Natural Convection of Dusty Hybrid Nanofluids in an Enclosure Including Two Oriented Heated Fins

Zehba A.S. Raizah

Department of Mathematics, Faculty of Science, King Khalid University, Abha 61421, Saudi Arabia; zaalrazh@kku.edu.sa; Tel.: +9-661-7241-4970

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Abstract: In the current work, the natural convection of dusty hybrid nanofluids in an enclosure including two inclined heated fins has been studied via mathematical simulation. The inclined heated fins are arranged near to the enclosure center with variations on their orientations and lengths. The present simulation is represented by two systems of equations for the hybrid nanofluids that are dusty. The pressure distributions for the dusty phase and hybrid nanofluids phase are evaluated using a SIMPLE algorithm based on the finite volume method. The numerical results are examined using contours of the streamlines, isotherms for the hybrid nanofluids and velocity components for the dusty phase. In addition, the graphical illustrations for profiles of the local and average Nusselt numbers are presented. The main results reveal that an increase in the mixture densities ratio and dusty parameter reduces the rate of the heat transfer. Both the local and average Nusselt numbers are supported as the fins lengths increase regardless of the fins’ rotation. In addition, the nanoparticles volume fraction enhances the thermal boundary layer near the top wall.

Keywords: enclosure; dusty fluids; heated fins; hybrid nanofluids; natural convection

1. Introduction

The studies of the flow of fluids with suspended dust particles have been attracted the attention of numerous researchers due to their practical applications in various problems of atmospheric, engineering and physiological fields [1]. Farbar and Morley [2] were the first researchers who analyzed the gas-particulate suspension on experimental grounds. Singleton [3] studied the boundary layer analysis for the dusty fluid. Gireesha et al. [4] had been considered the geometry of the laminar flow of an unsteady viscous fluid with a uniform distribution of dust particles through a rectangular channel under the influence of the pulsatile pressure gradient. Later, numerous authors [5–10] investigated the dynamics of the two-phase flow under different physical circumstances.

The method of the suspending dissimilar nanoparticles in either mixture or composite form was known as the hybrid nanofluid. Hybrid nanofluids are a new type of enhanced working fluids, engineered with enhanced thermophysical properties. The hybrid nanofluids profit from the thermo-physical properties of more than one type of nanoparticles. In recent years, many authors have used hybrid nanofluids [11–13]. Mehryan et al. [14] studied the free convective heat transfer of the Al$_2$O$_3$-Cu water hybrid nanofluid in a cavity filled with a porous medium. They considered two types of important porous media, glass ball and aluminum metal foam, for the porous matrix. Hussain et al. [15] performed computational analyses in a horizontal channel with an open cavity filled with a hybrid nanofluid of Al$_2$O$_3$-Cu/water. They studied the effect of Richardson number, nanoparticle volume fraction and Hartmann number on the streamlines, isotherms, average Nusselt number and entropy generation. Mansour et al. [16] investigated the effects of magnetic field on the natural convection flow of hybrid, Al$_2$O$_3$-water, or Cu-water nanofluids, in a square cavity with two pairs of heat source-sink. Ashorynejad and Shahriari [17] adopted the lattice Boltzmann method...
to study the natural convection heat transfer of Al$_2$O$_3$-Cu/water hybrid nanofluid within an open wavy cavity subjected to a uniform magnetic field. The magnetohydrodynamic (MHD) flow and heat transfer of non-Newtonian micropolar dusty fluid suspended with Cu-Al$_2$O$_3$ hybrid nanoparticles past a stretching sheet were investigated by Ghadikolaei et al. [18]. Sheikholeslami et al. [12] presented the analysis of magnetizable hybrid nanofluid of multi-walled carbon nanotubes (MWCNT)-Fe$_3$O$_4$/H$_2$O inside a circular cavity with two circular heaters. Chamkha et al. [19] adopted the Control volume finite element method (CVFEM) method to study the effects of thermal radiation and shape factor of nanoparticles’ impacts on magnetohydrodynamic nanofluid natural convection in a cavity.

On the other hand, Mozaﬀari et al. [20] studied the effects of asphaltene aggregation on rheological properties of diluted athabasca bitumen. Darjani et al. [21] derived the equation of state of lattice gases based on the random sequential adsorption simulations. Mozaﬀari et al. [22,23] introduced capillary driven flow to probe liquids’ rheology in nanofluidic systems and adopted an engine model consisting of a spherical Janus colloid coated with a symmetrical catalyst cap, which converts fuel into a product solute. Hosseini et al. [24] studied numerically the effects of poly disperse particle size, particle mass and flow velocity on the deposition ratio in fin channels.

