Discussion on Spontaneous Parametric Down-conversion (SPDC) Based on Parametric Oscillator Model

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Abstract. In this article, we use the theory of simple harmonic motion to estimate the vibration energy of Boron atom. Subsequently we employ the parametric oscillator model to discuss photon pairs with wavelength of 710 nm. When a laser with wavelength of 355nm illuminates the barium meta-borate crystal, parametric resonance could lead to the effect of spontaneous parametric down-conversion.

1. Introduction
Since spontaneous parametric down-conversion (SPDC) was demonstrated in 1970, more and more scholars are interested in applications of SPDC photon pairs [1-3]. Italian researchers used the pump laser with the wavelength of 355nm passing through the BBO (Barium meta-borate) crystal for two correlated photons with degeneracy frequency whose wavelength is 710nm [4]. We should pay more attention to the role of the boron atom in the BBO crystal.

2. Energy Estimation of Atom Vibration from Simple Harmonious Motion
Assuming that the kinetic energy and potential energy of molecules are equal, we get

\[ E_{vib} = kA^2 = M\omega_0^2 \]  

M stands for the mass of the atom.\( \omega_0 \) is the vibration frequency.

The energy of a certain atom is the same order as that of the hydrogen atom.

\[ M\omega_0^2 = 13.6eV = \hbar^2 / (8m_e\pi^2 a_0^2) \]

\[ m_e = 9.1 \times 10^{-31} \text{kg}, a_0 = 0.53 \times 10^{-10} \text{m} \]

where \( m_e \) is the rest mass of the electron,\( a_0 \) is the Bohr radius.

So we get

\[ \omega_0^2\hbar^2 / (4\pi^2) = \hbar^4 / (32m_e\pi^4a_0^4M) = 2m_e\hbar^4 / (64\pi^4m_e^2a_0^4M) \]

The above formula shows

\[ E_{osc}^2 = 2m_e\hbar^4 / (64\pi^4m_e^2a_0^4M) = 2m_e(13.6eV)^2 / M \]

For the boron atom, the oscillation energy is

\[ E_{oscB} = 0.136eV \]
We know the rules of transition of molecular vibration,

\[ \Delta n = 1 \]  

(6)

Energy of the light emission is at most

\[ E_0 = 0.136 \text{eV} = h\nu_0 \]  

(7)

The frequency is in the range of infrared light.

\[ \nu_0 = 3.4 \times 10^{13} \text{Hz} \]  

(8)

The vibration model of two boron atoms is shown in Figure 1.

\[ E = 0.136 \text{eV} \]

Boron atom

**Figure 1.** Vibration model of two Boron atoms connected by an interaction

The model of Boron atoms is shown in Figure 1. We know that it is impossible to emit red light whose wavelength is 710nm by boron atom vibration. It is also difficult to emit red light by increasing the quantum number because the quantum number is no more than 15. In general, the oscillator takes small quantum numbers. This impossibility leads us to put forward a corrected model about parametric oscillator.

3. **Light Emission from Parametric Down-conversion (PDC)**

The vibration of a Boron atom in x direction is in a forced oscillation. There is a photon with wavelength of 710nm as external force in x direction, so the expression of forced oscillation is written as

\[ \ddot{x} + \beta_{0x} \dot{x} + \omega_{0x}^2 x = f_{0x} \cos(2\pi \nu_{ins} t) \]  

(9)

\( \beta_{0x} \) is the damping coefficient and \( \omega_{0x} = 2\pi \nu_{0x} \) is the natural frequency in x direction, \( f_{0x} \) is the amplitude related to the external force and \( \nu_{ins} \) is the frequency of spontaneous photon with wavelength of 710 nm from environment.

Because frequency \( \nu_{ins} \) is much greater than that of the natural frequency of the boron atom \( \nu_{0x} \), we get its steady state solution:

\[ x = A \cos(2\pi \nu_{ins} t) \]  

(10)

This is the red light with same wavelength of incident light.
In a similar way, in view that illuminated laser with the wavelength of 355 nm leads to forced oscillation of a Boron atom in y direction, we can write as:

\[ \ddot{y} + \beta_{0y} \dot{y} + \omega_{0y}^2 y = f_{0y} \cos(2 \pi v_{inp} t) \]  \hspace{1cm} (11)

The forced oscillation of a Boron atom in y direction is valid. We get stable solution of y direction.

