Mass Hierarchy of Collisional Energy Loss

Rodion Kolevatov
(on work together with Urs Wiedemann, CERN)

Department of High-Energy Physics
Saint-Petersburg State University

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Outline

1. Introduction
   - Brief outlook
   - Motivation

2. The model of collisional parton energy loss
   - Generalities
   - Calculation details

3. Variation of mass hierarchy
   - $dE/dx$
   - Heavy to light
   - In combination with radiative
A snapshot of medium created in a collision
- We know how high $p_t$ partons are produced (we think so)
- Those partons hadronize outside the medium created
  ⇒ We make conclusions on what happens between creation and hadronization

Partons loose energy

Energy loss is sensitive to medium properties
- Radiative energy loss – parameterized by $\hat{q} \propto$ density (Escola et al. ’04, Armesto et al. ’05)
- Collisional energy loss – not so definite...
  (a) massless partons, thermal motion (Djordjevic’06)
  (b) massive scatterers at rest, mass taken to fit $dE/dx$ of (a) (Gyulassy, Wicks ’07).
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Introduction

The model of collisional parton energy loss
Variation of mass hierarchy
Summary

Hard Probes in Relativistic Heavy Ion Collisions

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Highlights on Radiative Energy Loss

Radiative E-loss:
- Fairly explains data on light hadron supression at RHIC (Dainese, Loizides’05)
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Radiative E-loss sets up projectile mass hierarchy at intermediate energies: $\Delta E_Q \ll \Delta E_q$ for $m_Q \gg m_q$.

Can collisional (elastic) energy loss change the situation?
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Can collisional (elastic) energy loss change the situation?
Collisional Energy Loss on a Classical Level

Let us consider elastic collision of two non-relativistic balls...

| $m_p \ll m_t$ | $\approx$ No loss |
|----------------|-------------------|
| $m_p \gg m_t$ | $\approx$ No loss |
| $m_p \approx m_t$ | Significant loss! |

Mass $m_t$ is the simplest possibility to modify the capability of the medium to absorb recoil.
Collisional Energy Loss on a Classical Level

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Let us consider elastic collision of two non-relativistic balls...

\[
\begin{align*}
    m_p \ll m_t & \quad \Rightarrow \quad |p'| \approx |p| \\
    m_p \gg m_t & \quad \Rightarrow \quad |k'| \ll |p'|
\end{align*}
\]

Approximately no loss.

\[
\begin{align*}
    m_p \approx m_t & \quad \Rightarrow \quad |k'| \ll |p|
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Mass Hierarchy of Collisional Energy Loss
Collisional Energy Loss on a Classical Level

Let us consider elastic collision of two non-relativistic balls...

\[
\begin{align*}
\text{Case 1: } & \quad m_p \ll m_t \\
\Rightarrow & \quad |p'| \approx |p| \\
\Rightarrow & \quad |k'| \ll |p'|
\end{align*}
\]

\[
\begin{align*}
\text{Case 2: } & \quad m_p \gg m_t \\
\Rightarrow & \quad p' \approx p \\
\Rightarrow & \quad |k'| \ll |p|
\end{align*}
\]

\[
\begin{align*}
\text{Case 3: } & \quad m_p \approx m_t \\
\Rightarrow & \quad |p'| < |p| \\
\Rightarrow & \quad |k'| \sim |p'|
\end{align*}
\]

Mass \(m_t\) is the simplest possibility to modify the capability of the medium to absorb recoil.
Energy loss due to elastic collisions could have mass hierarchy different from the radiative depending on what (parton effective mass) we take for the medium!

**NB:** Arguments above are based solely on conservation laws, thus should be valid irrespective of the scattering matrix element. (Provided projectile parton is not too relativistic)
The Model

We take medium as a set of independent scattering centers in thermal motion.

