1. Introduction

Dark matter is a crucial ingredient of modern cosmology. It is needed to account for the anisotropy of the cosmic microwave background (Wright et al. 1992; Spergel et al. 2003), large-scale structure formation and evolution (see, e.g., Springel et al. 2006), dynamics of galaxy clusters and groups (see, e.g., Zwicki 1993), flattened rotation curves of late-type galaxies (see, e.g., Bosma 1981), and mismatch between mass centers revealed by gravitational lensing and X-ray emission, respectively (see, e.g., Markevitch et al. 2002). While the mass density of dark matter is about six times higher than that of ordinary matter, its nature remains a mystery. Regardless of its composition, the microscopic properties of dark matter directly affect its cosmic mass distribution. Therefore, through a comparison between theoretical calculation and observations of dark matter distributions, the properties of dark matter can be constrained. Since the 1970s, N-body simulations have become an indispensable theoretical tool for dealing with dark matter’s nonlinear clustering. In the 1980s, N-body simulations were used to rule out a neutrino-dominated universe (White et al. 1983) and establish the canonical cold dark matter (CDM) model (Peebles 1982; Blumenthal et al. 1984; Davis et al. 1985; Frenk et al. 1985). With the improvements in the resolution of N-body simulations, the inner structure of CDM halos was first resolved in the 1990s (Dubinski & Carlberg 1991; Navarro et al. 1996b). These high-resolution simulations revealed a central cusp in CDM halos, characterized by a power-law density profile \( \rho(r) \propto r^{-\alpha} \), with \( \alpha = 1 \), independent of the halo mass. The cuspy density profile results in a steeply rising rotation curve as \( V_{\text{cir}} \propto r^{1/2} \), which is incompatible with the observed solid-body rotation, \( V_{\text{cir}} \propto r \), of dark matter-dominated dwarf and low surface brightness ( LSB) galaxies (Flores & Primack 1994; Moore 1994; Moore et al. 1999b). This tension is referred to as the core-cusp problem (de Blok 2010). Together with other small-scale problems it challenges the standard CDM model (Bullock & Boylan-Kolchin 2017).

Within the CDM framework, the resolution of the core-cusp problem relies on baryonic feedback. The idea is that the energy released from starburst radiation or supernovae explosions is indirectly transferred to dark matter particles via frequent fluctuations of potential well driven by repeated and fast gas outflows from the Galactic center (Navarro et al. 1996a; Pontzen & Governato 2012). The extra energy gained by dark matter particles expands their orbits and converts an initial cuspy density profile into a cored one. The degree of conversion depends on the total energy released from feedbacks measured by stellar mass \( M_* \), the total baryonic matter \( M_b \) opposing the expansion, and the energy transfer efficiency. However, the last one is found to be very sensitive to the star formation gas density threshold \( n \), a numerical parameter commonly adopted in sub-grid models of galaxy formation (Benitez-Llambay et al. 2019; Dutton et al. 2019). Cosmological simulations with a large \( n \) report core formation in simulated dwarf galaxies (Governato et al. 2010; Chan et al. 2015; Tollet et al. 2016), while the dark matter cusp remains in simulations using a smaller \( n \) (Sawala et al. 2016). To date, a consensus has not been reached about whether baryonic processes can solve the core-cusp problem, and it continues to cast doubt on the CDM model.

Apart from the cored profile, many dwarf and LSB galaxies also have a similar shape of rotation curves, implying a self-similar dark matter structure (Kravtsov et al. 1998; Salucci & Burkert 2000). This conformity is unlikely to be due to chaotic and dramatic baryonic processes (Burkert 1995; Navarro 2019), but rather a clue to the nature of dark matter beyond CDM. In this work we study a decaying dark matter (DDM) model, which describes two-body decay of dark matter:

\[
\psi^* \to \psi + l, \tag{1}
\]

where \( \psi^* \) stands for the unstable mother dark matter particle, \( \psi \) is a massive stable daughter dark matter particle, and \( l \) is a light and relativistic particle. Dark matter therefore has multiple components. The dynamics of the two-body decay in this model are fully controlled by two parameters: the decay rate \( \Gamma \), or half-life \( \tau^* = \ln 2/\Gamma \), of mother particles and \( \epsilon \) the energy of \( l \) in the unit of the mother particle’s rest mass. The recoil velocity, \( V_s \), of daughter particles in the center-of-mass frame...
of their mothers can be expressed as $V_k = c c$, where $c$ is the speed of light.

The DDM model has diverse behaviors, depending on the values of $V_k$ and $\tau^*$. For a very small $\tau^*$ such that almost all mother particles decay into daughter particles before any nonlinear structures have formed, the DDM model is similar to the warm dark matter model with a free-streaming length determined by the recoil velocity $V_k$ (Kaplinghat 2005; Strigari et al. 2007). For a large $V_k$ such that the decay converts a fair amount of energy from the matter component to the relativistic species, the expansion history of the whole universe would be altered (see, e.g., Vattis et al. 2019). For a broad range of $\tau^*$ while $V_k \lesssim 40$ km s$^{-1}$, the 1D Ly$\alpha$ forest power spectrum predicted by the DDM model is shown to be consistent with observation data (Wang et al. 2013), implying that the model behaves like CDM at large scale. Inside a parameter region outlined by $10.0 \lesssim V_k \lesssim 40.0$ km s$^{-1}$ (or equivalently $3.0 \times 10^{-5} \lesssim c \lesssim 1.3 \times 10^{-4}$) and $0.1 \lesssim \tau^* \lesssim 14$ Gyr, decay significantly heats up the dark matter inside dwarf-sized halos, and is relevant for the small-scale problems of CDM (Abdellegader & Melia 2008; Peter & Benson 2010; Wang et al. 2014). In light of this, we revisit the core-cusp problem and study the density profiles of dwarf halos in the DDM model using high-resolution cosmological N-body simulations, which is absent from previous studies.

This paper is structured as follows: we first review previous DDM algorithms for N-body simulations and then introduce our new DDM algorithm, test its performance, and calibrate numerical parameters for high-resolution zoom-in simulations in Section 2. An overview of our highest-resolution simulation suite is given in Section 3. Section 4 details our mathematical modeling of the evolution of an isolated halo in the DDM cosmology. We present our main results in Section 5, followed by a more extensive discussion of the DDM model in Section 6. We summarize in Section 7.

2. Methodology

2.1. Overview of DDM N-body Algorithms

In the DDM model, light particle $l$ does not take part in the structure formation directly, and is ignored in the following discussion. For the mother particle $\psi^*$ and daughter particle $\psi$, their Boltzmann equations are

$$\frac{df_{\psi^*}}{dt} = -\frac{\ln 2}{\tau^*} f_{\psi^*} , \quad (2)$$

and

$$\frac{df_{\psi}}{dt} = \frac{\ln 2}{4\pi^* V_k} \int f_{\psi^*}(r, v', t) \delta(|v - v'| - V_k) d^3v' , \quad (3)$$

respectively, where $f_{\psi^*}$ and $f_{\psi}$ are the corresponding phase-space mass densities. The evolution of matter density can be simulated by using the N-body method once the collision terms in Equations (2) and (3) are properly handled. The first DDM N-body simulation was presented in Peter et al. (2010, hereafter PMK10), where the DDM model (1) was realized on a simulation particle basis. In PMK10, each mother simulation particle has a decay probability. Once chosen for decay, the mother simulation particle is flagged to be a daughter simulation particle and receives a random velocity kick at the same time. This Monte Carlo sampling of decay is carried out at each simulation timestep. Therefore, the global decay rate $\ln 2/\tau^*$ is sampled continually and precisely for the whole system, while the local decay rate fluctuates around the global value with an amplitude depending on the local number density of mother simulation particles. As time goes on, the total number of mother simulation particles drops and the matter density field becomes increasingly nonlinear. It can be seen that the decay sampling precision of PMK10 is not uniform in both the space and time domains. It is also an intrinsic challenge for the PMK10 algorithm to resolve the central structures of dark matter halos due to the limited number of mother simulation particles there.

