The resummation approach to evolution equations

Hsiang-nan Li

Department of Physics, National Chung-Cheng University,
Chia-Yi, Taiwan, Republic of China

PACS numbers: 12.38.Cy, 11.10.Hi

Abstract

We derive the evolution equations of parton distribution functions appropriate in different kinematic regions in a unified and simple way using the resummation technique. They include the Gribov-Lipatov-Altarelli-Parisi equation for large momentum transfer $Q$, the Balitskii-Fadin-Kuraev-Lipatov equation for a small Bjorken variable $x$, and the Ciafaloni-Catani-Fiorani-Marchesini equation which unifies the above two equations. We propose a modified Balitskii-Fadin-Kuraev-Lipatov equation from the view point of the resummation. This modified version possesses an intrinsic $Q$ dependence and its predictions for the structure function $F_2(x, Q^2)$ are consistent with the HERA data.
I. INTRODUCTION

Perturbative QCD (PQCD), as a gauge field theory, involves large logarithms from radiative corrections at each order of the coupling constant $\alpha_s$, such as $\ln Q$ in the kinematic region with a large momentum transfer $Q$ and $\ln(1/x)$ in the region with a small Bjorken variable $x$. These logarithms, spoiling the perturbative expansion, must be organized. To organize the logarithmic corrections to a parton distribution function, the various evolution equations have been derived. For example, the Gribov-Lipatov-Altarelli-Parisi (GLAP) equation sums single logarithms $\ln Q$ for a large $x$ to all orders, the Balitskii-Fadin-Kuraev-Lipatov (BFKL) equation sums $\ln(1/x)$ for a small $x$, and the Ciafaloni-Catani-Fiorani-Marchesini (CCFM) equation, appropriate for both large and small $x$, unifies the above two equations.

The conventional derivation of the evolution equations usually requires complicated diagrammatic analyses. The idea is to locate the region of the loop momenta flowing through the rungs (radiative gluons) of a ladder diagram, in which leading logarithmic corrections are produced. For the GLAP, BFKL, and CCFM equations, the important regions are those with the strong transverse momentum ordering, the strong rapidity ordering, and the strong angular ordering, respectively. Summing the ladder diagrams with the different kinematic orderings to all orders, we obtain the evolution equations that organize the leading logarithms. The derivation will become very difficult, if one intends to improve the accuracy of the evolutions to next-to-leading logarithms.

In this paper we shall propose an alternative approach to the all-order summation of the various large logarithms. This approach is based on the resummation technique, which was developed originally for the organization of double logarithms $\ln^2 Q$. Recently, we applied this technique to some hard QCD processes, such as deep inelastic scattering, Drell-Yan production, and inclusive heavy meson decays, and demonstrated how to resum the double logarithms contained in parton distribution functions into Sudakov form factors. It has been shown that the resummation technique is equivalent to the Wilson-loop formalism for the summation of soft logarithms. Here we shall further show that it can also deal with the single-logarithm cases, i.e. the evolution equations mentioned above. Therefore, the resummation technique indeed has a wide application in PQCD.
The procedures of the resummation are briefly summarized below. The derivative of a parton distribution function with respect to \( Q \) or \( x \) is first related to a new function, which contains a special gluon vertex. Expressing this new function as the convolution of the subdiagram involving the special vertex with the original parton distribution function, we arrive at the evolution equation. The subdiagram, after factorized out of the new function, is identified as the corresponding kernel in the different kinematic limits. We show explicitly that the lowest-order subdiagram gives the kernel for the leading-logarithm summation exactly. Using our approach, it is not necessary to go into the detailed analysis of the orderings of radiative gluons in a ladder diagrams, since all the possible orderings have been included in the new function. Hence, the resummation technique provides a unified and simple derivation of the known evolution equations in the literature.

In the region with both large \( Q \) and small \( x \) many gluons are radiated in scattering processes with small spatial separation among them, and a new effect from the annihilation of two gluons into one gluon becomes important. Taking into account this effect, a nonlinear evolution equation, the Gribov-Levin-Ryskin (GLR) equation \([7]\) is obtained. Using the resummation technique, the annihilation effect is introduced through next-to-leading-twist contributions to the subdiagram containing the special vertex, and the GLR equation can also be derived easily \([8]\). In the present work we shall not address this subject, because it is not very relevant.

It is known that the BFKL equation is independent of \( Q \), and thus its predictions are insensitive to the variation of \( Q \). However, the recent HERA data \([9]\) for the structure function \( F_2(x, Q^2) \) of deep inelastic scattering exhibit a stronger \( Q \) dependence. It is plausible that the experiments have not yet explored the region with a low enough \( x \) such that the BFKL equation is applicable. Hence, to explain the data, the \( Q \) dependent GLAP equation is combined in some way. One can employ either the improved splitting function, which embeds the BFKL summation, in the GLAP equation \([10]\), or the CCFM equation directly \([11]\), which unifies the BFKL and GLAP equations. In this paper we shall not rely on the GLAP summation of \( \ln Q \), but propose a modified BFKL equation from the viewpoint of the resummation, which contains an intrinsic \( Q \) dependence. It will be shown that the gluon distribution function derived from the modified BFKL equation gives the predictions of \( F_2(x, Q^2) \), that are well consistent with the data.

We derive the GLAP, BFKL and CCFM equations in Sections II, III, and
IV, respectively, by means of the resummation technique. We explain how the subdiagram containing the special vertex reduces to the corresponding evolution kernels in the different kinematic regions. The modified BFKL equation along with its analytical solution of the gluon distribution function are presented in Section V. We then evaluate the structure function $F_2(x, Q^2)$ using the high-energy $p_T$-factorization theorem \[12\], and compare them with the HERA data. Section VI is the conclusion.

II. THE GLAP EQUATION

We derive the GLAP equation in this section. Consider deep inelastic scattering (DIS) of a hadron with a light-like momentum $p = p^+\delta^{\mu+}$ in the large $x$ limit, where $x = -q^2/(2p \cdot q) = Q^2/(2p \cdot q)$ is the Bjorken variable with $q$ the momentum transfer to the hadron through a virtual photon. It is known that the collinear region with loop momenta of radiative gluons parallel to $p$ is important, from which large logarithms arise \[5\]. The other important regions are soft, with loop momenta much smaller than $Q$, and hard, with loop momenta being of order $Q$. In the covariant gauge $\partial A = 0$ the scattered quark line the collinear gluons attach can be replaced by an eikonal line along an arbitrary direction $n$, $n^2 \neq 0$ \[5\]. The associated Feynman rules are $1/(n \cdot l)$ for the eikonal propagator, and $n^\mu$ for an vertex on the eikonal line, $l$ being the momentum flowing through it. With the eikonalization, the collinear gluons are factorized out of the hard scattering subamplitude and absorbed into a quark distribution function $\phi(\xi, p^+, \mu)$ with $\xi$ the momentum fraction, and $\mu$ a factorization or renormalization scale. The argument $p^+$ denotes the large logarithms $\ln(p^+/\mu)$ appearing in $\phi$, which will be organized below.

