Electrified Higher-Dimensional Brane Intersections

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Abstract

We discuss the duality of intersecting D1-D5 branes in the low energy effective theory in the presence of electric field. This duality is found to be unbroken. We also deal with the solutions corresponding to two and three excited scalars in the D3-brane theory in the absence and the presence of an electric field. The solutions are given as a spike which is interpreted as an attached bundle of a superposition of coordinates of another brane given as a collective coordinate along which the brane extends away from the D3-brane. The lowest energy in both cases is higher than the energy found in the case of D1⊥D3 branes.

1 Introduction

The Born-Infeld (BI) action [1] governs the D-branes world volume which has many fascinating features. Among these there is the possibility for D-branes to morph into other D-branes of different dimensions by exciting some of the scalar fields [2, 3]. The subject of intersecting branes in string theory is very rich and has been studied for a long time [4]. One of the main sections of the present paper will be devoted to discuss the

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duality of intersecting D1-D5 branes and another section studies the intersection of D2- and D3-branes with a D3-brane.

One of the interesting subjects we discuss in this work concerns the duality of D1⊥D5 branes in the presence of electric field. By considering the energy of our system from D1 and D5 branes descriptions we find that the energies obtained from the two theories match. Consequently, the presence of an electric field doesn’t spoil this duality as seen in D1⊥D3 branes case we showed in the reference [5].

In the case of D1⊥D3 branes, it is known in the literature that there are many different but physically equivalent descriptions of how a D1 brane may end on a D3 brane. From the point of view of the D3 brane the configuration is described by a monopole on its world volume. From the point of view of the D1 brane the configuration is described by the D1 opening up into a D3 brane where the extra two dimensions form a fuzzy two sphere whose radius diverges at the origin of the three-brane. These different viewpoints are the stringy realization of the Nahm transformation [6].

By following the same mechanism used for D1⊥D3 branes, this paper is devoted to finding the funnel solutions when two and three excited scalars are involved. Thus, we restrict ourselves to using BPS arguments to find some solutions which have the interpretation of a D2-brane ending on D3-brane and D3-brane ending on other D3-brane. In these studies we consider the absence and the presence of an electric field. We found that the investigation of excited D3-brane in each case leads to the fact that by exciting 2 and 3 of its transverse directions in the absence or the presence of electric field, the brane develops a spike which is interpreted as an attached bundle of a superposition of coordinates of another brane given as a collective coordinate along which the brane extends away from the D3-brane. Then, by considering the lowest energy states of our system we remark that the lowest energy in the intersecting branes case is obtained by the D1-D3 branes intersection and the energy is higher if we excite more scalar fields and even more in the presence of an electric field.

This paper is organized as follows: In section 2, we review in brief the funnel solutions of D1⊥D3 branes and its broken duality in the dyonic case. In section 3, we discuss the unbroken duality of intersecting D1-D5 branes in the presence of an electric field. In section 4, we investigate the two and three excited scalars in D3-brane theory and we conclude in section 5.

2 D1⊥D3 Branes in Dyonic Case

In this section, we review in brief, the funnel solutions for D1⊥D3 branes. First, we give the funnel solution by using abelian Born-Infeld action for the world volume gauge field and one excited transverse scalar in the dyonic case. It was shown in [7] that the BI action, when taken as the fundamental action, can be used to build a configuration with a semi-infinite fundamental string ending on a D3-brane [8]. The dyonic system is given by using D-string world volume theory and the fundamental strings are introduced
by adding a $U(1)$ electric field. The system is described by the following action

$$S = \int dt L = -T_3 \int d^4\sigma \sqrt{-\det(\eta_{ab} + \lambda^2 \partial_a \phi^i \partial_b \phi^i + \lambda F_{ab})}$$

$$= -T_3 \int d^4\sigma \left[ 1 + \lambda^2 \left( |\nabla \phi|^2 + \vec{B}^2 + \vec{E}^2 \right) \right]$$

$$+ \lambda^4 \left( (\vec{B} \cdot \nabla \phi)^2 + (\vec{E} \cdot \vec{B})^2 + |\vec{E} \wedge \nabla \phi|^2 \right)^{\frac{1}{2}},$$

in which $F_{ab}$ ($a, b = 0, \ldots, 3$) is the Maxwell field strength and the electric field is denoted by $F_{0a} = E_a$. $\sigma^a$ denote the world volume coordinates while $\phi^i$ ($i = 4, \ldots, 9$) are the scalars describing transverse fluctuations of the brane and $\lambda = 2\pi \ell_s^2$ with $\ell_s$ is the string length. In our case we excite just one scalar so $\phi^i = \phi^9 \equiv \phi$. The energy of the present dyonic system accordingly to (1) is given by

$$E = T_3 \int d^4\sigma \left[ \lambda^2 |\nabla \phi + \vec{B} + \vec{E}|^2 + (1 - \lambda^2 \nabla \phi \cdot \vec{B})^2 - 2\lambda^2 \vec{E} \cdot (\vec{B} + \nabla \phi) \right]$$

$$+ \lambda^4 \left( (\vec{E} \cdot \vec{B})^2 + |\vec{E} \wedge \nabla \phi|^2 \right)^{1/2}. \quad (2)$$

