Analysis of trilinear Higgs self coupling in Two Higgs Doublet model at lepton collider

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Abstract

Reconstruction of Higgs potential relies on trilinear Higgs self-coupling which further depends upon the Higgs boson masses. In this study, computational analysis of neutral Higgs bosons $h$, $H$ and $A$ is carried out within the parameter space of Two Higgs Doublet Model (2HDM) type-I in exact alignment limit of $\sin(\beta - \alpha) = 1$ at compact linear collider. Assuming Compact Linear Collider (CLIC) scenario where the electron-positron beams will be collided at $\sqrt{s} = 1.5$ TeV. Two different values of predicted integrated luminosities are used in our calculations, 1000 $fb^{-1}$ and 2500 $fb^{-1}$. Three signal processes $e^+e^- \rightarrow Zhh \rightarrow jjb\bar{b}$, $e^+e^- \rightarrow ZHH \rightarrow jjb\bar{b}$ and $e^+e^- \rightarrow Zhh \rightarrow jjb\bar{b}$ are selected for analysis. The polarized colliding beams are used to enhance the signal production cross-section. Low $\tan\beta$ and equal Higgs masses regime is chosen in favor of Hadronic decay of $Z$ boson and $H \rightarrow b\bar{b}$. Different signal scenarios are selected and event generation is performed for each process separately. The $W$ and $Z$ boson, as well as top quark backgrounds are analyzed. Signal significance is computed at each Higgs mass hypothesis for three signal processes. After a comprehensive study, we did not find sizeable amount of signal events on top of the background but still, signal significance is found to be larger than 1, without taking into account the detector effects and systematics. The maximum calculated value of signal significance is observed to lie in the range of $150 \leq m_H \leq 250$ GeV for $Zhh$ while for $ZHH$ the range is $175 \leq m_H \leq 200$ GeV at 1000 $fb^{-1}$ and $175 \leq m_H \leq 225$ GeV region at 2500 $fb^{-1}$. Similarly, for $ZAA$ process, the significance is highest at $m_A = 175$ GeV for both 1000 and 2500 $fb^{-1}$.

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I. INTRODUCTION

Giving mass to fermions by Electroweak Symmetry Breaking (EWSB) mechanism is the key feature of SM Higgs sector [1–3]. Quadratic scalar potential along with scalar Higgs field handles the EWSB mechanism. In the Higgs mechanism, the neutral constituent of an iso-doublet scalar field attains some vacuum expectation value (VEV) which results in masses of fermions and vector bosons while preserving the gauge symmetry of $SU(2) \times SU(1) \times U(1)$ transformation. In SM, coupling of fermions and vector bosons with Higgs is a function of their corresponding masses that’s why their mass is the only free parameter in the Higgs sector. Higgs-pair production is important, since it can be used for the detection of Higgs self-coupling. Discovering deviations from its SM prediction provides indirect possibilities for finding new physics and the existence of heavy new particles. Measurement precision in the range of 20% could allow observation of deviations from the SM and can reveal extended Higgs sectors [4]. The Higgs sector of SM is extended by incorporating another Higgs doublet of symmetry $SU(2)$. This modification steers to a 2HDM [5–12] that shows very interesting phenomenological attributes. Such as, in the parameter region of 2HDM, in which the theory of Electroweak Baryogenesis is possible, a minimal expected deviation from the SM is 20% [13]. Two presumed $SU(2)$ Higgs doublets of 2HDM comprise of eight degrees of freedom. Vector bosons absorb three of them and the remaining five steer towards the supposition of five Higgs bosons. One is considered as the uncovered SM Higgs boson and all others are treated as free parameters that are not yet discovered. In 2HDM, there are five Higgs bosons: two charged ones ($H^\pm$), two CP-even neutral Higgs $h$, $H$ and one CP-odd neutral $A$. Due to the addition of extra scalar doublet, 2HDM has richer phenomenology than SM.

In order to establish the EWSB mechanism, the scalar potential of the Higgs field needs to be constructed such as to obey orthogonality. However, that requires making measurements of the triple and quadratic self-couplings, $g_{\phi\phi\phi}$ and $g_{\phi\phi\phi\phi\phi}$ respectively. The double Higgs-strahlung ($e^-e^+ \rightarrow Z\phi\phi$) along with WW double-Higgs fusion makes it possible to measure the triple Higgs self-coupling with significant precision in the SM [14–17]. On the other hand, measuring Higgs self-coupling allows us to reconstruct the Higgs potential, which is the most conclusive test of the EWSB mechanism. If the scalar sector is extended like the 2HDM, determining the self-couplings, as well as the Higgs potential, could be a complicated task. In 2HDM, there are in a total of 8 trilinear Higgs self-couplings. A similar attempt was made earlier, to find out at what extent the trilinear Higgs couplings could be probed by studying various Higgs boson pairs associated with the $Z$ boson in reference [18]. However, the processes and the region of interest differ from our
study and most importantly, the motivation for the free parameters of the model does not hold the basic theoretical constraints (perturbativity and unitarity) of the model which is claimed otherwise. Some of the couplings are studied through the double and triple Higgs boson production in reference [19][20]. Besides, triple and quartic Higgs couplings have been studied at the linear colliders in the context of the MSSM in reference [21][25]. In this work, we analyze various scattering processes in $e^-e^+$ collider to conclude whether all these Higgs self-couplings can be determined. In this aim, the correlation between these couplings and scattering processes is examined. Cross-sectional values of double Higgs production in SM are insignificant, but Higgs self-coupling measurement required by new physics is quite challenging and enhanced by sizeable factors. Because of multi-jet final states, the measurement places many challenges on detector technologies and event reconstruction techniques. Hence, it is challenging to investigate the possibilities of measuring the Higgs self-coupling at particle colliders. Many experiments were carried out at LHC that gave many beneficial results. But a more precise machine like a lepton collider is required for the thorough study of Higgs particle and its properties. Since, the initial state is well defined in a lepton collider, we can easily determine the four momenta from the products which helps in reconstructing the event.

There are a variety of lepton colliders around the world: the Circular Electron-Positron Collider (CEPC) based in China, having beam energy of 120 GeV [26][28], the proposed Future Circular Collider (FCC-ee) [29] by CERN [30], the International Linear Collider (ILC) with beam energy of 500 GeV extendable to 1TeV, based in Japan [31] and Compact Linear Collider (CLIC) having beam energy of 380 GeV and proposed beam energies of 1.5 and 3 TeV based in CERN [32]. All of these lepton colliders will be Higgs factories and thus the best choice to study Higgs and its properties and can also interpret LHC results.

This study is related to $h$, $H$, and $A$ Higgs bosons with focus on the visibility of these particles in 2HDM parameter space in a SM-like regime. The 2HDM form contains flavor-changing neutral currents (FCNCs) which is in contradiction with the experimental results. These FCNCs can be avoided by imposing $Z_2$-symmetry. Four types of 2HDM arise whereas different couplings of fermions with Higgs result in flavor conservation. This $Z_2$-symmetry is broken by a dimension two operator $\phi_1^\dagger \phi_2$ or $\phi_2^\dagger \phi_1$ with the addition of a soft symmetry breaking term $m_{12}^2$. The exact alignment limit and equal mass scenario make signal processes independent of $\tan \beta$. Low $\tan \beta$ value is chosen in favor of hadronic decay of $Z$ boson and $H \to b\bar{b}$. For these scattering processes, contributing backgrounds are $WW$, $ZZ$, $t\bar{t}$ and $ZZZ$. Different benchmark points are chosen in 2HDM parameter space and event generation is done for each scenario. We expect this proposed analysis, a tool to serve those who are working on the observability of neutral Higgs boson via
Triple Higgs coupling. In section 2, a brief description of two Higgs doublet model (2HDM) is given. In section 3, trilinear Higgs coupling in SM and 2HDM is discussed. In section 4, details of computational tools are given. In sections 5 and 6, Higgs decays and procedure for selection of signal scenarios are discussed, respectively. In section 7, signal and background processes are discussed. Results are shown in sections 8 to 11 and section 12 gives the conclusion.

