Nonequilibrium phenomena encountered in steady-state quantum transport through nanoscale or mesoscopic conductors are presently attracting a lot of interest. Recent experiments on mesoscopic wires have shown that the nonequilibrium distribution function evolves from a two-step to a broad single-step distribution as the effective interaction strength increases \([1]\).\footnote{Nonequilibrium effects are captured by the Keldysh formulation [9, 18], where the total system evolves from the initial time \(-t_0/2\) after which the interaction is smoothly switched on, to time \(+t_0/2\) and back; one finally takes the limit \(t_0 \to \infty\). By a sequence of standard steps [9, 12, 19], we obtain a Keldysh functional integral representation as}

\[
\langle \hat{X} \rangle \equiv \sum_{W \in \mathbb{Z}} \int \mathcal{D}(\phi, Q) e^{iS_{\phi}(\phi, Q)+iS_{\text{sum}}[\phi]} X[Q, \phi],
\]

where \(X[Q_{\sigma}, \phi_{\sigma}]\) may be any observable expressed in terms of the charge on the dot (\(Q_{\sigma}(t)\)) and its phase (\(\phi_{\sigma}(t)\)), both defined on the upper/lower (\(\sigma = \pm\)) branch of the standard Keldysh contour [13]. The phase fields obey the boundary conditions, \(\sum_{\alpha=\pm} \sigma \phi_{\sigma}(-t_0/2) = 2\pi W, \sum_{\alpha=\pm} \sigma \phi_{\sigma}(+t_0/2) = 0\), where the integer \(W\) is summed over [20]. Here, \(W\) is the real-time analogue of the winding numbers central to establishing Coulomb blockade physics in imaginary-time theories [14, 15]. The charging energy, \(E_c\), of the dot enters the theory through

\[
S_{c} = \sum_{\sigma} \int_{-t_0/2}^{t_0/2} dt \left[ -E_c(Q_{\sigma} - Q_{\sigma})^2 + Q_{\sigma} \partial_t \phi_{\sigma} \right],
\]

where the constant \(Q_{\sigma}\) defines the electrostatically preferred charge configuration. The coupling to the source and drain \((\alpha = \pm)\) electrodes, biased by \(aV/2\), respectively, leads to the tunnel action [12, 16]

\[
S_{\text{tun}} = \frac{igT}{2} \sum_{\alpha \sigma \sigma'} \int dt dt' e^{-i\phi_{\sigma}(t)} L_{\sigma\sigma'}(t-t') e^{i\phi_{\sigma'}(t')},
\]

where, for simplicity, we assume identical tunneling conductances \(g_T\) for both contacts; the generalization to asymmetric cases is straightforward. The complex-valued functions \(L_{\sigma\sigma'}(t)\) appearing in Eq. \((3)\) are

\[
L_{\sigma\sigma'}(t) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} e^{-i\omega t} [\sigma' L'_{\omega} + (\sigma - \sigma') \omega],
\]

where the applied voltage \(V\) and the temperature \(T\) enter only via the real part,

\[
L'_{\omega} = \omega \coth \left( \frac{\omega}{2T} \right) + \sum_{\alpha=\pm} \frac{\omega + aV}{2} \coth \left( \frac{\omega + aV}{2T} \right).
\]

Our observable of main interest, the energy-dependent TDoS \(\nu(\epsilon, V, T)\) (technically, \((-\pi^{-1})\) times the imaginary
The hole contribution obtains by exchange (1 − n) ↔ n and \( \phi_\pm \leftrightarrow \phi_T \). We finally note that the current flowing through the dot is given by

\[
I(V) = \frac{g_T}{2} \int d\tau \int d\sigma [n_\sigma(\epsilon - V/2) - n_f(\epsilon + V/2)],
\]

where \( n = n_\sigma \). The phase factors in Eq. (6) encapsulate the effects of particle interactions. We next explore the effect of these phase fluctuations on the TDoS for the case \( g_T \gg 1 \), but return to the opposite limit, \( g_T \ll 1 \), as in the corresponding SEB problem. The phase factors in Eq. (6) then lead to the partition function

\[
\langle \xi(t) \rangle_x = 0 \text{ is correlated as } \langle \xi(t) \xi(t') \rangle_x = g_T L'(t - t'),
\]

