Optimization of energy expenses at convective and conductive heating of the liquid in cylindrical capacity

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Abstract. The mathematical model (MM) is developed for the description of evaporation process of model liquid with a surface of the cylindrical capacity (CC) for application of power expenses optimization at convective and conductive influence. Also processes of the changes of liquid and steam-gas mixture (SGM) temperature were considered in the MM. The optimization of power expenses was based on the Pontryagin maximum principle with application to process of liquid evaporation. For the modeling of heat and mass transfer processes with evaporation in cylindrical capacity at the combined influence, the system of the ordinary differential equations was used. At some admission, the MM allows to perform optimization of the cyclogram of a convective expulsion (and heating systems operation) and to define a tendency of power expenses reduction at consecutive inclusions and switching off the SGM expulsion and heating of walls of cylindrical capacity with a liquid film on its walls. Power expenses of conductive heating of liquid and a expulsion of SGM from cylindrical capacity are accepted for an optimality criterion. Time intervals of heating the walls with liquid and expulsions of SGM providing minimization of power expenses for evaporation of the set mass of liquid are defined with the help of numerical modeling. The result of calculations without optimization show the increased power expenses are carried out.

Keywords: convective and conductive heating, liquid, evaporation, power expenses, optimization, maximum principle

1. Introduction

Technologies of drying materials play an important role for a number of productions and in various fields, for example, as producing aviation and space equipment (fuel tanks and highways). It is connected with use of technological liquid for washing-up, durability tests, calibrating.

The task of the efficiency of heat and mass exchange processes, occurring in technical systems of evaporation, is important. In this work the example of such evaporation technical system, the system of convective and conductive evaporation of cylindrical capacity with a liquid film is considered.

Numerous theoretical-and-pilot studies including development of mathematical models (MM) on desiccation of materials when using of various feeders of heat, a expulsions, change of pressure, acoustic influence [1, 2].

Works on optimization of evaporation processes with change of designs [3], processes of heat exchange [4, 5], with change of pressure and heat [6] were carried out.

This work is further work with optimization of power expenses at evaporation of liquid at thermal influence [6]. The increase of efficiency of heat and a mass exchange process is considered at evaporation of liquid with a free surface at joint thermal and convective influences with application of the maximum principle of Pontryagin [7]. Specifics of application of the principle which is widely...
used in problems of optimum control are that it is applicable for the objects described by systems of the ordinary differential equations, for example, given in [8].

Thus, on the basis of the maximum principle of Pontryagin, for power expenses optimization it is necessary to develop simplified MM, describing the process of liquid evaporation from a free surface at joint thermal and convective influence on the basis of the ordinary differential equations that, naturally, will lead to methodical errors, but will allow to estimate a tendency of reduction of power expenses due to optimization of the cyclogram of systems of an air expulsions and heating operation.

2. Problem statement

On the basis of the review of works which is carried out above it is possible to formulate the following problem definition including:

– development of MM of liquid evaporation in the cylindrical capacity (CC) at convective and conductive heating on a basis of ordinary differential equations;

– development of optimum control of convective and conductive heating of liquid in CC on the basis of the maximum principle of Pontryagin.

Boundary conditions are the following: liquid is placed in the form of a thin layer with a free surface on a lateral surface of the cylinder which longitudinal axis is placed perpendicular to a gravity vector, there is a liquid surface mirror.

Initial conditions are the following: convective heat in the form of the warmed-up air with a constant volume speed, temperature and humidity, and conductive heat in the form of heating of CC walls are supplied into CC. The criterion is the total amount of the given heat for evaporation of the set mass of liquid taking into account costs of expulsion of the hot air given to CC.

The optimized parameters are the following: time of switching on (switching off) the supply of convective and conductive heat in CC with liquid.

The assumptions are the following:

– in the course of interaction of the given CC of hot air and the evaporated liquid in CC the steam-gas mixture deleted from CC at the expense of the given hot air from the subsequent expulsion of SGM is formed;

– the steady process of interaction of SGM with metal CC and liquid.

3. Development of physical and mathematical model of liquid evaporation

Convective heat moves in CE in the form of hot air with a constant pressure of \( P_0 \) with a volume speed \( U_{V0} \) through \( S_0 \) section temperature \( T_{V0} \) and concentration of vapors of \( C_W \) liquid in SGM.

