MSSM from AdS/CFT

Paul H. Frampton\textsuperscript{(a)} and Thomas W. Kephart\textsuperscript{(b)}

\textsuperscript{(a)}Department of Physics and Astronomy,
University of North Carolina, Chapel Hill, NC 27599.

\textsuperscript{(b)}Department of Physics and Astronomy,
Vanderbilt University, Nashville, TN 37325.

Abstract

We develop a robust version of the MSSM from a $Z_{12}$ orbifolded AdS/CFT, with natural low scale unification and briefly discuss cosmology in this testable model.
Introduction

Our collective understanding of particle physics is now called the standard model. Similarly, in cosmology, the sum of our knowledge is contained in the standard cosmological model (or standard big bang model). What these two standard models have in common is their lack of derivation from fundamental principles. String and M theory provide hope we can understand both the micro world and the macro world at a much deeper level and replace the standard models with standard theories. Here we present a modest step toward such a synthesis.

While the standard model (SM) was still in its infancy, the theoretical observations of the crossing of the running gauge couplings [1] as well as other facts led to attempts to unify the SM into one grand theory [2]. Although minimal SU(5) has been excluded experimentally, variants and extensions thereof [3] remain viable. Low-energy supersymmetry is natural in models derived from string theory, improves the accuracy of unification of the couplings [4,5] and raises the GUT scale to \( \sim 10^{16} \) GeV.

Recent progress in string theory [6–8] suggests a different path to unification where the standard gauge group \( SU(3)_C \times SU(2)_L \times U(1)_Y \) is embedded in a semi-simple product gauge group \( G = \Pi_i G_i \), reminiscent of the Pati-Salam [9] and Trinification [10] models, as inspired by gauge theories arising from compactification of the IIB superstring on an orbifold. In such quiver gauge theories the unification of couplings happens not by the logarithmic evolution over a enormous desert covering many orders of magnitude in energy scale, but occurs abruptly at a much lower scale \( M_U \) through the diagonal embeddings of \( SU(3)_C \times SU(2)_L \times U(1)_Y \) in \( G \). If \( M_U \) is sufficiently low (say \( \sim \)TeV) then the theory becomes testable, since additional highly constrained and patterned particle content is required at \( M_U \).

Our focus is on particle physics models with measurable consequences, derivable from orbifolded \( AdS \otimes S^5 \), i.e., \( AdS_5 \times S^5/\Gamma \) where \( \Gamma \) is a finite group. Viable models of this type are rare [11,12] because of the numerous constraints imposed on the spectrum. These include:
(1) bifundamental matter due to orbifolding, (2) TeV unification to allow experimental tests at accelerators, (3) consistent values for $\alpha_s$, $\alpha$ and $\sin^2 \theta_W$, (4) three families, and (5) low energy $\mathcal{N} = 1$ supersymmetry. The two models we will present arise from the only known abelian $Z_n$ orbifolded $AdS \otimes S^5$ with all these desirable traits. We believe that it will be difficult to find many more such models, and our work leads us to conclude that it is very unlikely to find even one more with $n \leq 12$. Hence, the scarcity of TeV scale unification models derivable from string theory is strong motivation to investigate the two models presented here. We would like to have as complete a model as possible, and this means inclusion of gravity. However, gravity in general, and cosmology in particular, within the $AdS/CFT$ scenario is a delicate issue. In order to introduce gravity, conformal symmetry must be broken and this must be provided by corrections to the basic field theoretic model. This is an important topic, but we defer further discussion until we have introduced our models.

We will show that the standard model may be unified at a scale of approximately 4 TeV in a gauge theory based on the group $SU(3)^{12}$. There are two such models, and in each the gauge hierarchy problem is ameliorated. At the same time the models predict $\alpha_3(M_Z)$ and $\sin^2 \theta(M_Z)$ both fall within experimental bounds. Finally, the models predict the existence of precisely three chiral families of quarks and leptons, and possesses $\mathcal{N} = 1$ supersymmetry above the 4 TeV scale. At this scale the models become distinguishable, but below 4 TeV we can still treat their renormalization group behaviors the same.

