Acoustic solitons in semiconductor nanostructures

A V Akimov1, A V Scherbakov1, P J S van Capel2, J I Dijkhuis2, T Berstermann3, D R Yakovlev1,3 and M Bayer3

1A.F. Ioffe Physico-Technical Institute, 194021 St. Petersburg, Russia
2Debye Institute, Department of Physics and Astronomy, Utrecht University, P.O. Box 80000, 3508 TA Utrecht, The Netherlands
3Experimentelle Physik II, University of Dortmund, D-44221 Dortmund, Germany,

E-Mail: akimov.os@mail.ioffe.ru

Abstract. The present lecture gives the review of recent studies where acoustic solitons strongly influence the electron states in a semiconductor nanostructure. The amplitude of soliton pulses is so high that the electron states in a quantum well make temporal excursions in energy up to 10 meV. The subpicosecond duration of the solitons is less than the coherence time of the optical transition between the electron states and a frequency modulation of emitted light during the coherence time (chirping effect) is observed.

1. Introduction

The nonlinear properties of wave propagation in the media can result in the formation of a solitary wave called a soliton [1]. The soliton is a propagating wavepacket which is stable due to the delicate balance between nonlinear properties and dispersion in the media. First the soliton was observed in water by John Scott Russell in 1834. Later, in 1895, Korteweg and de Vries (KdV) derived a partial differential equation, which describes solitons in shallow water. Apart from waves in water the solitons were observed in plasma and in optical fibers.

Favorable conditions for the formation of an acoustic soliton (AS) at low temperatures do exist in many crystals. Indeed, the elastic nonlinearity in the crystals for acoustic waves is quite strong. The formation of shock waves caused by nonlinear effects is a well known phenomenon. However, the formation of an ultrashort AS pulse at a realistic propagation distance requires additionally the dependence of the phase velocity on the phonon energy (i.e. nonlinear phonon dispersion), which in crystals starts to be distinct for phonons with frequencies ~0.1-1 THz.

The first experimental observation of the AS has been performed by Hao and Maris in 2001 [2]. To generate the initial strain impact the authors used the experimental methods of picosecond acoustics, developed earlier in several research groups [3,4]. Figure 1 shows the typical experimental scheme (inset) and calculated using KdV equation temporal (left panels) and spectral (right panels) evolution of a strain wave packet propagating in GaAs crystal. A thin metal film, deposited on the sample surface as an opto-elastic transducer, is excited by optical pulses from a femtosecond laser. Ultrafast optical excitation results in rapid heating of the metal film, which expands due to the thermoelastic effect. The expansion and further relaxation of the film to the initial state results in the formation of a bipolar strain pulse [left panel figure 1(a)] which is injected into the crystal and propagates with the velocity of longitudinal (LA) sound. The pulse is followed by low amplitude ringing due to the strain pulse reflections on the metal/crystal boundary. The net duration of the bipolar pulse lies in a
The amplitude of the injected strain pulse may be essentially higher than $10^{-4}$. At such strain nonlinearity plays a significant role. The velocity increases with strain and because of this the top of the negative strain pulse starts to catch up with the base while the top of the positive part slows down. Such behavior leads to the formation of an N-shaped pulse [left panel figure 1(b)]. The sharpening of the edges in the N-shaped pulse results in the appearance of high-frequency components in the spectrum of the wave packet [right panel figure 1(b)]. Due to the dispersion these high frequency modes have phase velocities slower than in the linear low frequency regime. The balance between fastening of the leading edge and slowing down of high frequency phonons in the wave packet leads to the formation of an AS which propagates slightly faster than the low amplitude LA sound [figure 1(c)]. Further the pulse shape and the wave packet spectrum change while propagating through the crystal in time and space, and an AS train consisting of several isolated ASs is formed [figure 1(d)].

