Weighing neutrinos in dynamical dark energy cosmology with the logarithm parametrization and the oscillating parametrization

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We revisit the constraint results of different dynamical dark energy models including the Chevallier-Polarski-Linder (CPL) model with \( w(z) = w_0 + w_1 \frac{1}{1+z} \) and the other two models with the logarithm parametrization of \( w(z) = w_0 + w_1 \ln \left( \frac{1+z}{2} \right) - \ln 2 \) and the oscillating parametrization of \( w(z) = w_0 + w_1 \left( \frac{\sin(1+z)}{1+z} - \sin(1) \right) \). The advantage over the CPL model is that the latter two parametrizations for dark energy can explore the whole evolution history of the universe properly. Using the current latest mainstream observations including the cosmic microwave background and the baryon acoustic oscillation as well as the type Ia supernovae, we perform the \( \chi^2 \) statistic analysis to global fit these models, finding that the logarithm parametrization and the oscillating parametrization are slightly preferred against the CPL scenario. We constrain the total neutrino mass in these dynamical dark energy models. We find that, compared with those in the CPL model, much looser constraints on \( \sum m_\nu \) are obtained in the logarithm model and the oscillating model. Consideration of the possible mass ordering of neutrinos reveals that the most stringent constraint on \( \sum m_\nu \) appears in the degenerate hierarchy case. In addition, we confirm that the normal hierarchy case is slightly favored over the inverted one.

I. Introduction

The fact that neutrinos have masses [1, 2] has drawn significant attention from physicists. The squared mass difference between different neutrino species have been measured, i.e., \( \Delta m^2_{21} \approx 7.5 \times 10^{-5} \text{ eV}^2 \) in solar and reactor experiments, and \( |\Delta m^2_{21}| \approx 2.5 \times 10^{-3} \text{ eV}^2 \) in atmospheric and accelerator beam experiments [2]. The possible mass hierarchies of neutrinos are derived to be \( m_1 < m_2 \ll m_3 \) and \( m_3 < m_1 < m_2 \), which are called the normal hierarchy (NH) and the inverted hierarchy (IH), respectively. When the mass splittings are neglected, we treat this case as the degenerate hierarchy (DH) with \( m_1 = m_2 = m_3 \).

Some famous particle physics experiments, such as tritium beta decay experiments [3–6] and neutrinoless double beta decay \( (0\nu\beta\beta) \) experiments [7, 8], have been designed to measure the absolute masses of neutrinos. However, there are no methods or probes that can directly measure the mass of a neutrino eigenstate. Cosmological observations are considered to be a more promising approach to measuring the total neutrino mass \( \sum m_\nu \). Massive neutrinos can leave rich imprints on the cosmic microwave background (CMB) anisotropies and the large-scale structure (LSS) formation in the evolution of the universe. Thus, the total neutrino mass \( \sum m_\nu \) is likely to be derived from these available cosmological observations.

In the standard \( \Lambda \) cold dark matter (\( \Lambda \)CDM) model with the equation-of-state parameter of dark energy \( w = -1 \), the Planck Collaboration gave \( \sum m_\nu < 0.26 \text{ eV} \) [9] from the full Planck TT, TE, EE power spectra data, assuming the NH case with the minimal mass \( \sum m_\nu = 0.06 \text{ eV} \). Adding the Planck CMB lensing data slightly tightens the constraints to \( \sum m_\nu < 0.24 \text{ eV} \). When the baryon acoustic oscillations (BAO) data are considered on the basis of the Planck data, the neutrino mass constraint is significantly tightened to \( \sum m_\nu < 0.12 \text{ eV} \). Further adding the type Ia supernovae (SNe) data marginally lowers the bound to \( \sum m_\nu < 0.11 \text{ eV} \), which put pressure on the inverted mass hierarchy with \( \sum m_\nu > 0.10 \text{ eV} \).

The impacts of dynamical dark energy on the total neutrino mass have been investigated in past studies [10–27]. In the simplest dynamical dark energy model with \( w = \text{Constant} \) (abbreviated as \( w\text{CDM} \)) model, the fitting results of \( \sum m_\nu \) are \( \sum m_\nu < 0.195 \text{ eV} \) (NH) and \( \sum m_\nu < 0.220 \text{ eV} \) (IH) [26], using the full Planck TT, TE, EE power spectra data and the BAO data as well as the SNe data. From the same data combination, \( \sum m_\nu < 0.129 \text{ eV} \) (NH) and \( \sum m_\nu < 0.163 \text{ eV} \) (IH) [26] in the holographic dark energy (HDE) model [28–35]. The constraints on \( \sum m_\nu \) are different from those in the standard \( \Lambda \)CDM model because of the different parametrization forms of \( w \) in these cosmological models.

