A model calculation of double parton distribution functions of the pion

Matteo Rinaldi\textsuperscript{1}, Sergio Scopetta\textsuperscript{2}, Marco Traini\textsuperscript{3}, and Vicente Vento\textsuperscript{1}

\textsuperscript{1} Departament de Fisica Teòrica, Universitat de València and Institut de Fisica Corpuscular, Consejo Superior de Investigaciones Científicas, 46100 Burjassot (València), Spain
\textsuperscript{2} Università degli Studi di Perugia, and Istituto Nazionale di Fisica Nucleare, Sezione di Perugia, Perugia Via A. Pascoli, I-06123, Italy
\textsuperscript{3} INFN - TIFPA, Dipartimento di Fisica, Università degli Studi di Trento, Via Sommarive 14, I-38123 Povo (Trento), Italy

June 27, 2018

Abstract

Two-parton correlations in the pion are investigated in terms of double parton distribution functions. A Poincaré covariant Light-Front framework has been adopted. As non perturbative input, the pion wave function obtained within the so-called soft-wall AdS/QCD model has been used. Results show how novel dynamical information on the structure of the pion, not accessible through one-body parton distribution, are encoded in double parton distribution functions.

1 Introduction

Double parton scattering (DPS), the simplest form of multiple parton interaction (MPI), has been observed at the LHC (see, e.g., Ref. \cite{1}). The DPS cross section can be written in terms of double parton distribution functions (dPDFs) \cite{2,3}, which represent the number density of two partons located at a given transverse separation in coordinate space and with given longitudinal momentum fractions. This is an information complementary to the tomography accessed through electromagnetic probes in terms of generalized parton distributions (GPDs) \cite{4,5}. If measured, dPDFs would therefore represent a novel tool to access the three-dimensional hadron structure. However, since dPDFs describe soft Physics, they are non perturbative objects and have not been evaluated in QCD. It is therefore useful to estimate them at low momentum scales ($\sim \Lambda_{QCD}$), for example using quark models as has been proposed in Refs. \cite{6,7,8,9,10,11}. In order to match theoretical predictions with future experimental analyses, the results of these calculations are then evolved using perturbative QCD to reach the high momentum scale of the data \cite{12,13}.

In a previous work, use has been made of the AdS/QCD framework to study dPDFs in proton-proton collisions \cite{11}. The AdS/QCD approach establishes a correspondence between conformal field theories and gravitation in an anti-de-Sitter space \cite{14,15}. The so-called bottom-up approach implements important features of QCD, generating a theory in which conformal symmetry is restored asymptotically \cite{16,17,18,19}. This approach has been successfully applied to the description of the spectrum of hadrons, of their form factors (ffs) and parton distributions (PDFs) \cite{20,21,22,23,24,25}. In particular, the structure of the pion is an
interesting subject which has attracted much attention from the point of view of AdS/QCD [20, 26, 27]. In this scenario we proceed here to generalize the formalism developed for nucleon dPDFs to mesons and apply it to pion wave functions defined via the AdS/QCD correspondence. This analysis has been partially motivated by a first estimate of moments of quantities related to pion dPDFs in the lattice, recently reported [28].

In section 2 we describe the meson dPDF in terms of the light-front (LF) wave function (w.f.), and introduce an approximation which relates dPDFs to GPDs and ffs. Furthermore we introduce a quantity relevant to DPS phenomenology, the effective cross section $\sigma_{\text{eff}}$, in terms of dPDFs and PDFs. In section 3 AdS/QCD model calculations of dPDFs are summarized and their properties analyzed. In sec. 4 the evolution of the dPDFs to high momentum scale is calculated and its implications discussed. Conclusions are collected in sec. 5.

\section{Double PDF and the meson light-front wave function}

In this section we describe how to express dPDFs in terms of the LF meson wave function. The formalism we use has been also presented in Ref. [29], where dPDFs have been studied for a dressed quark target treated as a two body system. Formally, dPDFs are defined by means of the light-cone correlator [3],

\begin{equation}
f^I_2(x_1, x_2, k_\perp) = \frac{P^+}{4} \int d^2y_\perp e^{-iy_\perp \cdot k_\perp} \int dy^- \int dz^- dz^+ \nonumber \times \frac{e^{-ix_1 P^+ z_1^- - ix_2 P^+ z_2^-}}{(2\pi)^2} \langle A, 0| O(0, z_1) O(y, z_2) | A, 0 \rangle \bigg|_{x_1^+, x_2^+, z_1^+, z_2^+ = 0},
\end{equation}

where, for generic 4-vectors $y$ and $z$, the operator $O(y, z)$ reads:

\begin{equation}
O(y, z) = \bar{q} \left( y - \frac{1}{2}z \right) \gamma^+ q \left( y + \frac{1}{2}z \right),
\end{equation}

