Tetraneutron resonance in the presence of a dineutron

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Background: Several previous studies provided contradicting results for the four-neutron system, some claiming the existence of a $0^+$ near-threshold resonance, others denying presence of any observable resonant states.

Purpose: Since most of the studies employed enhanced two-neutron interactions to follow the evolution of an artificially bound state into a continuum one, we examine several enhancement schemes that produce a bound dineutron as well.

Methods: We study the four-neutron system by solving exact four-particle equations. By varying the interaction enhancement factor we calculate two-dineutron scattering phase shifts and cross sections.

Results: When the same enhancement factor is used in all partial waves, a bound tetraneutron emerges together with a strongly bound dineutron. Furthermore, such a $0^+$ tetraneutron evolves not into a resonance but into a virtual state. Weak enhancement of $S$ waves together with strongly enhanced higher waves is needed for the emergence of the resonant state. Anyhow the resonant behavior disappears well before reaching the physical interaction strength.

Conclusions: The interaction enhancement scheme using the same factor for all waves, employed in several previous works, is misleading for the search of $0^+$ resonance as only a virtual state can emerge. Evolution of a bound tetraneutron into a resonance via an intermediate virtual state is possible with strong enhancement of higher two-neutron waves.

I. INTRODUCTION

A possible experimental observation of the tetraneutron resonance \cite{1} triggered a number of theoretical studies \cite{2-4}. Their results are highly contradictory: a resonance with total angular momentum and parity $J^\pi = 0^+$ just 1 - 2 MeV above threshold was obtained using a harmonic oscillator representation of the continuum \cite{2} and the bound-state quantum Monte Carlo with extrapolation to the continuum \cite{4}. Calculations based on the no-core Gamow shell model \cite{5} and the density matrix renormalization group \cite{6} approaches have not found any narrow resonant states and tolerate only the presence of broad structures with a width of at least 4 MeV, while their positions could not be determined accurately.

On the other hand, rigorous solution of four-particle Faddeev-Yakubovsky (FY) equations in the coordinate- \cite{7} and momentum-space \cite{8} representations predict no observable tetraneutron resonance, consistently with the pioneering studies on this subject \cite{4,8}. The aim of the present work is to shed some light on the origin of these disagreements.

We do not discuss the details of our four-particle bound-state and scattering calculations that can be found in Refs. \cite{4,8} and in references therein. In Sec. II we present the study of the artificial four-particle system using two different force schemes. The summary and conclusions are contained in Sec. III.

II. RESULTS

Apart from the work \cite{2}, none of the aforementioned theoretical studies were able to identify tetraneutron resonances by performing direct calculations considering the physical values of the nuclear interaction. In the studies \cite{4,8} no tetraneutron complex-energy states were observed sufficiently close to the physical energy axis, whereas Monte-Carlo techniques used in \cite{8} are unsuited to determine the positions of unbound states directly. All of the aforementioned studies \cite{8} therefore tried to enhance the nuclear interaction to make the four-neutron (4n) system bound, and then follow its evolution to the physical strength of the interaction.

However, important differences were present in the details: Refs. \cite{4,8} enhanced the neutron-neutron (nn) potential in all partial waves by the same factor $f$, Refs. \cite{4} added an attractive 3n or 4n force, Ref. \cite{8} enhanced the nn potential in partial waves with orbital momentum $L > 0$ by the same factor $f_L$ but kept the physical strength $f_0 = 1$ for the $^1S_0$ force. The strategies of Refs. \cite{4,8} had a particular goal to avoid binding dineutrons. In contrast, when the nn force is enhanced also in the $^1S_0$ wave as in Refs. \cite{4}, very soon a bound dineutron is generated. The presence of a bound dineutron with energy $E_d < 0$ relative to the free-particle threshold has several consequences for the 4n system. First, the lowest scattering threshold is no longer at the energy $E_{4n} = 0$ but at the two-dineutron threshold $E_{4n} = 2E_d < 0$. Thus, negative energy 4n states at $2E_d < E_{4n} < 0$ are not bound but continuum states that may be realized in the scattering of two dineutrons.
only those with $E_{4n} < 2E_d$ are bound states. Second, the presence of an additional threshold changes the structure of the complex energy plane of $4n$ states, further complicating trajectories of $4n$ states.

