TetraSphere: A Neural Descriptor for O(3)-Invariant Point Cloud Classification

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Abstract

Rotation invariance is an important requirement for the analysis of 3D point clouds. In this paper, we present a learnable descriptor for rotation- and reflection-invariant 3D point cloud classification based on recently introduced steerable 3D spherical neurons and vector neurons. Specifically, we show that the two approaches are compatible, and we show how to apply steerable neurons in an end-to-end method for the first time. In our approach, we perform TetraTransform—which lifts the 3D input to an equivariant 4D representation, constructed by the steerable neurons—and extract deeper rotation-equivariant features using vector neurons, subsequently computing pair-wise O(3)-invariant inner products of these features. This integration of the TetraTransform into the VN-DGCNN framework, termed TetraSphere, is used to classify synthetic and real-world data in arbitrary orientations. Taking only 3D coordinates as input, TetraSphere sets a new state-of-the-art classification performance on randomly rotated objects of the hardest subset of ScanObjectNN, even when trained on data without additional rotation augmentation. Our results reveal the practical value of spherical decision surfaces for learning in 3D Euclidean space.

1. Introduction

Automatic processing of 3D data obtained with sensors such as LiDARs, sparse stereo, and sparse time-of-flight is a central problem for many autonomous systems [18] [12] [34]. Point clouds—in the form of an array of a fixed number of 3D coordinates and corresponding optional features (e.g., color or intensity)—are a common representation of such data in various 3D vision tasks.

Consider, for example, the task of 3D object classification, where the goal is to predict the correct class given a point cloud. Importantly, the order of the points and different orientations of the shape do not alter its class membership. This imposes the requirements of permutation and rotation invariance on the classifier. Furthermore, in certain real-world scenarios (such as left- and right-hand traffic), global reflection invariance is desired. For instance, a vehicle designed for either type of traffic may be considered the same.

Fulfilling the first requirement is commonly done by constructing a model using shared multilayer perceptrons (MLPs) and a global aggregation function, producing permutation-invariant features, as in, e.g., PointNet [32].

To attain rotation invariance [40], a common approach is to augment available data by performing random rotations and training the model in the hope that it can generalize to other, possibly unknown, orientations during inference. However, such an approach relies heavily on augmentation and requires an increased model capacity. Such methods are commonly referred to as rotation-variant (rotation-sensitive), e.g., [33] [41].

There are also rotation-equivariant methods [43] [38] [9], in which the learned features rotate correspondingly with the input, and rotation-invariant (RI) techniques [47] [24] [5] [7] [16], in which the central trend is to construct RI low-level geometric features and use them instead of point coordinates. An alternative approach is to compute a canonical pose and then de-rotate the input point cloud and perform processing on it [11] [37] [22].

Our method is a combination of SO(3)-equivariant steer-
able 3D spherical neurons \[28\] and VN-MLPs \[9\], where deep rotation-equivariant features are learned using vector neurons and invariant predictions are obtained by taking the inner product of these features point-wise. However, unlike the original SO(3)-equivariant framework \[9\], we propagate equivariant features through the network by constructing a specific 4D space spanned by what we call a tetra-basis, as shown in Figure 1. Our main hypothesis is that features from learned rotation-equivariant TetraTransform projections are more expressive than the points themselves.

We summarize our contributions as follows:
1. We identify the compatibility of vector neurons \[9\] and steerable 3D spherical neurons \[28\], and propose TetraSphere—a learnable O(3)-invariant descriptor for 3D point cloud classification, built upon VN-DGCNN \[9\].
2. We unveil the practical utility of the steerable neurons, which, to the best of our knowledge, have never previously been used in an end-to-end framework.
3. We demonstrate the effectiveness of TetraSphere by using it to classify the most challenging subset of ScanObjectNN \[39\] real-world data and setting a new state-of-the-art classification performance.

2. Related work

2.1. Rotation-sensitive 3D point cloud learning

PointNet \[32\] is the pioneering work for learning on raw point sets as input data for the tasks of classification, part segmentation, and semantic segmentation. Its limited ability for recognizing fine-grained patterns was addressed in the PointNet++ method \[33\] that recursively applies PointNet on a nested partitioning of the input point cloud. Other noteworthy methods include PointCNN \[25\] with a special type of convolution operator applied to the input points and features before they are processed by an ordinary convolution, and dynamic graph CNN (DGCNN) \[41\], where a graph convolution is applied to edges of the k-nearest neighbor graph of the point clouds. Xiang et al. \[45\] introduced CurveNet based on a sequence-of-points (curve) grouping operator and a curve aggregation operator. A more geometrically inspired approach was presented by Melnyk et al. \[27\], who revisited modeling spherical decision surfaces with conformal embedding \[30\] in the context of learning 3D point cloud representations.

