Does the $^{12}\text{C}+^{12}\text{C}$ fusion reaction trigger superburst?

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Abstract. Superbursts are long, energetic, rare explosions, probably triggered by the $^{12}\text{C}+^{12}\text{C}$ burning in the crust of neutron stars. However, with the currently adopted $^{12}\text{C}+^{12}\text{C}$ fusion reaction rate, it is impossible for the current model to explain observations. Therefore, a strong resonance at $E_{c.m.}=1.5$ MeV has been proposed to enhance the carbon fusion reaction rate. By comparing the cross sections of the three carbon isotope fusion reactions, $^{12}\text{C}+^{12}\text{C}$, $^{12}\text{C}+^{13}\text{C}$ and $^{13}\text{C}+^{13}\text{C}$, we have established an upper limit for the $^{12}\text{C}+^{12}\text{C}$ fusion reaction rate. Our preliminary results show that the proposed strong resonance might not be realistic. The superburst puzzle is still unsolved.

1. Introduction
Superbursts are long, energetic, rare thermonuclear flashes on accreting neutron stars in low-mass X-ray binaries[1]. These bursts are considered to be triggered by the unstable $^{12}\text{C}$ burning in the ash left over from the rp-process on the surface of a neutron star. The ash is heated up with the heat generated in the crust of a neutron star and ignites the carbon in the ash. However, with the currently adopted $^{12}\text{C}+^{12}\text{C}$ fusion rate, superburst modes fail to explain the ignition of carbon at the column depth inferred from observations [2].

Studies of the $^{12}\text{C}+^{12}\text{C}$ fusion reaction started in early 1960s using the first tandem accelerator at Chalk River. For at least three energies, $E_{c.m.}=5.68$, 6.00 and 6.32 MeV, resonances were observed in the yields of collisional byproducts: $p$, $\alpha$, $n$ and $\gamma$[3]. These resonances have characteristic widths of about 100 keV. This measurement triggered several decades of studies on both experimental and theoretical sides. As measurements were pushed down towards lower energies, the discoveries of such resonances continued as far as the lowest measured energies. For instance, the most recent published measurement of the $^{12}\text{C}+^{12}\text{C}$ fusion reported a strong resonance at $E_{c.m.}=2.14$ MeV[4]. However, the important energy range for astrophysical interest lies between 1 to 3 MeV. For the superburst problem, the energy range is between 1 and 2 MeV, which is beyond the reach of the current measurements.

There are a number of theoretical models proposed to explain the unique resonance feature presented in $^{12}\text{C}+^{12}\text{C}$[5, 6]. However, none of them is able to provide a quantitative description of the $^{12}\text{C}+^{12}\text{C}$ resonances. With the presence of resonances in the measured and the potential resonances in the unmeasured energy ranges, it is quite ambiguous to determine the gross structure of the fusion cross section and provide an extrapolation down to the lower energies which are crucial for astrophysical application. The currently adopted carbon fusion reaction
rate was determined using the modified S* factor introduced based on the penetrability of a simple square model,

\[ S^*(E) = \sigma(E) E \frac{8.721}{\sqrt{E}} + 0.46E. \]  

(1)

An averaged S* factor of \( 3 \times 10^{16} \) obtained in the range of sub-barrier energies was recommended as the S* factors in the energy range of astrophysical relevant[7].

The strong resonance found at \( E_{c.m.} = 2.14 \) MeV has totally different characters than the other resonances we have observed. The width of this resonance is less than 12 keV which is much narrower than the 100 keV width observed before. The S* factor at the resonant energy is about 2 orders of magnitude higher than others. It is clear that the recommended averaged S* factor is not sufficient to accommodate such a strong resonance. Inspired by this strong resonance observed in experiment, a hypothetical resonance at \( E_{c.m.} = 1.5 \) MeV with a similar strength was proposed to enhance the fusion reaction rate at \( T = 0.5 \times 10^9 \) K by a factor of over two orders of magnitude and thereby alleviate the discrepancy between superburst models and observations[8].

