t-channel production of heavy charged leptons

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We study the pair production of heavy charged exotic leptons at $e^+e^-$ colliders in the $\text{SU}(2)_L \times \text{SU}(2)_I \times U(1)_Y$ model. This gauge group is a subgroup of the grand unification group $E_6$; $\text{SU}(2)_I$ commutes with the electric charge operator, and the three corresponding gauge bosons are electrically neutral. In addition to the standard $\gamma$ and $Z$ boson contributions, we also include the contributions from extra neutral gauge bosons. A t-channel contribution due to $W_I$-boson exchange, which is unsuppressed by mixing angles, is quite important. We calculate the left-right and forward-backward asymmetries, and discuss how to differentiate different models.

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1 Introduction

Many extensions of the Standard Model (SM) contain exotic fermions. Strongly interacting exotics, such as heavy quarks, can be produced in abundance at the Tevatron or the LHC. However, particles which are not strongly interacting, such as heavy charged leptons, can best be produced at an electron-positron collider. In general, studies of heavy charged leptons at such colliders focus on s-channel production, through a $\gamma$, $Z$, $Z'$, etc. The phenomenology of exotic particles has been considered widely [1-8]. A good report can be found in Ref. [9].

In this paper, we note that a model which arises from superstring-inspired $E_6$ grand unification models will allow pair production of heavy charged leptons in the t-channel. We discuss this model, and study the forward-backward and left-right asymmetries at linear colliders. For simplicity, we neglect mixing between extra particles (bosons or fermions) and the normal particles of the SM, since such mixing angles are generally small.

2 The model

There are many phenomenologically acceptable low energy models which arise from $E_6$.

\begin{align*}
(a) E_6 & \rightarrow \ SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{Y'} \\
(b) E_6 & \rightarrow \ SO(10) \times U(1)_{\psi} \rightarrow SU(5) \times U(1)_{\chi} \times U(1)_{\psi} \\
(c) E_6 & \rightarrow \ SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_L \times U(1)_R \\
(c') E_6 & \rightarrow \ SU(3)_C \times SU(2)_L \times SU(2)_I \times U(1)_Y \times U(1)_{Y'}
\end{align*}

where $U(1)_{\psi}$ and $U(1)_{\chi}$ can be combined into $U(1)_{\theta}$ in model (b), reducing it to the effective rank-5 model $SU(3)_C \times SU(2)_L \times U(1)_{Y'} \times U(1)_{\theta}$, which is most often considered in the literature. Models (c) and (c') come from the subgroup $SU(3)_C \times SU(3)_L \times SU(3)_R$.
The 27-dimensional fundamental representation has the branching rule

\[ 27 = (3^c, 1) + (\overline{3}, 3) + (1^c, \overline{3}, 3) \]  \hspace{1cm} (1)\]

and the particles of the first family are assigned as

\[
\begin{pmatrix}
    u \\
    d \\
    h
\end{pmatrix}
+ \begin{pmatrix}
    u^c \\
    d^c \\
    h^c
\end{pmatrix}
+ \begin{pmatrix}
    E^c & \nu & N \\
    N^c & e & E \\
    e^c & \nu^c & S^c
\end{pmatrix}
\]

where \( SU(3)_L \) operates vertically and \( SU(3)_R \) operates horizontally. (Different symbols for these particles may be used in the literature.)

The most common method of breaking the \( SU(3)_R \) factor is to break the 3 of \( SU(3)_R \) into 2+1, so that \((u^c, d^c)\) forms an \( SU(2)_R \) doublet with \( h^c \) as a \( SU(2)_R \) singlet. This gives Model (c), the familiar left-right symmetric model \[10\]. Model (c) can be reduced further to an effective rank-5 model with \( U(1)^{V=L+R} \). Another possibility, resulting in Model (c'), is to break the 3 of the \( SU(3)_R \) into 1+2 so that \((d^c, h^c)\) forms an \( SU(2) \) doublet with \( u^c \) as a singlet. In this option, the \( SU(2) \) doesn’t contribute to the electromagnetic charge operator and it is called \( SU(2)_I \) (I stands for Inert). Then the vector gauge bosons corresponding to \( SU(2)_I \) are neutral. Model (c') can be reduced to an effective rank-5 model \( SU(3)_C \times SU(2)_L \times U(1)_Y \times SU(2)_I \). Both of them will be considered in this paper.

