The authors consider planar Hamiltonian differential systems of the form
\[ \dot{x} = -H_y(x, y), \quad \dot{y} = H_x(x, y) \]
with a non-degenerated center at the origin. Without loss of generality the center is associated to \( h = 0 \), where \( \{H(x, y) = h\} \). The period function associated to the period annulus is denoted by \( T(h) \) and, under the assumptions of the paper, it is an analytic function of the energy \( h \) in a neighborhood of \( h = 0 \).

The area function which associates to \( h \) the area enclosed by each periodic orbit of the energy level \( h \) is denoted by \( A(h) \). By previous results it is known that \( A'(h) = T(h) \).

The first main result of the paper provides a systematic procedure to compute the Taylor expansion of \( T(h) \). It is based on transforming the system to polar coordinates. Quoting the authors, this procedure is new, totally different of the known methods and applicable to all integrable planar systems with a non-degenerated center. It is tedious, but so systematic that can be used with any computer algebra system. The authors apply this method to several examples in order to illustrate it.

In a second part of the paper, the authors consider a particular model of capillarity and they use this method to study the period function and the number of limit cycles that can bifurcate from the period annulus. After a change of variables, the authors study the system:
\[ \dot{x} = -y + \varepsilon P(x), \quad \dot{y} = x + x^2 + \varepsilon Q(x), \]
defined on \( x + 1 > 0, \varepsilon \) is a small parameter and \( P(x) \) and \( Q(x) \) are polynomials. Note that the system with \( \varepsilon = 0 \) is a Hamiltonian system with Hamiltonian
\[ H(x, y) = \frac{x^2}{2} + \frac{x^3}{3} + \frac{y^2}{2}. \]
The authors show that the bifurcation function that controls the appearance of limit cycles at first order of \( \varepsilon \) can be written in terms of the period function \( T(h) \) and the area function \( A(h) \).

Reviewer: Maite Grau (Lleida)

MSC:
- 34C08 Ordinary differential equations and connections with real algebraic geometry (fewnomials, desingularization, zeros of abelian integrals, etc.)
- 34C25 Periodic solutions to ordinary differential equations
- 37J46 Periodic, homoclinic and heteroclinic orbits of finite-dimensional Hamiltonian systems

Keywords:
period function; limit cycles; abelian integrals; extended complete Chebyshev systems; Picard–Fuchs differential equations

Software:
CASA

Full Text: DOI arXiv

References:
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