INTRODUCTION

Energy consumption is continuously increasing worldwide and thus, in the sense of sustainability and environmentalism, focus on renewable energy sources has been strongly enhanced. As far as the exploitation of solar energy is concerned, the electricity-generating capacity of photovoltaics (PV) has experienced a considerable growth over the last decades and the European Union established the objective of a 12% share of its total electricity demand until 2020.

The fundamental building block of any PV-system is the solar cell. Being basically a diode whose p-n junction is exposed to light its functioning is explained in detail by semiconductor theory. For practical purposes however, the microscopic processes are modeled by equivalent circuit diagrams allowing to obtain the cell’s current-voltage characteristics (IV-curve) with sufficient accuracy and within reasonable calculational effort. Determining the parameters of the single cell’s circuit model is essential for evaluation, dimensioning, and manufacturing of PV-modules and entire PV-systems. Moreover, the exact knowledge of the model parameters allows to draw conclusions about inner cell processes and can serve as starting point for further research and cell optimization.

In the literature, two lumped circuit models are prevalent, the single-diode model and the two-diode model. Obtaining the respective model parameters by fitting measured data points to the theoretical IV-curve is aggravated by...
the fact that these IV-curves are given by nonlinear, implicit equations. Various refined techniques have been developed, reviewed in Refs.14 requiring full IV-curve data in the majority of cases. In some approaches,8,9,15 the parameters are directly calculated using only selected data points. However, these methods necessitate the knowledge of slopes at short-circuit current8,9 and at open-circuit voltage,15 an information that is usually not provided by the manufacturer. In case of the single-diode model, methods to determine the model parameters from the limited information provided by manufacturer’s data sheets have already been developed,16,17 but require the solution of high-dimensional nonlinear equation systems. Here, we focus our investigations on the double-diode model, since for conventional silicon cells, it provides better accuracy in fitting measured data, especially in the vicinity of the maximum power point.18,19 A first scheme for the double diode parameter extraction relying solely on data points available from cell data sheets (short circuit current, open circuit voltage, current and voltage at maximum power point) was presented by Refs.,11 but suffers from some impracticalities and incompleteness of the analysis.

We reduce the two-dimensional equation system of11 to a single equation and can thus specify what is possible to gain from the limited informations of the data sheet with certainty and where approximations become necessary. We present different methods to extract the double diode model parameter set from the key data points and show that for present day high-quality cells the reliability of the newly derived extraction schemes surpasses the approach of.11

The paper is organized as follows: First, the double-diode model is recapitulated and the equations derived from the data sheet values are solved. Then, approximations are presented which allow to uniquely determine the double-diode parameters. In the next sections, the methods are illustrated in a case study and tested for further real cells with known double-diode parameters under varying temperature and illumination conditions and for numerically generated parameter sets. Conclusive remarks and acknowledgments complete the contribution.

2 | DOUBLE-DIODE EQUATION

In Figure 1, the equivalent circuit of the double-diode model is shown. From Kirchhoff’s nodal rule, the basic equation relating the cell’s output current I and voltage V is given by

\[ f(I, V) = I + I_1 \left( \frac{V}{n_1 k T} - 1 \right) + I_2 \left( \frac{V}{n_2 k T} - 1 \right) \]

where \( I_{ph} \) denotes the photo current, \( I_a \) and \( n_1 \) stand for the saturation current and ideality factor of the first diode, \( I_s \) and \( n_2 \) stand for the saturation current and ideality factor of the second diode, \( R_s \) and \( R_{sh} \) account for serial and parallel resistances, and \( V_T \) is the thermal voltage defined as \( V_T = k_B T / e \) where \( k_B \) is the Boltzmann constant (~1.38×10⁻²³ J/K), \( e \) is the elementary charge (~1.6×10⁻¹⁹ C), and \( T \) is the p-n junction’s absolute temperature. The first diode models the diffusion current and the second diode takes recombination currents into account. Thus, according to Shockley’s diffusion theory,20,21 we can set the ideality factors \( n_1 = 1 \) and \( n_2 = 2 \) in Equation (1). The possibility of directly relating physical processes inside the cell and corresponding efficiency loss mechanisms with model parameters is another reason for choosing the more elaborate double-diode model instead of the single-diode model.

The remaining five parameters \((R_s, R_{sh}, I_{ph}, I_s, I_a)\) shall now be recovered from data sheet information only, that is, from short-circuit current \( (I_{sc}) \), open-circuit voltage \( (V_{oc}) \) and current and voltage at maximum power point \( (I_m \text{ and } V_m) \). These parameters fulfill Equation (1) such that

\[ f(0, V_{oc}) = 0 \]
\[ f(I_m, V_m) = 0 \]
\[ f(I_{sc}, 0) = 0 \]

from which we can evaluate

\[ I_{ph} = \frac{V_{oc}}{R_{sh}} + I_a (A_{oc}^2 - 1) + I_s (A_{sc} - 1) \]

\[ I_s = \frac{a - I_{sc} (A_{oc} - A_{sc})}{A_{oc}^2 - A_{sc}^2} \]

\[ I_{sc} = \frac{b (A_{oc}^2 - A_{sc}^2) - a (A_m^2 - A_{sc}^2)}{(A_m - A_{oc}) (A_m - A_{sc}) (A_{sc} - A_{oc})} \]

where

\[ A_{oc} = \exp \left( \frac{V_{oc}}{2V_T} \right), \quad A_{sc} = \exp \left( \frac{I_{sc} R_s}{2V_T} \right) \]
\[ A_m = \exp \left( \frac{V_m + I_m R_s}{2V_T} \right) \]  
(9)

\[ a = I_{sc} \left( 1 + \frac{R_s}{R_{sh}} \right) - \frac{V_{oc}}{R_{sh}} \]  
(10)

\[ b = (I_{sc} - I_m) \left( 1 + \frac{R_s}{R_{sh}} \right) - \frac{V_m}{R_{sh}} \]  
(11)

