Empirical Period-Height Relationship for reinforced Concrete Moment Resisting Buildings in India

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Abstract. The fundamental period of vibration of buildings is the unique parameter that has a major influence on the earthquake induced lateral forces. Many design codes provide simple empirical period-height relationships for the determination of the fundamental period of vibration but using these equations time period and the resulted base shear are conservatively estimated. In most design cases the results obtained using conservative estimates formulae are largely deviating from the required engineering accuracy. Therefore a relationship is sought that will give accurate prediction of period of vibration for structural assessment of Indian reinforced concrete moment resisting frame buildings. On the basis of fundamental period of vibration of an Indian reinforced concrete moment resisting frame (RC MRF) building, obtained by finite element analysis, new equation has been proposed. In this study 21 existing reinforced concrete moment resisting frame buildings in India, with varying length, width and height are considered. The fundamental time periods of all the selected buildings were calculated using finite element method and IS 1893(Part 1) : 2002 based empirical formula. The comparison of proposed equation with Indian code shows that finite element analysis yielded 100 % higher periods as compared to the code approach.

1. Introduction

The period of vibration is a key parameter in the seismic design of structures. The spectral acceleration to be resisted by structure is calculated using the fundamental period of vibration. Moreover, the estimation of fundamental period of buildings is useful to recognize possible resonance phenomena between structure and soil vibration. Therefore the reliable evaluation of the fundamental period is an essential step in calculating the seismic response mutually in seismic design and assessment.

The fundamental time period depends upon mass, strength and stiffness of structure [1]. It is affected by many other factors such as structural irregularity, number of storeys and bays, and section properties including dimension. Generally, empirical formula is given as function of only height since even if other factors such as horizontal dimension has been used in regression analysis, height plays important role in fundamental period prediction [2]. For seismic design of a reinforced concrete frame, the period of vibration will not be identified a prior and hence simplified equations are employed in the seismic design codes to express the fundamental period to the height of the structure. Equations employed in the seismic design codes have been acquired by carrying out regression analysis on the periods of vibration measured during earthquakes [6].
For determination of the fundamental period of vibration many design codes provide simple empirical period-height relationships, which leads into conservative estimates of period and hence resulting base shear force will be conservatively evaluated. The results obtained using conservative estimates formulae are largely deviating from the required engineering accuracy. Therefore a relationship is sought that will give accurate prediction of period of vibration for structural assessment of Indian RC buildings.

The objective of this study is to find an empirical relationship between fundamental period-height with analytical methods instead of measured periods of vibration during earthquakes. This pragmatic choice stems from two major causes; (i) deficient of an adequate number of Indian buildings equipped with frequency measuring devices as well as exposure to earthquakes of adequate intensity and (ii) the significantly increased flexibility and control of structural response accessible when using analytical methods. The present work deals with determination of an improved empirical period height relationship to estimate the fundamental period of vibration of Indian reinforced concrete moment resisting frame multi storey buildings for structural assessment in India. First an evaluation of the empirical equation from current building codes IS 1893(Part I):2002 [15] for the calculation of fundamental periods of buildings was carried out. Then fundamental period of vibration was calculated using dynamic analysis. On the basis of these investigations new period-height relationship is proposed.

2. Empirical formulae

The empirical formula relating height of structure and period of building was first time presented in building code ATC3-06 (1978) and had the form of following Eq.(1):

\[ T = C_t H^{0.75} \]  

(1)

where \( C_t \) was a regression coefficient , was taken to be 0.025 for reinforced concrete moment resisting frame and \( H \) represented the height of the building in feet. It was theoretically derived using the Rayleigh’s method and with the assumption of linear distribution of equivalent static lateral forces over the height of the building and generation of uniform drift by the distribution of stiffness with height along with the linearly distributed lateral forces. The base shear force was supposed to be inversely proportional to \( T^{2/3} \). The numerical value of the constant \( C_t \) was acquired from the measured periods of buildings in the 1971 San Fernando earthquake. In SEAOC88(1996) it was found that a value of \( C_t = 0.03 \) (where \( H \) is measured in feet) was more appropriate for reinforced concrete buildings and has been implemented in various design codes since 1978, for instance in UBC-97 (1997), SEAOC-96 (1996), NEHRP-94 (1994). The NEHRP-94 provisions also suggested an another formula as mentioned in Eq. (2) for reinforced concrete moment resisting frame buildings.