Then if we require the condition that $\nabla \phi + \vec{B} + \vec{E} = 0$, the energy $E$ reduces to the following positive energy

$$E_0 = T_3 \int d^4\sigma \left[ (1 - \lambda^2 (\nabla \phi) \cdot \vec{B})^2 + 2\lambda^2 \vec{E} \cdot \vec{B} + \lambda^4 ((\vec{E} \cdot \vec{B})^2 + |\vec{E} \wedge \nabla \phi|^2) \right]^{1/2}, \quad (3)$$

as a minimum energy of the system. Thus, by solving the proposed condition $\nabla \phi + \vec{B} + \vec{E} = 0$ the funnel solution is found to be

$$\phi = \frac{N_m + N_e}{2r}, \quad (4)$$

with $N_m$ is magnetic charge and $N_e$ electric charge.

Now, we consider the dual description of the D3$\perp$D1 system. In D-string theory, we use non-abelian BI action given by the following to get a D3-brane from N D-string

$$S = -T_1 \int d^2\sigma Str \left[ -\det(\eta_{ab} + \lambda^2 \partial_a \phi^i \partial_b \phi^j)\det Q^{ij} \right]^{\frac{1}{2}} \quad (5)$$

where $Q_{ij} = \delta_{ij} + i\lambda[\phi_i, \phi_j]$, $i = 1, 2, 3$ and $a, b = \tau, \sigma$.

Expanding this action to leading order in $\lambda$ yields the usual nonabelian scalar action

$$S \simeq -T_1 \int d^2\sigma \left[ N + \lambda^2 Tr(\partial_a \phi^i + \frac{1}{2}[\phi_i, \phi_j][\phi_j, \phi_i]) + \ldots \right]^{\frac{1}{2}}.$$

The solution of the equation of motion of the scalar fields $\phi_i$ represent the $N$ D-string expanding into a D3-brane analogous to the bion solution of the D3-brane theory $[2, 3]$. The solutions are

$$\phi_i = \pm \frac{\alpha_i}{2\pi}, \quad [\alpha_i, \alpha_j] = 2i\varepsilon^{ijk}\alpha_k,$$

with the corresponding geometry is a long funnel where the cross-section at fixed $\sigma$ has the topology of a two-sphere.
The dyonic case is taken by considering \((N, N_f)-\text{string}\). We have \(N\) D-strings and \(N_f\) fundamental strings \([9]\). The theory is described by the action

\[
S = -T_1 \int d^2 \sigma \text{Str} \left[ - \text{det}(\eta_{ab} + \lambda^2 \partial_a \phi^i Q_{ij}^{-1} \partial_b \phi^j + \lambda F_{ab}) \text{det} Q^{ij} \right]^{\frac{1}{2}}.
\]

(6)

This action can be rewritten as

\[
S = -T_1 \int d^2 \sigma \text{Str} \left[ - \text{det} \left( \eta_{ab} + \lambda F_{ab} \frac{\lambda}{Q} \partial_a \phi^i \partial_b \phi^j \right) \right]^{\frac{1}{2}}.
\]

(7)

The bound states of D-strings and fundamental strings are made simply by introducing a background \(U(1)\) electric field on the D-strings corresponding to fundamental strings dissolved on the world sheet. Then we replace the field strength \(F_{\tau \sigma}\) by \(EI_{N_m}\) (\(I_{N_m}\) is \(N_m \times N_m\)-matrix) and the following ansatz is inserted

\[
\phi_i = \hat{R} \alpha_i.
\]

(8)

For non-abelian scalars and at fixed \(\sigma\), this ansatz describes a non-commutative two-sphere with a physical radius given by

\[
R(\sigma)^2 = \frac{\lambda^2}{N_m} \sum_{i=1}^{3} \text{Tr}[\phi_i(\sigma)^2] = \lambda^2 C \hat{R}(\sigma)^2,
\]

with \(C\) is the quadratic Casimir of the particular representation of the generators \(\alpha_i\) defined by the identity

\[
\sum_{i=1}^{3} (\alpha_i)^2 = CI_{N_m}
\]

where \(I_{N_m}\) is the \(N_m \times N_m\) identity matrix and \(C = (N_m)^2 - 1\) for the irreducible \(I_{N_m}\) is the \(N_m \times N_m\) representation.