Thus, using equations (2-4), three of the five parameters have been eliminated and \( I_{ph}, I_{s1}, \) and \( I_{s2} \) can be expressed as functions of \( R_s \) and \( R_{sh}. \) From \( P = I \cdot V, \) the derivative of \( I(V) \) at the maximum power point (MPP) can be deduced

\[ \frac{dP}{dV} = \frac{dI}{dV} \bigg|_{MPP} = -\frac{I_m}{V_m} \]  
(12)

yielding the fourth condition contained in the data sheet values

\[ \frac{d(I, V)}{dV} \bigg|_{MPP} = 0 \]  
(13)

Implicit differentiation of Equation 1 gives

\[ \frac{dI}{dV} = -\frac{D}{1 + R_s D} \]  
(14)

with

\[ D = \frac{1}{R_{sh}} + \frac{I_{s1}}{V_T^2} e^{\frac{V_s + I_{s1}}{V_T}} + \frac{I_{s2}}{2V_T} e^{\frac{V_s + I_{s2}}{2V_T}} \]  
(15)

Inserting the values at MPP, condition Equation 13 reads

\[ \frac{I_m}{V_m} = \left( 1 - R_s \frac{I_m}{V_m} \right) \left( \frac{1}{R_{sh}} + \frac{I_{s1} A_m^2 + I_{s2} A_m}{2V_T} \right) \]  
(16)

where \( A_m \) is taken from Equation 9. Using \( I_{s1} \) (Equation 6) and \( I_{s2} \) (Equation 7) in Equation (16) it is possible to express \( R_{sh} \) as

\[ R_{sh} = \frac{1 + q + s}{I_m e^{I_{s1}} - p - r} \]  
(17)

where

\[ q = \frac{h A_m^2}{V_T}, s = \frac{l A_m}{2V_T}, p = \frac{g A_m^2}{V_T}, r = \frac{k A_m}{2V_T} \]  
(18)

\[ c = 1 - R_s \frac{I_m}{V_m} \]  
(19)

\[ h = \frac{d - (A_{oc} - A_{sc}) l}{A_{oc}^2 - A_{sc}^2} \]  
(20)

\[ l = \frac{e (A_{oc}^2 - A_{sc}^2) - d (A_m^2 - A_{sc}^2)}{N} \]  
(21)

\[ d = I_{sc} R_s - V_{oc}, \quad e = (I_{sc} - I_m) R_s - V_m \]  
(22)

\[ N = (A_m - A_{oc}) (A_m - A_{sc}) (A_{sc} - A_{oc}) \]  
(23)

\[ g = \frac{I_{sc} - (A_{sc} - A_{oc}) k}{A_{oc}^2 - A_{sc}^2} \]  
(24)

\[ k = \frac{(I_{sc} - I_m) (A_{oc}^2 - A_{sc}^2) - I_{sc}^2 (A_m^2 - A_{sc}^2)}{N} \]  
(25)

With the above abbreviations, we can write \( I_{s1} \) and \( I_{s2} \) as

\[ I_{s1} = g + \frac{h}{R_{sh}}, \quad I_{s2} = k + \frac{l}{R_{sh}} \]  
(26)

which proves to be beneficial when using the algorithm in practice. Note, that \( R_{sh} \) (Equation 17) is a function of \( R_s \) only which also holds for \( I_{s1}, I_{s2}, \) and \( I_{ph} \) if \( R_s \) is inserted. Thus, as to be expected, four of the five unknown parameters are determined by the conditions imposed by Equations (2-4) and Equation 13.

To delimit the possible range of the remaining free parameter \( R_s \) additional physical constraints can be exploited. Diode saturation currents and ohmic resistances have positive values. Thus, the zeros of \( R_{sh}, I_{s1}, \) and \( I_{s1} \) (being functions of \( R_s \)) have to be determined.

We start with \( I_{s1} \) and search for a root in the interval \( R_s \in [0, (V_{oc} - V_m)/I_m]. \) The choice of the upper bound can be argued from

\[ R_s < -\frac{1}{P(V_{oc})} < \frac{V_{oc} - V_m}{I_m} \]  
(27)

where the first inequality is derived from Equation 14 recognizing that \( D \) is a positive number. The second inequality holds since the tangent of \( I(V) \) at \( V_{oc} \) is steeper than the secant intersecting \( (V_m, I_m) \) and \( (V_{oc}, 0). \) The obtained root constitutes an upper limit \( R_{sUPP} \) for \( R_s, \) since \( I_{s1} \) becomes negative when \( R_s \) is further increased. Alternatively, the case \( I_{s1} = 0 \) can be seen as switching to the single-diode model (with diode ideality factor \( n = 1 \)) whose \( R_s \) must be larger than the serial resistance of the double-diode model to result in the same circuit parameters \( (V_{oc}, I_m, V_m, I_m). \)

Since \( I_{s1} \) monotonously increases for \( R_s \in [0, R_{sUPP}] \) checking the sign of \( I_{s1}(0) \) is sufficient to ensure the positivity of \( I_{s1}. \) Only if \( I_{s1}(0) < 0 \) a root exists which has to be determined numerically yielding a lower limit \( R_{sLOW} \) for \( R_s, \) otherwise \( R_{sLOW} = 0. \)

In most cases, \( I_{s1} \) does not supply a lower limit different from zero. A restriction coming into effect more frequently is provided by the shunt resistance’s positivity. The sign change of \( R_{sh} \) does not happen continuously but at a pole of \( R_{sh}. \) Therefore, the root of the inverse \( 1/R_{sh} \) has to be determined
within $[R_s^\text{low}, R_s^\text{ppp}]$. The obtained value of $R_s$ serves as new lower bound $R_s^\text{low}$ for the actual double diode model’s $R_s$. If there’s no root in $[R_s^\text{low}, R_s^\text{ppp}]$ the lower bound for $R_s$ remains unaltered.

