Tracing Magnetic Fields with Spectroscopic Channel Maps

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Abstract
We identify velocity channel map intensities as a new way to trace magnetic fields in turbulent media. This work makes use of both the modern theory of magnetohydrodynamic (MHD) turbulence, which predicts that magnetic eddies are aligned with the local direction of the magnetic field, and also the theory of spectral line position–position–velocity (PPV) statistics, which describes how velocity and density fluctuations are mapped onto PPV space. In particular, we use the fact that the fluctuations of the intensity of thin channel maps are mostly affected by the turbulent velocity, while the thick maps are dominated by density variations. We study how contributions of the fundamental MHD modes affect the Velocity Channel Gradients (VChGs), and demonstrate that the VChGs arising from Alfvén and slow modes are aligned perpendicular to the local direction of the magnetic field, while the VChGs produced by the fast mode are aligned parallel to the magnetic field. The dominance of Alfvén and slow modes in interstellar media will therefore allow reliable magnetic field tracing using the VChGs. We explore ways of identifying self-gravitating regions that do not require polarimetric information. In addition, we also introduce a new measure, termed “Reduced Velocity Centroids” (RVCGs), and compare its abilities with those of VChGs. We employed VChGs in analyzing GALFA 21 cm data and successfully compared the magnetic field directions with the Planck polarization observations. The applications of the suggested techniques include both tracing the magnetic field in diffuse interstellar media and star-forming regions, and removing the galactic foreground in the framework of cosmological polarization studies.

Key words: ISM: magnetic fields – ISM: structure

1. Introduction
The interstellar medium (ISM) of spiral galaxies is both magnetized and turbulent (see Armstrong et al. 1995; Chepurnov & Lazarian 2010), with turbulent magnetic fields playing a critical role in many key processes, including star formation (see Mac Low & Klessen 2004; McKee & Ostriker 2007), the propagation and acceleration of cosmic rays (see Jokipii 1966; Yan & Lazarian 2008), and regulating the heat and mass transfer between different ISM phases (see Draine 2009 for a list of different ISM phases). In addition, galactic magnetic fields are responsible for the polarized radiation that presents a serious obstacle for the studies of polarization of cosmological origin. Therefore, it is essential to have a reliable way to study the properties of magnetic fields in the ISM.

Magnetic fields make turbulence anisotropic, with turbulent eddies elongated along the magnetic field (see Brandenburg & Lazarian 2013 for a review). As a result, the observed velocity correlations are expected to be elongated along the underlying magnetic field because of this property, which was demonstrated with synthetic observations in previous studies (Lazarian et al. 2002; Esquivel & Lazarian 2005, henceforth EL05). Later, anisotropy was studied using Principal Component Analysis (PCA) and applied to observations (Heyer et al. 2008).1

There is another way to employ the properties of magnetohydrodynamic (MHD) turbulence in order to study magnetic fields. The aforementioned turbulent eddies are aligned with the magnetic field, which entails that velocity gradients calculated in the direction perpendicular to the magnetic field should have larger values. This property of magnetic turbulence was employed in González-Casanova & Lazarian (2017, henceforth GL17), in which the approach of using velocity centroid gradients (VCGs) to trace magnetic field orientations was proposed. The technique was further extended and elaborated in Yuen & Lazarian (2017a, henceforth YL17a), and the new way of magnetic field tracing was successfully compared with polarization observations from Planck. The subsequent study by Yuen & Lazarian (2017b, henceforth YL17b) revealed the synergy of simultaneously using VCGs and intensity gradients (IGs).2 These papers introduced a new way of studying magnetic fields with spectroscopic data as well as studying other key processes taking place in the ISM.

In a separate development, the same idea of tracing magnetic fields with gradients was employed with synchrotron intensity maps in Lazarian et al. (2017, henceforth LYLCL). Using synthetic data, LYLCL showed that synchrotron intensity gradients (SIGs) can reliably trace the magnetic field in the ISM, and the study also confirmed this conclusion by comparing Planck synchrotron polarization maps with the SIG maps. The synergies of SIGs, IGs, and VCGs in self-gravitating media were all demonstrated in Yuen & Lazarian (2017b), showing that the relative rotations between different types of gradient vectors are informative of the stage of collapse of a piece of self-gravitating cloud.

1 The idea behind the studies employing PCA is the same, and, in fact, our study in Correia et al. (2016) shows that there is no practical advantage in using PCA compared to velocity centroids. On the contrary, the anisotropies of centroids, unlike the PCA eigenimages, are analytically related to the properties of the underlying turbulence, i.e., to the properties of the Alfvén, slow, and fast modes that constitute the MHD cascade (see Kandel et al. 2017b). This opens up prospects of separating the contributions of the compressible (slow and fast) and incompressible (Alfvén) modes using velocity centroids.

2 IGs have been shown to be inferior to VCGs in their ability to trace the magnetic field (GL17, YL17a, YL17b), but IGs and VCGs are synergistic in terms of studying shocks and regions dominated by self-gravity.
Velocity centroid is a way of representing ISM velocities in observations, and this has its limitations. For example, centroids reflect the contributions arising from both the velocity and density fluctuations along the line of sight (see EL05). However, the density fluctuations are not as well aligned with the magnetic fields as the velocity (see Cho & Lazarian 2003; Beresnyak et al. 2005; Kowal et al. 2007, or the comparison of VCGs to IGs in YL17b). At the same time, the analytical study in Lazarian & Pogosyan (2000, 2004) revealed that channel maps are sensitive only to velocity fluctuations if the corresponding turbulent density fluctuations are dominated by large-scale contributions. In addition, studies of different velocity channels in some cases may allow separate contributions coming from spatially different regions along the line of sight, and therefore enabling one to study the 3D structure of the magnetic field as well as other effects that the velocity gradients are sensitive to. This suggests that in a number of cases, velocity channel gradients (VChGs) may have advantages compared to VCGs. This motivates our present study to explore the ability of VChGs in tracing the orientations of the magnetic field.

In some aspects, VChGs have similarities to the technique based on the study of filaments in the velocity H I channel maps in Clark et al. (2015), where these filaments were shown to be correlated with magnetic field directions as revealed by Planck polarimetry. On the basis of Lazarian & Pogosyan (2000), one can conclude that the filaments observed by Clark et al. (2015) in thin channel maps can be identified with caustics caused by velocity crowding. The relationship between VChGs and the underlying velocity gradients is more straightforward than that for the filaments, and therefore, we expect a better correlation between the VChGs and the magnetic field than with the filaments. Furthermore, the gradient technique has shown its capability to identify shock and self-gravitating regions (YL17b), while no similar conclusion has ever been reported using filaments. Whether or not there can be any synergistic use of VChGs and tracing channel filaments simultaneously should be answered by further additional studies.

In what follows, we discuss in Section 2 the theoretical motivation of this work. Section 3 discusses the numerical methods in the simulations and the ways of doing analysis. We explore the gradients from turbulent velocities in the channel maps in Section 4. We examine the performance of channel map gradients and correlation anisotropies in Section 6. In Section 7, we compare the performance of density fluctuations and velocities from channel maps. We explore the reduced centroid gradient in Section 8. We test our method with observational data in Section 9. We discuss our result in Section 11, and make our conclusion in Section 12.

2. Theoretical Motivation and Expectations

2.1. Anisotropy of MHD Turbulence: Illustration

MHD turbulence theory is an old subject that has been boosted recently by the ability to perform high-resolution 3D numerical simulations. Before that, there was no way of testing theoretical constructions, and many competing theories describing MHD turbulence were able to coexist. For instance, the original studies of Alfvénic turbulence by Iroshnikov (1964) and Kraichnan (1965) were based on a hypothetical model of isotropic MHD turbulence, while later studies (see Montgomery & Turner 1981; Matthaeus et al. 1983; Shebalin et al. 1983; Higdon 1984) demonstrated the anisotropic nature of the MHD cascade.

