Summary: Every “x”-adjustment in the so-called xVA financial risk management framework relies on the computation of exposures. Considering thousands of Monte Carlo paths and tens of simulation steps, a financial portfolio needs to be evaluated numerous times during the lifetime of the underlying assets. This is the bottleneck of every simulation of xVA.

In this article, we explore numerical techniques for improving the simulation of exposures. We aim to decimate the number of portfolio evaluations, particularly for large portfolios involving multiple, correlated risk factors. The usage of the Stochastic Collocation (SC) method \[ L. A. Grzelak et al., Quant. Finance 19, No. 2, 339–356 (2019; Zbl 1428.62048) \] together with \[ S. A. Smolyak, Dokl. Akad. Nauk SSSR 148, 1042–1045 (1963; Zbl 0202.39901) \]; translation from Dokl. Akad. Nauk SSSR 148, 1042–1045 (1963); \[ K. L. Judd et al., J. Econ. Dyn. Control 44, 92–123 (2014; Zbl 1402.91368) \] sparse grid extension, allows for a significant reduction in the number of portfolio evaluations, even when dealing with many risk factors. The proposed model can be easily applied to any portfolio and size. We report that for a realistic portfolio comprising linear and non-linear derivatives, the expected reduction in the portfolio evaluations may exceed 6000 times, depending on the dimensionality and the required accuracy. We give illustrative examples and examine the method with realistic multi-currency portfolios consisting of interest rate swaps and swaptions.

MSC:

91Gxx Actuarial science and mathematical finance
41Axx Approximations and expansions
65Dxx Numerical approximation and computational geometry (primarily algorithms)

Keywords:

stochastic collocation; SC; xVA; valuation adjustment; expected exposures; Smolyak’s sparse grids; Chebyshev polynomials; Clenshaw-Curtis

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