ON EFFECTS OF THE LARGE NEUTRINO B-TERM ON LOW ENERGY PHYSICS

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To embed the seesaw mechanism in the MSSM, two or three right-handed neutrino supermultiplets, \( N_i \), have to be added to the model. In this framework, the supersymmetry breaking potential will include a new term called neutrino B-term: \( M_{\nu} N \bar{N}/2 \). In this talk, we present a toy model that generates a large neutrino B-term keeping other supersymmetry breaking parameters small. We then review the consequences of having a large neutrino B-term on the electroweak symmetry breaking parameters and electric dipole moments (EDMs) of elementary particles.

1 Introduction

The Standard Model (SM) of elementary particles has been so far able to explain the accelerator data. Despite its remarkable achievements, the SM suffers from some shortcomings: i) In order to cancel the quadratic divergences appearing in the radiative corrections to the Higgs mass, the parameters of the theory have to be highly fine-tuned. (ii) In the framework of SM with zero neutrino mass, we are unable to explain the solar and atmospheric neutrino data. We also need physics beyond the SM to explain the KamLAND and K2K results.

To overcome the former shortcoming, several extensions of the SM have been developed among which the Minimal Supersymmetric Standard Model (MSSM) is one of the most elegant models. The superpotential of this model is

\[
W = Y_{\ell}^{ij} \epsilon_{\alpha \beta} H_{d}^{\beta} E_{i} L_{j}^{\beta} + \mu H_{u} H_{d}.
\]

where \( L_{j}^{\beta} \) is the supermultiplet corresponding to the doublet \((\nu_{Lj}, l_{Lj})\) and \( E_{i} \) is the superfield
associated with the charged lepton $\ell^+_i$. $Y_\ell$ is the Yukawa matrix of the charged leptons and the last term is the famous mu-term.

The MSSM in its most general form contains several sources of Lepton Flavor Violation (LFV) which in principle can give rise to LFV rare decays of $\mu$ and $\tau$ exceeding the present experimental bounds. Motivated by this observation, the constrained MSSM (CMSSM) has been proposed which assumes that at high energies, which we will loosely call $M_{GUT}$, the masses of sfermions are universal. That is, at $M_{GUT}$, the soft supersymmetry breaking potential is

$$-\mathcal{L}_{soft} = m_0^2(\tilde{L}_i^\dagger \tilde{L}_i + \tilde{E}_i^\dagger \tilde{E}_i + H_d^i H_d + H_u^i H_u) + \frac{1}{2} m_{1/2}(\tilde{B}^\dagger \tilde{B} + \tilde{W}^a \tilde{W}^a)$$

where the $A$-coupling is supposed to be proportional to the corresponding Yukawa couplings, $A_\ell = a_0 Y_\ell$.

Although there are strong upper bounds on neutrino mass from beta decay experiments and cosmological considerations, the observation of neutrino oscillation guarantees that at least two neutrinos have nonzero mass. That is while in the framework of both SM and MSSM the masses of neutrinos are zero. One of the most economic ways to attribute a tiny but nonzero mass to neutrinos is the famous seesaw mechanism which involves three very heavy right-handed neutrinos. To embed the seesaw mechanism in the MSSM, three right-handed neutrino supermultiplets, $N_i$, have to be added to the model. In the presence of these new supermultiplets the superpotential includes new terms

$$\Delta W^N = Y^{ij}_{\nu} \epsilon_{\alpha \beta} H_u^\alpha N_i L_j^\beta + \frac{1}{2} M_{ij} N_i N_j,$$

where the first term is the Yukawa coupling of neutrinos and the second term is the mass term for $N_i$. Without loss of generality we can rotate and re-phase the fields to make both $Y_\nu$ and $M_{ij}$ real diagonal. Throughout this paper, we work in such basis: $Y^{ij}_\nu = \text{diag}(Y_e, Y_\mu, Y_\tau)$ and $M^{ij} = \text{diag}(M_1, M_2, M_3)$. In order to make neutrino masses tiny, $M_i$ have to be very large: $M_i / M_{susy} \gg 1$. In the presence of $\tilde{N}_i$, also the soft supersymmetry breaking potential includes new terms:

