Polychromatic Hilbert diagnostics of phase and temperature disturbances, induced by candle flame in air

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Abstract. The work is motivated by the scientific and practical significance of the problem of non-disturbing diagnostics of phase and temperature fields induced in a gas medium by a flame of a torch (candle). The spatial conditions in which the fields are studied satisfy the model of axial symmetry of the torch associated with the vertical orientation of the candle. A method adequate to the problem to be solved has been developed, based on polychromatic Hilbert visualization of phase optical density fields, measurement of the temperature profile in selected sections of the medium under study, registration and selection of RAW images recorded by the photomatrix in RGB channels. The visualized Hilbert structures contain information on the phase optical density perturbations induced by the temperature field. The reliability of the results is confirmed by comparing the experimentally obtained hilbertograms and those reconstructed from phase structures using the Abel transform.

1. Introduction

Optical methods are the most adequate for non-disturbing diagnostics of reacting jets and flames. They make it possible to obtain information on the thermodynamic and structural parameters of the medium under study without disturbing its state. The methods of Hilbert optics and interferometry are one of the directions of non-disturbing diagnostics of reacting jets and flames [1, 2]. The paper [3] describes a method for assessing the temperature distribution in a flame using high-contrast stereoscopic photography. Spectral reconstruction of temperature fields using pyrometry of color ratios and interferometric tomography [4] is reported; optical diagnostics based on Hilbert optics methods in combination with pixel-by-pixel processing of the dynamic structure of visualized phase fields is being developed [5]. However, tomographic diagnostics of the spatial optical phase structure of the flame remains an unsolved problem. The aim of this work is to develop Hilbert methods for the study of flames with the reconstruction of the spatial phase and temperature structure. The work is aimed at solving the scientific and practical problem of finding methods for controlling the structural and thermodynamic parameters of the torch.
2. Research method
The diagnostic complex is based on the IAB-451 device [6] with modified modules for optical filtration, light emitter, registration and processing of information. The lighting module consists of a light source (RGB LED with operating wavelengths $\lambda_1 = 626$ nm, $\lambda_2 = 525$ nm and $\lambda_3 = 461$ nm), a collimator lens and a slit diaphragm located in the front Fourier plane of the objective, which forms the sensing field. The Fourier spectrum of phase disturbances induced in the probing field by a candle flame is localized in the frequency plane of the Fourier lens, where the Hilbert quadrant filter is located. Depending on the spectral characteristics of the light source, the Hilbert filter converts the light field into an optical analytical signal or a Hilbert-conjugated signal. These signals are recorded by a digital video camera connected to the computer. [7, 8]

The Hilbert image obtained with simultaneous shooting at three different wavelengths $\lambda_1 = 626$ nm, $\lambda_2 = 525$ nm and $\lambda_3 = 461$ nm is shown in figure 1(a), chromatic frame selection by RGB channels is shown in figure 1–1(d).

3. Theoretical background
The phase structure of the probing light field, disturbed by the medium under study, is defined as

$$\Delta \psi(x, y) = k \int_{x_1}^{x_2} [n(x, y, z) - n_0] dz,$$

where $k = 2\pi/\lambda$ is the wave number of the probing field; $n(x, y, z)$ is the refractive index of the medium in the spatial structure of the flame; $n_0$ is the refractive index of the medium undisturbed by the flame. The axis $z$ is set by the direction of the probing light beam, the torch cross section is described in coordinates $x, z$. Coordinates $z_1, z_2$ limit the size of the flame section in the direction of the probe beam, and $y$ – its location relative to the end of the wick.

Formula (1) is transformed into the Abel equation for an axisymmetric flame structure

$$\Delta \psi(r, y) = 2k \int_x^R [n(r, y) - n_0] \frac{rdr}{\sqrt{r^2 - x^2}},$$

where $r^2 = x^2 + z^2$; $R$ is the radius of the section of the zone under consideration; $n(r, y)$ is the refractive index at a distance $r$ from the plume axis. The total phase shift $\Delta \psi(r, y)$ for a light beam in the cross section $y = const$ depends on the refractive index $n(r, y)$ in the segment $(z_1, z_2)$, (see figure 2).

