Constraint Correlation Dynamics of SU(N) Gauge Theories*†

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Abstract

A constraint correlation dynamics up to 4-point Green functions is proposed for SU(N) gauge theories which reduces the N-body quantum field problem to the two-body level. The resulting set of nonlinear coupled equations fulfills all conservation laws including fermion number, linear and angular momenta as well as the total energy. Apart from the conservation laws in the space-time degrees of freedom the Gauss law is conserved as a quantum expectation value identically for all times. The same holds for the Ward identities as generated by commutators of Gauss operators. The constraint dynamical equations are highly non-perturbative and thus applicable also in the strong coupling regime, as e.g. low-energy QCD problems.

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1 Introduction

A lot of problems in QCD at zero temperature, finite temperature, and in the case of non-equilibrium situations require the use of non-perturbative methods. Important examples are the mechanism of confinement, probably related to the vacuum structure [1], the investigation of hadron spectra and hadronic matter, the understanding of the nature of the deconfinement transition and the chiral symmetry restoration. Furthermore, the mechanism of hadronization in relativistic heavy-ion collisions, – following the possible preformation of a quark-gluon plasma – or possible non-perturbative effects in a quark-gluon plasma above the critical temperature cannot be treated perturbatively as well [2]. Besides numerically very involved lattice calculations [3] there are only preliminary attempts to incorporate non-perturbative effects by using the Dyson-Schwinger equation or variational calculations [4] - [7]. In addition, considering the formation, evolution, and decay of a quark-gluon plasma in relativistic heavy-ion collisions, a transport theory based on gauge covariant Wigner functions has been proposed by Elze et al. [8, 9]. It is, in fact, a many-body theory completely equivalent to the Heisenberg equations for the quarks and gluon fields, however, as a reformulation of the SU(N) gauge theory in phase space not manageable in practice. The standard applicable limit is the semi-classical, abelian approximation which corresponds to lowest order perturbation theory in the high temperature approximation. A more promising approach may be provided by the use of equal-time Wigner functions [10].

In this paper we propose a different approach along the line of relativistic two-body correlation dynamics [11]-[15], which has proven to provide the genuine basis for the formulation of non-perturbative transport theories for baryons and mesons [14]. It is already known that this approach obeys all conservation laws concerning space-time degrees of freedom, i.e. fermion number, linear and angular momentum as well as the total energy [14]. The novel phenomena in the application to quarks and gluons is the non-abelian dynamics of the gluon fields as well as a SU(N) internal gauge symmetry.

The paper is organized as follows: In Section 2 we first give a brief reminder of the SU(N) gauge theory in the temporal gauge and introduce the necessary notations. Section 3 is devoted to the derivation of equations of motion and suitable truncation schemes for linked n-point Green functions. In Section 4 we show the conservation of the Gauss law in time (as a quantum expectation value) within the approximation scheme adopted and investigate the higher order Ward identities (as generated by commutators of Gauss operators), whereas Section 5 provides a summary and discussion of open problems. The explicit form of the final constraint dynamical equations is in part shifted to the Appendices in view of their length.
2 SU(N) gauge theories in the temporal gauge

Here we repeat the basic concepts and equations of SU(N) gauge theories [16], which build the starting point for the correlation dynamics in QCD.

We begin with the SU(N) gauge Lagrangian

\[ \mathcal{L} = -\bar{\Psi} \gamma_\mu D_\mu \Psi - \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu}, \]  

(2.1)

where the gauge-covariant derivative \( D_\mu \) is defined as

\[ D_\mu = \partial_\mu - ig T^a A^a_\mu. \]  

(2.2)

The fields \( A^a_\mu \) are the gauge potentials and \( T^a \) are the \( N \times N \) generators of SU(N) following

\[ T^a \dagger = T^a, \]  

(2.3)

\[ \text{Tr} \{ T^a T^b \} = \frac{1}{2} \delta^{ab}, \]  

(2.4)

\[ [T^a, T^b] = if^{abc} T^c, \]  

(2.5)

whereas the field strength tensor is given by

\[ F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc} A^b_\mu A^c_\nu, \]  

(2.6)

or in terms of color electric and magnetic fields as

\[ E^a_j = i F^a_{j0} = -\dot{A}^a_j + i \partial_j A^a_0 + ig f^{abc} A^b_j A^c_0, \]  

(2.7)

\[ B^a_k = \frac{1}{2} \epsilon^{ijk} F^a_{ij} = \frac{1}{2} \epsilon^{ijk} (\partial_i A^a_j - \partial_j A^a_i + gf^{abc} A^b_i A^c_j). \]  

(2.8)

The Euler-Lagrange equations of motion for massless quarks then read

\[ \gamma_\mu D_\mu \Psi = \gamma_\mu (\partial_\mu - ig T^a A^a_\mu) \Psi = 0 \]  

(2.9)

and

\[ \partial_\mu F^a_{\mu\nu} + g(f^{abc} A^b_\mu F^c_{\mu\nu} + I^a_\nu) = 0 \]  

(2.10)

for the gluon fields, where \( I^a_\nu \) is the quark-color current

\[ I^a_\nu = i \bar{\Psi} \gamma_\nu T^a \Psi. \]  

(2.11)

In order to perform actual calculations we have to fix the gauge. In view of the standard Hamiltonian formulation of correlation dynamics we choose a special axial gauge, i.e. the temporal (or Weyl) gauge [17]

\[ A^a_0 = 0, \]  

(2.12)

such that the conjugate field of \( A^a_i \) is (except for a sign) the color electric field

\[ \Pi^a_i = \dot{A}^a_i = -E^a_i. \]  

(2.13)
In the temporal gauge we obtain the canonical quantization conditions

\[ [A^a_i(r, t), \Pi^b_j(r', t)] = i\delta_{ij} \delta^{ab} \delta^3(r - r'), \quad (2.14) \]
\[ \{\Psi(r, t), \Psi^\dagger(r', t)\} = \delta^3(r - r'), \quad (2.15) \]

while all other commutators or anticommutators vanish. The Hamiltonian and its density then is given by \((x = (r, t))\)

\[ H = \int d^3r \mathcal{H}(x), \quad (2.16) \]

with

\[ \mathcal{H}(x) = \frac{1}{2} (\Pi^a_i \Pi^a_i + B^a_i B^a_i) - g I^a_i A^a_i + \bar{\Psi} \gamma_i \partial_i \Psi \quad (2.17) \]

and the Heisenberg equations of motion for \(\Pi^a_i\) read

\[ \dot{\Pi}^a_i = \frac{1}{i} [\Pi^a_i, H] = D^a_{ij} F^c_{ji} + g I^a_i \quad (2.18) \]

with

\[ D^a_{ij} = \delta^{ac} \partial_k + g f^{abc} A^b_k, \quad (2.19) \]

while the equations of motion for \(A^a_i\) reduce to (cf. (2.13))

\[ \dot{A}^a_i = \frac{1}{i} [A^a_i, H] = \Pi^a_i. \quad (2.20) \]

Since the Euler-Lagrange equations with \(\nu = 0\) cannot be derived from the Heisenberg equations of motion, we have to deal with additional constraints on the gauge fields, i.e. the Gauss law \[16, 17\]

\[ g^a(x) = J^a(x) + \bar{\Psi}^\dagger(x) T^a \Psi(x) = 0 \quad (2.21) \]

with

\[ J^a(x) = \frac{1}{g} D^a_{ij} \Pi^c_j. \quad (2.22) \]

In quantum physics (2.21) cannot be fulfilled as an operator equation but has to be imposed as constraints on the physical states \(|p\rangle\),

\[ g^a(x) |p\rangle = 0. \quad (2.23) \]

These equations are similar to Slavnov-Taylor-Ward identities within the temporal gauge, which impose relations between different Green functions \[18\].

The basic idea of correlation dynamics now is the use of dynamical equations of motion for equal time Green functions, which is sufficient to describe the time evolution of the system in a local field theory. For this purpose, we consider the operators \(\hat{G}^{(a, \alpha)}_n(x_1, \ldots, x_n)\), which are products of the gluon-field operator \(A^a_\mu\) and its canonically conjugate momentum \(\Pi^a_\mu\) as well as quark field operators \(\Psi^\dagger\) and \(\Psi\),

\[ \hat{G}^{(a, \alpha)}_n(x_1, \ldots, x_n) = \hat{G}^{(a, \alpha)}_n(A^a_\mu, \Pi^a_\nu, \Psi^\dagger, \Psi), \quad (2.24) \]
where \( \{a, \alpha\} \) denotes a set of color and spinor indices, respectively. \( n \) is the number of field operators and all time arguments are taken to be equal. The equal time Green functions are given by the quantum expectation value within the physical state \(|p\rangle\)

\[
G^{(a,\alpha)}_n(x_1, \cdots, x_n) = \langle p|\hat{G}^{(a,\alpha)}_n|p\rangle
\]  

(2.25)

and the equations of motion for these Green functions follow from the Heisenberg equations

\[
\frac{d}{dt} G^{(a,\alpha)}_n(x_1, \cdots, x_n) = \frac{1}{i} \langle p|\{\hat{G}^{(a,\alpha)}_n, H\}|p\rangle.
\]  

(2.26)

Within the canonical quantization conditions (2.14) and (2.15) equation (2.26) leads to a coupled set of equations for the various \( n \)-point functions, which for practical purposes has to be truncated.

### 3 Equations of motion

The derivation of dynamical equations of motion is straightforward, however, somewhat lengthy. We start from the Heisenberg equations of motion (2.26) for the quantum expectation value of the Green functions (2.25) and first only show the lowest order equations within the notation adopted. We use \( \Psi = \psi_\beta \) with \( a \) denoting the color index and \( \beta \) the spinor index, respectively. For the Dirac matrices \( \gamma_i, \alpha_i = \gamma_0 \gamma_i \) we use the greek indices \( \alpha, \beta \) to specify the spinor components, i.e. \( \gamma^j_{\alpha\beta} = (\gamma^j)_{\alpha\beta}, \alpha^i_{\alpha\beta} = (\alpha^i)_{\alpha\beta} \), and define \( t_{bac} \) by

\[
it_{bac} = (T^b)^{ac}
\]  

(3.1)

where \( b = 1, \ldots, (N^2 - 1) \) runs over all generators, while \( a \) and \( c \) run from 1 to \( N \). This gives for the 2-point quark Green function

\[
i \frac{d}{dt} \langle \bar{u}_a^\alpha(x)u_\alpha^\alpha(x') \rangle = \\
\alpha^i_{\beta\alpha}[\partial_{x_i}\langle \bar{u}_a^\beta(x)u_\alpha^\alpha(x') \rangle - gt_{bac}\langle \bar{u}_a^c(x)u_\alpha^\beta(x')A_b^c(x) \rangle] \\
+\alpha^i_{\alpha\beta}[\partial_{x_i}\langle \bar{u}_a^\alpha(x)u_\alpha^\alpha(x') \rangle - gt_{bac}\langle \bar{u}_a^c(x)u_\alpha^\beta(x')A_b^c(x') \rangle].
\]  

(3.2)

For 1- and 2-point gluon Green functions we get

\[
\frac{d}{dt}\langle A^a_i(x) \rangle = \langle \Pi^a_i(x) \rangle,
\]  

