Abstract

We present time-dependent exact solutions composed of F-strings intersecting D2-branes in type IIA supergravity. The F-strings tend to approach each other with time in the six-dimensional space transverse to both F-strings and D2-branes. In general, a singularity appears before collision. An exceptional case is that the charges are the same and five directions in the transverse space are smeared out. Then we argue some applications of the solutions in building cosmological models. The possible models are classified based on compactifications of the internal space. All of them give rise to the Friedmann-Robertson-Walker universe with a power-law expansion.
1 Introduction

A fascinating issue in general relativity and string theory is to study time-dependent brane solutions. The time evolution of them is intimately related to the D-brane dynamics in string theory and also there are some potential applications to cosmology and black hole physics. We are concerned here with colliding brane solutions. The original colliding brane solutions have been found by Gibbons et al. [1]. For some generalizations of the solutions, see [1–19]. A remarkable point is that the solutions are time-dependent but still exact and analytic.

The colliding brane solutions are constructed basically by generalizing static brane solutions in supergravities [1–5, 7, 11, 13–19]. As a general property, the solutions give rise to Friedmann-Robertson-Walker (FRW) universes typically by regarding a homogeneous and isotropic part of the solution as our four-dimensional spacetime. Also, some colliding solutions may provide black holes in a FRW universe when three of the transverse directions to the D-branes are regarded as the three spatial directions of our world.

Although the colliding brane solutions have interesting aspects, there are some problems. As a feature of the colliding solutions, for example, a warp factor in the metric is a linear function of time for a nontrivial dilaton. Hence, even in the case of the fastest expansion, the power is too small to exhibit a realistic expansion law as in the matter dominated era or in the radiation dominated era. A possible resolution is to include some additional matter fields.

As another application of time-dependent solutions, one may consider condensed matter systems. An example is a Lifshitz point [20]. The static solutions dual to a Lifshitz point have originally been shown in [21]. The embedding of them into string theory has been carried out in [22, 23]. There exist D3-brane solutions that give rise to the Lifshitz space as a near-horizon geometry [24]. In the previous work [25], we have considered a time-dependent generalization of the solution found in [24]. Then dynamical Lifshitz-type solutions [25] have been constructed with a compactification [23]. The solutions describe the time evolution from the Lifshitz space to an AdS space and it may be interpreted as an aging phenomenon in condensed matter systems.

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1 When the dilaton is constant or there exists no dilaton in the theory as in the eleven-dimensional supergravity, the time-dependence is not restricted to be linear and higher-order dependence is possible.
In this paper, we first generalize the D3-brane solutions constructed in \cite{25} to multi-center D3-brane solutions with multiple waves. Then we consider a T-dual picture of the generalized D3-brane solutions (without the compactification to the Lifshitz space). With a slight extension, we obtain a brane system composed of dynamical F-strings intersecting D2-branes. In the six-dimensional space transverse to both F-strings and D2-branes (called the overall transverse space), the positions of the D2-branes are fixed at some points while F-strings are dynamical and tend to approach each other with time. In general, a singularity appears before collision. An exceptional case is that the F-string charges are the same and five directions of the overall transverse space are smeared out. We also discuss some cosmological aspects of the solutions towards building a realistic cosmological model. The possible models are classified based on compactifications of the internal space. All of them give rise to the FRW universe with a power-law expansion.

This manuscript is organized as follows. In Sec. 2, we first present time-dependent multicenter D3-brane solutions with multiple waves in type IIB supergravity. In Sec. 3 by performing a T-duality, we construct dynamical solutions composed of F-strings intersecting D2-branes in type IIA supergravity. The solutions describe collision of F-strings when the F-string charges are the same and five directions of the six-dimensional transverse space are smeared out. In Sec. 4 we argue cosmological implications of the solutions. We also give a classification of lower-dimensional theories with compactifications. Section 5 is devoted to conclusion and discussion.

2 Time-dependent D3-brane solutions with waves

Let us present time-dependent D3-brane solutions with multicenters and multiple waves in type IIB supergravity.

We restrict ourselves to the solutions that contain the dilaton $\phi$, the axion $\chi$ and the self-dual five-form field strength $F_{(5)}$. Then the relevant field equations are given by

$$
R_{MN} = \frac{1}{2} \partial_M \phi \partial_N \phi + \frac{1}{2} e^{2\phi} F_M F_N + \frac{1}{4 \cdot 4!} F_{MA_2 \cdots A_5} F_{N A_2 \cdots A_5},
$$

$$
F_{(1)} \equiv d\chi, \quad d \left[ e^{2\phi} * F_{(1)} \right] = 0, \quad dF_{(5)} = 0, \quad F_{(5)} = *F_{(5)}.
$$

Here $*$ is the Hodge operator in ten dimensions.
Now let us suppose the following ansatz:

\[
\begin{align*}
ds^2 &= h^{-1/2}(t, x, y^i, z^a) \left[ -\left\{ 2 - h_W(t, x, y^i, z^a) \right\} dt^2 + 2 \left\{ 1 - h_W(t, x, y^i, z^a) \right\} dt dx \\
&\quad + h_W(t, x, y^i, z^a) dx^2 + \gamma_{ij}(Y) \, dy^i dy^j \right] + h^{1/2}(t, x, y^i, z^a) \, u_{ab}(Z) \, dz^a dz^b, \\
\phi &= \phi_0, \\
F_1 &= -\frac{k}{\sqrt{2}} \left( dt - dx \right), \\
F_5 &= (1 \pm *) d \left[ h^{-1}(t, x, y^i, z^a) \land dt \land dx \land \Omega(Y) \right].
\end{align*}
\] (2.2a)

The two-dimensional metric \(\gamma_{ij}(Y)\) and the six-dimensional metric \(u_{ab}(Z)\) depend only on \(y^i (i = 1, 2)\) and \(z^a (a = 1, \ldots, 6)\), respectively. Then \(\phi_0\) and \(k\) are constant. The volume two-form \(\Omega(Y)\) is given by

\[
\Omega(Y) = \sqrt{\gamma} \, dy^1 \land dy^2, \quad \gamma \equiv \det(\gamma_{ij}).
\] (2.3)

The constant \(k\) is interpreted as the momentum of wave on the D3-branes in our later discussion.

