Covariance in the Thermal SZ-Weak Lensing Mass Scaling Relation of Galaxy Clusters

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ABSTRACT

The thermal Sunyaev-Zel’dovich (tSZ) effect signal is widely recognized as a robust mass proxy of galaxy clusters with small intrinsic scatter. However, recent observational calibration of the tSZ scaling relation using weak lensing (WL) mass exhibits considerably larger scatter than the intrinsic scatter predicted from numerical simulations. This raises a question as to whether we can realize the full statistical power of ongoing and upcoming tSZ-WL observations of galaxy clusters. In this work, we investigate the origin of observed scatter in the tSZ-WL scaling relation, using mock maps of galaxy clusters extracted from cosmological hydrodynamic simulations. We show that the inferred intrinsic scatter from mock tSZ-WL analyses is considerably larger than the intrinsic scatter measured in simulations, and comparable to the scatter in the observed tSZ-WL relation. We show that this enhanced scatter originates from the combination of the projection of correlated structures along the line of sight and the uncertainty in the cluster radius associated with WL mass estimates, causing the amplitude of the scatter to depend on the covariance between tSZ and WL signals. We present a statistical model to recover the unbiased cluster scaling relation and cosmological parameter by taking into account the covariance in the tSZ-WL mass relation from multi-wavelength cluster surveys.

Key words: galaxies: clusters: general — galaxies: clusters: intracluster medium — gravitational lensing: weak — cosmology: observations — method: numerical

1 INTRODUCTION

In recent years, the Sunyaev-Zel’dovich (SZ) effect observations of galaxy clusters have emerged as a powerful probe of the growth of cosmic structure and cosmology. The thermal SZ (tSZ) effect is the inverse Compton scattering of the CMB photons off of energetic electrons in the intracluster medium (ICM) (Sunyaev & Zeldovich 1972). Since the SZ effect signal is independent of redshift, it offers a powerful way of detecting galaxy clusters out to high redshift with the current generation of microwave experiments, such as the Atacama Cosmology Telescope (ACT), the South Pole Telescope (SPT), and the Planck satellite (e.g., Hasselfield et al. 2013; Bleem et al. 2013; Planck Collaboration et al. 2015a). These cluster samples have been used to measure the evolution of cluster abundance over the cosmic time and constrain cosmological parameters (e.g., Sievers et al. 2013; Planck Collaboration et al. 2015b; de Haan et al. 2016).

Cosmological constraints derived from these surveys rely critically on the calibration of the relationship between the observable and mass of galaxy clusters. Numerical simulations predict that the tSZ effect signal is a robust proxy of cluster mass with intrinsic scatter of $\lesssim 10\%$ as it directly probes the thermal energy content of the virialized ICM (e.g., Motl et al. 2005; Nagai 2006; Kay et al. 2012; Sembolini et al. 2013; Yu, Nelson & Nagai 2015).

However, the cluster-based cosmological constraint hinges on the still poorly understood calibration of the relationship between the observable and cluster mass (e.g., Bocquet et al. 2015; Sifón et al. 2015). As such, the tSZ-mass scaling relation has been calibrated observationally, based on the assumption that the cluster gas is in hydrostatic equilibrium with the gravitational potential of galaxy clusters. However, the hydrostatic mass estimate derived from X-ray observations is shown to produce biased estimates of cluster mass at the level of 5 – 30% depending on their dynamical states (e.g., Rasia et al. 2006; Nagai, Vikhlinin & Kravtsov 2007), and
it is one of the dominant sources of astrophysical uncertainties in cosmological constraints from SZ surveys (e.g., Planck Collaboration et al. 2014a, 2015b).

Weak lensing (WL) mass measurements, which directly probe the projected mass distribution of the cluster, provide a promising way to measure cluster mass independently of their dynamical states (e.g., Marrone et al. 2009; McNamara et al. 2009; High et al. 2012; Hoechst et al. 2012; Miyatake et al. 2013; von der Linden et al. 2014; Jee et al. 2014; Gruen et al. 2014; Battaglia et al. 2013; Smith et al. 2013). However, recent tSZ and WL measurements suggest that the scatter in the tSZ-WL mass scaling relation is on the order of ~20% (e.g., Marrone et al. 2012), which is considerably larger than the intrinsic scatter predicted by numerical simulations. This raises a question as to whether the WL mass calibration of the SZ-selected clusters can realize the full statistical power of the ongoing and upcoming SZ surveys to test cosmological models.

In this work, we investigate the origin of the large discrepancy between the intrinsic scatters in the tSZ-mass scaling relation from simulations and observations, by using mock tSZ and WL analyses of galaxy clusters extracted from high-resolution cosmological hydrodynamic simulations. We show that most of the scatter in the observed tSZ-WL mass relation is driven by the combination of the enhanced scatter in tSZ due to projections of correlated structures in the outskirts of individual clusters and the bias in WL determined cluster radius, within which the tSZ signal is measured. Most importantly, our results demonstrate the importance of the covariance between tSZ and WL due to the correlated structures along the line of sight. We present a statistical model to recover the unbiased Y − M relation from a set of tSZ and WL measurements, by taking into account covariances among clusters’ observables.

The paper is organized as follows. In Section 2, we describe our simulations and mock tSZ and WL analyses of simulated clusters. We first examine the nature of scatters in tSZ and WL measurements in Section 3 and the covariance between tSZ and WL observables and its impact on cluster-based cosmological analyses in Section 4. Section 5 explores the systematic uncertainties associated with baryonic effects. Conclusions are summarized in Section 6.

2 METHODS

2.1 Hydrodynamic Simulations

In this work, we analyze the mass-limited sample of 33 galaxy clusters extracted from the *Omega500* non-radiative (NR) hydrodynamics (Nelson et al. 2014) in a flat ΛCDM model with the WMAP five-year results (Komatsu et al. 2009): Ω_m0 = 0.27 (matter density), Ω_b0 = 0.0469 (baryon density), H_0 = 100h km s^{-1} Mpc^{-1} (Hubble constant), and σ_8 = 0.82 (the mass variance within a sphere with a radius of 8 h^{-1} Mpc). The simulation is performed using the Adaptive Refinement Tree (ART) N-body+gas-dynamics code (Kravtsov et al. 1999; Kravtsov, Klypin & Hoffman 2002; Rudd, Zentner & Kravtsov 2008), which is an Eulerian code that uses adaptive refinement in space and time and non-adaptive refinement in mass (Klypin et al. 2001), to achieve the dynamic range necessary to resolve the cores of halos formed in self-consistent cosmological simulations. The simulation volume has a comoving box length of 500 h^{-1} Mpc, resolved using a uniform 512^3 root grid and 8 levels of mesh refinement, implying a maximum comoving spatial resolution of 3.8 h^{-1} kpc. While the effects of baryonic physics, such as radiative gas cooling, star formation and energy feedback from supernovae and active galactic nuclei are important in the cluster core regions (r ≤ 0.15Rvir), these additional physics are shown to have negligible (≤ 2%) impact on the scatter in the tSZ-mass scaling relation (Nagai 2006; Battaglia et al. 2012; Kay et al. 2012). In Section 6 we assess the impact of baryonic physics with the *Omega500* simulation that includes radiative cooling, star formation, and supernova feedback.

