FeO2: Federated Learning with Opt-Out Differential Privacy

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Abstract

Federated learning (FL) is an emerging privacy-preserving paradigm, where a global model is trained at a central server while keeping client data local. However, FL can still indirectly leak private client information through model updates during training. Differential privacy (DP) can be employed to provide privacy guarantees within FL, typically at the cost of degraded final trained model. In this work, we consider a heterogeneous DP setup where clients are considered private by default, but some might choose to opt out of DP. We propose a new algorithm for federated learning with opt-out DP, referred to as FeO2, along with a discussion on its advantages compared to the baselines of private and personalized FL algorithms. We prove that the server-side and client-side procedures in FeO2 are optimal for a simplified linear problem. We also analyze the incentive for opting out of DP in terms of performance gain. Through numerical experiments, we show that FeO2 provides up to 9.27% performance gain in the global model compared to the baseline DP FL for the considered datasets. Additionally, we show a gap in the average performance of personalized models between non-private and private clients of up to 3.49%, empirically illustrating an incentive for clients to opt out.

1 Introduction

The abundance of data and advances in computation infrastructure have enabled the training of high-quality machine learning models. On the other hand, the data is distributed over many devices that are typically power-constrained and have limited computational capabilities. To reduce the amount of data transmission over networks and maintain the privacy of raw data, [1] proposed the federated learning (FL) framework for training a central server-side model using decentralized data at clients. See the recent surveys [2]; [3] for more. Federated learning frameworks aim to train a global model iteratively and collaboratively using clients’ data. During each round, the server has access to a select number of clients, each of whom has a local dataset. The server broadcasts the current model to such clients, who train the model by taking gradient steps using their local data on the model and return the gradient-based update back to the server. The server then aggregates the updates and produces the new global model for the next round.

Despite the clients’ data being kept on device in federated learning, the deployed model at the central server is still vulnerable to various privacy attacks, such as membership inference attacks, model inversion attacks, and others. To resolve this issue, many works in the literature propose privacy-preserving variations of federated learning algorithms. One approach to privacy-preserving FL algorithms utilizes differential privacy to provide privacy guarantees. Differential privacy is a widely studied and accepted mathematical notion that describes privacy-preserving algorithms where the information leakage of private data is bounded. Differential privacy is defined as follows

Definition 1 (differential privacy (DP) [4]). A randomized algorithm $A(\cdot)$, whose image is denoted as $O$, is said to be $(\epsilon, \delta)$-DP if for any two inputs $D_1$ and $D_2$ that differ in just one entry, and all
In federated learning, instead of targeting the privacy of individual samples of a client, one can apply client-level differential privacy by having the adjacent datasets describe the case where the data removed is all the samples associated with a single client \[^5\]. It is worth noting that using differential privacy in federated learning causes unavoidable degradation in performance. Although the trained global model in FL can perform well on average, it is not straightforward to guarantee the global model performs well for all clients on their local data, especially when considering heterogeneity in client data. This has encouraged many different works in the literature to obtain global models that work well for all clients or personalization techniques for federated learning setups to improve the performance of models at clients.

### 1.1 Contributions

This work examines heterogeneity in privacy requirements in federated learning setups. It considers a new setup of privacy-preserving federated learning where privacy is enabled by default and the clients may choose to opt out. We desire to understand the implications of the setting where a fraction of the clients choose to opt out of privacy, even if they represent a small percentage of the overall population, to improve the performance of the global model as well as the personalized local models. Specifically,

1. We propose a new setup for privacy in federated learning frameworks. The proposed setup considers heterogeneity in privacy choices of clients in FL. Instead of enabling privacy for all clients, each client is by default opted in privacy, and given the option to opt out, making it easier for clients to decide their privacy choices. We consider the client-level notion of privacy for the set of private clients rather than the entire set of clients.

