Experimental constraints on the free fall acceleration of antimatter

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In light of recent experimental proposals to measure the free fall acceleration of antihydrogen in the earth’s gravitational field, we investigate the bounds that existing experiments place on any asymmetry between the free fall of matter and antimatter. We conclude that existing experiments constrain any such asymmetry to be less than about $10^{-7}$. First we consider contributions to the inertial masses of atoms that encode the presence of antimatter and use precision Eötvös experiments to establish the level at which they satisfy the equivalence principle. In particular we focus on vacuum polarization effects and the antiquark content of nucleons. Second we consider a class of theories that contain long range scalar and vector forces that cancel with one another to some high precision. By construction such theories would be able to evade detection in Eötvös experiments that utilize matter while still allowing for a signal in antimatter experiments. Even taking such cancellation for granted, however, we show that the radiative damping of binary pulsar systems constrains these forces to be significantly weaker than gravity. Furthermore we show that there are limits to the accuracy with which such cancellation can be arranged: first by determining the precision to which scalar charges can track vector charges in the best candidate theories; and, second, by showing that the different velocity dependence of scalar and vector forces necessarily introduces non-cancellation at a quantifiable level.

1. INTRODUCTION

Experimentalists and theorists alike have long considered the possibility that matter and antimatter fall differently in the gravitational field of the earth (e.g. see [1, 2]). Early experimental endeavors began with Fairbank, who attempted to measure the differential free fall acceleration of electrons and positrons. These efforts, however, did not result in any conclusive measurement because of the extreme difficulty of isolating the test particles from stray electric fields. Recently an experiment has been proposed at Fermilab that aims to directly measure the free fall acceleration of antihydrogen in the field of the earth, $g_H$, with an expected precision of 1% or better [3]. Likewise, another experiment (AEGIS [4]) to measure $g_H$ has been proposed at CERN. In light of these experimental proposals, it is only reasonable to consider what sort of bounds existing experiments place on the inequality of $g_H$ and $g_T$. Although some of the arguments we make can be found elsewhere in the literature, we include them here to stress the point that existing experiments already place stringent bounds on any gravitational asymmetry between matter and antimatter.

There are two broad classes of theoretical possibilities for how gravitational asymmetry might be realized. The first is a modification of general relativity itself. Any such theory in which matter and antimatter gravitate differently will necessarily do violence to fundamental principles of general relativity and quantum field theory. As such, we are not aware of any concrete, self-consistent theoretical formulation—whether well-motivated or not—in which such an asymmetry exists. Nevertheless, we can still establish bounds on any such asymmetry, since existing experiments already tell us something about how antimatter gravitates. This will be the subject of section 3. The essential point is that the composite nature of atoms implies that precision Eötvös experiments, which have been done with a variety of elements, are sensitive to the gravitational coupling of antimatter.

The second possibility is to leave gravity itself untouched and introduce long range forces of (sub)gravitational strength mediated by scalar and/or vector particles. From a theoretical perspective this "fifth force" scenario is more tractable, since in this case we have a well-defined and predictive theory that does not violate any of the general principles that underpin the framework of quantum field theory cum general relativity. There are two distinct ways in which such forces could have evaded detection in all existing experiments. The first is simply that they could be incredibly weak, many orders of magnitude weaker than gravity. The second [1] is that such forces could be of gravitational strength but would have evaded detection in all existing (matter-matter) experiments because they cancel among themselves to a sufficiently high precision. This latter scenario can lead to a measurable asymmetry between $g_H$ and $g_T$, since while scalar-mediated forces are universally attractive, vector-mediated forces can be either attractive or repulsive, depending on the relative sign of the charges. Thus any cancellation of these
TABLE I: Constraints on the Eötvös parameter for various test bodies falling in the gravitational field of the earth or the sun

| Experiment            | Test bodies     | Measurement          |
|-----------------------|-----------------|----------------------|
| Lunar laser ranging   | Earth - Moon   | $\eta_{\odot,\oplus}$ = $(-1.0 \pm 1.4) \times 10^{-13}$ |
| Braginsky and Panov   | Al - Pt        | $\eta_{\odot,\text{Al-Pt}}$ = $(3 \pm 4) \times 10^{-13}$ |
| Eötvös                | Be - Ti        | $\eta_{\odot,\text{Be-Ti}}$ = $(0.3 \pm 1.8) \times 10^{-13}$ |
| Eötvös                | Be - Al        | $\eta_{\odot,\text{Be-Al}}$ = $(-1.5 \pm 1.5) \times 10^{-13}$ |
| Eötvös                | Be - Cu        | $\eta_{\odot,\text{Be-Cu}}$ = $(-1.9 \pm 2.5) \times 10^{-12}$ |

