Generating large misalignments in gapped and binary discs

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ABSTRACT

Many protostellar gapped and binary discs show misalignments between their inner and outer discs; in some cases, \( \sim 70^\circ \) misalignments have been observed. Here, we show that these misalignments can be generated through a secular resonance between the nodal precession of the inner disc and the precession of the gap-opening (stellar or massive planetary) companion. An evolving protostellar system may naturally cross this resonance during its lifetime due to disc dissipation and/or companion migration. If resonance crossing occurs on the right time-scale, of the order of a few million years, characteristic for young protostellar systems, the inner and outer discs can become highly misaligned, with misalignments \( \gtrsim 60^\circ \) typical. When the primary star has a mass of order a solar mass, generating a significant misalignment typically requires the companion to have a mass of \( \sim 0.01–0.1 \, M_\odot \) and an orbital separation of tens of astronomical units. The recently observed companion in the cavity of the gapped, highly misaligned system HD 142527 satisfies these requirements, indicating that a previous resonance crossing event misaligned the inner and outer discs. Our scenario for HD 142527's misaligned discs predicts that the companion’s orbital plane is aligned with the outer disc’s; this prediction should be testable with future observations as the companion’s orbit is mapped out. Misalignments observed in several other gapped disc systems could be generated by the same secular resonance mechanism.

Key words: accretion, accretion discs – protoplanetary discs – binaries: general – stars: individual: HD 142527.

1 INTRODUCTION

Newly formed binary stars often undergo a period of co-evolution with a circumstellar and circumbinary disc system. In this scenario, a circumbinary disc is disrupted into accreting streams by the binary, and feeds on to circumstellar discs around individual stars (e.g. Shi et al. 2012; D’Orazio, Haiman & MacFadyen 2013; Muñoz & Lai 2016). Young binary systems with both circumstellar and circumbinary discs have been observed for some time (e.g. Andersen et al. 1989; Mathieu et al. 1997). Interestingly, recent imaging observations with high angular resolution have revealed that the circumstellar and circumbinary discs are often misaligned, rather than sharing a common orbital plane.

In a picture where binaries form from an isolated rotating molecular cloud core, strongly misaligned discs are rather strange where one would expect everything to be aligned. Furthermore, if the binary companion forms through disc fragmentation (e.g. Adams, Ruden & Shu 1989; Matzner & Levin 2005; Rice, Lodato & Armitage 2005; Kratter & Matzner 2006; Clarke 2009; Meru & Bate 2011), then one would naturally expect it to share the same orbital plane as the disc from which it formed. Simulations of turbulent star formation, where the cores do not appear to be isolated from the surrounding molecular clouds, indicate that accretion from different directions can cause the angular momentum vector of the disc to evolve in single star systems (e.g. Bate, Lodato & Pringle 2010), and a similar process is expected to happen in binary systems. Accretion through the binary/disc is generally expected to drive the system towards alignment (Foucart & Lai 2013), although small misalignments are possible (Foucart & Lai 2014).

There are several observations of young binary systems that indicate misalignments between the circumbinary disc, circumstellar disc and the binary orbital plane (e.g. Winn et al. 2004; Hioki et al. 2011; Kennedy et al. 2012; Avenhaus et al. 2014; Brinch et al. 2016; Kraus et al. 2017; Takakuwa et al. 2017). Additionally, several protoplanetary discs with large gaps/cavities (often called ‘transition discs’ – see Casassus 2015; Owen 2016 for recent reviews), which could contain massive companions, are observed to contain misaligned inner and outer discs. Of particular interest is the system HD 142527, which contains an extended, massive disc (hundreds of astronomical units, \( \sim 0.1 \, M_\odot \)) with a large cleared dust gap from \( \sim 10 \) to \( \sim 100 \, \text{au} \) (Fukagawa et al. 2006); an accreting M-dwarf companion (with mass about 10–20 times smaller than...
the $2 \, M_\odot$ Herbig Ae/Be primary has been found to orbit within the cavity (Biller et al. 2012; Lacour et al. 2016). Recent NIR scattered light observations (e.g. Avenhaus et al. 2014) have revealed notches and shadows, indicating that the inner disc and the outer disc are misaligned by $\sim 70^\circ$ (Marino, Perez & Casassus 2015). This misalignment is confirmed by the gas kinematics from resolved CO observations (Casassus et al. 2015a). HD 100453 shows similar features to HD 142527 in scattered light observations, and have been interpreted as a misalignment between the inner and outer discs of $72^\circ$ (Benisty et al. 2016). Yet another system, HD 135344B, shows features in its image that indicate a weaker misalignment between its inner and outer disc of $22^\circ$ (Stolker et al. 2016).

In this paper, we explore a dynamical mechanism to generate large misalignments in gapped protoplanetary discs starting from a nearly co-planar configuration. The prototypical system we study consists of an inner disc and an outer disc, both assumed to be nearly flat (see Section 2), with a gap produced by a low-mass binary companion. Such a setup includes ‘transition’ discs if they do contain massive companions. The gravitational interaction between the discs and the companion generates mutual nodal precession of the disc’s and binary’s orbital planes. As the system evolves in time (e.g. due to accretion or dissipation of the discs), these precession frequencies match and may lead to inclination excitation. Such ‘secular precession resonance’ has long been studied in the context of planetary spin dynamics (e.g. Ward 1973). Recently, Batygin & Adams (2013) considered the effect of secular precession resonance as a mechanism of generating misalignment between the stellar spin and protoplanetary disc in the presence of an external binary companion. Lai (2014) presented a simple way (based on vector equations) of studying this resonance during the disc evolution and also included the effects of accretion and magnetic fields (see also Spalding & Batygin 2014). Matsakos & Königl (2017) studied a variant of this ‘primordial disc misalignment’ model in which the binary companion is a giant planet orbiting in the gap between the inner and outer disc.

Our motivation in this paper is the misaligned disc systems like HD 142527 and HD 100453. Our system is similar to that studied by Matsakos & Königl (2017); however, we ignore the stellar spin (since it has a negligible angular momentum compared to the discs and the embedded companion), and we focus on generating misalignments between the circumstellar (inner) and circumbinary (outer) discs, rather than on misalignments of the star’s spin. The basic setup is described in Section 2, and the concept of secular precession resonance is discussed further in Section 3. In Section 4, we numerically investigate various possible scenarios. In Section 5, we put forward a simple qualitative understanding of our results and discuss them in the context of observations. Finally, we summarize our findings in Section 6.

