Spacing of Intersections in Hierarchical Road Networks with Inward, Outward, and Through Traffic

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Abstract

This paper develops a continuous network model for examining the spacing of intersections that connect different levels of roads in a hierarchical network. The model extends a previous model to incorporate inward, outward, and through traffic, thus providing a more appropriate framework for analyzing the intersection spacing. An analytical expression for the total travel time is obtained for a grid road network with two road types. The travel time is defined as the sum of the free travel time and the delay at intersections. The analytical expression leads to a clear understanding of the tradeoff between the accessibility to higher level roads and the delay at intersections. The optimal pattern of intersections that minimizes the total travel time is then obtained. The result shows how the road length, the intersection delay, the travel speed, and the traffic composition affect the optimal intersection pattern.

Keywords: Network design, Grid network, Total travel time, Accessibility, Intersection delay

1 Introduction

Road networks have hierarchy consisting of different levels of roads such as major arterial roads, minor arterial roads, and access roads. The spacing of intersections connecting different levels of roads is significant for efficient hierarchical networks. If few intersections exist, the transfer between lower and higher level roads would be difficult. On the other hand, too many intersections increase the travel time due to the intersection delay. This tradeoff between the accessibility to higher level roads and the delay at intersections should be considered in hierarchical road network design.

There exist two main approaches for designing hierarchical networks: discrete and continuous network models. The discrete models aim to develop efficient algorithms applicable to actual networks. The discrete models then use detailed traffic data and yield numerical solutions. Current et al.1) formulated the hierarchical network design problem for identifying the minimal cost two-level network. The problem was extended by Balakrishnan et al.2) to multi-level networks. A number of solution methods for the problem and its extension have been proposed.3)–8) Santos et al.9) presented a multi-objective approach to multi-level road network planning considering efficiency, equity, and robustness. Bigotte et al.10) developed an integrated model of urban hierarchy and transportation network planning.

The continuous models, in contrast, aim to find fundamental relationships between variables. The continuous models then use approximated travel demand on idealized networks such as grid

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and radial-arc networks and often yield analytical solutions. The role of the continuous models is to supplement the discrete models. Creighton et al.\textsuperscript{11)} found the optimal spacing of a square grid network with three road types that minimizes the sum of the travel and construction costs. Tanner\textsuperscript{12)} evaluated networks of parallel motorways and rectangular grid motorways from the average travel time. Fawaz and Newell\textsuperscript{13,14)} obtained the optimal spacing of two, three, and four level rectangular grid networks. Miura\textsuperscript{15)} estimated the minimum length of high-speed roads required to manage traffic flow on a grid network. Miyagawa\textsuperscript{16)} proposed a model for finding the optimal road area in a hierarchical grid network. The model was extended by Miyagawa\textsuperscript{17)} to incorporate inter-city traffic and Miyagawa\textsuperscript{18)} to determine the total road area. Fujita and Suzuki\textsuperscript{19)} considered the optimal pattern of a network consisting of high-speed radial-arc roads and low-speed arc roads.

Despite a large number of studies concerning the hierarchical road network design, few have considered the effect of intersections. Although Miyagawa,\textsuperscript{20)} as an exception, proposed a model for finding the optimal spacing of intersections, the model considered only city traffic. That is, both origin and destination were assumed to be inside the city. The travel time and the optimal intersection spacing, however, depend not only on the volume of city traffic but also on the volumes of inward, outward, and through traffic. For example, a few intersections will be sufficient for cities with much through traffic because through traffic mainly uses major arterial roads.

In this paper, we develop a continuous network model for examining the spacing of intersections that connect different levels of roads in a hierarchical network. The model extends the model by Miyagawa\textsuperscript{20)} to incorporate inward, outward, and through traffic. This extension allows us to consider the effect of the traffic composition on the optimal intersection spacing. The model uses a grid road network with two road types and yields an analytical expression for the total travel time. The analytical expression leads to a clear understanding of the tradeoff between the accessibility to higher level roads and the delay at intersections.

The remainder of this paper is organized as follows. The next section develops a grid network model. Section 3 gives an analytical expression for the total travel time. Section 4 obtains the optimal intersection pattern that minimizes the total travel time. Section 5 provides a numerical example. The final section presents concluding remarks.

## 2 Grid network model

Consider a square city with side length $A$, as shown in Fig. 1. The city has a grid road network with two road types: minor and major roads. Minor roads exist everywhere, whereas major roads are on a square grid with spacing $a$. Let $\Lambda$ be the length of major roads, i.e., $\Lambda = 2A \cdot A/a = 2A^2/a$. The spacing is then expressed in terms of the road length as $a = 2A^2/\Lambda$.

