A Weakly Coupled Ultraviolet Completion of the Littlest Higgs with T-parity

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Abstract

We construct a weakly coupled, renormalizable ultraviolet completion of the Littlest Higgs model with T-parity (LHT), based on an $SU(5) \times SU(2) \times U(1)$ gauge theory with a discrete $Z_2$ symmetry. Our model reproduces the complete structure of the LHT below the 10 TeV scale, including the collective symmetry breaking mechanism which solves the little hierarchy problem. The model is manifestly free of anomalies, including both gauge/gravitational anomalies and anomalies involving T-parity. At the TeV scale, the model contains additional states not present in the LHT. We estimate the impact of these states on precision electroweak observables, and show that the model is realistic. We also discuss how our model can be embedded into a supersymmetric theory or a five-dimensional setup with a warped extra dimension, stabilizing the hierarchy between the 10 TeV and the Planck scale.
1 Introduction

One of the most pressing issues facing particle theory is the little hierarchy problem. On the one hand, electroweak precision measurements at LEP and the Tevatron seem to indicate the existence of a weakly coupled light (below 200 GeV) Higgs boson. This Higgs would be unstable against large radiative corrections, and one would expect new physics at or below the TeV scale to stabilize the Higgs potential. On the other hand, the same electroweak precision measurements have failed to provide any indirect evidence for such physics. For the case of supersymmetry (SUSY), a natural minimal model should have already been discovered at LEP2 or the Tevatron: null results of superpartner and Higgs searches imply that a fine-tuning of order 1% or worse is required to accommodate the data, which is the particular incarnation of the little hierarchy problem for SUSY.

The motivation for Little Higgs (LH) models is to solve this issue by pushing the scale of new physics that solves the “large” (weak/Planck) hierarchy problem up to 10 TeV, and provide a rationale for the cancelation of the remaining quadratic divergences in the Higgs mass between 1 TeV and 10 TeV. This is achieved by interpreting the Higgs as an approximate Goldstone boson corresponding to a spontaneously broken global symmetry of the electroweak sector. Gauge and Yukawa couplings of the Higgs must break the global symmetry explicitly; however, if this breaking is “collective” (meaning that no single coupling breaks all of the symmetry responsible for keeping the Higgs light), the extended theory can remain perturbative until the 10 TeV scale without fine-tuning [1]. Several explicit realizations of this idea have appeared in the literature [2]. Models with T-parity are especially promising, since they can be consistent with precision electroweak constraints without need for fine tuning in the Higgs mass [3]. In this paper, we will focus on the Littlest Higgs model with T-parity (LHT) [4], which is a fully realistic example of this class.

Like all existing Little Higgs models, the LHT has been constructed as an effective field theory, valid below the cutoff scale of order 10 TeV. This is sufficient to discuss the model’s consistency with precision electroweak data [5,6], its signatures at the Tevatron [7] and the LHC [8,9], and the dark matter candidate that naturally emerges in this model [6,8,10]. However, in order to really complete the program outlined above one needs to find the ultraviolet (UV) completion of these models, i.e. embed it into a more fundamental theory valid at higher scales, possibly all the way up to the scale of grand unification (GUT) or the Planck scale. The main aim of this paper is to present such a construction. As with most BSM models, there are two possibilities. The UV completion may be a strongly coupled theory, which happens to produce the LHT as its effective theory below the confinement scale of 10 TeV, or the UV completion remains perturbative, and the LHT emerges as a low-energy description of a renormalizable weakly coupled gauge theory. Here we choose to follow the second possibility, that is we present a linear UV completion of the LHT. In this approach, one needs to introduce supersymmetry to stabilize the hierarchy between the 10 TeV scale and the GUT/Planck scale; however, since SUSY is broken at 10 TeV, the model is free of the fine-tuning plaguing the MSSM. Alternatively one can have a Kaluza-Klein (KK) tower of a warped extra dimension starting at 10 TeV, which would also stabilize the large hierarchy. Our model explains the appearance and radiative stability of the global symmetry
structure of the LHT, which at first sight appears rather unnatural. Furthermore, the model is manifestly free of anomalies, including both the familiar gauge/gravitational anomalies and the anomalies involving T-parity. Thus, the anomaly-induced T-parity violating operators, which recently received some attention in the literature [11,12], are completely absent in our model and T-parity is an exact symmetry, at least as long as gravitational effects can be ignored. This illustrates the point that the existence of these operators depends crucially on the nature of the ultraviolet completion of the LH model. This has also been emphasized very recently in [13], where it was also pointed out the UV completions with anomalous T-parity are unlikely to have the correct vacuum alignment.

Before presenting our model, let us briefly comment on its relation to previous work in this area. UV completions of the Littlest Higgs model have been until now based on either a strongly interacting theory or equivalently a warped extra dimension at the 10 TeV scale. Models without T-parity have been constructed [14,15], while recently an attempt to incorporate a discrete parity based on two throats of warped dimensions was presented in [16]. Our model is based on conventional, four-dimensional and perturbative physics, making it much easier to incorporate T-parity and to analyze anomalies. Supersymmetric ultraviolet completions of an alternative LH model, the “simplest” little Higgs, have also appeared in the literature [17,18]. However, in those models the electroweak precision constraints are so strong that one has to assume that SUSY is broken at the weak scale, and the LH scale is much higher. The role of the Little Higgs mechanism is to solve the little hierarchy problem within SUSY. In contrast, in our model the LH partners appear first, and SUSY is irrelevant until the 10 TeV scale. At the LHC, our model would look like the familiar LHT, with a few extra states. We will also present an extra dimensional model that is reminiscent of the structure of the minimal composite Higgs (MCH) models of [19], in which the Higgs will appear as the zero mode of the $A_5$ bulk gauge fields, which will pick up a finite radiatively generated potential. The main difference between the model presented here and the MCH models is that we will have the T-odd little Higgs partners appearing at the 1 TeV scale, which will allow us to push the KK mass scale of the theory to 10 TeV without fine-tuning. Thus the KK tower only plays a role of UV completing the theory above 10 TeV and stabilizing the hierarchy between 10 TeV and the Planck scale, but it is not used to cut off the 1-loop quadratic divergences between 1 and 10 TeV.

The paper is organized as follows. We first construct a four-dimensional, non-supersymmetric, renormalizable model which reduces to the LHT (plus a few extra states) below the 10 TeV scale. We discuss the bosonic (gauge and scalar) sector of the model in section 2 and show how to incorporate fermions in section 3. In section 4 we extend the model to achieve complete anomaly cancelation, including anomalies involving T-parity. In section 5 we discuss how the hierarchy between the 10 TeV scale and the Planck scale can be stabilized by either supersymmetrizing the model or embedding it into a theory with a warped fifth dimension à la Randall and Sundrum [20]. In section 6 we estimate the precision electroweak constraints on the model, and show that the model is realistic. In section 7 we show by an explicit diagrammatic calculation how the little Higgs cancelations occur in our renormalizable model. Finally, section 8 contains our conclusions.
2 The Scalar/Gauge Sector for $SU(5) \times SU(2) \times U(1)$

The bosonic (scalar and gauge) degrees of freedom of the LHT model are described by a gauged non-linear sigma model (nlσm). The scalars are the Goldstone bosons of the global symmetry breaking $SU(5) \rightarrow SO(5)$. The symmetry-breaking vev (or condensate) is in the symmetric representation $15$ of the $SU(5)$. The symmetry breaking scale $f_S$ is assumed to be about 1 TeV. To incorporate the gauge degrees of freedom, an $[SU(2) \times U(1)]^2$ subgroup of the $SU(5)$ is gauged; for the fundamental representation, the gauged subgroup of $SU(5)$ is spanned by the generators

$$Q_1^a = \begin{pmatrix} \tau^a \\ 0 \\ 0 \end{pmatrix}, \quad Y_1 = \frac{1}{10} \begin{pmatrix} 3 \\ 3 \\ -2 \\ -2 \\ -2 \end{pmatrix}$$  \hspace{1cm} (2.1)$$

and

$$Q_2^a = \begin{pmatrix} 0 \\ 0 \\ -\tau^a T \end{pmatrix}, \quad Y_2 = \frac{1}{10} \begin{pmatrix} 2 \\ 2 \\ 2 \\ -3 \\ -3 \end{pmatrix}$$  \hspace{1cm} (2.2)$$

where $\tau^a = \sigma^a / 2$. Below $f_S$, the gauge symmetry is reduced to the diagonal $SU(2) \times U(1)$, which is identified with the Standard Model (SM) electroweak gauge group $SU(2)_L \times U(1)_Y$. Under this group, the physical (uneaten) Goldstones decompose into a weak doublet, identified with the SM Higgs, and a weak triplet. The Higgs mass is protected from a one-loop quadratic divergence by the collective symmetry breaking mechanism. The nlσm is an effective theory valid up to the scale $\Lambda \sim 4\pi f_S \sim 10$ TeV. For a more detailed description of the LHT model, see Refs. [4, 5, 8].

The first step to a weakly coupled UV completion of the LHT is to replace the nlσm with a linear sigma model with the same symmetry breaking structure. This model contains a single scalar field $S$, transforming as $15$ of $SU(5)$, which is assumed to get a vev

$$\langle S \rangle = f_S \begin{pmatrix} 1 \\ 1 \\ \hat{1} \end{pmatrix},$$  \hspace{1cm} (2.3)$$

where $f_S \sim 1$ TeV. The Lagrangian is simply

$$\mathcal{L}_{\text{lin}} = \frac{1}{8} |D_\mu S|^2 - V(S),$$  \hspace{1cm} (2.4)$$

where $D_\mu$ is the covariant derivative, and the renormalizable potential $V(S)$ is assumed to lead to an $S$ vev of the form $\langle S \rangle$. We will not need to specify this and other scalar potentials explicitly in this paper, for an example of a possible potential for $S$ see eq. (2.3).
The excitations around the vacuum \((2.3)\) can be parametrized as

\[
S = \langle S \rangle + i \left( \begin{array}{ccc}
\phi_S & \sqrt{2} h_S & \chi_S + \frac{n_S}{\sqrt{5}} \\
\sqrt{2} h_S^T & -i \frac{4n_S}{\sqrt{5}} & \sqrt{2} h_S^T \\
\chi_S & \frac{n_S}{\sqrt{5}} & \phi_S^\dagger 
\end{array} \right) + \text{(radial modes)},
\]

where \(\chi_S\) is a hermitan, complex \(2 \times 2\) matrix, \(n_S\) a real singlet, \(\phi_S\) a complex, symmetric \(2 \times 2\) matrix and \(h_S\) a complex doublet, which will be identified with the SM Higgs. These fields are pseudo-Goldstone bosons (they would be exact Goldstone bosons, if the gauge couplings were taken to zero). They contain 14 degrees of freedom, corresponding to the number of \(SU(5)\) generators broken by the \(S\) vev. The other 16 degrees of freedom in \(S\), the “radial” modes, obtain masses \(\sim cf_S\), where \(c\) are order-one numbers determined by the coupling constants in \(V(S)\). Integrating out the radial modes reproduces the \(\nu \sigma m\) description of the LHT, independent of the details of \(V(S)\). This is guaranteed by the Coleman-Wess-Zumino theorem \([21]\). In particular, the crucial feature of the LHT \(\nu \sigma m\) is the special structure of the Higgs coupling to gauge fields, which guarantees the absence of a quadratic divergence in the Higgs mass at one loop. In section \([7]\) we show by an explicit calculation how this structure emerges from the linear sigma model.