Somewhat surprisingly, similar to the projective method for 3D semantic segmentation by Järemo Lawin et al. \[21\], it was shown by Goyal et al. \[14\] that on a point cloud classification task, a simple projection-based baseline called SimpleView performs on par with 3D approaches. Moreover, the authors designed a protocol for a fair comparison between point cloud learning methods revealing the importance of many factors orthogonal to the method architectures, such as evaluation procedure and hyperparameter tuning. Recently, a transformer-based approach combining local and global attention mechanisms was presented by Berg et al. \[1\].

Notably, the aforementioned approaches are rotation-variant, i.e., they require data augmentation if rotation invariance is desired. This also entails the model having an increased number of parameters for memorizing the data in various orientations.

2.2. Rotation-aware models

As an alternative, approaches have been proposed for learning rotation equivariant features, in which learned representations rotate in accordance with the input \[38, 13, 31\]. Among these are quaternion-based models \[52, 35\] and methods that perform a projection of the 3D input to a unit sphere \[8, 10\] and realize convolutions in the spherical harmonic domain.

The work of Deng et al. \[9\] introduced vector neurons by extending neurons from 1D scalars to 3D vectors, and thereby enabling a simple mapping of SO(3)-actions to latent spaces in the general rotation-equivariant framework. In the context of equivariant methods, Melnyk et al. proposed steerable 3D spherical neurons \[28\], which are SO(3)-equivariant filter banks obtained by virtue of conformal modeling \[30, 27\] and the symmetries of spheres as geometric entities \[28\].

Other methods make use of group representation theory and transform the inputs into a space in which it is easier to express rotation-equivariant maps \[38, 13, 31\], and after that obtain rotation-invariant prediction, e.g., when performing classification. This is achieved using filters constrained to be combinations of spherical harmonics, which limits their expressiveness. Therefore, such methods have naturally limited learning capability, and their performance falls short compared to rotation-sensitive methods for tasks that do not require rotation invariance.

There is a plethora of conceptually different works on hand-crafting low-level RI geometric features for arbitrary pairs of points (PPF) based on angles and distances \[48, 51, 47, 4, 16\], proposed to be used instead of the input point coordinates. For instance, similar to the triplets used by Granlund et al. \[15\], Zhang et al. \[49\] introduced a convolution operator that uses a point neighborhood constructed with triple-point (reference-neighbor-centroid) local triangles. In contrast, vector norm and relative angles between points were used by Chen et al. \[4\]. A robust RI representation, capturing both local and global shape structures, and region relation convolution, alleviating global information loss, was presented by Li et al. \[24\].

Recently, the pose information loss problem was revealed and addressed by introducing a pose-aware RI convolution (PaRI-Conv) with compact and efficient kernels by Chen and Cong \[4\]. Therein, a lightweight augmented PPF (APPF)
is proposed, encoding the local pose of each point in a local neighborhood in an ambiguity-free manner. Notably, their approach is also invariant under reflections, i.e., $O(3)$-invariant, and they use local reference frames (LRFs) as input. However, utilizing principal component analysis (PCA) to construct the LRF for RI point cloud learning, as done by Kim et al. [20] and Xiao et al. [46], is sensitive to perturbations. This is why Chen and Cong [41] proposed to build the LRFs upon local geometry only.

Input canonicalization is another category of methods that includes both rotation-variant (e.g., variants of [32] [42] that use spatial transformers), -equivariant (e.g., [11] [37] [36]), and -invariant [22] methods. The key idea in these approaches is to bring the input to a computed or predicted canonical reference frame and process it there.

Our approach builds upon the equivariant framework [9]: we apply steerable 3D spherical neurons [28] to learn $SO(3)$-equivariant 4D features from the 3D input point coordinates and then compute inner product of these features in the equivariant feature space. This way, we can learn a learnable $O(3)$-invariant descriptor, encoding both ambiguity-free pose information and local and global context.

### 3. Preliminaries

In this section, we introduce the necessary notation and recap the notion of equivariance and invariance and theoretical results from prior work, which will enable us to realize the compatibility of steerable 3D spherical neurons and vector neurons.

We define a 3D point cloud $X \in \mathbb{R}^{N \times (3+C)}$ as a collection of $N$ points, represented by their coordinates $x \in \mathbb{R}^3$ concatenated with the corresponding optional features $q \in \mathbb{R}^C$: $X = \{x_n \oplus q_n\}_{n=1}^N$. In the scope of this paper, we focus only on the point coordinates and assume that the optional features are rotation- and reflection-invariant.

