Reconfigurable Intelligent Surface Aided Cell-Free MIMO Communications

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Abstract—Cell-free system is an efficient solution to eliminate inter-cell interference by serving users without cell boundaries. To further improve the spectrum efficiency, in this letter, we consider a reconfigurable intelligent surface (RIS) aided cell-free MIMO system. Multiple RISs are employed to create favorable propagation conditions via configurable reflection from the BS to users. Benefited from multiple transmission paths the spatial resources can be better utilized. We develop a hybrid beamforming (HBF) scheme consisting of the digital beamforming at the BS and the RIS-based beamforming. A sum rate optimization (SRO) problem is formulated and an iterative algorithm is designed to solve this problem. Simulation results show that the proposed system can achieve a significant performance gain compared to conventional single cell systems without RISs and distributed antenna systems.

Index Terms—Reconfigurable intelligent surface, cell-free, MIMO, hybrid beamforming.

I. INTRODUCTION

CELL-FREE systems have drawn a great attention where multiple users are served without cell boundaries, thereby alleviating inter-cell interference efficiently [1]. To further improve the network capacity, a potential technique called reconfigurable intelligent surface (RIS) is developed to create favorable propagation conditions [2]. Specifically, benefited from a large number of elements whose phase shifts are controllable by a controller, the RIS can reflect incident signals and generate directional beams [3]. Hence, each RIS can extend coverage for the users which are centered the RIS [4], [5]. By deploying multiple RISs over the area of interest and coordinated by a central BS, multiple users are served without any cell boundaries, forming an RIS aided cell-free MIMO system. Such a system can greatly enhance the link quality and coverage, owing to the favorable propagation and dispersed distribution of RISs.

In this letter, we consider an RIS aided cell-free MIMO system where a single BS and multiple RISs are coordinated to serve multiple users. To create reflected waves to serve multiple users, we consider a hybrid beamforming (HBF) scheme where the digital beamforming is performed at the BS, and the RIS-based beamforming is conducted at RISs. Challenges have arisen in such a system. First, unlike conventional cell-free systems, the propagation environment is more complicated due to the extra reflected links via RISs, rendering the optimal scheme very hard to be obtained. Second, since multiple RISs are employed to serve users cooperatively, the phase shifts of different RISs are coupled with each other which should be jointly optimized. Therefore, a coordinating mechanism is required to fully exploit the spatial diversity gain brought by the cooperation of multiple RISs. Third, each RIS element is controlled by the ON/OFF switching of PIN diodes, and thus, only a limited number of discrete phases shifts can be achieved, leading to a non-trivial scheme design.

In the literature, some existing works have considered the RIS aided communication systems in either a single RIS case [2], [6], or a multi-RIS case with continuous phase shifts [7]. Authors in [2] proposed a HBF scheme for a multi-user RIS assisted MIMO system together with a limited phase shifts optimization algorithm to maximize the sum rate. In [7], multiple RISs were employed to assist a single secondary user (SU) in a cognitive radio system. The data rate of the SU is maximized subject to the interference constraints on primary users via digital beamforming and continuous phase shift design of each RIS. Different from the above works, we consider the coordination of multiple RISs with discrete phase shifts to improve the rate performance of all users. A higher degrees of freedom (DoF) achieved by multipath for each transmitted signal is also investigated.

The rest of this letter is organized as follows. In Section II, we provide the system model. Following that, we present the HBF scheme and problem formulation in Section III. An iterative algorithm and simulation results are given in Section IV and Section VI, respectively. The performance analysis is provided in Section V. Finally, we draw our conclusions in Section VII.