The model defined by eq. \((2.4)\) is of course renormalizable, and can be valid up to an arbitrarily high scale, for example the Planck scale. In this sense, it is a viable UV completion of (the bosonic sector of) the LHT. However, it has two significant shortcomings:

- The symmetry structure of this model is very unnatural. Because gauge interactions break the global \(SU(5)\) explicitly, renormalization-group evolution generates \(SU(5)\)-violating operators in the Lagrangian. In the LHT model, the global \(SU(5)\) has to be a good symmetry at the 10 TeV scale. This would require the linear model to contain a very special combination of \(SU(5)\)-violating terms at the Planck scale, finely tuned just so that the \(SU(5)\) is miraculously restored at 10 TeV.

- SM fermions cannot be incorporated in this model in a way consistent with T-parity. T-parity requires that for every field transforming under one of the two \(SU(2) \times U(1)\) gauge groups of the LHT model, there must be another field transforming in the same way under the other \(SU(2) \times U(1)\). Since the SM weak group is the diagonal combination of the two \(SU(2)\) factors, this means that the model must have an even number of weak doublets of the same hypercharge and color charge. Therefore this model cannot lead to the chiral fermion content of the SM in the low energy limit.

To avoid the first problem, we would like to start at high energies with a model in which the full \(SU(5)\) is promoted to a gauge symmetry. Further, to incorporate chirality, we must enlarge the gauge structure to contain an odd number of gauged \(SU(2)\) factors. The most obvious and easiest choice is to add one extra gauge \(SU(2)\). As we will see below, obtaining the correct hypercharge assignments for all SM fermions also requires an additional \(U(1)\) gauge group.
Thus, the full gauge group of our model, at high energies, is

$$SU(5) \times SU(2)_3 \times U(1)_3,$$

(2.6)

where we labeled the extra $SU(2) \times U(1)$ factor with a subscript “3” to distinguish it from the $[SU(2) \times U(1)]^2$ subgroup of the $SU(5)$ that survives below 10 TeV. To break the $[SU(2) \times U(1)]^3$ subgroup to the SM electroweak gauge group, we also need additional bifundamental scalars under $SU(5) \times SU(2)_3$, $K_1$ and $K_2$, which will acquire the appropriate vevs (see eq. (2.9)).

To reproduce the symmetries of the LHT model at low energies, we introduce a set of scalar fields, summarized in Table 1. At the 10 TeV scale, the $\Phi$ fields get vevs of the form

$$\langle \Phi_1 \rangle = f_\Phi \begin{pmatrix} -3 \\ -3 \\ 2 \\ 2 \\ 2 \end{pmatrix}, \quad \langle \Phi_2 \rangle = f_\Phi \begin{pmatrix} 2 \\ 2 \\ 2 \\ -3 \\ -3 \end{pmatrix},$$

(2.7)

where $f_\Phi \sim 10$ TeV. These vevs break the $SU(5)$ down to $[SU(2) \times U(1)]^2$, the gauge group of the LHT model, and leave the $SU(2)_3 \times U(1)_3$ unbroken. If the scalar potential has the form

$$V = V(\Phi_1, \Phi_2) + V(S, K_1, K_2),$$

(2.8)

so that there are no direct couplings between $\Phi$’s and other scalars, the model will possess an $SU(5)$ global symmetry below 10 TeV, broken only by gauge interactions. This is the idea that was first employed in the context of $SU(6)$ GUT models in [22], and also in the ”simplest little Higgs” model in [23]. With this assumption, the full gauge/global symmetry structure of the LHT is reproduced. Of course, this construction is only natural, if there is a symmetry reason for the absence of direct potential couplings between $\Phi$’s and the other scalars. In section 5, we will show that the $\Phi$-vevs can be stabilized at the 10 TeV scale, either by supersymmetrizing the model or by embedding it into a five-dimensional model with warped geometry. In both cases, the couplings between $\Phi$ and the other scalars can be naturally suppressed.
Figure 1: The gauge symmetries and scalar field content of the model below the 10 TeV scale.

At the 1 TeV scale, the field $S$ gets a vev given in eq. (2.3), while the bifundamental fields get vevs

$$
\langle K_1 \rangle = f_K \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \\
\langle K_2 \rangle = f_K \begin{pmatrix} 1 \\ 1 \end{pmatrix},
$$

(2.9)

where $f_K \sim 1$ TeV. Together, these vevs break the $[SU(2) \times U(1)]^3$ gauge symmetry down to a single $SU(2) \times U(1)$, identified with the SM. The unbroken generators are simply $Q_D = Q_1^0 + Q_2^0 + Q_3^0$ and $Y_D = Y_1 + Y_2 + Y_3$.

The global symmetry breaking by the $K$-vevs results in additional pseudo-Goldstone bosons. We will assume that the tree-level scalar potential does not contain direct couplings between the fields: $V = V(S) + V(K_1, K_2)$. With this assumption, the Goldstones contained in different fields do not mix. Most of the Goldstones are not protected by the collective symmetry breaking mechanism. They will therefore receive quadratically divergent masses at the one-loop level from gauge loops, and their masses are in the TeV range. The only exceptions are the SM Higgs $h_S$, and a set of three real Goldstones transforming as a real triplet under the SM $SU(2)$ gauge group. Two of these triplets are eaten by the heavy $SU(2)$ gauge bosons, while the third one remains physical. The physical mode is a linear combination of the Goldstones coming from $S$, $K_1$ and $K_2$. In fact, one can think of our model below 10 TeV as a three-site deconstruction of a five-dimensional model, with the moose diagram shown in Fig. 1. In this picture, the light triplet mode is simply the counterpart of $A_5$, and can only receive a mass from non-local effects due to compactification. However, the Yukawa couplings of our model (discussed in the following section) do not have such an “extra-dimensional” structure, and the triplet mass is not protected from the one-loop diagrams involving the Yukawas. Thus, this mode will also receive a TeV-scale mass. The only pseudo-Goldstone protected by the collective symmetry mechanism is the SM Higgs.

In addition to the gauge symmetries, we impose that the model is invariant under a
discrete T-parity, which acts on the gauge and scalar fields as follows:

\[
\begin{align*}
W_{SU(5)} & \rightarrow \Omega(W_{SU(5)})\Omega^\dagger, \\
W_{SU(2)} & \rightarrow \omega(W_{SU(2)})\omega^\dagger = W_{SU(2)}, \\
B_{U(1)} & \rightarrow B_{U(1)}, \\
\Phi_1 & \leftrightarrow \Omega\Phi_2\Omega^\dagger, \\
S & \rightarrow \Omega S^\dagger\Omega^T, \\
K_1 & \leftrightarrow \Omega K_2\omega^T,
\end{align*}
\]

where \(W_{SU(5)}\), \(W_{SU(2)}\) and \(B_{U(1)}\) are the SU(5), SU(2)\(_3\) and U(1)\(_3\) gauge fields, respectively, and

\[
\Omega = \begin{pmatrix} -1 & \phantom{-}1 \\ 1 & -1 \end{pmatrix} \quad \text{and} \quad \omega = -1.
\]  

Note that \(\Omega \in SU(5)\) and \(\omega \in SU(2)\). The kinetic terms are automatically invariant under this parity, while the scalar potential must be restricted to the terms consistent with it. The vevs in eqs. (2.3), (2.7) and (2.9) do not break T-parity. It is easy to check that the T-parity defined in this way acts in the desired way on the fields of the LHT model: the two \(SU(2) \times U(1)\) factors inside the SU(5) are interchanged, the Higgs boson \(h_S\) is T-even, while the weak triplet is T-odd, as required by precision electroweak fits.

Now, let us discuss the spectrum of the bosonic states. Sixteen out of the 24 SU(5) gauge bosons get masses at the 10 TeV scale. These states are too heavy to have any phenomenological consequences, and we will not discuss them further. Below 10 TeV, we have three sets of SU(2) gauge bosons:

\[
\begin{align*}
m^2_{W_{SM}} & = 0 : \\
m^2_{W_{even}} & = \frac{g_5^2 + 2g_3^2}{4}f_K^2 : \\
m^2_{W_{odd}} & = \frac{g_5^2}{4}(2f_S^2 + f_K^2) : \\
m^2_{B_{SM}} & = 0 : \\
m^2_{B_{even}} & = \frac{g_5'^2 + 2g_3'^2}{4}f_K^2 : \\
m^2_{B_{odd}} & = \frac{g_5'^2}{10}(10f_S^2 + f_K^2) :
\end{align*}
\]

as well as three U(1) bosons:

\[
\begin{align*}
m^2_{B_{SM}} & = 0 : \\
m^2_{B_{even}} & = \frac{g_5'^2 + 2g_3'^2}{4}f_K^2 : \\
m^2_{B_{odd}} & = \frac{g_5'^2}{10}(10f_S^2 + f_K^2) :
\end{align*}
\]

Here \(g_5\), \(g_3\) and \(g_3'\) are the SU(5), SU(2)\(_3\) and U(1)\(_3\) coupling constants, respectively, and in proper normalization \(g_5' = \sqrt{5/3}g_5\).

Note that the model contains a set of T-even gauge bosons at the TeV scale, due to the presence of an extra SU(2) \(\times U(1)\) gauge factor, which is T-even. These states can be
problematic for electroweak precision constraints, but are inevitable in our model. However, they do not participate in the cancelation of the quadratic divergences in the Higgs boson mass. Therefore, they can be substantially heavier than the T-odd states, without spoiling naturalness. This occurs if $g'_3, g_3 \gg g_5$; if the T-odd states are at 1 TeV, requiring that $g'_3, g_3 \sim 3-5 g_5$ is sufficient to avoid precision electroweak constraints, and the model remains weakly coupled, but for these parameters, the Weinberg angle is fixed at a wrong value: $\sin^2 \theta_W = 5/8$ in the limit $g'_3, g_3 \gg g_5$. However, as we will discuss in section 3.2, reproducing the top sector of the LHT from a renormalizable model will require introduction of additional scalar vevs at the TeV scale, which will affect the gauge boson spectrum. It turns out that in the full model the correct value of the Weinberg angle can be easily reproduced without conflict with precision electroweak data, as we will show in detail in section 6.