Nowadays, the AS has been experimentally observed at helium temperatures in a number of crystals: Al$_2$O$_3$ [2, 7-9], Si [2, 10, 11], MgO [2,12], SiO$_2$ [2] and GaAs [13]. The detection of picosecond ASs has been realized by: pump-probe [2, 10]; Brillouin scattering [7]; superconducting bolometer [11]; photoluminescence [8,9]. The simple theory based on KdV equation successfully describes the main properties of ASs. However many basic problems of the AS behavior are still the subject of intensive experimental and theoretical studies nowadays [14].

The present paper describes our recent experiments where the AS meets a semiconductor quantum well (QW) [15, 16]. The main reason for carrying out this work was an idea to develop a new technique for an ultrafast control of electron states in semiconductor nanostructures. The perturbation, which an AS brings to the electron state in a semiconductor nanostructure is enormously high. The corresponding shift of the energy of an electron (hole) state may exceed 10 meV. While moving in the crystal, the AS occupies a space $\sim 10$ nm at each point of time which is the same order as the typical size of semiconductor nanostructures and tunneling devices. Thus the AS can induce huge electron spectrum changes on a picosecond time scale. Such energy control has not been realized earlier.

2. Experiment

To demonstrate the effect of AS on the electron spectrum in a QW a (001)-oriented GaAs slab with a thickness $l_0 \approx 100$ μm was used as the medium for AS propagation. The QW, grown by molecular beam epitaxy on the front side of the slab after 20-nm-wide ZnSe buffer layer, was a single 8-nm-wide ZnSe layer, surrounded by Zn$_{0.89}$Mg$_{0.11}$S$_{0.18}$Se$_{0.82}$ barriers [17].

The experimental setup is shown schematically in figure 2(a). The sample was immersed in pumped liquid helium at $T=1.8$ K. The strain pulses were generated in the GaAs slab from the side opposite to the QW heterostructure using the opto-elastic transducer technique [inset in figure 1(a)]. For this a 113-nm-thick Al film was deposited on the back side of GaAs slab. The laser was a mode-locked Ti-sapphire laser (wavelength 800 nm) with regenerative amplifier. The output pulses had duration less than 200-fs and a repetition rate 250 kHz. The laser beam was split into pump and probe beams. Pump pulses passed through the variable delay line (1) and were focused to a 100 μm diameter spot on the Al film, creating energy densities ($W$) up to 10 mJ/cm$^2$ per pulse. The temporal evolution of the injected strain pulse [figure 1(a)] was calculated using the known methods of ultrafast acoustics [5,6] and coincides with results from direct conventional pump-probe measurements [18].

In order to obtain the information about the effect of strain pulses and ASs on the electrons in the QW, we monitor the spectrum of the optical transitions between the quantized electron and hole states in the QW by measuring the reflectivity spectrum. The stationary reflectivity spectrum of the light from a halogen lamp in the studied heterostructure is shown in figure 2(b). The lower energy resonance at photon energy $E=E_{\text{hh}}$ corresponds to the coupled electron-heavy hole transition (HH exciton). The higher energy resonance at $E=E_{\text{lh}}$ corresponds to the electron and light hole (LH exciton). The exact spectral profile of the resonances in reflectivity spectrum is governed by optical
interference effects [19,20]. For our particular sample the reflectivity spectrum of each resonance, normalized to the background may be described by:

\[ r(E) = 1 + A_0 \frac{E - E_0}{\Gamma^2 + (E - E_0)^2} \]

where \( A_0 \) is a constant, \( E_0 \) is the energy of the resonance, and \( \Gamma \) is the width of the resonance, which is determined by the homogeneous and inhomogeneous broadening and also by the spectral resolution of the experimental setup. For the studied structure \( E_0 = 2.809 \) (2.836) eV for HH (LH) resonance. The measured value of \( \Gamma = 0.5 \) meV in our sample is in good agreement with earlier works [17].