II. TWO HIGGS DOUBLET MODEL

In 2HDM, another \(SU(2)_L\) doublet with same hypercharge as the first one, i.e. \(Y = 1/2\) is added. Two \(SU(2)\) Higgs doublets in general form are given as [33],

\[
\langle \phi_i \rangle = \begin{pmatrix} \phi_i^\dagger \\ \frac{1}{\sqrt{2}} (\nu_i + \rho_i + i\eta_i) \end{pmatrix}
\]

where \(i = 1, 2\). Let us assume that direction of vacuum expectation value is along CP-even neutral Higgs field [34], then \(\langle \rho_1 \rangle = \nu_1\) and \(\langle \rho_2 \rangle = \nu_2\), which defines \(\tan \beta = \nu_2/\nu_1\) and \(\nu = \sqrt{\nu_1^2 + \nu_2^2} = (\sqrt{2}G_F)^{-1/2} = 2M_W/g = 246\) GeV. Both vacuum expectation values are real due to \(U(1)_\gamma\) invariance and taken as positive without losing generality [45]. The most general form of the gauge invariant and renormalized Higgs potential in terms of generic basis is [36],

\[
V(\phi_1, \phi_2) = m_{11}^2 (\phi_1^\dagger \phi_1) + m_{22}^2 (\phi_2^\dagger \phi_2) - \{m_{12}^2 (\phi_1^\dagger \phi_2) + h.c\} + \frac{1}{2} \lambda_1 (\phi_1^\dagger \phi_1)^2 + \frac{1}{2} \lambda_2 (\phi_2^\dagger \phi_2)^2 + \lambda_3 (\phi_1^\dagger \phi_1)^2 (\phi_2^\dagger \phi_2)^2 + \lambda_4 (\phi_1^\dagger \phi_2)^2 (\phi_2^\dagger \phi_1)^2 + \left\{ \frac{1}{2} \lambda_5 (\phi_1^\dagger \phi_1)^2 + \lambda_6 (\phi_1^\dagger \phi_1) + \lambda_7 (\phi_2^\dagger \phi_2) \right\} (\phi_1^\dagger \phi_2) + h.c
\]

where, \(\phi_1, \phi_2\) are two Higgs doublets. Because of the addition of another doublet, 2HDM comprises of five physical Higgs that include CP-even neutral Higgs \(h\) and \(H\), CP-odd neutral Higgs \(A\) and the charged Higgs \(H^\pm\). Parameters used in the above expressions are \(m_{11}, m_{22}, \lambda_1, \lambda_2, \lambda_3, \lambda_4, m_{12}^2, \lambda_5, \lambda_6, \lambda_7\) and h.c stands for hermition conjugate. First six parameters are real and last four parameters can be both real or complex. Complex parameters violate CP which is not desirable, that’s why only real case is considered here. After inserting doublets, quadratic, cubic and quartic terms are obtained. Quadratic terms give us information about masses, while cubic and quartic terms give us information about couplings. This scalar potential includes Flavor Changing Neutral Currents (FCNCs). These can be avoided by imposing \(Z_2\) symmetry i.e. \(\phi_1 \to \phi_1\), and \(\phi_2 \to -\phi_2\) or in another term as \(\phi_1 \to -\phi_1\) and \(\phi_2 \to \phi_2\), in Lagrangian. If potential \(Z_2\) is
symmetric then $m_{12}^2$, $\lambda_6$, $\lambda_7$ are forbidden \cite{37}. However, parameter $m_{12}^2 \neq 0$ is considered because $Z_2$ symmetry is softly broken by dimension two operators $\phi_1^\dagger \phi_2$ or $\phi_2^\dagger \phi_1 \cite{34}$. After imposing $Z_2$ symmetry, Higgs bosons couplings to fermions are restricted. Various Higgs boson couplings to fermions are presented in Table \cite{Various scenarios of Higgs boson couplings to fermions obeying Z_2 symmetry}.

| PType | Description | $Q_u$ | $Q_d$ | $l^\pm$ | Detail |
|-------|-------------|-------|-------|-------|--------|
| I     | Fermi-phobic | $\phi_2$ | $\phi_2$ | $\phi_2$ | Charged fermions couple to second doublet only |
| II    | MSSM-like   | $\phi_2$ | $\phi_1$ | $\phi_1$ | Up and down type couple to different doublets |
| X     | Lepton- specific | $\phi_2$ | $\phi_2$ | $\phi_1$ | Charged lepton couple to first doublets |
| Y     | Flipped     | $\phi_2$ | $\phi_1$ | $\phi_2$ | Charged lepton couple to second doublet as flipped form type II |

TABLE I: Various scenarios of Higgs boson couplings to fermions obeying $Z_2$ symmetry

In SM, coupling of fermions ($f$) with Higgs boson can be given as $m_f/\nu$. So Yukawa Lagrangian in physical basis is given as \cite{38},

$$L_{\text{Yukawa}}^{2\text{HDM}} = - \sum_{f=u,d,l} \frac{m_f}{\nu} \left( \xi_f^f \bar{f} f h + \xi_f^H \bar{f} H + i \xi_f^A \bar{f} \gamma_5 A + \right)$$

$$- \left\{ \sqrt{2} \frac{V_{ud}}{\nu} \bar{u} d\xi_u^u P_L + m_d \xi_A^d P_R d H^+ + \sqrt{2} m_l \xi_A^l \frac{V_{UL}}{\nu} l H^+ + h.c. \right\} \quad (3)$$

where $V_{qq'}$ represents CKM matrix and $P_{L,R} = \frac{1}{2} (1 \pm \gamma_5)$ are projection operators for left and right-handed fermions. $\xi_f^f$, $\xi_A^f$, $\xi_f^H$ are coupling parameters whose values are given in Table \cite{Values of Yukawa coupling for Type-I of 2HDM}.

Finally, free parameters used in this model are Higgs masses $(m_h$, $m_H$, $m_A$, $m_{\pm}^2)$, ratio of vacuum expectation values ($\tan \beta$), soft discrete symmetry breaking ($m_{12}^2$) and basis independent quantities $\sin \beta \alpha = \sin (\beta - \alpha)$, $\cos \beta \alpha = \cos (\beta - \alpha)$ which are related to the CP-even neutral scalar mass eigenstate and mixing angle $(\beta - \alpha)$ that shows mixing between CP-even neutral Higgs states \cite{39}. These free parameters

| Type-I | $\xi_u^u$ | $\xi_u^H$ | $\xi_u^A$ | $\xi_u^l$ | $\xi_h^u$ | $\xi_h^H$ | $\xi_h^A$ | $\xi_h^l$ | $\xi_H^u$ | $\xi_H^H$ | $\xi_H^A$ | $\xi_H^l$ |
|--------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
|        | $\cos \alpha$ | $\cos \alpha$ | $\sin \alpha$ | $\sin \alpha$ | $\cos \beta$ | $\sin \alpha$ | $\sin \alpha$ | $\sin \alpha$ | $\sin \alpha$ | $\sin \beta$ | $\sin \beta$ | $\sin \beta$ |

TABLE II: Values of Yukawa coupling for Type-I of 2HDM.

need to be constrained somehow. Following constraints defined on a theoretical base in 2HDM are applied.
• **Stability:** Potential must be positive everywhere. This will provide lower bound to potential [34, 36, 40–46].