new forces in matter-matter interactions will be undone when considering matter-antimatter interactions, since the vector force switches from repulsive to attractive. Thus such a theory predicts that $g_H \neq g_{\Pi}$. Bounds from existing experiments on this scalar-vector scenario will be discussed in section 3. The bottom line is that composition dependence of free fall acceleration, which is tightly constrained by precision Eötvös experiments, is generic in this scenario due to the compositeness of atoms and the nature of scalar and vector interactions, both of which act to spoil any would-be cancellation.

2. EXPERIMENTAL INPUT

A number of very precise experiments have been done to measure the fractional differential acceleration, $\eta \equiv \Delta a/a$, of test bodies of various compositions falling in the gravitational field of the earth or the sun. For Eötvös experiments sensitive to the gravitational field of the sun, the most precise bounds on the Eötvös parameter $\eta_{\odot}$ come from lunar laser ranging (LLR) experiments \[5\], which measure the differential acceleration between the earth and moon towards the sun, and free torsion pendulum experiments performed by Braginsky and Panov using multiple aluminum and platinum test bodies \[6\]. Since we are mainly interested in the free fall acceleration of antihydrogen in the earth’s gravitational field, the most relevant experimental input for us will be the bounds obtained by the Eötvos Wash Group at the University of Washington. Their torsion balance experiments have tightly constrained $\eta_{\oplus}$ between several pairs of elements \[7\]-\[8\]. It is on the basis of these bounds, which are collected in Table I, that we will be able to tightly constrain any asymmetry between the free fall of matter and antimatter.

3. ATOMS HAVE MANY PARTS

When considering the possibility that antimatter gravitates differently from ordinary matter, one is really raising the more general possibility that different forms of energy gravitate differently. Existing free fall experiments, which have been performed with a wide variety of elements, put very stringent limits on any such non-universality of gravity, since the fractional contributions of various forms of energy to the inertial masses of atoms—nuclear binding energies, atomic binding energies, kinetic energies of the constituents, etc.—vary from element to element. What can these experiments tell us about how antimatter gravitates? The essential point is that nuclei and atoms are composite states. Although one can make a distinction between matter and antimatter at the level of quarks and electrons, that distinction is blurred when one considers bound states like nuclei and atoms. And because antimatter plays a quantifiable role in the physics of nuclei and atoms by contributing to their inertial masses, precision Eötvös experiments utilizing matter continue to be relevant when considering the possibility of gravitational asymmetry between matter and antimatter.

In particular we will focus on two ways in which antimatter enters the physics of nuclei and atoms. First, in sections 3.1 and 3.2 we will consider contributions to the inertial masses of nuclei and atoms due to vacuum polarization effects. Since these effects reflect the screening of electric charges by virtual pairs of electrons and positrons, we interpret these contributions to the inertial masses of nuclei and atoms as encoding their antimatter content. Second, in section 3.3 we will consider the sea antiquark content of nucleons as established by deep inelastic scattering experiments. In both cases we will quantify the degree to which existing Eötvös experiments require these forms of energy to satisfy the equivalence principle. We then make the assumption that any deviation of $g_H$ from $g_{\Pi}$ would manifest itself as a violation of the equivalence principle in these forms of energy at the same level. This reasoning will then allow us to place bounds on $|g_H - g_{\Pi}|/g_H$. It remains an interesting challenge to see whether it is possible to construct a theory for which the resulting bounds would not hold. Such a theory would require the effective gravitational coupling of antimatter as probed by fermion loops and sea antiquarks to be decoupled from the gravitational coupling of antihydrogen. In the absence of such a theory, however, our task is to establish the consequences of our basic assumption.

Having outlined our approach, it remains to quantify
the effects we are interested in. In section \[3.1\] we consider the Lamb shift in atoms and its implications for the universality of gravity. In section \[3.2\] we consider the analogous and much larger effect in the electrostatic self-energies of nuclei. Finally in section \[3.3\] we quantify the antiquark content of nucleons as well as the antimatter fractions of atoms, which will allow us to place further constraints on any gravitational asymmetry between matter and antimatter.