Cluster-sized halos are identified in the simulation using a spherical overdensity halo finder described in Nelson et al. (2014). We define the three-dimensional (3D) mass of a cluster using the spherical overdensity criterion: M_{500c} = 500 \rho_{crit}(z)(4\pi/3)R_{500c}^3, where ρ_{crit}(z) is the critical density of the universe at a given redshift z. In the following, we denote this 3D mass as M_{3D}. We select clusters with M_{3D} ≥ 3 × 10^{14} h^{-1} M_{⊙} at z = 0 and re-simulate the box with higher resolution dark matter particles in regions of the selected clusters with the “zoom-in” technique, resulting in an effective mass resolution of 2048^3, corresponding to a dark matter particle mass of 1.09 × 10^9 h^{-1} M_{⊙}, inside spherical region with cluster-centric radius of three time the virial radius for each cluster.

In this work, we work mainly with a mass-limited sample of 33 clusters with M_{500c} ≥ 2.3 × 10^{14} h^{-1} M_{⊙} at z = 0.33, which is comparable to the typical redshift of recent WL cluster observations (e.g., High et al. 2012; Battaglia et al. 2013).
κ = 3
θ
position into shear using Eq. (6). Throughout this paper, we con-
vergence field using Eq. (5) and transform convergence

\[ \kappa \] = \int \frac{\delta(\chi, \chi')}{a} \, d\chi \, d\chi' \]

where \( \kappa \) is convergence and \( \gamma \) is shear.

One can relate each component of \( A_{ij} \) to the second
derivative of the gravitational potential \( \Phi \) of the lens ob-
ject as follows (Bartelmann & Schneider 2001; Munshi et al.

\[ A_{ij} = \delta_{ij} - \phi_{ij}, \]

\[ \phi_{ij} = \frac{2}{c^2} \int_0^\chi \, d\chi' \, g(\chi, \chi') \partial_i \partial_j \Phi(\chi'), \]

\[ g(\chi, \chi') = \frac{r(\chi - \chi') r(\chi')}{r(\chi)}, \]

where \( \chi \) is the comoving distance and \( r(\chi) \) is the comoving
angular diameter distance. Gravitational potential \( \Phi \) can
then be related to the matter density perturbation \( \delta \) by the
Poisson equation.

The convergence can then be expressed as the weighted
integral of \( \delta \) along the line of sight,

\[ \kappa = \frac{3}{2} \left( \frac{H_0}{c} \right)^2 \Omega_m \int_0^\chi \, d\chi' \, g(\chi, \chi') \delta(\chi') a. \]

The relation between convergence and shear in Fourier space
is given by

\[ \tilde{\gamma}(k) = \tilde{\gamma}_1(k) + i\tilde{\gamma}_2(k) = \frac{k_z^2 - k_x^2 + i k_x k_z}{k^2} \tilde{r}(k), \]

\[ \tilde{\kappa}(k) = \tilde{\gamma}_1(k) \cos 2\phi_k + \tilde{\gamma}_2(k) \sin 2\phi_k, \]

where \( \tilde{X}(k) \) is the Fourier coefficient of \( X(\theta) \) and \( k = (k_x, k_y, k_z) = (\cos \phi_k, \sin \phi_k). \)

To simulate WL cluster mass measurement, we first cre-
ate projected mass density maps of each cluster viewed along
three orthogonal projections, \( (x, y, z) \). We then derive the
convergence field using Eq. (4) and transform convergence
into shear using Eq. (6). Throughout this paper, we con-
sider a single source redshift \( z_s = 1 \) for lensing calculations.

We then generate the projected mass density map on the
2048×2048 two-dimensional mesh points by extracting all parti-
cles around each cluster in a comoving box with volume of
15.6 × 15.6 × L\text{depth} (h^{-1}\text{Mpc})^3, \text{where } L\text{depth} \text{ is the projection}
depth along the line of sight. We vary the projection
depth \( L\text{depth} = 10, 20, 100, \) and 500 h^{-1}\text{Mpc} to explore the
effects of correlated structures along the line of sight, while
keeping the transverse size of the analysis volume fixed. Note
that we ignore two “past lightcone” effects associated with
(1) the evolution of large-scale structure and (2) the increasing
transverse size with redshift, which requires ray-tracing
simulation (e.g., White & Hu 2000) and left for future work.

Because the dark matter particles come in different
masses in our zoom-in simulations, the mass density maps
with a large projection depth get contribution from low reso-
lution dark matter particles, which appear as localized, point
like masses. We alleviate this effect by smoothing the mass
associated with these particles uniformly over the mesh in
which these particles reside. We confirmed that the aver-
age WL signal in the radial range of 0 < R/R_{500c} < 2
converges to better than 1% in the four different cases of
\( L\text{depth}. \) We, therefore, conclude that the low resolution dark
matter particles do not affect the resulting mean value of the
WL-inferred mass in the radial range of our interest.
Table 1. Summary of our \( \chi^2 \) fitting for WL and tSZ measurement of clusters.

| Model | Free parameters | Fitting range |
|-------|----------------|---------------|
| WL NFW (Eq. 11) | \( M_{2D} \) and \( c_{500c,h} \) | 0.3’ – 10’ |
| tSZ gNFW (Eq. 13) | \( M_{500p} \) and \( c_{500p} \) | 0.1’ – 5’ |

2.2.2 Compton-\( y \) maps

The tSZ effect is a spectral distortion of CMB caused by inverse Compton scattering of CMB photons off of electrons in the high-temperature plasma in the ICM. The temperature change at frequency \( \nu \) of the CMB is given by \( \Delta T_\nu/T_{\text{CMB}} = f_\nu \langle x \rangle y \), where \( f_\nu \langle x \rangle = [x(e^{x+1}/(e^x-1)-4(1+\delta_{\text{SZ}}(x,T_c))] \) is a frequency dependent factor, \( \delta_{\text{SZ}}(x,T_c) \) is the frequency dependent relativistic correction and \( x \equiv h\nu/k_BT_{\text{CMB}} \). The amplitude of the SZE signal is given by the Compton-\( y \) parameter:

\[
y(\ell) = \frac{\sigma_T}{m_e c} \int dl P_e(\ell),
\]

where \( \sigma_T \) is the Thomson cross-section, \( m_e \) is the electron rest mass, \( c \) is the speed of light, \( P_e \) is the electron pressure, and the integral is performed along the line of sight \( \ell \).

We generate the tSZ maps of each cluster viewed along three orthogonal projections on 2048 \( \times \) 1024 \( \times \) 512 mesh points by integrating Eq. (8) in a comoving box with volume of 15.6 \times 15.6 \times L_{\text{depth}} (h^{-1} \text{Mpc})^3, where \( L_{\text{depth}} = 10, 20, 100, \) and 500 h^{-1} Mpc. Because of the AMR nature of the simulation, the gas in the low density region is not refined as aggressively as it is designed to appear as a grid-like feature in the map. However, we checked that most grid-like features are found in the outer region of clusters (\( R \geq 2R_{500c} \)) and hence do not affect our analyses. An example of the resulting Compton-\( y \) map is shown in Figure 2.

2.3 Profile Fitting

From both WL and tSZ maps, we measure the azimuthally averaged, logarithmically spaced radial profiles in the radial range of \( \theta = 0.1’ – 30’ \) around the center of each cluster. Note that the angular size of \( R_{500c} \) corresponds to \( 2’ – 3’ \) for clusters at \( z = 0.33 \), which is well within the range of our angular bins.

