Nearly Perfect Raman Self-Cleaning in Graded-Index Multimode Fibers Using Pearcey-Gaussian Pulses

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ABSTRACT We propose a temporal manipulation scheme for the inner-mode interactions in graded-index multimode fibers (GRIN-MMF). In this scheme, a so-called Pearcey-Gaussian (PeG) wave packet is constructed for the pulse profile, which allows the manipulation of the self-compressing control by the second- and third-order dispersions. Our simulations show that such linear compressing cooperates with the nonlinear compressing of the Kerr effect when the wavelength is close to the zero-dispersion point of the fiber, which enables the manipulation of the nonlinear energy transfer from the higher modes to the fundamental one. Such cooperation of the linear and nonlinear effects triggers a Stokes soliton in the fundamental mode while the Raman red-shifts in the higher modes are negligible. After the proper filter in the spectrum, a nearly perfect Raman self-cleaning with the energy of the fundamental mode reaching 97.2 % is observed.

INDEX TERMS Raman self-cleaning, Pearcey-Gaussian pulse, multimode fibers (MMF).

I. INTRODUCTION
In recent years, the multimode optical fibers (MMF) have drawn a great deal of attention for their potential to break the Shannon limit through spatial-division multiplexing, and the MMF are served as a convenient platform to study complex space-time nonlinear dynamics. Various nonlinear propagation phenomena have been theoretically predicted to occur in multimode fibers, and recently some of them were actually observed. For instance, multimode optical solitons [1], [2], dispersive waves, geometric parametric instabilities [3], four-wave mixing [4] between modes, and spatial beam self-cleaning [5].

The self-cleaning effect refers to the complex spatiotemporal nonlinear coupling between modes, which eventually results in the energy of high-order modes transferring and concentrating in the fundamental mode [6]. Specifically, a speckled output intensity pattern will transfer into a bell-shaped beam close to the fundamental mode through the self-cleaning effect. So far, there is some research focusing on the nonlinear beam reshaping and self-cleaning effect in graded-index multimode fiber (GRIN-MMF) for the reason that the self-cleaning effect shows many potential applications like high-power fiber lasers. Recently, the simulation results based on the coupled-mode theory reported by Krupa et al. are consistent with experiments, which shows a transfer of energy from higher-order modes (HOMs) to the fundamental mode through the Kerr nonlinearity [7], [8]. Meanwhile, they have also shown that the previously unrecognized process of disorder-induced acceleration of condensation and experimentally demonstrate the analogous hydrodynamic 2D turbulence [9] can be utilized to explain the phenomenon of beam self-cleaning. Besides, the Frank Wise et al. present observations of beam self-cleaning in experiments performed with femtosecond-duration pulses in the normal-dispersion regime [10], [11]. However, when the power is high enough, stimulated Raman scattering (SRS) [12], [13] will also cause a Raman self-cleaning process which is different from the Kerr self-cleaning [14]. Among them, SRS is a nonlinear optical process.
When the intensity of pump light exceeds a certain threshold, the pump photons will generate Stokes photons at longer wavelengths, and it has been regarded as a method to generate lightwave new wavelengths. In recent years, the self-cleaning of light beams caused by SRS has attracted extensive attention. According to a report by Terry et al. [15], SRS only shows excellent beam cleaning characteristics in GRIN-MMF, which makes the multimode pump beam produce a Stokes beam with good beam quality. Thus, a multimode pump beam in GRIN-MMF can be used to generate a Stokes beam which near-single mode Stokes beam [16].

Additionally, Frank Wise et al. have also mentioned that with the decrease of core-diameter of MMF, the interaction between nonlinear modes will be accelerated [17]. When injecting a beam into the large core side of a multimode fiber taper, the self-imaging period is directly proportional to the core diameter, which contributes to accelerating the formation of the self-imaging. Kerr beam self-cleaning can obtain high brightness and high beam quality in MMF. In addition, the usage of tapered fibers also enhances the nonlinear effect from a spatial perspective and improves the quality of Kerr self-cleaning [18].