### 3.1. Lamb shift

Among the most precisely verified predictions of quantum electrodynamics is the Lamb shift in hydrogenlike atoms. Historically the term “Lamb shift” refers to the splitting between the \(2s_{1/2}\) and \(2p_{1/2}\) energy levels in the hydrogen atom; here we use it to refer to any correction to hydrogenlike energy levels from the values obtained by solving the Dirac equation for the Coulomb potential. One contribution to the Lamb shift is given by the vacuum polarization diagram of Figure 1. The electron loop in this diagram contributes to the running of the QED coupling constant at energies above the electron mass \(m_e\), which results in an effective electrostatic potential, the Uehling potential, that differs from the usual Coulomb \(1/r\) potential at distances shorter than \(m_e^{-1}\). This modification of the Coulomb potential at short distances can be interpreted as screening of the nuclear charge by pairs of virtual electrons and positrons. In an abuse of terminology, we will refer to the energy shift due to this effect as the Lamb shift, even though it constitutes only a fraction of the total Lamb shift (about 2–30% of the total depending on \(Z\)). The Lamb shift for the \(n^{th}\) energy level is given by

\[
E_{\text{Lamb}}^n = -\frac{\alpha(Z\alpha)^4}{n^3\pi^2} F(Z\alpha) m_e
\]

where \(Z\) is the atomic number and \(F(Z\alpha)\) varies slowly with \(Z\) (from about 0.25 to 1 as \(Z\) goes from 1 to 100) [9]. The total Lamb shift has been measured for high-

3.2. Electrostatic self-energy of the nucleus

Whereas the Lamb shift typically constitutes an \(\mathcal{O}(10^{-14} - 10^{-12})\) fraction of an atom’s inertial mass, electron loops make a much larger contribution to the electrostatic self-energy of the nucleus [10]. The classical electrostatic self-energy of the nucleus scales like

\[
E_{\text{EM}} \simeq -\frac{3\alpha}{5} \frac{Z(Z-1)}{A^{1/3} R_0} \approx 0.72 Z(Z-1) A^{1/3} \text{(MeV)}
\]

where \(R_0 = 1.2 \text{ fm}\). The leading correction (Figure 2) to this electrostatic energy comes from the insertion of a vacuum polarization loop analogous to that in the Lamb shift. This contribution amounts to a relative correction to the electrostatic self-energy

\[
\frac{E_{\text{Loop}}}{E_{\text{EM}}} \simeq \frac{\alpha}{4\pi} \log(m_e^2 R_0^2) \approx 10^{-3}
\]

There are of course additional (and potentially much larger) corrections coming from QCD and quark loops. Nonetheless, the above contribution from QED alone, which constitutes an \(\mathcal{O}(10^{-6})\) fraction of the inertial mass of the nucleus, is enough to provide a significant bound:

\[
|\eta_{\oplus,\text{Be-Ti}}| = \Delta \left(\frac{E_{\text{Loop}}}{m_{\text{atom}}} \frac{|g - g_{\text{Loop}}|}{g}\right) \lesssim 10^{-13}
\]
from element to element, we can establish a bound on electrons, as well as the nuclear binding energy, varies fractions simply because the ratio of nucleons to carry different nuclear (and therefore antimatter) energy nucleon. Making use of the fact that different elements as characterizing the antiquark energy fraction of a fractions of elements 

moment been measured at the percent level. We can take the corresponding parton distribution functions have 

This reasoning then yields a bound $|g - g_{\text{Loop}}|/g \lesssim 10^{-7}$ (7)

since the difference in the fractional contribution of $E_{\text{Loop}}$ to the inertial masses of beryllium and titanium is $\Delta (E_{\text{Loop}} / m_{\text{atom}}) \approx 10^{-6}$. Therefore the electron loop contribution to the electrostatic self-energy of the nucleus satisfies the equivalence principle to about one part in $10^9$. In exact analogy to the previous section, we expect $|g - g_{\text{Loop}}|/g$ to be related to $|g_1 - g_{\Pi}|/g_1$ by an $O(1)$ factor. This reasoning then yields a bound $|g_1 - g_{\Pi}|/g_1 \lesssim 10^{-7}$.