To reveal shortcomings of the approach of enhancing the $1S_0$ $nn$ force, we study the $4n$ system using rigorous FY equations in the coordinate-space and FY equivalent momentum-space Alt, Grassberger, and Sandhas (AGS) equations for transition operators. We start with the model enhancing all partial waves by the same factor, i.e., $f_0 = f_L = f$. As it is well known, a bound dineutron emerges in the $1S_0$ partial wave at $f_0 \approx 1.1$; this critical value of $f_0$ slightly varies with the $nn$ potential, see Ref. [10] for a number of realistic potentials. The energy of the dineutron $E_d$ relative to the free neutron threshold rapidly decreases with increasing $f_0$, i.e., it becomes bound more tightly. To support a bound tetranucleon, i.e., the one with $E_{4n} < 2E_d$, significantly larger enhancement is needed; it also depends on the potential. A bound tetranucleon emerges first in the $J^P = 0^+$ state at $f \approx 2.4$ for several local realistic potentials such as Nijmegen II or Argonne V18, where the dineutron energy is below $-20$ MeV [11]. In order to make comparison with Refs. [2–4] that used soft nonlocal potentials, our following results will be based on the next-to-leading order (NLO) chiral potential [12] as in Ref. [3]. With this model and the space of Ref. [6] the tetranucleon becomes bound at $f = 2.665$ where $E_d = -12.01$ MeV and $E_{4n} = -24.02$ MeV. At lower $f$ values the $4n$ system cannot be bound, the lowest-energy $4n$ state is the dineutron state. However, Refs. [2, 4] report a bound tetranucleon at threshold $E_{4n} \to -0$ with $f$ being as low as 1.3 [3] or 1.6 [4]. Those $4n$ states in Refs. [3, 4] have zero width, which is not compatible with the fact that they are embedded in the two-dineutron continuum. Nevertheless, one might still expect that those are resonant states that evolved from the bound state with decreasing $f$, and thereby could provide estimation for the real part of the $4n$ resonance energy. To verify this conjecture we study the $f$-evolution of two-dineutron scattering states using rigorous treatment of the four-particle continuum as provided by AGS equations.

In Fig. 1 we show dineutron-dineutron $0^+$ phase shift $\delta_0$ as a function of the kinetic energy $E_k$ in the center-of-mass (c.m.) frame at $f = 2.7, 2.65, 2.6, 2.5, 2.3, 2.0$, and 1.3. There is a clear qualitative difference between the $f = 2.7$ and $2.65$ phase shift results at low energies: the former monotonically decreases from 180 deg at threshold whereas the latter rapidly increases from 0 deg at threshold exhibiting a bump of 70 deg. This signals the emergence of a bound state at $f$ between 2.65 and 2.7, fully consistent with the result of 2.665 from the direct bound state calculation. However, the most important observation is that at $f = 2.65, 2.6$, and 2.5 where one could expect the bound state to evolve into a resonance, no resonant phase shift behavior is observed. Note that qualitatively the same behavior is seen in the realistic $1S_0$ two-nucleon phase shift, reflecting the presence of the well-known virtual state near threshold. Indeed, at $f = 2.65, 2.6$, and 2.5 the two-dineutron phase shift energy dependence is consistent with the presence of a virtual $4n$ state that with decreasing $f$ moves away from the two-dineutron threshold into the unphysical sheet of the complex energy plane. Reducing $f$ further, the virtual state becomes too far from the threshold to have a visible effect, while the phase shifts approach the universal ones obtained for two fermionic dimers in the unitary

![Fig. 1. $J^P = 0^+$ phase shift for the scattering of two artificially bound dineutrons as a function of the center-of-mass kinetic energy $E_k$. NLO potential enhanced by a factor $f$ as indicated in the plot was used.](image1)

![Fig. 2. $J^P = 0^+$ phase shift for the scattering of two artificially bound dineutrons as a function of the center-of-mass kinetic energy $E_k$. Enhanced NLO potential with $S$- and higher-wave factors $(f_0, f_L)$ as indicated in the plot was used.](image2)
limit [13], signaling an effective repulsion between the two dineutrons. For example, at $f = 1.3$ the deviation from the universal results [13] is below 5%. Thus, in the $nn$ force enhancement scheme $f_0 = f_L = f$ the $0^+$ tetraneutron bound state evolves not into a resonance but into a virtual state, rendering this scheme entirely misleading for the $0^+$ resonance study.

