Modelling and Simulation for Small Signal Stability of Multi-machine Power System under Various Disturbance Conditions

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Abstract — This paper presents the small signal stability of multi-machine power system over the 58-Bus, 26-Machine, Yangon Distribution Network and is validated with MATLAB software under various disturbance conditions. Time-domain solution analysis is employed to determine the small signal dynamic behavior of test system. Transtability model is used to perform time-domain simulation in SIMULINK. The simulation is carried out for normal condition, reference voltage of regulator (V_{ref}), disturbance, mechanical torque (T_m) disturbance and network (fault) disturbance and the conditions of change in center of inertia for rotor angle (delta COI), slip for center of inertia (slip COI), field current and mechanical torque are observed. According to the simulation results, perturbation of V_{ref} shows only instability on the system. But ramping of T_m and network disturbance can cause large disturbance on the system and unstable conditions can be observed.

Keywords—load flow, small signal stability analysis, time-domain simulation, multi-machine power system, system disturbances

I. INTRODUCTION

Increased requirement of electric power together with the restraints to build new transmission lines causes the transmission systems to operate touching the stability limits. To meet the demand, such stressed power system requires advance monitoring and control system to detect and respond to any disturbances immediately so that stability can be preserved. Stability study is divided in two parts, transient stability and steady state or small signal stability. Transient stability is concerned with large external disturbances such as fault on transmission line. Small signal stability is concerned with the rotor angle or speed deviation of generators which varies due to random load variations [1].

Small-signal rotor angle stability analysis mainly deals with a study of electromechanical oscillations-related performance of the system about an operating point when the system is subjected to sufficiently small magnitude of disturbance that will not trigger non-linear behavior of the system. Thus, this study is mainly concerned with the ability of the power system to maintain synchronism under small disturbances. The disturbances are considered to be sufficiently small that linearization of system equations is possible for analysis purposes. This permits linear system theory to be applied for system analysis even though the system is inherently non-linear [2].

If the power system at a given operating condition may be large disturbance unstable, still such a system can be operated, though unsecurely. However, if the system is small-signal unstable at a given operating condition, it cannot be operated. Therefore, small signal stability is a fundamental requirement for the satisfactory operation of power systems [3].

When a dynamic system such as power system is perturbed from its steady state condition, the system variables trace out a flow, referred to as trajectories. These trajectories may exhibit oscillatory or monotonic behavior. For the system to be stable, these trajectories must remain bounded and converge to an acceptable operating point.

In this paper a realistic multi-machine power system has been taken as a test case and simulation is conducted considering full-order linearized model including all type of network buses.

II. MODELLING FOR SMALL SIGNAL STABILITY STUDY OF POWER SYSTEM

A. Load Flow Studies

Load flow analysis provides the initial operating values for the system variables, before any dynamic analysis is carried out. Thus, it avoids long duration time-domain simulation to arrive at a pre-fault operating condition. Load flow problem generally involves the simultaneous solution of many non-linear algebraic equations using numerical iterative procedure. Most commonly used iterative methods are the Gauss-Seidel, the Newton-Raphson (NR) and fast decouple load flow (FDLF) methods. In this study, FDLF method is employed for the load flow solution [4].

B. Generator Model

A 3 phase synchronous machine is modelled in the rotor reference frame as shown in Fig. 1. The figure also depicts two rotor windings, including the field winding ‘f’ along d-axis and two rotor coils along q-axis. The short circuited coils, one along d-axis (‘h’) and two along q-axis (‘g’ and ‘k’) represent the effect of damper windings and eddy currents induced in the rotor mass [3].
The distance to center of inertia (COI) in terms of rotor angles and rotor speed deviations are defined as follows:

\[ \delta_{\text{COI}} = \frac{1}{H_T} \sum_{i} H_i \delta_i \]  
\[ \omega_{\text{COI}} = \frac{1}{H_T} \sum_{i} H_i \omega_i \]  
\[ \delta_i = \delta - \delta_{\text{COI}} \]  
\[ \omega_i = \omega - \omega_{\text{COI}} \]  

In the time-domain simulation, the stability of a system is inferred by plotting \( \delta \) against time [6].

