Josephson current through a Kondo molecule

Rosa López,1 Manh-Soo Choi,2,3 and Ramón Aguado4

1Departament de Física, Universitat de les Illes Balears, E-07122 Palma de Mallorca, Spain
2Department of Physics, Korea University, Seoul 136-701, Korea
3Department of Physics and Astronomy, University of Basel, Klingelbergstrasse 82, 4056 Basel, Switzerland
4Teoría de la Materia Condensada, Instituto de Ciencia de Materiales de Madrid (CSIC) Cantoblanco,28049 Madrid, Spain.

(Dated: March 23, 2022)

We investigate transport of Cooper pairs through a double quantum dot (DQD) in the Kondo regime and coupled to superconducting leads. Within the non-perturbative slave boson mean-field theory we evaluate the Josephson current for two different configurations, the DQD coupled in parallel and in series to the leads. We find striking differences between these configurations in the supercurrent as a function of the ratio \( t/\Gamma \), where \( t \) is the interdot coupling and \( \Gamma \) is the coupling to the leads: the critical current \( I_c \) decreases monotonously with \( t/\Gamma \) for the parallel configuration whereas \( I_c \) exhibits a maximum at \( t/\Gamma = 1 \) in the serial case. These results demonstrate that a variation of the ratio \( t/\Gamma \) enables to control the flow of supercurrent through the Kondo resonance of the DQD.

PACS numbers: 73.23.-b,72.15.Qm,74.45.+c

Introduction.— A localized spin (magnetic impurity) forms a many-body singlet with the itinerant electron spins surrounding it, and hence is screened. This, known as Kondo effect, happens at temperatures \( T \) lower than the Kondo temperature \( T_K \), which is the binding energy of the many-body singlet state \([1]\). It manifests itself as a quasi-particle resonance, namely, a peak of width \( T_K \) at the local density of states (LDOS) at the impurity site. In mesoscopic structures, such as a quantum dot (QD) coupled to normal electrodes, it leads to the unitary conductance as predicted theoretically \([2]\) and demonstrated experimentally \([3]\). Ever since, nanoscale systems have been of particular interest as they allow ultimate tunability and enable thorough investigations of the Kondo physics.

It is more intriguing when one couples localized spins to superconducting reservoirs and brings about the competition between superconductivity and Kondo effect \([4]\). The subgap transport through the Andreev bound states \([5, 6]\) (ABSs) in nanostructures between superconducting contacts is strongly affected by Kondo physics, leading to the phase transition in the sign of Josephson current \([7]\). The large Coulomb interaction prevents the tunneling of Cooper pairs into QD; electrons in each pair tunnel one by one via virtual processes \([8]\). Due to Fermi statistics, it results in a negative supercurrent (i.e., a \( \pi \)-junction). However, this argument is only valid in the weak coupling limit, when the gap \( \Delta \) is larger than \( T_K \). In the opposite strong coupling limit \((\Delta/T_K \ll 1)\) the Kondo resonance restores the positive Josephson current \([9]\). Lately, a S-QD-S system (S denotes superconductor) has been experimentally realized, confirming some of these physical aspects \([1]\).

The ongoing progress in semiconductor nanotechnology has brought more complicated nanostructures such as molecule-like double quantum dots \([9]\) (DQD’s). The latter is an unique tunable system for studying interactions between localized spins \([10]\). On one hand, they are basic building blocks for quantum information processing \([11]\) as a source of entangled states \([12]\). On the other hand, they are a prototype of the tunable two-impurity Kondo model \([13]\), as demonstrated experimentally \([12, 14]\). A natural step forward is to study the artificial molecule coupled to superconductors. Choi, Bruder, and Loss \([15]\) studied the spin-dependent Josephson current through DQD in parallel in the Coulomb blockade (CB) regime and generation of entanglement between the dot spins. Here, we examine the strong coupling regime, both in parallel and in series. We find striking differences between these configurations in the supercurrent as a function of the ratio \( t/\Gamma \): the critical current \( I_c \) decreases monotonously with \( t/\Gamma \) for the parallel configuration whereas \( I_c \) exhibits a maximum at \( t/\Gamma = 1 \) in the serial case.