At the \( SU(2)_L \times SU(2)_I \times U(1)_Y \times U(1)_Y' \) level, a single generation of fermions can be represented as

\[
\begin{pmatrix}
    \nu \\
    e^c \\
    N
\end{pmatrix}^L, \begin{pmatrix} 
    u \\
    d \\
    h
\end{pmatrix}^L, \begin{pmatrix}
    d^c \\
    h^c
\end{pmatrix}^L, \begin{pmatrix}
    E^c \\
    N^c
\end{pmatrix}^L, \begin{pmatrix}
    \nu^c \\
    S^c
\end{pmatrix}^L, h_L, e^c_L, u^c_L
\]

where \( SU(2)_{L(I)} \) acts vertically (horizontally). Note that additional heavy leptons \( \begin{pmatrix} N \\
E \end{pmatrix}^L \) and its conjugate \( \begin{pmatrix} E^c \\
N^c \end{pmatrix}^L \) form two new isodoublets under \( SU(2)_L \).
3 Cross Section Production and Asymmetries

The relevant interactions for the process $e^+e^- \rightarrow E^+E^-$ are

$$\mathcal{L} = \sum_{f=e,E} Q_f \overline{\psi}_f \gamma^\mu f_\alpha A_\mu + \frac{g}{\cos \theta_W} \overline{\psi}_e \gamma^\mu (T^3_e - Q_e \sin^2 \theta_W) e_\alpha Z_\mu$$

$$+ \frac{g}{2 \cos \theta_W} \overline{\psi}_E \gamma^\mu (1 - 2 \sin^2 \theta_W) E_\alpha Z_\mu$$

$$+ \frac{g_I}{2 \sqrt{2}} \overline{\psi}_E \gamma^\mu (1 - \gamma_5) E W_{I\mu} + H.c.$$

$$+ \frac{g_I}{4} (\overline{\psi}_E \gamma^\mu (1 - \gamma_5) E - \overline{\psi}_E \gamma^\mu (1 - \gamma_5) E) Z_{I\mu}$$

$$+ \sum_{f=e,E} g_{Y'} \frac{Y'}{2} \overline{\psi}_f \gamma^\mu f_\alpha Z'_{I\mu}$$

(2)

where $\alpha = L$ or $R$. $g$, $g_I$ and $g_{Y'}$ are coupling constants and $\theta_W$ is the electroweak mixing angle. For simplicity, we will assume that $g_I = g$ and $g_{Y'} = g_Y$ in our numerical results, it is straightforward to relax this assumption. The first two lines are couplings between fermions and standard $\gamma$ and Z. The rest are couplings with extra neutral gauge bosons.

The $e^+e^- \rightarrow E^+E^-$ process can proceed via s-channel exchange of a $\gamma$, $Z$, $Z'$ or $Z_I$, and can also proceed via t-channel exchange of a $W_I$. Each amplitude can be written as the form of

$$C_i \overline{\psi}_e \gamma^\mu (1 - a_i \gamma_5) u \overline{\psi}_E \gamma^\mu (1 - b_i \gamma_5)v_E.$$  

(3)

Note that the $W_I$ leads to a t-channel process unsuppressed by small mixing angles. This is unique to this model. Note that if one considered production of the heavy charged leptons which form an $SU(2)_I$ doublet with the muon or the tau, then the processes would be identical except that the t-channel process would be absent.

The differential cross section for this process is given by

$$\frac{d\sigma}{d \cos \theta} = \frac{1}{8\pi s} \sqrt{\frac{1}{4} - m_E^2/s} \left\{ D_1(m_E^2 - u)^2 + D_2(m_E^2 - t)^2 + 2D_3m_E^2 s \right\}$$

(4)

s, t and u are the Mandelstam variables, and with

$$D_1 = \sum_{i,j=1}^{5} C_i C_j \{(1 + a_i a_j)(1 + b_i b_j) + (a_i + a_j)(b_i + b_j)\}$$
\[ D_2 = \sum_{i,j=1}^{5} C_i C_j \{(1 + a_i a_j)(1 + b_i b_j) - (a_i + a_j)(b_i + b_j)\} \]

\[ D_3 = \sum_{i,j=1}^{5} C_i C_j \{(1 + a_i a_j)(1 - b_i b_j)\} \]  

where the \( C_i, a_i \) and \( b_i \) are given in Table 1.

