Regional Synchronization during Economic Contraction: The Case of the U.S. and Japan

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ABSTRACT
Two decades of studies have found significant regional differences in the timing of transitions in national business cycles and their durations. Earlier studies partly detect regional synchronization during business cycle expansions and contractions in Europe, the United States, and Japan. We examine this possibility by applying a sophisticated method for identifying the time-varying degree of synchronization to regional business cycle data in the U.S. and Japan. The method is prominent in nonlinear sciences but has been infrequently applied in business cycle studies. We find that the degree of synchronization in regional business cycles increased during contractions and decreased during expansions throughout the period under study. Such asymmetry between the contraction and expansion phases of a business cycle will contribute to our better understanding of the phenomenon of business cycles.

I. Introduction
National economies consist of interlinked regional economies that react differently to changing macroeconomic forces, government policies, imported materials prices, and technological innovation. Thus, national business cycles (BCs) are an admixture of regional cycles fluctuating diversely. Earlier studies of regional BCs surveyed by Domazlicky (1980) examined how and why cycles differ. By contrast, the advent of the Economic and Monetary Union (EMU) in Europe has renewed research interest in similarities and synchronization among EU states’ BCs because synchronization facilitates intra-EMU’s fiscal and monetary policies. However, empirical studies often reach divergent conclusions (Massmann and Mitchell 2004; Grayer 2007; De Haan et al. 2008; Montoya and de Haan 2008), likely because they use different raw data or methods for estimating cycles and gauging synchronization.

For instance, Artis and Zhang (1999) found that the degree of synchronization intensified during the European Exchange Rate Mechanism period (1973–1995), but Massmann and Mitchell (2004) determined periods of synchronization and desynchronization using identical but updated data. Meanwhile, after conducting a comprehensive study using six estimation methods and three measures of synchronization, Kappler and Sachs (2013) found little support for BC synchronization, and the degree of synchronization fluctuated over time.

Euro-area studies of synchronization aroused interest in regional cycles within a country, such as the United States (U.S.) and Japan. For example, Clark and Wincoop (2001) specified that BCs of nine U.S. Census regions are significantly more synchronized than those of EU countries. Moreover, Artis and Okubo (2011) found that the degree of regional BC synchronization within Japan is strikingly higher than that in the U.S. and the Euro area. These findings imply that national borders may dampen synchronization between regional BCs.

Numerous studies regarding the synchronization of regional BCs, whether they cover inter- or intra-national BCs, found significant regional differences in the timing of BCs’ transitions and duration. Among them, several studies have announced noteworthy results. For instance, Grayer (2007) determined a recurring pattern of declining synchronization during expansions in Europe. Hamilton and Owyang (2012) and Chung (2016)
noted that co-movement across states characterizes economic contractions in the U.S. Meanwhile, Wall (2007) concluded that contractions tend to be experienced across most Japanese prefectures.

These results garnered via different datasets and methods, when being put all together, suggest that the degree of synchronization between regional BCs intensifies during contractions and diminishes during expansions. We examine this possibility by applying a sophisticated method for identifying the time-varying degree of synchronization to regional BC data in the U.S. and Japan. As noted above, national borders may dampen synchronization between regional BCs, and therefore this study concentrates on analysing the synchronization of regional BCs within a single country as a first step. The method is prominent in nonlinear sciences (e.g. Pikovsky et al. 2001: ch. 6) but has been infrequently applied in BC studies.1 Before describing the method, we note how to extract BCs from raw data. Following recent studies, we acknowledge that BCs are relative to a trend and focus on its deviation.

