Adaptive OFDM with Index Modulation for Two-Hop Relay-Assisted Networks

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Abstract—In this paper, we propose an adaptive orthogonal frequency-division multiplexing with index modulation (OFDM-IM) for two-hop relay networks. In contrast to the traditional OFDM-IM with a deterministic and fixed mapping scheme, in this proposed adaptive OFDM-IM, the mapping schemes between a bit stream and indices of active subcarriers for the first and second hops are adaptively selected by a certain criterion. As a result, the active subcarriers for the same bit stream in the first and second hops can be varied in order to combat slow frequency-selective fading. In this way, the system reliability can be enhanced. Additionally, considering the fact that a relay device is normally a simple node, which may not always be able to perform mapping scheme selection due to limited processing capability, we also propose an alternative adaptive methodology in which the mapping scheme selection is only performed at the source and the relay will simply utilize the selected mapping scheme without changing it. The analyses of average outage probability, network capacity and symbol error rate (SER) are given in closed form for decode-and-forward (DF) relaying networks and are substantiated by numerical results generated by Monte Carlo simulations.

Index Terms—Index modulation, OFDM, two-hop relay system, adaptive modulation, decode-and-forward relay.

I. INTRODUCTION

Classical modulation schemes, e.g., phase-shift keying (PSK) and quadrature amplitude modulation (QAM), map a sequence of information bits to a modulated data symbol with a certain amplitude and phase, and such a modulated data symbol can be represented by a point in the two-dimensional constellation diagram [1]. With rapid increasing demands of reliability and data transmission rate, however, these classic modulation schemes might not be able to satisfy the challenging communication requirements for next generation networks in terms of spectrum and energy efficiency [2]. As a result, there has been a growing interest in design and optimization of modulation schemes which can achieve a high spectrum and/or energy efficiency [3]. Two of the most promising schemes are spatial modulation (SM) and orthogonal frequency-division multiplexing with index modulation (OFDM-IM), which are capable of adding one extra modulation dimension in addition to the conventional amplitude and phase dimensions [4]–[7]. SM is designed for multiple-input-multiple-output (MIMO) systems and exploits the spatial dimension, while OFDM-IM is mainly applied to OFDM systems by conveying information via the frequency dimension. In practice, OFDM-IM is easier to be implemented, since the deployment of MIMO under a limited correlation level among all antennas may not be affordable for most communication systems in terms of cost and physical size [8], [9], for example device-to-device (D2D) communication networks in which all nodes are nothing more than idle mobile devices [10]. Therefore, OFDM-IM is now being regarded as an attractive modulation methodology for 5G networks [11].

At the meantime, relay-assisted communications have been proved to be an effective approach to provide reliable communication service against propagation attenuation, multipath fading as well as shadowing, and are also regarded as one of the most promising technologies for next generation cellular networks [12]. Moreover, with recent research achievements on multicarrier relay systems, the performance of relay-assisted communications can be further enhanced in terms of multiplexing and/or diversity gains with a moderate system complexity [13]–[15]. Due to the maturity and benefits of multicarrier relay systems, it is of an increasingly high interest to integrate it with other popular communication paradigms [16]. In recent years, some works related to SM also considered involving relays to further improve system performance and proposed the concepts of distributed SM (DSM) and dual-hop SM (Dh-SM) [17], [18].

However, to the best of the authors’ knowledge, the link between two-hop relay networks and OFDM-IM is still lacking, which motivates us to construct an analytical framework of OFDM-IM in two-hop relay-assisted networks in order to integrate the merits of both. Also, motivated by a variety of adaptive modulation and transmit antenna selection schemes proposed for SM and proved to be capable of significantly enhancing system performance [19]–[21], we would also like to apply the adaptive modulation mechanism in OFDM-IM for two-hop relay networks. Based on these considerations, we propose the adaptive OFDM-IM for two-hop relay networks in this paper. In particular, a two-hop relay network with one source, one relay and one destination is considered and the mapping scheme for bit sequence to the indices of active subcarriers is dynamically selected according to the instantaneous channel state information (CSI). By such an adaptive mapping scheme selection process, the system reliability can be improved.

In spite of mapping scheme adaptation, our proposed scheme is different from those traditional OFDM-IM schemes, in which only a fixed number of subcarriers are active in each instant and thus will result in a low spectral efficiency [6], [22], [23]. Instead, an efficient activation mechanism with multiple active subcarriers is used in our proposed scheme to map the incoming bit stream to a corresponding subcarrier activation pattern. On the other hand, despite the higher spectral efficiency provided by adopting such an efficient activation mechanism with an indeterministic number of active subcarriers, there is a small possibility that all subcarriers are inactive. In this special case, the amplitude
and phase modulation (APM) data symbol cannot be transmitted. We term this **zero-active subcarrier dilemma**, which hinders the utilization of a variable number of active subcarriers. In this paper, an additional benefit brought by the subcarrier adaptation is that we can employ a dual-mode transmission protocol to solve the zero-active subcarrier dilemma, which is different from the subcarrier-index modulation (SIM) OFDM relying on error control techniques in higher layers [24]. Meanwhile, it is worth noting that the adaptive OFDM-IM proposed in this paper is different from the OFDM with generalized index modulation (OFDM-GIM) proposed in [25], as the diversity-multiplexing trade-off is provided in different manners. Theformer is dependent on the number of selected subcarriers, while the latter is related to the design of the set of all allowed numbers of active subcarriers, which will lead to a much higher complexity than ours.

Following by these core ideas, the contributions of this paper are listed infra:

1) We propose an adaptive OFDM-IM scheme for two-hop relay networks, by which the mapping schemes are dynamically selected based on instantaneous CSI and a mechanism allowing reducing throughput for gaining reliability can be constructed. Meanwhile, for implementation in practice, we propose two feasible mapping scheme selection methodologies, which can be applied depending on the design specification of the relay node, i.e. whether the relay is capable of performing mapping scheme selection or not.

2) We propose a dual-mode transmission protocol which can solve the zero-active subcarrier dilemma and transmit the APM symbol when all selected subcarriers are inactive.

3) We analyze the outage performance of the proposed modulation schemes and derive the exact expressions and asymptotic expressions in the high signal-to-noise ratio (SNR) region for the average outage probability in closed form; we analyze the average network capacity and derive its closed-form expression; we also investigate the error performance and give a closed-form approximation of the average symbol error rate (SER).

The rest of this paper is organized as follows. In Section II, the system model of adaptive OFDM-IM in two-hop networks is introduced. After that, we specify the mapping scheme selection and performance evaluation metrics in Section III. The analyses of average outage performance, network capacity and error performance of the proposed systems are carried out in Section IV. Then, all analyses are numerically verified and the simulation results are discussed in Section V. Finally, Section VI concludes the paper.

**II. System Model**

**A. System framework**

Consider a two-hop OFDM-IM system with one source, one relay, one destination and \( N_T \) subcarriers, which is operating in slow frequency-selective Rayleigh fading channels. Also, \( N_S \) out of the \( N_T \) subcarriers will be selected by a certain criterion to form a mapping scheme (a.k.a. a codebook) for OFDM-IM, where \( 1 \leq N_S < N_T \). In each instant, there is a variable-length equiprobable stream \( \{b_S(k), b_M(m_1), b_M(m_2), \ldots, b_M(m_{N_A(k)})\} \), required to be transmitted at the source, where \( m_t \in M = \{1, 2, \ldots, M\} \) denotes the index of the chosen APM constellation symbol for the \( t \)th active subcarrier and \( M \) is the order of APM; \( k \in K = \{1, 2, 3, \ldots, 2^{N_s}\} \) denotes the index of the subcarrier activation pattern and \( N_A(k) \) is the corresponding number of active subcarriers. Supposing that the lengths of \( b_S(k) \) and \( b_M(m_t) \) are \( B_S \) and \( B_M \), the length of the entire variable-length stream is \( B(k) = B_S + N_A(k)B_M \). The \( B_S \)-bit stream \( b_S(k) \) is modulated by the subcarrier activation pattern, and \( N_A(k) \) out of \( N_S \) subcarriers will be activated to convey the APM symbols generated according to these \( B_M \)-bit streams \( b_M(m_1), b_M(m_2), \ldots, b_M(m_{N_A(k)}) \) by the conventional \( M \)-ary APM scheme (e.g., \( M\)-PSK or \( M\)-QAM). Note that, normally the mapping scheme is preset in classic OFDM-IM systems and remains fixed, but in this paper, we assume it can be dynamically selected according to the instantaneous CSI. More details of the mapping scheme and its adaptation mechanisms will be given latter in Section III.

