Mathematical modelling of torque vectoring differentials

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Abstract. Active safety systems which distribute forces between the wheels are an integral part of a modern vehicle. The vast majority of systems are based on the brake systems, and reducing engine power, which certainly reduces the vehicle speed. However, the needs of car owners are constantly changing, there is a need for devices capable of ensuring the distribution of traction between the wheels without having to reduce speed. With this task perfectly cope with the main transmission containing the mechanisms of power distribution (MPD). Power distribution mechanisms have a greater impact on vehicle handling. More often meet term, Torque Vectoring Differential (TVD). They began to be used for the first time since the mid-1990s, and found their application in sports cars, and then in executive cars. The described main gears are an integral part of dynamic stabilization systems. During the vehicle development, these systems are developed and investigated using mathematical modeling methods. Therefore, the paper describes the design, operation principle and kinematic design of the most common. The equations systems describing the dynamics of the links and the mechanism as a whole are also composed. The mathematical modeling of vehicles equipped with TVD is modelled.

1. Introduction

Modern cars are equipped with various dynamic stabilization systems based on the operation of the brake system or reducing engine power [1]. However, since the mid-1990s, Mitsubishi has been using main drives with power distribution mechanisms known as Torque Vectoring Differential (TVD). Their distinctive feature compared to conventional main gears, is that they are able to transmit more torque to the high-speed wheel, thus creating an increased turning torque of the vehicle. Power distribution mechanisms, in turn, contain additional links and controls that ensure the distribution of torques [1–10].

PDM containing friction elements which activated by an electronic control system. However, to ensure effective operation of the control system, first, at the design stage, conducting the mathematical simulation of the vehicle motion [11–15], equipped with such a mechanism. In the development of mathematical models of multilink final drives made the following assumptions: the links of the final drive perfectly rigid, the losses in the gearing can be neglected, the ratio auxiliary links taken greater than one.

Thus, the paper analyzes the existing mechanisms, their comparison and writing of equations systems for mathematical modeling in the program MATLAB & Simulink.

2. Final drive Mitsubishi AYC
The final drive of Mitsubishi AYC was first used in 1996 on the Mitsubishi Lancer Evolution IV. [2] This final drive is based on using bevel differential. PDM is located on the right and is connected by one link to the differential housing and the right axle through friction clutches.

2.1. Final drive design

Mitsubishi AYC final drive design shown on the figure 1.

![Figure 1. Mitsubishi AYC final drive design:](image)

1 – first auxiliary link;  
2 – second auxiliary link;  
3 – differential housing;  
4 – left half-axis;  
5 – right half-axis;  
CL1 – right clutch;  
CL2 – left clutch.

In the final drive, depending on right friction clutch CL1 engaged, part of the torque from the differential housing 3 is transmitted to the first auxiliary link 1 through a multiplier that accelerates the rotation of the right half-axis 5. The increase in torque is due to the summation of torques from the first auxiliary link 1 and the half-axial differential gear. Depending on left friction clutch CL2 engaged, part of the torque from the differential housing 3 will be transferred to the second auxiliary link 2 through the gearbox, slowing down the angular speed of the right half-axis 5. The resistance torque, which occurs on the left clutch CL2, creates a braking torque relative to the differential housing 3.

2.2. Final drive equations system

To compile an equations system revealed that the mechanism of five moving links and three gearing. Thus, it is necessary to make five dynamic equations and three kinematic equations. Let the first two equations (1, 2) describe the dynamics of the auxiliary links:

\[ J_1 \cdot \dot{\omega}_1 = M_{\text{link}1} - M_{\text{CL1}} \]  
\[ J_2 \cdot \dot{\omega}_2 = M_{\text{link}2} - M_{\text{CL2}} \]  

\[ J_1, \ J_2 \] first and second auxiliary links inertias, \( \text{kg} \cdot \text{m}^2 \);  
\[ \dot{\omega}_1, \ \dot{\omega}_2 \] first and second auxiliary links angular accelerations, \( \text{rad} / \text{s}^2 \);  
\[ M_{\text{link}1}, \ M_{\text{link}2} \] first and second auxiliary links torques \( \text{N} \cdot \text{m} \);  
\[ M_{\text{CL1}}, \ M_{\text{CL2}} \] first and second clutches friction torques, \( \text{N} \cdot \text{m} \).

