Low-energy limit of the $O(4)$ quark-meson model

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Motivation: QCD at low energies

Chiral symmetry

Linear realization
- Linear sigma model (LSM)
- Extended LSM (eLSM)
- Quark-meson model

Nonlinear realization
- Chiral perturbation theory (ChPT)

Integration
- Tree level
- Functional renormalization group (FRG)

Low-energy limit same as for QCD?
Chiral perturbation theory (ChPT)

- Systematic analysis of **hadronic n-point functions** of QCD [Gasser, Leutwyler ’84,’85; Leutwyler ’94; etc.]

- Pion interactions dominate the low-energy regime

- **Chiral expansion**: Most general chiral-invariant Lagrangian with terms coupled by the low-energy couplings of QCD,

\[
\mathcal{L}_{\text{ChPT}} = \frac{1}{2} (\partial_\mu \vec{\pi})^2 - \frac{1}{2} m_\pi^2 \vec{\pi}^2 + C_{1,\text{ChPT}} (\vec{\pi}^2)^2 + C_{2,\text{ChPT}} (\vec{\pi} \cdot \partial_\mu \vec{\pi})^2 \\
+ C_{3,\text{ChPT}} (\partial_\mu \vec{\pi})^2 (\partial_\nu \vec{\pi})^2 + C_{4,\text{ChPT}} (\partial_\mu \vec{\pi} \cdot \partial_\nu \vec{\pi})^2 \\
+ \mathcal{O} (\pi^6, \partial^6)
\]
Extended linear sigma model (eLSM)

- Linear realization of chiral symmetry ("chiral partners")
- Applications at nonzero temperature and chemical potential
- Contains all $J^P = 0^\pm, 1^\pm$ mesons up to 2 GeV in mass
- **Representations** of $U(N_f)_r \times U(N_f)_l$:

  \[ \Phi \sim \bar{q}_r q_l \rightarrow U_l \Phi U_r^\dagger, \]

  \[ R_\mu \sim \bar{q}_r \gamma_\mu q_r \rightarrow U_r R_\mu U_r^\dagger, \]

  \[ L_\mu \sim \bar{q}_l \gamma_\mu q_l \rightarrow U_l L_\mu U_l^\dagger. \]

- Vector and axial-vector mesons:

  \[ V_\mu = \frac{1}{2} (L_\mu + R_\mu), \quad A_\mu = \frac{1}{2} (L_\mu - R_\mu) \]

- **Disclaimer**: No attempt to compete with ChPT
\[ \mathcal{L}_{\text{eLSM}} = \text{tr} \left[ (D^\mu \Phi)^\dagger D_\mu \Phi \right] - m_0^2 \text{tr} \left( \Phi^\dagger \Phi \right) \\
- \lambda_1 \left[ \text{tr} \left( \Phi^\dagger \Phi \right) \right]^2 - \lambda_2 \text{tr} \left[ (\Phi^\dagger \Phi)^2 \right] \\
- \frac{1}{4} \text{tr} \left( L_{\mu \nu}^2 + R_{\mu \nu}^2 \right) + \text{tr} \left[ \left( \frac{m_1^2}{2} + \Delta \right) (L_{\mu}^2 + R_{\mu}^2) \right] \\
+ \text{tr} \left[ H (\Phi + \Phi^\dagger) \right] - c_A \left( \text{det} \Phi + \text{det} \Phi^\dagger \right) \\
+ i \frac{g_2}{2} \left( \text{tr} \left\{ L^{\mu \nu} [L_\mu, L_\nu] \right\} + \text{tr} \left\{ R^{\mu \nu} [R_\mu, R_\nu] \right\} \right) \\
+ \frac{h_1}{2} \text{tr} \left( \Phi^\dagger \Phi \right) \text{tr} \left( L_{\mu}^2 + R_{\mu}^2 \right) + h_2 \text{tr} \left( |L_\mu \Phi|^2 + |\Phi R_\mu|^2 \right) \\
+ 2h_3 \text{tr} \left( \Phi R^\mu \Phi^\dagger L_\mu \right) + g_3 \left[ \text{tr} \left( L^{\mu \nu} L_\mu L_\nu \right) + \text{tr} \left( R^{\mu \nu} R_\mu R_\nu \right) \right] \\
+ g_4 \left[ \text{tr} \left( L^\mu L_\mu L_\nu L_\nu \right) + \text{tr} \left( R^\mu R_\mu R_\nu R_\nu \right) \right] + g_5 \text{tr} \left( L^\mu L_\mu \right) \text{tr} \left( R^{\nu} R_\nu \right) \\
+ g_6 \left[ \text{tr} \left( L^\mu L_\mu \right) \text{tr} \left( L^{\nu} L_\nu \right) + \text{tr} \left( R^\mu R_\mu \right) \text{tr} \left( R^{\nu} R_\nu \right) \right], \]

