Quantum vacuum fluctuations

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The existence of irreducible field fluctuations in vacuum is an important prediction of quantum theory. These fluctuations have many observable consequences, like the Casimir effect which is now measured with good accuracy and agreement with theory, provided that the latter accounts for differences between real experiments and the ideal situation considered by Casimir. But the vacuum energy density calculated by adding field mode energies is much larger than the density observed around us through gravitational phenomena. This “vacuum catastrophe” is one of the unsolved problems at the interface between quantum theory on one hand, inertial and gravitational phenomena on the other hand. It is however possible to put properly formulated questions in the vicinity of this paradox. These questions are directly connected to observable effects bearing upon the principle of relativity of motion in quantum vacuum.

Observation of movement forces us to conceive the notion of a space in which movement takes place. The absence of resistance opposed to movement by space then leads to the concept of empty space, or vacuum. These logical statements were formulated by Leucippus and Democritus more than 2000 years ago [1]. They raise paradoxes which have shown their pertinence from that time until today.

The question of relativity of motion played an important role in the birth of modern physics. Galileo emphasized that motion with uniform velocity in empty space is indistinguishable from rest but, also, that this property is only true when resistance of air to motion can be ignored [2]. Newton stressed this fact [3] by identifying empty space in which motion takes place with the experimental vacuum which was studied by Pascal, von Guericke and Boyle after the pioneers Galileo and Torricelli. The same question played anew a key role in Einsteinian theory of relativity which freed space from the absolute character it had in classical Newtonian theory [4], in contradiction with the Galilean principle of relativity (named in this manner by Einstein).

The emergence of quantum theory has profoundly altered our conception of empty space by forcing us to consider vacuum as the realm of quantum field fluctuations. This solves old paradoxical questions about the nature of vacuum. In quantum theory, vacuum becomes a well-defined notion and the question of relativity of motion is given a satisfactorily understanding. However, a new difficulty arises which involves the energy density of vacuum fluctuations and, precisely, the contradiction between the large values predicted by quantum theory and the evidence of its weak, if not null, value in the world around us. These questions are discussed in more detail below after a few extra historical remarks devoted to the birth of quantum theory.

The classical idealization of space as being absolutely empty was already affected by the advent of statistical mechanics, when it was realized that space is in fact filled with black body radiation which exerts a pressure onto the boundaries of any cavity. It is precisely for explaining the properties of black body radiation that Planck introduced his first quantum law in 1900 [5]. In modern terms, this law gives the energy per electromagnetic mode as the product $E$ of the energy of a photon $\hbar \omega \equiv \hbar \nu$ by a number of photons $n$ per mode

$$E = n \hbar \omega \quad n = \frac{1}{\exp \left( \frac{\hbar \omega}{k_B T} \right) - 1} \quad (1)$$

Unsatisfied with his first derivation, Planck resumed his work in 1912 and derived a different result where the energy contained an extra term [6]

$$E = \left( \frac{1}{2} + n \right) \hbar \omega \quad (2)$$

The difference between the two Planck laws just corresponds to what we now call “vacuum fluctuations”. Whereas the first law describes a cavity entirely emptied out of radiation at the limit of zero temperature, the second law tells us that there remain field fluctuations corresponding to half the energy of a photon per mode at this limit.

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The story of the two Planck laws and of the discussions they raised is related in a number of papers, for example [8,9]. Let us recall here a few facts: Einstein and Stern noticed in 1913 that the second law, in contrast with the first one, has the correct classical limit at high temperature [10]

\[
\left(\frac{1}{2} + n\right) \hbar \omega = k_B T + O\left(\frac{1}{T}\right) \quad T \to \infty
\]

Debye was the first to insist on observable consequences of zero-point fluctuations in atomic motion, by discussing their effect on the intensities of diffraction peaks [11] whereas Mulliken gave the first experimental proof of these consequences by studying vibrational spectra of molecules [12]. Nernst is credited for having first noticed that zero-point fluctuations also exist for field modes [13] which dismisses the classical idea that absolutely empty space exists and may be attained by removing all matter from an enclosure and lowering the temperature down to zero. At this point, we may emphasize that these discussions took place before the existence of these fluctuations was confirmed by fully consistent quantum theoretical calculations.