We assume that supersymmetry is broken at the unification scale, so, in a bottom-up approach, we should examine the running of all three of the SM couplings with length scale $\mu$ as discussed in e.g. [13,14]. At the Z-pole [15] $\alpha_Y(M_Z) = 0.0101, \alpha_2(M_Z) = 0.0338, \alpha_3(M_Z) = 0.118 \pm 0.003$ (the errors in $\alpha_Y(M_Z)$ and $\alpha_2(M_Z)$ are less than 1%) and run between $M_Z$ and $M$ according to the SM equations

$$\alpha_Y^{-1}(M) = (0.01014)^{-1} - (41/12\pi)y$$

$$= 98.619 - 1.0876y$$

(1)
\[ \alpha_2^{-1}(M) = (0.0338)^{-1} + (19/12\pi)y \]
\[ = 29.586 + 0.504y \quad (2) \]

\[ \alpha_3^{-1}(M) = (0.118)^{-1} + (7/2\pi)y \]
\[ = 8.474 + 1.114y \quad (3) \]

where \( y = \ln(M/M_Z) \).

From Eqs.(1,2) we find the scale [16–18] where \( \sin^2\theta(M) = \alpha_Y(M)/(\alpha_2(M) + \alpha_Y(M)) \) satisfies \( \sin^2\theta(M) = 1/4 \) to be \( M \simeq 4 \) TeV. Furthermore, from Eqs.(2,3), the ratio \( R(M) \equiv \alpha_3(M)/\alpha_2(M) \) is \( R(M) \simeq 3.5, 3, 2.5, \) and \( 2 \) correspond to \( M = M_Z, M_3, M_{5/2}, M_2 \simeq 91\)GeV, 400GeV, 4TeV, and 140TeV respectively. The proximity of \( M_{5/2} \) and \( M \), accurate to a few percent, suggests strong-electroweak unification at \( M_U \simeq 4 \) TeV. This scale will be taken as input in the model presented below.

Since the required ratios of couplings at \( M_U \simeq 4 \) TeV is: \( \alpha_3 : \alpha_2 : \alpha_Y :: 5 : 2 : 2 \) it is natural to examine \( Z_{12} \) orbifolds with CFT gauge groups \( SU(3)^{12} \) (There are a large number of such theories [19,20], but the constraints (1) through (5) above elminates all but one orbifold choice.) and diagonal embeddings of Color (C), Weak (W) and Hypercharge (H) in \( SU(3)^2, SU_W(3)^5, SU_H(3)^5 \) respectively. [To be precise, the hypercharge must come partially \( (\frac{1}{7}) \) from the same diagonal \( SU(3) \) of \( SU_W(3)^5 \) where \( SU(2)_L \) arises, and partially \( (\frac{2}{3}) \) from the diagonal \( SU(3) \) of \( SU_H(3)^5 \). As both diagonal \( SU(3) \)s arise from \( SU(3)^5 \)s, the ratios are as above.]

Both our \( \mathcal{N} = 1 \) models arise from the same orbifold choice and will have a top-down construction starting from the AdS/CFT correspondence [6–8] which suggests that the model satisfies conformality at an infra-red fixed point; in fact, for the present model this appears even more likely than in [13] because \( \mathcal{N} = 1 \) supersymmetry implies the presence of non-renormalization theorems.
Description of the Model

Maintaining $\mathcal{N} = 1$ supersymmetry at the 4 TeV scale, requires an embedding of the abelian finite orbifolding group $Z_{12}$ in the SU(4) isotropy of the 5-sphere of $AdS_5 \times S^5$ of type $4 = \{a_\mu\}$ with $a_\mu = (i, j, k, 0)$ where $a_\mu$ is a shorthand for $exp(2\pi ia_\mu/12)$, and $i, j$, and $k$ are integers between 1 and 11.

Out of the 10 possible choices for an $\mathcal{N} = 1$ embedding we must choose $a_\mu = (1, 2, 9, 0)$ if we are to agree with constraints (1), (2), (3), and (5). This choice of $a_\mu$ is necessary to have three chiral families and to have simultaneously a scalar sector which allows breakdown from $SU(3)^{12}$ to the standard gauge group. Constraint (4) then restricts the embedding of $SU(3)_C \times SU(2)_L \times U(1)_Y$ in $SU(3)^{12}$ and the allowed patterns of spontaneous symmetry breaking to arrive at $SU(3)_C \times SU(2)_L \times U(1)_Y$ from $SU(3)^{12}$, since changing the embedding changes the arrangement of scalar fields relative to the SM gauge group. Note, the corresponding $6 = (4 \times 4)_A = (1, 2, 3, -3, -2, -1)$ is real as required by consistency of the theory [21], and this provides yet further constraint.

