Erratum: Incipient order in the $t$-$J$ model at high temperatures

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Previously, we reported calculations of the high-temperature series for thermodynamical susceptibilities towards a number of possible ordered states in the $t$-$J$-$V$ model. Due to an error in the calculation, the series for $d$-wave superconducting and extended $s$-wave superconducting orders were incorrect. We give the replacement figures. In agreement with our earlier findings, we still find no evidence of any strong enhancement of the superconducting susceptibility with decreasing temperature. However, because different Padé approximants diverge from each other at somewhat higher temperatures than we originally found, it is less clear what this implies concerning the presence or absence of high-temperature superconductivity in the $t$-$J$ model.

Previously, we reported calculations of the high-temperature series for thermodynamical susceptibilities towards a number of possible ordered states in the $t$-$J$-$V$ model. Due to an error in the calculation, the series for $d$-wave superconducting ($d$-SC) and extended $s$-wave superconducting ($s$-SC) orders were incorrect. The first terms of the correct series are

$$\chi_{s\text{-SC}}^{(1)} = \chi_{d\text{-SC}}^{(1)} = \frac{\beta}{8} \frac{(1 + x)(1 - 3x)}{\ln((1 - x)/2x)}.$$

Here, $\beta = 1/T$ is the inverse temperature, and the doping $x$ is the average number of holes per site. The SC susceptibilities at small $x \to 0$ are suppressed not quadratically but only logarithmically in $x$. Note also that the value $x = 1/3$ is not singular, as the zeros in the numerator and the denominator cancel.

We performed the corrected calculations of the series for thermodynamical susceptibilities with an extra order in powers of $\beta$ up to the term with $\beta^3$. The coefficients of the corrected series are available upon request. Figs. 1 and 3 represent the analyzed results for the corrected series and should replace Figs. 2 and 3 in Ref. 1.

For physically plausible parameters, $t > J$, the tendency of the superconducting susceptibility to decrease with decreasing temperature is not quite as prominent as in our original work. The increased variance between different Padé relatives to our former results indicate that the series can only be trusted down to temperatures $T \sim J$, whereas for $t = 2J$, the mean susceptibility reaches a maximum at $T/J$ ranging from $T/J \sim 1.2$ at $x = 6\%$ to $T/J \sim 1.3$ at $x = 26\%$. Fig. 3 shows that at $t = 3J$ the maximum is shifted up to temperatures $T \sim 2J$. We also note that the effect of the n.n. repulsion $V$ on the superconducting susceptibility is much less dramatic than we originally reported.

As a result, although the corrected series still suggest
the absence of high-temperature superconductivity in the $t$-$J$-$V$ model at $t \gtrsim 2J$, this conclusion cannot be drawn as strongly as in our original Letter [1].

We should also note the papers by Koretsune and Ogata [2] and by Putikka and Luchini [3] which appeared after Ref. 1. In these papers, the authors use a similar technique to study not the thermodynamical susceptibilities but the equal-time correlation functions for the $t$-$J$-$V$ model with the special value $V = J/4$. They computed the high-temperature series for both the uniform and real-space equal-time pairing correlation functions up to 12th order. These authors derive an instantaneous superconducting correlation length which increases with decreasing temperature, but which never grows as large as one lattice constant in the accessible range of $T/J$. Nevertheless, they interpreted their results as an indication that the $t$-$J$ model has a superconducting ground state. In our opinion, the failure of the thermodynamical susceptibility we have computed to show any strong tendency to grow with decreasing temperature in the accessible range, as well as the extremely short correlation lengths involved, call into question the strength of the conclusions drawn in Refs. 2, 3.