(3.3)

\[
\frac{d}{dt}\langle \Pi^a_i(x) \rangle = \partial_x^2\langle A^a_i(x) \rangle - \partial_{x_i}\partial_{x_k}\langle A^a_{ik}(x) \rangle \\
+2g f^{abcd}\partial_{x_k}\langle A^b_k(x)A^c_i(x) \rangle|_{x=y} + g^2 f^{abcde}\langle A^b_k(x)A^c_d(x)A^e_i(x) \rangle \\
+gf^{abc}\partial_{x_k}\langle A^b_k(x)A^c_i(x) \rangle - \partial_{y_k}\langle A^b_k(x)A^c_i(y) \rangle|_{x=y} \\
+gt_{bac}\langle \bar{u}_a^\alpha(x)u_\alpha^\beta(x) \rangle,
\]  

(3.4)

\[
\frac{d}{dt}\langle A^a_i(x)A^a_{i'}(x') \rangle = \langle \Pi^a_i(x)A^a_{i'}(x') \rangle + \langle A^a_i(x)\Pi^a_{i'}(x') \rangle,
\]  

(3.5)
\[ \frac{d}{dt} \langle \Pi^a_\alpha(x) A^a_{\beta'}(x') \rangle = \langle \Pi^a_\alpha(x) \Pi^a_{\beta'}(x') \rangle + \partial^2_x \langle A^a_k(x) A^a_\beta(x') \rangle \\
- \partial_x \partial_x \langle A^a_k(x) A^a_\beta(x') \rangle + 2g f^{abc} \partial_y \langle A^b_k(x) A^a_\gamma(x) A^a_\beta(x') \rangle |_{y=x} \\
+ g^2 f^{abc} f^{cde} \langle A^b_k(x) A^d_k(x) A^a_\gamma(x) A^a_\beta(x') \rangle \\
+ g f^{abc} \partial_y \langle A^b_k(y) A^a_\gamma(x) A^a_\beta(x') \rangle - \partial_y \langle A^b_k(x) A^a_\gamma(y) A^a_\beta(x') \rangle |_{y=x} \\
+ g t^{abc} \gamma^i_{\alpha \beta} \langle \bar{u}^b_\alpha(x) u^b_\beta(x') A^a_\gamma(x') \rangle \]
and
\[ \frac{d}{dt} \langle \Pi^a_\alpha(x) \Pi^a_{\beta'}(x') \rangle = \partial^2_x \langle A^a_k(x) \Pi^a_{\beta'}(x') \rangle + \partial^2_x \langle \Pi^a_\alpha(x) A^a_\beta(x') \rangle \\
- \partial_x \partial_x \langle A^a_k(x) \Pi^a_{\beta'}(x') \rangle - \partial_x \partial_x \langle \Pi^a_\alpha(x) A^a_\beta(x') \rangle \\
+ 2g f^{abc} \partial_y \langle A^b_k(x) \Pi^a_{\gamma}(y) \Pi^a_{\beta'}(x') \rangle |_{y=x} \\
+ 2g f^{abc} \partial_y \langle \Pi^a_\alpha(x) A^b_k(x') A^a_\gamma(y) \Pi^a_{\beta'}(x') \rangle |_{y=x} \\
+ g^2 f^{abc} f^{cde} \langle A^b_k(x) A^d_k(x') A^a_\gamma(x) A^a_\beta(x') \rangle \\
+ g^2 f^{abc} f^{cde} \langle \Pi^a_\alpha(x) A^b_k(x') A^d_k(x') A^a_\gamma(x') \rangle \\
+ g f^{abc} \partial_y \langle A^b_k(y) A^a_\gamma(x) \Pi^a_{\beta'}(x') \rangle - \partial_y \langle A^b_k(x) A^a_\gamma(y) \Pi^a_{\beta'}(x') \rangle |_{y=x} \\
+ g f^{abc} \partial_y \langle \Pi^a_\alpha(x) A^b_k(x') A^a_\gamma(y) \Pi^a_{\beta'}(x') \rangle - \partial_y \langle \Pi^a_\alpha(x) A^b_k(x') A^a_\gamma(y) \Pi^a_{\beta'}(x') \rangle |_{y=x} \\
+ g t^{abc} \gamma^i_{\alpha \beta} \langle \bar{u}^b_\alpha(x) u^b_\beta(x') \Pi^a_{\gamma}(x') \rangle + g t^{abc} \gamma^i_{\alpha \beta} \langle \Pi^a_\alpha(x) \bar{u}^b_\alpha(x') u^b_\beta(x') \rangle. \] (3.6)

For the lowest order vertices one obtains
\[ i \frac{d}{dt} \langle \bar{u}^b_\alpha(x) u^a_{\alpha'}(x') A^b_\beta(y) \rangle = i \langle \bar{u}^b_\alpha(x) u^a_{\alpha'}(x') \Pi^b_\beta(y) \rangle \\
+ g^k \partial_{z_k} \langle \bar{u}^b_\alpha(x) u^a_{\alpha'}(x') A^b_\beta(y) \rangle - g t^{b a c} \langle \bar{u}^c_\beta(x) u^a_{\alpha'}(x') A^b_\beta(x) A^b_\beta(y) \rangle \\
+ g^k \partial_{z_k} \langle \bar{u}^a_\alpha(x) u^a_{\alpha'}(x') A^b_\beta(y) \rangle - g t^{b a c} \langle \bar{u}^b_\alpha(x) u^a_{\alpha'}(x') A^b_\beta(x) A^b_\beta(y) \rangle, \] (3.8)

\[ i \frac{d}{dt} \langle \bar{u}^b_\alpha(x) u^a_{\alpha'}(x') \Pi^b_\beta(y) \rangle = \\
+ g^k \partial_{z_k} \langle \bar{u}^b_\alpha(x) u^a_{\alpha'}(x') \Pi^b_\beta(y) \rangle - g t^{b a c} \langle \bar{u}^c_\beta(x) u^a_{\alpha'}(x') A^b_\beta(x) \Pi^b_\beta(y) \rangle \\
+ g^k \partial_{z_k} \langle \bar{u}^a_\alpha(x) u^a_{\alpha'}(x') \Pi^b_\beta(y) \rangle - g t^{b a c} \langle \bar{u}^b_\alpha(x) u^a_{\alpha'}(x') A^b_\beta(x) \Pi^b_\beta(y) \rangle \\
+ i (\partial^2_{y} \langle \bar{u}^b_\alpha(x) u^a_{\alpha'}(x') A^b_\beta(y) \rangle - \partial_y \partial_y \langle \bar{u}^b_\alpha(x) u^a_{\alpha'}(x') A^b_\beta(y) \rangle) \\
+ 2g f^{b b c} \partial_{z_k} \langle \bar{u}^b_\alpha(x) u^a_{\alpha'}(x') A^b_\beta(y) A^b_\beta(z) \rangle |_{z=y} \\
+ g^2 f^{b b c} f^{c d e} \langle \bar{u}^b_\alpha(x) u^a_{\alpha'}(x') A^b_\beta(y) A^b_\beta(z) \rangle |_{z=y} \\
+ g f^{b b c} \partial_{z_k} \langle \bar{u}^b_\alpha(x) u^a_{\alpha'}(x') A^b_\beta(y) A^b_\beta(z) \rangle |_{z=y} \\
+ i g t^{b b c} \gamma^i_{\beta \beta} \langle \bar{u}^b_\alpha(x) u^a_{\alpha'}(x') A^b_\beta(y) A^b_\beta(z) \rangle + \frac{d}{dt} \langle \Pi^a_\alpha(x) \Pi^a_{\beta'}(x') \rangle. \] (3.9)

In general the equations of motion for n-point Green functions couple to (n+1)- and (n+2)-point Green functions, as can be seen e.g. from commuting \((\Pi^a_\alpha)^n\) with the cubic and quartic terms of \(A^a_k\) in \(H\), such that the equations of motion form an infinite set that has to be truncated for practical purposes.
A truncation scheme which has been quite successful in the nuclear physics context is based on the cluster expansion of Green functions in terms of connected (correlated) Green functions since the connected Green functions become of minor importance with increasing order \cite{11}. This has been shown explicitly in the nuclear physics context in \cite{19}. A nonperturbative truncation scheme that leads to a resummation of loop and ladder diagrams in infinite order and observes the space-time conservation laws as well as the weak Gauss law (see next Section) is a truncation up to 4-point Green functions.

For the case of ground state Green functions the explicit expressions for the cluster expansions can be derived from the representation of the theory in terms of the generating functionals of full and connected Green functions, \( Z[\bar{\eta}, \eta, J] \) and \( W[\bar{\eta}, \eta, J] \) respectively, which are given by \cite{20}

\[
Z[\bar{\eta}, \eta, J] = \int d\mu \left( \bar{\Psi}, \Psi, A_\mu \right) \exp \left( i \int d^4x \left( L[\bar{\Psi}, \Psi, A_\mu] + \bar{\Psi} \eta + \bar{\eta} \Psi + A_\mu J_\mu \right) \right) \quad (3.10)
\]

and

\[
Z[\bar{\eta}, \eta, J] = \exp(W[\bar{\eta}, \eta, J]) , \quad (3.11)
\]

where ghosts can be omitted according to our choice of gauge. As an example we derive the lowest order expansions in order to present the general concept. We start with the cluster expansions for the time-ordered Green functions with different time arguments:

\[
\langle \mathcal{T} A_i^a(x) A_j^{a'}(x') \rangle = \frac{\delta}{\delta J_i^a(x)} \frac{\delta}{\delta J_j^{a'}(x')} \exp(W[\bar{\eta}, \eta, J]) |_{\bar{\eta} = \eta = J = 0} = \left\{ \left( \frac{\delta}{\delta J_i^a(x)} \frac{\delta}{\delta J_j^{a'}(x')} \right) W[\bar{\eta}, \eta, J] \right\} |_{\bar{\eta} = \eta = J = 0} \exp(W[\bar{\eta}, \eta, J]) \\
= \langle \mathcal{T} A_i^a(x) A_j^{a'}(x') \rangle_c + \langle A_i^a(x) \rangle_c \langle A_j^{a'}(x') \rangle_c ; \quad (3.12)
\]

\[
\langle \mathcal{T} \bar{u}_\alpha^a(x) u_{\alpha'}^{a'}(x') A_i^b(y) \rangle = \frac{\delta}{\delta \bar{\eta}_{\alpha}^a(x)} \frac{\delta}{\delta \eta_{\alpha'}^{a'}(x')} \frac{1}{\bar{i} \delta J_i^b(y)} \exp(W[\bar{\eta}, \eta, J]) |_{\bar{\eta} = \eta = J = 0} \\
= \left\{ \left( \frac{\delta}{\delta \bar{\eta}_{\alpha}^a(x)} \frac{1}{\bar{i} \delta J_i^b(y)} \right) \left( \frac{\delta}{\delta \eta_{\alpha'}^{a'}(x')} \right) W[\bar{\eta}, \eta, J] \right\} |_{\bar{\eta} = \eta = J = 0} \exp(W[\bar{\eta}, \eta, J]) \\
= \langle \mathcal{T} \bar{u}_\alpha^a(x) u_{\alpha'}^{a'}(x') A_i^b(y) \rangle_c + \langle \mathcal{T} \bar{u}_\alpha^a(x) u_{\alpha'}^{a'}(x') \rangle_c \langle A_i^b(y) \rangle_c , \quad (3.13)
\]
where all Green functions containing an unequal number of $u$ and $\bar{u}$ are assumed to vanish. The generalization to Green functions containing conjugate gluon field momenta is straightforward \cite{21}. The expressions for equal time ground state Green functions are obtained by taking the well-defined equal time limit which yields the appropriate operator ordering in the cluster expansions (3.12, 3.13). For the general non-equilibrium case we then simply define the connected Green functions by using the same form of the cluster expansion as in the ground state case.

