Emergency Autonomous Steering Control Research Based on Receding Horizon $H_{\infty}$ Control and Minimax Criteria

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ABSTRACT Aiming at the problems of model uncertainty, external interference of road conditions, and inherent nonlinearity of lateral movement of autonomous driving vehicles, this paper proposes an effective control technology for an automatic steering system, namely, the receding horizon $H_{\infty}$ control, which has good path tracking performance and can overcome external interference. First, a two-degree-of-freedom vehicle model and a road model are established. Then, the Hamilton equation is established using Pontriagin’s minimum principle, and the optimal control solution and the most serious disturbance are solved based on the minimax principle. Finally, in order to evaluate the receding horizon $H_{\infty}$ autonomous steering control method, lane change simulation and hardware in the loop simulation are developed to describe the effect of receding horizon $H_{\infty}$. The simulation results show that the model predictive control (MPC) and linear matrix inequality (LMI) controller will exhibit oscillation and instability under the condition of external interference. To improve the stability, the $H_{\infty}$ controller reduces part of the performance, while the rolling horizon $H_{\infty}$ strategy can ensure the robustness of automatic steering and has a certain anti-interference performance.

INDEX TERMS Intelligent transport systems, autonomous steering system, receding horizon $H_{\infty}$ control, min-max optimization, vehicle dynamics.

I. INTRODUCTION Sensing and automotive electronic technologies have achieved significant development, and the commercialization of several advanced driving assist systems has been gradually realized over the last two decades. Such advanced driving assist systems include lane keeping assist systems (LKA) for path-tracking control [1], emergency steering assist system (ESA) [2], adaptive cruise control (ACC) [3], and collision avoidance systems [4], in which LKA and ESA are autonomous steering control systems. The main aim of autonomous steering control is to find a control strategy that minimizes the lateral displacement between the vehicle and the center of the planning lane [1], [2].

Many aspects of autonomous steering control for path tracking have been researched since the 1980s, in which environment sensing and vehicle positioning can be implemented using a magnetic center line [5], vision systems such as Mobileye [6], or by inertial navigation and global positioning systems [7]. At present, there are already some autonomous steering products, such as LKA, that are all working under normal driving conditions [8]. However, owing to the external disturbances and nonlinearity of the steering system and vehicle dynamics, research on vehicle autonomous steering control for emergency obstacle avoidance in extreme driving situations still does not meet the requirements of commercialization [2]. In emergency situations, vehicle dynamics are subjected to severe external disturbances, which cause huge difficulties for autonomous steering control path tracking and cause a conflict between the controller’s robustness and performance [9]. Robust stability against external disturbances and tracking performance are both very important for realizing autonomous steering control. Therefore, the design of an autonomous steering system must solve the tracking and robustness problem.
Various control theories have been proposed for the design of autonomous steering control methods. Since the last century, the proportional-integral-differential (PID) control method has often been employed to track a target path by minimizing the lateral deviation at the preview point [10], [11]. With the development of control theory, some more advanced control algorithms have been proposed to solve these problems, including optimal algorithms [12]–[14], model predictive control (MPC) [15]–[17], robust control [18]–[21], and adaptive control [22], [23].

The actual control system is inevitably influenced by external interference, and $H_{\infty}$ control is one of the most effective methods for dealing with the robust problem. Currently, various robust control strategies are applied to path-tracking and autonomous steering systems. In [19], a steering controller was designed to track the current lane center of straight and curved sections when the curvature radius of the road was not clear. The controller was designed using an H∞-based loop shaping design procedure based on McFarlane and Glover. The results demonstrated that this method not only met the expected performance but also had good robustness. A gain-scheduling $H_{\infty}$ controller design considering disturbances and uncertainties was proposed to enhance the robustness of vehicle autonomous steering systems [20]. The lane-keeping and lane-changing control techniques were presented in Section V.

The structure of this paper is as follows: Section II introduces the establishment of the two-degree-of-freedom vehicle model and road model. Section III introduces in detail the derivation process of the horizon $H_{\infty}$ control decision based on the robust and minimax control theory. To verify the feasibility of the control strategy, the simulation and HIL experiments are designed, and the experimental data are analyzed in detail in Section IV. Finally, conclusions are presented in Section V.

**IV. VEHICLE AND PATH TRACKING MODEL**

**A. VEHICLE MODEL**

In this study, the classic two-degree-of-freedom vehicle model is adopted to describe the yaw-sidelip characteristics, and the detailed derivation process is described at the expense of performance, and MPC control has difficulty resisting disturbances. In [24], [25], to guarantee good robustness and control performance, the external disturbance was considered into the traditional rolling horizon as input considering the idea of $H_{\infty}$ control.

To solve the conflicts between path tracking performance and robustness in complex environments and extreme working conditions, inspired by the literature [24], [25], a receding horizon $H_{\infty}$ model predictive control framework based minimax criteria is proposed for vehicle trajectory tracking in this study.

The three innovations of this paper are:

1) A new model predictive control method for vehicle trajectory tracking is proposed, which is based on a robust principle to realize road tracking technology. Then the problem can be solved that the linearized prediction correction mechanism cannot essentially apply the traditional predictive control technology to the model uncertainty control of autonomous vehicles.