Model— A superconducting Kondo molecule is modelled as a two-impurity Anderson model where the normal leads are replaced by standard BCS s-wave superconductors. The Hamiltonian reads
\[ \mathcal{H} = \sum_{k \sigma \in \{L,R\} \sigma} \xi_{k \sigma} c_{k \sigma}^\dagger c_{k \sigma} + \sum_{k \sigma} \{ \Delta_\alpha \exp^{i \phi_\alpha} c_{k \sigma \alpha^\dagger} c_{-k \sigma \alpha} + h.c. \} \\
+ \sum_{i \in \{1,2\} \sigma} (\varepsilon_i n_\sigma + U i n_\sigma n_{i \bar{\sigma}}) + U_{12} n_1 n_2 + t \sum_i \left\{ d_{i \sigma}^\dagger d_{i \sigma} + h.c. \right\} + \sum_{i, k, \sigma, \sigma'} \left\{ V_{i,k \sigma} c_{i \sigma}^\dagger d_{i \sigma'} + h.c. \right\} . \] (1)

\[ d_{i \sigma} (d_i \sigma) \text{ describes the electrons with momentum } k \text{ and spin } \sigma \text{ in the lead } \alpha \text{ (the dot } i) ; n_{i \sigma} = d_{i \sigma}^\dagger d_{i \sigma} \text{. } \phi_\alpha (\Delta_\alpha) \text{ denotes the superconducting phase (gap) for the lead } \alpha \text{.} \]

\[ t (V_{i,k \sigma}) \text{ is the tunneling amplitude between the dots (the dots and leads).} \]

For a DQD in series \( V_{1,k L} = V_{2,k R} = 0 \) with \( V_{1,k L} = V_{2,k R} = V_0 \); for a DQD in parallel \( V_{1,k L} = V_{2,k L} = V_{2,k R} = V_0 \). \( \varepsilon_i \) are the single-particle energies on the dots tuned by gate voltages. \( U_i \) is the on-site Coulomb interaction on the \( i \text{th dot, and } U_{12} \) is the interdot Coulomb interaction.

We will be interested in the limit \( U_1, U_2 \to \infty, U_{12} = 0 \), and \( -\varepsilon_i \gg \Gamma \) so that \( n_{i \sigma} = 1 \) for each \( i = 1, 2 \). The model in this limit is well described in the slave-phonon language \( 14 \). We first write the physical fermionic operator as \( d_{i \sigma} = b_{i \sigma}^\dagger f_{i \sigma} \), where \( f_{i \sigma} (b_{i \sigma}) \) is the pseudofermion (boson) which destroys (creates) one “occupied (empty) state” on the dot \( i \). We then introduce two constraints which prevent double occupancy in each dot \( (U_1, U_2 \to \infty) \) by means of Lagrange multipliers, \( \lambda_1 \) and \( \lambda_2 \). The resulting model is solved within the mean-field (MF) approach, namely, replacing \( b_i (t) \to \bar{b}_i (t) = b_i \) and thereby neglecting charge fluctuations in the dots. This approach has been applied successfully to the Kondo molecules coupled to normal leads \( 10 \).

The MF version of \( \mathcal{H} \) is now quadratic and contains four parameters, i.e., \( b_{i \sigma}, \bar{b}_{i \sigma} \) that renormalizes the tunneling amplitudes, \( V_{i,k \sigma} = b_i V_{i,k \sigma} \) and \( \bar{t} = t b_i b_\sigma \) and \( \lambda_1 (2) \) that renormalizes the energy levels \( \bar{\varepsilon}_1 \) and \( \bar{\varepsilon}_2 \). They are determined from the solution of the MF equations in a self-consistently fashion. The MF equations become simpler in the Nambu-Keldysh space where \( \Psi_{i \sigma}^\dagger = \left( c_{i \sigma}^\dagger, c_{-i \bar{\sigma}} \right) \) and \( \Phi_{i \sigma}^\dagger = \left( f_{i \sigma}, f_i \right) \) are the spinors for the conduction and localized electrons.