In addition to the \( w\text{CDM} \) model and the HDE model, the constraints results of \( \sum m_\nu \) are investigated in the CPL model [36, 37] with \( w(z) = w_0 + w_1 \frac{1}{1+z} \) (where \( w_0 \) and \( w_1 \) are two free parameters). Over the years, the CPL parametrization has been widely used and explored extensively. In the model, \( \sum m_\nu < 0.247 \text{ eV} \) (NH) and \( \sum m_\nu < 0.290 \text{ eV} \) (IH) [26] are obtained by using the full Planck TT, TE, EE power spectra data combined the BAO data with the SNe data. The limit upper values of

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\( \sum m_\nu \) are larger than those in the wCDM model and the HDE model, confirming that the constraint results of \( \sum m_\nu \) can be changed as the different parametrization forms of \( w \). However, the CPL model has a drawback that it only explores the past expansion history, but cannot describe the future evolution, meaning that the CPL parametrization does not genuinely cover the scalar field models as well as other theoretical models. Such a problem makes the fitting results of \( \sum m_\nu \) untenable in the CPL model.

To overcome the shortcoming of the CPL model, Ref. [38] proposed two novel parametrization forms of \( w(z) \), i.e., the logarithm parametrization \( w(z) = w_0 + w_1 \left( \ln (2+z) - \ln 2 \right) \) and the oscillating parametrization \( w(z) = w_0 + w_1 \left( \sin(1+z) - \sin(1) \right) \), which are correspondingly called the Log model and the Sin model in this paper. They can inherit the advantages of the CPL model and explore the whole evolution history of the universe properly. More detailed analysis of the two models, see Ref. [38]. In our present work, the constraints on \( \sum m_\nu \) will be investigated in the two models. In fact, there are also some other well known dark energy parametrizations, such as the Jassal-Bagla-Padmanabhan (JBP) parametrization [39] and the Barboza-Alcaniz parametrization [40]. They will be explored with other research motivations in our next work.

On the other hand, in order to better match the current observational result of \( w = -1 \), we assume the case of \( w_0 = -1 \) in the CPL parametrization, the logarithm parametrization, and the oscillating parametrization. The forms of \( w \) in these models are modified as \( w(z) = -1 + w_1 \left( \ln (2+z) - \ln 2 \right) \), and \( w(z) = -1 + w_1 \left( \sin(1+z) - \sin(1) \right) \) with a free parameter \( w_1 \), in which the three dynamical dark energy models are called the MCPL model, the MLog model, and the MSin model. We also investigate the constraint results of them using the current mainstream observations.

In our present work, the three mass hierarchies of neutrinos (NH, IH, and DH) are also considered in the investigation of constraints on \( \sum m_\nu \). We have two main aims as follows: (1) Check the consistency of the dynamical dark energy models described above with current mainstream observational data; (2) Weigh neutrinos and diagnose neutrino mass ordering in these dynamical dark energy models. This paper has been structured in the following way. In Sect. II, we provide a brief description of the data and method used in our work. In Sect. III, we show the constraint results of different dynamical dark energy models and discuss the physical meaning behind these results. At last, we make some important conclusions in Sect. IV.

II. DATA AND METHOD

Throughout this paper, we only employ the data combination of the CMB data, the BAO data, and the SNe data, which is abbreviated as the CMB+BAO+SNe data. The usage of the data combination facilitates comparison with the results of Refs. [9, 12, 26], in which this typical data combination has also been used to constrain cosmological models. For the CMB data, we use the Planck 2018 temperature and polarization power spectra data at the whole multipole ranges, together with the CMB latest lensing power spectrum data [9]. For the BAO data, we use the 6dFGS and SDSS-MGS measurements of \( D_V/r_{\text{drag}} \) [41, 42] plus the final DR12 anisotropic BAO measurements [43]. For the SNe data, we use the “Pantheon” sample [44], which contains 1048 supernovae covering the redshift range of 0.01 < z < 2.3.

For the dynamical dark energy models with the CPL parametrization, the logarithm parametrization, and the oscillating parametrization, they all have eight free parameters, i.e., the present baryons density \( \omega_0 \equiv \Omega_0 h^2 \), the present cold dark matter density \( \omega_\cd \equiv \Omega_c h^2 \), the usage of the data combination facilitates comparison with the results of Refs. [46–48]. All constraint parameters, i.e., the present baryons density \( \omega_0 \equiv \Omega_0 h^2 \), the present cold dark matter density \( \omega_\cd \equiv \Omega_c h^2 \), an approximation to the angular diameter distance of the sound horizon at the decoupling epoch \( \theta_\text{DEC} \), the reionization optical depth \( \tau \), the amplitude of the primordial scalar power spectrum \( A_s \) at \( k = 0.05 \) Mpc\(^{-1} \), the primordial scalar spectral index \( n_s \), and the model parameters \( w_0 \) and \( w_1 \). For the MCPL model, the MLog model, and the MSin model, we fix the value of \( w_0 \) to −1, thus there are seven free parameters in the three models. The prior ranges of all these parameters are described explicitly in table 1 of Ref. [45].

