Vacuum angle theta in a strongly interacting hidden sector and its effect

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Abstract

Introducing a vacuum angle $\theta_h$ in a strongly interacting hidden sector like QCD, we study how $\theta_h$ affects physical quantities in a scale invariant extension of the standard model with that hidden sector. In this model the dynamical chiral symmetry breaking occurs in the hidden sector and generates a scale $|\langle \bar{q}q \rangle|$ which triggers electroweak symmetry breaking. We find that the expression for a vacuum expectation value of the Higgs field depends on $\theta_h$, while the condition which determines the Higgs particle mass does not. This model contains the hidden-sector pions $\pi^0$ and $\pi^\pm$ which are candidates for cold dark matter, and we find the mass difference between the hidden-sector $\pi^0$ and $\pi^\pm$ will not be negligible for $\theta_h \approx \pi$.


1 Introduction

Attempts of scale invariant extension of the standard model (SM) are significant because there arises the hierarchy problem in many models beyond the SM. In Ref. [1], Hur and Ko proposed a scale invariant extension of the SM with a new QCD-like strong interaction with $N_{h,c} = 3$ colors in the hidden sector. In their extension, the dynamical chiral symmetry breaking occurs in the hidden sector and its dynamical scale $\langle \bar{q}q \rangle^{1/3} \sim O(\text{TeV})$ is transmitted to the Higgs sector in SM by a real singlet scalar $S$ introduced. This $S$ connects the hidden sector and the SM, triggering the electroweak (EW) symmetry breaking. Their model also includes candidates of cold dark matter (CDM), i.e., the lightest mesons and baryons in the hidden sector. In Ref. [1], however, a vacuum angle $\theta_h$ was not considered in the hidden sector with a new QCD-like strong interaction.

In this paper we shall consider the vacuum angle $\theta_h$ in the hidden sector [2] of this model [1] and study its effects on physical quantities such as the Higgs field and hidden-sector mesons. Before dealing with $\theta_h$ in the QCD-like strong interaction (hidden sector), it would be helpful to recall the vacuum angle $\theta$ in ordinary QCD [3]. The rich structure of QCD vacuum gives rise to the following effective Lagrangian term,

$$\frac{\theta}{16\pi^2} \text{tr} F_{\mu\nu} \tilde{F}^{\mu\nu},$$

(1)

where $\theta$ is called a vacuum angle in QCD and this term violates P and CP conservation. In order to investigate $\theta$ dependence in QCD, effective models such as the Di-Vecchia-Veneziano model have been widely used [4, 5, 6, 7]. Di Vecchia and Veneziano constructed an effective Lagrangian [5] in the large-$N_{color}$ limit of QCD to resolve the U(1) problem [8]. The Di-Vecchia-Veneziano model is a low-energy effective theory of Nambu-Goldstone (NG) bosons $\pi_i (i = 1, 2, \ldots, 8)$ including the flavor-singlet pseudoscalar particle $\eta^0$, which will mix with $\pi_8$ and $\pi_3$ to form the mass eigenstates $\eta', \eta$, and $\pi^0$. While authors in Ref. [5] studied their effective theory for arbitrary $\theta$, the experimental upper limit on an electric dipole moment for the neutron or the CP-violating decay of NG boson $\eta \rightarrow \pi^+ \pi^-$ bounds the $\theta$ in QCD, $|\theta| < 10^{-10}$. Why the vacuum angle $\theta$ in QCD is nearly zero, and this is the strong CP problem which is not solved yet.

Returning to the discussion of the scale invariant extension of the SM [1], we shall consider a vacuum angle $\theta_h$ in the hidden sector with a new QCD-like strong interaction. Contrary to the case of QCD, there is no reason to restrict the value of $\theta_h$ in the hidden sector because no particle in the hidden sector is detected in experiment. As a low-energy effective theory of the hidden sector, we use the Di-Vecchia-Veneziano model with two flavors, containing the hidden-sector pions $\vec{\pi} = (\pi_1, \pi_2, \pi_3)$, the hidden-sector flavor-singlet pseudoscalar $\eta^0$, and the hidden-sector vacuum angle $\theta_h$. In the model of Ref. [1], as noted before, the SM sector is connected with the hidden sector by the messenger $S$ (a real scalar). Since the ground state energy of the Di-Vecchia-Veneziano Lagrangian in the hidden sector depends on $\theta_h$, the Higgs field will be affected by
\( \theta_h \), which we shall study. Other effects of \( \theta_h \) on physical quantities will be seen, for example, in the CP-violating interactions involving the hidden-sector pions\(^9\), or in the mass difference of the hidden-sector isospin multiplet, i.e., the hidden-sector pions that are candidates of CDM. We examine how the mixing term between \( \pi_3 \) and \( \eta_0 \) depends on \( \theta_h \), and calculate the mass difference between the hidden-sector \( \pi^0 \) and \( \pi^\pm \).