In order to find the best representation of an observable \( X(\theta) \), where \( X \) can be our WL shear or Compton-\( y \), we use a \( \chi^2 \)-fitting metric and the non-linear least-squares Levenberg-Marquardt algorithm [Press et al. 1992]. Suppose that the expected signal is expressed as \( X_{\text{model}}(\theta; p) \) with a set of parameters \( p \), a \( \chi^2 \) metric can then be defined as

\[
\chi^2 = \sum_{i=1}^{N_{\text{bin}}} (X(\theta_i) - X_{\text{model}}(\theta_i; p))^2, \tag{9}
\]

where \( N_{\text{bin}} \) is the number of angular bins.

For WL maps, the observable is the tangential component of reduced shear around each cluster, defined as

\[
g_T = -\frac{\gamma_1}{1-\kappa} \cos \phi - \frac{\gamma_2}{1-\kappa} \sin \phi, \tag{10}
\]

where \( \phi \) is the azimuthal angle on each WL map. To model \( g_T \), we use the NFW profile for matter density profile, which is given by

\[
\rho_s(r) = \frac{\rho_s}{(r/r_s)(1+r/r_s)^2}, \tag{11}
\]

where \( \rho_s \) and \( r_s \) are the scale density and the scale radius, respectively, and the concentration parameter is defined as \( c_{500c,h} \equiv R_{500c}/r_s \) [Navarro, Frenk & White 1997]. The corresponding convergence and shear can then be obtained analytically [Wright & Brainerd 2000]. We denote \( M_{2D} \) as \( M_{500c} \) inferred from \( \chi^2 \) fitting to \( g_T \). When performing \( \chi^2 \) fitting, we consider the angular range of \( 0.5’ – 10’ \), because it is difficult to simulate gravitational lensing effect with our method in the inner region of \( \theta \lesssim 0.5’ \), while the correlated matter contribution dominates at \( \theta \gtrsim 10’ \).

The observable in tSZ maps is the azimuthally averaged Compton-\( y \) profile around each cluster. To model this profile, we use the generalized NFW (gNFW) pressure profile [Nagai, Kravtsov & Vikhlinin 2007]. Since our ultimate goal is to apply the method developed in this paper to real cluster observations, we adopt the universal pressure profile calibrated by X-ray observations of nearby clusters [Arnaud et al. 2010], which is given by

\[
P_e(\ell) = 1.88 \times E(z)^{8/3} \left( \frac{M_{500p,500}}{10^{14} M_\odot} \right)^{0.787 + \phi(\ell/R_{500p})} \times c_{500p,500}^{\alpha_p} \approx 10^{-3} \text{eV cm}^{-3}, \tag{12}
\]

where \( E(z) = (\Omega_m(1+z)^3 + 1 - \Omega_m)^{1/2} \) and \( R_{500p} \) is defined by the relation of \( M_{500p} = 4\pi R_{500p}^3 \times 500 \rho_{500p}(z)/3 \). In Eq. (12), the functional form of \( p(x) \) and \( \phi(\ell) \) are specified by

\[
p(x) = \frac{P_0 h^{-3/2}}{(c_{500p,500}x^3 + 1)^{\gamma} (\gamma - 1)/\alpha_p}, \tag{13}
\]

\[
\phi(\ell) = 0.1 - 0.22 \left( \frac{x}{0.5} \right)^{3} \frac{1}{1 + \left( x/0.5 \right)^{3}}. \tag{14}
\]

Throughout our analysis, we use the best-fit parameters derived from all the EXCESS data set in [Arnaud et al. 2010]: \( P_0 = 4.921, \gamma = 0.3081, \alpha = 1.0510 \) and \( \beta = 5.4905 \) but float the parameters \( M_{500p} \) and \( c_{500p} \). A \( \chi^2 \)-fitting with Eq. (12) is performed in the angular range of \( 0.1’ – 5’ \). Note that our results are insensitive to the choice of the assumed pressure profile; e.g., the fractional change in the scatter in \( Y \) is less than 2\% if we use the pressure profile calibrated based on the NR simulations [Nagai, Kravtsov & Vikhlinin 2007]. Table 1 summarizes the parameters of the \( \chi^2 \)-fitting to the mock WL and tSZ maps of the simulated clusters. Figure 3 shows an example of profile fitting results to our sample.

3 SCATTERS IN TSZ AND WL MEASUREMENTS

3.1 3D \( Y – M \) Relation

First, we quantify the intrinsic scatter in the tSZ-WL mass scaling relation, using the spherically integrated global tSZ

Note that our results slightly depend on the fitting range. The fractional change in the scatter in \( Y_{2D} \) is of order 5\% when using the fitting range of \( 0.1’ – 7’ \).
Figure 3. An example of profile fitting to reduced shear (top panel) and Compton-$y$ (bottom panel) profile. We work on the simulated cluster with $M_{500c} = 9.1 \times 10^{14} h^{-1} M_\odot$ (as same as shown in Figure 2). The projection depth is set to 500$ h^{-1}$ Mpc. In each panel, the red points represent the measured profile and black open circles show the bins used for profile fitting. The solid line in upper portion is the best-fitted profile and the residual is shown in bottom portion of each panel. The dash line in each panel corresponds to the radius obtained from $M_{2D}$ or $M_{500p}$. The best-fitted value of parameters is summarized in each panel.

signal and true cluster mass computed directly from the simulation. The global tSZ signal is represented by the integrated Compton-$y$ parameter $Y_{3D}$, which is the volume integrated electron pressure in the ICM within a sphere with a radius $R_{\text{ref}}$:

$$Y \equiv \frac{\sigma_T m_e c^2}{\epsilon} \int_0^{R_{\text{ref}}} P_e(r) 4\pi r^2 dr,$$

where $R_{\text{ref}}$ is a reference radius to define the boundary of clusters. We evaluate $Y$ using the spherically averaged electron pressure profile $P_e$ of each simulated cluster, and we set $R_{\text{ref}} = R_{500c}$, which is obtained from the true 3D mass, $M_{3D}$, computed directly from simulation. Hereafter, we denote this spherically averaged $Y$ as $Y_{3D}$. Performing a linear least square fitting to 33 clusters in our sample at $z = 0.33$, the best-fit scaling relation between $\log Y_{3D}$ and $\log M_{3D}$ is

$$\log \left( \frac{Y_{3D}}{(h^{-1} \text{Mpc})^2} \right) = 1.71 \log \left( \frac{M_{3D}}{10^{14} h^{-1} M_\odot} \right) - 5.51,$$

where the 1σ errors in the normalization and slope are found to be 0.013 and 0.025, respectively. Hence, the best-fit slope is consistent with the self-similar prediction of $5/3$ within 2σ. The best-fit relation is shown as hatched region in Figure 4.

We quantify the intrinsic scatter of the $Y_{3D} - M_{3D}$ relation as

$$\sigma_{\log Y_{3D}}^2 = \frac{1}{N_s - 1} \sum_{i=1}^{N_s} (\log Y_{3D,i} - \log Y_{3D,\text{fit}}(M_{3D,i}))^2,$$

where $N_s = 33$ and $Y_{3D,\text{fit}}$ denotes the best-fit relation given by Eq. (16). The intrinsic scatter is $\sigma_{\log Y_{3D}} = 0.030$ or $\sigma_{\ln Y_{3D}} = \sigma_{\log Y_{3D}} \times \ln 10 = 6.9\%$ for our sample at $z = 0.33$, and it is consistent with previous results based on cosmological hydrodynamic simulations (e.g., Nagai 2006; Yang, Bhattacharya & Ricker 2010; Stanek et al. 2010; Krause et al. 2012; Battaglia et al. 2012; Yu, Nelson & Nagai 2015).