2. An Empirical relationship among the carbon isotope fusion reactions

In contrast to the striking resonances in the \( ^{12}\text{C}+^{12}\text{C} \) fusion reaction, other carbon isotope fusion reactions, such as \( ^{12}\text{C}+^{13}\text{C} \) and \( ^{13}\text{C}+^{13}\text{C} \), behave more regularly. Only minor resonance features have been observed in these two systems [9, 10]. The shape of the fusion cross sections at sub-barrier energies is primarily dominated by the Coulomb barrier penetration effect. To remove this effect and reveal more details of nuclear interaction, the cross sections of all three carbon fusion systems are converted into the S* factors using eq.1. The advantage of using this conversion over the traditional astrophysical S factor is that the cross section ratios among the three systems are preserved.

The experimental S* factor for all three carbon isotope fusion systems are shown in Fig.1. There are several important features in these carbon isotope fusion systems:

(i) The \( ^{12}\text{C}+^{12}\text{C} \) cross sections are bound from above by the cross sections of the other two carbon isotope fusion systems.

(ii) The major resonant cross sections of \( ^{12}\text{C}+^{12}\text{C} \) (\( E_r = 3.1, 4.3, 4.9, 5.7, 6.0 \) and \( 6.3 \) MeV) match remarkably well with the other two carbon isotope fusion cross sections, especially \( ^{13}\text{C}+^{13}\text{C} \).

(iii) Overall, the \( ^{12}\text{C}+^{13}\text{C} \) cross sections are the highest among the three carbon isotope fusion systems. The deviation between \( ^{12}\text{C}+^{13}\text{C} \) and \( ^{13}\text{C}+^{13}\text{C} \) occurs in the energy of 3.5 to 5 MeV (excluding the 4.25 MeV resonance in \( ^{12}\text{C}+^{13}\text{C} \)) and is less than 30%.

(iv) The 4.25 MeV resonance in \( ^{12}\text{C}+^{12}\text{C} \) is also present in \( ^{12}\text{C}+^{13}\text{C} \).

In the \( ^{12}\text{C}+^{13}\text{C} \) and \( ^{13}\text{C}+^{13}\text{C} \) fusion reactions, the valence neutron of \( ^{13}\text{C} \) introduces an additional degree of freedom for excitation, besides the \( ^{12}\text{C} \) core excitation. Despite this complication, the maximum deviation among the \( ^{12}\text{C}+^{13}\text{C} \) peak cross sections and the \( ^{12}\text{C}+^{13}\text{C} \) and \( ^{13}\text{C}+^{13}\text{C} \) fusion cross sections is less than 30%. Based on this strong correlation, we claim:

(i) In the \( ^{12}\text{C}+^{12}\text{C} \) fusion reaction, all the major resonances with maxima matching the \( ^{12}\text{C}+^{13}\text{C} \) and \( ^{13}\text{C}+^{13}\text{C} \) cross sections should have the same reaction mechanism;

(ii) This reaction mechanism should also be present in the fusion of \( ^{12}\text{C}+^{13}\text{C} \) and \( ^{13}\text{C}+^{13}\text{C} \);

(iii) The effect arising from the valence neutron in \( ^{13}\text{C} \) is not strong compared to this reaction mechanism but it is strong enough to make the resonances disappear in the fusion of \( ^{12}\text{C}+^{13}\text{C} \) and \( ^{13}\text{C}+^{13}\text{C} \).
Figure 1. The experimental S* factors of three carbon isotope fusion reactions, $^{12}\text{C}+^{12}\text{C}$ (red and blue points) [4, 12], $^{12}\text{C}+^{13}\text{C}$ from this work (black points) and Ref.[9] (green points), and $^{13}\text{C}+^{13}\text{C}$[10](triangle points). The averaged S* factor of $^{12}\text{C}+^{12}\text{C}(S^*=3\times10^{16}$ MeVb) labeled as CF88 (red dashed line) is recommended[7] based on the data of from Ref.[11, 12, 13]. The systematic uncertainties, 15% for the $^{13}\text{C}+^{13}\text{C}$ data from Ref.[10](13C+(Trentalange)) and 30% for the $^{12}\text{C}+^{13}\text{C}$ data from Ref.[9]($^{12}\text{C}+^{13}\text{C}$(Dayras)), are not shown in the graph. The $^{12}\text{C}+^{13}\text{C}$ data reported in this paper ($^{12}\text{C}+^{13}\text{C}$(ND)) is dominated by a 20% systematic uncertainty. The data from a recent $^{12}\text{C}+^{12}\text{C}$ measurement[4] ($^{12}\text{C}+^{12}\text{C}$(Spillane)) are shown by blue circles. The hypothetical resonance proposed in Ref. [8] with a smaller resonance strength ($\omega\gamma=3.4\times10^{-9}$ eV) is shown as Cooper 1 while the one with a larger resonance strength is shown as Cooper2 ($\omega\gamma=3.4\times10^{-7}$ eV).

