Nano-bridge Superconducting Quantum Interference Devices: beyond the Josephson limit

Dibyendu Hazra
Department of Physics, Indian Institute of Technology Kanpur, Kanpur-208016, India
(Dated: September 26, 2018)

Nano-scale superconducting quantum interference devices (nano-SQUIDs) where the weak-links are made from nano-bridges — i.e., nano-bridge-SQUIDs (NBSs) — are one of the most sensitive magnetometers for nano-scale magnetometry. Because of very strong non-linearity in the nano-bridge–electrode joints, the applied magnetic flux ($\Phi_a$) – critical current ($I_c$) characteristics of NBSs differ very significantly from conventional tunnel-junction-SQUIDs, especially when nano-bridges are long and/or the screening parameter is large. However, in most of the theoretical descriptions, NBSs have been treated like conventional tunnel-junction-SQUIDs, which are based on d.c. Josephson effect. Here, I present a model demonstrating that for long nano-bridges and/or large screening parameter the $I_c(\Phi_a)$ of a NBS can be explained by merely considering the fluxoid quantization in the NBS loop and the energy of the NBS; it is not necessary to take the Josephson effect into consideration. I also demonstrate that using the model, we can derive useful expressions like modulation depth and transfer function. I also discuss the role of kinetic inductance fraction ($\kappa$) in determining $I_c(\Phi_a)$.

I. INTRODUCTION

Nano-SQUIDs are the most sensitive magnetometers to measure the magnetic properties of individual nanoparticles or to probe the local magnetic properties of a sample in the sub-micron scale \[1-8\]. The other applications of nano-SQUIDs include measuring persistent current in a phase coherent ring \[4, 10\], single-photon detection \[14\], detecting motion of a nano-mechanical oscillator \[12\], and as non-linear circuit-elements in quantum bits \[13\]. Consequently, nano-SQUIDs have been developed from versatile methods and by using different types of weak-links (WLs) \[6, 14\], like, nano-bridges (NBs) \[15-22\], superconductor–normal-metal–superconductor (SNS) proximity junctions \[23-25\], tunnel junctions (TJs) \[26-28\], and carbon nano-tube \[29\] to mention only a few. Out of these, NBSs have been most commonly used primarily because of their easy fabrication method \[2, 6\].

Conventionally, a d.c. SQUID operation has been understood based on two phenomena: The d.c. Josephson effect and the fluxoid quantization in a superconducting loop \[30\]. An ideal d.c. Josephson effect predicts the flow of a lossless current — the supercurrent, $I_s$ — between two superconductors interrupted by a WL. $I_s$ follows the relation: $I_s = I_c \sin(\phi)$, where $I_c$ is the critical current and $\phi$ is the phase of the WL. This relation holds provided most of the phase across the superconductor–WL–superconductor drops between the WL, resulting in a well-defined phase of the WL, for instance, as it happens in TJs \[31-32\]. In case of a NB, the phase of the bridge is not well-defined in most of the cases \[15-17, 31-32\].

The ideal Josephson relation in NBs, therefore, only manifests in limiting cases, e.g., where bridge dimensions are smaller than the temperature dependent Ginzburg-Landau coherence length ($\xi_T$) \[15, 31-32\]. Consequently, in NBSs, various features in the $I_c(\Phi_a)$ have been observed — for instances, triangular-shaped \[15, 20, 21, 32-45\], double-branched \[15, 35-39, 41-46\] and a diamond-shaped $I_c(\Phi_a)$ \[15, 21, 36, 46\] — which are not conceivable by a conventional d.c. SQUID theory \[6, 30\]. Thus, alternative theories \[15, 33\] have been developed which describe some of the features, like, the non-sinusoidal $I_c(\Phi_a)$.

Here, I present a model that explains all of the above mentioned experimental features. More importantly, unlike the previous models, here, I demonstrate that for a NBS with long nano-bridges and/or large screening parameter, the fluxoid quantization in the NBS loop and the energy of the NBS can explain all the experimental features of $I_c(\Phi_a)$, without considering the Josephson effect. Moreover, the model presented here derives the expression for modulation depth and transfer function.