2 PROBLEM DESCRIPTION

We consider a primary star of mass $M_\star$ surrounded by a circumstellar disc of mass $M_{cs}$ and radial extent $R_{cs}$. The primary is then orbited by a lower mass companion with mass $M_c$ and separation $a_c$. The companion could be a lower mass star, brown dwarf or giant planet. The primary star and companion are then surrounded by a circumbinary disc of mass $M_{cb}$ and radial extent from $R_{cs}$ to $R_{out}$. Similar to previous works (Batygin & Adams 2013; Lai 2014; Lubow & Martin 2016; Matsakos & Königl 2017), we take the two discs to be flat, each having a single orientation. This assumption is reasonable because different regions of the disc can communicate with each other efficiently by internal stresses (such as bending waves$^1$), so that any disc warp is small (Lubow & Ogilvie 2001; Foucart & Lai 2014). Thus, our system is specified by three vectors: the angular momentum of the circumstellar disc ($\hat{L}_{cs}$), the angular momentum of the companion ($\hat{L}_{c}$) and the angular momentum of the circumbinary disc ($\hat{L}_{cb}$). There are six torques between the various components of which three are independent. We calculate the torques using the quadrupole approximation and assume that the primary star is much more massive than the companion and the discs (e.g. Lai 2014; Tremaine & Davis 2014).

The torque on the circumstellar disc due to the companion ($\tau_{cs,c}$) is

$$\tau_{cs,c} = \frac{3GM_c}{4a_c^3} (\hat{L}_{cs} \cdot \hat{L}_{c}) \hat{L}_{cs} \times \hat{L}_{c} \int_0^{R_{cs}} R^2 dM_{cs}, \quad (1)$$

where $\hat{L}_i$ are unit vectors. To get a sense of the magnitude and form of this torque, we can crudely evaluate the integral (assuming, as in most protostellar discs that the disc mass is dominated at large radius, but without specifically specifying its form – we will do this later in Section 2.1) to find (neglecting the $(\hat{L}_{cs} \cdot \hat{L}_{c})\hat{L}_{cs} \times \hat{L}_{c}$ factor):

$$|\tau_{cs,c}| \sim \frac{GM_c M_{cs} R_{cs}^3}{a_c^3}. \quad (2)$$

The torque on the companion due to the circumbinary disc ($\tau_{c,cb}$) is

$$\tau_{c,cb} = \frac{3GM_c a_c^2}{4} (\hat{L}_{c} \cdot \hat{L}_{cb}) \hat{L}_{c} \times \hat{L}_{cb} \int_{R_{in}}^{R_{out}} dM_{cb} \frac{R_{out}^3}{R^3}. \quad (3)$$

Similarly, the magnitude of this torque is approximately:

$$|\tau_{c,cb}| \sim \frac{GM_c M_{cb} R_{cb}^3}{R_{in}^3}, \quad (4)$$

where $M_{cb}^2$ is the disc mass ‘locally’ in the region near the inner edge of the circumbinary disc, or $M_{cb}^2 \sim 2\pi R_{in}^2 \Sigma_{in}$ with $\Sigma_{in}$ a representative surface density at the inner edge of the disc, this could be the unperturbed surface density or the peak surface density just outside the truncation radius. It is a useful quantity as it is observationally measurable with mm-imaging. Finally, the torque on the circumstellar disc due to the circumbinary disc ($\tau_{cs,cb}$) is

$$\tau_{cs,cb} = \int_0^{R_{cs}} R^2 dM_{cs} \int_{R_{in}}^{R_{out}} \frac{3GM_{cb}}{4R_{out}^3} (\hat{L}_{cs} \cdot \hat{L}_{cb}) \hat{L}_{cs} \times \hat{L}_{cb} \quad (5)$$

which has a magnitude of order:

$$|\tau_{cs,cb}| \sim \frac{GM_c M_{cb} R_{cs} R_{cb}^2}{R_{in}^3}. \quad (6)$$

With these torques, the evolution of the angular momenta in the system can be written as

$$\frac{dL_{cs}}{dt} = \tau_{cs,c} + \tau_{cs,cb} + \frac{d|L_{cs}|}{dt} L_{cs}, \quad (7)$$

$$\frac{dL_c}{dt} = -\tau_{cs,c} + \tau_{c,cb} + \frac{d|L_c|}{dt} L_c, \quad (8)$$

$$\frac{dL_{cb}}{dt} = -\tau_{cs,cb} - \tau_{c,cb} + \frac{d|L_{cb}|}{dt} L_{cb}. \quad (9)$$

$^1$ We note protostellar discs are in the ‘wave-like’ regime where bending waves propagate with speeds of order half the sound speed, rather than the diffusive regime (e.g. Papaloizou & Lin 1995).
This system contains many different precession frequencies allowing for the possibility of a ‘secular precession resonance’ that could result in large misalignments between the two discs. We note, as we will discuss later, a resonance does not necessarily result in a large misalignment; in these models a resonance is a necessary, but not a sufficient condition.

2.1 Precession frequencies

To evaluate the precession frequencies, we consider a power-law surface density profile of the form:

\[
\Sigma = \Sigma_0 \left( \frac{R}{R_0} \right)^{-1.5},
\]

(10)

Such a choice is the typical surface density profile obtained from mm-images of protoplanetary discs (e.g. Andrews et al. 2009, 2010) and is expected for a constant ‘alpha-disc’ model, with a flared structure (e.g. Hartmann et al. 1998, where temperature \( \propto R^{-1/2} \) - Kenyon & Hartmann 1987). With our choice of density profile, the angular momenta of the circumstellar and circumbinary discs are \( (2/3)M_\odot\Omega_K(R_\odot)^3 \) and \( (2/3)M_\odot\Omega_K(R_{\odot})R_\odot^2 \), respectively [with \( \Omega_K(R) \) the Keplerian angular velocity at radius \( R \)]. The precession frequencies are defined via \( \tau_{i,j} = \Omega_{i,j} \times L_i \times L_j \), and note that \( \Omega_{i,j} = \Omega_{j,i} |L_i|/|L_j| \). The six individual precession frequencies are

\[
\Omega_{cs,c} = -\frac{3}{8} \Omega_K(R_\odot) \left( \frac{M_\odot}{M_c} \right) \left( \frac{R_\odot}{a_c} \right)^2 \hat{L}_{cs} \cdot \hat{L}_c
\]

(11)

\[
\Omega_{cb,cb} = -\frac{3}{16} \Omega_K(R_\odot) \left( \frac{M_\odot}{M_b} \right) \left( \frac{R_\odot^3}{R_{out}^2} \right) \hat{L}_{cb} \cdot \hat{L}_{cb}
\]

(12)

\[
\Omega_{cb,cs} = -\frac{3}{8} \Omega_K(a_c) \left( \frac{M_b}{M_c} \right) \left( \frac{a_c^2}{R_{out}^2} \right) \hat{L}_{cs} \cdot \hat{L}_c
\]

(13)

\[
\Omega_{cs,cb} = -\frac{1}{4} \Omega_K(a_c) \left( \frac{M_b}{M_c} \right) \left( \frac{a_c^2}{R_{out}^2} \right) \hat{L}_{cs} \cdot \hat{L}_{cs}
\]

