Abstract—This paper establishes the system of differential equations model of Electronic Power Steering (EPS), which is based on the characteristics of EPS system, and then establishes the simulation model of simulink. According to the state space model of the system that have obtained, researchers get controllability and view of the system. The simulation shows that the effect of EPS is obvious, which improves steering portability and flexibility of the vehicle, simultaneously the control of back to the positive and damping improve the performance of vehicles running in a straight line.

Keywords—Electronic Power Steering; Stability; Modeling; Feedback; simulink

I. INTRODUCTION

ELECTRIC Power Steering (EPS) was first installed in Japan's SUZUKI cars in 1988. EPS replaced traditional Hydraulic Power Steering (HPS) has become an inevitable trend, because it has many advantages than that. Many vehicles are equipped with electric power steering overseas, at the same time, Changhe Plough cars, Guangzhou Honda Fit and Geely Pride has installed Electric Power Steering at home. Compared with Hydraulic Power Steering (HPS), Electric Power Steering (EPS) has the following advantages[1-4]:

1. Good dynamical characteristics, steering easiness, good road feeling, higher control stability.
2. Dynamical characteristics of EPS can quickly match models which through the software settings and modification.
3. Motor provides power only when EPAS turns, energy-saving 3% ~ 5% than HPS.
4. Cramped construction, modular installation easily.
5. Canceled the hydraulic pipeline, no pollution to the environment.
6. Good performance of low-temperature.

II. MODELING AND STABILITY ANALYSIS

A. EPS Mathematical Model

The typical structure of EPS is shown in Figure 1. Steering wheel torque transducer is mounted on the steering shaft. DC power motor is connected to the steering shaft through the speed reducer. ECU constantly receives signals from torque transducer and vehicle speed sensor, and controls power motor work through the internal control strategy[5-7].

The modeling of the EPS system is modeled by Simulink in this paper. The dynamics model of EPS system properly is simplified in order to study comfortably. The quality of tires are equivalent to the rack, ignore the influence of the steering universal joint. The equations of wheel, torque sensor and the dynamic are as follow[8-9].

\[ T_s = J_s \ddot{\theta}_s + B_s \dot{\theta}_s + K_s (\theta_h - \theta_s) \]

\[ M_s \ddot{p} + B_s \dot{p} + F_{sat} + F_{fric} \text{sgn}(p) = (T_s + T_{ass})/r_p \]

\[ \dot{\theta}_s = p / r_p \]

In the above formula, \( T_s \) is the rotating torque of steering gear working on steering wheel; \( T_s \) is the torque value of torque transducer; \( J_s \) is the rotary inertia of steering wheel; \( B_s \) is the damping factor of steering wheel; \( \theta_{sat} \) is the angle of steering wheel; \( K_s \) is the tensional rigidity of torque sensor; \( \theta_s \) is the steering angle of torque sensor; \( M_s \) is the equivalent mass of rack and wheels; \( B_s \) is the damping factor of rack; \( F_{sat} \) is the force from rack through steering return-to-center torque; \( P \) is the rack displacement; \( F_{fric} \) is the system friction equitable to the mast; \( T_{ass} \) is the torque of reducer; \( r_p \) is the radius of
pinion; \( G \) is the transmission ratio of turbine worm reducer.

1) **Steering aligning torque modeling.**

Steering aligning torque transformed into force on the rack is calculated by the following formula.

\[ F_w = K_p \]

In the above formula, \( K_p \) is the aligning torque elasticity. This value varies with changes in vehicle speed or motor modeling.

