INTERACTING DARK ENERGY AND COSMOLOGICAL EQUATIONS OF STATE

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Abstract

Interactions within the cosmic medium modify its equation of state. We discuss implications of interacting dark energy models both for the spatially homogenous background and for the perturbation dynamics.

Keywords: Cosmology; dark energy; non-adiabatic perturbations.

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I. INTRODUCTION

According to our current understanding the presently observable Universe is dynamically dominated by two so far unknown substances, Dark Matter (DM) and Dark Energy (DE). Of major interest is a precise knowledge of the DE equation of state (EOS) including its possible time dependence. A definitely time dependent EOS would rule out a “true” cosmological constant which still is the most favored DE candidate. A related problem is to understand why the energy densities of both components are of the same order of magnitude today (the “coincidence problem”).

Most approaches in the field rely on an independent evolution of DE and DM. Given the unknown nature of both DE and DM one may argue, however, that an entirely independent behavior is a very special case [1, 2]. At least, there is no compelling reason to exclude interactions from the outset. Unified models such as the Chaplygin gas models even try to understand DE and DM as different manifestations of one single fluid [3].

Our aim here is to point out the role of interactions within the dark sector for the cosmological dynamics. As far as the coincidence problem is concerned, an interaction may naturally lead to a fixed ratio of the energy densities of DM and DE. An interaction will also influence the perturbation dynamics. In particular, it modifies the adiabatic sound speed and the large scale perturbation behavior. It affects the time dependence of the gravitational potential and hence the Integrated Sachs-Wolfe effect (ISW) which is observable through the lowest multipoles of the anisotropy spectrum of the cosmic microwave background. Furthermore, a coupling between DM and DE is expected to be relevant for holographically determined DE and for the big rip scenario.

II. INTERACTING DARK ENERGY

An interaction between DM and DE is most conveniently modelled as a decay of the latter into the former,

\[
\dot{\rho}_M + 3H\rho_M = \Gamma \rho_X , \quad \dot{\rho}_X + 3H (1 + w_X) \rho_X = -\Gamma \rho_X ,
\]

where \( \rho_M \) and \( \rho_X \) are the energy densities of DM and DE, respectively, \( H \) is the Hubble expansion rate and \( w_X \) is the EOS parameter of the DE. The (not necessarily constant) decay rate is denoted by \( \Gamma \). Einstein’s field equations for spatially flat FRLW cosmologies
are
\[ H^2 = \frac{8}{3} \pi G \rho, \quad \dot{H} = -4 \pi G (\rho + p_X), \] (2)
where \( \rho = \rho_M + \rho_X \) is the total energy density. The quantity \( \dot{H} \) is related to the deceleration parameter \( q \) by \( q = -1 - \frac{\dot{H}}{H^2} \). It is obvious, that neither the Hubble parameter \( H \) nor the deceleration parameter is directly affected by the interaction (there is an indirect influence, however, since the coupling modifies the ratio \( \frac{\rho_M}{\rho_X} \)). It is only the second derivative of the Hubble parameter, equivalent to the third derivative of the scale factor \( a \) (where \( H \equiv \frac{\dot{a}}{a} \)) in which the decay rate enters explicitly,
\[ \frac{\ddot{H}}{H^3} = \frac{9}{2} + \frac{9}{2} w_X \frac{\rho_x}{\rho} \left[ 2 + w_X + \frac{1}{3H} \left( \Gamma - \frac{\dot{w}_X}{w_X} \right) \right]. \] (3)

This corresponds to the fact that \( \Gamma \) enters the luminosity distance \( d_L = (1 + z) \int \frac{dz}{H} \) only in third order of the redshift \( z \) (cf. Refs. [4, 5]). Consequently, the interaction becomes observable if the cosmological dynamics is not only described by the present values \( H_0 \) and \( q_0 \) of \( H \) and \( q \), respectively, but additionally by parameters that involve at least \( \ddot{H}_0 \), the present value of \( \ddot{H} \). The recently introduced “statefinder” parameters [6, 7] are of this type and represent useful tools to characterize interacting models [8].