For the lowest order Green functions one arrives at

\begin{align}
\langle A^a_i(x)A^{a'}_i(x') \rangle &= \langle A^a_i(x) \rangle_c \langle A^{a'}_i(x') \rangle_c + \langle A^a_i(x) A^{a'}_i(x') \rangle_c \tag{3.14} \\
\langle \Pi^a_i(x)A^{a'}_i(x') \rangle &= \langle \Pi^a_i(x) \rangle_c \langle A^{a'}_i(x') \rangle_c + \langle \Pi^a_i(x) A^{a'}_i(x') \rangle_c \tag{3.15} \\
\langle \Pi^a_i(x)\Pi^{a'}_i(x') \rangle &= \langle \Pi^a_i(x) \rangle_c \langle \Pi^{a'}_i(x') \rangle_c + \langle \Pi^a_i(x) \Pi^{a'}_i(x') \rangle_c \tag{3.16} \\
\langle \bar{u}^a_i(x)u^{a'}_i(x')A^b_i(y) \rangle &= \langle \bar{u}^a_i(x)u^{a'}_i(x') \rangle_c \langle A^b_i(y) \rangle_c + \langle \bar{u}^a_i(x)u^{a'}_i(x') A^b_i(y) \rangle_c \tag{3.17} \\
\langle \bar{u}^a_i(x)u^{a'}_i(x')\Pi^b_i(y) \rangle &= \langle \bar{u}^a_i(x)u^{a'}_i(x') \rangle_c \langle \Pi^b_i(y) \rangle_c + \langle \bar{u}^a_i(x)u^{a'}_i(x') \Pi^b_i(y) \rangle_c, \tag{3.18}
\end{align}

where now the time arguments are again taken to be equal. The explicit expressions for the relevant cluster expansions up to sixth order are presented in Appendix A.

In reducing the dynamics to the 4-point level we assume that all correlated terms of order $n \geq 5$ can be neglected, i.e.

\begin{equation}
G^{\{a,a\}}_{nc} = 0 \text{ for } n \geq 5. \tag{3.19}
\end{equation}

Thus the set of equations of motion becomes closed on a finite level.

The equations of motion for the connected Green functions are obtained by inserting the relevant cluster expansions into the equations for the equal time Green functions ((3.2)-(3.9) a.s.f.). As an example we present here the resulting equations of motion for the connected 1-point gluon Green functions,

\begin{equation}
\frac{d}{dt} \langle A^a_i(x) \rangle_c = \langle \Pi^a_i(x) \rangle_c, \tag{3.20}
\end{equation}

\begin{align}
\frac{d}{dt} \langle \Pi^a_i(x) \rangle_c &= \partial_x^2 \langle A^a_i(x) \rangle_c - \partial_{x_i} \partial_{x_k} \langle A^a_i(x) \rangle_c \\
&+ 2gf^{abc} \partial_{y_k} [A^b_k(x)A^c_i(y)]_c + \langle A^b_k(x) \rangle_c \langle A^c_i(y) \rangle_c |_{x=y} \\
&+ g^2f^{abc}f^{cde} [\langle A^b_k(x)A^d_k(x)A^c_i(x) \rangle_c \\
&+ \langle A^b_k(x) \rangle_c \langle A^d_k(x)A^c_i(x) \rangle_c + \langle A^d_k(x) \rangle_c \langle A^b_k(x) \rangle_c \langle A^c_i(x) \rangle_c] \\
&+ gf^{abc} \partial_{y_k} [\langle A^b_k(y) \rangle_c \langle A^c_i(x) \rangle_c |_{x=y}] \\
&- gf^{abc} \partial_{y_i} [\langle A^b_k(x) \rangle_c \langle A^c_i(y) \rangle_c |_{x=y}] \\
&+ g\gamma_{\alpha\beta} \langle u^a_\alpha(x)u^b_\beta(x) \rangle_c, \tag{3.21}
\end{align}
and the equations of motion for the connected 2-point gluon Green functions,

\[
\frac{d}{dt}(A_i^q(x)A_k^q'(x'))_c = \langle \Pi_i^q(x)A_k^q'(x') \rangle_c + \langle A_i^q(x)\Pi_k^q(x') \rangle_c, \tag{3.22}
\]

\[
\frac{d}{dt}(\Pi_i^q(x)A_k^q'(x'))_c = \langle \Pi_i^q(x)\Pi_k^q(x') \rangle_c
+ \partial_x^2(A_i^q(x)A_k^q'(x'))_c - \partial_x\partial_y(A_i^q(x)A_k^q'(x'))_c
c
+2g f^{abc}\partial_y[A_i^q(x)A_k^q(x)A_l^q(x)](A_i^q(y)_c)
+(A_i^q(y)A_i^q(x)A_k^q(x'))_c + (A_i^q(x)A_k^q(y)A_k^q(x'))_c \rangle_{x=y}
\]

\[
+g f^{abc}f^{fde}[(A_k^d(x)A_k^q(x)A_l^q(x)](A_i^q(x)A_k^q(x'))_c
+(1 + T_{cd} + T_{eb}) \{ (A_k^d(x)_c)(A_i^q(x))_c(A_l^q(x))_c
+(A_k^d(x)A_k^q(x))_c(A_i^q(x))_c + (A_k^d(x)A_k^q(x))_c \rangle_{x=y}
\]

\[
-g f^{abc}\partial_y[A_i^q(x)A_k^q(x')_c(A_k^q(y)_c)
+(A_i^q(y)A_i^q(x)A_k^q(x'))_c + (A_i^q(x)A_k^q(y)A_k^q(x'))_c \rangle_{x=y}
\]

\[
+g f^{abc}\partial_y[A_i^q(x)A_k^q(x')_c(A_i^q(x)A_k^q(x'))_c,
\tag{3.23}
\]

\[
\frac{d}{dt}(\Pi_i^q(x)\Pi_k^q'(x'))_c = (1 + P_{ii'})[\partial_x^2(\Pi_i^q(x)\Pi_k^q(x'))_c - \partial_x\partial_y(\Pi_i^q(x)\Pi_k^q(x'))_c]
\]

\[
+2g (1 + P_{ii'})f^{abc}\partial_y[A_i^q(x)\Pi_k^q(x')_c(A_i^q(y)_c)
+(A_i^q(y)A_i^q(x)\Pi_k^q(x'))_c + (A_i^q(x)A_i^q(y)\Pi_k^q(x'))_c \rangle_{x=y}
\]

\[
+g^2 (1 + P_{ii'})f^{abc}f^{fde}[(A_k^d(x)A_k^q(x)A_l^q(x)](A_i^q(x)\Pi_k^q(x'))_c
+(1 + T_{cd} + T_{eb}) \{ (A_k^d(x)_c)(A_i^q(x))_c(\Pi_k^q(x'))_c
+(A_k^d(x)A_k^q(x))_c(A_i^q(x))_c + (A_k^d(x)A_k^q(x))_c \rangle_{x=y}
\]

\[
+g (1 + P_{ii'})f^{abc}\partial_y[A_i^q(x)\Pi_k^q(x')_c(A_i^q(x)_c)
+(A_i^q(x)A_i^q(x)\Pi_k^q(x'))_c + (A_i^q(x)A_i^q(y)\Pi_k^q(x'))_c \rangle_{x=y}
\]

\[
-g (1 + P_{ii'})f^{abc}\partial_y[A_i^q(x)\Pi_k^q(x')_c(A_i^q(x)_c)
\]
In (3.23) and (3.24) we have introduced two kinds of permutation operators \( P_{iv'} \) and \( T_{ed} \) in order to achieve an unambiguous compactification. To demonstrate the action of these operators we look as an example at parts of the \( g^2 \)-terms in (3.24)

\[
g^2 (1 + P_{iv'}) f^{abc} f^{cde} (1 + T_{ed} + T_{eb}) \langle A^b_k(x) \rangle_c \langle A^d_k(x) \rangle_c \langle A^c_k(x) \rangle_{\Pi^f_i(x')} \]

\[
= g^2 (1 + P_{iv'}) f^{abc} f^{cde} [\langle A^b_k(x) \rangle_c \langle A^d_k(x) \rangle_c \langle A^c_k(x) \rangle_{\Pi^f_i(x')} + \langle A^b_k(x) \rangle_c \langle A^d_k(x) \rangle_c \langle A^c_k(x) \rangle_{\Pi^f_i(x')} + \langle A^b_k(x) \rangle_c \langle A^d_k(x) \rangle_c \langle A^c_k(x) \rangle_{\Pi^f_i(x')}]
\]

\[
= g^2 f^{abc} f^{cde} [\langle A^b_k(x) \rangle_c \langle A^d_k(x) \rangle_c \langle A^c_k(x) \rangle_{\Pi^f_i(x')} + \langle A^b_k(x) \rangle_c \langle A^d_k(x) \rangle_c \langle A^c_k(x) \rangle_{\Pi^f_i(x')} + \langle A^b_k(x) \rangle_c \langle A^d_k(x) \rangle_c \langle A^c_k(x) \rangle_{\Pi^f_i(x')}]
\]

\[
= g^2 f^{abc} f^{cde} [\langle A^b_k(x) \rangle_c \langle A^d_k(x) \rangle_c \langle A^c_k(x) \rangle_{\Pi^f_i(x')} + \langle A^b_k(x) \rangle_c \langle A^d_k(x) \rangle_c \langle A^c_k(x) \rangle_{\Pi^f_i(x')} + \langle A^b_k(x) \rangle_c \langle A^d_k(x) \rangle_c \langle A^c_k(x) \rangle_{\Pi^f_i(x')}]
\]

\[
= g^2 f^{abc} f^{cde} [\langle A^b_k(x) \rangle_c \langle A^d_k(x) \rangle_c \langle A^c_k(x) \rangle_{\Pi^f_i(x')} + \langle A^b_k(x) \rangle_c \langle A^d_k(x) \rangle_c \langle A^c_k(x) \rangle_{\Pi^f_i(x')} + \langle A^b_k(x) \rangle_c \langle A^d_k(x) \rangle_c \langle A^c_k(x) \rangle_{\Pi^f_i(x')}]
\]

Obviously \( P_{iv'} \) and \( T_{ed} \) act in the same way by interchanging the fields or conjugate momenta labeled by the spatial components \( i, i' \) (in case of \( P_{iv'} \)) or by the color indices \( c, d \) (in case of \( T_{ed} \)). Due to their length we have shifted the remaining equations of motion for the connected 3- and 4-point gluon Green functions to Appendix B.

For the connected 2- and 4-point quark Green functions we obtain

\[
i \frac{d}{dt} \langle \bar{u}_\alpha^a(x) u_{\alpha'}^a(x') \rangle_c =
\]

\[
\alpha_{i'\beta} \partial_{i'} \langle \bar{u}^a_{\beta}(x) u_{\alpha'}^a(x') \rangle_c - g t^{bac} \{ \langle \bar{u}_{\beta}^a(x) u_{\alpha'}^a(x') A_{\gamma}^{b}(x) \rangle_c + \langle \bar{u}_{\beta}^a(x) u_{\alpha'}^a(x') \rangle_c \langle A_{\gamma}^{b}(x) \rangle_c \}
\]

\[
+ \alpha_{i'\beta} \partial_{i'} \langle \bar{u}_{\alpha}^a(x) u_{\beta}^a(x') \rangle_c - g t^{bde} \{ \langle \bar{u}_{\alpha}^a(x) u_{\beta}^a(x') A_{\gamma}^{b}(x) \rangle_c + \langle \bar{u}_{\alpha}^a(x) u_{\beta}^a(x') \rangle_c \langle A_{\gamma}^{b}(x) \rangle_c \}
\]

(3.26)
which represents all equations on the pure quark sector.

