From F-theory GUTs to the LHC

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Abstract

This paper provides an overview to three recent papers on the bottom up approach to GUTs in F-theory. We assume only a minimal familiarity with string theory and phenomenology. After explaining the potential for predictive string phenomenology within this framework, we introduce the ingredients of F-theory GUTs, and show how these models naturally address various puzzles in four-dimensional GUT models. We next describe how supersymmetry is broken, and show that in a broad class of models, solving the $\mu/B\mu$ problem requires a specific scale of supersymmetry breaking consistent with a particular deformation of the gauge mediation scenario. This rigid structure enables us to reliably extract predictions for the particle spectrum of the MSSM. A brief sketch of expected LHC signals, as well as ways to falsify this class of models is also included.

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1 Introduction

The landscape of semi-realistic string vacua continues to grow. This embarrassment of riches is an obstacle in the way of extracting definite predictions for present and future experiments. While one might have hoped that a more complete formulation of the theory would reveal some inconsistency with these vacua, this has proven not to be the case. Faced with this state of affairs, extracting concrete predictions from string theory requires further selection criteria to ascertain which of these many possibilities could be consistent with present observations. In this regard, we believe it is important to note that of the many known vacua of the landscape which satisfy all global constraints imposed by gravity, not a single one reproduces every feature of the Standard Model.

Precisely because of its success as a quantum theory of gravity, most efforts in realizing the Standard Model within string theory have focussed on the development of a single unified framework which couples this gauge theory to gravity. Even so, gravity only weakly couples to the Standard Model, and can for the most part be ignored in discussions of particle physics. At a technical level, it is also far more cumbersome to include the effects of gravity in string based models. For example, besides pure gravity, such models typically also contain many fields which describe the shapes of the internal directions of a compactification. Giving all of these moduli fields an appropriate large mass is indeed a topic of current research, which in practice can be quite involved. In limits where gravity decouples, many of the main issues related to moduli are also absent.

Whereas to the experimentalist, the energy scales probed by the LHC are at the high energy frontier, to the string theorist, this is the low energy limit of a string compactification! Precisely because the low energy limits of string theory are those which are most likely to confront experiment, it is therefore natural to ignore the effects of gravity and instead focus on those aspects of a string based construction which can replicate the correct matter content and interactions of the Standard Model. Nevertheless, there are potentially a vast number of different UV completions of the same IR physics. To a certain extent, this is borne out by the existence of the string theory landscape.

The primary aim of the present paper is to argue that certain areas of the landscape are quite constrained, and that within a bottom up approach to string phe-
nomenology, it is in fact possible to make definite predictions for current and future experiments in such a framework. The essential point is that although we may not know every detail of either the UV, or IR physics, we may nevertheless have a crude characterization of both regimes. For example, the IR behavior of the theory could in principle be incompatible with a given set of UV boundary conditions. Using the renormalization group equations to repeatedly iterate between the UV and IR, the bottom up approach can effectively constrain the form of both sectors. Indeed, programs such as SOFTSUSY \cite{1} which compute the soft masses of the MSSM based on specified UV boundary conditions adjust various inputs at the weak scale such as $\mu$ and $B\mu$ to remain in accord with electroweak symmetry breaking. More broadly speaking, this approach is quite commonplace in phenomenology, but is somewhat foreign to the usual mindset adopted in string theory.

This paper provides an overview to the specific approach to string phenomenology detailed in the three recent papers \cite{2-4}. Combining recent insights on local model building in string theory with elements of Grand Unified Theories (GUTs), the resulting framework turns out to be rigid enough to extract definite predictions for phenomena seemingly far removed from the realm of string theory. As a general disclaimer, in this paper we will not provide citations to related work of potential interest. We refer the reader to the primary papers \cite{2-4} for a more detailed list of references and general guides to the literature.

To be sure, the aims of this approach are more modest than what was perhaps originally envisaged in earlier bold attempts to connect string theory with phenomenology. Indeed, by giving up on a complete description of ultraviolet physics, it is in principle possible that important constraints could be missed. Nevertheless, we will see that in certain cases, even in the absence of gravity, compatibility with embedding in a string based model imposes rather stringent conditions which are not easy to satisfy!

The rest of this paper is organized as follows. In section 2 we state the basic assumptions which we shall make throughout this paper. Next, in section 3 we describe the basic ingredients of F-theory GUT models. In section 4 we show how these models realize the exact spectrum of the MSSM. Section 5 shows that simply achieving the correct MSSM spectrum typically addresses some vexing problems of four-dimensional GUT models. Supersymmetry breaking turns out to also be quite predictive in this framework, and in section 6 we review the particular deformation
of gauge mediated scenarios in F-theory which solve the $\mu/B\mu$ problem. The precise region of MSSM parameter space and a discussion of how this class of models can be ruled out, or partially verified at the LHC is presented in section 7. Section 8 contains our conclusions and potential avenues of further investigation.

2 Basic Assumptions

Before proceeding to an overview of F-theory based models, we first spell out in greater detail the main assumptions which we shall make. The first assumption is perhaps the most crucial for bottom up string phenomenology:

- Low scale four-dimensional $\mathcal{N} = 1$ supersymmetry is present at energy scales which can be probed by the LHC. Moreover, this should be interpreted as evidence for the MSSM.

Within the MSSM, the unification of the coupling constants at an energy scale $M_{\text{GUT}} \sim 3 \times 10^{16}$ GeV also provides circumstantial evidence for the hypothesis of Grand Unification. Moreover, the chiral matter content of the Standard Model organizes into three copies of the $5 \oplus 10$ of $SU(5)$, which can further unify to the 16 of $SO(10)$ once right-handed neutrinos are included. For these reasons, we shall also make the additional assumption:

- At high energy scales, the matter content of the MSSM unifies into a GUT.

By this we do not necessarily mean that the resulting GUT will admit a four-dimensional interpretation. Indeed, in all known string theory based GUT models, the effective theory descends from a higher-dimensional gauge theory description.

Traditionally, the proximity of the GUT scale to the Planck scale $M_{\text{pl}} \sim 10^{19}$ GeV has been interpreted as circumstantial evidence that a further unification will occur where the gauge theory degrees of freedom unify with gravity, perhaps through some model based on string theory. Note, however, that there is in fact an imperfect alignment between $M_{\text{GUT}}$ and $M_{\text{pl}}$. Indeed, $M_{\text{GUT}}/M_{\text{pl}} \sim 10^{-3}$ is a small parameter, and from the perspective of GUT model building, this is perhaps a quite welcome feature! Indeed, the small value of this ratio is also consistent with the conjecture
that in any quantum theory of gravity, gravity is the weakest force \[5\]. At a practical level, if \(M_{\text{GUT}}\) had turned out to be close to \(M_{\text{pl}}\), there would be no regime of validity for effective field theory at the GUT scale. Moreover, in minimal incarnations of GUT models, the total amount of matter in the theory is sufficiently small that the gauge coupling of the GUT group is asymptotically free. In principle, then, this field theory admits an ultraviolet completion which does not require gravity. For these reasons, we shall also require that:

- There exists a limit where in principle, \(M_{\text{GUT}}/M_{\text{pl}} \rightarrow 0\).

We note that for realistic purposes, such a limit should not be taken, because gravity has certainly been observed. Nevertheless, one of the perhaps surprising outcomes of the recent work developed in [2–4] is that simply requiring the existence of a GUT as well the existence of a decoupling limit severely restricts the ultraviolet behavior of the theory. In fact, pushing this framework to its logical ends will allow us to make contact with the LHC!

