We study Fermi transitions and isospin mixing in an isotopic chain ($^{70-78}$Kr) considering various approximations that use the same Skyrme-Hartree-Fock single particle basis. We study Coulomb effects as well as the effect of BCS and quasiparticle random phase approximation (QRPA) correlations. A measure of isospin mixing in the approximate ground state is defined by means of the expectation value of the isospin operator squared in $N=Z$ nuclei (which is generalized to $N \neq Z$ nuclei). Starting from strict Hartree-Fock approach without Coulomb, it is shown that the isospin breaking is negligible, on the order of a few per thousand for $(N-Z)=6$, increasing to a few percent with Coulomb. Pairing correlations induce rather large isospin mixing and Fermi transitions of the forbidden type ($\beta$ for $N \leq Z$, and $\beta^+$ for $N \geq Z$). The enhancement produced by BCS correlations is compensated to a large extent by QRPA correlations induced by isospin conserving residual interactions that tend to restore isospin symmetry.

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I. INTRODUCTION

It is well known [1] that the selfconsistent mean field hamiltonian breaks the symmetries of the exact hamiltonian. The best known example is the breaking of rotational invariance by the selfconsistent mean field of deformed nuclei. Rotational invariance breaking often leads to ground states of Hartree-Fock (HF) or Hartree-Fock-Bogoliubov (HFB) type with large expectation values of the squared angular momentum operator. For heavy well deformed even-even nuclei one typically has [2,3]:

$$\langle J^2_{\perp} \rangle \gtrsim 100$$

where $J_{\perp}$ is the angular momentum operator component perpendicular to the symmetry axis and $\langle \rangle$ means expectation value in the HF or HFB ground state. In these instances the HF or HFB wave function can be interpreted as a wave packet from which the ground state rotational band can be obtained by angular momentum projection. As a matter of fact, the angular momentum projection can be carried out through an expansion in powers of $1/\langle J^2_{\perp} \rangle$ [4] which, to lowest order, provides a factorization approximation formally identical to that of Bohr and Mottelson [5].

Unlike rotational invariance in ordinary space, rotational invariance in isospin space is not an exact symmetry of the actual total nuclear hamiltonian. The Coulomb force is non-isoscalar and thus, the actual nuclear states may have isospin mixing. Isospin mixing in the ground state may allow for Fermi (F) transitions in $N=Z$ nuclei, and it is a point of present debate because of its implications on parity violation experiments on $^{12}$C [6], as well as in the analysis of superallowed Fermi transitions as a test of the unitarity of the Cabbibo-Kobayashi-Maskawa matrix [7].

On the other hand, even with isoscalar nuclear hamiltonians, the selfconsistent mean field may break isospin invariance, and particularly so the selfconsistent quasiparticle mean field (HF+BCS or HFB) type with large expectation values of the squared angular momentum operator. For heavy well deformed even-even nuclei one typically has [2,3]:

$$\langle J^2_{\perp} \rangle \gtrsim 100$$

where $J_{\perp}$ is the angular momentum operator component perpendicular to the symmetry axis and $\langle \rangle$ means expectation value in the HF or HFB ground state. In these instances the HF or HFB wave function can be interpreted as a wave packet from which the ground state rotational band can be obtained by angular momentum projection. As a matter of fact, the angular momentum projection can be carried out through an expansion in powers of $1/\langle J^2_{\perp} \rangle$ [4] which, to lowest order, provides a factorization approximation formally identical to that of Bohr and Mottelson [5].

Unlike rotational invariance in ordinary space, rotational invariance in isospin space is not an exact symmetry of the actual total nuclear hamiltonian. The Coulomb force is non-isoscalar and thus, the actual nuclear states may have isospin mixing. Isospin mixing in the ground state may allow for Fermi (F) transitions in $N=Z$ nuclei, and it is a point of present debate because of its implications on parity violation experiments on $^{12}$C [6], as well as in the analysis of superallowed Fermi transitions as a test of the unitarity of the Cabbibo-Kobayashi-Maskawa matrix [7].

On the other hand, even with isoscalar nuclear hamiltonians, the selfconsistent mean field may break isospin invariance, and particularly so the selfconsistent quasiparticle mean field (HF+BCS or HFB). Therefore in studying Fermi transitions and isospin mixing in the nuclear ground state, it is important to know to what extent the theoretical results respond to realistic properties of the interactions used in the calculations, or rather to spurious mean field contributions.

It is important in many respects to know the value of the quantity

$$\langle T^2_{\perp} \rangle = \langle T^2 \rangle - \left( \frac{N-Z}{2} \right)^2 .$$

If $\langle T^2_{\perp} \rangle$ is large when $N=Z$ one may consider, in analogy to the case $\langle J^2_{\perp} \rangle \gg 1$, that the mean field ground state is a superposition of several $T$-eigenstates. In such a case, one may generate a corresponding isospin rotational band by isospin projection [8], in analogy to the above mentioned angular momentum projection method, or one may even use an isospin-cranked mean field approximation. The latter method has been discussed by Wyss et al. [9] in a somewhat different context. On the contrary, if $\langle T^2_{\perp} \rangle$ is small ($\langle T^2_{\perp} \rangle \lesssim 1$ for $N=Z$), it means that isospin mixing is small. In
this case the mean field ground state is nearly an eigenstate of total isospin, no isospin projection may be required, and the influence of isospin mixing forces can be reliably studied.

