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Alpha Beta Gamma Filter for Cascaded PID Motor Position Control

Khin Hooi Ng*, Che Fai Yeong, Eileen Lee Ming Su, Liang Xuan Wong

Faculty of Electrical Engineering, Universiti Teknologi Malaysia, 81310 Skudai, Johor, Malaysia

Abstract

PID controller is normally used in position control e.g. motor position control. However, in real application, a motor is non-linear due to factors like friction and power saturation. Therefore, in this paper, cascaded PID controller is proposed to reduce these nonlinearities. The cascaded PID controller was used because of the fast response and better disturbance rejection. The error from the motor position is fed into the first PID, and the output from the PID controller is used as velocity reference for the second PID. With these characteristics of a cascaded control, the nonlinearity in a motor can be reduced. Implementation of PID controller is done in a digital form and usually, the derivative elements are subject to noise from sampling time error and quantization error. To solve this problem, the alpha beta gamma filter is used to estimate the velocity of the motor. Experiment was conducted using a microcontroller to show the effectiveness of cascaded control in reducing nonlinearity of a motor, and the alpha beta gamma filter was implemented to reduce sampling time and quantization error. Results show an overdamped system with no steady state error was produced.

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1. Introduction

The permanent magnet direct current motor is popular because of the ease of control and good torque regulation [1,2]. In applications like robotics, the direct current motor is used to manipulate a robotic arm. The dc motor can comes in many shapes and sizes, makes the development of dc motor for robotics application quite easy and flexible.

In amateur robotics, the dc motor is often used because it is easily control with a H-bridge. By changing the polarity of the H-bridge, the motor direction can be changed. And by manipulating the applied voltage, the speed of motor can be varied.

In the field of amateur robotics, all the robots are usually self-fabricated. This makes the load of the motor is highly nonlinear mainly due to the friction. As example of robotics application, in the Asia Pacific Broadcasting Union Robot Contest (ABU ROBOCON), mobile robots were constructed to complete given tasks in specific time [3].

The problem of motor position control is nontrivial in the presence of such nonlinearities. Given example of motor in paper like [2,4], these plants were manufactured with high accuracy, thus reducing the nonlinearities. Besides, the load is non-varying and the friction can be easily observed.

Compared it to a robotic arm manipulator in Fig. 5, different position of the slider might give different value of friction. Besides, the load might different because of pick and place operation. This makes linear controller like the PID controller does not perform well in general.

* Corresponding author. Tel.: +60-12-8055021
E-mail address: khng8@live.utm.my
Besides, the whole mobile robot task has to be programmed in microcontrollers. Advance nonlinear controller like [5] might not be an attractive solution due to its complexity. The solution for this problem must be computational cheap in terms of memory space and execution time.

In this paper, the cascaded controller was proposed to overcome the high nonlinearity in the process plant. By closing internal dynamics with additional feedback loop, the whole system can be improved.

Besides that, PID controller can be easily implemented in a microcontroller because of its simple algorithm. But to add another feedback loop requires additional sensor. Fortunately for the nature of a dc motor, the velocity can be used to close the loop of the internal dynamics.

By estimating the velocity from the motor position reading, another PID can be implemented in cascade. But by differentiating the position numerically usually cause noises. To solve this, the alpha beta gamma filter was used [6–8].

Alpha beta filter was developed in the 1960’s and it has been used widely in radar track-while-scan task. It is famous in the early ages because complex computational facilities and the Kalman filter have not yet been invented [9]. The modified alpha beta gamma filter is used to estimate the motor velocity because of additional acceleration state that gives better estimation than an alpha beta filter.

Section 2 describes the direct current motor model and the internal dynamics which enables the cascaded control to be applied. Section 3 describes the alpha beta gamma filter and the estimation problem it can overcome. Section 4 describes the cascaded PID controller where the position controller is cascaded with a velocity controller. Section 5 describes the experiment setup. Section 6 describes the experiment setup and results from those experiments. This paper is concluded in Section 7.

2. Direct Current Motor Model

The model used for the PMDC motor is presented here. Fig. 1 shows the motor schematic. The second order model is used for the controller design for this paper. Because the inductance of the armature is generally much smaller than the resistance, \( L_a \ll R_a \), it can be assumed zero [10].