The above ansatz reduces the field equations to the following form:

\[
\begin{align*}
R_{ij}(Y) &= 0, \quad R_{ab}(Z) = 0, \\
h &= h(z^a), \quad h_W = w_0(t) + w_1(x) + w_2(y^i, z^a), \\
\Delta_Z h &= 0, \quad \partial_t^2 w_0 = 0, \quad \partial_x^2 w_1 = 0, \quad \Delta_Z w_2 + h \left( \Delta_Y w_2 + \frac{k^2}{2} e^{2\phi_0} \right) = 0,
\end{align*}
\] (2.4a)

where \(\Delta_Y\) and \(\Delta_Z\) denote the Laplace operators on \(Y\) and \(Z\) spaces, respectively.

From now on, we shall focus upon a special case by imposing the conditions,

\[
\gamma_{ij} = \delta_{ij}, \quad u_{ab} = \delta_{ab}, \quad \Delta_Y w_2 = 0, \quad k \neq 0,
\] (2.5)

where \(\delta_{ij}, \delta_{ab}\) are the two-, six-dimensional Euclidean metric, respectively.

Then, by solving the field equations, \(h_W\) and \(h\) are given by

\[
\begin{align*}
h_W(t, x, y^i, z^a) &= a_0 + a_1 t + a_2 x + b_i y^i + \sum_{\alpha=1}^{N} \frac{M_\alpha}{2|z - z_\alpha|^4} \\
&\quad + \frac{k^2}{8} e^{2\phi_0} \sum_{\ell=1}^{N'} \left( \frac{L_\ell}{|z - z_\ell|^2} - \frac{|z - z_\ell|^2}{3} \right), \quad M_\alpha = \frac{(2\pi)^3 g_s^2 N_\alpha l_s^8}{R_x V_Y},
\end{align*}
\] (2.6a)

3
\[ h(z^a) = 1 + \sum_{\ell=1}^{N'} \frac{L_{\ell}}{|z - z_{\ell}|^4}, \quad L_{\ell} = 2^2 \pi \Gamma(2) g_s N_{\ell} l_s^4, \]

(2.6b)

where \( a_0, a_1, a_2, b_i, N_\alpha \) and \( N_\ell \) are constant parameters; \( g_s \) is the asymptotic string coupling constant; \( l_s \) is the string length, related to the string tension \((2\pi \alpha')^{-1}\) as \( \alpha' = l_s^2 / 2 \pi \); \( R_x \) is the radius of \( x \); and \( V_Y \) is the volume of \( Y \). Here \( M_\alpha (\alpha = 1, \cdots, N) \) are masses of traveling waves located at \( z_\alpha \) and \( L_\ell (\ell = 1, \cdots, N') \) are masses of D3-branes located at \( z_\ell \) and the \( x \) direction is compact with period \( 2\pi R_x \).

The distances in the six-dimensional space with \( z^1, \ldots, z^6 \) are defined as

\[
|z - z_{\alpha}| \equiv \sqrt{(z^1 - z_{\alpha}^1)^2 + (z^2 - z_{\alpha}^2)^2 + \cdots + (z^6 - z_{\alpha}^6)^2},
\]

(2.7a)

\[
|z - z_{\ell}| \equiv \sqrt{(z^1 - z_{\ell}^1)^2 + (z^2 - z_{\ell}^2)^2 + \cdots + (z^6 - z_{\ell}^6)^2}.
\]

(2.7b)

The solutions with (2.6a) and (2.6b) describe \( N \)-center D3-branes with \( N' \) waves. The directions along which the D3-branes and the waves extend are listed in Table 1. The time-dependent D3-brane solutions found in \[25\] correspond to the case with \( N = N' = 1 \).

| Branes | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|--------|---|---|---|---|---|---|---|---|---|---|
| D3     | o | o | o | o |   |   |   |   |   |   |
| Wave   | o | * |   |   |   |   |   |   |   |   |
| xN     | t | x | y^1 | y^2 | z^1 | z^2 | z^3 | z^4 | z^5 | z^6 |

Table 1: The D3-branes solutions with waves. The symbols \( o \) and \( * \) denote the directions along which the D3-branes and the waves extend, respectively.

**Simpler solutions with \( k = 0 \)**

It is worth noting a relation to other solutions by considering a special case of the solution with (2.6a) and (2.6b). Let us impose the conditions,

\[
\gamma_{ij} = \delta_{ij}, \quad u_{ab} = \delta_{ab}, \quad \triangle_Y w_2 \neq 0, \quad k = 0.
\]

(2.8)

Then we can obtain simpler solutions given by \[19, 26, 27\]

\[
h_W(t, x, y^i, z^a) = a_0 + a_1 t + a_2 x + \sum_{\beta=1}^{N} M_\beta \left( |y - y_\beta|^2 + L |z - z_0|^2 \right),
\]

(2.9a)

\[
h(z^a) = \frac{L}{|z - z_0|^4}.
\]

(2.9b)
Here $L$ is a constant parameter. The positions of wave and D3-branes are represented by $y_\beta$ and $z_0$, respectively. Then the distance in the two-dimensional space with $y^1$ and $y^2$ is defined as

$$|y - y_\beta| \equiv \sqrt{(y^1 - y^1_\beta)^2 + (y^2 - y^2_\beta)^2}.$$  \hspace{1cm} (2.10)

Thus the solutions with (2.6a) and (2.6b) can be regarded as a generalization of the solution found in [19, 26, 27].

### 3 Dynamical F-strings intersecting D2-branes

We shall consider a T-dual picture of the solutions presented in Sec. 2. The T-dualized solutions describe a brane system which consists of dynamical F-strings intersecting D2-branes. We also discuss collision of F-strings and the near-horizon geometries.

#### 3.1 Time-dependent F-strings intersecting D2-branes

The starting point is the D3-brane solutions with (2.6a) and (2.6b). To perform a T-duality along the $x$ direction, we have to set $a_2 = 0$. Hence $h_W$ does not depend on $x$.