The pulse dynamics is also abundant in the fibers when the high-order dispersions [19] or a specially-constructed pulse profile [20] is considered. Very recently, the Pearcey wavepacket has been described in the pulse dynamics in the single-mode fibers [21], [22]: the so-call Pearcey-Gaussian (PeG) pulses possess a remarkable self-compressing property under the action of the second-order dispersion: it reaches the tight compressing point, then undergoes an inversion and mirrors its previous dynamics. Further, it has been found that the self-compressing property can be modulated by the additional third-order dispersion. For a given multimode fiber, therefore, one can tune the wavelength of the PeG pulse near the zero-dispersion point of the fiber to achieve the modulation of the self-compressing. Such specially-constructed temporal wave packet could cooperate with the nonlinear compressing from the Kerr effect, which could enable the manipulation of the nonlinear energy transfer from the higher modes to the fundamental one.

In this paper, we propose a novel manipulation scheme for the inner-mode interaction in the temporal domain in the GRIN-MMF by introducing the PeG pulses. Our simulations show that such cooperation of the linear and nonlinear effects triggers a Stokes soliton in the fundamental mode while the Raman red-shifts in the higher modes are negligible. After the proper filter in the spectrum, a nearly perfect Raman self-cleaning with the energy of the fundamental mode reaching 97.2 % is observed.

**II. PROPAGATION MODAL**

In this section, we briefly review the model of the nonlinear pulse propagation in an MMF, as mentioned by Poletti and Horak [23]. Their approach is based on the pioneering work of Kolesik and Moloney [24]. And then we adopt this model to simulate PeG pulse propagation in the GRIN-MMF. The multimode generalized nonlinear Schrodinger equation (MM-GNLSE) for pulse propagation in MMF can be written as:

\[
\frac{\partial A_p(z,t)}{\partial z} = i \left( \beta_0^{(p)} - \beta_0^{(0)} \right) A_p(z,t) - \left( \beta_1^{(p)} - \beta_1^{(0)} \right) \frac{\partial A_p(z,t)}{\partial t} + \sum_{n \geq 2} \frac{\beta_n^{(p)}}{n!} \left( i \frac{\partial}{\partial t} \right)^n A_p(z,t) + \left( 1 + \frac{i \tau_{plmn}^{(1)}}{\partial t} \right) Q_{plmn}^{(1)}(\omega_0) P_{lmn}^{(1)} + \left( 1 + \frac{i \tau_{plmn}^{(2)}}{\partial t} \right) Q_{plmn}^{(2)}(\omega_0) P_{lmn}^{(2)}
\]

\[
D^{(p)}(z,t) + N^{(p)}(z,t)
\]

where \(A_p(z,t)\) is slowly varying amplitude of pulse envelope, \(\beta_0^{(0)}\) is propagation constant, \(n_2\) is nonlinear coefficient, \(\omega_0\) is the carrier frequency of the initial pulse. \(D^{(p)}(z,t)\) and \(N^{(p)}(z,t)\) refer to the dispersive and nonlinear part of the MM-GNLSE for the \(p\) mode, respectively. The nonlinear filed terms are given by

\[
P_{lmn}^{(1)}(\omega_0) = 2A_l(\zeta,t) \int d\tau R(\tau) A_m(\zeta,t-\tau) A_n^*(\zeta,t-\tau)
\]

\[
P_{lmn}^{(2)}(\omega_0) = A_l^*(\zeta,t) \int d\tau R(\tau) A_m(\zeta,t-\tau) A_n^*(\zeta,t-\tau) e^{2i\omega_0 t}
\]

and the nonlinear coupling terms are given by

\[
Q_{plmn}^{(1)}(\omega) = \frac{1}{3} \int dx dy F_m^*(\omega) F_l(\omega) F_n(\omega)
\]

\[
Q_{plmn}^{(2)}(\omega) = \frac{1}{3} \int dx dy F_m^*(\omega) F_l^*(\omega) F_n(\omega)
\]

the nonlinear response function can be written as

\[
R(\tau) = (1-f_h)\delta(\tau) + f_h h(\tau)
\]

where \(f_h \approx 0.18\) is the fractional contribution of the Raman response to the total nonlinearity and \(h(\tau)\) is the delayed Raman response function. The shock time constants are given by

\[
\tau_{plmn}^{(1,2)} = \frac{1}{\omega_0} + \left\{ \frac{\partial}{\partial \omega} \ln \left[ Q_{plmn}^{(1,2)}(\omega) \right] \right\}
\]