Number of collisions per unit time:

\[
\frac{dN_{pi}}{dt} = \int d^3k n_i(k) |\mathcal{M}_{pi}(p, q, k)|^2 \delta(p + k - p_f - k_f) \frac{d^3k_f}{2k_0^0} \frac{d^3p_f}{2p_0^0}
\]

Energy loss per unit path:

\[
\frac{dE}{dx} = \frac{dN^{tot}}{dt} \langle \Delta E \rangle_1 \frac{1}{dx/dt} = \frac{E}{p} \frac{dN}{dt} \langle \Delta E \rangle_1
\]

Definition:

\[
\frac{d\sigma^{\text{int}}}{dp_f} = 2\pi \int d\cos\psi \frac{1}{4p^0k^0} |\mathcal{M}|^2 d\Phi
\]
Calculation Details and Parameters

\[
\frac{dE_Q}{dx} = \frac{1}{v_Q} \int dp_f(E_0 - E_f) \int k^2 dk \left( n_q(k) \frac{d\sigma^\text{int}_{Qq}(k)}{dp_f} + n_g(k) \frac{d\sigma^\text{int}_{Qg}(k)}{dp_f} \right)
\]

\[
\approx \frac{1}{v_Q} \int dp_f(p - p_f) \int k^2 dk (n_q(k) + \frac{9}{4} n_g(k)) \frac{d\sigma^\text{int}_{Qq}(k)}{dp_f}
\]

where \( n_q(k) \) and \( n_g(k) \) – thermal momentum distributions (Fermi and Bose respectively).

- We take HTL-regularized propagator (Braaten ’91, Kalashnikov, Klimov ’79) for \(|\mathcal{M}|^2\)
- We keep all the mass dependence both in \(|\mathcal{M}|^2\) and phase space
- Perform calculations for \( T = 225 \text{ MeV} \) with light (\( m_p = 200 \text{ MeV} \)), \( c \) (\( m_p = 1200 \text{ MeV} \)) and \( b \) (\( m_p = 4750 \text{ MeV} \)) quarks and target parton mass \( m_t = 200 \text{ MeV } \div 1 \text{ GeV} \)
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Calculation Details and Parameters

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\frac{dE_Q}{dx} = \frac{1}{\nu_Q} \int dp_f(E_0 - E_f) \int k^2 dk \left( n_q(k) \frac{d\sigma^{\text{int}}_{Qq}(k)}{dp_f} + n_g(k) \frac{d\sigma^{\text{int}}_{Qg}(k)}{dp_f} \right)
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Energy loss for $m_t = 200$ MeV is consistent with other authors...
Light Quark

\[ \frac{dE}{dx} \]

Heavy to light
Variation of mass hierarchy

\[
L = 5 \text{ fm}, T = 225 \text{ MeV}, m_p = 200 \text{ MeV}
\]

Djordjevic ‘06, massless partons in the medium
**Light Quark**

\[ \frac{dE}{dx} \]

Heavy to light

Variation of mass hierarchy

In combination with radiative

\[ L = 5 \text{ fm}, T = 225 \text{ MeV}, m_p = 200 \text{ MeV} \]

\[ m_t = 200 \text{ MeV} \]

\[ m_t = 450 \text{ MeV} \]
Light Quark

\[ \frac{dE}{dx} \]

Heavy to light

In combination with radiative

\[ L = 5 \text{ fm}, \ T = 225 \text{ MeV}, \ mp = 200 \text{ MeV} \]

\[ \Delta E/E \]

\[ p, \text{ MeV} \]

\[ m_t = 200 \text{ MeV}, \ m_t = 450 \text{ MeV}, \ m_t = 680 \text{ MeV} \]
Light Quark

\[
\frac{dE}{dx} \quad \text{Heavy to light} \\
\text{Variation of mass hierarchy} \\
\text{In combination with radiative}
\]
**Introduction**

The model of collisional parton energy loss

Variation of mass hierarchy

**Summary**

$dE/dx$

Heavy to light

In combination with radiative

---

**Charm Quark**

$L = 5 \text{ fm}, T = 225 \text{ MeV}, mp = 1200 \text{ MeV}$

$\Delta E/E$

$p, \text{ MeV}$

$mt = 200 \text{ MeV}$
Introduction
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\[ \frac{dE}{dx} \]

Heavy to light
In combination with radiative

**Charm Quark**

\[ L = 5 \text{ fm}, \ T = 225 \text{ MeV}, \ m_p = 1200 \text{ MeV} \]

\[ \Delta E/E \]

\[ p, \text{ MeV} \]

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$dE/dx$
Heavy to light
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\[ L = 5 \text{ fm}, T = 225 \text{ MeV}, mp = 1200 \text{ MeV} \]

\[ \frac{\Delta E}{E} \]

\[ mt = 200 \text{ MeV} \]
\[ mt = 450 \text{ MeV} \]

\[ p, \text{ MeV} \]

\[ \frac{\Delta E}{E} \]

\[ 0, 5000, 10000, 15000, 20000 \]

