Compressive Spectrum Sensing Using Blind-Block Orthogonal Least Squares in Cognitive Sensor Networks

Liyang Lu, Member, IEEE, Wenbo Xu, Senior Member, IEEE, and Yue Wang, Senior Member, IEEE

Abstract—In cognitive sensor networks (CSNs), compressive spectrum sensing (CSS) has been shown as a promising candidate for improving the recovery performance based on the signal sparsity. However, most current works need to know the sparsity or the noise information in advance for reliable reconstruction, where such information is usually unavailable in practical applications. In this article, we propose a blind-block orthogonal least squares-based CSS (B-BOLS-CSS) algorithm, which utilizes a novel blind stopping rule to cut the cords to these prior information. Specifically, we first present both the noiseless and noisy recovery guarantees for the BOLS algorithm based on the mutual incoherence property (MIP). Motivated by them, we then formulate the blind stopping rule, which exploits an \(\ell_{2,\infty}\) sufficient statistic to blindly test the support atoms in the remaining measurement matrix. We further evaluate the theoretical performance analysis of the holistic B-BOLS-CSS algorithm by developing a lower bound of the signal-to-noise ratio (SNR) to ensure that the probability of recovery is no lower than a given threshold. Simulations not only demonstrate the improvement of our derived theoretical results but also illustrate that B-BOLS-CSS works well in both low and high SNR environments.

Index Terms—Blind stopping rule, block orthogonal least squares (OLSs), block sparsity, cognitive sensor networks (CSNs), compressive spectrum sensing (CSS).

I. INTRODUCTION

SPECTRUM sensing [1], [2], [3], [4], [5] is important for efficient utilization of spectrum resources in the cognitive sensor networks (CSNs) [6], [7], [8], [9], [10]. It dynamically provides the spectrum occupancy status for the cognitive sensors. In this way, secondary users (SUs) is able to avoid harmful interference to primary users (PUs) due to SUs’ access. Currently, with the increasing deployment of wide spectrum in wireless transmission, wideband spectrum sensing...
has received great attention. One challenge in the implementation of wideband spectrum sensing lies in its prohibitive sampling rate. As a real-time processing desired problem, wideband spectrum sensing thus calls for recovery technology with fast implementation. A remarkable choice is arguably compressive sensing (CS), which is used instead of traditional Nyquist sampling to reduce hardware complexity and cost.

Although compressive spectrum sensing (CSS) is able to successfully sense the wideband spectrum with sampling rate close to the information rate, it requires sparsity or noise information in advance. Otherwise, the CS algorithms cannot stop iteration in time, causing performance loss. To address this issue, a well-known method is the so-called blind CS (BCS) [11]. The primitive BCS contains the sparse coding and the measurement matrix update steps, which is similar to K-SVD dictionary learning [12]. Its later extension [13] considers the issue of simultaneous signal reconstruction with the assumption that the original signals come from the union of a few disjoint subspaces. Other application variants can be referred to [14], [15], [16]. The drawback of BCS lies in the complex dictionary learning procedure and the subspace constraints, which limits the applicability of BCS in wideband spectrum sensing.

Recently, Chen et al. [17] develop a blind stopping rule for orthogonal matching pursuit (OMP) by blindly testing the support signal in the remaining measurement matrix per iteration, lessening the dependence on prior information. The recovery performance of OMP with this blind stopping rule is comparable to the alternative with prior information. Hence, it is indeed a favorable choice for wideband spectrum sensing. Other research studies, such as [1], [18], address the unknown sparsity issue effectively by utilizing statistical analysis to estimate the sparsity level, and hence applied to spectrum sensing.

Though OMP has been well developed for spectrum sensing, the utilization of other greedy algorithms remains barely explored in this field. Orthogonal least squares (OLSs) [19], [20], [21] is a typical CS greedy algorithm with better convergence properties than OMP [22], [23]. Meanwhile, OLS depends less on the amplitude distribution of nonzero entries. These advantages motivate us to consider OLS as a more reliable candidate for spectrum sensing.

Furthermore, the spectrum to be recovered in CSS usually exhibits multiband property [24], [25], and thus the nonzero entries appear in blocks [26], [27], [28], [29]. Research studies have proved that exploiting block sparsity during sparse signal recovery generally brings better performance gain and convergence than using conventional sparsity [26], [29], [30], [31], [32], [33]. Such gain has also been verified in a number of applications including block spectrum sensing [4], [27], face recognition [34], [35] and so on. Zhang and Rao [36] and Yu et al. [37] propose blind-block sparse recovery methods based on the Bayesian framework of neighborhood statistics, which do not need any prior information of the sparsity structure. However, they are constrained by the huge consumption of computing resources. In [38], a deterministic iterative neighborhood-based blind-block algorithm, which is better than those in [36], [37] in both running time and recovery performance, is provided via a predefined sparsity-inducing function. This function excessively relies on the selection of the neighborhood radius and thus leads to the instability of the algorithm. Therefore, to improve the efficiency and stability of the CSS procedure while considering block sparsity, it is necessary to develop a practical blind-block sensing algorithm.

The OLS-related blind algorithm design depends on both the reliable recovery condition analysis and the blind mechanism development. Firstly, compared to the more mature theoretical analysis of the OMP algorithm, the theoretical recovery analysis of OLS is still in its infancy due to the sophisticated normalization factor in OLS [21], which makes the derivation of the reconstructible sparsity bound more involved. Meanwhile, since the block-structure should be considered in the reliable recovery conditions for BOLS, the theoretical results based on the utilization of the correlation of various block subspaces need to be further developed. Secondly, the blind stopping rule of the BOLS algorithm requires the enhanced matching detection idea to measure if there exists the block signal component in the residual. To this end, the discriminative mechanism based on mixed norm should be considered for the optimal iterative stopping moment, which is completely different from that of the blind OMP algorithm using the conventional matched filtering energy detection [17].

In this article we propose a novel blind-block OLSs-based CSS (B-BOLS-CSS) algorithm for realizing the real-time and accurate CSS. Specifically, we first formulate the exact recovery conditions (ERCs) for BOLS by tightening the matrix eigenvalue bounds via mutual incoherence property (MIP) [21], [26], which is a widely used metric for sparse signal recovery, in the noiseless scenario. Then, based on the derived recovery conditions, the blind stopping rule for BOLS algorithm is developed, followed by its theoretical performance guarantees in the noisy scenario. The B-BOLS-CSS algorithm is proposed by combining the blind stopping rule and BOLS algorithm. The main contributions are given as follows.

