An Improved MUSIC Algorithm for DOA Estimation of Non-Coherent Signals with Planar Array

Liu Yaning¹,a, Fu Juntao¹,b, Ran Xinghao¹,c and Ming Le¹,d

¹Air and Missile Defence College, Air Force Engineering University

¹13231126178@163.com; b762050486@qq.com; c1275465401@qq.com; d2390043575@qq.com

Abstract-To reduce the computational complexity of two-dimensional (2-D) direction of arrival (DOA) estimation of non-coherent signals, this paper proposes a semi-real-valued MUSIC algorithm with planar array. Firstly, the real part of the received signal covariance matrix is derived and performed an eigen-decomposition to obtain the noise subspace. Then use the reduced-dimensional MUSIC and the least-square method to evaluate the 2-D DOA estimates. The proposed algorithm can avoid the high computational cost and have very close performance to conventional 2-D MUSIC algorithm. Simulation results verify the availability of the algorithm.

1. Introduction

Recently, two-dimensional (2-D) direction of arrival (DOA) has become a research hotspot of array signal processing. It can not only be applied to antenna, but also be a key parameter estimation technique of location of signal source.

The commonly used signal array models of 2-D DOA estimation include L-shape array, planar array and parallel linear array [1-3]. The conventional 2-D DOA estimation algorithms like MUSIC[4], ESPRIT[5], Capon[6] are complex. Therefore, more researches are focused on how to reduce the complexity of these algorithms. An ESPRIT algorithm with one single snapshot is derived in [7] to achieve the 2-D DOA estimation. [8,9] use one-dimensional MUSIC and Capon algorithm to reduce calculation amount based on the characteristic of Kronecker.

According to [10], the planar array can do better in resisting disturbance and more accurate estimation can be obtained. Based on the planar array, this paper proposes a semi-real-valued MUSIC algorithm which can significantly reduce calculation amount. The proposed algorithm can avoid the high computational cost and have very close performance to conventional 2-D MUSIC algorithm. What’s more, the automatic pairing of angle can be realized.

2. Signal model

As shown in Figure.1, we consider a planar array (PA) with N elements on each row and M elements on each column. The inter-element spacing is d. Suppose that K narrowband source signals impinge on the PA from directions (θ₁,φ₁),(θ₂,φ₂),…,(θₖ,φₖ), respectively. θ and φ represent azimuth and elevation of the kth signal source θₖ ∈ [−90°,90°], φₖ ∈ [−90°,90°]. αₖ and βₖ represent the angle between the incident direction and X axis and Y axis.
The received noise is white Gaussian noise which is uncorrelated with source signals. The output of the element is given by

$$y_{k,nm}(t) = \sum_{k=1}^{K} \delta_{k,nm} s_k(t - \tau_{k,nm}) + n_{nm}(t)$$  \hspace{1cm} (1)

where $y_{k,nm}(t)$ is the output of the $n$th row and $m$th column element. $\delta_{k,nm}$ is the gain of the element and $\tau_{k,nm}$ is the delay relative to the coordinate origin. $s_k(t) = \exp(2\pi f_k t)$ represent the $k$th signal with the frequency $f_k$. $n_{nm}(t)$ is white Gaussian noise with a mean value of 0 and a variance of $\sigma^2$.

According to the signal model, we can obtain the delay as:

$$\tau_{k,nm} = (n \cos \theta_k + m \sin \theta_k \sin \phi_k) \cdot d / \lambda$$  \hspace{1cm} (2)

As shown in fig 1, we can have the following relation:

$$\cos \alpha_k = \cos \theta_k \sin \phi_k$$  
$$\cos \beta_k = \sin \theta_k \sin \phi_k$$  \hspace{1cm} (3)

Put (3) into (2),

$$\tau_{k,nm} = (n \cos \alpha_k + m \cos \beta_k) \cdot d / \lambda$$  \hspace{1cm} (4)

We can know from (3) that:

$$\theta_k = \arctan(\cos \beta_k / \cos \alpha_k)$$
$$\phi_k = \arcsin(\alpha_k \cos \beta_k + \cos^2 \beta_k)^{1/2}$$  \hspace{1cm} (5)