This paper is organized as follows. In Sect.2, as a low-energy effective theory of the hidden sector the Di-Vecchia-Veneziano model with two flavors is introduced and its ground state energy depending on a vacuum angle \( \theta_h \) is presented\(^5\)\(^10\). In Sect.3, we discuss physical effects of \( \theta_h \) on the Higgs field in SM sector and also on the mass difference between the hidden-sector pions \( \pi^0 \) and \( \pi^\pm \), which are dark matter candidates. The last section is devoted to the conclusion.

### 2 Ground state in a strongly interacting hidden sector with vacuum angle

The hidden sector Lagrangian\(^1\) with the vacuum angle \( \theta_h \) is given by

\[
\mathcal{L}_H = -\frac{1}{2} \text{tr} F_{\mu\nu}F^{\mu\nu} + \sum_{k=1}^{N_{h,f}} \text{tr} \bar{q}_k(i\gamma^\mu D_\mu - y_k S)q_k + \frac{\theta_h}{16\pi^2} \text{tr} F_{\mu\nu}\tilde{F}^{\mu\nu},
\]

where \( F_{\mu\nu} \) is the field strength for a hidden \( SU(N_{h,c}) \) gauge sector and \( q \) the hidden-sector quark transforming as a fundamental representation of \( SU(N_{h,c}) \). We choose the number of flavors \( N_{h,f} \) to be two, \( N_{h,f} = 2 \). A real singlet scalar \( S \) is introduced in this Lagrangian as a messenger between the hidden sector and SM sector. The scalar potential part for the Higgs doublet field \( H \) of the SM Lagrangian is modified as follows\(^1\)\(^12\):

\[
V_{SM+S} = \lambda_H (H^\dagger H)^2 + \frac{1}{4} \lambda_S S^4 - \frac{1}{2} \lambda_{HS} S^2 (H^\dagger H).
\]

Here we assume \( \lambda_{HS} > 0 \). For the stability of this potential with \( H = (0, h/\sqrt{2})^T \), it is required that \( \lambda_H > 0, \lambda_S > 0, \) and \( 4\lambda_H \lambda_S - \lambda_{HS}^2 > 0 \). With this modified SM Lagrangian, \( \mathcal{L}_{SM+S} \), the total Lagrangian is \( \mathcal{L}_T = \mathcal{L}_H + \mathcal{L}_{SM+S} \). In the hidden sector with strong interaction the hidden-sector quarks will condensate, \( \langle \bar{q}q \rangle \neq 0 \), by non-perturbative effect and this scale \( \langle \bar{q}q \rangle \) will be transmitted to the Higgs sector in SM by the real scalar \( S \). If the field \( S \) gets a vacuum expectation value (VEV) \( \langle S \rangle \neq 0 \), the hidden-sector quarks will obtain its mass, \( m_1 = y_1 \langle S \rangle, m_2 = y_2 \langle S \rangle \), through the Yukawa coupling in \( \mathcal{L}_H \). We will make use of the Di-Vecchia-Veneziano model\(^5\) as a low-energy effective theory of the hidden-sector Lagrangian \( \mathcal{L}_H \) with the hidden-sector
quark mass, \( m_1 \) and \( m_2 \),

\[
\mathcal{L}_{DVV} = \frac{f^2}{4} \text{Tr}\{\partial_\mu U \partial^\mu U^\dagger\} + \frac{|\langle \bar{q}q \rangle|}{2} \text{Tr}\{\mathcal{M} U^\dagger + U \mathcal{M}\} - \frac{\tau}{2} \left\{ \theta_h + \frac{i}{2} (\log \det U - \log \det U^\dagger) \right\}^2 ,
\]

where

\[
U = \exp \left[ \frac{i}{f_\pi} \left\{ \sum_{j=1}^{3} \pi_j \sigma^j + \eta^0 \cdot 1 \right\} \right] ,
\]

and

\[
\mathcal{M} = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} .
\]

The \( \pi_j(j = 1, 2, 3) \) are the hidden-sector pions with decay constant \( f_\pi \), \( \eta^0 \) the hidden-sector flavor-singlet pseudoscalar meson, \( |\langle \bar{q}q \rangle| \) the absolute value of the hidden-sector quark condensate in the massless theory, and \( \tau \) the topological susceptibility of the pure \( SU(N_{h,c}) \) Yang-Mills theory[3].