\[ T = 0.1 N \]  

(2)

where \( N \) = number of stories. This formula is constrained to buildings not greater than 12 stories in height and having a least story height of 10ft. This expression is given in many of the codes, e.g. IS 1893:1984 (1984), NEHRP-94 (1994), Korea 2005 (2005), ASCE 7-05 (2006), ASCE 7-10 (2010). Indian Code IS 1893-84 (1984) recommended using this Eq. (3) only for regular outline building having even allotment of mass and stiffness in horizontal and vertical direction.

\[ T = 0.075 H^{0.75} \]  

(3)

In EC8(2004), this \( C_t \) coefficient has simply been altered considering that the height is measured in meters \((0.03\times3.281^{0.75} = 0.073 \) and has also been rounded up for simplicity), leading to \( C_t = 0.075 \).

Taiwan and Iran have used more conservative value by rounding the value on the lower side and hence as \( C_t = 0.07 \) in their seismic codes [19, 14]. Goel and Chopra [5] collected data measured from eight Californian earthquakes, with 1971 (San Fernando earthquake) ending with 1994 (Northridge earthquake) and different formulas as given in Eq.(4)-(5) were proposed follow-on from semi empirical analysis, by the best-fit plus 1 standard deviation recommended for displacement-based
evaluation, while the best-fit minus 1 standard deviation recommended for conservative force-based design.

\[ T_u = 0.067 H^{0.9} \]  
\[ T_L = 0.0466 H^{0.9} \]

where \( H \) is the height of the structure in meter. The latter period-height formula has been included in ASCE 7-05 (2006).

3. Period of vibration of reinforced concrete moment resisting frame buildings

Goel and Chopra carried out the regression analysis of measured data to develop formulas to estimate fundamental periods of the buildings [5]. Database contained a total of 106 Californian buildings including 21 of them with peak ground acceleration (PGA) higher than 0.15 g. It was noticed that the calculated code periods were shorter than the measured periods from the recorded motions. For buildings up to 36 meters high, the code formulas produced lower-bound values compared to measured period data, on the other hand, the same formulas resulted in 20% to 30% shorter periods compared to the measured ones for buildings taller than 36 meters. For many buildings, measured period values were greater than 1.4 \( T \), where \( T \) was described as the fundamental period obtained from the empirical equation.

Hong and Hwang experimentally determined the fundamental time period of 21 reinforced concrete moment resisting frame buildings, located in Taiwan through vibration measuring instruments [2]. Based on the experimental results, an empirical relationship between building period and height was derived. However, the obtained relation was different from that of U.S. building code formula. On comparing the numerical values of fundamental time period it was observed that the formula based on data pertinent to Taiwanese buildings under predicted the period value as compared to U.S. Code proposed formulae. This implies that Taiwanese buildings are stiffer than the Californian buildings.

Helen Crowley et al. studied existing buildings from five different European countries exposed to earthquake forces [9]. The majority of the existing buildings were designed and built between 1930 and 1980. The study led to a simplified period versus height equation in the assessment of existing reinforced concrete framed buildings taking due account of the presence of infill panels.

Pinho and Crowley took a critical look at the way in which seismic design codes around the world have endorsed the designer to estimate the period of vibration for use in both linear static and dynamic analysis [6]. Based on this review, some preliminary proposals are made for updating the clauses related to the estimation of the periods of vibration in EC8 (2004). They suggested that when presenting the period-height equations for reinforced concrete moment resisting frames there should be two formulae for the infill panels are to be isolated from the moment resisting frame and the infill panels are to be rigidly connected to the moment resisting frame for the estimation of the periods of vibration in EC8 (2004).

Paolo et al. carried out modal analyses on 3D numerical RC MRF building models, varying structure morphology (height, surface area, ratio between plan dimensions) and infill characteristics. Simplified formulas based on regression analysis of obtained numerical data were presented and discussed [23]. These relationships were also compared with similar literature numerical expressions and empirical data from experimental measurements on existing buildings.