The standard definition of $\phi$ in the covariant gauge is then given by

$$
\phi(\xi, p^+, \mu) = \int \frac{dy^-}{2\pi} e^{-i\xi p^+ y^-} \langle p|\bar{q}(y^-)\gamma^+ P e^{i \int_{0}^{y^-} dzn \cdot A(zn)} q(0)|p \rangle, \quad (1)
$$

as shown in Fig. 1(a), where $\gamma^+$ is a Dirac matrix, and $|p\rangle$ denotes the incoming hadron. The path-ordered exponential $P e^{i \int dzn \cdot A}$ represents exactly the eikonal line collecting the collinear gluons. It is easy to confirm that this exponential generates the Feynman rules stated above. Though $\phi$ constructed in this way contains an artificial dependence on $n$, the cross section
of the scattering process, being a physical quantity, should be \( n \) independent. It has been shown that the \( n \) dependence of \( \phi \) is canceled by those of other involved subprocesses \[5\]. This vector \( n \) is, however, essential at the intermediate stage of the resummation.

In the axial gauge \( n \cdot A = 0 \) the path-ordered exponential is equal to the identity, and \( \phi \) is defined by Fig. 1(b), implying that the collinear gluons are decoupled from the eikonal line. The \( n \) dependence then goes into the gluon propagator, \((-i/l^2)N^{\mu\nu}(l)\), with

\[
N^{\mu\nu} = g^{\mu\nu} - \frac{n^\mu l^\nu + n^\nu l^\mu}{n \cdot l} + n^2 \frac{l^\mu l^\nu}{(n \cdot l)^2}.
\] (2)

The above decoupling can also be understood from the relation \( n^\mu N^{\mu\nu} = 0 \) associated with the attachment of a radiative gluon to the eikonal line. We have shown that the resummation results derived in the axial and covariant gauges are the same \[5\], and thus we can work in either of them. In this section we adopt the axial gauge.

The key step of the resummation is to obtain the derivative \( p^+ d\phi/dp^+ \). Because of the scale invariance of \( \phi \) in the vector \( n \) as indicated by Eq. \((1)\) or \((2)\), \( \phi \) must depend on \( p \) and \( n \) through the ratio \( (p \cdot n)^2/n^2 \). Hence, we have the chain rule relating \( p^+ d/dp^+ \) to \( d/dn \):

\[
p^+ \frac{d\phi}{dp^+} = -\frac{n^2}{v \cdot n} v_\alpha \frac{d}{dn_\alpha} \phi ,
\] (3)

with \( v_\alpha = \delta_\alpha + \) a vector along \( p \). In the axial gauge \( d/dn \) applies only to the gluon propagator, giving

\[
\frac{d}{dn_\alpha} N^{\mu\nu} = -\frac{1}{n \cdot l}(l^\mu N^{\alpha\nu} + l^\nu N^{\mu\alpha}) .
\] (4)

The loop momentum \( l^\mu \) \((l^\nu)\) flowing through the differentiated gluon line contracts with the vertex the gluon attaches, which is then replaced by a special vertex

\[
\hat{v}_\alpha = \frac{n^2 v_\alpha}{v \cdot n n \cdot l} .
\] (5)

This special vertex can be simply read off the combination of Eqs. \((3)\) and \((4)\).
The contraction of \( l^\mu (l^\nu) \) hints the application of the Ward identity, from which it is found that the special vertex moves to the outer end of the valence quark line. For a given \( x \), we obtain the formula,

\[
p^+ \frac{d}{dp^+} \phi(x, p^+, \mu) = 2 \tilde{\phi}(x, p^+, \mu) ,
\]

shown in Fig. 2(a), where \( \tilde{\phi} \), the new function mentioned in the Introduction, contains one special vertex represented by a square. The coefficient 2 comes from the equality of the new functions with the special vertex on either of the two valence quark lines. Note that Eq. (6) is an exact consequence of the Ward identity without specifying the ordering of radiative gluons. All the possible orderings are embedded in Eq. (6).

We show that Eq. (6) leads to the GLAP equation, when \( p^+ \), or equivalently \( Q \) in the center-of-mass frame of the virtual photon and the incoming quark, is large. The important regions of the loop momentum flowing through the special vertex are soft and hard, since the vector \( n \) does not lie on the light cone, and the collinear enhancements are suppressed. In the important soft and hard regions \( \tilde{\phi} \) can be factorized into the convolution of the subdiagram containing the special vertex with the original distribution function \( \phi \),

\[
\tilde{\phi}(x, p^+, \mu) = \int_x^1 d\xi [K(x, \xi, p^+, \mu) + G(x, \xi, p^+, \mu)] \phi(\xi, p^+, \mu) .
\]

The function \( K \), absorbing the soft divergences of the subdiagram, corresponds to Fig. 2(b), where the eikonal approximation for the valence quark propagator has been made. The function \( G \), absorbing the ultraviolet divergences, corresponds to Fig. 2(c), where the subtraction of the second diagram ensures that the involved loop momentum is of order \( Q \).

We identify the factor \( K + G \) as the GLAP kernel. According to Figs. 2(b) and 2(c), \( K \) and \( G \) are written as

\[
K = \frac{i g^2 C_F \mu^e}{(2\pi)^{4-\epsilon}} \int \frac{d^{4-\epsilon} l}{v \cdot l} \left[ \frac{\delta(\xi - x)}{l^2} + 2\pi i \delta(l^2) \delta(\xi - x - l^+/p^+) \right] N^{\mu\nu} - \delta K ,
\]

\[
G = -\frac{i g^2 C_F \mu^e}{(2\pi)^{4-\epsilon}} \int \frac{d^{4-\epsilon} l}{v \cdot l} \left[ \frac{\xi \cdot \bar{l} - \bar{l} \cdot l}{(\xi p - l)^2} \gamma_\nu v_{\nu} \right] \frac{N^{\mu\nu}}{l^2} \delta(\xi - x) - \delta G ,
\]
where $C_F = 4/3$ is the color factor. The $\delta$ functions come from the final state cut, and $\delta K$ and $\delta G$ are additive counterterms. The virtual and real gluon emissions correspond to the first and second terms in the integral of $K$, respectively. The second term in the integral of $G$ is the soft subtraction. We emphasize that the factorization formula in Eq. (7) is not an exact relation, but holds only up to the leading logarithms $\ln p^+/\mu$. As explained later, the approximation equivalent to the strong transverse momentum ordering in the conventional approach has been applied to Eq. (8).

A straightforward calculation gives

$$
K = \frac{\alpha_s(\mu)}{\pi \xi} C_F \left[ \frac{1}{(1-x/\xi)_+} + \ln \frac{\nu p^+}{\mu} \delta(1-x/\xi) \right],
$$

$$
G = -\frac{\alpha_s(\mu)}{\pi \xi} C_F \ln \frac{\xi \nu p^+}{\mu} \delta(1-x/\xi),
$$

(10)

where constants of order unity have been dropped, and $\nu = \sqrt{(v \cdot n)^2/n^2}$ is the gauge factor. Equation (10) confirms our argument that $\phi$ depends on $p$ and $n$ through the ratio $(p \cdot n)^2/n^2 = (\nu p^+)^2$. In the considered region with $x \to 1$ the logarithm $\ln(\xi \nu p^+/\mu)$ in $G$ can be replaced by $\ln(\nu p^+/\mu)$.

