Relativistic Effects in the Transverse Flow in the Molecular Dynamics Framework

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Abstract

In order to investigate relativistic effects we compare the transverse flow calculated by using the four versions of the QMD approaches with that of the full covariant RQMD approach. From the comparison we conclude that the simplified RQMD (RQMD/S), which uses the common time coordinate to all particles, can be used instead of RQMD up to 6 GeV/u.
The Quantum Molecular Dynamics (QMD) approach is one of the most powerful models to describe heavy-ion [1] and light-ion [2] reactions in the several tens to the several hundreds MeV/u energy region. In the high energy region above about 1 GeV/u this approach is also useful though relativistic effects become significant. The relativistic effects mean not only the relativistic kinematics, but also the Lorentz covariance of the interactions.

In the relativistic energy region we have the following problems when we introduce the relativistic kinematics and the Lorentz transformation into the non-covariant QMD approach. First, the increase of the initial density due to the Lorentz contraction makes an additional repulsion through the density-dependent force, which causes the spurious excitation and the unphysical instability of initial nuclei. Second, we cannot correctly evaluate the internal energies of fast-moving fragments at the end of the QMD calculation because the non-relativistic mean-field used in QMD is variant under the Lorentz transformation. In fact, these effects clearly appear in the transverse flow [3, 4] and in the multiplicity of alpha particles in the heavy-ion collisions even at $E_{\text{lab}} \sim 1$ GeV/u [5].

At the relativistic energy, therefore, the Lorentz covariant transport approach is desirable to make all nuclei and fragments hold the consistent phase-space distribution under the Lorentz transformation.

The Relativistic QMD (RQMD) approach [6, 7] is the most useful theoretical model for this purpose; it is formulated to describe the interacting $N$-body system in a fully Lorentz covariant way based on the Poincare-Invariant Constrained Hamiltonian Dynamics [8]. The position and momentum coordinates of the $i$-th particle, $q_i$ and $p_i$, are defined as four-dimensional dynamical variables and the functions of the time evolution parameter $\tau$. The on-mass-shell constraints are given by

$$H_i \equiv p_i^2 - m_i^2 - 2m_iU_i = 0,$$

where $m_i$ and $U_i(q_j,p_j)$ are the mass and the Lorentz scalar quasi potential for the $i$-th particle. The explicit form of the quasi-potential is determined by the requirement that it corresponds to the non-relativistic mean-field in the low energy limit [6, 7]. Whereas in the non-relativistic framework the argument of the potential is a square of the relative distance $r_{ij}^2$ between two particles, in RQMD we take it as a square of the relative distance in the rest frame of their CM system as

$$- q_{Tij}^2 = -q_{ij}^2 + \frac{(q_{ij} \cdot p_{ij})^2}{p_{ij}^2}$$

with

$$q_{ij} = q_i - q_j, \quad p_{ij} = p_i + p_j.$$

In this formulation the time coordinate $q_i^0$ is distinguished from the time evolution parameter $\tau$ and constrained by the following time-fixation as

$$\chi_i \equiv \sum_{j(\neq i)} \frac{e^{q_{Tij}/L_c}}{q_{Tij}^2/L_c} q_{ij}p_{ij} = 0.$$

Under this time-fixation the colliding two particles have equal values of their time coordinate at the their CM frame in the dilute gas limit.
Using the above on-mass-shell and the time-fixation constraints, we can describe the Lorentz covariant motions of particles. These constraints are chosen to be completely consistent in the non-relativistic framework at the non-relativistic limit of $m_i \to \infty$.

The above expression includes some important features of the Lorentz covariance as follows. First, the quasi-potential is given as the Lorentz scalar while it is given as a time coordinate of the Lorentz vector in the usual QMD approach. Second, the change from $r_{ij}^2$ to $-q_{ij}^2$ causes the direction dependent forces. The attractive force in the fast moving nuclei is stronger in the moving direction than in the transverse direction. The above two effects keep the Lorentz contracted phase-space distribution of fast moving matter stable.

The mean-field affects only low energy behaviors in the reaction. Without the Lorentz covariance in the mean-field, however, we cannot correctly give intrinsic motions of nucleons in the fast moving nuclei. Namely the above expression can describe relatively low energy phenomena in the fast moving system.

There are, however, two kinds of problems in applying the RQMD approach to the analysis of experimental data. One is the fact that it needs too long CPU time, and the other is the energy conservation problem. In RQMD, it is not easy to satisfy the energy conservation after the meson production and absorption. The change of the particle number due to the meson production and absorption breaks the time fixation. Thus we have to resolve the values of $q_0^i$ and $p_0^i$ to impose the constraints (the on-mass shell condition and the time-fixation) again. This procedure leads to the discontinuity of particle coordinates and causes the change of the potential energy. Particularly the latter problem is very serious to study the fragmentation process because IMF multiplicities are dominantly determined by the excitation energy of the residual nucleus.

In order to avoid these difficulties we propose the simplified version of RQMD (RQMD/S) in the following way. Equation (4) gives the time-fixation requiring equal time coordinates of two colliding particles in their center-of-mass system in the dilute gas limit. However a choice of the time fixation is completely arbitrary except for the non-relativistic limit and the cluster separability. We thus adopt the time fixations to equalize the time evolution parameter $\tau$ and all time coordinates of baryons and mesons as

$$\chi_i \equiv \hat{a} \cdot (q_i - q_N) = 0 \quad (i = 1 \sim N - 1),$$

where $\hat{a}$ is a four-dimensional unit-vector taken as $(1; \vec{0})$ in the reference frame. By this simplified choice of the time fixations, the energy conservation after the change of particle number is always satisfied. Though the time-coordinate has a physical meaning only in the reference frame, the above time-fixations are still defined in particle-motions Lorentz covariantly by the mean-field.