In equation (3) torques \( M_{\text{link}1} \) and \( M_{\text{link}2} \) written below:

\[ J_w \cdot \dot{\omega}_{\text{right}} = \frac{M_{\text{diff}}}{2} - M_{\text{link}1} + M_{\text{link}2} - M_{\text{dr right}} \]  
\[ J_w \cdot \dot{\omega}_{\text{left}} = \frac{M_{\text{diff}}}{2} - M_{\text{dr left}} \]  

\[ J_w, \ \text{wheel inertia, \( \text{kg} \cdot \text{m}^2 \)}; \ \dot{\omega}_{\text{right}}, \ \dot{\omega}_{\text{left}}, \ \text{right and left wheel angular accelerations \( \text{rad} / \text{s}^2 \)}; \ M_{\text{diff}}, \ \text{differential housing torque, \( \text{N} \cdot \text{m} \)}; \ M_{\text{dr right}}, \ M_{\text{dr left}}, \ \text{right and left half-axis drag torques, \( \text{N} \cdot \text{m} \).} \]

Equations (5), (6), (7) describe links interaction which included in the final drive:
\[
\dot{\omega}_{\text{input}} = i_{FD} \cdot \frac{\dot{\omega}_{\text{right}} + \dot{\omega}_{\text{left}}}{2} 
\]  
(5)

\[
\dot{\omega}_{\text{input}} = i_{FD} \cdot \frac{\dot{\omega}_1}{u_1} 
\]  
(6)

\[
\dot{\omega}_{\text{input}} = i_{FD} \cdot \dot{\omega}_2 \cdot u_2 
\]  
(7)

\[\dot{\omega}_{\text{input}}, \text{ input shaft angular acceleration, } \text{rad} / \text{s}^2; \ i_{FD}, \text{ final drive gear ratio; } u_1, \ u_2, \text{ ratio between differential housing and first and second auxiliary links.}\]

The equation of dynamics of the differential housing (8), reduced to the input shaft, is written below:

\[
J_{\text{input}} \cdot \dot{\omega}_{\text{input}} = M_{\text{input}} - \frac{M_{\text{diff}}}{i_{FD}} - \frac{M_{\text{link}1}}{i_{FD} \cdot u_1} - \frac{M_{\text{link}2} \cdot u_2}{i_{FD}} 
\]  
(8)

The considered final drive equations system written bellow:

\[
\begin{align*}
J_1 \cdot \dot{\omega}_1 &= M_{\text{link}1} - M_{\text{CL1}}; \\
J_2 \cdot \dot{\omega}_2 &= M_{\text{link}2} - M_{\text{CL2}}; \\
J_w \cdot \dot{\omega}_{\text{right}} &= \frac{M_{\text{diff}}}{2} - M_{\text{link}1} + M_{\text{link}2} - M_{\text{dr}}; \\
J_w \cdot \dot{\omega}_{\text{left}} &= \frac{M_{\text{diff}}}{2} - M_{\text{left}}; \\
\dot{\omega}_{\text{input}} &= i_{FD} \cdot \frac{\dot{\omega}_{\text{right}} + \dot{\omega}_{\text{left}}}{2}; \\
\dot{\omega}_{\text{input}} &= i_{FD} \cdot \frac{\dot{\omega}_1}{u_1}; \\
\dot{\omega}_{\text{input}} &= i_{FD} \cdot \dot{\omega}_2 \cdot u_2; \\
J_{\text{input}} \cdot \dot{\omega}_{\text{input}} &= M_{\text{input}} - \frac{M_{\text{diff}}}{i_{FD}} - \frac{M_{\text{link}1}}{i_{FD} \cdot u_1} - \frac{M_{\text{link}2} \cdot u_2}{i_{FD}}
\end{align*}
\]  
(9)

3. Final drive Magna

The Magna final drive was first used in 2009 on the Audi S4 [2]. It contains a symmetrical conical differential and multipliers located on the right and left.

3.1. Final drive design

Magna final drive design shown on the figure 2.

