Features of implementation of adaptive signals processing for a cylindrical antenna array with a horizontal generatrix

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Abstract. The ways of overcoming the problems caused by the specific asymmetry of the side lobes of the channels formed at the output of cylindrical antenna arrays with a horizontal generatrix, which are relevant for underwater robots, are considered. An algorithmic solution is proposed based on the use of adaptive signal power limitation for solving detection-direction finding problems in combination with non-adaptive reception or the diagonal loading Capon algorithm for assessing the spectral and correlation characteristics of strong signals.

1. Introduction
The current moment can be characterized as the moment of intensive development of hydroacoustic devices for underwater robots, i.e. autonomous underwater unmanned vehicles (AUUV). These tools are used to solve a wide range of tasks [1], including maritime safety, positioning [2], communications [3], environmental sensing, etc. At the same time, the conditions for the use of AUUV require that the solution of the listed tasks be carried out in the presence of interference of a various nature - from technogenic to biological. In the case of an AUUV, the adaptive suppression of hydroacoustic interferences should provide:

- resolution of signals against the background of interference;
- minimization of the adaptation time interval;
- implementation of interference suppression in the form of an adaptive filter, which makes it possible to implement subsequent spectral or correlation signal processing.

From the viewpoint of hydrodynamics, with all the variety of design options for underwater robots, their most natural form is cylindrical. As a result, the desire to maximize the aperture makes the use of cylindrical antennas with a horizontal generatrix many times larger than the diameter most relevant in AUUVs.

The specificity of such antennas, which complicates the interpretation of the results of hydroacoustic survey, is that, on one hand, the width of the radiation pattern in elevation angle (EA) can be several times greater than in azimuth. On the other hand, when using a grid of viewing directions, with constant values of the step in EA and azimuth, the lines of the side lobes are curves with a variable slope, depending on the direction of the signal arrival both in the EA and in azimuth.

Azimuth deviation of signal responses when scanning on different EAs can lead to false detection and direction finding errors due to signal responses deviation in the scans when the EA of scan does not match the actual signal arrival EA, especially in the case of multipath signals.
Therefore, the requirements listed above should be supplemented with requirements specific to the type of antenna in question, namely:

- suppression of side lobes of interference by EA, including that for relatively weak interference;
- suppression of side lobes of signals arriving at the antenna from directions outside the viewing sector, in particular, by EA.

The purpose of this work is to compare a number of adaptive algorithms for receiving signals against background interference that meet the above requirements, under conditions of receiving signals by a cylindrical antenna array with a horizontal generatrix, while solving three problems:

1) resolution of a weak signal against background high-, medium- and low-power noise;
2) resolution of beams of a strong signal, significantly differing in power;
3) resolution of a weak signal against the background clutter by side lobes of interference, the beams of which enter the antenna from directions outside the viewing sector along the EA.

2. Algorithms

The generally accepted standard for adaptive algorithms is the Capon algorithm [4,5]. The row vector \( y \) of complex spectra, according to the output of this algorithm for \( K \) consecutive spectral analysis intervals at one frequency \( f_n \) (the designation \( f_n \) is omitted here and in the expressions below) is calculated in accordance with the expression:

\[
y = A^*X,
\]

where \( A \) – is the column vector of adaptive weight coefficients with the dimension of the number of antenna hydrophones \( L \), \( + \) – is the sign of the Hermitian conjugation, \( X \) – is the matrix of the input sample of dimension \( L \) by \( K \), composed of \( K \) column vectors containing the values of the complex spectra at the output of the antenna array hydrophones at \( K \) successive spectral analysis intervals. The vectors making up the matrix \( X \) will be called sampling elements. Vector \( A \) is defined by the expression

\[
A = \frac{Q^{-1}V}{V^*Q^{-1}V}, \tag{1}
\]

where \( Q = XX^* \) – a sample estimate of the correlation matrix (here and below, up to a multiplicative parameter), \( V \) – steering vector. Energy at the output of the Capon algorithm is

\[
p = yy^* = \frac{1}{V^*Q^{-1}V}.
\]

The problem that makes the application of the Capon's algorithm in the above classical form unrealizable is that when evaluating the correlation matrix \( Q \), the sample size \( K \) to ensure noise immunity must be at least four times the number of hydrophones \( L \), which is unacceptable for reasons of minimizing the adaptation interval.