To achieve a uniform decay sampling in both the space and time domains, Cheng et al. (2015, hereafter CCT15) proposed a DDM algorithm based on a discretization of the Boltzmann Equations (2) and (3). In CCT15, decay is only sampled at certain simulation times, i.e., when each mother simulation particle is split partly to generate a new daughter simulation particle, which is kicked randomly at its birth. At other simulation times, all simulation particles are evolved according to the collisionless Boltzmann equation:

$$\frac{df_{\psi^*}}{dt} = 0 . \quad (4)$$

The number of mother particles is kept unchanged, and the mass-splitting procedure is the same for all mother simulation particles. Therefore, the decay can be sampled uniformly both in space and time domains. Two numerical parameters are introduced in this algorithm: $f_s$, the number of mass splittings throughout a simulation, and $N_s$, the number of daughter simulation particles produced per mother simulation particle at each mass splitting.

Consider a simulation following the CCT15 algorithm with $f_s$ splittings. Initially it has $N$ mother simulation particles. As each mass-splitting procedure generates $N_s N$ daughter simulation particles, the total number of daughter simulation particles will increase to $f_s N_s N$ by the end of the simulation. For typical values that achieve satisfactory numerical convergence, such as $f_s = 10$ and $N_s = 1$, the final number of daughter simulation particles is larger than that of mother simulation particles by an order of magnitude. Generally the number of simulation particles serves as a measure of the precision of an N-body simulation. However this measure does not apply to the CCT15 algorithm, because the phase-space distribution of daughter particles is inferred from N discrete mother simulation particles, not from an underlying continuous distribution function. The sampling resolution of daughter simulation particles is limited by that of the mother simulation particles, which is set in the initial condition. This is also true for the PMK10 algorithm. Hence, increasing the number of daughter simulation particles contributes little to improving the simulation’s resolution. On the other hand, the large demand for computing resources of the CCT15 algorithm hinders its practical usage in large cosmological simulations. Therefore, improvement of the CCT15 algorithm is needed for our purpose of running high-resolution zoom-in simulations.

2.2. An Improved DDM Algorithm

We follow the framework outlined in CCT15. In our algorithm, the mother simulation particles decay and give birth to daughter simulation particles only at a limited number of decay instances, called breakpoints. The breakpoints are ordered
on the time axis in such a way that in each phase (i.e., the time interval between two adjacent breakpoints), the same mass of mother simulation particles decay into daughter simulation particles. When a breakpoint is reached, each mother simulation particle is split into a less-massive mother simulation particle and a daughter simulation particle according to the decayed fraction. The newly generated daughter simulation particles are called auxiliary daughters. The mass-splitting procedure increases the total number of simulation particles by $N$. To save memory as well as to speed up the whole simulation, we only track the motion of these $N$ auxiliary daughters for each phase. When the simulation arrives at the next breakpoint, the auxiliary daughters born at the last breakpoint will undergo a random selection process such that only a fraction of them $\eta$ survive and are renamed as permanent daughters. The remaining auxiliary daughter particles are eliminated so that the memory occupied by them is released. All permanent daughters will be traced to the end of the simulation. Therefore, there are two states of daughter simulation particles in our scheme: auxiliary and permanent.

The auxiliary daughters help to conserve the local matter mass when a decay occurs, a basic conservation observed in both CCT15 and PMK10. After diffusing into the environment, these auxiliary daughters are replaced by permanent daughters, which have a smaller population, and hence heavier. The transition from the auxiliary state to permanent state decreases the resolution of daughter particles. Though numerical side effects can be introduced from the degradation of the resolution, it is controllable by tuning the survival fraction $\eta$. Suppose there are $N$ mother simulation particles initially and they go through $f_s$ breakpoints. When the simulation is finished, there are $n_f N$ permanent daughters in total, with $n_f = n_f s$. Setting $\eta = 1$ brings our algorithm back to CCT15. We implement this algorithm in an individual module named DDMPLUGIN, see Appendix A for details. It is designed to be compatible with other $N$-body codes such that the physics associated with dark matter decay is self-contained. This is achieved as follows: during each phase, the whole system is evolved by an $N$-body CDM code. When a breakpoint is reached, the system’s final state is output to the DDMPLUGIN module, which implements the physical effects of dark matter decay. Hence, an updated system state is generated. The DDMPLUGIN module then outputs this new state to be the initial condition for next phase’s evolution, which is again tracked by the $N$-body CDM code. Therefore, our DDM simulation can be easily implemented in any $N$-body code, an advantage rooted in the CCT15 algorithm.

### 2.3. Cosmological Zoom-in Simulation

To study the density profiles of dark matter halos in the DDM cosmology, we run cosmological zoom-in simulations (Oñorbe et al. 2014) using the DDMPLUGIN module with the $N$-body code P-GADGET3, a descendant of the public TreePM code GADGET2 (Springel 2005). In all simulations, a flat geometry with a cosmological constant is assumed for the background cosmology, where the cosmological parameters are taken from the final results of Planck TT,TE, and EE+lowE measurements: $\Omega_m = 0.3166$, $\Omega_k = 0.6834$, $h = 0.6727$, $n_s = 0.9649$, and $\sigma_8 = 0.8120$ (Planck Collaboration et al. 2020a). The initial conditions are generated using the code MULTI-Scale Initial Conditions (MUSIC; Hahn & Abel 2011) with the Bardeen–Bond–Kaiser–Szalay (BBKS) transfer function (Bardeen et al. 1986). The uncertainties induced by the choice of transfer functions are quantified in Appendix B.1. The effect turns out to be negligible. Dark matter halos are identified by the AMIGA Halo Finder (AHF) (Gill et al. 2004; Knollmann & Knebe 2009), with Bryan and Norman’s fitting formula (Bryan & Norman 1998) for the calculation of overdensity $\Delta_c$. For our adopted cosmology, dark matter halos at redshift $z = 0$ are defined by the virial radius $R_{vir}$ within which the mean overdensity is about 103.4 times the critical density $\rho_{\text{crit}}$. Based on the position of halo center, the virial radius, and the best-fit Navarro–Frenk–White (NFW) profile provided by AHF, we collect all particles bound to the target halo and use our own code to measure its radial mass distribution $\bar{\rho}(r)$, the average matter density inside radius $r$. The two-body relaxation time $t_{\text{rel}}$ is estimated for certain radii using the method documented in Binney & Tremaine (2008). Only those radii with $t_{\text{rel}}$ larger than the Hubble time $H_0^{-1}$ are considered to be reliably resolved (Fukushige & Makino 2001).