1) Both the noiseless and noisy recovery conditions for BOLS algorithm are derived, which are better than the existing ones. Specifically, we develop a tighter eigenvalue bounds than those in [39] by utilizing the block orthogonality. Then, based on these eigenvalue bounds, an extended upper bound of the reconstructible sparsity level is derived, which is better than that in [21] and acts as a solid foundation for the subsequent derivation of the blind stopping rule.

2) We propose a blind stopping rule, which is incorporated with $\ell_{2,\infty}$ norm, for BOLS by using the aforementioned theoretical results. A novel B-BOLS-CSS algorithm is then proposed by combining this blind stopping rule and BOLS algorithm. The lower bound of the $\text{SNR}_{\text{min}}$ required for reliable recovery of B-BOLS-CSS is developed, which is lower than that in [17]. These analyses reveal that B-BOLS-CSS algorithm performs better than the CSS using blind OMP algorithm.

3) The simulations demonstrate the superiority of our derived theoretical results compared with the existing ones. Meanwhile, we evaluate the effectiveness and
feasibility of our proposed B-BOLS-CSS algorithm. The performance of B-BOLS-CSS with unknown number of active PUs and the noise variance is close to that of CSS using the algorithms, e.g., conventional block OMP (BOMP) and BOLS, with these prior information, and is more robust than the one without utilizing block structure.

The rest of this article is organized as follows. In Section II, we introduce notations, CSS model and basic definitions, which facilitate the subsequent study of theoretical analysis and algorithm proposal of B-BOLS-CSS in Section III. We present simulation results in Section IV, followed by conclusions in Section V.

II. PRELIMINARIES

A. Notations and Assumptions

The notations used in this article are summarized in Nomenclature, where $r$ is a general vector and $D$ is a general matrix. In addition, throughout the article, the measurement matrix is assumed to be normalized, i.e., the $\ell_2$-norm of each column in $D$ is equal to 1.

B. Block Compressive Spectrum Sensing

At a cognitive sensor node, the received spectrum is denoted by $s \in \mathbb{R}^s$, which is sparse based on a certain basis $\Psi \in \mathbb{R}^{s \times n}$. Let $s = \Psi x$, where $x$ is a $k$ block-sparse spectrum that only contains $k$ spectrum blocks with nonzero $\ell_2$-norm. Define $\Phi \in \mathbb{R}^{m \times n}$ as the sampling matrix, where $m$ and $n$ are the numbers of sub-Nyquist-rate and Nyquist-rate samples, respectively. Denoting the additive noise as $\epsilon \in \mathbb{R}^{m \times 1}$, the low-dimensional measurement vector $y \in \mathbb{R}^m$ is given by

$$y = \Phi s + \epsilon = \Phi \Psi x + \epsilon = Dx + \epsilon \quad (1)$$

where $D = \Phi \Psi \in \mathbb{R}^{m \times n}$ is the measurement matrix with $n \gg m$. The number of the nonzero blocks $k$ in the vector $x$ is called the block sparsity [26]. The spectrum sensing algorithms are designed to accurately recover $x$ from the given measurement vector $y$. Then, the SUs can perform interference-free access with the help of the recovered $\hat{x}$.

Denote the block length as $d$, and the total number of blocks in $x$ as $N_B$. The block-sparse spectrum $x$ is defined as

$$x = [x_1, \ldots, x_d, x_{d+1}, \ldots, x_{2d}, \ldots, x_{n-d+1}, \ldots, x_n]^T \quad (2)$$

where $n = N_Bd$ and $x[i] \in \mathbb{R}^{d \times 1}$ denotes the $i$th block of $x$. The measurement matrix can be rewritten as a concatenation of $N_B$ column blocks, i.e.,

$$D = [D_1, D_2, \ldots, D_{kd}] = [D[1], D[2], \ldots, D[k]] \quad (3)$$

where $D[i] \in \mathbb{R}^{m \times d}$ is the $i$th block of $D$.

C. Definitions

In this section, we first give the definition of MIP, which contains the concepts of matrix coherence, block-coherence and sub-coherence, followed by the definitions of signal-to-noise ratio (SNR) and component SNR.

Definition 1: The matrix coherence of a matrix $D$, which measures the similarity of its entries, is defined as

$$\mu = \max_{i,j \neq i} |D^T D_j| \quad (4)$$

where $|\cdot|$ means the absolute value of its target.

Definition 2: The block-coherence of $D$ is defined as

$$\mu_B = \max_{i,j \neq i} \frac{|D[i,j]|}{d} \quad (5)$$

where $D[i,j]$ is the $D$th entry of $D$. The sub-coherence of $D$ is given by

$$\nu = \max_{i,j \neq i} |D[i,j]|, D[i], D[j] \in D[i]. \quad (6)$$

Definition 4: The SNR is defined as

$$SNR = \frac{\mathbb{E}(|\|Dx\|_2^2|)}{\mathbb{E}(|\|e\|_2^2|)} \quad (7)$$

where $\mathbb{E}(\cdot)$ represents the expectation of its objective. The component SNR is given by

$$SNR_q = \frac{|x_q D_q|}{M d^2}, \quad q = 1, 2, \ldots, n \quad (8)$$

and the minimum component SNR is the minimum value of the component SNRs [17].

III. RECOVERY ANALYSIS AND BLIND STOPPING RULE FOR BOLS

In this section, we first derive the extended MIP-based condition for the ERC of BOLS algorithm. Then, based on the derived condition, we present the blind stopping rule for BOLS and provide the B-BOLS-CSS algorithm.

A. MIP Condition in the Noiseless Scenario

Without loss of generality, assume that the first $kd$ entries of the sparse spectrum $x$ are nonzero and the set containing selected indices during the first $t$ iterations is $S^t = \{1, \ldots, td\}$ ($1 \leq t < k$). Then, in the $t+1$th iteration, define $D_0 = [D_1, D_2, \ldots, D_{kd}] = [D[1], D[2], \ldots, D[k]]$, $D_{0:S} = D_0 \setminus D_S$ and $\overline{D}_0 = D \setminus D_0$. $R_{0:S}$ and $\overline{R}_0$ corresponding to $D_{0:S}$ and $\overline{D}_0$ are defined by

$$R_{0:S} = f(D_{0:S}) = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ \|P^*_S D_1\|_2 & 1 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots \\ \|P^*_S D_{k-td}\|_2 & \cdots & 1 & 0 \end{bmatrix} \quad (9)$$

and $\overline{R}_0 = f(\overline{D}_0)$.

The condition

$$\gamma = \rho_s[D_{0:S} R_{0:S}^T(\overline{D}_{0:S} \overline{R}_{0:S})] < 1 \quad (10)$$

is called the ERC for BOLS algorithm [21], where $\rho_s(A) = \max_j \sum_i |A[i,j]|_2$ and $A[i,j]$ is the $(i,j)$th block of $A$. 