The equations of motion for the connected 3-point quark-gluon vertex Green functions are given by

\[
\begin{align*}
\frac{i}{\hbar} \frac{d}{dt} \langle u^{a}_{\alpha}(x) u^{\alpha'}_{\alpha'}(x') A^{b}_{i}(y) \rangle_{c} &= i \langle \bar{u}^{a}_{\beta}(x) u^{\alpha'}_{\alpha'}(x') \Pi^{b}_{i}(y) \rangle_{c} \\
&+ \alpha_{\beta}^{k} [\partial_{x k} \langle \bar{u}^{a}_{\beta}(x) u^{\alpha'}_{\alpha'}(x') A^{b}_{i}(y) \rangle_{c} - g^{b a c} \{ \langle \bar{u}^{a}_{\beta}(x) u^{\alpha'}_{\alpha'}(x') A^{b}_{k}(x) A^{b}_{i}(y) \rangle_{c} \\
&+ \langle \bar{u}^{a}_{\beta}(x) u^{\alpha'}_{\alpha'}(x') \Pi^{b}_{k}(x) \rangle_{c} \{ A^{b}_{i}(x) \Pi^{b}_{i}(y) \} \} ] \\
&\quad + \alpha_{\alpha'}^{k} \partial_{x k} \langle \bar{u}^{a}_{\alpha'}(x) u^{\alpha'}_{\alpha'}(x') A^{b}_{i}(y) \rangle_{c} - g^{b a c} \{ \langle \bar{u}^{a}_{\alpha'}(x) u^{\alpha'}_{\alpha'}(x') A^{b}_{k}(x) A^{b}_{i}(y) \rangle_{c} \\
&+ \langle \bar{u}^{a}_{\alpha'}(x) u^{\alpha'}_{\alpha'}(x') \Pi^{b}_{i}(x) \rangle_{c} \{ A^{b}_{i}(x) \Pi^{b}_{i}(y) \} \} ] \\
&\quad + i \partial_{y} \langle \bar{u}^{a}_{\alpha'}(x) u^{\alpha'}_{\alpha'}(x') A^{b}_{i}(y) \rangle_{c} - i \partial_{y} \partial_{x k} \langle \bar{u}^{a}_{\alpha'}(x) u^{\alpha'}_{\alpha'}(x') A^{b}_{k}(y) \rangle_{c} \\
&\quad + i 2 g f^{b d c} \partial_{x k} [(\bar{u}^{a}_{\alpha'}(x) u^{\alpha'}_{\alpha'}(x') A^{b}_{i}(y) A^{b}_{i}(z)]_{c} \\
&\quad + \{ \bar{u}^{a}_{\alpha'}(x) u^{\alpha'}_{\alpha'}(x') A^{b}_{i}(y) \} \{ A^{b}_{i}(y) A^{b}_{i}(y) \} ] \\
&\quad + i g^{2} f^{b d c} f^{c d e} (1 + T_{0 d i} + T_{0 c e}) \{ \langle \bar{u}^{a}_{\alpha'}(x) u^{\alpha'}_{\alpha'}(x') A^{b}_{k}(y) A^{b}_{i}(y) \rangle_{c} \\
&\quad + \langle \bar{u}^{a}_{\alpha'}(x) u^{\alpha'}_{\alpha'}(x') A^{b}_{k}(y) \rangle_{c} \{ A^{b}_{i}(y) A^{b}_{i}(y) \} ] \\
&\quad + i g f^{b c d} \partial_{x k} [(\bar{u}^{a}_{\alpha'}(x) u^{\alpha'}_{\alpha'}(x') A^{b}_{i}(y) A^{b}_{i}(z))_{c} \\
&\quad + \{ \bar{u}^{a}_{\alpha'}(x) u^{\alpha'}_{\alpha'}(x') A^{b}_{k}(z) \} \{ A^{b}_{i}(y) A^{b}_{i}(y) \} ]_{(y-z)} \\
&\quad - i g f^{b d c} \partial_{x} [(\bar{u}^{a}_{\alpha'}(x) u^{\alpha'}_{\alpha'}(x') A^{b}_{k}(y) A^{b}_{i}(z))_{c} \\
&\quad + \{ \bar{u}^{a}_{\alpha'}(x) u^{\alpha'}_{\alpha'}(x') A^{b}_{k}(z) \} \{ A^{b}_{i}(y) A^{b}_{i}(y) \} ]_{(y-z)} \\
&\quad + \{ \bar{u}^{a}_{\alpha'}(x) u^{\alpha'}_{\alpha'}(x') A^{b}_{i}(y) A^{b}_{i}(z) \} \{ A^{b}_{i}(y) A^{b}_{i}(y) \} ]_{(y-z)} \quad (3.27)
\end{align*}
\]
\[ +igt^{\alpha'\beta'} \gamma_i^{\alpha'\beta'} \delta(x' - y) \langle \bar{u}^a_\alpha(x) u^c_\beta(y) \rangle_c \]
\[ +igt^{\alpha'\beta'} \gamma_i^{\alpha'\beta'} \langle [\bar{u}^a_\alpha(x) \bar{u}^b_\beta(y) u^c_\beta'(x')]_c - \langle \bar{u}^a_\alpha(x) u^c_\beta(y) \rangle_c \langle \bar{u}^b_\beta(y) u^c_\beta'(x') \rangle_c \] \tag{3.29}

while the explicit equations of motion for the connected 4-point mixed quark and gluon Green functions are shifted to Appendix B again.

The closed set of equations (3.20) - (3.24) and (3.26) - (3.29) including (B.1) to (B.12) will be denoted as Constraint QCD (CQCD) equations furtheron.

4 Compatibility with Gauss law and Ward identities

The aim of the present section is to show the compatibility of the CQCD equations with the Gauss law and the Ward identities, respectively (cf. [22]). For this purpose, we first note that the operators \( g^a(x) \) at equal times constitute a set of local generators for the SU(N) gauge group because of \[ [g^a(r; t), g^b(r', t)] = if^{abc} g^c(r, t) \delta^b(r - r'). \] \tag{4.1}

Furthermore, the Gauss law operator commutes with the Hamiltonian \[ [g^a(r, t), H] = 0 \] \tag{4.2}

which expresses the conservation of \( g^a \) in time

\[ \frac{d}{dt} g^a(r, t) = \frac{1}{i} [g^a(r, t), H] = 0. \] \tag{4.3}

Equations (4.2) and (4.3) imply that within the temporal gauge the system evolved in time by \( H \) has a residual gauge symmetry and the Gauss operators \( g^a(r, t) \) are the local generators of the residual gauge symmetry.

Now we assume the actual realization of the Gauss law as a quantum expectation value

\[ \langle p | g^a(r, t) | p \rangle = 0 \] \tag{4.4}

instead of (2.23). This may be considered as a weak form of the Gauss law, since it is less restrictive for the physical states than (2.23). For example, the perturbative vacuum \( |0 \rangle \) fulfills (4.4) but not (2.23) [23]. Also (4.4) allows for the local propagation of colored objects, whereas (2.23) restricts to color-singlet objects. These consequences of (4.4) are exactly what we aim at for describing properties of the quark-gluon plasma.

Furthermore, since all \( g^a(x) \) are conserved quantities and the field equations of motion are generated by \( H \), the weak Gauss law is conserved identically in time provided that it is fulfilled initially. This is just what we have been aiming at: The Hamiltonian dynamics are compatible with the conservation of the weak Gauss law, and the problem is shifted to an initial value problem. This statement still holds within 4-point
correlation dynamics, since the Gauss law operator contains two-point operators at most.

Whereas the weak Gauss law is a constraint imposed on the lowest order Green functions of the fields by the residual gauge symmetry, the Ward identities impose constraints on higher order Green functions due to the same residual gauge symmetry.

We start defining a Lie operation \( L_{g^a(r,t)} \) by

\[
L_{g^a(r,t)} g^b(r',t) =: [g^a(r,t), g^b(r',t)]. \tag{4.5}
\]

From the algebraic structure of \( g^a(r,t) \) (4.1) we get

\[
L_{g^a_1(r_1,t)} L_{g^a_2(r_2,t)} g_{a_3}(r_3,t) = (i)^2 f^{a_1 a_2 a_3} (r_2 - r_3) \delta^a_a (r_1 - r_2) g^c(r_1,t), \tag{4.6}
\]

or

\[
L_{g^a_1(r_1,t)} L_{g^a_2(r_2,t)} \cdots L_{g^{a_{n-1}}(r_{n-1},t)} g^{a_n}(r_n,t) = F^{a_1 \cdots a_{n+1}}(r_1, \cdots, r_n) g^{a_{n+1}}(r_1,t), \tag{4.7}
\]

where \( F^{a_1 \cdots a_{n+1}}(r_1, \cdots, r_n) \) is a complex function of \( \{r_1, \cdots, r_n\} \). Thus we obtain from (4.4)

\[
\langle p | [g^{a_1}(r_1,t), g^{a_2}(r_2,t)] | p \rangle = 0 \tag{4.8}
\]

\[
\langle p | L_{g^{a_1}(r_1,t)} L_{g^{a_2}(r_2,t)} g^{a_3}(r_3,t) | p \rangle = 0 \tag{4.9}
\]

and

\[
\langle p | L_{g^{a_1}(r_1,t)} L_{g^{a_2}(r_2,t)} \cdots L_{g^{a_{n-1}}(r_{n-1},t)} g^{a_n}(r_n,t) | p \rangle = 0. \tag{4.10}
\]

The relation between (4.8), (4.9), (4.10) and gauge invariance can be established as follows. Since \( H \) due to the temporal gauge has a residual gauge symmetry generated by the local algebra \( \{g^a(r,t)\} \), the residual group \( U(g) \) will generate a set \( \mathcal{H}_{gp} \) of physical states \( |p\rangle_g \) from any physical state \( |p\rangle \)

\[
|p\rangle_g = U(g)|p\rangle; \text{ for } |p\rangle_g \in \mathcal{H}_{pg}, \tag{4.11}
\]

where the residual group element is given by

\[
U(g) = \exp \left( i \int d^3r \ g^b(r) \omega^b \right), \tag{4.12}
\]

where \( \omega^b = \omega^b(r) \) are the local gauge transformation angles. The state \( |p\rangle_g \) has the same energy as \( |p\rangle \) since

\[
g \langle p | H | p \rangle_g = \langle p | U^{-1}(g) H U(g) | p \rangle = \langle p | H | p \rangle. \tag{4.13}
\]

The weak Gauss law is fulfilled as well

\[
g \langle p | g^a | p \rangle_g = \langle p | \exp(-i \int g^b \omega^b d^3r) g^a \exp(i \int g^c \omega^c d^3r) | p \rangle
= \langle p | U_{ab} g^b | p \rangle = 0, \tag{4.14}
\]

where \( U_{ab} \) is a color matrix containing the gauge transformation angles.
From the theory of Lie groups it is evident that (4.10) and (4.14) are equivalent. Furthermore, since \(U(g)\) generates a gauge transformation for the \(SU(N)\) gauge fields, any gauge invariant physical observable \(O\) has the same expectation value within the set \(\mathcal{H}_{pg}\).