We then treat $K$ and $G$ by renormalization group (RG) methods:

$$
\mu \frac{d}{d\mu} K = -\lambda_K = -\mu \frac{d}{d\mu} G.
$$

(11)

The anomalous dimension of $K$ is defined by $\lambda_K = -\mu d \delta K/d\mu$, whose explicit expression is not essential here. When solving Eq. (11), we allow the variable $\mu$ to evolve from the scale of $K$ to the scale of $G$. The RG solution of $K + G$ is given by

$$
K(x, \xi, p^+, \mu) + G(x, \xi, p^+, \mu) = K(x, \xi, p^+, p^+) + G(x, \xi, p^+, p^+)

- \int_{p^+}^{p^+} \frac{d\mu}{\mu} \lambda_K(\alpha_s(\mu)) ,

= \frac{\alpha_s(p^+)}{\pi \xi} C_F \frac{1}{(1-x/\xi)_+}.
$$

(12)

It is obvious that the source of double logarithms, i.e. the integral containing $\lambda_K$, vanishes.
A remark is in order. The function $\delta(\xi - x - l^+/p^+)$ in Eq. (8) will be replaced by $\delta(\xi - x) \exp(i l_T \cdot b)$, $b$ being the conjugate variable of the transverse momentum carried by the valence quark, if the transverse degrees of freedom are considered [5]. The integration over $\xi$ can thus be performed trivially, and the convolution form in Eq. (7) is simplified to a multiplication form,

$$\tilde{\phi}(x, b, p^+, \mu) = [K(x, b, \mu) + G(x, p^+, \mu)] \phi(x, b, p^+, \mu).$$

(13)

This simplification will be implemented in Sec. V to solve the modified BFKL equation. In this case the scale $1/b$, instead of $p^+$, serves as the infrared cutoff of $K$. Consequently, $1/b$ is substituted for the lower bound of $\bar{\mu}$ in Eq. (12). Double logarithms then exist, implying that the soft logarithms in $\phi$ do not cancel completely. Therefore, the resummation technique can deal with both the single-logarithm and double-logarithm problems.

Inserting Eq. (12) into (7) and solving (6), we obtain

$$\phi(x, \bar{\mu}, \mu) = \phi(x, \bar{\mu}, \mu) + \int_{x(\bar{\mu})}^{1} \frac{d\xi}{\xi} \left[ \frac{2}{(1 - x/\xi)} \phi(\xi, \bar{\mu}, \mu) \right],$$

(14)

with $\Lambda$ an arbitrary cutoff. We have defined $\phi(x, \mu) \equiv \phi(x, \mu)$, which does not contain large logarithms. According to the formalism in [5], the upper bound of $\bar{\mu}$ should be set to the hadron momentum. However, it is equivalent to set it to $\mu$ here. It is known that a physical quantity such as the DIS cross section is $\mu$ independent. When applying the RG analysis to the factorization formula for the cross section, which is a convolution of the hard scattering subamplitude with $\phi$, $\mu$ evolves to the characteristic scale of the hard part at last. This hard scale is exactly the large hadron momentum.

Differentiating Eq. (14) with respect to $\mu$, and substituting the RG equation $\mu d\phi(x(\xi), \Lambda(\bar{\mu}), \mu)/d\mu = -2\lambda_q(x(\xi), \Lambda(\bar{\mu}), \mu)$, $\lambda_q = -\alpha_s/\pi$ being the quark anomalous dimension in the axial gauge, we have

$$\mu \frac{d}{d\mu} \phi(x, \mu) = \frac{\alpha_s(\mu)}{\pi} C_F \int_{x(\bar{\mu})}^{1} \frac{d\xi}{\xi} \left[ \frac{2}{(1 - x/\xi)} \phi(\xi, \bar{\mu}, \mu) - \lambda_q(\mu) \phi(x, \mu) \right].$$

(15)

The above equation can be reexpressed as

$$Q \frac{d}{dQ} \phi(x, Q) = \frac{\alpha_s(Q)}{\pi} \int_{x}^{1} \frac{d\xi}{\xi} P(x/\xi) \phi(\xi, Q),$$

(16)
where $\mu$ has been set to $Q$ and the kernel $P$ is

$$P(x) = C_F \left[ \frac{2}{(1-x)_+} + \frac{3}{2} \delta(1-x) \right]. \quad (17)$$

It is trivial to identify $P$ as the splitting function $P_{qq}$ in the limit $x \to 1$,

$$P_{qq}(x) = C_F \left[ \frac{1+x^2}{(1-x)_+} + \frac{3}{2} \delta(1-x) \right]. \quad (18)$$

Hence, Eq. (18) leads to the GLAP equation (16).

In fact, only the terms of $P_{qq}$, which are singular at $x \to 1$, can be reproduced in the resummation formalism. We emphasize that the factorization of the subdiagram containing the special vertex makes sense only in the leading soft and hard regions, and thus the functions $K$ and $G$ collect only the most important contributions to the splitting function. This conclusion applies to other cases that involve a soft approximation, such as the CCFM equation in the conventional derivation [3], where the finite part of the relevant splitting function was also missing and put in by hand at last.

### III. THE BFKL EQUATION

In this section we demonstrate that the resummation technique reduces to the BFKL equation for the gluon distribution function in the small $x$ region. It is convenient to adopt the covariant gauge $\partial A = 0$, under which the unintegrated gluon distribution function $F(x, p_T)$ is defined by Fig. 1(a) with the valence partons being gluons. The path-ordered exponential in Eq. (1), and thus the eikonal line in the direction $n$, which collects radiative gluons, appear. $F(x, p_T)$ describes the probability of a gluon carrying a longitudinal momentum fraction $x$ and a transverse momentum $p_T$. We have made explicit that $F$ depends on $p_T$, instead of the large hadron momentum $p^+$ appearing in the GLAP case. It will be shown later that $p^+$ disappears as $x \to 0$, and thus the $p_T$ dependence is not negligible. For a unified treatment of the large-$x$ and small-$x$ summation, the $p^+$ dependence should be included, and the resummation technique reduces to the CCFM equation discussed in Sec. IV.

To sum large logarithms $\ln(1/x)$, the strong rapidity ordering of radiative gluons in a ladder diagram is assumed in the conventional approach. In
the resummation formalism the BFKL equation can be derived simply by reinterpreting the derivative with respect to $p^+$ in Eq. (3) and by modifying the expression of $K$ in Eq. (8). Though $F$ does not depend on $p^+$ explicitly, it can vary with $p^+$ through the momentum fraction implicitly, which is proportional to $(p^+)^{-1}$ for a fixed parton momentum. For a similar reason, $F$ depends on the ratio $(p \cdot n)^2/n^2$, and thus Eq. (3) holds. In the covariant gauge the operator $d/dn$ applies to the Feynman rules for the eikonal line, giving

\[ \frac{d}{dn} n_\alpha \frac{n}{l} = \frac{1}{n \cdot l} \left( g^{\mu \alpha} - \frac{n^{\mu} l^\alpha}{n \cdot l} \right) . \]  

(19)

Combining Eqs. (3) and (19), we find that the differentiation with respect to $p^+$ generates a special vertex on the eikonal line,

\[ \hat{n}_\alpha = \frac{n^2}{v \cdot n} \left( \frac{v \cdot l}{n \cdot l} n_\alpha - v_\alpha \right) . \]  

(20)

The derivative of $F$ is then expressed as

\[ p^+ \frac{d}{dp^+} F(x, p_T) \equiv -x \frac{d}{dx} F(x, p_T) = 4 \tilde{F}(x, p_T) , \]  

(21)

described by Fig. 3(a), where the new function $\tilde{F}$ contains one special vertex denoted by the symbol $\times$. Note that the coefficient 4 in front of $\tilde{F}$ is twice of the corresponding coefficient in the GLAP case. Since the gluon interacts with the virtual photon through a quark box, two quark lines, and thus two eikonal lines after factorizing out the gluon distribution function, are adjacent to the parton vertex. Hence, there is one more attachment of the special vertex to the eikonal line on each side of the final state cut. It is trivial to show that these attachments give the same results. Starting with Eq. (21), we need not to go into the detailed analysis of the rapidity ordering of radiative gluons, since all the possible orderings have resided in it.

