IIB horizons

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Received 18 June 2013, in final form 23 August 2013
Published 19 September 2013
Online at stacks.iop.org/CQG/30/205004

Abstract
We solve the Killing spinor equations for all near-horizon IIB geometries which preserve at least one supersymmetry. We show that generic horizon sections are eight-dimensional almost Hermitian spin$^c$ manifolds. Special cases include horizon sections with a Spin$^7$ structure and those for which the Killing spinor is pure. We also explain how the common sector horizons and the horizons with only 5-form flux are included in our general analysis. We investigate several special cases mainly focusing on the horizons with constant scalars admitting a pure Killing spinor and find that some of these exhibit a generalization of the 2-SCYT condition that arises in the horizons with 5-form fluxes only. We use this to construct new examples of near-horizon geometries with both 3-form and 5-form fluxes.

PACS numbers: 04.65.+e, 11.30.Pb, 04.70.-s

1. Introduction

In understanding the physics of black holes and branes, and in AdS/CFT, horizons have played a central role. In most applications, the horizons are Killing\textsuperscript{4}, and in the extreme case can be investigated in a unified way because there is a local model for the near-horizon fields [1, 2]. Typically, some additional assumptions are made depending on the problem. The most common assumption is that the metric and all the other form field strengths\textsuperscript{5} are taken to be smooth. This assumption includes many black hole horizons, as well as the horizons of some branes, like the D3-, M2- and M5-branes. Therefore it is expected that a systematic understanding of all near-horizon geometries will have applications in AdS/CFT [3] as well as in the construction of IIB back hole solutions which exhibit horizons with exotic topology and geometry, see e.g. [4–16] for earlier related works.

\textsuperscript{4} This means that they admit a time-like Killing vector field which becomes null at a spacetime hyper-surface.
\textsuperscript{5} Non-smooth near-horizon geometries can also be investigated in the special case for which there exists a frame in which the metric is smooth. This class includes all branes. In this case, one works in a frame with respect to which the metric is smooth, and allow singularities in the other fields, such as the scalars of the theory. If all fields are smooth, the choice of frame is not essential.
In this paper, we solve the Killing spinor equations (KSEs) of IIB supergravity for all near-horizon geometries preserving at least one supersymmetry and determine the geometry of the horizon sections. This extends the results on M-horizons [17] and heterotic horizons [18] to IIB supergravity. In the investigation of the KSEs for IIB horizons three cases arise depending on the choice of Killing spinor: (i) the generic horizons, (ii) the Spin(7) horizons and (iii) the pure SU(4) horizons. We find that the sections $S$ of generic near-horizon geometries are eight-dimensional almost Hermitian spin $c$ manifolds, and the sections of the Spin(7) horizons admit a Spin(7) structure. The sections of the pure SU(4) horizons, which admit a pure Killing spinor, are again eight-dimensional almost Hermitian spin, manifolds. The parallel transport equation on the horizon sections may mix all three types globally. In all the above cases, we also examine the implications on the topology and geometry of horizon sections depending on whether the IIB scalars take values in the hyperbolic upper half-plane or the hyperbolic upper half-plane after an SL(2, Z) U-duality identification.

We further investigate near-horizon geometries with constant scalars. The structure group of horizon sections both for generic and pure SU(4) horizons reduces further to a subgroup of SU(4). Therefore in these two cases, the near-horizon sections are almost Hermitian spin manifolds with an SU(4) structure. In all IIB horizons without further restrictions on the fields, the KSEs do not impose additional restrictions on the geometry of the horizon sections. We also explore several examples mostly focusing on the pure SU(4) horizons with constant scalars. We have shown in [20] that such horizons with only 5-form flux are 2-strong Calabi–Yau manifolds with torsion (2-SCYT), i.e. they are Hermitian manifolds equipped with the unique compatible connection $\tilde{\nabla}$ with skew-symmetric torsion $\tilde{\mathcal{H}}$ such that $d(\omega \wedge \tilde{\mathcal{H}}) = 0$, or equivalently $\delta \delta \omega^2 = 0$, and $\rho(\tilde{\nabla}) = 0$, where $\omega$ is the Hermitian form and $\rho(\tilde{\nabla})$ is the Ricci form of $\tilde{\nabla}$. For horizons with both 3- and 5-form fluxes, we find a modification of the 2-SKT condition, $\delta \delta \omega^2 = 0$, by a source term which depends on the 3-form fluxes, see (6.5). We use this to give new examples of horizons with 3- and 5-form fluxes. We also describe how the horizons with only 5-form flux and the horizons of the common sector arise as special cases of our IIB horizons.

To prove the above results, we first demonstrate that the KSEs are integrable along the lightcone directions giving rise to a system of differential and algebraic equations on the horizon section $S$. We find using the field equations, Bianchi identities and the bilinear matching condition that this system can be considerably simplified giving rise to a parallel transport equation and an algebraic equation on the horizon sections associated with the gravitino and dilatino KSEs, respectively. These two equations are solved using spinorial geometry [21], and as we have already mentioned, there are three cases to consider: the generic horizons, the Spin(7) horizons, and the horizons with a pure Killing spinor. The geometry of the horizon sections is next investigated by computing the Killing spinor bilinears, and in particular those which are associated with nowhere vanishing forms on the horizon sections. In the analysis above, the compactness properties of section $S$ have not been used. Because of this, the results apply to both brane and black hole horizons. However in the construction of examples, like those associated with the deformation of the 2-SKT condition, we take $S$ to be compact without boundary as expected for black hole horizon sections.

This paper has been organized as follows. In section 2, we state the near-horizon metric and the form fluxes of IIB supergravity and decompose the field equations and Bianchi identities

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6 This is similar to the three local geometries that arise in the solution of the KSEs for IIB backgrounds preserving one supersymmetry in [19].

7 Of course to find solutions, one has to impose in addition the Bianchi identities and field equations.

8 The bilinear matching condition is the identification of the stationary Killing vector field of a black hole with the Killing spinor vector bilinear.
along the lightcone and horizon section $S$ directions. Moreover we derive which equations are independent. In section 3, we integrate the KSEs along the lightcone directions and use the field equations, Bianchi identities and the bilinear matching condition to derive the independent KSEs. In section 4, we solve the independent KSEs using spinorial geometry and express the fluxes in terms of the geometry. Moreover, we derive the geometry of the horizon section and describe some aspects of their topology. In section 5, we explain how the horizons with only 5-form flux and those of the common sector are included as special cases in our analysis. In section 6, we explore the geometry of horizon sections admitting a pure Killing spinor and demonstrate that in the complex case the 2-SKT conditions of 5-form horizons is deformed with the 3-form fluxes. In section 7, we state our conclusions, and in the appendix we give the solution of the linear system associated with IIB horizons.

2. Fields and dynamics near a horizon

2.1. Fields and KSEs

To describe the form field strengths of IIB supergravity, consider the complex line bundle $\lambda$ over the spacetime. This is the pull-back of the canonical bundle of either the coset space $\mathbb{H} = SU(1,1)/U(1)$ or after a U-duality identification of $SL(2,\mathbb{Z})\backslash \mathbb{H}$ with respect to the dilaton and axion, the two scalar fields of the theory, $\mathbb{H}$ is identified with the hyperbolic upper half-plane which is a manifold and $SL(2,\mathbb{Z})\backslash \mathbb{H}$ is identified with the fundamental domain of the moduli space of complex structures of a 2-torus which is an orbifold.

The field strengths of the bosonic fields of IIB supergravity are a $\lambda^2$-valued complex 1-form $P$, a $\lambda$-valued complex 3-form $G$ and a self-dual 5-form $F$. $P$, $G$ and $F$ are the field strengths of two scalars, two 2-form gauge potentials and a 4-form gauge potential, respectively. Assuming that the horizons under investigation are killing horizons, and under some regularity assumptions, the IIB fields in the extreme near-horizon limit can be expressed as

$$\mathrm{d}s^2 = 2\epsilon^+ \epsilon^- + \delta_{ij} \epsilon^i \epsilon^j = 2 \, \mathrm{d}u (\mathrm{d}r + rh - \frac{1}{2} r^2 \Delta \, \mathrm{d}u) + \gamma_{ij} \, \mathrm{d}y^i \wedge \mathrm{d}y^j,$$

$$F = \epsilon^+ \wedge X + \epsilon^- \wedge Y + *_{\mathbb{R}} Y,$$

$$G = \epsilon^+ \wedge L + \epsilon^- \wedge \Phi + H,$$

$$P = \xi,$$

where we have introduced the frame

$$\epsilon^{+} = du, \quad \epsilon^{-} = dr + rh - \frac{1}{2} r^2 \Delta \, du,$$

$$\epsilon^{i} = e^{i} \, \mathrm{d}y^{i},$$

and used the self-duality $^9$ of $F$,

$$F_{M_1 M_2 M_3 M_4 M_5} = -\frac{1}{5!} \epsilon_{M_1 M_2 M_3 M_4 M_5} N_{I_1 N_1 N_2 N_3 N_4} N_{I_2 N_1 N_2 N_3 N_4} F_{I_1 N_1 N_2 N_3 N_4},$$

which also requires that

$$X = - *_{\mathbb{R}} X.$$

IIB supergravity also admits a $U(1)$ connection of $\lambda$, denoted by $Q$. Upon restriction to the near-horizon geometry,

$$Q = \Lambda,$$

where $\Lambda$ is $r, u$-independent, i.e. $Q_+ = Q_- = 0$, and so $\Lambda$ is a connection of $\lambda$ restricted to $S$.

$^9$ We choose $\epsilon_{123456789} = 1$ and the spacetime volume form is related to that of the horizon section as $\mathrm{dvol}(M) = \epsilon^+ \wedge \epsilon^- \wedge \mathrm{dvol}(S)$. In particular, $(*_{\mathbb{R}} Y)_{IJ_1 J_2 J_3 J_4 J_5} = \frac{1}{5!} \epsilon_{I_1 J_1 J_2 J_3 J_4 J_5} N_{I_2 N_1 N_2 N_3 N_4} Y_{I_1 N_1 N_2 N_3 N_4}$.
The dependence on the coordinates $u$ and $r$ is explicitly given, and so $h$, $\Delta$, $X$, $Y$, $L$, $\Phi$, $H$, $\Lambda$ and $\xi$ depend only on the coordinates $y^i$ of the horizon section $S$ which is the co-dimension 2 subspace given by $r = u = 0$. In particular, the scalars depend only on $y$, and $\xi$ is a section of $\lambda^2 \otimes \Lambda^1(S)$, where we have restricted $\lambda$ to $S$. Similarly, $L$, $\Phi$ and $H$ are sections of $\lambda \otimes \Lambda^4(S)$, $\lambda \otimes \Lambda^1(S)$ and $\lambda \otimes \Lambda^3(S)$, respectively, and $X$ and $Y$ are sections of $\Lambda^4(S)$ and $\Lambda^3(S)$, respectively. Observe that $V = \frac{\partial}{\partial u}$ is the horizon timelike Killing vector which becomes null at $r = 0$. The derivation of (2.1)–(2.4) is similar to that of other near-horizon geometries with form fluxes and we shall not elaborate here.