For the ground state of this system, \( U \) takes the form

\[
U_g = \begin{pmatrix} e^{i\varphi_1} & 0 \\ 0 & e^{i\varphi_2} \end{pmatrix} ,
\]

where \( \varphi_1 \) and \( \varphi_2 \) can be determined modulo \( 2\pi \). The ground state energy \( E_{DVV}(\varphi_1, \varphi_2) \) for \( \mathcal{L}_{DVV} \) becomes

\[
E_{DVV}(\varphi_1, \varphi_2) = -|\langle \bar{q}q \rangle| (m_1 \cos \varphi_1 + m_2 \cos \varphi_2) + \frac{\tau}{2} \left\{ \theta_h - (\varphi_1 + \varphi_2) \right\}^2 ,
\]

and the angles \( \varphi_1 \) and \( \varphi_2 \) should satisfy

\[
m_1 \sin \varphi_1 = \left( \frac{\tau}{|\langle \bar{q}q \rangle|} \right) \left\{ \theta_h - (\varphi_1 + \varphi_2) \right\} ,
\]

\[
m_2 \sin \varphi_2 = m_1 \sin \varphi_1 .
\]

We shall consider the region of \( \theta_h, -\pi < \theta_h < \pi \). Now let us assume that the hidden-sector quark mass \( m_j(j = 1, 2) \) is small,

\[
m_j \ll \left( \frac{\tau}{|\langle \bar{q}q \rangle|} \right) , \quad (j = 1, 2) ,
\]

throughout this paper. Under this assumption the ground state energy \( E_{DVV} \) can be represented explicitly by \( \theta_h \)[5,10,11] as discussed below. We seek solutions having the following form,

\[
\varphi_1 = \frac{\theta_h}{2} + \alpha + O\left( \frac{|\langle \bar{q}q \rangle| m_j}{\tau} \right) , \quad \varphi_2 = \frac{\theta_h}{2} - \alpha + O\left( \frac{|\langle \bar{q}q \rangle| m_j}{\tau} \right) .
\]
In the limit, $|\langle \bar{q}q \rangle| m_j/\tau \to 0$, the $\alpha$ should satisfy
\[ m_1 \sin \left( \frac{\theta_h}{2} + \alpha \right) = m_2 \sin \left( \frac{\theta_h}{2} - \alpha \right), \tag{12} \]
and a solution of this equation was obtained by
\[ \cos \alpha = \frac{(m_1 + m_2) \cos \frac{\theta_h}{2}}{\sqrt{(m_1 + m_2)^2 \cos^2 \frac{\theta_h}{2} + (m_1 - m_2)^2 \sin^2 \frac{\theta_h}{2}}}, \]
\[ \sin \alpha = \frac{-(m_1 - m_2) \sin \frac{\theta_h}{2}}{\sqrt{(m_1 + m_2)^2 \cos^2 \frac{\theta_h}{2} + (m_1 - m_2)^2 \sin^2 \frac{\theta_h}{2}}}. \tag{13} \]

Using this solution, the ground state energy becomes
\[ E_{DVV}(\varphi_1, \varphi_2) \approx -|\langle \bar{q}q \rangle| \left( m_1 \cos \left( \frac{\theta_h}{2} + \alpha \right) + m_2 \cos \left( \frac{\theta_h}{2} - \alpha \right) \right) \]
\[ = -|\langle \bar{q}q \rangle| \sqrt{(m_1 + m_2)^2 \cos^2 \frac{\theta_h}{2} + (m_1 - m_2)^2 \sin^2 \frac{\theta_h}{2}}. \tag{14} \]
If one wishes to deal with the parameters $y_1$ and $y_2$ instead of $m_1$ and $m_2$, one can represent
\[ E_{DVV}(\varphi_1, \varphi_2) \approx -|\langle \bar{q}q \rangle| \langle S \rangle \sqrt{(y_1 + y_2)^2 \cos^2 \frac{\theta_h}{2} + (y_1 - y_2)^2 \sin^2 \frac{\theta_h}{2}} \]
\[ \equiv -|\langle \bar{q}q \rangle| Y(\theta_h) \langle S \rangle, \tag{15} \]
where
\[ Y(\theta_h) \equiv \sqrt{(y_1 + y_2)^2 \cos^2 \frac{\theta_h}{2} + (y_1 - y_2)^2 \sin^2 \frac{\theta_h}{2}}. \tag{16} \]
The $Y(\theta_h)$ is a monotone decreasing function of $\theta_h$ for $0 \leq \theta_h < \pi$, and $Y(\theta_h = 0) = y_1 + y_2$, $Y(\theta_h = \pi) = |y_1 - y_2|$.