C. IEEE-Type AC4A Exciter Model

The differential equations are written in the state-space form as follows:

\[ \frac{d\psi_f}{dt} = \frac{1}{T_d} \left[ -\psi_f + \left( \frac{x_d}{x_d - x_d} \right) E_b \right] \]  
\[ \frac{d\psi_h}{dt} = \frac{1}{T_a} \left[ -\psi_h + \psi_q \right] \]  
\[ \frac{d\psi_q}{dt} = \frac{1}{T_q} \left[ -\psi_g + \psi_q \right] \]  
\[ \frac{d\psi_k}{dt} = \frac{1}{T_k} \left[ -\psi_k + \psi_g \right] \]

2) Stator Equations:

Neglecting stator transients and ignoring speed variations, the stator d- and q-axes voltage equations are given by,

\[ v_d = -i_d R_a - v_q \]  
\[ v_q = -i_q R_a + v_d \]

3) Swing Equations:

The rotor mechanical equations are given by,

\[ \frac{d\delta}{dt} = S_m \omega_n \]  
\[ \frac{ds}{dt} = \frac{1}{2H}[T_m - T_s - DS_n] \]

\[ S_m = \text{per unit slip} \]  
\[ \omega_n = \text{base speed in rad/s} \]  
\[ H = \text{inertia constant of a generator in per unit} \]  
\[ T_m = \text{mechanical input torque in per unit} \]  
\[ D = \text{mechanical damping in per unit} \]

The electromagnetic torque can be written as

\[ T_e = E_q' i_d + E_d' i_q + (x_q' - x_q) i_d i_q \]

4) Centre of Inertia Reference

The distance to center of inertia (COI) in terms of rotor angles and rotor speed deviations are defined as follows:

\[ \delta_{\text{COI}} = \frac{1}{H_T} \sum_{i} H_i \delta_i \]  
\[ \omega_{\text{COI}} = \frac{1}{H_T} \sum_{i} H_i \omega_i \]  
\[ \delta_i = \delta - \delta_{\text{COI}} \]  
\[ \omega_i = \omega - \omega_{\text{COI}} \]

In the time-domain simulation, the stability of a system is inferred by plotting \( \delta \) against time [6].

IEEE-Type AC4A excitation system is given in fig. 2. Its differential equations are as follows:

\[ \frac{dx}{dt} = \frac{1}{T_b} \left[ -x + (1 - T_e) \left( V_{\text{ref}} + V_s - V_i \right) \right] \]  
\[ \frac{dV}{dt} = \frac{1}{T_A} \left[ -E_a + K_x (x_a + T_c (V_{\text{ref}} + V_s - V_i)) \right] \]

\[ V_c = \text{output of the voltage transducer} \]  
\[ V_s = \text{output of the power system stabilizer} \]  
\[ V_{\text{ref}} = \text{voltage regulator reference} \]  
\[ T_A = \text{time constant associated with the regulator and firing of thyristors} \]  
\[ K_x = \text{overall gain} \]

D. Hydro Turbine and its Speed Governor Model

The hydro turbine and its speed governor model are shown in fig. 3.

\[ \frac{dx_1}{dt} = \frac{K_0 + s T_1}{1 + s T_1} \frac{V}{V_{\text{ref}}} \]  
\[ \frac{dx_2}{dt} = \frac{K_1}{1 + s T_2} \]

\[ x_1 = \text{output of the governor} \]  
\[ x_2 = \text{output of the power system stabilizer} \]  
\[ V_{\text{ref}} = \text{voltage regulator reference} \]  
\[ K_0 = \text{overall gain} \]  
\[ T_1 = \text{time constant associated with the governor and firing of thyristors} \]

E. Speed Governor Model for Steam Turbine

The speed governor model for steam turbine is given in fig. 4.