Conductive heat moves in CC by means of heating CC walls with area \( S \) and thickness \( H_S \) by heating elements \( Q_T \) power per unit area.

Taking into account evaporation of liquid at the exit from CC the volume speed of the outflow of SGM will be equal to \( U_F = U_{V0} + SV_W \), where \( V_W \) is the volume speed of liquid evaporation from the unit of area, \( C_U(t) \) is the switching on (= 1) and switching off (= 0) function of hot air supply (blowing in), \( C_T(t) \) is the switching on (= 1) and switching off (= 0) function of conductive heating of CC walls.

Let us consider the following sizes for the studied processes parameters:

\( V_i, P_o, T_i, m_i, C_{pi} \) are the volumes, pressure, temperatures, masses and thermal capacities for a wall of CC, liquid, SGM, hot air and steam of liquid at \( i = s, w, v, Ai \) p, respectively.

Further, on the basis of the considered assumptions we will offer MM, describing the process of heat and mass exchange between elements in CC, in the form of the ordinary differential equations.

Similar to work [6] for representation of MM, the describing process of heat and mass exchange at convective and conductive heating, a number of designations is used:
\[ W_W(t,T,P) = K_0 \left( P_W(T_W) - P_P(m_P,T_V) \right) \] is the speed of evaporation of liquid from unit of area, \[ V_W(t,T,P) = \frac{W_W(t,T,P)RT_W(t)}{\rho_W} \] is the volume speed of evaporation of liquid from unit of area, \[ \rho_0(T_{V_0}) = \frac{RT_{V_0}(1-C_W)/\mu_A + C_W/\mu_W}{(RT_{V_0})} \] is the density of the given hot air from shares of vapors of liquid \( C_W \) with temperature \( T_{V_0} \), \( F_{A0}(t,m_A) = U_V(1-C_W)\rho_{V0}(T_{V0}) \) is the function of speed of supply of hot air in CC, \( F_{P0}(t,m_P) = U_{VW}C_WP_{V0}(T_{V0}) - \) function of speed of supply of steam with hot air in CC, \( U_V = C_U(t)U_{V0} + SW(t,T,P) \) is the function of volume speed of an exit of SGM from CC, \( F_A(t,m_A) = U_V \frac{m_A}{V_{V0}} \) is the function of speed of decrease of dry air from CC, \( F_m(t,m) = U_V \frac{m_P}{V_{V0}} \) is the function of speed of decrease of steams from CC, \( F_m(t,T) = \frac{(L_W-C_PW_T_W)}{m_SC_P+mwCP_W} \) is the function of change of average and mass temperature of liquid from its evaporation, \( F_T(t) = \frac{q_T}{m_SC_P+mwCP_W} \) is the function of change of average and mass temperature of liquid from the power of thermal (conductive) influence, \( F_{V_A}(t) = (C_U(t)U_{V0} + SK_0(174727.8 - 1312) \cdot T_W + 2.47 \cdot T_W^2 - \left( \frac{m_PRT_W}{\mu_WV_{V0}} \right))RT_W(t)/(P\mu_W) \) is the function of an exit of dry air from CC, \( F_{V_P}(t) = (C_U(t)U_{V0} + SW(t,T,P)) \frac{m_P}{V_{V0}} \) is the function of an exit of vapors of liquid from CC, \( F_{V_0}(t,T) = \frac{(L_W-C_PW_T_W)}{m_ACP_A+mpCP_P} \) is the function of energy of the entering hot air, \( F_{mV}(t,T) = \frac{(L_W-C_PW_T_W)}{m_ACP_A+mpCP_P} \) is the function of change of mean-mass temperature of SGM from liquid evaporation, \( F_{mW}(t,m,\mu) = \frac{(m_A(t) + m_P(t))}{\mu_A + \mu_W} \) is the molar ratio of mass of dry air and vapors of liquid.

