Fast domain wall propagation in uniaxial nanowires with transverse fields

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Under a magnetic field along its axis, domain wall motion in a uniaxial nanowire is much slower than in the fully anisotropic case, typically by several orders of magnitude (the square of the dimensionless Gilbert damping parameter). However, with the addition of a magnetic field transverse to the wire, this behaviour is dramatically reversed; up to a critical field strength, analogous to the Walker breakdown field, domain walls in a uniaxial wire propagate faster than in a fully anisotropic wire (without transverse field). Beyond this critical field strength, precessional motion sets in, and the mean velocity decreases. Our results are based on leading-order analytic calculations of the velocity and critical field as well as numerical solutions of the Landau-Lifshitz-Gilbert equation.

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Introduction

The dynamics of magnetic domain walls in ferromagnetic nanowires under external magnetic fields \cite{1} and spin-polarised currents \cite{12,20} is a central problem in micromagnetics and spintronics, both as a basic physical phenomenon as well as a cornerstone of magnetic memory and logic technology \cite{3,16–18}. From the point of view of applications, it is desirable to maximise the domain wall velocity in order to optimise switching and response times.

Partly because of fabrication techniques, attention has been focused on nanowires with large cross-sectional aspect ratio, typically of rectangular cross-section. In this case, even if the bulk material is isotropic (e.g., permalloy), the domain geometry induces a fully anisotropic magnetic permeability tensor, with easy axis along the wire and hard axis along its shortest dimension \cite{21,22}. Nanowires with uniaxial permeability, characteristic of more symmetrical cross-sectional geometries (e.g., square or circular), have been less studied \cite{23,26}. Here we investigate domain wall (DW) motion in uniaxial wires in the presence of transverse fields. We show that the DW velocity in uniaxial wires depends strongly on the longitudinal applied field $H_1$, increasing with $H_1$ up to a certain critical field and thereafter falling off as precessional motion sets in. We employ a systematic asymptotic expansion scheme, which differs from alternative approaches based on approximate dynamics for the DW centre and orientation; a detailed account of this scheme, also including anisotropy and current-induced torques, will be given separately \cite{24}.

We employ a continuum description of the magnetisation. For a thin nanowire, this is provided by the one-dimensional Landau-Lifshitz-Gilbert (LLG) equation \cite{22}

\begin{equation}
\dot{M} = \gamma M \times H - \alpha M \times (M \times H).
\end{equation}

Here $M(x,t)$ is a unit-vector field specifying the orientation of the magnetisation, which we shall also write in polar form $M = (\cos \Theta, \sin \Theta \cos \Phi, \sin \Theta \sin \Phi)$. The effective magnetic field, $H(m)$, is given by

\begin{equation}
H = A m'' + K_1 m_1 \hat{x} - K_2 m_2 \hat{y} + H_o.
\end{equation}

Here $A$ is the exchange constant, $K_1$ is the easy-axis anisotropy, $K_2 > 0$ is the hard-axis anisotropy, $H_o$ is the applied magnetic field (taken to be constant), $\gamma$ is the gyromagnetic ratio, and $\alpha$ is the Gilbert damping parameter. For convenience we choose units for length, time and energy so that $A = K_1 = \gamma = 1$. Domains correspond to locally uniform configurations in which $M$ is aligned along one of the local minima, denoted $m_+$ and $m_-$, of the potential energy

\begin{equation}
U(m) = -\frac{1}{2}(m_1^2 - K_2 m_2^2) - m \cdot H_o.
\end{equation}

Two distinct domains separated by a DW are described by the boundary conditions $M(\pm \infty, t) = m_\pm$.

For purely longitudinal fields $H_o = H_1 \hat{x}$ and for $H_1$ below the Walker breakdown field $H_W = \alpha K_2 / 2$, the DW propagates as a travelling wave \cite{1}, the so-called Walker solution $\Theta(x,t) = \theta_W(x - V_W t)$, $\Phi(x,t) = \phi_W$, where $\theta_W$ and $\phi_W$ are given by

\begin{equation}
\theta_W(\xi) = 2 \tan^{-1}(e^{-\xi/\gamma}), \quad \sin 2 \phi_W = H_1 / H_W.
\end{equation}