2 Throughout the paper, we use $\log = \log_{10}$ to compute scatter in scaling relations unless noted otherwise.
is consistent with the local model of the Malmquist bias [White, Cohn & Smidt 2010; Stanek et al. 2010; Rozo et al. 2014], highlighting the importance of understanding the selection function of the observed cluster samples and correcting the Malmquist bias.

3.3 Source of scatter in \( Y_{2D} \) and \( M_{2D} \)

3.3.1 Projection effect

Projection of line-of-sight structures is one of the primary sources of scatter in \( Y_{2D} \) and \( M_{2D} \) (e.g., Hallman et al. [2007]; Meneghetti et al. 2010; Battaglia et al. 2012). To quantify this effect, we compute \( Y_{2D} \) from the tSZ maps using the four different projection depths \( L_{\text{depth}} = 10, 20, 100, \) and \( 500 \) h\(^{-1}\) Mpc. The pressure profile fitting is performed in the angular range of 0.1 to \( \theta_{500c} \), where \( \theta_{500c} \) is the angle corresponding to \( R_{500c} \). In this section, we compute \( Y_{2D} \) within the true cluster radius \( R_{\text{cl}} \). Note, however, that the uncertainty in the halo radius \( R_{500c} \) can introduce additional scatter, which will be examined separately in Section 3.3.2.

We quantify the scatter between \( Y_{2D} \) and \( Y_{3D} \) for our sample of 33 clusters as

\[
\sigma_{\log Y_{2D} - 3D}^2 = \frac{1}{N_m} \sum_{i=1}^{N_m} (\log Y_{2D,i} - \log Y_{3D,i})^2,
\]

where \( Y_{2D,i} \) and \( Y_{3D,i} \) are the 3D and 2D integrated Compton-\( y \) values of the \( i \)-th cluster and \( N_m = 33 \).

Table 5 shows how the projection effect introduces additional scatter in \( Y_{2D} \) relative to the intrinsic scatter in \( Y_{3D} \) as we increase the projection depth, \( L_{\text{depth}} \). For all three projections, we find a general trend that the scatter increases monotonically with \( L_{\text{depth}} \), from 20 h\(^{-1}\) Mpc to 500 h\(^{-1}\) Mpc, except for one case between \( L_{\text{depth}} = 10 - 20 \) h\(^{-1}\) Mpc in the \( x \)-axis projection. In the case of \( L_{\text{depth}} = 10 \) h\(^{-1}\) Mpc, we find a cluster with \( \log Y_{2D}/Y_{3D} \sim -0.2 \), making this 7\( \sigma \) outlier in the population. We confirm that this is a merging cluster with \( M_{500c} = 4.1 \times 10^{14} \) h\(^{-3}\) M\(_{\odot} \) at \( z = 0.33 \). The projected Compton-\( y \) profile of this cluster has a flat core at \( \theta < 1' \), which makes the gNFW model a poor fit. We find that this merging cluster affects the estimation of scatter up to 30% (see the right portion in Table 5 for the result without the outlier). When removing this cluster, the scatter increases monotonically with \( L_{\text{depth}} \) as expected in absence of such an outlier. We also find that the scatter in the fitting range of 0.1' to \( \theta_{500c} \) is consistently smaller than the scatter based on the fitting range of 0.1' to 5' for any given \( L_{\text{depth}} \), suggesting that the tSZ signal from \( \theta > \theta_{500c} \) is responsible for the additional scatter in \( Y_{2D}(R_{500}) \).

We find that the scatter in \( M_{2D} \) increases monotonically with the projection depth, \( L_{\text{depth}} \) (Hoekstra 2003; Dodelson 2004; Hoekstra et al. 2012), because of the increased contribution from the uncorrelated matter distribution along the line of sight (Gruen et al. [2013]). Moreover, the scatter in \( \ln M_{2D} \) with \( L_{\text{depth}} = 500 \) h\(^{-1}\) Mpc is 0.24, which is similar to that in the scatter of 0.22 reported in the previous study based on a large cosmological N-body simulation with a box size of 1 h\(^{-1}\) Gpc (Becker & Kravtsov [2011]).
Another major source of the scatter in the estimated halo radius from WL

3.3.2 Uncertainties in estimated halo radius from WL

Another major source of the scatter in \( Y_{2D} \) is the biased estimation of the halo radius \( R_{2D} \) resulting from the bias in the WL mass, which enters into our calculation of the integrated \( Y \) in Eq. (15).

To quantify this effect, we compare \( Y_{2D}(R_{2D})/Y_{3D} \) with \( Y_{2D}(R_{2D})/Y_{3D} \), where \( Y_{2D}(R_{2D}) \) and \( Y_{2D}(R_{2D}) \) are computed within the true radius \( R_{2D} \) and the WL estimated radius \( R_{2D} \), respectively. In both cases, we compute the scatter in \( Y_{2D}/Y_{3D} \) using Eq. (19).

Figure 5 shows the distribution function of the deviation of projected \( Y_{2D} \) from the true \( Y_{3D} \) for 33 clusters obtained from the WL and tSZ maps with the projection depth of \( L_{\text{depth}} = 500 h^{-1} \text{Mpc} \) along the \( x \)-axis. The red line represents the distribution where the projected \( Y_{2D} \) is measured within \( R_{2D} \) (i.e., \( \Delta \log Y(R_{2D}) = \log (Y_{2D}(R_{2D})/Y_{3D}) \)), while the green line shows the distribution where the projected \( Y_{2D} \) is measured within \( R_{2D} \) estimated from the WL mass (i.e., \( \Delta \log Y(R_{2D}) = \log (Y_{2D}(R_{2D})/Y_{3D}) \)). The distribution of \( \Delta \log Y(R_{2D}) \) is broader than that of \( \Delta \log Y(R_{2D}) \), indicating that WL mass measurements of \( M_{2D} \) introduce additional scatter in \( Y_{2D} \) by 11.6%, which is larger than 4.5% increase in scatter due to projection effects discussed in the Section 3.3.1. Note that similar results are obtained for the other two projection axes, where the additional scatter in \( Y_{2D} \) are found to be 9.0% and 10.4% for \( y \)-axis and \( z \)-axis, respectively. This shows that the uncer-

