The instability driven by the ponderomotive electron current in the skin layer of the inductively coupled plasma.

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The stability theory of the skin layer plasma of the inductive discharge is developed for the case when the electron quiver velocity in RF wave is of the order of or is larger than the electron thermal velocity. The theory predicts the existence of the non-modal Buneman instability in the skin layer driven by the current formed by the accelerated motion of electrons relative ions under the action of the ponderomotive force.

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I. INTRODUCTION

The regime of the anomalous skin effect (or nonlocal regime) is typical for the low pressure inductive plasma sources employed in material processing applications. It occurs when the frequency \( \omega_0 \) of the operating electromagnetic (EM) wave is much above the electron-neutral collision frequency, but less than the electron plasma frequency. In this regime, the interaction of the EM field with electrons is governed by the electron thermal motion. For this reason, the EM wave absorption, the formation of the anomalous skin layer near the plasma boundary, and the anomalous electron heating require the kinetic description which involves the well known mechanism of collisionless power dissipation - Landau damping. It stems from the resonant wave-electron interaction under condition that the electron thermal velocity \( v_{Te} \) is comparable with (or is larger than) the EM phase velocity. The theory of the anomalous skin effect is developed as a rule, employing the linear approximation to the solution of the Vlasov equation for the electron distribution function. It is assumed in this theory that the equilibrium electron distribution function depends only on electron kinetic energy and does not involve the electron motion in the time dependent spatially inhomogeneous EM wave. This approximation is valid when the quiver velocity of electron in the EM wave is negligible in comparison with the electron thermal velocity.

It was found experimentally and analytically that at the low driving frequency of an inductive discharge, at which RF Lorentz force acting on electrons becomes comparable to or larger than the RF electric field force, the nonlinear effects in the skin layer becomes essential. The theoretical analysis has shown that electron oscillatory motion in the inhomogeneous RF field in the skin layer leads to the ponderomotive force. This force is regarded as the responsible for the reduction of the steady state electron density distribution within the skin layer and for the formation of experimentally observed and analytically predicted second harmonics which was found to be much larger than the electric field on the fundamental frequency. It was found that the low frequency/high-amplitude portion of the anomalous skin effect regime changes behaviour so completely that it was systematized as the nonlinear skin effect regime.

Our paper is devoted to the analytical investigations of the nonlinear processes which may be developed in the skin layer in the high frequency case for which the RF electric field force acting on electrons prevails over the RF Lorentz force. In this case a situation can occur that the electron quiver velocity in a skin layer under the action of the electromagnetic wave approaches or is larger than the electron thermal velocity. Under such conditions it is reasonable to talk about free oscillations of a plasma particle under the action of the RF field (at least, in the zero approximation in the ratio of the collision frequency to the field frequency). The relative oscillatory motion of the electrons and ions in the RF field is a potential source of numerous instabilities of the parametric type (see, for example, Refs. ) with frequencies \( \omega \) comparable with or less than the frequency \( \omega_0 \) of the applied RF wave. It is clear that in such a situation an essentially nonlinear dependence of the plasma conductivity on the RF field as well as the anomalous absorption of the RF energy due to the development of the plasma turbulence and turbulent scattering of electrons arise. This is just the case which interest us in the present paper.

It is usually accepted in the theoretical investigations of the parametric instabilities excited by the strong electromagnetic wave in the unbounded uniform plasmas, that the approximation of the spatially homogeneous pump wave may suffice since the parametrically excited waves have the wave number much larger than the wave number of the pump wave. The presence of the skin layer at the plasma boundary near the RF antenna where RF electromagnetic wave decays into plasma requires the development of new approach to the theory of the instabilities of the parametric type in which the spatial inhomogeneity of the pumping wave should be accounted for.
This new kinetic approach, grounded on the methodology of the oscillating modes, is developed in Sec. II. We found that in the skin layer electrons experience the oscillating motion in RF field jointly with the uniformly accelerated motion under the action of the ponderomotive force resulted from the spatial inhomogeneity of the RF field in this layer. The basic equation for the perturbed electrostatic potential which determines the stability of the inductively coupled plasma against the development of the electrostatic instabilities in skin layer is derived in Sec. II. We found, that the accelerated motion of the electrons in the skin layer is the dominant factor in the instabilities development. The linear theory of the Buneman instability with growing with time growth rate, driven by the accelerated motion of electrons relative to ions under the action of the ponderomotive motion was developed in Sec. IV. Conclusions are presented in Sec. V.

