Identification and estimation of acoustic signals parameters in telecommunication systems of audio exchange

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Abstract. The paper studies the formation of models of stationary signals by linear systems with constant parameters and nonlinear systems described by second-order and third-order functional series. It is also shown that the nonstationary nature of signals, in general, can be represented by a system with variable parameters. The equations obtained in the work solve the problem of identifying the parameters of a model of a non-stationary signal with a time-varying mean value and variance. The object of the study are stationary and nonstationary processes and their statistical characteristics. The aim of the work is to develop methods for identifying and evaluating process parameters describing the acoustic signal in telecommunication systems.

1. Introduction

Solving the problem of identifying the parameters of stationary and non-stationary acoustic signals makes it possible to develop algorithms for processing and compressing signals for transmission over telecommunication channels and in audio exchange systems.

2. Estimates of the characteristics of stationary acoustic signals in telecommunications

Characteristics of stationary signals include correlation functions, initial and centered moments of high orders, seven invariants (or cumulants), as well as their spectra - first-order and high-order spectra. Other characteristics are also known, such as the one-dimensional distribution function, as well as Kullback information, entropy and noise quality factor [1].

By the form of a one-dimensional distribution function, for example, it is possible to estimate the nonlinear distortions in the transmission channels, as well as those arising from the conversion of acoustic energy into an electrical signal. This is understandable if we take into account the differences in the nonlinear and inertial characteristics of the converters.

Similarly, it can be expected that the covariance functions of the signals at the outputs of different transducers are not required to be described by the same functions. In this case, the nonlinearity of the converter can manifest itself particularly at a high level of interference. The change in the characteristics of the output signal is also due to the movements of the converter relative to the source of the acoustic signal. In any case, regardless of the cause, the possible influence of the characteristics of the converter on the results of processing the acoustic signal should not be left without attention.

As a measure of the deviation of the distribution of a random variable \( x \) characterized by the probability density \( f(x, \varphi) \) from the distribution with density \( f(x, \theta) \), Kullback information is often used [2].
\[ I(\varphi, \theta) = E_\varphi \left[ \log \frac{f(X, \varphi)}{f(X, \theta)} \right] = \int f(x, \varphi) \log \frac{f(x, \varphi)}{f(x, \theta)} \, d\mu(x). \] (1)

This function is equal to zero if the probability densities \( f(x, \varphi) \) and \( f(x, \theta) \) on a measure \( \mu(x) \) coincide. In other cases \( I(\varphi, \theta) > 0 \). If the random variable \( x \) with independent values at the output of the linear converter has a probability density \( f(x, \theta) \), and at the output of the nonlinear converter there is a density \( f(x, \varphi) \), then the Kullback information

\[ I(\varphi, \theta) = \int f(x(\xi), \theta) \left| \frac{dx(\xi)}{d\xi} \right| \log \left| \frac{dx(\xi)}{d\xi} \right| \, d\xi. \] (2)

It is assumed here that a random variable \( \xi = \xi(x) \) at the output of a nonlinear converter is a monotonic nonlinear output function of a linear converter. Accordingly, \( x(\xi) \) – this is the inverse function.

As a measure of deviation from the normal distribution, an expression of the form

\[ H = -\int f(x, \theta) \log f(x, \theta) \, dx, \] (3)

called the entropy of a random variable [3]. Its maximum value attains the entropy on the Gaussian distribution \( f(x, \theta) \).

By means of entropy, the noise quality factor

\[ \eta = \frac{1}{2\pi} e^{2H(x)}, \] (4)

which satisfies the inequality \( \eta \leq 1 \) if the random variable \( x \) deviates from the Gaussian distribution.

Moments and high-order spectra allow us to estimate the asymmetry of distributions, to isolate non-Gaussian signals against Gaussian noise, since for Gaussian noise all the second-order and higher-order semi-invariants are zero. The corresponding spectra of a high order are also zero.

As a measure of the proximity of the distribution of experimental data to the adopted model, various agreement criteria are also used. However, due to the complexity of the implementation, they are usually limited to only some distribution parameters, such as those listed above. In order to identify the distributions, the moment functions are also used, including moments and spectra of the second and higher order [4]. Traditional is the use in assessing the accuracy of models of mathematical expectations and variances.

Electroacoustic transducers are characterized, as a rule, by non-uniform amplitude-frequency characteristics. In Figure 1, these characteristics are represented by a transfer function \( H_1 \).