• **Unitarity:** Partial wave amplitude $a_0$ for $l=0$ must be less than unity at High energies [39, 47].

• **Perturbativity:** This condition puts an upper bound on quartic couplings, according to which $|\lambda_i| \leq 4\pi$ (i=1,2,...,7) [34] for quartic coupling.

These theoretical constraints are checked for parameter space using 2HDMC v1.8.0.

There is another set of constraints, inspired by the previous experiments. Experimental constraints mentioned on page 3 of ref. [46] are used for the measurement of trilinear Higgs coupling.

All computations are carried out in the exact alignment limit, i.e. $\sin(\beta - \alpha) = 1$, where $h$ behaves like a SM Higgs and becomes indistinguishable from the SM Higgs boson in terms of mass and couplings. Inspired by the reference [48] and the current experimental results at the LHC [49], masses of all the extra Higgs bosons are set to be $m_{\phi} = m_{H}^0 = m_{A}^0 = m_{H}^\pm$. This selection also minimizes the oblique parameters [50–55], so all the electroweak observables are close to the SM ones. In the exact alignment limit, the decay of the $H^0$ to vector boson pairs is suppressed. We performed all computations in Type-I of 2HDM. To avoid meson mixing $\Delta M_s$ and $B_0 \to \mu^+\mu^-$ in Type-I, the region defined by $2 < \tan\beta < 40$ is considered. For this scenario, theoretically allowed region is determined and then central region is considered for further analysis [37, 56–61]. One of the allowed regions is shown in Figure 1. Theoretical allowed region at Higgs mass 150 GeV.

**FIG. 1:** Theoretical allowed region at Higgs mass 150 GeV
III. PROBING TRILINEAR HIGGS COUPLING IN 2HDM

Trilinear Higgs coupling can be probed easily in SM. Since coupling of electron and positron with Higgs $g_{e^+e^-H}$ is very weak that’s why the diagrams with Higgs as mediating particles does not contribute. Also, there is no contribution of quartic Higgs coupling $g_{ZZHH}$. Feynman diagrams for $e^+e^-\rightarrow ZHH$ scattering process are given in Figure 2. All possible Feynman diagrams for the scattering process $ZHH$ in SM. Only diagram 1 has trilinear Higgs coupling and gives significant contribution.

![Feynman Diagrams](image)

**FIG. 2:** All possible Feynman diagrams for the scattering process $ZHH$ in SM. Only diagram 1 has trilinear Higgs coupling and gives significant contribution.

This case is quite challenging in 2HDM because there is not only one trilinear Higgs coupling but there are several others, which will be discussed. As exact alignment limit is used, that’s why all conditions of SM also hold in 2HDM. In 2HDM, following Equations from (4) till (9) represent Higgs self-couplings as a function of $\Lambda_i$, where we selected two independent parameters to be $s_{\beta\alpha} = 1$ and $c_{\beta\alpha} = 0$ respectively. Among all the Higgs self couplings $g_{hhH}$ vanishes, as well as, due to exact alignment limit and equal masses of all extra Higgs bosons, two other parameters $\Lambda_6$ and $\Lambda_7$ vanish. In addition $g_{hhH}$ and $g_{hAA}$ are equal and $g_{HHH}/g_{HAA} = 3$. Given that $\Lambda_345 = \Lambda_3 + \Lambda_4 + \Lambda_5$, we can write

\[
\begin{align*}
    g_{h^0H^0H^0} &= -3\nu((\Lambda_7c_{\beta\alpha}^2 + 3\Lambda_6s_{\beta\alpha}^2)c_{\beta\alpha} + (\Lambda_345c_{\beta\alpha}^2 + \Lambda_1\delta_{\beta\alpha}^2)s_{\beta\alpha}) \\
    &= -3\nu\Lambda_1 \\
    \quad c_{\beta\alpha}\rightarrow 0 \\
    \quad (4) \\

    g_{h^0H^0H^0} &= -\nu((\Lambda_345(1 - 3s_{\beta\alpha}^2) + 3\Lambda_1s_{\beta\alpha}^2)c_{\beta\alpha} + 3(\Lambda_6(2 - 3s_{\beta\alpha}^2) - \Lambda_7c_{\beta\alpha}^2)s_{\beta\alpha}) \\
    &= 0 \\
    \quad c_{\beta\alpha}\rightarrow 0 \\
    \quad (5) \\

    g_{h^0H^0H^0} &= -\nu((3\Lambda_1c_{\beta\alpha}^2 + \Lambda_345(3s_{\beta\alpha}^2 - 2))s_{\beta\alpha} + 3(\Lambda_6 + \Lambda_7\%s_{\beta\alpha}^2 - 3\Lambda_6s_{\beta\alpha}^2)c_{\beta\alpha}) \\
    &= -\nu\Lambda_3 \\
    \quad c_{\beta\alpha}\rightarrow 0 \\
    \quad (6) \\

    g_{h^0A^0A^0} &= -\nu(\Lambda_7c_{\beta\alpha} + (\Lambda_3 + \Lambda_4 - \Lambda_5)s_{\beta\alpha}) \\
    &= -\nu\Lambda_3 \\
    \quad c_{\beta\alpha}\rightarrow 0 \\
    \quad (7)
\end{align*}
\]
The measurement of Higgs self-coupling within 2HDM is difficult due to the presence of more than one Higgs. The couplings where $h^0$, $H^0$ and $A^0$ are intermediated, do not make a noticeable contribution because of their absolute value, which is less than $10^{-6}$, so they can be neglected. Significant contributions are found to be from $Z^0$ coupling only, that is why only those Feynman diagrams are taken into account in which $Z$ boson is intermediated.

The scattering processes with various combinations of trilinear Higgs self-couplings need to be considered, i.e., $Zhh$, $ZAh$, $HHH$, $hhh$, $Ahh$, $AAh$, $AAH$, $HHh$, and $Hhh$. The Feynman diagrams of possible processes are shown in Figure 3. Their cross-section is less than $10^{-11}$ pb; therefore it is not possible to detect them and they can be easily neglected. To calculate the Higgs self-coupling in two Higgs doublet model, we use the scattering processes shown in Table II. These scattering processes are the only ones which can give the cross-section greater than attobarn.

In Equation 3.5, $g_{h^0 h^0 H^0}$ approaches zero so this coupling vanishes. The cross-section of scattering process $e^- e^+ \rightarrow ZAA$ makes it possible to determine the coupling $g_{h^0 A^0 A^0}$. The coupling $g_{h^0 H^0 H^0}$ can be determined by measuring the cross-section of process $e^- e^+ \rightarrow ZHH$. The cross-section of $e^- e^+ \rightarrow Zhh$ extracts the coupling $g_{h^0 h^0 h^0}$ which could be the same as determined in SM. The coupling $g_{H^0 A^0 A^0}$ can be determined by two processes, $e^- e^+ \rightarrow AHh$ and $e^- e^+ \rightarrow AHH$, whereas the last mentioned process can also give $g_{H^0 H^0 H^0}$.

