Engagement Maximization

Benjamin Hébert (Stanford and NBER) and Weijie Zhong (Stanford)*

July 12, 2024

Abstract

We study a Bayesian agent receiving signals over time and then acting. The agent chooses when to stop and act, and prefers to act earlier all else equal. The signals are determined by a principal, who maximizes engagement (the total attention paid by the agent). We show that engagement maximization minimizes the agent’s welfare, induces excessive information acquisition (relative to an agent-optimal benchmark), and leads to extreme beliefs. The principal optimally sends only “suspensive signals” that lead the agent to become “less certain than the prior” and “decisive signals” that lead the agent to stop immediately.

Key Words: Information Acquisition, Recommendation Algorithms, Polarization, Rational Inattention

JEL Codes: D83, D86

*The authors would like to thank Peter DeMarzo, Sebastian Di Tella, Darrell Duffie, Emir Kamenica, and Michael Woodford for helpful comments, and Ian Ball for an excellent discussion. All remaining errors are our own. bhebert@stanford.edu and weijie.zhong@stanford.edu
1 Introduction

Free-to-use online platforms such as Facebook, Instagram, Youtube, and Pinterest are used by billions of people worldwide and earn substantial profits by displaying advertisements to their users. Their business models are powered by personalized recommendation algorithms that seek to maximize the “engagement” of each user by selectively displaying content (Lada et al. [2021], Sequoia [2018]). Large quantities of computing and other resources have been invested by these firms to develop algorithms that can predict billions of users’ preferences in real time.

A key challenge for these algorithms is to manage users’ incentives. Content is presented sequentially in “news feeds,” “recommendations,” or “timelines,” and users can choose freely what to pay attention to and when to stop using the platform based on the entire history of content presented thus far. From the platform’s perspective, providing the most useful content immediately is suboptimal, as a user might become satisfied and choose to stop using the platform, limiting the quantity of advertisements the platform can display to the user. On the other hand, if the user anticipates that the platform will never provide useful content, they will never begin to use the platform in the first place.

In this paper, we study the optimal design of sequential information presentation from a principal-agent perspective. The principal (the platform) provides the agent (the user) with information. The agent derives value from the information, and chooses when to stop engaging with the platform. The rate at which the agent can process information is limited, and the agent experiences an opportunity cost of time spent on the platform net of any utility from using the platform. The principal’s goal is to maximize the attention the agent allocates to the platform (which under some conditions will be equivalent in equilibrium to maximizing the stopping time), and the principal accomplishes this by choosing the nature of the information provided to the agent. The key modeling assumptions we impose are that the principal knows perfectly the agent’s preferences and that the principal can flexibly manipulate the entire information flow. These assumptions capture the state platforms aim to reach with unlimited data and computing power.

Our main result is the characterization of an optimal strategy for the principal. First, we completely characterize the optimal “overall” information structure (i.e. the signal-state joint distribution that is unconditional on time). It is characterized by the solution to an

---

1We show that it is without loss of generality to assume the agent does not process the information selectively; that is, the agents attends to the information the principal provides in equilibrium.
augmented static rational inattention (RI) problem: the information structure maximizes a linear combination of the instrumental value of information and the measure of information with endogenous weights. Second, we identify one optimal sequential information structure for the principal: the principal sends a “dilution” of the overall information structure (a compound Poisson process such that a signal arrives at a Poisson rate and upon arrival, the signal is distributed according to the overall information structure).

Moreover, we show that any optimal strategy has the following features:

- **Agent welfare minimization**: Under the principal’s optimally chosen information structure and the agent’s optimal stopping policy, the agent engages substantially with the platform but is no better off than if she did not use the platform and made her decision without acquiring any information. That is, engagement maximization by the principal minimizes the welfare of the agent. The agent’s ability to stop using the platform at any time does not allow her to extract any surplus. This result depends critically on the principal’s ability to send arbitrary signals to the agent; we provide an example in which the principal can choose from a more limited set of signals and show that such limits can allow the agent to extract positive surplus.

- **Belief polarization**: When the agent chooses to stop and act after engaging with the platform, the agent will hold extreme beliefs, relative to a benchmark model in which the agent can choose both what information to receive and when to stop and act.

- **Learning paths**: An optimal strategy contains (up to) two types of signals almost surely. The first type are *decisive* signals that are so informative that the agent makes a decision immediately after receiving the signal. The second type are *suspensive* signals that causes the agent’s beliefs to move to a posterior that is “more uncertain than the prior”.

After presenting our main results of welfare-minimization and belief polarization, we consider several extensions of our baseline model. First, we consider the possibility that the agent does not fully attend to the signals sent by the principal. We show that it is without loss of generality to assume the agent pays full attention to the signals provided. Second, we show that the usual Bayesian persuasion assumption of commitment to a signal structure is without loss in our setting. Third, we consider the case in which there is an increasing opportunity cost of time and/or decreasing marginal utility from using the platform. This case is less tractable; to make progress, we impose additional symmetry assumptions, and
derive conditions under which our main results continue to hold. Fourth, we discuss the importance of the role of jumps in beliefs in our model. The optimal belief process in our baseline model will involve rare but informative signal realizations that cause beliefs to jump. This result speaks to the sensational nature of the information that the principal optimally provides the agent. To clarify this point, we extend our analysis to the case in which the size of jumps in beliefs is arbitrarily restricted. We characterize how the restriction on jump size restricts the set of implementable strategies, and show that under certain circumstances, if beliefs must follow a continuous process, the principal and agent are in fact fully aligned and it is instead the agent who receives the full surplus. Fifth, we consider a modification of the model in which the user has unlimited information processing capacity and the platform maximizes time spent (as opposed to attention). We show that some of our results continue to apply to this modified model. Lastly, we apply our framework to a seemingly quite different setting, in which a teacher seeks to maximize the engagement of a student who cares only about passing a test.\footnote{We are grateful to Emir Kamenica for suggesting this alternative setting as an application of our model.}

1.1 Example

We illustrate the main model and the logic of our main result in a simple example. An agent faces a choice between two actions, \( A = \{l, r\} \). The payoffs from the actions are uncertain and depend on the state of the world \( x \in X = \{L, R\} \). The agent assigns equal prior probability to both states. The agent gets utility one when the chosen action matches the state (\( l|L \) or \( r|R \)) and utility negative one otherwise (\( l|R \) or \( r|L \)). The agent is impatient and pays a constant cost of delay of two utils per unit of time.

Information & capacity constraint:

For this example, we restrict attention to the following type of signals. The signal has two possible realizations \( \{s_l, s_r\} \), which arrives at Poisson rates that depend on the state (symmetrically):

|       | L  | R     |
|-------|----|-------|
| \( s_l \) | \( \alpha \cdot p \) | \( \alpha \cdot (1 - p) \) |
| \( s_r \) | \( \alpha \cdot (1 - p) \) | \( \alpha \cdot p \) |

The parameters \( \alpha \in \mathbb{R}_+ \) and \( p \in \left[ \frac{1}{2}, 1 \right] \) represents the arrival frequency and precision of the signal, respectively. In the context of an internet platform sequentially presenting
news articles, tweets, or other kinds of content to a user, we will think the platform as presenting content to the user at a constant rate. The parameter $\alpha$ represents the likelihood that the content presented over an instant time is relevant to the decision facing the user. The parameter $p$ represents the depth or informativeness of the content conditional on its relevance.

We assume that the agent is limited in her information processing capacity such that $\alpha$ and $p$ must satisfy

$$\alpha \cdot \left( H^S \left( \frac{1}{2} \right) - H^S(p) \right) \leq 1,$$

where $H^S(\cdot)$ is Shannon’s entropy. Note that $H^S \left( \frac{1}{2} \right) - H^S(p)$ is an increasing and convex function of $p$. Intuitively, both more frequent and more precise signals require processing “more information.” This constraint, which is a specialized version of the standard mutual information constraint used in the rational inattention literature, creates a trade-off between the frequency and the precision of signals. Formally, the constraint is equivalent to requiring that the mutual information of the state and signals received up to any time $t + \delta$, conditional on the signal history up to time $t$, is bounded by $\delta$.

In the context of an internet platform sequentially presenting content to a user, we interpret this constraint as a breadth-vs.-depth trade-off. A news feed can initially present articles on a broad range of potentially relevant topics, increasing the likelihood that one is relevant to the user. Alternatively, a news feed can concentrate its initial coverage on a narrower range of topics, increasing its informativeness (conditional on relevance) but risking the possibility that none of the initial articles are relevant for the user. The former corresponds to a high $\alpha$, low $p$ strategy, the latter to a low $\alpha$, high $p$ strategy. The platform can choose between these strategies, but cannot present in-depth information about a large range of topics in a short period of time and expect the user to process all of the information.

**The agent-optimal strategy:**

Consider the signal with $p = \frac{e}{1+e}$ and $\alpha = \frac{1}{H^S(\frac{1}{2}) - H^S(p)}$. This signal is feasible and the capacity constraint is exactly binding. Suppose the signal arrives: it is straightforward to show that the agent should stop and follow the recommendation of the signal—choosing $l$ ($r$) upon the arrival of $s_l$ ($s_r$). The agent’s expected utility at time zero under this strategy is

$$V^B = \frac{e - 1}{1+e} - \frac{2\alpha^{-1}}{\text{Expected Cost of Delay}} \approx 0.24.$$
We now explain why the agent is willing to wait for the arrival of the signal. If the agent has not observed either signal by time $t$, her beliefs will remain equal to her prior (and hence assign equal probability to each state). As a result, if the agent were to stop and act before observing either of the Poisson signals, her expected payoff would be zero. Because the problem is stationary, $V^B$ is also the agent’s expected utility at any time $t > 0$, conditional on not having received either Poisson signal yet; it follows that the agent will strictly prefer waiting for the arrival of one of these signals to stopping. In fact, the results of Hébert and Woodford [2023] show that this signal structure and stopping strategy is optimal for the agent, both within the class of signals considered in this example and within the larger class of signals satisfying the mutual information bound.

The principal’s payoff depends on the total amount of information acquired by the agent, which we refer to as engagement. For the purpose of this example, we use Shannon’s entropy to both measure engagement and govern the rate at which the agent can process information. Our more general analysis will consider cases in which the principal’s engagement measure and the agent’s processing constraints differ. Under the proposed strategy, because the information capacity constraint always binds, the expected engagement is $\alpha^{-1}$, the expected decision time.

**A better (but not optimal) strategy for the principal:**

Let us now consider whether there are signals that are better for the principal, exactly satisfy the agent’s information processing constraint, and which the agent is willing to receive. Because such signals exactly satisfy the information processing constraint, maximizing engagement requires increasing the expected decision time. Within the class of Poisson signals, this is only possible if the signals become less frequent but more informative. Such signals are less desirable for the agent than the agent-optimal signals (by definition); but as established above, the agent strictly benefits from receiving the agent-optimal signals, and therefore might be willing receive somewhat less desirable signals as well.

Specifically, consider the signal with $\tilde{p} = \frac{4e}{1+4e}$ and $\tilde{\alpha} = \frac{1}{H^2(\frac{1}{2}) - H^2(\tilde{p})}$. Evidently, the signal is more precise conditional on arrival ($\tilde{p} > p$) but arrives less frequently ($\tilde{\alpha} < \alpha$). The signal exactly satisfies the information processing constraint. If the agent still stops and follows the recommendation of the signal upon signal arrival, her expected utility will be

$$\tilde{V}^B = \frac{4e - 1}{1 + 4e} - \frac{2\tilde{\alpha}^{-1}}{\text{Expected Cost of Delay}} \approx 0.02 > 0.$$
From the agent’s perspective, waiting for this signal is only slightly better than stopping—but it is strictly better, and by the same stationarity arguments given above, the agent will be willing to wait.

Using these less frequent, more informative signals allows the principal to increase expected engagement ($\bar{\alpha}^{-1} > \alpha^{-1}$) at the agent’s expense. It is straightforward in this example to see that using an even less frequent, even more informative signal would allow the principal to drive the agent to her indifference point (welfare minimization) while further increasing expected engagement. Our results will demonstrate that this is in fact an optimal strategy within a much more general class of sequential information structures. Note that utility is not transferable in this example (or in our general model); thus, the necessity of welfare minimization under the principal’s optimal strategy is not immediate from the setup of the model, and in fact will not hold when the principal is restricted to a subset of strategies (see Section 5).

This example illustrates the forces behind our main result. By assumption, the principal benefits from sending the agent signals that the agent will attend to, as it is assumed that this is what generates revenue for the principal. The agent perceives delay as costly, and will only continue to attend to the principal’s signals if she values the information being provided. The agent will in general be willing to receive “too much” information, relative to what she would choose for herself, provided that this still leaves her with some surplus. The principal can take advantage of this willingness by sending signals that always result in the agent either learning too much (generating what we call “extreme beliefs”) or leaving beliefs unchanged. These signals generate larger expected stopping times, and thus more revenue for the principal. In the context of an internet platform providing content sequentially to a user, it is optimal for the platform to provide a stream of mostly-irrelevant pieces of content, a few of which will be highly informative, as opposed to providing a stream of brief and somewhat-relevant content.

1.2 Related Literature

Our paper contributes to several strands of literature on the dynamic provision of information. Viewing our principal as a media company, our model is related to work on models of media bias (see [Gentzkow et al. 2015] for a survey). We share with [Kleinberg et al. 2022] an interest in explaining why the users of internet platforms would engage heavily with those platforms while perceiving themselves as gaining little from doing so. We
derive this outcome as a result of strategic behavior by rational agents with conflicting incentives; those authors emphasize the time-inconsistency of user preferences. We share with Acemoglu et al. [2021] an emphasis on explaining what kind of information is available on internet platforms; our analysis focuses on content selection algorithms, whereas their analysis focuses on information sharing between users.