To enhance the heat transfer rate within the cavities, some of the researchers added fins to the cavity walls. Liu et al. [25] studied experimentally the natural convection flow adjacent to a finned sidewall of a differentially heated cavity using the shadowgraph technique. Varol and Ozgen [26] studied the natural convection in a square cavity including an inclined fin on the wall. Kolsi et al. [27] analyzed the natural convection and entropy generation in a three-dimensional cavity with an inclined fin. They observed that the inclination angle significantly affect the heat transfer and the fluid flow. Elatar et al. [28] studied the laminar natural convection in a square cavity attached with a horizontal fin. They investigated the effect of different aspects of the fins such as length, thickness, conductivity ratio, and position. Ma and Xu [29] performed a numerical study on the unsteady natural convection of a differentially heated finned cavity. It was shown that the heat transfer is a function of the Rayleigh number and the fin positions. Azimifar and Payan [30] analyzed the natural convection in a finned cavity heated from one side. They found that applying optimal thin fins reduces the heat transfer at the cavity wall up to 8%. Torabi et al. [31] analyzed the natural convection inside a partially differentially heated cavity with a thin fin. Imani [32] studied a three-dimensional, steady and transient natural convection in a finned cubical cavity. Hatami [33] reviewed the natural convection of nanofluid flow in a rectangular cavity with two heated fins. The results indicated that the long fins give higher values of heat transfer. Gao et al. [34] used a lattice Boltzmann model to simulate melting of the phase change materials in the porous media with a conducting fin. Ghalambaz et al. [35] investigated the natural convection heat transfer and the fluid flow inside a square cavity affected by an oscillating fin on the hot wall. Alnaqi et al. [27] investigated the effects of the radiation and magnetic field on the heat transfer rate and the nanofluid entropy generation in a diagonal square cavity with a conductor fin. Abdi et al. [36] investigated the effects of the vertical fins on the heat transfer rate and energy density of a latent heat thermal energy storage system. They used the vertical fins on the bottom surface to enhance the charging rate.

The main objective of this study is to investigate the heat transfer and fluid flow of dusty hybrid nanofluids inside an enclosure that includes two inclined heated fins using one phase model. The governing equations are presented, separately, for the hybrid nanofluid phase and the dusty phase. Comparing the heat transfer rate in cases of pure dusty fluid and hybrid nanofluid is considered one of the objectives. In addition, this study aims to show the best length and best orientation of the fins to enhance the heat transfer rate inside the enclosure.

2. Problem Description

Consider a two-dimensional and laminar flow of incompressible Al$_2$O$_3$-Cu hybrid nanofluid in a rectangle enclosure, as depicted in Figure 1. The following assumptions are taken into account through the study of this phenomenon:
The $x$-axis is along the bottom wall and $y$-axis is along the left wall and the origin is intersection point of these two walls. $L_1$ and $L_2$ are the lengths of left and right fins.

Two inclined fins are located in the enclosure at $x = s_1$ and $x = s_2$ and the relation between $(x', y')$, $(x'', y'')$ and $(x, y)$ (Note that the left fin rotates in clockwise direction and the right fins rotates in anticlockwise direction) are given as follows: $(x', x'') = x \cos \gamma_1 - y \sin \gamma_1$, $(y', y'') = x \sin \gamma_1 + y \cos \gamma_1$.

The flow is laminar and incompressible.

A one-phase model is applied for the hybrid nanofluids in which the nanoparticles volume fraction and the thermophysical properties of the base fluid and nanoparticles' volume fraction are constants.

Table 1 shows the values of the thermophysical properties of the base fluid and nanoparticles at 20 °C.

The Boussinesq approximation is taken into account.

Effects of the gravity are considered in the vertical direction.

The viscous dissipation is neglected in the present study.

A system of equations is performed for the hybrid nanofluid and another one is presented for the dusty phase.

**Table 1.** Thermo-physical properties of the base fluid and nanoparticles.

| Property      | Water | Copper (Cu) | Alumina (Al$_2$O$_3$) |
|---------------|-------|-------------|------------------------|
| $\rho$ (kg/m$^3$) | 997.1 | 8933        | 3970                   |
| $C_p$ (J/kg·K) | 4179  | 385         | 765                    |
| $k$ (W/m·K)   | 0.613 | 401         | 40                     |
| $\beta$ (1/K) | $21 \times 10^{-5}$ | $1.67 \times 10^{-5}$ | $0.85 \times 10^{-5}$ |

![Figure 1. Physical model and coordinates system.](image)

Under all the above assumptions, the governing equations (continuity, momentum and energy equations for both the hybrid nanofluid and dusty phases) are expressed as follows; see [7,24,29,30]:

### 2.1. Dusty Hybrid Nanofluid

$$\frac{\partial u_f}{\partial x} + \frac{\partial v_f}{\partial y} = 0$$  \hspace{1cm} (1)
\[
\frac{\partial u_f}{\partial t} + u_f \frac{\partial u_f}{\partial x} + v_f \frac{\partial u_f}{\partial y} = -\frac{\partial p_f}{\partial x} - \alpha_f (u_s - u_f) + H_1(\phi_{Al2O_3}, \phi_{Cu}) \left( \frac{\partial^2 u_f}{\partial x^2} + \frac{\partial^2 u_f}{\partial y^2} \right) + H_2(\phi_{Al2O_3}, \phi_{Cu}) D_s \alpha_s (u_s - u_f)
\]