By computing the determinant, the action (7) becomes

\[
S = -T_1 \int d^2 \sigma \text{Str} \left[ (1 - \lambda^2 E^2 + \alpha_i \alpha_i \hat{R}^2)(1 + 4 \lambda^2 \alpha_i \alpha_j \hat{R}^4) \right]^{\frac{1}{2}}.
\]

(9)

Hence, we get the funnel solution for dyonic string by solving the equation of motion of \(\hat{R}\) as follows

\[
\phi_i = \frac{\alpha_i}{2\sigma \sqrt{1 - \lambda^2 E^2}}.
\]

(10)

Now, we see if the two descriptions discussed above match or not. As we showed in \([5]\), the two descriptions D1 and D3 don’t have a complete agreement in the presence of a world volume electric field since the energies don’t match as we will see in the following, even if their profiles match very well.

The energy is easily derived from the action (9) for the static solution (10). The minimum energy condition is

\[
\frac{d\phi_i}{d\sigma} = \pm \frac{i}{2} \epsilon_{ijk} [\phi_j, \phi_k],
\]

which can be identified as the Nahm equations \([10]\). We insert the ansatz (8) and this implies

\[
\hat{R}' = \pm 2\sqrt{1 - \lambda^2 E^2} \hat{R}.
\]
Using this condition and evaluating the Hamiltonian, \( \int d\sigma (DE - L) \), for the dyonic funnel solutions (\( L \) is the Lagrangian), the energy is expressed as

\[
E_1 = T_1 \int d\sigma STr \left[ \frac{\lambda^2 E^2}{\sqrt{1 - \lambda^2 E^2}} + \sqrt{1 - \lambda^2 E^2} |1 + 4\lambda^2 \alpha_j \alpha_j \dot{R}^4| \right].
\]

We can manipulate this result by introducing the physical radius \( R = \lambda \sqrt{C |\dot{R}|} \) and using \( T_1 = 4\pi^2 \ell_s T_3 \). It’s also useful to consider the electric displacement \( D \)

\[
D \equiv \frac{1}{N_m} \frac{\delta S}{\delta E} = \frac{\lambda^2 T_1 E}{\sqrt{1 - \lambda^2 E^2}} = \frac{N_e}{N_m}.
\]

Consequently, the energy from the D1 brane theory is found to be

\[
E_1 = T_1 \int d\sigma \sqrt{N_m^2 + g_s N_e^2} + T_3 (1 - \frac{1}{N_m^2}) \frac{\lambda}{N_m^4} \int dR 4\pi R^2,
\]

(11)

with \( g_s \) is the string coupling with \( T_1 = (\lambda g_s)^{-1} \). The first term comes from collecting the contributions independent of \( \dot{R} \). The second term gotten from the terms containing \( \dot{R} \) and is used to put these in the form \( \dot{R}^2 |\dot{R}| \). Then we have repeatedly applied \( \dot{R}' = \pm 2\sqrt{1 - \lambda^2 E^2} \dot{R}^2 \) in producing the second term.

If we consider large \( N_m \) the energy is reduced to the following

\[
E_1 = T_1 N_m \int d\sigma,
\]

(12)

which can be rewritten in terms of physical radius \( R \) as

\[
E_1 = T_3 N_m \int 4\pi R^2 dR,
\]

(13)

with \( T_3 = \frac{T_1}{4\pi^2 \ell_s^2} \). In D3-brane description the energy (3) becomes

\[
E_3 = T_3 \int d^3 \sigma \sqrt{1 + \lambda^4 \left[ N_m(N_m + N_e)^2 + N_e^2 \right] \frac{16r^8}{16r^8} + 2\lambda^2 N_m(N_m + N_e) \frac{4r^4}{4r^4} + 2\lambda^2 N_e^2 \frac{4r^4}{4r^4},
\]

(14)

such that the magnetic and the electric fields are given by

\[
\begin{align*}
\dot{B} = \frac{N_m}{2r^2} \frac{r}{r}, & \quad \dot{E} = \frac{N_e}{2r^2} \frac{r}{r}.
\end{align*}
\]

(15)

In the large \( N_m \) limit and fixed \( N_e \), the energy (14) of the spherically symmetric BPS configuration is reduced to

\[
E_3 = T_3 N_m \sqrt{(N_m + N_e)^2 + N_e^2} \int 4\pi r^2 dr.
\]

(16)

Again we consider large \( N_m \) limit and fixed \( N_e \) and we get

\[
\frac{\sqrt{(N_m + N_e)^2 + N_e^2}}{N_m + N_e} \rightarrow 1.
\]
Consequently, for fixed \(N_e\) and large \(N_m\) limit we have agreement from both sides (D1 and D3 descriptions) and the energy is

\[
E_3 = T_3 N_m \int 4\pi r^2 dr,
\]

(17)
in which we identify the physical radius \(R\) from D1 description and \(r\) from D3 description.