Let us consider the degenerate case $m_1 = m_2 \equiv m$. In this case one has $\sin \alpha = 0$ and $\cos \alpha = 1$ because of the condition, $-\pi < \theta_h < \pi$, and a solution $\alpha = 0$ leads to $\varphi_1 = \varphi_2 \equiv \varphi$. The ground state energy in the degenerate case is then
\[ E_{DVV}(\varphi) \approx -2m|\langle \bar{q}q \rangle| \cos \frac{\theta_h}{2} + O \left( \left( \frac{|\langle \bar{q}q \rangle| m_j}{\tau} \right)^2 \right), \tag{17} \]

$^2$The $m(\theta)$, $m(\theta) \equiv \sqrt{(m_1 + m_2)^2 \cos^2 \frac{\theta}{2} + (m_1 - m_2)^2 \sin^2 \frac{\theta}{2}}$, is the one which plays an important role in Ref.[11]. The $m(\theta)$ is a monotone decreasing function of $\theta$ ($0 \leq \theta < \pi$), and $m(\theta = 0) = m_1 + m_2$, $m(\theta = \pi) = |m_1 - m_2|$.
which becomes zero if $\theta_h = \pi$. If we solve the equation of motion for $\varphi$ up to $O(|\langle \bar{q}q \rangle|m/\tau)$, we get

$$\varphi = \frac{\theta_h}{2} - \frac{1}{2} \left( \frac{|\langle \bar{q}q \rangle|}{\tau} \right) \sin \frac{\theta_h}{2} + O \left( \left( \frac{|\langle \bar{q}q \rangle| m_j}{\tau} \right)^2 \right), \quad (18)$$

and

$$E_{DVV}(\varphi) \approx -2m|\langle \bar{q}q \rangle| \cos \frac{\theta_h}{2} - \frac{\tau}{2} \left( \frac{|\langle \bar{q}q \rangle| m}{\tau} \right)^2 \sin^2 \frac{\theta_h}{2} + O \left( \left( \frac{|\langle \bar{q}q \rangle| m_j}{\tau} \right)^3 \right), \quad (19)$$

which does not become zero when $\theta_h = \pi$. In the degenerate case if one restricts $\cos (\theta_h/2)$ as

$$\frac{m|\langle \bar{q}q \rangle|}{\tau} \ll \cos \frac{\theta_h}{2}, \quad (20)$$

the second higher term in the ground state energy $E_{DVV}$ can be neglected.

3 Physical effects of the vacuum angle $\theta_h$

3.1 On electroweak symmetry breaking —specially on Higgs field—

In the preceding section we see that the ground state energy for a strongly interacting hidden sector, $E_{DVV}$, Eq.(15), depends on the vacuum angle $\theta_h$. Since a linear term of $S$ in this $E_{DVV}$ leads to VEV of $S$[12], the vacuum angle $\theta_h$ will affect EW symmetry breaking which is triggered by this $S$. When $y_1 \neq y_2$, the tree level scalar potential is

$$V = V_{SM+S} + V_{DVV}$$

$$\approx \lambda_H (H^\dagger H)^2 + \frac{1}{4} \lambda_S S^4 - \frac{1}{2} \lambda_{HS} S^2 (H^\dagger H) - |\langle \bar{q}q \rangle| Y(\theta_h) S. \quad (21)$$

With $H = (0, h/\sqrt{2})^T$, a linear term in a real singlet scalar $S$ leads to VEV of $S$, and then to VEV of the Higgs field $h$[12],

$$\langle S \rangle = \left( \frac{4\lambda_H}{4\lambda_H \lambda_S - \lambda_{HS}^2} \right)^{\frac{1}{4}} |\langle \bar{q}q \rangle| Y(\theta_h) \right\}^{\frac{1}{4}}$$

$$= \left( \frac{4\lambda_H}{4\lambda_H \lambda_S - \lambda_{HS}^2} \right)^{\frac{1}{4}} |\langle \bar{q}q \rangle|^{\frac{1}{4}} \left\{ (y_1 + y_2)^2 \cos^2 \frac{\theta_h}{2} + (y_1 - y_2)^2 \sin^2 \frac{\theta_h}{2} \right\}^{\frac{1}{4}},$$

$$\langle h \rangle = \langle S \rangle \sqrt{\frac{\lambda_{HS}}{2\lambda_H}}, \quad (22)$$

