Quantum state transfer in spin chains via isolated resonance of terminal spins

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We propose a quantum state transfer protocol in a spin chain that requires only the control of the spins at the ends of the quantum wire. The protocol is to a large extent insensitive to inhomogeneity caused by local magnetic fields and perturbation of exchange couplings. Moreover, apart from the free evolution regime it allows one to induce an adiabatic spin transfer, which provides the possibility of performing the transfer on demand. We also show that the amount of information leaking into the central part of the chain is small throughout the whole transfer process (which protects the information sent from being eavesdropped) and can be controlled by the magnitude of the external magnetic field.

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I. INTRODUCTION

For practical quantum information processing, one needs not only controllable and decoherence-resistant qubits but also short-range communication channels providing inter-qubit communication during more complex computation processes [1]. Chains of coupled qubits, referred to as spin chains, have been proposed as such a channel [2, 3] which does not require conversion between different physical encodings and therefore presents an interesting alternative to photonic flying qubits [4, 5]. This approach has recently been extended to include the transfer of higher-dimensional states [6] and through qutrit (spin-1) chain. [7].

A useful information channel must work with high fidelity. Strictly perfect transfer is possible in chains with individually tuned XY couplings [8–13], which may assure commensurability of the spectrum [14]. However, this is not admitted by chains coupled with isotropic Heisenberg interactions [15] (which are typical for spins confined in quantum dots [16]) and an arbitrarily perfect state transfer can only be achieved for particular lengths of the spin chain [17]. As an alternative, applying a magnetic field with a simple spatial variation along the chain has been proposed [18]. It is also possible to assure high-fidelity transfer by appropriately switching the couplings [19] in order to induce adiabatic transfer by applying global [20] or local [21, 22] external fields. While achieving specific control over the whole spin chain may be quite demanding, protocols involving only local manipulation seem to be feasible. Such proposals include transfer procedures involving logical gates on the terminal bits [12, 23], using a receiver with memory [24] or local switching of a perturbation [25]. The transfer fidelity can also be improved in a uniformly coupled chain or ring by multi-qubit encoding [26–29]. An interesting solution, involving very limited control resources and not requiring any particular design of the couplings, is to weakly couple the terminal (sender and receiver) qubits to a spin chain [11, 30–34].

Clearly, the feasibility of a transfer protocol depends on the required degree of control over the spin chain. From this point of view, the solutions involving precisely engineered couplings along the chain or local time dependent control are less favorable than those based on controlling only the terminal qubits. Another desirable possibility is a conclusive transfer, such that the receiving party can be sure that the transfer has been completed. In order to achieve this, dual rail protocols have been proposed [35]. An information transfer protocol should also be resilient to environmental noise [36–38] and to imperfections and fluctuations of the chain parameters [39–43].

In our previous work [44] we proposed a quantum dot implementation of a quantum state transfer channel that supported high-fidelity transfer of the state of a terminal dot. In this paper we generalize and develop our idea to propose a spin chain based protocol that requires only the control of terminal qubits and can lead to almost perfect on-demand transfer of quantum information along a uniformly or randomly coupled spin chain. Moreover, our protocol provides protection against eavesdropping through low leakage of quantum information to the central part of the chain available to the third party. The key idea is to shift the energy of the states with the excitation located at one of the terminal qubits away from the spectral range of the states with the central part of the chain excited and exploit the dynamics in the approximately invariant two-dimensional subspace of terminal states coupled indirectly via the central part of the chain. While this concept is to some extent similar to the idea of weakly coupled terminal qubits [30], where decoupling of the terminal states is due to weak coupling between the terminal sites and the remaining part of the chain, it has a two-fold advantage over that proposal. First, it is much more resilient to fluctuations in the couplings, as the terminal states are isolated and not located in the narrow gap of the spectrum of a finite homogeneous chain. Second and more importantly, apart from the free dynamics, it admits an adiabatic evolution in which the qubit state is transferred from one terminal
to the other by sweeping the energy of one of the terminal states through resonance by a local field applied on either end of the chain. This opens the possibility of effecting the transfer on demand, either by the sender or by the receiver.

The paper is organized as follows. In Sec. II, we describe the system and the methods of simulation used. Next, in Sec. III, we discuss two versions of our protocol: free evolution and adiabatic transition driven by magnetic field sweep at the terminal sites of the chain. In Sec. IV we study the leakage of quantum information from the terminal nodes of the chain (controlled by communicating parties) to the central ones (which may be available to eavesdropper). Finally, Sec. V concludes the paper.