Meanwhile, we assume that a single bit in \( b_S(k) \) represents the activation state of a given subcarrier in an on-off keying (OOK) manner. In other words, for a given \( B_S \)-bit stream \( b_S(k) \), the selected subcarriers having the same sequence numbers with bits ‘1’ will be activated, on which \( N_A(k) \) different APM symbols are transmitted. That is to say, \( N_S = B_S \) and the number of active subcarriers \( N_A(k) \) is equal to the \textit{hamming weight} of the \( B_S \)-bit stream \( b_S(k) \). More specifically, the activation state matrix can be designed as

\[
\mathbf{S}(k) = \text{diag}\{b_S(k, 1), b_S(k, 2), \ldots, b_S(k, N_S)\},
\]

where \( b_S(k, n) \) is either ‘0’ or ‘1’ denoting the \( n \)th entry in \( b_S(k) \), \( \forall n \in \{1, 2, \ldots, N_S\} \). As a result, by OFDM-IM, the information contained in the entire \( B(k) \)-bit stream is now conveyed by both of \( N_A(k) \) \( M \)-ary constellation symbols \( \{x_m\} \) and the activation state matrix \( \mathbf{S}(k) \). Both will be jointly estimated by maximum-likelihood (ML) detection method at relay and destination\(^1\).

Here we further assume that there is no direct transmission link between source and destination due to deep fading or severe propagation attenuation and a half-duplex forwarding protocol is adopted at the relay, so that two orthogonal temporal phases are required for a complete symbol transmission from source to destination. In the first phase, the generated OFDM-IM symbol at the source is transmitted to the relay in slow frequency-selective Rayleigh fading channels. Because decode-and-forward (DF) relaying protocol and ML detection are adopted at the relay, once the distorted symbol is received by the relay, it will be first decoded and regenerated according to the relay’s own mapping scheme for the second hop. Then the relay transmits the regenerated symbol to the destination. Finally, the distorted version of the new symbol is received by the destination and then decoded to retrieve the complete \( B(k) \)-bit stream. For clarity, the system block diagram of the source is illustrated in Fig. 1, which is also highly similar to the transmission part of the relay.

**B. Channel model**

We assume the wireless channels in the first and second hops to be slow frequency-selective faded and their gains to be

\(^1\)Although the complexity of the ML detection will increase exponentially with the APM order \( M \) and the number of selected subcarriers \( N_S \), it can provide the optimal error performance and serves as a good reference to other suboptimal detection methods [26]. Therefore, we adopt the ML detection method at the relay and destination to demodulate the received signal and retrieve the transmitted bits.
exponentially distributed but with different means $\mu_1$ and $\mu_2$ due to different propagation attenuations. Note that, the slow or quasi-static property referred to here indicates that the channel gains are random but would remain unchanged for a sufficiently large period of time [27]. Based on such an assumption, the overheads caused by transmitting the selected mapping scheme via feedforward links to relay and destination for decoding purposes are negligible [19]. This justifies the feasibility of our proposed adaptive OFDM-IM in practice. Denoting the set of all subcarriers as $\mathcal{N} = \{1, 2, \ldots, N_T\}$, the probability density function (PDF) and the cumulative distribution function (CDF) of the channel gain $h_i(n)$, $\forall n \in \mathcal{N}$, can be written as

$$f_i(\epsilon) = \exp(-\epsilon/\mu_i)/\mu_i \iff F_i(\epsilon) = 1 - \exp(-\epsilon/\mu_i)$$

where $i \in \{1, 2\}$ denotes the index of the first or second hop. Meanwhile, because of the fundamental assumptions of OFDM, it is supposed that these $N_T$ channel gains in each hop are statistically independent$^2$.

C. APM scheme

In this paper, we adopt $M$-PSK as the APM scheme$^3$. Meanwhile, the analytical results in the following section can also be used for $M$-QAM cases if some precoding schemes (e.g., dynamic power allocation on constellation points) can be applied on each data symbol so that a constant envelope is achieved [30], [31]. Without loss of generality, we normalize the data symbol by $\chi_{m_1} \chi_{m_2}^* = 1$, $\forall m_\chi \in \mathcal{M}$.

III. MAPPING SCHEME SELECTIONS AND PERFORMANCE EVALUATION METRICS

Now let us focus on the key property of the proposed system, i.e. dynamical mapping scheme selection according to CSI. First, here we assume CSI can be perfectly known at source and relay without delay and overhead. Then, the set of all mapping schemes is denoted by $\mathcal{C}$ and the cardinality of $\mathcal{C}$, i.e. the total number of possible mapping schemes is given by $\text{Card}(\mathcal{C}) = (N_T M^*)_S$, where $\gamma_i$ denotes the binomial coefficient. It should be noted that the OFDM blocks are normally generated by inverse fast Fourier transform (IFFT), which will require $N_T = 2^\zeta$, $\zeta \in \mathbb{N}^+ = \{1, 2, \ldots\}$.

A. Signal model and dual-mode transmission protocol

In order to perform mapping scheme selection, we have to construct the signal model and transmission protocol as well as some important performance metrics. Now, let us first focus on the construction of the transmit OFDM block (a.k.a. transmit signal vector) before adopting a certain mapping scheme. Assuming a cyclic prefix (CP) with sufficient length is applied to the time-domain transmit OFDM block produced by a $N_T$-point IFFT, the OFDM signaling model can be considered at a subcarrier level [32]. A general form of the $N_T \times 1$ transmit OFDM block in frequency domain without interblock interference can be written as

$$\mathbf{x} = [x(1), x(2), \ldots, x(N_T)]^T \in \mathbb{C}^{N_T \times 1}$$

where $(-)^T$ denotes the matrix transpose operation.

When selecting an arbitrary $cth$ mapping scheme, $c \in \mathcal{C}$, which selects $N_S$ out of $N_T$ subcarriers to form a subset $\mathcal{N}_S(c) \subset \mathcal{N}$ for OFDM-IM, the reduced OFDM block determined by the $B(k)$-bit stream is given by

$$\mathbf{x}(k) = [x(m_1, 1), x(m_2, 2), \ldots, x(m_{N_S}, N_S)]^T \in \mathbb{C}^{N_S \times 1}$$

where

$$x(m_i, n) = \begin{cases} \chi_{m_i}, & n \in \mathcal{N}_S(k) \\ 0, & \text{otherwise} \end{cases}$$

corresponds to the data symbol transmitted on the $n$th subcarrier, and $\mathcal{N}_S(k) \subseteq \mathcal{N}_S(c)$ is the subset of $\mathcal{N}_S(k)$ active subcarriers out of $N_S$ selected subcarriers when $\mathbf{b}_S(k)$ is transmitted.

Now, in order to avoid the zero-active subcarrier dilemma, which is defined as the case when all selected subcarriers are required to be inactive (i.e. $\mathbf{b}_S(1) = \{0, 0, \ldots, 0\}$ and thus $\mathcal{N}_A(1) = 0$), we propose a dual-mode transmission protocol, which consists of the regular transmission mode and the complementary transmission mode.