Feynman diagrams with $Z$ as mediating particle are considered because only $g_{e^+ e^- Z}$ coupling has a major contribution. Then $ZAA$, $ZHH$, and $Zhh$ processes are selected for further analysis because they have reasonable cross-section. The presence of trilinear Higgs coupling can be seen in Feynman diagrams for these processes given in Figure 3. Measuring the cross section of $e^+ e^- \rightarrow Zhh$ will allow us to determine the coupling $g_{Hhh}$ and $g_{hhh}$. The couplings $g_{hHH}$ and $g_{HHH}$ can be determined by analyzing $e^+ e^- \rightarrow ZHH$. Similarly, the presence of coupling $g_{hAA}$ and $g_{HAA}$ can be determined by studying $e^+ e^- \rightarrow ZAA$. In Table II, trilinear Higgs self coupling which contributes to the signal processes given in first column are marked by a plus sign.

\[
g_{H^0 H^0 H^0} = -3 \nu ((\Lambda_1 c_{\beta \alpha}^2 + \Lambda_3 s_{\beta \alpha}^2) c_{\beta \alpha} - \Lambda_7 s_{\beta \alpha}^2 - 3 \Lambda_6 c_{\beta \alpha}^2) s_{\beta \alpha} \approx 3 \nu \Lambda_7 \]  
\[
g_{H^0 A^0 A^0} = -\nu ((\Lambda_3 + \Lambda_5) c_{\beta \alpha} - \Lambda_7 s_{\beta \alpha}) \approx \nu \Lambda_7 \]
FIG. 3: Feynman diagrams involved in scattering process $Zh\bar{h}$, $ZHH$ and $ZAA$ in 2HDM.

\[
\begin{align*}
(a) & \quad e^- & Z & H_{i,j} & H_{i,j} \\
(b) & \quad e^- & Z & A & H_{i,j} \\
(c) & \quad e^- & Z & H_{i,j} & A \\
(d) & \quad e^- & Z & H_{i,j} & A \\
(e) & \quad e^- & Z & H_{i,j} & A \\
(f) & \quad e^- & Z & A & H_{i,j} \\
(g) & \quad e^- & Z & A & H_{i,j}
\end{align*}
\]

TABLE III: Signal processes and their corresponding couplings at Higgs mass of 150 GeV and $\tan \beta = 10$

|       | $g_{hHH}$ | $g_{Hhh}$ | $g_{hAA}$ | $g_{HAA}$ | $g_{hhh}$ | $g_{HHH}$ |
|-------|-----------|-----------|-----------|-----------|-----------|-----------|
| $Zhh$ |           | +         |           |           |           |           |
| $ZHH$ | +         |           |           | +         |           |           |
| $ZAA$ |           |           | +         | +         |           |           |

IV. ANALYSIS TOOLS

For the calculation of branching ratios and decay widths for selected benchmarks points within 2HDM Type-I, 2HDMC 1.8.0 is used. The theoretically allowed region is also computed for each signal scenario. Generation of events for signal as well as background is carried out through CalcHEP 3.8.5 [62] and then these events are saved in LHE file for further compilation using PYTHIA 8244[63]. Relative efficiencies at each benchmark are computed. Then HepMC 2.06.09 [64] interface is used for event record. Jet definition and reconstruction is carried out with FastJet
3.3.3 [65,66] interface. The output is analyzed using ROOT 6.14.06 [67], which helps to draw all graphs and plots.

V. NEUTRAL HIGGS DECAYS

Decay widths and branching ratios of different Higgs decays are calculated at tan $\beta = 10$ for different values of Higgs masses.

![Branching ratios of H decays in Type-I of 2HDM at tan $\beta = 10$](image)

**FIG. 4:** Branching ratios of H decays in Type-I of 2HDM at tan $\beta = 10$

From Figure 4 it can be seen that prominent decay channels of $H$ are $b\bar{b}$ and $t\bar{t}$. In a mass region above 270 GeV, $t\bar{t}$ is a prominent decay and in low mass region $b\bar{b}$ is a prominent decay. Branching ratio of $\tau^+\tau^-$ and $c\bar{c}$ is quite small that is below 0.1, and for $g\gamma$ it increases with the increase in Higgs mass. Higgs decay branching ratios for $\mu^+\mu^-$, $s\bar{s}$, $e^+e^-$, $\gamma\gamma$ and $Z\gamma$ is very small and approaches to zero. Higgs decay into $WW$ and $ZZ$ vanishes when $\cos(\alpha - \beta) = 0$ is assumed.

Trilinear Higgs self coupling vertices depend upon $\sin\alpha\beta$ and $\cos\alpha\beta$. And in exact alignment limit, these become independent of tan $\beta$. So any value of tan $\beta$ can be chosen in favor of the branching ratio. To check the distribution of BR, plots are drawn between branching ratio and tan $\beta$ at various Higgs masses.

Figure 5 shows that there is no significant change in BR with the change in the value of tan $\beta$. It is
important to check change in branching ratio with the change in the value of $m^2_{12}$. Figure 5 Distribution of branching ratio at various Higgs masses as a function of $m^2_{12}$ at $\tan \beta = 9$ (right) shows that value of BR changes slightly as we move towards the upper limit of the allowed region. But this change is not prominent. So, central value of theoretically allowed region and $\tan \beta = 10$ is selected for further computations.

VI. BENCHMARK POINTS, SIGNAL AND BACKGROUND PROCESSES

Various processes are selected as signal processes for the analysis. In all these processes double Higgs is produced along with $Z$ boson by Higgs strahlung process generating a trilinear Higgs vertex. All these processes are generated by the annihilation of electron and positron along with $Z$ as mediating particle at Compact Linear Collider (CLIC) at centre of mass energy $\sqrt{s} = 1.5$ TeV. Signal processes have $ZHH$, $ZhH$, and $ZAA$ as decay products where $Z$ decays into di-jets while $H$, $h$ and $A$ decay into $b$ quark di-jets. So, in each process, there are six jet final states comprising of two light jets and four $b$ quark jets.

$$e^+ e^- \rightarrow Z^* \rightarrow ZHH, ZhH, ZAA \rightarrow jjb\bar{b}b\bar{b}$$

After analyzing the results given in section V, different benchmark points are selected within parameter space of 2HDM which are given in Table IV Benchmark points for different parameters.
The production cross-section of signal processes is very small, in the range of $\approx 0.1 - 0.2 \text{ fb}$ despite applying all constraints. This difficulty is further increased by the SM background. There are a number of background processes that can suppress the signal, some of which are $t\bar{t}$, $ZZ$, $Zb\bar{b}$ and $ZZZ$. The Feynman diagrams for which are given in Figure 6. Feynman diagrams for $t\bar{t}$, $ZZ$, $Zb\bar{b}$ and $ZZZ$ backgrounds processes represented by $p_1$, $p_2$, $p_3$ and $p_4$ respectively. Where $j_s$ represent jets of $u$, $d$, $s$, $c$ and $b$ quarks. It can be seen that $Z$ bosons decay into jets resulting in 4 jets and 6 jets final state from $ZZ$, $Zb\bar{b}$ and $ZZZ$ processes, respectively. Top quark decays into $W$ boson and $b$ quark, where $W$ boson further decays into jets resulting in 6 jets final state from $t\bar{t}$ process. For suppression of background, reconstruction of Higgs boson mass in every event will be beneficial. Because if $b$ quark pair was not produced from neutral Higgs boson then it will not lie within the Higgs mass range and these events will be excluded.