3.3. Antiquarks in nucleons

Deep inelastic scattering experiments have thoroughly established that the constituents of the proton and neutron include the antiquarks $\bar{u}, \bar{d}$, and $\bar{s}$. Furthermore, the corresponding parton distribution functions have been measured at the percent level. We can take the moment

$$\int_0^1 x\{\bar{u}(x) + \bar{d}(x) + \bar{s}(x)\}dx \approx 0.1 \quad (8)$$

as characterizing the antiquark energy fraction of a nucleon. Making use of the fact that different elements carry different nuclear (and therefore antimatter) energy fractions simply because the ratio of nucleons to electrons, as well as the nuclear binding energy, varies from element to element, we can establish a bound on $|g - g_\Pi|/g$. Consider two different elements A and B with inertial masses $m_A$ and $m_B$, respectively. The inequality of $g$ and $g_\Pi$ will drive the Eötvös parameter $\eta_{\Xi, A-B}$ away from zero:

$$|\eta_{\Xi, A-B}| = F_\Xi |F_A^\Xi - F_B^\Xi| \frac{|g - g_\Pi|}{g} \quad (9)$$

where $F_\Xi \sim 0.1$ is the antiquark energy fraction carried by a nucleon, and $F_A^\Xi$ and $F_B^\Xi$ are the nuclear mass fractions of elements $A$ and $B$, respectively. Since we have $|F_A^\Xi - F_B^\Xi| \sim 10^{-3}$, we obtain a bound $|g - g_\Pi|/g \lesssim 10^{-9}$. Therefore the antiquarks in atoms satisfy the equivalence principle to about one part in $10^9$. Since we are assuming that $|g - g_\Pi|/g$ is related to $|g_1 - g_{\Pi}|/g_1$ by an $O(1)$ factor, this yields a bound $|g_1 - g_{\Pi}|/g_1 \lesssim 10^{-9}$.

4. A FIFTH FORCE CANCELLED BY A SIXTH FORCE

Here we consider the possibility that there exist long range forces of (sub)gravitational strength mediated by scalar and/or vector particles. Since any new long range scalar or vector force by itself is constrained to be extremely weak (see e.g. [12, 13]), we consider situations in which a scalar-mediated force is approximately canceled by a vector-mediated force [11]. As discussed in the introduction, this approximate cancellation would still allow for a large deviation from $g_1 = g_{\Pi}$, since the force due to vector exchange becomes attractive when the test particle is an antiparticle.

In the following we leave aside any questions about the theoretical plausibility of this scenario—e.g. the level of fine-tuning required or whether some symmetry might enforce approximate cancellation—and establish bounds on the theory as given. In section 4.1 we obtain bounds on $|g_1 - g_{\Pi}|/g_1$ by considering the radiative damping of binary pulsar systems. These bounds have the virtue of holding irrespective of any precise cancellation. In section 4.2 we investigate scalar-vector scenarios in which approximate cancellation is possible and quantify the degree to which that cancellation fails. In section 4.3 we demonstrate that scalar forces cannot be arranged to exactly cancel against vector forces as a consequence of their different velocity dependence and quantify the degree of non-cancellation which necessarily results. In both sections 4.2 and 4.3 we use the computed degree of non-cancellation in conjunction with input from precision Eötvös experiments to place bounds on $|g_1 - g_{\Pi}|/g_1$.

Note that although the physical effects considered in section 3 have bearing on the composition dependence of scalar- and vector-mediated forces (since they contribute to the renormalization of the effective coupling constants), we do not explicitly explore this connection here, since the arguments presented below are better suited to constraining general combinations of long range scalar and vector forces.

4.1. Radiative damping of binary pulsar systems

Even supposing that scalar and vector forces could somehow be fine-tuned so as to evade detection in precision Eötvös experiments, no amount of fine-tuning can circumvent the fact that particles charged under long range forces can radiate energy. In particular binary pulsar systems can radiate off enough energy in the form of scalar or vector waves to modify their orbital decay at an observable level, provided that the
range of the scalar or vector force is somewhat larger than their orbital period, \( \lambda \gtrapprox P_b \), where typically \( P_b \approx 10^{12}\text{m} \approx 10^5R_\oplus \). The possibility of using binary pulsar systems to constrain long range scalar and vector forces was considered in [13], since which time the number of precisely measured binary systems has increased considerably. We consider two separate cases: (i) the vector couples to baryon number \( B \); and (ii) the vector couples to lepton number \( L \). In both cases the scalar force is assumed to cancel against the vector force, so that both forces couple to the same charge with identical strengths. By appealing to recent observational input we will be able to tightly constrain these two scenarios.