The leading regions of the loop momentum $l$ flowing through the special vertex are also soft and hard. If $l$ is collinear, the first term $v \cdot l$ in $\hat{n}_\alpha$ vanishes, and the second term $v_\alpha$, as contracted with a vertex in the distribution function which is dominated by momenta parallel to $p$, gives a small contribution. The soft divergences of the subdiagram are collected by Fig. 3(b), and the ultraviolet divergences by Fig. 3(c). We employ the relation $f_{abc}t_b t_c = (i/2)N t_a$ for the color structure, $t$ being the color matrices.
and $N = 3$ being the number of colors. Absorbing $t_a$ into the parton vertex, Fig. 3(b) leads to

\[
\tilde{F}_{\text{soft}}(x, p_T) = -\frac{i}{2}Ng^2 \int \frac{d^4l}{(2\pi)^4} \frac{\Gamma^{\mu\nu\lambda}n_\nu}{n \cdot l} \left[ 2\pi i \delta(l^2) F(x, |p_T + l_T|) + \theta\left(p_T^2 - l_T^2\right) \frac{l_T^2}{l^2} F(x, p_T) \right], \tag{22}
\]

where the triple-gluon vertex for vanishing $l$ is given by

\[
\Gamma^{\mu\nu\lambda} = -g^{\mu\nu}v^\lambda - g^{\nu\lambda}v^\mu + 2g^{\lambda\mu}v^\nu. \tag{23}
\]

The first term in the above integral corresponds to the real gluon emission, where $F(x, |p_T + l_T|)$ indicates that the parton coming out of the hadron carries a transverse momentum $p_T + l_T$ in order to radiate a real gluon of momentum $l_T$. The second term corresponds to the virtual gluon emission, where the $\theta$ function sets the upper bound of $l_T$ to $p_T$ to ensure a soft momentum flow. Therefore, it is not necessary to introduce a renormalization scale $\mu$ into $F$. There is not the $\xi$ integration in Eq. (22), since we have included the transverse degrees of freedom of the parton as a soft regulator, and the final state cut associated with the real gluon can be approximated by $\delta(\xi - x - l^+/p^+) \approx \delta(\xi - x)$ as stated in the previous section.

It can be easily shown that $v^\lambda$ in Eq. (23), contracted with a vertex in the quark box diagram, leads to a contribution smaller by a power $1/s$ with $s = (p + q)^2$, compared to the contribution from the last term $v^\nu$. Similarly, the term $v^\mu$ is also contracted with a vertex in the box diagram through the metric tensor associated with the gluon distribution function. Hence, we drop the first two terms of $\Gamma^{\mu\nu\lambda}$, and absorb $g^{\lambda\mu}$ into $F$. Evaluating the integral straightforwardly, Eq. (22) reduces to

\[
\tilde{F}_{\text{soft}}(x, p_T) = \frac{\bar{\alpha}_s}{4} \int \frac{d^2l_T}{\pi l_T^2} \left[ F(x, |p_T + l_T|) - \theta(p_T^2 - l_T^2) F(x, p_T) \right], \tag{24}
\]

with $\bar{\alpha}_s = N\alpha_s/\pi$.

It was argued that when the fractional momentum of a parton vanishes, the associated collinear enhancements are suppressed [5]. The vanishing of the contribution from the first diagram of Fig. 3(c)

\[
G^{(1)} = \frac{\bar{\alpha}_s}{4} \int \frac{d^2l_T}{\pi} \frac{1}{l_T^2 - \frac{1}{l_T^2 + (xp^+v)^2}}
\]
\[-\frac{1}{2} \frac{xp^+\nu}{[l^2_T + (xp^+\nu)^2]^{3/2}} \ln \frac{\sqrt{l^2_T + (xp^+\nu)^2} - xp^+\nu}{\sqrt{l^2_T + (xp^+\nu)^2} + xp^+\nu} \] 

(25)

at \(x \to 0\), reflects this argument. Hence, \(F\) does not acquire a dependence on the large scale \(p^+\), and the transverse degrees of freedom must be taken into account, differing from the GLAP case for a large \(x\). This is the basic idea of the so-called high-energy \(p_T\)-factorization theorem \([12]\). Therefore, the introduction of \(p_T\) and the disappearence of \(p^+\) are built in the resummation technique naturally. It is obvious that our formalism is applicable to the distribution functions constructed according to the collinear factorization (the GLAP case) and according to the \(p_T\)-factorization (the BFKL case).

Neglecting \(G^{(1)}\) along with its soft subtraction (the second diagram in Fig. 3(c)), that is, adopting \(\tilde{F} = \tilde{F}_{\text{soft}}\), Eq. (21) becomes

\[
\frac{F(x, p_T)}{d\ln(1/x)} = \bar{\alpha}_s \int \frac{d^2l_T}{\pi l^2_T} \left[ F(x, |p_T + l_T|) - \theta(p_T^2 - l_T^2)F(x, p_T) \right],
\]

(26)

which is exactly the BFKL equation. It is then understood that the subdiagram containing the special vertex plays the role of the BFKL kernel.

In summary, the BFKL equation is appropriate for the multi-Regge region, where the transverse momenta flowing through the rungs of a ladder diagram are of the same order, i.e. \(l_T \approx p_T\). Hence, the loop momentum \(l_T\) flowing through the parton distribution function is not negligible, and the final state cut \(\delta(\xi - x - l^+/p^+)\) can be approximated by \(\delta(\xi - x)\) as in Eq. (22).

While the GLAP equation is appropriate for the transverse momentum ordered region, in which we have \(l_T \ll p_T\), i.e. \(F(x, |p_T + l_T|) \approx F(x, p_T)\) for the real gluon emission. The \(p_T\) dependence of the parton distribution function then decouples, and can be integrated out from both sides of Eq. (24). Hence, a parton distribution function in the GLAP equation does not involve the transverse degrees of freedom. In this case the loop momentum \(l^+\) should be maintained in the final state cut as in Eq. (8), which then leads to the splitting function. If not, the right-hand side of Eq. (8) will be identical to zero. Therefore, the subdiagram containing the special vertex reduces to the corresponding evolution kernels in the different kinematic regions.