The T-duality relations from type IIB to type IIA are given by

\begin{align}
g^{(A)}_{xx} & = \frac{1}{g^{(B)}_{xx}}, \quad g^{(A)}_{\mu\nu} = g^{(B)}_{\mu\nu} - \frac{g^{(B)}_{x\mu} g^{(B)}_{x\nu} - B^{(B)}_{x\mu} B^{(B)}_{x\nu}}{g^{(B)}_{xx}}, \quad g^{(A)}_{x\mu} = - \frac{B^{(B)}_{x\mu}}{g^{(B)}_{xx}}, \\
e^{2\phi^{(A)}} & = \frac{e^{2\phi^{(B)}}}{g^{(B)}_{xx}}, \quad C_{\mu} = C_{x\mu} + \chi B^{(B)}_{x\mu}, \quad C_{x} = - \chi, \\
B^{(A)}_{\mu\nu} & = B^{(B)}_{\mu\nu} + 2 \frac{B^{(B)}_{x[\mu} g^{(B)}_{\nu]x}}{g^{(B)}_{xx}}, \quad B^{(A)}_{x\mu} = - \frac{g^{(B)}_{x\mu}}{g^{(B)}_{xx}}, \quad C_{x\mu\nu} = C_{\mu\nu} + 2 \frac{C_{x[\mu} g^{(B)}_{\nu]x}}{g^{(B)}_{xx}}, \\
C^{(A)}_{\mu\nu\rho} & = C^{(B)}_{\mu\nu\rho} + 3 \left( C^{(B)}_{x[\mu} B^{(B)}_{\nu\rho]} - B^{(B)}_{x[\mu} C^{(B)}_{\nu\rho]} - 4 \frac{B^{(B)}_{x[\mu} C^{(B)}_{x[\nu} g^{(B)}_{\rho]x}}{g^{(B)}_{xx}} \right), \hspace{1cm} (3.1)
\end{align}

where $x$ is the coordinate to which the T-duality is performed. The indices $\mu$, $\nu$ and $\rho$ denote the other coordinates.

With the relations in (3.1), the type IIA metric is given by

\begin{align}ds^2_{(E)} & = h^{3/8}(z^a) h^{-1/4}_W(t, y^i, z^a) \left[ - h^{-1}(z^a) h^{-1}_W(t, y^i, z^a) dt^2 + h^{-1}_W(t, y^i, z^a) dx^2 \\
& + h^{-1}(z^a) \gamma_{ij}(Y) dy^i dy^j + u_{ab}(Z) dz^a dz^b \right], \hspace{1cm} (3.2a)
\end{align}
\[ C_{(3)} = h^{-1}(z^a) dt \wedge \Omega(Y), \quad B_{(2)} = \left( h^{-1}_W(t, y^i, z^a) - 1 \right) dt \wedge dx, \]
\[ C_{(1)} = -\chi dx, \quad e^{2\theta(A)} = h^{1/2}(z^a) h^{-1}_W(t, y^i, z^a). \]  

(3.2b)

Here \( ds^2_{\text{E}} \) is the ten-dimensional metric with the Einstein frame. Then \( B_{(2)} \), \( C_{(1)} \) and \( C_{(3)} \) are the Neveu-Schwarz-Neveu-Schwarz two-form, the Ramond-Ramond (RR) one-form and the RR three-form, respectively. The field strengths of them are defined as \( H_{(3)} \equiv dB_{(2)} \), \( F_{(2)} \equiv dC_{(1)} \) and \( F_{(4)} \equiv dC_{(3)} \). The resulting metric describes a brane system composed of dynamical F-strings intersecting D2-branes. The directions along which the F-strings and the D2-branes extend are listed in Table 2.

| Branes | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|--------|---|---|---|---|---|---|---|---|---|---|
| D2     | o | o | o |   |   |   |   |   |   |   |
| F1     |   | o | o |   |   |   |   |   |   |   |
| xN     |   | t | x | y^1 | y^2 | z^1 | z^2 | z^3 | z^4 | z^5 | z^6 |

Table 2: Dynamical D2-F1 system in the metric (3.2). The symbol o denotes the directions along which the world volumes extend. The brane configuration obeys the intersection rule.

Let us check the equations of motion as a consistency check and consider a further generalization. First of all, the Bianchi identities for \( H_{(3)} \), \( F_{(2)} \) and \( F_{(4)} \) are automatically satisfied due to the T-duality relations in (3.1). Then the equation of motion for \( H_{(3)} \) is decomposed into a set of the five equations as follows:

\[ \partial_t \partial_i h_W = 0, \quad \partial_x \partial_i h_W = 0, \quad \partial_t \partial_a h_W = 0, \quad \partial_x \partial_a h_W = 0, \quad (3.3a) \]
\[ \triangle Y h_W + h^{-1} \triangle Z h_W = 0. \]  

(3.3b)

Note that the second and the fourth equations in (3.3a) are trivially satisfied because there is no \( x \) dependence now. The equation of motion for \( F_{(2)} \) is reduced to

\[ k d (h h_W) \wedge \Omega(Y) \wedge \Omega(Z) = 0, \]  

(3.4)

where \( \Omega(Y) \) and \( \Omega(Z) \) denote the volume forms, defined as

\[ \Omega(Y) \equiv \sqrt{\gamma} dy^1 \wedge dy^2, \]  

(3.5a)
\[ \Omega(Z) \equiv \sqrt{u} dz^1 \wedge dz^2 \wedge \cdots \wedge dz^6. \]  

(3.5b)

Now that \( \partial_t h_W \neq 0 \) is assumed, the condition (3.4) leads to the constraint \( k = 0 \). This means that \( F_{(2)} = 0 \). This is consistent with the equation of motion for \( H_{(3)} \) given in (3.3).
Finally, from the field equation for $F_{(4)}$, we obtain the following equation:

$$\Delta_Z h = 0.$$ (3.6)

Let us suppose that $\Delta_Y h_W = 0$. Then, from (3.3b), we obtain the relation, $\Delta_Z h_W = 0$.

The solutions of $h$ and $h_W$ are given by, respectively,

$$h_W(t, x, y^i, z^a) = a_0 + a_1 t + a_2 x + b_i y^i + \sum_{\alpha=1}^{N} \frac{M_\alpha}{2|z - z_\alpha|^4}, \quad M_\alpha = \frac{(2\pi l_s)^6 g_s^2 N_\alpha}{4\Omega_5}, \quad (3.7a)$$

$$h(z^a) = 1 + \sum_{\ell=1}^{N'} \frac{L_\ell}{|z - z_\ell|^4}. \quad (3.7b)$$

Here the lengths are defined in (2.7), and the volume of five-sphere with the unit radius is given by $\Omega_5$. The parameters $b_i$ are constant. Then $M_\alpha (\alpha = 1, \cdots, N)$ and $L_\ell (\ell = 1, \cdots, N')$ are mass parameters of F-strings and D2-branes, respectively. The F-strings (D2-branes) are located at $z = z_\alpha (z_\ell)$.