II. MODEL OF A NANO-BRIDGE–SQUID BEYOND THE JOSEPHSON LIMIT

I start by presenting a qualitative comparison between a TJ and a NB — how the phase ($\theta$) of the superconducting order parameter is distributed in these two cases, in presence of a finite $I_s$. In presence of a finite $I_s$, $\theta$ is spatially non-uniform and the phase gradient is related to the supercurrent density ($j_s$) and the Cooper-pair density ($n_s$): $\nabla \theta \propto j_s/n_s$ \[32\]. In a TJ, the insulating layer has negligible Cooper-pair density: $n_s \to 0$, the most of the $\theta$ drops across the insulating layer, yielding a well-defined $\theta$, as shown in Fig.1. In case of a NB, the bridge and the electrodes being made of the same superconductors, $n_s$ is almost same in NBs and in electrodes. The enhancement of the phase gradient in the NB is the result of the enhancement of $j_s$ due to the smallness of the width of the NB in comparison to the adjacent electrodes. In practical NBSs, the width of the NB is made typically \(2-3\) times smaller than the adjacent electrodes (much wider electrodes are not desirable in order to avoid vor-
The net current flowing across two NBs are symmetric of the net current flow in two branches—now, quantization formula \( I \) ically shown both a the NBS loop, another current, \( I \) ically spread almost uniformly across the whole structure, resulting in a poorly defined \( \Theta \). 

For a given \( \Phi \), \( I \) ical current, \( I \) ical current density, \( j \) FIG. 1. Schematics showing the spatial distribution of supercurrent density, \( j_s \), Cooper pair density, \( n_s \) and phase, \( \Theta \), for a tunnel junction (TJ) and for a nano-bridge (NB). In case of a TJ, there is a sharp drop of \( \Theta \) across the junction, making \( \Theta \) well-defined. In case of a NB, \( \Theta \) spreads almost uniformly across both the NBs become \( I \). That apart, due to the fluxoid quantization in \( \Phi \); \( I \) ical case can be straightforwardly generalized. When both NBs have the identical critical current, \( I_c \), the phase-drop across the NB is of the same order as \( \Phi \). Now, let me consider a standard NBS geometry, as shown in Fig.2. Here, I juxtapose a NB alongside a TJ in order to compare the spatial variation of \( j_s \), \( n_s \), Cooper pair density, \( L \) is the flux quanta. The magnitude and sign (sense of circulation) of \( I_cir \) depend on \( n \).

The origin of the \( L_k \) is the kinetic energy due to the motion of the Cooper pairs \([41, 43, 44]\). \( n \) is an integer and \( \Phi \) is the flux quanta. The magnitude and sign (sense of circulation) of \( I_cir \) depend on \( n \).

For a given \( \Phi \), \( n \) can have multiple values— the most probable \( n \) corresponds to the minimum energy (E) of the NBS which can be written as 

\[
E = \frac{1}{2} L_k \left[ \left( \frac{I_b}{2} + I_{cir} \right)^2 + \left( \frac{I_b}{2} - I_{cir} \right)^2 \right] + \frac{1}{2} L_l I_{cir}^2. \tag{3}
\]

The first term within square bracket is the kinetic energy of the Cooper pairs; the second term is the magnetic energy due to the circulation current. Moreover, to remain in the superconducting (zero-voltage) state, \( |I_{cir}| \) cannot exceed \( I_c \). This imposes restrictions on \( n \), following Eq.2

\[
\left| \frac{n\Phi_0 - \Phi_a}{I_l} \right| \leq I_c. \tag{4}
\]

Eqs.\(1\)–\(4\) lay the foundation to understand \( I_c(\Phi_a) \) of NBSs beyond the Josephson limit. It is convenient to express Eqs.\(1\)–\(4\) in terms of dimensionless units. I normalize currents by the maximum critical current of the NBS \( I_0 = 2I_c \), magnetic flux by \( \Phi_0 \), and the energy by \( \frac{1}{2} L_l I_0^2 \).