If $\mathcal{S}(k) \neq \mathbb{0}_{N_S \times N_S}$, i.e. $\mathbf{b}_S(k)$ is not an all-zero bit stream, consequently, the regular transmission mode is activated and the received signal for the $ith$ hop can be expressed by [33]

$$\mathbf{y}_i(k) = [y_i(m_1, 1), y_i(m_2, 2), \ldots, y_i(m_{N_S}, N_S)]^T = \frac{P_i}{\mathcal{N}_A(k)} \mathbf{H}_i(c) \mathbf{x}(k) + \mathbf{w}_i \in \mathbb{C}^{N_S \times 1}$$

where

$$\mathbf{w}_i = [w_i(1), w_i(2), \ldots, w_i(N_S)]^T \in \mathbb{C}^{N_S \times 1}$$

is the vector of independent complex additive white Gaussian noise (AWGN) samples on each subcarrier at the $ith$ hop, whose entries obey $\mathcal{CN}(0, P_n)$, and $P_n$ is the noise power; $\mathbf{H}_i(c) = \text{diag}\{h_i(c, 1), h_i(c, 2), \ldots, h_i(c, N_S)\} \in \mathbb{C}^{N_S \times N_S}$ is a $N_S \times N_S$ diagonal channel state matrix for the $ith$ hop when the $ith$ mapping scheme is selected; $P_i$ is a uniform transmit power adopted at both source and relay. Due to the normalizations of APM symbols, we obtain the received SNR in the $ith$ hop for the $nth$ subcarrier as

$$\gamma_i(k, n) = \frac{P_i}{\mathcal{N}_A(k) P_n} b_S(k, n) |h_i(c, n)|^2, \forall n \in \mathcal{N}_S(c).$$

However, for $\mathcal{S}(1) = \mathbb{0}_{N_S \times N_S}$ (we denote $k = 1$ for this case), i.e. $\mathbf{b}_S(1)$ is an all-zero bit stream, one of those $N_T - N_S$
unselected subcarriers \( \hat{n}_i \) will all be activated to undertake the transmission of data symbol \( \chi_{m_a} \) in the \( i \)th hop. In this case, the received signal transmitted on the complementary subcarrier in the \( i \)th hop becomes

\[
y_i(1, \hat{n}_i) = h_i(\hat{n}_i)\chi_{m_a} + w_i(\hat{n}_i),
\]

where \( w_i(\hat{n}_i) \) is a complex AWGN on the complementary subcarrier, and the SNRs of the complementary subcarrier as well as other selected subcarriers become

\[
\gamma_i(1, \hat{n}_i) = \frac{P_i}{P_0}|h_i(\hat{n}_i)|^2 \quad \text{and} \quad \gamma_i(1, n) = 0, \forall n \in N_S(e).
\]

We term this special case complementary transmission mode, so that at least one data symbol \( \chi_{m_a} \) can still be transmitted on the standby subcarrier \( \hat{n}_i \) when all selected subcarriers are inactive\(^4\). Based on the above specification for the dual-mode transmission protocol, we can have the average transmission rate in bit per channel use (bpcu) as follows

\[
\bar{B} = \log_2 \left( 2^{N_S} \right) + \mathbb{E}\{ \max \{ 1, N_A(k) \} \log_2(M) \}
\]

\[
\approx N_S + \log_2(M) \left( 1 + \frac{N_S}{N} \sum_{n=1}^{N_S} \left( \frac{N_S}{n} \right) \right) \quad \text{[bpcu]} \tag{10}
\]

\[
\approx N_S + \log_2(M) \left( 1 + 2^{N_S-1} N_S \right) \quad \text{[bpcu]},
\]

where \( \mathbb{E}\{ \cdot \} \) denotes the expected value of the random variable enclosed.

B. Decentralized mapping scheme selection

Here, we propose two feasible mapping scheme selection methodologies, which can be applied depending on the design specification of the relay node, i.e. whether the relay is capable of performing mapping scheme selection or not. We first introduce the decentralized mapping scheme as follows. When both source and relay can get access to perfect CSI and carry out the subcarrier adaptation process, in order to optimize the outage performance and error performance, the mapping scheme selections are independently performed at the source and relay according to the criterion\(^5\)

\[
\hat{c}_i = \arg \max_{c \in C} \left\{ \sum_{n \in N_S(c)} \gamma_i(2^{N_S}, n) \right\}, \tag{11}
\]

where \( \hat{c}_i \) denotes the index of the optimal mapping scheme selected for the \( i \)th hop. Because the mapping scheme selections are independently performed at both source and relay, we term this selection methodology the decentralized mapping scheme selection.

Subsequently, for the \( i \)th hop, the selection criterion of the complementary subcarrier is given by

\[
\hat{n}_i = \arg \max_{n \in N \setminus N_S(c_i)} |h_i(n)|^2. \tag{12}
\]

\(^4\)The information conveyed by \( \chi_{m_a} \) can be crucial for system coordination, e.g., synchronization.

\(^5\)We denote the index of all-one \( B_S \)-bit stream as \( k = 2^{N_S} \), i.e. \( b_S(2^{N_S}) = [1, 1, \ldots, 1] \). In this case, all \( N_S \) selected subcarriers will be active, so that a comprehensive mapping scheme selection considering the qualities of all channels can be performed. Obviously, by this selection criterion, it is equivalent to selecting the \( N_S \) best subcarriers out of the \( N_T \) subcarriers in terms of SNR for the \( i \)th hop.

C. Centralized mapping scheme selection

However, the decentralized mapping scheme selection methodology will result in a high system complexity at the relay node, which might not be practical as the relay is normally an idle user mobile device with relatively limited processing capability [34]. We thereby propose an alternative suboptimal mapping scheme selection methodology which is performed only at the source and thus relaxes the design requirements of the relay. Because the mapping scheme selection is only performed at the source, we term this the centralized mapping scheme selection. Specifically, the mapping scheme is only selected at the source and will be simply followed by the relay. The mapping scheme selection criterion is given by

\[
\hat{c} = \arg \max_{c \in C} \left\{ \sum_{n \in N_S(c)} \min \left\{ \gamma_2(2^{N_S}, n), \gamma_A(2^{N_S}, n) \right\} \right\}, \tag{13}
\]

which is equivalent to choosing the \( N_S \) best link from the \( N_T \) links in terms of end-to-end SNR.

In a similar manner to the decentralized case, we can select the contemporary subcarrier for a whole link by the criterion

\[
\hat{n} = \arg \max_{n \in N \setminus N_S(c)} \left\{ |h_1(n)|^2, |h_2(n)|^2 \right\}. \tag{14}
\]

By either the decentralized or centralized mapping scheme selection methodology, as long as the \( c \)th mapping scheme is selected, the \( B_S \)-bit stream \( b_S(k) \) will be mapped to activate a subset of \( N_A(k) \) from \( N_S(c) \) subcarriers. To achieve this, we first need to relabel the subcarriers in \( N_S(c) \) by the new indices in ascending order. To be clear, the indices in the original full set \( N \) are termed absolute indices, whereas the indices in the selected subset \( N_S(c) \) are termed relative indices\(^6\). An example of the mapping scheme selection process from \( N \) to \( N_S(c) \) can be observed in Fig. 2, given \( N_T = 8 \) and \( N_S = 4 \). Furthermore, for clarity, an example of the OFDM-IM mapping table when \( B_M = 1 \) and \( B_S = 2 \) is given in Table I (Because \( B_M = 1 \), BPSK is adopted for APM and here we stipulate that data symbols +1 and -1 correspond to bit ‘1’ and ‘0’, respectively).