To select a candidate halo for zoom-in simulations, we run a full-box CDM cosmological simulation starting from redshift 99. It uses 256$^3$ simulation particles inside a periodic cubic box with a width of 6.73 $h^{-1}$ Mpc, corresponding to a physical length of 10.0 Mpc at $z = 0$. Each CDM simulation particle has a mass of $1.59 \times 10^9 h^{-1} M_\odot$, which is the coarse resolution of the full-box simulation. We select dark matter halos at redshift 0 by two criteria: small Lagrangian volume and being isolated from larger structures. The selected dark matter halo has a virial mass of $M_{\text{vir}} = 5.17 \times 10^9 h^{-1} M_\odot$, and an NFW concentration of $c_{\text{crit}} = 21.6$, closely following the theoretically motivated $c-M$ relation of Diemer & Joyce (2019). Its large-scale environment is illustrated in Figure 1, showing that the selected halo is far away from surrounding larger structures.

For zoom-in simulations centered at the selected halo, the initial density fluctuations inside its Lagrangian volume are sampled by high-resolution particles, while its large-scale environment is represented by coarse resolution particles. A buffer volume is created between the above two regions to avoid large resolution gradient. We label the resolution level of
A zoom-in simulation by an integer \( l \). A level-\( l \) zoom-in simulation has a mass resolution equivalent to that of a full-box \( N \)-body simulation using \( (2^l)^3 \) CDM particles in its initial condition. We adopt the empirical formula recommended by Power et al. (2003) to calculate the gravitational softening lengths for high-resolution CDM zoom-in simulation particles. As for DDM zoom-in simulations, the decay of mother simulation particles are only switched on inside the high-resolution volume. It is reasonable since we do not consider decay parameters that result in significant deviations from the CDM large-scale matter distribution. Gravitational softening lengths for high-resolution DDM particles are set to be the same as those of CDM zoom-in particles that have the same resolution level. This reflects the fact that the resolution of a DDM simulation is constrained by its initial condition. We also require that zoom-in halos are free of contamination by lower resolution particles within their virial radii for a clean analysis. In the following subsection, we test and calibrate the parameter \( f_s \) and \( n_f \) of our DDM algorithm to study the dark matter halo density profile.

2.4. Numerical Parameters Calibration

The parameter \( f_s \) controls the decay sampling frequency, and a larger value of it leads to a finer sampling of the decay history. In CCT15, the total number of daughter particles is linearly proportional to \( f_s \). Hence, it cannot be arbitrarily large, otherwise the computational workload would be huge. However, this constraint on \( f_s \) is released in our algorithm as the total number of daughter particles is solely determined by \( n_f \). For the DDM zoom-in simulations used in this study, we set \( f_s = 10 \) by default. It proves to be large enough for acceptable numerical convergences (see Appendix B.2 for details).

In our algorithm, \( n_f \) controls the total number of simulation particles. To test its effects on the halo average density profile, we tried three values: \( n_f = 1, 3, \) and 5. A simulation using the CCT15 algorithm (equivalent to \( n_f = f_s \)) is also carried out for comparison. All simulations are run in level-11 resolution using the same decay parameters \( V_\chi = 20.0 \, \text{km} \, \text{s}^{-1} \) and \( \tau^* = 3.0 \, \text{Gyr} \). The density profiles at \( z = 0 \) are measured down to the innermost resolved radii and are shown in the left panel of Figure 2. The density profiles with different values of \( n_f \) all converge to CCT15’s density profile within 2% at all resolved radii. It implies that the radial mass distribution \( \rho(r) \) is insensitive to the value of \( n_f \) when \( n_f \geq 1 \). We further run a level-12 simulation using the same decay parameters for \( n_f = 1 \). In the right panel of Figure 2, we compare all level-11 profiles with the level-12 one. For \( r \lesssim 1.0 \, h^{-1} \, \text{kpc} \), all level-11 profiles are systematically lower than the level-12 profile. The differences continue to grow when \( r \) approaches to the halo center. A large \( n_f \) does help to narrow the differences down, however, the computational cost is also huge: the gain in precision is only about 3% by increasing \( n_f \) from 1–10 (the CCT15 algorithm). As a larger value of \( n_f \) brings forth a larger number of simulation particles, the two-body relaxation converged radius \( r_{\text{rel}} \) decreases as the value of \( n_f \) increases. However, for \( n_f > 1 \), \( r_{\text{rel}} \) cannot be as the innermost resolved radii anymore, because the deviation from the level-12 profile at \( r_{\text{rel}} \) also grows with the \( n_f \) and reaches 10% level for \( n_f \geq 5 \). Hence, for the level-11 runs, increasing the value of \( n_f \) does not decrease the innermost resolved radius reliably. On the contrary, the innermost resolved limit set by the \( r_{\text{rel}} \) of the \( n_f = 1 \) run is worthy: the density profile of the level-11 resolution converges to that of the level-12 resolution within 5% for all radii larger than 0.8 \( h^{-1} \, \text{kpc} \). Then we can safely use \( n_f = 1 \) without worrying about possible degradation of resolution and take \( r_{\text{rel}} \) as the innermost resolved radius. For our highest-resolution zoom-in simulations, the level-12 runs, we use \( n_f = 1 \) and \( f_s = 10 \). The results are presented in the next section.

3. High-resolution Zoom-in Simulations

Our level-12 simulation suite comprises 10 cosmological zoom-in simulations: one CDM and nine DDM realizations. All 10 simulations use the same initial condition at redshift 99. The high-resolution volume in the initial condition is a cuboid with three edge lengths of 0.09, 0.09, and 0.15 \( h^{-1} \, \text{Mpc} \), centering at its parent simulation box. The mass of each high-resolution particle is \( 3.89 \times 10^2 \, h^{-1} \, \text{M}_\odot \). The gravitational

![Figure 2](image-url)
softening length of high-resolution CDM particles is set to be 33.7 \( h^{-1} \) pc, frozen to a physical length of about 50 pc after \( z = 10 \). All DDM realizations follow the same softening length assignment scheme such that the force resolution of our simulation suite is uniform. All our zoom-in halos are free of contamination by low-resolution particles within their virial radii. The nine DDM realizations differ from each other in the combination of \( V_k \) and \( \tau^* \). We used three values of \( \tau^* \): 3.0, 6.93, and 14.0 Gyr, with corresponding decayed fractions 0.959, 0.748, and 0.495, respectively. For each \( \tau^* \), we used three different values of \( V_k \): 20.0, 30.0, and 40.0 km s\(^{-1}\). These nine realizations constitute a rough sampling of the region of interest in the \( \tau^* - V_k \) parameter space. Halo expansion has been observed in the nine DDM zoom-in halos as they generally have smaller virial masses but lower concentrations compared to the corresponding CDM halo. The virial masses, virial radii, \( \nu_e \) and other global properties of all zoom-in halos are summarized in Table 1. We study the physics accounting for the DDM halo expansion in next section.