## 3 Elements From F-theory

Having sketched in rough terms the bottom up approach to string phenomenology, in this section we introduce the basic ingredients for model building in F-theory. To this end, we now define in broad terms what is meant by “F-theory”, which may be viewed as a strongly coupled formulation of type IIB string theory. In most cases, it is assumed that the inverse coupling constant, \(1/g_{\text{string}}\) assumes a constant profile over the entire ten-dimensional spacetime. In fact, this “constant”, as well as its complexified counterpart which we refer to as \(\tau_{\text{IIB}}\) corresponds to the vev of a dynamical field and as such, it can in principle have a more complicated profile in the ten-dimensional spacetime. In F-theory, this more general case is encoded in terms of a twelve-dimensional geometry. In addition to the four usual large spacetime directions, this includes six internal spatial directions as well as two additional directions which parameterize the value of the real and imaginary parts of \(\tau_{\text{IIB}}\) as they vary from point to point in the internal directions.

The basic ingredient for model building in F-theory is a spacetime filling seven-brane which wraps a four-dimensional internal subspace of the six internal directions.
of the compactification. Such seven-branes can in general form intersections over two-dimensional Riemann surfaces, and can also form triple intersections at points of the internal geometry. Each lower-dimensional subspace provides an important model building element, as in the following table:

| Dimension | Ingredient      |
|-----------|-----------------|
| 10d       | Gravity         |
| 8d        | Gauge Theory    |
| 6d        | Matter          |
| 4d        | Yukawa Couplings|

We now explain in greater detail the origin of the bottom three entries of this table.

### 3.1 Higher-Dimensional Gauge and Matter Content

Near a seven-brane, the profile of $\tau_{IIB}$ becomes singular. There are only a few distinct ways that $\tau_{IIB}$ can become singular near a seven-brane which have been found by mathematicians to be in correspondence with the $ADE$ Lie groups $SU(N)$, $SO(2N)$, and $E_6, E_7$ and $E_8$. The crucial point for model building is that this singularity type also corresponds to the gauge group of the seven-brane! We note that depending on the details of the geometry, it is also possible to engineer non-simply-laced gauge groups such as $USp(2N)$ and $SO(2N + 1)$. This flexibility in achieving a wide range of different gauge groups in F-theory stands in contrast to the more limited possibilities available in perturbative type IIA and IIB models where $g_{string}$ is infinitesimal. Indeed, such models can only accommodate $SU, SO$ and $USp$ type gauge groups.

At low energies, a spacetime filling seven-brane which wraps a four-dimensional subspace $S$ gives rise to a four-dimensional gauge theory with coupling constant:

$$\frac{1}{g_Y^2} \sim M_s^4 \cdot Vol(S). \quad (3.2)$$

where $M_s$ is a characteristic mass scale which relates the volume of the six-dimensional
internal space $B$ to the Planck scale via the relation:

$$M_{pl}^2 \sim M_8^8 \cdot Vol(B).$$

(3.3)

The precise form of the effective action for a seven-brane with gauge group of $ADE$ type has been determined in detail in section 3 and appendix C of [2]. One of the main results of [2] is that detailed properties of the geometry such as the ways in which the internal directions can be deformed correspond to gauge theory quantities in the seven-brane theory. Indeed, this tight correspondence provides strong evidence that the resulting effective action accurately captures the local behavior of the seven-brane.

Proceeding down in dimension, the chiral matter of the MSSM originates from configurations where two seven-branes with gauge groups $G$ and $G'$ intersect. In such configurations, additional light degrees of freedom described by six-dimensional fields localize along these intersections. In terms of four-dimensional $\mathcal{N} = 1$ superspace, these six-dimensional fields can be described as a collection of vector-like pairs of fields $X$ and $X^c$ labeled by points on the corresponding Riemann surface. It can be shown on general grounds that the field $X$ transforms in a representation $R$ of the gauge group $G$ and $R'$ of the gauge group $G'$. Similarly, $X^c$ transforms in the complex conjugate\(^1\) representation $(\bar{R}, \bar{R}')$. The representations $R$ and $R'$ are completely determined by the profile of $\tau_{IIB}$ near the intersection locus of the two seven-branes. Starting from a gauge theory with gauge group $G_S$ along the six-dimensional Riemann surface, the basic point is that off of this curve, the original theory is Higgsed down to either the gauge group $G$, or $G'$, depending on which seven-brane occupies the same location. The adjoint representation of $G_S$ decomposes into irreducible representations of the subgroup $G \times G'$ as:

$$G_S \supset G \times G'$$

$$ad(G_S) \rightarrow (ad(G), 1) + (1, ad(G')) + (R, R') + (\bar{R}, \bar{R}') + \ldots$$

(3.4)\hspace{1cm}(3.5)

This analysis of local Higgsing in the higher-dimensional theory shows that we should expect light degrees of freedom in the representation $(R, R')$ localized on the Riemann surface.

\(^1\)More precisely, the fields in $X^c$ transform in the dual representation to $X$.\[7\]
In perturbative type IIB string theory, the limitations on the types of gauge groups also applies to the available types of matter content, which must descend from the adjoint representation of either a $SU$, $SO$ or $USp$ type gauge group. This has the important consequence that the resulting representations always have two tensor indices. In F-theory, the adjoint representations of the $E$-type groups provide a small but important set of additional possibilities. As a simple example, note that the adjoint representation of $E_6$ decomposes into irreducible representations of the subgroup $SO(10) \times U(1)$ as:

$$E_6 \supset SO(10) \times U(1) \quad (3.6)$$

$$78 \rightarrow 45_0 + 1_0 + 16_{-3} + 16_{+3} \quad (3.7)$$

where the 16 is the spinor representation of $SO(10)$. Similar examples show that other common building blocks of GUT models such as the 27 of $E_6$ are also available. As hinted at above, the role of an $E$-type singularity is the major reason for these additional possibilities. Note, however, that the available matter content is still rigidly determined as a descendant of an adjoint representation. For example, in order for an $SO$ gauge group to admit massless matter in the spinor representation, it must embed as a subgroup of $E_8$.

The detailed form of the effective action for a configuration of two intersecting seven-branes may be found in subsection 4.2 and appendix D of [2]. To test the form of this effective action, one can consider a larger class of geometric deformations which describe compactifications of F-theory in more elaborate geometries. As shown in [2], in all cases, the degrees of freedom in the intersecting seven-brane configuration exactly match to geometric quantities in the F-theory compactification. The detailed form of these consistency checks are presented in subsection 4.3 and appendix F of [2]. We emphasize that in general, these deformations are highly non-trivial. For example, parameterizing the form of one such deformation fills an entire page in [6].

3.1.1 Massless Matter Content

The massless particle content is obtained by expanding the higher-dimensional field theory about a given background field configuration. In general, massless modes can originate from both eight-dimensional fields associated with the worldvolume of a seven-brane, or from six-dimensional fields localized at the intersection of seven-
branes. The wave functions in the internal directions satisfy a wave equation which is of the schematic form:

$$(\partial + A) \psi = 0 \quad (3.8)$$

in either the two dimensions spanned by the Riemann surface, or the four internal dimensions wrapped by a seven-brane. In the above, $A$ is the background gauge field, and $\psi$ is the corresponding wave function. Although in the constructions of [3] all of the chiral matter of the MSSM localizes on Riemann surfaces, it is in principle possible that some of this matter could originate from eight-dimensional fields in the seven-brane. The precise zero mode content from eight-dimensional fields is given in subsections 3.3.1 and 3.3.2, and the zero mode content of six-dimensional fields is derived in subsection 4.4.1 of [2]. As an explicit example, we can view the modes originating from $X$ as left-moving particles on the Riemann surface, and the zero modes from $X^c$ as right-moving particles. The net number of left-movers minus right-movers is then given by the usual result available in many textbooks:

$$n_L - n_R = \int_{\Sigma} F_{\Sigma} \quad (3.9)$$

where in the above, $F_{\Sigma}$ denotes the background gauge field strength on the Riemann surface $\Sigma$.

