Assessment of Mechanical Crack Fault Dependent on Wavelet Finite Element Model

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Abstract. Given that cracks in the forging and heat treatment process of switch actuator rod can affect its service life, damage mechanics is combined with the finite element analysis method to calculate the fatigue crack propagation life by using the Paris equation. The wavelet finite element model is used to establish the GM (1,1) grey differential equation. The identification parameter of the model is determined by experimental data calculation to assess the fatigue crack life of the switch actuator rod and compare it with the measured data in the experiment. The results show that the application of the wavelet finite element model to assess the fault of switch actuator rod is simple, feasible, and highly accurate, with excellent application value.

Keywords: Switch, Wavelet Finite Element Model, Grey Model, Fatigue Crack

1. Introduction

S600 internal locking switch machine is a new type of switch developed by a company to adapt to the construction of railway automation. It mainly realizes the switch conversion through the actuator rod linkage switch [1-2]. As high-speed railway construction develops rapidly, the train speed has been continuously improved, and higher requirements are placed on the safety and reliability of the switch [3-4]. The actuator rod is a crucial component, which is very important for the accurate assessment and estimation of its safe service life. The fatigue life of the actuator rod is analyzed and studied, which provides the design basis for realizing the requirements of long life and high reliability. However, there are inevitably various defects in the forging and heat treatment process of the actuator rod [5-6]. The occurrence of the initial crack is the main factor affecting the service life of the switch actuator rod.

The problem of fatigue crack propagation life is very complex, so it is difficult to establish an analytical and universally applicable fault assessment model. The fatigue crack propagation life is defined by the fracture mechanics method. The crack propagation characteristics of components are determined by the following factors: the stress (load) - time history of components. The stress intensity factor corresponding to the geometry and crack shape of components. The basic crack propagation rate corresponding to the materials and relevant environment of components, etc. However, materials are not uniform and continuous, and the failure behavior of materials is often random. Hence, it is not rational to use the isolated deterministic fracture mechanics analysis method or the damage mechanics method to solve the reliability issue of the expected components. Although probabilistic fracture mechanics can address the uncertainty problem in fracture mechanics, the probabilistic method also
has defects, because the calculation of probability requires large samples. Wavelet finite element model method can solve the issue of the small sample size and inadequate information and has the advantage of less data modeling. The practice has verified its high calculation accuracy. The finite element method is widely used in fatigue research to simulate the fatigue crack propagation of components. The finite element method can significantly save the test cost and shorten the research period. The finite element method is used to calculate the stress around the crack in the process of bearing the load, and then the maximum crack length of the switch machine is calculated. Finally, the fatigue crack life of the actuator rod is determined by the Paris equation integration method. The grey theory is used to establish the GM (1,1) model and assess the fatigue crack life of the switch actuator rod.

2. Calculation of fatigue crack propagation life

2.1. Calculation method of fatigue crack propagation life based on Paris
The modern calculation method of fatigue crack propagation life mainly follows the principles of fracture mechanics and damage mechanics. The judgment basis is that the brittle fracture stress reaches the lowest value of material fragile fracture toughness, and then the crack propagation life is calculated according to the cumulative damage equation. According to different stress amplitudes, different fatigue life estimation equations are selected.

Where \( m \) is 2, the following can be obtained

\[
N_p = \frac{1}{\frac{1-m}{2}C_1(\Delta\sigma)^m \left( \frac{a_c}{a_0} - \frac{1}{2} \right)}
\]  

(1)

Where \( m=2 \), the following can be obtained

\[
N_p = \frac{1}{C_1(\Delta\sigma)^2} \ln \frac{a_c}{a_0}
\]  

(2)

\[
C_1 = C\alpha^m\pi^{m/2}
\]  

(3)

2.2. Establishment of the wavelet finite element model
The wavelet finite element assessment refers to the use of a dynamic model to assess the number of time series of the system, that is, to assess the main behavior characteristics or a specific index of the system, from the development and change to a certain extent in the future, time-related gray process.