To overcome this problem, it is customary [6,7] to implement the Capon algorithm for the output of a limited number of non-adaptive receiving channels (Beamspace Capon) formed in a sector (working window) covering the observation direction. This ensures a high resolution of the signals simultaneously entering the working window. However, such a solution also cannot be used to solve the formulated problems for two reasons.

Firstly, for observation directions in which the direction to the interference source does not fall into the working window, the interference signal level in the working window channels decreases to the extent of their directionality and, sooner or later, turns out to be insufficient to provide adaptive suppression of the interference signal.
And secondly, the need to suppress interference side lobes requires that the working window has two dimensions - both in azimuth and in elevation, which significantly increases the matrix \( Q \) dimension and, therefore, increases the adaptation interval.

In connection with the unsuitability of the mentioned well-known algorithms, to solve the problem under consideration, the possibility of using a number of algorithms was considered, the common feature of which is the implementation of signal processing in the space of sampling elements.

2.1. Algorithm for limiting the power of interference at the output of antenna elements (IPL-E - Interference Power Limitation in Elements)

In the classical form, the projection noise suppression algorithm has the form \[ y = V^\top(I - U_0 U_0^\top)X, \] (2)

where \( U_{0L} \) - the matrix of left eigenvectors corresponding to the \( M \) largest singular values of the matrix \( X : X = U_L \Lambda U_R^\top = U_L \Lambda_0 U_R^\top + U_L \Lambda_1 U_R^\top \), \( \Lambda \) - diagonal singular value of \( X \), \( U_L, U_R \) - matrices of the corresponding left and right eigenvectors, \( \Lambda_0, U_{L0}, U_{R0}, \Lambda_1, U_{L1}, U_{R1} \) - matrices of the \( M \) largest and of the \( K-M \) smallest eigenvalues and eigenvectors \( X \), respectively.

The matrix \( U_{0L} \) is also the matrix of the highest eigenvectors of the estimation of the correlation matrix \( XX^\top \) of the antenna element signals, obtained by averaging over the number of sampling elements \( K \). It is also the matrix of basis vectors of the estimate of the signal subspace of the signal space of the antenna elements.

Expression (2) can also be represented as

\[ y = V^\top X(I - U_0 U_0^\top). \] (3)

In this case, \( U_{0R} \) is the matrix of the highest eigenvectors of the correlation matrix \( X^\top X \) of dimension \( K \) by \( K \). This matrix, composed of estimates of pairwise correlation of sampling elements obtained by averaging over antenna elements, will be called the correlation matrix of sampling elements. Matrix \( U_{0R} \) is also a matrix of basis vectors for estimating the signal subspace, but already a space that can be called the space of sampling elements.

The formal transition from (2) to (3) can be considered as transition from signal processing in the antenna array element space to signal processing in the sample space. In expression (3), the product \( V^\top X \) corresponds to the formation of the receiving channel according to the Bartlett algorithm, i.e. the formation of the output of the traditional non-adaptive receiving channel in the direction of observation, and multiplication by the projector \( I - U_{0R} U_{0R}^\top \) provides adaptive rejection of strong signals that form the signal subspace, i.e. coherent interference. Nonzero eigenvalues \( XX^\top \) are the same as for \( X^\top X \), which makes it possible to estimate the number of interference signals \( M \) by analyzing the eigenvalues [9].

The problem of using algorithms (2) and (3) in practice is that when the interference is suppressed, a "puncture" occurs - a zone of zero sensitivity of the formed observation channels in the directions of interference arrival.

One of the ways [10] to overcome this problem is based on the principle of limiting the interference power [11, 12], which consists in the fact that the interference rejection should be carried out only partially - so that the response of the interference at the survey output would no longer be an interference, but would become the response of a conventional signal, the level of which exceeds the level of the distributed noise by no more than a relatively small given value.
The limitation of the interference power can be realized by replacing the projection matrix \((I - U_{0R}U_{0R}^+)\) in (3) with a quasi-projection matrix:

\[
y = V^+X(I - U_{0R}(I - Z\lambda_{M+1}^{-1}A_0^{-1})U_{0R}^+),
\]

where \(Z\) – parameter of interference suppression level control, \(\lambda_{M+1}\) – largest eigenvalue of matrix \(A_0\).