\[ D_\mu \Phi = \partial_\mu \Phi - ig_1 \left( L_\mu \Phi - \Phi R_\mu \right), \quad L_{\mu \nu} = \partial_\mu L_\nu - \partial_\nu L_\mu \]
eLSM publications

• Introduction of the eLSM
  [Parganlija, Giacosa, Rischke ’10; Parganlija, Kovacs, Wolf, Giacosa, Rischke ’13]

• Chiral partner of the nucleon
  [Gallas, Giacosa, Rischke ’10; Lakaschus, Mauldin, Giacosa, Rischke ’18]

• Incorporating scalar glueball
  [Janowski, Giacosa, Rischke ’14; Giacosa, Sammet, Janowski ’17]

• Baryon multiplets
  [Olbrich, Zetenyi, Giacosa, Rischke ’16,’18]

• Nonzero-temperature study within the FRG
  [JE, Grahl, Rischke ’15]
• Two-flavor model, $N_f = 2$
• Specific limit of the eLSM
• Euclidean action:

\[
S = \int_x \left\{ \frac{1}{2} (\partial_\mu \sigma) \partial_\mu \sigma + \frac{1}{2} (\partial_\mu \vec{\pi}) \cdot \partial_\mu \vec{\pi} + U(\rho) - h_{ESB} \sigma \\
+ \bar{\psi} (\gamma_\mu \partial_\mu + y \Phi_5) \psi \right\},
\]

\[
\Phi_5 = \sigma t_0 + i \gamma_5 \vec{\pi} \cdot \vec{t},
\]

\[
\rho = \sigma^2 + \vec{\pi}^2
\]

• Explicit and spontaneous symmetry breaking implemented,

\[
h_{ESB} \neq 0, \quad \sigma \rightarrow \phi + \sigma
\]
Integration at tree level
Low-energy limit of the eLSM

• Integrate out all fields, except pions, cf. [Divotgey, Kovacs, Giacosa, Rischke ’18]

• Low-energy limit of the eLSM at tree level:

\[ \mathcal{L}_{\text{eLSM}} = \frac{1}{2} (\partial_\mu \vec{\pi})^2 - \frac{1}{2} m_\pi^2 \vec{\pi}^2 + \mathcal{O}_1,\text{eLSM} (\vec{\pi}^2)^2 + \mathcal{C}_2,\text{eLSM} (\vec{\pi} \cdot \partial_\mu \vec{\pi})^2 + \mathcal{O}_3,\text{eLSM} (\partial_\mu \vec{\pi})^2 (\partial_\nu \vec{\pi})^2 + \mathcal{C}_4,\text{eLSM} (\partial_\mu \vec{\pi} \cdot \partial_\nu \vec{\pi})^2 + \mathcal{O} (\pi^6, \partial^6) \]

• Match coefficients to ChPT,

\[ \mathcal{L}_{\text{ChPT}} = \frac{1}{2} (\partial_\mu \vec{\pi})^2 - \frac{1}{2} m_\pi^2 \vec{\pi}^2 + \mathcal{O}_1,\text{ChPT} (\vec{\pi}^2)^2 + \mathcal{C}_2,\text{ChPT} (\vec{\pi} \cdot \partial_\mu \vec{\pi})^2 + \mathcal{C}_3,\text{ChPT} (\partial_\mu \vec{\pi})^2 (\partial_\nu \vec{\pi})^2 + \mathcal{C}_4,\text{ChPT} (\partial_\mu \vec{\pi} \cdot \partial_\nu \vec{\pi})^2 + \mathcal{O} (\pi^6, \partial^6) \]
Table 1: Comparison of the low-energy couplings at tree level, cf. [Bijnens, Ecker ’14; Divotgey, Kovacs, Giacosa, Rischke ’18].

