Indistinguishability and Energy Sensitivity of Asymptotically Gaussian Compressed Encryption

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Abstract—The principle of compressed sensing (CS) can be applied in a cryptosystem by providing the notion of security. In information-theoretic sense, it is known that a CS-based cryptosystem can be perfectly secure if it employs a random Gaussian sensing matrix updated at each encryption and its plaintext has constant energy. In this paper, we propose a new CS-based cryptosystem that employs a secret bipolar keystream and a public unitary matrix, which can be suitable for practical implementation by generating and renewing the keystream in a fast and efficient manner. We demonstrate that the sensing matrix is asymptotically Gaussian for a sufficiently large plaintext length, which guarantees a reliable CS decryption for a legitimate recipient. By means of probability metrics, we also show that the new CS-based cryptosystem can have the indistinguishability against an adversary, as long as the keystream is updated at each encryption and each plaintext has constant energy. Finally, we investigate how much the security of the new CS-based cryptosystem is sensitive to energy variation of plaintexts.

Index Terms—Compressed encryption, Hellinger distance, indistinguishability, linear feedback shift register (LFSR), probability metrics, self-shrinking generators, total variation distance.

I. INTRODUCTION

Compressed sensing (CS) [1]–[3] is to recover a sparse signal from the measurements that are believed to be incomplete. A signal $x \in \mathbb{R}^N$ is called $K$-sparse if it has at most $K$-nonzero entries, where $K \ll N$. A sparse signal is linearly measured by $y = \Phi x \in \mathbb{R}^M$, where $\Phi$ is an $M \times N$ sensing matrix with $M \ll N$. In CS theory, if $\Phi$ obeys the restricted isometry property (RIP) [1][4], a stable and robust reconstruction of $x$ can be guaranteed from the incomplete measurement $y$. The CS reconstruction is accomplished by solving an $l_1$-minimization problem with convex optimization or greedy algorithms [4]. With efficient measurement and stable reconstruction, the CS technique has been of interest in a variety of research fields, e.g., communications [5]–[7], sensor networks [8]–[10], image processing [11]–[13], radar [14], etc.

The CS principle can be applied in a cryptosystem for information security. A CS-based cryptosystem encrypts a plaintext through a CS measurement process, where the sensing matrix is kept secret. The ciphertext can then be decrypted through a CS reconstruction process by a legitimate recipient with the knowledge of the sensing matrix. In [15], Rachlin and Baron proved that a CS-based cryptosystem cannot be perfectly secure in itself, but might be computationally secure. Orsdenir et al. [16] showed that it is computationally secure against a key search technique via an algebraic approach. In [17], Bianchi et al. analyzed the security of a CS-based cryptosystem employing a random Gaussian sensing matrix updated at each encryption. Precisely, they showed that the cryptosystem with the one-time sensing random Gaussian matrix can be perfectly secure, as long as each plaintext has constant energy. A similar analysis has been made for a CS-based cryptosystem having a circulant sensing matrix for efficient CS processes [18][19]. In [20] and [21], wireless channel characteristics could be exploited for wireless security of CS-based cryptosystems. The CS technique can also be applied in database systems [22], where random noise has been intentionally added to CS measurements for differential privacy. In practice, a variety of CS-based cryptosystems concerning the security of multimedia, imaging, and smart grid data have been suggested in [23]–[29].

In this paper, we propose a new CS-based cryptosystem that employs a secret bipolar keystream and a public unitary matrix, which can be suitable for practical implementation by generating and renewing the keystream in a fast and efficient manner. The keystream generator, based on a linear feedback shift register (LFSR), plays a crucial role in the efficient implementation. We demonstrate that the entries of the sensing matrix are asymptotically Gaussian distributed if the plaintext length is sufficiently large. With the sensing matrix, it is obvious that the new CS-based cryptosystem, named as the asymptotically Gaussian one-time sensing (AG-OTS) cryptosystem, theoretically guarantees a stable and robust CS decryption for a legitimate recipient.

For security analysis, we study the indistinguishability [30] of the AG-OTS cryptosystem. The total variation (TV) distance [31][32] between probability distributions of ciphertexts conditioned on a pair of plaintexts is examined as a security measure for the indistinguishability, where the upper and lower bounds on the TV distance are developed by the Hellinger distance [33][34]. With the probability metrics, we examine the success probability of an adversary to distinguish a pair of potential plaintexts from a given ciphertext. By proving that the success probability of any kind of attack is at most that of a random guess, we demonstrate that the AG-OTS cryptosystem can have the indistinguishability, as long as each plaintext has constant energy. Therefore, the AG-OTS cryptosystem, if it has a normalization step before encryption for equalizing the plaintext energy, can be computationally secure.

Finally, we investigate how much the security of the AG-OTS cryptosystem is sensitive to energy variation of plaintexts. It is worth studying the energy sensitivity, since one might need to assign unequal energy for plaintexts in the presence of noise, depending on the reliability demands. As a consequence,
we develop sufficient conditions on the minimum energy ratio, the plaintext length, and the maximum plaintext-to-noise power ratio, respectively, to achieve the asymptotic indistinguishability of the AG-OTS cryptosystem having unequal plaintext energy. Since the analysis relies on the Gaussianity of the sensing matrix, the results of the energy sensitivity can also be applicable to the Gaussian one-time sensing (G-OTS) cryptosystem in [17].

This paper is organized as follows. In Section II, we propose a new CS-based cryptosystem employing a secret bipolar keystream, where the sensing matrix turns out to be asymptotically Gaussian. Also, we discuss an LFSR-based efficient keystream generator for the cryptosystem. Section III introduces the indistinguishability along with the probability metrics of total variation (TV) and Hellinger distances. For security analysis, Section IV studies the indistinguishability and the energy sensitivity of the new CS-based cryptosystem in the presence of noise. Section V presents numerical results to demonstrate the security of the new CS-based cryptosystem. Finally, concluding remarks will be given in Section VI.

