Mixed Weibull distribution model of DC protection system based on entropy weight method

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Abstract. By analyzing the blocking event of DC converter station of the State Grid Corporation in recently ten years, it can be concluded that the sampling system failure, secondary circuit fault, board fault, host fault, etc. of the protection system caused more than 10% of the DC system blockings, which have seriously threatened the normal operation of DC system. Towards this end, this paper makes a statistical analysis of the operation defects of high-voltage direct current (HVDC) transmission protection in Hubei Company in the past five years. A mixed Weibull distribution model method based on entropy weight method is proposed. By using this method, the reliability curve and average defect interval time (MTBF) of the DC protection system are obtained. Combined with the actual situation of the converter station, the DC protection system of the converter station is evaluated, which provides an important guiding basis for the subsequent maintenance, system transformation, and replacement of the converter station.

1. Introduction

As the core of the DC power transmission system, DC protection is directly related to the safe and reliable operation of the DC power transmission system. By the end of 2018, there were 23 HVDC projects under construction and operation by the State Grid Corporation (11 ultra high voltage direct current (UHVDC) projects and 13 HVDC projects).

Scholars in China and overseas have carried out much research on the evaluation method of AC protection reliability. At present, the widely accepted and widely used relay protection reliability evaluation methods are Markov model method, fault tree analysis method and GO probability analysis method. For example, The reliability model of safety control management system based on fault tree [1], Reliability Model of MMC Converter Valve based on Fault Tree [2], Running State and Its Risk Evaluation of Transmission Line Based on Markov Chain Model [3], Reliability Analysis Model for Protective Relaying System of UHV Power Network Based on Markov State-Space Method [4], Reliability Assessment of Protections for HVDC Transmission System Based on Extended Goal-Oriented Method [5], Application of GO Methodology in Reliability Assessment of Protective Relays [6], etc. These methods have been applied in AC / DC system, but compared with AC protection; the research of DC protection system is less. Because of the different interactions between DC protection...
and AC system, there are many common signals in DC protection system, and the sample of DC transmission protection systems is relatively small. Therefore, the feasibility of applying existing reliability evaluation methods to HVDC protection reliability evaluation needs to be further studied.

In this paper, the mixed Weibull distribution model based on the entropy weight method is used to analyze the DC protection defect data, and the reliability of the DC protection system is determined. Combined with the actual situation of the converter station, the effectiveness of the method is verified, which provides an important guide for the evaluation, operation, and maintenance of the DC protection system of the converter station and the transformation of the system.

2. Mixed Weibull reliability evaluation method based on entropy weight method

Weibull distribution is widely used in reliability engineering. Because the failure mode of the protection device of the DC protection system is different, and the various devices are independent of each other, the mixed Weibull model composed of single Weibull distribution of various protective devices [7] is more realistic. In order to obtain the mixed Weibull distribution of DC protection system, first of all, the single Weibull distribution of all kinds of equipment in DC protection system is obtained. Combined with the weight coefficient of all kinds of equipment calculated by entropy weight method, the mixed Weibull distribution of DC protection system is obtained, finally, the reliability of DC protection system is evaluated.

2.1. Determination of weight coefficient by entropy weight method

Entropy is a measure of the degree of system disorder. If the information entropy of the index is smaller, the more information the index provides, the greater the role in the evaluation, and the greater the weight. The entropy weight method is an objective weighting method. The greater the difference of an index, the smaller the entropy weight, the greater the amount of information provided by the index, the greater the weight of the index [8-9]. The specific steps of entropy weight method are as follows:

1) For a relational matrix \((x_{ij})_{m \times n}\), first of all, the data preprocessing is carried out, and the standard matrix \((a_{ij})_{m \times n}\) is obtained, \(m\) is the number of defective components, \(n\) is the number of evaluation indexes. The index data in the matrix are divided into forward index, reverse index and interval index, wherein the forward index is that the larger the evaluation value is, the better the index is, the reverse index is that the smaller the evaluation value is, the better the index is, the interval index is that the closer the evaluation value is to the middle of the interval, the better. The processing of the index data is as follows:

   The consistent treatment formula of forward index is:
   \[
   a_{ij} = \frac{x_{ij} - x_{\min}}{x_{\max} - x_{\min}}
   \]  
   (1)

   The consistent treatment formula of reverse index is:
   \[
   a_{ij} = \frac{x_{\max} - x_{ij}}{x_{\max} - x_{\min}}
   \]  
   (2)

   The consistent treatment formula of interval index is:
   \[
   a_{ij} = \frac{|x_{\text{mid}} - x_{ij}|}{\max(|x_{\text{mid}} - x_i|)}
   \]  
   (3)

   In the formula, \(x_{\max}\) is the maximum value in the same type of indicator data, \(x_{\min}\) is the minimum value in the same kind of indicator data; \(x_{\text{mid}}\) is the intermediate value in the same kind of indicator data.