In the shell model context [10] large isospin mixing forces (or matrix elements) have been considered. In this paper we restrict our consideration to the Coulomb force as treated in standard mean field calculations [11].

We study Fermi transitions and isospin properties of ground states of several nuclei, from stable to proton rich isotopes, at various levels of approximation. We first consider mean field ground states with and without pairing correlations and with and without isospin breaking interactions (Coulomb force). Next we take into account isospin dependent residual interactions and consider QRPA-correlated ground states. The isospin restoring effect of QRPA-correlations due to isospin conserving residual interactions is discussed.

Other HF, Tamm-Dancoff approximation (TDA) or RPA studies of isospin impurities in the ground states of several $N = Z$ nuclei, as well as studies of the effect of such impurities on superallowed Fermi and Gamow-Teller (GT) $\beta$-decay can be found in Refs. [12,13] and references therein. The latter works show that a simple perturbative treatment of the Coulomb interaction using the same unperturbed wave functions for neutrons and protons will lead to an overestimate of isospin mixing and that the use of selfconsistent solutions is essential in calculating the values of isospin mixing probabilities. The consistency between the interaction producing the single particle spectrum and the strength of the residual particle-hole interaction is an important ingredient in our calculations. This consistency has been shown to be essential in many RPA calculations of giant resonances [12,14,15], as well as in double beta-decay [16].

The paper is organized as follows: In Sec. II we present results on Fermi strength distributions. In Sec. III we compare $\langle T_2^+ \rangle$ values to $\langle J_2^\perp \rangle$ values in the mean field approach and we discuss $\langle T_2^\perp \rangle$ values in different approaches. In Sec. IV we summarize the main conclusions.

II. FERMI STRENGTH DISTRIBUTIONS

In previous publications we studied $\beta$-decay strength functions [17,18] of several isotopic chains in the $A \approx 60 – 80$ region, within the mean field context and beyond. We performed QRPA calculations on top of a quasiparticle basis obtained from a selfconsistent deformed Hartree-Fock approach with density-dependent Skyrme forces. In this work we use the Skyrme force SG2 [19] as a representative of these forces. We use Gamow-Teller and Fermi residual interactions consistent with the mean field single particle basis, both being derived from the same two-body Skyrme interactions. In those works attention was focused on Gamow-Teller strength distributions which are dominant (see Fig. 1). At variance with standard shell-model calculations [20], in our selfconsistent mean field based calculations [17,18], isospin is not an exact quantum number, and one may wonder how large can be the effect of isospin breaking. We investigate this question here focusing on Fermi transitions where isospin breaking effects can be expected to be of most importance.

The Fermi strength distribution can be written as

$$B_f^{\pm} (E) = \sum_f \delta(E - E_f) B_f^{\pm},$$  \hspace{1cm} (2.1)

with

$$B_f^{\pm} = |F_f^{\pm}|^2; \hspace{1cm} F_f^{\pm} = \langle f \mid T^{\pm} \mid 0 \rangle,$$  \hspace{1cm} (2.2)

where $T^{\pm}$ are the rising and lowering isospin operators, $\langle 0 \rangle$ represents the ground state and $\langle f \rangle$ are the proton-neutron (pn) excited states.

For the even-even system both initial and final states are represented by factorized wave functions of the Bohr-Mottelson type with a common collective wave function $D_{00}^0$ and intrinsic wave functions $\langle 0 \rangle$ and $\langle f \rangle$. In the deformed mean field approximation $\langle f \rangle$ represents a one-particle one-hole or a two-quasiparticle excitation, connected to the HF or HFB ground state of the parent nucleus by the $T^{\pm}$ operator,

$$F_f^{+} = 2 \sum_{ip \ i'p'} \langle i' \mid i \rangle u_{ip} u_{i'p} \langle f \mid \alpha_{ip}^{+} \alpha_{i'p}^{+} \mid 0 \rangle,$$  \hspace{1cm} (2.3)

$$F_f^{-} = 2 \sum_{ip \ i'p'} \langle i' \mid i \rangle^* v_{i'p} v_{ip} \langle f \mid \alpha_{ip}^{+} \alpha_{i'p}^{+} \mid 0 \rangle.$$  \hspace{1cm} (2.4)
The single particle states $|i\rangle$ are characterized by the eigenvalues $\Omega_i$ of $J_z$ and by parity $\pi_i$. They are expanded in terms of the eigenstates of an axially symmetric harmonic oscillator in cylindrical coordinates, given in terms of Hermite and Laguerre polynomials \cite{11,17}:

\begin{equation}
|i\rangle = \sum_{N} \frac{(-1)^N + \pi_i}{2} \sum_{n_r,n_z,\Lambda \geq 0,\Sigma} C_{Nn_r,n_z,\Lambda,\Sigma}^i \left| Nn_r,n_z,\Lambda,\Sigma \right\rangle
\end{equation}

with $\Omega_i = \Lambda + \Sigma \geq 1/2$. For each $N$, the sum over $n_r,n_z,\Lambda$ is extended to the quantum numbers satisfying $2n_r + n_z + \Lambda = N$. The sum over $N$ goes from $N = 0$ to $N = 10$. The overlap $\langle i'|i\rangle$ are given by

\begin{equation}
\langle i'|i\rangle = \sum_{Nn_\Lambda\Sigma} C_{Nn_r,n_z,\Lambda,\Sigma}|i\rangle C_{Nn_r,n_z,\Lambda,\Sigma}^*|i'\rangle
\end{equation}

In Eqs. (2.3,2.4) $ip'i'n$ indicates a proton, neutron state with quantum numbers $i,i'$ with $\Omega_i = \Omega_{i'} > 0$ and $v_{i\tau}(u_{i\tau})$ the probability amplitude that the single particle state $i\tau$ be occupied (empty) in the ground state. For each 2-quasiparticle excitation $i'n,ip$ the value of the energy is $E_{i'n} + E_{ip} = E_f$. In the limit of no pairing correlations in the ground state (i.e., in the HF limit) $v_{i\tau}$ goes to 1 and 0 for $i\tau$ levels below and above the $\lambda_i$ Fermi level, respectively. In this limit $E_f = \epsilon_{i'n} - \epsilon_{ip}$ for $\beta^+$ and $N < Z$; $E_f = \epsilon_{i'p} - \epsilon_{ip}$ for $\beta^-$, $N > Z$. We use the convention $\tau = p = -1/2$, $\tau = n = 1/2$ (i.e., $t^+_+|p\rangle = |n\rangle, t^-_|n\rangle = |p\rangle$).

In the QRPA method $|f\rangle$ represents a pn-QRPA phonon state and $|0\rangle$ the QRPA correlated ground state. The expression for the Fermi strength is then given by

\begin{equation}
F^+_f = 2 \sum_{ip'i'n} \langle i'|i\rangle \left[ u_{i'n}v_{ip}X_{i'n,ip}^f + v_{i'n}u_{ip}Y_{i'n,ip}^f \right],
\end{equation}

with $X_{i'n,ip}^f$ and $Y_{i'n,ip}^f$ the forward and backward amplitudes for the two-quasiparticle component $i'n,ip$ of the QRPA mode $|f\rangle = \Gamma^+_f \left| 0 \right\rangle$, of energy $E_f = \omega_f$.

An analogous expression is obtained for $F^-_f$ changing $X-$amplitudes by $Y-$amplitudes. The calculations of these amplitudes have been done as described in ref. \cite{17}. We use separable residual interactions as those derived in ref. [17]. We note that the residual Fermi interaction $(\vec{\tau}_1 \cdot \vec{\tau}_2)$ (as well as the GT $(\vec{\tau}_1 \cdot \vec{\tau}_2)(\vec{\tau}_1 \cdot \vec{\tau}_2)$) is an isospin restoring force and largely eliminates the spurious contributions due to isospin breaking that may be present in mean field calculations.

In Fig. 1 we compare Fermi strengths to GT strengths taking as an example $^{74}$Kr. The Fermi strength given by Eq. (2.1), as well as the corresponding expression for the Gamow-Teller strength, have been transformed into continuous curves by a folding procedure using $\Gamma = 1$ MeV width Gaussians. The strength distributions are plotted versus the excitation energy of the daughter nucleus. The dominance of Gamow-Teller strengths over Fermi strengths is apparent for both $\beta^+$ and $\beta^-$. One clearly sees that the already small mean field Fermi strengths get further reduced in RPA. The general reduction of the strengths in QRPA can be traced back to the isospin restoring role of the residual interactions, being maximal for the $\beta^+$ Fermi strength which is strongly reduced. The results shown in this figure are obtained taking into account the Coulomb interaction in the mean field.

In the next figures (Fig. 2 to Fig. 5) we show the role of Coulomb interaction, as well as that of pairing and RPA correlation effects, on Fermi strengths calculated for the Kr isotopic chain for $A=70$ to $A=74$.

In Fig. 2 we show the Fermi strengths obtained in the limit of small pairing ($\Delta = 0.1$ MeV) and no Coulomb force. Starting with $^{70}$Kr on the left, we observe that the $\beta^+$ strength (plotted in the upper panels) rapidly decreases with increasing A while the $\beta^-$ strength (plotted in the bottom panels), rapidly increases with $A$. We also observe that the reduction of the strength in going from mean field (MF) to QRPA is much stronger in the isospin-forbidden cases.