| Coupling          | ChPT         | eLSM         |
|-------------------|--------------|--------------|
| $C_1$             | $-0.28 \pm 1.9$ | $-0.268 \pm 0.021$ |
| $C_2 \times 10^5 \text{ MeV}^2$ | $5.882 \pm 0.013$ | $5.399 \pm 0.081$ |
| $C_3 \times 10^{11} \text{ MeV}^4$ | $-5.61 \pm 0.89$ | $-9.302 \pm 0.591 + \text{corr.}^a$ |
| $C_4 \times 10^{11} \text{ MeV}^4$ | $2.51 \pm 0.41$ | $9.448 \pm 0.589 + \text{corr.}^a$ |

$^a$ further corrections from undetermined model parameters

- **ChPT:** $C_1 = -M^2/(8f_\pi^2)$, $C_2 = 1/(2f_\pi^2)$, $C_3 = l_1/f_\pi^4$, $C_4 = l_2/f_\pi^4$
- **eLSM:** functions of model parameters $m_0^2$, $\lambda_1$, $\lambda_2$, $h_1$, $h_2$, $h_3$, $g_1$, ...
Open questions

- Tree-level calculation sufficient? **Resonance saturation** in low-energy constants, cf. [Ecker, Gasser, Pich, de Rafael ’89]
- How do these values change if we include loop corrections?
- Can we produce similar values within the FRG formalism?

  **First step:** Exploratory study of higher-derivative couplings within the FRG formalism, cf. [JE, Divotgey, Mitter, Rischke, PRD 98 (2018) no.1, 014024]

- Following analysis restricted to the **$O(4)$ quark-meson model** as specific limit of the eLSM
Integration within the FRG
Functional renormalization group (FRG)

- Implementation of the *Wilsonian* RG idea
- Renormalization scale \( k \)-dependent effective action \( \Gamma_k \)
- FRG flow equation: [Wetterich ’93]

\[
\partial_k \Gamma_k = \frac{1}{2} \text{tr} \left[ \partial_k R_k \left( \Gamma_k^{(2)} + R_k \right)^{-1} \right] = \frac{1}{2}
\]

- Regulator function \( R_k \) provides correct integration limits
- *Nonperturbative* continuum method
Flow in theory space

Figure 1: Theory space spanned by generic couplings.
O(4) quark-meson model – truncations

- Local potential approximation (LPA), i.e., consider scale-dependent effective potential,

\[
\Gamma_k = \int_x \left\{ \frac{1}{2} (\partial_\mu \sigma) \partial_\mu \sigma + \frac{1}{2} (\partial_\mu \pi) \cdot \partial_\mu \pi + U_k(\rho) - h_{ESB} \sigma \\
+ \bar{\psi} \left( \gamma_\mu \partial_\mu + y \Phi_5 \right) \psi \right\}
\]

- LPA', i.e., include wave-function renormalization,

\[
\Gamma_k = \int_x \left\{ \frac{Z_\sigma}{2} (\partial_\mu \sigma) \partial_\mu \sigma + \frac{Z_\pi}{2} (\partial_\mu \pi) \cdot \partial_\mu \pi + U_k(\rho) - h_{ESB} \sigma \\
+ \bar{\psi} \left( Z_\psi^\psi \gamma_\mu \partial_\mu + y \Phi_5 \right) \psi \right\}
\]
Flow of the effective potential – LPA

Figure 2: Scale evolution of the effective potential; grid discretization.
Remark: Technical details

- Analytic derivation of flow equations can be “expensive”
- Self-written routines and additional software used, 
  DoFun [Huber, Braun ’12]
  FormTracer [Cyrol, Mitter, Strodthoff ’17]