For monthly observations of the Composite Index (CI) of coincident indicators in the U.S. and the Index of Industrial Production (IIP) in Japan, we employ a band-pass filter to extract time series indicating regional BCs in both countries. The Hodrick–Prescott filter (Hodrick and Prescott 1997), a high-pass filter often used in economics literature, removes only trends with low frequencies. The Baxter–King (BK) (Baxter and King 1999) and Christiano–Fitzgerald (CF) (Christiano and Fitzgerald 2003) band-pass filters are also frequent in the literature, but we employ the Fourier band-pass filter that is mathematically and computationally simpler.2

This study’s method comprises the following three procedures. First, we convert time series fluctuations into two-dimensional oscillations using the Hilbert transform.3 This enables us to identify ‘phases’ of circular oscillations, defined as a position of a cyclically oscillating variable within one period. Converted oscillations include more information than the original one-dimensional time series and better assess synchronization. Second, we take the ‘phase difference’ between two cycles to indicate their synchronization. Third, we use the phase difference to calculate a synchronization index that measures the constancy of the phase difference. If the phase difference of two cycles is nearly constant over time, this index indicates a value near 1, and we designate this situation as (phase) synchronization. In this sense, synchronization does not depend on the level of the phase difference but on its constancy. Our use of this method supports the hypothesis that the degree of synchronization between regional BCs intensifies during contractions and diminishes during expansions both in the U.S. and Japan.

An overview of other synchronization measures distinguishes our method from others. The most popular measure of synchronization, namely, Pearson correlation coefficient, provides in one number the degree of similarity between series over a sampled period. It measures static relations between the series, whereas synchronization is a dynamic phenomenon with a varying degree over time. Meanwhile, the moving window correlations and new time-varying indexes overcome that deficiency. However, as the European Commission (2006) mentioned, correlation with a moving window is sensitive to the window’s length. Mink et al. (2012) proposed a multivariate, time-varying measure of synchronization based on an output gap. It gauges the percentage of regions over time whose output gap has the same sign as that of the reference region. However, this synchronization measure is nondifferentiable because of its absolute values, and graphs of the calculated series exhibit numerous nonessential spikes.

By contrast, our method focuses on phase differences between two time series. Calculated using phase differences, the synchronization index of Rosenblum et al. (2001) captures the time-dependent degree of synchronization even if phase differences between two time series are

1Another way to identify synchronization in time series is through the wavelet transform. Examples of previous studies using the cross-spectrum of wavelet coefficients to gauge synchronization of BCs include Aguiar-Conraria and Soares (2011) for EU countries and Aguiar-Conraria et al. (2017) for U.S. states.
2Ikeda et al. (2013) also use this filter.
3The Hilbert transform is discussed in detail in Section III. It has been used occasionally in the economics literature (see Ikeda et al. 2013).
large. The correlation coefficient fails to measure the degree of synchronization because its absolute value can be small in such a case.

The study proceeds as follows. Section II and III describe this study’s data and methods, respectively. Section IV presents empirical results. Section V concludes the paper.

II. Data

We employ two datasets frequently used to investigate regional BCs. One is monthly, seasonally adjusted CI data (2007 average = 100) in the U.S., spanning from April 1979 to April 2021 (505 months) for all 50 states compiled by the Federal Reserve Bank of Philadelphia. The other is monthly raw (i.e., not seasonally adjusted) IIP data (2010 average = 100) in Japan, spanning from January 1978 to August 2018 (488 months) for all 47 prefectures compiled by the Ministry of Economy, Trade, and Industry. It does not matter whether the data used in the analysis are seasonally adjusted or not because our settings of the band-pass filter can remove the high-frequency component corresponding to the seasonal variation.

Figure 1 graphs the time series of CI and IIP data for sampled regions. The panel (u1) compares the time series of CI in New York with those in Pennsylvania, New Jersey, and Illinois, the data for which exhibit the greatest synchronization with New York from the viewpoint of our analysis. By contrast, the panel (u2) compares the time series in New York with those in Louisiana, Hawaii, and Utah, the data for which exhibit the least synchronization with New York. Meanwhile, the panel (j1) compares the time series of IIP in Tokyo with those in Yamagata, Nara, and Akita Prefectures, which exhibit the greatest synchronization with Tokyo. The panel (j2) compares the time series in Tokyo with those in Okinawa, Miyagi, and Nagasaki Prefectures, showing the least synchronization with Tokyo.

