Fine Grained Dataflow Tracking with Proximal Gradients

Gabriel Ryan, Abhishek Shah, Dongdong She, and Suman Jana
Columbia University
{gabe,dongdong,suman}@cs.columbia.edu, abhishek.shah@columbia.edu

Koustubha Bhat
Vrije Universiteit
k.bhat@vu.nl

Abstract—Dataflow tracking with Dynamic Taint Analysis (DTA) is an important method in systems security with many applications, including exploit analysis, guided fuzzing, and side-channel information leak detection. However, DTA is fundamentally limited by the boolean nature of taint labels, which provide no information about the significance of detected dataflows and lead to false positives/negatives on complex real world programs.

We introduce proximal gradient analysis (PGA), a novel theoretically grounded approach that can track more accurate and fine-grained dataflow information than dynamic taint analysis. We observe that the gradients of neural networks precisely track dataflow and have been used widely for different data-flow-guided tasks like generating adversarial inputs and interpreting their decisions. However, programs, unlike neural networks, contain many discontinuous operations for which gradients cannot be computed. Our key insight is that we can efficiently approximate gradients over discontinuous operations by computing proximal gradients, a mathematically rigorous generalization of gradients for discontinuous functions. Proximal gradients allow us to apply the chain rule of calculus to accurately compose and propagate gradients over a program with minimal error.

We compare our prototype PGA implementation to two state of the art DTA implementations, DataFlowSanitizer and libdft, on 7 real-world programs. Our results show that PGA can improve the F1 accuracy of data flow tracking by up to 33% over taint tracking without introducing any significant overhead (<5% on average). We further demonstrate the effectiveness of PGA by discovering 23 previously unknown security vulnerabilities and 2 side-channel leaks, and analyzing 9 existing CVEs in the tested programs.

I. INTRODUCTION

Dataflow analysis with dynamic taint analysis (DTA) is a fundamental building block in many common systems security tasks, such as automated vulnerability analysis, guided fuzzing, discovering information leaks, and malware analysis [13], [40], [19], [37], [3], [52]. DTA analyzes dataflow between a specified set of sources and sinks in a program either statically, by analyzing the source code or binary, or dynamically, by instrumenting the program and tracking taint as it executes [33], [30].

All existing DTA techniques propagate dataflow information based on a set of rules for every executed operation. The final taint results are computed by propagating and composing the per-statement taint rules together. However, DTA is fundamentally limited by the boolean information contained in taint labels: data either is tainted by a given source or not, there are no intermediate states or other sources of information. This means there is no way to identify and prioritize which dataflows are most significant, or quantify what effect changing the values of the taint sources will have on the program. Moreover, it limits the ability of DTA frameworks to account for dataflows that cancel out over multiple operations. Detecting all possible canceling dataflows is extremely difficult in practice and they often result in false positives. For example, while most DTA frameworks may have a special case to avoid propagating incorrect taints for operations like \( y = x_1 - x_1 \);, they may not be able to correctly handle a sequence of operations like \( x_1 = *ptr; \) \( x_2 = *ptr; \) \( y = x_1 - x_2 \);. As observed by Slowinska et al. [45], [46], false positives caused by these types of errors have prevented DTA from being successfully applied for detecting keyloggers and certain memory corruption attacks.

The limitations of DTA have long been recognized in the community and led several researchers to propose
influence as a more fine grained form of dataflow, where influence measures the extent to which the inputs to each operation effect its output. This approach is based on Quantitative Information Flow (QIF) and uses channel capacity to measure influence. However, while QIF is able to track data more precisely using information measures such as channel capacity, computing these measures is a fundamentally difficult problem that limits the scalability of these approaches.

In this paper, we propose an alternate measure of influence inspired by the fields of machine learning and continuous optimization that addresses the limitations of DTA while retaining its advantages in scalability. We observe that gradient, a multi-variate generalization of derivatives from elementary calculus, is a popular method for tracking the influence of inputs on outputs through differentiable models. In particular, gradients have been used with neural networks to perform a variety of tasks that are analogous to the applications of DTA in program analysis, including generating inputs to trigger errors, explaining output behaviors, and maximizing test coverage. In these applications, gradients serve as a measure for how changes to a differentiable model affect its output, making it possible to explain a model’s observed behavior, or generate new inputs that alter the behavior.

The additional information provided by gradients confer two crucial advantages: (i) Fine-grained tracking. Gradients measure both the magnitude and direction of influence, which indicate how changes to an operations input will effect its output. This means gradients can be used to identify which marked sources are most influential, and how they will effect program behavior. This is illustrated in Figure 1 in which the magnitude of the gradient identifies the most influential of three inputs, and the direction of the gradient indicates how that input can be changed to reach a vulnerability. (ii) Precise composition. Due to the additional magnitude and direction information they incorporate, gradients compose precisely over multiple differentiable operations (e.g., floating point arithmetic) without introducing any inaccuracies due to the chain rule of calculus. For $x_1 = *ptr$; $x_2 = *ptr$; $y = x_1 - x_2$; the gradients of $x_1$ and $x_2$ will cancel out due to the subtraction, resulting in a gradient of 0 for $y$, correctly indicating that there is no dataflow.

However, in general, programs contain many discrete operations with different types of non-smooth behavior (i.e., bitwise operations, integer arithmetic, and branches as shown in Figure 2) that cannot be differentiated directly. These non-differentiable operations break the chain rule and prevent the gradient from being composed accurately. In this paper, we build on the rich non-smooth calculus literature to define generalized gradients for programs that satisfy weaker forms of chain rule. We observe that most operations performed by real-world hardware share a property called Lipschitz continuity due to the finite bit-width of the operands. Lipschitz continuity enforces that the output of a function will not change too drastically as long as the input changes are small (see Section II for a more formal definition). Therefore, in this paper, we use proximal gradients, a type of gradient approximation particularly suited for Lipschitz continuous functions, which convert the problem of computing the gradient to finding the local minima for a non-differentiable operation.

Proximal gradients provide a theoretically grounded framework for gradient evaluation that allows us to precisely track dataflow across real-world programs with minimal compositional errors.

We implement a prototype of Proximal Gradient Analysis (PGA) as an LLVM pass that instruments programs during compilation to compute and propagate proximal gradients. We compare the results of PGA to those of DataFlowSanitizer, LLVM’s state-of-the-art DTA implementation, on 7 widely used applications and show that PGA achieves up to 33% better F1 accuracy than DataFlowSanitizer without incurring any significant (<5%) extra overhead. We further compare the usefulness of PGA by applying it to gradient-guided fuzzing and show that using PGA
achieves up to 56% higher edge coverage than DTA and improves the rate of new edge discovery by 10% on average. Finally, we demonstrate PGA’s usefulness by using it to discover 23 previously unknown security vulnerabilities and 2 side-channel leaks, and analyzing 9 existing CVEs in our tested programs.

