Snowmass2021 Cosmic Frontier White Paper: Numerical relativity for next-generation gravitational-wave probes of fundamental physics

Francois Foucart\textsuperscript{1}, Pablo Laguna\textsuperscript{2}, Geoffrey Lovelace\textsuperscript{3}, David Radice\textsuperscript{4}, and Helvi Witek\textsuperscript{5}

\textsuperscript{1}Department of Physics & Astronomy, University of New Hampshire, Durham, New Hampshire 03824, USA
\textsuperscript{2}Center for Gravitational Physics and Department of Physics, The University of Texas at Austin, Austin, TX 78712, USA
\textsuperscript{3}Nicholas and Lee Begovich Center for Gravitational-Wave Physics and Astronomy, California State University, Fullerton, Fullerton, CA 92834, USA
\textsuperscript{4}Institute for Gravitation & the Cosmos, The Pennsylvania State University, University Park PA 16802, USA
\textsuperscript{5}Illinois Center for Advanced Studies of the Universe and Department of Physics, University of Illinois at Urbana-Champaign, Urbana, Illinois 61801, USA

Abstract

The next generation of gravitational-wave detectors, conceived to begin operations in the 2030s, will probe fundamental physics with exquisite sensitivity. These observations will measure the equation of state of dense nuclear matter in the most extreme environments in the universe, reveal with exquisite fidelity the nonlinear dynamics of warped spacetime, put general relativity to the strictest test, and perhaps use black holes as cosmic particle detectors. Achieving each of these goals will require a new generation of numerical relativity simulations that will run at scale on the supercomputers of the 2030s to achieve the necessary accuracy, which far exceeds the capabilities of numerical relativity and high-performance computing infrastructures available today.

Contents

1 Motivation 2

2 Gravitational waveform modeling 4

3 Nuclear physics and neutron stars 7

4 Modeling high-precision gravitational-wave observations 9

5 Testing gravity in the nonlinear regime 11

6 Black holes as cosmic particle detectors 12

7 Summary and future directions 12
Gravitational waves are ripples of warped spacetime that travel at the speed of light. In 2015, a century after Einstein predicted their existence, the Laser Interferometer Gravitational-Wave Observatory (Advanced LIGO) discovered gravitational waves from a merging binary black hole as the waves passed through Earth [1]. Two years later, LIGO and Virgo observed gravitational waves from merging neutron stars [2], a collision also observed by telescopes spanning the electromagnetic spectrum [3]. These events, together with the dozens of gravitational waves that LIGO and Virgo have observed, have inaugurated the era of gravitational-wave astronomy [4–6].

The next generation of gravitational-wave detectors will use gravitational waves from sources throughout the cosmos to probe fundamental physics with unprecedented sensitivity, as discussed in a separate Snowmass White Paper [7]. Proposed detectors on Earth include LIGO Voyager [8], Cosmic Explorer [9], Einstein Telescope [10], and NEMO [11]; future ground-based gravitational-wave facilities are described in a separate Snowmass [12]. In space, the Laser Interferometer Space Antenna (LISA) [13], the DECi-hertz Interferometer Gravitational-wave Observatory (DECIGO) [14], and TianQin [15] will observe gravitational waves at frequencies too low to ever detect on Earth because they would be obscured by seismic noise.

Next-generation gravitational-wave detectors are anticipated to begin observations in the 2030s. Their observations of coalescing binary neutron stars and black-hole/neutron-star binaries will measure the equation of state of dense nuclear matter in the most extreme environments in the universe, and their observations of gravitational waves from merging black holes—which contain the strongest spacetime curvature in the universe—will put general relativity to the strictest tests. These future gravitational-wave detectors might also enable observations that use black holes as cosmic particle detectors, potentially giving new, complementary insight into the nature of dark matter.

Accurate theoretical models of gravitational waves are critical for interpreting gravitational-wave observations—specifically, for inferring the nature and behavior of their sources. Long before the time of coalescence, the gravitational waves from merging black holes and neutron stars can be well modeled using the post-Newtonian approximation, which approximates general relativity in the limit of weak gravity and small velocities. Long after the time of coalescence, the gravitational waves from a black hole remnant resulting from merging black holes and neutron stars can be well approximated using perturbation theory. But near the time of coalescence, when the spacetime curvature and (if present) neutron-star matter are the most nonlinear and dynamic, all known analytic approximations break down: the emitted gravitational waves and the strong-gravity dynamics of their source can only be calculated with numerical relativity.

Numerical relativity amounts to numerically solving the equations of general relativity or, for simulations involving neutron stars, the equations of general relativistic radiation magnetohydrodynamics (fluid dynamics, magnetic fields, and radiation transport); the techniques of numerical relativity are reviewed, e.g., in Refs. [16–20], and briefly discussed in Sec. 2. Numerical-relativity calculations are technically challenging, in part because the equations are strongly nonlinear and, in the presence of neutron-star matter, because the solutions contain small scale features that are especially challenging to resolve, such as shocks, neutron-star surfaces, and turbulence. These cal-
Figure 1: Simulated gravitational-wave detector strain measurements of gravitational waves from two merging black holes. The signal is similar to GW150914 [1], the first directly detected gravitational waves. The strain is shown as a function of time for the signal superimposed on both simulated Cosmic Explorer noise (blue) and simulated LIGO A+ noise (yellow). Taken from Fig. 5.2 of Ref. [21].

Calculations are also computationally expensive, requiring high-performance computing to achieve the necessary accuracy.

How much accuracy is enough? The answer depends on the signal-to-noise ratio of an observation: roughly speaking, avoiding any bias in gravitational-wave interpretation requires numerical uncertainties smaller than the observation’s measurement uncertainty. The observations with the most potential to reveal new fundamental physics are those with the highest signal-to-noise ratios—precisely those observations that demand the most accuracy from numerical-relativity models. And the observations with the highest signal-to-noise ratios will come from next-generation detectors: their loudest observations will have signal-to-noise ratios in the thousands, more than an order of magnitude beyond the strongest signals observed to date. Figure 1 illustrates this gain in sensitivity by showing two simulated gravitational-wave detections of the same gravitational-wave source, one using an upgraded LIGO detector, and the other using Cosmic Explorer, a next-generation detector.

Extracting information from such high-fidelity signals while limiting systematic biases will require models with an order of magnitude increase in accuracy over today’s state of the art. Achieving this accuracy will require a new generation of numerical-relativity software, designed to run at scale on the exascale supercomputers that will be available in the 2030s. New (typically open-source) numerical-relativity codes under development today will help meet this goal by producing publicly available catalogs of simulated gravitational waveforms for coalescing compact binaries (i.e., binary black holes, binary neutron stars, and black-hole/neutron-star binaries).

The rest of this whitepaper is organized as follows. Sec. 2 briefly summarizes the methods of gravitational waveform modeling, especially numerical relativity. Then, Sec. 3 discusses progress and challenges in applying numerical relativity to probe nuclear physics and the nature of neutron
Figure 2: Numerical relativity waveform modeling GW150914, the first gravitational wave signal detected by LIGO. The inset shows the black holes’ horizons during the inspiral, merger and ringdown. Taken from Fig. 2 of Ref. [1].

stars. Sec. 4 explains in more quantitative detail the challenge that high-precision gravitational-wave observations pose to numerical-relativity waveform modeling. In Sec. 5, we discuss the importance of numerical-relativity waveform modeling in using gravitational-wave observations to seek physics beyond general relativity, and in Sec. 6 we discuss numerical relativity’s role in the possibility of using black holes as cosmic particle detectors. Finally, in Sec. 7 we present a brief summary and discuss the future work needed to fully realize the potential of gravitational waves as probes of fundamental physics.

2 Gravitational waveform modeling

A gravitational wave signal encodes vital information about its sources, such as the masses and spins of the companions in a compact binary, the equation of state of dense matter if one of them is a neutron star, and the underlying theory of gravity. These parameters are identified by using matched filtering techniques, in which the observed signal is compared against a catalog of theoretical gravitational waveform models, called templates. The templates need to accurately cover the different phases of a binary’s evolution consisting of the inspiral, merger and ringdown illustrated in Fig. 2.

Models of a binary’s evolution typically rely on two core methods: (i) approximations, such
as Post-Newtonian (PN) or Post-Minkowskian expansions, that are suitable for modeling the early inspiral of a compact binary using a weak-field and small velocity expansion; and (ii) numerical relativity, which numerically calculates a binary’s late inspiral, merger and ringdown by solving Einstein’s equations (or extensions of them) in the nonlinear regime. Both core methods feed into the production of full inspiral-merger-ringdown templates using either phenomenological models that directly combine PN and numerical relativity waveforms (e.g. [22]), effective-one-body models (e.g. [23]) that are a resummation of the PN expansion and are calibrated against numerical relativity, or surrogate models (e.g. [24]) that directly interpolate numerical-relativity waveforms. To ensure that the gravitational-wave interpretation is not limited by modeling errors, even as future gravitational wave detectors achieve ten to hundred times better sensitivity than today’s detectors, highly accurate waveform templates are crucial. In this white paper, new advances and challenges in numerical relativity are discussed, while new developments in using scattering amplitudes and effective field theory for gravitational-wave modeling are presented in a separate Snowmass White Paper [25]. A more extensive review on waveform modeling for future gravitational wave detectors can be found in the LISA Waveform Working Group White Paper [26].

Before outlining new physical applications and computational challenges below, we here give a brief summary of the current status of numerical relativity. Numerical relativity refers to solving Einstein’s equations, or extensions of them, possibly coupled to matter or additional fundamental fields, in four spacetime dimensions. This typically requires high-performance computing, because the equations form a system of more than ten coupled, nonlinear, partial differential equations (PDEs) of mixed character. By applying a spacetime decomposition into three dimensional, spatial hypersurfaces that are then propagated in time, the equations can be formulated as a time-evolution problem, subject to a set of constraints. Using this approach, a numerical-relativity calculation is divided into three stages:

1. Construction of initial data that represents the initial configuration (e.g., two compact objects orbiting each other in equilibrium). This requires solving the constraint equations, a set of coupled, elliptic-type PDEs in three dimensions. The bulk of contemporary numerical relativity software uses either the Bowen-York conformal approach or the conformal thin-sandwich method.

2. Time evolution, i.e., a binary’s development in time that is encoded in a set of coupled, hyperbolic-type PDEs and must be complemented by suitable gauge conditions. The majority of the numerical relativity codes uses either a variant of the generalized harmonic formulation of Einstein’s equations together with the damped harmonic gauge [27–29] or a variant of the Z4 [30–34] or Baumgarte-Shapiro-Shibata-Nakumara (BSSN) formulations [35, 36] that are complemented by the moving puncture approach [37, 38].

3. Extraction of physical information such as the gravitational and additional radiation or, in case of black hole spacetimes, the apparent horizons. Note that for modeling observations in distant gravitational-wave detectors, the gravitational radiation must be propagated to future null infinity (see Ref. [39] for a review), either by extrapolation [40], by evolving it, e.g. Cauchy-Characteristic Evolution (CCE) [41], or through perturbative techniques [42], and gauge conditions and transformations at future null infinity must be treated with care to yield well-behaved numerical waveforms.