Since the positivity of the other parameters is ensured for $R_s \in [R_s^\text{low}, R_s^\text{ppp}]$, the photo current $I_{ph}$ fulfills $I_{sh} > I_s$ as can be checked from Equation 4. $I_{ph}$ therefore provides no further restriction on the possible range of $R_s$.

### 3 | APPROXIMATE CONDITIONS

Any serial resistance $R_s \in [R_s^\text{low}, R_s^\text{ppp}]$ together with the corresponding $R_{sh}$ (Equation 17), $I_{s1}$ (Equation 7), $I_{s2}$ (Equation 6), and $I_{ph}$ (Equation 5) forms a physically meaningful set of double diode parameters reproducing the input circuit parameters $(V_{oc}, I_{sc}, V_m/I_m)$. Without further consideration, one could therefore readily chose for example the arithmetic mean $R_s^\text{half} \equiv (R_s^\text{ppp} + R_s^\text{low})/2$ as $R_s$ to be consistent with the data sheet specifications. We will call this approach $R_s^\text{half}$ method.

For reasons that will be explained in more detail in chapter 1, we will also investigate the parameter set arising from the lowest possible serial resistance which will consequently be called $R_s^\text{low}$-method.

Aside from randomly choosing a serial resistance within $[R_s^\text{low}, R_s^\text{ppp}]$, one could also try to find an additional, fifth condition to determine $R_s$. In $^{11}$ an approximate expression for the slope of the IV-curve at $V = 0$ is used

$$\frac{dl}{dV}|_{V=0} \approx -\frac{1}{R_{sh}}$$

which can be applied to Equation 14 to yield the condition

$$0 = (R_{sh} - R_s) \left( \frac{1}{R_{sh}} + \frac{I_{s1}}{V_T} e^{\frac{I_{s1}}{V_T}} + \frac{I_{s2}}{2V_T} e^{\frac{I_{s2}}{2V_T}} \right) \equiv C^{1/R_s}$$

Inserting the explicit expressions of $R_{sh}$ (Equation 17), $I_{s1}$ (Equation 6), and $I_{s2}$ (Equation 7) gives a nonlinear equation for $R_s$ whose solution uniquely determines the double diode model parameter set.

It is worth noting that the determination of the double diode parameters using Equation 28 gives the same numerical values as the method of $^{11}$ but the algorithm of the present work differs in various aspects. In $^{11}$ $R_{sh}$ is not explicitly expressed as function of $R_s$ from Equation (16), but a two-dimensional equation system for $R_{sh}$ and $R_s$, consisting of Equations (16) and (28) is solved with the Newton-Raphson method. Finding convergent initial values for both $R_{sh}$ and $R_s$ is therefore essential and requires some effort and case distinctions. We prefer the single equation for $R_s$ whose roots are restricted to a finite range by physical considerations and can then determined by bracketing procedures (e.g., bisection or Brent’s method).

The authors of $^{11}$ also use the approximation $e^{V_{oc}/(n_i V_T)} \gg e^{R_s I_{ph}/(n_i V_T)}$, $i = 1, 2$ from $^{15}$ which considerably simplifies the expressions for $I_{sh}$, $I_s$, and Equations (16) and (28). Since the nonlinear equation for $R_s$ anyway can only be solved numerically we do not apply these approximations. Moreover, though not mentioned in Ref. $^{11}$ the two-dimensional equation system exhibits a trivial solution for $R_s = (V_{oc} - V_m)/I_m$ and arbitrary $R_{sh}$ which has to be considered an artifact of the above approximations.

Determining a unique value of $R_s$ relied on demanding the approximate condition Equation (28) to be exactly fulfilled. In what follows, we will refer to this approach as $1/R_{sh}$-method. One may assume that other approximate conditions can also be used to determine $R_s$. We therefore consider the following:

In the double-diode model as described by Equation (1), the IV-curve $I(V)$ and its derivative $I'(V)$ are monotonously decreasing. Thus, the slope of a secant is bounded by the slope of the tangents at its intersection points, that is, for $V_1 < V_2$

$$I'(V_1) \leq \frac{I(V_1) - I(V_2)}{V_1 - V_2} \leq I'(V_2)$$

(30)

In the nearly linear regime of the IV-curve around $V = 0$ we can approximate

$$\frac{I(V_1) - I(V_2)}{V_1 - V_2} \approx \frac{I'(V_1) + I'(V_2)}{2}$$

(31)

For $V_1 = -I_{ph} R_s$, $I(-I_{ph} R_s) = I_{ph}$ and $V_2 = 0$, $I(0) = I_{sc}$ we get

$$\frac{I_{ph} - I_{sc}}{-I_{ph} R_s} \approx \frac{I'(0) - I_{ph} R_s}{2}$$

(32)

Although this approximation is usually very well fulfilled for all $R_s \in [R_s^\text{low}, R_s^\text{ppp}]$, it turns out that reaching an exact equality is often impossible. However, exchanging $I_{ph}$ in the denominator of the left hand side with $I_{sc}$ results in a solvable condition for $R_s$ within the physically allowed range

$$0 = \frac{I'(0) - I_{ph} R_s}{2} \equiv C^{2 \tan}$$

(33)

which we denote $2\text{tangs}$-method.