2) Characteristic weight
According to the standard matrix, the characteristic weight of the j index of the i equipment is obtained.

$$P_{ij} = \frac{a_{ij}}{\sum_{j=1}^{n} a_{ij}}$$

(4)

When $P_{ij}=0$, the calculated entropy value is meaningless. Therefore, $P_{ij}$ is amended [10] as

$$P_{ij} = \frac{1+a_{ij}}{\sum_{j=1}^{n} (1+a_{ij})}$$

(5)

3) Calculate the entropy value of the jth index

$$H_j = -\frac{1}{\ln(m)} \sum_{i=1}^{m} (P_{ij} \ln(P_{ij}))$$

(6)

In the formula: $j=1, 2, \cdots, n$.

4) Calculate the entropy weight of the j index

$$W_j = \frac{1-H_j}{n-\sum_{j=1}^{n} H_j}$$

(7)

In the formula: $j=1, 2, \cdots, n$.

5) Weight coefficient of various components

The $Z_i$ calculation method of defect data weight coefficient of various components in DC transmission protection system is as follows:

$$Z_i = \sum_{j=1}^{n} x_{ij} W_j$$

(8)

In the formula: $i=1, 2, \cdots, m$.

6) Weight coefficient of all kinds of equipment

In the above Formula (8), the weight coefficient of each component is obtained, and the weight coefficient of each type of equipment is obtained according to the weight coefficient of the components contained in the various devices in Table 1.

2.2. Single Weibull distribution

Weibull distribution is one of the most common distribution forms to describe product life. For the reliability analysis of power equipment, the existing research shows that the Weibull distribution with two parameters is the most common, and its probability density function, cumulative failure distribution function, reliability function and failure rate function [11-14] are as follows:

$$f(t) = \frac{\beta}{\eta} \left( \frac{t}{\eta} \right)^{\beta-1} \exp\left( -\left( \frac{t}{\eta} \right)^{\beta} \right)$$

(9)

$$F(t) = 1 - \exp\left( -\left( \frac{t}{\eta} \right)^{\beta} \right)$$

(10)

$$R(x) = \exp\left( -\left( \frac{x}{\eta} \right)^{\beta} \right)$$

(11)

$$\lambda(t) = \frac{f(t)}{R(t)}$$

(12)

The maximum likelihood estimation method is used to obtain the shape parameter $\beta$ and the shape parameter $\eta$. The likelihood function [15] is as follows:
\[ L(\beta, \eta)=\left\{ \frac{\beta}{\eta}(\frac{t}{\eta})^{\beta+1} \exp \left[ -\left( \frac{t}{\eta} \right)^{\beta} \right] \right\} \]  

(13)

For each parameter in the above formula, the partial derivative is obtained. The equations obtained are as follows:

\[
\frac{\partial \ln L}{\partial \beta} = \frac{n}{\beta} + \sum_{i=1}^{n} \ln \left( \frac{t_i}{\eta} \right) - \sum_{i=1}^{n} \frac{\beta}{\eta} \ln \left( \frac{t_i}{\eta} \right) = 0
\]

\[
\frac{\partial \ln L}{\partial \eta} = -\frac{n\beta}{\eta} + \beta \sum_{i=1}^{n} \left( \frac{t_i}{\eta} \right)^{\beta} = 0
\]

(14)

Formula (14) is a transcendental system, which cannot directly obtain the parameters. In order to solve the single Weibull distribution of all kinds of equipment, the Newton-Raphson iterative method is used on Matlab.