IV. THE CCFM EQUATION
Based on the discussion in the previous two sections, it is not difficult to demonstrate that Eq. (21) reduces to the CCFM equation [3], which is appropriate for both large $x$ and small $x$. It embodies the GLAP equation and the BFKL equation, and depends on the longitudinal momentum $p^+$ and the transverse momentum $p_T$ of a parton at the same time. For the conventional derivation of the CCFM equation, assuming the angular ordering of radiative gluons in a ladder diagram, refer to [3]. By means of the resummation technique, the complicated diagrammatic analysis can be avoided, and the physical meaning of each factor in the CCFM equation is clearer.

Consider Eq. (21) but with the unintegrated gluon distribution function $F$ depending on $p_T$ and $p^+$,

$$p^+ \frac{d}{dp^+} F(x, p_T, p^+) = 4 \tilde{F}(x, p_T, p^+)\ ,$$

which manifests the attempt to unify the GLAP and BFKL equations. It is not necessary to introduce a renormalization scale $\mu$ here, because the transverse degrees of freedom will not be integrated out. Again, the new function $\tilde{F}$ involves one special vertex on the eikonal line in the direction $n$.

If following the standard procedures of the resummation, we should factorize the subdiagram containing the special vertex by absorbing its soft and hard contributions into the functions $K$ and $G$, respectively. The $p^+$ dependence of $F$ then comes from $G$, which collects the virtual gluon emissions. This idea leads to a new unified evolution equation, which will be studied elsewhere. To reproduce the CCFM equation, however, the inclusion of the virtual corrections must be performed in a different way: We organize the virtual gluons embedded in $K$, instead of those in $G$. Hence, the subdiagram is factorized into Fig. 4(a), where the two jet functions $J$ group all the possible virtual corrections, and the real gluon between them is soft. It will be shown below that this subdiagram gives the CCFM kernel.

First, we resum the double logarithms contained in $J$ by considering its derivative

$$p^+ \frac{d}{dp^+} J(p_T, p^+) = 2 \tilde{J}(p_T, p^+),$$

$$= 2[K_J(p_T, \mu) + G_J(p^+, \mu)]J(p_T, p^+).$$

At lowest order the function $K_J$ comes from the first diagram of Fig. 3(b), and $G_J$ from Fig. 3(c). The coefficient 2 counts the two eikonal lines adjacent to the parton vertex. The relation between $K_J + G_J$ and $J$ is simply
multiplicative, since \( J \) groups only virtual gluons. We have set the infrared cutoff of \( K_J \) to \( p_T \), as indicated by its argument. This cutoff is necessary here due to the lack of the corresponding real gluon emission, which serves as a soft regulator. The one-loop \( K_J \) can be easily obtained by working out the second integral in Eq. (24) without the \( \theta \) function. The anomalous dimension of \( K_J \) is then found to be \( \gamma_J = \tilde{\alpha}_s/2 \). The function \( G_J \) can also be computed, but its explicit expression is not important. The standard RG analysis gives

\[
K_J(p_T, \mu) + G_J(p^+, \mu) = - \int_{p_T}^{p^+} \frac{d\tilde{\mu}}{\tilde{\mu}} \gamma_J(\alpha_s(\tilde{\mu})) ,
\]

(29)

with the initial conditions \( K_J(p_T, p^+) = G_J(p^+, p^+) = 0 \). Of course, we have neglected the constants of order unity in \( K_J \) and \( G_J \).

Substituting Eq. (29) into (28), we solve for

\[
J(p_T, Q) = \Delta(Q, p_T) J^{(0)} ,
\]

(30)

with the double-logarithm exponential

\[
\Delta(Q, p_T) = \exp \left[-\tilde{\alpha}_s \int_{p_T}^{Q} \frac{dp^+}{p^+} \int_{p_T}^{p^+} \frac{d\tilde{\mu}}{\tilde{\mu}} \right] .
\]

(31)

We have chosen the upper bound of \( p^+ \) as \( Q \), and ignored the running of \( \tilde{\alpha}_s \). The initial condition \( J^{(0)} \), regarded as the tree-level gluon propagator, will appear in the integrand for the real gluon emission in Fig. 4(b). We split the above exponential into

\[
\Delta(Q, p_T) = \Delta^{1/2}_S(Q, zq) \Delta^{1/2}_{NS}(z, q, p_T) ,
\]

(32)

with \( z = x/\xi \) and \( q = l_T/(1 - z) \), where \( \xi \) is the momentum fraction entering \( J \) from the bottom, and \( l_T \) is the transverse loop momentum carried by the real gluon. The so-called “Sudakov” exponential \( \Delta_S \) and the “non-Sudakov” exponentials \( \Delta \) are given by

\[
\Delta_S(Q, zq) = \exp \left[-2\tilde{\alpha}_s \int_{zq}^{Q} \frac{dp^+}{p^+} \int_{p_T}^{p^+} \frac{d\tilde{\mu}}{\tilde{\mu}} \right] = \exp \left[-\tilde{\alpha}_s \int_{(zq)^2}^{Q^2} \frac{dp^2}{p^2} \int_{0}^{1-p_T/p} \frac{dz'}{1 - z'} \right]
\]

14
\[
\Delta_{NS}(z, q, p_T) = \exp \left[ -2\bar{\alpha}_s \int_{p_T}^{zq} \frac{dp^+}{p^+} \int_{p_T}^{p^+} \frac{d\bar{\mu}}{\bar{\mu}} \right].
\]

where the variable changes \(\bar{\mu} = (1 - z')p\) and \(p^+ = p\) for \(\Delta_S\), and \(\bar{\mu} = p\) and \(p^+ = z'q\) for \(\Delta_{NS}\) have been employed. The inserted scale \(zq\) reflects the combination of the rapidity ordering for the BFKL equation and the transverse momentum ordering for the GLAP equation [3, 11].

Picking up the last term of the triple-gluon vertex in Eq. (23), \(\tilde{F}\) is written, in terms of Fig. 4(b), as

\[
\tilde{F}(x, p_T, p^+) = -i \frac{N g^2}{2x} \int_x^1 d\xi \int d^4l \frac{v^\mu \hat{n}_\mu}{(2\pi)^4 v \cdot l} \frac{2\pi i \delta(l^2) \Delta^2(Q, p_T)}{l^T} \times \delta(\xi - x - l^+/p^+) \theta(Q - zq) F(\xi, |p_T + l_T|, p^+),
\]

with \(l\) the gluon momentum. The propagator \(1/v \cdot l\) comes from the eikonalized tree-level \(J^{(0)}\) on the right-hand side of Fig. 4(b). The left-hand side \(J^{(0)}\) has been absorbed into \(F\). The argument of \(F\) in the integrand is \(p_T + l_T\), because it is for the real gluon contribution. Basically, the above formula is similar to the real gluon part of the BFKL equation (22) except for the exponential \(\Delta^2\) from the two jet functions \(J\), and \(\delta(\xi - x - l^+/p^+)\) associated with the final state cut, which is restored due to the inclusion of the \(p^+\) dependence as in Eq. (8). Through this \(\delta\) function, the GLAP splitting function is generated. The \(\theta\) function guarantees that the Sudakov exponential \(\Delta_S\) is meaningful. Hence, the expression of Eq. (34) manifests the combination of the GLAP and BFKL features.