Using Equation (14) the derivatives are explicitly given by

$$I'(-I_{ph} R_s) = \frac{-N_1}{1 + R_s N_1}, \quad I'(0) = \frac{-N_2}{1 + R_s N_2}$$

(34)

with

$$N_1 = \frac{1}{R_{sh} + \frac{I_{s1}}{V_T}} + \frac{I_{s2}}{2V_T}$$

(35)
Inserting these expressions into Equation (33) together with the formulas for \( R_{\text{sh}} \) (Equation 17), \( I_{s1} \) (Equation 6), \( I_{s2} \) (Equation 7), and \( I_{\text{ph}} \) (Equation 5) we obtain a nonlinear equation determining \( R_s \).

It has to be noted that different approximations involving the derivative of the IV-curve, for example simply equating the formulas for \( \frac{1}{V} \) together with \( \frac{1}{V} \) resulting in condition Equation (33) yielded the best results when testing the algorithms.

4 | CASE STUDY

To illustrate and compare the parameter extraction schemes, we apply them to the S’tile sunrays quarter cell, a \( 39 \times 156 \) mm\(^2\) multicrystalline silicon PV cell. The IV-curve consisting of about 60 points (see Figure 2) was recorded under standard testing conditions (STC, cell temperature of 25°C, irradiance of about 60 points (see Figure 2) was recorded under standard testing conditions (STC, cell temperature of 25°C and an irradiance of 1000 W/m\(^2\) with an air mass 1.5 spectrum) and has been kindly provided by the S’tile company, Poitiers, France.

From the full curve we can extract the double diode parameters by a Levenberg-Marquart fit. All data points are equally weighted in the fit routine since their measurement uncertainties can be assumed to be of the same order of magnitude. The optimal parameter set is thus determined by minimizing the quadratic distances of the theory curve to the measured data points. The normalized root mean square error percentage (nRMSE [%]) is given by

\[
\text{nRMSE} = \frac{1}{N} \sum_{i=1}^{N} (t_i - m_i)^2 \times 100
\]

where \( N \) is the number of data points, \( m_i \) are the measured values and \( t_i \) the theoretically expected values. As shown in Table 1, the double diode parameters obtained from the fit routine indeed give the lowest nRMSE. From these optimal double diode parameters, the circuit parameters \( (V_{\text{oc}}, I_{\text{sc}}, V_{\text{m}}, I_{\text{m}}) \) can be calculated and used as input for the extraction algorithms. The quality of the different extraction methods will be judged by how close each of them can reproduce the underlying “optimal” double diode parameters.

By inserting the circuit parameters \( (V_{\text{oc}}, I_{\text{sc}}, V_{\text{m}}, I_{\text{m}}) \) into Equations (26), (17), and (5), \( I_{s1}, I_{s2}, R_{\text{sh}}, \) and \( I_{\text{ph}} \) can be expressed as functions of \( R_s \) only. In Figure 3, we display the graphs of these functions. \( I_{s2} \) is plotted from zero to \( (V_{\text{oc}} - V_{\text{m}})/I_{\text{m}} = 57.49 \) m\( \Omega \) and exhibits a root at \( R_s = 19.11 \) m\( \Omega \) serving as upper bound \( R_{\text{sup}} \). \( I_{s1} \) is positive within the range \([0, R_{\text{sup}}]\) and does therefore not contribute to a restriction on \( R_s \). The shunt resistance \( R_{\text{sh}} \) however possesses a pole at \( R_s = 13.28 \) m\( \Omega \) which defines the physical lower bound \( R_{\text{low}} \).

Remarkably, \( R_{\text{sh}} \) decreases with increasing \( R_s \) within the physically allowed range. Low serial/high shunt resistance pairs yield the same circuit parameters and the same fill

**FIGURE 2** IV-curve (data points + fit), circuit parameters, fill factor (FF) and efficiency (N) of the case study cell (S’tile sunrays quarter cell) at standard testing conditions

**TABLE 1** Double diode parameters, root mean square deviations from the measured data points, and error measures \( E_1 \) and \( E_2 \) of the different extraction schemes applied to the S’tile quarter cell

| Parameters | Full IV-curve | \( R_{\text{s sup}} \) | \( 1/R_{\text{sh}} \) | 2tangs | \( R_{\text{low}} \) |
|------------|---------------|------------------|-----------------|--------|----------------|
| \( R_s \) [m\( \Omega \)] | 14.00 | 16.19 | 16.80 | 14.20 | 13.28 |
| \( R_{\text{sh}} \) [\( \Omega \)] | 103.3 | 21.8 | 17.2 | 79.3 | \( \infty \) |
| \( I_{\text{ph}} \) [A] | 2.160 | 2.161 | 2.162 | 2.160 | 2.160 |
| \( I_{\text{ph}} \) [mA] | 0.0453 | 0.0509 | 0.0529 | 0.0458 | 0.0434 |
| \( I_{\text{ph}} \) [\( \mu \text{A} \)] | 3.02 | 1.80 | 1.44 | 2.91 | 3.40 |
| Errors | nRSME [%] | 1.13 | 1.57 | 1.90 | 1.14 | 1.15 |
| | \( E_1 \) [%] | 0.0 | 29.63 | 34.54 | 5.9 | \( \infty \) |
| | \( E_2 \) [%] | 0.0 | 2.05 | 2.70 | 0.17 | 0.58 |
methods, parameter reproduction can be evaluated by the mean relative distance between derived \(p_i\) and full IV-curve (\(p_i^{\text{full}}\)) parameters which we denote as error \(E_1\)

\[
E_1 = \frac{1}{5} \sum_{i=1}^{5} \left| \frac{p_i - p_i^{\text{full}}}{p_i^{\text{full}}} \right|
\]

