NONLINEAR EVOLUTION OF THE MAGNEToHYDRODYNAMIC RAYLEIGH-TAYLOR INSTABILITY

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Draft version February 1, 2008

ABSTRACT

We study the nonlinear evolution of the magnetic Rayleigh-Taylor instability using three-dimensional MHD simulations. We consider the idealized case of two inviscid, perfectly conducting fluids of constant density separated by a contact discontinuity perpendicular to the effective gravity $g$, with a uniform magnetic field $B$ parallel to the interface. Modes parallel to the field with wavelengths smaller than $\lambda_c = B \cdot B / (\rho_h - \rho_l) g$ are suppressed (where $\rho_h$ and $\rho_l$ are the densities of the heavy and light fluids respectively), whereas modes perpendicular to $B$ are unaffected. We study strong fields with $\lambda_c$ varying between 0.01 and 0.36 of the horizontal extent of the computational domain. Even a weak field produces tension forces on small scales that are significant enough to reduce shear (as measured by the distribution of the amplitude of vorticity), which in turn reduces the mixing between fluids, and increases the rate at which bubbles and fingers are displaced from the interface compared to the purely hydrodynamic case. For strong fields, the highly anisotropic nature of unstable modes produces ropes and filaments. However, at late time flow along field lines produces large scale bubbles. The kinetic and magnetic energies transverse to gravity remain in rough equipartition and increase as $t^4$ at early times. The growth deviates from this form once the magnetic energy in the vertical field becomes larger than the energy in the initial field.

We comment on the implications of our results to Z-pinch experiments, and a variety of astrophysical systems.

1. INTRODUCTION

When a light fluid accelerates (or supports against gravity) a heavier fluid, the interface between the two is subject to the Rayleigh-Taylor instability (RTI). The idealized case of two incompressible, inviscid, unmagnetized fluids of constant density separated by a contact discontinuity perpendicular to the effective gravity $g$ has been extensively studied through theory, experiment, and numerical simulation. In the linear regime, short wavelength perturbations of the interface grow fastest. Once the perturbation amplitude is comparable to the wavelength, the interface can be characterized as rising “bubbles” of light fluid between descending “fingers” of heavy fluid. Secondary Kelvin-Helmholtz instabilities are induced by the shear between the rising and descending columns. In the fully nonlinear phase of a multimode spectrum of perturbations, smaller bubbles merge into larger structures, which then break up due to secondary instabilities, and a turbulent mixing layer results.

From self-similarity arguments, the time evolution of the height of the bubbles $h$ above the initial interface is expected to be

$$h = \alpha A g t^2$$

(1)

where $\alpha$ is a dimensionless constant, and $A$ is the Atwood number

$$A \equiv \frac{\rho_h - \rho_l}{\rho_h + \rho_l}$$

(2)

($\rho_h$ and $\rho_l$ are the densities of the heavy and light fluids respectively). Recently, a comparison of the values of $\alpha$ measured from laboratory experiments with the results of high resolution, three-dimensional numerical simulations of multimode RTI with strong mode coupling, computed with a variety of numerical methods, has been presented by Dimonte et al (see also Ref. 4) as part of a validation effort for numerical methods for hydrodynamics. Detailed analysis of the self-similar bubble dynamics, energy balance, and spectral properties of the resulting turbulent mixing layer demonstrated there is reasonable agreement between the simulations, theory and experiment, except in that the experimentally determined value of $\alpha$ is $0.057 \pm 0.008$, whereas most of the numerical simulations give a value for $\alpha$ which is about a factor of two smaller. It would appear this discrepancy is primarily due to mixing at small (close to the grid) scales, since the use of specialized front tracking algorithms, or correcting the numerically measured growth rates for the observed density dilution produced by small scale mixing, or comparison to experiments which use miscible fluids, all give better agreement.

It is important to validate numerical algorithms in experimentally accessible parameter regimes (such as the hydrodynamic RTI) so that these methods can be used with confidence to explore new physics in regimes which may be hard to realize in the laboratory. For example, there are a number of applications where magnetic fields may play an important role in the linear evolution and nonlinear saturation of the RTI. The axial compression of plasma produced by the ablation of wires in Z-pinch experiments is subject to the magnetic RTI. Since the instability can limit the maximum compression achieved in the pinch, understanding and controlling it is of critical importance. Furthermore, since most astrophysical plasmas are magnetized, the RTI associated with accretion onto compact objects, supernova remnants, magnetic flux emerging from the solar photosphere, and at the surface of synchrotron nebulae expanding into supernova ejecta is strongly influenced by the presence of magnetic fields.

For the ideal case introduced above, the linear growth rate $n$
of the RTI with a uniform magnetic field of strength $B$ parallel to the interface is given by:

$$n^2 = g k \left( \frac{\rho_h - \rho_l}{\rho_h + \rho_l} - \frac{(\mathbf{B} \cdot \mathbf{k})^2}{2\pi g k (\rho_h + \rho_l)} \right)$$

(3)

where instability occurs when $n^2 > 0$. Here and throughout we have chosen a system of units in which the magnetic permeability $\mu = 1$. In the above, $k$ is the wavenumber of a perturbation, and $k^2 = \mathbf{k} \cdot \mathbf{k}$. For perturbations perpendicular to the magnetic field the linear growth is identical to the purely hydrodynamic case; these modes are often referred to as interchange modes. For perturbations parallel to the field, wavelengths smaller than the critical value $\lambda_c = 2\pi/k_c = B^2/\left(\rho_h - \rho_l\right)g$ are stable, and the growth rate of modes at all scales larger than $\lambda_c$ is reduced compared to the non-magnetic case. Equivalently, for instability to occur on scales smaller than $L$, the magnetic field must be smaller than the critical value

$$B_c = \left(\left(\rho_h - \rho_l\right)gL\right)^{1/2}.$$  

(4)

Since the growth rate is zero at both large and small $k$, there must be a wavelength $\lambda_{\text{max}}$ where the growth rate is maximum; it occurs at $\lambda_{\text{max}} = \lambda_c$. The presence of a critical wavelength, a peak in the growth rate at $\lambda_{\text{max}}$, and the anisotropic nature of the dispersion relation for perturbations parallel versus perpendicular to field lines suggests the nonlinear evolution of the magnetic RTI will be much different than the non-magnetic case.

Due to the anisotropic nature of the linear modes, it is critical to study the magnetic RTI in full three dimensions. Two-dimensional ideal MHD studies in which the magnetic field is perpendicular to the domain are in fact equivalent to 2D hydrodynamics with the gas pressure $P$ replaced by the total pressure $P^* \equiv P + (\mathbf{B} \cdot \mathbf{B})/2$. On the other hand, two-dimensional studies in which the magnetic field is in the plane of the computation miss the interchange modes, which artificially suppresses the instability. For example, in 2D with $B > B_c$, the interface is completely stable, whereas in 3D it will be strongly unstable due to the interchange modes (which will act like 2D hydrodynamic RTI in the plane perpendicular to the field).