(2)

\[
\frac{\partial v_f}{\partial t} + u_f \frac{\partial v_f}{\partial x} + v_f \frac{\partial v_f}{\partial y} = -\frac{\partial p_f}{\partial y} - \alpha_f (v_s - v_f) + H_1(\phi_{Al2O_3}, \phi_{Cu}) \left( \frac{\partial^2 v_f}{\partial x^2} + \frac{\partial^2 v_f}{\partial y^2} \right) + H_2(\phi_{Al2O_3}, \phi_{Cu}) \frac{\text{Ray}}{Pr} \theta_f
\]

(3)

\[
\frac{\partial \phi}{\partial t} + u_f \frac{\partial \phi}{\partial x} + v_f \frac{\partial \phi}{\partial y} = \frac{H_1(\phi_{Al2O_3}, \phi_{Cu})}{\tau_f} \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + H_2(\phi_{Al2O_3}, \phi_{Cu}) \left[ \frac{2}{\text{Ray}} D_s \alpha_s (\theta_s - \theta_f) \right]
\]

(4)

2.2. For the Particle Phase

\[
\frac{\partial u_s}{\partial x} + \frac{\partial v_s}{\partial y} = 0
\]

(5)

\[
\frac{\partial u_s}{\partial t} + u_s \frac{\partial u_s}{\partial x} + v_s \frac{\partial u_s}{\partial y} = -\frac{\partial p_s}{\partial x} - \alpha_s (u_s - u_f)
\]

(6)

\[
\frac{\partial v_s}{\partial t} + u_s \frac{\partial v_s}{\partial x} + v_s \frac{\partial v_s}{\partial y} = -\frac{\partial p_s}{\partial y} - \alpha_s (v_s - v_f)
\]

(7)

\[
\frac{\partial \theta_s}{\partial t} + u_s \frac{\partial \theta_s}{\partial x} + v_s \frac{\partial \theta_s}{\partial y} = -\frac{2}{3 \gamma Pr} \alpha_s (\theta_s - \theta_f)
\]

(8)

In Equations (1)–(8), \((u_f, v_f, u_s, v_s)\) are components of the velocity in \((x, y)\) directions where \(f\) and \(s\) refer to the hybrid nanofluid phase and particle phase respectively, \(\theta\) is the temperature \(\tau\) is the time and \(g\) is the gravity. \(\alpha\) is the thermal diffusivity, \(p\) is the density, \(C_p\) is the specific heat and \(\nu\) is the kinematic viscosity. \(\phi\) is the nanoparticle volume fraction, \(\rho\) is the pressure, \(k\) is the thermal conductivity.

\[RaE = \frac{g \Delta T \theta^3}{\nu \alpha_f}\]

is the Rayleigh number, \(Pr = \frac{\nu_f}{\alpha_f}\) is the Prandtl number, \(D_s = \frac{\rho_s}{\rho_f}\) is ratio of the mixture densities, \(\gamma = \frac{C_p}{C_f}\) is ratio of the specific heat of the mixture, \(\alpha_s = \frac{\rho F}{\tau_f}\) is the dusty parameter and \(\tau_f = \frac{\gamma_T n}{\nu_f} \frac{\rho_f}{\rho_m}\) is mass concentration of the particle phase. It is important to mention here that for different mixtures, the interaction term \(\gamma\) may vary between 0.1 and 10.0. Also, more information on the kinds of the dusty particle, its size, mass concentration of particle phase and the dust parameters can be found in the valuable study presented by Rudinger [1]. Moreover, \(H_i(i = 1, 2, 3, 4, 5)\) are functions, those are expressed as follows [17,37]:

\[
\phi = \phi_{Al2O_3} + \phi_{Cu} \rho_{Cu}
\]

(9)

\[
H_1(\phi_{Al2O_3}, \phi_{Cu}) = \frac{\mu_{h\phi}}{\mu_f} \frac{P_f \rho_{h\phi}}{\phi_f} = \frac{1}{[1 - \phi]^{2.5}} \left[ \frac{1}{1 - \phi} \frac{\phi_{Al2O_3} \phi_{Al2O_3} + \phi_{Cu} \rho_{Cu}}{\rho_f} \right]
\]

(10)

\[
H_2(\phi_{Al2O_3}, \phi_{Cu}) = \frac{\rho_f}{\rho_{h\phi}} = \frac{1}{[1 - \phi]} \frac{\phi_{Al2O_3} \phi_{Al2O_3} + \phi_{Cu} \rho_{Cu}}{\rho_f}
\]

(11)
The corresponding boundary conditions are given by:

\[
H_3(\phi_{Al_{2}O_{3}}, \phi_{Cu}) = \frac{\rho_f}{p_{mf}} \left( \frac{\eta_f y}{\eta_f f} \right) \left[ 1 - \phi + \frac{\phi_{Al_{2}O_{3}} (\rho f y_f)_{Al_{2}O_{3}} + \phi_{Cu} y_y}{\rho f} \right] \]