Figure 1 shows the visualization of our numerical simulations. In particular, the iso-contours of the velocity in sub-Alfvénic turbulence are shown. It is obvious that the velocity gradients are directed perpendicular to the magnetic field, and therefore, by studying the direction of velocity gradients, one can study the direction of the magnetic field. In what follows, we explain that, in fact, one is expected not only to be able to trace the mean magnetic field in sub-Alfvénic turbulence, but to also trace the magnetic field in its complexity in both sub-Alfvénic and super-Alfvénic turbulence.

2.2. Anisotropy of MHD Turbulence: Local Directions of the Magnetic Field

The modern theory of MHD turbulence originates from the prophetic work of Goldreich & Sridhar (1995, henceforth GS95). Originally given rather lukewarm acceptance by the MHD turbulence community, this theory nevertheless was supported by further theoretical and numerical studies (Lazarian & Vishniac 1999, henceforth LV99; Cho & Vishniac 2000; Cho et al. 2001; Lithwick & Goldreich 2001; Maron & Goldreich 2001; Cho & Lazarian 2002, 2003; Kowal & Lazarian 2010; see Brandenburg & Lazarian 2013 for a review) that extended the theory and provided its rigorous testing. Our present study is based on the modern understanding of the MHD turbulence cascade, and the statistical properties of MHD turbulence that are confirmed numerically.

In the sub-Alfvénic regime, i.e., for the injection velocity \( V_A \) being less than the Alfvén velocity \( V_A \), the Alfvén modes initially evolve by increasing the perpendicular wavenumber while keeping the parallel wavenumber the same (see LV99, Galtier et al. 2005). The increase of the perpendicular

\(^3\) We are talking about realistic 3D MHD turbulence theories. MHD turbulence in 2D is very different from the one in 3D (see Eyink et al. 2011).

\(^4\) Produced using VisIt 2.8.1: https://wci.lnl.gov/simulation/computer-codes/visit.
wavenumber makes the Alfvénic wave vectors more and more perpendicular to magnetic field. Therefore, the gradients of velocity structures tend to be perpendicular to the magnetic field direction. Eventually, at a scale $l_{\text{trans}} \approx L M_A^2$, where $L$ is the turbulence injection scale and $M_A = V_L / V_A$ is the Alfvén Mach number (see LV99, Lazarian 2006), the parallel scale of the eddies starts changing, signifying the start of the GS95 regime. In the GS95 cascade, both parallel and perpendicular wavenumbers of Alfvénic perturbations increase, but the eddies get more and more elongated. To quantify this, one should adopt the system of reference aligned with the local magnetic field. In such a system of reference, the fluid motions perpendicular to magnetic fields are not constrained by magnetic tension. This is the consequence of the fast turbulent reconnection that was shown to change the topology of the interacting magnetic flux tubes within the eddy turnover time (LV99). Therefore, it is not surprising that the turbulent energy is channeled along this path of least resistance. It is not surprising either that in the absence of magnetic resistance to the mixing motions, the perpendicular eddies evolve along the Kolmogorov cascade with the Kolmogorov scaling $v_1 \sim l^{-3/5}$. However, the motions mixing the magnetic field in the perpendicular direction also induce wave propagation along the magnetic field. The period of these waves, $l_\perp / V_A$, is equal to the period of the magnetized plasma mixing in the perpendicular direction. $l_\perp / V_1$. The equality of the two timescales is usually referred to as the critical balance, and the scales $l_\parallel$ and $l_\perp$ are associated with the parallel and perpendicular scales of the eddies. The relation between these scales for sub-Alfvénic turbulence is (LV99)

$$l_\parallel \approx L \left( \frac{l}{L} \right)^{2/3} M_A^{-4/3},$$

which testifies that, for $l_{\text{box}}$ much smaller than the injection scale $L$, the eddies are strongly elongated. Note that these eddies are aligned with the local magnetic field. For trans-Alfvénic turbulence, $M_A = 1$, and Equation (1) provides the original GS95 scaling. Note that the alignment of the motions with respect to the local system of reference is an essential part of MHD turbulence as we understand it now. It is easy to see that first, due to this peculiar property of MHD turbulence, the velocity gradients are expected to be aligned with the local magnetic field. It is easy to show that the gradients of the smallest eddies produce the strongest signal. From Equation (1), we expect that the gradient of the aligned eddy mixing up the local magnetic field lines is proportional to $v_1 / l_\parallel \sim l_{\text{box}}^{-2/5}$. The smallest eddies are the eddies at the resolution scale of the telescope.

2.3. Intensity Fluctuations in Thin Channel Maps: Effect of Velocity Fluctuations

While velocity gradients are not directly available from astrophysical observations of diffuse media, a number of measures can be constructed using observational data. In GL17, we explored Velocity Centroid Gradients (VCGs) of the first order and considered the higher order VCGs in Yuen & Lazarian (2017a). In this paper, we prove another way to probe turbulent velocities, namely, using velocity channel maps that can be constructed using spectroscopic observations of Doppler-shifted lines. The statistics of these maps has been described in Lazarian & Pogosyan (2000, henceforth LP00) for optically thin data and in Lazarian & Pogosyan (2004) for observations in the presence of absorption. In what follows, we concentrate on the optically thin case and only mention some of the possible effects of optically thick data. Note that when we discuss thin and thick velocity slices, we do not mean the effects of absorption but the thickness of the channel maps. The minimum thickness of the latter is determined by the spectral resolution, $\Delta v$, of the instrument, and it can be increased by integrating the spectroscopic data over larger $\Delta v$. An important prediction in LP00 is that velocity caustics create fluctuations of intensity in the channel maps, and the relative importance of the velocity and density fluctuations changes with the thickness of the channel maps. In particular, LP00 identified a regime of “thin velocity slices” and found that in this regime, the intensity fluctuations in the slice are dominated by the velocity fluctuations, provided that the three-dimensional density spectrum is steep, i.e., most of the energy is concentrated at the large scales. In what follows, we use the terms “velocity slices” and “channel maps” interchangeably.

The aforementioned statement about the steep density spectrum can be expressed in terms of the 3D power spectrum

$\text{1}\text{.}\text{ }$In the works that followed the groundbreaking GS95 paper (namely, LV99; Cho & Vishniac 2000; Maron & Goldreich 2001; Cho & Lazarian 2002), it was shown that to describe MHD turbulence, one should use the local magnetic field rather than the mean magnetic field.

$\text{2}\text{.}\text{ }$If we are talking about the projected density with the Kolmogorov power spectrum, then the power spectrum in 2D would be $K^{-11/3}$, where $K$ is a two-dimensional wavenumber. The structure function for such a spectrum scales as $\sim l^{11/3}$ and the gradient would scale as $\sim l^{-11/3}$, still giving preference to small scales. However, according to Lazarian & Pogosyan (2000), the fluctuations within thin velocity channels have a modified spectrum, $K^{-3m/2}$, where $m$ is the spectral index of the velocity structure function. For the Kolmogorov turbulence, $m = 2/3$, which gives the spectrum of $K^{-8/3}$. The structure function of the intensity in the thin velocity channels scales as $\sim l^{11/3}$, which produces gradients that scale as $\sim l^{-8/3}$, definitely zooming into the physics of the small-scale fluctuations aligned with magnetic fields.