Origins and destinations are assumed to be uniformly distributed in the city. The uniform travel demand supplies building blocks for further analysis with more realistic travel demand. In fact, the uniform demand has frequently been used in continuous transportation models.\textsuperscript{18,21,22)} The traffic in the city is classified into four groups according to the location of origin and destination. If both origin and destination are inside the city, the traffic is called city traffic. If origin is outside (inside) the city and destination is inside (outside) the city, it is called inward (outward) traffic. If both origin and destination are outside the city, it is called through traffic.

Travelers are assumed to follow the nearest intersection routing suggested by Miyagawa.\textsuperscript{16)} Every traveler always uses both minor and major roads. The transfer between minor and major roads are only allowed at intersections shown by white circles in Fig. 1. The movement of a traveler is
shown in the figure. First, the traveler moves from origin along minor roads to the nearest intersection of minor and major roads. At the intersection, s/he transfers to major roads and moves to the intersection nearest to destination. Using again minor roads, s/he arrives at destination. Obviously, the travel time under this routing is not always the minimum travel time because whether travelers should use major roads depends on the location of origin and destination. Miyagawa, however, demonstrated that there exists a close relationship between the two travel times.

Figure 1. Grid road network in a square city.

To find the optimal pattern of intersections that connect minor and major roads, five patterns of intersections discussed by Miyagawa are considered, as shown in Fig. 2. In pattern (i), a intersection is located on the midpoint of a side of each square block. In patterns (ii), (iii), and (iv), an intersection is located on the midpoint of two, three, and four sides of each square block, respectively. In pattern (v), two intersections are located at equal intervals on four sides of each square block. These patterns are assumed to continue over the city. The theoretical results of these patterns will give an insight into empirical analysis on actual intersection patterns. The density of intersections (number of intersections per unit length), denoted by $\lambda$, is

$$\lambda = \begin{cases} \frac{5}{4\pi} = \frac{5\lambda}{8\pi^2}, & \text{(i)}, \\ \frac{3}{2\pi} = \frac{3\lambda}{4\pi^2}, & \text{(ii)}, \\ \frac{7}{4\pi} = \frac{7\lambda}{8\pi^2}, & \text{(iii)}, \\ \frac{2}{\pi} = \frac{2\lambda}{\pi}, & \text{(iv)}, \\ \frac{3}{2\pi} = \frac{3\lambda}{2\pi^2}, & \text{(v)}, \end{cases}$$

(Miyagawa). Note that $\lambda$ includes not only intersections connecting minor and major roads but also intersections of two major roads. The average spacing of intersections is given by $1/\lambda$.

3 Total travel time

Let $v_1$ and $v_2$ be the travel speeds on minor and major roads, respectively. Both $v_1$ and $v_2$ represent free travel speeds and assumed to be constant irrespective of traffic volume. The travel distance is approximated by the rectilinear distance. The rectilinear distance between two points $(x_1, y_1)$ and $(x_2, y_2)$ is defined as $|x_1 - x_2| + |y_1 - y_2|$. The rectilinear distance is a good approximation for the actual travel distance in cities with a grid road network. The average travel distance on minor roads is approximated by the average rectilinear distance from a uniformly distributed point.
in the square with side length $a$ to the nearest intersection. The average rectilinear distance to the nearest intersection is (i) $3a/4$, (ii) $a/2$, (iii) $5a/12$, (iv) $a/3$, and (v) $43a/162$. For example, the average rectilinear distance for pattern (i) is

$$\frac{2}{a^2} \int_0^a \int_0^{a/2} (x + y) \, dx \, dy = \frac{3}{4} a,$$

where the intersection is at (0,0), as shown in Fig. 3. Note that the average distance can be obtained by considering the rectangle with side lengths $a$ and $a/2$. The average rectilinear distance for the other patterns can be similarly obtained. The average travel time on minor roads, denoted by $T_1$, is then given by

$$(3)$$

Figure 2. Patterns of intersections.

$$T_1 = \begin{cases} 
\frac{3a}{4v_1} = \frac{3A^2}{2v_1A}, & (i), \\
\frac{a}{2v_1} = \frac{A^2}{v_1A}, & (ii), \\
\frac{5a}{12v_1} = \frac{5A^2}{6v_1A}, & (iii), \\
\frac{a}{3v_1} = \frac{2A^2}{3v_1A}, & (iv), \\
\frac{43a}{162v_1} = \frac{43A^2}{81v_1A}, & (v), 
\end{cases}$$

(Miyagawa\textsuperscript{20}).