II. SYSTEM MODEL

A. Scenario Description

Consider a downlink multi-user system in Fig. 1(a) where $K$ single-antenna users are served by a BS equipped with $N$ antennas with the assisting of multiple RISs. As shown in Fig. 1(b), each RIS can serve multiple users around it by controlling the phase shifts of its elements and reflecting signals from the BS to users. To improve the capacity and achieve a broader coverage, multiple RIS are deployed over the area of interest and coordinated by a central BS. Unlike the traditional
cell-centric implementation, an user-centric transmission is supported where users are served without any cell boundaries, forming an RIS aided cell-free MIMO system. Unlike the traditional cell-free systems, the RIS technique enables signals to be transmitted via both the direct and reflected links, and thus, a more diverse transmission is obtained to improve the sum rate performance.

In such a system, both the BS and all the RISs are coordinated to serve all users. The BS is responsible for network control and planning, and decides the transmission scheduling based on the locations of RISs and users. To characterize the optimal performance of the proposed system, without loss of generality [2], we assume that the channel state information (CSI) of all channels is perfectly known at the BS. After receiving the signals transmitted from the BS, multiple RISs operating at the same frequency are employed to reflect these signals to the users synchronously. Specifically, users receive both the signals transmitted from the BS directly and various reflection signals via multiple RISs.

**B. RIS Reflection Model**

RIS is an artificial thin film of electromagnetic and reconfigurable materials, consisting of \( L \) RIS elements. With a \( b \)-bit re-programmable meta-material, each RIS element can be configured into \( 2^b \) possible amplitudes/phase shifts to reflect signals. Generally, we assume that each RIS element is configured to maximize the reflected signal, i.e., the amplitude of each element is set as 1 [5]. Let \( \theta_{m,l} \) denote the phase shift of the \( l \)-th element of the \( m \)-th RIS, the corresponding frequency response of each RIS element can be given by \( g_{m,l} = e^{j\theta_{m,l}} \), where \( \theta_{m,l} = \frac{im_{l}\pi}{2^{b-1}}, \quad m,l \in \{0, 1, \ldots, 2^b - 1\} \). Without loss of generality, we ignore the coupling between any two RIS elements [2].

**C. Channel Model**

As shown in Fig. 1, each user receives signals from both the BS directly and \( M \) RISs which reflect the signals sent by the BS. The channel between each antenna \( n \) of the BS and each user \( k \) consists of a direct link \( H_D \in \mathbb{C}^{K \times N} \) from the BS to users and \( M \times L \) reflected links since there are \( M \) RISs each of which relies on \( L \) elements to reflect the received signals. Therefore, the equivalent channel \( H \in \mathbb{C}^{K \times N} \) can be given by \( H = H_D + H_{RBR}^{H}QH_{RBR} \), where the latter denotes the reflected links. Specifically, \( H_{RBR} \in \mathbb{C}^{ML \times N} \) and \( H_{RBR}^{H} \in \mathbb{C}^{K \times ML} \) represent the reflect channel between the BS and RISs and that between RISs and users, respectively. We assume that all channels are statistically independent and follow Rice distribution. Each RIS \( m \) reflects incident signal based on an equivalent \( M_L \times M_L \) diagonal frequency response matrix \( Q \). Though we consider a downlink communication system in this letter, the results can be easily extended to an uplink case.

**III. HYBRID BEAMFORMING AND PROBLEM FORMULATION**

In this section, we first present a HBF scheme based on the system model in Section II. A sum rate optimization problem is then formulated and decomposed for the HBF scheme.

**A. Hybrid Beamforming Scheme**

To create reflected waves towards preferable directions to serve multiple users, we present a HBF scheme where the digital beamforming and RIS-based beamforming are performed at the BS and RISs, respectively.

1) **Digital Beamforming**: For \( K \) users, the BS encodes \( K \) different data streams via a digital beamformer \( V_D \in \mathbb{C}^{N \times K} \), and then up-converts the encoded signals over the carrier frequency and allocates the transmit powers. We assume that \( K \leq N \). The users’ signals are sent directly through \( N \) antennas of the BS, which can be given by \( x = V_D s \), where \( s \in \mathbb{C}^{K \times 1} \) denotes the incident signal vector for \( K \) users.

