Bayesian versus Classical Econometric Inference to Revisit the Role of Human Capital in Economic Growth

Muhammad Akbar,1 Abdulmohsen Obied Alshamari,2 Muhammad Tariq,1 Alhelali Marwan,3 Basim S. O. Alsaedi,3 and Ishfaq Ahmad1

1Department of Mathematics and Statistics, International Islamic University Islamabad, Islamabad, Pakistan
2Department of Mathematics, University of Hail, Hail, Saudi Arabia
3Department of Statistics, Faculty of Science, University of Tabuk, Tabuk, Saudi Arabia

Correspondence should be addressed to Muhammad Akbar; muhammad.akbar@iiu.edu.pk

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Application of Bayesian inference to analyze real economic phenomena is rare in the literature on applied economics. This study contributes in two ways. Firstly, it contributes to methodological advancement in the literature on applied economic modeling by estimating a structural model using the classical econometric framework as well as the Bayesian two-stage econometric framework. The performance of the two approaches is compared due to the small sample size and the best model is selected. Secondly, the study is used to get fresh evidence about the impact of human capital upon economic growth in the form of Bayes mean estimates along with their Highest Posterior Density Intervals (HPDIs) which give certain ranges of estimates within which the parameters are likely to lie. Annual data on the Pakistan economy ranging from 1965 to 2019 is used for the estimation of the model. Classical estimates are obtained using the efficient GMM method. Bayes mean estimates are simulated using a Bayesian two-stage procedure assuming multivariate normal-Wishart informative priors. Results show that the Bayesian econometric framework gives more precise parameters’ estimates as compared to the classical econometric framework, and hence, the Bayesian inference may be preferred over classical inference, especially in the case of a small sample size. The Bayes estimates show that a 1% increase in education capital and health capital causes raising economic growth by 0.0091% and 0.1778%, respectively, with a 0.95 probability that the estimates are likely to lie within the intervals 0.0085%–0.0097% and 0.1606%–0.1952%, respectively. Hence, human capital might be considered a vital factor to achieve economic growth in Pakistan. Moreover, health capital shows strong effects as compared to education capital in the process of economic growth.

1. Introduction and Literature Review

Classical and Bayesian econometrics are two parallel approaches for statistical analyses. The two approaches differ because Bayesian inference requires explicitly stating the priors. Classical or Frequentist econometrics leave the priors unspecified and the model is considered an incomplete model from a Bayesian viewpoint. Bayesian statistical inference is based on the Bayes theorem published in 1763. However, the Bayesian econometric inference was derived by Zellner [1]. The bayesian inferential technique may provide more efficient estimates as compared to classical econometric techniques due to certain reasons. Firstly, the Bayesian inference may perform well even in a small sample size (see [2, 3] and [4]). On the other hand, small sample properties of classical econometric techniques like least square methods and GMM are doubtful (see [5]; p#302, p#107, 477). Secondly, the Bayesian inferential procedure is based on the idea that the model’s parameters are random variables, and hence, their posterior distribution is derived by merging prior knowledge about parameters with the information contained in the sample data. The posterior distribution is then used for parameter inference, hypothesis testing, prediction, etc. Appropriate incorporation of prior information into the model may significantly improve understanding of the phenomenon. Thirdly, the Highest
Posterior Density Interval (HPDI) of the Bayes estimate is another distinct feature of the Bayesian inference because it can be directly interpreted as the region within which the parameter is likely to lie with some probability statement [6]. Such type of probability statements cannot be attached to confidence intervals of classical estimates. Hence, econometric modeling based on the Bayesian framework may provide more efficient estimates of a model. Furthermore, estimates obtained by the application of classical econometric techniques may be criticized on the basis of the Lucas critique (1976). Lucas claims that under alternative policy formulations, because all the economic agents base their decisions on full information and rational expectations, therefore, “Any change in policy will systematically alter the structure of econometric models” ([7], P-41). It implies that the estimated coefficients of an econometric model are likely to vary. The Bayesian inference might be appropriate to tackle the Lucas critique by assuming parameters as random variables. However, the literature on economics has limited studies containing formal application of Bayesian econometric techniques (except Bayesian model averaging) to analyze real economic phenomena [8, 9]. Selecting an important topic of economic growth, the present study is conducted to revisit the phenomenon by comparing the performance of classical and Bayesian econometric frameworks.