Figure 3. Rectilinear distance to the intersection.
The average travel distance between intersections on major roads is approximated by the average rectilinear distance in the square with side length $A$. The average travel distance of city traffic is approximated by the average rectilinear distance between two uniformly distributed points in the square (Fig. 4a). The average travel distance of inward/outward traffic is approximated by the average rectilinear distance between a uniformly distributed point in the square and a side of the square (Fig. 4b). The average travel distance of through traffic is approximated by the average rectilinear distance between two sides of the square (Fig. 4c). These average rectilinear distances are $2A/3$, $A/2$, and $A$, respectively (Fairthorne (26)). For example, the average rectilinear distance between two uniformly distributed points in the square is obtained as follows. The average distance between two uniformly distributed points in the line segment of length $A$ is

$$\frac{1}{A^2} \int_0^A \int_0^A |x-y| \, dx \, dy = \frac{A}{3}. \tag{4}$$

Since the rectilinear distance is the sum of the horizontal and vertical distances, the average rectilinear distance is $2A/3$. The average travel time on major roads, denoted by $T_2$, is defined as the sum of the free travel time and the delay at intersections and given by

$$T_2 = \begin{cases} \frac{2A}{3v} + \tau \lambda \frac{2A}{3}, & \text{(city traffic)}, \\ \frac{A}{v} + \tau \lambda A, & \text{(inward/outward traffic)}, \\ \frac{A}{v} + \tau \lambda A, & \text{(through traffic)}, \end{cases} \tag{5}$$

where $\tau$ is the delay at one intersection. The rationale for this expression was discussed by Koshizuka and Imai (27). Examining the relationship between the density of signalized intersections and the average travel time in Tsukuba, they estimated the intersection delay $\tau$ at 21 seconds. It can be seen from (3) and (5) that as the density of intersections increases, the travel time on minor roads decreases but the travel time on major roads increases. Thus, there exists a tradeoff between the travel time on minor and major roads.

Let $q$ be the total traffic volume and $\alpha$, $\beta$, and $\gamma$ ($\alpha + \beta + \gamma = 1$) be the proportions of city, inward/outward, and through traffic, respectively. City traffic uses minor roads twice (near origin and destination) and major roads once, and the total travel time is

$$\alpha q (2T_1 + T_2). \tag{6}$$

Inward and outward traffic use minor roads and major roads once, and the total travel time is

$$\beta q (T_1 + T_2). \tag{7}$$

![Figure 4. Rectilinear distance in a square.](image)
Through traffic uses only major roads, and the total travel time is

$$\gamma qT_2.$$ \hspace{1cm} (8)

The total travel time $T$ is the sum of these travel time and expressed as

$$T = \begin{cases} 
(2\alpha + \beta)\frac{3qA^2}{2v_1A} + (4\alpha + 3\beta + 6\gamma)\left(\frac{5\tau A}{8A^2} + \frac{1}{v_2}\right)\frac{qA}{6}, & (i), \\
(2\alpha + \beta)\frac{4qA^2}{5v_1A} + (4\alpha + 3\beta + 6\gamma)\left(\frac{7\tau A}{8A^2} + \frac{1}{v_2}\right)\frac{qA}{6}, & (ii), \\
(2\alpha + \beta)\frac{2qA^2}{3v_1A} + (4\alpha + 3\beta + 6\gamma)\left(\frac{2\tau A}{2A^2} + \frac{1}{v_2}\right)\frac{qA}{6}, & (iii), \\
(2\alpha + \beta)\frac{2qA^2}{6v_1A} + (4\alpha + 3\beta + 6\gamma)\left(\frac{3\tau A}{2A^2} + \frac{1}{v_2}\right)\frac{qA}{6}, & (iv), \\
(2\alpha + \beta)\frac{6qA^2}{7v_1A} + (4\alpha + 3\beta + 6\gamma)\left(\frac{3\tau A}{2A^2} + \frac{1}{v_2}\right)\frac{qA}{6}, & (v). 
\end{cases} \hspace{1cm} (9)$$

4 Optimal intersection pattern

The total travel time is shown in Fig. 5, where $A = 10$ km, $v_1 = 20$ km/h, $v_2 = 40$ km/h, $\tau = 0.5/60$ h, $q = 100$, $\alpha = 0.4$, $\beta = 0.3$, $\gamma = 0.3$. As the length of major roads increases, the total travel time first decreases and then increases. This is because the increase in the road length reduces the distance to major roads but causes more intersection delay. The optimal intersection pattern that minimizes the total travel time is also shown in the figure. It can be seen that many intersections are required for cities with a few major roads, whereas a few intersections are sufficient for cities with many major roads.