| \( L_{\text{depth}} [h^{-1} \text{Mpc}] \) | 0.1' - 5' | 0.1' - \( \theta_{500c} \) | 0.1' - 5' | 0.1' - \( \theta_{500c} \) |
|-----------------|---------------|-----------------|---------------|-----------------|
| \( x \)-axis projection | mass-limited sample | without the outlier | mass-limited sample | without the outlier |
| 10 | (3.91 ± 0.04) × 10^{-2} | (2.74 ± 0.02) × 10^{-2} | (2.97 ± 0.02) × 10^{-2} | (2.53 ± 0.02) × 10^{-2} |
| 20 | (3.99 ± 0.03) × 10^{-2} | (2.64 ± 0.02) × 10^{-2} | (3.10 ± 0.02) × 10^{-2} | (2.65 ± 0.02) × 10^{-2} |
| 100 | (3.93 ± 0.03) × 10^{-2} | (2.74 ± 0.02) × 10^{-2} | (3.31 ± 0.03) × 10^{-2} | (2.76 ± 0.02) × 10^{-2} |
| 500 | (3.50 ± 0.03) × 10^{-2} | (2.80 ± 0.02) × 10^{-2} | (3.40 ± 0.03) × 10^{-2} | (2.83 ± 0.02) × 10^{-2} |
| \( y \)-axis projection | mass-limited sample | without the outlier | mass-limited sample | without the outlier |
| 10 | (3.25 ± 0.03) × 10^{-2} | (2.29 ± 0.01) × 10^{-2} | (2.75 ± 0.02) × 10^{-2} | (2.10 ± 0.01) × 10^{-2} |
| 20 | (3.40 ± 0.03) × 10^{-2} | (2.38 ± 0.02) × 10^{-2} | (2.90 ± 0.02) × 10^{-2} | (2.18 ± 0.01) × 10^{-2} |
| 100 | (3.70 ± 0.03) × 10^{-2} | (2.49 ± 0.02) × 10^{-2} | (3.23 ± 0.03) × 10^{-2} | (2.30 ± 0.01) × 10^{-2} |
| 500 | (3.94 ± 0.04) × 10^{-2} | (2.63 ± 0.02) × 10^{-2} | (3.45 ± 0.03) × 10^{-2} | (2.43 ± 0.02) × 10^{-2} |
| \( z \)-axis projection | mass-limited sample | without the outlier | mass-limited sample | without the outlier |
| 10 | (3.88 ± 0.04) × 10^{-2} | (2.65 ± 0.02) × 10^{-2} | (2.87 ± 0.02) × 10^{-2} | (2.34 ± 0.01) × 10^{-2} |
| 20 | (4.18 ± 0.04) × 10^{-2} | (2.94 ± 0.02) × 10^{-2} | (3.22 ± 0.03) × 10^{-2} | (2.65 ± 0.02) × 10^{-2} |
| 100 | (4.34 ± 0.05) × 10^{-2} | (3.05 ± 0.02) × 10^{-2} | (3.43 ± 0.03) × 10^{-2} | (2.76 ± 0.02) × 10^{-2} |
| 500 | (4.40 ± 0.05) × 10^{-2} | (3.10 ± 0.03) × 10^{-2} | (3.52 ± 0.03) × 10^{-2} | (2.82 ± 0.02) × 10^{-2} |
tainty in $M_{2D}$ leads to significant scatter in the WL calibration of the $Y - M$ relations, and this effect must be taken into account in the cosmological parameter estimation based on WL mass calibration of SZ-selected cluster samples.

In order to account for this effect, we develop a model to predict the distribution of $\log(Y_{2D}(R_{2D})/Y_{3D})$ for a given $\log(Y_{2D}(R_{2D})/Y_{3D})$. Assuming that the underlying pressure profile is given by the gNFW pressure profile with the best-fit parameters $M_{200}$, and $c_{200}$, the uncertainty in WL mass $M_{2D}$ is translated into the uncertainty in $R_{\text{ref}}$ through Eq. (15). Note that the integral in Eq. (15) scales with the two-dimensional log-normal distribution with the uncertainty in $M_{2D}$.

$\Delta \log 10 Y = \log 10(Y_{2D}(R_{2D})/Y_{3D})$ for a given $\log(Y_{2D}(R_{2D})/Y_{3D})$. The probability distribution of $\Delta \log 10 Y$ is given by the gNFW pressure profile with the best-fit parameters $M_{200}$, and $c_{200}$, the uncertainty in WL mass $M_{2D}$ is translated into the uncertainty in $R_{\text{ref}}$ through Eq. (15). Note that the integral in Eq. (15) scales with the uncertainty in $M_{2D}$.

$\int_{0}^{x_{\text{out}}} p(x) 4\pi x^2 dx \quad (20)$

where $x_{\text{out}} = R_{\text{ref}}/R_{200}, p(x)$ is given by Eq. (14) and $\alpha_0$ is assumed to play a minor role in the evaluation of this integral. Since $\delta \log I_0 \simeq \delta \log x_{\text{out}}$ at $x_{\text{out}} = 1$, the uncertainty in $\log M_{2D}$ introduces the scatter in $\log Y_{2D}$ by $\delta \log Y_{2D} \sim \delta \log M_{2D}/3$. We can then model the probability distribution of $\log Y_1 = \log(Y_{2D}(R_{2D}))$ based on the probability distribution of $\log Y_2 = \log(Y_{2D}(R_{3D}))$ as

$\varphi(\log Y_1) = \int d \log Y_2 \varphi(\log Y_2) \varphi(\log Y_1 | \log Y_2). \quad (21)$

where $\varphi(\log Y_1 | \log Y_2)$ is the distribution of $\log Y_1$ for a given $\log Y_2$. We assume $\varphi(\log Y_1 | \log Y_2)$ to be the log-normal distribution with the scatter of $(1/3) \sigma_{\log M_{2D} - 3D}$, where $\sigma_{\log M_{2D} - 3D}$ is the scatter of $\log(M_{2D}/M_{3D})$. The blue histogram in Figure 6 is the result of our model, which provides a good description of our simulation results.

$\Delta \log Y = \log(Y_{2D}(R_{2D})/Y_{3D})$ for a given $\log(Y_{2D}(R_{2D})/Y_{3D})$. The blue histogram corresponds to our modeling with the log-normal distribution of $\log(M_{2D}/M_{3D})$.

**Figure 6.** The probability distribution of $\Delta \log Y$ is given by the gNFW pressure profile with the best-fit parameters $M_{200}$, and $c_{200}$, the uncertainty in WL mass $M_{2D}$ is translated into the uncertainty in $R_{\text{ref}}$ through Eq. (15). Note that the integral in Eq. (15) scales with the uncertainty in $M_{2D}$.

**Figure 7.** The covariance between $M_{2D}$ and $Y_{2D}$ derived from WL mass and tSZ maps of 33 simulated clusters. The gray point shows the scatter plot of $\log(M_{2D}/M_{3D})$ and $\log(Y_{2D}/Y_{3D})$. The red solid and dashed lines indicate the 1σ and 2σ contours of the two-dimensional log-normal distribution with the measured covariance, respectively.

### 4 Covariance Between tSZ and WL Signals

The scatter in $Y_{2D}$ is likely correlated with the scatter in $M_{2D}$, as they are both affected by the projection effects and the uncertainties in the estimation of $M_{2D}$. Therefore, the covariance between $Y_{2D}$ and $M_{2D}$ must be taken into account in order to derive the unbiased estimate of the underlying $Y_{3D} - M_{3D}$ relations from tSZ and WL measurements.

#### 4.1 Covariance in the $Y - M$ relation

In order to characterize the nature of scatter in the observed $Y_{2D} - M_{2D}$ scaling relation, we quantify the correlation between the scatter in $Y_{2D}$ and $M_{2D}$ with the covariance matrix $C$ of the two-dimensional variable $X = (\log(M_{2D}/M_{3D}), \log(Y_{2D}/Y_{3D}))$ as follows:

$C_{ij} = \frac{1}{N_m - 1} \sum_{k=1}^{N_m} (X_{ki} - \bar{X}_i)(X_{kj} - \bar{X}_j), \quad (22)$

$\bar{X}_i = \frac{1}{N_m} \sum_{k=1}^{N_m} X_{ki}, \quad (23)$

where $X_{ki}$ represents the $i$-th component of $X$ for the $k$-th map.

