Gauge invariant field strength correlators from RG smoothing and color correlations between topological charge clusters

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Using the renormalization group based smoothing technique we have studied the gauge invariant field strength correlator at $T \neq 0$ and $T = 0$ in pure $SU(2)$ gauge theory. In conjunction with a cluster analysis, the field strength correlator is used to study correlations between the clusters in space and color orientation.

1. Introduction

With the rise of perfect lattice gauge actions, inverse blocking \cite{1} has become a viable tool in numerical lattice field theory. A natural requirement for the action is that a classical solution, if interpolated in this way, remains classical on all finer lattices. Equilibrium configurations are not classical, nevertheless, a topological charge can be unambiguously assigned to the interpolating field \cite{1} for generic lattice configurations. This has led to the hope to describe the topological (instanton) structure of Monte Carlo (MC) configurations with the help of the new method. Our variant of RG smoothing \cite{2} consists of (1) coarse graining a MC configuration (blocking in order to strip off UV fluctuations of scale $a$) and (2) inverse blocking to interpolate it by a smooth configuration (SM) on the original lattice.

RG-smoothing by our method does not lead to classical configurations. Clustering of topological charge and action in dominantly (anti)selfdual clusters \cite{3} is a property of SM lattice fields inherited from the MC vacuum. Smoothed fields preserve the string tension, its Abelian dominance and contain a topological susceptibility of the right magnitude. In earlier work we were concentrating on the monopole content \cite{4} of SM configurations in certain gauges, relating it to the topological charge distribution and to other gauge-invariant signatures. In our present study on SM samples at $T = 0$ we found the dimensionless ratio $m^2/\chi^2 \approx 1$, remarkably independent of $\beta$, in the maximal Abelian and Laplacian gauges ($m =$ monopole density, $\chi =$ topological susceptibility). Coming back to the topological structure of the $SU(2)$ vacuum, we have explored new capabilities of the RG smoothing technique: (1) to study the gauge invariant field strength correlators at finite temperature \cite{5} and, more recently, at $T = 0$, (2) to investigate further the clustering of topological charge (started in \cite{6} with a study of correlations in Euclidean and in color space). For the perfect action used in our studies see \cite{3}.

2. Field strength correlator

The gauge invariant field strength correlator \cite{7},

$$D_{\mu\nu,\rho\sigma}(x_1 - x_2) = \langle 0 | \text{tr} \{ G_{\mu\rho}(x_1) S(x_1, x_2) G_{\nu\sigma}(x_2) S^\dagger(x_1, x_2) \} | 0 \rangle \quad (1)$$

describes the non-perturbative vacuum structure in a model-independent way. Decomposing it into two basic structure functions,

$$D_{\mu\nu,\rho\sigma}(x) = (\delta_{\mu\nu} \delta_{\rho\sigma} - \delta_{\mu\rho} \delta_{\nu\sigma}) (D(x^2) + D_1(x^2) \right)$$
$$+ (x_{\mu} x_{\nu} \delta_{\rho\sigma} + x_{\rho} x_{\sigma} \delta_{\mu\nu} - ...) \frac{\partial D_1(x^2)}{\partial x^2},$$

exposes a confining part $D$, while $D_1 \ll D$ is related to the dominance of (anti)selfdual local excitations \cite{8} contributing to it. Using smoothing (instead of cooling \cite{9}) we want to study eventual effects of smoothing as a noise reduction method.
on the topological content. The "Schwinger line" $S(x_1, x_2)$, a path dependent transporter, requires (for distances not on-axis on the lattice) to perform a random sampling of paths of shortest length. We have measured at finite $T$ electric and magnetic structure functions $D^E_T = \left(D^E + D^E_1 + x^2 \frac{\partial D^E}{\partial x^2}\right)$, $D^B_T = \left(D^B + D^B_1\right)$ and $D^E_T = \left(D^E + D^E_1\right)$ at space-like distances for samples of 2000 smoothed $D^E_1 \times 4$ configurations at $\beta = 1.4, 1.5, 1.55, 1.6, 1.7$ and $1.8$ (a temperature interval $T/T_c = 0.62, 0.86, 1.03, 1.25, 1.97, \approx 4$). The lattice spacing $a(\beta)$ has been calibrated using the zero temperature $SU(2)$ string tension by comparison with the monopole Creutz ratios which reach a plateau early on a $12^4$ lattice (except for $\beta = 1.8$). For $T = 0$ the same functions have been studied on $12^4$ SM configurations at $\beta = 1.54$ ($a = 0.17$ fm) corresponding to the deconfinement $\beta_c$ found on $12^4 \times 4$ for this action.

In the confinement phase, the finite temperature correlators have an almost perfect electric-magnetic symmetry, even on our unsymmetric lattice. The signal of the deconfining transition is the breakdown of $D^E_T$ (Fig. 1). A fit with exponential structure functions $D^{E,B}_T$ and $D^{E,B}_1$ doesn’t give a satisfactory description for all $T$. In Fig. 2 we solve the equations defining $D^{E,B}_1$ and $D^E_B$. This leads to condensates and integrated correlation lengths given in Table 1. For $T = 0$ we find $\xi^E$ and $\xi^B$ compatible with the lowest temperature there. The values of the correlation lengths are in the expected ballpark. In terms of $D$ and $D_1$ the transition appears as the result of continuous changes. The degree of

![Figure 1. Electric and magnetic correlators (a) below, (b) above the deconfinement transition.](image)

Table 1

| $\beta$ | $D^E(0)+$ | $D^E(0)+$ | $\xi_{int}$ | $\xi_{int}^B$ |
|---------|------------|------------|-------------|-------------|
| 1.40    | 0.010(1)   | 0.010(1)   | 0.32(1)     | 0.34(2)     |
| 1.50    | 0.029(2)   | 0.028(3)   | 0.24(1)     | 0.27(1)     |
| 1.55    | 0.041(7)   | 0.047(3)   | 0.16(2)     | 0.24(1)     |
| 1.60    | 0.066(8)   | 0.083(12)  | 0.15(3)     | 0.23(2)     |
| 1.70    | 0.312(20)  | 0.312(81)  | 0.07(2)     | 0.13(2)     |

(anti)selfduality begins to decrease already below the deconfining transition 3. For a better understanding a detailed comparison with the cooling method (presently not available for $SU(2)$) is under way.