#### 3.1. Equivariance and invariance

Given a group $G$ and a set of transformations $T_g : X \rightarrow X$ for $g \in G$, a function $f : X \rightarrow Y$ is said to be $G$-equivariant if for every $g$, there exists a transformation $V_g : Y \rightarrow Y$ such that

$$V_g[f(x)] = f(T_g[x]) \quad \text{for all } g \in G, \quad x \in X,$$

where $g$ represents transformation parameters.

Invariance is a particular type of equivariance. A function $f : X \rightarrow Y$ is said to be $G$-invariant if for every $g \in G$, the transformation $V_g : Y \rightarrow Y$ is the identity, i.e.,

$$f(x) = f(T_g[x]) \quad \text{for all } g \in G, \quad x \in X.$$  

In particular, we consider invariance under 3D orthogonal transformations (rotations and reflections), i.e., the group $O(3)$, and, as an intermediate step, equivariance under 3D rotations—the group $SO(3)$. In order to act as a transformation $T_g$ on a 3D vector $x \in \mathbb{R}^3$, the elements $g \in SO(3)$ are often represented by $3 \times 3$ rotation matrices $R$ [6]. However, this representation is not unique [53].

Our proposed descriptor, which we present in Section 4, is $O(3)$-invariant and equivariant under permutations of the input points. That is, permuting point indices $1, \ldots, N$ results in the corresponding permutation of the descriptor outputs.

In the remainder of the manuscript, we use the same notation to represent a 3D rotation matrix $R$ in the Euclidean space $\mathbb{R}^3$, the projective (homogeneous) space $P(\mathbb{R}^3) \subset \mathbb{R}^4$, and $\mathbb{RP}^3$, by appending the required number of ones to the diagonal of the original rotation matrix without changing the transformation itself [27].

#### 3.2. Conformal embedding

Geometric transformations are represented in a uniform way in conformal space. Given the Euclidean space $\mathbb{R}^3$, its conformal counterpart is constructed as $\mathbb{ME}^n \equiv \mathbb{R}^{n+1,1} = \mathbb{R}^n \oplus \mathbb{R}^{1,1}$, where $\mathbb{R}^{1,1}$ is the Minkowski plane [23]. A Euclidean vector $x \in \mathbb{R}^n$ is embedded in the conformal space $\mathbb{ME}^n$ by means of the following non-linear transformation:

$$X = C(x) = x + \frac{1}{2} ||x||^2 \infty + e_0,$$

where $X \in \mathbb{ME}^n$ is normalized (since the coefficient of $e_0$ is 1), $\{e_0, \infty\}$ is the Minkowski plane $\mathbb{R}^{1,1}$ null basis, representing the origin $e_0 = \frac{1}{2}(e_- - e_+)$ and point at infinity $\infty = e_- + e_+$, and $\{e_+, e_-\}$ is an orthonormal basis in $\mathbb{R}^{1,1}$ [23]. Geometrically, the conformal embedding [3] corresponds to a stereographic projection of $x$ onto a projection sphere in $\mathbb{ME}^n$ and is homogeneous, i.e., all $\mathbb{ME}^n$ vectors

$$[X] = \{\tilde{X} \in \mathbb{ME}^{n+1,1} : \tilde{X} = \gamma X, \gamma \in \mathbb{R} \setminus \{0\}\}$$

represent the same Euclidean vector $x$.

The scalar product in the conformal space has an entrancing interpretation in the Euclidean space: given two conformal embeddings $X$ and $Y = y + \frac{1}{2}||y||^2 \infty + e_0$, their scalar product in the conformal space is their (scaled) distance in $\mathbb{R}^n$, $X \cdot Y = -\frac{1}{2}||x - y||^2$. This is the key component in the design and implementation of the spherical neurons, which we review in Section 3.3.

For more details, we refer the reader to Li et al. [23] and Section 3 in the work of Melnyk et al. [27].

#### 3.3. Spherical neurons

Spherical neurons are defined as neurons with (hyper)spherical decision surfaces [30] [27]. Following Perwass et al. [30], we embed both a data vector $x \in \mathbb{R}^n$ and a
In Section 3.4, we review where, inter alia, it is demonstrated that the spherical neuron $\gamma$ where

$$ c = (c_1, \ldots, c_n) \in \mathbb{R}^n $$

is the hypersphere center and $r \in \mathbb{R}$ is its radius. Their conformal space $\mathcal{M}^E_n$ scalar product $X \cdot S$ can be equivalently computed in $\mathbb{R}^{n+2}$ as

$$ X \cdot S = X^T S = -\frac{1}{2} \|x - e\|^2 + \frac{1}{2} r^2. $$

The sign of this scalar product depends on the relative position of the point to the sphere in the Euclidean space $\mathbb{R}^n$: inside the sphere if positive, outside of the sphere if negative, and on the sphere if zero [30].