and $q(z)$ is the LF quark field operator. In order to find a suitable expression of the dPDF, we make use of the LF wave function representation approach [30, 31]. In particular, by taking into account only the “valence” contribution in the LF intrinsic frame, the pion state is written as

\begin{equation}
|A, P_\perp\rangle = \sum_{h, \tilde{h}} \int \frac{dx_1 \, dx_2 \, d^2k_{1\perp} \, d^2k_{2\perp}}{\sqrt{x_1 x_2} \, 2(2\pi)^3} \delta(2) \langle k_{1\perp} + k_{2\perp} | x_1, k_{1\perp} + x_1 P_\perp, h \rangle \nonumber \times | x_1, k_{1\perp} + x_1 P_\perp, h \rangle | x_2, k_{2\perp} + x_2 P_\perp, \tilde{h} \rangle \nonumber \times \delta(1 - x_1 - x_2) \psi_{h, \tilde{h}}(x_1, x_2, k_{1\perp}, k_{2\perp}).
\end{equation}

Here, $h$ and $\tilde{h}$ represent parton helicities, $x_i = k_i^+/P^+$ and $k_{1\perp}$ the quark longitudinal momentum fraction and its transverse momentum, respectively, $P^\mu$ the meson 4-momentum. The light cone components are defined by $t^\perp = P^0 \pm i \xi$. In Eq. (3), $\psi_{h, \tilde{h}}(x_1, x_2, k_{1\perp}, k_{2\perp})$ is the LF meson wave-function, whose normalization is chosen as

\begin{equation}
1 = \frac{1}{2} \sum_{h, \tilde{h}} \int \frac{dx_1 dx_2 \, d^2k_{1\perp} \, d^2k_{2\perp}}{16\pi^3} \delta(1 - x_1 - x_2) \nonumber \times \delta(2) \langle k_{1\perp} + k_{2\perp} | \psi_{h, \tilde{h}}(x_1, x_2, k_{1\perp}, k_{2\perp}) \rangle^2.
\end{equation}
The w.f. \( \psi_{h,\bar{h}}(x_1, x_2, k_{1\perp}, k_{2\perp}) \) determines the structure of the state and is not known.

However, one can obtain the dPDF by using a standard procedure (see e.g. Ref. [8] for the proton) which makes use of the quark-antiquark field operator [20], the definition of the meson state Eq. (3), of Eq. (1) and the anticommutation relations between creation-annihilation operators (see Ref. [20] for details). The result of the calculation for the pion dPDF is

\[
f'_2(x_1, x_2, k_{\perp}) = \frac{1}{2} \sum_{h, \bar{h}} \int \frac{d^2k_{1\perp}}{(2\pi)^d} \psi_{h,\bar{h}}(x_1, x_2, k_{1\perp} \mp k_{\perp}) \times \delta(1 - x_1 - x_2). \tag{5}
\]

The physical object of interest here is \( f_2(x_1, k_{\perp}) \), obtained as integral over \( x_2 \) of \( f'_2(x_1, x_2, k_{\perp}) \) and given by

\[
f_2(x, k_{\perp}) = \frac{1}{2} \sum_{h, \bar{h}} \int \frac{d^2k_{1\perp}}{(2\pi)^d} \psi_{h,\bar{h}}(x, k_{1\perp}) \psi^*_{h,\bar{h}}(x, k_{1\perp} + k_{\perp}). \tag{6}
\]

Notice that for \( k_{\perp} = 0 \), the usual LF PDF expression is recovered [27]. We will calculate the quantity \( f_2(x, k_{\perp}) \) encoding the relevant dynamical information.

Since the LF meson wave function is evaluated under the conditions \( x_2 = 1 - x_1 \) and \( k_{2\perp} = -k_{1\perp} \), due to momentum conservation, for simplicity, we use the notation

\[
\psi_{h,\bar{h}}(x_1, k_{1\perp}) = \psi_{h,\bar{h}}(x_1, 1 - x_1, k_{1\perp}, -k_{1\perp}). \tag{8}
\]

We are mainly interested in non-perturbative aspects of the dPDFs, so that, in order to emphasize the role of correlations between \( x \) and \( k_{\perp} \), in the next sections the following ratio will be calculated:

\[
r_{k}(x, k_{\perp}) = \frac{f_2(x, k_{\perp})}{f_2(0.4, k_{\perp})}; \tag{9}
\]

in fact, if a factorized ansatz, e.g. \( f_2(x, k_{\perp}) \sim f_{2,\perp}(x)f_{2,k_{\perp}}(k_{\perp}) \), is used, \( r_k(x, k_{\perp}) \) does not depend on \( k_{\perp} \) [8, 7, 32]. The factorization ansatz is often used in experimental analyses for the proton target.