$$-\Delta \mathcal{L}_{soft}^N = m_0^2 \tilde{N}_i^\dagger \tilde{N}_i + A^{ij}_{\nu} \epsilon_{\alpha \beta} H_u^\alpha \tilde{N}_i \tilde{L}_j^\beta + \frac{1}{2} B_{\nu} M_i \tilde{N}_i \tilde{N}_i + \text{h.c.},$$

where at the GUT scale $A_{\nu} = a_0 Y_{\nu}$. The last term is the neutrino B-term which violates the lepton number by two units. Since $\tilde{N}_i$ are singlets of $SU(3) \times SU(2) \times U(1)$, in general the $B_\nu$ can be much higher than the electroweak scale, $m_{EW}$. However for the range of parameters that $m_{EW} \ll B_\nu \ll M_i$, the contribution of the neutrino B-term to neutrino masses can be significant. Also, if $B_\nu$ is complex, it can be considered as a new source of CP-violation, inducing EDMs for elementary particles.

This paper is organized as follows. In Sec. 2, we review the theoretical prediction for the order of magnitude of $B_\nu$ in the context of mSUGRA and we then suggest a toy model that allows large values of $B_\nu$ while keeping other supersymmetry breaking parameters low ($\approx 1 \text{ TeV}$). In Sec. 3 we review the effects of $B_\nu$ on electroweak symmetry breaking parameters. In Sec. 4, we study the effects of an imaginary $B_\nu$ on the EDMs of the elementary particles. In Sec. 5 we summarize our conclusions.
2 Theoretical Expectation for $|B_\nu|$ 

In the context of the mSUGRA, the soft supersymmetry breaking terms originate from the interaction of a chiral superfield $S$ with the super-potential:

$$\int d^2\theta S(\theta)W(\theta).$$

(4)

The scalar and $F$-components of $S$ develop vacuum expectation values $\langle S \rangle = 1 + F_S \theta^2$ and $\langle F_S \rangle$ determines the scale of the soft supersymmetry breaking terms. Within this model we expect $B_\nu \sim a_0 \sim m_{\text{susy}}$. Remember that we have parameterized the neutrino $B$-term as $MB_\nu N\bar{N}/2$ so, in this model, we expect $\sqrt{B_\nu M} \gg m_{\text{susy}}$.

Let us now suppose that besides $S$ which couples to the lepton number conserving part of the superpotential, there is a spurion field, $X$, that carries lepton number equal to two. We can then write the following term in the superpotential

$$\int d^2\theta \lambda \chi X N_i N_i.$$  

(5)

However, terms such as $\int d^2\theta X H_u H_d$ are forbidden by lepton number conservation. Moreover, terms such as $\int X^\dagger X \Phi^\dagger \Phi d^4\theta$ in the Kähler potential are suppressed by powers of $M^{-1}_{pl}$. Let us assume that the self-interaction of the hidden sector is such that both the scalar- and $F$-components of $X$ develop nonzero vacuum expectation values. The vacuum expectation values of the components of $X$ break the lepton number symmetry of the model. The vacuum expectation value of the scalar component of $X$, $\langle \tilde{X} \rangle$, corresponds to the Majorana mass term of the right-handed neutrinos while the vacuum expectation value of the $F$-component, $\langle F_X \rangle$, gives the neutrino $B$-term. With our parametrization of the neutrino $B$-term,

$$B_\nu = \frac{\langle F_X \rangle}{\langle X \rangle}.$$  

(6)

Both $\langle F_X \rangle$ and $\langle \tilde{X} \rangle$ can be large, giving rise to large right-handed neutrino masses and $B_\nu$, while other supersymmetry breaking terms, which are given by $\langle F_S \rangle$, are at the TeV scale or smaller. Notice that in this model, in the basis that the mass matrix of the right-handed neutrinos is real diagonal, the neutrino $B$-term is also diagonal so the parametrization that we are using for the neutrino $B$-term is the appropriate one.

3 Effects of the Neutrino $B$-term on the Higgs Mass Parameters 

In this section, we study the effects of a large $B_\nu$ on the Higgs mass parameters and derive bounds on its value from the fulfillment of the electroweak symmetry breaking condition.

