Research Based on The Improved Grey Wolf Optimization Algorithm

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Abstract. With the continuous progress of machine learning methods, more and more methods are widely used in various fields. Grey Wolf Optimization Algorithm (GWO), as a kind of population algorithm, has become a research hotspot in recent years because of its good optimization ability. Similar to other population algorithms, gray wolf optimization algorithm itself has the problem of imbalance between global search and local search capabilities. In order to solve this problem, the following improvements are proposed to the standard gray wolf optimization algorithm: First, change the decrement method of the convergence factor and adopt a non-linear decrement method to meet the actual search process; At the same time, a weighting strategy is introduced to dynamically assign weights to the guide wolves. It can ensure that the population jumps out of the local optimal solution. In order to verify the effectiveness of the improved algorithm, an international general test function is selected for simulation. The simulation results show that the improved gray wolf optimization algorithm has faster convergence speed, higher solution accuracy and better stability.

1. Introduction

The grey wolf optimizer (GWO) was proposed by S Mirhalili in 2014. The algorithm is obtained by modeling the prey-predation behavior of grey wolves[1]. Its main characteristics are strong convergence performance, few parameters, and easy implementation. It is often used in the field of parameter optimization.

Although the population algorithm has good optimizing ability, it also has some problems. The grey wolf optimization algorithm has a slower convergence speed in the later stage, and it is easy to fall into a local optimum, which leads to a decrease in the final solution accuracy[2]. In order to solve the above problems, scholars have proposed many methods to improve it. ZHU A pointed out in his paper that the search results can be more in line with the actual situation by changing the mode of change of the convergence factor, replacing linear with nonlinearity[3]. Wei Zhenglei uses the method of controlling variables to compare the results produced by the convergence factors of different decreasing methods. The results show that the test results are better with the decreasing convergence factors of the sinusoidal method.

In addition to changing the convergence factor, some scholars have improved the GWO algorithm by adjusting the position update formula of the algorithm. Castillo O proposed a weighted average method to assign corresponding weights $\alpha$, $\beta$ and $\delta$ wolves according to their social ranks, and achieved good results[4]. Zhang Mingxin proposed a weighting method based on the fitness value to calculate the fitness value of individual wolves. The first three are assigned to $\alpha$, $\beta$ and $\delta$ wolves. The weight of the position formula is dynamically adjusted to speed up the search.
Based on the research of the above scholars, this article has made the following improvements to the standard GWO: 1) Change the decrement method of the convergence factor, by introducing a trigonometric function, the convergence factor is reduced from 2 non-linearly to 0; 2) By adding dynamic weights, in this way, the guiding wolves assign more weight, and the guiding role of the guiding wolves is highlighted. Finally, eight test functions are selected for testing to verify the effectiveness of the improved algorithm.

2. Gray wolf optimization algorithm

Inspired by the prey-preying behavior of grey wolves, the grey wolf optimization algorithm was developed. In this algorithm, the population is divided into 4 levels, the alpha wolf has the highest level, the beta wolf is the second highest, the delta wolf is the third highest, and the remaining ω wolf has the lowest level. The algorithm uses high-level wolves to guide low-level wolves to search for targets for optimization. The mathematical model is as follows.

\[ D = |C \cdot X_\alpha(t) - X(t)| \]  
\[ X(t+1) = X_\rho(t) - A \cdot D \]

The distance between grey wolves and its prey is represented by formula (1), and the position update formula of the grey wolf is represented by formula (2). Among them, the current iteration number is represented by t, and the position of the prey is represented by \( X_\beta(t) \). Indicates that the position of the grey wolf is determined by \( X, A \) and \( C \) are the synergy vector, and the calculation formula is:

\[ A = 2a \cdot r_1 - a \]  
\[ C = 2 \cdot r_2 \]

Where: The convergence factor a changes linearly, decreasing from 2 to 0; \( r_1, r_2 \) are random vectors, and the variation range is 0 to 1.