3. Coupled-channel calculations using the Incoming Wave Boundary Condition (IWBC)

In this section we discuss coupled-channels calculations for the three systems using two different potentials. Before we present our results, it is important to briefly review the ingoing wave boundary condition (IWBC) used in our calculations[14]. In this approach, the fusion system is treated as a “black body” once the two nuclei in the fusion process overlap significantly. At the minimum position of the Coulomb plus nuclear potential pocket inside the barrier, the wave function for each partial wave has only ingoing components. This approximation is valid for most heavy-ion fusion reactions in which there is a strong absorption inside the Coulomb barrier. There is no provision within the IWBC for describing the resonant structure in the $^{12}\text{C}+^{12}\text{C}$ reaction. This approximation has been used to describe the average trend in the $^{12}\text{C}+^{12}\text{O}$ and $^{16}\text{O}+^{16}\text{O}$ fusion reactions. Reasonable fits have been achieved for these three systems by adjusting the interaction potential[14]. However, with the presence of resonances, the choice of the nuclear potential is somewhat ambiguous.

The first potential used in the calculations is the Akyüz-Winther potential [15] which is an empirical global potential frequently used for fusion reactions at energies around the Coulomb barrier. The coupled-channels calculation (CC-AW) was done using the CCFULL code[16]. In the calculation, both the $^{12}\text{C}(2^+, 4.44\text{ MeV})$ and the $^{13}\text{C}(3/2^-, 3.684\text{ MeV})$ states are included. The results are shown by the dot-dashed lines in Fig. 2. At $E_{c.m.}>5\text{ MeV}$, the CC-AW calculation provides a good description of the fusion cross sections for $^{12}\text{C}+^{12}\text{C}$, $^{12}\text{C}+^{16}\text{O}$ and $^{16}\text{O}+^{16}\text{O}$ fusion reactions. Reasonable fits have been achieved for these three systems by adjusting the interaction potential[14]. However, with the presence of resonances, the choice of the nuclear potential is somewhat ambiguous.

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seems to be a universal phenomenon at energies significantly below the Coulomb barrier[17, 18].

An improved coupled-channels calculation (CC-M3Y+Rep) has recently been performed using the M3Y potential with a repulsive core[18]. The $^{12}\text{C}+^{13}\text{C}$ and $^{13}\text{C}+^{13}\text{C}$ data were used to constrain the effective nuclear potential which was then used for the calculation of the fusion cross sections of the $^{12}\text{C}+^{12}\text{C}$ system within the CC formula. The coupling effects from all the possible excited states with large deformation parameters as well as the transfer channels are included. Details of the calculation are presented in a separate paper[19]. Here we focus on the results shown by the solid red lines in Fig. 2. It is clear that the CC-M3Y+Rep calculation provides an excellent description for the $^{12}\text{C}+^{13}\text{C}$ and $^{13}\text{C}+^{13}\text{C}$ systems. The overall deviations are less than 30% which is comparable with the experimental uncertainties. Using this constrained $^{12}\text{C}+^{12}\text{C}$ nuclear potential, the coupled-channels calculation is able, for the first time, to predict all the major $^{12}\text{C}+^{12}\text{C}$ peak cross sections reported in Ref. [12] with deviations of less than 30%.