3 The Fermion Sector

In this section we describe the fermion sector of our model that contains the SM fermions plus a number of heavier states. Our convention is to write all fermion fields as left-handed two-component spinors.

3.1 The SM fermions

It is straightforward to include the SM $SU(2)_L$ singlets as T-even fermionic singlets, $u_R, d_R$ and $e_R$. (The SM generation index will be omitted throughout this paper.) For each SM doublet, we introduce two fermions in the representations $\mathbf{5}$ and $\mathbf{\bar{5}}$ of $SU(5)$

$$\Psi_1 = \begin{pmatrix} \psi_1 \\ U_{L1} \\ \chi_1 \end{pmatrix} \quad \text{and} \quad \Psi_2 = \begin{pmatrix} \chi_2 \\ U_{L2} \\ \psi_2 \end{pmatrix}. \quad (3.1)$$

A linear combination of $\psi_1$ and $\psi_2$ will become the SM doublet. To decouple the extra components, we need 5 extra fermions: $\psi_3, \psi_4$ and $\psi_5$ are $SU(2)_3$ doublets, and $U_{R1}$ and $U_{R2}$ are singlets. We also need two extra scalar fields, $F_1 \in \mathbf{5}$ and $F_2 \in \mathbf{\bar{5}}$ of $SU(5)$. Both are uncharged under $SU(2)_3 \times U(1)_3$. Under T-parity,

$$\Psi_1 \leftrightarrow \Omega^\dagger \Psi_2$$
$$\psi_3 \rightarrow \omega \psi_3$$
$$\psi_4 \leftrightarrow \omega \psi_5$$
$$U_{R1} \leftrightarrow U_{R2}$$
$$u_R \rightarrow u_R$$
$$d_R \rightarrow d_R$$
$$F_1 \leftrightarrow \Omega F_2. \quad (3.2)$$
The Yukawa couplings allowed by gauge symmetries and T-parity are:

\[
\mathcal{L}_{\text{Yuk}} = \kappa_1 [\Psi_1 K_1 \psi_3 + \Psi_2 K_2 \psi_3] + \kappa_2 \left[ \Psi_1^\dagger K_2 \psi_4^\dagger + \Psi_2^\dagger K_1 \psi_5^\dagger \right] + \kappa_3 [\Psi_1 F_1 U_{R1} + \Psi_2 F_2 U_{R2}] + \text{h.c.}
\]

(3.3)

The invariance under T-parity can be easily shown using \(\Omega^\dagger \Omega = \mathbb{1}\) and \(\omega^\dagger \omega = \mathbb{1}\). This form of the Yukawas, together with the requirement of the correct hypercharges for the SM fields, unambiguously fixes the \(U(1)_3\) charges for all fermions. The gauge quantum numbers of the fermions are summarized in Table 2.

The fundamental scalars get vevs consistent with T-parity:

\[
\langle F_1 \rangle = \langle F_2 \rangle = (0, 0, f_F, 0, 0)^T,
\]

(3.4)

where \(f_F \sim \text{TeV}\). These vevs break \(Y_1\) and \(Y_2\) seperately, but leave \(Y_1 + Y_2 + Y_3\) unbroken, so that no gauge symmetries not already broken by \(S\) and \(K\) vevs are broken.

For each SM doublet, our model contains five massive Dirac fermions at the TeV scale\(^\dagger\), three T-odd and the other two T-even. Their masses are \(m_{1-} = \sqrt{2}\kappa_1 f_K\), \(m_{2\pm} = \kappa_2 f_K\) and \(m_{3\pm} = \kappa_3 f_F\), where the signs denote the T-parity of each state. There is one massless T-even doublet, \(\psi_{SM} = \frac{1}{\sqrt{2}} (\psi_1 - \psi_2)\), which is identified with the SM quark or lepton doublet. In the next subsection, we will explain how the SM Yukawa couplings can be generated in this model.

### 3.2 The Yukawa couplings

We will start with the top Yukawa. Due to the large value of this coupling in the SM, naturalness requires it to be implemented in a way that only breaks the global symmetries of the LHT collectively. It is straightforward to incorporate the top Yukawas of the LHT

\(^\dagger\)Note that the T-odd fermion masses are bounded from above by constraints on four-fermion operators \([5]\), and cannot be much heavier than a TeV.
model in our linear model. For the third generation quarks, we use the set of fields listed in Table 2. In addition to the terms in (3.3), we include the following operators:

$$L_t = \frac{1}{M} \left[ \epsilon^{ijk} \epsilon^{xy} \Psi_i S^t_{jx} S^t_{ky} + \epsilon_{i'j'} \epsilon_{x'y'z'} \Psi_{i}^{x'} S^{y'} S^{z'} \right] u_R + \text{h.c.}$$

where we restrict the summation to $i, j, k \in \{1, 2, 3\}$, $x, y \in \{4, 5\}$ and $i', j' \in \{1, 2\}$, $x', y', z' \in \{3, 4, 5\}$ and $M$ is the mass scale suppressing this dimension-5 operator. Note that eq. (3.5) is T-parity invariant, although this is not immediately manifest; taking the T-parity transformation of the first term yields

$$\epsilon^{ijk} \epsilon^{xy} \Psi_i S^t_{jx} S^t_{ky} \rightarrow \epsilon^{ijk} \epsilon^{xy} (\Omega^i S \Omega^j S \Omega^k S \Omega^*)_{ky} = [\epsilon^{ijk} \epsilon^{xy} \Omega^i_{x} \Omega^j_{y} \Omega^k_{k} \Omega^*_{x'} \Omega^*_{y'} \Omega^*_{x} \Omega^*_{y} \Omega^*_{k} S^{x'} S^{y'} S^{z'}],$$

which together with $\det \Omega = 1$ gives exactly the second term in eq. (3.5). The expansion to summing over 1 to 5 (and then restricting again to partial summation as in eq. (3.5)) in this derivation is possible due to the special structure of $\Omega$. After the $S$ field gets a vev and the radial modes are integrated out, eq. (3.5) reduces to the top Yukawa term of the usual nσm LHT model (see e.g. [4,5,8]). These Yukawa couplings incorporate the collective symmetry breaking mechanism, which protects the Higgs mass from large renormalization by top loops.

We now want to obtain the operators in eq. (3.5) from an $SU(5)$-invariant, renormalizable Lagrangian. To restore $SU(5)$ invariance, let us introduce two scalar fields,

$$A_1 \in \overline{10}, \quad A_2 \in 10,$$

with T-parity action

$$A_1 \leftrightarrow \Omega^i A_2 \Omega^*. $$

These fields get vevs

$$\langle A_1 \rangle = f_A \begin{pmatrix} 0 & 0 \\ \varepsilon & 0 \end{pmatrix}, \quad \langle A_2 \rangle = f_A \begin{pmatrix} \varepsilon & 0 \\ 0 & 0 \end{pmatrix}, \quad \text{where} \quad \varepsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. $$

These vevs do not break T-parity or the gauged $SU(2)$s, but break the $Y_1$ and $Y_2$ gauged generators. So, the $A$'s need to be charged under $U(1)_3$ with charges chosen such that the broken linear combinations are orthogonal to the one identified with hypercharge, $Y_1 + Y_2 + Y_3$. This requires $Q_3(A_1) = Q_3(A_2) = -1$. In addition to their role in the top sector, the antisymmetric fields also help resolve the problem with the correct value of the Weinberg angle mentioned earlier. For a discussion of this issue, see section 6.

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2By convention fundamental $SU(5)$ indices are upper, antifundamental are lower. $SU(2)$ indices are raised and lowered with $\epsilon^{ab}$ and $\epsilon_{ab}$ as usual.
Eq. (3.5) can now be thought of as the low-energy limit of the following ($SU(5)$-invariant, but still non-renormalizable) Lagrangian:

\[
\mathcal{L}_t \propto \epsilon^{abcde} \Psi_1 a S^i_{b} S^j_{c} \epsilon_x (A_1)^{xy} + \epsilon^{abcde} \Psi_2 a S^{bx} S^{cy} (A_2)^{de} (A_1)^{xy} \right] u_R + \text{h.c.,} \tag{3.10}
\]

where the summations are no longer restricted and run from 1 to 5.

One possible way to obtain a renormalizable model is to introduce four scalar fields, $\eta, \eta', \xi, \xi'$. These are uncharged under $SU(2)_3 \times U(1)_3$, and transform under $SU(5)$ as follows:

\[
\eta \in \Box, \quad \eta' \in \Box, \quad \xi, \xi' \in \text{Adj}. \tag{3.11}
\]

T-parity acts in by-now familiar way:

\[
\eta \leftrightarrow \Omega \eta', \quad \xi \leftrightarrow \Omega^\dagger \xi' \Omega. \tag{3.12}
\]

The renormalizable Lagrangian is then given by

\[
\mathcal{L}_t \propto \left( \epsilon^{abcde} \Psi_1 a \eta \epsilon_x (A_1)^{xy} + \epsilon^{abcde} \Psi_2 a \eta' \epsilon_x (A_2)^{xy} \right) u_R + \text{h.c.} \tag{3.13}
\]

plus mass terms for the scalars. Assuming that the scalars are heavier than $f$, integrating them out reproduces eq. (3.10).

With the above quantum numbers there is no Yukawa coupling possible for the leptons and the down quarks, which resembles the top Yukawa in eq. (3.5). However, it is possible to write down a dimension-6 operator to generate these Yukawa couplings. For the down quarks, this operator has the form

\[
\mathcal{L}_d \sim \frac{\lambda}{M_d^2} \left( \epsilon_{ijk} \epsilon_{xy} \Psi_2 K^i_{1a} K^j_{1a} S^{kxy} + \epsilon_{ij'} \epsilon_{x'y'} \Psi_1 a K_{2x} K_{2y'} a S^i_{x'y'} \right) d_R + \text{h.c.,} \tag{3.14}
\]

where the summation is restricted to $i, j, k \in \{1, 2, 3\}, \ x, y \in \{4, 5\}$ and $i', j' \in \{1, 2\}, \ x', y', z' \in \{3, 4, 5\}$, and $M_d$ is the mass scale at which this operator is generated. The lepton Yukawas are of the same form. In complete analogy to the top sector, the desired operators can be obtained from a renormalizable and $SU(5)$ invariant lagrangian by introducing new heavy states (scalars or fermions) and integrating them out.