Since the breakthroughs in numerical relativity in 2005 [27] and 2006 [37, 38], that saw the
| Code             | Open Source | Catalog | Formulation   | Hydro | Beyond GR |
|------------------|-------------|---------|---------------|-------|-----------|
| AMSS-NCKU [43–46] | Yes         | No      | BSSN/Z4c      | No    | Yes       |
| BAM [47–49]      | No          | [18]    | BSSN/Z4c      | Yes   | No        |
| BAMPS [50, 51]   | No          | No      | GHG           | Yes   | No        |
| COFFEE[52, 53]   | Yes         | No      | GCFE          | No    | Yes       |
| Dendro-GR [54–56] | Yes         | No      | BSSN/CCZ4     | No    | Yes       |
| Einstein Toolkit [57, 58] | Yes | No | BSSN/Z4c | Yes | No |
| *Canuda [59–62]  | Yes         | No      | BSSN/CCZ4     | No    | Yes       |
| *IllinoisGRMHD    | Yes         | No      | BSSN          | Yes   | No        |
| *LazEv [37, 64]  | No          | [65–68] | BSSN/CCZ4     | No    | No        |
| *Lean [69, 70]   | Partially   | No      | BSSN          | No    | Yes       |
| *MAYA [71]       | No          | [71]    | BSSN          | No    | Yes       |
| *NRPy+ [72]      | Yes         | No      | BSSN          | Yes   | No        |
| *SphericalNR [73, 74] | No | No | spherical BSSN | Yes | No |
| *THC [75–77]     | Yes         | [18]    | BSSN/Z4c      | Yes   | No        |
| ExaHyPE [78]     | Yes         | No      | CCZ4          | Yes   | No        |
| FIL[79]          | No          | No      | BSSN/Z4c/CCZ4 | Yes   | No        |
| FUKA [80, 81]    | Yes         | No      | XCTS          | Yes   | No        |
| GR-Athena++ [82] | Yes         | No      | Z4c           | Yes   | No        |
| GRChombo [83–85] | Yes         | No      | BSSN/CCZ4     | No    | Yes       |
| HAD [86–88]      | No          | No      | CCZ4          | Yes   | Yes       |
| Illinois GRMHD [89, 90] | No   | Yes | BSSN | Yes | No |
| MANGA/NRPy+ [91] | Partially   | No      | BSSN          | Yes   | No        |
| MHDuet [92, 93]  | No          | No      | CCZ4          | Yes   | Yes       |
| SACRA-MPI [94]   | No          | No      | BSSN/CCZ4     | Yes   | No        |
| SpEC [95, 96]    | No          | [96, 97]| GHG           | Yes   | Yes       |
| SpECTRE [98, 99] | Yes         | No      | GHG           | Yes   | No        |
| SPHINCS_BSSN [100]| No         | No      | BSSN          | No    | SPH       |

Table 1: List of numerical relativity codes. We indicate if a code is open-source, if it has been used to produce gravitational waveform catalogs, the formulation of Einstein’s equation used (GHG: generalized harmonic, BSSN: Baumgarte-Shapiro-Shibata-Nakamura, CCZ4 / Z4c variants of the Z4 formulation, GCFE: generalised conformal field equations ), if a code implements general relativistic hydrodynamics, and if it is capable to simulate compact binaries beyond general relativity. An asterisk indicates codes that are either (partially) based on the open-source Einstein Toolkit or are co-funded by its grant. Credit: Deidre Shoemaker; taken from Ref. [26].
very first simulations of the last orbits of a black hole binary and its merger, the field has matured into a state-of-the-art tool to investigate extreme gravity. A large variety of numerical relativity cyberinfrastructures for computational astrophysics is available. In Table 1, we present a list of currently available numerical-relativity software, indicating for each if it is open-source, the formulation of Einstein’s equations used, if it is capable of performing general relativistic hydrodynamics simulations (and not just vacuum simulations), and if it is capable to perform simulations in alternative theories of gravity. This list is adapted from the LISA Waveform Working Group White Paper [26].

A number of the numerical relativity codes (collaborations) have constructed catalogs of simulated gravitational waveforms as indicated in Table 1. For black-hole binaries, the combined catalogs contain more than 5,700 waveforms that cover mass ratios $q = m_1/m_2 = 1, \ldots, 15$ up to $q = 128$, where $m_1$ ($m_2$) is the mass of the heavier (lighter) black hole, and spins magnitudes up to 0.998 [65–68, 71, 96, 101, 102]. There are also the first numerical relativity waveform catalogs for binary neutron stars [18], and a recent study [103] used head-on collisions (in which the black holes begin at rest) to demonstrate that numerical-relativity techniques can in principle model gravitational-wave emission at mass ratios as high as 1000.

Given the wealth of available simulations, what future development is needed? The answer is two-fold and concerns the waveform accuracy as well as the physics included in the models. Each of these items will be discussed in detail in the following sections.

3 Nuclear physics and neutron stars

When two neutron stars, or a black hole and a neutron star, coalesce, they emit gravitational waves that encode the behavior of the densest matter in the universe. The cold cores of neutron stars are expected to have densities $\rho \sim 10^{15} \text{g/cm}^3$. In that regime, the strength of nuclear interactions between densely packed particles is uncertain, and even the composition of the core is unknown. The properties of dense matter are however tightly correlated with the size of neutron stars, their maximum mass, and their response to external gravitational fields. Dense matter’s presence in a merging binary, as a finite size object distorted by the gravitational field of its companion, leads to more gravitational wave emission than for black hole binaries and a faster evolution towards merger [104–111]. The size of a neutron star also determines if and when it can be tidally disrupted by a black hole companion (for black hole-neutron star binaries) and when two neutron stars collide and merge (for neutron star-neutron star binaries) [112–115]. Finally, the post-merger evolution of a neutron star-neutron star binary is strongly impacted by the properties of dense matter: unknown nuclear physics determines whether the remnant collapses to a black hole, as well as the frequency of post-merger gravitational waves driven by oscillations in the remnant [116–132]. Recovering this information from gravitational-wave observations requires an accurate theoretical understanding of the emitted waves and thus high-accuracy numerical relativity simulations.

These simulations are challenging and expensive, yet they must be sufficiently accurate to avoid introducing systematic biases into the interpretation of gravitational-wave observations. The accuracy required (cf. Sec. 4) increases with the square of the observation’s signal-to-noise ratio [133]. The first (and loudest) gravitational wave observation from coalescing binary neutron stars to date, GW170817 [2], had a signal-to-noise ratio (SNR) $\sim 30$. Recent studies [134–136] find that systematic uncertainties from inaccurate waveform models would be substantial at SNRs $\gtrsim 70$,
which could be achieved if a signal as loud as GW170817 were observed in current-generation detectors when they achieve their design sensitivities. The tremendous sensitivity gains that future gravitational-wave detector concepts [8–10] would achieve means that they would observe a GW170817-like signal with an SNR in the thousands [137], requiring vastly more accurate theoretical waveform models.

Simulations modeling the tidal response of the neutron stars during the last stages of the inspiral need to decrease their phase errors by more than two orders of magnitude. With current simulation technology, this is expected to require the grid resolution to be decreased by at least a factor 10 compared to the highest resolution simulations available to date [138, 139] (assuming second order convergence), leading to a $\sim 10^4$ increase in computational cost. Even if numerical relativity codes could scale efficiently to millions of CPU cores, a single simulation would still require several years to complete and tens of billions of CPU hours.

Neutron star mergers also power bright electromagnetic counterparts that carry additional information about the merging objects, including the properties of dense matter. Neutron-rich matter ejected during and after merger undergoes r-process nucleosynthesis, making neutron star mergers one of the lead candidates for the production site of r-process elements [140]. The radioactive decay of the ashes of the r-process powers kilonovae, UV/optical/infrared transients observable days to weeks after the merger [141–143]. Some neutron star mergers also result in the formation of massive accretion disks around a compact object that power narrow jets of highly-relativistic material observed from Earth as short gamma-ray bursts [144–146]. Both types of signals were observed following the first neutron star merger detection (GW170817) [147]. Numerical simulations are required to understand which mergers power electromagnetic signals and to connect the observable properties of these signals to the properties of the merging compact objects and to the equation of state of dense matter.

Simulations aiming to model the post-merger gravitational wave signal of neutron star binaries and to study their electromagnetic counterparts face the additional challenge of having to resolve high Reynolds number magneto-hydrodynamics turbulence and to model complex neutrino-radiation effects, which might impact the postmerger gravitational wave signal and will certainly impact the electromagnetic counterparts and nucleosynthesis yields of these events [125]. Extremely strong magnetic fields ($\sim 10^{16}$ G) are likely grown from small scale magneto-hydrodynamics instabilities in neutron star mergers, and even the highest resolution simulations performed to date (with grid spacing an order of magnitude smaller than what is typically affordable with current codes) have not been able to converge to a well-defined answer for the post-merger magnetic field [148]. As magnetic fields are likely a crucial ingredient in the production of short gamma-ray bursts [149–151] and in the ejection of the material producing r-process elements and kilonovae [151, 152], this represents a major limitation in our ability to model these systems. Neutrino-matter interactions are less important to the dynamics of the post-merger remnant, but they play a major role in setting the composition (neutron-richness) of matter outflows, which largely determines the outcome of r-process nucleosynthesis [153]. Properly including all relevant neutrino processes is a daunting challenge. At the very least, we will need to solve the 7-dimensional transport equations; but even that may not be enough. For example, neutrino oscillations due to fast-flavor instabilities may significantly impact the composition of matter outflows [154] and can only be captured by evolving the quantum kinetics equations with grid resolution orders of magnitude smaller than what is used in merger simulations.

A direct approach using current codes and numerical methods cannot be successful. Instead,
the numerical relativity community will need to develop more accurate numerical schemes to model tidally interacting neutron stars, sophisticated algorithms for neutrino-radiation hydrodynamics, and subgrid turbulence models. First steps in these directions have been made [76, 98, 155–157], but significant more work needs to be done in preparation for next-generation gravitational wave experiments. Several next-generation numerical relativity code are currently in development, employing novel methods that will enable high accuracy and performance on the supercomputers that will be available in the next decade (e.g. [82, 98, 155]), but none of them have yet matured to the point where they can calculate gravitational waves or electromagnetic signals from merging neutron-star binaries.

4 Modeling high-precision gravitational-wave observations

The next generation of gravitational-wave detectors on Earth and in space will yield observations of coalescing binary black holes with signal-to-noise ratios in the thousands, enabling high-fidelity observation of the behavior of the curved spacetime near stellar-mass black-hole horizons, the most strongly curved spacetime known. Gravitational wave signals will be so plentiful they will sometimes overlap.

As they have for current observations [2, 158, 159], numerical-relativity simulations will play crucial roles in the detection and interpretation of gravitational waves from merging black holes and neutron stars. In particular, waveforms from these simulations have been used to construct and validate approximate, phenomenological models necessary for interpreting observations (since numerical relativity is too costly to produce every model waveform needed) [158, 160–164], have featured in direct analysis of observations [165], and have helped validate our methods for detecting faint gravitational waves in detector data [166].

But to model high-precision observations, numerical-relativity calculations will have to be significantly more accurate than today’s state of the art. Qualitatively, the increase in accuracy is necessary to ensure that numerical errors are smaller than experimental uncertainty given the much lower noise level that next-generation gravitational-wave detectors will achieve (cf. Fig. 3). Section 4 of Ref. [96] gives a quantitative estimate of how much the accuracy must improve in terms of the improvement in signal to noise ratio, based on a sufficient condition [133, 167–169] for a model waveform and an observed gravitational waveform to be indistinguishable. Specifically, two gravitational waveforms are indistinguishable if their mismatch $\mathcal{M}$ (a noise-weighted inner product, defined, e.g., by Eq. (24) of Ref. [96]) is no larger than an amount proportional to the inverse square of the signal-to-noise ratio $\rho$:

$$\mathcal{M} < D / (2\rho^2),$$

where $D$ is a constant that depends on the number of parameters needed to specify the gravitational waveform (e.g., in Ref. [96], for binary-black-hole waveforms, $D = 8$). Thus the accuracy required scales as the square of the signal-to-noise ratio $\rho$.

With today’s detectors, the loudest gravitational waves from binary black holes have $\rho \sim 24$, whereas future detectors will observe binary black holes with $\rho$ in the thousands. The estimate in the previous paragraph suggests that future detectors will demand more than an order of magnitude more accuracy from numerical-relativity codes, even for modeling binary black holes (a less challenging case than the case of simulations involving dense matter, cf. Sec. 3). Studies using more sophisticated variants of the estimate sketched in the previous paragraph [170, 171] give comparable conclusions: numerical-relativity waveforms will need to be significantly more sensitive to
Figure 3: Predicted waveform accuracy for current second generation and future third generation detectors. The mismatch between the waveform models is shown as a function of the detector signal-to-noise ratio (SNR). Solid lines indicate results for pure numerical-relativity simulated signals, while dashed lines come from numerical-relativity signals extended (“hybridized”) with post-Newtonian (PN) waveforms in the inspiral. The blue lines and data points show how the mismatch falls with rising SNR. Horizontal red lines show the mismatch of the signal against the IMRPhenomPv2 phenomenological template waveform at the signal parameters for LIGO’s design sensitivity. Taken from Fig. 2 of Ref. [170].
avoid introducing bias into interpretation of high-precision gravitational-wave observations.

5 Testing gravity in the nonlinear regime

A consistent theory of quantum gravity is a major goal of modern physics. General relativity (GR) itself is not consistent with quantum mechanics, because it breaks down at high-energy scales: it is non-renormalizable and exhibits physical singularities, such as those inside black holes and at the big bang. Candidate quantum-gravity theories include well-motivated extensions of GR, typically involving additional fields, higher curvature corrections, or symmetry breaking [172–176].

Studies focusing on the formation or evolution of single black holes were considered in Lorentz-violating theories [177, 178], massive gravity [179–184], quadratic gravity [185–194], or higher curvature effective theories [195]. These have shown that black holes may develop scalar hair during their collapse, e.g., if described in quadratic and higher derivative gravity. In contrast, in scalar-tensor theories it is neutron stars that can develop a scalar hair, while black holes may remain the same as their general-relativistic counterparts. Given the extended phase-space of allowed, possibly hairy solutions one might expect new signatures during the inspiral, merger and ringdown such as additional (scalar) radiation channels, a phase-shift of the gravitational wave emission as compared to the GR signal or new nonlinear effects during the merger.