$\text{3}\text{.}\text{ }$This is not true for the relativistic MHD turbulence where the coupling between fast and Alfvénic fundamental modes can be significant (Takamoto & Lazarian 2016).
P(k). In terms of P(k), the Kolmogorov cascade corresponds to \( k^{-11/3} \), and it is steep. The borderline spectrum is \( k^{-5} \), with the turbulence having spectrum \( k^{-4} \), \( \alpha < 3 \) containing more energy at the small scale and therefore being shallow. For subsonic flows, the density spectra in MHD turbulence are steep (see Kowal et al. 2007).\(^8\) Thus, for such flows, the intensity fluctuations in thin velocity slices of position–position–velocity (PPV) data are influenced only by the turbulent velocity statistics. Naturally, thermal broadening interferes with the minimum slice of the PPV for which the fluctuations can be studied. This means that to study thin slices of subsonic flows, one should use species heavier than those of the main hydrogen astrophysical flow. On the other hand, if the density spectrum is shallow, i.e., \( \alpha < 3 \), the contribution of density and velocities to the statistics of the intensity fluctuations of the velocity slices was evaluated in LP00. For thin slices, both velocity and density are important, while the contribution of velocity decreases as the slice thickness increases. For instance, when the integration is performed over the entire line, only density fluctuations determine the fluctuations of the resulting intensity distribution. In fact, no matter how steep the spectrum is, the velocity contribution can be enhanced by reducing the channel width, which significantly simplifies the interpretation of the VChGs’ directions in terms of magnetic field tracing.

In view of the above, it is important to perform the study of the gradients of the intensities within PPV slices. In terms of separating compressible and incompressible components in (Kandel et al. 2016, hereafter KLP16), such a study provides an additional test that the anisotropies are actually caused by turbulence. By varying the thickness of the slice, one can study the variation of gradients as the relative contribution of density and velocity changes. This may be important as the density and velocity, in general, have different statistics, and the fluctuations of these fields are aligned with the magnetic field to different degrees. For instance, the strongly supersonic flows demonstrate an isotropic spectrum of density. In addition, for studies of galactic flows, the regular galactic shear opens up a way to study the different turbulent regions separately, and therefore, different channel maps can be associated with different locations within the galactic volume. In addition, velocity and density gradients can behave differently, e.g., in self-gravitating regions and shocks (see YL17b). In such cases, comparing the alignments of VChGs from different channel widths opens a new way of locating self-gravitating media, regardless of the steepness of the density spectrum.

In what follows, the advancements in understanding the theory of PPV anisotropies in KLP16 guide us in studying the gradients in the PPV velocity slices. Note that the criterion for the velocity slice being thin or thick as given in LP00 is as follows: the velocity slice is thin if the square root of the turbulent velocity dispersion on the scales that the slice is being studied, \( \sqrt{\langle \delta v^2 \rangle} \), is greater than the thickness of the slice \( \Delta v \), i.e.,

\[
\sqrt{\langle \delta v^2 \rangle} > \Delta v,
\]

(2)

where \( R \) is the separation of the correlating points over the plane of the sky, i.e., the PP separation. In the following, we distinguish thin and thick slices depending on whether the ratio \( \Delta v / \sqrt{\langle \delta v^2 \rangle} \) is smaller or larger than unity. In observations, the thickness \( \Delta v \) can be constrained either by the velocity resolution of the instrument or by the thermal line width \( \sigma_{th} \) that, according to LP00 acts similarly. Therefore, to make sure that the gradients are measured in the thin-slice regime, one should make sure that the scale over which the velocity gradients are calculated is sufficiently large.\(^9\) For an individual turbulent volume, \( \delta v_{th} \) can be measured using the dispersion of the structure functions.

We note that if \( \sqrt{\langle \delta v^2 \rangle} < \sigma_{th} \), then it is advantageous to use velocity centroids that can represent velocity statistics for subsonic turbulence (see Esquivel & Lazarian 2005; Kandel et al. 2017a). The traditional centroids, however, have disadvantages compared to channels, e.g., one cannot isolate a particular region of the galaxy according to the rotation curve. To address this problem in the paper, we discuss the use of a new construction that we term “reduced velocity centroids,” which make use of part of the line only. Gradients of the reduced velocity centroids and the VChGs can be used together, with the VChGs applied to scales \( R \) for which the criterion given by Equation (2) is valid and with the reduced velocity centroid gradients applied to smaller scales.

This all suggest that the scale of the data blocks for calculating the gradients is another important parameter for calculating the gradients. In YL17a, when we dealt with the velocity centroids, we showed that the uncertainty of the calculation of the velocity centroid gradients (VCGs) decreases with the increase of the data block size. This was augmented in YL17b by the finding that there was an optimal size for the VCGs, which provided maximal resolution without compromising accuracy. Below we prove that this property is also a part of the VChG technique.

3. Numerical Simulations

3.1. MHD Turbulence Simulations and Mode Decomposition

The numerical data are obtained by 3D MHD simulations using the single-fluid, operator-split, staggered-grid MHD Eulerian code ZEUS-MP/3D (Hayes et al. 2006) to set up a 3D, uniform, isothermal turbulent medium. Periodic boundary conditions are applied to emulate a part of the interstellar cloud. Solenoidal turbulence injections are employed. Our simulations employ various Alfvénic Mach numbers \( M_A = V_L / V_A \) and sonic Mach numbers \( M_s = V_L / V_s \), where \( V_L \) is the injection velocity, while \( V_A \) and \( V_s \) are the Alfvén and sonic velocities, respectively, which are listed in Table 1. The domain \( M_s < M \) corresponds to the simulations of plasma with magnetic pressure larger than the thermal pressure, i.e., plasma with low \( \beta = 2 V^2_L / V^2_A < 1 \), while the domain \( M_s > M \) corresponds to the pressure-dominated plasma with \( \beta > 1 \). Further, we refer to the simulations in the table by their model name. For instance, our figures will have the model name indicating which data cube was used to plot the figure. The simulations are named with respect to a variation of \( M_s \) and \( M_A \).

\(^8\) The traditionally used definition of the Kolmogorov spectrum is obtained via spatial integration of \( P(k) \) in \( k \)-space, which is equivalent to multiplying the spectrum by \( k^2 \). Thus, the usual value literature quoted for (shell-integrated) Kolmogorov spectrum is \( k^{-5/3} \), and the borderline spectrum between shallow and steep is \( k^{-1} \).

\(^9\) The LP00 study shows that velocity fluctuations are still important for slice thicknesses larger than that given by Equation (2). However, their relative contribution compared to density is gradually decreasing. Incidentally, this regime was termed in LP00 the “thick slice regime”. In the present paper, we do not use this terminology and refer to “thick slice” only in situations where the velocity information is integrated out.
in ascending values of $\beta$. The ranges of $M_s$, $M_A$, $\beta$ are selected so that they cover different possible scenarios of astrophysical turbulence from very subsonic to supersonic cases. In this study, we devoted much of our analysis to the sub- and trans-Alfvénic cases only, and postpone the discussion on super-Alfvénic simulations to our next paper (K. H. Yuen & A. Lazarian, in preparation). We expect the velocity gradients to successfully trace the magnetic field in super-Alfvénic turbulence after the appropriate filtering of low-frequency spatial modes. The practical difficulty of such a study is that the inertial range of the MHD turbulence is rapidly shrinking with the increase of $M_A$. If for $M_A > 1$ the injection scale is $L$, the transition to the MHD regime happens at the scale $l_* \approx L M_A^{-3}$ (see Lazarian 2006), which requires very large cubes to study the velocity gradients arising from MHD turbulence.

The numerical simulations that we employ for the study are listed in Table 1. The names of the simulations reflect both the sonic and Alfvén Mach numbers. For instance, $M_{0.4}M_{0.04}$ corresponds to $M_s = 0.4$ and $M_A = 0.04$.