Sizeable energy shifts \( \Delta E \) of an optical resonance in bulk semiconductors have been demonstrated by applying uniaxial stress [21, 22]. In our QWs \( \Delta E \) is governed by the deformation potential mechanism. In case when the stress is perpendicular to the plane of the QW grown on the (001), \( \Delta E = c \varepsilon_{xx} \), where \( \varepsilon = \varepsilon_{xx} \) (\( \varepsilon_{xx} \) is the corresponding component of the strain tensor) and \( c \) is a net parameter which is a combination of the deformation potentials for electrons and holes [23].

The main idea of the experiment was to measure the resonance energy shift induced by the dynamical strain \( \varepsilon(x,t) \) when the AS hits the QW. To monitor the AS induced effects we measure the reflectivity spectra with a femtosecond temporal resolution. For this the probe pulse was passed through a sapphire plate (figure 2) in order to generate a white light fs pulse, which is used for probing the QW resonant optical transition. Then the probe beam was delayed on \( t_0 = l_0 / s_{GaAs} \approx 20 \) ns.

![Figure 1.](image1.png)  
**Figure 1.** Temporal (left panels) and spectral (right panels) evolutions while moving in the GaAs crystal: (a) initially generated strain pulse for \( W=10 \) mJ/cm\(^2\); (b) N-shape pulse; (c) a pulse with an AS; (d) a pulse with a train of ASs. The inset: a scheme for strain pulse generation.

![Figure 2.](image2.png)  
**Figure 2.** (a) Experimental setup; (b) reflectivity spectrum of ZnSe/ (Zn,Mg)(S,Se) quantum well. The inset in (b) shows the band diagram and optical transitions in the QW.
\( v_{\text{GaAs}} = 4.8 \times 10^3 \text{ m/s} \) is LA sound velocity in GaAs) by a fixed delay line (2). This delay is required to give time for the LA strain pulse to travel from the metal elasto-optic transducer to the QW. After that the probe beam was focused on the front side of the sample to a spot less than 50-μm-diameter exactly opposite to the spot of the pump beam. The reflected probe beam was collected to the slit of an optical spectrometer followed by a CCD camera. The time resolution of reflectivity spectra measurements was realized by the scanning delay line of the pump beam, which was synchronized with a readout of CCD camera. The overall time resolution of our experimental scheme was better than 300 fs.

The left panels in figure 3 (a) and (b) show measurements of reflectivity HH exciton spectra for values of \( W = 0.5 \text{ mJ/cm}^2 \) and 9.2 mJ/cm² respectively, in spectral/temporal contour plots. The color scale is a measure of the time-dependent reflected spectral intensity in the spectral range around the electron-HH optical resonance, normalized to the off-resonant value. The value \( t=0 \) corresponds to the arrival time of the center of the initial bipolar wave packet [figure 1 (a)] at the QW. The time intervals \( t<25 \text{ ps} \) and \( t>100 \text{ ps} \) correspond to the situation prior to the arrival of the strain wavepacket and after full passage of the heterostructure, respectively. No temporal modulation is observed and the reflectivity spectrum is equal to the one in the absence of strain, shown in figure 2(b).

Between -25 and 100 ps, the measurements show a clear optical response, that moreover depends on the pump excitation density \( W \). In the linear regime (\( W<1 \text{ mJ/cm}^2 \), left panel figure 3 (a)) and upon arrival of the bipolar acoustic pulse at the QW, the reflectivity spectrum shifts smoothly to higher energies, back to low energies, and recovers the original position. After reflection of the pulse from the sample surface the same sequence occurs, but in the opposite direction. We have observed the same time evolution in similar experiments with GaAs/AlGaAs QWs [15]. The important point here is that the shape of the exciton resonance does not change and the energy of the exciton resonance simply shifts in time [compare solid and dashed line in the inset of the left panel in figure 3(a)]. Therefore, the strain-induced shift of the exciton resonance \( \Delta E(t) \) can be approximated by \( \Delta E(t) = c \varepsilon_{\text{QW}}(t) \), where \( \varepsilon_{\text{QW}}(t) \) is the time-dependent strain in the QW, which consists of the sum of the pulses incident from the GaAs substrate (taking into account impedance mismatch), and its reflection from the sample/LHe interface [15]:

\[
\varepsilon_{\text{QW}}(t) = \varepsilon_0(t) - \varepsilon_0(t-t_r).
\]

