The study of special positions of a crawling robot when changing its configuration

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Abstract. The paper considers the movement of a crawling robot, consisting of three modules, along a horizontal rough surface when changing its configuration. The fundamental characteristic of the research object is its equipping with support elements with a controlled coefficient of friction, due to which the movement along the plane can occur with the periodic fixation of two supports on the surface, which makes the device more maneuverable and high-speed. Another feature of the proposed construction is the controlled length of each module due to the prismatic pairs. This allows the robot to adapt to space limitations, such as corridors, obstacles, etc. The paper proposes a system for controlling the lengths of the robot modules, as well as the angles of their rotation when moving while fixing on the surface the two extreme supports of the side links. This system is sequential, first the central module is controlled, then one of the side modules, and only after that the second side module. Transitions from one control stage to another are carried out when the module length reaches its maximum permissible value. The paper analyzes the operation of the control system, and also examines in detail the issue of the robot transition to special positions.

1. Introduction
Among the various areas of the crawling robots use there can be singled out their use for reconnaissance and search operations after emergencies [1-4]. To do this, they must 1) be maneuverable and highly passable when moving on a surface with obstacles, 2) have a high speed of movement and 3) have the ability to move in limited spaces. For the crawling robot under study, the first two points are discussed in [5-8]. So, in [5, 6], to increase the speed of the device movement, the original design of the support elements and their control are proposed, due to which these supports can be periodically fixed on the surface. The system for detecting obstacles on the surface and determining the method of overcoming them is described in [7, 8]. Therefore, in order to provide the robot with the ability to move in limited spaces, for example, when avoiding obstacles, in this paper a control system for the lengths of its modules is proposed, and the problem of achieving special positions by the robot at which the object stops is considered in detail.

2. Description of a three-link crawling robot
The object of the study is a crawling robot, the design scheme of which in the horizontal plane of Oxyz is shown in Figure 1. The device consists of three modules i=1-3, each of which is formed by two
links of the prismatic pair, the lengths of the modules are \( l_i \in [l_{\text{min}}, l_{\text{max}}] \), where \( l_{\text{min}}, l_{\text{max}} \) — minimum and maximum module lengths, defined as follows:

\[
    l_{\text{max}} = l_i^0 + \Delta l_{\text{im}}, \quad l_{\text{min}} = l_i^0 - \Delta l_{\text{im}},
\]

where \( l_i^0 \) — initial value of the module length, \( \Delta l_{\text{im}} \) — the maximum permissible value of the extension / shortening of the module due to a change in the length of the prismatic pair. The modules are connected to each other by rotary motion actuators installed at points \( O_2 \) and \( O_3 \) and providing their turns at absolute angles \( \varphi_i \). At the extreme points of the modules \( O_1-O_4 \) there are support elements with a controlled coefficient of friction [5, 6], and the height of the supports on the central link 2 is greater than on the side links: \( h_2 > h_{1,3} \).

![Figure 1](image1.jpg)

**Figure 1.** The scheme of the three-link crawling robot.

The motion of the robot on the plane \( Oxy \) is described by the vector of generalized coordinates

\[
    \mathbf{q} = (x_{O1}, y_{O1}, l_1, l_2, l_3, \varphi_1, \varphi_2, \varphi_3)^T,
\]

where \( x_{O1}, y_{O1} \) — coordinates of the point \( O_1 \).

### 3. Control system for module lengths and rotation angles

Let us consider the movement of the robot during a change in its configuration with two supports \( O_1 \) and \( O_4 \) fixed on the surface: \( x_{O_{1n}} = x_{O_{1k}} = \text{const} \), \( y_{O_{1n}} = y_{O_{1k}} = \text{const} \), \( x_{O_{4n}} = x_{O_{4k}} = \text{const} \), \( y_{O_{4n}} = y_{O_{4k}} = \text{const} \), where the index \( n \) corresponds to the moment of the movement beginning (Figure 2), and the index \( k \) — to the moment of the movement completion (Figure 3). The supports fixed on the surface are shown in black. At the beginning of the movement the links 2 and 3 are located in one line \( \varphi_{2n} = \varphi_{3n} \), and the links 1 and 2 — at such relative angle \( \varphi_{21n} = \varphi_{2n} - \varphi_{1n} \), that \( \varphi^* = \pi - \varphi_{21n}, 10^0 \leq \varphi^* \leq 170^0 \). At the moment of the movement completion the links 1 and 2 are located in one line \( \varphi_{1k} = \varphi_{2k} \), and the links 2 and 3 — at such relative angle \( \varphi_{32k} = \varphi_{3k} - \varphi_{2k} \), that \( \varphi^* = \pi + \varphi_{32k} \).