(14)

\[
\Omega_{cb,cb} = -\frac{3}{16} \Omega_K(R_{out}) \left( \frac{M_\odot}{M_b} \right) \left( \frac{a_c^2}{R_{in}^2} \right) \hat{L}_{cb} \cdot \hat{L}_c
\]

(15)

\[
\Omega_{cb,cs} = -\frac{3}{16} \Omega_K(R_{out}) \left( \frac{M_\odot}{M_b} \right) \left( \frac{R_\odot|R_{in}|}{R_{out}^2} \right) \hat{L}_{cb} \cdot \hat{L}_{cs}
\]

(16)

For systems that are close to being aligned initially, there are three possible precession resonances: a resonance between the circumstellar disc and the companion (\( \Omega_{cs} = \Omega_c \), where \( \Omega_{cs} = \Omega_{cs,c} + \Omega_{cs,cb} \) and \( \Omega_c = \Omega_{cs,c} + \Omega_{cs,cb} \)); a resonance between the circumstellar disc and the companion disc (\( \Omega_{cs} = \Omega_{cb} \), where \( \Omega_{cb} = \Omega_{cb,cs} + \Omega_{cb,cb} \)) and finally a resonance between the companion and the circumbinary disc (\( \Omega_c = \Omega_{cb} \)). Not all these resonances can occur in realistic systems or lead to large misalignments as we shall demonstrate below.

3 SECULAR PRECESSION RESONANCES

To assess the importance of various resonances, we consider a set of specific examples before performing numerical integrations. In each example, we assume one component of the system has so much angular momentum that it precesses so slowly that the corresponding angular momentum unit vector can be considered constant.

3.1 A massive circumbinary disc (\( L_{cb} \gg L_{cs}, L_c \))

Perhaps the most natural setup consists of a massive extended circumbinary disc that dominates the angular momentum budget of the system. Therefore, only the circumstellar disc and the companion precess, and a secular precession resonance occurs when \( \Omega_{cs} = \Omega_c \), or:

\[
\Omega_{cs,c} + \Omega_{cs,cb} = \Omega_{cb,cs} + \Omega_{cs,cb}.
\]

For a massive, large circumbinary disc, \( \Omega_{cb,cs}/\Omega_{cs,cb} \gg 1 \), unless the circumbinary disc is abnormally far away from the companion. In the scenario we are envisaging, it is the companion itself that truncates the circumstellar and circumbinary disc, so we would expect the inequality to be readily satisfied. Furthermore, we would also expect \( \Omega_{cs,cb}/\Omega_{cs,cb} \gg 1 \), as to truncate the circumbinary disc, the tidal torque from the binary should dominate over that from the circumbinary disc. Hence, the resonance condition is simplified to

\[
\Omega_{cs,c} \approx \Omega_{cs,cb}
\]

or,

\[
\left( \frac{M_c}{M_{cb}} \right)^{3/2} \approx \left( \frac{a_c}{R_{cs}} \right) \left( \frac{M_b}{R_{in}} \right)^{3/2}.
\]

(19)

When a circumbinary disc accretes on to a companion, starting with \( M_b \gg M_c \), to \( M_b \ll M_c \), we would expect the criterion in equation (19) could be satisfied at some point during the evolution, leading to resonant behaviour and possibly large changes in the alignment angles. Setting equation (19) to have parameters similar to those observed in HD 142527, we find

\[
M_{cb} \approx 2 M_f \left( \frac{a_c}{0.1 M_\odot} \right)^{3/2} \left( \frac{R_{in}}{10 \text{ au}} \right)^{3/2}.
\]

(20)

Local disc masses of order Jupiter masses are observed in many gapped systems (e.g. Andrews et al. 2010). In the case of HD 142527, Casassus et al. (2015b) measure a local disc mass of approximately a Jupiter mass, assuming a gas-to-dust mass ratio of 100. Therefore, we anticipate it is likely a resonance between the precession of the companion and circumstellar disc can occur in real systems.

To determine whether the resonance (equation 18) can lead to a significant production of misalignment, we can use a ‘geometric picture’ following Lai (2014). In the limit of \( |L_{cb}| \gg |L_c| \gg |L_{cb}| \), the unit angular momentum vectors evolve according to

\[
\frac{d\hat{L}_{cs}}{dt} \approx \Omega_{cs,c} \hat{L}_c \times \hat{L}_{cs}
\]

(21)

\[
\frac{d\hat{L}_{cb}}{dt} \approx \Omega_{cb,cs} \hat{L}_{cs} \times \hat{L}_{cb}
\]

(22)

We can transform into a frame rotating with the precession of the companion at a rate \( \Omega_c \), such that \( \hat{L}_c \) remains fixed in time. In this rotating frame, the evolution of \( \hat{L}_{cs} \) is governed by

\[
\left( \frac{d\hat{L}_{cs}}{dt} \right)_{\text{rot}} \approx \Omega_{cs,cb} \hat{L}_c \times \hat{L}_{cb}.
\]

(23)

This tells us that \( \hat{L}_{cs} \) precesses around a vector \( \hat{L}_c \) with precession rate \( \Omega_c \), given by

\[
\hat{L}_c = \Omega_{cs,c} \hat{L}_c - \Omega_{cs,cb} \hat{L}_{cb}
\]

(24)

As the system evolves from being far from resonance with \( \Omega_{cs,cb} \gg \Omega_{cs,c} \ll \Omega_{cs,cb} \), the circumbinary disc goes from precessing about \( \hat{L}_c \) to precessing around \( \hat{L}_{cb} \). However, close to resonance when \( \Omega_{cs,c} \approx \Omega_{cs,cb} \), the vector \( \hat{L}_c \) about which \( \hat{L}_{cs} \) is precessing
deviates from \( \hat{L}_{cb} \) by a large angle. This is obvious if one considers the case where \( \hat{L}_c \) and \( \hat{L}_{cb} \) are misaligned by some small angle \( \theta \), the one finds

\[
\arccos (\hat{L}_r \cdot \hat{L}_{cb}) \approx \frac{\pi}{2} + \frac{\theta}{2}. \tag{25}
\]

Thus, in resonance \( \hat{L}_{cs} \) is precessing around an axis that is almost orthogonal to \( \hat{L}_{cb} \). For a large misalignment to be generated, a necessary (but not sufficient) condition is \( \Omega_{cs}^{-1} \) is comparable to, or shorter than the time-scale on which the system is evolving. If resonance crossing is too fast, \( \hat{L}_{cs} \) is unable to precess far enough around \( \hat{L}_r \) to generate a large misalignment. Finally, it is clear from equation (22) that the misalignment between the companion and circumbinary disc is constant.