4. Explanation to the observed correlation
The Nogami-Imanishi model suggests that the effect of the coupling to the $^{12}\text{C}(2^+, 4.44\text{ MeV})$ state is crucial for the formation of the $^{12}\text{C}+^{12}\text{C}$ molecular resonances. The coupling effect leading to the resonances only takes place when the energy matches the condition of the molecular states[20]. This model was subsequently generalized to provide a qualitative description for the molecular resonances at energies both below and above the Coulomb barrier[5, 6]. Kondon, Matsuse, and Abe took into account the excitation of both $^{12}\text{C}$ nuclei and
extended the calculation to the energy range of 4 MeV<\(E_{c.m.}\)<7 MeV[21]. Their model could reproduce the characteristic features of the total reaction cross section, i.e. resonance widths, peak heights, level densities of the resonances and the energy dependence of the averaged total reaction cross sections. In this calculation, only three rotational eigenstates with \(L=0, 2\) and 4 were included in elastic, single and mutual excited channels. However, with these coupling effects, they obtained a rich resonant structure with 14 resonances in the range of 4 MeV<\(E_{c.m.}\)<7 MeV. The level spacing is about 200 keV which is wider than the resonance widths (~100 keV).

In the other two carbon fusion systems, such a coupling effect should also exist. The presence of valence neutron significantly increases the level density of compound nuclei[22]. As a result, the matching condition becomes valid at any energies and the coupling effect can take place at any energy. Therefore, these two systems are expected to behave like strong absorption systems and exhibit a similar behavior in the fusion cross sections over a wide energy range. However, for the \(^{12}\text{C} + ^{12}\text{C}\) system, the fusion cross sections are suppressed because of its low level density and are upper bounded by the cross sections of \(^{12}\text{C}+^{13}\text{C}\) and \(^{13}\text{C}+^{13}\text{C}\).

5. Strong resonances at low energies

The recently published low-energy data points from Ref. [4] are also included in Fig.2(c). It is obvious that our CC-M3Y+Rep calculation (see red solid line in Fig.2c) provides an excellent upper limit for almost all the data except for the two data points around 2.14 MeV. These two points are also well above the CC-AW calculation which seems to always over-predict fusion cross sections at deep sub-barrier energies[17].

At deep sub-barrier energies, the number of the reaction channels which are important for the coupling effect is very limited. According to our calculation, the enhancement from this effect is about a factor of 2~4 at deep sub-barrier energies. It is impossible for this effect to enhance the cross section by one or two order of magnitude. To produce strong resonances such as the one reported at 2.14 MeV, it is mandatory to introduce a sudden change in nuclear potential to lower the Coulomb barrier. It is a great challenge for theory to produce such a kind of nuclear potential and still maintain a good description to the observed correlation among the three carbon systems.

It should be mentioned, however, that the \(^{12}\text{C}+^{12}\text{C}\) reaction has been remeasured at Naples by the same group in the energy range of 2.1 MeV<\(E_{c.m.}\)<4MeV[24]. Their preliminary result shows a much smaller cross section at 2.15 MeV which agrees with the upper limit shown in Fig.2(c).

If the 2.14 MeV resonance is eventually confirmed to be overestimated, our extrapolated upper limit would put a useful constraint on the carbon fusion cross sections at lower energies which are important for astrophysical application. For \(E_{c.m.}<2\) MeV, with the CC-M3Y+Rep prediction, the fusion rate in the range of 0.1 to 1.5 GK could increases only by a factor less than 2. Therefore, the enhancement of \(\geq 25\) or \(\geq 250\) proposed in Ref.[8] could not be realistic. It is likely that some unknown physics process is not included in the current superburst models. For example, an additional heating process in the neutron star crust could raise the peak temperature in the ash to meet the required ignition condition. It is also possible that some process other than carbon burning triggers the burst.

6. Summary and outlook

To summarize, an empirical relationship among the fusion cross sections for the three carbon isotope systems, \(^{12}\text{C}+^{12}\text{C}\), \(^{12}\text{C}+^{13}\text{C}\) and \(^{13}\text{C}+^{13}\text{C}\), has been found. After calibrating the M3Y+Rep effective potential, the coupled-channels calculation with the IWBC (CC-M3Y+Rep) is able to provide a quantitative description for the peak cross sections in \(^{12}\text{C}+^{12}\text{C}\). This finding can be qualitatively understood within the Nogami-Imanishi model and the level density
differences in the compound nuclei. For the first time, we set an upper limit for the $^{12}$C+$^{12}$C fusion cross section in the range of $E_{c.m.}>2.1$ MeV which agrees with all the experimental data except for the strong resonance observed at 2.14 MeV. This system has recently been re-measured yielding a smaller cross section which agrees with our predictions. The extrapolated upper limit puts a useful constraint on the carbon fusion cross sections at lower energies which are important for astrophysical application.

