Study on Friction/Wear and Contact Stress of Variable-lead Spiral Pair Substructure

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Abstract. The variable-lead spiral pair changes the moving speed by the change of lead, which can improve structural stability. The structure has the zero self-locking property due to the change of the lead. Firstly, this paper analyses the whole structure, and then studies the contact problem of the variable-lead spiral pair, including the roller rotation problem and the analysis of friction and wear, finally analyses the structural stress. Through the analysis, the damage type of the structure and the structural stress which meets the allowable stress of the material can be obtained. Then it verifies the structure is reasonable.

1. Introduction
The variable-lead spiral pair substructure is always applied to subway gates, it has high stability and maintainability, and the self-locking function at the zero lead. It can improve the reliability of the subway gate and reduce the production cost. The research on the variable-lead spiral drive mainly focuses on the analysis of the motion characteristics, the design of spiral line, and the fitting of the transition curve at the lead change. Shen-Tarng Chiou [1] study the motion characteristics of a variable-lead screw nut mechanism for high-speed shuttleless looms. Ming J.T [2] compare the variable-lead screw mechanism by point contact hypothesis and used the comparison results to optimize the relevant motion characteristics. HS Yan [3] study the meshing characteristics of the involute and helicoid surfaces of various shape curved surfaces. Zhang Wei [4] apply two methods to design the curve of the transition part of the variable-lead spiral line. Liu Lizhen [5] design the variable-lead spiral at the filling production line. Tang Wei [6] proposed a mathematical modeling for the variable pitch of the variable-lead spiral pair groove which used in aerospace liquid engines. Shan Jihong [7] study the parametric equations of the spiral surface and the spiral line of the variable-lead, and gained the 3-D modeling method for the parts.

This paper mainly studies the motion characteristics of the structure of the variable-lead spiral pair, and then analysis of friction and wear. Finally, calculating the stress of the structure.

2. Analysis of the Motion Characteristics of Variable-lead Spiral Pair
When the roller of the variable-lead spiral pair moves in the screw, the movement form of the roller has rolling and sliding motion. The contact surfaces of the roller and the lead screw are free rigid bodies. The movement of the variable-lead spiral pair is a linear movement under the rotation of the screw. The rotation motion analysis of the roller is shown in Fig.1.
Fig. 1 Schematic diagram of the contact movement between the roller and the screw and nut

When the screw rotates at a certain speed, the nut remains stationary, and point B is the motion instant of the contact point between the roller and the nut. \( v_o \) is the linear velocity of the roller center, \( v_A \) is the tangential speed of the roller at point A, \( n_l \) is the absolute rotation speed of the roller around the screw, \( n_s \) is the rotation speed of the screw, and \( r_i \) is the contact radius of the roller, \( \alpha \) is the contact angle, and \( r_o \) is the nominal radius of the variable-lead screw.

\[
v_o = n_s \times 2\pi \times r_o \tag{1}
\]
\[
v_A = n_s \times 2\pi \times r_i \tag{2}
\]

There is only one roller in the variable-lead spiral pair, and the roller plays the role of transmitting motion. When the variable-lead screw is rotated one turn and the roller drives the nut to move a pitch, the calculation formula is as follows.

The spiral line length \( L_1 \) of a lead:

\[
L_1 = \sqrt{(2\pi r_a)^2 + L^2} \tag{3}
\]

The number of turns \( N_1 \) that a lead roller can rotate is:

\[
N_1 = \frac{L_1}{2\pi r_a} \tag{4}
\]

The roller radius is \( r_a \), which can be obtained by combining formulas (3) and (4):

\[
N_1 = \sqrt{(r_o/r_a)^2 + (L/2\pi r_a)^2} \tag{5}
\]

When the rotational speed of the variable-lead screw is \( n_s \), the rotation speed \( n_b \) of the roller is:

\[
n_b = n_s\sqrt{(r_o/r_a)^2 + (L/2\pi r_a)^2} \tag{6}
\]

By the equation (6), the rotation of the roller is related to the variable-lead screw rotation speed \( n_s \) and the lead \( L \) where the roller radius and the screw radius are constant.

