The lattice QCD simulation of the quark-gluon mixed condensate $g \langle \bar{q} \sigma_{\mu\nu} G_{\mu\nu} q \rangle$ at finite temperature and the phase transition of QCD

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The thermal effects on the quark-gluon mixed condensate $g \langle \bar{q} \sigma_{\mu\nu} G_{\mu\nu} q \rangle$, which is another chiral order parameter, are studied using the SU(3) lattice QCD with the Kogut-Susskind fermion at the quenched level. We perform the accurate measurement of $g \langle \bar{q} \sigma_{\mu\nu} G_{\mu\nu} q \rangle$ as well as $\langle \bar{q} q \rangle$ for $0 \leq T \leq 500$ MeV. We observe the sharp decrease of both the condensates around $T_c \approx 280$ MeV, while the thermal effects below $T_c$ are found to be weak. We also find that the ratio $m_0^2 \equiv g \langle \bar{q} \sigma_{\mu\nu} G_{\mu\nu} q \rangle / \langle \bar{q} q \rangle$ is almost independent of the temperature even in the very vicinity of $T_c$, which indicates that the two condensates have nontrivial similarity in the chiral behaviors. We also present the correlation between the condensates and the Polyakov loop to understand the vacuum structure of QCD.

In order to understand the nature of QCD and the relation to hadron phenomenology, the nonperturbative nature is one of the most important characteristics of QCD. The nonperturbative phenomena, such as spontaneous chiral-symmetry breaking and color confinement, also bring the rich structure to the QCD phase diagram. For example, at high temperature, QCD is believed to exhibit phase transition into QGP, where chiral symmetry is restored and the color is deconfined. To realize these phenomena, the RHIC experiments are in progress, which attempt to produce QGP in the laboratory. For the theoretical study of the finite temperature QCD and the phase transition of QCD, we focus on the thermal effects on the condensates. In fact, condensates directly characterize the nontrivial QCD vacuum and thus can indicate the change of the vacuum structure at finite temperature.

Among various condensates, we study the quark-gluon mixed condensate $g \langle \bar{q} \sigma_{\mu\nu} G_{\mu\nu} q \rangle$ at finite temperature as a relevant physical quantity on the chiral structure of the QCD vacuum. Here, the mixed condensate is another chiral order parameter of the QCD vacuum, since the chirality of the quark in $g \langle \bar{q} \sigma_{\mu\nu} G_{\mu\nu} q \rangle$ flips as $g \langle \bar{q} \sigma_{\mu\nu} G_{\mu\nu} q \rangle = g \langle \bar{q} R (\sigma_{\mu\nu} G_{\mu\nu} ) q L \rangle + g \langle \bar{q} L (\sigma_{\mu\nu} G_{\mu\nu} ) q R \rangle$. We emphasize that $g \langle \bar{q} \sigma_{\mu\nu} G_{\mu\nu} q \rangle$ characterizes different aspect of the QCD vacuum from $\langle \bar{q} q \rangle$, because $g \langle \bar{q} \sigma_{\mu\nu} G_{\mu\nu} q \rangle$ reflects the direct correlation between color-octet components of $q-\bar{q}$ pairs and the spontaneously generated gluon field, while $\langle \bar{q} q \rangle$ reflects only the color-singlet $q-\bar{q}$ components.

We further note that the mixed condensate is important quantity in hadron phenomenology through the QCD sum rule framework. In fact, it is known that $g \langle \bar{q} \sigma_{\mu\nu} G_{\mu\nu} q \rangle$ has large effects in the QCD sum rule especially for baryons, such as $N-\Delta$ splitting and parity splitting. Recently, it is also shown that $g \langle \bar{s} \sigma_{\mu\nu} G_{\mu\nu} s \rangle / \langle \bar{s} s \rangle$ is a key quantity for the prediction on the parity of the penta-quark baryon, $\Theta^+(1540)$. To study the thermal effects on $g \langle \bar{q} \sigma_{\mu\nu} G_{\mu\nu} q \rangle$, we use lattice QCD Monte Carlo simulation, which is the direct and nonperturbative calculation from QCD. So far, the lattice QCD studies for $g \langle \bar{q} \sigma_{\mu\nu} G_{\mu\nu} q \rangle$ have been performed at zero temperature by a pioneering but rather preliminary work, and by the recent works of our group using the KS fermion, and of another group using the Domain-Wall fermion. At finite temperature, however, there has been no result on $g \langle \bar{q} \sigma_{\mu\nu} G_{\mu\nu} q \rangle$ except for our early reports. Therefore, we present the extensive results of the thermal effects on $g \langle \bar{q} \sigma_{\mu\nu} G_{\mu\nu} q \rangle$ with the analysis near the critical temperature.