There have only been a few previous investigations of the magnetic RTI in full three dimensions. Jun et al.\textsuperscript{10} presented a few 3D simulations, although at much lower resolution than is currently possible. Their focus was in potential field amplification and dynamo action produced by turbulent mixing in the saturated state, thus most of their 3D simulations began with a field weak compared to $B_c$ so that $\lambda_c$ was initially unresolved. Zhu et al.\textsuperscript{11} performed simulations of single mode interchange instabilities in three dimensions at very low $\beta \equiv P/P_m$, where $P$ and $P_m$ are the gas and magnetic pressures respectively, with the goal of testing the predictions of perturbation theory on the nonlinear structure that emerges for a single mode. More recently, Isobe et al.\textsuperscript{8} showed that the magnetic RTI can produce filamentary structures during the buoyant rise of flux in the solar photosphere.

In this paper, we use numerical MHD simulations, using methods that have been validated in the hydrodynamic regime, to study the nonlinear evolution of the magnetic RTI with strong fields in three dimensions. By strong we mean $\lambda_c$ is comparable to the size of the computational domain. Our goal is to explore how strong fields affect the formation, structure, and evolution of bubbles and fingers in the nonlinear regime, and how they affect the turbulent mixing layer (and vice-versa). Since we do not use front tracking methods, our calculations are limited by the numerical diffusion that occurs near the grid scale\textsuperscript{3,4}. By comparison of hydrodynamic and MHD simulations computed with the same parameters, numerical resolution, and algorithm, we can assess the relative rate of mixing between the magnetic and non-magnetic RTI. Interestingly, we find that even with an initial field too weak to resolve $\lambda_c$ (so that one might expect it to evolve more like a hydrodynamic model), the mixing rate between the light and heavy fluids is substantially reduced, and the rate of rise of the bubbles as measured by the $\alpha$ parameter is substantially increased with MHD. Although such fields may be too weak to stabilize resolved modes, they still add a significant “surface tension” at very small scales, which supports the idea that the discrepancy between the experimentally measured value of $\alpha$ in the hydrodynamic RTI and numerical simulations that lack front tracking is due to small scale mixing. Most of our calculations are motivated by the astrophysical applications of the magnetic RTI, thus we study inviscid fluids in the ideal MHD approximation (without any explicit resistivity) and in a planar geometry. The latter requires the thickness of the mixing zone between the heavy and the light fluids be much smaller than the radius of curvature of the interface $R$, and that $\lambda_c/R \ll 1$. The magnetic RTI associated with Z-pinch experiments occurs at a very low magnetic Reynolds number, and in a cylindrical geometry. The inclusion of non-ideal MHD effects\textsuperscript{12}, and the appropriate geometry, will be important for future studies with application to Z-pinch\textsuperscript{13}.

The organization of this paper is as follows. In the next section, we describe the equations we solve, our numerical algorithm, and the initial conditions. In section 3 we present most of our results, while in section 4 we summarize and discuss the application of our results.

2. Method

We solve the equations of ideal MHD with a constant vertical acceleration $g = (0, 0, -g)$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$  

(5)

$$\frac{\partial \mathbf{v}}{\partial t} + \nabla \cdot (\rho \mathbf{v} \mathbf{v} - \mathbf{B} \mathbf{B}) + \nabla P^* = \rho \mathbf{g}$$  

(6)

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \times (\mathbf{v} \times \mathbf{B}) = 0$$  

(7)

$$\frac{\partial E}{\partial t} + \nabla \cdot ((E + P^*) \mathbf{v} - \mathbf{B} (\mathbf{B} \cdot \mathbf{v})) = \rho \mathbf{v} \cdot \mathbf{g}$$  

(8)

In these equations, $P^*$ is the total pressure (gas plus magnetic), and $E$ is the total energy density, which is related to the internal energy density $\epsilon$ via

$$E \equiv \epsilon + \rho \mathbf{v} \cdot \mathbf{v}/2 + (\mathbf{B} \cdot \mathbf{B})/2.$$  

(9)

We use an ideal gas equation of state for which $P = (\gamma - 1) \epsilon$, where $\gamma$ is the ratio of specific heats. We take $\gamma = 5/3$. Although we are solving the equations of compressible gas dynamics, we choose a sound speed which is so large that the
resulting motions are highly subsonic and nearly incompressible. Thus, varying the adiabatic index should have little effect on the results reported here.

The calculations are performed in a three-dimensional domain of size $L \times L \times 2L$, where $L = 0.1$. The vertical coordinate spans $-0.1 \leq z \leq 0.1$. The interface between heavy and light fluids is initially at $z = 0$. The upper half of the domain ($z > 0$) is filled with heavy fluid of density $\rho_h = 3$, while in the lower half ($z < 0$) the density of the light fluid is $\rho_l = 1$. The Atwood number is $A = 1/2$. The profile of the pressure is given by the condition of hydrostatic equilibrium, while the amplitude is chosen so that the sound speed in the light fluid is unity at the interface, thus

$$P^*(z) = \frac{3}{5} - g \rho z + B^2/2$$

(10)

We choose $g = 0.1$. The sound crossing time $t_s$ in the light fluid at the interface is 0.1. We will report evolutionary times in our computations in terms of $t_s$. Periodic boundary conditions are used in the transverse ($x$- and $y$-) directions, and reflecting boundary conditions are used at the top and bottom.

The magnetic field is initialized to be uniform and along the $x$-axis $B = (B_0, 0, 0)$. From the linear analysis (eq. [4]), if $B_0 > B_c = 0.14$, then there will be no unstable wavelengths shorter than the size of the computational domain $L$. We study the nonlinear evolution for a variety of field strengths between 0.1 and 0.6$B_c$. For the strongest field $B_0 = 0.6B_c$, the ratio of the Alfvén speed to the sound speed in the light fluid at the interface is 0.024, which corresponds to $\beta = 1.2(c_s/V_A)^2 \approx 2 \times 10^3$. For the weakest field $B_0 = 0.1B_c$, $\beta = 7.5 \times 10^4$. Thus, we study a regime where the magnetic energy density is small compared to thermal pressure (high $\beta$), and the vertical hydrostatic equilibrium is determined by gas pressure alone. Nonetheless, we study strong fields in the sense that modes parallel to $B$ are nearly completely suppressed. The magnetic RTI in plasmas with $\beta \approx 1$, which is often referred to as the Parker instability in astrophysics, has been extensively studied in the literature.

An important dimensionless parameter which characterized our simulations is the ratio of the critical unstable wavelength to the size of the computational domain $\lambda_c/L$. For the strongest fields studied here, $\lambda_c/L = 0.36$. Thus, only a few unstable modes are present in the domain in this case. This regime is appropriate to physical systems in which some scale in the system (e.g. the diameter of a wire in a Z-pinch) is comparable to the critical unstable wavelength. In §4 we discuss the interpretation of our results in terms of this parameter.

