Designing gears with minimal effective transmission error by optimizing the parameters of tooth modification

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Abstract. This paper is dedicated to the development of a method for optimizing the parameters of modification of gears. An experimental study of the effectiveness of the modification, calculated according to the current standard, has been carried out. Within the framework of the project, a software has been developed that implements an algorithm for designing three-dimensional models of gears with a modified topology of contacting side surfaces, which are used for the subsequent modeling by the finite element method of the process of teeth meshing. The results obtained were used to solve the one-criterion optimization problem. As an example of using the method, the optimization of the parameters of the modification of the teeth was carried out in order to minimize the amplitude of the transmission error harmonics.

1. Introduction

Aircraft gears operate under high loads and peripheral speeds. The experience of designing such gears has shown the need to use profile, crowning and end relief modification of the teeth. Parameter assignment is a laborious task and often requires experimental refinement. In some cases, a small change in the values of the modification parameters can lead to the transmission breakdown. For example, [1] presents a case of experimental refinement of spiral tooth modification bevel gears. The value of modification in one of the designs is $C_b = 3..4 \, \mu m$, which led to the destruction of the gear transmission, but the manufacture of gears with the modification $C_b = 10..15 \, \mu m$ ensured the operability of the gear transmission. The difference in the modification parameters, between ensuring the operability of the transmission and leading to the destruction of the gear transmission, is $6 \, \mu m$, which indicates high accuracy requirements when assigning modification parameters.

2. Development of methods for finding the optimal parameters of the tooth modification

2.1. Tooth modification

In a general sense, modification is an intentionally introduced error in the tooth flank. The value of the profile error and its shape have a significant effect on the nature of the gear train and determine the dynamic component of the load in the gearing. GOST 1643-81 gives the following definition of the profile error: "The normal distance between the two nearest to each other nominal face profiles of the tooth, between which the real face effective profile of the gear tooth is located" [3].
Figure 1. Determination of the geometric location of the points of the tooth profile, taking into account the error.

Figure 1 shows the scheme for determining the value of the profile error. As can be seen from the figure, there are several zones on tooth profile. The first zone is located in the section of the true involute of the $AB$ profile. In this zone, each point $M$ of the profile corresponds with the point of the ideal involute profile $i$ with an error. The coordinates of the point $i$ are known and can be determined by the formulas in [4]. The value of the error $f_i$ at point $M$ can be determined from the angle of $\nu_i$ and measured along the normal from point $i$ to point $M$. The second zone is located on the $VA$ section. The limiting point $V_r$, located on the tip diameter, will be at a distance $f_v$ from the involute at point $V$, defined by the radius $OV$. Therefore, when measuring the error in this section, the line of the nominal involute profile must be extended to point $V$. Diameter $D_v$ is currently missing on the drawings. The position of point $F$ must also be resolved to determine the beginning of the modification line. Nevertheless, the approach shown in Figure 1 allows us to create a mathematical model, in which for each point of tooth flank, the function of the error value from the angle $\nu_i$ can be determined. In some cases, the radius of curvature of the involute or the radius $R_i$ from the center of the gear $O$ to point $i$ can be used as an argument to the function [5]. By setting a value of intentionally introduced profile error over the entire area of the tooth flank (including zero values), we can obtain the desired modification shape. The resulting geometric model can be used in the manufacture of a gear.

GOST 13755-2015 provides recommendations on the shape of the modification line, depending on the purpose of the gear transmission. Each shape is used by a set of standard parameters, for example, a crowning is characterized only by depth. Flanking is determined by both the depth and the length of the modification line. The standard recommends determining the parameters by calculation or experiment.

2.2. Experimental study of standard tooth modification

Currently, there is a standardized method for analytical estimation of the optimal modification value (ISO 21771), implemented in the KISSsoft software package. To assess the effectiveness of the assignment of a modification according to this standard, an experimental study was carried out at CIAM. The tests were carried out at the State Scientific Center of the Russian Federation Federal State Unitary Enterprise "Central Institute of Aviation Motors" (CIAM) at the U-394 test bench, designed for
experimental research and life tests of cylindrical gears. The test bench permits realizing high gear ratios, loads and operation of gears at a speed of up to 10,000 rpm.

