Forces acting on particles in a Pelton bucket and similarity considerations for erosion

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Abstract. High sediment transport rates cause severe erosion issues in hydropower plants leading to interruptions in power generation, decrease in efficiency and shutdown for repair and maintenance. For Pelton turbines operating at high head, the issue of erosion is severe, especially in components like buckets, nozzle rings and needles. Goal of the study is to develop erosion focussed guidelines for both designing as well as operating hydropower plants with Pelton runners. In this study, the flow of sediment inside a Pelton bucket with respect to forces acting on solid particles is analysed with an analytical approach by considering different dynamic forces originating from the rotation of the turbine, the curvature of the buckets, and the Coriolis effect. Further, the path of sediment particles and its effect on erosion phenomena are analysed based on the process of separation of different sized sediment particles from streamlines.

The data relating to head, power, discharge, number of jet and efficiency of 250 hydropower plants installed all over the world were analysed in this study to find the major factors related to erosion in Pelton turbine bucket. From analysis of different force ratios, it is found that an increase of D/B, i.e. the ratio of pitch circle diameter and bucket width, and/or decrease of specific speed (nq) enhances erosion. As the erosion process depends significantly on non-dimensional parameters D/B and nq, these are considered as similarity measures for scaling of the erosion process in the Pelton buckets of various sizes.

1. Introduction

Sediment erosion in hydropower components e.g. turbine blades, buckets, needles, penstokes etc. is a widely encountered problem in hydropower plants. This leads to loss in power generation due to decrease in efficiency and shutdowns for repair and maintenance. Catastrophic failures of components due to reduced mechanical strength because of erosion may lead to higher losses/consequences. Due to the high acceleration and curvature of the flow in injectors and buckets, Pelton turbine components are very susceptible to erosion. Though the main parameters, like sediment properties (concentration, size, shape and hardness), turbine surface properties (hardness, toughness etc.) and flow properties (velocity, angle of attack etc.) contributing to erosion are known, the quantitative influence is still not fully understood [1, 2].

It is essential to analyse the flow inside Pelton buckets to understand the basic cause and underlying phenomenon of erosion in Pelton buckets. Flow in Pelton buckets has been analysed using different approaches by researchers. Brekke et al. [3] used stroboscopic photos to analyse the flow and the shape of the water surface in a Pelton bucket. They evaluated the resultant acceleration normal to the surface. They reported accelerations normal to the surface in the bucket with magnitude as high as 50 000 to 100 000 m/s² and claimed that this high acceleration causes separation of the sediment particles’ trajectories from streamlines bringing them in a collision course towards the bucket surface [4]. Bovet [5] considered the frictional effect to be the prominent contributor to erosion and developed...
a mathematical expression for the same. He analytically explained the separation of sediment particles from streamlines and explained with case studies from hydropower plants.

Neopane [6,7] used a numerical approach whereas Thapa [8] used an experimental approach to analyse the forces on contained sediment particles and the subsequent erosion effect in Francis turbines. Recently, researchers [6, 9, 10] have numerically simulated erosion in a Francis turbine to study the erosion on guide vanes, turbine blades etc. Erosion on Pelton turbine parts is mainly studied by observation and measurements in HPPs (at full-scale/at prototypes) [11, 12].

Thapa and Brekke [13] experimentally supported the concept of sediment particle separation considering particles of various sizes. They used, in stationary experiments, samples of different curvatures to simulate flow inside Pelton bucket. They identified that the splitter was eroded due to larger particles whereas smaller particles eroded the outer part of the buckets. Visual observation and surface roughness were used to quantify the erosion in the different zones of the buckets. They supported these findings from three case studies in hydropower plants with particles of various sizes. Padhy and Saini [14] supported these findings using analysis from scanning electron microscope (SEM) photographs.

The erosion presented in this article is from force consideration on sediment particles in Pelton bucket. The sediment properties taken into account in this study are concentration and the size of the particles, which are considered spherical in shape. Other sediment properties like shape and hardness are not considered. The erosion of a surface depends significantly on the velocity of the flow/particle on/over the surface. In case of a Pelton turbine, the velocity representative for erosion is the relative velocity of the flow in the buckets, which is approximately constant in magnitude and amounts to about half the absolute jet velocity at the nozzle exit.

In this study, an analytical approach is adopted to support and add to the findings of Thapa and Brekke [13]. The forces on sediment particle in a Pelton bucket are analysed to investigate hydro-abrasive erosion. For this purpose data related to turbine design parameters from around 250 hydropower plants are collected and studied to derive the parameters contributing significantly to the erosion process.