The RTI is seeded by small amplitude, random, zone-to-zone perturbations to the vertical velocity $v_z$ added throughout the volume. The amplitude of the perturbations is smoothed toward the vertical boundaries; $v_z(z) = A_0 R (1 + \cos 2\pi z/L)$ where $A_0 = 0.005$, and $R$ is a random number between -1 and 1. The peak perturbations are therefore 1% of $c_s$. We have found that perturbing the vertical velocity is superior to perturbing the position of the interface, since the latter requires smoothing at the grid scale when the perturbation amplitude is smaller than a grid zone. Dimonte et al. were careful to introduce a spectrum of multimode perturbations which is truncated at high wavenumbers equivalent to 32 gridpoints, so that all linear modes are initially resolved. Instead, our perturbation spectrum is white noise down to the grid scale. This may mean that at very early times, when modes are non-interacting, and the high-$k$ modes dominate, there may be differences between our hydrodynamical simulations. However, since we follow the multimode evolution deep into the nonlinear regime, where mode interactions should erase memory of the initial conditions, we do not expect this to limit comparisons at late times.

The computations presented in this paper are computed using a recently developed Godunov method for compressible MHD that combines the piecewise parabolic method and the directionally unsplit corner transport upwind (CTU) integrator with the constrained transport algorithm for enforcing the divergence free constraint. A complete description of the algorithm, including the results of an extensive series of test problems, is given in Ref. [19]. Adding source terms (vertical gravity) to a Godunov scheme requires particular care, by adding them to both the PPM reconstruction step, as well as to the transverse flux differences in the CTU integrator, we find our method holds the vertical equilibrium state (in which the pressure gradient balances gravity) exactly.

When run in hydrodynamic mode (which utilizes the same algorithms as when the code is configured for MHD, except for the Riemann solver), the algorithm is similar to the FLASH and WP/PPM codes used by Dimonte et al, except for the use of the unsplit CTU integrator. All of the simulations use a grid of $256 \times 256 \times 512$, the highest resolution reported in Dimonte et al. In fact, apart from the perturbation spectrum, we use the same grid and parameters in order to more easily facilitate comparisons. We have tested for convergence of our numerical solutions by running a few simulations at one half this resolution ($128 \times 128 \times 256$). Although such details as the number and location of bubbles and fingers in the flow are substantially different at lower resolution, none of the diagnostics which are the focus of this work are changed to a significant degree. This indicates our solutions are converged with respect to these quantities. We provide a much more comprehensive investigation of the effect of mass diffusion at the grid scale due to numerical effects on our results in §3.1.

The use of a fully compressible code to study low Mach number flows such as investigated here is not optimal, and requires many more timesteps to be taken compared to the strictly incompressible case (typically about $4 \times 10^4$ timesteps are required to reach $t/t_s = 60$). The numerical algorithms used here are second order in both space and time, which helps to reduce the accumulation of error. The convergence study presented in §3.1 provides insight into the effect of temporal errors on our results. Although approximate methods for low Mach number flows might be more cost effective, it is not clear they will be substantially more accurate.

3. RESULTS

We describe the results from four MHD simulations using field strengths of $B_0 = 0.1, 0.2, 0.4$, and $0.6 B_c$. We shall refer to each of these calculations as runs R1, R2, R4, and R6.
respectively. For comparison purposes, we also describe the results of a hydrodynamic calculation computed with the same code, hereafter referred to as run RH. Table 1 lists important properties of the calculations. In particular, note that for the weakest field simulation, R1, the critical wavelength $\lambda_c$ is only about 2.5 grid cells, therefore it is unresolved. For the smallest well resolved modes (requiring about 16 grid cells per wavelength), the growth rate is reduced by only about 15% at this field strength compared to the non-magnetic case. Thus, we expect run R1 to be similar to the purely hydrodynamic calculation RH. Our discussion will focus on the strong field run R6, the weak field run R2 (the weakest field for which the critical wavelength $\lambda_c$ is resolved), and the hydrodynamic run RH. Each calculation is continued until the rising bubbles or sinking fingers reach the vertical boundaries, which typically requires about $60\tau_t$.

### 3.1. Convergence Study in Two Dimensions

Before presenting the complex nonlinear evolution of the magnetic RTI in fully three dimensions, it is important to first assess the extent to which mixing of mass and momentum due to numerical effects at the grid scale affect our results. We have used a series of simulations of the growth of the magnetic RTI in two dimensions, computed with the identical algorithms and using the same parameters as used for the three-dimensional runs listed in Table 1, to investigate the convergence of our results. The calculations are performed in the $x-z$ plane, so that the magnetic field is in the plane of the computation. We perturb the interface with a single mode with a wavelength equal to the horizontal size of the domain $L_x$ to investigate how numerical grid effects alter the evolution of a single, smooth interface.

To track the amount of mixing between the heavy and light fluids, we define a mixing parameter

$$\Theta = 4 f_h f_l$$

(11)

where $f_h$ is the fraction of each grid cell occupied by the heavy fluid (of density $\rho_h$), and $f_l = 1 - f_h$. For a purely incompressible flow, with $\rho_h = 3$ and $\rho_l = 1$, then $f_h = (\rho - 1)/2$. In regions with no mixing, $\Theta = 0$, while the maximum value occurs for $f_h = f_l = 1/2$, when $\Theta = 1$. A useful diagnostic is the volume averaged mixing parameter, which in two dimensions is

$$\langle \Theta \rangle_V = \frac{\int_{x,z} \int 4 f_h f_l dx dz}{2L^2}$$

(12)

A comparison of the time evolution of $\langle \Theta \rangle_V$ measured in our simulations with simple analytic expectations allows us to measure the effect of mixing at the grid scale.

Figure 1 presents images of the mixing parameter in the evolution of the single mode RTI in two dimensions using a strong field (run R6) at time $t/t_s = 30$ for four different resolutions corresponding to 32, 64, 128, and 256 grid points per $L$. Note the highest resolution is the same as is used in all of the three-dimensional results presented in this paper. The images show that $\Theta$ is non-zero only at the interface, which is smooth and has the same shape at every resolution. Clearly, the growth rate and nonlinear structure of a single mode is captured correctly even at the lowest resolution. Although the physical width of the interface over which mass mixing occurs is larger at low resolution, this is simply due to the increase in the size of grid cells $\delta x = \delta z$; it has not affected the rate of growth or structure of the mode. Figure 1 gives the visual impression that the amount of mass diffusion at the grid scale converges at first order (linear in $\delta x$), we quantify this dependence below.