In the Magna final drive, when the right clutch CL1 is activated, part of the torque from the differential housing 3 is transmitted to the first auxiliary link 1 through the first stage of the multiplier, then through the friction right clutch CL1 to the second stage of the multiplier. This accelerates the rotation of the right half-axis 5. The increase in torque is due to the summation of moments from the first auxiliary link 1 and the right half-axis 5. In the case of activation of the left clutch CL2 part of the torque from the differential housing 3 will be similarly transmitted to the left axle 4.
3.2. Final drive equations system

The equations describing the auxiliary units dynamics do not differ from the equations (1, 2) written above. Next, dynamics equations of the half-axis are written bellow:

\[
J_n \cdot \dot{\omega}_{n \text{ right}} = \frac{M_{\text{diff}}}{2} - \frac{M_{\text{link} 1}}{u_1} - M_{\text{dr right}}, \tag{10}
\]

\[
J_n \cdot \dot{\omega}_{n \text{ left}} = \frac{M_{\text{diff}}}{2} - \frac{M_{\text{link} 2}}{u_2} - M_{\text{dr left}}, \tag{11}
\]

\[ u_1, \quad u_2, \] accordingly ratio between right and left half-axis and first and second auxiliary links.

Next, kinematic equations for auxiliary links are written bellow:

\[
\dot{\omega}_{\text{input}} = i_{\text{FD}} \cdot \dot{\omega}_1 \cdot u_{\text{diff}}; \tag{12}
\]

\[
\dot{\omega}_{\text{input}} = i_{\text{FD}} \cdot \dot{\omega}_2 \cdot u_{\text{diff}}; \tag{13}
\]

\[ u_{\text{diff}} \] the gear ratio between the differential housing and supporting links. The dynamics equation of the differential housing, reduced to the input shaft, is written, as well as (8).

\[
\begin{align*}
J_1 \cdot \dot{\omega}_1 &= M_{\text{link} 1} - M_{\text{CL1}}; \\
J_2 \cdot \dot{\omega}_2 &= M_{\text{link} 2} - M_{\text{CL2}}; \\
J_n \cdot \dot{\omega}_{n \text{ right}} &= \frac{M_{\text{diff}}}{2} - \frac{M_{\text{link} 1}}{u_1} - M_{\text{dr right}}; \\
J_n \cdot \dot{\omega}_{n \text{ left}} &= \frac{M_{\text{diff}}}{2} - \frac{M_{\text{link} 2}}{u_2} - M_{\text{dr left}}; \\
\dot{\omega}_{\text{input}} &= \frac{i_{\text{FD}}}{2} \left( \dot{\omega}_{n \text{ right}} + \dot{\omega}_{n \text{ left}} \right); \\
\dot{\omega}_{\text{input}} &= i_{\text{FD}} \cdot \dot{\omega}_1 \cdot u_{\text{diff}}; \\
\dot{\omega}_{\text{input}} &= i_{\text{FD}} \cdot \dot{\omega}_2 \cdot u_{\text{diff}}; \\
J_{\text{inpu}} \cdot \dot{\omega}_{\text{input}} &= M_{\text{inpu}} - \frac{M_{\text{diff}}}{i_{\text{FD}}} - \frac{M_{\text{link} 1}}{i_{\text{FD}} \cdot u_{\text{diff}}} - \frac{M_{\text{link} 2}}{i_{\text{FD}} \cdot u_{\text{diff}}},
\end{align*}
\]
4. Final drive ZF Torque Vectoring

The final drive ZF Torque Vectoring was first introduced in 2008 \[2\] and was installed on BMW cars. Unlike the above-mentioned mechanisms, the power distribution is due to the planetary multipliers located on the right and left.

4.1. Final drive design

ZF Torque Vectoring final drive design shown on the figure 3.

Figure 3. ZF Torque Vectoring final drive design:
1 – right planetary multiplier carrier;
2 – left planetary multiplier carrier;
3 – differential housing;
4 – left half-axis;
5 – right half-axis;
6 – satellite unit;
7 – satellite unit;
B1 – right friction break;
B2 – left friction break.

In ZF Torque Vectoring through the activation of the right friction break B1, the right planetary multiplier carrier. Due to this, the gear right half-axis 5 is accelerated with respect to the differential housing 3. The increase in moments occurs by summing the moments from the half-axis gear and the multiplier. However, in this mechanism, the closing of the brake link occurs on the final drive housing, so part of the moment is transferred to the final drive housing. That is, the torque on the input shaft, reduced to the drive axle, is the sum of the moments on the half-axis 4 and 5 minus the friction moment in the brake friction. Similarly, there is an increase in speed and torque on the left axle 4.

