Precision determination of band offsets in strained InGaAs/GaAs quantum wells by C-V-profiling and Schrödinger-Poisson self-consistent simulation

V. I. Zubkov, M. A. Melnik, A. V. Solomonov, and E. O. Tsvelev
St. Petersburg State Electrotechnical University, Prof. Popov str. 5, 197376, St.-Petersburg, Russia

F. Bugge, M. Weyers, and G. Tränkle
Ferdinand-Braun-Institut für Höchstfrequenztechnik, Albert-Einstein-Str. 11, D-12489 Berlin, Germany

(Dated: October 29, 2018)

The results of measurements and numerical simulation of charge carrier distribution and energy states in strained quantum wells In$_x$Ga$_{1-x}$As/GaAs (0.06 ≤ x ≤ 0.29) by C-V-profiling are presented. Precise values of conduction band offsets for these pseudomorphic QWs have been obtained by means of self-consistent solution of Schrödinger and Poisson equations and following fitting to experimental data. For the conduction band offsets in strained In$_x$Ga$_{1-x}$As/GaAs - QWs the expression ∆$E_C(x) = 0.814x - 0.21x^2$ has been obtained.

PACS numbers: 73.21.Fg, 81.07.St

I. INTRODUCTION

Since the development of semiconductor heterostructures the determination of energy band discontinuities of various semiconductor pairs has been a very important task. Energy band offsets dominantly control the electronic states in heterostructures and, hence, the output parameters of semiconductor devices. The importance of getting true values of band offsets as well as the difficulties in obtaining and, even more, in interpreting the relevant data have been attracting attention for the last 30 years. R. Dingle was one of the first who reported in 1974-75 the value of band offsets for isoperiodic heterosystem (Al-Ga)As/GaAs (“Dingle” rule 85:15). Then H. Kroemer, G. Duggan and Yu et al comprehensively reviewed the understanding of band offsets before 1991 and provided an overview of the methods commonly used in experimental band offset determination, mostly optical at that time. At the same time, the authors and others showed that a low sensitivity of the optical transition energies to the band offsets made its determinations rather confusing. Up to now a great number of papers has been published on this subject (see bibliography in recent comprehensive review). So far, however, as was pointed out in the review, among the ternary alloys used in quantum electronics, only the AlGaAs/GaAs system has generally accepted values of band offsets.

For one of the most important used heteropairs – In$_x$Ga$_{1-x}$As/GaAs – as yet no clear picture about the dependence of band offsets on alloy composition has been obtained, despite the very intensive investigations in last years. The data collected by P. Bhattacharya show a great scatter of the values of relative conduction band offset ∆$E_C$ between 35% and 85% for x < 0.35. Above mentioned review reports relative conduction band offsets for the In$_x$Ga$_{1-x}$As/GaAs – system in the range 57–90% and recommends as a rule of thumb ∆$E_C$[eV] = x for x < 0.5. They conclude that no detailed study has yet been carried out on InGaAs-based heterojunctions. Recent publications on this subject only present partial results for different compositions, more or less agreed with “recommended” in Ref. 9. Theoretic calculations give the valence band offset for the end combination InAs/GaAs ∆$E_V$ = 0.06 eV, which is in serious disagreement with experimental data.

One important device application of the heterosystem InGaAs/GaAs is high power laser diodes with strained quantum wells. In these structures thin quantum-size layers of InGaAs grow pseudomorphically, i.e. having the lattice constant of the underlying GaAs-layer in the plane of the heterojunction. The elastic energy, accumulated due to crystal cell distortion, causes the band structure of the thin InGaAs layer to be modified, altering particularly its energy gap. Hence, in strained InGaAs/GaAs quantum wells one should expect another band offsets than in heterostructures with thick layers of the solid solution. In cases between pseudomorphic growth and full strain relaxation (occurs in thick layers) the band offset in InGaAs/GaAs will have, obviously, some intermediate values. This fact explains, we suppose, the variety of data found in literature.