Now, if we take large \(N_m\) limit keeping \(N_e/N_m = K\) fixed at any arbitrary \(K > 0\) the result will be different. Thus, from D1 description the energy becomes

\[
E_1 = T_3 N_m \sqrt{1 + g_s K^2} \int 4\pi R^2 dR,
\]

(18)
and from D3 description the energy is

\[
E_3 = T_3 \frac{N_m \sqrt{(1 + K)^2 + K^2}}{1 + K} \int 4\pi R^2 dR.
\]

(19)
Then we have disagreement. Consequently, the presence of electric field spoils the duality between D1 and D3 descriptions of intersecting D1-D3 branes.

### 3 Intersecting D1-D5 Branes at the Presence of an Electric Field

Although D1⊥D5 branes system [11] is not supersymmetric, the fuzzy funnel configuration in which the D-strings expand into orthogonal D5-branes shares many common features with the D3-brane funnel. Thus, we are interested in establishing whether a similar result holds also in the case of D1⊥D5 branes meaning the presence of a world volume electric field leads to broken duality or not.

From the D1 description the system is described by the action (6). We consider static configurations involving five (rather than three) nontrivial scalars, \(\phi_i\) with \(i = 1, ..., 5\). The proposed ansatz for the funnel solution is

\[
\phi_i(\sigma) = \pm \hat{R}(\sigma) G_i,
\]

(20)
with \(\hat{R}(\sigma)\) is the (positive) radial profile and \(G_i\) are the matrices constructed in [12] to provide a fuzzy four-sphere and to construct the string funnel [11]. Also, \(G_i\) are given by the totally symmetric \(n\)-fold tensor product of \(4 \times 4\) gamma matrices, and that the dimension of the matrices is related to the integer \(n\) by

\[
N = \frac{(n + 1)(n + 2)(n + 3)}{6}.
\]

(21)
Then the solution is a fuzzy 4-sphere with the physical radius

\[
R(\sigma) = \lambda \sqrt{\frac{Tr(\phi_i \phi_i)}{N}} = \sqrt{c \lambda \hat{R}(\sigma)},
\]

(22)
c is the "Casimir" associated with the \(G_i\) matrices, i.e., \(G_i G_i = c 1_N\), given by

\[
c = n(n + 4).
\]

(23)
By inserting the ansatz into (6) the action becomes

\[ S = -NT_1 \int d^2 \sigma \sqrt{1 - \lambda^2 E^2 + (R')^2 \left[ 1 + \frac{4R^4}{c\lambda^2} \right]}, \]  

(24)

where the prime indicates the derivative with respect to \( \sigma \).

The electric field is fixed by the quantization condition on the displacement field, \( D = \frac{N_f}{N} \), where

\[ D = \frac{1}{N} \frac{\delta S}{\delta E} = \frac{\lambda^2 T_1 E}{\sqrt{1 - \lambda^2 E^2}} \]  

(25)

after using the equations of motion, the energy \( \tilde{E}_1 = \int d\sigma (DE - L) \) of the system is evaluated to be

\[ \tilde{E}_1 = \sqrt{N^2 + g_s^2 N_f^2 T_1} \int d\sigma + \frac{6N}{c} \int T_5 \int \Omega_4 R^4 dR + NT_1 \int dR + \Delta E, \]  

(26)

with \( T_5 = \frac{T_1}{(2\pi \ell_s)^4} \) and the first and the second terms correspond to the energies of \( N \) semi-infinite strings stretching from \( \sigma = 0 \) to infinity and of \( 6N/c \) D5-branes respectively. The contribution of the last terms to the energy indicates that the configuration is not supersymmetric. The last contribution is a finite binding energy \( \Delta E = 1.0102Nc^{1/4}T_1\ell_s \).

In the dual point of view of the D5-brane world volume theory, the action describing the system is the Born-Infeld action

\[ S = -T_5 \int d^6 \sigma STr \sqrt{-det(G_{ab} + \lambda^2 \partial_a \phi \partial_b \phi + \lambda F_{ab})} \]  

(27)

with \( a, b = 0, 1, ..., 5 \) and \( \phi \) the excited transverse scalar. To get a spike solution with electric field switched on from D5-brane theory we follow the analogous method of bion spike in D3-brane theory as discussed in [11, 12]. We use spherical polar coordinates and the metric is

\[ ds^2 = G_{ab} d\sigma^a d\sigma^b = -dt^2 + dr^2 + r^2 g_{ij} d\alpha^i d\alpha^j, \]  

(28)

with \( r \) the radius and \( \alpha^i, i = 1, ..., 4, \) Euler angles. \( g_{ij} \) is the diagonal metric on a four-sphere with unit radius

\[ g_{ij} = \begin{pmatrix} 1 & \sin^2(\alpha^1) & \sin^2(\alpha^1)\sin^2(\alpha^2) & \sin^2(\alpha^1)\sin^2(\alpha^2)\sin^2(\alpha^3) \\ \sin^2(\alpha^1) & 1 & \sin^2(\alpha^2) & \sin^2(\alpha^2)\sin^2(\alpha^3) \\ \sin^2(\alpha^1)\sin^2(\alpha^2) & \sin^2(\alpha^2) & 1 & \sin^2(\alpha^3) \\ \sin^2(\alpha^1)\sin^2(\alpha^2)\sin^2(\alpha^3) & \sin^2(\alpha^2)\sin^2(\alpha^3) & \sin^2(\alpha^3) & 1 \end{pmatrix}. \]  

