Gyrofluid simulation on the nonlinear excitation and radial structure of geodesic acoustic modes in ITG turbulence

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Abstract. The nonlinear excitation and saturation mechanism of geodesic acoustic mode (GAM), as well as its radial structure, in tokamak plasmas are investigated by applying a newly well-benchmarked gyrofluid model. At first, an empirical closure relation for the conventional three-field gyrofluid modeling is presented for ion temperature gradient (ITG) fluctuations and the GAMs. The zonal flow damping is precisely examined by comparing with theoretical predictions and other kinetic calculations. Then, a local code and the global version are advanced to simulate the nonlinear excitation of the GAMs by ITG fluctuations through the Reynolds stress. It is found that the GAM instability can be nonlinearly excited under the competition between the nonlinear driving and the collisionless damping. The pump amplitude threshold of the GAM instability is higher than that of the zonal flow instability. Meanwhile, the unstable GAMs are mainly saturated by the intrinsic Landau damping, which is different from the stationary zonal flow counterpart. It is testified that the sound waves are damped fluctuations in ITG turbulence. Furthermore, the radial structure of the GAMs is shown as $k_{r} \rho_{i} \leq 1.0$, which is shorter than that of the pure zonal flows.

1. Introduction

Geodesic acoustic mode (GAM) is a class of toroidal eigenmode with finite low frequency [1]. It has attracted increasing attention recently due to the combination with the stationary zonal flows (zero frequency) through the geodesic curvature effects [2-4]. The GAMs are characterized in spatial structure by poloidally and toroidally symmetric potential and poloidally asymmetric density or pressure fluctuations. The latter gives rise to a time-dependent zonal flow in toroidal plasmas. GAM dynamics is important in turbulent transport studies since their low-frequency radial structure can modulate or scatter drift-wave fluctuations, which are primarily responsible for collisionless transport. On the other hand, the GAM is a damped oscillator with finite frequency $\omega_{gas} = \sqrt{2(\Gamma + \tau)T_{eq}} a/R$ coupling with axisymmetrical static potential (i.e., the stationary zonal flows). Here $\Gamma = 5/3$, $\tau = T_{i}/T_{e}$, $T_{eq}$ is the equilibrium ion temperature normalized by the central value $T_{i,c}$, $a$ and $R$ are the minor and major radius of tokamak plasma, respectively. The frequency is normalized by $v_{s}/R$ with $v_{s} = \sqrt{T_{i}/m_{i}}$. The level of zonal flows in toroidal ITG turbulence is strongly influenced by the collisionless damping of the GAMs [5]. Hence, the GAM dynamics have been intensively studied in toroidal plasma experiments and large-scale parallel simulations in light of theoretical analyses [6-20].
The GAM is a damped eigenmode so that it is believed to be mainly excited nonlinearly similar to the stationary zonal flows in turbulence[2-4, 21], except for the case linearly driven by energetic particle[22]. It may also be degenerate with the Beta-induced Alfven eigenmode(BAE) in the long wavelength limit[23]. In this work, we numerically study the nonlinear excitation, saturation mechanism and the radial structure of the GAMs in tokamak plasmas by using newly well-benchmarked gyrofluid model. At first, an empirical closure relation of gyrofluid modeling is presented for the ion temperature gradient (ITG) fluctuations and the GAM components. The zonal flow damping is precisely testified by comparing with theoretical predictions and other kinetic calculations. Then, the model is applied to simulate the nonlinear excitation of the GAMs by ITG fluctuations. Furthermore, the saturation mechanism and the radial structure of the GAMs are investigated.

2. Gyrofluid ITG-zonal flow model with new closure relation
It is well-known that fluid and gyrofluid models possess advantages of clear/simplified physics picture in theoretical analyses and of fast calculation in large-scale simulations. The gyrofluid modelling has been well developed in plasma turbulence simulation, especially in the studies of zonal flow dynamics and turbulent transport [24-26]. However, a precise closure relation for lower order moment equations is still under improvement. Especially, the residual level of the zonal flows in ITG turbulence due to the collisionless damping is sensitively depended on the closure relation of the higher order moments in gyrofluid theory [27]. A new zonal flow closure relation has been derived systematically by writing the parallel heat flux as a summation of long and short wavelength parts. Here, we present an empirical closure relation for the GAM components by considering the Landau damping and its parallel wavenumber, which may implicitly involve the effect of finite orbit width. The new gyrofluid model and closure relation are testified by comparing the linear eigen values of ITG mode with other kinetic calculations under the standard Cyclone parameters [28] and the zonal flow damping in ITG turbulence with the theoretical prediction [29].