\[ \frac{dx}{dt} = \frac{1}{T_1} \left( P_{GV} - x_1 \right) \]  

\[ x_1 = \text{output of the governor} \]  
\[ P_{GV} = \text{output of the governor} \]  
\[ T_1 = \text{time constant associated with the governor and firing of thyristors} \]  
\[ K_0 = \text{overall gain} \]
steam turbines. Further, constant impedance type load turbine, and the rest generators are with non-reheat type with hydro-turbine, generator 22 is with reheat type steam turbines. Bus-1 is treated as a slack bus. All generators transformers and 23 numbers of 2-windings load network includes 14 numbers of 3-windings line lines are 66kV, and the rest 1 line is 33 kV. The whole system has 58 buses, 26 lines, 37 transformers, 23 loads and 8 capacitor banks. Out of 26 lines, 10 lines are 230 kV, 15 lines are 66kV, and the rest 1 line is 33 kV. The whole network includes 14 numbers of 3-windings line transformers and 23 numbers of 2-windings load transformers. Bus-1 is treated as a slack bus. All generators are provided with IEEE-type AC4A exciters. Generator 1 is with hydro-turbine, generator 22 is with reheat type steam turbine, and the rest generators are with non-reheat type steam turbines. Further, constant impedance type load model has been employed for both real and reactive components of loads.

\[ \text{Conductance } G \text{(pu)} = \frac{1}{1+st_{CH}} \] (20)

\[ \text{Susceptance } B \text{(pu)} = \frac{1}{1+st_{CO}} \] (21)

\[ \text{dy}_1 = \frac{1}{T_1} [K(1-T_2)S_m - y_1] \] (22)

\[ \frac{dP_{GV}}{dt} = \frac{1}{T_3} [P_0 - K T_2 S_m - y_1 - P_{GV}] \] (23)

T_{CH}, T_{RH} and T_{CO} are the time constants of the steam chest, re-heater and crossover piping.

2) Non-reheat type
The non-reheat type of steam turbine model is shown in fig. 6.

![Non-reheat type steam turbine model](image)

Fig. 6. Non-reheat type steam turbine model
The state variable of this type is P_{GV}.

F. Network Model
The linearized network equation can be expressed using admittance matrix (and DQ variables) as follows:

\[ [Y_{DO}] \Delta V_{DO} = \Delta \Delta \] (24)

G. Time-domain Solution Analysis
Time-domain techniques provide an exact determination of stability of non-linear systems both for small and large disturbances. The choice of disturbance and selection of variables to be observed in time response are critical. The input, if not chosen properly, may not provide substantial excitation of the important modes. The observed response may contain many modes and the poorly damped modes may not be dominant. Number of modes depends on modeling details employed for different dynamic components. Larger systems may have a number of inter-area modes of similar frequencies, and it is quite difficult to separate them from a response in which more than one is excited [7].

III. CASE STUDY
Case study is carried out for a 26 machine system at Yangon Distribution Network as shown in Fig. 7. This system has 58 buses, 26 lines, 37 transformers, 23 loads and 8 capacitor banks. Out of 26 lines, 10 lines are 230 kV, 15 lines are 66kV, and the rest 1 line is 33 kV. The whole network includes 14 numbers of 3-windings line transformers and 23 numbers of 2-windings load transformers. Bus-1 is treated as a slack bus. All generators are provided with IEEE-type AC4A exciters. Generator 1 is with hydro-turbine, generator 22 is with reheat type steam turbine, and the rest generators are with non-reheat type steam turbines. Further, constant impedance type load

![Single Line Diagram for Yangon Distribution Network](image)

Fig. 7. Single Line Diagram for Yangon Distribution Network
The capacitor bank data and the synchronous generator data are given in Table I and Table II. All data are calculated on 100MVA base.

| Bus No | Conductance G(pu) | Susceptance B(pu) |
|--------|------------------|------------------|
| 1      | 0                | 0.929            |
| 26     | 0.0031           | 0.5071           |
| 38     | 0.0008           | 0.4066           |
| 41     | 0.06             | 1.06             |
| 46     | 0                | 0                |
| 49     | 0.00392          | 0.67888          |
| 52     | 0                | 0.5              |
| 58     | 0                | 0.16             |

IV. SIMULATION AND RESULTS
The Simulink model for small signal stability study is illustrated in Fig. 8. The modeling is carried out by Matlab/Simulink. For simulation of model, the matlab programs for initial condition (initcond.m), load flow program (fdlf_loadflow.m), small signal stability program (small_sig.m) and transient stability program (transtability.mdl) are formerly executed and the results are stored in workspace. For the simulation of this model, the data of 26 machines are input from Matlab workspace. The responses of each machine are recorded by using scopes.
A time-domain simulation is carried out with four cases by using transtability programme.