Notations: A matrix (or a vector) is represented by a bold-face upper (or lower) case letter. $U^T$ and $|U|$ denote the transpose and the determinant of a matrix $U$, respectively. $U(k, t)$ is an entry of an $M \times N$ matrix $U$ in the $k$th row and the $t$th column, where $0 \leq k \leq M - 1$ and $0 \leq t \leq N - 1$. Also, $U(k, :)$ denotes the $k$th row vector of $U$, while $U(:, t)$ is the $t$th column vector of $U$. $\text{diag}(s)$ is a diagonal matrix whose diagonal entries are from a vector $s$. An identity matrix is denoted by $I$, where the dimension is determined in the context. $D$ denotes an $N \times N$ discrete-cosine transform (DCT) matrix, where $DD^T = D^T D = NI$. For an $N$-dimensional vector $x = (x_1, \cdots, x_N)^T \in \mathbb{R}^N$, the $l_p$-norm of $x$ is denoted by $||x||_p = \left( \sum_{k=1}^{N} |x_k|^p \right)^{\frac{1}{p}}$, where $1 \leq p < \infty$. If the context is clear, $||x||$ denotes the $l_2$-norm of $x$. A vector $n \sim \mathcal{N}(0, \sigma^2 I)$ is a Gaussian random vector with mean $0 = (0, \cdots, 0)^T$ and covariance $\sigma^2 I$. Finally, $\mathbb{E}[\cdot]$ denotes the average of a random vector or a random matrix.

II. SYSTEM MODEL

In [17], the authors presented the Gaussian one-time sensing (G-OTS) cryptosystem, where a random Gaussian sensing matrix is used only once for each encryption, and renewed for the next. In information-theoretic sense, they showed that if each plaintext has constant energy, the G-OTS cryptosystem can be perfectly secure, which implies the indistinguishability [30] that will be discussed in next section.

In practice, generating the Gaussian entries at each encryption may require high complexity and large memory for CS encryption and decryption. For efficient implementation, this section proposes a new CS-based cryptosystem in which the sensing matrix employs a bipolar keystream.

A. Asymptotically Gaussian Sensing Matrices

Definition 1: Let $U \in \mathbb{R}^{N \times N}$ be a public unitary matrix, i.e., $U^T U = UU^T = NI$, where each element of $U$ has the magnitude of $O(1)$. Let $S$ be a secret $M \times N$ matrix, where we assume that each element takes $\pm 1$ independently and uniformly at random. Then, a new CS-based cryptosystem has the sensing matrix of 

$$\Phi = \frac{1}{\sqrt{MN}}SU.$$

Theoretically, each element of $S$ can be taken from the random Bernoulli distribution. In practice, however, we consider a keystream generator of stream ciphers to generate it in a fast and efficient manner. Employing an efficient keystream generator allows us to construct and update $S$ at each encryption with low complexity and small memory. Since a keystream for a stream cipher is designed to have nice pseudorandomness properties [33], such as balance, large period, low autocorrelation, large linear complexity, etc., we assume that each entry of $S$ from the keystream takes $\pm 1$ independently and uniformly at random, which facilitates the reliability and security analysis of the new CS-based cryptosystem.

Theorem 1: In Definition 1 the elements of $\Phi$ follow the Gaussian distribution asymptotically for a sufficiently large $N$.

Proof: Each row of $\Phi$ is represented by

$$\Phi(k, :) = \frac{1}{\sqrt{MN}}S(k, :)U$$

$$= \sqrt{\frac{N}{M}} \cdot \frac{1}{\sqrt{N}} \text{diag}(S(k, :)) \frac{U}{\sqrt{N}}, \quad k = 1, \cdots, M$$

(1)

where $S(k, :)$ is the $k$th row vector of $S$, and $1 = (1, \cdots, 1)$ is all one row vector of length $N$, respectively. In $|U|$, $\frac{1}{\sqrt{N}}$ is a row of a unit-norm row matrix with absolute magnitude of all entries of $O\left(\frac{1}{\sqrt{N}}\right)$. Also, $\frac{U}{\sqrt{N}}$ is a unit-norm column matrix with the maximum absolute magnitude of entries of $o(1)$. With the structure, Theorem III.1 of [33] shows that the elements of $\Phi(k, :)$ are asymptotically Gaussian if $N$ is sufficiently large, which completes the proof.

The asymptotic Gaussianity of Theorem 1 also holds if the elements of $S$ are generated by an efficient keystream generator, under the assumption that each one takes $\pm 1$ independently and uniformly at random. The assumption will be validated by the numerical results of Section V.

B. Keystream Generation

For the secret matrix $S$ of Definition 1 we employ a keystream generator based on a linear feedback shift register (LFSR), to generate the elements in a fast and efficient manner. As an example, we introduce the self-shrinking generator (SSG) [33].

Definition 2: Assume that an $L$-stage LFSR generates a binary $m$-sequence of $a = (a_0, a_1, \cdots)$. With a clock-controlled operation, the self-shrinking generator outputs $b_t = a_{2t+1}$ if $a_{2t+1} = 1$, and discards $a_{2t+1} = 0$. Then, we obtain a bipolar keystream of $s = (s_0, s_1, \cdots)$, where $s_t = (-1)^b_t$ for $t = 0, 1, \cdots$, which will be arranged as the elements of $S$.