2) **Motor modeling.**

In this paper, researchers study the EPS system of the steering column power type (C-EPS), using the permanent magnet DC brush motor, 30 A rated current, rated voltage 12 V. DC motor dynamics and electromagnetism equations as follows.

\[ T_m = K_t i \]

\[ J_m \dot{\theta_m} + B_m \dot{\theta_m} + T_{ass} = T_m \]

\[ L_i + R_i K_s \dot{\theta_m} = U \]

\[ T_{ass} = K_m (\theta_m - \theta_i) \]

In the above formulas, \( J_m \) is the moment of inertia of the motor and the worm; \( B_m \) is the motor damping coefficient; \( \theta_m \) is the motor rotation; \( T_{ass} \) is the Input torque of the motor reducer; \( T_m \) is the motor electromagnetic torque; \( K_t \) is the motor electromagnetic torque coefficient; \( i \) is the motor current; \( K_m \) is the motor shaft stiffness; \( L_i \) is the motor inductance; \( R_i \) is the electrical resistance; \( K_s \) is the counter electromotive force coefficient for the motor; \( U \) is the voltage for the motor.

B. **Simulink Modeling**

According to the above mathematical model, simulation model is built, based on Matlab/Simulink, as shown in Figure 2.

![EPS simulink model](image)

C. **The Analysis of System Stability**

The establishment of the EPS system states space model. Each component is defined as a state variable \( x \).

\[ x_1 = \theta_d , x_2 = \dot{\theta}_d , x_3 = p , x_4 = \dot{p} , x_5 = \theta_m , x_6 = \dot{\theta}_m , x_7 = i \]

The variables are substituted into the differential equation.

\[ \dot{x} = Ax + Bu \]

State variables, \( x = [x_1, x_2, x_3, x_4, x_5, x_6, x_7] \).

The input variable, \( u = [T_d, U] \).

The output variable, \( y = [T_s, T_{ass}] \).

Ignoring the nonlinear friction coefficient, the state-space model of EPS system is as follow.

\[
\begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 \\
-\frac{K_t}{J_m} & -\frac{B_m}{J_m} & \frac{K_t}{J_m} & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3 \\
\dot{x}_4 \\
\dot{x}_5 \\
\dot{x}_6 \\
\dot{x}_7
\end{bmatrix}
= \begin{bmatrix}
K_t & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{r_p} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4 \\
x_5 \\
x_6 \\
x_7
\end{bmatrix}
+ \begin{bmatrix}
K_m & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{r_p} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
K_t \theta_m \\
K_t \dot{\theta}_m \\
K_t \ddot{\theta}_m \\
K_t \dot{\theta}_m \\
K_t \dot{\theta}_m \\
K_t \dot{\theta}_m \\
K_t \dot{\theta}_m
\end{bmatrix}
\]

On the basis of the state space model of the EPS, you can get the system controllability and view matrix, and both of matrix rank is 6. So it is the conclusion that EPS is not controlled, not concept system.

III. EPS DYNAMICAL CHARACTERISTICS RESEARCH

A. **The Determination of Dynamical Characteristics Curve**

Electric power steering system dynamical characteristics have a variety of curve forms. But power trends and power critical point are the same, usually divided into linear power mode, fold-assisted manner and the curve dynamical mode. Each curve is formed by three regions, divided into power area, power area and dynamical saturation area. As shown in Figure 3. Three kinds of curve in the application of electric power-assisted steering have their respective advantages and disadvantages. The following respectively introduce three kinds of curve modeling and performance analysis. First shows three curve structure is similar. When steering wheel torque reaches 1 Nm, power motor starts to provide power, power to the motor current, the biggest power current for 28A. When the steering wheel input torque reaches 6N·m, power torque no longer increases, that booster has been reached a saturation region. In the region of the power, steering assist current increases as the input torque increases. In a certain moment, when the steering wheel input, power current decreases with the increase of the speed of the car[10].
Comparing with the above three kinds of assisted characteristics curve that is not hard to find, the characteristic of linear power is the simplest, easy to adjust. Curve type power feature is complex and adjustment is not convenient. Broken line type power feature is somewhere in between. In the condition of the smaller front axle load, the steering resistance torque is small, steering portability issue is not prominent, linear assist characteristic can provide good road feel without sacrificing portability, thus better to reconcile the contradiction between portability and road feel. This experiment mainly adopts linear assist characteristics.

