Research of various inductors configurations for heating titanium slabs

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Abstract. In this paper various inductors configurations for heating a titanium slab, as well as the influence of frequency changes on efficiency of inductors are considered. The calculations are carried out by means of finite element method. In this research a three-dimensional time-harmonic model in A-formulation of Maxwell’s equations is used. In the framework of this work, only electromagnetic problem was calculated, since the main aim of this work is to assess the effectiveness of inductor configurations in terms of electromagnetic problem. Efficiency of inductor and the uniformity coefficient introduced herein have been chosen as the comparison parameters. Ultimately, distribution of electromagnetic power density in the workpiece is shown, also conclusions are drawn for each of inductor configurations.

1. Introduction
The use of titanium and its alloys is a promising area in modern industry. Titanium has optimal parameters: the mechanical strength approximately equals to steel, but density is less (7.8 $\frac{g}{cm^3}$ is the density of steel; 4.54 $\frac{g}{cm^3}$ is the density of titanium; 2.7 $\frac{g}{cm^3}$ is the density of aluminium), due to these physical properties it is one of the most important material in aerospace engineering. These industry requires high quality of fabrication operation, that increases the level of production knowledge intensity.

Titanium and its alloys go to production in the shape of finished blanks (slabs). A slab is a thick blank of rectangular cross section with a large ratio of width to height. To process the workpieces, it is necessary to heat them for further plastic deformation. The main requirement for heating is minimum temperature difference between center and surface of workpiece, as well as uniform heating. One way to meet this requirement is application of electric resistance furnaces. But the productivity of such furnaces is low, since heating is due to radiation from heaters. An alternative solution is application of induction heating, which provides heating of workpiece by induced current in it, thereby it is possible to carry out rapid heating by increasing power [1, 2, 3, 4].

Induction heating is a complex multiphysics problem (coupled thermal and electromagnetic problem) from the simulation point of view. Therefore, in modern realities, in the most cases the simulation of induction heating is performed by finite element method, which allows solving complex systems of partial differential equations by solving systems of linear algebraic equations [1, 2].
2. Formulation of the problem
The main aim of this research is to consider various configurations of inductors. The comparison is taken place according to the efficiency of the inductor and by analyzing the uniformity of electromagnetic power density distribution.

The object of heating is a slab with dimensions 4.1 m x 1.12 m x 0.28 m. The configurations of inductors are presented in fig. 1. The properties of titanium, electrical steel and copper which are necessary for the electromagnetic problem, are summarized in tab. 1.

| Material       | Property                          | Value  |
|----------------|-----------------------------------|--------|
| Titan          | Electrical conductivity, MSm/m    | 1.798  |
|                | Relative magnetic permeability    | 1      |
| Electrical steel| Electrical conductivity, Sm/m     | 10     |
|                | Relative magnetic permeability    | 500    |
| Copper         | Electrical conductivity, MSm/m    | 60     |
|                | Relative magnetic permeability    | 1      |

Figure 1. Inductor configurations: a) C-type inductor, b) six flat-type inductors, c) solenoid-type inductor

The cross section of the inductors is made so that the skin effect is clearly expressed, therefore, the calculation of the active resistance of the inductor is performed according to eq. (5).

3. Mathematical description
In this paper, the magnetic field is calculated based on the Maxwell’s equations in A-formulation:

\[ \Delta A - j\mu\sigma \omega A = -\mu_0 j_{ext}, \]  
\[ (1) \]
where \( \mathbf{A} \) is the value of the magnetic vector potential, \( \sigma \) is the electrical conductivity of the material, \( \mu \) is the magnetic permeability of the material, \( \mathbf{j}_{\text{ext}} \) is the current density vector.

The induced current density in workpiece is calculated as follow

\[
\sigma \mathbf{E} = \mathbf{j}_i 
\]  

(2)

where \( \mathbf{E} \) is the induced electric field strength.

Electromagnetic power in workpiece is calculated by eq. (3):

\[
Q = \iiint \frac{1}{2} \mathbf{E} \cdot \mathbf{j}_i \, dV, 
\]  

(3)

where \( Q \) is the electromagnetic power in workpiece.

