SUPERNova PcEl BEAM Ssurvey

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ABSTRACT

Type Ia supernovae (SNe Ia) can be calibrated to be good standard candles at cosmological distances. We propose a supernova pencil beam survey that could yield between dozens and hundreds of SNe Ia in redshift bins of 0.1 up to \( z = 1.5 \), which would complement space-based supernova searches and enable the proper consideration of the systematic uncertainties of SNe Ia as standard candles, in particular, luminosity evolution and gravitational lensing. We simulate SNe Ia luminosities by adding weak lensing noise (using empirical fitting formulae) and scatter in SN Ia absolute magnitudes to standard candles placed at random redshifts. We show that flux averaging is powerful in reducing the combined noise due to gravitational lensing and scatter in SN Ia absolute magnitudes. The supernova number count is not sensitive to matter distribution in the universe; it can be used to test models of cosmology or to measure the supernova rate. The supernova pencil beam survey can yield a wealth of data which should enable accurate determination of the cosmological parameters and the supernova rate, and provide valuable information on the formation and evolution of galaxies.

The supernova pencil beam survey can be accomplished on a dedicated 4 m telescope with a square degree field of view. This telescope can be used to conduct other important observational projects compatible with the supernova pencil beam survey, such as QSOs, Kuiper belt objects, and, in particular, weak lensing measurements of field galaxies, and the search for gamma-ray burst afterglows.

Subject headings: distance scale — gravitational lensing — supernovae: general

1. INTRODUCTION

Toward the end of the millennium, cosmology has matured into a phenomenological science. Observational data now dominate aesthetics in the evaluation of cosmological models. Of fundamental importance is the determination of cosmological parameters, in particular, the Hubble constant \( H_0 \), the matter density fraction \( \Omega_m \), and the density fraction contributed by the cosmological constant \( \Omega_{\Lambda} \). Observation of distant Type Ia supernovae (SNe Ia) has become an increasingly powerful means of measuring cosmological parameters (Perlmutter et al. 1997, 1999; Riess et al. 1998; Schmidt et al. 1998), because SNe Ia can be calibrated to be good standard candles at cosmological distances (Riess, Press, & Kirshner 1995).

A Type Ia supernova (SN) is the thermonuclear explosion of a carbon-oxygen white dwarf in a binary when the rate of the mass transfer from the companion star is high. The SN explosion blows the white dwarf completely apart. The radioactive decay of the isotopes \( ^{56}\text{Ni} \) and \( ^{56}\text{Co} \) is responsible for much of the light emitted. The SN light curve reaches a maximum about 15 days after the explosion and then declines slowly over years. A SN can outshine the galaxy in which it lies.

Two independent groups (Riess et al. 1998; Perlmutter et al. 1999) have made systematic searches for SNe Ia for the purpose of measuring cosmological parameters. Their preliminary results seem to indicate a low matter density universe, possibly with a sizable cosmological constant. Even though the uncertainty in these results is large, they clearly demonstrate that the observation of SNe Ia can potentially become a reliable probe of cosmology. However, there are important systematic uncertainties of SNe Ia as standard candles, in particular, luminosity evolution and gravitational lensing. To constrain the evolution of SN Ia peak absolute luminosities, we need a large number of SNe Ia at significantly different redshifts (low-\( z \) and \( z > 1 \)), which are not available at present. Both groups have assumed a smooth universe in their data analysis, although they include lensing in their error budgets. Since we live in a clumpy universe, the effect of gravitational lensing must be taken into account adequately for the proper interpretation of SN data. At present, the small number of observed high-\( z \) SNe Ia prevents adequate modeling of gravitational lensing effects.

In this paper, we propose a pencil beam survey of SNe Ia that could yield between dozens and hundreds of SNe Ia in redshift bins of 0.1 up to \( z = 1.5 \) (see § 3), which would allow the proper modeling of gravitational lensing, as well as a quantitative understanding of luminosity evolution. Such a survey would yield a wealth of data which can be used to make accurate measurement of cosmological parameters and the SN rate, and provide powerful constraints on various aspects of the cosmological model. In § 2 we consider the weak lensing of SNe Ia. We simulate SN Ia luminosities by adding weak lensing noise and scatter in SN Ia absolute magnitudes to standard candles placed at random redshifts. We show how flux averaging reduces the combined noise of gravitational lensing and scatter in SN Ia absolute magnitudes. In § 3 we show that the number count of SNe from a pencil beam survey is not sensitive to matter distribution in the universe; it can be used as a test of models of cosmology and SN progenitors, or to measure the SN rate accurately. In § 4 we discuss the observational feasibility of the SN pencil beam survey. Section 5 contains conclusions.

2. WEAK LENSING OF SUPERNOVAE

In a SN Ia Hubble diagram, one must use distance-redshift relations to make theoretical predictions. Unlike
angular separations and flux densities, distances are not directly measurable, but they are indispensable theoretical intermediaries. The distance-redshift relations depend on the distribution of matter in the universe.