The evolution of a bound into a virtual state is typical for $S$-wave two-body systems [13] unless there is a sufficiently high potential barrier leading to the appearance of a resonance, as shown in the example of Ref. [3] based on a two-Gaussian potential. The angular-momentum barrier is always present in $L > 0$ waves, resulting in the evolution of a bound state into a resonance. In the $4n$ systems both $L = 0$ and $L > 0$ waves are present, but their balance at $f_0 = f_L$ clearly favors a virtual state, not a resonance. The $S$-wave dominance in the physical $f = 1$ case is demonstrated also in Ref. [3].

In the following we will show a counterexample where the $0^+$ tetraneutron bound state evolves into a resonance seen in the two-dineutron scattering. To achieve this goal one needs to break the $S$-wave dominance, using larger enhancement factor $f_L$ for $L > 0$ waves as compared to $f_0$. In our example we take $f_0 = 1.3$. In that case the dineutron energy is $E_d = -0.316$ MeV. To support a bound tetraneutron with $E_{4n} < 2E_d$, significantly stronger enhancement $f_L = 3.898$ is needed for higher waves. Results for two-dineutron phase shifts obtained around these values of $(f_0, f_L)$ are presented in Fig. 2. There is clear qualitative difference between $f_L = 3.9$ and 3.895 results, signaling the presence of a bound state in the former case and a resonance in the latter case, consistently with direct bound-state calculations. With decreasing $f_L$ the resonance rapidly moves to higher energy and becomes broader, as is evident from the energy dependence of $f_L = 3.895, 3.89, 3.885$ two-dineutron phase shifts shown in Fig. 2. Thus, under these conditions of $f_L$ being considerably larger than $f_0$, the $0^+$ tetraneutron bound state evolves into a resonance as there is a strong contribution of higher partial waves creating an effective repulsive barrier via centrifugal terms. However, with decreasing $f_L$ the dominance of $S$-waves is restored well before reaching the $f_L = f_0$ such that the tetraneutron pole on the unphysical sheet moves deeply below the two-dineutron threshold, without having a visible effect on the scattering processes, as already known from $f = 1.3$ results in Fig. 1. In fact, already at $f_L = 2f_0 = 2.6$ the phase shift results are very close to those of $f_L = f_0 = 1.3$ and thereby also to the universal ones.

The regime very close to the critical point where the tetraneutron becomes unbound deserves special consideration. In Fig. 3 we show two-dineutron phase shifts $\delta_0$ obtained with $f_0 = 1.3$ and $f_L = 3.898, 3.8975, 3.897, 3.896$ in a narrow energy regime $E_k < |E_d|/10$ very close to the threshold and in Fig. 4 the corresponding $0^+$ total cross sections $\sigma_0$; the latter are normalized by the two-neutron zero-energy cross section $\sigma_{nn}^0$ calculated with $f_0 = 1.3$. Clearly, $f_L = 3.898$ corresponds to a bound state, while $f_L = 3.897$ and 3.896 cases appear to be resonant. However, at $f_L = 3.8975$ the energy-dependence of phase shift and cross section is consistent with the presence of a virtual $4n$ state very near to the threshold, not a bound state or resonance. Thus, in the case of the strong $L > 0$ wave enhancement the bound

FIG. 3. $J^\Pi = 0^+$ phase shift for the scattering of two artificially bound dineutrons as a function of the center-of-mass kinetic energy $E_k$. Enhanced NLO potential with $f_0 = 1.3$ and different higher-wave factors $f_L$ indicated in the plot was used.

FIG. 4. $J^\Pi = 0^+$ total cross section $\sigma_0$ for the scattering of two artificially bound dineutrons as a function of the center-of-mass kinetic energy $E_k$. Enhanced NLO potential with $f_0 = 1.3$ and different higher-wave factors $f_L$ indicated in the plot was used.
imaginary axis when reducing the attraction. In this way, the bound state with $\text{Im}(k) > 0$ crosses the threshold becoming a virtual state with $\text{Im}(k) < 0$. Nevertheless, in some particular cases, quite soon this S-matrix pole may move into the fourth quadrant of the complex momentum plane: first, as long as $-\text{Im}(k) > \text{Re}(k)$, appearing as a subthreshold resonance ($\text{Re}(E) < 0$) and then, once $\text{Re}(k) > -\text{Im}(k)$, turning into a resonance above the

state first evolves into a virtual state that, however, further evolves into a resonance.

