The influence of electromagnetic waves on conductivity tensor in the presence of laser field in quantum wells with parabolic potential for the case of electrons-acoustic phonon scattering

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Abstract. In quantum well studies, the conductivity tensor problem is one of the fundamental problems. In this work the topic covered here is the effect of electromagnetic waves on the conductivity tensor with the presence of a laser field in a quantum well with a parabolic potential considering the case of electrons-acoustic phonon scattering. By using quantum kinetic equations for electrons in quantum wells with parabolic potential, the author has calculated conductivity tensor with electrons–acoustic phonon scattering in the presence of an electromagnetic wave field and a laser field. The expression tensor for conductivity shows its dependence on the frequency of the electromagnetic wave field, the frequency of the laser field, and other parameters specific to the system. From the conductivity tensor expression plotted the effect of the electromagnetic wave field on the conductivity tensor in the presence of the laser field. The quantum well is discussed and plotted here is the GaAs/GaAsAl quantum well.

1. Introduction

In recent years, research on low-dimensional semiconductors in general and quantum wells in particular has brought many benefits to science and technology, emerging new effects strongly applied in engineering [1-5]. Low-dimensional semiconductors are semiconductors in which electrons are confined in 1, 2 or all 3 directions. It is this motion restriction that the energy of electrons is interrupted according to the confinement methods [6-12]. A quantum well is a two-dimensional structure in which electrons are restricted to move in one direction. Since the electrons are confined in one direction, the wave functions, energies, and other quantities that characterize the system have many differences from the bulk semiconductor [3-5].

There are many researches on low-dimensional semiconductor in general and quantum well in particular [13-19]. Conductivity tensor is a familiar concept in materials research, conductivity tensor is also an important concept that forms other fundamental concepts. This work investigates the effect of electromagnetic waves on conductivity tensors in the presence of a laser wave field in a quantum well, but only considers the case of electrons–acoustic phonon scattering and quantum wells with parabolic potential.

Conductivity tensor expression shows the dependence of tensor on electromagnetic wave frequency, laser field frequency, laser field amplitude and characteristic parameters for low-dimensional
semiconductor system. The influence of conductivity tensor on electromagnetic wave frequency, laser field frequency, laser field amplitude will be investigated and plotted for the quantum well GaAs/GaAsAl.

2. Calculating conductivity sensor for the case electrons–acoustic phonon scattering

In case of electrons–acoustic phonon scattering, the time-independent component of distribution function of acoustic phonon:

$$N_i = \frac{k_B T}{v_s q_{lz}}$$  \hspace{1cm} (1)

and the electrons-acoustic phonon interaction constant:

$$C_i^2 = \frac{\xi^2 q_{lz}}{2p v_s V}$$  \hspace{1cm} (2)

here $V$, $\rho$, $v_s$ and $\xi$ are volume, the density, the acoustic velocity and the deformation potential constant.

The parabolic potential in a quantum well is given by:

$$\epsilon_{n, p_z} = \frac{\hbar}{2m} \left( n + \frac{1}{2} \right) + \frac{\hbar^2 p_z^2}{2m}$$  \hspace{1cm} (3)

with $\omega_b$ is the frequency of the parabolic potential.

and wave function:

$$\psi(x) = \sum_{n,i} \psi_{n,i}(\vec{R}_i) W(\tilde{r} - \vec{R}_i)$$  \hspace{1cm} (4)

Electromagnetic wave field with wave vectors:

$$\mathbf{H}(t) = \mathbf{E}(t)e^{-i\omega t} + c.c.$$  \hspace{1cm} (5)

here $\mathbf{E}(t)$ is the electromagnetic wave frequency;

and a laser field:

$$\mathbf{F}(t) = \tilde{F}_0 \sin(\Omega t)$$  \hspace{1cm} (6)

with $F_0$ is the laser field amplitude, $\Omega$ is the laser field frequency.

