We look at buildings’ competition over space in cities through the lens of ecology. Adopting the convex hull of the building’s footprint perimeter as a definition of species yields parallels to forest trees’ competition, which we expound on. Their perimeter distribution $p(r)$ follows a power-law behavior beyond a critical threshold of the density of the built environment. In this regime, the species coexistence likelihood $p(d)$, where $d$ is the distance to the nearest competitor, which we define to be a building with a larger $r$, bifurcates with the buildings’ number $n$. This reveals two different predation laws: a vicious predatory one which is linked spatial homogeneity and segregation, as opposed to another favoring spatial diversity and intermixing between species.

**One Sentence Summary:** We reveal different predation rules governing the spatial distribution of buildings’ sizes seen as contending species in cities.
Introduction

The notion of scaling has been extensively invoked in order to gain a quantitative understanding of out-of-equilibrium systems and to infer relations between their underlying structures and dynamics (1–11). It has been examined in a wide range of systems, namely biological organisms and cities. Further, the analogy between the latter two was explored; cities were described as living systems with corresponding metaphorical urban ecosystems and metabolisms (12–18). This established correspondence raised the question of whether urban systems are governed by similar principles and evolve under similar constraints as their biological counterparts (19, 20). Endeavors to answer this question showed that the interplay between size and abundance in ecological communities as well as in urban environments is manifested through the emergence of scaling laws or self-similar patterns relating growth metrics to size (21–25). However, this systems-of-cities approach blurs the spatial structure within cities and zooms out to study them as points in the size versus population/income/employment space (26). Conversely to this inter-cities approach there are numerous measures to assess the city’s internal spatial organization, which serve as proxies to determine its level of connectivity, resilience, accessibility, sustainability, and livability (10, 27–31). These span: (i) information theoretic metrics, such as Shannon’s information entropy, which reveal the spatial embedding of urban design, (ii) measures of self-similarity and fractal dimensions, which uncover the interdependence between physical structure and topological arrangement, (iii) geometrical and topological characteristics of urban networks. Notions like atomic-scale structure, borrowed from condensed matter physics, are also used as identifiers to provide additional insight into a city’s geometrical patterns and texture (32–34). The ensuing spatial order of buildings, which is of particular interest to us in this work and is quantified by the above measures, was shown to be associated with the interaction of microeconomic forces, urban design, as well as geometrical constraints (29, 35).
City growth, an example of what Mitzenmacher terms ‘multiplicative processes’, can generate lognormal or power-law distributions and scaling laws (36, 37). When resources are limited, sustainable distributions of city components, buildings in our case, seem to be those which permit the presence of a small number of large buildings as opposed to a large number of small ones. Particularly, the buildings’ size distribution, which is a power-law, is a signature of such an evolving competitive process (38).

Our study of intra-city dynamics, inspired by (22) and by similarities of form between urban and ecosystems systems drawn by Wilson (39, 40), investigates competition rules between buildings’ species. Speciation is defined as footprint perimeter, and a competitor is understood as a building with a larger perimeter. Competition is studied through the distribution of the distance to the nearest competitors. A city where competition for space is weak entails a mixture of building species across the urban fabric, whereas fierce competition tends to homogenize species distribution into segregated neighborhoods. We also compute buildings’ orientations and consequently the entropy associated with their directionality, and the length of the roads. Relying on these metrics we identify two distinct scaling regimes characterizing two different “predatory rules” between buildings.