2) The online min problem of predictive control during automatic driving road tracking is changed into a minimax description, and the control law is solved to minimize the control objective function value under the worst case of the vehicle uncertainty set.

3) The receding horizon $H_{\infty}$ controller designed in the paper is solved by the form of iterative equation, and has a higher online computational efficiency.

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**FIGURE 1.** Vehicle path tracking model based on vehicle-fixed axes.
in [1], [2], and [3].

\[
\begin{bmatrix}
\dot{y}(t) \\
\dot{\omega}(t) \\
\dot{y}(t) \\
\dot{\psi}(t)
\end{bmatrix}
= \begin{bmatrix}
-(C_f + C_r) & -(aC_f - bC_r) & -U & 0 \\
\frac{U_m}{U} & \frac{-aC_f - bC_r}{U} & 0 & 0 \\
1 & 0 & 0 & U \\
0 & 1 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
y(t) \\
\omega(t) \\
y(t) \\
\psi(t)
\end{bmatrix}
\times
\begin{bmatrix}
\frac{C_f}{mG} \\
\frac{aC_f}{IG} \\
0 \\
0
\end{bmatrix}
\delta(t)
\tag{1}
\]

The eq. (1) can be rewritten in the form of a state space:

\[
\dot{x}(t) = A_c x(t) + B_c \delta(t)
\]

with the state equation of the model. The lateral displacements viewing information of road tracking can be augmented to control framework based on multi-point preview. The pre-rate information Np steps in advance. The state variables of the vehicle road model based on vehicle fixed axes are defined as \( x_{\psi y}(k) = [y, \psi, y_1, \cdots, y_{N_p}]^T \), and those based on ground-fixed axes are defined as \( x_{y\psi}(k) = [y, \psi, y_1, \cdots, y_{N_p}]^T \).

For the convenience of subsequent calculation, Equation (2) is discretized here:

\[ x_{i+1} = A_i x_i + B_i \delta_i \]

The variables and parameters of the Eq. 1 are shown in Table 1.

### Table 1. Variable symbol and its definition in vehicle model.

| Variables and parameters | Variable Name                          |
|--------------------------|---------------------------------------|
| \( y \)                  | The lateral velocity of the vehicle   |
| \( \omega \)             | The yaw velocity of the vehicle       |
| \( y \)                  | The lateral displacement of the vehicle |
| \( \psi \)               | The steering angle of the front wheel |
| \( U \)                  | The velocity of the vehicle           |
| \( I \)                  | The inertia about the z-axis          |
| \( a \)                  | Distance from CM to the front axle    |
| \( b \)                  | Distance from CM to the rear axle     |
| \( C_f \)                | Cornering stiffness of the front tyres |
| \( C_r \)                | Cornering stiffness of the rear tyres  |

**B. AXIS TRANSFORMATION AND VEHICLE-ROAD MODEL**

In actual driving, the driver’s vision is usually based on the vehicle coordinate system, as shown in Figure 1. The designed vehicle road model, the sensor perceives the road information Np steps in advance. The variables of the vehicle road model based on vehicle fixed axes are defined as \( x_{\psi y}(k) = [y, \psi, y_1, \cdots, y_{N_p}]^T \), and those based on ground-fixed axes are defined as \( x_{y\psi}(k) = [y, \psi, y_1, \cdots, y_{N_p}]^T \).

In Figure 2, the distance between the preview point \( i \) and the path of expectation is given as follows:

\[ y_i = y + jU_T \psi + y_{rij} \tag{3} \]

The axis transformation matrix is derived as follows:

\[ x_{y\psi}(k) = \begin{bmatrix} I & 0 \end{bmatrix} x_{\psi y}(k) \tag{4} \]

where, \( M = \begin{bmatrix} 1 & UT \\ 1 & 2UT \\ \vdots & \vdots \\ 1 & N_p UT \end{bmatrix} \)

**Remark 1:** The multi-point previewing road tracking control algorithm has been widely used in driver modeling or lateral motion control [18]. However, there is little research on the receding horizon control theory with respect to the road tracking of autonomous driving vehicles.

**C. SHIFT REGISTER BASED ON VEHICLE ROAD PREVIEW MODEL**

This paper first expounds that the basis of rolling horizon rightmost steering control is a path-following steering control framework based on multi-point preview. The previewing information of road tracking can be augmented to the state equation of the model. The lateral displacements \( y_0, y_1, y_2, \cdots, y_{N_p-1}, \cdots, y_{N_p} \) of road ahead can be obtained with the concept of “shift register” as follows:

\[ y(k+1) = D_\psi y(k) + E_y y_{rij}(k) \tag{5} \]
where,

\[
D = \begin{bmatrix}
0 & 1 & 0 & \cdots & 0 \\
0 & 0 & 1 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & 1 \\
0 & 0 & 0 & \cdots & 0
\end{bmatrix}
\quad E = \begin{bmatrix}
0 \\
0 \\
\vdots \\
0 \\
1
\end{bmatrix}
\]

where y is the control output, in the preview road array of \((N_P+1)\), and \(N_P\) is the preview field of view.