Closely related to our work is the literature on dynamic Bayesian persuasion (e.g. Ely [2017], Renault et al. [2017], Ely and Szydlowski [2020], Orlov et al. [2020], Che et al. [2020]) that build on the static model of Kamenica and Gentzkow [2011]. Our setting differs from most of these papers in that our principal’s payoff solely depends on the total attention paid by the agent, while the agent obtains pure instrumental value of information from solving a general decision problem. Most of these papers focus on the “Bayesian persuasion” settings where the principal’s payoffs directly depend on the agent’s action and the maximization of “engagement” is a side effect. Two exceptions are Knoepfle [2020] and Koh and Sanguanmoo [2022], who also focus on maximizing “engagement” or “attention,” but without an information processing constraint. In both papers, the lack of an information constraint leads to the optimal persuasion strategy fully revealing the state. Our analysis emphasizes the beliefs the agent will arrive at, and how these differ between the principal’s optimal strategy and an agent-optimal benchmark, a comparison that is not possible absent information processing constraints. We elaborate further on the connection between our model and these models in Section 5.5. We assume both players are long-lived and have commitment power, in contrast to the limited commitment settings in Orlov et al. [2020], Che et al. [2020] or the myopic agent settings in Ely [2017], Renault et al. [2017]. Interestingly, unlike the findings in these papers, neither commitment power nor forward-lookingness of the agent is necessary for sustaining our equilibrium strategy (see detailed discussions in Section 4).

Our model predicts gradual information revelation over time, which is a feature shared by many of the dynamic information design models (e.g. Ely et al. [2015], Hörner and Skrzypacz [2016], Che et al. [2020], Orlov et al. [2020]). However, unlike these papers, the gradual nature of belief evolution in our model arises from the agent’s information processing constraint, as opposed to a desire to maximize suspense or address problems of

---

3 Our model is equivalent to one where the principal’s only goal is to ensure the agent continues to pay attention, as in Kawamura and Le Quement [2019]. However, our principal presents unbiased information, and hence is not engaged in cheap talk (Crawford and Sobel [1982], Cheng and Hsiaw [2022]).

4 Knoepfle [2020]’s main focus is on the competition between multiple senders. The single sender case of Knoepfle [2020] is the closest to ours.
limited commitment. In particular, Che et al. [2020] predicts that the agent’s belief involves Poisson jumps and a drift, while our optimal strategy admits Poisson jumps without a drift.

Formally, our approach is a principal-agent version of Hébert and Woodford [2023]. Those authors consider a model in which a single decision maker chooses both what information to acquire and when to stop and act, whereas in our model the principal chooses the information and the agent chooses when to stop and act. We compare our model to a benchmark in which the agent chooses both the information and when to stop and act; this benchmark is characterized by results found in Hébert and Woodford [2023]. We follow Hébert and Woodford [2023] in assuming that the principal can choose any stochastic process for the agent’s beliefs, subject only to the martingale requirement (which is imposed by Bayesian updating) and the upper bound on the agent’s attention. We model this upper bound using a “uniformly posterior-separable” information cost, in the terminology of the rational inattention literature [Caplin et al. 2022].

The rest of the paper is organized as follows. We begin in section 2 by describing the basic environment of our model. Section 3 characterizes optimal policy in our baseline model. Section 4 discusses the key implications, welfare minimization and belief polarization. Section 5 discusses several extensions of our baseline model, and in Section 6 we conclude.

2 The Environment

2.1 The Agent’s Problem

We study the problem of a rational, Bayesian agent receiving signals about an underlying state for the purpose of taking an action. We model information acquisition as a continuous time process, building on results in Hébert and Woodford [2023] and Zhong [2022].

Let $X$ be a finite set of possible states of nature. The state of nature is determined ex ante, does not change over time, and is not known to the agent. Let $q_t \in \mathcal{P}(X)$ denote the agent’s beliefs at time $t \in [0, \infty)$, where $\mathcal{P}(X)$ is the probability simplex defined on $X$. We will represent $q_t$ as vector in $\mathbb{R}^{|X|}_+$ whose elements sum to one, each of which corresponds to the likelihood of a particular element of $X$, and use the notation $q_{t,x}$ to denote the likelihood under the agent’s beliefs at time $t$ over the true state being $x \in X$.

---

5 Examples of such information costs include mutual information, as applied in Sims [2010], as well as other proposed alternatives (Hébert and Woodford [2021], Bloedel and Zhong [2020]).
At each time $t$, the agent can either stop and choose an action from a finite set $A$, or continue to acquire information. Let $\tau$ denote the time at which the agent stops and makes a decision, with $\tau = 0$ corresponding to making a decision without acquiring any information. The agent receives utility $u_{a,x}$ if she takes action $a$ and the true state of the world is $x$, and pays a flow cost of delay per unit time, $\bar{\kappa} > 0$, until an action is taken. This flow cost captures the opportunity cost of the agent’s time net of whatever utility is gained using the principal’s platform. In [5.3], we provide an extension where there is an increasing time-dependent flow cost of delay $\kappa(t)$.

Let $\hat{u}(q')$ be the payoff (not including the cost of delay) of taking an optimal action under beliefs $q' \in P(X)$:

$$\hat{u}(q') = \max_{a \in A} \sum_{x \in X} q'_x u_{a,x}.$$ 

In what follows, the convex function $\hat{u} : P(X) \rightarrow \mathbb{R}$ will summarize the value the agent places on information. This function, as opposed to the utility function $u_{a,x}$, can be viewed as the primitive “value of information when acting” in our model; nothing in our analysis will depend on the nature of the action space $A$ or on the predicted joint distribution of states and actions. As a result, our analysis extends without modification to the case in which the agent values information for its own sake (i.e. for non-instrumental reasons).

The agent’s beliefs, $q_t$, will evolve as a martingale. This property follows from “Bayes-consistency.” In a single-period model, Bayes-consistency requires that the expectation of the posterior beliefs be equal to the prior beliefs. The continuous-time analog of this requirement is that beliefs must be a martingale.

Formally, let $\Omega$ be the set of $P(X)$-valued càdlàg functions, let $q : \Omega \times \mathbb{R}_+ \rightarrow P(X)$ be the canonical stochastic process on this space, let $\{\mathcal{F}_t\}$ be the natural filtration associated with this canonical process, and let $\mathcal{F} = \lim_{t \to \infty} \mathcal{F}_t$. Let $\mathcal{I} \subset \Omega \rightarrow \mathbb{R}_+$ be the set of non-negative stopping times with respect to $\{\mathcal{F}_t\}$.

Given a probability measure $P$ defined on $(\Omega, \mathcal{F})$, $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, P)$ defines a probability space. The agent’s problem, given this probability space, is to choose a stopping time to solve

$$V(P) = \sup_{\tau \in \mathcal{I}} \mathbb{E}^P[\hat{u}(q_\tau) - \bar{\kappa}\tau | \mathcal{F}_0].$$

\footnote{Note that we assume that the agent does not discount the utility from acting. Discounting, in our context, has the effect of changing the relative values of acting and never acting, and for this reason would complicate our analysis; see the more extensive discussions in [Hebert and Woodford, 2023] and [Zhong, 2022] on this point.}
2.2 The Principal’s Problem

The principal chooses the information the agent receives so as to maximize engagement. In the background, we have in mind that the principal is showing ads (unrelated to the agent’s current decision problem) or otherwise profiting from each additional increment of attention the agent pays to the information the principal provides.

We begin with defining a measure for the agent’s information processing using a continuous time version of what Caplin et al. [2022] call a “uniformly posterior-separable” cost function, as described in Hébert and Woodford [2023]. Uniformly posterior-separable cost functions are defined in terms of a “generalized entropy function,” $H : \mathcal{P}(X) \to \mathbb{R}_+$. We assume that $H$ is strongly convex and twice continuously-differentiable. Then, define

$$\frac{d}{dt} I^H_t = \limsup_{h \downarrow 0} \frac{1}{h} \mathbb{E}^P[H(q_{t+h}) - H(q_t)|\mathcal{F}_t].$$

Here, $I^H_t$ denotes the cumulative information acquired by the agent measured by $H$. We assume the agent can process information at a rate defined by the constraint

$$\frac{d}{dt} I^H_t \leq \chi,$$

where $\chi > 0$ is a finite constant. This constraint can be understood as requiring that $E^P[H(q_{t+\delta}) - H(q_t)|\mathcal{F}_t] \leq \chi \delta$ for all times $t$ and time intervals $\delta > 0$.

The principal’s goal, on the other hand, is to maximize “engagement.” Specifically, we assume that the principal earns profits in proportion to the information acquired by the agent, as measured by the generalized entropy function $G : \mathcal{P}(X) \to \mathbb{R}_+$ (a convex and twice continuously-differentiable function). Note that the function $G$ is not necessarily equal to the function $H$. The former determines the information that the principal would like the agent to acquire; the latter governs the rate at which the agent can acquire different kinds of information.

The principal’s goal is to design the agent’s belief process so as to maximize profits, taking into account the fact the agent will know the nature of the beliefs process and optimally choose when to stop paying attention. Let $\bar{q}_0 \in \mathcal{P}(X)$ be the agent’s prior. The

---

Footnotes:

7 Network effects, in particular the possibility that this agent’s attention increases utility other agents receive from the platform and thus the profits the principal can extract from those agents, are one relevant engagement maximization motive.

8 We write the constraint using (2) so that we can directly invoke results in Hébert and Woodford [2023], who consider a more general class of information costs.
principal chooses his policies from the set $\mathcal{A}(\bar{q}_0)$, which is the set of probability measures on $(\Omega, \mathcal{F})$ such that $q$ is martingale belief processes with $q_0 = \bar{q}_0$, $I^H$ is an increasing adapted process with $H_0^H = 0$, and non-negative stopping times $\tau$, such that (I) and (2) are satisfied for all $t \in [0, \tau)$. Formally, the principal chooses the probability measure $P$, which is equivalent to choosing the law of of the belief process $q$. In this sense, the principal can choose any càdlàg martingale belief process with $q_0 = \bar{q}_0$, subject to the constraint imposed by the agent’s information processing capacity.

**Definition 1.** The principal’s problem given initial belief $\bar{q}_0 \in \mathcal{P}(X)$ is to maximize engagement,

$$J(\bar{q}_0) = \sup_{(P, I^H, \tau) \in \mathcal{A}(\bar{q}_0)} \mathbb{E}_P^\tau[G(q_\tau) - G(\bar{q}_0)|\mathcal{F}_0]$$

subject to the agent’s stopping decision,

$$\tau \in \arg\max_{\tau' \in \mathcal{F}} \mathbb{E}_P^\tau[\hat{u}(q_{\tau'}) - \bar{\kappa}\tau'|\mathcal{F}_0].$$

### 2.3 Discussion

The generalized entropy functions $G$ and $H$ play conceptually distinct roles in our model. The function $H$ governs the agent’s information processing constraint. It determines which kinds of information are relatively easy or difficult for the agent to process. The function $G$, on the other hand, measures the kind of engagement that generates profits for the principal.

One leading case of interest is when $H = G$. We interpret this case as one in which the principal displays advertisements to the user interspersed with content, and the principal is paid “per impression” by advertisers without any direct measurement of whether the user paid attention to the advertisement. The principal’s profits are maximized by maximizing the time the agent spends attending to the principal’s content. Arrangements similar to this assumption are observed in a variety of media markets (see, e.g., Gentzkow et al. [2024]).

---

9 An alternative interpretation is that the principal can send more information than the agent can process, in which case the agent chooses which information to attend to. Allowing the agent this choice cannot benefit the principal, and it is therefore without loss of generality to suppose the principal chooses a process that satisfies the agent’s information processing constraint.

10 The authors would like to thank Emir Kamenica for suggesting this interpretation. Note that when $H = G$, the principal’s objective is not to maximize the stopping time $\tau$ directly; however, in equilibrium the constraint (2) will bind, and the principal will maximize time spent subject to (2) holding with equality.
A second leading case is one in which $G$ measures advertisement-related information acquisition. This information can be partially or completely irrelevant to the agent’s decision problem, which is to say that there are beliefs $q, q' \in \mathcal{P}(X)$ that are identical from the agent’s perspective ($\hat{u}(q) = \hat{u}(q')$) but distinct from the principal’s perspective ($G(q) \neq G(q')$). We interpret this case as one which the principal cares about whether or not the agent pays attention to the advertisements that are embedded in the content the principal provides, perhaps because part of the principal’s profits (now or in the future) are based on the effectiveness of the advertisements. Section 5.6 offers examples of these two cases, in a different interpretative context (engaging test-motivated students). Note also that these two cases are not mutually exclusive; a principal could be paid per impression at a rate that is linked to the historical ad effectiveness, which could be modeled in our framework as setting $G$ equal to the sum of $H$ and another entropy function measuring the attention paid to advertisements.

Our formulation of the principal’s problem (Definition 1) embeds several assumptions. First, we assume that the principal acts first and can commit to law $P$ before the agent chooses her stopping strategy $\tau$. Given that the agent faces a pre-determined $P$, her problem is dynamically consistent and the agent’s commitment or lack thereof is irrelevant. In the context of an internet platform and a user, we interpret this assumption as implying that the internet platform designs its recommendation algorithms before the user can commit to a usage pattern. We discuss the role of commitment for the principal in Section 4.

Second, we assume that the belief process is a martingale under both the principal and the agent’s initial information set $(\mathcal{F}_0)$. This assumption is standard in the literature on Bayesian Persuasion (following Kamenica and Gentzkow [2011]) and is usually interpreted as assuming that the principal commits to a particular information design before learning the true state. In the typical Bayesian persuasion setting, this assumption is important; a sender whose preferences are ex-post aligned with the receiver would like to truthfully reveal the state, and might be able to do so by signaling via the choice of information structure. As a result, the lack of commitment to the information structure strictly restricts what the sender can implement (see Lipnowski et al. [2022], Min [2021]). In contrast, in our model, an informed principal would never want to reveal the true state, as doing so would cause the agent to immediately stop, and for this reason our main results apply to the case of an informed principal, as discussed in Section 5.2.

Third, we assume that the agent attends fully to the information the principal provides (i.e. that the $P$ measure chosen by the principal is that one that generates the agent’s ex-
pectations). We show in Section 5.1 that this is without loss of generality for an agent who faces an information processing constraint.