2.1. Gyrofluid ITG model with new closure relation
Following the standard procedure, a set of three-field fluid equations of the normalized potential \( \phi \); parallel ion velocity \( v_\parallel \) and ion pressure \( p_i \) can be derived as follows under the assumption of adiabatic electron response \( n_e = (n_e / T_e)(\phi - <\phi>) = (n_e / T_e)(1 - \delta \phi) \) [30]

\[
\begin{align*}
(1 - \delta - \nabla^2)v_\parallel &= -(a / L_a)\nabla_\parallel\phi - (1 + \eta_i)(a / L_a)\nabla_\parallel^2\phi - 2\Gamma_\phi\phi + p_i, \\
&\quad -\frac{1}{2}\omega_n\nabla_\parallel^2\phi - \partial_t <\phi > \nabla_\parallel\phi - \nabla_\parallel v_\parallel + \left[\phi_\parallel, \nabla_\parallel^2\phi\right] - \mu_n^2\nabla_\parallel^2\phi, \\
\partial_t v_\parallel &= -\nabla_\parallel(\phi + p_i) - \left[\phi_\parallel, v_\parallel\right] + \eta_i\nabla_\parallel^2v_\parallel, \\
\partial_t p_i &= -(1 + \eta_i)(a / L_a)\nabla_\parallel\phi - \left(\Gamma - 1 / 3\right)(1 + \eta_i)(a / L_a)\nabla_\parallel^2\phi + 4\Gamma_\phi p_i, \\
&\quad + \left(\Gamma - 1 / 2\right)\omega_n\nabla_\parallel^2\phi - \Gamma_\phi \nabla_\parallel v_\parallel - \gamma_{LD} \sqrt{8T_{\parallel e}/\pi k_\parallel} (p_i - \phi - \left[\phi_\parallel, p_i\right] + \chi_\parallel \nabla_\parallel^2p_i)
\end{align*}
\]

The normalized perturbed quantities are conventionally defined as [15]

\[
\begin{align*}
(r; \nabla_\parallel; v_\parallel; t) &\rightarrow \left(r / \rho_i; \rho_i \nabla_\parallel; a\nabla_\parallel; tv_\parallel / a\right); \\
(n; \phi; v_\parallel; p_i) &\rightarrow \left(n_i / L_a; e\phi / T_e; v_\parallel / v_\parallel; p_i / n_i T_e\right).
\end{align*}
\]

Here the magnetic drift term \( \omega_B f = 2\epsilon (\cos\theta V_\parallel - \sin\theta V_\perp) f \) for any perturbed quantity \( f \) with \( \epsilon = a / R \), Heaviside step function \( \delta = 0(1) \) for ITG fluctuations (the zonal flow component \( <\phi> \), \( \cdots \) denotes the flux surface average), which represents appropriately the adiabatic electron response to ITG fluctuations and the zonal flow; \( \eta_i = L_i / L_a \) with \( L_a = (d \ln n_i / dr)^{-1} \) and \( L_i = (d \ln T_e / dr)^{-1} \), i.e., the
characteristic length of equilibrium density (and ion temperature); ion Larmor radius $\rho_i = v_i/\omega_i$, with ion cyclotron frequency $\omega_i = e B_i/m_i$; $\mu_\perp$, $\eta_\perp$ and $\chi_\perp$ are the numerical normalized cross-field viscosities and thermal conductivity, which absorb the energy cascaded to short wavelength region. The dominant nonlinear terms come from the $\hat{E} \times \hat{B}$ convective nonlinearity, which are expressed by the Poisson bracket $[f,g] = \left( \partial_r f \rho_r g - \partial_\theta f \rho_\theta g \right) r$ in circular tokamak geometry $(r,\theta,\phi)$ with the radius of the magnetic surface $r$, the poloidal and toroidal angles $\theta$ and $\phi$, respectively. In this work, $\tau = 1$ is used.