![Test bench U-394](image)

**Figure 2.** Test bench U-394.

The test bench is shown in Figure 2. The test bench is constructed according to the “mechanically closed loop” scheme and consists of working and auxiliary gearboxes with an axle distance $a_w = 201$ mm, connected by springs to eliminate the mutual influence of alternating loads.

| Parameter | Value   |
|-----------|---------|
| $\alpha$  | 20 deg  |
| B         | 0 deg   |
| $m_n$     | 4       |
| $z_1$     | 49      |
| $z_2$     | 51      |
| $x_1$     | 0.0561  |
| $x_2$     | 0.1985  |
| $P$       | 0.38    |
| $h_{a1}$  | 1       |
| $h_f$     | 1.38    |

**Table 1.** Gear parameters.
For the experiment, 3 sets of gears were manufactured (parameters are given in Table 1) with a 24 mm face width. The material of the gears is steel 12Cr2Ni4A (ME quality according to ISO 6336-5, electroslag remelting). The gears from set No. 1 do not have a profile modification of the teeth. The gears from sets No. 2 and No. 3 are made with tip relief linear modification in accordance with Appendix 2 GOST 8889-88. The gears from set No. 2 have a tip relief modification value of 30 μm, which corresponds to the results of the choice of the parameters of the optimal modification to reduce the excited vibrations in accordance with the methodology given in [7]. The gears from set No. 3 have a tip relief modification value of 62 μm in accordance with the recommendations of the ISO 21771 standard and the KISSsoft software package.

The macrogeometry parameters profile of all three sets provide the value of the gear ratio $\xi = 1.78$, which makes these gears more sensitive to the values of the profile modification parameters.

The appearance of the experimental gears is shown in Figure 3. The estimation of the value of the dynamic stresses in the engagement was carried out using strain gauges with a base of 1 mm. The dynamic loads excited in the engagement of the studied pair of gears were also estimated by vibration sensors mounted on the working gearbox housing in three directions: vertical, horizontal, and longitudinal. The records of the torquemeter and the drive shaft speed sensor were also kept. The experiment was carried out according to the developed test program for pre-resonance, resonance and over resonance modes. Each set of gears has worked at least $2 \times 10^5$ loading cycles.

The results of vibration measurement of the studied sets of gears were evaluated by analyzing the vibroacceleration signals spectrum measured in the radial direction. The analysis in the vibration spectrum highlights the components of the harmonics of tooth mesh frequency, as the main source of vibration excitation in a mechanical system.

The results of vibroacceleration signal measurements of all three experimental gear sets are summarized in histograms presented in Figure 4 - Figure 6. The sets are presented from left to right - set No. 1 (black bars), set No. 2 (dark gray bars), set No. 3 (gray bars).
Figure 4. Spectral composition of the vibration signal of three sets of gears at a rotation speed of \( n = 1500 \) rpm (pre-resonance mode).

Figure 5. Spectral composition of the vibration signal of three sets of gears at a rotational speed of \( n = 1900 \) rpm (resonance mode).
Figure 6. Spectral composition of the vibration signal of three sets of gears at a rotation frequency of n = 4000 rpm (resonance mode).

Figure 4 - Figure 6 shows a comparison between the amplitude values for each of the three sets of gears of the first five harmonics of tooth mesh frequency in the vibration signal, measured in the radial direction, at different transmission modes. Figure 5 shows a sharp increase in the amplitude of the first harmonic of tooth mesh frequency when the transmission is operating in the resonant mode and the absence of significant changes in the amplitudes of the 2nd and higher harmonics of tooth mesh frequency of the vibration spectrum in comparison with the results of measurements at the pre-resonance and over-resonance modes. Over the entire frequency range of the gears in the vibration spectrum of the three sets of gears, there is a distinct difference in the levels of the amplitudes of the gear harmonics in the vibration signals, depending on the modification of the teeth. The gears from set No. 2 with optimal modification parameters have the lowest vibration activity, and when operating in a resonant mode, the vibration amplitude at all harmonics of tooth mesh frequency is 40% less (1st harmonic) and higher (64% less for the second harmonic) in comparison with vibration levels of the gears from set No. 1 without profile modification.