2.2. Algorithm for limiting the power of interference at the output of the viewing channels (IPL-Ch – Interference Power Limitation in Channels)

The IPL-E algorithm, implemented at the output of the antenna elements, does not use any a priori information about the assumed shape of the interference wave fronts, which ensures its universality. The price paid for versatility is the limited rejection capability of relatively weak interference, i.e. interference, the level of which is equal to or less than the noise level at the output of the Bartlett algorithm. To overcome this limitation, a variant of the interference power limitation algorithm can be used, which is implemented not at the input, but at the output of the Bartlett algorithm. Let us assume that the view sector in the azimuthal and elevation planes is uniformly covered by \(J\) observation channels formed by the Bartlett algorithm. Row vector \(Y_j\) of complex spectra output channel with number \(j\) for \(K\) successive intervals of spectral analysis is computed in accordance with the expression \(Y_j = V_jX\), where \(V_j\) – channel \(j\) steering vector. If we consider the set of outputs of \(J\) channels for \(K\) implementations as an adaptive sample represented by the matrix

\[
Y = [y_1^T, ..., y_J^T, y_J^T]^T,
\]

then, by analogy with the IPL-E algorithm, the IPL-Ch algorithm can be formulated as limiting the interference power in the \(j\)-th channel using the matrix \(Y\):

\[
y_j = V_j^+X(I - \tilde{U}_{0R}(I - Z\tilde{\lambda}_{M+1}^{-1}\tilde{A}_0^{-1})\tilde{U}_{0R}^+)XV_j,
\]

where \(\tilde{\lambda}_0\) – diagonal matrix of the highest \(M\) singular values of the matrix \(Y\), and \(\tilde{U}_{0R}\) - the matrix of the corresponding eigenvectors. The matrix \(Y^+Y\), in turn, like \(X^+X\), is an estimate of the correlation matrix of the sample elements, but already in relation to the sample matrix \(Y\), in which the component level of the signals arriving at the antenna is amplified with respect to noise by the channel shaping procedure.

2.3. Capon algorithm with diagonal loading.

Diagonal loading of the matrix \(Q\) from (1) is one of the widely used techniques for increasing the robustness of the Capon algorithm [13]. The use of this technique for the implementation of the Capon algorithm in the case of extremely small size of training samples [14], as in the case of IPL-E and IPL-Ch, leads to operations in the space of sampling elements of the Bartlett's algorithm output data. In this case, if we replace the ill-conditioned matrix \(Q\) in (1) by its diagonal loaded version \(Q + \mu I\), where \(\mu = z\hat{\sigma}^2\), \(\hat{\sigma}^2\) - estimation of hydrophones noise power, \(z\) – regularization parameter, considering that

\[
Q^{-1} = (XX^* + \mu I)^{-1} = \frac{1}{\mu}(I - X(X^*X + \mu I)^{-1}X^*) \text{ in } V^*V = L,
\]

then it will allow us to obtain an expression for \(A\):

\[
A = \frac{V - X(X^*X + \mu I)^{-1}X^*V}{L - V^*X(X^*X + \mu I)^{-1}X^*V},
\]

and for the output of the adaptive channel:
1. Introduction

2. Theoretical foundations

2.1. Capon algorithm

2.2. Borgiotti-Lagunas algorithm with diagonal loading (BL algorithm)

3. Simulation results

4. Conclusion

Table 1. Parameters of simulated signals

| Signal number | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  |
|---------------|----|----|----|----|----|----|----|----|
| Beam number   | 1  | 2  | 3  | 1  | 1  | 1  | 1  | 1  |
| Azimuth, degrees | -50 | -44 | -19 | -15 | 20  | 23.5 | 45  | 60 |
| EA, degrees   | -50 | -44 | -19 | -15 | 20  | 23.5 | 45  | 60 |
| S / N at the output of the non-adaptive channel, dB | 27  | 17  | 13  | -6  | -6  | 3   | -6  | 20 |

The signals are grouped into four groups to demonstrate the features of the described algorithms functioning when solving three formulated problems.

The first group (signals with numbers 1 and 2) is characterized by the presence of strong interference coming along three beams and a weak signal, the level of which in the output of the Bartlett algorithm is one fourth of the distributed noise level. The first beam prevails in power, which makes it possible to judge on the specific form of signal responses when the signal arrives at an angle to the normal of the generating antenna using the Bartlett's algorithm scan (Fig. 1a). As can be seen from the figure, the side lobes provide interference background in a wide sector of angles in azimuth and EA, and also generate local maxima in the scan, which can cause false detections.