The width of the DW, $\gamma$, is given by $\gamma = (1 + K_2 \cos^2 \phi_W)^{-1/2}$, and the velocity is given by

\begin{equation}
V_W = -\gamma (1 + 1/\alpha) H_1.
\end{equation}

For $H_1 > H_W$, the DW undergoes non-uniform precession and translation, with mean velocity decreasing with
The effects of additional transverse fields have been examined recently [11]. If the cross-sectional geometry is sufficiently symmetrical (e.g., square or circular), the permeability tensor becomes uniaxial, so that $K_2 = 0$ [21, 22]. The dynamics in this case is strikingly different. The LLG equation has an exact solution, $\Theta(x, t) = \theta_0(x - V_t t)$, $\Phi(x, t) = -H_1 t$, in which the DW propagates with velocity

$$V_P = -\alpha H_1$$

and precesses about the easy axis with angular velocity $-H_1$ [23, 24]. The precessing solution persists for all $H_1$ – there is no breakdown field – but becomes unstable for $\alpha_2 = 1$. For typical values of $\alpha$ (0.01–0.1), the uniaxial velocity $V_P$ is less than the fully anisotropic velocity $V_W$ by several orders of magnitude. As we show below, applying a transverse field $H_2 > 0$ to a uniaxial wire dramatically changes its response to an applied longitudinal field $H_1$. The transverse field, analogous to hard-axis anisotropy, inhibits precession and facilitates fast DW propagation. For $H_1$ less than an $H_2$-dependent critical field $H_{1c}$, given in the linear regime by (25) below, there appears a travelling wave, while for $H_1 > H_{1c}$, there appears an oscillating solution, as in the Walker case. The DW velocity of travelling wave exceeds that of oscillating solution.

### Velocity of travelling wave

We first obtain a general identity, of independent interest, which relates the velocity of a travelling wave $\mathbf{M}(x, t) = \mathbf{m}(x - V t)$ (assuming one exists) to the change in potential energy across the profile (for zero transverse field, this coincides with results of [1] and [10]). Noting that $\mathbf{M} = -V \mathbf{m}'$, we take the square of (11) and integrate over the length of the wire to obtain

$$V^2||\mathbf{m}'||^2 = (1 + \alpha^2)||\mathbf{m} \times \mathbf{H}||^2.$$  

(7)

Here we use the notation

$$||\mathbf{u}||^2 = \langle \mathbf{u}, \mathbf{u} \rangle, \quad \langle \mathbf{u}, \mathbf{v} \rangle = \int_{-\infty}^{\infty} \mathbf{u} \cdot \mathbf{v} \, dx$$

for the $L^2$-norm and inner product of vector fields (analogous notation for scalar fields is used below). Next, we take the inner product of (11) with $\mathbf{H}$ to obtain

$$V \langle \mathbf{m}', \mathbf{H} \rangle = -\alpha||\mathbf{m} \times \mathbf{H}||^2.$$  

(9)

Noting that $\mathbf{m}' \cdot \mathbf{H} = \left(\frac{1}{2} \mathbf{m}' \cdot \mathbf{m}' - U(\mathbf{m})\right)'$, we combine (7) and (9) to obtain

$$V = \frac{1}{2}(\alpha + 1/\alpha)||\mathbf{m}'||^{-2} \left( U(\mathbf{m}_-) - U(\mathbf{m}_+)\right).$$  

(10)

The identity (10) has a simple physical interpretation; the velocity is proportional to the potential energy difference across the wire, and inversely proportional to the exchange energy of the profile.

From now on, we consider the uniaxial case $K_2 = 0$ and applied field with longitudinal and transverse components $H_1, H_2 > 0$ (by symmetry, we can assume $H_2 = 0$ with $|H_2| < 1$. An immediate consequence of (10) is that, in the uniaxial case, the velocity must vanish as $H_1$ goes to zero. When $H_1 = 0$, the local minima $\mathbf{m}_\pm$ are related by reflection through the 23-plane, and $U(\mathbf{m}_+) = U(\mathbf{m}_-)$. The identity (10) has a simple physical interpretation; the velocity is proportional to the potential energy difference across the wire, and inversely proportional to the exchange energy of the profile.

### Small transverse field

In order to understand travelling wave and oscillating solutions as well as the transition between them, we first carry out an asymptotic analysis in which both $H_1$ and $H_2$ are regarded as small, writing $H_1 = \epsilon h_1$, $H_2 = \epsilon h_2$ and rescaling time as $\tau = \epsilon^2 t$ (a systematic treatment including current-induced torques will be given in [29]). We seek a solution of the LLG equation (11) of the following asymptotic form:

$$\Theta(x, t) = \theta_0(x, \tau) + \epsilon \theta_1(x, \tau) + \ldots,$$  

(11)

$$\Phi(x, t) = \phi_0(x, \tau) + \epsilon \phi_1(x, \tau) + \ldots.$$  

(12)

