Quenched Hadron Spectroscopy with Improved Actions

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A variety of different combinations of improved gluon and fermion actions is tested for quenched hadron spectroscopy.

1. INTRODUCTION

The improvement of the lattice QCD action has become recently a main focus of research in lattice field theory. Improved actions for lattice QCD are designed to remove lattice-spacing artifacts in numerical estimates of physical observables. This allows to conduct the numerical calculations on much coarser and smaller lattices and thereby to substantially reduce the simulation costs. In this contribution a variety of different combinations of gauge and fermion actions is tested for quenched hadron spectroscopy. For the gluon action we used the tadpole improved L"uscher-Weisz (T-LW) \cite{1–3} and the Iwasaki (IW) action \cite{4}. The LW action has been obtained by extending Symanzik’s perturbative improvement program to lattice gauge theories, demanding $\mathcal{O}(a^2)$ improvement of on-mass-shell quantities \cite{1}. The use of tad-pole improvement is essential to enhance the convergence of the perturbative expansion and extend perturbation theory to larger distances \cite{2}. The IW action on the contrary is a renormalization group improved action. Scaling tests carried out recently for both models have been very encouraging \cite{3,5,6}. The fermion actions which we shall consider in this contribution are the Sheikholeslami-Wohlert (SW) action \cite{7}, the D234 action \cite{8} and for comparison also the standard Wilson action (W). Symanzik improvement of the Wilson action leads to the SW action \cite{7}. The D234 action one attempts to go even one step beyond the $\mathcal{O}(a)$ improvement and to eliminate partially some of the cut-off effects to $\mathcal{O}(a^2)$. Also in the case of the improved fermion actions it is essential to couple the fermions to a "tadpole improved" (T) lattice link variable \cite{9}. We computed the hadron spectrum for the following five combinations of improved gluon and fermion actions: T-LW&W, T-LW&T-SW, T-LW&T-D234, IW&T-SW and IW&T-D234.

2. DETAILS OF THE SIMULATION

In order to be able to compare the results obtained with the different gluon actions we have conducted the simulations for the T-LW and IW actions at the critical $\beta$ of the deconfining phase transition for $N_f = 2$ \cite{6}. We performed the simulations on a $6^3\times16$ lattice using a standard Hybrid Monte Carlo algorithm. For the determination of the hadron spectrum we generated about 500 configurations, which were separated by 50 trajectories. For the inversion of the fermion matrix we used the BICG$\gamma_5$ algorithm which differs from the CG algorithm by the insertion of a $\gamma_5$ in the inner products. We find that the BICG$\gamma_5$ is, within the interval $m_\pi = 0.7-1.0$, by a factor 3-4 times faster than the BICGstab and by a factor 6-8 faster than the CG algorithm. The BICG$\gamma_5$ algorithm is not guaranteed to converge as a division by zero can occur during the iteration process. We have however never encountered a breakdown of the algorithm although we performed in total about $2.5 \times 10^5$ inversions of the fermion matrix.

For the determination of the hadron spectrum we implemented the standard correlation functions for the $\pi$, $\rho$, $a_0$, $a_1$ and $b_1$ mesons, the nucleon (N) and the $\Delta$, using local operators for the quark field, both at the source and the sink. The meson and baryon propagators have been fitted
Figure 1. The mass ratios $m_N/m_\rho$ (a) and $m_\Delta/m_\rho$ (b) as a function of $m_\pi/m_\rho$. The $T$-LW&W, $T$-LW&T-SW, T-LW&T-D234, IW&T-SW and IW&T-D234 data are marked by the filled triangles, open circles, crosses, open triangles and squares. The horizontal error bars are dropped as they are smaller than the symbol size.

Figure 2. The masses $m_\rho$, $m_N$ and $m_\Delta$ as a function of $m_\pi^2$ for the IW&D234 action.