The second (signals 3 and 4) and the third (signals 5 and 6) groups include weak signals and low- and medium-power interference, exceeding the noise in the output of the Bartlett algorithm by a factor of two and twenty, respectively.
The fourth group (signals 7 and 8) is characterized by the presence of medium-power interference, the beams of which arrive at the antenna from directions outside the viewing sector along the EA, moreover, the side lobes of two interference beams arriving at an azimuth of 60 degrees are displayed in scans by two arcs ranging from 42 to 46.5 degrees for the first beam and 49 to 54 for the second one. As a result, a weak signal arriving at the zero EA in an azimuth of 45 degrees is completely masked in the sweep of the Bartlett algorithm.

![Image](image_url)

**Fig. 1** The result of a space survey using Bartlett (a), IPL-E (b), IPL-Ch (c), Capon algorithm (d), BL algorithm (e).

It can be seen from Fig. 1 that the application of Bartlett algorithm does not provide a solution to any of the three formulated problems, except for the unstable resolution of a weak signal against weak background interference (in azimuth -15).

Solution of the first problem, i.e. resolution of weak signals against the background interference, with a significant advantage in relation to others, is provided by the IPL-Ch algorithm (Fig. 1c). The IPL-E algorithm (Fig.1b) also ensures the detection of all weak signals, but with a lower resolution margin, especially in the case of weak interference, which is due to the advantage of the IPL-Ch algorithm in its ability to suppress low-power interference.
The resolving power of the Capon (Fig.1d) and BL (Fig.1e) algorithms for weak signals is approximately the same and is determined by the ability to suppress interference, which manifests itself to the greatest extent, the greater their power is. It is this property that determines the resolution of a weak signal against the background powerful interference (in azimuth -50) and the impossibility of resolution against the background medium-power interference (in azimuth 20) due to incomplete suppression of side lobes. The ability of these algorithms to resolve a weak signal against weak background interference, in full accordance with their basic properties, is the same as that of the non-adaptive Bartlett algorithm, with the only advantage that quite effective suppression of distant strong interference is carried out.

The solution of the second problem, i.e., the resolution of the beams, in the simulated situation is also provided in the best way by the IPL-Ch algorithm (multipath signal in azimuth -50). The IPL-Ch algorithm, parametrically optimized for maximum interference suppression, ensures reliable resolution of two weaker beams, but at the same time weakens the response of the most powerful first beam up to the loss of its resolution. The Capon and BL algorithms, on the contrary, in accordance with their properties, do not provide the resolution of the third, weak, beam against the second beam that is slightly more powerful, but at the same time they reliably resolve the second beam against the very powerful first one. In the latter case, in accordance with the properties of these algorithms, the BL algorithm forms an attenuated response along the first beam, while the Capon algorithm maintains its high level in accordance with the signal strength.

Solution of the third problem, i.e. resolution of a weak signal in an azimuth of 45 degrees against the interference side lobes coming from outside the viewing sector by the EA is provided with acceptable quality only by the IPL-E algorithm. On the one hand, the IPL-Ch algorithm provides resolution of a weak signal, albeit it is unstable. But on the other hand, the insufficient and uneven EA level of interference suppression leads to the appearance of false marks indistinguishable from the signal ones. The algorithms of Capon and BL allow to weaken the level of interference side lobes, but not enough to detect a weak signal due to their limited effectiveness in suppressing relatively low-power interference.

Conclusions

1. When cylindrical antennas with a horizontal generatrix are used, the specific form of the signal responses in the scans of the noise direction finder complicates their automatic analysis. Under these conditions, the purpose of adaptive signal processing is not only to suppress interference side lobes, but also to reduce the responses of the entire set of signals arriving at the antenna, including relatively low-power ones, acting as interference for receiving even more weak signals.

2. To solve this problem, one can use algorithms for limiting the interference power IPL-E IPL-Ch, based on the use of projectors on the subspace of signals in the sampling space. The IPL-Ch algorithm, implemented by the output of a set of formed channels of the survey system, provides the best resolution of signals arriving in the survey sector.

3. Suppression of interference coming from directions outside the sector of view is best ensured by the IPL-E algorithm, which implements signal processing via the direct output of the antenna array hydrophones.

4. A universal solution can be based on the simultaneous use of several algorithms, in particular, IPL-E and IPL-Ch for the detection and direction-finding of weak signals in combination with the Capon algorithm, implemented either traditionally, based on the output of a limited number of viewing channels, or in accordance with the option, considered in this work.

5 As a trend for further research, it is advisable to find the ways to combine the IPL-E and IPL-Ch algorithms into a single algorithm that combines their advantages and is free from drawbacks.

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