### 3.3 A non $SU(5)$ invariant theory

One might wonder if the rich structure of the model we built is just due to the requirement of $SU(5)$ gauge invariance at high energies. If one is willing to assume that the $SU(5)$ global symmetry accidentally emerges at the 10 TeV scale, a model with ungauged $SU(5)$ can be considered. Could this dramatically simplify the particle content needed to reproduce the LHT? A detailed look at the previous section reveals that only very few states could actually be omitted in such a non-$SU(5)$ invariant model:
• We could use incomplete \( SU(5) \) representations in (3.1) and omit the states \( \chi_{1,2} \).

• We would not need the scalars \( F_{1,2} \) to give mass to the \( U_{L1,2} \) states.

• We would not need the scalars \( A_{1,2} \), whose role is to make the coupling (3.5) \( SU(5) \) invariant.

• Fewer massive scalars would be necessary to obtain the top Yukawa (3.5) from a renormalizable theory.

In total one would end up with a slightly smaller particle content, but overall the model would not simplify significantly.

4 Anomaly Cancellation

While the model presented above suffers from gauge anomalies, in this section we will present a simple extension of the model which is anomaly free. Furthermore, we will show that T-parity is an anomaly free symmetry of the quantum theory.

4.1 Gauge anomalies

First, we examine the gauge anomalies of the model. The chiral fermion content of a single generation is summarized in Table 2, where \( Y = 1/6 \) for quarks and \( Y = -1/2 \) for leptons. Note that the \( SU(5) \) group is vectorlike, while \( SU(2) \) representations are real, so all anomalies involving only these two groups vanish. However, anomalies involving \( U(1)_3 \) are not canceled with this fermion content. The simplest way to achieve anomaly cancelation is to extend the model in such a way that it contains a sector which is vectorlike under the full \( SU(5) \times SU(2)_3 \times U(1)_3 \) gauge group, plus a sector which is chiral under \( SU(2)_3 \times U(1)_3 \), but with charges identical to one generation of the SM fermions. This guarantees anomaly cancelation as in the SM. Since at low energies the matter content of our model coincides with the SM, this is in fact possible. In order to achieve this, we need to introduce mirror partners for all fields that don’t already have SM quantum numbers. In particular for the quark sector we introduce the mirror partners \( Q'_1, Q'_2, q'_3, q'_5, U'_{R1}, U'_{R2} \) and two fields \( q'_3, q''_3 \). The two \( q_3 \) partners are necessary in order to exactly reproduce the chiral SM matter content under \( SU(2)_2 \times U(1)_3 \), guaranteeing complete anomaly cancelation. The total anomaly-free fermion content in the quark sector is summarized in Table 3 in the columns (a) and (b).

The additional states acquire TeV-scale masses through a Lagrangian of the form

\[
\mathcal{L} \propto Q'_1 K_2 q'_3 + Q'_2 K_1 q''_3 + Q_1^\dagger K_1^* q'_4 + Q_2^\dagger K_2^* q'_5 + Q'_1 F'_1 U'_{R1} + Q'_2 F'_2 U'_{R2} . \tag{4.1}
\]

Note that this is almost the same as eq. (3.3), except that the presence of the two different fields \( q'_3 \) and \( q''_3 \) guarantees that there is no light mode.

For the lepton sector with \( Y = -1/2 \) in Table 2 we automatically have a charge assignment that produces the SM chiral matter content under \( SU(2)_3 \times U(1)_3 \), so no additional
Table 3: The complete fermion sector (single generation) and the gauge charge assignments for the anomaly-free version of the model.

| a) | SU(5) | SU(2)_3 | U(1)_3 | b) | SU(5) | SU(2)_3 | U(1)_3 | c) | SU(5) | SU(2)_3 | U(1)_3 |
|----|-------|---------|--------|----|-------|---------|--------|----|-------|---------|--------|
| \(Q_1\) | \(
\begin{array}{c}
\text{SU(5)} \\
1
\end{array}
\) | +2/3 | \(\begin{array}{c}
\text{SU(5)} \\
Q'_1
\end{array}\) | \(-2/3\) |
| \(Q_2\) | \(
\begin{array}{c}
\text{SU(5)} \\
1
\end{array}
\) | +2/3 | \(\begin{array}{c}
\text{SU(5)} \\
Q'_2
\end{array}\) | \(-2/3\) |
| \(q_3\) | 1 | -1/6 | \(q'_3, q''_3\) | +1/6 | \(\ell_3\) | 1 | 0 | +1/2 |
| \(q_4\) | 1 | -7/6 | \(q'_4\) | +7/6 | \(\ell_4\) | 1 | 0 | -1/2 |
| \(q_5\) | 1 | -7/6 | \(q'_5\) | +7/6 | \(\ell_5\) | 1 | 0 | -1/2 |
| \(U_{R1}\) | 1 | -2/3 | \(U'_{R1}\) | +2/3 | \(E_{R1}\) | 1 | 0 |
| \(U_{R2}\) | 1 | -2/3 | \(U'_{R2}\) | +2/3 | \(E_{R2}\) | 1 | 0 |
| \(u_R\) | 1 | -2/3 | \(\nu_R\) | +1 |
| \(d_R\) | 1 | 1 | +1/3 |

Table 4: The chiral matter content for one generation of the anomaly-free version of the model.

| | SU(5) | SU(3)_c | SU(2)_3 | U(1)_3 |
|----|-------|---------|---------|--------|
| \(q''_3\) | 1 | | | +1/6 |
| \(u_R\) | 1 | 1 | -2/3 |
| \(d_R\) | 1 | 1 | +1/3 |
| \(\ell_5\) | 1 | 1 | 0 | -1/2 |
| \(e_R\) | 1 | 1 | +1 |
| \(\nu_R\) | 1 | 1 | 0 | +1 |
mirror fields are needed. The matter content in the lepton sector is summarized in Table 3 (c).

The chiral matter content of one generation of the model is summarized in Table 4. Here $SU(3)_c$ denotes the color gauge group. As anticipated above, the quantum numbers of these fermions under $SU(3)_c \times SU(2)_3 \times U(1)_3$ are exactly the same quantum numbers as for the usual SM fermions under $SU(3)_c \times SU(2)_L \times U(1)_Y$. Hence all gauge and gravitational anomalies cancel.

The above construction should be viewed as a proof of principle, showing that it is possible to add a set of spectator fermions to our model to cancel all gauge and gravitational anomalies, and to give them large masses in a way consistent with the symmetries. The particular set of spectators chosen here is rather large, but has the advantage that the anomalies cancel in exactly the same way as in the SM. Its disadvantage is that the QCD $\beta$-function will become very large and the theory would rapidly develop a Landau pole. The exact location of the pole depends on the values chosen for the Yukawa couplings and vevs in eqs. (4.1) and (3.3). In the supersymmetric version of this model, which we will describe in section 5.1, this implies that once the Landau pole is hit an appropriate Seiberg duality [24] has to be performed and the theory will be a cascading gauge theory as in [25]. It would be interesting to see if a more minimal anomaly-free matter content can be found.

4.2 T-parity anomalies

Whenever physical Goldstone bosons appear in a theory, one has to check whether the global symmetries whose spontaneous breaking produces the Goldstones are anomalous. The presence of such anomalies would produce new couplings for the Goldstones, of the general form

$$\frac{1}{f} \pi^a \partial_\mu J^{a\mu}.$$  (4.2)

If the global current $J^{\mu a}$ is anomalous with respect to a gauge symmetry, then

$$\partial_\mu J^{a\mu} = \frac{Ag^2}{16\pi^2} \text{Tr} FF,$$  (4.3)

where $F$ is the gauge field, and the anomaly coefficient $A$ can be calculated from the triangle diagrams involving fermion loops. In the low energy effective theory after the fermions are integrated out, a term involving the light gauge fields and the Goldstones has to be present, whose variation reproduces the anomalies of the global current. This is the Wess-Zumino-Witten (WZW) term [26], whose coefficient can be found by matching to the triangle diagrams in the high energy theory. This WZW term may break discrete symmetries of the Goldstone sector. The canonical example is the $\pi^a \rightarrow -\pi^a$ symmetry of the pseudoscalar octet of QCD. The effect of the $SU(2)_3 \times U(1)_{em}$ anomaly in the quark picture will imply the presence of the $\pi_0 F\tilde{F}$ coupling in the effective low-energy theory, which breaks the $\pi \rightarrow -\pi$ reflection symmetry. Using similar arguments Hill and Hill [11] argued that T-parity will also be broken in a similar way in little Higgs models. They have discussed several examples based both on more complicated versions of the $SU(3) \times SU(3) \rightarrow SU(3)_D$ breaking pattern,
as well as the $SU(5) \to SO(5)$ and other little Higgs-type models, and have calculated the form of the Wess-Zumino-Witten terms in a variety of examples. However, whether these T-parity breaking terms are ultimately present in the low-energy effective theory or not depends on the UV completion of the theory. If the global symmetries (and T-parity itself) are not anomalous, then the coefficient of the Wess-Zumino term vanishes, and T-parity remains a good symmetry at the quantum level. Therefore, in a complete model with T-parity one has to show that T-parity is not broken by any of the global anomalies present in the theory. While in an effective low-energy theory one may only speculate whether such anomalies are present or not, our UV completion allows us to address this issue straightforwardly. Since the $SU(5)$ global symmetry responsible for producing the Goldstones is also gauged, it has to be anomaly free. Indeed we have shown above that it is possible to choose the matter content such that all anomalies involving $SU(5)$ will disappear. Therefore there can be no Wess-Zumino-Witten term from $SU(5)$ anomalies present in this theory that would give rise to T-parity violation.

A final worry might be that the T-parity itself as a discrete symmetry might be anomalous. However, as we have seen before, T-parity is a combination of an $SU(5) \times SU(2)_3$ gauge transformation element with a discrete exchange symmetry. We have seen that the gauge transformations are anomaly free, but what about the exchange symmetry (which is a symmetry similar to charge conjugation)? Could that possibly be anomalous? The answer is clearly negative. The exchange symmetry in the path integral language merely corresponds to a relabeling of the integration variables. The integration measure is invariant under this relabeling. So, if the Lagrangian is invariant under the exchange symmetry, then the whole path integral is invariant. Therefore we do not expect T-parity violating anomalous terms to show up anywhere in the model.

5 Solutions to the Large Hierarchy Problem

We constructed a weakly coupled, four-dimensional UV completion of the LHT model, with T-parity exact at the quantum level. However, the model assumes a large hierarchy between the scale of scalar vevs (1 or 10 TeV), and the Planck scale. This hierarchy needs to be stabilized. In this section, we will explore two possible ways this can be achieved: by embedding the model into a supersymmetric theory above 10 TeV, and by promoting it to a warped-space five-dimensional model with the Planck scale at the infrared (IR) boundary of order 10 TeV.