D. Performance evaluation metrics

1) Average outage probability: We define the outage event of the proposed systems as follows [35].

\[^6\]Because the values of indices per se do not affect the mathematical analysis, with a slight abuse of notation, we do not specifically distinguish them and denote a generic subcarrier index as \( n \) in this paper.
TABLE I: An example of the OFDM-IM mapping table when $B_M = 1$ and $B_S = N_S = 2$.

| $b_S(k)$ | $N_A(k)$ | $\sum_{r=1}^{N_A(k)} p_M(m_r)$ | $S(k)$ | $X(k)$ | $\chi_{m_a}$ |
|----------|----------|---------------------------------|--------|--------|-------------|
| [0, 0]   | $\emptyset$ | [0]                            | diag(0, 0) | $[0, 0]^T$ | -1          |
| [0, 0]   | $\emptyset$ | [1]                            | diag(0, 0) | $[0, 1]^T$ | +1          |
| [0, 1]   | [1]      | [0]                            | diag(0, 1) | $[0, -1]^T$ | 0           |
| [0, 1]   | [0]      | [1]                            | diag(1, 0) | $[+1, 0]^T$ | 0           |
| [1, 0]   | [0]      | [2]                            | diag(1, 0) | $[-1, 0]^T$ | 0           |
| [1, 0]   | [2]      | [0]                            | diag(1, 1) | $[-1, -1]^T$ | 0           |
| [1, 1]   | [1, 2]   | [0, 0]                          | diag(1, 1) | $[-1, -1]^T$ | 0           |
| [1, 1]   | [0, 1]   | [2, 0]                          | diag(1, 1) | $[-1, -1]^T$ | 0           |
| [1, 1]   | [1, 2]   | [1, 0]                          | diag(1, 1) | $[+1, -1]^T$ | 0           |
| [1, 1]   | [1, 2]   | [2, 0]                          | diag(1, 1) | $[+1, -1]^T$ | 0           |

* The complementary transmission mode will be activated in this case.

Definition 1: An outage occurs when the SNR of any of the active subcarriers in either the first or the second hop falls below a preset outage threshold $s$.

By fundamental probability theory, we can determine the conditional outage probability when $b_S(k)$ is transmitted by

$$P_o(s|k) = \begin{cases} \mathbb{P}\left\{ \bigcup_{i=1,2} \left\{ \gamma_i(1, \tilde{n}_k) < s \right\} \right\}, & \text{if } k = 1 \\ \mathbb{P}\left\{ \bigcup_{i=1,2} \bigcup_{n \in N_A(k)} \left\{ \gamma_i(k, n) < s \right\} \right\}, & \text{if } k > 1 \end{cases}$$

where $\mathbb{P}\{\cdot\}$ denotes the probability of the event enclosed. Therefore, the average outage probability over $k$ can be written as

$$\bar{P}_o(s) = \mathbb{E}_{k \in K} \left\{ P_o(s|k) \right\}.$$  \hspace{1cm} (16)

which is an important metric used in this paper to characterize the system reliability.

2) Average network capacity: To jointly consider throughput and reliability, we should also take network capacity into consideration to further investigate the performance of the proposed systems. According to the max-flow min-cut theorem [36], the average network capacity in two-hop networks is given by

$$C = \mathbb{E}_{\{\hat{c}_m|H_k, h_k\}} \left\{ C(k) \right\} \text{ [bit/s/Hz]},$$

where $C(k)$ is the network capacity when $b_S(k)$ is transmitted and can be written as

$$C(k) = \begin{cases} \frac{1}{2} \min \left\{ \log_2 (1 + \gamma_1(1, \tilde{n}_k)), \log_2 (1 + \gamma_2(1, \tilde{n}_2)) \right\}, & \text{if } k = 1 \\ \sum_{n \in N_A(k)} \frac{1}{2} \min \left\{ \log_2 (1 + \gamma_1(k, n)), \log_2 (1 + \gamma_2(k, n)) \right\}, & \text{if } k > 1 \end{cases}$$

3) Average symbol error rate: To analyze the error performance, we have to first specify the ML detection method used in our proposed systems. Due to the dual-mode transmission protocol adopted in this paper, the ML detection method has to be tailored accordingly. In order to effectively perform detection, we first need to construct a concatenated symbol block by $X(k)$ and $\chi_{m_a}$ as $X(k) = (x(k), \chi_{m_a})$, where $\langle \cdot, \cdot \rangle$ represents the vector/diagonal matrix concatenation operation$^7$. The set of all possible $X(k)$ is denoted as $X$ and $\text{Card}(X) = M = \sum_{n=1}^{N_S} (N^2) M^n$. Subsequently, the detection criterion of the ML detector for the received concatenated signal $Y(\tilde{k}) = (\tilde{y}_1(\tilde{k}), y_2(1, \tilde{n}))$ distorted from the authentic transmitted concatenated OFDM-IM symbol $\tilde{X}(k)$ in the $i$th hop is given by

$$\tilde{X}(k) = \arg \min_{X(k)} \left\| Y(\tilde{k}) - \sqrt{\frac{P}{N_X(k)}} H_c(\tilde{c}, h_k) X(k) \right\|^2_F,$$  \hspace{1cm} (19)

where $\left\| \cdot \right\|^2_F$ denotes the Frobenius norm of the enclosed matrix/vector.

Hence, the conditional SER for the $i$th hop can be written as

$$P_{e,i}(\tilde{X}(k)|H_c(\tilde{c}), h_k) = \sum_{X(k) \neq \tilde{X}(k)} P_{e,i}(X(k)|H_c(\tilde{c}), h_k).$$

Subsequently, the unconditional SER can be expressed as

$$P_e(\tilde{X}(k)) = \mathbb{E}_{H_c(\tilde{c}), h_k} \left\{ P_{e,i}(\tilde{X}(k)|H_c(\tilde{c}), h_k) \right\}.$$  \hspace{1cm} (20)

Due to the bottleneck effect of two-hop DF relay networks, the end-to-end SER can be approximated by$^6$

$$P_e(\tilde{X}(k)) \approx P_{e,1}(\tilde{X}(k)) + P_{e,2}(\tilde{X}(k)) = P_{e,1}(\tilde{X}(k)) P_{e,2}(\tilde{X}(k)).$$

Now, we can define the average SER as

$$\bar{P}_e = \mathbb{E}_{X(k) \in K} \left\{ P_e(X(k)) \right\},$$  \hspace{1cm} (23)

which will be used in this paper to evaluate the fidelity of the proposed systems.

E. Comparison benchmarks

1) Conventional OFDM-IM: Without mapping scheme adaptation, the conventional OFDM-IM system will activate $N_T/2$ out of $N_T$ subcarriers$^9$ in each instant to convey information [6]. The transmission rate of the conventional OFDM-IM is given by

$$B_{\text{classic}} = \frac{N_T}{2} \log_2(M) + [\log_2 \left( \frac{N_T}{N_T/2} \right)] \text{ [bpcu]},$$

where $\lfloor \cdot \rfloor$ represents the floor function.

It can be seen by comparing (10) with (24) that the transmission rate of the conventional OFDM-IM scheme without mapping scheme adaptation is higher than that with mapping scheme adaptation. Another merit of the conventional OFDM-IM is that it always activates $N_T/2$ subcarriers, so that the zero-active subcarrier dilemma does not exist. However, because the

$^7$Here we define: for arbitrary vectors $u_1 = [u_1(1), u_1(2), \ldots, u_1(n)]^T$ and $u_2 = [u_2(1), u_2(2), \ldots, u_2(m)]^T$, $u_1$ and $u_2$ are for arbitrary diagonal matrices $U_1 = \text{diag}(u_1(1), u_1(2), \ldots, u_1(n))$ and $U_2 = \text{diag}(u_2(1), u_2(2), \ldots, u_2(m))$. $U_1$ and $U_2$ are for arbitrary matrices $U_1 = \text{diag}(u_1(1), u_1(2), \ldots, u_1(n), u_2(1), u_2(2), \ldots, u_2(m)) = \text{diag}(u_1, u_2)$.