The basic expression of the forward and the backward Pearcey-Gaussian (PeG±) pulses are in the following form [22]:

\[
PeG^\pm(t) = Pe^\pm(t) \exp \left[ \frac{-\left( t - \tau \right)^2}{2T_G^2} \right]
\]

where \(\tau\) and \(T_G = 0.5\) ps are respectively the delay of the Gaussian function and the pulse width of the Gaussian profile. For the positive third-order dispersion, we should consider the PeG± pulse as the initial profile, which is given by

\[
A_p(0,t) = \sqrt{P_p} PeG^\pm(t),
\]
where \( P = 30kW \) is the initial peak power of PeG\(^+\) pulse for the \( p\)-th mode. In our simulation, we only consider the first five cylindrical symmetric modes [25] \( L_{P_0}(x = 1, 2, 3, 4, 5) \) for simplicity and to ensure reasonable computational time. We use Eq. (1) to describe the propagation of the PeG\(^+\) pulses close to the zero-dispersion point wavelength (1330nm) in a GRIN-MMF with a core diameter of 62.5\( \mu \)m and a length of 5m, \( D^{(p)}(\tau, t) \) and \( N^{(p)}(\tau, t) \) denotes dispersion and nonlinearity terms of the MM-GNLSE [26], [27]. Here, we only consider \( \beta_2^{(p)} \) and \( \beta_3^{(p)} \), of which the values are shown in Table 1.

**TABLE 1. Dispersion value of different modes.**

| \( \beta_2(\text{ps}^2/\text{m}) \) | \( \beta_3(\text{ps}^3/\text{m}) \) |
|-----------------|-----------------|
| \( L_{P_0} \)   | \( L_{P_2} \)   | \( L_{P_3} \)   | \( L_{P_4} \)   | \( L_{P_5} \)   |
| \(-5.6289\)     | \(-5.4121\)     | \(-5.560\)      | \(-5.7252\)     | \(-5.8318\)     |
| 87.2501         | 87.5560         | 87.8834         | 88.1954         | 87.5984         |

The temporal and spectral distributions of the PeG\(^\pm\) pulses are shown in Figs. 1(a) and (b) (solid lines), we also introduced Gaussian pulses for comparison (dash lines). It can be seen that the PeG pulses consist of a main lobe and several minor lobes.

**III. RESULTS AND ANALYSIS**

In Fig. 2, we carry out respectively the numerical simulation of the PeG\(^+\) and Gaussian pulses at the wavelength close to the zero-dispersion point in GRIN-MMF. With the increasing of propagation distance, it is observed that the energy transfer appears on both PeG\(^+\) and Gaussian pulses. As shown in Fig. 2(a) (energy ratio diagram), the energies of higher-order modes significantly transfer to the fundamental mode, and eventually most of the energy populates in the fundamental mode. The similarly important features have been predicted for the so-called classical wave condensation [28]. It is clear that the energy transfer is saturated for the PeG\(^+\) pulse, which indicates that the Raman effect occurs to suppress the Kerr self-cleaning [14]. In Fig. 2(b), we also present the evolutions of the pulse intensity for different modes as the function of the propagation distance. It is interesting that the intensity of the fundamental mode for the PeG\(^+\) pulse is higher than that for the Gaussian pulse when the pulses propagate to \( z = 1.8m \), which the energies for the higher modes are lower than those of the Gaussian pulse. Consider the evolutions of the pulse energies and intensities for different modes, we can speculate that the Raman effect only occurs in the fundamental modes for the PeG\(^+\) pulse, while that occurs in the higher mode(s) for the Gaussian pulse.