III. CONSTANT DENSITY RATIO

Assuming \( w_X = \text{const} \) there exists a special solution of the system (1) for which \( \kappa \equiv \frac{\rho_M}{\rho_X} = \kappa_0 = \text{const} \). This solution which is of interest for the coincidence problem [2] requires a decay rate
\[ \Gamma = -3H \frac{\kappa_0}{1 + \kappa_0} w_X. \] (4)

The EOS parameter \( w_X \) has to be negative to obtain \( \Gamma > 0 \), i.e., a decay of the component \( X \) into the matter component. Under this condition the energy densities scale as
\[ \rho_M, \rho_X, \rho \propto a^{-3(1+w)}, \quad w = \frac{w_X}{1 + \kappa_0}. \] (5)

Here, \( w \) is the overall EOS which, compared with \( w_X \), is reduced in magnitude by a factor \( 1 + \kappa_0 \). The time dependence of the scale factor in the spatially flat case is \( a \propto t^{\frac{2}{3(1+w)}} \).

In a next step we realize that the chain of arguments leading to (5) (via (1) and (4)) may be reversed. The resulting statement is that any cosmic EOS \( w \) can be interpreted as an
effective EOS of an interacting mixture of a component with EOS $w_X$ and a dust component (a generalization to an arbitrary constant EOS is possible). With other words, for any one-component description of the cosmic medium there exists an equivalent interacting two-component model which generates the same background dynamics. This equivalence (which implies a degeneracy with respect to observational results) will turn out to be useful on the perturbative level. It provides us with an internal structure for any one-component cosmic medium and represents a systematic way to generate non-adiabatic perturbations which we shall consider later on.

An example of how a solution $\kappa_0 = \text{const}$ may be approached as the result of a dynamical evolution of the energy density ratio $\kappa$ can be sketched as follows [9]. For a decay rate $\Gamma = 3Hb^2 (1 + \kappa)$ with a parameter $b^2 = \text{const}$, the set of equations (1) admits two stationary solutions, $\kappa_+$ and $\kappa_-$, for the ratio $\kappa$ of the energy densities. These have the properties $\kappa_+ - \kappa_- \geq 0$ and $\kappa_+\kappa_- = 1$. For $\kappa = \kappa_+ = \kappa_-^{-1}$ we have $\rho_M > \rho_X$, i.e., matter dominance, while for $\kappa = \kappa_-$ the reverse relation $\rho_M < \rho_X$ holds, equivalent to a DE dominated phase. There exists a dynamical solution $\kappa(t)$ according to which the density ratio evolves from $\kappa = \kappa_-^{-1}$ for small values of the scale factor (matter dominance) towards a stable, stationary solution $\kappa = \kappa_-$ for large values of the scale factor (DE dominance). The deceleration parameter changes from positive values at $\kappa_-^{-1}$ to negative values at $\kappa_-$. This means, the interaction drives the transition from a matter dominated era to a subsequent phase with accelerated expansion. The solution $\kappa = \kappa_-$ can be identified with the solution $\kappa_0$, given by (5) for $b^2 = -\kappa_0 w_X (1 + \kappa_0)^{-2}$. Consequently, a stationary ratio for a density ratio which generates accelerated expansion is obtained as the result of a dynamical evolution.

As already mentioned, the overall equation of state $w$ in (5) is reduced in magnitude compared with $w_X$. In particular, one may ask whether it is possible to have $w \geq -1$ while $w_X < -1$. To this purpose it is useful to consider the basic equations (1) for the simple case $\Gamma = \text{const}$. Even if at the start of the decay the evolution is dominated by $\rho_X$, after a short time $\rho_M$ will be approximately

$$\rho_M \approx \frac{2}{3} \frac{\Gamma H^{-1}}{1 - W_X^2} \rho_X .$$

(6)

The calculations leading to this result are similar to those at the end of an out-of-equilibrium decay of (effectively) dust matter into radiation in inflationary scenarios [10, 11]. It is obvious, that the ratio $\frac{\omega_M}{\rho_X}$ in (6) can only be constant for $H = \text{const}$, i.e., for exponential
expansion. Combining now (6) with (4) we obtain a direct relation between $w_X$ and $\kappa_0$,

$$w_X = -\frac{1 + \kappa_0}{1 - \kappa_0}.$$  

(7)

This configuration has $w_X < -1$ necessarily. It approaches $-1$ only in the limit of a vanishing matter component. While a non-interacting DE component with $w_X < -1$ is known to lead to a singularity in a finite time [12, 13] (see, however, Ref. [14]), our example demonstrates that a suitable interaction makes the expansion de Sitter like, thus avoiding the big rip. Similar conclusions for interacting phantom energy have been obtained in Refs. [15] - [17]. (Alternative settings with interactions that lead to a transition from $w_X > -1$ to $w_X < -1$ were discussed in Refs. [18] and [19]).