With these normalizations, Eqs.\(1\)–\(4\) take the form:

\[
i_{cs} = \left( 1 - 2|I_{cir}| \right), \tag{5}
\]

\[
i_{cir} = \frac{n - \phi_a}{\beta L}. \tag{6}
\]

Note that, maximum \( I_{cs} \) is \( 2I_c \), i.e., when \( I_{cir} = 0 \) and the net current flow across both the NBs becomes \( I_c \).

For a given \( \Phi_a \), \( I_{cir} \) can be evaluated from the fluxoid quantization formula

\[
L_l I_{cir} + \Phi_a = n\Phi_0, \tag{2}
\]
$$
\epsilon = \left[ \left( \frac{i_b}{2} + i_{cir} \right)^2 + \left( \frac{i_b}{2} - i_{cir} \right)^2 \right] + \frac{(1 - \kappa)^2}{\kappa} i_{cir}^2, \tag{7}
$$

and

$$|n - \phi_a| \leq \frac{\beta L}{2}, \tag{8}$$

respectively.

Here, $i_{cs} = I_{cs}/I_0$, $\kappa = \frac{\Phi_0}{\Phi} = \frac{\Phi_0}{\Phi_0}$, $\epsilon = E/\frac{1}{2} L_k I_0^2$, $\beta_L = L_1 I_0 / \Phi_0 = 2 L_1 I_c / \Phi_0$ and $\kappa = \frac{L_k}{L_t}$. $\beta_L$ is the well-known screening parameter and $\kappa$ is the kinetic inductance fraction: $0 \leq \kappa \leq 1$. Here, instead of $I_0/L_k$, I have preferred to express energy in terms of $\kappa$, as this is more commonly used in literature (see, e.g., Ref. [39] and references therein).

III. RESULTS, ANALYSIS AND DISCUSSION

A. Variation of $i_{cs}$ and $\epsilon$ as a function of $\phi_a$

In this section, first, I analyze the variation of $i_{cs}(\phi_a)$ and $\epsilon(\phi_a)$, for different values of $\beta_L$ and $\kappa$. In Fig. 3, I show the variation of $i_{cs}(\phi_a)$ and $\epsilon(\phi_a)$ for $\beta_L = 2.0$ and for three different $\kappa$. Since, $I_{cs}$ is periodic in $\Phi_0$, i.e., $i_{cs}$ is periodic in $1$, I restrict myself in the range $-0.5 \leq \phi_a \leq 0.5$. For this particular $\beta_L$, Eq. 8 suggests that the allowed $n$ are $n = 0$ for the entire range of $\phi_a$: $-0.5 \leq \phi_a \leq 0.5$, and 1 and -1 for positive and negative flux axis, respectively. The corresponding $i_{cs}$ are plotted in different colours, as indicated in the figure, by solid lines. For this particular $\beta_L$, therefore, maximum two $I_t$ branches are possible. Out of these two, to understand, whether only one or both should be observable in an experiment, I also plot corresponding $\epsilon$ on the right-hand panel—keeping in mind that the probability to occupy the lowest energy branch is more than the higher one. For a given $\phi_a$, to determine the threshold energy difference, $\Delta \epsilon_{th}$, between two branches, below which both the $I_{cs}$ branches should be experimentally observable, one requires a detailed thermodynamical analysis, which is not the aim of this article. Instead, first, I shall analyze the expected experimental $i_{cs}(\phi_a)$ qualitatively and subsequently discuss whether a single- or double-branched $i_{cs}(\phi_a)$ would appear for an arbitrarily chosen $\Delta \epsilon_{th}$ quantitatively.