To investigate the detailed structure of gradients from different wave modes, we employ the wave mode decomposition method in Cho & Lazarian (2002, 2003, hereafter CL02 and CL03, respectively) to extract Alfvén, slow, and fast modes from velocity data. The corresponding equations determining the basis for the decomposition into modes are

$$\hat{\zeta}_f \propto \left( \frac{1 + \beta}{2} + \sqrt{D} \right) \hat{k}_f \hat{k}_\perp + \left( -1 + \frac{\beta}{2} + \sqrt{D} \right) \hat{k}_f \hat{k}_l, \quad (3a)$$

$$\hat{\zeta}_s \propto \left( 1 + \frac{\beta}{2} - \sqrt{D} \right) \hat{k}_s \hat{k}_\perp + \left( -1 + \frac{\beta}{2} - \sqrt{D} \right) \hat{k}_s \hat{k}_l, \quad (3b)$$

$$\hat{\zeta}_a \propto -\hat{k}_a \times \hat{k}_l, \quad (3c)$$

where $D = (1 + \beta/2)^2 - 2\beta \cos^2 \theta$, $\beta = \frac{\rho}{\rho_s} = \frac{2M_A^2}{M_s^2}$, and $\cos \theta = \hat{k}_|| \cdot \hat{B}$. We only used the LOS component of the decomposed velocities for velocity channel calculations. That is to say, the three velocity modes can then be acquired from

$$v_{(f,s,a),z} = [\mathcal{F}^{-1}(\mathcal{F}(v) \cdot \hat{\zeta}_{f,s,a})](\hat{\zeta}_{f,s,a} \cdot \hat{\zeta}_{\text{LOS}}), \quad (4)$$

where $\mathcal{F}$ is the Fourier transform operator.

The upper panel of Figure 2 illustrates the decomposition procedure that takes place in Fourier space. The resulting three data cubes are dominated by the Alfvén, slow, and fast modes. In the middle and lower panels of Figure 2, we show the results of the decomposed velocity cube, which is projected along the $x$-axis. We illustrate the decomposition method using two cubes with low and high $\beta$. The properties of the fast and slow modes differ in low- and high-$\beta$ plasmas; therefore, we study these two cases separately in the following sections.

### 3.2. Calculations of Gradients in Channel Maps

Gradients were calculated following the procedures described in YL17a, including the gradient calculation method and sub-block averaging. In short, the gist of the sub-block averaging method is to define the most probable direction within a block by finding a local Gaussian-fitting peak for the distribution of gradients. The fitting error provides a quantitative estimate of whether the block size is large enough to reliably determine the direction.

The concept of thin channel maps was introduced in LP00, where it was shown that the fluctuations within thin channel maps are mostly determined by velocity fluctuations. To determine whether the gradients are probing thin or thick channel maps, we use the following criterion: the channel is thin if the gradient is calculated over the scale of size $R$ for which the criterion given by Equation (2) is satisfied; otherwise, the channel is thick. To study velocity gradients over a scale larger than $R$, we construct the velocity channel map:

$$C(x, y) = \int dv \rho_{pp}(x, y, v) e^{-\frac{v_{||}^2 - v_0^2}{2}}.$$ (5)

As the thickness $v_0$ should not be less than the maximum of the thermal line width and spectrometer resolution, it means for images with high resolution, the gradients at the smallest scales should be calculated using velocity centroids.

Readers should be reminded that the channel map carries intensity information within a velocity channel. The reference velocity slice is convenient to define at the center of the spectral line. However, if the spectral line is broadened by galactic shear, as is the case with Galactic atomic hydrogen, different velocity channels carry information about the 3D distribution of turbulence, and therefore, it is advantageous to study gradients in different bunches of channels in order to obtain the 3D distribution of magnetic field in the galactic disk. Similarly, in the presence of absorption, the wings of the lines, rather than entire absorption lines, are advantageous to use. The latter are saturated and thus not informative at the center of the line.

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10 The proportional constant for $\cos \theta$ is correct only in CL02, i.e., $\cos^2 \theta$.

11 This procedure is very different from that employed for gradient calculation by other authors, e.g., Soler et al. (2013). It is very advantageous to apply this procedure for calculating other types of gradients.
To produce channel maps that contain only spatial frequencies for which the slice is thin, we provide filtering of the high spatial frequencies for which the criterion given by Equation (2) is not satisfied.

3.3. Uncertainties of Magnetic Field Tracing

Unlike the traditional technique of calculating gradients (see Soler et al. 2013), our approach provides us with the uncertainty of the determination of the magnetic field direction. To test how well this works, in Figure 3 we show the error estimate for the VChG in the cube \( M_1 \cdot M_0 / 0.53 \) that we calculate using our technique. As we fit the Gaussian into the distribution of the gradient directions, we get an estimate of the fitting error, which is the lowest curve in Figure 3. The half width of the Gaussian provides a significantly larger uncertainty shown by the black curve in Figure 3. In numerical simulations, we also can calculate the difference between the projected magnetic field and the measured gradient directions. This measurement is given by the black curve in Figure 3. These measures that are changing are the function of the block size, and we can observe that our procedure of evaluating our error for magnetic field tracing is in reasonable correspondence with the actual measurements of the differences of the projected magnetic field direction and the direction given by the velocity gradients.

4. Alignment Measure and Density Effects for the VChGs

Before showing the gradient maps, we would like to illustrate the difference between supersonic and subsonic simulation in terms of density spectra. Figure 4 shows the normalized density spectra from the two simulations, one supersonic and the other one subsonic. The normalization is done by making the amplitudes of the spectra the same at the injection. The subsonic spectrum of the density in strongly magnetized media scales as the pressure and therefore follows not the Kolmogorov \( k^{-5/3} \) law, but a steeper \( k^{-2/3} \) law (see Kowal et al. 2007). For this definition of spectra, the borderline between the shallow and the steep spectra corresponds to \( k^{-1} \). It is very clear that the high \( M_j \) spectra is shallow compared to the steep density spectra in subsonic systems. Our results on the density studies agree well with those in Beresnyak et al. (2005) and Kowal et al. (2007).

As the velocities, unlike the densities, are directly related to MHD turbulence, we first consider only intensity fluctuations that arise only from turbulent velocity. For this purpose we create data cubes using the velocity field obtained from our 3D numerical simulations but substitute the actual densities with a constant density, which is just the number density channel map constructed above. Figure 5 illustrates the relative orientation of the velocity channel gradients (VChGs) and the projected magnetic field on a thin slice setting. The gradients are all rotated by 90\(^\circ\), which will be annotated as rotated gradients. The rotated gradients according to the theoretical considerations above correspond to the magnetic field directions. In the following sections, gradients are assumed to be rotated, unless emphasized otherwise. One can see from Figure 5 that both number density and velocity channel maps behave very well in terms of gradient alignment. This gives us confidence in using the velocity channel maps in our later analysis.

The fact that the VChGs for thin maps are strongly influenced by the velocity field suggests that the tracing of magnetic fields using thin channel maps may be more accurate due to velocity being a better tracer of MHD turbulence. Below, we consider the gradients arising from the Alfvén, slow, and fast modes separately. The theoretical considerations guiding this study are provided in KLP16, where the anisotropies of the correlation of the intensities arising from velocities were studied.

We used the AM to express quantitatively how well a certain type of gradient vector traces the magnetic field. The AM is defined by the expression\(^{12}\):

\[
AM = (2 \cos^2 \theta - 1),
\]

and it ranges from \(-1\) to 1. \( \theta \) in the equation stands for the angle between the VChG and the projected magnetic field. The physical meaning of AM is as follows: if AM \( \sim 1 \), then the average angular difference in the directions of two vectors across the map is negligibly small; if AM \( \sim 0 \), then there is no alignment between the two vectors. If the two directions tend to be perpendicular to each other, then AM \( \sim -1 \).

5. Applying VChGs to Basic MHD Modes

It is important to understand the effect of different modes on the VChGs. The composition of MHD turbulence in terms of the basic modes changes with \( M_A \) and the plasma \( \beta \) (see Cho & Lazarian 2003).