It is straightforward to check that the boundary conditions, namely that $\mathbf{m}$ approach distinct minima of $U$ as $x \to \pm \infty$, imply that

$$\mathbf{m}(\pm \infty, \tau) = (-1, \epsilon h_2, 0) + O(\epsilon^2).$$  

(13)

The leading-order equations for $\Theta$ and $\Phi$ become

$$\theta_{0,xx} - \frac{1}{2}(1 + \phi_{0,xx}^2)\sin 2\theta_0 = 0,$$  

(14)

$$\sin^2 \theta_0 \phi_{0,xx} = 0.$$  

(15)

The only physical (finite-energy) solutions of (14) and (15), consistent with the boundary conditions (13) are of the form

$$\phi_0(x, \tau) = \phi_0(\tau)$$  

(16)

$$\theta_0(x, \tau) = 2 \arctan \exp(-x - x_*(\tau)),$$  

(17)

where $\phi_0$ and $x_*$ respectively describe the DW orientation and centre, and are functions of $\tau$ alone. It is convenient to introduce a travelling coordinate $\xi = x - x_*(\tau)$ and rewrite the ansatz (11)–(12) as

$$\Theta(x, t) = \theta_0(\xi, \tau) + \epsilon \theta_1(\xi, \tau) + \ldots,$$  

(18)

$$\Phi(x, t) = \phi_0(\xi, \tau) + \epsilon \phi_1(\xi, \tau) + \ldots.$$  

(19)

To obtain equations for $\phi_0(\tau)$ and $x_*(\tau)$ we must proceed to the next order. It is convenient to introduce new
variables at order $\epsilon$ which, in light of the boundary conditions \(12\), vanish at $x = \pm \infty$, as follows:

\[
\Theta_1 := \theta_1 - h_2 \cos \phi_0 \cos \theta_0, \quad \tag{20}
\]

\[
u := \phi_1 \sin \theta_0 + h_2 \sin \phi_0. \quad \tag{21}
\]

These satisfy the linear inhomogeneous equations

\[
L\Theta_1 = f, \quad \tag{22}
\]

\[
u = g. \quad \tag{23}
\]

Here $L$ is the self-adjoint Schrödinger operator given by

\[
L = -\frac{\partial^2}{\partial \xi^2} + W(\xi), \quad \tag{24}
\]

where

\[
W = \frac{\theta''_0}{\theta'_0} = 1 - 2 \text{sech}^2 \xi, \quad \tag{25}
\]

and $f(\xi, \tau)$ and $g(\xi, \tau)$ are given by

\[
f = (1 + \alpha^2)^{-1} \sin \theta_0 (-\alpha \dot{x}_* - \dot{\phi}_0) - h_1 \sin \theta_0, \quad \tag{26}
\]

\[
g = (1 + \alpha^2)^{-1} \sin \theta_0 (\dot{x}_* - \alpha \phi_0) + 2 h_2 \sin^2 \theta_0 \sin \phi_0.
\]

The DW position $x_*$ and orientation $\phi_0$ are determined from the solvability conditions for \(22\) – \(23\). According to the Fredholm alternative, given a self-adjoint operator $L$ on $L^2(\mathbb{R})$, a necessary condition for the equation $L\Theta_1 = f$ to have a solution $\Theta_1$ is that $f$ be orthogonal to the kernel of $L$. If this is the case, a sufficient condition is that the spectrum of $L$ is isolated away from 0. From \(24\) and \(25\) it is clear that $\theta'_0$ belongs to the kernel of $L$, and since the eigenvalues of a one-dimensional Schrödinger operator are non-degenerate, it follows that $\theta'_0$ spans the kernel of $L$. Moreover, since $W(\xi) \to 1$ as $\xi \to \pm \infty$, it follows that the spectrum of $L$ is discrete near 0. (In fact, $W$ is a special case of the exactly solvable Pöschl-Teller potential, but we won’t make use of this fact.)

Requiring $f$ and $g$ in \(22\) and \(23\) to be orthogonal to $\theta'_0$ and noting that $\langle \theta'_0, \theta'_0 \rangle = 0$, $\langle \theta'_0, \sin \theta_0 \rangle = -2$, $\langle \theta'_0, 1 \rangle = -\pi$, and $\langle \theta'_0, \cos \theta_0 \rangle = 0$, we obtain the following system of ODEs for $\phi_0$ and $x_*$:

\[
\dot{\phi}_0 = -h_1 - \frac{\alpha \pi}{2} h_2 \sin \phi_0, \quad \tag{27}
\]

\[
\dot{x}_* = -\alpha h_1 + \frac{\pi}{2} h_2 \sin \phi_0. \quad \tag{28}
\]