Here, \( \varepsilon_0(t) \) is the strain pulse arriving directly at the QW and \( -\varepsilon_0(t-t_r) \) the one arriving after reflection at time \( t_r = 2l_r/\pi \) (\( l_r = 50 \text{ nm} \) is the distance from the QW to the surface in our sample, and \( \pi = 4 \text{ km/s} \) is the mean longitudinal sound velocity in the ZnSe/(Zn,Mg)(S,Se) heterostructure [24]). The minus sign between the two strain components in equation (2) accounts for the fact that the phase of the strain pulse jumps by \( \pi \) upon reflection at the free surface.

The response at high power [left panel figure 3(b)] is highly distorted when comparing to the low power trace. In figure 3 (c)-(f) we show some characteristic features of spectra measured at specific times. We can distinguish: (i) sharp features in the temporal signal appear, (ii) the leading edge of the detected signal arrives earlier with the increase of \( W \) [compare left panels in figure 3 (a) and (b)], (iii) the reflectivity spectrum broadens strongly [figure 3 (d) and (e)] and (iv) doublet structures appears at certain times [figure 3 (f)]. Features (i) and (ii) point to AS arriving at the QW. From the simulations we know that the soliton pulses may become shorter than 1 ps [see figure 1(d)], explaining the sharp features in the temporal evolution of the detected signal at high \( W \). Furthermore, the AS velocity is supersonic, which results in the early arrival of the front of the strain wavepacket as shown separately in the inset of the right panel in figure 3(a).
3. Theoretical analysis and discussion

The theoretical analysis of the experimental results can be divided into three steps: (1) the evaluation of the KdV equation and the simulations of spatial/temporal evolution of the strain pulse $\varepsilon(t,x)$ as a function of pump fluence $W$; (2) the calculation of the strain induced temporal evolution of the energy $\Delta E(t)$ of the quantized electron (hole) state in a QW; (3) the effect of a finite relaxation time of the optical transition on AS probing in the QW.

3.1. KdV equation and computer simulations

As was already mentioned in the introduction, the KdV equation is a well-known differential equation that has been widely used to describe a propagation of a strain wave packet through a nonlinear, dispersive medium $[2, 7, 12, 13]$. We have explicitly assumed that damping can be neglected at LHe temperatures. Therefore, we evaluate the KdV equation

\[
\frac{\partial \varepsilon}{\partial t} = -\frac{\alpha}{2\rho_s_{GaAs}} \varepsilon \frac{\partial \varepsilon}{\partial x} - \beta \frac{\partial^3 \varepsilon}{\partial x^3} \tag{3}
\]

Figure 3. (a,b) Temporal evolution of the reflectivity spectrum measured (left panels) and calculated (right panels) for $W=0.5$ mJ/cm$^2$ (a) and 9.2 mJ/cm$^2$ (b). Arrows indicate the arrival times of ASs at the QW center. (c-f) Measured (red, solid line) and calculated (black, short dash) reflectivity spectra for $W=9.2$ mJ/cm$^2$ at different times. Arrows in (c) and (d) indicate the shifted spectral lines due to AS. Blue, long dashed lines are unperturbed spectra. Insets in (a): (left) reflectivity spectra before arrival of the strain at the QW (blue dashed line), and when the line shift is maximum (red line); (right) measured (symbols) and calculated (solid line) dependence of the arrival time of the strain pulse on pump fluence.
with the relevant GaAs parameters: \( \rho = 5.31 \times 10^3 \text{ kg/m}^3 \) the mass density, and \( \alpha \) and \( \beta \) nonlinear and dispersive terms respectively (\( \alpha = C_{111} + 3C_{11} = -3.9 \times 10^{10} \text{ N/m}^2 \) and \( \beta = 0.74 \times 10^{-17} \text{ m}^3/\text{s} \) [25]). The input pulse can be described by a Gaussian derivative as shown in figure 1(a) and the amplitude of it linearly varies with pump density \( W \). The examples for the strain temporal evolution for the three distances at highest \( W = 10 \text{ mJ/cm}^2 \) are shown in figure 1(b-d). One can clearly see the transformation of the initial strain pulse to a N-shape pulse [figure 1(b)] under the influence of nonlinearity, and due to the arising of dispersion further to ASs [figure 1(c-d)]. For the high excitation density a distance of only 50 \( \mu \text{m} \) is required for the formation of AS. The dispersive tail shows an increasing number of fast oscillations due to dispersion. Note that dispersion occurs far earlier than would be expected from pure linear propagation [2], since the nonlinear propagation has increased the frequencies in the wave packet significantly in the first microns of propagation.