Grey modeling is to use the known data series to accumulate and weaken the randomness of the series, to find the change rule of the system, and then to establish the differential equation model, namely the wavelet finite element assessment model. Its modeling and assessment process are as follows:

(1) Sequence \( x^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \cdots, x^{(0)}(n)\} \) is set, which is the original non-negative...
sequence, and a new sequence is generated by the one-time accumulation as follows

\[ x^{(i)} = \{x^{(i)}(1), x^{(i)}(2), \ldots, x^{(i)}(n)\} \]  
\[ x^{(i)}(k) = \sum_{j=1}^{k} x^{(0)}(k) \quad (k = 1, 2, \ldots, n) \]  

(2) The adjacent mean series \( y^{(i)} \)

\[ y^{(i)}(k) = \frac{1}{2}(x^{(i)}(k) + x^{(i)}(k-1)) \quad (k = 2, 3, \ldots, n) \]  

(3) The model is established as follows:

\[ x^{(0)}(k) = ay^{(i)}(k) \quad (k = 2, 3, \ldots, n) \]  

The corresponding whitening differential equation can be obtained as follows:

\[ \frac{dx^{(i)}}{dt} + ax^{(i)}(i) = u \]  

Where \( A \) and \( u \) represent the identification parameters of the model. In the model, \( -a \) represents the development coefficient, which reflects the development trend of the sequence; \( u \) represents the grey action quantity, which size and change reflect the changing relationship between the data and the behavior model, equivalent to the action quantity \( a \) in the system.

\[ \begin{bmatrix} a \\ u \end{bmatrix} = (B^T B)^{-1} B^T Y(1) \]  

\[ B = \begin{bmatrix} -y^{(i)}(2) & 1 \\ -y^{(i)}(3) & 1 \\ \vdots & \vdots \\ -y^{(i)}(n) & 1 \end{bmatrix}, \quad Y = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix} \]  

Identify the grey differential equation of assessment model value as follows

\[ x^{(0)}(k) + ay^{(i)}(k) = u \]  

The corresponding time series is as follows
\[ x(k+1) = x(k+1) - \frac{u}{a} + \frac{u}{a} e^{-ak} \] (9)

(5) The reduced value can be obtained as follows

\[ x^{(0)}(k+1) = x^{(1)}(k) - x^{(1)}(k) \] (10)

(6) Error test

\[ \varepsilon(k) = x^{(0)}(k) - x^{(0)}(k) \] (11)

3. Assessment of switch actuator rod crack life based on wavelet finite element

45 steel is used as the structure of switch motor-driven rod. The heat treatment condition is 840 °C quenching, 550 °C tempering, \( \sigma_s = 513 \text{mpa} \), \( \sigma_B = 803 \text{mpa} \). The minimum value of fracture toughness \( K_{IC} = 96.8 \text{mpa} \cdot \text{m}^{1/2} \), material constant \( C = 9.59 \times 10^{-12} \), \( M = 2.75 \).

Based on the actual measurement, the main bearing load of the actuator rod varies from 4800n to 5000n. During the experiment, 8 samples of switch actuator rod with different initial crack lengths are taken to obtain the fatigue life corresponding to the initial crack length A0 of the observed sample, as shown in Table 2 below.

3.1. Fault assessment of the crack of the switch actuator rod based on Paris equation

Using ANSYS finite element software, a solid element of solid95 is selected to divide finite element mesh. Simulate the actual working load, fix the pinhole, load at the connection hole of the front end of the actuator rod, and the load direction is along the axis of the actuator rod. In the process of forging and machining, there is an initial crack in the actuator rod. A semi-elliptical crack is simulated at the pinhole connection of the actuator rod to calculate the stress around the crack under the action of fluctuating cyclic load.

The stress around the crack is \( \sigma = 429 \text{MPa} \). Consider the fluctuating cyclic stress of the switch motor lever in the working process, namely:

\[ \Delta \sigma = \sigma = 429 \text{ MPa} \]

According to fracture mechanics:

\[ K_I = \frac{2}{\pi} \sigma \sqrt{\pi a} \] (12)

When \( k_i = K_{IC} \), \( a = AC \), and the critical crack size \( AC \) can be obtained as follows

\[ a_c = \frac{\left( \frac{K_{IC} \pi}{2 \sigma} \right)^2}{\pi} = 0.01 \text{m} = 10 \text{mm} \]
Since \( \alpha = \frac{2}{\pi} \), through the calculation of material parameters of 45 steel, the following can be obtained,

\[
C_1 = C \alpha^n \pi^{m/2} = 9.59 \times 10^{-12} \left( \frac{2}{\pi} \right)^{2.75} \pi^{2.75/2} = 1.3368 \times 10^{-11}
\]

The components are subject to the action of equal stress amplitude, and the Paris equation method is used to calculate the life of the switch motor pole, as shown in Table 1:

| Initial crack length / mm | 0.096 | 0.249 | 0.498 | 0.997 |
|---------------------------|-------|-------|-------|-------|
| Fatigue life NP / time    | 298960| 193220| 134190| 86875 |
| Initial crack length / mm | 2.002 | 3.499 | 4.001 | 4.998 |
| Fatigue life NP / time    | 53577 | 31190 | 26513 | 19190 |