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Incipient order in the $t$-$J$ model at high temperatures

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We analyze the high-temperature behavior of the susceptibilities towards a number of possible ordered states in the $t$-$J$-$V$ model using the high-temperature series expansion. From all diagrams with up to ten edges, reliable results are obtained down to temperatures of order $J$, or (with some optimism) to $J/2$. In the unphysical regime, $t < J$, large superconducting susceptibilities are found, which moreover increase with decreasing temperatures, but for $t > J$, these susceptibilities are small and decreasing with decreasing temperature; this suggests that the $t$-$J$ model does not support high-temperature superconductivity. We also find modest evidence of a tendency toward nematic and $d$-density wave orders.

The discovery of high temperature (high-$T_c$) superconductivity in the cuprate perovskites launched a renewed effort to develop an understanding of the physics of highly correlated electronic systems. It is clearly significant that superconductivity arises in these materials upon doping a nearly ideal, spin 1/2 antiferromagnetic insulating “parent” state. Indeed, there is a prominent school of thought\[1\] that holds that high-activity arises in these materials upon doping a nearly ideal, spin electronic systems. It is clearly significant that superconductivity in the cuprate perovskites launched a renewed effort to develop an understanding of the physics of highly correlated two dimensional (2D) antiferromagnet; for this reason enormous effort has been focused on studies of the $t$-$J$ model [Eq. (1)], as the simplest model of a doped antiferromagnet. However, while the existence of antiferromagnetic order in the spin 1/2 Heisenberg model (the zero doping limit of the $t$-$J$ model) is well established\[3\], it remains uncertain whether the 2D $t$-$J$ model, by itself, supports high-$T_c$ superconductivity. In addition to antiferromagnetism and superconductivity, various other types of order\[4, 5, 6, 7\] have been or may have been observed in the cuprates, including nematic\[8\] (or spontaneous breaking of the point-group symmetry), charge-stripe\[9, 10\], spin-stripe\[11\] and $d$-density\[12\] wave (also called staggered flux or orbital antiferromagnetic) order.

It is thus interesting to determine which if any of these orders are generic features of a doped antiferromagnet.

In this paper we report the results of an extensive high-temperature series (HTS) study of the susceptibilities of the 2D $t$-$J$ model toward various short-period orders. (Long-period stripe order cannot be readily studied using these methods.) Naturally, the results obtained in this way are only reliable at moderately high temperatures. However, corresponding to any low-temperature broken symmetry state there must be a susceptibility which diverges at the ordering transition; unless the transition is strongly first order, this susceptibility will be large, and an increasing function of decreasing temperature even at temperatures well above any ordering transition.

Put another way, the HTS is sensitive to relatively short-distance physics (the range is determined by the order to which the series is computed). However, since the superconducting coherence length in the cuprates is thought to be around two lattice constants, it is reasonable to expect that the 10-12 terms we have computed in this series are sufficient to probe the physics of the model on length-scales relevant to superconductivity. It is important to stress that HTS\[8\] is free of finite size effects that plague other computational techniques.

Specifically, we have computed the HTS for antiferromagnetic (AF), $d$-wave superconducting ($d$-SC), extended $s$-wave superconducting ($s$-SC), nematic (N), and orbital antiferromagnetic ($d$-DW) susceptibilities, as defined in Eq. (2). To extend the temperature range over which the results can be trusted, we have made use of standard methods of resummation,\[9\] which we describe explicitly below. In outline, what we do is to construct a set of Padé approximants of related Euler-transformed series [see Eq. (3)] with different values of the parameter $\beta_0$, eliminate all “defective” members of this set, and then average over the remaining series. The variance in this average gives an estimate for associated errors; where the variance gets large we conclude that the results can no longer be trusted. It is possible that by biasing the series, using additional information about the low temperature state obtained from other methods, one might be able to extend the results to lower temperatures. However, without such additional information, it is our experience that using different prescriptions for resummation, or even adding a few additional terms to the series, does not significantly change the results or increase their range of validity.

There have been previous high quality series studies\[1, 2, 3\] of this same model which inspired the present work, but they primarily focused on extrapolating the results to $T = 0$. What distinguishes the present study from these earlier studies is (a) we have obtained susceptibilities that were not previously computed\[3, 4\] and (b) we have contented ourselves with studying the high temperature behavior of the model.