One of the most recent important topics in economic growth is to analyze the impact of human capital on economic growth. Empirical literature analyzing the impact of human capital upon economic growth relies on classical econometric inference. In the cases of developing countries, the models were estimated using small sample sizes due to limited time-series data availability of the required variables. Hence, the validity of those estimates may be doubtful. Although some studies used a large sample size as quarterly data were used for estimation of error correction model to obtain long-run and short-run estimates separately, annual data were converted into quarterly data by following Arby (2008). This manipulation of annual data is highly criticized. The small sample properties of classical estimation techniques are ambiguous. It requires the application of Bayesian econometric techniques to get more precise and valid estimates using a small sample of annual time series.

Theoretically, the literature on economic growth takes human capital as an important determinant of economic growth. Different growth theories explained by Romer [10], Lucas [11], Mankiw et al. [12], and Hoeffler [13] suggest that human capital plays a vital role in the process of economic growth. Empirical pieces of evidence support the theoretical discussion in explaining cross-country differences in economic growth. In spite of widespread belief about the positive role of human capital, the literature reveals that studies based on cross-country data have produced surprisingly mixed results (see Table 1). The reason for this uncertainty may be that the role of human capital varies due to the fact that institutions, labor markets, and education quality vary across countries [28, 29]. Funke and Strulik [30] emphasize the existence of varying effects of human capital at the different stages of the development of the country. It implies that the development stage of an economy may determine the role of human capital in an economy.

A number of empirical studies have been conducted which show the vital importance of human capital for economic growth in different countries including Pakistan, e.g., Abbas and foreman [27], Imran et al. [31], Khattak and Khan [18], and Asghar et al. [32]. A brief review of the studies is given in Table 1.

Most of the studies found positive and significant impacts of human capital on economic growth. But some studies reveal contradictions to these findings. It implies that the empirical literature studies found varying impacts of human capital in different economies. However, all these studies were carried out using classical econometric inference. Moreover, all of the reviewed empirical studies conducted for individual economies use a small sample size while taking annual data. Hence, the present study is conducted to achieve two major objectives. Firstly, it compares the performance of classical and Bayesian econometric frameworks in the analysis of a real economic phenomenon while using small sample data. Secondly, the study is used to extract fresh evidence on the basis of the Bayesian econometric framework about the impact of human capital on economic growth in Pakistan. It may, therefore, be a significant contribution to make a formal application of Bayesian econometric techniques in order to get fresh evidence about the varying impact based upon posterior estimates along with HPDI of human capital on economic growth in Pakistan. The remaining part of the study consists of material and methodology, results, discussion, and concluding remarks.
2. Material and Methods

2.1. Theoretical Specification of the Model. In order to test the effect of human capital on economic growth, the methodology of Mankiw et al. [12], Barro and Sala [33], and Diaz-Bautista (2003) is followed, and the standard neoclassical model of economic growth is specified by augmenting human capital to the production function. GDP per worker is taken as the dependent variable representing economic growth in line with Mankiw et al. [12], Acemoglu et al. [34], and Temple and Wömann [35]. The final form of the model is derived from Rahman [17].

Assuming Cobb-Douglas production function, i.e.,

\[ Y(t) = K(t)^\alpha A(t)L(t)^{1-\alpha}. \] (1)

Dividing L(t) on both sides gives us output per unit of labor; i.e.,

\[ y(t) = k(t)^\alpha A(t). \] (2)