It is preferable to employ regional CI data for both countries. However, we use IIP for Japan

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Figure 1. Time series comparison of the composite index and the index of industrial production data for sampled regions. Note: Comparison of the composite index data between New York and other three states whose data exhibit the greatest synchronization with New York (i.e., Pennsylvania, New Jersey, and Illinois) (u1), and the least synchronization with New York (i.e., Louisiana, Hawaii, and Utah) (u2). Comparison of the index of industrial production data between Tokyo and other three prefectures whose data exhibit the greatest synchronization with Tokyo (i.e., Yamagata, Nara, and Akita) (j1), and the least synchronization with Tokyo (i.e., Okinawa, Miyagi, and Nagasaki) (j2).

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4The CI data for 50 states in the U.S. are from the website of the Federal Reserve Bank of Philadelphia. The IIP data for Japan’s 47 prefectures are from NIKKEI NEEDS.
because CI data are unavailable for all prefectures. Figure 2 supports our use of IIP data for Japan. It includes monthly time series of IIP (2015 average = 100: spanning from January 1978 to August 2018) and CI (2015 average = 100: spanning from January 1985 to August 2018, which are compiled by the Economic and Social Research Institute (ESRI) of the Cabinet Office) for Japan overall. The shadowed area corresponds to business contractions, whereas the white area corresponds to business expansions. Figure 2 demonstrates that the timing of peaks and troughs in IIP data duplicate those of CI data even though their deviations from 1991 to 2012 can be large. In short, IIP data still capture the Japanese BCs adequately.

III. Measuring synchronization

We apply three procedures to measure synchronization between two scalar (i.e., one-dimensional) time series. First, we convert fluctuations in each scalar series into two-dimensional oscillations using the Hilbert transform to identify a ‘phase’ at each time. Second, we take ‘phase differences’ between two cycles as an indicator of their synchronization. Third, using the phase differences, we calculate a synchronization index proposed by Rosenblum et al. (2001).

**Phase synchronization**

Synchronization is a phenomenon in which multiple oscillations adjust their individual rhythms through mutual interactions to maintain a constant phase difference for a time. This phenomenon is strictly called phase synchronization and also called phase-locking or frequency entrainment. Remark that phase synchronization does not depend on the amplitudes of oscillations. If their amplitudes are identical, it is called complete synchronization (Pikovsky et al. 2001, p. 23). In the present paper, when we use the term synchronization, we mean phase synchronization.

Synchronization is exemplified with the aid of simple oscillators $s_1^t = \sin(2\pi t)$ and $s_2^t = 2\sin(2\pi t - \pi/2)$ in Figure 3. The phases of $s_1^t$ and $s_2^t$ are $2\pi t$ and $2\pi t - \pi/2$, respectively, rendering their phase difference as $\pi/2$. Time series $s_1^t$ and $s_2^t$ are synchronized because their phase difference is constant over time.

**Hilbert transform and the instantaneous phase**

Phases are crucial in synchronization analysis. However, it is impossible to extract time-varying amplitudes and phases of BCs just from scalar time series data. Therefore, we construct a complex-valued time series $s_t^H$ whose real part is actual data $s_t$ and imaginary part $s_t^I$ is generated from $s_t$ via the

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5To be exact, the phase value is restricted to $[-\pi, \pi)$ by taking mod 2$\pi$. “Phase” is defined in Section III.
Hilbert transform:

\[ \hat{s}_t = s_t + is_t^H, \]  

(1)

The Hilbert transform of \( s_t \) is given by

\[ s_t^H = \frac{1}{\pi} \text{P.V.} \int_{-\infty}^{\infty} \frac{s_r}{t-r} \, dr, \]

where \( \text{P.V.} \) denotes Cauchy principal value integrals. Intuitively, the Hilbert transform provides a phase shift of \( -\pi/2 \) radian for every Fourier component of a function. For example, the Hilbert transform of \( s_t = \cos(2\pi t) \) is \( s_t^H = \cos(2\pi t - \pi/2) = \sin(2\pi t) \).