At the considered assumptions in a problem definition the system of the ordinary differential equations considering an exchange of masses (liquid evaporation, air blown and dumping of SGM) and heat exchange for liquid and SGM, will have the following form:

\[
\begin{align*}
\frac{\partial m_W}{\partial t} &= -SW_W(t,T,P), \\
\frac{\partial m_A}{\partial t} &= C_U(t)F_{A0}(t,m_A) - F_{V_A}(t,m_A) = C_U(t)F_{A0}(t,m_A) - (C_U(t)U_{V0} + SW_W(t,T,P))\frac{m_A}{V_{V0}}, \\
\frac{\partial m_P}{\partial t} &= C_U(t)F_{P0}(t,m_P) - F_{V_P}(t,m_P) + SW_W(t,T,P), \\
\frac{\partial m_P}{\partial t} &= F_{V_0}(t,T) + SW_W(t,T,P)F_{mV}(t,T), \\
\frac{\partial T_W}{\partial t} &= \frac{m_W}{\rho_WC_W}RT_W(t), \quad \frac{\partial T_V}{\partial t} = \frac{m_A(m_A(t) + m_P(t))}{\mu_A + \mu_W}RT_W(t)
\end{align*}
\]
management of switching on and switching off of giving of SGM in CC (1 or 0), $C_p_s$ is the thermal capacity of CC (J/(kg · K)), $C_{pw}$ is the a liquid thermal capacity (J/(kg · K)), $C_{pp}$ is the  thermal capacity of vapors of liquid (J/(kg · K)), $C_p$ is the therm al capacity of dry air (J/(kg · K)).

At temperatures of liquid state $T_w$ from 273.1 °K and up to 303.1 °K we will approximately present the partial pressure of saturation of the evaporated liquid in the form, according to tabular data:

$$P_w(T_w) = 174727.8 - 1312.1 \times T_w + 2.47 \times T_w^2 \text{ (Па)}.$$  \hspace{2cm} (6)

On the basis of Klapeyron- Mendeleyev's equation for mix of gases let us present the partial pressure of liquid in $P_{\text{air}}$ in the form:

$$P_{\text{air}}(t) = \frac{P_w(T_w) V_{\text{V}}}{P_w(T_w) V_{\text{V}} + (1 - C_W) C_p A + C_W C_p (T_w - T_V)}.$$  \hspace{2cm} (7)

For the system of the equations (1) – (5) for the required functions initial and regional conditions from time of $t_0 = 0$ to $t_k$ can be set in the following form:

$$t_0 = 0: m_W = m_{W0}, m_A = m_{A0}, m_P = m_{P0}, T_W = T_{W0}, T_V = T_{V0}$$  \hspace{2cm} (8)

where $t_k$ is the uncertain size.

4. Optimization criterion

For formation of an optimality criterion of the heat and a mass exchange process we will consider minimization of power costs of evaporation of a certain mass of liquid with $m_0$ to $m_k$ during time periods from $t_0 = 0$ to $t_k$ with change of all energy in the considered system of elements (CC+ liquid + SGM).

Total costs of convective energy of heating of the air given to CC, costs of a expulsions of the given $E_U$ air and conductive energy of heating walls of CC should correspond to the change of energy of the whole system of material bodies with liquid and SGM there.

Total energy that can be presented in the form of integrals on time:

$$E_{\Sigma}(t) = E_U + E_{U1} + E_T = \frac{1}{2} \int_{t_0}^{t_k} C_U U_{V0} S_0^2 \rho V_0 (T_{V0}) dt + \int_{t_0}^{t_k} C_U U_{V0} \rho V_0 (T_{V0}) ((1 - C_W) C_p A + C_W C_p P)(T_{V0} - T_V) dt + \int_{t_0}^{t_k} C_T Q_T dt,$$

where $E_U$ is the corresponds to work of forces on supply of hot air $\frac{1}{2} C_U U_{V0} S_0^2 \rho V_0$ (kinetic energy of SGM) during $t_k$ with energy of $E_{U1}$, where $U_{V0}$ is the giving speed through section Square $S_0$, and its preliminary heating to temperature $T_{V0}$, $E_T$ is the energy of conductive heating of walls and liquid power $C_T Q_T$.

Numerical results of modeling of system of the equations are given in 5 (1) – (5) as well as the comparison of power expenses with the results without management at $C_U = 1$ and $C_T = 1$, i.e. at simultaneous supply of convective and conductive energy.