Human capital includes health, education, and any other investments that increase an individual’s productivity. However, most of the studies have put their focus on education as human capital. This study includes both health and education factors while analyzing the role of human capital in the process of economic growth. It is because education and better health of individuals make efficient use of basic factors of production which improve economic growth (see, e.g., [36]). Since 1990, the country-wise human development index has been constructed by UNDP on the basis of three indicators, i.e., “average life” of a newborn person, literacy rate, and living standard. According to [36] (p. 128), the concept of human capital includes information, skills, abilities, and experiences as well as the physical and mental fitness or strength of the individuals. These views are supported by many other studies in the literature (e.g., [37]). Hence, human capital can be included by introducing education capital and health capital in the above production function. That is,

\[ y(t) = k(t)^\alpha E(t)^\delta h(t)^{\gamma} A(t). \] (3)

Taking natural logarithm after considering the evolution of physical capital, education capital, and health capital and adding trade openness as the control variable in order to capture the effect of the foreign sector, the following structural model is specified as equation (2). Following the standard practice, we have used natural logarithmic transformation to all the variables for estimating the specified model because it helps to linearize the exponential trend of the series which is common in macroeconomic variables. Moreover, the application of logarithmic transformation allows regression coefficients to be interpreted as elasticities.

\[ y_{it} = \beta_0 + \beta_1 X_{it} + \beta_2 X_{it} + \beta_3 X_{it} + \beta_4 X_{it} + \nu_{it}. \] (4)

Here, \( y_{it}, X_{it}, X_{it}, X_{it}, \) and \( X_{it} \) represent GDP per worker, physical capital stock per worker, education capital, health capital, and trade openness, respectively.

There are two major objectives of this study. Firstly, we have to compare the performance of classical versus Bayesian econometric inference using small sample data of a real phenomenon. Secondly, we have to get fresh evidence in the form of random elasticities along with HPDI explaining the varying role of human capital in economic growth. Hence, this study adopts model specifications based upon standard economic growth theories by following a number of previous studies (see, e.g., [14, 26, 30–32]).

2.2. Data Description and Construction of Variables. The study uses data of real GDP per worker, physical capital stock per worker derived using the perpetual inventory method, literacy rate as a proxy for education capital, real health expenditure per capita as a proxy for health capital, and trade as a percentage of GDP for trade openness. Annual data (million rupees) of all the above variables are taken from Pakistan Economic Survey. The range of the data is 1965–2019 while the financial year 2005–06 is the base year of the real variables.

2.3. Estimation Methods. The specified model is estimated using classical as well as Bayesian econometric frameworks. Unlike a number of previous studies which have estimated error correction model (ECM) to obtain short-run and long-run elasticities of human capital, this study is conducted to estimate only long-run estimates of the structural model due to two reasons. Firstly, human capital is the long-run determinant of economic growth (see [14]) and hence short-run estimates of ECM may not have much significance from a theoretical point of view. Moreover, ECM requires high-frequency data such as monthly or quarterly data, which are not available in the case of developing economies like Pakistan. Hence, estimation of ECM using annual data may not be useful from a methodological point of view.

Most of the studies in economic growth literature consider the above explanatory variables in (4) as correlated to the error term and hence endogeneity is an established fact in growth models. We require such classical and Bayesian estimation techniques that may tackle the problem of endogeneity. An efficient generalized method of moments based on 2SLS consistent estimates is used to get estimates of parameters under a classical econometric framework. It may be a valid classical estimation technique for the model as it properly tackles problems of unknown forms of heteroskedasticity, nonlinearity, endogeneity, and biasedness of estimates due to nonstationarity (see [38]; p.42 and [39]; p.303). White heteroskedasticity test, Breusch-Godfrey LM test of autocorrelation, J test of moment conditions, and Jarque Bera test of residuals’ normality are used as diagnostic tests to establish the validity of classical estimates.