Results and Discussion

The distributions $p(r)$, where $r$ is the perimeter of the convex hull of a building’s footprint, shown in Figure S1, were followed for a sample of 1,500 cities in the US using OpenStreetMap data. For a critical number of buildings $n_c$ the distributions followed a power law-behavior. We restrict our analysis to those cities whose $n > n_c$ where the power law holds. For those cities the distributions to the nearest competitor $p(d)$, where $d$ is the inter-competitors distance taken here to be a building with a radius larger than $r$ where computed. There cumulative distributions $P(d)$ followed a power law whose lower cutoff we denote by $d_{\text{min}}$ and exponent
\( \gamma + 1 \), which turns out to be clustered at 2.9 and 1.8. This correspond to a super-linear and sub-linear probability distributions \( p(d) \) with \( \gamma = 1.9 \) and 0.8 respectively as shown in Figure 1. The lower cluster is commensurate with the results of forest trees’ scaling laws of (22), which we suspect to be an “organic setting” where species intermix in space whereas the higher value of \( \gamma \) is a more “discriminatory arrangement”. To validate our claim we follow \( \phi = d_{\text{min}}^\gamma \) as a function of \( n \) as shown in Figure 2a. The red dashed line is the threshold \( n_c = e^{4.7} \) below which \( p(r) \) and \( p(d) \) fail the power-law test. Beyond that, the bifurcation curve’s upper branch, with high values of \( \phi \) compared to the lower branch, corresponds to a setting where competing species are distant. Large \( \phi \) can be traced back to, although not solely, a large value of \( d_{\text{min}} \) which is indicative of local spatial homogeneity in building size where inter-competitor distance is large or is due to a large city extent equally leading to high \( d_{\text{min}} \). The first is confirmed in the example of Arcadia and Tuscon, where the buildings sizes cluster homogeneously in space leading to a high value of \( d_{\text{min}} \); they both belong to the upper branch of \( \phi \). Conversely, Miami and Largo, belonging to the lower branch of \( \phi \), exhibit high spatial mixing between species reflected by a low value of \( d_{\text{min}} \). Their buildings’ footprints are shown in the Supplementary Material in Figures S3-S10, while the annotated version of Figure 2a is shown in Figure S2. Additionally, the effect of city size is measured by renormalizing \( d_{\text{min}} \) by \( L \), where \( L \) is the city’s street length; it serves as a proxy to inter-buildings’ distance. \( (d_{\text{min}}/L)\gamma \) exhibits a linear dependence on \( n \) on a log-log scale as shown in Figure 2b, which confirms the additional dependence of the branchings on city size.

Further, the behavior of buildings’ size and orientation entropies, defined in the Supplementary Material, as a function of \( n \), are shown in Figures 3a and 3b respectively. It reveals that buildings’ size entropy \( S_{\text{size}} \), is maximum at \( n_c \), beyond which the constraints on allowed buildings sizes increase; this reflects a tendency towards size homogeneity. For this range the buildings’ orientation entropy \( S \) is also near constant. Below \( n_c \), \( S_{\text{size}} \) and \( S \) increase, which
corresponds to sparse cities with small number of buildings free to orient along any direction with no constraints on their sizes.

Moreover, $\dot{S}_{size}$ is plotted on a semi-logarithmic scale as shown in Figure 4. We note that the $d\dot{S}_{size}/dn = -\mu/T$, where $\mu$ is the chemical potential and $T$ is the city’s “temperature”; it is given by:

$$d\dot{S}_{size}/dn = -\mu/T = \begin{cases} 0.54 \log n + 0.92 & n < n_c \\ -0.22 \log n + 4.38 & n \geq n_c \end{cases} \quad (1)$$

Since $\mu$ measures the necessary work to change the number of “particles”, in this case buildings, by $dn$, or equivalently the system’s resistance to adding an extra building, we conclude that below $n_c$, when $d\dot{S}_{size}/dn$ increases, $\mu/T$ decreases; beyond that the city becomes resistant to the addition of buildings; that is to say the construction of more buildings injects order into the city, which might be local or global.

Conclusively, this discontinuity in entropy, is signature of a second-order phase transition, which is a measure that differentiates between “nascent-planning-free towns” and “planned cities” in both their mixed and segregated states corresponding to both branches of $\phi$. However, the observed increase in orientation entropy beyond $n_c$ does not reflect the spatial distribution of order and thus is not able to reflect the difference between the upper and lower branches of $\phi$.

