Quantum Gunn effect: Zero-resistance state in 2D electron gas.

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Usually, the conductivity is quantized as the inverse of the resistivity, \( \rho = \frac{hc}{ie^2} \), \( \sigma = \frac{ie^2}{hc} \), and the velocity versus the electric field is linear, \( v = \mu E \), where \( \mu \) is the mobility of the electrons. However, when the applied electric field exceeds a certain value, microwaves are emitted and the relation \( v = \mu E \) breaks down so that the velocity actually reduces as \( E \) increases. In this region, when magnetic field is applied, the conductivity quantizes like the magnetic field, i.e., in multiples of \( hc/e \) which is different from the usual quantization. Because of the flux quantization, the resistivity will touch zero in the region of high electric field. Factors like \( 4/5 \) arise due to new spin dependence of the effective charge.

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1. Introduction

Recently, Mani et al\(^1\) have measured the resistivity of GaAs/AlGaAs which is linear at small magnetic fields and shows de Haas-van Alphen oscillations for fields larger than \( (4/5)B_f \) where \( eB_f/m^*c = \omega_c \) is the cyclotron frequency. When the sample is irradiated with microwaves of frequency \( \sim 103 \text{ GHz} \), the linear resistivity below the field of \( (4/5)B_f \) comes down and touches the zero value. We wish to understand this reduction in the resistivity and why it approaches zero value. Zudov et al\(^2\) also found that the diagonal resistivity touches the zero value when sample is irradiated with microwaves of frequency \( \sim 57 \text{ GHz} \). We have reported\(^3\) that spin determines the fraction which appears before the magnetic field, \( B_f \) and flux quantization is important to get zero resistivity. Volkov\(^4\) and Bergeret et al\(^5\) find that Gunn effect is important to obtain negative resistivity which is the cause of reduction with respect to the value without the microwave radiation and this type of negative resistance is in agreement with the experimental work of Willett\(^6\) where observation of the negative resistance was mentioned.

At small electric fields, the velocity is linearly related to the electric field, \( v = \mu E \). However, at large fields, the velocity does not increase with increasing field but it decreases. In this region, it is interesting to quantize the conductivity. Since the system radiates microwaves, it is called the Gunn effect. The quantization of the magnetic field in the region of Gunn effect leads to quantized conductivity. Thus, we report the “quantum Gunn effect” (QGE) which is the quantized conductivity in the region of electric field relevant to Gunn effect. Usually, the conductivity is quantized in units of \( e^2/hc \) but
in the present case, it is quantized in units of $\hbar c/e$ with a suitable multiplier so that the units are correct. Another aspect of the QGE is that as $n$ is varied, several microwave frequencies are emitted instead of only one frequency of the Gunn effect.

2. Theory

Usually the conductivity, $\sigma$, is defined by, $j = \sigma E$ where $j$ is the electrical current and $E$ is the electric field. The current is defined in terms of velocity, $v = \mu E$ where $\mu$ is called the mobility. The electron as well as the hole currents can thus be written in terms of velocity as,

$$j = n_i e v_e + n_h e v_h = n_i e \mu_i E + n_h e \mu_h E$$  \hspace{1cm} (1)

This means that $v$ is a linear function of $E$. Actually, at large electric fields, $v$ as a function of $E$ ceases to be linear and when there is sufficient heating, energy is emitted in the form of microwaves. Such an emission of microwaves for electric fields larger than a certain characteristic value, $E_o$ is called the Gunn effect. For $E > E_o$, the velocity actually decreases with increasing $E$. Therefore, the region, $E > E_o$, offers an unusual opportunity to investigate the flux quantization. The usual quantization of resistivity or that of conductivity, is based on the region $E < E_o$, where $\rho = \hbar / ie^2$ or $\sigma = ie^2 / h$.

We therefore expect a new type of quantization for fields $E > E_o$. The number of particles with velocity in the range of $dv$ at $v$ is given by Maxwell velocity distribution but conversion of velocity into electric field requires the knowledge of linearity between the velocity and the electric field. For $E > E_o$, such a linear formula is not valid. Therefore, we use a simple form which has a peak at $E_o$. For $E < E_o$, this function has a linear region so that velocity increases with increasing electric field up to $E = E_o$. Once $E > E_o$, the linear behavior is not needed and the system emits microwaves. Therefore, the distribution should have a peak at $E_o$ and should fall for $E > E_o$. Therefore we use,

$$v = \mu E \frac{\Delta^2}{(E - E_o)^2 + \Delta^2}$$  \hspace{1cm} (2)