### 2.2. Bianchi identities and field equations

Before proceeding to an analysis of the KSEs, it is useful to evaluate the Bianchi identities and the field equations of IIB supergravity [22, 23] on the near-horizon fields (2.1)–(2.4). First consider the Bianchi identity

$$dF - \frac{i}{8} G \wedge \tilde{G} = 0. \quad (2.9)$$

This implies that

$$X = d_0 Y - \frac{i}{8} (\Phi \wedge \tilde{H} - \tilde{\Phi} \wedge H), \quad d_0 Y = d Y - h \wedge Y, \quad (2.10)$$

or equivalently

$$d \ast_8 Y = \frac{i}{8} H \wedge \tilde{H}, \quad (2.11)$$

Next, the Bianchi identity

$$dG - i Q \wedge G + P \wedge \tilde{G} = 0, \quad (2.13)$$

implies that

$$L = d_0 \Phi - i \Lambda \wedge \Phi + \xi \wedge \tilde{\Phi}, \quad (2.14)$$

and

$$dH = i \Lambda \wedge H - \xi \wedge \tilde{H}. \quad (2.15)$$

So again $L$ is not independent and can be expressed in terms of the other fluxes.

The Bianchi identity

$$dP - 2i Q \wedge P = 0, \quad (2.16)$$

implies

$$d\xi = 2i \Lambda \wedge \xi, \quad (2.17)$$

Note that $\lambda$ is topologically trivial if it is the pull-back of a line bundle over the hyperbolic upper half-plane which is a contractible space. This will also be the case after an $SL(2, \mathbb{Z})$ U-duality identification of the IIB scalars as the $j$-function maps the fundamental domain homeomorphically to $\mathbb{C}$, unless two copies are glued together to a sphere as in the case of 24 cosmic strings [25]. In both cases, $\lambda$ may be equipped with a connection with non-vanishing curvature.
and the Bianchi identity
\[ dQ = -iP \wedge \tilde{P}, \] (2.18)
implies
\[ d\Lambda = -i\xi \wedge \tilde{\xi}. \] (2.19)

We remark that Bianchi identities of IIB supergravity (2.9) and (2.13) imply two additional identities on the near horizon fields involving \( dX \) and \( dL \), however, these are in fact implied by the remaining Bianchi identities (2.10), (2.11), (2.14), (2.15), (2.17), (2.19). As a consequence, the latter are necessary and sufficient conditions imposed by the IIB Bianchi identities on the near horizon fields.

Next we turn to the bosonic field equations. Substituting the near-horizon fields into the 2-form gauge potentials field equations
\[ \nabla^C G_{ABD} - iQ^C G_{ABD} - P^C \hat{G}_{ABD} + \frac{2i}{3} F_{ABCD} G^{CD} = 0, \] (2.20)
one finds that
\[ \tilde{\nabla}^i \Phi_i - i\Lambda'^i \Phi_i - \xi'^i \Phi_i + \frac{2i}{3} Y_{ij\ell\ell'} X^{ij\ell\ell'} = 0, \] (2.21)
\[ \Rightarrow \tilde{\nabla}^i L_{ij} = i\Lambda'^i L_{ij} + h^i L_{ij} - \frac{1}{2} \delta h^i H_{ij} + \xi'^i \tilde{L}_{ij} + \frac{2i}{3} (X_{m\ell\ell'} H_{m\ell\ell'} - 3Y_{m\ell\ell'} L_{m\ell\ell'}) = 0, \] (2.22)
and
\[ \tilde{\nabla}^i H_{ij} - i\Lambda'^i H_{ij} + h^i H_{ij} + \xi'^i \tilde{H}_{ij} + \frac{2i}{3} (Y_{ij\ell\ell'} H_{ij\ell\ell'} - 3Y_{ij\ell\ell'} \Phi^\ell) = 0. \] (2.23)
Similarly, the field equation of the scalars
\[ \nabla^A P_A - 2iQ^A P_A + \frac{1}{3} G_{N_i N_j N_k} G^{N_i N_j N_k} = 0, \] (2.24)
implies
\[ \tilde{\nabla}^i \xi_i - 2i\Lambda'^i \xi_i - h^i \xi_i + \frac{1}{3} (\xi^i - 6 \Phi^i \Phi_i + H_{i\ell\ell'} H^\ell\ell') = 0. \] (2.25)
Note that there is no independent field equation for \( F \) because \( F \) is self-dual.

It remains to investigate the Einstein equation
\[ R_{AB} - \frac{1}{6} F_{AC} F_{BD} - \frac{1}{4} G_{(A} N_{B) N_i N_j} + \frac{1}{36} g_{AB} G_{N_i N_j N_k} G^{N_i N_j N_k} - 2P_{(A} \tilde{P}_{B)} = 0. \] (2.26)
Substituting the near-horizon fields, one finds that \( + \) - component gives
\[ \frac{1}{2} \tilde{\nabla}^i h_{i\ell} - \Delta = - \frac{1}{2} h^2 + \frac{2}{3} X_{i\ell\ell'} X^{i\ell\ell'} + \frac{2}{3} \Phi^i \Phi_i + \frac{1}{36} H_{i\ell\ell'} \tilde{H}^i\ell\ell' = 0. \] (2.27)
Similarly, the \( + i, j \) and \( + + \) components imply that
\[ - \frac{1}{2} \tilde{\nabla}^i d_h_{j\ell} - \frac{1}{2} h_{j\ell} h^i - \tilde{\nabla}^i \Delta + \Delta h_i + \frac{1}{3} X_{i\ell\ell'} X^{i\ell\ell'} - \frac{1}{2} (L_{i\ell\ell'} \tilde{H}^i\ell\ell' - 2 \Phi^i \Phi_i + L_{i\ell\ell'} H^i\ell\ell' - 2 \tilde{\Phi}^i \Phi_i) = 0, \] (2.28)
\[ \tilde{R}_{ij} + \tilde{\nabla}^i (h_{i\ell}) - \frac{1}{2} h_{i\ell} h_{j\ell} + 4 Y_{i\ell\ell'} Y^{i\ell\ell'} + \frac{1}{3} \Phi^i \Phi_i - 2 \tilde{\xi}_{i\ell} \tilde{\xi}_{j\ell} - \frac{1}{2} H_{i\ell\ell'} \tilde{H}_{j\ell\ell'} + \frac{1}{36} H_{i\ell\ell'} \tilde{H}^i\ell\ell' = 0, \] (2.29)
and
\[ \frac{1}{2} \tilde{\nabla}^2 \Delta - \frac{1}{2} h^\ell \tilde{\nabla}^\ell \Delta - \frac{1}{2} \Delta \tilde{\nabla}^i h_i + \Delta h^2 + \frac{1}{4} d_h_{i\ell} d_h_{j\ell} - \frac{1}{2} X_{i\ell\ell'} X^{i\ell\ell'} - \frac{1}{4} L_{i\ell\ell'} L_{j\ell\ell'} = 0, \] (2.30)
respectively, where \( \tilde{R} \) is the Ricci tensor of \( \tilde{S} \).
Not all of the above field equations are independent. In particular, on taking the divergence of the field equation (2.23), and making use of the Bianchi identities (2.10), (2.11), (2.14), (2.15), (2.17), the field equations (2.19), (2.23), and the anti-self-duality of $X$ (2.7), one obtains (2.22). Also, on computing the Einstein tensor of $S$ using the field equations (2.29) and (2.27), and evaluating the Einstein tensor Bianchi identity, one obtains the field equation (2.28) upon making use of the Bianchi identities (2.10), (2.11), (2.14), (2.15), (2.17), and the field equations (2.21), (2.23), (2.25). Furthermore, on taking the divergence of the field equation (2.28), one also obtains the field equation (2.30), after making use of anti-self-duality condition (2.7) and Bianchi identity (2.10) to compute $d * X$, the Bianchi identity (2.14) to compute $d L$, and also the field equations (2.28) and (2.23). Hence it follows that the field equations (2.22), (2.28) and (2.30) are implied by the other field equations and Bianchi identities.

3. KSEs on IIB horizons

3.1. Lightcone integrability of KSEs

The gravitino and dilatino KSEs of IIB supergravity [22, 23] are

\[ \left( \nabla_M - \frac{i}{2} Q_M + \frac{i}{48} \nabla_M N_1 N_2 N_3 N_4 \right) \epsilon - \frac{1}{96} \left( \Gamma_M^{N_1 N_1} G_{N_1 N_2 N_3} - 9 G_{M N_1 N_2} \Gamma^{N_1 N_2} \right) C * \epsilon = 0, \]

\[ (P_M \Gamma^M C * \epsilon) + \frac{1}{32} G_{N_1 N_2 N_3} \Gamma^{N_1 N_2 N_3} \epsilon = 0. \]

For our conventions see [19].

As in the analysis of KSEs for near-horizon geometries in other supergravity theories, the IIB KSEs can be integrated along the lightcone directions. In particular solving the—component of the gravitino KSE, one finds

\[ \epsilon_+ = \phi_+ \]

\[ \epsilon_- = \phi_- + r \Gamma_0 \left( \frac{1}{4} h^i \Gamma^i + \frac{i}{12} Y_M^{(N_{1234})} \Gamma^{N_{1234}} \right) \phi_+ + r \Gamma_0 \left( \frac{1}{96} H_{i_{1234}} \Gamma^{i_{1234}} + \frac{3}{16} \Phi_i \Gamma^i \right) C * \phi_+, \]

where $\phi_\pm$ do not depend on $r$. Similarly, the solution of the $+$ component of the gravitino KSE gives

\[ \phi_+ = \eta_+ + u \Gamma_0 \left( \frac{1}{4} h^i \Gamma_i - \frac{i}{12} Y_M^{(N_{1234})} \Gamma^{N_{1234}} \right) \eta_- + u \Gamma_0 \left( \frac{1}{96} H_{i_{1234}} \Gamma^{i_{1234}} - \frac{3}{16} \Phi_i \Gamma^i \right) C * \eta_- \]

\[ \phi_- = \eta_- , \]

where $\eta_\pm$ do not depend on the $u$ and $r$ coordinates. Furthermore, $\eta_+$ and $\eta_-$ must also satisfy a number of algebraic conditions. To simplify these, we shall first identify the 1-form associated with the stationary Killing vector field $V = \partial_u$ of horizons with the 1-form $Z$ constructed as a Killing spinor bilinear.