Note that we allowed the solution $h_W$ (3.7a) to depend on $x$ again. In fact, this generalization is possible. The second and the fourth equations in (3.3a) are also satisfied. Thus we will discuss a T-dual picture of the D3-brane solutions with a slight generalization.

### 3.2 Smearing out some of the transverse directions

Let us next see the behavior of the solutions with (3.7a) and (3.7b). First of all, the solutions are time-dependent. It is obvious from the expression of the metric. By putting $h_W$ (3.7a) into the metric (3.2a), the following metric is obtained:

$$ds^2 = h^{3/8}(z^a) \left[ a_0 + a_1 t + a_2 x + b_i y^i + w_2(z^a) \right]^{1/4} \left\{ a_0 + a_1 t + a_2 x + b_i y^i + w_2(z^a) \right\}^{-1}$$

$$\times \left\{ -h^{-1}(z^a) dt^2 + dx^2 \right\} + h^{-1}(z^a) \delta_{ij}(Y) dy^i dy^j + \delta_{ab}(Z) dz^a dz^b, \quad (3.8)$$

where the function $w_2$ is defined by

$$w_2(z^a) = \sum_{\alpha=1}^{N} \frac{M_\alpha}{2|z - z_\alpha|^4}. \quad (3.9)$$

The time dependence appears through the function $h_W$. Hence the next task is to study the time evolution of the solutions carefully. Here we shall consider it by focusing upon collision of F-strings.
Here, in order to decrease the number of transverse dimensions to D2-F1 brane effectively, let us smear out some of the directions in the Z space.

When the number of the smeared direction is given by $d_s$, the functions $w_2$ and $h$ are rewritten as, respectively,

\[ w_2(z^a) = \sum_{\alpha=1}^{N} \frac{M_\alpha}{2|z - z_\alpha|^{4-d_s}}, \]  
\[ h(z^a) = 1 + \sum_{\ell=1}^{N'} \frac{L_\ell}{|z - z_\ell|^{4-d_s}}. \]  
\hspace{1cm} (3.10a, 3.10b)

Hereafter, we will use the smeared solutions.

### 3.3 The behavior of the solutions

Let us see here the asymptotic behavior of the solutions. The harmonic function $w_2$ dominates in the limit of $z \to z_\alpha$ (near a position of F-strings) and the time dependence can be ignored. Thus the system becomes static. On the other hand, in the limit of $|z| \to \infty$, $w_2$ vanishes. Then $h_W$ depends only on time $t$ in the far region from F-strings and the resulting metric is given by

\[ ds^2 = \left( a_0 + a_1 t + a_2 x + b_i y^i \right)^{1/4} \left[ \left( a_0 + a_1 t + a_2 x + b_i y^i \right)^{-1} \right. \left. \{ -h^{-1}(z^a)dt^2 + dx^2 \} + h^{-1}(z^a)\delta_{ij}(Y)dy^idy^j + \delta_{ab}(Z)dz^a dz^b \right]. \]  
\hspace{1cm} (3.11)

The metric becomes singular at $h_W = 0$ or $h = 0$. Therefore the spacetime is regular when it is restricted inside the domain specified by the conditions,

\[ h_W(t, x, y^i, z^a) = a_0 + a_1 t + a_2 x + b_i y^i + w_2(z^a) > 0, \quad h(z^a) > 0. \]  
\hspace{1cm} (3.12)

Here the function $w_2$ is defined in (3.10a). The spacetime cannot be extended beyond this region because the spacetime evolves into a curvature singularity. Note that the regular spacetime with two F-strings ends up with the singular hypersurfaces.

The dynamics of the spacetime depends on the signature of $a_1$. The system with $a_1 > 0$ has the time reversal one of $a_1 < 0$. In the following, we will study the case with $a_1 < 0$. Then the function $h_W$ is positive everywhere for $t < 0$ and the spacetime is nonsingular. In the limit of $t \to -\infty$, the solution is approximately described by a time-dependent uniform
spacetime (apart from a position of branes, near which the geometry takes a cylindrical form of infinite throat).

Let us study the time evolution for $t > 0$. At the initial time $t = 0$, the spacetime is regular everywhere and has a cylindrical topology near each brane. As time slightly goes, a curvature singularity appears as $|z - z_\alpha| \to \infty$. The singular hypersurface cuts off more and more of the space as time goes further. When $t$ continues to increase, the singular hypersurface eventually splits and surrounds each of the F-string throats individually. The spatial surface is composed of two isolated throats. For $t < 0$, the time evolution of the spacetime is the time reversal of $t > 0$.

For any values of fixed $z^a$ in the regular domain, the metric (3.11) implies that the overall transverse space tends to expand asymptotically like $t^{3/8}$. Thus, the solutions describe static intersecting brane systems composed of F-strings and D2-branes near the positions of the branes. On the other hand, in the far region as $|z - z_\alpha| \to \infty$, the solutions approach FRW universes with the power law expansion $t^{3/8}$. The emergence of FRW universes is an important feature of the dynamical brane solutions.