2) **RIS-Based Beamforming**: In the RIS aided cell-free MIMO system, the analog beamforming is achieved by determining the phase shifts of all RIS elements, i.e., \( Q \). Specifically, RIS \( m \) with \( L \) elements performs a linear mapping from the incident signal vector to a reflected signal vector. Therefore, the received signal at user \( k \) can be expressed by

\[
    z_k = h_k v_{D,k} s_k + \sum_{k' \neq k} h_k v_{D,k'} s_{k'} + \omega_k, \quad \text{inter-user interference}
\]

where \( \omega_k \sim \mathcal{C}\mathcal{N}(0, \sigma^2) \) is the additive white Gaussian noise. \( h_k \) and \( v_{D,k} \) denote the \( k \)-th row and the \( k \)-th column of matrices \( H \) and \( V_D \), respectively.

**B. Sum Rate Optimization Problem Formulation**

Based on the RIS reflection model, the channel model, and the HBF scheme, the data rate of user \( k \) can be given by

\[
    R_k = \log_2 \left( 1 + \frac{|h_k v_{D,k}|^2}{\sum_{k' \neq k} |h_k v_{D,k'}|^2 + \sigma^2} \right).
\]

The sum rate optimization problem can be formulated as

\[
    \max_{V_D, Q} \sum_{1 \leq k \leq K} R_k, \quad \text{s.t.} \quad \text{Tr}(V_D V_D^H) \leq P_T, \quad q_{m,l} = e^{j\theta_{m,l}}, \quad \theta_{m,l} = \frac{im_{l}\pi}{2^{b-1}}, \quad m,l \in \{0, 1, \ldots, 2^b - 1\}, \quad \text{where } P_T \text{ is the total transmit power of the BS.}
\]

**C. Problem Decomposition**

Problem (3a) is NP-hard due to the complicated interference items. To solve this problem efficiently, we decouple it into two subproblems, i.e., the digital beamforming subproblem and RIS-based beamforming subproblem.

1) **Digital Beamforming Subproblem**: Given the fixed RIS-based beamformer \( Q \), this subproblem can be formulated by

\[
    \max_{V_D} \sum_{1 \leq k \leq K} \tilde{R}_k, \quad \text{(4a)}
\]
Algorithm 1: Digital Beamforming Design Algorithm

Input: RIS-based beamformer matrix \( \mathbf{Q} \)
Output: Digital beamformer matrix \( \mathbf{V}_D \)
1. Solve power allocation problem (6);
2. Obtain the optimal power allocation solution using water-filling, i.e., \( p_k = \frac{1}{\sigma_k^2} \max \left\{ \frac{1}{\lambda} - v_k \sigma_k^2, 0 \right\} \);
3. Derive the digital beamformer from the optimal power allocation solution by \( \mathbf{V}_D = \mathbf{H}^H (\mathbf{H} \mathbf{H}^H)^{-1} \mathbf{P}^{1/2} \).

\[
\begin{align*}
\text{s.t.} \quad & \text{Tr}(\mathbf{V}_D \mathbf{V}_D^H) \leq P_T. \\
\end{align*}
\]

2) RIS-Based Beamforming Subproblem: Similarly, the RIS-based beamforming subproblem with fixed digital beamformer \( \mathbf{V}_D \) can be written by

\[
\begin{align*}
\max_{\mathbf{Q}} & \sum_{1 \leq k \leq K} R_k, \\
\text{s.t.} & \quad (3c) \text{ and } (3d). \\
\end{align*}
\]

IV. Sum Rate Optimization Algorithm Design

In this section, we develop a SRO algorithm by iteratively solving the above two subproblems.

A. Digital Beamforming Design

In this letter, we consider the zero-forcing (ZF) beamforming with power allocation as the digital beamformer to manage the inter-user interference [9]. Specifically, the digital beamformer can be given as \( \mathbf{V}_D = \mathbf{H}^H (\mathbf{H} \mathbf{H}^H)^{-1} \mathbf{P}^{1/2} = \mathbf{V} \mathbf{P}^{1/2} \), where \( \mathbf{P} \triangleq \text{diag}(p_1, \ldots, p_K) \) is the power matrix where the \( k \)-th diagonal element denotes the received power at user \( k \) [2].

