Search for Short-Term Periodicities in the Sun’s Surface Rotation: A Revisit

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Abstract We have used the daily values of the equatorial rotation rate determined from the Mt. Wilson daily Doppler velocity measurements during the period 3 December 1985 – 5 March 2007 to search for periodicities in the solar equatorial rotation rate on time-scales shorter than 11 years. After the daily values have been binned into 61-day intervals, a cosine fit with a period of one year was applied to the sequence to remove a seasonal trend. The spectral properties of this sequence were then investigated using a standard Fourier analysis, the maximum-entropy methods, and the Morlet-wavelet analysis. From the analysis of the Fourier power spectrum we detected peaks with periodicities around 7.6 years, 2.8 years, 1.47 years, 245 days, 182 days, and 158 days, but none of them were at a statistically significant level. In the Morlet-wavelet analysis the ≈ 1.47 year periodicity is detected only for 1990, i.e. near the maximum of cycle 22, and near the end of cycle 22 in 1995. From the same wavelet analysis we found some evidence for the existence of a 2.8-year periodicity and a 245-day periodicity in the equatorial rotation rate around the years 1990 and 1992, respectively. In the data taken during the period 1996 – 2007, when the Mt. Wilson spectrograph instrumentation was more stable, we were not able to detect any signal from the wavelet analysis. Thus, the detected periodicities during the period before the year 1996 could be artifacts of frequent changes in the Mt. Wilson spectrograph instrumentation. However, the temporal behavior of most of the activity phenomena during cycles 22 (1986 – 1996) and 23 (after 1997) is considerably different. Therefore, the presence of the aforementioned short-term periodicities during the last cycle and absence of them in the current cycle may, in principle, be real temporal behavior of the solar rotation during these cycles.
1. Introduction

Temporal variations in the Sun occur on many time scales. The time scales relevant for dynamical processes range from minutes (lifetime of small convective elements such as granules) to years/decades/centuries (solar cycles) to billion of years (evolution). Studies of short-term variations in solar activity and their solar cycle dependence may greatly help for better understanding the basic process of solar activity cycle and for predicting the level of solar activity (Krivova and Solanki, 2002; Knaack, Stenflo, and Berdyugina, 2005; Atac and Özgc, 2006; Javaraiah, 2007; Javaraiah, 2008; Forgács-Dajka and Borkovits, 2007; Obridko and Shelting, 2007).

The current model is that the solar dynamo, thought to be responsible for the solar magnetic-activity cycle, operates near the base of the convection zone near the interface between convection and radiation zones (Rosner and Weiss, 1992; Ossendrijver, 2003). This so-called Tachocline is a layer of strong radial shear where the solar rotation profile changes from having a latitude dependency in the convection zone to a pure radial profile in the radiative zone (Spiegel and Zahn, 1992).

The variation of solar rotation with the 11-year solar cycle is well-established by now. The 11-year torsional oscillations were discovered by Howard and LaBonte (1980), using Mt. Wilson Doppler velocity measurements, and have been confirmed using different data sets and methods. Helioseismic observations show that the torsional oscillations are not just a superficial phenomenon but they extend to at least the upper third of the solar convection zone (Howe et al., 2000a).

Variations on the few other time scales in the coefficients of solar differential rotation (Javaraiah and Gokhale, 1995; Javaraiah and Gokhale, 1997; Javaraiah, 1998; Javaraiah, 1999; Javaraiah, 2003; Javaraiah, 2005; Javaraiah and Komm, 1999; Javaraiah, Bertello, and Ulrich, 2005) and the residual rotation (Brajsa, Ruzdjak, and Wohlf, 2006), including one which is approximately equal to the Gleissberg cycle, have been found using sunspot group data. However, variations in the solar rotation on shorter time scales are more difficult to determine because several observational and instrumental effects can produce spurious peaks with similar periodicities.