II. BASIC TRANSFORMATIONS AND GOVERNING EQUATIONS

We consider a model of a plasma occupying region \( z \geq 0 \). The RF antenna which launches the RF wave with frequency \( \omega_0 \) is assumed to exist to the left of the plasma boundary \( z = 0 \). The electric, \( E_0 (z, t) \), and magnetic, \( B_0 (z, t) \), fields of a such RF wave, are directed along the plasma boundary and attenuate along \( z \) due to the skin effect. We assume that these fields are exponentially decaying with \( z \), and sinusoidally varying with time,

\[
E_0 (z, t) = E_{0y} e^{-\kappa z} \sin (\omega_0 t - k_{0z} z) e_y
\]

and

\[
B_0 (z, t) = E_{0y} \frac{c \kappa}{\omega_0} e^{-\kappa z} \cos (\omega_0 t - k_{0z} z),
\]

where \( E_0 \) and \( B_0 \) satisfy the Faradays law, \( \partial E_0 / \partial z = \partial B_0 / \partial t \). Usually the real part \( k_{0z} \) of the complex wave number \( k_0 = k_{0z} + i \kappa \) in the anomalous skin layer is much less than the imaginary part \( \kappa \) and will be neglected here. In this paper, we consider the effect of the relative motion of plasma species in the applied RF field on the development the short scale electrostatic perturbations in the skin layer near the antenna with wavelength much less than the skin layer thickness. Our theory bases on the Vlasov equation for the velocity distribution function \( F_\alpha \) of \( \alpha \) species (\( \alpha = e \) for electrons and \( \alpha = i \) for ions),

\[
\frac{\partial F_\alpha}{\partial t} + \frac{\mathbf{v}}{m_\alpha} \frac{\partial F_\alpha}{\partial \mathbf{r}} + \frac{e_\alpha}{m_\alpha} \left( \mathbf{E}_0 (z, t) + \frac{1}{c} [\mathbf{v} \times \mathbf{B}_0 (z, t)] \right) - \nabla \varphi (\mathbf{r}, t) \frac{\partial F_\alpha}{\partial \mathbf{v}} = 0.
\]

This equation contains the potential \( \varphi (\mathbf{r}, t) \) of the electrostatic plasma perturbations which is determined by the Poisson equation

\[
\nabla^2 \varphi (\mathbf{r}, t) = -4\pi \sum_{\alpha=e,i} e_\alpha \int f_\alpha (\mathbf{v}, \mathbf{r}, t) \, d\mathbf{v}_\alpha,
\]

where \( f_\alpha \) is the perturbation of the equilibrium distribution function \( F_{0\alpha} \), \( F_\alpha = F_{0\alpha} + f_\alpha \). The equilibrium distribution function \( F_{0\alpha} \) is a function of the canonic momentums \( p_z = m_\alpha v_z \) and \( p_y = m_\alpha v_y - \frac{e}{2} A_{0y} (z, t) \), which are the integrals of the Vlasov equation (3) without potential \( \varphi (\mathbf{r}, t) \). It will be assumed to have a form

\[
F_{0\alpha} (v_y, v_z, t) = \frac{n_\alpha}{2\pi \tau_\alpha} \exp \left[ \frac{v_y^2}{2\tau_\alpha^2} \right] - \frac{1}{2}\frac{v_y}{\tau_\alpha} \left( v_y - \frac{e}{cm_\alpha} A_{0y} (z, t) \right)^2, \tag{5}
\]

where the electromagnetic potential \( A_{0y} (z, t) \), for the electromagnetic field (1) and (2) is equal to

\[
A_{0y} (z, t) = \frac{eE_{0y}}{\omega_0} e^{-\kappa z} \cos \omega_0 t. \tag{6}
\]

The kinetic theory of the plasma stability with the time dependence of the \( F_{0\alpha} \) caused by the strong spatially homogeneous oscillating electric field \( E_0 (t) = \hat{E}_{0y} \sin \omega_0 t e_y \) was developed by employing the transformation \( \mathbf{v} = \mathbf{v}_\alpha + \mathbf{V}_{0\alpha} (t) \) of the velocity variable \( \mathbf{v} \) in the Vlasov equations for ions and electrons to velocity \( \mathbf{v}_\alpha \) determined in the frame of references which oscillates with velocity \( \mathbf{V}_{0\alpha} (t) \) of particles of species \( \alpha \) in velocity space, leaving unchanged position coordinates. With new velocity \( \mathbf{v}_\alpha \) the explicit time dependence which stems from the RF field is excluded from the Vlasov equation. In this paper we employ more general transformation of the velocity and position coordinates to the convected-oscillating frame of references determined by the relations

\[
\mathbf{v}_\alpha = \mathbf{v} - \mathbf{V}_{\alpha} (\mathbf{r}, t),
\]

\[
\mathbf{r}_\alpha = \mathbf{r} - \mathbf{r}_{\alpha} (\mathbf{r}, t) = \mathbf{r} - \int_0^t \mathbf{V}_{\alpha} (\mathbf{r}, t_1) \, dt_1. \tag{7}
\]