Based on the properties of the ZF beamforming, i.e., \( |\mathbf{h}_k \mathbf{v}_{D,k} | = \sqrt{p_k} \) and \( |\mathbf{h}_k \mathbf{v}_{D,k'} | = 0, \forall k' \neq k \), the digital beamforming subproblem (4a) can be reduced to a power allocation problem which can be given by

\[
\begin{align*}
\max_{\{p_k \geq 0\}} & \sum_{1 \leq k \leq K} \log_2 \left( 1 + \frac{p_k}{\sigma_k^2} \right), \\
\text{s.t.} & \quad \text{Tr}(\mathbf{P}^{1/2} \mathbf{V}_D \mathbf{V}_D^H \mathbf{P}^{1/2}) \leq P_T. \\
\end{align*}
\]

The optimal solution of this power allocation problem can be obtained by water-filling [9], expressed by \( p_k = \frac{1}{\sigma_k} \max \left\{ \frac{1}{\lambda} - v_k \sigma_k^2, 0 \right\} \), where \( v_k \) is the \( k \)-th diagonal element of \( \mathbf{V} \mathbf{P}^{1/2} \) and \( \lambda \) is a normalized factor which is chosen to satisfy that \( \sum_{1 \leq k \leq K} \max \left\{ \frac{1}{\lambda} - v_k \sigma_k^2, 0 \right\} = P_T \). The algorithm can be summarized in Algorithm 1.

B. RIS-Based Beamforming Design

Note that we optimize the sum rate via performing the digital and RIS-based beamforming iteratively. Based on the designed digital beamforming, problem (3) is reduced to a power allocation problem in (6) where the optimization objective (6a) is independent of the RIS-based beamformer \( \mathbf{Q} \). Hence, problem (6) depends on \( \mathbf{Q} \) only through the power constraint (3b). Therefore, the subproblem (5a) can be reformulated as a power minimization problem, expressed by

\[
\begin{align*}
\min_{\{\theta_{m,l}\}} & \quad f(\mathbf{Q}), \\
\text{s.t.} & \quad (3c) \text{ and } (3d), \\
\end{align*}
\]

where \( f(\mathbf{Q}) = \text{Tr}(\mathbf{V}_D \mathbf{V}_D^H) \). However, problem (7a) is still difficult to solve due to the large-scale of \( \mathbf{Q} \). For simplicity, we ignore the items about direct link \( \mathbf{H}_D \) in \( f(\mathbf{Q}) \) which is unrelated to the optimization objective \( \mathbf{Q} \). For \( K = N, f(\mathbf{Q}) \) can be rewritten by

\[
\begin{align*}
\begin{split}
\end{align*}
\]

By setting phase shifts of all other RIS elements fixed except \((m, l), f(\mathbf{Q})\) can be viewed as a function of \( \theta_{m,l} \) below:

\[
\begin{align*}
\begin{split}
\end{align*}
\]

where \( a_1 \sim a_8 \) are defined as in Appendix A and independent of \( \theta_{m,l} \). The minimum value of \( f(\mathbf{Q}) \) can be obtained with respect to \( \theta_{m,l} \) which satisfies \( \frac{\partial f(\mathbf{Q})}{\partial \theta_{m,l}} = 0 \). Based on the result in Appendix B, the optimal value of \( \theta_{m,l} \) can be obtained by \( \theta_{m,l}^* = 2 \arctan \chi \), where \( \chi \) is defined as in Appendix B.\(^2\) Moreover, since only a limited number of discrete phase shifts are available at the RIS-based beamforming, the optimal value \( \theta_{m,l}^* \) should be quantized to the nearest points in the set \( \mathcal{F} = \{ \frac{\text{im}_{\chi}}{2\pi l} \}, i_m, l \in \{0, 1, \ldots, 2^b - 1\} \).