and \( L'(t) \) is the Fourier transform of the kernel \( \xi \). Integration over \( \phi_q \) then constrains fluctuations of the voltage \( U \equiv \partial_t \phi_q \) (relative to the stationary value \( V/2 \)) to solutions of the semiclassical RC-Langevin equation

\[
C \partial_t U(t) + R^{-1} U(t) = \xi(t),
\]

where \( R = 1/(2g_T) \) is the parallel resistance of the circuit in Fig. 1 and the fluctuating noise current \( \xi(t) = 2i \Delta t^{-1} \int_{t - \Delta t}^t dt' \xi(t') \), averaged over a characteristic time window \( \Delta t \), gives rise to the dc noise power \( S \equiv 2 \Delta t \text{ var}(I) \). At finite temperatures and zero bias, the classical limit of Eq. (1), \( \xi(t) = 4T \delta(t) \), describes a thermal equilibrium situation, where the voltage \( U(t) \) relaxes to the Boltzmann distribution \( P(U) \sim \exp(-CU^2/2T) \), and Johnson-Nyquist thermal noise, \( S = 4T(g_T/2) \), is recovered. For high voltages and low temperatures, \( T \ll V \), the noise correlator \( g_T \) instead asymptotes to \( L'(t) = V \delta(t) \), which implies shot noise, \( S = 2FI_0 \), with the expected Fano factor \( F = 1/2 \): upon increasing \( V \), transport through the quantum dot undergoes a crossover from an equilibrium thermal to a steady-state nonequilibrium shot-noise dominated regime.

Employing the TDoS \( \nu_e \) as a reference observable, we next explore the dephasing influence that this noise has on the quantum physics of the system. To this end, we use \( \phi_0(t_2) - \phi_0(t_1) = \int_{t_1}^{t_2} dt' U(t') \) in Eq. (6) and integrate over \( \phi_q \). As a result, the (electron contribution to the) TDoS assumes the form

\[
\frac{\nu_e(\epsilon)}{\nu_0} = \frac{g_T}{2} \int d\tau \int d\sigma [n_\sigma(\epsilon - V/2) - n_f(\epsilon + V/2)] \frac{\nu(\epsilon)}{\nu_0},
\]

where \( U_c \) is the solution to a variant of the previous Langevin equation,

\[
(C \partial_t + R^{-1}) U_c = \xi + E_c \delta(t - \bar{t}) + \delta(t - (\bar{t} + \tau)).
\]

Equation (8) affords an intuitive interpretation: the TDoS \( \nu_e \) probes tunneling processes where an electron enters the dot at time \( \bar{t} \), remains at the dot for a characteristic time \( \tau \), leaves at time \( \bar{t} + \tau \), and enters the dot again at time \( \bar{t} + \tau + \Delta \). In the present structureless environment (a “dot”), the dynamical phase controlling these processes obtains by integration of the time-dependent voltage on the dot. The latter is governed by a superposition of (i) a voltage pulse of height \( E_c \) upon the entry of the external particle, as described by the second term on the r.h.s. of Eq. (9) [23], and (ii) the background voltage noise \( \xi \) on the dot. Averaging the exponentiated solution of Eq. (9) over the \( \xi \) and adding the hole contribution, \( \nu_h \), we obtain

\[
\frac{\nu(\epsilon)}{\nu_0} = 1 - \frac{1}{4\pi g_T} \int_0^\infty d\tau \sum_{\alpha = \pm} \frac{\cos(\epsilon + \alpha V/2)\tau}{\tau} \times (1 - e^{-\Omega\tau}) e^{-\xi},
\]

where non-singular contributions of higher order in \( g_T^{-1} \) have been neglected. In Eq. (10), the first term under the integral is the temporal Fourier transform of the distribution function, the factor \( (1 - e^{-\Omega\tau}) \) accounts for...
for the relaxation of the initial voltage pulse, and the last term defines the average noise-action (cf. Eq. (5)),

$$S(\tau) = \frac{\Omega^2}{4\pi g_T} \int_0^\infty d\omega \frac{1 - \cos(\omega \tau)}{\omega^2 (\omega^2 + \Omega^2)^2} L'(\omega).$$

(11)

Note that in this weak Coulomb blockade limit, the TDoS is independent of the reference charge \(Q_g\). In what follows, we consider the limit \(T = 0\) where the influence of nonequilibrium dephasing on the TDoS is strongest.