Chiauzzi et al. focused on twelve reinforced concrete framed buildings, located in the cities of Victoria and Vancouver (British Columbia, Canada) and carried out a field campaign to estimate their fundamental periods using in-situ ambient vibration measurements [10]. They concluded that building periods estimated based on simple equations provided by earthquake design codes in Europe (EC8) and North America (UBC-97 and NBCC-2005) are significantly greater than the periods computed using ambient vibration records on the monitored buildings.

Ditommaso et al. investigated 68 reinforced concrete moment resisting frame buildings in Italy with different characteristics, such as age, height and damage level, by performing ambient vibration measurements that provided their fundamental translational periods [4]. Four different damage levels
(DL0, DL1, DL2, DL3) were considered according to the definitions in EMS 98 (European Macro seismic Scale), trying to regroup the estimated fundamental periods versus building heights using damage level as a key parameter. The two suggested experimental period height relationships were obtained.

Elgohary carried out a parametric study using finite element analysis to study the effect of the major parameters influencing in the fundamental period [8]. He concluded that the code's formulae, in most cases, underestimate the fundamental period with a large deviation from finite element results. This large deviation is a result of considering only the effect of frame height and neglecting of remaining major parameters in the codes formulae.

Al-Nimry et al. investigated the fundamental period of vibration of infilled reinforced concrete frame buildings using measurements of ambient vibrations and numerical analyses [22]. Ambient vibrations were measured at the roof level of 29 selected buildings with heights of one to six stories. Using Nakamura technique, the horizontal-to-vertical spectral ratio curves were obtained in the two orthogonal building directions. The estimated period values ranged between 44% and 91% of elastic periods suggested by the local code. Preliminary period–height relations were proposed using regression analysis of the measured periods. Analytical period values showed large differences with both the measured and code values.

Asteris et al. investigated the fundamental period of vibration of reinforced concrete buildings by means of finite element macro-modelling and modal eigenvalue analysis [24]. 14-storey RC buildings were considered "according to code designed" and "according to code non-designed". They studied various parameters including the number of spans; the span length in the direction of motion; the stiffness of the infills; the percentage openings of the infills and; the location of the soft storeys. From the analysis of the results it has been found that the span length, the stiffness of the infill wall panels and the location of the soft storeys are crucial parameters influencing the fundamental period of reinforced concrete buildings.

Magdy proposed improved formulas for estimating the fundamental period of vibration of reinforced concrete moment-resisting frame buildings by taking the effect of both building height \((H)\) and number of stories together \((N)\) [3]. The improved formula is based on regression analysis of the existing data for the fundamental vibration period of reinforced concrete moment resisting frame buildings measured from their motions recorded during eight California earthquakes.

Prakash and Dubey predicted the fundamental period of vibration of reinforced concrete buildings with moment resisting frames by considering the effect of building base width in both the directions and stiffness of the structure [7]. Values of the time period of vibration differ substantially, and the variation comes out to be on higher side than those derived from code base formula, which results in reduction in base shear and makes the structure economical.

4. Existing period- height relationship in India

The fundamental natural period of vibration for a moment resisting frame building without brick infill panels may be estimated by the empirical expression as mentioned in Eq. (3) as per of IS 1893 (Part 1): 2002.

The fundamental natural period of vibration of all other buildings, as well as moment resisting frame buildings with brick infill panels, estimated by the empirical expressions per IS 1893 (Part 1):2002 is shown in Eq. (6):

\[
T_a = \frac{0.9h}{\sqrt{d}}
\]

where \(h\) = height of the building in meter, \(d\) = Base dimensions of the building at the plinth level in meter, along the considered direction of lateral force.

5. Structural modeling
Modeling a building involves the modeling and assemblage of its various load-carrying elements. The model must ideally represent the mass distribution, strength, stiffness and deformability. The Finite element analysis of the selected building is carried out using ETABS 2015 (Integrated Building Design Software).