Thus, Perwass et al. [30] suggested to use the scalar product (6) as a classifier, i.e., a spherical neuron in $\mathcal{M}^E_n$ is implemented using the standard dot product in $\mathbb{R}^{n+2}$ simply as $f_s(X; S) = X^T S$, with learnable parameters $S \in \mathbb{R}^{n+2}$. Importantly, as noted by Melnyk et al. [27], spherical neurons do not necessarily require an activation function, due to the natural non-linearity of the embedding (5).

During training, the components of $S$ in (5) are treated as independent learnable parameters. Therefore, a spherical neuron effectively learns non-normalized hyperspheres of the form $S = (s_1, \ldots, s_{n+2}) \in \mathbb{R}^{n+2}$. Due to the homogeneity (4) of the conformal embedding (3), both normalized and non-normalized hyperspheres represent the same decision surface, and the spherical neuron can thus be written as

$$ f_s(X; S) = X^T S = \gamma X^T S, $$

where $\gamma := s_{n+2}$ is the (learned) normalization parameter and $S \in \mathbb{R}^{n+2}$ is the normalized sphere defined in (5). From this point, we will write $S$ when referring to a spherical decision surface, specifying its normalization if needed.

Further details are found in the work of Melnyk et al. [27], where, inter alia, it is demonstrated that the spherical neuron activations are isometries in 3D. That is, rigid transformations commute with the application of the spherical neuron. This result is a necessary condition to design rotation equivariant feature extractors based on spherical neurons [28], that we review in Section 3.5.

### 3.4. Steerable 3D spherical neurons

Under certain conditions, equivariant operators can be steered. A steerable 3D spherical neuron, recently introduced by Melnyk et al. [28], is a filter bank consisting of one learnable spherical decision surface $S \in \mathbb{R}^5$ (5) and three copies, formed by rotating the original sphere into the other three vertices of the regular tetrahedron, as defined in (8).

To construct this filter bank, the original (learned) sphere center $c_0$ is first rotated to $\|c_0\| (1, 1, 1)$ with the corresponding (geodesic) rotation denoted as $R_O$. The resulting sphere is then rotated into the other three vertices of the regular tetrahedron. This is followed by rotating all four spheres back to the original coordinate system. One steerable 3D spherical neuron is thus composed as the $4 \times 5$ matrix

$$ B(S) = \left[ R_O R_{T_1} R_O S \right]_{i=0,1}, $$

where each of $(R_{T_i})^3_{i=0}$ is the isomorphism in $\mathbb{R}^5$ corresponding to a 3D rotation from $(1, 1, 1)$ to the vertex $i + 1$ of the regular tetrahedron. Hence, $R_{T_0} = I_5$, i.e., $S$ remains at $c_0$.

We can view the steerable spherical neuron (8) as a function $f_s(\cdot; S) : \mathbb{R}^5 \rightarrow \mathbb{R}^4$ with five learnable parameters as a vector $S$. Crucially for our work, Melnyk et al. [28] proved that it is equivariant under 3D rotations:

$$ V_R B(S) X = B(S) RX, $$

where $X \in \mathbb{R}^5$ is a properly embedded 3D input point, $R$ is a representations of the 3D rotation in the conformal space $\cong \mathbb{R}^3$, and $V_R \in G < SO(4)$ is the 3D rotation representation in the filter bank output space:

$$ V_R = M^T R O K R_O^T M, $$

where $M \in SO(4)$ is a change-of-basis matrix that holds the homogeneous coordinates of the tetrahedron vertices (scaled by 1/2) in its columns as

$$ M = \frac{1}{2} \begin{bmatrix} 1 & 1 & -1 & -1 \\ -1 & 1 & 1 & -1 \\ -1 & -1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}. $$

We will use the equivariant filter bank output as a replacement for 3D points, e.g., in vector neural networks (VNNs) by Deng et al. [9].

### 3.5. Vector neurons

Two important properties of a VNN [9] are that 1) it is $SO(3)$-equivariant and produce RI features at the later layers, and 2) the local interaction between the points is modeled by exploiting edges, i.e., edge convolution introduced in DGCNN [41] are designed for processing data embedded in $\mathbb{R}^3$ and produce an ordered set of 3D vectors $y \in \mathbb{R}^3$ as output. Taking a point cloud $X \in \mathbb{R}^{N \times 3}$ as input, a VN extracts vector-list features $Y = \{Y_n\}_{n=1}^N \in \mathbb{R}^{N \times C \times 3}$, where $Y \in \mathbb{R}^{C \times 3}$ is a vector-feature and $C$ is the number of latent channels.