The rest of this paper is organized as follows. First, Section II summarizes the background on different generalizations of gradients to non-smooth analysis including proximal gradients. Next, we describe our methodology for computing proximal gradients on real-world programs in Section III. We describe the details of our implementation of proximal gradient analysis in Section IV, and Section V contains the details of our experimental setup and detailed results. Finally, we summarize related work in section VI and conclude in section VII.

Our main contributions are:

1) We are the first, to the best of our knowledge, to use non-smooth analysis for dataflow tracking in real-world programs. Specifically, we design, implement, and evaluate Proximal Gradient Analysis (PGA), a novel, theoretically grounded technique for measuring fine grained influence with minimal errors in real-world programs.

2) We implement our PGA framework for automatically computing and tracking proximal gradients as an LLVM Pass. We are currently working on making our code available as open-source software.

3) We perform extensive experimental evaluation of PGA and compare it to two state-of-the-art DTA implementations, DataFlowSanitizer and libdft, on 7 popular, real-world programs. Our experimental results show that PGA achieves up to 33% higher F1 accuracy than DTA without introducing significant additional overhead (on average <5%). PGA also achieves up to 56% improvement in new edge coverage relative to DTA for data-flow-guided fuzzing of our tested programs.

4) We demonstrate that PGA’s fine-grained tracking is helpful for finding and analyzing different types of security vulnerabilities. In our experiments, PGA found 23 previously unknown bugs and 2 side-channel leaks in our tested programs. PGA also detected the exploitable dataflow in 9 known CVEs including 3 where DTA fails.

II. BACKGROUND

Our approach to gradient-based dataflow analysis draws on several techniques from the mathematical analysis and optimization literature. We provide a summary of the relevant methods below.

A. Smooth Analysis

Gradients. The derivative for a smooth scalar function $f(x)$ is defined as $f'(x) = \lim_{\delta x \to 0} \frac{f(x+\delta x)-f(x)}{\delta x}$. The gradient is a generalization of the derivative to multi-variate functions that can be understood as the slope of the function at the point where it is evaluated. The directional derivative evaluates the gradient in a specific direction; formally, it is defined as $D_v f(x) = \lim_{h \to 0} \frac{f(x+hv)-f(x)}{h}$, where $v$ is the direction of the derivative and $x$ is the point at which it is evaluated.

Chain Rule. Gradients over differentiable functions have the useful property that gradients of compositions of functions can be computed precisely from the gradients of the individual functions. This is known as the chain rule of calculus and is defined as follows where $\circ$ indicates the composition of two functions $f$ and $g$ and $f'$ and $g'$ are their respective gradients.

\[ (f \circ g)' = (f' \circ g) \cdot g' \] (1)

Automatic Differentiation. Gradients of functions which have no analytical form but are composed of many simple and analytically differentiable functions are usually computed with a process called Automatic Differentiation (AutoDiff). Autodiff uses the chain rule to compute the gradient over a program as a series of gradients of individual operations multiplied together, each of which can be computed analytically. Autodiff has been a longstanding tool in computational modeling and is a core component of deep learning frameworks such as Tensorflow [49], [1]. However,
existing AutoDiff methods and frameworks are limited to working with mostly continuous functions with limited discontinuity (e.g., ReLUs in neural networks).

B. Non-smooth Analysis

Extensive work has been done in the field of mathematical analysis on different methods for approximating gradients over discrete and non-smooth functions. The exact type of approximation depends on the nature of the underlying function. We describe two major subclasses of non-smooth functions below that are most relevant to our work.

**Lipschitz Continuity.** A function is intuitively Lipschitz Continuous if its output does not change too much for small changes in the input. Formally, a function \( f \) is Lipschitz Continuous if for all pairs of points \( x_1 \) and \( x_2 \) in the domain of \( f \) the following property holds:

\[
|f(x_1) - f(x_2)| \leq K|x_1 - x_2|
\]

(2)

Figure 3a shows a simple Lipschitz continuous function along with the corresponding Lipschitz constant. All useful functions that can be computed in reasonable time by a computer are Lipschitz continuous due to the finite bit widths of intermediate computation, although the Lipschitz constant may become very large for some operations.

**Convexity.** Intuitively, a function is convex if the straight line connecting any two points on the graph of the function lies entirely above or on the surface of the function. Figure 3b shows a simple convex function. More formally, a function \( f \) is called convex if the following property is satisfied by all pairs of points \( x \) and \( y \) in its domain: \( f(tx + (1-t)y) \geq tf(x) + (1-t)f(y), \forall t \in [0, 1]. \)

All convex functions are locally (i.e., over some interval of their domain) Lipschitz continuous but the converse is not true. While operations like addition and multiplication, or multiplying a variable by itself, are convex, bitwise operations and discontinuities from branching in computer programs are generally not convex. Therefore, programs in general are not convex, although they may be convex over many local neighborhoods.

For convex non-smooth functions, subgradients are a popular approach for approximating gradients that have several desirable composition and global convergence properties as described below. For nonconvex functions, an extension of subgradients called generalized gradients may be used that allow optimization to still be performed, albeit with further relaxed composition and convergence properties \[12\], \[39\]. We summarize both of these approaches below.

**Subgradients.** For convex non-smooth functions, subgradients present a generalization of gradients that follows the chain rule \[7\]. A subgradient of a convex function at a point is a vector that only intersects the function at that point. Formally, a vector \( v \) is a subgradient of a function \( f \) at a point \( x \) if:

\[
f(y) \geq f(x) + v \cdot (y - x) \text{ for all } y \in \text{dom}(f)
\]

(3)

Intuitively, this means that the vector \( v \) either touches or is less than the function \( f \) for all points on \( f \). The dot product \( v \cdot (y - x) \) projects points along the subgradient \( v \). Unlike gradients, which tend to be unique for a given point on a function, there can be multiple valid subgradients for a given point of a discrete convex function, as shown in Figure 4a.

Subgradients follow the chain rule and therefore can be efficiently computed over complex composite functions.