instead of $v = \mu E$. At $E = E_o$, there is a peak in the velocity. For $E < E_o$, the velocity becomes,

$$v = \mu E \frac{\Delta^2}{E_o^2 + \Delta^2}$$  \hspace{1cm} (3)

which is linear in $E$ but is smaller than $\mu E$. At the peak $E = E_o$, $v = \mu E$ is established and for $E > E_o$, $v = \mu E[\Delta^2 / (E^2 + \Delta^2)]$ which falls with increasing $E$. This is the region where Gunn effect is known to occur. When the distribution of velocity as a function of electric field (2) is used the current becomes,

$$j = n_i e \mu_i E \frac{\Delta^2}{(E - E_o)^2 + \Delta^2} + n_h e \mu_h E \frac{\Delta^2}{(E - E_o)^2 + \Delta^2}$$  \hspace{1cm} (4)

where $n_i (n_h)$ are the electron (hole) concentrations and $\mu_i$ and $\mu_h$ are the electron and hole mobilities, respectively. The conductivity is determined by two terms, one due to electrons, $\sigma_e$ and the other due to holes, $\sigma_h$, with,

$$\sigma_e = n_i e \mu_i \Delta^2 \left[ \frac{1}{(E - E_o)^2 + \Delta^2} - \frac{2E(E - E_o)}{((E - E_o)^2 + \Delta^2)^2} \right] + e \mu_i \Delta^2 \frac{d n_i}{dE} \frac{E}{(E - E_o)^2 + \Delta^2}$$  \hspace{1cm} (5)
\[ \sigma_h = n_h e \mu_h \Delta^2 \left[ \frac{1}{(E - E_o)^2 + \Delta^2} - \frac{2E(E - E_o)}{(E - E_o)^2 + \Delta^2} \right] + \mu_h e \Delta^2 \frac{dn_h}{dE} \frac{E}{(E - E_o)^2 + \Delta^2} \]  

(6)

When the electric field is increased, if the electron concentration increases, that of the holes must reduce so that,

\[ \frac{dn_e}{dE} = - \frac{dn_h}{dE} \]  

(7)

Substituting this result in (5) and (6) and adding the two terms,

\[ \sigma = \sigma_e + \sigma_h = \sigma_o + (\mu_i - \mu_h)e \Delta^2 \frac{dn_i}{dE} \frac{E}{(E - E_o)^2 + \Delta^2} \]  

(8)

where

\[ \sigma_o = e \Delta^2 (n_i \mu_i + n_h \mu_h) \left[ \frac{1}{(E - E_o)^2 + \Delta^2} - \frac{2E(E - E_o)}{(E - E_o)^2 + \Delta^2} \right] \]  

(9)

In the Gunn effect, microwaves are emitted once the applied electric field is larger than \( E_o \) but there is no need of any magnetic field. There are many calculations of the application of crossed electric and magnetic fields to the semiconductors but in the present problem, \( \nu = \mu E \) has been avoided and peaked distribution has been used so that it will be of interest to see the effect of magnetic field when \( E > E_o \). We can apply the magnetic field by considering the Lorentz force so that the effective electric field becomes,

\[ E' = E + \frac{1}{c} (\vec{v} \times \vec{B}) \]  

(10)

In the present case \( \vec{v} \) is a peaked function so that the electric field becomes,

\[ E' = E + \frac{1}{c} \mu (\vec{E} \times \vec{B}) \frac{\Delta^2}{(E - E_o)^2 + \Delta^2} \]  

(11)

where \( \mu \) is the mobility. The effective electric field then becomes,

\[ E' = |E||1 + \frac{\mu \Delta^2 B}{c (E - E_o)^2 + \Delta^2}] \]  

(12)

The second negative term of (9), upon the application of magnetic field to electrons becomes,

\[ \sigma_{o-} \sim -\frac{2e \Delta^2 n_i \mu_i |E|^2}{(E - E_o)^2 + \Delta^2} \left[ 1 + \frac{\mu_i \Delta^2 B}{c (E - E_o)^2 + \Delta^2} \right] \left[ 1 - \frac{E_o}{|E|} \right] \]  

\[ \left[ (E - E_o)^2 + \Delta^2 \right]^2 \]  

(13)

which shows that when the applied field is larger than certain value, the conductivity can reduce. We can write the above conductivity as,

\[ \sigma_{o-} \sim -a_1 [1 + a_2 B] \]  

(14)
Where we consider the flux quantization, so that the magnetic field in the area A can be written as $B.A=n\phi_o$ where $\phi_o=hc/e$ which means that the conductivity in the Gunn region becomes,

$$\sigma_o \simeq -a_1[1 + a_2n\phi_o/A] \simeq -a_1[1 + a_2(n/A)hc/e]$$  \hspace{1cm} (15)

This formula is very different from the $ie^2/h$ type quantization of conductivity but this is a “quantized Gunn effect”. Without the quantization, where only a single frequency microwave was emitted, now many more microwave frequencies will be emitted due to different values of $n$ in (15). However, the expression (13) is quite approximate so that actual behaviour is much more complicated than by only one term in (15).