5. Development of optimum control of thermoprocess of evaporation on the basis of the maximum principle of Pontryagin

Let us consider optimization of power costs of evaporation of the set mass of liquid by means of Pontryagin’s method [7].

According to this theory, at the solution of a problem of optimization with function of Hamilton

$$H = \lambda_1 \frac{\partial m_W}{\partial t} + \lambda_2 \frac{\partial m_A}{\partial t} + \lambda_3 \frac{\partial m_P}{\partial t} + \lambda_4 \frac{\partial T_W}{\partial t} + \lambda_5 \frac{\partial T_V}{\partial t} - f_H$$

it is necessary to introduce (introduce ипн use) the interfaced functions $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5$ and the $f_H$ function for minimization of power expenses.

Expression for $f_H$ can be written down in the following form:

$$f_H = \frac{\partial E_{\Sigma}}{\partial t} = \frac{1}{2} C_U U_{V0} S_0^2 \rho V_0 (T_{V0}) + C_U U_{V0} \rho V_0 (T_{V0}) ((1 - C_W) C_p A + C_W C_p P)(T_{V0} - T_V) + C_T Q_T.$$  \hspace{2cm} (11)
On (1) – (5) and (12) final expression for the function (10) determining the maximizing functionality with minimization of expenses of energy by function (11) during time \([0, t_k]\) is based:

\[
H = \lambda_1 \frac{\partial m_w}{\partial t} + \lambda_2 \frac{\partial m_a}{\partial t} + \lambda_3 \frac{\partial m_p}{\partial t} + \lambda_4 \frac{\partial T_w}{\partial t} + \lambda_5 \frac{\partial T_v}{\partial t} - \frac{1}{2} C_U U_0^3 \rho_0 T_0^2 \rho_0(T_0) - \\
- C_{U0} U_0 \rho_0(T_0)((1 - C_W)C_{PA} + C_W C_{Pp})(T_0 - T_V) - C_T Q_T = \\
= \lambda_1 [\text{SW}_w(t, T, P)] + \\
+ \lambda_2 [C_{U}(t) F_{A0}(t, m_A) - F_{V_A}(t, m_A)] + \\
+ \lambda_3 [C_{U}(t) F_p(t, m_p) - F_{Vp}(t, m_p) + \text{SW}_w(t, T, P)] + \\
+ \lambda_4 [\text{SW}_w(t, T, P) F_{m_w}(t, T) + S C_r(t) F_{T}(t)] + \\
+ \lambda_5 [F_{V0}(t) + \text{SW}_w(t, T, P) F_{mV}(t, T)] - \\
- \frac{1}{2} C_U U_0^3 \rho_0(T_0) - C_{U0} U_0 \rho_0(T_0)((1 - C_W)C_{PA} + C_W C_{Pp})(T_0 - T_V) - C_T Q_T.
\]

For build of the interfaced system of the equations for definition of functions \(\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5\) originally are determined particular derivative of expressions (1) – (5) and (10) by the chosen independent variables \((m_w, m_a, m_p, T_w, T_v)\). After their use in derivative of function \(H\) of Hamilton (derivative of \(H\), including \(f_{ji}\)) the interfaced system of the equations to (1) – (5) in the following form is written [7]:

\[
\dot{\lambda}_1 = - \frac{\partial H}{\partial m_w} \dot{\lambda}_2 = - \frac{\partial H}{\partial m_a} \dot{\lambda}_3 = - \frac{\partial H}{\partial m_p} \dot{\lambda}_4 = - \frac{\partial H}{\partial T_w} \dot{\lambda}_5 = - \frac{\partial H}{\partial T_v} = (12),(13),(14),(15),(16)
\]

