photon-number correlations between the solitons can be generated. For this case, a simple polarization beam splitter will be sufficient to separate the two entangled solitons into different channels.

Such new photon-number-correlated soliton pairs feature unique entanglement properties, which may offer new possibilities for applications to quantum communications and computation. The applications will be considered in detail elsewhere.

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