Efficient solution of a multi objective fuzzy transportation problem

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Abstract. In this paper we present a methodology for the solution of multi-objective fuzzy transportation problem when all the cost and time coefficients are trapezoidal fuzzy numbers and the supply and demand are crisp numbers. Using a new fuzzy arithmetic on parametric form of trapezoidal fuzzy numbers and a new ranking method all efficient solutions are obtained. The proposed method is illustrated with an example.

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1. Introduction
The transportation problem is a special type of linear programming problem where the objective is to minimize the cost of distributing a product from a number of sources to a number of destinations. In many real world problems there are situations where several objectives are to be considered and optimized at the same time. Such problems are called multi objective problems. Moreover the input data or parameters are imprecise because of incomplete or non-obtainable information. Hence traditional Mathematical techniques cannot solve all type of problems. The term fuzzy was proposed by Zadeh [9] in 1965. Large number of Methods has been developed by several researchers to solve multi objective fuzzy transportation problem.

Zimmermann [10] gave the fuzzy programming technique for the multi-objective transportation problem. Sathy Prakash [6] provides the new algorithm to obtain the set of Pareto optimal solution of the multi objective transportation problem. Doke et.al [3] has discussed a three objective transportation problem using fuzzy compromise programming approach. Sukhveer Singh [7] convert bi-objective transportation problem to a single objective transportation problem and find the fuzzy optimal solution for fuzzy cost and fuzzy time. Annie Christi [1] transformed multi-objective transportation model with intuitionistic fuzzy values transformed into a multi-objective linear programming model. Mirmohseni et. al. [4] proposed a fuzzy interactive possibilistic approach for solving fuzzy multi objective Solid Transportation Problem. Sungeeta Singh et.al [8] has proposed a simple method to obtain the efficient cost-time trade-off pairs in Multi-Objective Bulk Transportation Problem. Osuji et.al [5] obtained the solution of multi-objective transportation problem via fuzzy programming algorithm. In this paper we obtained the efficient solution of multi-objective fuzzy transportation problem involving trapezoidal fuzzy numbers and all the cost – time trade off pairs.
The rest of the paper is organized as follows: Section 2 gives the basic concepts of trapezoidal fuzzy numbers, rank, and their arithmetic operators. In Section 3, we define formulation of multi-objective fuzzy transportation problem and in Section 4, we propose an algorithm for the solution of multi-objective fuzzy transportation problem involving trapezoidal fuzzy numbers. The method is illustrated by means of numerical example and the solution is obtained in Section 5.

2. Preliminaries

Definition 2.1: A fuzzy set \( \tilde{A} \) defined on the set of real numbers \( \mathbb{R} \) is said to be a fuzzy number if its membership function \( \tilde{A}: \mathbb{R} \rightarrow [0,1] \) has the following characteristics:

(i) \( \tilde{A} \) is convex (i.e.) \( \tilde{A}(\lambda x_1 + (1-\lambda)x_2) \geq \min\{\tilde{A}(x_1), \tilde{A}(x_2)\}, \quad \lambda \in [0,1] \) for all \( x_1, x_2 \in \mathbb{R} \)

(ii) \( \tilde{A} \) is normal (i.e.) there exists an \( x \in \mathbb{R} \) such that \( \tilde{A}(x) = 1 \).

(iii) \( \tilde{A} \) is piece-wise continuous.

Definition 2.2: A fuzzy number \( \tilde{A} \) is a trapezoidal fuzzy number denoted by \( \tilde{A}(a_1, a_2, a_3, a_4) \), where \( a_1, a_2, a_3, a_4 \) are real numbers and its membership function is given by

\[
\tilde{A}(x) = \begin{cases} 
\frac{x-a_1}{a_2-a_1} & \text{if } a_1 \leq x \leq a_2 \\
1 & \text{if } a_2 \leq x \leq a_3 \\
\frac{a_4-x}{a_4-a_3} & \text{if } a_3 \leq x \leq a_4 \\
0 & \text{otherwise}
\end{cases}
\]