- Flow equations: Set of coupled partial differential equations
- Numerical solution: Grid discretization, Taylor expansion, ...

\[ U_k (\rho) = \sum_{n=1}^{N} \frac{\alpha_{n,k}}{n!} (\rho - \chi)^n \]
Remark: Grid vs. Taylor

**Figure 3**: Scale evolution of meson and quark masses and the pion decay constant; lines: Taylor; crosses: grid.
Figure 4: $(\mu, T)$ dependence of the v.e.v. $\phi$ in the chiral limit ($h_{ESB} = 0$); grid method. Consistent with [Schaefer, Wambach ’05].
Remark: Nonzero-temperature analysis

- Study of chiral-symmetry breaking in the eLSM with vector and axial-vector mesons, cf. [JE, Grahl, Rischke ’15]

- Different symmetry aspects investigated (chiral limit, axial anomaly, ...)

- Phase-transition order consistent with other studies, cf. [Pisarski, Wilczek ’84, etc.]
Effective action

- Introduce **low-energy couplings**,

\[
\Gamma_k = \int_x \left\{ \frac{Z_k^\sigma}{2} (\partial_\mu \sigma) \partial_\mu \sigma + \frac{Z_k^{\bar{\pi}}}{2} (\partial_\mu \bar{\pi}) \cdot \partial_\mu \bar{\pi} + U_k(\rho) - h_{ESB} \sigma \\
+ C_{2,k} (\bar{\pi} \cdot \partial_\mu \bar{\pi})^2 + Z_{2,k} \bar{\pi}^2 (\partial_\mu \bar{\pi}) \cdot \partial_\mu \bar{\pi} \\
- C_{3,k} [(\partial_\mu \bar{\pi}) \cdot \partial_\mu \bar{\pi}]^2 - C_{4,k} [(\partial_\mu \bar{\pi}) \cdot \partial_\nu \bar{\pi}]^2 \\
- C_{5,k} \bar{\pi} \cdot (\partial_\mu \partial_\mu \bar{\pi}) (\partial_\nu \bar{\pi}) \cdot \partial_\nu \bar{\pi} \\
- C_{6,k} \bar{\pi}^2 (\partial_\mu \partial_\nu \bar{\pi}) \cdot \partial_\mu \partial_\nu \bar{\pi} \\
- C_{7,k} (\bar{\pi} \cdot \partial_\mu \partial_\mu \bar{\pi})^2 - C_{8,k} \bar{\pi}^2 (\partial_\mu \partial_\mu \bar{\pi})^2 \\
+ \bar{\psi} \left( Z_k^{\psi} \gamma_\mu \partial_\mu + y \Phi_5 \right) \psi \right\}
\]

- **State-of-the-art truncation**

- **Goal:** Compute the IR values of all \(k\)-dependent quantities
Renormalized quantities

- Renormalized fields:
  \[ \tilde{\sigma} = \sqrt{Z_k^{\sigma}} \sigma, \quad \tilde{\pi} = \sqrt{Z_k^{\pi}} \pi, \quad \tilde{\psi} = \sqrt{Z_k^{\psi}} \psi \quad \tilde{\bar{\psi}} = \sqrt{Z_k^{\bar{\psi}}} \bar{\psi} \]

- (Squared) renormalized masses:
  \[ M_{\sigma,k}^2 = \frac{m_{\sigma,k}^2}{Z_k^{\sigma}}, \quad M_{\pi,k}^2 = \frac{m_{\pi,k}^2}{Z_k^{\pi}}, \quad M_{\psi,k}^2 = \frac{m_{\psi,k}^2}{\left(Z_k^{\psi}\right)^2} \]

- Renormalized low-energy couplings:
  \[ \tilde{C}_{i,k} = \frac{C_{i,k}}{\left(Z_k^{\pi}\right)^2}, \quad i = 1, \ldots, 8, \quad \tilde{Z}_{2,k} = \frac{Z_{2,k}}{\left(Z_k^{\pi}\right)^2} \]
Low-energy limit

- Use equation of motion in the IR,

\[ \frac{\delta \Gamma}{\delta \tilde{\sigma}} = 0 \]