2. To address this heterogeneity in privacy requirements, we propose the federated learning with opt-out differential privacy algorithm, referred to as FeO2. The FeO2 algorithm is designed to take advantage of the newly introduced setup to improve the performance of the global model. FeO2 comes with a personalization add-on, where we employ a recently introduced algorithm, known as Ditto \[^6\], within FeO2 to examine the effect of FeO2 on the local models.

3. We theoretically investigate FeO2 in a simplified setup of federated point estimation, first introduced by \[^6\]. We show that the FeO2 algorithm can significantly improve the estimation noise variance at the server while providing better personalized estimates at clients. We also establish the Bayes optimality of FeO2 when combined with Ditto.

4. Finally, we provide experimental results of the FeO2 algorithm using various synthetic and realistic federated datasets from TensorFlow Federated (TFF) \[^7\].

### 1.2 Related Work

Federated learning has been garnering a huge amount of efforts in the literature over the past few years. Works on federated learning algorithms have been proposed in the literature to overcome various issues that arise in realistic federated learning setups, e.g., \[^8\]–\[^17\] to name a few. In federated learning, the data generated by clients is not considered to be independent and identically distributed (IID) as in distributed or centralized learning setups; therefore, the resulting model at the server may not necessarily perform well on the local data at each client. Personalization is a solution to this issue. By modifying algorithms to provide clients with better models that perform well on their local data, see for example, \[^18\]–\[^25\], \[^26\]–\[^28\].

The trained models in federated learning are vulnerable to various privacy attacks. Differential privacy has been employed with federated learning in various works in the literature to provide privacy guarantees for such algorithms. For example, \[^26\]–\[^28\] apply DP mechanisms at clients to ensure sample-level DP guarantees. On the other hand, \[^29\]–\[^31\] apply DP mechanisms at the server to ensure client-level DP guarantees. Applying DP typically causes some degradation in performance, i.e., the model’s performance degrades as the privacy budget gets smaller. These approaches to privacy-preserving federated learning use fixed privacy budgets for all clients, an approach that can be overly strict and cause unnecessary degradation in performance. Our work is focused on giving clients the option to opt out and utilizing such information at the server to produce better models. A variation of DP setups is proposed in the literature, for scenarios with clients and a server, to use a hybrid model by combining sample-level DP with client-level DP and giving clients the option to opt in either \[^32\]–\[^33\].
2 Privacy & Personalization In Federated Learning

The federated learning setup consists of a central server and a set of clients. The central server coordinates the training of the model using the clients over multiple training rounds. The set of all clients, denoted by \( C \), contains all clients that wish to cooperate in training the model. Each client \( c_i \in C \) has a local dataset, denoted by \( D_i \), and a local loss function \( f_i(\cdot) \). During each communication round \( t \), the server sends the current model state, i.e., \( \theta_t \), to the set of available clients in that round, denoted by \( C^t \), who train the model on their local datasets to minimize their local loss functions \( f_i(\cdot) \). The clients then return the updated model to the server who aggregates them, e.g., averages them, to produce the next model state \( \theta^{t+1} \). Next, we describe an approach for privacy-preserving federated learning as well as an approach to personalized federated learning. The proposed algorithm \( FeO2 \) employs a version of both approaches, and they are considered the two baselines against which \( FeO2 \) is examined.

2.1 Privacy-Preserving Federated Learning: \( DP-FedAvg \)

To design privacy-preserving federated learning algorithms using differential privacy, a few modifications are required to the baseline federated averaging algorithm. Specifically, two modifications are introduced: clipping and noising. Considering client-level privacy, the averaging operation at the server is the target of such modifications. Assume clients are selected each round with probability \( q \) from the population of all clients of size \( N \). First, each client update is clipped to have norm at most \( S \), then the average is computed and additive Gaussian noise is added with mean zero and covariance \( \sigma^2 I = z^2 (\frac{2}{qN})^2 I \). The variable \( z \) is referred to as the noise multiplier, which dictates the achievable values of \((\epsilon, \delta)\)-DP. Training the model via multiple rounds increases the amount of leaked information, the moment accountant method, introduced by [34], can be used to provide a tighter estimate of the resulting DP parameters \((\epsilon, \delta)\).