Travelling wave solutions appear provided \(27\) has fixed points; this occurs for $h_1$ below a critical field $h_{1,c}$ given by

\[
h_{1,c} = \frac{\alpha \pi h_2}{2}, \quad \tag{29}
\]

The velocity and orientation of the travelling wave are given by

\[
\dot{x}_* = -\left(\frac{\alpha + 1}{\alpha}\right) h_1, \quad \tag{30}
\]

\[
\sin \phi_0 = -\frac{h_1}{h_{1,c}}. \quad \tag{31}
\]

There are two possible solutions for $\phi_0 \in [0, 2\pi)$, only one of which is stable. Oscillating solutions appear for $h_1 > h_{1,c}$, and are given by

\[
h_1 \tan \frac{\pi}{2} \phi_0 = -h_{1,c} - \sqrt{h_1^2 - h_{1,c}^2} \tan \left(\frac{\pi}{2} \sqrt{h_1^2 - h_{1,c}^2} \right), \quad \tag{32}
\]

with the period $T = 2\pi / \sqrt{h_1^2 - h_{1,c}^2}$. The mean precessional and translational velocities are obtained by averaging over a period, with result

\[
\langle \dot{\phi}_0 \rangle = -\frac{1}{\alpha} h_1 \sqrt{h_1^2 - h_{1,c}^2}, \quad \tag{33}
\]

\[
\langle \dot{x}_* \rangle = -\left(\alpha h_1 + \frac{1}{\alpha} \right) h_1 \sqrt{h_1^2 - h_{1,c}^2}. \quad \tag{34}
\]

Note that for $h_1 = h_{1,c}$, \(34\) coincides with the travelling wave velocity \(30\), whereas for $h_1 \gg h_{1,c}$, \(34\) reduces to the velocity of the precessing solution given by \(30\).

The behaviour is similar in many respects to the Walker case (i.e., $K_2 \neq 0$ and $H_2 = 0$). Here, the transverse field rather than hard-axis anisotropy serves to arrest the precession of the DW (provided the longitudinal field is not too strong). There are differences as well; in the transverse-field case there is just one stable travelling wave, whereas in the Walker case there are two. Also, in the transverse-field case the asymptotic value of the magnetisation has a transverse component, whereas in the Walker case it has none.

**Moderate transverse field**

We can extend the travelling wave analysis to the regime where $H_2$ is no longer regarded as small. We continue to regard $H_1$ as small, writing $H_1 = \epsilon h_1$ and $V = \epsilon V$, and expand the travelling wave ansatz $\Theta(x, t) = \Theta(x - V t), \Phi(x, t) = \phi(x - V t)$ to first order in $\epsilon$, writing $\theta = \theta_0 + \epsilon \theta_1$, $\phi = \phi_0 + \epsilon \phi_1$. Substituting into the LLG equation, we obtain the $O(\epsilon^0)$ equations

\[
\theta'_0 = (H_2 - \sin \theta_0), \quad \phi_0 = 0, \quad \tag{35}
\]

with boundary conditions $\sin \theta_{0+} = H_2, \theta_{0+} > \pi/2$ and $\theta_{0-} < \pi/2$. Thus, for $H_2 = O(\epsilon^0)$, azimuthal symmetry is broken at leading order, and the static profile is parallel to the transverse field (the alternative solution with $\phi_0 = \pi$ is unstable). The solution of \(35\) is given by

\[
\tan \frac{\theta_0}{2} = \kappa \tanh \left[ \frac{1}{\kappa} \left( \frac{H_2 - 1}{\kappa^2} \right) - \frac{\kappa \epsilon}{2} \right] + \frac{1}{H_2}, \quad \tag{36}
\]
where $\kappa = \sqrt{1 - H_2^2}$.

At order $\epsilon$ we obtain the linear inhomogeneous equations

$$L\theta_1 = \frac{\alpha}{1 + \alpha^2} \eta \theta_0 - \eta V \sin \theta_0, \quad (37)$$

$$M\phi_1 = \frac{1}{1 + \alpha^2} \theta_0', \quad (38)$$

where

$$L = -d^2/dx^2 + \frac{\eta''}{\eta'}, \quad M = -d \sin^2 \theta_0 \frac{d}{d\xi} + H_2 \sin \theta_0. \quad (39)$$

Here $\theta_0$ is given by (36), and $\theta_1, \phi_1$ are required to vanish as $\xi \to \pm \infty$. As above, the Fredholm alternative implies that the right-hand side of (37) must be orthogonal to $\theta_0'$ in order for a solution to exist. Calculation yields

$$V = -\left(\alpha + \frac{1}{\alpha}\right) (1 - (H_2/\kappa) \cos^{-1} H_2)^{-1} H_1. \quad (40)$$

For $H_2 = 0$, this coincides with (30); thus, (40) gives $H_2$-nonlinear corrections to the velocity. Moreover, it is straightforward to show that (40) is consistent with the general identity (10). Finally, one can also show that $M$ has trivial kernel with spectrum bounded away from zero, so that (35) is automatically solvable.