3.2. Fatigue crack propagation fault assessment of switch actuator rod based on wavelet finite element model

According to the measured life of the experiment, it is the original non-negative sequence

\[ x(0) = \{301064;195121;132838;96275;53577;30294;28213;18397\} \]

Generating a new sequence by a single accumulation

\[ x(1) = \{301064;496185;629023;725298;778875;809169;837382;855779\} \]

According to equation (5), the next mean value sequence is obtained:

\[ y(1) = \{398625, 562604, 677166; 752098, 794028, 823286, 846586\} \]

MATLAB is used to obtain the model sequence and identification parameters \( a, u \):

\[
B = \begin{bmatrix}
3.98E5 & 1 \\
5.62E5 & 1 \\
6.77E5 & 1 \\
7.52E5 & 1 \\
7.94E5 & 1 \\
8.23E5 & 1 \\
8.46E5 & 1
\end{bmatrix}
\quad Y = \begin{bmatrix}
1.95E5 \\
1.32E5 \\
9.62E5 \\
5.35E5 \\
3.02E5 \\
2.82E5 \\
1.84E5
\end{bmatrix}
\]

\[
\begin{bmatrix}
ad \\
u\end{bmatrix} = (B^T B)^{-1} B^T Y (1) = \begin{bmatrix}
0.4039 \\
3.5876
\end{bmatrix}
\]

The time series of the grey differential equation (5) is as follows
\[
\hat{x}^{(1)}(k + 1) = \left(\hat{x}^{(0)}(k + 1) - 8.8824\right)e^{-0.4039k} = 8.8824
\]

Based on the reduced value in equation (10), the following can be obtained by progressive reduction:

\[
\hat{x}^{(0)}(k + 1) = (301064, 496064, 626657, 713891, 772152, 811076, 837079, 854441)
\]

Through Equation (11), the results are compared with the experimental data to check accuracy: as shown in Table 2 below.

**Table 2.** Wavelet finite element model error on the fatigue life of actuator rod

| Initial crack length/mm | Experimental fatigue life | Assessment value | Assessment model residual | relative error% |
|-------------------------|---------------------------|-------------------|---------------------------|-----------------|
| 0.096                   | 301064                    | 301064            | 0                         | 0               |
| 0.249                   | 195121                    | 195000            | 121                       | 0.06            |
| 0.498                   | 132838                    | 130593            | 2245                      | 1.69            |
| 0.997                   | 96275                     | 87234             | 9041                      | 9.39            |
| 2.002                   | 53577                     | 58261             | -4684                     | 8.74            |
| 3.499                   | 30294                     | 38924             | -8630                     | 28.4            |
| 4.001                   | 28213                     | 26003             | 2210                      | 7.83            |
| 4.998                   | 18397                     | 17362             | 1035                      | 5.62            |

(6) The comparison results are shown in Figure 1 below by Paris equation:

\[
x^{10^5} = \begin{cases} 
3.5 & \text{Measured value in the test} \\
3.0 & \text{Paris calculated value} \\
2.5 & \text{GM (1, 1) estimate}
\end{cases}
\]

**Figure 1.** Comparison of fatigue crack life

The analysis results suggest that the fatigue crack propagation life of the switch actuator rod can be assessed when the initial crack length changes between 0.1mm and 4.9mm. For example, when the initial crack of the switch actuator rod is 3mm, the fatigue life is estimated to be 45763 times by the
Lagrange interpolation equation according to the wavelet finite element model. Which is consistent with the experimental data and Paris calculation results, and the calculation is simple and feasible with high assessment accuracy.

4. Conclusions
In this paper, the damage mechanics is combined with the finite element analysis method to calculate the fatigue crack propagation life of the switch actuator at different initial cracks by using the Paris equation. The wavelet finite element model is used to establish the grey differential equation. The identification parameter of the model is determined based on the measured data in the experiment to assess the fatigue crack life of the switch actuator rod and compare the assessed value with the calculation result from the Paris equation and the experimental data. The results show that the assessment accuracy of the wavelet finite element model is relatively high, and the results are correct and reliable, laying a foundation for the preliminary study and exploration of applying the wavelet finite element model to the fatigue life assessment of switch machine. The research method is of excellent application value for the reliability design and fatigue failure assessment of critical components of the switch machine.

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