Dark matter Admixed Neutron Stars

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Neutron Rich Matter on Heaven and Earth
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DM Admixed NSs

- Dark Matter can accumulate in stars (the Sun, white dwarfs and neutron stars)

- Significant amounts (few percent of NS total mass) of DM can exist in stable equilibrium configurations of neutron stars

- Lead to observable effects through GW (tidal deformability) and EM (X-ray) radiation
Dark matter Accumulation

- **Accretion** from DM galactic halo
  
  $$M_{\text{accretion}} \sim 10^{-16} \left( \frac{\rho \chi}{\text{GeV/cm}^3} \right) \left( \frac{\sigma \chi n}{10^{-45} \text{cm}^2} \right) \left( \frac{t}{10^8 \text{ yrs}} \right) M_\odot$$

  - mostly considered too inefficient for observable effects
    (Goldman & Nussinov 1989), (Kouvaris 2007), (Kouvaris & Tinyakov 2010, 2011), (Nelson et al. 2019), etc.

- **Produced from** SM species and trapped within NS
  $$(\sim 0.01 \ M_\odot)$$

  - Due to high core densities
    (Mckeen et al. 2018), (Baym et al. 2018), (Motta et al. 2018a, 2018b)

  - Due to high temperatures in progenitor supernovae or proto-NS
    (Nelson et al. 2019), (Reddy & Zhou 2022), (Collier et al. 2022)
Dark matter Accumulation

- Mergers with dark compact objects
  - self-interacting DM can form dark stars ($\sim 0.1 \, M_\odot$)

(Kouvaris & Nielsen 2015, Collier et al. 2022)

Rest of the talk – Assume DM exists in NSs in equilibrium configurations and explore observable effects.
Dark Core or Halo

Dark Core

Dark Halo
Depending on
i. DM particle mass ($m_\chi$)
ii. self-interaction strength ($\gamma$)
iii. DM mass fraction in the star ($f_\chi$)
DANS Model Generation

- **Single fluid description**
  - Compute equation of state with **SM couplings** between DM and baryonic matter
    - (Panotopoulos & Lopes 2017), (Quddus et al. 2020), (Das et al. 2019), (Das et al. 2021), (Lopes et al. 2022)

- **Two-fluid description**
  - No **SM couplings** between DM and baryonic matter
  - Only gravitational interaction between the two

  Our method of choice since it is more relevant for NICER X-ray observations!
Two-Fluid TOV equations

Energy-momentum of each fluid is conserved separately

\[ G_{\mu\nu} = 8\pi (T_{\mu\nu,B} + T_{\mu\nu,D}) \]

\[ \frac{dP_B}{dr} = -\frac{(\epsilon_B+P_B)(M+4\pi r^3P)}{r(r-2M)} \]

\[ \frac{dM_B}{dr} = 4\pi r^2 \epsilon_B \]

\[ \frac{dP_D}{dr} = -\frac{(\epsilon_D+P_D)(M+4\pi r^3P)}{r(r-2M)} \]

\[ \frac{dM_D}{dr} = 4\pi r^2 \epsilon_D \]

\[(M = M_B + M_D, \quad P = P_B + P_D, \quad \epsilon = \epsilon_B + \epsilon_D)\]

4 coupled equations with 6 unknowns. Need 2 EOSs relating pressure and energy density of each fluid.
Equation of State

- **Baryonic EOS** – NL3$\omega\rho$L55
  - choosing stiff EOS just to easily illustrate effects of DM in MR profile

- **DM EOS**
  - **bosonic** *(requires repulsive self-interaction)*
    - without self-interaction collapses to a black hole at the centre of the NS
  - **fermionic** *(no self-interaction required due to degeneracy pressure)*
    - we choose to include self-interaction to explore effects
Dark Matter EOS

\[ \epsilon_D = \epsilon_{D,\text{kin}} + m_\chi n_D + \frac{n_D^2}{m_I^2} \]

Assume ideal Fermi gas at zero temperature (Nelson et al. 2019)

\[ \epsilon_{D,\text{kin}} = \frac{1}{\pi^2} \int_0^{k_F} dk \ k^2 \left( \sqrt{k^2 + m_\chi^2} - m_\chi \right) \]

\[ P_D = -\epsilon_D + \frac{\mu_D}{\sqrt{g_{tt}}} n_D \]

\[ \frac{\mu_D}{\sqrt{g_{tt}}} = \frac{\partial \epsilon_D}{\partial n_D} \]

\[ n_D = \frac{k_F^3}{3\pi^2} \]
Dark Matter EOS

\[ \epsilon_D = \frac{m_\chi^4}{8\pi^2} \left[(2x^3 + x)\sqrt{1 + x^2} - \text{arcsinh}(x)\right] + \frac{m_\chi^4 y^2 x^6}{(3\pi^2)^2} \]

\[ P_D = \frac{m_\chi^4}{24\pi^2} \left[(2x^3 - 3x)\sqrt{1 + x^2} + 3 \text{arcsinh}(x)\right] + \frac{m_\chi^4 y^2 x^6}{(3\pi^2)^2} \]

\[ x = \frac{k_F}{m_\chi} : \text{relativity parameter} \]

\[ y = \frac{m_\chi}{m_i} : \text{self-interaction strength} \]
Dependence on $m_\chi$