**Generalized Gradients.** Non-convex functions (e.g., \( y = x \% 4 \) in Figure 2) may have points where no valid subgradient exists. For such non-convex but locally Lipschitz functions, **generalized gradients** are used to approximate gradients \[12\], \[39\]. Generalized gradients consist of generalized directional derivatives, which work like subgradients, but only project in a single direction, starting at the point where they are evaluated and always remaining under the function, much as ordinary directional derivatives operate on smooth functions. Generalized directional derivatives can be computed even at points where there is no valid subgradient as shown in Figure 4b.

A generalized directional derivative in a direction \( v \) is denoted by \( f^\circ(x; v) \), which is defined as follows:

\[
f^\circ(x; v) = \lim_{\lambda \downarrow 0} \sup_{y \rightarrow x} \frac{f(y + \lambda v) - f(y)}{\lambda}
\]

(4)

Here \( x \) is the point at which the generalized derivative is evaluated, \( y \) can be any other point in the domain of \( f \), and \( \lambda \) is a distance along the vector \( v \) that the derivative is taken in. The \( \lim sup \) notation indicates that the generalized derivative takes on the

![Convex function](image1.png) ![Lipschitz function](image2.png)

Fig. 3: Classes of functions

![Class of functions](image3.png)

**Fig. 4:** Generalized gradients.
The notation \( \argmin \) indicates that the operator selects the value of \( y \) that minimizes both the value of function \( f(y) \) and the distance cost \( \frac{1}{2} \|x - y\|^2 \).

Evaluating the proximal operator will give the minimum point near \( x \). This point can then be used to compute the largest directional gradient in the region near the point.

\[
prox_{\nabla f}(x) = \frac{f(x) - f(prox f(x))}{x - prox f(x)} 
\]  

One of the key advantages of proximal gradients is that even in the non-convex and non-smooth case, the accuracy of the gradient computation usually increases with more samples from the bounded region. For the convex case, proximal gradients provide strong convergence guarantees.

### III. Methodology

At a high level, our gradient propagation framework, PGA, is similar to Autodiff, computing the gradient of each operation and using the results as inputs to the next gradient computation. However, unlike Autodiff, we approximate the gradients of non-smooth functions with their proximal gradients. The accuracy of the proximal gradients for non-smooth, non-convex operations depend on the sampling rate and the underlying function’s local Lipschitz constants. As noted in Section III, all practical programs are composed of Lipschitz continuous operations. For more precise approximations of local Lipschitz constants of different operations, we propagate them throughout the program execution trace along with the gradient.

#### A. Principled Program Gradient Evaluation

To compute gradients over programs with PGA, we model a program as an arbitrary function \( P \) that transforms the system state from \( x \) to \( x' \). Since the program \( P \) usually is composed of simple individual operations, we model \( P \) as a composition of \( N \) individual simple functions on the system state that represent each operation in the program:

\[
P(x) = P_N \circ P_{N-1} \circ \cdots \circ P_2 \circ P_1(x)
\]

Since these operations, in general, are not differentiable, we apply proximal gradients to compute an approximated gradient over each operation \( \nabla_{\text{sub}} P(x) \). This, by definition, results in a valid subgradient on convex operations and a valid generalized gradient on operations that are non-convex. We then apply the chain rule to estimate the overall program gradient as a product of the individual proximal gradients.
Moreover, even on non-convex functions where the non-smooth chain rule does not hold, as mentioned in Section II, we can still minimize error by increasing the number of samples.

**Bounded Proximal Gradient.** To guarantee that a proximal gradient is computed correctly, the region in which the local minima may occur must be fully sampled. We derive an exact bound for this region based on the function’s Lipschitz Constant $K$.

$$||\text{prox}_f(x) - x||_2 \leq 2K$$ (7)

We use this bound to define the bounded proximal gradient, $\text{prox}_f(x, K)$, which takes the Lipschitz Constant of the function as a second input and samples within the bounded region. A full derivation of the bound and definition of the bounded proximal gradient are provided in Appendix A.

Figure 2 gives a concrete example of evaluating the bounded proximal gradient. The bitwise function in the example, $x \& 4$, has a Lipschitz Constant of $K = 4$, so the sampling distance bound is $2K = 8$. Figure 5a shows the value of the cost function near $x = 7$. There is a clear minimum at $x = 8$, so the proximal gradient is evaluated as $\text{prox}_{\nabla f}(7) = \frac{f(7) - f(8)}{7 - 8} = -4$ as shown in figure 5b.

**B. Gradient Propagation Rules**

Our approach to approximating gradients on programs with minimal error requires that both the gradient itself and the Lipschitz Constant of each operation $f$ is propagated as $K_f$. For most operations we define specific rules for updating the $K_f$, but in some cases the Lipschitz constant cannot be determined analytically. In these cases, we estimate $K_f$ by sampling the function based on the input $x_i$ and its Lipschitz Constant, $x_i \pm K_i$, and taking the maximum difference.

We organize the propagation rules for program operations into four categories based on their behavior: floating point operations, integer operations, load and store operations, and branching operations.

**Floating Point Operations.** Floating point operations are handled analytically using standard forward auto-differentiation methods and the chain rule [22].

**Integer Operations.** Unlike floating point operations, integer operations exhibit a variety of behaviors that require special handling. Some functions are convex with analytical solutions, while some non-smooth functions have well-defined behavior that makes it possible to derive specific rules to update Lipschitz Constants.

Addition, multiplication, and subtraction are relaxed to their corresponding continuous functions (i.e. floating point operations) then handled analytically.

For non-smooth integer operations, we define the following special cases for determining Lipschitz Constants.

1) **Modulo Operations.** In cases where the modulo operand is constant, the Lipschitz Constant for the modulo function is simply the modulo operand (i.e. right hand operand).

2) **Shift Operations.** When the shift operand (i.e. the right hand operand) is constant, the Lipschitz constant be determined by raising 2 to the operand value and applying chain rule.

3) **Bitwise Operations.** When one of the operands of a bitwise operation is a constant, or at least has a 0 derivative with regard the marked input, the Lipschitz Constant can be set the value of the highest set bit in the constant operand.

4) **Integer Division.** To compute the Lipschitz Constant on integer division, we use the quotient rule and consider the case that causes the maximum possible change in $f$, where $\frac{dg}{dx_i} = K_g$, and $\frac{dh}{dx_i} = -K_h$.

In cases where any one of these nonsmooth operations have a non-constant operand, we apply the sampling procedure defined above in order to estimate sampling bounds for the proximal gradient.