In closing this section, we note that the dPDFs depend on two momentum scales, corresponding to the mass of the states produced in the two parton-parton scattering in the DPS process, which have not been explicitly shown.

### 2.1 An approximation in terms of one body quantities

An ansatz commonly used to describe the unknown dPDFs makes use of ffs and GPDs (in the case of the proton some experimental knowledge is available). Following the strategy of Refs. [3, 33, 34], we consider the correlator (11) and insert a complete set of states assuming that the pion is dominant. The formal expression for this approximated quantity, \( f'_{2,A}(x_1, x_2, k_{\perp}) \), is:
\[ f_{2, A}'(x_1, x_2, k_\perp) = \frac{P^+}{4} \int d^2 y_\perp e^{-i y_\perp \cdot k_\perp} \int dy^- \]
\[ \times \int dz_1 dz_2 \int \frac{dP^{+\perp} dP'^{+\perp}}{(2\pi)^3} \frac{e^{-ix_1 P^+ z_1 - ix_2 P^+ z_2}}{(2\pi)^2} \]
\[ \times \langle A, 0 | O(0, z_1) | A', P'_\perp \rangle \langle A', P'_\perp | O(y, z_2) | A, 0 \rangle \bigg|^{z_1 = z_2 = 0}_{y^+ = z_1^+ = z_2^+ = 0}. \]

In this scenario, the approximation relies on the assumption
\[ f_{2, A}'(x_1, x_2, k_\perp) \sim f_{2, A}'(x_1, x_2, k_\perp). \]

At this point, using again the strategy already discussed in the previous section, we find:

\[ f_{2, A}'(x_1, x_2, k_\perp) = H(x_1, k_\perp) H(1 - x_2, k_\perp), \quad (11) \]

where \( H(x, k_\perp) = H(x, \xi = 0, k_\perp) \), is the pion GPD at zero skewness. The integral over \( x_2 \) of Eqs. (5) and (11) leads approximately to

\[ f_2(x, k_\perp) \sim \int_0^1 dx_2 f_{2, A}'(x, x_2, k_\perp) = H(x, k_\perp) F(k_\perp), \quad (12) \]

where \( F(k_\perp) \) is the standard pion e.m. form factor. The difference between \( f_2(x, k_\perp) \) and \( H(x, k_\perp) F(k_\perp) \) addresses the presence of unknown parton correlations that can not be studied by means of one-body distributions. In order to emphasize such effects, the relation (12) will be discussed in the next section.

The GPD for the pion \([23]\) might be written also in terms of the wave function \([35, 36]\).

\[ H(x, \xi = 0, \Delta^2_\perp) = \frac{1}{2} \sum_{h, \bar{h}} \int \frac{d^2 k_\perp}{16\pi^3} \]
\[ \times \psi_{h, \bar{h}}(x, k_\perp) \bar{\psi}_{h, \bar{h}}(x, k_\perp + (1 - x) \Delta_\perp), \quad (13) \]

an expression well suited for model calculations which will be used in the next section.

### 2.2 The effective cross section

A relevant observable for DPS proton studies is the so called effective cross section, \( \sigma_{\text{eff}} \), see e.g. Ref. [37]. It is defined as the ratio of the product of two single parton scattering process cross sections to the DPS with the same final states. It is extracted from data using model assumptions, and it can be expressed in terms of PDFs and dPDFs [3]. For proton-proton collisions, this quantity has been also studied within the AdS/QCD soft-wall model [11]. In Refs. [9, 11] it has been shown how a dependence of \( \sigma_{\text{eff}} \) on the longitudinal momentum fractions of the acting partons reflects the presence of non trivial double parton correlations. In the present study we use the definition of \( \sigma_{\text{eff}} \) for a meson target in order to make new predictions.

The effective cross section for a DPS process, involving meson-meson collisions, generally depends on four variable \( x_1, x_2 \) and \( x'_1, x'_2 \), i.e. the longitudinal momentum fractions of the partons involved in the process. Nevertheless in the zero rapidity region, i.e. \( x_1 = x'_1 \) and \( x_2 = x'_2 \), \( \sigma_{\text{eff}} \) reads:
\[ \sigma_{\text{eff}}(x_1) = \frac{(f_1(x_1)f_1(1-x_1))^2}{\int \frac{d^2k_{\perp}}{(2\pi)^2} f_2(x_1, k_{\perp})^2}, \] (14)

where \( f_1(x) \) is the single PDF. Furthermore, one can define an average value as follows:

\[ \overline{\sigma}_{\text{eff}} = \frac{1}{\int \frac{d^2k_{\perp}}{(2\pi)^2} F_2(k_{\perp})F_2(-k_{\perp})}, \] (15)