(12)

\[
H_4(\phi_{Al_{2}O_{3}}, \phi_{Cu}) = \frac{\alpha_{mf}}{H_f} = \left[ \frac{1}{1 - \phi + \frac{\phi_{Al_{2}O_{3}} (\rho f y_f)_{Al_{2}O_{3}} + \phi_{Cu} y_y}{\rho f}} \right] \left[ \phi_{Al_{2}O_{3}} k_{Al_{2}O_{3}} + 2k_f \right] + 2 \phi k_f \left( \phi_{Al_{2}O_{3}} k_{Al_{2}O_{3}} + \phi_{Cu} k_{Cu} \right) - 2 \phi k_f \left( \phi_{Al_{2}O_{3}} k_{Al_{2}O_{3}} + \phi_{Cu} k_{Cu} \right) + 2 k_f \left( \phi_{Al_{2}O_{3}} k_{Al_{2}O_{3}} + \phi_{Cu} k_{Cu} \right) + \phi k_f \]

(13)

\[
H_5(\phi_{Al_{2}O_{3}}, \phi_{Cu}) = \left( \frac{\rho C_f}{\eta_f f} \right) = \left[ \frac{1}{1 - \phi + \frac{\phi_{Al_{2}O_{3}} (\rho f y_f)_{Al_{2}O_{3}} + \phi_{Cu} y_y}{\rho f}} \right] \]

(14)

The previous equations are non-dimensionalized using the following quantities:

\[
\tau = \frac{\nu H}{\nu_f}, (x, y) = \left( \frac{x', y'}{H} \right), \left( u_f, v_f, u_s, v_s \right) = \left( \frac{u_f, v_f, u_s, v_s}{v_f} \right), \left[ \theta_f, \theta_s \right] = \left( \frac{\theta_f, \theta_s}{\theta_f} \right) = \left( \frac{\theta_f, \theta_s}{p_{mf}} \right)
\]

(15)

The corresponding boundary conditions are given by:

On the top wall: \( u_f = v_f = u_s = v_s = 0, \theta_f = \theta_s = 0 \).

(16)

On the left and right walls: \( u_f = v_f = u_s = v_s = 0, \frac{\partial \theta_f}{\partial n} = \frac{\partial \theta_s}{\partial n} = 0 \).

(17)

On the bottom wall: \( u_f = v_f = u_s = v_s = 0, \frac{\partial \theta_f}{\partial n} = \frac{\partial \theta_s}{\partial n} = 0 \).

(18)

On the inclined fins \( \theta_f = \theta_s = 1 \).

(19)

The local Nusselt number in case of hybrid nanofluid that is computed at the cold wall is defined as:

\[
Nu = -\frac{H_4(\phi_{Al_{2}O_{3}}, \phi_{Cu}) \partial \theta_f}{H_5(\phi_{Al_{2}O_{3}}, \phi_{Cu}) \partial \eta_y}
\]

(20)

Moreover, the average Nusselt number is expressed as:

\[
\overline{Nu} = \frac{1}{A} \int_0^A Nu \, dX
\]

(21)
3. Numerical Method and Validation

The time-dependent partial differential Equations (1)–(8) with dimensionless boundary conditions (16) are solved numerically using the FVM (finite volume method) with the SIMPLE algorithm [38–44]. This treatment starts with re-writing the systems of equations in the following general forms:

\[
\int_\Omega \partial_t \Phi dV + \int_\Omega (\hat{\partial} \Phi - \Gamma_\Phi \nabla \Phi) \cdot \nabla dS = \int_\Omega S_\Phi dV
\]  

(22)

where \( \Omega \) is the control volume with volume \( V \) and \( \Phi \) refers to \( u_f, v_f, \theta_f, u_s, v_s \) and \( \theta_s \). In addition, the UDS (upwind differences scheme) is applied to the convective terms while the diffusive fluxes are approximated using the CDS (central differences scheme) then the following algebraic system is obtained:

\[
\frac{a^p_\Phi}{\alpha_\Phi} \Phi_p = a_E \Phi_E + a_W \Phi_W + a_N \Phi_N + a_S \Phi_S + \sum_n + \frac{(1 - \alpha_\Phi)}{\alpha_\Phi} a^p_\Phi \Phi_p^{n-1}.
\]

(23)

The previous systems are solved using the ADI (alternating direction implicit) technique. Here, it should be mentioned that two SIMPLE algorithms are performed for the hybrid nanofluid system and dusty systems. Also, the collected grid-system is used and the interface velocity at cell face \( e \) (east) is evaluated using the following interpolation:

\[
u_e = f^+ u_E + (1 - f^+) u_p - \frac{\alpha_u \Delta Y [P_E - P_p]}{(a^p_\nu)_E} + f^+ \frac{\alpha_u \Delta Y [P_E - P_w]}{(a^p_\nu)_E} + (1 - f^+) \frac{\alpha_u \Delta Y [P_e - P_w]}{(a^p_\nu)_p}.
\]

(24)

A home code is performed and the convergence criteria of \( 10^{-6} \) are applied. In order to choose the suitable grid size, a study of a grid independency is carried out and presented in Figure 2. It is found that 100 \( \times \) 121 is suitable for the systems of equations. Also, a validation test is conducted and presented in Figure 3 to check the accuracy of the present results. In this figure, comparison of the streamlines and isotherms contours between the present results and those obtained by Hatami [33] is presented. A well agreement is observed between the results. Moreover Table 2 presents a quantitative comparison between the present results, in special cases (\( Ra = 10^5, \phi_{Al_2O_3} = \phi_{Cu} = 0\%, D_s = 0, \alpha_s = 0 \)) and those reported in Grosan et al. [45]. It is found that the percentages of the errors in values of \( \psi_{max} \) and \( \theta_{max} \) are 0.58\% and 1.16\%, respectively, which confirms accuracy of the present results.