Vibration levels for the gears from set No. 3 with ISO 21771 recommended profile modification parameters exceed the vibration levels of sets No. 1 and No. 2 by double or more. A sharp increase in the amplitude of the first harmonic of tooth mesh frequency is observed almost over the entire range of operating frequency in the spectral composition of vibration signals for set No. 3. However, in a narrow range in the pre-resonance mode of operation (1450 ... 1520 rpm), an increase in the amplitude of the second harmonic of tooth mesh frequency (Figure 4) with a corresponding decrease in the amplitude of the first harmonic of tooth mesh frequency is found.
Thus, the use of the standard method for calculating the parameters of the modification ISO 21771 does not allow minimizing the dynamic loads of gears, and the value of dynamic loads obtained using the modification calculated according to the standard gave a worse result compared to a gear pair without modification of the teeth.

In most cases, the strength calculation according to standard methods does not allow evaluating the shape of the modification line. The standards consider the effect of transmission error only approximately. For example, in the GOST 21354-87 standard, the influence of errors on the dynamic load is taken into account by the error of the pitch of the engagement and the error of the profile in the form of refinement coefficients BP and BF [2].

For this reason, it becomes necessary to develop a new method for calculating the optimal modification parameters. It is important that when designing gears of aircraft in which dynamic loads can have a greater effect than static loads, it is necessary to have more complete information about the nature of dynamic processes and account for the deformation of the teeth during the change of teeth. One of the methods of such calculation is the use of the finite element method.

2.3. 3D models

One of the key components in performing a high-quality finite element analysis is the availability of a three-dimensional model that is highly accurate in the areas of stress and strain calculation.

Most of the current 3D CAD systems support parametric modeling. Parameter-based modeling can significantly save time when creating new models. It is possible to enter both direct values of the parameters and to create various relationships between them. This tool is highly efficient and allows to quickly create various designs options, but is nevertheless practically inapplicable when creating solid models of gears with modified tooth flank. This is due to high standards for the geometric accuracy of the model and, as a consequence, the need to calculate a significant number of points describing the shape of the side surfaces. It should also be borne in mind that algorithms that require an iterative approach can be used to calculate some values. For example, the Cheng method [10] can be used to determine the value of the angle from its involute function, but the Cheng method has limitations on the value of the angles, and in the general case, this value can be obtained with high accuracy by an iterative method by minimizing the error function when calculating this value. Thus, the use of internal tools for parameterization of CAD systems will, on the one hand, lead to the creation of extremely parameters overloaded files, to the loss of flexibility in changing the geometry (for example, to change the number of points to reduce the size of the model file, or, on the contrary, to increase accuracy) and, on the other hand, to significant restrictions on the calculation methods available for use. Therefore, a special software was developed to create three-dimensional models.

![Figure 7. The result of calculating the geometric positions of the points of the tooth profiles with modification.](image-url)
Figure 7 shows the result of calculating the coordinates of the points of the tooth profiles with modification. The dash-dotted lines from top to bottom show the addendum circle, the pitch circle, and the dedendum circle. On the left side of the tooth, the modification is shown - flanking. The modification value has been increased for clarity. On the right side of the tooth, a thin line shows the gear backlash. An arc joining adjacent teeth is used when constructing a tooth to form a tooth space body. The points show the junction of the tooth profile and the root fillet, as well as the beginning and end of the root fillet along the dedendum circle. The geometric calculation program uses the parameters of the tooth modification as parameters. The macro geometry parameters are saved in the gear database. The calculation results are transferred to a 3D CAD system using CAM technology.

