Stirring of liquid metal in electrovortex flow in a hemispherical volume under the influence of external magnetic field

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Abstract. The results of calculations of the stirring rate of liquid metal in a system simulating a DC arc furnace are presented. The equations of motion and transfer of impurity were solved for the electric vortex flow of a liquid metal between two hemispherical electrodes under the influence of an external axial magnetic field. The quality of mixing was characterized by the deviation of the impurity concentration from the mean and was calculated at each time step. It is shown that the effect of an external magnetic field can both accelerate and slow down mixing. These results can be used in electrometallurgy to assess the effectiveness of electromagnetic stirring.

1. Introduction

In metallurgy, stirring is used to equalize the chemical composition and temperature by the volume of the melting bath [1]. In principle, an electric arc furnace consists of a melting pool with connected electrodes. The electric arc between the electrode and the surface heats the metal and the flow caused by the electromagnetic force appears inside the bath (electrovortex flow, EVF [2]) The intensity of this flow depends only on the electric current passing through the bath and it is interesting to consider the problem of controlling the flow with an external magnetic field and to study its effect on stirring. We will consider a model problem about flow in a hemispherical container with a central small electrode because this geometry is very close to the configuration of the industrial DC-arc furnaces [3-9].

Let us consider the system with the electric current propagating from the small hemispherical electrode through the liquid metal to the wall of the big hemispherical electrode. Electromagnetic force \( F = J \times B \) (where \( J \) is the current density and \( B \) is the own magnetic field) leads the liquid to move.

![Figure 1. Elecrovortex structure. 1 - small electrode, 2 - liquid metal, 3 - big electrode. a) Toroidal EVF; b) occurrence of the azimuthal swirl; c) occurrence of the secondary vortex; d) double-vortices rotating EVF.](image-url)
In such an axisymmetric system, without the influence of external axial magnetic fields, EVF has the form of a toroidal vortex (Fig. 1a). The influence of the axial external magnetic field causes the azimuthal swirl of the flow (Fig. 1b). Rotation tends to throw the liquid to the periphery, but the wall (including bottom area in hemisphere) breaks the flow. The regions in the middle of the volume and at the free surface remain not braked, thus a flow directed from the bottom upwards and to the side wall develops. This flow is vortex rotating in the opposite direction to the main EVF (figure 1d). This secondary vortex pushes the EVF towards the periphery of the bath. The azimuthal velocity leads to significant redistribution of axial and radial velocity components in liquid metal [10].

The present paper is devoted to the investigation of the stirring process in the liquid metal volume under action of the electrovortex flow with the external magnetic field.

2. Calculation method
To study stirring, we used the neutral impurity method. The essence of the method lies in the fact that we introduce impurity (In-Ga-Sn-2, Fig. 2a) with physical properties similar to the properties of the main substance (In-Ga-Sn-1, Fig. 2a) into the investigated volume. This approach makes it possible to concentrate solely on hydrodynamics to solve motion equations with constant properties. By changing the impurity concentration, conclusions can be drawn about the effectiveness of the mixing process.

It is necessary to solve the impurity transfer equation:

\[
\frac{\partial C}{\partial t} + \nabla \cdot (U C) = D \Delta C. \tag{1}
\]

Here \( D \) is the self-diffusion coefficient, and \( C \) is the mass fraction of the added substance. The self-diffusion coefficient of the alloy In-Ga-Sn can be estimated as the average of the self-diffusion coefficients of the components and was taken as \( D = 5 \times 10^{-9} \text{ m}^2/\text{s} \). The boundary condition for concentration is zero flux on the walls and surface, \((\partial C/\partial n)=0\).

To determine quality of stirring, we use the following criterion [11]:

\[
K_{st} = \frac{1}{C(1-C)V} \sum_{i=1}^{N} V_i (C_i - \overline{C})^2, \overline{C} = \frac{1}{V} \sum_{i=1}^{N} C_i V_i, \tag{2}
\]

Where \( C \) is the average mass fraction, \( V_i \) is the volume of \( i \)-cell, \( V \) is the full volume of the liquid. This criterion is essentially normalized variance.

The velocity field was determined by solving the motion equation with added electromagnetic force. The Navier-Stokes equation has the form:

\[
\rho \left( \frac{\partial U}{\partial t} + (U \nabla)U \right) = -\nabla p + \rho \nu \Delta U + F, \tag{3}
\]

where \( U \) is the velocity of the liquid, \( \rho \) is the density of the liquid, \( \nu \) is the kinematic viscosity coefficient, \( p \) is the pressure, and \( F \) is the electromagnetic force.

Electromagnetic force is:
\[ F = J \times B, \] (4)

where \( J \) is a current density and \( B \) – a magnetic field. Here \( B \) is the sum of the self-magnetic field of current \( (B_{\text{EVF}}) \) and external magnetic fields \( (B_{\text{ext}}) \) in a case when they are present.

\[ B = B_{\text{EVF}} + B_{\text{ext}}, \] (5)

\[ F = J \times (B_{\text{EVF}} + B_{\text{ext}}), \] (6)

and also we use so-called electrodynamic approximation when we neglect induced currents \( \sigma U \times B \) in the Ohm’s law.