At first sight there is too much arbitrariness in identification of the quiver nodes as C, W or H. The twelve nodes must be identified as two C’s, and five each of W and H. (The notation refers to the intermediate trinification $SU(3)_C \times SU(3)_W \times SU(3)_H$.) The numbers of C, W, H nodes is dictated by prediction of the correct $\alpha_3(M_Z)$ and $\sin^2\theta(M_Z)$. However, there is only one ambiguity, as we will show there are only two possible embedding assignments. Consider first the choice of two C nodes. There are six inequivalent choices where the two C’s are separated by between 1 and 6 places on the dodecagonal quiver. Now consider the positioning of the W’s and H’s. For each assignment of the two C’s there are precisely 252 ways of assigning W’s and H’s: this is the number of unordered partitions of the integer 6. This degeneracy is almost completely removed by the requirements of spontaneous symmetry breaking to the $3_C3_W3_H$-model and then to the standard model, where we require there are exactly three chiral families. In counting families it is convenient to bear in mind anomaly cancellation (which is guaranteed by the construction). This means that under...
$3_W 3_H$ the only bi-fundamental combination that can occur is

$$(3, ar{3}, 1) + (3, 1, 3) + (1, 3, ar{3})$$

and so it is sufficient to count the $(3, ar{3}, 1)$’s.

The first observation is that the two C’s cannot be separated by more than 3 places on the dodecagon because the complex scalars available to break the $SU(3) \times SU(3)$ to the required $SU(3)_C$ diagonal subgroup do not exist for these cases.

Suppose that the two C’s are next-to-nearest neighbors. Then by considering all the ten possible nodes among which to distribute the five W’s one can easily see that not more than two chiral families are possible.

Now assign the two C’s to be neighboring nodes. The VEVs of the scalars must be able to break the sets of five W’s and five H’s to their respective diagonal subgroups, there is just one out of all 252 unordered partitions which works. The quiver is:

$$-C - C - W - W - H - W - H - W - H - H - H -$$

with the ends identified. The chiral fermions in $(3, ar{3}, 1)$ are shown in Figure 1 which shows how three chiral families survive, arising from four families and one anti-family.

This then is an explicit model with (i) $\mathcal{N}=1$, $SU(3)^{12}$ symmetry; (ii) successful predictivity for $\alpha_3(M_Z)$ and $\sin^2\theta(M_Z)$ (see [14]); (iii) three families; (iv) an ameliorated gauge hierarchy where the ratio of the GUT scale to the weak scale is less than 2 orders of magnitude rather than the greater than 12 orders of magnitude as in the MSSM [5].

To be complete, we should give the only alternative dodecagonal quiver with similar properties. In it the C nodes are next-to-next-to-nearest neighbors and the quiver is

$$-C - H - W - C - W - W - H - W - H - W - H -$$

with the ends identified. These two models are in fact distinguishable at $M_U$ since the patterns of spontaneous symmetry breaking can be used to reveal one of the two inequivalent embeddings. The symmetry breaking of the present model [21] can be summarized as follows.
We first let \((1,1,...,1,3,1,...,1,3,1,...,1)\) be defined as \((3,\bar{3})_{i,j}\) for the 3 and \(\bar{3}\) in the \(i^{th}\) and \(j^{th}\) positions. Then we can use \((3,\bar{3})_{1,2}\) to break \(SU(3) \times SU(3)\) to \(SU(3)_C\). Similarly, we need \((3,\bar{3})_{3,4}, (3,\bar{3})_{4,6}, (3,\bar{3})_{6,8}, \) and \((3,\bar{3})_{8,10}\) to break the W sector to \(SU(3)_L\), and \((3,\bar{3})_{5,7}, (3,\bar{3})_{7,9}, (3,\bar{3})_{9,11}, \) and \((3,\bar{3})_{11,12}\) to break the H sector to \(SU(3)_R\). Next, VEVs for octets resulting from \((3,\bar{3})_{3,6} \rightarrow 1 + 8\) and \((3,\bar{3})_{9,12} \rightarrow 1 + 8\) can break the symmetry to \(SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_L \times U(1)_R\). Finally, it is easy to show that the appropriate doublets and charged singlets exist to complete the breaking to \(SU(3)_C \times SU(2)_L \times U(1)_Y\).