Since changes in the parameters are not linearly reflected by the curve we also introduce another error measure \(E_2\). It is evaluated from the difference between the IV-curves calculated from the reproduced parameters \((I(V))\) and the full curve fit \((I^{\text{full}}(V))\). The behavior near the maximum power point (roughly \(V_m \pm 10\%\)) is of particular interest, for example when calculating the maximum power point for an entire module consisting of an arbitrary number of serially or parallel connected cells. We define

\[
E_2 = \frac{1}{0.2V_m} \int_{0.9V_m}^{1.1V_m} \frac{|I(V) - I^{\text{full}}(V)|}{I^{\text{full}}(V)} \, dV
\]
reproduced parameters. For the investigated cell, the second error measure of the 2tangs-method lies one order of magnitude below the results for the $R_{sh}^{half}$- and $1/R_{sc}$-method (see last row of Table 1).

5 | EVALUATION OF THE EXTRACTION SCHEMES

5.1 | Real cell data

It cannot be guaranteed that the results of the case study are representative for the majority of PV cells. To reach a more objective conclusion we use already published double-diode model parameters\textsuperscript{10,15,22} to further test the algorithms. From these “official” double diode parameters, the electrical circuit parameters are deduced which serve as input for the extraction methods. The approximated double diode values are calculated and compared to the original ones. The general scheme is depicted in Figure 5. The two error measures $E_1$ and $E_2$ defined in the previous chapter quantify the quality of the different methods. The published double diode parameters have to be taken as reference quantities $p_i^{(full)}$ in Equation (38). The IV-curve $I^{(full)}(V)$ in Equation (39) is calculated from the published double diode parameters as well.

In\textsuperscript{10} a $2 \times 2$ cm\textsuperscript{2} silicon cell was investigated for different illumination levels (40\%, 100\%, and 140\% of AM1) and temperatures (299.4\,K, 317.5\,K, and 330\,K). The full results are given in the appendix and allow the conclusion that the qualification of an extraction method for a particular cell is not influenced by temperature or illumination conditions.

In\textsuperscript{22} double diode parameters for three different commercial cells measured under standard testing conditions are listed. In\textsuperscript{15} double diode parameters for different cells at $T = 323.15\,\text{K}$ under AM1 illumination are given. In addition, mono- and multicrystalline cells available at our research facility were measured under standard testing conditions and the double diode parameters were derived from a fitting procedure as described in the case study. The results of all these investigations for a total number of 16 cells are summarized in Figures 6 and 7.
The mean values of errors $E_1$ and $E_2$ displayed in Table 2. It can be concluded that the best parameter reproduction is achieved by the 2tangs-method. The only exception is the TL1-900-50 cell from. It is therefore investigated in detail in Appendix B revealing that the double diode parameters obtained from the 2tangs-method are practically identical to the parameters of the $R_{s}^{low}$-method. In such cases, the parameters from the 2tangs-method have to be treated with care and it is recommended to switch to the $R_{s}^{half}$ parameters. Despite the worse parameter reproduction, the deviation of the IV-curve (error measure $E_2$) is still below that of the $1/R_{sh}$-method. It can be assumed that the double diode parameter sets belonging to lower $R_s$ still give good IV-curves, even if the parameter guess is inaccurate. This observation motivated us to include the $R_{s}^{low}$-method in our investigations. The $R_{s}^{low}$-method requires just as much effort as the $R_{s}^{half}$-method since no fifth equation has to be solved. Obviously, it performs badly if quantified by error measure $E_1$ since it yields an infinite shunt resistance $R_{sh}$ for the majority of cells. Nevertheless, if the value of the parameters is irrelevant and if only a reliable IV-curve is required the quality of results is comparable to the 2tangs-method for our investigated cells (see Table 2).

5.2 | Numerically generated parameters

The number of available real cell data is limited and still rather low. A single outlier already caused a high standard deviation of the 2tangs-method’s error measure $E_1$ (see Table 2). Therefore, we also generated cells numerically. Data from the real cell are used to estimate meaningful boundaries for double diode parameters of $156 \times 156$ mm$^2$ cells under standard temperature and illumination conditions (see Table 3).

A parameter set randomly generated within these boundaries is accepted if the derived fill factor exceeds 70% as can be expected from commercially produced crystalline silicon solar cells. The corresponding circuit parameters are fed into the different parameter extraction schemes.

Histograms of the two error measures for 10 000 generated parameter sets are shown in Figures 8 and 9. Mean value and standard deviation are listed in Table 4.

The histograms and average performances allow no statements about the reliability of an extraction method for an actual parameter set. We therefore also provide probabilities for the error of an extraction scheme to lie below the error of the other schemes. In Table 5 we compare error measures $E_1$ and in Table 6 error measures $E_2$. For example, the second entry in the first row of Table 5 indicates that for 9966 of the 10 000 generated double diode parameter sets the error measure $E_1$ of the $R_{s}^{half}$-method was smaller than the error measure $E_1$ of the $1/R_{sh}$-method.

It can be concluded that the low cost $R_{s}^{half}$-method already surpasses the $1/R_{sh}$-method, both for parameter (error $E_1$) and
IV-curve (error \( E_2 \)) reproduction. The reproduction quality can be further improved using the 2tangs-method which outdoes the \( R_{\text{half}} \)-method. It has to be noted that in the rare cases (less than 1‰) where multiple roots occurred for the 2tangs-method we took the largest one for \( R_s \). Despite the good performance of the 2tangs-method, for reproducing the original IV-curve, the \( R_{\text{low}} \)-method beats all the other approaches, but behaves badly as far as parameter reproduction is concerned.