The SSG keystream generation requires a simple structure of an $L$-stage LFSR along with a clock-controlled operator.
Moreover, the SSG keystream possesses nice pseudorandomness properties \cite{33}, such as balance, large period, and large linear complexity. Meier and Staffelbach \cite{33} showed that the SSG keystream is balanced, and has the period of at least \(2^{L/2}\) and the linear complexity of at least \(2^{L/2-1}\), respectively. Although the SSG keystream generator is considered in this paper, any other LFSR-based keystream generator can also be applied for the new CS-based cryptosystem.

When each element of \(S\) is obtained by a keystream generator, the initial seed (or state) of the generator is essentially the key of the new CS-based cryptosystem. Therefore, the key should be kept secret between a sender and a legitimate recipient, while the structure of the keystream generator can be publicly known.

C. AG-OTS Cryptosystem

From Definition 1, the new CS-based cryptosystem encrypts a \(K\)-sparse plaintext \(x\) by producing a ciphertext \(y = \Phi x = \frac{1}{\sqrt{MN}}SUx\), where \(S\) is updated at each encryption. Under the presence of noise, a legitimate recipient and an adversary have a noisy ciphertext \(r = \Phi x + n\), where \(n \sim N(0, \sigma^2 I)\). As \(\Phi\) is asymptotically Gaussian and \(S\) is updated at each encryption, the new CS-based cryptosystem will be called the asymptotically Gaussian one-time sensing (AG-OTS) cryptosystem throughout this paper. Table II summarizes the symmetric-key AG-OTS cryptosystem proposed in this paper.

The reliability and stability of the AG-OTS cryptosystem for a legitimate recipient is straightforward from the RIP result \cite{30} of a random Gaussian matrix, under the fact that \(\Phi\) is Gaussian for a sufficiently large \(N\).

**Proposition 1:** \cite{30} For a legitimate recipient, if \(N\) is sufficiently large, the AG-OTS cryptosystem theoretically guarantees a stable and robust CS decryption with bounded errors of a \(K\)-sparse plaintext, as long as \(M = O(K \log(N/K))\).

III. SECURITY MEASURE

This section introduces a security measure of the indistinguishability of a CS-based cryptosystem. To examine the indistinguishability, we also discuss the probability metrics of total variation (TV) and Hellinger distances.

A. Indistinguishability

Assume that a cryptosystem produces a ciphertext by encrypting one of two possible plaintexts of the same length. Then, the cryptosystem is said to have the indistinguishability \cite{30}, if no adversary can determine in polynomial time which of the two plaintexts corresponds to the ciphertext, with probability significantly better than that of a random guess. In other words, if a cryptosystem has the indistinguishability, an adversary is unable to learn any partial information of the plaintext in polynomial time from a given ciphertext.

Table II describes the indistinguishability experiment \cite{30} in the presence of an eavesdropper, which will be used to investigate the indistinguishability of a CS-based cryptosystem in this paper.

\footnote{In general, \(x\) can be \(K\)-sparse in an arbitrary orthonormal basis \(\Psi\), i.e., \(x = \Psi \theta\) with \(||\theta||_0 \leq K\), where \(\Psi \neq \frac{1}{\sqrt{N}} U^T\). For simplicity, we assume \(\Psi = I\) in this paper.}

B. Total Variation (TV) and Hellinger Distances

In this paper, we make use of the total variation (TV) distance \cite{31} to evaluate the performance of an adversary in the indistinguishability experiment of Table II. In the experiment, let \(d_{TV}(p_1, p_2)\) be the TV distance between the probability distributions \(p_1 = \Pr(r|x_1)\) and \(p_2 = \Pr(r|x_2)\). Then, it is readily checked from \cite{32} that the probability that an adversary can successfully distinguish the plaintexts by any kind of test \(D\) is bounded by

\[
p_d \leq \frac{1}{2} + \frac{d_{TV}(p_1, p_2)}{2}
\]  

(2)

where \(d_{TV}(p_1, p_2) \in [0, 1]\). Therefore, if \(d_{TV}(p_1, p_2)\) is zero, the probability of success is at most that of a random guess, which leads to the indistinguishability \cite{30}.

Since computing \(d_{TV}(p_1, p_2)\) directly is difficult \cite{37}, we may employ an alternative distance metric to bound the TV distance. In particular, the Hellinger distance \cite{31}, denoted by \(d_{H}(p_1, p_2)\), is useful by giving both upper and lower bounds on the TV distance \cite{33}, i.e.,

\[
d_{H}^2(p_1, p_2) \leq d_{TV}(p_1, p_2) \leq d_{H}(p_1, p_2) \sqrt{2 - d_{H}^2(p_1, p_2)}
\]  

(3)

where \(d_{H}(p_1, p_2) \in [0, 1]\). Moreover, if a ciphertext \(r\) conditioned on \(x_h\), is a jointly Gaussian random vector with zero mean and the covariance matrix \(C_h\), where \(h = 1\) and \(2\), the Hellinger distance between the multivariate Gaussian distributions \(p_1\) and \(p_2\) is given by \cite{39}, \cite{40}.

\[
d_{H}(p_1, p_2) = \sqrt{1 - \frac{|C_1|^{+} \cdot |C_2|^{+}}{|C_3|^2}}
\]  

(4)

where \(C_3 = C_1 + C_2\). For the formal definitions and properties of the TV and the Hellinger distances, readers are referred to \cite{31}, \cite{32}, and \cite{37}.

Throughout this paper, we use (2) - (4) to examine the success probability of the indistinguishability experiment for the AG-OTS cryptosystem, by taking the Gaussian distributed ciphertexts into account.

IV. SECURITY ANALYSIS

In this section, we show that the AG-OTS cryptosystem can be indistinguishable, as long as each plaintext has constant energy. Moreover, we study how much the security of the AG-OTS cryptosystem is sensitive to energy variation of plaintexts.