The initial conditions used in all of our simulations consist of a density (and temperature) discontinuity at the interface between the heavy and light fluids. Initially this discontinuity is aligned with the grid. For the numerical algorithms used in this paper (Godunov method with a Roe solver), there is no numerical diffusion of this discontinuity as long as it remains at rest parallel to the grid. (We have confirmed that if the interface is not perturbed, our code holds the initial equilibrium state, and the mixing parameter remains $\Theta = 0$ everywhere.) Once the interface is perturbed and the RTI begins to grow, however, motion of the interface produces mass diffusion at the grid scale. This is an inevitable consequence of using a control volume approach without interface tracking: when the interface crosses the middle of the cell, the volume averaged density contains contributions from both the heavy and light fluids, and will therefore be intermediate between the two. Figure 1 demonstrates that for the algorithms used here, the spread of the contact discontinuity is confined to a few cells (several $\delta x$), Hence, at early times (when the length of the interface is just $L_x$) we expect that

$$\langle \Theta \rangle_V \sim \frac{L_x M \delta x}{L_x L_z} \Theta = \frac{M \tilde{\Theta}}{N_z}$$

(13)

where $M$ is the number of cells over which the interface spreads due to numerical effects, $\tilde{\Theta}$ is the normalized average of $\Theta$ over the width of the mixing region, and $N_z$ is the number of grid cells in $L_z$. If the variation of the heavy and light fluid fractions is linear over the width of the mixing region, then $\tilde{\Theta} = 2/3$. As the RTI grow, the length of the interface grows (e.g. figure 1). Thus, equation 13 expresses the expectation that the time evolution of $\langle \Theta \rangle_V$ should be proportional to the time evolution of the length of the interface, and that at any time $\langle \Theta \rangle_V$ should converge with $\delta x$ at first order (as expected for any discontinuous solution using a fixed grid).

Figure 2 plots the time evolution of $\langle \Theta \rangle_V$ for the single mode RTI in two dimensions (run R6) at the same resolutions shown in figure 1. There is a rapid rise in $\langle \Theta \rangle_V$ to $t/t_s = 5$, as the interface begins to move across the grid, and numerical diffusion causes it to spread to a size of a few $\delta x$. The amplitude of $\langle \Theta \rangle_V$ at $t/t_s = 5$ is in excellent quantitative agreement with the expectation of equation 13, if the width of the interface $M \approx 2$. Thereafter, $\langle \Theta \rangle_V$ shows slow linear growth as the interface lengthens. At $t/t_s = 20$, the mode begins to go nonlinear, the length of the interface begins to grow more rapidly, and $\langle \Theta \rangle_V$ increases more rapidly. The fractional rate of increase $(1/\langle \Theta \rangle_V) d\langle \Theta \rangle_V/dt$ is the same in each case, indicating the increase in time is simply due to the lengthening of the interface. At $t/t_s = 20$, the convergence rate of $\langle \Theta \rangle_V$ is 0.8, close to our expectation of first order.

The analysis presented thus far has considered an interface which remains smooth, unaffected by secondary KH instabili-
ties which are present with weaker magnetic fields. By twisting the interface on small scales, these secondary instabilities can increase the mass mixing at the grid scale. Figure 3 presents images of the mixing parameter $\Theta$ at $t/t_s = 30$ in the evolution of a single mode RTI computed with a resolution of 256 grid points per $L$, but with a variety of field strengths corresponding to runs R6 (strong field), R2 (intermediate field), R1 (weak field), and RH (hydrodynamic); see Table 1. The growth of an increasingly larger number of vortices at the interface is evident as the magnetic field strength is decreased. As these vortices wind up the interface, mass mixing occurs due to numerical effects when the radius of curvature approaches the grid scale.

Figure 4 plots the time evolution of the volume averaged mixing parameter $\langle \Theta \rangle_V$ for these runs. Interestingly, until the appearance of the first KH roll at $t/t_s = 20$, the amplitude and evolution of $\langle \Theta \rangle_V$ is identical. Thereafter, the values of $\langle \Theta \rangle_V$ increase rapidly in runs R2, R1, and RH, reflecting the increased importance of the small scale distortion of the interface due to KH instabilities.

The fact that the time evolution of $\langle \Theta \rangle_V$ is identical regardless of the field strength for $t/t_s < 20$ (until the first KH roll forms) indicates the intrinsic spread of a smooth interface in our numerical methods is independent of the field strength. This gives us confidence that the relative amount of mixing we observe in three dimensions between hydrodynamic and MHD simulations with different field strengths will be due to physical differences in the amount of geometric distortion of the interface, rather than a change in the intrinsic numerical mixing inherent in the algorithm between hydrodynamics and MHD. Moreover, increasing the resolution will not reduce this mixing, rather it will simply introduce more small scale structure that produces a similar amount of mixing, unless a physical process such as surface tension or viscosity is introduced to create a scale on which such motions are suppressed. We conclude the relative rate of mass mixing we observe between hydrodynamic and MHD is robust.

The analysis above has focused on quantifying the amount of mass mixing due to numerical effects in our algorithms. Since we do not include an explicit viscosity, momentum diffusion at the grid scale is also dominated by grid effects. For the algorithms used here, it has been shown that the properties of the numerical diffusion of momentum produces proper convergence to a solution of the Navier-Stokes equations. The effective Reynolds number of the flow is determined by the grid resolution. Since we adopt the same resolution for all runs, the simulations reported here are equivalent to a study of the change in the flow due to the effect of the magnetic field at fixed Reynolds number. Comparison to a specific experiment, rather than comparative results as reported here, would require simulations that achieve the same Reynolds number as the experiment.

Finally, the analysis presented above has focused on the evolution of the RTI in two dimensions. In fully three dimensions, the value of the volume averaged mixing parameter $\langle \Theta \rangle_V$ will be given by equation 13 with the length of the interface replaced by the surface area. The time evolution of $\langle \Theta \rangle_V$ will then depend on the rate of growth of this surface area, which will be more rapid than in two dimensions.

### 3.2. Three-dimensional structure

Figure 5 is a comparison of the three-dimensional structure at both early ($t/t_s = 29.6$) and late ($t/t_s = 60$) times from runs RH, R2, and R6 using vertical slices of the density at the edges of the computational domain, as well as a horizontal slice at the midplane $z = 0$. The hydrodynamic calculation RH (top two panels) can be compared directly to the results in Ref. [3]. The classic evolution of the hydrodynamic RTI is evident; at early times most structure is at small scales. Bubbles of light fluid have detached from the interface, and are dominated by a “mushroom cap” appearance. At late times, the dominant structures are on larger scales. Secondary Kelvin–Helmholtz instabilities have produced vortices and mixing along the edges of the bubbles, and a fully developed turbulent mixing layer is evident.

In the weak field calculation R2 (middle panels), the critical wavelength at which the magnetic field can suppress the RTI is small, only $0.01L$, much smaller than the largest bubbles seen at late times in RH. Thus, we might expect the structure produced in the nonlinear regime in R2 to be similar to the hydrodynamic case RH. At early times, R2 does show the classic bubble and finger morphology of the hydrodynamic RTI. There is no evidence for anisotropy; structures are identical parallel and perpendicular to the field. However, even at the early time there is clearly much less mixing between the light and heavy fluids; the horizontal slice shows that at the midplane most of the fluid is either at the highest or lowest density, whereas in RH most of the fluid is at intermediate (grey) densities (for online version in color, high density is red, low density is blue, intermediate density is green). At late times, the structure of the RTI is much different in R2 in comparison to RH. Rather than a turbulent mixing layer, in R2 the bubbles have become long columns which have extended far above the midplane. These structures show no anisotropy. There continues to be little mixing between the fluids, thus, even a weak field has strongly affected the structure.