![Figure 2](image2.jpg)

**Figure 2.** The initial position of the robot.

![Figure 3](image3.jpg)

**Figure 3.** The final position of the robot.
For the implementation of the considered motion, it is necessary to control the lengths of the robot modules and their rotation angles, for this purpose a sequential control system is proposed, shown in Figure 4. In this scheme, the abbreviation K.P. is used to indicate the end position, and the length of the robot module, enclosed in square brackets, corresponds to the permissible module length, lying in the range from minimum to maximum values calculated by formulas (1).

![Figure 4. Three-stage control system for link lengths and angles of rotation.](image)

At the first stage of control, the lengths of the side modules are considered equal to the initial ones $l_{i1,3} = l_{0,i1,3}$, the length of the central module and its rotation angle vary in accordance with the formulas

$$l_i = \sqrt{(x_{Oi1} - x_{Oi})^2 + (y_{Oi1} - y_{Oi})^2},$$

$$\varphi_i = \arctg\left(\frac{y_{Oi1} - y_{Oi}}{x_{Oi1} - x_{Oi}}\right)$$

provided that the side modules rotate at angular speeds $\dot{\varphi}_i = -\dot{\varphi}_3$.

When the length of the central module reaches its maximum permissible value $l_2 = l_{2,m}$, where $l_{2,m} = (l_{2,max} \lor l_{2,min})$ until the movement is completed (before reaching the end position) the second stage of control begins, at which the length of the side module 1 and its angle of rotation change according to the formulas (3) and (4), the length of the module 3 remains equal to the initial value $l_3 = l_{3,0}$, and its rotation occurs at the same angular velocity. The angle of rotation of the central module is determined by the formula (4), in which the coordinates of the points $O_2$ and $O_3$ are calculated for the initial lengths of the side modules $l_{i1,3} = l_{0,i1,3}$.

When the module 1 reaches its maximum length $l_1 = l_{1,m}$ the third stage of control begins, during which the length of the module 3 and the angle of rotation are calculated by the formulas (3) and (4). The angles of rotation of the modules 1 and 2 are also determined by the formula (4), only for the module 2, when calculating the coordinates of the points, the initial link lengths are used, and for the module 1 – the initial length of the module 3 and the maximum permissible lengths of the modules 1 and 2.

The description of the three stages the robot links control is presented more clearly in Table 1, the number of the formula, according to which a particular value is calculated, is indicated in parentheses. In addition, Table 2 shows the formulas by which the coordinates of the points $O_2$ and $O_3$ used in the formula (3) are determined.

If the lengths of all modules become equal to the maximum permissible values $l_{i1,3} = l_{i1,3,m}$ until the end of the movement, the robot moves to a special position at which there will be a stop of the device with the loss of all degrees of mobility. With one of them this position is undesirable, because the
object does not reach the specified final position. On the other hand, this option of the orientation change from the initial position to the special one has a right to exist, because for the further implementation of the robot movement the special position becomes the initial for the next stage of movement.

### Table 1. Stages of the lengths and angles control of the robot modules.

| Stage of the control | Lengths of the modules | Rotation angles of the modules |
|----------------------|------------------------|-------------------------------|
| 1                    | $l_1^0$ (3) $l_3^0$    | $\int \dot{\phi}_1 dt$ \(\text{(4)}\) $\int \dot{\phi}_3 dt$ |
| 2                    | (3) $l_{2n}$ $l_3^0$   | (4) at $l_1^0$ and $l_3^0$ $\int \dot{\phi}_3 dt$ |
| 3                    | $l_{1n}$ $l_{2n}$ (3)  | (4) at $l_{2n}$ and $l_3^0$ (4) at $l_1^0$ and $l_3^0$ (4) |