Case-1: Without any disturbance
Case-2: Perturbation of $V_{\text{ref}}$ for generator-1
Case-3: Ramping of $T_m$ for generator-1
Case-4: Network disturbances

![Simulink Model for Small Signal Stability Study](image)

**Fig. 8.** Simulink Model for Small Signal Stability Study

Case-1: Without any disturbance

In this case, all machines are operated under normal condition without any disturbance. For this simulation, in program execution of small signal (small_sig.m), network disturbances, perturbation of $V_{\text{ref}}$ and ramping of $T_m$ are set to zero. The important parameters for the machines are shown in Fig. 9 through Fig. 12.

![Variation of rotor angles with respect to COI reference](image)

**Fig. 9.** Variation of rotor angles with respect to COI reference (26 m/c system)

![Variation of slip with respect to COI reference](image)

**Fig. 10.** Variation of slip with respect to COI reference (26 m/c system)

![Variation of I_{fd}(26 m/c system)](image)

**Fig. 11.** Variation of $I_{fd}$ (26 m/c system)

![Variation of Turbine output (26 m/c system)](image)

**Fig. 12.** Variation of Turbine output (26 m/c system)

Case-2: Perturbation of $V_{\text{ref}}$ for generator-1

For this simulation, in program execution of small signal (small_sig.m), network disturbances is set to zero. Then the perturbation of $V_{\text{ref}}$ is set as one. The generator number whose $V_{\text{ref}}$ needs to be perturbed is given as 1 i.e. machine 1. The variation of the rotor angle, slip, field current, and turbine output obtained are taken up for analysis as shown in Fig. 13 through Fig. 16.
Fig. 17. Variation of rotor angles with respect to COI reference (26 m/c system)

Fig. 18. Variation of slip with respect to COI reference (26 m/c system)

Case-3: Ramping of $T_m$ for generator-1

For this simulation, in program execution of small signal (small_sig.m), network disturbances and perturbation of $V_{ref}$ are set to zero. Then the ramping of $T_m$ is set as one. The generator number whose $T_m$ needs to be ramped-up/down is given as 1 i.e. machine 1. The Turbine-governor on the chosen generator is disabled using TURB selector in the program. The simulation results for this case are shown in Fig. 17 through Fig. 20.

Fig. 19. Variation of $I_d$ (26 m/c system)

Fig. 20. Variation of Turbine output (26 m/c system)

Case-4: Network disturbances

For this simulation, in program execution of small signal (small_sig.m), network disturbance is set as one. In this case, a three phase fault is applied at bus 46 at time $t = 5s$ and is cleared by tripping the line 24. For a fault clearing time of 0.1s, the variation of the rotor angle, slip, field current, and turbine output are shown in Fig. 21 through Fig. 24.

Fig. 21. Variation of rotor angles with respect to COI reference (26 m/c system)
Similarly, the field current of machine 16 is slightly reduced significantly increased from 4 to 4.4 pu at 11 second. Under network disturbance condition, turbine outputs of all other machines are under steady condition but machine 1 is significantly dropped within fault instant but machine 1 decrease at 8 seconds. Therefore, the system is unstable.

V. CONCLUSIONS

In this paper a multi-machine small-signal stability analysis programme has been developed using time-domain simulation. The programmes have been tested employing 58-bus, 26-machine power system of Yangon Distribution Network. The detail study is carried out for delta COI, slip COI, field current and mechanical torque. According to the simulation results, perturbation of \( V_{\text{ref}} \) shows only instability on the system. But ramping of \( T_m \) causes large disturbance on the system and unstable conditions can be observed. To maintain the small signal stability of the system, the application of power system stabilizer (PSS) is needed and should be done in further study.

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