DYNAMICAL PROCESSES IN THE SOLAR RADIATIVE INTERIOR

A. Palacios, S. Talon, S. Turck-Chièze, and C. Charbonnel

1 CEA/DSM/DAPNIA/Service d’Astrophysique, CE Saclay bât 709, F-91191 Gif-sur-Yvette (France)
2 Département de physique, Université de Montréal, C.P. 6128, succ. centre-ville, Montréal (Québec) H3C 3J7 Canada
3 CEA/DSM/DAPNIA/Service d’Astrophysique - CE Saclay bât 709 - F-91191 Gif-sur-Yvette (France)
4 Observatoire de l’Université de Genève, 51 chemin des Maillettes, CH-1290 Sauverny (Switzerland)
5 LATT- Observatoire Midi-Pyrénées, 14 av. E. Belin, F-31400 Toulouse (France)

ABSTRACT

Recent seismic observations coming from acoustic and gravity modes clearly show that the solar standard model has reached its limits and can no longer be used to interpret satisfactorily seismic observations. In this paper, we present a review of the non-standard processes that may be added to the solar models in order to improve our understanding of the helioseismic data. We also present some results obtained when applying “non-standard” stellar evolution to the modelling of the Sun.

Key words: Stellar evolution; hydrodynamics; rotation; internal gravity waves.

1. INTRODUCTION

Dynamical processes are thought to occur in stars all across the Hertzsprung-Russell diagram. The development of advanced and dedicated observational facilities have greatly improved the constraints that we can derive from observations. In particular, helioseismology, and more recently asteroseismology, have given a new insight to our understanding of solar and stellar physics in both the convective and radiative regions. Spectroscopic and (spectro-)polarimetric data collected for stars of different masses, metallicities and evolutionary stages, including the Sun, also demonstrate the actual presence and action of dynamical processes such as rotation and magnetic fields from direct measurements as well as from indirect evidence provided by the abundance anomalies.

However, while processes such as rotation, magnetic fields, turbulence and internal waves appear to play a crucial role in many aspects of stellar evolution, they are not well understood and their effects are seldom accounted for in stellar and solar evolution models. At the dawn of a new era for stellar physics, these processes can no longer be neglected.

The dynamical processes occurring in stellar interiors may have a direct impact on the fundamental parameters of stars (modification of the basic equations of stellar structure by rotation or magnetic fields, see for instance [22]). They also lead to transport of matter and/or angular momentum. Last but not least, they operate on very different scales at which their action has to be understood if one wants to properly take them into account in the stellar modelling. Two complementary approaches have emerged:

(1) the multi-dimensional simulation of the physical process in restricted areas on dynamical time-scales
(2) the incorporation of non-standard physical processes in stellar evolution codes in order to follow their integrated effects over evolutionary timescales.

In the following, we will adopt the second point of view and evaluate importance of various physical processes in the radiative stellar zones. The first approach has been essentially applied to the convective zones that evolve on dynamical scales, but few studies of radiative zones also exist (see e.g. [41]).

2. DYNAMICAL PROCESSES IN STELLAR RADIATIVE ZONES

To understand the observed stars we need to be able to correctly model their evolution up to their present status. This implicitly assumes that the integrated effect of the dynamical processes can be accounted for over large scales, both spatial and temporal. At present, numerical and computational restrictions prevent the simultaneous follow up of evolutionary and dynamical scales, so that we are bound to use prescriptions for the dynamical processes that can be integrated in a 1D stellar evolution code.

Atomic diffusion, which consists of gravitational separation, thermal diffusion and radiative accelerations [1], [37], has been the first process added to improve classical stellar evolution model. The prescriptions for atomic diffusion can be derived from first principles. In the past decade, the implementation of atomic diffusion established the new standard stellar evolution model.

By non-standard, we mean all processes different from convection.
which has been applied with success to various types of stars (Chemically peculiar stars, horizontal branch stars,...). In particular, it achieved important improvement of the predicted density and sound speed profiles in the case of the solar model [2, 4, 38].