Now, we can define a (instantaneous) phase at time \( t \) using a point \( P_t(s_t, s_t^H) \) on the complex plane as an angle \( \phi_t \) that is formed between \( \text{OP}_t \) and the horizontal axis in Figure 4:

\[ \phi_t = \begin{cases} \tan^{-1}\left(\frac{s_t^H}{s_t}\right) & (s_t > 0) \\ \tan^{-1}\left(\frac{s_t^H}{s_t}\right) + \pi & (s_t < 0) \end{cases} \]

The phase value ranges from \( -\pi \) to \( \pi \); hence, it can be discontinuous over time. Using phase \( \phi_t \), we can rewrite Equation 1 as

\[ \hat{s}_t = s_t + iA_t \sin \phi_t, \]

where the time-varying amplitude is represented as \( A_t = \sqrt{s_t^2 + (s_t^H)^2} \). The earlier discussion assumes that \( t \) is continuous; however, we apply the procedure to discrete time series data of CI and IIP in Section IV.

**Synchronization index**

To measure the degree of synchronization between two series for a discrete time interval \( 1 \leq i \leq W \), we use the synchronization index \( \gamma^2 \in [0, 1] \) proposed by Rosenblum et al. (2001):

\[ \gamma^2 = \left( \frac{1}{W} \sum_{i=1}^{W} \cos \psi_i \right)^2 + \left( \frac{1}{W} \sum_{i=1}^{W} \sin \psi_i \right)^2, \]

(2)

where \( \psi_i \) denotes the phase difference defined by the difference between phases of two time series and \( W \) denotes a length of the moving window. This index \( \gamma^2 \), also known as the phase-locking value, was first used in the economics literature by Bruzda (2015). When \( \psi_i \) is nearly constant over time, the value of \( \gamma^2 \) is close to 1. This situation is defined as (phase) synchronization. When \( \psi_i \) is chosen randomly from the uniform distribution of \( [-\pi, \pi] \), \( \gamma^2 \) approaches 0 as \( W \) increases.

To examine time evolution of \( \gamma^2 \), we presume \( W \) is an odd number of discrete time points, and \( \gamma_i^2 \) defined below represents the strength of synchronization at time \( t \), which corresponds to the temporal centre point of the moving window of the length \( W \). Thus, instead of Equation 2, we calculate for each time \( t \)

\[ \gamma_i^2 = \left( \frac{1}{W} \sum_{i=-p}^{t-p} \cos \psi_i \right)^2 + \left( \frac{1}{W} \sum_{i=-p}^{t-p} \sin \psi_i \right)^2, \]

(3)

where \( p = (W - 1)/2 \) and \( 0 < p < t \). Throughout the analysis, we set \( W = 13 \) for the U.S. and \( W = 17 \) for Japan because the window’s length \( W = 13 \) (\( W = 17 \) for Japan) is approximately half of the shortest duration of 28 (36) months of the past BCs of the U.S. (Japan) (see Tables 1 and 2). A small change in the length of \( W \) hardly affects the analysis results.
identify the frequency bands corresponding to the time scale of BCs under consideration. Tables 1 and 2 list the reference dates for cycles in the U.S. announced by the National Bureau of Economic Research (NBER) and those in Japan by ESRI, from which we identify a band spanning 28–130 months for the U.S. and 36–86 months for Japan, respectively. We extract time series for the U.S. with the frequency band of that range using the lower cutoff frequency of \( k_l = 4 \) and the upper cutoff frequency of \( k_u = 18 \), which correspond to 126 (\( \approx 505/k_l \)) and 28 (\( \approx 505/k_u \)) months, respectively. For the notations \( k_l \) and \( k_u \), see Appendix B. We also extract time series for Japan with the lower cutoff frequency of \( k_l = 6 \) and the upper cutoff frequency of \( k_u = 14 \), corresponding to 81 (\( \approx 488/k_l \)) and 35 (\( \approx 488/k_u \)) months, respectively. In Appendix D, we perform a robustness check of our analysis against frequency band selection. Both ends of band-pass-filtered data contain artificial information. Thus, for the robustness of our results, we eliminate data points at both ends corresponding to the period of the band’s highest frequency.