From the equation-of-motion (EOM) of the boson fields and the constrains the MF equations read:

\[ \hat{b}_{1(2)}^\dagger (t) + \frac{1}{N} \sum_{\sigma} \left( \Phi_{1(2) \sigma}^\dagger (t) \hat{\sigma}_z \Phi_{1(2) \sigma} (t) \right) = \frac{1}{N}, \] (2a)

\[ \frac{1}{N} \sum_{\alpha \in \{L,R\} \sigma} \hat{V}_{1,k \sigma}^\dagger \Psi_{1 \sigma}^\dagger (t) \hat{\sigma}_z \Phi_{1 \sigma} (t) + \bar{t} \sum_{\sigma} \left( \Phi_{1 \sigma}^\dagger (t) \hat{\sigma}_z \Phi_{2 \sigma} (t) \right) + \lambda_1 (2) \hat{b}_{1(2)}^\dagger (t) = 0, \] (2b)

Hereforth, for simplicity we assume a symmetric structure with \( \bar{\varepsilon}_{1(2) \sigma} = \varepsilon_{0 \sigma}, \Delta_L (R) = \Delta, \) and \( \phi_L = -\phi_R = \phi. \)

The system of MF equations \( 2 \) is now written in terms of the \( 2 \times 2 \) matrix lesser Green function (GF) for the dots \( G_{i,j \sigma} (t, t') = i \langle \Psi_{i \sigma}^\dagger (t') \Phi_{j \sigma} (t) \rangle \) and the lead-dot matrix lesser GF \( \hat{G}_{i,k \sigma} (t, t') = i \langle \Psi_{i \sigma}^\dagger (t') \Phi_{i \sigma} (t) \rangle \) (with components \( \hat{G}^{11} = G \), “electron-like” GF \( \hat{G}^{21} = F \) is the anomalous propagator, \( \hat{G}^{21} = F^1 \), and \( \hat{G}^{11} = \hat{G} \) corresponds to the “hole-like” propagator). Following the standard procedure, the lesser GF’s are obtained applying rules of analytical continuation along a complex time contour to the EOM of the time-ordered GF (for details, see Ref. \( 12 \)). The two off-diagonal GF’s, namely, \( \hat{G}_{i,j \sigma} \) (with \( i \neq j \)) lead-dot GF \( \hat{G}_{i,k \sigma} \) can be cast in terms of the diagonal dot GF \( \hat{G}_{i,i \sigma} = \hat{G}_{i \sigma} \) using the EOM technique. By doing this Eq. \( 2 \) becomes (for simplicity we omit the spin indices)

\[ \hat{\Gamma} + \int_{-\infty}^{\infty} \frac{d\epsilon}{2\pi i} \left( G_{1(2)} (\epsilon) + \hat{G}_{1(2)}^< (\epsilon) \right) = 0, \] (3a)

\[ \hat{\Gamma}_i (\bar{\epsilon}_{1(2)} - \bar{\epsilon}_{1(2)}') + \int_{-\infty}^{\infty} \frac{d\epsilon}{2\pi i} \left( G_{1(2)} (\epsilon) - \bar{\epsilon}_{1(2)}' \right) \]

\[ + \bar{G}_{1(2)}^< (\epsilon')(\epsilon') = 0, \] (3b)