Numerical fitting of C-V-curves by means of self-consistent solution of Schrödinger and Poisson equations is one of the most promising approaches to measure band offsets of quantum well structures. This approach correctly takes the quantization of carriers in a quantum well into consideration and yields very accurate results. However, well-defined heterostructures are necessary for this.

Complex multilayer structures like multiQWs etc. and unknown dopant profile or the presence of deep levels add sources of uncertainties. Therefore, in order to be sure to get precise values for band offsets at heterojunctions simple structures with a minimum of unknown parameters or parameters to be fitted should be used.

This work presents accurate data for band offsets in heterostructures with strained pseudomorphic...
In$_x$Ga$_{1-x}$As/GaAs ($0 < x < 0.3$) quantum wells. To obtain these values we have carried out a systematic cycle of C-V-measurements on specially fabricated structures. Details of sample preparation and measurements are described in Section II. In Section III the model for simulating measured concentration profiles and deriving the values of conduction band offsets based on self-consistent numerical solution of the Schrödinger and Poisson equations is described. The carrier concentration in the quantum well region is calculated on the base of a quantum-mechanical approach. Mathematical aspects of the computations are presented in Sections III A and III B. To increase the accuracy of the numerical calculations a non-uniform mesh with the mesh step inside the quantum well 10 times smaller than in the other regions has been used. Finally, in Section IV we present the results of numerical fitting of experimentally measured C-V curves. The dependence of conduction band offset for strained pseudomorphically grown In$_x$Ga$_{1-x}$As/GaAs - QWs has been obtained as a function of quantum well composition in the range $0.06 \leq x \leq 0.29$.

II. SAMPLE STRUCTURE AND MEASUREMENT PROCEDURE

A special set of high quality samples with a simplified structure (Fig. 1) containing In$_x$Ga$_{1-x}$As/GaAs quantum wells of different width ($w = 6.0 - 9.5$ nm) and composition ($x = 0.065 - 0.29$) was grown on $n^+$-GaAs substrates by MOVPE at deposition temperatures of 650°C and 770°C. The GaAs cladding layers were uniformly doped with Si, except for the QWs themselves and thin (5 nm) spacer layers on both sides of the quantum well. To get the best experimental results and to eliminate possible uncertainties in the subsequent numerical fitting, the cap GaAs layer was designed to be 300 nm thick and have a constant doping level of $(6 - 7) \times 10^{16}$ cm$^{-3}$. The width and composition of the QWs and cladding layers have been determined by high resolution X-ray diffraction (HRXRD). All QWs were fully strained without any relaxation seen in X-ray area maps. Ag-Schottky barriers were fabricated on top of the structures and Ohmic contacts were formed on the substrate.

The parameters of the grown structures are listed in Table II.

The measurements of capacitance-voltage characteristics and profiling of majority carriers in the quantum wells have been carried out with the help of a computer-controlled C-V-profilometer at a testing frequency of 1 MHz and with an amplitude of the probing signal of 15 or 50 mV.

At zero bias the width of the space charge region under the Schottky-barrier in the samples was less than the thickness of the cap GaAs-layer. With increasing reverse bias the space charge region was broadened and its border crossed the quantum well. The C-V-characteristics of all samples clearly exhibit a plateau in the range of $0.06 \leq x \leq 0.29$.

![Layer sequence of the grown samples with In$_x$Ga$_{1-x}$As/GaAs – quantum wells.](image1)

![C-V-characteristics of In$_{0.225}$Ga$_{0.775}$As/GaAs QW at different temperatures (Sample #307).](image2)
TABLE I: Characteristics of the structures and results of numerical simulation of conduction band discontinuities in In$_x$Ga$_{1-x}$As/GaAs strained quantum wells grown by MOVPE.