5.1 A supersymmetric version

It is straightforward to supersymmetrize our model by promoting all fields to superfields, and assuming that the components that do not appear in our model receive soft masses at the 10 TeV scale. In addition, one needs to introduce a superfield $\bar{S}$, which has the same quantum numbers as $S^\dagger$. This fields gets interchanged with $S$ under T-parity in the familiar way $S \leftrightarrow \Omega \bar{S} \Omega^T$. It ensures that it is possible to write down a superpotential that allows for the
vev in eq. (2.3) and generates the Yukawa couplings (3.13). We assume the superpotential of the form

\[ W = W_\Phi(\Phi_1, \Phi_2) + W_{\text{Yuk}}(S, \bar{S}, K_1, K_2, \ldots), \]

where \( W_\Phi \) generates \( SU(5) \) breaking vevs as in eq. (2.7) without breaking SUSY, and \( W_{\text{Yuk}} \) includes the Yukawa couplings of our model. This superpotential allows for the adjoint vevs in Eq. (2.7), with \( \langle \sigma \rangle = 0 \). At the same time, since the Yukawa couplings do not contain the \( \Phi \) fields, it does not lead to direct couplings between \( \Phi \) and the other fields in the F-term scalar potential. As a result, the global \( SU(5) \) symmetry below the scale \( f_\Phi \sim 10 \text{ TeV} \) is preserved at this level. Note that this structure of the F-term potential is technically natural, due to the standard non-renormalization theorems of SUSY.

The scalar potential also receives a D-term contribution. Since both \( \Phi \) and the other scalar fields, including \( S \) and \( \bar{S} \), are charged under \( SU(5) \), the D-term potential will in general couple them, violating the global \( SU(5) \). This can give a large contribution to the Higgs mass, potentially of order \( g_5 f_\Phi \). However, it can be shown that this effect is suppressed in the limit when the soft masses for the adjoint fields are small compared to \( f_\Phi \), and the Higgs mass can remain at the weak scale without fine-tuning.

The argument is based on the following observation [27, 28]: In the limit of unbroken SUSY, the effective theory below the scale \( f_\Phi \) is a supersymmetric theory with reduced gauge symmetry. This SUSY theory does not contain any D-terms for \( S \) or \( \bar{S} \) corresponding to the broken generators, and does not contain any \( \Phi \) fields as they are either eaten or get masses at the scale \( f_\Phi \). So, in this limit we are only left with D-terms for \( S \) and \( \bar{S} \) corresponding to the unbroken subgroup. These terms do not generate a tree-level \( S \) or \( \bar{S} \) mass, and moreover they break the \( SU(5) \) in exactly the same pattern as the unbroken gauge symmetries themselves. In particular, the Higgs (contained in \( S \) and \( \bar{S} \)) would still remain a Goldstone if only one of the two \( SU(2) \) subgroups was gauged. Thus, in the unbroken-SUSY limit, the D-terms do not spoil the symmetries responsible for keeping the Higgs light.

Let us see explicitly how this works. Since for the protection of the higgs mass only the interactions between \( S, \bar{S} \) and \( \Phi_{1,2} \) are relevant, we will only focus on these fields on the following discussion. Above \( f_\Phi \), the D-term potential has the form

\[ V_D = \frac{g_5^2}{2} \sum_a (D^a_\Phi + D^a_S + \ldots)^2, \]

with

\[ D^a_\Phi = \sum_i \text{Tr} \Phi_i^\dagger [T^a, \Phi_i], \quad D^a_S = 2 \text{Tr} S'^\dagger T^a S - 2 \text{Tr} S'^\dagger T^a T^b S. \]

After the \( \Phi \)'s get vevs, this potential includes \( SU(5) \) symmetry breaking terms for \( S \) and \( \bar{S} \). However, to obtain the correct low-energy potential, we have to carefully integrate out the heavy “radial” modes of the \( \Phi \) fields. The important radial modes are \( R^a \) along the generators \( T^a \) broken by \( \langle \Phi_{1,2} \rangle \). These modes are the real parts of the superfield containing the Goldstones, and as such they must be F-flat directions. But since the Goldstones are non-linearly realized Goldstones are completely F-flat. If realized linearly, however, one will encounter quartic and higher interactions in the F-term potential.

\[ ^3 \text{Non-linearly realized Goldstones are completely F-flat. If realized linearly, however, one will encounter quartic and higher interactions in the F-term potential.} \]
eaten by the broken gauge bosons, the $R^{\hat{a}}$ fields will get masses from the D-terms, which
must be precisely equal to the gauge boson masses in order to preserve SUSY. Furthermore,
they are the only radial modes that receive a mass from the D-terms. The scalar potential
has the form

$$V_{\text{SUSY}} = F^* F + \frac{g^2}{2} D^a D^a = \frac{1}{2} \sum_{\hat{a}} (M_{\hat{a}} R^{\hat{a}} + \ldots + g_5 D_S^{\hat{a}})^2 + \ldots ,$$  \hspace{1cm} (5.3)$$

where $\hat{a}$ labels the broken generators, $M_{\hat{a}}$ are the gauge boson masses and the dots denote
terms that do not contain either $D_S^{\hat{a}}$ or $R^{\hat{a}}$. The equations of motion yield

$$R^{\hat{a}} = - \frac{g_5 D_S^{\hat{a}}}{M_{\hat{a}}},$$  \hspace{1cm} (5.4)$$

which exactly cancels the unwanted D-terms for $S$ and $\bar{S}$ corresponding to the broken
generators.

In a realistic model, SUSY must be broken. Consider a situation when the SUSY-breaking
soft masses for the $\Phi$ fields are lower than the $SU(5)$ breaking scale $f_\Phi$. Assume that the
soft breaking are of the form

$$V_{\text{SUSY}} \sim \frac{1}{2} \sum_{\hat{a}} m_{\hat{a}}^2 R^{\hat{a}}^2 + \ldots ,$$  \hspace{1cm} (5.5)$$

with $m_{\hat{a}} \ll f_\Phi$, and dots denote terms not containing $R^{\hat{a}}$. The important feature of these
soft terms is that they do not contain a linear term in $R^{\hat{a}}$, and thus only affect the SUSY
 cancellation of the D-terms at subleading order in $m_{\hat{a}}/M_{\hat{a}}$. The equations of motion for $R^{\hat{a}}$ now yield

$$R^{\hat{a}} = - \frac{g_5 D_S^{\hat{a}}}{M_{\hat{a}}} M_{\hat{a}}^2 + \ldots \approx - \frac{g_5 D_S^{\hat{a}}}{M_{\hat{a}}} \left(1 + \frac{m_{\hat{a}}^2}{M_{\hat{a}}^2} + \ldots \right).$$  \hspace{1cm} (5.6)$$

The resulting low-energy potential has the form

$$V_{\text{eff}} \sim \sum_{\hat{a}} \frac{m_{\hat{a}}^2}{M_{\hat{a}}^2} (g_5 D_S^{\hat{a}})^2 + \ldots$$  \hspace{1cm} (5.7)$$

where the dots denote terms of higher order in $m_{\hat{a}}/f_\Phi$. This potential gives a mass to the
Goldstones in $S$ and $\bar{S}$ (including the SM Higgs) of the order

$$m_{h}^2 \sim \frac{m_{\hat{a}}^2}{M_{\hat{a}} f_\Phi^2}.$$  \hspace{1cm} (5.8)$$

This is phenomenologically acceptable as long as $m_{\hat{a}}/M_{\hat{a}} \lesssim 0.1$. One possibility is that
$f_\Phi \sim M_{\hat{a}} \sim 10$ TeV as previously assumed, but the soft masses for $\Phi$ are an order of
magnitude smaller than the other soft masses in the theory, $m_{\hat{a}} \sim 1$ TeV. This small mass
hierarchy would be radiatively stable. Another possibility is that $m_{\hat{a}} \sim 10$ TeV along with
the other soft masses, but $f_\Phi \sim 100$ TeV. In this case, all quadratic divergences are still
cut off at 10 TeV due to SUSY, but SU(5)-violating logarithmic corrections are enhanced by running between 10 and 100 TeV scales. This leads to an additional contribution to the Higgs mass of order \( \sim \frac{g^2}{16\pi^2} f_\Phi^2 \log \frac{100 \text{ TeV}}{10 \text{ TeV}} \), which is of the same order as the top contribution.

The above discussion is completely general and does not depend on any particular representation of the SU(5) breaking fields and their vevs, the specific form of the superpotential \( W_\Phi \), or the soft breaking potential \( V_{\text{SUSY}} \). As an example consistent with our model, we can use a T-parity invariant superpotential of the form

\[
W = \kappa \sigma (\text{Tr} \Phi_1 \Phi_1 + \text{Tr} \Phi_2 \Phi_2 - 60 f_\Phi^2) + W_{\text{Yuk}}(S, \bar{S}, K_1, K_2, \ldots),
\]

(5.9)

with \( \sigma \) a gauge-singlet chiral superfield, and the soft breaking terms

\[
V_{\text{SUSY}} = M_\Phi^2 \left( \text{Tr} \Phi_1 \Phi_1 + \text{Tr} \Phi_2 \Phi_2 \right) + M_\sigma^2 |\sigma|^2.
\]

(5.10)

This potential has an extended SU(5)\(^2\) global symmetry, and thus not all Goldstone bosons are eaten by the heavy gauge field. However, the uneaten Goldstones will receive a contribution to their mass of order \( f_\Phi^4 / 4\pi \) at one loop, which is of order \( 1 - 10 \text{ TeV} \).

### 5.2 A five-dimensional version

A popular alternative to supersymmetry for solving the weak/Planck hierarchy problem is the warped-space five-dimensional (5D) setup pioneered by Randall and Sundrum [20]. It is straightforward to embed our model into such a setup.

The five-dimensional version of the model is illustrated in Fig. 2. We assume that the extra dimension has a warped AdS\(_5\) gravitational background given by the metric

\[
ds^2 = \left( \frac{R}{z} \right)^2 \left( \eta_{\mu\nu} dx^\mu dx^\nu - dz^2 \right),
\]

(5.11)

The extra dimension is an interval bounded at \( z = R \) by the “ultraviolet” (UV) boundary (or brane), and at \( z = R' \) by the “infrared” (IR) brane. The AdS curvature \( R \) is assumed to be \( 1/R \sim O(M_{\text{Pl}}) \), while \( 1/R' \) is of order a few TeV.

The 5D theory should reproduce at \( \sim 1 \text{ TeV} \) the T-odd particle spectrum necessary for the little Higgs mechanism. The cutoff scale of the 4D little Higgs theory is usually at around 10 TeV. In the 5D theory this will be identified with the scale \( m_{KK} \) where the additional KK resonances appear, thus UV completing the theory above 10 TeV. The cutoff scale of the 5D theory can be estimated via NDA to be of the order \( \Lambda_{5D} \sim 24\pi^3 / (g^2 R' \log R'/R) \), while the scale \( f \) is given by \( f = 2/(g R' \sqrt{\log R'/R}) \). In our case we want \( f \sim 1 \text{ TeV} \), then the cutoff scale is of order 100 TeV, while the KK mass scale is \( m_{KK} \sim 2/R' \sim 10 \text{ TeV} \).