Authorized licensed use limited to the terms of the applicable license agreement with IEEE. Restrictions apply.
In the following, we present a detailed analysis of the improved MIP-based sufficient condition for the establishment of this ERC.

Lemma 1: Suppose \((k - 1)dμ_B < 1\) and \(v = 0\), then

\[
λ_{k, d, μ_B} = 1 - (k - 1)dμ_B ≤ λ_{min} ≤ λ_{max} ≤ 1 + (k - 1)dμ_B = \lambda_{k, d, μ_B}
\]

where \(μ_k\) is the block-coherence, \(v\) is the sub-coherence, \(d\) is the block length, \(λ_{min}\) and \(λ_{max}\) denote the minimum and maximum eigenvalues of \(D_{k}^{T}D_{0}\).

Proof: See Appendix I.

Remark 1: Our derived result in Lemma 1 is much tighter due to the utilization of block sparsity and block orthogonality than the well-known result [39, Lemma 2], i.e., \(1 - (k - 1)dμ_B ≤ λ_{min} ≤ λ_{max} ≤ 1 + (k - 1)dμ_B\). This is because the block-coherence satisfies \(μ_B ≤ μ\) and \(dμ_B ≥ μ\) [21], which indicates that \(1 - (k - 1)dμ_B ≥ 1 - (k - 1)μ\) and \(1 + (k - 1)dμ_B ≤ 1 + (k - 1)μ\).

Remark 2: Denote the compression rate as a fixed constant \(τ\). Then \(\lim_{n/m = τ, m,n → ∞} ν = 0\), which indicates the assumption \(ν = 0\) in Lemma 1 is reasonable in high-dimensional application, e.g., bandpass spectrum sensing.

To compare the derived ranges of eigenvalues using block-structure property with the one using conventional structure, we present Corollary 1.

Corollary 1: Suppose \((K - d)μ_B < 1\) and \(ν = 0\), then \(1 - (K - d)μ_B ≤ λ_{min} ≤ λ_{max} ≤ 1 + (K - d)μ_B\), where \(μ_B\) is the block-coherence, \(ν\) is the sub-coherence, \(d\) is the block length, \(λ_{min}\) and \(λ_{max}\) denote the minimum and maximum eigenvalues of \(D_{k}^{T}D_{0}\).

Remark 3: Corollary 1 is derived by replacing \(kd\) with \(K\) in Lemma 1. Compared with [39, Lemma 2], i.e.,

\[
1 - (K - 1)μ ≤ λ_{min} ≤ λ_{max} ≤ 1 + (K - 1)μ
\]

the bounds of eigenvalues in Corollary 1 are tighter since \(d ≥ 1\) and \(μ_B ≤ μ\). This tightness can be attributed to the consideration of block-structure [21, 26].

In order to present the extended MIP-based condition for ERC of the BOLS algorithm, we first provide the following lemma, which describes a new lower bound for \(||P_S^{T}D_i||_2\) \(∀i \in \{1, 2, \ldots, n\}\{S\}'.

Lemma 2: Suppose \((k - 1)dμ_B < 1\) and \(v = 0\), then \(||P_S^{T}D_i||_2 ≥ \sqrt{1/B}\) for \(i \in \{1, 2, \ldots, n\}\{S\}'\), where \(B = (1 - (kdμ_B^2)/(1 + (k - 1)dμ_B))/((1 - (k - 1)dμ_B)^2)^{-1}\), \(μ_B\) is the block-coherence, \(ν\) is the sub-coherence, and \(d\) is the block length.

The proof of Lemma 2 is similar to that of [21, Lemma 3] but with our derived Lemma 1 in this article. It is useful to compare our derived result in Lemma 2 with the existing ones, which is presented in the following Remark 4.

Remark 4: There are two existing lower bounds for \(||P_S^{T}D_i||_2\). Those are, the bound using [39, Lemmas 2 and 5]

\[
||P_S^{T}D_i||_2 ≥ \sqrt{1 - kdμ}
\]

and the bound in our previous work [21]

\[
||P_S^{T}D_i||_2 ≥ \frac{1}{\sqrt{1 + (kd - 1)dμ} \sqrt{kdμ^2}} \frac{1}{1 - (k - 1)dμ}.
\]

Similar to the analysis given in remark 1, our derived result is tighter than these existing ones due to the extra analysis about block sparsity.

Based on Lemma 2, the MIP-based sufficient condition for the ERC of the BOLS is presented in the following Theorem 1.

Theorem 1: The ERC in (10) is satisfied if

\[
k < \frac{\sqrt{Q}/2 + \sqrt{∆} + √{Q}/2 - √{∆} - β/3α}{2}
\]

where \(Q = (27α^2δ - 9αβω + 2β^3)/27α^3\), \(P = (3αω - β)/3α^2\), \(α = -d^3μ_B^2 + 3d^3μ_B^2\), \(β = d^3μ_B^2 + d^2μ_B^2 - 7d^2μ_B^2 - 6d^2μ_B^2 - 2d^2μ_B^2\), \(ω = 5d^3μ_B^2 - 2d^2μ_B^2 + 8d^2μ_B^2 + 4d^2μ_B^2 + 2d^2μ_B^2 + 3dμ_B^2 + 4dμ\), \(δ = -d^3μ_B^2 - 2d^2μ_B^2 - 2d^2μ_B^2 - d^2μ_B^2 - 4dμ - 2\) and \(∆ = (Q/2)^2 + (P/3)^3\).

Proof: Based on [21, eq. (100)] and Lemma 2, we have

\[
a^3κ^3 + βκ^2 + ακδ + δ < 0
\]

where \(α, β, ω,\) and \(δ\) are defined in Theorem 1. The condition (15) is obtained by solving the cubic inequality (16).

Remark 5: The preliminary lemmas for Theorem 1, i.e., Lemmas 1 and 2, are tighter than the existing ones. Therefore, the MIP-based condition for ERC of BOLS in Theorem 1 is better than [21, Th. 4]. This indicates a higher reconstructible sparsity level for BOLS.

Based on Theorem 1, the following lemma holds:

Lemma 3: Suppose \((k - 1)dμ_B < 1\) and \(ν = 0\), then \(γ\) in (10) is constrained by\( γ ≤ (2Bkνdμ_B^2)/(2 - (k - B)dμ_B)\) < 1, \(μ_B\) is the block-coherence, \(ν\) is the sub-coherence, and \(d\) is the block length.