Performing the integration over \(l^+\) and \(l^-\), we obtain

\[
\tilde{F}(x, p_T, p^+) = \frac{\bar{\alpha}_s}{4} \int_x^1 d\xi \int \frac{d^2l_T}{\pi} \frac{2n^2(\xi - x)p^{+2}}{[n^+ l_T^2 + 2n^-(\xi - x)^2p^{+2}]^2} \Delta^2(Q, p_T) \times \theta(Q - zq) F(\xi, |p_T + l_T|, p^+),
\]

where we have assumed \(n = (n^+, n^-, 0)\) for convenience. Eq. (35) is then substituted into (27) to find the solution of \(F\). We adopt the variable changes \(\xi = x/z\) and \(l_T = (1 - z)q\), and integrate Eq. (35) from \(p^+ = 0\) to \(Q\). To work out the \(p^+\) integration, \(F(x/z, |p_T + l_T|, p^+)\) is approximated by
\( F(x/z, |p_T + l_T|, l_T) \). This approximation is fine, if one does not intend to obtain the \( \delta \)-function terms of the splitting function \( P_{gg} \), written as

\[
P_{gg} = \tilde{\alpha}_s \left[ \frac{1}{1 - z} + \frac{1}{z} + z(1 - z) + \left( \frac{11}{12} N - \frac{1}{6} n_f \right) \delta(1 - z) \right],
\]  

(36)

\( n_f \) being the number of quark flavors. Eq. (35) then becomes

\[
F(x, p_T, Q) = F(0) + \tilde{\alpha}_s \int_x^1 dz \int \frac{d^2 q}{\pi q^2} \theta(Q - zq) \Delta_S(Q, zq) \Delta_{NS}(z, q, p_T)
\times \frac{1}{z(1 - z)} F(x/z, |p_T + (1 - z) q|, l_T),
\]  

(37)

where the nonperturbative initial condition \( F(0) \) corresponds to the lower bound of \( p^+ \), and the term suppressed by \( 1/Q^2 \) in the integral has been dropped.

Eq. (37) can be rewritten as

\[
F(x, p_T, Q) = F(0) + \tilde{\alpha}_s \int_x^1 dz \int \frac{d^2 q}{\pi q^2} \theta(Q - zq) \Delta_S(Q, zq) \tilde{P}(z, q, p_T)
\times F(x/z, |p_T + (1 - z) q|, l_T),
\]  

(38)

with the splitting function

\[
\tilde{P} = \tilde{\alpha}_s \left[ \frac{1}{1 - z} + \Delta_{NS}(z, q, p_T) \frac{1}{z} + z(1 - z) \right].
\]  

(39)

To arrive at the above expression we have employed the identity \( 1/(z(1 - z)) \equiv 1/(1 - z) + 1/z \), and neglected the non-Sudakov form factor \( \Delta_{NS} \) in front of \( 1/(1 - z) \), because \( \Delta_{NS} \) has no pole as \( z \to 1 \) as shown in Eq. (33). Obviously, Eq. (38) is the CCFM equation (3). The last term \( z(1 - z) \) of \( \tilde{P} \) is put in by hand, which is hinted by Eq. (36). This term, finite at \( z \to 0 \) and at \( z \to 1 \), can not be obtained in the conventional approach either (3) as stated at the end of Sec. II.

V. A MODIFIED BFKL EQUATION

We have mentioned in the Introduction that the recent HERA data of the DIS structure function \( F_2(x, Q^2) \) can be explained by the CCFM equation

16
in which the Sudakov exponential $\Delta S$ collects the summation of $\ln Q$ for a large $x$. In this section we propose a modified BFKL equation based on the resummation technique. This modified equation, much simpler than the CCFM equation, contains an intrinsic $Q$ dependence, instead of that from the $\ln Q$ summation. It will be shown that the resultant predictions for $F_2$ match the HERA data. With this success, we demonstrate the power of the resummation technique.

An alert reader may have noticed that to derive the BFKL equation, we must extend the loop momentum $l^+$ to infinity in Eq. (22). Strickly speaking, the real gluon emission in fact involves the distribution function $F(x + l^+ / p^+, |p_T + l_T|)$ as part of the integrand, which is then approximated by $F(x, |p_T + l_T|)$ in the soft $l$ region. Therefore, the behavior of $F$, vanishing at the momentum fraction equal to unity, should introduce an upper bound of $l^+$. To obtain a more reasonable BFKL kernel, we truncate $l^+$ at some scale, and a plausible choice of this scale is of order $Q$. We then derive a modified BFKL equation for a $Q$ dependent gluon distribution function,

$$\frac{dF(x, p_T, Q)}{d\ln(1/x)} = \hat{\alpha}_s \int \frac{d^2l_T}{\pi l_T^2} \left[ F(x, |p_T + l_T|, Q) - \theta(Q_0^2 - l_T^2)F(x, p_T, Q) \right]$$

$$- \hat{\alpha}_s \int \frac{d^2l_T}{\pi} \frac{F(x, |p_T + l_T|, Q)}{l_T^2 + Q^2}, \tag{40}$$

in the $x \to 0$ limit, where the last term comes from the upper bound of $l^+$. The gauge vector $n$ has been chosen to render the coefficient of $Q^2$ equal to unity. It can be shown that our predictions are insensitive to this coefficient, as long as it is of order unity. Obviously, Eq. (40) approaches Eq. (22) in the $Q \to \infty$ limit.

Another modification is that the loop momentum $l_T$ in the virtual gluon emission is truncated at an arbitrary scale $Q_0$ of order 1 GeV, instead of $p_T$ as in the conventional BFKL equation. This modification is reasonable, since the virtual gluon contribution plays the role of a soft regulator for the real gluon emission only, and setting the cutoff to $Q_0$ serves the same purpose. Furthermore, the replacement of $p_T$ by $Q_0$ allows us to solve Eq. (40) analytically, and the solution of $F$ maintains the essential BFKL features. It will be shown that the first term of the integral is responsible for the rise of $F$, and the last term acts to slower the rise. We then expect that $F$ ascends faster at a larger $Q$, for which the effect of the last term is weaker.
We observe that the CCFM equation will be identical to the modified BFKL equation, if the exponential $\Delta^2$ is “unfolded” into the lowest-order virtual gluon contribution, $\theta(Q-zq)$ is dropped, $F(\xi, |p_T+1_T|, Q)$ is replaced by $F(x, |p_T+1_T|, Q)$, and the $\xi$ integration is performed to produce the $Q$ dependent term. Therefore, the $Q$ dependence of the latter is not attributed to the all-order $\ln Q$ summation in $\Delta$, but to the boundary of the phase space for radiative corrections. In fact, this $Q$ dependent term also appears in the CCFM equation, but was dropped in the derivation of Eq. (37).

The Fourier transform of Eq. (40) leads to

$$\frac{d\tilde{F}(x, b, Q)}{d\ln(1/x)} = \tilde{\alpha}_s(1/b) \int \frac{d^2l_T}{\pi} \left[ \frac{e^{-il_T \cdot b} - \theta(Q_0^2 - l_T^2)}{l_T^2} \tilde{F}(x, b, Q) - \frac{e^{-il_T \cdot b}}{l_T^2 + Q^2} \tilde{F}(x, b, Q) \right],$$

with

$$S(b, Q) = 2\tilde{\alpha}_s(1/b) [\ln(Q_0b) + \gamma - \ln 2 + K_0(Qb)].$$

The Bessel function $K_0$ comes from the last term of the integral, and $\gamma$ is the Euler constant. The argument of $\alpha_s$ has been chosen as $1/b$, since we work in the conjugate $b$ space.