Specifically, a linear layer $f_{lin}(\cdot; W)$ comprised of VNs is defined by means of a weight matrix $W \in \mathbb{R}^{C \times C}$ acting
4.1. Learning SO(3)-equivariant features

TetraTransform The first layer $Y^{(0)}$ is formed by the TT layer $l_{\text{TT}}(\cdot; S): \mathbb{R}^{N \times 3} \rightarrow \mathbb{R}^{N \times 4 \times K}$, consists of $K$ steerable spherical neurons $B(S_k)$, representing a $K \times 5$ learnable weight matrix $S$.

TT first takes in a point cloud of 3D points $X \in \mathbb{R}^{N \times 3}$ and embeds them in the conformal space $\mathbb{R}^5$ according to [3], resulting in $(X_n)_n \in \mathbb{R}^{N \times 5}$.

Following the structure of point cloud processing networks [32, 41], the subsequent application of the steerable spherical neurons [8] is shared across points, thus making the output

$$Y^{(0)} = l_{\text{TT}}(X; S) \in \mathbb{R}^{N \times 4 \times K}$$

both rotation- and permutation-equivariant. Importantly, thanks to the conformal embedding of vectors [3], $l_{\text{TT}}(\cdot; S)$ is a non-linear layer, which is essential for neural networks.

Tetra-basis projections Note that each of the $K$ steerable spherical neurons [8] in $l_{\text{TT}}(\cdot; S)$ has its own representation of a 3D rotation $R$, given as $V^k_R \in G < \text{SO}(4)$, $k \in \{1, \ldots, K\}$, due to the rotation $R_O$ in (10) and (8) being computed from a learnable $S_k$. This must be taken into consideration when transforming $Y^{(0)}$, especially when obtaining invariant representations. We, therefore, keep all operations in the following sections $K$-independent. In fact, we see $Y^{(0)}$ as a collection of $N$ rotation-equivariant 4D vectors in $K$ different tetra-bases.

Deeper equivariant propagation We are free to add more SO(3)-equivariant layers on top of $l_{\text{TT}}$, if desired. For this, we can use the framework of VNs [9], reviewed in Section 3.5, and apply VNs to the first layer output $Y^{(0)}$, which we, therefore, need to view as a list of vector-features $Y^{(0)} = \{Y_n\}_{n=1}^N \in \mathbb{R}^{N \times 4 \times K}$.

We can thus extend VNs [9] to operate on 4D vectors, such as contained in $Y^{(0)}$. Obviously, a linear layer comprised of VNs $f_{\text{lin}}(\cdot; W)$ is also equivariant under $V^k_R \in G < \text{SO}(4)$. By replacing $R$ in (12) with $V^k_R$ in (10), and keeping in mind that vector-features $Y$ contain now 4D vectors, we see that (12) holds. The same applies to other VN-layers (e.g., non-linearities, batch norm) since they are specifically designed to be rotation-equivariant, see Deng et al. [9].

We denote a consequent application of SO(3)-equivariant and non-linear) edge convolution (13) and pooling (14) layers as $l_{\text{VN}}(\cdot; \Theta, \Phi): \mathbb{R}^{N \times C \times 4 \times K} \rightarrow \mathbb{R}^{N \times C' \times 4 \times K}$. In general, the $d$-th VN-layer taking $Y^{(d)} \in \mathbb{R}^{N \times C \times 4 \times K}$ as input produces an SO(3)-equivariant and permutation-equivariant feature map

$$Y^{(d+1)} = l_{\text{VN}}(Y^{(d)}; \Theta, \Phi) \in \mathbb{R}^{N \times C' \times 4 \times K},$$

on a vector-feature $Y \in \mathcal{Y}$ as $f_{\text{lin}}(Y; W) = WY$, and is SO(3)-equivariant since

$$Y' = l_{\text{VN}}(Y; W) = f_{\text{lin}}(Y; W) R = Y' R,$$ (12)

where $R \in \text{SO}(3)$ and $Y' \in \mathbb{R}^{C' \times 3}$.