Taking the subway gate as example, selecting the relevant dimensions of the variable-lead spiral pair to study the variation law of the roller rotation with the lead \( L \). And the radius \( r_o \) of the variable-lead screw is 20 mm, the roller radius \( r_a \) is 2.5 mm, the rotation speed \( n_s \) of the variable-lead screw is 280 r/min, and the lead \( L \) is selected to be 10 to 70 mm. The rotation speed \( n_b \) of the roller is calculated as shown in Fig. 2.
3. Analysis of Friction and Wear of Variable-lead Spiral Pair

3.1. Analysis of the Type of Wear of Variable-lead Spiral Pair

The variable-lead spiral pair has the contact and the relative motion, so friction is inevitable. With long-term used, the spiral pair will come to wear and cause structural failure. At present, wear can be divided into four types, abrasive wear, adhesive wear, fatigue wear and corrosion wear. Its classification is shown in Table 1.

| Classification    | Wear form                                                                 | Typical application                        |
|-------------------|---------------------------------------------------------------------------|--------------------------------------------|
| Abrasive wear     | Hard particles or surface protrusions on the outside cause the surface material to fall off | Excavator tooth, machine guide, turbine blade |
| Adhesive wear     | Grinding wear on the sliding surface due to the adhesive effect           | High speed heavy duty friction pair         |
| Fatigue wear      | Two mutually moving surfaces cause surface flaking due to cyclic stress to form pits | Gear transmission, rolling bearing          |
| Corrosion wear    | Surface damage due to the external environment has a lot to do with the medium | Environmental medium                       |

According to the above analysis, in the variable-lead spiral pair substructure, the rollers move in the spiral groove which is a rolling accompanying sliding. The wear is caused by the contact friction between the roller and the spiral groove that is subjected to the cyclic load. Therefore, the main type of wear of the variable-lead spiral pair is fatigue wear.

Under the cyclic stress, cracks occur on the contact surface between the roller and the screw, and the crack propagates for a long time, eventually causing the surface of the material to peel off and form pits. The material of the variable-lead spiral pair is quenched and tempered steel, so the wear form is mainly pitting corrosion.

3.2. Analysis of Rolling Friction in Variable-lead Spiral Pair Substructure

By the previous analysis, the friction between the roller and the spiral groove in the variable-lead spiral pair is rolling friction. When the variable-lead screw rotates, the roller drives the nut to move. Under the interaction stress, the roller and the spiral groove generate deforms. When the roller rolls, the front of the contact is raised, and the supporting surface acts as a point of action of the resultant stress of the roller. This stress is equivalent to the moment at the center of the roller that hinders the movement of the roller. As shown in Fig. 3, the contact pressure distribution is asymmetrical due to the deformation of the contact zone in the contact deformation zone C, and the reaction stress of the contact surface produces an offset e, rolling friction creates a torque on the point of contact.
The dimensioned rolling friction coefficient \( k \) is defined as the ratio of the rolling friction torque \( FR \) to the normal load \( W \), namely:

\[
k = \frac{FR}{W} = e
\]  

(7)

Among them, \( k \) is a dimensioned rolling friction coefficient, which is related to factors such as material hardness and humidity. The dimensionless rolling resistance coefficient \( f_r \) is sometimes used to indicate the magnitude of the rolling friction, which is equal to the ratio of the work done by the rolling drive stress to the unit distance and the normal load.

\[
f_r = \frac{Fr \theta}{W} = \frac{F}{W} = \frac{k}{r}
\]  

(8)

Where \( \theta \) is the angle of rolling and \( k \) is the dimensioned rolling friction coefficient.

It can be seen from equation (8) that \( f_r \) is related to a dimension rolling friction coefficient \( k \) and a roller radius. When selecting a roller, the material hardness should be considered and select a suitable roller radius.

4. Analysis of Contact Stress of Variable-lead Spiral Pair

By the analysis, the contact between the roller and the helicoid can be regarded as a point or line contact, and the Hertz Contact Theory can be used to analyze and calculate the contact stress.

4.1. Hertz Contact Theory

In 1881, Hertz used mathematical methods to derive calculation formulas for contact problems. When analyzing the contact stress on objects, the following assumptions are made:

(1) The material is homogeneous;
(2) The deformation of the contact object is a small strain;
(3) The surfaces in contact with each other are sufficiently smooth;
(4) Do not consider friction.