Since the weak Gauss law is conserved in time within a 4-point truncation scheme, all Ward identities which follow from the group properties (1.1) of the Gauss law operators in combination with the weak Gauss law, such as (1.8), (1.9) and (1.11), will also be conserved. This is due to the fact that correlation dynamics in general conserves the commutation and anticommutation relations of all field operators within Green functions, which leads to a conservation of the expectation values of all operator equations generated by these relations. The problem of the Ward identities thus is again shifted to an initial value problem.

5 Summary and discussion

In this work we have derived a closed set of equations of motion for connected Green functions of quark and \(SU(N)\) gluon fields up to the 4-point level. The resulting set of equations, which we for abbreviation will denote by Constraint \textit{QCD} (CQCD) equations furtheron, fulfill the conservation of linear momentum, angular-momentum and total energy throughout time as well as the weak Gauss law and the Ward identities - as generated by commutators of Gauss operators - provided that the initial conditions follow the gauge constraints. The latter may be achieved, for example, by a functional containing the chromomagnetic field (2.8), which obeys the Gauss law [24]. It might be used as a trial wave function in a variational calculation within a finite number of basis states. Alternatively, one might start from perturbative gluon-field configurations and increase the coupling \(g\) from zero adiabatically for the preparation of an initial state [1].

We note that the CQCD equations can easily be transformed into transport equations by means of a Wigner transformation with respect to the space-time degrees of freedom, which is nothing but a unitary transformation. Since the latter transformation has been performed quite often in the literature, we do not present the final expressions for reasons of missing compactness.

The actual goal is a numerical integration of the CQCD equations within a finite basis set in analogy to the works performed in the nuclear physics context [25] - [29] which appear to quite naturally explain a variety of non-perturbative nuclear phenomena [30]. Though the actual expressions in Appendix B appear very cumbersome, we note that the reduction of the quark-gluon many-body problem – as e.g. studied by lattice QCD in thermal equilibrium – to a nonlinear two-body problem provides a numerical task of lower complexity than the original QCD equations on the lattice. Furthermore, the CQCD equations allow to study the dynamical evolution of non-equilibrium configurations of quarks and gluons and to explore the low-energy QCD dynamics, especially the linear response of systems close to the ground state. However, it is not clear if the actual solutions of our approach will reproduce all the physical phenomena of the full

\[^1\text{Note that the perturbative vacuum } |0\rangle \text{ is a physical state in our realization of the gauge constraint because it fulfills (4.4).}\]
QCD equations. Before speculating about the convergence of the CQCD equations we prefer to analyse the numerical results first. Such work is in progress.

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Appendix

A Cluster expansions

By taking the functional derivative of the generating functional (3.11) with respect to the different sources (cf. Sect. 3) we obtain the relevant cluster expansions of Green functions up to sixth order in the gluon, quark, and mixed sector of the SU(N) gauge theory. For reasons of compactness we introduce the general particle exchange operators

\[ S^k_l = (1 + \sum_{n=l}^k [P_{1n} + P_{2n}] + \sum_{n=l}^{k-1} \sum_{j=n+1}^k P_{1n}P_{2j}), \quad (A.1) \]

where \( P_{ij} \) is the two-body permutation operator. Here \( i \) and \( j \) represent the set of quantum numbers characterizing the gauge fields, the corresponding conjugate momenta or the quark spinors.

We note that any application of the \( P_{ij} \) operators has to ensure the original order of the field operators within the connected Green functions.

A.1 Gluon sector

In the following the integer numbers between brackets stand for gauge fields \( A_i^a \) or conjugate momenta \( \Pi_i^a \). The cluster expansions then can be written in compact form as:

\[ \langle 1 \rangle = \langle 1 \rangle_c, \quad (A.2) \]

\[ \langle 12 \rangle = \langle 12 \rangle_c + \langle 1 \rangle \langle 2 \rangle, \quad (A.3) \]

\[ \langle 123 \rangle = \langle 123 \rangle_c + (1 + \sum_{n=2}^3 P_{1n}) \langle 1 \rangle \langle 23 \rangle_c + \langle 1 \rangle \langle 2 \rangle \langle 3 \rangle, \quad (A.4) \]

\[ \langle 1234 \rangle = \langle 1234 \rangle_c + (1 + \sum_{n=2}^4 P_{1n}) \langle 1 \rangle \langle 234 \rangle_c + (1 + \sum_{n=3}^4 P_{2n}) \langle 12 \rangle_c \langle 34 \rangle_c + S^4_3 \langle 1 \rangle \langle 2 \rangle \langle 34 \rangle_c + \langle 1 \rangle \langle 2 \rangle \langle 3 \rangle \langle 4 \rangle, \quad (A.5) \]

\[ \langle 12345 \rangle = (1 + \sum_{n=2}^5 P_{1n}) \langle 1 \rangle \langle 2345 \rangle_c + S^5_3 \langle 12 \rangle_c \langle 345 \rangle_c + \langle 1 \rangle \langle 2 \rangle \langle 345 \rangle_c + \langle 1 \rangle \langle 2 \rangle \langle 3 \rangle \langle 4 \rangle \langle 5 \rangle + \langle 1 \rangle \langle 2 \rangle \langle 3 \rangle \langle 4 \rangle \langle 5 \rangle, \quad (A.6) \]
\begin{align}
\langle 123456 \rangle &= (1 + \sum_{n=4}^{6} \sum_{j=1}^{3} P_{jn})[(123)\langle 456 \rangle_c + (1 + P_{14}P_{25}P_{36})\langle 123 \rangle_c\langle 4 \rangle_c\langle 5 \rangle_c\langle 6 \rangle_c] \\
&+ S_3^6[(12)\langle 3456 \rangle_c + \langle 1 \rangle_c\langle 2 \rangle_c\langle 3456 \rangle_c + \langle 12 \rangle_c\langle 3 \rangle_c\langle 4 \rangle_c\langle 5 \rangle_c\langle 6 \rangle_c] \\
&+ S_3^5(1 + \sum_{n=5}^{6} P_{4n})(1\langle 2 \rangle_c\langle 34 \rangle_c\langle 56 \rangle_c + (1 + \sum_{n=1}^{5} P_{6n})S_3^5\langle 12 \rangle_c\langle 345 \rangle_c\langle 6 \rangle_c \\
&+ (1 + \sum_{n=1}^{4} P_{5n})(1 + \sum_{n=3}^{4} P_{2n})\langle 12 \rangle_c\langle 34 \rangle_c\langle 56 \rangle_c + \langle 1 \rangle_c\langle 2 \rangle_c\langle 3 \rangle_c\langle 4 \rangle_c\langle 5 \rangle_c\langle 6 \rangle_c]. \tag{A.7}
\end{align}

The number of terms for the 1-, 2-, ..., 5-, and 6-point gluon Green functions consist out of 1, 2, 5, 15, 52, and 203 different contributions, respectively. Due to the applied truncation scheme we have neglected the connected 5-point function \( \langle 12345 \rangle_c \) in (A.6) and the terms \( (1 + \sum_{n=2}^{6} P_{1n})\langle 1 \rangle_c\langle 23456 \rangle_c \) and \( \langle 123456 \rangle_c \) in (A.7).

### A.2 Quark sector

The argument \( i \) of a quark spinor \( u(i) \) in this subsection stands for the spatial coordinates and the Dirac, flavor and color indices of a quark \( i \). The cluster expansions then can be written as:

\begin{align}
\langle \bar{u}(1)u(1') \rangle &= \langle \bar{u}(1)u(1') \rangle_c, \tag{A.8} \\
\langle \bar{u}(1)\bar{u}(2)u(2')u(1') \rangle &= \langle \bar{u}(1)\bar{u}(2)u(2')u(1') \rangle_c + (1 - P_{1'2'})\langle \bar{u}(1)u(1') \rangle_c\langle \bar{u}(2)u(2') \rangle_c, \tag{A.9} \\
\langle \bar{u}(1)\bar{u}(2)\bar{u}(3)u(2')u(1') \rangle &= \langle \bar{u}(1)\bar{u}(2)\bar{u}(3)u(2')u(1') \rangle_c \\
&= (1 - P_{1'2'} - P_{1'3'} - P_{2'3'} + P_{23}(P_{1'3'} + P_{1'2'}))\langle \bar{u}(1)u(1') \rangle_c\langle \bar{u}(2)u(2') \rangle_c\langle \bar{u}(3)u(3') \rangle_c \\
&+ (1 - P_{1'2'} - P_{1'3'})\langle \bar{u}(1)u(1') \rangle_c\langle \bar{u}(2)\bar{u}(3)u(3')u(2') \rangle_c \\
&+ (1 - P_{1'3'} - P_{2'3'})\langle \bar{u}(2)u(2') \rangle_c\langle \bar{u}(1)\bar{u}(3)u(3')u(1') \rangle_c \\
&+ (1 - P_{1'3'} - P_{2'3'})\langle \bar{u}(3)u(3') \rangle_c\langle \bar{u}(1)\bar{u}(2)u(2')u(1') \rangle_c. \tag{A.10}
\end{align}

The number of terms for the 2-, 4-, and 6-point quark Green functions consist out of 1, 3, and 16 contributions, respectively. Due to the truncation on the 4-point level we have neglected the connected 6-point function \( \langle \bar{u}(1)\bar{u}(2)\bar{u}(3)u(3')u(2')u(1') \rangle_c \) in (A.10).

### A.3 Mixed sector

In the mixed sector of the SU(N) gauge theory the Green functions contain quark and gluon field operators simultaneously. Here the \( u(i) \) represent the quark spinors while the numbers 2, 3, 4, and 5 stand for gauge fields or conjugate momenta as in Subsection A.1. Since the explicit cluster expansions in the gluon sector are presented in (A.2) - (A.7) we have not specified the expansions of the pure gluonic Green functions in (A.11) - (A.14). The expansions up to the 4-point connected level read:

\begin{align}
\langle \bar{u}(1)u(1')2 \rangle &= \langle \bar{u}(1)u(1')2 \rangle_c + \langle \bar{u}(1)u(1') \rangle_c\langle 2 \rangle, \tag{A.11}
\end{align}
\[ \langle \bar{u}(1)u(1')23 \rangle = \langle \bar{u}(1)u(1')23 \rangle_c + \langle \bar{u}(1)u(1') \rangle_c (23) + (1 + \mathcal{P}_{23})\langle \bar{u}(1)u(1')23 \rangle_c (3), \quad (A.12) \]

\[ \langle \bar{u}(1)u(1')234 \rangle = \langle \bar{u}(1)u(1') \rangle_c (234) + (1 + \mathcal{P}_{23} + \mathcal{P}_{24})\langle \bar{u}(1)u(1')2 \rangle_c (34) + (1 + \mathcal{P}_{24} + \mathcal{P}_{34})\langle \bar{u}(1)u(1')23 \rangle_c (4), \quad (A.13) \]

\[ \langle \bar{u}(1)u(1')2345 \rangle = \langle \bar{u}(1)u(1') \rangle_c (2345) + (1 + \sum_{n=3}^{5} \mathcal{P}_{2n})\langle \bar{u}(1)u(1')2 \rangle_c (345) + (1 + \sum_{n=4}^{5} \mathcal{P}_{2n} + \mathcal{P}_{24}\mathcal{P}_{35})\langle \bar{u}(1)u(1')23 \rangle_c (45), \quad (A.14) \]

\[ \langle \bar{u}(1)\bar{u}(2)u(2')u(1')3 \rangle = \langle \bar{u}(1)\bar{u}(2)u(2')u(1') \rangle_c (3) + (1 - \mathcal{P}_{12'})\langle \bar{u}(1)u(1')c\rangle (23) + \langle \bar{u}(1)u(1')3 \rangle_c \langle \bar{u}(2)u(2') \rangle_c + \langle \bar{u}(1)u(1')c\rangle (2345), \quad (A.15) \]

\[ \langle \bar{u}(1)\bar{u}(2)u(2')u(1')34 \rangle = \langle \bar{u}(1)\bar{u}(2)u(2')u(1') \rangle_c (34) + (1 - \mathcal{P}_{12'})\langle \bar{u}(1)u(1')3 \rangle_c (2345) + (1 + \sum_{n=3}^{5} \mathcal{P}_{2n})\langle \bar{u}(1)u(1')3 \rangle_c (2345) + (1 + \mathcal{P}_{34})\langle \bar{u}(1)\bar{u}(2)u(2')u(1')3 \rangle_c (345) + (1 + \mathcal{P}_{34})\langle \bar{u}(1)\bar{u}(2)u(2')u(1')3 \rangle_c (45) \quad (A.16) \]