2. Methodology

2.1 Data Treatment to Calculate Pelton Bucket Dimensions

Raw data like head, power, rotational speed, number of jets, discharge, pitch circle diameter (D), turbine axis orientation (horizontal/vertical) and year of commissioning of a Pelton turbine were obtained by contacting different manufacturers, power plant operators, searching relevant literature and brochures of HPPs. The 150 hydropower plant cases, where design discharge was not available within the raw data mentioned above, the missing discharge was obtained initially by assuming an efficiency of 88% as to get an initial estimate of design discharge. This initial estimate of design discharge is then used in method proposed by Gordon [17] to obtain better estimate of design efficiency and discharge.

\[
\eta_{\text{peak}} = A - \left( \frac{1998 - y}{240} \right) - \left( \frac{17.5 - n_q}{185} \right) + 0.1d_{\text{jet}}^2 \quad (1)
\]

where,

\[
d_{\text{jet}} = \frac{0.544Q^{0.5}}{H^{0.25}} \quad (2)
\]

\[
n_q = n_{q1\text{jet}} = \frac{n}{H_{\text{rated}}^{0.75}} \quad (3)
\]

\[
\eta_{\text{peak}} = 0.917 \quad \text{and} \quad 0.9085 \quad \text{for vertical and horizontal axis units respectively.} \quad y = \text{Year of installation/commissioning (After 1998, all years must be 1998).} \quad \eta_{\text{peak}} \quad \text{is the efficiency at the peak point,} \quad n_q = n_{q1\text{jet}} \quad \text{is the specific speed of the Pelton turbine (defined with the discharge of one jet),} \quad d_{\text{jet}} \quad \text{is the jet diameter in m,} \quad H = H_{\text{rated}} \quad \text{is rated head in m,} \quad Q \quad \text{is rated flow in m}^3/\text{s,} \quad Z_0 \quad \text{is number of jets and} \quad n \quad \text{is rotational speed of the turbine in rev/min.} \]

2
The obtained design discharge and efficiency values are used for basic dimensions/parameters calculations, such as jet diameter, jet velocity, peripheral velocity and bucket width – as per hydraulic design equations for Pelton turbines [18].

Jet velocity (m/sec), 
\[ C_1 = \sqrt{2gH} \]  
(4)

Peripheral velocity, 
\[ \frac{Q_{jet}}{\pi C_1 Z_0} \]  
(5)

where \( g \) is acceleration due to gravity, the value of it is taken as 9.81 m/sec\(^2\).

Circumferential velocity at the pitch circle diameter (PCD) is given by Eq. (6).

Circumferential velocity, \( \nu = \frac{\pi Dn}{60} \)  
(6)

where \( D \) is the PCD of the Pelton runner in m.

Bucket width [18], \( B = 3.1d_{jet} \) for \( Z_0 = 1 \), \( B = 3.2d_{jet} \) for \( Z_0 = 2 \), \( B = 3.3d_{jet} \) for \( Z_0 > 2 \)  
(7)
(8)
(9)

2.2 Force calculation inside a Pelton Bucket

In an absolute system, the forces acting on individual sediment particles in the fluid flow are due to the deceleration and the curvature of the absolute flow. Due to the velocity difference between a particle and the surrounding fluid, also called slip velocity, a drag force has to be considered also. In this article, the forces are considered in the relative system rotating around the runner axis and moving with the flow in a bucket cross section, where additionally Coriolis forces have to be taken into account, but no deceleration. For simplification, it is assumed that the fluid flows along a streamline at a given distance from the runner axis with accordingly constant angular velocity \( \omega \). Steady flow (constant relative velocity \( w \)) and constant radius of curvature (\( r_1 \)) are assumed along the streamline.

Forces acting on a particle of given mass are the centrifugal force due to the rotation of the runner (\( F_{cent} \)), the Coriolis force (\( F_{cori} \)), and the force due to the path curvature (\( F_{curve} \)). Vector addition of these forces gives the resulting force vector (\( F_R \)). The direction of resulting force vector results in the direction difference of the streamlines and the particle path lines. Since the mass of the particles is assumed to be small compared to the water mass flow, the influence of the particles on the fluid flow can be neglected.

With the motion of the particles relative to the fluid motion, a drag force on the particles arises. This drag force must be in equilibrium with the resulting force: \( F_{drag} = F_R \). For estimation of the drag forces spherical particles are assumed. From this equilibrium condition, the particle velocity (\( V_P \)) relative to the fluid velocity can be determined. The relative velocity vector is parallel to the resulting force and accordingly perpendicular to the streamline. Once the particle velocity is determined the separation angle \( \alpha \) of the particles and the streamlines can be calculated.