For the equations (12) – (16) initial data \(\lambda_{10}, \lambda_{20}, \lambda_{30}, \lambda_{40}, \lambda_{50}\), final data \(\lambda_{1k}, \lambda_{2k}, \lambda_{3k}, \lambda_{4k}, \lambda_{5k}\) are unknown, though final values for four functions \(m_{a_k}, m_{p_k}, T_{w_k}, T_{v_k}\), are not set yet and therefore short circuit of boundary conditions in systems of the equations with the unknown final time \(t_k\) will require five conditions of transversality at \(t = t_k\), at which with the variables \(y_1 = m_{a_k}, y_2 = m_{a_k}, y_3 = m_{p_k}, y_4 = T_{w_k}, y_5 = T_{v_k}\) the first variation of the optimizing functionality

\[
J = \int_0^k H(y_i, \dot{y}_i, t) dt
\]

at the requirement of achievement of an extremum has to be equal to zero:

\[
\delta H|_{t_k} = \sum_{i=1}^5 \frac{\partial H}{\partial y_i} \delta y_i(t_k) + H(y_i, \dot{y}_i, t_k) \delta y_k = 0.
\]

The equation (18) taking into account that already \(\delta y_i(t_k) = 0\), will be carried out at equalities to zero coefficients at other variations on unknown variables on the right border, i.e. at

\[
\frac{\partial H}{\partial y_2} = 0, \frac{\partial H}{\partial y_3} = 0, \frac{\partial H}{\partial y_4} = 0, \frac{\partial H}{\partial y_5} = 0, H(y_i, t_k) = 0,
\]

or

\[
\left. \frac{\partial H}{\partial m_a} \right|_{t_k} = 0, \left. \frac{\partial H}{\partial m_p} \right|_{t_k} = 0, \left. \frac{\partial H}{\partial T_w} \right|_{t_k} = 0, \left. \frac{\partial H}{\partial T_v} \right|_{t_k} = 0,
\]

\[
f_H \left. \frac{\partial H}{\partial T_w} \right|_{t_k} = -m_p \frac{\partial H}{\partial m_p} - m_w \frac{\partial H}{\partial m_w} - T_w \frac{\partial H}{\partial T_w}, \left. \frac{\partial H}{\partial T_v} \right|_{t_k} = -T_v \frac{\partial H}{\partial T_v} = -H|_{t_k} = 0.
\]

These conditions are simplified as follows:

\[
\lambda_2 = 0, \lambda_3 = 0, \lambda_4 = 0, \lambda_5 = 0, f_H - \lambda_5 m_w = 0
\]

or

\[
\frac{1}{2} C_U U_0^3 \rho_0(T_0) + C_{U0} U_0 \rho_0(T_0)((1 - C_W)C_{PA} + C_W C_{Pp})(T_0 - T_V) + C_T Q_T +
\]

\[ + \lambda_1 \left[ SK_0 \left( 174727.8 - 1312.1 \cdot T_W + 2.47 \cdot T_W^2 - \frac{m_r R T_V}{m_P V_0} \right) \right], \quad (19) \]

For obtaining the maximum value of functionality from \( H(t) \) on an interval \([0, t_i] \) management of switching on and switching off of an expulsion of SGM on the \( C_U(t) \) function will be carried out from conditions:

\[
\begin{align*}
U_U &= \frac{\partial H}{\partial C_U} \Bigg|_{C_U = 0} > 0 \text{ at } C_U(t) = 1, \\
U_U &= \frac{\partial H}{\partial C_U} \Bigg|_{C_U = 1} < 0 \text{ at } C_U(t) = 0.
\end{align*}
\]

From here, the supply of hot air will be under a condition

\[
U_U = \lambda_2 \left[ U_{v0} (1 - C_W) \rho v_0 (T_{V0}) \right] + \lambda_3 \left[ \frac{U_{v0} C_W \rho v_0 (T_{V0}) - U_{v0} m_P}{m_P v_0} \right] - \lambda_5 \left[ \frac{U_{V0} \rho v_0 (C_{PA} + C_{PP} C_W)(T_{V0} - T_{V})}{m_A C_{PA} + m_P C_{PP}} \right] - \frac{1}{2} \frac{U_{V0} S_0^2 \rho v_0 (T_{V0}) - U_{V0} \rho v_0 (T_{V0}) (1 - C_W) C_A}{U_{V0} S_0 \rho v_0 (T_{V0}) - U_{V0} \rho v_0 (T_{V0}) (1 - C_W) C_A + C_W C_{PP} (T_{V0} - T_{V})}. \]

For \( C_T \) the condition of switching on and switching off the external heating (convective component) is defined in the following form:

\[
\begin{align*}
U_T &= \frac{\partial H}{\partial C_T} > 0 \text{ at } C_T(t) = 1, \\
U_T &= \frac{\partial H}{\partial C_T} < 0 \text{ at } C_T(t) = 0.
\end{align*}
\]

The requirement of positivity of functions of management of \( U_U, U_T \) indicates increase in coefficients of \( C_U, C_T \) (here only with 0 to 1) that for functionality of \( J \) the maximum values were reached.