We now come to a serious difficulty of quantum theory which was noticed by Nernst in his 1916 paper. Vacuum is permanently filled with electromagnetic field fluctuations propagating with the speed of light, as any free field, and corresponding to an energy of half a photon per mode. In modern words, quantum vacuum is the field state where the energy of field fluctuations is minimal. This prevents us from using this energy to build up perpetual motions otherwise. In other words, the reference level setting the zero of energy for gravitation theory appears to be finely tuned to fit the mean value of vacuum energy. However, this can not lead to dismiss the effects of vacuum energy. As a matter of fact, even for an exact cancellation of the contribution of mean vacuum energy, energy differences and energy fluctuations still have to contribute to gravitation. This point will be discussed in more detail below.

Certainly, we have to acknowledge with Pauli that the mean value of vacuum energy does not contribute to gravitation as an ordinary energy. This is just a matter of evidence since the universe would look very differently otherwise. In other words, the reference level setting the zero of energy for gravitation theory appears to be finely tuned to fit the mean value of vacuum energy. However, this cannot lead to dismiss the effects of vacuum energy. As a matter of fact, even for an exact cancellation of the contribution of mean vacuum energy, energy differences and energy fluctuations still have to contribute to gravitation. This point will be discussed in more detail below.

It is no more possible to uphold, as Pauli did, that vacuum fluctuations cannot “be emitted, absorbed, scattered ... or contained within walls”. Vacuum fluctuations of the electromagnetic field have well known observable consequences on atoms [22] and, more generally, microscopic scatterers [23]. An atom interacting only with vacuum fields suffers
spontaneous emission processes induced by these fields. When fallen in its ground state, the atom can no longer emit photons but its coupling to vacuum still results in measurable effects like the Lamb shift of absorption frequencies. Two atoms located at different locations in vacuum experience an attractive Van der Waals force. While studying this effect which plays an important role in physico-chemical processes, Casimir discovered in 1948 that a force, now named after him, arises between two mirrors placed in vacuum and is then a macroscopic analog of the microscopic Van der Waals forces [24]. This effect is discussed below.

We also want to recall that vacuum fluctuations of electromagnetic fields are, in a sense, directly detected through the study of photon noise in quantum optics. In simple words, photon noise reflects field fluctuations. This idea was first formulated by Einstein during his early efforts to build up a consistent theory of light [25–27]. Here we will illustrate it in a simple manner by discussing in the words of modern quantum optics a gedanken experiment where an electromagnetic field impinges a beam splitter [28].

![FIG. 1. Schematic description of a simple quantum optical experiment: a beam of photons enters the input port 'a' of a beam splitter; photon noise is analyzed through intensity fluctuations of the beams in the output ports 'c' and 'd'; its statistics can be understood as determined by the field fluctuations entering the usually disregarded input port 'b'.](image)

This experiment can be analyzed in terms of photons, each one having probabilities of 50% for being either transmitted or reflected. The random character of this process results in the photon noise observed behind the beam splitter as fluctuations of the number of photons in the output ports. If we now think in terms of electromagnetic fields, we first represent the field $\mathcal{E}$ in any of the involved modes, with a frequency $\omega$, as a sum over two quadrature components $\mathcal{E}_1$ and $\mathcal{E}_2$ obeying Heisenberg inequality

$$\mathcal{E} = \mathcal{E}_1 \cos \omega t + \mathcal{E}_2 \sin \omega t \quad \Delta \mathcal{E}_1 \Delta \mathcal{E}_2 \geq \mathcal{E}_0^2$$

(5)

This is basically the reason for the necessity of quantum fluctuations of electromagnetic fields, $\mathcal{E}_0$ being a constant characterizing the level of vacuum fluctuations. The vacuum state corresponds to the case where fields have a null mean value and a minimum energy, which leads to $\Delta \mathcal{E}_1^2 = \Delta \mathcal{E}_2^2 = \mathcal{E}_0^2$. But Heisenberg inequality allows one to obtain “squeezed” states with a noise on a given quadrature component smaller than in vacuum [29].