To clarify what we mean by isospin-forbidden cases let us recall that if the ground state $|0\rangle$ is the exact ground state of the system, in the absence of Coulomb force, it is an eigenstate of the isospin operator with eigenvalues $T_z = (N - Z)/2$ and $T = |T_z|$. Hence, it is clear that in this case

\begin{equation}
T^+_+|0\rangle = T^+_+|T T_z\rangle = \sqrt{(Z - N)} \left | T T_z + 1 \right \rangle \quad \text{for } N < Z ,
\end{equation}

\begin{equation}
T^-^-|0\rangle = T^-^-|T T_z\rangle = \sqrt{(N - Z)} \left | T T_z - 1 \right \rangle \quad \text{for } N > Z ,
\end{equation}

for $N \geq Z$.

\begin{equation}
T^-^-|0\rangle = T^-^-|T T_z\rangle = \sqrt{(N - Z)} \left | T T_z + 1 \right \rangle \quad \text{for } N < Z ,
\end{equation}
So that in Eqs. (2.1,2.2) there is a single \( |f\) state \( |f\rangle = |T T_{z} \pm 1\rangle\) (with \( E_f = E_0 \)) which is accessible by the \( \beta_+ \) (if \( N < Z \)) or the \( \beta_- \) (if \( N > Z \)) operators. Accordingly, we call isospin-forbidden transitions to the \( \beta^+ \) transitions in Kr isotopes with \( A \geq 72 \) and to the \( \beta^- \) transitions with \( A \leq 72 \). The fact that all isospin-forbidden transitions in Fig. 2 are small indicates that isospin breaking is small in the mean field approximation and gets further reduced in QRPA.

As we include the Coulomb interaction, \( T \) is no longer an exact quantum number, and isospin breaking (although very small) becomes an actual property of the system. In the presence of the Coulomb interaction Eqs. (2.8,2.9) are no longer strictly valid, allowing for \( \beta^\pm \) transitions in the isospin-forbidden cases. When we compare the results in Fig. 3, which we obtained including the Coulomb force, with the corresponding ones in Fig. 2 we observe an increase of the \( \beta^+ \) \( 72 \text{Kr}, \) \( 74 \text{Kr} \) strengths, as well as of the \( \beta^- \) \( 70 \text{Kr}, \) \( 72 \text{Kr} \) strengths. This small increase of the isospin-forbidden transitions of Fig. 2 is solely due to the effect of the Coulomb force on Fermi transition and it is a signature of the corresponding small isospin breaking.

One may also observe that in the absence of Coulomb force (Fig. 2) the \( \beta^+ \) and \( \beta^- \) strength distributions are identical in \( 72 \text{Kr} \), but the two distributions become somewhat different when the Coulomb force is taken into account (Fig. 3).

Finally in Fig. 4 we show the \( \beta^+ \) and \( \beta^- \) strengths for the same nuclei obtained with realistic pairing gaps (\( \Delta \sim 1.5 \) MeV, see Table 2). For the allowed transitions (\( \beta^+ \) in \( 70 \text{Kr}, \) \( \beta^- \) in \( 74 \text{Kr} \)) one can see by comparison with Fig. 2 that the main effect of pairing is to open up more transition channels and to moderately reduce the strengths of the dominant peaks. More dramatic is the effect of pairing on the isospin-forbidden transitions which grow up by orders of magnitude in all cases, even though they remain considerably smaller than the allowed ones. This growth of the isospin-forbidden strengths is mainly connected to the stronger isospin breaking of the quasiparticle mean field with increasing pairing gaps. It is present irrespective of whether Coulomb interaction is included or not in the calculation (see also Fig. 5). One also sees that the QRPA results strongly reduce the strengths of the isospin-forbidden \( \beta^\pm \) transitions as compared to their mean field values.

### III. ISOSPIN MIXING AND EXPECTATION VALUES OF ISOSPIN OPERATORS

As mentioned in the Introduction the amount of angular momentum mixing in the mean field ground state of axially symmetric deformed nuclei is measured by the expectation value of the squared angular momentum operator perpendicular to the symmetry axis \( z \).

\[
\langle J_z^2 \rangle \equiv \langle J^2 \rangle - \langle J_z \rangle^2 = \frac{1}{2} \sum_f |\langle f | J_+ | 0 \rangle|^2 + |\langle f | J_- | 0 \rangle|^2 .
\]

(3.1)

Similarly, taking the \( z \) axis in isospin space in the standard way (\( \hat{T}_z = (\hat{N} - \hat{Z})/2 \)), or \( T_z = (N - Z)/2 \), we can measure the amount of isospin mixing by the expectation value of \( T_{z}^2 = T_{x}^2 + T_{y}^2 \), or

\[
\langle T_{z}^2 \rangle = \frac{1}{2} \sum_f \left( |\langle f | T_+ | 0 \rangle|^2 + |\langle f | T_- | 0 \rangle|^2 \right).
\]

(3.2)

Using the definition of Fermi transition amplitudes in Eqs. (2.3,2.4) one immediately gets the identity

\[
\langle T_{z}^2 \rangle = \frac{1}{2} (S_{F+} + S_{F-}) ,
\]