B. Power Gain of the Match

To maintain the stability of the car travels in a straight line, to avoid the steering wheel in the middle position too sensitive, in the middle of the sensor position [-1,1]Nm range for the dead zone, this interval motor power current is zero. The steering wheel torque changes within this range, motor without power; each of the power curve has saturated interval. When the torque reaches a certain value, power current no longer changes with torque, main effect is to limit current in a certain range, avoid the current exceeding the drive circuit and motor to withstand the limit state; Reference abroad on EPS system dynamical characteristics vary with the speed of the relationship, when the speed is more than 80 km/h, power characteristics do not change, maintain 80 km/h speed of dynamical characteristics; When the speed is shown in figure between two curves, using the curve interpolation on both sides, it determines the target power torque at this time (which is as shown in Figure 2).

In the process of driving, the steering wheel input actually comes in two forms: On the one hand, to the steering wheel an angular displacement, it calls angular displacement input, abbreviation angle input. On the other hand, to the steering wheel function a torque, it calls input torque, abbreviation power input. When the driver actually drives vehicles, the two inputs are joined at the same time.

A. Current Loop Feedback Control System

Simulation test using current loop feedback control in the first place is the force control system, the control principle as shown in Figure 6. The current is given according to the characteristic table. The steering wheel inputting square wave, amplitude is 180°, frequency of 0.5 Hz. Using proportional feedback, the ratio is 100, and the power in the process of current tracking characteristics is shown in Figure 7. In Figure 7, actual current curve and the given current curve almost overlap, and electrical current tracking performance is good. It can satisfy the system requirements.
B. The Design of the Transition Process

To reduce the error caused by the high frequency signal, to ensure the trajectory tracking performance, a given signal requires careful design. The specific measure to reduce initial error is: to arrange a suitable transition process, and then let the system output tracking this arrangement of the transition process to ultimately achieve control target. The actual application often uses first-order low-pass filter, $G(s) = \frac{1}{0.01s + 1}$, and compares with step response characteristic, Figure 8. Under the condition of the same response time, the transition from figure can be found in the design process after the system realizing fast without overshoot ideal trajectory tracking.

![Figure 8. The comparison of steering wheel angle is given and response curve.](image)

C. Back to the Positive and Active Damping Control

When the vehicle is driven at low speed, due to the existence of friction and damping in the system, and the damping effect of the tire, the steering wheel will appear unable to accurately return, residual steering wheel angle and residual yawing velocity, they reduce the straight line of the vehicle driving performance. Therefore, the electric power steering should set the return-to-center control algorithm in the control strategy, in order to improve the return-to-center performance of the vehicle.

In addition, when the vehicle is driven at highway speed, the process of the steering wheel in the back will appear steering wheel angle overshoot, severe cases will cause the vehicle’s yaw oscillations. Therefore, electric power steering should increase active damping at high speed. In order to avoid the yaw oscillation occurs. Aligning torque and damping torque is mainly determined by the size of the steering wheel angle and speed, when $\dot{\theta}_w \cdot \dot{\theta}_w \geq 0$, showing the steering process; when $\dot{\theta}_w \cdot \dot{\theta}_w < 0$, representation back to the positive process. That is:

$$T_{ed} = K_1\alpha + K_2\int\alpha dt + K_3\alpha \cdot$$

In the formulas, $T_{ed}$ is aligning torque motor or damping torque, along with the different of torque direction change its nature; $\alpha$ is the steering wheel angle speed; $K_1$, $K_2$, $K_3$ is the PID controller parameters; Differential control corresponds to the damping torque.

V. CONCLUSIONS

In this paper, through the establishment of the state-space model, EPS is not controllably understood, unobservable system. Simulation results show that EPS can significantly reduce the burden on the driver's steering, and then booster effect is obvious. Improve the vehicle’s steering portability and flexibility. The positive control and damping control shows a positive return of good performance, and the straight running performance is improved.

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