In a smooth Friedmann-Robertson-Walker (FRW) universe, the metric is given by $ds^2 = dt^2 - a^2(t)[dr^2/(1 - kr^2) + r^2(d\theta^2 + \sin^2 \theta d\phi^2)]$, where $a(t)$ is the cosmic scale factor, and $k$ is the global curvature parameter ($\Omega_k = 1 - \Omega_m - \Omega_\Lambda = -k/H_0^2$). The comoving distance $r$ is given by

$$r(z) = \frac{cH_0^{-1}}{\Omega_k^{1/2}} \sinh \left\{ \sqrt{\frac{\Omega_k}{1 + \Omega_k}} \left[ \frac{1}{1 + \Omega_k} \frac{dz}{dz'} + \frac{\Omega_\Lambda}{1 + \Omega_k} \frac{1}{1 + z'} - \frac{r^2}{1 + z'} \right]^{1/2} \right\}, \quad (1)$$

where "sinn" is defined as sinh if $\Omega_k > 0$ and as sin if $\Omega_k < 0$. If $\Omega_k = 0$, the sinn and $\Omega_k$'s disappear from equation (1), leaving only the integral. The angular diameter distance is given by $d_A(z) = r(z)/(1 + z)$, and the luminosity distance is given by $d_L(z) = (1 + z)^2d_A(z)$.

However, our universe is clumpy rather than smooth. According to the focusing theorem in gravitational lens theory, if there is any shear or matter along a beam connecting a source to an observer, the angular diameter distance of the source from the observer is smaller than that which would occur if the source were seen through an empty, shear-free cone, provided the affine parameter distance (defined such that its element equals the proper distance element at the observer) is the same and the beam has not gone through a caustic. An increase of shear or matter density along the beam decreases the angular diameter distance and consequently increases the observable flux for given $z$ (Schneider, Ehlers, & Falco 1992).

The observation of SNe Ia at $z > 1$ is important for the determination of cosmological parameters, but the dispersion in SN Ia luminosities due to gravitational lensing can become comparable to the intrinsic dispersion of SNe Ia absolute magnitudes because the optical depth for gravitational lensing increases with redshift.

2.1. Direction-dependent Smoothness Parameter

If only a fraction $\tilde{\alpha}$ (known as the smoothness parameter) of the matter density is smoothly distributed, the largest possible distance (for given redshift) for light bundles that have not passed through a caustic is given by the approximate solution to the following equation:

$$g(z) \frac{d}{dz} \frac{dD_A}{dz} + \frac{3}{2} \tilde{\alpha} \Omega_\Lambda (1 + z)^2 D_A = 0, \quad (2)$$

where $g(z) \equiv (1 + z)^3[1 + \Omega_m z + \Omega_\Lambda[(1 + z)^{-2} - 1]]^{1/2}$ (Kantorowski 1998). The $\Omega_\Lambda = 0$ form of equation (2) has been known as the Dyer-Roeder equation (Dyer & Roeder 1973; Schneider et al. 1992).

Figure 1a shows magnitude versus redshift for the three cosmological models considered by Riess et al. (1998), SCDM ($\Omega_m = 1, \Omega_\Lambda = 0$), OCDM ($\Omega_m = 0.2, \Omega_\Lambda = 0$), and $\Lambda$CDM ($\Omega_m = 0.2, \Omega_\Lambda = 0.8$). For each cosmological model, the upper curve represents the completely clumpy universe (empty beam, $\tilde{\alpha} = 0$), while the lower curve represents the completely smooth universe (filled beam, $\tilde{\alpha} = 1$). Figure 1b shows the same models relative to smooth OCDM (filled beam, $\tilde{\alpha} = 1$), the middle curve for each model now represents a universe with half of the matter smoothly distributed (half-filled beam, $\tilde{\alpha} = 0.5$). Clearly, at $z > 1$, there is degeneracy of distances in a flat clumpy universe and an open smooth universe, and also in an open clumpy universe and a flat smooth universe with a sizable cosmological constant, as has been noted by a number of previous authors (Kantorowski, Vaughan, & Branch 1995; Kantorowski 1998; Linder 1998; Holz 1998).

We can generalize the angular diameter distance $D_A(z)$ by allowing the smoothness parameter $\tilde{\alpha}$ to be direction dependent, i.e., a property of the beam connecting the observer...
and the standard candle. The smoothness parameter $\tilde{z}$ essentially represents the amount of matter that causes weak lensing of a given source. Since matter distribution in our universe is inhomogeneous, we can think of our universe as a mosaic of cones centered on the observer, each with a different value of $\tilde{z}$. This reinterpretation of $\tilde{z}$ implies that we have $\tilde{z} > 1$ in regions of the universe in which there are above-average amounts of matter which can cause magnification of a source (Wang 1999).