B. Recovery Conditions in the Noisy Scenario

Due to the noise in the actual CSS scenario, we present the performance analysis for BOLS with block-blind stopping rule under Gaussian noise, i.e., \(ε ∼ N(0, σ^2I_m)\), in this section. Specifically, we first derive the theorem of exactly selecting a correct support in the current iteration for BOLS based on the ERC analysis in Section III-A. Then, the performance analysis for BOLS with block-blind stopping rule under Gaussian noise is investigated by developing a lower bound of the \(ℓ_2\)-norm of the spectrum support for reliable recovery.

Theorem 2: Suppose that the condition in (15) holds and the remaining nonzero vector \(x_{0/S}\) in the \((t + 1)\)th iteration satisfies

\[
||x_{0/S}||_2 > \frac{2\sqrt{k - τ(2 - (k - B)dμ_B)N_1}}{K_{d, μ_B}2 - (k - B)dμ_B - 2Bkdμ_B}
\]

then the BOLS algorithm selects one correct block in the \((t + 1)\)th iteration with the probability at least \(1 - 1/m\), where \(K_{d, μ_B}\) can be obtained by Lemma 1.

Proof: See Appendix II.

The result in Theorem 2 indicates that if the power of the remaining atoms in the current iteration is large enough, the
BOLS algorithm chooses a correct block. Based on Theorem 2, the following corollary holds.

**Corollary 2:** Suppose that the condition in (15) holds and all the nonzero blocks in the (t + 1)th iteration satisfy
\[
||x_{0,S}||_2 > \frac{2\sqrt{k - t(2 - (k - B)d\mu_B)}\sqrt{d\sigma\sqrt{m + 2\sqrt{m}\log m}}}{\Delta_{k,d,\mu_B}(2 - (k - B)d\mu_B - 2Bk\mu_B)}
\]
then the BOLS algorithm selects one correct block in the (t + 1)th iteration with the probability at least \(1 - 1/m\), where \(\Delta_{k,d,\mu_B}\) can be obtained by Lemma 1.

**Proof:** This corollary is proved by using Theorem 2 and [21, Lemma 7].

The following corollary is derived from Corollary 2.

**Corollary 3:** Suppose that the condition in (15) holds and all the nonzero blocks \(x[i]\) satisfy
\[
||x[i]||_2 > \frac{2(2 - (k - B)d\mu_B)}{\Delta_{k,d,\mu_B}(2 - (k - B)d\mu_B - 2Bk\mu_B)}
\]
then the BOLS algorithm selects the true support set with the probability at least \(1 - 1/m\), where \(\Delta_{k,d,\mu_B}\) can be obtained by Lemma 1.

There exists a trade-off between the bounds in Corollaries 2 and 3. It can be observed that the condition in Corollary 3 is stricter than that in Corollary 2, but the result in Corollary 3 is universally valid for all iterations during the algorithm operation. The decision-making according to the condition in Corollary 3 is only implemented one time, while the bound in Corollary 2 should be verified per iteration.

**C. B-BOLS-CSS Algorithm**

In this section, we propose the blind stopping rule for BOLS and formulate the corresponding B-BOLS-CSS algorithm. We denote the right-hand side of (15) as \(C\) and begin with the following Lemma 4.

**Lemma 4:** For Gaussian noise \(\epsilon \sim \mathcal{N}(0, \sigma^2 I_m)\) and the measurement matrix \(D\) with the block-coherence \(\mu_B\), we have
\[
\Pr(||D^T\epsilon||_2 \leq \sqrt{\eta\mu_B}\eta\sigma) \geq 1 - \frac{n}{\sqrt{2\pi\xi^2\mu_B^2\eta^2\eta^2e^{\frac{\xi^2\mu_B^2\eta^2}{4}}}}
\]
where \(\xi > 0\) and
\[
\eta = \sqrt{4(m - C) - 2} - \sqrt{m - C + 2\sqrt{(m - C)\log(m - C)}}.
\]

**Proof:** It is known that \(||D^T\epsilon||_2 \leq \sqrt{\eta}||D^T\epsilon||_1\). The lemma then follows by using [17, Lemma 2].

**Remark 6:** Since the variance of the element in \(D^TP_{S}\epsilon\) is less than \(\sigma^2\), the result in Lemma 4 can be extended to a new one
\[
\Pr(||D^TP_{S}\epsilon||_2 \leq \sqrt{\eta\mu_B}\eta\sigma) > 1 - \frac{n}{\sqrt{2\pi\xi^2\mu_B^2\eta^2\eta^2e^{\frac{\xi^2\mu_B^2\eta^2}{4}}}}.
\]

**Theorem 3:** Suppose that the condition in (15) holds and \(\epsilon \sim \mathcal{N}(0, \sigma^2 I_m)\). Then, if the minimum component SNR satisfies (23), as shown at the bottom of the next page, with the probability
\[
P_{\xi} > 1 - \frac{C}{m} - \frac{1}{m - C} - \frac{n}{\sqrt{2\pi\xi^2\mu_B^2\eta^2\eta^2e^{\frac{\xi^2\mu_B^2\eta^2}{4}}}}
\]
the BOLS algorithm using the stopping rule
\[
\frac{||D^T\epsilon||_2}{||\epsilon||_2} \leq \sqrt{\eta\mu_B}
\]
can reconstruct the given \(k\) block-sparse signal.

**Proof:** See Appendix III.

By further utilizing Lemma 4, we obtain the following corollary, which provides a tighter bound for SNR than that in [17].

**Corollary 4:** Suppose that the condition in (15) holds and \(\epsilon \sim \mathcal{N}(0, \sigma^2 I_m)\). Then, if the minimum component SNR satisfies (26), as shown at the bottom of the next page, with the probability
\[
P_{\xi} > 1 - \frac{C}{m} - \frac{1}{m - C} - \frac{n}{\sqrt{2\pi\xi^2\mu_B^2\eta^2\eta^2e^{\frac{\xi^2\mu_B^2\eta^2}{4}}}}
\]
the BOLS algorithm using the stopping rule (25) can reconstruct the given \(k\) block-sparse signal, where \(\xi > 0\) and \(\eta\) is provided in (21).

The rule in (25) is called the blind-block stopping rule for BOLS, which does not depend on the sparsity level or the noise information. The \(\xi, \eta, C, \epsilon\) term in (25) can be regarded as the energy sum of the outputs of multiple matched filters for one signal block. In this way, the stopping rule can blindly decide whether there exist matrix blocks corresponding to support blocks of the sparse spectrum in the remaining measurement matrix. The probability in (24) is actually the target probability of detection. With a given \(P_{\xi}\), \(\xi\) can be calculated according to (24). Based on these descriptions, the B-BOLS-CSS algorithm is given in Algorithm 1.