Thus we learn that conductivity has positive terms and electric field dependent negative terms so that when the electric field is increased the negative term in the conductivity can become large so that conductivity reduces. Thereafter, upon application of magnetic field, plateaus can arise due to “flux quantization”. This conductivity quantization is different from the usual quantization in terms of $ie^2/h$. For different values of the integer several plateaus can arise which means that instead of a single microwave frequency emission in the Gunn effect, several frequencies can arise. These predicted features of the conductivity are qualitatively in accord with the present day experimental data. The prediction of emission of multiple frequencies in the Gunn effect has not yet been verified by the experiments. However, such multiple frequencies in the Gunn effect must occur. The flux quantization measures the product of the charge and the magnetic field. Therefore, fractional numbers arise in the product $eB$. These fractions are well predicted in a previous paper$^3$ where it was found that the flux quantization depends on spin.

3. Discussions.

Anderson and Brinkman$^7$ have pointed out the importance of observation of zero resistivity when the sample is irradiated with microwaves at a frequency somewhat higher than the cyclotron frequency. They have suggested that it depends on the structure of the energy levels in crossed electric and magnetic fields. The old calculations of the crossed electric and magnetic fields have positive resistance. We have found that the factors like $4/5$ arise only when spin is involved due to the spin dependent flux quantization. The resistivity is zero for the flux-quantized state. The negative resistance arises due to heating of the sample above an electric field of $E_o$. This negative resistance phenomenon is called the Gunn effect where microwaves are emitted by the same. The emission of radiation is associated with negative resistance with absorption associated with positive resistance. The microwave frequency somewhat higher than the cyclotron frequency is thought to be due to scattering by impurities by the Yale group$^8$. However, there is no need of impurities in our calculation and we obtain the correct values without impurities in the pure sample$^9,10$.

Cheremisin$^{11}$ has correctly pointed out that $j = \sigma_{xy}E$ is violated. In the N shaped current(voltage) characteristics, if peak occurs for negative $E$, then it serves not much purpose. Similarly, in the S-shaped $I(V)$ characteristics there is no peak for positive electric field. We need a peak in the current(voltage) characteristics for positive value of the electric field as we have introduced by eq.(2). In this case Gunn effect leads to
reduced or negative resistivity needed by the experiment. Similarly, Shi and Xie\textsuperscript{12} used the current linear in $E_o$ along with integral of current-current correlations. This will not produce a Gunn effect which we obtain because of peaked relationship between velocity and the electric field.

When microwaves are emitted by the Gunn effect\textsuperscript{13–18}, it is important that Goldstone theorem is obeyed. That means that Goldstone bosons are emitted. These Goldstone bosons are the heat waves or phonons. The emission of phonons must be accompanied by charge-density waves. This can be seen, for example, by factorizing the electron-phonon interaction. Making factors of electron number density automatically produces soft phonons. Therefore, there are charge density waves which actually emit electromagnetic waves or microwaves which is another way of saying that there is Gunn effect which leads to negative or reduced resistance. We can obtain zero resistance by having spin singlets as in superconductivity or triplet states as in $^3$He. The charge density waves are paramagnetic and hence have small resistance. When energy is emitted, we call it “negative resistance”. In the present phenomenon, there are spin singlets with one component leading to zero resistance accompanied by flux quantization.

4. Conclusions.

We propose a flux-quantized Gunn effect as the interpretation of negative resistance which also has plateaus. The fractions before the cyclotron frequency arise due to “spin-dependent flux quantization”. The zero-resistance plateaus are due to flux quantization. The zero as well as negative resistance thus is well understood. There is a striking similarity between the resistance in the present samples and the diagonal resistance in the quantum Hall effect. In fact, in the correct theory\textsuperscript{9} which produces the same fractional charges as the experimental data, the requirement of the Hall geometry is not strictly needed. The fractional charge depends on the spin and the orbital state of the quasiparticles. The fractional charges, such as 4/3, found in the data are the same as in the calculated tables. As far as the emission of microwaves is concerned, more than one frequency will be generated due to integer, $n$ in $n\phi_o$ which quantizes the magnetic field.

5. References

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