A likelihood function for the *Gaia* Data

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**ABSTRACT**

When we perform probabilistic inferences with the *Gaia* Mission data, we technically require a *likelihood function*, or a probability of the (raw-ish) data as a function of stellar (astrometric and photometric) properties. Unfortunately, we aren’t (at present) given access to the *Gaia* data directly; we are only given a Catalog of derived astrometric properties for the stars. How do we perform probabilistic inferences in this context? The answer—implicit in many publications—is that we should look at the *Gaia* Catalog as containing the *parameters of a likelihood function*, or a probability of the *Gaia* data, conditioned on stellar properties, evaluated at the location of the data. Concretely, my recommendation is to assume (for, say, the parallax) that the Catalog-reported value and uncertainty are the mean and root-variance of a Gaussian function that can stand in for the true likelihood function. This is the implicit assumption in most *Gaia* literature to date; my only goal here is to make the assumption explicit. Certain technical choices by the Mission team slightly invalidate this assumption for *DR1* (*TGAS*), but not seriously. Generalizing beyond *Gaia*, it is important to downstream users of any Catalog products that they deliver likelihood information about the fundamental data; this is a challenge for the probabilistic catalogs of the future.

**Keywords**: astrometry — catalogs — methods: statistical — parallaxes — proper motions — stars: distances

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1. INTRODUCTION

The age of Gaia is also (perhaps coincidentally) the age of principled probabilistic inference in astrophysics. For this reason, the Gaia (Gaia Collaboration et al. 2016b) data are being used in many probabilistic inferences (for example, Astraatmadja & Bailer-Jones 2016b; Hawkins et al. 2017; Sesar et al. 2017; Lin et al. 2018). These probabilistic inferences take many forms, and have different levels of hierarchical complexity, but all of them require that there be, at base, a likelihood function, or a probability for the Gaia data as a function of model parameters. One confusing question investigators face is: What constitutes the Gaia data? And how do I write a probability over it, when all I get to see is the official Catalog release, with photometric and astrometric parameters and associated uncertainties?

There is a standard answer, but in most inferences it appears only implicitly: The inferences presume that the Gaia Catalog entries can be used to construct a likelihood function approximation, which is appropriate for use in inferences. In almost all work so far, this likelihood function has been given a Gaussian form, with mean and variance set to Catalog values (for example, this is clearly stated in the introductory remarks in Astraatmadja & Bailer-Jones 2016a). The implicit assumptions underlying this choice are what I am attempting to make explicit in this Note.

An investigator can take one of (at least) two attitudes towards the Gaia data: The investigator can think of the Gaia Catalog as being the data, in which case the assumption is that the generative process for each Gaia Catalog entry itself is Gaussian. Or the investigator can think of the Gaia data as taking some raw form, from which the Catalog has been derived (by, say, pipelines), in which case the Catalog contains parameters of a Gaussian approximation to the likelihood function for those raw data. It turns out it doesn’t matter which attitude the investigator takes; the proposal for the Gaia likelihood function made here works in either case; the two attitudes are identical in the limit that the Catalog contains sufficient statistics of the raw data. Indeed, it is almost a definition of sufficient statistics that—if you have them—you can use them to construct a good approximation to the likelihood function for the raw data. All that said, we are going to take the latter attitude: That is, that the Gaia data are raw data, and the Catalog is delivering statistics that can be used to construct an approximation to the likelihood function.