In order to ensure the speculation above, we show the temporal and the spectral distributions of both the PeG\(^+\) pulse and the Gaussian pulse in Fig. 3, respectively. It is clear shown in the Figs. 3(a) and (b) that a significant first-order Stokes soliton is observed in the fundamental mode for the PeG\(^+\) pulse, while the Raman effect is negligible in other high-order modes. For the case of the Gaussian pulse, the Raman effect occurs in the fundamental and the second mode, as shown in Figs. 3(c) and (d). This can be revealed in the evolution of the pulse intensity shown in Fig. 2(b) that the intensity of the fundamental mode for the PeG\(^+\) pulse abruptly increases at the beginning of the pulse propagation and then fluctuates rapidly while those of the high-orders modes vary slowly, and such rapid fluctuation of the pulse intensity occurs not only in the fundamental mode (\( L_{P_0} \)) but also in the second high-order mode (\( L_{P_2} \)). We also present in Fig. 4 the evolutions of the spectral distributions of the fundamental and the second high-order mode for the PeG\(^+\) and the Gaussian pulses. It is clear that the Raman effect in the pulse propagation of the second mode can hardly be observed, which can be attributed to the asymmetric distribution of the PeG pulse [22], where the Raman threshold is much higher than the Gaussian or Gaussian-like distributions.
The above discussions indicate that a Stokes soliton with the energy occupied in the fundamental mode can be obtained via a proper spectral filter. The results are shown in Fig. 5. The energy of the Stokes soliton in the fundamental mode is 2.82nJ while that in the second high-order mode is only 0.08nJ, and those of other high-order modes are even lower than 0.01nJ. Therefore, the energy ratio of the fundamental mode of the Stokes soliton reaches 97.2%, showing a near-perfect self-cleaning during modal interaction. The filtered mode of the Stokes soliton reaches 97.2%, showing a near-perfect self-cleaning during modal interaction. The filtered light spot and its transverse distribution are also presented, which reveals the near-perfect Gaussian profile.

FIGURE 4. The spectral evolution of Stokes soliton for a PeG\(^+\) pulse. (a) LP\(_{01}\), (b) LP\(_{02}\), respectively. The spectral evolution of Stokes soliton for a Gaussian pulse. (c) LP\(_{01}\) (d) LP\(_{02}\), respectively.

In our simulations above, the wavelength of 1330nm is specially chosen, which is very closed to the zero-dispersion point of the fiber at 1270nm. It has been mentioned that such manipulation scheme of the modal interaction is due to the self-compressing control by the cooperation of the second- and the third-order dispersions, therefore the wavelength for the given fiber would affect the modal interaction. Here we would like to show two examples at two other different wavelengths of 1270nm and 1550nm which are respectively at the zero-dispersion point of the fiber and far away from it. The spectral distributions of the five modes at z = 5m for the cases of 1270nm and 1550nm are respectively shown in Figs. 6(b) and (d). It is shown that the energy of high-order mode of the Stokes soliton can be ignored for the case of 1270nm, while that for the case of 1550nm is observable.

FIGURE 5. The temporal and spectral evolution of Stokes soliton along the propagation distance of 5m. (a) The temporal Stokes soliton for a PeG\(^+\) pulse. (b) The spectral distribution of energy of five modes filtered Stokes soliton for a PeG\(^+\) pulse. (c) Near-filed beam profiles for filter five modal Stokes soliton. (d) The transverse intensity distribution of unfiltered and filtered Stokes soliton.

FIGURE 6. (a) The second-order dispersion of the given fiber. (b) The spectral distribution of five modes for PeG\(^+\) pulse at the wavelength of 1270nm. (c) The spectral distribution of five modes for PeG\(^+\) pulse at the wavelength of 1550nm.

In conclusion, the propagation of the Pearcey-Gaussian (PeG) pulse in a GRIN-MMF is investigated. Since the self-compressing of the PeG pulses can be manipulated by the cooperation of the second- and the third-order dispersions, we can carefully adjust the central wavelength of the PeG pulse near the zero-dispersion point of the given fiber to control its nonlinear propagation and then further influence the inter-mode interaction. The numerical results show that the PeG\(^+\) pulse of the fundamental mode is compressed faster compared with the Gaussian pulse with the same energy, and the Raman effect is predominant in the fundamental mode while those in the higher-order modes are negligible, which results in a nearly perfect Raman self-cleaning with the energy of the fundamental mode reaching 97.2%. Note that this effect can only be observed for the PeG\(^+\) pulse since the third-order dispersion is positive for the given fiber in our simulation. It is known from our previous work that the TOD accelerates/deaccelerates the self-compressing for the PeG\(^+/\)PeG\(^-\) pulse when the signs of SOD and TOD are opposite. For the PeG\(^-\) pulse, therefore, the negative third-order dispersion should be required to achieve such results. It is also worth mentioning that only the cylindrical symmetric modes are considered in this work, and the radially asymmetric modes would make the results much more complicated, which will be discussed in our future work.

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