Simple calculations then lead to the result that number fluctuations in one of the output ports are determined by the field fluctuations corresponding to one of the quadrature components, say $B_1$, in the usually disregarded input port ‘b’ (see Figure 1)

$$\delta n = \delta n_c - \delta n_d \sim |A| \delta B_1$$

$$\Delta n^2 \sim |A|^2 \Delta B_1^2$$

(6)

Usually the field fluctuations entering the port ‘b’ are in the vacuum state, so that the variance $\Delta B_1^2$ is just the constant $\mathcal{E}_0^2$ which leads to the ordinary Poissonian statistics of photon noise $\Delta n^2 = \langle n_a \rangle$. But this statistics may be modified by manipulating the fluctuations of $B_1$. In particular, letting a squeezed field with $\Delta B_1^2 < \mathcal{E}_0^2$ enter the input port ‘b’ allows one to obtain a sub-Poissonian statistics on the variable $n$.

This means that photon noise merely reflects quantum field fluctuations entering the optical system and can therefore be controlled by manipulating the components responsible for fluctuations of the signal of interest. For the experiment sketched on Figure 1, the random scattering of photons from the input beam ‘a’ into the two output beams ‘c’ and ‘d’ may be made more regular by squeezing the fluctuations of $B_1$. More generally, this idea may be used for improving the sensitivity of ultra-high performance optical sensors such as the interferometers designed for gravitational wave detection [30]. It may be applied to a measurement apparatus as sophisticated as the cold damping accelerometer developed at ONERA for testing the equivalence principle in space experiments. In this case, fluctuations entering all electrical or mechanical ports have to be taken into account, including ports involved in active elements [31].

The analysis can be pushed further, up to the problem of a satellite actively controlled to follow a dragfree geodesic trajectory. In this situation of interest for the future spatial missions devoted to fundamental physics, the remaining fluctuations can be calculated up to the ultimate quantum level [32].
We now come back to the discussion of the Casimir force, i.e. the mechanical force exerted by vacuum fluctuations on macroscopic mirrors \^[3]. Casimir calculated this force in a geometrical configuration where two plane mirrors are placed a distance $L$ apart from each other, parallel to each other, the area $A$ of the mirrors being much larger than the squared distance. Casimir considered the ideal case of perfectly reflecting mirrors and obtained the following expressions for the force $F_{\text{Cas}}$ and energy $E_{\text{Cas}}$

$$F_{\text{Cas}} = \frac{\hbar c \pi^2 A}{240 L^4} \quad E_{\text{Cas}} = \frac{\hbar c \pi^2 A}{720 L^4} \quad (A \gg L^2)$$

(7)

This faint attractive force ($\sim 0.1 \mu N$ for $A = 1 \text{cm}^2$ and $L = 1 \mu m$) has been observed in a number of experiments \^[34-39]. The accuracy in the first “historical” experiments was of the order of 100% : according to \^[35] for example, the experimental results did “not contradict Casimir’s theoretical prediction”. It has been greatly improved in recent measurements \^[34-39] and this is important for at least two reasons.

First, the Casimir force is the most accessible experimental consequence of vacuum fluctuations in the macroscopic world. Due to the difficulties with vacuum energy evoked above, it is of course crucial to test with great care the predictions of Quantum Field Theory. Any pragmatic definition of vacuum necessarily involves a region of space limited by some enclosure and the Casimir force is nothing but the physical manifestation of vacuum when it is enclosed in this cavity. Vacuum fluctuations are thus modified and vacuum energy depends on the distance $L$ so that mirrors are attracted to each other. In the ideal case of perfectly reflecting mirrors in vacuum, the force only depends on the distance and on two fundamental constants, the speed of light $c$ and Planck constant $\hbar$. This is a remarkably universal feature in particular because the Casimir force is independent of the electronic charge in contrast to the Van der Waals forces. In other words, the Casimir force corresponds to a saturated response of the mirrors which reflect 100% of the incoming light in the ideal case, but cannot reflect more than 100%. However most experiments are performed at room temperature with mirrors which do not reflect perfectly all field frequencies and this has to be taken into account in theoretical estimations \^[40-43].