A. Indistinguishability

Recall the indistinguishability experiment of Table II. Given a plaintext \(x_h\), \(E[r|x_h] = E[\Phi|x_h] + E[n] = \frac{1}{\sqrt{MN}} E[S] \cdot Ux_h\) from \(E[S] = 0\), where \(h = 1\) and \(2\). In the following, Lemma I derives the covariance matrix of \(r\) conditioned on \(x_h\), by exploiting the independency and the uniformity of the entries of \(S\).

**Lemma 1:** In the AG-OTS cryptosystem, the covariance matrix of \(r\) conditioned on \(x_h\) is given by

\[
C_h = E[rr^T|x_h] = \left(\frac{|x_h|^2}{M} + \sigma^2\right) I
\]  

(5)
Proof: Let \( \xi_h = U \mathbf{x}_h = (\xi_{h,1}, \cdots, \xi_{h,N})^T \) for \( h = 1 \) and \( 2 \), respectively, where \( ||\xi_h||^2 = N ||x_h||^2 \). Also, let \( s_k = S(:, k) \) and \( s_l = S(:, l) \) be the \( k \)th and the \( l \)th column vectors of \( S \), respectively. Since the elements of \( S \) and \( \mathbf{n} \) are independent to each other,

\[
C_h = \mathbb{E} \left[ \frac{1}{MN} \mathbf{S} \mathbf{U} \mathbf{x}_h \cdot \mathbf{x}_h^T \mathbf{U}^T \mathbf{S}^T | \mathbf{x}_h \right] + \mathbb{E} [\mathbf{m} \mathbf{m}^T] = \frac{1}{MN} \sum_{k=1}^{N} \sum_{l=1}^{N} \mathbb{E} [\xi_{h,k} \xi_{h,l}^T] + \sigma^2 \mathbf{I} \tag{6}
\]

where

\[
\mathbb{E} [s_k s_l^T] = \begin{cases} 
1, & \text{if } k = l, \\
0, & \text{if } k \neq l
\end{cases}
\]
as the entries of \( s_k \) and \( s_l \) take \( \pm 1 \) independently and uniformly at random. Thus, \( \Box \) yields

\[
C_h = \frac{1}{MN} \left( \sum_{k=1}^{N} ||\xi_{h,k}||^2 \right) \cdot \mathbf{I} + \sigma^2 \mathbf{I}
\]

which completes the proof. \( \Box \)

In Lemma\( \Box \) note that the derivation of covariance matrices does not rely on the asymptotic Gaussianity of \( \Phi \). Instead, the covariance matrices are non-asymptotic results, obtained by exploiting the independency and the uniformity of the elements of \( S \).

Using the covariance matrices of Lemma\( \Box \) we can develop upper and lower bounds on the TV distance in the AG-OTS cryptosystem, which is the main contribution of this paper.

**Theorem 2:** In the AG-OTS cryptosystem, assume that the plaintext length \( N \) is sufficiently large such that \( \Phi \) can be asymptotically Gaussian by Theorem 1. In the indistinguishability experiment, let \( d_{TV}(p_1, p_2) \) be the TV distance between probability distributions of ciphertexts conditioned on a pair of plaintexts in the AG-OTS cryptosystem. Let \( x_{\min} \) and \( x_{\max} \) be the plaintexts that have the minimum and maximum possible energies, respectively, where \( \gamma = \frac{||x_{\min}||^2}{||x_{\max}||^2} \) is the minimum energy ratio and \( \text{PNR}_{\max} = \frac{1}{\gamma + 1} \) is the maximum plaintext-to-noise power ratio, respectively, of the cryptosystem. Then, the worst-case lower and upper bounds on \( d_{TV}(p_1, p_2) \) are given by

\[
d_{TV, low} = 1 - \left( \frac{4 \gamma_e}{(\gamma_e + 1)^2} \right)^{4}, \\
d_{TV, up} = \sqrt{1 - \left( \frac{4 \gamma_e}{(\gamma_e + 1)^2} \right)^{4}},
\]

respectively, where

\[
\gamma_e = \frac{1 + \gamma \cdot \text{PNR}_{\max}}{1 + \text{PNR}_{\max}}.
\]

**Proof:** In the indistinguishability experiment of Table\( \Box \) let us consider a pair of plaintexts \( x_1 \) and \( x_2 \), where \( \text{PNR}_h = \frac{||x_h||^2}{MN} \) for \( h = 1 \) and \( 2 \). From the covariance matrices of Lemma\( \Box \)

\[
|C_h| = \left( \frac{||\xi_h||^2}{M} + \sigma^2 \right)^M = \sigma^{2M} \cdot (\text{PNR}_h + 1)^M
\]

for each \( h \). Obviously,

\[
|C_3| = \sigma^{2M} \cdot \left( \frac{\text{PNR}_1 + \text{PNR}_2 + 1}{2} \right)^M.
\]
In [4],
\[ \Gamma = \frac{|C_1|^{\frac{1}{2}} \cdot |C_2|^{\frac{1}{2}}}{|C_3|^{\frac{1}{2}}} = \left( \frac{(\text{PNR}_1 + 1)(\text{PNR}_2 + 1)}{(\text{PNR}_1 + \text{PNR}_2) + 1} \right)^{\frac{1}{2}} \]
\[ = \left( \frac{1 + \text{PNR}_1}{1 + \text{PNR}_2} \cdot \text{PNR}_2 \right) \]
where
\[ \gamma_e = \frac{1 + \text{PNR}_1}{1 + \text{PNR}_2} \cdot \text{PNR}_2 \]
and \( \gamma = \frac{||x_2||^2}{||x_1||^2} \). With \( d_{TV}(p_1, p_2) = \sqrt{1 - \Gamma} \), [3] yields the lower and upper bounds of the form of (7). Without loss of generality, we may assume \( ||x_1||^2 \leq ||x_2||^2 \), which yields \( 0 \leq \gamma \leq 1 \). As the lower and upper bounds turn out to be monotonically decreasing over \( \gamma \in [0, 1] \), we can redefine \( \gamma = \frac{||x_2||^2}{||x_1||^2} \) and \( \gamma_e = \frac{1 + \gamma \cdot \text{PNR}_1}{1 + \text{PNR}_2} \), \( 0 \leq \gamma_e \leq 1 \) with \( x_1 = x_{\text{min}} \) and \( x_2 = x_{\text{max}} \), to obtain the worst-case bounds, which completes the proof. \( \square \)