Diagrams shown in Fig. 1 give a correction to $m_{H_u}^2$ which is equal to

$$-i \Delta m_{H_u}^2 = 2 \sum_k \int \frac{M_k^2 \text{Re}[B_\nu \sum_i (Y_{\nu})_{ki}(A_{\nu}^*)_{ki}]}{k^2(k^2 - M_k^2)^2} \frac{d^4k}{(2\pi)^4} = -i2 \sum_{k,i} \text{Re}[B_\nu \text{Tr}(Y_{\nu} A_{\nu}^*)] \frac{1}{16\pi^2}. $$

(7)

Presence of a large neutrino $B$-term also induces non-negligible corrections to $b_H$ as it is shown in Fig 2. The correction is finite and is equal to

$$-i \Delta b_H = -B_\nu \sum_k \int \frac{M_k^2 \text{Tr}[(Y_{\nu})_{ki}(Y_{\nu})_{ki}]}{k^2(k^2 - M_k^2)^2} \frac{d^4k}{(2\pi)^4} = \frac{iB_\nu \mu \text{Tr}[Y_{\nu} Y_{\nu}^\dagger]}{16\pi^2}.$$  

(8)

By dimensional analysis we can show that any correction due to $B_\nu$ to the quadratic Higgs
interaction is suppressed by $B_\nu/M$ which is negligible. The contribution to the cubic Higgs term is also zero. So the potential of $H_u^0$ and $H_d^0$ is

$$V = (|\mu|^2 + m_{H_u}^2 + \Delta m_{H_u}^2)|H_u^0|^2 + (|\mu|^2 + m_{H_d}^2)|H_d^0|^2$$

$$+ [(b_H + \Delta b_H)H_u^0 H_d^0 + \text{H.c.}] + \frac{g^2}{8}(|H_u^0|^2 - |H_d^0|^2)^2.$$  

Note that here we have not included the one-loop effective potential terms $^4$; however, since our analysis is based on an order of magnitude consideration, including those terms cannot alter our conclusions.

Requiring $m_Z^2 = (g^2 + g'^2)(\langle H_u \rangle^2 + \langle H_d \rangle^2)/2$ and $\partial V/\partial H_u^0 = \partial V/\partial H_d^0 = 0$, we find

$$|\mu|^2 + m_{H_d}^2 = |b_H + \Delta b_H| \cot \beta + (m_Z^2/2) \cos 2\beta$$

and

$$|\mu|^2 + m_{H_u}^2 + \Delta m_{H_u}^2 = |b_H + \Delta b_H| \cos \beta + (m_Z^2/2) \cos 2\beta$$

where $\tan \beta = \langle H_u \rangle/\langle H_d \rangle$. Assuming $|\mu|^2 \sim m_{H_u}^2 \sim m_{\text{susy}}$, Eq. (10) gives

$$|b_H - B_\nu \mu \frac{\text{Tr}[Y_\nu Y_\nu^\dagger]}{16\pi^2}| \sim m_{\text{susy}}/\tan \beta.$$  

From the LEP data $^5$, we know that $\tan \beta > 2$ and the data favors large values of $\tan \beta$ ($\tan \beta > 10$). Based on the naturalness condition, it seems quite unlikely that $b_H$ and $\Delta b_H$ cancel each other, so we expect that

$$B_\nu Y_\nu^2/(16\pi^2) < m_{\text{susy}}/\tan \beta.$$  

Notice that if $Y_\nu \ll 1$, $B_\nu$ can still be several orders of magnitude larger than $m_{\text{susy}}$. 

Figure 1: Diagrams contributing to $m_{H_u}^2$. $F_N^k$ represents the auxiliary field associated with the right-handed neutrino, $N_k$. The $A_\nu$ vertices are marked with black circles. The neutrino $B$-term and $M$ insertions are indicated by $\otimes$ and $\Delta$, respectively.

Figure 2: Diagrams contributing to the Higgs $B$-term. $F_N^k$ represents the auxiliary field associated with the right-handed neutrino, $N_k$. The two vertices marked with black circles are given by $\mu Y_\nu^*$ and $\mu^* Y_\nu$. The neutrino $B$-term and $M$ insertions are indicated by $\otimes$ and $\Delta$, respectively.
In the CMSSM, in addition to the phase of the CKM matrix, there are two sources of CP-violation which are usually attributed to the phases of $\mu$ and $a_0$. These two phases can induce EDMs for the electron, neutron and mercury. Combining the bounds on $d_e$ and $d_{Hg}$, one can derive strong bounds on $\text{Im}[a_0]$ and $\phi_{\mu}$. Adding the three heavy right-handed neutrinos to the model new sources of CP-violation emerge; six physical phases associated with $Y_\nu$ and the phase of $B_\nu$. The effects of the phases of $Y_\nu$ on the EDMs of charged leptons have been studied in a series of papers.\cite{30,31,32}. The effects of an imaginary $B_\nu$ on $d_e$ has been first noticed in Ref.\cite{3}. In this section, we briefly review the latter effect.