| Sample # | x   | T deposition (°C) | d cap layer (µm) | QW width (nm) | E of bound level at 0V (meV) | ∆E$_C$ (meV) |
|----------|-----|------------------|-----------------|---------------|-------------------------------|-------------|
| 298      | 0.065 | 770             | 0.304           | 9.5           | -10.9                         | 57          |
| 299      | 0.14  | 770             | 0.304           | 8.0           | -29.0                         | 110         |
| 308      | 0.145 | 650             | 0.302           | 6.0           | -25.0                         | 110         |
| 303      | 0.145 | 650             | 0.305           | 7.5           | -32.1                         | 120         |
| 309      | 0.145 | 650             | 0.302           | 9.5           | -35.1                         | 120         |
| 306      | 0.2   | 770             | 0.298           | 6.5           | -39.5                         | 155         |
| 307      | 0.225 | 770             | 0.308           | 7.4           | -48.8                         | 175         |
| 300      | 0.23  | 770             | 0.304           | 7.2           | -48.5                         | 175         |
| 301      | 0.27  | 770             | 0.300           | 6.5           | -54.6                         | 210         |
| 305      | 0.29  | 650             | 0.300           | 6.0           | -55.3                         | 220         |

Here $U$ is the applied voltage, and $\varphi_{bi}$ is the built-in potential.

In addition, the matching conditions for the potential at both heterointerfaces have to be fulfilled

$$\epsilon_{\text{barr}} \frac{d\varphi_{\text{barr}}}{dz} = \epsilon_{\text{well}} \frac{d\varphi_{\text{well}}}{dz}.$$  

The indexes "barr" and "well" correspond to the regions of GaAs barrier and InGaAs quantum well, respectively.

The free carrier concentration $n(z)$ in the far from the QW can be calculated as in the case of a homogeneous structure through the Fermi integral

$$n(z) = N_C \frac{2}{\sqrt{\pi}} F_{1/2} \left( -\frac{E_C - E_F - e\varphi(z)}{kT} \right),$$

where $N_C$ is the effective density of states in the conduction band, $E_F$ is the Fermi level, $T$ is the temperature, and $k$ is the Boltzmann constant. In contrast, in the vicinity of a quantum well the carrier concentration should be calculated by solving Schrödinger’s equation. The needed spatial distribution of the electrostatic potential was derived using a procedure of self-consistency of the Schrödinger and Poisson equations. The essence of the procedure is the sequential (step-by-step) solution of Schrödinger and Poisson equations until convergence. As a criterion of convergence we took an increment of the potential of less than $10^{-8}$ V in the next iteration.

Quantum size effects are important only inside the quantum well and in its immediate vicinity. So for numerical solution of the Schrödinger equation we used a "quantum box", a narrow region containing the QW (Fig. 4). The optimal width of the quantum box was chosen experimentally during simulations, in order to achieve a high precision in determination of the quantized carrier concentration, and, in balance, to reduce the computing time. The quantum box width was chosen as

\[ d = \frac{\epsilon_0 A}{C}. \]
FIG. 3: C-V-profiling of In$_x$Ga$_{1-x}$As/GaAs quantum wells. Samples #296–#309. a) Common view of apparent carrier concentrations for several samples, T=300K; b) Apparent concentration peak values as a function of $x$.

Note that near $x = 0.14$ there are 4 samples with different width of QW.

According to the boundary conditions, there must be nodes of the wave functions at the edges of the quantum box. For this reason the data calculated close to the quantum box boundaries are dropped. The length of this region is no more than 1 $W$, as can be seen from Fig. 4. On the other hand, at distances about 2–3 $W$ from the QW the quantization effect is very weak, and there we can use the Fermi integral (7) for deriving the free carrier concentration. The coincidence of the concentration profiles at this part derived from quantum-mechanical approach and from (7) was used as the proof for a true solution.

A. Solving the Poisson equation

The Poisson equation has been solved numerically by Newton’s method relative to the correction term.

The great difference (tenfold) in the values of electron and hole effective masses in GaAs and nearby ternary InGaAs alloys makes the Fermi level shift toward the bottom of the conduction band. Therefore, a significant part of donors (up to 30%) remains non-ionized at room temperature, even despite the very low ionization energy of Si donors in GaAs (about 5 meV). Because of this, the incomplete donor ionization has to be taken into account.