3When $\theta_h = 0$, this potential is equal to the one in Ref.[1].
where $Y(\theta_h)$ has been defined in Eq. (16). Note that $\langle h \rangle$ depends on the vacuum angle $\theta_h$ as well as $\langle S \rangle$ does. On the other hand, the $\langle h \rangle$ should be 246GeV. Therefore, the six parameters, $(\lambda_H, \lambda_S, \lambda_{HS}, |\langle q\bar{q} \rangle|, y_1, y_2)$, appeared in Eq. (22) should satisfy the condition, $\langle h \rangle = 246$GeV, for a given value of $\theta_h$. Since $Y(\theta_h)$ has different values for different values of $\theta_h$, the set of the six parameters $(\lambda_H, \lambda_S, \lambda_{HS}, |\langle q\bar{q} \rangle|, y_1, y_2)$ for a $\theta_h$ is different from that for another $\theta_h(\neq \theta_h)$. If we first give the values of $|\langle q\bar{q} \rangle|$, $y_1$, and $y_2$, the three parameters, $(\lambda_H, \lambda_S, \lambda_{HS})$, become the remaining parameters which are determined for a given value of $\theta_h$ by the condition, $\langle h \rangle = 246$GeV. This means that for different values of $\theta_h$, we take different values of $(\lambda_H, \lambda_S, \lambda_{HS})$. In the degenerate case $m_1 = m_2 \equiv m$, or $y_1 = y_2 \equiv y$, though, we should pay attention when we use the expression, Eq. (22). The VEV's, $\langle S \rangle$ and $\langle h \rangle$, will be given in the same form, Eq. (22), as in the case of $m_1 \neq m_2$ if we can use the form of the tree level scalar potential, Eq. (21), for the degenerate case. In order for the tree level scalar potential to be represented by Eq. (22) also in the degenerate case, the second term in the right-hand side of Eq. (19) should be neglected. So as to be neglected that second term, we need to restrict the vacuum angle as discussed in Eq. (20), $m|\langle q\bar{q} \rangle|/\tau \ll \cos(\theta_h/2)$. The factor $\cos(\theta_h/2)$ will not be so small unless the value of $m|\langle q\bar{q} \rangle|/\tau$ is nearly zero.

Next, we shall examine whether the condition which determines the Higgs particle mass depends on the vacuum angle $\theta_h$ or not. With $H = (0, \langle h \rangle + \hat{h})/\sqrt{2}$ and $S = \langle S \rangle + \hat{S}$, the potential $V = V_{SM+S} + V_{DVV}$ leads to the mass matrix,

$$
\frac{1}{2} \left( \begin{array}{c} \hat{h} \\ \hat{S} \end{array} \right) \left( \begin{array}{cc} \frac{3}{2} \lambda_H \langle h \rangle^2 - \frac{1}{2} \lambda_{HS} \langle S \rangle^2 & -\lambda_{HS} \langle h \rangle \langle S \rangle \\ -\lambda_{HS} \langle h \rangle \langle S \rangle & 3 \lambda_S \langle S \rangle^2 - \frac{1}{2} \lambda_{HS} \langle h \rangle^2 \end{array} \right) \left( \begin{array}{c} \hat{h} \\ \hat{S} \end{array} \right) = \frac{1}{2} \langle h \rangle \langle S \rangle \left( \begin{array}{cc} 2 \lambda_H \\ -\sqrt{2} \lambda_H \lambda_{HS} \\ -\sqrt{2} \lambda_H \lambda_{HS} \\ 6 \lambda_S \lambda_H - \frac{\lambda_{HS}}{2} \end{array} \right) \left( \begin{array}{c} \hat{h} \\ \hat{S} \end{array} \right),
$$

(23)

where we have used $\langle h \rangle/\langle S \rangle = \sqrt{\lambda_{HS}/2\lambda_H}$. Diagonalizing the mass matrix, one must obtain the mass eigenstate of the Higgs particle whose eigenvalue should be 125.1GeV. Namely, the matrix,

$$
\left( \begin{array}{cc} 2 \lambda_H \\ -\sqrt{2} \lambda_H \lambda_{HS} \\ -\sqrt{2} \lambda_H \lambda_{HS} \\ 6 \lambda_S \lambda_H - \frac{\lambda_{HS}}{2} \end{array} \right),
$$

(24)

should give the eigenvalue $(125.1$GeV$)^2/\langle h \rangle^2 = 0.2586$ of the mass eigenstate of the Higgs particle. This condition does not depend on $\theta_h$, and the mixing angle between $\hat{h}$ and $\hat{S}$ also does not. If, however, one involves the next leading order correction, $O(|\langle q\bar{q} \rangle| m_j/\tau)^2)$, in the ground state energy $E_{DVV}$, the mass matrix will depend on $\theta_h$.