Eq. (41) can be trivially solved to give

$$\tilde{F}(x, b, Q) = \tilde{F}(x_0, b) \exp[-S(b, Q) \ln(x_0/x)],$$

$x_0$ being the initial momentum fraction below which $F$ begins to evolve according to the BFKL summation. Transforming Eq. (43) back to the momentum space, we derive the analytical solution to the modified BFKL equation,

$$F(x, p_T, Q) = \int_0^\infty db J_0(pTb) \tilde{F}(x, b, Q),$$

and the gluon density $xg$ by integrating Eq. (44) over $p_T$,

$$xg(x, Q^2) = \int \frac{d^2p_T}{\pi} F(x, p_T, Q) = 2Q \int_0^\infty db J_1(Qb) \tilde{F}(x, b, Q).$$

In the above expressions $J_0$ and $J_1$ are the zeroth and first order Bessel functions, respectively.
To proceed with the numerical analysis, we assume a “flat” gluon distribution function \[11, 13\]
\[
\tilde{F}^{(0)}(0)(x, b) = 3N_g(1 - x)^5 \exp(-Q_0^2 b^2 / 4),
\]
for \(x \geq x_0\), \(N_g\) being a normalization constant, which is the Fourier transform of
\[
F^{(0)}(0)(x, p_T) = \frac{6}{Q_0^5} N_g(1 - x)^5 \exp(-p_T^2 / Q_0^2)
\]
in momentum space. The initial condition \(\tilde{F}(x_0, b)\) in Eq.(43) is then equated to \(\tilde{F}(x_0, b) = \tilde{F}^{(0)}(0)(x_0, b)\). We set \(Q_0 = 1\) GeV and \(x_0 = 0.1\) arbitrarily. It can be shown that the predictions vary only slightly for other choices of the parameters \(Q_0\) and \(x_0\) of the same order. \(N_g\) will be determined by the data of the structure function \(F_2(x, Q^2)\) at a specific value of \(Q^2\), and then employed to make predictions for other values of \(Q^2\).

An advantage of the \(b\) space is that the infrared sensitivity of the BFKL solution from the \(p_T\) diffusion is moderated. The divergence of \(\alpha_s(1/b)\) from a large \(b\) is suppressed by the exponential \(e^{-Q_0^2 b^2 / 4}\) in the initial condition \(\tilde{F}^{(0)}\). In the momentum space, however, the divergence of \(\alpha_s(p_T)\) from small \(p_T\) is not suppressed by \(\exp(-p_T^2 / Q_0^2)\) in \(F^{(0)}\), and thus the solution is sensitive to the infrared cutoff of \(p_T\). We have confirmed that our predictions of \(F_2\) almost remain the same for the cutoff of \(b\) at 2-4 GeV\(^{-1}\).

We compute the structure function \(F_2\), whose expression, according to the \(p_T\)-factorization theorem, is given by
\[
F_2(x, Q^2) = \int_x^1 \frac{d\xi}{\xi} \int_0^{p_c} \frac{d^2p_T}{\pi} H(x/\xi, p_T, Q)F(\xi, p_T, Q),
\]
\(p_c\) being the upper bound of \(p_T\) which will be specified later. The hard scattering subamplitude \(H\) denotes the contribution from the quark box diagrams, where both the incoming photon and gluon are off shell with \(q^2 = -Q^2\) and \(p^2 = -p_T^2\), respectively. For simplicity, we consider only the contraction \(-g^\mu\nu W_{\mu\nu}\) in the calculation of \(H\), and neglect the contribution from \(p^\mu p^\nu W_{\mu\nu}\), which is less important, \(W_{\mu\nu}\) being the DIS hadronic tensor. We also assume that a charm quark is massless, and that a \(b\bar{b}\) quark pair is not involved in the box diagram, \(i.e.\) the active flavor number \(n_f\) in the running coupling constant \(\alpha_s\) is equal to 4. Following these assumptions, we concentrate on the range of \(Q^2\) between 8 and 20 GeV\(^2\).
A simple calculation gives

\[ H(z, p_T, Q) = e^2_q \frac{Q_s}{2\pi} z \left\{ \left[ z^2 + (1 - z)^2 - 2z(1 - 2z)\frac{p_T^2}{Q^2} + 2z^2 p_T^4 \right] \right\} \times \frac{1}{\sqrt{1 - 4z^2 p_T^2 / Q^2}} \ln \frac{1 + \sqrt{1 - 4z^2 p_T^2 / Q^2}}{1 - \sqrt{1 - 4z^2 p_T^2 / Q^2}} - 2 \right\} . \] (49)

with \( e_q \) the electric charge of the quark \( q \). Note that the terms in the braces approach the splitting function

\[ P_{qg}(z) = \frac{1}{2} [z^2 + (1 - z)^2] \] (50)

in the \( p_T \to 0 \) limit. To require a meaningful \( H \), we modify the upper bound of \( p_T \) in Eq. (49) from \( p_c = Q \) to

\[ p_c = \min \left( Q, \frac{\xi}{2x} Q \right) . \] (51)

We evaluate the integral in Eq. (49) straightforwardly for \( Q^2 = 15 \text{ GeV}^2 \), and then determine the normalization constant \( N_g = 3.656 \) from the data fitting. When \( Q^2 \) varies, we adjust \( N_g \) such that \( xg \) has a fixed normalization \( \int_0^1 x g dx \). It is found that \( N_g \) changes only by about 5% in the considered range of \( Q^2 \). \( F_2 \) for \( Q^2 = 8.5, 12, \) and \( 20 \text{ GeV}^2 \) are then computed, and results along with the HERA data [9] are displayed in Fig. 5. It is obvious that our predictions agree with the data well. The curve has a steeper rise at a larger \( Q \), which is the consequence of the \( Q \) dependent modified BFKL equation. The curves descend rapidly at \( x \) close to 0.1, since the contributions from other kinds of partons, such as the valence quarks, which are more important in this intermediate \( x \) region, are not included. For comparison, we also present the results from the conventional BFKL equation, which can be obtained simply by substituting \( l_T^2 + M^2 \) for the denominator \( l_T^2 + Q^2 \) in Eq. (49), or equivalently, \( Mb \) for the argument \( Qb \) of the Bessel function \( K_0 \) in Eq. (12) with an extremely large \( M = 10^3 - 10^4 \text{ GeV} \). In this case the normalization constant determined from the best fit to the data for \( Q^2 = 15 \text{ GeV}^2 \) is \( N_g = 2.908 \). It is found that the shape of the curves is almost independent of \( Q \), and thus the match with the data is not very satisfactory.
At last, we show in Fig. 7 the behavior of the gluon density $xg$ computed from Eq. (45). The rise of $xg$ at small $x$ is due to the term $\ln Q_0 b$ in the exponent $S$ as mentioned before, which leads to an integrand proportional to