Then, evaluating Casimir force is a key point in a lot of very accurate force measurements in the range between nanometer and millimeter. These experiments are motivated either by tests of Newtonian gravity at short distances \^[50] or by searches for new short range weak forces predicted in theoretical unification models \^[51-53]. Basically, they aim at putting limits on deviations from present standard theory through a comparison of experimental results with theoretical expectations. Casimir force is the dominant force between two neutral non-magnetic objects in the range of interest and, since a high accuracy is needed, it is therefore important to account for differences between the ideal case considered by Casimir and real situations studied in experiments. For theory-experiment comparisons of this kind in fact, the accuracy of theoretical calculations becomes as crucial as the precision of experiments \^[53].

Let us first discuss the effect of imperfect reflection by introducing scattering amplitudes which depend on the frequency of the incoming field and obey general properties of unitarity, high-frequency transparency and causality. The Casimir force can thus be given a regular expression, free from the divergences usually associated with the infiniteness of vacuum energy \^[54].

$$F = \frac{A}{c^3} \sum_p \int \frac{d\omega}{2\pi} e^{-2\pi \frac{d \cos \theta}{2\pi}} \cos^2 \theta \; h\omega (1 - g_p)$$

$$g_p = \frac{1 - |r_p(\omega, \kappa)|^2}{|1 - r_p(\omega, \kappa) e^{2\pi L}|^2}$$

(8)

The force is an integral over field modes of vacuum radiation pressure effect with a sum over the two polarization states ‘p’: $h\omega$ represents the energy per mode, $\cos^2 \theta$ an incidence factor with $\theta$ the incidence angle; the parenthesis shows that the Casimir force is the difference between the radiation pressures on outer and inner sides of the cavity with the factor 1 representing outer side while the Airy function $g_p$ describes the transformation of field energy from the outer to the inner side. The Airy function depends on the product $r_p = r_{1,p} r_{2,p}$ of the reflection amplitudes associated with the two mirrors at a given frequency $\omega$ and longitudinal wavevector $\kappa = \frac{c}{2\hbar} \cos \theta$. The ideal Casimir result is recovered at the limit where mirrors may be considered as perfect over the frequency range of interest, that is essentially over the first few resonance frequencies of the cavity. Since metals are perfect reflectors at frequencies lower than their plasma frequency, deviation from the ideal result \^[7] becomes significant at distances $L$ shorter than a few plasma wavelengths, typically $L \leq \sim 0.3 \mu m$ for gold or copper \^[57].

A second important correction is due to the radiation pressure of thermal fluctuations which are superimposed to vacuum fluctuations as soon as the temperature differs from zero. The expression \^[5] of the force has then to be modified by replacing the vacuum energy $h\omega$ per mode by the thermally modified energy $h\omega \times \coth \frac{h\omega}{2kBT}$. The modification becomes large at low frequencies so that the thermal deviation from the ideal formula is significant at large distances, typically $L \geq \sim 3 \mu m$ at room temperature \^[58].
The corrections to the ideal Casimir energy are shown on Figure 2 where we have plotted the factors \( \eta_E = \frac{E}{E_{Cas}} \) corresponding respectively to imperfect reflection and to temperature correction as well as the whole correction when both effects are taken into account. Since the two corrections are significant in non-overlapping distance ranges, the whole correction is approximately the product of the two effects evaluated separately [59].

![Figure 2](image)

**FIG. 2.** Variation of the factor \( \eta_E = \frac{E}{E_{Cas}} \) as a function of distance \( L \); the long-dashed and short-dashed curves respectively represent the corrections associated with imperfect reflection and non-zero temperature; the solid curve is the whole correction; these curves have been evaluated for a plasma model with the plasma wavelength corresponding to copper or gold.

