The decay $\tau \rightarrow f_1\pi\nu_\tau$ in the Nambu–Jona-Lasinio model

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The width of the decay $\tau \rightarrow f_1\pi\nu_\tau$ was recently obtained from the data of BaBar experiment. The theoretical estimations for this process are given in the framework of two chiral models: the standard NJL model, and the extended NJL model, which includes radially excited meson states. The first test of the extended NJL model with radially excited axial-vector resonances is performed here. Both predictions are in agreement with the experimental data. The differential decay widths for each model is also provided in the present paper.

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I. INTRODUCTION

The decays of $\tau$-leptons are intensively studied in various experiments, for instance, BaBar, Belle, etc. As far as $\tau$ decays occur at the energies below 1.8 GeV, the QCD perturbation theory cannot be applied. Thus, one has to use various phenomenological models. These models are generally based on the chiral symmetry and using the vector meson dominance model for the intermediate states description. One of the most successful models of this type is the Nambu–Jona-Lasinio (NJL) model. For the description of the radially excited mesons the extended NJL model is used. Unlike the other models, the NJL model does not require additional arbitrary parameters for each process considered, thus, it was successfully applied for description of a number of $\tau$ decays and $e^+e^- \rightarrow h\bar{h}$ processes, e.g. $\tau \rightarrow \pi\pi\nu_\tau$, $\pi\omega\nu_\tau$, $\pi\eta\nu_\tau$, $\pi\pi\nu_\tau$, $\pi\pi\pi\nu_\tau$, $\pi\pi\pi\nu_\tau$, $\pi\pi\nu_\tau$, $\pi\omega\nu_\tau$, $\pi\nu_\tau$, $\pi\rho\nu_\tau$, $\pi\pi(\pi')$, $\eta(\eta')\gamma\nu_\tau$, $\omega\nu_\tau$, $\rho\nu_\tau$, $\pi\pi(\pi')$, $\eta\pi\nu_\tau$ (see [19, 20] and the references therein). In all these processes intermediate vector mesons take place. Let us note that the processes with intermediate vector mesons are described in the same way for $e^+e^-$ collisions and $\tau$ decays.

However, there is a number of $\tau$ decays with intermediate axial-vector mesons, unlike $e^+e^-$ annihilation processes. For instance, $\tau \rightarrow 3\pi\nu_\tau$, $\tau \rightarrow \pi\pi\nu_\tau$, $\tau \rightarrow \pi\pi\pi\nu_\tau$, $\tau \rightarrow \pi\pi\pi\nu_\tau$, $\tau \rightarrow \pi\pi\pi\nu_\tau$, $\tau \rightarrow \pi\pi\pi\nu_\tau$. In all these works only the ground-state intermediate axial-vector meson $a_1(1260)$ was taken into account. However, the mass of $\tau$-lepton allows one also to consider an intermediate radially excited $a_1(1640)$ meson in these processes. In a recent paper [18] radially excited axial-vector mesons in the extended NJL model were described. It allows us to estimate the contribution of the radially-excited axial-vector intermediate mesons in decays of $\tau$-leptons. In the present work, this method is demonstrated for the decay $\tau \rightarrow f_1\pi\nu_\tau$.

In the present paper, this decay is described in the framework of the standard NJL model with only intermediate $a_1(1260)$ meson, and in the extended NJL model with $a_1(1260)$ and $a_1(1640)$ intermediate mesons. We also take into account $\pi - a_1$ transitions in the intermediate states. Our estimates are compared with previous theoretical predictions [12] and BaBar experimental data [26].

II. THE LAGRANGIAN OF THE STANDARD AND THE EXTENDED NJL MODEL

In the standard NJL model, the quark-meson interaction Lagrangian for pseudoscalar and axial-vector mesons takes the form:

$$\Delta L_s^{\text{int}}(q, \bar{q}, a, \pi) = \bar{q}(k) \left[ g_{\rho\pi} a_0^{\mu+} \gamma^\mu \gamma^5 + i g_{\pi} \pi_0^{\mu} \gamma^\mu \gamma^5 \right] q(k),$$

where $q$ and $\bar{q}$ are u- and d-quark fields, $a$ and $\pi$ are the axial-vector and pseudoscalar meson fields in the ground state, $\pi^0 = I$ and $\tau^+ = \sqrt{2} \left( \begin{array}{c} 1 \\ 0 \\ 0 \end{array} \right)$, $\tau^- = \sqrt{2} \left( \begin{array}{c} 0 \\ 1 \\ 0 \end{array} \right)$.