4.2. Final drive equations system

The dynamics equations for links 15 and 16 are written below:

\[ J_{carr} \cdot \ddot{\omega}_{carr1} = M_{carr1} - M_{B1} \]  
\[ J_{carr} \cdot \ddot{\omega}_{carr2} = M_{carr2} - M_{B2} \]  

\[ J_{carr}, \text{ planetary multiplier carrier inertia, kg \cdot m}^2; \]  
\[ \dot{\omega}_{carr1}, \dot{\omega}_{carr2}, \text{ first and second planetary multiplier carrier angular accelerations, rad / s}^2; \]  
\[ M_{carr1}, M_{carr2}, \text{ first and second planetary multiplier carrier torques, N} \cdot \text{m}; \]  
\[ M_{B1}, M_{B2}, \text{ first and second planetary multiplier carrier friction breaks torques, N} \cdot \text{m}. \]

Next, dynamics equations of the half-axes are written below:

\[ J_w \cdot \ddot{\omega}_{w\text{right}} = \frac{M_{\text{diff}}} {2} - M_{\text{halfaxis sun right}} - M_{\text{dr right}} \]  
\[ J_w \cdot \ddot{\omega}_{w\text{left}} = \frac{M_{\text{diff}}} {2} - M_{\text{halfaxis sun left}} - M_{\text{dr left}} \]  

\[ M_{\text{halfaxis sun right}}, \text{ right half-axle shafts sun gear torque 5}; \]  
\[ M_{\text{halfaxis sun left}}, \text{ left half-axle shafts sun gear torque 4}; \]  
\[ N \cdot \text{m}. \]
Below are the kinematics equations for planetary mechanisms:

\[
\dot{\omega}_{\text{input}} = i_F D \cdot \frac{\dot{\omega}_{\text{right}} + k \cdot \dot{\omega}_{\text{car1}}}{1 + k} \tag{19}
\]

\[
\dot{\omega}_{\text{input}} = i_F D \cdot \frac{\dot{\omega}_{\text{left}} + k \cdot \dot{\omega}_{\text{car2}}}{1 + k} \tag{20}
\]

The equations system describing ZF Torque Vectoring is written below:

\[
\begin{align*}
J_{\text{car1}} \cdot \dot{\omega}_{\text{car1}} &= M_{\text{car1}} - M_{B1}; \\
J_{\text{car2}} \cdot \dot{\omega}_{\text{car2}} &= M_{\text{car2}} - M_{B2}; \\
J_w \cdot \dot{\omega}_{\text{right}} &= \frac{M_{\text{diff}}}{2} - M_{\text{halfass right}} - M_{\text{dr right}}; \\
J_w \cdot \dot{\omega}_{\text{left}} &= \frac{M_{\text{diff}}}{2} - M_{\text{halfass left}} - M_{\text{dr left}}; \\
\dot{\omega}_{\text{input}} &= i_F D \cdot \frac{\dot{\omega}_{\text{right}} + \dot{\omega}_{\text{left}}}{2}; \\
\dot{\omega}_{\text{input}} &= i_F D \cdot \frac{\dot{\omega}_{\text{right}} + k \cdot \dot{\omega}_{\text{car1}}}{1 + k}; \\
\dot{\omega}_{\text{input}} &= i_F D \cdot \frac{\dot{\omega}_{\text{left}} + k \cdot \dot{\omega}_{\text{car2}}}{1 + k}; \\
J_{\text{input}} \cdot \dot{\omega}_{\text{input}} &= M_{\text{input}} \frac{M_{\text{diff}}}{i_F D} - M_{\text{car1}} \frac{M_{\text{car1}}}{i_F D} - M_{\text{car2}} \frac{M_{\text{car2}}}{i_F D}.
\end{align*}
\tag{21}
\]

5. Final drive mathematical modelling in MATLAB\&Simulink

In this section, the simulation of the vehicle motion [16–20] with AWD system, on the drive axis of which is the final drive with PDM. The final drive Mitsubishi AYC is used as an example. The total vehicle weight is 1700 kg.

5.1. Study of the movement of vehicles with and without PDM
The vehicle motion on the road "ice with snow" (with the coefficient of friction of the wheel with the support \( \mu_{s,\text{max}} = 0.35 \)). The front wheels are steering. Start speed \( v = 25 \text{ km} / \text{h} \), and limited \( v = 35 \text{ km} / \text{h} \). Figure 4 shows graphs of vehicle yaw rate.

![Figure 4. Vehicle yaw rate [rad/s]:
1 – equipped with conventional final drive,
2 – equipped with conventional final drive Mitsubishi AYC](image-url)
6. Conclusion
In the paper presents the analysis of kinematic designs of power distribution mechanisms, their purpose, design and principle of operation. Also derived equations systems that can be used for mathematical modeling and study of the vehicle motion. The vehicles yaw rates are presented.

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