Background processes can also be suppressed by using kinematic selection cuts. Efficiencies of background processes are given in Table V, which shows that efficiency is maximum for $t\bar{t}$.

| BP1 | 130 | 130 | 130 | 1433-1688 |
| BP2 | 140 | 140 | 140 | 1700-1956 |
| BP3 | 150 | 150 | 150 | 1987-2243 |
| BP4 | 125.09 | 175 | 175 | 2792-3047 |
| BP5 | 200 | 200 | 200 | 3720-3975 |
| BP6 | 225 | 225 | 225 | 4772-5027 |
| BP7 | 250 | 250 | 250 | 5948-6203 |

TABLE IV: Benchmark points for different parameters within the allowed region.
FIG. 6: Feynman diagrams for $t\bar{t}$, $ZZ$, $Zb\bar{b}$ and $ZZZ$ backgrounds processes represented by $p1$, $p2$, $p3$ and $p4$ respectively.

|          | $t\bar{t}$ | $ZZ$  | $Zb\bar{b}$ | $ZZZ$ |
|----------|------------|-------|-------------|-------|
| $\sigma$ [fb] | 80.44     | 84.29 | 32.82       | 0.739 |
| $N_{jets} \geq 5$ | 0.823    | 0.079 | 0.090       | 0.814 |
| $N_{bjets} \geq 3$ | 0.327    | 0.009 | 0.230       | 0.016 |
| $N_{bb} \geq 1$ | 0.184    | 0.148 | 0.202       | 0.116 |
| Total Efficiency | **0.049** | **0.000** | **0.004** | **0.002** |
| Six jets final state | 0.000   | 0.000 | 0.004       | 0.002 |

TABLE V: Efficiencies of SM background processes at different kinematic selection cuts.

variables studied to define the selection criteria include transverse momentum ($p_T$), pseudo-rapidity ($\eta$), azimuthal angle ($\phi$), jet multiplicity ($N_{jet}$), b-jet multiplicity ($N_{b-jet}$), $\Delta R$ (Jet cone size) and invariant mass ($m_{bb}$).

The first step of analysis requires, writing of a simulation code in C++ and linking the libraries of simulation packages to it. After the initialization of the LHE file, desired decays are allowed. A particle loop is applied to identify the signal process. All this information is then stored in an event file. This event record is provided as an input to FastJet. In this mechanism, some of the jets lie beyond the cone size due to many factors such as the detector’s impairment, magnetic field, and material influence. All the jets are sorted out by a built-in class of PYTHIA in order of highest to
FIG. 7: Pseudorapidity distributions of jets for signal and background processes.

FIG. 8: Comparison of pseudorapidity distributions of various signal process.

lowest transverse momentum $P^\text{jet}_T$. First of all, a loop on sorted jets is applied and then resulting jets are passed through kinematic selection cuts.

$$P^\text{jet}_T \leq 10 \text{ GeV} \quad \text{and} \quad |\eta^\text{jet}| \leq 3$$

(10)

Where, $\eta \equiv -\ln[tan(\theta/2)]$ is the angle between the particle and beam axis and $\theta$ is the angle between three components of momentum and positive beam axis. Figures 7 and 8 show the distribution of pseudorapidity for signal and background processes.

Particles with $\eta = 0$ are perpendicular to the beam axis and particles having higher $\eta$ values are lost that is why the condition given in Equation 7 is applied to find events with true particles. This condition will also exclude background events with high pseudorapidity values. Further cuts are applied on the resulting good jets. As six jets final state is the main feature of
FIG. 9: Number of jets for signal and background processes.

FIG. 10: Transverse momentum of jets for signal and background processes.

FIG. 11: Comparison of transverse momentum of jets produced in various signal processes.

the signal process, another requirement is imposed to fulfill this condition,

$$N_{jets} \geq 5$$  \hspace{1cm} (11)$$

where $N_{jets}$ is the number of jets. The number of jets produced in each event are plotted as jet multiplicity which is shown in Figure 9 Number of jets for signal and background processes figure. In Figures 10 Transverse momentum of jets for signal and background processes figure and 11 Comparison of transverse momentum of jets produced in various signal processes.
parison of transverse momentum of jets produced in various signal processes

Figure 11: Distribution of transverse momentum of jets for signal and background processes, and their comparison in case of different signal processes is given. Now a particle loop is applied on the list of sorted jets to identify the jets produced from Z boson decay. Then angular separation ∆R of jets with respect to quarks is calculated. Jets with ∆R < 0.4, $P_{\text{jet}}^T \geq 10$ GeV and $|\eta_{\text{jet}}| \leq 3.0$ are named as light jets. The light jets are compared amongst themselves and two of them with maximum transverse momentum are selected.

The next step is the identification of b-jets from the list of sorted jets. The identification of b-jets is carried out through b-tagging where only bottom and charm quarks are allowed to pass the selection criteria. Then ∆R between jets and bottom or charm quark is computed. In Figure 12, ∆R of b-jets for signal and background processes is given for signal and background processes. Jets with ∆R < 0.4, $P_{\text{jet}}^T \geq 20$ GeV and $|\eta_{\text{bjet}}| \leq 3.0$ are tagged as b-jets. The resulting number of b-jets are plotted as b-jet multiplicity, shown in Figure ???. The possibility of b-jets arising from b-quark is taken to be 60% and from c-quark is taken as 10%. These values are considered as b-tagging efficiencies.

As each scattering process in the analysis contains two Higgs and each one of them decays into a b-quark pair, that is why four b-quarks must be present in the final state. This condition is attained by applying a b-jet selection cut,

$$N_{b-\text{jets}} \geq 3$$ (12)

where $N_{b-\text{jets}}$ represents the numbers of b-jets. The b-jets resulting from the above condition are then analyzed and true b-jet pairs are found, which are produced from Higgs boson decays.

VIII. RECONSTRUCTION OF INVARIANT MASSES

In particle physics, invariant mass of a particle is defined as its mass in rest frame and given as,

$$(m_0 c)^2 = \left( \frac{E}{c} \right)^2 - |p|^2$$ (13)

Using natural units, set $c = 1$,

$$(m_0)^2 = (E)^2 - |p|^2$$ (14)

After the selection of desired events from randomly generated events, the next step is the reconstruction of invariant masses of parent particle from its decay products. As a first step, the
invariant mass of neutral Higgs is reconstructed. Events with at least four b-jets are selected and invariant mass is reconstructed for all possible b-jet pairs using the following relation,

\[ m_{b_1b_2} = \sqrt{(E_{b_1} + E_{b_2})^2 - (p_{x_{b_1}} + p_{x_{b_2}})^2 - (p_{x_{b_1}} + p_{x_{b_2}})^2 - (p_{x_{b_1}} + p_{x_{b_2}})^2} \]  

Then difference of invariant mass \(|m_{b_1b_2} - m_{b_3b_4}|\) is calculated for each and every combination. The b-jet pairs with least mass difference are selected as true b-jet pairs provided they fulfill the condition,

\[ N_{b\bar{b}} \geq 1 \]  

Where, \(N_{b\bar{b}}\) is the number of true b-jet pairs. Next invariant mass of Z boson is reconstructed using the remaining jets that do not take part in Higgs boson mass reconstruction.