In the strong field calculation R6 (bottom panels) the anisotropic nature of linear modes is clearly evident: the density fluctuations at the midplane are aligned with the direction of the field (along the $x$-axis). As in R2, very little mixing is evident at the midplane in R6. At late times in the strong field case, the dominant structures are smooth and highly anisotropic. The slice in the $y-z$ plane shows columns and bubbles which result from interchange modes that act like the hydrodynamic RTI in two dimensions. In the $x-z$ plane, however, extended loop-like structures are evident which result from suppression of short wavelength modes along the field. These structures are unlike any of the bubbles seen in the hydrodynamic RTI (run RH, top panel).

To further illustrate the nonlinear structure at late time, Figure 6 shows isosurfaces of the density at $\rho = 1.1$ and 2.9, along with vertical slices of the density at the faces of the computation domain for runs RH and R2 at $t/t_s = 56$. A complex network of bubbles is evident in the shape of the density isosurface in
RH, whereas in R2 the bubbles are much larger and smoother. Note the circular ring near the rear edge of the domain in RH resulting from the roll-up of the KH instability around a spherical bubble. The slices at the edge of the domain also make clear the turbulent mixing in RH, whereas in R2 there is far less mix.

Isosurfaces and slices of the density in the strong field run R6 are shown at two times in Figure 7: the left panel is at $t/t_\ast = 20$ and the right at $t/t_\ast = 56$. At the earlier time, the isosurface reveals the formation of filaments and tubes rather than bubbles and fingers as in the hydrodynamic case. The anisotropy of linear modes is evident in the comparison of the slices in the $x-z$ and $y-z$ planes. In the former, the magnetic field has suppressed short wavelengths, and the density structures are highly elongated in the $x$-direction. In the latter, the magnetic field is perpendicular to the plane, and thus has no effect. Bubbles and fingers on short wavelengths have formed, reminiscent of the 2D hydrodynamic RTI. The formation of flux tubes seen at early times is very similar to that observed in the solar photosphere, as shown in Ref. [8]. At late times (right panel of Figure 7), the density isosurface is remarkably smooth. Rather than sheets, more rounded bubbles have been formed at the tips of the columns by the flow of fluid along loops until it collects at the tips. Similar evolution is observed in the nonlinear regime of the Parker instability\(^{15}\).

An important diagnostic of the instability is the rate at which bubbles and fingers are displaced from the initial interface. To quantify this rate, we define the horizontally averaged mass fraction of heavy fluid as (following Ref. [3])

$$\langle f_h \rangle \equiv \int\int f_h \, dx \, dy / L^2.$$  

(14)

Since our simulations are not purely incompressible, the maximum (minimum) densities can be slightly larger (smaller) than 3 (one). To account for this, we define the height of bubbles to be the location where $\langle f_h \rangle = 0.985$, while the height of fingers is the location where $\langle f_h \rangle = 0.015$.

Figure 8 is a plot of the height of bubbles and fingers in runs RH, R2, and R6 versus $t^2$. From the self-similar arguments that lead to eq. [1], we expect at late time a straight line with a slope of $\alpha$. In each case, this expectation is confirmed. In the hydrodynamic run RH, the best fit slope at late time is $\alpha = 0.021$, whereas for MHD we obtain $\alpha = 0.035$ for both R4 and R6. Although not plotted to avoid cluttering the figure, we have also confirmed that for the weakest field run R1, the height of bubbles follows a straight line when plotted versus $t^2$, with a slope of $\alpha = 0.031$. Thus, we find that (1) the rate at which bubbles rise in the hydrodynamic RTI measured in our simulations agrees with the results of Dimonte et al for non-front tracking methods, and (2) the addition of even a small magnetic field significantly increases the slope. We show below that there is far less mixing in the MHD RTI simulations, which may account for this increase.

3.3. Mixing

Figure 9 plots the vertical profile of the fraction of heavy fluid $\langle f_h \rangle$ for runs RH and R6 at $t/t_\ast = 56$, where the horizontal axis has been scaled by the bubble height at that time. The profiles are remarkable similar to each other. We also find the profiles are the same at different times in the evolution, as is expected due to the self-similar evolution. The similarities in the profiles of $\langle f_h \rangle$ between the magnetized and unmagnetized RTI, despite the lack of a turbulent mixing layer in the former, suggests that this profile is not sensitive to mixing.

Figure 10 plots the vertical profile of the horizontally averaged mixing parameter

$$\langle \Theta \rangle = 4 \langle f_h f_i \rangle$$  

(15)

for runs RH, R1, R2 and R6 at $t/t_\ast = 56$. There is a monotonic decrease in the maximum value of $\langle \Theta \rangle$ with the field strength. The hydrodynamic case RH is closest to reaching $\langle \Theta \rangle = 1$, the maximum value possible. Even the weakest field case, run R1, in which the critical wavelength suppressed by the magnetic field is unresolved initially, has a significantly lower value of $\langle \Theta \rangle$, indicating far less mixing is occurring compared to the hydrodynamic case.

It is also of interest to compare the integral of the mixing parameter over the vertical coordinate between different runs. For the hydrodynamic and weak field runs RH and R1 respectively, $\int \langle \Theta \rangle dz \approx 0.06$, with smaller values obtained for the stronger field cases R2 and R6. This quantity has dimensions of length and so can be interpreted as the effective width of a completely mixed region (for which $\langle f_h f_i \rangle = 1$). The volume averaged value $\langle \Theta \rangle_{V} = \int \langle \Theta \rangle dz / L \approx 0.6$, which can be interpreted as the fractional height of the domain over which mixing occurs. Thus, not only is the mixing local to the initial interface larger for hydrodynamics compared to MHD, but also even in an integral sense the effective width is reduced.

We have found the variance of the density is also a sensitive diagnostic of the mixing region. At each vertical position $z$, we compute the horizontally averaged density $\langle \rho \rangle$, where the $\langle \rangle$ denotes an average over a horizontal plane as in eq. 11. The variance is then

$$\delta \rho (z) = (\langle \rho - \langle \rho \rangle \rangle^2 / 2)^{1/2}$$  

(16)

Regions which are well mixed have a smaller variance. If the heavy and light fluids remain completely unmixed (that is, if the density at every location can only be either 3 or one) then the largest value of the variance is $\delta \rho / \langle \rho \rangle = 1 / \sqrt{3} \approx 0.57$ and occurs for $\langle f_h \rangle = 1 / 4$.