We use \( F(\mathbb{R}) \) to denote the set of all trapezoidal fuzzy numbers defined on \( \mathbb{R} \). We represent the trapezoidal fuzzy number \( \tilde{A} = (a_1, a_2, a_3, a_4) \) in terms of its core \( [a_2, a_3] \), left spread \( \alpha = (a_2 - a_1) \) and right spread \( \beta = (a_4 - a_3) \) as \( \tilde{A} = (a_1, a_2, a_3, a_4) = ([a_2, a_3], \alpha, \beta) \). Also, the core \( [a_2, a_3] \) can be expressed as \( (m, w) \), where \( m = \left( \frac{a_2 + a_3}{2} \right) \) and \( w = \left( \frac{a_4 - a_3}{2} \right) \) are the midpoint and width of the core \( [a_2, a_3] \) respectively. Without loss of generality, we represent the trapezoidal fuzzy number \( \tilde{A} = (a_1, a_2, a_3, a_4) \) as \( \tilde{A} = (a_1, a_2, a_3, a_4) = ([a_2, a_3], \alpha, \beta) = ((m, w), \alpha, \beta) \).

2.1 Ranking of Trapezoidal Fuzzy Numbers

An efficient approach for comparing the fuzzy numbers is the ranking function based on the graded means. For every \( \tilde{A} = (a_1, a_2, a_3, a_4) \in F(\mathbb{R}) \), define the ranking function \( R: F(\mathbb{R}) \rightarrow \mathbb{R} \) by its graded mean as

\[
R(\tilde{A}) = \left[ \left( \frac{a_2 + a_3}{2} \right) + \frac{\beta - \alpha}{4} \right] = \left[ m + \frac{\beta - \alpha}{4} \right].
\]
2.2. Arithmetic Operations on Trapezoidal Fuzzy Numbers

For arbitrary trapezoidal fuzzy numbers \( \tilde{A} = (m(A), w(A), \alpha_i, \beta_i) \) and \( \tilde{B} = (m(B), w(B), \alpha_j, \beta_j) \) and \( * = (+, - , \times , /) \), the arithmetic operations on trapezoidal fuzzy numbers are defined by

\[
\tilde{A} * \tilde{B} = (\langle m_1, w_1, \alpha_i, \beta_i \rangle * \langle m_2, w_2, \alpha_j, \beta_j \rangle) = \langle m_1 * m_2, w_1 \vee w_2, \alpha_1 \vee \alpha_j, \beta_1 \vee \beta_j \rangle
\]

That is the midpoint is taken in the ordinary arithmetic, whereas the width, left and right spread are considered to follow the lattice rule. That is for \( a, b \in L \), define \( a \vee b = \max\{a, b\} \) and \( a \wedge b = \min\{a, b\} \).

3. Multi-objective fuzzy transportation problem

Consider a multi-objective fuzzy transportation problem with \( m \) sources and \( n \) destinations. Let \( a_i \) (\( i = 1, 2, ..., m \)) and \( b_j \) (\( j = 1, 2, ..., n \)) be the unit availability at source \( i \) and the unit demand at the destination \( j \) respectively. Let \( C \) and \( T \) represents the total fuzzy cost and total fuzzy time of transportation respectively. The mathematical formulation of the problem is as follows:

Determining \( x_{ij} \)'s which minimize the two objective functions

\[
\hat{C} = \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{C}_{ij} x_{ij} \quad \hat{T} = \max\{\tilde{t}_{ij}, i = 1, 2, ..., m, j = 1, 2, ..., n\}
\]

Subject to the constraints

\[
\sum_{j=1}^{n} b_j x_{ij} \leq a_i \quad (i = 1, 2, ..., m)
\]

\[
\sum_{i=1}^{m} x_{ij} = 1 \quad (j = 1, 2, ..., n) \quad \text{and} \quad x_{ij} = 0 \quad \text{(or) 1} \quad (i = 1, 2, ..., m; j = 1, 2, ..., n),
\]

where \( \tilde{C}_{ij} (i = 1, 2, ..., m, j = 1, 2, ..., n) \) be the fuzzy cost of transportation of the product from \( i \)th source to \( j \)th destination, \( \tilde{t}_{ij} \) be the fuzzy time of transportation of the product from \( i \)th source to \( j \)th destination and \( x_{ij} \) is the variable assuming 0 or 1 depending upon the entire requirement of destination \( j \) is fulfilled from source \( i \).