Fourth, we assume that the agent is willing to follow the principal’s recommended stopping rule whenever she is indifferent between that rule and another equally optimal stopping rule. However, it will become clear that the equilibrium strategy can be slightly modified to induce strong incentive to follow the stopping rule (see the discussion following Proposition 2).

Lastly, note that our setup says almost nothing about the agent’s motives for acquiring information. The function $\hat{u}$ describes the value the agent places on the information she receives, and the only meaningful restriction we have imposed as that it be convex (reflecting the fact that all information is at least weakly beneficial). The function $\hat{u}$ could arise from the rational anticipation of the information’s instrumental value, as described above, but could also arise from mistaken beliefs about the value of information, or non-instrumental preferences for information acquisition (e.g. a preference for the early resolution of uncertainty).

3 Optimal Policy

Throughout this section, we assume that it is prohibitively costly for the agent to perfectly learn the true state, to the point that the agent would prefer to learn nothing at all if confronted with only these two possibilities.

Assumption 1. We assume that

$$\hat{u}(\bar{q}_0) - \frac{\bar{\kappa}}{\mathcal{K}} H(\bar{q}_0) > \sum_{x \in X} \bar{q}_{0,x}(\hat{u}(e_x) - \frac{\bar{\kappa}}{\mathcal{K}} H(e_x)),$$

where $e_x \in \mathcal{P}(X)$ has full support on state $x \in X$.

This assumption will imply that perfectly learning the state with probability one cannot occur in our principal-agent problem, as the principal cannot simultaneously satisfy the agent’s participation constraint and the restriction on the rate of information acquisition, (2).
3.1 A Relaxed Problem

We start by defining a relaxed version of the principal’s problem. Any probability measure and stopping rule the principal can implement will induce a probability measure over beliefs the agent will hold when she chooses to stop (i.e. a law for $q_\tau$). We study a version of the principal’s problem in this space. Let $\mathcal{P}(\mathcal{P}(X))$ denote the space of probability measures of beliefs on $\mathcal{P}(X)$, and, given any feasible policy chosen by the principal, let $\pi \in \mathcal{P}(\mathcal{P}(X))$ denote the probability measure of the stopped belief $q_\tau$. The following lemma describes the date-zero participation constraint of the agent and an upper bound on the total engagement associated with the measure $\pi$.

**Lemma 1.** $\forall (P, I^H, \tau) \in \mathcal{A}(q_0)$ satisfying the agent’s optimal stopping in Definition 1 the following conditions are satisfied:

1. $\mathbb{E}_\pi[\bar{u}(q)] - \kappa \mathbb{E}_P[\tau|\mathcal{F}_0] \geq \bar{u}(\bar{q}_0)$, and
2. $\mathbb{E}_\pi[H(q) - H(\bar{q}_0)] \leq \chi \mathbb{E}_P[\tau|\mathcal{F}_0]$.

**Proof.** Condition (1): let $\tau' \equiv 0$, then the agent’s utility from stopping is $\bar{u}(q_0)$. The optimality of $\tau$ implies condition (i).

Condition (2): $dI^H_t \leq \chi dt \implies \mathbb{E}_P[I^H_t|\mathcal{F}_0] = \mathbb{E}_P[\int_0^\tau dI^H_t|\mathcal{F}_0] \leq \mathbb{E}_P[\int_0^\tau \chi dt|\mathcal{F}_0] = \chi \mathbb{E}_P[\tau|\mathcal{F}_0]$.

Lemma 1 presents a necessary condition for any admissible policy for the principal in the optimization problem (Definition 1). The first condition states that the agent’s optimal stopping utility is weakly greater than the utility from stopping immediately. The second condition states that the cumulative information acquired by the agent is weakly less than $\chi \mathbb{E}_P[\tau|\mathcal{F}_0]$—the maximal attention permitted by the information constraint (2).

Combining these two constraints,

$$\mathbb{E}_\pi[\bar{u}(q) - \hat{u}(\bar{q}_0)] \geq \kappa \mathbb{E}_P[\tau|\mathcal{F}_0] \geq \frac{\bar{\kappa}}{\chi} \mathbb{E}_\pi[H(q) - H(\bar{q}_0)].$$

Let us define the principal’s relaxed optimization problem as incorporating only this combined constraint:

$$\tilde{J}(q_0) = \sup_{\pi \in \mathcal{P}(\mathcal{P}(X)): \mathbb{E}_\pi[q] = q_0} \mathbb{E}_\pi[G(q) - G(\bar{q}_0)] \quad \text{s.t.} \quad \frac{\bar{\kappa}}{\chi} \mathbb{E}_\pi[H(q) - H(\bar{q}_0)] \leq \mathbb{E}_\pi[\bar{u}(q) - \hat{u}(\bar{q}_0)].$$

14
Because this combined constraint must hold in the original principal’s problem, we must have $\bar{J}(\bar{q}_0) \geq J(\bar{q}_0)$.

Equation [3] is a convex optimization problem that satisfies the conditions in Theorem 4 of Zhong [2018], which immediately implies:

**Proposition 1.** Equation [3] has a solution $\pi^*$ with finite support. Either $\bar{J}(q_0) = 0$ or $\bar{J}(q_0) > 0$ and there exists $\bar{\lambda} > 0$ s.t. all $\pi^*$ satisfy

$$\pi^* \in \arg \max_{\pi \in \mathcal{P}(\mathcal{P}(X)): \mathbb{E}_\pi[q] = \bar{q}_0} \mathbb{E}_\pi^\pi \left[ \hat{u}(q) - \frac{\tilde{k}}{\chi} H(q) + \bar{\lambda} G(q) \right].$$

In both cases, $\frac{\tilde{k}}{\chi} \mathbb{E}_\pi^\pi \left[ H(q) - H(\bar{q}_0) \right] = \mathbb{E}_\pi^\pi \left[ \hat{\mu}(q) - \hat{\mu}(\bar{q}_0) \right]$.

This proposition demonstrates that $\pi^*$ is the solution to a static rational inattention problem, with a general UPS cost function (of the sort studied by Caplin et al. [2022]). Those authors show that a necessary condition for $\pi^*$ is that it concavifies the function $\hat{\mu} - \frac{\tilde{k}}{\chi} H + \bar{\lambda} G$.[11] Engagement is infeasible ($\bar{J}(q_0) = 0$) whenever the only $\pi$ satisfying the constraint are ones with $\mathbb{E}_\pi^\pi [G(q) - G(\bar{q}_0)] = 0$, which is to say that it is not possible for the principal to induce information acquisition in the dimensions he cares about. When $G$ is strictly convex (meaning that all information acquisition is at least somewhat beneficial to the principal, as in the $G = H$ case), this implies $\text{Supp}(\pi) = \{\bar{q}_0\}$. When $\text{Supp}(\pi) = \{\bar{q}_0\}$, we will say that $\pi^*$ is degenerate, and otherwise say that $\pi^*$ is non-degenerate[12].

Let us take as given a solution $\pi^*$ to this relaxed problem, and consider how it might be implemented in an incentive compatible way in the original principal’s problem.

### 3.2 Implementation

Take any non-degenerate $\pi \in \mathcal{P}(\mathcal{P}(X))$ s.t. $\mathbb{E}_\pi^\pi[q] = \bar{q}_0$, and define the stochastic process $q_t$ as:

$$q_t = \bar{q}_0 + 1_{N_\alpha(t) \geq 1} \cdot (Q - \bar{q}_0), \quad (4)$$

[11] Related results appear in earlier working papers by those authors and in the Bayesian persuasion literature.

[12] When $G$ is not strictly convex, meaning that there are some dimensions of potential information acquisition the principal does not value, it is possible to have $J(\bar{q}_0) = 0$ with a non-degenerate $\pi^*$. However, the information acquired in this case generates no surplus for either the principal or the agent.
where $Q \in \mathcal{P}(X)$ is a random variable distributed according to $\pi$ and $N_\alpha(t)$ is an independent Poisson counting process with parameter $\alpha$. Here, $q_t$ is a compound Poisson process that jumps according to $\pi$ at rate $\alpha$. We call such process $q_t$ an $\alpha$-dilution of $\pi$.\footnote{Pomatto et al. \cite{pomatto2018} first introduce the notion of dilution. They define the dilution of an information structure $\pi$ as “producing $\pi$ with probability $\alpha$ and uninformative signal with probability $1 - \alpha$.” (the same notion appeared in Bloedel and Zhong \cite{bloedel2020}). Our notion of $\alpha$-dilution is essentially the repetition of a dilution in continuous time.}

**Proposition 2.** For all non-degenerate $\pi \in \mathcal{P}(\mathcal{P}(X))$ that satisfy the constraint in \cite{3}, let $\alpha = \frac{\chi}{\mathbb{E}[H(q) - H(q_0)]}$ and let $q_t$ be the $\alpha$-dilution of $\pi$. Then $q_t$ is feasible in the problem of Definition \cite{2} and implements utility level $\mathbb{E}^\pi[G(q) - G(q_0)]$.

**Proof.** Let $P$ be the law of the process defined in \cite{3}, and define the stopping time $\tau = \inf \{ t \in \mathbb{R}_+ | q_t \neq q_0 \}$. Evidently, conditional on continuation, $\lim_{h \downarrow 0} \frac{1}{h} \mathbb{E}^P[H(q_t) - H(q_{(t-h)^-}) \mid \mathcal{F}_{(t-h)^-}] = \lim_{h \downarrow 0} \frac{1}{h}(1 - e^{-\frac{\chi}{h}[H(q) - H(q_0)]})\mathbb{E}^\pi[H(q) - H(q_0)] = \chi$. Thus, the information constraint is satisfied. Since $q_\tau \sim \pi$, the principal’s utility is $\mathbb{E}^\pi[G(q) - G(q_0)]$. What remains to be verified is the agent’s optimality condition.

Take any admissible stopping time $\tau'$,

$$
\mathbb{E}^P[\widehat{\mu}(q_{\tau'}) - \kappa \tau' \mid \mathcal{F}_0] = \text{Prob}(\tau' < \tau) \left( \mathbb{E}^P[\widehat{\mu}(q_0) - \kappa \tau' \mid \tau' < \tau] + \text{Prob}(\tau' \geq \tau) \left( \mathbb{E}^P[\widehat{\mu}(q_\tau) - \kappa \tau' \mid \tau' \geq \tau] \right) \right) \\
\leq \text{Prob}(\tau' < \tau) \left( \mathbb{E}^\pi[\widehat{\mu}(q) - \kappa \mathbb{E}^\pi[\tau' \mid \mathcal{F}_0] - \kappa \mathbb{E}^P[\tau' \mid \tau' < \tau] \right) + \text{Prob}(\tau' \geq \tau) \left( \mathbb{E}^\pi[\widehat{\mu}(q) - \kappa \mathbb{E}^\pi[\tau] \mid \tau' < \tau] \right) \\
= \text{Prob}(\tau' < \tau) \left( \mathbb{E}^\pi[\widehat{\mu}(q) - \kappa \mathbb{E}^\pi[\tau] \mid \tau' < \tau] \right) + \text{Prob}(\tau' \geq \tau) \left( \mathbb{E}^\pi[\widehat{\mu}(q) - \kappa \mathbb{E}^P[\tau] \mid \tau' \geq \tau] \right) \\
= \mathbb{E}^P[\widehat{\mu}(q) - \kappa \tau \mid \mathcal{F}_0].
$$

The first equality is from the definition of conditional expectations and the process defining $q_t$, \cite{3}. The first inequality is from the constraint $\mathbb{E}^\pi[\widehat{\mu}(q) - \widehat{\mu}(q_0)] \geq \kappa \mathbb{E}^P[\tau] \mid \mathcal{F}_0]$ in the relaxed problem \cite{3} and $\mathbb{E}[\tau \mid \tau' \geq \tau] \leq \mathbb{E}[\tau' \mid \tau' \geq \tau]$. The second equality is from the memoryless property of $\tau$\footnote{Specifically, $\mathbb{E}^P[\tau] = \mathbb{E}^P[\tau - \tau' \mid \tau' < \tau] + \mathbb{E}^P[\tau' \mid \tau' < \tau] = \mathbb{E}^P[\tau \mid \mathcal{F}_0] + \mathbb{E}^P[\tau' \mid \tau' < \tau]$.} and the last from the definition of conditional expectations. Note that if the constraint in \cite{3} is slack, the first inequality is strict if $\tau'$ is not equal to $\tau$ $P$-a.e. In this case, the $\alpha$-dilution of $\pi$ strongly implements principal’s utility level $\rho \mathbb{E}^\pi[G(q_t) - G(q_0)].$ \hfill \boxed
Combining Lemma 1, Proposition 1, and Proposition 2, we obtain the main characterization of the optimal policy:

**Theorem 1.** ∀ ¯q₀ ∈ ℙ(X), there exists π* ∈ ℙ(ℙ(X)) with finite support solving Equation 3. If Supp(π*) = {¯q₀}, the agent will immediately stop and any feasible policy is optimal. Otherwise, let α* = χE π* [H(q) − H(¯q₀)] and let (P*, τ*) be the law and jumping time of the α*-dilution of π*. Then, (P*, H* = χt, τ*) is an optimal solution to the principal’s problem (Definition 1).

There are generally many optimal policies in the principal’s problem. First, there may be multiple π* that solve the relaxed principal’s problem (although uniqueness in static rational inattention problems is guaranteed under certain additional assumptions). Second, there are many stochastic processes qₜ (equivalently, laws P) and stopping times τ that induce the same law for qₜ; provided that this law is equal to π* and that incentive compatibility and the bounds on information acquisition are satisfied, all such policies are optimal. However, it is not the case that anything goes, as we show next.

### 3.3 Characterization of optimal policies

For the purpose of discussing optimal policies, we assume for this subsection that the optimal policy is to acquire some information (J(¯q₀) > 0 in the context of Proposition 1).