**Rotation** is a dynamical process at play in all stars. Although the effects of rotation on stellar structure and evolution have been studied since the early days of stellar evolution [10], it has long been considered a second order effect and as such it was neglected in stellar computations. In the last decades, the increasing number of discrepancies between observations and model predictions have motivated the introduction of rotation and rotation-induced transport in stellar evolution, as well as the further development of suitable prescriptions for 1D modelling (see contribution by S. Mathis, this volume). The main effects of rotation are the modification of the effective gravity due to the centrifugal forces [22, 32], and the transport of angular momentum and chemical species. In the stellar radiative zones, this transport is ensured by meridional circulation, which is a consequence of the thermal imbalance existing in rotating stars, and by turbulence (see [23] for more details).

Various approximations and formalisms are currently used in order to take these effects into account, with two approaches for the rotational transport of angular momentum. The **turbulent diffusion approach** was first used by Endal & Sofia [18]. It describes the evolution of angular momentum according to a diffusion equation considering several linear criteria for hydrodynamical instabilities in order to derive a turbulent diffusion coefficient (see also Heger et al. [20]). The second formalism is the **meridional circulation approach**, proposed by Zahn [48]. It describes the angular momentum evolution according to an advection/diffusion equation. The advection term is related to the large-scale meridional circulation resulting from the thermal imbalance existing in a rotating star. The diffusion term is associated with turbulence generated by only one hydrodynamical source: the shear instability in its non-linear regime.

We use the latter description in its first order formulation for the transport of angular momentum [48, 23], and the Chaboyer & Zahn [11] formalism to account for the transport of nuclides. In the following, we will refer to this as the **“rotational mixing of type I”** (see also S. Mathis, this volume).

Let us note that the coupling of rotation and associated transport processes with stellar structure results in additional sets of dependent differential equations that need to be solved at each time step. In the case of **“rotational mixing of type I”**, the transport of angular momentum is described by a non-linear system of 5 differential equations, the resolution of which can be particularly intricate in the dynamical phases of the stellar evolution (i.e. beyond the main sequence).

**Magnetic fields** are also observed in a large variety of stars. Transport processes associated to such fields are particularly complex, as can be inferred from the intricate field patterns observed at the surface of solar-type stars. Their incorporation in stellar evolution models is particularly difficult due to the intrinsic three-dimensional nature of magnetic fields and the everlasting problem of their generation. At present, the account for magnetic fields in stellar structure and evolution consists in treating magnetic perturbations of the pressure and energy density profiles [24], and/or considering the magnetic transport of angular momentum and chemical species via magnetohydrodynamical instabilities [39, 20, 27, 47]. More recently, Mathis & Zahn [50] have also derived a formalism to treat self-consistently the interplay between meridional circulation, shear-induced turbulence and an axisymmetric magnetic field (see also S. Mathis, this volume).

The introduction of magnetic fields in stellar evolution codes implies an additional increase of the number of equations to be solved at each time step, as well as the addition of terms (Lorentz force) in the equation for the evolution of angular momentum.

**Internal gravity waves** are the travelling counterpart of the standing gravity modes of helioseismology (g-modes). They are excited at the base of the convective zone by Reynolds stresses and/or convective plumes, but the efficiency of each mechanism is not yet clearly assessed. They transport angular momentum with a null net contribution if the flux of prograde and retrograde waves is the same. In a differentially rotating medium however, the filtering of one of the families of waves can result in a non-null net transport of angular momentum in the region where they fade away. The spectrum of internal gravity waves strongly depends on the structure of the stellar convective envelope, and Talon & Charbonnel [43] showed that the angular momentum transport by internal gravity waves is important in the Sun and in main sequence stars with $T_{\text{eff}} \lesssim 6700$ K (see S. Talon, this volume for details).

At present, the full spectrum of internal gravity waves cannot be computed in a stellar evolution code. It needs to be determined in an independent module using predictions of the stellar evolution models as inputs (extension of the convective zone, pressure height scales, ...). The feedback on stellar evolution appears via additional terms in the equations for the transport of angular momentum and chemicals solved in the rotating case [45].

### 3. DYNAMICAL MODELS OF THE SUN

In the previous section, we briefly reviewed the main dynamical processes that will influence the stellar structure and evolution. Rotating models have proved to largely improve our understanding of massive stars [20] (see also Maeder & Meynet [25] and references therein).