Conventional greedy algorithms, e.g., OMP [39] and MOLS [23], usually exit iteration if the energy of the residual is smaller than a predefined threshold, when there is no sparsity information as an auxiliary parameter. The threshold consists of the product of a hyper-parameter and the noise power, i.e., \(\eta||\epsilon||_2\) [23]. However, this threshold highly depends on the appropriate setting of \(\eta\) and the acquisition of \(||\epsilon||_2\), which is not parameter-free and lacks rigorous theoretical interpretability.

Moreover, when there is no prior knowledge of the noise information, there are three typical types of algorithms, i.e., the greedy B-OMP-CSS algorithm [17], the sparse Bayesian learning algorithm [40], and the convex/nonconvex optimization algorithm [30]. Compared with the B-OMP-CSS, the SNR for reliable recovery of our proposed B-BOLS-CSS is lower than that of the B-OMP-CSS, which is proved in the aforementioned theoretical results. Since further considerations of the least squares selection mechanism and the block-structure of the sparse spectrum, B-BOLS-CSS provides more reliable performance in various MIP scenarios. Compared with the sparse
Bayesian learning and the optimization algorithm, B-BOLS-CSS exhibits a natural advantage of not relying on complex hyper-parameter settings. Meanwhile, the sparse Bayesian learning and the convex/nonconvex optimization algorithms are burdened by high computational complexity due to the existence of optimization procedures, which contain numerous matrix inversion or embedded iterative solving approaches.

**Algorithm 1 B-BOLS-CSS Algorithm**

**Input:** D, y, detection probability \( P_d \) and the block length \( d \)

**Output:** The recovered spectrum \( \hat{x} \in \mathbb{R}^n \) and the set of the support indices \( \hat{S} \subseteq \{1, 2, \ldots, n\} \)

1: **Initialization:** \( t = 0, \hat{r}^0 = y, \hat{S}^0 = \emptyset, \hat{x}^0 = 0 \)
2: Calculate \( \xi \) according to (24) with the given \( P_d \)
3: Calculate the block-coherence \( \mu_B \) of \( D \)
4: while \( \|D^t r^t\|_2 \equiv > \sqrt{d} \xi \mu_B \) do
5: Set \( t^{i+1} = \arg \min_{t \in \{1, \ldots, n\} \cap \hat{S}^t} \|P_{\hat{S}^t \cap \{i(t^t + 1) \}} y\|_2^2 \)
6: Augment \( \hat{S}^{i+1} = \hat{S}^t \cup \{(i(t^t + 1) + 1 : t^t + 1) d\} \)
7: Estimate \( \hat{x}^{i+1} = \arg \min_{x: \text{support}(x) = \hat{S}^{i+1}} \|y - D x\|_2^2 \)
8: Update \( r^{i+1} = y - D \hat{x}^{i+1} \)
9: \( t = t + 1 \)
10: end while
11: return \( \hat{S} = \hat{S}^t \) and \( \hat{x} = \hat{x}^t \)

**IV. SIMULATION RESULTS**

In this section, we first perform simulations to illustrate our theoretical results presented in Section III and compare them with the existing ones. Then, we compare our proposed B-BOLS-CSS algorithm with the state-of-the-art ones in CSN.

**A. Simulations for Theoretical Results**

1) **Comparison Between Lemma 1 and [39, Lemma 2]:** The lower bound and the higher bound in Lemma 1 are called “Lemma 1 lower” and “Lemma 1 higher”, respectively. Accordingly, the lower bound and the higher bound in [39, Lemma 2] are called “Existing lower” and “Existing higher”. As shown in Fig. 1, our derived bounds are tighter than the existing ones, i.e., they are much closer to 1. With the increase of matrix coherence \( \mu \) or the block sparsity \( k \), the bounds become away from 1 but our derived ones still keep considerable tightness, which indicates that the follow-up theoretical analysis based on Lemma 1 may provide better results.

2) **Comparison Among Lemma 2, the Existing Bounds in (13) and (14):** The existing bounds in (13) and (14) are called “Existing bound 1” and “Existing bound 2”, respectively. As illustrated in Fig. 2, our derived result in Lemma 2 is always much closer to 1, which indicates that this result is tighter than the existing ones. Although a larger \( \mu \) or a larger \( k \) causes degradation of the bound, the result in Lemma 2 performs the best.

3) **Comparison Between Theorem 1 and [21, Th. 4]:** The result in [21, Th. 4] is called “Existing bound”. As presented in Fig. 3, our derived result of the reconstructible sparsity level is higher than the existing one. This indicates that our result presents a more relaxed theoretical reconstructible sparsity level for the BOLS algorithm. Furthermore, this improved theoretical analysis lays a beneficial foundation for the subsequent analysis of the noisy recovery performance and the blind stopping mechanism for BOLS.

4) **Comparison of the Lower Bounds of SNR\(_{\text{min}}\) in Theorem 3 and the SNR\(_{\text{min}}\) in [17, Th. 1]:** The lower bound of \( \text{SNR}_{\text{min}} \) in [17, Th. 1] is called “Existing bound”. The dimensions of the measurement matrix are set as \( m = 1024, n = 8192 \), and \( m = 2048, n = 8192 \), which are the same as those in

\[
\text{SNR}_{\text{min}} > \max \left\{ \frac{(2(2 - (k - \gamma))d \mu_B) \sqrt{d} \sqrt{m + 2 \sqrt{m \log \frac{m}{h}}}}{m(\lambda_{h,d,\mu_B}(2 - (k - \gamma))d \mu_B - 2 \gamma k d^2 \mu_B))^2}, \frac{(\sqrt{d} \xi \mu_B \sqrt{m + 2 \sqrt{m \log \frac{m}{h}}})^2}{m(\lambda_{h,d,\mu_B}(2 - (k - \gamma))d \mu_B - 2 \gamma k d^2 \mu_B))^2} \right\}
\]

\[
\text{SNR}_{\text{min}} > \max \left\{ \frac{(2(2 - (k - \gamma))d \mu_B) \sqrt{d} \xi \mu_B \eta^2}{m(\lambda_{h,d,\mu_B}(2 - (k - \gamma))d \mu_B - 2 \gamma k d^2 \mu_B))^2}, \frac{(\sqrt{d} \xi \mu_B \sqrt{m + 2 \sqrt{m \log \frac{m}{h}}})^2}{m(\lambda_{h,d,\mu_B}(2 - (k - \gamma))d \mu_B - 2 \gamma k d^2 \mu_B))^2} \right\}
\]
the simulations of [17]. The corresponding coherence $\mu$ are 0.135 and 0.109, respectively. The probabilities of recovery $P_\xi$ in (24) are fixed to 0.9:0.01:0.99.