The kinetic Landau damping physics is also represented by Hammett-Perkins closure model for the parallel heat flux moment $q_\parallel = -i \gamma L_D \sqrt{8 T_e / \pi k_b T_i} \left| k_r \right|$ [24]. However, the coefficients $\gamma_{L_D}$ for the ITG fluctuations and for the GAM components are different. It is empirically determined as

$$\gamma_{L_D} = \begin{cases} \Gamma - 1 & \text{for ITG} \\ 3\Gamma & \text{for GAMs} \end{cases} \quad (4)$$

Further, the parallel wavenumber for the GAMs is chosen with the parametric dependence of the safety factor $q$ and the inverse aspect ratio $\epsilon$, which may implicitly involve the finite orbit width effects.

$$k_\parallel = i k_\parallel = \frac{\varepsilon \left( \partial_\theta / q - \partial_r \right)}{(3 + \Gamma) (q / 1.6)^{\varepsilon^2} \left( \partial_\theta / q \right)}$$

for ITG

$$k_\parallel = i k_\parallel = (3 + \Gamma) \left( q / 1.6 \right)^{\varepsilon^2} \left( \partial_\theta / q \right)$$

for GAMs \quad (5)

It is noticed that the coefficients in equations (4) and (5) for the GAMs are still adjustable, which depend on the nonlinear modification of pressure profile.

2.2. Benchmark of toroidal ITG mode with Cyclone base case parameters

The standard Cyclone base case parameters have been extensively applied to benchmark newly developed kinetic codes. However, it is difficult to precisely benchmark a three-field fluid and/or gyrofluid model due to the low-order moment closure relation for finite Larmor radius effects. In equations (1)-(3), several additional terms, which represent more precise finite Larmor radius effects, have been incorporated to have a correct spectral structure of linear ITG growth rate. A local spectral code with fixed and/or periodic boundary conditions in radial direction and corresponding global version have been advanced to calculate the nonlinear evolution equations (1)-(3). Figure 1 displays the eigen frequency and growth rate of toroidal ITG mode under the standard Cyclone base case parameters: $L_T / R = 6.9$, $L_\phi / R = 2.2$, $a/R = 0.18$, $q = 1.4$, $s = 0.78$, $\mu_\perp = \eta_\perp = \chi_\perp = 0.8$.

![Figure 1](image-url)

**Figure 1.** Growth rate and real frequency of toroidal ITG modes under the standard Cyclone base case parameters: $L_T / R = 6.9$, $L_\phi / R = 2.2$, $a/R = 0.18$, $q = 1.4$, $s = 0.78$, $\mu_\perp = \eta_\perp = \chi_\perp = 0.8$. 

parameters: \( L_{\parallel}/R = 6.9, L_{\rho}/R = 2.2, \quad a/R = 0.18, \quad q = 1.4, \quad \dot{s} = r q' / q = 0.78 \). Here the characteristic lengths of density and temperature and the magnetic shear \( \dot{s} \) are taken at local region with maximum pressure gradient. Note that the maximum growth rate and corresponding frequency are much close to the kinetic values near the spectral peak \( k_{\parallel} \rho_i = 0.4 \). The global mode structure near the surface of maximum pressure gradient exhibits strong ballooning characteristics, as shown in figure 2. The profiles of the safety factor \( \eta \), the density and ion temperature are assumed as [31]

\[
q(r) = 0.854 - 2.184(r/a)^2; \\
n(r) = n_0 \exp \left\{ 0.667 \exp \left( (r/a - 0.5)/0.3 \right) \right\}; \\
T_i(r) = T_{i0} \exp \left\{ -2.076 \exp \left( (r/a - 0.5)/0.3 \right) \right\}.
\]

**Figure 2.** Eigenfunctions of toroidal ITG mode for different toroidal mode number \( n = 20 \) (left); \( n = 40 \) (center) and \( n = 60 \) (right). The corresponding wavenumbers are \( k_{\parallel} \rho_i = 0.175; 0.35 \) and 0.525. The standard Cyclone base case parameters are used: \( L_{\parallel}/R = 6.9, L_{\rho}/R = 2.2, \quad a/R = 0.18, \quad q = 1.4, \dot{s} = 0.78, \quad \mu = 0.18, \quad \chi = 0.8, \quad a/\rho_i = 320 \).

