Arterial Branching in Man and Monkey

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ABSTRACT Vessel diameters and branching angles are measured from a large number of arterial bifurcations in the retina of a normal human subject and in that of a rhesus monkey. The results are compared with each other and with theoretical results on this subject.

INTRODUCTION

Physiological principles governing the geometry of arterial branching in the cardiovascular system have been investigated by several authors (Murray, 1926a and b; Leopold, 1971; Kamiya and Togawa, 1972; Kamiya et al., 1974; Rodbard, 1975; Zamir, 1976a and b; Uylings, 1977; Zamir, 1978). It is generally agreed now that the branching angles and vessel diameters at an arterial bifurcation are governed by certain fluid dynamic principles that optimize the bifurcation in its physiological function.

These so-called “optimality principles” derive mainly from theoretical considerations of the cardiovascular system as a fluid-conducting network. They are based on theoretical models of arterial junctions and involve certain simplifying assumptions regarding the flow pattern in the bifurcation region. The aim of these principles is not to duplicate with accuracy the flow pattern and branching angles in an arterial bifurcation. Their aim is to duplicate the “design rules” that govern arterial branching in the cardiovascular system and this aim has been met with reasonable success so far. Although opinions may differ on the exact details of these rules, there is fairly wide agreement on their general form.

A major difficulty in this field is the lack of experimental or observational data. It is not an easy matter to measure the diameters and branching angles of blood vessels in vivo, and if the vessels are removed from the body it is not easy to avoid some changes in their diameters and branching angles in the process. These difficulties are compounded by the need for a sufficiently large amount of data to provide good statistical evidence.

Recently Zamir et al. (1979) obtained a set of such data from photographs of arterial bifurcations in the human retina. The results gave good support to the theoretical predictions but the data were characterized by a considerable amount of scatter. Also, this was the first extensive test of the theoretical...
results and as such it could not provide definitive conclusions. Scatter was attributed in part to possible uncertainties in the measurements and in part to natural biological scatter. It could not be decided on the basis of that experiment alone which of these was the dominant factor.

The purpose of the present paper is to report the results of another set of measurements that were obtained with the aim of resolving some of the above uncertainties and providing a wider base of experimental data on arterial branching. The new data consist of two parts. The first part contains measurements of arterial branching in the retina of a normal human eye that were undertaken with the aim of testing the reproducibility of the previous data and the degree of scatter in particular. Measurement techniques were improved to reduce any scatter arising from this source and, perhaps equally important, a different subject was chosen with the retinal photography being performed in a different laboratory by different workers. The second part of the data contains a similar set of measurements of arterial branching in the

Figure 1. Photograph of the vascular bed of a normal human retina from which branching measurements were taken. Individual bifurcations were enlarged and traced, and measurements were taken from the tracings.
retina of a monkey, with fluorescein being injected into the blood stream before the retinal photograph is taken. The effect of this is to improve considerably the sharpness of blood vessel images and hence, again, to improve the accuracy of measurements. Also, since this part of the data is derived from a different species, it serves to test and supplement the human data to provide a much wider test of the theoretical results.

**Figure 2.** Fluorescein photograph of the vascular bed of the retina of a rhesus monkey from which branching measurements were taken. Individual bifurcations were enlarged and traced, and measurements were taken from the tracings.

**Methods**

Retinal photographs were made by Mr. B. Hochheimer in the laboratory of Dr. Arnal Patz, Department of Ophthalmology, Johns Hopkins Hospital, Baltimore, Md. We are grateful to Mr. Hochheimer for making the photographs available to us. The photographs were made with a Zeiss fundus camera (Carl Zeiss, Inc., New York) using Kodak Tri-X film (Eastman Kodak Co., Rochester, N. Y.). The subjects were a young adult human and an adult rhesus monkey with normal eyes in both cases. The human eye was photographed in normal light with normal blood stream. In the case of the monkey, a solution of sodium fluorescein dye was injected into the blood stream and the eye was illuminated with blue light, which excites fluorescein and causes it to fluoresce.
Measurements were taken from arterial bifurcations in the central area of the retina, an area containing the optic disc and the fovea (see Figs. 1 and 2). Individual bifurcations were magnified ~100 times by projecting their images onto a distant background and in each case a tracing was made of the enlarged image of that particular bifurcation. Vessel diameters and branching angles were measured from these tracings. Vessel diameters were measured by laying a straight line along the tracing of each wall to integrate local irregularities. Branching angles were measured by constructing the center line of each vessel in the vicinity of the junction point and then fitting the best straight line through it. These straight lines were then regarded as the longitudinal axes of the vessels and the angles between them were taken as the branching angles, as illustrated in Fig. 3.