3.1.1. Vector bilinear matching condition. The 1-form Killing spinor bilinear\(^{11}\) is

\[ Z = \langle B(\epsilon^*)^\dagger, \Gamma_A \epsilon \rangle e^A = \langle \Gamma_{0\epsilon}, \Gamma_A \epsilon \rangle e^A, \]

\(^{11}\)There are other $\lambda$-valued 1-form bilinears which can be constructed from the Killing spinor $\epsilon$. However, these cannot be identified with $V$ as they are twisted 1-forms and generically are not associated with a Killing vector.
where \( \epsilon \) is a Killing spinor and \( \langle \cdot, \cdot \rangle \) is the standard Hermitian inner product, see [19]. We require that \( Z \) should be proportional to \( V \), where

\[
V = -\frac{1}{2} r^2 \Delta e^\tau + e^-.
\]

First, evaluate \( Z \) at \( r = u = 0 \), for which \( \epsilon = \eta_+ + \eta_- \). Requiring that \( Z_+ = 0 \) at \( r = u = 0 \) forces

\[
\eta_- = 0.
\]

Then, using \( r, u \) independent Spin(8) gauge transformations of the type considered in [19], one can, without loss of generality, take

\[
\eta_+ = p + q e^{1234},
\]

for \( p, q \) complex valued functions. Furthermore, on computing the component \( Z_- \), one finds that \(|p|^2 + |q|^2\) must be a (non-zero) constant, or equivalently,

\[
\langle \eta_+, \eta_+ \rangle = \text{const}.
\]

Next, evaluate \( Z_t \) at \( r \neq 0 \). As this component must vanish, one finds the condition

\[
(p^2 + |q|^2)Y_{\alpha \beta} + \frac{3}{16} Y_{\alpha \beta} - \frac{1}{12} Y_{\alpha \beta} = 0,
\]

where the eight-dimensional indices are split as \( i = (\alpha, \ell) \).

It will be convenient to define

\[
\tau_+ = \left( \frac{3}{16} \Phi_i \Gamma^i + \frac{1}{96} H_{i134} \Gamma^i_{134} \right) C \eta_+.
\]

Then the condition (3.10) implies that one can write

\[
\tau_+ = q \mu^\alpha e_{\alpha} - \frac{1}{6} p (\mu^\beta) e_{\alpha}^a, (\alpha, \mu) \in \Gamma_{\alpha_2 \beta_2},
\]

with

\[
(p^2 + |q|^2)\mu^\alpha = \frac{3}{8} \sqrt{2} (\langle q \rangle^2 \Phi^\alpha - p^2 \bar{\Phi}^\alpha) + \sqrt{2} m \bar{q} Y^\mu \mu^\alpha
\]

\[
- \frac{1}{3} \sqrt{2} (p^2 + |q|^2) Y_{\mu_1 \mu_2 \mu_3} e^{\mu_1 \mu_2 \mu_3} + \frac{1}{8} \sqrt{2} (\langle q \rangle^2 H^\mu \mu^\alpha - p^2 \bar{H}^\mu \mu^\alpha)
\]

\[
+ \frac{\sqrt{2}}{48} (\bar{p} q H_{\mu_1 \mu_2 \mu_3} + q \bar{H}_{\mu_1 \mu_2 \mu_3}) e^{\mu_1 \mu_2 \mu_3}.
\]

Also, noting that

\[
\Delta = -2 r^2 Z_+ \frac{Z_+}{Z_-},
\]

one finds

\[
\Delta = 4 \delta_{\alpha \beta} \mu^\alpha \bar{\mu}^\beta,
\]

so \( \Delta \geq 0 \), as expected.

**3.1.2. Algebraic conditions.** Returning to the solution of the + and - components of the gravitino KSE, the Killing spinor \( \epsilon \) can be written in terms of \( \eta_+ \), \( \tau_+ \) as

\[
\epsilon = \eta_+ + r \Gamma^\tau \tau_+.
\]
Moreover, one also finds the algebraic conditions
\[
\left( \frac{1}{2} \Delta - \frac{1}{8} \partial h_i \Gamma^i \right) \eta_+ + \frac{1}{16} L_{ij} \Gamma^i*C \eta_+ + \left( \frac{1}{2} h_i \Gamma^i - \frac{i}{6} Y_{i \ell \ell', \ell'} \Gamma^{i \ell \ell'} \right) \tau_+ + \left( \frac{3}{8} \Phi \Gamma^i + \frac{1}{48} H_{i \ell \ell'} \Gamma^{i \ell \ell'} \right) C \eta_+ = 0 \tag{3.17}
\]
and
\[
(\Delta h_i - \partial i \Delta) \Gamma^i \eta_+ + \left( -\frac{1}{2} \partial h_i \Gamma^i + \frac{i}{12} X_{i \ell \ell', \ell'} \Gamma^{i \ell \ell'} \right) \tau_+ + \frac{1}{2} L_{ij} \Gamma^j \eta_+ = 0. \tag{3.18}
\]
This completes the integration of the gravitino KSEs along the light-cone directions.

### 3.2. The S components of the gravitino KSE

Substituting the Killing spinor $\epsilon$, (3.16), into the gravitino KSE and evaluating the resulting expression along the directions transverse to the light cone, one finds
\[
\tilde{\nabla}_i \eta_+ + \left( -\frac{i}{2} \Lambda_1 - \frac{1}{4} h_i \eta_+ + \frac{i}{4} Y_{i \ell \ell', \ell'} \Gamma^{i \ell \ell'} \right) \eta_+ + \left( -\frac{1}{16} \Gamma_i^j \Phi^j + \frac{3}{16} \Phi_i \right) \eta_+ = 0,
\]
and
\[
\tilde{\nabla}_i \tau_+ + \left( -\frac{i}{2} \Lambda_1 - \frac{3}{4} h_i + \frac{i}{12} Y_{i \ell \ell', \ell'} \Gamma^{i \ell \ell'} \Gamma^{\ell' \ell} \right) \eta_+ + \left( \frac{1}{16} \Gamma_i^j \Phi^j + \frac{3}{16} \Phi_i \right) \tau_+ + \left( -\frac{1}{4} \partial h_i \Gamma^j \right) \eta_+ = 0.
\]
Both the above equations are parallel transport equations along $S$.

### 3.3. The dilatino KSE

It remains to evaluate the dilatino KSE (3.2) on the spinor (3.16). A direct substitution reveals that
\[
\left( -\frac{1}{2} \Phi \Gamma^i + \frac{1}{72} H_{i \ell \ell', \ell'} \Gamma^{i \ell \ell'} \right) \eta_+ + \xi_i \Gamma^i \eta_+ = 0, \tag{3.21}
\]
and
\[
\left( -\frac{1}{2} \Phi \Gamma^i - \frac{1}{22} H_{i \ell \ell', \ell'} \Gamma^{i \ell \ell'} \right) \tau_+ - \xi_i \Gamma^i \tau_+ + \frac{1}{8} L_{ij} \Gamma^j \eta_+ = 0. \tag{3.22}
\]
This concludes the evaluation of the KSEs in the IIB near-horizon geometries and their integration along the lightcone directions.

### 3.4. Independent KSEs

It is well known that the KSEs imply some of the Bianchi identities and field equations. Because of this, to find solutions, it is customary to solve the KSEs and then impose the remaining field equations and Bianchi identities. However, we shall not do this here because of the complexity of solving the KSEs (3.17), (3.18), (3.20), and (3.22) which contain the $\tau$ spinor as expressed in (3.11) and (3.12). Instead, we shall show that all the KSEs which contain $\tau$ are actually implied from those containing only $\eta$, i.e. (3.19) and (3.21), and some of the field equations and Bianchi identities.
3.4.1. The (3.20) KSE condition. The (3.20) component of KSEs is implied by (3.19) and (3.11) together with a number of bosonic field equations and Bianchi identities. To see this, first evaluate the LHS of (3.20) by substituting in (3.11) to eliminate $\tau_+$, and use (3.19) to evaluate the supercovariant derivatives of $\eta_+$ and $\Gamma * \eta_+$. Also evaluate

$$
\left( \frac{1}{2} \tilde{R}_{ij} \Gamma^{ij} - \frac{1}{2} \Gamma^{ij} (\tilde{\nabla}_j \tilde{\nabla}_i - \tilde{\nabla}_i \tilde{\nabla}_j) \right) \eta_+ - \frac{1}{8} \xi \Gamma \ast A_1 - \left( \frac{1}{192} \Gamma^{\ell_1 \ell_2 \ell_3} \tilde{H}_{\ell_1 \ell_2 \ell_3} \right) \left( \frac{3}{64} \tilde{H}^{\ell_1 \ell_2} \Gamma^{\ell_2 \ell_3} - \frac{3}{32} \tilde{\Phi}_i + \frac{3}{32} \tilde{\Phi}_j \right) A_1 = 0,
$$

(3.23)

where

$$
A_1 = \left( - \frac{1}{2} \Phi \Gamma^i - \frac{1}{32} H_{\ell_1 \ell_2 \ell_3} \Gamma^{\ell_1 \ell_2 \ell_3} \right) \eta_+ + \xi L \Gamma \ast \eta_+.
$$

(3.24)

The expression in (3.23) vanishes on making use of the dilatino KSE (3.21), as $A_1 = 0$ is equivalent to (3.21). However a non-trivial identity is obtained by expanding out the supercovariant derivative terms again using (3.19), and expanding out the $A_1$ terms using (3.24). Then, on adding (3.23) to the LHS of (3.20), with $\tau_+$ eliminated in favor of $\eta_+$ using (3.11) and (3.19) as mentioned above, one obtains, after some calculation, a term proportional to (2.29).

Therefore, it follows that (3.20) is implied by KSEs (3.19) and (3.21), the expression for $\tau_+$ (3.11), and the bosonic field equations and Bianchi identities. We remark that in addition to using field equation (2.29) in establishing this identity, we also make use of Bianchi identities (2.10), (2.7), (2.11), (2.14), (2.15), (2.19), and the field equations (2.21) and (2.23).

