Wake-induced vibration of two-phase conductors connected by spacers

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Abstract. Considering finite element approach applied to modelling of wind-induced split-phase overhead power line conductor oscillations caused by aerodynamic wake. Model represents an aeroelastic system including twin bundled conductor with spacers. Modified Simpson’s theory is used for conductor wake coupling simulation. Assuming small oscillating conductor deflection from element ends connection line compared to element length, thus the element tension and element tensile strain can be considered as constant values. Strain, as function of transverse displacements, defines as quadratic approximation. Element absolute displacements and twisting angles, together with Ritz’s method coefficients is taken as generalized coordinates, in which conductor move equations are compiled with non-linear elastic, inertial and aerodynamic forces. Appropriate linearized small oscillation equations near the static equilibrium are derived – those can be used for defining system flutter critical wind velocity. An example with split-phase twin bundled conductor, three-sub-span system is presented.

1. Introduction
Wind-induced overhead power line conductor oscillation phenomena can be subdivided on three groups depending of frequencies and amplitudes.

First one – high frequency low amplitude vibrations, also called “Aeolian vibrations”, occurring when wind are weak and stable and conductor itself is strained. Those vertical plane oscillations in form of steady-waves have frequencies about 5 to 100 Hz with initial wind speed of 0.5 to 8 m·s⁻¹. The cause of vibration excitement is the air flow interruption behind the conductors in forms of Karman vortex streets [1, 2].

Next group contains low frequency (0.2 to 3 Hz) high amplitude oscillations, so called “Conductor galloping”. This phenomenon occurs with combination of quasi-stable wind (speeds of 5-20 m·s⁻¹) and conductor ice accretion [3, 4]: due to asymmetrical iced conductor cross-section various aerodynamic forces can arise and change, leading to oscillation. Conductor galloping is a variety of flutter oscillations, “peak-to-peak” type, amplitude of which can reach several meters vertically.

Last group contains split-phase power line bundled conductors oscillations, usually called sub-span oscillations (figure 1). In this case most common frequencies are 0.7 to 5 Hz. Necessary initial conditions for this phenomenon to occur consists approximately close location of two-bundled conductors in one horizontal plane and a wind velocity of 6-15 m·s⁻¹; thus, one conductor (leeward) is in the wake of another (windward), which leads to aerodynamic lift and drag changes, causing windward conductor and, due to spacers connections, leeward, to oscillate [5-7]. Sub-span oscillations
(in combination with constant vibrations) can lead to various power line structural elements (spacer clamps, dampers, etc.) damages and wear-outs.

![Figure 1](image1.png)

**Figure 1.** Sub-span oscillations: (a) wake-induced oscillations scheme; (b) case of oscillations.

2. **Basic hypotheses, defining relations and equations**

A strained conductor in the span of a power line is considered, assuming conductor having only tensile elasticity with rigidity of $EF$, where $E$ – is a Young’s modulus, $F$ – effective cross-section area.

Initial surplus of the conductor length:

$$\Delta^0 = L^0(1 + \alpha T) - L$$  \hspace{1cm} (1)

where $L^0$ – initial conductor length in normal temperature conditions, $L$ – span length, $T$ – temperature increment, $\alpha$ – linear temperature expansion coefficient.

In purpose of dimensional coordinates discretisation finite element approach is using. With each $i$ element a local coordinate system $O^{(i)}x^{(i)}y^{(i)}z^{(i)}$, which moving relatively to global (inertial) system OXYZ, is connected (next we will neglect upper $(i)$ index for the case of convenience). Local axes are parallel to global axe OX and in model only rotations around OX axe are considered (figure 2).

![Figure 2](image2.png)

**Figure 2.** Global and local coordinate systems.
By the influence of gravitation and wind conductor is oscillating near static position. Nodes displacements along OX, OY, OZ axes and twisting angle around OX axe denote as \( u, v, w \) and \( \phi \).