Fig. 1 shows a schematic of a Pelton runner with a pitch circle diameter \( D (= 2R) \) rotating at angular velocity \( \omega \). The directions of the different force contributions are shown in red color. The Pelton bucket is assumed to have two semi-circular parts with internal radius \( r_1 \) as shown in the enlarged view of the bucket in Fig. 1. The impinging jet is divided into two equal parts and an axial outflow from the buckets is assumed as shown by green arrows.

With the assumption that the flow is only considered on a radius \( R \), the centrifugal force (\( F_{cent} \)) due to rotation is constant in magnitude and actsradially outwards. The Coriolis force, which is perpendicular to the relative velocity \( w \) and the vector of the angular velocity direction (\( z \)-axis), changes in magnitude as well as direction along the streamline as shown in Fig. 1. At the inlet of the jet in the bucket, the Coriolis force is opposite to the centrifugal force with maximum magnitude. From this condition, the magnitude of Coriolis force decreases to zero at the bottom of the bucket and attains the same value at the outlet of the flow from bucket as at the inlet. However, the direction will be opposite at the outlet as indicated in Fig. 1. The force due to the path curvature is basically a centrifugal force due to the assumed circular motion of flow in the bucket, and is perpendicular to the
bucket surface, and thus to the streamlines. The magnitude of this force will remain constant, because also the relative velocity \( w \) is assumed constant in the bucket flow.

![Figure 1](image-url)

**Figure 1.** Simplified flow inside a Pelton bucket and directions of acting forces in the flow

Fig. 2a shows the different forces acting on a particle in the flow. The magnitudes of these forces can be calculated with the following equations:

- Centrifugal force, \( F_{\text{cent}} = m \omega^2 R \) (10)
- Coriolis force, \( F_{\text{cori}} = 2m\omega w \sin \varphi \) where \((-90^\circ \leq \varphi \leq 90^\circ)\) (11)
- Force due to the path curvature, \( F_{\text{curve}} = \frac{w^2}{r_1} \) (12)

The resultant force \( F_R \) is given by the following expression.

\[
F_R = \sqrt{(F_{\text{cent}} + F_{\text{cori}})^2 + (F_{\text{curve}})^2}
\]

which finally takes the form:

\[
F_R = m \sqrt{\left(\omega^2 R + 2\omega w \sin \varphi\right)^2 + \left(\frac{w^2}{r_1}\right)^2}
\]

where \( \varphi \) is the effective mass of the solid particle considering the buoyancy effect. Assuming spherical particles, \( m \) is determined as:

\[
m = \frac{4}{3} \pi \left(\frac{d_{50}}{2}\right)^3 \left(\rho_{\text{solid}} - \rho_{\text{fluid}}\right)
\]

where \( d_{50} \) is the median size, widely used to represent the average size of the particles of samples [19]. \( \rho_{\text{solid}} \) and \( \rho_{\text{fluid}} \) are sediment and fluid densities.

The drag force expression for a spherical particle contained in a fluid is given by:

\[
F_{\text{drag}} = c_{\text{drag}} \pi \left(\frac{d_{50}}{2}\right)^2 \frac{V_p^2}{2} \rho_{\text{fluid}}
\]

where \( V_p \) is the magnitude of the velocity difference of the particle and the fluid flow. The associated vector is perpendicular to the streamline and approximately perpendicular to the bucket surface. This velocity \( V_p \) will be termed as separation velocity in text here onwards.
**Figure 2.** Forces acting on a particle at any instant and its separation from the fluid streamline

c_{drag} is a function of the particle Reynolds number Re_p which is given by:

\[
Re_p = \frac{\rho_{fluid} d_{50} V_p}{\eta}
\]  

(17)

where \( \eta \) is the dynamic viscosity.

The equations for \( c_{drag} \) as a function of the particle Reynolds number Re_p are given in Table 1 for various ranges of Re_p [6].

**Table 1: Values of coefficient of drag and corresponding flow zones**

| \( c_{drag} = \frac{24}{Re_p} \) | Re < 0.3 | Stroke’s law valid, laminar flow zone |
|---------------------------------|----------|-------------------------------------|
| \( \frac{24}{Re_p} \left(1 + 0.15 Re_p^{0.687}\right) \) | 0.3<Re<500 | Transition flow zone from laminar flow to turbulent flow |
| 0.44 | 500<Re<2\times10^5 | Turbulent zone |

2.3 Separation angle of flow and particles

The velocity \( V_p \) is small compared to the relative velocity \( w \) and it allows estimating the angle between the streamline and the particle path, as schematically shown in Fig. 2b.