In order to derive a unique mapping between the distribution in distances and the distribution in the direction-dependent smoothness parameter for given redshift $z$, we define the direction-dependent smoothness parameter $\tilde{z}$ to be the solution of equation (2) for given distance $D_d(z)$.

At given redshift $z$, the magnification of a source can be expressed in terms of the apparent brightness of the source, $f(\tilde{z} | z)$, or in terms of the angular diameter distance to the source, $D_d(\tilde{z} | z)$:

$$\mu = \frac{f(\tilde{z} | z)}{f(\tilde{z} = 1 | z)} = \left[ \frac{D_d(\tilde{z} = 1 | z)}{D_d(\tilde{z} | z)} \right]^{-1},$$

where $f(\tilde{z} = 1 | z)$ and $D_d(\tilde{z} = 1 | z)$ are the flux of the source and angular diameter distance to the source in a completely smooth universe (filled beam), and $\tilde{z}$ is the direction-dependent smoothness parameter. Since distances are not directly measurable, we should interpret equation (3) as defining a unique mapping between the magnification of a standard candle at redshift $z$ and the direction-dependent smoothness parameter $\tilde{z}$ at $z$; $\tilde{z}$ parameterizes the direction-dependent matter distribution in a well-defined manner.

From the magnification distributions of standard candles at various redshifts, $p(\mu | z)$, with $z = 0.5, 1, 1.5, 2, 2.5, 3, 5$, found numerically by Wambsganss et al. for $\Omega_m = 0.4$, $\Omega_L = 0.6$ (Wambsganss et al. 1997; J. Wambsganss 1999, private communication), Wang (1999) has obtained simple empirical fitting formulae for the distribution of $\tilde{z}$:

$$p(\tilde{z} | z) = C_{\text{norm}} \exp \left[ -\left( \frac{\tilde{z} - \tilde{z}_{\text{peak}}}{w(z)} \right)^2 \right],$$

where $C_{\text{norm}}$, $\tilde{z}_{\text{peak}}$, $w$, and $q$ depend on $z$ and are independent of $\tilde{z}$. They are given by

$$\tilde{z}_{\text{peak}}(z) = 1.01350 - 1.07857 \left( \frac{1}{5z} \right) + 2.05019 \left( \frac{1}{5z} \right)^2 - 2.14520 \left( \frac{1}{5z} \right)^3,$$

$$w(z) = 0.06375 + 1.75355 \left( \frac{1}{5z} \right) - 4.99383 \left( \frac{1}{5z} \right)^2 + 5.95852 \left( \frac{1}{5z} \right)^3,$$

$$q(z) = 0.75045 + 1.85924 \left( \frac{z}{5} \right) - 2.91830 \left( \frac{z}{5} \right)^2 + 1.59266 \left( \frac{z}{5} \right)^3.$$

$C_{\text{norm}}(z)$ is the normalization constant for given $z$. The parameter $\tilde{z}_{\text{peak}}(z)$ indicates the average smoothness of the universe at redshift $z$; it increases with $z$ and approaches $\tilde{z}_{\text{peak}}(z) = 1$ (filled beam) at $z = 5$. The parameter $w(z)$ indicates the width of the distribution in the direction-dependent smoothness parameter $\tilde{z}$; it decreases with $z$. The $z$ dependences of $\tilde{z}_{\text{peak}}(z)$ and $w(z)$ are as expected because, as we look back to earlier times, lines of sight become more filled in with matter, and the universe becomes smoother on the average. The parameter $q(z)$ indicates the deviation of $p(\tilde{z} | z)$ from Gaussianity (which corresponds to $q = 0$).

Models with different cosmological parameters should lead to somewhat different matter distributions $p(\tilde{z} | z)$. In the context of weak lensing of standard candles, we expect the cosmological parameter dependence to enter primarily through the magnification $\mu$ to direction-dependent smoothness parameter $\tilde{z}$ mapping at given $z$ (the same $\tilde{z}$ corresponds to very different $\mu$ in different cosmologies).

### 2.2. Flux Averaging of SN Luminosities

Gravitational lensing noise in the Hubble diagram can be reduced by appropriate flux averaging of SNe Ia in each redshift bin. Because of flux conservation, the average flux of a sufficient number of SNe Ia at the same $z$ from the same field should be the same as the true flux of the SNe Ia without gravitational lensing if the sample is complete.