### 4. A Simplified Semi-analytic Model of DDM Density Profiles

The halo expansion in the DDM model is driven by two primary physical processes. The first one is the decay itself. On average, the kinetic energy of newly born daughter particles are greater than those of their mothers. This extra kinetic energy drives the orbits of daughters outwards, hence expanding the whole halo. We refer to this as the Step-1 expansion. A consequence of this expansion is the weakening of the gravitational potential, triggering the Step-2 expansion: the bulk particles’ orbits expand outwards to rebalance the weakened gravity with the inertial force seen in the orbits’ rotating frames. \( \nu_e \) (2001) considered a special case of the two-step expansion: \( V_k \gg \nu_e \) and \( \tau > t_{\text{dyn}} \), where \( \nu_e \) and \( t_{\text{dyn}} \) are the halo’s typical escape velocity and dynamical time, respectively. The large \( V_k \) unbinds all daughter particles during the Step-1 expansion, while the slow decay simplifies the Step-2 evolution to an adiabatic expansion. Starting from an NFW density profile, the resulting density profile turns out to remain an NFW shape, but with a smaller concentration and a lower normalization density (see also the relevant discussion in Peter 2010). Sánchez-Salcedo (2003) considered the situation where most of daughter particles are bound to the halo (\( V_k \ll \nu_e \)) and the halo expands adiabatically (\( \tau > t_{\text{dyn}} \)). Through simple semi-analytic calculations, he argued that the cored profile is a natural result of the two-step expansion. In this section, we present a general formalism to implement the two-step expansion.

Our model assumes that a dark matter halo forms at a high redshift when only a small fraction of mother particles have decayed. Initially, the halo is CDM-like with an NFW density profile. Then the decay proceeds and the density profile evolves. The decay of a mother particle is a random process and does not have a preferential direction. On average, each newborn daughter particle acquires an additional amount of kinetic energy from the mass deficit of its mother particle:

\[
\langle E_{k, \text{dau}} \rangle = E_{k, \text{mom}} + \frac{1}{2} m_{\text{dau}} V_k^2,
\]

where \( E_{k, \text{mom}} \) is the kinetic energy of the decayed mother particle and \( \langle E_{k, \text{dau}} \rangle \) is the expected kinetic energy of its daughter particle with mass \( m_{\text{dau}} \). Similarly, the mean angular momentum of a daughter particle \( \langle L_{\text{dau}} \rangle \) is the same as that of its mother, \( L_{\text{mom}} \):

\[
\langle L_{\text{dau}} \rangle = L_{\text{mom}}.
\]

Based on Equations (5) and (6), we build our simplified model using Schwarzschild’s orbit-based method (Schwarzschild 1979) (see Chaname 2010 for a short overview), which represents a collisionless system by a large library of particle orbits. Physical quantities, such as mass distribution, are then derived through constructing superpositions of these orbits.

Given a gravitational potential \( \Phi(\mathbf{r}) \), a particle’s orbit is a function of its total energy \( E \) and angular momentum \( L \). The joint distribution of \( E \) and \( L \) provides the whole library of particle orbits. To make the model as simple as possible, we assume all mother particles take circular orbits around the halo center, with random orientations of orbital planes. The daughter particles take up rosette orbits.

Now we consider the enclosed mass profile \( M(R) \) as a superposition of orbits. Mathematically, all orbits in the library form a set. We refer to it as Olib. For each element \( x \) in Olib, a weighting factor \( g_x(R) \) is assigned such that \( M(R) \) is the summation of all particle masses weighted by \( g_x(R) \) over the set

### Table 1

| Name | \( V_k \) (km s\(^{-1}\)) | \( \tau^* \) (Gyr) | \( M_{\text{nf}} \) (10\(^6\) \( h^{-1} \) M\(_{\odot}\)) | \( R_{\text{vir}} \) (\( h^{-1} \) kpc) | \( \rho_{\text{vir}} \) (\( h^{-1} \) kpc) | \( R_{0.3} \) (\( h^{-1} \) kpc) | \( V_{0.3} \) (km s\(^{-1}\)) | \( c_{\text{dau}} \) | \( N_p \) (10\(^3\)) |
|------|-----------------|--------------------|--------------------------|------------------|---------------------|-----------------|-----------------|----------------|----------------|
| CDM  | ...             | ...                | 5.17                     | 35.0             | 0.254               | 0.407           | 27.7            | 21.6           | 1.33           |
| V20T3| 20.0            | 3.00               | 4.22                     | 32.7             | 0.291               | 0.904           | 26.0            | 15.0           | 2.35           |
| V20T7| 20.0            | 6.93               | 4.41                     | 33.2             | 0.224               | 0.805           | 27.4            | 17.3           | 2.36           |
| V20T14| 20.0          | 14.0               | 4.70                     | 33.9             | 0.200               | 0.684           | 28.7            | 19.3           | 2.41           |
| V30T3| 30.0            | 3.00               | 2.89                     | 28.8             | 0.366               | 1.72            | 20.1            | 6.99           | 1.93           |
| V30T7| 30.0            | 6.93               | 3.32                     | 30.2             | 0.254               | 1.23            | 23.3            | 11.4           | 1.91           |
| V30T14| 30.0          | 14.0               | 4.05                     | 32.3             | 0.215               | 0.810           | 26.4            | 16.8           | 2.08           |
| V40T3| 40.0            | 3.00               | 0.350                    | 14.3             | 0.470               | 3.58            | 10.0            | 4.64           | 0.377          |
| V40T7| 40.0            | 6.93               | 1.79                     | 24.6             | 0.278               | 1.69            | 18.1            | 9.93           | 1.19           |
| V40T14| 40.0          | 14.0               | 3.26                     | 30.0             | 0.234               | 1.00            | 23.7            | 13.5           | 1.67           |

**Note.** Including the Name of Each Level-12 Zoom-in Simulation (Column 1), Recoil Velocity of Daughter Particles (Column 2), Decay Half-life (Column 3), Virial Mass (Column 4), Virial Radius (Column 5), Innermost Resolved Radius (Column 6), Characteristic Scale (Column 7) and the Corresponding Characteristic Velocity (Column 8), Concentration Obtained from an NFW Fitting (Column 9), and Total Particle Number Inside the Virial Radius (Column 10). See Section 5 for details.
Olib:  

$$M(R) = \sum_{x \in \text{Olib}} m_x g_x(R),$$  

(7)

where $m_x$ is the mass of the particle moving in the orbit $x$. The weighting factor $g_x(R)$ can be constructed as the fraction of time the particle spends inside the sphere $r = R$:  

$$g_x(R) = \Delta t_x(R)/T_x,$$  

(8)

where $\Delta t_x(R)$ is the duration that a particle travels inside the sphere $r = R$ within an orbital period $T_x$. The weighting factors for circular orbits $g_{\text{cir}}(R, r)$ are step functions since a circle with radius $r$ is either totally inside or totally outside the sphere $R$:  

$$g_{\text{cir}}(R, r) = \begin{cases} 0 & \text{if } r > R, \\ 1 & \text{if } r \leq R. \end{cases}$$  

(9)

Similarly for a rosette orbit, if its perihelion $r_{\text{min}}$ (aphelion $r_{\text{max}}$) is larger (smaller) than $R$, then its weighting factor $g_{\text{ros}}(R, r_{\text{min}}, r_{\text{max}})$ is 0 (1). If $r_{\text{min}} < R < r_{\text{max}}$, the weighting factor is calculated as follows:  

$$g_{\text{ros}}(R, r_{\text{min}}, r_{\text{max}}) = \frac{\int_{r_{\text{min}}}^{r_{\text{max}}} \sqrt{R^2 - V_\text{eff}(r)} \, dr}{\int_{r_{\text{min}}}^{r_{\text{max}}} \sqrt{R^2 - V_\text{eff}(r)} \, dr},$$  

(10)

where $V_\text{eff}(r)$ is the effective potential.