One contemporary trend in astrophysics is to think about replacing rigid catalogs with something more probabilistic, possibly representing uncertainties through a sampling in catalog space (for example, Brewer et al. 2013; Portillo et al. 2017). This idea is also informing some of the expected high-level outputs from the Gaia Mission too (Bailer-Jones et al. 2013). These ideas are interesting and new, and connect to the age of principled probabilistic inference in which we find ourselves. However, these ideas also come with substantial risk: In many cases, a posterior sampling or posterior probability information does not successfully encode sufficient likelihood information to permit downstream analyses (with, say, different priors). That is, investigators generally want—from an experiment or data source—likelihood information, not posterior information. This is because different investigators can have very different priors,
even qualitatively different priors; they won’t agree on anything about the data except what new information those data bring. Also, if they want to combine \(N\) stars in an inference, they obtain dangers of taking some prior to the \(N\)th power if they can’t get back to pure likelihood information. These concerns all flow from two principles: The first is the likelihood principle, which states that new knowledge comes in likelihood form. The second is the subjectivity of inference, or the principle that an experiment ought to produce likelihood updates for all investigators, no matter what their prior beliefs.

2. A LIKELIHOOD FUNCTION FOR GAIA

In the simplest possible case, imagine that we are trying to infer the true\(^1\) distance \(d_n\) to a star \(n\) given the Gaia data \(y_n\) that pertain to star \(n\), and nothing else (which is the goal, for example, of Astraatmadja & Bailer-Jones 2016a) The inference looks like this:

\[
p(d_n \mid y_n) = \frac{1}{Z_n} p(y_n \mid d_n) p(d_n),
\]

where \(p(d_n \mid y_n)\) is the posterior pdf for the true distance \(d_n\) given the data \(y_n\), \(Z_n\) is a normalization constant, \(p(y_n \mid d_n)\) is the likelihood (or the pdf for the data given the distance), and \(p(d_n)\) is the prior pdf for the true distance. In order to perform this inference, we need the likelihood and the prior.

Note that, in order to write down these schematic equations, we don’t need to be perfectly specific here about what, exactly, is the data \(y_n\). It could be all the raw Gaia data pertaining to this particular star \(n\), or it could be the catalog entry in the Gaia Catalog on star \(n\). Nothing about this formalism changes with these different choices, although there might be some implicit marginalizations over nuisance parameters in some of these particular cases. That is, what you explicitly put in for the likelihood function \(p(y_n \mid d_n)\) will depend on what you consider to be the data \(y_n\), but the formal structure will be identical.

My goal here is to promote a particular choice for this likelihood function. Getting straight to the point, in this simplest possible case,

\[
p(y_n \mid d_n) = p(y_n \mid \varpi_n)
\]

\[
\varpi_n = \frac{A_n \text{A.U.}}{d_n}
\]

\[
p(y_n \mid \varpi_n) = A_n \mathcal{N}(\varpi_n \mid \hat{\varpi}_n, \hat{\sigma}_{\varpi n}^2)
\]

where \(\varpi_n\) is the true parallax to star \(n\) at true distance \(d_n\) (implicitly \(\varpi_n\) is measured in radians here), \(p(y_n \mid \varpi_n)\) is the likelihood as a function of true parallax (rather than distance), \(A_n\) is a normalization (and units-conversion) constant, \(\mathcal{N}(\xi \mid \mu, V)\) is the Gaussian pdf for \(\xi\) given mean \(\mu\) and variance \(V\), \(\hat{\varpi}_n\) is the (noisy) value given for the parallax of star \(n\) in the Gaia Catalog, and \(\hat{\sigma}_{\varpi n}\) is the value given for the uncertainty on that parallax. The amplitude \(A_n\) is not directly given in the Gaia

\(^1\) There are many possible meanings for the word “true”. In this context, we say the “true distance” because it is not the measured distance, but rather the distance that the star truly has in some model.
Catalog but it turns out—because of that factor of $1/Z_n$ in equation (1)—it isn’t needed for parameter-estimation-like inferences. If you do find that you are doing an inference for which the amplitude $A_n$ matters—for instance some inferences that might involve comparing certain kinds of fully marginalized likelihoods—then $A_n$ can probably be reconstructed from a goodness-of-fit statistic in the Catalog (that we expect in Gaia DR2).