The active resistance of the inductor is calculated taking into account the fact that there is a clearly narrowed skin effect in it, therefore, the calculation is performed according to the following formulas

\[
\delta = \sqrt{\frac{2}{\sigma \cdot \omega \cdot \mu}}, 
\]  

(4)

\[
R = \frac{l}{\sigma \cdot \delta \cdot a}, 
\]  

(5)

\[
\eta = \frac{Q}{Q + 0.5 \cdot R \cdot I^2}, 
\]  

(6)

where \( \delta \) is the penetration depth of the electromagnetic wave into the material, \( \omega \) is the angular frequency, \( R \) is the active resistance of the inductor, \( l \) is the length of the inductor, \( a \) is the width of the cross section, \( I \) is the amplitude of the current in the conductor of the inductor.

4. Results

All calculations were performed at a constant value of the current in inductor. Due to the symmetry of the arrangement, only one part of geometry is used, thereby reducing the required computation power [3]. Due to the skin effect of the electromagnetic field, the surface mesh of titanium billet is finer than that of the center [3].

Also, to assess the uniformity of electromagnetic power distribution in workpiece, we introduce a uniformity coefficient according to the following formula

\[
k_u = \frac{Q_{\text{avg}}}{Q_{\text{max}}}, 
\]  

(7)

where \( k_u \) is the uniformity coefficient, \( Q_{\text{avg}} \) is the average density of electromagnetic energy on the surface facing the inductor, \( Q_{\text{max}} \) is the maximum density of electromagnetic energy on the surface facing the inductor.

4.1. C-type inductor

Calculation of electromagnetic field for this configuration is performed only for a quarter of the model.
4.1.1. Configurations with different arrangement of magnetic cores

![Image 1](image1.png)  

**Figure 2.** The distribution of electromagnetic power in the workpiece: a) all magnetic cores, b) there is no middle magnetic core, c) half of the middle magnetic core, d) all magnetic core and three-phase power supply of the inductor

| Parameter | a) | b)  | c)  | d)  |
|-----------|----|-----|-----|-----|
| $\eta$    | 0.63 | 0.55 | 0.57 | 0.434 |
| $k_u$     | 0.337 | 0.45 | 0.505 | 0.308 |

4.1.2. Different inductor supply frequencies

![Image 2](image2.png)  

**Figure 3.** The distribution of electromagnetic power in the workpiece: a) 50 Hz, b) 100 Hz, c) 250 Hz, d) 500 Hz

| Parameter | a) | b)  | c)  | d)  |
|-----------|----|-----|-----|-----|
| $\eta$    | 0.63 | 0.64 | 0.646 | 0.65 |
| $k_u$     | 0.337 | 0.273 | 0.203 | 0.163 |
4.2. *Solenoid-type inductor*
Calculation of electromagnetic field for this configuration was also performed only for a quarter of the model.

4.2.1. **Different inductor supply frequencies**

![Image](image1)

**Figure 4.** The distribution of electromagnetic power in the workpiece: a) 50 Hz, b) 100 Hz, c) 250 Hz, d) 500 Hz

| Parameter | a) | b) | c) | d) |
|-----------|----|----|----|----|
| $\eta$    | 0.607 | 0.613 | 0.62 | 0.623 |
| $k_w$     | 0.485 | 0.391 | 0.29 | 0.23 |

**Table 4:** Efficiency values for various cases

4.2.2. **Without side magnetic core with different inductor supply frequencies**

![Image](image2)

**Figure 5.** The distribution of electromagnetic power in the workpiece: a) 50 Hz, b) 100 Hz, c) 250 Hz, d) 500 Hz
Table 5: Efficiency values for various cases

| Parameter | a) | b) | c) | d) |
|-----------|----|----|----|----|
| $\eta$    | 0.6 | 0.606 | 0.612 | 0.615 |
| $k_u$     | 0.568 | 0.457 | 0.338 | 0.268 |

4.3. Six flat-type inductors

Calculation of electromagnetic field for this configuration was performed only for a half of the model.