The nonlinear regime of gravity that unfolds during the collision of compact objects is a particularly promising target to probe for extensions of GR, both because new phenomena are expected to be most prominent in that case [196] and because candidate theories can be confronted with gravitational-wave observations [172, 175, 197–199]. However, current gravitational-wave based tests of gravity have either been limited to the weak-field regime or to null-tests against GR, because complete inspiral-merger-ringdown waveform models that capture these truly nonlinear beyond-GR effects are lacking. Numerical relativity has produced first proof-of-principle simulations beyond GR in scalar-tensor theories [70, 200–203], Einstein-Maxwell-Dilaton models [204, 205], cubic Horndeski theories [206], effect field theories for dark energy, namely k-essence [207–209], dynamical Chern-Simons gravity [210–212], or scalar Gauss-Bonnet gravity [62, 213–216]. A second body of work has studied the nonlinear dynamics of black-hole mimickers such as boson stars [217–222]. The effect of fluctuations near black holes’ horizons, mimicking for example microstate geometries, was modelled in Ref. [228].

An important difficulty when attempting simulations of binary mergers in beyond-GR theories is to devise mathematically well-posed and numerically stable formulations of the evolution equations. The development of formulations of Einstein’s equations amenable to numerical simulations took decades to come to fruition [27, 35, 36, 229–231], and repeating that work for every possible theory of gravity beyond general relativity is a daunting task. For Brans-Dicke type scalar tensor theories it was proven that the resulting time evolution equations are indeed well-posed [232]. More general scalar tensor theories of the Horndeski class can be cast in well-posed form if they are complemented with a modified generalized harmonic gauge as long as coupling parameters remain small [233]. Other theories for which hyperbolic formulations are available include $f(R)$ gravity [234], or Einstein-Aether theory [235]. On the other hand, there are a number of gravity theories that involve higher derivative terms that lead to (Ostrogradski) ghost instabilities and ill-posed evolution equations if they are treated as a complete theory. For example, this has been shown for dynamical Chern-Simons gravity, and one “cure” is to treat it as an effective field the-
ory [236]. Another remedy, proposed in Refs. [237, 238], and tested for a sixth-order model in [239], is a reformulation of the evolution equations in the spirit of Israel-Stewart theory for hydrodynamics.

Therefore, existing beyond-GR simulations have so far mainly focused on theories that can be recast as the evolution of a scalar or vector field coupled to the usual equations of general relativity (scalar-tensor, Maxwell-dilaton, boson stars), or on treating beyond-GR effects perturbatively [62, 210, 212–214]. That said, the calculations in the decoupling limit have already identified new dynamical effects that have been missed with weak-field approximations. This includes burst of scalar radiation during the merger of black holes in dynamical Chern-Simons gravity [210] or dynamical scalarization and descalarization of black holes in scalar Gauss-Bonnet gravity [214]. The gravitational waveform typically exhibits a phase-shift, compared to the vacuum GR case, due to additional radiation channels [62, 213, 216].

Enabling high precision tests of gravity and searches for signatures of new physics will likely require innovative theoretical avenues to devise well-posed formulations of beyond-GR theories, and their application to creating high-precision catalogs of simulated waveforms.

6 Black holes as cosmic particle detectors

Although dark matter makes up more than 80% of all matter in the universe, its nature, composition and properties have remained elusive. Black holes might shed light on the dark matter question and also ultralight beyond-standard model particles in general. Massive bosonic fields scattering off rotating black holes might form condensates around them if the fields’ Compton wavelength is comparable to the black holes’ size [240–242]. That is, astrophysical black holes in the mass range \( 5M_\odot \ldots 10^{10}M_\odot \) are sensitive to ultralight particles in the mass range \( 10^{-21}eV \ldots 10^{-8}eV \) [240, 241, 243]. This range includes popular dark matter candidates [244], the QCD axion [245] and axion-like particles of the string axiverse [246], as well as higher-spin fields such as vector fields [61, 243, 247–251] or massive spin-2 fields [252]; see also the companion Snowmass White Paper [7].

Because the underlying phenomenon of black hole superradiance only relies on gravitational interactions, it facilitates searches for new particles independently from their specific coupling to the standard model and thus complements traditional collider physics or direct detection experiments. The single black hole scenario has been studied extensively, and there are first computations of binary black-hole systems in the weak-field regime [253–255], for extreme mass ratio inspirals [256–258] and in the fully nonlinear regime modeling the last orbit before merger, the merger and ringdown [259]. How these light fields impact the nonlinear dynamics of the late inspiral and coalescence of black-hole binaries endowed with scalar condensates and what its observational signatures are remain open questions. Addressing them will enable gravitational-wave based searches for new particles but will require significant advances in numerical relativity.

7 Summary and future directions

The next generation of gravitational-wave detectors will probe fundamental physics with exquisite sensitivity. Observations with far higher signal-to-noise ratios than the loudest gravitational waves
observed to date will use neutron-star mergers to probe the nuclear physics of dense matter, use
the loudest observations of binary black holes to seek physics beyond general relativity, and will
perhaps enable a search for new particles that complements existing experimental searches.

Realizing these goals will require model waveforms that will rely on a new generation of
numerical-relativity codes capable of achieving dramatically improved accuracy. These codes will
need to use novel techniques (such as task-based parallelism) that enable them to scale to make
effective use of the exascale computing resources expected to be available in the coming decade.
Active development of such codes is already underway. Examples of next-generation numerical-
relativity codes include NMesh [260], Dendro-GR [54], GR-Athena++ [82], bamps [50], GR-
Chombo [83, 84] and SpECTRE [261].

Future studies will need to determine (more precisely than estimates such as in Refs. [170,
171]) how the challenges of extremely high signal-to-noise ratios and overlapping signals will
impact the accuracy required to prevent numerical-relativity simulations from biasing the interpre-
tation of next-generation gravitational-wave observations. One approach to such a study would
be to use numerical-relativity simulations to create simulated gravitational-wave detections and
then checking how much inaccuracies in model waveforms used to interpret those signals bias the
inferred properties.

These calculations will require significant computational resources to complete. A typical
numerical-relativity model waveform today typically require weeks to months of runtime on tens
to thousands of compute cores. Future waveforms will require additional computational cost, in
part because of higher accuracy requirements (which will require higher resolution) and in part be-
cause future detectors will have more sensitivity at lower frequencies, so that simulations will have
to be much longer to span the detectors’ sensitive frequency spaces. And many simulations will
be necessary to span the parameter space of potential signals. Binary-black-hole waveforms, for
instance, are characterized by at least 7 parameters (the mass ratio and the black-hole spin angular
momenta); even spanning this space requires thousands of simulations (for instance, choosing 3
distinct possible values for each parameter would yield $3^7 \approx 2,000$ simulations). Simulations in-
volving neutron stars depend on even more parameters, including the parameters characterizing the
(not yet well understood) neutron-star matter’s equation of state. Simulations in theories beyond
general relativity also introduce additional parameters.

By meeting the challenges ahead, numerical relativity will play a crucial role in realizing the
science goals of future gravitational-wave observatories, by enabling accurate, unbiased interpre-
tations of their high-fidelity observations.

8 List of Endorsers

Cosimo Bambi (Fudan University) [bambi@fudan.edu.cn]
Sambaran Banerjee (University of Bonn) [sambaran@astro.uni-bonn.de]
Enrico Barausse (SISSA and INFN Sezione di Trieste, Italy) [barausse@sissa.it]
Wayne Barkhouse (University of North Dakota) [wayne.barkhouse@und.edu]
Daniele Bertacca (University of Padova and INFN Sezione di Padova, Italy) [daniele.bertacca@pd.infn.it]
Emanuele Berti (Johns Hopkins University) [berti@jhu.edu]
Miguel Bezares (SISSA and INFN Sezione di Trieste, Italy) [mbezares@sissa.it]
Alexander Bonilla (Universidade Federal de Juiz de Fora) [abonilla@fisica.ufjf.br]
Richard Brito (CENTRA, Instituto Superior Técnico, Portugal) [richard.brito@tecnico.ulisboa.pt]
Liam Brodie (Washington University in Saint Louis) [b.liam@wustl.edu]
Marco Bruni (University of Portsmouth) [marco.bruni@port.ac.uk]
Tomasz Bulik (University of Warsaw) [tb@astrouw.edu.pl]
Manuela Campanelli (Rochester Institute of Technology) [manuela@astro.rit.edu]
Zhoujian Cao (Beijing Normal University) [zjcao@bnu.edu.cn]
Pedro R. Capelo (University of Zurich) [pcapelo@physik.uzh.ch]
Marco Cavaglià (Missouri University of Science and Technology) [cavagliam@mst.edu]
Jose A. R. Cembranos (Universidad Complutense de Madrid and IPARCOS, Spain) [cembrana.ucm.es]
Philip Chang (University of Wisconsin-Milwaukee) [chang65@uwm.edu]
Maria Chernyakova (Dublin City University) [masha.chernyakova@dcu.ie]
Cecilia Chirenti (University of Maryland) [chirenti@umd.edu]
Katy Clough (Queen Mary, University of London) [k.clough@qmul.ac.uk]
Lucas G. Colodel (University of Tübingen) [lucas.gardai-colodel@uni-tuebingen.de]
Mesut Çalışkan (Johns Hopkins University) [caliskan@jhu.edu]
Saurya Das (University of Lethbridge) [saurya.das@uleth.ca]
Tim Dietrich (University of Potsdam) [tim.dietrich@uni-potsdam.de]
Daniela Doneva (University of Tübingen) [daniela.doneva@uni-tuebingen.de]
Francisco Duque (CENTRA, IST, University of Lisbon) [francisco.duque@tecnico.ulisboa.pt]
Scott Field (University of Massachusetts Dartmouth) [sfield@umassd.edu]
Robert Eisenstein (Massachusetts Institute of Technology) [reisenst@mit.edu]
Zachariah B. Etienne (University of Idaho) [zetienne@uidaho.edu]
Pedro G. Ferreira (University of Oxford) [pedro.ferreira@physics.ox.ac.uk]
Pau Figueras (Queen Mary University of London) [p.figueras@qmul.ac.uk]
Giacomo Fragione (Northwestern University) [giacomo.fragione@northwestern.edu]
JörgFrauendiener (University of Otago) [joerg.frauendiener@otago.ac.nz]
Mandeep S. S. Gill (Stanford University) [msgill@slac.stanford.edu]
Andreja Gomboc (University of Nova Gorica) [andreja.gomboc@ung.si]
Leonardo Gualtieri (University of Rome “La Sapienza”) [leonardo.gualtieri@roma1.infn.it]
Roland Haas (University of Illinois) [rhaas@illinois.edu]
Alexander Haber (Washington University in Saint Louis) [ahaber@physics.wustl.edu]
Wen-Biao Han (Shanghai Astronomical Observatory, CAS) [wbhan@shao.ac.cn]
Mark Hannam (Cardiff University) [hannammd@cardiff.ac.uk]
Ian Hawke (University of Southampton) [I.Hawke@soton.ac.uk]
Lavinia Heisenberg (Heidelberg University/ETH Zurich) [laviniah@ethz.ch]
Thomas Helfer (Johns Hopkins University) [thelfer1@jhu.edu]
Shang-Jie Jin (Northeastern University, China) [jinshangjie@stumail.neu.edu.cn]
Cristian Joana (University of Louvain) [cristian.joana@uclouvain.be]
Atul Kedia (Rochester Institute of Technology) [asksma@rit.edu]
Antoine Klein (University of Birmingham) [antoine@star.sr.bham.ac.uk]
Shiho Kobayashi (Liverpool John Moores University) [s.kobayashi@ljmu.ac.uk]
Kostas Kokkotas (University of Tübingen) [kostas.kokkotas@uni-tuebingen.de]
Savvas M. Koussiappas (Brown University) [koussiappas@brown.edu]
Dicong Liang (Peking University) [dlchang@pku.edu.cn]
Steven L. Liebling (Long Island University) [steve.liebling@liu.edu]
References

[1] B. P. Abbott et al. Observation of Gravitational Waves from a Binary Black Hole Merger. Phys. Rev. Lett., 116(6):061102, 2016.

[2] B. P. Abbott et al. GW170817: Observation of Gravitational Waves from a Binary Neutron Star Inspiral. Phys. Rev. Lett., 119(16):161101, 2017.

[3] B. P. Abbott et al. Multi-messenger Observations of a Binary Neutron Star Merger.Astrophys. J. Lett., 848(2):L12, 2017.

[4] R. Abbott et al. GWTC-2: Compact Binary Coalescences Observed by LIGO and Virgo During the First Half of the Third Observing Run. Phys. Rev. X, 11:021053, 2021.