It is also interesting to see how a solution (5) with $\kappa = \kappa_0$ is related to the holographic bound on the DE [20]. With an infrared cutoff scale $L$ the DE density is $\rho_\mathrm{hol}^X = 3c^2M_P^2L^{-2}$ where $M_P^2 = 8\pi G$ (cf. Ref. [21]). It seems natural to identify $L$ with the Hubble scale $H^{-1}$. With this choice it follows from Friedmann’s equation that both $\rho_X$ and $\rho_M$ are proportional to $H^2$. While this property was recently used to discard a cutoff $L = H^{-1}$ for independently evolving components [21], the situation is entirely different in the context of interacting models [22, 23]. Namely, the same dependence of both $\rho_X$ and $\rho_M$ on $H$ can be regarded as the result of a suitable interaction between these components in the sense described by (4) and (5). With a cutoff $L = H^{-1}$ we have $\kappa = \frac{\rho_M}{\rho_X} = \kappa_0 = \text{const}$ from the outset. Moreover, it immediately relates the parameter $c^2$ in the expression for $\rho_\mathrm{hol}^X$ to the ratio $\kappa_0$ by $\kappa_0 = \frac{1-c^2}{c^2}$. It follows that $c^2 < 1$, different from the case of non-interacting models [21] where $L$ is identified with the future event horizon (for an interacting model in the latter context see Ref. [24]). The condition for accelerated expansion, $-1 < w \leq -1/3$, then translates into

$$-(\kappa_0 + 1) < w_X < -\frac{1}{3} \ (\kappa_0 + 1) \quad \text{equivalent to} \quad -\frac{1}{c^2} < w_X < -\frac{1}{3c^2}.$$  

(8)

We emphasize that the interaction is essential here to have acceleration. There is no non-interacting limit. Without interaction acceleration is impossible, i.e., acceleration is a pure interaction phenomenon in this picture.

IV. PERTURBATIONS

Adiabatic perturbations of the cosmic fluid are characterized by $\ddot{p} = \frac{\dot{p}}{p}\dot{\rho}$, where $\dot{p}$ and $\dot{\rho}$ denote the total pressure and energy density perturbations, respectively. Both quantities
are related by the adiabatic sound speed $\dot{\rho}/\dot{p}$. If the substratum is understood as composed of two subcomponents with component $X$ decaying into component $M$, the decay rate $\Gamma$ may fluctuate. Fluctuating decay rates are known to produce curvature perturbations in certain inflationary scenarios \[25, 26, 27\]. Here, in a different context, they are used to characterize internal structure in the dark sector of the cosmic substratum \[28\]. For the medium discussed here we find

$$\dot{\rho} - \frac{\dot{p}}{\rho} \dot{\rho} = \rho \left( \frac{\Gamma}{3H} \right)^\hat{},$$

(9)

where $\left( \frac{\Gamma}{3H} \right)^\hat{}$ denotes fluctuations about the (constant) background ratio $\frac{\Gamma}{3H}$. A fluctuating fractional decay rate $\frac{\Gamma}{3H}$ necessarily modifies the adiabatic sound speed. In particular, for a fluctuation of the type $\left( \frac{\Gamma}{3H} \right)^\hat{} = \lambda \frac{\dot{\rho}}{\rho}$ we obtain an effective sound speed square $c_{\text{eff}}^2 = \frac{\dot{p}}{\rho} + \lambda$. Even if $\frac{\dot{p}}{\rho}$ is negative, a non-adiabatic contribution due to an internal structure may give rise to a physically acceptable effective sound speed. On the perturbative level the cosmic EOS may be different from $\dot{\rho} = w \dot{\rho}$. Fluctuations of the ratio $\frac{\Gamma}{3H}$ can be regarded as fluctuations of the EOS which give rise to pressure perturbations $\dot{\rho} = w \dot{\rho} + \dot{w} \rho$, where $\dot{w} = \left( \frac{\Gamma}{3H} \right)^\hat{}$. Thus, the equivalent two-component picture represents a simple method to construct variations of the cosmic EOS while keeping $w_M = 0$ and $w_X = \text{const}$. On this basis one may reconsider fluid models of the cosmic substratum which occasionally are abandoned since a purely adiabatic treatment does not correctly reflect the observational situation. Taking into account an internal structure of the medium, which is equivalent to go beyond the limits of an adiabatic analysis, will generally result in a different picture.