### 3.2 Interaction Terms

Gauge invariance of the higher-dimensional theory descends to the usual condition that gauge fields interact with the chiral matter of the MSSM. On the other hand, there are also additional interaction terms in the MSSM which originate from the superpotential of the MSSM. In F-theory, fields localized on Riemann surfaces will yield a cubic superpotential term when the corresponding wave functions form a triple overlap in the internal directions. The geometric condition for this to occur is that the corresponding Riemann surfaces should all meet at a point in the geometry. Near this point, the profile of $\tau_{IIB}$ becomes even more singular. Indeed, we can effectively treat this more complicated system as an eight-dimensional gauge theory of higher rank which locally Higgses to lower rank along each Riemann surface, and even lower rank along the worldvolumes of the various seven-branes. In this way, the
The precise form of the resulting interaction terms was obtained in section 5 of [2].

The flexibility in achieving various chiral matter representations also extends to interaction terms. For example, in perturbative type IIB string theory, there is a well-known obstruction to engineering the cubic coupling $5_H \times 10_M \times 10_M$ in an $SU(5)$ GUT. Here, the subscripts indicate whether the GUT field is a Higgs or chiral matter field. The reason this interaction term is perturbatively forbidden is that the actual gauge group which is perturbatively realized is $U(5)$ rather than $SU(5)$. As a consequence, this interaction term violates the net $U(1) \subset U(5)$ of the gauge group. This difficulty can again be traced to the limited ways in which the profile of $\tau_{IIB}$ can change over points in perturbatively realized configurations.

When the value of the string coupling deviates away from the strict $g_{string} \to 0$ limit, a far greater range of possibilities are available which again underscores the necessity of passing to F-theory where $g_{string}$ is not required to be small. In subsection 5.3 of [2], it is shown that when the bulk gauge group $SU(5)$ enhances at points of the four-manifold to the singularity $E_6$, the low energy superpotential contains the standard cubic term $5_H \times 10_M \times 10_M$ of an $SU(5)$ GUT. Similarly, other common interaction terms such as the $27^3$ in $E_6$ GUT models can also be achieved from local enhancements from $E_6$ to $E_8$.

### 3.3 Geometric Meaning of the Decoupling Limit

One of the important features of the above ingredients is that whereas gravity propagates in ten dimensions, in F-theory based models, all gauge theory ingredients localize on subspaces of the compactification. We now use this fact to sharpen the meaning of the “decoupling limit” described in broad terms in section [2]. Geometrically, gravity can decouple when some of the internal directions expand to infinite size. Equations (3.2) and (3.3) imply that the gauge dynamics of the MSSM do not decouple in a limit where $Vol(B) \to \infty$, but $Vol(S)$ remains finite. Another way to state this condition is that the four-dimensional subspace wrapped by the GUT model seven-brane must admit a limit where it contracts to zero size while $B$ remains of fixed size. It turns out that this condition is quite stringent, and there is

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In more perturbative treatments of GUT models, non-perturbative effects in $g_{string}$ are often invoked as a means to generate such interaction terms. Note that this already requires $g_{string}$ to be an order one number.
essentially a single type of four-dimensional subspace available which satisfies these requirements known as the “del Pezzo eight surface”. Here, “surface” refers to a subspace which has two complex dimensions. For further review on del Pezzo surfaces, we refer the reader to appendix A of \cite{2} and section 8 of \cite{3}.

Requiring a geometric decoupling limit is also in accord with expectations based on moduli stabilization. Insofar as the volume of the surface $S$ is stabilized by Planck scale physics, it is natural to expect $M_{GUT}$ to be perhaps not too far from $M_{pl}$. However, if $S$ had not been contractible, there would have been an obstruction to making $M_{GUT}$ smaller than $M_{pl}$. This provides additional motivation for assuming the existence of a decoupling limit. In actual applications to GUT models, note that the value of the fine structure constant $\alpha_{GUT} = g_{GUT}^2/4\pi \sim 1/25$. As a consequence, $Vol(S)$ must also be large, although still finite.

The relevant length scales of the compactification are crudely characterized by the radii $R_S \equiv Vol(S)^{1/4}$, $R_B \equiv Vol(B)^{1/6}$, as well as a measure of the distance scale normal to the seven-brane, $R_\perp$. As estimated in section 4 of \cite{3}, the corresponding energy scales are:

$$\frac{1}{R_S} \sim M_{GUT} \sim 3 \times 10^{16} \text{ GeV} \quad (3.10)$$

$$\frac{1}{R_B} \sim M_{GUT} \times \varepsilon^{1/3} \sim 10^{16} \text{ GeV} \quad (3.11)$$

$$\frac{1}{R_\perp} \sim M_{GUT} \times \varepsilon^{\gamma_\perp} \sim 5 \times 10^{15\pm0.5} \text{ GeV} \quad (3.12)$$

where:

$$\varepsilon \equiv \frac{M_{GUT}}{\alpha_{GUT} M_{pl}} \sim 7.5 \times 10^{-2} \quad (3.13)$$

and $1/3 \leq \gamma_\perp \leq 1$ is a measure of the eccentricity of the normal directions to the four-manifold $S$. The hierarchy $M_{GUT}/M_{pl} \sim 10^{-3}$ will appear repeatedly in estimates on the axion decay constant, the masses of neutrinos, as well as the $\mu$ term. Finally, note that as required to achieve a decoupling limit:

$$R_S < R_B < R_\perp. \quad (3.14)$$
4 Achieving the Exact Spectrum of the MSSM

Up to this point, we have simply given a broad set of general considerations for local model building in F-theory GUTs. Here, we show that the two conditions:

- A supersymmetric GUT exists;
- There exists a limit where gravity can in principle decouple,

impose surprisingly powerful restrictions on the ultraviolet behavior of the four-dimensional effective field theory. Indeed, as we explain in this section, simply achieving the correct matter spectrum will automatically address a number of puzzles present in four-dimensional GUT models. While any one solution might be considered at best only circumstantial evidence for this approach, we find it compelling that a single ingredient typically addresses several issues in traditional four-dimensional GUT models simultaneously.

The starting point for our discussion is that the chiral matter content of the MSSM should organize into representations of a GUT group. Achieving this requires that the gauge degrees of freedom descend from a seven-brane which contains the GUT group $SU(5)$. In these models, the chiral matter and Higgs fields of the MSSM correspond to zero modes of six-dimensional fields which localize on Riemann surfaces in the four-dimensional subspace wrapped by the GUT model seven-brane. For example, in the explicit minimal realizations of an $SU(5)$ GUT presented in section 17 of [3], the $5_M$, $5_H$ and $5_H$ localize on various Riemann surfaces where the singularity type enhances from $SU(5)$ to $SU(6)$. In addition, the $10_M$ fields localize on Riemann surfaces where the singularity type enhances to $SO(10)$.

In traditional four-dimensional GUT models, breaking the GUT group is typically achieved by allowing an adjoint-valued chiral superfield to develop a suitable vev in the $U(1)_Y$ hypercharge direction, breaking $SU(5)$ to $SU(3)_C \times SU(2)_L \times U(1)_Y$. In appendix E of [2], it is shown that in models which admit a decoupling limit, no such chiral superfields exist in the four-dimensional effective theory. As a consequence, the usual four-dimensional GUT scenario cannot be realized! This feature is also quite common to many other string based models which aim to realize the MSSM. In that context, another common approach is to consider gauge group breaking via Wilson lines. This requires that certain topological conditions must be met in the
internal directions of the compactification. As explained in section 5 of [3], models which admit a decoupling limit do not satisfy this criterion. Thus, the two main ways that have been attempted to even break the GUT group are simply unavailable in local F-theory models.

It turns out that there is another way to break the GUT group which is somewhat unique to F-theory. This corresponds to turning on an internal flux on the GUT model seven-brane which is aligned in the $U(1)_Y$ hypercharge direction. Given its simplicity, it may at first appear surprising that this mechanism is not used more frequently in string based constructions. Indeed, there is a well-known obstruction to utilizing this mechanism in contexts other than F-theory because the corresponding field strengths couple to axion-like fields in the four-dimensional effective theory. Via the Stückelberg mechanism, such couplings end up generating a large mass for the $U(1)_Y$ gauge boson, in sharp disagreement with observation.