### 3.2 On mass difference between dark matter $\pi^0$ and $\pi^\pm$

We will study how the vacuum angle $\theta_h$ contributes to mass difference between the hidden-sector pions $\pi^0$ and $\pi^\pm$, which are dark matter candidates. The masses of these hidden-sector pions $\pi^0$ and $\pi^\pm$ can be obtained using the effective theory $L_{DVV}$, Eq. (4),
as follows. Since both fields $\eta^0$ and $\pi_3$ have VEV’s, Eq.(17), provided $\theta_h \neq 0$, it is convenient to introduce new fields that have no VEV’s. We shall define $\hat{U}$ by

$$\hat{U} \equiv U U_g^{-1},$$

(25)

and the fields $\hat{\pi}_j$ and $\hat{\eta}$ by

$$\hat{U} = \exp \left[ i \int_{\Delta} \left\{ \sum_{j=1}^{3} \hat{\pi}_j \sigma^j + \hat{\eta} \cdot 1 \right\} \right],$$

(26)

where $U_g$ is defined in Eq.(7). These new fields $\hat{\pi}_j$ and $\hat{\eta}$ have no VEV’s and the Lagrangian $L_{DVV}$ can be rewritten as follows with the help of Eq.(9) [4, 5, 6, 7].

$$L_{DVV} = \frac{f_\pi^2}{4} \text{Tr}\{\partial_\mu \hat{U} \partial^\mu \hat{U}^\dagger\} + \frac{|\langle \bar{q} q \rangle|}{2} \text{Tr}\{M(\theta_h)(\hat{U} + \hat{U}^\dagger - 2)\} + \frac{\tau}{8} \left\{ \log \det \hat{U} - \log \det \hat{U}^\ast \right\}^2$$

$$-i \frac{\tau}{2} \{\theta_h - (\varphi_1 + \varphi_2)\} \text{Tr}\left\{ \log \frac{\hat{U}}{\hat{U}^\dagger} - (\hat{U} - \hat{U}^\dagger) \right\} + \text{const}.,$$

(27)

where $M(\theta_h) \equiv \text{diag} (m_1 \cos \varphi_1, m_2 \cos \varphi_2)$. Expanding the exponential in $\hat{U}$, we get

$$\frac{f_\pi^2}{4} \text{Tr}\{\partial_\mu \hat{U} \partial^\mu \hat{U}^\dagger\} + \frac{|\langle \bar{q} q \rangle|}{2} \text{Tr}\{M(\theta_h)(\hat{U} + \hat{U}^\dagger - 2)\} + \frac{\tau}{8} \left\{ \log \det \hat{U} - \log \det \hat{U}^\dagger \right\}^2$$

$$\approx \frac{1}{2} \left( \partial_\mu \hat{\pi} \right) \cdot \left( \partial^\mu \hat{\pi} \right) + \frac{1}{2} \left( \partial_\mu \hat{\eta} \right) \cdot \left( \partial^\mu \hat{\eta} \right) - \frac{1}{2} \left\{ \frac{|\langle \bar{q} q \rangle|}{f_\pi^2} \right\} \left( \hat{\pi} \cdot \hat{\pi} \right)$$

$$- \frac{1}{2} \left\{ \frac{4 \tau}{f_\pi^2} + \frac{m_+ (\theta_h) |\langle \bar{q} q \rangle|}{f_\pi^2} \right\} \hat{\eta}^2 - \frac{1}{2} \left\{ \frac{2 m_- (\theta_h) |\langle \bar{q} q \rangle|}{f_\pi^2} \right\} \left( \hat{\pi}_1 \cdot \hat{\pi}_2 \right)$$

$$= \frac{1}{2} \left( \partial_\mu \hat{\pi} \right) \cdot \left( \partial^\mu \hat{\pi} \right) + \frac{1}{2} \left( \partial_\mu \hat{\eta} \right) \cdot \left( \partial^\mu \hat{\eta} \right) - \frac{1}{2} \left\{ \frac{|\langle \bar{q} q \rangle|}{f_\pi^2} \frac{m_+ (\theta_h)}{f_\pi^2} \right\} \left( \hat{\pi}_1 \cdot \hat{\pi}_2 \right)$$

$$- \frac{1}{2} \left( \hat{\eta} \cdot \hat{\pi}_3 \right) \left( \frac{4 \tau}{f_\pi^2} + \frac{|\langle \bar{q} q \rangle| m_+ (\theta_h)}{f_\pi^2} \right) + \frac{|\langle \bar{q} q \rangle|}{f_\pi^2} \frac{m_- (\theta_h)}{f_\pi^2} \right\} \left( \hat{\eta} \cdot \hat{\pi}_3 \right) + \cdots ,$$