As shown in Fig. 4, the lower bounds of $\text{SNR}_{\text{min}}$ given in (23) are lower than those in [17], which indicates that BOLS performs better than OMP under low SNR conditions. Meanwhile, BOLS is more capable of achieving the target probability of recovery even if the number of measurements of BOLS is only half of that of OMP, leading to computing resource savings.

B. Simulations for CSS

In this section, we perform simulations to compare our proposed B-BOLS-CSS algorithms with the other CSS algorithms.

Consider two wideband CSNs with $m = 128$, $n = 512$ and $m = 256$, $n = 512$. The locations of the nonzero blocks of the sparse spectrum are selected uniformly at random. Two types of the measurement matrices are generated. The first type is the widely used Gaussian measurement matrix and the elements in this matrix are independently and identically distributed as $\mathcal{N}(0, 1/m)$ with block orthogonality, i.e., $\nu = 0$. The second one is the hybrid measurement matrix, which is used in [22]. The column of the measurement matrix is set as $D_i = a_i(h_i + g_i1)$, where $h_i$ satisfies the standard Gaussian distribution, $1$ is the all 1 vector and $g_i$ obeys the uniform distribution on $[0, G]$ with $G > 0$. Note that the MIP of a hybrid measurement matrix is bad, which is close to 1 and is much higher than that of a Gaussian measurement matrix. All realizations of the measurement matrix are normalized. The recovery is successful if the recovered spectrum vector is within a certain small Euclidean distance of the original spectrum. For each trial, we average over 1000 realizations of the sparse spectrum.

1) CSS Using Gaussian Measurement Matrix: The nonzero entries of the spectrum are independently and identically distributed as $\mathcal{N}(0, 1)$. The CSS schemes using OLS, BOLS, OMP, and BOMP are called “OLS-CSS,” “BOLS-CSS,” “OMP-CSS” and “BOMP-CSS.” They all iterate for exact $k$ or $kd$ times. The CSS using the OMP with blind stopping rule in [17] is called “B-OMP-CSS”.

In Figs. 5 and 6, we plot the probability of recovery as a function of the block sparsity $k$ of the generated spectrum where the SNR is fixed as 20 dB. The number of measurements of the measurement matrices is fixed as $m = 128$ and $m = 256$, respectively. It is observed that the CSS algorithms using block-structure outperform the ones without employing block characteristics. The sensing performance of our proposed B-BOLS-CSS is comparable with that of BOLS-CSS and BOMP-CSS, which indicates that B-BOLS-CSS iterates for appropriate times and thus can effectively deal with the unknown prior information issue. Meanwhile, the performance of B-BOLS-CSS is better than that of B-OMP-CSS, which reveals that the utilization of block property is an effective way in improving the accuracy and B-BOLS-CSS is really an attractive choice for wideband spectrum sensing.

In Figs. 7 and 8, we illustrate the performance of the CSS algorithms using Gaussian measurement matrix, while the conventional sparse learning via iterative minimization (SLIM)
Fig. 6. Probability of recovery versus $k$ using Gaussian measurement matrix with $m = 256$, $n = 512$ and $d = 8$.

Fig. 7. Probabilities of recovery versus SNR (dB) using Gaussian measurement matrix with $m = 256$, $n = 512$ and $d = 2$.

Fig. 8. Probabilities of recovery versus SNR (dB) using Gaussian measurement matrix with $m = 256$, $n = 512$ and $d = 4$.

In the existing works [41], [42], it can be observed that the performance of the block CSS algorithms is better than that of the SLIM in both low and high SNR scenarios due to the block-sparse consideration, and they perform slightly better than the BSLIM-CSS with the aforementioned parameter settings. The performance of the proposed B-BOLS-CSS always approaches that of the other block CSS algorithms, while the latter ones know perfect sparsity information in advance.

2) CSS Using Hybrid Measurement Matrix: The nonzero atoms of the spectrum are independently and identically distributed as either $\mathcal{N}(0,1)$ or $\mathcal{N}(1,0.01)$ to further see the influence of different amplitude distributions on the sensing performance.

In Fig. 9, the probabilities of recovery of BOLS-CSS are better than those of BOMP-CSS. It is known that the hybrid measurement matrix expresses a particularly poor MIP which is close to 1. It thus reveals that BOLS is more suitable for wideband CSS since BOLS-CSS exhibits reliable performance even if the MIP of the measurement matrix is unsatisfactory.
Meanwhile, for the blind CSS algorithm, B-BOLS-CSS is always better than B-OMP-CSS. This indicates that the atomic selection mechanism in BOLS and the consideration of block sparsity are indeed conducive to improve CSS performance.

In Fig. 10, the sensing performance of the OLS-type algorithms is better than that of the OMP-type algorithms, which reveals that OLS-type algorithms are able to deal with different distributions of the spectrum supports. Meanwhile, the sensing performance of BOLS-CSS is competitive with that of the BOLS-CSS, indicating the effectiveness and robustness of B-BOLS-CSS when prior information is absent.

Figs. 11 and 12 give the recovery performance when the number of measurements and block length are doubled to 256 and 8, respectively. When compared with Figs. 9 and 10, the similar conclusions can be obtained in Figs. 11 and 12.

V. CONCLUSION

In this article, we propose a B-BOLS-CSS algorithm to address the CSS challenge that arose with unknown prior information, e.g., sparsity and noise information. Our theoretical and empirical work demonstrates that the proposed B-BOLS-CSS algorithm performs well in wideband spectrum sensing when these prior information is absent. Our results provide several improvements over previous work on MIP-based recovery condition analyses, and they significantly reduce the required SNR bound for the implementation of the blind recovery algorithm. In other words, B-BOLS-CSS, which exploits the block-structure and the fast greedy property, is an effective and robust CSS algorithm.