\[ y = A_{0y} \cos[(2 \pi v_{inp} t + \phi_{f})] \]  \hspace{1cm} (12)

The velocity of the force oscillation is

\[ \dot{y} = 2 \pi v_{inp} A \cos[(2 \pi v_{inp} t + \phi_{f})] \]  \hspace{1cm} (13)

The two orthogonal scattering photons shown in Figure 2 are independent and no correlation in photon pairs. The rotation of an electron in z direction \( \omega z \) is intensified by illumination, forming a strong magnetic field. Although the displacement of the forced vibration in the y direction is small, but the speed is very large from (13), so the coupled oscillation in the x direction through the magnetic force is possible. Because the frequency of the forced oscillation in x direction is half of that in y direction, parametric resonance occurs which is in x direction. This is the model of parametric oscillator shown in Figure 3.

The model is as follows:
There are two photons with wavelength of 710 nm in the x direction in Figure 3. The magnetic force cannot be ignored when illumination lasts sometime, so the parametric resonance occurs. The two independent oscillators become a parametric oscillator. We have

\[ \ddot{x} + \omega_{\text{in}}^2 [1 + h \cos(2\pi \nu_{\text{in}} t + \epsilon)] x = 0 \]  

(14)

Where \( h \) is a small coupling amplitude from the fluctuating force. Equation (14) is the Matthew equation [5].

\[ \ddot{x} + \omega_{\text{in}}^2 [1 + h \cos(2\pi \nu_{\text{in}} t)] x = 0 \]  

(15)

When two frequencies have the following relationship \( \nu_{\text{in}} = 2 \nu_{\text{int}} \), the solution in the x direction changes exponentially with time.

\[ x = A_0 x \exp(s \Omega t) \cos((\pi \nu_{\text{in}} + \epsilon/2)t) \]  

(16)

The amplifying factor is

\[ S^2 = \frac{1}{4} \left( \frac{\omega_{\text{in}}}{2} + \frac{\epsilon}{2} \right)^2 \left( \frac{\omega_{\text{in}}^2 h}{2} \right)^2 \left[ -\left( \frac{\omega_{\text{in}}}{2} + \frac{\epsilon}{2} \right)^2 + \omega_{\text{in}}^2 \right]^2 \]  

(17)

\( \epsilon \) is the line width of variable frequency which depends on the amplitude of the weakly coupled force.

Value in square brackets is close to zero, since the half-frequency of the inputted laser photon is the same frequency as the forced vibration of the x direction excited by spontaneous photon. The parametric resonance leads to the stimulated absorption with the wavelength of 355 nm for emission of a photon with the wavelength of 710 nm and a stimulated phonon emission.

The transition happened from the high energy oscillator gives the energy expression:

\[ h \gamma_{\text{in}} \approx h \gamma_{\text{int}} - h \gamma_{\text{ox}} \]  

(18)

So, there are two correlated photons with the degeneracy frequencies.

Could the parametric resonance exist in the y direction if magnetic force is considered from the oscillation of the x direction?

\[ \ddot{y} + \omega_{\text{in}}^2 [1 + h \cos(2\pi \nu_{\text{in}} t)] y = 0 \]  

(19)
The answer is negative. If it is valid, we get

\[ S_\Omega^2 = \frac{1}{4} \left( \frac{\omega_{in}}{2} + \frac{\epsilon}{2} \right)^2 \left\{ \left( \frac{\omega_{in}}{2} + \frac{\epsilon}{2} \right)^2 - \left( \frac{\omega_{in}}{2} + \frac{\epsilon}{2} \right)^2 \right\} \]  

(20)

The relationship \( v_{inp} = 2v_{in} \) leads to \( h > \frac{3}{2} \) which contradicts the assumption of weak parameter resonance (\( h << 1 \)). We conclude that only when the frequency of the periodic motion from the external world is twice the frequency of the forced vibration, the amplitude of the parametric resonance increases exponentially with time for a phonon emission.

4. Summary and Conclusion

The energy of boron atoms is estimated by simple harmonic motion. Our estimation indicates that red light with wavelength of 710nm couldn’t emit from the simple harmonious motion of the boron atom. However, when the BBO crystal is illuminated by a laser with wavelength of 355nm, parametric resonance could lead to a photon with a wavelength of 710nm, and the two photons are correlated to the degeneracy frequency. We explain the effect of spontaneous parametric down conversion (SPDC) by employing the parametric oscillation model.

5. References

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