Figure 2. Grid study at \( \phi_{Al_2O_3} = \phi_{Cu} = 0.02, D_s = 10, \alpha_s = 0.01, Ra = 10^5, L_1 = L_2 = 0.5, \gamma_1 = 45, \) and \( \gamma = 0.1. \)
4. Results and Discussion

This section discusses results of the natural convection for dusty hybrid nanofluids in an enclosure including two inclined heated fins. The streamlines and isotherms are represented in contours forms. The local and average Nusselt numbers are represented in curves profiles under effects of the current physical parameters. The variations of parameters are summarized as: the inclination angle \( \sim \sim \sim \) for \( \sim \sim \sim \) fins, lengths \( \sim \sim \sim \), nanoparticles volume fraction \( \sim \sim \sim \) and dusty parameter \( \sim \sim \sim \). The fixed parameters are: Rayleigh number \( \sim \sim \sim \) and ratio of specific heat of the mixture \( \sim \sim \sim \). The present results.

4.1. Effects of Fins’ Geometric Parameters

Figures 4–7 depict effects of the inclination angle of the heated-fins on contours of the streamlines, isothermal, and dusty horizontal and vertical velocities at \( \sim \sim \sim \) \( \sim \sim \sim \), \( \sim \sim \sim \), \( \sim \sim \sim \), \( \sim \sim \sim \) and \( \sim \sim \sim \). Distributions of the isothermal lines are increased horizontally across the enclosure as the inclination angle of heated fins increases from \( \sim \sim \sim \) to \( \sim \sim \sim \). Also, the formation of streamlines inside the enclosure depends on the inclination angle of heated fins. There are four cells of the streamlines around the two heated fins, and the two wider cells are formed near to the enclosure walls. As the inclination angle increases, the wider cells of streamlines are shrinking and the center cells are expanding. The maximum values of the stream function are fluctuating as inclination angle increases from \( \sim \sim \sim \) to \( \sim \sim \sim \). In addition, the distributions of the horizontal and vertical dusty velocity contours are depending on the orientation of the heated fins. The maximum of the horizontal and vertical dusty velocities is slightly fluctuating according to an increase on the inclination angle of heated fins shapes. Moreover, the horizontal and vertical dusty velocities along the enclosure are distributed according to variations on the inclination angle.
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Figure 4. Isothermal contours under the effects of an inclination angle of heated-fins at $\phi_{Al_2O_3} = \phi_{Cu} = 0.02$, $Da = 10$, $\alpha_s = 0.01$, $Ra = 10^5$, $L_1 = L_2 = 0.5$ and $\gamma = 0.1$. (a) $\gamma_1 = 0^\circ$; (b) $\gamma_1 = 30^\circ$; (c) $\gamma_1 = 45^\circ$; (d) $\gamma_1 = 60^\circ$.

Figure 5. Streamlines contours under the effects of an inclination angle of heated-fins at $\phi_{Al_2O_3} = \phi_{Cu} = 0.02$, $Da = 10$, $\alpha_s = 0.01$, $Ra = 10^5$, $L_1 = L_2 = 0.5$ and $\gamma = 0.1$. (a) $\gamma_1 = 0^\circ$ ($|\psi|_{max} = 2.8446$); (b) $\gamma_1 = 30^\circ$ ($|\psi|_{max} = 2.7720$); (c) $\gamma_1 = 45^\circ$ ($|\psi|_{max} = 2.8013$); (d) $\gamma_1 = 60^\circ$ ($|\psi|_{max} = 2.7946$).

Figure 6. Horizontal dusty velocity contours under the effects of an inclination angle of heated-fins at $\phi_{Al_2O_3} = \phi_{Cu} = 0.02$, $Da = 10$, $\alpha_s = 0.01$, $Ra = 10^5$, $L_1 = L_2 = 0.5$ and $\gamma = 0.1$. (a) $\gamma_1 = 0^\circ$ ($|u_s|_{max} = 0.1468$); (b) $\gamma_1 = 30^\circ$ ($|u_s|_{max} = 0.1463$); (c) $\gamma_1 = 45^\circ$ ($|u_s|_{max} = 0.1464$); (d) $\gamma_1 = 60^\circ$ ($|u_s|_{max} = 0.1457$).
Figures 7 and 9 present the effects of the heated-fins lengths with two values of an inclination angle. Figures 10 and 11 show the horizontal and vertical dusty velocity contours under effects of the inclination angle of heated-fins at $\phi_{Al_2O_3} = \phi_{Cu} = 0.02$, $D_s = 10$, $\alpha_s = 0.01$, $Ra = 10^5$, $L_1 = L_2 = 0.5$ and $\gamma = 0.1$. (a) $\gamma_1 = 0^\circ$, $(|v_0|_{max} = 0.2195)$; (b) $\gamma_1 = 30^\circ$; (c) $\gamma_1 = 45^\circ$, $(|v_0|_{max} = 0.1463)$; (d) $\gamma_1 = 60^\circ$, $(|v_0|_{max} = 0.2229)$.