$^6$For simplicity, we neglect the circumstance that an erroneously estimated and transmitted signal at the relay can be again ‘erroneously’ estimated to the correct signal at the destination, as the probability is small.

$^9$In fact, the choice of the fixed number of active subcarriers can also provide a diversity-multiplexing trade-off. Therefore, to make the conventional OFDM-IM comparable with our proposed adaptive scheme (i.e. when $N_S = N_T - 1$, the rates will be almost the same, so that we can observe the difference in outage probability, network capacity and SER etc.), we choose $N_T/2$ as the fixed number of active subcarriers for the conventional OFDM-IM in this paper.
conventional OFDM-IM utilizes the index of subcarrier set to convey information, it is expected that some possible subcarrier sets will never be used in this scheme and therefore the spectral efficiency is degraded without any benefit [6]. Also, without subcarrier selection, the reliability of the systems will be much lower than that with our proposed adaptive scheme.

2) **FPSK**: As a well known modulation scheme [22], we also take FPSK as a comparison benchmark in this paper. In FPSK, only one subcarrier will be activated and the index of the active subcarrier is used for conveying information. Meanwhile, one should note that FPSK has a different mapping scheme compared to the conventional OFDM-IM when there is only one active subcarrier. The transmission rate is given by

$$B_{FPSK} = \log_2(M) + \lfloor \log_2(N_T) \rfloor \text{ bpcu}. \quad (25)$$

The FPSK scheme would lead to a low-complexity system, since the detection process is much simpler than our proposed adaptive OFDM-IM scheme and the zero-active subcarrier dilemma does not exist either. On the other hand, the transmission rate of the systems using FPSK might be significantly lower than that of systems using our proposed schemes, because the spectral efficiency of FPSK in frequency dimension is too low by activating only a single subcarrier in each instant.

Detailed numerical comparisons in terms of outage performance, network capacity and error performance for the adaptive OFDM-IM scheme and these two benchmarks will be given in Section V. The transmission rates of these three schemes are plotted in Fig. 3 for illustration purposes. From this figure, it can be found that the adaptive OFDM-IM has a much better rate with a moderate network capacity and error performance for the adaptive OFDM-IM without selection. Therefore, statistically, the rate of the adaptive OFDM-IM with $N_S = N_T - 1$ will be the same as the conventional OFDM-IM. As a consequence, there will be no diversity anymore.

**IV. PERFORMANCE ANALYSIS**

A. **Outage Performance Analysis**

1) **Decentralized mapping scheme selection**: When $b_S(1) = [0, 0, \ldots, 0]$ is required to be transmitted, the complementary transmission mode will be activated and the standby subcarrier $\bar{n}_i$ will be employed to convey the APM symbol $\chi_{m_i}$. By (12), the standby subcarrier $\bar{n}_i$ is the $(N_T - N_S)$th worst among all $N_T$ subcarriers and the only active subcarrier in the complementary transmission mode. Therefore, the outage probability regarding the subcarrier $\bar{n}_i$ in the $i$th hop is given by

$$P_{o,i}(s|k = 1) = F_i(N_T - N_S) \left( s \frac{P_n}{P_t} \right), \quad (26)$$

where

$$F_i(\xi) (s) = \sum_{n=\xi}^{N_T} \left( \binom{N_T}{n} (F_i(s))^n (1 - F_i(s))^{N_T - n} \right) \quad (27)$$

denotes the outage probability of the $\xi$th order statistic of the channel gain among $N_T$ subcarriers in the $i$th hop [37]. Hence, the outage probability of the complementary transmission mode is

$$P_o(s|k = 1) = P_{o,1}(s|k = 1) + P_{o,2}(s|k = 1) - P_{o,1}(s|k = 1)P_{o,2}(s|k = 1). \quad (28)$$

In order to analyze the outage performance when $k \neq 1$, we propose and prove a lemma as follows.

**Lemma 1**: For $k \neq 1$, the generic conditional outage probability for the $i$th hop can be determined by

$$P_{o,i}(s|k) = \sum_{\xi=N_T-N_S}^{N_T-N_S+1} \Upsilon(k,\xi) F_i(\xi) \left( s \frac{P_n N_A(k)}{P_t} \right), \quad (29)$$

where $\Upsilon(k,\xi) = \frac{N_T - \xi}{N_A(k) - \xi} \left( \frac{N_S}{N_A(k) - 1} \right)^{N_A(k)}$.

**Proof**: See Appendix A

Due to the DF forwarding protocol, the conditional outage probability considering two hops is derived by

$$P_o(s|k) = P_{o,1}(s|k) + P_{o,2}(s|k) - P_{o,1}(s|k)P_{o,2}(s|k). \quad (30)$$

Because $B_S(k)$ is equiprobable, the average outage probability is given by

$$P_o(s) = \frac{1}{2N_S} \sum_{k \in K} P_o(s|k). \quad (31)$$

By employing a power series expansion at $P_t/P_n \to \infty$, we can obtain the asymptotic expression of $P_o(s)$ as

$$P_o(s) \approx \left[ \left( \frac{1}{\mu_1} \right) \frac{N_T - N_S}{2N_S} + \left( \frac{1}{\mu_2} \right) \frac{N_T - N_S}{2N_S} \right] \left( s \frac{P_n}{P_t} \right)^{N_T - N_S}, \quad (32)$$

by which we can find the diversity order to be $N_T - N_S$. This aligns with our expectation that $N_T$ will always have a positive impact on the outage performance. The effects of $N_S$ is indeterminate and dependent on other factors, but asymptotically it yields a destructive impact on the outage performance.

2) **Centralized mapping scheme selection**: Because the mapping schemes used at the source and relay are the same by the centralized mapping scheme selection, we have to integrate the channels in two hops into a whole, which is termed a link. When DF forwarding protocol is adopted, due to the bottleneck effect, the PDF and CDF of the gain regarding a certain unsorted link can be determined from (2) to be

$$\phi(\epsilon) = \exp \left( -\epsilon/\mu_S \right) / \mu_S \quad \Leftrightarrow \quad \Phi(\epsilon) = 1 - \exp \left( -\epsilon/\mu_S \right), \quad (33)$$

where $\mu_S = \mu_1 + \mu_2$. In a similar manner to the decentralized case, we can have the outage probability of the complementary transmission mode as

$$P_o(s|k = 1) = \Phi(N_T - N_S) \left( s \frac{P_n}{P_t} \right), \quad (34)$$

where

$$\Phi(\xi)(s) = \sum_{n=\xi}^{N_T} \left( \binom{N_T}{n} (\Phi(s))^n (1 - \Phi(s))^{N_T - n} \right) \quad (35)$$

denotes the outage probability of the $\xi$th order statistic of the link gain among $N_T$ links.

Now, let us generalize the analysis to an arbitrary case when $k \neq 1$, and similarly obtain the generic conditional average outage probability by

$$P_o(s|k) = \sum_{\xi=N_T-N_S}^{N_T-N_S+1} \Upsilon(k,\xi) \Phi(\xi) \left( s \frac{P_n N_A(k)}{P_t} \right). \quad (36)$$

Then, by (16) and the assumption of equiprobable $b_S(k)$, the average outage probability can be calculated by (31) as well.
employing a power series expansion at $P_t / P_n \to \infty$, we can obtain the asymptotic expression of $P_o(s)$ as

$$P_o(s) \sim \frac{1}{2N^2s} \left( \frac{N_T}{N_T - N_S} \right)^{N_T - N_S} \left( \frac{sP_n}{\mu_s P_t} \right)^{N_T - N_S},$$  \hspace{1cm} (37)

by which we can find the diversity order to be $N_T - N_S$. Seemingly, the centralized mapping scheme selection method will result in the same diversity as its distributed counterpart, but with a lower coding shift.