### 3.4 Collision of F-strings

We shall argue whether two F-strings can collide or not. We put the two F-strings at $z_1 = (0, 0, \cdots, 0)$ and $z_2 = (\lambda, 0, \cdots, 0)$, where $\lambda$ is a constant parameter. In addition, we suppose that $N'$ D2-branes are sitting at a point, namely $z^a_1 = \cdots = z^a_{N'} \equiv z^a_0$ and introduce the total mass of the $N'$ D2-branes as

$$
L \equiv \sum_{\ell=1}^{N'} L_{\ell} .
$$

(3.13)

It is helpful to introduce the following quantity:

$$
z_\perp = \sqrt{ (z^2)^2 + (z^3)^2 + \cdots + (z^{6-d_s})^2 } .
$$

(3.14)

Then the proper distance at $z_\perp = 0$ between the two F-strings is represented by

$$
d(t, x, y^i) = \int_0^\lambda dz^{\perp} \left( 1 + \frac{L}{|z^{\perp} - z_0^{\perp}|^{4-d_s}} \right)^{3/16}
\times \left( a_0 + a_1 t + a_2 x + b_i y^i + \frac{M_1}{2|z^{\perp}|^{4-d_s}} + \frac{M_2}{2|z^{\perp} - \lambda|^{4-d_s}} \right)^{1/8} .
$$

(3.15)
Figure 1: Two plots of the proper distance between two F-strings. For both cases, the two charges are identical, $M_1 = M_2 = L = 1$ and the parameters are taken as $a_0 = 0$, $a_1 = -1$, $a_2 = 1$, $b_i = 0$, $z_0 = 0$, $\lambda = 1$. The difference is just the value of $d_s$. The result is also the same and a singularity develops before collision. The plots in (a) and (b) correspond to $d_s = 1$ and $d_s = 2$, respectively.

This is a monotonically decreasing function of time. Here $L$ is the mass of the D2-brane. The behavior of the proper distance is different depending on the number of the smeared directions $d_s$. We shall consider it for each of the values of $d_s$ below.

**The case with $d_s \leq 4$**

Let us consider the case with $d_s \leq 4$. The proper distance is plotted in Fig. 1 for the cases with $d_s = 1$ (left) and $d_s = 2$ (right). Both plots mean that a singularity appears before the proper distance becomes zero. Hence the singularity between two F-strings develops before collision. The two F-strings approach very slowly, and then the singular hypersurface suddenly appears at a finite distance. After that, the spacetime splits into two isolated brane throats. Therefore one cannot see collision of the F-strings in these examples. For the other cases with $d_s = 3$ and $d_s = 4$, the result is the same.

**The case with $d_s = 5$**

The next is to consider the case with $d_s = 5$. We assume that the $z^a$ directions apart from $z^1$ are smeared. The functions $h$ and $h_W$ are linear in $z$. Hence the behavior of the proper distance is different from the previous case. The metric is given by (3.8). By choosing
\( z = z^1 \), the harmonic functions \( h \) and \( w_2(z) \) are given by

\[
w_2(z) = \frac{1}{2} \sum_{\alpha=1}^{N} M_{\alpha} |z - z_{\alpha}|, \tag{3.16}
\]

\[
h(z) = 1 + \sum_{\ell=1}^{N'} L_{\ell} |z - z_{\ell}|. \tag{3.17}
\]

Let us consider the solutions in the case that one F-string charge \( M_1 \) is located at \( z = 0 \) and the other \( M_2 \) at \( z = \lambda \). The proper distance between the two F-strings is represented by

\[
d(t, x, y^i) = \int_0^{\lambda} dz \left[ a_0 + a_1 t + a_2 x + b_i y^i + \frac{1}{2} (M_1 |z| + M_2 |z - \lambda|) \right]^{1/8} \times (1 + L |z - z_0|)^{3/16}. \tag{3.18}
\]

For \( a_1 < 0 \), the proper length decreases with time. In the case that \( M_1 \neq M_2 \), a singularity appears again at a certain finite time \( t = t_c \), while the distance is still finite. Here \( t_c \) is defined as

\[
t_c \equiv -\frac{2a_0 + 2a_2 x + 2b_i y^i + M_1 |z| + M_2 |z - \lambda|}{2a_1} > 0.
\]

This is the same result as the case with \( d_s \leq 4 \).

However, in the same charge case that \( M_1 = M_2 = M \), the distance indeed vanishes at a certain finite time \( t = t_c \), where \( t_c \) is defined as

\[
t_c \equiv -(2a_0 + 2a_2 x + 2b_i y^i + M\lambda)/2a_1.
\]

Hence two F-strings can collide completely.

By using \( t_c \), the proper distance is expressed as

\[
d(t, x) = \frac{16}{19L} \left[ -1 + (1 + \lambda L)^{19/16} \right] [a_1(t - t_c)]^{1/8}. \tag{3.19}
\]

For the values as \( a_0 = 0 \), \( a_1 = -1 \), \( a_2 = 1 \), \( b_i = 0 \), \( z_0 = 0 \), \( \lambda = 1 \) and \( L = 1 \), the proper distance \( d(t, x) \) is plotted in Fig. 2 for the two cases (a) the same charges \( M_1 = M_2 = 1 \) and (b) different charges \( M_1 = 2, M_2 = 1 \). In the case (a) the two F-strings can collide completely, but in the case (b) a singularity appears before collision, as we have already seen analytically.
Figure 2: Two plots of the proper distance between the F-strings. For both cases, the parameters are set as $a_0 = 0$, $a_1 = -1$, $a_2 = 1$, $b_i = 0$, $z_0 = 0$, $\lambda = 1$ and $L = 1$. The difference is the values of the charges. For the plot in (a), $M_1 = M_2 = 1$. Then the complete collision occurs at finite time $t = t_c$. For the plot in (b), $M_1 = 2$ and $M_2 = 1$. Then a singularity suddenly appears before collision again.

3.5 Near-horizon geometry

Finally we shall see the near-horizon geometry of the solutions. When all of the F-strings and D2-branes are located at the origin of the Z spaces, the solutions are rewritten as

$$h_W(t, x, y^i, r) = a_0 + a_1 t + a_2 x + b_i y^i + \frac{M_W}{2r^4},$$

$$h(r) = 1 + \frac{L}{r^3}, \quad r^2 \equiv \delta_{ab} z^a z^b.$$  \hfill (3.20)

(3.21)

Here $L$ is introduced in (3.13) and $M_W$ is the total mass of F-strings

$$M_W \equiv \sum_{\alpha=1}^N M_\alpha.$$  \hfill (3.22)

In the near-horizon region $r \to 0$, the dependence on $t$ and $y^i$ in (3.20) is negligible and hence the metric is reduced to the following form:

$$ds^2 = L^{-\frac{5}{3}} \left( \frac{M_W}{2} \right)^{\frac{1}{3}} r^{-1} \left[ ds^2_{\text{AdS}_2} + \left( L dx^2 + \frac{M_W}{2} (dy^i)^2 \right) \right] \tilde{r}^\frac{2}{3} + \frac{M_W L}{2} d\Omega_5^2,$$  \hfill (3.22)

where $r^3 = \tilde{r}$ has been performed. The line elements of a two-dimensional AdS space (AdS$_2$) and a five-sphere with the unit radius ($S^5$) are given by $ds^2_{\text{AdS}_2}$ and $d\Omega_5^2$, respectively. Thus the metric (3.22) describes a warped product of AdS$_2$ and $S^5$.  

