MAGNETIC FIELDS AND THE POLARIZATION OF ASTROPHYSICAL MASER RADIATION: A REVIEW

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RESUMEN

Favor de proporcionar un resumen en español. If you are unable to translate your abstract into Spanish, the editors will do it for you. Basic aspects of the relationship between the magnetic field and polarized maser radiation are described with the emphasis on interpreting the observed spectra. Special attention is given to three issues—the limitations on the applicability of the classic solutions of Goldreich, Keeley & Kwan (1973), inferring the strength of the magnetic field from the circular polarization when the Zeeman splitting is much less than the spectral linebreadth (especially for SiO masers), and the significance of the absence of components of the Zeeman triplet in the spectra of OH masers in regions of star formation.

ABSTRACT

Basic aspects of the relationship between the magnetic field and polarized maser radiation are described with the emphasis on interpreting the observed spectra. Special attention is given to three issues—the limitations on the applicability of the classic solutions of Goldreich, Keeley & Kwan (1973), inferring the strength of the magnetic field from the circular polarization when the Zeeman splitting is much less than the spectral linebreadth (especially for SiO masers), and the significance of the absence of components of the Zeeman triplet in the spectra of OH masers in regions of star formation.

Key Words: circumstellar matter — ISM:clouds — magnetic fields — masers — polarization

1. INTRODUCTION

The extreme surface brightnesses and the narrow spectral line breadths of astrophysical masers make them ideal sources for high precision observational studies with very long baseline interferometry, as well as in studies with a single dish. Strong masing was first detected in the mid-1960’s—in the 18 cm transitions of the OH molecule. Strong masing was subsequently detected in the transitions of H$_2$O, SiO and CH$_3$OH. Masing occurs in star-forming regions, in the circumstellar gas of evolved stars, in the gaseous disks at the nuclei of active galaxies, and in distant galaxies. The radiation often is polarized so that conclusions can be drawn about the direction and strength of the magnetic fields.

One might think that information about the magnetic fields in masers would not be generally useful since the density, temperature, and history of the gas in masers is so different from that of the bulk of the gas of interstellar clouds. However, there is evidence that this is not the case. The directions of ordered magnetic fields of the OH masers in star forming regions are similar to the directions of the field in the larger volume surrounding the masers (Fish et al. 2003). The field strengths inferred for the OH and H$_2$O masers in star forming regions lie on the same curve of magnetic field $B \propto (\text{density})^{1/2}$ as those obtained for the interstellar gas where the density is lower by many orders of magnitude (Fiebig & Gusten 1989). The patterns in the directions of the linear polarization of circumstellar masers strongly suggests that they reflect the direction of a larger scale magnetic field (Kemball et al. 2008) and the variation of the inferred field strengths with distance from the star has been interpreted as the variation of a large scale field (Vlemmings 2007). Upper limits to the field strength in the masing disk around the massive black hole at the nucleus of the galaxy NGC4258 have been used to restrict the mass accretion rate and the nature of the accretion process (Modjaz et al. 2005). Magnetic field strengths in distant galaxies have also been inferred from the circular polarization of maser radiation (Robishaw et al. 2008).

In the review here, I will discuss issues involved in extracting information about the magnetic field from the observed polarization characteristics of astrophysical maser radiation. After a general orientation, I will focus mainly on three topics: limitations in applying the classic work of Goldreich, Keeley & Kwan (1973; designated here as GKK) to the
observations, the validity of using the Zeeman effect to infer strong magnetic fields from the circular polarization of SiO masers, and the significance of the characteristics of the Zeeman components of OH maser spectra for turbulence in star forming regions. More comprehensive reviews of the observations include those of Lo (2005) and of Vlemmings (2007).

2. GENERAL CONSIDERATIONS

2.1. Basic Ideas

Astrophysical masers are considered as tubular with the consequence that the radiation is beamed, perhaps highly beamed. The solid angle $\Delta \Omega$ into which the radiation is beamed is quite uncertain. Unless the stimulated emission rate $R$ due to the maser radiation is negligible in comparison with the loss rate $\Gamma$ of the molecular states due to all other processes, the maser radiation must be included in the calculation of the molecular populations and the radiative transfer calculation is thus nonlinear. The ratio $R/\Gamma$ is then the degree of saturation. Even when the angular size of the masering feature can be resolved and the surface brightness is determined, $R/\Gamma$ is still uncertain because $\Delta \Omega$ cannot be measured. Estimates can be made. From these it seems that $R/\Gamma$ probably is less than about 10, but is not much less than 1—typically, at least for prominent masers.