Remark 2: In the course of road tracking, the purpose of vehicle lateral motion control is to minimize the deviation of road tracking (e.g., lateral offset), and a shift register based on the vehicle road preview model can generate the \((N_P+1)\) road path lateral displacements \(y_0, y_1, y_2, \ldots, y_{N_P-1}, \ldots, y_{N_P}\) ahead of the vehicle. The \((N_P+1)\) road-path lateral displacement sequence can be used as the input for the control system.

To ensure that the system satisfies the optimal control under interference, the Pontryagin minimum principle was first introduced, as shown in Fig. 3.

Remark 3: Although the feedback correction mechanism is introduced for the model prediction control to reduce the uncertainty in the model to the greatest extent, the use of a linear prediction correction mechanism in the traditional sense of predictive control technology cannot be used for the control process with strong nonlinearity and uncertainty. The robust predictive control based on min-max criteria takes on the idea of \(H_\infty\) control, namely, the online minimization problem of predictive control is changed into min-max optimization, and the control law can be solved by minimizing the objective function value under the uncertainty worst case.

### A. PONTRYAGIN’S MINIMUM PRINCIPLE

The discrete time system is given as,

\[
x_{i+1} = f(x_i, u_i, \omega_i, i), \quad x_0 = x_0
\]

And the performance indicators is,

\[
J(x_0, u_0, \omega_0, i_0) = \sum_{i=0}^{i_0-1} g(x_i, u_i, \omega_i, i) + h(x_{i_0}, i_0)
\]

The performance index of the augmented form is build,

\[
J_a = \sum_{i=i_0}^{i_0-1} \left[ g(x_i, u_i, \omega_i) + p_{i+1}^T [f(x_i, u_i, i) - x_{i+1}] \right] + h(x_{i_0}, i_0)
\]

Further simplification \(g(x_i^*, u_i^*, i)\) can be replaced by \(g\) and \(f(x_i^*, u_i^*, i)\) can be replaced by \(f\). Then, the increment of the augmented performance index \(J_a\) can be described as, where \(p_{i+1}\) is the Lagrange multiplier,

\[
\Delta J_a = \sum_{i=i_0}^{i_0-1} \left[ \frac{\partial g}{\partial x_i} - p_{i+1}^T + \frac{\partial f}{\partial x_i} p_{i+1}^T \right] \delta x_i + \left[ \frac{\partial g}{\partial u} \right] \delta u
\]

Therefore, all the coefficients are zero,

\[
x_{i+1}^* = f(x_i^*, u_i^*, i)
\]

\[
p_{i+1}^* = \frac{\partial g}{\partial x_i} + \frac{\partial f}{\partial x_i} p_{i+1}^T
\]

\[
x_{i_0}^* = x_0
\]

\[
p_{i_0}^* = \frac{\partial h}{\partial x_{i_0}}
\]

Note that the variables \(\delta x_i\) for \(i = i_0 + 1, \ldots, i_f\) are arbitrary. The Hamiltonian function \(H_i\) is defined as follows.

\[
H(x_i, u_i, p_{i+1}, i)g(x_i, u_i, i) + p_{i+1}^T f(x_i, u_i)
\]
then we have

$$\Delta J_u = \sum_{i=0}^{j-1} \left[ \frac{\partial H}{\partial u^i} (x^e_i, u^e_i, p^e_i, i) \right]^T \delta u_i + \text{higer order terms}$$

(15)

**Lemma 1 (W. H. Kwon et al. [25]):** For the discrete-time system in (6) and the performance criterion in (7). According to the Hamiltonian equations, the optimal control $u^e_i$ can be obtained when (16), (17), and (18) hold.

$$x^e_{i+1} = \frac{\partial H}{\partial p_{i+1}} (x^e_i, u^e_i, p^e_{i+1}, i)$$

(16)

$$p^e_i = \frac{\partial H}{\partial x^e_i} (x^e_i, u^e_i, p^e_{i+1}, i)$$

(17)

$$H(x^e_i, u^e_i, p^e_{i+1}, i) \leq H(x^e_i, u^e_i, p^e_{i+1}, i)$$

(18)

This lemma is Pontryagin’s minimum principle.

**Remark 4:** In Figure 3, it can be concluded that the minimax criterion is the theoretical basis of receding horizon robust control, in which the performance and robustness are considered simultaneously. A robust control strategy based on minimax theory ensures the robustness of the system on the premise of meeting the performance index.