[5] B. P. Abbott et al. GWTC-1: A Gravitational-Wave Transient Catalog of Compact Binary Mergers Observed by LIGO and Virgo during the First and Second Observing Runs. Phys. Rev. X, 9(3):031040, 2019.
[6] R. Abbott et al. GWTC-3: Compact Binary Coalescences Observed by LIGO and Virgo During the Second Part of the Third Observing Run. 11 2021. https://arxiv.org/abs/2111.03606.

[7] Emanuele Berti, Vitor Cardoso, Zoltán Haiman, Daniel E. Holz, Emil Mottola, Suvodip Mukherjee, Bangalore Sathyaprakash, Xavier Siemens, and Nicolás Yunes. Snowmass2021 Cosmic Frontier White Paper: Fundamental Physics and Beyond the Standard Model. 3 2022. Submitted to the Proceedings of the US Community Study on the Future of Particle Physics (Snowmass 2021), eprint https://arxiv.org/abs/2203.06240.

[8] Rana X. Adhikari et al. Astrophysical science metrics for next-generation gravitational-wave detectors. Class. Quant. Grav., 36(24):245010, 2019.

[9] David Reitze et al. Cosmic Explorer: The U.S. Contribution to Gravitational-Wave Astronomy beyond LIGO. Bull. Am. Astron. Soc., 51:035, 7 2019.

[10] M. Punturo et al. The Einstein Telescope: A third-generation gravitational wave observatory. Class. Quant. Grav., 27:194002, 2010.

[11] K. Ackley et al. Neutron Star Extreme Matter Observatory: A kilohertz-band gravitational-wave detector in the global network. Publ. Astron. Soc. Austral., 37:e047, 2020.

[12] Stefan Ballmer, Rana Adhikari, Leonardo Badurina, Duncan A. Brown, Swapan Chattopadhyay, Peter Fritschel, Evan Hall, Jason M. Hogan, Karan Jani, Tim Kovachy, Kevin Kuns, Ariel Schwartzman, Daniel Sigg, Salvatore Vitale, Bram Slagmolen, and Christopher Wipf. Snowmass2021 cosmic frontier white paper: Future gravitational-wave detector facilities. 2022. Submitted to the Proceedings of the US Community Study on the Future of Particle Physics (Snowmass 2021), eprint https://arxiv.org/abs/2203.08228.

[13] Pau Amaro-Seoane et al. Laser Interferometer Space Antenna. 2 2017.

[14] Seiji Kawamura et al. Current status of space gravitational wave antenna DECIGO and B-DECIGO. PTEP, 2021(5):05A105, 2021.

[15] Jun Luo et al. TianQin: a space-borne gravitational wave detector. Class. Quant. Grav., 33(3):035010, 2016.

[16] Joshua A. Faber and Frederic A. Rasio. Binary Neutron Star Mergers. Living Rev. Rel., 15:8, 2012.

[17] Luca Baiotti and Luciano Rezzolla. Binary neutron star mergers: a review of Einstein’s richest laboratory. Rept. Prog. Phys., 80(9):096901, 2017.

[18] Tim Dietrich, David Radice, Sebastiano Bernuzzi, Francesco Zappa, Albino Perego, Bernd Brügmann, Swami Vivekanandji Chaurasia, Reetika Dudi, Wolfgang Tichy, and Maximiliano Ujevic. CoRe database of binary neutron star merger waveforms. Class. Quant. Grav., 35(24):24LT01, 2018.
[19] Matthew D. Duez and Yosef Zlochower. Numerical Relativity of Compact Binaries in the 21st Century. Rept. Prog. Phys., 82(1):016902, 2019.

[20] Tim Dietrich, Tanja Hinderer, and Anuradha Samajdar. Interpreting Binary Neutron Star Mergers: Describing the Binary Neutron Star Dynamics, Modelling Gravitational Waveforms, and Analyzing Detections. 4 2020.

[21] Matthew Evans et al. A Horizon Study for Cosmic Explorer: Science, Observatories, and Community. 9 2021.

[22] Geraint Pratten et al. Computationally efficient models for the dominant and subdominant harmonic modes of precessing binary black holes. Phys. Rev. D, 103(10):104056, 2021.

[23] Serguei Ossokine et al. Multipolar Effective-One-Body Waveforms for Precessing Binary Black Holes: Construction and Validation. Phys. Rev. D, 102(4):044055, 2020.

[24] Vijay Varma, Scott E. Field, Mark A. Scheel, Jonathan Blackman, Davide Gerosa, Leo C. Stein, Lawrence E. Kidder, and Harald P. Pfeiffer. Surrogate models for precessing binary black hole simulations with unequal masses. Phys. Rev. Research, 1:033015, Oct 2019.

[25] Alessandra Buonanno, Mohammed Khalil, Donal O'Connell, Radu Roiban, Mikhail P. Solon, and Mao Zeng. Snowmass white paper: Gravitational waves and scattering amplitudes. 2022. To be submitted to the Proceedings of the US Community Study on the Future of Particle Physics (Snowmass 2021).

[26] LISA Waveform Working Group. The LISA Waveform Working Group White Paper. 2022. in prep.

[27] Frans Pretorius. Evolution of binary black hole spacetimes. 95:121101, 2005.

[28] Lee Lindblom, Mark A. Scheel, Lawrence E. Kidder, Robert Owen, and Oliver Rinne. A New generalized harmonic evolution system. Class. Quant. Grav., 23:S447–S462, 2006.

[29] Bela Szilagyi, Lee Lindblom, and Mark A. Scheel. Simulations of Binary Black Hole Mergers Using Spectral Methods. Phys. Rev. D, 80:124010, 2009.

[30] C. Bona, T. Ledvinka, C. Palenzuela, and M. Zacek. General covariant evolution formalism for numerical relativity. Phys. Rev. D, 67:104005, 2003.

[31] Sebastiano Bernuzzi and David Hilditch. Constraint violation in free evolution schemes: Comparing BSSNOK with a conformal decomposition of Z4. Phys. Rev. D, 81:084003, 2010.

[32] Andreas Weyhausen, Sebastiano Bernuzzi, and David Hilditch. Constraint damping for the Z4c formulation of general relativity. Phys. Rev. D, 85:024038, 2012.

[33] Daniela Alic, Carles Bona-Casas, Carles Bona, Luciano Rezzolla, and Carlos Palenzuela. Conformal and covariant formulation of the Z4 system with constraint-violation damping. Phys. Rev. D, 85:064040, 2012.
[34] Daniela Alic, Wolfgang Kastaun, and Luciano Rezzolla. Constraint damping of the conformal and covariant formulation of the Z4 system in simulations of binary neutron stars. *Phys. Rev. D*, 88(6):064049, 2013.

[35] T. W. Baumgarte and S. L. Shapiro. On the numerical integration of Einstein’s field equations. 59:024007, 1999. gr-qc/9810065.

[36] M. Shibata and T. Nakamura. Evolution of three-dimensional gravitational waves: Harmonic slicing case. 52:5428–5444, 1995.

[37] Manuela Campanelli, C. O. Lousto, P. Marronetti, and Y. Zlochower. Accurate evolutions of orbiting black-hole binaries without excision. *Phys. Rev. Lett.*, 96:111101, 2006.

[38] John G. Baker, Joan Centrella, Dae-II Choi, Michael Koppitz, and James van Meter. Gravitational wave extraction from an inspiraling configuration of merging black holes. *Phys. Rev. Lett.*, 96:111102, 2006.

[39] Nigel T. Bishop and Luciano Rezzolla. Extraction of Gravitational Waves in Numerical Relativity. *Living Rev. Rel.*, 19:2, 2016.

[40] Michael Boyle and Abdul H. Mroue. Extrapolating gravitational-wave data from numerical simulations. *Phys. Rev. D*, 80:124045, 2009.

[41] Nigel T. Bishop, Roberto Gomez, Luis Lehner, and Jeffrey Winicour. Cauchy-characteristic extraction in numerical relativity. *Phys. Rev. D*, 54:6153–6165, 1996.

[42] Hiroyuki Nakano, James Healy, Carlos O. Lousto, and Yosef Zlochower. Perturbative extraction of gravitational waveforms generated with Numerical Relativity. *Phys. Rev. D*, 91(10):104022, 2015.

[43] Zhoujian Cao, Hwei-Jang Yo, and Jui-Ping Yu. Reinvestigation of moving punctured black holes with a new code. *Phys. Rev. D*, 78:124011, Dec 2008.

[44] Zhoujian Cao, Pablo Galaviz, and Li-Fang Li. Binary black hole mergers in $f(r)$ theory. *Phys. Rev. D*, 87:104029, May 2013.

[45] Zhoujian Cao and David Hilditch. Numerical stability of the z4c formulation of general relativity. *Phys. Rev. D*, 85:124032, Jun 2012.

[46] David Hilditch, Sebastiano Bernuzzi, Marcus Thierfelder, Zhoujian Cao, Wolfgang Tichy, and Bernd Brügmann. Compact binary evolutions with the z4c formulation. *Phys. Rev. D*, 88:084057, Oct 2013.

[47] Bernd Brügmann, Jose A. Gonzalez, Mark Hannam, Sascha Husa, Ulrich Sperhake, and Wolfgang Tichy. Calibration of Moving Puncture Simulations. *Phys. Rev. D*, 77:024027, 2008.

[48] Marcus Thierfelder, Sebastiano Bernuzzi, and Bernd Brügmann. Numerical relativity simulations of binary neutron stars. *Phys. Rev. D*, 84:044012, 2011.
[49] Tim Dietrich, Sebastiano Bernuzzi, Maximiliano Ujevic, and Bernd Brügmann. Numerical relativity simulations of neutron star merger remnants using conservative mesh refinement. *Phys. Rev. D*, 91(12):124041, 2015.

[50] Marcus Bugner, Tim Dietrich, Sebastiano Bernuzzi, Andreas Weyhausen, and Bernd Brügmann. Solving 3D relativistic hydrodynamical problems with weighted essentially nonoscillatory discontinuous Galerkin methods. *Phys. Rev.*, D94(8):084004, 2016.

[51] David Hilditch, Andreas Weyhausen, and Bernd Brügmann. Pseudospectral method for gravitational wave collapse. *Phys. Rev. D*, 93(6):063006, 2016.

[52] Georgios Doulis, Jörg Frauendiener, Chris Stevens, and Ben Whale. COFFEE – An MPI-parallelized Python package for the numerical evolution of differential equations. 3 2019.

[53] Jörg Frauendiener and Chris Stevens. The non-linear perturbation of a black hole by gravitational waves. I. The Bondi–Sachs mass loss. *Class. Quant. Grav.*, 38(19):194002, 2021.

[54] Milinda Fernando, David Neilsen, Hyun Lim, Eric Hirschmann, and Hari Sundar. Massively Parallel Simulations of Binary Black Hole Intermediate-Mass-Ratio Inspirals. *SIAM J. Sci. Comput.*, 41:C97, 2018.

[55] Milinda Fernando, David Neilsen, Hyun Lim, Eric Hirschmann, and Hari Sundar. Massively parallel simulations of binary black hole intermediate-mass-ratio inspirals. *SIAM Journal on Scientific Computing*, 41(2):C97–C138, 2019.

[56] Milinda Fernando, David Neilsen, Eric W Hirschmann, and Hari Sundar. A scalable framework for adaptive computational general relativity on heterogeneous clusters. In *Proceedings of the ACM International Conference on Supercomputing*, pages 1–12, 2019.

[57] Frank Löffler et al. The Einstein Toolkit: A Community Computational Infrastructure for Relativistic Astrophysics. *Class. Quant. Grav.*, 29:115001, 2012.

[58] Steven R. Brandt, Gabriele Bozzola, Cheng-Hsin Cheng, Peter Diener, Alexandru Dima, William E. Gabella, Miguel Gracia-Linares, Roland Haas, Yosef Zlochower, Miguel Alcubierre, Daniela Alic, Gabrielle Allen, Marcus Ansorg, Maria Babiuc-Hamilton, Luca Baiotti, Werner Benger, Eloisa Bentivegna, Sebastiano Bernuzzi, Tanja Bode, Brockton Brendal, Bernd Bruegmann, Manuela Campanelli, Federico Cipolletta, Giovanni Covino, Samuel Cupp, Roberto De Pietri, Harry Dimmelmeier, Rion Dooley, Nils Dorband, Matthew Elley, Yaakoub El Khamra, Zachariah Etienne, Joshua Faber, Toni Font, Joachim Frieben, Bruno Giacomazzo, Tom Goodale, Carsten Gundlach, Ian Hawke, Scott Hawley, Ian Hinder, E. A. Huerta, Sascha Husa, Sai Iyer, Daniel Johnson, Abhishek V. Joshi, Wolfgang Kastaun, Thorsten Kellermann, Andrew Knapp, Michael Koppitz, Pablo Laguna, Gerd Lanferman, Frank Löffler, Joan Masso, Lars Menger, Andre Merzky, Jonah Maxwell Miller, Mark Miller, Philipp Moesta, Pedro Montero, Bruno Mundim, Andrea Nerozzi, Scott C. Noble, Christian Ott, Ravi Paruchuri, Denis Pollney, David Radice, Thomas Radke, Christian Reisswig, Luciano Rezzolla, David Rideout, Matei Ripeanu, Lorenzo Sala, Jascha A Schewtschenko, Erik Schnetter, Bernard Schutz, Ed Seidel, Eric Seidel, John Shalf, Ken Sible, Ulrich Sperhake, Nikolaos Stergioulas, Wai-Mo Suen, Bela Szilagyi,
Ryoji Takahashi, Michael Thomas, Jonathan Thornburg, Malcolm Tobias, Aaryn Tonita, Paul Walker, Mew-Bing Wan, Barry Wardell, Leonardo Werneck, Helvi Witek, Miguel Zilhão, and Burkhard Zink. The einstein toolkit, December 2021. To find out more, visit http://einsteintoolkit.org.

