Abstract

In the color string picture with fusion and percolation the dependence of the flow coefficients $v_n$ on the transverse momentum is studied for pp collisions the LHC energy respectively. Monte-Carlo simulations are used to locate simple strings and their fused clusters. The results favorably agree with the CMS data in the region $0.2 \leq p_t \leq 3$ GeV/c appropriate for the string scenario.

1 Introduction

One of the most impressing discoveries at LHC is observation of strong azimuthal correlations in nucleus-nucleus collisions [1, 2, 3]. It can be characterized by the non-zero flow coefficients $v_n$ governing the correlation function of the azimuthal distribution of secondaries as

$$C(\phi) = A(1 + 1 + 2 \sum_{n=1}^{n} v_n \cos(n\phi)).$$

Several approaches tried to understand this effect. The simplest approach was to match the initial anisotropic distribution of participant nucleons with the following hydrodynamical evolution [4]-[6]. More sophisticated treatments tried to relate the initial anisotropy with the preasymptotic partonic distributions in models based on the Regge exchanges [7], in the Color Glass Condensate framework [8]-[12] or in the color string scenario [13]-[15].

In the latter case both the flow coefficient and angle-rapidity correlations (“ridge”) were studied averaged over transverse momenta in the region 0–4 GeV/c. In this paper we study the dependence of the flow coefficients on the transverse momenta, as experimentally observed in [3]. Color strings picture has been able to successfully explain many observable phenomena in the soft dynamics domain. One expects it to be also applicable to the flow problem in so far as one is dealing with relatively modest transverse momenta.
Note that on very general grounds in the color string framework the $p_t$ dependence of flow coefficients in AA collisions exhibits a striking scaling behavior, in a very good agreement with the data \[16, 17\].

As experimental data show, the behavior of azimuthal correlations in pp collisions is very similar to AA collisions provided one separates events of unusually high multiplicities. In the string picture this corresponds to events in which the number of strings generated in a collision is much greater than the average. Since in this scenario the dynamics totally depends on the number of strings and clusters formed in their fusion then one indeed expects to eventually find a picture quite similar to AA collisions. In our paper \[15\], in the framework of Monte-Carlo simulations, we indeed found that at the LHC energy with 50 formed strings as compared to the average 18 strings the azimuthal correlations are very similar to those for AA collisions and to a good approximation agree with the experimental data in \[18\].

In this paper we continue to study this phenomenon in a more detailed aspect. Namely we study the $p_t$ dependence of the flow coefficients $v_n$ as calculated by Monte-Carlo simulations in the color string framework. Our results can be compared with the recent data on $v_2(p_t)$ published in \[19\]. We find a rather good agreement, which we interpret as confirmation of the validity of the color string approach to soft phenomena.

## 2 String picture

The color string model was proposed some time ago to describe multiparticle production in the soft region. Its basic ideas can be found in original papers and in a review \[20, 21\]. Later the version of the model with string fusion and percolation \[22\], in line with the ideas proposed in \[23\], was suggested. Its application to the flow problem was developed in our previous paper \[14\]. Here we only reproduce the main points necessary to understand the technique. It is assumed that in a high-energy collision between the partons of the participants color strings are stretched, which may be visualized as a sequence of $q\bar{q}$ pairs created from the vacuum or alternatively as a strong gluonic field generated by the participant partons. The strings are assumed to possess a certain finite dimension in the transverse space related to confinement. Each string then breaks down in parts several times until its energy becomes of the order of GeV and it becomes an observed hadron. The number of strings in the interaction area depends on the total available energy and partonic structure of the colliding particles: it grows with energy and atomic number. When the number of strings is small they occupy a small part of the whole interaction area like drops of liquid at considerable distance from one another. However when the number of string grows they begin to overlap and fuse giving rise to strings with more color and covering more space in the interaction area. At a certain critical density strings begin to fuse forming clusters of the dimension comparable to that of the interaction area (string percolation). The basic assumptions which lie at the basis of the color string picture are supported by its very successful application to multiparticle production in the soft region. It describes well the multiplicity and transverse momentum distributions and many other details of the particle spectra. The color string picture has a certain similarity with the saturation (Color Glass Condensate or Glasma) models, where the dynamics is explained by the classical gluon field stretched between the
colliding hadrons. The effective number of independent color sources in string percolation can be put in correspondence with the number of color flux tubes in the Glasma. It is found that they indeed have the same energy and number of participants dependence. As a consequence predictions of both approaches for most of the observables are similar.