We evaluate the condensates $g \langle \bar{q} \sigma_{\mu\nu} G_{\mu\nu} q \rangle$ as...
well as \langle \bar{q}q \rangle using the SU(3)_c lattice QCD at the
quenched level. The Monte Carlo simulations are performed with the
standard Wilson action for \( \beta = 6.0, 6.1 \) and 6.2. The lattice units are
obtained as \( a^{-1} = 1.9, 2.3 \) and 2.7GeV for \( \beta = 6.0, 6.1 \) and 6.2,
respectively, so as to reproduce the string tension \( \sqrt{\sigma} = 427 \text{MeV} \) [8]. We
perform the calculation at various temperatures as
\( 0 \lesssim T \lesssim 500 \text{MeV} \) using the following lattices,

(i) \( \beta = 6.0, 16^3 \times N_t \ (N_t = 16, 12, 10, 8, 6, 4) \),
(ii) \( \beta = 6.1, 20^3 \times N_t \ (N_t = 20, 12, 10, 8, 6) \),
(iii) \( \beta = 6.2, 24^3 \times N_t \ (N_t = 24, 16, 12, 10, 8) \).

We generate 100 gauge configurations for each
lattice. Moreover, we generate 1000 gauge configurations
which are continuously connected to the trivial
vacuum \( U_\mu = 1 \). For the fermion action, we employ
the KS fermion to preserve the explicit chiral symmetry for the quark mass \( m = 0 \), which
is desirable for the study of chiral order parameters.

In the calculation of the condensates on
the lattice, we use the formula as
\( a^3 \langle \bar{q}q \rangle = -\frac{1}{4} \sum_f \text{Tr} \left[ \{ \bar{q}f(x)qf(x) \} \right] \),
\( a^3 \langle \bar{q}q \rangle G_{\mu\nu} = -\frac{1}{4} \sum_f, \mu, \nu \text{Tr} \left[ \{ \bar{q}f(x)qf(x) \} \right] G_{\mu\nu}^\text{lat}(x) \),
where SU(4)_f quark-spinor fields, \( q \) and \( \bar{q} \), are
converted into spinless Grassmann KS fields and
the gauge-link variable in the actual calculations.

The more detailed formula are given in Ref. [6].

In each configuration, we measure the condensates
on 16 (\( \beta = 6.0 \)) or 2 (\( \beta = 6.1, 6.2 \)) different
points which are taken so as to be equally spaced
on the 4-dimensional lattice [33]. Therefore, we
achieve high statistics as 1600 data at \( \beta = 6.0 \)
and 200 data at \( \beta = 6.1 \) and 6.2. Note that for
the lattices of \( 20^3 \times 8 (\beta = 6.1) \) and \( 24^3 \times 10 \)
(\( \beta = 6.2 \)), we obtain 2000 data, which guarantees
reliability of the results even in the vicinity of \( T_c \).

We calculate the condensates at each quark
mass of \( m \simeq 20, 35, 50 \text{MeV} \). At each temperature,
we observe that both the condensates show
a clear linear behavior against \( m \), and therefore
we fit the data with a linear function and determine
the condensates in the chiral limit. We es-
imate the statistical error to be 5-7% level using
jackknife error method. The finite-volume arti-
fact is estimated to be about 1% level, through a
check of boundary condition effects [6,9].