[59] Helvi Witek, Miguel Zilhão, Gabriele Bozolla, Matthew Elley, Giuseppe Ficarra, Taishi Ikeda, Nicolas Sanchis-Gual, and Hector Silva. Canuda: a public numerical relativity library to probe fundamental physics, October 2021.

[60] Hirotada Okawa, Helvi Witek, and Vitor Cardoso. Black holes and fundamental fields in Numerical Relativity: initial data construction and evolution of bound states. Phys. Rev. D, 89(10):104032, 2014.

[61] Miguel Zilhão, Helvi Witek, and Vitor Cardoso. Nonlinear interactions between black holes and Proca fields. Class. Quant. Grav., 32:234003, 2015.

[62] Helvi Witek, Leonardo Gualtieri, Paolo Pani, and Thomas P. Sotiriou. Black holes and binary mergers in scalar Gauss-Bonnet gravity: scalar field dynamics. Phys. Rev. D, 99(6):064035, 2019.

[63] Zachariah B. Etienne, Vasileios Paschalidis, Roland Haas, Philipp Mösta, and Stuart L. Shapiro. IllinoisGRMHD: An Open-Source, User-Friendly GRMHD Code for Dynamical Spacetimes. Class. Quant. Grav., 32:175009, 2015.

[64] Y. Zlochower, J. G. Baker, Manuela Campanelli, and C. O. Lousto. Accurate black hole evolutions by fourth-order numerical relativity. Phys. Rev. D, 72:024021, 2005.

[65] James Healy, Carlos O. Lousto, Yosef Zlochower, and Manuela Campanelli. The RIT binary black hole simulations catalog. Class. Quant. Grav., 34(22):224001, 2017.

[66] James Healy, Carlos O. Lousto, Jacob Lange, Richard O’Shaughnessy, Yosef Zlochower, and Manuela Campanelli. Second RIT binary black hole simulations catalog and its application to gravitational waves parameter estimation. Phys. Rev. D, 100(2):024021, 2019.

[67] James Healy and Carlos O. Lousto. Third RIT binary black hole simulations catalog. Phys. Rev. D, 102(10):104018, 2020.

[68] James Healy and Carlos O. Lousto. The Fourth RIT binary black hole simulations catalog: Extension to Eccentric Orbits. 1 2022.

[69] Ulrich Sperhake. Binary black-hole evolutions of excision and puncture data. Phys. Rev. D, 76:104015, 2007.

[70] Emanuele Berti, Vitor Cardoso, Leonardo Gualtieri, Michael Horbatsch, and Ulrich Sperhake. Numerical simulations of single and binary black holes in scalar-tensor theories: circumventing the no-hair theorem. Phys. Rev. D, 87(12):124020, 2013.

[71] Karan Jani, James Healy, James A. Clark, Lionel London, Pablo Laguna, and Deirdre Shoemaker. Georgia Tech Catalog of Gravitational Waveforms. Class. Quant. Grav., 33(20):204001, 2016.
[72] Ian Ruchlin, Zachariah B. Etienne, and Thomas W. Baumgarte. SENR/NRPy+: Numerical Relativity in Singular Curvilinear Coordinate Systems. *Phys. Rev. D*, 97(6):064036, 2018.

[73] Vassilios Mewes, Yosef Zlochower, Manuela Campanelli, Ian Ruchlin, Zachariah B. Etienne, and Thomas W. Baumgarte. Numerical relativity in spherical coordinates with the Einstein Toolkit. *Phys. Rev. D*, 97(8):084059, 2018.

[74] Vassilios Mewes, Yosef Zlochower, Manuela Campanelli, Thomas W. Baumgarte, Zachariah B. Etienne, Federico G. Lopez Armengol, and Federico Cipolletta. Numerical relativity in spherical coordinates: A new dynamical spacetime and general relativistic MHD evolution framework for the Einstein Toolkit. *Phys. Rev. D*, 101(10):104007, 2020.

[75] David Radice and Luciano Rezzolla. THC: a new high-order finite-difference high-resolution shock-capturing code for special-relativistic hydrodynamics. *Astron. Astrophys.*, 547:A26, 2012.

[76] David Radice, Luciano Rezzolla, and Filippo Galeazzi. Beyond second-order convergence in simulations of binary neutron stars in full general-relativity. *Mon. Not. Roy. Astron. Soc.*, 437:L46–L50, 2014.

[77] David Radice, Luciano Rezzolla, and Filippo Galeazzi. High-Order Fully General-Relativistic Hydrodynamics: new Approaches and Tests. *Class. Quant. Grav.*, 31:075012, 2014.

[78] Sven Köppel. Towards an exascale code for GRMHD on dynamical spacetimes. *J. Phys. Conf. Ser.*, 1031(1):012017, 2018.

[79] Elias R. Most, L. Jens Papenfort, and Luciano Rezzolla. Beyond second-order convergence in simulations of magnetized binary neutron stars with realistic microphysics. *Mon. Not. Roy. Astron. Soc.*, 490(3):3588–3600, 2019.

[80] L. Jens Papenfort, Samuel D. Tootle, Philippe Grandclément, Elias R. Most, and Luciano Rezzolla. New public code for initial data of unequal-mass, spinning compact-object binaries. *Phys. Rev. D*, 104(2):024057, 2021.

[81] https://kadath.obspm.fr/fuka/ and ETK thorns at https://bitbucket.org/fukaws/.

[82] Boris Daszuta, Francesco Zappa, William Cook, David Radice, Sebastiano Bernuzzi, and Viktoria Morozova. GR-Athena++: Puncture Evolutions on Vertex-centered Oct-tree Adaptive Mesh Refinement. *Astrophys. J. Supp.*, 257(2):25, 2021.

[83] Katy Clough, Pau Figueras, Hal Finkel, Markus Kunesh, Eugene A. Lim, and Saran Tunyasuvunakool. GRChombo : Numerical Relativity with Adaptive Mesh Refinement. *Class. Quant. Grav.*, 32(24):245011, 2015.

[84] https://www.grchombo.org.
[85] Tomas Andrade et al. GRChombo: An adaptable numerical relativity code for fundamental physics. *J. Open Source Softw.*, 6:3703, 2021.

[86] http://had.liu.edu/.

[87] Steven L. Liebling. The Singularity threshold of the nonlinear sigma model using 3-D adaptive mesh refinement. *Phys. Rev. D*, 66:041703, 2002.

[88] Luis Lehner, Steven L. Liebling, and Oscar Reula. AMR, stability and higher accuracy. *Class. Quant. Grav.*, 23:S421–S446, 2006.

[89] Zachariah B. Etienne, Yuk Tung Liu, and Stuart L. Shapiro. Relativistic magnetohydrodynamics in dynamical spacetimes: A new AMR implementation. *Phys. Rev. D*, 82:084031, 2010.

[90] Lunan Sun, Milton Ruiz, Stuart L. Shapiro, and Antonios Tsokaros. Jet Launching from Binary Neutron Star Mergers: Incorporating Neutrino Transport and Magnetic Fields. 2 2022.

[91] Philip Chang and Zachariah Etienne. General relativistic hydrodynamics on a moving-mesh I: static space–times. *Mon. Not. Roy. Astron. Soc.*, 496(1):206–214, 2020.

[92] Carlos Palenzuela, Borja Miñano, Daniele Viganò, Antoni Arbona, Carles Bona-Casas, Andreu Rigo, Miguel Bezares, Carles Bona, and Joan Massó. A Simflowny-based finite-difference code for high-performance computing in numerical relativity. *Class. Quant. Grav.*, 35(18):185007, 2018.

[93] Steven L. Liebling, Carlos Palenzuela, and Luis Lehner. Toward fidelity and scalability in non-vacuum mergers. *Class. Quant. Grav.*, 37(13):135006, 2020.

[94] Kenta Kiuchi, Kyohei Kawaguchi, Koutarou Kyutoku, Yuichiro Sekiguchi, and Masaru Shibata. Sub-radian-accuracy gravitational waves from coalescing binary neutron stars in numerical relativity. II. Systematic study on the equation of state, binary mass, and mass ratio. *Phys. Rev. D*, 101(8):084006, 2020.

[95] Lawrence Kidder, Harald Pfeiffer, Mark Scheel, Matthew Duez, Francois Foucart, Béla Szilágyi, Dan Hemberger, Lee Lindblom, Andy Bohn, Michael Boyle, Luisa Buchman, M. Brett Deaton, Nils Deppe, Roland Haas, Francois Hebert, Kate Henriksen, Stephen Lau, Geoffrey Lovelace, Curran Muhlberger, Sergei Ossokine, Rob Owen, Saul Teukolsky, Will Throwe, Kevin Barkett, Thomas Baumgarte, Jonathan Blackman, Wyatt Brege, Jean-drew Brink, Tony Chu, Michael Cohen, Gregory Cook, Tim Dietrich, Matt Giesler, Jason Grigsby, Casey Handmer, Frank Herrmann, Ian Hinder, Jeff Kaplan, Rez Khan, Prayush Kumar, Adam Lewis, François Limousin, Jonas Lippuner, Keith Matthews, Abdul Mroué, Lydia Nevin, Fatemeh Nouri, Maria Okounkova, David Radice, Oliver Rinne, Olivier Sarbach, Deirdre Shoemaker, Leo C. Stein, Nick Tacik, Nick Taylor, Manuel Tiglio, Olivier Sarbach, Trevor Vincent, John Wendell, Catherine Woodford, Anil Zenginoglu, Fan Zhang, Aaron Zimmerman, Nousha Afshari, Aliya Babul, Adam Bartnik, Deshpriit Bedi, Darius Bunandar, Iryna Butsky, Patrick Calhoun, Sourabh Chakraborty, Cameron Cogburn,
Nick Demos, Patrick Fraser, Alyssa Garcia, Bryant Garcia, Yi Chen Hu, Daniel Jones, Haroon Khan, Dave Kotfis, Dongjun Li, Yor Limkumnerd, Ian MacCormack, Tamin Mansour, Robert McGehee, Dmitry Meyerson, Adam Neumann, Amin Nikbin, Hiroaki Oyaizu, Daniel Parada, Jennifer Seiler, Haolin Shi, Keara Soloway, Alexandre Streicher, and Allen Sussman. Spectral einstein code. \url{http://www.black-holes.org/SpEC.html}, 2021.

[96] Michael Boyle, Daniel Hemberger, Dante A.B. Iozzo, Geoffrey Lovelace, Serguei Ossokine, Harald P. Pfeiffer, Mark A. Scheel, Leo C. Stein, Charles J. Woodford, Aaron B. Zimmerman, Nousha Afshari, Kevin Barkett, Jonathan Blackman, Katerina Chatziioannou, Tony Chu, Nicholas Demos, Nils Deppe, Scott E. Field, Nils L. Fischer, Evan Foley, Heather Fong, Alyssa Garcia, Matthew Giesler, Francois Hebert, Ian Hinder, Reza Katebi, Haroon Khan, Lawrence E. Kidder, Prayush Kumar, Kevin Kuper, Halston Lim, Maria Okounkova, Teresita Ramirez, Samuel Rodriguez, Hannes R. Rüter, Patricia Schmidt, Bela Szilagyi, Saul A. Teukolsky, Vijay Varma, and Marissa Walker. The SXS Collaboration catalog of binary black hole simulations. \textit{Class. Quant. Grav.}, 36(19):195006, 2019.

[97] \url{http://www.black-holes.org/waveforms}.

[98] Lawrence E. Kidder et al. SpECTRE: A Task-based Discontinuous Galerkin Code for Relativistic Astrophysics. \textit{J. Comput. Phys.}, 335:84–114, 2017.