However, as recently shown in [3, 7], such problematic couplings to axion-like fields are absent when the internal flux obeys a certain set of topological conditions. For discussion on the precise form of this condition, as well as some explicit examples where it can be satisfied, see section 9 of [3].

Usual expectations from string theory might suggest that the mechanism for GUT group breaking detailed above is present in some dual formulation of the theory. Indeed, there is a well known duality between certain compactifications of F-theory and the heterotic string. In fact, the geometry of the compactification must be of a rather special type in order for this duality to hold. As explained in section 9.1 of [3], the mechanism detailed above turns out to be unavailable in those models which possess a heterotic dual!

The presence of an internal hypercharge flux, or “hyperflux” has immediate repercussions throughout the rest of the model. Just at the level of the spectrum, this background flux can sometimes generate matter fields in exotic representations. The precise conditions for avoiding such exotica are discussed in detail in section 10 of [3]. One consequence of this work is that the higher the rank of the GUT group, the more difficult it is to remove the exotics. In this way, it was shown there that if all of the matter of the MSSM localizes on Riemann surfaces, the GUT group must correspond either to $SU(5)$ or $SO(10)$ in eight dimensions. In the latter case, the corresponding model descends to a flipped $SU(5)$ GUT model in four dimensions. One interesting feature of this scenario is that typical problems with embedding four-dimensional
flipped GUT models in $SO(10)$ gauge groups are absent in this higher-dimensional approach.

The background hyperflux also provides a conceptual explanation for why the Higgs fields do not organize into full GUT multiplets, whereas the rest of the chiral matter of the MSSM does. Recall that the chiral matter of the MSSM descends from zero modes of six-dimensional fields which localize on Riemann surfaces inside of the GUT model seven-brane. These six-dimensional fields couple to $U(1)$ gauge fields associated with seven-branes which intersect the GUT model seven-brane, as well as the activated hyperflux. The individual components of a representation charged under the GUT group $SU(5)$ will couple differently to the $U(1)_Y$ hyperflux. For example, the 5 of $SU(5)$ decomposes to the Standard Model gauge group as:

$$SU(5) \supset SU(3)_C \times SU(2)_L \times U(1)_Y$$

$$5 \rightarrow (1,2)_{1/2} + (3,1)_{-1/3}. \quad (4.1)$$

In a suitable integral normalization of charge, this implies that the total number of left-movers minus right-movers in the $(1,2)_{1/2}$ versus $(3,1)_{-1/3}$ is:

$$(1,2)_{1/2} : n_L - n_R = 3 \int \Sigma F_{U(1)_Y} + q \int \Sigma F_{U(1)_\perp} \quad (4.3)$$

$$(3,1)_{-1/3} : n_L - n_R = -2 \int \Sigma F_{U(1)_Y} + q \int \Sigma F_{U(1)_\perp} \quad (4.4)$$

where in the above, $U(1)_\perp$ refers to a $U(1)$ from a seven-brane which intersects the GUT model seven-brane, and $q$ denotes the charge of the six-dimensional field under this gauge group. As is apparent from equations (4.3) and (4.4), when the net hyperflux is non-zero, the resulting matter content cannot organize into full GUT multiplets. Conversely, the zero mode content will always form GUT multiplets when the net hyperflux vanishes. This yields the simple criterion:

Higgs: $\int \Sigma F_{U(1)_Y} \neq 0 \quad (4.5)$

Matter: $\int \Sigma F_{U(1)_Y} = 0. \quad (4.6)$
Note that in order to achieve a chiral matter spectrum, the net flux from the $U(1)_{\perp}$ factor must therefore be non-trivial. Explicit examples which achieve the exact spectrum of the MSSM based on $SU(5)$ GUTs are given in section 17, and models based on flipped $SU(5)$ GUTs are presented in section 18 of [3].

5 Addressing Four-Dimensional GUT Puzzles

It may appear somewhat perplexing that our ultimate goal is a realization of a GUT model, because there are also well-known difficulties with traditional four-dimensional GUTs. In this section we show that issues such as proton decay, GUT mass relations, and large masses for vector-like pairs, as well as more positive features such as variants on the seesaw mechanism for neutrino masses are naturally addressed in local F-theory GUTs. As we now explain, the essential reason for this is that the $U(1)$ hyperflux which plays a prominent role in achieving the correct matter spectrum also enters more indirectly into other aspects of the low energy physics.

5.1 Proton Decay and Doublet-Triplet Splitting

One obstacle in realizing semi-realistic four-dimensional GUT models is the so-called doublet-triplet splitting problem. In its mildest form, this is the fact that as opposed to the chiral matter of the MSSM, the Higgs fields do not fit into complete GUT multiplets. While any string based model which realizes the MSSM must provide a solution to this problem, the fully story is more complicated. This is because heavy Higgs triplets can still mediate proton decay. Indeed, heavy triplet exchange could still generate the superpotential term:

$$W \supset \eta \frac{Q}{M} QQQL,$$

(5.1)

where $Q$ denotes a quark doublet superfield, and $L$ denotes a lepton doublet superfield. Even if $M$ is on the order of the Planck scale, the parameter $\eta$ must be quite small in order to avoid rapid nucleon decay. This operator is generated when the Higgs triplet associated with $H_u$ and the one associated with $H_d$ obtain a large mass.
through the superpotential term:

\[ W \supset MT_u T_d \]  \hspace{1cm} (5.2)

in the obvious notation. In higher-dimensional theories, it is important to note that even if the zero mode spectrum does not contain such fields, their Kaluza-Klein modes will still be present.

This potential problem is absent in local F-theory GUT models. It follows from equation (4.3) that a given matter curve will typically only support one type of Higgs field. As a consequence, the Higgs up and down fields must localize on distinct matter curves. Instead of equation (5.2), the resulting mass terms are therefore of the form:

\[ W \supset MT_u T'_d + MT'_d T_u \]  \hspace{1cm} (5.3)

where the \( T' \)'s are additional triplet chiral superfields corresponding to Kaluza-Klein modes of the theory. Hence, the operator \( QQQL \) is not generated by heavy Higgs triplet exchange. Further discussion on this point, as well as a more general explanation based on the presence of background \( U(1) \) symmetries may be found in sections 12 and 13 of [3]. See [7] for some additional recent discussion on proton decay in local F-theory models.

### 5.2 GUT Mass Relations

In a strictly four-dimensional model, note that because the chiral matter of the MSSM organizes into GUT multiplets, gauge invariance of the higher rank gauge group requires that the cubic interaction terms of the superpotential must assume a far more constrained form than what is necessary to realize the Yukawa couplings of the MSSM. For example, the right-handed down type quarks and lepton doublets organize into the \( \overline{5} \) of \( SU(5) \). As a consequence, we can expect the mass relation:

\[ m_b = m_\tau \]  \hspace{1cm} (5.4)

to hold at the GUT scale. While this relation holds fairly well for the heaviest third generation, it is violated for the lighter two generations. Various elaborate

\[ ^3 \text{In the four-dimensional GUT model building literature, this is known as the missing partner mechanism.} \]
mechanisms have been proposed to circumvent this problem based on either including higher dimension operators in the superpotential, or by including fields in larger dimension representations of the GUT group which can induce further structure in the Yukawa couplings once they develop suitable vevs. In F-theory based models, the second option is not even available. Indeed, a rough classification of the possible representations available from minimal rank enhancements can be found in appendix C of [3].

A priori, the presence of a background hyperflux will distort these GUT mass relations. As reviewed near equation (4.6), the net hyperflux through a Riemann surface supporting a chiral generation must vanish in order for the number of irreducible components of a GUT multiplet to remain equal. This, however, is a discrete quantity, and in particular is not sensitive to detailed properties of the flux at individual points of the Riemann surface. Indeed, it is a far more severe requirement to demand that the flux vanish pointwise on the Riemann surface. Because different irreducible representations of the Standard Model gauge group have distinct $U(1)$ hypercharges, the resulting wave functions on the Riemann surface will also be dissimilar. In particular, the overlap of these wave functions with the Higgs field wave functions will in general be different. For this reason, it would at first seem that there is no reason to expect any relation of the type given by equation (5.4).