II. MODEL AND METHOD

We consider a model consisting of $N$ linearly arranged spins with the nearest neighbour XY couplings $J_k$ and a local magnetic field at each spin site $B_k$. The model is restricted to at most one spin-flip excitation in the chain. Then, the Hamiltonian of the system is given by

$$ H = \sum_{k=1}^{N-1} J_k \langle k|k + 1| + \text{h.c.} \rangle + \sum_{k=1}^{N} B_k |k|k], $$

where $|k\rangle$ denotes the state with an inverted (excited) spin at the $k$th site of the chain. We assume that only the terminal spins ($k = 1, N$) are controlled by external fields $B_{k}^{(\text{ext})}$, while the fields on all the nodes (including the terminal ones) are random (but time-independent), reflecting the effects of an inhomogeneous environment. We include the inhomogeneity of the spin chain by using the normal distribution for both the local magnetic fields, $B_k$, and the couplings $J_k$, with the standard deviations $\sigma_B$ and $\sigma_J$, respectively. The average value of $J_k$ is fixed and equal to $J$ for all $k$ and the average value of the local field, $B_k$, is non-zero (and equal to the applied control field) only at the terminal ends.

In this paper, we present two possible kinds of the state transfer protocol. In the first case the free transfer with time-independent external magnetic fields is assumed, modeled by numerical diagonalization of the Hamiltonian and finding the evolution in the basis of eigenstates. In the second case the transfer takes place adiabatically with time-dependent external field and is simulated numerically to find the numerical solution of the Schrödinger equation.

III. QUANTUM STATE TRANSFER

We consider a generic initial spin state localized at the first (sender’s) node of the chain,

$$ |\Psi(0)\rangle = \cos \frac{\theta}{2} |0\rangle + e^{i\phi} \sin \frac{\theta}{2} |1\rangle, $$

where $|0\rangle$ denotes state of the chain with no spins inverted. After time $t$ the system state evolves into

$$ |\Psi(t)\rangle = \cos \frac{\theta}{2} |0\rangle + e^{i\phi} \sin \frac{\theta}{2} \sum_{k=1}^{N} c_k |k\rangle, $$

and we are interested in the fidelity between initial state $|\Psi(0)\rangle$ and a state $|\Psi(t)\rangle$ after time $t$.

A. Exploiting isolated resonance

Due to the external magnetic field applied at the terminal sites, the energies of the spin-up states localized on those sites are higher than for those localized inside the chain. If this external magnetic field is strong enough ($\gtrsim 2J$), it results in the formation of two eigenstates $|\Psi_N\rangle$ and $|\Psi_{N-1}\rangle$ that are energetically separated from all the others and mainly composed of the states $|1\rangle$ and $|N\rangle$. Due to indirect coupling via the spin chain, these two eigenstates form an antiresonance when the on-site energies of the states $|1\rangle$ and $|N\rangle$ are brought to resonance. While the position of this resonance as a function of magnetic field $\delta B = \bar{B}_i^{(\text{ext})} - \bar{B}_N^{(\text{ext})}$ is randomly shifted in the presence of parameter fluctuations in the central part of the chain (due to coupling-induced energy shifts), its width remains almost constant [44]. The two states can be brought to resonance by applying an asymmetric external magnetic field $\bar{B}_i^{(\text{ext})} = \bar{B}_N^{(\text{ext})} + \delta B$ which compensates the shift of the resonance and leads to the resonantly enhanced tunnelling [46, 47] (we will refer to this situation as “compensated inhomogeneous chain”). Close to resonance we can restrict, to a good approximation, the free evolution of the system to the subspace
\[
\{ |\Psi_N\rangle, |\Psi_{N-1}\rangle \}. \quad \text{Then the evolution corresponds to the oscillation of the spin-inverted state between the terminal sites of the chain. In the simulations described in the following sections we set } B_N/J = 5.
\]

### B. Free transfer

The oscillation period is determined by the width of the resonance \(2V\), which critically depends on \(N\). This is reflected in the free transfer time dependence \(\tau_f = \pi h/(2V)\). In Fig. 1 we show dependence of \(\tau_f\) and maximum obtained fidelity on \(N\) for a few values of the normalized standard deviations \(\bar{\sigma}_J = \sigma_J/J\) and \(\bar{\sigma}_B = \sigma_B/J\). As long as the inhomogeneity is not too strong, it weakly affects the width of the resonance and the free transfer time for a compensated inhomogenous chain is of the order of the free transfer time in the homogeneous chain [Fig. 1(a)]. Since the fidelity depends mainly on the degree of localization of \(|\Psi_N\rangle\) and \(|\Psi_{N-1}\rangle\) on the terminal sites (|1\rangle and |\(N\rangle\)), it is well characterized by the parameter \(\Delta = 1 - |\langle 1|\Psi_N\rangle|^2 - |\langle 1|\Psi_{N-1}\rangle|^2\). In our model, \(\Delta\) does not depend on the chain length \(N\), so the transfer fidelity should also weakly depend on \(N\). This is in fact confirmed by our simulations [Fig. 1(b)]. We have observed also that the inhomogeneity of the magnetic field affects both transfer time and achieved fidelity stronger than the inhomogeneity of the couplings.