To reduce the computation time, it is desirable to use some approximation of the Fermi integral (7). The simplest exponential approximation is not applicable here because of the close position of the Fermi level to the bottom of the conduction band. There is another well known approximation for (7) by the expression

$$n(z) = \frac{N_C}{C_n + \exp \left( \frac{E_C - E_F - e\varphi(z)}{kT} \right)},$$

that better matches the Fermi integral. The constant $C_n$ here usually falls between 0.17 and 0.35.

In order to minimize the approximation error and to fulfill the electroneutrality condition on the right-hand side of the simulated region (i.e. in the GaAs substrate) we used the following procedure: by solving the electroneutrality equation and using expression (7) the Fermi
level position is determined at \( \varphi = 0 \). Then equating (7) to (8) one can derive the current adaptivity constant \( C_n \). At another \( \varphi \) the maximum relative error of such approximation does not exceed \( 3 \times 10^{-4} \).

The electrostatic potential was written as an initial approximation \( \varphi_0(z) \) and a correction term \( \Delta \varphi(z) \):

\[
\varphi(z) = \varphi_0(z) + \Delta \varphi(z).
\]

To linearize the Poisson equation (8) the expression for \( \varphi_0(z) \) was decomposed into a Taylor series including linear term relative to the correction \( \Delta \varphi(z) \).

Then a finite-difference analog of the Poisson equation (9) was obtained using the condition of normalizing the wave functions inside and outside the quantum box. The number of points in the mesh was 8000, including about 1500 in the quantum box. The Gauss method was applied to solve the system with some modifications based on obvious symmetry of the equations.

After getting the correction \( \Delta \varphi(z) \) a new potential was obtained according to (9).

### B. Solving the Schrödinger Equation

The effective mass, one-dimensional Schrödinger equation can be written as:

\[
-\frac{\hbar^2}{2m^*(z)} \frac{d^2 \psi_i(z)}{dz^2} + V(z) \psi_i(z) = E_i \psi_i(z),
\]

where \( E_i \) are the eigenvalues, \( \psi_i \) are the corresponding eigenvectors, \( m^* \) is the coordinate-dependent electron effective mass. \( V(z) \) is the effective potential energy:

\[
V(z) = \begin{cases} 
    e\varphi(z) + \Delta E_C & \text{inside QW}, \\
    e\varphi(z) & \text{outside QW}.
\end{cases}
\]

\( \Delta E_C \) is the conduction band offset.

We used boundary conditions of Neuman’s type at the ends of the quantum box.

The finite-difference analog for (10) was obtained using the three-point formula

\[
-\frac{\hbar^2}{2m_J} \frac{\psi_{i,j-1} + \psi_{i,j+1} - 2\psi_{i,j}}{h_J^2} + V_J \psi_{i,j} = E_i \psi_{i,j},
\]

where \( j \) identifies the point on the one-dimensional mesh, and \( h_J \) is the distance between the mesh nodes (a step of the mesh).

In addition to boundary conditions, at the heterojunctions the following matching conditions should be maintained between the derivative of wave functions inside and outside the quantum well:

\[
\frac{1}{m_{\text{barr}}} \frac{\Delta \psi_{\text{barr}}}{h_{\text{barr}}} = \frac{1}{m_{\text{well}}} \frac{\Delta \psi_{\text{well}}}{h_{\text{well}}},
\]

The Schrödinger equation (10) was solved numerically by the well known "shooting" method with some improvements aimed to reduce the computation time.

The number of points in the mesh should be enough to eliminate the error due to substitution of the derivative with the finite-difference approximation (12). We compared the results of numerical solution of (10) by the shooting method with the well known analytical solution for a rectangular quantum well. It was found that the mesh size of about 1500 points yields quite good accuracy with a relative error in eigenvalue determination less than \( 10^{-3} \) for almost all levels.