The resulting covariance matrix for the 33 simulated clusters viewed along the $x$ projection axis is

$C = \begin{pmatrix} 1.12 \times 10^{-2} & 5.67 \times 10^{-3} \\ 5.67 \times 10^{-3} & 3.55 \times 10^{-3} \end{pmatrix}. \quad (24)$

Figure 7 shows the covariance between $Y_{2D}$ and $M_{2D}$ for the 33 simulated clusters viewed along the $x$ projection axis, where the grey points represent the resulting $X$ from a
Various covariance in the tSZ–WL mass scaling relation

\( \chi^2 \) fitting, and the red lines are the 1σ and 2σ contours of the log-normal distribution with the covariance matrix \( \mathbf{C} \) in Eq. (24). The points trace the log-normal contours quite well. We also find that the scatter in log(\( M_{2D}/M_{3D} \)) is tightly correlated with that of log(\( Y_{2D}/Y_{3D} \)). The correlation coefficients for our simulated clusters are 0.902, 0.769 and 0.828 for the x, y, and z projection axes, respectively. Removing the outlier discussed in Section 3.3.1 changes the correlation coefficient by \( \sim 0.02 \). The significant covariance between the scatter in \( Y_{2D} \) and \( Y_{3D} \) we found is consistent with previous theoretical studies on covariance between cluster observables (White, Cohn & Smit 2010; Stanek et al. 2010; Angulo et al. 2012; Noh & Cohn 2012) and observational work (e.g., Rozo et al. 2009).

Another important correlation in the tSZ and WL measurement is the covariance between \( M_{2D} \) and \( M_{3D} \) at a given \( M_{3D} \). This covariance \( \mathbf{C}' \) is defined by the two-dimensional variable of \( \mathbf{X}' = (\log(\frac{M_{2D}}{M_{3D}}), \log(\frac{Y_{2D}}{Y_{3D,scal}})) \), where \( Y_{3D,scal} \) is given by Eq. (16) at a given \( M_{3D} \). For the 33 simulated clusters viewed along the x projection axis, we found that

\[
\mathbf{C}' = \begin{pmatrix} 1.12 \times 10^{-2} & 6.29 \times 10^{-3} \\ 6.29 \times 10^{-3} & 4.45 \times 10^{-3} \end{pmatrix}.
\]

Figure 4 shows that our model is able to recover the 95% confidence level of the posterior distribution of the parameters of the \( Y_{2D} - M_{2D} \) relation for 33 simulated clusters. The red filled circle shows the best-fit parameters derived from the likelihood analysis with the covariance between \( M_{2D} \) and \( Y_{2D} \). The black filled circle is the best-fit parameters when \( Y_{2D} \) and \( M_{2D} \) are assumed to be independent, while the black star symbol represents the best-fit parameters of the \( Y_{3D} - M_{3D} \) relation. The hatched region shows the 95% confidence level of the posterior distribution.

### 4.2 Recovering the unbiased 3D \( Y - M \) Relation

With the covariance between \( Y_{2D} \) and \( M_{2D} \) in hand, we can develop a statistical model to recover the underlying \( Y_{3D} - M_{3D} \) relation. The posterior distribution of the parameters of the \( Y_{2D} - M_{2D} \) relation is given by the likelihood function of number counts of \( M_{2D}, Y_{2D} \) using the Bayesian framework as follows.

\[
dN(M_{2D}, Y_{2D})/dM_{2D}dY_{2D} = \int dY_{3D} dM_{3D} \times \varphi(M_{3D}) \varphi(Y_{3D}|M_{3D}) \varphi(M_{2D}, Y_{2D}|M_{3D}, Y_{3D}),
\]

where \( \varphi(Y_{3D}|M_{3D}) \) represents the probability distribution of the underlying \( Y_{3D} - M_{3D} \) relation and \( \varphi(M_{2D}, Y_{2D}|M_{3D}, Y_{3D}) \) is the probability distribution function of a set of \( (M_{2D}, Y_{2D}) \) for a given set of \( (M_{3D}, Y_{3D}) \). Assuming that they follow the log-normal distributions, we have

\[
\varphi(Y_{3D}|M_{3D}) = A \exp\left\{ -\frac{1}{2} \frac{[\log Y_{3D} - \log Y_{model}]^2}{\sigma} \right\},
\]

where \( A = \frac{1}{\sqrt{2\pi}\sigma}, \sigma = \sigma_{\log Y_{3D}} \) and \( \log Y_{model} = \alpha_0 + \alpha_1 \log \left(M_{3D}/(10^9 h^{-1} M_\odot)\right) \), and

\[
\varphi(M_{2D}, Y_{2D}|M_{3D}, Y_{3D}) = B \exp\left\{ -\frac{1}{2} \mathbf{X}' \mathbf{C}'^{-1} \mathbf{X} \right\},
\]

where \( \mathbf{X} = (\log(M_{2D}/M_{3D}), \log(Y_{2D}/Y_{3D})) \), \( B = \sqrt{2\pi} \sigma \), and \( \mathbf{C} \) represents the covariance matrix of \( \mathbf{X} \).

The red points show the expected distribution of the model and the best-fit parameters \( \alpha_0, \alpha_1, \sigma_{\log Y_{3D}}, \) and \( \mathbf{C} \). The red error bars represent the 68% confidence level of \( \log Y_{2D} \) for a given \( M_{2D} \). The red points recover our 2D measurements indicated by grey points, demonstrating that our model provides a good description of the \( Y_{2D} - M_{2D} \) relation from tSZ-WL mock analyses. We stress that the covariance is an essential ingredient in explaining the scatter in the \( Y_{2D} - M_{2D} \) relation. The scatter of \( \sim 14\% \) in \( \log(Y_{2D}/Y_{3D}) \) alone is not enough to explain the total scatter of \( \sim 23\% \). One also has to include the covariance between \( \log(Y_{2D}/Y_{3D}) \) and \( \log(M_{2D}/M_{3D}) \).

Next, we recover the \( Y_{3D} - M_{3D} \) relation from our model by estimating the parameters \( \alpha_0 \) and \( \alpha_1 \) in Eq. (24). To do this, we first construct the likelihood function of number density of clusters in the \( Y_{2D} - M_{2D} \) assuming the Poisson distribution:

\[
\mathcal{L} = \prod_i^{N_{\log Y}} \prod_j^{N_{\log M}} \frac{\lambda^{N_{ij}} \exp(-\lambda)}{N_{ij}!},
\]

where \( N_{ij} \) is the number count of clusters found in \((i,j)-th\) grid in the \( Y_{2D} - M_{2D} \) plane, \( N_{\log Y} \) and \( N_{\log M} \) represent the number of bins in log \( Y_{2D} \) and log \( M_{2D} \), respectively. The best-fit parameters \( \alpha_0 \) and \( \alpha_1 \) are then found by maximizing the likelihood \( \mathcal{L} \). We test our method with measured values of \( Y_{2D} \) and \( M_{2D} \) over \( 33 \times 3 = 99 \) realizations of projected
cluster maps (by combing simulated clusters viewed along three orthogonal projections) with \( L_{\text{depth}} = 500 h^{-1} \) Mpc.

The likelihood function is calculated over 100 logarithmically spaced bins in \( 10^{15} < M_{2D} [h^{-1} M_\odot] < 10^{18} \) and \( 10^{-5.5} < Y_{2D} [h^{-1} \text{Mpc}]^2 < 10^{-4} \). For simplicity, we set \( \sigma_{\log Y,3D} = 0.030 \) and adopt the distribution of \( M_{3D} \) measured from our simulations (see the black hatched histogram in Figure 1).