### 3.2 A massive companion, \((L_c \gg L_{cs}, L_{cb})\)

One can imagine late in the evolution of a protostellar system, when the discs contain very little mass, the companion dominates the total angular momentum budget. Therefore, a ‘resonance’ can occur between the precessing circumstellar and circumbinary discs when \( \Omega_{cs} = \Omega_{cb} \), or

\[
\Omega_{cs,c} + \Omega_{cs,cb} = \Omega_{cb,c} + \Omega_{cb,cs}. \tag{26}
\]

In this case, the massive companion predominately drives the precession of both the inner and outer discs and equation (26) reduces to

\[
\Omega_{cs,c} \simeq \Omega_{cb,c} \tag{27}
\]

or,

\[
a_c \sim 70 \text{ au} \left( \frac{R_{cs}}{10 \text{ au}} \right)^{3/10} \left( \frac{R_{in}}{100 \text{ au}} \right)^{2/5} \left( \frac{R_{out}}{300 \text{ au}} \right)^{3/10}. \tag{28}
\]

This criterion is independent of mass and could be satisfied in real systems. In this scenario, the disc angular momentum unit vectors evolve according to

\[
\frac{d\hat{L}_{cs}}{dt} \approx \Omega_{cs,c} \hat{L}_c \times \hat{L}_{cs} \tag{29}
\]

\[
\frac{d\hat{L}_{cb}}{dt} \approx \Omega_{cb,c} \hat{L}_c \times \hat{L}_{cb}. \tag{30}
\]

Obviously, both discs are precessing around the same axis \((L_c)\) independently as the gravitational coupling between the two discs is negligible. As such, in ‘resonance’ a large misalignment cannot be generated from an initially small misalignment.

### 3.3 A massive circumstellar disc \((L_{cs} \gg L_c, L_{cb})\)

While a circumstellar disc that contains most of the angular momentum seems strange, it is not so unusual from the perspective of formation of the companion by fragmentation. Fragmentation is expected to occur in the outer regions of massive circumstellar discs (e.g. Clarke 2009). At the point of formation, the angular momentum could still be dominated by the circumstellar disc. The companion is then expected to migrate through the disc to smaller separations (e.g. Li et al. 2015; Stamatellos 2015), wherein the more usual hierarchy of a circumbinary disc that contains most of the angular momentum is restored (Section 3.1). Here, we assume that the circumstellar disc precesses so slowly that we can consider it to be fixed, thus a resonance can occur when the precession frequencies of the companion and circumbinary disc become equal, \( \Omega_c = \Omega_{cb,c} \) or

\[
\Omega_{cs,c} + \Omega_{cs,cb} = \Omega_{cb,c} + \Omega_{cb,cs}. \tag{31}
\]

In this case of a massive inner disc, \( \Omega_{cs,c}/\Omega_{cb,c} \gg 1 \). For the precession of the circumbinary disc, it is not clear that either the circumstellar disc or companion will dominate its precession. Arguments similar to those discussed in Sections 3.1 and 3.2 indicate a strong resonance will occur if \( \Omega_{cb,c}/\Omega_{cb,cs} \gg 1 \). In this case, the equation (31) simplifies to

\[
\Omega_{cs,c} \simeq \Omega_{cb,c} \tag{32}
\]

or,

\[
M_{cs} \sim 3 M_c \left( \frac{a_c^3}{R_{out}^2} \right) \left( \frac{a_c^3}{R_{in}^2 R_{in}} \right). \tag{33}
\]

Now requiring that the companion forms by fragmentation out of the disc that will make up the circumbinary disc means that \( M_c \lesssim M_{cb} \). For fragmentation to take place, one normally requires the outer regions to have a mass \( \sim 0.1 M_c \). Combing all these criteria, along with \( M_{cs} \gg M_{cb} \) and \( M_{cs} \gg M_c \), implies a circumbinary disc more massive than the central star. We conclude that the precession resonance between the companion and circumbinary disc is unlikely to occur in realistic binary-disc systems. This inference is confirmed by our numerical investigation, where we can only get resonant behaviour for unphysically large circumbinary disc masses.

### 4 AN EVOLVING SYSTEM

We consider two basic scenarios. In the first (Section 4.1) case, the companion remains at a fixed separation and we allow the discs to evolve and accrete mass on to the companion. In the second scenario (Section 4.2), we allow the companion to migrate through the disc while the total mass of the two discs remains fixed. It is easiest to work in an inertial Cartesian frame, where we initialize the angular momentum unit vectors of the components to

\[
\hat{L}_{cs} = [0, 0, 1] \tag{34}
\]

\[
\hat{L}_c = [0, -\sin (\theta_0), \cos (\theta_0)] \tag{35}
\]

\[
\hat{L}_{cb} = [-\sin (\theta_0), 0, \cos (\theta_0)] \tag{36}
\]

where \( \theta_0 \) is a small angle that we set to 5°. We have checked that our results are qualitatively independent of the initial choice of orientation of the three vectors and the choice of \( \theta_0 \), provided it is small but non-zero.

#### 4.1 Evolving discs

In the ‘evolving disc’ scenario, we assume the companion remains on an orbit with a fixed separation, but the discs deplete due to accretion. Simple, self-similar viscous accretion theory (Lynden-Bell & Pringle 1974; Hartmann et al. 1998) suggests that for \( \Sigma \propto R^{-1} \) the disc mass \( (M_d) \) declines as

\[
M_d = \frac{M_d^0}{(1 + t/t_v)^{3/2}}. \tag{37}
\]

where \( t_v \) is the ‘viscous time-scale’ that sets the time-scale for global disc evolution. Note, this form differs from the temporal evolution of the disc mass used by previous authors (e.g. Batygin & Adams 2013; Lai 2014; Spalding & Batygin 2014; Matsakos & Königl...

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\[ \text{MNRAS 469, 2834–2844 (2017)} \]
We take equation (37) to specify how the circumbinary disc evolves. In the case where the companion accretes, we assume it accretes some fraction ($f$) of the material that is depleted from the circumbinary disc. Therefore, the circumbinary disc evolves as

$$M_{cb}(t = 0) = \frac{M_{cb}(t = 0)}{(1 + t/t_\nu)^{3/2}}$$

(38)

and the companion evolves as

$$M_c = M_c(t = 0) + fM_{cb}(t = 0) \left[ 1 - \frac{1}{(1 + t/t_\nu)^{3/2}} \right].$$

(39)

The numerical value of $f$ is uncertain; for example, it is not clear whether the circumstellar disc will receive most of the material ($f < 0.5$) or whether the companion will accrete most of the material ($f > 0.5$). Thus, we will keep $f$ as a free parameter. The remaining mass that does not accrete on to the companion resupplies the inner disc. Since the viscous time of the inner disc is considerably shorter than that of the circumbinary disc, the circumstellar disc can be considered to be in a steady state with a constant mass supply, where the accretion rate through the circumstellar disc is $M_{cs} = (1 - f)M_{cb}$. As a steady disc has surface density proportional to $M_c$, then we can write

$$M_{cs} = \frac{M_{cs}(t = 0)}{(1 + t/t_\nu)^{3/2}}.$$  

(40)