The quantum kinetic equation for electron in quantum well $(\hbar = 1)$ [13-14]:

$$\frac{\partial f_{n,i} (\vec{p}_{lz}, t)}{\partial t} = \frac{\epsilon_{n, p_z}}{m} \left[ \mathbf{H}(t) + \omega_b \hat{\mathbf{n}} \times \mathbf{B} \right]$$  \hspace{1cm} (7)

where $f_{n,i}(\vec{p}_{lz},t)$ is distribution function of electrons;

$$f_{i,n} (\vec{p}_{lz}, t) = f_{i,n} (\epsilon_{n, p_z} + \frac{1}{2} \mathbf{E}(t) \cdot \vec{p}_{lz} + \frac{1}{2} \mathbf{B}(t) \times \vec{p}_{lz}) - f_{i,n} (\epsilon_{n, p_z})$$  \hspace{1cm} (8)

$$f_{i,n}^* (\epsilon_{n, p_z}) \frac{\partial \epsilon_{n, p_z}}{\partial \epsilon_{n, p_z}} \hat{\mathbf{n}} \cdot \mathbf{B} \right]$$  \hspace{1cm} (9)

$$\tilde{\mathbf{Z}}(t) = \tilde{\mathbf{Z}} e^{-i\omega t} + \tilde{\mathbf{Z}}^* e^{i\omega t} \right]$$  \hspace{1cm} (10)

$$\frac{\partial f_{n,i} (\vec{p}_{lz}, t)}{\partial t} = \frac{\partial}{\partial \epsilon_{n, p_z}} \left[ f_{i,n} (\epsilon_{n, p_z} + \frac{1}{2} \mathbf{E}(t) \cdot \vec{p}_{lz} + \frac{1}{2} \mathbf{B}(t) \times \vec{p}_{lz}) - f_{i,n} (\epsilon_{n, p_z}) \right]$$  \hspace{1cm} (11)

$$\hat{\mathbf{n}}(t) = \frac{\mathbf{H}(t)}{\mathbf{H}}$$  \hspace{1cm} (12)

is the unit vector in the magnetic field direction; $\hat{\mathbf{q}}$ is momentum of phonon;

$\hat{\mathbf{p}} = \hat{\mathbf{p}}_{lz} + \hat{\mathbf{p}}_z$ is momentum of electron;
\[ e_{n,p} = \frac{1}{2m} \left( \frac{\hbar}{a} \right)^2 + \hbar \omega_0 (n + \frac{1}{2}); e_{n,p,q} = \frac{1}{2m} \left( \frac{\hbar}{a} + \frac{\hbar}{q} \right)^2 + \hbar \omega_0 (n + \frac{1}{2}) \]

\( J_1^2 (\bar{a}, \bar{q}) \) is the Bessel function of real argument; \( \bar{a} = \frac{eF}{ma^2}; \) \( \omega_0 = \frac{eH}{mc} \)

\[ M_{n,a}(\bar{q}) = \left| C_n \right|^2 I_{n,a}^2 \left( \bar{q} \right) \]

\[ I_{n,a} = \sum_{j=0}^d \exp (iq, z) \psi_n (z - jd) \psi_n (z - jd) d \]

Calculate with simple case: \( l = 0, \pm 1; J_0^2 \approx 1; J_0^2 = \frac{(\bar{a}, \bar{q})}{4} \)

Proceed to multiply both sides Eq. (1) by \( -\frac{e}{m} \bar{p} \delta (\varepsilon - e_{n,p}) \) and then sum over \( \bar{p} \) to get expression:

\[ -\frac{e}{m} \sum_{n,p} \bar{p} \left[ e \bar{E}(t) + \omega_0 \left[ \frac{\partial}{\partial \bar{p}} \right] \right] \delta (\varepsilon - e_{n,p}) = \]

\[ = -\frac{e}{m} \sum_{n,a,q} M_{n,a}(\bar{q}) \sum_{p} \left[ 2\pi \sum_{p} J_1^2 (\bar{a}, \bar{q}) \left[ \frac{\partial}{\partial \bar{p}} \right] \right] \delta (\varepsilon - e_{n,p}) \]

Calculate the left side of equation (14):