It is convenient to compare the distance modulus of SNe Ia, $\mu_0$, with the theoretical prediction

$$\mu_0 = 5 \log \left( \frac{d_L}{\text{Mpc}} \right) + 25,$$

where $d_L(z)$ is the luminosity distance. Before flux averaging, we convert the distance modulus $\mu_0(z)$ of SNe Ia into “fluxes,” $f(z) = 10^{-\mu_0(z)/2.5}$. We then obtain “absolute luminosities,” $\langle L(z) \rangle$, by removing the redshift dependence of the “fluxes,” i.e.,

$$\langle L(z) \rangle = 4\pi d_L^2(z) \int H_0, \Omega_m, \Omega_L f(z) \, dz,$$

where $(H_0, \Omega_m, \Omega_L)$ are the best-fit cosmological parameters derived from the unbinned data set $\{f(z)\}$. We then flux-average over the “absolute luminosities” $\{\langle L(z) \rangle \}$ in each redshift bin. The set of best-fit cosmological parameters derived from the binned data is applied to the unbinned data $\{f(z)\}$ to obtain a new set of “absolute luminosities” $\{\langle L \rangle \}$, which is then flux-averaged in each redshift bin, and the new binned data are used to derive a new set of best-fit cosmological parameters. This procedure is repeated until convergence is achieved. This iteration should lead to the optimal removal of gravitational lensing noise and the accurate determinations of the cosmological parameters. Wang (2000) has applied this method to analyze the combined data from the two groups (Riess et al. 1998; Perlmutter et al. 1999).

To illustrate how flux averaging can reduce the dispersion in SN Ia luminosities caused by weak lensing, let us simulate the data by drawing $N_{SN}$ random points from the redshift interval $[z_{\min}^i, z_{\max}^i]$; each point represents a standard candle. We add weak lensing noise by giving each standard candle a direction-dependent smoothness parameter $\tilde{z}$ (corresponding to $\mu = [D_d(\tilde{z} = 1)/D_d(\tilde{z})]^2$) drawn at random from the distribution $p(\tilde{z} | z)$ given in § 2.1. The scatter in SN Ia absolute magnitudes, $\Delta m_{\text{abs}}$, can be written as

$$\Delta m_{\text{abs}} = \Delta m_{\text{int}} + \Delta m_{\text{obs}},$$

where $\Delta m_{\text{int}}$ is the intrinsic scatter and $\Delta m_{\text{obs}}$ is the observational noise. We assume that both intrinsic scatter and
The luminosity of each SN Ia extracted from the data is with We take The absolute luminosity of each SN Ia extracted from the data is

\[ \Phi(z) = 10^{-\Delta m_{\text{abs}} / 2.5} \mu(x|z) \Phi(z) \]

\[ = 10^{-\Delta m_{\text{abs}} / 2.5} \mu(x|z) \Phi(z) \]

\[ = \Phi(x = 1|z) \mu(x|z) \sigma_{\text{abs}} = 0.20. \]

The flux-averaged noise is

\[ \Delta m = -2.5 \log \left( \frac{\Phi(z)}{\Phi(x = 1|z)} \right). \]

Let us average the fluxes of all SNe Ia in the redshift bin \([z_1, z_2] \):

\[ \overline{\Phi} = \frac{1}{N_{\text{SN}}} \sum_{i=1}^{N_{\text{SN}}} \Phi(z_i). \]

The flux-averaged noise is

\[ \langle \Delta m \rangle_{\text{avg}} = -2.5 \log \left( \frac{\overline{\Phi}}{\Phi(x = 1|z)} \right), \]

where

\[ \bar{z} = \frac{\sum_{i=1}^{N_{\text{SN}}} z_i}{N_{\text{SN}}} \].

Table 1 lists the means and dispersions (in the form mean \pm dispersion) of \( \Delta m \) [which are \( \langle \Delta m \rangle \) and \( \sigma = (\langle \Delta m \rangle - \langle \Delta m \rangle^2 \rangle^{1/2} \), and \( \langle \Delta m \rangle_{\text{avg}} \) which are \( \langle \langle \Delta m \rangle_{\text{avg}} \rangle \) and \( \sigma_{\text{avg}} = (\langle \langle \Delta m \rangle_{\text{avg}} \rangle - \langle \langle \Delta m \rangle_{\text{avg}} \rangle^2 \rangle^{1/2} \) for various redshift bins with \( N_{\text{SN}} = 2, 4, \) and 9 SNe Ia in each bin, for 4\(^4\) random samples. We have taken \( \Omega_m = 0.4, \Omega_{\Lambda} = 0.6. \)

The dispersion decreases roughly as \( 1/N_{\text{SN}}^{1/2} \). The dispersion would decrease by 30\% if the sample contains 2 SNe Ia and 50\% if the sample contains 4 SNe Ia. Even though gravitational lensing noise increases with redshift, the combined gravitational lensing and SN Ia absolute magnitude scatter noise in the redshift interval \( z = [4.5, 5] \) can be reduced to the same level as in the absence of lensing by flux averaging over two SNe Ia.

Note that the flux-averaged luminosities are biased toward slightly higher luminosities. This is as expected, because we have assumed that the intrinsic scatter and observational noise in the SN Ia absolute magnitude are Gaussian in luminosity. It is straightforward to show that the mean of \( 10^{-\Delta m_{\text{abs}} / 2.5} \) is \( \exp (\ln 10 / 2.5) \sigma_{\text{abs}}^2 / 2 \), which corresponds to a bias of \( -\sigma_{\text{abs}}^2 \ln 10 / 2.5 \approx -0.018 \) for \( \sigma_{\text{abs}} = 0.2 \). If we assume that the intrinsic scatter and observational noise are Gaussian in luminosity, the flux-averaged luminosities become unbiased.