As the companion grows in mass, the truncation radii of the circumstellar and circumbinary discs due to the companion’s torque evolve. For high-mass ratio binaries (with $M_c/M_* \sim 60$), this disc is typically truncated at several Hill radii away from the companion (e.g. Artymowicz & Lubow 1994; Crida, Morbidelli & Masset 2006). For concreteness, we adopt the following prescription for the disc truncation radii:

$$R_{cs} = a_c \left[ 1 - 3 \left( \frac{M_c}{3M_*} \right)^{1/3} \right]$$

(41)

and

$$R_m = a_c \left[ 1 + 3 \left( \frac{M_c}{3M_*} \right)^{1/3} \right].$$

(42)

4.1.1 Results of numerical integrations

As our canonical example, we adopt the parameters that are typical of gapped discs: $M_* = 2M_\odot$, $a_c = 40$ au, $M_c(t = 0) = 10^{-3}M_\odot$, $M_{cs}(t = 0) = 3 \times 10^{-3}M_\odot$, $M_{cb}(t = 0) = 0.25M_\odot$ and $R_{out} = 300$ au, with evolution parameters $f = 0.5$ and $t_\nu = 2 \times 10^5$ yr. In this example, the companion will reach a mass of $\sim 0.1 M_\odot$ after 5 Myr of accretion.

In Fig. 1, we show the evolution of the mutual inclinations between the discs and companion as well as their precession frequencies. In this scenario, the circumbinary disc contains enough angular momentum such that its precession time-scale is long compared to the evolution of the system. In Fig. 2, we show the evolution of various system parameters. We see in Fig. 1 that the precession frequencies of the circumbinary disc and companion become comparable after about $\sim 0.25$ Myr of evolution. This causes a resonant response where the circumstellar and circumbinary disc strongly misalign, with a misalignment of $\sim 60^\circ - 70^\circ$ found. After the discs have misaligned, the circumbinary disc precesses on a time-scale $\sim 0.5$ Myr with a nutation amplitude of about $10^\circ$, driven by the torque from the circumbinary disc.

As can be seen in Fig. 2, the resonance occurs in the case where the outer disc dominates the angular momentum and mass budget of the system, and is representative of the case discussed in Section 3.1.

We can investigate how the evolution parameters affect the results in Fig. 3, where we vary the viscous time and $f$. The evolution curves show that the same evolution is achieved if the disc evolution parameters are varied: rapid evolution towards resonance crossing (either by a shorter viscous time or larger $f$) causes a large rapid change in the misalignment angle, whereas for a slower approach towards resonance crossing results in a smoother change over several precession periods of the circumstellar disc. In all cases, we find that resonance crossing can produce large misalignments between the circumstellar and circumbinary disc, while aligning the companion with the circumbinary disc.

Finally, if the companion does not accrete in this scenario, a resonance between the circumstellar disc and companion’s precession never occurs and no large misalignment is generated. This is demonstrated in Fig. 4 where we drop the accretion efficiency to $f = 0.01$, so that the companion only reaches a mass of $\sim 1.5 M_J$ after 3 Myr. A similar outcome is obtained by beginning the evolution with much less mass in the outer disc, such that the companion cannot become massive enough to permit resonance crossing. In general, we do not expect misalignments to be generated in planet hosting protoplanetary discs, this conclusion is specifically relevant to whether or not our mechanism operates in ‘transition’ discs.
4.2 Migrating companion

In this ‘migrating companion’ scenario, we consider a companion of fixed mass that has formed in the outer regions of the disc, and is now migrating to smaller separations. We prescribe the migration rate as

\[ \frac{\dot{a}_c}{a_c} = \frac{1}{t_{\text{mig}}} \]

(43)

where the migration time-scale \( t_{\text{mig}} \) is a free parameter. The truncation radii of the discs then evolve according to equations (41) and (42). For simplicity, we do not allow the discs to deplete due to viscous accretion, instead the total mass in the circumstellar and circumbinary discs remains fixed, but the individual mass of each disc evolves in response to the migrating companion. For a disc with a \( \Sigma \propto R^{-1} \) surface density profile, the disc mass scales linearly with its outer radius. Thus,

\[ \frac{dM_{cs}}{dt} = M_{cs} \left( \frac{R_{cs}}{R_{cs}} \right) = -\frac{M_{cs}}{t_{\text{mig}}} \]

(44)

and, by mass conservation:

\[ \frac{dM_{cb}}{dt} = -\frac{dM_{cs}}{dt} = \frac{M_{cs}}{t_{\text{mig}}} \]

(45)

4.2.1 Results of numerical integrations

Again, we motivate our choice of initial parameters by those similar to the observed systems: \( M_* = 2 M_\odot \), \( a_\odot (t = 0) = 200 \) au, \( M_1 = 0.2 M_\odot \), \( M_2 (t = 0) = 0.1 M_\odot \), \( M_{c,b} (t = 0) = 0.05 M_\odot \) and \( R_{\text{in}} = 1.2 R_\odot (t = 0) \), with a migration time of \( t_{\text{mig}} = 7.5 \times 10^5 \) yr. The outer edge of the circumstellar and inner edge of the circumbinary discs are again set using equations (41) and (42).

Fig. 5 shows the evolution of the misalignment angles and precession frequencies for the migrating companion scenario. The evolution of the system components are shown in Fig. 6. We see that after about 2 Myr, the precession frequencies of the circumstellar disc and companion cross, by this time the companion has migrated to \( \sim 10 \) au. Furthermore, the circumbinary disc now contains more mass than the circumstellar disc and dominates the angular momentum budget of the system. The resonance crossing again causes a large misalignment angle between the two discs (\( \sim 70^\circ - 80^\circ \)). This evolution is similar to that of the accreting companion scenario discussed in Section 4.1, where the secular resonance is dominated by \( \Omega_{cs,c} \sim \Omega_{c,cb} \). Again, we find that the circumbinary disc and the companion align slightly, meaning the circumstellar disc and companion are also highly misaligned. However, unlike the accreting companion scenario, as the companion continues to migrate to smaller separations, the precession frequency of the circumstellar disc gets shorter as its outer edge and mass becomes smaller. In Fig. 7, we show how varying the migration time between \( 5 \times 10^5 \) yr (left-hand panel) and \( 1 \times 10^6 \) yr (right-hand panel) affects the result. The faster migration time leads to a sharper change in the misalignment angles. Similar to the canonical example, after a several million years of evolution, the two discs are misaligned by \( > 70^\circ \).