The result of our likelihood analysis is summarized in Figure 5. The black star symbol represents the parameters of the underlying 3D \( Y - M \) relation. The red point is for the best-fit parameters obtained from our likelihood analysis. The red dashed line shows the 95% confidence region, demonstrating that our maximum likelihood analysis can recover the true 3D scaling relation reasonably well. We emphasize that it is critical to include the covariance \( C \) between \( \log(Y_{2D}/Y_{3D}) \) and \( \log(M_{3D}/M_{3D}) \). Ignoring it leads to biases in the estimated parameters of the 3D scaling relation, as illustrated by the black point and hatched region in Figure 5. Note that the bias in the estimated slope \( \alpha \) of the \( Y - M \) relation is on the order \( \sim 0.10 \), which is comparable to the statistical uncertainty in the current observations (e.g., Planck Collaboration et al. 2015; de Haan et al. 2016). Thus, the covariance among cluster observables must be taken into account in order to take advantage of the statistical power of current and future tSZ and WL cluster surveys.

After recovering the unbiased 3D \( Y - M \) relation, one can reduce the uncertainty in the estimate of \( Y_{2D}(R_{2D}) \) by an iterative approach as follows (see also Liu et al. 2015).

Using the \( Y_{3D} - M_{3D} \) relation, one can compute a new estimate of \( M_{3D} = f(Y_{3D}) \) to re-define the boundary of a cluster \( R_{3D} \) through \( M_{3D} = 500 h^{-1} \) Mpc, one can then iterate to obtain a new estimate of \( Y_{3D} \) within the new radius \( R_{3D} \). This iterative approach is expected to be efficient because the scatter in WL mass is larger than the scatter in \( Y \) at a given \( M_{3D} \). We tested this iterative approach by using the mock measurements of \( Y_{2D} \) and the \( Y_{3D} - M_{3D} \) relation in Eq. (16). In the case of \( L_{\text{proj}} = 500 h^{-1} \) Mpc, we found that the scatter in \( \log(Y_{2D}/Y_{3D}) \) changes from 5.9% to 4.5% for \( x \)-axis after ten iterations, which was sufficient for convergence of results. Note that similar results are obtained for the other two axes, where the scatter decreases from 5.6% to 5.1% and from 6.3% to 5.5% for \( y \)-axis and \( z \)-axis, respectively. While this iterative approach is useful to obtain a more accurate estimate of \( Y_{2D} \), it still does not completely remove the uncertainty in \( R_{2D} \) in measurement of \( Y_{2D} \); i.e., we cannot reduce the scatter of \( \log(Y_{2D}/R_{2D})/Y_{3D} \) to that of \( \log(Y_{2D}/R_{2D})/Y_{3D} \) through this iterative approach.

4.3 Implications for Cosmological Inferences

Finally, we assess the impact of the biased \( Y - M \) relation on cluster-based cosmological constraints. Here, we consider the cumulative number count of galaxy clusters as a function of the angular integrated Compton-\( y \) parameter \( Y_{\text{ang}} = 1/D_A^2(z)Y \), where \( D_A(z) \) is the angular diameter distance for redshift of \( z \). The number count per solid angle in the redshift range of \( z_{\text{min}} \) to \( z_{\text{max}} \) is given by

\[
N(Y_{\text{ang},3}\text{hre}; z_{\text{min}}, z_{\text{max}}) = \int_{z_{\text{min}}}^{z_{\text{max}}} \frac{dV}{dz} \int_{Y_{\text{ang},3}\text{hre}(M,z)} dM \frac{dn}{dM} \varphi(Y_{\text{ang}}|M,z),
\]

where \( \frac{dn}{dM} \) is the halo mass function and \( \varphi(Y_{\text{ang}}|M,z) \) expresses the scaling relation between \( Y_{\text{ang}} \) and mass \( M \) at redshift \( z \). We use the halo mass function by Tinker et al. (2008), and \( \varphi(Y_{\text{ang}}|M,z) \) is set to be the log-normal function with the scatter of 0.18 (Angulo et al. 2012). As a fiducial model, we consider the self-similar \( Y - M \) relation as shown in Eq. (15) with cosmological parameters set to the WMAP nine-year results (Hinshaw et al. 2013). We consider two additional scenarios where the \( Y - M \) relation is biased when the covariance between the clusters in \( Y \) and \( M \) are ignored, as shown by the black point in Figure 5. In one scenario, we set our cosmological parameters to the fiducial WMAP9 values, while in the other we increase \( \Omega_m \) higher by 2.5%, which corresponds to the 1\( \sigma \) error in the WMAP9 value. Note that we take into account changes in both halo mass function and angular diameter distance when varying cosmological parameters.

For illustration, we consider the redshift range of \( z = 0.2 - 0.4 \), which is the relevant redshift range for recent WL measurements of tSZ-selected clusters (e.g., High et al. 2014; Battaglia et al. 2013). Figure 8 shows the expected cluster number counts for the three different models. The red points represent our fiducial case, the black dashed line corresponds to the biased \( Y - M \) relation with the fiducial cosmology, and the black solid line corresponds to the case with...
the biased $Y - M$ relation and with higher $\Omega_{m0}$. The red open and hatched boxes show the Poisson error for a hypothetical survey with the sky coverage of 1,500 and 27,000 square degrees, which correspond to the coverage of ongoing imaging surveys (such as the Hyper Suprime-Cam) and the full-sky coverage with masking of the galactic plane, respectively. For a fixed cosmology, the biased $Y - M$ relation leads to reduction in the number count in the survey area of 27,000 square degrees, which is comparable to the sample size of the Planck tSZ cluster catalog (Planck Collaboration et al. 2014a, 2015a). Increasing $\Omega_{m0}$ leads to higher cluster counts, suggesting that the biased $Y - M$ relation can introduce biases in cosmological parameters, such as $\Omega_{m0}$ and $\sigma_8$. In this case, 10% bias in the $Y - M$ relation leads to an increase of 2.5% in $\Omega_{m0}$, or an increase of 6.6% in $\sigma_8$ for a fixed initial curvature perturbation amplitude.

5 BARYONIC EFFECTS

So far, the simulations we have treat the ICM as a non-radiative gas and ignored additional baryonic physics, such as radiative cooling, star formation, and feedback from active galactic nuclei. These baryonic physics can in principle induce additional scatter in the observed $Y_{2D} - M_{2D}$ relation by changing the level of gas pressure in the correlated structure along the line of sight. While these effects are expected to be small (Nagai 2006; Battaglia et al. 2012; Kay et al. 2014), further scrutiny is still useful in order to assess to what extent the impact of the uncertain baryonic physics on the $Y_{2D} - M_{2D}$ relation.

In order to examine the effects of baryonic physics on the scatter of $Y - M$ relation, we analyzed re-simulation of Omega500 with radiative cooling, star formation, and supernova feedback (CSF). This CSF run includes metallicity-dependent radiative cooling, star formation, thermal supernova feedback, metal enrichment and advection, which are based on the same subgrid physics modules in [Nagai, Kravtsov & Vikhlinin (2007)], which we refer the reader for more details. In the following, we work with a mass-limited sample of 46 clusters with $M_{500c} \geq 2.8 \times 10^{14} h^{-1} M_{\odot}$ at $z = 0.33$. Note that our CSF simulation suffers from the well-known “overcooling” problem, where the simulation over-predicts the amount of central stellar mass by a factor of $\sim 2$. As such, the results of our NR and CSF run can be used to bracket systematic uncertainties associated with baryonic effects.