Our principal findings are as follows: 1. The results we obtain are reliable, without apology, for temperatures $T > J$, and are probably qualitatively correct down to $T \sim J/2$. However, we have not found any method of analyzing the series that we trust to any lower temperatures. 2. In the “unphysical” range of parameters, $t \leq J$, with $V = 0$, the strongest incipient order is AF—the AF susceptibility, $\chi_{AF}$, is large and shows a strong tendency to increase with decreasing temper-
nature, although this tendency gets gradually weaker with increasing doping \(x\) (Fig. 1). The SC susceptibilities are largest at values of doping \(x \lesssim 16\%\) (both \(d\)-SC shown in Fig. 2 and \(s\)-SC, not shown[15]); typically \(\chi_{d-SC} \geq \chi_{s-SC}\). However, the inclusion of an additional nearest-neighbor (n.n.) repulsion, \(V = J/4\), is sufficient to strongly suppress the pairing fluctuations. In the physical range of parameters, \(t > J\), where the bare interactions between electrons are truly repulsive, both \(\chi_{d-SC}\) (Fig. 3) and \(\chi_{s-SC}\) (not shown[15]) are small and decreasing with decreasing temperature already for \(T \lesssim 2J\); the pairing fluctuations are further suppressed by the addition of small n.n. repulsion. Based on these observations we conclude that the 2D t-J model with \(t > J\) probably does not support high temperature superconductivity. For \(t > J\), commensurate AF fluctuations are moderate for \(x = 1\%\) but are dramatically suppressed already at \(x \geq 6\%\) (Fig. 1). At larger \(x\), we find that the \(d\)-DW (Fig. 2) and the nematic (Fig. 3) susceptibilities are both moderate, show a weak tendency to increase with decreasing temperature, and remain virtually unaffected by the addition of a weak n.n. repulsion, \(V = J/4\).

We compute the high-temperature series for the 2D t-J model defined on the square lattice,

\[
H = -\sum_{\langle ij \rangle} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} + J \left( S_i \cdot S_j - \frac{1}{4} n_i n_j \right) + V n_i n_j, \tag{1}
\]

where \(t_{ij} = t\) is the hopping matrix element, \(c_{i\sigma}\) is the electron annihilation operator, \(S_i \equiv (1/2) c_{i\sigma}^\dagger \tau_{\sigma\sigma'} c_{i\sigma'}\) and \(n_i \equiv c_{i\sigma}^\dagger c_{i\sigma}\) are on-site spin and charge operators, and the doubly-occupied sites are projected out. The canonical \(t\)-\(J\) model[11] can be obtained from Eq. (1) by setting the n.n. repulsion \(V = 0\), while the version of the model used in previous high-temperature series studies[3, 11, 14, 15] can be obtained by setting \(V = J/4\). All results presented in this

![FIG. 1: Temperature dependence of the AF susceptibility for \(t = J/2\) (dashes, \(x = 1\%, 16\%, 31\%\)) and \(t = 2J\) (dot-dashed lines, \(x = 1\%, 6\%\)) from the HTS to \(1/T\) (diagrams with \(N_E \leq 10\)). Shading represents the standard deviation of non-defective Padé approximants as discussed in text.](image1)

![FIG. 2: Superconducting (\(d\)-wave) susceptibility \(\chi_{d-SC}\) for \(t = J/2\) with \(V = 0\) (dashes) and \(V = J/4\) (dot-dash) from the HTS to \(1/T\) (diagrams with \(N_E \leq 10\)). Shading as in Fig. 1.](image2)

![FIG. 3: Same as Fig. 2 for \(t = 2J\). Pairing fluctuations are weak already at \(V = 0\) (dashes), and decrease further with introduction of weak n.n. repulsion (dot-dash).](image3)

![FIG. 4: Staggered-flux susceptibility \(\chi_{d-DW}\) for \(t = 2J\), at \(V = 0\) (dashes) and \(V = J/4\) (dot-dash) obtained from HTS to \(1/T\) (diagrams with \(N_E \leq 10\)). Shading as in Fig. 1.](image4)
paper refer to these two values of $V$.