### B. MINIMAX BASED $H_\infty$ ROBUST CONTROL METHOD

In this section, we present the $H_\infty$ robust tracking control method.

$$x_{k+j+1|k} = Ax_{k+j|k} + Bu_{k+j|k} + Bw_{k+j|k}$$

$$\hat{z}_{k+j|k} = \begin{bmatrix} Q^2 x_{k+j|k} \\ T^2 u_{k+j|k} \end{bmatrix}$$

$$z_{k+j|k} = C(z) x_{k+j|k}$$

(19)

$k$ and $j$ are the initial time and step size, respectively. In the system, $x_{k+j|k}$ represents the system state, $w_{k+j|k}$ represents the interference, $u_{k+j|k}$ represents the input, $\hat{z}_{k+j|k}$ and $z_{k+j|k}$ represent the auxiliary output. Then, the infinite norm of the transfer function can be expressed as

$$\left| \begin{array}{c} T \left( z \right) \\ w \end{array} \right|_\infty = \sup_{w_{k+j|k}} \left( T \left( z \right) \right)$$

$$= \sum_{j=0}^{j-1} \left[ \begin{array}{c} x^T_{k+j|k} Q x_{k+j|k} + u^T_{k+j|k} R u_{k+j|k} \end{array} \right]$$

$$\leq \sup_{w_{k+j|k}} \left[ \sum_{j=0}^{j-1} \left[ \begin{array}{c} x^T_{k+j|k} Q x_{k+j|k} + u^T_{k+j|k} R u_{k+j|k} \end{array} \right] \right]$$

(20)

where $T \left( z \right)$ is the transfer function. The suboptimal cost function (21) is given by:

$$\sum_{j=0}^{j-1} \left[ \begin{array}{c} x^T_{k+j|k} Q x_{k+j|k} + u^T_{k+j|k} R u_{k+j|k} \end{array} \right] < \gamma^2$$

(21)

Further transformation, we can obtain,

$$\sum_{j=0}^{j-1} \left[ x^T_{k+j|k} Q x_{k+j|k} + u^T_{k+j|k} R u_{k+j|k} - \gamma^2 w^T_{k+j|k} w_{k+j|k} \right]$$

(22)

It can be seen from equation (21) that all $w_i$ satisfy equation (23), so the left side of equation (23) should be negative.

$$\sup_{w_{k+j|k}} \left\{ \sum_{j=0}^{j-1} \left[ x^T_{k+j|k} Q x_{k+j|k} + u^T_{k+j|k} R u_{k+j|k} - \gamma^2 w^T_{k+j|k} w_{k+j|k} \right] \right\} < 0$$

(23)

According to the (23), we can list a weighted matrix, (24), as shown at the bottom of the next page.

The solution can be derived from the minimax theory,

$$\min \max_w J(u, w)$$

(25)

where,

$$J(u, w) = \sum_{j=0}^{j-1} \left[ x^T_{k+j|k} Q x_{k+j|k} + u^T_{k+j|k} R u_{k+j|k} - \gamma^2 w^T_{k+j|k} w_{k+j|k} \right] + x^T_{f} Q_f x_f$$

(26)

Then, the following function can be obtained,

$$J(z_k|k, z'_{0:k}, u_{0:k}, w_{0:k+j|k})$$

$$= \sum_{j=0}^{j-1} \left[ (z_k+j|k - z'_{k+j|k})^T Q (z_k+j|k - z'_{k+j|k}) + u^T_{k+j|k} R u_{k+j|k} - \gamma^2 w^T_{k+j|k} w_{k+j|k} \right] + (z_i - z'_{i})^T Q (z_i - z'_{i})$$

(27)

where $z_{k+j|k}$ is the autopilot preview path, which is expected to approach the target variable.

**Remark 5:** In this paper, we discuss the relationship between the minimax criterion and the difference game. Optimal control is to minimize the state quantity in the performance index, and the objective of the disturbance is to maximize the performance index. $u^*$ is the best control and $w^*$ is the worst disturbance. The control $u^*$ makes the performance criterion (27) a local minimum for all admissible controls. Under the condition of ensuring that the system meets the performance index (27), the maximum and minimum principle is used, and the system has good robustness under external interference.

In the lateral motion control of an automated driving vehicle, the target of steering control mainly minimizes the tracking errors of the road-path lateral displacement sequence in the future. Therefore, according to the characteristics of autonomous path tracking, minimax-based $H_\infty$ control will be extended to the receding horizon prediction control described in the next section.
C. MINIMAX BASED RECEeding HORIZON H∞ CONTROL

To derive the Receding Horizon $H_\infty$ control based on minimax criterion path tracking control strategies by using Lemmal 1, Corollary 1 is introduced.

Corollary 1: For the discrete-time system as (19) and minimax performance criterion as (27), in which $z(k+j|k) = C(z)x(k+j|k)$ is expected to approach the autopilot preview path variable $z^*_p(k+j|k)$. In the presence of external interference, the optimal solution is:

\[ u^*_k(k+j|k) = -R^{-1}B^T\Lambda^{-1} \]  
\[ \times \left[ M_{k+j+1|k} + N_{k+j+1|k} \right] \]  
\[ A_k x_{k+j|k} + g_{k+j+1|k} \]  
\[ w^*_k(k+j|k) = \gamma^2 R_{w}^{-1} B_{w}^T \Lambda^{-1} \]  
\[ \times \left[ M_{k+j+1|k} + N_{k+j+1|k} \right] \]  
\[ A_k x_{k+j|k} + g_{k+j+1|k} \]  
\[ \]  
where $M(k+j|k,k+N|k)$ satisfies the backward recurrence relation. In which,

\[ M_{k+j+1|k,k+N|k} = A^T \Lambda^{-1} \]  
\[ M_{k+N|k,k+N|k} = \bar{Q}_f \]  
and