Of course these time-fixations break the Lorentz covariance in the two-body collisions at the ultra-relativistic energy. In our work, however, we are interested in the phenomena around several GeV/u, where our simplification does not make any significant troubles.

To reduce the computation time, furthermore, we make an approximation that the momentum coordinate $p_0^i$ in the argument of the quasipotential is replaced by the kinetic energy $\varepsilon_i = \sqrt{p_i^2 + m_i^2}$. This approximation does not affect the final results much because the quasi-potential is much smaller than the kinetic energy in the relativistic energy region.

Now we must examine the above simplified RQMD (RQMD/S) before applying it to the data analysis. For this purpose we arrange the five kinds of methods as follows.
The first is the standard QMD, so called QMD/G, which does not include the Lorentz contraction of the initial distribution. The second is the QMD/L, which is QMD including the Lorentz contracted initial distribution. The third is the QMD/R, where the mean-field is treated as a time-component of the vector type, but the argument is varied along the eqs. (3, 4). The forth is the RQMD/S which is the simplified RQMD explained above. The last one is the full RQMD. Then we calculate the directed transverse momentum, which is most sensitive to the relativistic effects [3], defined as

\[ <P_{x}^{\text{dir}} > = \frac{1}{A} \sum_{i=1}^{A} \text{sign}[Y_{CM}(i)] p_{x}(i), \] (6)

where \( Y_{CM}(i) \) and \( p_{x}(i) \) are the center-of-mass rapidity and the transverse momentum in the reaction plane of the \( i \)-th nucleon.

In the numerical calculations the predictor-corrector method is used to integrate the equation of motion. For the two-body effective interaction, we use a Skyrme-type interaction with HARD EOS (the incompressibility \( K = 380 \) MeV). The widths of the wave packets are taken from the values for Ca + Ca in Ref. [10]. The initial nuclei are given with the cooling method [11]. We omit the Coulomb force and two-body collision term for a simplicity because our purpose is the examination of the relativistic effects.

In Fig. 1 we show the energy dependence of \( <P_{x}^{\text{dir}} > \). We plot the result of the QMD/G, QMD/L, QMD/R and RQMD/S simulation as a difference from that of RQMD [7] for \( ^{40}\text{Ca} + ^{40}\text{Ca} \) reactions at \( b = 2 \) fm, for the energy range from 150 MeV/u to 6 GeV/u.

As mentioned before, the Lorentz contraction of the initial phase space distribution increases the flow, which is shown by the change from the dot-dashed line to the dashed line. By the full covariant treatment of the interaction, however, this effect is counterbalanced with the Lorentz covariance of the mean-field, but still remains an increased flow from QMD/G value [1].

As seen in Fig. 4, the prescription of QMD/R does not deviate so much from the full covariant treatment up to 3 GeV/u. At much higher energy, however, the result of QMD/R is decreasing linearly from that of RQMD. On the other hand, RQMD/S gives results very similar to RQMD up to 6 GeV/u and more.

The difference between the RQMD (RQMD/S) and QMD/R comes from the different treatment of the potential; a Lorentz scalar type in the former, while a time-component of the vector type in the latter, respectively. This is understood qualitatively by considering a single particle motion under a fixed external potential \( U \). In the Lorentz scalar treatment of the potential \( U \), the single particle energy \( p_{i}^{0} \) is expressed in this simple case as

\[ p_{i}^{0} = \sqrt{p_{i}^{2} + m_{i}^{2} + 2m_{i}U}. \] (7)

Accordingly the equation of motion is

\[ \dot{p}_{i} = -\sum_{j} \frac{m_{j}}{p_{j}^{2}} \frac{\partial U_{j}}{\partial r_{i}}. \] (8)

On the other hand, in QMD/R they are

\[ p_{i}^{0} = \sqrt{p_{i}^{2} + m_{i}^{2} + U_{i}}, \] (9)
and
\[ \dot{p}_i = - \sum_j \frac{\partial U_j}{\partial r_i}. \] (10)

The form of \( \partial U_j/\partial r_i \), which is attractive at the early time stage of nucleus-nucleus collisions, is almost the same in the QMD/R and in the RQMD (RQMD/S). In the high energy region the single particle energy \( p_i^0 \) has approximately a same value for all nucleons in the early time stage. Thus the force of QMD/R becomes larger and deviates linearly from that of the RQMD (RQMD/S) as energy increases. Above 3 GeV/u, hence, the difference between the Lorentz scalar and a time component of the Lorentz vector becomes significant, and the full covariant prescription for the mean-field is necessary to describe the reactions, particularly the nucleus-nucleus collisions.

In summary we have calculated the directed transverse momentum using the five kinds of (R)QMD approaches and have discussed their relativistic effects. It has been shown that the Lorentz covariance of the mean-field is very important above about 1 GeV energy region. The RQMD/S can give almost the same results as the full RQMD in the transverse flow which is thought to be the most sensitive observables to the relativistic effects at present. Hence we can conclude that the RQMD/S approach can be used up to 6 GeV instead of the full RQMD approach.

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Figure 1: The directed transverse momentum as a function of energy per nucleon for $^{40}\text{Ca} + ^{40}\text{Ca}$ reaction at $b = 2$ fm. The results are shown as the differences from that of the RQMD [7]. The dot-dashed line with open circles, the dashed line with open boxes, the dotted line with full boxes and the solid line with full boxes denote the results of QMD/G, QMD/L, QMD/R and RQMD/S, respectively.