**Figure 3.** Time evolution of an initial static zonal flow \( V_{ZF0}(t = 0) = \sin(0.19 \pi x) \) in the zonal flow damping tests for different \( q \) values with \( \varepsilon = 0.18 \) (a) and for different \( \varepsilon \) values with \( q = 1.4 \) (b).
2.3. Benchmark of the zonal flow damping
The level of the zonal flows in turbulent fluctuations determines the regulation of turbulence and the anomalous transport. It is sensitively dependent on the collisionless Landau damping due to the toroidal coupling with the GAMs. To have a suitable coefficient of the closure relation for the GAMs, an initial value test for the zonal flow damping is performed by comparing the residual zonal flows with the analytical theory under the Cyclone base case parameters. Figure 3 illustrates the residual level of the zonal flows for different parameters $q$ and $\varepsilon$. It can be seen that the new closure relation shows the zonal flow damping and a correct residual level of the zonal flows, which are well in agreement with the theoretical prediction $V_{ZF} = \left( 1 + 1.6q^2 / \varepsilon v^2 \right) V_{ZF0}$ by Rosenbluth and Hinton [29]. The wavelet energy analysis as shown in figure 4 also clearly exhibits the zonal flow damping process and the parametric dependence of the GAM frequency on $\varepsilon = a/R$.

![Figure 4](image1.png)

Figure 4. Wavelet energy analysis for the zonal flow damping. It produces clearly the parametric dependence of the GAM frequency on the inverse aspect ratio, i.e., $\omega_{GAM} \propto \varepsilon (= a/R)$. The initial static zonal flow is assumed as $V_{ZF}(t=0) = \sin(0.19x)$, $q=1.4$. The contours indicate the spatially averaged $\ln(\langle \phi_{ZF}^2 / 2 \rangle)$ in the frequency-time plane.

3. GAM dynamics in ITG turbulence
The GAM is a damped eigenmode in toroidal plasmas, which is coupled with axi-symmetrical static potential through the geodesic curvature. Since the GAM is important in determining the regulation role of the stationary zonal flows and the transport level in ITG turbulence, the generation mechanism and/or the excitation process of the GAMs become an interesting topic. Generally, the GAM oscillation can be kicked through a static potential like an initial stationary zonal flow or a stationary pressure ripple. It is quickly damped through the Landau damping and the coupling with ion sound wave(SW). The decay rate is roughly estimated as $\propto q^5 \exp(-\alpha q^2)$ with constant $\alpha$ for small drift orbits [20, 23]. It is believed that the GAMs can be excited nonlinearly through the Reynolds stress like the stationary zonal flows. Based on equations (1)-(3), the GAM oscillator and the nonlinear source terms are expressed as follows

$$-V_\phi^2 \frac{\partial \phi(0,0)}{\partial t} = -2\Gamma \omega_p (\phi + p_\perp)(x_\perp,0) \cdot \left( -\frac{1}{2} \omega_q V_\phi^2 \phi(\pm 1,0) + \left[ \phi, V_\phi^2 \phi \right]_{(0,0)} \right),$$

(6)
The equations of the G.A.M.s is formed with the frequency and $0.3$ (a); $0.12$ (b); $0.047$ (c). The radial differential field as well as the coupling with the $p$ and $v$ and $\phi$ in $\omega$ and $\omega$. Meanwhile, it also suffers from the collisionless damping due to the coupling $\omega_{GAM}$. Here we will simulate the non-linear excitation of the G.A.M.s due to the ITG fluctuations.

The G.A.M.s are the combination of the perturbation of the G.A.M.s, ion SWs and the zonal flows. When the non-linear terms are ignored, the damped G.A.M. oscillator accompanied by the ion SW is formed with the frequency $\omega_{GAM}$. Here we will simulate the non-linear excitation of the G.A.M.s.