In this study 21 reinforced concrete moment resisting frame buildings in India with varying length, width and height (ranging from 6.6m to33m) are considered. Compressive strength for M20 and M25 grade of concrete are 20MPa and 25 MPa respectively. Modulus of Elasticity for M20 and M25 grade of concrete are 19.3364 x10³ MPa and 22.364x10³ MPa respectively. Modulus of Elasticity for Fe 415 grade of steel is 2 X 10⁵MPa. Density of Reinforced Concrete is 25 KN/m³ and density of steel is 76.82 KN/m³. Live load intensity is considered as 3kN/m² and floor finishes intensity as 1kN/m². Live loads were considered as 25 % of imposed load, as a part of the seismic weight.

Beams and columns were modeled as 3D frame elements. The members were represented through the assignment of properties like cross sectional area, reinforcement details and the type of material used. It has 6 Degrees of Freedom (Ux, Uy, Uz, Rx, Ry, and Rz) for each node. Slabs have been modeled as rigid diaphragms. Shell type elements were used to model slabs. Auto meshing was carried out. Auto Line Constraints were imposed on these slabs. Supports were considered as fixed at the base of the structure. Buildings under consideration do not include infill walls.

6. Estimation of fundamental period for Reinforced concrete moment resisting frame buildings
In this study 21 existing reinforced concrete moment resisting frame buildings in India with varying length, width and height as shown in Table 1 are considered. The fundamental time periods of all the 21 selected buildings were calculated using finite element method and code based empirical formula. The analytical results of fundamental period of these structures using analytical methods are compare with those using empirical formula IS 1893(Part 1): 2002 . The results are presented in table 1.

| Model number | Dimensions (m) | Analytical Time period (s) | Fundamental Natural period obtained from IS 1893 (Part 1):2002 (s) | % Conservati ve |
|--------------|----------------|---------------------------|---------------------------------------------------------------|----------------|
|              | Height | Length | Width | Longitudinal Direction (T₁) | Transverse Direction (T₂) | Fundamental Time Period (T) | |
| 1             | 6.6    | 12.78  | 9.735 | 0.622                        | 0.501                         | 0.622                       | 0.309                       | 101 |
| 2             | 8      | 10.14  | 9.085 | 0.645                        | 0.591                         | 0.645                       | 0.357                       | 81  |
| 3             | 8.6    | 6.045  | 28    | 0.638                        | 0.764                         | 0.764                       | 0.377                       | 103 |
| 4             | 10.5   | 15.05  | 9.98  | 0.902                        | 0.785                         | 0.902                       | 0.437                       | 106 |
| 5             | 10.5   | 7.48   | 11.26 | 0.732                        | 0.838                         | 0.838                       | 0.437                       | 92  |
| 6             | 10.8   | 14.87  | 16.15 | 0.793                        | 0.661                         | 0.793                       | 0.447                       | 77  |
| 7             | 11     | 5.698  | 13.43 | 0.886                        | 1.115                         | 1.115                       | 0.453                       | 146 |
| 8             | 11.5   | 7.06   | 13.44 | 1.004                        | 0.864                         | 1.004                       | 0.468                       | 115 |
| 9             | 13     | 7.21   | 18.1  | 1.152                        | 0.856                         | 1.152                       | 0.513                       | 125 |
| 10            | 13     | 16.27  | 8.77  | 1.043                        | 0.897                         | 1.043                       | 0.513                       | 103 |
| 11            | 15.05  | 7.11   | 11.41 | 1.062                        | 1.052                         | 1.062                       | 0.573                       | 85  |
| 12            | 14     | 10.5   | 7.25  | 0.906                        | 0.807                         | 0.906                       | 0.543                       | 67  |
| 13            | 15.1   | 12.6   | 12    | 0.785                        | 1.134                         | 1.134                       | 0.575                       | 97  |
| 14            | 15.65  | 17.39  | 8.37  | 0.995                        | 0.831                         | 0.995                       | 0.59                        | 69  |