Deng et al. [9] also presented how common neural network operations, such as batch norm [19], pooling, and non-linearities, can be adopted for VNs, and how VNs can be used in other point cloud processing networks. In particular, their VN-DGCNN modifies the permutation-equivariant edge convolution of the predecessor DGCNN [41] by computing adjacent edge features $E'_{nm} \in \mathcal{E}$ of vector-list representations $Y_n \in \mathbb{R}^{C \times 3}$, followed by a local SO(3)-equivariant pooling as

$$E'_{nm} = l_{\text{VN-nonlin}}(\Theta(Y_m - Y_n) + \Phi Y_n),$$ (13)

$$Y' = l_{\text{VN-pool}} r;(n,m) \in \mathcal{E}(E'_{nm}),$$ (14)

where $\Theta$ and $\Phi$ are learnable weight matrices, VN-nonlin and VN-pool are the respective equivariant non-linear and pooling layers (see Section 3 in Deng et al. [9] for details).

Notably, average pooling, being a linear operation, maintains rotation-equivariance and helps to achieve higher performance [9].

4. TetraSphere

In this section, we present TetraSphere—a learnable descriptor for O(3)-invariant point cloud processing—based on steerable 3D spherical neurons and the VN-framework.

Our overall approach consists of two steps: 1) we extract SO(3)-equivariant features, and 2) we obtain O(3)-invariant representations from them. As the first step, we perform TetraTransform (TT), i.e., lift the 3D input to a specific 4D space spanned by what we call a tetra-basis (see Figure 1).

Transforming points in the tetra-basis implies embedding a 3D rotation into a subgroup of SO(4) [28]. Since the entire theory of VNs [9] applies to $\mathbb{R}^4$ and SO(4) exactly the same way it does to $\mathbb{R}^3$ and SO(3), we plug our TetraTransform into VNs of dimension 4 and achieve rotation invariance. Note, however, that the VN layers operating on 4D vectors in our model are still equivariant only under 3D rotations.
where $C'$ are the latent channels. Given the TT output \( Y^{(0)} \in \mathbb{R}^{N \times 4 \times K} \), a VN-layer outputs a feature map \( Y^{(1)} \in \mathbb{R}^{N \times C \times 4 \times K} \).

### 4.2. O(3)-invariant representations

The TetraSphere architecture (see Figure 4), presented in this section, performs TT \((15)\) as the first step.

To obtain RI features, we follow related work (e.g., \([9]\) and \([47]\)) and exploit the fact that the inner product of two roto-equivariant vectors, rotated in \(\mathbb{R}^n\) with the same \(R\), is invariant:

\[
UR (TR)^T = URR^T T^T = UT^T = H, \tag{17}
\]

where \(U \in \mathbb{R}^{C \times n}\), \(T \in \mathbb{R}^{C' \times n}\), and \(H \in \mathbb{R}^{C \times C'}\). Note that \(H\) is O\((n)\)-invariant since the sign of \(\det(R)\) does not change the equality \(17\).

If we take \((17)\) and consider \(U \in \mathbb{R}^{C \times 3}\) and \(T \in \mathbb{R}^{C' \times 3}\) to be 3D vector-features of the same 3D point, but at two different layers with \(C\) and \(C'\) channels, respectively, we will get the VN-framework approach (see Section 3.5 in Deng et al. \([9]\)). In this case, we refer to \((17)\) as a point-wise inner product of features. We adopt this procedure to our 4D vectors and perform this for each \(K\).

In the first step, TT \((15)\) produces \(Y^{(0)} \in \mathbb{R}^{N \times 4 \times K}\). We then apply a desired number of VN-layers \((16)\) to it, obtaining \(Y^{(d)} \in \mathbb{R}^{N \times C \times 4 \times K}\).

To produce RI features, we follow Deng et al. \([9]\) and concatenate \(Y^{(d)}\) with its global mean (over \(N\)), \(\bar{Y}^{(d)} = \frac{1}{N} \sum_n Y^{(d)}_n \in \mathbb{R}^{C \times 4 \times K}\), and propagate the result through \(m\) additional VN-layers to obtain \(Y^{(d+m)} \in \mathbb{R}^{N \times C' \times 4 \times K}\). We then extract matrices \(U \in \mathbb{R}^{C \times 4}\) from \(Y^{(d)}\) and \(T \in \mathbb{R}^{C' \times 4}\) from \(Y^{(d+m)}\) and perform \((17)\) for all \(N\) and \(K\).

We denote the propagation from VN-layer \(d\) to layer \(d + m\) with the subsequent point-wise product as a block \(l_{mn}(\cdot; \Theta, \Phi) : \mathbb{R}^{N \times C \times 4 \times K} \rightarrow \mathbb{R}^{N \times C' \times 4' \times K}\), where \(\Theta\) and \(\Phi\) denote the learnable parameters of the VN-layers. In practice, we select \(C' = 4\), motivated by choice of \(C' = 3\) in the original VN-approach \([9]\).