Define the steering vector of the $k$th signal is $a_k$. The output of the array can be expressed by:

$$Y(t) = \sum_{k=1}^{K} \sum_{m=0}^{M} \sum_{n=0}^{N} (a_n(\alpha_k) \otimes a_m(\beta_k)) s_k(t) + N(t)$$  \hspace{1cm} (6)

where

$$a_n(\alpha_k) = (1, \exp(j2\pi d / \lambda) \cos \alpha_k, ..., \exp(j2\pi(N-1)d / \lambda) \cos \alpha_k)^T$$

$$a_m(\beta_k) = (1, \exp(j2\pi d / \lambda) \cos \beta_k, ..., \exp(j2\pi(M-1)d / \lambda) \cos \beta_k)^T$$  \hspace{1cm} (7)

3. Conventional 2-D MUSIC algorithm

2-D MUSIC is widely used to estimate azimuth and elevation of the signals. The covariance matrix of received signals can be described as:

$$R = \frac{1}{L} \sum_{t=1}^{L} Y(t_1) * Y(t_1)^H$$  \hspace{1cm} (8)

where $L$ is the number of snapshots

Do the eigenvalue decomposition and (8) can be written by:
\[
R = E_S U_S E_S^H + E_N U_N E_N^H
\]  \hspace{1cm} (9)

where \(U_S\) is a diagonal matrix whose diagonal elements contain the largest \(K\) eigenvalues and \(U_N\) stands for a diagonal matrix whose diagonal elements contain the smallest \(MN-K\) eigenvalues. \(E_S\) is the matrix composed of the eigenvectors corresponding to the largest \(K\) eigenvalues of \(R\), while \(E_N\) represents the matrix including the rest eigenvectors.

According to [11], the 2-D MUSIC spectrum function can be constructed as:

\[
P_{2D-MUSIC} = \frac{1}{a(\alpha, \beta)^H E_S E_N^H a(\alpha, \beta)}
\]  \hspace{1cm} (10)

\(\alpha \in (-90^\circ, 90^\circ], \beta \in (-90^\circ, 90^\circ]\)

Using the characteristic of Kronecker, (10) can be described as:

\[
P_{2D-MUSIC} = \frac{1}{[a_n(\alpha) \otimes a_n(\beta)]^H E_S E_N^H [a_n(\alpha) \otimes a_n(\beta)]}
\]  \hspace{1cm} (11)

This algorithm can effectively estimate 2-D DOA, but searching the peak in the whole space costs lots of calculation amounts. Therefore, we proposed an improved algorithm to solve the problem.

4. Improved 2-D MUSIC algorithm

4.1 2-D DOA estimation via semi-real-valued MUSIC(srv MUSIC)

Define the denominator in [11] is \(\Phi\):

\[
\Phi = [a_n(\alpha) \otimes a_n(\beta)]^H E_S E_N^H [a_n(\alpha) \otimes a_n(\beta)]
\]  \hspace{1cm} (12)

From (7),(10),(11), we can obtain that \(a(\alpha, \beta) = a_n(\alpha) \otimes a_n(\beta), a_n(\alpha) = a_n(\alpha), a_n(\beta) = a_n(-\beta)\). So:

\[
a(\alpha, \beta) = a_n(\alpha) \otimes a_n(\beta) = a_n(-\alpha) \otimes a_n(-\beta) = a(-\alpha, -\beta)
\]

\[
a(\alpha, \beta) = a_n(\alpha) \otimes a_n(\beta) = a_n(\alpha) \otimes a_n(-\beta) = a(\alpha, -\beta)
\]

\[
a(\alpha, \beta) = a_n(\alpha) \otimes a_n(\beta) = a_n(-\alpha) \otimes a_n(-\beta) = a(-\alpha, \beta)
\]  \hspace{1cm} (13)

It can be known from (13) that the search domain of 1-D DOA can be halved, \(\alpha \in [0, 90^\circ), \beta \in [0, 90^\circ]\). Therefore, the domain of 2-D DOA can be reduced to \([0, 4\]\). According to [12], the noise subspace can be obtained by the eigenvalue decomposition of the real covariance matrix \(\text{Re}(R)\).

