Open charm production in central Pb+Pb collisions at the CERN SPS: statistical model estimates

R. V. Poberezhnyuk, 1 M. Gazdzicki, 2, 3 and M. I. Gorenstein 1, 4

1 Bogolyubov Institute for Theoretical Physics, Kiev, Ukraine
2 Goethe–University, Frankfurt, Germany
3 Jan Kochanowski University, Kielce, Poland
4 Frankfurt Institute for Advanced Studies, Frankfurt, Germany

Abstract

Charm particle production in nucleus-nucleus collisions at the CERN SPS energies is considered within a statistical approach. Namely, the Statistical Model of the Early Stage is used to calculate mean multiplicity of charm particles in central Pb+Pb collisions. A small number of produced charm particles necessitates the use of the exact charm conservation law. The model predicts a rapid increase of mean charm multiplicity as a function of collision energy. The mean multiplicity calculated for central Pb+Pb collisions at the center of mass energy per nucleon pair $\sqrt{s_{NN}} = 17.3$ GeV exceeds significantly the experimental upper limit. Thus, in order to describe open charm production model parameters and/or assumptions should be revised.

PACS numbers: 15.75.Ag, 12.40.Ee

Keywords: Charm production, nucleus-nucleus collisions
Model predictions concerning mean multiplicity of $\bar{c}c$-pairs produced in central lead-lead collisions at the top CERN SPS energy, $E_{\text{lab}} = 158A$ GeV, differ significantly. Perturbative-QCD calculations for proton-proton (p+p) interactions were done in Ref. [1]. Extrapolation of these results to central Pb+Pb collisions led to estimate $\langle N_{c\bar{c}} \rangle \approx 0.17$ [2]. The hadron resonance gas model with chemical freeze-out temperature $T \approx 170$ MeV gives $\langle N_{c\bar{c}} \rangle = 0.3 \div 0.45$ [3]. The ALCOR hadronization model [4] predicted $\langle N_{c\bar{c}} \rangle \approx 3.6$. Even a larger yield, $\langle N_{c\bar{c}} \rangle \approx 8$, was expected in the Statistical Model of the Early Stage (SMES) [5] which assumes statistical creation of charm quarks in a quark gluon plasma (QGP). Thus, the model predictions vary almost by two orders of magnitude.

The model predictions for the system size dependence are also very different. In the perturbative-QCD inspired models, $\langle N_{c\bar{c}} \rangle$ is proportional to $N_p^{4/3}$, where $N_p$ is the number of nucleon participants in Pb+Pb collisions. The dependence $\langle N_{c\bar{c}} \rangle \sim N_p$ is expected in both, the SMES [5] and hadron-resonance gas model [3]. A behavior of $\langle N_{c\bar{c}} \rangle \sim N_p^{1.7}$ was suggested within the statistical coalescence model [3].

The NA49 Collaboration published [6] an upper limit of 2.4 for mean multiplicity of $D^0 + \bar{D}^0$ mesons produced in central Pb+Pb collisions at 158A GeV. This gives $\langle N_{c\bar{c}} \rangle < 3.6$ if one assumes that, like in p+p interactions, about one third of $\bar{c}$ and $c$ quarks hadronizes into $D^0$ and $\bar{D}^0$ mesons.

The aim of the present paper is to calculate collision energy dependence of open charm within the SMES and discuss its dependence on model parameters related to charm production. The SMES describes the transition between confined and deconfined phases of strongly interacting matter created in nucleus-nucleus collisions. The model has predicted several signals of the deconfinement phase transition [5][10], which were observed experimentally.

The SMES assumes that nucleons slow down and lose the fraction $\eta < 1$ of their initial energy in A+A central collisions. They fly away carrying their baryonic and electric charges. Therefore, the newly created matter with all conserved charges equal to zero is considered. This matter is assumed to be statistically produced in longitudinally contracted fireball with volume:

$$V = \frac{4\pi r_0^3 N_p/3}{\sqrt{s_{NN}/2m_N}},$$

(1)
where \( m_N = 939 \text{ MeV} \) is the nucleon mass, \( \sqrt{s_{NN}} \) is the center of mass energy of the nucleon pair, \( N_p \) is the number of participant nucleons from a single nucleus (\( N_p = 207 \) for central Pb+Pb collisions is assumed). The \( r_0 \) parameter is taken to be 1.30 fm in order to fit the mean baryon density in the nucleus, \( \rho_0 = 0.11 \text{ fm}^{-3} \). The energy used for particle creation (inelastic energy) is assumed to be:

\[
E = \eta (\sqrt{s_{NN}} - 2m_N) N_p ,
\]

where parameter \( \eta = 0.67 \) \cite{5}.