Figure 11 plots the vertical profile of $\delta \rho / \langle \rho \rangle$ in Runs RH, R2, and R6 at $t/t_\ast = 56$. Both the weak and strong field runs are similar to each other, and have a much larger variance than the unmagnetized case, indicating less mixing. For $\langle f_h \rangle = 1 / 2$, which from a visual inspection of the horizontal slice plane in Figure 5 is a rough approximation to the volume fraction of the heavy fluid there, the ideal case of no mixing gives $\delta \rho / \langle \rho \rangle = 1 / 2$. From Figure 11 the peak values in runs R2 and R6 approach this value, an indication that very little mix occurs in the MHD RTI. Note from Figure 9 that $\langle f_h \rangle = 1 / 4$ occurs at $Z/H \approx -0.4$. At the time of this plot, the height of the bubbles $H \approx 0.7$, thus we should expect the peak in the variance to occur at $z \approx -0.03$, which is in good agreement with Figure 11. Thus, the asymmetry in the variance (with larger values occurring for negative $z$) is a consequence of the lower $\langle f_h \rangle$ there.
To investigate the dynamical processes that can lead to reduced mixing in the MHD RTI compared to hydrodynamics, we have calculated the distribution of the amplitude of the vorticity $|\mathbf{W}| = |\nabla \times \mathbf{v}|$. Figure 12 plots the number of cells $N$ in runs RH and R1 which have a given value of the amplitude of vorticity, $N(|\mathbf{W}|)$, versus $|\mathbf{W}|$. Clearly there is a shift in the distribution to lower values of $|\mathbf{W}|$ in the weak field run R1 (dotted line). Large values of the vorticity are associated with lower motion implied by the change in the distribution $N(|\mathbf{W}|)$ in the weak field run RH (vertical line). The shift in the distribution to lower $|\mathbf{W}|$ with weak fields is an indication that even if the critical wavelength $\lambda_c$ is not resolved in the initial conditions (R1), the tension forces associated with small scale bending of the field lines is sufficient to affect the flow, and suppress the shear. We postulate that the suppression of small scale shearing motion implied by the change in the distribution $N(|\mathbf{W}|)$ is responsible for the reduced mixing observed in the MHD RTI as shown by figures 10 and 11.

### 3.4. Evolution of the magnetic field

To investigate the three-dimensional structure of the magnetic field, Figure 13 plots slices of the magnetic energy in the fluctuating part of the field,

$$
\delta B^2 / 2 = (B_z - B_0)^2 / 2 + B_y^2 / 2 + B_x^2 / 2
$$

(17)

at the edges of the domain, and at the midplane $z = 0$ in runs R2 and R6 at $t/t_i = 60$. The slices are made at the same locations and the same times as the slices of the density shown in the right hand panels of figure 5. There is a direct correspondence to the structures visible in the magnetic field and the density. The fluctuations in $\delta B^2$ occur at smaller scales in R2 in comparison to R6. Most of the magnetic energy is concentrated in rope and sheet-like structures associated with the bubbles and fingers in the density. The maximum of the magnetic energy is quite similar in the two plots (the maximum is 0.021 in R2 and 0.015 in R6), despite the fact the energy in the background field is nearly an order of magnitude larger in R6. In fact, at $t/t_i = 60$, the energy in fluctuations is larger in the weaker field run. As discussed below, this is likely due to the fact that the fingers and bubbles are larger in R2 at this time, thus more gravitational energy has been released that can be tapped to amplify the magnetic field.

Figure 14 is a plot of the the vertical profile of the horizontally averaged magnetic energy at two times in both runs R2 and R6. The energy in the vertical component $\langle B_z^2 / 2 \rangle$ is separated from the horizontal components $\langle (B_x - B_0)^2 / 2 + B_y^2 / 2 \rangle$. The energy in the vertical field always dominates, a consequence of the dominance of vertical motions. The amplitude of the energy in the vertical field is remarkably similar between the two runs; at $t/t_i = 56$ it is about 0.009 in R2 and 0.0075 in R6. This is consistent with the amplitude of the energies being similar in the slices shown in figure 13. The total energy in fluctuations (the integral over vertical height of the lines plotted in figure 14) must therefore be similar between the two runs; we discuss this further below. The ratio of energy in the vertical versus horizontal fields is larger in the weak field case R2.

Simple energy arguments can be used to interpret the amplitude of the magnetic energies observed in figures 13 and 14. The gravitational potential energy released by descending heavy fluid is converted into kinetic energy by the RTI and secondary KH instabilities. In turn, these result in twisting and amplification of the magnetic field. Some kinetic and magnetic energy is converted into internal energy by viscous and resistive dissipation. Although our simulations do not include explicit dissipation, our method conserves total energy exactly so that whatever energy is lost by numerical viscosity and reconnection is explicitly captured as an increase in internal energy. The simplest expectation is that the kinetic and magnetic energies will remain in approximate equipartition, and that the amount of energy released when the tip of the fingers reaches the same height (as measured by the vertical location where $\langle f_h \rangle = 0.995$) will be the same amongst all runs. To test these ideas, table 2 compares the volume averaged energies from all the runs when the tips of the fingers reaches $z/H = 0.5$. The third column gives the time at which this happens, the fourth is the magnetic energy in fluctuations, the fifth is the kinetic energy, the sixth is the change in internal energy at that time. The seventh column is the total change in energy, and the last is the amplification factor of the magnetic energy in fluctuations. Since the data is collected not at the same physical time, but at the point where the fingers have reached the same height (and therefore the same amount of gravitational energy has been released), then the total of the kinetic, magnetic, and thermal energies should be the same amongst all the runs. Indeed, this expectation is confirmed, the difference in total energy is only 15% amongst all the simulations. Note also the magnetic and kinetic energies are in rough equipartition, regardless of the initial field strength. The increase in internal energy is a non-negligible contribution to the total change, especially in the weaker field simulations. In fact the total gravitational potential energy released depends on the detailed distribution of density in the interface region and not just the location of the tip of the fingers, thus we do not expect the total energy to be identical between the different runs.

These same energy arguments can be used to predict the time evolution of the magnetic and kinetic energies. The gravitational energy released depends on the total mass and the distance it drops, both of which are proportional to the height of the bubbles $h$. Thus, the energy released $E \propto \dot{m}^2 \propto t^4$. Figure 15 plots each component of the magnetic and kinetic energies in runs R1, R2, R4, and R6 versus $t^4$. Our expectation is that each should be a straight line, with rough equipartition between the transverse magnetic and kinetic energies, and with the vertical components dominating. This expectation is clearly borne out by the figure, at least so long as the energy in the vertical component of the field $B_z^2 / 2$ (which grows the fastest) is less than the magnetic energy associated with the initial field $B_0^2 / 2$. Thus, during the evolution of strong field case R6 (figure 15a), the ratio $B_z^2 / B_0^2$ is always less than one, and the growth of each component of the energy closely follows $t^4$. With increasingly weaker fields, the amplification of $B_z^2 / B_0^2$ becomes larger and larger, and the time evolution of each component diverges farther and farther from $t^4$. For the very weak field case, R1 (figure 15d), the ratio $B_z^2 / B_0^2 = 1$ is reached very early in the evolution,
and the maximum amplification at late time is more than 15. In this case, the time evolution of the energy is lower than $t^4$. As discussed in the next section, we expect deviation from the simple $t^4$ scaling when the bubbles and fingers have moved a distance much larger then $\lambda_c$.

