A Prediction Model of Tunnel Surrounding Rock Deformation Based on Induced Ordered Weighted Averaging Theory and its Application

FAN Fangfang, LI Jian

CCCC Second Highway Consultants Co., Ltd, Wuhan, Hubei 430056, China

*Corresponding author’s e-mail: 45844894@qq.com

Abstract. For the different prediction accuracy of tunnel surrounding rock deformation prediction model are different. Based on induced ordered weighted averaging theory (IOWA), IOWA modified combined forecasting model of tunnel surrounding rock deformation has been put forward with the relationship between the accuracy and the weight of a single prediction model. With the monitoring data of Mila Mountain tunnel of highways from Lasa to Linzhi, coupling the time series prediction model of deformation and exponential smoothing model of two kinds of single tunnel, IOWA modified combination prediction model for tunnel surrounding rock deformation has been built up. The result shows that predicted displacement of IOWA modified combination prediction model for tunnel surrounding rock deformation is closer to the measured value and has higher accuracy. The combined model for the prediction of tunnel surrounding rock deformation is feasible.

1. Introduction

The monitoring and prediction of deformation for tunnel surrounding rock are an important research content of information design and construction in tunnel engineering. Therefore, domestic and foreign scholars have proposed many practical prediction models and methods for tunnel surrounding rock deformation prediction, such as neural network analysis, gray prediction model, time series analysis, fuzzy theory [1-6]. Throughout these results of research, the prediction model of tunnel surrounding rock deformation is mainly a single prediction model, and the combined prediction model is less. The prediction values of surrounding rock deformation obtained by different single tunnel surrounding rock deformation prediction models are quite different from actual values. That is to say, the prediction accuracy of different single prediction models is quite different. Each single prediction model of tunnel surrounding rock deformation has its own advantages and disadvantages. It can be seen that the modeling and prediction of tunnel surrounding rock deformation monitoring using a single model can not meet the prediction accuracy requirements in the actual engineering of tunnel surrounding rock deformation monitoring and forecasting. Therefore, in order to give full play to the advantages of each single prediction model of tunnel surrounding rock deformation, the combined prediction model for predicting and predicting the surrounding rock deformation of tunnels is the future development trend of tunnel engineering monitoring and prediction research.

In this paper, based on induced ordered weighted averaging (IOWA), an IOWA combination prediction model for tunnel surrounding rock deformation is proposed. The model obtains the relationship between the prediction accuracy and the weight value of a single prediction model. The correction value of the weight and the combination model is established. Combined with the
surrounding rock monitoring data of the Milashan Tunnel of Lalin Expressway, two single prediction models of tunnel surrounding rock deformation, time series model and exponential smoothing model which are commonly used are selected. The results show that the predicted value of the IOWA combined prediction model for tunnel surrounding rock deformation is closer to the measured value and the accuracy is higher. The combined model provides a feasible method for predicting and forecasting the surrounding rock deformation of the tunnel.

2. Prediction model of Tunnel deformation

Based on the induced ordered weighted average theory, the IOWA combination prediction model for tunnel surrounding rock deformation can be established.

\[ f_{w}(\{a_1(x_1), a_2(x_2), \ldots, a_n(x_n)\}) = \sum_{i=1}^{n} k_i x_{it} \]  

(1)

Where, \(a_{it}\) is the accuracy of the i prediction method relative to the actual tunnel deformation at the time t; \(k_i\) is the weight of the prediction value of the deformation of the i model tunnel at the moment in the IOWA combination prediction model of the tunnel surrounding rock deformation; \(x_{it}\) is the prediction model for the i model tunnel deformation at time t for each prediction model ordered by size of \(a_{it}\). \(f_w\) is the deformation prediction value of the IOWA combination prediction model for tunnel surrounding rock deformation.

In order to obtain the IOWA combined prediction model of the tunnel surrounding rock displacement of formula (1), it is first necessary to determine the accuracy value in the combined prediction model. The calculation formula is

\[ a_{it} = \begin{cases} 1 - \frac{(x_{it} - x_t)}{x_t}, & \frac{(x_{it} - x_t)}{x_t} < 1 \\ 0, & \frac{(x_{it} - x_t)}{x_t} \geq 1 \end{cases} \]  

(2)

In the formula, the value of \(x_t\) is monitored for the actual tunnel deformation at time t.