In low-mass stars the picture is somewhat more complex. In these stars, the surface convective zone is much deeper than for more massive objects, and dynamical processes such as magnetic transport and internal gravity waves may become important. Their interaction with meridional circulation and turbulence must be accounted for. Observational evidence for non-standard transport processes of both angular momentum and chemicals exist in the case of late-type stars, and the most striking ones are...
the lithium abundances and the helioseismic data. Main sequence F-type stars with $T_{\text{eff}} \in [6300 \text{ K}; 7100 \text{ K}]$ exhibit lithium depletion at their surface, and lie in the so-called “lithium dip” [5, 3]. On the other hand, sharp transition in rotational velocities and stellar activity also occur in the range of effective temperature of the Li dip, suggesting a strong interplay between Li abundances and the action of dynamical processes.

The other key observation is the helioseismic rotation inversion which reveals that, below the convective zone, differential rotation in latitude disappears and that the solar radiative zone rotates more or less as a solid body down to at least $r \sim 0.3 R_{\odot}$ (see Garcia et al., this volume). This suggests that efficient transport processes of angular momentum are at play in the solar radiative interior.

In short, on the main sequence, Pop I low-mass stars experience efficient enough transport of angular momentum so as to flatten out the rotation profile in the solar interior, but mild transport of chemicals that allows for moderate destruction of lithium in their convective envelope.

In the following, we present results for dynamical solar models, and illustrate how stellar evolution together with observational constraints can provide clues for the understanding of dynamical processes along the evolutionary path of stars.

3.1. Rotation

The first rotating models that were applied to the solar case used the turbulent diffusion approach. In their early work, Pinsonneault et al. [36] explored the parameter space of their formalism and adjusted the 6 main parameters to observational constraints from both the present Sun and other stars that can be used to trace the young Sun (pre-main sequence and early main sequence) and the Sun to come (sub-giant). Among these parameters, three were arbitrary factors that allow the fine tuning of (1) the transport of angular momentum with respect to chemicals efficiency ($f_c$), (2) the inhibiting effect of mean molecular weight gradients on the angular momentum evolution ($f_\mu$), (3) and the efficiency of angular momentum transport by hydrodynamical turbulence ($f_\omega$).

The authors could find a combination of parameters that reproduced the present Sun in terms of luminosity, radius, surface rotation rate and lithium abundance. However, they obtained a considerable amount of differential rotation in the inner 60% in radius of their model. Last but not least underline that they had to reduce by a factor 20 the efficiency of nuclides transport in order for their model to have the required lithium depletion. Updated versions of this work was published by Chaboyer et al. [10], yielding very similar results.

Later on during the 90’s, models using a simplified version of the meridional circulation approach were computed by Richard et al. [38]. In that work, the transport of chemicals was solved for atomic diffusion and rotation-induced mixing. The transport of angular momentum was not fully solved, but considering the asymptotic regime of the moderate wind case and given solid-body rotation, the authors could derive an estimate of the meridional circulation velocity and of the associated effective diffusion coefficient for the equation for the transport of chemicals. The best model that was obtained with this approach included transport of chemicals by atomic diffusion and simplified rotation-induced mixing, and fitted nicely the present Sun in terms of radius, luminosity, lithium and beryllium surface abundances. It was however lacking consistency, not only because angular momentum evolution was not followed, but also because of tuned mean molecular weight feedback on the chemicals transport.

Matias & Zahn [31] and Talon [40] applied for the first time the rotational mixing of type I as described in [48] in order to evaluate the transport of angular momentum
in the Sun. Assuming a solid-body rotation on the ZAMS and associated initial equatorial velocity of 100 km s\(^{-1}\), and magnetic torquing at the surface in order to have \(v_{\text{surf,}0} = 2\) km s\(^{-1}\), they found that a steep gradient of angular velocity develops in the solar interior and remains at the age of the Sun. The rotation profiles that they obtained are reproduced in Fig. 1 from Talon [40].