In the case of a single VN-layer \((16)\) following after the TT-layer \((15)\), we describe TetraSphere operating on \(X \in \mathbb{R}^{N \times 3}\) as

\[
H = l_{VN}(l_{TT}(X; S)), \tag{18}
\]

where \(H \in \mathbb{R}^{N \times C' \times 4 \times K}\) is an O\((3)\)-invariant and permutation-equivariant descriptor of \(X\).

TetraSphere encodes densely-connected angle and distance information of 4D vectors. However, it does so on the latent 4D features of each point point-wise, relying on edge convolutions \((15)\) in the VN-layers to model the interaction between the points.

**Aggregating over tetra-bases** The feature \(H \tag{18}\) is O\((3)\)-invariant, which is why we can safely manipulate its last, tetra-basis, dimension \(K\). While it is possible to combine it with other dimensions, e.g., reshaping the output to be \(H \in \mathbb{R}^{N \times C \times 4' \times K}\), it will evidently increase the complexity of the model, requiring \(K\) times more parameters in the subsequent layer in the network (considering a fully-connected layer).

In this initial study, we will only consider the case \(K = 1\) to investigate the practical utility of TetraSphere, which corresponds to a non-linear change of the coordinate system from 3D to a 4D space spanned by the tetra-basis.

### 5. Experiments

In this section, we conduct experiments with TetraSphere based on the rotation-equivariant VN-DGCNN architecture \([9]\). We evaluate our model on the task of classifying synthetic and real-world 3D objects and compare it with the results from VN-DGCNN \([9]\) and other methods.

#### 5.1. Datasets

**ModelNet40** We use the ModelNet40 dataset \([44]\) provided by \([32]\) that consists of 12,311 CAD models of 40 classes and has been widely employed for the task of synthetic 3D shape classification \([4] [52] [24]\). The dataset is split into 9843 training shapes and 2468 shapes for testing. We follow the common preprocessing routine and randomly sample 1024 points from each 3D model, center them at the origin, and normalize them to a unit sphere. A shape from the test split can be seen in Figure 1.
Methods | $z/z$ | $z/\text{SO}(3)$ | $\text{SO}(3)/\text{SO}(3)$ |
|---|---|---|---|
| **Rotation-sensitive** | | | |
| PointNet++ [33] | 91.8 | 28.4 | 85.0 |
| PointCNN [25] | 92.5 | 41.2 | **84.5** |
| DGCNN [42] | 90.3 | 33.8 | 88.6 |
| ShellNet [50] | **93.1** | 19.9 | 87.8 |
| **Rotation-equivariant** | | | |
| Spherical-CNN [10] | 88.9 | 76.7 | 86.9 |
| TFN [38] | 88.5 | 85.3 | 87.6 |
| TFN [31] | 89.7 | 89.7 | 89.7 |
| VN-DGCNN [9] | 89.5 | 89.5 | **90.2** |
| TetraSphere (Ours) | **90.1** | **90.1** | 90.0 |

Table 1. Classification accuracy (%) on the ModelNet40 shapes under different train/test settings of rotation augmentation. The listed methods use no additional input information, such as normals or features. The best results per method category are presented in **bold**.

ScanObjectNN We also evaluate our method on real-world indoor scenes. For this, we use ScanObjectNN [39], which consists of approximately 15,000 objects with 2902 unique objects belonging to 15 classes. We employ its hardest subset called PB_T50_RS, in which perturbations introduce various levels of partiality and background to the objects. Besides, the objects undergo 50% bounding box translation, random rotation around the gravity axis, and random scaling. Some examples are presented in Figure 3.

We preprocess the dataset the same way as ModelNet40, obtaining 1024 points per object instance, and follow the train/test split provided by the original repository.

Rotation setup In general, we employ the following train/test rotation settings: $I/\text{SO}(3)$, $z/z$, $z/\text{SO}(3)$ and $\text{SO}(3)/\text{SO}(3)$, with the first one being the most challenging and the most practical. Here, “I” means no rotation augmentation, $z$ denotes vertical-axis rotation augmentation, and $\text{SO}(3)$ stands for arbitrary 3D rotations, all generated and applied to the input shapes during training/testing.

https://github.com/hkust-vgd/scanobjectnn

5.2. Architecture and implementation details

We use VN-DGCNN as the backbone, with the standard choice of $k = 20$ (nearest-neighbor graph computation parameter) for all layers and the dropout in the last two fully-connected layers of 0.5. Thus, the VN-MLP and the $l_{m}$-block in TetraSphere (see Figure 2) reformulate the structure of the baseline [9]. We apply VN-LeakyReLU as the learnable equivariant non-linearity in (13), and use average pooling in VN-layers [14], given its better performance for the baseline. We refer to the resulting model with $K = 1$ simply as TetraSphere.