On the basis of (10) function of inclusion of \( U_T \) in a look turns out:

\[
U_T = \lambda_4 \left[ SF_T(t) \right] = Q_T. \]

6. Numerical modelling of process

Numerical modeling of system (1) – (5), with an initial mass of liquid \( m_{w0} = 10 \text{ g} \) to final \( m_{w} = 5 \text{ g} \) and 1 g from capacity the mass of \( m_s = 52 \text{ kg} \) and the area of walls of \( S = 3.32 \text{ m}^2 \) with minimization of expenses of energy when using heating with power \( Q_T = 35.7 \text{ kW/m}^2 \) and air supply with speed \( U_{V0} = 1.0 \text{ m/s} \) on \( T_{v0} = 353.1 \text{ °K} \) or 413.1 °K was carried out to CC with volume \( V_0 = 0.022 \text{ m}^3 \text{ with SGM pressure } P_0 = 105.6 \text{ kPa} \) and temperature \( T_0 = 273.1 \text{ °K} \) for air in CC, liquid, CC walls from iron the mass of \( m_r = 52 \text{ kg} \).

In an initial timepoint in CC air with humidity of vapors of liquid of 0% or 90% will be considered. The second case will demand, as the most intense case, bigger expenses of energy or an expulsion of drier SGM (that is confirmed by the carried-out calculations).

For the numerical decision of system of the differential equations (1) – (5) and the system conjugated to them (12) – (16) Runge-Kutt’s method of the 4th order of accuracy on discrete steps of time was used. Calculations were carried out before evaporation of 5 and 9 g from 10 g of liquid, and when using conditions of optimization updating of initial data for variables of the conjugate system (at the beginning was: \( \lambda_{10} = 1, \lambda_{20} = 1, \lambda_{30} = -1, \lambda_{40} = -1, \lambda_{50} = -1 \)) with the requirement of approach to (18). In calculations with optimization conditions (19) – (23) were applied.

By analogy with data in experiments [9] the following physical quantities were taken: \( L_w = 2256.0 \text{ kJ/kg} \), \( R = 8.314 \text{ J/(mol  · K)} \), \( K_0 = 1.39 \times 10^{18} \text{ kg} / (\text{Pas}  · \text{m}^2  · \text{c}) \), \( q = 1.18 \), \( \mu_w = 0.0182 \text{ kg/mol} \), \( \mu_A = 0.02897 \text{ kg/mol} \), \( C_{PA} = 460 \text{ J/kg  · J} \), \( C_{PP} = 4180 \text{ J/kg  · J} \), \( C_{P0} = 2020 \text{ J/kg  · J} \), \( C_{P_A} = 1005 \text{ J/kg  · J} \).

For the accounting of heat exchange between PGS, liquid and CC the following sizes were considered: the CC case sizes – \( L = 0.969 \text{m} \) – length, \( r = 0.39 \text{m} \) – the radius, \( h = 0.002 \text{m} \) – thickness, a thermal capacity of walls of the CC – \( C_{PK} = 460 \text{ J/kg  · J} \), coefficients of heat conductivity of liquid – \( \lambda_w = 0.63 \text{ W/(m  · J)} \), SGM – \( \lambda_v = 0.022 \cdot (P_{v0}/P_0) \text{ W/ (m  · J)} \), walls of the CC – \( \lambda_s = \lambda_K = 92.0 \text{ W/ (m  · J)} \) modeling of heat exchange was carried out by m according to the methods presented in [9].
The case of joint continuous influence of the heater and giving of SGM and with management for achievement of minimization of total power expenses of $E_{ΣV}$ according to expression (9) was considered.