(28)

where we have defined

$$m_+ (\theta_h) \equiv m_1 \cos \varphi_1 + m_2 \cos \varphi_2,$$

$$m_- (\theta_h) \equiv m_1 \cos \varphi_1 - m_2 \cos \varphi_2.$$

(29)

In the case of $m_1 \neq m_2$, we have $m_- (\theta_h) = (m_1^2 - m_2^2) / m_+ (\theta_h)$ because of $m_2 \sin \varphi_2 = m_1 \sin \varphi_1$. We see that masses of the hidden-sector pions and flavor-singlet pseudoscalar depend on the hidden-sector vacuum angle $\theta_h [5, 11, 2]$. Note that the factor $m_+ (\theta_h)$ in the diagonal term of $\hat{\pi}_3$ appears in the right-hand side of Eq.(14) and is a monotone decreasing function of $\theta_h (0 \leq \theta_h \leq \pi)$, while the absolute value of the factor $m_- (\theta_h)$ in
the mixing term between $\hat{\pi}_3$ and $\hat{\eta}$ is a monotone increasing function of $\theta_h$. The mass eigenstates $\hat{\eta}'$ and $\hat{\pi}^0$,

$$
\begin{pmatrix}
\hat{\eta}' \\
\hat{\pi}^0
\end{pmatrix} =
\begin{pmatrix}
\cos \alpha & -\sin \alpha \\
\sin \alpha & \cos \alpha
\end{pmatrix}
\begin{pmatrix}
\hat{\eta} \\
\hat{\pi}_3
\end{pmatrix},
$$

(30)
can be obtained by diagonalizing the mass matrix and resultant mixing angle $\alpha$ is small, $|\sin \alpha| \approx |\langle \bar{q} q \rangle| |m_-(\theta_h)|/(4\tau) \ll 1$. One obtains the mass eigenstate $\hat{\eta}'$ with the eigenvalue(squared),

$$
m(\eta')^2 = \frac{4\tau}{f^2_\pi} + \frac{|\langle \bar{q} q \rangle| m_+(\theta_h)}{f^2_\pi} + \frac{|\langle \bar{q} q \rangle|^2 m_-^2(\theta_h)}{4\tau f^2_\pi},
$$

(31)and the mass eigenstate $\hat{\pi}^0$ with the eigenvalue(squared),

$$
m(\pi^0)^2 = \left\{ \frac{|\langle \bar{q} q \rangle| m_+(\theta_h)}{f^2_\pi} - \frac{|\langle \bar{q} q \rangle|^2 m_-^2(\theta_h)}{4\tau f^2_\pi} \right\}.
$$

(32)
The mass difference between the hidden-sector pions $\hat{\pi}^0$ and $\hat{\pi}^\pm$ can be represented,

$$
\frac{m(\pi^\pm)^2 - m(\pi^0)^2}{m(\pi^\pm)^2} \bigg|_{\theta_h} \approx \frac{\langle \bar{q} q \rangle |m_-(\theta_h)|^2}{4\tau m_+(\theta_h)} \approx \frac{\langle \bar{q} q \rangle}{4\tau} \left( \frac{(m_1 + m_2)^2(m_1 - m_2)^2}{(m_1 + m_2)^2 \cos^2 \frac{\theta_h}{2} + (m_1 - m_2)^2 \sin^2 \frac{\theta_h}{2}} \right)^{\frac{1}{2}}.
$$

(33)
Note that the right-hand side of the above equation is a monotone increasing function of $\theta_h$ ($0 \leq \theta_h \leq \pi$), then the mass difference between $\hat{\pi}^0$ and $\hat{\pi}^\pm$ becomes large as the $\theta_h$ changes from 0 to $\pi$. When $\theta_h = 0$, the right-hand side of the above equation is very small,

$$
\frac{m(\pi^\pm)^2 - m(\pi^0)^2}{m(\pi^\pm)^2} \bigg|_{\theta_h=0} \approx \frac{\langle \bar{q} q \rangle |m_+(m_1 + m_2)|}{4\tau} \times \left( \frac{m_1 - m_2}{m_1 + m_2} \right)^2 \ll 1,
$$