To begin, we define, given a measure π ∈ ℙ(ℙ(X)), the value of information acquisition in a hypothetical restricted static rational inattention problem:

\[
V_R(q, π) = \max_{π' ∈ ℙ(\text{Supp}(π))} \mathbb{E}_{π'}[\mu(q') - \hat{u}(q) - \bar{κ}(H(q') - H(q))].
\]

This problem maximizes the agent’s expected utility subject to the usual constraint of Bayes consistency and the additional constraint that the posteriors lie in the support of π. We use the notation V_R(q, π) to emphasize that the problem is restricted relative to the usual rational inattention problem. Note that the problem is feasible only for q that lie in in the convex hull of the support of π, which we denote by Conv(\text{Supp}(π)).

We interpret V_R(q, π) as describing the value of information acquisition, in the sense that V_R(q, π) is the difference between the utility the agent achieves with and without information acquisition.\(^{15}\) We define the set of “suspensive” beliefs given π as the set for which

\(^{15}\)The value of information acquisition is related to, but not the same as, the notion of uncertainty defined in Frankel and Kamenica [2019].
the value of information acquisition is weakly positive.

**Definition 2.** Given $\pi \in \mathcal{P}(\mathcal{P}(X))$, the belief $q \in \mathcal{P}(X)$ is decisive if $q \in \text{Supp}(\pi)$; the belief $q \in \mathcal{P}(X)$ is suspensive if $q \in \text{Conv}(\text{Supp}(\pi)) \setminus \text{Supp}(\pi)$ and satisfies $V_R(q, \pi) \geq 0$. Let $\Delta(\pi) \subseteq P(X)$ denote the set of all suspensive beliefs given $\pi$.

Given a measure $\pi$ that describes the stopping beliefs, it is intuitive that beliefs in $\text{Supp}(\pi)$ are “decisive” because they directly lead to stopping. At the suspensive beliefs in $\Delta(\pi)$, the agent benefits, relative to the prior consistent with $\pi$, from the signals that the principal is offering (which always result in some posterior in the support of $\pi$). Therefore, $\Delta(\pi)$ characterizes the beliefs at which the principal can possibly induce “suspense” before eventually leading the agent to stopping beliefs in $\text{Supp}(\pi)$.

Suppose the optimal policy in (3), $\pi^*$, is unique. In this case, the stochastic process for beliefs $q_t$ cannot leave the set of suspensive beliefs $\Delta(\pi^*)$ before $t = \tau$. If $q_t$ left the convex hull of the support of $\pi^*$, then $\pi^*$ would not describe the law of $q_\tau$. If $q_t$ was decisive or if $V_R(q_t, \pi^*) < 0$, the agent would choose to stop. In the case of multiple optimal policies, this argument applies for some optimal $\pi^*$, which leads to the following result.

**Proposition 3.** Given $\tilde{q}_0 \in \mathcal{P}(X)$, let $\Pi^*$ be the set of solutions to (3). Then, in all solutions to the principal’s problem, for all $t \in [0, \tau^*)$, beliefs are suspensive given some $\pi^* \in \Pi^*$,

$$\text{Supp}(q_t) \subseteq \bigcup_{\pi^* \in \Pi^*} \Delta(\pi^*),$$

and at $t = \tau^*$, beliefs are decisive given some $\pi^* \in \Pi^*$,

$$\text{Supp}(q_\tau) \subseteq \bigcup_{\pi^* \in \Pi^*} \text{Supp}(\pi^*).$$

**Proof.** Let $\pi^*$ be a solution to the problem defined in (3). We first show that for any $q' \in \text{Supp}(\pi^*)$, $\tilde{J}(q') = 0$. Let $\pi^{**}$ be the optimal policy associated with $q'$. The policy $\pi^{***}(q) = \pi^*(q) \mathbf{1}_{\{q \neq q'\}} + \pi^*(q') \pi^{**}(q)$ is feasible in (3) given the prior $\tilde{q}_0$ and would achieve strictly higher utility than $\pi^*$ if $\tilde{J}(q') > 0$. It follows in the principal-agent problem that if $q_t \in \text{Supp}(\pi^*)$, then $t = \tau^*$, because there is no way to induce the agent to continue.

By Theorem 1, there is some $\pi^* \in \Pi^*$ that is the law of $q_\tau^*$ under the optimal policies, and the second claim follows. By the martingale property of the belief process, $q_t = E_{\pi^*}[q_\tau|F_t, t < \tau^*]$ for all $t \in [0, \tau^*)$, and thus $q_t$ must lie in the convex hull of the support of $\pi^*$. The belief $q_t$ cannot be decisive if $t < \tau^*$, by the argument above. By the
definition of (3), \( \frac{k}{\kappa} \mathbb{E}^{\pi^*} [H(q) - H(\bar{q}_0)] \leq \mathbb{E}^{\pi^*} [\hat{\mu}(q) - \hat{\mu}(\bar{q}_0)] \), from which it follows immediately that \( V_R(q, \pi^*) \geq 0 \).

Solving (3) generically leads to the IC constraint binding, which suggests that the value of acquiring \( \pi^* \) is exactly zero at the prior for the agent. The set of suspensive beliefs \( \Delta(\pi^*) \) thus defines beliefs that are “more uncertain” than the prior. Therefore, the interpretation of Proposition 3 is that two types of signals appear in any optimal policy almost surely. The first type are \textit{decisive} signals that are so informative that the agent makes a decision immediately after receiving the signal. The second type are \textit{suspensive} signals that causes the agent’s beliefs to move to a posterior that is “more uncertain” than the prior.

### 3.4 The agent-optimal benchmark

We will compare our results to a benchmark in which the agent and not the principal chooses the probability space and martingale belief process (subject to (2)). This benchmark is a special case of the more general models described in [Hébert and Woodford (2023)] and [Zhong (2022)].

Those authors show that the optimal policies of this benchmark dynamic model are also equivalent to the solution to a static rational inattention problem. That is, under the optimal policies given the initial belief \( \bar{q}_0 \in \mathcal{P}(X) \),

\[
\mathbb{E}^{\mathcal{P}} [\hat{\mu}(q_{\tau}) - \bar{k}_\tau | \mathcal{F}_0] = V^B(\bar{q}_0) = \max_{\pi \in \mathcal{P}(\mathcal{P}(X)) : \mathbb{E}^\pi[\bar{q}_0] = \bar{q}_0} \mathbb{E}^{\pi} [\hat{\mu}(q) - \frac{k}{\kappa} (H(q) - H(\bar{q}_0))].
\]

This solution can be implemented, as above, by a compound Poisson process, but also by a diffusion (and many other processes).

This static rational inattention problem is essentially identical to the one that characterizes optimal policy in our principal-agent framework, except that the UPS information cost is \( \frac{k}{\kappa} H(q) \) in the benchmark problem and \( \frac{k}{\kappa} H(q) - \lambda G(q) \) in the principal agent case. The similarity in structure between these two problems will allow us to highlight the implications of engagement maximization.

Before proceeding, we should note that the principal is unable to induce any information acquisition whenever the only feasible \( \pi \) in the relaxed problem places full support on \( \bar{q}_0 \); this corresponds to the case in the benchmark model in which the agent strictly prefers to
not gather information. In all other cases, the principal can induce information acquisition in the principal-agent model.

### 3.5 Example (continued)

We illustrate Theorem 1 by continuing the numerical example in Section 1.1. Note in this example that $H = G$ (both are the negative of Shannon’s entropy). We maintain the same model primitives, with the exceptions that we now vary the agent’s prior belief and focus our analysis on the agent’s posteriors when stopping ($q_\tau$) as opposed to the compound Poisson signals that generate those posteriors. Figure 1 illustrates the optimal $q_\tau$ for every prior belief: When $\bar{q}_0 \leq q$ or $\bar{q}_0 \geq \bar{q}$, the agent stops immediately no matter what information she will receive. When $\bar{q}_0 \in (q, \bar{q})$, the dashed lines illustrate the support of the agent-optimal policy. The lines are flat, meaning that the agent’s stopping beliefs do not change with the prior belief, consistent with the prediction of the standard RI models. The solid curves illustrate the support of the unique optimal $\pi^*$. They are closer to the boundaries zero and one, indicating that the posterior beliefs become more polarized under the principal’s optimal policy.

![Figure 1: Supp($q_\tau$) as a correspondence of $q_0$](image)

Figure 2 illustrates the engagement level for every prior belief. The dashed curves represent the agent-optimal policy and the solid curves represent the principal-optimal policy. The principal-optimal policy induces higher engagement level than the agent-optimal policy.

---

This property is known as the “locally invariant posteriors” [Caplin et al. 2022].

---
icy. However, the engagement level converges to zero when the prior belief goes to the boundary of the continuation region \((\underline{q}, \bar{q})\).

Figure 2: Engagement as a function of \(q_0\)

Now, focus on the \(\bar{q}_0 = \frac{1}{2}\) case. As is illustrated in Figure 1, the unique optimal \(\pi^*\) involves two posterior beliefs \(\{q^1, q^2\}\), where \(q^1 < q\) and \(q^2 > \bar{q}\). Per Proposition 2, the dilution of \(\pi^*\) implements an optimal policy. In this case, the dilution of \(\pi^*\) is the unique optimal policy. It is apparent from the symmetry of the problem that the value of information acquisition, \(V_R(q, \pi^*)\), is maximized at \(q = \frac{1}{2}\). As a result, the set of suspensive beliefs, \(\Delta(\pi^*)\), is a singleton, \(\{\bar{q}_0\}\). By Proposition 3, beliefs will remain at \(\bar{q}_0\) until they jump to either \(q^1\) or \(q^2\).

When \(\bar{q}_0 < \frac{1}{2}\), the optimal policy is not unique. In this case, again by the symmetry of the problem, it is apparent that the set of suspensive beliefs is \(\Delta(\pi^*) = [q_0, 1 - q_0]\). Beliefs will remain in this interval until eventually jumping to either \(q^1\) or \(q^2\). A symmetric argument applies when \(\bar{q}_0 > \frac{1}{2}\). Figure 3 illustrates the sample paths of one optimal policy, in which beliefs jump between \(\bar{q}_0\) and \(1 - \bar{q}_0\) before eventually jumping to \(q^1\) or \(q^2\).

Figure 3: Sample paths of an optimal policy
4 Implications

In this section, we consider several implications of our main result.

Implication 1: Agent Welfare Minimization. An immediate implication of Theorem 1 is that the agent’s participation constraint binds. That is,

$$\mathbb{E}[\hat{u}(q_{\tau^*}) - \bar{\kappa}_{\tau^*}|\mathcal{F}_0^*] = \hat{u}(\bar{q}_0).$$

Strikingly, the agent is no better off receiving information from an engagement-maximizing principal than if she could not receive any information at all. The principal extracts the full surplus generated by the ability to produce information and apply it in the agent’s decision problem, despite the agent’s ability to choose when to stop and act. The principal, in maximizing the engagement of the agent, minimizes the agent’s welfare subject to a participation constraint.

A different way of viewing this result is as showing that paying the principal for the information using attention as opposed to a monetary payment is inefficient. If the principal could commit to a signal structure in exchange for a payment from the agent (the transferable utility case), he would of course provide the optimal signal structure from the agent’s perspective, as this would maximize revenue. Our model assumes that the principal is instead paid via attention, which distorts the principal’s incentives for information provision.

In practice, there are likely good reasons why platforms such as Facebook and Twitter choose to make their services free to users. Providers that charge users in exchange for information (the New York Times, for example) generally have far fewer users than freely accessible, advertising-supported platforms. Our baseline model does not speak to this trade-off, but instead offers an explanation for why the information available on free-to-use platforms differs in nature from the information available from paid information providers.

Implication 2: Commitment is Unnecessary. Consider a modified version of our model in which the principal lacks commitment with respect to the law of the stochastic process $q_t$ and instead simply chooses signals for the agent to receive at each instant. In this case, we also assume that the agent lacks commitment with respect to her stopping rule. It is immediate that if both agents expect the policies of the solution with commitment to played

22
going forward, they will be willing to adopt those policies at the current moment.\footnote{That is, there are no profitable one-shot deviations from the commitment solution. To claim that as a result, the commitment solution is an equilibrium even in the absence of commitment, we would have to define sub-game perfection in the context of our continuous-time game.}

The intuition behind this statement is that, under the optimal policy, the principal never promises the agent anything other than the minimum possible utility starting from any belief $q \in \mathcal{P}(X)$. That is, although it is in theory possible for a principal with commitment to induce the agent to continue by promising a larger-than-minimum level of utility in the future, such promises are not necessarily a feature of the optimal policy with commitment, and consequently commitment is unnecessary.

However, a lack of commitment can restrict the set of optimal policies. If beliefs change in a way that is strictly suspensive, $V_R(q_t, \pi^*) > 0$, then the agent’s participation constraint becomes slack. The principal at this point would be tempted to re-optimize (inducing a different $\pi^{**} \neq \pi^*$). In our example, the set of suspensive beliefs was $[q_0, 1 - q_0]$ for any $q_0 < \frac{1}{2}$. But it is apparent from Figure 2 that if $q_t \in (q_0, 1 - q_0)$, the optimal posteriors given the initial belief $q_t$ are no longer in the support of $\pi^*$. Commitment is thus necessary for the agent to tolerate moving to strictly suspensive beliefs. In the absence of commitment, the only optimal policies are the kinds of dilutions described in Proposition 2 and policies that mix such dilutions with jumps to other equally suspensive beliefs (such as the policies illustrated in Figure 3).