In the following, we adopt a similar approach, and present new results for the rotating Sun, obtained with the best available formulation for the transport of angular momentum within the framework of 1D stellar evolution. We have implemented the rotational mixing of type I in the STAREVOL V2.81 stellar evolution code. The reader is referred to Palacios et al. [34, 33] for a description of standard and non-standard inputs. We have implemented an update of the equation of state and currently use the FreeEOS-2.0.0 package by Allan Irwin\(^2\). We used the solar chemical composition by Grevesse & Noels [19], but the differential effects presented here would be little affected by the use of the new solar chemical composition. The reader is referred to N. Grevesse’s and J. Guzik’s contributions in this volume for more details concerning the new solar abundances.

The characteristics of the models presented here are given in Table 1. We adopted the following solar values: \(R_\odot = 6.9599 \times 10^{10}\) cm, \(L_\odot = 3.846 \times 10^{33}\) erg s\(^{-1}\), \(Z/X_\odot = 0.0245\). Considering the Sun as a typical G star, it might have undergone strong magnetic braking in its early years on the main sequence. We follow Kawaler [21] and use a braking law of the form \(dJ/dt \propto \Omega^2\). We start with a solid-body rotation on ZAMS, with an initial rotational velocity typical of late-type stars of 50 km s\(^{-1}\).

We then let the angular momentum evolve under the combined action of meridional circulation and turbulence in the radiative region (the convective region are assumed to have rigid rotation), so that its surface velocity reaches 2 km s\(^{-1}\) at 4.6 Gyr. Table 1 presents the main characteristics of three models: a \textit{classical} model, with no rotation and no atomic diffusion, a \textit{standard} model, with atomic diffusion only, and a \textit{rotating} model, that also includes atomic diffusion. All models reproduce the radius and luminosity of the present Sun, as well as the metal fraction for the Grevesse & Noels [19] chemical composition. In addition, the \textit{standard} model leads to a radius at the base of the convective envelope that is compatible with helioseismology.

The rotating model presents some striking features that are worth commenting with further details.

Figure 2 shows the evolution of the angular velocity profile inside our calibrated rotating Sun at different ages on the main sequence. This figure is very similar to Fig. 1: during the first 600 My spent on the main sequence, the large torque applied at the surface is very efficient at slowing down the surface convective zone. In the radiative zone, this creates a large gradient of angular velocity. As evolution proceeds, the profile changes slowly, and we can see that a fraction of angular momentum is extracted from the core. However, advection by meridional flow dominates over the turbulent diffusion, and since it is not an efficient process, so that at the age of the Sun, the gradient of angular velocity is still very strong.

This result confirms the previous calculations. The strong gradient of angular momentum is mainly built up due to the angular momentum losses at the surface. The strong extraction of angular momentum in the Sun is related to the fact that in solar-type stars, the convective envelope is deep enough so as to allow the magnetic torque to get rooted in it and to be more efficient. This is constrained by the evolution of rotation velocities of solar-type stars.

\[\Delta R/R_\odot = 4.17 \times 10^{-6}\]
\[\Delta L/L_\odot = 2.35 \times 10^{-6}\]
\[\alpha = 1.6746\]
\[\gamma = 0.27188\]
\[Z_0 = 0.01740\]
\[\langle Z/X \rangle_p = 0.01793\]
\[\langle Z/X \rangle_\odot = 0.01741\]
\[\rho_\odot (g cm^{-3}) = 1.511 \times 10^2\]
\[R_{\text{BCZ}} (R_\odot) = 0.7302\]
\[\rho_c (g cm^{-3}) = 1.526 \times 10^2\]
\[R''_c (R_\odot) = 0.7067\]

Table 1. Characteristics of our calibrated solar models at 4.6 Gyr.