Almost all theoretical discussions of maser polarization in the literature (a) consider the maser geometry to be completely linear, (b) consider only the two energy states of the masing transition (these are split into substates by interaction with the magnetic field), and (c) consider only maser transitions between molecular states of low angular momentum (esp. J=1-0 transitions). The premise is, of course, that the results are indicative for actual astrophysical masers.

With the above estimates for $R$ and the strengths of the magnetic fields that seem to be present, it is almost always (if not always) the case that the Zeeman splitting $g\Omega \gg R, \Gamma$ where $g\Omega$ is the splitting in ordinary frequency units multiplied by $2\pi$. It follows that the magnetic field direction is an axis of symmetry and the ordinary magnetic substates are “good quantum states”. A key simplification is then valid: (d) the calculation can be performed with radiative transfer equations that are of the same form as those for ordinary thermal spectral lines and the populations of the magnetic substates can be found by solving ordinary rate equations. Otherwise, it would be necessary to use the more involved formulation in terms of quantum mechanical density matrices as done by GKK. We are fortunate. If $g\Omega \gg R$ were not satisfied, it is unlikely that useful information about the magnetic field could be inferred from the polarization data. The behavior of the polarization becomes quite complicated when $R$ approaches and exceeds $g\Omega$, as can be seen from density matrix calculations for this regime (Nedoluha & Watson 1990,1994). Because the magnetic field provides an axis of symmetry when $g\Omega \gg R$, the direction of the linear polarization of the radiation emitted by the molecules must be either parallel or perpendicular to the direction of the magnetic field as projected on the sky. The direction of the linear polarization that we observe may be altered from this by propagation effects (especially Faraday rotation), though in practice such effects do not seem to be important except possibly for the 18 cm OH masers.

Conveniently, astrophysical masers divide into two classes according to whether the Zeeman splitting is much weaker (SiO, CH$_3$OH and H$_2$O masers) or much stronger (mainline OH masers) than the spectral linebreadth.

2.2. The Formulation of Goldreich, Keeley and Kwan (GKK)

GKK commonly has served as the starting point for calculations and discussions about the polarization of astrophysical maser radiation—and rightly so! It is thus important to be clear about what GKK did and did not do, and what are the restrictions on the expressions that they obtain as solutions to their equations. GKK consider a linear maser permeated by a constant magnetic field and involving an angular momentum $J=1$ radiative transition. The pumping and the loss of excitation due to causes other than the maser transition are not specifically treated, but are expressed as “phenomenological” pump $\Lambda$ and loss $\Gamma$ rates—a procedure that often is used in laboratory as well as in astrophysical maser theory. GKK obtain solutions only in certain limits—including the limits of strong and weak splitting. I will discuss the solutions of GKK for weak splitting since this is where most of the confusion has arisen in recent years. GKK consider only the center of the spectral line in obtaining solutions for weak splitting. Understanding the GKK equations requires considerable effort because they are expressed in terms of quantum mechanical density matrices for the magnetic substates of the masering molecules. GKK do this so that the equations can be used when $R \approx g\Omega$ and $R \gg g\Omega$, as well as when $R \ll g\Omega$ which we now realize to be the regime of chief (if not exclusive) importance for astrophysical masers.
The key point about the GKK solutions to their equations is that the solutions are obtained in certain asymptotic limits of the degree of saturation $R/\Gamma$—though still subject to certain other restrictions that define various regimes. In the regime $R \ll g\Omega$ which is of practical interest and when $R/\Gamma \gg 1$, GKK obtain the simple formula for the fractional linear polarization (Stokes $Q/I$) at line center that is probably the most quoted result of their paper,

$$Q/I = (3\sin^2 \theta - 2)/3\sin^2 \theta \text{ for } \sin^2 \theta \geq 1/3$$

and

$$-1 \text{ for } \sin^2 \theta \leq 1/3.$$

where $\theta$ is the angle between the propagation direction and the magnetic field.

This is a valuable result, but it should not be compared directly with the observations because the $R/\Gamma$ for astrophysical masers are not so large that solutions for $R/\Gamma \gg 1$ are a good approximation—as we have shown in a number of papers by numerically integrating the GKK equations to provide solutions for $Q/I$ as a function of $R/\Gamma$ as shown in Figure 1.