The first term:

\[ -\frac{e}{m} \sum_{n,p} \bar{p} \left[ e \bar{E}(t) + \omega_0 \left[ \frac{\partial}{\partial \bar{p}} \right] \right] \delta (\varepsilon - e_{n,p}) = -i\omega \tilde{R} (\varepsilon) e^{-i\omega t} + i\omega \tilde{R}^* (\varepsilon) e^{i\omega t} \]

here

\[ \tilde{R} (\varepsilon) = -\frac{e}{m} \sum_{n,p} \bar{p} \left[ e \bar{E}(t) + \omega_0 \left[ \frac{\partial}{\partial \bar{p}} \right] \right] \delta (\varepsilon - e_{n,p}) \]

\[ \tilde{R}^* (\varepsilon) = -\frac{e}{m} \sum_{n,p} \bar{p} \left[ e \bar{E}(t) + \omega_0 \left[ \frac{\partial}{\partial \bar{p}} \right] \right] \delta (\varepsilon - e_{n,p}) \]

The second term:

\[ -\frac{e}{m} \sum_{n,p} \bar{p} \left[ e \bar{E}(t) + \omega_0 \left[ \frac{\partial}{\partial \bar{p}} \right] \right] \delta (\varepsilon - e_{n,p}) = -\bar{Q} (e^{-i\omega t} + e^{i\omega t}) \]

here

\[ \bar{Q} = \frac{e^2}{m} \sum_{n,p} \left( \bar{E} \left[ \frac{\partial}{\partial \bar{p}} \right] \right) \bar{p} \delta (\varepsilon - e_{n,p}) \]

The third term:

\[ -\frac{e}{m} \sum_{n,p} \bar{p} \left[ \omega_0 \left[ \frac{\partial}{\partial \bar{p}} \right] \right] \delta (\varepsilon - e_{n,p}) = \]

\[ = -\omega_0 \bar{R} (\varepsilon) e^{2i\omega t} - \omega_0 \bar{R}^* (\varepsilon) e^{-2i\omega t} \]

Calculate the right side of equation (14):

Consider the case that \( l = 0; \)
\[-\frac{e}{m} \sum_{n,a,\beta_1} 2\pi \sum_{\vec{q}} M_{n,a}^{\beta_1} (\vec{q}) \tilde{J}_n^{\beta_1,\beta_2} (\vec{a}, \vec{q}) \left[ f_n(\vec{p}_2 + \vec{q}, t) - f_n(\vec{p}_2, t) \right] \delta \left( \epsilon_{n,\beta_2} - \epsilon_{n,\beta_1} - e_{n,\beta_1} \right) = \]

\[-\frac{1}{\tau(\epsilon)} \left( \tilde{R}(\epsilon) e^{-i\omega t} + \tilde{R}^*(\epsilon) e^{i\omega t} \right) \]  
(21)

Consider the case that \( \lambda = \pm 1 \):

\[-\frac{e}{m} \sum_{n,a,q} M_{n,a}(q) \sum_{p_\perp} \left[ 2\pi \sum_{p_{\parallel}} J^2 (\vec{a}, \vec{q}) \left[ f_n(\vec{p}_\perp + \vec{q}, t) - f_n(\vec{p}_\perp, t) \right] \delta \left( \epsilon_{n,p_{\parallel} + \vec{q}} - \epsilon_{n,p_{\parallel}} \pm \Omega \right) \delta \left( \epsilon - \epsilon_{n,p_{\parallel}} \right) \right] = \]

\[= \tilde{S}(\epsilon) e^{-i\omega t} + \tilde{S}^*(\epsilon) e^{i\omega t} \]  
(22)

with

\[ \tilde{S}(\epsilon) = -\frac{e}{m} 2\pi \sum_{n,a,q} M_{n,a}^{\beta_1} (\vec{q}) \left( \frac{\tilde{a}, \vec{q}}{4} \right)^2 \sum_{p_\perp} f_n(\vec{p}_\perp) \times \]

\[ \times \left[ \delta \left( \epsilon_{n,p_{\parallel} + \vec{q}} - \epsilon_{n,p_{\parallel}} - \Omega \right) + \delta \left( \epsilon_{n,p_{\parallel} + \vec{q}} - \epsilon_{n,p_{\parallel}} + \Omega \right) \right] \times \]

\[\times \left[ (\vec{p}_{\perp} + \vec{q}) \delta (\epsilon - \epsilon_{n,p_{\parallel}}) - \vec{p}_{\perp} \delta (\epsilon - \epsilon_{n,p_{\parallel}}) \right] \]

\[= -2\pi n_0 \tilde{S}(\epsilon) \lambda \delta (\epsilon - \Omega) - A \delta (\epsilon - \epsilon_{\beta_1}) \]  
(23)