The best handle for finding the right setup is to use the dictionary of the AdS/CFT correspondence. From that point of view we would be looking for the dual of a CFT with an SU(5) global symmetry, where the SU(2)\(^2\) \( \times U(1)^2 \) subgroup is gauged. As we discussed in

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\(^4\)A 5D version of the original Littlest Higgs model was given in [15].
In this paper, this symmetry needs to be extended to $SU(5) \times SU(2)_3 \times U(1)_3$, with $[SU(2) \times U(1)]^3$ gauged, in order to incorporate T-parity in the (chiral) fermion sector. So, the 5D setup we start with is an $SU(5) \times SU(2)_3 \times U(1)_3$ bulk gauge group. The action of T-parity on the gauge bosons is again given by eq. (2.10). We assume that the gauge symmetry is broken by boundary conditions (BC’s) for the gauge fields, as in [29]: on the UV brane,

$$SU(5) \times SU(2) \times U(1) \rightarrow [SU(2) \times U(1)]^3 \quad (UV),$$

while on the IR brane

$$SU(5) \times SU(2) \times U(1) \rightarrow SO(5) \times SU(2) \times U(1) \quad (IR).$$

In the language of the 4D model, this is equivalent to placing the $\Phi_{1,2}$ fields on the UV brane and the $S$ field on the IR brane, and integrating out the radial models of these fields after they get vevs. (Note that this geometric separation of $\Phi$ and $S$ automatically guarantees the absence of the direct potential couplings between them, as needed in our model.) These BC’s result in an unbroken $[SU(2) \times U(1)]^3$ gauge group at low energies and leave T-parity unbroken. The gauge fields in $[SU(2) \times U(1)]^3$ which are only broken by BC’s on the IR brane will get a mass of order $f \sim 1$ TeV. These fields correspond to the T-odd gauge bosons of the LHT model. As discussed above, the full Kaluza-Klein (KK) tower starts at the somewhat higher scale $m_{KK} \sim 10$ TeV.

To reduce the group further (down to just the SM) we will assume that the scalars $K_1, K_2$ live on the IR brane, getting vevs of order $m_{KK} \sim 10$ TeV. Furthermore, to incorporate fermion masses in an $SU(5)$ invariant way, we also add the scalars $A_1, A_2$ on the IR brane, with

Figure 2: Geometric setup, gauge symmetries and matter content of the five-dimensional model.
vevs of order $m_{KK}$. (We will not need to introduce the scalars $F_{1,2}$ to give masses to $U_{L1,2}$.) Note that $m_{KK} \sim 10$ TeV is the natural scale for the vevs on the IR brane. It is an order of magnitude larger than the vevs for these fields in the 4D version of the model. However, these larger vevs do not lead to larger masses for the corresponding massless gauge bosons: in fact, their contribution to the masses is at most of order $gf \sim 1$ TeV. This can be seen by observing that the limit of very large vevs is equivalent to breaking gauge symmetries by BC’s on the IR brane, which produce masses of order $gf$.

The $A_5$ components of the gauge fields corresponding to the broken $SU(5)/SO(5)$ generators develop zero modes. These modes, which are scalars from the 4D point of view, include the weak doublet identified with the SM Higgs. The Higgs mass is protected by the collective symmetry breaking mechanism. To see this, consider a variation of the symmetry breaking pattern in eqs. (5.12), (5.13), with $SU(5)$ broken down to a single $SU(2) \times U(1)$ subgroup on the UV brane. This theory possesses an $SU(3)$ global symmetry, broken down to $SU(2)$ by the BC’s on the IR brane. The $A_5$ components identified with the Higgs are the Goldstone bosons of this global symmetry breaking, and as such are exactly massless. Thus, the Higgs can only get a mass if both $SU(2) \times U(1)$ factors in $SU(5)$ are unbroken at the UV brane. That is, zero modes for at least two different gauge fields must enter into any diagram contributing to the Higgs mass. Just as in the 4D LHT, this implies cancelation of the quadratic divergence in the Higgs mass between the SM gauge bosons and their T-odd counterparts at scale $f$. The remaining logarithmic divergence is canceled by the KK states at the scale of order $1/R' \sim 10$ TeV, and a finite Higgs mass is generated, as guaranteed by non-locality and 5D gauge invariance. Note that there may be additional light states among the $A_5$ modes due to the large vevs of $K_{1,2}, A_{1,2}$ on the IR brane. However, those would not be protected by the collective breaking mechanism, but only by the 5D non-locality, so their masses would be of the order of $m_{KK}/4\pi \sim 1$ TeV, rather than the 100 GeV range for the doubly protected physical Higgs.

It is useful to compare this structure to that of the “minimal” holographic composite Higgs model of Agashe, Contino and Pomarol [19]. In that model, all divergences in the Higgs mass are canceled at the same scale, the KK scale $1/R'$ Precision electroweak (PEW) constraints push this scale up to at least 3 TeV, and some amount of fine-tuning is needed to obtain consistent EWSB. In contrast, in our theory, the quadratic divergence is canceled at the 1 TeV scale by the Little Higgs mechanism, without any tension with PEW constraints thanks to T parity. This allows us to push the KK scale to 10 TeV without fine-tuning. At this scale, the KK states themselves are completely safe from PEW constraints. Thus, the tension between fine-tuning and PEW constraints is eliminated. Of course, the price to pay is a larger symmetry group and matter content.

In principle, the fermion content of the five-dimensional model could be simplified compared to the 4D $SU(5)$-invariant model, if one were to take advantage of the symmetry breaking BC’s and simply project out some of the unwanted zero modes for the fermions (such as, for example, $U_i$ and $\chi_i$ components of the $\Psi_i$ fields) instead of introducing new states for them to marry. However, one needs to be careful with this, if T-parity is to be maintained as an exact symmetry. 5D theories are automatically anomaly free in the sense that every bulk fermion is actually a 4D Dirac fermion, and so the theory is always vectorlike.
However, once orbifold projections are introduced, localized anomalies can be generated on the boundaries, which would be locally canceled by an anomaly flow corresponding to the bulk Chern-Simons (CS) term [30]. These bulk CS terms would contain the $A_5$ field and thus could violate T-parity similarly to the WZW operators in the 4D case. In order to avoid such terms, we need to make sure that there are no localized anomalies in our theory. The most obvious way of achieving this is by putting a separate bulk fermion field for every field in Table 3, with a zero mode forming a complete $SU(5)$ representation. This would imply that we pick a $(+, +)$ boundary condition for all the left handed components, and a $(-, -)$ BC for all the right handed components. This choice ensures that all localized anomalies cancel in the same way as in the 4D theory (see section 4), and there would be no bulk CS term appearing. The terms corresponding to the Lagrangian in eqs. (3.3) and (4.1) can then be mimicked by brane localized Yukawa terms involving the $K_1, K_2$ fields on the IR brane, and via UV brane localized mass terms of the form $U_{L1}U_{R1} + U_{L2}U_{R2}$ (remember that on the UV brane $SU(5)$ is broken and so these mass terms are not violating gauge invariance, so we do not need to introduce $F_{1,2}$). If we were to try to simplify the spectrum by using $(-, +)$ type boundary conditions for some of the fermions (and introducing fewer bulk fields), we would end up with a consistent theory, but with a bulk CS-term breaking T-parity.

In order to obtain Yukawa couplings, we need to make sure that the zero modes for the right-handed quarks also partly live in the right-handed component of $U_{L1,2}$. This can be achieved via the IR brane localized scalars corresponding to $\eta, \eta', \xi, \xi'$ in eq. (3.11). A Lagrangian corresponding to eq. (3.13) can be also added to the IR brane, except for adding mass terms along the pattern of the $\langle S \rangle$ instead of the complete $S$ field (which is allowed due to the symmetry breaking BC’s). The effect of those boundary terms will be to partially rotate the $u_R$ zero mode into $Q_1$, and thus generate our effective Yukawa coupling. Note, that since all global $SU(3)_{1,2}$ violating effects are non-local (as they need to involve both branes), the radiatively generated Higgs potential will be completely finite. We leave the detailed study of the EWSB and the phenomenology of the holographic T-parity models to future investigations.

6 Constraints from the Weinberg Angle, Precision Electroweak Fits, and Dark Matter

The model constructed in sections 2 and 3 correctly reproduces the particle content of the SM at low energies. At the TeV scale, the model reproduces the particle content and couplings of the LHT. This sector eliminates the little hierarchy problem, and is consistent with precision electroweak fits as long as $f_S \geq 500$ GeV, and the T-odd partners of the SM fermion doublets are not too far above the TeV scale [5]. In addition, our model contains a number of states at the TeV scale that were not present in the LHT. These states can produce additional contributions to precision electroweak observables. While a detailed analysis of the resulting constraints is outside the scope of this paper, we would like to briefly discuss the most salient constraint and show that it can be satisfied.
Most TeV-scale non-LHT states in our model are vectorlike fermions, and their contributions to PEW observables are small. The dominant new contribution is from the massive T-even gauge bosons. As discussed in section 2, these states can be significantly heavier than the T-odd gauge bosons, if the gauge couplings of the $SU(2)_3 \times U(1)_3$ gauge groups are stronger than that of the $SU(5)$ group. Since the SM Higgs does not couple to the $SU(2)_3 \times U(1)_3$ gauge bosons, the little hierarchy problem is still solved in this limit, provided that the T-odd gauge bosons remain sufficiently light. However, as mentioned at the end of section 2, the potential problem with this limit is the Weinberg angle prediction: the SM coupling are related to the $SU(5) \times SU(2)_3 \times U(1)_3$ gauge couplings via

$$\frac{1}{g^2} = \frac{2}{g_5^2} + \frac{1}{g_3^2} \quad \text{and} \quad \frac{1}{g'^2} = \frac{6}{5g_5^2} + \frac{1}{g_3^2}, \quad (6.1)$$

so that $\sin^2 \theta = 5/8$ in the limit $g_5, g_3 \gg g_5$. Is it possible to satisfy precision electroweak constraints and at the same time reproduce the experimental value of the Weinberg angle, $\sin^2 \theta_{\text{exp}} \approx 0.2315$?