APPENDIX I

PROOF OF LEMMA 1

We first present the proof for the lower bound of $\lambda_{\min}$. It suffices to prove that the matrix $D_0^T D_0 - \lambda I$ is nonsingular under the condition $\lambda < 1 - (k - 1)d \mu_B$ when $(k - 1)d \mu_B < 1$. The proof is equivalent to proving that for any nonzero vector $r = (r_1, r_2, \ldots, r_d)^T \in \mathbb{R}_d^d$, $(D_0^T D_0 - \lambda I)r \neq 0$. Without loss of generality, we assume $||r||_1 \geq ||r||_2 \geq \cdots \geq ||r||_d$. The $\ell_2$-norm of the first block of $(D_0^T D_0 - \lambda I)r$ satisfies

$$||((D_0^T D_0 - \lambda I)r)||_2$$

$$= ||(D_0[1]^T D_0[1] - \lambda I)r[1] + \sum_{i=2}^{k} D_0[1]^T D_0[i]r[i]||_2$$

$$\geq ||D_0[1]^T D_0[1]r[1]||_2 - \lambda ||r[1]||_2 - d \mu_B \left( \sum_{i=2}^{k} ||r[i]||_2 \right)$$

$$(a) 

\geq (1 - \lambda) ||r[1]||_2 - d \mu_B \left( \sum_{i=2}^{k} ||r[i]||_2 \right)$$

$$\geq (k - 1)d \mu_B ||r[1]||_2 - d \mu_B \left( \sum_{i=2}^{k} ||r[i]||_2 \right)$$

$$\geq 0$$

(28)

where $(a)$ is obtained because of $v = 0$. Afterward, $(D_0^T D_0 - \lambda I)r \neq 0$ and we obtain $\lambda_{\min} \geq 1 - (k - 1)d \mu_B$. $\lambda_{\max} \leq 1 + (k - 1)d \mu_B$ can be obtained similarly.

APPENDIX II

PROOF OF THEOREM 2

In the $t$th iteration of BOLS, the residual is

$$r' = (I - P_S) y = s' + n'$$

(29)

where $s' = (I - P_S) D x$ and $n' = (I - P_S) e$ are called the signal and noise parts of the residual, respectively. Denote $\gamma(t, 1) = ||(D_{0,b} R_{0,b}^T s')||_{2,\infty}$, $\gamma(t, 2) = ||(D_{0,b} R_{0,b}^T s')||_{2,\infty}$ and $N_t = ||(D_{0,b} R_{0,b}^T n')||_{2,\infty}$. It is known that the condition

$$||(D_{0,b} R_{0,b}^T r')||_{2,\infty} \geq ||(D_{0,b} R_{0,b}^T r')||_{2,\infty}$$

(30)

guarantees that the BOLS algorithm selects a correct block in the $(t + 1)$th iteration. Since $||(D_{0,b} R_{0,b}^T r')||_{2,\infty} \geq \gamma(t, 1) - N_t$ and $||(D_{0,b} R_{0,b}^T r')||_{2,\infty} \leq \gamma(t, 2) + N_t$, we obtain that

$$\gamma_{t, 1} - \gamma_{t, 2} > 2N_t$$

(31)

is a sufficient condition for the establishment of (30). By using [39, Lemma 4] and the similar analyses in [21], we have

$$\gamma_{t, 1} - \gamma_{t, 2} \geq (1 - \gamma) \gamma_{t, 1}$$

(32)
Combining (31), (32) and Lemma 3 yields
\[
\gamma(t, 1) \geq \frac{2(2 - (k - B)d_{\mu_B})N_i}{2 - (k - B)d_{\mu_B} - 2Bkd_{\mu_B}}. \tag{33}
\]
In addition
\[
\gamma(t, 1) = \frac{||\{(Q_{0, S}R_{0, S})^T(I - P_S)D_{0, S}x_{0, S}\}|_2}{\sqrt{k - t}} \geq \frac{\lambda_{k, d, \mu_B}||x_{0, S}\|_2}{\sqrt{k - t}}. \tag{34}
\]
The above equation and (33) indicate that
\[
||x_{0, S}\|_2 > \sqrt{\frac{2\lambda_{k, d, \mu_B}^2(2 - (k - B)d_{\mu_B} - 2Bkd_{\mu_B})}{\lambda_{k, d, \mu_B}^2(2 - (k - B)d_{\mu_B} - 2Bkd_{\mu_B})}} \tag{35}
\]
guarantees the correct block selection of BOLS in the (t+1)th iteration.

**APPENDIX III**

**PROOF OF THEOREM 3**

The proof of the theorem contains three steps: 1) BOLS chooses a correct nonzero entry during each iteration; 2) BOLS does not stop in the rth step (t < k); and 3) BOLS stops after k iterations.

The condition of the first step follows from Corollary 2. After some simple calculations, we have
\[
\frac{||x_{0}\|_2}{\sigma} = \sqrt{\sum_{i \in S} \frac{|x_i|^2}{\sigma^2} = \sqrt{\sum_{i \in S} m \times SNR_i}} > \frac{2\sqrt{k - t - 2(2 - (k - B)d_{\mu_B})N_i}}{\sqrt{\lambda_{k, d, \mu_B}^2(2 - (k - B)d_{\mu_B} - 2Bkd_{\mu_B})}}. \tag{36}
\]
This means BOLS chooses a correct block if
\[
\text{SNR}_{\text{min}} > \left( \frac{2(2 - (k - B)d_{\mu_B})\sqrt{\sigma/m + 2m\log m}}{\lambda_{k, d, \mu_B}(2 - (k - B)d_{\mu_B} - 2Bkd_{\mu_B})} \right)^2. \tag{37}
\]
Then, for t < k, with the probability \( \text{Pr}(|D^TP_i\epsilon|_{2, \infty} \leq \sqrt{\delta \epsilon \mu_B} \eta \sigma, ||\epsilon||_2 \leq (m + 2m \log m)^{1/2} \sigma) \), we obtain
\[
\frac{||D^TP_i\epsilon||_{2, \infty}}{||\epsilon||_2} \geq \frac{1}{\lambda_{k, d, \mu_B}^2} ||D_{0, S}^TP_{0, S}D_{0, S}x_{0, S}\|_2 - \sqrt{\delta \epsilon \mu_B \eta \sigma} \geq \frac{||P_{0, S}^TD_{0, S}x_{0, S}\|_2 + \sigma \sqrt{m + 2m \log m}}{\lambda_{k, d, \mu_B}^2 ||x_{0, S}\|_2 / \sigma + \sqrt{m + 2m \log m}} \geq \sqrt{\delta \epsilon \mu_B} \tag{38}
\]
with the SNR\textsubscript{min} satisfying
\[
\text{SNR}_{\text{min}} > \left( \frac{\sqrt{\delta \epsilon \mu_B} \eta \sigma / \sqrt{m + 2m \log m}}{m (\lambda_{k, d, \mu_B} - \sqrt{\delta \mu_B \lambda_{k, d, \mu_B}})} \right)^2. \tag{39}
\]
For t = k, with the probability \( \text{Pr}(|D^TP_i\epsilon||_{2, \infty} \leq \sqrt{\delta \epsilon \mu_B} \eta \sigma, ||P_{0, S}\epsilon||_2 \geq \eta \sigma) \), we have \( (||D^TP_i\epsilon||_{2, \infty})/(||\epsilon||_2) \leq \sqrt{\delta \epsilon \mu_B} \).
Finally, the probability in (24) can be obtained by using \( k < C \).
[25] L. Lu, W. Xu, Y. Wang, and Z. Tian, “Compressive spectrum sensing using sampling-controlled block orthogonal matching pursuit,” IEEE Trans. Commun., vol. 71, no. 2, pp. 1096–1111, Feb. 2023.