Since the system of newly created particles has all conserved charges equal to zero, the pressure \( p \) and energy density \( \varepsilon = E/V \) are assumed to be functions of temperature \( T \) only. These functions in the confined (W-phase) and deconfined (Q-phase) phases are equal to the (almost) ideal gas ones, where massless non-strange hadron and quark-gluon degrees of freedom have the degeneracy factors \( g_W = 16 \) and \( g_Q = 37 \). Strange constituents are considered as massive with \( m_{qs}^* \sim 200 \text{ MeV} \) and \( g_{qs}^* = 12 \) in the quark-gluon phase, and \( m_{W}^* = 500 \text{ MeV} \), \( g_{W}^* = 14 \) in the confined phase. For Q-phase the bag model equation of state is used \cite{11}:

\[
p_Q = p_{id}^Q - B \quad \text{and} \quad \varepsilon_Q = \varepsilon_Q + B .
\]

Thus, the bag constant is added to the ideal gas of quarks and gluons. It is chosen to fix the value of the phase transition temperature \( T_c = 200 \text{ MeV} \). Note that the lattice QCD data suggests the crossover transition temperature \( T_c = 150 - 170 \text{ MeV} \). However, in the present paper we keep the value \( T_c = 200 \text{ MeV} \), as used in the original SMES formulation \cite{5}. A revision of the SMES with self-consistent changes of all model parameters is outside of the scope of the present paper.

The entropy densities in the pure phases \( (i = W, Q) \) read:

\[
s_i(T) = \frac{p_i(T) + \varepsilon_i(T)}{T} .
\]

The energy and entropy densities in the mixed phase are \( (0 < \xi < 1) \):

\[
\varepsilon_{\text{mix}}(T_c) = \xi \varepsilon_Q(T_c) + (1 - \xi) \varepsilon_W(T_c) ,
\]

\[
s_{\text{mix}}(T_c) = \xi s_Q(T_c) + (1 - \xi) s_W(T_c) .
\]

The mixed phase starts at collision energy \( \sqrt{s_{NN,1}} \) and ends at \( \sqrt{s_{NN,2}} \):

\[
\sqrt{s_{NN,1}} = 7.42 \text{ GeV} , \quad \sqrt{s_{NN,2}} = 10.83 \text{ GeV} .
\]
We introduce now charm degrees of freedom assuming that mean multiplicity of charm carriers is small ($\lesssim 1$). This assumption has two consequences:

(i) one can neglect the contribution of charm degrees of freedom to energy density and pressure of the system (thus, the phase transition location remains unchanged);

(ii) one has to consider the canonical ensemble (CE) for charm particles that assures an equal number of charm and anti-charm charges in each microscopic state of the system.

The CE was used previously to calculate mean multiplicity of strange particles in p+p interaction [12]. Within the SMES, similarly to the strangeness case, the CE formulation for charm leads to a suppression of charm yield with respect to the grand canonical ensemble (GCE) yield by a factor equal to the ratio of the Bessel functions $I_1$ and $I_0$:

$$n_{\bar{c}}^{c(CE)}(T, V) = n_{\bar{c}}^{c}(T) \frac{I_1}{I_0} \left[ \frac{V n_{\bar{c}}^{c}(T)}{I_0} \right],$$

where the number density of the sum of charm and anti-charm particles in the GCE for pure phases can be calculated as

$$n_{\bar{c}}^{c}(T) = \frac{g_{\bar{c}}^{c} \pi}{2} \int_{0}^{\infty} dk k^2 \exp \left[ -\frac{\sqrt{k^2 + (m_{\bar{c}}^{c})^2}}{T} \right],$$

with $m_{\bar{c}}^{c} \approx 1.9$ GeV being a $D$-meson mass, $m_{\bar{c}}^{c} \approx 1.3$ GeV being a charm quark mass. The degeneracy factor for the (anti-)charm quarks is $g_{\bar{c}}^{c} = 12$, whereas the degeneracy factor for the (anti-)charm particles in the confined phase, $g_{\bar{c}}^{c}$, is a free parameter. In the mixed phase Eq. (7) should be replaced by

$$n_{\bar{c}}^{c(CE)}(T, V, \xi) = X \frac{I_1}{I_0} [X],$$

where

$$X = X(T, V, \xi) = \xi V n_{\bar{c}}^{c}(T) + (1 - \xi) V n_{\bar{c}}^{c}(T)$$

is the mean number of charm and anti-charm particles in the mixed phase calculated within the GCE. At each $\sqrt{s_{NN}}$ one calculates $V$ and $\epsilon = E/V$ according to Eqs. (1) and (2), and then mean multiplicity of $c\bar{c}$-pairs is calculated as

$$\langle N_{c\bar{c}} \rangle = \frac{1}{2} n_{c}^{c(CE)} V,$$
where \( n^{c(CE)} \) is given by Eq. (7) in the pure phases or by Eq. (9) in the mixed phase.