B. Network Capacity Analysis

1) Decentralized mapping scheme selection: By (18), we can reduce the expression of the network capacity of the complementary transmission mode to

$$C(1) = \frac{1}{2} \log_2 \left( 1 + \min \{ \gamma_1(1, \tilde{n}_1), \gamma_2(1, \tilde{n}_2) \} \right),$$  \hspace{1cm} (38)

in which the PDF of $\min \{ \gamma_1(1, \tilde{n}_1), \gamma_2(1, \tilde{n}_2) \}$ after performing mapping scheme selections can be expressed as

$$\psi(\gamma) = \int_{x_{\min}}^{x_{\max}} \frac{dF_\gamma(t)}{dt} \left( 1 - F_\gamma(t) \right) dx,$$

where $F_\gamma(t) = \int_{x_{\min}}^{t} dF_\gamma(s)$. Therefore, the average capacity over $h_1(\tilde{n}_1)$ and $h_2(\tilde{n}_2)$, for the complementary transmission mode can be obtained as

$$C(1) = \sum_{h_1(\tilde{n}_1), h_2(\tilde{n}_2)} \{ C(1) \} = \int_{0}^{\infty} \frac{1}{2} \log_2 \left( 1 + s \psi(\gamma) \right) dx,$$

where $\psi(\gamma) = \psi(\gamma_1, \gamma_2) = \int \psi(\gamma_1) \psi(\gamma_2) d\gamma_1 d\gamma_2$. Therefore, the average capacity over $h_1(\tilde{n}_1)$ and $h_2(\tilde{n}_2)$, for the complementary transmission mode can be obtained as

$$C(1) = \frac{1}{2} \log_2 \left( 1 + \psi(\gamma) \right).$$  \hspace{1cm} (41)

For $k \neq 1$, there will be $N_A(k)$ active subcarriers at each hop, whose orders are denoted by $\Xi_1(k) = \{ \xi_1, \xi_2, \ldots, \xi_{N_A(k)} \}$ and $\Xi_2(k) = \{ \eta_1, \eta_2, \ldots, \eta_{N_A(k)} \}$, where $\xi_i \neq \xi_j$ and $\eta_i \neq \eta_j$ for $i \neq j$ and $N_T - N_S + 1 \leq \xi_i, \eta_i \leq N_T$, $\forall 1 \leq i \leq N_A(k)$. The set of permutations of $\Xi_1(k)$ and $\Xi_2(k)$ is denoted as $\mathcal{D}(k)$ and $\text{Card}(-\mathcal{D}(k)) = N_S!/(N_S - N_A(k))!$. Therefore, in order to determine the conditional average capacity over $H_1(\tilde{c}_1)$ and $H_2(\tilde{c}_2)$, we propose and prove a lemma as follows.

Lemma 2: For $k \neq 1$, the conditional average capacity over $H_1(\tilde{c}_1)$ and $H_2(\tilde{c}_2)$ is given by

$$\hat{C}(k) = \mathbb{E}_{H_1(\tilde{c}_1), H_2(\tilde{c}_2)} \{ C(k) \} = \sum_{\xi, \eta} \sum_{k \in \mathcal{D}(k)} \left( \frac{(N_S - N_A(k))!}{N_S!} \right)^2 \left( \frac{P_n N_A(k)}{P_t} \right).$$  \hspace{1cm} (45)

Proof: See Appendix B.

As a result of the equiprobable assumption of $b_S(k)$, the average network capacity can be obtained by

$$C = \mathbb{E}_{k \in \mathcal{K}} \{ \hat{C}(k) \} = \frac{1}{2N^2s} \sum_{k \in \mathcal{K}} \hat{C}(k).$$  \hspace{1cm} (46)

2) Centralized mapping scheme selection: We can express the network capacity of the complementary mode by modifying (38) and determine the PDF of $\min \{ \gamma_1(1, \tilde{n}), \gamma_2(1, \tilde{n}) \}$ by $\phi(\gamma) = \phi(\gamma_1, \gamma_2) = \int \phi(\gamma_1) \phi(\gamma_2) d\gamma_1 d\gamma_2$. Therefore, the average capacity of the complementary mode can be determined by

$$C(1) = \frac{1}{2} \log_2 \left( 1 + \psi(\gamma) \right),$$  \hspace{1cm} (41)

where $\psi(\gamma) = \psi(\gamma_1, \gamma_2) = \int \psi(\gamma_1) \psi(\gamma_2) d\gamma_1 d\gamma_2$. Therefore, the average capacity of the complementary mode can be obtained as

$$C(1) = \int_{0}^{\infty} \frac{1}{2} \log_2 \left( 1 + s \phi(\gamma) \right) dx,$$

where $\phi(\gamma) = \phi(\gamma_1, \gamma_2) = \int \phi(\gamma_1) \phi(\gamma_2) d\gamma_1 d\gamma_2$. Therefore, the average capacity of the complementary mode can be obtained as

$$C(1) = \frac{1}{2} \log_2 \left( 1 + \phi(\gamma) \right).$$  \hspace{1cm} (41)

Here, $\Lambda_{\gamma}(\xi, \eta, x)$ is a defined special function, which can be explicitly written in (42). The expressions $\lambda_i(\xi, x)$ and $\nu_{ij}(\xi, \eta, x)$ can be derived in closed form in (43) and (44), where $(i, j) \in \{ (1, 2), (2, 1) \}$; $E(x) = -\int_{-\infty}^{\infty} \frac{\phi(x)}{\phi(x)} dx$ is the exponential integral function and $\kappa_{ij}(\xi, n, d, b) = \frac{N_t - 1 + n}{P_n} + \frac{N_t + d - n}{P_t}$ represents an intermediate coefficient.
\[ \Lambda_G(\xi, \eta, x) = \int_0^\infty \frac{x}{2} \log_2(1 + s) f_1(\xi)(xs) ds - \int_0^\infty \frac{x}{2} \log_2(1 + s) f_2(\eta)(xs) ds \]

\[ + \int_0^\infty \frac{x}{2} \log_2(1 + s) f_1(\xi)(xs) ds - \int_0^\infty \frac{x}{2} \log_2(1 + s) f_2(\eta)(xs) ds \]

\[ \lambda_i(\xi, x) = \frac{N_T!}{(2 \ln 2)(\xi - 1)!} \int_0^{\frac{x}{\mu_i}} \left( \frac{\xi - 1}{b} \right)^{b+1} \exp \left( \frac{x}{\mu_i} (\xi - \xi + 1 + b) \right) Ei(- (\xi - \xi + 1 + b)) \]

\[ \nu_{i,j}(\xi, x, k) = \frac{N_T!}{\mu_i(2 \ln 2)(\xi - 1)!} \int_0^{\frac{x}{\mu_i}} \left( \frac{\xi - 1}{n} \right)^n \left( \frac{\xi - 1}{b} \right)^{b+1} \exp \left( \frac{x}{\mu_i} (\xi - \xi + 1 + b) \right) Ei(- (\xi - \xi + 1 + b)) \]

For \( k \neq 1 \), we can derive the conditional average capacity similarly as for the decentralized case by

\[ \hat{C}(k) = \frac{E}{H_i(\xi)} \sum_{\xi(k) \in D(k)} \{C(k)\} = \sum_{\xi(k) \in D(k)} \left( \frac{(N_S - N_A(k))!}{N_S!} \right) \sum_{n \in N_A(k)} \Lambda_L \left( \xi, \frac{P_n N_A(k)}{P_i} \right) \]

(49)

Then, by (46), the average network capacity for the centralized case can be derived.