2.3. Mixed Weibull model

The mixed Weibull model of the DC protection system is as follows:

\[ R(t) = \sum_{i=1}^{5} P_i R_i(t) \]  

(15)

\[ f(t) = \sum_{i=1}^{5} P_i f_i(t) \]  

(16)

\[ P = P_1 + P_2 + P_3 + P_4 + P_5 \]  

(17)

\[ \lambda(t) = \frac{f(t)}{R(t)} \]  

(18)

In the formula, \( R(t) \) is a reliability function of DC protection system, \( R_i(t) \) is the reliability function of class i equipment, \( f(t) \) is a probability density function of DC protection system, \( f_i(t) \) is the probability density function of Class i equipment, \( P_i \) is the weight coefficient corresponding to the reliability function of class i equipment, it is obtained by entropy weight method, \( P \) is the total weight coefficient, the value is 1, \( \lambda(t) \) is the failure rate function of DC protection system.

**Table 1. Classification of DC protection system equipment.**

| Device name                  | Name of component contained in equipment                                                                 |
|------------------------------|----------------------------------------------------------------------------------------------------------|
| measurement equipment        | Photoelectric CT, Pure optical CT, Photoelectric PT, Zero flux transformer, Conventional transformer        |
|                              | processor board, switching value interface board, analog interface board, communication board, power supply module, chassis backplane |
| measurement interface device | Host type \[ CPU board, PCI board, power module, software \]                                              |
|                              | Device type \[ processor plug-in, communication plug-in, input/output plug-in, power supply plug-in, Chassis backplane, software \] |
| DC protection device         | Independent protection type \[ processor plug-in, input/output plug-in, MMI plug-in, A/D(VFC) plug-in, communication plug-in, AC plug-in, power supply plug-in, software \] |
| three-out-of-two device      | Switch operation box, cable loop, fiber optic loop, communication channel, accessories                     |
| trip outlet and secondary circuit | Switch operation box, cable loop, power supply board, communication board, independent module          |
3. Mixed Weibull evaluator for DC protection system

3.1. Defect classification of DC protection system

DC protection refers to the protection equipment related to DC transmission, including AC and DC bus protection, AC and DC line protection, AC and DC filter protection, converter protection and switch protection, etc. Combined with the characteristics of DC protection system, according to the location of the defects, DC protection system defects are divided into the following five categories: measuring equipment defects, measuring interface device defects, DC protection device defects (main type, device type and independent type), three take two device defects, trip outlet and secondary circuit defects. The components contained in all kinds of equipment are shown in Table 1.

3.2. Defect index of DC protection system

The defect data of the HVDC protection system in Hubei Power Grid from January 2014 to June 2019 and the equipment operation information of five converter stations are collected, and three index data of DC protection system defects are determined. These indexes are component defect-free time rate MTTF, component average maintenance time rate MTTR and component defect rate EDR, they reflect the running time of DC protection system equipment, defect maintenance time and the probability of defects. The specific calculation methods are as follows:

\[
MTTF = \frac{\sum T_{ND}}{N_D} \quad (19)
\]

\[
T_{ND} = \frac{T_{TNO}}{N_D} \quad (20)
\]

\[
MTTR = \frac{\sum T_{AC}}{N_R} \quad (21)
\]

\[
T_{AC} = \frac{T_{TDM}}{N_T} \quad (22)
\]

\[
EDR = \frac{N_D}{N_T} \quad (23)
\]

In this formula, \(T_{ND}\) is the defect-free time of the component, \(N_D\) is the number of defects in the given time period, \(T_{TNO}\) is the total time for the normal operation of the component, \(\sum T_{ND}\) is the sum of defect-free time for all kinds of components, \(T_{AC}\) is the average maintenance time of components, \(N_R\) is the number of times the component defects are repaired in a given time period, \(T_{TDM}\) is the total time for defect maintenance of the component, \(\sum T_{AC}\) is the sum of the average maintenance time of all kinds of components, \(N_T\) is the total number of DC protection system defects in a given time period.

3.3. Mixed Weibull model of Longquan converter station

Taking the Longquan converter station as an example, this method is used to evaluate the reliability of the Longquan converter station.

3.3.1. Weight coefficient of defect index of DC protection system. Using the entropy weight method, the defect index data of the Longquan converter station as shown in Table 2 is obtained.