In order to calculate the full effect on the QW we take into account the first and higher-order reflections at the barrier layer/GaAs substrate interface (due to impedance mismatch, there is an 8% amplitude reflection) and the fact that the QW can simultaneously be influenced by the strain and of its reflection from the surface [see equation (2)]. Due to the small distance between the QW and the surface, the temporal shapes of the incident and reflected strain pulses are taken identical except for the phase jump. In this way, we obtain the full temporal strain profile \( \varepsilon_{QW}(t, x') \) in the QW (with \( x' = 0 \) at the center of the QW).

3.2. The effect of a finite QW width

Figure 1(d) shows the presence of THz frequencies in the wave packet, corresponding to wavelengths of \( 1 \text{ ps} \times 4 \text{ km/s} = 4 \text{ nm} \), smaller than the QW width. In the AS regime we therefore cannot consider the QW as an infinitively narrow object as it was assumed in the linear regime [15]. To include into the finite QW width into the analysis, we use the approximation of infinitely high barriers and take the electron wave function \( \varphi(x') = \sqrt{2/a} \cos(\pi x'/a) \). In addition, we assume that the strain does not change the potential profile of the QW significantly. Using perturbation theory the energy changes of the exciton resonance in the QW in the presence of the strain pulse are found to be equal to

\[
\Delta E(t) = c \int_{-a/2}^{a/2} \varphi(x')^2 \varepsilon_{QW}(t, x') \, dx'.
\]  

(4)

Here we note that the equation (4) is reminiscent of the equation of matrix elements for the exciton-phonon interaction in a QW [26] that indeed shows the cutoff for high-frequency phonons. This effectively decreases the sensitivity of a QW as a detector for very short soliton pulses (phonon frequencies > 500 GHz), where the energy is concentrated mostly in the high-frequency components.

3.3. Chirping of optical transition

The interaction of the probe light with the exciton system is inherently not instantaneous but takes the corresponding exciton coherence time \( T_e \), which in high-quality QWs at low temperatures is close to twice the value of the radiative exciton lifetime [27], of the order of several picoseconds. This is an order of magnitude larger than the probe pulse length and the typical AS widths. Thus QW essentially integrates all ultrafast strain modulations occurring within a coherence time \( T_e \) after the incident probe pulse. This leads to an acoustically induced chirping effect: the coherent optical frequency modulation of the reflected probe light by a picosecond strain pulse. We approximate the effect by convoluting the modulated at time \( t \) reflectivity \( r(E - \Delta E(t)) \) with an exponentially decaying coherent emission:

\[
R(E, t) = \frac{1}{T_e} \int_{t}^{\infty} r[E - \Delta E(\tau)] \exp\left(-\frac{\tau - t}{T_e}\right) d\tau
\]

(5)

where \( r(E) \) and \( \Delta E(\tau) \) are given by equations (1) and (4) respectively.
To evaluate all the steps, and optimize the correspondence between measured and calculated, we have developed an automated fitting procedure in which the width and height of the input wave and the coherence time $T_e$ are the free parameters. The input wave width of 6.8 ps [figure 1 (a)] and amplitude $(7.5 \pm 0.2) \times 10^{-5}$ W were in good agreement with the values obtained in the pump-probe measurements mentioned before. The best agreement for the values of the shift values are obtained for a deformation potential value $c = -8$ eV. Further, we find $T_e = 4.6 \pm 1.3$ ps, close to the measured dephasing time of excitons in similar QWs [28].