The invariant mass distributions for neutral Higgs bosons, shown in Figure 14, are obtained from selected \(b\bar{b}\) combinations from events remaining after passing through many selection and kinematic cuts, that were generated at center of mass energy of 1.5
TABLE VI: The generated ($m_{Gen.}$), reconstructed ($m_{rec.}$) and corrected reconstructed ($m_{corr.rec.}$) Higgs masses along with relative uncertainties at $\sqrt{s} = 1.5$ TeV.

| Benchmarks | $m_{Gen.}$ [GeV] | $m_{rec.}$ [GeV] | $m_{corr.rec.}$ [GeV] | Benchmarks | $m_{Gen.}$ [GeV] | $m_{rec.}$ [GeV] | $m_{corr.rec.}$ [GeV] | Benchmarks | $m_{Gen.}$ [GeV] | $m_{rec.}$ [GeV] | $m_{corr.rec.}$ [GeV] |
|------------|-----------------|-----------------|------------------------|------------|-----------------|-----------------|------------------------|------------|-----------------|-----------------|------------------------|
| BP1        | 200 ± 1.12      | 198.9 ± 0.07    | 198.9 ± 0.04            | BP3        | 143.4 ± 0.18    | 143.3 ± 0.15    | 143.2 ± 0.15            | BP5        | 142.1 ± 0.11    | 142.1 ± 0.11    | 142.1 ± 0.11            |
| BP2        | 200 ± 1.12      | 198.9 ± 0.07    | 198.9 ± 0.04            | BP4        | 143.4 ± 0.18    | 143.3 ± 0.15    | 143.2 ± 0.15            | BP7        | 142.1 ± 0.11    | 142.1 ± 0.11    | 142.1 ± 0.11            |
| BP3        | 143.4 ± 0.18    | 143.3 ± 0.15    | 143.2 ± 0.15            | BP5        | 142.1 ± 0.11    | 142.1 ± 0.11    | 142.1 ± 0.11            | BP7        | 142.1 ± 0.11    | 142.1 ± 0.11    | 142.1 ± 0.11            |
| BP4        | 143.4 ± 0.18    | 143.3 ± 0.15    | 143.2 ± 0.15            | BP6        | 142.1 ± 0.11    | 142.1 ± 0.11    | 142.1 ± 0.11            | BP7        | 142.1 ± 0.11    | 142.1 ± 0.11    | 142.1 ± 0.11            |
| BP5        | 142.1 ± 0.11    | 142.1 ± 0.11    | 142.1 ± 0.11            | BP7        | 142.1 ± 0.11    | 142.1 ± 0.11    | 142.1 ± 0.11            | BP7        | 142.1 ± 0.11    | 142.1 ± 0.11    | 142.1 ± 0.11            |

FIG. 14: Reconstructed masses of neutral Higgs bosons from b-quark pairs produced from signal and background processes.

The contribution of signal, as well as background, is also shown. It can be seen that there is some contribution of SM backgrounds $t\bar{t}$, $ZZ$, and $ZZZ$. Contribution of $t\bar{t}$ background is prominent but under control. All other backgrounds appear below the signal.

All distributions are normalized on the basis of $L \times \sigma \times \epsilon$, where $L$ is the integrated luminosity, $\sigma$ represents the total cross-section and $\epsilon$ represents the total efficiency. Calculations are performed for two values of integrated luminosity i.e. 1000 $fb^{-1}$ and 2500 $fb^{-1}$. The $\sigma \times BR$ is obtained by multiplying cross-section values in Table VII with the corresponding branching ratios of decay products, $Z \rightarrow jj$ and $H \rightarrow b\bar{b}$. The selection efficiencies of Higgs mass distributions are calculated by dividing the number of events used for reconstruction of Higgs mass by the total number of events generated. Fit functions are applied on the obtained Higgs mass distributions. A Gaussian function is used to fit these distributions, where the Mean parameter represents the fitted value of Higgs mass. It gives the value at the center of the peak of the distribution. Values obtained from Mean are named as reconstructed mass ($m_{Rec.}$). Generated mass ($m_{Gen.}$) of Higgs mass distributions.
are also given for contrast. It can be seen from Table VII the generated ($m_{Gen.}$), reconstructed ($m_{rec.}$) and corrected reconstructed ($m_{Corr.rec.}$) Higgs masses along with relative uncertainties at $\sqrt{s} = 1.5$ TeV that there is a difference between generated and reconstructed masses. These variations may be caused due to different sources such as unreliability arising from jet clustering algorithm or wrong jet identification, jet tagging efficiency, selection of the fit function, fitting method, and inaccuracy in the measured energy and momentum of the particles. These errors can be resolved by improving the jet clustering algorithm, b-tagging efficiency, and fitting method. However, these improvements are out of the purviews of this study. A simple offset correction is implemented in this study to minimize errors. This off-set correction is tried as follows. It can be seen from Table VII the generated ($m_{Gen.}$), reconstructed ($m_{rec.}$) and corrected reconstructed ($m_{Corr.rec.}$) Higgs masses along with relative uncertainties at $\sqrt{s} = 1.5$ TeV that, on average, values of reconstructed masses of $h$, $H$, and $A$ are approximately 6.12, 8.04, and 8.03 lower than generated masses respectively. To remove this error, same values are added to the reconstructed masses and values thus obtained are named as corrected reconstructed mass($m_{Corr.rec.}$). It can be seen from the Table VII the generated ($m_{Gen.}$), reconstructed ($m_{rec.}$) and corrected reconstructed ($m_{Corr.rec.}$) Higgs masses along with relative uncertainties at $\sqrt{s} = 1.5$ TeV that corrected reconstructed masses are in accordance with generated masses. Hence it can be deduced that signal candidate masses are observable at these benchmarks points.

IX. EVENT SELECTION EFFICIENCIES

In this study, 85000 events are generated and compiled for signal processes to get a better result from simulation for selected benchmarks points. The signal efficiency corresponding to each selection cut is then computed and at the end, total efficiency for all cuts is calculated. The efficiency obtained at the end of the whole simulation process corresponds to six jet final state which comprises of two light jets coming from Z boson decays and four b-jets coming from decays of Higgs boson. All these efficiencies mentioned in Table VII efficiencies of signal processes $Zhh$, $ZHH$ and $ZAA$ at different kinematic and selection cut table are calculated at the center of mass energy of 1.5 TeV. It can be seen from Table VII efficiencies of signal processes $Zhh$, $ZHH$ and $ZAA$ at different kinematic and selection cuttable that signal processes have significant efficiencies at all benchmark points for selected BPs. Mostly six jets are present in generated events of signal processes. This is happening because of $H \rightarrow b\bar{b}$ and $Z \rightarrow jj$
| Process → Zhh | BP1 | BP2 | BP3 | BP4 | BP5 | BP6 | BP7 |
|--------------|-----|-----|-----|-----|-----|-----|-----|
| $\sigma$ [fb] | 0.207 | 0.207 | 0.207 | 0.207 | 0.207 | 0.207 | 0.207 |
| $N_{jets} \geq 5$ | 0.710 | 0.710 | 0.832 | 0.832 | 0.833 | 0.835 | 0.831 |
| $N_{bjets} \geq 3$ | 0.970 | 0.967 | 0.957 | 0.957 | 0.957 | 0.956 | 0.957 |
| $N_{bb} \geq 1$ | 0.683 | 0.688 | 0.676 | 0.674 | 0.673 | 0.672 | 0.672 |
| Total Efficiency | 0.470 | 0.472 | 0.538 | 0.537 | 0.536 | 0.535 | 0.535 |
| Six jets final state | 0.469 | 0.471 | 0.538 | 0.536 | 0.536 | 0.535 | 0.534 |
| Expected Events at 1000 $fb^{-1}$ | 83 | 83 | 95 | 95 | 95 | 94 | 94 |
| Expected Events at 2500 $fb^{-1}$ | 207 | 208 | 237 | 236 | 236 | 236 | 236 |