From investigating both real and numerically generated cells the following observations from the case study can be confirmed: The conditions provided by the electrical circuit parameters can be used to express \( R_{\text{sh}}, I_{\text{ph}}, I_{s1}, \) and \( I_s \) as functions of \( R_s \) whose roots can be found numerically. Within the physically allowed range of \( R_s \) thereby determined, shunt resistance and recombination current decrease with increasing \( R_s \), photo current and saturation current increase. The 2tangs-method yields a serial resistance in the lower range of the physically allowed values accompanied by high, but still realistic shunt resistance whereas the serial resistance obtained from the \( 1/R_{\text{sh}} \)-method usually lies in the upper range of the spectrum and goes along with a lower shunt resistance. Cell design is no trivial task and not all cell parameters can be independently optimized. However, shunt resistances typically arise from the non-ideal manufacturing process and grown-in material defects, rather than poor solar cell design. Therefore, the higher shunt resistances values obtained from the 2tangs-method can be assumed to be closer to the actual parameters for present-day cells.

### 6 | CONCLUSION

The present work was motivated by a straightforward task: Gaining access to the full current-voltage characteristics of a photovoltaic cell from the limited information provided by the manufacturer’s data sheet. While in case of the single diode model for photovoltaic cells a complete solution has already been published, the double diode model, which is more accurate for silicon cells, lacked a thorough analysis of this problem.

After the diode ideality factors have been set to \( n_1 = 1 \) and \( n_2 = 2 \) to account for diffusion and recombination currents respectively, the double-diode model of the PV cell consists of five parameters. The cell’s data sheet values \( V_{\text{oc}}, I_{\text{sc}}, V_{\text{m}}, \) and \( I_m \) yield four independent equations (three data points of the IV-curve and its derivative at MPP), thus allowing to express four of these parameters \((R_{\text{sh}}, I_{\text{ph}}, I_{s1}, I_s)\) as functions of the last \( (R_s) \). The positivity of all model parameters, which is a physical necessity, can be exploited to further delimit the possible range of the parameters.

Thus, if \( V_{\text{oc}}, I_{\text{sc}}, V_{\text{m}}, \) and \( I_m \) are known the formulas presented in this contribution give boundaries within which the double diode parameters must lie with certainty. These
boundaries can be used to restrict the search region or provide suitable initial values for fitting procedures if additional IV-curve data points are available.

If no additional data are available various methods can be exploited to guess the double diode parameters. In the most simple approach, the remaining parameter $R_s$ is arbitrarily chosen within the physically allowed range. Alternatively, an approximate fifth condition can be demanded to hold exactly yielding a nonlinear equation for $R_s$. The different methods are tested for cells whose double diode parameters are known, either from literature or from fits to full IV-curve data. These “approved” parameters are compared to the extracted. Two error measures have been introduced, accounting for the accuracy of either parameter or IV-curve reproduction. As benchmark for the newly introduced methods, an adapted version of an elsewhere published method relying on approximating the slope of a secant in the v-i-plot can be exploited to guess the double diode parameters.

When cells are connected to modules an exact calculation of the maximum power output requires the knowledge of the full IV-curves of the constituent single cells. The actual values of the double diode parameters are less important in this case. The parameter set gained from the lowest possible value of $R_s$ yields a further reduced curve reproduction error compared to the $2tangs$-method, though it is often accompanied by an unphysical, infinite shunt resistance. Since no additional equation has to be solved, an easily implementable, fast procedure for a reliable parameter extraction from cell data sheet values only is achieved.

When cells are connected to modules an exact calculation of the maximum power output requires the knowledge of the full IV-curves of the constituent single cells. The actual values of the double diode parameters are less important in this case. The parameter set gained from the lowest possible value of $R_s$ yields a further reduced curve reproduction error compared to the $2tangs$-method, though it is often accompanied by an unphysical, infinite shunt resistance. Since no additional equation has to be solved, an easily implementable, fast algorithm to obtain reliable IV-curves from cell data sheet values only is achieved.

ACKNOWLEDGMENTS

The authors would like to thank A. Malinge and E. Terrace from the S’tile company in Poitiers, France for providing the full IV-curve of the case study cell. This work was supported by the EU-horizon 2020 project ID 737884.

CONFLICT OF INTEREST

None declared.