Returning to Fig. 3 for $\kappa = 0.01$ and 0.45, we see that the energy is always much smaller for $n = 0$ in comparison to $n = 1$ and $-1$, except at the boundary: $\phi_a = \pm 0.5$. Thus, in this case, the probability of $n = 0$ configuration is much more than $n = 1$ and $-1$ for the entire range of $-0.5 < \phi_a < 0.5$. Thus, in $i_{cs}(\phi_a)$, experimentally, only $n = 0$ branch should be observable, with maxima at $\phi_a = 0$, as has been observed quite commonly in several experiments, for instances, in Refs. [15, 20, 21, 33, 45]. The above scenario, quite interestingly, changes for $\kappa = 0.9$. In this case, the energy is almost same for $n = 0$ and $+1$ and $-1$. Thus, in $i_{cs}(\phi_a)$, experimentally, all three $n = 0, 1$, and $-1$ are accessible, and $i_{cs}(\phi_a)$ should look like an incomplete-diamond-shaped, as has been observed, for instances, in Refs. [15, 21, 33, 45]. We note that the energy difference, $\Delta \epsilon_{th}$, between two branches becomes smaller and smaller as we move from center, i.e., at $\phi_a = 0$, towards the edges, i.e., $\phi_a = \pm 0.5$. Thus, the possibility of double-valued $i_{cs}(\phi_a)$ near $\phi_a = \pm 0.5$ is more than near $\phi_a = 0$, leading to an incomplete-diamond-shaped $i_{cs}(\phi_a)$, as has been observed, for instances, in Refs. [33, 37, 39, 41].

With increasing $\beta_L$, more features appear. In Fig. 4, I show the variation of $i_{cs}(\phi_a)$ and $\epsilon(\phi_a)$ for $\beta_L = 5.0$ for three different $\kappa$, identical to ones used in Fig. 3. For this particular $\beta_L$, the allowed $n$ are $0, \pm 1$ and $\pm 2$ for the entire range of $\phi_a$. Thus, as the figures indicate, five $i_{cs}(\phi_a)$ branches are possible, in principle. Here, I would like to mention that experimentally, with best of my knowledge, more than two branches of $i_{cs}(\phi_a)$ has never been observed in NBSs [4]. This indicates that the probability to occupy the third or any of the higher branches is very small. Following the discussion of the previous paragraph, i.e., $\beta_L = 2.0$ case, here also, we can qualitatively understand whether single or two branches of $i_{cs}(\phi_a)$ is likely to be observed in experiments. Instead, I shall discuss the other important salient features, assuming that only single-branched $i_{cs}(\phi_a)$, corresponding to the minimum energy, is observable. For $\kappa = 0.01$, $n = 0$ corresponds to minimum energy and accordingly we get...
are indicated schematically by dashed lines. All three curves are for the same screening parameter, \( i \). Double-branched, otherwise it leads to single-branched.

**FIG. 4.** Left panel: The normalized critical current, \( i_{cs} \), of a NBS as a function of normalized flux, \( \phi_a \), for different kinetic inductance fraction, \( \kappa \). All possible \( i_{cs} \) branches, corresponding to different allowed fluxoid number, \( n \), as per Eq. 8, are shown. The values of \( n \) are represented by different colours: black (0), red (-1), blue (1), green (2) and brown (-2), and also indicated in the figures. The expected experimental \( i_{cs} \) are indicated schematically by dashed lines. All three curves are for the same screening parameter, \( \beta_L = 5.0 \). Right panel corresponding to the normalized energy, \( \epsilon \), for the identical parameters of the left panel.

A \( i_{cs} \) with maxima at \( \phi_a = 0 \). The scenario changes quite dramatically for \( \kappa = 0.45 \). In this case, \( n = 1 \) and \( -1 \) correspond to minimum energy for positive and negative flux axis, respectively. Accordingly, we get a single-branched \( i_{cs} \) with minima at \( \phi_a = 0 \). So, we see that, even for a symmetric NBS, \( \phi_a = 0 \) can correspond to minima of \( i_{cs} \). This has been experimentally observed, for instances, in Refs. [21, 42, 46]. The scenario turns even more dramatic for \( \kappa = 0.9 \). Here, like \( \kappa = 0.45 \), the minimum energy is governed by \( n = \pm 1 \); but, \( n = -1 \) corresponds to minimum energy for the positive flux axis whereas \( n = 1 \) corresponds to minimum energy for the negative flux axis. Accordingly, we get a single-branched \( i_{cs} \) with maxima at \( \phi_a = 0 \). It, therefore, recovers the \( i_{cs} \) pattern of \( \kappa = 0.01 \) case, despite the fact that different \( n \) are stabilized in these two cases.