In (8), \( \gamma_e \) is a general definition of the energy ratio covering noisy cases, which will be called the effective energy ratio in this paper. For security analysis, we assume that both a legitimate recipient and an adversary have the same energy ratio \( \gamma \) and the same \( \text{PNR}_{\text{max}} \) in the AG-OTS cryptosystem.

Theorem 2 shows that the indistinguishability of the AG-OTS cryptosystem depends on the ciphertext length \( M \), the minimum energy ratio \( \gamma \), and the maximum plaintext-to-noise ratio \( \text{PNR}_{\text{max}} \), irrespective of the plaintext length \( N \) and the sparsity \( K \). In particular, if \( \gamma = \gamma_e = 1 \), the indistinguishability can be guaranteed for the AG-OTS cryptosystem, regardless of \( M \) and \( \text{PNR}_{\text{max}} \).

**Corollary 1:** If each plaintext has constant energy or \( \gamma = 1 \), the AG-OTS cryptosystem has the indistinguishability, since the success probability of the indistinguishability experiment is at most 0.5 from [2], thanks to \( d_{TV}(p_1, p_2) = 0 \) for \( d_{TV, \text{low}} = d_{TV, \text{up}} = 0 \).

In the AG-OTS cryptosystem, Corollary 1 ensures that no adversary can learn any partial information about the plaintext from a given ciphertext, as long as each plaintext has constant energy, which is also the case in the G-OTS cryptosystem of [17]. To achieve the indistinguishability, therefore, a normalization step for equalizing the plaintext energy is implicitly required before CS encryption in the AG-OTS cryptosystem of Table 1. Since it also offers a practical benefit from the efficient keystream generation, the AG-OTS cryptosystem can be a promising option for information security, by guaranteeing the indistinguishability, reliability, and efficiency in a CS framework.

**B. Energy Sensitivity**

Theorem 2 implies that the indistinguishability of the AG-OTS cryptosystem can be sensitive to the minimum energy ratio \( \gamma \). Figure 1 sketches the upper and lower bounds of (7) over \( \gamma \) at \( \text{PNR}_{\text{max}} = \infty \) in the noiseless AG-OTS cryptosystem. It indicates that the TV distance increases as \( \gamma \) gets away from 1. In particular, if \( M \) gets larger, the TV distance approaches to 1 more quickly as \( \gamma \) decreases. Such a behavior of the TV distance suggests that if \( \gamma \) is far less than 1, an adversary may be able to detect a correct plaintext in the indistinguishability experiment with a significantly high probability of success, which implies that the AG-OTS cryptosystem may not be indistinguishable.

In addition, Figure 2 shows the upper bounds of (7) over \( \gamma \) for various \( \text{PNR}_{\text{max}} \) in the noisy AG-OTS cryptosystem, where \( M = 64 \). In the figure, the bounds are sensitive to \( \gamma \) for each \( \text{PNR}_{\text{max}} \), as in the noiseless case of Figure 1. Moreover, the bound itself is smaller at less \( \text{PNR}_{\text{max}} \), which implies that an adversary may have a difficulty in distinguishing plaintexts at low \( \text{PNR}_{\text{max}} \) due to the low TV distance. As a result, it appears that the security of the AG-OTS cryptosystem would...
be more sensitive to the energy ratio \( \gamma \) at higher PNR\(_{\text{max}}\).

In summary, the AG-OTS cryptosystem may not be able to achieve the indistinguishability, unless each plaintext has constant energy. In what follows, we study how much the security of the AG-OTS cryptosystem is sensitive to energy variation of plaintexts. It is worth studying the energy sensitivity, since one might need to assign unequal energy for each plaintext in the presence of noise, depending on the reliability demands.

Theorems 3–5 present sufficient conditions for the minimum energy ratio \( \gamma \), the plaintext length \( N \), and the maximum plaintext-to-noise ratio PNR\(_{\text{max}}\), respectively, to guarantee the asymptotic indistinguishability for the AG-OTS cryptosystem.

**Theorem 3:** When \( M \) and PNR\(_{\text{max}}\) are given, let \( \varphi = (1 - 4\epsilon_N^2)^{-\frac{1}{2}} \) and \( \gamma_{e,\text{min}} = 2\varphi - 1 - 2\sqrt{\varphi(\varphi - 1)} \). If the minimum energy ratio \( \gamma \) satisfies

\[
\gamma \geq \gamma_{e,\text{min}} - (1 - \gamma_{e,\text{min}}) \cdot \text{PNR}\(_{\text{max}}\) = \gamma_{\text{min}},
\]

the success probability of the indistinguishability experiment is \( p_d \leq 0.5 + \epsilon_N \), where \( \epsilon_N \) vanishes as the plaintext length \( N \) increases. In other words, the AG-OTS cryptosystem is asymptotically indistinguishable for a sufficiently large \( N \), as long as \( \gamma \geq \gamma_{\text{min}} \) for given \( M \) and PNR\(_{\text{max}}\).