Therefore, starting from a randomly initiated RIS-based beamformer, the optimal \( \mathbf{Q} \) can be obtained by sequentially updating the phase shift of each RIS element \((m, l)\) in an iterative manner, until the algorithm converges to a local minimum of \( f(\mathbf{Q}) \). The algorithm is summarized in Algorithm 2.

C. Overall Algorithm Description

The proposed SRO algorithm solves problem (3a) in an iterative manner. We first design the digital beamforming \( \mathbf{V}_D \) by water-filling with fixed RIS-based beamforming. Based on that, the RIS-based beamforming is optimized by Algorithm 2. The SRO algorithm converges if the value difference of the objective functions between two adjacent iterations is less than a threshold \( \varepsilon \).

V. Performance Analysis

We now analyze convergence of the SRO algorithm and the influence of the number of RISs on the sum rate, respectively.

A. Convergence

In the \( n \)-th iteration, a better digital beamformer \( \mathbf{V}^{(n)}_D \) can be achieved by the water-filling scheme with a fixed RIS-based beamforming \( \mathbf{Q}^{(n-1)} \). Given that, the RIS-based beamforming is optimized to maximize the sum rate by Algorithm 2. Hence, we can obtain \( \mathcal{R}(\mathbf{V}^{(n)}_D, \mathbf{Q}^{(n)}) \geq \mathcal{R}(\mathbf{V}^{(n-1)}_D, \mathbf{Q}^{(n-1)}) \).\(^2\) Since \( 0 \leq \theta_{m,l} \leq 2\pi \), only the smaller of the two solutions of \( \arctan \chi \), i.e., the solution which less than \( \pi \), is taken into account.
This indicates that the objective value of the problem (3a) is non-decreasing after each iteration. Since the maximum value of the sum rate is bounded, the proposed SRO algorithm is guaranteed to converge.

B. Influence of Number of RISs

To improve the sum rate performance, multiple RISs are deployed to create favorable propagation conditions. Therefore, it is important to analyze the influence of the number of RISs.

By ignoring the items about direct link \( H_D \) which is unrelated to the number of RISs \( M \), the transmit power allocated to the signals intended for the user \( k \) can be expressed as

\[
p_k = |H_k V_{D,k}|^2 = V^H_{D,k} H_{BR}^H Q^H_H H_{RU,k}^H R U_k^H Q_H H_{BR} V_{D,k}
\]

\[
= V^H_{D,k} H_{BR}^H Q^H_H H_{RU,k}^H H_{RU,k}^H H_{RU,k} Q_H H_{BR} V_{D,k}
\]

\[
\approx M^2 L^2 V^H_{D,k} H_{BR} V_{D,k}, \quad (10)
\]

where \( (a) \) is obtained by the well-known channel hardening effect in MIMO communication systems [9]. Specifically, as the number of receive (or transmit) antennas increases while keeping the number of transmit (or receive) antennas constant, the column-vectors (or row-vectors) of the propagation matrix are asymptotically orthogonal. Hence, we have

\[
\lim_{M \times L \to \infty} H_{BR}^H H_{BR} \approx (M \times L)I,
\]

\[
\lim_{M \times L \to \infty} H_{RU}^H H_{RU} \approx (M \times L)I, \quad (11)
\]

where \( I \) denotes the identity matrix. Each RIS relies on the combination of a large number of elements to realize a desired transformation on the transmitted, received, or reflected waves, and thus \( M \times L \to \infty \) can be satisfied. Moreover, the phase shift matrix \( Q \) satisfies \( QQ^H = I \), since \( Q \) is a diagonal matrix where the module of each element is 1, i.e., \( |q_{m,l}| = 1, \forall m, l \). Therefore, the approximation \( (a) \) in (10) can be obtained. Based on (6a) and (10), the sum rate \( R \) at a high SNR region can be rewritten as

\[
R = 2K \log_2 M + \sum_{k=1}^{K} \log_2 (L^2 V^H_{D,k} V_{D,k} \sigma^{-2}). \quad (12)
\]

From this equation, we can know that the derivative of the sum rate with respect to \( M \) is positive, and gradually drops to zero as \( M \) grows. Therefore, the sum rate grows rapidly with a small number of RISs and slows down as the number of RISs continues to increase.