5.1. Alfvén Modes

Most important for the MHD turbulence are Alfvén modes (see CL03). We will first demonstrate some properties of VChGs with these modes.

The dynamics of Alfvénic modes in strong MHD turbulence is very different from the dynamics of freely propagating Alfvén waves. The modes cascade on the scale of the order of one period, with the wave vector of the Alfvénic perturbations in strong turbulence being nearly perpendicular to the local direction of the magnetic field. As a result, the anisotropy and the iso-contours of intensity correlation are both elongated parallel to the magnetic field. The latter is essential for tracing magnetic fields, as the gradients that these modes are inducing are also perpendicular to the local direction of the magnetic field.

For Alfvén modes, the theoretical expectation is very natural: the anisotropy of the intensity correlations in a thin slice increases with the decrease of the Alfvén Mach number, i.e., the weaker the velocity perturbations, the more turbulent motions are dominated by the magnetic field, which causes the eddies to be more anisotropic. Figure 6 illustrates the channel maps for low and high \( \beta \), and their gradients within a thin slice that are induced by the Alfvénic modes. According to Cho & Lazarian (2003), the change of the Alfvén modes with \( M_s \) is marginal. The corresponding change in terms of gradients is shown in the left panel of Figure 7.

As we increase the thickness of the channel maps, the AM of the Alfvén modes is decreasing, which is shown by the blue curves in Figure 8 for both low- and high-\( \beta \) cases. This is because the contribution from velocities decreases as the the channel width increases. However, we observe that the change

\(^{12}\) This 2D measure is analogous to the 3D measure that is used in the theory of grain alignment (see Lazarian 2007 for a review).
is gradual (see also the theory in LP00) rather than a sharp jump, which indicates that even thin slices satisfying Equation (2) may not be available for some data; the selection of slices as thin as possible can already help enhance the contribution from velocity information in observation data.

We can double check the above statement by testing the gradient variation dependence on the channel width. In this experiment, we use the density-weighted PPV cubes to illustrate the relative dominance of velocity over density fluctuations when one changes the channel width, as illustrated in Figure 9. When we increase the channel width, the VChG variations, given by $\sqrt{\langle \xi_{\text{VChG}}^2 \rangle / \langle \xi_{\text{VChG}} \rangle}$, are decreasing. Due to this decrease in gradient deviations, the role of velocity fluctuations in the channel map fluctuations is decreasing, and the thick

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**Figure 2.** (Top) Illustration of the decomposition method. From Cho & Lazarian (2003). (Middle and bottom) The resultant projected maps from the CL03 decomposition from two sample cubes with low (middle panel) and high (low panel) $\beta$. The Astrophysical Journal, 853:96 (23pp), 2018 January 20 Lazarian & Yuen
channels get dominated by density fluctuations. A more detailed discussion on the contribution of the density effect will be given in Section 7.

One should remember, however, that whether the channels are thin or thick depends on the size of the eddies that are being studied (LP00). In our gradient studies, the block size is an additional factor limiting the size of the eddies that we can study, which usually have a larger velocity dispersion than a small block of sampled channel maps. The verification of the thin-slice criterion (see Equation (2)) has to be done every time in the case of small blocks.

According to LP00, the thermal line width acts similarly to the channel thickness. Here we used the data of the cube Ms1.6Ma0.53 by adding the additional thermal dispersion of the velocities before constructing the PPV cube. Figure 10 shows the relationship between the AM versus the thermal width ratio over a channel width of $\delta v = 0.2 \sqrt{\delta v^2}$ and the AM. The decreasing trend resulting from an increase in the thermal width is similar to the effect of increasing the channel width illustrated in Figure 8. Moreover, when the ratio between the thermal width and the channel width is larger than 1, the thermal velocities dominate over the contribution from turbulent motions. Different from the effect from increasing channel width, the increase of the thermal line width retains the velocity information yet washes away the fluctuations. This explains why the AM oscillates around zero once the noise

Figure 3. Average error estimate from the Gaussian-fitting function (Stderr), the angular differences between the magnetic field directions from polarization and VChG orientation ($\phi_{pol} - \phi_{VChG}$), and the dispersion of VChGs ($\delta \phi_{VChG}$) for the velocity centroid map of cube $M_s1.6M_a0.53$ plotted against the change in block size.

Figure 4. 3D density spectrum from the supersonic (blue, b15) and subsonic (red, b11) runs. While the subsonic spectrum has a steep slope, the supersonic one has a shallow one. The straight line shows the theoretically expected $-7/3$ density slope in subsonic strongly magnetized turbulence (see Kowal & Lazarian 2010).

Figure 5. Intensity gradients (left panel, green) and VChGs (right, red) from a thin-slice map with respect to the projected magnetic field (blue). The maps prior to the CL03 decomposition into modes are used.

Figure 6. Velocity channel map for low (left) and high (right) $\beta$ from Alfvén modes in a thin-slice map. The rotated 90° gradients (red for VChGs, green for IGs) are overplotted over the projected magnetic field (blue).

Figure 7. Change in the AM of the Alfvén mode VChGs with respect to $M_s$.
width is larger than the channel width. In spite of this, such an effect would not prevent the application of our technique in most regions of the sky, which is mostly supersonic. This effect also sets a lower bound for observers to select the channel width when applying the method.

5.2. Slow Modes

Slow waves present perturbations that propagate along magnetic field lines. In the limit of incompressible media, slow waves are pure magnetic compressions that propagate along magnetic field lines. Formally, the incompressible case corresponds to $\beta = \infty$, and in this limit, the slow modes are frequently called pseudo-Alfvén modes. By contrast, for $\beta \ll 1$, the slow waves are density perturbations propagating along magnetic field lines.

In the presence of Alfvénic turbulence, slow modes do not evolve on their own, but are sheared by Alfvén modes. As a result, the features of Alfvénic turbulence, e.g., spectrum and anisotropies, are imprinted on the slow modes (see GS95, Lithwick & Goldreich 2001; Cho & Lazarian 2002, 2003). This also means that the perpendicular velocity gradients characteristic of Alfvén modes are also inherited by the slow modes.

Figure 11 illustrates the channel maps for a thin slice for the case of low $\beta$ (left panel) and high $\beta$ (right panel). One can see that low-$\beta$ VChGs perform better, which is also shown in Figure 8. In fact, when $\beta$ is small, the gradients of the corresponding channel maps are similar to those of the Alfvén mode channel maps. On the other hand, the slow-mode channel maps from high-$\beta$ systems are not as highly structured as the maps with low $\beta$. Moreover, the AM of slow modes from both cases decreases faster than that of Alfvén modes as channel width increases. In reality, both modes contribute to the gradients of the channel maps. The result in Figure 8 suggests that as the velocity channel thickness increases, the AM decreases. The infinite channel thickness corresponds to using total intensities. It is obvious that in terms of magnetic field tracing, the intensities (thick channels) are inferior to the thin channels.
5.3. Fast Modes

Similar to the properties of the slow modes, the properties of fast modes are different in low-$\beta$ and high-$\beta$ plasmas. For high-$\beta$ plasmas, the fast waves are similar to sound waves that propagate with sound speed irrespective of the magnetic field direction. Similar to acoustic turbulence, the corresponding turbulence is expected to be isotropic. In low-$\beta$ plasmas, the fast modes correspond to magnetic field compressions that propagate with Alfvén velocity. In terms of the correlation function of the anisotropy of fast modes, the elongation of the iso-contours of correlation is perpendicular to the magnetic field. Therefore, for fast-mode dominated environments, one should expect the gradients to be parallel instead of perpendicular to the local magnetic field. We note that the alignment of the velocity gradients from the actual MHD turbulence contain contributions from the three modes, with the fast modes decreasing the alignment induced by the Alfvén and slow modes.