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Mass Hierarchy of Collisional Energy Loss
The model of collisional parton energy loss
Variation of mass hierarchy
Summary

Charm Quark

\[ L = 5 \text{ fm}, T = 225 \text{ MeV}, m_p = 1200 \text{ MeV} \]

\[ \frac{dE}{dx} \]

Heavy to light
In combination with radiative

\[ \Delta E/E \]

\[ p, \text{ MeV} \]

\[ m_t = 200 \text{ MeV} \]
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**Charm Quark**

\[ L = 5 \text{ fm}, T = 225 \text{ MeV}, m_p = 1200 \text{ MeV} \]

\[ \frac{\Delta E}{E} \]

- \( m_t = 200 \text{ MeV} \)
- \( m_t = 450 \text{ MeV} \)
- \( m_t = 680 \text{ MeV} \)
- \( m_t = 1 \text{ GeV} \)
Bottom Quark

$L = 5 \text{ fm}, T = 225 \text{ MeV}, m_p = 4750 \text{ MeV}$

$\Delta E/E$

$p, \text{ MeV}$

$\nabla \quad m_t = 200 \text{ MeV}$
**Introduction**

The model of collisional parton energy loss

**Variation of mass hierarchy**

**Summary**

\[ \frac{dE}{dx} \]

Heavy to light

In combination with radiative effects

---

**Bottom Quark**

\[ L = 5 \text{ fm}, T = 225 \text{ MeV}, m_p = 4750 \text{ MeV} \]

\[ \Delta E/E \]

\[ p, \text{ MeV} \]

\[ 0 \leq p \leq 30 \text{ MeV} \]

\[ 0 \leq \Delta E/E \leq 0.2 \]

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**Mass Hierarchy of Collisional Energy Loss**
**Bottom Quark**

$L = 5\, \text{fm}, T = 225\, \text{MeV}, m_p = 4750\, \text{MeV}$

- $m_t = 200\, \text{MeV}$
- $m_t = 450\, \text{MeV}$
Bottom Quark

$dE/dx$

Heavy to light

In combination with radiative

$\frac{\Delta E}{E} = 5 \text{ fm}, T = 225 \text{ MeV}, m_p = 4750 \text{ MeV}$

- $m_t = 200 \text{ MeV}$
- $m_t = 450 \text{ MeV}$
- $m_t = 680 \text{ MeV}$
**Bottom Quark**

\[ \frac{dE}{dx} \]

Heavy to light

In combination with radiative

**Introduction**

The model of collisional parton energy loss

**Variation of mass hierarchy**

**Summary**

\[ L = 5 \text{ fm}, T = 225 \text{ MeV}, m_p = 4750 \text{ MeV} \]

- \( m_t = 200 \text{ MeV} \)
- \( m_t = 450 \text{ MeV} \)
- \( m_t = 680 \text{ MeV} \)
- \( m_t = 1 \text{ GeV} \)
Mass hierarchy for collisional E-loss can be inverted by appropriate medium parton mass.
We assume that collisional and radiative energy loss add incoherently.

For instance we take radiative $\Delta E_{\text{rad}}$ using ADSW’05 quenching weights with $\hat{q} = 1 \text{ GeV}^2/\text{fm}$ and $L = 5 \text{ fm}$.
Combining With Radiative Energy Loss

Ratios of absolute values of E-loss

\[ L = 5 \text{ fm}, \; \hat{q} = 1 \text{ GeV}^2/\text{fm} \]

\[ \Delta E/Q/\Delta E_q \]

\[ p, \text{ MeV}/c \]

- **Rad. only**
- **Coll+Rad, \( mt = 200 \text{ MeV} \)**
- **Coll+Rad, \( mt = 450 \text{ MeV} \)**
- **Coll+Rad, \( mt = 680 \text{ MeV} \)**
- **Coll+Rad, \( mt = 1 \text{ GeV} \)**
Collisional energy loss is sensitive to the model we take for the medium, strongly depends on medium partons’ mass. Its mass hierarchy at intermediate energies can be varied with this parameter. Collisional E-loss effectively reduces mass hierarchy set by the radiative.

Outlook
- What should be taken for $m_t$?
- Application to single electron suppression in AA still to follow.