In the mixed phase, the temperature \( T \) and the parameter \( \xi \) are obtained by solving the equations:

\[
\xi = \frac{\varepsilon(\sqrt{s_{NN}})}{\varepsilon_Q(T_c)} - \frac{\varepsilon_W(T_c)}{\varepsilon_Q(T_c)}, \quad p_Q(T_c) = p_W(T_c).
\]  

(12)

\begin{align*}
\text{Figure 1:} \quad \text{Collision energy dependence of the charm to entropy ratio (a) and of the strangeness to entropy ratio (b) calculated within the SMES for central Pb+Pb collisions. Dashed lines denote the mixed phase region.}
\end{align*}

The charm to entropy ratio calculated for central Pb+Pb collisions for \( g_W = 10 \) is plotted in Fig. 1(a) as a function of collision energy. The strangeness to entropy ratio is plotted in Fig. 1(b) for a comparison. While the behavior of strangeness to entropy ratio is plotted in the horn structure \([5]\), the charm to entropy ratio is a monotonous function of collision energy.

Figure 2 shows a collision energy dependence of results on open charm for \( m_Q^c = 1.3 \) GeV and 1.5 GeV, and \( g_W^i = 10 \) and 50. As seen from Fig. 2, the horn structure is absent in the charm to entropy ratio for \( g_W^i = 10 \) and \( m_Q^c = 1.3 \) GeV, but appears for large (unphysical) values of \( g_W^i = 50 \) and \( m_Q^c = 1.5 \) GeV.

To estimate the proper value of parameter \( g_W^i \) let us consider charm and anti-charm in the white phase as a sum of all \( D \)-meson states,

\[
n^{c}_W (T) = \sum_i \frac{g_{W}^i}{2\pi^2} \int_{0}^{\infty} dk \, k^2 \exp \left[ - \frac{\sqrt{k^2 + (m^c_T)^2}}{T} \right].
\]

(13)
Figure 2: Collision energy dependence of the charm to entropy ratio (a, b) and a mean multiplicity of $c\bar{c}$-pairs (c, d) calculated within the SMES for central Pb+Pb collisions with the mass of the charm quark 1.3 GeV (left) and 1.5 GeV (right) and charm particle degeneracy factor $g^c_W = 10$ (lower lines) and $g^c_W = 50$ (upper lines).

In sum (13) we include 7 non-strange charm mesons from $D^0$ ($i = 1$) with $m^c_1 = 1.86$ GeV and $g^c_1 = 2$ and $D^\pm$ ($i = 2$) with $m^c_2 = 1.87$ GeV and $g^c_2 = 2$ up to $D^*_2$ and $D^*_2$ ($i = 6, 7$), both with $m^c_{6,7} = 2.46$ GeV and $g^c_{6,7} = 10$. In Fig. 3 the results of Eq. (13) for energy dependence of $\langle N_{c\bar{c}} \rangle$ in the CE are compared with those obtained from Eq. (8) for $n^c_W$ with mass $m^c_W = 1.9$ GeV and effective degeneracy factors $g^c_W = 10$ and $g^c_W = 50$. The results for the full spectrum of charm mesons are higher (by a factor 1.9 in the beginning of the mixed phase) than the results for $g^c_W = 10$, considered as lower limit, whereas they are significantly lower (by a factor 5.8 in the beginning of the mixed phase) than the results for $g^c_W = 50$. 

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Figure 3: Collision energy dependence of a mean multiplicity of \( c\bar{c} \)-pairs calculated within the SMES for central Pb+Pb collisions with the mass of the charm quark 1.3 GeV. Black and blue lines correspond to the assumption of \( D \)-meson mass \( m_{cW}^c = 1.9 \) GeV and effective degeneracy factors \( g_{W}^c = 10 \) and \( g_{W}^c = 50 \), respectively. Red line is obtained by accounting contributions from all non-strange \( D \)-mesons with corresponding masses and degeneracy factors \( \{m_{c_i}^c, g_{c_i}^c\} \).

In summary, collision energy dependence of mean number of \( c\bar{c} \)-pairs in central Pb+Pb collisions was calculated within the Statistical Model of the Early Stage. Mean charm multiplicity and its ratio to entropy exhibit a rapid growth as functions of collision energy in the considered energy region. In central Pb+Pb collisions at \( \sqrt{s_{NN}} = 17.3 \) GeV, \( \langle N_{c\bar{c}} \rangle \cong 20 \) for \( m_Q^c = 1.3 \) GeV and \( \langle N_{c\bar{c}} \rangle \cong 8 \) for \( m_Q^c = 1.5 \) GeV. These values are significantly larger than the experimental bound, \( \langle N_{c\bar{c}} \rangle \cong 3.6 \), reported in Ref. [6]. The SMES predictions are sensitive to assumed value of charm quark mass and charm degeneracy factor in the confined matter. But even for extreme values of these parameters the SMES predictions disagree with the experimental data. Thus, a quantitative description of charm production within SMES requires a revision of parameters and/or assumptions of the model.
Acknowledgments

The work of M.I.G. is supported by the Goal-Oriented Program of the National Academy of Science of Ukraine, by the European Organization for Nuclear Research (CERN), Grant CO-1-3-2016, and by the Program of Fundamental Research of the Department of Physics and Astronomy of the National Academy of Science of Ukraine.

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