**External library functions.** External library function calls without source code access can be modeled
\[ y = 2 \times x \]
\[ y_{\text{shad}} = \text{alloc\_shadow}() \]
\[ y_{\text{grad}} = \text{gradient}(2 \times x) \]

**Application**
- **Memory**
  - Gradient Table
- **Application Code**
- **Shadow Memory**
  - Instrumentation Code

![Fig. 6: GRSan architecture illustrating how gradients are propagated.](image)

as arbitrary nonsmooth functions \( f(g) \). Since we cannot derive the Lipschitz Constant \( K_f \) as we do not know the inner details of the function, we estimate it by sampling the function at \( g \pm g' \) and taking the maximum difference.

**Load and Store Operations.** Gradients on store operations are dispersed to each byte used to store the variable in memory, and the corresponding gradients on load operations are aggregated to compute the gradient of the result. When a loaded value is more than a byte in width, the gradient and Lipschitz constant associated with each byte in memory is left shifted according to its offset and summed together. Similarly, when a value of more than a byte is stored, the gradient and Lipschitz constant of the value are right shifted and stored.

**Branches.** In our current prototype implementation we do not take branches into account and simply propagate gradients along the current execution path. However, for completeness, we describe two possible approaches to handling branches with PGA in Appendix B.

**IV. Implementation**

We implement PGA as a new sanitizer in the LLVM framework [27] called Gradient Sanitizer (GRSan). We use LLVM because it allows us to instrument a program during compilation after it has been converted to LLVM’s intermediate representation. This means that GRSan can be used to instrument any program written in a language supported by LLVM and reduces runtime overhead due to our compile-time instrumentation.

**Overall Architecture.** We base GRSan on LLVM’s taint tracking implementation, DataFlowSanitizer. DataFlowSanitizer uses shadow memory to track taint labels. For each byte of application memory, there are two corresponding bytes of shadow memory that store the taint label for that byte. At compile time, DataFlowSanitizer instruments the original program with additional instructions that update the taint information in the shadow memory.

We modify DataFlowSanitizer in the following two ways: First, we add additional metadata associated with each label that is used to store the gradient information in a gradient table as shown in Figure 6. Second, we change the dataflow propagation rules to compute gradients over each operation. Figure 6 gives an example of how the instrumentations for propagation rules work. Given an operation \( y=2\times x \), the instrumentation first allocates space in the shadow memory and gradient table for \( y \). The instrumentation then computes the derivative of \( 2\times x \) and stores it in the corresponding gradient table entry.

**Gradient Propagation Rules.** When a differentiable binary operator such as multiplication instruction is visited, GRSan uses the chain rule to update the gradients. For nondifferentiable binary operators such as the binary \( \text{And} \) operator, GRSan approximates the proximal operator and lipschitz bound by sampling based on the input gradient and taking the closest nonzero sampled gradient. In practice, we found this approximation picked the same values that the proximal operator would select and was computationally lighter (i.e., does not require computing exponents). A code sample of gradient instrumentation for Mul and And operations is provided in Appendix C.

We leave most external function calls uninstrumented, but some operations in glibc are given special instrumentation. We set the gradients for any buffer overwritten by \text{fread} or \text{memset} to 0, and the gradients of buffers copied by \text{memcpy} or \text{strcpy} are also copied. Type casting instructions are handled by simply copying labels from the original value to the result.

**V. Evaluation**

We evaluate PGA by comparing its performance directly to DTA, and in direct applications for vulnerability detection and analysis. Specifically, we run experiments to answer the following questions:

1) Is PGA more precise than DTA in tracking dataflows?
2) How does the overhead introduced by PGA compare to DTA?
3) Does using PGA to guide fuzzing lead to better edge coverage?
TABLE I: Test programs used in our evaluation.

| Library       | Test Command | SLOC | File Format |
|---------------|--------------|------|-------------|
| zlib-1.2.11   | minigzip -d  | 3228 | GZ/ZIP      |
| libjpeg-9c    | djpeg        | 8,857| JPEG        |
| mupdf-1.14.0  | mutool show  | 123,562| PDF        |
| libxml2-2.9.7 | xmllint      | 73,920| XML         |
| binutils-2.30 | objdump -xD | 72,955| ELF         |
|               | strip        | 56,330|             |
|               | size         | 52,991| ELF         |

4) Can PGA detect and analyze recent CVEs that taint is typically used to detect?
5) Is PGA an effective tool for vulnerability discovery?
6) Can PGA detect and analyze memory and timing-based information leaks?

A. Experimental Setup

Test Programs. We perform tests on a set of 5 widely used file parsing libraries and 7 total programs. We use file parsers because these programs often must process files from untrusted sources, making them a common target for attacks. Table I shows the test programs and SLOC associated with each executable tested. In total the programs have 391,883 SLOC.

Test Environment. All of our evaluations are performed on an Ubuntu 16.04 server with an Intel Xeon E5-2623 v4 2.60GHz CPU and 192G of memory.

B. Performance

We first evaluate the performance of PGA as a tool for dynamic dataflow analysis. In our experiments, we compare PGA to DataFlowSanitizer, LLVM’s state-of-the-art DTA implementation. Since our implementation of PGA is based on the DataFlowSanitizer architecture, our setup ensures that any differences in performance between PGA and DTA are due to the respective performance of gradient and taint and not due to differences in the underlying architectures.

We compare performance in three areas: first, we estimate the accuracy of the dataflows predicted by PGA and DTA. Second, we evaluate the overhead introduced by the PGA instrumentation. Third, we compare the edge coverage achieved by a dataflow-guided fuzzer using either PGA or DTA to guide its mutation strategy.

1) Dataflow Accuracy: We evaluate the accuracy of PGA in comparison to DTA against an estimate of ground truth dataflows. This comparison setting favors DTA since it does not take the fine grained dataflow information from taint into account (i.e., only considers binary 0/1 influence), but still illustrates the benefits of PGA’s increased precision. In addition to comparing against DFSan, which we consider a fair comparison since both systems are based on LLVM, we also compare against Libdft, which is another widely used DTA framework. Libdft uses Intel PIN to instrument the binary directly, and therefore requires running a parallel experiment in which ground truth dataflows are also estimated separately using PIN instrumentation.

Ground truth estimation. We first estimate ground truth for each potential dataflow. We record the value of the source and sink variable for a single execution and then modify the value of source variable and record whether the value of the sink changes during execution. If any modification of the source variables value causes a change in the value of the sink while following the same execution path, the source and sink are considered a valid dataflow. This approach may miss some dataflows, but provides a reasonably fair basis for evaluation.