The spectrum of the TeV-scale gauge bosons has been discussed in section 2, see eqs. (2.12) and (2.13). However, these equations did not take into account the effect of the additional breaking of the $U(1)$ gauge bosons by the vevs of $A_{1,2}$ and $F_{1,2}$. Including these vevs, the $U(1)$ gauge boson masses are

$$m_{B_{\text{even}}}^2 = \frac{g_5^2 + 2g_3^2}{4} (f_K^2 + 16f_A^2) \quad \text{and} \quad m_{B_{\text{odd}}}^2 = \frac{g_5^2}{100} (10f_S^2 + f_K^2 + 16f_A^2 + 32f_F^2), \quad (6.2)$$

(6.4)

where $g_5' = \sqrt{\frac{5}{3}} g_5$, while the $SU(2)$ gauge boson masses are still given by eq. (2.12).

It is convenient to rewrite the gauge boson spectrum and the Weinberg angle in terms of dimensionless ratios:

$$\sin^2 \theta = \left[ 1 + \frac{1}{5} \cdot \frac{6 + 5/r'}{2 + 1/r} \right]^{-1}$$

$$\frac{m_{W_{\text{even}}}^2}{m_{W_{\text{odd}}}^2} = \frac{1 + 2r}{1 + 2r_S}$$

$$\frac{m_{B_{\text{odd}}}^2}{m_{W_{\text{odd}}}^2} = \frac{1 + 10r_S + 16r_A + 32r_f}{60(1 + 2r_S)}$$

$$\frac{m_{B_{\text{even}}}^2}{m_{W_{\text{odd}}}^2} = \left[ \frac{5}{3} + 2r' \right] \frac{1 + 16r_A}{1 + 2r_S}, \quad (6.3)$$

(6.4)

where the ratios are defined as

$$r = \frac{g_3^2}{g_5^2}, \quad r' = \frac{g_3^2}{g_5^2}, \quad r_S = \frac{f_S^2}{f_K^2}, \quad r_A = \frac{f_A^2}{f_K^2}, \quad r_F = \frac{f_F^2}{f_K^2}. \quad (6.4)$$

Tree-level shifts in precision electroweak observables can be computed in terms of the T-even gauge boson masses and the coupling constant ratios, $r$ and $r'$. For example, taking
the Z mass, the Fermi constant $G_F$ and the fine structure constant $\alpha$ as inputs, the shift in the $W$ boson mass with respect to the reference value is given by

$$\Delta m_W \equiv m_W - c_w^\text{ref} m_Z = \frac{m_W}{4} \frac{\pi \alpha}{c_w - s_w} \left( \frac{1}{r} \frac{v^2}{m_{W_{\text{even}}}^2} + \frac{5}{3} \frac{v^2}{r' m_{B_{\text{even}}}^2} \right),$$

(6.5)

where $c_w^\text{ref}$ is the reference value of the cosine of the Weinberg angle, and $v \approx 246$ GeV is the Higgs vev. The structure of corrections to all observables is the same as in eq. (6.5): the contributions of the heavy $SU(2)$ states are proportional to $r^{-1} m_{W_{\text{even}}}^{-2}$, while those due to the heavy $U(1)$ states are proportional to $r'^{-1} m_{B_{\text{even}}}^{-2}$. This is because both the light-heavy gauge boson mixing, and the couplings of the heavy gauge bosons to light fermions, are inversely proportional to $\sqrt{r}$ or $\sqrt{r'}$.

This structure can be exploited to find the region of parameter space where the corrections are suppressed without fine-tuning. To avoid large corrections to the Higgs mass from the $SU(2)$ sector, the $W_{\text{odd}}$ gauge bosons should be light, preferably around 1 TeV or below. At the same time, the $W_{\text{even}}$ can be much heavier, if the parameter $r$ is large. In this regime, the contribution to precision electroweak observables from the $SU(2)$ sector is suppressed both by the $W_{\text{even}}$ mass and by its small mixing and couplings to the SM fermions, as noted above. The PEW constraint on the mass of an extra $SU(2)$ boson with SM-strength couplings (such as the KK gauge bosons in models with extra dimensions) is typically around 3 TeV. Using this value and assuming $m_{W_{\text{odd}}} = 1$ TeV and $f_S = f_K$, we estimate that the $SU(2)$ contributions in our model are sufficiently suppressed if $r > 2$. The $r$ parameter is limited from above by the requirement that the $SU(2)$ not be strongly coupled:

$$\frac{g_3^2}{4\pi} \lesssim 0.3 \quad \Leftrightarrow \quad r \lesssim 5.$$  

(6.6)

There is a wide range of values where the model is perturbative and consistent with data.

Once $r$ is fixed, the requirement of getting the correct Weinberg angle fixes $r'$; the range $2 < r < 5$ corresponds to $0.14 \lesssim r' \lesssim 0.16$, so that the $U(1)$ mixing angle is essentially fixed. Thus, the $B_{\text{even}}$ boson cannot be decoupled by assuming large $g'_3$. Moreover, the couplings of the heavy $U(1)$ gauge boson to the SM fermions are actually enhanced compared to the SM hypercharge coupling. However, its mass is essentially a free parameter, and it can be heavy provided that $f_A \gg f_S, f_K$. For example, assuming again $m_{W_{\text{odd}}} = 1$ TeV and $f_S = f_K$, the value of $f_A = 3f_S$ gives $m_{B_{\text{even}}} \approx 10$ TeV, which should be completely safe for precision electroweak fits even with the enhanced coupling. At the same time, for the same parameters and $f_F = f_S$, the T-odd $U(1)$ boson $B_{\text{odd}}$ has a mass just above 1 TeV, so that the Higgs mass divergence is still canceled at 1 TeV and there is no fine-tuning. Thus, we estimate that in the region

$$2 \lesssim r \lesssim 5, \quad r' \approx 0.15, \quad r_A \gtrsim 10,$$

(6.7)

and all other dimensionless ratios of order one, our model should be consistent with precision electroweak data without fine-tuning in the Higgs mass.

An interesting phenomenological feature of the spectrum needed to satisfy the constraints is that the $B_{\text{odd}}$ boson is not necessarily the lightest T-odd particle (LTP), in contrast to the
situation typical in the original LHT model. Cosmological considerations require that the LTP not be strongly interacting or electrically charged. In our model, the T-odd partner of the SM neutrino can also play the role of the LTP. The T-odd neutrino LTP has not been considered in the previous studies of Little Higgs dark matter, which focused on the $B_{\text{odd}}$ as the dark matter candidate. Our model provides a motivation to analyze this alternative possibility.

In addition to the gauge bosons, several new scalar states appear at the TeV scale in our model. These include pseudo-Goldstone bosons which receive a mass at the one-loop order, as well as the radial excitations of the fields $S$ and $K_{1,2}$. Several of these states are triplets with respect to the SM weak SU(2). If allowed by T-parity and hypercharge conservation, gauge interactions will generate terms of the form $h^i \phi_i h$, where $\phi_i$ are the triplets, in the one-loop Coleman-Weinberg potential. Such terms do indeed arise for some of the triplets in our model. Those triplets are forced to acquire vevs, which can give large corrections to precision electroweak observables. For example, this effect played an important role in constraining the original littlest Higgs model without T-parity [31]. In our model, the triplet vevs are not directly related to the magnitude of the Higgs quartic coupling, as was the case in the LH without T-parity. We expect that it should be possible to find phenomenologically consistent regions of parameter space where the triplet vevs are small.

7 Little Higgs Mechanism in the Linear Sigma Model

A key feature of little Higgs models is the protection of the SM Higgs mass from quadratic divergence at the one-loop level through collective symmetry breaking. We argued in sections 2 and 3 that, since our model below the 10 TeV scale reproduces the nlσm LHT, the same cancelations will occur. While our model has extra states at the TeV scale, the symmetric scalar field $S$, which contains the SM Higgs, has no direct couplings to those states. (It is uncharged under the extra gauge group $SU(2)_3 \times U(1)_3$ and has no Yukawa couplings other than the top Yukawa already present in the LHT.) Thus, no new one-loop quadratic divergences arise. This argument ensures that in our model the little hierarchy problem is resolved in exactly the same manner as in the LHT. Nevertheless, it is interesting and instructive to see explicitly how the little Higgs cancelations occur in our weakly-coupled, UV-complete model. We will do so in this section.

First, let us consider the renormalization of $h_S$ mass by gauge boson loops. We will focus on the $SU(2)$ gauge bosons; the analysis for the $U(1)$ bosons is essentially identical. In our model, the Higgs coupling to the gauge bosons includes the terms

$$\mathcal{L} \supset \frac{1}{8} h^i_3 h_s (g_1^2 W^1_1 + g_2^2 W^2_2),$$

where $g_t$ denotes the gauge coupling to the $SU(2)_i$ subgroup of $SU(5)$ (which are the same in our model, but potentially different in the original Littlest Higgs). These terms arise from the covariant derivative in eq. (2.4) and are required by gauge invariance. These couplings produce a quadratic divergence in the Higgs mass via the “bow-tie” diagrams in Fig. 3 (a).
Figure 3: The Feynman diagrams contributing to the effective gauge couplings of the Higgs boson at low energies.

Recall that in the Littlest Higgs model, the structure of the four-point Higgs-gauge boson coupling is different [32]:

\[ \mathcal{L}_{\text{LHT}} \supset \frac{1}{4}g_1 g_2 W_1 W_2 (h^\dagger h), \]  

which does not lead to a quadratic divergence at one loop. Since our model must reduce to the LHT below the 10 TeV scale, there seems to be a contradiction.

This issue is resolved when the full set of diagrams contributing to the Higgs mass at one-loop in our linearized model is included. Specifically, the relevant diagrams are the ones involving two radial (heavy) modes of \( S \), coupling to the Higgs and the gauge bosons. These diagrams are shown in Fig. 3(b). Let us assume that a potential for \( S \) has the form

\[ V = -M^2 \text{Tr} SS^\dagger + \lambda_1 (\text{Tr} SS^\dagger)^2 + \lambda_2 \text{Tr} SS^\dagger SS^\dagger, \]  

where \( M^2 = 2(5\lambda_1 + \lambda_2) f_S^2 \). This potential produces the desired pattern of symmetry breaking at scale \( f_S \). It leads to the following pieces in the Lagrangian containing the heavy radial modes \( R_1 \) and \( R_2 \) (amongst others):

\[ \mathcal{L} \supset -\frac{1}{2}M_{R_1}^2 R_1^2 - \frac{1}{2}M_{R_2}^2 R_2^2 + \frac{1}{\sqrt{3} f_S} \left( \frac{1}{2} M_{R_1}^2 R_1 + 2 M_{R_1}^2 R_2 \right) h_S^\dagger h_S + \frac{f_S}{4\sqrt{3}} (R_1 - 2 R_2) \left( g_1^2 W_1^2 + g_2^2 W_2^2 - 2 g_1 g_2 W_1 W_2 \right), \]  

where the radial modes have masses \( M_{R_1}^2 = 32\lambda_2 f_S^2 \) and \( M_{R_2}^2 = 32(5\lambda_1 + \lambda_2) f_S^2 \). Note that the couplings of the radial modes to \( h_S^\dagger h_S \) are proportional to their masses. The effective Lagrangian below the scale \( f_S \) is obtained by integrating out the radial modes \( R_{1,2} \) in eq. (7.4). The resulting Lagrangian contains terms that exactly cancel the gauge-Higgs four-point couplings in eq. (7.1). The remaining coupling has the form

\[ \mathcal{L}_{\text{eff}} \supset \frac{1}{4}g_1 g_2 W_1 W_2 (h_S^\dagger h_S), \]  

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which exactly matches the non-linear Littlest Higgs Lagrangian and does not lead to quadratic divergences at one loop. Note that this result is independent of the couplings $\lambda_{1,2}$, as expected from the Coleman-Wess-Zumino theorem.