The internal structure, modelled here through a fluctuating decay rate, will also influence the large scale perturbation behavior. Large scale perturbations are conveniently described by the quantity $\zeta$ which represents a curvature perturbation on hypersurfaces of constant energy density \[29\]. On large perturbation scales $\zeta$ obeys the equation (cf. Refs. \[30 - 32\])

$$\dot{\zeta} = - \frac{H \dot{\rho} - \frac{\dot{p}}{\rho} \dot{\rho}}{\rho + p}. \quad (10)$$

For adiabatic perturbations $\dot{\rho} = \frac{\dot{p}}{\rho} \dot{p}$ the quantity $\zeta$ is conserved. As soon as there are non-adiabatic perturbations, $\zeta$ will no longer be constant. For our model with $w > -1$ (and, for simplicity, time independent perturbations $\dot{w} = \left( \frac{\Gamma}{3H} \right)^\hat{}$) we find

$$\zeta = \zeta_i - \frac{1}{1 + w} \left( \frac{\Gamma}{3H} \right)^\hat{} \ln \left( \frac{a}{a_i} \right). \quad (11)$$
A fluctuating fractional decay rate gives rise to a change in the curvature perturbation which is logarithmic in the scale factor. The subscript \( i \) denotes some initial value. The circumstance that \( \zeta \) is time dependent has consequences for the time dependence of the (gauge-invariant) gravitational potential for which we obtain
\[
\Phi = -3 \frac{1 + w}{5 + 3w} \zeta_i + \frac{9}{25 + 3w} \left( \frac{\Gamma}{3H} \right)^\hat{H} \left[ \ln \left( \frac{a}{a_i} \right) - \frac{3}{5 + 3w} \right].
\tag{12}
\]
While the first constant term on the right-hand side is a standard result for adiabatic perturbations \[33]\), the fluctuating decay rate introduces a logarithmic dependence on the scale factor. The time dependence of \( \Phi \),
\[
\dot{\Phi} = 9 \frac{1}{25 + 3w} \left( \frac{\Gamma}{\Theta} \right)^\hat{H} H,
\tag{13}
\]
determines the ISW. Different from the pure adiabatic case where \( \dot{\Phi} = 0 \) there is an ISW in this case. It is interesting to compare the result (12) with the corresponding expression of the \( \Lambda \)CDM model. In the long time limit one finds for the latter \( \Phi_{\Lambda\text{CDM}} \approx -\frac{3}{4} \frac{\rho_{M_0}}{\rho_\Lambda} \frac{a_0}{a} \) where \( \rho_{M_0} \) is some initial value and \( \rho_\Lambda = \text{const.} \) This means, \( \Phi_{\Lambda\text{CDM}} \) shows a stronger time dependence than \( \Phi \) in (12). Apparently, our interacting DE model predicts a smaller ISW compared with the non-interacting \( \Lambda \)CDM model. A smaller ISW is equivalent to a suppression (compared with the prediction of the \( \Lambda \)CDM model) of the lowest multipoles in the anisotropy spectrum of the cosmic microwave background. Such kind of suppression is required by the data and different mechanisms have been invented to account for this phenomenon \[34, 35, 36, 37]\). Here we argue that it may well be the consequence of an interaction in the dark sector.

V. SUMMARY

Interacting DE models are more general than non-interacting ones. A coupling between DE and DM enriches the cosmological dynamics. It reveals potential degeneracies in interpreting the observational data and provides a method to refine the equation of state of the cosmic substratum. An interaction is particularly useful to address the coincidence problem. It also opens new perspectives on holographic DE and on the big rip scenario. Furthermore, it introduces a non-adiabatic feature into the perturbation dynamics with consequences for
the effective sound speed and the time dependence of the gravitational potential. The interaction should be detectable by suitably refined luminosity distance measurements and by certain features in the anisotropy spectrum of the cosmic microwave background.

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