Classical inference of the model follows the Bayesian inference using informative priors. The Bayesian two-stage estimation procedure given by Lancaster [40] and Rossi et al. [41] is employed to estimate the specified model (4) as it tackles the problem of endogeneity (see also [42]). The structural model can be written in matrix form as follows.
Instrumental variable equations for endogenous regressors are given by

\[ y_1 = \pi Z_1 + v_0. \]  
\[ W_{1t} = \gamma_{11} + \gamma_{12} W_{1t-1} + v_{1t}, \]  
\[ W_{2t} = \gamma_{21} + \gamma_{22} W_{2t-1} + v_{2t}, \]  
\[ W_{3t} = \gamma_{31} + \gamma_{32} W_{3t-1} + v_{3t}, \]  
\[ W_{4t} = \gamma_{41} + \gamma_{42} W_{4t-1} + v_{4t}. \]

Equations (4-7) are merged into matrix form, i.e.,

\[ X = \pi Z_2 + V^*. \]

Hence, the reduced form of (5) is derived as

\[ y_{1t} = \theta_0 + \theta_1 W_{1t-1} + \theta_2 W_{2t-1} + \theta_3 W_{3t-1} + \theta_4 W_{4t-1} + u_t, \]

where \( \theta_0 = \beta_0 + \beta_1 y_{11} + \beta_2 y_{21} + \beta_3 y_{31} + \beta_4 y_{41} \), \( \theta_1 = \beta_1 y_{11} \), and \( \theta_2 = \beta_2 y_{21} \), \( \theta_3 = \beta_3 y_{31} \), \( \theta_4 = \beta_4 y_{41} \) and \( u_t = \beta_1 v_{1t} + \beta_2 v_{2t} + \beta_3 v_{3t} + \beta_4 v_{4t} + v_0 \).

The compact matrix form of the above equation is

\[ y_1 = \pi Z_3 + U. \]

Equations (10) and (13) can be written as a system of equations; i.e.,

\[ \begin{bmatrix} y_1 \end{bmatrix} = \begin{bmatrix} \pi \ & \ 0 \ & \ 0 \\ Z_3 & 1 & 0 \end{bmatrix} \begin{bmatrix} \pi Z_2 & U \end{bmatrix}, \]

\[ Y = \pi Z + \varepsilon. \]

The likelihood function of equation (10) is derived while assuming \( \varepsilon \sim MN(0, \Omega^{-1}) \); i.e., \( L(\pi, \pi Y / Z) \propto |\Omega|^{n/2} \exp(-1/2 tr(S_0 \Omega)) \), where \( S_0 = \varepsilon' \varepsilon \).

The final form of likelihood would be as follows:

\[ L \propto |\Omega|^{n/2} \exp\left(-\frac{1}{2} \Omega (\pi - \hat{\pi}) Z' Z (\pi - \hat{\pi})\right). \]

The next step of the Bayesian inference is the specification of prior distributions. Here, we assume that \( \Omega \sim W(n_0, \Sigma_0^{-1}) \), i.e., the Wishart distribution, and \( \pi \sim MN(\pi_0, \Omega_0^{-1}) \), i.e., multivariate normal distribution. These priors are multiplied with the likelihood function to obtain posterior density. A detailed derivation of the likelihood function and posterior distribution may be seen in Akbar [9]. The final form of posterior density to be used for inference is derived as follows.

\[ P\left(\frac{\pi, \Omega}{Y, Z, X}\right) \propto |\Omega|^{n+m+1/2} \exp\left(-\frac{1}{2} \Omega V^*\right) \]

\[ \times \exp\left(-\frac{1}{2} (\pi - \pi') \Omega^* (\pi - \pi)\right). \]

where \( n^* = n_t + n \), \( V^* = S + \Sigma_0 \), \( \pi = (\pi_0 \Omega_0 + \hat{\pi} (Z' Z) \Omega) \)

(\( \Omega_0 + (Z' Z)^{-1} \Omega)^{-1} \), and \( \Omega^* = (\pi_0 + (Z' Z)^{-1} \Omega) \).