Howe et al. (2000b) detected a 1.3 year periodicity in the Sun’s internal rotation rate near the base of the convection zone, using GONG and SOHO/MDI helioseismic data over the period May 1995 to November 1999. A similar periodicity is found in sunspot activity (Krivova and Solanki, 2002) and in solar-wind speed (Richardson et al., 1994), suggesting that this periodicity may be associated with the basic process of the solar dynamo. In addition, a similar periodicity is also found in the interplanetary magnetic field (Lockwood, 2001) and many other related solar and geomagnetic data, suggesting that, possibly, there exists a direct connection between the basic mechanism of solar activity and that of interplanetary planetary magnetic field (for a recent and comprehensive review see Obridko and Shelting, 2007). Such a connection may be possible through the mediation of the solar wind (Georgieva et al., 2005). Javaraiah and Komm (1999) analysed the sun’s surface “mean” rotation rate determined from the Mt. Wilson velocity data (1986–1994) and found, beside a few other short-term periodicities, a statistically significant 1.2 ± 0.2-year periodicity. This periodicity in the surface rotation data may be related to the 1.3-year periodicity in the solar activity and the internal rotation. However, Howe et al. (2000b) did...
not find a 1.3-year periodicity in the sun's surface rotation rate determined from the MDI and the GONG data. The aim of this paper is to improve the analysis described in Javaraiah and Komm (1999) by using relatively longer and better calibrated daily data of the equatorial rotation rate determined from the Mt. Wilson Doppler velocity measurements. In addition, we investigated other short-term periodicities and their solar cycle dependence.

In the next two sections we describe in some detail the data and the analysis performed. The power spectra of the time series were calculated using both the standard Fourier analysis and the maximum entropy method (MEM). In addition, we have applied the Morlet wavelet analysis to study the solar-cycle dependence of these periodicities. In Section 4 we present conclusions and briefly discuss them.

2. Data and Analysis

It is standard practice to summarize solar differential rotation profiles by fitting them to minimize least squares of the residuals with the functional form:

$$\omega(\phi) = A + B\sin^2 \phi + C\sin^4 \phi,$$

where $\omega(\phi)$ is the solar angular velocity at latitude $\phi$, the coefficient $A$ represents the equatorial rotation rate and the coefficients $B$ and $C$ measure the latitudinal gradient in the rotation rate, with $B$ representing mainly low latitudes and $C$ representing largely higher latitudes. This formula is chosen largely for historical reasons but can be related to a spherical harmonic expansion.

We have used the daily values of the equatorial rotation rate ($A$) derived from the Mount Wilson Doppler measurements during the period 3 December 1985 to 5 March 2007. This period covers solar cycle 22 and most of cycle 23. In the present analysis we have used the data corrected for the scattered-light effect (for a detail see Ulrich, 2001) and removed the very large spikes ($\text{viz.} > 2\sigma$, where $\sigma$ is the standard deviation of the original time series). Figure 1 shows both the original (dotted curve) and the corrected (solid curve) time series of $A$. The time series has data gaps which vary in size, with a maximum gap of 49 days during Carrington rotation numbers 1560 – 1608. In order to produce an uninterrupted time series, we have binned the above corrected daily data into 120 consecutive 61-day intervals. The 61-day interval was chosen in order to have a sufficient amount of data in the intervals near a large gap. Figure 2 shows the 61-day binned time series of the sidereal $A$ values. In the same figure the horizontal dotted lines indicate the rms deviation in the rotation rate. The one-year periodicity is found in several solar-activity indices, but its origin is doubtful. That is, it is difficult to rule out the possibility that this periodicity is not due to influence of seasonal effects. Therefore, we removed the 1.0 year periodicity from the times series to determine the short-term periodicities in $A$ shown in Figure 2. Figure 3 shows the corresponding time series after subtracting a cosine fit of 1.0 year from the time series shown in Figure 2. From the analysis of Figures 2 and 3 we can see that there are systematic variations in $A$ on time
Figure 1. Variations in \( A \) derived from the Mt. Wilson daily Doppler measurements during 3 December 1985 to 5 March 2007. The dotted curve represents the uncorrected data, and the solid curve represents the corrected data after removal of the large spikes which are greater than \( 2\sigma \), where \( \sigma = 0.071 \) deg/day is the standard deviation of the uncorrected data (\( \sigma = 0.056 \) deg/day for the corrected data). The \( 1\sigma \) uncertainty of an individual measurement is about 0.01 deg/day, much smaller than the aforementioned \( \sigma \) values of the original and the corrected data.

scales of the order of a few years to few days. The amplitudes of these variations are significantly larger than the rms deviation at some epochs, before the year 1996.