Selecting the clipping threshold as well as the noise multiplier is instrumental to getting useful models with desirable privacy guarantees. During training, the norm of updates can either increase or decrease, if the norm increases or decreases significantly compared to the clipping norm, the algorithm may slow down or diverge. Hence, [30] presented a solution to privately and adaptively update the clipping norm during each round of communication in federated learning based on the feedback from clients on whether or not their update norm exceeded the clipping norm. We consider this as the baseline for privacy-preserving federated learning algorithm and refer to it in the rest of the paper as \( DP-FedAvg \). The case where no noise is added is the baseline for non-private federated learning algorithm, which is referred to simply as \textit{non-private}.

2.2 Personalized Federated Learning: \( Ditto \)

There are multiple approaches to personalization in federated learning as mentioned in the previous section. In this paper, we consider a recent work by [6], referred to as \( Ditto \), due to its simplicity and modularity. \( Ditto \) is a bi-level optimization objective, which does not modify the global FL training process, but adds a local training task to personalize a model locally with the following local loss function:

\[
    h_i(\theta_i, \theta^*) = f_i(\theta_i) + \frac{\lambda_i}{2} \| \theta_i - \theta^* \|^2,
\]

where \( \theta_i \) is the personalized local model for client \( c_i \), \( \lambda_i \) is the regularization parameter for client \( c_i \), and \( \theta^* \) is the global server-side model. As \( Ditto \) can simply be added onto the previously described learning algorithms, we combine it with them to train personalized local models for each client. This forms the baseline for the personalized learning in both \( DP-FedAvg \) and \textit{non-private}.

3 Federated Learning with Opt-Out Differential Privacy: \( FeO2 \)

In this section, we describe the \( FeO2 \) algorithm that is designed to take advantage of the aforementioned heterogeneous privacy setup. The proposed algorithm is described in Algorithm 1. As mentioned before, the \( FeO2 \) algorithm utilizes differential privacy with adaptive clipping, and optionally \( Ditto \) for personalized learning.

First, we provide the notation of the variables in the algorithm. We split the set of clients \( C \) into two subsets containing private and non-private clients denoted by \( C_p \) and \( C_{np} \), respectively. Assume the number of private and non-private clients is \( N_p = (1 - \rho_{np})N \) and \( N_{np} = \rho_{np}N \), where \( \rho_{np} \) is the fraction of clients who opt out. The rest of the hyperparameters in the algorithm are as follows: the ratio hyperparameter \( r \), the noise multipliers \( z, z_{np} \), the clipping sensitivity \( S \), the learning rate at clients \( \eta \), the personalized learning rate at clients \( \eta_{np} \), quantile \( \kappa \), and factor \( \eta_{np} \). The superscript \((\cdot)^t\) is used to denote a parameter during the \( t \)-th training round. During round \( t \) of training, no additional
As for the personalization part, we have the hyperparameter $\lambda_p$, ratio $r$, noise multipliers $z_1, z_2$, quantile $\kappa$, and factor $\eta_1$.

