Hydrodynamical analysis of hadronic spectra in the 130 GeV/nucleon Au+Au collisions

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We study one-particle spectra and a two-particle correlation function in the 130 GeV/nucleon Au+Au collisions at RHIC by making use of a hydrodynamical model. We calculate the one-particle hadronic spectra and present the first analysis of Bose-Einstein correlation functions based on the numerical solution of the hydrodynamical equations which takes both longitudinal and transverse expansion into account appropriately. The hydrodynamical model provides excellent agreement with the experimental data in the pseudorapidity and the transverse momentum spectra of charged hadrons, the rapidity dependence of anti-proton to proton ratio, and almost consistent result for the pion Bose-Einstein correlation functions. Our numerical solution with simple freeze-out picture suggests the formation of the quark-gluon plasma with large volume and low net-baryon density.

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Relativistic heavy ion collisions are very attracting problems which provide us the nature of hot and dense hadronic matter [1]. Creation of a new state of the matter, the quark-gluon plasma (QGP), and many kinds of new phenomena are expected to be found in the Relativistic Heavy Ion Collider (RHIC) experiments at BNL of which the collision energy is much higher than any other accelerator. However, the complicated processes during the many-body interactions and multiparticle productions are quite hard to catch clear. Therefore, a simple phenomenological description is indispensable for the better understanding of the phenomena. The aims of this paper are, based on a hydrodynamical model, to draw a simple and clear picture of the space-time evolution of the hot and dense matter produced in the high energy heavy ion collisions at RHIC and to give a possible explanation for the recent experimental results.

We use a (3+1)-dimensional hydrodynamical model [2] to describe the space-time evolution assuming the local thermal and chemical equilibrium. Several authors have already discussed RHIC results based on hydrodynamical models. Kolb et al. [3] discussed anisotropic flow by making use of a (2+1)-dimensional hydrodynamic model in which Bjorken’s ansatz [4] was used for the beam direction. Hence, their discussion is limited only in the midrapidity region. Zschiesche et al. [5] discussed HBT radii based on a hydrodynamical model with use of the Bjorken’s scaling solution in the collision direction. One of the authors (T.H.) has already reproduced the both the pseudorapidity and the transverse momentum spectra of hadrons by using a full (3+1)-dimensional hydrodynamical model in Ref. [6], where main theme of the analysis is also anisotropic flow. In this paper, we focus our discussion on central collisions by assuming the cylindrical symmetry of the system. We calculate the one-particle hadronic spectra and present the first analysis of Bose-Einstein correlation functions based on the numerical solution of the hydrodynamical equations which takes both longitudinal and transverse expansion into account appropriately.

The hydrodynamical equations are given as $\partial_\mu T^{\mu\nu}(x) = 0$ with the baryon number conservation law $\partial_\mu n_B^0(x) = 0$. We numerically solve these coupled equations for the perfect fluid by the method described in Ref. [2]. Our numerical solution preserves entropy, energy and net baryon number conserved within 5% of accuracy throughout the calculation with the time step $\delta \tau = 0.01$ fm/c. As for an equation of state (EOS), we adopt a bag model EOS in which phase transition of first order takes place [3]. The QGP phase is a free gas with a bag constant $B$. The gas consists of massless quarks of three flavor and gluons. The hadronic phase is a free resonance gas with an excluded volume correction in which all resonances up to 2 GeV/$c^2$ of mass are included. These two phases are connected by the condition of pressure continuity [3]. Putting the initial time as $\tau = 1.0$ fm/c, we parameterize the initial energy density distribution and net baryon number distribution as simple gaussian forms (Fig. 1). The parameters in the model should be chosen so that calculated single-particle spectra reproduce the experimental results of Au+Au central collisions in the 130 GeV/nucleon at RHIC. The parameter set is summarized in Table. 1. Note that we use Bjorken’s solution $Y_L = \eta$ only as an initial condition for the longitudinal flow velocity. Initial transverse flow is simply neglected. Once the freeze-out hypersurface is fixed, one can calculate the single particle spectra via Cooper-Frye formula [8]. We assume that the freeze-out process occurs a specific temperature $T_f$. This condition corresponds to...
FIG. 1: Initial energy density (left) and net baryon number density (right) distribution.