4.3 Massive companions and low-mass discs

Towards the end of the disc’s evolution, most of the angular momentum of the system will be contained in the binary companion. In this case, the companion’s orbit will remain unchanged, but could result in nodal precession of the circumstellar and circumbinary discs (Section 3.2). In order to briefly explore this case, we consider the migrating companion scenario as described in Section 4.2, but reduce the mass of both discs to \( 5 \times 10^{-3} M_\odot \), while keeping the companion’s mass to \( 0.2 M_\odot \). While it is of course questionable whether the companion could still migrate rapidly, our example is illustrative. Fig. 8 shows the evolution of such a system. We see that in this case the companion always dominates the angular momentum budget of the system. The precession frequencies of the circumstellar and circumbinary discs (both of which are completely dominated by driving due to the companion) become equal after...
Figure 3. The evolution of the misalignment angle for the system shown in Fig. 1, but with the viscous time ($t_\nu$) varied from $1 \times 10^5$ to $5 \times 10^5$ yr and the accretion efficiency of the companion ($f$) varied from 0.25 to 0.75.

about 1.5 Myr of evolution. This causes a modest increase in the misalignment angle (to about 20°) between the circumstellar and circumbinary discs. In general, a ‘resonance’ crossing between the circumstellar and circumbinary discs’ precession frequencies do not lead to large misalignments from initially aligned configurations.

5 DISCUSSION

We have investigated the possibility that secular precession resonances can generate large misalignments between circumstellar and circumbinary discs in young binary systems that have mass ratios of the order of 0.1. For realistic systems, we find that only resonant interactions between the precession of the circumstellar disc and companion, or between the precession circumstellar and circumbinary disc can occur.

The resonance between the circumstellar disc and companion is the most robust and the only one that can generate large misalignments (>60°) between the two discs, from small (few degree) primordial misalignments. In a frame precessing with the companion, the circumstellar disc precesses around an axis almost orthogonal to the disc’s angular momentum axis when in resonance, with a precession rate $\Omega_r \approx \sqrt{20}/\Omega_{0,c}$, where $\theta_0$ is the small initial misalignment. In the examples depicted in Figs 1 & 5, $\Omega_{0,c} \sim 50$ Myr$^{-1}$ in resonance, and the rate of change of our system is $\sim 1$–5 Myr$^{-1}$ for both the accreting and migrating companion cases. Thus, the evolution time-scale of the system is comparable to the precession time $\Omega^{-1}$ (for $\theta_0 \sim 0.1$). This means that the circumstellar disc can precess a significant fraction of the way around $\hat{L}_r$ before the system is no longer in resonance.

Now, we can naturally see why we often get large misalignments if resonance crossing occurs. Young systems will always evolve on ~Myr time-scales in the radial range of 10–100 au. For a companion to star mass ratio ($q$) in the range 0.01–0.1, equations (11), (24) and (41) tell us $\Omega_c \sim 0.1 q \theta_0 \Omega_{0,c}/(a_c)$ which for the orbital periods found at tens of astronomical units of a few hundred years, we get precession time-scales about $\hat{L}_r$ of ~Myr. Therefore, it is not surprising in the case of high-mass ratio binaries with separations of order tens of astronomical units resonance crossing leads to large misalignments. If the evolutionary time-scales were significantly shorter (e.g. $\lesssim 10^5$ yr), hence the resonance crossing time-scale is shorter, smaller misalignments would be generated; this result is verified by changing $t_\nu$ to $3 \times 10^4$ yr in our numerical models where more modest ~20–40° misalignments are generated. Finally, we note that while it is easy to construct initial and evolutionary parameters that cause our systems to undergo resonance, it is also easy to find examples that do not (see Fig. 4). Therefore, depending on their parameters, real systems may or may not experience resonant misalignment excitations during their lifetimes.

5.1 Assumptions

In this work, we have made several simplifications in order to make the problem simple and to reveal the basic dynamics. The two most important assumptions are that we ignore explicit angular
Figure 4. The same as Fig. 1, expect that \( f = 0.01 \) so that \( \Omega_c \) and \( \Omega_{cs} \) to not become comparable during the evolution of system.

Figure 5. The top panel shows the angles between the angular momentum vectors of the circumstellar disc, circumbinary disc and the companion as a function of time. The bottom panel shows the precession frequencies of the circumstellar disc and the companion as well as the individual components. These plots are for the case of a migrating companion, starting at 200 au and migrating inwards with a time-scale of \( (\dot{a}/a)^{-1} = 7.5 \times 10^3 \) yr.

Figure 6. Evolution of the disc/companion system as a function of time in the migrating companion scenario. The top panel shows the truncation radii of the circumstellar and circumbinary discs. The middle panel shows the evolution of the masses of the circumstellar disc, companion and circumbinary disc and finally the bottom panel shows the evolution of the magnitude of the angular momenta of the circumstellar disc, companion and circumbinary disc. The evolving system shown is for the migrating companion scenario depicted in Fig. 5.

In the construction of our evolving system, we have followed previous work and made no effort to ensure that our disc-companion system explicitly conserves angular momentum. In the case of the accreting disc system, we do not allow the circumbinary disc to expand as it accretes, or lose mass (and angular momentum) in a photoevaporative wind. In the case of viscous accretion with \( R_{out} \gg R_{in} \), we would actually expect \( R_{out} \) to expand such that the
circumbinary disc maintains approximately constant angular momentum in the absence of mass-loss. Since the resonance of interest between the circumbinary disc and companion occurs when most of the angular momentum is in the circumbinary disc, this assumption does not have any important consequences on our calculations.

In our models, we have implicitly assumed that as disc material moves from the circumbinary disc to the circumstellar disc (in the case of the accreting system) or from the circumstellar disc to the circumbinary disc (in the case of the migrating companion), it joins the new disc’s orbital plane. In reality, gas parcels may join the new disc in its original orbital plane. The interaction of material flowing between the two discs is mediated by the companion and it is still uncertain how much angular momentum is absorbed by the companion (and in what orbital plane) during this interaction. However, since we are interested in starting from systems that are initially close to alignment, which then approach a resonance, this assumption should not significantly affect the resonant misalignment excitation. In the case of the accreting system, after the large misalignment has occurred, one would expect accretion from the circumbinary disc on to the circumstellar disc to possibly re-align the two discs on of order Myr time-scales (this is similar to the alignment of a binary’s orbit that is accreting from a misaligned circumbinary disc discussed by Foucart & Lai 2013, 2014). Therefore, we imagine large misalignments should readily be observable after the resonance has occurred, even if accretion does slowly realign the discs afterwards.

In all our models, we treat the discs as rigidly precessing plates. In general, the discs are warped and twisted (Papaloizou & Pringle 1983). Warp propagation in accretion discs occupies two distinctly different regimes (e.g. Papaloizou & Lin 1995), when the viscous α parameter is larger than the aspect ratio (H/R) of the disc, the warps propagate diffusively; however, if α is smaller than the aspect ratio, the warp is wave-like and propagates with a speed approximately half of the sound speed. With aspect ratios of the order of 0.1 in protostellar discs at tens of astronomical units (e.g. Chiang & Goldreich 1997) and observationally inferred viscous ‘alphas’ at least an order of magnitude smaller (e.g. Hartmann et al. 1998), protoplanetary discs are in the wave-like warp regime (e.g. Lubow & Ogilvie 2000; see also Lubow & Ogilvie 2001), where bending waves propagate at approximately half of the sound speed (e.g. Papaloizou & Lin 1995). On time-scales longer than the warp propagation time, the disc evolves towards and precesses as a rigid body (e.g. Foucart & Lai 2014, in the linear regime). Since our systems evolve on the long, secular evolutionary time-scales, much longer than the warp propagation time over hundreds of astronomical units, the rigid plate approximation is likely to be valid.