Sort the fitted values, take the top three, and assign them to \( \alpha, \beta \) and \( \delta \) wolves respectively, and obtain their position update formula. The remaining grey wolves update their position according to the position of the guide wolf. The process is modeled as:

\[ D_\alpha = |C_1 \cdot X_\alpha - X| \]  
\[ D_\beta = |C_2 \cdot X_\beta - X| \]  
\[ D_\delta = |C_3 \cdot X_\delta - X| \]  
\[ X_1 = X_\alpha - A_1 \cdot D_\alpha \]  
\[ X_2 = X_\beta - A_1 \cdot D_\beta \]  
\[ X_3 = X_\delta - A_1 \cdot D_\delta \]  
\[ X(t+1) = \frac{X_1 + X_2 + X_3}{3} \]

Equation (5) expresses the distance between \( \alpha, \beta \) and \( \delta \) wolves and other grey wolves. Equation (6) indicates that grey wolves update their positions under the guidance of \( \alpha, \beta \) and \( \delta \) wolves. Finally, the position is synthesized by formula (7).

3. Improved grey wolf optimization algorithm

To measure the quality of a group optimization algorithm, the key is to balance the global search and local development capabilities of the algorithm. Global search emphasizes a wider range of exploration in the early stage of evolution; local optimization emphasizes faster and more accurate optimization in the later stage of evolution[5]. GWO, as a kind of group intelligence optimization algorithm, in order to obtain better optimal results, while ensuring a wider global search scale, it also needs to ensure local development capabilities. In order to balance the relationship between the above
two, an improved grey wolf optimization algorithm (improved grey wolf optimization, IGWO) is represented by this.

In the standard GWO, the convergence factor $a$ varies linearly from 2 to 0. In the actual search and development process, most of them are non-linear and do not conform to the actual situation, and it is easy to fall into the local optimum. The literature pointed out that changing, the change mode of the convergence factor $a$, using a non-linear change strategy instead of a linear change strategy can get better optimization results. Therefore, the convergence factor formula in this paper adopts the trigonometric function expression to achieve the purpose of nonlinear change. The formula is:

$$a = \sin(\pi \cdot \frac{t}{t_{\text{max}}} + \frac{\pi}{2}) + 1$$

(8)

Where: the current iteration number is determined by $t$; the maximum number of iterations is determined by $t_{\text{max}}$.

![Figure 1. Convergence factor a comparison.](image)

It can be seen from Figure 1 that the convergence factor $a$ varies nonlinearly from 2 to 0. In the early stage of evolution, a change slowly to ensure that a wider area is detected in the early stage; In the middle of evolution, the rate of change of $a$ is accelerated, which increases the speed of algorithm optimization; in the later stage of evolution, the rate of change of $a$ decreases to ensure that the algorithm is accurately optimized.

In addition, when the grey wolf population is hunting prey, the proportion of each guide wolf is the same, and the leading role of the head wolf is not highlighted, so it is easy to obtain a local optimal solution. Therefore, consider assigning different weights to wolves to dynamically adjust the search ability. By means of dynamic weighting, it can jump out of the local optimal solution and get better optimization results.

$$W_\alpha = \frac{|X_1| + |X_2| + |X_3|}{|X_1|} - 1$$

$$W_\beta = \frac{|X_1| + |X_2| + |X_3|}{|X_2|} - 1$$

$$W_\delta = \frac{|X_1| + |X_2| + |X_3|}{|X_3|} - 1$$

(9)

The final location update method is:

$$X(t+1) = \frac{X_1 \cdot W_\alpha + X_2 \cdot W_\beta + X_3 \cdot W_\delta}{W_\alpha + W_\beta + W_\delta}$$

(10)

Where: $W_\alpha$, $W_\beta$, and $W_\delta$ respectively represent the weights assigned by the wolves of $\alpha$, $\beta$, and $\delta$. 
4. Experimental verification analysis

4.1. Test function

In order to verify the effectiveness of the improved grey wolf algorithm (IGWO), it demonstrates its good optimization ability. The international benchmark function is selected for testing, and the experimental platform is Matlab 2014. Here select 8 test functions, and their specific information is shown in Table 1. The first 5 functions in the table are unimodal functions, and the last 3 are multimodal functions. Experiment independently for each function 30 times to reduce the error caused by the experimental data.