where \( \bar{\epsilon}_i = \varepsilon_0 + \lambda_i \) are the renormalized levels and \( \hat{\Gamma}_i = \hat{\Gamma}_i \Gamma \) the renormalized hybridization, which are equal to the Kondo temperature \( T_K \) for the coupled system \( 13 \) \[ \Gamma = 2\pi \rho \nu V_{12}^2 \] with \( \rho \nu \) being the normal-state DOS. At equilibrium we can employ \( \hat{G}_{1(2)} = \frac{1}{2} f_i (\epsilon) \hat{G}_{1(2)}^{11} \) with \( f_i (\epsilon) \) being the Fermi function. \( \hat{G}_{1(2)} \) is determined by the lead-dot \( \hat{\Sigma}_{1(2)} \) and the interdot tunneling \( \hat{\Sigma}_{1(2)} \) self-energies. Thus, the matrix elements of the lead-dot self-energy are:

\[ \hat{\Sigma}_{i \sigma}^{11,22} = -i \hat{\Gamma}_i \rho \hat{S} (\epsilon), \quad \hat{\Sigma}_{i \sigma}^{12} = (\hat{\Sigma}_{i \sigma}^{21})^* = -i \hat{\Gamma}_i \rho \hat{S} (\epsilon) \Delta \exp (i \hat{\phi}_\sigma) / |\epsilon| = \hat{\Gamma}_i \rho \hat{S} (\epsilon) \Delta \exp (i \phi_\sigma) / |\epsilon| \]

as the superconducting DOS and \( \hat{\Gamma}_i = 2\pi \rho \nu V_{i,k \sigma}^2 \rho \nu \). The interdot tunneling self-energy is \( \hat{\Sigma}_{12} = i \hat{\Gamma}_i \hat{\sigma}_z \hat{g}_{2(1)} \), where \( \hat{g}_{2(1)} \) is the matrix GF for an isolated QD. Since we deal with a symmetric structure \( \lambda_1 = \lambda_2 = \hat{\varepsilon}_0 = \hat{\varepsilon}_0 \) and \( \hat{\Gamma} = \hat{\Gamma}_i \). Thus, for example for the serial configuration the diagonal components of the GF read

\[ G^{\sigma \sigma} (\epsilon) = \frac{[(A + \hat{\varepsilon}_0) (A - \hat{\varepsilon}_0) - s(\epsilon) \epsilon^2 + \hat{\Gamma}_i^2 (A \mp \hat{\varepsilon}_0)]}{D(\epsilon)}, \] (4)

where \( D(\epsilon) = [A^2 - \hat{\varepsilon}_0^2 - Y^2 - (\hat{\varepsilon}_0 - \hat{\varepsilon}_i)^2 - 4\hat{\Gamma}_i^2 (Y^2 + \hat{\varepsilon}_i^2)] \) with \( X = s(\epsilon) \cos(\phi), Y = s(\epsilon) \sin(\phi), A = \)
The Josephson current for the DQD in series (see Eq. 6) describes the discrete Andreev bound states in the subgap region ($|\epsilon| < \Delta$) as well as the continuum spectrum above the gap ($|\epsilon| > \Delta$). The Andreev states appear as poles of the GF; i.e., the solutions of $D(\epsilon) = 0$ (see Fig. 1). Accordingly, the Josephson current has two contributions, $I_{\text{tot}} = I_{\text{dis}} + I_{\text{con}}$. $I_{\text{dis}}$ from the discrete Andreev states and $I_{\text{con}}$ from the continuum (see Figs. 2 and 3). The two parts of the Josephson current for the DQD in series (see below and Ref. 10 for the DQD in parallel) are given by (hereafter currents are expressed in units of $2|\epsilon|\Delta/h$):

$$I_{\text{dis}} = -2t^2 \sin(2\phi) \sum_{E_p} \frac{s^2(\epsilon)}{\Delta D(\epsilon)} \bigg|_{\epsilon = E_p}, \quad (5a)$$

$$I_{\text{con}} = -2t^2 \sin(2\phi) \text{Im} \int_{-\Delta}^{-\infty} dc \frac{1}{\Delta D(\epsilon)}. \quad (5b)$$

In Eq. 5a, the summation is over all Andreev states $E_p \in [-\Delta, \epsilon_F]$ ($\epsilon_F$ is the Fermi energy). Interestingly, we will see below [Eqs. 6 and 7] that in the deep Kondo limit ($\Delta \ll T_K$), we recover the short-junction limit for the Josephson current through non-interacting resonance levels 12. In this limit the continuum contribution is almost negligible. For $\Delta \lesssim T_K$ the contributions from the continuum part becomes considerable and the behavior deviates from the non-interacting case.