This transformation was decisive in the development of the parametric weak turbulence theory, and the theory of the stability and turbulence of plasma in pumping wave with finite wavelength, and admits the solution of the Vlasov equation in the case of the oscillating spatially inhomogeneous RF field. The transformation of Eq. (3) for \( F_e \) to velocity \( \mathbf{v}_e \), and coordinate \( \mathbf{r}_e \) variables determined by Eq. (7) transforms Eq. (3) to the form

\[
\frac{\partial F_e (\mathbf{v}_e, \mathbf{r}_e, t)}{\partial t} + \mathbf{v}_e \frac{\partial F_e}{\partial \mathbf{r}_e} - \mathbf{v}_e \int_{t_0}^t \frac{\partial \mathbf{V}_{ek} (\mathbf{r}, t_1)}{\partial r_j} \, dt_1 \frac{\partial F_e}{\partial r_{ek}}
\]

\[
- \mathbf{v}_e \frac{\partial \mathbf{V}_{ek}}{\partial r_j} \frac{\partial F_e}{\partial \mathbf{r}_e} - \mathbf{V}_{ek} (\mathbf{r}, t) \int_{t_0}^t \frac{\partial \mathbf{V}_{ek} (\mathbf{r}, t_1)}{\partial r_j} \, dt_1 \frac{\partial F_e}{\partial r_{ek}}
\]

\[
+ \frac{e}{m_e} \left( \nabla \varphi (\mathbf{r}, t) - \frac{1}{c} [\mathbf{v}_e \times \mathbf{B}_0 (z, t)] \right) \frac{\partial F_e}{\partial \mathbf{v}_e}
\]

\[
- \left( \frac{\partial \mathbf{V}_{ek} (\mathbf{r}, t)}{\partial t} + \mathbf{V}_{ek} (\mathbf{r}, t) \frac{\partial \mathbf{V}_{ek} (\mathbf{r}, t)}{\partial \mathbf{r}_e} \right)
\]
\[ + \frac{e}{m_e} \left( E_{0y} (z, t) + \frac{1}{c} \left[ V_e (r, t) \times B_0 (z, t) \right] \right) \left\{ \right. \]
\[ \left. \times \frac{\partial F_e (v_e, r_e, t)}{\partial v_{ej}} \right\} = 0. \tag{8} \]

In the approximation of the spatially uniform RF field (i.e., for \( \kappa = 0 \) in our case), the time dependent RF electric field is excluded from Eq. (8) for the velocity \( V_e \) for which the expression in braces vanishes. In the case of the spatially inhomogeneous RF fields, this selection of the velocity \( V_e \) provides the derivation of the solution for \( F_e \) in the form of power series in the small parameter \( \delta r_e \ll 1 \), where \( \delta r_e \) is the amplitude of the displacement of electron in the RF field. For the electric field \( \vec{E} \) and magnetic field \( \vec{B} \), this velocity is determined by the equations

\[ \frac{\partial V_{ey} (z, t)}{\partial t} + V_{ez} (z, t) \frac{\partial V_{ey} (z, t)}{\partial z} = - \frac{e}{m_e} \left( E_{0y} (z, t) + \frac{1}{c} V_{ez} (z, t) B_{0x} (z, t) \right), \tag{9} \]

\[ \frac{\partial V_{ez} (z, t)}{\partial t} + V_{ez} (z, t) \frac{\partial V_{ez} (z, t)}{\partial z} = \frac{e}{m_e} V_{ey} (z, t) B_{0x} (z, t). \tag{10} \]

With new variables \( \delta z, t' \), determined by the relations\(^{22}\)

\[ z = z_e + \int_0^{t'} V_{ez} (z_e, t'_1) dt'_1, \quad t = t', \tag{11} \]

Eqs. (9) and (10) becomes

\[ \frac{\partial V_{ey} (z_e, t')}{\partial t'} = - \frac{eE_{0y}}{m_e} \left[ \left( z_e + \int_0^{t'} V_{ez} (z_e, t'_1) dt'_1 \right) \right] \]
\[ \times \left( \sin \omega_0 t' + \frac{\kappa V_{ez} (z_e, t')}{\omega_0} \cos \omega_0 t' \right), \tag{12} \]

\[ \frac{\partial V_{ez} (z_e, t')}{\partial t'} = \frac{eE_{0y}}{m_e \omega_0} V_{ey} (z_e, t') \cos \omega_0 t', \tag{13} \]

where

\[ \xi_e = \frac{eE_{0y}}{m_e \omega_0} \xi_e \]

(14)

is the amplitude of the displacement of an electron along the coordinate \( y \) at \( z_e = 0 \). For the collisionless plasma \( \Re \kappa^{-1} = L_s \) is the skin depth for the anomalous skin effect,

\[ L_s = \left( \frac{v_T e^2}{\sqrt{\pi \omega_0 m_e}} \right)^{1/3}. \tag{15} \]

We find the approximate solutions to nonlinear Eqs. (12), (13) for \( V_{ey} (z_e, t) \) and \( V_{ez} (z_e, t) \) assuming that the parameter \( \kappa \xi_{e0} \) is much less than unity. In this paper, we consider the case of the high frequency \( \omega_0 \) RF wave for which the RF electric field force acting on electrons in the skin layer prevails over the RF Lorentz force. In this case the electron cyclotron frequency formed by the magnetic field \( B_{0x} \) is much less than \( \omega_0 \). The procedure of the solution of system (12), (13) for the opposite case of the low frequency RF wave, for which the RF Lorentz force dominates over the RF electric field force, is different and will be considered in the separate paper. It follows from Eq. (13) that \( V_{ez} \) is constant in zero-order approximation and without loss of the generality we put it to be equal to zero. In this approximation, we obtain from Eq. (12) the equation for \( V_{ey} \),

\[ \frac{\partial V_{ey} (z_e, t)}{\partial t} = - \frac{eE_{0y} (z_e)}{m_e} \sin \omega_0 t, \tag{16} \]

with solution

\[ V_{ey} (z_e, t) = \frac{eE_{0y} (z_e)}{m_e \omega_0} \cos \omega_0 t \]