\[ g_{k+j+1|k,k+N|k} = A^T \Lambda^{-1} \]  
\[ g_{k+N|k,k+N|k} = -\bar{Q}_f x_{k+j+1|k} \]  
\[ \]  
Proof: Similar to the steering control strategy of the minimax $H_\infty$ control, the cost functions of the receding horizon $H_\infty$ control based on the minimax criterion should be given. Considering $z_{k+j|k} = C_z x_{k+j|k}$ is expected to approach $z^*_p(k+j|k)$, $J(u,w)$ can be rewritten as

\[ J(x_{k+j|k}, x^*, u_{k+j|k}, w_{k+j|k}) \]  
\[ = \sum_{j=0}^{y-1} \left[ \right] \]  
\[ + (x_{j+j} - x^*_{j+k})^TQ_f(x_{j+j} - x^*_{j+k}) \]  
\[ + (x_{j} - x^*_j)^TQ_f(x_{j} - x^*_j) \]  
\[ \]  
where $Q = C^T_z \bar{Q}_C z$ and $Q_f = C^T_z \bar{Q}_C z$.

First, the receding horizon of optimal control $[k,k+N]$ is given, where $k$ represents the current time and $N$ represents the step size. Therefore, the prediction horizon can be expressed in the following form:

\[ J(x_{k+j|k}, x^*, u_{k+j|k}, w_{k+j|k}) \]  
\[ = \sum_{j=0}^{y-1} \left[ \right] \]  
\[ + (x_{k+N|k} - x^*_{k+N|k})^TQ_f(x_{k+N|k} - x^*_{k+N|k}) \]  
\[ \]  
Define the Hamilton equation to find the optimal solution,

\[ H = \left[ \right] \]  
\[ \]  
According to Lemma 1, the conditions of the minimax solution are,

\[ x_{k+j+1|k} = \frac{\partial H}{\partial u_{k+j+1|k}} = A x_{k+j|k} + B_w w_{k+j|k} + B_{u_k} u_{k+j|k} \]  
\[ \]  
\[ p_{k+j|k} = \frac{\partial H}{\partial x_{k+j|k}} = 2 Q (x_{k+j|k} - x^*_{k+j|k}) + A^T p_{k+j+1|k} \]  
\[ \]  
\[ 0 = \frac{\partial H}{\partial w_{k+j|k}} = 2 R_{w} w_{k+j|k} + B_{w}^T p_{k+j+1|k} \]  
\[ \]  
\[ 0 = \frac{\partial h}{\partial x_{k+j|k}} = 2 Q_f (x_{k+j|k} - x^*_{k+j|k}) \]  
\[ \]  
Assume,

\[ p_{k+j|k} = 2 M(k+j|k,k+N|k)x_{k+j|k} + 2 g(k+j|k,k+N|k) \]  
\[ \]  
From (32), (34), and (38), we have

\[ \frac{\partial H}{\partial u_{k+j|k}} = 2 R_{u} u_{k+j|k} + B_{u}^T p_{k+j+1|k} \]  
\[ \]  
\[ 2 R_{u} u_{k+j|k} + 2 B_{u}^T M_{k+j+1|k,k+N+1} x_{k+j+1|k} \]  
\[ + 2 B_{u}^T g_{k+j+1|k,k+N|k} \]  
\[ = 2 R_{u} u_{k+j|k} + 2 B_{u}^T M_{k+j+1|k,k+N+1} (A x_{k+j|k}) \]  
\[ + B_{u}^T w_{k+j|k} + B_{u}^T g_{k+j+1|k,k+N|k} \]  
\[ \]  
Similarly, we have

\[ \frac{\partial H}{\partial u_{k+j|k}} = 2 y^2 R_{w} w_{k+j|k} + 2 B_{w}^T M_{k+j+1|k,k+N+1} (A x_{k+j|k}) \]  
\[ + B_{w}^T w_{k+j|k} + B_{w}^T g_{k+j+1|k,k+N|k} \]  
\[ \]  
The methods of transforming $u_i$ and $w_i$ into $w^*_i$ and $u^*_i$ are described as

\[ x(k+j+1|k) = Ax(k+j|k) \]  
\[ + \frac{1}{2} (-B_{u} R_{u}^{-1} B_{u}^T + \gamma^2 B_{w} R_{w}^{-1} B_{w}^T) p_{k+j+1|k} \]  
\[ \]  
\[ \max_{w_{k+j|k}} \left\{ \sum_{j=0}^{y-1} \right\} \]  
\[ + \frac{1}{2} (-B_{u} R_{u}^{-1} B_{u}^T + \gamma^2 B_{w} R_{w}^{-1} B_{w}^T) p_{k+j+1|k} \]  
\[ \]  
\[ < 0 \]  
\[ (24) \]
From (38) and (41) we obtain

\[
p_{k+j+1|k} = 2M(k+j+1|k, k+N|k)\xi_{k+j+1|k} + 2g(k+j+1|k, k+N|k)
\]

\[
= 2M(k+j+1|k, k+N|k)A_k + M(k+j+1|k, k+N|k)
\]

\[
(-BR^{-1}B^T + \gamma^{-2}B_sR_w^{-1}B^Tw)(k+j+1|k)
\]

\[
+ 2g(k+j+1|k, k+N|k)
\]  

