COMPASS measurement of the $P_T$ weighted Sivers asymmetry

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The SIDIS transverse spin asymmetries weighted with powers of $P_T$, the hadron transverse momentum in the $\gamma N$ reference system, have been introduced already twenty years ago and are considered quite interesting. While the amplitudes of the modulations in the azimuthal distribution of the hadrons are the ratios of convolutions over transverse momenta of the transverse-momentum dependent parton distributions and of the corresponding fragmentation functions, and can be evaluated analytically only making assumptions on the transverse-momentum dependence of these functions, the weighted asymmetries allow to solve the convolution integrals over transverse-momenta without those assumptions. Using the high statistics data collected in 2010 on transversely polarized proton target COMPASS has evaluated in $x$-bins the $P_T$ weighted Sivers asymmetry which is proportional to the product of the first transverse moment of the Sivers function and of the fragmentation function. The results are compared to the standard unweighted Sivers asymmetry.

Keywords: transverse spin effects, Sivers asymmetry, COMPASS experiment

In the generalized parton model, admitting a finite intrinsic transverse momentum $k_T^\perp$ for the quarks, a total of eight Transverse Momentum Dependent (TMD) Parton Distribution Functions (PDFs) are needed for a full description of the structure of the nucleon at leading twist \cite{4}. These functions lead to asymmetries in the azimuthal distributions of hadrons produced in Semi-Inclusive measurements of Deeply Inelastic Scattering (SIDIS) processes off polarised and unpolarised nucleons and can be disentangled by measuring the amplitudes of the different angular modulations. Among the eight TMD PDFs, the T-odd Sivers function \cite{4} is of particular interest. This function arises from a correlation between the transverse momentum of an unpolarised quark in a transversely polarised nucleon and the nucleon spin. It is responsible for the Sivers asymmetry, $A_{Siv}$, which is proportional to the convolution of the Sivers function $f_{1T}^\perp$ and of the unpolarised fragmentation function $D_1$. This asymmetry consists in a $\sin \Phi_{Siv}$ modulation in the number of the produced hadrons, where $\Phi_{Siv} = \phi_h - \phi_S$ is the difference of the azimuthal angle of the hadron and of the nucleon spin. The azimuthal angles are defined in a reference system in which the $x$ axis is the virtual photon direction and the $xz$ plane is the lepton scattering plane.

The Sivers effect was experimentally observed in SIDIS on transversely polarised proton targets, first by the HERMES Collaboration \cite{5, 6} and then by the COMPASS Collaboration \cite{7, 8}. A small effect was also observed at JLab for positive hadrons produced on a $^3$He target \cite{10}, while the COMPASS measurements on a transversely polarised deuteron target \cite{11, 13} gave asymmetries compatible with zero. Combined analysis of the proton and deuteron data soon allowed for first extractions of the Sivers function for u- and d-quarks \cite{14, 17}, which turned out to be different from zero, with similar strength and opposite in sign, a most important result in TMD physics. In the standard Amsterdam notation the Sivers asymmetry can be written as

\[
A_{Siv}(x, z) = \frac{\sum_q e_q^2 x f_{1T}^{\perp q}(x) \otimes D_1^q(z)}{\sum_q e_q^2 x f_1^{q}(x) \cdot D_1^q(z)}
\]

where $x$ is the Bjorken variable, $z$ is the fraction of the available energy carried by the hadron, and $\otimes$ indicates a convolution over the transverse momenta of the Sivers function $f_{1T}^\perp$ and the fragmentation function $D_1$. In all those analyses, due to the presence of the convolution, some functional form had to be assumed for the transverse momentum dependence of both the quark distribution functions and of the fragmentation functions. In most of the analyses these functions were assumed to be Gaussian, and as a result the first transverse moment of the Sivers function

\[
f_{1T}^{\perp(1)}(x) = \int d^2 k_T \frac{k_T^2}{2M^2} f_{1T}^{\perp}(x, k_T^2)
\]

could be determined from the asymmetry data.

Already twenty years ago an alternative method was proposed to determine $f_{1T}^{\perp(1)}$ without making any assumption on the functional form neither of the distribution functions nor of the fragmentation functions. The method consists in measuring asymmetries weighted by the transverse momentum of the hadron $P_T$ \cite{18, 20} as described in the following. For some reasons the method was not pursued: the only results (still preliminary) came from HERMES \cite{21} and have already been used to extract the transverse moment of the Sivers function \cite{16}. Recently, much interest has been dedicated again to the weighted asymmetries (see, f.i. \cite{22}). In this contribution the first COMPASS results \cite{23} on the weighted Sivers asymmetries from the data collected in 2010 on a transversely polarized proton target are presented.
Keeping only the relevant terms, the SIDIS cross-section can be written as
\[
\frac{d\sigma}{dx dy dz d\Phi_{\text{Siv}}} = \frac{\alpha^2}{s} \left( 1 - y + \frac{y^2}{2} \right) x^2 y^2 \\
\times \left[ F_{UU}^2 + ST F_{UU}^\text{sin}\Phi_{\text{Siv}} \sin \Phi_{\text{Siv}} + \cdots \right] \tag{3}
\]
where \(\alpha\) is the fine structure constant, \(s = Q^2/xy\) is the center-of-mass energy squared, and terms of the order of \(\gamma^2 = (2Mx/Q)^2\), where \(M\) is the nucleon mass, have been neglected. The quantities \(y\) and \(Q^2\) are the fraction of lepton energy carried away by the virtual photon and the photon virtuality respectively.