To logarithmic accuracy, the equilibrium \((V = 0)\) limit of Eq. (11) can be approximated by \(S_0(\tau) \approx (2\pi g_T)^{-1} \ln(\Omega \tau)\). The resulting TDoS is symmetric in \(\epsilon\) and displays a characteristic dip at \(\epsilon = 0\) – the ZBA – which vanishes in the limit \(E_c \to 0\), and also for \(g_T \to \infty\). At finite voltage, the TDoS remains symmetric, while the double-step profile of the distribution \(\mu(\epsilon)\) implies a splitting of the \(\epsilon = 0\) ZBA into two minima at \(\epsilon = \pm V/2\), see Fig. 2. The strict positivity of \(S(\tau) - S_0(\tau) > 0\), previously identified as a manifestation of enhanced noise levels for \(V \neq 0\), then causes a suppression of the ZBA. At the same time, its line shape broadens. In the limit \(V \to \infty\), the linear growth \(S(\tau) \sim V \tau\) implies a vanishing ZBA. Similar phenomena were recently studied [8] for tunneling into a 2D dissipative metal. Eq. (10) may serve to define a nonequilibrium dephasing rate \(\Gamma(V)\) from the voltage-induced broadening of the ZBA dips at \(\epsilon = \pm V/2\) [8]. Writing \(\delta \epsilon = \epsilon - V/2\) with \(|\delta \epsilon| \ll \Omega\), the ZBA dip is described by \(\delta \nu(\delta \epsilon) = \nu_0 - \nu(\epsilon)\). Parameterizing small deviations off the dip at \(V/2\) in terms of the dephasing rate \(2\Psi\), \([\delta \nu(0) - \delta \nu(\delta \epsilon)]/\delta \nu(0) \approx 1/2[\delta \epsilon/\Gamma(V)]^2\), Eq. (10) yields

$$\Gamma^2(V) = \int_0^\infty \frac{d\tau e^{-S(\tau)}(1 - e^{-\Omega \tau})}{\int_0^{\infty} d\tau e^{-S(\tau)}(1 - e^{-5\Omega \tau})}. \quad (12)$$

Analytical results for \(\Gamma(V)\) can then be extracted from Eq. (12) in various limiting regimes, while the full curve is obtained numerically, see Fig. 2.

Let us first discuss the asymptotic regime of weak interactions, \(E_c \to 0\). (Technically, this means that we take the limit \(\Omega \to 0\) prior to \(\delta \epsilon \to 0\).) Eq. (12) is then dominated by contributions \(\Omega \tau \ll 1\), which implies \(S(\tau) \approx V E_c \tau^2/4\) and \(\Gamma(V) = \sqrt{E_c V/2}\). The \(g_T\) independence of \(\Gamma\) reflects the diverging RC-time: particles tunneling onto the dot do have time to realize the dissipative nature of their environment. Turning to the complementary regime, \(\delta \epsilon \to 0\) at fixed \(\Omega\), we note that the leading contribution to an expansion of the TDoS in \(g_T^2\) \(\ll 1\), obtained by setting \(S = 0\), is given by the logarithmically singular expression \(\delta \nu(\delta \epsilon)/m = (8\pi g_T)^{-1} \ln[1 + (\Omega/\delta \epsilon)^2]\). Using Eq. (7), this recovers the well-known result [13] [17] for the differential conductance, \((2/g_T) dI/dV = 1 - (4\pi g_T)^{-1} \ln[1 + (\Omega/V)^2]\). However, in order to define the dephasing rate, one needs to retain a finite noise action \(S\). Specifically, for \(V > \Omega\), \(S(\tau) \approx V \tau/(4\pi g_T)\), implying

$$\frac{\Gamma(V)}{V} \approx \frac{1}{4\pi g_T} \left[ \frac{\ln(1 + 4\pi g_T \Omega/V)}{1 - (1 + 4\pi g_T \Omega/V)^{-2}} \right]^{1/2}, \quad (13)$$

while for \(\Omega e^{-\pi g_T} < V < \Omega [29], we estimate

$$\frac{\Gamma(V)}{V} \approx \frac{c}{\sqrt{2\pi g_T}} (V/\Omega)^{-1/(4\pi g_T)}.$$  