Figures 8 and 9 present the effects of the heated-fins lengths with two values of an inclination angle $\gamma_1 = 0^\circ$ and $45^\circ$ on the streamlines and isothermal contours. It is observed that, as expected, the increase in lengths of the heated fins enhances the heated area inside the enclosure and consequently both of the temperature distributions and thermal boundary layers are supported. Also, the inclination of the heated fins makes higher heated area inside the enclosure. In addition, the maximum of the stream function decreases as the lengths of the heated fins are increased. The streamlines form different cells according to lengths and orientation of the heated fins. Figures 10 and 11 show the horizontal and vertical dusty velocity contours under effects of the inclination angle of the heated-fins at $\phi_{Al_2O_3} = \phi_{Cu} = 0.02$, $D_s = 10$, $\alpha_s = 0.01$, $Ra = 10^5$ and $\gamma = 0.1$. From these figures, the maximum of the horizontal and vertical dusty velocities increases as the heated fins’ lengths increase from 0.3 to 0.6.
fraction enhances the viscosity of the fluid and hence the fluid activity is reduced. This can be proved by observing the maximum value of the stream function, at \(\phi_{Al_2O_3} = \phi_{Cu} = 0.02, D_s = 10, \alpha_s = 0.01, Ra = 10^5\) and \(\gamma = 0.1\). (a) \(L_1 = L_2 = 0.3, (|\psi|_{\text{max}} = 2.9405)\); (b) \(L_1 = L_2 = 0.6, (|\psi|_{\text{max}} = 2.8480)\); (c) \(L_1 = L_2 = 0.9, (|\psi|_{\text{max}} = 2.8227)\); (d) \(L_1 = L_2 = 0.3, (|\psi|_{\text{max}} = 3.0054)\); (e) \(L_1 = L_2 = 0.6, (|\psi|_{\text{max}} = 2.7263)\); (f) \(L_1 = L_2 = 0.9, (|\psi|_{\text{max}} = 2.5927)\).

Figure 10. Horizontal dusty velocity contours under the effects of heated-fins lengths at inclination angle \(\gamma_1 = 0^\circ, \phi_{Al_2O_3} = \phi_{Cu} = 0.02, D_s = 10, \alpha_s = 0.01, Ra = 10^5\) and \(\gamma = 0.1\). (a) \(L_1 = L_2 = 0.3, (|\psi|_{\text{max}} = 0.1993)\); (b) \(L_1 = L_2 = 0.6, (|\psi|_{\text{max}} = 0.1477)\).

Figure 11. Vertical dusty velocity contours under the effects of heated-fins lengths at inclination angle \(\gamma_1 = 0^\circ, \phi_{Al_2O_3} = \phi_{Cu} = 0.02, D_s = 10, \alpha_s = 0.01, Ra = 10^5\) and \(\gamma = 0.1\). (a) \(L_1 = L_2 = 0.3, (|\psi|_{\text{max}} = 0.1993)\); (b) \(L_1 = L_2 = 0.6, (|\psi|_{\text{max}} = 0.2169)\).

4.2. Effects of Hybrid Nanofluids

Figure 12 shows effects of the nanoparticle volume fraction for hybrid nanofluid on contours of the isotherms, streamlines, horizontal and vertical velocities of the dusty fluid at \(L_1 = L_2 = 0.5, D_s = 10, \alpha_s = 0.01, Ra = 10^5, \gamma_1 = 45^\circ\) and \(\gamma = 0.1\). Generally, an increase in the nanoparticle volume fraction enhances the viscosity of the fluid and hence the fluid activity is reduced. This can be proved from observing the maximum value of the stream function, at \(\phi_{Al_2O_3} = \phi_{Cu} = 1\%\), \((|\psi|_{\text{max}} = 2.8613)\) and at \(\phi_{Al_2O_3} = \phi_{Cu} = 4\%\), \((|\psi|_{\text{max}} = 2.7907)\). Similar trends occur on the maximum values of horizontal and vertical velocities, when \(\phi_{Al_2O_3} = \phi_{Cu} = 1\%\) \((|u|_{\text{max}} = 0.1485, |v|_{\text{max}} = 0.2422)\) and at \(\phi_{Al_2O_3} = \phi_{Cu} = 4\%\) \((|u|_{\text{max}} = 0.1414, |v|_{\text{max}} = 0.2192)\). The isotherms are slightly affected by adding the nanoparticles inside the enclosure.
The variations of mixture densities ratio from 1 to 100 have slight effects on the isothermal dusty contours. From these investigations, the inclination angle, fins lengths with two values of an inclination angle, nanoparticle volume fraction for hybrid nanofluids have slight effects on the isothermal dusty contours appear only at low dusty parameter. Generally, an increase in the nanoparticle volume fraction for hybrid nanofluid on contours of isothermal dusty contours. From this figure, the distributions of the isothermal dusty contours are slightly affected by adding the nanoparticles inside the enclosure.

