The hypothesis of CP-violation in the strong sector of the Standard Model arises from the analysis of the quantum structure of the QCD vacuum. At a classical level, the QCD Hamiltonian possesses an infinite set of degenerate gauge-dependent vacua, each of which is characterized by its winding number $n$. The gauge-invariant definition of the ground-state is given in terms of a linear superposition of such classical vacua (θ-vacuum) [1]:

$$ |\theta\rangle = \sum_n e^{i\theta n} |n\rangle, \quad \theta \in \mathbb{R}. $$

(1)

The structure of the QCD vacuum, combined with the observation of weak CP-violation in neutral kaon systems, gives rise to the so-called θ-term in the QCD action (in the Euclidean formulation):

$$ S \rightarrow S + S_\theta, \quad S_\theta = i\theta \frac{1}{32\pi^2} \int d^4z F_{\mu\nu} \tilde{F}_{\mu\nu}, \quad (2) $$

where $\theta = \theta + \text{argdet}(M)$, and $M$ is the complex, non-hermitian quark mass matrix, arising from the spontaneous breaking of the electro-weak gauge symmetry. The real constant $\theta$ is an additional parameter of the Standard Model, which has to be fixed by experiment. At the moment, the most constraining bounds on this quantity come from measurements of the neutron electric dipole moment (EDM) [2], which indicate that $\theta < 10^{-9}$ [3, 7].

In this Rapid Communication we explore some dynamical consequences of the θ-term at the microscopic level. In particular, we identify the mechanisms which give rise to the tunneling between the degenerate classical vacua, quarks and anti-quarks in a neutron migrate in opposite directions, giving rise to an oscillating electric dipole moment. We discuss a possible phenomenological implication of this effect.

The correlation function (3) measures the probability to find a quark of flavor $f$ at the point $\mathbf{y}$, in a system with neutron quantum numbers.

$$ \Sigma(\tau) = \int d^3y \, y_3 \, G_{e/m}(\mathbf{y}, \tau), \quad (4) $$

which, in the large time $\tau$ limit, reads:

$$ \Sigma(\tau) \to D_z \times \int \frac{d^4p}{(2\pi)^3} \frac{-2\Lambda_N^2}{e^{2\mu p_z} + (p^2 + M^2)}, \quad (5) $$

where $D_z$ is the component of the neutron EDM along the spin direction, $M$ is the neutron mass and $\Lambda_N$ is the coupling of the neutron to the interpolating operator, $\langle 0 | J_N(0) | N \rangle = \Lambda_N u_p$.

In QCD, a finite dipole can only arise from the CP-breaking interaction [2]. To lowest order in $\theta$, we can write (in an obvious notation):

$$ G_f(\mathbf{y}, \tau) = \langle 0 | Tr [ J_N \, J_N^\dagger \, \tilde{J}_N \, \Gamma_3 ] \int d^4z \, F \tilde{F} \, | 0 \rangle_{\theta=0}. \quad (6) $$

In the semi-classical limit, the topological charge is condensed around instantons and anti-instantons ($Q = N_I - N_A$), and the physics of the quantum mixing of
the \( \theta \)-vacuum can be formulated in terms of an intuitive pseudo-particle picture. The path-integral can then be computed by summing over the configurations of a statistical grand-canonical ensemble of pseudo-particles. In such an approach, a quantitative estimate of the matrix element \( \langle \theta \rangle \) can only be performed in a model-dependent way, as we do not know from first principles the density and size distribution of pseudo-particles in the vacuum. This is the starting point of the Instanton Liquid Model (IIM) - for a review see [8] - which has been proved to be very successful in describing the phenomenology of the QCD vacuum and of light hadrons.

In the present study, we choose to avoid introducing model-dependent parameters and we focus on the qualitative effects generated by the dynamical interplay of the strong CP-breaking interaction \( (2) \) with the quantum structure of the \( \theta \)-vacuum, at semi-classical level. A quantitative, albeit model-dependent, prediction of the neutron EDM in this model will be presented in a separate publication [9].

In the instanton vacuum, matrix elements with one insertion of the topological charge operator can be written as a sum over the contributions of the different topological sectors [11]:

\[
\langle O \rangle = \frac{1}{32\pi^2} \int d^4 z \, F_{\mu \nu} \tilde{F}_{\mu \nu} = \sum_Q \mathcal{P}(|Q|) \langle O \rangle_Q \quad (7)
\]

where \( \mathcal{P}(Q) \) denotes the relative occurrence of configurations with topological charge \( Q \) and \( \langle O \rangle_Q \) is the average performed in a canonical ensemble with such a total topological charge. This equation expresses the fact that CP-breaking interactions arise from topologically non-trivial gauge configurations. Physically, it implies that these forces are triggered only during tunneling between distinct classical vacua.