In the case where these assumptions are true, the contact stress can be calculated using the Hertz Contact Theory. The deformation of the point contact object after being stressed is elliptical, the stress change and the contact body should become linear, and the maximum stress appears at the center of the contact surface, and the contact surface is reasonably equal to the external load. The line contact object can be seen as a rectangle after being deformed. In addition, the Hertz Contact Theory assumes that the compressive stress distribution is a semi-ellipsoid, as shown in Fig. 4.
As can be seen from Fig. 4, the pressure $P$ acts along the Z axis, $a$ and $b$ represent the long and short halves of the ellipse. The pressure in the deformation zone is related to deformation, and the maximum stress occurs at the center of the ellipse.

According to the Hertz Contact Theory, the calculation formula of the contact stress $p$ is:

$$p = p_0 \left(1 - \frac{x^2}{b^2} - \frac{y^2}{a^2}\right)^{\frac{1}{2}}$$

(9)

The maximum contact stress is:

$$p_0 = \frac{3P}{2\pi ab}$$

(10)

Where $P$ is the load and $p_0$ is the maximum contact stress.

If the two objects are in line contact as shown in Fig. 5, when subjected to the pressure $P$, deformation occurs, and the contact stress is also an elliptical distribution that has a width $2a$ and a maximum contact stress occurring at the center of the contact.

According to the stress in the contact zone distributed in the light of the ellipse, the formula is:
\[ p = p_0 \left(1 - \frac{x^2}{a^2}\right)^{\frac{1}{2}} \]  

(12)

Where \( p \) is the contact stress and \( p_0 \) is the maximum contact stress.

\[ p_0 = \frac{2P}{\pi ab} = \left(\frac{PE'}{2\pi RL}\right)^{\frac{1}{2}} = \frac{E'b}{4R} \]  

(13)

4.2. Contact Stress of Variable-lead Spiral Pair

According to the Hertz Contact Theory, the contact stress of the deformation zone is distributed as the ellipse. The contact formula can be calculated by equation (10). Among them, \( a \) and \( b \) are calculated by the elastic theory, and the calculation formula is as follows:

\[ a = a \left(\frac{3PR}{2E'}\right)^{\frac{1}{3}} \quad b = b \left(\frac{3PR}{2E'}\right)^{\frac{1}{3}} \]  

(14)

The equivalent curvature radius \( R \) and equivalent elastic modulus \( E' \) can be calculated by the formula (11). \( \alpha \) and \( \beta \) can be obtained by consulting related literature. Among them, the contact stress of the variable-lead screw and the pin roller is shown in Fig. 6:

![Fig. 6 Schematic diagram of the contact stress between the roller and the screw](image)

\[ F_n = F_Q \cos \alpha \]

\[ \alpha = \tan^{-1} \left(\frac{dv}{dt}\right) = \tan^{-1} \frac{v}{w_r} = \tan^{-1} \frac{L}{2\pi r} \]  

(15)

Where \( \alpha \) is the pressure angle between the spiral groove and the roller.

The radius \( r_0 \) of the variable-lead screw is 20mm, the roller radius \( r_a \) is 2.5mm, the rotational speed \( n_s \) of the variable-lead screw is 280r/min, the \( F_Q \) is 30N. By analyzing the increase phase of the lead, the change of the contact stress of the spiral pair is obtained, the result is shown in Fig. 7.

![Fig. 7 Maximum contact stress at different contact points](image)
As can be seen from Fig. 7, the point $\sigma_{\text{max}} = 203$ MPa which has the largest contact stress appears at the position where the roller and the screw motion start, because the position has a flexible impact due to the motion start.

Through the above analysis, it can be found that the contact stress at each point is smaller than the allowable contact stress of the material, which meets the design requirements.

5. Conclusion

In this paper, the characteristics of the structure are studied by analyzing the variable-lead spiral pair. The motion characteristics of the variable-lead spiral pair are studied. In order to analyze the contact problem of the structure, the friction and wear problems and the rolling friction were analyzed. The failure type of the structure was fatigue pitting. Finally, the contact stress of the structure is analyzed. The contact stress of the variable-lead spiral pair is analyzed by Hertz Contact Theory. The structure meets design requirements and operation requirements of the subway.

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