[26] Y. C. Eldar, P. Kuppinger, and H. Bolcskei, “Block-sparse signals: Uncertainty relations and efficient recovery,” IEEE Trans. Signal Process., vol. 58, no. 6, pp. 3042–3054, Jun. 2010.

[27] F. Li and X. Zhao, “Block-structured compressed spectrum sensing with Gaussian mixture noise distribution,” IEEE Wireless Commun. Lett., vol. 8, no. 4, pp. 1183–1186, Aug. 2019.

[28] L. Lu, W. Xu, Y. Wang, and Z. Tian, “Supervised dictionary learning for block threshold feature in compressive spectrum sensing,” IEEE Trans. Cognit. Commun. Netw., vol. 8, no. 4, pp. 1632–1646, Dec. 2022.

[29] L. Lu, W. Xu, Y. Cui, Y. Dang, and S. Wang, “Gamma-distribution-based logit weighted block orthogonal matching pursuit for compressed sensing,” Electron. Lett., vol. 55, no. 17, pp. 959–961, Aug. 2019.

[30] Z. Zeinalkhani and A. H. Banihashemi, “Iterative reweighted ℓ1 recovery algorithms for compressed sensing of block sparse signals,” IEEE Trans. Signal Process., vol. 63, no. 17, pp. 4516–4531, Sep. 2015.

[31] S. Daei, F. Haddadi, and A. Amini, “Exploiting prior information in block-sparse signals,” IEEE Trans. Signal Process., vol. 67, no. 19, pp. 5093–5102, Oct. 2019.

[32] R. Qi, D. Yang, Y. Zhang, and H. Li, “On recovery of block sparse signals via block generalized orthogonal matching pursuit,” Signal Process., vol. 153, pp. 34–46, Dec. 2018.

[33] X. Zhang, W. Xu, Y. Cui, L. Lu, and J. Lin, “On recovery of block sparse signals via block compressive sampling matching pursuit,” IEEE Access, vol. 7, pp. 175554–175563, 2019.

[34] Y. Wang, Y. Y. Tang, L. Li, and X. Zheng, “Block sparse representation for pattern classification: Theory, extensions and applications,” Pattern Recognit., vol. 88, pp. 196–209, Apr. 2019.

[35] C. Zou, K. I. Kou, Y. Wang, and Y. Y. Tang, “Quaternion block sparse representation for signal recovery and classification,” Signal Process., vol. 179, Feb. 2021, Art. no. 107849.

[36] Z. Zhang and B. D. Rao, “Extension of SBL algorithms for the recovery of block sparse signals with intra-block correlation,” IEEE Trans. Signal Process., vol. 61, no. 8, pp. 2009–2015, Apr. 2013.

[37] L. Yu, C. Wei, J. Jia, and H. Sun, “Compressive sensing for cluster structured sparse signals: Variational Bayes approach,” IET Signal Process., vol. 10, no. 7, pp. 770–779, Sep. 2016.

[38] D. Lazzaro, L. B. Montefusco, and S. Papi, “Blind cluster structured sparse signal recovery: A nonconvex approach,” Signal Process., vol. 109, pp. 212–225, Apr. 2015.

[39] T. T. Cai and L. Wang, “Orthogonal matching pursuit for sparse signal recovery with noise,” IEEE Trans. Inf. Theory, vol. 57, no. 7, pp. 4680–4688, Jul. 2011.

[40] J. Fang, Y. Shen, H. Li, and P. Wang, “Pattern-coupled sparse Bayesian learning for recovery of block-sparse signals,” IEEE Trans. Signal Process., vol. 63, no. 2, pp. 360–372, Jan. 2015.

[41] X. Tan, W. Roberts, J. Li, and P. Stoica, “Sparse learning via iterative minimization with application to MIMO radar imaging,” IEEE Trans. Signal Process., vol. 59, no. 3, pp. 1088–1101, Mar. 2011.

[42] A. Aubry, V. Carotenuto, A. De Maio, and M. A. Govoni, “Multi-snapshot spectrum sensing for cognitive radar via block-sparsity exploitation,” IEEE Trans. Signal Process., vol. 67, no. 6, pp. 1396–1406, Mar. 2019.

[43] A. Aubry, V. Carotenuto, A. De Maio, and M. A. Govoni, “ Spectrum sensing for cognitive radar via model sparsity exploitation,” in Compressed Sensing in Radar Signal Processing. Cambridge, U.K.: Cambridge Univ. Press, 2019, pp. 257–283.

Liyang Lu (Member, IEEE) received the B.S. degree in communication engineering from the Beijing University of Posts and Telecommunications (BUPT), Beijing, China, in 2005, and the Ph.D. degree from the School of Information and Communication Engineering, BUPT, 2010.

Since 2010, she has been with BUPT, where she is currently a Professor with the School of Artificial Intelligence. Her current research interests include sparse signal processing, machine learning, and signal processing in wireless networks.

Wenbo Xu (Senior Member, IEEE) received the B.S. degree from School of Information Engineering, Beijing University of Posts and Telecommunications (BUPT), Beijing, China, in 2005, and the Ph.D. degree from the School of Information and Communication Engineering, BUPT, 2010.

Since 2010, she has been with BUPT, where she is currently a Professor with the School of Artificial Intelligence. Her current research interests include sparse signal processing, machine learning, and signal processing in wireless networks.

Yue Wang (Senior Member, IEEE) received the Ph.D. degree in communication and information system from the School of Information and Communication Engineering, Beijing University of Posts and Telecommunications (BUPT), Beijing, China, in 2011.

He is currently an Assistant Professor with the Department of Computer Science, Georgia State University, Atlanta, GA, USA. Previously, he was a Research Assistant Professor with the Department of Electrical and Computer Engineering, George Mason University, Fairfax, VA, USA. His general interests lie in the interdisciplinary areas of machine learning, signal processing, wireless communications, and their applications in cyber-physical systems. His specific research focuses on trustworthy AI, compressive sensing, massive MIMO, millimeter-wave communications, wideband spectrum sensing, cognitive radios, direction of arrival estimation, high-dimensional data analysis, and distributed optimization and learning.