The original cluster expansions \((A.11) - (A.16)\) consist out of 2, 5, 15, 52, 8, and 17 terms. Because of the truncation scheme on the 4-point level we have neglected \(\langle \bar{u}(1)u(1')234 \rangle_c\), \(\langle \bar{u}(1)u(1')2345 \rangle_c\), and \((1 + \sum_{n=3}^{5} \mathcal{P}_{2n})\langle \bar{u}(1)u(1')345 \rangle_c (2)\) in \((A.13)\), \(\langle \bar{u}(1)\bar{u}(2)u(2')u(1')3 \rangle_c\) in \((A.13)\), and \(\langle \bar{u}(1)\bar{u}(2)u(2')u(1')34 \rangle_c\) and \((1 + \mathcal{P}_{34})\langle \bar{u}(1)\bar{u}(2)u(2')u(1')3 \rangle_c (4)\) in \((A.16)\).

## B Equations of motion for connected Green functions

### B.1 Gluon sector

Whereas the time evolution of the 1-point and 2-point gluon Green functions is given in \((B.20) - (B.24)\) we here continue with the 3- and 4-point functions. In order to allow for a compact presentation we again use the permutation operators \(\mathcal{P}_{ii'}\) and \(\mathcal{T}_{ed}\) as defined in Section 3. The equations of motion for the connected 3-point gluon Green functions read:

\[
\frac{d}{dt}(A_i^a(x)A_i^{a'}(x')A_i^{a''}(x''))_c = \langle \Pi_i^a(x)A_i^{a'}(x')A_i^{a''}(x'') \rangle_c \\
+ \langle A_i^a(x)\Pi_i^{a'}(x')A_i^{a''}(x'') \rangle_c + \langle A_i^a(x)A_i^{a'}(x')\Pi_i^{a''}(x'') \rangle_c, \quad (B.1)
\]
\[
\frac{d}{dt} \langle \Pi^a_\gamma(x) A^\gamma_\delta(x') A^{\alpha''}_\nu(x'') \rangle_c = \langle \Pi^a_\gamma(x) \Pi^\alpha_\delta(x') A^{\alpha''}_\nu(x'') \rangle_c + \langle \Pi^a_\gamma(x) A^\gamma_\delta(x') \Pi^{\alpha''}_\nu(x'') \rangle_c
\]
\[
+ \partial^2_x \langle A^\gamma_\delta(x') A^{\alpha''}_\nu(x'') \rangle_c - \partial_x \partial_{x_k} \langle A^\gamma_\delta(x') A^{\alpha''}_\nu(x'') \rangle_c
\]
\[
+ 2g f^{abc} \partial_{y_i} \langle A^a_k(x) A^b_l(y) A^c_k(x') A^{\alpha''}_\nu(x'') \rangle_c
\]
\[
+ (1 + \mathcal{T}_{eb}) \{ \langle A^a_k(x) A^b_l(x') A^{\alpha''}_\nu(x'') \rangle_c \langle A^c_k(y) \rangle_c
\]
\[
+ (1 + \mathcal{T}_{d' a''}) \langle A^a_k(x) A^b_l(x') \rangle_c \langle A^c_k(y) A^{\alpha''}_\nu(x'') \rangle_c \} |_{x=y}
\]
\[
+ g^2 f^{abc} \langle A^a_k(x) A^b_l(x') A^{\alpha''}_\nu(x'') \rangle_c
\]
\[
+ (1 + \mathcal{T}_{eb}) \{ \langle A^a_k(x) A^b_l(x') A^{\alpha''}_\nu(x'') \rangle_c \langle A^c_k(y) \rangle_c
\]
\[
+ (1 + \mathcal{T}_{d' a''}) \langle A^a_k(x) A^b_l(x') \rangle_c \langle A^c_k(y) A^{\alpha''}_\nu(x'') \rangle_c \} |_{x=y}
\]
\[
- g f^{abc} \partial_{y_i} \langle A^a_k(x) A^b_l(x') A^{\alpha''}_\nu(x'') \rangle_c
\]
\[
+ (1 + \mathcal{T}_{eb}) \langle A^a_k(x) A^b_l(x') A^{\alpha''}_\nu(x'') \rangle_c \langle A^c_k(y) \rangle_c
\]
\[
+ (1 + \mathcal{T}_{d' a''}) \langle A^a_k(x) A^b_l(x') \rangle_c \langle A^c_k(y) A^{\alpha''}_\nu(x'') \rangle_c \} |_{x=y}
\]
\[
+ gt^{abc} \gamma_i (u^b_\alpha(x) u^c_\beta(x') A^{\alpha''}_\nu(x'') \rangle_c,
\]
\[
\text{(B.2)}
\]
\[
\frac{d}{dt} \langle \Pi^a_\gamma(x) \Pi^\alpha_\delta(x') A^{\alpha''}_\nu(x'') \rangle_c = \langle \Pi^a_\gamma(x) \Pi^\alpha_\delta(x') \Pi^{\alpha''}_\nu(x'') \rangle_c
\]
\[
+ (1 + \mathcal{P}_{iv'})[ \partial^2_x \langle A^\gamma_\delta(x') \Pi^\alpha_\delta(x') A^{\alpha''}_\nu(x'') \rangle_c - \partial_x \partial_{x_k} \langle A^\gamma_\delta(x') \Pi^\alpha_\delta(x') A^{\alpha''}_\nu(x'') \rangle_c]
\]
\[
+ 2g (1 + \mathcal{P}_{iv'}) f^{abc} \partial_{y_i} \langle A^a_k(x) A^b_l(y) \Pi^\alpha_\delta(x') A^{\alpha''}_\nu(x'') \rangle_c
\]
\[
+ (1 + \mathcal{T}_{eb}) \{ \langle A^a_k(x) \Pi^\alpha_\delta(x') A^{\alpha''}_\nu(x'') \rangle_c \langle A^c_k(y) \rangle_c
\]
\[
+ (1 + \mathcal{T}_{d' a''}) \langle A^a_k(x) \Pi^\alpha_\delta(x') \rangle_c \langle A^c_k(y) A^{\alpha''}_\nu(x'') \rangle_c \} |_{x=y}
\]
\[
+ g^2 (1 + \mathcal{P}_{iv'}) f^{abc} \langle A^a_k(x) \Pi^\alpha_\delta(x') \Pi^{\alpha''}_\nu(x'') \rangle_c \Pi^\alpha_\delta(x') A^{\alpha''}_\nu(x'') \rangle_c
\]
\[
+ (1 + \mathcal{T}_{eb}) \langle A^a_k(x) \Pi^\alpha_\delta(x') \rangle_c \langle A^c_k(y) A^{\alpha''}_\nu(x'') \rangle_c
\]
\[
+ (1 + \mathcal{T}_{d' a''}) \langle A^a_k(x) \Pi^\alpha_\delta(x') \rangle_c \langle A^c_k(y) A^{\alpha''}_\nu(x'') \rangle_c \} |_{x=y}
\]
\[
+ (1 + \mathcal{T}_{d' a''}) \langle A^a_k(x) A^b_l(x') \Pi^\alpha_\delta(x') \rangle_c \langle A^c_k(y) A^{\alpha''}_\nu(x'') \rangle_c \}
\]
\[
\text{20}
\]
The equations of motion for connected 4-point gluon Green functions are the most
cumbersome of our approach and read:

\[
\frac{d}{dt} \langle A_i^\alpha(x)A_{ij}^\beta(x')A_{ij}^\gamma(x'')A_{ij}^\delta(x''') \rangle_c = \langle \Pi_i^\alpha(x)A_{ij}^\beta(x')A_{ij}^\gamma(x'')A_{ij}^\delta(x''') \rangle_c + \langle A_i^\alpha(x)\Pi_i^\beta(x')A_{ij}^\gamma(x'')A_{ij}^\delta(x''') \rangle_c + \langle A_i^\alpha(x)A_{ij}^\beta(x')A_{ij}^\gamma(x'')\Pi_i^\delta(x''') \rangle_c,
\]

(B.5)

\[
\frac{d}{dt} \langle \Pi_i^\alpha(x)A_{ij}^\beta(x')A_{ij}^\gamma(x'')A_{ij}^\delta(x''') \rangle_c = \langle \Pi_i^\alpha(x)\Pi_i^\beta(x')A_{ij}^\gamma(x'')A_{ij}^\delta(x''') \rangle_c + \langle \Pi_i^\alpha(x)A_{ij}^\beta(x')\Pi_i^\gamma(x'')A_{ij}^\delta(x''') \rangle_c + \langle \Pi_i^\alpha(x)A_{ij}^\beta(x')A_{ij}^\gamma(x'')\Pi_i^\delta(x''') \rangle_c
\]

\[+ \partial_x^2 \langle A_i^\alpha(x)A_{ij}^\beta(x')A_{ij}^\gamma(x'')A_{ij}^\delta(x''') \rangle_c - \partial_x \partial_x \langle A_i^\alpha(x)A_{ij}^\beta(x')A_{ij}^\gamma(x'')A_{ij}^\delta(x''') \rangle_c - 2g f^{abc} \partial_y \{ (1 + \mathcal{T}_{cb})[\langle A_k^a(x)A_{ij}^b(x')A_{ij}^c(x'')A_{ij}^d(x''') \rangle_c \rangle \langle A_i^e(y) \rangle_c + (1 + \mathcal{T}_{a'w} + \mathcal{T}_{a''w}) \langle A_k^a(x)A_{ij}^b(x')A_{ij}^c(x'')A_{ij}^d(x''') \rangle_c \langle A_i^e(y) \rangle_c \},(x=y) \]