The next step is the construction of priors and obtaining prior estimates of the parameters in equations (3-7). Hyperparameters of the prior densities of the structural model are elicited on the basis of experts’ information. Elicitation is done by using the PV method. Five researchers who have wide experience in research related to human capital and economic growth and are serving as Professors or Associate Professors of economics at different universities in Pakistan are selected as experts. For each parameter, the five experts \( j = 1, 2, 3, 4, 5 \) are queried to provide expected quantile values of each parameter at the 10th, 25th, 50th, 75th, and 95th quantiles. The quantile values given by the experts are their expected guesses based upon their researches and these estimates are presented in Table 2. Elicitation of hyperparameters is done by estimating a linear regression model taking quantile values as the dependent variable while standard normal points \( (zm) \) as explanatory variable. Hence, estimates of the intercepts represent mean values of the parameters while estimates of slopes will be the standard errors of the parameters. Elicited hyperparameters are given in Table 3. Parameters of instrumental variable equations (4)-(7) are also assumed to be normally independently distributed and their means and variances are obtained by estimating the instrumental equations (4)-(7) using the least square method. As these equations contain only lagged terms of the dependent variables and hence incorporation of prior information is not necessary, the least square estimates of variances are then converted into precision. Values of these hyperparameters are also presented in Table 3.
The posterior distribution is derived by assuming that \( \Omega \sim \text{Wishart}(n_0, \Sigma_0^{-1}) \). Here, \( n_0 \) is the prior degrees of freedom that are taken as 50 in this study, i.e., sample size minus the number of structural parameters to be estimated. \( \Sigma_0 \) is the \( 5 \times 5 \) variance-covariance matrix of residuals. Off-diagonal elements are taken zero whereas diagonal elements consist of residuals’ variances or precisions from (3) to (7). Elicitation of these hyperparameters on the basis of prior knowledge may be done. As no prior information is available, and hence, we use classical estimates of residuals’ variances of (3)–(7), i.e.,

\[
\tau_{i} = \frac{1}{\sigma_{i}^2} = 1.00E+3, 1.00E+3, 1.00E+3, 1.00E+3, 1.00E+3
\]

for \( i = 1, 2, 3, 4, 5 \).

### Table 3: Elicited estimates of hyperparameters.

| Variable         | Parameters | Mean     | Variances         | Precision |
|------------------|------------|----------|-------------------|-----------|
| Intercept        | \( \beta_0 \) | -1.507285 | 0.01199484        | 83.369    |
| Physical capital | \( \beta_1 \) | 0.84563  | 0.00242516        | 412.342799|
| Education capital| \( \beta_2 \) | 0.0054685 | 0.00000088387     | 113138.4388|
| Health capital   | \( \beta_3 \) | 0.1692637 | 0.003664927       | 2728.567  |
| Trade openness   | \( \beta_4 \) | 0.008812 | 0.0000009385      | 1065446.727|
| Physical capital | \( \gamma_{11} \) | 0.3404   | 0.017798575       | 11.62576  |
| Physical capital | \( \gamma_{12} \) | -0.3572369 | 0.086015857     | 56.18427  |
| Education        | \( \gamma_{21} \) | 14.205   | 14.2053           | 0.07041   |
| Education        | \( \gamma_{22} \) | 0.5432   | 0.014096718       | 70.9385   |
| Health capital   | \( \gamma_{31} \) | 0.676406 | 0.08278113        | 12.08005  |
| Health capital   | \( \gamma_{32} \) | 0.8844875 | 0.002643276      | 378.3184 |
| Trade openness   | \( \gamma_{41} \) | 0.0026187 | 0.0470118        | 2.127125  |
| Trade openness   | \( \gamma_{42} \) | 1.030388 | 0.000293129       | 341.471   |

### Table 4: Classical and Bayesian estimation results.

| Parameters          | Classical estimates (standard error) [P value] | Posterior mean estimates (standard deviation) [95% HPDI] |
|---------------------|-----------------------------------------------|---------------------------------------------------------|
| Intercept           | 4.971068 (3.232) [0.1327]                      | -1.445 (0.1081) [-1.654 -1.235]                          |
| Physical capital    | 0.488922 (0.231550) [0.0410]                   | 0.8993 (0.01043) [0.8788 0.9198]                         |
| Education capital   | 0.005674 (0.004728) [0.2372]                   | 0.009121 (0.000309) [0.008522 0.009728]                  |
| Health capital      | 0.059269 (0.019084) [0.0035]                   | 0.1778 (0.008874) [0.1606 0.1952]                        |
| Trade openness      | 0.003240 (0.001552) [0.0432]                   | 0.008335 (0.001867) [0.004735 0.01203]                   |
| Precision           | 1628.405                                       | 1749                                                    |
respectively. Hence, true prior information as the spirit of Bayesians is incorporated only for parameters of the structural equation model.