This kind of theoretical evaluations should make possible accurate comparison, say at the 1% level, between experiment and theory, provided other discrepancies between ideal and real situations are also mastered. Recent experiments have not been performed in the configuration of two plane plates but of a sphere and a plane. The Casimir force in this geometry is usually estimated from the proximity theorem [60]. Basically this amounts to obtaining the force as the integral of contributions of the various inter-plate distances as if they were independent. In plane-sphere geometry, the force evaluated in this manner is just given by the Casimir energy obtained in plane-plane configuration for the distance \( L \) of closest approach in plane-sphere geometry. Then, the factor \( \eta_E \) drawn on Figure 2 is used to infer the factor for the force measurement in plane-sphere geometry. The accuracy of this approximation however remains to be mastered in a more reliable manner. This is also the case for surface roughness corrections which play an important role for short distance measurements [61].

Clearly, these points have to be inspected with great care before any claim for an accurate agreement between theory and experiments. It is however clear that the existence of the Casimir force, its sign and magnitude have now been experimentally demonstrated, to within a few % for the magnitude. Hence, it is certainly no longer possible to dismiss the mechanical effects of vacuum. Then the question arises of the consistency of these effects with the principles of relativity theory. As argued above, it is possible to formulate well-defined questions and to answer at least some of them. Let us discuss here one such question which is directly connected to the question of the validity of Einstein equivalence principle.

As already discussed, even if the mean value of vacuum energy does not contribute to gravity, energy differences have to contribute. This is in particular the case for the Casimir energy, a variation of the vacuum energy with the length of the cavity. If Einstein equivalence principle is obeyed for such energy contributions, Casimir energy should also contribute to the inertia of the Fabry-Perot cavity. Clearly, this effect has a small influence on the motion of a macroscopic cavity but is of fundamental importance, since it is a quantum version, at the level of vacuum fluctuations, of Einstein argument for the inertia of a box containing a photon bouncing back and forth [62,63]. Here, Einstein law of inertia of energy has to be applied to the case of a stressed body, where it is read [64]

\[
F_{mot} = -\mu a \quad \mu = \frac{E_{Cas} - F_{Cas}L}{c^2}
\]  

A detailed investigation of this question requires an evaluation of Casimir forces when the mirrors of the cavity are allowed to move [65]. When specifying a global motion of the cavity with uniform acceleration, the obtained force perfectly fits the contribution of Casimir energy to inertia [66]. This confirms that variations of vacuum energy effectively contribute to gravitation and inertia as expected from the general principles of relativity.
Going further along the same lines, fluctuations of vacuum energy should also contribute to gravitation. This implies that quantum fluctuations of stress tensors and of spacetime curvatures are necessarily coupled to each other. This problem can be treated by linear response techniques, in analogy with the study of fluctuations of moving objects coupled to the quantum fluctuations of forces acting upon them. The latter study has important consequences for the question of relativity of motion.

Even a single mirror at rest in vacuum is submitted to the effect of vacuum radiation pressure. The resulting force has a null mean value due to the balance between the contributions of opposite sides but still has fluctuations since the vacuum fluctuations impinging on both sides are statistically uncorrelated. When the mirror is moving, the balance is also broken for the mean force which entails that vacuum exerts a radiation reaction against motion. The dissipative force is described by a susceptibility allowing one to express the force $F_{\text{mot}}$ as a function of motion $q$, in a linear approximation and in the Fourier domain,

$$F_{\text{mot}}[\Omega] = \chi[\Omega]q[\Omega]$$

The motional susceptibility $\chi[\Omega]$ is directly related to the force fluctuations evaluated for a mirror at rest through quantum fluctuation-dissipation relations. The damping of mechanical energy is associated with an emission of radiation and the radiation reaction force is just the consequence of momentum exchange.