3.4.2. The (3.22) KSE condition. Next consider (3.21) and (3.22). On defining

$$
A_2 = \left( - \frac{1}{2} \Phi \Gamma^i - \frac{1}{32} H_{\ell_1 \ell_2 \ell_3} \Gamma^{\ell_1 \ell_2 \ell_3} \right) \tau_+ - \xi L \Gamma \ast \tau_+ + \frac{1}{2} L \Gamma \ast \eta_+,
$$

(3.25)

one obtains the following identity

$$
A_2 = \left( \frac{3i}{4} \Lambda \Gamma^i \ast \eta_+ + \frac{3}{8} \tilde{\eta}_i \Gamma^i \ast \eta_+ \right) A_1,
$$

(3.26)

where $A_1$ is given by (3.24), and we have made use of KSE (3.19) in order to evaluate the covariant derivative in the above expression. In addition, we also have made use of the Bianchi identities (2.14), (2.15), (2.17), and the field equations (2.21), (2.23) and (2.25). It follows that these conditions, together with KSE (3.21) imply (3.22).

3.4.3. The (3.17) KSE condition. To show that (3.17) is also implied as a consequence of the KSEs (3.19) and (3.21), and the field equations and Bianchi identities, contract the KSE (3.20) with $\Gamma^i$ and use (3.11) to rewrite the $\tau_+$ terms in terms of $\eta_+$. Then subtract $(\frac{1}{2} \Phi \Gamma^i + \frac{1}{32} H_{\ell_1 \ell_2 \ell_3} \Gamma^{\ell_1 \ell_2 \ell_3})A_1$ from the resulting expression to obtain KSE (3.17). In order to obtain (3.17) from these expressions, we also make use of Bianchi identities (2.10), (2.11), (2.23), (2.24), the field equations (2.21), (2.27), and the anti-self-duality condition for $X (2.7)$. It follows, from section 3.4.1 above, that (3.17) follows from the above mentioned Bianchi identities and field equations, together with KSEs (3.19) and (3.21).

3.4.4. The (3.18) KSE condition. The (3.18) condition is obtained from KSE (3.17) as follows. First act on (3.17) with the Dirac operator $\Gamma^i \tilde{\nabla}_i$, and use the bosonic field equations and Bianchi identities to eliminate the $d \ast s_h$, $d h$, $d L$, $d \ast s L$, $d \ast s h$, $d Y$, $d \ast s Y$, $d H$ and $d \ast s H$ terms, and rewrite $d \Phi$ in terms of $L$. Then use the algebraic KSEs (3.21) and (3.22) to eliminate the $\xi$-terms from the resulting expression. The terms involving $\Lambda$ then vanish as a consequence of algebraic KSE (3.17).
Next consider the $d\eta$-terms; after some calculation, these can be rewritten as
\[
\begin{aligned}
\frac{1}{2} d h_{ij}/\gamma_{ij} \tau_+ &= -\frac{7}{32} h_{\ell}/\gamma_{\ell} \gamma_{ij} \tau_+ + \frac{1}{384} \gamma_{1\ell} \gamma_{ij} \eta_+ + \left(-\frac{1}{12} \Phi_1 \gamma^\ell + \frac{1}{384} H_{\ell_1 \ell_2 \ell_3} \gamma^{\ell_1 \ell_2 \ell_3 \ell_4} \right) d h_{ij}/\gamma_{ij} C \eta_+.
\end{aligned}
\]

The $d\eta$ terms involving $\eta_+$ and $C \eta_+$ in the above expression are then eliminated, using (3.17). On collating the remaining terms, one finds that those involving $\Delta_1$ (but not $d\Delta_1$) are
\[
-\Delta h_1 \gamma^{ij} \eta_+.
\]

(3.27)

It is also straightforward to note that the terms involving $\tilde{L}$ vanish, whereas the terms involving $X$ and $L$ can be rewritten as
\[
-\frac{1}{2} L_{ij} \gamma^{ij} C \eta_+ - \frac{i}{12} X_{\ell_1 \ell_2 \ell_3 \ell_4} \gamma^{\ell_1 \ell_2 \ell_3 \ell_4} \tau_+,
\]
where the anti-self-duality of $X$ has been used to simplify the expression. The remaining terms which are linear in $\tau_+, C \eta_+$ and quadratic in $h, Y, \Phi, \Phi, H, \bar{H}$ can be shown to vanish after some computation. After performing these calculations, the condition which is obtained is (3.18).

4. Solution of KSEs and geometry of horizons

4.1. The linear system

It is straightforward to derive the linear system associated with the (3.19) and (3.21) KSEs evaluated on the spinor $\eta_+=p^1 + q e_{1234}$,
\[
\eta_+ = p^1 + q e_{1234},
\]
where we have used a local Spin(8) · U(1) transformation to arrange such that $p$, $q$ are real functions on $S$, and as required from the bilinear matching condition take
\[
p^2 + q^2 = 1.
\]

(4.2)

Substituting this spinor into (3.19) and (3.21), one obtains a linear system which is explicitly given in the appendix. This can be used to express some of the fluxes in terms of geometry and find the conditions on the geometry of the horizon sections $S$ imposed by supersymmetry.

There are three different types of backgrounds which arise naturally in the investigation of the solutions of the linear system in the appendix.

- The generic horizons for which the Killing spinor is chosen such that $p^2 + q^2 = 1$, $p^2 - q^2 \neq 0$, and $p$ and $q$ not identically zero at the horizon though they may vanish at some points.
- The Spin(7) horizons for which $p^2 + q^2 = 1$ and $p = q$.
- The pure SU(4) horizons for which either $p = 1$, $q = 0$ or $p = 0$, $q = 1$.

Clearly, the generic horizons include the pure SU(4) horizons. Typically, as the Killing spinor is parallel transported along the horizon section with (3.19), it will change type as the only requirement is that $p^2 + q^2 = 1$. In what follows, we shall solve the KSEs locally assuming that each type is preserved at an open set and we shall give a description of the geometry. Of course if each type is preserved everywhere, the local description of the geometry can be extended globally. The solution of the linear system for all three cases is given in the appendix. In what follows, we shall explore the consequences which arise on the topology and geometry of horizon sections $S$ from the solution of the linear system and hence from the solution of the KSEs for each of the three types.
4.2. Generic horizons

4.2.1. Solution of the linear system. The general solution of the linear system is described in the appendix under the assumption that the functions $p$ and $q$ may vanish at isolated points. The main properties of the solution are the following. All complex fluxes $\xi$, $\Phi$ and $H$ can be expressed in terms of the real fluxes $Y$ and the geometry $\Omega$. Moreover, the solution of the linear system does not contain a condition which involves only the spin connection of $S$.

4.2.2. Topology and geometry of $S$. The gravitino KSE as reduced to $S$ in (3.19) is a parallel transport equation of sections of a $\lambda^2 \otimes \Sigma^+$ bundle over $S$, where $\lambda^2 \otimes \Sigma^+$ is a Spin$(8)$ bundle and $\Sigma^+$ is the complexified spin bundle over $S$ associated with the positive chirality Majorana–Weyl representation of Spin$(8)$. The real rank of $\lambda^2 \otimes \Sigma^+$ is 16. An application of [24] implies that the supercovariant connection as restricted on $S$ given in (3.19) has holonomy contained in $GL(16, \mathbb{R})$. The existence of a solution to the gravitino KSE requires that the holonomy of the supercovariant connection reduces to $GL(15, \mathbb{R})$, and so $\lambda^2 \otimes \Sigma^+$ must admit a nowhere vanishing section spanned by the parallel spinor. The existence of nowhere vanishing sections typically impose a topological restriction on the spin bundle and so on a manifold. However, a priori this may not be the case here as $\lambda^2 \otimes \Sigma^+$ has rank much bigger than the dimension of $S$ and so it always admits such a nowhere vanishing section.

Nevertheless, the existence of a solution to the KSEs requires that the structure group of $S$ reduces form $SO(8)$ to a subgroup of $U(4)$. To see this, recall that $S$ admits a nowhere vanishing spinor $\eta_+ = p1 + qe_{1234}$ which is a section of $\lambda^2 \otimes \Sigma^+$. One can consider the bilinears of $\eta_+$. There are two kinds of bilinears which can be constructed from $\eta_+$. First the bilinears constructed from $\eta_+$ and itself with respect to the standard Hermitian inner product of Spin$(8)$. These bilinears are forms on $S$. In addition one can also consider the bilinears constructed from $\eta_+$ and its complex conjugate $C \ast \eta_+$. Such bilinears are not forms on $S$. Instead they are forms twisted by $\lambda$ as they carry a $U(1)$ charge.

Consider first the bilinears of $\eta_+ = p1 + qe_{1234}$ which are forms on $S$. We find that a basis in the ring of these bilinears is

$$\omega = -i(p^2 - q^2)\delta_{\alpha\beta}e^\alpha \wedge e^\beta, \quad \psi = pq \, \text{Re}(e^1 \wedge e^2 \wedge e^3 \wedge e^4),$$

with $p^2 + q^2 = 1$. Since in the generic case $p^2 - q^2 \neq 0$, $\omega$ vanishes nowhere, it is an almost Hermitian form on $S$. In addition $\psi$ defines a (4,0)-form on $S$. But as $p$ and $q$ can vanish at some points on $S$, $\psi$ is not nowhere vanishing. As a result, the structure group of $S$ reduces from $SO(8)$ to a subgroup of $U(4)$ rather than to a subgroup of $SU(4)$. $S$ is an almost Hermitian eight-dimensional manifold.

Next, the form bilinears of $\eta_+ = p1 + qe_{1234}$ which are twisted by $\lambda$ are

$$\rho = -ipq\delta_{\alpha\beta}e^\alpha \wedge e^\beta, \quad \tau = p^2 e^1 \wedge e^2 \wedge e^3 \wedge e^4 + q^2 e^1 \wedge e^3 \wedge e^3 \wedge e^3.$$  \hspace{1cm} (4.4)

As a result $\lambda \otimes (\Lambda^4(\mathcal{S}) \oplus \Lambda^{0,4}(\mathcal{S}))$ admits a nowhere vanishing section.

Furthermore, as has been mentioned, the KSEs solved in the appendix do not impose any additional geometric restrictions on $S$. As a result, the horizon sections $S$ are spin, almost Hermitian eight-dimensional manifolds. The only remaining potential restrictions on $S$ are those imposed by the field equations and Bianchi identities which we shall investigate in some special cases below.

So far, we have performed our analysis without distinguishing whether the IIB scalars take values in the upper half plane or in the fundamental domain. Now we return to address this issue. If the IIB scalars take values in the upper half plane, then the line bundle $\lambda$ is topologically
trivial as it is the pull-back of a trivial bundle over the upper half plane with respect to a smooth map. Nevertheless, if the IIB scalars are non-constant\footnote{It is not apparent that there are smooth solutions for which $S$ is compact and the IIB scalars are non-trivial.} $\lambda$ will be geometrically twisted as it will have non-vanishing curvature.