As long as we are interested in qualitative phenomena, we do not need detailed knowledge of the positive-definite weight factor \( \mathcal{P}(|Q|) \) in \( (7) \). We shall therefore concentrate on the contribution from each topological sector (factor \( Q \langle O \rangle_Q \) in \( (7) \)). This can be done by evaluating averages in different canonical ensembles in which the number of instantons and anti-instantons is fixed. In order to do so, we have used the Interacting Instanton Liquid Model (IIIM), developed in [11]. We have averaged over 1000 configurations of an ensemble of 130 pseudo-particles in a periodic box of volume \( V = 3.2 \times 4 \) fm\(^3\). For example, the contribution of the topological sector \( Q = 10 \) was obtained inserting 70 instantons and 60 anti-instantons in the box. For each configuration, the fermionic determinant and the quark propagator have been calculated by diagonalizing numerically the Dirac operator (for a detailed discussion of the method see Section VI in [8]). In topologically non-trivial sectors, the quark propagator receives contribution from \( N_f |Q| \) exact zero-modes (semi-classical realization of the Index Theorem). The ensemble of collective coordinates of the pseudo-particles was then generated dynamically, through an usual accept/reject Metropolis algorithm. To improve the signal-to-noise ratio, we have used rather large quark masses, \( m_u = m_d = 75 \) MeV and \( m_s = 150 \) MeV, and a rather small Euclidean time, \( \tau = 0.7 \) fm. We note that, in the present approach, we do not make use of an anomalous Ward identity to relate the topological charge to the divergence of a gauge-invariantly defined axial current. Hence, the results of the present EDM calculation do not allow to distinguish a spontaneous \( \theta \) from an anomalous \( \theta \) instanton-induced breaking of the axial symmetry (for an example in which the two types of breaking lead to different predictions, see [4]).

In Fig. \( \text{I} \) we report the results of the averages of the baryon density correlators \( (3) \) performed in canonical ensembles with topological charge \( Q = -28, Q = 0, Q = +28 \) (with \( y \) chosen along the \( \hat{z} \) direction ). Calculations in ensembles with other positive and negative total topological charge have also been performed and
The results were found to follow the same trend. Hence, at a qualitative level, Fig. 1A (Fig. 1C) can be regarded as representing all terms $\langle O \rangle_{y}$ in (4) with $Q < 0$ ($Q > 0$). (We choose to plot the results of the topological sectors with a large topological charge, because they have a better signal-to-noise ratio.)

These results have several interesting implications. First of all, we note that the baryon density at zero net topology (Fig. 1B) is even under parity transformation $y_{3} \to -y_{3}$, as expected. On the other hand, contributions from the topologically non-trivial sectors are $P$-odd and lead to a non-vanishing EDM.

The most interesting feature of these results is the separation of positive and negative baryonic charges in physical systems with neutron quantum numbers. Fig. 1A and Fig. 1C imply that in sectors with non-vanishing topological charge, the baryonic charge density of $u$ quarks is positive for $y_{3} > 0$ and negative for $y_{3} < 0$. Conversely, the baryonic charge density of $d$ quarks is negative for $y_{3} > 0$ and positive for $y_{3} < 0$. Notice that this qualitative effect is the same in all positive and negative topological sectors, due to the sign carried by the topological charge factor $Q$ in (7).

The physical interpretation of these results is the following. Any time the quantum QCD ground-state rearranges itself by tunneling, the $\theta$-term generates an interaction which effectively shifts the $u$ quarks and the $d$ anti-quarks along the positive direction of the neutron spin. At the same time, $\bar{u}$ anti-quarks and $d$ quarks are shifted toward the opposite direction. The net effect is the creation of an electric current inside the neutron, pointing in the direction of the spin.

From these correlators we can construct the contribution to the baryonic number density (Fig. 2A). We conclude that the tunneling produces a local separation of the baryonic charge, with an accumulation of matter in the $y_{3} < 0$ hemisphere and of anti-matter in the $y_{3} > 0$ hemisphere. Notice that the contribution to the total baryon charge of the neutron coming from configurations with $Q \neq 0$ vanishes, due to the odd symmetry of the correlators (4) under $y_{3} \to -y_{3}$ transformations. In other words, the neutron total baryon number comes entirely from the topological sector with $Q = 0$. Hence, the baryon and electric charge asymmetries are associated with the sea quarks.

In Fig. 2B we show that the separation of the baryonic charge induced by the $\theta$-term generates a disentanglement of positive and negative electric charge in the neutron. This is the microscopic dynamical mechanism underlying the EDM formation in QCD, at the semi-classical level.

