

**RESEARCH ARTICLE**

**INFLUENCE OF ANISOTROPIC TURBULENT PLASMA ON THE PARTIALLY COHERENT FOUR-PETAL GAUSSIAN VORTEX BEAMS**

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Received: 05 August 2022 / Accepted: 20 September 2022 / Published online: 30 September 2022

**Abstract**

Based on the extended Huygens-Fresnel principle, the analytical expressions for partially coherent four-petal Gaussian (FPG) vortex beams propagating in turbulent plasma are obtained, the influence of the turbulent plasma parameters on beam profile and coherence of the beams is discussed in detail using numerical examples. It is found that a four-petal Gaussian-shaped intensity distribution will eventually transform into a Gaussian distribution after propagating in turbulent plasma. Meanwhile, turbulent plasma parameters will also influence the coherence characteristics of the beams.

**Keywords:** Partially coherent, Four-petal Gaussian vortex beams, Anisotropic turbulent plasma.

**1-Introduction**

Hypersonic turbulence (due to turbulence in the flow about a high-speed flight vehicle) is an important factor in understanding optical propagation and optical communication [1]. In fact, when the aircraft or spacecraft pierces the Earth’s atmosphere with exceedingly high speed (hypersonic), the gas environment surrounding a hypersonic aircraft will rub with an aircraft body and causes a hypersonic plasma sheath surrounding an aircraft. The presence of anisotropic turbulent plasma sheaths around the vehicles can have a strong influence on the communication characteristics between the vehicles and radars. It can be perturb the communication and in some circumstances may lead to a disconnection [2, 3].

In 2005, four-petal Gaussian beams are first introduced [4]. The new beams have four equal petals at the source plane. The four-petal Gaussian beams may find potential applications in micro-optics, optical communications, beam splitting techniques, etc. With the development of laser optics, a new laser beam called four-petal Gaussian vortex beams has been introduced, where the wave-front phase of four-petal Gaussian vortex beams can modulate the beam profile and carry the orbital angular momentum by the passage of a four-petal Gaussian vortex beam through a spiral phase plate. This beam has potential application in the fields of optical micro-manipulation, nonlinear optics, etc. Since then, the evolution properties of four-petal Gaussian vortex beams have been widely studied [5, 6, 7, 8].

However, to the best of our knowledge, the intensity distribution and coherence properties of partially coherent four-petal Gaussian vortex beams propagating in turbulent plasma have not been studied and reported. In this work, we studied the influence of turbulent plasma on beam profile and coherence of partially coherent four-petal Gaussian vortex beams using numerical examples.

**2- Theoretical Model**

In the Cartesian coordinate system, the z-axis is taken to be the propagation axis. The general four-petal Gaussian vortex beam in the source plane z=0 takes the form [8]:

\[
E(x', y', 0) = \left(\frac{x'y'}{w_0^2}\right)^2 \exp\left[-\frac{x'^2 + y'^2}{w_0^2}\right] \left[\begin{array}{l}
x' \\
+i \text{sgn}(l) y' \\
\end{array}\right]^{[l]},
\]

where \(n\) denotes the order of the four-petal Gaussian beams, \(\text{sgn}(\cdot)\) is specifies the sign function \(l\) is the topological charge of the spiral phase plane and \(w_0\) is the waist width of the Gaussian beam. Figure 1 shows the normalized intensity distribution of the four-petal Gaussian vortex beam for different values of \(n\), when \(n = 0\), Eq.(1) will reduce to electric field of ordinary fundamental Gaussian beam.

The cross-spectral density of a partially coherent vortex beam at the source plane is defined as a two-point correlation function, i.e. [9]:

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\[
W(r_1', r_2', 0) = \langle E(r_1', 0)E^*(r_2', 0) \rangle
= A(r_1')A(r_2')g(r_1' - r_2'),
\]