Consider the Step-1 expansion during a small time interval $\Delta t$ such that the gravitational potential $\Phi(r)$ remains static. Consider a pair of mother and daughter particles: a circular orbit with radius $r_0$ before decay and the corresponding rosette orbit after decay. According to Equation (6), they share the same specific angular momentum $l_0$, and thus the same effective potential:  

$$V_\text{eff} = \frac{l_0^2}{2r^2} + \Phi(r).$$  

(11)

The value of $l_0$ can be obtained by considering the circular motion at radius $r_0$, where $V_\text{eff}$ reaches its minimum. As for the rosette orbit, its perihelion and aphelion are the two roots of the following equation from Equation (5):  

$$V_\text{eff}(r) - V_\text{eff}(r_0) - \frac{1}{2} \frac{l_0^2}{r^2} = 0,$$  

(12)

They both depend on the radius $r_0$ of its mother particle’s orbit. If Equation (12) has only one root, the daughter particle is considered to be unbound and makes no contribution to the mass profile $M(R)$.

After $\Delta t$, the mass increment of daughter particles inside the sphere $r = R$ can be calculated by considering the contributions from the daughter particles that are born during this time interval:  

$$\Delta M_{\text{dau}}(R, \Delta t, t_i) = \Delta f \int_0^\infty 4\pi r^2 \rho_{\text{mom}}(r, t)$$  

$$\times g_{\text{ros}}(R, r_{\text{min}}(r, V_k), r_{\text{max}}(r, V_k), t_i) \, dr,$$  

(13)

where $\Delta f$ is the fraction of mother particles decayed during $\Delta t$:  

$$\Delta f = \ln(2) \Delta t/\tau^*.$$  

(14)

$\rho_{\text{mom}}(r, t)$ is the density profile of mother particles at time $t$. The mass profile of mother particles declines uniformly by a factor of $\Delta f$:  

$$\Delta M_{\text{mom}}(R, \Delta t, t_i) = -\Delta f \rho_{\text{mom}}(R, t).$$  

(15)

Then after the Step-1 expansion, the total mass profile $M(R)$ changes by the amount of  

$$\Delta M(R, \Delta t, t_i) = \Delta M_{\text{dau}}(R, \Delta t, t_i) - \Delta f \rho_{\text{mom}}(R, t_i).$$  

(16)

We proceed to consider the Step-2 expansion in the time interval $(t_i, t_i + \Delta t)$. For the sphere $r = R$ at $t_i$, the adiabatic expansion reads as  

$$R_M(R, t_i) = R_f(t_{i+1})[M(R, t_i) + \Delta M(R, \Delta t, t_i)],$$  

(17)

from which $R_f(t_{i+1})$, the radius at $t_{i+1} = t_i + \Delta t$, can be derived. Initially the total mass inside $R$ is $M(R, t_i)$. After $\Delta t$, the total mass inside $R_f(t_{i+1})$ is $M(R, t_i) + \Delta M(R, \Delta t, t_i)$. Therefore, the mass profile $M(R, t)$ evolves a bit.

Through the two-step expansion we evolved the whole system from $t_i$ to $t_{i+1}$. We loop the procedures listed above and evolve the whole system from the initial time $t_0$ to the final time $t_f$.

5. Results

The simplified semi-analytic model presented in Section 4 is implemented with the well-tested numerical code SemiCore. Starting from the best-fit NFW profile of the level-12 CDM halo ($M_{\text{vir}} = 5.17 \times 10^9 \, h^{-1} \, M_{\odot}$ and $c_{\text{vir}} = 21.6$), we run SemiCore with the combinations of $V_k$ and $\tau^*$ listed in Table 1, with the initial and final times being the same as those of cosmological N-body simulations. In Figure 3, we show the present ($z = 0$) average density profile ratios, $\rho_{\text{ddm}}(r)/\rho_{\text{cdm}}(r)$, between the DDM-CDM halo pairs, which evolve from the same primordial local overdensity field. We compare the results of the SemiCore model with those from the level-12 N-body simulations. For the SemiCore model, $\rho_{\text{ddm}}(r)$ is calculated from the input NFW profile.

The simulation data reveals that dark matter decay reduces the mass profile throughout the dwarf halos. The global reduction in amplitude increases as $V_k$ increases or $\tau^*$ decreases. For a given pair of decay parameters, the difference between DDM and CDM haloes becomes more pronounced as $r$ approaches the halo center. These trends are well reproduced by the SemiCore model. Furthermore, the average density ratios $\rho_{\text{ddm}}/\rho_{\text{cdm}}$ predicted by SemiCore agree with those from sophisticated N-body simulations to better than 40%, for all resolved radii and for all combinations of $V_k$ and $\tau^*$ considered in this study. The success of using the SemiCore model confirms the two-step expansion scenario for explaining the halo expansion induced by dark matter decay.

Figure 3 also shows that SemiCore systematically overpredicts the mass reduction in the inner region ($r \lesssim 0.1 \, r_{\text{vir}}$). It may be related to the simplified assumption of circular orbits for all mother particles. The same effect was observed in modeling adiabatic contraction of dark matter by circular orbits, which leads to an enhancement of the central density relative to the results of high-resolution simulations (Gnedin et al. 2004). In the outer region ($r \gtrsim 0.8 \, r_{\text{vir}}$), N-body simulations show that the ratios $\rho_{\text{ddm}}/\rho_{\text{cdm}}$ continue to grow while the SemiCore model predicts a flattened or slightly declining shape. Note that the decay of a mother particle in a rosette orbit generally produces daughter particles that reach
larger $r$, compared to those from a circular mother orbit. Also, there is mass accretion from the environment in $N$-body simulations, which is absent in the SemiCore model. Both factors could be responsible for the discrepancy seen in the outer halo region.

In Figure 3, the average density ratios from $N$-body simulations display a common shape: rising inner and outer regions connected by an extended plateau. A positive slope of the $\rho_{DM}/\rho_{DM}$ curve implies the flattening of DDM density profile compared to that of its CDM counterpart. We calculate the rotation curve $V_{circ}(r)$ scaled by $V_{circ,0.3}$, the circular velocity at radius $R_{0.3}$, where $d \ln V_{circ}/d \ln r = 0.3$ (Hayashi & Navarro 2006). In Figure 4, we plot the scaled rotation curves for all level-12 DDM halos, and their values of $R_{0.3}$ and $V_{circ,0.3}$ are listed in Table 1. The scaled rotation curves based on the cuspy NFW profile and cored Burkert profile (Burkert 1995) are also plotted together for comparison. The DDM curves spread between the two theoretical curves in two groups. One is made up of four DDM halos: V20T14, V30T14, V40T14, and V20T7. Their scaled rotation curves are closer to the NFW curve than the Burkert one. The remaining five halos form the second group, which features a significant deviation from the NFW curve and is much closer to the Burkert curve. For V20T14, a member of the pro-NFW group, and V40T3, a member of the pro-Burkert group, we further plot their scaled rotation curves of mother and daughter components in Figure 4. Surprisingly, the scaled daughter (mother) rotation curve of V40T3 (V20T14) follows the Burkert (NFW) shape. Since V40T3 has the strongest decay effect, while V20T14 has the weakest, the simulation results show that the halo expansion due to dark matter decay can flatten the halo density profile and transform it from a cuspy shape to the cored shape, depending on the combination of $V_\kappa$ and $\tau^*$ for a given dwarf halo mass.