Output: $\theta^0 \cdot \{\theta_j \}_{j \in [N]}$

At server:
- for round $t = 0, 1, 2, \ldots, T - 1$ do
  - $C^t \leftarrow \text{Sample } N^t \text{ clients from } C$
  - for client $c_j \in C^t$ in parallel do
    - $\Delta \theta_j^{t+1} \leftarrow \text{ClientUpdate}(\theta^t, c_j, S^j)$
  - end for
  - $N^p_r \leftarrow |C^t|^r$, $N^p \leftarrow |C^t| - N^p_r$. $z^t \leftarrow z_1^t / N^p_r$
  - $\Delta \theta^{t+1} \leftarrow \frac{1}{N^p} \sum_{j \in C^t} \Delta \theta_j^{t+1} + N(0, (z^t)^2) I$
  - $\Delta \theta^{t+1} \leftarrow \frac{1}{N^p} \sum_{j \in C^t} \Delta \theta_j^{t+1}$
  - $\theta^{t+1} \leftarrow \theta^t + \frac{1}{r N^p_r + z_2^t} \left[ N^p_r \Delta \theta^{t+1} + r N^p_r \Delta \theta^{t+1} \right]$
  - $S^{t+1} \leftarrow S^t - \eta_1 \left( \frac{1}{N^p_r} \sum_{j \in C^t} b_j - N(0, z_2^t)^2 \right)$
- end for

In this simplified setup, we consider a single round of communication and assume that all clients have the same number of samples, and that the effect of clipping is negligible. Let $n_s$ denote the number of samples on each client.

### 4 Analyzing Federated Point Estimation

In this section, we provide some insights into the proposed FeO2 algorithm through a simplified setup, inspired by the one proposed by [6], known as federated point estimation. We first start by considering the global estimation on the server and show that FeO2 is Bayes optimal when using the appropriate value of the ratio $r$. Then we consider personalization using Ditto for both private and non-private clients and show that combining the optimal FeO2 with Ditto is Bayes optimal for private and non-private clients when using appropriate values of the regularization parameters $\lambda_p$ and $\lambda_{np}$.

Finally, we show the gain of opting out in the proposed personalized setup compared to the baseline. The federated point estimation is presented for clarity of discussion, refer to the extended version for additional discussions and an extension of the setup to linear regression problems.

#### 4.1 Setup

In this simplified setup, we consider a single round of communication and assume that all clients have the same number of samples, and that the effect of clipping is negligible. Let $n_s$ denote the number of samples on each client.
Algorithm 1. In this setup, the server and clients’ goals is to minimize the Bayes risk (i.e., test error), where
the server’s goal is to combine the updates received from clients such that the resulting noise variance
values of both the server and clients are optimal. Due to the server aggregation being different in these two algorithms, the
resulting optimum will also be different for clients in each algorithm. Additionally, as
Ditto aims to produce the global model, while in FeO2 with Ditto the server utilizes the side-information available to it about the privacy choices of clients to optimally aggregates their updates. Due to the server aggregation being different in these two algorithms, the
resulting optimum λ’s will also be different for clients in each algorithm. Additionally, as FeO2 aims to provide optimal performance for both private and non-private clients, we need to find the optimal values of both λ
and λp. Next, we discuss the optimality of Ditto combined with FeO2 using the estimate θ∗ for both private and non-private clients in the federated point estimation problem. To this end, we have the following lemma.

Lemma 1 (Global estimate optimality). FeO2 from the server’s point of view, with ratio r∗ = σ2
σ2 + N p γ2, is Bayes optimal (i.e., θ converges to θ∗) in the considered federated point estimation problem. Furthermore, the resulting variance is σ2
σ2 + N p γ2.

4.3 Personalized Local Models On Clients
In this part, we consider using Ditto to train personalized local models at clients. We show that combining Ditto with FeO2 provides the Bayes optimal local model for clients. We first note that in vanilla Ditto, the server applies vanilla FedAvg to produce the global model, while in FeO2 with Ditto the server utilizes the side-information available to it about the privacy choices of clients to optimally aggregates their updates. Due to the server aggregation being different in these two algorithms, the
resulting optimum λ’s will also be different for clients in each algorithm. Additionally, as FeO2 aims to provide optimal performance for both private and non-private clients, we need to find the optimal values of both λ
and λp. Next, we discuss the optimality of Ditto combined with FeO2 using the estimate θ∗ for both private and non-private clients in the federated point estimation problem. To this end, we have the following lemma.