After elicitation of hyperparameters, Monte Carlo Markov Chain (MCMC) simulations are conducted using the posterior density to simulate parameters’ estimates. The simulations are based on the Gibbs sampling algorithm. Various diagnostic tests are employed to establish the validity of the simulated estimates. These tests are Kernel densities, trace plots, quantile plots, autocorrelation plots, and the Brooks Gelman-Rubin test.

3. Results and Discussion

The classical inferential procedure is applied to estimate parameters of the structural model, i.e., (5). Classical estimates are obtained using the efficient GMM method by plugging 2SLS consistent estimates in EViews. Lag terms of the explanatory variables and the dependent variable are taken as instruments. Parameters’ estimates are presented in Table 4. Diagnostic tests are applied to check the validity of the estimated model. The P value of the White test statistic is 0.99984 which shows homogeneity of residual variance. The P value of the J statistic is reported as 0.522717 which establishes the validity of the instruments. The value of the Breusch-Godfrey LM test is 1.0489 which accepts the null hypothesis of no autocorrelation. The P value of the JB test statistic is 0.6051 which shows the normality of the estimated residuals. The value of the F test statistic is 575 and $R^2$ is 0.99. It indicates that the specified model explains a significant amount of variation in the dependent variable. Hence, diagnostic tests establish the validity of GMM estimates based on the classical econometric framework.

Classical inference of the model follows the Bayesian inference. For this purpose, a code in WinBUGS is developed where MCMC simulations are conducted using Gibbs sampling to obtain the Bayesian estimates from the joint posterior distribution (equation (11)). Two chains of MCMC simulations are conducted where each chain consists of 1500000 MCMC simulations with 200 thinning intervals while discarding the first 500000 initial simulation results. The two chains are two independent streams of simulations that help to achieve convergence to the desired distribution. Discarding of first 500000 initial simulations means that the estimates of those initial simulations are not considered to obtain final estimates. Thinning interval is fixed in order to remove the problem of autocorrelation in the simulated estimates. The final posterior mean estimates are provided on the basis of 10000 simulated samples. Diagnostic tests establish the validity of the Bayesian estimates. Normality of Kernel densities presented in Figure 1 implies that results converged to the desired distribution. Trace plots presented in Figure 2 show convergence of all simulated estimates. Quantile plots presented in Figure 3 also show that distributions of the parameters are stationary as medians move in the center of the 95% credible interval. Autocorrelation disappears quickly as shown in Figure 4, and hence, there is no problem with autocorrelation. A graphical presentation of the Brooks Gelman-Rubin test for all parameters is presented in Figure 5. Values of $R$ are close to one, and hence, each of the chains moves near to the central line. It proves the convergence of each of the chains in all cases.

The two estimated models may be compared on the basis of the precision of parameters’ estimates and the overall precision of the models. Results presented in Table 4 show that the standard errors of all the Bayesian estimates are smaller than those of the classical estimates and hence the Bayesian estimates are more precise as compared to classical estimates. Moreover, the precision of the Bayesian estimated model is larger than that of the model estimated under the classical framework. It implies that the application of the Bayesian econometric techniques improves the precision of the results. It also implies that the incorporation of prior information through the Bayesian framework improves the precision of estimates as well as the overall model. Hence,
Figure 3: Quantile points.

Figure 4: Autocorrelation plots.
the Bayesian inference may be considered a competitive alternative to classical inference in econometric modeling. Moreover, the results of econometric modeling may be improved by incorporating useful information. As much as prior information is useful, it may be used to improve the precision of the model’s estimates. Therefore, applied researchers in economics may improve the precision and scope of their research by employing the Bayesian econometric inference. The significance of posterior mean estimates is tested by considering zero null hypotheses. On the basis of 95% HPDI, all posterior estimates given in Table 3 are significant. Moreover, the signs of all estimates are positive. It implies that all variables including human capital significantly and positively affect economic growth in Pakistan’s economy.