**Implication 3: Extreme Beliefs.** Let $Q^i(\bar{q}_0) \subseteq \mathcal{P}(X)$ be the union of the support of all optimal stopping beliefs in the benchmark ($i = a$) and principal-agent ($i = p$) models. Let $\text{Conv}Q^i(\bar{q}_0)$ denote the convex hull of $Q^i(\bar{q}_0)$. The following proposition demonstrates that the beliefs the agent will hold when stopping after using the principal’s platform are more extreme than the beliefs the agent would choose to acquire in the benchmark model. This result follows from the observation that in both models, stopping beliefs are characterized by the solution to a static rational inattention problem, with a lower information cost in the principal-agent case than in the benchmark case. There are two possible exceptions to this result. One is the case in which the principal cannot induce any engagement ($\bar{J}(q_0) = 0$ in Proposition 1), in which case there is a principal-optimal policy in which no information is acquired, $\bar{q}_0 \in Q^p(\bar{q}_0)$. The second is the case in which the agent’s beliefs lie on the boundary of the simplex, in which case there may not be “room” for beliefs to become more extreme.
Proposition 4. Assume that $\bar{J}(q_0) > 0$ and that $Q^a(\bar{q}_0)$ is a subset of the relative interior of $\mathcal{P}(X)$. Then for all $q \in Q^p(\bar{q}_0), q \notin \text{Conv} Q^a(\bar{q}_0)$.

Proof. See the appendix, section A.1.

This proposition is illustrated in Figure 2. For all priors $\bar{q}_0 \in (q, \bar{q})$, which is the continuation region in the benchmark model defined by $V^B(\bar{q}_0) > \hat{u}(\bar{q}_0), q^1 < q < \bar{q} < q^2$, and consequently $Q^p(\bar{q}_0) = \{q^1, q^2\} \not\subset \hat{Q}^a(q_0) = [q, \bar{q}]$.

Extreme beliefs are a natural consequence of the tradeoffs facing the principal. By forcing the agent to ultimately acquire more information than she would choose for herself (the extreme beliefs), the principal can simultaneously delay the agent’s stopping decision while providing information the agent is willing to attend to. Interpreted in the context of an internet platform and a user described earlier, our extreme beliefs result implies that the platform should provide in-depth content on a narrower set of topics than would be preferred by the user. We can interpret this loosely as encouraging the user to go down “rabbit holes” that lead to extreme beliefs.

Implication 4: The Necessity of Jumps in Beliefs. We have shown in Proposition 3 that, at all times under an optimal policy, beliefs are either suspensive or decisive. This result immediately implies that the optimal policy cannot involve continuous sample paths for beliefs (we assume the initial prior lies in the continuation region). If sample paths were continuous, the belief process $q_t$ would have to exit $\Delta(\pi^*)$, which would immediately induce the agent to stop. Consequently, beliefs must jump discontinuously from the set of suspensive beliefs (which lie in the continuation region of the benchmark model) to the set of decisive beliefs, which lie in the strict stopping region of the benchmark model (by Proposition 4).

Relatedly, our result that the agent’s value function is equal to $\hat{u}(q)$ everywhere implies that a pure diffusion process is infeasible almost everywhere in the continuation region. Because the agent recognizes that the principal will leave her with no surplus, only information that causes her to change her beliefs about the currently optimal action is valuable. If the belief $q_t$ is such that one action is strictly optimal given those beliefs (which will be true generically), the principal must offer at least the possibility of jumping to a different region in which another action is optimal; otherwise, the agent will perceive no benefit from the information provided. We interpret this result as suggesting that the principal provides that agent with news articles containing extreme or sensational claims, which should
cause the agent to either move her beliefs a lot or not at all, as opposed to providing more nuanced or qualified information.

5 Extensions

In this section, we consider several extensions to the model analyzed above. We first discuss the possibility that the agent does not fully attend to the principal’s signals, and show that this is never optimal for the agent. Second, we show that commitment to the signal structure plays a minor role in our model. We next consider the case in which the cost of delay is increasing over time, representing either an increasing opportunity cost of time or diminishing utility from using the platform. We then discuss an alternative version of our model in which agents are not capacity constrained and the principal’s objective is to maximize the time until a decision is made. Lastly, we provide an alternative interpretation of our model in the context of a teacher and student.

5.1 Incentive Compatibility

In our main model, the agent solves a stopping problem given the principal’s chosen information structure. In practice, the agent may choose to process the principal’s chosen information structure in a selective way, which leads to an extra “incentive compatibility” requirement for the principal. We argue that the optimal policy is “incentive compatible” as is defined in Proposition 5.

Proposition 5. Let \((P^*, \tau^*)\) be defined as in Theorem 7. For all stopping times \(\hat{\tau}\) and all \(\hat{\pi} \in \mathcal{P}(\mathcal{P}(X))\) that are a mean preserving contraction of \(q_{\hat{\tau}}\), \(\mathbb{E}^\hat{\pi}[\mu(q)] - \mathbb{E}^{P^*}[\hat{\kappa}_{\hat{\tau}}|\mathcal{F}_0] \leq \mu(\bar{q}_0)\).

Proof. Observe that

\[
\mathbb{E}^\hat{\pi}[\mu(q)] - \mathbb{E}^{P^*}[\hat{\kappa}_{\hat{\tau}}|\mathcal{F}_0] \leq \mathbb{E}^{P^*}[\hat{\mu}(q_{\hat{\tau}}) - \hat{\kappa}_{\hat{\tau}}|\mathcal{F}_0] \leq \mathbb{E}^{P^*}[\hat{\mu}(q_{\tau^*}) - \hat{\kappa}_{\tau^*}|\mathcal{F}_0] = \mu(\bar{q}_0)
\]

The first inequality is from the convexity of \(\hat{\mu}\) and \(\hat{\pi}\) being a mean preserving contraction of \(q_{\hat{\tau}}\). The second inequality is the same as the main inequality of Proposition 2. The last equality is from the constraint in Equation 3 being binding for \(\pi^*\). \(\square\)
Any deviation of the agent leads to an alternative belief process \( \hat{q}_t \) and a corresponding stopping time \( \hat{\tau} \). Instead of modeling \( \hat{q}_t \) explicitly, observe that since \( \hat{q}_t \) is derived from a garbling of the signals that generate \( q_t \), the law of \( \hat{q}_t \) (i.e. \( \hat{\pi} \)) is a mean preserving contraction of the law of \( q_t \). Proposition 5 therefore implies that the agent’s expected utility from such a deviation is weakly less than \( \mu(\tilde{q}_0) \)—the utility of fully attending the principal’s information.

The intuition of the result is simple: in the optimal policy, the principal is providing valuable information to the agent, to ensure that the agent does not stop using the platform. If the agent fails to attend to this information while continuing to use the platform, she incurs the cost of delay without fully benefiting from the information provided.

In the particular case in which the principal would like the agent to acquire decision-irrelevant information, the agent is indifferent between attending to all of the information provided by the principal and attending only to its decision-relevant components. Ignoring the decision-irrelevant information (i.e. ads) does not allow the agent to more rapidly acquire decision-relevant information; the arrival rate of decision-relevant information is determined by the principal, not the agent.

### 5.2 An Informed Principal

In this subsection, we modify our model by assuming that the principal knows \( x \in X \) before choosing \( P \). For the sake of brevity, we will discuss this case without providing a formal definition of equilibrium, and argue that our results continue to apply.

The key difference between this case and our main analysis is that the state \( x \in X \) is now the principal’s “type,” and there is a set of conditional measures, \( \{P_x\}_{x \in X} \), describing the belief processes the principal might induce in the agent conditional on each state \( x \in X \). We will continue to think of these measures as being generated from signals the principal sends to the agent via Bayesian updating, and we assume that the agent observes the signal structures generating these signals. As a result, there is the potential for the principal to “signal via signal structures,” by selecting signal structures that signal the principal’s type, independently of the information conveyed by the signal structure itself.

To simplify our argument, we will assume that the agent observes, at date zero, the set of future signal structures the principal will offer.\[\text{\footnote{It should be clear from the argument that follows that this assumption is not essential.}}\] We claim that there is a pooling equilibrium in which all types of principal offer the signal structures that induce \( P^* \), as in
our main analysis, and the agent acts accordingly. The intuition is as follows: suppose a principal of type $x \in X$ deviated and offered a different set of signal structures. The agent would observe this deviation, and form off-path beliefs. Suppose that after such a deviation the agent places full support on a single type, $x' \in X$. Given that the agent now believes she knows the true state, she will stop immediately, and therefore such a deviation is not beneficial for any principal.

Note that this argument holds even if the agent places the full support of her off-path beliefs on the type with the most reason to deviate (in the spirit of the D1 refinement). This point highlights a key difference between our model and standard Bayesian persuasion models. In standard Bayesian persuasion models, in which the principal cares about the action the agent ultimately chooses, an informed principal whose preferences are aligned with the agent’s has a strong incentive to reveal the true state, which gives refinements like D1 bite. In contrast, in our framework, no principal ever wants to reveal the true state, because each type of principal cares only about maximizing engagement and not about the action the agent ultimately chooses.

5.3 Increasing Costs of Delay

In this subsection, we consider the case in which the cost of delay is increasing over time. We denote the flow cost of delay at period $t$ by $\kappa(t)$. This case is of interest because it is natural to assume that the utility gained from using a platform declines, and the opportunity cost of time rises, the more time is spent on the platform.

This case is not as tractable as the constant cost case. For this subsection only, we limit our analysis to the case of $H = G$. We also impose additional symmetry assumptions, not present in our main model, to allow us to prove our results. We should emphasize that we have no reason to believe, absent these symmetry assumptions, that our main results fail—but we have not been able to prove that they continue to hold, either. We will conjecture and verify that an equilibrium without commitment exists in which our characterization of optimal policy, suitably modified, continues to hold. Note that, that unlike the constant cost case, we will prove only the existence of an such an equilibrium in the absence of commitment, as opposed to proving the stronger claim that all equilibria share these properties.

Specifically, we will prove that there exists a $t_0 \in \mathbb{R}_+$ such that if $q_{t_0}$ is uniform, then the optimal policy in the increasing cost case can be constructed from the optimal policies of the constant cost case.
Let $\pi^*(\kappa)$ be a solution to Proposition 1 under a constant cost of delay $\bar{\kappa} = \kappa$, given the uniform prior $t \in \mathcal{P}(X)$. The essence of our solution is that in each time $t$ the principal offers a belief process that either jumps to an element of the support of $\pi^*(\kappa(t))$ or remains constant. Define the stochastic process $\hat{q}_s$ as:

$$\hat{q}_s = \iota + 1_{N_\alpha(s) \geq 1} \cdot (Q_s - t),$$

(5)

where $Q_s \in \mathcal{P}(X)$ is a random variable distributed according to $\pi^*(\kappa(s))$ and $N_\alpha(s)$ is an independent Poisson counting process with an arbitrary parameter $\alpha > 0$ and $N_\alpha(s) = 0$ for $s \leq t_0$. Now define a time-changed version of this process,

$$q_t = \hat{q}_{s(t)},$$

with $s(t) = t$ for $t \in [0, t_0]$ and

$$\frac{ds}{dt} = \frac{\chi}{\alpha E^{\pi^*(\kappa(t))}[H(q') - H(\tilde{q}_0)]}$$

for all $t > t_0$.

We will prove that if $q_{t_0} = t$, the law of this time-changed process from $t_0$ onward, $P^*$, along with $I_{t}^{H*} = \chi(t - t_0)$ and the stopping rule $\tau^* = \inf\{t \in [t_0, \infty) : \hat{q}_t \neq t\}$, is part of a solution to

$$\sup_{(P, I_{t}^{H}, \tau) \in \mathcal{A}(t)} \mathbb{E}^{P}[H(q_{\tau}) - H(\tilde{q}_0) | \mathcal{F}_{t_0}]$$

subject to

$$\tau \in \arg\max_{\tau' \in \mathcal{F}} \mathbb{E}^{P}[\hat{u}(q_{\tau'}) - \int_{t_0}^{\tau'} \kappa(t)dt | \mathcal{F}_{t_0}].$$

(Recall here our assumption that $H = G$). That is, the principal-optimal policy $(P^*, I_{t}^{H*}, \tau^*)$ can be implemented via this kind of signal structure.

**Additional assumptions.** We introduce here a set of additional assumptions for the purpose of tractability. The first assumption states that the delay cost is non-decreasing, truncated at finite time, and high enough to guarantee interior solutions.

**Assumption 2.** We assume that $\kappa(t) : \mathbb{R}^+ \to \mathbb{R}^+$ is non-decreasing, $\exists T$ s.t. $\kappa(t)$ is constant for $t \geq T$, and $\text{Supp}(\pi^*(\kappa(0)))$ is in the interior of $\mathcal{P}(X)$. 28
The non-decreasing aspect of this assumption is natural, given that we would like to model an increasing opportunity cost of time or diminishing utility from using the platform. The existence of a $T$ after which the cost is constant is a technical device; our results will hold with $T$ arbitrarily large. The assumption that the optimal constant-cost policy is interior at time zero ensures that this remains true for all $t > 0$, and simplifies our analysis. We view these assumptions as relatively innocuous.

Our substantive assumption is rotational symmetry. Let $R$ denote a matrix in $\mathbb{R}^{|X|^2}$. We say $R$ is a rotation if $Re_x \in \mathcal{P}(X)$, $R$ is full rank, stochastic, and satisfies $R^J = I$ for some $J \in \mathbb{N}^+$. A function $f : \mathcal{P}(X) \to \mathbb{R}$ is rotationally symmetric if $f(q) \equiv f(R(q))$ (for some rotation $R$).

**Assumption 3.** $H$ and $\hat{\mu}$ are rotationally symmetric.

This kind of symmetry holds in our example, as the utility function and Shannon’s entropy are both symmetric about $q = \frac{1}{2}$.

**Theorem 2.** Under Assumptions 2 and 3, there exists a $t_0 < T$ s.t. if $q_{t_0}$ is uniform, then $(P^*, I^*, \tau^*)$ is an optimal policy in the principal’s problem from $t_0$ onward.

**Proof.** See the appendix, section A.2

The role of the initial time $t_0$ is analogous to the role of Assumption 1 in our main analysis. At $t = t_0$, the agent’s cost of delay is high enough to ensure that the incentive compatibility constraint is binding for the principal.