The Sivers asymmetry \(A_{\text{Siv}}\) is the ratio of the structure functions
\[
F_{UU}^\text{sin}\Phi_{\text{Siv}} = \int d^2 \vec{P}_T \, P_T F(P_T^2) \\
F_{UU} = \int d^2 \vec{P}_T \int d^2 \vec{k}_T \int d^2 \vec{p}_\perp \, \delta^2(\vec{P}_T - \vec{p}_\perp - z\vec{k}_T) \\
\times f_1(k_T^2)D_1(p_\perp^2) \tag{4}
\]
\[
F(P_T^2) = \int d^2 \vec{k}_T \int d^2 \vec{p}_\perp \, \delta^2(\vec{P}_T - \vec{p}_\perp - z\vec{k}_T) \\
\times \frac{\vec{P}_T \cdot \vec{k}_T}{MP_T^2} f_1(k_T^2)D_1(p_\perp^2) \tag{5}
\]
and \(\vec{p}_\perp\) is the hadron transverse momentum relative to the fragmenting quark. Although it has not been explicitly indicated the quark distribution functions depend also on \(x\) and \(Q^2\) and the fragmentation functions on \(z\) and \(Q^2\). Also, the sum over the quark flavor is not indicated.

The convolution which appears in \(F_{UU}\) can be easily calculated (the usual relation among the transverse momenta of the quark and of the hadron is guaranteed by the delta function), but in general this is not the case for \(F_{UU}^\text{sin}\Phi_{\text{Siv}}\). It is possible to calculate it by assuming specific functional forms for the transverse momentum dependence of \(f_1^{(1)}\) and \(D_1\), and a common choice is the Gaussian ansatz
\[
f_1^{(1)}(x, k_T^2) = f_1^{(1)}(x) e^{-k_T^2/(k_T^2)_S} \frac{\pi (k_T^2)_S}{\pi (k_T^2)_S}, \\
D_1(z, p_\perp^2) = D_1(z) e^{-p_\perp^2/(p_\perp^2)_S} \frac{\pi (p_\perp^2)_S}{\pi (p_\perp^2)_S}, \tag{6}
\]
which gives
\[
A_{\text{Siv},G}(x, z) = a_G \frac{\sum_q e_q^2 x f_1^{(1)}(q) \cdot D_1^2(z)}{\sum_q e_q^2 x f_1^{(1)}(q) \cdot D_1^2(z)}. \tag{7}
\]
where \(a_G = \sqrt{\pi M/\sqrt{(k_T^2)_S + (p_\perp^2)_S}}\). From eq. \(7\) it is clear that in the Gaussian ansatz the first moment of the Sivers function \(f_1^{(1)}\) can be evaluated directly from the measured Sivers asymmetries using some assumption for the transverse momenta which appear in \(a_G\).

As stated above, the first moment of the Sivers function can be accessed without making any hypothesis on the specific functional forms for \(f_1^{(1)}\) and \(D_1\) by measuring the \(P_T\) weighted Sivers asymmetries. These asymmetries are obtained by weighting the events by the measured transverse momentum of the hadron. Only the spin-dependent part of the cross-section has to be weighted, leaving unweighted the unpolarised cross-section. In this work the weighting is done with \(P_T/zM\), where \(M\) is the nucleon mass. After some algebra one gets the simple result
\[
F_{UU}^\text{sin}\Phi_{\text{Siv}},w = \int d^2 \vec{P}_T \frac{P_T^2}{zM} F(P_T^2) = 2 f_1^{(1)}(1) D_1 \tag{8}
\]
namely the convolution becomes the product of the first transverse moment of the Sivers function and the fragmentation function \(D_1\), so that the weighted Sivers asymmetry is
\[
A_{\text{Siv},w}(x, z) = 2 \frac{\sum_q e_q^2 x f_1^{(1)}(q) D_1^2(z)}{\sum_q e_q^2 x f_1^{(1)}(q) D_1^2(z)}. \tag{9}
\]
An interesting remark is that, knowing the unpolarised distributions and fragmentation functions, both \(f_1^{(1)}\) and \(f_1^{(1)}(d)\) could easily be obtained following the procedure of Ref. \cite{24}. Also, assuming u-dominance for positive hadrons produced on a proton target, the fragmentation function cancels out and the asymmetry simply becomes
\[
A_{\text{Siv},w}(x, z) \simeq 2 f_1^{(1)}(1) \frac{u(x)}{f_1(x)}. \tag{10}
\]