(B.6)
\[ + g^2 (1 + \mathcal{P}_{iv}) f^{abc} f^{cde} (1 + \mathcal{T}_{eb} + \mathcal{T}_{ed}) \times \left[ (A_k^b(x) x \Pi_i^a (x'))_{\epsilon} + (A_k^b(x) A_k^a(x))_{\epsilon} \langle A_k^a(x) \Pi_i^a (x') A_i^a (x''') A_i^a (x''') \rangle_{\epsilon} + (1 + \mathcal{T}_{db}) \langle A_k^b(x) \Pi_i^a (x')_{\epsilon} A_k^a(x) A_i^a (x'') \rangle_{\epsilon} \langle A_k^a(x) A_i^a (x''') \rangle_{\epsilon} + (1 + \mathcal{T}_{a' a''} + \mathcal{T}_{a'' a'} \rangle \langle A_k^a(x) \Pi_i^a (x') A_i^a (x'') \rangle_{\epsilon} \langle A_k^a(x) A_i^a (x''') \rangle_{\epsilon} \langle A_k^a(x) A_i^a (x'') \rangle_{\epsilon} \right] \]

\[ + g (1 + \mathcal{P}_{iv}) f^{abc} \partial_{yk} (1 + \mathcal{T}_{eb}) \left[ (A_k^b(x) y \Pi_i^a (x') A_i^a (x'') A_i^a (x''') \rangle_{\epsilon} \langle A_k^a(x) \rangle_{\epsilon} \right. \]

\[ + (1 + \mathcal{T}_{a' a''} + \mathcal{T}_{a'' a'} \rangle \langle A_k^a(x) \Pi_i^a (x') A_i^a (x'') \rangle_{\epsilon} \langle A_k^a(x) A_i^a (x''') \rangle_{\epsilon} \langle A_k^a(y) A_i^a (x''') \rangle_{\epsilon} \right]_{x=y} . \quad (B.7) \]

\[ \frac{d}{dt} \langle \Pi_i^a (x) \Pi_i^a (x') \Pi_i^a (x'') \rangle_{\epsilon} = \langle \Pi_i^a (x) \Pi_i^a (x') \Pi_i^a (x'') \Pi_i^a (x''') \rangle_{\epsilon} \]

\[ + (1 + \mathcal{P}_{iv} + \mathcal{P}_{iv'}) f^{abc} f^{cde} (1 + \mathcal{T}_{eb} + \mathcal{T}_{ed}) \times \left[ (A_k^b(x) x \Pi_i^a (x'))_{\epsilon} + (A_k^b(x) A_k^a(x))_{\epsilon} \langle A_k^a(x) \Pi_i^a (x') A_i^a (x'') \rangle_{\epsilon} \langle A_k^a(x) A_i^a (x'') \rangle_{\epsilon} + (1 + \mathcal{T}_{db}) \langle A_k^b(x) \Pi_i^a (x')_{\epsilon} A_k^a(x) A_i^a (x'') \rangle_{\epsilon} \langle A_k^a(x) A_i^a (x'') \rangle_{\epsilon} + (1 + \mathcal{T}_{a' a''} + \mathcal{T}_{a'' a'} \rangle \langle A_k^a(x) \Pi_i^a (x') A_i^a (x'') \rangle_{\epsilon} \langle A_k^a(x) A_i^a (x''') \rangle_{\epsilon} \langle A_k^a(x) A_i^a (x'') \rangle_{\epsilon} \right] \]

\[ + g (1 + \mathcal{P}_{iv} + \mathcal{P}_{iv'}) f^{abc} \partial_{yk} (1 + \mathcal{T}_{eb}) \left[ (A_k^b(x) y \Pi_i^a (x') A_i^a (x'') A_i^a (x''') \rangle_{\epsilon} \langle A_k^a(x) \rangle_{\epsilon} \right. \]

\[ + (1 + \mathcal{T}_{a' a''} + \mathcal{T}_{a'' a'} \rangle \langle A_k^a(x) \Pi_i^a (x') A_i^a (x'') \rangle_{\epsilon} \langle A_k^a(x) A_i^a (x''') \rangle_{\epsilon} \langle A_k^a(y) A_i^a (x''') \rangle_{\epsilon} \right]_{x=y} . \quad (B.8) \]

\[ \frac{d}{dt} \langle \Pi_i^a (x) \Pi_i^a (x') \Pi_i^a (x'') \Pi_i^a (x''') \rangle_{\epsilon} = \]

\[ (1 + \mathcal{P}_{iv} + \mathcal{P}_{iv'} + \mathcal{P}_{iv''}) [\partial_x A_i^a(x) \Pi_i^a (x') A_i^a (x'') \Pi_i^a (x''') \rangle_{\epsilon} \]
\[ -\partial_x, \partial_{x'} \langle A_k^a(x) \Pi_{\nu'}^a(x') \Pi_{\nu''}^a(x'') \Pi_{\nu'''}(x''') \rangle_c \]

\[ + 2g(1 + P_{i'v} + P_{i''v} + P_{i'''v}) f^{abc} \partial_{y_k} (1 + T_{eb}) [ \langle A_k^b(x) \Pi_{\nu'}^b(x') \Pi_{\nu''}^b(x'') \Pi_{\nu'''}(x''') \rangle_c \langle A_i^c(y) \rangle_c + (1 + T_{a'v} + T_{a''v}) [ \langle A_k^b(x) \Pi_{\nu'}^b(x') \Pi_{\nu''}^b(x'') \Pi_{\nu'''}(x''') \rangle_c \langle A_i^c(y) \rangle_c ]_{(x=y)} \]

\[ + g^2(1 + P_{i'v} + P_{i''v} + P_{i'''v}) f^{abc} f^{cde} (1 + T_{eb} + T_{ed}) \]

\[ \times \left[ \langle (A_k^b(x) \Pi_{\nu'}^b(x'))_c \langle A_i^c(y) \rangle_c + \langle A_k^b(x) \Pi_{\nu'}^b(x') \Pi_{\nu''}^b(x'') \Pi_{\nu'''}(x''') \rangle_c \langle A_i^c(y) \rangle_c + (1 + T_{db}) \{ \langle A_k^b(x) \Pi_{\nu'}^b(x') \rangle_c \langle A_i^c(y) \rangle_c \langle A_i^c(y) \rangle_c \} \right]_{(x=y)} \]

\[ + g(1 + P_{i'v} + P_{i''v} + P_{i'''v}) f^{abc} \partial_{y_k} (1 + T_{eb}) [ \langle A_k^b(x) \Pi_{\nu'}^b(x') \Pi_{\nu''}^b(x'') \Pi_{\nu'''}(x''') \rangle_c \langle A_i^c(y) \rangle_c + (1 + T_{a'v} + T_{a''v}) [ \langle A_k^b(x) \Pi_{\nu'}^b(x') \Pi_{\nu''}^b(x'') \Pi_{\nu'''}(x''') \rangle_c \langle A_i^c(y) \rangle_c ]_{(x=y)} \]

\[ - g(1 + P_{i'v} + P_{i''v} + P_{i'''v}) f^{abc} \partial_{y_k} (1 + T_{eb}) [ \langle A_k^b(x) \Pi_{\nu'}^b(x') \Pi_{\nu''}^b(x'') \Pi_{\nu'''}(x''') \rangle_c \langle A_i^c(y) \rangle_c + (1 + T_{a'v} + T_{a''v}) [ \langle A_k^b(x) \Pi_{\nu'}^b(x') \Pi_{\nu''}^b(x'') \Pi_{\nu'''}(x''') \rangle_c \langle A_i^c(y) \rangle_c ]_{(x=y)} \]  

(B.9)

**B.2 Mixed sector**

The remaining equations of motion - not specified in Section 3 - read as follows:

\[ i \frac{d}{dt} \langle \bar{u}_a^\alpha(x) u_{\alpha'}^{\alpha'}(x') \Pi_i^\alpha(y) A_i^\nu(y') \rangle_c = \]

\[ i \langle \bar{u}_a^\alpha(x) u_{\alpha'}^{\alpha'}(x') \Pi_i^\alpha(y) A_i^\nu(y') \rangle_c + i \langle \bar{u}_a^\alpha(x) u_{\alpha'}^{\alpha'}(x') A_i^b(y) A_i^\nu(y') \rangle_c \]

\[ + g^2 \left[ \langle \bar{u}_a^\alpha(x) u_{\alpha'}^{\alpha'}(x') \Pi_i^\alpha(y) A_i^\nu(y') \rangle_c + \langle \bar{u}_a^\alpha(x) u_{\alpha'}^{\alpha'}(x') A_i^b(y) A_i^\nu(y') \rangle_c \right] \]

\[ + g(1 + P_{i'v} + P_{i''v} + P_{i'''v}) f^{abc} \partial_{y_k} (1 + T_{eb}) [ \langle A_k^b(x) \Pi_{\nu'}^b(x') \Pi_{\nu''}^b(x'') \Pi_{\nu'''}(x''') \rangle_c \langle A_i^c(y) \rangle_c + (1 + T_{a'v} + T_{a''v}) [ \langle A_k^b(x) \Pi_{\nu'}^b(x') \Pi_{\nu''}^b(x'') \Pi_{\nu'''}(x''') \rangle_c \langle A_i^c(y) \rangle_c ]_{(x=y)} \]

\[ + g(1 + P_{i'v} + P_{i''v} + P_{i'''v}) f^{abc} f^{cde} (1 + T_{eb} + T_{ed}) \]

\[ \times \left[ \langle (A_k^b(x) \Pi_{\nu'}^b(x'))_c \langle A_i^c(y) \rangle_c + \langle A_k^b(x) \Pi_{\nu'}^b(x') \Pi_{\nu''}^b(x'') \Pi_{\nu'''}(x''') \rangle_c \langle A_i^c(y) \rangle_c + (1 + T_{db}) \{ \langle A_k^b(x) \Pi_{\nu'}^b(x') \rangle_c \langle A_i^c(y) \rangle_c \langle A_i^c(y) \rangle_c \} \right]_{(x=y)} \]

\[ + g(1 + P_{i'v} + P_{i''v} + P_{i'''v}) f^{abc} \partial_{y_k} (1 + T_{eb}) [ \langle A_k^b(x) \Pi_{\nu'}^b(x') \Pi_{\nu''}^b(x'') \Pi_{\nu'''}(x''') \rangle_c \langle A_i^c(y) \rangle_c + (1 + T_{a'v} + T_{a''v}) [ \langle A_k^b(x) \Pi_{\nu'}^b(x') \Pi_{\nu''}^b(x'') \Pi_{\nu'''}(x''') \rangle_c \langle A_i^c(y) \rangle_c ]_{(x=y)} \]

\[ - g(1 + P_{i'v} + P_{i''v} + P_{i'''v}) f^{abc} \partial_{y_k} (1 + T_{eb}) [ \langle A_k^b(x) \Pi_{\nu'}^b(x') \Pi_{\nu''}^b(x'') \Pi_{\nu'''}(x''') \rangle_c \langle A_i^c(y) \rangle_c + (1 + T_{a'v} + T_{a''v}) [ \langle A_k^b(x) \Pi_{\nu'}^b(x') \Pi_{\nu''}^b(x'') \Pi_{\nu'''}(x''') \rangle_c \langle A_i^c(y) \rangle_c ]_{(x=y)} \]  