Figure 12 shows the channel maps for low- and high-$\beta$ plasmas. The low-$\beta$ map carries more structures than the high-$\beta$ map, but when examining the contours in the maps, it is obvious that they are carrying different anisotropy directions. In the language of fast-mode gradients, the maximal gradient is parallel to the local magnetic field direction. Figure 13 illustrates the AM of fast-mode VChGs.

In contrast to the slow decreasing AM for rotated Alfvén and slow-mode VChGs, the AM of rotated fast-mode VChGs decreases more significantly when the channel width passes through unity for both low- and high-$\beta$ cases. This suggests that a decrease of channel width suppresses the unwanted contribution of fast modes in velocity gradient calculations, which makes thin channels preferable for tracing magnetic fields. The difference in the behavior of the gradients from fast mode on one side and the Alfvén and slow modes on the other side makes it possible to separate the contributions from these modes. This issue will be discussed elsewhere.

Numerical simulations (e.g., Cho & Lazarian 2002, 2003) indicate that fast modes are subdominant at least for the cases of incompressible driving of turbulence. Our study indicates that in terms of VChGs, one can expect a further suppression of the fast-mode contribution in thin channels.

6. Comparison with the Correlation Anisotropies within Channel Maps

It was demonstrated in Lazarian et al. (2002) that the correlation anisotropies in channel maps can trace magnetic fields. Further studies of this effect are provided in Esquivel & Lazarian (2005) and Esquivel et al. (2015). We believe that correlation anisotropies can be very informative in terms of determining the relative contribution of compressible versus incompressible modes (see Kandel et al. 2016, 2017b). However, in terms of tracing the detailed magnetic field structure, they are inferior to the velocity gradients. Indeed, for the correlation function to be informative, one requires a large number of sampling points to acquire enough statistics. As a result, one can expect the correlation anisotropy to require a lot more data points than the VChGs to trace the magnetic field vector. Incidentally, a similar effect was demonstrated in LYL17c for a test using the SIGs and in YL17c for a test using the VCGs. Here we test this statement for the VChGs.

In Figure 14, we show the VChG AM versus the AM of the directions of the maximal anisotropy (elongation) of the correlation functions of the channel map intensities. We clearly see that the channel map intensities are not good for detailed magnetic field tracing, which is consistent with our previous studies (see YL17a, LYL1, YL17b).

The relative efficiency of velocity gradients is easy to understand. The velocity gradients correspond to the angular velocities of turbulent eddies. These individual eddies are aligned with the local magnetic field. On the contrary, the correlation functions are the statistical measures that are well-defined only after averaging. Thus, averaging over small patches does not produce the good statistics necessary for the correlation functions to be well-defined. The shortcomings of the magnetic field tracing technique based on the correlation function anisotropies are also seen when the corresponding modifications of the technique are compared with the VCGs and SIGs (see LYL1, YL17b).

7. Relative Importance of Velocity versus Density Fluctuations, and Scale-dependent Gradients

Both velocities and densities contribute to the intensity fluctuations within channel maps, but their relative contribution changes with the change in the channel map thickness (LP00). To study the relative importance of density and velocity fluctuations, we use velocity and density data cubes obtained through our 3D MHD compressible simulations. The structures in these cubes are elongated in the same direction, which we take to be the $x$-direction. For our study, we create the PPV cubes of the synthetic spectral line by turning the density cube.
90° with respect to the velocity data cube, so that the elongated structures in the density cube become perpendicular to the magnetic field. Mathematically,

\[ C_r(y, z) = \int_{|v_z| < \Delta v} dx \rho(x, y, z) v_z(x, z, y). \] (7)

In simple words, we are creating synthetic maps by transposing the velocity and density data by 90° within the plane of sky, i.e., through the rotation along z-direction.

In the PPV cube created this way, the directions of anisotropy for the velocity and density are orthogonal, and the resulting anisotropy determines which contribution dominates. When examining the anisotropy using the correlation function \( R_2(r) = \langle C_r(x) C_r(x + r) \rangle \), in the centroid map, the ratio between the axis perpendicular (the elongation direction of \( C_r \)) to the magnetic field direction and that of the parallel one (the elongation direction of \( I \)) will reflect the relative importance of the velocity to the density fluctuations to the centroids measured this way.

Figure 15 shows the decreasing trend of the aforementioned axis ratio as the velocity slice thickness increases. It is evident that even though the density fluctuations can contaminate the contribution of the velocity fluctuations within the channel maps, the overall anisotropy comes from the velocities provided the channel map is sufficiently thin. Therefore, using a thin slice, one can reduce the contribution from density data (see Figure 15.)

We use the same set-up employing density and velocity cubes turned 90° to each other to explore the effects of channel thickness for the VChGs. We study the transposed channel map by picking the slices satisfying Equation (2) in a PPV cube constructed with transposed density. Figure 16 illustrates a thick and a thin slice result from such a construction. The thin slice uses the contribution of PPV cubes with a channel width \(~0.8 \) of \( \delta v \), while the thick slice is just the integration of the transposed density on the full spectral line. It is very obvious from visual inspection that thin-slice structures are parallel to the magnetic field in the velocity cube, while those in the thick slice reflect the direction of the magnetic field in the density cube. The latter is reflected in the AM being negative. In agreement with Figure 15, we see that the velocity information dominates over the density information when the velocity channel is thin.

A more quantitative result can be seen in Figure 17, which is a plot of the AM. When the channel width \( \Delta v / \sqrt{\delta v^2} \) is smaller than unity, velocity contribution is dominant and therefore the AM is positive. On the other hand, when the AM for thick channel width maps is negative, it indicates that the contribution from density is dominant. The point where \( AM = 0 \) corresponds to the channel width being unity.

8. Reduced Velocity Centroids Obtained from the Data

Our study above suggests that the decrease in channel width improves the alignment between the gradient direction and the magnetic field. However, as we discussed earlier, the thermal line width provides a lower limit on the effective thickness of the velocity channels. To study velocities on scales less than the thermal velocity broadening, one may invoke velocity centroids, the properties of which have been studied in, e.g.,
The potential disadvantage of traditional centroids is that the entire spectral line is used while in some cases different parts of the line reflect magnetic fields in spatially different regions. This is the case, for instance, for the H I measurements where the galactic rotation curve provides rough information about the location of the emitting material. In addition, if the center of the spectral line is saturated due to absorption effects, it is also good to use only the informative part of the line. Thus, it is important to explore whether velocity centroids can be made a more flexible tool that can be used to study only part of the spectral line. Below we experiment with such a measure.

Given the channel map data, we introduce a new measure, i.e., the Reduced Velocity Centroids, which contain only part of the line:

\[ C_{\Delta v, n}(x) = \int_{\Delta v} dv v^n n_{ppv}, \]  

where the index \( n \) determines the order of the centroid. Increasing \( n \) enhances the effects of velocity, but, in practice, it also increases the noise. Reduced Velocity Centroids are useful in studying both gradients from extended galactic disk data and also the data from the wings of the absorption lines. In this study, we explore how Reduced Velocity Centroid Gradients (RVCGs) trace the magnetic field. To study the new measure, we separate the line into three regions, namely, the central part, the middle part, and the wing. Figure 18 illustrates the \( n = 1 \) reduced centroid from a selected Alfvén mode velocity data cube. The central portion is very much the same as the thin-slice result, giving good alignment with respect to the magnetic field. The middle portion of the reduced centroid also shows a fair alignment, but the number of data points becomes limited due to the limitations of the numerical resolution and the increase in shot noise. Nevertheless, the gradient map after sub-block averaging still provides a nice fit to the magnetic field direction. The wing part also has a good alignment with the magnetic field. This actually tells us that one can use only part of the line to study magnetic fields. This can be the portion of the line broadened by galactic shear, e.g., the 21 cm line from the galactic plane hydrogen we discuss below. This provides us a way to study the 3D structure of magnetic fields and other phenomena (e.g., self-gravity and shocks) that are being traced by gradients. Using only wings potentially provides a way to study gradients using lines with high optical depth, e.g., the \(^{12}\)CO line. All of this should be discussed in detail in other publications; here we provide a preliminary study of this new measure.