6. Discussion

The flattening of the central density profile of dwarf halos is needed to resolve the core-cusp problem of CDM. We show DDM and CDM’s rotation curves together with observational data in Figure 5. Halo V20T3 represents the flattened DDM rotation curves. Two dwarf galaxies UGC07866 and UGCA444 are selected from the Spitzer Photometry & Accurate Rotation Curves (SPARC) database (Lelli et al. 2016) for comparison as they have roughly the same mass as the halo V20T3 (Li et al. 2020). In order to cover the possible mass range where the two SPARC galaxies reside, we use SemiCore to calculate the rotation curves of two DDM halos, with current virial masses of $4.12 \times 10^9$ $h^{-1} M_\odot$ and $2.0 \times 10^9$ $h^{-1} M_\odot$, respectively. The two SemiCore runs both use $V_\kappa = 20.0 \, km \, s^{-1}$ and $\tau^* = 3.0$ Gyr. Their initial halos are set to follow the CDM $c$–$M$ relation (Diemer & Joyce 2018). For the CDM rotation curve, we assume an NFW profile and a fixed virial mass $4.22 \times 10^9$ $h^{-1} M_\odot$, the same as the halo V20T3. In the left panel of Figure 5, the dark matter contributed rotation curves of dwarf galaxies UGC07866 and UGCA444 are shown with observational uncertainties. The simulated or modeled DDM and CDM rotation curves have been scaled by a factor of $\sqrt{1 + f_{b,0}}$, where $f_b$ is the cosmic baryon fraction of global matter density. It shows that the DDM rotation curves naturally follow the observational data, while the CDM ones give a good fit to data only with a small concentration, which is unusual for halos formed in CDM $N$-body simulations (Dutton & Macciò 2014). The data points at small radii ($R \lesssim 0.7$ kpc) favor DDM curves as the NFW curves systematically overpredict the rotation velocities there. In the right panel of Figure 5, we plot the corresponding scaled rotation curves. With considerably large uncertainties, the observational data distribute between the NFW and Burkert curves at small radii, in good agreement with the DDM scaled rotation curves for dwarf galaxies. Our results agree with Sánchez-Salcedo (2003) in that the core-cusp problem can be solved in the DDM
model with a recoil velocity $V_k$ smaller than the typical escape velocities of dwarf halos, provided that the decay half-life is $\tau^* \lesssim 7.0$ Gyr.

In Figure 6, we show the positions of the nine zoom-in dwarf halos in the DDM parameter space. They reside inside a region, shown in gold, where dark matter decay has prominent effects on the number density and inner structure of dwarf galaxies (Peter & Benson 2010). The five zoom-in halos with a much more flattened central density are indicated by blue squares and the remaining four by blue triangles. For $V_k = 20.0$ km s$^{-1}$ and $\tau^* = 6.93$ Gyr, Wang et al. (2014) ran a zoom-in simulation on a Milky Way sized host halo with the PMK10 algorithm. They found that the circular velocity profiles of the 15 most massive subhalos pass through most of the data points from the nine classical Milky Way dSphs, and therefore the too-big-to-fail problem (Boylan-Kolchin et al. 2011) is potentially resolved. The brown strip is the parameter region, shown by Abdelqader & Melia (2008) using a semi-analytic model that incorporates dark matter decay in the hierarchical formation history of dark matter halos, which can account for the deficit of dwarf galaxies in our local group, a puzzle closely related to the missing-satellites problem (Klypin et al. 1999; Moore et al. 1999a). It can be seen that several different CDM problems can be solved by a common parameter subspace in the DDM model.

In Figure 7, we show that $R_{0.3}$ and $V_{0.3}$ have a power-law relation for DDM zoom-in dwarf halos:

$$\frac{R_{0.3}}{R_{\text{vir}}} \propto \left( \frac{V_{0.3}}{V_k} \right)^{-\beta(\tau^*)},$$  

(18)

where the power index $\beta$ increases as $\tau^*$ decreases. Given the values of $R_{0.3}/R_{\text{vir}}$ and $V_{0.3}$, both being observable quantities, a relation between $V_k$ and $\tau^*$ can be derived from relation (18), implying a possible degeneracy in the DDM parameter space as far as the halo’s inner structure is concerned. As $V_k$ and $\tau^*$ are constants in the DDM model, from relation (18) we also find

$$\frac{V_{0.3}}{V_{\text{vir}}} \propto \left( \frac{R_{0.3}}{R_{\text{vir}}} \right)^{-1/3} M_{\text{vir}}^{-1/3}.$$  

(19)

Since dwarf halos have a narrow virial mass spectrum and Equation (19) depends only weakly on $M_{\text{vir}}$, an approximate universal density profile is expected. The observed value of $\beta$...
method for mass pro-
halos have a lower mass concentration and shallower density
to about 700 pc. Compared to CDM counterparts, DDM dwarf
and we succeeded in resolving the halo structure robustly down
DDM algorithm. Good numerical convergence was achieved,
at certain times and evolves the whole
semi-quantitatively with resolved simulation pro-
s. The two SPARC dwarf galaxies favor
les. The two SPARC dwarf galaxies are shown with red
dashed, green dotted–dashed and dotted line respectively. The best-fit power
indices are also shown near each line for reference.
can then be used to measure \( \tau^* \). We will study this possibility in
future work.

7. Conclusions

In this work we improved the DDM N-body algorithm by combining the advantages of the PMK10 and CCT15 algo-
rithms. The new algorithm outperforms PMK10 in accuracy, while requiring much less computing resources than CCT15. Like CCT15, the new algorithm samples dark matter decay only at certain times and evolves the whole N-body system in a collisionless way at other times. This feature enables the algorithm to be implemented in a plug-in module called DDMPLUGIN, which can be used with any CDM N-body code.

We carried out high-resolution cosmological N-body simu-
tions to study the density profiles of dwarf halos with the new
DDM algorithm. Good numerical convergence was achieved, and we succeeded in resolving the halo structure robustly down to about 700 pc. Compared to CDM counterparts, DDM dwarf
halos have a lower mass concentration and shallower density profile at the inner region. Adopting the orbit-superposition method for mass profile construction, we developed a simplified semi-analytic model for the DDM halo mass profile, which features rosette orbits for daughter particles and incorporates the effects of dark matter decay and adiabatic expansion. Although simple, the model predicts DDM halo mass profiles that agree semi-quantitatively with resolved simulation profiles. It therefore illustrates clearly the physics mechanisms involved in the transformation from cusp to core density profiles.

We also calculated the scaled rotation curves for DDM
simulation halos and compared them with two dwarf galaxies from the SPARC database. The shape of the DDM rotation curve is shallower than that based on the NFW profile but steeper than Burkert’s. The two SPARC dwarf galaxies favor the DDM shape of the rotation curve. Furthermore, we show that there is an approximate universal power-law relation between \( V_{0.3}/V_k \) and \( R_{0.3}/R_{\text{vir}} \) for dwarf halos, which can be used to extract DDM parameters from observation data. Together with previous studies, this work supports the DDM
cosmology, which maintains the success of CDM at large scale and reconciles the differences between observations and predictions from N-body simulations at small scale.