We focus the accuracy evaluation on dataflows between the program inputs and branch constraints because they ultimately determine the behavior of a program, and because many security vulnerabilities in a program can only be exploited when certain branches are taken. We consider each byte in the input file to be a taint source and each branch condition to be sink. For each byte, we generate sample inputs by setting the byte to 0, 255, and toggling each bit for a total of 10 samples. We found that this sampling strategy usually triggered a change in the sink variable when there was a valid dataflow.

Accuracy evaluation. We perform the accuracy evaluation on the programs shown in table I using a set

| Library | Prec. | Rec. | F1   |
|---------|-------|------|------|
| minigzip | 0.42  | 0.29 | 0.39 |
| djpeg   | -     | 0.60 | 0.39 |
| mutool  | 0.70  | 0.61 | 0.62 |
| xmllint | -     | 0.60 | 0.50 |
| objdump | 0.47  | 0.50 | 0.33 |
| strip   | 0.26  | 0.50 | 0.33 |
| size    | 0.20  | 0.50 | 0.33 |

TABLE II: Summary of accuracy comparison results for taint and gradient analysis. Best F1 scores for each program are highlighted. Experiments with Libdft on djpeg and xmllint timed out after 24hrs. PGA outperforms DTA on all programs.
of small seed files (<1Kb) to make sampling each byte feasible. Since the actual number of input bytes with valid dataflows to a given branch might be often very small compared to the total number of input bytes, we use F1 accuracy as a performance metric, which evaluates predictions on imbalanced classes. F1 accuracy is computed as the harmonic mean of precision and recall, \( F1 = \frac{2 \times \text{precision} \times \text{recall}}{\text{precision} + \text{recall}} \). Precision indicates the proportion of bytes with predicted dataflows that are correct (i.e. not false positives), while recall indicates the proportion of valid dataflows that were correctly predicted (i.e. not false negatives). Results are shown in table II.

Generally, PGA achieves a significant improvement in precision, achieving up a 0.37 increase in precision and 0.33 increase in F1 accuracy compared to DFSan. Overall PGA gets higher F1 scores for all programs.

### Binary vs. floating point gradients

To measure the importance of the gradient information to determining accurate dataflows, we perform an ablation on gradients by limiting them to binary values. The ablation uses the same proximal gradient propagation rules, but rounds all gradients to 0 or 1 depending on if they are nonzero, in effect converting PGA into DTA with PGA propagation rules. Results of the comparison are shown in table III. PGA with full gradient information performs better than PGA with binary gradients for every program, and in some cases, such as minizip, the large errors caused by limiting the gradient to binary values render the dataflow predictions almost completely invalid. The higher error rates caused by binary gradients indicate that using precise gradients that compose accurately over multiple operations is key to the performance gains achieved by PGA.

### Zero gradient analysis

Overtainting is a serious problem in DTA that creates many false positives and causes excessive overhead. PGA is able to avoid overtainting when it computes a zero gradient on an instruction DTA would mark as tainted. Therefore we investigate the distribution of zero gradients across programs and instruction types to determine where and how PGA is more precise than DTA.

For each program and each type of instruction, we count how many times the instruction had zero gradient in the execution traces from the accuracy evaluation. The left hand side in table IV shows the results of this analysis for each program.

We hypothesize that the programs that have the largest differences in precision and recall will have large numbers of zero gradients, while programs with minimal differences will have very few zero gradients. Generally, the counts of zero gradients support this hypothesis. objdump, strip, and djpeg show larger differences in accuracy between DFSan and PGA. Conversely, xmllint also has a high ratio of zeros but has a relatively smaller difference between DFSan and grsan. We found that in this case many of the branch dataflow effects we observed appeared to be caused by implicit dataflow effects that both DFSan and grsan could not detect.

We also examine which instructions are most likely to have 0 gradients across all programs. The right side of table IV shows the top 7 instructions with the most zero gradients. We find the instructions most likely to have zero gradients are intuitively more likely always have the same output. For example, right shifts remove bytes from values, which often causes them to output 0 regardless of their input. Similarly, And and Mul operations will always output 0 if one of their inputs is 0.

### Table III: Summary of precision comparison results for binary vs floating point gradient comparison. Best F1 scores for each program are highlighted.

| Program   | Prec. | Recall | F1   | Prec. | Recall | F1   |
|-----------|-------|--------|------|-------|--------|------|
| minizip   | 0.41  | 0.15   | 0.22 | 0.63  | 0.51   | 0.57 |
| jpeg      | 0.62  | 0.63   | 0.62 | 0.60  | 0.83   | 0.69 |
| mutool    | 0.87  | 0.50   | 0.63 | 0.86  | 0.51   | 0.64 |
| xmllint   | 0.91  | 0.87   | 0.89 | 0.94  | 0.91   | 0.92 |
| objdump   | 0.51  | 0.66   | 0.58 | 0.66  | 0.77   | 0.71 |
| strip     | 0.42  | 0.72   | 0.53 | 0.50  | 0.86   | 0.63 |
| size      | 0.54  | 0.76   | 0.63 | 0.62  | 0.91   | 0.74 |

### Table IV: Analysis of operations from execution traces where gradient drops to 0, aggregated for each program and for each type of instruction across all programs. Outputs of these operations will have 0 gradient but still be marked as tainted by DTA.

| Program   | Instrs | %Zeros |
|-----------|--------|--------|
| minizip   | 3012   | 28.2   |
| jpeg      | 703    | 38.7   |
| mutool    | 401    | 40.4   |
| libxml    | 430    | 39.5   |
| objdump   | 1070   | 39.0   |
| strip     | 3089   | 41.0   |
| size      | 659    | 19.3   |

| Instruction   | Total | %Zeros |
|---------------|-------|--------|
| And           | 6756  | 30.2   |
| URem          | 214   | 29.0   |
| Sub           | 1214  | 21.0   |
| Mul           | 875   | 15.9   |
| LShr          | 2377  | 14.4   |
| AShr          | 149   | 6.0    |
| Add           | 895   | 5.7    |
Program Libdft Dfsan Grsan Grsan rel. to Dfsan

| Program | Libdft Overhead | Dfsan Overhead | Grsan Overhead | Grsan rel. to Dfsan |
|---------|-----------------|----------------|----------------|---------------------|
| minigzip | 2379.5%         | 54.7%          | 61.5%          | 4.4%                |
| djpeg   | -               | 70.5%          | 73.7%          | 1.9%                |
| mupdf   | 853.5%          | 198.4%         | 262.1%         | 21.5%               |
| xmlint  | 231.4%          | 5.5%           | 0.0%           | -5.2%               |
| size    | 152.5%          | 101.1%         | 107.1%         | 3.0%                |
| objdump | 180.0%          | 133.2%         | 131.2%         | -0.9%               |
| strip   | 142.5%          | 12.0%          | 11.4%          | -2.2%               |