(29)

In this theory, we add the electric field \( E \) as a static radial field in the \( U(1) \) sector. The scalar \( \phi \) is only a function of the radius by considering bion spike solutions with a "nearly spherically symmetric" ansatz. In the same time to compare with the radial profile obtained in D1-brane theory we identify the physical transverse distance as \( \sigma = \lambda \phi \), and the radius \( r = R \) which fixes the coefficients. Thus the scalar is found to be

\[ \phi(r) = \pm \int \frac{dr}{\lambda(1 - \alpha^2)(\frac{r^2}{N} + 1)^2 - 1} \]  

(30)

where \( \alpha = \frac{g_s N_f}{\sqrt{N^2 + g_s^2 N_f^2}} \). The energy of the system is evaluated to be

\[ \tilde{E}_5 = \sqrt{\frac{NT_1}{1 - \left( \frac{g_s N_f}{\sqrt{N^2 + g_s^2 N_f^2}} \right)^2}} \int d\sigma + \frac{6N}{c} \int T_5 \int \Omega_4 R^4 dR + NT_1 \int dR + \Delta E, \]  

(31)
with $\Delta E$ is the same one found above from D1 description. In the absence of electric field, it’s clear that by identifying the profiles of D1 and D5 descriptions in the limit of $N$ we could get complete agreement for the geometry and the energy determined by the two dual approaches. Now, in the presence of an electric field it seems there is also agreement. We compare the energy from the D1 description (26) and the energy from the D5 description (31). If we consider the large $N$ limit and fixed $N_f$ the first term of $\tilde{E}_1$ becomes

$$\sqrt{N^2 + g_s^2 N_f^2} T_1 \int d\sigma \rightarrow NT_1 \int d\sigma,$$

and the first term of $\tilde{E}_5$ goes to the following value

$$NT_1 \sqrt{1 - \left(\frac{g_s N_f}{\sqrt{N^2 + g_s^2 N_f^2}}\right)^2} \int d\sigma \rightarrow NT_1 \int d\sigma,$$

which proves the agreement at large $N$.

Now, let’s fix the value $\frac{g_s N_f}{N}$ to be one value $M$ which can’t be neglected at large limit of $N$. Thus, if $N$ is large the last two limits (32) and (33) become

$$\sqrt{N^2 + g_s^2 N_f^2} T_1 \int d\sigma \rightarrow NT_1 \sqrt{1 + M^2} \int d\sigma,$$

and

$$NT_1 \sqrt{1 - \left(\frac{g_s N_f}{\sqrt{N^2 + g_s^2 N_f^2}}\right)^2} \int d\sigma \rightarrow \frac{NT_1}{\sqrt{1 - M^2}} \int d\sigma.$$

The right hand term of Eq(35) is equal to the right hand term of Eq(34) $\frac{NT_1}{\sqrt{1 + M^2}} \frac{NT_1}{\sqrt{1 + M^2 - M^2}} = NT_1 \sqrt{1 + M^2}$. Then, this implies agreement of the two duals at the level of energy of the two descriptions. Consequently, the duality in D1 $\perp$ D5 branes is unbroken by switching on the electric field.

4 Two and Three Excited Scalars

In this section, we use the abelian Born-Infeld action for the world volume gauge field and transverse displacement scalars to explore some aspects of D3-brane structure and dynamics. We deal with magnetic and dyonic cases.

4.1 Absence of Electric Field

We consider the case where D3-brane has more than one scalar describing transverse fluctuations. We denote the world volume coordinates by $\sigma^a$, $a = 0, 1, 2, 3$, and the transverse directions by the scalars $\phi^i$, $i = 4, ..., 9$. In D3-brane theory construction, the low energy dynamics of a single D3-brane is described by the BI action by using static gauge

$$S_{BI} = \int L = -T_3 \int d^4 \sigma \sqrt{-\det(\eta_{ab} + \lambda^2 \partial_a \phi^i \partial_b \phi^i + \lambda F_{ab})}$$
with $F_{ab}$ is the field strength of the $U(1)$ gauge field on the brane. By exciting two scalar fields and setting to zero the other scalars, the energy is evaluated for the fluctuations through two directions and for static configurations as follows

$$\zeta = -L = T_3 \int d^3 \sigma \left[ 1 + \lambda^2 \left( | \nabla \phi_4 |^2 + | \nabla \phi_5 |^2 + \hat{B}^2 \right) + \lambda^4 \left( (\hat{B} \cdot \nabla \phi_4)^2 + (\hat{B} \cdot \nabla \phi_5)^2 \right) \right] \frac{1}{2} + \lambda^4 \left| \nabla \phi_4 \wedge \nabla \phi_5 \right|^2.$$  