$\epsilon$ [MeV/fm$^3$] vs $r$ [km]

- **Solid lines**: Core
- **Dashed lines**: Halo

- **Pure baryonic NS (NL3ωρL55)**
- $m_\chi = 1.0$ GeV, $y = 0$
- $m_\chi = 0.3$ GeV, $y = 0$
- $m_\chi = 0.15$ GeV, $y = 0$
- $m_\chi = 0.1$ GeV, $y = 0$

$M_T = 1.4M_\odot$

$f_\chi = 0.05$

$\epsilon_B$ [MeV/fm$^3$] vs $r$ [km]

$\epsilon_B$ values are shown in the inset graph.
Dependence on $m_\chi$
Dependence on $\gamma$

- Pure baryonic NS (NL3$\omega$L55)
- $M_T = 1.4M_\odot$
- $f_\chi = 0.05$
- $m_\chi = 1$ GeV

- Solid: core
- Dashed: halo

- PSR J0740+6620
- PSR J0030+0451
Dependence on $f_\chi$

Cores

Halos

$M_T [M_\odot]$ vs $R_B [\text{km}]$

- pure baryonic NS (NL3$\omega$pL55)
- $m_\chi = 1 \text{ GeV}, y = 0, f_\chi = 0.05$
- $m_\chi = 1 \text{ GeV}, y = 0, f_\chi = 0.1$
- $m_\chi = 1 \text{ GeV}, y = 0, f_\chi = 0.2$
- $m_\chi = 1 \text{ GeV}, y = 0, f_\chi = 0.3$

solid: core
dashed: halo

$M_T [M_\odot]$ vs $R_B [\text{km}]$

- pure baryonic NS (NL3$\omega$pL55)
- $m_\chi = 1 \text{ GeV}, y = 1000, f_\chi = 0.05$
- $m_\chi = 1 \text{ GeV}, y = 1000, f_\chi = 0.1$
- $m_\chi = 1 \text{ GeV}, y = 1000, f_\chi = 0.2$
- $m_\chi = 1 \text{ GeV}, y = 1000, f_\chi = 0.3$

solid: core
dashed: halo

PSR J0740+6620
PSR J0030+0451
Gravitational Self-Lensing

\[
\cos \psi = \cos \zeta \cos \theta_c + \sin \zeta \sin \theta_c \cos \phi
\]

\[
\psi = \int_R^\infty \frac{dr}{r^2} \left[ \frac{1}{b^2} - \frac{1}{r^2} \left( 1 - \frac{R_S}{r} \right) \right]^{-1/2}
\]

\[
b = \frac{R \sin \alpha}{\sqrt{1 - \frac{R_S}{R}}}
\]

\[
F = \delta^5 g_{tt}(R_B) \cos(\alpha) \frac{d \cos(\alpha)}{d \cos(\psi)}
\]

This can be used to calculate pulse profiles (flux vs time plots)

Credit: Bogdanov et al. 2022
Gravitational Self-Lensing due to Dark Halo

- Pure NS
- Dark Halo
- emitted X-rays
- extra light-bending
Flux from NS with Dark Halo

![Graph showing flux from a neutron star with dark halo](image)

Normalized Flux

\[ M_T = 1.4M_\odot \]

\[ f_\chi = 0.05 \]

- Pure baryonic NS (NL3$\omega$PL55)
- \( m_\chi = 0.3 \text{ GeV}, y = 0 \)
- \( m_\chi = 0.15 \text{ GeV}, y = 0 \)
- \( m_\chi = 0.1 \text{ GeV}, y = 0 \)
Flux from NS with Dark Halo

Normalized Flux

\[ M_T = 1.4M_\odot \]

\[ f_\chi = 0.05 \]

\[ m_\chi = 1.0 \text{ GeV}, y = 25 \]

\[ m_\chi = 1.0 \text{ GeV}, y = 100 \]

\[ m_\chi = 1.0 \text{ GeV}, y = 1000 \]
Flux from NS with Dark Halo

Note: Largest difference in flux, $F$, between pure NS and dark halo is at the peak ($F_{\text{peak}}$).
NS Mass-Radius Measurements

- Mass measurements through radio observations measures \( M_T = M_T(R_B) + M_{\text{halo}} \)

- NICER measures \( M_T(R_B) \)
  - Since light is emitted from \( R_B \) and most of light bending occurs near \( R_B \) due to strongest gravitational potential
Approximate $\Delta F_{\text{peak}}$

Pure NS

$M_T(R_B)$

$R_B$

emitted X-rays

Dark Halo

$M_T(R_B)$

$R_B$

$M_{\text{halo}}$
Approximate $\Delta F_{\text{peak}}$

- Try to approximate the pulse profile of a NS with dark halo by that of a pure NS of mass $M_T(R_B)$.

$$\log\left(\frac{|\Delta F_{\text{peak}}|}{F_{\text{peak}}}\right) = 0.998\log\left(\frac{M_{\text{halo}}}{R_D}\right) + 0.854 \quad (\text{Miao et al. 2022})$$

For sufficiently small $\left(\frac{M_{\text{halo}}}{R_D}\right)$, $\Delta F_{\text{peak}}$ is small.