In addition, the equations of $\phi_{(l=1,0)}$ and $v_{(l=1,0)}$ should be also incorporated to have a coupled system of the G.A.M.s, ion SWs and the zonal flows. When the non-linear terms are ignored, the damped G.A.M. oscillator accompanied by the ion SW is formed with the frequency $\omega_{GAM}$. Here we will simulate the non-linear excitation of the G.A.M.s due to the ITG fluctuations.

The G.A.M.s are the combination of the perturbation of $(m,n)=(0,0)$ and $k_r \neq 0$ in potential field and the perturbation of $(m,n)=(1,0)$ and $k_r \neq 0$ in the density and/or pressure fields. The former is mixed by the stationary zonal flows and the components with finite low frequency $\omega_{GAM}$. The radial structure of the G.A.M.s may also be different from the counterpart of the zonal flows. Hence, the zonal flow component $\phi(0,0)$ is composed in a general sense by two parts with different frequencies, i.e.,

$$\phi(0,0) = \lambda_{ZF}\phi(0,0)(\omega = 0) + \lambda_{GAM}\phi(0,0)(\omega = \omega_{GAM})$$

The ratio of the two parts is determined by the balance between the driving force and damping sink of the G.A.M.s. The stationary part of the zonal flows is commonly generated nonlinearly through a modulational instability. Meanwhile, it also suffers from the collisionless damping due to the coupling with the G.A.M.s. For the part with G.A.M. frequency, it may be driven nonlinearly through a parametric instability, namely, 3-wave interaction, due to the finite frequency. The parametric instability may involve the non-linear terms $[\phi, p]_{(l=1,0)}$, $[\phi, v_{//}]_{(l=1,0)}$ and $[\phi, \nabla^2 \phi]_{(l=1,0)}$ as well as the coupling with the Reynolds stress $[\phi, \nabla^2 \phi]_{(l=0)}$. The parametric instability results from the balance between the non-linear driving and the Landau damping as well as the coupling with the ion SW. To have a direct simulation of the non-linear excitation of the G.A.M.s in ITG turbulence, several simulations have been designed to investigate the detailed physics processes.

![Image](Figure 5. Wavelet energy analyses for the non-linear excitation of the G.A.M.s in high q plasmas by the ITG fluctuation with different pump amplitude level $<\phi_{ZF}^2>/2≈0.3$ (a); $0.12$ (b); $0.047$ (c). The contours exhibit the spatially averaged $\ln(<\phi_{ZF}^2>/2)$ in the frequency-time plane. Note that the non-linear excitation of G.A.M. instability depends on the ITG pump amplitude. The inset graphs are the corresponding time evolution of total zonal flow energy and the pump wave. The standard Cyclone base case parameters are used here except for $q = 2.6$.)
3.1. Nonlinear excitation of the GAMs

The GAMs are robust in toroidal plasmas with higher $q$ [15, 16]. To observe the parametric instability of the GAMs, ITG fluctuations with different amplitude are provided as the pump wave. In these simulations, linear ITG modes are initially excited to some level and then artificially controlled to keep a constant amplitude. In such a quasi-steady ITG fluctuation, the nonlinear excitation of the zonal flows with the GAMs can be analyzed by choosing different ways for the comparison. Figure 5 shows the dependence of the nonlinear excitation of the GAMs on the ITG pump amplitude. The inset graphs illustrate the time evolution of total zonal flow energy and the pump waves. It can be seen that the GAM instability becomes weaker as the ITG pump amplitude decreases. Further, the GAM fluctuation may be stabilized or damped even if the zonal flows still grow up for lower ITG pump amplitudes, indicating that the pump amplitude threshold for the GAM instability is higher than that of the zonal flow instability. This is understandable at least since the GAMs suffer from strong Landau damping. As the ITG pump amplitude becomes lower, weak GAMs are initially produced only through the beat wave of the pump fluctuations and strongly damped as shown in figure 5(c). In addition, the zonal flow energy decreases after the saturation due to the collisionless damping of the GAMs. Hence, the GAMs are difficult to survive in turbulent fluctuations if there exists no enough strong nonlinear driving force.