The results of the numerical calculations for the two values of $W$ are shown in the right panels in figure 3 (a) and (b) (for the low and high power, respectively). The agreement at low power is excellent. At first sight, the correspondence at high powers seems to be less. The individual spectra, as shown in figure 3 (c)-(f), show better agreement than the contour plots suggest. The sharpening of the leading edges at elevated $W$ signifies the formation of ultrashort strain pulses, i.e. ASs, indicated by arrows in figure 3 (b). The arrival time of the ASs decreases with the increase of $W$, as correctly described by the simulations and shown by a solid line in the inset of figure 3(a, right panel). In general, the passage of incident and reflected ASs leads to a broadening of the reflectivity spectrum over a broad range of times, visible as an increase in green areas in figure 3 (b). The reason for this is that in the present case the duration of individual AS pulses as well as the delay between different soliton pulses are shorter than $T_e$. Thus when the AS train arrives, the exciton resonance is chirped several times back and forth during its coherence time $T_e$. For the first soliton, the shifted exciton line is positioned at the dip indicated by the arrow in figure 3 (c). In figure 3 (d), the dip caused by the reflection of the second soliton is more clearly visible in the measured spectrum, since the shift is smaller and the soliton broader in time.

An even more elucidating example is presented in the figure 3 (f), which shows a clear doublet structure that can be observed around times 25 ps and 52 ps. We can explain this by the tensile part of the incident wave [figure 1 (d)]. Here, a dispersive tail develops of both high frequency and high amplitude. Since the typical oscillation time is 2 ps, within time $T_e \sim 5$ ps, the excitons are swept over almost 10 meV in energy a couple of times. One can qualitatively understand the origin of the doublet by making the analogy with a pendulum, which spends most of the time in its extreme positions.

4. Conclusions

To conclude we demonstrate that the AS has a strong effect on the electron spectrum in the semiconductor QW. The energy of the exciton resonance shifts up to 10 meV, which is over 5 times the detected linewidth. The experimental results are in an agreement with the theoretical analysis based on KdV equation and basic principles for the electron-phonon interaction in the semiconductor nanostructures.

We observe strong chirping probing the exciton resonance when ASs pass a QW. The soliton pulses are so sharp in space (~3 nm) and time (~750 fs) that the exciton resonance shifts up and down in energy over several meV during the exciton coherence time. More generally, the presence of features occurring at timescales shorter than the exciton coherence time results in complex behavior (e.g. doublet structures) of the probed exciton spectrum and forms a qualitatively new dynamical non-stationary quantum mechanical state.

The presented work uses the QW as the archetypal object of semiconductor nanostructures. The observed constructive interference between incident and reflected strain pulses is analogous to coherent strain control [29] and realizes manipulation of nanostructures by AS pulses. The AS-induced effects in more sophisticated nanostructures (e.g. tunneling devices, shallow QWs and delta layers, quantum wires, dots and dot molecules), where the adiabatic approximation for electron and lattice systems is not valid anymore may lead to the discovery of new ultrafast phenomena at constant carrier densities. The experiments and theoretical analysis show that the effect of acoustic solitons on the electron spectrum in a QW may be used as an ultrafast method for modulating the optical response in nanostructures. The large value of the energy resonance shift may become a basis for picosecond
control of emission from nanophotonic devices (semiconductor microcavities, 2D arrays, etc) and other switching principles in nanoelectronics and photonics.