(42)

Therefore,

\[
p_{k+j+1|k} = 2\left[I + M_{k+j+1|k, k+N|k}(-BR^{-1}B^T + \gamma^{-2}B_sR_w^{-1}B^Tw)\right]^{-1}
\]

\[
\times (M_{k+j+1|k, k+N|k}A_k + g_{k+j+1|k, k+N|k})
\]

(43)

Let

\[
A_{k+j+1|k, k+N|k}
\]

\[
= I + M_{k+j+1|k, k+N|k}(-BR^{-1}B^T + \gamma^{-2}B_sR_w^{-1}B^Tw)
\]

(44)

Then \(p_{k+j+1|k}\) is rewritten as

\[
p_{k+j+1|k} = 2A_{k+j+1|k, k+N|k}^{-1}
\]

\[
\times \left[I + M_{k+j+1|k, k+N|k}A_k + g_{k+j+1|k, k+N|k}\right]
\]

(45)

If we substitute (45) into (33), then we obtain

\[
p_{k+j|k} = 2Q(x_{k+j|k} - \lambda^T_{k+j|k}) + 2A^T\lambda_{k+j+1|k, k+N|k}
\]

\[
\times \left[I + M_{k+j+1|k, k+N|k}A_k + g_{k+j+1|k, k+N|k}\right]
\]

\[
= 2\left[+A^T\lambda_{k+j+1|k, k+N|k}M_{k+j+1|k, k+N|k}A + Q\right]x_{k+j|k}
\]

\[
+ 2A^T\lambda_{k+j+1|k, k+N|k}g_{k+j+1|k, k+N|k} - 2Qx_{k+j|k}
\]

(46)

Therefore, from (36) and the assumption (38), we have

\[
M_{k+j|k, k+N|k}
\]

\[
= A^T\lambda_{k+j+1|k, k+N|k}M_{k+j+1|k, k+N|k}A + Q
\]

(47)

And

\[
g_{k+j|k, k+N|k}
\]

\[
= A^T\lambda_{k+j+1|k, k+N|k}g_{k+j+1|k, k+N|k} - Qx_{k+j|k}
\]

\[
= -Q_{x_{k+j|k}}
\]

(48)

From (34) and (35), the \(H_\infty\) control can be given by

\[
u_{k+j|k} = -R^{-1}B^T\lambda_{k+j+1|k, k+N|k}
\]

\[
\times \left[I + M_{k+j+1|k, k+N|k}A_k + g_{k+j+1|k, k+N|k}\right]
\]

\[
w_{k+j|k} = \gamma^{-2}B_sR_w^{-1}B^Tw\lambda_{k+j+1|k, k+N|k}
\]

\[
\times \left[I + M_{k+j+1|k, k+N|k}A_k + g_{k+j+1|k, k+N|k}\right]
\]

\[
+ g_{k+j+1|k, k+N|k}
\]

(49)

Finally, according to (44), (46), (47), and (48), the control solutions that satisfy the minimax-based rolling horizon robust control in Corollary 1 can be derived.

IV. RESULTS AND ANALYSIS

In this section, a driving scenario of single-lane change (SLC) is used to verify the designed control method. The path to be tracked is shown in Fig. 4. The ideal lane change trajectory can be found in [20], \(L_1 \leq X \leq L_1 + L_2\) where \(L_1 = 70\), \(L_2 = 40\), and \(L_3 = 150\). The ideal lane-change trajectory is shown in Fig. 4.

![FIGURE 4. Ideal lane change trajectory.](image)

\[X\text{ and } Y\text{ are the abscissa and ordinate of the curve of AB, respectively. The parameters of the control model were set as } m = 1385 \text{ kg}, I = 2065 \text{ kg-m}^2, l_f = 1.114 \text{ m}, l_r = 1.436 \text{ m}, C_f = 85000 \text{ N-rad}, C_r = 123000 \text{ N-rad}, U = 110 \text{ km/h}, \text{ and the discretization simulation step length was set as } T_s = 0.01 \text{ s}.

To evaluate the effect of lateral motion control of an automated driving vehicle, a normal SLC driving scenario with an extreme situation was considered. Based on the actual road conditions, the width of the road was set to 3m and the road adhesion coefficient was set to 0.80.

To verify the effect of the receding horizon robust control, the three control strategies proposed in this paper were tested by computer simulation and Hardware-In-Loop (HIL) experiments.

A. SLC DRIVING SCENARIO SIMULATION

Figure 5 shows the simulation results for the different control strategies. They are respectively the results of MPC control strategy, receding horizon \(H_\infty\) control strategy and LMI control strategy for vehicle control path tracking.

![FIGURE 5. Simulation results for the different control strategies.](image)

Figure 5(a) shows a comparison of the three control strategies to control the lateral displacement of the vehicles. It can be clearly seen that the three different control strategies can effectively control the vehicles to track the target path. It can be seen from the figure that the receding horizon \(H_\infty\) controlled vehicle starts tracking at the target path at 1.5s. The other two algorithms start tracking at 1.9s, so the receding horizon \(H_\infty\) controller responds faster and more smoothly.