$$(Q_0 b)^{-2\alpha_s \ln(x_0/x)}.$$  

The small $b$ region then gives a huge contribution. When $x$ approaches zero such that $2\alpha_s \ln(x_0/x) > 1$, the integration over $b$ diverges. However, the last term of $S$, $K_0(Qb) \propto -\ln(Qb) b$ in the $b \to 0$ limit, which comes from the upper bound of the loop momentum $l^+$, cancels the divergence. This is the reason $l^+$ must be truncated. On the other hand, the variation of $xg$ with $x$ for different $Q$ can also be understood from the combination of $\ln(Q_0 b)$ and $K_0(Qb)$ in the $b \to 0$ limit, written as,

$$(x/x_0)^{-2\alpha_s \ln(Q/Q_0)}.$$  

The above expression indicates that the exponent $\lambda$, characterizing the rise of $xg \sim x^{-\lambda}$ at small $x$, increases with $Q$. The values $\lambda \approx 0.36$ for $Q^2 = 8.5$ GeV$^2$ and $\lambda \approx 0.51$ for $Q^2 = 20$ GeV$^2$ are deduced from Fig. 6, which are consistent with that obtained from a phenomenological fit to the HERA data ($\lambda \approx 0.3$ for $Q^2 = 4$ GeV$^2$) and with that in [10] from solving the conventional BFKL equation numerically ($\lambda \approx 0.5$ for a wide range of $Q^2$).

**VI. CONCLUSION**

In this paper we have shown that the resummation technique provides a unified and simple viewpoint to the organization of the various large logarithms, and reduces to the GLAP equation, the BFKL equation, and the CCFM equation in different kinematic regions. The main idea is to relate the derivative of a parton distribution function to a new function involving a special vertex. The summation of the large logarithms is then embedded in the new function without resort to the complicated diagrammatic analyses. When expressing the new function as a factorization formula, we obtain the evolution equation, and the subdiagram containing the special vertex is exactly the corresponding kernel. By means of the resummation technique, the
derivation of the evolution equations is simpler. Furthermore, to improve
the accuracy of the kernel to next-to-leading logarithms, we only need to
evaluate the $O(\alpha_s^2)$ subdiagram. Such an evaluation can be performed in a
straightforward way. The BFKL equation including the summation of the
next-to-leading $\ln(1/x)$ will be published elsewhere.

We have also calculated the DIS structure function $F_2(x, Q^2)$ using the
analytical solution of the unintegrated gluon distribution function from the
modified BFKL equation. Our predictions exhibit a stronger $Q$ dependence,
and are in a good agreement with the HERA data, compared to those from
the conventional BFKL equation. Note that the $Q$ dependence in the modi-
fied BFKL equation is intrinsic, which arises from the boundary of the phase
space for radiative corrections, instead of from the $\ln Q$ summation in the
CCFM equation. In this sense we argue that the current experiemnts may
have explored the multi-Regge region with $\ln(1/x) \gg \ln Q$. Certainly, this
issue still needs to be clarified by further theoretical and experimental studies
[15].

This work is supported by National Science Council of R.O.C. under the
Grant No. NSC-86-2112-M-194-007.
References

[1] V.N. Gribov and L.N. Lipatov, Sov. J. Nucl. Phys. 15, 428 (1972); G. Altarelli and G. Parisi, Nucl. Phys. B126, 298 (1977); Yu.L. Dokshitzer, Sov. Phys. JETP 46, 641 (1977).

[2] E.A. Kuraev, L.N. Lipatov and V.S. Fadin, Sov. Phys. JETP 45, 199 (1977); Ya.Ya. Balitskii and L.N. Lipatov, Sov. J. Nucl. Phys. 28, 822 (1978); L.N. Lipatov, Sov. Phys. JETP 63, 904 (1986).

[3] M. Ciafaloni, Nucl. Phys. B296, 49 (1988); S. Catani, F. Fiorani, and G. Marchesini, Phys. Lett. B 234, 339 (1990); Nucl. Phys. B336, 18 (1990); G. Marchesini, Nucl. Phys. B445, 49 (1995).

[4] J.C. Collins and D.E. Soper, Nucl. Phys. B193, 381 (1981).

[5] H-n. Li, Phys. Rev. D 55, 105 (1997).

[6] H-n. Li, Phys. Lett. B 369, 137 (1996).

[7] L.V. Gribov, E.M. Levin and M.G. Ryskin, Nucl. Phys. B188, 555 (1981); Phys. Rep. 100, 1 (1983).

[8] H-n. Li, Report no. hep-ph/9607256.

[9] ZEUS Collaboration, M. Derrick et al., Z. Phys. C 65, 379 (1995); H1 Collaboration, T. Ahmed et al., Nucl. Phys. B439, 471 (1995).

[10] A. J. Askew, J. Kwieciński, A.D. Martin, and P.J. Sutton, Phys. Rev. D 49, 4402 (1994); J.R. Forshaw, R.G. Roberts, and R.S. Thorne, Phys. Lett. B 356, 79 (1995).

[11] J. Kwieciński, A.D. Martin, and P.J. Sutton, Phys. Rev. D 53, 6094 (1996).

[12] T. Jaroszewicz, Acta. Phys. Pol. B 11, 965 (1980); S. Catani, M. Ciafaloni, and F. Hautmann, Phys. Lett. B 242, 97 (1990); Nucl. Phys. B366, 657 (1991); S. Catani and F. Hautmann, Nucl. Phys. B427, 475 (1994).
[13] P.D.B. Collins and F. Gault, Phys. Lett. B 112, 255 (1982); A. Donnachie and P.V. Landshoff, Nucl. Phys. B244, 322 (1984); Nucl. Phys. B267, 690 (1986).

[14] A.D. Martin, R.G. Roberts, and W.J. Stirling, Phys. Rev. D 50, 6734 (1994); Phys. Lett. B 354, 155 (1995).

[15] A.H. Mueller, in Proceeding of the Topical Workshop on the Small-x Behavior of Deep Inelastic Scattering Structure Functions in QCD, Hamburg, Germany, 1990, edited by A. Ali and J. Bartels [Nucl. Phys. B (Proc. Suppl.) 18C, 125 (1990)]; J. Kwieciński, S.C. Lang, and A.D. Martin, Phys. Rev. D 54, 1874 (1996).
**Figure Captions**

**FIG. 1.** Definition of a parton distribution function in (a) the covariant gauge and in (b) the axial gauge.

**FIG. 2.** (a) The derivative $p^+d\phi/dp^+$ in the axial gauge. (b) The $O(\alpha_s)$ function $K$. (c) The $O(\alpha_s)$ function $G$.

**FIG. 3.** (a) The derivative $-xF/dx$ in the covariant gauge. (b) The soft structure and (c) the ultraviolet structure of the $O(\alpha_s)$ subdiagram containing the special vertex.

**FIG. 4.** (a) The subdiagram containing the special vertex for the CCFM equation. (b) The subdiagram for the CCFM equation after resumming the double logarithms in $J$.

**FIG. 5.** The dependence of $F_2$ on $x$ derived from the modified BFKL equation (solid lines) and from the conventional BFKL equation (dashed lines). The HERA data [9] are also shown.

**FIG. 6.** The dependence of $xg$ on $x$ derived from the modified BFKL equation for, from bottom to top, $Q^2 = 8.5, 12, 15, \text{and } 20 \text{ GeV}^2$. 