C. Error Performance Analysis

1) Decentralized mapping scheme selection: According to [38], we can determine the conditional SER in the \( t \)th hop as

\[ P_{c,i}(\hat{X}(k)) = \hat{X}(k) | H_i(\hat{c}), h_i(\hat{n}) \]

\[ = Q \left( \sqrt{ \frac{P_i}{P_n} } \left\| H_i(\hat{c}), h_i(\hat{n}) \right\| \frac{\hat{X}(k)}{\sqrt{N_A(k)}} - \frac{\hat{X}(k)}{\sqrt{N_A(k)}} \right\|_F^2 \right) \]

(50)

where \( Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp \left( -u^2/2 \right) du \) is the Q-function. Subsequently, in order to remove the conditions on \( H_i(\hat{c}), h_i(\hat{n}) \) and obtain an accurate closed-form approximation, we first need to approximate the Q-function by [39]

\[ Q(x) \approx \frac{1}{12} \exp \left( -\frac{x^2}{2} \right) + \frac{1}{4} \exp \left( -\frac{2x^2}{3} \right) \]

(51)

which becomes accurate when \( x \) is large, and has been proved to be effective in obtaining the approximate SER for traditional OFDM-IM as shown in [6]. Also, we define a function \( \Theta_{L_i}(\hat{X}(k), \hat{X}(k), \Xi) \) and simplify it in (52), where \( \Xi \in D \) is the vector of ordered indices of the channel gains regarding \( N_S \) selected subcarriers in the \( t \)th hop and \( D \) is the permutation set; \( \Gamma(x) = \int_0^x u^{-1} \exp(-u) du \) is the gamma function.

By (20), (21) and (22), we can obtain the unconditional end-to-end SER by

\[ P_r(\hat{X}(k)) = \Omega_{G_1}(\hat{X}(k)) + \Omega_{G_2}(\hat{X}(k)) - \Omega_{G_1}(\hat{X}(k)) \Omega_{G_2}(\hat{X}(k)) \]

(53)

where \( \Omega_{G_i}(\hat{X}(k)) \) is defined as follows:

\[ \Omega_{G_i}(\hat{X}(k)) = \frac{1}{N_S!} \sum_{\hat{X}(k) \neq \hat{X}(k) \in D} \Theta_{G_i}(\hat{X}(k), \hat{X}(k), \Xi) \]

(54)

Therefore, by (23), the average SER can be determined by

\[ P_e = \sum_{\hat{X}(k) \in X} \varphi(\hat{X}(k)) P_r(\hat{X}(k)) \]

(55)

where \( \varphi(\hat{X}(k)) \) represents the probability that \( \hat{X}(k) \) is transmitted, and can be written as

\[ \varphi(\hat{X}(k)) = \left( \frac{N_S}{N_A(k)} \right) \left( \frac{2^{N_S} M^{\max(1,N_A(k))}}{2^{N_S} M^{\max(1,N_A(k))}} \right) \]

(56)

2) Centralized mapping scheme selection: Following the concept of a link as explained in Section IV-A, we can let

\[ L(\hat{c}) = \text{diag}\{l(\hat{c}, 1), l(\hat{c}, 2), \ldots, l(\hat{c}, N_S)\} = \text{diag}\{\min\{h_1(\hat{c}, 1), h_2(\hat{c}, 1)\}, \min\{h_1(\hat{c}, 2), h_2(\hat{c}, 2)\}, \ldots, \min\{h_1(\hat{c}, N_S), h_2(\hat{c}, N_S)\}\} \]

(57)

and

\[ \hat{l}(n) = \min\{h_1(\hat{n}), h_2(\hat{n})\} \]

(58)

Then, when applying the centralized mapping scheme selection, we can similarly obtain the unconditional end-to-end SER by

\[ P_r(\hat{X}(k)) = \Omega_{L}(\hat{X}(k)) \]

(59)

where

\[ \Omega_{L}(\hat{X}(k)) = \frac{1}{N_S!} \sum_{\hat{X}(k) \neq \hat{X}(k) \in D} \sum_{\Xi} \Theta_L(\hat{X}(k), \hat{X}(k), \Xi) \]

(60)

and \( \Theta_L(\hat{X}(k), \hat{X}(k), \Xi) \) can be similarly defined and approximated as (52) by replacing \( \mu_i \) with \( \mu_L \). As a result, substituting (59) into (55) yields the average SER for the centralized mapping scheme selection case.
 Meanwhile, from these two figures, some important features of schemes were numerically simulated by Monte Carlo methods and repeated trials. The simulation results are shown and discussed as follows. Note that, all results regarding the comparison benchmarks, i.e. conventional OFDM-IM and FPSK, are numerical and generated by averaging over a large number of repeated trials.

### A. Simulations of outage performance

The relations between average outage probability and the ratio of transmit power to noise power $P_t / P_n$ for different transmission schemes were numerically simulated by Monte Carlo methods and shown in Fig. 4. From this figure, the numerical results match the analytical results, which verifies our analysis in Section IV-A. Meanwhile, from these two figures, some important features of adaptive OFDM-IM are revealed. First, the systems with decentralized and centralized mapping scheme selections share the same diversity order, but have different coding gains. This aligns with our analytical results given in (32) and (37).

Second, our proposed schemes always have a better outage performance compared to the scheme without adaptation, and this performance advantage is brought by reducing the number of subcarriers used for OFDM-IM. Besides, our proposed schemes with any $N_S$ will enjoy a better outage performance than that of FPSK, as long as $P_t / P_n$ is relatively large. Also, a larger $N_S$ will result in a larger average number of active subcarriers, which will degrade the outage performance, as all active subcarriers need to be ensured not in outage as specified in Definition 1. Therefore, when $N_S$ approaches $N_T$, the outage performance of the adaptive schemes will be close to that of the conventional OFDM-IM. This reflects a diversity-multiplexing trade-off in terms of $N_S$ in adaptive OFDM-IM, which is worth further investigating as
For the effects of selection always outperforms the system using the centralized adaptive OFDM-IM in two-hop networks. Meanwhile, it is presented the results in Fig. 6. The numerical results verify our average network capacity for different transmission schemes and the asymptotic relations in Section IV-A and the diversity discussion above.

### B. Simulations of network capacity

By taking the same settings for the simulations of outage probability, we carried out Monte Carlo simulations to study the average network capacity for different transmission schemes and present the results in Fig. 6. The numerical results verify our analysis in Section IV-B. Therefore, they can be used as effective tools to estimate the average network capacity of systems applying adaptive OFDM-IM in two-hop networks. Meanwhile, it is expected that the system using the decentralized mapping scheme selection always outperforms the system using the centralized mapping scheme selection in terms of average network capacity. For the effects of $N_S$, the situation becomes complicated and interesting. On the one hand, with a small $P_t/P_n$, a smaller $N_S$ will lead to a higher average network capacity, since the power allocated in each subcarrier matters and has a significant impact on the capacity. Therefore, when $N_S$ is small, the average transmit power allocated to each active subcarrier will be large, which yields a positive impact on the network capacity. On the other hand, when $P_t/P_n$ is large, because of the logarithmic function (c.f. (18)), the power allocated in each subcarrier only has a small impact on the capacity, while the number of terms taken in the summation operation matters. Therefore, as long as $P_t/P_n$ is sufficiently large, a larger $N_S$ will lead to a higher network capacity.