Finally, even though the rigid plate approximation is good, the discs are still likely to maintain a small warp, such a warp can viscously dissipate the mutual inclination between the disc’s and companion’s orbital planes. This viscous damping rate is highly uncertain (e.g. Ogilvie & Latter 2013 and see discussion in Lai 2014); however, Foucart & Lai (2014) suggest that large misalignments (> 20°) can be maintained for time-scales similar to a disc’s lifetime.

5.2 Observational implications

One of the motivations for our study was the observed misalignment between the circumbinary and circumbinary discs in HD 142527. HD 142527 is a well-known gapped disc with a dust cavity extending from a few tens of au to ~100 au (e.g. Fukagawa et al. 2006). Scattered light imaging of the circumbinary disc at NIR wavelengths (Casassus et al. 2012; Canovas et al. 2013; Avenhaus et al. 2014) were interpreted by Marino et al. (2015) as a tilt of 70° ± 5° between the inner (circumstellar) and outer (circumbinary) discs. Biller et al. (2012) discovered a low-mass companion in the cavity of HD 142527 that is now known to be a M dwarf with a mass of the order of 0.1–0.2M⊙ (Lacour et al. 2016) and hence HD 142527 is a high-mass ratio binary system with a mass ratio q = 0.05–0.1. Therefore, the secular resonance model presented in this work is a likely explanation for the observed misalignment in HD 142527. The precession resonance could have been triggered due to either the companion’s migration to smaller separations, or accretion of mass on to the companion from the disc, or some combination of both.

Figure 7. The evolution of the misalignment angle for the migrating companion scenario. The system is the same as in Figs 5 and 6, except $t_{\text{mig}} = 5 \times 10^5$ yr in the left-hand panel and $t_{\text{mig}} = 1 \times 10^6$ yr in the right-hand panel.
Massive and extended outer disc (e.g. Andrews et al. 2011), which could have been generated by this mechanism and is in the process of crossing a precession resonance between the circumstellar disc and companion. For some cases, the generated misalignments can be large (\( \sim 0.1 \))

There are several other well-known gapped discs that show evidence for large misalignments: HD 100453 (Benisty et al. 2016) exhibits a similar misalignment between the inner and outer discs of \( \sim 70^\circ \), and HD 135344B (Stolker et al. 2016) shows a smaller misalignment of \( \sim 20^\circ \). No companion has been detected in either system to date. The misalignment in HD 100453 could have been generated by resonance crossing and such a scenario would imply a low-mass \( \sim 0.01–0.1 \) M\(_{\odot} \) companion residing in the cavity that is aligned with the outer disc. The misalignment of HD 135343B could have been generated by this mechanism and is in the process of realigning due to accretion, or viscous dissipation. We can probably rule out a resonance between the inner and outer disc in the case of HD 135433B as it requires the companion to dominate the angular momentum budget of the system. HD 135433B has a particularly massive and extended outer disc (e.g. Andrews et al. 2011), which would require the companion to be solar mass or above to induce a large misalignment.

A precession resonance between the circumstellar disc and companion, starting from a nearly aligned configuration will keep the companion close to alignment with the circumbinary disc. Small arc analysis of approximately 2 yr of observations of the companion in HD 142527 has provided weak constraints on the orbital elements of the companion (Lacour et al. 2016). These constraints show the companion is on an eccentric orbit, but the orbital inclination is more uncertain. Lacour et al. (2016) suggest (see their fig. 8) that in order for the companion’s apocentre to correspond to the inner edge of the circumbinary (outer) disc, it is likely that the companion is aligned with the inner disc. However, we caution that the apocentre distance is not where a companion would truncate the disc (e.g. Artymowicz & Lubow 1994; Miranda & Lai 2015). Companions with \( q \sim 0.1 \) are in between the regime at low masses where they truncate the disc at several Hill radii (e.g. Crida et al. 2006) and comparable mass ratios where they truncated the disc at a distance of a few times the companions separation (e.g. Miranda & Lai 2015). Furthermore, the companion will truncate the gas disc, whereas the disc’s inner edge is measured from dust emission. It is well known that the gas extends further in than the gas in HD 142527 (e.g. Casassus et al. 2015a). Therefore, it is plausible that the companion is aligned with the outer disc and still play a role in carving the circumbinary disc. For example, applying the results of Artymowicz & Lubow (1994) indicates an aligned binary with mass ratio \( q \sim 0.1 \), an eccentricity of \( \sim 0.5 \) will truncate the circumbinary disc between three and four times the binary separation. HD 142527’s binary separation is \( 20^{+17}_{-10} \) au and eccentricity is \( 0.5 \pm 0.2 \); thus, the companion can orbit in the same plane as the circumbinary disc and still truncate the disc’s dust component at \( \sim 100 \) astronomical units. Further monitoring of the companion should be able to determine its orbital plane, and test the resonant scenario presented here.

\[ 6 \text{ SUMMARY} \]

We have shown that secular precession resonances can operate in protostellar gapped/binary discs, which can result in a misalignment between the circumstellar (inner) and circumbinary (outer) discs. For some cases, the generated misalignments can be large (\( \gtrsim 60^\circ \))

(i) We identify two secular precession resonances in gapped/binary-disc systems. First, a resonance between the precession of the circumstellar and circumbinary discs. Secondly, a resonance between the precession of the circumstellar disc and a low-mass companion (or massive planet) as the system evolves. Our main results are summarized below:

(ii) The resonance between the circumstellar and circumbinary disc cannot lead to large misalignments as both discs precess independently, even in ‘resonance’.

(iii) The resonance between the circumstellar disc and companion can lead to significant misalignments, even from systems that are close to being co-planar initially, provided that the resonance crossing occurs on the right time-scale. This typically requires that the companion has a mass \( \sim 0.01–0.1 \) M\(_{\odot} \), separation \( \sim 10–100 \) au for an approximately solar mass primary and that the circumbinary disc dominates the angular momentum budget of the system. Such requirements are not satisfied by giant – of order Jupiter mass – planets in ‘transition’ discs.

Figure 8. The evolution of the misalignment angles (top), precession frequencies (middle) and angular momenta (bottom) of a system where most of the angular momentum is in the companion.

6 SUMMARY

We have shown that secular precession resonances can operate in protostellar gapped/binary discs, which can result in a misalignment between the circumstellar (inner) and circumbinary (outer) discs. For some cases, the generated misalignments can be large (\( \gtrsim 60^\circ \)) resulting from crossing a precession resonance between the circumstellar disc and companion residing in a gap between the circumstellar disc and a low-mass companion (or massive planet) as the system evolves. Our main results are summarized below:

(i) We identify two secular precession resonances in gapped/binary-disc systems. First, a resonance between the precession of the circumstellar and circumbinary discs. Secondly, a resonance between the precession of the circumstellar disc and a low-mass companion residing in a gap between the circumstellar and circumbinary disc.