**Figure 1.** Structure of the converter with an inverse correction system.

The transfer function \( H_2 \) characterizes the linear distortion of the signal \( x(t) \) transmitted against the background of the interference \( n(t) \). Therefore, under certain conditions, the converter should be considered as a nonlinear inertial system, in which it is possible to describe the processes by means of nonlinear differential equations. Also this converter system can be represented in the form of a nonlinear Volterra system [5]. Accordingly, the inverse system can also be found in the class of
Volterra nonlinear systems described by Volterra functional series. In this case, the nonlinear system of the converter in Fig. 1 can be described quite fully by the Voltaire functional series of the second order in the form

$$x(t) = \sum_{k=0}^{K} W_k(y, t), \quad (5)$$

where $K$ - is the order of the system, and the term $W_k(y, t)$ has the form

$$W_k(y, t) = \int_{t_0}^{t} \int_{t_0}^{t} w_k(t, \tau_1, \Lambda, \tau_k) y(\tau_1) \Lambda y(\tau_k) d\tau_1 d\tau_k. \quad (6)$$

An example of a second-order Volterra nonlinear system is shown in the figure. 2.

**Figure 2.** Structure of a second-order Volterra nonlinear system.

In Figure 2, the impulse functions $F_k$ correspond to the transfer functions $f_k(t)$. Moreover, in the expansion (5) all the terms with the index $k > 2$ are assumed to be zero. The expression (6) with the value $k = 2$ has the form

$$W_2(y, t) = \int_{t_0}^{t} \int_{t_0}^{t} f_2(t-\theta) f_1(0-\tau_1) f_2(\theta-\tau_2) y(\tau_1) y(\tau_2) d\tau_1 d\tau_2 d\theta. \quad (7)$$

Or, taking into account that since $f_k(t) = 0$, if $t < 0$, then

$$W_2(y, t) = \int_{t_0}^{t} \int_{t_0}^{t} f_2(t-\theta) f_1(0-\tau_1) f_2(\theta-\tau_2) y(\tau_1) y(\tau_2) d\theta d\tau_1 d\tau_2. \quad (8)$$

The Volterra kernel in expression (8) can be written in the form

$$w_2(t, \tau_1, \tau_2) = \int_{t_0}^{t} f_3(t-\theta) f_1(0-\tau_1) f_2(0-\tau_2) d\theta. \quad (9)$$

This approach can in particular be used to take into account nonlinear distortions in communication channels.

For discrete systems, expression (6) can be written in the form

$$F_k(y, t) = \sum_{n_1=1}^{L} \sum_{n_k=1}^{L} h_k(t, n_1, \Lambda, n_k) y(n_1) \Lambda y(n_k). \quad (10)$$

The task of identifying such systems, consisting in determining the coefficients $h_k(t, n_1, \Lambda, n_k)$, can be considered in this case an equivalent problem of multidimensional regression [6].

3. Estimation of non-stationary signals in telecommunication audio exchange systems

By their nature, nonstationary processes are divided into classes [6,7]:

1) with a variable in time mean value -

$$y(t) = a(t) + u(t), \quad E[y(t)] = a(t) \quad (11)$$

2) with a time-varying variance -
\[ y(t) = b(t)u(t), \quad E\{y(t)\} = 0, \quad E\{y^2(t)\} = b^2(t)E\{u^2(t)\} \]  
(12)

3) with variable frequency structure -
\[ y(t) = u(\varphi(t)), \]  
(13)

4) a periodically stationary or cyclo-stationary process is a process whose characteristics on intervals \([kT, (k+1)T]\) vary according to the same laws,
5) the process formed by the dynamic system -
\[ \frac{dx(t)}{dt} = A(t)x(t) + B(t)u(t), \quad y(t) = c(t)u(t), \quad x \in R^n, \quad u \in R^m, \quad y \in R. \]  
(14)

Almost periodic and cyclical processes are known, with which all possible situations are not exhausted. In information systems, the stationarity intervals of a process may depend on the nature of the transmitted data in such a way that the statistical characteristics of the process will be synchronized with the data.

A similar situation has, for example, a place in the case of an acoustic speech signal, which is stationary at intervals caused by the transmitted information. Something similar can apparently be observed in the transmission of information through dynamic chaos.

Estimates of mathematical expectation, variance and correlation functions are also used in problems of detecting changes in the properties of signals and dynamical systems. In particular, they are used for segmentation of an acoustic speech signal. In similar problems, we can assume that the character of the nonstationarity signal refers to the first or second class. The necessary estimates can be obtained by the method of least squares [8,9].