TABLE V: Program overhead measurements averaged over five runs. Libdft overhead is measured relative to running a program only with PIN. DFSan and GRsan are measured relative to uninstrumented programs. Libdft programs ran in seconds to minutes, while DFSan and GRsan instrumented programs ran in milliseconds. Libdft execution timed out on djpeg after 6 hours. On average GRsan adds 3.22% additional overhead relative to DFSan.

| Instruction | Taint (ns) | Grad (ns) | Increase |
|-------------|------------|-----------|----------|
| Add         | 0.075      | 0.082     | 9.33%    |
| Mult        | 0.077      | 0.079     | 2.60%    |
| Div         | 0.143      | 0.143     | 0.0%     |
| And         | 0.076      | 0.083     | 9.21%    |
| Shift       | 0.079      | 0.081     | 2.53%    |
| Urem        | 0.144      | 0.144     | 0.0%     |
| FAdd        | 0.076      | 0.080     | 5.26%    |
| FMult       | 0.077      | 0.080     | 3.90%    |
| FDiv        | 0.077      | 0.081     | 5.96%    |

TABLE VI: Average overhead for different types of instructions measured over 100B executions. GRSan adds 4.1% overhead relative to DFSan on average.

2) Overhead: We evaluate the overhead introduced by our implementation of PGA next and compare it to DataFlowSanitizer. To measure overhead, we execute each program 5,000 times while recording runtime. We perform each measurement 5 times and average the measured runtime.

Table V shows the results of the overhead comparison. For each program, we compute the overhead of DTA and PGA individually based on the DFSan and GRSan implementations, as well as the relative overhead of PGA compared to DTA. In the worst case PGA has 21% greater overhead relative to DTA, but on average only adds 2.8% relative overhead. We also provide overhead measurements for Libdft, although it is significantly slower due to the overhead involved in binary instrumentation.

Additionally, we perform microbenchmarks on individual instructions that require gradient computation to determine how much overhead PGA instrumentation adds. For each instruction, we run a program that executes it 100B times and compare measured CPU time for taint and gradient instrumented versions of the program. Table VI shows results for a representative subset of operations. Generally PGA instrumentation adds a small amount of overhead to each instruction relative to taint tracking, with 9.33% in the worst case on Add instructions, and 4.1% on average.

3) Dataflow-Guided Fuzzing: Since dynamic dataflow analysis is often used as a tool to guide fuzzing, we also evaluate PGA in comparison to DTA as a method for guiding fuzzer mutations. Unlike our evaluation of dataflow precision, this experiment emphasizes the dataflow magnitude information provided by the program gradient, since bytes with the largest derivatives are selected for fuzzing.

Mutation Algorithm. We use the following procedure to guide mutations: First, we execute the program with all inputs set as sources and all branches set as sinks. When selecting bytes with PGA, the bytes with the greatest gradients are chosen first; while bytes selected with DTA are chosen randomly. This approach utilizes the additional information provided by PGA to improve the mutation strategy.

The fuzzer performs a deterministic set of mutations on the selected 128 bytes, in which each byte in turn is set to all 256 possible values. We use this simple strategy to ensure an unbiased comparison of the information from gradient and taint. More sophisticated strategies used by dataflow guided fuzzers such as Vuzzer or NEUZZ may achieve better coverage but would likely bias the comparison between PGA and DTA [37], [42], [54]. This strategy is summarized in Appendix D.

Edge coverage comparison. We execute the fuzzer with both PGA and DTA for 100,000 mutations, and record coverage every 10,000 mutations. Figure 7 shows the relative edge coverage achieved by each method over 100,000 mutations. On average the gradient guided fuzzing outperforms taint in increasing edge coverage by 10% per 10,000 mutations. The gradient guided fuzzer achieves higher coverage on all programs, with the greatest improvement in overall edge coverage of 56% on strip. On other programs the difference between gradient and taint guided fuzzing is smaller: on minigzip and djpeg, the taint guided fuzzer eventually closes the gap, while on objdump there are relatively small differences. We hypothesize that the taint guided fuzzing is able to eventually reach to a similar level of edge coverage on minigzip and...
djpeg because they are relatively simple programs. We also note that for some programs such as xmlint, there is a significant disparity between the results of the guided fuzzing and precision evaluations. We believe this difference is caused by two factors: the magnitude of the gradient was more important than its accuracy in guiding the fuzzer on these programs, and that even small differences in accuracy can be significant if they allow the fuzzer to precisely target key branches in the program.

C. Bug Finding

Next, we show the additional information provided by PGA makes it very effective for discovering and analyzing different types of bugs in real world programs. We test PGA in three applications: detecting and analyzing known vulnerabilities, guiding discovery of new vulnerabilities, and discovering information leaks.

1) Analysis of known CVEs: We first evaluate PGA as a tool for detecting and analyzing dangerous dataflows in known CVEs. To detect these dataflows, we instrument the programs so that the gradients of instruction operands involved in the attacks are recorded. We select 9 CVEs that cover a range of vulnerability types in our evaluation programs, including stack and heap overflows, integer overflows, memory allocation errors, and null pointer dereferences. As shown in table VII, PGA can detect and trace the relevant dataflows in these CVEs.

Additionally, we show PGA can trace dataflows for several CVEs that DTA cannot. In the case of CVE-2018-11214 and CVE-2018-11212, overtainting on the inputs that trigger the error cause DataFlowSanitizer to run out of labels and crash, while PGA is able to precisely identify the relevant dataflows without overtainting. In the case of CVE-2017-15996, an out of memory allocation error triggered by the dataflow from an input byte, PGA can predict by how much the memory allocation size will change with respect to the input byte and is thus able to correctly identify which input byte values will trigger the out-of-memory error. In contrast, DTA can identify the dataflow but cannot predict the rate of change of the memory allocation and thus cannot distinguish between an input byte value triggering a vulnerable or normal program execution.

2) Bug Discovery: After evaluating PGA as a tool for detecting known attacks, we next evaluate the utility of PGA in discovering new bugs in programs. To do so we add additional instrumentation to record gradients for instruction and function arguments that can potentially trigger program errors, such as memory allocations, copy instructions, indexing operations, and shift operators. We then execute the programs on a corpus of files generated by running AFL on each program for 24 hours, as well as a selection of files generated from other programs to further extend coverage. For each file, if any input bytes have a nonzero derivative with an instrumented function, we generate new inputs using the function gradient as a guide. For most functions we generate inputs setting the bytes with a gradient to either 0 or 255, although for instructions that can potentially trigger a division by 0 we also search all possible values.