When \( \sum m_\nu \) is included as a free parameter in the above models, we further consider the different mass hierarchies of neutrinos, i.e., the NH case, the IH case, and the DH case. The lower bounds of \( \sum m_\nu \) are \( \sum m_\nu \geq 0.06 \) eV for the NH case, \( \sum m_\nu \geq 0.10 \) eV for the IH case, and \( \sum m_\nu \geq 0 \) eV for the DH case. Correspondingly, the neutrino mass spectrum is described as

\[
(m_1, m_2, m_3) = (m_1, \sqrt{m_1^2 + \Delta m_{21}^2}, \sqrt{m_1^2 + |\Delta m_{31}^2|})
\]

with a free parameter \( m_1 \) for the NH case,

\[
(m_1, m_2, m_3) = (\sqrt{m_3^2 + |\Delta m_{31}^2|}, \sqrt{m_3^2 + |\Delta m_{31}^2| + \Delta m_{21}^2}, m_3)
\]

with a free parameter \( m_3 \) for the IH case, and

\[
m_1 = m_2 = m_3 = m
\]

with a free parameter \( m \) for the DH case.

In order to check the consistency between these dynamical dark energy models and the CMB+BAO+SNe data, we employ the \( \chi^2 \) statistic to do the cosmological fits. A model with a lower value of \( \chi^2 \) is more favored by the CMB+BAO+SNe data combination. Regarding the usage of the \( \chi^2 \) statistic, see Refs. [46–48]. All constraint
III. RESULTS AND DISCUSSIONS

We constrain the total mass of neutrinos \( \sum m_\nu \) in these dynamical dark energy models by using the CMB+BAO+SNe data. In what follows, we will present the fitting results with the \( \pm 1\sigma \) errors of cosmological parameters. But for the constraints on \( \sum m_\nu \), we only provide the \( 2\sigma \) upper limit. Meanwhile, we also list the values of \( \chi^2_{\text{min}} \) for different dark energy models.

A. Comparison of dynamical dark energy models

We constrain the models parameterized by \( w(z) = w_0 + w_1 \frac{z}{1+z} \), \( w(z) = w_0 + w_1 \left( \frac{\ln(1+z)}{1+z} - \ln 2 \right) \) and \( w(z) = w_0 + w_1 \left( \frac{\sin(1+z)}{1+z} - \sin(1) \right) \). The fitting results are listed in Tab. I, showing that the current CMB+BAO+SNe data favor the constraint results of \( w_0 = -1 \) and \( w_1 = 0 \) in the three models. For the CPL model, we obtain \( \Omega_m = 0.3059 \pm 0.0077 \) and \( H_0 = 68.37 \pm 0.83 \text{ km/s/Mpc} \), with \( \chi^2_{\text{min}} = 3821.214 \). For the Log model, we have \( \Omega_m = 0.3060 \pm 0.0075 \) and \( H_0 = 68.37 \pm 0.81 \text{ km/s/Mpc} \), with \( \chi^2_{\text{min}} = 3821.150 \). For the Sin model, we have \( \Omega_m = 0.3056 \pm 0.0077 \) and \( H_0 = 68.41 \pm 0.83 \text{ km/s/Mpc} \), with \( \chi^2_{\text{min}} = 3821.164 \). We find that the fitting results of \( \Omega_m \) and \( H_0 \) are similar in the three models. According to the \( \chi^2_{\text{min}} \), the Log model and the Sin model with smaller values of \( \chi^2_{\text{min}} \) are indicated to be more favored by the CMB+BAO+SNe data.

As described in Sect. I, when \( w_0 = -1 \) is fixed in the above models, the form of \( w(z) \) is modified with a free parameter \( w_1 \). The fitting results are also given in the last three columns of Tab. I for the MCPL model with \( w(z) = -1 + w_1 \frac{z}{1+z} \), the MLog model with \( w(z) = -1 + w_1 \left( \frac{\ln(1+z)}{1+z} - \ln 2 \right) \), and the MSin model with \( w(z) = -1 + w_1 \left( \frac{\sin(1+z)}{1+z} - \sin(1) \right) \). We obtain \( w_1 = -0.12^{+0.13}_{-0.11} \), \( w_1 = 0.52^{+0.39}_{-0.48} \), and \( w_1 = 0.22^{+0.16}_{-0.21} \), showing a slight deviation to \( w_1 = 0 \) in the MLog model and the MSin model. This is because \( w_1 \) is intrinsically correlated with \( w_0 \), as shown in Fig. 1 \( (w_1 \) is anticorrelated with \( w_0 \) in the CPL model, but the correlation between them is opposite in the Log model and the Sin model). When the value of \( w_0 \) is fixed to \(-1\), the fitting value of \( w_1 \) will be changed to a certain extent.