Deterministic functions \(a(t)\) and \(b(t)\) non-stationary processes \(y(t) = a(t) + b(t)u(t)\) are separately found by the method of least squares as a function of linear regression
\[ a(t) = \sum_{k=1}^{n} \alpha_k \varphi_k(t) = \varphi^T(t)\alpha \quad \text{and} \quad b^2(t) = \sum_{k=1}^{n} \beta_k \varphi_k(t) = \varphi^T(t)\beta. \]  
(15)

Here vectors \(\varphi^T(t) = (\varphi_1(t) \Lambda \varphi_n(t))\), \(\alpha = (\alpha_1 \Lambda \alpha_n)^T\) and \(\beta = (\beta_1 \Lambda \beta_n)^T\) are introduced. The coefficient vector \(\alpha\) is, provided that the function \(b(t)\) is known, as a result of minimizing the loss function
\[ Q(\alpha) = \frac{1}{2} \sum_{k=0}^{N} \frac{1}{b^2(t_k)} \left(y(t_k) - \varphi^T(t_k)\alpha\right)^2. \]  
(16)

If we introduce a matrix \(\Phi = (\varphi(t_0) \varphi(t_1) \Lambda \varphi(t_N))\), a vector of observable data \(y = (y(t_0) \ y(t_1) \Lambda y(t_N))^T\), and a diagonal matrix \(P = diag\left(\frac{1}{b^2(t_0)} \frac{1}{b^2(t_1)} \Lambda \frac{1}{b^2(t_N)}\right)\), then the loss function can be written in the form
\[ Q(\alpha) = \frac{1}{2} \left(y - \Phi^T\alpha\right)^TP(y - \Phi^T\alpha). \]  
(17)

Then the coefficient vector ensuring the minimum of this function is found from expression
\[ \alpha = \left(\Phi P \Phi^T\right)^{-1} \Phi Py. \]  
(18)

Similarly, if the expectation \(a(t)\) is known, then the coefficient vector \(\beta\) of the function \(b^2(t)\) is found from the condition of the minimum of the function
\[ Q(\alpha) = \frac{1}{2} \sum_{k=0}^{N} \left(\frac{(y(t_k) - a(t_k))^2}{\sigma_u^2} - \varphi^T(t_k)\beta\right)^2. \]  
(19)
To extend this approach to the case when both functions $a(t)$ and $b(t)$ are unknown, it is suggested to use the method of successive approximations, based on the sequential one, until the specified accuracy is obtained, the calculation of these functions. In the same place, recursive algorithms for calculating the coefficients vectors $\alpha$ and $\beta$, as well as statistical characteristics of the solutions obtained, are considered.

Since the solutions obtained in this way are effective only on limited time intervals, the problem can be supplemented by the conditions of conjugation of individual local solutions, for example, by the conditions of smooth conjugation [10, 11, 12]. When solving this problem, one can use the recursive algorithm that provides updating of the regression coefficients as the sliding window of the final data set is displaced. In principle, this approach is more consistent with the task of processing non-stationary signals than the update algorithm as the sample size increases [13,14].

However, the solution of problem (19) does not guarantee that the function $b^2(t) = \Phi^T(t)\beta$ will be nonnegative on the interval of its definition and, accordingly, that this method allows us to evaluate the function $b(t)$. Therefore, the problem of finding a function $b(t)$ must be modified. For example, to formulate it as a task of minimizing the objective function (19) with an additional constraint $\Phi^T(t)\beta \geq 0$.

4. Conclusion

It is shown that linear systems with constant parameters and nonlinear Volterra systems can be used as forming models of stationary signals. The method of least squares is also a common means of identifying systems. In this regard, its application in the identification of dynamic, described by differential or difference equations, acoustic signal model also looks natural. If we assume that, for example, a room in which an acoustic signal can propagate is characterized by a certain number of vibration modes or resonance frequencies, then the identification task will in particular be the estimation of the number and parameters of these modes. It is also shown that the non-stationary character of a signal can have a different description—in the form of a time-varying mathematical expectation, variance or frequency, and in general can be represented by a system with variable parameters. The equations obtained in this paper generally solve the problem of identifying the parameters of a model of a non-stationary signal with a time-varying mean value and a time-varying variance.

The developed algorithms for estimating the characteristics of processes can, in particular, be used to control the parameters of technical means for the acoustic signals emitted by them. The tasks of estimating the parameters of signals and identifying systems with variable slowly varying parameters provide, respectively, signal segmentation, their classification according to the level of activity and the possibility of adaptation, and the determination of the level of suppressed or compensated interference.

5. References

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