Table VIII summarizes our results. Overall we find 23 bugs in our evaluated programs, including arithmetic errors, out-of-memory allocations, and integer overflows. Figure 8 illustrates how large gradients are used to find an arithmetic error in djpeg. By altering an input byte with a large gradient to a shift operand, an overflow is triggered that results in an invalid operation. Similarly, identifying inputs with large gradients to memory operations was key finding memory errors. We are in contact with developers of these programs to fix the issues discovered with PGA.

3) Information Leak Discovery: Finally, we investigate PGA’s potential utility in finding side channel information leaks. Side channel leaks often occur when an amplification effect is present, such that small
Fig. 7: Comparison of guided fuzzer edge coverage achieved by PGA and DTA over 100k mutations from a single seed. Overall gradient-guided fuzzing achieves up to 56% higher coverage and improves the rate of new edge discovery by 10% on average.

```
1  INPUT_BYTE(cinfo, &c);
2 3  GRSAN_MARK_BYTE(c, 1.0);
4  /* c marked with gradient 1.0 */
5  cinfo->Al = (c) & 15;
6  /* cinfo->Al gradient = 1.0 */
7  ...
8
9  (*block)[natural_order[k]] = 
10     (JCOEF) (v << cinfo->Al);
11  /* block[0] gradient = 8.0 */
12  ...
13
14  void jpeg_idct_islow(int * block)
15  {
16    ...
17    int * inptr = block;
18    /* inptr[0] gradient = 8.0 */
19    z2 = (int) inptr[0] * quantptr[0]
20    /*z2 gradient = 2040.0 can overflow*/
21    z2 = z2 << 13;
22    /* negative z2 triggers error */
23    ...
24  }
```

Fig. 8: Arithmetic Error in djpeg.

changes to an internal program value cause large changes in the program behavior that can be observed externally. Prior work has shown that by correlating a set of known inputs with external measurements, it is possible to leak information about other user’s private inputs to a program [10]. PGA is well suited to detecting the amplification effects associated with these side channels, since operations that cause large changes to program behavior typically have large gradients.

We provide two case studies to demonstrate how PGA can be used to detect side channel leaks: one example of a side channel leak in memory usage in objdump and one example of a leak in execution time in cjpeg. To identify each leak, we marked the input file headers as sources and relevant program values as sinks, either memory allocation operands for memory side channels or comparison operands in loops for execution time side channels.

Gradient magnitude was key to identifying the memory based side channel in objdump, which had a gradient of 1 million to a malloc instruction from the ELF section header for program size. Figure 9a shows the effect of incrementing the value from 46 to 59 on the program’s total memory usage. The memory consumption is linear in the byte value if the byte is in range from 48 to 57, which can be converted to a valid number ’0’ to ’9’ in ASCII. For byte values out of this range, the memory consumption drop sharply to 7K bytes.

The timing side-channel in cjpeg was identified by a non-zero gradient from the height field in the jpeg header to the operand of a while loop condition. Figure 9b illustrates the effect of incrementing the field value on the program execution time.

While information leaks like ELF file size and JPEG file dimensions may seem harmless at first glance, prior side channel attacks have demonstrated that these types of leaks can be exploited to learn sensitive information about a user [24]. For example, one can imagine a malicious Android app that uses JPEG dimensions leaked from a browser to determine which websites the device user is visiting.

VI. Related Work

Dynamic Taint Analysis. Dynamic Taint Analysis (DTA) tracks data flow from taint sources to taint sinks at runtime. Common applications of DTA include software vulnerability analysis and information leak
Fig. 9: Memory and timing side channel leaks
detection [33], [53], [14], [17], [54]. DTA typically
overestimates the tainted bytes which contribute to a
large performance overhead. Therefore, much of the
recent work in DTA has focused on developing more
efficient systems [25], [6], [29]. PGA is similar to DTA
in that it dynamically propagates dataflow information
through a program, but provides more fine-grained
information in the form of gradients. Moreover, PGA
is more precise than DTA, which reduces overtainting
in large programs.

Some DTA systems use bit level taint tracking to
improve precision at the cost of higher overheads [51],
[50]. Although we have not implemented it in our cur-
cent prototype, gradients can also be propagated over
individual bits based on functional boolean analysis,
and we expect it to offer similar tradeoffs in improved
accuracy for higher overheads [34].

Recently, automatically learning taint rules has been
used to reduce the approximation errors in DTA [11].
This approach is orthogonal to ours and could also
potentially be applied to learn gradient propagation
rules, especially for the functions with large Lipschitz
constants in new architectures.

Quantitative Information Flow. Quantitative In-
f ormation Flow (QIF) measures the potential transmis-
sion of information through a program using entropy
based measures such as channel capacity and min-
entropy [28], [17], [18]. QIF has primarily been used for
detecting information leaks and ensuring the integrity
of program secrets [23], [2], [16], but has also been
proposed as a way of enhancing taint tracking [32].
PGA adds a different type of information in the form
of gradients, and does not have the high computational
complexity involved in estimating information flows
accurately.

Gradient-guided fuzzing. Two recent fuzzers have
used gradient approximations to guide their mutation
process. Angora estimates finite differences, an ap-
proximation of gradients with many known limitations
especially for high-dimensional problems, by executing
the program on modified inputs and recording the
changes in the outputs [9], [38]. NEUZZ incorporates a
neural network that predicts program branch behavior
and then uses the gradient of that network to guide
its mutations [12]. Neutaint is concurrent work that
further develops this approach by using neural
networks specifically to approximate data flows [41].
This incurs less overhead than instrumentation based
methods but is also less exact since it operates on an
approximate model of the program. By contrast, PGA
computes gradients directly over individual instruc-
tions on the entire program and therefore produces
precise gradients.

Program Smoothing. Prior work has explored
smooth interpretation of a program as another ap-
proach to computing gradients by applying Gaussian
smoothing to derive a differentiable program approxi-
mation [7], [8]. These methods use symbolic reasoning
on static programs to compute their approximations
and do not scale to large programs. PGA’s approxima-
tion methods are more efficient and scale to real world
programs.