The coupling constants

$$g_{\rho\pi} = \left( \frac{2}{3} Z I_2(m_u) \right)^{-1/2}, \quad g_{\pi} = \left( \frac{4}{Z} I_2(m_u) \right)^{-1/2},$$

where $Z = (1 - 6m_u^2/m_{a_1}^2)^{-1}$ is the factor corresponding to the $\pi - a_1$ transitions, and the integrals $I_m$ have the following form:

$$I_m(m_q) = \frac{N_c}{(2\pi)^4} \int \frac{d^4k}{(m_q^2 - k^2)^m} \Theta(\Lambda_4^2 - k^2),$$

where $m_q = m_u = m_d = 280$ MeV is the quark mass, and the cut-off parameter $\Lambda_4 = 1.24$ GeV. The pion coupling constant can be also obtained from the Goldberger-Treiman relation: $g_{\pi_\gamma} = m_u/F_\pi$, where $F_\pi = 93$ MeV is the pion weak decay constant.
In paper [18], the corresponding Lagrangian in the extended NJL model was obtained:

\[
\Delta L^{\mu\nu}_E (q, \bar{q}, \alpha, \alpha', \pi, \pi') = \\
= \bar{q}(k') \left[ (A_1 a^{\mu}_{0,\pm} - A_2 a^{\mu}_{0,\mp}) \tau^{0,\pm} \gamma^5 + \\
+ (P_1 \pi_0 \pm P_2 \pi_0') \tau^{0,\pm} \right] q(k),
\]

where \(a'\) and \(\pi'\) are the axial-vector and pseudoscalar meson fields in the excited state,

\[
A_1 = g_{p2} \frac{\sin(\beta + \beta_0)}{\sin(2\beta_0)} + g_{p2} f(k_{\perp}^2) \frac{\sin(\beta - \beta_0)}{\sin(2\beta_0)},
\]

\[
A_2 = g_{p1} \frac{\cos(\beta + \beta_0)}{\sin(2\beta_0)} + g_{p2} f(k_{\perp}^2) \frac{\cos(\beta - \beta_0)}{\sin(2\beta_0)},
\]

\[
P_1 = g_{p1} \frac{\sin(\alpha + \alpha_0)}{\sin(2\alpha_0)} + g_{p2} f(k_{\perp}^2) \frac{\sin(\alpha - \alpha_0)}{\sin(2\alpha_0)},
\]

\[
P_2 = g_{p1} \frac{\cos(\alpha + \alpha_0)}{\sin(2\alpha_0)} + g_{p2} f(k_{\perp}^2) \frac{\cos(\alpha - \alpha_0)}{\sin(2\alpha_0)},
\]

where \(\beta = 79.85^\circ, \beta_0 = 61.44^\circ\) and \(\alpha = 59.38^\circ, \alpha_0 = 59.06^\circ\) are the mixing angles for axial-vector and pseudoscalar mesons, respectively. The form factors \(f(k_{\perp}^2)\) used in the extended NJL model have the form \(f(k_{\perp}^2) = (1 + dk_{\perp}^2)\), where \(k_{\perp}\) is the transverse quark momentum, for light quarks \(d = -1.784\) GeV\(^{-2}\). The quark-meson coupling constants are

\[
g_{p2} = \left( \frac{2}{3} I_{p2}^L (m_u) \right)^{-1/2}, \quad g_{p2} = \left( 4 I_{p2}^L (m_u) \right)^{-1/2},
\]

where the integrals \(I_{p2}^L\) read

\[
I_{p2}^L (m_u) = \frac{N_c}{(2\pi)^4} \int d^4 k (k_{\perp}^2)^n (m_u - k_{\perp}^2)^m \Theta(A_3 - k_{\perp}^2),
\]

and the cut-off parameter \(A_3 = 1.03\) GeV [13, 19].

### III. THE DECAY \(\tau \to f_1 \pi \nu_\tau\)

In this section, we present the calculations for the branching ratio of the process \(\tau \to f_1 \pi \nu_\tau\).

The decay is described with the diagrams on Figs. 1-3.