Furthermore, we focus on the \( \chi^2_{\text{min}} \) values of the three models with only a free parameter \( w_0 \) for dark energy. We obtain \( \chi^2_{\text{min}} = 3821.310 \) in the MCPL model, \( \chi^2_{\text{min}} = 3821.288 \) in the MLog model, and \( \chi^2_{\text{min}} = 3821.290 \) in the MSin model. Among the three models, the MLog model with a slightly smaller \( \chi^2_{\text{min}} \) value is most favored by the CMB+BAO+SNe data. In Fig. 1 and 2, we also provide the one-dimensional marginalized distributions and two-dimensional contours at \( 1\sigma \) and \( 2\sigma \) level for these dynamical dark energy models. The fitting results of the parameter \( \Omega_m, H_0, \) and \( \sigma_8 \) hardly change in these models with different \( w(z) \) parametrizations.

B. Constraint on neutrino masses

Through the analysis of the consistency of these dynamical dark energy models with current mainstream observational data, we confirm that the logarithm parametrization and the oscillating parametrization for dynamical dark energy have been underestimated in the past studies. We investigate the constraints on total neutrino mass in these models in our present work. For the neutrino mass measurement, we consider the NH case, the IH case, and the DH case. The fitting results are listed in Tabs. II– IV.

In the CPL+\( \sum m_\nu \) model, we obtain \( \sum m_\nu < 0.285 \text{ eV} \) for the NH case, \( \sum m_\nu < 0.304 \text{ eV} \) for the IH case, and \( \sum m_\nu < 0.254 \text{ eV} \) for the DH case (see Tab. II). In the Log+\( \sum m_\nu \) model, we have \( \sum m_\nu < 0.302 \text{ eV} \) for the NH case, \( \sum m_\nu < 0.317 \text{ eV} \) for the IH case, and \( \sum m_\nu < 0.282 \text{ eV} \) for the DH case (see Tab. III), showing that much looser constraints are obtained than those in the CPL+\( \sum m_\nu \) model. In the Sin+\( \sum m_\nu \) model, the constraint results become \( \sum m_\nu < 0.327 \text{ eV} \) for the NH case, \( \sum m_\nu < 0.336 \text{ eV} \) for the IH case, and \( \sum m_\nu < 0.311 \text{ eV} \) for the DH case (see Tab. IV), which are looser than those in the Log+\( \sum m_\nu \) model. All the above fitting upper value of \( \sum m_\nu \) are larger than those obtained in the standard LCDM model (in the LCDM model, the constraint results are \( \sum m_\nu < 0.156 \text{ eV} \) for the NH case, \( \sum m_\nu < 0.184 \text{ eV} \) for the IH case, and \( \sum m_\nu < 0.121 \text{ eV} \) for the DH case [26, 51]), indicating that the dynamical dark energy with the logarithm form and the oscillating form can increase the fitting value of \( \sum m_\nu \).

Considering the same neutrino mass ordering, the fitting value of \( \sum m_\nu \) is smallest in the CPL model and largest in the Sin model, confirming that the fitting values of \( \sum m_\nu \) can be changed by modifying the \( w(z) \) forms. In Fig. 3, we provide two-dimensional marginalized contours (68.3% and 95.4% confidence level) in the \( m_\nu-w_0 \) plane of the CPL, Log, and Sin models, considered mass hierarchy cases of NH, IH, and DH. We see that the positive correlations between \( m_\nu \) and \( w_0 \) are shown in the three two-parametrization models. When we compare the constraint results of \( \sum m_\nu \) in the three different cases of neutrino mass orderings, we find that the smallest value of \( \sum m_\nu \) is obtained in the DH case, and the largest value of \( \sum m_\nu \) corresponds to the IH case, which mean that considering the mass hierarchy can affect the fitting values of \( \sum m_\nu \).