At a qualitative level, this distortion also decreases as the mass of a field increases because the kinetic term for a field localized on a Riemann surface $\Sigma$ is proportional to its volume, $Vol(\Sigma)$. In a canonical normalization of all fields, the mass therefore scales as:

$$m \propto \frac{1}{\sqrt{Vol(\Sigma)}}.$$ (5.5)

As $Vol(\Sigma) \to 0$, the cost in energy to maintain a large imbalance in flux increases. Hence, for smaller curves, the hyperflux also diminishes in strength. In this case, the wave functions of different components of a GUT multiplet will have similar profiles over the Riemann surface. The corresponding mass relation of equation (5.4) will therefore become qualitatively more accurate for heavier fields, just as is observed. See section 14.3 of [3] for a slightly more technical version of this same discussion.
5.3 Singlet Wave Functions and Neutrino Masses

One of the successes of the GUT paradigm is the seesaw mechanism. This provides a qualitative explanation for why neutrino masses can in general be far lighter than the weak scale. This is usually taken as evidence for $SO(10)$ GUTs, and the fact that with right-handed neutrinos, the matter content of the MSSM unifies into the spinor 16 of $SO(10)$. In local F-theory GUT models, such spinors can be accommodated even when the bulk gauge group is only $SU(5)$ when the bulk singularity type enhances by more than one rank along a Riemann surface. Some related examples of this type are presented in section 7 of [4].

A variant of the seesaw mechanism is also available in $SU(5)$ GUTs even when the singularity type enhances by a minimal amount. This is because singlet fields will generically interact with lepton doublets in such a way that they can be consistently identified with right-handed neutrinos. In these cases, the Majorana mass of the heavy neutrinos is typically somewhat lighter than in a traditional four-dimensional GUT model. Recall that in the MSSM, the lepton doublet and Higgs up field form a vector-like pair. As explained in section 15.1 of [3], vector-like pairs of fields $\rho$ and $\rho'$ can interact with a $SU(5)$ GUT group singlet $X_\perp$ through an interaction term of the form:

$$W \supset \kappa X_\perp \rho \rho'. \quad (5.6)$$

As a singlet, the Riemann surface supporting $X_\perp$ does not reside inside of the GUT model seven-brane, but rather will only intersect it at distinct points. In particular, this implies that the behavior of the $X_\perp$ wave function will behave in a qualitatively different fashion from $\rho$ and $\rho'$. This singlet wave function was analyzed in detail in subsections 15.1 and 15.2 of [3] where it was shown that the local positive curvature of the four-dimensional surface wrapped by the seven-brane can either repel or attract the $X_\perp$ field wave function away from the GUT model seven-brane. When it is attracted towards the seven-brane, $\kappa$ is typically suppressed by a small overall volume effect related to the small hierarchy of scales $M_{GUT}/M_{pl} \sim 10^{-3}$. On the other hand, when $X_\perp$ is repelled away from the seven-brane, $\kappa$ will be exponentially suppressed, providing a potential mechanism for generating large hierarchies in energy scales. As specific applications, it was proposed in [3] that this type of suppression term could generate a very small $\mu$ term, or light Dirac neutrino masses. Some potential drawbacks are that this type of exponential suppression provides a mostly qualitative
picture, which is in itself not very predictive.

In the more predictive case where \( \kappa \) is only suppressed by a mild volume related factor, it can be shown that the superpotential for the neutrinos is of the rough form:

\[
W = \alpha_{GUT}^{3/4} \left( H_u L N_R + M_{GUT} \varepsilon^4 \gamma_{\perp} \cdot N_R N_R \right).
\] (5.7)

where \( L \) denotes the lepton doublet, \( H_u \) the Higgs up and \( N_R \) the right-handed neutrino chiral superfield. Here, \( \varepsilon \) is the same volume suppression factor defined by equation (3.13), and \( 1/3 \leq \gamma_{\perp} \leq 1 \) is again a measure of the eccentricity in directions normal to the GUT model seven-brane. The resulting value for the Majorana masses of the right-handed neutrinos is typically close to \( 10^{12} \) GeV, and the light neutrinos have mass:

\[
m_{\text{light}} \sim \alpha_{GUT}^{3/4} \varepsilon^{-4} \frac{(H_u)^2}{M_{GUT}} \sim 2 \times 10^{-1\pm1.5} \text{ eV}. \] (5.8)

This provides a small enhancement over the usual value \( (H_u)^2 / M_{GUT} \) of the simplest GUT based seesaw mechanism. This value is in slightly better accord with expectations based on neutrino oscillations. Further discussion on neutrinos in local F-theory models may be found in subsection 15.4 of [3]. Of course, this analysis should only be viewed as a first step towards a more complete theory of neutrinos.

### 5.4 Towards a Theory of Flavor

Flavor physics is an important component of any model which aims to incorporate the Standard Model which has so far met with limited success in string based models. The interplay between the geometry of the matter curves and the Yukawas of the four-dimensional effective theory is described in section 14 of [3]. At zeroth order, the most important requirement is that the top quark should have much larger mass than the other quarks of the Standard Model. The geometric conditions for semi-realistic textures are described in subsections 14.1 and 14.2 of [3].

Some general speculations on realizing a hierarchical CKM matrix via a geometric realization of the Froggatt-Nielsen mechanism are presented in subsection 14.4 of [3]. Additional speculations on possible discrete flavor symmetries originating from the symmetry groups of del Pezzo surfaces are presented in subsection 14.5 of [3].
5.5 Addressing the Crude $\mu$ Problem

Achieving the correct matter spectrum has another important consequence in the Higgs sector of the theory. In subsection 5.1 we have emphasized the role of the heavy Higgs triplets in doublet triplet splitting. On the other hand, it is also important that the Higgs doublets remain light. As before, the main point is that achieving the correct matter spectrum requires that the Higgs up and Higgs down fields must localize on different Riemann surfaces. This already addresses the crudest version of the $\mu$ problem which is the puzzling fact that in the MSSM, the vector-like pair of Higgs fields could develop a large mass through the superpotential term:

$$W \supset \mu H_u H_d.$$  \hspace{1cm} (5.9)

In local F-theory constructions, the Higgs fields originate at the intersection of the GUT model seven-brane with other seven-branes. This implies that the Higgs fields are charged under additional $U(1)$ gauge group factors which forbid bare $\mu$ terms. When the Higgs curves do not intersect, the $\mu$ term is absent. When these curves do meet, the Higgs fields could interact with a GUT group singlet $X$, either through an F- or D-term. These contributions can induce an effective $\mu$ term once $X$ develops a suitable vev. In principle, when the resulting singlet wave function is exponentially suppressed, very small values for the $\mu$ term are also possible. In the next section we will address a more refined version of this same issue where the value of $\mu$ correlates with the scale of supersymmetry breaking.

6 Supersymmetry Breaking

One of the main themes of this paper is that imposing only a few qualitative assumptions on the behavior of F-theory GUT models is enough to tightly constrain the ultraviolet behavior of the effective field theory. Even so, any semi-realistic model which aims to incorporate the MSSM must also address the origin of supersymmetry breaking. In most viable scenarios, there are three sectors corresponding to the visible sector, the hidden sector where supersymmetry is broken, and the messenger sector which communicates these effects to the visible sector. In most models, the mediation sector proceeds either through Planck suppressed operators, as in gravity mediation, or through the gauge fields of the Standard Model, as in gauge mediation,
and there are many variants on these two basic possibilities. In this section we review
some of the analysis of [4] showing that crude considerations based on correlating
the weak scale with the scale of supersymmetry breaking determine to a remarkable
extent the IR behavior of this class of models. In fact, this information is sufficiently
precise that in section 7 we will be able to determine the primary characteristics of
the sparticle spectrum for a broad class of F-theory GUTs.