Table 2 above is used as the relational matrix \((x_{ij})_{9 \times 3}\), first of all, the data preprocessing is carried out, and the standard matrix \((a_{ij})_{9 \times 3}\) is obtained. The weight coefficients of various components in the defect data and the weight coefficients of all kinds of equipment in DC protection system are obtained by entropy weight method, as shown in Table 3 below.
### Table 2. Defect index data of the Longquan converter station.

| device name           | defect element          | component defect free time rate $MTTF$ | component average maintenance time rate $MTTR$ | component defect rate $EDR$ |
|-----------------------|-------------------------|----------------------------------------|-----------------------------------------------|-----------------------------|
| measurement equipment | Photoelectric CT        | 0.145014                               | 0.205993                                      | 0.303797                    |
|                       | processor board         | 0.105065                               | 0.157303                                      | 0.050633                    |
|                       | switching value interface board | 0.125934                               | 0.089888                                      | 0.025316                    |
|                       | analog interface board  |                                        |                                               |                             |
|                       | communication board     |                                        |                                               |                             |
|                       | power supply module     |                                        |                                               |                             |
| measurement interface device |                  | 0.105084                               | 0.089888                                      | 0.025316                    |
| Host type DC protection | CPU board,             | 0.118252                               | 0.089888                                      | 0.240506                    |
|                       | PCI board               | 0.115507                               | 0.097378                                      | 0.151899                    |
| Independent type DC protection | processor plug-in    | 0.069765                               | 0.089888                                      | 0.012658                    |

### Table 3. Weight coefficient of the Longquan converter station.

| device name           | defect element          | Weight coefficient of various components | Weight coefficient of equipment |
|-----------------------|-------------------------|-------------------------------------------|---------------------------------|
| measurement equipment | Photoelectric CT        | 0.2287496                                 | 0.2287496                       |
|                       | processor board         | 0.1026067                                 |                                |
|                       | switching value interface board | 0.0738034                               |                                |
|                       | analog interface board  |                                           |                                |
|                       | communication board     |                                           |                                |
|                       | power supply module     |                                           |                                |
| measurement interface device |                  | 0.0685575                               | 0.4376350                       |
|                       | communication board     |                                           |                                |
|                       | power supply module     |                                           |                                |
| Host type DC protection | CPU board,             | 0.1556975                               |                                |
|                       | PCI board               | 0.1231778                               |                                |
| Independent type DC protection | processor plug-in    | 0.0547401                               | 0.3336154                       |

3.3.2. **Mixed Weibull model for DC protection system.** Taking the time interval of each defect in the Longquan converter station as a sample, the single Weibull distribution of all kinds of equipment is obtained, and then the mixed Weibull model is obtained. The parameters and weights of Weibull distribution of all kinds of equipment in the Longquan converter station are shown in Table 4 below.

From the mixed Weibull model of the Longquan converter station, the corresponding reliability function $R(x)$, probability density function $f(x)$ and failure rate function $\lambda(x)$ can be obtained. According to these functions, the reliability of DC protection system, the average defect interval time $MTBF$ and the probability of the next defect can be obtained.
Table 4. Weibull distribution parameters and weights for each type of equipment in the Longquan converter station.

| device name            | shape parameter | scale parameter | Weight coefficient of equipment |
|------------------------|-----------------|-----------------|---------------------------------|
| measurement equipment  | 34.2053         | 1.0364          | 0.2287                          |
| measurement interface device | 17.7369   | 1.3273          | 0.4376                          |
| DC protection device   | 29.2556         | 0.9702          | 0.3336                          |

The average defect interval time [11-13] is:

\[ MTBF = \int_0^\infty f(x)dx \]  

(24)

The average defect interval time \( MTBF \) of the Longquan converter station is 24.73 days. From January 2014 to June 2019, 79 defects occurred in the Longquan converter station, and the actual average defect interval was 25.39 days. There is little difference between the two values, which verifies the correctness of the method. The probability of the next defect can be judged to be 0.0456 by the failure rate function and the average defect interval time. The average defect interval time \( MTBF \) and the actual defect interval time of all kinds of equipment in the Longquan converter station are compared as shown in Table 5 below.

Table 5. Comparison of defect interval time of all kinds of equipment in the Longquan converter station.

| device name            | \( MTBF/\text{day} \) | actual defect interval time/ \( \text{day} \) |
|------------------------|------------------------|-----------------------------------------------|
| measurement equipment  | 33.70                  | 30.92                                         |
| measurement interface device | 16.30          | 14.81                                         |
| DC protection device   | 29.63                  | 28.72                                         |

The defect interval time of the measuring interface device of the Longquan converter station is the shortest, the defect rate is the highest, the maintenance time of the measuring interface device should be shortened, and the maintenance and storage of the spare parts shall be strengthened, and the defect repair time shall be reduced.