(14)

where \(c\) is a numerical prefactor of order unity. Importantly, in all parameter regimes, we find \(\Gamma(V) < V\), and the double peak structure in the nonequilibrium TDoS is reasonably well resolved. As a function of \(V\), the relative strength of the dephasing rate (i.e. the ratio \(\Gamma/V\)) increases with decreasing \(V\). Furthermore, Eq. (13) shows that for very large voltage, \(V > g_T \Omega\), the rate \(\Gamma(V) \sim V/g_T\) is directly determined by the shot noise \(S = 2\Omega I_0\) discussed above. The analytical results (13) and (14) describe the numerical solution to Eq. (12) rather well, as indicated in Fig. 2. With increasing \(g_T\), the dephasing rates are gradually suppressed, and for \(V = 0\), as expected, the dephasing rate is zero.

Finally, we briefly turn to the opposite case of strong Coulomb blockade, \(g_T \ll 1\), where the dual charge representation is appropriate. Expanding \(e^{i\sum_{\alpha=+}^n S_\alpha}\) in Eq. (1) into a Taylor series, we can integrate out the phase fields \(\phi_\alpha\). Now the \(\sum_{\alpha=1}^n\) factors appearing in the tunnel action (3) can be interpreted as charge raising or lowering operators, i.e. they change the corresponding \(Q_\sigma(t)\) variable by \(\pm 1\). In order \(m\) of the tunnel expansion, we thus have \(2n\) jumps at times \(t_j, t'_j\) \((j = 1, \ldots, m)\), with jump directions \(\sigma_j, \sigma'_j = \pm 1\). The charge fields are then expressed in terms of these jump times and directions, \(Q_\sigma(t) = N - \sum_{k=1}^m \left[ \delta_{\sigma \sigma_k} \Theta(t - t_k) - \delta_{\sigma' \sigma_k} \Theta(t - t'_k) \right]\), where \(\Theta(t)\) is the Heaviside function and the \(W\)-summation in Eq. (1) implies the boundary condition \(Q_{\pm}(-t_0/2) = N \in \mathbb{Z}\).
The charge representation of Eq. (1) is then given by

\[ Z = \sum_{m=1}^{\infty} \frac{1}{m!} \left(-\frac{g}{2}\right)^m \sum_{N_i(\sigma_i\sigma'_i)} \prod_{j=1}^{m} dt_j dt'_j \]

(15)

\[ \times e^{-i \sum \sigma_i E_{\sigma_i}} \int dt (Q_+ - Q_0) \prod_{k=1}^{m} L_{\sigma_k, \sigma_k'} (t_k - t'_k) \]

Away from charge degeneracies, i.e. for \( Q_g \) not half-integer, Eq. (15) admits a solution in terms of a “noninteracting-blep approximation” (NIBA) [27], where \( Z \) is dominated by short “blips” of length \(|t_j - t'_j|\) during which \( Q_+ (t) \neq Q_0 (t) \), with long time intervals between subsequent blips where \( Q_+ (t) = Q_0 (t) \). Defining the optimal charge state \( Q \in \mathbb{Z} \) according to \( Q_g - 1/2 < Q < Q_g + 1/2 \), the NIBA transition rate from \( Q_{\pm} = Q \) to any other charge state vanishes for \( T = 0 \) and \( V < 2 E_c (Q - Q_g + 1/2) \). In this regime, the TDoS is thus given by (see Fig. 2)

\[ \frac{\nu(\epsilon)}{\nu_0} = 1 - \sum_{s=\pm, \alpha=\pm} \frac{s}{2} \Theta (\epsilon + s + 2(Q - Q_g)E_c + \alpha V) \]

(16)

which recovers the equilibrium result of Ref. [15] for \( V = 0 \). Note that \( \nu(\epsilon) \) is generally asymmetric; Eq. (7) implies well-known Coulomb blockade expressions for the IV curve. Vanishingly small (noise) current levels imply that nonequilibrium dephasing is strongly suppressed in the Coulomb blockade regime and may leave traces only beyond NIBA. The near-degeneracy case, on the other hand, is related to a many-channel variant of the Kondo effect [19], where frequent transitions between two charge states occur. NIBA does not hold in this regime, but a real-time renormalization-group approach [25] can be constructed and will be described elsewhere.

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