Figure 12 shows effects of the nanoparticle volume fraction for hybrid nanofluid on contours of isothermal dusty contours. From this figure, the distributions of the isothermal dusty contours are slightly affected by adding the nanoparticles inside the enclosure. The third results showed that the local Nusslet number is enhanced by adding the nanoparticles inside the enclosure.

Figure 13. Effects of nanoparticle volume fraction for hybrid nanofluids on contours of isothermal dusty contours. From this figure, the distributions of the isothermal dusty contours are slightly affected by adding the nanoparticles inside the enclosure. The third results showed that the local Nusslet number is enhanced by adding the nanoparticles inside the enclosure. The variations of mixture densities ratio from 1 to 100 have slight effects on the isothermal dusty contours. From these investigations, the inclination angle, fins lengths with two values of an inclination angle, nanoparticle volume fraction for hybrid nanofluids have slight effects on the isothermal dusty contours appear only at low dusty parameter. Generally, an increase in the nanoparticle volume fraction for hybrid nanofluid on contours of isothermal dusty contours. From this figure, the distributions of the isothermal dusty contours are slightly affected by adding the nanoparticles inside the enclosure. The third results showed that the local Nusslet number is enhanced by adding the nanoparticles inside the enclosure.

Figure 13 presents the isothermal dusty contours under the effects of the mixture densities ratio at inclination angle $\gamma_1 = 45^\circ$, $\phi_{Al_2O_3} = \phi_{Cu} = 0.02$, $\alpha_s = 0.001$, $Ra = 10^5$, at $L_1 = 0.5$, $Ra = 0.1$. From this figure, the distributions of the isothermal dusty contours decrease when the mixture densities ratio $D_s$ increases from 100 to 1000. The variations of mixture densities ratio from 1 to 100 have slight effects on the isothermal dusty contours.
Figure 13. Isothermal dusty contours under the effects of mixture densities ratio at inclination angle $\gamma = 45^\circ$, $\phi = 0.02$, $Ra = 10^5$, at $L_1 = L_2 = 0.5$ and $\gamma = 0.1$. (a) $D_s = 1$; (b) $D_s = 10$; (c) $D_s = 100$; (d) $D_s = 1000$.

4.3. Local and Average Nusselt Number

Figure 14 shows the local Nusselt number along $X$-axis under the effects of (a) an inclination angle, (b) fins lengths with two values of an inclination angle, (c) nanoparticle volume fraction for hybrid nanofluid, and (d) mixture densities ratio with two values of dusty parameter, at $Ra = 10^5$ and $\gamma = 0.1$. It is observed that, the inclination angle affects the local Nusselt number along the $X$-axis without uniform tendency. Also, there are maximum values of the local Nusselt number that occurred at different locations along $X$-axis particularly, at the range $0.5 \leq X \leq 2.5$. As the fins lengths increase from 0.1 to 0.6, the local Nusselt number increases with three different peaks near to the location of the heated fins. The third results showed that the local Nusslet number is enhanced with a small rate as the nanoparticles volume fraction of hybrid nanofluid increase from 1% to 4%. Finally, in case of the low dusty parameter ($\alpha_s = 0.001$), effects of the mixture densities ratio on the local Nusselt number appear only when the mixture densities ratio $D_s$ equals 1000 and then the local Nusselt number decreases clearly as $D_s$ increases from 100 to 1000. While in case of the higher dusty parameter ($\alpha_s = 0.01$), the effects of $D_s$ on the local Nusselt number appear soon from $D_s = 10$ to 100 and lowest values of local the Nusselt number at $\alpha_s = 0.01$ with $D_s = 1000$. The physical explanation of this behavior is due to the fact that the carrier fluid loses kinetic energy by loading the dust particles and this factor leads to a reduced rate of the heat transfer. Figure 15 depicts the average Nusselt number under the effects of (a) an inclination angle, (b) fins lengths with two values of an inclination angle, (c) solid volume fraction for hybrid nanofluid, and (d) mixture densities ratio with two values of dusty parameter. As the inclination angle increases from $0^\circ$ to $180^\circ$, the average Nusselt number is fluctuating between increasing or decreasing. The average Nusselt number is increasing between $0^\circ$ to $45^\circ$, decreasing between $45^\circ$ to $100^\circ$ and increasing between $100^\circ$ to $180^\circ$. As the heated fins lengths increase from $L_1 = L_2 = 0.1$ to 0.9 this enhances the average Nusselt number. The average Nusselt number is higher at an inclination angle $45^\circ$ compared to $0^\circ$ when fins lengths vary from 0.1 to 0.6 only. The third results exposed that the average Nusselt is enhanced as the nanoparticles volume fraction increases due to the increase in the thermal conductivity of the nanofluid. At the end, the average Nusselt number is decreasing as the mixture densities ratio increases especially at a higher dusty parameter ($\alpha_s = 0.01$) and a slight decrease on the average Nusselt number occurs at a lower dusty parameter ($\alpha_s = 0.001$).
consequently the thermal boundary layers are supported. In addition, an increase of the fin lengths enhances the heated area inside the enclosure and increases the Nusselt number at $\alpha_0 = 0.01$, $0.3$, and $0.9$. In this figure, the formation of hot layers around the heated fins are increase as the times $t$ increases from 0.0 to 3.0.

