Branes in the 5D Abelian Higgs Model

P. Dimopoulos\(^a\), K. Farakos\(^b\), C. P. Korthals-Altes\(^b\), G. Koutsoumbas\(^c\) and S. Nicolis\(^d\)

\(^a\)Physics Department, National Technical University, 15780 Zografou Campus, Athens, Greece
\(^b\)CNRS–Centre de Physique Théorique, Luminy, BP 907, 13288 Marseille, France
\(^c\)CNRS (UMR 6083)–Laboratoire de Mathématiques et Physique Théorique, Université de Tours, Parc Grandmont, 37200 Tours, France

We find 3-brane Higgs and Coulomb phases in the 5D Abelian Higgs Model and determine the transition surfaces that separate them from the usual bulk phases.

1. INTRODUCTION

Anisotropy has been shown to be a relevant perturbation for gauge theories.\(^1\) The coupling to fermions allows us to put chiral fermions on the lattice in either the overlap or the domain wall variants.\(^2\) The continuum limit may be taken, since a new phase of “layers”, or 3-branes, appears,\(^3\) and the transition from this phase to the 5D confining phase turns out to be second order.\(^4\)

In the past year we have studied what happens, when scalar fields are included.\(^5\) We have found that an additional “layered” phase appears. It consists of a stack of 3-branes, one lattice spacing apart, that are in the Higgs phase and the fields are confined on each layer. The phase diagram comprises thus of five phases, three bulk and two layered: a confining phase (\(5\)), a bulk Higgs phase (\(H_5\)) a bulk Coulomb phase (\(C_5\)), a layered Coulomb phase (\(C_4\)) and a layered Higgs phase (\(H_4\)). We obtained the phase diagram by Monte Carlo simulations and mean field theory calculations. Indeed the phase of “layers” is found also as a solution of the mean field (i.e. classical) equations of motion.

2. THE MODEL

We use the standard, compact, Abelian Higgs action, with provision made for different couplings along a single (\(5\)) direction from the other four.

\[ S = b_\varphi \sum_x \sum_{1 \leq \mu < \nu \leq 4} (1 - \cos F_{\mu\nu}(x)) + b_\psi \sum_x \sum_{1 \leq \mu \leq 4} (1 - \cos F_{\mu5}(x)) + b_h \sum_x \text{Re}[\varphi^*(x)\varphi(x) - \sum_{1 \leq \mu \leq 4} \varphi^*(x)U_{\mu}(x)\varphi(x + \hat{\mu})] \]

\( + b_h' \sum_x \text{Re}[\varphi^*(x)\varphi(x) - \varphi^*(x)U_{\mu}(x)\varphi(x + \hat{\mu})] \]

\( + \sum_x [(1 - 2b_h - 4b_h' - b^2_h)\varphi^*(x)\varphi(x) + b_h(\varphi^*(x)\varphi(x))^2] \]

The order parameters we will use are the expectation values of the plaquette in the bulk \(P_S\) and the plaquette in the transverse direction, \(P_T\) defined by

\[ P_S = \left\langle \frac{1}{6N^2} \sum_x \sum_{1 \leq \mu < \nu \leq 4} \cos F_{\mu\nu}(x) \right\rangle \]

\( P_T = \left\langle \frac{1}{4N^2} \sum_x \sum_{1 \leq \mu \leq 4} \cos F_{\mu5}(x) \right\rangle \]

and the susceptibility of the link in the bulk

\[ L_S = \frac{1}{4N^2} \sum_x \sum_{1 \leq \mu \leq 4} \cos(\chi(x + \hat{\mu}) + A_{\mu}(x) - \chi(x)) \]

\[ S(L_S) = N^5 \left( \langle L_S^2 \rangle - \langle L_S \rangle^2 \right) \]

where \(\chi(x)\) is the phase of the Higgs field, \(\varphi(x) = \rho(x) \exp(i\chi(x))\) and \(U_{\mu}(x) = \exp(iA_{\mu}(x)).\)

The phase diagram of the 5D, anisotropic, compact \(U(1)\) theory is, displayed, for reference purposes, in fig. 1.\(^5\) Including the Higgs adds
two more dimensions. We fix $\beta_h' = 0.001$ and $\beta_R, \beta_g$ and vary $\beta_h$ and $\beta_g'$. For $\beta_g = 4$ (weak 4d gauge coupling) we find the following “snapshot” cf. fig. 2, while for $\beta_g = 0.5$ (strong 4d gauge coupling) we find the snapshot in fig. 3. As expected, at strong coupling the $C_4$ phase is no longer there. For generic values of the Higgs parameters, $\beta_h$ and $\beta_h'$ we find that the phase transitions are 1st order: we display typical hysteresis loops for the “bulk” and “transverse” plaquettes, $P_S$ and $P_T$ in figs. for the transitions from the bulk confining phase $S$ to the Higgs phases $H_4$ and $H_5$. However, we also find hints of continuous transitions, for some subsets of parameter values—cf. the susceptibility in fig. 3.

Figure 1. Phase diagram of the 5D, anisotropic, compact $U(1)$ theory.

3. CONCLUSIONS-PERSPECTIVES

We have found 3-brane configurations in the 5D, anisotropic, Abelian Higgs model. They may be in either the Coulomb or the Higgs phases. The fields in these configurations are confined on these layers. This confinement is not put in by hand—it is the defining characteristic of the layered phase(s). The transitions between these layered phases and the usual bulk phases are, generically, 1st order; however it is also possible to find subsets in the space of parameter values, that lead to second order transitions. This points to the possibility of new, strongly coupled, continuum theories, whose elucidation is of major interest. It is to be noted that Yang-Mills theories should exhibit $m$-brane configurations with $m > 3$, since they have a Coulomb phase in more than four dimensions. Another alternative, relevant for four
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