Results of calculations with initial humidity of air $C_{W0} = 0\%$ and 90% with constant heating are given in Table 1 at $Q_T = 35.7$ KW/ m² and continuous supply of hot air with $U_{V0} = 1.0$ m³/s (line 1-3). Here time of evaporation is less, than at application of optimization of switching on (line 4-5), but thus power expenses are considerable, and at the increased humidity quickly set mass of liquid in view of considerable overheating of walls of CC and liquid at inertial delay on evaporation cannot evaporate. But at increase in power expenses with $Q_T = 357.0$ KW/ m² when giving more heated-up SGM ($T_{V0} = 413.1$ °K) occur quicker evaporation on time that is more effective on a final interval of process of evaporation (Table 2, 3 calculation).

Reduction of heating of the given hot air ($T_{V0} = 353.1$ °K) or speeds ($U_{V0} = 0.4$ m³/s) leads to reduction of power expenses (Table 2, calculations 1 and 5) for evaporation of mass of liquid from 10 g to 1 g. As calculations with optimization show, heating of CC walls is effective at the initial interval for the subsequent increase in speed of evaporation after humidity reduction in CC after supply of rather dry hot air (0–30%). Thus, economy of power expenses to 2,5 times (Table 1, line 4-5 at $m_{wk} = 5$ g) or to 10–20% (Table 2, calculations 2,4,6 at $m_{wk} = 1$ g) can be obtained.

**Table 1.** Energy consumption at evaporation of liquid at an expulsion and heating with optimization and without optimization.

| №  | Time $t$ (s) | $Q_T$ (KW/ m²) | $U_{V0}$ (m³/s), $T_{V0}$ (K) | $λ_{t0}$ | $λ_{20}$ | $λ_{30}$ | $λ_{40}$ | $λ_{50}$ | $Δm$ (g), $C_{W0}$, $C_{W00}$ | $E_{ΣV}$ (KJ) |
|----|--------------|----------------|-------------------------------|---------|---------|---------|---------|---------|-------------------------|-------------|
| 1  | $t = 16.1$   | 35.7           | 1.0, 413                      | –       | –       | –       | –       | –       | 0% 0%                   | 4982.2      |
| 2  | $t = 16.2$   | 35.7           | 1.0, 413                      | –       | –       | –       | –       | –       | 0% 90%                  | 4984.6      |
| 3  | $t = 16.3$   | 35.7           | 1.0, 413                      | –       | –       | –       | –       | –       | 30% 90%                 | 5006.1      |
| 4  | $t = 32.4$   | 35.7           | 1.0, 413, 413                 | 5.62    | -9.89   | -67.2   | 31.78   | 0.00339 | 5.0 0%                   | 2201.0      |
| 5  | $t = 21.4$   | 35.7           | 1.0, 413, 0.17                | -6.29   | 13.2    | -66.9   | 30.96   | 0.00354 | 5.0 30% 90%              | 3104.0      |

When using interruptions in heating and an expulsion (comparing lines 4-5 with lines 1 and 3) it follows from Table 1 that by means of the set mass of liquid evaporation management power expenses are significantly reduced (to more than 2 times). $c_{W00}$ is the initial humidity in CC.
Figure 1. Diagram of liquid evaporation with the cyclogram of switching on – lines 1 and 4 from Table 1.

Figure 2. Diagram of liquid evaporation with the cyclogram of switching on – lines 3 and 5 from Table 1.
In Figure 1 diagram of evaporation of mass of liquid with 10g to 5g on time and cyclograms of switching on and switching off of expulsion out and heating by the calculations given in Table 1 in lines 1 (the diagram of mass of evaporation 1 without optimization) and 4 are shown (the diagram of mass of evaporation 2 with optimization of inclusions on the Cu and Ct functions).

In Figure 2 similar diagram and cyclograms of switching on and switching off of expulsion out and heating by calculations in lines 3 and 5 (Table 1) are shown.

