**ABSTRACT**

In the Internet of Things (IoT), the internet-connected objects send the Collected data and act on the received data. Encryption controls a large number of structured and unstructured data protection during transmission. Inadequate memory and processing capacity of IoT devices demand Elliptic Curve Cryptography (ECC) for simple, secure functionalities. Scalar Multiplication frequently uses Modular Inversions that impact significantly on ECC-based applications with low resource usage with the enhancement of reliable IoT System availability. The Point Inversion algorithm for Elliptic Curve Cryptology (PIECC) enhances security and reduces the Computation time of Modular Point Inversion of Elliptic Curve using High-Speed Split Multiplication and Squaring. The use of limited intermediate registers for Cryptographic functions optimizes the Storage. The proposed algorithm reduces the Computation Time of the Cryptographic operations in terms of Clock cycles using chain Fermat-based Inversion compared with High-Speed multiplication and Product Scanning algorithms with lower Space Complexity.

**Keywords**

Elliptic Curve Cryptography, Internet of Things, Modular Inversion, Montgomery Curve, Security of data

1. **INTRODUCTION**

The varying worldwide development in Modern Technology shows the converging of computation and communication. The distributed Smart devices with a remote connection substitute the Personal Computers with a wired network in almost all the sectors. It necessitates information protection and security measures. Data security and confidentiality are currently the requisite for Banking applications (mobile, SMS, UPI, etc.) on Phones, wearable Healthcare devices, work-from-home, etc. Internet of Things (IoT) drives the concept of bringing the world together via universal connectivity. The fundamental requirement of IoT is to secure information and data communication providing robust availability with optimized resource usage. The embedded systems are highly domain-specific; The domains expand for such systems. The purpose of the washing machine is to inlet and outlet water at a controlled time using a programmed microcontroller. The fully automatic washing machines provide the option of fuzzy, pre-set hot water washing and other fabric-dependent programming features. The cell phone acts as a router and the Television screen as a Smart screen using Hotspot and WifiDirect facility. Every device viz., laptop, printer, refrigerator provides functionality and maintenance status. The internet of Things provides connectivity of the underlying devices with least or without human intervention. The connected embedded devices transfer information and need network security in a resource-constrained environment. The small key size with a high level of protection causes Elliptic Curve Cryptography [?] effective in an IoT scenario with reduced Storage, and Time overhead minimizes the power consumption. It increases the life span of IoT devices.

The primary component of IoT is Device-to-Device communication. Wireless sensor networks are application-specific. The sensor nodes or motes design in compliance with the application. IoT and Device-to-Device communication use existing motes. The functionality of sensors integrates into the chipset of the specific device leads to a device-specific implementation. The Sensor nodes of the IoT and WSN nodes vary in IP connectivity. Cooja simulator implements an IP stack into sensor nodes e.g., TELOS, Tmote, Micaz, etc., Sensor nodes in IoT are IP-enabled with a 6lowPAN IP address allowing them to interact with remote Edge-to-Edge motes. The characteristics of the sensor node or mote are non-replaced battery, low data transfer capacity, restricted computational and operational efficiency. IoT environment requires the highest performance of the nodes consuming the least energy.

**Motivation:** Finite Field Arithmetic is extensively used in numerous fields viz., Combinatorics, Coding, String Theory, and Cryptology, Logic Gate Theory, etc. A wide range of Public-Key Crypto Applications implemented are over a high-order Finite Fields [?]; accordingly, multiplying and dividing operations influence the processing time of encrypting and decrypting functionality. Thus, designing and developing a faster approach to carry out such operations is essential. High Speed Split Multiplier (HSSM) for Elliptic Curve Cryptography [?] uses Split Multiplier with confined registers to carry out Cryptographic operations in IoT. The faster Inversion algorithm accelerates the implementation.
of the Scalar Multiplication. The reduction in computation cost and storage complies with Micaz mote of 4KB RAM. The Point Inversion algorithm for Elliptic Curve Cryptography (PIECC) to secure Data Communication in IoT reduces the Computation Time. It thereby decreases the processing time of the Encryption and Decryption in the Public Key Infrastructure of IoT. Laporta et al., [2] described a Modular Multiple Word Inversion enhancing Stein Greatest Common Divisor with De Bruijn Contiguous Subsequence and Montgomery arithmetic. Hyper Thread [?] and Reverse Product Scanning [?] enhanced the existing systems. Many researchers worked on Binary Edward, Twisted Edward, and Montgomery curve [?], [?], [?] over modular Non-CoPrime [?], Gaussian Integer [?] and Prime Field [?].

Contributions: The key contributions of this work are to enhance:

i) security by reducing the Computation time of Modular Point Inversion of Elliptic Curve using High-Speed Split Multiplication and Squaring.

ii) performance by optimizing the Storage using limited intermediate registers of notes for Cryptographic functions.