6.1 Addressing the Refined $\mu/B\mu$ Problem

Correlating the scale of supersymmetry breaking with the weak scale requires that
the $\mu$ term should somehow be sensitive to the effects of supersymmetry breaking.
Parameterizing the effects of supersymmetry breaking by a chiral superfield $X$ which
develops a supersymmetry breaking vev:

$$\langle X \rangle = x + \theta^2 F,$$

(6.1)

the scale of supersymmetry breaking in the hidden sector is given by $\sqrt{F}$. In order
to spontaneously break supersymmetry, $X$ should be treated as a dynamical field
rather than as a spurion field, and we shall therefore only consider this possibility.
In this section we demonstrate that there are a broad class of vacua in F-theory
where $\mu(F, \overline{F})$ is a non-constant function. One of the main results of [4] is that
the resulting value of the $\mu$ term typically requires $F \sim 10^{17}$ GeV$^2$ to solve the $\mu$
problem. This turns out to imply that gravity mediation would generate a value for
$\mu$ far above the weak scale. Assuming that supersymmetry breaking communicates
to the visible sector via gauge mediation, this also requires that $x \sim 10^{12}$ GeV.

These crude restrictions stem from requiring that the vev of $X$ determines $\mu(F, \overline{F})$.
As implicitly assumed throughout [4], this is naturally realized when the wave
function for $X$ overlaps with the wave functions for the Higgs fields. Matter fields which
localize on Riemann surfaces interact most strongly at points of maximal wave func-
tion overlap. Geometrically, this requires that the corresponding Riemann surfaces
touch at some point. As reviewed in subsection 3.2, when two matter curves inter-
sect, there will always be a third matter curve which also touches at this same point.[4]
For these reasons, it is perhaps most straightforward to assume that $X$ localizes on a

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[4]This follows by analyzing the local profile of $\tau_{IIB}$ near such a point of intersection.
Riemann surface which forms a triple intersection with the matter curves supporting the Higgs up and Higgs down fields.

In this case, section 4 of [3] establishes that either the following F- or D-term will be present in the low energy theory:

\[ \int d^2\theta X H_u H_d \quad \text{or} \quad \int d^4\theta \frac{X^\dagger H_u H_d}{M_X} \]

(6.2)

where the \( U(1) \) symmetries of the background seven-branes prevent both terms from appearing in the same action. In the above, the D-term is generated by integrating out Kaluza-Klein modes of the \( X \) field which have mass \( M_X \approx 10^{15.5} \) GeV.

First consider the case where the F-term is allowed. If present, this operator would exacerbate the \( \mu/B\mu \) problem for typical values of \( x \) and \( F \). Moreover, \( \mu \) would depend on neither \( F \) nor \( \overline{F} \). On the other hand, the D-term is potentially more promising for generating a potential correlation because it realizes a variant of the Giudice-Masiero mechanism [8]. When \( X \) develops a supersymmetry breaking vev, the D-term will induce an effective \( \mu \) term with:

\[ \mu \approx \gamma \frac{F}{M_X}. \]

(6.3)

In practice, \( \gamma \) is typically an order 10 number so that for \( F \approx 10^{17} \) GeV\(^2\), \( \mu \) is close to the weak scale. In gravity mediated scenarios, \( F \approx 10^{21} - 10^{22} \) GeV\(^2\), which would generate too large a value for \( \mu \) in the present class of models. Finally, the phenomenological requirements of gauge mediation also imply \( F/x \approx 10^5 \) GeV so that \( x \approx 10^{12} \) GeV. When we discuss the explicit supersymmetry breaking scenario based on a Fayet-Polonyi model, we will show that this value for \( x \) naturally emerges from string based considerations.

It may at first seem somewhat perplexing that this natural mechanism has not previously been more exploited in the phenomenology literature. In a generic effective field theory, the suppression scale \( M_X \) is naturally identified with \( x \). This would not solve the \( B\mu \) problem, however, because the D-term:

\[ \int d^4\theta \frac{X^\dagger X X^\dagger H_u H_d}{M_X^2} \]

(6.4)
generates a $B\mu$ term when $X$ develops a supersymmetry breaking vev:

$$B\mu \sim \frac{\overline{x}|F|^2}{M_X^3}. \quad (6.5)$$

Hence, when $x \sim M_X$, we find $B\mu \sim |F/x|^2 \sim (10^5 \text{ GeV})^2$ at the messenger scale, which is problematic. Note, however, that in the present case, the value of $B\mu$ is far smaller at the messenger scale because $x/M_X \sim 10^{-3}$, solving the $B\mu$ problem. For further discussion on this point, see section 4 of [4]. An explicit realization of the gauge mediation scenario which solves the $\mu/B\mu$ problem through the described variant of the Giudice-Masiero mechanism was constructed in section 5 of [4] and is referred to as the ‘diamond ring model’.

At the messenger scale, $B\mu = 0$, and all of the $A$-terms of the soft supersymmetry breaking Lagrangian also vanish. One consequence of this is that the argument of all of these terms at lower scales are correlated, thus preventing any extraneous CP violation in the low energy Lagrangian. This point was already briefly mentioned in section 16 [3] and was implicitly assumed throughout [4].

### 6.2 Consequences of an Anomalous $U(1)$ Peccei-Quinn Symmetry

#### 6.2.1 $E_6$ Embedding

From the perspective of the effective field theory, the bare $\mu$ and $B\mu$ terms are forbidden by requiring that all fields of the MSSM have appropriate charges under a $U(1)$ Peccei-Quinn symmetry. In local F-theory models, this symmetry originates from the gauge theory of seven-branes which intersect the GUT model seven-brane. A potential refinement on the ‘diamond ring model’ which emphasizes the central role of $U(1)_{PQ}$ is given in section 7 of [4]. This refinement is based on the observation that $U(1)_{PQ}$ naturally embeds in the group $E_6$ as the abelian factor of the maximal subgroup $SO(10) \times U(1)_{PQ}$. Indeed, at the level of representation theory, the 27 of $E_6$ decomposes into irreducible representations of $SO(10) \times U(1)_{PQ}$ as:

$$E_6 \supset SO(10) \times U(1)_{PQ} \quad (6.6)$$

$$27 \rightarrow 1_{+4} + 10_{-2} + 16_{+1}. \quad (6.7)$$
In this case, $X$ and the messenger fields respectively embed in the $1_{-4}$ and $10_{+2}$ of $27$. In local F-theory models where the bulk gauge group is given by $SU(5)$, a local enhancement in singularity type to $E_7$ along a Riemann surface will contain matter fields in the $27$ of $E_6$. This flexibility allows these local models to avoid much of the baggage of four-dimensional GUT models with larger rank gauge groups. Nevertheless, the construction presented in section 7 of [4] contains some residual exotic matter fields so that the resulting matter spectrum is only semi-realistic. Improving this type of construction is one avenue of investigation which would be important to pursue in this approach.

6.2.2 PQ Deformation of Gauge Mediation

Leaving behind such aesthetic considerations, the matter content of the effective theory generates an anomaly for the $U(1)_{PQ}$ gauge theory which is canceled through a variant of the Green-Schwarz mechanism. As a consequence, the $U(1)_{PQ}$ gauge boson develops a large mass through the St"uckelberg mechanism. These results are well-known in the string theory literature, but for further discussion on this point in the specific context of these local F-theory models, see section 8.2 of [4].