Strong bounds on the baryon number case can be obtained by considering dipole radiation. The ratio of the energy loss due to dipole radiation from a vector interaction, \( \langle \dot{E}_V \rangle \), to the energy loss due to gravitational quadrupole radiation as predicted by general relativity, \( \langle \dot{E}_{GR} \rangle \), is given by

\[
\frac{\langle \dot{E}_V \rangle}{\langle \dot{E}_{GR} \rangle} = \chi(m_V, \epsilon) \frac{\alpha_V}{\alpha_{GR}} \left[ \frac{\Delta (B/\mu)}{a^2 \omega^2} \right]^2 \frac{1}{a^2 \omega^2} \tag{10}
\]

where \( \alpha_V \equiv g_V^2/4\pi \) characterizes the strength of the vector interaction, \( \alpha_{GR} \equiv Gm_H^2/2 \) with \( m_H \) the mass of the hydrogen atom, \( B \) is the star’s baryon number, \( \mu \) is the star’s mass in units of \( m_H \), \( a \) is the semimajor axis of the relative orbit, \( \omega = 2\pi/P_b \) is the characteristic frequency of the system, and \( \chi(m_V, \epsilon) \) is a geometric factor that depends on the orbital eccentricity \( \epsilon \) and the mass of the vector particle \( m_V \) [14]. Here the term \( \Delta(B/\mu) \) characterizes the size of the baryonic dipole moment, and the factor of \( a^2 \omega^2 \) reflects the fact that this is a ratio of dipole radiated power to quadrupole radiated power. An analogous expression holds for scalar radiation

\[
\frac{\langle \dot{E}_S \rangle}{\langle \dot{E}_{GR} \rangle} = \frac{1}{2} \chi'(m_S, \epsilon) \frac{\alpha_S}{\alpha_{GR}} \left[ \frac{\Delta (B/\mu)}{a^2 \omega^2} \right]^2 \frac{1}{a^2 \omega^2} \tag{11}
\]

where the geometric factor \( \chi'(m_S, \epsilon) \) reduces to \( \chi(m_V, \epsilon) \) in the limit \( m_{V,S} \to 0 \).

In order to constrain the energy loss due to scalar and vector dipole radiation we relate the observed change of the orbital period (after correcting for various kinematics effects [15]) to the GR prediction:

\[
\frac{\langle \dot{E}_S \rangle + \langle \dot{E}_V \rangle}{\langle \dot{E}_{GR} \rangle} = 1 - \frac{\dot{P}_b^{GR}}{\dot{P}_b^{\text{intrinsic}}} \tag{12}
\]

which is valid for \( \dot{E}_{V,S} \ll \dot{E}_{GR} \). Going to the massless limit \( m_{V,S} \to 0 \) (which is a good approximation whenever the range of the scalar and vector interactions is somewhat larger than the orbital period), letting \( \alpha_S = \alpha_V = \alpha_{SV} \), and rewriting equations (10)-(12) in terms of the orbital period \( P \), and the advance of periastron \( \dot{\omega}_{GR} \), we obtain a bound

\[
\frac{\alpha_{SV}}{\alpha_{GR}} \leq \frac{16}{15\pi} f(\epsilon) \dot{\omega}_{GR} P_b \left( 1 - \frac{\dot{P}_b^{GR}}{P_b^{\text{intrinsic}}} \right) \left[ \Delta \left( \frac{B}{\mu} \right) \right]^{-2} \tag{13}
\]

where \( f(\epsilon) \) is a monotonic function that ranges from 1 to 2.95 as \( \epsilon \) varies from 0 to 1. As is clear from [13], the best bounds will come from binary systems that have large baryonic dipole moments. For that reason the binary system J1141-6545, which consists of a neutron star with a white dwarf companion, is a good candidate for our purposes. It has a large baryonic dipole moment because neutron stars have significant gravitational binding energy \( (B/\mu \approx 1.1) \) while white dwarves have negligible gravitational binding energy \( (B/\mu \approx 1) \). Using (13) and the observational input of [13], summarized in Table II, we arrive at a bound \( |g_H - g_B|/g_H \lesssim 10^{-4} \).

Bounds for the lepton number case can also be obtained in the same way, with \( B \) replaced by \( L \) in [13]. Here a system consisting of two neutron stars does not provide a good bound, since neutron stars carry few electrons. That neutron stars are electron poor, however, becomes useful when considering a system consisting of a neutron star and white dwarf, since for the white dwarf \( L/\mu \approx 0.5 \), which results in a large leptonic dipole moment. Appealing to the observational input from J1141-6545 we arrive at a bound \( |g_H - g_B|/g_H \lesssim 10^{-5} \).