12
A simpler case

Let us consider a simpler case that the spacetime metric and $h_W$ satisfy (2.8). It is convenient to perform a coordinate transformation for $y^i$ and $z^a$ as follows [27]:

$$
y^1 = \frac{1}{r} \cos \theta \cos \alpha, \quad y^2 = \frac{1}{r} \sin \theta \cos \alpha, \quad z^a = \frac{r L^{1/2}}{\sin \alpha} \mu^a. \tag{3.23}
$$

Here the unit vector $\mu^a$ parametrizes $S^5$ and satisfies the following conditions:

$$\mu_a \mu^a = 1, \quad d\Omega^2_{(5)} = d\mu_a d\mu^a. \tag{3.24}
$$

In terms of the new coordinates (3.23), $h_W$ and $h$ are rewritten as

$$h_W(t, x, r) = a_0 + a_1 t + a_2 x + \frac{M_W}{r^2}, \tag{3.25a}
$$

$$h(r) = \frac{\sin^4 \alpha}{L r^4}, \tag{3.25b}
$$

where $M_W$ is a constant parameter and the F-strings are assumed to be at the origin of the Y space and the Z space.

In the near-horizon region $r \to 0$, the metric is described by [19, 27]

$$ds^2 = M_W^{1/4} L^{5/8} (\sin \alpha)^{-5/2} \left[ -\frac{r^4}{M_W} dt^2 + \frac{dr^2}{r^2} + d\alpha^2 + \cos^2 \alpha d\theta^2 + \sin^2 \alpha d\Omega^2_{(5)} + (M_W L)^{-1} \sin^4 \alpha (dx)^2 \right]. \tag{3.26}
$$

This is a warped product of a two-dimensional Lifshitz spacetime with an eight-dimensional internal space. Note that the Lifshitz part can be rewritten as an AdS$_2$ with a coordinate transformation $r^2 = u$.

### 4 Cosmological aspects of the solutions

Let us consider cosmological aspects of the solution describing dynamical F-strings intersecting D2-branes. We first study the time dependence of the scale factors in the solutions. Then, by compactifying the extra directions, our dynamical Universe is discussed.

#### 4.1 The scale factors

First of all, let us see the scale factors in the solutions. It is helpful to introduce a new time coordinate $\tau$ as

$$\left( \frac{\tau}{\tau_0} \right) \equiv (a_1 t)^{5/8}, \quad \tau_0 \equiv \frac{8}{5a_1}. \tag{4.1}
$$
where $a_1 > 0$ is assumed for simplicity. The ten-dimensional metric is thus given by

$$ds_{10}^2 = h^{-\frac{3}{5}} \left[ 1 + \left( \frac{\tau}{\tau_0} \right)^{-\frac{8}{5}} W(x, y^i, z^a) \right]^{-\frac{2}{5}} \left[ -d\tau^2 + \left( 1 + \left( \frac{\tau}{\tau_0} \right)^{-\frac{5}{8}} W(x, y^i, z^a) \right) \left( \frac{\tau}{\tau_0} \right)^{\frac{2}{5}} \right]$$

$$\times \left\{ \delta_{ij}(\tilde{Y})dy^i dy^j + h \left( \delta_{\mu\nu}dz^\mu dz^\nu + \delta_{mn}(\tilde{Z})dz^m dz^n \right) \right\} + \left( \frac{\tau}{\tau_0} \right)^{-\frac{8}{5}} hdx^2 \right], \quad (4.2)$$

where $W(x, y^i, z^a)$ is defined by

$$W(x, y^i, z^a) \equiv a_0 + a_2 x + b_i y^i + \sum_{\alpha=1}^{N} \frac{M_\alpha}{2|z - z^\alpha|^4}. \quad (4.3)$$

Recall that $d_s$ is the number of smeared dimensions and should satisfy $0 \leq d_s \leq 5$. Here we assume that at least one direction of $z^a$ ($a = 1, \ldots, 6$) is not smeared in order to specify the location of our Universe in the transverse space.

Let us consider a simplified case that $(\tau/\tau_0)^{-8/5}W = 0$. This geometry is realized in the limit $\tau \to \infty$. Then the scale factor of the four-dimensional space is proportional to $\tau^{1/5}$. For the remaining spaces $\tilde{X}$ and $\tilde{Y}$, it is proportional to $\tau^{-3/5}$ and $\tau^{1/5}$, respectively. Thus, unfortunately, the solutions do not to give a realistic universe such as an accelerating expansion, a matter dominated era or a radiation dominated era \[32,35\]. However, there is a possibility that appropriate compactification and smearing of the extra directions may lead to a realistic expansion. In the next subsection we will discuss this possibility.

### 4.2 Classifying dynamical universes

The next is to consider some compactification and smearing of the extra directions of the solutions. First of all, our Universe has to be specified in the solutions with ten directions $t, x, y^i, z^a$ ($i = 1, 2, a = 1, \ldots, 6$). The time direction is identified with $t$. The remaining task is to identify the three spatial directions from the other coordinates.

In the usual approach such as the brane-world scenario, three spatial directions are supposed to be on the branes. In the present case, however, it does not work because the spatial directions are specified with $x, y^1$ and $y^2$ and then the space is not isotropic from the expression of the metric. Thus we have to look for another way to realize an isotropic and homogeneous three-dimensional space in the present solutions.

The remaining choice is to take the three-dimensional from the overall transverse space with $z^a$. That is, our Universe is spanned by $t, z^2, z^3$ and $z^4$, for example. The $z^1$ direction
is preserved to measure the position of our Universe in the overall transverse space. Note that the metric depends on $z^a$ explicitly. Hence we have to smear out $z^2$, $z^3$ and $z^4$ so as to define our Universe. Thus the number of the smeared directions $d_s$ should satisfy the bound $3 \leq d_s \leq 5$. The remaining degrees of freedom to be smeared out are $z^5$ and $z^6$.