VI. SIMULATION RESULTS

In this section, we provide the simulation results of our SRO algorithm. To evaluate the performance, the SRO algorithm is compared with 1) No-RIS case [9, Chapter 7]: Conventional single cell networks without RISs. 2) Conventional distributed antenna system (DAS) [10]: Multiple distributed antennas are deployed remotely rather than centrally at the BSs. 3) Random Phase Shift Algorithm: The phase shifts of each RIS element are selected randomly from the feasible set \( \mathcal{F} \). 4) Exhaustive Search Algorithm: We consider the exhaustive search as a upper bound to evaluate the sum rate performance of the SRO algorithm where the phase shifts of each RIS element are selected by traversing the feasible set \( \mathcal{F} \).

Simulation parameters are as follows. For propagation parameters, we use the UMa path loss model in [11] as the distance-dependent channel path loss model, and the Rician factor \( \kappa \) is set as 4 [2]. Users are uniformly distributed on the ring around the BS within a radius of 80m and 100m. Moreover, both the number of users and the number of antennas of the BS are set as 8, i.e., \( K = N = 8 \). The noise power is \( \sigma^2 = -90 \text{ dBm} \), and the threshold of the proposed algorithm is \( \varepsilon = 10^{-4} \) [8].

The DoF obtained by different schemes is evaluated in Fig. 2(a) with \( M = 3, L = 64, \) and \( b = 3 \). Following the definition of DoF, i.e., the slope of the achievable rate v.s. SNR (at high SNR), we add some auxiliary lines (i.e., the light blue dashed lines in Fig. 2(a)) parallel to the curve of the RIS aided cell-free system at high SNR. Our proposed SRO algorithm achieves a higher sum rate than the random phase shift algorithm and performs very close to that of the exhaustive search algorithm. We can observe that our proposed scheme can achieve a higher DoF compared with the conventional DAS and no-RIS case. This is because signals are transmitted via a larger number of independent paths, i.e., the direct
and reflected links, in the RIS aided cell-free system than the conventional ones.

Fig. 2(b) depicts how the sum rate is influenced by the number of quantization bits $b$ and the number of RISs $M$ with $P_T = 30$ dBm and $L = 64$. As $b$ increases, the sum rate with discrete phase shifts gradually approaches that in the continuous case. The gap between the discrete and continuous cases is insignificant when $b \geq 3$, indicating that a small value of $b$ is already enough to obtain a satisfactory performance. Fig. 2(b) also shows that the sum rate grows rapidly with a small number of RISs and slows down as the number of RISs continues to increase.

Fig. 2(c) shows the influence of the distance between the BS and RISs on the sum rate performance with different sizes of each RIS $L$. The transmit power of the BS $P_T$ and the number of deployed RISs are set as 30 dBm and 3, respectively. We assume that the RISs are uniformly distributed on the circle around the BS. It can be seen that as the distance between the BS and RISs increases, the sum rate decreases first and then increases, which verifies the results in our previous work [8].

This implies that the RISs should be deployed in close proximity of the BS/users to provide high data rate services. We can also observe that the sum rate grows with the size of each RIS $L$.

VII. Conclusion

In this letter, we have considered an RIS aided cell-free MIMO system where a BS and multiple RISs are coordinated to serve multiple users. To maximize the sum rate, we have proposed an HBF scheme where the digital beamforming and the RIS-based beamforming are performed at the BS and RISs, respectively. The sum rate optimization problem has been formulated and solved by the proposed SRO algorithm in an iterative manner. Both theoretical analysis and simulation results reveal that the sum rate increases first and then saturates when as the number of RISs grows. We can also conclude that the RIS aided cell-free system achieves a better sum rate performance compared with the conventional ones. Moreover, the RISs should be deployed in close proximity of the BS/users for sum rate maximization.