4. SUMMARY AND DISCUSSION

We have studied the RTI in ideal MHD and full three dimensions. We have restricted our analysis to uniform fields parallel to the interface. We study the high-$\beta$ regime, where the energy density in the magnetic field is small compared to the thermal energy in the fluid. Nonetheless, we study strong fields in the sense that the initial field strength $B_0 \lesssim B_c$, where $B_c$ is the critical field strength at which all modes parallel to $B$ are completely suppressed. We use numerical methods that have been validated in the sense that they reproduce the growth rate of fingers and the amount of mixing between light and heavy fluids in the hydrodynamic RTI, as reported in previous high resolution numerical simulations\(^3\), and laboratory experiment\(^1,3\).

Uniform magnetic fields do not suppress the RTI, but rather make modes strongly anisotropic. Along the field, the growth rate of modes is reduced, and wavelengths below a critical value are stable. Perpendicular to the field, the dispersion relation of the magnetic RTI is identical to the dispersion relation in hydrodynamics. Even if the field is made arbitrarily strong to suppress all modes parallel to $B$, the density interface will still be RTI unstable in 3D due to the interchange modes perpendicular to $B$. It is therefore critical to study the MHD RTI in full three dimensions.

Although uniform magnetic fields can not suppress the RTI in three dimensions, they significantly change the nonlinear evolution and structure. Even weak fields reduce the mix between light and heavy fluids, resulting in fingers and bubbles which rise much more quickly compared to the hydrodynamics case. Such fields are weak in the sense that $\lambda_c / L$ is small, however they still are strong enough to influence the flow through tension forces at small scales, as evidenced by the change in distribution of the vorticity between very weak field, and purely hydrodynamical, simulations (figure 12). A turbulence mixing zone is not produced with strong fields. Instead, at early times the bubbles and fingers are elongated along $B$, forming flux ropes and tubes. Fluid drains along these tubes, pooling as bubbles at the tips and valleys, eventually forming the usual bubble and finger morphology. Interchange instabilities wrinkle the surfaces of the bubbles perpendicular to the field. Several diagnostics, including the variance of the density, that are good diagnostics of the amount of mixing between heavy and light fluids show that there is a monotonic decrease in mixing with increasing field strength.

The evolution of the MHD RTI follows the same self-similar evolution as in hydrodynamics. The vertical profile of the volume fraction of heavy fluid is self-similar. The vertical extent of the bubbles and fingers $h \propto t^2$. Simple energy arguments suggest the magnetic and kinetic energies should grow in time as $t^4$, and our results confirm this expectation. The energy in transverse motions and field are always in rough equipartition.

The total energy in fluctuations is independent of the initial field strength, but depends only on the extent of the bubbles and fingers (which in turn determines the amount of gravitational energy released that is available for field amplification). Since the magnetic energy density in the mixing layer grows to the same value in every case (determined by the amount of gravitational binding energy released by descending heavy fluid and the thickness of the mixing layer), this leads to a much larger fractional increase in magnetic energy for initially weak fields (more than a factor of 10). However, this dynamo action cannot lead to strong fields with $\beta \lesssim 1$, since the total magnetic energy is limited to equipartition with the kinetic rather than thermal energy, and the flows induced by the RTI are highly subsonic. One explanation for the larger increase in magnetic energy with initially weak fields is that the critical wavelength at which the RTI is stable is smaller for weak fields, thus weak fields can be folded, twisted, and amplified on smaller scales than strong fields.

The self-similar evolution of the magnetic and kinetic energies diverges from the simple expectation above once the magnetic energy in the vertical component of the field exceeds that associated with the original field, i.e. $B_z^2/B_0^2 \gtrsim 1$. This occurs when the bubbles and fingers have moved a large distance compared to $\lambda_c$. The evolution of the energies in the strong field simulations described here (R6), which has $\lambda_c/L = 0.36$, closely follows a $t^4$ scaling throughout. However, for the very weak field simulation (R1), which has $\lambda_c/L = 0.01$, this scaling is broken at early times. This difference can be interpreted as being due to the very different dimensionless length and time scales, $\lambda_c/L$ and $\sqrt{\lambda_c/g} / t_r$ respectively, in each case. If the very weak field case R1 were repeated with identical parameters but in a computational domain of size $L/36$, then it would have the same ratio $\lambda_c/L$, and it would evolve identically to the strong field case R6 on a time scale which is 6 times shorter. Similarly, if the strong field case R6 were repeated in a computational domain 36 times larger, it would be identical to run R1 at times a factor of 6 longer than R1. Thus, run R1 samples a much later stage of evolution of the magnetic RTI than R6.

We conclude the self-similar evolution which predicts energy growth at $t^4$ is only applicable for $t \leq \tau_c = \sqrt{\lambda_c/g}$. Although we have reported simulations of the magnetic RTI with different field strengths and the same sized computational domain $L$ in this paper, this is equivalent to simulations with the same field strength but with different sized computational domains.

The fact that the saturated magnetic energy is independent of the initial field strength provides a simple explanation for the break in the $t^4$ scaling of the magnetic energy at late times $t > \tau_c$. In this case, since the magnetic field strength and magnetic energy density $B^2$ reaches a constant value in the mixing layer, then the total energy will increase further only because of the thickening of the mixing layer, thus in the saturated regime $E \propto h \propto t^2$.

The MHD RTI is relevant to a number of astrophysical systems, for example density interfaces in the ISM, to the formation of filaments in the Crab nebulae\(^5\), and to the contact discontinuity in supernovae blast waves\(^7\). As discussed above, a uni-
form field will not suppress the RTI in these systems, nor even reduce the growth rate of modes perpendicular to $B$. It will, however, result in highly anisotropic structures, and reduce the mixing between fluids. As reported elsewhere (Stone & Gardiner 2007, submitted to the Astrophys. J.), we have also found that rotation of the direction of the field with vertical position (which is equivalent to currents parallel to the interface) significantly affects the nonlinear evolution of the the MHD RTI.

The MHD RTI is also relevant to Z-pinch experiments, where implosion is driven by low density, highly magnetized plasma ablated from wires. Non-ideal MHD effects (resistivity, and Hall currents) are important at the densities and temperatures realized in the plasma in these experiments. Studies of the MHD RTI including resistive dissipation and Hall currents in a cylindrical geometry with higher $\beta$ are needed.

We thank John Hawley for useful discussions, and for assistance in running the simulations, and the anonymous referees for suggestions that improved the paper. Simulations were performed on the Teragrid cluster at NCSA, the IBM Blue Gene at Princeton University, and on computational facilities supported by NSF grant AST-0216105. Financial support from DoE grant DE-FG52-06NA26217 is acknowledged.