Notice that in the present discussion we have assumed that the quarks in the correlator (3) form a bound-state. Indeed, these arguments hold also for short-sized Euclidean correlators, in which the lowest-lying pole is not isolated from its excitations. This means that the dynamical mechanism analyzed here does not only concern neutrons, but applies to all systems with nucleon quantum numbers. In particular, it should be effective also in the its excited states and in the proximity of the de-confinement phase transition, where the nucleon is melted into its partonic components.

We insist on the fact that the outcome of our analysis does not depend on the particular values of the model parameters which define the ILM. These results rely only on the working assumption that the quantum mixing of the QCD vacuum can be described in terms of isolated tunneling events. On the one hand, there is no a priori guarantee that a semi-classical description may lead to quantitatively realistic predictions. On the other hand, such an approach contains the correct ingredients to point out explicitly the connection between the quantum structure of the vacuum and the effect of CP-breaking interactions. We believe that it is unlikely that further quantum corrections would completely destroy the qualitative mechanism which emerges from the present semi-classical analysis.

An interesting question to ask is how this microscopic
QCD dynamics is translated in the language of a low-energy effective theory, with hadronic degrees of freedom. In [7] Crewther et al. performed the calculation of the lowest-order contribution to the EDM in chiral perturbation theory. In their approach, the EDM arises from the dissociation of the electric charge of the neutron associated with the quantum fluctuation, \( n \rightarrow \pi^- p \) (see Fig. 3).

In order to produce a finite EDM, the neutron must dissociate into charge components, in a way that breaks the spherical symmetry of the electric charge distribution. In particular, in order to generate a positive EDM, the virtual \( \pi^- \) cloud must be mostly localized in the southern hemisphere \( (z < 0) \), relative to the direction of the spin of the nucleon. The same effect is achieved at the microscopic level by the non-perturbative instanton-induced effective repulsion discussed above. In fact, we have seen that the topological fluctuations drive the \( \bar{u} \) and \( d \) sea quarks towards the region \( z < 0 \). (see Fig. 1C).

Let us now discuss some phenomenological implications of the non-perturbative CP-breaking dynamics discussed in this work. One immediate consequence is that \( \theta > 0 \) would imply an asymmetry in the photon-production of charged pions, with an excess of \( \pi^- \) produced in the \( z < 0 \) emisphere. This effect is of the same type of the asymmetries discussed in [12], in the context of relativistic heavy ion collisions.

A second phenomenological implication of the present semi-classical description is that the neutron EDM must have characteristic frequency of oscillation. We have seen that the dissociation of the electric charge is realized through periodic currents, induced by local topological fluctuations in the vacuum. When the \( Q = 0 \) condition is restored, the electric charge distribution is relaxed to its symmetrical equilibrium state. Hence, we expect a natural frequency of oscillations of the EDM, of the order of the inverse of the topological screening length. Such a frequency can be estimated in different ways. For example, in [12] we analyzed some combination of meson point-to-point correlators which relate directly to the amplitude for chirality-flips in a quark-antiquark state induced by topological interactions. From a spectral analysis of such a correlation function it was shown that the information about the initial chiralities of the quark and antiquark is exponentially destroyed in time with a characteristic decay-constant given by \( \tau_\chi \sim \frac{1}{m_{q'}} \sim \frac{1}{m_{a_0}} \) (\( a_0 \) being the lightest \( I = 1, J^P = 0^+ \) meson). Hence, we can argue that the characteristic semi-classical frequency of oscillation of the neutron EDM is of the order \( 1/m_{q'} \) GeV.

A natural consequence is that, for \( \bar{\theta} \neq 0 \), we expect a resonance in the neutron Compton scattering cross section, at center of mass energy \( \sqrt{s} \sim M + m_{q'} \simeq 2 \) GeV. The corresponding final photon will be emitted in a state such that the total parity will not be conserved in the scattering process. The observability of such a resonance depends on the magnitude of \( \bar{\theta} \). Since the measurements of the EDM indicate that this parameter is at least extremely small (if not vanishing), it is hard to imagine that any sizable effect could be seen in a foreseeable scattering experiment. On the other hand, it would be interesting to study what constraints on \( \bar{\theta} \) could be set from Compton scattering data. Such a quantitative analysis would require a model-dependent choice of the instanton size and density parameters and is outside the scope of the present work.

Summarizing, we have studied the dynamical mechanism for EDM formation in QCD, in the presence of a \( \theta \)-term. We have found that the baryon number carried by the sea quarks in the neutron periodically undergoes a local rearrangement. This fact can be interpreted as due to a flavor-dependent quark-antiquark repulsion, triggered by tunneling events in the \( \theta \)-vacuum. This is the microscopic origin of the breaking the spherical symmetry of the electric charge distribution, at the semi-classical level. The same mechanism can be effective in excitations of the nucleon and in the proximity of the QCD phase transition. We have estimated the natural frequency oscillation of the EDM to be of the order of the \( 1/m_{q'} \) and argued on possible phenomenological implications.

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