5.3.1 Example (continued)

We illustrate Theorem 2 by continuing the numerical example in Sections 1.1 and 3.5. We maintain the same model primitives except that $\kappa(t)$ no longer constant. The top panel of Figure 4 depicts $\kappa(t)$: $\kappa(t)$ strictly increasing from $[0, 2]$ and stays constant afterwards. We choose $\kappa(t)$ take values close to 2 for the purpose of facilitating comparisons with our previous example, which is illustrated by the lower panel of Figure 4. The two black curves depict $\text{Supp}(\pi^*(\kappa(t)))$. The optimal policy involves the same dilution structure as in Theorem 1 but with posterior beliefs varying over time. If one interpret how extreme the posterior belief is as the decision quality, then decision quality falls over time under increasing time cost.

\[19\text{For example, any permutation matrix is a rotation. Therefore, any exchangeable function is rotationally symmetric.}\]
5.4 Bounds on Belief Revision

In this subsection, we consider the case in which there is an exogenous bound on how much the agent can revise his posterior belief. Formally, for $d > 0$, let $D^d$ denote the set of $\mathcal{P}(X)$-valued càdlàg functions whose jump size is bounded by $d$,

$$D^d = \{ f \in \Omega | \forall t \in \mathbb{R}_+, |f(t) - f(t^-)| \leq d \}.$$ 

Let $A^d(\bar{q}_0)$ denote the subset of $A(\bar{q}_0)$ whose strategies are supported within $D^d$. Consider the optimization problem:

$$J^d(\bar{q}_0) = \sup_{(P^H, \tau) \in A^d(\bar{q}_0)} \mathbb{E}^P[G(q_{\tau}) - G(\bar{q}_0)|\mathcal{F}_0]$$

s.t. $\tau \in \arg \max_{\tau' \in \mathcal{T}} \mathbb{E}^P[\hat{u}(q_{\tau'}) - \kappa \tau'|\mathcal{F}_0].$

Equation (6) is the same as Definition 1 except that the set of admissible strategies is restricted to have no jumps of size greater than $d$.

We begin our analysis of this restricted problem by observing that the agent-optimal benchmark, $V^B$, is unchanged. This follows from results in Hébert and Woodford [2023], who show that the agent-optimal policy can be implemented by a pure diffusion process. Let $E^A = \{ q \in \mathcal{P}(X) | V^B(q) > \hat{u}(q) \}$ be the continuation region of the agent-optimal
benchmark.

Now define the set of beliefs that can be reached by a jump of size no greater than $d$ from this continuation region:

$$\tilde{Q}^d = \left\{ q \in \mathcal{P}(X) \mid \inf_{q' \in E^A} |q - q'| \leq d \right\}.$$ 

Note that the definition of $\tilde{Q}^d$ is independent of the prior $\tilde{q}_0$.

As we argued previously, the beliefs $q_t$ must lie in the continuation region of the agent-optimal benchmark if $t < \tau$. Given that the maximum possible jump size is $d$, it follows that the stopping beliefs must lie in $\tilde{Q}^d$.

**Lemma 2.** If $(P, I^H, \tau)$ is admissible in Equation 6 then $\text{Supp}(q_\tau) \subset \tilde{Q}^d$.

**Proof.** See the appendix, section A.3.

Trivially, if the bound $d$ is sufficiently large, it does not restrict the principal, and the problem collapses to the problem considered in our main analysis.

**Proposition 6.** $\lim_{d \to \infty} J^d(\tilde{q}_0) = J(\tilde{q}_0)$.

**Proof.** For $d$ larger than the diameter of $\mathcal{P}(X)$, $\mathcal{A}^d(\tilde{q}_0) = \mathcal{A}(\tilde{q}_0)$.

Let us next consider the opposite case, in which the beliefs process must be continuous ($d = 0$). In this case, the only possible stopping beliefs are the ones on the boundary of the continuation region in the agent-optimal problem ($\tilde{Q}^0$ is the closure of $E^A$). If in addition there are only two states ($|X| = 2$), the locally invariant posteriors property implies that the stopping beliefs will be identical to those the agent would choose in the agent-optimal problem; the only alternative the principal could choose would involve less information acquisition by the agent.

**Proposition 7.** If $|X| = 2$, then $J^0(\tilde{q}_0)$ is achieved by an agent-optimal policy.

**Proof.** See the appendix, section A.4.

When there are more than two states ($|X| > 2$), the principal’s and agent’s optimal policies need not coincide. Consider as an example the case of two actions ($|A| = 2$). The agent-optimal policy will in this case involve a diffusion on a line segment within the probability simplex (see Hébert and Woodford [2023]). The principal cannot induce the agent to follow this line segment beyond its endpoints, but can send the agent signals that cause the agent’s beliefs to move orthogonal to this line segment.
5.5 Optimal Policy without Capacity Constraints

In this subsection, we consider a modified version of our model with a constant cost of delay in which the agent has an unlimited capacity to acquire information, and the principal’s goal is to maximize the time spent by the agent on the platform. This modified model is the continuous-time analog of the discrete-time models studied by Knoepfle [2020] and Koh and Sanguanmoo [2022]. The purpose of this subsection is to illustrate the connection between these models and a special case of our more general framework. The agent-optimal policy in the modified model is for the agent to learn the optimal action with certainty immediately, as this avoids entirely the cost of delay.

The principal in this case chooses his policies from the set \( \mathcal{A}(q_0) \), which is the set of probability measures on \((\Omega, \mathcal{F})\) such that \( q_0 = \bar{q}_0 \) and non-negative stopping times \( \tau \). Note that this set does not impose the constraint on the rate of information acquisition, (2), that was imposed in our main analysis. The principal solves

\[
J(\bar{q}_0) = \sup_{(P, \tau) \in \mathcal{A}(\bar{q}_0)} \mathbb{E}^P[\tau | \mathcal{F}_0]
\]

subject to the same constraint with respect to the agent’s stopping decision,

\[
\tau \in \arg \max_{\tau' \in \mathcal{T}} \mathbb{E}^P[\hat{u}(q_{\tau'}) - \int_0^{\tau'} \bar{k}dt | \mathcal{F}_0].
\]

The first part of Lemma 1 remains applicable: if \( \pi \) is the law of \( q_\tau \), we must have \( \mathbb{E}^\pi[\hat{u}(q) - \hat{u}(\bar{q}_0)] \geq \bar{k}\mathbb{E}^P[\tau | \mathcal{F}_0] \). Now observe that the expected utility under \( \pi \) is bounded above by the utility of fully learning the state. Let \( \pi_{\max} \in \mathcal{P}(X) \) be the unique probability measure that places full support on the extreme points of \( \mathcal{P}(X) \) (i.e. the \( e_x \) basis vectors), with \( \pi_{\max}(e_x) = \bar{q}_{0,x} \).

By the convexity of \( \hat{u} \), for all \( \pi \) such that \( E^\pi[q] = \bar{q}_0 \),

\[
\mathbb{E}^{\pi_{\max}}[\hat{u}(q) - \hat{u}(\bar{q}_0)] \geq \mathbb{E}^\pi[\hat{u}(q) - \hat{u}(\bar{q}_0)] \geq \bar{k}\mathbb{E}^P[\tau | \mathcal{F}_0] \geq \bar{k}J(\bar{q}_0).
\]

It follows that \( \bar{k}^{-1}\mathbb{E}^{\pi_{\max}}[\hat{u}(q) - \hat{u}(\bar{q}_0)] \) is an upper bound on the utility achievable in this problem.

But now observe that the \( \alpha \)-dilution of \( \pi_{\max} \), as defined in (5), with intensity \( \alpha = \bar{k}(\mathbb{E}^{\pi_{\max}}[\hat{u}(q) - \hat{u}(\bar{q}_0)])^{-1} \), achieves this bound. Moreover, the policy is incentive-compatible: at each instant, the agent compares the utility benefit of the signal’s arrival, \( \alpha\mathbb{E}^{\pi_{\max}}[\hat{u}(q) - \hat{u}(\bar{q}_0)] \).
\( \hat{u}(\tilde{q}_0) \), against the cost of delay, \( \bar{\kappa} \), and is willing to continue. It follows that this policy is optimal.

This policy is the optimal policy in a special case of our main model. Consider the case of our main model in which \( \bar{\kappa} = \chi \) and \( G(q) = H(q) = \hat{u}(q) \). In this special case, \( \pi^{\text{max}} \) is an optimal policy in our relaxed problem \([3]\), because the constraint in that problem satisfied for any policy, and the process described in Proposition \( \text{2} \) is exactly the process above. The intuition behind this equivalence is that in our main model, it is always optimal for the principal to exhaust the agent’s information processing capacity. If exhausting this capacity necessarily involves satisfying the incentive compatibility constraint, then the principal is free to choose the process that simultaneously exhausts the agent’s capacity and takes as long as possible to reach a decisive belief.

### 5.6 Engaging Test-Motivated Students

Our model was developed to describe the process of engagement maximization by an internet platform. However, the problem of engagement maximization appears in many contexts. In this subsection, we provide an alternative interpretation of our engagement maximization model in the context of a teacher (the principal) and a student (the agent). We first discuss this interpretation in the context of our general framework, and then provide a concrete example.

Let \( R \) be a set of possible responses to a test. Let the states of nature be \( X = T \times Q \), where \( T \) denotes the true state (a finite set) and \( Q \) is the set of possible questions (functions \( T \to R \) describing the correct response conditional on the true state). We will think of the test question realization \( t \in Q \) as unknown to both the student and teacher (e.g. the test is a standardized test outside the control of the teacher)\(^{20}\).

The action \( A \) is a function \( Q \to R \) that describes the student’s responses given the question. The student’s utility given action \( a \in A \) in state \( x = (\omega, t) \in T \times Q \) is

\[
 u_{a,x} = v \left( \sum_{i=1}^{N} 1\{a_i(t) = t_i(\omega)\} \right),
\]

where \( a_i(t) \) denotes the student’s response to question \( i \) and \( t_i(\omega) \) is the correct response to question \( i \) given the state \( \omega \), and \( v(\cdot) \) is an increasing function. That is, the student’s utility

\(^{20}\)A natural extension of this framework would be to consider the case in which the teacher designs and can reveal information about the test questions, in addition to revealing the truth. Exploring this case is beyond the scope of the present paper.
is an increasing function of the number of questions answered correctly.

The teacher commits to a teaching strategy without regards to the realization of \( \omega \), which determines the sequence of signals the student will receive\(^{21}\). The teacher’s objective is to maximize the student’s learning. Let \( q_\omega = \sum_{i \in Q} q_{i,\omega} \) denote the total probability of the true state \( \omega \in T \) under \( q \in \mathcal{P}(X) \). We will assume initially that the teacher’s goal is to maximize the expectation of the log of this probability.

The expected utility for the teacher when the teaching strategy is designed (prior to the state being revealed, so that the expectation is taken over stopping beliefs and \( \omega \)) is

\[
E^P[\ln(q_{\tau,\omega})] = E^\pi[\sum_{\omega \in T} q_{\tau,\omega} \ln(q_{\tau,\omega})]
\]

where \( \pi \) is the law of the posterior stopping beliefs \( q_{\tau} \) under \( P \). The rate at which the student can learn the truth is governed by the mutual information between her prior and posterior over the truth, and it is impossible for the student to acquire any information about the test questions. Define

\[
H(q) = \begin{cases} 
\sum_{\omega \in T} q_{\tau,\omega} \ln(q_{\tau,\omega}) & \text{proj}(q, \mathcal{P}(Q)) = \text{proj}(\bar{q}_0, \mathcal{P}(Q)) \\
\infty & \text{otherwise} 
\end{cases}
\]

This function is infinite when the projections of \( q \) and \( \bar{q}_0 \) onto \( \mathcal{P}(Q) \) are not equal, ensuring that the student will not acquire information about the test questions until she takes the test, and is otherwise equal to the negative of the Shannon’s entropy of the projection of \( q \in \mathcal{P}(X) \) onto \( \mathcal{P}(T) \). Under these assumptions, the information processing constraint is exactly that (2) in our main model, and the teacher’s objective is to maximize \( E^P[H(q_{\tau})] \).

Note that we have constructed this example so that the constraint and principal’s objective are governed by the same \( H \) function, in keeping with our main example\(^{22}\). If we assume that the student has a constant opportunity cost of time spent on the course equal to \( \bar{\kappa} \), then this teacher-student model is equivalent to our principal-agent model.

The inherent conflict in this model is that the student is concerned only about the test-relevant portions of the truth, whereas the teacher would like the student to learn both the

\(^{21}\)Committing to a teaching strategy before \( \omega \in T \) is revealed is analogous, in this context, to the assumption of commitment in Bayesian persuasion models. It avoids the need to consider strategic interactions between teachers with different “types” (knowledge of the truth).

\(^{22}\)This equivalence is a consequence of assuming the log-probability objective and is well-known in the proper scoring rule literature.
test-relevant and test-irrelevant aspects of the truth. The teacher must balance covering all aspects of the subject with the need to keep students engaged by providing test-relevant information. Our main results demonstrate that the teacher will drive the student to her reservation value (so that she is indifferent between studying and not) and cause the student to learn more (arrive at more extreme beliefs) than the student would choose for herself.

More subtly, our results show that the information the teacher provides is equivalent to the information the student would choose for herself if she had a lower opportunity cost of time (by Proposition 1 and Theorem 1 under the assumption that $G = H$). One implication of this result, which we highlight in the next example, is that the teacher will never induce the student to acquire test-irrelevant information (i.e. the teacher will “teach to the test”). The teacher optimally prefers to induce the student to acquire more test-relevant information than the student would choose to acquire on her own as opposed to inducing the student to acquire some test-irrelevant information. We illustrate this in the following example. We also show that this conclusion arises as a consequence of the alignment between the teacher’s objective and the student’s information processing constraint (i.e. that $G = H$).