| Process → ZHH | BP1 | BP2 | BP3 | BP4 | BP5 | BP6 | BP7 |
|--------------|-----|-----|-----|-----|-----|-----|-----|
| $\sigma$ [fb] | 0.162 | 0.156 | 0.150 | 0.136 | 0.123 | 0.110 | 0.098 |
| $N_{jets} \geq 5$ | 0.765 | 0.832 | 0.952 | 0.969 | 0.979 | 0.984 | 0.987 |
| $N_{bjets} \geq 3$ | 0.971 | 0.978 | 0.980 | 0.984 | 0.987 | 0.990 | 0.992 |
| $N_{bb} \geq 1$ | 0.704 | 0.760 | 0.797 | 0.825 | 0.841 | 0.858 | 0.874 |
| Total Efficiency | 0.523 | 0.619 | 0.743 | 0.787 | 0.812 | 0.836 | 0.855 |
| Six jets final state | 0.523 | 0.616 | 0.741 | 0.784 | 0.810 | 0.833 | 0.852 |
| Expected Events at 1000 $fb^{-1}$ | 92 | 103 | 117 | 107 | 93 | 80 | 67 |
| Expected Events at 2500 $fb^{-1}$ | 230 | 258 | 292 | 267 | 236 | 200 | 168 |

| Process → ZAA | BP1 | BP2 | BP3 | BP4 | BP5 | BP6 | BP7 |
|--------------|-----|-----|-----|-----|-----|-----|-----|
| $\sigma$ [fb] | 0.162 | 0.156 | 0.150 | 0.136 | 0.123 | 0.110 | 0.098 |
| $N_{jets} \geq 5$ | 0.764 | 0.834 | 0.953 | 0.969 | 0.978 | 0.983 | 0.987 |
| $N_{bjets} \geq 3$ | 0.972 | 0.979 | 0.980 | 0.985 | 0.987 | 0.990 | 0.992 |
| $N_{bb} \geq 1$ | 0.709 | 0.759 | 0.796 | 0.824 | 0.842 | 0.858 | 0.872 |
| Total Efficiency | 0.527 | 0.619 | 0.744 | 0.787 | 0.813 | 0.835 | 0.853 |
| Six jets final state | 0.525 | 0.617 | 0.742 | 0.784 | 0.811 | 0.832 | 0.850 |
| Expected Events at 1000 $fb^{-1}$ | 77 | 84 | 93 | 80 | 66 | 53 | 41 |
| Expected Events at 2500 $fb^{-1}$ | 192 | 210 | 233 | 201 | 165 | 132 | 101 |

TABLE VII: Efficiencies of signal processes $Zhh$, $ZH H$ and $ZAA$ at different kinematic and selection cuts.

decays, where two jets are coming from $Z$ decay and four jets are coming from double Higgs decays. The efficiency of jets production (efficiency is detector parameter while production is accelerator parameter, both are unrelated) increases with the increase in neutral Higgs boson mass.
To check the visibility of the neutral Higgs boson at Compact Linear Collider for selected benchmarks, the signal significance is computed for each neutral Higgs mass distribution by incorporating total number of signal and background events of neutral Higgs bosons within selected mass limit. Signal significance is computed for the two integrated luminosities i.e. 1000 \( fb^{-1} \) and 2500 \( fb^{-1} \). The computation results including signal \( S \) and background \( B \), signal to background ratio \( S/B \), signal significance \( S/\sqrt{B} \) and total efficiency \( \epsilon \) at \( \sqrt{s} = 1.5 \) TeV are given in Table VIII. To be specific, for \( h \) it can be seen from Table VIIIValues of total signal efficiency \( \epsilon_{\text{Total}} \), signal and background events, signal significance, signal to background ratio for processes \( Zhh, ZHH \) and \( ZAA \) for integrated luminosities of 1000 \( fb^{-1} \) and 2500 \( fb^{-1} \) at \( \sqrt{s} = 1.5 \) TeV.

### Table VIII: Values of total signal efficiency \( \epsilon_{\text{Total}} \), signal and background events, signal significance, signal to background ratio for processes \( Zhh, ZHH \) and \( ZAA \) for integrated luminosities of 1000 \( fb^{-1} \) and 2500 \( fb^{-1} \) at \( \sqrt{s} = 1.5 \) TeV.

#### Process \( \rightarrow Zhh \)

| Process | \( \mathcal{L}_{\text{int.}} \) [\( fb^{-1} \)] | 1000 | 2500 |
| --- | --- | --- | --- |
| \( S \) | 42.1 | 44.3 | 764.8 | 762.3 | 679.7 | 771.9 | 765.9 | S | 99.4 | 104.7 | 1753.4 | 1750.1 | 1699.3 | 1713.8 | 1727.4 |
| \( B \) | 1755.6 | 1755.6 | 64508.8 | 64508.8 | 58205.9 | 65781.9 | 64913.6 | B | 2218.1 | 2218.1 | 145515.0 | 145515.0 | 145515.0 | 145515.0 | 145515.0 |
| \( S/B \) | 0.02 | 0.03 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | S/B | 0.04 | 0.05 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 |
| \( S/\sqrt{B} \) | 1.01 | 1.06 | 3.01 | 3.00 | 2.82 | 3.01 | 3.01 | S/\sqrt{B} | 2.11 | 2.22 | 4.60 | 4.59 | 4.45 | 4.49 | 4.53 |
| \( \epsilon_{\text{Total}} \) | 0.24 | 0.25 | 4.34 | 4.33 | 3.86 | 4.38 | 4.35 | \( \epsilon_{\text{Total}} \) | 0.23 | 0.24 | 3.98 | 3.97 | 3.86 | 3.89 | 3.92 |

#### Process \( \rightarrow ZHH \)

| Process | \( \mathcal{L}_{\text{int.}} \) [\( fb^{-1} \)] | 1000 | 2500 |
| --- | --- | --- | --- |
| \( S \) | 56.4 | 75.8 | 36.0 | 53.5 | 52.9 | 35.7 | 35.2 | S | 137.5 | 153.8 | 162.8 | 136.4 | 107.4 | 85.5 | 68.3 |
| \( B \) | 2942.0 | 5684.9 | 1246.4 | 308.2 | 294.7 | 300.3 | 287.1 | B | 4159.6 | 8026.6 | 2867.4 | 1105.2 | 669.7 | 442.4 | 475.9 |
| \( S/B \) | 0.02 | 0.03 | 0.03 | 0.17 | 0.18 | 0.12 | 0.12 | S/B | 0.03 | 0.03 | 0.06 | 0.12 | 0.16 | 0.19 | 0.14 |
| \( S/\sqrt{B} \) | 1.04 | 1.00 | 1.02 | 3.05 | 3.08 | 2.02 | 2.08 | S/\sqrt{B} | 2.13 | 2.22 | 3.04 | 4.10 | 4.15 | 4.07 | 3.13 |
| \( \epsilon_{\text{Total}} \) | 0.32 | 0.45 | 0.23 | 0.40 | 0.46 | 0.37 | 0.45 | \( \epsilon_{\text{Total}} \) | 0.13 | 0.37 | 0.41 | 0.40 | 0.37 | 0.36 | 0.35 |