Next we present our results for the DQD in parallel and in series, respectively. We will choose $\varepsilon_0 = -3.25$, $D = 100$ (bandwidth), $\epsilon_F = 0$ and different values for the rest of parameters. For these values $T_K^0 = D \exp -\pi|\varepsilon_0|/\Gamma = 0.0036$. (All energies are given in units of $\Gamma$).

**DQD in parallel**—The problem is greatly simplified by the transformation $c_{k\varepsilon\sigma} = (c_{k\sigma} \pm c_{k\sigma})\sqrt{2}$ and

$$f_{1(2)\sigma} = (f_{\sigma\sigma} \pm f_{\sigma\sigma})\sqrt{2}. \quad \text{The MF Hamiltonian is mapped into two independent Josephson junctions ("even" and "odd") through effective resonant levels at } \varepsilon_0 \pm \tilde{t}. \quad \text{Each of the two resonant levels accommodates an Andreev state } E_{\varepsilon_0} \text{ and carries Josephson current } I_{\text{tot}}(\phi) = -2e\Delta/h (I_e + I_o) \quad \text{(see Ref. 10 for their explicit expressions). Typical profiles of Andreev states on DQD in parallel are shown in the left panels of Fig. 1. Josephson currents are shown in the left panels of Fig. 2 (t \leq \Gamma) and Fig. 3 (t \geq \Gamma). We note that in the deep Kondo limit } \Delta \ll T_K \text{ the DQD in parallel behaves in effect like two non-interacting resonance levels 12: The Andreev states and the Josephson currents are given by the corresponding expressions for the short-junction limit } E_{\varepsilon_0} = \Delta[1 - T_{\varepsilon_0} \sin^2(\phi)]^{1/2} \text{ and }$$

$$I_{\varepsilon_0}(\phi) = T_{\varepsilon_0} \sin 2\phi \left[1 - T_{\varepsilon_0}(\phi)\sin^2(\phi)\right]^{-1/2}, \quad (6)$$

respectively, with $T_{\varepsilon_0} = 4\tilde{t}^2/(\varepsilon_0^2 + \tilde{\Gamma})$. For very small $t/\Gamma$ both dots have their own Kondo resonances at $\epsilon_F$.
and the Josephson current resembles that of a ballistic junction. As $t/\Gamma$ increases the even and odd Kondo resonances $\bar{\varepsilon}_0 \pm \bar{t}$ move away from $\epsilon_F$ and as a result $I_{\text{tot}}(\phi)$ diminishes becoming more sinusoidal $I_{\text{tot}}(\phi) \approx \mathcal{T}_{\epsilon_F} \sin 2\phi$.

Experimentally more accessible is the critical current $I_c$ \cite{21}, which is given by $I_c(\phi) \approx 2e\Delta/\hbar \sum_{\beta \in \{e,o\}} [1 - (1 - T_\beta)^{1/2}]$ and hence decreases with $t/\Gamma$. Notice that in spite of the simple formal expression for $I_{\text{tot}}$, $I_c$ depends on the many-body parameters, $T_K$ and $\bar{\varepsilon}_0 \pm \bar{t}$, in a nontrivial manner through the solution of Eq. (4).