As it is depicted in Fig. 3 the neutrino $B$-term can induce a correction to the $A$-term of charged leptons. If $B_\nu$ is imaginary, the correction which is proportional to $B_\nu$ will also be imaginary, contributing to the EDMs of corresponding charged lepton:

$$d_{\ell_i} = -\frac{2\alpha}{(4\pi)^3} \sum_{\alpha k} \left( \frac{V_{\alpha 1 a}}{c_w} \right) \left( \frac{V_{\alpha 2 a}}{s_w} \right) \text{Im}[B] \sum_{\ell} \left( Y_{\nu}^k \right)^* Y_{\nu}^{k\ell} f \left( \frac{m_e^2}{m_a^2}, \frac{m_e^2}{m_a^2} \right) \tilde{S}$$ \hspace{1cm} (14)

where $\tilde{S}$ is the spin of the particle; $V_{\alpha i a}$ and $m_a$ are respectively the mixing and masses of the neutralinos and

$$f(\nu_L, \nu_E) = \frac{1}{2} \left( \frac{1}{x_E - x_L} \right) \left( 1 - \frac{x_L^2 + 2x_L \log x_L}{(1 - x_L)^3} - \frac{1 - x_E^2 + 2x_E \log x_E}{(1 - x_E)^3} \right).$$ \hspace{1cm} (15)

If we assume that the imaginary $B_\nu$ is the dominant source of CP-violation contributing to $d_e$, the present strong bound\cite{33} on $d_e$ ($d_e < 1.4 \times 10^{-27}$) implies $\text{Im}[B_\nu] \sum_i \left| (Y_{\nu})_{\alpha i} \right|^2 / (16\pi^2) \lesssim 0.1 m_{\text{susy}}$. This bound can be improved significantly in the near future.

Recently, it has been shown that an imaginary $B_\nu$ gives an imaginary correction to $A_\mu$, inducing a contribution to the EDMs of $d_{Hg}$ and $d_a$, too\cite{34}.

In principle, the contribution of the different CP-violating phases to EDMs can cancel each other. According to Ref.\cite{35}, if $\mu$ and $a_0$ are the only sources of CP-violation, it will not be possible to satisfy the upper bounds on $d_e$ and $d_{Hg}$ by cancelation scenario and as a result the phases of $\mu$ and $a_0$ indeed have to be very small. Now, if we turn on the imaginary $B_\nu$, there will be enough parameters to satisfy the experimental bounds even if $\phi_\mu, \phi_{a0} \sim 1$. This can have novel experimental implications in accelerator physics\cite{36}.

5 Conclusions

The condition for the electroweak symmetry breaking implies $|b_H - B_\nu \text{Tr}[Y_{\nu}^\dagger Y_{\nu}] / 16\pi^2| \sim m_{\text{susy}}^2 / \tan \beta$. Assuming that the other supersymmetry breaking parameters are all of order
of a few hundred GeV, this puts an upper bound on $B_\nu$ which is stronger than the bound derived from the radiative correction of the $B$-term to $m_{l_i}$. Furthermore, unlike the bound derived in Ref. 1, the bound discussed in this paper does not depend on the values of the right-handed neutrino masses.

Even within this bound, an imaginary $B_\nu$ can induce a significant contribution to the charged lepton EDMs:

$$d_{l_i} \sim 10^{-27} \frac{\text{Im}[B_\nu]}{m_L} Y^2 \left( \frac{200 \text{ GeV}}{m_L} \right)^2 \frac{m_{l_i}}{m_e} e \text{ cm.} \quad (16)$$

Note that if $\text{Im}[B_\nu] \sim m_{\text{susy}}$ (as it is expected in the framework of mSUGRA) this contribution can saturate the present bound on $d_e$.

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