Following the analyses in Section3 we first measure the $Y - M$ relation and its intrinsic scatter in the CSF run. We find that the best-fit scaling relation between $\log Y_{3D}$ and $\log M_{3D}$ is

$$\log \left( \frac{Y_{3D}}{(h^{-1} \text{Mpc})^2} \right) = 1.88 \log \left( \frac{M_{3D}}{10^{14} h^{-1} M_{\odot}} \right) - 5.84,$$

where the best-fit slope of $1.88 \pm 0.030$ (1$\sigma$ error; see the best-fit relation shown as the hatched region in Figure 10) is different from the self-similar prediction of $5/3$ because of the increasingly larger reduction in the gas mass fraction at the low-mass clusters (e.g., Nagai 2006). The intrinsic scatter in the CSF run is $\sigma_{\log Y_{3D}} = 0.050$, suggesting that gas cooling and star formation can increase the intrinsic scatter of the $Y_{3D} - M_{3D}$ relation by up to 70%. We find that the increased scatter originates from the enhanced fluctuations in gas pressure in the CSF run relative to the NR run (see also Khedekar et al. 2013). Note that the scatter changes by only $\lesssim 4\%$ when excising the core region ($R \leq 0.15 R_{500c}$), concluding that the cluster core makes a minor contribution to the scatter.

Table4 reports the scatter between $Y_{2D}$ and $Y_{3D}$ in the CSF run. Analogous to the NR case, we find that the scatter of the CSF run increases with the projection depth $L_{p_{proj}}$ from 10 to 500 $h^{-1}$Mpc for three different projections. For $L_{p_{proj}} = 500 h^{-1}$Mpc and fitting range of 0.1'-5', we find that the baryonic effects change the scatter between $Y_{2D}$ and $Y_{3D}$ by about 10%, except for the $x$-axis projection. In the $x$-axis projection, we find two clusters with $\log Y_{2D}/Y_{3D} \sim 0.3$ and $-0.5$, making them $6\sigma$ and $7\sigma$ outliers in the population, respectively. The $6\sigma$ outlier has two high-pressure cores within $R_{500c}$. One of the cores is located around $\theta_{500c}$ in the projected Compton-$y$ map, causing a poor gNFW model fit. The $7\sigma$ outlier has a flat core at $\theta < 1'$, making the gNFW a poor fit. When removing these outliers, there is a clearer trend of increasing scatter with $L_{p_{proj}}$.

Finally, we measure the covariance between tSZ and WL signals to be

$$C = \begin{pmatrix} 1.33 \times 10^{-2} & 1.06 \times 10^{-3} \\ 1.06 \times 10^{-2} & 1.22 \times 10^{-2} \end{pmatrix},$$

for the $x$-axis projection in the CSF run. We also confirmed that the two-dimensional variable $X = (\log(M_{2D}/M_{3D}), \log(Y_{2D}/Y_{3D}))$ follows the bivariate Gaussian distribution with the covariance matrix for the CSF run. The correlation coefficients are found to be 0.838, 0.706 and
0.690 for the $x$, $y$, $z$ projections, respectively, which differ from the NR values at the level of $\lesssim 20\%$.

In summary, baryonic effects can alter the statistical property of tSZ and WL signals at some level. However, we show that our model can accommodate the baryonic effects, by taking into account changes in the $Y_{2D} - M_{2D}$ relation, its intrinsic scatter, and the covariance matrix of the two-dimensional variable, $X = (\log(M_{2D}/M_*)$, $\log(Y_{2D}/Y_\text{obs}))$. In Figure 11, the gray points show the measured $Y_{2D}$ and $M_{2D}$ of the CSF clusters, and the red points with error bar represent our modeling as shown in Section 4.4. Our model shows that the scatter in the WL calibrated $Y - M$ relation is 28% in the CSF run, compared to 23% in the NR run. Since the NR and CSF runs should bracket the range of baryonic effects, we expect that the realistic model should lie within the range explored in this work.

### 6 CONCLUSIONS

The tSZ effect is widely recognized as a robust mass proxy of galaxy clusters with small intrinsic scatter. However, recent observational calibration of the tSZ-WL mass relation shows that the observed scatter is considerably larger than the intrinsic scatter predicted by numerical simulations. This raises a question as to whether we can exploit the full statistical power of upcoming SZ and WL cluster surveys. In this work, we investigated the origin of observed scatter in the $Y - M$ relations, using mock tSZ and WL maps of galaxy clusters extracted from high-resolution cosmological hydrodynamical simulations. Our main findings are summarized as follows:

(i) We showed that the scatter in the WL calibrated $Y - M$ relation is 23%. This is significantly larger than the intrinsic scatter of $\lesssim 10\%$ predicted by simulations, and it is consistent with the observed scatter of about 20%.

(ii) The uncertainty in the integrated Compton-y, $Y$, inferred from the projected Compton-y profile originates from the combination of (a) the projection effect in the tSZ maps and (b) the uncertainty in the cluster radius determined from the WL mass measurements, with each effect contributing to the total scatter by 5% and 10%, respectively.

(iii) The scatter in the tSZ-WL mass relation can be explained by the combination of uncertainties associated with $Y$ and WL mass measurements. Namely, the amplitude of the scatter is determined by the covariance between tSZ and WL signals. In the presence of the uncertainty in the WL mass, the distribution of clusters in the $Y_{2D} - M_{2D}$ plane is smeared in both $Y_{2D}$ and $M_{2D}$, where its scatter is different from the scatter in log $Y_{2D}$ alone.

(iv) We show that the covariance between tSZ and WL signals is important for recovering the true $Y - M$ relation. Ignoring the covariance would lead to 10% bias in the $Y - M$ relation, which leads to the biases in $M_{2D}$ by 2.5%, and $\sigma_Y$ by 6.6%. Thus, this covariance must be taken into account for cosmological constraints with ongoing and future cluster surveys.

(v) We show that the covariance of the $Y - M$ relation depends on the input baryonic physics at a level of $\lesssim 20\%$, by using two sets of simulations that bracket a broad range of astrophysical uncertainties. We further demonstrate that our statistical model to describe the $Y_{2D} - M_{2D}$ relation can provide a reasonable description of the simulation results, provided that the proper modeling of the true $Y - M$ relation and the covariance in the $Y_{2D} - M_{2D}$ plane are performed.

(vi) We present a statistical model to recover the unbiased $Y - M$ relation from a set of tSZ and WL measurements which enables us to obtain the unbiased tSZ-mass scaling relation from a simultaneous measurement of tSZ and WL, and opens up the possibility of extracting cosmological information from upcoming multi-wavelength surveys that will provide a large statistical sample of galaxy clusters out to the high-redshift ($z \lesssim 1$) universe.

Future work should focus on developing and analyzing a larger sample of simulated clusters and tSZ and WL mocks in order to characterize the mass and redshift dependence of the $Y_{2D} - M_{2D}$ relation, its covariance matrix, and the impact of the outlier populations due to mergers. Addressing these issues is the critical step for understanding the remaining astrophysical uncertainties and hence accurate and
robust interpretations of current and upcoming SZ and lensing surveys, including cluster counts, SZ power spectrum and higher-order moments, and cross-correlations between tSZ and WL maps.

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