### Table 2. Formulas for calculating the coordinates of the points $O_2$ and $O_3$.

| Stage of the control | The coordinates of the point $O_2$ | The coordinates of the point $O_3$ |
|----------------------|-----------------------------------|-----------------------------------|
| 1                    | for $l_2$ and $\varphi_2$         | for $l_2$ and $\varphi_2$         |
|                      | $x_{O2} = x_{O1n} + l_1^0 \cos \varphi_1$, | $x_{O3} = x_{O4n} - l_1^0 \cos \varphi_3$, |
|                      | $y_{O2} = y_{O1n} + l_1^0 \sin \varphi_1$ | $y_{O3} = y_{O4n} - l_1^0 \sin \varphi_3$ |
| 2                    | for $l_1$ and $\varphi_1$         | for $\varphi_2$, $l_1$ and $\varphi_1$ |
|                      | $x_{O2} = x_{O4n} - l_1^0 \cos \varphi_3 - l_{2n} \cos \varphi_2$, | $x_{O3} = x_{O4n} - l_1^0 \cos \varphi_3$, |
|                      | $y_{O2} = y_{O4n} - l_1^0 \sin \varphi_3 - l_{2n} \sin \varphi_2$ | $y_{O3} = y_{O4n} - l_1^0 \sin \varphi_3$ |
| 3                    | for $\varphi_3$                   | for $\varphi_2$, $l_1$ and $\varphi_3$ |
|                      | $x_{O2} = x_{O1n} + l_1^0 \cos \varphi_1$, | $x_{O3} = x_{O4n} - l_1^0 \cos \varphi_3$, |
|                      | $y_{O2} = y_{O1n} + l_1^0 \sin \varphi_1$ | $y_{O3} = y_{O4n} - l_1^0 \sin \varphi_3$ |
|                      | for $\varphi_1$, $l_3$ and $\varphi_3$ | for $\varphi_1$, $l_3$ and $\varphi_3$ |
|                      | $x_{O2} = x_{O4n} + l_{3n} \cos \varphi_1$, | $x_{O3} = x_{O1n} + l_{3n} \cos \varphi_1 + l_{2n} \cos \varphi_2$, |
|                      | $y_{O2} = y_{O4n} + l_{3n} \sin \varphi_1$ | $y_{O3} = y_{O1n} + l_{3n} \sin \varphi_1 + l_{2n} \sin \varphi_2$ |

### 4. Special positions study

Let us conduct a numerical simulation of the robot movement during a change in its configuration, provided that the lengths of the modules in dimensionless quantities are $l_{1,3}=1$, the angles corresponding to the initial and final positions of the object are $\varphi_{2n}=\varphi_{3n}=0^\circ$, $\varphi_{2k}=\varphi_{3k}=0^\circ$, the angle $\varphi^\circ \in[10^\circ, 170^\circ]$, for the convenience of the analysis all the angles will be measured in degrees. We will also assume that the values of the maximum permissible elongation / shortening of all modules are equal to each other: $\Delta l_m = \Delta l_{1-3n}$. The purpose of the simulation is to define ranges of the values $\Delta l_m$ and $\varphi^*$, at which one or another stage of adjustment is required to reach the final position, or there is a transition to a special position. Also, the issue of determination the dependences of the links rotation angles at which the special position of the device is observed will be considered.

#### 4.1. Diagram of the control stages
Figure 5 shows the diagram $\Delta l_m(\phi^*)$, which highlights the areas 1-3 of the lengths control stages of the robot modules and their rotation angles, as well as the area 4, where the object before the completion of the movement goes into special positions. According to this diagram, for large values of $\Delta l_m$, it is sufficient to control the central module to change the configuration of the robot; as $\Delta l_m$ decreases, it will be necessary to control one side module, and then the second. For small values of $\Delta l_m$ in the angle range $10^0 \leq \phi^* \leq 160^0$, the device will move to special positions, and for $10^0 \leq \phi^* < 60^0$, the boundary between regions 3 and 4 is an oblique straight line that decreases with increasing $\phi^*$, with $\phi^*=60^0$, its minimum is observed, then increasing, reaching a maximum at $\phi^*=100^0$ and decreasing along a curve close to a parabola with an upward convexity.

Figure 5. Diagram of the control stages.

4.2. Building time dependencies

Figure 6 (a) shows the time dependences of changing the lengths of the robot modules until the devise reaches a special position, and Figure 6 (b) – similar dependences of the rotation angles, each graph also shows the regions of 1-3 control stages and the moment of transition to the region 4.

According to Figure 6 (a) it can be seen that at each control stage the length of one module changes, while the lengths of the other two modules remain constant. By the time of reaching a special position, the lengths of all modules have maximum permissible values $l_{2kr}=l_{2max}, l_{1kr}=l_{1min}, l_{3kr}=l_{3min}$. The rotation angles vary according to curvilinear laws, the moments of transitions from one control stage to another on the are not particularly reflected on the graphs $\phi_i(t)$ (Figure 6 (b)).