![Figure 5. Length of major roads and the total travel time.](image)

Comparing the total travel time (9) yields the optimal intersection pattern and the condition that the pattern outperforms the others, as shown in Table 1. Note that the optimal spacing of intersections increases with the road length $\Lambda$, the intersection delay $\tau$, and the travel speed on minor roads $v_1$. Note also that pattern (iii) cannot be optimal and that the optimal pattern is independent of the total traffic volume $q$.

The optimal intersection pattern is depicted in a ternary plot in Fig. 6, where $A = 10$ km, $v_1 = 20$ km/h, $\tau = 0.5/60$ h. It can be seen that the four, three, and two patterns are optimal for $\Lambda = 50$ (Fig. 6a), $\Lambda = 100$ (Fig. 6b), and $\Lambda = 200$ (Fig. 6c), respectively. If $\Lambda = 300$ (Fig. 6d), only pattern (i) is optimal. This figure shows how the traffic composition affects the optimal intersection pattern. The optimal spacing of intersections decreases with the proportion
Table 1. Optimal intersection pattern.

| Pattern | Condition |
|---------|-----------|
| (i)     | $\Lambda > \frac{2\sqrt{b(2\alpha+\beta)A^{1/2}}}{\sqrt{(4\alpha+3\beta+6\gamma)\tau v_1}}$ |
| (ii)    | $\frac{2\sqrt{b(2\alpha+\beta)A^{1/2}}}{\sqrt{(4\alpha+3\beta+6\gamma)\tau v_1}} < \Lambda \leq \frac{2\sqrt{6(2\alpha+\beta)A^{1/2}}}{\sqrt{(4\alpha+3\beta+6\gamma)\tau v_1}}$ |
| (iv)    | $\frac{2\sqrt{11(2\alpha+\beta)A^{1/2}}}{3\sqrt{3(4\alpha+3\beta+6\gamma)\tau v_1}} < \Lambda \leq \frac{2\sqrt{2(2\alpha+\beta)A^{1/2}}}{\sqrt{(4\alpha+3\beta+6\gamma)\tau v_1}}$ |
| (v)     | $\Lambda \leq \frac{2\sqrt{11(2\alpha+\beta)A^{1/2}}}{3\sqrt{3(4\alpha+3\beta+6\gamma)\tau v_1}}$ |

of city traffic $\alpha$ and increases with the proportion of through traffic $\gamma$. If all the traffic is through traffic, i.e., $\gamma = 1$, pattern (i) is optimal. It follows that cities with much through traffic require fewer intersections than cities where city traffic is dominant, even though the total traffic volume is the same. This finding suggests that considering only the total traffic volume is insufficient.

Figure 6. Traffic composition and optimal intersection pattern: (a) $\Lambda = 50$; (b) $\Lambda = 100$; (c) $\Lambda = 200$; (d) $\Lambda = 300$.

5 Numerical example

As an example, let us consider the optimal intersection pattern for a region of $n \times n$ square cities, as shown in Fig. 7. Two types of travel demand are examined: uniform and many-to-one demand.
Since actual travel demand can be regarded as a combination of these two types of demand, the result serves as a basis for further analysis with actual demand.

Assume first that origins and destinations are uniformly distributed in the region. Assume also that inward, outward, and through traffic minimize the number of turns in choosing the shortest route. If there exist two shortest routes with an equal number of turns, the traffic volume is equally divided between the two routes. The volumes of city, inward, outward, and through traffic of city $(i, j)$, denoted by $q^C_{ij}$, $q^I_{ij}$, $q^O_{ij}$, and $q^T_{ij}$, are

\begin{align*}
q^C_{ij} &= A^4, \quad (10) \\
q^I_{ij} &= q^O_{ij} = A^4(n^2 - 1), \quad (11) \\
q^T_{ij} &= A^4\{(2i - 1)(n - i)n + 2(j - 1)(n - j)n + (j - 1)(n - 1) + (n - j)(n - 1)\}, \quad (12)
\end{align*}

respectively (Miyagawa\textsuperscript{17}). For example, through traffic of city $(i, j)$ consists of four types, as shown in Fig. 8. The volume of through traffic $q^T_{ij}$ is the sum of the four traffic volumes. The total traffic volume and the proportions of city, inward/outward, and through traffic are

\begin{align*}
q &= q^C_{ij} + q^I_{ij} + q^O_{ij} + q^T_{ij}, \quad \alpha = \frac{q^C_{ij}}{q}, \quad \beta = \frac{q^I_{ij} + q^O_{ij}}{q}, \quad \gamma = \frac{q^T_{ij}}{q}, \quad (13)
\end{align*}

respectively.