That this GKK solution is not directly relevant can also be seen simply by comparing with the observations. The prominent $H_2O$ and CH$_3$OH masers typically exhibit fractional linear polarizations less than a few percent and the weakly split OH masers, no more than about ten percent. Such low polarization is obtained in the above GKK solutions only at an angle $\theta$ of almost exactly 55 degrees. It is clearly implausible that the radiation from all such masers propagates at only this angle relative to the magnetic field! Reasonable estimates for the beaming angles $\Delta\Omega$ together with observed brightnesses also indicate that $R/\Gamma$ is not so large that GKK solutions are directly applicable. The spectral line widths and profiles of maser radiation provide additional evidence that $R/\Gamma$ is not large (Nedoluha & Watson 1991; Watson, Sarma, & Singleton 2002).

It is true that one prominent class of masers with weak Zeeman splitting—the circumstellar SiO masers—does exhibit high fractional linear polarization. However, the problem with these is that the linear polarization is too high! Too high, at least for plausible $R/\Gamma$ (see subsection 3.2).

To summarize, the GKK solution in the weak splitting regime is not directly applicable to the observations mainly because (a) it is only for asymptotically large $R/\Gamma$. Other limitations are that GKK (b) consider only isotropic pumping and (c) consider only a J=1-0 transition. The limitation (b) is critical for the SiO masers (subsection 3.2). Any electric dipole transition has the asymptotic $Q/I$ of the GKK solution. However, with increasing angular momentum of the molecular states in the transition, larger and larger values of $R/\Gamma$ are required for the actual (numerical) solution to approach the GKK solution as indicated in Figure 1. Finally, (d) in the weak splitting regime, GKK consider only line center and thus have nothing to say about the circular polarization. The circular polarization is always zero at line center in this idealization.

3. THE WEAK SPLITTING REGIME AND SIO MASER POLARIZATION

3.1. Masers Other than SiO Masers

The GKK solution reproduced above demonstrates the key feature of intense ($R/\Gamma \gtrsim 1$), beamed radiation in the presence of a magnetic field in the weak splitting regime when $g\Omega \gg R$—that the intense radiation will depopulate the magnetic substates in a way that leads to polarization of the beam itself. Though the $R/\Gamma$ for astrophysical masers is not large enough that the GKK solution for the linear polarization is quantitatively accurate, the GKK solution does accurately tell us the direction of the linear polarization as a function of the angle of propagation of the maser beam including the angle of 55 degrees (the “van Vleck angle”) at which the linear polarization changes abruptly from parallel to perpendicular relative to the projected direction of the magnetic field—as long as the pumping is isotropic.

The fractional linear polarizations observed for the $H_2O$ and CH$_3$OH masers typically are no more than a few percent and are consistent with the low polarizations expected from the numerical solutions to the GKK equations shown in Figure 1 for masers with little or no radiative saturation. As mentioned in the previous Section, estimates for $R/\Gamma$ are consistent with the low values implied by the fractional polarization. For these transitions which involve molecular states with higher angular momentum, the calculated fractional polarization at a specific $R/\Gamma$ also tends to be lower than in Figure 1 (see Nedoluha & Watson 1990, 1992). The weakly split OH masers associated with supernova shells exhibit somewhat higher fractional linear polarizations ($Q/I \approx 10$ percent) which would be consistent with modest saturation ($R/\Gamma \approx 1$) for the low angular momentum of the states involved and a plausibly wide range of possible propagation angles $\theta$.

For these masing species, the fractional circular polarization that is detected, together with the Zeeman interpretation, leads to magnetic field strengths that seem to be “reasonable”. That is, the field strength lies near $B \propto (\text{density})^{1/2}$ when this relationship is extrapolated from lower densities and the
magnetic pressures are similar to the gas thermal pressures. For the H$_2$O and CH$_3$OH masers, the low fractional linear polarization and the low degree of saturation are additional factors that tend to make it difficult for non-Zeeman causes to be responsible for the circular polarization. Hence, the Zeeman interpretation is expected to be at least approximately correct for the circular polarization of these masers.