(17)

\[ = \frac{e}{m_e} A_{0y} (z_e, t), \tag{17} \]

where \( E_{0y} (z_e) = E_{0y} e^{-\kappa z_e} \) is the local value of the amplitude of the \( E_{0y} \) field. We employ this local approximation for the electric field \( E_{0y} \), assuming small amplitude of the electron oscillation in RF field along coordinate \( z_e \).

Accounting for the terms of the first order in \( \kappa \xi_{e0} \ll 1 \) in Eq. (13), we find from the equation for \( V_{ez} \) that

\[ \frac{\partial V_{ez} (z_e, t)}{\partial t} = \frac{e \kappa E_{0y} (z_e)}{m_e \omega_0} V_{ey} (z_e, t) \cos \omega_0 t \]

(18)

\[ = \frac{e \kappa \xi_e E_{0y} (z_e)}{m_e \omega_0} \xi_e \cos \omega_0 t, \tag{18} \]

where

\[ \xi_e = \frac{eE_{0y}}{m_e \omega_0} \xi_e \]

(19)

is the amplitude of the local displacement of electron along the coordinate \( y \) at \( z_e \). Equation (18) is similar to the equation of the electron motion under the action of the ponderomotive force\(^{23}\),

\[ \frac{\partial V_{ez} (z_e, t)}{\partial t} = \frac{\kappa \xi_e}{2 m_e} E_{0y} (z_e) t \]
\[ + 1 \frac{\kappa \xi_e}{4 m_e} E_{0y} (z_e) \sin 2 \omega_0 t. \tag{20} \]

Solutions (17) and (19), derived in the local approximation, are valid only on the finite time interval at which \( - \kappa \int_0^{t'} V_{ez} (z_e, t'_1) dt'_1 \) in exponentials is much less than unity. By employing the method of successive approximations for the solution of the nonlinear Eqs. (12) and (13), with \( V_{ez} (z_e, t) \) determined by Eq. (20) as the starting approximation, we obtain for the time \( t \gg \omega_0^{-1} \) the following equation for \( V_{ey} (z_e, t) \)

\[ \frac{\partial V_{ey} (z_e, t)}{\partial t} = - \frac{eE_{0y} (z_e)}{m_e} e^{-\frac{\kappa^2 \omega_0^2 t^2}{2}} \sin \omega_0 t \tag{21} \]
with solution
\[ V_{ey}(z_e, t) = \frac{e E_{0y}(z_e)}{m_e \omega_0} e^{-\frac{1}{4} \kappa^2 \xi^2 \omega_0^2 \epsilon^2 t^2} \cos \omega_0 t \] (22)
which is valid for \( \kappa \xi \ll 1 \). In this approximation, Eq. \[ (23) \]
becomes
\[ \frac{\partial V_{xz}(z_e, t)}{\partial t} = \frac{e}{m_e} \kappa \xi E_{0y}(z_e) e^{-\frac{1}{4} \kappa^2 \xi^2 \omega_0^2 \epsilon^2 t^2} \cos \omega_0 t \] (23)
with solution
\[ V_{xz}(z_e, t) = \kappa \xi \frac{e}{2m_e} E_{0y}(z_e) t e^{-\frac{1}{4} \kappa^2 \xi^2 \omega_0^2 \epsilon^2 t^2} + \frac{1}{4} \kappa \xi e \frac{m_e \omega_0}{E_{0y}(z_e)} e^{-\frac{1}{4} \kappa^2 \xi^2 \omega_0^2 \epsilon^2 t^2} \sin \omega_0 t. \] (24)
Equation (24) reveals that the accelerated motion of electron in the spatially decaying RF field is limited by the time interval for which
\[ t < t^* = \frac{\sqrt{2}}{\kappa \xi \omega_0}, \] (25)
and it is slowing down after this time.