According to the characteristic of Kronecker, \(a_n(\alpha) \otimes a_n(\beta)\) can be transformed into \([a_n(\alpha) \otimes I_M] a_n(\beta)\). (12) can also be expressed as:

\[
\Phi = a_n(\beta)^H [a_n(\alpha) \otimes I_M]^H E_S E_N^H [a_n(\alpha) \otimes I_M] a_n(\beta)
\]  \hspace{1cm} (14)

Define \(P(\alpha) = [a_n(\alpha) \otimes I_M]^H E_S E_N^H [a_n(\alpha) \otimes I_M]\). It’s easy to calculate \(a_n(\beta)^H a_n(\beta) = M\). So (14) can be transformed to:

\[
\Phi = M \cdot a_n(\beta)^H P(\alpha) a_n(\beta)
\]  \hspace{1cm} (15)

According to Rayleigh-Ritz theory, the minimum of (15) is \(\lambda_{\min}(\alpha)\) which is the minimum eigenvalue of \(P(\alpha)\). It can be described by:

\[
\min a_n(\beta)^H P(\alpha) a_n(\beta) = \lambda_{\min}(\alpha)
\]  \hspace{1cm} (16)

So the estimation of \(\alpha\) can be obtained by:
The estimation of \( k \) received signals’ \( \alpha \) are recorded separately as \( (\alpha_1, \alpha_2, ..., \alpha_k) \). Then estimate \( \beta \).

Define \( \phi_k = \text{angle}(a, (\beta_i)) \), Combining with (7), we can know that:

\[
\phi_k = [0, (2\pi d / \lambda) \cos \beta_k, ..., (2\pi (M-1)d / \lambda) \cos \beta_k]^T
\]

According to the least-square method, define

\[
C = \begin{bmatrix}
1 & 0 \\
1 & 1 \\
M & M \\
1 & M-1
\end{bmatrix}
\]

Then solve the minimum of \( \|C d_k - \phi_k\|^2 \), and the result is \( d_k^T = (C^T C)^{-1} C^T \phi_k \). \( \beta_k \) can be sovled by following formula:

\[
\beta_k = \arcsin(d_k \left( \frac{2}{2\pi d / \lambda} \right))
\]

4.2 steps of algorithm

Step 1: Solve the covariance matrix \( R \) of received signals and derive the real part \( \text{Re}(R) \). Do the eigenvalue decomposition on it to obtain the real number domain noise subspace.

Step 2: Construct \( P(\alpha) \) based on the characteristic of Kronecker and solve the minimum eigenvalue \( \lambda_{\min}(\alpha) \). Put it in (17) to solve the estimation of \( \alpha \).

Step 3: Put the estimation of \( \alpha \) in (16) and obtain the eigenvectors. Then following (18)-(20), use least-square method to get the estimation of \( \beta \).

Step 4: Obtain the 2-D DOA estimation according to (5).

4.3 Analysis on computation complexity

Analyses the computation complexity according to the steps above and compare it with conventional 2D-MUSIC. Assume the number of signals is \( K \), the snapshots are \( L \), the search points are \( n \). Total calculation is \( \mathcal{O}(MN^2 + (MN)^3 / 4 + n(MN - K)(M^2N + M^2) + 10M) \). And the calculation of 2D-MUSIC is \( \mathcal{O}(MN^2 + (MN)^3 + 2n(MN - K)(M^2N + M^2) + N^2) \).