- Yields effective pion action,

\[ \Gamma_k = \int_x \left\{ \frac{1}{2} \left( \partial_\mu \tilde{\pi} \right) \cdot \partial_\mu \tilde{\pi} + \frac{1}{2} M_{\pi, k}^2 \tilde{\pi}^2 - \tilde{C}^{\text{total}}_{1, k} \left( \tilde{\pi}^2 \right)^2 
\]
\[ + \tilde{C}^{\text{total}}_{2, k} \left( \tilde{\pi} \cdot \partial_\mu \tilde{\pi} \right)^2 + \tilde{Z}^{\text{total}}_{2, k} \tilde{\pi}^2 \left( \partial_\mu \tilde{\pi} \right) \cdot \partial_\mu \tilde{\pi} 
\]
\[ - \tilde{C}^{\text{total}}_{3, k} \left[ \left( \partial_\mu \tilde{\pi} \right) \cdot \partial_\mu \tilde{\pi} \right]^2 - \tilde{C}^{\text{total}}_{4, k} \left[ \left( \partial_\mu \tilde{\pi} \right) \cdot \partial_\nu \tilde{\pi} \right]^2 
\]
\[ - \tilde{C}^{\text{total}}_{5, k} \tilde{\pi} \cdot \left( \partial_\mu \partial_\mu \tilde{\pi} \right) \left( \partial_\nu \tilde{\pi} \right) \cdot \partial_\nu \tilde{\pi} 
\]
\[ - \tilde{C}^{\text{total}}_{6, k} \tilde{\pi}^2 \left( \partial_\mu \partial_\nu \tilde{\pi} \right) \cdot \partial_\mu \partial_\nu \tilde{\pi} 
\]
\[ - \tilde{C}^{\text{total}}_{7, k} \left( \tilde{\pi} \cdot \partial_\mu \partial_\mu \tilde{\pi} \right)^2 - \tilde{C}^{\text{total}}_{8, k} \tilde{\pi}^2 \left( \partial_\mu \partial_\mu \tilde{\pi} \right)^2 \} \]
Flow equations – examples

\[ \partial_k U_k = \mathcal{V}^{-1} \left( \begin{array}{c}
\frac{1}{2} \sigma \\
+ \frac{1}{2} \pi \\
- \psi
\end{array} \right), \]

\[ \partial_k Z_{k}^{\pi} = \mathcal{V}^{-1} \left. \frac{d}{dp^2} \right|_{p^2=0} \left( \begin{array}{c}
\frac{1}{2} \pi \\
\sigma \\
3 \\
\pi \\
\sigma \\
3 \\
\pi - \frac{1}{2} \pi \\
\pi + \cdots
\end{array} \right), \]

\[ \partial_k C_{2,k} = \mathcal{V}^{-1} \left. \frac{d}{dp^2} \right|_{p^2=0} \left( \begin{array}{c}
-\frac{1}{2} \pi \\
\sigma \\
3 \\
\pi \\
\sigma \\
3 \\
\pi \\
\pi + \cdots
\end{array} \right), \]

\vdots

Masses and pion decay constant

**Figure 5:** Scale evolution of the renormalized meson and quark masses and the pion decay constant; grid method; [JE, Divotgey, Mitter, Rischke, PRD 98 (2018) no.1, 014024].
Wave-function renormalization

Figure 6: Scale evolution of the wave-function renormalization factors; grid method; [JE, Divotgey, Mitter, Rischke, PRD 98 (2018) no.1, 014024].
Low-energy couplings

Figure 7: Scale evolution of the renormalized low-energy couplings (l); grid method; [JE, Divotgey, Mitter, Rischke, PRD 98 (2018) no.1, 014024].
Low-energy couplings

Figure 8: Scale evolution of the renormalized low-energy couplings (II); grid method; [JE, Divotgey, Mitter, Rischke, PRD 98 (2018) no.1, 014024].
Figure 9: Scale evolution of the renormalized low-energy couplings (III); grid method; [JE, Divotgey, Mitter, Rischke, PRD 98 (2018) no.1, 014024].
Table 2: Low-energy couplings \((f_\pi = 93\ \text{MeV})\);
[JE, Divotgey, Mitter, Rischke, PRD 98 (2018) no.1, 014024].