After the set of eigenvalues \( E_i \) and corresponding eigenvectors \( \psi_i(z) \) had been obtained, the carrier concentration in the region of QW was calculated via local density of states from the expression:

\[
n(z) = \frac{m^*(z)kT}{\pi\hbar^2} \sum_i \ln \left[ 1 + \exp \left( \frac{E_F - E_i}{kT} \right) \right] |\psi_i(z)|^2,
\]

using the condition of normalizing the wave functions:

\[
\int_{-\infty}^{+\infty} |\psi_i(z)|^2 dz = 1.
\]

Summation in (14) runs over all subbands.

The prefactor in the expression (14) before the square of wave function is considered as the number of electrons per unit area in the \( i \)th subband.

In the quantum well region the concentrations of both bound and free electrons were calculated from the Schrödinger equation. We consider this a more correct approach than the simple summation of bound (inside QW) and free carriers (above QW) used, for example, in Ref. 22. But, due to the finite width of the quantum box the continuum of free electron states in this scheme of computations is represented by a set of discrete levels with energies determined by the size of the quantum box.

In order to sum up all charge carriers in the quantum well region we took into account the 16 lowest energy levels. Fig. 6 shows the occupation of the 16 energy levels at different bias. The carrier concentration in the first subband \( n_1 \) was at least ten times greater than in the second one, and the concentration in 16th subband (and all higher subbands) is below \( 10^{-7} \) of \( n_1 \) and can be neglected.

The results of the computation of energy states in dependence of applied reverse bias for the sample #300 (\( \Delta E_C = 175 \text{ meV} \)) are shown in Fig. 6(a), and a lineup of the conduction band bottom for this structure is depicted in Fig. 6(b). As can be seen, a single bound level is observed in the structure with an energy of 49 meV in equilibrium. (The bottom of the conduction band in the electroneutrality region was taken as zero). Starting approximately at \(-2.5 \text{ V} \) the space charge regions of the Schottky barrier and the QW merge (Fig. 7), and the penetrating electric field bends the conduction band bottom near the QW, forcing the bound level to lift up. At \( U = -5 \text{ V} \) the level becomes unbound.
To calculate the C-V characteristics so called "quasistatic approach" was applied. The capacitance of a structure is the first derivative of the total charge. The latter can be derived via the flow of electric field across the surface according to the Gauss theorem. The spatial distribution of electrostatic potential $\varphi$ is calculated during solution of the Poisson equation, so one can derive the value of the electric field at the surface at different applied biases:

$$E_{\text{surf}} = \frac{\varphi_1 - \varphi_0}{h_{\text{bulk}}}, \quad (16)$$

and, hence, build up the capacitance-voltage characteristic (or restore the apparent concentration profile) using $\mathbf{I}$.

IV. RESULTS OF SIMULATION AND DISCUSSION

As has been established earlier there exists a certain discrepancy between the true and "apparent" concentration profiles of free charge carriers near a heterojunction, a quantum well or a quantum dot. An apparent profile, obtained in experiment, is more smeared in comparison to the true one and has a shift in the peak position (see Fig. 1). The general reason for this discrepancy is the indirect and non-equilibrium procedure of concentration profile restoration from C-V-measurements. Generally, this technique involves differentiation of the C-V-curve $\mathbf{I}$ in the approximation of fully depleted space charge region and does not take into account the problem of Debye smearing. In the case of QW profiling, where one expects sub-Debye resolution, this standard technique leads to an essential distortion of the apparent profile. So, for the goal of adequate fitting, during simulations we must accomplish just the same procedure of restoring the apparent profile as in real experiment and, particularly, the bias voltage increment in the simulation must be equal to the voltage step used in the experiment.

The results of fitting for two samples are presented in Figs. 8 and 9. As can be seen, excellent matching is obtained. This proves the correctness of the used model. One should underline again that due to the high quality of the specially fabricated for C-V-measurements samples no additional adjustable parameters like an impurity concentration gradient or a charge at the heterojunction had...
to be used in the fitting procedure. The only fitting parameter was the conduction band offset, $\Delta E_C$. The value of majority carrier concentration was taken on the shoulders of the measured concentration profiles. Parameters for In$_x$Ga$_{1-x}$As, needed for the calculations, were taken from.\textsuperscript{25,34} Fig. 8 also demonstrates the resolution of our fitting. For a medium In-content ($x = 0.14$) the error was less than 10 meV. It was found that the resolution is approximately directly proportional to the alloy composition of the quantum well. In general, we estimate the relative error in the determination of band offsets as less than 10% within the measured range of $x$.