**Figure 8.** Algorithm of the program for building three-dimensional models.

In the second stage of building the model, a part blank is created. For example, for cylindrical gears, the part blank will be a cylinder with a diameter equal to the tip diameter. The model is formed by cutting out the volumes of the tooth spaces from the part blank. The next step is the analysis of coordinates. The permissible deviation of the coordinates of the surface control points from the coordinates obtained from the initial data file should not exceed 1 μm. At the next stage, for the subsequent calculation using the finite element method, the surfaces are assigned a marks to use in the calculation, as well as auxiliary identifiers for forming the assembly. The algorithm of the program is shown in more detail in Figure 8.
Figure 9. The result of modeling of a gear with modification of the teeth.

Figure 9 shows the result of modeling. The darker areas on the sides of the tooth space show the modified faces.

Figure 10. Fragment of the stage of comparing the coordinates of the points of the model.

The obtained models were compared with three-dimensional models obtained in the KissSoft software without modification and zero tolerances for the values of the addendum and dedendum diameters. The models were aligned along the axes of the gears, along the end face, and along the involute section of one of the faces of the teeth. The error of the models was estimated by the difference in distances between points located on the same diameter of each of the models. Figure 10 shows a snippet of measuring the distance between points. Satisfactory results were obtained (the distance between the points did not exceed 1 μm for the section of the tooth profile without modification). The accuracy of the modification assignment was verified by sketching the tooth profiles. Distance measurements on the sketch were carried out along the normal between the modified and true involute profile. The error should not exceed $10^{-5}$ mm.
2.4. Dynamic model of a gear transmission

The accuracy of the results of the calculation by the finite element method [6] depends on many factors: the quality of the three-dimensional model, the correspondence of the initial and boundary conditions to physical processes, etc. Gears are a nonlinear system with periodically changing teeth stiffness. The accuracy of the calculation is largely determined by the conditions of contact and application of loads, including inertial ones. Thus, to ensure a high quality of the calculation, it is necessary to select the correct dynamic gear model.

For optimization, a dynamic model was used, described in [7] for a planetary gear mechanism. This model is presented as a nonlinear system with 6 degrees of freedom, consisting of a gear pair, represented by solid disks with masses $m_1$ and $m_2$ and moments of inertia $J_1$ and $J_2$, respectively. The meshing of the teeth is described by an elastic-damping connection with variable stiffness $k_d(t)$ directed along the line of action of the force. The bearing arrangements of the toothed shafts are characterized by the bearing stiffness $k_{bx}$ and $k_{by}$ in accordance with the directions of the selected orthogonal coordinate system for each gear. The system is balanced by torque applied in opposite directions to the gears.

![Figure 11. Dynamic model of two gear pairs in planetary set [7].](image)

Figure 11 shows a schematic diagram of the dynamic model used. 1 - ring gear, 2 - sun gear, 3 – planetary gear, $\alpha$ - pressure angle. The combination of analytical methods and the finite element method used in the dynamic model made it possible to obtain accurate estimates of the dynamic forces in gears at low time costs, achieved by simulating the process of gearing of gears in the finite element method and taking into account the possible loss of contact between the teeth. Since only a pair of gears was considered in the calculation, changes related to the kinematic scheme of the gearbox were introduced in the used dynamic model - there is no elastic-damping connection with variable stiffness $k_e(t)$. The $x$, $y$, coordinate system is motionless.

The combination of a dynamic model and a system for constructing solid models of gears with modification allows optimization of modification parameters by the finite element method. It should be
noted that the used dynamic model has been successfully verified experimentally at the U-394 test bench at CIAM [7]. Based on the calculation results using this dynamic model, the vibration level of the gears has been successfully assessed. To implement this approach, a group of computer programs (modules) was developed that interact with each other.

2.5. Optimization scheme

Optimization input data includes: a set of parameters, a method for calculating the objective function connecting the values of the parameters with the optimization criterion and imposed constraints. The developed software and dynamic model are sufficient for optimization. The implementation of the method is achieved by providing links between and the modules involved in the calculation.

