Mathematical diagnostic model of brake master cylinder of hydraulic brake system of automobile

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Abstract. The article considers a dynamic mathematical model of a new method of componentwise diagnosis of the brake master cylinder of a “tandem” type of the hydraulic brake system. We propose a variant of how simulated various faults affect the output parameters of brake process on the basis of mathematical modeling.

1. Introduction
The hydraulic brake actuator is now widely used in passenger cars. Faults in the brake actuator lead to serious traffic accidents. Fault diagnosis before a pre-failure or a failure state of elements of the braking system is an important task for engineers, involved in the service and design of vehicles. Researches, aimed at increasing the range of detected faults and improving the diagnosis of the hydraulic brake system, are relevant.

Currently, the diagnosis of the hydraulic brake system is carried out mainly on the basis of the general signs of “in good-in bad order”. Carrying out componentwise diagnosis of the entire system and, in particular, its individual units and elements will help to determine specific faults, identify a pre-failure state and reduce the labor intensity, diagnosing the brake system.

In connection with the labor intensity and costs of conducting experiments, it is better to use a theoretical experiment [1, 2]. Simulation of faults on a computer will subsequently reduce the number of natural experiments; help to choose a set of informative, single-value and stable diagnostic parameters.

The greatest labor intensity of fault diagnosis in a hydraulic brake system is found in the brake master cylinder (BMC), which controls braking systems, equipped with electronic auxiliary braking systems such as ABS / ESC. The componentwise diagnosis of BMC is an important task for vehicle owners and diagnosticians.

2. Material and methods
Recommendations and methods for determining faults and mathematical modeling of dynamic processes in the brake master cylinder, proposed in the literature [3, 4, 5], in some cases make it difficult to use them for componentwise diagnosis of BMC. Therefore, in the mathematical model, some formulas have been put in a form that allows revealing the dependence of specific structural parameters on the technical state of individual elements.

The processes, occurring during the retardation of the hydraulic brake actuator, are studied insufficiently. In early works brake diagrams and diagrams of the dependence of the force of pressing the brake pedal or deceleration because of time are presented with a great degree of simplification [7, 8, 10, etc.]. At the same time, the increasing force on the brake pedal during the retardation of the brake
The control of the brake system is due to the change in fluid pressure which is going on because of the actuation of the pedal; this change in pressure is created by the movement of the pistons in the brake master cylinder. In this case, an input force is applied to the first piston rod, which is a function of time.
A mathematical model of the hydraulic circuit with two separately concentrated volumes of fluid describes the dynamic processes, occurring in such a circuit with a known mode of pressing the brake pedal. This model by a system, consisting of the following nonlinear differential equations [5, 3]:

\[
\begin{aligned}
\frac{d^2 x_1}{dt^2} + a_1 \frac{dx_1}{dt} + a_2 \frac{dx_1}{dt} + a_3 \left( \frac{dx_1}{dt} \right)^2 + \left( a_1 + a_2 + a_3 \right) \frac{dx_1}{dt} + \left( a_1 + a_2 + a_3 \right) \frac{dx_1}{dt} + p_3 = p_1; \\
\frac{d^2 x_2}{dt^2} + a_1 \frac{dx_2}{dt} + a_2 \frac{dx_2}{dt} + a_3 \left( \frac{dx_2}{dt} \right)^2 + \left( a_1 + a_2 + a_3 \right) \frac{dx_2}{dt} + p_4 = p_2; \\
\end{aligned}
\]

where \( F_{HT} \) – control force on the master cylinder rod, 
\( F_{tp1} \) – first piston spring force, 
\( F_{HT1} \) – inertial force of the first piston, 
\( F_{p1} \) – force from pressure in the primary area, 
\( F_{dp1} \) – damping force (force of resistance to the movement of the first piston created by the fluid), H; 
\( F_{mp1} \) – friction force from the first type compression ring, 
\( F_{mp2} \) – friction force from the second type compression ring, 
\( R_1 \) – first cylinder support pressure, 
\( x_1 \) – movement of the primary piston, 
\( t \) – current time of system operation, 
\( F_{tp2} \) – second piston spring force, 
\( F_{HT2} \) – inertial force of the second piston, 
\( F_{p2} \) – force from pressure in the secondary area, 
\( F_{dp2} \) – damping force of the second piston, 
\( F_{mp2} \) – friction force from the second type compression ring, 
\( R_2 \) – second cylinder support pressure, 
\( x_2 \) – movement of the secondary piston, 
\( m_{mp1} \) – mass of moving parts, connected to the support piston, 
\( F_{sup} \) – force of resistance to the movement of the support piston, 
\( F_{mp4} \) – inertial force of support piston, 
\( z_1 \) – movement of the support piston, 
\( p_3 \) and \( S_3 \) – pressure and surface area of the support piston. 
\( m_{mp2} \) – mass of moving parts, connected to the piston of the rear brake cylinder, 
\( F_{mp3} \) – inertial force of the piston of the rear brake cylinder, 
\( F_{ztc} \) – force of resistance to the movement of the piston of the rear brake cylinder, 
\( z_2 \) – movement of the piston of the rear brake cylinder, 
\( p_4 \) and \( S_4 \) – pressure and surface area of the rear brake cylinder. 
\( m_{mp3} \) – mass of moving parts, connected to the support piston, 
\( p_3 \) – pressure in the support piston. 