(37)

If we introduce a complex scalar field $C = \phi_4 + i \phi_5$, we can rewrite the energy as

$$\zeta = T_3 \int d^3 \sigma \left[ \lambda^2 (\nabla C + \hat{B})(\nabla C + \hat{B})^* + (1 - \lambda^2 (\nabla C \cdot \hat{B}))(1 - \lambda^2 (\nabla C \cdot \hat{B}))^* \right] \frac{1}{2}.$$  

(38)

$$+ \frac{1}{2} \lambda^4 \left| \nabla C \wedge \nabla C^* \right|^2.$$  

In this case, we observe that to get minimum energy we can set the first term to zero and this will lead to

$$\nabla C = - \hat{B} = \nabla \phi_4 + i \nabla \phi_5.$$  

(39)

We know that $\hat{B}$ is real, then $\nabla \phi_4 = 0$ and thus in the "bi-excited scalar" system the lowest energy is identified to $D3\perp D1$ system. This suggests that to study the minimum energy configuration of the D3-brane system it is only worthwhile to excite just one scalar.

Now, requiring $\nabla \phi_5 \neq 0$ we get different energy bound. The energy (38) can be rewritten as follows

$$\zeta = T_3 \int d^3 \sigma \left[ \lambda^2 \left( | \nabla \phi_4 + \nabla \phi_5 \pm \hat{B}|^2 + (1 \pm \lambda^2 \hat{B} \cdot (\nabla \phi_4 + \nabla \phi_5))^2 \right) \right] \frac{1}{2} + 2 \lambda^2 | \nabla \phi_4 \wedge \nabla \phi_5 |^2.$$  

(40)

The new bound is now found with the following constraint

$$\nabla C \wedge \nabla C = \pm \hat{B}.$$  

(41)

and $\pm 2 \lambda^2 | \nabla \phi_4 \wedge \nabla \phi_5 \geq 0$ should also be satisfied. Then the energy is

$$\zeta = T_3 \int d^3 \sigma \left[ (1 \pm \lambda^2 \hat{B} \cdot (\nabla \phi_4 + \nabla \phi_5))^2 + \lambda^4 | \nabla \phi_4 \wedge \nabla \phi_5 |^2 \right] \frac{1}{2}.$$  

(42)

We choose without loss of generality $\nabla \phi_4 \perp \nabla \phi_5$.

Thus we get $\nabla^2 \phi_4 + \nabla^2 \phi_5 = 0$ (by using the Bianchi identity), and the solution is

$$\phi_4 + \phi_5 = \pm \frac{N_m}{2r}.$$  

(43)

This solution could be generalized by considering three excited scalars. The energy is then found to be

$$\zeta_3 = T_3 \int d^3 \sigma \left[ 1 + \lambda^2 \left( \sum_{i=4}^{6} | \nabla \phi_i |^2 + \hat{B}^2 \right) + \lambda^4 \left( \sum_{i=4}^{6} | \nabla \phi_i \cdot \hat{B} \cdot \nabla \phi_i |^2 + \frac{1}{2} \sum_{i,j=4}^{6} | \nabla \phi_i \wedge \nabla \phi_j |^2 \right) \right] \frac{1}{2} + \lambda^4 \left| \nabla \phi_4 \wedge \nabla \phi_5 \right|^2.$$  

(44)
We should also consider $\pm \lambda^2 \sum_{i,j=4}^{6} \nabla \phi_i \nabla \phi_j \geq 0$ to get the lowest energy configuration. This should be found by canceling some of the terms in the second line of the expression given in (44). The simplest way is to require the orthogonality of $\nabla \phi_i$ and $\nabla \phi_j$. Then the lowest energy in the static gauge is

$$\tilde{\zeta}_3 = T_3 \int d^3\sigma \left[ (1 \pm \lambda^2 \vec{B} \cdot \sum_{i=4}^{6} \nabla \phi_i)^2 + \frac{1}{2} \sum_{i,j=4}^{6} | \nabla \phi_i \wedge \nabla \phi_j |^2 \right]^{\frac{1}{2}},$$

with the constraints

$$\sum_{i=4}^{6} \nabla \phi_i = \pm \vec{B},$$

and the solution is similar to the "bi-excited system", we find

$$\sum_{i=4}^{6} \phi_i = \pm \frac{N_m}{2r}.$$ (47)

The solution obtained for each excited scalar field has one collective coordinate in the D3-brane world volume theory. This is the direction along which the brane extends away from the D3-brane. Thus, this collective coordinate represents a "ridge" solution in the D3-brane theory.

Now, we will look at another case in which the electric field is present, and to see how the energy of the system could be minimized and what kind of solutions we could obtain.