In the case that the IIB scalars take values in the fundamental domain, $\lambda$ may not be topologically trivial. This arises from the analysis of stringy cosmic string solutions in \cite{25} where there are solutions for which the horizon section of the cosmic strings is a 2-sphere, e.g. the configuration of 24 cosmic strings. Of course, our case here is more general and $S$ is not a complex manifold. Nevertheless, it is an indication that $\lambda$ over $S$ may not be topologically trivial in the fundamental domain case.

\section*{4.3. Spin(7) horizons}

\subsection*{4.3.1. Solution of the linear system.} The spinor $\eta_+$ can be chosen as $\eta_+ = \frac{1}{\sqrt{2}}(1 + e_{1234})$. The solution of the linear system is given as in the appendix after setting $p = q = 1/\sqrt{2}$ in the solution of the generic case. Furthermore, the expression for $\mu$ can be simplified as

$$\frac{\mu}{\sqrt{2}} = -\frac{1}{2} \Lambda_{\lambda} + i Y_{\lambda\lambda', \lambda, \lambda'},$$

which is consistent with our previous results on IIB horizons with only 5-form flux.

\subsection*{4.3.2. Topology and geometry of horizon sections.} The analysis of the consequences for the existence of a parallel $\eta_+$ spinor on the holonomy of the supercovariant connection as reduced on $S$ is similar to the generic case. To find whether the structure group of $S$ reduces to a subgroup of $SO(8)$, we again compute the form bilinears associated with $\eta_+$. In this case, there is a single nowhere vanishing 4-form bilinear\footnote{The almost Hermitian form $\omega$ and the $(4,0)$-form $\chi$ may not be globally defined on $S$ but $\phi$ is. Observe that we have normalized $\chi$ differently in \cite{19}.} $\phi = -\frac{1}{2} \omega \wedge \omega + 4 \text{Re} \chi$ (4.6)

which is a representative of the fundamental form of Spin(7). Therefore the structure group of $S$ reduces to Spin(7), where $\chi = \frac{1}{3!} \epsilon_{\alpha \beta \gamma \delta \epsilon \zeta \lambda \mu} e^\alpha \wedge e^\beta \wedge e^\gamma \wedge e^\delta \wedge e^\epsilon \wedge e^\zeta \wedge e^\lambda \wedge e^\mu$. It is known that the structure group of an eight-dimensional compact spin manifold reduces from $SO(8)$ to Spin(7) provided that $[26]$

$$\pm \epsilon - \frac{1}{8} p_2 + \frac{1}{5} p_1^2 = 0,$$

(4.7)

where $\epsilon$ is the Euler class, and $p_2$ and $p_1$ are the second and first Pontryagin classes, respectively. This condition restricts the topology of $S$. Furthermore, a direct inspection of the solution of the linear system reveals that there are no additional geometric conditions on the Spin(7) structure of $S$. Thus $S$ can be any manifold with a Spin(7) structure.

\section*{4.4. Pure SU(4) horizons}

\subsection*{4.4.1. Solution of the linear system.} The $\eta_+$ spinor in this case can be chosen as $\eta_+ = 1$. The solution of the linear system has been given in the appendix. In particular, many components of the complex fields, like $H$, can be expressed in terms of the components of the 5-form and components of the spin connection of spacetime. Again, the solution of the linear system in the appendix does not yield a condition which restricts the spin connection of $S$.
4.4.2. Topology and geometry of horizon sections. A similar analysis to that presented for the generic case reveals that the structure group of $S$ reduces to a subgroup of $U(4)$. In this case, it is instructive to also consider the mixed bilinears of $\eta_+$ with $\tilde{\eta}_+ = C \eta_+ = e_{1234}$ in (4.4). In addition to the almost Hermitian form $\omega$ which arises as a bilinear of $\eta_+$, as in (4.3), there is a $\lambda$-twisted (4,0)-form bilinear given locally by $r$ for $p = 1$, $q = 0$. Therefore the line bundle $\lambda \otimes \Lambda^{4,0}$ on $S$ admits a nowhere vanishing section, and so it is topologically trivial. Thus $\lambda$ can be identified with the anti-canonical bundle of $S$. Furthermore, the solution of the linear system in the appendix does not reveal any additional geometric constraints on $S$. As a result, $S$ is any almost Hermitian spin$_c$ manifold.

Next let us turn to examine the upper half plane and fundamental domain cases. As in the generic class, if the IIB scalars take values in the upper half plane, $\lambda$ is topologically trivial. The structure group of $S$ then reduces to a subgroup of $SU(4)$. However, $\lambda$ can be geometrically non-trivial as its curvature may not vanish provided that the IIB scalars are non-trivial functions. On the other hand, if the IIB scalars take values in the fundamental domain, as has been explained in the generic case, $\lambda$ may not be topologically trivial on $S$.

5. Special cases

5.1. Horizons with only 5-form flux

It is well known that IIB supergravity has two consistent truncations. One truncation leads to a sector with only 5-form fluxes active, while for the other truncation the 5-form field strength vanishes and both the 3-form and 1-form field strengths are real. The latter is the common sector. In both cases, the near-horizon geometries are compatible with both these truncations. It is straightforward to see this in the 5-form sector. The near-horizon geometries with only 5-form fluxes can be recovered from the general IIB horizons that we have investigated by setting

$$P = G = 0.$$  \hfill (5.1)

Conversely, the 5-form horizons investigated in [20] can be embedded into general IIB horizons exhibiting only non-vanishing 5-form fluxes. As a result, all the examples of IIB horizons constructed in [20] can be embedded into the general IIB horizons.

5.2. Common sector horizons

The embedding of the common sector horizons into general IIB horizons is not as straightforward as that described in the previous section for horizons with only 5-form fluxes. To describe the embedding of common sector horizons, let us recall how the common sector is obtained from the general IIB theory. For this set $F = 0$, and as has already been mentioned, choose both $P$ and $G$ to be real. Observe that $Q = 0$, and the Bianchi identities of IIB supergravity can be solved after setting

$$P = \frac{1}{2} \phi, \quad G = -e^{-\phi} \mathcal{H},$$  \hfill (5.2)

provided that $d\mathcal{H} = 0$. These field definitions yield the common sector in the string frame provided that in addition we relate the IIB metric, $ds_{\text{IIB}}^2$, with the common sector metric, $ds_{\text{CS}}^2$, as

$$ds_{\text{IIB}}^2 = e^{-\phi} ds_{\text{CS}}^2,$$  \hfill (5.3)

and identify $\phi$ and $\mathcal{H}$ with the dilaton and the NS-NS 3-form field strength, respectively.

\textsuperscript{14} It is not apparent that there exist smooth solutions with $S$ compact and non-trivial scalars.
Furthermore, the KSEs of the common sector are obtained from those of IIB provided that the IIB supersymmetry parameter is related to that of the common sector as
\[ \epsilon_{\text{CS}} = e^{i\phi} \epsilon_{\text{IIB}}. \] (5.4)
In particular, we find that the KSEs of the common sector are
\[ \nabla^\pm e_{\pm}^{\pm} = 0, \]
\[ (\Gamma^{M} \partial_{M} \phi \mp \frac{1}{27} \Gamma^{M N R} \mathcal{H}_{M N R}) e_{\pm}^{\pm} = 0, \] (5.5)
where \( \nabla^\pm = \nabla \pm \frac{1}{2} \mathcal{H}, \) \( \nabla \) the Levi-Civita connection of the common sector metric, and \( C * e_{\pm}^{\pm} = \pm e_{\pm}^{\pm}. \)

It remains to show how the common sector near-horizons are embedded into general IIB horizons. For this, one has to demonstrate that after making the field redefinitions (5.2) and (5.3), one can adapt a coordinate system such that the common sector near-horizon fields give rise to IIB near-horizon fields. Indeed consider the metric. If the common sector metric is in near-horizon form, the IIB metric can also be put in near-horizon form provided that we make a coordinate transformation
\[ r_{\text{IIB}} = e^{-\frac{i\phi}{2}} r_{\text{CS}}, \] (5.6)
the 1-form \( h_{\text{CS}} \) is replaced with \( h_{\text{IIB}} = h_{\text{CS}} + \frac{1}{2} d\phi, \) and the metric of the horizon section \( S \) is conformally scaled as \( g_{\text{IIB}} = e^{-\frac{i\phi}{2}} g_{\text{CS}}. \)

Next turn to the 3-form field strength. Suppose that \( \mathcal{H} \) is in a near-horizon form. \( G \) in (5.2) can also be put into a near-horizon form as well since the conformal factor in the definition of \( G \) is absorbed in the coordinate transformation for \( r \) in (5.6) and the redefinition of \( h. \) One also has to re-scale \( H \) as \( H_{\text{IIB}} = e^{-\frac{i\phi}{2}} H_{\text{CS}}. \) Observe now that \( H_{\text{CS}} \) is closed as expected. To make connection with our results for heterotic horizons, one can compensate the overall sign in the definition of \( H \) by changing the sign of \( L, \Phi \) and \( H. \)

A consequence of our analysis above is that we can embed all the near-horizon geometries we have found in heterotic theory into IIB. We remark that all the horizons we had investigated in [18] have \( d\mathcal{H} = 0 \) and so they can be thought as solutions of the common sector as well. In this way, all the explicit heterotic near-horizon geometries found in [18] can be utilized to give explicit examples of IIB horizons. Note also that all the IIB horizons with non-trivial fluxes that arise from the embedding of heterotic horizons in IIB preserve more than two supersymmetries and admit additional isometries.