**DQD in series**—A completely different physical scenario is found for the serial configuration. Here the even/odd channels are no longer decoupled and cause novel interference. The manifestation of the interference can be first seen in the profiles of the Andreev states as depicted in Fig. 11. An important difference from the parallel case is the almost flat spectrum with values close to $E_{\epsilon_F/\phi}$ (reflecting a very small supercurrent as seen below). As $t/\Gamma$ increases the spectrum possesses larger amplitude, and for $t/\Gamma \approx 1$ we eventually recover the spectrum of a ballistic junction $E_{\epsilon_F/\phi} \approx \Delta \cos(\phi)$. For $t/\Gamma \gtrsim 1$ gaps are opened again (suggesting that $I_{\text{tot}}(\phi)$ diminishes, see below).

Formally, the supercurrent, for $\Delta \ll T_K$, is still given by the expression

$$I(\phi) = \mathcal{T} \sin 2\phi \left[ 1 - T \sin^2 \phi \right]^{-1/2},$$

but now the transmission $\mathcal{T}$ is $\mathcal{T} = 4\tau^2 \bar{t}^2/[(\bar{\varepsilon}_0 - \bar{t})^2 + \bar{T}^2][(\bar{\varepsilon}_0 + \bar{t})^2 + \bar{T}^2)]$. We can interpret that for small $t/\Gamma$ the Cooper pairs hop directly between the two Kondo resonances and $\mathcal{T} \propto t^2$. The supercurrent presents a sinusoidal-like behavior as shown in Fig. 2. With $t/\Gamma$ increasing, the physical situation changes drastically around $t/\Gamma = 1$, where the Kondo singularities of each dot hybridize into a correlated state as a result of the coherent superposition of both Kondo states. We find $\mathcal{T} \approx 1$ and consequently the supercurrent-phase relation exhibits a typical ballistic-junction behavior (see Fig. 4). Further increasing $t/\Gamma$ makes $I_{\text{tot}}(\phi)$ smaller, which is attributed to the formation of bonding and antibonding Kondo resonances. This results in a nonmonotonous behavior of $I_c$ as a function of $t/\Gamma$ (shown in Fig. 4) for $\Delta/T_K^0 = 0.1, 0.25, 0.5$, from top to bottom), with a maximum at $t = \Gamma$ (coherent superposition of both Kondo resonances). For the lowest gap (in the short junction regime) the maximum critical current reaches the universal value of $2e\Delta/\hbar$ as expected \cite{21}.

The physics of the tunability of $I_c$ as a function of $t/\Gamma$ is similar to the transistor-like control of supercurrents in a carbon nanotube quantum dot connected to superconducting reservoirs recently reported by Jarillo et al. \cite{21}. These experiments demonstrate that the supercurrent flowing through the QD can be varied by means of a gate voltage which tunes on- and off-resonance successive discrete levels of the QD with respect to $\epsilon_F$ of the reservoirs. In our case, the Kondo resonances play the role of the discrete levels in the experiments of Ref. 21 whereas $t/\Gamma$ is the extra knob that tunes the position of the Kondo resonances and thus modulates the critical current. Interestingly, in our case, i) the supercurrent is mediated by a coherent many-body state (the Kondo resonance) instead of a single particle one and ii) the maximum supercurrent at $t = \Gamma$ corresponds to coherent transport of Cooper pairs through the whole device whereas an increase of $t/\Gamma \geq 1$ splits the Kondo resonance into two (bonding and antibonding) resulting in a splitting of the Cooper pair into two electrons (one on each resonance) and thus a reduction of the supercurrent. This suggest the use of the Kondo effect as an alternative to previous proposals using DQDs \cite{15} for generating and manipulating entangled pairs in a controlled way.

For the experimental realization of the superconducting Kondo DQD, we propose carbon nanotubes since (i) they show Kondo physics \cite{22}, (ii) it is possible to fabricate tunable double quantum dots \cite{23}, and (iii) they are ideal systems to attach new material as electrodes \cite{24}.