We thank Volker Springel for offering us the P-GADGET3 code. We thank the anonymous referee for the constructive comments. We also benefited from the discussions with Hantao Liu, Sze-Him Lee, Kiu-Ching Leung, and Wai-Cheong Lee. We thank Hoi-Tim Cheung and Hoi-Tung Yip for help in checking the SemiCore results. We acknowledge the support of the CUHK Central High Performance Computing Cluster, on which the computation in this work has been performed. We used the public Python package Cosmology, haLO, and large-
Scale StrUcture tools (COLOSSUS; Diemer 2018) and the NASA’s Astrophysics Data System. The work described in this paper was partially supported by a grant from the Research Grants Council of the Hong Kong Special Administrative Region, China (Project No. AoE/P-404/18).

Appendix A

DDMPLUGIN Implementation

A.1. Decay of Mother Particles

At each breakpoint, the mother simulation particles will decay and release auxiliary daughter particles. Due to the random nature of dark matter decay, the N-body mother particles do not receive velocity kicks when decay occurs. Only their masses are affected and reduced by \( m_{\text{aux}} \):

\[
m_{\text{mom}}' = m_{\text{mom}} - m_{\text{aux}},
\]

where \( m_{\text{mom}} \) and \( m_{\text{mom}}' \) are the masses of a mother simulation particle just before and after the decay. Other information associated with the post-decay mother are exactly the same as those of the pre-decay mother. As for the newly generated auxiliary daughter particles, the decay kicks them away from their mother particles:

\[
x_{\text{aux}} = x_{\text{mom}},
\]

\[
v_{\text{aux}} = v_{\text{mom}} + V_k n,
\]

where \( V_k \) is the recoil velocity, and \( n \) is a unit vector pointing to a random direction. The IDs of the auxiliary daughters are assigned as follows:

\[
I_{\text{aux}} = N_{\text{tot.ini}} + I_{\text{ini}} + I_{\text{off}},
\]

where \( N_{\text{tot.ini}} \) is the total number of particles in the initial condition, \( I_{\text{off}} \) is the smallest particle ID taken by the simulation, and \( I_{\text{off}} \) is an integer ranging from 0 to \( N-1 \). Each auxiliary daughter particle gets a different value of \( I_{\text{off}} \) randomly such that their IDs are different from each other. The masses of auxiliary daughter particles are determined by the decay half-life \( \tau^* \) and the number of breakpoints \( f_i \):

\[
m_{\text{aux}} = m_{\text{mom.ini}} \frac{1 - \exp[-\ln(2)t/\tau^*]}{f_i},
\]

where \( m_{\text{mom.ini}} \) is the initial mass of a mother particle and \( t \) is the time span of the whole simulation.

A.2. Auxiliary-permanent Transition

The auxiliary-permanent transition of a daughter simulation
particle is implemented as follows in the DDMPLUGIN

Figure 7. Relation between \( R_{0.3} \) and \( V_{0.3} \) for DDM zoom-in dwarf halos with the same half-life \( \tau^* \). The red circles, green diamonds, and gray hexagons refer to halos with \( \tau^* = 3.00, 6.93, \) and 14.0 Gyr, respectively. There is a 2% uncertainty in \( R_{0.3} \) and a 0.6% uncertainty in \( V_{0.3} \). The best-fit power-law curves for data points with \( \tau^* = 3.00, 6.93, \) and 14.0 Gyr are shown with red
dashed, green dotted–dashed and dotted line respectively. The best-fit power
indices are also shown near each line for reference.
module. First, the particle ID \( I_{\text{aux}} \) of an auxiliary daughter is decomposed into a pair of integers \((q, p)\):

\[
I_{\text{aux}} = q f_s + p,
\]

where \( q \) is the quotient of \( I_{\text{aux}} \) by \( f_s \) and \( p \) the remainder. Auxiliary daughters satisfying \( p < n_f \) survive and are flagged as permanent daughters. Auxiliary daughters with \( p \geq n_f \) are eliminated from the simulation. This scheme ensures that the survival fraction \( \eta_f \) is \( n_f / f_s \). Once an auxiliary daughter is flagged to be permanent, its mass and ID have to be modified, while its position and velocity are retained. The particle mass needs modification such that the daughter particles’ total mass is unaffected by the transition:

\[
m_{\text{pmt}} = m_{\text{aux}} f_s / n_f,
\]

where \( m_{\text{pmt}} \) and \( m_{\text{aux}} \) are the particle masses of permanent and auxiliary daughters, respectively. A new ID \( I_{\text{pmt}} \) is assigned to the permanent daughter in order to distinguish it from its pre-transition auxiliary state:

\[
I_{\text{pmt}} = N_{\text{tot}} + I_{\text{ini}} + (q - q_{\text{min}}) n_f + (p - p_{\text{min}}),
\]

where \( N_{\text{tot}} \) is the total number of particles in the simulation just before the transition, \( I_{\text{ini}} \) is the smallest particle ID taken by the simulation, and \((q_{\text{min}}, p_{\text{min}})\) is the integer pair derived from the minimum ID of the auxiliary daughters that are flagged to be permanent daughters. The ID assignment scheme \( (A8) \) ensures that the particle IDs in the simulation are continuous and there is no spatial bias in the selection process. Position and velocity are invariant under the auxiliary-permanent transition:

\[
x_{\text{pmt}} = x_{\text{aux}},
\]

\[
v_{\text{pmt}} = v_{\text{aux}}.
\]

The auxiliary-permanent transition will also be applied when the state of the system is output at certain simulation times. This operation keeps the consistency that only permanent daughter particles appear in the output file(s) of the simulation.

### A.3. Gravitational Softening Length

In our implementation of the DDMPLUGIN module, the mother simulation particles, auxiliary daughter particles, and permanent daughter particles are all labeled as halo particles, which are type 1 according to Gadget’s classification. As they have the same particle type, their gravitational softening lengths are also the same.

### Appendix B

#### Numerical Tests

##### B.1. Transfer Function

Here we test the effects of uncertainties in the transfer function on the dark matter halo’s average density profile \( \bar{\rho}(r) \). Two different transfer functions, the default BBKS and Eisenstein–Hu (Eisenstein & Hu 1998) with baryonic features, are used in the test. We run level-11 resolution CDM zoom-in simulations and measure the resulting profiles at \( z = 0 \). The results are shown in the left panel of Figure 8. The uncertainties on the average density profile are well within 4% for most resolved radii. Hence, the choice of transfer functions has little impact on our results.

##### B.2. \( f_s \)

We used three values to test the effects of \( f_s \) on the halo average density profile \( \bar{\rho}(r); f_s = 5, 10, \) and 15, with \( n_f = 1 \) being 1. The decayed mass fraction per phase are 19.2%, 9.59%, and 6.39% for \( f_s = 5, \) 10, and 15, respectively. From the right panel of Figure 8, it can be seen that the average densities near a halo’s center are more easily affected by varying \( f_s \). It is clear that convergence can be achieved by increasing the value of \( f_s \). The relative differences in the average density profiles between \( f_s = 10 \) and \( f_s = 15 \) are within 3%.