**B. Determining whether single- or double-branched \( i_{cs}(\phi_a) \) should be observable**

From Fig. 3 and 4 it is apparent that depending upon the values of \( \beta_L \) and \( \kappa \), \( i_{cs}(\phi_a) \) can be single- or double-branched. In this section, I determine which combinations of \( \beta_L \) and \( \kappa \) yield single-branched and which ones yield double-branched \( i_{cs}(\phi_a) \). To do so, I calculate the energy difference, \( \Delta \epsilon \), between the first two branches, close to the edge (i.e., \( \phi_a = \pm 0.5 \)), at an arbitrarily chosen \( \phi_a = \pm 0.35 \). I assume that \( \Delta \epsilon \leq \Delta \epsilon_{th} \) yields double-branched, otherwise it leads to single-branched \( i_{cs}(\phi_a) \). In Fig. 5 I show the possibility of single- or double-branched \( i_{cs}(\phi_a) \) for four different choices of \( \Delta \epsilon_{th} \) — 1.0, 0.1, 0.01, and 0.001 — respectively, as a function of \( \beta_L \) and \( \kappa \). We see that for \( \beta_L \rightarrow 1 \), irrespective of the values of \( \kappa \) and for \( \kappa \rightarrow 0 \), irrespective of the values of \( \beta_L \), yield single-branched \( i_{cs}(\phi_a) \), independent to the choice of \( \Delta \epsilon_{th} \). For \( \Delta \epsilon_{th} = 1 \) and \( 0.1 \), at a fixed \( \beta_L \), higher \( \kappa \) values increase the probability of double-branched \( i_{cs}(\phi_a) \). In these cases, the most of the area in the \( \beta_L \)-\( \kappa \) space favours the double-branched \( i_{cs}(\phi_a) \). With decreasing \( \Delta \epsilon_{th} \), \( \beta_L \)-\( \kappa \) space is divided in different domains: certain combinations of \( \beta_L \) and \( \kappa \) favours single- and the remaining combinations favours double-branched \( i_{cs}(\phi_a) \), as expected. Furthermore, with decreasing \( \Delta \epsilon_{th} \), more area of \( \beta_L \)-\( \kappa \) space favours single-branched \( i_{cs}(\phi_a) \). We also note that with increasing \( \beta_L \) and \( \kappa \), the area of the double-branched \( i_{cs}(\phi_a) \) domains increases. For materials with higher \( \kappa \), like niobium and niobium nitride [50], typically also have higher critical current density compared to materials with lower \( \kappa \), for instance, Al. Thus, for identical nano-SQUID geometries, \( \beta_L \) is also higher for high-\( \kappa \) materials, making the appearance of double-branched \( i_{cs}(\phi_a) \) more probable compared to low-\( \kappa \) ones, as has been reported in several publications, for instances, in Refs. [13, 35, 36, 41, 46].

**C. Calculating modulation depth and transfer function**

In this section, I shall calculate two important parameters, namely, the modulation depth and the transfer func-
tion. For simplicity, first, let me consider the case where only $n = 0$ is accessible. From Eq.8 it is clear that maximum $i_{cs}^{\text{max}}$, corresponds to minimum $|i_{\text{cir}}|$ whereas, minimum $i_{cs}^{\text{min}}$, corresponds to maximum $|i_{\text{cir}}|$. For $n = 0$, Eq.8 tells that minimum $|i_{\text{cir}}|$ is 0 whereas maximum $|i_{\text{cir}}|$ is $0.5/\beta_L$ (corresponding to $\phi_n = 0$ and $\pm 0.5$, respectively). This leads $i_{cs}^{\text{max}} = 1$ and $i_{cs}^{\text{min}} = 1 - 1/\beta_L$, yielding a modulation depth 

$$i_{cs}^{\text{max}} - i_{cs}^{\text{min}} = \frac{I_{cs}^{\text{max}} - I_{cs}^{\text{min}}}{I_0} = \frac{1}{\beta_L}.$$  

in normalized unit.