With velocity \( \mathbf{V}_e \) determined above, the Vlasov equation \[ (6) \]
becomes
\[ \frac{\partial F_e(v_e, \mathbf{r}_e, t)}{\partial t} + \mathbf{v}_e \frac{\partial F_e}{\partial \mathbf{r}_e} + \frac{e}{m_e} \nabla \phi (\mathbf{r}_e, t) \frac{\partial F_e}{\partial v_e} \\
+ \kappa \xi e \frac{m_e \omega_0}{e E_{0y}(z_e)} v_{xz} \sin \omega_0 t \frac{\partial F_e}{\partial v_e} \\
+ \kappa \xi e \frac{m_e \omega_0}{E_{0y}(z_e)} v_{y0} \omega_0 \sin \omega_0 t \frac{\partial F_e}{\partial \mathbf{v}_{xz}} = 0. \] (26)

Equation (26) and the Vlasov equation for ions jointly with the Poisson equation \[ (4) \] for the potential \( \phi (\mathbf{r}_e, t) \) compose basic system of equations. It is important to note, that the spatial inhomogeneity and time dependence in the zero order in \( \kappa \xi \) is excluded from the Maxwellian distribution \[ (5) \] in convective coordinates with velocity \( V_{ey} \) determined by Eq. \[ (17) \]. At the same time, the transition from \( v_z \) to \( v_{ez} \) introduces spatial inhomogeneity and time dependence of the first order in \( \kappa \xi \) to \( F_{0e} \). Therefore, the solution of the Vlasov equation \[ (20) \] for \( F_{e0}(v_{ez}, v_{ey}, z_e, t) \) may be presented in the form of power series in \( \kappa \xi \ll 1 \),
\[ F_{e0}(v_{ez}, v_{ey}, z_e, t) = F_{e0}^{(0)}(v_{ez}, v_{ey}) \\
+ F_{e0}^{(1)}(v_{ez}, v_{ey}, z_e, t), \] (27)
where
\[ F_{e0}^{(0)}(v_{ez}, v_{ey}) = \frac{n_{0e}}{2\pi v_T^2} \exp \left[ -\frac{v_{xz}^2}{2v_T^2} - \frac{v_{y0}^2}{2v_T^2} \right]. \] (28)
With expansion \[ (28) \] the spatial inhomogeneity and time dependence of \( F_{e0}(v_{ez}, v_{ey}, z_e, t) \) in the convective coordinates is determined by \( F_{e0}^{(1)} \), which is the solution of Eq. \[ (21) \] with \( \phi (v_e, \mathbf{r}_e, t) = 0 \),
\[ \frac{\partial F_{e0}^{(1)}}{\partial t} + v_{ee} \frac{\partial F_{e0}^{(1)}}{\partial v_e} \]
\[ = -\kappa \xi \omega_0 e^{-\frac{1}{4} \kappa^2 \xi^2 \omega_0^2 \epsilon^2 t^2} \frac{v_{ey}}{2} \left( e^{-i\omega_0 t} + e^{i\omega_0 t} \right) \frac{\partial F_{e0}^{(0)}}{\partial \mathbf{v}_{xz}}. \] (29)
With new characteristic variable \( \xi' = z_e - v_{ez} t \), the derivative over \( z_e \) is excluded from Eq. \[ (24) \] and the solution to Eq. \[ (24) \] becomes
\[ F_{e0}^{(1)}(v_{ey}, \xi', t) = -\kappa \xi \omega_0 e^{-\frac{1}{4} \kappa^2 \xi^2 \omega_0^2 \epsilon^2 t^2} \frac{v_{ey}}{2} \frac{\partial F_{e0}^{(0)}}{\partial \mathbf{v}_{xz}} \]
\[ \times \left( \frac{e^{i\omega_0 t}}{\omega_0 - \kappa \xi \mathbf{v}_{xz}} - \frac{e^{-i\omega_0 t}}{\omega_0 + \kappa \xi \mathbf{v}_{xz}} + \Psi (v_{ez}, v_{ey}) \right). \] (30)
The function \( \Psi (v_{ez}, v_{ey}) \) is determined by employing simple boundary conditions determined for different values of coordinate \( z_e \). The first condition is applied at \( z_e = \infty \) for the electrons moving from \( z_e = \infty \) toward plasma boundary \( z_e = 0 \), i.e. for electrons with velocity \( v_{ez} < 0 \). Because the electric field \( E_{0y}(z_e) \) vanishes at \( z_e = \infty \), the boundary condition \( F_{e0}^{(1)}(v_{ey}, 0, t) = 0 \) determines \( \Psi (v_{ez} < 0, v_{ey}) = 0 \) and
\[ F_{e0}^{(1)}(v_{ey}, v_{ez} < 0, \xi', t) = -\kappa \xi \omega_0 \frac{v_{ey}}{2} \frac{\partial F_{e0}^{(0)}}{\partial \mathbf{v}_{xz}} \]
\[ \times \left( \frac{e^{i\omega_0 t}}{\omega_0 - \kappa \xi \mathbf{v}_{xz}} - \frac{e^{-i\omega_0 t}}{\omega_0 - \kappa \xi \mathbf{v}_{xz}} + \frac{4 \kappa \xi \omega_0 v_{ez}}{\omega_0^2 + \kappa^2 \xi^2} \cos \omega_0 t \right). \] (31)
The second boundary condition is the condition of the specular reflection of electrons at the plasma boundary \( z = 0 \),
\[ F_{e0}^{(1)}(v_{ey}, v_{ez} < 0, 0, 0) = F_{e0}^{(1)}(v_{ey}, v_{ez} > 0, 0, 0). \] (32)
This condition determines the solution for electron distribution function \( F_{e0}^{(1)}(v_{ey}, v_{ez} > 0, \xi', t) \) in a form
\[ F_{e0}^{(1)}(v_{ey}, v_{ez} > 0, \xi', t) = -\kappa \xi \omega_0 e^{-\frac{1}{4} \kappa^2 \xi^2 \omega_0^2 \epsilon^2 t^2} \frac{v_{ey}}{2} \frac{\partial F_{e0}^{(0)}}{\partial \mathbf{v}_{xz}} \]
\[ \times \left[ \frac{\kappa \xi \omega_0 v_{ez}}{\omega_0^2 + \kappa^2 \xi^2} + \frac{4 \kappa \xi \omega_0 v_{ez}}{\omega_0^2 + \kappa^2 \xi^2} \cos \omega_0 t \right]. \] (33)