**Figure 3.** Definition of vessel diameters and branching angles. Subscript 0 refers to the parent artery, subscript 1 refers to the larger branch, and subscript 2 refers to the smaller branch.

**RESULTS AND DISCUSSION**

Measurements of vessel diameters for man and monkey are shown in Figs. 4 and 5. An important observation to be made first is that the data for man and monkey appear to be evenly distributed and fairly uniformly mixed. There appears to be no species-related bias.

Another important observation is that the degree of scatter is the same in both cases, even though the fluorescein images of the monkey's vessels were much sharper and hence amenable to more accurate measurement. These results, combined with those reported earlier by Zamir et al. (1979), strongly suggest that what is being observed here is normal biological scatter.
The shaded curves in Figs. 4 and 5 show the optimum curves predicted by the theoretical studies. The curves are given by

\[
\frac{d_1}{d_0} = \frac{1}{b} \quad \text{and} \quad \frac{d_2}{d_0} = \frac{a}{b}
\]

where

\[
a = \frac{d_2}{d_1} \quad \text{and} \quad b = (1 + a^3)^{1/3}.
\]

The agreement is reasonably good if it is recalled that what is being modeled here is not the actual diameters of individual vessels but the principles governing these diameters at arterial bifurcations. Thus the scatter of the data points does not express disagreement of these results with theory. Such disagreement would manifest itself in the form of bias of the data points away from the theoretical curves. As it is, the data points are fairly uniformly scattered about the theoretical curves, which indicates that the governing principles proposed by the theoretical models are the same as those that govern the data. It is significant that this is true of the data from the human eye as well as from that of the monkey.

Measurements of branching angles for man and monkey are shown in Figs. 6 and 7. Again, the data points appear to be fairly evenly mixed with no species-related bias. The degree of scatter is somewhat higher here, however,
than in the case of vessel diameters. One possible reason for this is that some arterial bifurcations may not lie perfectly flat within the thickness of the retina and hence our normal view of them may give rise to distorted branching angles (Zamir, 1981). This would not explain all the scatter, however. It would be more consistent with a systematic bias toward smaller angles as in Fig. 6 than with the more uniform scatter of Fig. 7. Another possible reason is that branching angles are not as well defined as vessel diameters because vessels are not always straight in the bifurcation region.

The optimum branching angles predicted by theory are represented by the shaded regions in Figs. 6 and 7. Each region is defined by two curves which outline its upper and lower boundaries and are based on different optimality principles (Zamir, 1978). The curves outlining the upper boundaries are given by

\[
\cos \theta_1 = \frac{b^2 + 1 - a^2}{2b}, \quad \cos \theta_2 = \frac{b^2 + a^2 - 1}{2ab}
\]

and those outlining the lower boundaries are given by

\[
\cos \theta_1 = \frac{b^4 + 1 - a^4}{2b^2}, \quad \cos \theta_2 = \frac{b^4 + a^4 - 1}{2a^2b^2}
\]

where \(a\) and \(b\) are as defined previously in Eq. 2. Despite the degree of scatter, the data in Figs. 6 and 7 give very clear support to the theoretical principle.
**Figure 6.** Measurements of the branching angle of the larger branch ($\theta_1$) from man (○) and monkey (□) plotted against the asymmetry ratio ($d_2/d_1$). The shaded area represents an optimum region suggested by theory.

**Figure 7.** Measurements of the branching angle of the smaller branch ($\theta_2$) from man (○) and monkey (□) plotted against the asymmetry ratio ($d_2/d_1$). The shaded area represents an optimum region suggested by theory. A flag on a data point indicates that there is another data point at the same position.
that at an arterial bifurcation the larger branch makes a smaller angle with
the direction of the parent vessel than does the smaller branch.

CONCLUSIONS

Measurements of vessel diameters and branching angles of arterial bifurcations
in the retinas of a human subject and a rhesus monkey show no difference in
the resulting range of data points or their distribution.

Data from both the human and the monkey give reasonable support to
theoretical proposals regarding the principles governing arterial branching in
the cardiovascular system.

The results indicate clearly, however, that the prevalence of these principles
in the cardiovascular system will generally be associated with a fair amount
of biological scatter.

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