A and \( \lambda \) are calculated depending on the electron-phonon scattering mechanism. Put the expressions. (15), (18), (20), (21), (22) on (14), then unify the terms of both sides to get the system of equations:

\[ \left[ i\omega + \frac{1}{\tau(\epsilon)} \right] \tilde{R}(\epsilon) = \tilde{Q} + \tilde{S}(\epsilon) \]  
(24)

and:

\[ \tilde{R}(\epsilon) = \frac{\tau(\epsilon)}{1 - i\omega \tau(\epsilon)} (\tilde{Q} + \tilde{S}(\epsilon)) \]  
(25)

\( n_0 \) is the particle density; \( m \) is the effective mass of electron; \( e = 1.6 \times 10^{-19} \) C; \( \tau(\epsilon) \) is the momentum relaxation time in absence of laser radiation.

At time \( t = 0 \), the density of current [7]:

\[ j(t = 0) = \int \left( \tilde{R}(\epsilon) + \tilde{R}^*(\epsilon) \right) d\epsilon = \tilde{j} + \tilde{j}^* \]

\[ = \frac{4e^2 n_0}{m} \tau(\epsilon_f) \left[ \epsilon_f - \epsilon_0 \left( n + \frac{1}{2} \right) - \frac{1}{2} \right] \left[ 1 + \frac{\tau(\Omega)}{1 + \omega^2 \tau(\Omega)} \right] + \frac{\tau(\epsilon_f)}{1 + \omega^2 \tau(\epsilon_f)} \cdot \mathcal{E} \]

\[\frac{1}{1 + \omega^2 \tau(\epsilon_f)} \]  
(26)

with electrone-acoustic phonon scattering:

\[ A = \frac{e^2 F^2}{2m \sqrt{\Omega}} \tilde{\kappa}_{\beta_1} T \sqrt{\Omega} \left[ 2m \left( \epsilon_f - \epsilon_0 \left( n + \frac{1}{2} \right) \right) \right] \]  
\[\frac{1}{1 + \omega^2 \tau(\epsilon_f)} \]  
(27)

\[ \lambda = \frac{e^2 F^2}{2m \sqrt{\Omega}} \tilde{\kappa}_{\beta_1} T \sqrt{\Omega} \left[ 2m \left( \epsilon_f - \epsilon_0 \left( n + \frac{1}{2} \right) \right) \right] \left[ 2m \left( \Omega - \epsilon_0 \left( n + \frac{1}{2} \right) \right) - 1 \right] \]  
(28)

here \( \tilde{j}(t = 0) = \sigma \tilde{E}(t = 0) = \sigma 2 \mathcal{E} \) so:
\[
\sigma_k = \frac{2e^2 n_0}{m} \frac{\tau(e_F)}{1 + \omega^2 \tau(e_F)} \left\{ e_F - \omega_0 \left( n + \frac{1}{2} \right) \right\} \delta_{ij} + \frac{\tau}{} \left( 1 - \frac{\omega^2 \tau(e_F)}{} \right) \Lambda + \frac{\tau(e_F)}{1 + \omega^2 \tau(e_F)} \Lambda \right\}
\]

with \( \delta_{ij} \) is the Kronecker symbol.

Equation (29) is an explicit expression of the conductivity tensor in a quantum well with a parabolic potential and considers the case of negative phonon electron scattering. The expression shows the dependence of tensors on electromagnetic wave fields, laser fields, and other parameters that characterize low-dimensional semiconductor systems.