With the equilibrium distribution function \( F_{0e} \), determined by Eq. \[ (24) \], the Vlasov equation \[ (20) \] for the function \( f_e \) becomes
\[ \frac{\partial f_e (v_e, \mathbf{r}_e, t)}{\partial t} + v_{ee} \frac{\partial f_e}{\partial \mathbf{r}_e} \\
+ \kappa \xi e^{-\frac{1}{4} \kappa^2 \xi^2 \omega_0^2 \epsilon^2 t^2} v_{ey} \sin \omega_0 t \frac{\partial f_e}{\partial v_e} \]
The Fourier transform of Eq. (41) may be found in the form of power series in $\kappa \xi_e \ll 1$. In this paper, we obtain the solution to Eq. (41) for $f_i$ and $f_e$ in the zero order in $\kappa \xi_e$ and use them in the Poisson equation for the potential $\phi(r_e, t)$. On this way, we obtain the basic equations of the theory of the instabilities which may be developed in the anomalous skin layer of the inductively coupled plasma.

III. ELECTRON CONVECTING-OSCILLATING MODE

In the zero order in $\kappa \xi_e$, the equilibrium distribution functions $F_{e0,0}$ in the convective coordinates are determined by the spatially homogeneous functions $F_{e0,0}(v_{e,i})$, and the Vlasov equation (33) for $f_e(v_e, r_e, t)$ and similar equation for $f_i(v_i, r_i, t)$ do not contain the RF electric field in their convective-oscillating frames. Therefore the the equations for $f_i$ and $f_e$ will be the same as for the plasma without RF field, as

$$
\frac{\partial f_i}{\partial t} + v_i \frac{\partial f_i}{\partial r_i} - \frac{e}{m_i} \nabla \phi_i(r_i, t) \frac{\partial F_{e0}}{\partial v_i} = 0,
$$

$$
\frac{\partial f_e}{\partial t} + v_e \frac{\partial f_e}{\partial r_e} - \frac{e}{m_e} \nabla \phi_e(r_e, t) \frac{\partial F_{e0}}{\partial v_e} = 0.
$$

The solution of the linearised equations for $f_i$ Fourier transformed over $r_i$ is

$$
f_i(v_i, k_i, t) = \frac{e_i}{m_i} k_i \frac{\partial F_{e0}}{\partial v_i} \int_0^t dt_1 \phi_i(k_i, t_1) e^{-ik_i v_i (t-t_1)},
$$

where $\phi_i(k_i, t_1)$ is the Fourier transform of the potential $\phi_i(r_i, t_1)$ over $r_i$,

$$
\phi_i(k_i, t_1) = \frac{1}{(2\pi)^3} \int dr_i \phi_i(r_i, t_1) e^{-ik_i r_i}.
$$

The ion density perturbation $n_i(k_i, t)$ Fourier transformed over $r_i$ with the conjugate wave vector $k_i$ is

$$
n_i(k_i, t) = \int f_i(v_i, k_i, t) dv_i = \frac{e_i}{m_i} k_i \frac{\partial F_{e0}}{\partial v_i} \int_0^t dt_1 \phi_i(k_i, t_1) e^{-ik_i v_i (t-t_1)},
$$

The Fourier transform of $n_e(k_e, t)$ of the electron density perturbation performed in the electron frame is given by

$$
n_e(k_e, t) = \int f_e(v_e, k_e, t) dv_e = i \frac{e}{m_e} k_e
$$

which is the same as Eq. (39) for $n_i(k_i, t)$ with changing ion on electron subscripts.

The perturbations of the ion, (39), and electron, (40), densities are used in the Poisson equation (41) which may be the equation for $\phi_i(k_i, t_1)$ by the Fourier transform of Eq. (41) over $r_i$,

$$
k_i^2 \phi_i(k_i, t_1) = 4\pi e (n_i(k_i, t) - \int dr_i n_e(r_i, t) e^{-ik_i r_i}),
$$

or as the equation for $\phi_e(k_e, t)$ by the Fourier transform of Eq. (41) over $r_e$.