| Parameter                  | Classical model | Standard model | Rotating model |
|----------------------------|-----------------|----------------|----------------|
| Atomic diffusion           | no              | yes            | yes            |
| \(v_{\text{ini}}\) (km s\(^{-1}\)) | 0               | 0              | 2 km s\(^{-1}\) |
| \(v_{\odot}\) (km s\(^{-1}\))    | 0               | 0              | 50 km s\(^{-1}\) |
| \(\Delta R/R_\odot\)          | \(4.17 \times 10^{-6}\) | \(1.2 \times 10^{-6}\) | \(2 \times 10^{-6}\) |
| \(\Delta L/L_\odot\)          | \(2.35 \times 10^{-6}\) | \(4.8 \times 10^{-5}\) | \(4.06 \times 10^{-4}\) |
| \(\alpha\)                   | 1.6746          | 1.9280         | 1.6042         |
| \(\gamma\)                  | 0.27188         | 0.28501        | 0.26668        |
| \(Z_0\)                     | 0.01740         | 0.02068        | 0.01620        |
| \(\langle Z/X \rangle_p\)    | 0.02448         | 0.02978        | 0.02259        |
| \(\langle Z/X \rangle_\odot\) | 0.2719          | 0.2521         | 0.2686         |
| \(\rho_\odot\)              | 0.01793         | 0.01789        | 0.0175         |
| \(\rho_c (g cm^{-3})\)       | 0.02461         | 0.02451        | 0.0245         |
| \(\langle Z/X \rangle_\odot\) | 0.62164         | 0.63202        | 0.60173        |
| \(\rho_c (g cm^{-3})\)       | 0.01793         | 0.02389        | 0.01669        |
| \(T_c \times 10^6 (K)\)      | 15.48           | 15.80          | 15.33          |
| \(\rho_c (g cm^{-3})\)       | 1.511 \times 10^2| 1.526 \times 10^2| 1.467 \times 10^2|
| \(R_{\text{BCZ}} (R_\odot)\) | 0.7302          | 0.7067         | 0.7405         |

\(^2\)The FreeEOS-2.0.0 package is the latest release of a FORTRAN source code developed by A. Irwin, and that is made publicly available at [http://freeeos.sourceforge.net](http://freeeos.sourceforge.net) under the GNU General Public License.
in open clusters (see Talon & Charbonnel [44]).

Let’s now examine the consequences of such a large $\Omega$-gradient on the internal stratification of the model. Table 1 indicates that, contrary to the standard solar model, in the rotating model the surface helium mass fraction increases as the model evolves on the main sequence. The large $\Omega$-gradient in the radiative interior allows indeed the shear instability to develop and become turbulent. The associated diffusion coefficient is large ($D \approx 10^2 - 10^4$ cm$^2$.s$^{-1}$) and more efficient than atomic diffusion, which is dominated by element settling in the Sun.

As diffusion is a “down gradient” process, $^4$He is transported from the inner to the outer regions. When the model reaches 4.6 Gyr, the helium gradient is smaller in the radiative zone, and the gradient that would have been built by element settling below the convective zone, has been smoothed out (see Fig. 3). The lithium abundance profile has a positive slope, and shear-induced turbulent diffusion, that dominates the transport of chemicals, induces inward transport down to regions where lithium is easily burnt. As a result of this efficient mixing, lithium is destroyed in the envelope and our rotating model is devoid of Li at 4.6 Gyr.

The diffusion of chemicals induces a modification of the opacity in the radiative zone that translates into shallower convective zone, as mentioned above.

Figures 4 and 5 present respectively the comparison between the predicted sound speed and density, and the profiles derived from helioseismic inversions. The differences between the actual Sun and the standard model are very small, mainly because of the helium gradient that appears in this model due to efficient settling below the convective zone (see Fig. 3). On the other hand, the rotating model predictions largely differ from the helioseismic profiles in the radiative region, where efficient transport of chemicals occurs. Below the convective zone, the peak in the $\delta c^2/c^2$ profile is larger than that in the classical model, and the mismatch is also larger in the inner regions. The diffusion of hydrogen and helium ($^3$He and $^4$He) modifies the nuclear energy generation, and the opacity. We find the same result as Brun [6], that is to say that diffusion of the $^3$He, which participates in the major reactions of the pp chains, increases the discrepancy between computed and inverted sound speed.

Concerning these figures, let us finally note here that in these first applications of the STAREVOL code to the solar case, we obtain a mismatch of the sound speed in the convective region that remains to be explained. Similarly, the obtained differences for the classical model appear to be large compared to what is commonly published for the Sun, but this should not affect the differences that between the rotating and the non-rotating models that we have discussed.