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**Authors’ Contributions**

S.N conceptualized the idea. The data was curated by T.T and S.N. The methodology was designed by M.G and S.N. The investigation was carried out by T.T and S.N., while the analysis was carried out by M.G and S.N. The authors equally contributed in the write up of the manuscript.
Supplementary materials

Material and Methods

We adopt the following definition for building species which are distinguished by the perimeter of the convex hull of its footprint $r$, which is equivalent to the notion of the tree-crown in (22). It is a preferable alternative to the footprint perimeter since the latter does not necessarily define a no contact space as shown in Figure S1. In what follows the terms perimeter, radius, and size will be used interchangeably and are equivalent as far as scaling relations are concerned.

We compute the buildings’ sizes and their distributions $p(r)$ and subsequently we evaluate the distribution of distances to nearest competitor for a sample of 1,500 cities in the US using their corresponding OpenStreetMap data, which is to the best of our computational capabilities. The size distribution of buildings, given by:

\[ p(r) \propto \left( \frac{r}{r_{\text{min}}} \right)^{-\alpha}, \]  

where $r_{\text{min}}$ and $\alpha$ are respectively the lower cutoff and the exponent of the cumulative distribution function $P(r)$ were computed using the poweRlaw package in R. In what follows, we restrict our analysis to the cities whose $P(r)$ and thus $p(r)$ pass the power law test.

Given the inter-competitors distances $d$, taken here to be a building with a radius larger than $r$, allows us to calculate $p(d|r)$ the conditional distribution and subsequently the non-conditional distributions of competitors $p(d)$. For cities with number of buildings $n > n_c$, $p(d)$ followed a power-law given by:

\[ p(d) \propto \left( \frac{d}{d_{\text{min}}} \right)^{-\gamma}, \]  

where $d_{\text{min}}$ is the lower-cutoff distance of the power law. These exponents were retrieved from the applying the power law test on the cumulative distribution $P(d)$. We test the effect of city size on $p(d)$ through $n, L$. We note that the combination $\phi = d_{\text{min}}^{-\gamma}$ completely characterizes
$p(d)$, which we follow as function of $n$ and $L$. Additionally, the effect of city size is measured by renormalizing $d_{\text{min}}$ by $L$, since the street length is a proxy to inter-buildings’ distance.

For each city we additionally compute its total streets length $L$, its buildings’ size and the orientation entropy respectively given by:

$$S_{\text{size}} = - \sum_{i}^{N} p_i \log p_i, \quad (4)$$

where $p_i$ is the probability of a buildings to have a size $i$.

$$S = - \sum_{i}^{N} p_i \log p_i, \quad (5)$$

where $p_i$ is the probability of a buildings be orientated along $i$.

We looked at the averaged $\bar{S}_{\text{size}}$ as a function of $n$ and $L$. We report the significant dependencies in the results section.

**Fig S1**

In red is the convex hull around a building’s footprint shown in black.

**Fig S2**

The figure shows the annotated cities’ bifurcation diagram of $d_{\text{min}}^{\gamma}$ versus $n$ on a log-log scale.

**Fig S3**

Tucson-Arizona’s buildings’ footprints.

**Fig S4**

Athens-Alabama’s buildings’ footprints.
Fig S5
Arcadia-California’s buildings’ footprints.

Fig S6
Miami-Florida’s buildings’ footprints.

Fig S7
Largo-Florida’s buildings’ footprints.

Fig S8
Roswell-Georgia’s buildings’ footprints.

Fig S9
Roseville-California’s buildings’ footprints.

Fig S10
Monterey-California’s buildings’ footprints.
Figure S1: In red is the convex hull around a building’s footprint shown in black.

Figure S2: The figure shows the annotated cities’ bifurcation diagram of $d_{\text{min}}$ versus $n$ on a log-log scale.