It is more stable numerically to do inferences with logarithms of probabilities. In the log,

$$\ln p(y_n \mid \varpi_n) = Q_n - \frac{1}{2} \frac{[\varpi_n - \hat{\varpi}_n]^2}{\hat{\sigma}^2_{\varpi_n}}$$

(5)

$$Q_n \equiv \ln \frac{A_n}{\sqrt{2\pi \hat{\sigma}^2_{\varpi_n}}}.$$  

(6)

This choice (4) or (5) for the likelihood function is Gaussian, and presumes that the Catalog values for the parallax and its uncertainty are accurate and represent a likelihood maximum and width. It is the choice made in many publications (for example, Astraatmadja & Bailer-Jones 2016a; Leistedt & Hogg 2017; Hawkins et al. 2017; Lin et al. 2018, among many others). It has the property that the likelihood peaks when the true parallax matches the Catalog-reported parallax, and that the function is symmetric in parallax space—not distance space—because Gaia measures geometric parallaxes, not distances. The likelihood function is a pdf for the data, evaluated at the data (which, for a Bayesian, are fixed; Jaynes & Bretthorst 2003), and therefore although it is a pdf over data, it is really a function of the true parallax: The likelihood function (4) returns the answer to the question: How probable are the observed data, if $\varpi_n$ is the true parallax of star $n$?

It is worth making a few technical notes related to dimensions or units: Because the data $y_n$ and the true parallax $\varpi_n$ have (in general, though not always) different units, the unknown amplitude $A_n$ will have (in general) non-trivial units. Also, you might think that the conversion from distance $d_n$ to parallax $\varpi_n$ in equation (2) would bring in some Jacobian factors of the form $||dd_n/d\varpi_n||$. However, because the true distance only parameterizes a function of the data, or because the likelihood function has units of per-data (and not per-parallax), the change of parameters doesn’t bring a change of units, at least not in the likelihood function (it would bring a change of units in the posterior pdf, or the prior pdf). I say more about these units issues elsewhere (Hogg 2012).

In a more general inference, it is not just the parallax (or distance) that the investigator is modifying, but the (say) $D = 5$ astrometric quantities (celestial positions, proper motions, and parallax) or a non-trivial subset of $D < 5$ of these. In this case the likelihood becomes

$$p(y_n \mid X_n) = A_n N(X_n \mid \hat{X}_n, \hat{C}_{Xn})$$  

(7)

where now $X_n$ is a $D$-vector of true values for the astrometric quantities for star $n$, $p(y_n \mid X_n)$ is the likelihood as a function of that vector of true quantities, $A_n$ is
again a normalization (and units-conversion) constant, \(N(\xi \mid \mu, V)\) is now the Gaussian function for \(D\)-vector \(\xi\) given \(D\)-vector mean \(\mu\) and \(D \times D\) covariance matrix \(V\), \(\hat{X}_n\) is the \(D\)-vector of values given for the \(D\) astrometric quantities for star \(n\) in the Gaia Catalog, and \(\hat{C}_{Xn}\) is the value given for the \(D \times D\) covariance matrix for that \(D\)-vector. Again, the assumption here is that the likelihood function has Gaussian form, peaked when the true values match the Catalog values, and symmetric in the quantities reported in the Catalog (which are celestial positions, proper motions, and parallaxes). In the log, this is

\[
\ln p(y_n \mid X_n) = Q_n - \frac{1}{2} [X_n - \hat{X}_n]^\top \cdot \hat{C}_{Xn}^{-1} \cdot [X_n - \hat{X}_n] 
\]

\[
Q_n \equiv \ln \frac{A_n}{\sqrt{||2\pi \hat{C}_{Xn}||}}, 
\]

where we have implicitly assumed that the \(D\)-vectors are column vectors.

One comment to make briefly here is that although the Gaia Catalog contains the covariance matrices \(\hat{C}_{n}\), those matrices are not in the data in a trivial form. They must be constructed from the Catalog entries. Read the manual for details.