Organization: The paper is structured as follows: Section 2 abstracts the Related Works of the modular Inversions for Elliptic Curve Cryptography in an IoT environment. Section 3 describes the Background Work, and Section 4 presents the Proposed Point Inversion algorithm for Elliptic Curve Cryptography (PIECC) for secure Data Communication in IoT Algorithm. The Performance Analysis of the proposed system is in Section 5 Section 6 contains the Conclusions.

2. RELATED WORKS

Fermat’s little theorem (FLT) over Prime Fields reduces Squaring and Multiplication in modular Inversion. Gallin et al., [2] presented a Pipelined Modular Multipliers for Hyper Elliptic Curve Cryptography of Field Programmable Gate Arrays with Digital Signal Processor blocks. The pipelines are filled using Dynamic Threads. The modified 128 bits algorithm computes parallel independent intermediate results and uses Block RAMs to produce smaller and faster circuits compared to the Multiplier with Threads of [2]. The performance is distinct and non-generalized, excluding the non-square quadrant blocks of the Digital Signal Processor, focusing on the single word parameters applicable for Digital Signal Processor functions of the Field Programmable Gate Array.

Ding et al., [2] presented a Two-stage pipelined Montgomery Modular Multiplication using a Two-stage Karatsuba multiplication [?] which is extended to Three and Four stage multipliers for unbounded positive operands [?]. The partitioning of the operands into small chunks reduces the core multiplication by occupying all the pipeline stages. The cropped multipliers and registers reduce the Look-Up Table (LUT) cost. Feng et al., [2] proposed a multi-path Number Theoretic Fourier Transformation algorithm using Quantum Computational cryptanalysis train and test over Polynomial Rings. The computational complexity is directly proportional to the degree and inversely proportional to the paths. The use of the modified Caching technique and the Round Constant speeds up the product computation.

Leelavathi et al., [2] implemented and processed ECC with 8 Montgomery Curve Point Product obtained using Urdhva Triyambhagayam Vedic Multiplication technique. The 194 bits implementation overflows the Spartan-3E resource limitations. Li et al., [2] presented a framework for a Koblitz Elliptic Curve Point Multiplier (ECPM) with Heterogeneous Scalar conversion. The parallel and pipelined executions with a three-level chained Finite Field accumulator of the functional unit increase the performance of the Addition operation. The heterogeneous delayed reduction and dual-?$\text{a}$-adic Nonadjacent Form method reduce the resource usage. Inversion is performed using continuous Addition operations.

Wenger et al., [2] implemented Four Elliptic Curve multiplier with an Accumulator and performed Elliptic Curve Multiplication of 4 Bytes accessed in 2 sets of a Word using eight registers accelerating the Functional Unit. Abdulrahman et al., [7] presented a Radix-Eight Multiplier method to assess the EC multiplier by pre-computing results of doubling and addition operations for three bits of the scalar in parallel without using the Look-Up Table. The Prime Extended Twisted Edward Curve with Eight multiplier implementation performs better compared to the Montgomery with resistance to Simple Power Analysis (SPA) attack. The scheme outperforms in a multiple processor parallel execution environment.

Salarifard et al., [2] proposed architecture for an Elliptic Curve Point Multiplier using a Fixed base Comb to decrease the quantity of Addition Operations performed and the time complexity. The Static base algorithm requires the Prior Multiplication operation to complete, introducing an extra operating cost. A Two-level pipelined Karatsuba Ofman Point Multiplier with overlapped operations decreases the complexity and latency, increasing the throughput. Ali et al., [7] designed a 521-bit Modular Multiplier using Matrices and Vectors applying dual and triple partitions over a Four Byte result for the static and varied base operands in two stages. The number of Arithmetic operations and the time complexity are reduced by avoiding the overflow compromising the efficiency of the algorithm.

Roy et al., [?] presented an improved Montgomery Elliptic Curve Point Multiplier [?] that incorporates the compressed circuitry allowing two sets of multiplier functionalities performed simultaneously using Carry links, LUTs, and the scheduler. Amiet et al., [?] designed an ECPM processing framework for the operands to the maximum of 0.5Kbits encompassing a Finite Field bounded by 1KBits. The implementation uses DSP blocks of FPGA, and parallelization of Addition and Doubling increases the efficiency. The Modular Arithmetic functionalities execute in parallel. Fermat’s method is applied in the Modular Inversion operation. The Elliptic Curves of Parameter Size more than 256bits are not supported.

Rashidi et al., [?] employed two-Field parallel Multiplier Selim et al., [?] NIST Montgomery Modular Multiplication Al-Asli et al., [?] resists Internal Attack. Loi et al., [?] designed and implemented a NIST ECC processing framework using DSP blocks and parallel execution to reduce the time complexity and memory usage. The multiplier and the Inverter use the same functional block without parallelization.