**Figure 8.** Left panel: effects of transfer function variation on dark matter halo average density profile \( \bar{\rho}(r) \). Densities are normalized by the current critical density \( \rho_{\text{crit}} \). The red solid line represents the BBKS transfer function, while the black dotted–dashed line represents the Eisenstein–Hu transfer function, and the black dashed line represents the relative differences between them normalized by BBKS’s results. The average density profile is not affected by the two-body relaxation for radii larger than \( 0.6 \ h^{-1} \) kpc. Right panel: effects of the numerical parameter \( f_s \) on \( \bar{\rho}(r) \). DDM simulations for this test all use \( V_\text{i} = 20.0 \ \text{km s}^{-1} \), \( \tau^* = 3.0 \ \text{Gyr} \) and \( n_f = 1 \). All profiles are measured at redshift 0 and plotted down to the innermost resolved radii. The black dashed, dotted–dashed, and red solid lines represent profiles for \( f_s = 5, 10, \) and 15 respectively. The profiles with \( f_s = 15 \) serve as the baselines for comparison.
ORCID iDs
M.-C. Chu @ https://orcid.org/0000-0002-1971-0403

References
Abdelqader, M., & Melia, F. 2008, MNRAS, 388, 1869
Bardeen, J. M., Bond, J. R., Kaiser, N., & Szalay, A. S. 1986, ApJ, 304, 15
Benítez-Llambay, A., Frenk, C. S., Ludlow, A. D., & Navarro, J. F. 2019, MNRAS, 488, 2387
Binney, J., & Tremaine, S. 2008, Galactic Dynamics (2nd ed.; Princeton, NJ: Princeton Univ. Press)
Blumenthal, G. R., Faber, S. M., Primack, J. R., & Rees, M. J. 1984, Natur, 311, 517
Bosma, A. 1981, AJ, 86, 1825
Boylan-Kolchin, M., Bullock, J. S., & Kaplinghat, M. 2011, MNRAS, 415, L40
Bryan, G. L., & Norman, M. L. 1998, ApJ, 495, 80
Bullock, J. S., & Boylan-Kolchin, M. 2017, ARA&A, 55, 343
Burkert, A. 1995, ApJL, 447, L25
Cen, R. 2001, ApJL, 546, L77
Chan, T. K., Kereš, D., Oñorbe, J., et al. 2015, MNRAS, 454, 2981
Chanamé, J. 2010, HiA, 15, 190
Cheng, D., Chu, M. C., & Tang, J. 2015, JCAP, 2015, 009
Davies, M., Efstathiou, G., Frenk, C. S., & White, S. D. M. 1985, ApJ, 292, 371
de Blok, W. J. G. 2010, AdAst, 2010, 789293
Diemer, B. 2018, ApJS, 239, 35
Diemer, B., & Joyce, M. 2019, ApJ, 871, 168
Dubinski, J., & Carlberg, R. G. 1991, ApJ, 378, 496
Dutton, A. A., & Macciò, A. V. 2014, MNRAS, 441, 3359
Dutton, A. A., Macciò, A. V., Buck, T., et al. 2019, MNRAS, 486, 655
Eisenstein, D. J., & Hu, W. 1998, ApJ, 496, 605
Flores, R. A., & Primack, J. R. 1994, ApJL, 427, L1
Frenk, C. S., White, S. D. M., Efstathiou, G., & Davis, M. 1985, Natur, 317, 595
Fukushige, T., & Makino, J. 2001, ApJ, 557, 533
Gill, S. P. D., Knebe, A., & Gibson, B. K. 2004, MNRAS, 351, 399
Gnedin, O. Y., Kravtsov, A. V., Klypin, A. A., & Nagai, D. 2004, ApJ, 616, 16
Governato, F., Brook, C., Mayer, L., et al. 2010, Natur, 463, 203
Hahn, O., & Abel, T. 2011, MNRAS, 415, 2101
Hayashi, E., & Navarro, J. F. 2006, MNRAS, 373, 1117
Kaplinghat, M. 2005, PhRvD, 72, 063510
Klypin, A., Kravtsov, A. V., Valenzuela, O., & Prada, F. 1999, ApJL, 522, 82
Knollmann, S. R., & Knebe, A. 2009, ApJS, 182, 608
Kravtsov, A. V., Klypin, A. A., Bullock, J. S., & Primack, J. R. 1998, ApJ, 502, 48
Lelli, F., McGaugh, S. S., & Schombert, J. M. 2016, A1, 152, 157
Li, P., Lelli, F., McGaugh, S., & Schombert, J. 2020, ApJS, 247, 31
Markevitch, M., Gonzalez, A. H., David, L., et al. 2002, ApJL, 567, L27
Moore, B. 1994, Natur, 370, 629
Moore, B., Ghigna, S., Governato, F., et al. 1999a, ApJL, 524, L19
Moore, B., Quinn, T., Governato, F., Stadel, J., & Lake, G. 1999b, MNRAS, 310, 1147
Navarro, J. F. 2019, in IAU Symp. 344, Dwarf Galaxies: From the Deep Universe to the Present, ed. K. B. W. McQuinn & S. Stierwalt (Cambridge: Cambridge Univ. Press), 455
Navarro, J. F., Eke, V. R., & Frenk, C. S. 1996a, MNRAS, 283, L72
Navarro, J. F., Frenk, C. S., & White, S. D. M. 1996b, ApJ, 462, 563
Oñorbe, J., Garrison-Kimmel, S., Maller, A. H., et al. 2014, MNRAS, 437, 1894
Peebles, P. J. E. 1982, ApJL, 263, L1
Peter, A. H. G. 2010, PhRvD, 81, 083511
Peter, A. H. G., & Benson, A. J. 2010, PhRvD, 82, 123521
Peter, A. H. G., Moody, C. E., & Kamionkowski, M. 2010, PhRvD, 81, 103501
Planck Collaboration, Aghanim, N., Akrami, Y., et al. 2020a, A&A, 641, A6
Planck Collaboration, Akrami, Y., Arroja, F., et al. 2020b, A&A, 641, A1
Pontzen, A., & Governato, F. 2012, MNRAS, 421, 3464
Power, C., Navarro, J. F., Jenkins, A., et al. 2003, MNRAS, 338, 14
Salucci, P., & Burkert, A. 2000, ApJL, 537, L9
Sánchez-Salcedo, F. J. 2003, ApJL, 591, L107
Sawala, T., Frenk, C. S., Fattahi, A., et al. 2016, MNRAS, 457, 1931
Schwarzschild, M. 1979, ApJ, 232, 236
Springel, D. N., Verde, L., Peiris, H. V., et al. 2003, ApJS, 148, 175
Springel, V. 2005, MNRAS, 364, 1105
Springel, V., Frenk, C. S., & White, S. D. M. 2006, Natur, 440, 1137
Strigari, L. E., Kaplinghat, M., & Bullock, J. S. 2007, PhRvD, 75, 061303
Tollet, E., Macciò, A. V., Dutton, A. A., et al. 2016, MNRAS, 456, 3542
Vattis, K., Koushiappas, S. M., & Loeb, A. 2019, PhRvD, 99, 121302
Wang, M.-Y., Croft, R. A. C., Peter, A. H. G., Zentner, A. R., & Purcell, C. W. 2013, PhRvD, 88, 123515
Wang, M.-Y., Peter, A. H. G., Strigari, L. E., et al. 2014, MNRAS, 445, 614
White, S. D. M., Frenk, C. S., & Davis, M. 1983, ApJL, 274, L1
Wright, E. L., Meyer, S. S., Bennett, C. L., et al. 1992, ApJL, 396, L13
Zwicky, F. 1933, AcHPh, 6, 110