It can be shown that Eq.9 is valid in general, irrespective of whether $i_{cs}(\phi_n)$ is single or double-branched. This is also evident from both Fig.3 and 4. Here, I would like to point out that Eq.9 can be derived approximately from conventional d.c. SQUID theory [6, 30] and has often been used in the context of NBSs.

For the transfer function($I_{cs}\Phi_a$), i.e., the slope of the $I_{cs}(\Phi_a)$, since the variation of $I_{cs}(\Phi_a)$ is linear, $I_{cs}\Phi_a$ can straight forwardly be derived as

$$I_{cs}\Phi_a = \frac{I_{cs}^{\text{max}} - I_{cs}^{\text{min}}}{\Phi_0/2} = \frac{2I_0}{\beta_L}\Phi_0.$$  

D. Limits of the model

I have shown that using the model presented here, which does not take the Josephson effect in NB−electrode joints into account, $I_{cs}(\Phi_a)$ of NBSs is derivable. The result is triangular-shaped $I_{cs}(\Phi_a)$ with one to two branches, as has been observed in several experiments [15, 20, 21, 22, 12]. Now, let me discuss the limits in which the model works. The central assumption of the model is that the phase drop across the NBs is not significantly higher than the overall phase drop across the electrodes of the NBS. This assumption is valid for NBs longer than $\xi_T$. A large number of NBSs reported in the literature fulfils this criterion (see for instance Ref. [8] and references therein). As the length of the NBs approaches $\xi_T$, a well-defined $\theta$ can be attributed to the NBs and they approximately behave like Josephson junctions [21, 32] — consequently, $I_{cs}(\Phi_a)$ of a NBS deviates from being triangular and becomes more sinusoidal [15, 19]. Another restriction comes from Eq.8 which imposes that $\beta_L$ must be $\geq 1.0$. Like a short NB, for $\beta_L \leq 1.0$ also, a NB behaves more like a Josephson junction.

IV. CONCLUSION

In conclusion, I have developed a model for NBSs beyond the Josephson limit, i.e., for long NBs and/or large screening parameter. In this limit, the $I_{cs}(\Phi_a)$ of a NBS can be understood by considering the fluxoid quantization in the NBS loop and the energy of the NBS. The model explains various experimental features — like, triangular-shaped, double-branched, and a diamond-shaped $I_c(\Phi_a)$ — reported in the literature. From the model, I derive the expression for the modulation depth and the transfer function. Using the model, I have shown that both the screening parameter and the kinetic inductance fraction play vital role in deciding the number of $I_{cs}(\Phi_a)$ branches.

V. ACKNOWLEDGEMENTS

I acknowledge the financial support from the CSIR India.