Figure 5 illustrates the band-pass-filtered time series for sampled regions. The panel (u1) compares the time series for New York and the same three states as in Figure 1(u1). Meanwhile, the panel (j1) compares the time series for Tokyo and the same three prefectures as in Figure 1(j1). The timing of peaks and troughs almost coincides because those regions in (u1) and (j1) are most synchronized. By contrast, because those regions in (u2) and (j2) are least synchronized with each other, the timing of peaks and troughs is considerably disordered. These four panels imply periods for which the degree of synchronization between regions in the U.S. and Japan is either high or low.

We next convert fluctuations in each band-pass-filtered scalar time series into two-dimensional oscillations using the Hilbert transform. The panels (u1, j1) of Figure 6 depict two-dimensional trajectories of the instantaneous phase \( \psi_t = \arctan \frac{a_t}{b_t} \) on the complex plane. The horizontal axis represents the variable \( \bar{s}_t \), that is, the band-pass-filtered CI (IIP) data for New York (Tokyo) with the aforementioned upper and lower cutoff frequencies, and

| Trough | Peak | Trough | Duration (months) |
|--------|------|--------|-------------------|
| 1975:03 | 1980:01 | 1980:07 | 64 |
| 1980:07 | 1981:07 | 1982:11 | 28 |
| 1982:11 | 1990:07 | 1991:03 | 100 |
| 1991:03 | 2001:03 | 2001:11 | 128 |
| 2001:11 | 2007:12 | 2009:06 | 91 |
| 2009:06 | 2020:02 | 2020:04 | 130 |

Note: Duration of cycles spans 28–130 months.

Table 2. Reference dates for Japanese business cycles announced by the Economic and Social Research Institute of the Cabinet Office.

| Trough | Peak | Trough | Duration (months) |
|--------|------|--------|-------------------|
| 1977:10 | 1980:02 | 1983:02 | 64 |
| 1983:02 | 1985:06 | 1986:11 | 45 |
| 1986:11 | 1991:02 | 1993:10 | 83 |
| 1993:10 | 1997:05 | 1999:01 | 63 |
| 1999:01 | 2000:11 | 2002:01 | 36 |
| 2002:01 | 2008:02 | 2009:03 | 86 |
| 2009:03 | 2012:03 | 2012:11 | 44 |
| 2012:11 | 2018:10 | |

Note: Duration of cycles spans 36–86 months.
the vertical axis represents $s_i^H$, that is, the Hilbert-transformed time series of $s_i$. Trajectories of $P_t(s_i, s_i^H)$ oscillate around the origin with certain frequencies and amplitudes. This finding implies that BC fluctuations are adequately extracted using the aforementioned lower and upper cutoff frequencies.

Comparing the panel (u1) with (u2) or the panel (j1) with (j2) of Figure 6 uncovers the significance of the band-pass filter and selection of a frequency band. The panels (u2, j2) show trajectories of $P_t(s_i, s_i^H)$ with $s_t$ de-trended but not band-pass-filtered. Trajectories in the panel (u2) slowly rotate and those in the panel (j2) consist of numerous irregular oscillations. These imply that the time series $s_t$ in the panels (u2, j2) contains lower (higher)-frequency fluctuations than in the panels (u1, j1) and that BC fluctuations are not adequately extracted. Moreover, in the panel (j2), trajectories sometimes pass by the origin, suggesting that phase movements exhibit abrupt jumps that may defeat our synchronization analysis.

The converted trajectory on the complex plane via the Hilbert transform allows us to identify the phase of circular oscillations and calculate phase differences between two trajectories as an indicator of the degree of synchronization at each time. Thus, we can compute the time evolution of synchronization index $\gamma_t^2$ between two trajectories using Equation 3 to gauge the constancy of phase differences.