It is interesting to compare the DW velocity with transverse field to the Walker case. From (5) and (10),

$$V_W/V = \gamma (1 - (H_2/\kappa) \cos^{-1} H_2) < 1. \quad (41)$$

Thus, to leading order in $H_1$, the DW velocity in a uniaxial wire with transverse field exceeds the Walker velocity.

Numerical results

To verify our analytical results, we solve the LLG equation (4) using a finite-difference scheme on a domain $-L \leq x \leq L$ where $L = 100$ (the DW has width of order 1). Neumann boundary conditions, $m' = 0$, are maintained at the endpoints. The damping parameter $\alpha$ is taken to be 0.1 throughout. As initial condition we take the stationary profile, with $\theta_0$ given by (36) and $\phi_0 = 0$. After an initial transient period, during which the asymptotic values of $m$ at $x \to \pm L$ converge to $m_\pm$, a stable solution emerges, in which the DW propagates with a characteristic mean velocity $V$. (For convenience, we have taken $H_1 < 0$, so that $V$ is positive.) In Figure 1, numerically computed values of $V$ are plotted as a function of $|H_1|$ for three fixed values of the transverse field: $H_2 = 0.2$, $H_2 = 0.1$, and the limiting case $H_2 = 0$, where the dynamics is given by the precessing solution. There is good quantitative agreement with the analytic results for small transverse fields, (30), for $|H_1| < H_{1c}$, and (31), for $|H_1| > H_{1c}$. In Figure 2 the analytic expressions for the velocity for small and moderate transverse fields are compared to numerical results for $H_2 = 0.2$ and $|H_1| \ll H_{1c}$. The moderate-field expression (40), which depends nonlinearly in $H_2$, gives excellent agreement for small driving fields. For nonzero $H_2$, the velocity exhibits a peak at a critical field $|H_{1c}|$, which depends on $H_2$. 

![FIG. 1: Average DW velocity $V$ as a function of the driving field $|H_1|$ for three values of the transverse field $H_2$. The analytic formulas (solid curves) (30), for $|H_1| < H_{1c}$, and (31), for $|H_1| > H_{1c}$, are plotted against numerically computed values (open circles). For $H_2 = 0$, the analytic formula is exact.](image1.png)

![FIG. 2: DW velocity $V$ as a function of the driving field $|H_1|$ for $H_2 = 0.2$. The expressions for small-transverse field (30) (red curve) and moderate-transverse field (40) (light blue curve) are plotted against numerically computed values (open circles).](image2.png)

Figure 3 shows the dependence of the critical field $|H_{1c}|$.
on $H_2$, in close agreement with the analytic result [29].

FIG. 3: The critical driving field $|H_{1c}|$ as a function of the transverse field $H_2$. A linear fit (blue curve) through the numerically computed data (blue diamonds) is plotted alongside the analytical result [29] (red curve).

FIG. 4: The magnetization distribution, $\theta(x, t)$ and $\phi(x, t)$, for two values of the driving field: $H_1 = -0.01$ in figures (a) and (b), and $H_1 = -0.05$ in figures (c) and (d). The transverse field is taken as $H_2 = 0.1$ throughout.

As in the Walker case, the properties of the propagating solution are qualitatively different for driving fields $|H_1| < |H_{1c}|$. This is confirmed in Figure 4 which shows contour plots of the magnetization in the $(x, t)$-plane. Figs. 4a and 4b, where $H_1 = -0.01$, exemplify the case $|H_1| < |H_{1c}|$. The magnetization evolves as a fixed profile translating rigidly with velocity $V$. For $|H_1| > |H_{1c}|$, as exemplified by Figs. 4c and 4d, where $H_1 = -0.05$, the magnetization profile exhibits a non-uniform precession as it propagates along the nanowire, with mean velocity in good agreement with [54].

**Summary**

We have established, both analytically in leading-order asymptotics and numerically, the existence of travelling wave and oscillating solutions of the LLG equation in uniaxial wires in applied fields with longitudinal and transverse components. We have obtained analytic expressions for the velocity, (30) and (40), and for the critical longitudinal field, (29), above which the travelling wave solution ceases to exist. We have also obtained the mean precessional and linear velocities (33) and (34) for oscillating solutions. The analytic results are confirmed by numerics.

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