In fact, in MHD turbulence, we expect the overall line profile to be determined by the largest eddies that produce most of the dispersion. The gradients are not expected to depend on this. Therefore, gradients in all three parts of the line reflect the alignment of the small-scale eddies with respect to the magnetic fields local to them.

\(^{13}\) This can also be a part of the line corresponding to clouds at velocities formally forbidden by the galactic rotation curve (see Stil et al. 2006), but arising from the turbulent stochastic nature of the velocity distribution in the galactic disk. High-velocity clouds also present a case where RVCGs can be useful.

\(^{14}\) Naturally, the mapping from velocity space to real space is distorted by turbulent velocities and therefore is accurate up to the turbulent velocity dispersion.
9. Comparison with the Observational Data

9.1. Application of VChGs and RVCGs

For observations of galactic \HI, the use of different portions of the line is no longer just a test of our theoretical concepts. Due to the galactic rotation, different parts of the galactic \HI have different velocities with respect to the observer. Therefore, one can trace the magnetic field directions using the VChGs and the RVCGs. We illustrate our techniques using three regions consisting of both diffuse \HI regions and also \(^{13}\)CO data from the self-gravitating molecular cloud Vela C (Fissel et al. 2016). The \HI region we compare with is from Clark et al. (2015) for studies of filamentary structures in velocity channel spaces coming from the Galactic Arecibo L-band Feed Array \HI (GALFA-H\textsubscript{I}) survey. GALFA-H\textsubscript{I} is a survey of the Galaxy in the 21 cm neutral hydrogen line. The data are obtained with the Arecibo Observatory 305 m telescope. With the large aperture of the telescope, one obtains the angular resolution \(\sim 4\text{'}\), which is similar to \textit{Planck}’s best resolution \(\sim 5\text{'}\). For Vela C, we compare our calculation with that from the Balloon-borne Large Aperture Submillimeter Telescope for Polarimetry (BLASTPol; Galitzki 2014; Fissel et al. 2016), which is a 1.8 m Cassegrain interferometric telescope detecting linearly submillimeter dust polarization.

The regions we selected here are across 209.7°–196.6°, and declination (decl.) across 22°5–35°3 for \HI and \(l \sim 264.75–265.60\) and \(b \sim 1.2–11.8\) for Vela C. For the spectroscopic \HI data, we stick with the channel with velocity ranges 0–2.944 km s\(^{-1}\), where the velocity spectral line is peaked. As a comparison, the dispersion of velocity across the whole map is 10.3 km s\(^{-1}\), which means our selection of velocity channel is both thin (with a relative channel thickness of \(\sim 0.3\)) and most representative compared to other channels. The analysis of Vela C is given in K. H. Yuen et al. (2018, in preparation). Here we reproduce one figure from that study to illustrate the ability of VChGs to trace the magnetic field in molecular clouds. Due to the thinner channels provided in the \(^{13}\)CO Vela C data (0.183 km s\(^{-1}\)), we use the relative channel thickness of \(\sim 0.5\) for Vela C, which is about 1 km s\(^{-1}\). We plot our VChGs for \HI against the \textit{Planck} 353 Ghz polarization in the respective regions by rotating both of them by 90°. For Vela C, as it is a self-gravitating region, we keep the center part of VChGs unrotated while rotating the boundary by 90° (see YL17b for the criteria and the method of determining the gradient rotation density threshold) by estimating the volume density around the core region. We vary our block size and Gaussian kernel width (see Figure 19) to acquire the optimal value ensuring the best combination of the alignment and the resolution. For the case of \HI, the Gaussian filters have their respective gradient errors converge at a block size of 100. For Vela C, there is no such convergence plot; therefore, we pick the lowest error point from the smallest Gaussian filter to retain the data quality. The result is shown in Figure 20 using the line integral convolution method (LIC; Cabral & Leedom 1993). We find that the AM is in general pretty good for the two regions.

Note that by taking the thickness of the channel map of the order of the velocity dispersion, one produces intensity-dominated channel maps, which provide a rough structure of the column densities with the grid size of the order of the large-scale turbulent eddies. The gradient study of such a map is equivalent to the study using IGs. Therefore, the corresponding channel map gradients can be associated with the IGs that trace the combination of effects arising from the magnetic field and shocks, as we discussed in YL17b. Combining this with the magnetic field information, e.g., obtained with the VChGs, is very synergistic for studying interstellar shocks propagating through the galactic disk (see Section 11.3 for a complete discussion).

10. Additional Effects and Prospects for Research

The technique of studying magnetic fields with velocity gradients shows many promising directions for further studies. We outline a few directions without getting into much detail within this publication.

10.1. Effect of Self-gravity

In the presence of self-gravity (see Table 2 for a summary of the directions of different types of gradients in various physical regimes), we expect the gradients to change their alignment in relation to the magnetic field. Naturally, if this happens over the block over which the calculation of the gradients is performed, this will result in a higher than average uncertainty in determining the direction of the gradient. This is what we observe in our simulations, as illustrated in
Figure 21, which shows how the raw gradients (i.e., gradients before block averaging) react when they come close to the self-gravitating core. Instead, we calculate the dispersion of raw gradients within a block of selected size, and move the block from the left of the core to the right. From Figure 21, we clearly see, as we come closer to the gravitational center (core center), that the dispersion of gradients significantly increases. We also observe that the increase of dispersion is smoothed out and the peak of the dispersion deviates from the core center as we are using a bigger block size. However, the average level of dispersion around the core region (about ~1.2 radians) is still significantly higher than that of the diffuse region, which is ~0.8 radians.

In addition, our earlier work, in particular YL17b, identified the synergistic way of using the VCGs and the IGs simultaneously in order to reveal both regions dominated by self-gravity and shocks and to trace the magnetic field in the presence of these phenomena. In particular, in the presence of self-gravity, both the VCGs and the IGs are changing their direction by 90°, thus becoming parallel to the magnetic field direction. However, the change in the direction of the IGs happens earlier than that of the VCGs, and this provides a way...
of, first of all, identifying regions dominated by self-gravity, and second, tracing magnetic fields within self-gravitating regions without any polarimetry data. Naturally, this effect is also present when we use VChGs. The thick velocity channels provide information about the IGs, while the thin velocity channels provide information about the velocities. Figure 22 illustrates the difference in the response of the gradients with time measured within the thin and thick channels. The time is measured in the simulations from the moment self-gravity is turned on. Due to this effect in observations, we expect the region where the gradients in the thick velocity channels are turned 90° with respect to the magnetic field to be more extended than the region over which the gradients in the thin velocity channels are turned 90° with respect to the magnetic field. Therefore, while at the regions far from the self-gravity center the gradients in the thin and thick velocity channels are going to be aligned, they will turn 90° with respect to each other at some distance from the self-gravity center; closer to the self-gravity center, they are expected to become aligned again. The detection of the relative orientation between thick and thin VChGs is a signature of a self-gravity effect that can be used to indicate when gradients rotated 90° trace magnetic fields.

The turning of gradients in thin velocity channels compared to thick velocity channels is also expected to happen within strong shocks. However, shocks can be distinguished from the self-gravity regions both through morphological differences and due to differences in the column densities. Thus, in most cases, one can trace magnetic fields and shocks and identify regions of gravitational collapse by combining the gradients within the thin and thick velocity channels.