The rotation and the surface abundances of light elements in the Sun cannot be reproduced by rotation-induced mixing alone. This provides evidence that additional transport processes act together with wind-driven meridian circulation, to extract more angular momentum. Additional extraction of angular momentum will result in lower differential rotation, and this will inhibit the shear-induced transport of chemicals.

The internal structure of the Sun provides some clues for the possible additional processes that could achieve the extra transport of angular momentum. We mentioned in the above that magnetic torquing is stronger in this region of the Hertzsprung-Russell diagram due to the extent of the convective zone, suggesting that magnetic fields could play a non-negligible role for the transport of angular momentum in solar-type stars. On the other hand, the flux of internal gravity waves becomes important for main sequence stars with effective temperatures cooler than $T_{\text{eff}} < 6700$ K, and net transport of angular momen-
3.2. Rotation and internal gravity waves

After the first studies by Talon [40] and Kumar et al. [23] on the potential of internal gravity waves generated by Reynolds stresses at the base of the solar convective zone, Talon & Charbonnel [43] re-investigated their interplay with rotation in stars within the lithium dip (see § 2). They computed the flux of momentum associated with the waves, depending on the position of the star in the dip, and found a strong correlation between this flux and the depth of the surface convective region. In the cooler stars (with the lower mass), where lithium is observed to be less depleted, they found larger flux of momentum than in hotter stars, laying in the hot side of the lithium dip, where the flux is almost null. In these hotter stars, the rotational mixing of type I has been shown to reproduce simultaneously surface velocities, lithium and beryllium abundances [42, 34], and waves were not expected to be significant for the transport of angular momentum. On the contrary, the stars in the cool side of the dip, with effective temperatures and masses close to the solar values, could not be explained via the rotational mixing of type I alone, because of the lack of additional processes to extract angular momentum. In [43], we showed that internal gravity waves had the good properties to account for this extra-extraction of momentum. In solar-type stars, the large flux of angular momentum associated with the internal gravity waves combines with the filtering of prograde waves at the base of the convective envelope, and leads to a net extraction of angular momentum from the radiative interior. Let us stress that the actual process is efficient only because differential rotation of the radiative zone underlying the convective region affect the shear oscillation layer so as to filter the prograde waves. The Sun having a similar structure as the Pop I stars on the cool side of the lithium dip, we computed a model of 1 \( M_\odot \) taking into account the transport of angular momentum by meridional circulation, shear-induced turbulence and internal gravity waves [13]. An additional contribution to the transport of chemicals in the oscillating shear layer (see S. Talon, this volume) was also added below the convective region. As in the rotating model presented in the previous section, we considered a solid-body rotation with an initial equatorial velocity of \( v_{ZAM} = 50 \, \text{km}\,\text{s}^{-1} \). Let us stress that this is not a calibrated solar model, but that this will not affect the behaviour we find. We will present results for a calibrated solar model in a forthcoming paper (Palacios et al. [35]).
The evolution of the internal rotation profile that we obtain is presented in Fig. 6. We can see that extraction of angular momentum by internal gravity waves occurs in several stages (right panel). During the first stage, (black solid lines) the bulk of the angular momentum is extracted from the core, which is decelerated. In the following successive stages, internal gravity waves complete the flattening of the angular velocity profile, until it is almost flat when the model reaches 4.6 Gy. The extraction is less efficient as the star evolves since the differential rotation, which causes the filtering of prograde waves and allows for negative deposition of angular momentum by the retrograde waves, is less important. At the same time, the effect of meridional circulation and shear-induced turbulence is attenuated in the presence of internal gravity waves. The lithium abundance of the model decreases much less than what we presented in Fig. 5 and we get the right amount of depletion at the age of the Sun.

Despite the uncertainties on the flux of internal gravity waves, due to the uncertainties on the excitation mechanism and mixing-length description of convection, the transport by internal gravity waves is very appealing, and certainly does play a major role in main sequence low-mass stars. It also gives a consistent framework to explain the evolution of angular momentum and chemicals in stars of all masses [43, 44, 13].