a constant energy density in the present calculation because of small baryonic chemical potential. By virtue of the Lagrangian hydrodynamics, we expect that contributions of the time-like hypersurface are small and the space-like hypersurface dominates the particle emission at freeze-out; we simply put \( k^\mu d\sigma_{\mu} \approx k^\tau d\sigma_\tau \). We take hadrons from decay of resonances into account as well as directly emitted particles from the freeze-out hypersurface. We include decay processes \( \rho \to 2\pi, \omega \to 3\pi, \eta \to 3\pi, K^* \to \pi K \), and \( \Delta \to N \pi \) \([10, 11]\). The resonances are also assumed to be emitted from the freeze-out hypersurface. Figure 2 shows pseudorapidity distribution of charged hadrons. Figure 3 shows pseudorapidity distribution of charged hadrons. Solid line shows our result (\( \pi, K, p \)) including resonance contribution. Dotted line denotes contribution of the directly emitted particles from the freeze-out hypersurface. Closed circles are preliminary result from the PHOBOS Collaboration \([21]\).

FIG. 2: Pseudorapidity \( \eta_p \) distribution of charged hadrons. Solid line shows our result (\( \pi, K, p \)) including resonance contribution. Dotted line denotes contribution of the directly emitted particles from the freeze-out hypersurface. Closed circles are preliminary result from the PHOBOS Collaboration \([21]\).

2 It is well known that the Cooper-Frye formula has an ambiguity in the treatment of the time-like hypersurface \([9]\).
shows transverse momentum spectrum of negatively charged hadrons. In both figures, our results well reproduce the experimental data. We display the transverse mass spectra of identified negatively charged hadrons ($\pi^-$, $K^-$, $\bar{p}$) in Fig. 4. All slopes of the spectra are well reproduced by our calculation. For $K^-$ and $\bar{p}$, our results fail to reproduce the particle numbers but slopes agree well. In Fig. 5, our result shows excellent agreement with the experimental data of anti-proton to proton ratio as a function of rapidity. However, the absolute numbers of anti-protons was not enough (Fig. 4). Though we here assume the same freeze-out condition for all particle species, the discrepancy may indicate the more complicated mechanism. From the parameter set in Table. I, we can see that the energy density is sufficient for the QGP production. However, the energy density, $\epsilon_{\text{max}} = 6.0$ GeV/fm$^3$ which corresponds to the temperature of 229 MeV, is only 5% higher than SPS [12]. Large collision energy at RHIC leads to very large volume of the hot matter. Longitudinal extension of the hot matter, $\sigma_\eta + \eta_0 = 2.47$, is about 2.3 times larger than the one of SPS [12]. Initial energy density itself depends strongly on the initial time $\tau_0$ which corresponds to the spatial size in the longitudinal direction. Hence, much higher energy density should be obtained if we assume an earlier thermalization time. Furthermore, we do not include the thickness of incident nuclei but use almost flat profile with gaussian smearing for the transverse direction (see Fig. 1). Therefore, maximum energy density of our model is smaller than other models which take the thickness into account [3, 5, 6]. The average energy density at $\eta = 0$ transverse plane in our model is 3.9 GeV/fm$^3$. Total energy of the fluid of our model corresponds to 25290 GeV,
TABLE I: Parameter set for the Au+Au collisions.

| Parameter                                                   | Value               |
|-------------------------------------------------------------|---------------------|
| Maximum initial energy density $E_0$                        | 6.0 GeV/fm$^3$      |
| Maximum initial net baryon density $n_{B0}$                 | 0.125 fm$^{-3}$     |
| Longitudinal gaussian width $\sigma_\eta$ of initial energy density | 1.47                |
| Longitudinal extension $\eta_0$ of the flat region in the initial energy density | 1.0                |
| Longitudinal gaussian width $\sigma_D$ of the initial net baryon density | 1.4                |
| Space-time rapidity $\eta_D$ at maximum of the initial net baryon distribution | 3.0                |
| Gaussian smearing parameter $\sigma_r$ of the transverse profile | 1.0 fm              |
| Freeze-out temperature $T_f$                                | 125 MeV             |

TABLE II: Output.

| Parameter                             | Value               |
|---------------------------------------|---------------------|
| Net baryon number                     | 131                 |
| Mean chemical potential at freeze-out $\langle \mu_B \rangle$ | 76.1 MeV            |
| Mean transverse flow velocity $\langle v_T \rangle$ of the fluid at $|\eta| < 0.1$ | 0.509c              |
| Lifetime of the QGP phase $\tau_{QGP}$ | 2.92 fm/c           |
| Lifetime of the mixed phase $\tau_{MIX}$   | 12.61 fm/c          |
| Total lifetime of the fluid $\tau_{HAD}$ | 18.94 fm/c          |