An interesting example of fitting for the sample with the smallest In-content in QW ($x = 0.065$, sample #298) is presented in Fig. 9. Here the apparent peak of enrichment in the QW is even smaller than the value of the impurity concentration, despite the spatial confinement inside the QW. The simulated profile in this case is very sensitive to the band offset (the error is about 5 meV), however, the fitting is not as good as for other compositions. For such a weak concentration peak the presence of residual impurities in the quantum well and in adjacent spacers begins to play an essential role. One also should bear in mind for explanation the increased relative value of the experimental noise.

In Table I we summarized the conduction band offsets in strained pseudomorphically grown In$_x$Ga$_{1-x}$As/GaAs ($0.06 \leq x \leq 0.29$) quantum wells obtained in our study. Only one bound level was observed in all samples. Its depth in equilibrium ($U = 0$ V) is depicted in Fig. 11 as a function of composition. One can see that for compositions $x < 0.25$ the level appears above the corresponding Fermi level. Nevertheless, the occupation in the subband

FIG. 7: Simulated concentration profiles of electrons in the region of QW at $U = -1$ V (solid) and $-2.5$ V (dashed line). Sample #300.

FIG. 8: Experimental (dots) and fitted (solid) apparent concentration profiles of In$_{0.23}$Ga$_{0.77}$As/GaAs quantum well. (Sample #300, $T=300$K). Best fit $\Delta E_C = 175$ meV.

FIG. 9: Resolution of the fitting. The results for the best fit ($\Delta E_C = 120$ meV, solid line) and for $\Delta E_C = 130$ meV (dashed line) are presented. Dotted curve — the experimental apparent profile for sample #303 ($x = 0.145$).
remains significant to provide an excess of apparent carrier concentration in QW region over the dopant value in all samples, except for $x = 0.065$.

From Fig. 11 it can be seen that there is no bound energy level for $x < 4\%$. Indeed, the weak doping in the adjacent to QW spacers leads to an additional conduction band bending near the QW, which lifts the energy level up. The effect of disappearing bound level does not exist if the spacers are absent (at least, down to extremely low $x$, about 1\%, when errors in numerical calculations begin to occur).

In Fig. 12 the results on conduction band offsets in strained In$_x$Ga$_{1-x}$As/GaAs-QWs obtained during the numerical fitting to the experimental C-V characteristics are presented. The ”recommended” curve from the above mentioned review is also depicted. The ”recommended” values of band offsets are higher by about 25% in comparison to our results for strained quantum wells. The origin of this difference most probably is the presence of significant degree of elastic strain in pseudomorphic InGaAs on GaAs. The deformation potential in compressively strained InGaAs modifies the energy line-ups of the heterostructure. Estimations on the base of model-solid theory predict an increase of the band gap of compressively strained InGaAs in comparison to the value $\Delta E_g$ for bulk material (this effect again is of the order of 25%). So, the absolute values of band discontinuities are smaller in compressively strained quantum wells than in relaxed single heterostructures or thick double heterostructures. Recent results of other researchers

![FIG. 10: Apparent concentration profile of In$_x$Ga$_{1-x}$As/GaAs quantum well with low In-content $x_{In} = 0.065$ (dotted) and fitted curve (solid) with $\Delta E_C = 55$ meV. (Sample #298, T=300K).](image)

![FIG. 11: Position of bound energy level $E_1$ in strained In$_x$Ga$_{1-x}$As/GaAs-QWs (triangles up for $w \leq 7.2$ nm and triangles down for $w > 7.2$ nm). Solid line — calculated dependence $E_1$ of $x_{In}$ in the assumption $w = 7.2$ nm and for the doping concentration as in the sample #300. Dash-dot line — the corresponding position of Fermi level.](image)

![FIG. 12: Conduction band offsets in strained In$_x$Ga$_{1-x}$As/GaAs QWs as a function of composition. Dashed line — as recommended in Ref. 9. Open circles — the latest results of capacitance and optical investigations on strained In$_x$Ga$_{1-x}$As/GaAs–QWs: 1 – Ref. 14; 2 – Ref. 18; 3 – Ref. 13; 4 – Ref. 11.](image)
\( \Delta E_C(x) = 0.814x - 0.21x^2 \) for the conduction band offsets in strained In\(_x\)Ga\(_{1-x}\)As/GaAs quantum wells in the composition range \(0 < x < 0.3\).