**Proof:** When \( M \) is given, we have \( \frac{4\gamma_{e,\text{min}}^2}{(\gamma_{e,\text{min}} + 1)^2} \geq (1 - 4\epsilon_N^2)^{\frac{1}{2}} \) from \( d_{TV,\text{up}} \leq 2\epsilon_N \). The inequality turns into \( \gamma_{e,\text{min}}^2 - 2(2\varphi - 1)\gamma_{e,\text{min}} + 1 \leq 0 \), which holds if \( \gamma \geq \gamma_{e,\text{min}} \), or equivalently if \( \gamma \geq \gamma_{\text{min}} \) is met, the success probability of the indistinguishability experiment is \( p_d \leq 0.5 + \epsilon_N \) by (2), which completes the proof. \( \square \)

In Theorem 3 \( \gamma_{\text{min}} \) is the minimum energy ratio required for the asymptotic indistinguishability of the AG-OTS cryptosystem. Figure 3 displays \( \gamma_{\text{min}} \) over \( M \) in the AG-OTS cryptosystem at PNR\(_{\text{max}}\) = 20 dB, where \( N = 512 \). In the figure, \( \gamma_{\text{min}} \) is sketched for various \( \epsilon_N = \frac{\log N}{\sqrt{N}} \).

\begin{center}
\textbf{Fig. 3.} Minimum energy ratio required for the asymptotic indistinguishability of the AG-OTS cryptosystem at PNR\(_{\text{max}}\) = 20 dB, where \( N = 512 \).
\end{center}

and \( \frac{1}{2} \). The figure reveals that the minimum energy ratio required for the asymptotic indistinguishability approaches to 1 as the ciphertext length \( M \) increases. In particular, if the AG-OTS cryptosystem allows larger energy variation for plaintexts, the asymptotic indistinguishability can be achieved at a lower rate over \( N \).

**Theorem 4:** When \( \gamma \) and PNR\(_{\text{max}}\) are given, let \( \gamma_{e} = 1 + \frac{1}{4\gamma\text{PNR}_{\text{max}}} \). Let \( C_{\gamma_{e}} = \log(\frac{4\gamma_{e}}{(\gamma_{e} + 1)^2}) \leq 0 \), where the equality holds if and only if \( \gamma = 1 \). If the ciphertext length \( M \) satisfies

\[
M \leq \frac{2}{C_{\gamma_{e}}} \log(1 - 4\epsilon_N^2) \triangleq M_{\text{max}},
\]

then \( p_d \leq 0.5 + \epsilon_N \), which implies that the AG-OTS cryptosystem is asymptotically indistinguishable for a sufficiently large \( N \), as long as \( M \leq M_{\text{max}} \) for given \( \gamma \) and PNR\(_{\text{max}}\).

**Proof:** When \( \gamma_{e} \) is given from \( \gamma \) and PNR\(_{\text{max}}\), the proof is similar to that of Theorem 3 from \( d_{TV,\text{up}} \leq 2\epsilon_N \). \( \square \)

Figure 4 depicts the maximum compression ratio \( \rho_{\text{max}} = \frac{M_{\text{max}}}{N} \) over \( N \) for the AG-OTS cryptosystem to be asymptotically indistinguishable at PNR\(_{\text{max}}\) = 20 dB with \( p_d \leq 0.5 + \epsilon_N \), where \( \epsilon_N = \frac{1}{N} \). We also sketch the minimum compression ratio \( \rho_{\text{min}} = \frac{2K\log(N/K)}{N} \) for reliable CS decryption\(^2\) from a random Gaussian sensing \( [41] \) with \( K = 8 \), to compare the requirements for the asymptotic indistinguishability and the reliability. Note that if \( \gamma = 1 \) or the plaintexts have constant energy, the indistinguishability can be achieved at any compression ratio. Meanwhile, if \( \gamma < 1 \), the compression ratio of the AG-OTS cryptosystem may be at most \( \rho_{\text{max}} \) for the asymptotic indistinguishability. In particular, if \( \rho_{\text{max}} < \rho_{\text{min}} \), the cryptosystem may not be valid at least in theory for the corresponding \( N \), since the indistinguishability cannot be compatible with the reliability. Thus, Figure 4 shows that if

\begin{center}
\textbf{Fig. 4.} Compression ratios for the asymptotic indistinguishability and the reliability of the AG-OTS cryptosystem at PNR\(_{\text{max}}\) = 20 dB, where \( K = 8 \).
\end{center}

\(^2\)This is a theoretical ratio in noiseless recovery.
\( \gamma = 0.98 \), the AG-OTS cryptosystem at \( \text{PNR}_{\text{max}} = 20 \ \text{dB} \) can achieve both reliability and security for the plaintexts of at most \( K = 8 \) nonzero entries only at the compression ratios of the achievable (shaded) region. It also shows that if \( \gamma \leq 0.96 \), the AG-OTS cryptosystem has no theoretically achievable region for \( N > 500 \), where the reliability and the indistinguishability cannot be guaranteed simultaneously.

In (10), note that
\[
\gamma_e = \gamma + \frac{1 - \gamma}{1 + \text{PNR}_{\text{max}}}
\]
where \( \gamma_e \geq \gamma \) for \( \gamma \in [0, 1] \). Since the upper bound of (10) is monotonically decreasing over \( \gamma_e \in [0, 1] \), (10) implies that the upper bound on the TV distance is lower in noisy case (\( \text{PNR}_{\text{max}} \ll \infty \)) than in noiseless case (\( \text{PNR}_{\text{max}} = \infty \)). Ultimately, it points out that the presence of noise improves the security of the AG-OTS cryptosystem by lowering the success probability of an adversary in the indistinguishability experiment. Moreover, one can increase \( \gamma_e \) by reducing \( \text{PNR}_{\text{max}} \) in (10) for a given \( \gamma \), which indicates that the AG-OTS cryptosystem will be more secure for less \( \text{PNR}_{\text{max}} \).