[99] Nils Deppe, William Throwe, Lawrence E. Kidder, Nils L. Vu, François Hébert, Jordan Moxon, Cristóbal Armaza, Gabriel S. Bonilla, Yoonsoo Kim, Prayush Kumar, Geoffrey Lovelace, Alexandra Macedo, Kyle C. Nelli, Eamonn O’Shea, Harald P. Pfeiffer, Mark A. Scheel, Saul A. Teukolsky, Youshia Afshari, Kevin Barkett, Jonathan Blackman, Katerina Chatziioannou, Tony Chu, Nicholas Demos, Nils Deppe, Scott E. Field, Nils L. Fischer, Evan Foley, Heather Fong, Alyssa Garcia, Matthew Giesler, Francois Hebert, Ian Hinder, Reza Katebi, Haroon Khan, Lawrence E. Kidder, Prayush Kumar, Kevin Kuper, Halston Lim, Maria Okounkova, Teresita Ramirez, Samuel Rodriguez, Hannes R. Rüter, Patricia Schmidt, Bela Szilagyi, Saul A. Teukolsky, Vijay Varma, and Marissa Walker. Spectre, June 2022. \url{https://doi.org/10.5281/zenodo.6335350}.

[100] S. Rosswog and P. Diener. SPHINCS_BSSN: A general relativistic Smooth Particle Hydrodynamics code for dynamical spacetimes. \textit{Class. Quant. Grav.}, 38(11):115002, 2021.

[101] Abdul H. Mroue et al. Catalog of 174 Binary Black Hole Simulations for Gravitational Wave Astronomy. \textit{Phys. Rev. Lett.}, 111(24):241104, 2013.

[102] Carlos O. Lousto and James Healy. Exploring the Small Mass Ratio Binary Black Hole Merger via Zeno’s Dichotomy Approach. \textit{Phys. Rev. Lett.}, 125(19):191102, 2020.

[103] Carlos O. Lousto and James Healy. Study of the Intermediate Mass Ratio Black Hole Binary Merger up to 1000:1 with Numerical Relativity. 3 2022.

[104] Thibault Damour, Alessandro Nagar, and Loic Villain. Measurability of the tidal polarizability of neutron stars in late-inspiral gravitational-wave signals. \textit{Phys. Rev. D}, 85:123007, 2012.
[105] Jocelyn S. Read, Luca Baiotti, Jolien D. E. Creighton, John L. Friedman, Bruno Giacomazzo, Koutarou Kyutoku, Charalampos Markakis, Luciano Rezzolla, Masaru Shibata, and Keisuke Taniguchi. Matter effects on binary neutron star waveforms. *Phys. Rev. D*, 88:044042, 2013.

[106] Walter Del Pozzo, Tjonnie G. F. Li, Michalis Agathos, Chris Van Den Broeck, and Salvatore Vitale. Demonstrating the feasibility of probing the neutron star equation of state with second-generation gravitational wave detectors. *Phys. Rev. Lett.*, 111(7):071101, 2013.

[107] Sebastiano Bernuzzi, Alessandro Nagar, Tim Dietrich, and Thibault Damour. Modeling the Dynamics of Tidally Interacting Binary Neutron Stars up to the Merger. *Phys. Rev. Lett.*, 114(16):161103, 2015.

[108] Kenta Hotokezaka, Koutarou Kyutoku, Yu-ichiro Sekiguchi, and Masaru Shibata. Measurability of the tidal deformability by gravitational waves from coalescing binary neutron stars. *Phys. Rev. D*, 93(6):064082, 2016.

[109] Tanja Hinderer et al. Effects of neutron-star dynamic tides on gravitational waveforms within the effective-one-body approach. *Phys. Rev. Lett.*, 116(18):181101, 2016.

[110] Soumi De, Daniel Finstad, James M. Lattimer, Duncan A. Brown, Edo Berger, and Christopher M. Biwer. Tidal Deformabilities and Radii of Neutron Stars from the Observation of GW170817. *Phys. Rev. Lett.*, 121(9):091102, 2018. [Erratum: Phys.Rev.Lett. 121, 259902 (2018)].

[111] B. P. Abbott et al. GW170817: Measurements of neutron star radii and equation of state. *Phys. Rev. Lett.*, 121(16):161101, 2018.

[112] Francois Foucart. Black Hole-Neutron Star Mergers: Disk Mass Predictions. *Phys. Rev. D*, 86:124007, 2012.

[113] Francois Foucart, M. Brett Deaton, Matthew D. Duez, Lawrence E. Kidder, Ilana MacDonald, Christian D. Ott, Harald P. Pfeiffer, Mark A. Scheel, Bela Szilagyi, and Saul A. Teukolsky. Black hole-neutron star mergers at realistic mass ratios: Equation of state and spin orientation effects. *Phys. Rev. D*, 87:084006, 2013.

[114] Koutarou Kyutoku, Kunihito Ioka, Hirotada Okawa, Masaru Shibata, and Keisuke Taniguchi. Dynamical mass ejection from black hole-neutron star binaries. *Phys. Rev. D*, 92:044028, 2015.

[115] Koutarou Kyutoku, Masaru Shibata, and Keisuke Taniguchi. Coalescence of black hole–neutron star binaries. *Living Rev. Rel.*, 24(1):5, 2021.

[116] Yuichiro Sekiguchi, Kenta Kiuchi, Koutarou Kyutoku, and Masaru Shibata. Effects of hyperons in binary neutron star mergers. *Phys. Rev. Lett.*, 107:211101, 2011.

[117] Kenta Hotokezaka, Koutarou Kyutoku, Hirotada Okawa, Masaru Shibata, and Kenta Kiuchi. Binary Neutron Star Mergers: Dependence on the Nuclear Equation of State. *Phys. Rev. D*, 83:124008, 2011.
[118] A. Bauswein, H. T. Janka, K. Hebeler, and A. Schwenk. Equation-of-state dependence of the gravitational-wave signal from the ring-down phase of neutron-star mergers. Phys. Rev. D, 86:063001, 2012.

[119] Kentaro Takami, Luciano Rezzolla, and Luca Baiotti. Constraining the Equation of State of Neutron Stars from Binary Mergers. Phys. Rev. Lett., 113(9):091104, 2014.

[120] David Radice, Sebastiano Bernuzzi, Walter Del Pozzo, Luke F. Roberts, and Christian D. Ott. Probing Extreme-Density Matter with Gravitational Wave Observations of Binary Neutron Star Merger Remnants. Astrophys. J. Lett., 842(2):L10, 2017.

[121] Elias R. Most, L. Jens Papenfort, Veronica Dexheimer, Matthias Hanauske, Stefan Schramm, Horst Stöcker, and Luciano Rezzolla. Signatures of quark-hadron phase transitions in general-relativistic neutron-star mergers. Phys. Rev. Lett., 122(6):061101, 2019.

[122] Andreas Bauswein, Niels-Uwe F. Bastian, David B. Blaschke, Katerina Chatzioannou, James A. Clark, Tobias Fischer, and Micaela Oertel. Identifying a first-order phase transition in neutron star mergers through gravitational waves. Phys. Rev. Lett., 122(6):061102, 2019.

[123] Steven L. Liebling, Carlos Palenzuela, and Luis Lehner. Effects of High Density Phase Transitions on Neutron Star Dynamics. Class. Quant. Grav., 38(11):115007, 2021.

[124] Andreas Bauswein, Sebastian Blacker, Georgios Lioutas, Theodoros Soulntas, Vimal Vijayan, and Nikolaos Stergioulas. Systematics of prompt black-hole formation in neutron star mergers. Phys. Rev. D, 103(12):123004, 2021.

[125] David Radice, Sebastiano Bernuzzi, and Albino Perego. The Dynamics of Binary Neutron Star Mergers and GW170817. Ann. Rev. Nucl. Part. Sci., 70:95–119, 2020.

[126] Elias R. Most and Carolyn A. Raithel. Impact of the nuclear symmetry energy on the post-merger phase of a binary neutron star coalescence. Phys. Rev. D, 104(12):124012, 2021.

[127] Aviral Prakash, David Radice, Domenico Logoteta, Albino Perego, Vsevolod Nedora, Ignazio Bombaci, Rahul Kashyap, Sebastiano Bernuzzi, and Andrea Endrizzi. Signatures of deconfined quark phases in binary neutron star mergers. Phys. Rev. D, 104(8):083029, 2021.

[128] A. Perego, D. Logoteta, D. Radice, S. Bernuzzi, R. Kashyap, A. Das, S. Padamata, and A. Prakash. Probing the incompressibility of nuclear matter at ultra-high density through the prompt collapse of asymmetric neutron star binaries. 12 2021.

[129] Matteo Breschi, Sebastiano Bernuzzi, Daniel Godzieba, Albino Perego, and David Radice. Constraints on the neutron star’s maximum densities from postmerger gravitational-waves with third-generation observations. 10 2021.

[130] Yong-Jia Huang, Luca Baiotti, Toru Kojo, Kentaro Takami, Hajime Sotani, Hajime Togashi, Tetsuo Hatsuda, Shigehiro Nagataki, and Yi-Zhong Fan. Merger and post-merger of binary neutron stars with a quark-hadron crossover equation of state. 3 2022.
[131] Elias R. Most, Anton Motornenko, Jan Steinheimer, Veronica Dexheimer, Matthias Hanauske, Luciano Rezzolla, and Horst Stoecker. Probing neutron-star matter in the lab: connecting binary mergers to heavy-ion collisions. 1 2022.

[132] Atul Kedia, Hee Il Kim, In-Saeng Suh, and Grant J. Mathews. Binary neutron star mergers as a probe of quark-hadron crossover equations of state. 3 2022.

[133] Lee Lindblom, Benjamin J. Owen, and Duncan A. Brown. Model Waveform Accuracy Standards for Gravitational Wave Data Analysis. Phys. Rev. D, 78:124020, 2008.

[134] Anuradha Samajdar and Tim Dietrich. Waveform systematics for binary neutron star gravitational wave signals: effects of the point-particle baseline and tidal descriptions. Phys. Rev. D, 98(12):124030, 2018.

[135] Rossella Gamba, Matteo Breschi, Sebastiano Bernuzzi, Michalis Agathos, and Alessandro Nagar. Waveform systematics in the gravitational-wave inference of tidal parameters and equation of state from binary neutron star signals. Phys. Rev. D, 103(12):124015, 2021.

[136] Yiwen Huang, Carl-Johan Haster, Salvatore Vitale, Vijay Varma, Francois Foucart, and Sylvia Biscoveanu. Statistical and systematic uncertainties in extracting the source properties of neutron star - black hole binaries with gravitational waves. 5 2020.

[137] Michele Maggiore et al. Science Case for the Einstein Telescope. JCAP, 03:050, 2020.

[138] Tim Dietrich, Sebastiano Bernuzzi, and Wolfgang Tichy. Closed-form tidal approximants for binary neutron star gravitational waveforms constructed from high-resolution numerical relativity simulations. Phys. Rev. D, 96(12):121501, 2017.

[139] Kenta Kiuchi, Kyohei Kawaguchi, Koutarou Kyutoku, Yuichiro Sekiguchi, Masaru Shibata, and Keisuke Taniguchi. Sub-radian-accuracy gravitational waveforms of coalescing binary neutron stars in numerical relativity. Phys. Rev. D, 96(8):084060, 2017.

[140] J. M. Lattimer and D. N. Schramm. The tidal disruption of neutron stars by black holes in close binaries. 210:549–567, December 1976.

[141] Li-Xin Li and Bohdan Paczynski. Transient events from neutron star mergers. 507:L59, 1998.

[142] L. F. Roberts, D. Kasen, W. H. Lee, and E. Ramirez-Ruiz. Electromagnetic transients powered by nuclear decay in the tidal tails of coalescing compact binaries. Astrophys. J. Lett., 736:L21, July 2011.

[143] J. Barnes and D. Kasen. Effect of a High Opacity on the Light Curves of Radioactively Powered Transients from Compact Object Mergers. 775:18, September 2013.

[144] D. Eichler, M. Livio, T. Piran, and D. N. Schramm. Nucleosynthesis, neutrino bursts and gamma-rays from coalescing neutron stars. Nature, 340:126, July 1989.

[145] R. Narayan, B. Paczynski, and T. Piran. Gamma-ray bursts as the death throes of massive binary stars. 395:L83, August 1992.

27
[146] W. H. Lee and E. Ramirez-Ruiz. The progenitors of short gamma-ray bursts. *New J. Phys.*, 9:17, January 2007.

[147] B. P. Abbott, R. Abbott, T. D. Abbott, F. Acernese, K. Ackley, C. Adams, T. Adams, P. Addesso, R. X. Adhikari, V. B. Adya, and et al. Gravitational Waves and Gamma-Rays from a Binary Neutron Star Merger: GW170817 and GRB 170817A. *Astrophys. J. Lett.*, 848:L13, October 2017.

[148] K. Kiuchi, P. Cerdá-Durán, K. Kyutoku, Y. Sekiguchi, and M. Shibata. Efficient magnetic-field amplification due to the Kelvin-Helmholtz instability in binary neutron star mergers. 92(12):124034, December 2015.