| i   | \( C_i \)                                | \( a_i \) | \( b_i \) |
|-----|------------------------------------------|-----------|-----------|
| 1   | \( e^2 \)                                | 0         | 0         |
| 2   | \( \frac{g'(1 - 4 \sin^2 \theta_W)(1 - 2 \sin^2 \theta_W)}{8 \cos^2 \theta_W(s - m_Z^2)} \) | \( \frac{1}{1 - 4 \sin^2 \theta_W} \) | 0         |
| 3   | \( \frac{-g_Y'}{16(s - m_Z^2)} \)        | 1         | 1         |
| 4   | \( \frac{-9 g_Y',}{144(s - m_Z^2)} \)    | \( \frac{-1}{3} \) | \( \frac{-1}{5} \) |
| 5   | \( \frac{g_Y'}{8(t - m_W^2)} \)          | 1         | 1         |

The forward-back asymmetry is defined by

\[ A_{FB} = \frac{\int_{-1}^{1} \frac{d\sigma}{d\cos \theta} d\cos \theta \int_{-1}^{1} \frac{d\sigma}{d\cos \theta} d\cos \theta}{\int_{-1}^{1} \frac{d\sigma}{d\cos \theta} d\cos \theta} \]  

\[ A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R}, \]  

Note that the \( C_i, a_i \) and \( b_i \) will be somewhat different for \( \sigma_L \) and \( \sigma_R \) due to the insertion of the projection operator in Eq. (3). Both \( A_{FB} \) and \( A_{LR} \) at \( e^+e^- \) colliders were studied in Ref. [11, 12], but only s-channel contributions were considered.

## 4 Results

The electroweak part \( SU(2)_L \times U(1)_Y \) has been measured precisely. Let us first consider the rank 5 case. Setting \( g_Y' = 0 \), we have two gauge boson mass parameters \( m_{W_i} \) and \( m_{Z_i} \). We will assume that these masses are equal and thus there is only one mass parameter remaining, which we choose to be near the experimental lower bound for direct
production, $m_{Z_I} = 650$ GeV. This is basically the same as assuming that the gauge bosons do not substantially mix with each other. The numerical results for cross section, forward-backward and left-right asymmetries are shown in Figs. 1-3. We have plotted the results for $E^+E^-$ and $M^+M^-$ production, where $M$ is the $SU(2)_I$ partner of the muon or tau (the only difference will be due to the t-channel process). For comparison, we also include the standard model results for both a vectorlike heavy lepton and a chiral heavy lepton. Although we have assumed that the $Z_I$ mass ($E, M$ mass) is 650 GeV (200 GeV), it is easy to see how the figures will be qualitatively modified if these assumptions are relaxed.

In the rank 6 model, one has an additional mass scale and additional coupling. If we assume that the $g_{Y'}$ coupling is the same as $g_Y$, and that the mass of the $Z'$ is $\frac{5\alpha_Y}{3g_Y} M_{Z_I}$, then one can recalculate the cross section, forward-backward and left-right asymmetries. We find that there is not a substantial difference from the rank 5 case, except in the immediate vicinity of the $Z'$ mass.

## 5 Conclusions

How does one detect these leptons? The main decay modes depend sensitively on the masses and mixing angles. Since the $E$ and its isodoublet partner $N$ are degenerate in the limit of no mixing, one expects the $E \to NW^*$ to be into a virtual $W$, leading to a three-body decay. Since the allowed three-body phase space is very small, this decay will be negligible unless the mixing with the lighter generations is extremely small. In the more natural case, in which such mixing is not very small, the two-body decays $E \to \nu_e W$ and $E \to eZ$ would dominate. A detailed analysis of the lifetimes and the decay modes can be found in Ref. [14]. There, it was shown that the ratio of $\Gamma(E \to eZ)$ to $\Gamma(E \to \nu_e W)$ is given by the ratio of $|U_{Ee}|^2$ to $|U_{E\nu_e}|^2$. This is very model-dependent.
Certainly, the signature for \( E \to eZ \) would be quite dramatic. Even if the \( Z \) decays hadronically or invisibly, the monochromatic electron, plus the invariant mass of the \( Z \) decay products, would allow for virtually background-free detection. The signature for \( E \to \nu_eW \) is less dramatic, but would lead to \( W^+W^- \) plus missing transverse momentum. As discussed in Ref. [8], requiring that the \( W \)'s decay leptonically gives a signal of \( l^+l^- \), where \( l = (e, \mu) \). The backgrounds, due to \( e^+e^- \to \tau^+\tau^- \), \( W^+W^- \) and \( ZZ \), can be eliminated by calculating the invariant mass of the charged fermion pair. The signal would be striking since it would consists of a pair of \( l^+l^- \) with approximately the same invariant mass.