Figure 15 depicts the average Nusselt number occurs at a lower dusty parameter $\gamma = 0.1$. The cooling rate of the heat transfer. Figure 16 presents the time-dependent of isothermal contours for three values of heated-fin lengths $L_1 = L_2 = 0.1$, $0.3$, and $0.9$ at an inclination angle $\gamma_1 = 45^\circ$, $\phi_{Al_2O_3} = \phi_{Cu} = 0.02$, $D_s = 10$, $\alpha_s = 0.01$, $Ra = 10^5$ and $\gamma = 0.1$. In this figure, the transition of the isothermal distributions across the cavity from the initial to the final was introduced for three values of heated-fin lengths $L_1 = L_2 = 0.3$, 0.5 and 0.9. In this figure, the formation of hot layers around the heated fins are increase as the times increase. In addition, an increase of the fin lengths enhances the heated area inside the enclosure and consequently the thermal boundary layers are supported.
Figure 15. Average Nusselt number under the effects of (a) an inclination angle, (b) fins lengths with two values of an inclination angle, (c) solid volume fraction for hybrid nanofluid, and (d) mixture densities ratio with two values of dusty parameter, at $Ra = 10^5$ and $\gamma = 0.1$.

Figure 16 presents the time-dependent of isothermal contours for three values of heated-fin lengths $L_1 = L_2 = 0.3, 0.5$ and $0.9$ at an inclination angle $\gamma_1 = 45^\circ$, $\phi_{(Al_2O_3, Cu)} = 0.02$, $\alpha_0 = 0.01$, $Ra = 10^5$ and $\gamma = 0.1$. In this figure, the transition of the isothermal distributions across the cavity from the initial to the final was introduced for three values of heated-fin lengths $L_1 = L_2$. In this figure, the formation of hot layers around the heated fins are increase as the times increase. In addition, an increase of the fin lengths enhances the heated area inside the enclosure and consequently the thermal boundary layers are supported.

Figure 16. Cont.
5. Conclusions

Numerical simulations have been performed to study the natural convection flow of dusty hybrid nanofluids inside shallow cavities; those include two rotating heated fins. One of the fins rotates in the clockwise direction and the other rotates in the opposite direction (anticlockwise). This phenomenon was presented by two systems of partial differential equations for the hybrid nanofluid and dusty phase. Two SIMPLE algorithms were performed to evaluate the pressures for the hybrid nanofluid and dusty phases. The main results of this simulation can be summarized as follows:

1. There are minimum values of the local Nusselt number that occur at the center of the top wall. Also, the increases in the fins lengths enhance the rate of heat transfer regardless of the values of the inclination angle.
2. For the low values of the lengths of the fins, the vertical fins are the best for the heat transfer compared with the rotating fins.
3. A clear reduction in the heat transfer rate is obtained as a ratio of the mixture densities increases. Also, like the effects of the mixture density ratio, the increase in the dusty parameter reduces the average Nusselt number.
4. A good support in profiles of the average Nusselt number is observed as the nanoparticles’ volume fraction increases.
5. The maximum values of the streamlines and dusty velocity components are diminished as values of the nanoparticles’ volume fraction decreases.

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Nomenclature

$C_p$ specific heat
$D_s$ ratio of the mixture densities
$g$ gravitational acceleration, m/s$^2$
$k$ thermal conductivity, W m$^{-1}$K$^{-1}$
$L_1$, $L_2$ lengths of left and right fins
$Nu$ Nusselt number
$p$ pressure, N/m$^2$
$Pr$ Prandtl number
$Q$ heat generation parameter
$Ra$ Rayleigh number
$s_1, s_2$ fin location in the enclosure
$T$ temperature, K
$t$ time, s
$u$, $v$ dimension velocity components, m/s
$x$, $y$ Cartesian coordinates, m

Figure 16. Time-dependence of isothermal contours for two values of heated-fin lengths $L_1 = L_2 = 0.3$, 0.5 and 0.9 at an inclination angle $\gamma = 45^\circ$, $\phi_{Al_{2}O_{3}} = \phi_{Cu} = 0.02$, $D_s = 10$, $\alpha_s = 0.01$, $Ra = 10^5$ and $\gamma = 0.1$. (a) $L_1 = L_2 = 0.3$; (b) $L_1 = L_2 = 0.5$; (c) $L_1 = L_2 = 0.9$. 

(c)
Greek Symbols

\(\alpha\) thermal diffusivity, \(m^2/s\)
\(\alpha_s\) dusty parameter
\(\gamma\) ratio of the specific heat
\(\gamma_1\) Inclined angel
\(\phi\) nanoparticle volume fraction
\(\mu\) viscosity
\(\theta\) dimensionless temperature
\(\nu\) kinematic viscosity, \(m^2/s\)
\(\rho\) density, \(kg/m^3\).
\(\tau\) mass concentration of the particle
\(\tau_T\) dimensionless time

Subscripts

\(f\) hybrid nanofluid
\(H\) hot
\(C\) cold
\(s\) particles

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