Xu et al., [?] constructed a Modular Inversion over 192 bit NIST and Chinese Elliptic Curves resistive to SPA and Heuristic Lattice-based Decoding Attacks. The Montgomery Point reduction decreased the number of multiplication operations performed to minimize the Time complexity. Choi et al., [?] Hardware-based NIST ECC processor. A repeated Modular Prime number reduction over multiple intermediate products of size greater than the operand size and half the size of Final Result used. The Computation cost reduced is directly proportional to the size of the Intermediate results using a 0.5Kbits register. Gu et al., [?] generalized the McLaughlin Multiplier of Residue Number System with unpaired Secondary Prime applying
3. BACKGROUND WORK

Researchers have designed Scalar Multiplication with Inversion algorithms for specific Elliptic Curves providing mathematical models and validations for various motes in an IoT scenario.

Liu et al. [7] implemented MoTE Curves to secure IoT device communication. The Curve models resist the Power Analysis attack. Table 1 gives the comparison of related background works on the Elliptic Curve Cryptographic operations.

### 3.1 Elliptic Curve Cryptography

The Elliptic Curve Point representation of the data provides tamper-resistance data transfer. The secure ECC is assured by the Discrete Logarithmic Property (DLP). An Elliptic Curve is a tamper-resistance data transfer. The secure ECC is assured by the Elliptic Curves. The resource-restricted IoT devices demand the utilization of Elliptic Curve Cryptographic (ECC) techniques. The Scalar Multiplication significantly affects the ECC usage in minimizing the Computational Complexity and the Storage overhead of Modular Inversions, as the ECC operations involve the Fundamental Modular Arithmetic. Thus, upgrading an ECC enhances the Security and Privacy of the IoT Application. Given a collection of IoT devices of specified configuration, to design a Point Inversion algorithm for Elliptic Curve Cryptology to secure IoT Data Communication, the objectives are as follows:

i) Enhance the security with decreased computation time using the High-Speed Split Multiplication based Squaring algorithm for Modular Point Inversion of Elliptic Curve Cryptography.

ii) Optimize the data and code Storage, improving the performance of IoT devices using limited intermediate registers to carry out Cryptographic functions.

### 3.2 Fermat-Euler theorem

The Fermat-Euler theorem is given in the equation (3), (4) is used to Inverse the Prime number exponentiation of an Elliptic Curve.

\[
\forall_{a \in \mathbb{N}^+} \exists_b \{a = \rho \times b\}
\]

where \( \rho \) is Prime, \( \mathbb{N}^+ \) is the set of Positive Integer Numbers and \( a, b, \rho \in \mathbb{N}^+; i.e., \)

\[
\forall_{a \in \mathbb{N}^+} \begin{cases} a^\rho \equiv a \mod \rho & \iff a \mod \rho \neq 0 \\ a^\rho \equiv 1 \mod \rho & \iff (a-1) \mod \rho = 0 \end{cases}
\]

### 3.3 Problem Statement

The Encryption/Decryption technique provides secure information exchange in a stable framework. The resource-restricted IoT devices demand the utilization of Elliptic Curve Cryptographic (ECC) techniques. The Scalar Multiplication significantly affects the security of the IoT Application. Given a collection of IoT devices of specified configuration, to design a Point Inversion algorithm for Elliptic Curve Cryptology to secure IoT Data Communication, the objectives are as follows:
Table 1. Comparison of Recent Related works

| Article                        | Approaches                                      | Advantage                                                                 | Disadvantage                                                                 |
|-------------------------------|------------------------------------------------|---------------------------------------------------------------------------|------------------------------------------------------------------------------|
| Gallin et al., [7] (2019)     | Pipelined Modular Multipliers Dynamic Threads   | computes parallel independent intermediate results; uses Block RAMs       | Distinct, non-generalized; exclude non-square quadratic blocks; focus on the single word parameters |
| Leelavathi et al., [7] (2019) | ECC using Urdhva Triyambyagam Vedic Multiplication | 8 Montgomery Curve Point Product                                           | 194 bits implementation overflows the Spartan-3E resource limitations.       |
| Amiet et al., [7] (2016)      | Finite Field bounded by 1Kbits                 | ECPM for the operands to the maximum of 0.5Kbits; DSP blocks and parallelization of Addition and Doubling increases the efficiency. | Parameter Size more than 256bits not supported.                               |
| Loi et al., [7] (2015)        | NIST ECC processing framework using DSP blocks | parallel execution reduce the time complexity and memory usage           | Multiplier, and Inverter use the same functional block without parallelization. |

Table 2. Table of Notations

| Symbol | Definition                               |
|--------|------------------------------------------|
| p      | Prime Field number                       |
| ω      | mote Word-length                         |
| $E_m$X | X bit Montgomery Curve                   |
| $E_o$X | X bit Twisted Edwards curve              |
| $P_I$  | Twisted Edwards curve                    |
| $P_1$  | Operand of η-words                      |
| $P_2$  | Product of 2η-words                      |
| $T_1$  | Temporary variables                     |
| $μ, ν$ | Iteration control variables             |
| $α$    | Finite Field element α                   |

Function 1 Squaring of two numbers.