What about transformations of the astrometric quantities? If the Gaia likelihood is Gaussian in the Catalog values of celestial position, parallax, and proper motion, is it also Gaussian in other functions of those variables? The answer is that it is only Gaussian in strictly linear transformations of these parameters. It isn’t Gaussian, therefore, in distance (which is inverse parallax), nor is it Gaussian in velocity (which is proper motion over parallax). That is, the likelihood function in equation (4) has a Gaussian form when plotted against parallax, but it doesn’t when plotted against distance. That said, The proper-motion likelihood will be Gaussian in any celestial coordinate system you prefer (equatorial or ecliptic or Galactic, for example), because these transformations (coordinate changes) are simply rotations, which themselves are linear transformations. Of course, if you transform the \(D\)-vector \(\hat{X}_n\) by a linear operator \(R\) to make a new vector \(R \cdot \hat{X}_n\), you must also transform the covariance matrix to the new matrix \(R \cdot \hat{C}_{Xn} \cdot R^\top\).

The likelihood functions of (4) and (7) are probabilities of the observed Gaia data as a function of astrometric parameters. But there are many more parameters, including photometric, point-spread-function, and spacecraft-attitude parameters as well, all of which contribute to the probability for the data. How do we deal with these nuisance parameters, or how can we ignore them? Implicitly, I am assuming here that the Gaia Catalog team has either optimized or marginalized out these nuisance parameters. Since the Gaia Mission delivers immense signal-to-noise on these nuisance parameters in most cases (Holl et al. 2012), it won’t matter much, technically, to this discussion whether they optimize the nuisance parameters or marginalize them out.

Of course, most inferences are not as simple as the inference described by equation (1). Except in rare cases, we aren’t just trying to find out the distance to a single star! We usually are trying to fit some model of the Galaxy, or of some set of stars, or calibrate a color-luminosity relationship, or something like that (for example, Sesar et al. 2017; Hawkins et al. 2017; Leistedt & Hogg 2017; Oh et al. 2017;
In these cases, there is a model $p(X_n \mid \theta)$ that says what we expect for star $n$’s true astrometric quantities $X_n$, given a set of parameters $\theta$ of our larger model. Within that larger model, the likelihood for a single star becomes

$$p(y_n \mid \theta) = \int p(y_n \mid X_n) p(X_n \mid \theta) \, dX_n,$$  \hfill (10)

where we have marginalized out the true astrometric properties $X_n$ for star $n$. These marginalized likelihoods $p(y_n \mid \theta)$ can be multiplied together (or, better, added in the log) to make likelihoods for collections of stars. None of that hierarchical structure or marginalization changes the story here, which is that the internal likelihood $p(y_n \mid X_n)$ should be given the Gaussian form in equation (7).

Related to this, sometimes a model very specifically determines the astrometric properties (celestial position, parallax, and proper motions) of the star or a subset of those. Sometimes a model only determines distributions over those parameters. If the model is deterministic in this sense, then the investigator can take the $D$-slice (the subset of $D$) of the five astrometric parameters in the Gaia Catalog that are determined, and the $D \times D$ sub-part of the $5 \times 5$ covariance matrix and use that in the likelihood form given in equation (7). An example of this is given by Astraatmadja & Bailer-Jones (2016a). If the model is not deterministic, but rather produces distributions over some set of parameters, then the investigator will have to do integrals like that shown in equation (10). An example of this is given by Oh et al. (2017).

Finally, you might want to do inference not purely on the astrometric properties (celestial position, parallax, and proper motion) but also on photometric properties (magnitudes, colors, reddenings). In this Note, I don’t take a position on the best likelihood function approximations for these other observables, but in general my recommendations would be similar: Use the Catalog entries to construct a Gaussian function to stand in for the likelihood function. In general, the magnitudes will have uncertainties that are closest to Gaussian in flux space (not magnitude space) but I expect that the photometric information from the Mission is so precise that it won’t matter that much what you assume here for most kinds of stars.