In Fig. 2(a) we present an exemplary evolution of the spin-up state in a compensated inhomogenous chains and show that the transfer fidelity \(F\) achieved for the perfect compensation is very high (over 0.95). However, since the weak indirect coupling between the two terminal states leads to a very narrow resonance, \(F\) is very sensitive to small variations of the compensating field, i.e., a small deviation of magnetic field results in a big decrease of the maximum fidelity obtained [Fig. 2(b)].

### C. Adiabatic transfer

The requirement of a very precise control of the magnetic field makes it a rather demanding task to obtain a free transfer. Moreover, in the presence of randomness, the optimal transfer time is unpredictable. To overcome this problem, we propose an adiabatic variation of our protocol. By slowly changing the asymmetry of the external magnetic field \(\delta B\) one can sweep the energy levels of \(|\Psi_N\rangle\) and \(|\Psi_{N-1}\rangle\) through the resonance. It is worth noting that this procedure can be done either by the sender or by the receiver. The system evolution in this case can be described by an effective 2-level model including the states \(|\Psi_N\rangle\) and \(|\Psi_{N-1}\rangle\), with the coupling \(V\) taken as half of the energy splitting at the resonance. Then, using the Landau-Zener formula [48] for nonadiabatic transition probabilities,

\[
P_{na} = \exp \left( \frac{-2\pi |V|^2}{h \alpha} \right), \tag{4}
\]

we found the dependence of the speed of magnetic field sweep \(\alpha = dB/dt\) as a function of the desired fidelity \(F = 1 - P_{na}\).

Formally, the field is swept over \((-\infty, \infty)\), which is unrealistic. To achieve a finite transfer time we narrow the limits of the magnetic field sweep to the area where the energy separation of the states is smaller than \(\beta V\) for a certain parameter \(\beta\) (we assume that interaction is negligible for \(\Delta E > \beta V\)). In this way we obtain the adiabatic transfer time \(\tau_\alpha = \left[ h\beta/(\pi V) \right] \ln (1 - F)\), whose ratio to the free transfer time \(\tau_f\) (for a given \(\beta\)) depends only on the desired fidelity, \(\tau_\alpha/\tau_f = -2\beta/\pi^2 \ln (1 - F)\).

To confirm the agreement of the effective two-level model with the Landau-Zener result we simu-
enough values of $\beta$ and simulated results (Fig. 3) we get a good agreement between the Landau-Zener
$\beta$ formula and Landau-Zener theory. It remains true also for a considerable improvement. It
remains true also for a decreased speed of the magnetic field sweep [Fig. 4(b)]. This can be explained
by noting that the total occupation probability of the terminal nodes at the end of the evolution is
lower than 1, which in turn indicates that the origin of the problem with achieving very high fidelity lies
in the leakage to the nodes inside the chain.

IV. LEAKAGE OF QUANTUM INFORMATION

To estimate the amount of leaking information let us define

$$\epsilon = 1 - |c_k|^2 - |c_N|^2,$$

i.e., the total occupation probability for all the states $|k\rangle$, $2 \leq k \leq N - 1$ (coefficients $c_k$ as defined in Eq. (3)). In both cases, adiabatic and free transfer, simulations show that $\epsilon$ remains very small throughout the whole evolution (around 10%) and is mainly determined by the magnitude of the external magnetic field responsible for
isolating terminal states [Fig. 5(a)]. Moreover it is observed that the value of $\epsilon$ does not strongly depend on
the chain length or the inhomogeneity parameters $\sigma_j^x$, $\sigma_j^z$ [Fig. 5(b)].

The low value of $\epsilon$ not only allows for obtaining high fidelity transfer, but is also important from the point
of view of protection against eavesdropping. In order to show this, let us consider the following scenario. The
sender prepares one of the two pure states from an arbitrarily chosen basis, i.e., a generic qubit state defined
in Eq. (2) or a state orthogonal to it. Now, the essential question is: how well can the eavesdropper distinguish
which of the two states have been sent? In order to answer this question one may define a measure of the
information available to the eavesdropper as

$$D := \frac{1}{2} |\rho - \rho^\perp|_1 = \frac{1}{2} \text{Tr} \left( |\rho - \rho^\perp| \right),$$

where $\rho$ and $\rho^\perp$ are the reduced density matrices of the part of the chain that eavesdropper has access to and
that correspond to two orthogonal initial sender’s states. Such a choice of measure is justified by its operational
meaning, as $(1 + D)/2$ is the average success probability for distinguishing between the states $\rho$ and $\rho^\perp$ when they are
equally probable to occur [49] (see also [50]).