At the same time, using the predicted values of the prediction models of the surrounding rock deformation of each single tunnel, the prediction error of the i-th deformation prediction value at time t is obtained as

\[ e_{it} = x_t - x_{it} \]  

(3)

From the prediction error \(e_{it}\) of the i-th deformation prediction value at the time t of the above formula, the sum of squared errors generated during the deformation prediction process of the entire tunnel surrounding rock can be obtained as:

\[ S = \sum_{i=1}^{n} \sum_{j=1}^{n} k_i k_j (x_i - x_{it})^2 \]  

(4)

\[ = \sum_{i=1}^{n} \sum_{j=1}^{n} k_i k_j (x_i - x_{it})(x_i - x_{jt}) \]

\[ = \sum_{i=1}^{n} \sum_{j=1}^{n} k_i k_j e_{it} e_{jt} \]

Among them, \(E_{ij} = \sum_{i=1}^{n} \sum_{j=1}^{n} e_{it} e_{jt}\) is also called the error information matrix [3], and \(k_i\) is the weight of the deformation prediction value of the tunneling rock deformation IOWA combination prediction model for the accuracy ranking i. In order to obtain the weight of each single tunnel deformation prediction model, the minimum criterion of the sum of squared errors can be obtained.
\[
\min S(k_i, k_j) = \sum_{i=1}^{n} \sum_{j=1}^{n} k_i k_j e_i e_j
\]

\[s.t. = \left\{ \begin{array}{l}
\sum_{i=1}^{n} k_i = 1 \\
k_i \geq 0, \ (i = 1, 2, \ldots, n)
\end{array} \right. \] (5)

From the above formula, the weighted optimal solution \(k_1, k_2, k_3, \ldots, k_n\) of each tunnel deformation prediction model can be obtained, so the IOWA combined prediction model of tunnel surrounding rock deformation at time \(t\) is obtained.

\[f_w = k_1 x_{1t} + k_2 x_{2t} + \cdots + k_n x_{nt} = \sum_{i=1}^{n} k_i x_{it} \] (6)

If the accuracy sequence of each tunnel deformation prediction model is \(\alpha = \{\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_n\}\), each tunnel deformation prediction model weight sequence \(k = \{k_1, k_2, k_3, \ldots, k_n\}\), therefore, the correction weight of each tunnel deformation prediction model is:

\[k'_i = \begin{cases} 
\frac{k_i + \Delta k}{2} \, \alpha_i \geq \alpha_j, \\
\frac{k_i - \Delta k}{2} \, \alpha_j > \alpha_i
\end{cases} \] (7)

Where \(k'_i\) is the corrected weight; \(\Delta k\) is the weight difference and satisfies the following formula:

\[\Delta k = m \left[ (\alpha_i + \alpha_j) \cdot \frac{(\alpha_i - \alpha_j)}{2} \right] \] (8)

Where \(m\) is a multiple of the weight difference and satisfies the largest integer of \(0 \leq k'_i \leq 1\). When \(\alpha_i = \alpha_j\), the prediction accuracy values are equal, and no correction need made. Therefore, it can be seen from the above that the final IOWA modified combination prediction model of tunnel surrounding rock deformation is:

\[f'_w = k'_{1t} x_{1t} + k'_{2t} x_{2t} + \cdots + k'_{nt} x_{nt} = \sum_{i=1}^{n} k'_{it} x_{it} \] (9)

3. Engineering Applications

3.1 Project Overview
The Milashan Tunnel of Lalin Expressway is 5,720 m long, with an average elevation of over 4,700 meters and a maximum altitude of 5020 m. It is currently the highest highway tunnel in the world. The surrounding rocks of the Milashan Tunnel are mainly Grade IV and Grade V tuffs, accounting for about 85%. The rock mass is relatively broken, the joint fissures are developed, and the integrity is poor. This paper selects the monitoring data of the tunnel arch displacement of the YK4480+120 section of the Milashan Tunnel, and analyzes the application effect of the IOWA combination prediction model of the tunnel surrounding rock deformation in detail.

The actual measured data of the dome displacement of the tunnel section from January 4, 2016 to January 31, 2016 is shown in Table 1.
Table 1. Vault accumulated displacement of tunnel

| Time  | Total displacement of the dome /mm | Time  | Total displacement of the dome /mm | Time  | Total displacement of the dome /mm |
|-------|----------------------------------|-------|----------------------------------|-------|----------------------------------|
| 2016/1/4 | 0.00                             | 2016/1/14 | 9.47                         | 2016/1/24 | 13.76                         |
| 2016/1/5 | 2.47                             | 2016/1/15 | 9.75                          | 2016/1/25 | 13.91                         |
| 2016/1/6 | 4.25                             | 2016/1/16 | 10.19                         | 2016/1/26 | 14.33                         |
| 2016/1/7 | 5.57                             | 2016/1/17 | 10.37                         | 2016/1/27 | 14.66                         |
| 2016/1/8 | 6.53                             | 2016/1/18 | 10.90                         | 2016/1/28 | 15.16                         |
| 2016/1/9 | 7.37                             | 2016/1/19 | 11.43                         | 2016/1/29 | 15.41                         |
| 2016/1/10| 7.88                             | 2016/1/20 | 11.87                         | 2016/1/30 | 15.61                         |
| 2016/1/11| 8.21                             | 2016/1/21 | 12.25                         | 2016/1/31 | 15.72                         |
| 2016/1/12| 8.55                             | 2016/1/22 | 12.44                         |         |                                |
| 2016/1/13| 9.10                             | 2016/1/23 | 13.16                         |         |                                |