Figure 7 illustrates time evolution of synchronization index $\gamma_t^2$ between sampled regions. States in the panels (u1, u2) correspond to those in the panels (u1, u2) of Figure 1, respectively. Likewise, prefectures in the panels (j1, j2) correspond to those in the panels (j1, j2) of Figure 1, respectively. Although states and prefectures in the panels (u1, j1) belong to the most synchronized group with New York and Tokyo,
those panels display some intervals during which $\gamma_2^2$ takes a low value. States and prefectures in the panels (u2, j2) belong to the least synchronized group with New York and Tokyo, so that the degree of synchronization in those panels is lower than that in the panels (u1, j1). These four panels imply that synchronization is generally high during most periods and tends to decline almost concurrently during economic expansions (white areas).

To scrutinize the degree of BC synchronization between regions, we calculate 1,225 ($=50C_2$) series of $\gamma_2^2$ for all two-tuples between 50 states in the U.S., and 1,081 ($=47C_2$) series of $\gamma_2^2$ for all two-tuples between Japanese 47 prefectures. By $R(\gamma_2^2 \geq r)$, we denote the ratio of two-tuples for which $\gamma_2^2$ exceeds or equals the threshold $r$ at each time $t$. By definition, $R(\gamma_2^2 \geq r) \in [0, 1]$. The larger the portion of prefectures synchronized, the greater the value of $R(\gamma_2^2 \geq r)$.

Figure 8 illustrates the time evolution of $R(\gamma_2^2 \geq r)$ for $r = 0.7$ and 0.8. It implies that, in both countries, $R(\gamma_2^2 \geq r)$ is inclined to be low during expansions (white areas), whereas it is inclined to be high during contractions (shadowed areas). These observations support the hypothesis that the degree of synchronization between regional BCs increases during contractions and decreases during expansions.

However, Figure 8(j) reveals two discrepancies between our results and the hypothesis concerning Japan. One is that $R(\gamma_2^2 \geq r)$ shows relatively low
values for the contraction from 2012:03 to 2012:11. This is because a band-pass filter may fail to extract an adequate trajectory if the duration of an expansion or contraction is too short compared to the period corresponding to the cutoff frequency. The contraction in question is actually a short period of 8 months. The other is that $R(\gamma^2_t \geq r)$ shows relatively low value during the contraction from 1981:08 to 1983:02. This is because observations may disparage our hypothesis during periods when expansions and contractions coexist, that is, when Japan’s economy does not expand or contract unidirectionally. To see this in detail, we inquire into the Diffusion Index (DI) of coincident indicators. Figure 9 illustrates the time evolution of the normalized DI data in Japan. The original DI data $x_t$ that takes a value from 0 to 100 is normalized as $(x_t - 50)/50$; hence, the normalized DI tends to be positive in expansions and negative in contractions. During the contraction from 1980:02 to 1983:02, the normalized DI moves back and forth between positive and negative regions several times, implying some expansions during
an economic contraction. Therefore, in Figure 8, $R(y_t^2 \geq r)$ during that contraction period exhibits relatively low values. In a nutshell, our observations might deviate from our hypothesis because our method captures interims of expansions within contractions and vice versa sensitively. This does not constitute a defect in our method.

Finally, we offer a conjecture about why our hypothesis holds, that is, why the degree of synchronization between regional BCs increases during economic contractions and decreases during economic expansions. Under prospect theory (Kahneman and Tversky 1979), people prefer avoiding losses over acquiring equivalent gains. This suggests industrial firms behave asymmetrically when BCs enter contractions or expansions. When entering an economic contraction, firms trim production to avoid losses, and that behaviour synchronizes well. When entering an economic expansion, firms' behaviour tends to be poorly synchronized because some firms increase production while others increase employment or make capital investments before increasing production. Thus, loss-averse behaviour by firms engenders synchronization in production during contractions.