Then, local displacements and conductor cross-section twisting angle within finite element length of \( l \) can be described using Ritz method as:

\[
\tilde{u} = (u_1 - u_0) \frac{x}{l}, \\
\tilde{v} = (v_1 - v_0) \frac{x}{l} + \sum_k q_k \sin \left( \frac{k \pi x}{l} \right), \\
\tilde{w} = (w_1 - w_0) \frac{x}{l} + \sum_k r_k \sin \left( \frac{k \pi x}{l} \right), \\
\tilde{\phi} = (\phi_1 - \phi_0) \frac{x}{l} + \sum_k p_k \sin \left( \frac{k \pi x}{l} \right),
\]

(2)

where \( u_k(t), v_k(t), w_k(t), \phi_k(t) \) are boundary values of displacements and twisting angle given that \( x = 0 \) \( (k = 0) \) and \( x = l \) \( (k = 1) \), that is a generalized coordinates. Besides, problem generalized coordinates are coefficients \( p_i(t), q_m(t), r_n(t) \), that defines finite element inner freedom degrees relative to sine forms.

According to Hook’s law, given that \( \varepsilon > \frac{\Delta \phi}{l} \) and considering (1), tensile force \( N \) denotes:

\[
N = EF \left( \varepsilon - \frac{\Delta \phi}{l} \right),
\]

(3)

where \( \varepsilon \) – longitudinal strain, which will be denoted in the form of quadratic dependence as:

\[
\varepsilon = \frac{du}{dx} + 0.5 \left( \frac{d\phi}{dx} \right)^2 + \left( \frac{dw}{dx} \right)^2.
\]

(4)

Torque of the arbitrary conductor cross-section is \( M_t = GJ \frac{d\phi}{dx} \), where \( GJ \) is torsional rigidity.

EF and GJ rigidities are assuming to be constant along the span.

Because of longitude inertial forces neglection tensile force will be constant along the conductor length, \( N(x,t) \approx N(t) \). Thus by using (3) same can be applied to the strains, \( \varepsilon(x,t) \approx \varepsilon(t) \). By taking integral of (4) considering (2) we will get:

\[
\varepsilon = \left\{ \left( u_1 - u_0 \right) + \frac{1}{2l} \left[ \left( v_1 - v_0 \right)^2 + \left( w_1 - w_0 \right)^2 \right] + \frac{1}{2} \sum_k \left( \pi k \right)^2 \left( q_k ^2 + r_k ^2 \right) \right\},
\]

(5)

then potential energy will be denoted as:

\[
U = \frac{1}{2} \int_0^l \left[ EF \left( \varepsilon - \frac{\Delta \phi}{l} \right)^2 + GJ \frac{d\phi}{dx} ^2 \right] dx = \frac{1}{2} \int_0^l \left[ EF \left( \varepsilon - \frac{\Delta \phi}{l} \right)^2 + \frac{GJ}{2l} \left[ \left( \phi_1 - \phi_0 \right)^2 + \frac{1}{4} \sum_k \left( \pi k \right)^2 p_k ^2 \right] \right].
\]

(6)

Virtual work principle is using in form of \( \delta U = \delta A_g + \delta A_a + \delta A_{int} \) to get the nonlinear move equations of the system. Potential energy variation:

\[
\delta U = \sum_{i=0} \left( \frac{\delta u}{\delta u_i} \delta u_i + \frac{\delta v}{\delta v_i} \delta v_i + \frac{\delta w}{\delta w_i} \delta w_i \right) + \sum_k \left( \frac{\delta q_k}{\delta q_k} \delta q_k + \frac{\delta r_k}{\delta r_k} \delta r_k + \frac{\delta p_k}{\delta p_k} \delta p_k \right),
\]

(7)

and the gravity, aerodynamic and inertial work variations are:

\[
\delta A_g = -\mu g \int_0^l \delta (w + \eta_c \sin \phi + \theta_c \cos \phi) dx, \delta A_a = \int_0^l (Y \delta v + Z \delta w) dx,
\]

\[
\delta A_i = \int_0^l \mu [\tilde{u} \delta u + (\tilde{v} - \eta_c \phi - \theta_c \phi^2) \delta v + (\tilde{w} + \theta_c \phi - \eta_c \phi^2) \delta w + (\eta_c \tilde{u} + \theta_c \tilde{w} + \rho_c \phi) \delta \phi] dx,
\]

(8)

Where \( \mu \) – linear mass, \( \theta_c \) and \( \eta_c \) – cross-section center-of-mass coordinates (according to cross-section coordinate system), \( \mu \rho_c ^2 \) – polar moment of inertia; \( Y \) and \( Z \) – aerodynamic load components.