By using Kirchoff’s Voltage Law and Newton’s Second Law, it is not difficult to derive the transfer function of the motor where the input is applied voltage and output is the motor angle [10]

\[
G(s) = \frac{\theta_m}{v_a} = \frac{K_i}{R_a J_m} \left[ 1 + \frac{K_i K_e}{R_a} \right] \quad (1)
\]

where \( \theta_m \) is the motor angle, \( v_a \) is the applied voltage, \( K_i \) is the torque constant, \( K_e \) is the speed constant. \( R_a \) is the armature resistance, \( J_m \) is the load inertia, and \( B_m \) is the viscous damping coefficient.

Transfer function in (1) can be split into two using Eq. (2) where the motor velocity, \( \omega_m \), can be used as feedback for a cascaded motor control system.
\[
\theta_m = \frac{1}{\omega_m} s
\]  

(2)

3. Alpha Beta Gamma Filter

The alpha beta gamma filter was used to estimate the position and velocity of the motor. This method is used to smooth out the position and velocity reading of the motor due to its simplicity. Compared to a Kalman filter, this method is computational cheaper.

The algorithm to estimate the position and velocity is [11,12]

\[
x_p(k+1) = x_p(k) + T v_p(k) + \frac{1}{2} T^2 a_p(k)
\]  

(3)

\[
v_p(k+1) = v_p(k) + T a_p(k)
\]  

(4)

\[
a_p(k+1) = a_p(k) + \alpha (x_o(k) - x_p(k))
\]  

(5)

\[
v_s(k) = v_p(k) + \frac{\beta}{T} (x_o(k) - x_p(k))
\]  

(6)

\[
a_s(k) = a_p(k) + \frac{\gamma}{2T^2} (x_o(k) - x_p(k))
\]  

(7)

where (3), (4) and (5) are the one step ahead prediction and (6), (7) and (8) are the smoothing equations. \( q_p = [x_p, v_p, a_p]^T \) is prediction state while \( q_s = [x_s, v_s, a_s]^T \) is the estimated state.

Equation (3) through (8) was arranged in z-domain, and the transfer function for \( x_p/x_o \) is [12]

\[
G(z) = \frac{\left(\alpha + \beta + \frac{\gamma}{4}\right) z^{-2} + \left(-2\alpha - \beta + \frac{\gamma}{4}\right) z + \alpha}{z^3 + \left(\alpha + \beta + \frac{\gamma}{4} - 3\right) z^{-2} + \left(-2\alpha - \beta + \frac{\gamma}{4} + 3\right) z + \alpha - 1}
\]  

(9)

and the stable range for \( \alpha, \beta, \) and \( \gamma \) are

\[
0 < \alpha < 2
\]

\[
0 < \beta < 4 - 2\alpha
\]

\[
0 < \gamma < \frac{4\alpha\beta}{2 - \alpha}
\]

There are two reasons to use a smoothing function like the alpha beta gamma filter. Firstly, it is used to smooth out white noise from the measurements. And due to the attachment of the encoder to the motor actuated slider, bumps might occur which can gives false reading. Secondly, reading from the encoder is in digital form. Therefore, it is prone to sampling time and quantization error.

Simulations were done to show the effects of changing sampling time and quantization level. In this simulation, the true measures is \( x = A \sin(t) \), the derivative \( v = A \cos(t) \) and double derivative, \( a = -A \sin(t) \). Normalized root mean squared error (NRMSE) was used to evaluate the performance of filter given different sampling time and quantization error.

Note that the purpose of this simulation is to show the effect of sampling time and quantization error to the NRMSE. There is not attempt to search for the optimal value. For this simulation, all the filter gains can be set to any arbitrary value as long as they stay in the range of stability. And for example here, they were set to \( \alpha = 0.8 \), \( \beta = 0.3 \), and \( \gamma = 0.01 \). The NRMSE equation for position and velocity estimate is in (10) and (11).
The first simulation is to evaluate the performance of alpha beta gamma filter in presence of different sampling time. In this simulation, the sampling times were varied from 0.001 to 1 second in logarithm scale. The level of quantization was fixed at 200. Quantization was approximated with the rounding function.

The second simulation is to evaluate the performance of alpha beta gamma filter in presence of different quantization level. For the second simulation, the quantization levels were varied from 10 to 10000. The sampling time was fixed at 0.001 second.

As comparison, the position estimate will be used directly from the quantized value as in equation (12). The three-point first derivative formula (13) is used to obtain the velocity from the position measure. This is performed in the third and fourth simulation.