(3.3)

with

\[
S_{F^\pm} = \sum_f |F_f^\pm|^2 = \sum_f B_f^{F^\pm} .
\]

(3.4)

In Tables 1 and 2 we compare \( \langle T_{z}^2 \rangle \) values to \( \langle J_z^2 \rangle \) values. The \( \langle J_z^2 \rangle \) values are calculated [2] using the expression

\[
\langle J_z^2 \rangle = \sum_{\tau=p,n, i, \tau k} \sum_{\tau' k'} (u_{i\tau} v_{k\tau'} - u_{k\tau} v_{i\tau'})^2 \left[ |\langle i\tau | j_+ | k\tau' \rangle|^2 + \frac{1}{2} |\langle i\tau | j_+ | k\tau' \rangle|^2 \right].
\]

(3.5)

Table 1 corresponds to calculations in the no pairing limit (\( \Delta = 0 \)). Table 2 contains the results obtained taking into account BCS pairing correlations with realistic values of the pairing gaps (listed in the tables).
To see the amount of isospin mixing introduced by the Coulomb force we give $\langle T^2_\perp \rangle$ values obtained with and without Coulomb force.

As seen in the tables, the values of $\langle J_z^2 \rangle$ are large for the deformed isotopes. They decrease with decreasing deformation and become 0 ($\langle J_z^2 \rangle = 0$) for spherical isotopes ($^{76}$Kr, $^{78}$Kr).

Comparing the $\langle J_z^2 \rangle$ values in Tables 1 and 2, one can see that $\langle J_z^2 \rangle$ decreases with pairing. In the no pairing limit for the prolate shape of $^{74}$Kr, where the deformation value reaches the value $\beta \approx 0.4$, one gets $\langle J_z^2 \rangle \approx 90$. Pairing correlations with realistic gap values reduce this large $\langle J_z^2 \rangle$ value by one third.

This is in contrast to $\langle T^2_\perp \rangle$ values which are increased when pairing correlations are included. The large $\langle J_z^2 \rangle$ values for the well deformed shapes are in contrast with the small values of $\langle T^2_\perp \rangle_0$, with which they are to be compared. The $\langle T^2_\perp \rangle_0$ values, given in the tables along with $\langle T^2_\perp \rangle$ values, are defined as

$$\langle T^2_\perp \rangle_0 = \langle T^2_\perp \rangle - \left| \frac{N - Z}{2} \right| .$$

(3.6)

To clarify the meaning of this magnitude, we recall that in the limit in which the ground state is an isospin eigenstate, with $T = |T_z| = |(N - Z)/2|$, we have that

$$\langle T^2_\perp \rangle = [T(T + 1) - T^z_z] = |T_z| = \left| \frac{N - Z}{2} \right| .$$

(3.7)

Therefore $\langle T^2_\perp \rangle_0$ gives a better measure of the isospin mixing than $\langle T^2_\perp \rangle$ in isotopes with $|N - Z| \neq 0$.

We see in Table 1 that without pairing and Coulomb $\langle T^2_\perp \rangle = \langle T^2_\perp \rangle_0 = 0$ in $N = Z$ isotopes, i.e., there is no isospin mixing in HF approach when isospin conserving two-body force is used. The amount of isospin mixing introduced by the inclusion of the Coulomb force in the Hartree-Fock approximation can be seen comparing the $\langle T^2_\perp \rangle_0$ values with and without Coulomb in Table 1. For the $N \neq Z$ isotopes in the HF approach the amount of isospin mixing as measured by $\langle T^2_\perp \rangle_0$ is on the order of a few percent with no Coulomb and gets somewhat increased when Coulomb force is included. It is interesting to see that the $\langle T^2_\perp \rangle_0$ value tends to decrease with increasing $|T_z| = |(N - Z)/2|$ value. This is in agreement with spectroscopic observations and confirms the idea that the larger is the value of $|N - Z|$ the better is the isospin quantum number.

As already mentioned, pairing correlations increase the isospin mixing, which as seen in Table 2 reaches a maximum in the $N = Z$ $^{72}$Kr isotope where $\langle T^2_\perp \rangle_0 \approx 2.2$.

Yet if we compare this latter value with the $\langle J_z^2 \rangle$ value ($\langle J_z^2 \rangle \approx 32$) for the same nucleus, we see that one cannot really talk of an isospin rotational band. Indeed if we write the HF+BCS state $\phi$ of $^{72}$Kr as a linear combination of isospin eigenstates $\phi_T$

$$\phi = \sum_T C_T \phi_T , \quad \sum_T |C_T|^2 = 1 ,$$

(3.8)

one sees that the mixing of $T \neq 0$ components is quite small. Indeed, $\langle T^2_\perp \rangle \approx 2$ means that

$$\langle T^2_\perp \rangle = 6 |C_2|^2 + 20 |C_4|^2 + .... \approx 2 ,$$

(3.9)

where one sees that $\sum_{T>0} |C_T|^2 \lesssim 0.3$ and $|C_{2(n+1)}|^2 \approx |C_{2n}|^2$.