| Coupling | Tree level | Tree | Trunc | Total |
|----------|------------|------|-------|-------|
| \(C_1\) | -0.2514 | 2.1063 | -2.3550 | -0.2487 |
| \(C_2\ [1/f^2_\pi]\) | 0.4054 | 0.3598 | -0.0426 | 0.3172 |
| \(Z_2\ [1/f^2_\pi]\) | - | - | -0.2068 | -0.2068 |
| \(C_3\ [1/f^4_\pi] \times 10^2\) | 1.7311 | 1.5364 | 3.3946 | 4.9310 |
| \(C_4\ [1/f^4_\pi] \times 10^2\) | - | - | 1.3752 | 1.3752 |
| \(C_5\ [1/f^4_\pi] \times 10^2\) | 3.4621 | 3.0728 | 5.8294 | 8.9023 |
| \(C_6\ [1/f^4_\pi] \times 10^2\) | - | - | -2.2697 | -2.2697 |
| \(C_7\ [1/f^4_\pi] \times 10^2\) | 1.7311 | 1.5364 | 0.8439 | 2.3804 |
| \(C_8\ [1/f^4_\pi] \times 10^2\) | - | - | 1.1828 | 1.1828 |
Figure 10: Renormalized masses and pion decay constant within the LPA’ truncation. Dashed lines show the related results from Fig. 5; [JE, Divotgey, Mitter, Rischke, PRD 98 (2018) no.1, 014024].
Figure 11: Wave-function renormalization factors within the LPA’ truncation. Dashed lines show the related results from Fig. 6; [JE, Divotgey, Mitter, Rischke, PRD 98 (2018) no.1, 014024].
Work in progress
Effective action – full $O(4)$ symmetry

- $O(4)$ vector $\varphi = (\vec{\pi}, \sigma)$
- Include momentum-dependent mixed $\sigma\pi$ vertices and $\sigma$ self-interactions,

$$
\Gamma_k = \int_x \left\{ \frac{Z_k}{2} (\partial_\mu \varphi) \cdot \partial_\mu \varphi + U_k(\rho) - h_{ESB}\sigma \\
+ C_{2,k} (\varphi \cdot \partial_\mu \varphi)^2 + Z_{2,k} \varphi^2 (\partial_\mu \varphi) \cdot \partial_\mu \varphi \\
- C_{3,k} [(\partial_\mu \varphi) \cdot \partial_\mu \varphi]^2 - C_{4,k} [(\partial_\mu \varphi) \cdot \partial_\nu \varphi]^2 \\
- C_{5,k} \varphi \cdot (\partial_\mu \partial_\nu \varphi) (\partial_\nu \varphi) \cdot \partial_\nu \varphi \\
- C_{6,k} \varphi^2 (\partial_\mu \partial_\nu \varphi) \cdot \partial_\mu \partial_\nu \varphi \\
- C_{7,k} (\varphi \cdot \partial_\mu \partial_\mu \varphi)^2 - C_{8,k} \varphi^2 (\partial_\mu \partial_\mu \varphi)^2 \\
+ \bar{\psi} \left( Z_{k}^{\psi} \gamma_\mu \partial_\mu + y_k \Phi_5 \right) \psi \right\}
$$

- Flow of Yukawa coupling taken into account
Figure 12: Renormalized masses and pion decay constant; Taylor-expansion method. Dashed lines show the related results from Fig. 5; [Eser, Divotgey, Mitter, in preparation].
Yukawa couplings – full $O(4)$ symmetry

Figure 13: Scale evolution of the renormalized Yukawa couplings; Taylor-expansion method; [Eser, Divotgey, Mitter, in preparation].
Low-energy couplings – full $O(4)$ symmetry

Figure 14: Renormalized low-energy couplings (I); Taylor-expansion method; Dashed lines show the related results from Fig. 8; [Eser, Divotgey, Mitter, in preparation].
Figure 15: Renormalized low-energy couplings (II); Taylor-expansion method; Dashed lines show the related results from Fig. 9; [Eser, Divotgey, Mitter, in preparation].
Numerical results – full $O(4)$ symmetry