From \( F_R = F_{drag} \), the velocity \( V_p \) can be estimated:

\[
V_p = \sqrt{\frac{4}{3} \frac{d_{50}}{c_{drag} \rho_{fluid}} \left(\frac{\rho_{solid}}{\rho_{fluid}} - 1\right) \left(\omega^2 R + 2\omega w \sin \phi\right)^2 + \left(\frac{w^2}{r_i}\right)^2}
\]

where \((-90^\circ \leq \phi \leq 90^\circ\)

The equation for the separation angle \( \alpha \) is given by the following expression:

\[
\text{Separation angle, } \alpha = \arctan \left(\frac{V_p}{w}\right)
\]

(19)
The calculations for the separation velocity, the particle Reynolds number, and the coefficient of drag are done iteratively as shown in Fig. 3 as these quantities are dependent on each other. A Pelton turbine is initially selected and these parameters are calculated for a specific value of sediment size. Once the iteration converges, the obtained values of parameters are accepted.

![Diagram](image)

**Figure 3.** Iterative calculation of separation velocity, particle Reynolds number, and drag coefficient

### 2.4 Similarity considerations for erosion in Pelton buckets

The separation of sediment particles from streamlines can be quantified by determining the angle of separation as per Eq. (19). This angle of separation will be larger for bigger particles than for the smaller ones as evident from Eq. (18) and (19). With increase of this angle of separation, the probability of a particle hitting the Pelton bucket surface increases as shown in Fig. 4a, which in turn increases the proneness to erosion. Further, the local flow around a sediment particle and its laminar or turbulent nature may also play role in the erosion process. If particles are in the turbulent regime then they will perform superimposed random movements due to the unsteady vortex shedding in their wakes. This could eventually add to the harmfulness of the particles. Based on these reasoning, it is assumed that the separation angle and the particle Reynolds number are important measures for erosion.

![Diagram](image)

**Figure 4.** Schematic showing different separation angles and forces causing erosion in different zones of the bucket
The grinding of particles, which is caused by the frictional force (F\text{\_friction}) between bucket and particle, is responsible for hydro-abrasive erosion. The impact force (F\text{\_impact}) causes erosion in splitter zone. The effect of grinding increases with the above defined separation angle between the particles and the streamlines (\(\alpha\)) and the particle Reynolds number. Thus, this angle and the particle Reynolds number are considered as major similarity measures to compare Pelton buckets and their potential for being eroded by sediment laden flows.

3. Results and discussions

3.1 Effect of forces on particles in Pelton buckets

The forces on particles of various size were calculated for each investigated Pelton turbine and the corresponding accelerations were analysed. As an example, the accelerations calculated for the turbine in the power plant named Toss were plotted in Fig. 5. The Toss HPP with 10 MW capacity is located in Kullu district of Himachal Pradesh, India, on the river Tosh, a tributary to Parbati River. There are two Pelton turbine units with a designed head of 174 m and total discharge of 7 m\(^3\)/s. The D and bucket width (=4r\(_1\)) of turbine are 1.089 m and 375 mm respectively. It is evident from the plot that the resulting acceleration and the associated force on a particle remain almost constant from inlet to bottom portion of the bucket and then increases to maximum at the outlet. This increase of forces supports the previous statement that Pelton buckets are more prone to erosion at the outlet zone. The Pelton bucket can be divided into 3 zones as per the erosion intensity/severity as shown in Fig 4b. Brekke [4] attributed this erosion behaviour inside Pelton bucket to the small radii of curvature in outside zone due to result from local previous erosion (self-intensifying process). In addition to this fact, the particle concentration near the wall is increasing during the passage through the bucket due to the curvature and the particle concentrations near the bucket surface become highest at the outer zone. Increased concentration goes along with higher erosion.

3.2 Dependency of the separation angle on the D/B ratio

Fig. 6a shows the plot of the separation angle for a 30 \(\mu\)m sized particle for all of the 250 investigated Pelton turbines with respect to the D/B ratio. Here the separation angle is shown in the bottom of the bucket where the Coriolis forces disappear. Most of the considered turbines have a D/B ratio < 6, however a few show higher values. The general observation is that the increase in the D/B ratio results in an increase of the separation angle. Similar plots were also obtained for particles of size 90 \(\mu\)m, 200 \(\mu\)m, 300 \(\mu\)m, and 500 \(\mu\)m, leading to the finding that an increase in the D/B ratio leads always to higher separation angles. Moreover, the larger the particles are and the higher the D/B ratio is, the chance for turbulent drag forces on particles gets higher, as it is evident from Fig. 6b. Turbulent drag forces and the associated oscillatory motion of the particles might further increase the particle erosion potential.