3.3. Rotation and magnetic fields

The effects of magnetic torquing on the transport of chemicals and angular momentum have been studied in detail by Charbonneau & MacGregor [12], Barnes et al. [4], who managed to reproduce the internal solar rotation profile, and underline the importance of the field configuration on the derived surface abundance patterns. They did not take into account the possible dynamical instability of the field in their approach, that were recently investigated by Spruit [39]. In the Tayler-Spruit dynamo theory, azimuthal magnetic fields can be generated in the radiative interior of differentially rotating stars. The wound up of the toroidal field by the differential rotation leads to a magneto-hydrodynamical instability that contributes to the transport of angular momentum and chemicals. The formalism first derived by Spruit [39] was revised by Maeder & Meynet [27] and recently applied to the solar case by Eggenberger et al. [15]. They consider an initial solid-body rotation, with a surface equatorial velocity of 50 km.s⁻¹, as we did in the rotating model presented in the previous section, and end up with a flat rotation profile at the age of the Sun, in very good agreement with helioseismology. Yang & Bi [12] also computed solar model with rotation and magnetic transport of angular momentum and chemicals. They use the turbulent diffusion approach for the transport of angular momentum instead of the meridional circulation, and also obtain a flatter angular rotation profile when accounting for magnetic instabilities. In terms of internal rotation, the difference with non-magnetic models is however not as striking as in [17].

These recent computations are promising and show that magnetic fields are a well-behaved candidate for the additional transport of angular momentum required in the Sun in order to explain its internal rotation profile. However they rely on still controversial prescriptions (the Tayler-Spruit dynamo theory destroys the agreement between rotating models and observations in the case of massive stars [28]), and should be regarded with the appropriated caution.

4. TOWARDS A DYNAMICAL VISION OF THE HERZSPRUNG-RUSSELL DIAGRAM

In this paper, we have reviewed the main advances in stellar evolution modelling concerning the implementation of dynamical processes. Rotation and associated transport processes has been applied with success to a large number of stars. It is important to note that in the case of meridional circulation approach, where transport of angular momentum is ensured by meridional circulation and shear-induced turbulence, the same set of input physics describing the rotational transport have been successfully applied to low-mass and massive stars at different evolutionary phases. This makes us confident in the prescriptions used in what we call the rotational mixing of type I. This formalism is however essentially successful when applied to fast rotators, and one encounters some difficulties when trying to apply it to slow rotators.

In the case of the Sun, the tachocline and the flat internal rotation profile revealed by helioseismic inversions can not be accounted for with this processes alone. In solar-type stars, we have also seen that lithium depletion/preservation requires additional processes to counteract the effect of rotational mixing when the free parameters that describe turbulence are calibrated on massive stars. Other indirect observations, relying mainly on abundance anomalies and reconstruction of the global evolution of angular momentum during the evolution of low-mass and massive stars also provide clues on the action of additional transport processes and on the weaknesses of the commonly adopted prescriptions.

An outstanding effort has been done in the last years to understand the physics of magnetic fields and internal gravity waves, and their interplay with rotation. This effort led to the development of adapted formalisms as well as direct numerical simulations, and provides us with reliable tools for the interpretation of the high quality observational data to come.

We saw that, considering the Sun as a star and using different observational constraints on pre-main sequence as well as evolved late-type stars to pin down the solar evolution, has led to great improvement of our understanding of its angular momentum and surface chemical abundances evolution.

The point we want to make in this contribution is that dialogue between observations, theory and direct numerical simulations and modelling is a key ingredient to improve our understanding of stellar physics. The theoretical efforts presented in these proceedings (see S. Mathis, S.
Talon), as well as their application in models and numerical simulations represent a pool of tools that are being validated, and will have to be used in the interpretation of the missions to come.