As can be seen in Figure 1 there are some large jumps in the original time series (dotted-curve) of \( A \). A few of these large jumps are associated with the time-intervals during which the following changes were made in the Mount Wilson Spectrograph: During 14 February 1989 to 2 May 1989, fiber-optics were rearranged for “rubber-band” grams. During 1 May 1990 to 4 May 1990, exit slit box was rebuilt with inch-worm and exit slits were realigned. On 4 September 1990, a new grating box was installed. On 16 November 1990, entrance slit was made narrower and spectrograph optics were realigned. On 11 December 1991, Littrow lenses were rotated to reduce astigmatism and reduced the entrance slit width again. On 29 December 1996, Littrow lenses were removed and cleaned. In order to implement this log-book information in the analysis, we first arbitrarily removed some high spikes around these time-intervals. But we found that it increased the inconsistency of the time series, \( i.e. \) it increased the number of gaps in the data series and also the sizes of some gaps are found to be increased considerably. Hence, we did not implement the deletions. Moreover, many of those spikes are absent in the corrected time series (the solid-curve of Figure 1).
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Figure 2. Variation of A in the 61-day intervals of the corrected data obtained after removing the large spikes in the daily data. The horizontal solid line represents the mean and the horizontal dotted lines indicate the corresponding rms deviation, which is about 0.035 deg/day.

and the remaining ones are washed out in the aforementioned 61-day averages (Figures 2 and 3).

We have computed the fast Fourier transform (FFT), maximum-entropy method (MEM), and Morlet wavelet power spectra of the data shown in Figures 2 and 3. The MEM FORTRAN program was provided to us by A.V. Raveendran, and was also used in the earlier papers (Javaraiah and Gokhale, 1995; Javaraiah and Gokhale, 1997). We have used the wavelet program by Torrence and Compo (1998). The results of these analyses are described in the next section.

3. Power Spectrum Analysis

3.1. FFT Analysis

Figure 4 shows the FFT power spectrum of the variations in A shown in Figure 3. Before computing the FFT, the mean value of the series was subtracted from the data and a cosine bell function was applied to the first and the last 10% of the time series. Because of the requirement of the specific code we used for the calculation of the Morlet wavelet and to keep the consistency between the two analyses, we have padded the time series with eight zeros so that the number of data points (128) corresponds to an exact power of two.
Figure 3. Variation of $A$ in the 61-day intervals of the corrected data obtained after removing the large spikes in the daily data and also after subtracting a cosine fit of 1.0 year period. The horizontal solid line represents the mean and the horizontal dotted lines indicate the corresponding rms deviation, which is about 0.032 deg/day.

The significance level of the peaks in a FFT power spectrum can be computed by assuming that the mean power spectrum can be modeled using either a white-noise or a red-noise spectrum. For the case of a red-noise spectrum, the discrete Fourier power spectrum is given, after normalization, by

$$p_k = \frac{1 - \alpha^2}{1 + \alpha^2 - 2\alpha \cos(2\pi k/N)},$$

where $k = 0, ..., N/2$ is the frequency index, $N$ is the number of data points, and $\alpha$ is the lag-one auto-correlation coefficient (when $\alpha = 0$ we obtain the white-noise spectrum). In this case, for a peak to be significant at a given significant level, higher values of power are required compared to the white-noise background spectrum (Torrence and Compo, 1998). We find $\alpha = 0.62$ for the time series of $A$ shown in Figure 3. In the FFT power spectrum shown in Figure 4 there are noticeable peaks around the frequencies $21-7$ year$^{-1}$, $2.7$ year$^{-1}$, $1.4$ year$^{-1}$, $0.668$ year$^{-1}$ (250 day period), and $0.411$ year$^{-1}$ (150 day period). For a white-noise model only the peaks at frequency $21-7$ year$^{-1}$ are above $3\sigma$ level (> 99% confidence level). From the red-noise model we find no peak is significant at a 90% confidence level.

We have also determined the FFT power spectra of $A$ during the moving-time-intervals (MTI) successively shifted by 61 days and 365 days. We derived
these MTI series from the data time series shown in Figure 1, in which the spikes above the 2σ level are removed. We did not show these spectra because we found no notable difference between these spectra and the ones shown in Figure 4, in the low-frequency side correspond to the aforementioned periodicities.

3.2. MEM Analysis

A different approach for determining the value of the periodicities in a short time series with higher accuracy is to compute the power spectrum using a maximum entropy method (MEM). An important step in this method is the optimum selection of the order $M$ of the autoregressive process. If $M$ is chosen too low the spectrum is over-smoothed and the high-resolution potential is lost. If $M$ is chosen too high, frequency shifting and spontaneous splitting of the spectral peaks occurs. In order to find the correct values of the periodicities-particularly the $\approx 21–7$ year periodicity seen in the FFT power spectrum, we computed MEM power spectra choosing various values for $M$ in the range $(N/3, N/2)$ as suggested by Ulrych and Bishop (1975), where $N$ is the total number of intervals in the analyzed time series. We find that $M = N/3$ is suitable in the present MEM analysis (i.e., the peaks are sharp and well separated). Figure 5 shows the MEM power spectra of $A$. This spectrum shows the values $\approx 7.6$ year, 2.8 year,
1.47 year, 245 day, 182 day, and 158 day for the periodicities noticed from the FFT analysis.