(34)thereby $\hat{\pi}^0$ and $\hat{\pi}^\pm$ are almost degenerate. However, when $\theta_h = \pi$, the mass difference will not be so small,

$$
\frac{m(\pi^\pm)^2 - m(\pi^0)^2}{m(\pi^\pm)^2} \bigg|_{\theta_h=\pi} \approx \frac{\langle \bar{q} q \rangle |m_+(m_1 + m_2)|}{4\tau} \times \left( \frac{m_1 + m_2}{|m_1 - m_2|} \right).
$$

(35)
If the value of $(m_1 + m_2)/(|m_1 - m_2|)$ (the inverse of the hidden-sector isospin violation in $\theta_h = 0$) is rather large, the mass difference between $\hat{\pi}^0$ and $\hat{\pi}^\pm$ can not be neglected. Among the four particles, $(\hat{\pi}^0, \hat{\pi}^\mp, \hat{\eta}', \hat{\eta}')$, $\hat{\pi}^0$ is most light, while $\hat{\eta}'$ is most heavy[8, 34]. Since $\hat{\pi}^0$, $\hat{\pi}^+$, and $\hat{\pi}^-$ are stable ($\hat{\pi}^+$ and $\hat{\pi}^-$ have the Cartan subalgebra $U(1)$ charge +1 and -1, respectively when $y_1 \neq y_2$ [13]), they are dark matters.
In the degenerate case, \( m_1 = m_2 \equiv m \), we have restricted \( \theta_h \) in Sec. 3.1 as \( m|\langle \bar{q}q \rangle|/\tau \ll \cos(\theta_h/2) \). The coefficient of the mixing term \( (\hat{\pi}_3 \cdot \hat{n}) \) in Eq. (28) vanishes when \( m_1 = m_2 \) as \( m_-(\theta_h) = m_+ \cos \varphi - m_\cos \varphi = 0 \) and then all the three pions have the same mass,

\[
\frac{|\langle \bar{q}q \rangle|m_+(\theta_h)}{f_\pi^2} \approx \frac{2m|\langle \bar{q}q \rangle|}{f_\pi^2} \frac{\theta_h}{2},
\]

depending on the hidden-sector vacuum angle \( \theta_h \) \[1\].

4 Conclusion

Considering vacuum angle \( \theta_h \) in a strongly interacting hidden sector like QCD, we studied how the vacuum angle \( \theta_h \) affects physical quantities in a scale invariant extension of the standard model with that hidden sector proposed by Hur and Ko\[1\]. In their model, the dynamical chiral symmetry breaking occurs in the hidden sector and generates a scale \( |\langle \bar{q}q \rangle| \) which can trigger electroweak symmetry breaking. As the low-energy effective theory of the hidden sector with \( \theta_h \) like QCD, we have used the Di-Vecchia-Veneziano model with two flavors.

We find explicitly how the expression for VEV of the Higgs field which should be 246GeV depends on the vacuum angle \( \theta_h \) besides the parameters in our model. On the other hand, it is shown that the condition which determines the Higgs particle mass does not depend on \( \theta_h \) in a first approximation of small hidden-sector quark masses, \( m_1 \) and \( m_2 \). Next, we studied the mass difference between the hidden-sector pions, the hidden-sector isotriplet pions \( \vec{\pi} = (\pi_1, \pi_2, \pi_3) \) being candidates for cold dark matter. The arising mixing term between \( \pi_3 \) and a \( SU(2) \) flavor-singlet \( \eta^0 \) depends on \( \theta_h \), and this term leads to the mass difference between dark matter \( \pi^0 \) and \( \pi^\pm \). We find that while the mass difference between \( \pi^0 \) and \( \pi^\pm \) is negligible for \( \theta_h \approx 0 \), the mass difference will not be so small for \( \theta_h \approx \pi \) when \( (m_1 + m_2)/|m_1 - m_2| \) is rather large.

Since no hidden-sector particle is detected in experiment, the vacuum angle \( \theta_h \) in the strongly interacting hidden sector is not restricted. It is therefore worthwhile to study the scale invariant extension of the standard model\[1\] with any value of \( \theta_h \). While we have restricted ourselves to the two flavor case, \( N_{h,f} = 2 \), of the hidden sector, it will be interesting to study the case of \( N_{h,f} \geq 3 \).
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