In the CPL+\( \sum m_\nu \) model, we obtain \( \chi^2_{\text{min}} = 3822.102 \) for the NH case, \( \chi^2_{\text{min}} = 3822.516 \) for the IH case, and \( \chi^2_{\text{min}} = 3821.168 \) for the DH case (see Tab. II).
TABLE I: The fitting values for the six dynamical dark energy models

| Parameter | CPL    | Log    | Sin    | MCPL   | MLog   | MSin   |
|-----------|--------|--------|--------|--------|--------|--------|
| $w_0$     | $-0.968 \pm 0.079$ | $-0.968^{+0.065}_{-0.072}$ | $-0.973^{+0.059}_{-0.058}$ | $-1$   | $-1$   | $-1$   |
| $w_1$     | $-0.24^{+0.33}_{-0.27}$ | $0.93^{+0.79}_{-1.11}$ | $0.36^{+0.28}_{-0.40}$ | $-0.12^{+0.13}_{-0.11}$ | $0.52^{+0.39}_{-0.48}$ | $0.22^{+0.16}_{-0.21}$ |
| $\Omega_m$ | $0.3059 \pm 0.0077$ | $0.3060 \pm 0.0075$ | $0.3056 \pm 0.0077$ | $0.3048^{+0.0076}_{-0.0071}$ | $0.3045^{+0.0069}_{-0.0068}$ | $0.3044 \pm 0.0068$ |
| $H_0$ [km/s/Mpc] | $68.37 \pm 0.83$ | $68.37 \pm 0.81$ | $68.41 \pm 0.83$ | $68.47 \pm 0.76$ | $68.53 \pm 0.73$ | $68.55 \pm 0.73$ |
| $\sigma_8$ | $0.822 \pm 0.011$ | $0.822 \pm 0.011$ | $0.823 \pm 0.011$ | $0.822 \pm 0.011$ | $0.823 \pm 0.011$ | $0.824 \pm 0.011$ |
| $\chi^2_{\text{min}}$ | $3821.214$ | $3821.150$ | $3821.164$ | $3821.310$ | $3821.288$ | $3821.290$ |

**FIG. 1:** One-dimensional marginalized distributions and two-dimensional contours at $1\sigma$ and $2\sigma$ level for the CPL, Log, and Sin models.

In the Log+$\sum m_\nu$ model, we have $\chi^2_{\text{min}} = 3822.100$ for the NH case, $\chi^2_{\text{min}} = 3822.180$ eV for the IH case, and $\chi^2_{\text{min}} = 3821.048$ for the DH case (see Tab. III). In the Sin+$\sum m_\nu$ model, the constraint results become $\chi^2_{\text{min}} = 3822.408$ for the NH case, $\chi^2_{\text{min}} = 3823.456$ for the IH case, and $\chi^2_{\text{min}} = 3821.080$ for the DH case (see Tab. IV). Obviously, for the three parametrizations, the values of $\chi^2_{\text{min}}$ in the NH case are slightly smaller than those in the IH case. That indicates that the NH case is favored over the IH case, but the weak the difference $\Delta \chi^2_{\text{min}} < 2$ between them does not seem to be significant enough to distinguish between the mass hierarchies. When the mass splittings are neglected, the values of $\chi^2_{\text{min}}$ are smallest among the three cases of neutrino mass orderings.

We also discuss the constraints of $\sum m_\nu$ in the MCPL.
model, the MLog model, and the MSin model, in which $w(z)$ is parameterized with a single free parameter $w_1$. In the MCPL+$\sum m_\nu$ model, we obtain $\sum m_\nu < 0.250$ eV for the NH case, $\sum m_\nu < 0.276$ eV for the IH case, and $\sum m_\nu < 0.228$ eV for the DH case (see Tab. II).

In the MLog+$\sum m_\nu$ model, we have $\sum m_\nu < 0.268$ eV for the NH case, $\sum m_\nu < 0.288$ eV for the IH case, and $\sum m_\nu < 0.250$ eV for the DH case (see Tab. III). In the MSin+$\sum m_\nu$ model, the constraint results become $\sum m_\nu < 0.298$ eV for the NH case, $\sum m_\nu < 0.318$ eV

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**FIG. 2:** One-dimensional marginalized distributions and two-dimensional contours at 1$\sigma$ and 2$\sigma$ level for the MCPL, MLog, and MSin models.

**TABLE II:** The fitting values for the CPL+$\sum m_\nu$ and MCPL+$\sum m_\nu$ models considered mass hierarchy cases of NH, IH, and DH.