The optimal intersection pattern for a region of $5 \times 5$ cities is shown in Fig. 9, where $A = 10$ km, $v_1 = 20$ km/h, $\tau = 0.5/60$ h. If $\Lambda = 50$ (Fig. 9a), the optimal pattern is (iv) for all cities. If $\Lambda = 100$ (Fig. 9b), in contrast, the optimal pattern is (i) for the central city and (ii) for the other cities. This is because much through traffic passes the central city even though origins and destinations are uniformly distributed.
Figure 9. Optimal intersection pattern for uniform demand: (a) $\Lambda = 50$; (b) $\Lambda = 100$.

Assume next that origins are uniformly distributed in the region and destinations are uniformly distributed only in the central city. An example of this many-to-one demand is commuting to the city center. The volumes of city, inward, outward, and through traffic of city $(i, j)$ are

$$q_{ij}^c = \begin{cases} A^4, & i = j = \frac{n+1}{2}, \\ 0, & \text{otherwise}, \end{cases}$$

$$q_{ij}^i = \begin{cases} A^4(n^2 - 1), & i = j = \frac{n+1}{2}, \\ 0, & \text{otherwise}, \end{cases}$$

$$q_{ij}^o = \begin{cases} 0, & i = j = \frac{n+1}{2}, \\ A^4, & \text{otherwise}, \end{cases}$$

$$q_{ij}^T = \begin{cases} 0, & i = j = \frac{n+1}{2}, \\ A^4 \left( \frac{i-1}{2} + \frac{i-1}{2} \right), & i < \frac{n+1}{2}, j < \frac{n+1}{2}, \\ A^4 \left( i - 1 + \frac{(n-1)}{2} \right), & i < \frac{n+1}{2}, j = \frac{n+1}{2}, \\ A^4 \left( \frac{n-j+i}{2} \right), & i < \frac{n+1}{2}, j > \frac{n+1}{2}, \\ A^4 \left( j - 1 + \frac{(n-1)}{2} \right), & i = \frac{n+1}{2}, j < \frac{n+1}{2}, \\ A^4 \left( n - j + \frac{(n-j+1)(n-1)}{2} \right), & i = \frac{n+1}{2}, j > \frac{n+1}{2}, \\ A^4 \left( \frac{i-1}{2} + \frac{n-i}{2} \right), & i > \frac{n+1}{2}, j < \frac{n+1}{2}, \\ A^4 \left( \frac{n-i+1}{2} \right), & i > \frac{n+1}{2}, j = \frac{n+1}{2}, \\ A^4 \left( \frac{n-j+1}{2} + \frac{n-i}{2} \right), & i > \frac{n+1}{2}, j > \frac{n+1}{2}, \end{cases}$$

respectively (Miyagawa\textsuperscript{18}). The optimal intersection pattern for a region of $5 \times 5$ cities is shown in Fig. 10. It can be seen that the optimal spacing is long for cities with much through traffic. In fact, the optimal pattern for the cities adjacent to the central city is (ii) for $\Lambda = 50$ (Fig. 10a) and (i) for $\Lambda = 100$ (Fig. 10b). Since no through traffic passes the central city and the four corner cities, the optimal spacing of those cities is shorter than the other cities.

6 Conclusions

This paper has extended a previous model for determining the optimal intersection spacing to incorporate inward, outward, and through traffic. The optimal pattern of intersections that minimizes
the total travel time has been obtained. The model considers the effect of the traffic composition on the optimal intersection pattern, thus giving a more appropriate framework for analyzing the intersection spacing in hierarchical road networks.

The model is useful for further hierarchical network design models. The analytical expression for the total travel time leads to a clear understanding of the tradeoff between the accessibility to higher level roads and the delay at intersections. The result shows how the road length, the intersection delay, the travel speed, and the traffic composition affect the optimal intersection pattern. Note that finding these relationships by using discrete network models requires computation of the travel time for various combinations of the parameters. The optimal intersection pattern helps planners to evaluate actual intersection spacing and estimate the spacing required to achieve a certain level of service. For example, if actual spacing is much smaller than the optimal spacing provided in Table 1, consolidating some intersections should be considered to reduce the travel time.

The present model assumes that intersections are located at equal intervals. Future research should determine the optimal location of intersections. Finding the optimal intersection spacing in a radial-arc network is another topic for future research.

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