\[
\begin{aligned}
F_{HT} + R_1 = F_{tp1} + F_{HT1} + F_{p1} + (F_{dp1} + F_{mp1} + F_{mp2}) \cdot \text{sgn}\left( \frac{dx_1}{dt} \right); \\
F_{tp1} + F_{p1} + R_2 = F_{tp2} + F_{HT2} + F_{p2} + (F_{dp2} + 2 \cdot F_{mp2}) \cdot \text{sgn}\left( \frac{dx_2}{dt} \right); \\
m_{n1} \frac{d^2 z_1}{dt^2} + F_{sup} + F_{mp4} \cdot \text{sgn}\left( \frac{dz_1}{dt} \right) - p_3 S_3 = 0; \\
m_{a2} \frac{d^2 z_2}{dt^2} + F_{ztc} + P_{mp3} \cdot \text{sgn}\left( \frac{dz_2}{dt} \right) - p_4 S_4 = 0; \\
a_1 \frac{dz_1}{dt} + a_2 \frac{dz_1}{dt} + a_3 \left( \frac{dz_1}{dt} \right)^2 + \left( a_1 + a_2 + a_3 \right) \frac{dz_1}{dt} + p_3 = p_1; \\
a_1 \frac{dz_2}{dt} + a_2 \frac{dz_2}{dt} + a_3 \left( \frac{dz_2}{dt} \right)^2 + \left( a_1 + a_2 + a_3 \right) \frac{dz_2}{dt} + p_4 = p_2; \\
\frac{dx_1}{dt} = \frac{S_3}{S_1} \frac{dz_1}{dt} - \frac{dx_2}{dt} + \left[ (x_{\text{max}1} - x_1) + \frac{S_3}{S_1} (z_{\text{min}1} + z_1) + \frac{f_1 l_1}{S_1} \right] \psi_1(p_1) \frac{dp_1}{dt}; \\
\frac{dx_2}{dt} = \frac{S_4}{S_2} \frac{dz_2}{dt} + \left[ (x_{\text{max}2} - x_2) + \frac{S_4}{S_2} (z_{\text{min}2} + z_2) + \frac{f_2 l_2}{S_2} \right] \psi_2(p_2) \frac{dp_2}{dt}. \\
\end{aligned}
\]
$p_1$ – pressure in the first area of the brake master cylinder,
$\text{sgn}$ – piecewise constant function of the actual argument (the amount of movement of the first and second pistons),
$l_1$ and $f_1$ – length and cross-sectional area of the pipe,
$S_3$ – surface area of the support piston,
$k_e$ – approximation coefficient, the value of which depends on the relative roughness $\varepsilon$ of hydraulic lines,
$\rho$ – brake fluid density,
$\nu$ – kinematic viscosity coefficient,
$\zeta_1$ – local resistance coefficient.

$m_{a2}$ – mass of moving parts, connected to the piston of the rear brake cylinder,
$p_{4}$ – pressure in the piston of the rear brake cylinder,
$p_2$ – pressure in the second area of the brake master cylinder,
$l_2$ and $f_2$ – length and cross-sectional area of the pipe,
$S_4$ – surface area of the piston of rear brake cylinder,
$\psi_1(p_1)$ – non-linear coefficient of compliance of the primary circuit.
$\psi_2(p_2)$ – non-linear coefficient of compliance of the secondary circuit.

Let us consider the main faults and their mathematical implementation in a mathematical model.

The amount of air in the brake fluid affects the braking performance of a vehicle. In mathematical modeling, the amount of air in a liquid characterizes the compliance coefficient [5]. The determination of the coefficient of compliance depending on the percentage of air in the brake fluid has been made on the basis of experimental data. According to the researches, the curves of the compliance coefficient for the brake fluid have been constructed with air content of 1%, 5%, 10%, 15%, 20%. On the curves trend lines have been plotted with 99% accuracy. Formulas, reflecting the dependence of the compliance coefficient on the percentage of air in the brake fluid and pressure, are used in the mathematical model.

Changes in the stiffness of the springs of the primary and secondary circuits directly affect the compression forces of the springs in the circuits.