Largest difference in $F$ between pure NS and dark halo is at the peak flux $F_{\text{peak}}$. 
Relate $F_{\text{peak}}$ to $g_{tt}(R_B)$

$$ds^2 = -g_{tt}(r)dt^2 + g_{rr}(r)dr^2 + r^2 d\theta^2 + r^2 \sin^2(\theta)d\phi^2$$

$$F = \delta^5 g_{tt}(R_B) \cos(\alpha) \frac{d \cos(\alpha)}{d \cos(\psi)} \sim g_{tt}^2(R_B) \text{ (at peak)} \sim (1 + z)^{-4} \text{ (bolometric)}$$

$$\left| \frac{\Delta F_{\text{peak}}}{F_{\text{peak}}} \right| \sim \frac{2|\Delta g_{tt}(R_B)|}{g_{tt}(R_B)}$$

- Energy dependent flux will depend on a different power of $g_{tt}(R_B)$
  - Even if $M_{\text{halo}}/R_D$ relation does not hold, $\Delta g_{tt}(R_B)$ can be used to set maximum $\Delta F_{\text{peak}}$

- Both $M_{\text{halo}}/R_D$ and $g_{tt}(R_B)$ can be indicators towards if $\Delta F_{\text{peak}}$ is small
Can we use current NICER results to constrain Dark Halos?

- Current NICER results assume no DM exists in the NSs

- If $\Delta F_{\text{peak}}$ (or $M_{\text{halo}}/R_D$ or $g_{tt}(R_B)$) is large, a NICER reanalysis is required to constrain DM properties
  - Current $M_T(R_B)$ and $R_B$ measurements significantly differ from that of NS with dark halo

- If $\Delta F_{\text{peak}}$ is sufficiently small, we can use current NICER results to analyse the validity of baryonic EOSs and constrain $f_\chi$ in the NS
We find that 
\[ M_T(R_B) \approx M_{T,\text{pure}} \]
for large parameter space of dark halo cases

In these cases, we can analyse whether baryonic EOSs are valid if a dark halo exists
Case 1: No radio mass measurement. Only NICER measurement available.

Both baryonic and DM EOSs are valid, whether DM $M_T$ curve intersects NICER confidence region or not.

Both baryonic and DM EOSs are invalid, even if DM $M_T$ curve intersects NICER confidence region.
Case 2: **Only radio mass measurement available.**

No NICER measurement.

Trivial: Both baryonic and DM EOSs are valid.

Pure baryonic EOS ruled out by radio mass measurement. But adding DM makes it valid.
Case 3: Both Radio mass and NICER measurements available.

Trivial: Both baryonic and DM EOSs are valid.

Softer EOS is untested by NICER measurements due to strong radio mass priors.
Assume some maximum $f_\chi \sim 0.05$.

Reduce NICER mass priors by $f_\chi$.

Redo NICER analysis to measure new $M_T(R_B)$ and $R_B$.

Check which EOSs are valid now.

Softer EOS is untested by NICER measurements due to strong radio mass priors.
Constrain $f_\chi$ in NS

Allowed $f_\chi$

Not allowed

$M_T$: pure baryonic NS
$M_T$: $m_\chi = 1$ GeV, $y = 1000$
$M_T(R_B)$: $m_\chi = 1$ GeV, $y = 1000$
cartoon Radio mass measurement
cartoon NICER measurement

Allowed $R_B$ for NS
Constrain $f_\chi$ in NS

- Simulate dark halos
- Is $\Delta F_{\text{peak}}$ small?
- Is $M_T(R_B) \approx M_{T,\text{pure}}$?
- Constrain $f_\chi$ based on a particular baryonic EOS and DM $m_\chi$ and $y$. 

![Graphical representation of constraints on $f_\chi$ and $R_B$]
Conclusions

- Dark matter Admixed NSs can have M-R and X-ray pulse profiles significantly different from pure baryonic NSs

- If $\frac{|\Delta F_{\text{peak}}|}{F_{\text{peak}}}$ for DM halo is large, NICER reanalysis is required to constrain DM properties

- If $\frac{|\Delta F_{\text{peak}}|}{F_{\text{peak}}}$ for DM halo is small, we can use current NICER results to constrain both baryonic EOSs as well as DM fractions for a wide parameter space of DM $m_\chi$ and $\gamma$