REFERENCES

[1] Aller, L. H. & Chapman, S. 1960, ApJ, 132, 461
[2] Bahcall, J. N., Pinsonneault, M. H., & Wasserburg, G. J. 1995, Reviews of Modern Physics, 67, 781
[3] Balachandran, S. 1995, ApJ, 446, 203
[4] Barnes, G., Charbonneau, P., & MacGregor, K. B. 1999, ApJ, 511, 466
[5] Boesgaard, A. M. & Tripicco, M. J. 1986, ApJL, 302, L49
[6] Brun, A. 1998, Ph.D. Thesis
[7] Brun, A. S., Browning, M. K., & Toomre, J. 2005, ApJ, 629, 461
[8] Brun, A. S. & Toomre, J. 2002, ApJ, 570, 865
[9] Brun, A. S., Turck-Chièze, S., & Zahn, J. P. 1999, ApJ, 525, 1032
[10] Chaboyer, B., Demarque, P., & Pinsonneault, M. H. 1995, ApJ, 441, 865
[11] Chaboyer, B. & Zahn, J.-P. 1992, A&A, 253, 173
[12] Charbonneau, P. & MacGregor, K. B. 1993, ApJ, 417, 762
[13] Charbonnel, C. & Talon, S. 2005, Science, 309, 2189
[14] Christensen-Dalsgaard, J., Di Mauro, M. P., Schlattl, H., & Weiss, A. 2005, MNRAS, 356, 587
[15] Couvidat, S., García, R. A., Turck-Chièze, S., et al. 2003, ApJL, 597, L77
[16] Eddington, A. S. 1925, The Observatory, 48, 73
[17] Eggenberger, P., Maeder, A., & Meynet, G. 2005, A&A, 440, L9
[18] Endal, A. S. & Sofia, S. 1978, ApJ, 220, 279
[19] Grevesse, N. & Noels, A. 1993, Physica Scripta Volume T, 47, 133
[20] Heger, A., Langer, N., & Woosley, S. E. 2000, A&A, 528, 368
[21] Kawaler, S. D. 1988, ApJ, 333, 236
[22] Kippenhahn, R. & Thomas, H.-C. 1970, in IAU Colloq. 4: Stellar Rotation, ed. A. Slettebak, 20
[23] Kumar, P., Talon, S., & Zahn, J.-P. 1999, ApJ, 520, 859
[24] Lydon, T. J. & Sofia, S. 1995, ApJS, 101, 357
[25] Maeder, A. & Meynet, G. 2000, ARA&A, 38, 143
[26] Maeder, A. & Meynet, G. 2003, A&A, 411, 543
[27] Maeder, A. & Meynet, G. 2004, A&A, 422, 225
[28] Maeder, A. & Meynet, G. 2005, A&A, 440, 1041
[29] Maeder, A. & Zahn, J.-P. 1998, A&A, 334, 1000
[30] Mathis, S. & Zahn, J.-P. 2005, A&A, 440, 653
[31] Matias, J. & Zahn, J. P. 1997, in IAU Symp. 181: Sounding Solar and Stellar Interiors - Posters volume, ed. J. Provost & F.-X. Schneider, 103–104
[32] Meynet, G. & Maeder, A. 1997, A&A, 321, 465
[33] Palacios, A., Charbonnel, C., Talon, S., & Siess, L. 2006, A&A, 453, 261
[34] Palacios, A., Talon, S., Charbonnel, C., & Forestini, M. 2003, A&A, 399, 603
[35] Palacios, A., Turck-Chièze, S., Talon, S., & Charbonnel, C. 2007, A&A, in prep.
[36] Pinsonneault, M. H., Kawaler, S. D., Sofia, S., & Demarque, P. 1989, ApJ, 338, 424
[37] Proffitt, C. R. & Michaud, G. 1991, ApJ, 380, 238
[38] Richard, O., Vauclair, S., Charbonnel, C., & Dziembowski, W. A. 1996, A&A, 312, 1000
[39] Spruit, H. C. 2002, A&A, 381, 923
[40] Talon, S. 1997, Ph.D. Thesis
[41] Talon, S. 2005, in EAS Publications Series, ed. G. Alecian, O. Richard, & S. Vauclair, 187–196
[42] Talon, S. & Charbonnel, C. 1998, A&A, 335, 959
[43] Talon, S. & Charbonnel, C. 2003, A&A, 405, 1025
[44] Talon, S. & Charbonnel, C. 2004, A&A, 418, 1051
[45] Talon, S. & Charbonnel, C. 2005, A&A, 440, 981
[46] Turck-Chièze, S., Couvidat, S., Piau, L., et al. 2004, Physical Review Letters, 93, 211102
[47] Yang, W. M. & Bi, S. L. 2006, A&A, 449, 1161
[48] Zahn, J.-P. 1992, A&A, 265, 115