Bounds for the baryon number case can also be obtained by considering quadrupole radiation, since even in the case when the binary system has \( \Delta(B/\mu) = 0 \), there is a nonvanishing quadrupole moment that changes in time. When the scalar or vector forces are of gravitational strength, the energy loss due to scalar or vector quadrupole radiation is comparable to that due to gravitational wave emission, since they are all sourced by essentially the same multipole moment. Since the GR prediction accounts for the observed orbital decay in the binary pulsar system B1913+16 to within about 0.3% [10], we obtain a bound \( |g_H - g_Q|/g_H \lesssim 10^{-3} \).

Thus radiative damping of binary pulsar systems alone places tight bounds on \( |g_H - g_Q|/g_H \) in the scalar-vector scenario. These bounds are robust and can only be evaded (while simultaneously keeping experiments sourced by the earth relevant) by requiring the range of the scalar and vector forces to sit somewhere in the window \( R_\oplus \lesssim \lambda \lesssim 10^5R_\oplus \), which corresponds to a mass

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**TABLE II: Observed and inferred orbital parameters of J1141-6545 [15]**

| Parameter                        | Measured Value |
|----------------------------------|----------------|
| Orbital Period \( P_b \)         | 0.1976509593(1) days |
| Eccentricity \( \epsilon \)     | 0.171884(2)     |
| Advance of Periastron \( \dot{\omega}_{GR} \) | 5.3096(4) deg yr\(^{-1}\) |
| Observed Period Derivative \( \dot{P}_b^{obs} \) | \(-0.403(25) \times 10^{-12}\) |
| Intrinsic Period Derivative \( \dot{P}_b^{\text{intrinsic}} \) | \(-0.401(25) \times 10^{-12}\) |
| Ratio of \( \dot{P}_b^{\text{intrinsic}} \) to GR prediction | 1.04(6) |
range $10^{-19}$ eV $\lesssim m \lesssim 10^{-13}$ eV. Quite interestingly for scalar particles this range of masses can be probed by a super-radiant instability of rotating black holes [17].

4.2. Scalars charges are not vector charges

Let us return to the possibility of approximate cancellation between scalar and vector forces in matter-matter interactions. For concreteness, let us first consider a scalar particle coupled to the trace of the energy momentum tensor $T^\mu_\mu$ (so that the scalar couples to mass) and a vector particle coupled to baryon number $B$, where the strengths of both couplings have been adjusted so as to achieve approximate cancellation and both forces have the same range $\lambda \gtrsim R_{\phi}$. Approximate cancellation is possible, since across the periodic table the ratio $B/\mu$ (where $\mu$ is the inertial mass in atomic mass units) is approximately constant: $B/\mu \approx 1$. The typical variation from element to element is $\Delta(B/\mu) \approx O(10^{-3} - 10^{-4})$. This cancellation cannot be made exact, however, since there is no mechanism by which the scalar can couple precisely to $B$. Thus $\eta$ between any two given elements will be nonzero:

$$|\eta| = \Delta \left( \frac{B}{\mu} \right) \frac{|g_H - g_{\Pi}|}{2g_H}.$$  

Using the experimental input from section 2, this reasoning yields a bound $|g_H - g_{\Pi}|/g_H \lesssim 10^{-8}$. If the vector instead couples to lepton number $L$, one obtains a more stringent bound $|g_H - g_{\Pi}|/g_H \lesssim 10^{-10}$, since the ratio $L/\mu$ varies more strongly across the periodic table: $\Delta(L/\mu) \approx O(10^{-1} - 10^{-2})$. Bounds of similar order of magnitude or better will hold for other scenarios [12], e.g. if the scalar couples primarily to glue ($L \supset \phi \text{Tr} F_{\mu\nu}^2 \Phi_{CD}$). Even allowing for the possibility that one introduces additional adjustable parameters corresponding to additional interactions (e.g. non-renormalizable vector interactions or a scalar coupling to the photon field strength squared) to improve the cancellation between the elements that have been tested in Eötvös experiments, it is unlikely that the degree of cancellation will be such that the above bounds on $|g_H - g_{\Pi}|/g_H$ will be substantially weakened—even allowing for incredible fine-tuning. In any case we need not consider the entire spectrum of possible interactions, since the next section we will give a robust argument that does not depend on the particular form of the interactions.