Table 1. Period data for RC moment resisting frames.
The results of fundamental period calculation of reinforced concrete moment resisting frame buildings using IS 1893(Part I):2002 are shown in Fig. 1. It can be noticed that the fundamental period of vibration is proportional to the height of reinforced concrete moment resisting frame buildings as given by Eq.(3):

\begin{align*}
T &= 0.075h^{0.75}
\end{align*}

Figure 1. Relationship between fundamental period and height of reinforced concrete moment resisting frame buildings using IS 1893(Part I:2002).

\begin{align*}
T_x &= 0.111h^{0.844}
\end{align*}

Figure 2. Relationship between time period in longitudinal direction and height of reinforced concrete moment resisting frame buildings using finite element analysis.
The results of time period calculation in longitudinal ($x$) direction of reinforced concrete moment resisting frame buildings using Finite element analysis are shown in Fig. 2 with best fit curve. Dots in figure indicates the fundamental time period in longitudinal direction by analytical method. The straight line is drawn using best fit curve, which gives the relationship between fundamental time period and height of building as given by following Eq. (7):

$$T_X = 0.11 h^{0.844}$$  \hspace{1cm} (7)

The results of time period calculation in transverse ($y$) direction of reinforced concrete moment resisting frame buildings using Finite element analysis are shown in Fig. 3 with best fit curve. Dots in figure indicates the fundamental period in longitudinal direction by analytical method. The straight line is drawn using best fit curve which gives the relationship between fundamental time period and height of building as given by following Eq.(8):

$$T_Y = 0.107 h^{0.837}$$  \hspace{1cm} (8)

Figure 3. Relationship between time period in transverse direction and height of reinforced concrete moment resisting frame buildings using finite element analysis.

The fundamental time period is the highest time period among the two directions. The fundamental time period is plotted vs height of building as shown in Fig. 4. The dots in figure represents these values. Using best fit curve the straight line is drawn as shown in Fig. 4 which indicates the period-height relationship.

Figure 4. Relationship between fundamental period and height of reinforced concrete moment resisting frame buildings using finite element analysis.
Comparison of fundamental period between empirical relationship using IS 1893(Part I):2002 and finite element analysis method is shown in Fig. 5. It can be noticed that proposed equations yielded higher periods as compared to the code approach. The two curves are seen to be very similar so it provides confidence towards use of proposed equations. Therefore, the proposed period -height relationship could be utilized for the estimation of the fundamental vibration period of reinforced concrete moment resisting frame buildings in India.

Figure 5. Comparison of proposed equation results and IS 1893(Part I):2002 equation results for reinforced concrete moment resisting frame buildings in India.

Comparison of Empirical and Analytically calculated time periods for reinforced concrete moment resisting frame buildings in India is shown in Table 1. It can be noticed that IS 1893(Part I):2002 yielded average 100% conservative periods as compared to the analytically calculated results.

7. Proposed equation
In the present study of the fundamental period of various existing reinforced concrete moment resisting frame buildings in India with varying number of stories and number of bays is carried out for RC moment resisting frames. Analysis is done using finite element method using software ETABS 2015 V15.2. On the basis of these evaluation and investigation following new Eq. (9) is proposed for fundamental time period for reinforced concrete moment resisting frame buildings in India. Comparing this equation with IS 1893(Part1):2002 formula is highly conservative.

\[ T = 0.132 h^{0.795} \]  

where \( h \) = height of reinforced concrete moment resisting frames in meter, \( T \) = fundamental period of reinforced moment resisting frames in seconds

Proposed analytical - based relation provides an insight into the period-height relation but needs further investigation before being adopted in the local seismic code.

8. Conclusions
In this study 21 existing reinforced concrete moment resisting frame buildings in India with varying length, width and height are considered. The fundamental time periods of all the 21 selected buildings were calculated using finite element method and code based empirical formula. The analytical results of fundamental period of these structures using analytical methods are compare with those using empirical formula IS 1893(Part 1) : 2002. The code proposed equations are based on experimentally determined periods of certain set of buildings measured during earthquakes.

- It is observed from this study that the fundamental time period increases with increase in height of structures.
• New period -height relationship is proposed based on the study. Compare to this relationship IS1893(Part 1) : 2002 relationship is found to be conservative.

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