Input: Two operands $P_1 = (P_1[η − 1], \cdots, P_1[1], P_1[0])$  
Output: Product $P_2 = P_1 \times P_1 = (P_2[2η − 1], \cdots, P_2[1], P_2[0])$

1. $T_1 \leftarrow P_2[0] \leftarrow 0$
2. for $μ$ from 1 to $η$ do
3. for $ν$ from 0 to $μ/2 − 1$ do
4. $T_1 \leftarrow T_1 + P_1[ν]P_1[μ − ν − 1]$
5. end for
6. $P_2[μ − 1] \leftarrow T_1$ mod $2^ω$; $T_1 \leftarrow T >> ω$
7. end for
8. for $μ$ from $η$ to $2η − 3$ do
9. for $ν$ from $μ − η + 1$ to $(μ − 1)/2$ do
10. $T_1 \leftarrow T_1 + P_1[ν]P_1[μ − ν]$
11. end for
12. $P_2[μ] \leftarrow T_1$ mod $2^ω$; $T_1 \leftarrow T >> ω$
13. end for
14. $P_2[2η − 2] \leftarrow T_1$ mod $2^ω$; $P_2[2η − 1] \leftarrow 0$
15. $T_1 \leftarrow 0$
16. for $μ$ from 0 to $η − 1$ do
17. $T_1 \leftarrow T_1 + P_1[μ]P_1[μ] + (P_2[2μ + 1]2^ω + P_2[2μ])$
18. $P_2[2μ] \leftarrow T_1$ mod $2^ω$; $T_1 \leftarrow T >> ω$
19. $P_2[2μ + 1] \leftarrow T_1$ mod $2^ω$; $T_1 \leftarrow T >> ω$
20. end for
21. return $(P_2[2η − 1], \cdots, P_2[1], P_2[0])$

The Function 1 plays a vital role in the Modular Inversion algorithm as the frequency of multiplying in others is more. So the efficiency of this Function increases the efficiency of incorporated algorithms in many folds. The symmetrical factor of the large Integer Multiplicand and Multiplier in the Square Function, $P_1$ provides better performs compared to asymmetric Integer Multiplication function. In a regular Multiplication based Squaring, all interjacent outcomes of the form $P_1[κ] \times P_1[ν]$, with $ν = κ$ computed twice. The Function 1 computes these interjacent outcomes merely one time and later shifts words to the right to obtain the Doubled result by shrinking excess processing cost.

The two nested loops in the Function compute the Double of interjacent outcomes $P_1[κ] \times P_1[ν]$, equivalent loops in multiplication algorithm used in this HSSM. The initial and the final interjacent outcomes are attached to the interleaved blocks, and reduction in the frequency of inner blocks repetition differs the Squaring Function from the regular Doubling. The modified exit-control statements reduce the overall Multiplications carried out by the dual repetitive blocks from $η^2 − 2$ to $(η^2 − η)/2$.

4.3 Modular Reduction

The Multiplication and Squaring of two numbers of bit length $η$ results in $2η$ length of the result. Since the value of $η$ is large, subtracting the prime number from the result consumes much time. The Reduction is a process of minimizing the result of $2η$ bit length to $η$ bit length less than the prime number chosen, which is the same as reducing the result belong to the selected Prime Field.

Function 2 Reducing $2η$ bit length result.

Input: Prime number $p$, $Z = A ∗ A$, a $2η$-word.  
Output: Reduced product $Z'$ having $η$-word. $(A ∗ A)$ modulo $p$.

1. $Z_H \leftarrow A[2m], A[2m − 1], \cdots, A[m]$
2. $Z_L \leftarrow A[m − 1], A[m − 2], \cdots, A[0]$
3. Write the Prime number $p$ as $2p = 2(2^n − C) = 2^n − d$ where $d = 2 ∗ C$
4. $Z' \leftarrow Z_H \cdot d + Z_L$
5. while $Z' > p$ do
6. $Z' \leftarrow Z' − p$
7. end while
8. return $Z'$

The Modular Reduction Function can be implemented in two methods efficiently. In Function 2 the first method, the value of $Z_H$ is left-shifted $n$ times, where $n$ is the bit length of the integer $d$. The intermediate results and the value $Z_2$ are added and reduced.
further using subtraction. In Function 3 the second method, the usual multiplication algorithm is used instead of the left shift, and further reduction is obtained by subtraction.