APPENDIX A

DERIVATION OF (9)

For brevity, we define the index $j = (m - 1)L + l$, and thus $q_{m,l}$ can be abbreviated to $q_j$. To separate the optimization objective $q_j$ from other fixed elements, the $j$-th diagonal element of $Q$ is set to 0, denoted by $Q^{(-j)}$. Hence, $f(Q)$ can be rewritten as

$$f(Q) = M^2L^2 \text{Tr}[(A_j + B_j q_j)(A_j^H + B_j^H q_j)^{-1}] = M^2L^2 \text{Tr}(D_j + C_j q_j - 1),$$

where $A_j = \tilde{P} Q_j^{(-j)} H_{BR}$ and $B_j = \tilde{p}_j h_j^{(j)}$. Let $\tilde{P} = P^{-1/2} H_{BR}^H$, and the $j$-th column and $j$-th row of $\tilde{P}$ and $H_{BR}$ are represented by $\tilde{p}_j$ and $h_j^{(j)}$, respectively. Moreover, we have $C_j = A_j A_j^H + B_j B_j^H$ and $D_j = A_j + A_j^H$.

Note that $C_j$, $\tilde{T}_j$, and $C_j^{(-j)}$ are one-rank matrices and $D_j$ is a full-rank matrix. Therefore, $f(Q)$ can be rewritten according the Sherman Morrison formula, i.e., $(A + B)^{-1} = A^{-1} - A^{-1}B(A + B)^{-1}$, and can be expressed as (9). Due to the limited space, detailed definitions of $a_1 \sim q_6$ are not repeated here, which depend on the following variables. $E_1 = \text{eig}(A_j)$, $E_2 = \text{Tr}(D_j^2 C_j)$, $E_3 = \text{Tr}(D_j^2 C_j)$, $E_4 = \text{Tr}(D_j^2 H_{BR}^H)$, $E_5 = \text{Tr}(D_j^2 C_j H_{BR}^H + D_j^2 C_j H_{BR}^H)$, $E_6 = \text{Tr}(D_j^2 C_j H_{BR}^H)$, $E_7 = \text{Tr}(D_j^2 C_j H_{BR}^H)$, and $E_8 = \text{Tr}(D_j^2 C_j H_{BR}^H)$.

APPENDIX B

DERIVATION OF $\chi$

Let $\frac{\partial f(Q)}{\partial q_{m,l}} = 0$, we have $b_1 e^{j \theta_{m,l}} + b_2 e^{j \theta_{m,l}} + b_3 + b_4 e^{-j \theta_{m,l}} + b_5 e^{-2j \theta_{m,l}} = 0$, where $b_1 = j(a_1 a_2 - a_2 a_4)$, $b_2 = j(a_1 a_2 - a_3 a_4)$, $b_3 = j(a_1 a_2 + a_2 a_4 - a_3 a_4)$, $b_4 = j(a_2 a_4 - a_4 a_6)$, and $b_5 = j(a_3 a_4 - a_4 a_7)$. According to the Euler’s formula and the auxiliary angle formula, this equation can be rewritten as (14), shown at the top of the page, where $\tan 2\gamma_1 = \frac{2 \tan \gamma_1}{1 + \tan^2 \gamma_1}$ and $\tan 2\gamma_2 = \frac{2 \tan \gamma_2}{1 + \tan^2 \gamma_2}$. Based on $\sin \alpha = \frac{1}{1 + \tan^2 \alpha}$ and $\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$, (14) can be rewritten as a quartic equation of one unknown, i.e., $\tan \frac{\theta_{m,l}}{2}$, which can be easily solved. Denote the solution of this equation as $\chi$.

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