However, the circular polarization of even the ordinary Zeeman effect can be altered somewhat by maser saturation as shown in Figure 1. Although GKK do not consider frequencies away from line center in the weak splitting regime where the circular polarization would be non-zero, their equations can be generalized in a straightforward manner. The results of the numerical integration of these generalized GKK equations is what is shown in Figure 1. When $R/T \lesssim 0.1$, the relationship between $B$ and Stokes $V$ differs negligibly from that for ordinary thermal lines ($V \propto B \cos \theta$) as seen in Figure 1. When saturation is significant, the relationship changes so that $V$ can actually increase rather than decrease as $\theta$ increases—a variation first demonstrated in the context of the polarization of water masers (Nedoluha & Watson 1992). Nevertheless, $R/T$ is unlikely to be large enough for any of these masers to cause more than “factor of two” uncertainties in the magnetic fields that are inferred by using the standard relationship for thermal lines and omitting the uncertain $\cos \theta$ factor.

### 3.2. SiO masers

Understanding the polarization characteristics of the circumstellar SiO masers is more challenging. In the following, I will continue to assume that $g\Omega \gg R$ is satisfied as is indicated from “best estimates”. It is somewhat less clear, however, that this inequality is so strongly satisfied as for the other types of masers. The magnetic moment is somewhat less than for H$_2$O and the spectral line profiles of the SiO masers are less amenable for drawing conclusions about the degree of saturation. For the magnetic moment of the SiO transition, $g\Omega = 1.5B(\text{mG})$. Noticeable deviations from the “$g\Omega \gg R$ polarizations” begin to appear when $R \approx g\Omega/10$ (Nedoluha & Watson 1990,1994). Of course, if the magnetic fields are as strong as inferred from the Zeeman interpretation of the observed circular polarizations, $g\Omega \gg R$ should be well satisfied.

**Linear polarization.**

Observations have shown for many years (e.g., Clark, Troland, & Johnson 1982) that the fractional linear polarization of the prominent J=2-1 $v=1$ masing transition can be 50% and greater; it can also be as large for the weaker and less well studied masing transitions involving states of higher angular momentum or higher vibration. While such high polarization is a feature of the GKK solution, it would require that the degree of saturation be implausibly large. A saturation degree $R/T > 10^4$ would be required according to calculations in Figure 1. The natural resolution is that the polarization is at least partly a result of the pumping. That is, the magnetic substates are excited unequally because the angular distribution of the infrared radiation involved in the pumping (mainly that due to vibrational $v=1-0$ transitions) is anisotropic. This can happen if the direct infrared radiation from the star is important in exciting the $v=1$ states from the $v=0$ ground states or if the optical depths for the escape of the $v=1-0$ radiation emitted by the SiO molecules following excitation by collisions are anisotropic. Detailed calculations have been performed to show that such anisotropic pumping is likely (Deguchi & Iguchi 1976; Western & Watson 1983; Ramos et al. 2005).

Depending upon the details of the anisotropic pumping, the angle at which the change in sign occurs for $Q$ will be different from the 55 degrees of Figure 1. In fact, the sign of $Q$ can be the same at all angles. The approximately 90 degree difference in the direction of the linear polarization that is sometimes observed for SiO masers at nearby locations can be due to changes in the directions of the anisotropy relative to the magnetic field and the line of sight (Western & Watson 1983; Ramos et al. 2005). Since $g\Omega \gg R$ probably is satisfied, the direction of the linear polarization will be either parallel or perpendicular to the projected direction of the magnetic field—as long as it is unaffected by propagation effects. The observed patterns seem to indicate that such propagation effects are not a major factor for circumstellar SiO masers.

**Circular polarization.**

The original detections of the circular polarization of the circumstellar SiO masers (Barvainis, McIntosh & Predmore 1987) and the interpretation that this polarization is a straightforward consequence of the Zeeman effect yield magnetic field strengths of 10-100G! Reasonable estimates can be made for the kinetic temperature and the maximum gas density of the masing gas. With these and $B = 10G$, the magnetic pressure exceeds the thermal gas pressure by a factor of 1000! This may indicate the presence of interesting magnetic phenomena in the circumstellar envelopes, but the implication of such unusual conditions also suggests that the Zeeman in-
terpretation of the circular polarization should be severely scrutinized beyond the "factor of two" corrections that potentially follow simply from radiative saturation as given in Figure 1.