The data we used for this analysis are the data collected in 2010 with the transversely polarised proton target. Since the comparison with the standard Sivers asymmetry is also important, the data production and all the cuts to select the muons and the hadrons are the same as for the published data \cite{25}. In particular, the selected phase space is defined by \(0.004 < x < 0.7, Q^2 > 1\ GeV/c^2, 0.1 < y < 0.9, W > 5\ GeV/c^2, P_T > 0.1\ GeV/c,\) and \(z > 0.2\). Presently, the asymmetry has been measured only as a function of \(x\) and the results have been extracted in the nine \(x\)-bins of ref. \cite{25}.

The distributions of the weight factor \(P_T/zM\) in the different \(x\) bins are shown in Fig. \ref{fig:pt} for the positive hadrons. The corresponding distributions for the negative hadrons are very similar. The \(P_T/z\) acceptance of the spectrometer is about 60% and rather flat.

To extract \(A_{\text{Siv},w}\), we used an ad-hoc double-ratio method which utilizes the information coming from the different cells in which our target system is divided and insures cancellation of the target acceptance and of the beam flux. The method used in so far had to be modified since only the counts in the numerator of the expression of \(A_{\text{Siv},w}\) are weighted, while the counts at the denominator are unweighted. Three different estimators have been used, and it has been checked that the results
FIG. 1: Distributions of the weight factor $P_T/\mu_M$ in the nine $x$ bins.

FIG. 2: Full points: $A_{Siv}^w$ in the nine $x$ bins for positive (left panel) and negative (right panel) hadrons. The open crosses are the standard Sivers asymmetries $A_{Siv}$ from Ref. [8].

are essentially identical.

The results in the case of positive hadrons are given in Fig. 2 left, while Fig. 2 right gives the results for negative hadrons. In both cases the standard Sivers asymmetries published in [8] are also plotted for comparison. As expected, the trend of the asymmetries is similar both for positive and negative hadrons. Assuming $u$-dominance, the results for positive hadrons which are clearly different from zero in particular at large $x$, where $\langle Q^2 \rangle$ reaches ~20 GeV$^2$, constitute the first direct measurement of
To gain insight into the physics of the ratio $R^w$ it is useful to take for $A_{Siv}^w$, the expression given in eq. (7) obtained in the Gaussian model which from the past phenomenological analyses of the Sivers effect is known to work rather well. By comparison of eq. (7) and eq. (9) it is clear that their ratio provides some information on the quantity $a_G$. In particular

$$ R_G^w(x) = \frac{A_{Siv}^w(x)}{A_{Siv,G}(x)} $$

(11)

can be further simplified by making the (reasonable) assumption that $\sqrt{(P_T^2)_S} = \sqrt{z^2(k_T^2)_S + (p_T^2)}$ can be regarded as a constant since the dependence on $z$ of $\sqrt{(P_T^2)}$ is known to be much weaker than that of $D_1(z)$. Further simplifications on the ratios $R_G^w$ can be obtained by weighting the events by $P_T/zM$ rather than $P_T/zM$.

As a conclusion, COMPASS is extracting the weighted Sivers asymmetry in the SIDIS process of 160 GeV muons on transversely polarized protons extending the standard measurements already performed. Preliminary results from the data collected in 2010 have already been derived, using as weight $P_T/zM$ and are presented in this paper as a function of $x$. No major experimental problems have been encountered and the results look very promising in view of more precise extractions of the Sivers function, and further measurements are foreseen.

FIG. 3: The ratios $R^w$ between $A_{Siv}^w$ and $A_{Siv}$, in the nine $x$-bins for positive hadrons.

\[ \frac{f_{1T}^{(+)u}(x)}{f_{1T}^{(+)}(x)} \]

Given the similar trend of the weighted asymmetries $A_{Siv}^w$ and of the standard asymmetries $A_{Siv}$, we have evaluated in each $x$ bin their ratios $R^w = A_{Siv}^w/A_{Siv}$. The values of $R^w$ are given in Fig. 3 for positive hadrons, which exhibit a large Sivers asymmetry. In spite of the large statistical uncertainties the ratios are compatible with a constant value, which is rather well determined $(1.6 \pm 0.1)$. Also, the values in the different $x$ bins agree rather well with those of $(P_T/zM)$, as expected.

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