On the other hand, for the plasmas with lower $q$ values, weak GAM instability can be observed only in higher ITG pump amplitude case, and it is quickly damped after the zonal flow saturation, as shown in figure 6(a). For the ITG pump wave with low amplitude, the GAM fluctuation is almost damped. A reference simulation without the GAM components is performed for the comparison with figure 6(b). It shows that the damped GAMs can efficiently reduce the zonal flow instability and the suppression role in ITG fluctuations even if they are very weak.

![Figure 6](image)

**Figure 6.** Wavelet energy analyses for the nonlinear excitation of the GAMs in low $q$ plasmas by the ITG fluctuation with different pump amplitude level $\langle \phi_{\text{ITG}}^2 \rangle / 2 \approx 0.58$ (a); 0.082 (b). The standard Cyclone base case parameters are used here except for $q = 1.0$.

3.2. Saturation of the GAM instability

Generally, the zonal flow can be saturated through different mechanisms such as the tertiary instability (Kelvin-Helmholtz mode), spectral modulation of ambient turbulence and wave-packet scattering or trapping [4]. However, the GAMs themselves suffer from intrinsic Landau damping so that the GAM instability becomes weakened. While the GAM instability is determined by the competition between the nonlinear driving and the Landau damping as well as the coupling with sound wave, the saturation mechanism may be also attributed to the balance of the driving and damping. Careful comparison of the ITG pump amplitudes at the GAM saturation shows that the ITG fluctuation levels are the same at
the saturation of $t = 210$ in figure 5(a) and $t = 230$ in figure 5(b). Note that the zonal flows are saturated earlier than the GAMs in figure 5(a), indicating their saturation mechanisms are different. Figure 5(a) and (b) also show that the zonal flow saturation is strongly coupled with the GAMs if the ITG pumps are weaker. Whereas the zonal flows are saturated even if the GAMs are still unstable. Since the GAM damping depends on $q$ with the scaling $\propto q^4 \exp(-aq^2)$, the ITG pump amplitude at the GAM saturation should be higher in low $q$ plasma than that in the high $q$ case. This point is confirmed by the observation in figure 6(a). Hence, the GAM instability is mainly saturated by the intrinsic Landau damping.

3.3. GAMs and ion sound wave in quasi-steady ITG turbulence

Although the GAMs are highly damped fluctuations, they can be sustained in a quasi-steady ITG turbulence through the balance among the nonlinear driving, collisionless damping and the coupling with the ion SW. Several simulations have been performed for different $q$ values. In the case without the nonlinear modification of pressure profile, i.e., forbidding the quasi-linear flattening effects, the GAM fluctuations are continuously kept with the frequency $\omega_{\text{GAM}}$ after the ITG turbulence saturation. It is interestingly observed that the SWs with the frequency $\omega_{\text{SW}} = \omega_{\text{GAM}}/q$ are excited earlier than the GAMs and are damped after the ITG saturation even if the GAM fluctuations are sustained, as shown in figure 7. Note that the frequencies of the GAM fluctuations in the perturbed potential $\phi_{(\pm 1,0)}$, pressure $p_{(\pm 1,0)}$, and parallel velocity $v_{\parallel(\pm 1,0)}$ are downshift slightly from the GAM frequency in the zonal flow potential for all $q$ values. Meanwhile, the ion SWs are damped in the quasi-steady ITG turbulence. It is also observed that some fluctuations with much lower frequency about half sound wave frequency are excited. These SW-like fluctuations may correspond to the low frequency spectra observed in some theoretical analyses and experiments [9,19,20]. When the quasi-flattening effects of the pressure profile are allowed in the simulations, the frequencies of GAM fluctuations in all perturbed components with $(m,n) = (\pm 1,0)$ are downshift after the ITG turbulence saturation. This may show that the zonal flows have stronger coupling with the pressure profile through the GAM fluctuations.

![Figure 7](image_url)

**Figure 7.** Wavelet energy analyses for the GAM fluctuations with the ion SW in the quasi-steady ITG turbulence. The parameters are the same as in figure 1. The contours exhibit the spatially averaged $\ln(<v_{\parallel(\pm 1,0)}^2>/2)$ in the frequency-time plane. The standard Cyclone base case parameters are used here except for different $q$ values (a) $q=4$, (b) $q=3$ and (c) $q=2$. 
Wavelet energy analyses in an artificial simulation similar to the zonal flow damping test in figures 3 and 4 except that the static zonal flow is set to be constant in time. The contours exhibit the spatially averaged $\ln(<v_{\parallel}^2>)$ in the frequency-time plane. Note that the SW is excited and behaves as a damping fluctuation.