Let us first consider the simple case of a mirror moving with a uniform velocity. When this motion takes place in vacuum, the radiation reaction force vanishes, so that the reaction of vacuum cannot distinguish between inertial motion and rest, in full consistency with the principle of relativity of motion. But a friction force arises when a mirror moves in a thermal field, in analogy with the damping of motion by air molecules. With the notations of the present paper, this force corresponds for a perfectly reflecting mirror in the limits of large plane area and large temperature to

$$\chi[\Omega] \simeq \frac{i\hbar A}{240\pi^2c^4}\theta^4\Omega$$

$$F_{\text{mot}} \simeq \frac{\hbar A}{240\pi^2c^4}\theta^4q(t) \quad (A \gg \frac{c^2}{\Omega^2}, \theta \gg \Omega)$$

This expression vanishes when $\theta$ is set to zero, but this does not lead to the absence of any dissipative effect of vacuum on a moving scatterer. For a perfect mirror with an arbitrary motion in electromagnetic quantum vacuum, we obtain a susceptibility proportional to the fifth power of frequency or, equivalently, a force proportional to the fifth order time derivative of the position

$$\chi[\Omega] \simeq \frac{i\hbar A}{60\pi^2c^4}\Omega^5$$

$$F_{\text{mot}} \simeq -\frac{\hbar A}{60\pi^2c^4}q'''(t) \quad (A \gg \frac{c^2}{\Omega^2}, \theta = 0)$$

This result can be guessed from the previous one through a dimensional analysis: the factor $\theta^4$ appearing in (11) has been replaced by $\Omega^4$ in (12). This argument played a similar role in the discussion of the expression of vacuum energy density with however an important difference: the vacuum term is now perfectly regular and it corresponds to a well-defined physical effect.

This effect was first analyzed in 1976 by Fulling and Davies for the simpler case of a perfectly reflecting mirror moving in vacuum of a scalar field theory in two-dimensional spacetime. In this case, the dissipative force is proportional to the third order time derivative of the position since $\frac{1}{6}\Omega^4$ is replaced by $\Omega^3$ in the preceding dimensional argument. The vacuum reaction force has thus the same form as for an electron in electromagnetic vacuum and it raises the same causality and stability problems. This difficulty is however solved by taking into account the fact that any real mirror is certainly transparent at high frequencies. This leads to a completely satisfactory treatment of motion of a real mirror in vacuum. Furthermore, the stable equilibrium reached by the mirror’s motion coupled to vacuum radiation pressure fluctuations contains a consistent description of the quantum fluctuations of the mirror, generalizing the standard Schrödinger equation.

We may emphasize that the motional force does not raise any problem to the principle of special relativity. As a matter of fact, the reaction of vacuum vanishes in the particular case of uniform velocity. The quantum formalism gives an interesting interpretation of this property: vacuum fluctuations appear exactly the same to an inertial observer and to an observer at rest. Hence the invariance of vacuum under Lorentz transformations is an essential condition for the principle of relativity of motion to be valid and it establishes a precise relation between this principle and the symmetries of vacuum. More generally, vacuum does not oppose to uniformly accelerated motions and this property corresponds to conformal symmetry of quantum vacuum. In this sense, vacuum fluctuations set a class of privileged reference frames for the definition of mechanical motions.

At the same time, the existence of dissipative effects associated with motion in vacuum challenges the principle of relativity of motion in its more general acceptance. Arbitrary motion produces observable effects, namely the
resistance of vacuum against motion and the emission of radiation by the moving mirror, although there is no further reference for this motion than vacuum fluctuations themselves. This means that quantum theory has built up a theoretical framework where the questions raised in the introduction may find consistent answers. The space in which motion takes place can no longer be considered as empty since vacuum fluctuations are always present. These fluctuations give rise to real dissipative effects for an arbitrary motion. However these effects vanish for a large category of specific motions, including the cases of uniform velocity and uniform acceleration.

Clearly, it would be extremely interesting to obtain experimental evidence for the dissipative effects associated with motion in vacuum [86]. These effects are exceedingly small for any motion which could be achieved in practice for a single mirror, but an experimental observation is conceivable with a cavity oscillating in vacuum. In this case, the emission of motional radiation is resonantly enhanced [87], specific signatures are available for distinguishing the motional radiation from spurious effects [88] so that an experimental demonstration appears to be achievable with very high finesse cavities [89].

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