The expressions for \( J \) and \( B_{\text{EVF}} \) can be easily obtained analytically. In spherical coordinates \( J \) and \( B \) have only the components (7,8).

\[ J_R = \frac{l}{2\pi R^2}, \] (7)

\[ B_{\text{EVF}} = \frac{\mu_0 I (1 - \cos \theta)}{2\pi R \sin \theta}, \] (8)

so the only component of the electromagnetic force is:

\[ F_\theta = \frac{\mu_0 I^2 (1 - \cos \theta)}{4\pi R^2 \sin \theta}. \] (9)

The external axial field \( B_z \) (which can be created by a solenoid at our installation) interacts with the radial component of the current and leads to the appearance of an azimuthal force (10):

\[ F_\phi = -\frac{\mu_0 I B_z \sin \theta}{2\pi R^2}. \] (10)

We used the finite volume method on an unstructured 2D axisymmetric grid (figure 2a) in cylindrical coordinates \((\partial.../\partial \phi = 0, U_{\phi} = 0)\). The boundary conditions for velocity are: \( U = 0 \) at the wall (both electrodes), and for shear stresses: \( \partial U / \partial z = 0 \) at the surface. Calculation area was a quarter of circle with internal radius \( R_1 = 2.5 \) mm and external radius \( R_2 = 94 \) mm and consisted of 5000 quadrangular cells. The symmetry axis coincided with the \( z \) axis and free surface coincided with the \( r \) axis direction. This area corresponds to the experimental setup in JIHT RAS [10] consisting of the big hemispherical electrode filled with eutectic alloy In-Ga-Sn (weight content: \( \text{Ga-67\%}, \text{In-20.55\%}, \text{Sn-12.5\%} \), physical properties: melting point \(+10.5^\circ\text{C}, \rho = 6482 \text{ kg/m}^3, \nu = 4.3e^{-7} \text{ m}^2/\text{s}, \sigma = 3.3e6 \text{ S}, [12])\), and the immersed small electrode and solenoid around the container creating a magnetic field of up to 0.1 T. In our calculations we used the same physical properties.

Here are the expressions for the components of the force in the cylindrical coordinates (10) which were used in calculations:

\[ F_r = \frac{\mu_0 I^2 z \sqrt{r^2 + z^2} - z}{4\pi^2 r (r^2 + z^2)^{3/2}}, \] (11)
\[
F_\varphi = \frac{IB.r}{\pi(r^2 + z^2)^{3/2}}, \quad (12)
\]
\[
F_z = -\frac{\mu_0 I^2(\sqrt{r^2 + z^2} - z)}{4\pi^2 (r^2 + z^2)^2}. \quad (13)
\]

Figure 2. a) Geometric scheme of the working area and the position of the impurity at the initial moment of time, b) 2D axisymmetric grid of the working area

3. Results

3.1. Pure diffusion
As a test, the problem of pure diffusion (when \(U=0\)) was solved firstly, i.e. in the absence of flow. In the volume, a region filled with an impurity was set (alloy In-Ga-Sn-2 with the same physical properties), the impurity was immersed into the cylindrical region on the axis at a distance of 35 mm from the surface. Geometric characteristics of the area are shown in the Fig. 2b.

The concentration distribution during diffusion normalized to the maximum is shown in Fig. 3. It is obvious that the process is very slow and this problem has only methodological value.

3.2. Diffusion and convection. Constant external magnetic field.
A numerical study of the change in the impurity concentration was carried out at a current of 400 A. The area with impurities was taken as in the previous problem with pure diffusion. The picture on Figure 4 shows a dependence of the stirring criterion on time with different external magnetic fields.
Figure 3. Distribution of the concentration on the time. a) 0 s, b) 1e4 s, c) 2e4, d) 3e4, e) 4e4, f) 5e4.

Figure 4. Distribution of the concentration relative to time with EVF and without external magnetic field. a) 0 s, b) 5 s, c) 10 s, d) 20 s, e) 50 s, f) 100 s.

It can be seen from the Figure 5 that a weak external magnetic field (1e–4 T) suppresses EVF and slows down mixing compared to pure EVF, despite the created rotation. The relatively strong magnetic field $B_z=1e-2$ T creates a rotation and intense upward flow, which leads to acceleration of the stirring (the rate of reduction of the stirring criterion is the highest).
3.3. Pulsed external magnetic field.

The calculations were also carried out for the pulsed external field. The external field $B_z$ was set as a piecewise linear function, with a pulse duration equal to half of the entire period and with amplitude of $5 \times 10^{-3}$ T. The modes with different frequencies $f = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6$ Hz were calculated at a current of 400 A. These frequencies were chosen as the most interesting: effect of self oscillations can be observed at these frequencies [13]. Fig. 6 shows the dependence of the stirring criterion on time for all cases and a comparison with the mode of the constant external field and its absence.

4. Conclusions

The results of calculations showed that in our conditions the use of a constant external magnetic field is the most effective. It is the impact of a constant external field that ensures the most rapid leveling of the concentration of impurities in the volume. The field must be strong enough to change the main flow...
direction and avoid the formation of stagnant zones. Using the pulsed magnetic field does not increase stirring effectiveness.

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