Although a cause of non-Zeeman circular polarization that must be kept in mind is the possibility that $g\Omega \gg R$ is not sufficiently well satisfied (Nedoluha & Watson 1994), this does not now seem to be the most likely possibility. Magnetic fields of some 100 mG are detected from the water masers, and those at the SiO masers are expected to be at least this large since the SiO masers seem to be somewhat closer to the star. A more likely possibility is that a misalignment occurs within the masing gas between the directions of the linear polarization of the maser radiation and the projected direction of the magnetic field as the radiation propagates through the maser. Linearly polarized radiation will then be converted to elliptically polarized in much the same way as is well known in classical optics when linearly polarized radiation passes through a plate in which the phase velocity is different along the two orthogonal, optical axes (e.g., the "quarter wave plate"). The phase velocity near an absorption or emission feature depends on the strength of the feature and changes rapidly with frequency. We know that the strength of the emission feature is different along two orthogonal axes in the masing gas since it is just this difference that is creating the strong linear polarization of the observed maser radiation and, of course, the maser radiation itself is linearly polarized within the masing gas. We do not, however, know whether the third requirement for this non-Zeeman circular polarization is satisfied—that the optical axes and the direction of the linear polarization become misaligned within the masing region. The circumstellar medium appears quite irregular (due perhaps to convection, turbulence and/or shock waves) and it seems plausible that the magnetic field can change direction somewhat and cause the required misalignment. Faraday rotation within the masing gas would have a similar effect. The Stokes V that would result from this non-Zeeman cause would also be antisymmetric about line center because of the antisymmetric variation of the phase velocity. It can thus easily be confused with the antisymmetric profile of Stokes V due to the Zeeman effect.

What is the observational evidence? The SiO masers are believed to be somewhat closer to the stars and stronger fields are expected than for the H$_2$O masers where the fields are more reliably determined to be in the hundreds of milliGauss. It can be reasoned that much stronger fields of 10G or so at the location of the SiO masers represent a plausible extrapolation of the weaker fields at larger distances from the stars (Vlemmings 2007), though this is a large extrapolation. On the other hand, in single dish observations there seems to be a correlation between the fractional linear polarization and the fractional circular polarization of SiO maser radiation from star to star based on the observation of a large number of stars (Herpin et al. 2006). Since circular polarization in non-Zeeman processes is created from a small fraction of the linear polarization, a correlation tends to support a non Zeeman origin. However, the significance of single dish observations, which can be summing over numerous masing components around each star, is unclear. The fractional linear polarization in the weak splitting regime is independent of the strength of the magnetic field. Hence, there should be no correlation between the linear and circular polarizations if the variation in the circular polarization is due to variations in the magnetic field strength and the Zeeman effect. Because both polarizations can increase with angle and with saturation in Figure 1, an apparent correlation might occur if the range of propagation angles in Figure 1 is limited because of, perhaps, preferred viewing angles. Strong masing in both the J=1-0 and J=2-1 transitions of v=1 is widely observed and a clear prediction can be made for the ratio of their fractional circular polarizations based on the Zeeman effect—assuming that the degree of saturation either is small or is equal for the two transitions when the relationship in Figure 1 is applied. To be sure that the radiation of the two transitions is from the same masing environment, VLBI with good velocity resolution is required to separate the numerous masing spots in the circumstellar environments. The results of such studies are not yet available.

4. THE STRONG SPLITTING REGIME AND MHD TURBULENCE IN REGIONS OF STAR FORMATION

Soon after the discovery of astrophysical masers, it was recognized that the spectra of these OH 18 cm masers from regions of star formation tended to be highly circularly polarized. A small fraction of the features can be identified as pairs of Zeeman sigma components from the same spatial locations. The components are well separated in frequency due to Zeeman splitting and the magnetic field strength can then be reliably inferred. Except in one region, the Zeeman pi components do not seem to be present. The comprehensive study of Garcia-Barreto et al. (1988) established the statistics of the polarization
properties of the numerous masing features that tend be present at various locations in a cloud. Though the most extensive data come from the 18 cm transition, the absence of pi components also seems to be a feature of the analogous masing transitions of the excited states of OH in regions of star formation (e.g., Caswell & Vail 1995). I will thus accept as the observational characteristics to be understood: (a) only a single Zeeman component is usually observed and (b) though pairs of sigmas are sometimes detected, the pi components are essentially never present. This behavior is almost certainly a result of irregularities (turbulence, waves, etc.) in the velocities and magnetic fields of the gas.