3. DISCUSSION

In order to perform inferences with the Gaia data, we need a likelihood function. The main point of this Note is that a sensible likelihood function stand-in or surrogate can be constructed from the Gaia Catalog, under the assumption that the Catalog contains likelihood information, and accurate (and sufficient) statistics of the data. I give explicit forms for the likelihood function surrogate in equations (4) and (7). These likelihood functional forms are not new—as I have emphasized, they are used in multiple places in the literature (for example, Astraatmadja & Bailer-Jones 2016a; Leistedt & Hogg 2017; Oh et al. 2017; Delgado et al. 2018)—the point of this is to make the likelihood function assumptions explicit.

One of my soap-box issues is that inference is technically subjective. The investigator must make decisions about what constitutes the data, and details of the model
that generates those data, in order to make inferences about the world. In this case, I am recommending making certain decisions. The first and most important is to decide that the likelihood function has a Gaussian form, with mean and width set by \textit{Gaia} Catalog entries. That is, assuming that the data are generated from the world by some mechanism that is fundamentally Gaussian in the true parameters. I am also implicitly making another recommendation, but it is a bit more subtle: I am recommending making the assumption that the \textit{Gaia} Catalog contains nearly-sufficient statistics about the raw data, and that the likelihood function constructed here is some kind of approximation for a probability for the raw data from the Mission (which we currently don’t get to see). Another equally valid attitude to take is that the \textit{Gaia} Catalog is the \textit{Gaia} data, and then this is just an assumption about how the world (which is, apparently, full of true values for stellar positions and velocities; oh and a DPAC) generated the \textit{Gaia} Catalog. It is worthy of note and comment here that although I take the former view, there is no change to anything we do if we take the latter view. That is, this is a purely philosophical position.

Now, in detail, what have we really assumed about the \textit{Gaia} data? That is, what assumptions would make the Gaussian form accurate? There are a host of things to say here, so I will only say a few of the most important things. The first is that we are assuming that the \textit{Gaia} team is delivering accurate results. We are assuming that the Catalog is accurate, in terms of parameter measurements and associated error variances and covariances. Second, we have also assumed that the Catalog is a representation of likelihood information. That is, the \textit{Gaia} DPAC has optimized a likelihood, or something very much like that. Thirdly, we have assumed that the likelihood is close to Gaussian in form. The likelihood will be exactly Gaussian when the raw data are connected to the model by Gaussian noise, and the model is linear. These requirements, amazingly, are close to being met in the \textit{Gaia} global solution. But for central-limit-theorem-like reasons, any likelihood will become close to Gaussian when the data are good enough. So we are assuming something jointly about the simplicity of the model and the data constraining it.

The second assumption—that the Catalog is a representation of likelihood information—is in fact violated in detail by \textit{Gaia} DRI TGAS. This catalog is a set of posterior inferences, one per star (Michalik et al. 2015; Gaia Collaboration et al. 2016a). However, the posterior inference for the \textit{Gaia} data was made using a prior built from the Tycho likelihood outputs (with effectively very broad or nearly flat priors), so the TGAS Catalog can be seen as providing (nearly) pure likelihood information, but for the combined \textit{Gaia}+Tycho data. In principle, any user of the TGAS Catalog should build a likelihood function that is a ratio of the posterior Gaussian to the prior (or a difference of logs)! But this is a detail in the context of the very weakly informative priors used in the construction of the TGAS data set. The weak priors mean that the posterior pdf shape is very similar to (or perhaps identical to) the likelihood function shape for these data (that is, the combined \textit{Gaia}+Tycho data). This gets more challenging in the future, because the \textit{Gaia} Collaboration will probably have to use more informative priors in the future for many Catalog entries.
Another assumption we have made is that we can treat each star independently. That is, we can write down a likelihood for star $n$ without considering any other star $n'$. This assumption actually comprises two qualitatively different assumptions. The first is that the inference of the Gaia attitude model (and other nuisances) induces no covariances between stars. This is not true in detail, but becomes more true as the Mission proceeds (Holl et al. 2012). The second is that the raw data can be separated cleanly between star $n$ and star $n'$. This isn’t true for stars that overlap consistently on the focal plane; that is, for stars that are closer than a few tenths of an arcsecond. These close pairs (or K-tuples) of stars will remain covariant in the Catalog, no matter how much data are taken. For now we are ignoring all of these effects, because for most stars and projects these are probably small effects, and the Catalogs themselves ignore them anyway.