It should be noted, that the graphs shown in Figure 6 (a) are one of the options for elongation / shortening the modules. In the general case, the nature of the change in the lengths of the robot modules before it reaches a special position varies depending on the angle $\phi^*$ (Figure 7 (a)), which
determines the relative position of the modules relative to each other, and on the value \( \Delta l_m \) (Figure 7 (b)). The nature of the change in the time dependences of the rotation angles of the modules does not depend on the indicated values.

![Graphs](image)

**Figure 7.** Dependence graphs: \( l_i(t) \): (a) \( - \Delta l_m=0.04 \) and \( \phi^*=80^\circ \), (b) \( - \Delta l_m=0.02 \) and \( \phi^*=80^\circ \).

The graphs Figure 6 (a) and Figure 7 (a) are given for the same value \( \Delta l_m \) and two different angles \( \phi^* \). In this case, there is a different sequence of control stages before moving to a special position, as well as the maximum permissible lengths of modules achieved to this position: \( l_{2kr}=l_{2max}, l_{1kr}=l_{1min}, l_{3kr}=l_{3min} \) at \( \phi^*=45^\circ \) and \( l_{ikr}=l_{imin, i=1-3} \), at \( \phi^*=80^\circ \). The graphs Figure 7 (a) and Figure 7 (b) are built at the same angle \( \phi^* \), but with two different values \( \Delta l_m \). It can be noted that at a lower value of \( \Delta l_m \) (Figure 7 (b)) when the first transition to the module control stage 3 occurs, the robot enters a special position, because the lengths of all modules reach the limit values: \( l_{2kr}=l_{2min}, l_{1kr}=l_{1max}, l_{3kr}=l_{3max} \).

### 4.3. Determination of critical rotation angles of the links

In the study of the special positions of the mechanism, the most interesting are the dependences of the rotation angles \( \phi_{ikr} \) of the links, at which a transition to special positions is observed, on the module’s maximum permissible elongation / shortening and on the angle \( \phi^* \), that determines the orientation of the links relative to each other. Figure 8 shows the corresponding graphs built for one part of the region 4, separated from the region 3 by the oblique straight line (with small angles \( \phi^* \) and a large range of \( \Delta l_m \) values.

![Graphs](image)

**Figure 8.** Dependence graphs: \( \phi_{ikr}(\Delta l_m) \): (a) \( - i=1 \), (b) \( - i=2 \), (c) \( - i=3 \), 1 \(- \phi^*=10^\circ \), 2 \(- \phi^*=15^\circ \), 3 \(- \phi^*=20^\circ \), 4 \(- \phi^*=25^\circ \), 5 \(- \phi^*=30^\circ \), 6 \(- \phi^*=35^\circ \), 7 \(- \phi^*=40^\circ \), 8 \(- \phi^*=45^\circ \).

The rotation angle of the link 1 increases according to a curvilinear law with increasing \( \Delta l_m \), and at \( \phi^* \in [10^\circ, 25^\circ] \) two sections of the curve, connecting with a kink, are observed (Figure 8 (a)). At the
range $\varphi^* \in [30^0, 45^0]$ the first section of the curve ceases to exist; in the second section the nature of the graphs is the same as for the angles $\varphi^* \in [10^0, 25^0]$. The numerical values of the angle $\varphi_{kr}$ at the same values of $\Delta lm$ increase with growing $\varphi^*$. According to the graphs Figure 8 (b) it can be seen that at small angles $\varphi^* \in [10^0, 25^0]$ the dependence $\varphi_{kr}$ on the maximum elongation / shortening consists of two sections, each of which is an oblique straight line, the angle of which increases with growing $\varphi^*$. The first section corresponds to small values $\Delta lm$, the second – to large ones, and the value $\Delta lm$, at which a kink occurs decreases with increasing $\varphi^*$. At large values $\varphi^* \in [30^0, 45^0]$, the dependence $\varphi_{kr}(\Delta lm)$ has a curvilinear character. As $\Delta lm$ and $\varphi^*$ increase, the value of the critical angle $\varphi_{kr}$ increases.