For the deriving the Poisson equation for $\phi_i(k_i, t)$ the Fourier transforms $n_e^{(i)}(k_i, t)$ and $\phi_e^{(i)}(k_i, t)$ of $n_e(r_e, t)$ and $\phi_e(r_e, t)$ over $r_i$ should be determined. Using Eq. (41), which determines the relations among the coordinates in the laboratory, ion and electron frames, we find that the electron density perturbation $n_e(r_e, t)$ Fourier transformed over $r_i$ is

$$
n_e^{(i)}(k_i, t) = \int dr_i n_e(r_e, t) e^{-ik_i r_i} = \int dr_e n_e(r_e, t)
$$

$$
\times \exp \left( -ik_i r_e - i{k_1 t} \int_0^t dt_1 (V_{ey} (t_1) - V_{iy} (t_1))
\right.
$$

$$
-ik_{iz} \int_0^t dt_1 (V_{ez} (t_1) - V_{iz} (t_1))
\right)
$$

where velocities $V_{ey}(t_1)$ and $V_{ez}(t_1)$ are determined by Eqs. (22) and (24). The velocities $V_{iy}(t_1)$ and $V_{iz}(t_1)$, which are determined by the same Eqs. (22) and (24) with subscript $i$ instead of $e$, are in $m_e/m_i$ times less than $V_{ey}$ and $V_{ez}$ and are neglected in what follows. With velocities $V_{ey}(t)$ and $V_{ez}(t)$ determined by Eqs. (22) and (24) relation (42) for time $t < t_s$ becomes

$$
n_e^{(i)}(k_i, t) = n_e^{(e)}(k_i, t) \exp \left( -i k_{iz} \frac{\kappa \xi_e}{2} \sin \omega_0 t
$$

$$
-\frac{1}{2} k_{iz} a_e t^2 + i k_{iz} n_e \cos 2\omega_0 t \right)
$$

where $\xi_e$ is determined by Eq. (19),

$$
n_e = n_e(z_e) = \frac{1}{8} \kappa \xi_e^2 (z_e)
$$

is the amplitude of the electron displacement in the RF electric field along coordinate $z_e$, and

$$
a_e = \frac{1}{2} \frac{e^2 \kappa E_{0y}^2}{m_e^2 \omega_0^2}
$$

as the characteristic time of change of perturbations.
is the electron acceleration under the action of the ponderomotive force.

The relation between the Fourier transform $\varphi_e(r_\xi,t)$ of the potential $\varphi_e(r_\xi,t)$ over $r_\xi$, involved in Eq. (40) for $n_e(r_\xi,t)$, and the Fourier transform $\varphi_i(k_i,t)$ of the potential $\varphi_e(r_\xi,t)$ over $r_\xi$ when it is used in $n_e^{(i)}(k_i,t)$, is derived similarly and is determined by the relation

$$
\varphi_e^{(i)}(k_i,t_1) = \exp \left( i k_{iy} r_\xi \sin \omega_0 t_1 + i \frac{1}{2} k_{iz} a_e t_1^2 \right.
\left. -i k_{iz} \eta_e \cos 2\omega_0 t_1 \right) \varphi_i(k_i,t_1),
$$

(46)

which follows from the identity $\varphi_e(r_\xi,t_1) = \varphi_i(r_\xi,t_1)$.

With Eq. (49) for $n_i(k_i,t)$, and with Eq. (43) for $n_e^{(i)}(k_i,t)$ in which potential is determined by (41), the Poisson equation (41) gives the following equation

$$
k^2 \varphi_i(k_i,t) = \frac{4 \pi i e^2}{m_i} \int dt_1 \varphi_i(k_i,t_1) e^{-ik_i \varphi_i(t-t_1)}
\times \int_0^t dt_1 \varphi_i(k_i,t_1) e^{-ik_i \varphi_i(t-t_1)}
\times \int_{-\infty}^{\infty} \frac{4 \pi i e^2}{m_e} \int d\varphi_i \frac{\partial F_{0_\varphi}}{\partial \varphi_i} \int dt_1 \varphi_i(k_i,t_1)
\times \exp \left( -ik_i \varphi_i(t-t_1) - i k_{iy} r_\xi (\sin \omega_0 t - \sin \omega_0 t_1) \right.
\left. -i \frac{1}{2} k_{iz} a_e (t^2 - t_1^2) + i k_{iz} \eta_e (\cos 2\omega_0 t - \cos 2\omega_0 t_1) \right)
$$

(47)

which determines the evolution of the electrostatic potential $\varphi_i(k_i,\omega)$ in the skin layer of an inductively coupled plasma. For the Maxwellian distribution $F_{0_\varphi}(\varphi_\alpha)$,

$$
F_{0_\varphi}(\varphi_\alpha) = \frac{\pi_0}{(2 \pi T_4)^{3/2}} \exp \left( -\frac{\varphi_\alpha^2}{2 T_4} \right),
$$

(48)

this equation becomes

$$
\varphi_i(k_i,t) + \omega_{pe}^2 \int_0^t dt_1 \varphi_i(k_i,t_1) (t-t_1) e^{-ik_i^2 \varphi_i(t-t_1)^2}
+ \omega_{pe}^2 \int_0^t dt_1 \varphi_i(k_i,t_1) (t-t_1) e^{-ik_i^2 \varphi_i(t-t_1)^2}
\times \exp \left( -ik_{iz} a_e (t^2 - t_1^2) - i k_{iy} r_\xi (\sin \omega_0 t - \sin \omega_0 t_1) \right.
\left. + i k_{iz} \eta_e (\cos 2\omega_0 t - \cos 2\omega_0 t_1) \right) = 0.
$$