ENDNOTES

1. Haegel NM, Margolis R, Buonassisi T, et al. Terawatt-scale photovoltaics: trajectories and challenges. Science. 2017;356:141-143.
2. van Overstraeten R, Mertens R. Physics, Technology and Use of Photovoltaics. Boca Raton, FL: Taylor & Francis; 1986.
3. Fahrenbruch A, Bube R. Fundamentals of Solar Cells: Photovoltaic Solar Energy Conversion. New York, NY: Academic Press; 1983.
4. Datta SK, Mukhopadhyay K, Bandopadhyay S, Saha H. An improved technique for the determination of solar cell parameters. Solid-State Electron. 1992;35:1667-1673.
5. Chegaar M, Azzouzi G, Mialhe P. Simple parameter extraction method for illuminated solar cells. Solid-State Electron. 2006;50:1234-1237.
6. Chegaar M, Ouennoughi Z, Hoffmann A. A new method for evaluating illuminated solar cell parameters. Solid-State Electron. 2001;45:293-296.
7. Easwarakhanthan T, Bottin J, Bouchouh I, Boutrit C. Nonlinear minimization algorithm for determining the solar cell parameters with microcomputers. Int J Sol Energy. 1986;4:1-12.
8. Enebish N, Aghbabayar D, Dorjkhand S, Baatar D, Ylemj I. Numerical analysis of solar cell current-voltage characteristics. Sol Energy Mater Sol Cells. 1993;29:201.
9. Hovinen A. Fitting of the solar cell IV-curve to the two diode model. Phys Scr. 1994;1994:175-176.
10. Charles JP, Bordure G, Khoury A, Mialhe P. Consistency of the double exponential model with physical mechanisms of conduction for a solar cell under illumination. J Phys D: Appl Phys. 1985;18:2261-2268.
11. Hejri M, Mokhtari H, Azizian MR, Ghandhari M, Söder L. On the parameter extraction of a five-parameter double-diode model of photovoltaic cells and modules. IEEE J Photovolt. 2014;4:915-923.
12. Protogeropoulos C, Brinkworth BJ, Marshall RH, Cross BM. Evaluation of Two Theoretical Models in Simulating the Performance of Amorphous – Silicon Solar Cells. Dordrecht: Springer Netherlands; 1991:412-415.
13. Garrido-Alzar C. Algorithm for extraction of solar cell parameters from I-V curve using double experimental model. Renewable Energy. 1997;10:125-128.
14. Cotfas D, Cotfas P, Kaplanis S. Methods to determine the dc parameters of solar cells: a critical review. Renew Sustain Energy Rev. 2013;28:588-596.
15. Chan DSH, Phang JCH. Analytical methods for the extraction of solar-cell single- and double-diode model parameters from I-V characteristics. IEEE Trans Electron Devices. 1987;34:286-293.
16. Soto WD, Klein S, Beckman W. Improvement and validation of a model for photovoltaic array performance. Sol Energy. 2006;80:78-88.
17. Boyd MT, Klein SA, Reindl DT, Dougherty BP. Evaluation and validation of equivalent circuit photovoltaic solar cell performance models. J Sol Energy Eng. 2011;133:021005.
APPENDIX A: Silicon cell under different temperature and illumination conditions

The double diode parameters in the following tables are taken from \(^{10}\) where a 2 × 2 cm\(^2\) silicon cell has been investigated under different temperature and illumination conditions. From these double diode parameters, the electrical circuit parameters \(V_{oc}, I_{sc}, V_m,\) and \(I_m\) are calculated and fed into the different extraction methods yielding new double diode parameter sets. The quality of the methods is quantified by the error measures \(E_1\) and \(E_2\) related either to parameter or to IV-curve reproduction accuracy.

### TABLE A1

Double diode parameters and error measures for the cell studied in \(^{10}\) under AM1 illumination and with \(T = 299.4\) °K. The corresponding electrical circuit parameters fed into the algorithms are \(I_{sc} = 117.5\) mA, \(V_m = 470.7\) mV, \(I_m = 108.0\) mA, and \(V_{oc} = 583.7\) mV yielding a fill factor of 74.13%.

| Literature | \(R_{s}^{half}\) [\(\Omega\)] | \(1/R_{sh}\) | 2tangs | \(R_{s}^{low}\) [\(\Omega\)] |
|---|---|---|---|---|
| Parameters | 264 | 305 | 323 | 276 | 256 |
| \(R_s\) [\(\Omega\)] | 2550 | 352 | 239 | 977 | \(\infty\) |
| \(I_{ph}\) [mA] | 117.5 | 117.6 | 117.6 | 117.5 | 117.5 |
| \(I_s1\) [nA] | 0.0129 | 0.0147 | 0.0155 | 0.0134 | 0.0126 |
| \(I_s2\) [\(\mu\)A] | 0.38 | 0.22 | 0.14 | 0.33 | 0.41 |
| Errors | 0.0 | 31.7 | 39.3 | 16.4 | \(\infty\) |
| \(E_1\) [%] | 0.0 | 1.88 | 2.84 | 0.49 | 0.31 |

### TABLE A2

Double diode parameters and error measures for the cell studied in \(^{10}\) under AM1 illumination and with \(T = 317.5\) °K. The corresponding electrical circuit parameters fed into the algorithms are \(I_{sc} = 121.3\) mA, \(V_m = 415.8\) mV, \(I_m = 109.7\) mA, and \(V_{oc} = 529.3\) mV yielding a fill factor of 71.11%.

| Literature | \(R_{s}^{half}\) [\(\Omega\)] | \(1/R_{sh}\) | 2tangs | \(R_{s}^{low}\) [\(\Omega\)] |
|---|---|---|---|---|
| Parameters | 272 | 310 | 338 | 292 | 265 |
| \(R_s\) [\(\Omega\)] | 2040 | 258 | 140 | 459 | \(\infty\) |
| \(I_{ph}\) [mA] | 121.3 | 121.4 | 121.6 | 121.4 | 121.3 |
| \(I_s1\) [nA] | 0.0352 | 0.0402 | 0.0442 | 0.0378 | 0.0344 |
| \(I_s2\) [\(\mu\)A] | 2.08 | 1.18 | 0.45 | 1.61 | 2.23 |
| Errors | 0.0 | 31.7 | 44.4 | 22.9 | \(\infty\) |
| \(E_1\) [%] | 0.0 | 1.36 | 2.60 | 0.69 | 0.21 |
For the cell studied in Ref. 10 the 2tangs-method is most suited to reproduce the double diode parameters for all investigated combinations of temperature and illumination. The most reliable reproduction of the IV-curve is obtained from the \( R_{\text{low}} \)-method. This suggests that the results from the standard testing conditions (25°C, 1000 W/m²) for a silicon cell can be extrapolated to other temperature and illumination levels.