The threshold behavior of the cross section can be read off from the two-neutron effective range expansion

$$k \cot \delta_0 = -\frac{1}{a_{dd}} + \frac{1}{2} r_{dd} k^2 + o(k^4) \quad (1)$$

with the on-shell momentum $k$, the scattering length $a_{dd}$, and the effective range parameter $r_{dd}$. Namely, $\sigma_0 \approx 1/|k \cot \delta_0 - ik|^2$ is increasing (decreasing) at $E_k = 0$ if the ratio $r_{dd}/a_{dd}$ is above (below) 1. Since in the regime $f_L < 3.898$ with no bound tetranucleon $a_{dd}$ is negative, only negative $r_{dd}$ with $|r_{dd}| > |a_{dd}|$ leads to increasing $\sigma_0$ as seen for $f_L \leq 3.897$ resonances in Fig. 4. Though $r_{dd} < 0$ in the whole regime $f_L \in [3.885, 3.9]$, large $|a_{dd}|$ at $f_L = 3.8975$ leads to decreasing $\sigma_0$. The values are $a_{dd} \approx -29 a_0$ and $r_{dd} \approx -9 a_0$ at $f_L = 3.8975$, while $a_{dd} \approx -2 a_0$ and $r_{dd} \approx -14 a_0$ at $f_L = 3.897$, expressed in terms of the two-neutron scattering length $a_0 = 12.72$ fm (taken at $f_0 = 1.3$). Thus, in this regime $r_{dd}$ is not only negative, but also of unnaturally large absolute value. As will be discussed below, this feature is essential for the appearance of resonant behavior. Note, that in all studied $f_L = f_0$ cases $r_{dd}$ is positive, as in the universal regime [13].

This kind of bound state evolution into a resonance via the virtual state may also appear in two-body systems for the $L = 0$ ($S$-wave) state generated by the short range potential with special properties. A general feature of the $S$-matrix pole trajectory, represented in the momentum ($k$) manifold, is that it moves down along the imaginary axis when reducing the attraction. In this way, the appearance of resonant behavior. Note, that in all studied $f_L = f_0$ cases $r_{dd}$ is positive, as in the universal regime [13].

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threshold \((\text{Re}(E) > 0)\). Such a non-standard behavior is determined by the presence of a large negative effective range parameter \(r_0\) in the effective range expansion of the form \(V(r)\), which can be generated by employing a potential containing a repulsive barrier. To study this case, we adapt the two-Gaussian potential from Ref. 2:

\[
V(r) = f_0 \{-1000e^{-(r/0.4981)^2} + 865e^{-(r-0.9972)/0.2877)^2}\}.
\]

(2)

Here the potential \(V(r)\) is in units of MeV, whereas the distance between the neutrons \(r\) is in fm; neutron mass is fixed by \(\hbar^2/m_n = 41.4425\text{ MeV} \cdot \text{fm}^2\). We study the evolution of the S-matrix pole by varying the potential enhancement factor \(f_0\). The S-matrix pole trajectory generated in this way is presented in Figs. 5 and 6 along with the trajectory of unphysical antibound/antiresonant state. The physical and unphysical poles collide at \(k \approx i/r_0\), where \(r_0\) approaches half of the two-body scattering length \(a_0\). This can be understood from the effective range expansion, which is typically a good approximation near the unitary limit \(1/a_0 = 0\) and is suitable to estimate the positions of the near-threshold S-matrix poles given by

\[
k = \frac{1}{r_0}(i \pm \sqrt{2r_0/a_0 - 1}).
\]

(3)