5.6.1 Example (continued)

We adapt the example studied in Sections 1.1 and 3.5 to study engaging test-motivated students. Suppose the true state of the world $T$ consists of both a test-relevant and test-irrelevant dimension, $T = T_1 \times T_2 = \{L,R\} \times \{0,1\}$. The student and teacher know that the student will be asked a single question, $Q = \{Q_0\}$, whose responses are $R = \{l, r\}$, with $Q_0(L0) = Q_0(L1) = l$ and $Q_0(R0) = Q_0(R1) = r$. That is, the $\{L,R\}$ component of the true state is relevant, and the $\{0,1\}$ component is irrelevant. The student (agent) again faces a binary choice $A = \{l, r\}$. The student’s utility is one if she answers the question correctly and negative one otherwise, and therefore is identical (ignoring the test-irrelevant dimension) to that in Sections 1.1 and 3.5. The prior belief is uniform. We assume the agent’s information processing capacity is defined by the mutual information of state $i$ and the signal, and continue to use the parameters $\bar{\kappa} = 2, \chi = 1$ and $\rho = 1$.

**Engagement maximization.** We first study the benchmark case where the teacher maximizes the engagement of the student. This setting is a special case of our main model. By Proposition 1, the teacher’s optimal strategy maximizes
for some $\lambda < 2$. The student-optimal benchmark maximizes

$$\mathbb{E}^\pi [\hat{u}(q) - (2 - \lambda)(H(q) - H(\bar{q}_0))]$$

Both optimization problems can be written as:

$$\sup_{P(a|t_1,t_2)} \sum_{a,t_1,t_2} \frac{1}{4} u(a,t_1)P(a|t_1,t_2) - C \cdot (\sum_{a} \sum_{t_1,t_2} \frac{1}{4} P(a|t_1,t_2) \log(P(a|t_1,t_2))$$

$$- \sum_{a} (\sum_{t_1,t_2} \frac{1}{4} P(a|t_1,t_2)) \log(\sum_{t_1,t_2} \frac{1}{4} P(a|t_1,t_2))),$$

for some positive constant $C$. Note that replacing $P(a|t_1,t_2 = 0)$ and $P(a|t_1,t_2 = 1)$ with $\frac{P(a|t_1,0)+P(a|t_1,1)}{2}$ (denoted by $P(a|t_1)$) does not change the positive term while strictly reduces the negative term if $P(a|t_1,0)$ and $P(a|t_2,1)$ are not identical. Evidently, w.l.o.g., the optimization problem reduces to

$$\sup_{P(a|t_1)} \sum_{a,t_1} \frac{1}{2} u(a,t_1)P(a|t_1) - C \cdot (\sum_{a} \frac{1}{2} P(a|t_1) \log(P(a|t_1))$$

$$- \sum_{a,t_1} \frac{1}{2} P(a|t_1) \log(\sum_{t_1} \frac{1}{2} P(a|t_1))),$$

which is equivalent to a static rational inattention problem\footnote{This equivalence is special to mutual information and is the implication of a more general “compression invariance” property introduced by Caplin et al. [2022], Bloedel and Zhong [2020].} The analysis above implies that when there is a test-irrelevant state (which is costly to learn about), no information about the state will be acquired either in the student-optimal benchmark or in the principal-agent problem. This is interesting because the student learning about state $t_2$ enters the teacher’s payoff function (the measure of engagement), but the teacher optimally provides only test-related information to the student.

Figure 5 illustrates the analysis above. The tetrahedron depicts the probability simplex that lies in $\mathbb{R}^3$. Each vertex of the tetrahedron is a degenerate belief (labeled by the state that occurs with probability one). The red dot is the uniform prior. The hyperplane represents
the cutoff beliefs where the student is indifferent between actions \( l \) and \( r \). To the right of the hyperplane, \( q_R > q_L \) and \( r \) is optimal. Vice versa for the left of the hyperplane. The two blue dots are the student-optimal posterior beliefs. The two green dots are the solution to the principal-agent problem. Evidently, all the dots are one the same straight line, which illustrates that the teacher only releases test-related information to the student, but at a higher precision comparing to what the student prefers.

**Knowledge maximization.** Now, we turn to the case where the teacher only cares about how much the student knows about the state \( t_2 \in T_2 \) (the test-irrelevant dimension). This is a special case of \( G \neq H \) in the context of our more general framework. We assume that the teacher’s payoff is \( 2E[|q_{t_2} - 0.5|] \) if the measure of stopping belief is \( \pi \), where \( q_{t_2} \) denotes the probability of \( t_2 = 1 \) under the student’s stopping belief. Note that this payoff function is equivalent to the instrumental value of information in a hypothetical binary decision problem where the utility of matching (mismatching) the state \( y \) is 1 (-1). Proposition \( \square \) can be extended to show that the teacher’s optimal policy maximizes the following auxiliary problem (for some \( \lambda > 0 \)):

\[
\sup_{\pi} E[2|q_{t_2} - 0.5| + \lambda \hat{u}(q) - 2\lambda (H(q) - H(q_0))],
\]

which is equivalent to a hypothetical static RI problem where the teacher has four possible actions, whose utilities are
$u(a,t) \left| \begin{array}{ccccc} L0 & R0 & L1 & R1 \\ a1 & 1 + \lambda & 1 - \lambda & -1 + \lambda & -1 - \lambda \\ a2 & 1 - \lambda & 1 + \lambda & -1 - \lambda & -1 + \lambda \\ a3 & -1 + \lambda & -1 - \lambda & 1 + \lambda & 1 - \lambda \\ a4 & -1 - \lambda & -1 + \lambda & 1 - \lambda & 1 + \lambda \\ \end{array} \right.$

Per Matêjka et al. [2015], the solution is logit, given by

$$P(a|t) \equiv \frac{1}{1 + \lambda}$$

and the default rule is uniform $P(a) \equiv \frac{1}{1 + \lambda}$. Note that the conditional distribution,

$$P(a|t) \equiv \frac{1}{1 + \lambda}$$

is identical to the student-optimal solution (and independent to $\lambda$). The unknown parameter $\lambda$ can be pinned down by setting the student’s IC condition binding. The analysis above suggests that when the teacher only cares about the test-irrelevant knowledge, she will give the student exactly his preferred test-relevant information and extra knowledge such that the student gets barely enough welfare to be willing to participate.

Figure 6 illustrates the analysis above. Except the green dots, the figure is exactly the same as Figure 5. The green dots are the optimal posteriors of the principal-agent problem. For any pair of green dots on the same side of the hyperplane, they equal the blue dot in expectation. Their distance to the hyperplane is also the same as the blue dot, illustrating that the teacher provides the student his preferred test-relevant information together with extra test-irrelevant knowledge.
6 Conclusion

We have considered the problem of a principal who provides information to an agent so as to maximize the attention the agent pays to the principal’s information (engagement). The agent values this information for instrumental purposes, is rational and Bayesian, and faces a constraint on the rate at which she can process information. Our main results are (i) that by maximizing engagement, the principal leaves the agent no better off than if she could not receive any information at all, and (ii) that the agent will end up holding extreme beliefs, relative to a benchmark in which the agent could choose the information for herself. Our results highlight the pitfalls of presenting users with information for the purposes of maximizing engagement, a standard practice on popular internet platforms.

References

Daron Acemoglu, Asuman Ozdaglar, and James Siderius. Misinformation: Strategic sharing, homophily, and endogenous echo chambers. Technical report, National Bureau of Economic Research, 2021.

Alexander W Bloedel and Weijie Zhong. The cost of optimally-acquired information. Unpublished Manuscript, November, 2020.

Andrew Caplin, Mark Dean, and John Leahy. Rationally inattentive behavior: Characteriz-
ing and generalizing shannon entropy. *Journal of Political Economy*, 130(6):1676–1715, 2022.

Yeon-Koo Che, Kyungmin Kim, and Konrad Mierendorff. Keeping the listener engaged: a dynamic model of bayesian persuasion. *arXiv preprint arXiv:2003.07338*, 2020.

Ing-Haw Cheng and Alice Hsiaw. Bayesian doublespeak. *Available at SSRN*, 2022.

Vincent P Crawford and Joel Sobel. Strategic information transmission. *Econometrica: Journal of the Econometric Society*, pages 1431–1451, 1982.

Jeffrey Ely, Alexander Frankel, and Emir Kamenica. Suspense and surprise. *Journal of Political Economy*, 123(1):215–260, 2015.

Jeffrey C Ely. Beeps. *American Economic Review*, 107(1):31–53, 2017.

Jeffrey C Ely and Martin Szydlowski. Moving the goalposts. *Journal of Political Economy*, 128(2):468–506, 2020.

Alexander Frankel and Emir Kamenica. Quantifying information and uncertainty. *American Economic Review*, 109(10):3650–80, 2019.

Matthew Gentzkow, Jesse M Shapiro, and Daniel F Stone. Media bias in the marketplace: Theory. In *Handbook of media economics*, volume 1, pages 623–645. Elsevier, 2015.

Matthew Gentzkow, Jesse M Shapiro, Frank Yang, and Ali Yurukoglu. Pricing power in advertising markets: Theory and evidence. *American Economic Review*, 114(2):500–533, 2024.

Benjamin Hébert and Michael Woodford. Neighborhood-based information costs. *American Economic Review*, 111(10):3225–55, 2021.

Benjamin Hébert and Michael Woodford. Rational inattention when decisions take time. *Journal of Economic Theory*, 208:105612, 2023.

Johannes Hörner and Andrzej Skrzypacz. Selling information. *Journal of Political Economy*, 124(6):1515–1562, 2016.

Emir Kamenica and Matthew Gentzkow. Bayesian persuasion. *American Economic Review*, 101(6):2590–2615, 2011.
Kohei Kawamura and Mark T Le Quement. News begets news: A model of endogenously repeated costly consultation. 2019.

Jon Kleinberg, Sendhil Mullainathan, and Manish Raghavan. The challenge of understanding what users want: Inconsistent preferences and engagement optimization. *arXiv preprint arXiv:2202.11776*, 2022.

Jan Knoepfle. Dynamic competition for attention. *Working paper*, 2020.

Andrew Koh and Sivakorn Sanguanmoo. Attention capture. *arXiv preprint arXiv:2209.05570*, 2022.

Akos Lada, Meihong Wang, and Tak Yan. How machine learning powers Facebook’s news feed ranking algorithm, January 2021. URL https://engineering.fb.com/2021/01/26/ml-applications/news-feed-ranking/.

Elliot Lipnowski, Doron Ravid, and Denis Shishkin. Persuasion via weak institutions. *Journal of Political Economy*, 130(10):000–000, 2022.

Filip Matějka, Alisdair McKay, et al. Rational inattention to discrete choices: A new foundation for the multinomial logit model. *American Economic Review*, 105(1):272–98, 2015.

Daehong Min. Bayesian persuasion under partial commitment. *Economic Theory*, 72(3):743–764, 2021.

Dmitry Orlov, Andrzej Skrzypacz, and Pavel Zryumov. Persuading the principal to wait. *Journal of Political Economy*, 128(7):2542–2578, 2020.

Luciano Pomatto, Philipp Strack, and Omer Tamuz. The cost of information. *arXiv preprint arXiv:1812.04211*, 2018.

Jérôme Renault, Eilon Solan, and Nicolas Vieille. Optimal dynamic information provision. *Games and Economic Behavior*, 104:329–349, 2017.

Sequoia. News feed ranking drives engagement, 2018. URL https://medium.com/sequoia-capital/engagement-part-iv-activity-feed-ranking-40d786b9d479.
A Additional Proofs

A.1 Proof of Proposition 4

Proof. Observe first by Proposition 1 that \( J(\bar{q}_0) > 0 \) implies \( \lambda > 0 \) (where \( \lambda \) is defined as in that proposition).

Define

\[
U^a(q) = \hat{u}(q) - \frac{\bar{k}}{\chi} H(q)
\]

and define

\[
U^p(q) = \hat{u}(q) - \frac{\bar{k}}{\chi} H(q) + \lambda G(q).
\]

Define \( H^*(q) \) as the concave envelope of \( \frac{\bar{k}}{\chi} H(q) - \lambda G(q) \). Observe that any

\[
\pi^* \in \arg \max_{\pi \in \mathcal{P}(\mathcal{P}(X)): \mathbb{E}\pi[q] = \bar{q}_0} \mathbb{E}\pi \left[ \hat{u}(q) - \frac{\bar{k}}{\chi} H(q) + \lambda G(q) \right]
\]

places full support on points where \( H^*(q) = \frac{\bar{k}}{\chi} H(q) - \lambda G(q) \), by the convexity of \( \hat{u}(q) \), and for each \( q' \in \mathcal{P}(X) \) there exists a \( \pi' \in \mathcal{P}(\mathcal{P}(X)) \) with \( \mathbb{E}\pi'[q] = q' \) and

\[
\hat{u}(q') - H^*(q') \leq \mathbb{E}\pi' [\hat{u}(q) - \frac{\bar{k}}{\chi} H(q) + \lambda G(q)],
\]

and therefore

\[
\pi^* \in \arg \max_{\pi \in \mathcal{P}(\mathcal{P}(X)): \mathbb{E}\pi[q] = \bar{q}_0} \mathbb{E}\pi [\hat{u}(q) - H^*(q)].
\]
Now observe that $q^* \in Q^p(\bar{q}_0)$ and $\bar{J}(\bar{q}_0) > 0$ implies by Proposition[1] and Theorem[1] that there exists a $\pi^* \in \mathcal{P}(\mathcal{P}(X))$ with $q^* \in \text{Supp}\pi^*$ such that $\pi^*$ is a solution to a static rational inattention problem. By the Lagrangian lemma of Caplin et al. [2022] applied to the problem with the cost function $H^*$, there exists a linear function $L^p : \mathcal{P}(X) \rightarrow \mathbb{R}$ such that

$$\hat{u}(q) - H^*(q) - L^p(q) \leq \hat{u}(q^*) - H^*(q^*) - L^p(q^*)$$

with equality for all $q \in \text{Supp}\pi^*$. By $H^*(q) \geq \frac{k}{\lambda} H(q) - \lambda G(q)$, with equality for all $q \in \text{Supp}\pi^*$,

$$U^p(q) - L^p(q) \leq U^p(q^*) - L^p(q^*),$$

with equality for all $q \in \text{Supp}\pi^*$.