The critical angle of the link 3, unlike the angles of the other two links, decreases along the curvilinear law with increasing $\Delta lm$, and in the case of $\varphi^* \in [10^0, 25^0]$ the curve consists of two sections, on the first of them its convexity is directed down, on the second – up, as $\varphi^*$ increases, the convexity of the first section becomes more smoothed (Figure 8 (c)). For the angles range $\varphi^* \in [30^0, 45^0]$ only the second section of the curve with an upward bulge is observed.

The kinks $\varphi_{ikr}(\Delta lm)$ observed in the graphs, and the possibility of distinguishing a different number of sections in these graphs, arise from the differences in the conditions for the robot to move to special positions at different $\Delta lm$ and $\varphi^*$ values, that was shown earlier. For a more visual representation of the special positions of the robot Figure 9 shows the pictograms at three values of the angle $\varphi^*$, which demonstrate the relative angles $\varphi_{21}^*$ and $\varphi_{32}^*$ of the modules “opening”, which are sharp.

![Figure 9](image)

**Figure 9.** Pictograms of the robot positions at $\Delta lm=0.06$: (a) – $\varphi^*=10^0$, (b) – $\varphi^*=20^0$, (c) – $\varphi^*=30^0$.

According to the pictogram it can be seen, that at lower value of $\varphi^*=10^0$ the central module rotates in such a way, that goes “above” the link 1, which is permissible in the proposed construction of the robot (the central link is located above the side links). This is not observed while the angle $\varphi^*$ increases, but the opening angle $\varphi_{21}^*$ grows. The mutual arrangement of the modules 2 and 3 always has the same configuration regardless of $\varphi^*$, but with its growth, the opening angle $\varphi_{32}^*$ decreases.

### 5. Conclusion

The paper investigates the movement of a crawling robot on a horizontal surface during the change of its configuration when fixing on the plane of the two extreme supports of the side links. This movement is realized thanks to the work of the control system of module lengths and angles of rotation, proposed in this article. The algorithm placed to the control system provides sequential control of module lengths until they reach their limit values.

As a result of the simulation the diagram of values of elongation / shortening of the modules from the angle of their mutual arrangement at the initial and final moments of motion is constructed, on which three areas corresponding to the control stages are highlighted, as well as the area where the robot...
moves to special positions at which the lengths of all modules are equal to the limit values, and the object stops. This diagram can be used in the design of robot modules and the selection of ranges of acceptable values for their elongation / shortening, taking into account the alleged movements of the device. If the achievement of special positions is undesirable, then the values $\Delta_{lm}$ and $\varphi^*$ must be chosen so that they get to the area 1-3 of control. If the achievement of a special position is considered as an acceptable variant of movement, then the choice of the values $\Delta_{lm}$ and $\varphi^*$ in region 4 is possible. In addition, the paper analyzes the time dependences of module lengths and their rotation angles before reaching special positions, as well as the dependence of the rotation angles of the links, at which critical positions are observed, on the values of elongation / shortening of the modules and the angle of their mutual arrangement.

References

[1] Wright C et al 2007 Design of a modular snake robot *Proc. IEEE/RSJ Intern. Conf. on Intelligent Robots and Systems (San Diego, USA)* pp 2609-14
[2] Transeth A A et al 2008 Snake robot obstacle-aided locomotion: Modeling, simulations, and experiments *IEEE Transactions on Robotics* **24** pp 88-104
[3] Prautsch P et al 2000 Analysis and control of a gait of snake robot *IEEE Transactions on Industry Applications* **120** pp 372-81
[4] Paap K L et al 2000 A robot snake to inspect broken buildings *Proc. IEEE/RSJ Intern. Conf. on Intelligent Robots and Systems (Takamatsu, Japan)* vol 3 pp 2079-2082
[5] Vorochaeva L Yu et al 2015 Simulation of Motion of a Three-Link Robot with Controlled Friction Forces on a Horizontal Rough Surface *J. of Computer and Systems Sciences International* **54** pp 151–64
[6] Vorochaeva L Yu et al 2017 Simulation of the motion of a five-link crawling robot with controlled friction on a surface having obstacles *J. of Computer and Systems Sciences International* **56** pp 527–52
[7] Vorochaeva L Yu et al 2019 Development of a system to determine the mode of contact between link crawling robot with an obstacle *J. Vestnik BSTU* **8** pp 11-21
[8] Vorochaeva L et al 2019 Development of the motion correction system of the crawling robot link on the surface with obstacles *Proc. 9th Int. Conf. on Physics and Control: Physcon (Innopolis, Russia)* pp 300-5