[149] Luciano Rezzolla, Bruno Giacomazzo, Luca Baiotti, Jonathan Granot, Chryssa Kouveliotou, et al. The missing link: Merging neutron stars naturally produce jet-like structures and can power short Gamma-Ray Bursts. *Astrophys. J.*, 732:L6, 2011.

[150] Vasileios Paschalidis, Milton Ruiz, and Stuart L. Shapiro. Relativistic Simulations of Black Hole-Neutron Star Coalescence: the Jet Emerges. *Astrophys. J.*, 806(1):L14, 2015.

[151] I. M. Christie, A. Lalakos, A. Tchekhovskoy, R. Fernández, F. Foucart, E. Quataert, and D. Kasen. The Role of Magnetic Field Geometry in the Evolution of Neutron Star Merger Accretion Discs. *Mon. Not. Roy. Astron. Soc.*, 490(4):4811–4825, 2019.

[152] Daniel M. Siegel and Brian D. Metzger. Three-dimensional general-relativistic magneto-hydrodynamic simulations of remnant accretion disks from neutron star mergers: Outflows and r-process nucleosynthesis. *Phys. Rev. Lett.*, 119(23):231102, 2017.

[153] S. Wanajo, Y. Sekiguchi, N. Nishimura, K. Kiuchi, K. Kyutoku, and M. Shibata. Production of All the r-process Nuclides in the Dynamical Ejecta of Neutron Star Mergers. *Astrophys.J.Lett.*, 789:L39, July 2014.

[154] Xinyu Li and Daniel M. Siegel. Neutrino Fast Flavor Conversions in Neutron-Star Post-merger Accretion Disks. *Phys. Rev. Lett.*, 126(25):251101, 2021.

[155] Francesco Fambri, Michael Dumbser, Sven Köppel, Luciano Rezzolla, and Olindo Zanotti. ADER discontinuous Galerkin schemes for general-relativistic ideal magnetohydrodynamics. *Mon. Not. Roy. Astron. Soc.*, 477(4):4543–4564, 2018.

[156] Francois Foucart, Matthew D. Duez, Francois Hebert, Lawrence E. Kidder, Phillip Kovarik, Harald P. Pfeiffer, and Mark A. Scheel. Implementation of Monte Carlo Transport in the General Relativistic SpEC Code. *Astrophys. J.*, 920(2):82, 2021.

[157] C. Palenzuela, R. Aguilera-Miret, F. Carrasco, R. Ciolfi, J. V. Kalinani, W. Kastaun, B. Miñano, and D. Viganò. Turbulent magnetic field amplification in binary neutron star mergers. 12 2021.

[158] B.P. Abbott et al. GW190425: Observation of a Compact Binary Coalescence with Total Mass $\sim 3.4M_\odot$. *Astrophys. J. Lett.*, 892(1):L3, 2020.
[159] Masaru Shibata, Sho Fujibayashi, Kenta Hotokezaka, Kenta Kiuchi, Koutarou Kyutoku, Yuichiro Sekiguchi, and Masaomi Tanaka. Modeling GW170817 based on numerical relativity and its implications. *Phys. Rev. D*, 96(12):123012, 2017.

[160] Mark Hannam, Patricia Schmidt, Alejandro Bohé, Leïla Haegel, Sascha Husa, Frank Ohme, Geraint Pratten, and Michael Purrer. Simple Model of Complete Precessing Black-Hole-Binary Gravitational Waveforms. *Phys. Rev. Lett.*, 113(15):151101, 2014.

[161] Alejandro Bohé et al. Improved effective-one-body model of spinning, nonprecessing binary black holes for the era of gravitational-wave astrophysics with advanced detectors. *Phys. Rev. D*, 95(4):044028, 2017.

[162] Sebastian Khan, Sascha Husa, Mark Hannam, Frank Ohme, Michael Purrer, Xisco Jiménez Forteza, and Alejandro Bohé. Frequency-domain gravitational waves from non-precessing black-hole binaries. II. A phenomenological model for the advanced detector era. *Phys. Rev. D*, 93(4):044007, 2016.

[163] Jonathan Blackman, Scott E. Field, Mark A. Scheel, Chad R. Galley, Christian D. Ott, Michael Boyle, Lawrence E. Kidder, Harald P. Pfeiffer, and Béla Szilágyi. Numerical relativity waveform surrogate model for generically precessing binary black hole mergers. *Phys. Rev. D*, 96(2):024058, 2017.

[164] Sascha Husa, Sebastian Khan, Mark Hannam, Michael Purrer, Frank Ohme, Xisco Jiménez Forteza, and Alejandro Bohé. Frequency-domain gravitational waves from non-precessing black-hole binaries. I. New numerical waveforms and anatomy of the signal. *Phys. Rev. D*, 93(4):044006, 2016.

[165] J. Lange et al. Parameter estimation method that directly compares gravitational wave observations to numerical relativity. *Phys. Rev. D*, 96(10):104041, 2017.

[166] Patricia Schmidt, Ian W. Harry, and Harald P. Pfeiffer. Numerical Relativity Injection Infrastructure. 2017.

[167] Eanna E. Flanagan and Scott A. Hughes. Measuring gravitational waves from binary black hole coalescences: 2. The Waves’ information and its extraction, with and without templates. *Phys. Rev.*, D57:4566–4587, 1998.

[168] Sean T. McWilliams, Bernard J. Kelly, and John G. Baker. Observing mergers of non-spinning black-hole binaries. *Phys. Rev.*, D82:024014, 2010.

[169] Katerina Chatziioannou, Antoine Klein, Nicolás Yunes, and Neil Cornish. Constructing Gravitational Waves from Generic Spin-Precessing Compact Binary Inspirals. *Phys. Rev.*, D95(10):104004, 2017.

[170] Michael Purrer and Carl-Johan Haster. Gravitational waveform accuracy requirements for future ground-based detectors. *Phys. Rev. Res.*, 2(2):023151, 2020.

[171] Deborah Ferguson, Karan Jani, Pablo Laguna, and Deirdre Shoemaker. Assessing the readiness of numerical relativity for LISA and 3G detectors. *Phys. Rev. D*, 104(4):044037, 2021.
[172] Emanuele Berti et al. Testing General Relativity with Present and Future Astrophysical Observations. *Class. Quant. Grav.*, 32:243001, 2015.

[173] Timothy Clifton, Pedro G. Ferreira, Antonio Padilla, and Constantinos Skordis. Modified Gravity and Cosmology. *Phys. Rept.*, 513:1–189, 2012.

[174] Stephon Alexander and Nicolas Yunes. Chern-Simons Modified General Relativity. *Phys. Rept.*, 480:1–55, 2009.

[175] Nicolás Yunes and Xavier Siemens. Gravitational-Wave Tests of General Relativity with Ground-Based Detectors and Pulsar Timing-Arrays. *Living Rev. Rel.*, 16:9, 2013.

[176] Clifford M. Will. The Confrontation between General Relativity and Experiment. *Living Rev. Rel.*, 17:4, 2014.

[177] David Garfinkle, Christopher Eling, and Ted Jacobson. Numerical simulations of gravitational collapse in Einstein-aether theory. *Phys. Rev. D*, 76:024003, 2007.

[178] Alexander Adam, Pau Figueras, Ted Jacobson, and Toby Wiseman. Rotating black holes in Einstein-aether theory. 7 2021.

[179] Mikica Kocic, Anders Lundkvist, and Francesco Torsello. On the ratio of lapses in bimetric relativity. *Class. Quant. Grav.*, 36(22):225013, 2019.

[180] Francesco Torsello. The mean gauges in bimetric relativity. *Class. Quant. Grav.*, 36(23):235010, 2019.

[181] Mikica Kocic, Francesco Torsello, Marcus Högås, and Edvard Mörtsell. Spherical dust collapse in bimetric relativity: Bimetric polytropes. 4 2019.

[182] Francesco Torsello, Mikica Kocic, Marcus Högås, and Edvard Mörtsell. Covariant BSSN formulation in bimetric relativity. *Class. Quant. Grav.*, 37(2):025013, 2020. [Erratum: Class.Quant.Grav. 37, 079501 (2020)].

[183] Francesco Torsello. bimEX: A Mathematica package for exact computations in 3+1 bimetric relativity. *Comput. Phys. Commun.*, 247:106948, 2020.

[184] Mikica Kocic, Francesco Torsello, Marcus Högås, and Edvard Mörtsell. Initial data and first evolutions of dust clouds in bimetric relativity. *Class. Quant. Grav.*, 37(16):165010, 2020.

[185] Robert Benkel, Thomas P. Sotiriou, and Helvi Witek. Black hole hair formation in shift-symmetric generalised scalar-tensor gravity. *Class. Quant. Grav.*, 34(6):064001, 2017.

[186] Hector O. Silva, Jeremy Sakstein, Leonardo Gualtieri, Thomas P. Sotiriou, and Emanuele Berti. Spontaneous scalarization of black holes and compact stars from a Gauss-Bonnet coupling. *Phys. Rev. Lett.*, 120(13):131104, 2018.

[187] Daniela D. Doneva and Stoytcho S. Yazadjiev. New Gauss-Bonnet Black Holes with Curvature-Induced Scalarization in Extended Scalar-Tensor Theories. *Phys. Rev. Lett.*, 120(13):131103, 2018.
[188] Justin L. Ripley and Frans Pretorius. Gravitational collapse in Einstein dilaton-Gauss–Bonnet gravity. *Class. Quant. Grav.*, 36(13):134001, 2019.

[189] Justin L. Ripley and Frans Pretorius. Dynamics of a $\mathbb{Z}_2$ symmetric EdGB gravity in spherical symmetry. *Class. Quant. Grav.*, 37(15):155003, 2020.

[190] Daniela D. Doneva, Lucas G. Collodel, Christian J. Krüger, and Stoytcho S. Yazadjiev. Black hole scalarization induced by the spin: 2+1 time evolution. *Phys. Rev. D*, 102(10):104027, 2020.

[191] Daniela D. Doneva and Stoytcho S. Yazadjiev. Spontaneously scalarized black holes in dynamical Chern-Simons gravity: dynamics and equilibrium solutions. *Phys. Rev. D*, 103(8):083007, 2021.

[192] Hao-Jui Kuan, Daniela D. Doneva, and Stoytcho S. Yazadjiev. Dynamical Formation of Scalarized Black Holes and Neutron Stars through Stellar Core Collapse. *Phys. Rev. Lett.*, 127(16):161103, 2021.

[193] Daniela D. Doneva and Stoytcho S. Yazadjiev. Beyond the spontaneous scalarization: New fully nonlinear mechanism for the formation of scalarized black holes and its dynamical development. *Phys. Rev. D*, 105(4):L041502, 2022.

[194] Abhishek Hegade K. R., Elias R. Most, Jorge Noronha, Helvi Witek, and Nicolas Yunes. How do spherical black holes grow monopole hair? *Phys. Rev. D*, 105(6):064041, 2022.

[195] Aaron Held and Hyun Lim. Nonlinear dynamics of quadratic gravity in spherical symmetry. *Phys. Rev. D*, 104(8):084075, 2021.

[196] Vitor Cardoso et al. NR/HEP: roadmap for the future. *Class. Quant. Grav.*, 29:244001, 2012.

[197] Nicolas Yunes, Kent Yagi, and Frans Pretorius. Theoretical Physics Implications of the Binary Black-Hole Mergers GW150914 and GW151226. *Phys. Rev. D*, 94(8):084002, 2016.

[198] Kent Yagi and Leo C. Stein. Black Hole Based Tests of General Relativity. *Class. Quant. Grav.*, 33(5):054001, 2016.

[199] Leor Barack et al. Black holes, gravitational waves and fundamental physics: a roadmap. *Class. Quant. Grav.*, 36(14):143001, 2019.

[200] Enrico Barausse, Carlos Palenzuela, Marcelo Ponce, and Luis Lehner. Neutron-star mergers in scalar-tensor theories of gravity. *Phys. Rev. D*, 87:081506, 2013.

[201] Masaru Shibata, Keisuke Taniguchi, Hirotada Okawa, and Alessandra Buonanno. Coalescence of binary neutron stars in a scalar-tensor theory of gravity. *Phys. Rev. D*, 89(8):084005, 2014.

[202] James Healy, Tanja Bode, Roland Haas, Enrique Pazos, Pablo Laguna, DeirdreM. Shoemaker, and Nicolás Yunes. Late Inspiral and Merger of Binary Black Holes in Scalar-Tensor Theories of Gravity. *Class. Quant. Grav.*, 29:232002, 2012.
[203] Laura Sagunski, Jun Zhang, Matthew C. Johnson, Luis Lehner, Mairi Sakellariadou, Steven L. Liebling, Carlos Palenzuela, and David Neilsen. Neutron star mergers as a probe of modifications of general relativity with finite-range scalar forces. *Phys. Rev. D*, 97(6):064016, 2018.