6. Pure SU(4) horizons with constant scalars

6.1. Solution to the linear system

A special class of horizons are those for which the IIB scalars are constant. In this case, there is a significant simplification of the solution to the linear system because \( P = Q = 0. \)

The solution to the KSEs given in the appendix simplifies somewhat. In particular, one that the fluxes of pure SU(4) horizons with constant scalars can be expressed as
\[ Y = \frac{1}{2} (d\omega - \theta_{\text{Re} \chi} \wedge \omega) + \text{Im} \Phi \cdot \text{Re} \chi, \omega \cdot H^{1,2} = H^{0,3} = 0, \]
\[ H^{3,0} = (\theta_{\text{Re} \chi} - \theta_{\text{Im} \chi}) \cdot \chi, H^{\alpha \lambda \kappa \beta} = \frac{1}{2} \delta_{\alpha \lambda} \delta_{\kappa \beta} - 2 \delta_{\alpha \kappa} \theta_{\text{Re} \chi}, \]
where \( \theta_{\text{Re} \chi} \) and \( \theta_{\text{Im} \chi} \) are the Lee forms of the almost Hermitian form \( \omega \) and the (4,0) form \( \chi = \frac{1}{2} \delta^{\alpha \lambda} e^{\alpha \lambda} \wedge e^{\alpha \lambda} \wedge e^{\alpha \lambda} \) of the SU(4) structure, respectively, and
\[ N_{j k} = I_{m} \nabla_{m} I_{k} - I_{m} \nabla_{j} I_{k} = I_{m} (\nabla_{j} I_{k} - \nabla_{k} I_{j}), \]
where \( \theta_{\alpha \lambda} = -I_{j} \nabla_{k} \alpha_{k j} \) and \( \theta_{\text{Re} \chi} = \frac{i}{2} \text{Re} \chi_{j k} \).

15 If \( v \) is \( \epsilon \)-vector and \( \alpha \) a \( k \)-form, then \( (v \cdot \alpha)_{j_{1}...j_{k}} = \epsilon^{i_{1}...i_{k}}(\alpha)_{j_{1}...j_{k}}. \)

16 We define \( (\theta_{\alpha \lambda}) = -I_{k} \nabla_{j} \alpha_{j k} \) and \( (\theta_{\text{Re} \chi}) = \frac{i}{2} \text{Re} \chi_{j k} \).
is the Nijenhuis tensor of the almost complex structure \( I \). The fluxes \( \Phi^{1,0} \) and the traceless part of \( H^{1,2} \) are not restricted by the KSEs.

6.2. Topology and geometry of horizons

As the scalars are constant, the line bundle \( \lambda \) is trivial. As a consequence, the bilinears of both \( \eta_+ \) and \( \tilde{\eta}_+ = e^{1234} \) are forms on \( S \) and in particular \( \chi \) is a nowhere vanishing (4,0)-form on \( S \). Thus the structure group of \( S \) reduces to a subgroup of \( SU(4) \).

As in the Spin\( (7) \) case, there are topological obstructions to reduce the structure group of \( S \) from \( SO(8) \) to \( SU(4) \). Since Spin\( (7) \) and \( SU(4) \) have the same maximal torus the obstruction \( (4.7) \) is also an obstruction in the \( SU(4) \) case. In particular using the relation between the Pontryagin and Chern classes \( p_1 = c_1^2 - 2c_2 \) and \( p_2 = c_2^2 - 2c_1c_3 + 2c_4 \) on almost complex manifolds, see e.g. [27], and the identification of the Euler class \( e = c_4 \), \( (4.7) \) is satisfied provided \( c_1 = 0 \) which vanishes as a consequence of \( SU(4) \) structure. The orientation induced by the almost complex structure leads to the choice of the plus sign in \( (4.7) \). There may be additional obstructions to reduce the structure group from \( SO(8) \) to \( SU(4) \) which take values in cohomology with \( \mathbb{Z}_k \) coefficients.

The solution of the linear system does not impose any additional conditions on the geometry of \( S \). Therefore \( S \) is an almost Hermitian manifold with an \( SU(4) \) structure.

6.3. Magnetic 3-form flux deformations

6.3.1. Complex deformation of horizons with 5-form fluxes. A class of horizons with 3-form flux can be constructed as a deformation of horizons with 5-form fluxes. Recall that the horizons with only 5-form fluxes are 2-SCYT manifolds. The deformation we shall investigate here lifts the 2-strong condition but the horizon remains a complex manifold. For this, we take\(^\text{18}\)

\[
\Phi = H^{0,3} = H^{3,0} = H^{2,1} = 0, \quad Y^{0,3} = Y^{3,0} = 0. \tag{6.3}
\]

In this case, the conditions stated in the appendix imply that \( S \) is a Hermitian manifold and in addition

\[
Y = \frac{1}{4}(\omega \wedge \theta_\omega), \quad \omega \cdot H^{1,2} = 0, \quad \theta_\omega = \theta_{\Re \chi}, \quad h = \theta_\omega, \tag{6.4}
\]

where \( \theta_\omega \) and \( \theta_{\Re \chi} \) are the Lee forms of the Hermitian form \( \omega \) and \( \Re \chi \) is the real part of the (4,0)-form \( \chi \). Observe that the traceless part of \( H^{1,2} \) is not restricted by the KSEs. Since \( S \) is a Hermitian manifold with an \( SU(4) \) structure, the condition \( \theta_\omega = \theta_{\Re \chi} \) implies that \( S \) admits a connection \( \hat{\nabla} = \nabla + \frac{1}{2}H \) with skew-symmetric torsion \( \hat{H} \) which has holonomy \( SU(4) \), i.e. hol\( (\hat{\nabla}) \subseteq SU(4) \), where \( \hat{H} \) is uniquely determined in terms of the metric \( g \) and complex structure \( I \) of the horizon section as \( \hat{H} = -i \omega \wedge \theta_\omega \). Thus \( S \) admits a CYT structure.

It remains to solve the field equations and Bianchi identities. One can show after a long but straightforward calculation that all these are satisfied provided that

\[
d(\omega \wedge \hat{H}) = \frac{i}{2} H \wedge \hat{H}, \quad dH = 0, \quad h^\beta \hat{H}_{\gamma\delta\beta} + 2iH_{\alpha\gamma\delta\beta}Y_\beta^{\gamma\delta} = 0. \tag{6.5}
\]

The first condition is a generalization of the 2-strong condition found in [20] for horizons with only 5-form fluxes. In particular, the 3-form flux appears as a source term in the rhs of the 2-strong condition. Observe also that the last condition together with \( \omega \cdot H^{1,2} = 0 \) imply that \( H \cdot Y = 0 \) and so \( H \) and \( Y \) must be orthogonal.

\(^{17}\) We thank Simon Salamon for discussions on this.

\(^{18}\) Note that since \( \Phi = 0 \) it follows from (A.17), using (A.16), that \( r_+ = 0 \) and therefore the Killing spinor in this case is just \( \epsilon = \eta_+ \), which is a considerable simplification.
6.3.2. Examples. To construct examples, one begins with an eight-dimensional CYT manifold and then imposes the conditions (6.5). Examples of CYT manifolds can be constructed as toric fibrations over Kähler manifolds [28–30]. This construction has been generalized in [20] and adapted to solve the 2-SCYT condition of IIB horizons with only 5-form fluxes. Here we shall use the analysis of [20] to find geometries that solve (6.5).

First consider $S$ as a $T^3$ fibration over a six-dimensional Kähler manifold $X^6$ with Kähler form $\omega_{(6)}$. Let $(\lambda^1, \lambda^2)$ be a principal bundle connection on $S$. In order for $S$ to admit an $\text{SU}(4)$ structure compatible with a connection with skew-symmetric torsion, the curvature $\mathcal{F} = d\lambda^1$ must be identified with the Ricci form of $X^6$, i.e.

$$\rho_{(6)} = -\mathcal{F}, \quad (6.6)$$

and $\mathcal{F}^2 = d\lambda^2$ must be (1,1) and traceless. In the latter case, such connections always exist on complex line bundles. The metric and Hermitian form on $S$ is chosen as

$$d\lambda^2 = 2\left(\lambda^1\right)^2 + d\lambda^2 X^6 \quad \omega_{(8)} = -\frac{2}{k} \lambda^1 \wedge \lambda^2 + \omega_{(6)}, \quad (6.7)$$

where $\mathcal{F}_i^{(i)} \omega_{(6)} = k$ and $k$ is required to be constant\(^{19}\). This choice of Hermitian structure specifies both $\hat{H}$ and $Y$. It remain to choose $H$. For this write

$$H = \bar{\tau} \wedge \alpha + \beta, \quad \tau = \frac{1}{\sqrt{2}} (\lambda^1 + i\lambda^2), \quad (6.8)$$

where $\alpha$ and $\beta$ are complex (1,1)- and (1,2)-forms on $X^6$. The requirement that $H$ is traceless implies that

$$\omega_{(6)} \cdot \alpha = \omega_{(6)} \cdot \beta = 0. \quad (6.9)$$

Furthermore $dH = 0$ implies that

$$d\alpha = 0, \quad d\beta + \frac{1}{\sqrt{2}} \alpha \wedge [\mathcal{F}^1 - i\mathcal{F}^2] = 0. \quad (6.10)$$

Next the first equation in (6.5) is solved provided

$$\alpha \wedge \bar{\alpha} = 0, \quad \alpha \wedge \bar{\beta} = 0, \quad \omega_{(6)} \wedge [(\mathcal{F}^1)^2 + (\mathcal{F}^2)^2] = \frac{ik}{4} \beta \wedge \bar{\beta}. \quad (6.11)$$

The simplest case is to take $\alpha = 0$. Then since $\theta_\omega = -\lambda^2$ and

$$Y = \frac{1}{2k} (\lambda^1 \wedge \mathcal{F}^2 - \lambda^2 \wedge \mathcal{F}^1) + \frac{1}{4} \lambda^2 \wedge \omega_{(6)}, \quad (6.12)$$

the last condition in (6.5) is also satisfied.

Therefore to find examples of near-horizon geometries, one must find a Kähler manifold $X^6$ with positive constant scalar curvature that admits a traceless (1,2) closed form $\beta$ that satisfies the last equation in (6.11).

To find solutions, one possibility is to take $X^6$ to either be a Kähler–Einstein manifold or products of Kähler–Einstein manifolds. For example, one can take $X^6 = S^2 \times S^2 \times T^2$. Normalizing the metric of $S^2$ such that $\mathcal{F}^1 = \omega^1 + \omega^2$ where $\omega^1$ and $\omega^2$ are the Kähler forms on the $S^2$’s, $k = 4$, and choosing

$$\omega_{(6)} = -d\phi^1 \wedge d\phi^2 + \omega^1 + \omega^2, \quad H = m(d\phi^1 - i d\phi^2) \wedge (\omega^1 - \omega^2),$$

$$\mathcal{F}^2 = \ell (\omega^1 - \omega^2), \quad \ell \in \mathbb{Z}, \quad (6.13)$$

\(^{19}\) Since $X^6$ is Kähler, observe that $k$ is constant provided $\mathcal{F}^1$ is co-closed. Equivalently $X^6$ has positive constant scalar curvature.
all equations are solved provided that $2|m|^2 = \ell^2 - 1$, where $\varphi^1$ and $\varphi^2$ are the angular coordinates of $T^2$ in $X^6$. This generalizes the examples of 2-SCYT near-horizon geometries with only 5-form flux in [20].