**Function 3 Calculating Z in Reduction function.**

**Input:** \( Z_H, Z_L \) and \( d = 2 \times C \).

**Output:** \( Z' \), where \( Z' = Z_H \times d + Z_L \).

1. Convert to binary form.
2. while \( d > 0 \) do
3. left shift \( Z_H \times \log(d) \) times.
4. \( Z' \leftarrow Z' + Z_H \)
5. \( d = d/2 \)
6. end while
7. return \( Z' \)

### 4.4 Modular Inversion

Modular Inverse of an integer \( a \) is an integer \( x \) such that \( a \cdot x \equiv 1 \pmod{p} \). Since the elements of the Prime Field \( p \) are from 0 to \( p - 1 \), the modular Inversion, i.e., reducing the integer obtained after multiplying and squaring to an integer in the Prime Field \( p \).

Modular Inversion is the most consuming process in the encryption algorithm. The Extended Euclidean Algorithm and Inversion via exponentiation based on Fermat’s little theorem are two principal approaches for evaluating an inversion in \( F_p \). A Fermat-based inversion in \( F_p \) with \( p = 2^k - 1 \) costs about \( k - 1 \) squaring and no more than \( 2 \log_2(k - 1) \) multiplications.

For the 223 bit Prime Field with prime number \( p = 2^{223} - 235 \), the optimized chain Fermat-based inversion algorithm is shown in Algorithm 1.

**Algorithm 1 Calculating Inversion for \( p = 2^{223} - 235 \).**

**Input:** Finite Field element \( a \).

**Output:** The inversion \( a^{-1} \) in \( F_p \) as \( t = a^{p-2} \pmod{p}, p = 2^{223} - 235 \).

1. \( a_2 \leftarrow a^2 \)
2. \( a_0 \leftarrow (a_2)^2.a \)
3. \( x_1 \leftarrow 2; t \leftarrow (a_2.a_2)^{t_1}.a_9 \)
4. \( x_2 \leftarrow x_3 \equiv 2^{t} \pmod{t} \)
5. for \( i \) from 1 to 4 do
6. \( x_3 \leftarrow (x_3)^2; t \leftarrow (t)^{t_2} \)
7. end for
8. \( x_4 \leftarrow (x_4)^2; t \leftarrow ((t)^{t_3}.)^{t_2} \)
9. \( x_5 \leftarrow x_4; t \leftarrow ((t)^{t_4}.a)^{t_2} \)
10. return \( t \)

From the given integer \( a \), which belongs to the Finite Field, the algorithm has to calculate the value \( a^{p-2} \) where \( p = 2^{223} - 235 \). The algorithm calculates intermediate powers and reduces at each step by calling the modular reduction function. The intermediate results are stored in the array to prevent re-calculating the values calculated earlier, thus increasing speed. The modular Reduction of any number results in values being in the finite field save the memory required to store non-reduced values. The squaring and multiplications are obtained by the respective functions described before.

The total number of squaring and multiplications required to calculate Inversion for the prime number \( p = 2^{223} - 235 \) is 173 squaring and 12 multiplications. As the squaring of a number is considerably faster than multiplying the number twice, Squaring Function is called wherever feasible to make it more efficient. The algorithm described calculates the Inversion of any number belonging to the Finite Field chosen significantly faster than most previously implemented versions. The inversion algorithms for the prime numbers \( p = 2^{191} - 19 \), \( p = 2^{159} - 91 \) and \( p = 2^{255} - 19 \) follows the same methodology with few required changes required accordingly.

**Algorithm 2 Calculating Inversion for \( p = 2^{159} - 91 \).**

**Input:** Finite Field element \( a \).

**Output:** Inversion \( a^{-1} \) in \( F_p \) as \( t = a^{p-2} \pmod{p}, p = 2^{159} - 91 \).

1. \( a_4 \leftarrow a^2 \)
2. \( a_2 \leftarrow (a_2)^2 \)
3. \( a_0 \leftarrow (a_4)^2.a \)
4. \( a_15 \leftarrow (a_0.a_4.a_2) \)
5. \( a_31 \leftarrow (a_15)^2.a \)
6. \( x_1 \leftarrow 2^{t}; t \leftarrow (a_31)^{t_1}.a_15 \)
7. \( x_2 \leftarrow (x_1)^2 t \leftarrow (t)^{t_2} \)
8. \( x_3 \leftarrow (x_1)^2 t \leftarrow (t)^{t_3} t \)
9. \( x_9 \leftarrow (x_3)^2 t \leftarrow (t)^{t_4} t \)
10. \( x_10 \leftarrow (x_3)^2 t \leftarrow (t)^{t_5} t \)
11. \( x_11 \leftarrow (x_3)^2 t \leftarrow (t)^{t_6} t \)
12. \( x_12 \leftarrow (x_3)^2 t \leftarrow (t)^{t_7} a_15.a_2 \)
13. return \( t \)