[204] Eric W. Hirschmann, Luis Lehner, Steven L. Liebling, and Carlos Palenzuela. Black Hole Dynamics in Einstein-Maxwell-Dilaton Theory. *Phys. Rev. D*, 97(6):064032, 2018.

[205] Steven L. Liebling. Maxwell-dilaton dynamics. *Phys. Rev. D*, 100(10):104040, 2019.

[206] Pau Figueras and Tiago França. Black Hole Binaries in Cubic Horndeski Theories. 12 2021.

[207] Lotte ter Haar, Miguel Bezares, Marco Crisostomi, Enrico Barausse, and Carlos Palenzuela. Dynamics of Screening in Modified Gravity. *Phys. Rev. Lett.*, 126:091102, 2021.

[208] Miguel Bezares, Ricard Aguilera-Miret, Lotte ter Haar, Marco Crisostomi, Carlos Palenzuela, and Enrico Barausse. No Evidence of Kinetic Screening in Simulations of Merging Binary Neutron Stars beyond General Relativity. *Phys. Rev. Lett.*, 128(9):091103, 2022.

[209] Miguel Bezares, Lotte ter Haar, Marco Crisostomi, Enrico Barausse, and Carlos Palenzuela. Kinetic screening in nonlinear stellar oscillations and gravitational collapse. *Phys. Rev. D*, 104(4):044022, 2021.

[210] Maria Okounkova, Leo C. Stein, Mark A. Scheel, and Daniel A. Hemberger. Numerical binary black hole mergers in dynamical Chern-Simons gravity: Scalar field. *Phys. Rev. D*, 96(4):044020, 2017.

[211] Maria Okounkova, Leo C. Stein, Mark A. Scheel, and Saul A. Teukolsky. Numerical binary black hole collisions in dynamical Chern-Simons gravity. *Phys. Rev. D*, 100(10):104026, 2019.

[212] Maria Okounkova, Leo C. Stein, Jordan Moxon, Mark A. Scheel, and Saul A. Teukolsky. Numerical relativity simulation of GW150914 beyond general relativity. *Phys. Rev. D*, 101(10):104016, 2020.

[213] Maria Okounkova. Numerical relativity simulation of GW150914 in Einstein dilaton Gauss-Bonnet gravity. 1 2020.

[214] Hector O. Silva, Helvi Witek, Matthew Elley, and Nicolás Yunes. Dynamical Descalarization in Binary Black Hole Mergers. *Phys. Rev. Lett.*, 127(3):031101, 2021.

[215] William E. East and Justin L. Ripley. Evolution of Einstein-scalar-Gauss-Bonnet gravity using a modified harmonic formulation. *Phys. Rev. D*, 103(4):044040, 2021.

[216] William E. East and Justin L. Ripley. Dynamics of Spontaneous Black Hole Scalarization and Mergers in Einstein-Scalar-Gauss-Bonnet Gravity. *Phys. Rev. Lett.*, 127(10):101102, 2021.

[217] Steven L. Liebling and Carlos Palenzuela. Dynamical Boson Stars. *Living Rev. Rel.*, 20(1):5, 2017.
[218] Miguel Bezares, Carlos Palenzuela, and Carles Bona. Final fate of compact boson star mergers. *Phys. Rev. D*, 95(12):124005, 2017.

[219] Carlos Palenzuela, Paolo Pani, Miguel Bezares, Vitor Cardoso, Luis Lehner, and Steven Liebling. Gravitational Wave Signatures of Highly Compact Boson Star Binaries. *Phys. Rev. D*, 96(10):104058, 2017.

[220] Katy Clough, Tim Dietrich, and Jens C. Niemeyer. Axion star collisions with black holes and neutron stars in full 3D numerical relativity. *Phys. Rev. D*, 98(8):083020, 2018.

[221] Tim Dietrich, Francesca Day, Katy Clough, Michael Coughlin, and Jens Niemeyer. Neutron star–axion star collisions in the light of multimessenger astronomy. *Mon. Not. Roy. Astron. Soc.*, 483(1):908–914, 2019.

[222] James Y. Widdicombe, Thomas Helfer, and Eugene A. Lim. Black hole formation in relativistic Oscillatons collisions. *JCAP*, 01:027, 2020.

[223] Fabrizio Di Giovanni, Nicolas Sanchis-Gual, Pablo Cerdá-Durán, and José Antonio Font. Can fermion-boson stars reconcile multimessenger observations of compact stars? *Phys. Rev. D*, 105(6):063005, 2022.

[224] Víctor Jaramillo, Nicolas Sanchis-Gual, Juan Barranco, Argelia Bernal, Juan Carlos Degollado, Carlos Herdeiro, Miguel Megevand, and Darío Núñez. Head-on collisions of $\ell$-boson stars. 2 2022.

[225] Miguel Bezares, Mateja Bošković, Steven Liebling, Carlos Palenzuela, Paolo Pani, and Enrico Barausse. Gravitational waves and kicks from the merger of unequal mass, highly compact boson stars. 1 2022.

[226] Miguel Bezares and Carlos Palenzuela. Gravitational Waves from Dark Boson Star binary mergers. *Class. Quant. Grav.*, 35(23):234002, 2018.

[227] Thomas Helfer, Eugene A. Lim, Marcos A. G. Garcia, and Mustafa A. Amin. Gravitational Wave Emission from Collisions of Compact Scalar Solitons. *Phys. Rev. D*, 99(4):044046, 2019.

[228] Steven L. Liebling, Michael Kavic, and Matthew Lippert. Probing Near-Horizon Fluctuations with Black Hole Binary Mergers. *JHEP*, 03:176, 2018.

[229] Lee Lindblom, Mark A. Scheel, Lawrence E. Kidder, Robert Owen, and Oliver Rinne. A New Generalized Harmonic Evolution System. 23:S447, 2006.

[230] Olivier Sarbach and Manuel Tiglio. Continuum and Discrete Initial-Boundary-Value Problems and Einstein’s Field Equations. *Living Rev. Rel.*, 15:9, 2012.

[231] David Hilditch. An Introduction to Well-posedness and Free-evolution. *Int. J. Mod. Phys. A*, 28:1340015, 2013.

[232] Marcelo Salgado, David Martinez-del Rio, Miguel Alcubierre, and Dario Nunez. Hyperbolicity of scalar-tensor theories of gravity. *Phys. Rev. D*, 77:104010, 2008.

33
[233] Áron D. Kovács and Harvey S. Reall. Well-posed formulation of Lovelock and Horndeski theories. *Phys. Rev. D*, 101(12):124003, 2020.

[234] Bishop Mongwane. On the hyperbolicity and stability of $3 + 1$ formulations of metric $f(R)$ gravity. *Gen. Rel. Grav.*, 48(11):152, 2016.

[235] Olivier Sarbach, Enrico Barausse, and Jorge A. Preciado-López. Well-posed Cauchy formulation for Einstein-æther theory. *Class. Quant. Grav.*, 36(16):165007, 2019.

[236] Térence Delsate, David Hilditch, and Helvi Witek. Initial value formulation of dynamical Chern-Simons gravity. *Phys. Rev. D*, 91(2):024027, 2015.

[237] Juan Cayuso, Néstor Ortiz, and Luis Lehner. Fixing extensions to general relativity in the nonlinear regime. *Phys. Rev. D*, 96(8):084043, 2017.

[238] Gwyneth Allwright and Luis Lehner. Towards the nonlinear regime in extensions to GR: assessing possible options. *Class. Quant. Grav.*, 36(8):084001, 2019.

[239] Ramiro Cayuso and Luis Lehner. Nonlinear, noniterative treatment of EFT-motivated gravity. *Phys. Rev. D*, 102(8):084008, 2020.

[240] Sam R. Dolan. Instability of the massive Klein-Gordon field on the Kerr spacetime. *Phys. Rev. D*, 76:084001, 2007.

[241] Asimina Arvanitaki and Sergei Dubovsky. Exploring the String Axiverse with Precision Black Hole Physics. *Phys. Rev. D*, 83:044026, 2011.

[242] Richard Brito, Vitor Cardoso, and Paolo Pani. *Superradiance: Energy Extraction, Black-Hole Bombs and Implications for Astrophysics and Particle Physics*, volume 906. Springer, 2015.

[243] Helvi Witek, Vitor Cardoso, Akihiro Ishibashi, and Ulrich Sperhake. Superradiant instabilities in astrophysical systems. *Phys. Rev. D*, 87(4):043513, 2013.

[244] Lam Hui, Jeremiah P. Ostriker, Scott Tremaine, and Edward Witten. Ultralight scalars as cosmological dark matter. *Phys. Rev. D*, 95(4):043541, 2017.

[245] R.D. Peccei and Helen R. Quinn. CP Conservation in the Presence of Instantons. *Phys. Rev. Lett.*, 38:1440–1443, 1977.

[246] Asimina Arvanitaki, Savas Dimopoulos, Sergei Dubovsky, Nemanja Kaloper, and John March-Russell. String Axiverse. *Phys. Rev. D*, 81:123530, 2010.

[247] Joao G. Rosa and Sam R. Dolan. Massive vector fields on the Schwarzschild spacetime: quasi-normal modes and bound states. *Phys. Rev. D*, 85:044043, 2012.

[248] William E. East. Superradiant instability of massive vector fields around spinning black holes in the relativistic regime. *Phys. Rev. D*, 96(2):024004, 2017.
[249] William E. East and Frans Pretorius. Superradiant Instability and Backreaction of Massive Vector Fields around Kerr Black Holes. *Phys. Rev. Lett.*, 119(4):041101, 2017.

[250] William E. East. Massive Boson Superradiant Instability of Black Holes: Nonlinear Growth, Saturation, and Gravitational Radiation. *Phys. Rev. Lett.*, 121(13):131104, 2018.

[251] Zipeng Wang, Thomas Helfer, Katy Clough, and Emanuele Berti. Superradiance in massive vector fields with spatially varying mass. 1 2022.

[252] Richard Brito, Sara Grillo, and Paolo Pani. Black Hole Superradiant Instability from Ultralight Spin-2 Fields. *Phys. Rev. Lett.*, 124(21):211101, 2020.

[253] Emanuele Berti, Richard Brito, Caio F.B. Macedo, Guilherme Raposo, and Joao Luis Rosa. Ultralight boson cloud depletion in binary systems. *Phys. Rev. D*, 99(10):104039, 2019.

[254] Jun Zhang and Huan Yang. Dynamic Signatures of Black Hole Binaries with Superradiant Clouds. *Phys. Rev. D*, 101(4):043020, 2020.

[255] Dina Traykova, Katy Clough, Thomas Helfer, Emanuele Berti, Pedro G. Ferreira, and Lam Hui. Dynamical friction from scalar dark matter in the relativistic regime. *Phys. Rev. D*, 104(10):103014, 2021.

[256] Daniel Baumann, Horng Sheng Chia, and Rafael A. Porto. Probing Ultralight Bosons with Binary Black Holes. *Phys. Rev. D*, 99(4):044001, 2019.

[257] Daniel Baumann, Horng Sheng Chia, Rafael A. Porto, and John Stout. Gravitational Collider Physics. *Phys. Rev. D*, 101(8):083019, 2020.

[258] Otto A. Hannuksela, Kaze W. K. Wong, Richard Brito, Emanuele Berti, and Tjonnie G. F. Li. Probing the existence of ultralight bosons with a single gravitational-wave measurement. *Nature Astron.*, 3(5):447–451, 2019.

[259] Sunil Choudhary, Nicolas Sanchis-Gual, Anshu Gupta, Juan Carlos Degollado, Sukanta Bose, and José A. Font. Gravitational waves from binary black hole mergers surrounded by scalar field clouds: Numerical simulations and observational implications. *Phys. Rev. D*, 103(4):044032, 2021.

[260] Wolfgang Tichy, Ananya Adhikari, and Liwei Ji. Numerical relativity with the new nmesh code. *Bulletin of the American Physical Society*, 65, 2020. https://meetings.aps.org/Meeting/APR20/Session/D15.4.

[261] Nils Deppe, William Throwe, Lawrence E. Kidder, Nils L. Fischer, Cristóbal Armaza, Gabriel S. Bonilla, François Hébert, Prayush Kumar, Geoffrey Lovelace, Jordan Moxon, Eamonn O’Shea, Harald P. Pfeiffer, Mark A. Scheel, Saul A. Teukolsky, Isha Ananthpurkar, Michael Boyle, François Foucart, Matthew Giesler, Dante A. B. Iozzo, Isaac Legred, Dongjun Li, Alexandra Macedo, Denyz Melchor, Marlo Morales, Teresita Ramírez, Hannes R. Rüter, Jennifer Sanchez, Sierra Thomas, and Tom Wlodarczyk. Spectre, June 2021. https://doi.org/10.5281/zenodo.4913286.