Integrals in (8) are taking by using (2), (3) and (5).
3. Aerodynamics

While inside the aerodynamic wake, average flow speed is slower compared to free-stream. This phenomenon can be described by using empirical functions [8]. Implying \( d \) as diameter of conductor, we denote coordinates of \( \xi = y/d, \eta = z/d \) and also free-stream drag coefficient \( C_{D_{\text{Max}}} \) which for smooth cylinder is \( C_{D_{\text{Max}}} \approx 1.2 \). Ratio between the free-stream velocity and a local velocity inside the wake (correspondingly \( V \) and \( V_l(\xi, \eta) \)) is:

\[
b(\xi, \eta) = \frac{V_l(\xi, \eta)}{V} = 1 - \psi(\xi, \eta) \frac{\psi^2(\xi, \eta)}{\eta + 6},
\]

\[
\psi(\xi, \eta) = C_{D_{\text{Max}}} \exp \left( -\frac{\xi^2}{0.23C_{D_{\text{Max}}}(\eta+6)} \right)
\]

(Figure 3)

Figure 3. Distribution of \( b(\xi, \eta) \) function for various profiles along the aerodynamic wake.

References [9-13] contain coefficients for a wide range of various intervals. In this particular paper we use smooth cylinder coefficients of Price. Also by using analytical expressions of drag and lift coefficients (\( C_D \) and \( C_L \)) derivative taking problem are significantly simplified.

Our polynomial expressions are similar to [14] except the higher precision and covering of the wider aerodynamic wake area:

\[
C_D(\eta, \xi) = \sum_{i=0}^{5} a_i \xi^{2i},
\]

\[
a_i = b_{i0} + b_{i1} \eta + b_{i2} \eta^2,
\]

\[
C_L(\eta, \xi) = \sum_{i=0}^{4} p_{i+1} \xi^{2i+1},
\]

\[
p_i = q_{i0} + q_{i1} \eta + q_{i2} \eta^2,
\]

\[
|\xi| \leq 4; \quad 5 \leq \eta \leq 20.3.
\]

Outside the designated intervals of \( \xi, \eta \) drag and lift coefficients are \( C_D = 1.2, C_L = 0 \); values for \( b_{i0}, b_{i1}, b_{i2}, q_{i0}, q_{i1}, q_{i2} \) are given in [15].
4. Example. Split-phase twin bundle conductor, three sub-spans
Considering the case where the line supposed to be strongly prone to sub-span oscillations: large and equal sub-spans, tilted bundle plane. Wind velocity is 15 m·s⁻¹. Model scheme are in figure 5 and detailed model parameters are in table 1. Results illustrated in figures 6 and 7.

Figure 4. Aerodynamic coefficients surfaces (10) inside the designated intervals.

Figure 5. Example model scheme.

| **Table 1.** Model parameters. |
|-------------------------------|
| Span length                   | 240 m |
| Sub-spans (m)                 | 3 x 80 m |
| Conductor diameter (cm)       | 2.54 |
| Horizontal spacing (diameters) | 15 |
| Vertical spacing (diameters)  | 2.5 |
| Conductor tension (kN)        | Windward: 19; Leeward: 19.1 |
| Conductor linear mass (kg·m⁻¹) | 1.25 |
**Figure 6.** Red and black curves are representing correspondingly horizontal displacement of leeward sub-conductors in the middle of sub-spans 1 and 3.

**Figure 7.** Windward (left, red curve) and leeward (right, black curve) sub-conductor orbits due to sub-oscillations, middle of end sub-span.

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