where the effective form factor

\[ F_2(k_{\perp}) = \int_0^1 dx f_2(x, k_{\perp}) \] (16)

has been introduced (see Refs. [9, 38]). Equation (15) assumes factorization between the \( x \) and \( k_{\perp} \) in the dPDF. In this factorized scenario, one might notice that \( \sigma_{\text{DPS}} \), i.e. the DPS cross section (see, e.g., Refs. [2, 39]), depends on \( 1/\overline{\sigma}_{\text{eff}} \) [39]. Thanks to this feature, the value of \( 1/\overline{\sigma}_{\text{eff}} \) provides a rough estimate of the magnitude of \( \sigma_{\text{DPS}} \). In our model calculation, we provide predictions for hypothetical experiments with mesons, as illustrated in the next section.

### 3 Calculation of the pion dPDF using AdS/QCD models

In the present section we introduce and discuss the LF wave function then used to evaluate the dPDFs. In particular, we will make use of the approach based on the AdS/QCD soft-wall model, where the pion w.f. reads [20, 21]:

\[ \psi_{\pi o}(x, k_{1\perp}) = A_o \frac{4\pi}{\kappa_o \sqrt{x(1-x)}} e^{-\frac{k_{1\perp}^2 + m_o^2}{2\kappa_o (1-x) x}}, \] (17)

where \( m_o = m_u \sim m_d, x = x_1, x_2 = 1-x_1 \) and \( k_{2\perp} = -k_{1\perp} \). The parameters of the model have been recently fixed to reproduce the Regge behavior of the mass spectrum of mesons [26, 40]. They are \( \kappa_o = 0.523 \) GeV and \( m_o \sim 0.33 \) GeV. The constant \( A_o \) is fixed by the normalization condition (4) and it is found to be \( A_o = 3.0498 \). Several models of pion LF wave functions are available, [20, 21, 23, 40], the most straightforward and therefore suitable to show general properties of pion dPDFs is probably the first model proposed [20, 21], where the dPDF is analytically expressed by:

\[ f_{\pi o}^2(x, k_{\perp}) = A_o^2 e^{-\frac{4m_o^2 + k_{\perp}^2}{4\pi^2 (1-x)}}. \] (18)

In this paper, we calculate dPDF for \( \pi^+ \). The distributions for \( \pi^- \) and \( \pi^0 \) can be obtained by isospin and charge conjugation. As one can see in the left panel of Fig. 1, as happens in the proton case [7, 8], the dPDF decreases as \( k_{\perp} \) increases, and the factorization in the \( k_{\perp} \) and \( x \) is not supported by the model as can be observed in the right panel of Fig. 1 where the ratio of Eq. (9) shows a clear \( k_{\perp} \) dependence. We conclude the discussion of these results by reporting the mean value of \( \sigma_{\text{eff}} \) within the model at the hadronic scale \( \mu_0 \), \( \overline{\sigma}_{\text{eff}}(\mu_0) = 41.69 \).
Figure 1: Left panel: dPDF of the pion within the AdS/QCD model of Ref. [20] (cfr Eq. (17)) at different values of \( k_\perp \). Full line \( k_\perp = 0 \) GeV, dashed line \( k_\perp = 0.2 \) GeV, dot-dashed line \( k_\perp = 0.5 \) GeV and dotted line \( k_\perp = 0.6 \) GeV. Right panel: The ratio defined by Eq. (9) for the same parameters as in the left panel.

mb. This value is larger than the corresponding proton value [9, 33, 34], a feature related to the geometrical properties of the targets (see Ref. [38] for details).

For completeness, we report in Fig. 2 the pion GPD evaluated within the model. As one can see, the pion GPD is very similar to its dPDF. It is apparent that the expressions for the dPDF and GPD, Eqs. (11), in terms of the light-front pion wave function, are similar. However, in the dPDF, \( k_\perp \) represents an intrinsic imbalance of the parton momentum between the initial and the final states keeping the same pion momentum in both states, while in the GPDs, \( \Delta_\perp = k_\perp \) represents the difference in momentum between the initial and final state of the pion. Therefore, the dependence of the GPDs on the partonic momentum, i.e. \( k_1, k_2 \pm (1-x)k_\perp \) produces an asymmetry in the \( x \) dependence, which is not present in dPDF. Moreover, since in the GPDs the momentum imbalance in the wave function is multiplied by the pre-factor \( 1-x < 1 \), at variance with the dPDF, the latter goes to zero faster than the GPD. Let us stress that such a similarity between dPDFs and GPDs holds only for the valence component and at the hadronic scale, i.e. where only two valence particles are taken into account in the model. If higher Fock states were included in the LF wf representation of the pion, other non perturbative \( x_1 - x_2 \) correlations would appear. Moreover if one considers the pQCD evolution of dPDFs, also perturbative \( x_1 - x_2 \) correlations show up (see e.g. Ref. [10]). Analogously to the proton case, all these non trivial dependence of dPDFs on \( x_1 \) and \( x_2 \) cannot be accessed via GPDs, a confirmation of the rich three-dimensional structure accessible via dPDFs.