3. Correlations of topological clusters

The uncorrelated instanton liquid provides a good description of the QCD ground state. There are no indications from the field strength correlator at $T = 0$ 3 for strong correlations between (anti)instantons ($I$ and $A$). At higher temperatures interactions are expected to be important in the instanton ensemble. Contrary to fermionic interactions, Ansätze for gluonic interactions rely on specific $II$ and $IA$ superpositions. Analyzing the clustered SM configurations we hoped to learn about instantons in the real Yang Mills vacuum. Instead of supporting the instanton liquid picture our results are more suggestive for a different interpretation of the clusters exposed by smoothing.

The charge density has been measured by means of Lüscher’s charge which is inexpensive for SM configurations. We have defined topological clusters using a 4 – $D$ site percolation algorithm for assigning neighboring (same sign) lattice sites, marked to have $\left|q(x)\right| > q_{th}$ (threshold), to the same cluster. Clusters are characterized by a center (identified by $q_{max}$), a cluster volume $V_{cl}$ and cluster charge $Q_{cl}$. Eventual color correlations between different clusters can be studied by the normalized cluster overlap for a pair of clusters,

$$O = \frac{\left\langle \text{tr} \left( G_{\mu\nu}(1) S(1, 2) G_{\mu\nu}(2) S(2, 1) \right) \right\rangle_{\text{paths}}}{\left\langle \text{tr} \left( G_{\sigma\sigma}(1)^2 \right) \right\rangle_{\text{paths}} \left\langle \text{tr} \left( G_{\tau\lambda}(2)^2 \right) \right\rangle_{\text{paths}}}$$
Figure 2. Average cluster overlap $O$ vs. distance, in confinement (top) and deconfinement (bottom) for same sign and opposite sign clusters.

with the field strength correlator between the centers involved. Correlators of specified components would give, for instanton pairs, access to the relative color orientation matrix $U_{rel}$. The overlap $O$ (2) can be identified with the adjoint trace $(\text{tr}U_{rel})^2 - 1)/3$. For random orientations the average of $O$ over all pairs would vanish. Plotting the average for different pairs as a function of the distance shows that the clusters are not uncorrelated (Fig. 2). This average is sensitive to deviations of the histogram of $\text{tr}U_{rel}$ from the Haar measure. Maximizing the overlap by a gauge rotation one can find $U_{rel}$ for each pair. This method can be applied in all instanton finding studies. Our analysis has shown for II pairs strong aligning correlations in the confined phase (decreasing with distance) but none for IA pairs and aligning correlations similar for both types of pairs in the deconfined phase. A recent $T = 0$ study confirms strong II correlations and rather weak ones for IA. These results are difficult to reconcile with any instanton picture.

Therefore we have carefully studied the influence of the cutoff $q_{th}$ (over an interval of space filling fraction between 1 and 10 %) on the cluster composition. The cluster multiplicity goes through a maximum before a cluster percolation transition towards huge and multiply charged clusters is observed. With a cutoff keeping the cluster multiplicity below the maximum we get clustering properties being rather cutoff independent (apart from distance correlations between centers) with an almost Poissonian multiplicity distribution. For instance, on the $12^4$ lattice at $\beta = 1.54 (\langle Q^2\rangle \approx 11)$ we find for $q_{th} = 0.065$ an average number of clusters of $\approx 35$. A strong correlation exists relating charge and volume, $|Q_{cl}| \approx 0.01V_{cl}$ (with almost all $|Q_{cl}| < 0.5$).

4. Conclusion

If these clusters identified on SM configurations by Lüscher’s charge could be interpreted as subclusters of instantons (“instanton quarks” = half-instantons for $SU(2)$) this would explain the unexpected pattern of color correlations. Equal sign clusters would be bound into instantons (dipoles) in the confinement phase. They could dissociate in the deconfined phase with opposite charge correlations becoming of equal importance. Further investigations are worthwhile, relating this to the behavior of the field strength correlators above. If this picture can be consolidated this would be a step beyond the present search for instanton structure by various cooling techniques.

REFERENCES

1. T. A. DeGrand et al., Nucl. Phys. B475 (1996) 321; *ibid.* B478 (1996) 349.
2. M. Feurstein et al., *Nucl. Phys.* B511 (1998) 421.
3. E.-M. Ilgenfritz et al., *Phys. Rev.* D58 (1998) 094502.
4. E.-M. Ilgenfritz et al., hep-lat/9904010.
5. E.-M. Ilgenfritz and S. Thurner, hep-lat/9905012.
6. E.-M. Ilgenfritz and S. Thurner, hep-lat/9810010.
7. Yu. A. Simonov, *Phys. Usp.* 39 (1996) 313.
8. E.-M. Ilgenfritz et al., *Phys. Rev.* D58 (1998) 114508.
9. A. Di Giacomo et al., *Nucl. Phys.* B (Proc. Suppl.) 54A (1997) 343 (1997); *Nucl. Phys.* B483 (1997) 371; M. D’Elia et al., *Phys. Lett.* B408 (1997) 315.
10. M. Teper, plenary talk at this conference.
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\langle G(0)G(x) \rangle
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\beta = 1.5
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\beta = 1.55
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Distance \(r/a\)