Given a large negative \(a_0\) for a near-threshold virtual state, large negative \(r_0\) ( \(\sim -125\text{ fm}\) in the present example) ensures that the two poles collide on the negative imaginary momentum axis in close vicinity of the threshold and then scatter by angle \(\pi/2\), see Fig. 5. In Fig. 7 cross sections are plotted at two selected trajectory points: at \(f_0^{(1)} = 1.004325\) generating a virtual state at \(k = -0.0046i \text{ fm}^{-1}\), and at \(f_0^{(2)} = 1.0042\) generating a resonance at \(k = 0.0357 - 0.0078i\). The total cross section, dominated by the presence of the virtual state, decreases with energy from \(E = 0\). On the other hand, a near-threshold resonance (with \(\text{Re}(k) > -\text{Im}(k)\)) leads to a cross section which grows with energy at \(E = 0\). If the enhancement \(f_0\) is further reduced, the S-matrix pole will recede from the real energy axis at the same time losing impact on the scattering cross section. The S-matrix pole trajectory is even more complicated when presented in the energy manifold, see Fig. 7. At the critical point \(E = 0\) the physical pole of the S-matrix is reflected backward while being projected into the next Riemann sheet. At the point where physical and unphysical poles collide, their trajectories turn by \(\pi/2\) acquiring imaginary energy parts, whereas the real energy parts once again start to increase. Obviously, such a non-analytic behavior is highly non-linear and cannot be approximated by a simple polynomial as was naively tried in [3].

Returning back to the \(4n\) system, we would like to stress again the qualitative difference between the \(4n\) \(J^P = 0^+\) S-matrix pole trajectories when bound dineutrons are produced from the ones when dineutrons are kept unbound. In the first case the trajectory is dominated by the two-dineutron threshold and thus inherits features common to a two-body \(S\)-state one, i.e., the \(4n\) bound state turns into a two-dineutron virtual state. On the contrary, if no bound dineutron states are present, at the critical point the \(4n\) bound state evolves directly into a narrow resonance – this feature is determined by the presence of the repulsive \(1/r^2\) term in the effective \(4n\) hyperradial potential. Notably, such a behavior is also exhibited by three-body Efimov states. On the right side of the unitary limit (positive two-body scattering lengths) bound Efimov states evolve from the virtual atom-dimer states. On the contrary, on the left-hand side of the unitary limit (negative two-body scattering lengths) bound Efimov states appear from three-atom resonant states [15].

III. CONCLUSIONS

Using rigorous treatment of the four-particle continuum as given by FY and AGS equations, we investigated the \(4n\) system with artificially enhanced interaction. In contrast to our previous studies [3, 4, 5, 6] with no bound dineutrons, here we enhanced also the two-neutron \(1S_0\) potential, aiming to elucidate the \(0^+\) tetraneutron resonance behavior in the presence of bound dineutrons, naturally appearing in the \(1S_0\) partial wave once the factor \(f_0\) exceeds roughly 1.1. As even larger \(f_0\) has been used in previous works suggesting the existence of a near-threshold [3] or broad [4] \(0^+\) tetraneutron resonance, the present study was aiming to resolve the disagreement between Refs. [3, 4] and the \(f_0 = 1\) approach [2, 11] contesting the presence of an observable \(0^+\) tetraneutron resonance.

Following the approach of Refs. [2, 4] with \(f_0 = f_L = f\) we found that a significant enhancement \(f \approx 2.4\) to 2.7 is needed to support a truly bound tetraneutron below the two-dineutron threshold. However, when \(f\) is reduced the \(4n\) bound state evolves not into a resonance but into a virtual state. This is convincingly demonstrated by the energy dependence of the two-dineutron phase shifts. Thus, the negative energy \(4n\) states, considered in Refs. [3, 4] to be zero-width bound states evolving into resonances at positive energy, are neither bound states nor narrow resonances. From the point of view of rigorous four-particle theory the \(E_{4n} < 0\) states above the two-dineutron threshold are either two-dineutron or dineutron plus two-neutron continuum states without resonant character. With decreasing \(f\) the scattering observables approach the universal zero-range behavior of the four-fermion system with \(E_d = 0\). Note that on the way to the physical potential strength the unitary limit has to be crossed.

We have also shown that enhancing the higher-wave potential considerably stronger as compared to \(S\) wave may lead to a resonant \(4n\) behavior, as demonstrated by the example of the two-dineutron phase shift and cross section. Anyhow, well before reaching the \(f_0 = f_L\) limit, that resonance moves far away from the scattering re-
region, possibly below the two-dineutron threshold in the unphysical sheet of the complex energy plane, thereby becoming experimentally unobservable. This is consistent with the previous studies [5, 6, 8]. Another remarkable feature of this enhancement scheme is the evolution of the bound state into a resonance via the virtual state appearing in a very narrow transition regime. A simple two-body example exhibiting the same type of evolution was presented. In both cases the essential feature is the presence of a large negative effective range parameter.

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