Now our Universe is described by the coordinates $(t, z^2, z^3, z^4)$. Hence it is convenient to decompose the ten-dimensional metric of the solutions into the following form:

$$ds_{10}^2 = ds_4^2(\tilde{M}) + ds^2(\tilde{X}) + ds^2(\tilde{Y}) + ds^2(\tilde{Z}),$$  \hspace{1cm} (4.4)

where each part of the metric is given by

$$ds_4^2(\tilde{M}) = h^{3/8}(z^a) h_1^{1/4}(t, x, y^i, z^a) \left[-h^{-1}(z^a) h_{-1}^{1/4}(t, x, y^i, z^a) \, dt^2 + \delta_{\mu \nu} dz^\mu dz^\nu \right],$$  \hspace{1cm} (4.5a)

$$ds^2(\tilde{X}) = h^{3/8}(z^a) h_{-3/4}^{1/4}(t, x, y^i, z^a) \, dx^2,$$  \hspace{1cm} (4.5b)

$$ds^2(\tilde{Y}) = h^{-5/8}(z^a) h_1^{1/4}(t, x, y^i, z^a) \delta_{ij}(\tilde{Y}) \, dy^i dy^j,$$  \hspace{1cm} (4.5c)

$$ds^2(\tilde{Z}) = h^{3/8}(z^a) h_{1/4}^{1/4}(t, x, y^i, z^a) \delta_{mn}(\tilde{Z}) \, dz^m dz^n.$$  \hspace{1cm} (4.5d)

Here $ds_4^2(\tilde{M})$ is the metric of the four-dimensional spacetime with $t, z^\mu$ ($\mu = 2, 3, 4$). The space $\tilde{X}$ is one-dimensional with $x$. The space $\tilde{Y}$ is described by $y^i$ ($i = 1, 2$) and the space $\tilde{Z}$ is three-dimensional with $z^m$ ($m = 1, 5, 6$). Then $\delta_{\mu \nu}, \delta_{ij}, \delta_{mn}$ are the three-, two- and three-dimensional Euclidean metrics, respectively.

The next task is to derive the lower-dimensional effective theory by compactifying the extra directions. It is necessary to take that $a_0 = a_2 = b_i = 0$ in (4.2) to compactify the $x$ and $y^i$ directions. To find a realistic universe, let us compactify $d = (d_{\tilde{X}} + d_{\tilde{Y}} + d_{\tilde{Z}})$-dimensional space to be a $d$-dimensional torus, where $d_{\tilde{X}}, d_{\tilde{Y}}$ and $d_{\tilde{Z}}$ are the compactified dimensions for the direction of $x$, $\tilde{Y}$ and $\tilde{Z}$ spaces, respectively. The range of $d_{\tilde{Z}}$ is given by $0 \leq d_{\tilde{Z}} \leq 2$ because the $z^1$ direction is preserved to measure the position of the Universe in the overall transverse space. That is, the $z^5$ and $z^6$ directions may be compactified, where the direction to be compactified has to be smeared out before the compactification. On the other hand, for the compactification of the $x$ and $y^i$ directions, the smearing is not needed because we have set that $a_0 = a_2 = b_i = 0$.

We shall classify the compactifications of the solutions depending on the variety of the internal space. The remaining noncompact space is referred to as the external space. Then the metric (4.4) is recast into the following form:

$$ds_{10}^2 = ds_{\tilde{c}}^2 + ds_{\tilde{t}}^2,$$  \hspace{1cm} (4.6)
where $ds^2_e$ is the metric of $(10 - d)$-dimensional external spacetime and $ds^2_i$ is the metric of compactified dimensions.

Recall here that our four-dimensional spacetime should be described in the Einstein frame. The Einstein frame metric is obtained by multiplying the extra factor and the resulting metric, which leads to new exponents, is given by

$$
\begin{align*}
    ds^2_e &= h^\xi \left[ 1 + \left( \frac{\tau}{\tau_0} \right)^{-\frac{2}{\beta + 2}} W(z^a) \right]^{\beta} \left[ -d\tau^2 + \left( 1 + \left( \frac{\tau}{\tau_0} \right)^{-\frac{2}{\beta + 2}} W(z^a) \right) \left( \frac{\tau}{\tau_0} \right)^{2(\beta + 1)/(\beta + 2)} \right] \\
    &\times \left\{ \delta_{ij}(\tilde{Y'}) dy^i dy^j + h \left( \delta_{\mu\nu} dz^\mu dz^\nu + \delta_{mn}(\tilde{Z'}) dz^m dz^n \right) \right\} + \left( \frac{\tau}{\tau_0} \right)^{\frac{2\beta}{\beta + 2}} hdx'^2,
\end{align*}
$$

where $ds^2_e$ is the $(10 - d)$-dimensional metric in the Einstein frame. Here $x'$ and $Y'$ are the coordinates of $(1 - d_{\tilde{X}})$- and $(2 - d_{\tilde{X}})$-dimensional relative transverse spaces, respectively. Finally $Z'$ denotes $(3 - d_{\tilde{Z}})$-dimensional spaces.

The constant parameters $\xi$, $\beta$, $\tau_0$ and the cosmic time $\tau$ are defined as

$$
\begin{align*}
    \xi &\equiv \frac{1}{8 - d} (d - d_{\tilde{X}} - 5), & \beta &\equiv \frac{1}{8 - d} (d - d_{\tilde{X}} - 6), \\
    \frac{\tau}{\tau_0} &\equiv (a_1 t)^{\frac{\beta + 2}{2}}, & \tau_0 &\equiv \frac{2}{(\beta + 2) a_1}.
\end{align*}
$$

The power of $M$ of the solutions is given by

$$
\lambda_E \equiv \frac{\beta + 1}{\beta + 2} < 1 \quad \text{for} \quad 8 > d \quad \text{and} \quad d > 0.
$$

In Table 3, we list the power exponent of the fastest expansion of our $(10 - d)$-dimensional Universe in the Einstein frame. Unfortunately, every exponent in Table 3 is so small that the solutions do not give rise to our expanding Universe. To realize a realistic cosmological model such as in the inflationary scenario, it would be necessary to add some new ingredients.