Note that $\hat{u}$ is piecewise linear and convex and both $H$ and $G$ are continuously differentiable. It follows that $\hat{u}$ can not involve a convex kink at any $q \in \text{Supp}\pi^*$, and hence if such a $q$ lies in the relative interior of $\mathcal{P}(X)$,

$$\nabla U^p(q) = \nabla L^p(q).$$

The same conclusions apply for any $q^+ \in Q^a(\bar{q}_0)$: there must exist an associated $\pi^+ \in \mathcal{P}(\mathcal{P}(X))$ and linear function $L^a(q)$ such that for all $q$,

$$U^a(q) - L^a(q) \leq U^a(q^*) - L^a(q^*),$$

with equality for all $q \in \text{Supp}\pi^+$, and with $\nabla U^p(q) = \nabla L^p(q)$ for all such $q$ in the relative interior of $\mathcal{P}(X)$.

We first prove that, under the assumption that $Q^a(\bar{q}_0) \subseteq \text{relint}(\mathcal{P}(X))$, there cannot exist a $q^* \in Q^p(\bar{q}_0) \cap Q^a(\bar{q}_0)$. Proof by contradiction: suppose such a $q^*$ exists, as part of a solution to the relaxed principal’s problem $\pi^* \in \mathcal{P}(\mathcal{P}(X))$. We must have, by the linearity of the functions $L^a$ and $L^p$,

$$\nabla L^p(q) - \nabla L^a(q) = \nabla G(q^*)$$
for all $q \in \mathcal{P}(X)$. Therefore,

$$
\mathbb{E}^{\pi^*} [U^p(q)] = U^p(q^*) + (\bar{q}_0 - q^*)(\nabla G(q^*) + L^a(q^*)) \\
= U^a(q^*) + G(q^*) + (\bar{q}_0 - q^*)(\nabla G(q^*) + L^a(q^*)) \\
= \mathbb{E}^{\pi^*} [U^a(q) + q \cdot \nabla G(q^*)] + G(q^*) - q^* \cdot \nabla G(q^*) \\
\leq \mathbb{E}^{\pi^*} [U^a(q) + G(q)] = \mathbb{E}^{\pi^*} [U^p(q)].
$$

Thus, we must have

$$
G(q^*) + \mathbb{E}^{\pi^*} [(q - q^*) \nabla G(q^*)] = \mathbb{E}^{\pi^*} [G(q)],
$$

but this requires (by $\bar{J}(\bar{q}_0) = E^{\pi^*} [G(q)] - G(\bar{q}_0) > 0$) that

$$
G(q^*) + (\bar{q}_0 - q^*) \nabla G(q^*) > G(\bar{q}_0),
$$

contradicting the convexity of $G$. We conclude that $q^* \in Q^p(\bar{q}_0)$ implies $q^* \notin Q^a(\bar{q}_0)$.

Now suppose $q^* \in Q^p(\bar{q}_0) \cap \text{Conv} Q^a(\bar{q}_0)$. Definitionally, there is some $\pi' \in \mathcal{P}(Q^a(\bar{q}_0))$ such that $\mathbb{E}^{\pi'} [q] = q^*$, and by $q^* \notin Q^a(\bar{q}_0)$,

$$
U^a(q^*) < \mathbb{E}^{\pi'} [U^a(q)].
$$

It follows by the convexity of $G$ that

$$
U^p(q^*) < \mathbb{E}^{\pi'} [U^a(q) + G(q^*]) \leq \mathbb{E}^{\pi'} [U^p(q)],
$$

which contradicts $q^* \in Q^p(\bar{q}_0)$. We conclude that $q^* \in Q^p(\bar{q}_0)$ implies $q^* \notin \text{Conv} Q^a(\bar{q}_0)$.

A.2 Proof of Theorem 2

Proof. By Assumption 5, it is w.l.o.g. to consider strategies whose marginal distribution of $(q, \tau)$, which we denote $F(q, \tau) \in \Delta(\Delta(X) \times \mathbb{R}_+)$, is rotationally symmetric, because any implementable process $q^*_{t}$ can be rotated by $R$ and remains implementable. Therefore, $\mathbb{E}^{F(q,t)}[q|t \geq t'] = \mathbb{E}^{F(q,\tau)} \left[ \sum_{j=0}^{J-1} \frac{1}{2} R_j q | t \geq t' \right] = \bar{q}_0$ for all $t' \geq t_0$. Now, consider the following relaxed problem for the principal:
\[
\max_{F \in \Delta(\mathcal{X} \times \mathbb{R}^+)} \rho \int_{\mathcal{X}} \int_{\tau^*} H(q) F(dq, dt) \\
\text{s.t.} \int_{\mathcal{X}} \int_{s \leq t} (H(q) - H(q_0)) F(dq, ds) \leq \chi \int_{s \leq t} (1 - F(s)) ds; \\
\int_{\mathcal{X}} \int_{s > t} (\tilde{\mu}(q) - \tilde{\mu}(q_0)) F(dq, ds) \geq \int_{s > t} K_t(s) dF(s),
\]

where \(K_t(s) = \int_t^s \kappa(\tau) d\tau\) and \(F(s) = \int_{\mathcal{X}} \int_{\tau \leq s} F(dq, ds)\). Note that the first inequality in equation (7) is a necessary condition for the information constraint and the second inequality in equation (7) is a necessary condition for the agent's IC.

Let \(F^*\) be the joint distribution of \((q^*_\tau, \tau^*)\) under the policies \((P^*, I^*, \tau^*)\). We prove that \(F^*\) solves equation (7) and hence that the policies \((P^*, I^*, \tau^*)\) achieve weakly higher utility than any other policies.

We first write down the Lagrangian: let \(\lambda(t)\) and \(\gamma(t)\) be the multipliers of the first and second constraints, respectively,

\[
\mathcal{L} = \rho \int_{\mathcal{X}} \int_{0}^{\infty} H(q) F(dq, dt) + \chi \int_{\mathcal{X}} \int_{0}^{\infty} \left( \int_{0}^{t} \Lambda(s) ds \right) F(dq, dt) \\
- \int_{\mathcal{X}} \int_{0}^{\infty} \Lambda(t) H(q) F(dq, dt) \\
+ \int_{\mathcal{X}} \int_{0}^{\infty} \Gamma(t) \tilde{\mu}(q) F(dq, dt) - \int_{0}^{\infty} \left( \int_{0}^{t} \gamma(s) K_s(t) \right) F(dt) \\
+ \int_{\mathcal{X}} \int_{0}^{\infty} \xi(q, t) F(dq, dt) + \nu (1 - \int_{\mathcal{X}} \int_{0}^{\infty} F(dq, dt)),
\]

where \(\Lambda(t) = \int_{t}^{\infty} \lambda(s) ds\) and \(\Gamma(t) = \int_{0}^{t} \gamma(s) ds\). Note that we normalize \(H(q_0) = \tilde{\mu}(q_0) = 0\) to simplify notations. It is sufficient to show that \(F^*\) together with a set of multipliers jointly maximize the Lagrangian. Since the Lagrangian is linear in \(F\), it is sufficient to verify the FOC: \(\forall (\mu, t)\) in the support of \(F^*:\)

\[
\rho H(q) + \chi \int_{t_0}^{t} \Lambda(s) ds - \Lambda(t) H(q) + \Gamma(t) \tilde{\mu}(q) - \int_{t_0}^{t} \gamma(s) K_s(t) ds = \nu \quad (8)
\]

Note that the characterization applies to the constant \(\kappa\) case as well: there exist \(\Lambda_t\) and \(\Gamma_t\).
(both are constant fixing $t$) s.t.

\[
\begin{cases}
\nabla H(q)(\rho - \Lambda(t)) + \Gamma(t)\nabla \tilde{\mu}(q) = 0 \quad \forall q \text{ in the support of } \pi^*(\kappa(t)) \\
\chi \Lambda_t - \Gamma_t \kappa(t) = 0
\end{cases}
\]

The first equality is from concavification. The second equality is from the FOC. Now define $\Lambda(t)$ and $\Gamma(t)$:

\[
\begin{cases}
\frac{\rho - \Lambda(t)}{\Gamma(t)} = \frac{\rho - \Lambda_t}{\Gamma_t} \\
\chi \Lambda(t) - \Gamma(t) \kappa(t) + \gamma(t) \mathbb{E} \pi^*(\kappa(t))[\tilde{\mu}(q)] + \lambda(t) \mathbb{E} \pi^*(\kappa(t))[H(q)] = 0
\end{cases}
\]

Replace $\Lambda(t)$ with $\rho + \left(\frac{\kappa(t)}{\chi} - \frac{\rho}{\Gamma_t}\right) \Gamma(t)$, we get an ODE for $\Gamma(t)$:

\[
\rho \chi - \frac{\rho \chi}{\Gamma_t} \Gamma(t) - \mathbb{E} \pi^*(\kappa(t))[H(q)] \left(\frac{\kappa(t)}{\chi} + \frac{\rho \Gamma_t}{\Gamma_t^2}\right) \Gamma(t) + \\
\left(\mathbb{E} \pi^*(\kappa(t)) \left[\tilde{\mu}(q) - \left(\frac{\kappa(t)}{\chi} - \frac{\rho}{\Gamma_t}\right) H(q)\right]\right) \gamma_t = 0
\]

Note that the IC implies $\left(\mathbb{E} \pi^*(\kappa(t)) \left[\tilde{\mu}(q) - \frac{\kappa(t)}{\chi} H(q)\right]\right) = 0$. Reorganizing terms, we get:

\[
\gamma(t) = \frac{\Gamma_t}{\rho \mathbb{E} \pi^*(\kappa(t))[H(q)]} \left(\rho \chi \left(\frac{\Gamma(t)}{\Gamma_t} - 1\right) + \mathbb{E} \pi^*(\kappa(t))[H(q)] \left(\frac{\kappa(t)}{\chi} + \frac{\rho \Gamma_t}{\Gamma_t^2}\right) \Gamma(t)\right).
\]

The ODE with initial conditions $\gamma(T) = 0$, $\Gamma(T) = \Gamma_T$ satisfies the Lindelof condition and has a unique solution. Note that when $\Gamma(t) = \Gamma_t$, $\gamma(t) = \frac{\kappa(t)}{\rho \chi} \Gamma_t^2 + \Gamma_t > 0$, so $\Gamma(t)$ stays below $\Gamma_t$. When $\Gamma(t) = 0$, $\gamma(t) < 0$, so $\Gamma(t)$ stays above 0. Therefore, there exists $\tau < T$ s.t. on $[\tau, T]$, $\Gamma(t) \geq 0$ and weakly increases.

By the construction of $\tau$, for all $t_0 \geq \tau$, there exist multipliers $\lambda(t)$ and $\gamma(t)$ on $[t_0, T]$ s.t. the Lagrangian $\mathcal{L}$ is maximized jointly by $(\lambda(t), \gamma(t))$ and $F^*$.

\section{A.3 Proof of Lemma 2}

\textit{Proof.} Define:

\[
\tau' = \tau \land q_t \text{ first leaves } E^A.
\]
By definition $\tau' \leq \tau$. Since $\text{Supp}(P) \subset D^d$ and $\text{Supp}(q_{\tau'}) \subset E^A$, $\text{Supp}(q_{\tau'}) \subset \bar{Q}^d$. We prove by contradiction that $\tau' = \tau$. Suppose $\tau' < \tau$ on a positive measure, on which

$$
\mathbb{E}^P [\tilde{u}(q_{\tau}) - (\tau - \tau') \cdot \bar{k} | \tau' < \tau] = \mathbb{E}^P \mathbb{E} [\tilde{u}(q_{\tau}) - (\tau - \tau') \cdot \bar{k} | \tau' < \tau] < \mathbb{E}^P [\tilde{u}(q_{\tau'}) | \tau' < \tau].
$$

The inequality is from the fact that $\tau' < \tau \implies q_{\tau'} \not\in E^A$. Therefore, $\tau'$ strictly improves upon $\tau$ and $\tau$ is not incentive compatible. The contradiction implies that $\text{Supp}(q_{\tau'}) \subset \bar{Q}^d$.

\[QED\]

### A.4 Proof of Proposition 7

\textit{Proof.} As is discussed in Section [3] the agent-optimal policy can be solved by concavifying $\tilde{u}(q) - \frac{\bar{k}}{\bar{\kappa}} H(q)$. Therefore, there exists a linear function $L(q)$ that is weakly higher than $\tilde{u}(q) - \frac{\bar{k}}{\bar{\kappa}} H(q)$ and tangents it at two beliefs $q^1 < \bar{q}_0 < q^2$. WLOG, let $q^1$ and $q^2$ be the smaller and largest such beliefs, respectively. Since $\tilde{u} - \frac{\bar{k}}{\bar{\kappa}} H$ is piece-wise strictly concave, the interval $[q^1, q^2]$ is bounded away from the rest of $E^A$.

Proposition [2] implies that any admissible principal’s strategy has $\text{Supp}(q_{\tau}) \subset \bar{Q}^0$, which is the closure of $E^A$. Moreover, any continuous path that starts from $\bar{q}_0$ and ends outside of $[q^1, q^2]$ leaves $E^A$; hence, it is not admissible. Therefore, $\text{Supp}(q_{\tau}) \subset [q^1, q^2]$.

Therefore, $J^0(q^0)$ is bounded above by the following relaxed problem:

$$
\sup_{\pi \in \mathcal{P}(A([q^1, q^2]))} \mathbb{E}^\pi [\rho(G(q) - G(\bar{q}_0))].
$$

Since $G$ is strictly convex, the relaxed problem is solved by $\pi^*$ with support $\{q^1, q^2\}$ (such $\pi^*$ is unique). By Hébert and Woodford [2023], there exists a Gaussian process that implements $\pi^*$ and satisfies the information constraint. Note that this Gaussian process also implements the agent maximal continuation payoff (the upper concave hull of $\tilde{u}(q) - \frac{\bar{k}}{\bar{\kappa}} (H(q) - H(q_t))$) for every interim belief; hence, it is incentive compatible. \[QED\]