### 4.2 Addition of an Electric Field

First we start by exciting two transverse directions ($\phi_4$ and $\phi_5$) with the electric field $\vec{E}$ switched on. We consider as previous that $\nabla \phi_4 \perp \nabla \phi_5$. Then the energy of our system is

$$E_3 = T_3 \int d^3\sigma \left[ 1 + \lambda^2 \left( | \nabla \phi_4 |^2 + | \nabla \phi_5 |^2 + \vec{B}^2 + \vec{E}^2 \right) 
+ \lambda^4 \left( \vec{B} \cdot \nabla \phi_4 \right)^2 + \left( \vec{B} \cdot \nabla \phi_5 \right)^2 + (\vec{E} \cdot \vec{B})^2 + | \nabla \phi_4 \wedge \nabla \phi_5 |^2 + | \vec{E} \wedge \nabla \phi_4 |^2 + | \vec{E} \wedge \nabla \phi_5 |^2 \right]^{\frac{1}{2}} 
+ \lambda^6 | \vec{E} \cdot (\nabla \phi_4 \wedge \nabla \phi_5) |^2 \right]^{\frac{1}{2}}$$

$$= T_3 \int d^3\sigma \left( \lambda^2 | \nabla \phi_4 + \nabla \phi_5 \pm (\vec{B} + \vec{E}) |^2 + \left[ 1 \pm \lambda^2 (\nabla \phi_4 + \nabla \phi_5) \cdot (\vec{B} + \vec{E}) \right]^2 
- 2\lambda^2 B.E + \lambda^4 (\vec{E} \cdot \vec{B})^2 
+ \lambda^4 \left[ | \nabla \phi_4 \wedge \nabla \phi_5 |^2 + | \vec{E} \wedge \nabla \phi_4 |^2 + | \vec{E} \wedge \nabla \phi_5 |^2 \right] - (\vec{E} \cdot \nabla \phi_4)^2 - (\vec{E} \cdot \nabla \phi_5)^2 
- 2 \left( \nabla \phi_4 . (\vec{E} + \vec{B}) \nabla \phi_5 . (\vec{E} + \vec{B}) + (\nabla \phi_4 . \vec{B}) (\nabla \phi_5 . \vec{E}) + (\nabla \phi_5 . \vec{B}) (\nabla \phi_4 . \vec{E}) \right) 
+ \lambda^6 | \vec{E} \cdot (\nabla \phi_4 \wedge \nabla \phi_5) |^2 \right]^{\frac{1}{2}}.$$ (48)
By taking into consideration the previous analysis in the absence of an electric field, the energy $E_3$ becomes

$$E_3 \geq T_3 \int d^3 \sigma \left( [1 \pm \lambda^2 (\nabla \phi_4 + \nabla \phi_5). (\vec{B} + \vec{E})]^2 - 2 \lambda^2 \vec{B} \cdot \vec{E} + \lambda^4 (\vec{E} \cdot \vec{B})^2 \right),$$  

(49)

$$+ \lambda^4 \left[ \vec{E}^2 | \nabla \phi_4 |^2 + \vec{E}^2 | \nabla \phi_5 |^2 \right] + \lambda^6 | \vec{E} . (\nabla \phi_4 \wedge \nabla \phi_5) |^2 \right]^{\frac{1}{2}}$$

This expression is consistent with the fact that

$$\nabla \phi_4 + \nabla \phi_5 \pm (\vec{B} + \vec{E}) = 0$$  

(50)

and

$$- 2 \lambda^2 \vec{B} \cdot \vec{E} + \lambda^4 (\vec{E} \cdot \vec{B})^2 \geq 0.$$  

(51)

We also used the following expression

$$| \vec{E} \wedge \nabla \phi_5 |^2 = \vec{E}^2 | \nabla \phi_5 |^2 - (\vec{E} \cdot \nabla \phi_5)^2.$$  

(52)

Then to get the lowest energy in the presence of an electric field we require $- 2 \lambda^2 \vec{B} \cdot \vec{E} + \lambda^4 (\vec{E} \cdot \vec{B})^2 = 0$; i.e. $\vec{E} \cdot \vec{B} = \frac{2}{\lambda^2}$ or $\vec{E} \perp \vec{B}$. With these simplifications, the energy becomes

$$\tilde{E}_3 = T_3 \int d^3 \sigma \left( [1 \pm \lambda^2 (\nabla \phi_4 + \nabla \phi_5). (\vec{B} + \vec{E})]^2 + \lambda^4 \vec{E}^2 \left[ | \nabla \phi_4 |^2 + | \nabla \phi_5 |^2 \right] + \lambda^6 | \vec{E} . (\nabla \phi_4 \wedge \nabla \phi_5) |^2 \right]^{\frac{1}{2}}.$$  

(53)

In the following we consider $\vec{E} \cdot \vec{B} = \frac{2}{\lambda^2}$. We remark that the energy of D2-D3 brane is increased by the presence of the electric field and by switching it off we obtain the lowest energy configuration obtained previously.