Figure 5(b) shows a comparison of the steering wheel angle data of the three control strategies. It is obvious from the figure that the steering angle of the receding horizon robust control varies from \(-20^\circ\) to \(20^\circ\), while the angle of the other two algorithms varies from \(-26^\circ\) to \(26^\circ\). Therefore, the overshoot of the receding horizon robust control was smaller.

At high speeds, a large steering wheel angle is more likely to lead the vehicle to dangerous conditions. In addition, a larger steering wheel angle requires more energy. Therefore, the
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FIGURE 5. Vehicle status during road tracking.

proposed robust control can achieve the desired effect while consuming less energy. This proves that the proposed robust control algorithm exhibits optimal control performance.

Figure 5(c) compares the vehicle yaw rate controlled by the three control strategies. It can be clearly seen that the vehicle yaw rate value controlled by the receding horizon $H_\infty$ control algorithm varies within the range of $-6^\circ/s \sim -6^\circ/s$, and the vehicle yaw rate value controlled by the other two control strategies varies within the range of $-7^\circ/s \sim -8^\circ/s$ respectively. Therefore, the receding horizon $H_\infty$ control vehicle is more stable and safe during the driving process. In addition, it can be found that the vehicle controlled by receding horizon $H_\infty$ control converges the yaw faster, which proves that the controller has better control performance.

Figure 5(d) shows a comparison of the vehicle lateral acceleration data controlled by the three control strategies. The lateral acceleration value of the proposed robust control vehicle varies within the range of $-2.8m/s^2 \sim 2.8m/s^2$, whereas the lateral acceleration values of the other two control strategies vary within the range of $-3.2m/s^2 \sim 3.2m/s^2$. A smaller lateral acceleration ensures stable driving of the vehicle in receding horizon $H_\infty$ control.

Figure 5(e) shows a comparison of the data of the vehicle side slip angle controlled by the three control strategies. The vehicle controlled by the receding horizon robust control has smaller data oversets. The controller can ensure the control performance while improving the robustness of the system.

In summary, compared with the other two control algorithms, the receding horizon $H_\infty$ autonomous steering control strategy can achieve the desired control effect with lower energy consumption and better control performance.

B. SLC DRIVING SCENARIO HIL EXPERIMENTS

Owing to the high cost and inconvenience of real vehicle tests, Hardware-In-Loop (HIL) experiment is widely applied, in which the real-time vehicle-road model is substituted for real vehicle and environment information; however real steering and braking actuators are included to maintain the real characteristics of actuators. It can reduce the complicated work procedure compared with real vehicle tests and can efficiently perform repeated test validation. Therefore, in order to evaluate the receding horizon $H_\infty$ autonomous steering effect and real-time performance, high-speed autonomous lane change maneuver Hardware-In-Loop (HIL) experiments were performed for different test scenarios.

The HIL system used in this study is illustrated in Fig. 6. In the HIL system, Carsim is utilized to establish the vehicle and road model that can interact, and the real steering controller is realized by Labview RT software embedded in the PXI controller, and the Simulink program designed for autonomous steering control in this study can be recalled by the Labview MIT toolkits.

Fig.7 shows the entire structure of the receding horizon robust control autonomous steering strategy. The structure mainly includes the vehicle and road model, previewed road shift register, receding horizon $H_\infty$ upper controller, steering system and lower controller, and 27-DOF vehicle simulation model. The (NP+1) road-path lateral displacements $y_0, y_1, y_2, \ldots, y_j, \ldots, y_{N_p-1}, \ldots, y_{N_p}$ were designed through.
However, the receding horizon method can improve the robustness of the system. The data collected by the sensor fluctuated slightly owing to external interference. The car corner range controlled by the receding horizon $H_{\infty}$ is $-20^\circ \sim 20^\circ$, and the steering angle controlled by the other two control algorithms changes within the range of $-25^\circ \sim 25^\circ$. Therefore, the steering column angle overshoot of the receding horizon control $H_{\infty}$ is smaller, which consumes less energy while achieving the desired path tracking.

Figure 8(c) compares the vehicle yaw rate controlled by the three control strategies. It can be concluded that the yaw rate is controlled by the receding horizon $H_{\infty}$ control algorithm, which varies within the range of $-5.5^\circ/s \sim 5.5^\circ/s$, and the vehicle yaw rate value controlled by the other two control strategies varies within the range of $-7.5^\circ/s \sim 7.5^\circ/s$. The lower yaw rate of the receding horizon $H_{\infty}$ control proves that a more stable vehicle can guarantee the safety of the vehicle. The receding horizon $H_{\infty}$ control vehicle converges to zero and does not fluctuate at 5.5/s, which proves that the vehicle travels more stably. At 8s, the vehicle controlled by the other two control algorithms wobbles, which proves that the receding horizon $H_{\infty}$ control strategy has good robustness.

Figure 8(d) shows a comparison of the vehicle lateral acceleration data controlled by the three control strategies. It can be clearly seen that the lateral acceleration changes in the range of $-3m/s^2 \sim 3m/s^2$ under the control of the horizon $H_{\infty}$ and LMI, while the lateral acceleration value of the MPC control strategy varies within the range of $-3.6m/s^2 \sim 3.6m/s^2$.