The axisymmetric distributions of the refractive index obtained from the Abel equation make it possible to determine the radial temperature fields $T(r, y)$ in the torch cross section $y$ using the Gladstone-Dale equation [9]

$$T(r, y) = \left[ \frac{n_0 - 1}{n(r, y) - 1} \right] T_0,$$

where $T_0$ – temperature and $n_0$ – refractive index of the environment surrounding the flame.
4. Hilbertograms processing

Reconstruction using three RGB channels of the radial temperature distribution in the section at a distance of \( y = 26 \) mm from the end of the wick is shown in figure 3. The red line represents the phase function \( \Delta \psi(r, y) \) obtained from the Abel equation (2), the green and black lines represent the interferogram and the hilbertogram, respectively, reconstructed from the phase function \( \Delta \psi(r, y) \). Blue line – experimental hilbertogram in a given section. The radial temperature profile is reconstructed in the range from 0 to 25 mm. Experimental and reconstructed hilbertograms, interferograms and phase functions in the range from -25 to 25 mm.

The method consists in iterative sequential selection of the radial temperature profile, represented by the Bezier curve (a special case of B-splines), and the subsequent calculation of the refractive index \( n(r, y) \) and phase function \( \Delta \psi(r, y) \). Further, the hilbertogram (interferogram) is calculated from the phase function. The iterative algorithm is repeated until the points of the local minima of the Hilbert bands on the experimental and reconstructed hilbertograms coincide. The adequacy of the phase functions obtained from the solution of the Abel equation and the real ones is confirmed by the coincidence of the reconstructed temperature fields in the selected sections.

Figure 4(a) shows the reconstructed radial temperature distributions in the cross section of 26 mm for three RGB channels. Good matches are illustrated. Figure. 4(b) shows the distributions of the refractive index \( n(r, y) \) in the cross section of \( y = 26 \) mm, obtained from the solution of equation (3) for each of the three RGB channels. Figure 4(c) shows the corresponding distributions of the phase function \( \Delta \psi(r, y) \) in a cross section of \( y = 26 \) mm, obtained from the solution of the Abel equation (2). The maximum value of the phase function is achieved at a wavelength of \( \lambda_3 = 461 \) nm, the minimum is \( \lambda_4 = 626 \) nm, and at \( \lambda_2 = 525 \) nm there is an intermediate value. This corresponds to the number of Hilbert bands in the experimental and reconstructed hilbertograms: for the R-channel – 5, for the G-channel – 6, for the B-channel – 7 (see figure 3). Figure 5 shows the radial temperature field of an axisymmetric section of the air flow heated by a candle flame, obtained using piecewise-cubic Hermitian interpolation from the reconstructed temperature values in sections at a height of \( y = 26, 30, \) and 34 mm (sections are shown in figure 1(a)). Figures 6–7 illustrate the radial fields of the refractive index and phase function for the three RGB channels.
Figure 3. Reconstruction of the phase structure and temperature of the air flow from the candle flame in a cross section of \( y = 26 \) mm: (a) – R-channel \( \lambda_1 = 626 \) nm; (b) – G-channel \( \lambda_2 = 525 \) nm; (c) – B-channel \( \lambda_3 = 461 \) nm.
Figure 4. Section $y = 26$ mm ($r$ from -25 to 25 mm): (a) – radial temperature distribution for three RGB channels; (b) – refractive index for three RGB channels; (c) – phase function for three RGB channels.

Figure 5. The temperature field of the heated air flow from the candle flame, reconstructed from RGB Hilbert images ($r$ from -25 to 25 mm).

5. Verification
The verification of the results obtained is confirmed by the solution of the direct problem: from the reconstructed temperature field (figure 5), Hilbert images are reconstructed, which are compared to the images obtained in the experiment (figure 8). The structures obtained in the experiment and reconstructed have a similar character. This confirms the reliability of the results. Some discrepancy is due to the distortion of the axial symmetry of the flame in a real experiment due to the influence of dynamic disturbances of the air surrounding the flame.
Figure 6. Radial field of the refractive index (from -25 to 25 mm): (a) – R-channel $\lambda_1 = 626$ nm; (b) – G-channel $\lambda_2 = 525$ nm; (c) – B-channel $\lambda_3 = 461$ nm.

Figure 7. Phase function field ($r$ from -25 up to 25 mm): (a) – R-channel $\lambda_1 = 626$ nm; (b) – G-channel $\lambda_2 = 525$ nm; (c) – B-channel $\lambda_3 = 461$ nm.

6. Conclusion
The method of multiwave RGB optical Hilbert diagnostics of phase and temperature fields induced in air by a candle flame in the approximation of axial symmetry has been theoretically and experimentally substantiated. Verification of the results is provided by comparing the hilbertograms obtained in the experiment and reconstructed from phase structures using the Abel transform.
Figure 8. Comparison of experimental and simulated hilbertograms: (a) – R channel $\lambda_1 = 626$ nm; (b) – G-channel $\lambda_2 = 525$ nm; (c) – B channel $\lambda_3 = 461$ nm.

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