It is assumed that strings decay into particles ($q\bar{q}$ pairs) by the well-known mechanism for pair creation in a strong electromagnetic field. In its simplest version, the particle distribution at the moment of its production by the string is

$$P(p, \phi) = Ce^{-\frac{p_{0}^{2}}{T}}$$  \hspace{1cm} (2)

where $p_0$ is the particle initial transverse momentum, $T$ is the string tension (up to an irrelevant numerical coefficient) and $C$ is the normalization factor. However, as proposed in [14], $p_0$ is different from the observed particle momentum $p$ because the particle has to pass through the fused string area and emit gluons on its way out. So in fact in Eq. (2) one has to consider $p_0$ as a function of $p$ and path length $l$ inside the nuclear overlap: $p_0 = f(p, l(\phi))$ where $\phi$ is the azimuthal angle. Note that Eq. (2) describes the spectra only at very soft $p_0$. To extend its validity to higher momenta one may use the idea that the string tension fluctuates, which transforms the Gaussian distribution into the thermal one [24, 25]:

$$P(p, \phi) = Ce^{-\frac{p_{0}}{t}}$$  \hspace{1cm} (3)

with temperature $t = \sqrt{T/2}$. To describe the energy loss of the parton due to gluon emission one may use the corresponding QED picture for a charged particle moving in the external electromagnetic field [26]. This leads to the quenching formula [14]

$$p_0(p, l) = p\left(1 + \kappa p^{-1/3}T^{2/3}l\right)^{3},$$  \hspace{1cm} (4)

with the quenching coefficient $\kappa$ to be taken from the experimental data. We adjusted $\kappa$ to give the experimental value for the coefficient $v_2$ in mid-central Pb-Pb collisions at 5-13 TeV GeV, integrated over the transverse momenta, which gives $\kappa = 0.48$.

Of course the possibility to use electrodynamic formulas for the chromodynamic case may raise certain doubts. However in [27] it was found that at least in the $N = 4$ SUSY Yang-Mills case the loss of energy of a colored charge moving in the external chromodynamic field was given by essentially the same expression as in the QED.

### 3 Calculations

The general scheme of calculations repeats the one presented in our previous papers dedicated to flow coefficients [14, 15]. So here we only briefly describe the main points.

For a particular event, that is, for a given string configuration with a fixed azimuthal angle $\phi_0$ of the impact parameter vector, the inclusive cross-section to produce a particle with a given transverse momentum and azimuthal angle $p_t$ and $\phi$ at fixed rapidity is

$$I^t(p_t, \phi) = A^t(p_t) + 2\sum_{n=1}^{\infty} \left(B_n^t(p_t) \cos n(\phi - \phi_0) + C_n^t(p_t) \sin n(\phi - \phi_0)\right)$$
\[ A^e(p_t) = A^e(p_t) \left[ 1 + 2 \sum_{n=1}^{\infty} \left( a_n^e(p_t) \cos n\phi + b_n^e(p_t) \sin n\phi \right) \right]. \] (5)

The flow coefficients for this event are given by
\[ v_n^e(p_t) = \left[ (a_n^e)^2(p_t) + (b_n^e)^2(p_t) \right]^{1/2}. \] (6)

The experimentally observed flow coefficients, \( v_n(p_t) \), are obtained after averaging over different events
\[ v_n(p_t) = \langle v_n^e(p_t) \rangle. \] (7)

Integrating \( I^e(p_t, \phi) \) over \( p_t \) in a certain interval one obtains the integrated inclusive distribution for the event depending only on \( \phi \). It can also be presented as
\[ I^e(\phi) = A^e \left[ 1 + 2 \sum_{n=1}^{\infty} \left( a_n^e \cos n\phi + b_n^e \sin n\phi \right) \right]. \] (8)

and the integrated flow coefficients \( v_n \) are obtained after averaging
\[ v_n = \langle \left[ (a_n^e)^2 + (b_n^e)^2 \right]^{1/2} \rangle. \] (9)