Lemma 2 (Personalized local estimate optimality). Assuming using Ditto combined with FeO2 with ratio r∗ in Lemma 1 there exist λ∗ for non-private clients and λ∗ p for private clients such that FeO2 is Bayes optimal (i.e., θ converges to θ∗ for each client j ∈ [N]).

4.4 Incentive for Opting Out of DP
Earlier in this section, we have shown that for the problem of federated point estimation, the global estimate benefited greatly from the setup of opt-out differential privacy. A better global estimate enables better performance on clients’ devices in the federated point estimation setup. However, a question may arise on whether or not clients are actually going to opt out of privacy. In other words, what incentivizes clients to opt out other than helping the server produce better estimates?

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Figure 1: The effect of opting out on the personalized local model estimate as a function of $\lambda$.

Table 1: Summarized results of experiments on synthetic datasets: We compare the performance of the baseline algorithms against FeO2 with the hyperparameters that perform best. The variance of the performance metric across clients is between parenthesis.

| Setup | Global model | Personalized local models |
|-------|--------------|---------------------------|
| **Non-private** | $\lambda_p = 0.005$ | $\lambda_p = 0.005$ | $\lambda_p = 0.005$ | $\lambda_p = 0.005$ | $\lambda_p = 0.005$ |
| 3% | 93.8 | 93.8 | 93.8 | 93.8 | 93.8 |
| 5% | 93.6 | 93.6 | 93.6 | 93.6 | 93.6 |
| 10% | 93.5 | 93.5 | 93.5 | 93.5 | 93.5 |

To answer this question, we argue that opting out helps the server produce better global estimates for clients, in addition to helping clients produce better personalized local estimates. In other words, clients that opt out can produce better personalized local estimates compared to the ones that remain private. To illustrate the motivation of opting out for clients, we perform an experiment where we conduct the federated point estimation experiment for two scenarios. The first is the case where client $c_k$ remains private, and the second is the case where $c_k$ opts out of privacy and becomes non-private. The results of these experiments are shown in Figure 1. This illustrates the incentive for clients to opt out of privacy.

5 Experimental Evaluation

In this section, we present the results of a number of experiments to show the gain in performance of the proposed FeO2 algorithm compared to the baseline DP-FedAvg algorithm. The experiments show that FeO2 outperforms DP-FedAvg with the right choice of the hyperparameter $r$ in terms of the global model accuracy, as well as in terms of the average personalized local model accuracy.

5.1 Setup

The experiments are conducted on multiple federated datasets, synthetic and realistic. The synthetic datasets are manually created to simulate extreme cases of data heterogeneity often exhibited in federated learning scenarios. The realistic federated datasets are from TFF [7], where such datasets are assigned to clients according to some criteria. The synthetic dataset is referred to as the non-IID MNIST dataset, and the number of samples at a client is fixed across all clients. Each client is assigned samples randomly from the subsets of samples each with a single digit between 0 – 9. A skewed version of the synthetic dataset is one where non-private clients are sampled from the clients who have the digit 7 in their data. In the non-IID MNIST dataset, we have 2,000 clients and we randomly sample 5% of them for training each round. The realistic federated datasets are the FMNIST and FEMNIST from TFF datasets. The FMNIST and FEMNIST datasets contain 3,833 and 3,400 clients, respectively, and we sample 3% of them for training each round. TensorFlow Privacy (TFP) [35] is used to compute the privacy loss incurred during training. Refer to the extended version for more extended description of the used models and extended results.