V. Conclusion
We investigate CI data for all 50 states in the U.S. and IIP data for all 47 prefectures in Japan from the viewpoint of regional BC synchronization. Using a method distinguished in nonlinear sciences to analyse synchronization between data series, we converted one-dimensional time series into two-dimensional circular oscillations via the Hilbert transform. Our quantitative results indicate an increase (decrease) in synchronization of regional BCs during economic contractions (expansions) throughout the period under study. Such asymmetry between the contraction and expansion phases of a BC will contribute to our better understanding of the phenomenon of BCs. Among other things, our results provide important information to policymakers. This is because, during a contractionary period, regional BCs tend to be coherent, so counter-cyclical fiscal and monetary policies to avoid a severe downturn in the economy need to be implemented as quickly as possible. In contrast, during an expansionary period, regional cycles are less coherent and therefore counter-cyclical policies to avoid overheating of the economy are less urgent. Furthermore, as Figure 8(u) shows, the degree of synchronization can rise and fall significantly several times during a single expansionary period. This implies that the U.S. economy is not monotonically expanding in the period assigned as an expansionary period by the business cycle reference date. If this is the case, our method may allow us to subdivide the expansionary and contractionary periods.

Remark that our method concentrates on a specific frequency band and may fail to extract a good trajectory if the duration of an expansion or contraction is too short compared to the period corresponding to the cutoff frequency of the bandpass filter.
Future research should generalize our findings by applying our method to regional BCs in other countries and even to cross-border regions. In particular, synchronization of BCs in EU countries, which were excluded from the analysis in this study, is of primary importance. Furthermore, it would be interesting to analyse how the impact of COVID-19 has brought about changes in the appearance of the regional BCs compared to prior years. Incidentally, De Haan et al. (2022) found that the impact of COVID-19 was strengthened the synchronization of BCs in EU countries, but with large differences in amplitude. It would also be useful to re-examine our hypothesis via different methods such as wavelet analysis, and construct a macroeconomic dynamical model with firms’ loss-averse behaviour to explain our hypothesis.

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Appendices

Appendix A. Robustness against the Length of the Moving Window

When calculating the synchronization index $r^2$, we employ moving window $W$ in Equation 3 with a length of 13 for the U.S. CI data and that with 17 for Japan’s IIP data. We chose these lengths of $W$ to be approximately half of the shortest duration of the past BCs. Here, we discuss the robustness of our analysis against the selection of the window’s length.

Figure A1 compares time evolution of the ratio $R(y_t^2 \geq r)$ for different window’s length with respect to $r = 0.8$. Note: Time series of $R(y_t^2 \geq 0.8)$ for the U.S. composite index data with respect to window’s length $W = 11, W = 13,$ and $W = 15$ (u). Time series of $R(y_t^2 \geq 0.8)$ for Japan’s index of industrial production data with respect to $W = 15, W = 17,$ and $W = 19$ (j).

Appendix B. Fourier Filter

We briefly review the Fourier series of a function $f$. For simplicity, let $f$ be a real-valued continuous periodic function on $[0, L)$. The function $f$ can be represented as a Fourier series:

$$f(x) = \frac{a_0}{2} + \sum_{k=1}^{\infty} \left( a_k \cos \left( \frac{2\pi kx}{L} \right) + b_k \sin \left( \frac{2\pi kx}{L} \right) \right),$$

where

$$a_k = \frac{1}{L} \int_{0}^{L} f(x) \cos \left( \frac{2\pi kx}{L} \right) dx \quad (k = 0, 1, 2, 3, \ldots),$$

$$b_k = \frac{1}{L} \int_{0}^{L} f(x) \sin \left( \frac{2\pi kx}{L} \right) dx \quad (k = 1, 2, 3, \ldots).$$

We can obtain a Fourier series for a more general class of function $f$ (see, e.g., Körner 1989).