A coefficient estimate of physical capital per worker indicates that a 1% increase in physical capital per worker raises output per worker by 0.8993%. On the basis of the Bayesian HPDI, the probability is 0.95 where the estimate of physical capital is likely to be found within the interval from 0.8788% to 0.9198%. It implies that the probability is 0.95 where the positive impact of a 1% change in physical capital upon economic growth may vary from 0.8788% to 0.9198%. The Bayesian estimate of education capital is positive and significant. It shows that the rise in education capital raises productivity by 0.1778% and the probability is 0.95 where the estimate will be likely to be found within the range from 0.1606% to 0.1952%. It is because healthy people are more efficient in their productivity which leads to economic growth. A comparison of the posterior mean estimates of education capital versus health capital shows that health capital is more effective for economic growth. The reason might be that education capital requires more time to be effective than health capital. The health economics literature has several explanations for the health productive efficiency hypothesis in this regard [43]. A simulated estimate of trade openness is also positive and significant which shows that trade openness improves economic growth in the economy. On the basis of the Bayesian estimates of the study, it is concluded that human capital significantly and positively affects economic growth in Pakistan’s economy. Hence, human capital may be considered the driver of the economic growth engine in Pakistan.

4. Conclusions and Policy Implications

The Bayesian inferential approach is considered an alternative approach to classical inference. It has three distinct features, i.e., incorporation of prior information other than sample information, interpretation of estimates with probability statements on the basis of HPDI, and the
established validity of the Bayes estimates even in a small sample size. Literature on applied economics reveals that the application of the Bayesian econometric inference to analyze real economic phenomena is rare. An important topic of growth literature, i.e., human capital and economic growth, is selected for the Bayesian analysis. Standard neoclassical production function augmented with human capital is considered to estimate the model by annual data (1965–2019) of Pakistan’s economy and two major objectives are achieved. Firstly, the efficiency of classical estimates versus the Bayes estimates is compared. Classical estimates are obtained using the efficient GMM method. The Bayesian two-stage method is employed to get posterior mean estimates of the model. Posterior distribution of the reduced form structural model and the instrumental variable equations is derived assuming that parameters follow the multivariate normal distribution and variance-covariance matrix follows the Wishart distribution. Hyperparameters of the structural model are elicited by using the PV method on the basis of experts’ information. MCMC simulations using Gibbs sampling are conducted to estimate posterior estimates of the structural model. Results of both estimated models (i.e., using classical and Bayesian inference) show that estimates of all the regressors are significant with a positive sign. However, standard errors of Bayesian estimates are much smaller compared to those of classical estimates. It implies that Bayesian estimates are more precise than classical estimates. Moreover, the precision of the model estimated by the Bayesian inference is smaller than that of the estimated model using classical inference. It implies that the Bayesian econometric framework gives more precise results and, therefore, may be preferred over the classical econometric framework in order to analyze real economic phenomena. The second objective of the study is to get fresh estimates of human capital along with HPDI estimates for the specified model. Findings show that a 1% increase in physical capital causes a 0.89% increase in output per worker with a 0.95 probability that the impact may vary from 0.87% to 0.91%. A 1% increase in education capital and health capital may raise output per worker by 0.0091% and 0.1778%, respectively, with a 0.95 probability that the estimates are likely to lie within the intervals 0.0085%–0.0097% and 0.1606%–0.1952%, respectively. It may be concluded that the Bayesian econometric framework may be applied to get more precise results as compared to the classical econometric framework provided that proper informative priors are available to incorporate. The precision of estimates may be improved by incorporating all types of prior information other than sample data. Human capital may be considered a vital factor for economic growth in Pakistan’s economy on the basis of Bayesian as well as classical estimates of the model. The empirical results of this study are in line with a number of previous studies and growth theories with respect to sign and significance. However, this study contributes by providing random estimates along with an interval of each estimate within which the parameter is likely to lie with some probability statement.

Data Availability

The data used to support the findings of the study are freely available at http://www.pbs.gov.pk.

Conflicts of Interest

The authors declare that there are no conflicts of interest.

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