7. Concluding remarks

We have solved the KSEs for all IIB horizons that admit at least one supersymmetry. This has been done by first integrating the KSEs along the lightcone directions and then identifying the independent equations using the Bianchi identities, field equations and bilinear matching condition. Then the independent KSEs are solved using spinorial geometry which leads to three different cases, the generic horizons, the Spin(7) horizons and the pure SU(4) horizons. We have found that the requirement of IIB horizons to admit one supersymmetry puts rather weak geometric restrictions on the horizon sections $\mathcal{S}$. In particular for generic horizons and pure SU(4) horizons, the horizon sections can be any eight-dimensional almost Hermitian spin manifold. A similar result also applies for Spin(7) horizons. We have also described some topological aspects of the horizon sections and how the various fluxes are expressed in terms of the geometry.

We have also explained how horizons with only 5-form fluxes and common sector horizons, which had been investigated previously, are included in our analysis. As a result all the examples constructed in these two special cases can be embedded in the full IIB theory. Furthermore, we give some examples for which, in addition to the KSEs, we also solve the field equations and Bianchi identities. In particular, we focus on horizons with constant scalars which have complex horizon sections, and we find a generalization of the 2-SCYT structure which had appeared for horizons with 5-form fluxes only.

The construction of examples with non-trivial scalars taking values on the upper half-plane after a $SL(2, \mathbb{Z})$ identification is natural within the context of 10-dimensional type of F-theory [31]. In particular, one may consider $T^2$-fibrations over $\mathcal{S}$. However there are some differences. As we have mentioned, $\mathcal{S}$ are almost complex manifolds instead of Kähler, which mostly arise in the context of F-theory. In addition, the example of horizons with constant scalars that we have explicitly constructed indicates that $\mathcal{S}$ is not Kähler because of the presence of form fluxes. Nevertheless in the complex case, it may be possible to construct examples imitating techniques that have been employed in F-theory.

In most of our considerations, like the solution of the KSEs and the description of the geometry of the horizon sections, we have not used the compactness of $\mathcal{S}$. So our results apply to both black holes and brane horizons. For applications to black holes, it would be of interest to enforce compactness of $\mathcal{S}$. This has been done for M-horizons in [32], and after an application of the index theorem for the Dirac operator, it has led to the conclusion that all M-horizons preserve an even number of supersymmetries and admit an $\mathfrak{sl}(2, \mathbb{R})$ symmetry. A similar application may be possible in IIB. However, there are some differences between M-theory and IIB. One of them is that unlike the M-horizon sections which are odd dimensional, the IIB near-horizon sections are even dimensional and so the index of the Dirac operator is not expected to vanish. As the vanishing of the Dirac index has been instrumental in proving supersymmetry enhancement for M-horizons, a similar application in IIB will require some modification. Nevertheless, it is expected that even if one cannot prove supersymmetry enhancement for IIB black hole horizons, it may be possible to relate the number of supersymmetries preserved in terms of the index of a Dirac operator on the horizon sections. Such a relation will generalize the classic formula $N = \text{index}(D)$ which relates the number of parallel spinors $N$ on irreducible holonomy Spin(7), SU(4) and Sp(2) eight-dimensional manifolds, for $N = 1, 2$ and $3$ respectively, to the index of the Dirac operator.
In turn such a formula will provide a topological criterion for eight-dimensional manifolds to admit Killing spinors.

Acknowledgments

UG is supported by the Knut and Alice Wallenberg Foundation. JG is supported by the STFC grant, ST/1004874/1. GP is partially supported by the STFC rolling grant ST/J002798/1.

Appendix. Solution of the linear system

A.1. The linear system

It is straightforward to derive the linear system associated with the KSE (3.19) evaluated on the spinor $\eta_+ = p\eta + qe_{1234}$, where $p, q$ are real functions on $S$ and $p^2 + q^2 = 1$. Substituting this spinor into (3.19), one obtains,

$$\partial_\alpha p + p \left( -\frac{i}{2} \Lambda_\alpha - \frac{1}{4} h_\alpha + \frac{1}{2} \Omega_{a,\mu}^\mu - iy_{a,\mu}^\mu \right) + q \left( \frac{1}{4} H_{a,\mu}^\mu - \frac{1}{4} \Phi^\alpha \right) = 0, \quad (A.1)$$

$$p \left( \frac{1}{2} \Omega_{a,\mu}^\mu - i y_{a,\mu}^\mu + \frac{1}{2} \delta_{\alpha\beta} \epsilon_{\mu,\lambda_1 \lambda_2} \epsilon^{\lambda_1 \lambda_2 \lambda_3 \lambda_4 \alpha} H_{\lambda_1 \lambda_2 \lambda_3 \lambda_4} \right) + q \left( \frac{1}{4} \Omega_{a,\mu}^\mu - \frac{1}{8} H_{a,\mu}^\mu - \frac{1}{4} \Phi^\alpha \right) = 0, \quad (A.2)$$

$$\partial_\alpha q + q \left( -\frac{i}{2} \Lambda_\alpha - \frac{1}{4} h_\alpha + \frac{1}{2} \Omega_{a,\mu}^\mu + \frac{1}{24} H_{\lambda_1 \lambda_2 \lambda_3}^\lambda \epsilon^{\lambda_1 \lambda_2 \lambda_3 \alpha} \right) + p \left( \frac{1}{8} H_{a,\mu}^\mu - \frac{1}{4} y_{a,\mu}^\mu - \frac{1}{8} \Phi^\alpha \right) = 0, \quad (A.3)$$

$$\partial_\alpha p + p \left( -\frac{i}{2} \Lambda_\alpha - \frac{1}{4} h_\alpha + \frac{1}{2} \Omega_{a,\mu}^\mu + \frac{1}{24} H_{\lambda_1 \lambda_2 \lambda_3}^\lambda \epsilon^{\lambda_1 \lambda_2 \lambda_3 \alpha} \right) + q \left( \frac{1}{8} H_{a,\mu}^\mu - \frac{1}{3} y_{a,\mu}^\mu - \frac{1}{8} \Phi^\alpha \right) = 0, \quad (A.4)$$

$$p \left( \frac{1}{2} \Omega_{a,\mu}^\mu - \frac{1}{8} H_{a,\mu}^\mu + \frac{1}{16} \epsilon^{\lambda_1 \lambda_2 \lambda_3 \lambda_4} Y_{a,\lambda_1 \lambda_2 \lambda_3 \lambda_4} \right) + q \left( \frac{1}{16} H_{a,\mu}^\mu - \frac{1}{2} y_{a,\mu}^\mu - \frac{1}{8} \Phi^\alpha \right) = 0, \quad (A.5)$$

Similarly, the linear system associated with (3.21) is

$$\frac{1}{12} p e_\alpha \tilde{\gamma}_\gamma \tilde{\gamma}_\gamma H_{\tilde{\gamma}_\gamma \tilde{\gamma}_\gamma} - \frac{1}{4} q H_{a,\mu}^\mu - \frac{1}{4} q \Phi^\alpha + p \xi_\alpha = 0, \quad (A.7)$$
The above system can be solved to express some of the fluxes in terms of the geometry, and find the conditions on the geometry imposed by supersymmetry. In the analysis of the solutions, it is convenient to consider three different cases as described in section 4.1.

A.2. Solution of the linear system

A.2.1. Generic horizons. The solution of the linear system can be arranged in different ways. The procedure which we adopt here is to solve first for the complex field strengths and express them in terms of the real fields and functions \( p, q \), and then use the remaining equations to find the expression of the real fields in terms of the geometry, and to determine the conditions on the geometry. We shall demonstrate that, although the KSEs determine the complex fields in terms of the real fields and geometry, the real fields remain undetermined.

To solve the linear system, first recall that in the generic case \( p^2 - q^2 \neq 0 \) and \( p^2 + q^2 = 1 \), and let us assume that in some open set \( p, q \neq 0 \). If either \( p, q \) vanish in an open set then the linear system will be solved as a special case. Next we take the trace of (A.2), and after a re-arrangement, that of (A.5) to find

\[
q(-\Lambda_\alpha^\lambda - i Y_\alpha^\lambda - \frac{1}{8} \epsilon_{\lambda \beta \gamma} H_{\alpha \beta \gamma}) + p(\frac{1}{2} \epsilon_{\alpha \beta \gamma} \Omega_{\lambda \beta \gamma} + \frac{1}{8} H_{\alpha \lambda} - \frac{3}{8} \Phi_{\alpha}) = 0,
\]

(A.9)

\[
p(\Lambda_\alpha^\lambda + i Y_\alpha^\lambda + \frac{1}{8} \epsilon_{\alpha \beta \gamma} H_{\beta \gamma}) + q\left(\frac{1}{2} \epsilon_{\alpha \beta \gamma} \Omega_{\lambda \beta \gamma} + \frac{1}{8} H_{\lambda \alpha} + \frac{3}{8} \Phi_{\alpha}\right) = 0,
\]

(A.10)

respectively.

