Particle Image Velocimetry in Confined Vortex Flows

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Abstract. In this paper, the challenges of using Particle Image Velocimetry as a diagnostic tool for confined swirling flows are discussed. To highlight these challenges, results from a PIV experiment are compared to a pair of analytical models. While there is generally good qualitative agreement, the velocity measurements degrade in the core of the vortex. Techniques to mitigate the reduced fidelity in this region will be explored and alternative visualization methods will be presented.

1. Introduction

There has been a renewed interest in confined cyclonic motions in the context of aerospace engineering development. Applications for this technology range from hybrid rockets \cite{1, 2} to purely liquid-fueled engines \cite{3, 4}. The benefits of swirl driven flows are also realized in other ways: swirl injectors are often used to increase the combustion stability of flows \cite{5} and swirl-based combustors often have reduced thermal loads on the walls of the chamber \cite{6}.

Characterizing the velocity in such confined vortex devices is often a challenging endeavor. Early investigations relied on intrusive measurement devices such as Pitot probes \cite{7} to make measurements but these devices impact the ensuing flow field. The effect is especially significant in confined vortex flows where the fluid motion has components in all three orthogonal directions. Laser Doppler Velocimetry (LDV) and Particle Image Velocimetry (PIV) have reduced the intrusiveness of these measurements but challenges remain, especially in swirling flows where the seeding quality in the core is often inadequate \cite{8}. In this paper, we discuss some of the challenges of laser-based velocity measurement by way of a comparison with a pair of analytical models.

2. Experimental and Analytical Methods

For the sake of geometric flexibility, a modular cylindrical chamber is fabricated from quartz and provides a range of aspect ratios from 2.8 to 8.8. Gaseous nitrogen enters tangentially at the base of the chamber through configurable injectors to provide pressure drops ranging from 10\% to 30\% of the chamber pressure. A smoke generator seeds the flow with liquid particles tuned to a 0.2\,\mu m diameter.

The imaging system consists of a 250 mJ/pulse Nd:YAG laser with a typical pulse separation of 1\,\mu s to adequately resolve the velocities in the $r$--$\theta$ plane. A LaVision 1280 x 1024 Flowmaster 3 camera captures the particle images at 1\,\mu s increments spaced out over a thirty second run time. The images are then cross-correlated to deduce the swirl velocity in the chamber. The optics used to generate the laser sheet are movable, allowing investigation of the $r$ -- $\theta$ plane at
three separate axial locations. For additional information on the PIV experiment, the reader is referred to the study by Rom [9].

In order to assess the results of the PIV experiment, two analytical models are compared with the data. The first is a laminar core model derived from the tangential momentum equation with second-order viscous terms retained. Solving the perturbed boundary-layer equation [10] yields an approximate solution of the form

$$u_\theta \approx \frac{1}{r} \left[ 1 - \exp \left( -\frac{1}{4} V r^2 \right) \right]$$  \hspace{1cm} (1)

The second comparison is with the constant shear stress model [11] which assumes a balance between the shear and pressure forces in the core and a free vortex of the $u_\theta = 1/r$ type in the outer region. The solution is a piecewise function, reminiscent of Rankine’s [12], but with the benefit of matching both the value of the velocity and the slope where the inner and outer solutions meet. The resulting swirl velocity expression is

$$u_\theta = \begin{cases} 
\frac{r}{X^2} \left[ 1 - \ln \left( \frac{r^2}{X^2} \right) \right] & ; \quad r \leq X \\
\frac{1}{r} & ; \quad r > X 
\end{cases}$$  \hspace{1cm} (2)

A modified least-squares regression provides the location of the matching radius, $X$.

3. Results

The vortex Reynolds numbers observed in these experiments ($V \approx 35000$) are larger than a typical laminar solution. However, as the confined vortex still behaves as a laminar structure, it is possible to correlate the models to the experiment by way of an eddy viscosity ratio, $\ell_t = \mu/\mu_t$, with $\mu_t$ denoting a turbulent viscosity. For the constant shear stress model, we write an expression for the matching radius of the constant shear model in terms of the vortex Reynolds number [11]. The resulting vortex Reynolds number definition and matching radius relation may be written as

$$V = \frac{\rho U A_i}{\mu L} = \frac{\rho U A_i}{\mu_t \ell_t L} = \frac{V_t}{\ell_t}; \quad X = \frac{X_0}{\ell_t}$$  \hspace{1cm} (3)

A least-squares algorithm optimizes $\ell_t$ and $X_0$ to produce a matching radius that incorporates the experimental vortex Reynolds number. Average values of $X_0 \approx 50$ and $\ell_t \approx 150$ provide good agreement with the experimental data.

Fig. 1 depicts the experimental data with the analytical models. In the outer region, the data follows the free-vortex shape predicted by both models but the quality of agreement deteriorates in the core. To accommodate the scatter, the fits for both theoretical models are weighted such that they provide an envelope near the centerline rather than a strict regression fit through the data. This weighting is chosen specifically because of the increased scatter that artificially lowers the predicted velocities.

In most cases, the PIV technique constitutes a powerful non-intrusive method for measuring velocities, but the viscous core of swirling flows is difficult to accurately assess even with properly sized particles. There are two significant considerations in the core: increased drag owing to high azimuthal velocities and a tendency for the particles to separate via centrifugal entrainment. This is visible in Fig. 1, as the majority of the data points appear in the inviscid, outer vortex with markedly fewer points in the inner region. Adding to the uncertainty is the fact that the axial velocity is highest at the centerline, perpendicular to the measurement plane; the $r - \theta$ plane measurements are more difficult to correlate since particles that were illuminated in the
first exposure may have passed below the light sheet by the second. The combined effect of these factors is decreased confidence in the correlated data near the center of the vortex [13].

Aside from the measurement challenges near the centerline, fully characterizing the structure of this flow requires knowledge of all three velocities. Traditional PIV techniques are unable to simultaneously obtain all three velocity measurements since a traditional PIV setup cannot measure components perpendicular to the light sheet. This deficiency can be mitigated through the incorporation of a second PIV camera and additional optics, resulting in a stereoscopic PIV system. The second camera provides the system with depth of field information, but at the expense of increased equipment cost and longer processing times as images from both cameras require correlation. Additionally, the perpendicular velocity is usually captured at a lower resolution than its in-plane counterparts. Though not as common, it is possible for a single camera to behave as a stereoscopic PIV with the expense of additional optics, though research to improve the technique is ongoing [14].

Recent research has seen an increase in the use of plenoptic, or light-field, cameras in PIV experiments. A plenoptic camera adds an array of micro-lenses before the image sensor which allows the camera to capture both the position and angle of propagation of the incoming light [15]. While the concept is sound, improving the accuracy of the reconstructions from the light field data is still an active avenue of research [16]. Plenoptic PIV is a developing technology but as specialized cameras become more available and data reconstruction improves, it will begin to see more widespread use.

4. Conclusion
Despite its drawbacks, Particle Image Velocimetry offers a vital resource in the quantification of confined vortex flows. Since these flows require fully three-dimensional velocity measurement, improving existing stereoscopic or plenoptic techniques will provide researchers with valuable tools to characterize these structures. Though increased particle drag coupled with centrifugal separation make accurately assessing velocities in the center of these structures a challenge, it is possible to refine existing models to better predict measured swirl velocities.

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