There are other sources of non-Gaussianity: There are cosmic rays and there is binary (and triple) contamination and stellar blending and crowding and so on; won’t these induce non-Gaussianities? They will, probably, and there are empirical hints in the data of such issues. However, Gaussianity is the standard assumption across Gaia science at present (see, for example, all the previous citations for uses of a Gaia likelihood function), and it is the only sensible assumption in a context in which we are given nothing more than a mean value and a variance around that.

On that note: One thing I like to say about Gaussianity is that if you are given a mean and a variance for a distribution, and you believe those statistics to be accurate, then the Gaussian is the most conservative assumption one can make. It is the maximum-entropy distribution with that mean and variance! Of course this point is something of a red herring, because it is precisely when the noise is non-Gaussian that it will be impossible to obtain an accurate estimate of the mean and variance! The point here is that the Gaussianity assumption in this work is strongly connected to the assumption that the Gaia Catalog is delivering accurate statistics of the data.

I have listed many assumptions here; it is worthy of note that most assumptions can be tested. Testing assumptions about the likelihood function can be challenging, but there are classes of objects, like quasars, and co-moving binary stars, and Solar-System objects, that have very predictable astrometric properties, or properties that are highly constrained by external physical considerations. These objects can be used to test likelihood-function assumptions.

Right now, with the simple 5-parameter solutions for stars (that is, descriptions in terms of just a celestial position, parallax, and proper motion), it is possible to do likelihood optimization straightforwardly because the model is close to linear. Once the space opens up to binary companions, exoplanets, and hierarchical systems, there will be no straightforward way to optimize the likelihood, even for a single Gaia source, and the likelihood function will become (in general) multi-modal and non-Gaussian. The Gaia DPAC will have to face two issues: One is how to explore the full space or distribution of parameters that is consistent with the data. The other is how to represent and deliver that information to the user so that it is usable in inferences.
Even now, some of the post-processing of *Gaia* TGAS data to deliver improved distance and parallax estimates are producing not likelihood outputs but rather posterior outputs (for example, Astraatmadja & Bailer-Jones 2016b; Leistedt & Hogg 2017; Anderson et al. 2017). In order for these outputs to be usable in downstream inferences, they have to be convertible back into likelihood (or marginalized-likelihood) form. It isn’t sufficient to say “the prior of the next experiment will be the posterior of the previous experiment” because the down-stream user might have extremely different goals, or highly informative external data. In either case, that user needs likelihood information from the Mission. When the *Gaia* DPAC starts doing binary-star astrometric orbit inferences, the likelihood functions will become complex and multi-modal in the orbit-parameter space, and they will be faced with challenging questions about how to represent this information back to the user properly. Many choices here are either very computationally expensive, very complex, or limited in support (in the mathematical sense).

Finally, I want to end by answering the age-old question: *Why does the Gaia Catalog contain negative parallaxes?* The answer, in the context of these remarks, is that the Catalog is a description of the likelihood function, and for some stars, the peak of the Gaussian likelihood function happens to be at a negative parallax! In general, if a model is something like a linear fit in the presence of Gaussian noise (and the *Gaia* astrometric solution is something like this), then at low signal-to-noise the model will produce (at the maximum of its likelihood function) negative linear coefficients just as readily as positive coefficients. That is, linear fitting in a likelihood context will always produce negative model parameters. If the Catalog produced posterior information, rather than likelihood information, these negative parallaxes might get cut off by a prior. But for our purposes, this would be bad! We want the Catalog to translate into a simple description of the likelihood function so we can do simple inferences. Those requirements put us in a world in which *Gaia* reports many negative parallaxes.

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