A corresponding expansion of $\phi$ in orthonormalized angular momentum eigenfunctions

$$\phi = \sum_J A_J \varphi_J , \quad \sum_J |A_J|^2 = 1 ,$$

(3.10)

gives

$$\langle J_z^2 \rangle = 6 |A_2|^2 + 20 |A_4|^2 + .... \approx 32 \text{ for } \Delta = 1.5 \text{ MeV} ,$$

$$\approx 63 \text{ for } \Delta = 0 ,$$

(3.11)

which provides no constraint on the amount of $J = 2, 4$ angular momentum components. This exercise shows that while angular momentum projection can be used to build up a rotational band, isospin projection can hardly be used to construct an isospin rotational band out of $\phi$.  

5
Let us now consider what happens when we calculate the expectation values of $T_{\pm}^2$ in the pn-QRPA correlated ground state. The results obtained when we take into account QRPA correlations induced by Fermi residual interactions of the form $\chi_F \left( \vec{\tau}_1 \cdot \vec{\tau}_2 \right)$ can be seen in Tables 3 and 4. In these tables we give the values obtained for the total strengths $S_{F^+}$ and $S_{F^-}$ together with $\langle T_{\pm}^2 \rangle_0$ with and without Coulomb interaction.

In the QRPA approximation the total strengths for $\beta^+$ and $\beta^-$ Fermi transitions are given by Eq.(3.4), where the sum over $f$ runs over all the pn-QRPA solutions and the $\beta^+$ amplitudes $F_f^\pm$ are given by Eq.(2.7).

The QRPA solutions [17] satisfy the orthonormalization conditions

$$2 \sum_f \left( X_{i' n ip}^f X_{j' n ip}^{f*} - Y_{i' n ip}^f Y_{j' n ip}^{f*} \right) = 0, \quad (3.12)$$

$$\sum_f \left( X_{i' n ip}^f Y_{j' n ip}^{f*} - Y_{i' n ip}^f X_{j' n ip}^{f*} \right) = 0. \quad (3.13)$$

Using these orthonormalization conditions one can show that

$$S_{F^+} = Z - 2 \sum_{i'i'} \langle i' | i \rangle v_{i' n ip}^2 v_{i n ip}^2 - \sum_f \left( Y_{i' n ip}^f \left[ v_{i' n ip} F_f^+ + u_{i' n ip} F_f^- \right] \right), \quad (3.14)$$

$$S_{F^-} = N - 2 \sum_{i'i'} \langle i' | i \rangle v_{i' n ip}^2 v_{i n ip}^2 - \sum_f \left( Y_{i' n ip}^f \left[ v_{i' n ip} F_f^+ + u_{i' n ip} F_f^- \right] \right), \quad (3.15)$$

with $F_f^\pm$ as given in Eq.(2.7).

These equations explicitly show that, as pointed out in ref. [21], the total $\beta^+(\beta^-)$ strength splits into a one-body term that counts the total number of protons (neutrons), and a two-body term that depends on the two-body correlations included in the descriptions of the nuclear ground state. The two-body term is identical in $F^+$ and $F^-$ summed strengths, and as a result one gets the Ikeda sum rule

$$S_{F^-} - S_{F^+} = N - Z, \quad (3.16)$$

which is always satisfied in our calculations.

The last term in Eqs.(3.14,3.15) is the QRPA correlation term

$$C_{QRPA} = 2 \sum_f \sum_{i'ip'i n} \left( Y_{i' n ip}^f \right) \langle i' | i \rangle v_{i' n ip}^2 v_{i n ip}^2 \left[ v_{i' n ip} F_f^+ + u_{i' n ip} F_f^- \right]. \quad (3.17)$$

This term is zero when there are no QRPA correlations. If the strength of the residual interaction is zero $\langle \chi_F \rangle = 0$ the $Y-$amplitudes are zero.

The second term in Eqs.(3.14,3.15) contains the pure pairing correlation and it reaches its minimum value in the limit of no pairing and no Coulomb force. Indeed in the HF limit one has that

$$\left( -2 \sum_{i'i'} \langle i' | i \rangle v_{i' n ip}^2 v_{i n ip}^2 \right)_{HF} = -2 \sum_{i < \lambda_p, i' < \lambda_n} \langle i' | i \rangle^2. \quad (3.18)$$

If, in addition, there is no Coulomb force

$$\langle i' | i \rangle = \delta_{i'i}, \quad \text{for } N = Z, \quad (3.19)$$

and we have that

$$\left( -2 \sum_{i'i'} \langle i' | i \rangle v_{i' n ip}^2 v_{i n ip}^2 \right)_{HF} = -2 \sum_{i < \lambda_p, i' < \lambda_n} \delta_{i'i} = -Z \quad \text{for } N = Z. \quad (3.20)$$