4.3. Bounds from the velocity dependence of scalar and vector forces

Further limits on any cancellation between scalar and vector interactions can be obtained by considering how the corresponding forces transform differently under Lorentz boosts. In particular for two pointlike particles the magnitude of the inverse-square force mediated by the vector is larger than that mediated by the scalar by a relative factor $u_1 \cdot u_2$, where the $u_i$ are the four-velocities of the particles [19]. Hence any cancellation between the two forces in the static limit will be undone in the nonstatic case. Therefore in the case where the vector couples to some linear combination of $B$ and $L$ (and the scalar interaction somehow tracks the same linear combination to some high precision), the motion of nucleons within the nucleus and electrons within the atom places limits on the precision of the would-be cancellation, since the average velocities of the nucleons and electrons will vary from element to element. Within the Fermi gas model we can calculate the average kinetic energy of a nucleon, inside a nucleus consisting of $Z$ protons and $N$ neutrons, to be

$$\langle E_{\text{kin}} \rangle = (31 \text{ MeV}) \frac{Z^{5/3} + N^{5/3}}{(Z + N)^{5/3}}$$  

This corresponds to an average Lorentz boost factor

$$\langle \gamma - 1 \rangle = \left( \frac{1}{2} u^2 \right) = (3 \times 10^{-2}) \frac{Z^{5/3} + N^{5/3}}{(Z + N)^{5/3}}.$$  

After averaging over the velocities of the nucleons within a nucleus the effect of the factor $u_1 \cdot u_2$ is to introduce a relative factor of $\langle \gamma \rangle$ between the magnitudes of the scalar- and vector-mediated forces. Note that the quantity $Z^{5/3} + N^{5/3}$ varies at the $10^{-3}$ level between elements. Combined with the experimental input from section 3, this yields a bound $|g_H - g_{\Pi}|/g_H \lesssim 10^{-7}$ for the case where the vector couples primarily to $B$. The same reasoning applies in the case where the vector couples primarily to $L$. Here one obtains a bound $|g_H - g_{\Pi}|/g_H \lesssim 10^{-9}$, which is more stringent than in the former case because electron velocities vary more from element to element than nucleon velocities. These bounds are robust and cannot be simply evaded by postulating further scalar and/or vector interactions and further fine-tuning. Since these bounds are derived by considering the kinetic energies of nucleons and electrons within atoms, one might imagine that they could in principle be weakened somewhat with the addition of scalar couplings to field strengths squared, since these approximately track the binding energy and therefore (by virial theorems) the kinetic energies of the constituent nucleons and electrons [1]. But since the ratio of nuclear (and electronic) binding energy to kinetic energy varies significantly from element to element, the inclusion of such interaction terms does nothing to weaken the above bounds.

The above considerations are just one of the many effects that contribute to the renormalization of the effective scalar and vector charges of atoms. The implication of these effects is that even if cancellation could be achieved at the level of electrons, protons, and neutrons, that cancellation would necessarily be undone as one descends to the (relevant) effective theory in which atoms are the degrees of freedom. As a consequence composition dependence is generic in the scalar-vector scenario.
TABLE III: Summary of bounds. See section 3 for details on bounds on the scenario where GR is modified and section 4 for details on bounds on the scalar-vector scenario.

| Scenario                | Argument                          | Bound on $|\rho_H - \sigma_T|/\rho_H$ |
|-------------------------|-----------------------------------|-------------------------------------|
| Modification of GR      | Lamb shift                        | $\lesssim 10^{-2}$                   |
|                         | Electrostatic self-energies of nuclei | $\lesssim 10^{-7}$                   |
|                         | Antiquarks in nucleons             | $\lesssim 10^{-9}$                   |
| Scalar-vector           | Radiative damping of binary systems| $\lesssim 10^{-4}$                   |
|                         | Scalar charges are not vector charges | $\lesssim 10^{-8}$                   |
|                         | Velocity dependence                | $\lesssim 10^{-7}$                   |

5. SUMMARY

The discussion in this paper was motivated by recent experimental proposals to test for violations of the equivalence principle in the free fall acceleration of antihydrogen in the gravitational field of the earth. Focusing our attention on two different scenarios for how such gravitational asymmetry between matter and antimatter might be realized, we established a number of strict bounds, which are collected in Table III.

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