### TABLE A3

Double diode parameters and error measures for the cell studied in Ref. 10 under AM1 illumination and with \( T = 330 \)°K. The corresponding electrical circuit parameters fed into the algorithms are \( I_{sc} = 123.3 \) mA, \( V_{oc} = 379.4 \) mV, \( I_{m} = 109.9 \) mA, and \( V_{oc} = 491.8 \) mV yielding a fill factor of 68.78%.

| Parameters | \( R_s \) [\( \mu \Omega \)] | \( R_{sh} \) [\( \Omega \)] | \( I_{ph} \) [mA] | \( I_{s1} \) [nA] | \( I_{s2} \) [\( \mu \)A] | \( E_1 \) [%] | \( E_2 \) [‰] |
|------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| Literature | 272            | 306            | 341            | 294            | 260            | 0.0            | 0.0            |
| \( R_s \)  | 910            | 200            | 99             | 288            | 0.0            | 28.9           | 45.6           |
| \( R_{sh} \)| 123.3          | 123.5          | 123.7          | 123.4          | \infty         | 0.0            | 22.0           |
| \( I_{ph} \)| 2.72           | 3.09           | 3.50           | 2.95           | 2.60           | 0.0            | 0.0            |
| \( I_{s1} \)| 6.08           | 3.65           | 9.42           | 4.55           | 6.85           | 0.0            | 0.0            |
| Errors     | 0.0            | 2.27           | 0.62           | 0.30           | 0.0            | 0.0            | 0.0            |

### TABLE A4

Double diode parameters and error measures for the cell studied in Ref. 10 with \( T = 320.8 \)°K and illumination of 140% AM1. The corresponding electrical circuit parameters fed into the algorithms are \( I_{sc} = 162.8 \) mA, \( V_{oc} = 409.2 \) mV, \( I_{m} = 145.8 \) mA, and \( V_{oc} = 532.8 \) mV yielding a fill factor of 68.76%.

| Parameters | \( R_s \) [\( \mu \Omega \)] | \( R_{sh} \) [\( \Omega \)] | \( I_{ph} \) [mA] | \( I_{s1} \) [nA] | \( I_{s2} \) [\( \mu \)A] | \( E_1 \) [%] | \( E_2 \) [‰] |
|------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| Literature | 271            | 300            | 325            | 287            | 258            | 0.0            | 0.0            |
| \( R_s \)  | 625            | 163            | 89             | 259            | \infty         | 0.0            | 0.0            |
| \( R_{sh} \)| 162.9          | 163.1          | 163.4          | 163.0          | 162.8          | 0.0            | 0.0            |
| \( I_{ph} \)| 0.47           | 0.55           | 0.61           | 0.51           | 0.44           | 0.0            | 0.0            |
| \( I_{s1} \)| 3.32           | 2.06           | 0.87           | 2.66           | 3.84           | 0.0            | 0.0            |
| Errors     | 0.0            | 27.6           | 41.9           | 18.5           | \infty         | 0.0            | 0.0            |

### APPENDIX B: TL1-900-50 cell parameters

The following double diode parameters are from a 3-in-diameter silicon cell labeled TL1-900-50 and measured in Ref. 15. Original and reproduced double diode parameters are listed in Table B1. In contrast to the other investigated cells, the 2tangs-method has the largest parameter reproduction error \( E_1 \). The value of \( R_s \) found from the 2tangs-method’s condition Equation (33) is de facto identical to the lowest possible value of \( R_s \). In that case, the double diode parameters obtained from the 2tangs-method have to be treated with care. However, the reproduction of the IV-curve in the vicinity of the maximum power point expressed by error measure \( E_2 \) is still more reliable than for the \( 1/R_{sh} \)-method.

For the cell studied in Ref. 10, the 2tangs-method is most suited to reproduce the double diode parameters for all investigated combinations of temperature and illumination. The most reliable reproduction of the IV-curve is obtained from the \( R_{\text{low}} \)-method. This suggests that the results from the standard testing conditions (25°C, 1000 W/m²) for a silicon cell can be extrapolated to other temperature and illumination levels.
| Parameter | Literature | R_{\text{sh}} [m\Omega] | \text{1/R}_{\text{sh}} | 2\text{tangs} | R_{\text{low}} |
|-----------|------------|-------------------------|----------------------|----------------|----------------|
| Rs [m\Omega] | 31.17 | 26.14 | 44.74 | 7e-4 | 0 |
| Rs [\Omega] | 19.92 | 24.80 | 10.32 | 46.41 | 46.41 |
| Is \text{[A]} | 0.9072 | 0.9067 | 0.9097 | 0.9058 | 0.9058 |
| Is \text{[nA]} | 2.466 | 2.091 | 3.564 | 0.215 | 0.737 |
| Is \text{[\mu A]} | 28.31 | 33.91 | 11.33 | 60.83 | 60.83 |

| Errors | | | |
| E_1 [%] | 0.0 | 15.13 | 39.30 | 87.86 |
| E_2 [‰] | 0.0 | 0.61 | 2.55 | 1.91 |

*Table B1* Double diode parameters and error measures for the TL1-900-50 cell studied in 15 with $T = 323.15$ K and under AM1 illumination. The corresponding electrical circuit parameters fed into the algorithms are $I_{sc} = 905.8 \text{mA}$, $V_{mp} = 414.6 \text{mV}$, $I_{m} = 792.3 \text{mA}$, and $V_{oc} = 531.7 \text{mV}$ yielding a fillfactor of 68.2%