(49)

It follows from Eq. (40) for $k_{iz} \sim k_{iy}$, that because

$$
\frac{k_{iy} \xi_e}{k_{iz} \eta_e} \sim \frac{8}{\kappa_{ce}} > 1,
$$

(50)

and

$$
an_e^2 t^2 \sim \kappa_{ce} \omega_2^2 t^2 > 1,
$$

(51)

when

$$
1 \gtrsim \kappa_{ce} \omega_0 t > \frac{1}{\omega_0 t}.
$$

(52)

the uniformly accelerating motion of electrons, which stems from the ponderomotive force, dominates over their oscillating motion. Therefore the possible instability of the skin layer under condition $\kappa_{ce} < 1$ at time $t < t_*$ is the current driven instability with accelerated electron current velocity, instead of the supposed instability of the parametric type.

IV. THE NONMODAL BUNEMAN INSTABILITY OF THE SKIN LAYER DRIVEN BY THE ACCELERATED ELECTRONS

In this section, we derive the solution to Eq. (40) for potential $\varphi_i(k_i,t)$ under condition (52) for time $t < t_*$. We will find the solution to Eq. (40) in the WKB-like form

$$
\varphi(k_i,t_1) = \varphi_i(k_i) e^{-i \int_0^t \omega(k_i,t_2) dt_2},
$$

(53)

where $\varphi(k_i) = \int_{-\infty}^{\infty} e^{ik_r r_\xi} \varphi(r_\xi,0) dr_\xi$ is the Fourier transform of the initial perturbation of $\varphi(r_\xi,t_1)$ at $t_1 = 0$. Then, Eq. (40) becomes

$$
\varphi(k_i) \left[ 1 + \omega_{pe}^2 \int_0^t dt_1 (t-t_1) e^{-i \int_0^t \omega(k_i,t_2) dt_2 - \frac{1}{2} k_i^2 v_e^2 (t-t_1)^2}
+ \omega_{pe}^2 \int_0^t dt_1 (t-t_1)
\times e^{i \int_0^t (\omega(k_i,t_2)+ik_{iz} a_e) dt_2 - \frac{1}{2} k_i^2 v_e^2 (t-t_1)^2} \right] = 0.
$$

(54)

We derive the asymptotics of the ion and electron terms of Eq. (54) in the hydrodynamic limit corresponding to the weak electron and ion Landau damping, for which $|\omega(k_i,t_1)| \gg k_i v_T i$ and $|\omega(k_i,t_1) + ik_{iz} a_e t_1| \gg k_i v_T e$. By integration by parts employing the presentation

$$
- i \int_0^t \omega(k_i,t_1) dt_1 = \frac{i}{\omega(k_i,t_1)} \frac{d}{dt} \left( - i \int_0^t \omega(k_i,t_2) dt_2 \right)
$$

(55)

in the ion term and the similar presentation for $i \int_0^t (\omega(k_i,t_2)+ik_{iz} a_e) dt_2$ in the electron term, we derive the equation

$$
\left[ 1 - \frac{\omega_{pe}^2}{\omega^2(k_i,t)} - \frac{\omega_{pe}^2}{(\omega(k_i,t) + k_{iz} a_e t)^2} \right]
= Q(k_i,t,t=0),
$$

(56)
where $Q(k_z, t, t = 0) = e^{-i\int_0^t \omega(k_z, t_1) dt_1}$ originates from the limit $t = 0$ of the integration by parts of Eq. (54). For the potential exponentially growing with time, for which $\text{Im} \omega(k_z, t_1) > 0$, the function $Q(k_z, t, t = 0)$ is exponentially small and may be neglected. Then, the left hand side of Eq. (56) forms the equation for the time dependent frequency $\omega(k_z, t)$. This equation is similar to the well known dispersion equation for the inductive discharge plasma of the inductive discharge is developed for the argon plasma. The stability theory of the skin layer, the spatial structure and density profile of which will affect on the propagation and absorption of the RF field in skin layer, will be developed on the base of the solution to system of Eqs. (26), (34) and the Poisson equation (4) with accounting for the terms of the first and higher order in $\kappa \xi (\zeta)$. We developed in Sec. IV the linear theory for the Buneman instability driven by the accelerated electron current. This instability displays fast non-modal growth of the electrostatic potential with growing with time growth rate $\gamma_{\max}$. The development of this instability by RF wave will affect on the propagation and absorption of the RF wave in plasma, to the nonlinear evolution of the skin layer, the spatial structure and density profile of which will depend on the level of the turbulence powered by the considered Buneman instability. The theory of these nonlinear processes may be developed on the base of the solution to system of Eqs. (55) and (57) and the Poisson equation (1) with accounting for the terms of the first and higher order in $\kappa \xi (\zeta)$.

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