3.2 Modeling of surrounding rock deformation

At present, there are many single tunnel surrounding rock deformation prediction models. Here, two single prediction models of tunnel surrounding rock deformation are selected to establish the IOWA combined prediction model for tunnel surrounding rock deformation. According to the monitoring data of Table 1, the measurement data of the first 21 days is taken as the modeling sample data, and the time series model and exponential smoothing model of the tunnel surrounding rock deformation, and the IOWA combined prediction model of the tunnel surrounding rock deformation are obtained. The measured data of the last 7 days is used as the detection data of the tunnel surrounding rock deformation model, which proves the validity and accuracy of the IOWA combination prediction model for tunnel surrounding rock deformation.

- **Time series model of tunnel surrounding rock deformation**

From the data in Table 1, the time series model of tunnel surrounding rock deformation can be established \(^\text{[7]}\) as ARIMA \((1, 1, 2)\), ie

\[
\hat{X}_t = 0.7736X_{t-1} + e_t - 0.3692e_{t-1} + 0.0498e_{t-2}
\]

Where \(\hat{X}_t\) is the predicted displacement of the prediction model; \(e_t\) is the random variable of the predicted displacement.

- **Smooth model of tunnel surrounding rock deformation index**

From the data in Table 1, the smoothing model of tunnel surrounding rock deformation index can be established, and the smoothing parameter is \(\alpha = 1\) and \(\beta = 0.64\). The seasonally exponential smoothing model structure \(^\text{[8]}\) is:

\[
\hat{Y}_{t+k} = Y_t + b_t k
\]

Where \(\hat{Y}_{t+k}\) is the model predicted displacement; \(Y_t\) is the actual measured displacement; \(k\) is the predicted lead period; \(b_t\) is the predicted trend value, the expression is:

\[
b_t = 0.64(Y_t - Y_{t-1}) + 0.36b_{t-1}
\]

- **IOWA combined prediction model of tunnel surrounding rock deformation**

Using the displacement prediction values of the above two models and the formula (2), the accuracy values of the single model in the IOWA combination prediction model of the tunnel surrounding rock deformation shown in Table 2 can be obtained.

Table 2. Precision value of ARIMA \((1, 1, 2)\) model and no-season exponential smoothing model

| Time  | ARIMA(1,1,2) model | Non-seasonal exponential smoothing | Time  | ARIMA(1,1,2) model | Non-seasonal exponential smoothing | Time  | ARIMA(1,1,2) model | Non-seasonal exponential smoothing |
|-------|--------------------|----------------------------------|-------|--------------------|----------------------------------|-------|--------------------|----------------------------------|
After sorting each tunnel prediction model according to the precision value in Table 2, Combined with formula (3), the predicted displacement error and the error information matrix \( E_{ij} = \{E_{11}, E_{12}, E_{22}\} \) of the predicted displacement of the IOWA combination prediction model of tunnel surrounding rock deformation are obtained and obtained by formula (4).

\[
S = 4.3285k_1^2 + 10.321k_1k_2 + 4.3221k_2^2
\]

(13)

\( K_1, K_2 \) in equation (13) can be calculated from equation (5): \( k_1 = 0.5019, \ k_2 = 0.4981 \).

According to the weight correction formulas (7) and (8), the modified weight value \( \hat{k}_1 \) and the non-seasonal exponential smoothing model \( \hat{k}_2 \), of the time series model ARIMA (1, 1, 2) can be obtained as shown in Table 3.