| Parameter | CPL | MCPL |
|-----------|-----|------|
|           | NH  | IH  | DH  | NH  | IH  | DH  |
| $w_0$     | $-0.940 \pm 0.095$ | $-0.929 \pm 0.083$ | $-0.950 \pm 0.082$ | 1   | 1   | 1   |
| $w_1$     | $-0.49 \pm 0.33$ | $-0.50 \pm 0.48$ | $-0.39 \pm 0.47$ | $-0.24 \pm 0.18$ | $-0.30 \pm 0.18$ | $-0.17 \pm 0.19$ |
| $\sum m_\nu$ [eV] | $< 0.285$ | $< 0.304$ | $< 0.254$ | $< 0.250$ | $< 0.276$ | $< 0.228$ |
| $\Omega_m$ | $0.3094 \pm 0.0081$ | $0.3103 \pm 0.0081$ | $0.3074 \pm 0.0083$ | $0.3069 \pm 0.0073$ | $0.3078 \pm 0.0072$ | $0.3058 \pm 0.0072$ |
| $H_0$ [km/s/Mpc] | $68.27 \pm 0.82$ | $68.27 \pm 0.83$ | $68.32 \pm 0.83$ | $68.47 \pm 0.76$ | $68.49 \pm 0.75$ | $68.45 \pm 0.77$ |
| $S_8$ | $0.825 \pm 0.012$ | $0.823 \pm 0.012$ | $0.827 \pm 0.012$ | $0.824 \pm 0.011$ | $0.822 \pm 0.011$ | $0.826 \pm 0.012$ |
| $\chi^2_{\text{min}}$ | 3822.102 | 3822.516 | 3821.168 | 3822.144 | 3823.046 | 3821.112 |
FIG. 3: Two-dimensional marginalized contours (68.3% and 95.4% confidence level) in the $\Sigma m_{\nu}-w_0$ plane of the CPL, Log, and Sin models considered mass hierarchy cases of NH, IH, and DH.

for the IH case, and $\sum m_{\nu} < 0.277$ eV for the DH case (see Tab. IV). Not surprisingly, the constraint results of $\sum m_{\nu}$ are largest in the MSin model and smallest in the MCPL model.

Furthermore, comparing constraint results of $\sum m_{\nu}$ with those derived from the above two-parametrization models, we find that the values of $\sum m_{\nu}$ are smaller in these one-parametrization models, indicating that a model with less parameters tends to provide a smaller fitting value of $\sum m_{\nu}$. The two-dimensional marginalized contours in the $\sum m_{\nu}-w_1$ plane are shown in Fig. 4, we see that the correlations between $\sum m_{\nu}$ and $w_1$ are

TABLE III: The fitting values for the Log+$\sum m_{\nu}$ and MLog+$\sum m_{\nu}$ models considered mass hierarchy cases of NH, IH, and DH.

| Parameter | Log | MLog |
|-----------|-----|------|
| $w_0$     | $0.946^{+0.071}_{-0.080}$ | $0.955^{+0.069}_{-0.079}$ | $-1$ |
| $w_1$     | $1.96^{+1.00}_{-1.70}$ | $2.20^{+1.10}_{-1.70}$ | $1.59^{+0.95}_{-1.63}$ |
| $\sum m_{\nu}$ [eV] | $<0.302$ | $<0.317$ | $<0.282$ |
| $\Omega_m$ | $0.3094^{+0.0082}_{-0.0082}$ | $0.3080^{+0.0081}_{-0.0089}$ | $0.3066^{+0.0072}_{-0.0072}$ |
| $H_0$ [km/s/Mpc] | $68.31^{+0.83}_{-0.82}$ | $68.33^{+0.82}_{-0.81}$ | $68.54^{+0.74}_{-0.74}$ |
| $S_8$     | $0.825^{+0.012}_{-0.012}$ | $0.823^{+0.012}_{-0.012}$ | $0.824^{+0.011}_{-0.011}$ |
| $\chi^2_{min}$ | $3821.100$ | $3821.180$ | $3821.048$ |

TABLE IV: The fitting values for the Sin+$\sum m_{\nu}$ and MSin+$\sum m_{\nu}$ models considered mass hierarchy cases of NH, IH, and DH.

| Parameter | Sin | MSin |
|-----------|-----|------|
| $w_0$     | $-0.956^{+0.063}_{-0.070}$ | $-0.952^{+0.065}_{-0.066}$ | $-0.962^{+0.063}_{-0.069}$ |
| $w_1$     | $0.80^{+0.37}_{-0.70}$ | $0.91^{+0.41}_{-0.69}$ | $0.66^{+0.34}_{-0.69}$ |
| $\sum m_{\nu}$ [eV] | $<0.327$ | $<0.336$ | $<0.311$ |
| $\Omega_m$ | $0.3097^{+0.0083}_{-0.0090}$ | $0.3106^{+0.0082}_{-0.0083}$ | $0.3081^{+0.0084}_{-0.0085}$ |
| $H_0$ [km/s/Mpc] | $68.33^{+0.83}_{-0.84}$ | $68.32^{+0.84}_{-0.83}$ | $68.37^{+0.72}_{-0.73}$ |
| $S_8$     | $0.825^{+0.012}_{-0.012}$ | $0.823^{+0.012}_{-0.012}$ | $0.826^{+0.012}_{-0.012}$ |
| $\chi^2_{min}$ | $3821.408$ | $3821.456$ | $3821.080$ |

for the IH case, and $\sum m_{\nu} < 0.277$ eV for the DH case (see Tab. IV). Not surprisingly, the constraint results of $\sum m_{\nu}$ are largest in the MSin model and smallest in the MCPL model.