Acknowledgements
We are thankful to S. Tamura, A.A. Kaplyanskii, B. Perrin, E. Péronne and O.B. Wright for useful discussions, P. van der Straten for help with the simulation work, and D. Schemionek for sample preparation. We acknowledge for financial support the Deutsche Forschungsgemeinschaft (DFG), the BMBF, the Dutch Foundation for Fundamental Research on Matter (FOM), the Netherlands Organization for Scientific Research (NWO), and the Russian Academy of Sciences. The DFG financed research stays of AVA (Mercator professorship) and AVS (grant 436 RUS 17/101/06) in Dortmund.

References
[1] Drazin P G and Johnson R S 1989 *Solitons: an Introduction* (Cambridge University Press)
[2] Hao H-Y and Maris H J 2001 *Phys. Rev. B* **64** 064302
[3] Thomsen C, Grahn H T, Maris H J and Tauc J 1986 *Phys. Rev. B* **39** 4129
[4] Wright O B 1994 *Phys. Rev. B* **49** 9985
[5] Tas G and Maris H J 1994 *Phys. Rev. B* **49** 15046
[6] Saito T, Matsuda O and Wright O B 2003 *Phys. Rev. B* **67** 205421
[7] Muskens O L and Dijkhuis J J 2002 *Phys. Rev. Lett.* **89** 285504
[8] Muskens O L, Akimov A V and Dijkhuis J J 2004 *Phys. Rev. Lett.* **92** 035503
[9] Muskens O L and Dijkhuis J J 2005 *Phys. Rev. B* **71** 104304
[10] Daly B C, Norris T B, Chen J and Khurgin J B 2004 *Phys. Rev. B* **70** 214307
[11] Stanton N M and Kent A J 2005 *Phys. Rev. B* **73** 22030(R)
[12] Singhsonroje W and Maris H J 2004 *Phys. Rev. B* **69** 174303
[13] Péronne E and Perrin B Ultrasonics 2006 **44** e1203
[14] This volume to be added later
[15] Akimov A V, Scherbakov A V, Yakovlev D R, Foxon C T and Bayer M 2006 *Phys. Rev. Lett.* **97** 037401
[16] Scherbakov A V, P van Capel J S, Akimov A V, Dijkhuis J J, Yakovlev D R, Berstermann T and Bayer M 2007 *Phys. Rev. Lett.* in press
[17] Astakhov G V et al. 2002 *Phys. Rev. B* **65** 165335
[18] van Capel P J S and Dijkhuis J J 2006 *Appl. Phys. Lett.* **89** 139903
[19] Ivchenko E L, Kopev P S, Kochereshko V P, Uraltsev I N, Yakovlev D R, Ivanov S V, Meltser B Ya and Kalitievskii M A 1988 *Sov. Phys. Semicond.* **22** 495
[20] Ivchenko E L and Pikus E G 1997 *Superlattices and Other Heterostructures: Symmetry and Optical Phenomena* (Telos: Springer-Verlag)
[21] Gross E F and Kaplyanskii A A 1961 *Soviet Physics Solid State* **2** 1518
[22] Cardona M 1969 *Modulation Spectroscopy* (New York: Academic Press)
[23] Sander G D and Chang Yia-Chung 1985 *Phys. Rev. B* **32** 4282
[24] Lee B H 1970 *J. Appl. Phys.* **41** 2984
[25] Hao H-Y and Maris H J 2001 *Phys. Rev. B* **63** 224301
[26] Akimov A V 2003 *Exciton-phonon interaction in quantum wells: Electron-Phonon Interactions in Low-Dimensional Structures* ed L J Challis (Oxford University Press), pp 239-267
[27] Mayer E J, Pelekanos N T, Kuhl J, Magne N and Mariette H 1995 *Phys. Rev. B* **51** 17263
[28] Wagner H P, Schätz A, Maier R, Langbein W and Hvam J M 1997 *Phys. Rev. B* **55** 12581
[29] Özgür U, Lee C-W and Everitt H O 2001 *Phys. Rev. Lett.* **86** 5604