99% of total collision energy. We display the outputs from the fluid in Table II. In the present model, net baryon number is much smaller than total baryon number of incident nuclei. $\langle \mu_B \rangle$ means average of the chemical potential on the whole freeze-out hypersurface. $\langle v_T \rangle$ is the average transverse velocity of the fluid element in $|\eta| < 0.1$ at the freeze-out. “Lifetimes” of the each phase are also shown in Table II. Space-time evolution of the fluid is displayed in Fig. 6. The large area between $T = 160$ MeV and $T = 158$ MeV means large space-time volume of the mixed phase.

The two-particle correlation function for chaotic source is calculated through

$$C_2(q^\mu, K^\mu) = 1 + \frac{|I(q^\mu, K^\mu)|^2}{I(0, k^\mu_1)I(0, k^\mu_2)}$$  \hspace{1cm} (1)
where \( K^\mu = (k_1^\mu + k_2^\mu)/2 \), \( q^\mu = k_1^\mu - k_2^\mu \), respectively. \([13, 14]\). We put

\[
I(q^\mu, K^\mu) = \int K_\mu d\sigma^\mu(x) \sqrt{f(k_1, x)f(k_2, x)} e^{iq_\nu x^\nu},
\]

so that \( I(0, k^\mu) \) reduces to the Cooper-Frye formula with \( f(k, x) \) being the Bose-Einstein distribution function. As a first trial, we neglect the contributions from resonance decay for simplicity. Figure 7 shows the projected correlation functions for \( \pi^- \) with \( |Y| \leq 0.5 \) and \( 0.125 < K_T < 0.225 \text{ GeV/c} \). In each correlation function, our results are corrected by a common \( \lambda \) factor whose value is 0.6. The other origin of the reduction is integration with respect to other components of the relative momenta over the range \( 0 < q_i < 35 \text{ MeV/c} \). Despite the neglect of resonance contribution,\(^3\) both outward and longitudinal correlation functions show good agreements with the experiment. Finally, we compare pair transverse mass \( M_T = \sqrt{K_T^2 + m^2} \) dependence of the HBT radii extracted through the Gaussian fit to the three-dimensional correlation function (Fig. 8). We obtain almost consistent sideward HBT radii.

\(^3\) It has been discussed that the hadronic cloud significantly affects the HBT radii.\([23]\).
but obtained outward and longitudinal HBT radii are larger than experimental data. This tendency is common to the all hydrodynamical calculation \cite{11,12}. However, the difference between our results and data, 1 fm in HBT radii, corresponds to only 5 MeV in the correlation function (Fig. 6). As for $R_{out}$, a naive interpretation of the experimental result is that high $M_T$ pions come from high temperature source and the experimental results indicate short lifetime of the high temperature region. On the other hand, if the freeze-out picture works well, i.e., most of particles are emitted from the 3-dimensional hypersurface, the experimental data suggest very short freeze-out duration or strong opaqueness of the source \cite{13}. Though our model naturally shows opaque property due to hydrodynamical flow \cite{18}, the freeze-out time duration, $\Delta t = \sqrt{\langle t^2 \rangle - \langle t \rangle^2}$ where $A(x) = \int d^4 x A(x) S(x, K)/\int d^4 x S(x, K)$ with $S(x, K)$ being the source function \cite{14}, of our model is about $5 \sim 7$ fm/c which is longer than the SPS case \cite{12,18}. This long time duration of our model leads to the large $R_{out}$/side $R_{side}$.

In summary, we present a hydrodynamical-model calculation for the 130 GeV/nucleon Au+Au collisions data from the RHIC experiments. The present numerical solution indicates that the produced quark-gluon plasma in the RHIC has much lower baryon density, slightly higher energy density, and several times larger extension in the longitudinal direction than in the SPS case, if we compare at the same initial time, $\tau_0 = 1$ fm/c. We present the first analysis of the pion Bose-Einstein correlation data at RHIC based on the hydrodynamical model which takes into account both longitudinal and transverse expansion appropriately. The result is partly consistent with the experimental result but obtained $R_{out}$ and $R_{long}$ are slightly larger than the experimental data. In this work, we concentrate our discussion on the RHIC data. More detailed discussion including the comparison with the SPS data will be given in the forthcoming paper \cite{12}.

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