To see more intuitively, specific values are given to draw the Figure.2. Assume \( N = M \), \( K = 3 \), \( L = 200 \), \( n = 3 \times 10^7 \). It can be seen from Figure.2 that the algorithm in this paper can significantly reduce the computation complexity compared with 2-D MUSIC.
5. Simulation results and analysis

5.1 Algorithm evaluation
To evaluate the algorithm, simulation is carried out just as following. We present 500 Monte Carlo simulations to assess the angle estimation of the algorithm. Define root mean squared error (RMSE). We adopt the array with \( N = M = 8 \), and \( d = \frac{\lambda}{2} \). Let SNR is 20dB and \( L = 100 \). Assume that there are two non-coherent source signals located at angles \((\theta_1, \phi_1) = (10^\circ, 15^\circ), (\theta_2, \phi_2) = (25^\circ, 35^\circ)\). Figure 3 presents angle estimation results of the algorithm for all targets. It can be seen that 2-D DOAs can be clearly observed. So it is indicated that the algorithm has high accuracy in 2-D DOA estimation.

![Figure 3 DOA estimation with the algorithm for two targets](image)

5.2 RMSE versus SNR
The number of Monte Carlo tests is 500. The number of snapshots is set as 100. There are both 8 array elements on x axis and y axis. The range of SNR is from -4dB to 20dB with interval 4dB. Compare the proposed algorithm with 2-D MUSIC, and the results are shown in Figure 4.

![Figure 4 RMSE versus SNR](image)
It can be seen that the algorithm in this paper is a little worse than 2-D MUSIC when the SNR is low. But the difference will be narrowed with the SNR being higher, and the estimation accuracy will be mainly the same. What's more, the RMSE curves become lower and lower with the SNR increasing. It indicates that the estimation performance becomes better, however, the trend of becoming better slows down.

6. Conclusion
Based on the conventional 2-D MUSIC, this paper proposes an improved MUSIC algorithm to reduce the computation complexity. Compared with 2-D MUSIC, the search domain can be halved. Therefore, the calculation amounts can be reduced significantly. Through simulation verify, the proposed algorithm can have very close performance to 2-D MUSIC. All in all, this algorithm is an efficient 2-D DOA estimation algorithm.

References
[1] N. T, H. M. K. L-shape 2-dimension arrival angle estimation with propagator method[J]. IEEE Trans. on Antennas and Propagation. 2005, 53(1):1622-1630.
[2] N. Y, T. K. S. 2-D unitary matrix pencil method for efficient direction of arrival estimation[J]. Digital Signal Processing. 2006, 16(6):767-781.
[3] He Z Q, Liu Q H, Jin LN. Low complexity method for DOA estimation using array covariance matrix sparse representation[J]. Electronics Letters. 2013, 49(3):228-230.
[4] Schmidt R O. Multiple emitter location and signal parameter estimation[J]. IEEE Trans.on Antennas and Propagation. 1986, 34(3):276-280.
[5] Capon J. High-resolution frequency wavenumber spectrum analysis[J]. proceedings of the IEEE. 1987, 57(8):1408-1418.
[6] Cai J J, Li P, Zhao G Q. Two-dimensional DOA with reduced-dimension MUSIC[J]. Journal of Xidian University. 2013, 40(3):81-86.
[7] Zhang X, Xu D. Angle estimation in MIMO radar using reduced-dimension Capon[J]. Electronics Letters. 2010, 46(12):860-861.
[8] Zhang X, Xu L. DOD and DOA estimation in MIMO radar with reduced-dimension MUSIC [J]. IEEE Communications Letters. 2010, 14(12):1161-1163.
[9] Heidenreich P, Zoubir AM, Rubsamen M. Joint 2D DOA estimation and phase calibration for uniform rectangular arrays[J]. IEEE Transactions On Signal Processing. 2012, 60(9):4683-4693.
[10] Liu X D, Zhou X L, Xiao J G. Single Snapshot DOA Estimation Algorithm Based on Spatial Smoothing[J]. Journal of Detection & Control. 2015, 37(06):66-70.
[11] Yan F, Jin M, Liu S. Real-valued MUSIC for efficient direction estimation with arbitrary arrays[J]. IEEE Trans. on Signal Processing. 2014, 62(6):1548-1560.
[12] Golub GH, Loan CF. Matrix computations[M]: The John Hopkins University Press. 1996.