Finally we compare the complete \( f_\pi^2(x, k_\perp) \) with its approximation Eq. (12), i.e. \( f_\pi^2(x, k_\perp) \). If only the valence contribution were considered, the approximation to the dPDF would become a product of a GPD and a form factor, as seen in Eq. (12), at variance with the proton case, where the dPDF is written as a product of two GPDs. In Fig. 3 we compare the dPDF (11) and its approximation (12) as a function of \( x \) for three different values of \( k_\perp \). As one can see, at the hadronic scale, the pion dPDFs contains non trivial information different from that encoded in the GPDs and ffs, a feature also observed in the proton case (see e.g. Refs. [6, 7, 8, 10, 32, 39, 38, 41]).

4 Evolution

The next step in our scheme is to calculate the perturbative evolution of the dPDFs from the low momentum scale of the model, the so called hadronic scale \( \mu_0^2 \), to the high scale of the data \( Q^2 \). As stated in the introduction, dPDFs depend on two momentum scales. For simplicity, as it has been done in previous works, see e.g. Ref. [12], we assume here that two scales coincide. We follow here the same strategy developed in Refs. [8, 10] adapted to the use of
Figure 2: The pion GPD defined in Eq. (13) for $k_{\perp} = 0$ GeV (full line), $k_{\perp} = 0.2$ GeV (dashed line), $k_{\perp} = 0.5$ GeV (dot-dashed) line and $k_{\perp} = 0.6$ GeV (dotted line).

Figure 3: The pion dPDF, evaluated by means of its definition of Eq. (7), is shown in full lines, and its approximation, defined by Eq. (12), is plotted in dotted lines, for three values of $k_{\perp}$: $k_{\perp} = 0$ GeV, $k_{\perp} = 0.2$ GeV and $k_{\perp} = 0.5$ GeV. The quality of the approximation decreases as $k_{\perp}$ increases as shown by the bands emphasizing for the difference between the exact calculation and the approximation.
quark models to calculate the proton’s dPDFs. Historically, the evolution equations for dPDFs can be seen as a generalization of the usual DGLAP equations (see the original papers \cite{12, 13} and recent contributions in Refs. \cite{3, 53, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23}). This feature sets up the strategy which we are going to discuss next.

We start with the decomposition of the dPDF at a generic scale $Q^2$:

\[
F_{ud} = F_{(uv+u_{sea})(\bar{d}v+\bar{d}_{sea})} = F_{(uv+\bar{u})(\bar{d}v+\bar{d})} = F_{uv\bar{d}v} + F_{uv\bar{d}} + F_{\bar{u}d\bar{v}} + F_{\bar{u}\bar{d}}
\]

where, at the hadronic scale \cite{53},

\[
u_{V}^+ = \bar{\nu}_{V}^+ = \bar{\nu}_{V}^- = \nu^+ \equiv u_V;
\]

while at any scale $q_{sea} = \bar{q}_{sea} = \bar{q}$, with $q = u, d, s$ for $N_f = 3$ three active flavors. It is convenient to use the symmetrized form of dPDFs, $\bar{F}_{ab} = (F_{ab} + F_{ba})/2$ where $\bar{F}_{ab} \equiv \bar{F}_{ab}(x_1, x_2, k_\perp, Q^2)$ is symmetric in $x_1, x_2$.

### 4.1 Flavor decomposition

In order to proceed with the evolution equations one has to construct from the $\bar{F}_{ud}$ the Singlet and Non-Singlet components

\[
\Sigma = \sum_q q^+ = u_V + 2\bar{u} + d_V + 2\bar{d} + s + \bar{s}
\]

\[
T_3 = u^+ - d^+ = u_V + 2\bar{u} - d_V - 2\bar{d}
\]

\[
T_8 = u_V + 2\bar{u} + d_V + 2\bar{d} - 2(s + \bar{s})
\]

\[
V_i = q_i^+ ,
\]

where $q_i^+ = q_i \pm \bar{q}_i$.

The evolution equations involve different equations for the Singlet – Singlet component ($\Sigma \Sigma$), NonSinglet – Singlet components ($T_3 \Sigma + \Sigma T_8$, $d_V \Sigma + \Sigma d_V$, $u_V \Sigma + \Sigma u_V$), and NonSinglet – NonSinglet contributions (constructed from $V_i$, $T_3$, $T_8$).