3. Supernova Number Count

The SN number count is most sensitive to the SN rate as a function of \( z \), which depends on the specific rate of SNe, as well as on the number density of galaxies and their luminosity distribution. The frequency of SNe is a key parameter for describing the formation and the evolution of galaxies; the winds driven by SNe tune the energetics, and their production of metals determines the chemical evolution of galaxies and of clusters of galaxies (Ferrini & Poggianti 1993; Renzini et al. 1993). The SN II rate is related (for a given initial mass function) to the instantaneous stellar birthrate of massive stars because SNe II have short-lived progenitors; the SN Ia rate follows a slower evolutionary track and can be used to probe the past history of star formation in galaxies. Accurate measurements of the SN rates at intermediate redshifts are important for understanding galaxy evolution, cosmic star formation rate, and the nature of SN Ia progenitors (Madau 1998; Ruiz-Lapuente & Canal 1998; Sadat, Guiderdoni, & Silk 1998; Yungelson & Livio 1998; Madau, Della Valle, & Silk 1998; Partridge, Livio, & Silk 1998). The SN rates are very uncertain at present due to the small number of SNe discovered in systematic searches (van den Bergh & McClure 1994; Pain et al. 1996). Kolatt & Bartelmann (1998) have estimated the SN Ia average rate per proper time unit per comoving volume to be

\[ n_{\text{SN}}(z) = (A + B z)(100 \text{ yr})^{-1}(h^{-1} \text{ Mpc})^{-3}, \]

\[ A = 0.0136, \quad B = 0.067, \quad (15) \]

for \( q_0 = 0.5 \). Changing \( q_0 \) to lower values leads to lower comoving densities but higher luminosities of galaxies at high \( z \) (Lilly et al. 1995). Equation (15) has been derived assuming that all SNe reside in galaxies, neglecting the redshift dependence of the specific SN rate (number per unit

| \( z \) | \( \langle \Delta m \rangle \pm \sigma \) |
|---|---|
| [0.5, 0.6] | 0.001 \pm 0.200 |
| [1, 1.1] | 0.002 \pm 0.204 |
| [1.5, 1.6] | 0.003 \pm 0.210 |
| [1.5, 2] | 0.003 \pm 0.213 |
| [2, 2.5] | 0.005 \pm 0.218 |
| [2.5, 3] | 0.006 \pm 0.223 |
| [3, 3.5] | 0.007 \pm 0.228 |
| [3.5, 4] | 0.007 \pm 0.233 |
| [4, 4.5] | 0.008 \pm 0.238 |
| [4.5, 5] | 0.008 \pm 0.243 |
luminosity per unit time), and using the number density of galaxies (as a function of redshift) and the Schechter function parameters derived from the Canada-France Redshift Survey (Lilly et al. 1995), the APM Survey (Loveday et al. 1992), and the AUTOFIB Survey (Ellis et al. 1996). We use equation (15) for all cosmological models considered in this paper, because it is a very crude and conservative estimate, and we use it for the purpose of illustration only.

The expected number of SNe Ia in a field of angular area \( \theta^2 \) for an effective observation duration of \( \Delta t \) up to redshift \( z \) is

\[
N(z) = \int_0^z dz' r^2(z') \left( \frac{dr(z')}{a(t')} \right)^2 \frac{n_{\text{SN}}(z')}{(1 + z')^3} \frac{0.0136 + 0.067z'}{(1 + z')\sqrt{\Omega_m(1 + z')^3 + \Omega_\Lambda + \Omega_k(1 + z')^2}},
\]

where \( dr = -c \, dt \) is the proper distance interval, \( r \) is comoving distance, and the factor \( (1 + z)^{-1} \) accounts for the cosmological time dilation. Note that the number counts fundamentally probe a different aspect of the global geometry of the universe than do the distance measures (Carroll, Press, & Turner 1992).

Figure 2 shows the number of SNe Ia expected per 0.1 redshift interval as a function of redshift for the same three cosmological models as in Figure 1, SCDM \((\Omega_m = 1, \Omega_\Lambda = 0)\), OCDM \((\Omega_m = 0.2, \Omega_\Lambda = 0)\), and ΛCDM \((\Omega_m = 0.2, \Omega_\Lambda = 0.8)\). For a 1 square degree field, and an effective observation duration of 1 year, the total numbers of expected SNe Ia are 464, 899, and 1705 for SCDM, OCDM, and ΛCDM, respectively, for \( z \) up to 1.5.