For this configuration of six flat-type inductors, various schemes of connecting individual inductor are possible, these schemes are shown in fig. 6–8.

![Figure 6](image1)

**Figure 6.** With accordant connection of adjacent coils and opposite connection of opposite coils

![Figure 7](image2)

**Figure 7.** With opposite connection of adjacent coils and opposite connection of opposite coils
4.3.1. With accordant connection of adjacent coils and opposite connection of opposite coils

Figure 8. With opposite connection of adjacent coils and accordant connection of opposite coils

Figure 9. The distribution of electromagnetic power in the workpiece: a) 50 Hz, b) 100 Hz, c) 250 Hz, d) 500 Hz

Table 6: Efficiency values for various cases

| Parameter | a)    | b)    | c)    | d)    |
|-----------|-------|-------|-------|-------|
| $\eta$    | 0.396 | 0.416 | 0.433 | 0.443 |
| $k_u$     | 0.27  | 0.224 | 0.171 | 0.138 |
4.3.2. With opposite connection of adjacent coils and opposite connection of opposite coils

![Image](https://example.com/image1.png)

**Figure 10.** The distribution of electromagnetic power in the workpiece: a) 50 Hz, b) 100 Hz, c) 250 Hz, d) 500 Hz

| Parameter | a) | b) | c) | d) |
|-----------|----|----|----|----|
| $\eta$    | 0.462 | 0.479 | 0.494 | 0.502 |
| $k_\mu$   | 0.357 | 0.292 | 0.221 | 0.178 |

4.3.3. With opposite connection of adjacent coils and accordant connection of opposite coils

![Image](https://example.com/image2.png)

**Figure 11.** The distribution of electromagnetic power in the workpiece: a) 50 Hz, b) 100 Hz, c) 250 Hz, d) 500 Hz
Table 8: Efficiency values for various cases

| Parameter | a) | b)  | c) | d) |
|-----------|----|-----|----|----|
| $\eta$    | 0.51 | 0.522 | 0.533 | 0.54 |
| $k_u$     | 0.205 | 0.173 | 0.134 | 0.109 |

4.3.4. With three-phase power supply and accordant connection of opposite coils

![Image of electromagnetic power distribution]

**Figure 12.** The distribution of electromagnetic power in the workpiece: a) 50 Hz, b) 100 Hz, c) 250 Hz, d) 500 Hz

Table 9: Efficiency values for various cases

| Parameter | a) | b) | c) | d) |
|-----------|----|----|----|----|
| $\eta$    | 0.458 | 0.471 | 0.484 | 0.491 |
| $k_u$     | 0.212 | 0.178 | 0.138 | 0.113 |

5. Conclusions

Based on fig. 2–5, fig. 9–12, as well as tab. 2–9, we can conclude that the best variant in terms of efficiency and uniformity of electromagnetic power density is the solenoidal-type inductor without a side magnetic core (although its presence increases the efficiency, but the uniformity of the power distribution is worsened). This variant of the inductor is the most inconvenient for the operating procedure, because loading the workpiece into the inductor is complicated.

C-type inductor with all magnetic cores also has a good efficiency equals to 0.63 for 50 Hz (see tab. 2), but with this configuration, there is a significant nonuniformity in electromagnetic power density distribution (fig. 2). A compromise in this case can be a configuration with half of middle magnetic core, which provides efficiency equals to 0.57 and uniformity coefficient equals to 0.505.

In the case of six flat-type inductors, the best variant for optimal efficiency and uniformity would be a configuration with with opposite connection of adjacent coils and opposite connection of opposite coils (fig. 7).
The use of a three-phase power supply system for inductors of the C-type and six flat-type inductors, although it provides load symmetry, does not show satisfactory results in terms of efficiency and power distribution in the workpiece.

An increase in the supply frequency shows an increase in efficiency, but the distribution of power density becomes less uniform, the maximum power is on the edges of the workpiece, which can cause overheating in these places.

References
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