The observed integrated single inclusive cross section, obtained after averaging does not depend on \( \phi \):
\[ I = \langle I^e(\phi) \rangle = \langle A^e \rangle. \] (10)

If we neglect correlations for emissions from a single string, then the double inclusive cross-section for an event, integrated over the transverse momenta of both observed particles, is just the product of two \( I^e(\phi) \), Eq. (8). Its averaging over events gives the experimentally observed double inclusive cross-section
\[ I_2(\phi_{12}) = \langle A^e^2 \left( 1 + 2 \sum_n v_n^e \cos n\phi_{12} \right) \rangle. \] (11)

where \( \phi_{12} = \phi_1 - \phi_2 \). The correlation function
\[ C(\phi_{12}) = \frac{I_2(\phi_{12})}{I^2} - 1. \]

is then
\[ C(\phi_{12}) = \frac{1}{I^2} \left( 2A^2 \sum_{n=1}^{\infty} v_n^e \cos n\phi_{12} + A^2 - \langle A \rangle^2 \right). \] (12)

In the Monte-Carlo technique, in the framework of the color string approach, at each simulation one throws strings onto the interaction area and determines the event distribution \( I^e(p_t, \phi) \) and the related event flow coefficients \( v_n^e(p_t) \). Successive simulations and the following averaging give the observed flow coefficient \( v_n(p_t) \), the integrated ones \( v_n \) and correlation function \( C(\phi_{12}) \). Note that the experimental data mostly assume multiplying \( C(\phi) \) by the multiplicity minus unity, that is normalizing the correlation function according to
\[ C(\phi) \rightarrow \langle A - 1 \rangle C(\phi). \] (13)
Table 1: Integrated flow coefficients \( v_n \) for minimum bias events and events with a triple multiplicity and with temperature and Schwinger distributions in \( p_t \)

| \( n \) | min.bias | temperature | Schwinger |
|------|---------|-------------|-----------|
| 1    | 0.2528E-01 | 0.1901E-01 | 0.1878E-01 |
| 2    | 0.2884E-01 | 0.2308E-01 | 0.2240E-01 |
| 3    | 0.2470E-01 | 0.1668E-01 | 0.1648E-01 |
| 4    | 0.1941E-01 | 0.9273E-02 | 0.9024E-02 |
| 5    | 0.2023E-01 | 0.1130E-01 | 0.1115E-01 |
| 6    | 0.1869E-01 | 0.2775E-02 | 0.2671E-02 |
| 7    | 0.2007E-01 | 0.5928E-02 | 0.5855E-02 |
| 8    | 0.1854E-01 | 0.1343E-02 | 0.1254E-02 |
| 9    | 0.1801E-01 | 0.3718E-02 | 0.3669E-02 |
| 10   | 0.1824E-01 | 0.7131E-03 | 0.6819E-03 |
| 11   | 0.1912E-01 | 0.4231E-02 | 0.4177E-02 |
| 12   | 0.1828E-01 | 0.1558E-02 | 0.1477E-02 |
| 13   | 0.2108E-01 | 0.3833E-02 | 0.3787E-02 |
| 14   | 0.2036E-01 | 0.3478E-03 | 0.3344E-03 |
| 15   | 0.1633E-01 | 0.2464E-02 | 0.2437E-02 |
| 16   | 0.1511E-01 | 0.4720E-03 | 0.4425E-03 |

Presenting our results we use this normalization.

Following these lines a Monte-Carlo code was developed for proton-proton collisions. Each colliding nucleon was presented as a disk of the typical nucleon radius 0.8 fm and with the matter distributed inside according to the Gaussian density.