VII. Conclusion
In this paper we introduce proximal gradient
analysis (PGA), a novel theoretically-grounded
approach to measuring influence in a program that
uses non-smooth calculus techniques to compute
gradients over programs. PGA is more precise than
dynamic taint tracking and provides more fine grained
information about program behavior. We provide a
prototype implementation of PGA based on the LLVM
framework and show that it outperforms LLVM’s DTA
implementation and libdft in accuracy and guided
fuzzing while adding less than 5% overhead on average.
Finally, we show gradient analysis is an effective
tool for bug finding, detecting 9 different CVEs, 23
previously unknown bugs, and 2 side-channel leaks in
7 real world programs. We hope that our approach to
program analysis will motivate other researchers to
explore new techniques exploiting the rich non-smooth
analysis literature.

We are currently in the process of notifying the
developers of the affected programs. Please treat
the bugs as confidential and do not disclose any
information about them.

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APPENDIX A

DERIVATION OF BOUNDED PROXIMAL GRADIENT

Sampling bounds for proximal gradients are derived from two observations. First, by definition, the cost function of the proximal operator at \( y = \text{prox}_f(x) \) must be less than or equal to the cost function’s value at \( x \). This follows directly from the definition of the proximal operator, as it returns a point near \( x \) that minimizes the cost function relative to \( x \), leading to:

\[
f(y) + \frac{1}{2}\|y - x\|^2_2 \leq f(x)
\]

Where \( \| \cdot \|_2 \) refers to Euclidean Distance. Second, the maximum possible difference between \( f(y) \) and \( f(x) \) is bounded by the Lipschitz Constant \( K \). Hence, the minimum value \( f \) can take with \( \|y - x\|_2 \) distance away from \( x \) is \( f(x) - K \|y - x\|^2_2 \) which is upper bounded by \( f(y) \).

\[
f(x) - K \|y - x\|^2_2 \leq f(y) + \frac{1}{2}\|y - x\|^2_2 \leq f(x)
\]

This inequality can be simplified by dropping the middle term as well as subtracting \( f(x) \) from both sides:

\[-K \|y - x\|^2_2 \leq 0\]

Adding \( K \|y - x\|_2 \) to both sides, dividing by \( \|y - x\|_2 \), and multiplying by 2 gives a bound of \( 2K \) on the distance between the initial point \( x \) and the proximal operator result \( y \). We substitute \( y \) with \( \text{prox}_f(x) \) for the final result.

\[
\|\text{prox}_f(x) - x\|_2 \leq 2K
\]

This result means that the distance between the result of the proximal operator and its initial value \( x \) is bounded by at most twice the Lipschitz Constant of
the function, $2K$. We then define the bounded proximal gradient as

$$\text{prox}_K f (x) = \frac{f(x) - f(\text{prox}_f(x, K))}{x - \text{prox}_f(x, K)}$$  \hspace{1cm} (9)

where $K$ is the Lipschitz Constant of the evaluated function $f$. This definition of the proximal gradient allows us to determine the exact amount of sampling required to prevent error when the Lipschitz Constant is known. Fortunately, it is possible to determine the Lipschitz Constants for most operations in programs when these constants for the input operands are known. We therefore include Lipschitz Constants in the gradient propagation rules we define for operations in programs.

**Appendix B**

**METHODS FOR COMPUTING BRANCH GRADIENTS**

Although our current prototype implementation does not take branches into account, we describe two possible approaches to handling branches with PGA here for completeness.

Barrier functions provide a means to attenuate gradients that are likely to be incorrect because they would change the current execution path. For example, on the path $\text{if}(x<8) \ x=x\&4; \text{if} \ x=7$ initially, sampling $x=8$ for $x=x\&4$ would yield a gradient of $-4$. However, this gradient would be invalid because $x=8$ would follow a different execution path and skip the $x=x\&4$ operation. A barrier function on branch $\text{if}(x<8)$ could set the positive gradient of $x$ to $0$ such that the proximal gradient would not sample invalid values.

Sampling alternate execution paths is another possible approach to handling branches that could potentially be used to model implicit data flows. For example, upon encountering the branch $\text{if}(x<1) y=0; \text{else} \ y=1; \text{with the value x=0, analysis could fork execution and set x=1 to observe how the alternate execution affects the value of y. This could then used to estimate a gradient of y with regard to x, even though x has no direct effect on y.**

**Appendix C**

**GRADIENT INSTRUMENTATION CODE SAMPLE**

Figure 10 shows a sample of the instrumentation for propagating gradients over two Mul and And operations. The Mul operation gradient can computed analytically, while the And operation requires sampling.

```c
/* use chain rule to compute gradient */
for MULTIPLICATION operation */

7 case 15:
8 neg_gr = x1 * neg_gr2 + x2 * neg_gr1;
9 pos_gr = x1 * pos_gr2 + x2 * pos_gr1;
10 break;
11...
12 /* use sampling to approximate gradient */
for binary AND operation */

13 case 26: {
14 y = x1 & x2;
15 for (s=1... NSAMPLES) {
16  neg_y = ( x1 - s * neg_gr1 ) & ( x2 - s * neg_gr2);
17  if ( neg_gr == 0)
18     neg_gr = (y - neg_y )/ s;
19  
20  pos_y = ( x1 + s * pos_gr1 ) & ( x2 + s * pos_gr2);
21  if ( pos_gr == 0)
22     pos_gr = ( pos_y - y )/ s;
23  
24  break;
25 }}
26 }
```

**Fig. 10:** Simplified sample of gradient computation with both analytic integer multiplication and sampling bitwise and operation as examples.

**Appendix D**

**FUZZING COMPARISON MUTATION ALGORITHM**

Algorithm 1 formally defines the mutation strategy used in the guided fuzzing evaluation in section V-B3.

**Algorithm 1** Simple mutation algorithm for dataflow-guided fuzzing that focuses on influential bytes.

**Input:**

$k \leftarrow$ select $k$ influential bytes
seed \leftarrow$ initial seed
program \leftarrow$ targeted program
total_mut \leftarrow$ number of mutations

1: \textbf{while} cur\_mutations < total\_mutations \textbf{do}
2: \hspace{1cm} influential\_bytes \leftarrow$ program(seed)
3: \hspace{1cm} \textbf{for} byte \in influential\_bytes \textbf{do}
4: \hspace{2cm} \textbf{for} $v = 1$ to 255 \textbf{do}
5: \hspace{3cm} gen\_mutate(seed, byte, $v$)
6: \hspace{2cm} cur\_mutations \leftarrow cur\_mutations + 1
7: \hspace{2cm} \textbf{end for}
8: \hspace{1cm} \textbf{end for}
9: \textbf{end while}