By solving (50) in the static gauge, using the Bianchi identity, we obtain the solution

$$\phi_4 + \phi_5 = \mp \frac{N_m + N_e}{2r},$$  

(54)

with $r^2 = \sum_{a=1}^{3} (\sigma^a)^2$, $N_m$ and $N_e$ the magnetic and the electric charges respectively.

Now, by exciting three scalars, the energy is generalized to the following

$$\xi_3 = T_3 \int d^3 \sigma \left[ 1 + \lambda^2 \left( \sum_{i=4}^{6} | \nabla \phi_i |^2 + \vec{B}^2 + \vec{E}^2 \right) + \lambda^4 \left( (\vec{E} \cdot \vec{B})^2 + \sum_{i=4}^{6} | \vec{B} \cdot \nabla \phi_i |^2 + \sum_{i=4}^{6} | \vec{E} \wedge \nabla \phi_i |^2 + \frac{1}{2} \sum_{i,j=4}^{6} | \nabla \phi_i \wedge \nabla \phi_j |^2 \right) + \lambda^6 \frac{1}{2} | \vec{E} \cdot (\sum_{i,j=4}^{6} \nabla \phi_i \wedge \nabla \phi_j) |^2 \right]^{\frac{1}{2}}.$$  

(55)
We also consider as before $\nabla \Phi_i \perp \nabla \Phi_j$ and $\vec{E} \cdot \vec{B} = \frac{2}{\lambda}$ with $i, j = 4, 5, 6$. Then the energy will be reduced to the lowest energy for three excited directions

$$\bar{\xi}_3 = T_3 \int d^3 \sigma \left( [1 \pm \lambda^2 \sum_{i=4}^{6} \nabla \phi_i \cdot (\vec{B} + \vec{E})]^{\frac{2}{\lambda}} + \lambda^4 \sum_{i, j=4}^{6} \vec{E}^2 \left( |\nabla \phi_i|^2 + |\nabla \phi_j|^2 \right) \right)$$

$$+ \lambda^6 |\vec{E}| \left( \sum_{i, j=4}^{6} (\nabla \phi_i \wedge \nabla \phi_j)^2 \right)^{\frac{1}{2}},$$

with $i < j$ in the summations and where we assume the following condition

$$\sum_{i=4}^{6} \nabla \phi_i \pm (\vec{B} + \vec{E}) = 0.$$

The last equation is easily solved to give the solution

$$\sum_{i=4}^{6} \phi_i = \mp \frac{N_m + N_e}{2r}.$$

Again if we require that $\vec{E}$ is parallel to one of $\nabla \phi_i$ the energy will be minimized as the last term in the expression (56) of the energy vanishes. Then, accordingly to (56) and (58) with

$$|\vec{B}| = \left| \frac{N_m}{2r^2} \right|, \quad |\vec{E}| = \left| \frac{N_e}{2r^2} \right|$$

the energy becomes

$$\bar{\xi}_3 = T_3 \int d^3 \sigma \left( [1 + \lambda^2 \frac{(N_m + N_e)^2}{4r^4}]^{\frac{2}{\lambda}} + \lambda^4 \frac{N_e^2(N_m + N_e)^2}{8r^6} \right)^{\frac{1}{2}}$$

This is the lowest energy for both cases where two or three transverse scalars are excited since the solution is a superposition of the scalars.

5 Discussion and Conclusion

In this work we investigated the physics of intersecting D1-D5 branes in the presence of an electric field. We also studied D2$\perp$D3 branes and D3$\perp$D3 branes in presence/absence of an electric field.

Our main interest was the fate of the duality of D1$\perp$D5 branes. We found that the duality between the D1 and D5 descriptions is unbroken in the presence of an electric field. Then, the duality in D1-D5 case is still valid. By contrary, as we saw in [5], the duality in D1$\perp$D3 branes is no longer valid when we add an electric field. Then we have further argued that the D1-brane description, in D1-D3 case, breaks down. This is strengthened by the result discussed in [13] which have argued that the effective tension of the string goes to zero. Thus, excited strings modes will not be very heavy compared to massless string modes and one might question the validity of the Dirac-Born-Infeld action which retains only the massless modes.

The investigation of excited D3-branes through the two cases of absence or presence of an electric field lead to the fact that by exciting 2 and 3 of its transverse directions
the brane develops a spike which is interpreted as an attached bundle of a superposition of coordinates of another brane given as a collective coordinate along which the brane extends away from the D3-brane. In our study of D3-branes, by exciting 2 and 3 transverse directions we found that a magnetic monopole produces a singularity in the D3-branes transverse displacement which can be interpreted as a superposition of coordinates describing Dp-branes ($p = 2, 3$) attached to the D3-brane and the same for the dyonic case. We also obtained another important result that the lowest energy in the intersection branes case is obtained at the level of D1⊥D3 branes and the energy is higher if we excite more scalar fields and even more so in the presence of an electric field.

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