### 4.2 Mellin-Moments and inversion

The procedure follows by constructing the Mellin-moments which allow to solve the evolution equations easily. These quantities are

\[
\frac{1}{2} M_{n_1n_2}^{n_1n_2}(Q^2) = \frac{M_{n_1n_2}^{n_1n_2}(Q^2) + M_{n_1n_2}^{n_1n_2}(Q^2)}{2}
\]

\[
= \int_0^1 dx_1 \int_0^{1-x_1} dx_2 x_1^{n_1-1} x_2^{n_2-1} F_{ab}(x_1, x_2; Q^2).
\]

At the hadronic scale $\mu_0^2$, all the combinations of dPDFs with $\Sigma$, $T_8$, $T_3$ and $V_i$ will contain valence partons only. As a result the remaining term will be $F_{uv\bar{d}v}$, and the non vanishing moments at the hadronic scale will assume the form

\[
M_{uv\bar{d}v}^{n_1n_2}(\mu_0^2, k_\perp) = \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \delta(1 - x_1 - x_2) \times x_1^{n_1-1} x_2^{n_2-1} f_2^{xO}(x_1, x_2, k_\perp, \mu_0^2)
\]

\[
= \int_0^1 dx x^{n_1-1}(1 - x)^{n_2-1} f_2^{xO}(x, k_\perp, \mu_0^2).
\]
They enter the moments of the combinations directly depending on \( \Sigma, T_3, T_8 \) and \( V_i \), but each moment \( M_{n_1 n_2}^{ab}(\mu_0^2) \), defined at the hadronic scale, will evolve according to its specific flavor symmetry \([10]\). The moments are independent functions of the complex indices \( n_1, n_2 \) and the inversion of Eq. (23) will produce dPDFs defined in the whole \((x_1, x_2)\) domain with \( x_1 + x_2 \leq 1 \).

### 4.3 Evolution of the dPDFs: results

In Fig. 4, we plot the second moment of the double distribution \( x_1 x_2 \tilde{F}_{ud}(x_1, x_2, y, Q^2) \) defined by

\[
M_{ud}^{22}(y, Q^2) = \int_0^1 dx_1 \int_0^{1-x_1} dx_2 x_1 x_2 \tilde{F}_{ud}(x_1, x_2, y, Q^2),
\]

where \( y \) is the distance between the two correlated partons, obtained Fourier transforming the \( k_\perp \) dependent distribution \( f_2^\pi(x, k_\perp) \) given in Eq. \([18]\). This quantity incorporates the evolution to large \( Q^2 \) of the distribution \( \tilde{F}_{ud}(x_1, x_2, y, Q^2) \) starting from the initial scale \( \mu_0^2 = (0.523)^2 \) GeV\(^2\), already used in calculation of pion PDF and unpolarized transverse momentum dependent PDF in Ref. \([27]\).

At the hadronic scale since only two valence particles are present, the support condition, preserved within the Light-Front approach, forces the dPDF to exist only when \( x_1 + x_2 = 1 \). This is at variance with the proton case where the existence of a third particle allows complete freedom for \( x_1 \) and \( x_2 \) as long as momentum is conserved \( x_1 + x_2 < 1 \). The evolution procedure, described by Eq. \([22]\), where \( n_1 \) and \( n_2 \) are independent complex parameters allows to obtain \( F_{ud}(x_1, x_2, y, Q^2) \) for all values of \( x_1 \) and \( x_2 \) and \( x_1 + x_2 < 1 \). The creation, in the evolution process, of sea and glue partons allows \( x_1 \) and \( x_2 \) to free themselves from the valence condition.

As we mentioned in the Introduction, preliminary results for quantities related to moments of the pion dPDFs have been recently reported within a lattice QCD approach \([28]\). A comparison of results obtained in model calculations with lattice data would open interesting new perspectives.
A first important effect of the evolution procedure can be seen in Fig. 5, where the double distribution \( x_1 x_2 \bar{F}_{ud}(x_1, x_2, y, Q^2 = 100 \text{ GeV}^2) \) is shown in the domain \((x_1, x_2 = 1 - x_1)\) as a function of \( x_1 \) and for different values of \( y \). The comparison with the same distribution at the hadronic scale \( \mu_0^2 \) and \( y = 0 \) clearly emphasizes the effects of the evolution. The evolution from \( \mu_0^2 \) to \( Q^2 = 100 \text{ GeV}^2 \) produces a reduction of the distribution, a behavior physically interpretable as the creation of new partonic species carrying momentum, in particular gluon distributions. Recall that the latter are zero at the hadronic scale for the models considered.

In Fig. 6 the double distribution \( x_1 x_2 \bar{F}_{uv,g}(x_1, x_2, y, Q^2) \) is plotted. The upper panel shows the dependence of the distribution on the scale \( Q^2 \), while the lower panel illustrates its dependence on the parton distance \( y \).