We therefore find that in the limit of no pairing and no Coulomb force:

$$\langle T_{\pm}^2 \rangle = \langle T_{\pm}^2 \rangle_{HF} = 0 \quad \text{for } N = Z, \quad (3.21)$$
as we found numerically in Table 1. In the general case we can write

\[ \langle T^2 \rangle = \frac{1}{2} \left( S^+ + S^- \right) = \left| \frac{N - Z}{2} \right| + C_{BCS} + C_{QRPA}, \]  

(3.22)

with \( C_{QRPA} \) the QRPA correlation term defined in Eq.(3.17) and \( C_{BCS} \) the BCS correlation term defined as

\[ C_{BCS} = -2 \sum_{i' n' i p} |\langle i' | i \rangle|^2 v^2_{n', n} v^2_{i p} + \min(Z, N). \]  

(3.23)

Owing to Eq.(3.22) we see that the value of \( \langle T^2 \rangle_0 \), as defined in Eq.(3.6), is purely due to the correlation terms in Eqs.(3.17-3.23),

\[ \langle T^2 \rangle_0 = \langle T^2 \rangle - \left| \frac{N - Z}{2} \right| = C_{BCS} + C_{QRPA}, \]  

(3.24)

and that for \( N = Z \) and no Coulomb force

\[ \langle T^2 \rangle = \langle T^2 \rangle_0 = \langle T^2 \rangle_{HF}. \]  

(3.25)

The results in Tables 1 and 2 show that the BCS correlations increase always the \( \langle T^2 \rangle \) value from its limiting Hartree-Fock value. On the other hand if we compare the results of Tables 1 and 2 to the results of Tables 3 and 4, which include both correlation terms (see Eq.(3.24)), we see that the increase of \( \langle T^2 \rangle_0 \) due to the BCS correlations is largely reduced by the QRPA correlations, which tend to restore isospin invariance.

**IV. SUMMARY AND CONCLUDING REMARKS**

We have studied isospin mixing properties in several Kr isotopes around \( N = Z \) and analyzed their Fermi transitions at various levels of approximation.

We have considered first, selfconsistent deformed Skyrme HF mean fields with and without Coulomb and pairing interactions. Then, we take into account isospin dependent residual interactions and consider QRPA correlated ground states. We have studied the effect on the Fermi strength distributions of isospin breaking interactions (Coulomb force and pairing), as well as the effect of QRPA correlations including Fermi type residual interactions, whose particle-hole strengths are consistently fixed with the Skyrme force.

Taking as a reference the case of a selfconsistent mean field calculation, we have seen that in the absence of Coulomb interactions (and in the limit of small pairing correlations) the isospin-forbidden transitions (\( \beta^- \) in \( N \leq Z \) and \( \beta^+ \) in \( N \geq Z \)) are negligible. When the isospin breaking Coulomb interaction is switched on, there is an increase of isospin-forbidden Fermi transitions. Although this increase is small, it is a signature of isospin breaking. Pairing correlations increase by orders of magnitude the isospin-forbidden Fermi transitions, a fact that is related to the isospin breaking nature of the quasiparticle mean field, which increases with increasing pairing gaps. On the other hand, the isospin breaking effects and forbidden Fermi transitions are reduced when RPA correlations are taken into account.

In analogy with the quantity \( \langle J^2 \rangle \), which measures the amount of angular momentum mixing, we have introduced the quantity \( \langle T^2 \rangle_0 \) as a measure of the isospin mixing in the ground state.

In the extreme case of mean field approach without Coulomb interaction and without pairing correlations, we have \( \langle T^2 \rangle_0 = 0 \) for \( N = Z \). There is no isospin mixing in HF approach when isospin conserving two-body forces are used. The amount of isospin mixing introduced by the Coulomb force is small and the maximum mixing occurs when \( N = Z \). In contrast to \( \langle J^2 \rangle \), which decreases with increasing pairing, the amount of isospin mixing \( \langle T^2 \rangle_0 \) increases with pairing correlations and it is also maximum for \( N = Z \). The lowest isospin mixing, as measured by \( \langle T^2 \rangle_0 \), occurs in HF approximation. Pairing correlations increase the mixing and QRPA correlations reduce it.

One may wonder whether, in the deformed nuclei, the calculated Fermi strengths may contain spurious contributions from higher angular momentum components in the initial and final wave functions. As mentioned before, the Fermi strengths are calculated in the laboratory frame in the factorization approximation of Bohr and Mottelson [5]. Using angular momentum projection techniques [2], we find that an upper bound to such contributions is proportional to \( 1/ \langle J^2 \rangle^2 \), where the values of \( \langle J^2 \rangle \) can be found in Tables 1 and 2. Thus, exact angular momentum projection in the deformed cases would lead in all cases to less than 1% effect in the Fermi strengths.