The force of the first spring:

$$F_{np1} = C_{np1} \cdot (b_1 + x_1 - x_2),$$

where $C_{np1}$ – stiffness of the return spring of the primary piston, H/m;

$b_1$ – pre-compression of the primary piston spring, m;

$x_1$ – movement of the first piston, m;

$x_2$ – movement of the second piston, m.

The force of the second spring:

$$F_{np2} = C_{np2} \cdot (b_2 + x_2),$$

where $C_{np2}$ – stiffness of the return spring of the secondary piston, H/m;

$b_2$ – pre-compression of the secondary piston spring, m;

$x_2$ – movement of the second piston, m.

By introducing different stiffness into the mathematical model, it is possible to trace the change in the operation of the entire system in dynamics.

The wear of the compression rings of the primary and secondary circuits will lead to a change in two parameters in the operation of the BMC - the damping force and the friction force. The damping force given in formulas 4 and 5 is influenced by the fluid flow through the brake pipe and the fluid flow through the hole between the compression ring and the cylinder. The calculation of the damping force is detailed in the following literature sources [12, 13, 14, 15, 16].

Guided by the Poiseuille formula, the second rate of fluid flow through the pipe (provided a laminar fluid flow) will be equal to:
\[
Q = \frac{\pi \cdot R^4}{8 \cdot \eta \cdot l} \cdot (p_1 - p_2) = \frac{\pi \cdot d^4}{128 \cdot \eta \cdot l} \Delta p,
\]  
(4)

where \( p_1 - p_2 = \Delta p \) – pressure fall at the ends of the capillary, Pa;
\( Q \) – volumetric fluid flow rate, m³ / s;
\( R \) – capillary radius, m;
\( d \) – capillary diameter, m;
\( \eta \) – dynamic viscosity coefficient, Pa • s;
\( l \) – brake line length, m.

The damper is the pistons of the primary and secondary areas of the BMC. When the piston moves at a speed \( V \), it presses on the brake fluid located in the lower area of the cylinder and pushes it through the gap \( \delta \) between the piston and the cylinder into the overflow tank, and also pushes the fluid into the front and rear circuits of the brake system through tubes with a radius of 1.5 mm.

Dynamic viscosity for cooled brakes at a brake fluid temperature of 50°C is given in reference sources [19, 20].

The fluid flow through the gap comprises:

\[
Q_2 = \frac{\pi \cdot R_{CP} \cdot \delta^3 \cdot \Delta p}{6 \cdot \eta \cdot l_2},
\]  
(5)

where \( \Delta p \) – pressure difference at different ends of the compression ring, Pa.
\( \eta \) – dynamic viscosity coefficient, Pa • s;
\( R_{CP} = 0.5(R_1 + R_2) \) – the average radius of the annular slot between the cylinder radius and the piston radius, m;
\( l_2 \) – compression ring width, cm;
\( \delta \) – gap between the piston and the cylinder (slot width), cm.

If the pressure difference in the sealed system is zero, the flow rate \( Q_1 \) will also be zero. It can be assumed that if there are leaks of brake fluid through the working brake cylinders, the pressure difference will begin to differ from zero and \( Q_1 \) will become more than zero.

\[
Q = Q_1 + Q_2 = \Delta p \cdot \left[ \frac{\pi \cdot R^4}{8 \cdot \eta \cdot l_1} + \frac{\pi \cdot R_{CP} \cdot \delta^3}{6 \cdot \eta \cdot l_2} \right] = \Delta p \cdot (k_1 + k_2).
\]  
(6)

The amount of fluid, flowing from one part of the cylinder to another is equal to:

\[
Q = S_1 \cdot V_{\Pi},
\]  
(7)

where \( S_1 \) – cylinder surface area, m.
\( V_{\Pi} \) – piston speed, m/s.

\[
\Delta p = \frac{S_1 \cdot V_{\Pi}}{(k_1 + k_2)}
\]  
(8)

The fluid pressure creates force \( F_{\Pi} \), which opposes the movement of the piston and is equal to:

\[
F_{\Pi} = \Delta p \cdot S_2, \quad S_2 = \pi \cdot R^2,
\]  
(9)

where \( \Delta p \) – fluid pressure, influencing the piston;
\( R \) – piston radius.

\[
F_{\Pi} = \frac{S_2 \cdot S_1 \cdot V_{\Pi}}{(k_1 + k_2)} = \frac{(\pi \cdot R^2) \cdot V_{\Pi}}{(k_1 + k_2)}.
\]  
(10)

The opposing force with the consideration of formula (4.3) and the expression for the volume of the overflowing fluid, is equal to:
where $C_{\beta \Omega}$ – damping coefficient (specific damping force), defined as

$$
C_{\beta \Omega} = \frac{(\pi \cdot R^2)^2}{(k_1 + k_2)}; \quad k_1 = \frac{\pi \cdot r^4}{8 \cdot \eta \cdot l_1}; \quad k_2 = \frac{\pi \cdot R_{cp} \cdot \delta^3}{6 \cdot \eta \cdot l_2}.
$$

Coefficients $k_1$ and $k_2$ depend only on the construction sizes of the damper. In case of wear of the compression rings, coefficient $k_2$ is additionally calculated for the gap between the ring and the BMC body.