We adopt the official implementation of Deng et al. [9] to implement our model in PyTorch [29]. Following Melnyk et al. [28], we initialize the parameters in the TT layer (i.e., the spheres) using the standard initialization for the linear layers in PyTorch.

We use the same strategy for training TetraSphere as the baseline [9]. We employ SGD with an initial learning rate of 0.1 and momentum equal to 0.9, and a cosine annealing strategy for gradually reducing the learning rate to 0.001. Like the baseline, we augment the data with random translation in the range $[-0.2, 0.2]$ and scaling in the range $[2/3, 3/2]$ during training. For ModelNet40 classification, we train TetraSphere for 250 epochs, and for 1000 epochs for all ScanObjectNN experiments. The batch size is set to 32.

5.3. Results

The main results of our experiments are presented in Table 1 and Table 2, where for a fair comparison, we list methods that only use point clouds/meshes as input, and no additional information, such as normals or features.

TetraSphere is slightly outperformed by some recent RI methods [43, 22, 4] classifying the synthetic ModelNet40 data, as shown in Table 1. On the other hand, TetraSphere respects rotation equivariance, which broadens its applicability: among equivariant methods, including the baseline VN-DGCNN, TetraSphere demonstrates superior performance.

The marginally inferior result under the $\text{SO}(3)/\text{SO}(3)$ protocol indicates that our method does not benefit from additional input features.

Table 2. Classification accuracy (%) on the real-world objects from the PB_T50_RS subset of ScanObjectNN under different train/test settings of rotation augmentation. The listed methods only use 3D coordinates as input. The overall best results are presented in **bold**.
rotation augmentation when classifying synthetic shapes.

Classifying the real-world objects from the most challenging subset of ScanObjectNN (see Table 2), our model outperforms the most recent best result [16] by 2.8% under the more practical Z/SO(3) protocol, and by 3.7% under the SO(3)/SO(3) setting, achieving state-of-the-art classification performance.

We also verify that TetraSphere is exactly O(3)-invariant by applying random reflections during inference, as shown in Table 3. Note that even when training on data in a single orientation (“I”), TetraSphere achieves remarkably high performance classifying arbitrarily rotated objects.

Furthermore, we do an ablation study testing the importance of training the TT layer parameters. As presented in Table 3, having a learnable TT (the default choice) results in higher accuracy than when keeping the initial parameters of TT frozen.

5.4. Visualization

The TetraTransform layer $l_{TT}$ [15] parameters have a lucid geometric meaning since they constitute steerable 3D spherical neurons [28] $B(S_k)$ [8], which we illustrate in Figure 4.

For the visualization, we use a ModelNet40 test shape and our TetraSphere with $K = 1$, i.e., a single learnable sphere in $l_{TT}$, initialized (randomly) as

$$\tilde{S}_{\text{init}} = (-0.0920, 0.2274, 0.0621, -0.0547, 0.1240)$$

and, after the model training is complete, optimized to

$$\tilde{S}_{\text{optim}} = (0.0529, 0.2875, 0.4309, 0.3016, 0.3694).$$

Following Melnyk et al. [27], we perform the normalization of the spheres to bring them to the canonical form [5] and extract the centers and radii of $S_{\text{init}}$ and $S_{\text{optim}}$:

$$c_{\text{init}} = (-0.7417, 1.8342, 0.5012),$$

$$r_{\text{init}} = 2.2469, \quad \gamma_{\text{init}} = 0.1240,$$

and

$$c_{\text{optim}} = (0.1432, 0.7783, 1.1666),$$

$$r_{\text{optim}} = 0.5950, \quad \gamma_{\text{optim}} = 0.3694.$$ 

The resulting spheres $B(S_{\text{init}})$ and $B(S_{\text{optim}})$ are shown on the top and bottom in Figure 4, respectively.

6. Conclusion

The O(3)-invariant TetraSphere descriptor, proposed in this paper, is based on steerable 3D spherical neurons and builds upon VN-DGCNN. To the best of our knowledge, we use the steerable neurons in an end-to-end approach for the first time, thereby unveiling their practical utility. TetraSphere sets a new state-of-the-art performance on the task of classifying randomly rotated 3D objects from the challenging real-world ScanObjectNN dataset. This result demonstrates the effectiveness of spherical decision surfaces for learning in 3D Euclidean space. We believe our work can pave the path to geometrically justified and more robust handling of real-world 3D data.
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