### V. SUMMARY

Aiming to get accurate and precise values for conduction band offsets, a set of high quality samples containing strained In\(_x\)Ga\(_{1-x}\)As/GaAs quantum wells was grown in the composition range \(0.06 \leq x \leq 0.29\). Specially for C-V-measurements a constant impurity concentration in the cladding layers was maintained during the growth in order to eliminate uncertainties in subsequent numerical simulations. A fitting procedure of experimentally obtained apparent concentration profiles has been implemented using self-consistent solution of Schrödinger and Poisson equations. All important information about the properties of the quantum well structures was derived: the majority carrier profiles, the positions of energy levels, corresponding wave functions, profile of the conduction band bottom, as well as the dependencies of the above mentioned parameters on the applied electric field. The presence of only one bound level was discovered in all samples. The conduction band offsets in strained In\(_x\)Ga\(_{1-x}\)As/GaAs quantum wells follow the expression \( \Delta E_C(x) = 0.814x - 0.21x^2 \).

1. R. Dingle, W. Wiegmann, and C. H. Henry, Phys. Rev. Lett. 33, 827 (1974).
2. R. Dingle, A. C. Gossard, and W. Wiegmann, Phys. Rev. Lett. 34, 1327 (1975).
3. H. Kroemer, Surf. Sci. 132, 543 (1983).
4. H. Kroemer, J. Vac. Sci. Technol. B 2, 433 (1984).
5. H. Kroemer, Surf. Sci. 174, 299 (1986).
6. G. Duggan, J. Vac. Sci. Technol. B 3, 1224 (1985).
7. E. T. Yu, J. O. McCalpin, and T. C. McGill, Solid State Lett. 183, 2093 (1998).
8. R. Dingle, W. Wiegmann, and C. H. Henry, Phys. Rev. Lett. 33, 827 (1974).
9. H. Kroemer, Surf. Sci. 132, 543 (1983).
10. I. Vurgaftman, J. R. Meyer, and L. R. Ram-Mohan, J. Appl. Phys. 80, 864 (1996).
11. F. Stern, Phys. Rev. B 5, 4892 (1972).
12. F. Stern and S. D. Sarma, Phys. Rev. 30, 840 (1984).
13. S. Adachi, Physics Properties of III-V Semiconductors Compounds (John Wiley & Sons, New York, 1999).
14. E. V. Zubkov and A. V. Solomonov, in Ternary and quaternary III-V compounds, pp. 244–247.
15. C. M. A. Kapteyn, F. Heinrichsdorff, O. Stier, R. Heitz, M. Grundmann, N. D. Zakharov, D. Bimberg, and P. Werner, Phys. Rev. B 60, 14265 (1999).
16. V. I. Zhukov and A. V. Solomonov, in Int. Symp. Nanostructures: Physics and Technology (St.Petersburg, 1997), p. 236.
17. M. Levinshtein, S. Rumyantsev, and M. Shur, eds., Ternary and quaternary III-V compounds, vol. 2 of Handbook series on semiconductor parameters (World Scientific, London, 1999).
18. J. H. Davies, The physics of low-dimensional semiconductors: an introduction (Cambridge University Press, 1998).
19. X. Letartre, D. Stevenard, and E. Barber, Appl. Phys. Lett. 58, 1047 (1991).
20. B. M. Tschirner, F. Morier-Genoud, D. Martin, and F. K. Reinhart, J. Appl. Phys. 79, 7005 (1996).