![Figure 12. Modification parameters optimization scheme.](image)

According to the diagram presented in Figure 12, the developed method is highly flexible and allows the use of various solvers, optimization methods and geometric models. In the general case, the described method uses the modification parameters of all gears of the gearbox kinematic diagram as initial data. Parameters of the transmission gears macrogeometry remain unchanged and are determined by the parameters of the research object. The selected optimization method («Optimization» block on the diagram) generates a vector of microgeometry parameters («Parameters») and transfers them to the geometric calculation program («ImportGearGeometry») via the solver (FEM). In its geometric calculation program, it accepts macrogeometry parameters («GeomCalc») and, based on the data generated at the first stage of the gear modeling, at the second stage combines them into an assembly corresponding to the kinematic scheme involved in the calculation. Then it forms a geometric model («Model») through the CAD program, which is transferred to the program for calculating the objective function. The calculation results are returned to the optimization program. Based on the results obtained, either a new vector of parameters is created, or the optimization process is completed.

Adaptive response surface method (ARSM) was chosen as an optimization method for the considered object of research. The ARSM procedure starts at a single start design, and an initial Design of Experiments (DoE) scheme is built having the start design as center point. Based on the approximation of the model responses the optimal design is searched within the parameter bounds of the DoE scheme. Convergence of the algorithm is achieved if the change of the position and the objective value of the best design is below the specified tolerances or if the Design of Experiments scheme reaches a minimal size [11].

2.6. Optimization criterion
The result of optimization is the achievement of the highest or the lowest value of a certain criterion or group of criteria, subject to certain constraints. The goal of optimization is determined by the problem solved by the engineer during the design of the detail. When optimizing the parameters of the macrogeometry of gears, the optimization criteria can be the achievement of the maximum load-carrying capability of the tooth in terms of contact or bending stresses, the minimization of the mass or sliding speeds, the achievement of the highest overlap ratio, and other qualitative indicators. Constraints are design specific. For example, in some cases, the problem of determining the greatest load-carrying capability is solved while maintaining the mass and dimensions of the structure within the specified limits. Therefore, the choice of the optimization criterion when changing the parameters of the modification of the teeth is of great importance and is determined by the qualitative indicators, which are influenced by the parameters of the modification of the teeth.

The peculiarity of gears consists in the periodic meshing of the teeth and their subsequent release from the meshing. In an ideal gear train, each subsequent pair of teeth engages until the previous pair of teeth disengages. But in the manufacture of gears, errors inevitably arise associated with the transmission error of the machine, with errors in the manufacture, sharpening and installation of a gear cutting tool, with an error in the basing of the workpiece and inaccuracies in setting up and adjusting all technological equipment when cutting teeth, which together leads to errors in shape and dimensions that cause uneven movement, accompanied by increased dynamic loads, noise and uneven distribution of the load on the teeth flank [5].

Aircraft gears are typically highly accurate. However, this is not enough to reduce dynamic loads. This is due to the high loads that gear drives experience during the transmission of motion. Under load, deformation of the teeth occurs, the magnitude of which can exceed the tolerance for the manufacture of the gear train. As a result of deformation, the edges of the teeth can enter the off-design zone, which leads to increased contact stresses at the first point of contact. Resilient deformations arising in the process of persisting teeth lead to a periodic change in the stiffness of the engagement, which leads to increased dynamic excitation.

In some cases, the values of dynamic loads can significantly exceed the values of static loads. Therefore, reduction of dynamic loads can be an optimization goal in the design of aircraft gears.

This study examined the influence of various forms of modification on the stiffness of the gearing and the value of dynamic loads. An effective transmission error was proposed as criterion for choosing the optimal modification [9]. In accordance with GOST 1643-81, the transmission error is understood as the difference between the real and nominal (calculated) angles of rotation of the gear on its working axis [3]. Harmonic analysis methods allow to decompose the transmission error function into harmonic components. As an effective transmission error in [9], it is proposed to use the sum of the amplitudes of the first 12 harmonic components of the transmission errors. The contribution of each individual frequency component to the dynamic characteristic of the gear train loading was investigated, and in [9] it was found that the first 4 harmonics have the greatest influence on the dynamic load of the gear transmission. The value of the amplitude of the transmission error, as a rule, is exerted by the first harmonic, with a period equal to one revolution of the gear, harmonics with shorter periods determine the values of cyclic errors. In many studies, it is customary to distinguish between static (STE) and dynamic (DTE) transmission errors. The static transmission error, including both the technological component and the component from the deformation of the teeth. STE does not depend on the speed and dynamic state of the transmission but is determined only by the angular position of the drive gear and the value of the engagement load. Therefore, to assess the effectiveness of the effect of modification of the teeth on dynamic loads, the following relationship was adopted, which describes the effective transmission error:

\[ A_e = \sqrt{A^2_2 + A^2_3 + A^2_4 + A^2_5 + A^2_6 + A^2_7}, \]  

\( A_e \) – optimization criterion, effective transmission error

\( A_{2...7} \) – transmission error harmonic amplitude.
2.7. Results

To achieve the optimization result, 191 calculations were performed. Tables 2-4 present a small part of the calculation results. The depth (H1) and flank height (L1) of the driving gear have been optimized. The driven gear has not been modified. The parameterization scheme is shown in Figure 13. The "*" sign means that the value is specified in units of the module.

**Table 2.** Fragment of the calculation results, part 1.

| №  | L1, *module | H1, mm  | Contact Stress, MPa |
|----|-------------|---------|---------------------|
| 151 | 0,4766797   | 0,0094269 | 1006                |
| 152 | 0,4752302   | 0,0065616 | 1007                |
| 153 | 0,4752302   | 0,0094269 | 1006                |

**Table 3.** Fragment of the calculation results, part 2.

| №  | Amplitude, mm | Harmonic amplitude A1, mm | Harmonic amplitude A2, mm | Harmonic amplitude A3, mm | Harmonic amplitude A4, mm |
|----|----------------|---------------------------|---------------------------|---------------------------|---------------------------|
| 151 | 0,0061455      | 0,0012759                 | 0,0004727                 | 0,0002553                 | 0,0000906                 |
| 152 | 0,0060473      | 0,0012044                 | 0,0004898                 | 0,0002569                 | 0,0000888                 |
| 153 | 0,0061471      | 0,0012766                 | 0,0004725                 | 0,0002554                 | 0,0000906                 |

**Table 4.** Fragment of the calculation results, part 3.

| №  | Harmonic amplitude A6, mm | Harmonic amplitude A7, mm | Effective transmission error (Ae), mm |
|----|---------------------------|---------------------------|-------------------------------------|
| 151 | 0,0000328                 | 0,0000346                 | 0,0005471                           |
| 152 | 0,0000444                 | 0,0000515                 | 0,0005643                           |
| 153 | 0,0000327                 | 0,0000346                 | 0,0005470                           |

**Figure 13.** Parameterization scheme.
Figure 14. Convergence optimization process.

Figure 14 shows the optimization process. The goal of optimization is to minimize the objective function $A_e$, the value of which is determined by formula (1).

Figure 15. Transmission error graph.
For each point, the transmission error was calculated (an example of the calculation result is shown in Figure 15). Then, the obtained function of the transmission error was expanded into a Fourier series (Figure 16). In tables 2-4, the amplitude harmonic values are presented in columns A1… A7. Then the calculation bending stresses was carried out (Figure 17). The minimum value of the Ae function was reached at design point No. 153.

3. Conclusions

1. On the U-394 test bench, an experimental assessment of the effect of various modification parameters on the vibration values in gear pairs at pre-resonance, resonance and over-resonance modes was carried out.

2. Modification parameters, calculated according to the current ISO 21771 standard, have shown a negative effect on the value of dynamic loads and led to an increase in vibration, including in comparison with an unmodified gear set.

3. A method was developed for determining the optimal modification parameters. The finite element method is used for calculations.

4. During the optimization, a dynamic model was used, which has been validated by an experimental test.

5. As an optimization criterion for reducing dynamic loads, an effective transmission error can be used, calculated from the values of the amplitudes of the dynamic transmission error (DTE).

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