3. Numerical results and discussion

The parameters used in the calculations are as follows [2,3]: \( m = 0.0665 m_0 \) (\( m_0 \) is the mass of free electron); \( e_F = 50 \text{meV} \); and \( \tau(e_F) \sim 10^{-11} \text{ s}^{-1} \); \( v_i = 5220 \text{m/s} \); \( n_0 = 10^{22} \text{ m}^{-3} \); \( \rho = 5.3 \times 10^{3} \text{ kg/m}^3 \); \( \xi = 2.2 \times 10^{-8} \text{J} \); \( \omega_0 = 5 \times 10^{3} \text{ s}^{-1} \).

![Figure 1](image1.png)  
Figure 1. The dependence of \( \sigma \) on the frequency \( \omega \) with different values of \( \Omega \) while \( \omega < 10^{11} \).

![Figure 2](image2.png)  
Figure 2. The dependence of \( \sigma \) on the frequency \( \omega \) with different values of \( \Omega \) when \( \omega \) at frequencies from \( 10^{12} \) to \( 10^{13} \).

At frequencies of electromagnetic wave \( \omega < 10^{11} \), Figure 1 shows that the dependence of the conductivity tensor on the electromagnetic wave frequency is nonlinear. As the frequency of electromagnetic waves increases, the tensor of conductivity decreases. Given the three different values of the laser field frequency for the three graphs that are almost identical, it is said that the influence of the laser field frequency on the conductivity tensor in the presence of the electromagnetic wave is very small.

Figure 2 shows the dependence of the conductivity tensor on the electromagnetic wave frequency in the range \( 10^{12} - 10^{13} \) with 3 different values of the laser field frequency. From the graph it can be seen
that as the electromagnetic wave frequency increases, the conductivity tensor decreases but decreases very quickly. The three plots closely coincide with the three different values of the laser field frequency, which also shows that the laser field frequency image does not affect the conductivity tensor much.

In figure 3, the dependence of the conductivity tensor on the electromagnetic wave frequency is plotted at frequencies greater than 10. The figure shows that the conductivity tensor now has a very

Figure 3. The dependence of $\sigma$ on the frequency $\omega$ with different values of $\Omega$ while $\omega > 10^5$

Figure 4. The dependence of $\sigma$ on the amplitude $F$ of the laser radiation.

Figure 5. The dependence of $\sigma$ on the frequency $\Omega$ with different values of $\omega$ at very low frequencies.

Figure 6. The dependence of $\sigma$ on the frequency $\Omega$ with different values of $\omega$ when $\Omega$ at higher frequencies.
small value close to the value equal to zero, which that is to say, the effect of the electromagnetic wave frequency on the conductivity tensor is very small.

Figure 4 shows the dependence of the conductivity tensor on the laser field amplitude, as the laser field amplitude increases, the conductivity tensor also increases. With three different values of the electromagnetic wave frequency giving us three separate graphs, it proves that the influence of the electromagnetic wave field is very large in the presence of the laser field.

Figure 5 and 6 show the effect of the laser field frequency on the conductivity tensor. At too small frequencies are not suitable for laser fields, but at high frequencies we find that the plot is straight lines parallel to the horizontal axis, which shows that the conductivity tensor has very little effect on the laser field frequency.

4. Conclusions

The paper investigated the effect of the electromagnetic wave field on the conductivity tensor in a quantum well in the presence of a laser field, and only considered the case of electron-phonon scattering. From the conductivity tensor expression shows its dependence on characteristic parameters for electromagnetic waves, laser fields, and low dimensional systems.

A graph of the dependence of the conductivity tensor on electromagnetic wave frequency, laser field frequency and laser field amplitude was also plotted. When $\omega < 10^{15}$, the graph shows that the dependence of conductivity tensor on electromagnetic wave frequency is very large, in this frequency range conductivity tensor always decreases as frequency increases. But when the frequency $\omega > 10^{15}$, the conductivity tensor decreases very quickly and has a very small value, at this time the dependence of the conductivity tensor on the electromagnetic wave frequency is very small.

The graphs also show that the dependence of the conductivity tensor on the laser field frequency is very small, the conductivity tensor is strongly dependent on the laser field amplitude, the amplitude increases, the conductivity tensor also increases very rapidly.

5. References

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