We will focus on two limiting types of eavesdroppers: when the third party has access to all but terminal
spins (which we will refer to as the powerful eavesdropper) and when this access is limited only to a single $n$th node inside the chain (weak eavesdropper). In the first case the

FIG. 3: Occupation probabilities for the initial (red lines) and final (blue lines) nodes for an inhomogenous spin chain with $N = 6$, $\sigma_j^x = 0.1$, $\sigma_j^z = 0.5$ during adiabatic evolution. Horizontal dashed lines show the desired fidelity $F = 0.66$ obtained from the Landau Zener formula; vertical dotted lines indicate the end of the magnetic field sweep.

FIG. 4: Occupation probabilities for the initial (red lines) and final (blue lines) nodes for an inhomogenous spin chain with $N = 6$, $\sigma_j^x = 0.1$, $\sigma_j^z = 0.5$ during an adiabatic evolution for $\beta = 20$. Horizontal dashed lines show the desired fidelity $F = 0.95$. (a) Speed of the magnetic field sweep $\alpha$ calculated from the Landau-Zener formula; (b) Four times slower magnetic field sweep $0.25\alpha$. 

To estimate the amount of leaking information let us define

$$\epsilon = 1 - |c_k|^2 - |c_N|^2,$$
eavesdropper’s reduced states are given by

\[ \rho = \left( 1 - \epsilon \sin^2 \frac{\theta}{2} \right)|0\rangle \langle 0| + \epsilon \sin \frac{\theta}{2} \tilde{\Psi} \langle \tilde{\Psi} | \right. + \left. \cos \frac{\theta}{2} \sin \frac{\theta}{2} \left( \sqrt{\epsilon} e^{i\phi} \tilde{\Psi} \langle \tilde{\Psi} | + \text{h.c.} \right), \tag{7a} \]

\[ \rho^\perp = \left( 1 - \epsilon \cos^2 \frac{\theta}{2} \right)|0\rangle \langle 0| + \epsilon \cos \frac{\theta}{2} \tilde{\Psi} \langle \tilde{\Psi} | \right. - \left. \cos \frac{\theta}{2} \sin \frac{\theta}{2} \left( \sqrt{\epsilon} e^{i\phi} \tilde{\Psi} \langle \tilde{\Psi} | + \text{h.c.} \right), \tag{7b} \]

where tildes indicate states of the system without terminal spins, i.e., states of the central part of the chain of length \( N - 2 \), and

\[ |\tilde{\Psi} \rangle = \left( \sum_{k=2}^{N-1} c_k |\tilde{k} \rangle \right) / \left( \sum_{k=2}^{N-1} |c_k|^2 \right) = \frac{1}{\epsilon} \sum_{k=2}^{N-1} c_k |\tilde{k} \rangle. \]

For the weak eavesdropper expressions for reduced states are very similar to the ones given by Eqs. (7a)-(7b): one simply needs to replace \( \epsilon \) by \( |c_n|^2 \) by \(|0\rangle_n\) and \(|\tilde{\Psi} \rangle\) by \(|1\rangle_n\) (where \(|0\rangle_n\) and \(|1\rangle_n\) denote the ground and excited state of the \( n \)th spin, respectively). In both cases the information available to the third party is given by

\[ D = \sqrt{a^2 \cos^2 \theta + a \sin^2 \theta}, \tag{8} \]

where \( a = \epsilon \) for the powerful eavesdropper and \( a = |c_n|^2 \) for the weak one. As \( |c_n|^2 \leq \epsilon \) the low value of \( \epsilon \) throughout the whole evolution provides protection against both considered kinds of eavesdroppers. Note that \( D \) is minimized for states sent from the computational basis, as a consequence of the form of the considered Hamiltonian (for which \(|0\rangle\) is an eigenstate).

V. CONCLUSIONS

We have proposed two variations of the quantum state transfer protocol in a spin chain that fulfill two basic requirements: simplicity and resistance to perturbation. They are immune to the inhomogeneity caused both by the local magnetic fields and disorder in exchange couplings, and require only limited control of the external magnetic field at the terminal sites of the chain. Although the transfer time in the proposed protocol increases rapidly with the chain length, the fidelity of the transfer weakly depends on the chain length and it is possible to achieve state transfer on demand using a variable magnetic field (adiabatic protocol). This compares favorably with the earlier proposals. We have also shown that the amount of the information available in the central part of the chain is small during the whole transfer process and can be decreased even more by increasing the magnitude of the external magnetic field. This can be considered as an advantage, as it decreases the probability for the eavesdropper to distinguish between two orthogonal states sent and increases the time needed for the interception of the quantum state.
However, it also affects the information transmission time, therefore a trade-off is expected between security and the transfer speed. Finally, let us note that the proposed adiabatic protocol allows a trivial extension to a dual-rail protocol [35].

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