Here, \(E(r_1', 0)\) electric field of a fully coherent optical vortex beam and \(r_1'\) is the position vectors at the source plane, \(i = 1, 2\). The angular brackets \(\langle \cdot \rangle\) denote an ensemble average, while the asterisk * denotes the complex conjugate, \(A(r')\) represents the amplitude and \(g(r_1' - r_2')\) denotes the correlation function between two points \(r_1'\) and \(r_2'\) which can be written as [9]:

\[
g(r_1' - r_2') = \exp \left( - \frac{(r_1' - r_2')^2}{2\sigma^2} \right),
\]

where \(\sigma\) denotes the spatial coherence length. Then the cross-spectral density of a partially coherent four-petal Gaussian vortex beams at the source plane \(z = 0\) is written as:

\[
W(r_1', r_2', 0) = \frac{\left( x_1' y_1' \right)^{2n}}{w_0^2} \frac{\left( x_2' y_2' \right)^{2n}}{w_0^2} \exp\left[-\frac{r_1'^2 + r_2'^2}{2\sigma^2}\right] \times \left[ x_1' \right] + i \operatorname{sgn}(l) y_1' \left| \left| x_1' \right| \right| \times \left[ x_2' \right] - i \operatorname{sgn}(l) y_2' \left| \left| x_2' \right| \right| \exp\left[-\frac{(r_1' - r_2')^2}{2\sigma^2}\right]
\]

\[\text{(4)}\]

**Fig. 1:** Normalized intensity distribution at the source plane before beam passing through anisotropic turbulent plasma for different values of the order of the four-petal beams (a) \(n = 1\), (b) \(n = 2\), (c) \(n = 3\).

By using the extended Huygens-Fresnel integral formula, the cross-spectral density of the partially coherent four-petal Gaussian vortex beams in anisotropic turbulent plasma can be expressed as [10]:

\[
W_{out}(r_1, r_2, z) = \frac{k}{2\pi \lambda z} \int \int W_{in}(r_1', r_2', 0) \exp\left\{-\frac{ik}{2z} \left( (r_1' - r_1)^2 - (r_2' - r_2)^2 \right) \right\} 
\times \langle \psi^*(r_1', r_1, z) + \psi(r_2', r_2, z) \rangle d^2r_1' d^2r_2',
\]

\[\text{(5)}\]

where \(k = 2\pi/\lambda\) is the wave number, in which \(\lambda\) is the incident wave length, \(z\) is the beam propagation distance, \(\psi\) is the random part of the complex phase of a spherical wave propagating through anisotropic turbulent plasma. Over the ensemble of the statistical realization of the random medium, we can written [11]:

\[
\langle \psi^*(r_1', r_1, z) + \psi(r_2', r_2, z) \rangle = \exp \left( -4\pi^2k^2z \right) \int_0^1 \int_0^\infty \left[ \right.
\]

\[
\left. - J_0(kl)(1 - \xi)\phi(r_2 - r_1') + \xi(r_2 - r_1') \right] \phi_0(k\kappa)\kappa d\kappa d\xi
\approx \exp(-D(z))\left( (r_1 - r_2)^2 + (r_1 - r_2)(r_1' - r_2') + (r_1' - r_2')^2\right).
\]

\[\text{(6)}\]
Now, substituting Eqs. (4) and (6) into Eq. (5), and using the following expression [14, 15]:

\[(x + iy)^n = \sum_{t=0}^{n} \frac{n!}{t!(n-t)!} x^{n-t} (iy)^t, \]

\[f = -p x^n \exp(-px^2 + 2xq) dx = \]

\[n! \frac{1}{p} \frac{q^n}{p^n} \exp\left(\frac{q^n}{p^n}\right) \sum_{k=0}^{\lfloor n/2 \rfloor} \frac{1}{(k!)(n-2k)!} \left(\frac{p}{4q^2}\right)^k, \]

\[H_n(x) = \sum_{k=0}^{\lfloor n/2 \rfloor} \frac{(-i)^k n!}{k!(n-2k)!} (2x)^{n-2k}, \]

The integration in the Eq. (5) can be evaluated, and finally we obtain the cross spectral density for partially coherent four-petal Gaussian vortex beams propagating through turbulent plasma:

\[W_{out}(r_x, r_z) = \left( \frac{k}{2\pi^2} \right)^{2n} \frac{1}{w_0^2} \exp\left( -\frac{ik}{2z} (x_1^2 + y_1^2) \right) \times \exp(-D(z)(x_1 - x_2)^2 - D(z)(y_1 - y_2)^2) \times \frac{1}{\sqrt{\omega_0^2 + 1}} \left( \frac{1}{\sqrt{b}} \right)^{2n+1+l-t} H_{2n+1+l-t}(-\frac{c_1}{\sqrt{b}}) \times \frac{1}{\sqrt{\omega_0^2 + 1}} \left( \frac{1}{\sqrt{b}} \right)^{2n+4+l-t} H_{2n+4+l-t}(-\frac{c_2}{\sqrt{b}}). \]

\[c_2 = \frac{1}{\sqrt{\omega_0^2 + 1}} \frac{ik}{2z} y_1 - \frac{D(z)}{2} (y_1 - y_2) \left( \frac{1}{2\sigma^2} + D(z) \right) - \frac{ik}{2z} y_2 + \frac{D(z)}{2} (y_1 - y_2), \]