#### Process \( \rightarrow ZAA \)

| Process | \( \mathcal{L}_{\text{int.}} \) [\( fb^{-1} \)] | 1000 | 2500 |
| --- | --- | --- | --- |
| \( S \) | 44.1 | 50.5 | 27.0 | 41.0 | 30.3 | 28.6 | 20.7 | S | 61.9 | 123.3 | 133.3 | 101.2 | 75.7 | 72.9 | 42.7 |
| \( B \) | 1339.3 | 2294.4 | 726.7 | 344.4 | 228.0 | 187.5 | 344.5 | B | 3063.2 | 3183.8 | 4244.8 | 605.7 | 570.1 | 581.7 | 1798.9 |
| \( S/B \) | 0.03 | 0.02 | 0.04 | 0.12 | 0.13 | 0.15 | 0.06 | S/B | 0.02 | 0.04 | 0.03 | 0.17 | 0.13 | 0.13 | 0.02 |
| \( S/\sqrt{B} \) | 1.20 | 1.06 | 1.00 | 2.21 | 2.01 | 2.08 | 1.12 | S/\sqrt{B} | 1.12 | 2.14 | 2.05 | 4.11 | 3.17 | 3.02 | 1.00 |
| \( \epsilon_{\text{Total}} \) | 0.30 | 0.37 | 0.22 | 0.40 | 0.37 | 0.45 | 0.44 | \( \epsilon_{\text{Total}} \) | 0.17 | 0.36 | 0.43 | 0.40 | 0.37 | 0.46 | 0.36 |

X. SIGNAL SIGNIFICANCE

To check the visibility of the neutral Higgs boson at Compact Linear Collider for selected benchmarks, the signal significance is computed for each neutral Higgs mass distribution by incorporating total number of signal and background events of neutral Higgs bosons within selected mass limit. Signal significance is computed for the two integrated luminosities i.e. 1000 \( fb^{-1} \) and 2500 \( fb^{-1} \). The computation results including signal \( S \) and background \( B \), signal to background ratio \( S/B \), signal significance \( S/\sqrt{B} \) and total efficiency \( \epsilon \) at \( \sqrt{s} = 1.5 \) TeV are given in Table VIII. To be specific, for \( h \) it can be seen from Table VIIIValues of total signal efficiency \( \epsilon_{\text{Total}} \), signal and background events, signal significance, signal to background ratio for processes \( Zhh, ZHH \) and \( ZAA \) for integrated luminosities of 1000 \( fb^{-1} \) and 2500 \( fb^{-1} \) at \( \sqrt{s} = 1.5 \) TeV that it is observable at \( m_h = 125 \) GeV and \( 150 \leq m_H \leq 250 \) GeV for both 1000 \( fb^{-1} \) and 2500 \( fb^{-1} \) integrated luminosity. Also from Table VIIIValues of total signal
efficiency $\epsilon_{\text{Total}}$, signal and background events, signal significance, signal to background ratio for processes $Zhh$, $ZHH$ and $ZAA$ for integrated luminosities of 1000 $fb^{-1}$ and 2500 $fb^{-1}$ at $\sqrt{s} = 1.5$ TeV. Table VIII shows that $A$ is observable at $m_A = 175$ GeV for integrated luminosity of 1000 $fb^{-1}$ and at $150 \leq m_A \leq 225$ GeV for integrated luminosity of 2500 $fb^{-1}$. Signal to background candidate mass distributions are shown in Figures 15 and 16. Pseudo-scalar Higgs mass distribution of $H$ at integrated luminosity 1000 $fb^{-1}$ (left) and 2500 $fb^{-1}$ (right) at $\sqrt{s} = 1.5$ TeV.
FIG. 17: Pseudo-scalar Higgs mass distribution of $A$ at integrated luminosity 1000 $fb^{-1}$ (left) and 2500 $fb^{-1}$ (right) at $\sqrt{s} = 1.5$ TeV.

FIG. 18: Behavior of signal significance for processes $Zhh$, $ZHH$ and $ZAA$ respectively for each benchmark point at integrated luminosity 1000 $fb^{-1}$ and 2500 $fb^{-1}$.

$A$ at integrated luminosity 1000 $fb^{-1}$ (left) and 2500 $fb^{-1}$ (right) at $\sqrt{s} = 1.5$ TeV.

The Figure 18 Behavior of signal significance for processes $Zhh$, $ZHH$ and $ZAA$ respectively for each benchmark point at integrated luminosity 1000 $fb^{-1}$ and 2500 $fb^{-1}$ shows variation in signal significance with each benchmark point at an integrated luminosity of 1000 $fb^{-1}$ and 2500 $fb^{-1}$ for signal processes $Zhh$, $ZHH$ and $ZAA$ respectively. It can be seen that with the increase in integrated luminosity, the signal significance also increases. Signal significance increases up to BP3 then it remains almost constant for the process $Zhh$ and increases up to BP4 for the o processes $ZHH$ and $ZAA$, and then decreases for higher BPs.
XI. CONCLUSION

This study is carried out within Type-I of 2HDM in standard model like scenario. Different benchmark points are selected in the parameter space of 2HDM. Observability of standard model Higgs \( h \), CP-even heavy Higgs \( H \), and pseudoscalar Higgs \( A \) is evaluated. Three signal processes \( e^+e^- \rightarrow Zh \rightarrow j\bar{b}b \), \( e^+e^- \rightarrow Zhh \rightarrow j\bar{b}\bar{b} \) and \( e^+e^- \rightarrow Zh \rightarrow j\bar{b}\bar{b} \) are selected for analysis. Left-right-handed polarized incoming beams are used to enhance signal cross-section. Vertices involved in these processes depend upon \( \sin(\beta - \alpha) \) that is why in exact alignment and equal Higgs masses regime, trilinear Higgs vertices become simplified and these processes become independent of \( \tan\beta \). So lower value of \( \tan\beta \) can be taken in support of \( h, H, A \to b\bar{b} \) decay without leaving any destructive effects on \( Zhh, ZHH \) and \( ZAA \) production. Also in this study hadronic decay of \( Z \) boson is taken that can cause many errors and uncertainties due to mistagging and jet misidentification but this prominent decay channel will compensate for these errors and uncertainties. Various benchmark points are taken at the center of mass-energy of 1.5 TeV for CLIC and event generation is carried out at each point separately. Simulation results show that the proposed analysis can be used to observe signal scenarios considered in this study because candidate mass distributions have excess of events on the top of backgrounds. Fitting functions applied on the candidate mass distributions show that the corrected reconstructed masses are very close to the generated ones. The signal significance for all signal scenarios is calculated for integrated luminosity of 1000 and 2500 \( fb^{-1} \). Computation results show that there are a fewer numbers of events at integrated luminosity of 1000 \( fb^{-1} \), due to less scattering cross-section (or production cross-section)over total backgrounds and increases up to 1.4 times for integrated luminosity of 2500 \( fb^{-1} \). For \( Zhh \) at \( m_{h} = 125.09 \) GeV and \( 150 \leq m_{H} \leq 250 \) GeV signal significance is highest and almost constant. And in that region signal significance on an average increases by 51% from 1000 to 2500 \( fb^{-1} \). As \( h \) is the SM Higgs already observed that is why the observability of \( H \) in \( 150 \leq m_{H} \leq 250 \) GeV region is proposed. For \( ZHH \) signal significance is highest in the mass range \( 175 \leq m_{H} \leq 200 \) GeV at 1000 \( fb^{-1} \) and in the mass range \( 175 \leq m_{H} \leq 225 \) GeV at 2500 \( fb^{-1} \). Signal significance increases by 34% in the range \( 175 \leq m_{H} \leq 200 \) GeV region and by 100% at \( m_{H} = 225 \) GeV from 1000 to 2500 \( fb^{-1} \). For \( ZAA \) signal significance is highest at \( m_{A} = 175 \) GeV for both 1000 and 2500 \( fb^{-1} \). And at this value signal significance increases up to 86% from 1000 to 2500 \( fb^{-1} \). It is expected that the proposed analysis will help those who are working on the observability of neutral Higgs bosons in 2HDM.
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The authors have no relevant financial or non-financial interests to disclose.

Availability of data and materials
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