## 5 Conclusion and discussion

We have constructed gravitational solutions composed of dynamical F-strings intersecting D2-branes in type IIA supergravity and studied the time evolution focusing upon the collision of F-strings. We also have discussed some applications to cosmology.
| $10 - d$ | External spacetime | Internal space | $\lambda_E$ |
|---------|-----------------|----------------|-----------|
| 9       | $\tilde{M} & \tilde{X}' & \tilde{Y}' & \tilde{Z}'$ | $(d_{\tilde{X}}, d_{\tilde{Y}}, d_{\tilde{Z}}) = (0, 0, 1)$ | 2/9       |
| 9       | $\tilde{M} & \tilde{X}' & \tilde{Y}' & \tilde{Z}'$ | $(d_{\tilde{X}}, d_{\tilde{Y}}, d_{\tilde{Z}}) = (0, 1, 0)$ | 2/9       |
| 9       | $\tilde{M} & \tilde{Y}' & \tilde{Z}'$ | $(d_{\tilde{X}}, d_{\tilde{Y}}, d_{\tilde{Z}}) = (1, 0, 0)$ | 1/8       |
| 8       | $\tilde{M} & \tilde{X}' & \tilde{Y}' & \tilde{Z}'$ | $(d_{\tilde{X}}, d_{\tilde{Y}}, d_{\tilde{Z}}) = (0, 0, 2)$ | 1/4       |
| 8       | $\tilde{M} & \tilde{X}' & \tilde{Y}' & \tilde{Z}'$ | $(d_{\tilde{X}}, d_{\tilde{Y}}, d_{\tilde{Z}}) = (0, 1, 1)$ | 1/4       |
| 8       | $\tilde{M} & \tilde{Y}' & \tilde{Z}'$ | $(d_{\tilde{X}}, d_{\tilde{Y}}, d_{\tilde{Z}}) = (1, 0, 1)$ | 1/7       |
| 8       | $\tilde{M} & \tilde{X}' & \tilde{Z}'$ | $(d_{\tilde{X}}, d_{\tilde{Y}}, d_{\tilde{Z}}) = (0, 2, 0)$ | 1/4       |
| 8       | $\tilde{M} & \tilde{Y}' & \tilde{Z}'$ | $(d_{\tilde{X}}, d_{\tilde{Y}}, d_{\tilde{Z}}) = (1, 1, 0)$ | 1/7       |
| 7       | $\tilde{M} & \tilde{X}' & \tilde{Y}' & \tilde{Z}'$ | $(d_{\tilde{X}}, d_{\tilde{Y}}, d_{\tilde{Z}}) = (0, 1, 2)$ | 2/7       |
| 7       | $\tilde{M} & \tilde{X}' & \tilde{Y}' & \tilde{Z}'$ | $(d_{\tilde{X}}, d_{\tilde{Y}}, d_{\tilde{Z}}) = (1, 0, 2)$ | 1/6       |
| 7       | $\tilde{M} & \tilde{X}' & \tilde{Z}'$ | $(d_{\tilde{X}}, d_{\tilde{Y}}, d_{\tilde{Z}}) = (0, 2, 1)$ | 2/7       |
| 7       | $\tilde{M} & \tilde{Y}' & \tilde{Z}'$ | $(d_{\tilde{X}}, d_{\tilde{Y}}, d_{\tilde{Z}}) = (1, 1, 1)$ | 1/6       |
| 7       | $\tilde{M} & \tilde{Z}'$ | $(d_{\tilde{X}}, d_{\tilde{Y}}, d_{\tilde{Z}}) = (1, 2, 0)$ | 1/6       |
| 6       | $\tilde{M} & \tilde{X}' & \tilde{Z}'$ | $(d_{\tilde{X}}, d_{\tilde{Y}}, d_{\tilde{Z}}) = (0, 2, 2)$ | 1/3       |
| 6       | $\tilde{M} & \tilde{Y}' & \tilde{Z}'$ | $(d_{\tilde{X}}, d_{\tilde{Y}}, d_{\tilde{Z}}) = (1, 1, 2)$ | 1/5       |
| 5       | $\tilde{M} & \tilde{Z}'$ | $(d_{\tilde{X}}, d_{\tilde{Y}}, d_{\tilde{Z}}) = (1, 2, 2)$ | 1/4       |

Table 3: The classification of compactifications of the transverse directions and the maximum value of the power exponent $\lambda_E$ for $\tilde{M}$. The exponent is measured in the Einstein frame.
First, we have obtained time-dependent multicenter D3-brane solutions with multiple waves in type IIB supergravity by generalizing the solutions found in [25]. Then we have obtained the solutions composed of dynamical F-strings intersecting D2 branes by using a T-duality. The solutions can also be obtained from supersymmetric static solutions by replacing a constant in the harmonic function with a linear function of time. This procedure is applicable universally to construct a class of time-dependent brane solutions.

It is worth commenting on the time dependence of the solutions. The solutions admit F-strings to be dynamical, but D2-branes have to be static. The origin of this restriction is that the ansatz we imposed is too restrictive. In fact, in recent works on similar systems, both harmonic functions may depend on time [17,18]. We have also discussed collision of F-strings. Two F-strings approach each other with time. In general, a singularity appears before collision. An exceptional case is that five directions in the overall transverse space are smeared out.

The dynamical solutions give rise to a singularity when $h_W(t, x, y^i, z^a) = 0$. The appearance of the singularity indicates the two possibilities: (1) the ansatz is simply wrong or (2) just the supergravity approximation is not valid and it is necessary to include some stringy corrections from $\alpha'$ or $g_s$. It is difficult to decide which of them applies to the present case only from the supergravity perspective. It would be interesting to consider some generalizations to include stringy corrections.

We have also argued cosmological models obtained from the solutions by compactifying the internal space. The resulting FRW universes exhibit a power-law expansion. However, the power of the scale factor is less than $1/2$ and hence it is too small to realize a realistic expansion. To obtain a realistic expanding universe such as an inflationary scenario or matter or a radiation dominated era, we have to include additional matter fields. Thus it is interesting to try to construct a cosmological model by studying a more complicated setup.

After all, the solutions constructed here do not give a realistic cosmological model. However, we believe that more elaborated construction along this line would open a new window to realize realistic cosmological expansions in the context of string theory.
Acknowledgments

This work was also supported in part by the Grant-in-Aid for the Global COE Program “The Next Generation of Physics, Spun from Universality and Emergence” from MEXT, Japan.

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