A large part of the valence parton momentum is transferred to the gluons which increases dramatically at low-\( x \), while the relevance of the valence partons decreases.

5 Conclusions

Double parton distribution functions may represent a novel tool to access the three-dimensional structure of hadrons. It is therefore natural to study the dPDFs of the pions, specially now that the first estimates of quantities related to the dPDF of pion have been reported by lattice studies [28]. We have used here a Light Front formalism, for which the wave function of the system is required. The AdS/QCD correspondence has generated our LF wave function. Once the formalism has been set up we have calculated several quantities. Among them, we have obtained the mean effective cross section for pion-pion scattering at the hadronic scale, \( \bar{\sigma}_{\text{eff}}(\mu_0) = 41 \text{ mb} \), which turns out to be larger than the same cross section, evaluated with a similar approach, in the proton case. This quantity is very much independent on QCD evolution and provides us with an estimate of the magnitude of DPS [9, 11].

In the adopted AdS/QCD model, dPDFs turn out to be analytical. It has been found that an approximation in terms of generalized parton distributions, proposed in several approaches, is not reliable, as it happens also in the proton case [10]. Analogously, our calculations show that dPDFs do not factorize into \( x_{1,2} \)– and \( k_{\perp} \)– dependent terms. These facts expose
Figure 6: The quantity $x_1x_2F_{u/g}(x_1, x_2, y, Q^2)$ is plotted as a function of $x_1$ for $x_2 = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9$. In the upper panel the full lines represent the results at $Q^2 = 4$ GeV$^2$ and the dashed lines those ones at $Q^2 = 100$ GeV$^2$ for the same value of the parton distance $y = 0$ fm. The lower panel compares the distributions for different distances: $y = 0$ fm (full lines) and $y = 0.4$ fm (dashed lines) and $Q^2 = 4$ GeV$^2$. 
the presence of unknown double parton correlations in the pion not accessible from one-body distributions.

We have performed the evolution to high $Q^2$ using the conventional formalism, subject in this case, at the model momentum scale where only two valence constituents with momentum fractions $x_1$ and $x_2$ are present, to the $x_1 + x_2 = 1$ restriction. Expected results are obtained. For example, the second moment decreases as $Q^2$ increases, signalling the opening of new dPDFs associated with sea and gluons. A good example has been shown in Fig. 6 where the dPDF due to the correlation of valence and gluons is shown for different values of the parton distance and $Q^2$. At the hadronic scale such a distribution vanishes because no gluons are included. At higher scale $Q^2$ the radiative production of gluons from the valence system makes $F_{uvg} > 0$. The dPDFs show, both at the model scale and at a high momentum scale, also a strong dependence on the partonic distance, decreasing in magnitude as the distance increases.

While, at present, experiments designed to measure dPDFs of the pion cannot be imagined, lattice calculations have started to approach this problem and will be likely able, in the near future, to distinguish between predictions of different models of the pion structure, such as the one presented here, opening new perspectives.

Acknowledgements

This work was supported in part by the Mineco under contract FPA2013-47443-C2-1-P and SEV-2014-0398. M.T. and V.V. thank the University of Perugia and INFN, Perugia section, for warm hospitality and support. S.S. thanks the University of Valencia and the IFIC for warm hospitality and support.