By taking a partial sum in Equation 4 we can create a band-pass-filtered periodic function $\tilde{f}$ using a band $[k_l, k_u]$ with the lower and upper cutoff frequencies of $k_l$ and $k_u$ $(0 \leq k_l \leq k \leq k_u)$ from a given function $f$:

$$\tilde{f}(x) = \sum_{k=k_l}^{k_u} \left( a_k \cos \left( \frac{2\pi kx}{L} \right) + b_k \sin \left( \frac{2\pi kx}{L} \right) \right).$$

Appendix C. Robustness against Filter Selection

We see that qualitative results of our analysis using two different band-pass filters, Fourier and CF, are identical. Figure C1 shows the filtered time series of CI in New York and those of IIP in Tokyo by both Fourier and CF filters. The band-pass filters’ upper (lower) cutoff frequency for New York corresponds to 28 (126) months in the upper panel and that for Tokyo to 35 (81) months in the lower panel. Therefore, the two band-pass-filtered time series are similar except for both ends of the data period. Both ends of band-pass-filtered data contain artificial information; thus, we eliminate data points at both ends corresponding to the period of the highest frequency of the band for the robustness of results.
Figure C1: Comparison of Band-Pass-Filtered Time Series of the Composite Index and the Index of Industrial Production by Fourier and Christiano–Fitzgerald Filters. Note: The band-pass filters' upper (lower) cutoff frequency for New York corresponds to 28 (126) months in the upper panel and that for Tokyo to 35 (81) months in the lower panel.

Figure C2 depicts the time evolution of the ratio \( R(y_t^2 \geq r) \) for \( r = 0.7 \) and 0.8 of band-pass-filtered time series by CF filter (u1, j1) and those by Fourier filter (u2, j2). The lower panels are reprints from Figure 8. The qualitative results of the analysis are almost the same for both filters, although some differences exist in detail. Therefore, the filter selection robustness follows.
Appendix D. Robustness against Frequency Band Selection

When applying the Fourier band-pass filter, we employ the frequency band spanning 28–126 \((k_l = 4, k_u = 18)\) months for the U.S. CI data and 35–81 months \((k_l = 6, k_u = 14)\) for Japan's IIP data. Here, we discuss the robustness of our analysis against frequency band selection.

**Figure D1** illustrates time evolution of the ratio \(R(y_t^2 \geq r)\) with respect to \(r = 0.7\) and 0.8 for different frequency bands. The two panels in the middle are reprints from **Figure 8**, which depict time series of \(R(y_t^2 \geq r)\) with respect to frequency bands spanning 28–126 months \((k_l = 4, k_u = 18)\) (u2) and 35–81 months \((k_l = 6, k_u = 14)\) (j2). The two upper panels correspond to a shorter frequency band spanning 30–101 months \((k_l = 5, k_u = 17)\) (u1) and 38–70 months \((k_l = 7, k_u = 13)\) (j1), and the two lower panels correspond to a longer frequency band spanning 27–168 months \((k_l = 3, k_u = 19)\) (u3) and 33–98 months \((k_l = 5, k_u = 15)\) (j3).

Comparing these panels vertically, we show almost identical qualitative results for both the U.S. and Japan, although the shape of the graphs varies to some extent depending on the choice of frequency band.

*Figure D1*: Time Evolution of the Ratio \(R(y_t^2 \geq r)\) for Different Frequency Bands with respect to \(r = 0.7\) and 0.8. Note: Time series of \(R(y_t^2 \geq r)\) for the U.S. composite index data with respect to frequency bands spanning 30–101 months \((k_l = 5, k_u = 17)\) (u1), 28–126 months \((k_l = 4, k_u = 18)\) (u2), and 27–168 months \((k_l = 3, k_u = 19)\) (u3). Time series of \(R(y_t^2 \geq r)\) for Japan’s index of industrial production data with respect to frequency bands spanning 38–70 months \((k_l = 7, k_u = 13)\) (j1), 35–81 months \((k_l = 6, k_u = 14)\) (j2), and 33–98 months \((k_l = 5, k_u = 15)\) (j3). The two panels in the middle are reprints from **Figure 8**.