**Conclusions.**—In closing we have studied Cooper pair transport through an artificial Kondo molecule. We find remarkable differences in the phase-current relation when the DQD is built in series or in parallel. For a DQD in parallel, the supercurrent always decreases with $t/\Gamma$ whereas for a serial Kondo molecule the current behaves nonmonotonously. This fact allows an extra control of the critical current, and thus of Cooper pairs, through Kondo molecules by simply tuning the interdot tunneling coupling.

We acknowledge D. Sánchez for fruitful and long discussions. This work was supported by MAT2005-07369-C03-03 and the Spanish MEC through the program Ramon y Cajal program (R.L.), the SRC/ERC program (R11-2000-071), the KRF Grant (KRF-2005-070-C00055), the SK Fund, and the KIAS.
[1] A.C. Hewson, *The Kondo Problem to Heavy Fermions* (Cambridge University Press, Cambridge, UK, 1993)

[2] L.I. Glazman and M.E. Raikh, Pis’ma Zh. ksp. Teor. Fiz. 47, 378 (1988) [JETP Lett. 47, 452 (1988)]. T.K. Ng and P.A. Lee, Phys. Rev. Lett. 61, 1768 (1993).

[3] D. Goldhaber-Gordon et al, Nature 391, 156 (1998).

[4] S.M. Cronenwett et al, Science 281, 540 (1998).

[5] J. Schmid et al, Physica B 256-258, 182 (1998).

[6] M.R. Buitelaar, T. Nussbaumer, C. Schonenberger, Phys. Rev. Lett. 89, 256801 (2002). M.R. Buitelaar et al, Phys. Rev. Lett. 91, 057005 (2003).

[7] A.F. Andreev, Zh. ksp. Teor. Fiz. 47, 378 (1988) [JETP Lett. 47, 452 (1988)].

[8] S.L. Bravyi and A.A. Khveshchenko, Phys. Rev. A 57, 120 (1998).

[9] N.G. van Wees and H. Takayanagi, in *Mesoscopic Electron Transport*, edited by L.L. Schön et al, (Kluwer, Dordrecht, 1997).

[10] D. Loss and D.P. DiVincenzo, Phys. Rev. A 57, 120 (1998).

[11] N.J. Craig et al, Science 304, 365 (2004).

[12] C. Jayaprakash, H. R. Krishna-murthy, and J. W. Wilkins, Phys. Rev. Lett. 47, 737 (1981).

[13] H. Jeong et al, Science 293, 2221 (2001). J.C. Chen, A.M. Chang, M.R. Melloch, Phys. Rev. Lett. 92, 176801 (2004).

[14] M.S. Choi, C. Bruder, D. Loss, Phys. Rev. B 62, 13569 (2000).

[15] P. Coleman, Phys. Rev. B 29, 3035 (1984). For a review, see D.M. Newns and N. Read, Adv. Phys.

[16] R. Lopez, R. Aguado, G. Platero, Phys. Rev. B 69, 125327 (2004).

[17] T.K. Ng and K. Hamada, Phys. Rev. B 62, 235305 (2000).

[18] For a DQD connected in series to normal leads, $T_K$ has an exponential dependence on $t$: $T_K \approx T_{K,n} e^{(t/t_n^{-1})/(t/t_n^* - 1)}$.

[19] For the DQD in parallel, the ABSs and Josephson current are conveniently expressed in the even/odd basis. The ABSs are given as the solutions of $D_{e/o}(\epsilon) = e^{2i(\Delta^2 - \epsilon^2 + \tilde{E}) - (\tilde{E} \pm i \tilde{J})(\Delta^2 - \epsilon^2) - \Delta^2 \Gamma \cos^2(\phi)$.

[20] C. W. J. Beenakker and H. Van Houten, *Single-electron tunneling and Mesoscopic devices*, edited by H. Koch, and H. L. L. (Springer Berlin 1992).

[21] J. Nygård et al, Nature (London) 408, 342 (2000).

[22] N. Manson et al, Science 303, 655 (2004). S. Sapmaz et al, *cond-mat/0602421* (unpublished). M. R. Graeb et al, *cond-mat/0603367* (unpublished).