4. Evaluation of DC protection system in five converter stations

4.1. Basic situation of five converter stations

The basic conditions of each station are as follows:

1) The Gezhouba converter station is the first ±500kV HVDC transmission project in China. It was put into operation in August 1990. After three major revampings, the important equipment has been basically reformed.

2) The Longquan converter station is the first terminal of the "three Gorges-Jiangsu" ±500kV Longzheng DC transmission project, which was put into operation in June 2003.

3) The Jiangling converter station is the first terminal of the "three Gorges-Guangdong" ±500kV Jiangcheng DC transmission project, which was put into operation in June 2004.

4) The Yidu converter station is the first terminal of the "three Gorges-Shanghai" ±500kV Yihua DC transmission project, which was put into operation in January 2007.

5) The Tuanlin converter station is the first terminal of the "three Gorges-Shanghai" ±500kV Linfeng DC transmission project, which was put into operation in May 2011.
4.2. Mixed Weibull model of five converter stations

The mixed Weibull model and related functions of the other four converter stations are obtained by the same method as the Longquan converter station. The reliability function of the five converter stations is shown in Figure 1, and the calculated defect interval time is compared with the actual defect time interval as shown in Table 6.

![Figure 1](image_url)

Figure 1. Reliability functions of five converter stations.

| Name of converter station | MTBF/day | actual defect interval time/ day | Next defect probability/ % |
|--------------------------|----------|---------------------------------|---------------------------|
| Longquan converter station | 24.73    | 25.47                           | 4.56%                     |
| Jiangling converter station | 39.85    | 34.00                           | 2.31%                     |
| Tuanlin converter station | 93.70    | 91.18                           | 1.07%                     |
| Yidu converter station | 99.54    | 105.58                          | 0.86%                     |
| Gezhouba converter station | 102.59   | 105.58                          | 0.73%                     |

As can be seen from Figure 1 and Table 6, combined with the actual operation of the converter station, the following conclusions are obtained:

1) The average defect interval time of the Longquan converter station is the shortest, the system reliability is the lowest, the probability of the next defect is the highest, it is 8 times of the Gezhouba converter station. The Longquan converter station is put into operation in 2003, and the operation time is up to 16 years. Although the DC protection system does not have the equipment specified in the AC system for more than 10 years to replace the equipment, the operation time of the system is long, and the probability of the defect will be increased. The Longquan converter station has two defects in July and August 2019, and it is suggested to transform Longquan converter station as soon as possible.

2) The defect interval of Jiangling converter station is short and the reliability is low. Two defects occurred in July and August 2019. Jiangling converter station was put into operation in 2004 and has been in operation for 15 years. It is suggested that Jiangling converter station should be reformed.

3) The Tuan Lin converter station was put into operation in 2011, the operation time is short, the Yidu converter station is put into operation in 2007, and the equipment operation time is longer. However, according to the data in Figure 1 and Table 6, the reliability of the Yidu converter station is
slightly higher than that of the Tuan Lin converter station. Two defects occurred in July and August 2019, and no defects occurred in the Yidu converter station.

4) Although the Gezhouba converter station is the bipolar as-built power transmission in August 1990, the new protection equipment has been put into operation in 2006 and 2016, and the equipment is relatively new, and some equipment has only been operated for a few years. Therefore, the overall reliability is the highest, two defects occurred in July and August 2019, and no defects occurred in the Gezhouba converter station.

5) At present, the reliability of the five-seat converter station is from high to low in order of the Gezhouba converter station, the Yidu converter station, the cluster forest converter station, the Jiangling converter station and the Longquan converter station.

6) It is suggested that the equipment maintenance time should be adjusted according to the average defect interval time $MTBF$ of each station, but the operation and maintenance time of the equipment should be shortened as much as possible for the Longquan converter station and the Jiangling converter station, and more spare parts should be prepared for the equipment with high defect rate, so as to shorten the repair time of defects.

5. Conclusions
In this paper, the mixed Weibull distribution of DC protection system is obtained based on the entropy weight method, and then the reliability curve and average defect interval time of DC protection system is obtained to determine the reliability of DC protection system. Combined with the actual situation of each converter station, the effectiveness of this method is verified, which provides an important guidance for DC protection system evaluation, operation and maintenance, system transformation and replacement of converter station. However, since most of the defect data comes from the equipment which has been running for more than 10 years, it is not good to reflect the DC protection system just put into operation, and the next step should be as perfect as possible.

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