We conclude this section with some comments about the fermion mass spectrum. The three families are chiral and so guaranteed to be light, while all other representations are vectorlike after breaking to \(SU(3) \times SU(2) \times U(1)\); and since only the three families are protected by chiral symmetry, all other fermions should have masses substantially higher than the weak scale. While this latter point is likely to be true, we do not have a definitive proof that all the vectorlike fermions are heavy, but it would require an accidental symmetry to keep some of them light. Furthermore, even though the families are all light compared to the unification scale, the masses of the three families arise from different VEVs in the pattern of spontaneous symmetry breaking, as is easily seen from the quiver diagram. Thus in general, the masses should all be different, but relatively light. The individual members of the families must also get contributions to their masses in different ways, since again they come from a variety of VEVs. Hence the model has sufficient robustness to have a realistic mass spectrum. Finally, although the Yukawa couplings are initially fixed by conformality, soft conformal breaking terms which give masses to fermions directly are not constrained. Since we are using an orbifolded N=3 theory, it is possible that conformal invariance will not hold beyond first order. See, for instance, [22] where a large number of orbifolded AdS/CFT models were investigated. These all have vanishing one-loop beta functions, but somewhat less than ten percent of the models are two-loop finite, hence we expect two-loop contributions to lift any constraints on the Yukawa couplings. This along with the discussion above makes for an interesting albeit complex fermion mass and interaction structure. Mass terms for scalars are likewise unconstrained.
Cosmology and Summary

We have shown that there exists two $\mathcal{N}=1$ models that unify at $\sim 4$ TeV. In the quiver diagram the two color nodes must be nearest neighbors or next-to-next-to-nearest neighbors. For both choices the assignment of all other nodes is dictated by the required symmetry breaking pattern.

The motivation for supersymmetry is weakened by the amelioration of the GUT gauge hierarchy but alternatively it may be justified by the concomitant non-renormalization theorems which can translate one-loop conformality into all-order conformality.

All $\mathbb{Z}_n$ orbifolds have at least a $\mathbb{Z}_n$ symmetry amongst the particle irreps before the $SU(N)^n$ symmetry breaking. Breaking the gauge symmetry also breaks the $\mathbb{Z}_n$ and can lead to cosmic domain walls. Even though the breaking scale is low ($\sim$ TeV), a single infinite wall is still sufficiently massive to overclose the Universe if not inflated away. This requires an inflation scale $\langle \phi \rangle$ in the range $M_W \lesssim \langle \phi \rangle \lesssim 4$ TeV with subsequent weak scale baryogenesis. A cosmologically interesting possibility is for $\mathbb{Z}_n$ to break before inflation, but for the gauge group breaking to the standard model gauge group not to complete until after inflation. Then cosmic strings and light monopoles can arise and persist until the present. The monopoles [23,24] would be light enough ($\sim 100$ TeV) to avoid the cosmological monopole problem, would also be ultra relativistic with only electroweak interactions and be detectable at RICE. One of the major historic reasons for resorting to string theory is that it can potentially provide a consistent quantum gravity. While gravity is absent in the CFTs we have considered here, they should be thought of as a sector of the full theory where gravity must enter. We are just beginning to understand how gravity corrections come about, so here all we can do is just assume gravity arises in a natural way such that the effective theory below $\sim 4$ TeV is $\mathcal{N}=1$ SUSY gauge theory plus general relativity [25].

Baryon number generation is a concern for any model that requires inflation at a low scale. Weak scale baryogenesis has now been ruled out for the minimal standard model, but is still viable for the MSSM and its extensions [26]. The fact that our models must inflate
below 4 TeV means they are testable cosmologically as well as testable and distinguishable at the LHC. In analogy with the results of [14], it is easy to see the properties for the two, similar but experimentally distinguishable, models presented here are quite robust in the sense that fine tuning is not necessary for them to agree with experimental particle physics data.

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Figure Caption

Fig. 1.

Chiral fermions transforming as $(3, \overline{3}, 1)$ in dodecagonal quiver diagram.