The observation that the ion SW is a damped fluctuation in the quasi-steady ITG turbulence can be further tested and analyzed through an artificial simulation. Similar to the test simulation for the zonal flow damping in figures 3 and 4, the static initial flow is set to be constant in time. It is found from figure 8 that the GAM fluctuations are artificially suppressed, but the ion SWs are excited, which well match the SW frequency for different $q$ values. Note that these SWs are damped although the same static flow is always provided. This test result may be helpful to understand the observed damped sound wave in the ITG turbulence.

### 3.4. Radial spectrum of the GAMs

The GAM fluctuations are characterized by finite low frequency in the zonal flow potential and the perturbed components with $(m,n) = (\pm 1,0)$. They also possess a radial structure. Two local simulations are performed for the cases without and with the GAMs under the Cyclone base case parameters for the most unstable ITG mode $k_0 \rho_i = 0.35$ to compare the difference of the zonal flows and the GAMs. It is observed that the radial structure of the GAMs is shorter than that of the pure zonal flow fluctuation as shown in figure 9. This result approaches the observation $k_r \rho_i \sim 1.0$ in the global ITG simulation [16]. Note that some scaling of the radial structure of the GAMs has been also estimated as $k_r \rho_i \propto L_f^{1/3}$ or $L_T^{1/3}$ [21,32]. Some simulations with different $L_f/R$ have been done and the results show almost the same spectra of the GAMs.

![Figure 8](image1.png)

**Figure 8.** Wavelet energy analyses in an artificial simulation similar to the zonal flow damping test in figures 3 and 4 except that the static zonal flow is set to be constant in time. The contours exhibit the spatially averaged $\ln(<v_{\parallel}^2>/2)$ in the frequency-time plane. Note that the SW is excited and behaves as a damping fluctuation.

![Figure 9](image2.png)

**Figure 9.** Radial spectra of the zonal flows in the simulations without (closed circles) and with (closed squares) the GAMs. The curve with diamonds corresponds to that of $d\phi_{\parallel(\pm 1,0)}/dx$ component of the GAMs. The standard Cyclone base case parameters are used in both simulations except for $q = 2.0$ and $k_0 \rho_i = 0.35$. 

4. Summary
The precise gyrofluid model is helpful in large-scale turbulence simulations and physics analyses. In this work, a new empirical closure relation for the conventional three-field gyrofluid model is presented with emphasizing the zonal flow residual level due to the collisionless damping. It is shown that the model can reproduce the linear spectra of the toroidal ITG mode and the ballooning structures under the standard Cyclone base case parameters. The simulation tests for the zonal flow damping show that the key parametric dependence of the residual level of the static zonal flows on both the safety factor and the inverse aspect ratio is in agreement with the analytical prediction of the gyrokinetic theory.

As an application of the newly developed gyrofluid modelling, the nonlinear excitation of the GAMs is simulated in toroidal ITG turbulence. The spatio-temporal spectra of the zonal flows with the GAM components are analyzed by using time-dependent wavelet energy analysis. It is examined that the damped GAMs can be nonlinearly driven to excite an instability by an ITG pump wave with larger amplitude, which is higher than the pump amplitude threshold of the zonal flow instability. Meanwhile, the GAM instability is mainly saturated by the intrinsic Landau damping. It is also observed that the ion SW can be excited with the GAMs in the quasi-steady ITG turbulence, but it is characterized by a damped fluctuation even if the GAMs are sustained continuously. Meanwhile, some SW-like fluctuations with much lower frequency are observed in the simulations, which may correspond to the theoretically predicted low frequency spectra by the kinetic theory and the observation in some experiments. Furthermore, it is found that the radial structure of the GAMs is scaled as \( k, \rho_i \leq 1.0 \), which is shorter than that of the pure zonal flows.

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