It is also shown that the total \( \beta^\pm \) Fermi strengths can be separated into a one-body term counting basically the number of nucleons of a given type, and a two-body term that depends on the two-body correlations (BCS and RPA). The two-body term is identical in both \( \beta^+ \) and \( \beta^- \) summed strengths and the Ikeda sum rule is fulfilled as a result of their cancellation. The isospin mixing \( \langle T^2 \rangle_0 \) is purely due to the net effect of the correlation terms.
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TABLE I. Comparison of angular momentum and isospin mixing in the mean field approximation without pairing correlations (HF). The HF value of the $\beta^-$ deformation parameter is given in the second column.

| Isotope | $\beta$ | $\langle J_z^2 \rangle$ | $\langle T_z^2 \rangle_{\alpha}$ | $\langle T_z^2 \rangle_{\beta}$ | IKEDA |
|---------|---------|----------------|------------------|------------------|-------|
| $^{70}$Kr | -0.27 | 50.2 | 0.04 | 0.00 | 1.04 | 1.00 |
| $^{72}$Kr | -0.29 | 63.1 | 0.05 | 0.00 | 0.05 | 0.00 |
| $^{74}$Kr | -0.15 | 21.1 | 0.05 | 0.01 | 1.05 | 1.01 |
| $^{76}$Kr | 0.39 | 89.9 | 0.06 | 0.02 | 1.06 | 1.02 |
| $^{78}$Kr | 0.00 | 0.0 | 0.03 | 0.01 | 2.03 | 2.01 |

TABLE II. Same as Table 1 taking into account pairing correlations (HF+BCS) with fixed pairing gaps $\Delta_p = \Delta_n = \Delta$ given in the second column.

| $\Delta$ (MeV) | $\beta$ | $\langle J_z^2 \rangle$ | $\langle T_z^2 \rangle_{\alpha}$ | $\langle T_z^2 \rangle_{\beta}$ | IKEDA |
|----------------|---------|----------------|------------------|------------------|-------|
| $^{70}$Kr | 1.5 | -0.23 | 28.5 | 1.2 | 1.1 |
| $^{72}$Kr | 1.5 | -0.24 | 31.8 | 2.2 | 2.2 |
| $^{74}$Kr | 1.5 | -0.15 | 13.0 | 1.5 | 1.4 |
| $^{76}$Kr | 1.65 | 0.0 | 0.0 | 1.3 | 1.2 |
| $^{78}$Kr | 1.75 | 0.0 | 0.0 | 1.0 | 0.9 |

TABLE III. Results of QRPA calculations of total $\beta^+$ and $\beta^-$ Fermi strengths of Kr isotopes, with and without Coulomb, in the small pairing limit ($\Delta = 0.1$ MeV is used for all isotopes). Also listed are the values of $\langle T_z^2 \rangle_{\alpha}$. The values of the deformation parameter $\beta$ are as in Tables 1 and 2. The strengths of the particle-hole and particle-particle residual interactions are $\chi_{ph} = 0.5$ MeV and $\kappa_{pp} = 0.03$ MeV, respectively (see ref. [17]).

| Isotope | $S_{\beta^-}$ | $S_{\beta^+}$ | $\langle T_z^2 \rangle_{\alpha}$ |
|---------|-------------|-------------|------------------|
| $^{70}$Kr | 0.05 | 0.00 | 2.05 |
| $^{72}$Kr | 0.07 | 0.02 | 0.07 |
| $^{74}$Kr | 2.03 | 2.00 | 0.03 |
| $^{76}$Kr | 2.05 | 2.00 | 0.05 |
| $^{78}$Kr | 4.02 | 4.00 | 0.02 |
| $^{80}$Kr | 6.01 | 6.00 | 0.01 |

TABLE IV. Same as Table 3 but for standard values of the pairing gap ($\Delta = \Delta_n = \Delta_p \sim 1.5$ MeV, as listed in Table 2).

| Isotope | $S_{\beta^-}$ | $S_{\beta^+}$ | $\langle T_z^2 \rangle_{\alpha}$ |
|---------|-------------|-------------|------------------|
| $^{70}$Kr | 0.43 | 0.41 | 2.43 |
| $^{72}$Kr | 1.15 | 1.11 | 1.15 |
| $^{74}$Kr | 2.50 | 2.44 | 0.50 |
| $^{76}$Kr | 2.46 | 2.48 | 0.46 |
| $^{78}$Kr | 4.30 | 4.23 | 0.30 |
| $^{80}$Kr | 6.17 | 6.12 | 0.17 |
FIG. 1. Comparison of $\beta^\pm$ Gamow-Teller and Fermi strength distributions in both mean field (MF) and QRPA approximations in the case of $^{74}$Kr.
FIG. 2. Fermi strength distributions $\beta^{\pm}$ in $^{70,72,74}$Kr plotted as a function of the excitation energy of the daughter nucleus. We compare results obtained from Skyrme Hartree-Fock calculations (dotted lines) and QRPA (solid lines). The results correspond to the case of no Coulomb interaction and pairing gaps approaching zero ($\Delta = 0.1$ MeV).
FIG. 3. Same as in Fig. 2 but with Coulomb interaction.
FIG. 4. Same as in Fig. 2 without Coulomb interaction but including pairing correlations with realistic gaps (see Table 2).
FIG. 5. Same as in Fig. 4 but including Coulomb interaction.