Figure 8(e) shows a comparison of the data of the vehicle side slip angle controlled by the three control strategies. It is obvious that the overshoot of the receding horizon $H_{\infty}$ controlled vehicle is smaller and more stable, which can improve the robustness of the system.

To verify the anti-interference ability of rolling time-domain robust control in the presence of strong and slow disturbances, a disturbance was added to the HIL SLC in this experiment. It is worth noting that this disturbance acts on the CarSim vehicle model in the form of the front wheel rotation angle. An analysis of the data is presented in Figure 9.

Figure 9(a) shows the interference input data. It can be seen from the figure that it is a slowly changing data with a range of $-2^\circ \sim 2^\circ$, which will be magnified several times at a high speed, causing no small impact on the stability of the vehicle. The main fluctuation occurs in 3~5s, and then 5~7s shows a gradual convergence trend.

Figure 9(b) compares the path tracking data of the three schemes of vehicle MPC, LMI, and receding horizon $H_{\infty}$ control with the target path. It can be seen from the figure that the interference input has a great influence on the stability of the vehicle during 3~5s, in which the lateral displacement deviation of the vehicle is the largest under MPC control, and the deviation reaches 0.4 during 3.5~4.5s. However, LMI has the smallest error, but it still fluctuates after 4.5s, while the rolling time domain receding horizon $H_{\infty}$ control also has a small deviation between 3.5~4.5s, but it basically coincides with the target path after 4.5s, and there is no obvious jitter in the whole process.

**FIGURE 6.** Vehicle HIL system.

**FIGURE 7.** Control system structure diagram.

The test data are compared and analyzed in Figure 8. As there is a great influence of external interference on the test process in the bench test, it can be seen from the figure that the data are not as smooth as the simulated data.

Figure 8(a) compares the lateral displacement data of the vehicle controlled by the three control strategies. It can be clearly seen that the three different control strategies can effectively control the vehicle to track the target path. Owing to the influence of hardware delay, in the loop test, it takes a long time for the vehicle to return to the desired path. However, the receding horizon $H_{\infty}$ controlled vehicle has fully reached the target path at 6s, with a fast convergence speed and a small overdrive at 4.5s. Therefore, the rolling horizon method can improve the robustness of the system.

Figure 8(b) shows the actual steering column angles of the three control strategies. The data collected by the sensor include the steering resisting moment loading module by the servo motor and reducer, pinion-EPS motor, and lower motor controller. The resisting moment loading module produces the steering resisting moment calculated by the 27-DOF vehicle simulation model, which controls the pinion-EPS motor to track the target steering angle from the upper controller, and transmits the actual steering angle measured by the steering angle sensor to the 27-DOF vehicle simulation model of CarSim.

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However, the receding horizon method can improve the robustness of the system. The data collected by the sensor
Figure 9(c) shows the steering wheel angles of the three control schemes. It can be seen from the figure that the steering wheel angles have a large range and an obvious jangle under interference. In particular, the steering wheel angles of MPC and LMI exceeded 70 degrees during 2.5~4s and
their oscillations exceeded $-20^\circ \sim 20^\circ$ during 4~8s, while the receding horizon $H_\infty$ control has a smaller oscillation amplitude and faster convergence, which is sufficient to prove its strong robustness.

Figure 9(d) for the yaw angular velocity of the vehicle, the figure shows that the MPC control of the horizontal pendulum angular velocity amplitude is larger, especially in its strong robustness.

Figure 9(e)~9(f) show the lateral acceleration and the center of mass of the vehicle side-slip angle. The receding horizon $H_\infty$ control scheme has a minimum oscillation amplitude and faster convergence in the 7s, while both the MPC and LMI robust control oscillation amplitudes are two times higher and have slower convergence rates. This demonstrates the effectiveness of receding horizon $H_\infty$ control.

In summary, the receding horizon $H_\infty$ control strategy can guarantee tracking performance and improve the robustness of the system when it is used for vehicle path tracking.

V. CONCLUSION

In view of the vehicle model uncertainty, the state of the roads outside interference, and lateral movement of the inherent nonlinear problem, this paper proposes a minimax rolling time domain based on the theory of robust control method, first the two degrees of the freedom vehicle model and road model was set up; then, the ponte-aurous Hamilton equation was used to establish the principle of minimum value, and the optimal control input and maximum disturbance input were solved based on the minimax principle. Finally, the feasibility of the proposed scheme was verified by simulation and HIL experiments. The following conclusions were drawn:

1) Considering the influence of vehicle parameter uncertainties on the autonomous steering controller, the receding horizon robust control method was established and used for autonomous steering control in this study. The proposed receding horizon robust control method is an improvement from the receding horizon and robust control, and can reduce the influence of parameter uncertainties on the autonomous steering controller.

2) The MPC controller and LMI controller are stable, but at the cost of performance. Only the receding horizon $H_\infty$ controller overcomes the contradiction between performance and robustness, thus the steering stability and high performance can be guaranteed.

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