Furthermore, comparing constraint results of $\sum m_{\nu}$ with those derived from the above two-parametrization models, we find that the values of $\sum m_{\nu}$ are smaller in these one-parametrization models, indicating that a model with less parameters tends to provide a smaller fitting value of $\sum m_{\nu}$. The two-dimensional marginalized contours in the $\sum m_{\nu}-w_1$ plane are shown in Fig. 4, we see that the correlations between $\sum m_{\nu}$ and $w_1$ are
different in the three models because of them with the different \( w(z) \) parametrizations.

\section*{IV. CONCLUSION}

In this paper, we focus on some dynamical dark energy parameterized by two free parameters \( w_0 \) and \( w_1 \). They correspond to the CPL parametrization, the logarithm parametrization, and the oscillating parametrization, respectively. The difference from the CPL model is that the logarithm parametrization and the oscillating parametrization can overcome the future divergency problem, and successfully probe the dynamics of dark energy in all the evolution stages of the universe. We constrain these dynamical dark energy models by using current cosmological observations including the CMB data, the BAO data, and the SNe data. We find that the Log model and the Sin model slightly behave better than the CPL model in terms of consistency comparison with the CMB+BAO+SNe data.

The another important thing in our present work is to investigate the constraints on the total neutrino mass \( \sum m_\nu \) in these dynamical dark energy. Meanwhile, we consider the NH case, the IH case, and the DH case of three-generation neutrino mass. We confirm the fact that the different neutrino mass hierarchies can affect the constraint results of \( \sum m_\nu \) significantly. The smallest fitting value of \( \sum m_\nu \) is obtained in the DH case, and the largest value of \( \sum m_\nu \) corresponds to the IH case. We find that the value of \( \sum m_\nu \) in the Log and Sin parametrizations are larger than those in the CPL parametrizations. This is because an early phantom dark energy from \( w(z) < -1 \) to \( w(z) > -1 \) is slightly more favored in the two unconventional models. In addition, the result of the NH case more favored than the IH case in these dark energy models is consistent with the previous studies [23–26, 51, 52].

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[1] J. Lesgourgues and S. Pastor, Phys. Rept. 429, 307 (2006). doi:10.1016/j.physrep.2006.04.001 [astro-ph/0603494].
[2] K. A. Olive et al. [Particle Data Group], Chin. Phys. C 38, 090001 (2014). doi:10.1088/1674-1137/38/9/090001.
[3] A. Osipowicz et al. [KATRIN], arXiv:hep-ex/0109033 [hep-ex].
[4] C. Kraus et al., Eur. Phys. J. C 40, 447 (2005). doi:10.1140/epjc/s2005-02139-7 [hep-ex/0412056].
[5] E. W. Otten and C. Weinheimer, Rept. Prog. Phys. 71, 086201 (2008). doi:10.1088/0034-4885/71/8/086201 [arXiv:0909.2104 [hep-ex]].
[6] J. Wolf [KATRIN Collaboration], Nucl. Instrum. Meth. A 623, 442 (2010). doi:10.1016/j.nima.2010.03.030 [arXiv:0810.3281 [physics.ins-det]].
[7] H. V. Klapdor-Kleingrothaus and U. Sarkar, Mod. Phys. Lett. A 16, 2469 (2001). doi:10.1142/S0217732301005850 [hep-ph/0201224].
[8] H. V. Klapdor-Kleingrothaus, I. V. Krivosheina, A. Di etz and O. Chkvorets, Phys. Lett. B 586, 198 (2004). doi:10.1016/j.physletb.2004.02.025 [hep-ph/0404088].
[9] N. Aghanim et al. [Planck], Astron. Astrophys. 641, A6 (2020). doi:10.1051/0004-6361/201833910 [arXiv:1807.06209 [astro-ph.CO]].
[10] M. M. Zhao, Y. H. Li, J. F. Zhang and X. Zhang, Mon. Not. Roy. Astron. Soc. 469, no. 2, 1713 (2017). doi:10.1093/mnras/stx978 [arXiv:1608.01219 [astro-ph.CO]].
[11] X. Zhang, Phys. Rev. D 93, no. 8, 083011 (2016). doi:10.1103/PhysRevD.93.083011 [arXiv:1511.02651 [astro-ph.CO]].
[12] S. Roy Choudhury and S. Hannestad, JCAP 07, 037 (2020). doi:10.1088/1475-7516/2020/07/037 [arXiv:1907.12598 [astro-ph.CO]].
[13] H. Li and X. Zhang, Phys. Lett. B 713, 160 (2012). doi:10.1016/j.physletb.2012.06.030 [arXiv:1202.4071 [astro-ph.CO]].
[14] Y. H. Li, S. Wang, X. D. Li and X. Zhang, JCAP
S. Wang, J. J. Geng, Y. L. Hu and X. Zhang, Sci. China Phys. Mech. Astron. 58, 1 (2015). doi:10.1007/s11433-014-5628-5 [arXiv:1312.0184 [astro-ph.CO]].