The number of SNe which are strongly lensed by galaxies is given by

\[
N_{\text{lensed}}(z) = \int_0^z dz' \tau(z') \frac{dN}{dz'},
\]

\[
= 3.81 \times 10^{-2} \left( \frac{\theta}{1'} \right)^2 \left( \frac{F}{0.05} \right) \times \left( \frac{\Delta t}{1 \text{ yr}} \right) \int_0^z dz' \frac{r(z')}{cH_0^{-1}} \times \frac{0.0136 + 0.067z'}{(1 + z')\sqrt{\Omega_m(1 + z')^3 + \Omega_\Lambda + \Omega_k(1 + z')^2}}.
\]

We have used the optical depth for gravitational lensing by galaxies (Turner 1990; Fukugita & Turner 1991):

\[
\tau(z) \approx \frac{F}{30} \left( \frac{1 + z) d_A(z)}{cH_0^{-1}} \right),
\]

where \( F \) parameterizes the gravitational lensing effectiveness of galaxies (as singular isothermal spheres). Figure 3 shows the number of strongly lensed SNe as function of survey depth \( z \), for the same cosmological models as in Figures 1 and 2. For a 1 square degree field, and an effective observation duration of 1 year, the total numbers of strongly lensed SNe Ia are 0.2, 0.6, and 1.8 for SCDM, OCDM, and ΛCDM, respectively, for \( z \) up to 1.5.

Figures 2 and 3 show that the SN number counts from a pencil beam survey can be used to measure the SN rate at high redshifts and perhaps to probe cosmology. Because of the large number of SNe in each redshift bin and the smallness of gravitational lensing optical depth, the SN number count should be insensitive to matter distribution in the universe, and should therefore provide a robust probe of the cosmological model. Note that the strongly lensed SNe can
be easily removed from the survey sample, because they appear as unusually bright SNe. The number of strongly lensed SNe is very sensitive to the cosmological model and might be used to further constrain the cosmological model. But given the small number of strongly lensed SNe Ia expected in any realistic observational program, their usefulness may be limited.

The SN number counts provide a combined measure of the cosmological parameters and the SN rate. Figure 4 shows the parameter dependence of the SN Ia number count per 0.1 redshift interval \((z - 0.05, z + 0.05)\) as function of redshift \(z\). Note that the dependences of the SN number count on the SN rate (parameterized by \(A\) and \(B\)) and the cosmological parameters are not degenerate and can be differentiated in principle. In practice, we are ignorant of the functional form of the SN rate as a function of \(z\). Hence, we may apply the measurements of the cosmological parameters as priors to the SN number counts to obtain a direct measure of the SN rate in the universe for \(z \leq 1.5\), which can be used as a powerful constraint on the cosmological model. For a 1 square degree field, and an effective observation duration of 1 year, the SN rate per 0.1 redshift interval can be determined to 14%–16%, 9%–11%, and 7%–8% at \(1 < z < 1.5\) for SCDM, OCDM, and \(\Lambda\)CDM, respectively.

4. OBSERVATIONAL FEASIBILITY

A pencil beam survey would be efficient in the discovery of SNe at \(z \geq 1\) through the combination of data from successive nights and the comparison of the latest frame of images with all previous frames.

SNe Ia are quite faint at \(z \sim 1.5\), with AB magnitude in the \(I\) band of \(I_{AB} \sim 26\). Because of the UV suppression due to line blanketing and the apparent IR suppression in the rest frame SN Ia spectrum, one should use a passband that corresponds to the wavelength range of 3000–10000 Å in the SN rest frame; this means using the \(I, J, H,\) or \(K\) passbands to observe the \(z > 1\) SNe. SN searches from space (the Hubble Space Telescope [HST] and the Next Generation Space Telescope [NGST]) are limited by small fields of view; a large-scale SN search is only possible from the ground, where one is limited to the \(I\) band by the atmosphere. Here we discuss the observational feasibility of a pencil beam survey of SNe Ia up to \(z \sim 1.5\) in terms of exposure times in the \(I\) band, although multiple-band photometry should be obtained to constrain extinction and evolution of the SNe.

Using values for the photometric parameters from the Sloan Digital Sky Survey (1" seeing, effective sky brightness of 20.3 mag arcsec\(^{-2}\) in the \(I\) band, seeing-dominated point-spread function, etc), we find that the exposure time for a point source with AB magnitude of \(I_{AB}\) can be written as

\[
t = 13.94 \text{ hr} \left(\frac{S/N}{10}\right)^2 \left(\frac{4 \text{ m}}{D}\right)^2 10^{0.8(I_{AB}-26)},
\]

where \(S/N\) is the signal-to-noise ratio, \(D\) is the diameter of the aperture of the telescope. The above equation shows that the supernova pencil beam survey can be accomplished from a modest dedicated 4 m telescope, which can image the same fields (two pencil beams should be observed to keep the fields close to zenith) every night; this can lead to the discovery of SNe Ia up to \(z = 1.5\) via appropriate com-

![Figure 4](image-url)

**Fig. 4.—** Parameter dependence of the SN Ia number count per 0.1 redshift interval as a function of redshift. Note that the dependence of the SN number count on the SN rate is parameterized by \(A\) and \(B\).
bination of data from successive nights, and the light curves of SNe Ia at \( z < 1.5 \). It should be feasible to monitor the faintest SNe Ia from the Keck 10 m telescope or the HST.