**Figure 5.** MEM power spectrum of $A$ correspond to the corrected with one-year period filtered time series (shown in Figure 3). Near each considerably large peak, the value of the corresponding periodicity is mentioned.

3.3. Wavelet Analysis

Wavelet analysis is a powerful method for analyzing localized variations of the power within a time series at many different frequencies (Torrence and Compo, 1998). The short-term periodicities 1.3 year, 150–160 day, etc. in several solar activity phenomena have been analysed using this technique (e.g., Oliver, Ballester, and Baudin, 1998; Krivova and Solanki, 2002; Prabhakaran Nayar et al., 2002; Ballester, Oliver, and Carbonell, 2004; Knaack, Stenflo, and Berdyugina, 2004; Mendoza, Velasco, and Valdés-Galicia, 2006; Chowdhury and Ray, 2006; Forgács-Dajka and Borkovits, 2007). Hence, we used the Morlet-wavelet analysis, which also helps to determine the solar-cycle dependence of the short-term periodicities. Figure 6 shows the wavelet spectra, normalized by $1/s^2$, of the corrected with one-year period filtered time series of $A$ shown in Figure 3, where $s^2$ is the variance of the same time series. The wavelet spectrum suggests that the 1.47-year and 2.8-year periodicities might have occurred around the year 1990 (and 1995), and the 245-day periodicity might have occurred around 1992, in the variation of $A$. These periodicities are above the 95% confidence level (corresponding to the green contour level). The $\approx 7.6$ year periodicity is inside the cone-of-influence (COI). Therefore, this periodicity is not detected here unambiguously, although it is above the 99% confidence level in the
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FFT power spectrum (Figure 4). The available data are inadequate to accurately determine the correct value as well as the time dependence of this periodicity. On the other hand, such a periodicity seems present in the rotation rates of magnetic features. Singh and Prabhu (1985) have found a seven-year periodicity in the rotation rate derived from Ca$^+$ K plage data during the period 1951–1981. The existence of an approximate eight-year periodicity is found in both the equatorial rotation rate and the latitudinal gradient of the rotation determined from over 100 years of Greenwich and Solar Optical Observatory Network (SOON) sunspot group data (Javaraiah and Komm, 1999; Javaraiah, 2005).

**Figure 6.** Wavelet power spectrum and the corresponding global power spectrum of $A$ using the Morlet wavelet, normalized by $1/s^2$, where $s^2$ is the variance of the corresponding time series shown in Figure 3. The contours are at normalized variances of 1.5 (blue), 3.0 (green), 4.5 (yellow) and 6.0 (red). The dashed curve represent the 95% confidence level. Cross-hatched regions indicate the cone of influence, where edge effects become important (Torrence and Compo, 1998).

4. Conclusion and Discussion

The power spectral analyses of the Sun’s surface equatorial rotation rate determined from the Mt. Wilson daily Doppler velocity measurements during the period 3 December 1985–5 March 2007 suggests the existence of 7.6 year, 2.8
year, 1.47 year, 245 day, 182 day and 158 day periodicities in the surface equa-
torial rotation rate during the period before 1996. However, there is no variation
of any kind in the more accurately measured data during the period after 1995.
That is, the aforementioned periodicities in the data during the period before the
year 1996 may be artifacts of the uncertainties of those data due to the frequent
changes in the instrumentation of the Mt. Wilson spectrograph. Therefore, the
results of the present analysis are largely consistent with result of no variations
in the solar-surface equatorial rotation rate occur found by Ulrich and Bertello
[1996].

On the other hand, the temporal behavior of most of the activity phenomena
is considerably different during cycles 22 and 23. In several solar-activity phe-
nomena the 1.3 year periodicity has been found to be dominant during cycle 22
and weak or absent in the later period (see [Obridko and Shelting, 2007]). In
addition, recently, [Howe et al. (2007)] reported that the 1.3-year periodicity in
the rotation rate at the base of the convection zone does not persist after 2001.
Hence, presence of the short-term periodicities during cycle 22 (1986–1996) and
absence of them in cycle 23 (after 1996) may indicate that the temporal behavior
of the rotation is also considerably different between these cycles. Therefore, in
spite of the fact that quality of the data during the period 1986–1995 is poor,
the periodicities in A found in this data may represent the corresponding real
variations in the surface equatorial rotation rate during the period 1986–1995
and need to be confirmed from an independent data set.

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