Since the optical intensity of the partially coherent light is given by [9]:

\[I(r, z) = W(r, r, z) \]

, and the spectral degree of coherence for a partially coherent four-petal Gaussian vortex beams at a pair of field points \(r_1 = (x_1, y_1)\) and \(r_2 = (x_2, y_2)\) can be written as [16]:

\[\mu(r_1, r_2, z) = \frac{W(r_1, r_2, z)}{\sqrt{W(r_1, r_1, z)W(r_2, r_2, z)}}. \]

3- Numerical results and discussion

In this section, we will study the effect of the anisotropic turbulent plasma on the intensity distribution and coherence properties of partially coherent four-petal Gaussian vortex beams using numerical examples. The parameters of the beam and anisotropic turbulent plasma are chosen as (unless the other values of parameters are specified in the caption): the order of the four-petal Gaussian beams \(n = 1\), the topological charge \(l = 1\), the beam width \(w_0 = 1cm\), the wave length \(\lambda = 1550nm\), the correlation length \(\sigma = 1mm\), the outer scale \(L_0 = 0.1m\), the inner scale \(l_0 = 5 \times 10^{-6}m\), we use refractive index fluctuation variance of \((n^2) = 0.73 \times 10^{-20}\) and anisotropy parameters \(\xi_x = 2, \xi_y = 1 [12].\)

Figures 2-4 give the normalized intensity of partially coherent four-petal Gaussian vortex beams propagating in turbulent plasma for the different turbulent plasma parameters (anisotropy parameter \(\xi_x\), outer scale \(L_0\) and refractive index fluctuation variance \((n^2)\)) at the propagation distance \(z = 1\) m. The normalized intensity of partially coherent FPG beams propagating through turbulent plasma for different values of anisotropy parameter \(\xi_x\) and outer scale \(L_0\) are shown in Figs. 2-3, respectively. As can see from these figures, when the anisotropy parameter \(\xi_x\) and outer scale \(L_0\) are large, the distribution of the intensity keeps initial four-petal profile as shown in figure 2(c) and figure 3(c). With decreasing \(\xi_x\) and \(L_0\), the beam is gradually spread, and the shape of the beam will lose their initial four-petal profile. While Figure 4 shows the normalized intensity of partially coherent FPG vortex beams propagating through turbulent plasma for the different refractive index fluctuation variance \((n^2)\). It can be found that partially coherent FPG vortex beams in turbulent plasma with higher refractive index fluctuation variance \((n^2)\) will lose their initial four-petal profile rapidly.
**Fig. 2**: Normalized intensity of the partially coherent four-petal Gaussian vortex beams in anisotropic turbulent plasma for different values of the anisotropy parameter (a) $\xi_x = \xi_y = 1$ , (b) $\xi_x = 3$ , $\xi_y = 1$ , (c) $\xi_x = 6$ , $\xi_y = 1$ , ($z = 1$ m).

**Fig. 3**: Normalized intensity of the partially coherent four-petal Gaussian vortex beams in anisotropic turbulent plasma for different values of the outer scale (a) $L_0 = 0.1$ m, (b) $L_0 = 0.2$ m, (c) $L_0 = 0.4$ m, ($z = 1$ m).

**Fig. 4**: Normalized intensity of the partially coherent four-petal Gaussian vortex beams in anisotropic turbulent plasma for different values of refractive index fluctuation variance (a) $\langle n_1^2 \rangle = 0.2 \times 10^{-20}$, (b) $\langle n_1^2 \rangle = 0.6 \times 10^{-20}$, (c) $\langle n_1^2 \rangle = 1 \times 10^{-20}$, ($z = 1$ m).