Table 3: Low-energy couplings ($f_\pi = 93$ MeV); full $O(4)$ symmetry.

| Coupling | Tree level | FRG                  |
|----------|------------|----------------------|
|          |            | Tree | Trunc | Total  |
| $C_1$    | -0.2568    | 2.9993 | -3.2526 | -0.2533 |
| $C_2$    | 0.4290     | 0.4732 | -0.0671 | 0.4061  |
| $Z_2$    | -          | 0.2069 | -0.2244 | -0.0175 |
| $C_3$    | 1.2495     | -4.6110 | 2.7193 | -1.8917 |
| $C_4$    | -          | -     | 1.1660  | 1.1660  |
| $C_5$    | 2.4990     | -4.5794 | 4.4299 | -0.1494 |
| $C_6$    | -          | 1.9885 | -2.1565 | -0.1679 |
| $C_7$    | 1.2495     | 0.3885 | 0.0798  | 0.4684  |
| $C_8$    | -          | -0.9669 | 1.0485 | 0.0816  |
Figure 16: Bar representation of the low-energy couplings obtained from the tree-level and the FRG approaches.
Outlook
Functional QCD (fQCD)

• fQCD fluctuations:

\[ \partial_k \Gamma_k = \frac{1}{2} \]

• Dynamical-hadronization technique

[Gies, Wetterich '02; Mitter, Pawlowski, Strodthoff '15; Braun et al. '16]

• **Goal:** Determine low-energy couplings from fQCD

• **First approach:** Evaluate \( O(4) \) equations on fQCD solution
Figure 17: Renormalized meson and quark masses and the pion decay constant from fQCD (I).
Figure 18: Renormalized meson and quark masses and the pion decay constant from fQCD (II).
**Figure 19:** Pion wave-function renormalization from fQCD.
Figure 20: Quark wave-function renormalization from fQCD.
Figure 21: Low-energy couplings obtained from fQCD input (I).
Low-energy couplings from fQCD

Figure 22: Low-energy couplings obtained from fQCD input (II).
### Table 4: Low-energy couplings from fQCD input.

| Coupling | Tree level | FRG | O(4) | fQCD |
|----------|------------|-----|------|------|
| $C_1$    | $-0.2514$  |     | $-0.2487$ | $-0.3479$ |
| $C_2$ $[1/f_\pi^2]$ | $0.4054$ | | $0.3172$ | $0.4372$ |
| $Z_2$ $[1/f_\pi^2]$ | $-$ | | $-0.2068$ | $-0.2565$ |
| $C_3$ $[1/f_\pi^4] \times 10^2$ | $1.7311$ | | $4.9310$ | $7.1016$ |
| $C_4$ $[1/f_\pi^4] \times 10^2$ | $-$ | | $1.3752$ | $1.7625$ |
| $C_5$ $[1/f_\pi^4] \times 10^2$ | $3.4621$ | | $8.9023$ | $12.7534$ |
| $C_6$ $[1/f_\pi^4] \times 10^2$ | $-$ | | $-2.2697$ | $-2.9918$ |
| $C_7$ $[1/f_\pi^4] \times 10^2$ | $1.7311$ | | $2.3804$ | $3.6749$ |
| $C_8$ $[1/f_\pi^4] \times 10^2$ | $-$ | | $1.1828$ | $1.5770$ |
Conclusions
Conclusions

Summary:

• Low-energy couplings of the eLSM at tree level in reasonable agreement with ChPT, cf. [Divotgey, Kovacs, Giacosa, Rischke ’18]

• Low-energy limit of the $O(4)$ quark-meson model within the FRG confronted with a tree-level estimate, cf. [JE, Divotgey, Mitter, Rischke, PRD 98 (2018) no.1, 014024]

• Discrepancy between these approaches observed

Upcoming:

• Finalize analysis of the full $O(4)$-symmetric scenario
• Include vector mesons
• Compute low-energy limit from fQCD
• Confront the FRG calculation with ChPT