From \[31\] at 5-13 TeV the average number of strings is approximately 18. Our results for energies in this interval are very similar. Calculations have shown that for such small number of strings fluctuations are quite strong, so that reliable results can be obtained after no less that 1000 simulations. In Fig. 1 we show coefficients \( v_n(p_t) \) as functions of \( p_t \). Approximating the behavior of \( v_n(p_t) \) with the growth of \( p_t \) as \( A p_t^\alpha \) we find \( \alpha_n = 0.59, 0.70, 0.77 \) and 0.83 for \( n = 2, 3, 4, 5 \) respectively. The flow coefficients \( v_n \) corresponding to integration in the interval \( p_t < 4 \text{ GeV/c} \) are shown in Table 1. in columns 2-4 . In the second column we present \( v_n \) for minimum bias events with the temperature distribution \[3\]. It is remarkable that they do not diminish with \( n \) in contrast to the events with triple multiplicity (third and fourth columns). In Fig. 2 we present the correlation coefficient \( C(\phi) \) for these minimal-bias events. All the \( \phi \) dependence is found to be collimated to quite small angles \( \phi \leq 10^\circ \). Note that in this case the constant term \((< A^2 > - < A >^2)/ < A >^2 \) dropped in Fig. 2 is of the order unity, so that the ridge turns out to be only a small ripple against a constant background.

Next, following the experimental observations, we studied rare cases in which the multiplicity is three or more times greater than the average. The maximal number of string is then found to be 50. The resulting dependence of the flow coefficients \( v_n(p_t) \) on \( p_t \) is
Figure 1: Flow coefficients $v_n(p_t)$ for pp collisions at 5-13 TeV with average multiplicity

Figure 2: Correlation coefficient $C(\phi)$ for pp collisions at 5-13 TeV with average multiplicity
Figure 3: Flow coefficients $v_n(p_t)$ for pp collisions at 5-13 TeV with triple multiplicity.

Figure 4: Flow coefficient $v_2(p_t)$ for pp collisions with triple multiplicity compared to the experimental data at 5 TeV (lower) and 13 TeV (higher) from [19].

shown in Fig. 3. Taking again $v_n(p_t) \simeq A p_t^{\alpha n}$ we now have $v_n = 0.65, 0.79, 0.80$ and $0.82$ for $n = 2, 3, 4, 5$. So the rise of $v_n$ with $p_t$ on the whole turns out to be similar to the minimum bias events. In Fig. 4 we compare our results for $v_2(p_t)$ with the experimental data from [19], measured for two energies 5 and 13 GeV/c (which are not very different). As we see our results agree rather well with both sets of data. The flow coefficients $v_n$ corresponding to integration over $p_t > 4$ GeV/c are shown in Table 1. in the third column. The correlation coefficient is presented in Fig. 5. As one observes the correlation coefficient then becomes quite similar to the one in AA collisions in good agreement with the experimental findings in [18]. Still the dropped constant term is again of the order unity, so that the ridge stands on a large constant pedestal.

Just for comparison we present the coefficients $v_n(p_t)$ calculated with the original Schwinger distribution (2) in Fig. 6. The found coefficients rise to their maximum much faster than observed experimentally. The corresponding integrated $v_n$ are shown in Table in the fourth column. Curiously they are practically the same as for the temperature distribution, which
Figure 5: Correlation coefficient $C(\phi)$ for pp collisions at 5 and 13 TeV with triple multiplicity compared to the experimental data from [18] (with the ZYAM procedure at positive $\phi$).

Figure 6: Flow coefficients $v_n$ for pp collisions at 5-13 TeV with triple multiplicity, calculated with the Schwinger distribution [2].

obviously testifies that the Schwinger distribution describes emission at low momenta dominating the integrated quantities quite well. It fails to work at medium momenta, which is evident from the comparison of Figs. 3 and 6. As a result the correlation coefficient calculated with the Schwinger distribution is practically identical with the one calculated with the temperature distribution shown in Fig. 5.

4 Conclusions

Within the color string scenario via Monte-Carlo simulation the flow coefficients for proton-proton collisions at 5-13 TeV were calculated both as functions of the transverse momentum and integrated over these in the interval $0 < p_t < 4$ GeV/c. The found $v_2(p_t)$ and correlation coefficient favorably agree with the recent experimental data. It remains to be seen in which
degree the obtained agreement depends on the assumed form of quenching. This may shed light on the behavior of \( v_n(p_t) \) in AA collisions, which experimentally shows a remarkable simple dependence of \( \alpha_n = n/3 \) in the parametrization \( v_n(p_t) \propto p^{\alpha_n} \) [30].

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