**Algorithm 3 Calculating Inversion for \( p = 2^{191} - 19 \).**

**Input:** Finite Field element \( a \).

**Output:** Inversion \( a^{-1} \) in \( F_p \) as \( t = a^{p-2} \pmod{p}, p = 2^{191} - 19 \).

1. \( a_2 \leftarrow a^2 \)
2. \( a_4 \leftarrow (a_2)^2 \)
3. \( a_0 \leftarrow (a_4)^2.a \)
4. \( a_1 \leftarrow (a_4)^2.a \)
5. \( a_1 \leftarrow (a_0.a_4.a_2) \)
6. \( a_{31} \leftarrow (a_{15})^2.a \)
7. \( x_2 \leftarrow 2^{t}; t \leftarrow (a_{31})^{t_2}.a_{11} \)
8. \( x_3 \leftarrow 2^{t}; t \leftarrow (t)^{t_3} t \)
9. \( x_9 \leftarrow (x_3)^2 t \leftarrow (t)^{t_4} t \)
10. \( x_10 \leftarrow (x_3)^2 t \leftarrow (t)^{t_5} t \)
11. \( x_11 \leftarrow (x_3)^2 t \leftarrow (t)^{t_6} t \)
12. \( x_12 \leftarrow (x_3)^2 t \leftarrow (t)^{t_7} a_{15}.a_2 \)
13. return \( t \)

Fermat’s little theorem states that Any prime number perfectly divides one of the powers - 1 of any progression, i.e., \( \exists d \) such that \( d \equiv 1 \pmod{p} \). The exponent of this power is a sub-multiple of the given prime number - 1, i.e., \( d \equiv 1 \). The first power that satisfies the property, all those whose exponents are multiple of the exponent of the first, still satisfy the property.

Consider an Integer \( \overline{a} = 3 \) and a Prime \( \rho = 5 \), then, \( \overline{a}^5 = 243 \) and \( 243 - 3 = 240 = 5 \times 48 \), i.e., \( 3^5 \equiv 3 \pmod{5} \) and \( 4^8 = 81 \) and \( 81 - 1 = 80 = 5 \times 16 \), i.e., \( 3^4 \equiv 1 \pmod{5} \). Similarly, let \( a = 3 \) and \( \rho = 11 \), then, \( 3^{11} = 177147 \) and \( 177147 - 3 = 177144 = 11 \times 16104 \), i.e., \( 3^{11} \equiv 3 \pmod{11} \) and \( 3^{10} = 50949 \) and \( 50949 - 1 = 50948 = 11 \times 5368 \), i.e., \( 3^{10} \equiv 1 \pmod{11} \).
Thus, $31 = 2^5 - 1$, hence, $a^{31}$ is represented as $a^{2^5-1}$ given in the equation (7)

$$a^{31} = a^{2^5-1} \quad (7)$$

In general,

$$a^{(2^5-1)^2} \equiv a^{(2^{(5^2)}-2^5)+2^5-1} = a^{2^{10}-1} \quad (8)$$

$$a^{(2^5-1)^5} \equiv a^{(2^{(5^5)}-2^5)+2^5-1} = a^{2^{10}-1} \quad (9)$$

$$a^{(2^{60})-1} \equiv a^{(2^{(80+80)}-2^{80})+2^{80}-1} = a^{2^{(1600)}-1} \quad (10)$$

$$a^{(2^{600})-1} \equiv a^{(2^{(800+400)}-2^{400})+2^{400}-1} = a^{2^{(800)}-1} \quad (11)$$

$$(a^{2^{115}-1})^{2^7} \equiv a^{2^{115+7}-1} \equiv a^{2^{223}-1} \quad (12)$$

$$a^{2 \cdot 2^{223}-256+19} = a^{2 \cdot 223-237} \quad (13)$$

$$a^8 \equiv a^{2 \cdot 223-237} \quad (14)$$

For the prime number $p = 2^{159} - 91$, the algorithm follows the same procedure. The algorithm requires 155 squares and 15 multiplications to calculate all the intermediate powers to find the inverse of the given number. All the intermediate values contributed to reduced multiplication values and squared values calculated by the Brute-Force method. This comparison is to determine whether all the intermediate calculations and reducing perform correctly. The Inversion algorithm is then used to calculate the inversion of the denominator in point addition and point doubling. As there is no division arithmetic function for the prime group, a division is performed by finding Inversion of the denominator and then multiplying with the numerator.

Algorithm 4 Calculating Inversion for $p = 2^{255} - 19$

Input: Finite Field element a.
Output: The inversion $a^{-1}$ in $F_p$ as $t = a^{p-2} \mod p, p = 2^{255} - 19$.