As for comparison, in the third simulation, no filter was used. The first simulation was repeated where the position estimate were taken from the quantized position while velocity was estimated with three points first derivatives. The formulas are

\[
NRMSE, x = \frac{1}{n} \sqrt{\frac{1}{n} \sum_{i=1}^{n} (x_i - x_i^*)^2} 
\]

\[
NRMSE, v = \frac{1}{n} \sqrt{\frac{1}{n} \sum_{i=1}^{n} (v_i - v_i^*)^2} 
\]

In the third simulation, the quantization level is fixed at 200 and the sampling time is varied from 0.001 to 1 second, which is similar with the first simulation. For the final simulation, the sampling time is fixed at 0.001 second while the quantization levels were varied from 10 to 10000, which is similar to the second simulation.

![Fig. 2. Performance of position and velocity estimates with numerical method and alpha beta gamma filter given different sampling time](a) (b) (c) (d)
as small as possible. With the filter, although the minimum error in position estimate (Fig. 2(c)) is not as small as shown in Fig. 2(a), the error in velocity estimate is reduced significantly as shown in Fig. 2(d).

Likewise for different quantization levels as shown in Fig. 3, alpha beta gamma filter has better performance. Although there are not much different in the position estimate as shown in Fig. 3(a) and 3(c), the error in the velocity estimate can be reduced significantly.

Because a microcontroller is used in this research, only moderate sampling time can be achieved due to the slow processing speed of the microcontroller. Besides, due to the digital domain of the computer control, the measurements are subjected to quantization error. Therefore, alpha beta gamma filter is used to improve the estimation of position and velocity. Besides that, this filter is chosen because it is computational cheap, compared to other filters like a Kalman filter.

![Fig. 3. Performance of position and velocity estimates with numerical method and alpha beta gamma filter given different quantization level](image)

4. Cascaded PID Controller

A cascaded controller has the advantage of faster response and better at disturbance rejection [13,14]. This is because the internal dynamic was controlled by additional controller by adding more sensor feedback. For a motor, the velocity can be feedback to the controller forming the internal loop of the cascaded control system.

In this case, adding another sensor would be redundant because the velocity can be estimated from the position by differentiating it. But differentiating numerically (13) might produce noises. This is when the alpha beta gamma filter is used to have a better estimate of the velocity.

The proposed cascade controller is depicted in Fig. 4. And as performance measure, the NRMSE will be used again. In this case, the error is obtained by subtracting the desired motor position and the actual motor position. For comparison, a single loop controller was experimented too.
5. Experiment Setup

For this experiment, a mechanical slider actuated by a power window motor is used. A quadrature encoder with 500 pulses per revolution is attached to the slider. One revolution is equivalent to 10.5cm linear movement in the slider. The microcontroller used for this experiment is the dsPIC33FJ64GS610 microcontroller from Microchip Technology. The setup is shown in Fig. 5.

This slider is part of a mobile robot used for the ABU ROBOCON 2011 in Bangkok [3]. The slider is constructed out of pulleys, attached to the motor by couplings. Since this robot is self-fabricated, the friction of the mechanical slider is highly nonlinear and hard to be estimated. Different position of the slider might give different friction reading. This is when the cascaded controller can be used to reduce the effect of friction and other unmodelled nonlinearities. Four different settings of are used in this experiment where three of them are single loop controller while another one is a cascaded PID controller.

6. Results and Discussions

In the first experiment, a single loop PID is used to control the motor position and the Kp is set to 400 and Ki to 1. The result as shown in Fig. 6(a), there is some parts with large steady state error due to motor dead zone caused by friction. Increasing the Ki to 100 as depicted in Fig. 6(b) improves the system by reducing the steady state error but it does not totally eliminate them.

Again, by increasing the integral gain to 1000 as depicted in Fig. 6(c) will eliminate the dead zone but it causes the overshoot to increase. By using the cascaded PID control, the system has been improved. The position PID’s Kp is set to 8 and Ki to 0.3 while the velocity PID’s Kp is set to 8 and the Ki is set to 2.5. The result is depicted in Fig. 6(d). The dead zone is almost eliminated and it does not cause large overshoot. In fact, the system itself is overdamped, which is more preferable than a system that is underdamped.
7. Conclusion and Recommendations

Cascaded PID controller has been proposed to solve a motor position problem with high nonlinearity in the friction. It has the advantage of faster response and better disturbance rejection, in this case the dead zone caused by friction.

To implement the cascaded PID, additional velocity feedback is required. It can be achieved by adding a tachometer. However, in this project, the velocity is estimated from the position obtained from motor encoder by using a filter. This filter is shown to be better at velocity estimation than differentiation by numerical method.

Future works might include the parameter estimation of the motor position control system and to find the optimum gains for the PID controller. By optimizing the gain, the motor position control can be improved.

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