An important consequence of this fact is that heavy $U(1)_{PQ}$ gauge boson exchange will generate a correction to the usual soft mass terms present in gauge mediation scenarios. The precise value of this correction is fixed by the mass of the gauge boson $M_{U(1)_{PQ}}$, and the fine structure constant $\alpha_{PQ}$ of the $U(1)_{PQ}$ gauge theory so that at the messenger scale, the soft masses squared for the Higgs fields and chiral matter receive an additional correction of the form:

$$m^2(M_{muss}) = m^2_{GMSB}(M_{muss}) + q\Delta^2_{PQ}$$  \hspace{1cm} (6.8)

where $q = +2$ for the Higgs fields, $-1$ for all the other chiral superfields of the MSSM, and:

$$\Delta_{PQ}^2 = 16\pi\alpha_{PQ}\left|\frac{F}{M_{U(1)_{PQ}}}\right|^2.$$  \hspace{1cm} (6.9)

Section 4.2 of [4] contains additional details on the derivation of equation (6.8). When $M_{U(1)_{PQ}}$ is sufficiently low, this effect constitutes an important, predictive deformation away from the soft mass terms expected in the minimal gauge mediation scenario.
6.2.3 \(U(1)_{PQ}\) and the QCD Axion

Remarkably, the phase of \(X\) is also a viable candidate for the QCD axion. This is because the \(U(1)_{PQ}\) gauge symmetry is already Higgsed via the Stückelberg mechanism at high energy scales, leaving behind a nearly exact global symmetry at lower energies. The vev of the \(X\) field breaks this nearly exact global symmetry, and the associated Goldstone mode \(a\) is the phase of \(x\). As shown in section 6 of [4], this phase directly couples to the QCD instanton density. The Lagrangian density for \(a\) includes the terms:

\[
L_a \supset |x|^2 \partial_\mu a \partial^\mu a + \frac{a}{32\pi^2} \varepsilon^{\mu\nu\rho\sigma} Tr_{SU(3)} F_{\mu\nu} F_{\rho\sigma}.
\]  
(6.10)

This field mixes a small amount with other modes of the compactification. The end result of this is that the axion decay constant is to leading order:

\[
f_a = \sqrt{2} |x| \sim 10^{12} \text{ GeV}
\]  
(6.11)

which is within the standard allowed axion window with at most only mild fine tunings of various couplings. Further background on axions and the potential role of the phase of \(x\) as the QCD axion may be found in section 6 and appendix A of [4].

6.3 Explicit Supersymmetry Breaking Sector

For the most part, the effects of supersymmetry breaking in the hidden sector can be encapsulated entirely in terms of the vev of \(X\). Nevertheless, for completeness it is also important to develop models which realize the required mass scales in a natural fashion. Section 8 of [4] provides an explicit model of supersymmetry breaking based on a hybrid of a Fayet and Polonyi model of supersymmetry breaking. Using the results of [9] on instanton effects in the higher-dimensional anomalous \(U(1)_{PQ}\) gauge theory, it can be shown that a Polonyi-like linear term is generated:

\[
W \supset M_{PQ}^2 Q \cdot X
\]  
(6.12)

where \(Q\) is the instanton tunneling amplitude. For \(M_{PQ} \sim M_{GUT}\) we obtain \(Q \sim 5 \times 10^{-17}\). This contribution breaks supersymmetry and sets the value of \(F\) in equation (6.1) to the required value \(M_{PQ}^2 Q \sim 10^{17} \text{ GeV}^2\). The value of \(x\) in equation

25
(6.1) originates instead from the D-term potential of the anomalous $U(1)_{PQ}$ gauge theory due to a non-zero field dependent Fayet-Iliopoulos term $\xi_{PQ}$:

$$V_{\text{Fayet}} = \left( |x|^2 + \ldots - \xi_{PQ} \right)^2$$

(6.13)

where the “...” refers to all other contributions charged under $U(1)_{PQ}$. As shown in section 8 of [4], $\xi_{PQ}$ consists of a field dependent background value $\xi_*$ which is always present in anomalous $U(1)$ theories, as well as another contribution $\xi_{\text{flux}}$ from fluxes of the higher-dimensional theory through the Riemann surface supporting $X$ so that:

$$\xi_{PQ} = \xi_{\text{flux}} + \xi_*.$$  

(6.14)

These two quantities are both large but will approximately cancel in a scan over all available fluxes. The analysis of section 8.3 of [4] establishes that the minimal non-zero value of $\xi_{PQ}$ is:

$$\xi_{PQ} = \frac{M_X}{M_{pl}^2} \sim \left( 10^{12} \, \text{GeV} \right)^2.$$  

(6.15)

As a consequence, $|x| \sim 10^{12} \, \text{GeV}$, as required for both gauge mediation and axion physics! We note that just as in the case of neutrino physics reviewed in subsection 5.3 the small hierarchy $M_{GUT}/M_{pl} \sim 10^{-3}$ is again crucial for realizing this effect.

7 MSSM Parameter Space

In this section we determine detailed features of the soft supersymmetry breaking terms of the MSSM Lagrangian. The essential point is that the crude considerations of previous sections on the scale of supersymmetry breaking, the mediation mechanism, and the existence of a possible PQ deformation are actually sufficient to completely fix the IR behavior of the theory. While one might argue that this is simply a byproduct of the gauge mediation scenario, there is a priori no reason to expect the scale of supersymmetry breaking to be compatible with this scenario, or for various details of the messenger sector to be constrained at even a crude level, as we have done here.

To ensure compatibility with electroweak symmetry breaking, and to determine
the sparticle spectrum, in [4] we utilized the program SOFTSUSY [1]. After specifying some details of the messenger sector, this program adjusts the IR boundary conditions of the theory to remain in accord with electroweak symmetry breaking. These further considerations fully determine the remaining UV boundary conditions, which in turn constrain the remaining parameters of the MSSM.

Some sample scans over various regions of the constrained UV boundary conditions are provided in Section 9 of [4]. As representative examples, we mainly considered models with a single vector-like pair of messenger fields in the $5 \oplus \overline{5}$ of $SU(5)$. Because small changes in the messenger scale $M_{mess}$ only altered the resulting sparticle masses by a small amount, we took as fixed the value $M_{mess} = 10^{12}$ GeV. The scans presented in [4] primarily focused on the two parameter subspace defined by $\Lambda = F/x \sim 10^5 - 10^6$ GeV and the PQ deformation $\Delta_{PQ} \sim 0 - 10^3$ GeV. The final input necessary for determining the UV boundary conditions is given in terms of $\tan \beta \equiv \langle H_u \rangle / \langle H_d \rangle$ at the messenger scale. In order to remain in accord with electroweak symmetry breaking, SOFTSUSY adjusts the UV values of $\mu$ and $B\mu$. Scanning over a range of values for $\tan \beta$, it is then possible to recover the boundary condition $B\mu = 0$ at the messenger scale. We find that at the weak scale, $\tan \beta \sim 30 \pm 7$, where the particular numerical value depends on the specific UV boundary conditions imposed. While perhaps obvious, note that a large $\tan \beta$ scenario is quite compatible with the expectation that in GUT models, the Yukawa couplings for up and down type quarks are expected to be comparable, order one numbers. In such a situation, achieving a large hierarchy between the mass of the bottom and top quark, for example, requires that the vev $\langle H_u \rangle$ be at least an order of magnitude larger than $\langle H_d \rangle$.

As in nearly all gauge mediation models, the lightest supersymmetric particle (LSP) is the gravitino, and in our case its mass is $\sim 10 - 100$ MeV. Scanning over the UV boundary conditions reveals that for the most part, the bino is indeed the next lightest supersymmetric particle (NLSP). Large values of the PQ deformation can sometimes alter this story for sufficiently low values of $\Lambda$ by allowing the stau to decrease in mass to the point where it becomes first a co-NLSP with the bino, and then the NLSP. There are limits to how large the PQ deformation can become, because it will eventually cause the scalar effective potential to develop a tachyonic mode, other than the one present in the Higgs sector. Figure 1 shows a plot of the sparticle masses in a scenario with a low value of $\Lambda$ which is consistent with
the current bound on the mass of the Higgs for vanishing PQ deformation, and for a maximal stable PQ deformation. In this case, the lightest stau can become the NLSP. For larger values of \( \Lambda \), the PQ deformation induces an instability in the squark/slepton effective potential before the stau can become the NLSP. Many further details on the sparticle spectrum, as well as a discussion on the amount of fine-tuning in the Higgs sector can be found in section 9 of [1].