Besides, both decentralized and centralized adaptive schemes outperform FPSK in terms of average network capacity. However, the proposed adaptive schemes do not always outperform the conventional OFDM-IM when $P_t/P_n$ is large. This indicates that if the network capacity is regarded as a major system performance metric, our proposed adaptive schemes are particularly useful for cell-edge communications with a relatively small average SNR. The ratio of transmit power to noise power by which the capacity without adaptation is equal to that with adaptation (i.e. the crossing point) is termed critical power ratio. We have numerically simulated the relation between critical power ratio and $N_S$ with different $N_T$ for both decentralized and centralized cases and have presented the results in Fig. 7. From this figure, we can see that the system without adaptation would require a large transmit power in order to have a higher network capacity than that with adaptation, especially when $N_S$ is large. Therefore, this further confirms the applicability of our proposed adaptive schemes in practice.

### C. Simulations of error performance

Fixing the total number of subcarrier $N_T = 4$ and the APM order $M = 4$ (QPSK), we adopted different numbers of selected subcarriers $N_S$, and numerically simulated the relations between average SER and the ratio of transmit power to noise power $P_t/P_n$ for both decentralized and centralized mapping scheme selections. The simulation results are plotted in Fig. 8. From this figure, we can see that the derived approximations based on (51) can effectively estimate the SER at high SNR. Also, the diversity orders shown in the simulation results align with our expectation, and are equal to $N_T - N_S$, which is the same as average outage probabilities. A smaller $N_S$ will lead to a better error performance as expected. Compared with conventional OFDM-IM and FPSK schemes, our proposed schemes always exhibit a better error performance when $P_t/P_n$ is relatively large, because of their adaptive mechanisms. Also, OFDM-IM with the decentralized mapping scheme selection outperforms OFDM-IM with the centralized mapping scheme selection in terms of error performance, as an extra degree of freedom is obtainable by changing the mapping scheme at the relay in the former case.

Meanwhile, the assumption of perfect CSI would not be realistic, and the imperfect CSI could affect the mapping scheme selection and OFDM-IM symbol estimation processes. Therefore, we also numerically investigate the effects of imperfect CSI in this subsection. To quantify the effects of imperfect CSI, we involve the circularly-symmetric Gaussian random variables with variance $\delta_S$, $\delta_R$ and $\delta_D$ to mimic the estimation errors at the source, relay and destination, respectively. To be specific, we model the channel variable with estimation errors by $h_i(\tau(n)) = h_i(n) + \Delta_T(n)$ where $\Delta_T(n) \sim \mathcal{CN}(0, \delta_T)$ and $\tau \in \{S, R, D\}$. Because the system applying decentralized mapping scheme selection requires CSI at the relay, and is thereby more generic for exploring the effects of imperfect CSI, we simulate the case of decentralized mapping scheme selection and fix $N_T = 4$, $N_S = 1$ and $M = 4$. The simulation results are shown in Fig. 9. From these numerical results, we can observe that error floors are inevitable at high SNR for cases utilizing imperfect CSI. Also, the imperfect CSI at the relay will yield a severer impact on the fidelity, as the CSI is required for both mapping scheme selection and symbol estimation at the relay. Meanwhile, it can also be observed that the impact of imperfect CSI at the destination for symbol estimation is more significant than that at the source for mapping scheme selection. Therefore, the priority order for obtaining accurate CSI is: relay > destination > source, which provides a hint for analyzing and optimizing the proposed system with imperfect CSI.

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10A similar trade-off for MIMO systems has been found and analyzed in [40].
In this paper, we proposed an adaptive OFDM-IM scheme for two-hop DF relay networks, which is capable of obtaining diversity gains by adaptively selecting a portion of the total available OFDM subcarriers, instead of using all for OFDM-IM. This provides a possible mechanism allowing reducing throughput for gaining reliability. Considering the complexity and processing capability of relays, we also raised two mapping scheme selection methodologies, which can be applied to different network scenarios. We analyzed a series of important performance metrics, including average outage probability, network capacity and SER. The average outage probability and network capacity can be determined in closed form, while the average SER can be approximated by closed-form expressions, and the approximations are accurate at high SNR. The numerical results substantiated our analysis and also verified the feasibility of the proposed schemes in practice compared to conventional OFDM-IM without adaptation as well as PSK. By the analytical and numerical results provided in this paper, a comprehensive framework for analyzing adaptive OFDM-IM in two-hop relay-assisted networks has been constructed, which can be modified to analyze other extended cases with more complicated channel models and transmission protocols.

On the other hand, through the simulations regarding imperfect CSI, it is worth pointing out that the superior performance of the adaptive OFDM-IM proposed in this paper relies on the acquisition of accurate CSI, which is used for mapping scheme selection and symbol estimation. Consequently, the practicality of the adaptive OFDM-IM in fast fading environment would be weakened, since the instantaneous CSI would be difficult to obtain or a high signaling overhead will be caused. Therefore, the analysis vis-à-vis imperfect CSI, partial CSI and/or statistical CSI as well as the complexity-performance trade-off would be worth investigating as future works.

VI. CONCLUSION

In this paper, we proposed an adaptive OFDM-IM scheme for two-hop DF relay networks, which is capable of obtaining diversity gains by adaptively selecting a portion of the total available OFDM subcarriers, instead of using all for OFDM-IM. This provides a possible mechanism allowing reducing throughput for gaining reliability. Considering the complexity and processing capability of relays, we also raised two mapping scheme selection methodologies, which can be applied to different network scenarios. We analyzed a series of important performance metrics of the two-hop relay networks with OFDM-IM, including average outage probability, network capacity and SER. The average outage probability and network capacity can be determined in closed form, and the asymptotic expressions of average outage probability at high SNR were also given in closed form, by which the diversity gain is revealed. Meanwhile, the average SER can be approximated by closed-form expressions, and the approximations are accurate

APPENDIX A
PROOF OF LEMMA 1

When $b_S(k)$ is transmitted, we need to activate $N_A(k)$ subcarriers from $N_S$ selected subcarriers. Suppose that the worst subcarrier among these $N_A(k)$ active subcarriers in terms of channel gain is the $\xi$th worst in the total ordered $N_T$ subcarriers. Because of the mapping scheme selection, it is easy to derive the relation $N_T - N_S + 1 \leq \xi \leq N_T - N_A(k) + 1$. As a result, by (27), the outage probability when the worst subcarrier among these $N_A(k)$ active subcarriers is the $\xi$th worst in the total ordered $N_T$ subcarriers can be written as $F_{\xi}(\xi; \frac{sP_sN_A(k)}{P_n})$.

The total number of combinations of $N_A(k)$ active subcarriers is $\binom{N_S}{N_A(k)}$, and the number of possible combinations in which the worst subcarrier among these $N_A(k)$ active subcarriers is the $\xi$th worst is $\binom{N_S - \xi}{N_A(k) - 1}$. Consequently, the probability that the worst
Finally, when $b_S(k)$ is transmitted, the weighted average of the conditional outage probability over $\xi$ gives (29).

\[ p(i,j) = \left( \frac{1}{\text{Card}(D(k))} \right)^2 = \left( \frac{(N_S - N_A(k))!}{N_S!} \right)^2, \forall i,j, \]  

APPENDIX B

PROOF OF LEMMA 2

Because $\Xi_1$ and $\Xi_2$ are independently, discretely and uniformly distributed over domain $D(k)$, the joint probability mass function (PMF) with respect to $\Xi_1$ and $\Xi_2$, i.e. the probability that the $i$th element in $D(k)$ is chosen for $\Xi_1$ and the $j$th element in $D(k)$ is chosen for $\Xi_2$ is thereby

in which the corresponding network capacity can be written as

\[ C(k|i,j) = \sum_{n \in N_A(k)} \Delta_G \left( \xi_n, \eta_n, \frac{P_t N_A(k)}{P_t} \right). \]  

Hence, the weighted average of $C(k|i,j)$ over $\Xi_1$ and $\Xi_2$ is

\[ C(k) = \sum_{\xi_1(k) \in D(k)} \sum_{\xi_2(k) \in D(k)} p(i,j) C(k|i,j), \]  

which yields (45).

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