We evaluate the thermal effects on each condensate,
\( g(\bar{q}q_{\mu\nu}G_{\mu\nu}q)/\langle \bar{q}q \rangle \) or \langle \bar{q}q \rangle, by taking the ratio
between the values at finite and zero tempera-
tures. Note that the renormalization constants
cancel in these ratios. In figure 1 we plot the
thermal effects on \( g(\bar{q}q_{\mu\nu}G_{\mu\nu}q)/\langle \bar{q}q \rangle \) plotted against temperature \( T \).
The vertical dashed line denotes the critical tem-
perature \( T_c \). We per-
Figure 1. \( g(\bar{q}q_{\mu\nu}G_{\mu\nu}q)/\langle \bar{q}q \rangle \) plotted against temperature \( T \).

Figure 2. \( m_0^2(T) \equiv g(\bar{q}q_{\mu\nu}G_{\mu\nu}q)/\langle \bar{q}q \rangle \) plotted against \( T \).

We observe that \( m_0^2(T) \) is almost in-

\( m_0^2(T) \equiv g(\bar{q}q_{\mu\nu}G_{\mu\nu}q)/\langle \bar{q}q \rangle \) plotted against temperature \( T \).

This result indicates the same chiral behavior between
\( g(\bar{q}q_{\mu\nu}G_{\mu\nu}q)/\langle \bar{q}q \rangle \) and \langle \bar{q}q \rangle.
dependent of the temperature, even in the very vicinity of $T_c$. This nontrivial result can be interpreted that $g\langle \bar{q}q_{\mu\nu}G_{\mu\nu}\rangle_T$ and $\langle \bar{q}q\rangle_T$ have similarity in the chiral behaviors.

To understand this nontrivial similarity, we express the condensates as $\langle \bar{q}q\rangle = \frac{1}{V} \int d\lambda' \frac{m_{\mu\nu}(\lambda')}{\lambda'^2 + m^2}$, $g\langle \bar{q}q_{\mu\nu}G_{\mu\nu}\rangle = \frac{1}{V} \int d\lambda' \frac{m_{\mu\nu}(\lambda')}{\lambda'^2 + m^2} \langle \lambda' | \sigma_{\mu\nu}G_{\mu\nu} | \lambda' \rangle$, where $\rho(\lambda)$ denotes the eigenvector of the Dirac operator and $\rho(\lambda)$ the spectral density on $\Lambda$. Using these formula, the common thermal behavior is understood in the way that the thermal effects are dominated by $\rho(\lambda)|_{\lambda=0}$ and $\langle \lambda | \sigma_{\mu\nu}G_{\mu\nu} | \lambda \rangle|_{\lambda=0}$ has remarkably weak dependence on $T$.

This result further implies that the thermal effects hardly appear in the local gluon field strength, but appear in the global structure of the vacuum such as topological quantities. To proceed, we plot the time history of $g\langle \bar{q}q_{\mu\nu}G_{\mu\nu}\rangle$ and the Polyakov loop in figure 3, and find the strong correlation between $g\langle \bar{q}q_{\mu\nu}G_{\mu\nu}\rangle$ and the Polyakov loop. For the gauge configuration with a small (large) value of $g\langle \bar{q}q_{\mu\nu}G_{\mu\nu}\rangle$, the Polyakov loop takes a large (small) value. One of the possible explanation is that the thermal effects are dominated by the quark propagation making a circuit in time direction, which can directly reflect the topological structure of the vacuum.

In summary, we have studied thermal effects on $g\langle \bar{q}q_{\mu\nu}G_{\mu\nu}\rangle$ using the SU(3)$_c$ lattice QCD with the KS fermion at the quenched level. We have observed a clear signal of chiral restoration as a sharp decrease of $g\langle \bar{q}q_{\mu\nu}G_{\mu\nu}\rangle$ as well as $\langle \bar{q}q\rangle$, while the thermal effects have been found to be small for $T \lesssim 0.9T_c$. We have also found that $m_0^2(T) \equiv g\langle \bar{q}q_{\mu\nu}G_{\mu\nu}\rangle_T / \langle \bar{q}q\rangle_T$ is almost independent of $T$ in the entire region up to $T_c$, which indicates that these chiral condensates show a common thermal behavior. The strong correlation between the condensates and the Polyakov loop has been also observed. For further studies, full QCD lattice calculations are interesting to analyze dynamical quark effects on the condensates.

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