Spectra of SNe are required to determine whether they are Type Ia. SNe Ia have an Si II absorption line at \( \sim 4000 \) Å that may be used to identify the \( 1 < z \leq 1.5 \) SNe Ia in ground-based observations. The follow-up spectroscopy can be attempted on the Keck Low-Resolution Imaging Spectrometer (LRIS). Assuming that we use the 300 grooves mm\(^{-1}\) grating, which provides dispersions of 4.99 Å per 48 µm (2 pixels on the CCD), we find the exposure time for 0:5 seeing to be

\[
t = 28 \text{ hr} \left( \frac{S/N}{3} \right)^2 \left( \frac{W}{1 \text{ arcsec}} \right) 10^{0.4(2(I_{AB} - 26) - (I_{AB}^{sky} - 21))},
\]

\[
= 4.44 \text{ hr} \left( \frac{S/N}{3} \right)^2 \left( \frac{W}{1 \text{ arcsec}} \right) 10^{0.4(2(I_{AB} - 25) - (I_{AB}^{sky} - 21))},
\]

\[
= 12.33 \text{ hr} \left( \frac{S/N}{5} \right)^2 \left( \frac{W}{1 \text{ arcsec}} \right) 10^{0.4(2(I_{AB} - 25) - (I_{AB}^{sky} - 21))},
\]

(20)

where \( W \) is the width of the slit, and \( I_{AB}^{sky} \) is the sky brightness in the \( I \) band in units of mag arcsec\(^{-2}\). Note that the Si II absorption feature at \( \sim 4000 \) Å is not deep enough to be useful in a very noisy (S/N = 3) spectrum. Clearly, spectroscopic follow-up of the \( 1.5 \leq z \leq 2 \) SNe Ia will require substantial observational resources; NICMOS or NGST will be more suitable for the spectroscopy of the SNe Ia at the highest redshifts discovered from the ground. The Keck LRIS can be used to obtain the spectra of the SNe Ia at more modest redshifts (\( 1 < z \leq 1.3, I_{AB} \sim 25 \)).

A dedicated 4 m telescope with a square degree field of view can be used to conduct other important scientific projects compatible with the SN pencil beam survey, such as QSOs, Kuiper belt objects, and, in particular, weak lensing and the search of gamma-ray burst (GRB) afterglows.

Weak lensing is a powerful tool in mapping the mass distribution in the universe. The large field of view and the depth of a pencil beam survey would be ideal for weak lensing measurements of field galaxies, which can be used to constrain the large-scale structure in the universe.

GRBs are perhaps the most energetic astrophysical events in the universe. Currently, there are many competing theories to explain GRBs. GRB afterglows contain valuable information; a statistically significant sample of GRB afterglows can provide strong constraints on the GRB theory. If beaming is involved in GRBs, we expect most of the GRB afterglows not to be associated with observable bursts. Schmidt et al. (1998) have observed a few optical transients which were too short in duration to be SNe; these could be GRB afterglows. Since present observational data seem to indicate that GRBs have associated host galaxies, the detection of host galaxies associated with short optical transients would support the interpretation of the latter as GRB afterglows. Since the GRB host galaxies are typically fainter than \( R = 25 \), a pencil beam survey would be ideal in detecting these host galaxies of candidate GRB afterglows.

5. SUMMARY AND DISCUSSION

We have proposed a pencil beam survey of SNe Ia which can yield from tens to hundreds of SNe Ia per 0.1 redshift interval for \( z \) up to 1.5, which would enable the quantitative consideration of the systematic uncertainties of SNe Ia as standard candles, in particular, luminosity evolution and gravitational lensing. Using the Perlmutter et al. “batch” search technique repetitively over the same field in the sky, the pencil beam survey would be efficient in the discovery of SNe at \( z \geq 1 \) by allowing the comparison of the latest frame of images (which may consist of combined data from successive nights) with all previous frames.

The direct product of such a survey is the SN number count as a function of redshift (see § 3), which is a combined measure of the cosmological parameters and the SN rate. When the measurements of the cosmological parameters are applied as priors to the number count, we obtain a direct measure of the SN rate, which is a key parameter in the formation and evolution of galaxies. The non–Type Ia SNe discovered by the pencil beam survey may be comparable to the Type Ia SNe in number.