(ii) The resonance between the circumstellar and circumbinary disc cannot lead to large misalignments as both discs precess independently, even in ‘resonance’.

(iii) The resonance between the circumstellar disc and companion can lead to significant misalignments, even from systems that are close to being co-planar initially, provided that the resonance crossing occurs on the right time-scale. This typically requires that the companion has a mass \( \sim 0.01–0.1 \) M\(_{\odot} \), separation \( \sim 10–100 \) au for an approximately solar mass primary and that the circumbinary disc dominates the angular momentum budget of the system. Such requirements are not satisfied by giant – of order Jupiter mass – planets in ‘transition’ discs.
... (iv) In numerical calculations of realistic systems, we typically find misalignments of $\sim 70^\circ$, consistent with those observed in HD 142527 and HD 100453. This degree of misalignment is set by a resonance crossing time of the order of a million years. Our results indicate a secular resonance between a companion and the circumstellar (inner) disc in HD 142527 and HD 100453 occurred previously in their histories and misaligned their two discs. Like the companion present in HD 142527, we would suggest a companion exists in HD 100453 that is either a very massive planet, brown dwarf or low-mass star.

(v) For gapped disc systems that are initially close to being co-planar, the secular resonance mechanism would predict the companion to remain close to co-planar with the circumbinary (outer) disc, but misaligned to the circumstellar (inner) disc. This prediction is testable in HD 142527 with a longer baseline of observations of the companion’s orbit.

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REFERENCES

Adams F. C., Ruden S. P., Shu F. H., 1989, ApJ, 347, 959
Andersen J., Lindgren H., Hazen M. L., Mayor M., 1989, A&A, 219, 142
Andrews S. M., Wilner D. J., Hughes A. M., Qi C., Dullemond C. P., 2009, ApJ, 700, 1502
Andrews S. M., Wilner D. J., Hughes A. M., Qi C., Dullemond C. P., 2010, ApJ, 723, 1241
Artyomowicz P., Lubow S. H., 1994, ApJ, 421, 651
Avenhaus H., Quanz S. P., Schmid H. M., Meyer M. R., Garufi A., Wolf S., Dominik C., 2014, ApJ, 781, 87
Bate M. R., Lodato G., Pringle J. E., 2003, MNRAS, 340, 1505
Batygin K., Adams F. C., 2013, ApJ, 778, 169
Benisty M. et al., 2016, A&A, 597, A42
Biller B. et al., 2012, ApJ, 753, L38
Brinch C., Jørgensen J. K., Hogerheijde M. R., Nelson R. P., Gressel O., 2016, ApJ, 830, L16
Canovas H., Ménard F., Hales A., Jordán A., Schreiber M. R., Casassus S., Gledhill T. M., Pinte C., 2013, A&A, 556, A123
Casassus S., 2016, Publ. Astron. Soc. Aust., 33, e013
Casassus S., Perez M. S., Jordán A., Ménard F., Cuadra J., Schreiber M. R., Hales A. S., Ercolano B., 2012, ApJ, 754, L31
Casassus S. et al., 2015a, ApJ, 811, 9
Casassus S. et al., 2015b, ApJ, 812, 126
Chiang E. I., Goldreich P., 1997, ApJ, 490, 368
Clarke C. J., 2009, MNRAS, 396, 1066
Crida A., Morbidelli A., Masset F., 2006, Icarus, 181, 587
D’Orazi D. J., Haiman Z., MacFadyen A., 2013, MNRAS, 436, 2997
Foccart F., Lay D., 2013, ApJ, 764, 106
Foccart F., Lay D., 2014, MNRAS, 445, 1731
Fukagawa M., Tamura M., Itoh Y., Kudo T., Imaeda Y., Oasa Y., Hayashi S. S., Hayashi M., 2006, ApJ, 636, L153
Hartmann L., Calvet N., Gullbring E., D’Alessio P., 1998, ApJ, 495, 385
Hioki T., Itoh Y., Oasa Y., Fukagawa M., Hayashi M., 2011, PASJ, 63, 543
Kennedy G. M. et al., 2012, MNRAS, 421, 2264
Kenyon S. J., Hartmann L., 1987, ApJ, 323, 714
Kratter K. M., Matzner C. D., 2006, MNRAS, 373, 1563
Kraus S. et al., 2017, ApJ, 835, L5
Lacour S. et al., 2016, A&A, 590, A90
Lai D., 2014, MNRAS, 440, 3532
Li Y., Kouwenhoven M. B. N., Stamatellos D., Goodwin S. P., 2015, ApJ, 805, 116
Lubow S. H., Martin R. G., 2016, ApJ, 817, 30
Lubow S. H., Ogilvie G. I., 2000, ApJ, 538, 326
Lubow S. H., Ogilvie G. I., 2001, ApJ, 560, 997
Lynden-Bell D., Pringle J. E., 1974, MNRAS, 168, 603
Marino S., Perez S., Casassus S., 2015, ApJ, 798, L44
Mathieu R. D., Stassun K., Basri G., Jensen E. L. N., Johns-Krull C. M., Valenti J. A., Hartmann L. W., 1997, AJ, 113, 1841
Matsakos T., Königl A., 2017, AJ, 153, 60
Matzner C. D., Levin Y., 2005, ApJ, 628, 817
Meru F., Bate M. R., 2011, MNRAS, 410, 559
Miranda R., Lai D., 2015, MNRAS, 452, 2396
Mutoz D. J., Lai D., 2016, ApJ, 827, 43
Ogilvie G. I., Latter H. N., 2013, MNRAS, 433, 2420
Owen J. E., 2016, Publ. Astron. Soc. Aust., 33, e005
Papaloizou J. C. B., Lodato G., Armitage P. J., 2005, MNRAS, 364, L56
Shi J.-M., Krolik J. H., Lubow S. H., Hawley J. F., 2012, ApJ, 749, 118
Spalding C., Batygin K., 2014, ApJ, 790, 42
Stamatellos D., 2015, ApJ, 810, L11
Stolker T. et al., 2016, A&A, 595, A113
Takakuwa S., Saito M., Takahashi T., Saito M., Lim J., Hanawa T., Yen H.-W., Ho P. T. P., 2017, ApJ, 837, 86
Tremaine S., Davis S. W., 2014, MNRAS, 441, 1408
Triaud A. H. M. J. et al., 2010, A&A, 524, A25
Ward W. R., 1973, Science, 181, 260
Winn J. N., Holman M. J., Johnson J. A., Stanek K. Z., Garnavich P. M., 2004, ApJ, 603, L45
Winn J. N. et al., 2009, ApJ, 700, 302

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