Table 2. Energy consumption at evaporation of liquid at an expulsion and heating with optimization and without optimization.

| №  | Time t (s) | $Q_T$ (KW/m²) | $U_{p0}$ (m³/s), $T_{p0}$ (K) | $\lambda_{20}$ | $\lambda_{30}$ | $\lambda_{40}$ | $\lambda_{50}$ | $\Delta m$ (g), C_W0, C_W00 | $E_{zB}$ (KJ) |
|----|------------|---------------|----------------------|----------------|----------------|----------------|----------------|-----------------------------|---------------|
| 1  | $t = 37.5$ | 35.7          | 1.0, 353             | -              | -              | -              | -              | 9.0, 0% 0%                  | 1454.0        |
| 2  | $t = 50.1$ | 35.7          | 1.0 353, $t_0=14.3$, $t_k=42.4$ | 0.331          | 2.94           | -67.1          | 31.6           | 0.00337, 9.0, 0% 0%         | 1302.0        |
| 3  | $t = 19.4$ | 357.          | 1.0, 413             | -              | -              | -              | -              | 9.0, 0% 0%                  | 5047.0        |
| 4  | $t = 29.1$ | 357.          | 1.0 413, $t_0=5.2$, $t_k=16.1$ | 0.574          | 2.96           | -67.31         | 31.90          | 0.00324, 9.0, 0% 0%         | 4131.0        |
| 5  | $t = 52.5$ | 35.7          | 0.4 353              | -              | -              | -              | -              | 9.0, 0% 0%                  | 1131.0        |
| 6  | $t = 60.8$ | 35.7          | 0.4 353, $t_0=10.3$, $t_k=58.1$ | -0.334         | 11.72          | -67.02         | 31.52          | 0.00349, 9.0, 0% 0%         | 1077.0        |
Figure 3. Diagram of liquid evaporation with the cyclogram of switching on – lines 1 and 2 from Table 2.

Figure 4. Diagram of liquid evaporation with the cyclogram of switching on – lines 3 and 4 from Table 2.
In Figure 3 diagram of evaporation of mass of liquid with 10g to 1g the time and cyclograms of switching on and switching off of expulsion out and heating by the calculations given in Table 2 in lines 1 (the schedule of mass of evaporation 1 without optimization) and 2 are shown (the diagram of mass of evaporation 2 with optimization of switching on the Cu and Ct functions ).

In Figure 4 similar diagram and cyclograms of switching on and switching off of expulsion out and heating by calculations in lines 3 and 4 (Table 2) are shown. As appears from diagram, short-term influences of an expulsion and heating are sufficient for evaporation of liquid with big economy of power expenses.

7. Discussion of the received results
Two options of an evaporation of walls of CC from liquid from 10 g to 5 g and to 1 g that showed by optimization of processes different cyclograms of inclusions of an expulsion of SGM and similar cyclograms on heating of CC were considered.

So in the first case and without optimization of energy consumption (Table 1) follows from comparison of results of modeling of an evaporation (heating and an expulsion of SGM) with optimization that the prize at application of optimization occurs due to initial warming up of liquid (line 4 and 5) and later short-term expulsions of SGM. Thus with reduction of energy consumption more optimum control (line 4) leads to lengthening of time of process of evaporation of the set mass of liquid. There more optimum case is reached due to evaporation of liquid in nonsaturated SGM without inclusions of heat and expulsions on the second half of time. Calculations with the increased humidity of gas mixes in CC and for an expulsion lead to increase in power expenses.

In the second case at the evaporation to 1 g and zero humidity of gas mixes (Table 2) calculations are carried out at the different capacities of heating and different speeds of an expulsion of CC. And as showed calculations, for minimization of energy expenses it is expedient to use smaller capacities on heating (35,7 KW/m² instead of 357,0 KW/ m²) and the reduced expulsion speeds (0.4 m³/s instead of 1.0 m³/s).

The further direction of researches provides an assessment of the methodical errors caused by replacement of the differential equations in particular derivatives on the ordinary differential equations, and also experimental check of the received results of optimum control of process of evaporation of liquids with other parameters of system (volumes, masses, speeds of an expulsion, heat feeders in capacity).

8. Conclusion
• The mathematical model of liquid evaporation at conductive and convective influence on the basis of the ordinary differential equations is developed.
• The criterion of optimization of management of an expulsion and heating at the fixed values of the heater power and of expulsions productivity by heated air from a condition of minimization of energy for evaporation of the fixed mass of liquid is offered.
• Procedure of application of the theory of optimum control on the basis of the maximum principle of Pontryagin is developed for optimization of conductive and convective evaporation process.
• Efficiency of application of the optimum control theory for the considered problem definition is shown.

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