In a completely analogous way, one can show that the diagrams for canceling the top loop divergence are generated by integrating out $R_1, R_2$ properly. These diagrams are shown in Fig. 4. Especially, we also recover the sum rule from [33] for the Yukawa coupling of the top quark with itself $\lambda_t$ and with its heavy partner $\lambda_T$

$$\frac{M_T}{f_S} = \frac{\lambda_t^2 + \lambda_T^2}{\lambda_T},$$

which ensures that the one-loop quadratic divergence due to the top quark cancel.

8 Conclusions and Outlook

In this paper, we constructed a weakly coupled, renormalizable theory which reproduces the structure of the LHT model below the 10 TeV scale. This structure includes collective symmetry breaking mechanism to protect the Higgs mass from one-loop quadratic divergences, resolving the little hierarchy problem. The model is manifestly free of anomalies, and T-parity is an exact symmetry of the quantum theory. This leads to an exactly stable lightest T-odd particle, which can be either the T-odd hypercharge gauge boson or the partner of the neutrino. This particle can play the role of dark matter, and provide a missing energy signature at colliders. In addition, our model contains a few T-even extra states at the TeV scale, which can however be made sufficiently heavy to avoid conflict with precision electroweak data, without any fine tuning. Above the 10 TeV scale, our model can be embedded into either a supersymmetric theory or a five-dimensional setup with warped geometry, stabilizing the large hierarchy between 10 TeV and the Planck scale. A remaining concern regarding the fully anomaly free matter content is that due to the large numbers of states required for anomaly cancelation a Landau pole in the QCD $\beta$-function would develop. It would be very interesting to find a smaller anomaly canceling matter content that can avoid this issue.

In a weakly coupled UV completion of the LHT, a number of issues can be addressed which could not be analyzed in the original effective theory. One issue is gauge coupling...
unification, since in our model renormalization group evolution of all couplings is calculable all the way up to the Planck scale. The other one is flavor physics, in particular flavor-changing neutral currents (FCNCs). There are two sources of FCNCs in the LHT model. The first one is the effects generated by loops of heavy T-odd quarks and leptons, calculable within the effective theory. These effects have been considered in [34, 35]. The second class are the effects generated at or above the cutoff scale of the effective theory. These effects should be represented by local operators in the effective theory, with coefficients obtained by matching to the UV completion at the cutoff scale. If the UV completion does not contain any flavor structure, one expects such operators to appear suppressed by powers of the cutoff scale, with order-one coefficients. In the LHT, the cutoff scale is 10 TeV, so several of these operators would strongly violate experimental bounds on the FCNCs. This indicates that additional flavor structure (e.g. flavor symmetries) is a necessary part of the UV completion of the LHT. It would be interesting to extend our model to obtain realistic flavor physics.

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References

[1] N. Arkani-Hamed, A. G. Cohen and H. Georgi, Phys. Lett. B 513, 232 (2001) [arXiv:hep-ph/0105239]; N. Arkani-Hamed, A. G. Cohen, E. Katz and A. E. Nelson, JHEP 0207, 034 (2002) [arXiv:hep-ph/0206021].

[2] For reviews, see M. Schmaltz and D. Tucker-Smith, Ann. Rev. Nucl. Part. Sci. 55, 229 (2005) [arXiv:hep-ph/0502182]; M. Perelstein, Prog. Part. Nucl. Phys. 58, 247 (2007) [arXiv:hep-ph/0512128].

[3] H. C. Cheng and I. Low, JHEP 0309, 051 (2003) [arXiv:hep-ph/0308199]; JHEP 0408, 061 (2004) [arXiv:hep-ph/0405243].

[4] I. Low, JHEP 0410, 067 (2004) [arXiv:hep-ph/0409025].

[5] J. Hubisz, P. Meade, A. Noble and M. Perelstein, JHEP 0601, 135 (2006) [arXiv:hep-ph/0506042].

[6] M. Asano, S. Matsumoto, N. Okada and Y. Okada, Phys. Rev. D 75, 063506 (2007) [arXiv:hep-ph/0602157].

27
[7] M. S. Carena, J. Hubisz, M. Perelstein and P. Verdier, Phys. Rev. D 75, 091701 (2007) [arXiv:hep-ph/0610156].

[8] J. Hubisz and P. Meade, Phys. Rev. D 71, 035016 (2005) [arXiv:hep-ph/0411264].

[9] A. Belyaev, C. R. Chen, K. Tobe and C. P. Yuan, Phys. Rev. D 74, 115020 (2006) [arXiv:hep-ph/0609179].

[10] A. Birkedal, A. Noble, M. Perelstein and A. Spray, Phys. Rev. D 74, 035002 (2006) [arXiv:hep-ph/0603077]; M. Perelstein and A. Spray, Phys. Rev. D 75, 083519 (2007) [arXiv:hep-ph/0610357].

[11] C. T. Hill and R. J. Hill, Phys. Rev. D 76, 115014 (2007) [arXiv:0705.0697 [hep-ph]].

[12] R. J. Hill, arXiv:0710.5791 [hep-ph].

[13] D. Krohn and I. Yavin, arXiv:0803.4202 [hep-ph].

[14] E. Katz, J. y. Lee, A. E. Nelson and D. G. E. Walker, JHEP 0510, 088 (2005) [arXiv:hep-ph/0312287].

[15] J. Thaler and I. Yavin, JHEP 0508, 022 (2005) [arXiv:hep-ph/0501036].

[16] K. Agashe, A. Falkowski, I. Low and G. Servant, arXiv:0712.2455 [hep-ph].

[17] A. Birkedal, Z. Chacko and M. K. Gaillard, JHEP 0410, 036 (2004) [arXiv:hep-ph/0404197].

[18] Z. Berezhiani, P. H. Chankowski, A. Falkowski and S. Pokorski, Phys. Rev. Lett. 96, 031801 (2006) [arXiv:hep-ph/0509311]; T. Roy and M. Schmaltz, JHEP 0601, 149 (2006) [arXiv:hep-ph/0509357]; C. Csáki, G. Marandella, Y. Shirman and A. Strumia, Phys. Rev. D 73, 035006 (2006) [arXiv:hep-ph/0510294].

[19] K. Agashe, R. Contino and A. Pomarol, Nucl. Phys. B 719, 165 (2005) [arXiv:hep-ph/0412089].

[20] L. Randall and R. Sundrum, Phys. Rev. Lett. 83, 3370 (1999) [arXiv:hep-ph/9905221].

[21] S. R. Coleman, J. Wess and B. Zumino, Phys. Rev. 177, 2239 (1969); C. G. Callan, S. R. Coleman, J. Wess and B. Zumino, Phys. Rev. 177, 2247 (1969).

[22] Z. G. Berezhiani and G. R. Dvali, Bull. Lebedev Phys. Inst. 5, 55 (1989) [Kratk. Soobshch. Fiz. 5, 42 (1989)]; R. Barbieri, G. R. Dvali, A. Strumia, Z. Berezhiani and L. J. Hall, Nucl. Phys. B 432, 49 (1994) [arXiv:hep-ph/9405428]; Z. Berezhiani, C. Csaki and L. Randall, Nucl. Phys. B 444, 61 (1995) [arXiv:hep-ph/9501336].

[23] D. E. Kaplan and M. Schmaltz, JHEP 0310, 039 (2003) [arXiv:hep-ph/0302049]; M. Schmaltz, JHEP 0408, 056 (2004) [arXiv:hep-ph/0407143].
[24] N. Seiberg, Nucl. Phys. B 435, 129 (1995) [arXiv:hep-th/9411149].
[25] I. R. Klebanov and M. J. Strassler, JHEP 0008, 052 (2000) [arXiv:hep-th/0007191].
[26] J. Wess and B. Zumino, Phys. Lett. B 37, 95 (1971); E. Witten, Nucl. Phys. B 223, 422 (1983).
[27] Y. Kawamura, H. Murayama and M. Yamaguchi, Phys. Rev. D 51, 1337 (1995) [arXiv:hep-ph/9406245].
[28] A. Falkowski, “Note on Little SUSY”, (2005) unpublished.
[29] C. Csaki, C. Grojean, H. Murayama, L. Pilo and J. Terning, Phys. Rev. D 69, 055006 (2004) [arXiv:hep-ph/0305237]; C. Csaki, C. Grojean, L. Pilo and J. Terning, Phys. Rev. Lett. 92, 101802 (2004) [arXiv:hep-ph/0308038].
[30] N. Arkani-Hamed, A. G. Cohen and H. Georgi, Phys. Lett. B 516, 3 95 (2001) [arXiv:hep-th/0103135].
[31] C. Csaki, J. Hubisz, G. D. Kribs, P. Meade and J. Terning, Phys. Rev. D 67, 115002 (2003) [arXiv:hep-ph/0211124].
[32] G. Burdman, M. Perelstein and A. Pierce, Phys. Rev. Lett. 90, 241802 (2003) [Erratum-ibid. 92, 049903 (2004)] [arXiv:hep-ph/0212228].
[33] M. Perelstein, M. E. Peskin and A. Pierce, Phys. Rev. D 69, 075002 (2004) [arXiv:hep-ph/0310039].
[34] J. Hubisz, S. J. Lee and G. Paz, JHEP 0606, 041 (2006) [arXiv:hep-ph/0512169].
[35] M. Blanke, A. J. Buras, A. Poschenrieder, C. Tarantino, S. Uhlig and A. Weiler, JHEP 0612, 003 (2006) [arXiv:hep-ph/0605214]; M. Blanke, A. J. Buras, A. Poschenrieder, S. Recksiegel, C. Tarantino, S. Uhlig and A. Weiler, JHEP 0701, 066 (2007) [arXiv:hep-ph/0610298]; M. Blanke, A. J. Buras, B. Duling, A. Poschenrieder and C. Tarantino, Model JHEP 0705, 013 (2007) [arXiv:hep-ph/0702136].