With given \( \gamma \) and \( M \), Theorem 5 presents the largest possible \( \text{PNR}_{\text{max}} \) to guarantee the asymptotic indistinguishability for the AG-OTS cryptosystem, where the proof is straightforward from \( \gamma_e \geq \gamma_{e, \text{min}} \) in (10).

**Theorem 5:** In the AG-OTS cryptosystem, assume that the minimum energy ratio is given as \( \gamma < \gamma_{e, \text{min}} \) for a given \( M \), where \( \gamma_{e, \text{min}} \) is the minimum effective energy ratio defined in Theorem 3. Then, the asymptotic indistinguishability can be achieved for a sufficiently large \( N \), if
\[
\text{PNR}_{\text{max}} \leq \frac{1 - \gamma_{e, \text{min}}}{\gamma_{e, \text{min}} - \gamma}.
\]

Note that if \( \gamma \geq \gamma_{e, \text{min}} \), the AG-OTS cryptosystem is asymptotically indistinguishable, regardless of \( \text{PNR}_{\text{max}} \), due to \( \gamma_e \geq \gamma \geq \gamma_{e, \text{min}} \). Figure 5 displays the upper bounds on \( \text{PNR}_{\text{max}} \) of Theorem 5 for various \( \gamma < \gamma_{e, \text{min}} \), where \( N = 512 \) and \( \epsilon_N = \frac{1}{\sqrt{N}} \). From (10), it is clear that if \( \text{PNR}_{\text{max}} \) is sufficiently high, \( \gamma_e \approx \gamma < \gamma_{e, \text{min}} \) from which the asymptotic indistinguishability cannot be achieved from Theorem 3. Figure 5 points out that we need to increase \( \gamma_e \) by reducing \( \text{PNR}_{\text{max}} \) below the upper bound for each given \( \gamma \), to achieve the asymptotic indistinguishability of the AG-OTS cryptosystem. However, it appears that the largest possible \( \text{PNR}_{\text{max}} \) is relatively low for a reliable CS decryption. For the AG-OTS cryptosystem, therefore, it is an important issue to keep the energy variation of plaintexts as low as possible.

In conclusion, it turned out that the security of the AG-OTS cryptosystem is highly sensitive to the energy ratio of plaintexts. The indistinguishability can be achieved only if all the plaintexts have equal and constant energy. Therefore, if the AG-OTS cryptosystem is to be indistinguishable non-asymptotically, it is essential that each plaintext should be normalized before CS encryption to have constant energy. By analyzing the energy sensitivity, we presented the sufficient conditions of Theorems 3 − 5 for the asymptotic indistinguishability of the AG-OTS cryptosystem with unequal plaintext energy. However, we found that even the asymptotic indistinguishability can be achieved only if the plaintexts have low energy variation for most \( M \), \( N \), and \( \text{PNR}_{\text{max}} \). As the analysis technique utilizes the result of Theorem 2 based on the Gaussianity of the sensing matrix, the energy sensitivity of this paper can also be valid for the G-OTS cryptosystem, which has never been discussed in [17].

V. **Numerical Results**

This section presents numerical results to demonstrate the indistinguishability and the energy sensitivity of the AG-OTS cryptosystem. In numerical experiments, each plaintext \( x \) has at most \( K \) nonzero entries, where the positions are chosen uniformly at random and the coefficients are taken from the Gaussian distribution. In CS encryption, \( \Phi = \frac{1}{\sqrt{MN}} \text{SU} \), where \( U = D \) is the discrete cosine transform (DCT) matrix. Each element of the secret matrix \( S \) is taken from a bipolar keystream obtained by the self-shrinking generator (SSG) with a 128-stage LFSR. For comparison, we test with \( S \) whose elements are taken from the random Bernoulli distribution. We assume that a ciphertext is available for both an adversary and a legitimate recipient with the same \( \text{PNR} = \frac{||x||^2}{MN} \). For CS decryption, the CoSaMP recovery algorithm [42] is employed for a legitimate recipient to decrypt each ciphertext with the knowledge of \( S \). Meanwhile, we assume that an adversary can attempt any kind of detection in polynomial time, to pass the indistinguishability experiment by distinguishing a pair of plaintexts from a given ciphertext.

Figure 6 displays the quantile-quantile (QQ) plots of the entries of total 100 matrices of \( \sqrt{N} \Phi \) in the AG-OTS cryptosystem, where \( N = 512 \) and \( M = 64 \). In Figure 6(a), each entry of \( S \) is taken from the random Bernoulli distribution taking \( \pm 1 \) independently and uniformly at random, while \( S \) of Figure 6(b) is from the bipolar SSG keystream. Since both QQ-plots are linear with slope 1, it appears that the entries
of $\sqrt{M} \Phi$ follow the normal distribution in both cases of $S$. The figure gives a numerical evidence that $\Phi$ of the AG-OTS cryptosystem is asymptotically Gaussian for a sufficiently large $N$, even if $S$ is generated in a pseudorandom fashion by the SSG.

Figure 7 illustrates the covariance matrices of $M \cdot E[rr^T|x]$ in the AG-OTS cryptosystem at PNR = 20 dB, where $N = 512$, $M = 64$, and $K = 8$. In the experiment, total 10000 matrices of $S$ have been tested for the average with $||x||^2 = 1$. In the figure, the dark areas indicate the off-diagonal entries of each covariance matrix having very small magnitudes less than $0.04$, whereas the white cells represent the diagonal components of significant values, determined by the plaintext energy and the noise variance. Figure 7 numerically confirms that the covariance analysis of Lemma 1 is valid for the AG-OTS cryptosystem, whether $S$ is a random Bernoulli matrix or a matrix from the SSG keystream.