#### A. The standard NJL model

In the standard NJL model, the amplitude of this decay reads

\[
T_\pi^{\mu} = \frac{G_F |V_{ud}| L_\mu}{g_{p1}} (1 + (Q^2 - 6m_u^2) BW_{a_1}(Q^2)) T_{a_1 \to f_1 \pi}^{\mu
u},
\]

where \(G_F = 1.16637 \times 10^{-11}\) MeV\(^{-2}\) is the Fermi constant, \(|V_{ud}| = 0.97428\) is the cosine of the Cabibbo angle, \(L_\mu = \bar{\nu} \gamma_\mu (1 - \gamma^5) \tau\) is the lepton current.

The first two addenda correspond to the amplitudes obtained in [21, 23], and the third one describes \(\tau \to a_1\) transitions and is obtained the following way (see Eq. 2):

\[
FWQ BW_Q (Q^2) g_{\pi}, g_{p1}, m_u Q, A_{l2} \approx 6 m_u^2 / g_{p1}.
\]

In the pion Breit-Wigner function \(BW_Q (Q^2)\) the mass and the width of pion is small in comparison with the momentum \(Q\), thus we neglect them.

The contribution of the intermediate \(a_1(1260)\) is

\[
BW_{a_1}(Q^2) = \frac{1}{m_{a_1}^2 - Q^2 - im_{a_1} \Gamma_{a_1}},
\]

where \(Q\) is the \(a_1\) meson momentum, \(m_{a_1} = 1230\) MeV [28]. As far as the decay width of \(a_1\) meson is not known well, we use the following values from the recent experimental data: \(\Gamma_{a_1} = 367\) MeV [29], \(\Gamma_{a_1} = \)
The amplitude \( a_1 \rightarrow f_1 \pi \) was obtained in \([18]\) and reads

\[
T_{a_1 \rightarrow f_1 \pi}^{\mu\nu} = \frac{N_c}{8\pi^2 \Lambda^2} e^{\mu\nu\omega\rho} \rho_\omega q_\rho,
\]

where \( p \) and \( q \) are the pion and \( f_1 \) meson momenta. One can see that in this case the vector meson dominance model appears automatically. One can obtain the branching ratio \( B(\tau \rightarrow f_1 \pi) = 4.10 \cdot 10^{-4} \) for \( \Gamma_{a_1} = 367 \text{ MeV} \) and \( B(\tau \rightarrow f_1 \pi) = 3.97 \cdot 10^{-4} \) for \( \Gamma_{a_1} = 410 \text{ MeV} \).

The experimental values of the branching ratio were obtained recently by BaBar collaboration with measuring \( \Gamma \) where the branching ratio appears automatically. One can obtain the corresponding branching ratio \( B = 1.8 \) and reads

\[
\frac{3}{4},
\]

These values are \((4.73 \pm 0.28 \pm 0.45) \cdot 10^{-4} \) and \((3.60 \pm 0.18 \pm 0.23) \cdot 10^{-4} \), respectively.

**B. The extended NJL model**

In the extended NJL model, this amplitude becomes more complicated:

\[
T_E^{\mu\nu} = G_F |V_{ud}| L_\mu \left[ V_W + V_{a_1} BW_{a_1}(Q^2) \right] T_{a_1 \rightarrow f_1 \pi},
\]

where

\[
V_W = \frac{\sin(\beta + \lambda_0)}{\sin(2\beta_0)} g_{p_1} I_3(m_u)
\]

\[
+ \frac{\sin(\beta - \lambda_0)}{\sin(2\beta_0)} g_{p_2} I_3(m_u),
\]

\[
V_{a_1} = \left[ \frac{\sin(\beta + \lambda_0)}{\sin(2\beta_0)} \right]^2 g_{p_1}^2 I_3(m_u)
\]

\[
+ 2 \frac{\sin(\beta + \lambda_0) \sin(\beta - \lambda_0)}{\sin(2\beta_0)} g_{p_1} g_{p_2} I_3^2(m_u)
\]

\[
+ \left[ \frac{\sin(\beta - \lambda_0)}{\sin(2\beta_0)} \right]^2 g_{p_2}^2 I_3^2(m_u),
\]

\[
C_1 = \frac{1}{g_{p_1}} \left( \frac{\sin(\beta + \lambda_0)}{\sin(2\beta_0)} + \frac{\sin(\beta - \lambda_0)}{\sin(2\beta_0)} \right),
\]

\[
\Gamma = \sqrt{I_2 I_2^*}.
\]

The corresponding branching ratio is \( B_E(\tau \rightarrow f_1 \pi) \) for \( \Gamma_{a_1} = 367 \text{ MeV} \) and \( B_E(\tau \rightarrow f_1 \pi) = 6.3 \cdot 10^{-4} \) for \( \Gamma_{a_1} = 410 \text{ MeV} \).