The second component that is directly affected by the wear of the rings is the friction force of the compression rings against the cylinder walls, and the increase of the compression rings in diameter, due to swelling, will also affect the friction force. In the first case, with a decrease in the structural parameter of the diameter of the compression ring, the friction force will decrease almost to zero, in the second case it will increase until the seizure of the BMC pistons.

An increase in the outer diameter of the compression rings occurs due to the use of a brake fluid of a non-recommended type, the ingress of gasoline and other operating fluids into the brake fluid, increased rubber aging will be accompanied by a change in the size dimensions of the compression rings with a subsequent increase in the friction force.

The friction force in the compression fittings when starting from a place (friction at rest) is 1.5 ... 2.5 times greater than the friction force during steady motion; with an increase in the pressure of the compressed medium from 2 to 15 MPa, the friction force additionally increases from 0.2 to 0.25% [12, 21].

The frictional force of a circular rubber ring is defined as the component of two forces, the friction force arising from the pressure of the working fluid, and the friction force arising from the preliminary compression of the ring.

For round rubber rings, located in the piston groove, the friction force is:

$$
F_{tr} = T_e \cdot \pi \cdot D + T_n \cdot \pi \cdot \frac{(D^2 - D_k^2)}{4}. \quad (13)
$$

where $D$ – inner diameter of the cylinder in cm,

$D_k$ – ring groove diameter in cm,

$D_{sh}$ – outer diameter of the rod,

$T_e$ – the friction force, arising from the pressure of the working fluid per 1 cm² of the contact surface of the ring, is determined graphically.

$T_n$ – the friction force, arising from the preliminary compression of the ring by 1 cm of its length, is determined graphically, depending on the preliminary assembly compression.

$$
w = \frac{\gamma}{d} \cdot 100\%, \quad (14)
$$

where $\gamma' = d - b_1$, here $d$ – ring section diameter;

$b_1$ – groove depth, taking into account the gap between the sealing surfaces.

To determine forces $T_n$, $T_e$ plots were made in Excel and trend lines and formulas for theoretical calculation were selected, the error was less than 1%.

$$
T_n = 2.93 \cdot 10^{-7} \cdot p. \quad (15)
$$

$$
T_e = 0.0185 \cdot w + 0.032. \quad (16)
$$

To reconcile the data received with the SI system, we carry out the necessary transformations and obtain the following formula:

$$
F_{tr} = 9.81 \cdot (T_e \cdot \pi \cdot D + T_n \cdot \pi \cdot \frac{(D^2 - D_k^2)}{4}). \quad (17)
$$

These equations for calculating friction forces are applicable in a mathematical model corrected to friction of rest and friction of motion. When changing the diameter of the ring section within the
permissible limits indicated above, we obtain the effect of changes in structural parameters in real service conditions on the output diagnostic parameters.

The introduction of the subsystem of differential equations for correcting the numerical values of the structural parameters into the algebraic expressions gives a change in the characteristics of the dynamic system of differential equations at the output. By changing the diagnostic parameters, it is possible to determine the faults of the BMC.

4. Results
As a result of the conducted research, the following results have been received: we have updated the mathematical model of the operation of the main double-circuit brake cylinder with the fixation of the floating piston by a pre-compressed spring. The possibility to introduce the most common faults of BMC into the mathematical model has been obtained. The program for modeling BMC states has been developed and worked out. Optimal set of diagnostic parameters has been chosen and the algorithm for determining faults has been designed. A comparison of the mathematical and experimental models has been carried out, the general discrepancy was 1.42%.

5. Discussions
The application of the method of diagnosing the BMC of the hydraulic brake system by the drag force to the force of pressing the drive pedal at the actuation of the brake system, with various modes of influence on the brake pedal, shows the operation of the brake actuator most objectively. After choosing a methodology for evaluating the characteristics of a brake actuator with precise definitions of faults and developing an algorithm for determining faults in the BMC, we were able to determine faults directly in the nodes of the brake system and determine their service life to failure.

6. Conclusions
At present, research in this area has been minimized, much attention is paid to the development of new braking assist systems, although the whole service of the systems often depends on the correct and stable operation of the BMC itself. Studies of the patterns, occurring in the hydraulic brake actuator, help to identify faults in the brake system at all periods of service. The use of research results when diagnosing the brake system will reduce the labor intensity of diagnostic operations and will increase the reliability of the diagnosis.

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