1. $a_2 \leftarrow a^2$
2. $a_4 \leftarrow (a_2)^2 \cdot a$
3. $a_7 \leftarrow (a_4)^2 \cdot a$
4. $a_{15} \leftarrow (a_7)^2 \cdot a$
5. $x_1 \leftarrow 2^5 \cdot t \leftarrow (a_{15})^{x_1} \cdot a_2 \cdot a$
6. $x_2 \leftarrow (x_1)2^1 \cdot t \leftarrow (t)^{x_2} \cdot t \cdot a$
7. $x_3 \leftarrow x_1 \cdot x_2 \cdot t \leftarrow (t)^{x_3} \cdot t$
8. $x_3 \leftarrow x_3 \cdot x_2 \cdot t \leftarrow (t)^{x_3} \cdot t$
9. $x_3 \leftarrow x_3 \cdot x_2 \cdot t \leftarrow (t)^{x_3} \cdot t$
10. $x_3 \leftarrow x_3 \cdot x_2 \cdot t \leftarrow (t)^{x_3} \cdot t$
11. $x \leftarrow (x_1)2^1 \cdot t \leftarrow (t)^{x_3} \cdot t$
12. return $t$

The Algorithms 2, 3, 4, and 5 describes the Point Inversion for $p = 2^{259} - 91, p = 2^{255} - 19, p = 2^{223} - 255$ and $p = 2^{255} - 19$ respectively. These individual Algorithms call Multiplication, Squaring, and Reduction algorithms. The number of times multiplication and squaring called and efficiency of Multiplication and Squaring decides the efficiency of the Inversion algorithm. The intermediate calculations are stored, and complex calculations break into sub-problems to increase efficiency. All intermediate calculations are stored as computationally expensive and used when required. The algorithms for $p = 2^{259} - 19$ and $p = 2^{159} - 91$.

The number of Multiplication and Squaring in the Inversion Algorithms are as given in Table 3. PIECC uses 173 Squaring, and 12 Multiplication operations for 224 bit; and 122 Squaring and 16 Multiplying operations. A 50% reduction in the number of Squaring operations compared to LHS and RPM reduces the Computation time of PIECC algorithm.

4.5 Scalar Point Multiplication

Scalar point multiplication multiplies a fixed point on the selected elliptic curve and performs scalar multiplication $\kappa$ times. Scalar point multiplication is one of the time-consuming processes in elliptic curve cryptography. In this step, the message is represented by any point on the elliptic curve. Then the selected Point is multiplied $\kappa$ times, which is the Elliptic Curve Discrete Logarithm problem. Key generation and Diffie-Hellman key exchange require one Scalar multiplication. In Diffie-Hellman key exchange, $\kappa$ is kept secret. The computation of $\kappa \cdot P$ is the same for public and secret $\kappa$. The problem with the standard, straightforward implementation of scalar multiplication is safety. An attacker can measure time and can try and deduce information about $\kappa$. Montgomery Ladder Algorithm used for performing Scalar multiplication. The safety and execution of the algorithm depend on the value of $\kappa$. The steps involved are shown in algorithm 5.

Algorithm 5 Calculating Scalar Point Multiplication.

Input: An integer $\kappa > 0$ and a Point $P_0$.
Output: $P_3 = \kappa P_0$.

1. set $\kappa \leftarrow (\kappa_{n-1}, \kappa_{n-2}, \cdots, \kappa_1, \kappa_0)$
2. set $P_1 \leftarrow 0$; $P_2 \leftarrow P_0$
3. for $i$ from $l - 1$ downto 0 do
4. if $\kappa_i = 1$ then
5. $P_1 \leftarrow P_1 + P_2$; $P_2 \leftarrow 2P_2$
6. else
7. $P_2 \leftarrow P_2 + P_1$; $P_1 \leftarrow 2P_1$
8. end if
9. end for
10. $P_3 \leftarrow P_1$
11. return $P_3$

The algorithm calculates $\kappa \cdot P_0$ by repeated point addition and point multiplication. For point addition and point multiplication, the standard formula requires Addition, Subtraction, Multiplication, Reduction, and Inversion. Hence Scalar Point Multiplication calls corresponding arithmetic functions whenever required. The algorithm computes the point multiplication in a fixed time, thus providing immunity against time-based attacks. This algorithm performs the same number of point additions and doubles.
regardless of the value of $\kappa$. The algorithm does not leak any information through timing or power.

5. PERFORMANCE ANALYSIS

The Elliptic Curve Cryptosystem designed consists of Sensor devices or the mote as the fundamental component. These devices are equipped with 4-16 KiloBytes of main memory and 256 KiloBytes of nonvolatile memory. In Elliptic Curve Cryptosystem, the selected Curve Parameters and Prime numbers are in Binary representation. The system encodes data into a Point on the Elliptic Curve. The required number of Inversion and Reduction is performed over the Elliptic Curve Point. The Scalar multiplication is performed to encrypt the data in Elliptic Curve Diffie-Hellman Cryptography. The encrypted data is sent from the Sender to the Receiver.