Figure 8 displays the upper and lower bounds of Theorem 2 on the TV distance over $\gamma$ in the AG-OTS cryptosystem at PNR$_{\text{max}} = 20$ dB, where $N = 512$, $M = 64$, and $K = 8$. In the experiment, we computed the bounds of (3) using the covariance matrices obtained by testing total $10^6$ matrices of $S$, where each entry of $S$ is taken from the random Bernoulli distribution or the SSG keystream. In both cases of $S$, the figure shows that the bounds from the experiment are well matched to the theoretical results of Theorem 2. In summary, Figures 6-8 validate our assumption of the independency and the uniformity of the elements of $S$ from the SSG keystream through the numerical experiments.

Figure 9 displays the success probabilities over the ciphertext length $M$ in the AG-OTS cryptosystem at PNR$_{\text{max}} = 20$ dB, where $N = 512$. For an adversary, it sketches the upper bounds on the success probability of the indistinguishability experiment, obtained by (2) from the upper bound of Theorem 2. For comparison, we also sketch the empirical success probabilities of a legitimate recipient, where we tested total 10000 plaintexts each of which has at most $K = 8$ nonzero entries and the energy $||x||^2 = \alpha ||x_{\text{max}}||^2$ with $\alpha$ uniformly distributed in $[\gamma, 1]$. In CS encryption, each entry
observed that the decryption performance is similar to that of \( \hat{S} \).

The CS decryption of the secret matrix \( S \) in the AG-OTS cryptosystem at \( \text{PNR}_{\max} \) shows that if \( M = 512 \), \( N = 512 \), and \( K = 8 \). For a given \( x \), total \( 10^8 \) matrices of \( S \) have been tested.

![Success probabilities over \( M \) in the AG-OTS cryptosystem at \( \text{PNR}_{\max} = 20 \) dB, where \( N = 512 \).](image)

Fig. 9. Success probabilities over \( M \) in the AG-OTS cryptosystem at \( \text{PNR}_{\max} = 20 \) dB, where \( N = 512 \). For an adversary, the upper bounds on the success probability of the indistinguishability experiment are sketched.

![Upper and lower bounds on TV distance over \( \gamma \) in the AG-OTS cryptosystem at \( \text{PNR}_{\max} = 20 \) dB, where \( N = 512 \), \( M = 64 \), and \( K = 8 \).](image)

Fig. 8. Upper and lower bounds on TV distance over \( \gamma \) in the AG-OTS cryptosystem at \( \text{PNR}_{\max} = 20 \) dB, where \( N = 512 \), \( M = 64 \), and \( K = 8 \). For a given \( x \), total \( 10^8 \) matrices of \( S \) have been tested.

of the secret matrix \( S \) is from the SSG keystream, where we observed that the decryption performance is similar to that of \( S \) from the random Bernoulli distribution. The CS decryption is declared as a success if a decrypted plaintext \( \hat{x} \) achieves \( \frac{|x - \hat{x}|}{|x|} < 10^{-2} \). The figure shows that a legitimate recipient enjoys a reliable and stable CS decryption for a sufficiently large \( M \) at each \( \gamma \). Meanwhile, the upper bounds on the success probability of an adversary indicate that no detection test can be successful in the indistinguishability experiment with the probability more than the bounds. In particular, if \( \gamma = 1 \), no adversary can learn any information about the plaintext with the success probability higher than 0.5, which leads to the indistinguishability. However, if energy variation occurs in plaintexts with \( \gamma < 1 \), the figure reveals that an adversary may be able to distinguish the plaintexts in the experiment, with the probability higher than 0.5. It also shows that the success probability of an adversary becomes more significant as the minimum energy ratio \( \gamma \) decreases and the plaintext length \( M \) increases.

Figure 10 depicts the success probabilities over \( \text{PNR}_{\max} \) in the AG-OTS cryptosystem, where \( N = 512 \) and \( M = 64 \). For an adversary, the upper bounds on the success probability of the indistinguishability experiment are sketched.

![Success probabilities over \( \text{PNR}_{\max} \) in the AG-OTS cryptosystem, where \( N = 512 \) and \( M = 64 \).](image)

Fig. 10. Success probabilities over \( \text{PNR}_{\max} \) in the AG-OTS cryptosystem, where \( N = 512 \) and \( M = 64 \). For an adversary, the upper bounds on the success probability of the indistinguishability experiment are sketched.

VI. CONCLUSIONS

This paper has proposed a new CS-based cryptosystem, named as the AG-OTS cryptosystem, by employing a secret bipolar keystream and a public unitary matrix for efficient implementation in practice. We demonstrated that the elements of the sensing matrix are asymptotically Gaussian for a sufficiently large plaintext length, which guarantees a stable and robust CS decryption for a legitimate recipient. By means of the total variation (TV) and the Hellinger distances, we showed that the AG-OTS cryptosystem can have the indistinguishability against an adversary, as long as each plaintext
has constant energy. Therefore, it is essential that the AG-OTS cryptosystem should have a normalization step before CS encryption for equalizing the plaintext energy, which guarantees the computational security against any kind of polynomial time attack from an adversary. Finally, we found that the indistinguishability of the AG-OTS cryptosystem is highly sensitive to energy variation of plaintexts. To support the AG-OTS cryptosystem with unequal plaintext energy, we developed sufficient conditions on the minimum energy ratio, the plaintext length, and the maximum plaintext-to-noise power ratio, respectively, for the asymptotic indistinguishability. The results of the energy sensitivity can be directly applicable to the G-OTS cryptosystem of [17].

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