The most important and straightforward application of the data from the SN pencil beam survey is to reduce the gravitational lensing noise in a SN Ia Hubble diagram via flux averaging. We have simulated SN Ia luminosities by adding weak lensing noise (using empirical fitting formulae given by Wang 1999) and scatter in SN Ia absolute magnitudes to standard candles placed at random redshifts. We have shown that flux averaging is powerful in reducing the combined noise of gravitational lensing and SN Ia absolute magnitude scatter (see § 2). Because of the non-Gaussian nature of the luminosity distribution of SNe Ia at given \( z \) due to weak lensing, the large number of SNe Ia in a given redshift interval at high \( z \) is essential for the proper modeling and removal of the gravitational lensing effect. The SN Ia luminosity distribution in each redshift interval can be used to constrain the cosmological model (in particular, the fraction of matter in compact objects) by comparison with predictions of numerical ray-shooting (Holz & Wald 1998).

The completeness of the SN Ia sample determines the effectiveness of the removal of gravitational lensing noise from the SN Ia Hubble diagram and the amount of information contained in the SN Ia luminosity distribution in each redshift interval. Note that the magnitude limit of the survey can lead to observational bias against the most distant demagnified SN; therefore, the SNe which are close to the magnitude limit of the survey should not be used to probe cosmology in the manner described in this paper. To ensure the maximum usefulness of the data, scrupulous attention will have to be paid to photometric calibration, uniform treatment of nearby and distant samples, and an effective way to deal with reddening (Riess et al. 1998).

Note that although we can remove or reduce the effect of gravitational lensing on the SN Ia Hubble diagram, other systematics can affect the observed luminosity of SNe Ia, for example, gray dust, an evolution of the reddening law, or evolution in the peak absolute luminosity of SNe Ia. While it is possible to constrain gray dust and determine dust evolution through multiband photometry at significantly different redshifts, the dimming with \( z \) in peak absolute luminosity of SNe Ia is degenerate with the effect of low matter density (Aguirre 1999; Riess et al. 1999; Drell, Loredo, & Wasserman 1999; Wang 2000). Evolution will remain a caveat in the use of SNe Ia as cosmological standard candles, unless one can somehow correct for the effect of evolution. It is critical to obtain up to hundreds of SNe Ia at \( z > 1 \) to constrain luminosity evolution, because we expect the luminosity evolution and low matter density to
affect the distance modulus of SNe Ia through different functionals of $z$, which should become distinguishable at $z > 1$.

We have proposed a survey cutoff of $z = 1.5$ mainly for two reasons. First, going to higher redshift makes obtaining the spectra of the SNe (which are needed to distinguish different types of SNe) practically impossible from the ground; even obtaining the spectra of $z \sim 1.5$ SNe Ia may prove impractical from the ground (since it would require several clear nights per spectrum on the Keck; see § 4). Observers have already demonstrated that SNe Ia up to $z = 1$ can be found in the ongoing searches (Goobar & Perlmutter 1995; Garnavich et al. 1998; Perlmutter et al. 1999). At $z \sim 1.5$, the SNe Ia should be $\sim 2.5$ mag fainter than at $z \sim 1$; it should be possible to discover them on the ground through deep imaging, and the follow-up spectroscopy can be done on NICMOS or NGST (see § 4). Second, the predicted rest-frame SN Ia rate per comoving volume as a function of redshift seems to peak at $z \sim 1$ (Yungelson & Livio 1998; Sadat et al. 1998). A pencil beam survey of SNe up to $z = 1.5$ will enable the accurate determination of the SN rate as a function of redshift in the redshift region important for studying the cosmic star formation rate and the SN Ia progenitor models. Although the NGST can detect SNe at as high redshifts as they exist (Stockman et al. 1998), the estimated rate of detection is of order 20 SNe II per $4' \times 4'$ field per year in the interval $1 < z < 4$ (Madau et al. 1998), and the detection rate of SNe Ia is likely smaller. Thus a ground-based pencil beam survey of SNe is essential to complement the space-based SN searches.

The most challenging aspect of the SN pencil beam survey is obtaining spectra for the SNe Ia at redshifts close to 1.5. Instead of waiting for future space equipments, we may find innovative ways of obtaining spectra from the ground. The referee has pointed out that since we can find multiple SNe Ia at the same time on the same 1 square degree field (the number of SNe depends on cosmology), it may be possible to get multiple spectra at once with fibers.

The nominal numbers we have used for the proposed SN pencil beam survey, a 1 square degree field, a depth of $z = 1.5$, and an effective observation duration of 1 year (which is equivalent to several years of actual observation), are optimistic but not implausible. The large sky coverage and the long effective observation duration will probably require a large consortium of existing and new SN search teams through, for example, a dedicated 4 m telescope which can be used at the same time for other important observational projects compatible with the SN pencil beam survey (see § 4). The goal of going up to $z = 1.5$ in spectroscopy will require support from the Keck LRIS and NICMOS/NGST. We conclude by noting that a SN pencil beam survey can yield an enormous scientific return. The observational efforts directed toward a SN pencil beam survey should be very rewarding.

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