**Definition 3.1:** [Efficient solution / Cost-time pair]

Let \( \tilde{C}_1 \) and \( \tilde{T}_1 \) be the minimum cost and corresponding time of transportation respectively. Let \( \tilde{Z}_1 \) be the solution for the first efficient cost-time pair\( (\tilde{C}_1, \tilde{T}_1) \). Let \( \tilde{C}_2 (> \tilde{C}_1) \) be another cost of transportation and \( \tilde{T}_2 (< \tilde{T}_1) \) be the minimum time of transportation at cost \( \tilde{C}_2 \). Then the solution \( \tilde{Z}_2 \) be the second efficient cost-time trade-off pair solution, if there exists no other solution pair \( (\tilde{C}, \tilde{T}) \) such that \( \tilde{C}_1 < \tilde{C} < \tilde{C}_2 \) and \( \tilde{T}_1 < \tilde{T} < \tilde{T}_2 \). Similarly the ith efficient solution may be determined.
3.1 Algorithm
An algorithm to find the efficient solution of multi-objective fuzzy transportation problem is as follows:

Step 1: Represent each fuzzy data $\tilde{A} = (a_1, a_2, a_3, a_4)$ in terms of parametric form $\tilde{A} = (a_1, a_2, a_3, a_4) = ([a_2, a_4], \alpha, \beta) = (m, w, \alpha, \beta)$.

Step 2: Remove the cells $(i, j)$ from the initial multi-objective fuzzy cost transportation table for which requirement of destination $b_j (j = 1, 2, ..., n)$ exceeds the availability of source $a_i (i = 1, 2, ..., m)$.

Step 3: Using the proposed ranking function find the minimum cost for each row and subtract it from all the costs of corresponding row. Repeat the same process in column-wise and there will be at least one zero in each row and each column in the reduced cost table.

Step 4: For each zero in the reduced cost table, add the number of zeros in corresponding row and column except itself.

Step 5: Allocate 1 to the cell for which number of zeros in the reduced cost table is minimum. If there is tie, then select the zero for which sum of all the entries in the corresponding row and column is maximum. Again if there is tie then select the cell for which the maximum allocation is made.

Step 6: Delete the destination from the table whose demand is satisfied and also remove the source whose availability is zero or less than demand of each destination. Repeat the steps 2 to 5 unless all the demands are satisfied and all the supplies are exhausted. Then we get first efficient solution of cost and corresponding time.

Step 7: To find the next efficient cost-time trade off pair define the new cost-time fuzzy transportation table by removing the cells whose rank of time coefficients is greater than or equal to rank of $\hat{T}$ in the first efficient solution of cost-time trade off pair.

Repeating the same process, we find all the subsequent cost-time trade-off pair of multi-objective fuzzy transportation problem.

4. Numerical example
Consider a multi-objective fuzzy transportation problem in which the cost coefficients (upper entries) and time coefficients (lower entries) are trapezoidal fuzzy numbers.

| $S_i$ | $D_1$ | $D_2$ | $D_3$ | $D_4$ | $D_5$ |
|-------|-------|-------|-------|-------|-------|
| $S_1$ | (0,1,2.5,8) | (1,2,3,6) | (1,2,3,6) | (2,5,7,14) | (0,0.5,1.5,2) |
|       | (1,3,4,8) | (1,3,4,8) | (3,7,10,20) | (3,5,8,16) | (2,5,7,14) |
| $S_2$ | (1,3,4,8) | (0,0.5,1.5,2) | (0,0.5,1.5,2) | (0,1,2.5) | (3,5,8,16) |
|       | (1,3,4,8) | (2,5,7,14) | (5,7,12,24) | (5,9,14,28) | (3,5,8,16) |
| $S_3$ | (0,0.5,1.5,2) | (2,5,7,14) | (5,6,11,22) | (0,0.5,1.5,2) | (1,4,5,10) |
Table 2: Fuzzy Transportation Problem with first objective in parametric form