(B.10)
\[
\begin{align*}
&(\bar{u}_{\alpha}^a(x)u_{\alpha}^\alpha(x') A_k^\alpha(y))_c (A_k^{\nu}(x)\Pi_i^\beta(y))_c + (\bar{u}_{\beta}^a(x)u_{\alpha}^\alpha(x') \Pi_i^\beta(y) A_k^{\nu}(y'))_c (A_k^{\nu}(x')_c) \\
+\alpha_{\alpha\beta}^k[\partial_{x_k}(\bar{u}_{\alpha}^a(x)u_{\beta}^\beta(x') \Pi_i^\beta(y) A_k^{\nu}(y'))_c \\
- g^{\nu\alpha}c \{ (\bar{u}_{\alpha}^a(x)u_{\beta}^\beta(x')_c (A_k^\nu(x')\Pi_i^\beta(y) A_k^{\nu}(y'))_c + (\bar{u}_{\alpha}^a(x)u_{\beta}^\beta(x') \Pi_i^\beta(y) A_k^{\nu}(y'))_c (A_k^{\nu}(x'))_c \\
+ (\bar{u}_{\alpha}^a(x)u_{\alpha}^\alpha(x') A_k^{\nu}(y'))_c (A_k^{\nu}(x')_c (A_k^{\nu}(x'))_c \\
+ i \partial_{y_i}(\bar{u}_{\alpha}^a(x)u_{\alpha}^\alpha(x') A_k^\alpha(y) A_k^{\nu}(y'))_c - i \partial_{y_i}(\bar{u}_{\alpha}^a(x)u_{\alpha}^\alpha(x') A_k^\alpha(y) A_k^{\nu}(y'))_c \\
+ 2g f^{\beta\nu c} \partial_{x_k}[(\bar{u}_{\alpha}^a(x)u_{\alpha}^\alpha(x') A_k^{\nu}(z) A_k^\alpha(y))_c (A_k^\nu(y) A_k^\alpha(y'))_c \\
+ (1 + T_{de}) (\bar{u}_{\alpha}^a(x)u_{\alpha}^\alpha(x') A_k^{\nu}(y))_c (A_k^\nu(y) A_k^\alpha(y'))_c \\
+ (1 + T_{de}) (\bar{u}_{\alpha}^a(x)u_{\alpha}^\alpha(x') A_k^{\nu}(y))_c (A_k^\nu(y) A_k^\alpha(y'))_c \\
+ (1 + T_{de}) (\bar{u}_{\alpha}^a(x)u_{\alpha}^\alpha(x') A_k^{\nu}(y))_c (A_k^\nu(y) A_k^\alpha(y'))_c \\
+ i g f^{\beta\nu c} \partial_{x_k} [(\bar{u}_{\alpha}^a(x)u_{\alpha}^\alpha(x') A_k^{\nu}(y))_c (A_k^\nu(y) A_k^\alpha(y'))_c \\
+ (u_{\alpha}^\alpha(x)u_{\alpha}^\alpha(x') A_k^{\nu}(y))_c (A_k^\nu(z) A_k^\alpha(y')_c + (u_{\alpha}^\alpha(x)u_{\alpha}^\alpha(x') A_k^{\nu}(y))_c (A_k^\nu(z) A_k^\alpha(y')_c \\
+ (u_{\alpha}^\alpha(x)u_{\alpha}^\alpha(x') A_k^{\nu}(y))_c (A_k^\nu(z) A_k^\alpha(y')_c \\
- ig f^{\beta\nu c} \partial_{x_k} [(\bar{u}_{\alpha}^a(x)u_{\alpha}^\alpha(x') A_k^{\nu}(y))_c (A_k^\nu(z) A_k^\alpha(y')_c \\
+ (u_{\alpha}^\alpha(x)u_{\alpha}^\alpha(x') A_k^{\nu}(y))_c (A_k^\nu(z) A_k^\alpha(y')_c \\
+ (u_{\alpha}^\alpha(x)u_{\alpha}^\alpha(x') A_k^{\nu}(y))_c (A_k^\nu(z) A_k^\alpha(y')_c \\
+ ig t^{\alpha\beta \gamma} A_k^\alpha A_k^\beta \delta(x' - y)(\bar{u}_{\alpha}^a(x)u_{\alpha}^\alpha(y) A_k^{\nu}(y'))_c \\
- ig t^{\alpha\beta \gamma} [(\bar{u}_{\alpha}^a(x)u_{\alpha}^\alpha(y) A_k^{\nu}(y'))_c (\bar{u}_{\alpha}^a(y)u_{\alpha}^\alpha(x'))_c \\
+ (u_{\alpha}^\alpha(x)u_{\alpha}^\alpha(y))_c (u_{\alpha}^\alpha(x') A_k^{\nu}(y'))_c],
\end{align*}
\]
\[+(\bar{u}_\alpha^c(x)u^c_\alpha(x')\Pi^w_{\nu'}(y'))_c\langle A^y_k(z)\Pi^w_{\nu'}(y')\rangle_c+(\bar{u}_\alpha^c(x)u^c_\alpha(x')\Pi^y_{\nu'}(y'))_c\langle A^y_k(z)\Pi^y_{\nu'}(y')\rangle_c\]

\[+i(1+\mathcal{P}_{\nu'})(\partial^2_y(\bar{u}_\alpha^c(x)u^c_\alpha(x')A^y_k(z)\Pi^y_{\nu'}(y'))_c+\partial_y\partial_y(\bar{u}_\alpha^c(x)u^c_\alpha(x')A^y_k(z)\Pi^y_{\nu'}(y'))_c\]

\[+2g(1+\mathcal{P}_{\nu'}f^{bc'd}A^y_{\nu'}(y/\nu')\langle A^d_k(z)\Pi^y_{\nu'}(y')\rangle_c+2\bar{u}_\alpha^c(x)u^c_\alpha(x')A^y_k(z)\Pi^y_{\nu'}(y'))_c\]

\[+(\bar{u}_\alpha^c(x)u^c_\alpha(x')A^y_k(z))_c\langle A^y_k(z)\Pi^y_{\nu'}(y')\rangle_c+(\bar{u}_\alpha^c(x)u^c_\alpha(x')A^y_k(z))_c\langle A^y_k(y)\Pi^y_{\nu'}(y')\rangle_c\]

\[+(\bar{u}_\alpha^c(x)u^c_\alpha(x')A^y_k(z))_c\langle A^y_k(y)\Pi^y_{\nu'}(y')\rangle_c\]

\[+ig(1+\mathcal{P}_{\nu'}f^{bc'd}f^{cde}(1+\mathcal{T}_{\nu'd}+\mathcal{T}_{\nu'e})[(\bar{u}_\alpha^c(x)u^c_\alpha(x')A^y_{\nu'}(y))_c\langle A^d_k(z)\Pi^y_{\nu'}(y')\rangle_c\]

\[+(\bar{u}_\alpha^c(x)u^c_\alpha(x')A^y_k(z))_c\langle A^d_k(z)\Pi^y_{\nu'}(y')\rangle_c+(\bar{u}_\alpha^c(x)u^c_\alpha(x')A^y_k(z))_c\langle A^y_k(z)\Pi^y_{\nu'}(y')\rangle_c\]

\[+(\bar{u}_\alpha^c(x)u^c_\alpha(x')A^y_k(z))_c\langle A^y_k(z)\Pi^y_{\nu'}(y')\rangle_c\]

\[+ig(1+\mathcal{P}_{\nu'}f^{bc'd}f^{cde}(1+\mathcal{T}_{\nu'd}+\mathcal{T}_{\nu'e})[(\bar{u}_\alpha^c(x)u^c_\alpha(x')A^y_{\nu'}(y))_c\langle A^d_k(z)\Pi^y_{\nu'}(y')\rangle_c\]

\[+(\bar{u}_\alpha^c(x)u^c_\alpha(x')A^y_k(z))_c\langle A^d_k(z)\Pi^y_{\nu'}(y')\rangle_c+(\bar{u}_\alpha^c(x)u^c_\alpha(x')A^y_k(z))_c\langle A^y_k(z)\Pi^y_{\nu'}(y')\rangle_c\]

\[+(\bar{u}_\alpha^c(x)u^c_\alpha(x')A^y_k(z))_c\langle A^y_k(z)\Pi^y_{\nu'}(y')\rangle_c\]

\[+ig(1+\mathcal{P}_{\nu'}f^{bc'd}f^{cde}(1+\mathcal{T}_{\nu'd}+\mathcal{T}_{\nu'e})[(\bar{u}_\alpha^c(x)u^c_\alpha(x')A^y_{\nu'}(y))_c\langle A^d_k(z)\Pi^y_{\nu'}(y')\rangle_c\]

\[+(\bar{u}_\alpha^c(x)u^c_\alpha(x')A^y_k(z))_c\langle A^d_k(z)\Pi^y_{\nu'}(y')\rangle_c+(\bar{u}_\alpha^c(x)u^c_\alpha(x')A^y_k(z))_c\langle A^y_k(z)\Pi^y_{\nu'}(y')\rangle_c\]

\[+(\bar{u}_\alpha^c(x)u^c_\alpha(x')A^y_k(z))_c\langle A^y_k(z)\Pi^y_{\nu'}(y')\rangle_c\]

\[+ig(1+\mathcal{P}_{\nu'}f^{bc'd}f^{cde}(1+\mathcal{T}_{\nu'd}+\mathcal{T}_{\nu'e})[(\bar{u}_\alpha^c(x)u^c_\alpha(x')A^y_{\nu'}(y))_c\langle A^d_k(z)\Pi^y_{\nu'}(y')\rangle_c\]

\[+(\bar{u}_\alpha^c(x)u^c_\alpha(x')A^y_k(z))_c\langle A^d_k(z)\Pi^y_{\nu'}(y')\rangle_c+(\bar{u}_\alpha^c(x)u^c_\alpha(x')A^y_k(z))_c\langle A^y_k(z)\Pi^y_{\nu'}(y')\rangle_c\]

\[+(\bar{u}_\alpha^c(x)u^c_\alpha(x')A^y_k(z))_c\langle A^y_k(z)\Pi^y_{\nu'}(y')\rangle_c\]

\[-ig(1+\mathcal{P}_{\nu'}f^{bc'd}f^{cde}(1+\mathcal{T}_{\nu'd}+\mathcal{T}_{\nu'e})[(\bar{u}_\alpha^c(x)u^c_\alpha(x')A^y_{\nu'}(y))_c\langle A^d_k(z)\Pi^y_{\nu'}(y')\rangle_c\]

\[+(\bar{u}_\alpha^c(x)u^c_\alpha(x')A^y_k(z))_c\langle A^d_k(z)\Pi^y_{\nu'}(y')\rangle_c+(\bar{u}_\alpha^c(x)u^c_\alpha(x')A^y_k(z))_c\langle A^y_k(z)\Pi^y_{\nu'}(y')\rangle_c\]

\[+(\bar{u}_\alpha^c(x)u^c_\alpha(x')A^y_k(z))_c\langle A^y_k(z)\Pi^y_{\nu'}(y')\rangle_c\]

\[+ig(1+\mathcal{P}_{\nu'}f^{bc'd}f^{cde}(1+\mathcal{T}_{\nu'd}+\mathcal{T}_{\nu'e})[(\bar{u}_\alpha^c(x)u^c_\alpha(x')A^y_{\nu'}(y))_c\langle A^d_k(z)\Pi^y_{\nu'}(y')\rangle_c\]

\[+(\bar{u}_\alpha^c(x)u^c_\alpha(x')A^y_k(z))_c\langle A^d_k(z)\Pi^y_{\nu'}(y')\rangle_c+(\bar{u}_\alpha^c(x)u^c_\alpha(x')A^y_k(z))_c\langle A^y_k(z)\Pi^y_{\nu'}(y')\rangle_c\]

\[+(\bar{u}_\alpha^c(x)u^c_\alpha(x')A^y_k(z))_c\langle A^y_k(z)\Pi^y_{\nu'}(y')\rangle_c\]

This completes the nonlinear CQCD equations, which form a closed set of first order differential equations in time and can be integrated by standard numerical techniques for almost arbitrary initial conditions, provided that these fulfill the weak Gauss law as well as the Ward identities as discussed in Section 4.