| Time       | ARIMA(1,1,2) model | Non-seasonal exponential smoothing model | Time       | ARIMA(1,1,2) model | Non-seasonal exponential smoothing model | Time       | ARIMA(1,1,2) model | Non-seasonal exponential smoothing model |
|------------|--------------------|-----------------------------------------|------------|--------------------|-----------------------------------------|------------|--------------------|-----------------------------------------|
| 2016/1/6   | 0.5132             | 0.4868                                  | 2016/1/13  | 0.6374             | 0.3626                                  | 2016/1/20  | 0.5275             | 0.4725                                  |
| 2016/1/7   | 0.9118             | 0.0882                                  | 2016/1/14  | 0.6017             | 0.3983                                  | 2016/1/21  | 0.5086             | 0.4914                                  |
| 2016/1/8   | 0.8938             | 0.1062                                  | 2016/1/15  | 0.5844             | 0.4156                                  | 2016/1/22  | 0.5657             | 0.4343                                  |
| 2016/1/9   | 0.6976             | 0.3024                                  | 2016/1/16  | 0.5665             | 0.4335                                  | 2016/1/23  | 0.5401             | 0.4599                                  |
| 2016/1/10  | 0.7467             | 0.2533                                  | 2016/1/17  | 0.5831             | 0.4169                                  | 2016/1/24  | 0.5897             | 0.4103                                  |
| 2016/1/11  | 0.6661             | 0.3339                                  | 2016/1/18  | 0.5449             | 0.4551                                  |            |                    |                                         |
| 2016/1/12  | 0.5580             | 0.4420                                  | 2016/1/19  | 0.5826             | 0.4174                                  |            |                    |                                         |

### 3.3 Analysis of Model result

In order to compare the accuracy of the IOWA combination prediction model of the tunnel surrounding rock deformation model established by the sample data, the time series prediction model ARIMA (1, 1, 2) and the displacement prediction without the non-seasonal exponential smoothing model, the deformation of the surrounding rock of the tunnels of the three models on 2016/1/25–2016/1/31 is predicted. The comparison curve between the predicted displacement and the measured displacement is shown in Fig. 1. The error analysis of the predicted displacement and the measured displacement is shown in Table 4.
Fig 1  Comparison of measured data and predicted values of vault displacement of three models

| Time       | Actual monitoring value /mm | ARIMA(1,1,3) model Predictive value /mm | error /mm  | Non-seasonal exponential smoothing model Predictive value /mm | error /mm  | IOWA combined forecasting model Predictive value /mm | error /mm  |
|------------|-----------------------------|----------------------------------------|------------|-------------------------------------------------------------|------------|------------------------------------------------------|------------|
| 2016/1/25  | 13.91                       | 13.9790                                | 0.0690     | 14.3447                                                     | 0.4347     | 14.1045                                              | 0.1945     |
| 2016/1/26  | 14.33                       | 14.2640                                | -0.0740    | 14.2165                                                     | -0.1135    | 14.2413                                              | -0.0887    |
| 2016/1/27  | 14.66                       | 14.7569                                | 0.0969     | 14.7091                                                     | 0.0491     | 14.7320                                              | 0.072      |
| 2016/1/28  | 15.16                       | 14.9750                                | -0.185     | 15.0883                                                     | -0.0717    | 15.0369                                              | -0.1231    |
| 2016/1/29  | 15.41                       | 15.4776                                | 0.0676     | 15.4674                                                     | 0.0574     | 15.4724                                              | 0.0624     |
| 2016/1/30  | 15.61                       | 15.5976                                | -0.0124    | 15.8466                                                     | 0.2366     | 15.7003                                              | 0.0903     |
| 2016/1/31  | 15.72                       | 15.9043                                | 0.1843     | 16.2257                                                     | 0.5057     | 16.0258                                              | 0.3058     |

It can be seen from Figure 1 that whether it is ARIMA (1,1,2) prediction model, non-seasonal exponential smoothing model, or IOWA combined prediction model for tunnel surrounding rock deformation both can more accurately describe the development trend of tunnel surrounding rock dome displacement. However, before the 29th, the ARIMA (1,1,2) prediction model has a large prediction error, After 29 days, the error of the non-seasonal exponential smoothing model is even larger. Corresponding to this, the prediction error of the IOWA combination prediction model for tunnel surrounding rock deformation has been small. And from the predicted displacement value and error in Table 4, the overall accuracy of IOWA combined prediction model of tunnel surrounding rock deformation is higher than the other two models, and the prediction of tunnel surrounding rock displacement is more stable; It can predict and forecast the deformation of surrounding rock more accurately.

4. Conclusion
Based on the induced ordered weighted average theory, an IOWA combined prediction model of tunnel surrounding rock deformation is established, and the model is applied to the modeling and prediction of surrounding rock monitoring data of the Milashan tunnel. The results show that the predicted value of the IOWA combined prediction model of tunnel surrounding rock deformation is closer to the measured value and the accuracy is higher. The IOWA combined prediction model of tunnel surrounding rock deformation proposed in this paper is feasible for tunnel surrounding rock deformation prediction, and provides a new method for tunnel surrounding rock deformation prediction and prediction.
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