Next consider the equations (A.3) and (A.7), and (A.4) and (A.8) and solve them to express the components of the 3-form field strength \( H \) in terms of the rest of the fields yielding

\[
\frac{1}{8} H_{\alpha \lambda} = -\frac{1}{8} \Phi_{\alpha} + \frac{pq}{2} \bar{\xi}_{\alpha} - pq \bar{a} q - pq \left[ \frac{1}{2} \Lambda_{\alpha} + \frac{1}{4} h_{\alpha} + \frac{1}{2} \Omega_{\alpha, \lambda}\right] + \frac{i}{3} p^2 \epsilon_{\alpha \beta \gamma} \bar{Y}_{\beta \gamma},
\]

\[
\frac{1}{24} \epsilon_{\lambda \beta \gamma} H_{\alpha \beta \gamma} = -\frac{p^2}{2} \bar{\xi}_{\alpha} + q \bar{a} q - q^2 \left[ \frac{1}{2} \Lambda_{\alpha} + \frac{1}{4} h_{\alpha} + \frac{1}{2} \Omega_{\alpha, \lambda}\right] + \frac{i}{3} q^2 \epsilon_{\alpha \beta \gamma} \bar{Y}_{\beta \gamma}.
\]

(A.11)

\[
\frac{1}{8} H_{\lambda \alpha} = -\frac{1}{8} \Phi_{\alpha} - \frac{pq}{2} \bar{\xi}_{\alpha} - q \bar{a} p - pq \left[ -\frac{i}{2} \Lambda_{\alpha} - \frac{1}{4} h_{\bar{\alpha}} + \frac{1}{2} \Omega_{\bar{\alpha}, \lambda}\right] - \frac{i}{3} q^2 \epsilon_{\alpha \beta \gamma} \bar{Y}_{\beta \gamma},
\]

\[
\frac{1}{24} \epsilon_{\lambda \beta \gamma} H_{\alpha \beta \gamma} = -\frac{q^2}{2} \bar{\xi}_{\alpha} + p \bar{a} p + p^2 \left[ -\frac{i}{2} \Lambda_{\alpha} - \frac{1}{4} h_{\bar{\alpha}} + \frac{1}{2} \Omega_{\bar{\alpha}, \lambda}\right] + \frac{i}{3} q^2 \epsilon_{\alpha \beta \gamma} \bar{Y}_{\beta \gamma}.
\]

(A.12)

It remains to solve the trace conditions (A.9) and (A.10) and (A.1) and (A.6) of the linear system in terms of \( \phi \) and \( \xi \). After some straightforward computation, one finds that

\[
pq \Phi_{\alpha} = \partial_{\alpha} q^2 - (2 + q^2) \left( i \Lambda_{\alpha} + \frac{1}{2} h_{\alpha}\right) - (-2 + 5 q^2) \Omega_{\alpha, \lambda} + 2i(2 - q^2) Y_{\alpha, \lambda} + 2q^2 \Omega_{\lambda, \alpha}
\]

\[
+ 2pq \epsilon_{\alpha \beta \gamma} \bar{Y}_{\beta \gamma},
\]

\[
p^2 q^2 \xi_{\alpha} = q^2 \partial_{\alpha} q^2 - (1 + 2q^4) \left( \frac{i}{2} \Lambda_{\alpha} + \frac{1}{4} h_{\alpha}\right) - \frac{1}{2} (2q^4 + 2q^2 - 1) \Omega_{\alpha, \lambda} - iY_{\alpha, \lambda} + q^2 \Omega_{\lambda, \alpha}
\]

\[
+ pq(1 + 2q^2) \frac{i}{3} \epsilon_{\alpha \beta \gamma} \bar{Y}_{\beta \gamma} + \frac{pq}{2} \epsilon_{\alpha \beta \gamma} \bar{Y}_{\beta \gamma},
\]
for the fluxes given above, one gets identically zero. Conversely, any one of the solutions above

\[ p^2 q^2 \xi_a = p^2 q^2 \xi_a - (1 + 2p^4) \left[ \frac{1}{2} \Lambda_a + \frac{1}{4} h_a \right] + \frac{1}{2} (2p^4 + 2p^2 - 1) \Omega_{a, \lambda} + i Y_{a, \lambda} + p^2 \Omega_{a, \lambda} \]

Clearly, we can substitute the \( \Phi_s \) and the \( \xi_s \) into (A.11) and (A.12) to express the traces and (3,0) and (0,3) part of \( H \) in terms of the \( Y \) fluxes and the geometry. However, we shall not do this here as for our conclusions this is not utilized.

Next using the expressions in (A.11) and (A.12), one can determine the (2,1) and (1,2) components of \( H \) as

\[ \frac{p}{4} H_{a, \mu_1, \mu_2} = q \left[ -\frac{1}{2} \Omega_{a, \mu_1, \mu_2} + i Y_{a, \mu_1, \mu_2} + i \delta_{[a]}(Y_{a, \mu_1, \mu_2})^b \right] + \frac{p}{4} \Omega_{a, \nu_1, \nu_2} \epsilon_{\nu_1 \nu_2} \]

\[ -\delta_{[a]}(Y_{a, \mu_1, \mu_2}) q \left( \frac{1}{2} \Lambda_{\mu_1, \mu_2} - \frac{1}{4} h_{\mu_1, \mu_2} + \frac{i}{2} \Omega_{\mu_1, \mu_2} \right) + i \frac{p}{3} \delta_{[a]}(Y_{a, \mu_1, \mu_2}) \epsilon_{\nu_1 \nu_2} \]

Not that the expression for \( h \) in (3.10) is not independent, and so if one substitutes the solution for the fluxes given above, one gets identically zero. Conversely, any one of the solutions above can be used to express \( h \) in terms of the fluxes and geometry.

Furthermore, one can use the solution of the linear system to determine \( \mu \) in (3.13) in terms of the \( Y \) fluxes and geometry as

\[ \sqrt{2} p q \mu_a = -\frac{1}{2} \Lambda_a + \frac{1}{4} (p^2 - q^2) h_a + \frac{1}{2} (p^4 - q^2) \Omega_{a, \lambda} + i Y_{a, \lambda} + \frac{2i}{3} pq e_{a, \lambda} Y_{a, \lambda} \]

Observe that the spin connection \( \tilde{\Omega} \) of the horizon sections \( S \), which is given by \( \tilde{\Omega}_{a, \lambda} = \Omega_{a, \lambda} \), is not restricted. This indicates that there are no additional restrictions on the topology and geometry of \( S \) apart from those required for the global existence of certain forms which we describe in section 4.2. This concludes the solution of the KSEs for the generic case.

The solution of the linear system for Spin(7) horizons can be derived from that of the generic case after setting \( p = q = 1/\sqrt{2} \). This is straightforward to implement and we shall not carry out the substitution here.

A.2.2. Pure SU(4) horizons. For pure SU(4) horizons, one has either \( p = 1, q = 0 \) or \( p = 0, q = 1 \). The two cases are symmetric and without loss of generality, we choose \( p = 1, q = 0 \). A direct computation reveals that the solution can be written as

\[ \Phi_a = H_{a, \lambda} \]

\[ \Phi_a = -e_{a, \lambda} \lambda_{a, \lambda} \Omega_{a, \lambda} - \frac{2i}{3} e_{a, \lambda} \lambda_{a, \lambda} Y_{a, \lambda} + H_{a, \lambda} \]

\[ \frac{1}{8} e_{a, \lambda} \lambda_{a, \lambda} H_{a, \lambda} = -\Omega_{a, \lambda} \]

\[ \frac{1}{2} H_{a, \lambda} h_a = \frac{1}{2} \Omega_{a, \nu_1, \nu_2} \epsilon_{\nu_1 \nu_2} H_{a, \lambda} \]

\[ \frac{1}{2} H_{a, \lambda} \frac{1}{2} \Omega_{a, \nu_1, \nu_2} \epsilon_{\nu_1 \nu_2} H_{a, \lambda} = \frac{1}{2} \delta_{[a]}(Y_{a, \lambda} \epsilon_{\nu_1 \nu_2} Y_{a, \lambda} \epsilon_{\nu_1 \nu_2}) \]

\[ \frac{1}{8} e_{a, \lambda} \lambda_{a, \lambda} H_{a, \lambda} = -\Omega_{a, \lambda} \]

\[ \frac{1}{2} H_{a, \lambda} h_a = \frac{1}{2} \Omega_{a, \nu_1, \nu_2} \epsilon_{\nu_1 \nu_2} H_{a, \lambda} \]

\[ \frac{1}{2} H_{a, \lambda} \frac{1}{2} \Omega_{a, \nu_1, \nu_2} \epsilon_{\nu_1 \nu_2} H_{a, \lambda} = \frac{1}{2} \delta_{[a]}(Y_{a, \lambda} \epsilon_{\nu_1 \nu_2} Y_{a, \lambda} \epsilon_{\nu_1 \nu_2}) \]
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A.3. Spinor conventions

For our spinor conventions, we use those of [19]. In particular, we use a realization of spinors in terms of forms. We shall not give further details here as this has been concisely explained in the appendices A of the first and third papers in [19]. In addition to integrate the KSEs along the lightcone directions, we have decomposed the Spin(9, 1) spinors $\psi$ into positive and negative parts as

$$\psi = \psi_+ + \psi_-, \quad \Gamma_\pm \psi_\pm = 0, \quad (A.18)$$

which is a lightcone, or equivalently $\text{Spin}(1, 1)$, chiral decomposition. Furthermore, we have found the following identities useful

$$\Gamma_{\epsilon_1, \ldots, \epsilon_n} \psi_\pm = \pm (-1)^{[\frac{n}{2}]} \frac{1}{(8 - n)!} \epsilon_{\epsilon_1, \ldots, \epsilon_n j_1 \ldots j_n} \Gamma_{j_1, \ldots, j_n} \psi_\pm, \quad n \geq 4, \quad (A.19)$$

where $\psi$ is a positive chirality $\text{Spin}(9, 1)$ spinor. While, one has

$$\Gamma_{\epsilon_1, \ldots, \epsilon_n} \psi_\pm = \mp (-1)^{[\frac{n}{2}]} \frac{1}{(8 - n)!} \epsilon_{\epsilon_1, \ldots, \epsilon_n j_1 \ldots j_n} \Gamma_{j_1, \ldots, j_n} \psi_\pm, \quad n \geq 4, \quad (A.20)$$

provided that $\psi$ is a negative chirality $\text{Spin}(9, 1)$ spinor.

A.4. Several identities

Finally, we present the following identities which are found useful. For our spinor conventions, we use those of [19]. In particular, we use a realization of spinors in terms of forms. We shall not give further details here as this has been concisely explained in the appendices A of the first and third papers in [19]. In addition to integrate the KSEs along the lightcone directions, we have decomposed the Spin(9, 1) spinors $\psi$ into positive and negative parts as

$$\psi = \psi_+ + \psi_-, \quad \Gamma_\pm \psi_\pm = 0, \quad (A.18)$$

which is a lightcone, or equivalently $\text{Spin}(1, 1)$, chiral decomposition. Furthermore, we have found the following identities useful

$$\Gamma_{\epsilon_1, \ldots, \epsilon_n} \psi_\pm = \pm (-1)^{[\frac{n}{2}]} \frac{1}{(8 - n)!} \epsilon_{\epsilon_1, \ldots, \epsilon_n j_1 \ldots j_n} \Gamma_{j_1, \ldots, j_n} \psi_\pm, \quad n \geq 4, \quad (A.19)$$

where $\psi$ is a positive chirality $\text{Spin}(9, 1)$ spinor. While, one has

$$\Gamma_{\epsilon_1, \ldots, \epsilon_n} \psi_\pm = \mp (-1)^{[\frac{n}{2}]} \frac{1}{(8 - n)!} \epsilon_{\epsilon_1, \ldots, \epsilon_n j_1 \ldots j_n} \Gamma_{j_1, \ldots, j_n} \psi_\pm, \quad n \geq 4, \quad (A.20)$$

provided that $\psi$ is a negative chirality $\text{Spin}(9, 1)$ spinor.

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