Suppose these leptons are found. One would first learn the cross section. Unless one is in the vicinity of the \( Z_I \) resonance, the cross section in this model would be somewhat higher than the standard model. For example, at an NLC of \( \sqrt{s} = 500 \text{ GeV} \) and luminosity of \( 6 \times 10^4 \text{ pb}^{-1} \text{yr} \) and for a heavy lepton of 200 GeV, one expects approximately \( 2 \times 10^4 \) SM vectorlike fermion pairs produced per year, whereas one has \( 3 \times 10^4 E^+E^- \) pairs and \( 5 \times 10^4 M^+M^- \) pairs (note that the t-channel process destructively interferes). In the vicinity of the resonance, of course, the cross section can be much larger. As discussed in the previous paragraph, if the main decay is into \( \nu W \), then a very clear signature arises if both \( W \)'s decay into \( e\nu_e \) or \( \mu\nu_\mu \). This will occur approximately 5\% of the time, giving a few thousand such events per year. Necessary cuts on the transverse missing energy will reduce the number of usable events, but it should still be several hundred per year, with very low background. If the main decay is into \( eZ \) or \( \mu Z \), then the signature is even more dramatic.

There is no forward-backward asymmetry for the pair production of SM vectorlike fermions, while the polarization asymmetry for heavy SM chiral fermions is very small. Therefore, combining \( A_{FB} \) with \( A_{LR} \) would make it very straightforward to distinguish \( E^+E^- \) and \( M^+M^- \) pairs from SM fermions. The behavior of the asymmetries for each
of these is very different at high $\sqrt{s}$.

An important point is to note that the statistical uncertainty, $(1 - \frac{A^2}{N})^{1/2}$, is very small for this model. With the approximate number of reconstructed events being between several hundred and several thousand, this gives a statistical uncertainty of between 1 and 10 percent. This will be even smaller in the vicinity of the resonance. From the figures, it is clear that this uncertainty is small enough that the various models can be distinguished, even off-resonance.

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Figure 1: Total cross section for the process $e^+e^- \rightarrow L^+L^-$ as a function of $\sqrt{s}$, for a heavy lepton of 200 GeV. The solid and dotted lines correspond to Standard Model production of chiral and vectorlike fermions, respectively. The dashed and dot-dashed lines correspond to $L = E$ and $L = M$ in the $SU(2)_I$ model, respectively, where $E$ and $M$ are the $SU(2)_I$ partners of the electron and muon.
Figure 2: $A_{FB}$, the forward-backward asymmetry, for the process $e^+e^- \rightarrow L^+L^-$ as a function of $\sqrt{s}$, for a heavy lepton of 200 GeV. The solid and dotted lines correspond to Standard Model production of chiral and vectorlike fermions, respectively. The dashed and dot-dashed lines correspond to $L = E$ and $L = M$ in the $SU(2)_I$ model, respectively, where $E$ and $M$ are the $SU(2)_I$ partners of the electron and muon.
Figure 3: $A_{LR}$, the left-right asymmetry, for the process $e^+e^- \rightarrow L^+L^-$ as a function of $\sqrt{s}$, for a heavy lepton of 200 GeV. The solid and dotted lines correspond to Standard Model production of chiral and vectorlike fermions, respectively. The dashed and dot-dashed lines correspond to $L = E$ and $L = M$ in the $SU(2)_I$ model, respectively, where $E$ and $M$ are the $SU(2)_I$ partners of the electron and muon.