[1] W. Wernsdorfer, Advances in Chemical Physics, Volume 118, 99 (2007).
[2] W. Wernsdorfer, Superconductor Science and Technology 22, 064013 (2009).
[3] C. Foley and H. Hilgenkamp, Superconductor science and technology 22, 064001 (2009).
[4] D. Vasyukov, Y. Anahory, L. Embon, D. Halbertal, J. Cuppens, L. Neeman, A. Finkler, Y. Segev, Y. Myasoedov, M. L. Rappaport, et al., Nature nanotechnology 8, 639 (2013).
[5] E. Levenson-Falk, R. Vijay, N. Antler, and I. Siddiqi, Superconductor Science and Technology 26, 055015 (2013).
[6] C. Granata and A. Vettoliere, Physics Reports 614, 1 (2016).
[7] J. Gallop and L. Hao, ACS nano 10, 8128 (2016).
[8] G. Yue, L. Chen, J. Barreda, V. Bevara, L. Hu, L. Wu, Z. Wang, P. Andrei, S. Bertaina, and I. Ciorescu, Applied Physics Letters 111, 202601 (2017).
[9] D. Mailly, C. Chapelier, and A. Benoit, Physical review letters 70, 2020 (1993).
[10] W. Rabaud, L. Samimyadayar, D. Mailly, K. Hasselbach, A. Benoit, and B. Etienne, Physical Review Letters 86, 3124 (2001).
[11] L. Hao, J. Gallop, C. Gardiner, P. Josephs-Franks, J. Macfarlane, S. Lam, and C. Foley, Superconductor Science and Technology 16, 1479 (2003).
[12] S. Etaki, M. Poot, I. Mahboob, K. Onomitsu, H. Yamaguchi, and H. Van der Zant, Nature Physics 4, 785 (2008).
[13] R. Vijay, J. Sau, M. L. Cohen, and I. Siddiqi, Physical review letters 103, 087003 (2009).
[14] M. J. Martínez-Pérez and D. Koele, Physical Sciences Reviews 2 (2016).
[15] K. Hasselbach, D. Mailly, and J. Kirtley, Journal of applied physics 91, 4432 (2002).
[16] S. Lam and D. Tilbrook, Applied physics letters 82, 1078 (2003).
[17] A. G. Troeman, H. Derking, B. Borger, J. Pleikies, D. Veldhuis, and H. Hilgenkamp, Nano Letters 7, 2152 (2007).
[18] L. Hao, J. Macfarlane, J. Gallop, D. Cox, J. Beyer, D. Drung, and T. Schurig, Applied Physics Letters 92, 192507 (2008).
[19] R. Vijay, E. Levenson-Falk, D. Slichter, and I. Siddiqi, Applied Physics Letters 96, 223112 (2010).
[20] S. Mandal, T. Bautze, O. A. Williams, C. Naud, E. Busstarret, F. Omnes, P. Rodiere, T. Meunier, C. Bauerle, and L. Saminadayar, ACS nano 5, 7144 (2011).
[21] D. Hazra, J. R. Kirtley, and K. Hasselbach, Applied Physics Letters 103, 093109 (2013).
[22] L. Chen, H. Wang, X. Liu, L. Wu, and Z. Wang, Nano letters 16, 7726 (2016).
[23] L. Angers, F. Chiodi, G. Montambaux, M. Ferrier, S. Guérion, H. Bouchiat, and J. Cuevas, Physical Review B 77, 165408 (2008).
[24] A. Ronzani, M. Baillergeau, C. Altimiras, and F. Giazotto, Applied Physics Letters 103, 052603 (2013).
[25] S. Samaddar, D. Van Zanten, A. Fay, B. Sacépé, H. Courtois, and C. Winkelmann, Nanotechnology 24, 375304 (2013).
[26] R. Wölbing, J. Nagel, T. Schwarz, O. Kieler, T. Weimann, J. Kohlmann, A. Zorin, M. Kemmler, R. Kleiner, and D. Koelle, Applied Physics Letters 102, 192601 (2013).
[27] C. Granata, A. Vettoliere, R. Russo, M. Fretto, N. De Leo, and V. Lacquaniti, Applied Physics Letters 103, 102602 (2013).
[28] M. Schmelz, V. Zakosarenko, T. Schönau, S. Anders, S. Linzen, R. Stolz, and H. Meyer, Superconductor Science and Technology 30, 014001 (2016).
[29] J.-P. Cleuziou, W. Wernsdorfer, V. Bouchiat, T. Onoda, and M. Monthioux, Nature nanotechnology 1, 53 (2006).
[30] J. Clarke and A. I. Braginski, The SQUID handbook: Applications of SQUIDs and SQUID systems, John Wiley & Sons (2006).
[31] K. Likharev, Reviews of Modern Physics 51, 101 (1979).
[32] M. Tinkham, Introduction to superconductivity, Courier Corporation (1996).