**Computation Time:** The reduction in Computation Time of Multiplication, Squaring, Inversion, and Scalar Multiplication used in the Elliptic Curve Crypto-system shows the better performance of PIECC over the existing algorithms: Liu [2], and Reverse Product Scanning methods [2] with Fermat-based Inversion. The Computation time of the Functions and Algorithms is given in Table 5.

Computation time of Multiplication: Fig. 1 shows the time taken to perform Multiplication over Elliptic Curves of 160, 192, 224, 255 bits. The multiplication function of PIECC over the 192-bit curve consumes 2632 Clock Cycles, LHS consumes 2706 Clock Cycles, and RPSM consumes 3831 Clock Cycles for 192 bits. The processing time of multiplication in PIECC is 2.7% better than LHS and 36% better than RPSM for 256 bits. The use of the High-Speed Split multiplier reduces the computation Time of multiplication.

Computation time of Squaring: Fig. 2 shows the time taken to perform Scalar Multiplication over Elliptic Curves of 160, 192, 224, 255 bits. The Scalar multiplication algorithm of PIECC over the 192-bit curve consumes 2632 Clock Cycles, LHS consumes 2706 Clock Cycles, and RPSM consumes 3831 Clock Cycles for 192 bits. The processing time of multiplication in PIECC is 2.7% better than LHS and 36% better than RPSM for 256 bits. The use of the High-Speed Split multiplier reduces the computation Time of multiplication.

Computation time of Inversion: Fig. 3 depicts the time of Inversion over Elliptic Curves of 160, 192, 224, 255 bits. The Inversion algorithm over the 192-bit curve uses 309813 Clock Cycles, LHS uses 346162 Clock Cycles, and RPSM uses 497478 Clock Cycles for 192 bits. For 224 bits curve, the Point Inversion algorithm takes 399059 Clock Cycles, LHS takes 518504 Clock Cycles, and RPSM takes 791946 Clock Cycles. The processing time of Inversion in PIECC is 46.6% better than LHS and 64.5% better than RPSM for 256 bits. The use of limited High-Speed Split multiplier and HSSM-based Squaring reduces with reduced code decreases the computation Time of Inversion.

Computation time of Scalar Multiplication: Fig. 4 shows the time taken to perform Scalar Multiplication over Elliptic Curves of 160, 192, 224, 255 bits. The Scalar multiplication algorithm of PIECC over the 192-bit curve consumes 1603 Clock Cycles, LHS consumes 1651 Clock Cycles, and RPSM consumes 2457 Clock Cycles for 192 bits. The processing time of multiplication in PIECC is 49% better than LHS and 69% better than RPSM for 256 bits. The use of the High-Speed Split multiplier, HSSM based Inversion reduces the computation Time of Scalar Multiplication.
multiplier and HSSM based Inversion reduces the computation Time of Scalar Multiplication.

Fig. 4. Computation Time of Scalar Multiplication in LHS, RPSM and PIECE Elliptic Curves

Memory usage: The reduction in Memory usage of PIECC in Fig. 5 shows the better performance of PIECC over the existing algorithms: Liu [7], and Reverse Product Scanning methods [7] with Fermat-based Inversion. The PIECE uses 412Bytes, LHS uses 446Bytes, and RPSM uses 1174Bytes of RAM. The PIECE uses 11434Bytes, LHS uses 12156Bytes, and RPSM uses 19000Bytes of ROM. The PIECC uses ROM 5.5% lesser than LHS and 39.8% than RPSM. The RAM used by PIECE is 7.6% less than LHS and 64.9% less than RPSM. The memory usage is reduced by minimizing the intermediate memory usage for data and the code.

6. CONCLUSIONS

In an IoT environment, devices store and transmit data over communication media must be protected from illegitimate usage. Encryption controls for Data security at rest and in transit are essential. Elliptic Curve Cryptography enhances IoT security providing secure data communication in the Internet of Things Application. The proposed Point Inversion algorithm for Elliptic Curve Cryptology uses High-Speed Split Multiplier and HSSM based Squaring in Scalar Multiplication to secure IoT Data Communication. The PIECE optimizes the data and code Storage, improving the performance of IoT devices using limited intermediate registers to carry out Cryptographic functions. The proposed algorithm is almost two times faster than LHS and three times than RPSM. The PIECC uses 6% less memory than the LHS and 41% than RPSM by the reduction in intermediate memory usage and code. The Modular Inversion for other Curves can apply Factorization over Montgomery-friendly Prime Representation.

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