The susceptibilities (per unit site) $\chi$ can be expressed via the second derivatives of the free energy with respect to appropriately chosen perturbative parameters, or as the irreducible thermodynamic correlation functions of (properly projected) operators $O$,

$$\chi_O \equiv \frac{1}{N} T \beta \langle \langle e^{iH} O e^{-H} O^\dagger \rangle \rangle_{\beta}, \quad \beta \equiv 1/T. \quad (2)$$

We take the staggered magnetization $O_{AF} = \sum_x e^{iQ \cdot \sigma} S_x$, $Q \equiv (\pi, \pi)$, for AF ordering, the anisotropic part of the kinetic energy $O_N = \sum_x \left( c_{x+\pi,\sigma}^\dagger c_{x,\sigma} - c_{x,\sigma}^\dagger c_{x+\pi,\sigma} - h.c. \right)/2$ for nematic ordering, the staggered orbital currents $O_{d-DW} = i \sum_x e^{iQ \cdot \sigma} \left( c_{x+\pi,\sigma}^\dagger c_{x,\sigma}^\dagger - c_{x,\sigma}^\dagger c_{x+\pi,\sigma}^\dagger - h.c. \right)/2$ for $d$-DW ordering$[16]$, and the isotropic (anisotropic) part of the uniform pairing $O_{SC} = \sum_x \left( \Delta_x e^{i\pi \cdot \sigma} + \Delta_x e^{-i\pi \cdot \sigma} \right)/2$ for $s$-$SC$ (d-$SC$) ordering, where the pair annihilation operator $\Delta_{ij} \equiv c_{i\uparrow} c_{j\downarrow} + c_{i\downarrow} c_{j\uparrow}$. Respectively, the first coefficients of the corresponding series are $\chi_{AF}^{(1)} = \beta(1-x)/4$, $\chi_{N}^{(1)} = \chi_{d-DW}^{(1)} = \beta x(1-x)$, and $\chi_{s-SC}^{(1)} = \beta x^2$.

The definition of susceptibilities in terms of the derivatives of the free energy offers a convenient way for constructing the cluster expansion$[8]$. For each inequivalent lattice placement $C$ of a given diagram (connected graph) with $n_C$ edges, the relevant traces (grouped by the number of particles) are computed using block-diagonal matrices of the cluster Hamiltonian and the operator $O$. The traces are combined to produce the coefficients of the inverse-temperature expansion of the cluster susceptibility $\chi_{O}(\beta, y; C)$ and the thermodynamic potential $\Omega(\beta, y; C)$ dependent on the variable $y = z/(1+2z)$ related to fugacity $z = e^{\beta \mu}$. After the subcluster subtraction we obtain the irreducible weights of the cluster [whose expansion starts with $O(\beta^{n_C - 1})$ or higher power of $\beta$]. Then, combining the cluster weights for diagrams with up to $N_E$ edges, we generate the series exact in the thermodynamic limit to $\sim \beta^{N_E - 1}$ (for AF order since the operator $O_{AF}$ is defined on the vertices). As the last step, we perform the Legendre transformation to obtain the series for the free energy $F(x, \beta)$, and reexpress$[9]$ (b) the obtained series for $\chi$ in terms of the average hole density $x = 1 + (\partial \Omega/\partial \mu)_T$.