The strong tendency for only a single Zeeman component is the easier to understand—at least conceptually. If there are gradients in both the magnetic field and the velocity of the gas along the path of the radiation, the change in the magnetic field will cause the rest frequency of one of the Zeeman components to vary in the sense that tends to compensate for the apparent change in the frequency of the radiation due to the varying Doppler shift (Cook 1966). The maser optical depth will then be largest for one of the sigma components. In addition, if the velocity differences are large enough that the maser radiation from one Zeeman transition “sweeps through” the frequencies of other Zeeman transitions at other locations as it traverses masing gas, a single sigma component can also emerge (Deguchi & Watson 1986) even without a variation in the strength of the magnetic field along the path of the radiation. Detailed calculations demonstrate that the spectral line can remain narrow under these conditions as observed (Nedoluha & Watson 1990b). It is evident that this process can also mix the polarization characteristics of Zeeman components in the radiation at a specific observed frequency.

The absence of the pi components is more challenging. The presence of pairs of Zeeman sigmas indicates that velocity and magnetic field gradients are not always sufficient to reduce the spectrum to a single component. However, the angular distributions of pi and sigma radiation are different—with the peak maser optical depths for the sigmas being along the magnetic field and the peak for the pi components, perpendicular to the magnetic field. Gray and Field (1995) emphasized that there may be additional considerations that favor the propagation of maser radiation at small angles to the field lines so that we tend to observe only at propagation angles where the sigma components are favored. Because of the “exponential gain” associated with masers, the intensities of the pi components will be negligible at the smaller angles where their maser optical depths are smaller, consistent with the observations.

A reason for the OH maser optical depths to be greater in directions near the magnetic field lines may be found in the properties of MHD turbulence. Coherent regions in the turbulence tend to be elongated along the lines of the magnetic field (Goldreich & Shridar 1995). Hence the gradients in velocity tend to be smaller along the field lines, and the maser optical depths greater, than in other directions. Calculations have been performed that demonstrate the plausibility of this idea at a more quantitative level by calculating the maser optical depths at various angles to the mean magnetic field using the results of MHD simulations to represent the turbulence in the masing gas (Watson et al. 2004; also Wiebe & Watson 2006). The degree of anisotropy in MHD turbulence depends upon the ratio of the Alfvén velocity to the sound speed. The calculations indicate that this ratio must be at least 3 to understand the absence of the pi components of these OH masers.

5. SUMMARY

Prevailing estimates for the strengths of the magnetic fields in the environments of astrophysical masers indicate that essentially always the Zeeman splitting in frequency units is so much greater than the rates for other processes (stimulated emission, etc.) that “ordinary methods” can be used in calculations for the polarized maser radiation.

The classic paper of GKK provides us with the methods to perform the calculations in all regimes, but their widely cited result for the fractional linear polarization of masers with weak Zeeman splitting is not quantitatively applicable because it is obtained for maser saturation that is much higher than actually occurs. The formulation of GKK must also be extended to include anisotropic pumping (which seems to be essential for the SiO masers) and to calculate the circular polarization in the weak splitting regime (needed to infer field strengths). Neither is treated by GKK.

Whether the circular polarization of the SiO masers is due to the Zeeman effect, and hence whether the inferred magnetic fields of 10G or so actually are present in these circumstellar environments should be considered as uncertain at present.

The tendency for only one sigma component of the Zeeman triplet of OH masers to be detected in regions of star formation and (apparently) for the pi components to be absent is almost certainly related to irregularities in the velocities and magnetic fields.
Calculations of the masing in which simulations of MHD turbulence are used to describe these irregularities lead to spectra that are generally similar to what is observed. In this interpretation, the absence of pi components is related to the anisotropic nature of MHD turbulence.

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Fig. 1. Circular and linear polarization of maser radiation as a function of the cosine of the angle $\theta$ between the direction of the magnetic field and the line of sight. Polarizations are presented for angular momentum $J = 1 - 0$ and $J = 2 - 1$ masers, in separate panels as indicated by the label in each panel. The curves are labeled by the $\log_{10}$ of the saturation degree.

(upper three panels) The circular polarization is measured by the magnitude of $V/(pB\partial I/\partial v)$ which is equal to $\cos \theta$ in the unsaturated limit.

(lower three panels) The fractional linear polarization. Stokes-$Q$ is positive when the linear polarization is perpendicular to the direction of the magnetic field projected onto the plane of the sky. (from Watson & Wyld 2001)