Now let us take into account the contribution of \( a_1(1640) \) meson, which reads

\[
T_{a_1'}^{\mu\nu} = G_F |V_{ud}| L_\mu V_{a_1'} BW_{a_1'} (Q^2)
\]

\[
\times C_2 (Q^2 - 6m_\mu^2) T_{a_1 \rightarrow f_1 \pi},
\]

where

\[
BW_{a_1'}(Q^2) = \frac{Q^2 - 6m_\mu^2}{m_{a_1'}^2 - Q^2 - im_{a_1'}},
\]

\[
V_{a_1'} = - \frac{\sin(\beta + \lambda_0) \cos(\beta + \lambda_0)}{\sin(2\beta_0)} g_{p_1} I_3(m_u)
\]

\[
- \frac{\sin(\beta - \lambda_0) \cos(\beta - \lambda_0)}{\sin(2\beta_0)} g_{p_1} g_{p_2} I_3^2(m_u),
\]

\[
C_2 = \frac{1}{g_{p_1}} \left( \frac{\cos(\beta + \lambda_0)}{\sin(2\beta_0)} + \frac{\cos(\beta - \lambda_0)}{\sin(2\beta_0)} \right),
\]

where \( m_{a_1'} = 1647 \text{ MeV}, \Gamma_{a_1'} = 254 \text{ MeV} \).

Finally, the branching ratio of this process with the intermediate ground- and excited-state axial-vector mesons is \( B_{tot}(\tau \rightarrow f_1 \pi) = 5.26 \cdot 10^{-4} \) for \( \Gamma_{a_1} = 367 \text{ MeV} \) and \( B_{tot}(\tau \rightarrow f_1 \pi) = 4.72 \cdot 10^{-4} \) for \( \Gamma_{a_1} = 410 \text{ MeV} \).

Let us note that we do not take into account \( \pi \rightarrow a_1 \) transitions for radially excited mesons because of their negligible contribution \([13]\). All estimations for partial branching ratios are presented in Table I.

**Table I: Partial branching ratios, \( \Gamma_{a_1} = 410 \text{ MeV} \) (367 MeV)**

| Contribution | Standard NJL, \( \times 10^{-4} \) | Extended NJL, \( \times 10^{-4} \) |
|--------------|-------------------------------|----------------------------------|
| \( W + a_1 \) | 2.21 (2.22)                     | 3.53 (3.62)                       |
| \( W + a_1 + \pi \) | 3.97 (4.10)                     | 6.30 (6.57)                       |
| \( W + a_1 + a_1' \) | -                            | 2.28 (2.58)                       |
| \( a_1' \) | -                            | 0.94                             |
| \( W + a_1 + a_1' + \pi \) | -                            | 4.72 (5.26)                       |

We give the differential decay widths in the framework of both models on Fig. Unfortunately, they cannot be compared with the experimental data yet.

**IV. CONCLUSION**

In the present work, the branching ratio of the decay \( \tau \rightarrow f_1 \pi \) with intermediate \( a_1(1260) \) and \( a_1(1640) \) mesons is calculated. We applied both the standard NJL model with only ground-state intermediate mesons, and the extended NJL model, which allows one to take into account also radially excited mesons. Our calculations in both models give results, which are in satisfactory agreement with the recent experimental values.
We have shown the importance of taking into account $\pi - a_1$ transitions in the intermediate axial-vector meson states. Indeed, their contribution significantly increases the final results and leads to better agreement with the experimental values (see Table I).

This process was recently described theoretically in [1, 2]. In the first paper, there is the same approach as in the standard NJL model. The branching ratio obtained there is $2.91 \times 10^{-4}$. In the second paper, another phenomenological model based on the vector dominance is used. The result obtained equals to $1.30 \times 10^{-4}$. Both previous predictions include only $a_1(1260)$ without $\pi - a_1$ transitions as an intermediate meson.

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