All obtained series for $\chi$, $\Omega$, and $F$ were carefully compared with series computed analytically to $\beta^3$, and also with series to $\beta^5$ obtained by a direct differentiation of the free energy expression generated with an independently-written Mathematica$[7]$ program. Among other consistency checks, we verified the cancellation of low-order terms in the irreducible weights. We have also compared the obtained free energy and related specific heat series with those for the $t$-$J$ model with $V = 1/4$ from Refs.$[9, 11]$ and also the specific heat series at $x = 0$ with the corresponding series for the Heisenberg model$[13]$.

A standard way of extrapolating a power series in $\beta$ is to construct a ratio of polynomials $p_n(\beta)/q_m(\beta)$ with matching expansion in powers of $\beta$, refered to as an $(n,m)$ Padé approximant. However, when only a few first terms of the series are known, and without a detailed knowledge of the structure of the singularities of the function, the procedure is plagued by spurious divergences. Indeed, the coefficients of the power series are only weakly modified if the numerator and the denominator of the fraction have close roots. The accuracy of the extrapolation would not be affected by cancelation of such factors, which amounts to using smaller $n$ and $m$. Furthermore, it is not generally clear whether a series in $\beta$ would give better extrapolation than a series in a related variable $\tilde{\beta} = f(\beta)$. As a result, it has become standard practice to average$[9]$ over a large number of “non-defective” Padé approximants generated for a family of functions $f(\beta, \beta_0)$ with different $\beta_0$, and use the corresponding dispersion to estimate the errors. A specific difficulty of extrapolating the series for susceptibilities [compared to non-singular quantities such as $n(k)$] is that interesting susceptibilities can actually diverge at a critical temperature. The peaks in $\chi$ develop at relatively large spatial scale, meaning that they become pronounced only at sufficiently high orders.

To generate the curves in Figs. 1–5, for each $\chi(\beta)$ at a given set of parameters we constructed a large number of Padé approximants (all $n, m$ with $n + m \geq (n + m)_{\text{max}} - 4$) for both the original series and a number of series in terms of the Euler-transformed variable$[9]$

$$\tilde{\beta} = \beta/(\beta_0 + \beta), \quad (3)$$

using $0.3 \leq \beta_0 \leq 10$. We then eliminated as “defective” the approximants with positive real roots in denominators and numerators [susceptibilities are nonnegative, see Eq. (2)], and we do not expect an actual phase transition with a divergent $\chi$ at such high temperatures, as well as the approximants with close numerator-denominator complex root pairs, and calculated the averages and the corresponding rms deviations for different temperatures $T = 1/\beta$.

As an independent method of analysis, we have also looked at the behavior of the untransformed and most closely bal-
anced (m close to n) Padé approximants with even m. When they are non-defective, these “best” approximants produce curves that look qualitatively similar to the average, and lie well within the shaded regions of the plots. We have also tried a number of variations on the averaging procedure, testing on curves that look qualitatively similar to the average, and lie well within the shaded regions of the plots. We have also tried prominent peak at low temperatures. In particular, we found which was popular in previous HTS studies[9, 11], strongly suppresses all peaks, effectively flattening the extrapolated functions for all susceptibilities we computed.

To summarize, our results indicate that, although in the “unphysical” region, t < J, the t-J model displays a sharp increase in the pairing fluctuations as the temperature goes down, this is no longer the case for t > J. Also, superconducting fluctuations are strongly suppressed by introduction of a small n.n. repulsion V. Thus we conclude that high-temperature superconductivity is probably not a generic feature[20] of doped AFs. Apart from χAF at very small doping, none of the studied susceptibilities remarkably strongly fluctuations in the “physical” range t > J as the temperature goes down to T ≈ J/2, but χN and χd-DW do show a modest enhancement.

In the future, we intend to continue studying the t-J-V model within the high-temperature series approach, for a wider range of orders in hope of identifying the most relevant fluctuations in the pseudogap region. We also intend to study the interaction of order parameters, by looking at various susceptibilities in modified models where an ordering (e.g., hopping anisotropy) is imposed at the Hamiltonian level. Within this general approach, we also plan to study a range of related models, in particular an array of coupled t-J ladders.

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