5.2 Results

In this part, we provide the outcomes of the experiments on the datasets mentioned above. In these experiments, we provide results for an opt-out rate of 5% of the total client population. Clients that opt out are picked randomly from the set of all clients but fixed for a fair comparison across all experiments. The exception for this assumption is for the skewed non-IID MNIST dataset, where clients that opt out are sampled from the clients who have the digit 7. All other hyperparameters are
Table 2: Summarized results of experiments on realistic federated datasets: We compare the performance of the baseline algorithms against FeO2 with the hyperparameters that perform best. The variance of the performance metric across clients is between parenthesis.

| Setup | Global model | Personalized local models |
|-------|--------------|---------------------------|
| Algorithm | Hyperparameters | Accg (%) | Accgp (%) | Accgp (%) | △g (%) | Accg (%) | Accgp (%) | △g (%) |
| Non-private | λp = 0.05 | 89.99 | 77.64 (2.55) | 85.30 (1.68) | - | 90.04 (1.04) | 84.85 (0.99) | - |
| DP-FedAvg | r = 0.01 | 76.66 | 77.91 (1.67) | 83.90 (1.27) | 5.99 | 77.91 (1.22) | 79.15 (0.99) | 1.25 |
| FeO2 | λp = λnp = 0.05 | 86.88 | 85.36 (1.89) | 90.02 (1.28) | 4.66 | 93.76 (0.68) | 95.94 (0.41) | 2.18 |

Fixed. To evaluate the performance of each algorithm, we measure the following quantities for each dataset:

- **Accg**: the average test accuracy on the server test dataset using the global model.
- **Accgp, Accgp**: the average test accuracy of all private and non-private clients using the global model on their local test datasets, respectively.
- **Accgp, Accgp**: the average test accuracy of all private and non-private clients using their personalized local models on their local test datasets, respectively.
- **△g, △l**: the gain in the average performance of non-private clients over the private ones using the global model and the personalized local models on their local test datasets, respectively; computed as △g = Accgp - Accgp and △l = Accgp - Accgp.

A summary of the results, shown in Table 1 and Table 2, provides the best performance for each experiment along with their corresponding hyperparameters. More detailed results are shown in the extended version. If different values of the hyperparameters in FeO2 yield two different competing results, such as one with better global model performance at the server and one with better personalized local models at the clients, we show both.

We can see from Tables 1 and 2 that FeO2 allows the server to learn better global models while allowing clients to learn better personalized local models compared to the baseline private FL, i.e., DP-FedAvg. For example, the gain due to FeO2 compared to the DP-FedAvg in terms of global model performance is 3.8% on average and is up to 9.27%. For personalized local models, the gain in average accuracy for clients due to FeO2 compared to DP-FedAvg is up to 9.99%. The gap in the average performance of personalized local models between DP-FedAvg and non-private is 3.57%, which is reduced to 0.95% between FeO2 and non-private. Additionally, we can also see the gain in the average performance in personalized local models between clients who choose to opt out of privacy and clients who choose to remain private. This demonstrates the advantage of opting out, which provides clients with an incentive to opt out of differential privacy to improve their personalized local models, for example, non-private clients can gain up to 3.49% on average in terms of personalized local model performance compared to private clients. It is worth mentioning that opting out can also improve the global model’s performance on clients’ local data. We observe that there is up to 12.4% gain in the average performance of non-private clients in terms of the accuracy of the global model on the local data compared to the one of baseline DP-FedAvg.

### 6 Conclusion

In this paper, we considered a new aspect of heterogeneity in federated learning setups, namely heterogeneity in privacy requirements. We proposed a new setup for privacy heterogeneity between clients where privacy is no longer necessary for all clients, and some clients choose to opt out of privacy. We proposed a new algorithm called FeO2 for the considered setup. In FeO2, the aim is to employ differential privacy to maintain the privacy of clients who choose to remain private, and we proposed a two-step aggregation scheme at the server to improve utility. Additionally, we employed Ditto as a personalization scheme to examine whether FeO2 enhances the performance for personalized FL. We provided a treatment for the federated point estimation problem and showed the success of FeO2 on the central server as well as the personalized local models. Finally, we provided a set of experiments on synthetic and realistic federated datasets and showed that FeO2 outperforms the baseline private FL algorithm in terms of the global model as well as the personalized local models performance, and showed the incentive of becoming non-private compared to remaining private in such scenarios in terms of the gain in the average performance.
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