3. RESULTS

In fig. 1 we have plotted the mass ratios $m_N/m_\rho$ (a) and $m_\Delta/m_\rho$ (b) as a function of $m_\pi/m_\rho$. The filled circles to the left correspond to the physical point and the filled squares to the right to the heavy quark limit, where $m_\pi/m_\rho = 1$ and $m_N/m_\rho = m_\Delta/m_\rho = 3/2$. The solid lines connecting the two points represent the result of a phenomenological quark model formula derived in ref. 8. The graphs show that the points which correspond to the combinations T-LW&T-SW (open circle), T-LW&T-D234 (crosses), IW&T-SW (open triangles) and IW&T-D234 (squares) cluster in a narrow strip which is located in both cases above the solid line. The large gap between the standard Wilson action data (filled triangles) and all other data reveals very clearly the dramatic effect of the fermion improvement. A naive linear extrapolation of the T-LW&T-SW, T-LW&T-D234 and IW&T-D234 data in fig. 1a down to the physical value of $m_\pi/m_\rho$ leads to a $m_N/m_\rho$ value which is not too far off from the ex-
Figure 3. Mass ratios for the various actions. The symbols have the same meaning as in fig. 1.

Experimental point (filled circle). The extrapolation of the IW&T-SW data on the other hand results in a value that is significantly larger indicating that the cut-off effects are bigger for this particular action. We checked that our data obtained with the improved fermion actions are, within error bars, consistent with the standard Wilson action results which have been obtained during the past years on a much finer and bigger lattices.

Using chiral perturbation theory we extrapolated the hadron masses $m_\rho$, $m_{a_0}$, $m_{a_1}$, $m_{b_1}$, $m_N$ and $m_\Delta$ to the chiral limit, $m_\pi \to 0$. To this extend we fitted the meson mass data $m_M$ with $m_M^{(0)} + c_M m_\pi^2$, where $m_M^{(0)}$ designates the mass in the chiral limit and $M = \rho, a_0, a_1, b_1$ in our case. For the nucleon it is essential to include also a term in the chiral fit which is cubic in the pion mass, $m_N^{(0)} + c_2 m_\pi^2 + c_3 m_\pi^3$. Our results show that the coefficient $c_3$ depends strongly on the particular action. It is largest for the three combinations T-LW&T-SW, T-LW&T-D234 and IW&T-D234 and roughly by a factor two smaller for the other two actions [10]. In the case of the $\Delta$ we fitted the mass data with $m_\Delta^{(0)} + c_2 m_\pi^2$, as a cubic fit leads to a $c_3$ value which is consistent with zero [10]. As an example, we plotted in fig. 2 the $m_\rho$, $m_N$ and $m_\Delta$ data for the case of the IW&T-D234 action as a function of $m_\pi^2$. The results of the fits are represented in this graph by the solid lines.

The ratios of the extrapolated masses, $m_N^{(0)}/m_\rho^{(0)}$ and $m_\Delta^{(0)}/m_\rho^{(0)}$ are displayed in fig. 3 along with $\sqrt{\sigma}/m_\rho^{(0)}$. The horizontal lines represent the experimental values. We find that the T-LW&T-SW (open circle), T-LW&T-D234 (crosses) and IW&T-D234 (squares) mass ratios are very close to the experimental values. The gap is slightly bigger for the IW&T-SW data (open triangles), and largest for the standard fermion action data (filled triangles). The results for the mass ratios $m_{a_0}^{(0)}/m_\rho^{(0)}$, $m_{a_1}^{(0)}/m_\rho^{(0)}$ and $m_{b_1}^{(0)}/m_\rho^{(0)}$ are summarized in table 1 which also shows that the mass ratios obtained with the improved fermion action are much closer to the experimental values than the ones obtained with the standard fermion action.

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Table 1

|       | IW&W | IW&T-D234 | experiment |
|-------|------|-----------|------------|
| $m_{a_0}^{(0)}/m_\rho^{(0)}$ | 2.37(15) | 1.81(8) | 1.600(52) |
| $m_{a_1}^{(0)}/m_\rho^{(0)}$ | 2.37(17) | 1.70(10) | 1.279(2) |
| $m_{b_1}^{(0)}/m_\rho^{(0)}$ | 2.28(14) | 1.90(10) | 1.601(13) |