| Function expression | Dimension | Search interval | The optimal value |
|----------------------|-----------|-----------------|-------------------|
| $f_1(x) = \sum_{i=1}^{n} x_i^2$ | 30 | [-100,100] | 0 |
| $f_2(x) = \sum_{i=1}^{n} |x_i|$ | 30 | [-10,10] | 0 |
| $f_3(x) = \sum_{i=1}^{n} x_i^3$ | 30 | [-100,100] | 0 |
| $f_4(x) = \max \{ |x_i|, 1 \leq i \leq n \}$ | 30 | [-100,100] | 0 |
| $f_5(x) = \sum_{i=1}^{n} x_i^2 \cdot \prod_{i=1}^{n} (2x_i + 1)$ | 30 | [-30,30] | 0 |
| $f_6(x) = 3.20 \cdot 10^0 - \sum_{i=1}^{n} x_i^2 \cdot \prod_{i=1}^{n} \cos(2\pi x_i + 10)$ | 30 | [-5.12,5.12] | 0 |
| $f_7(x) = \frac{1}{\prod_{i=1}^{n} x_i}$ | 30 | [-600,600] | 0 |

4.2. Comparison with GWO and HGWO algorithms

For 8 test functions, use the IGWO algorithm to solve them one by one. The solution results are compared with the results obtained by the other two algorithms. The solution result of the HGWO algorithm comes from the literature. The experimental results illustrate the effectiveness of the IGWO algorithm. Among the three algorithms: the population size is 30, and the maximum number of iterations is 500. The solution experiment corresponding to each algorithm is performed 30 times, and the experimental results obtained are shown in Table 2.

| Test function | average value | Standard deviation | average value | Standard deviation | average value | Standard deviation |
|---------------|---------------|-------------------|---------------|-------------------|---------------|-------------------|
| $f_1$         | 1.07E-27      | 1.88E-27          | 1.12E-32      | 2.32E-32          | 2.29E-43      | 5.12E-43          |
| $f_2$         | 7.94E-17      | 5.55E-17          | 9.33E-20      | 6.92E-20          | 1.90E-24      | 3.44E-24          |
| $f_3$         | 2.07E-05      | 1.85E-05          | 3.18E-08      | 6.55E-08          | 6.22E-62      | 1.39E-61          |
| $f_4$         | 6.46E-07      | 5.67E-07          | 4.17E-08      | 4.56E-08          | 8.25E-158     | 0                 |
| $f_5$         | 27.0096       | 0.54298           | 26.4876       | 0.70271           | 28.8843       | 0.10533           |
| $f_6$         | 3.20E+00      | 4.00E+00          | 2.27E-01      | 9.20E-01          | 0             | 0                 |
| $f_7$         | 1.00E-13      | 5.90E-14          | 4.27E-14      | 4.37E-15          | 8.89E-16      | 0                 |
| $f_8$         | 7.19E-03      | 1.40E-02          | 1.37E-03      | 5.82E-03          | 0             | 0                 |

It can be concluded from Table 2 that the IGWO algorithm can uniformly converge to the global optimal solution of the problem on the seven test functions, and the function $f_6$ and function $f_8$ converge to the theoretical optimum solution $0$. The test function $f_5$ is an exception. The results obtained by the three algorithms are close. Among them, the average value obtained by the HGWO algorithm is better, and the standard deviation obtained by the IGWO algorithm is better. In summary, compared with the other two algorithms, the IGWO algorithm has more accurate convergence accuracy, faster convergence speed and superior stability in most test functions, which can prove the effectiveness of the IGWO algorithm.
Figure 2. Convergence curves of GWO and IGWO algorithms.

Figure 2 shows the optimization curves of the 8 benchmark functions using the GWO algorithm and the IGWO algorithm. From the convergence curve given in Figure 2, we can draw the following conclusions: Compared with the standard GWO algorithm, the GWO algorithm has higher solving accuracy and faster convergence speed of the curve, which verifies the effectiveness of the IGWO algorithm.

5. Conclusion
In order to make up for the shortcomings of the standard grey wolf optimization algorithm, this paper proposes a convergence factor based on sine change, and at the same time adjusts the position update formula based on the Euclidean distance combination. These two methods are used to improve the traditional grey wolf optimization algorithm. The test function is selected for simulation, and the results show that the improved algorithm has faster convergence speed and higher accuracy, which proves the effectiveness of the improved algorithm.

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