J. Cui, Y. Xu, J. Zhang and X. Zhang, Sci. China Phys. Mech. Astron. 58, 110402 (2015). doi:10.1007/s11433-015-5734-z [arXiv:1511.06956 [astro-ph.CO]].

D. Z. He, J. F. Zhang and X. Zhang, Sci. China Phys. Mech. Astron. 60, no.3, 039511 (2017). doi:10.1007/s11433-016-4047-1 [arXiv:1607.05643 [astro-ph.CO]].

Y. Y. Xu and X. Zhang, Eur. Phys. J. C 76, no.11, 588 (2016). doi:10.1140/epjc/s10052-016-4446-5 [arXiv:1607.06262 [astro-ph.CO]].

M. Chevallier and D. Polarski, Int. J. Mod. Phys. D 10, 213-224 (2001). doi:10.1142/S0218271801000822 [arXiv:gr-qc/0009008 [gr-qc]].

E. V. Linder, Phys. Rev. Lett. 90, 091301 (2003). doi:10.1103/PhysRevLett.90.091301 [arXiv:astro-ph/0208512 [astro-ph]].

J. Z. Ma and X. Zhang, Phys. Lett. B 699, 233-238 (2011). doi:10.1016/j.physletb.2011.04.013 [arXiv:1102.2671 [astro-ph.CO]].

H. K. Jassal, J. S. Bagla and T. Padmanabhan, Phys. Rev. D 72, 103503 (2005). doi:10.1103/PhysRevD.72.103503 [arXiv:astro-ph/0506748 [astro-ph]].

E. M. Barbosa, Jr. and J. S. Alcaniz, Phys. Lett. B 666, 415-419 (2008). doi:10.1016/j.physletb.2008.08.012 [arXiv:0805.1713 [astro-ph]].

F. Beutler et al., Mon. Not. Roy. Astron. Soc. 416, 3017 (2011). [arXiv:1106.3366 [astro-ph.CO]].

A. J. Ross, L. Samushia, C. Hewlett, W. J. Percival, A. Burden and M. M. Manera, Mon. Not. Roy. Astron. Soc. 449, no. 1, 835 (2015). doi:10.1093/mnras/stv154 [arXiv:1409.3242 [astro-ph.CO]].

S. Alam et al. [BOSS Collaboration], Mon. Not. Roy. Astron. Soc. 470, no. 3, 2617 (2017). doi:10.1093/mnras/stx721 [arXiv:1607.03155 [astro-ph.CO]].

D. M. Scolnic et al., Astrophys. J. 859, no. 2, 101 (2018). doi:10.3847/1538-4357/aab9bb [arXiv:1710.00845 [astro-ph.CO]].

P. A. R. Ade et al. [Planck], Astron. Astrophys. 571, A16 (2014). doi:10.1051/0004-6361/201321591 [arXiv:1303.5076 [astro-ph.CO]].

R. Y. Guo, L. Feng, T. Y. Yao and X. Y. Chen, JCAP 12, no.12, 036 (2021). doi:10.1088/1475-7516/2021/12/036 [arXiv:2110.02536 [gr-qc]].

L. Feng, R. Y. Guo, J. F. Zhang and X. Zhang, Phys. Lett. B 827, 136940 (2022). doi:10.1016/j.physletb.2022.136940 [arXiv:2109.06111 [astro-ph.CO]].

R. Y. Guo, J. F. Zhang and X. Zhang, JCAP 02, 054 (2019). doi:10.1088/1475-7516/2019/02/054 [arXiv:1809.02340 [astro-ph.CO]].

A. Lewis, A. Challinor and A. Lasenby, Astrophys. J. 538, 473-476 (2000). doi:10.1086/309179 [arXiv:astro-ph/0002117 [astro-ph]].

A. Lewis, Phys. Rev. D 87, no.10, 103529 (2013). doi:10.1103/PhysRevD.87.103529 [arXiv:1304.4473 [astro-ph.CO]].

S. J. Jin, R. Q. Zhu, L. F. Wang, H. L. Li, J. F. Zhang and X. Zhang, Commun. Theor. Phys. 74, no.10, 105404 (2022). doi:10.1088/1572-9494/ac7b76 [arXiv:2204.04689 [astro-ph.CO]].
[52] R. Y. Guo, J. F. Zhang and X. Zhang, Chin. Phys. C 42, no.9, 095103 (2018). doi:10.1088/1674-1137/42/9/095103 [arXiv:1803.06910 [astro-ph.CO]].