To provide a more accurate prediction at deep sub-barrier energies, it is urgent to improve the experimental techniques so that the measurements of the fusion reactions among carbon isotopes can be pushed towards lower energies. Right now, the Notre Dame group is building a new 5 MV accelerator with an ECR source to provide high current carbon beams. With improvements of the detection techniques [25], better experimental data can be expected in the near future. The proposed LUNA II upgrading project in Gran Sasso [26] and the DIANA project proposed at DUSEL[27] will offer the best approach to conquer this challenging problem.

7. acknowledgments

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References
[1] T. Strohmayer and L. Bildsten, Compact Stellar X-ray Sources (Cambridge University Press, 2004), p. 113.
[2] L. Keek et al., Astro & Astrophys., (2008) 479
[3] Almqvist, Bromley and Kuehner, Phys. Rev. Lett. V4, (1960) 515
[4] T. Spillane et al., Phys. Rev. Lett. 98 (2007) 122501
[5] K. A. Erb and D. A. Bromley, Heavy Ion Resonance (Plenum Preess, New York, 1985), vol. 3, chap. 3, pp. 201310.
[6] W. Greiner, Nuclear Molecules (World Scientific, 1994)
[7] G. R. Caughlan and W. A. Fowler, Atomic Data And Nuclear Data Tables 40 (1988)283
[8] R. L. Cooper, A. W. Steiner, and E. F. Brown, Ap. J. 702, (2009)660
[9] R. A. Dayras, R. G.Stokstad, Z.E.Switkowski, and R.M.Wieland, Nucl. Phys. A265, (1975) 153
[10] S. Trentalange, S. C. Wu, J. L. Osborne, and C. A. Barnes, Nucl. Phys. A483 (1988) 406
[11] J.R.Patterson and H.Winkler and C.S.Zaidins, Ap. J. 157 (1969) 367
[12] H.W.Becker, K.U.Kettner, C.Rolfs, and H.P.Tauftvetter, Z. Phys. A 303 (1981) 305
[13] H.Spinka and H.Winkler, Nucl. Phys. A233 (1974) 456
[14] P. Christensen and Z. Switkowski, Nucl. Phys. A 280, 205 (1977).
[15] Ö. Akyüz and A. Winther, in Proc. of the Énrico Fermi School of Physics, 1979, edited by R. A. Broglia, C. H. Dasso, and R. Ricci (1981).
[16] K. Hagino, N. Rowley, and A.T. Kruppa, Comput. Phys. Comm 123, 143 (1999).
[17] C. L. Jiang, K. E. Rehm, B. B. Back, and R. V. F. Janssens, Phys. Rev. C. 75, 015803 (2007).
[18] S. Misicu and H. Esbensen, Phys. Rev. C 75, (2007) 034606
[19] H. Esbensen, X.D. Tang and C. L. Jiang, Submitted to journal (2011).
[20] B. Imanishi, Phys. Lett. B 237, (1968) 267
[21] Y. Kondo, T. Matsuura, and Y. Abe, Prog. Theor. Phys. 59, 465 (1978).
[22] B. Heusch et al., Phys. Rev. C 23, (1981) 1527
[23] M. Notani et al., Nucl.Phys. A 834, (2010) 192c.
[24] J. Zickefoose, U. Conn. Thesis (2011)
[25] C. Broginni, D. Bennmerer, A. Guglielmetti, and R. Menegazzo, Annual Review of Nuclear and Particle Science, V60 (2010) 53-73.
[26] DUSEL White Paper, Working group A5, Underground Accelerator Laboratory for Nuclear Astrophysics, Organized by Daniela Leitner, Arthur Champagne and Michael Wiescher, http://www.lbl.gov/nsd/homestake/aprilworkshop/whitepapers/NuclearAstrophysicsAccelerator071102007.pdf.