References

[1] G. Aad et al. [ATLAS Collaboration], New J. Phys. 15 (2013) 033038
[2] N. Paver and D. Treleani, Nuovo Cim. A 70 (1982) 215.
[3] M. Diehl, D. Ostermeier and A. Schafer, JHEP 03, 089 (2012)
[4] M. Guidal, H. Moutarde and M. Vanderhaeghen, Rept. Prog. Phys. 76 (2013) 066202
[5] R. Dupré, M. Guidal and M. Vanderhaeghen, Phys. Rev. D 95 (2017) no.1, 011501
[6] H. M. Chang, A. V. Manohar and W. J. Waalewijn, Phys. Rev. D 87 (2013) no.3, 034009.
[7] M. Rinaldi, S. Scopetta and V. Vento, Phys. Rev. D 87, 114021 (2013)
[8] M. Rinaldi, S. Scopetta, M. Traini and V. Vento, JHEP 12, 028 (2014)
[9] M. Rinaldi, S. Scopetta, M. Traini and V. Vento, Phys. Lett. B 752, 40 (2016).
[10] M. Rinaldi, S. Scopetta, M. C. Traini and V. Vento, JHEP 16, 063 (2016).
[11] M. Traini, M. Rinaldi, S. Scopetta and V. Vento, Phys. Lett. B 768 (2017) 270
[12] R. Kirschner, Phys. Lett. 84B (1979) 266.
[13] V. P. Shelest, A. M. Snigirev and G. M. Zinovev, Phys. Lett. 113B (1982) 325.
[14] J. M. Maldacena, Int. J. Theor. Phys. 38 (1999) 1113 [Adv. Theor. Math. Phys. 2 (1998) 231]
[15] E. Witten, Adv. Theor. Math. Phys. 2 (1998) 253
[16] J. Polchinski and M. J. Strassler, hep-th/0003136.
[17] S. J. Brodsky and G. F. de Teramond, Phys. Lett. B 582 (2004) 211
[18] J. Erlich, E. Katz, D. T. Son and M. A. Stephanov, Phys. Rev. Lett. 95 (2005) 261602
[19] L. Da Rold and A. Pomarol, Nucl. Phys. B 721 (2005) 79
[20] S. J. Brodsky and G. F. de Teramond, Phys. Rev. D 77, 056007 (2008).
[21] S. J. Brodsky and G. F. de Teramond, Subnucl. Ser. 45, 139 (2009).
[22] J. R. Forshaw and R. Sandapen, Phys. Rev. Lett. 109, 081601 (2012)
[23] G. F. de Teramond et al. [HLFHS Collaboration], Phys. Rev. Lett. 120, no. 18, 182001 (2018)
[24] M. C. Traini, Eur. Phys. J. C 77 (2017) no.4, 246
[25] M. Rinaldi, Phys. Lett. B 771, 563 (2017)
[26] R. Swarnkar and D. Chakrabarti, Phys. Rev. D 92, no. 7, 074023 (2015)
[27] A. Bacchetta, S. Cotogno and B. Pasquini, Phys. Lett. B 771, 546 (2017).
[28] C. Zimmermann [RQCD Collaboration], PoS LATTICE 2016, 152 (2016)
[29] T. Kasemets and A. Mukherjee, Phys. Rev. D 94, no. 7, 074029 (2016)
[30] S. J. Brodsky, H. C. Pauli and S. S. Pinsky, Phys. Rept. 301, 299 (1998).
[31] S. J. Brodsky, M. Diehl and D. S. Hwang, Nucl. Phys. B 596, 99 (2001).
[32] M. Rinaldi and F. A. Ceccopieri, Phys. Rev. D 95, no. 3, 034040 (2017).
[33] B. Blok, Y. Dokshitzer, L. Frankfurt and M. Strikman, Eur. Phys. J. C 72, 1963 (2012).
[34] B. Blok, Y. Dokshitzer, L. Frankfurt and M. Strikman, Eur. Phys. J. C 74, 2926 (2014).
[35] M. Diehl, Phys. Rept. 388, 41 (2003).
[36] C. Mezrag, L. Chang, H. Moutarde, C. D. Roberts, J. Rodríguez-Quintero, F. Sabatié and S. M. Schmidt, Phys. Lett. B 741, 190 (2015)
[37] A. Del Fabbro and D. Treleani, Phys. Rev. D 63, 057901 (2001)
[38] M. Rinaldi and F. A. Ceccopieri, Phys. Rev. D 97, no. 7, 071501 (2018)
[39] F. A. Ceccopieri, M. Rinaldi and S. Scopetta, Phys. Rev. D 95, no. 11, 114030 (2017)
[40] M. Ahmady, F. Chishtie and R. Sandapen, Phys. Rev. D 95, no. 7, 074008 (2017)
[41] T. Kasemets and S. Scopetta, arXiv:1712.02884 [hep-ph].
[42] A. M. Snigirev, N. A. Snigireva and G. M. Zinovjev, Phys. Rev. D 90, no. 1, 014015 (2014)
[43] A. V. Manohar and W. J. Waalewijn, Phys. Rev. D 85, 114009 (2012)
[44] E. Cattaruzza, A. Del Fabbro and D. Treleani, Phys. Rev. D 72, 034022 (2005)
[45] M. Diehl and A. Schafer, Phys. Lett. B 698, 389 (2011)
[46] M. G. Ryskin and A. M. Snigirev, Phys. Rev. D 83, 114047 (2011)
[47] J. R. Gaunt and W. J. Stirling, JHEP 1106, 048 (2011)
[48] F. A. Ceccopieri, Phys. Lett. B 697, 482 (2011)
[49] J. R. Gaunt, JHEP 1301, 042 (2013)
[50] A. V. Manohar and W. J. Waalewijn, Phys. Lett. B 713, 196 (2012)
[51] W. Broniowski and E. Ruiz Arriola, Few Body Syst. 55, 381 (2014)
[52] M. Diehl, T. Kasemets and S. Keane, JHEP 1405, 118 (2014)
[53] M. Gluck, E. Reya and M. Stratmann, Eur. Phys. J. C 2, 159 (1998).