![Figure 5. Accelerations with respect to the particle position in Pelton bucket of Toss HPP](image-url)
3.3 Effect of the specific speed on erosion

Also the specific speed ($n_q$) of a turbine is found to have significant relation with the separation angle and the particle Reynolds number. Fig. 7a shows that small specific speeds $n_q$ mean a large angle between particle trajectories and streamlines ($\alpha$). Fig. 7b shows that the particle Reynolds number increases with a decrease of $n_q$. This implies that even finer particles remain in the turbulent drag zone potentially enhancing erosion.

![Figure 6. Effect of the D/B ratio on the separation angle and particle Reynolds number](image)

Both parameters the D/B ratio and the specific speed of the turbine potentially influence the erosion process. Fig. 8a shows the well known relation between these two parameters. It can be seen that after a decrease in the $n_q$ value below 5, there is a steeper rise in the D/B ratio. As observed in the previous paragraphs, either an increase in the D/B ratio or a decrease in $n_q$ value increases the separation angle and thus the endangerment of erosion. Hence, from a general perspective, the turbines having $n_q$ values smaller than 5 are assumed to be more prone to erosion with otherwise the same conditions and are marked red in Fig. 6 – 8. The hydropower plants with single turbine unit capacity more than 10 MW are marked with crosses so as to differentiate the range of designs of large and small turbines.
Figure 8. Relation of specific speed with D/B ratio and ratio of curvature and centrifugal acceleration

3.4 Acceleration ratio
Very high values of accelerations normal to the streamlines in the range 1 500 – 150 000 m/sec² were calculated for the investigated Pelton buckets, which are comparable with Brekke [4], where accelerations of 50 000 and 100 000 m/sec² have been reported. Acceleration ratios are plotted in Fig. 8b as a function of n_q. It is observed that the ratio of curvature acceleration (a_curve) and centrifugal acceleration (a_cent) increases with decreasing n_q values. The increase of the displayed ratio for small n_q emphasizes the growing influence of the curvature acceleration and accordingly the potential for erosion for turbines with small specific speed. Hence, this ratio a_curve/a_cent can be used for similarity considerations.

4. Conclusion
The presented study considers different types of forces acting on particles in sediment laden flow in Pelton buckets. These forces could be determined after a series of assumptions and simplifications by analysing data of Pelton turbines installed in about 250 hydropower plants. The main reason for the susceptibility of uncoated Pelton buckets to erosion is found to be the high accelerations due to curvature of the bucket, which separates the sediment particles from the streamlines leading to a grinding of the particles on the surface. It was found that the local flow in the wake of the larger particles becomes turbulent when they separate from the fluid stream lines. Such a turbulent flow causes unsteady loads on the particles and particle oscillations and may additionally enhance erosion. The findings from this study can be summarised as below:

1. The angle between the particle path and the fluid streamline resulting from the forces on particles is considered as major similarity measure for the erosion potential in Pelton buckets.
2. An increase in D/B ratios goes along with an increase of the separation angle of sediment particles from the streamlines.
3. The decrease in the specific speed is also accompanied by an increase of the separation angle.
4. The largest resulting separation angles and the largest separation velocity occur in the outer zone of the buckets, since the Coriolis and centrifugal forces add there and lead to the highest resultant forces on the particles.

For a potential hydropower site, an increase in the number of jets leads to a decrease in value of n_q; hence, the tendency of erosion of Pelton buckets will increase and vice versa. For a site with high erosive sediment flow conditions a design of a turbine with higher value of n_q is recommended. This finding is in agreement with the recommendations given by Brekke [9] that low number of jets is beneficial in erosive conditions. On the basis of the results of this study, it is hypothesised that erosion can be minimized with designs, which reduce the separation angles of particles from the streamlines. However, further investigation of flow, sediment and material properties and the relevant processes are required for prediction of hydro-abrasive erosion.

Acknowledgements
The authors would like to thank Ministry of Human Development Resource (MHRD), India for the financial support in form of a PhD scholarship.

\[ y = 22.515x^{0.985} \]

\[ R^2 = 0.9394 \]
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