Figure 5 shows the evolution of the intensity distribution of partially coherent FPG vortex beams propagating in turbulent plasma. From this figure, we can see that the beam will keep its initial four-petal profiles in the near field propagation (Fig. 5(a)), that it will lose its four-petal profiles and become a flat-topped-like beam with increasing propagation distance (Fig. 5(b)), and that it will evolve into a Gauss-like beam in the far-field (Fig. 5(c)) due to the influence of coherence length and plasma turbulence.
Fig. 5: Normalized intensity of the partially coherent four-petal Gaussian vortex beams in anisotropic turbulent plasma for different values of propagation distances (a) $z = 1\, m$, (b) $z = 2\, m$, (c) $z = 3\, m$.

The spectral degrees of coherence $\mu(x, 0, z)$ along the x-axis of the partially coherent four-petal Gaussian vortex beams propagating in a turbulent plasma for the different turbulence parameters (anisotropy parameter $\xi_x$, outer scale $L_0$ and refractive index fluctuation variance $\langle n_1^2 \rangle$) and propagation distances $z$ are investigated in Fig.6. In this numerical calculation, the point $(x_2, y_2)$ is chosen as $(0, 0)$. It can be seen from Fig.6(a-d), the spectral degree of coherence of FPG beams has an oscillatory behavior and exists zero points. The influence of anisotropy parameter $\xi_x$ on the spectral degree of coherence is shown in Fig.6(a). It is shown that with the increases in the distance from the source, the spectral degree of coherence decreases from unity firstly and then oscillates around zero. It is also found that the turbulent plasma with the smallest anisotropy parameter will become the least coherent. Figure 6(b) is plotted to illustrate the variation of the spectral degree of coherence against outer scale $L_0$. It is found that after propagating for a certain distance, the turbulent plasma with the smallest outer scale will become the least coherent. Figure 6(c) illustrates the behavior of the degree of coherence for refractive index fluctuation variance $\langle n_1^2 \rangle$. One sees that the degree of coherence alters significantly with the change in $\langle n_1^2 \rangle$ for turbulent plasma. For the large value of $\langle n_1^2 \rangle$, the faster the degree of coherence decays. Finally, Fig.6(d) is plotted to show how the spectral degree of coherence changes with the increasing propagation distance in turbulent plasma. We find that as the beam propagates, the coherence of the beam becomes better, what is more, the oscillation of the degree of coherence becomes weaker due to the beam has a larger beam spot.
Fig. 6 : The spectral degree of coherence $|\mu(x, 0, z)|$ of the partially coherent FPG vortex beams in anisotropic turbulent plasma for different values of (a) anisotropy parameter $\xi_x$, (b) outer scale $L_0$, (c) refractive index fluctuation variance $\langle n_1^2 \rangle$, (d) propagation distances $z$.

4- Conclusions

In this paper, the analytical formula for the cross-spectral density of partially coherent four-petal Gaussian vortex beams through turbulent plasma is derived. The results show that beam will lose its initial four-petal profile rapidly with decreasing turbulent plasma parameters anisotropy parameter $\xi_x$ and outer scale $L_0$ or increasing refractive index fluctuation variance $\langle n_1^2 \rangle$. We found that partially coherent four-petal Gaussian vortex beams can keep their initial four-petal profile in the short propagation distance, and the beam will evolve into the Gauss-like beam in the far-field due to the influence of turbulent plasma. The influences of turbulent plasma parameters on the coherence properties of partially coherent FPG vortex beams have been illustrated, it is found that the degree of coherence has an oscillatory behavior will decrease as the distance between two points increases, and decrease slower at the larger propagation distance, and it may disappear for small anisotropy parameter $\xi_x$ and outer scale $L_0$ or for large refractive index fluctuation variance $\langle n_1^2 \rangle$. It is to be noted that these results may be useful for optical communications.

*Disclosures

the authors declare no conflicts in interest

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How to cite this article:
A. A. Alkelly, M. A. H. Khaled and L. F. Hassan, “INFLUENCE OF ANISOTROPIC TURBULENT PLASMA ON THE PARTIALLY COHERENT FOUR-PETAL GAUSSIAN VORTEX BEAMS”, Electron. J. Univ. Aden Basic Appl. Sci., vol. 3, no. 3, pp. 145-151, Sept. 2022. DOI: https://doi.org/10.47372/ejua-ba.2022.3.179

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