7.1 Discovery Potential at the LHC

The rigid framework we have described in previous sections makes definite predictions for what should be seen at the LHC. That being said, given a particular experimental signal, it is a notoriously difficult problem to reconstruct from LHC data alone a given model of beyond the Standard Model physics. In this regard, it is perhaps more important to determine signals which could falsify this class of models.

In many gauge-mediated supersymmetry breaking scenarios, the NLSP is either a bino-like lightest neutralino, \( \tilde{\chi}_1^0 \), or the lightest stau \( \tilde{\tau}_1 \). Here, the \( \tilde{\ } \) indicates that these are sparticles. These sparticles will decay to the gravitino LSP through the respective processes \( \tilde{\chi}_1^0 \to \tilde{G}_{3/2} \gamma \) and \( \tilde{\tau}_1 \to \tilde{G}_{3/2} \tilde{\tau}_1 \). Although this is a gauge mediation scenario, the high scale of supersymmetry breaking implies that the NLSP decays outside of the detector, effectively behaving as an LSP. As reviewed for example, in [10][11], the average decay length for an NLSP with mass \( m \) produced with energy \( E \) is:

\[
L = \frac{1}{\kappa_\gamma} \left( \frac{m}{100 \text{ GeV}} \right)^{-5} \left( \frac{\sqrt{F}}{100 \text{ TeV}} \right)^4 \sqrt{\frac{E^2}{m^2}} - 1 \times 10^{-2} \text{ cm}, \tag{7.1}
\]

where in the above, \( \kappa_\gamma \) is a constant which depends on details of the NLSP. In a crude approximation, we can essentially set all of the above factors to unity except for the term involving \( \sqrt{F} \). Because \( \sqrt{F} \sim 10^{8.5} \text{ GeV} \), the resulting decay length is:

\[
L \sim 10^{12} \text{ cm}, \tag{7.2}
\]

which is well outside the particle detector.

The consequences of this for the LHC depend on whether the \( \tilde{\chi}_1^0 \) or the \( \tilde{\tau}_1 \) is the NLSP. When the \( \tilde{\chi}_1^0 \) is the NLSP, it will simply leave the detector as a neutral
Figure 1: Plot originally presented in [4] of the sparticle spectrum (sparticles denoted by a $\tilde{}$) in a gauge mediation scenario with $\Lambda = 1.3 \times 10^5$ for a single vector-like pair of messenger fields in the $5 \oplus \overline{5}$ of $SU(5)$ at vanishing PQ deformation (red), and for a maximal PQ deformation of $\Delta_{PQ} = 290$ GeV (blue). Beyond this point, a tachyon is present in the squark/slepton sector. This deformation causes the lightest stau ($\tilde{\tau}_1$) to become the NLSP. Note also that at large PQ deformations, the selectron and smuon ($\tilde{e}_R, \tilde{\mu}_R$) are comparable in mass to the bino-like lightest neutralino ($\tilde{\chi}_0^1$).
particle. This is in sharp contrast to many models of gauge mediated supersymmetry breaking where the scale of $\sqrt{F}$ is significantly lower. Indeed, one striking prediction of many gauge mediation scenarios are events with two hard photons from the decay of $\tilde{\chi}_1^0$’s, which is clearly not the case in the present class of models.

When the $\tilde{\tau}_1$ is the NLSP, we can expect events with two charged tracks through the detector calorimeter. Due to the large difference in mass between these particles and the muon, it is then possible to distinguish this signature from a generic muon event.\footnote{For an extensive study of a stau NLSP in the related, although ultimately different, sweet spot model of supersymmetry breaking, see \cite{12}. We caution, however, that the results of this reference are not directly applicable to the case at hand, because the sparticle spectrum is somewhat different there. For example, the heaviest neutralino ($\tilde{\chi}_4^0$) is primarily a wino in \cite{12}, although in the present class of models it is a higgsino. Some other differences include the mass of the gravitino, which is typically heavier at around 1 GeV, than in the models we consider.}

Using the program \textsc{Pythia} \cite{13}, we have determined the cross sections for such high scale gauge mediation models.\footnote{We thank T. Hartman for very helpful explanations on how to use \textsc{Pythia}.} Regardless of the particular details of the NLSP, it appears likely that the LHC will be able to produce a sufficient number of events to allow some crude features of these models to be either verified or falsified. For example, in the scenario considered earlier with a single vector-like pair of messenger fields, $\Lambda = 1.3 \times 10^5$ GeV, and $\Delta_{PQ} = 0$, the mass of the gluino is $\sim 1000$ GeV, and the mass of the lightest stop is $\sim 900$ GeV. In this case, the MSSM process with the largest total cross section is the quark gluon parton collision $qg \rightarrow \tilde{\tau}_R \tilde{g}$. We find that $\sigma (qg \rightarrow \tilde{\tau}_R \tilde{g}) \sim 3 \times 10^2$ fb. See figure 2 for a depiction of one decay chain of $qg \rightarrow \cdots \rightarrow E_T + jets$ which has a relatively large branching ratio at each vertex. Here, $E_T$ denotes missing transverse energy. A similar analysis can also be performed for models with a maximal PQ deformation turned on, although in this case the appearance of tracks in the calorimeter is likely to be a more reliable tool for discrimination. It is beyond the scope of this paper to present a more complete analysis of potential collider signatures. Indeed, while certain processes may have large cross sections, it is likely that signatures with less QCD background could be of greater utility. It would be potentially quite interesting to go beyond the quick sketch presented here to investigate this set of issues in detail.
Figure 2: Depiction of a sample decay chain in a scenario where the bino-like lightest neutralino ($\tilde{\chi}_1^0$) is the NLSP. Starting from the initial collision of partons, the end result is two $\tilde{\chi}_1^0$’s and some number of top quarks and anti-quarks. These tops will then decay further into jets which will sometimes also include leptons in the end result.

8 Conclusions and Future Directions

In this paper, we have given a short overview to our recent work on constructing GUT models in F-theory. One of the perhaps surprising outcomes of this analysis is that simple, qualitative considerations can have far-reaching consequences on both the UV, and IR behavior of the theory. This, rather than any particular mechanism endows these models with surprising predictive power. While we have reviewed the main threads of analysis which lead from the GUT scale all the way down to the weak scale, there are many additional ingredients which we have only briefly touched on, and which are covered in far greater detail in the three papers [2–4]. We have also presented a short description of potential ways that the LHC could discover evidence either in favor of, or against, these types of models.

There are potentially other ways that the ingredients described above could fit together to form a viable phenomenological model. One example would be to find refinements to our solution to the $\mu/B_\mu$ problem in gauge mediation models based on embedding the $U(1)_{PQ}$ symmetry into an $E_6$ GUT. In this class of models, the exact spectrum of the MSSM remains to be constructed, but further analysis of the associated geometries provides a potentially promising avenue of investigation.

Another important ingredient is the way that the small hierarchy between the
GUT scale and Planck scale $M_{\text{GUT}}/M_{\text{pl}} \sim 10^{-3}$ has appeared repeatedly throughout these F-theory models, and especially in [3,4]. In the most predictive models we have found, this mild hierarchy appears in the form of some power law dependence, which tethers the physics of the axion, neutrinos, and messengers to the scale $10^{12}$ GeV. There is also the possibility that a stronger hierarchy could be generated due to exponential suppression of wave functions near the GUT model seven-branes. By its nature, these effects lead to less predictive models, but they also provide additional flexibility, and this topic would be interesting to study in further detail.

Finally, the primary focus of the work presented in this paper has been on potential realizations of particle physics models. Cosmological constraints constitute another important avenue of investigation [14], which are likely to shed further light on details of both the UV and IR regimes of the theory.

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