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### Table 1
**Parameters of Runs**

| Run | $B/B_c$ | $\lambda_c/L$ | $\lambda_{\text{max}}/L$ | $\lambda_c/\Delta x$ |
|-----|---------|----------------|--------------------------|----------------------|
| $RH$ | 0.0     | -              | -                        | -                    |
| $R1$ | 0.1     | 0.01           | 0.02                     | 2.56                 |
| $R2$ | 0.2     | 0.04           | 0.08                     | 10.2                 |
| $R4$ | 0.4     | 0.16           | 0.32                     | 41.0                 |
| $R6$ | 0.6     | 0.36           | 0.72                     | 92.2                 |

### Table 2
**Volume Averaged Energies at $h/L = 0.5$.**

| Run | $b_0/b_c$ | $t/t_s$ | $(b^2 - b_0^2)/2$ | $\rho v^2/2$ | $e - e_0$ | total | $\delta b^2/b_0^2$ |
|-----|-----------|---------|-------------------|--------------|-----------|-------|-------------------|
| $RH$ | -         | 53      | -                 | 1.6          | 2.5       | 4.1   | -                 |
| $R1$ | 0.1       | 46      | 0.85              | 1.6          | 2.7       | 4.4   | 10.6              |
| $R2$ | 0.2       | 42      | 1.0               | 1.3          | 2.1       | 4.4   | 3.2               |
| $R4$ | 0.4       | 42      | 1.2               | 1.4          | 2.1       | 4.7   | 0.93              |
| $R6$ | 0.6       | 49      | 1.1               | 1.2          | 2.2       | 4.5   | 0.37              |
Fig. 1.— Images of the mixing parameter $\Theta$, defined in equation 12, in the two-dimensional version of run R6 at $t/t_s = 30.0$ at resolutions of $32 \times 64$ (left), $64 \times 128$ (middle left), $128 \times 256$ (middle right), and $256 \times 512$ (right). The color table runs blue to red (online version) over the range zero to one, with white corresponding to $\Theta = 0$. 
Fig. 2.— Time evolution of the volume-averaged mixing parameter at different resolutions for the two-dimensional version of the strong field run R6. Each curve is labelled by the number of grid points per $L$. 
Fig. 3.— Images of the mixing parameter $\Theta$ at $t/t_c = 30.0$, in two-dimensional version of runs R6 (left, strong field), R2 (middle left, intermediate field), R1 (middle right, weak field), and RH (right, hydrodynamic), all at a resolution of $256 \times 512$. The color table runs blue to red (online version) over the range zero to one, with white corresponding to $\Theta = 0$. 
FIG. 4.— Time evolution of the volume-averaged mixing parameter for runs R6 (strong field), R2 (intermediate field), R1 (weak field) and RH (hydrodynamic). The rapid increase in the mixing rate occurring at $t/t_s = 20$ in runs with weaker fields is due to the formation of KH rolls, as is evident in figure 3.
Fig. 5.— Slices of the density at $t/t_0 = 29.6$ (left panels) and $t/t_0 = 60.0$ (right panels) in runs RH (top, hydrodynamic), R2 (middle, weak field), and R6 (bottom, strong field). Note the decrease in mixing in the MHD RTI (as evidenced by reduced volume at intermediate densities, i.e. grey regions), and the elongation of structures along the magnetic field (z-axis) in the strong field case. (Online version in color.)
Fig. 6.— Isosurfaces of the density at $\rho = 2.9$ and $\rho = 1.1$ at $t/t_s = 56.0$ in runs RH (left, hydrodynamic) and R2 (right, weak field). Also shown are slices of the density at the edges of the domain. (Online version in color.)
Fig. 7.— Isosurfaces of the density at $\rho = 2.9$ and $\rho = 1.1$ at $t/t_s = 20.0$ (left) and $t/t_s = 56.0$ (right) in run R6 (strong field). Also shown are slices of the density at the edges of the domain. (Online version in color.)
FIG. 8.—Height of bubbles \((z > 0)\) and fingers \((z < 0)\) as a function of time in runs RH (solid line, hydrodynamic), R2 (dashed line, weak field) and R6 (dotted line, strong field). The height is defined as the location where the horizontally averaged mass fraction (eq. 11) is 0.985 and 0.015 respectively.
FIG. 9.— Vertical profile of the mass fraction $\langle f_h \rangle$ defined by eq. 11 in runs RH (dashed line, hydrodynamic) and R6 (solid line, strong field) at $t/t_s = 56$. The horizontal axis has been scaled by the height of the bubbles at that time.
Fig. 10.— Vertical profile of the horizontally averaged mixing parameter $\langle \Theta \rangle \equiv 4\langle f_i f_j \rangle$ at time $t_0 \approx 56$ in runs RH (solid line, hydrodynamic), R1 (dotted line, very weak field), R2 (dot-dash line, weak field), and R6 (dashed line, strong field). As measured by $\langle \Theta \rangle$, there is monotonically less mixing with increasing field strength.
Fig. 11.— Vertical profile of the variance of the density defined by eq. 13 in RH (solid line, hydrodynamic), R2 (dotted line, weak field) and R6 (dashed line, strong field). Both of the MHD runs show a significantly larger variance than the hydrodynamic case indicating less mixing.
Fig. 12.— Distribution of the amplitude of the vorticity $W$ in runs RH (solid line, hydrodynamic) and R1 (dotted line, very weak field) at $t / t_s = 40$. The vertical axis is the number of cells with the given value of $W$. The horizontal axis is scaled by the maximum of $W$ in run RH.
Fig. 13. — Slices of the magnetic energy in fluctuations, defined by eq. 14, at $t/t_s = 60$ in runs R2 (left, weak field) and R6 (right, strong field). The slices are taken at the same locations and at the same time as the right-hand panels in figure 1. (Online version in color.)
Fig. 14.— Profiles of the horizontally averaged magnetic energy in fluctuations for runs R2 (top, weak field) and R6 (bottom, strong field). The dashed lines in each plot correspond to the energy in the vertical component of the field $B^2_z$, while the solid lines correspond to the energy in the horizontal components of the field $(B_x - B_0)^2 + B_y^2$. The profiles are shown at $t/t_s = 28$ and $t/t_s = 56$ in each plot, the latter pair of lines extend over a larger horizontal range in both plots.
Fig. 15.—Time evolution of each component of the magnetic and kinetic energies in runs (a) R6, strong field, (b) R4, intermediate field, (c) R2, weak field, and (d) R1, very weak field. Solid lines show magnetic energy, dashed lines are kinetic. Each line is labeled by the associated field component. The energy in perturbations is shown for the $x$–component of the magnetic field, $\delta B_x^2/2 = (B_x^2 - B_0^2)/2$, and in each panel the values are scaled by the initial magnetic energy $B_0^2/2$. 