| \(S_i\) | \(D_j\) | \(D_k\) | \(D_m\) | \(D_n\) |
|--------|--------|--------|--------|--------|
| \((3,5,8,16)\) | \((0,1,2,5)\) | \((1,3,4,8)\) | \((1,3,4,8)\) | \((1,3,4,8)\) |
| \((9,11,20,40)\) | \((13,17,30,60)\) | \((3,7,10,20)\) | \((0,1,2,5)\) | \((1,4,5,10)\) |
| \((1,3,4,8)\) | \((2,4,6,12)\) | \((2,5,7,14)\) | \((0,0,5,15,2)\) | \((0,0,5,15,2)\) |
| 3 | 3 | 2 | 2 | 1 |

Apply step 2 to step 4 the reduced cost table is given in table 3.

Table 3: Fuzzy Transportation Problem with reduced cost table

| \(D_i\) | \(D_j\) | \(D_k\) | \(D_m\) | \(D_n\) |
|--------|--------|--------|--------|--------|
| \(S_1\) | 1 | 2 | 6 | 0 | 5 |
| \(S_2\) | 3 | 0 | 0 | 1 | 7 |
| \(S_3\) | 0 | 6 | 10 | 0 | 4 |
| \(S_4\) | - | - | 8 | 0 | 3 |

By using step 5 we get \(x_{15} = 1\). Repeat the same process the first efficient solution is given by \(X_1 = \{x_{15}, x_{22}, x_{13}, x_{31}, x_{14}\}\).

That is cost of transportation \(\tilde{C}_1 = \tilde{C}(X_1) = (5.5, 6.5, 7.5, 10.5)\) and the corresponding time of transportation \(\tilde{T}_1 = \tilde{T}(X_1) = (5, 7, 12, 24)\) and the first cost-time trade off pair is \((\tilde{C}_1, \tilde{T}_1)\).

By using step 7 we find the second efficient solution is \(X_2 = \{x_{22}, x_{13}, x_{15}, x_{31}, x_{14}\}\).

The second cost-time trade pair is \((\tilde{C}_2, \tilde{T}_2)\) where \(\tilde{C}_2 = \tilde{C}(X_2) = (7, 10, 13, 20)\) and \(\tilde{T}_2 = \tilde{T}(X_2) = (3, 7, 10, 20)\).

Using this procedure, the fuzzy efficient optimal solution is obtained and shown in table 4.
Table 4: Fuzzy Efficient Solution

| Optimal Solution | Total Fuzzy Cost | Total Fuzzy Time |
|------------------|------------------|-----------------|
| $X_1 = \{x_{11}, x_{21}, x_{31}, x_{41}\}$ | $\tilde{C}_1 = (5.5, 6.5, 7.5, 10.5)$ | $\tilde{T}_1 = (5, 7, 12, 24)$ |
| $X_2 = \{x_{12}, x_{13}, x_{32}, x_{42}\}$ | $\tilde{C}_2 = (7, 10, 13, 20)$ | $\tilde{T}_2 = (3, 7, 10, 20)$ |
| $X_3 = \{x_{13}, x_{22}, x_{13}, x_{33}, x_{34}\}$ | $\tilde{C}_3 = (8.5, 12.5, 14.5, 24.5)$ | $\tilde{T}_3 = (2, 5, 7, 14)$ |
| $X_4 = \{x_{12}, x_{43}, x_{21}, x_{33}, x_{34}\}$ | $\tilde{C}_4 = (22.19, 5, 20.5, 31.5)$ | $\tilde{T}_4 = (1.3, 4.8)$ |

5 Conclusion

We have proposed a simple method to find the efficient solution of multi-objective fuzzy transportation problem involving trapezoidal fuzzy numbers. The proposed method is easy to apply and also reduces the computational work.

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