10.2. Use of Interferometers and the Effect of Noise

We would also like to examine the effect of noise for both RVCGs and VChGs. After obtaining channel and reduced centroid maps from cube Ms1,6Ma0.53 at the relative channel width of 0.2, we add white noises as we did in Lazarian et al. (2017) on top of the maps, i.e., add white noises according to the root-mean-squared value of the map intensities. Figure 23 shows that the noise level is 0.1 of the map intensities. One may already observe from the map that there is already some dirt altering the alignment of the gradients to the projected magnetic field. We therefore add a Gaussian filter with width of 2 pixels according to Lazarian et al. (2017) to negate the effect of noise. Figure 24 shows the AM versus the amplitude of the noise for VChGs and RVCGs after the Gaussian filter is applied. The alignment remains fairly good until the ratio between the noise amplitude relative to the map intensities is close to 1, and significantly drops to zero when the ratio is around 3. This simple test illustrates the strength of VChGs and RVCGs in predicting alignments in strong noise environments with the aid of small-width Gaussian filters.

Our results also show that RVCGs and VChGs are comparable in their performance both when the turbulent broadening is larger and smaller than the turbulent line width. Revealing the relative advantages of these two techniques requires a more detailed study and is beyond the scope of the present paper.

In Figure 25, we show how the VChGs change in the presence of noise. We see that it is advantageous to remove spatial frequencies in order to decrease the noise aside from the Gaussian filter method. Incidentally, similar results have been
achieved in mimicking LYLC and YL17b with other types of gradients. We would like to note that the interferometric studies are frequently missing the low frequencies, and therefore the filtering of the low frequencies is happening naturally in this case.

### 10.3. Pure Velocity Caustics

Both velocity and density gradients in subsonic turbulence are directed perpendicular to the magnetic field. Therefore, to isolate the effect of velocity, we create PPV cubes using constant density. For such cubes, all intensity variations arise from velocity crowding. To mimic subsonic turbulence, we introduce the thermal line width, which produces a broadening larger than the turbulent velocities; in our case, the thermal line width is 3.78 times that of turbulent velocities. Our studies of velocity channel intensity correlations in Lazarian & Pogosyan (2000, 2004) suggest a significant suppression of the amplitude of the correlations in such a case and, as a result, the large influence of noise. Thus, in subsequent publications, e.g., Kandel et al. (2017b), we claimed that velocity centroids provide a better way to study velocity fluctuations when the thermal line width exceeds the turbulent line broadening. Nevertheless, for the gradients, we do not see such a loss of

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**Figure 23.** Alignment between gradients and projected magnetic field with white noise added to the original maps.

**Figure 24.** (Left) Alignment measure vs. amplitude of the noise for VChGs and RVCGs. A Gaussian filter of width 2 pixels has been applied to all maps with noises. (Right) The same procedure as in the left but we keep the density constant for the low $M_s$ case.
information (see the right panel of Figure 24), which we relate to the fact that for gradients, the differences of the velocities are important and therefore the constant thermal width does not affect them much. This is an interesting effect that calls for further studies of the relative advantages of the channel map gradients and velocity centroid gradients. In particular, the studies of the gradients of the velocity centroids may be advantageous in order to provide complementary information.

10.4. Physics of Velocity Gradients at Different Scales: Ways to Determine B Strength

Real astrophysical flows include both regular and turbulent motions. However, an interesting property of turbulence is that it increases the velocity gradients as the scale decreases. Therefore, the velocity gradients induced by turbulence are expected to dominate. In addition, to increase the signal-to-noise ratio, the contribution of large-scale motions can be removed using spatial filtering.

The properties of MHD turbulence are different at different scales. For instance, if turbulence is injected at scale $L$ with velocities $v_L$ larger than the Alfvén velocity $V_A$, up to the scale $l_A = L M_A^{-3}$, where $M_A = v_L / V_A$ is the Alfvén Mach number, the turbulent motions are marginally affected by the magnetic field presence. Therefore, if the velocity gradients are measured at scales larger than $l_A$, they cannot trace the magnetic field. As a result, for super-Alfvénic motions, the influence of gradients from scales larger than $l_A$ is pernicious in terms of magnetic field tracing. Motions at these large scales should be filtered out in order to increase the accuracy of VChGs as a magnetic field tracer.

At the same time, the change in the properties of the gradients around the scale $l_A$ can be used to find the fluid magnetization. Indeed, by observing the change in the dispersion of VChGs, which changes the scale at which the gradients are calculated, one can determine $l_A$ and then calculate $M_A = (L / l_A)^{3/2}$. The calculations of $L$ can be done spectroscopically as demonstrated by Chepurnov et al. (2010) using the Velocity Coordinate Spectrum (VCS) technique suggested in Lazarian & Pogosyan (2006). If $M_A$ is determined, the Alfvén velocity can be obtained by associating the $v_L$ with the velocity dispersion that is measured spectroscopically. Note that this technique is applicable to finding the magnetic field in super-Alfvénic turbulence, i.e., turbulence with $M_A > 1$ where the traditional Chandrasekhar–Fermi technique fails (see Falceta-Gonçalves et al. 2008).

In practical terms, we expect to observe the magnetic field to be organized over patches of scale $l_A$, with the mean magnetic field in different patches changing randomly from one patch to another. As gradients reflect the magnetic field direction, we expect a similar coherent organization of velocity gradients over patches of $l_A$.

The VChGs that we have discussed are tested with the simulations of MHD turbulence. The MHD approximation is applicable to partially ionized gas in scales significantly larger than the scale of the neutral–ion decoupling (see Lithwick & Goldreich 2001; Xu & Lazarian 2017). At such scales, the rate of turbulent variations is slow compared to the rate of neutral–ion collisions, and therefore, both species behave as one magnetized fluid. For Kolmogorov or GS95 scaling, the rate of the eddy rotations changes as $v_L / l ~ l^{2/3}$, i.e., it increases with the decrease of the scale. Since, at a sufficiently small scale $l_{\text{decoup}}$, the ions do not collide with the neutrals frequently enough and get decoupled. For $l < l_{\text{decoup}}$, neutrals can start their own unmagnetized cascade, and therefore, their motion will stop reflecting magnetic field.

We have discussed that the VChGs are tracing the magnetic field at the smallest scale that is resolved by the telescope. If this scale is larger than $l_{\text{decoup}}$, then the velocity gradients represent the magnetic field as we discussed in this paper. The change in the orientation of the velocity gradients in the vicinity of $l_{\text{decoup}}$ can be used to establish this important scale. In particular, we expect to see a change in the relative orientation of the gradients obtained with neutrals with maps smoothed over scales $l > l_{\text{decoup}}$ and the high-resolution maps resolving $l < l_{\text{decoup}}$. We also expect to see differences in the velocity gradient orientation of neutrals and ions at scales $l < l_{\text{decoup}}$. Both effects can be used to establish $l_{\text{decoup}}$. The latter scale is related to the magnetic field strength as discussed in Xu et al. (2016), which presents a new way of measuring magnetic field strength. In other words, the VChGs present a
tool that can be used both to trace the magnetic field in molecular clouds and, given enough resolution, to test the neutral–ion decoupling scale.

We believe that velocity gradients can trace the magnetic field in most turbulent astrophysical environments. In particular, we expect the velocity gradients to trace the magnetic fields in accretion disks, where recent polarimetry showed that grain alignment happens with respect to radiation (see Lazarian & Hoang 2007).

Naturally, the changes in velocity gradient properties that we described for the case of VChGs can also be studied with the VCGs and the RVCGs. We will present a detailed testing of the ability of the velocity gradient technique to obtain the magnetic field strength elsewhere.

10.5. Constraining VChG Directions Using Expectations from MHD Turbulence Theory

It is important to make magnetic field tracing as precise as possible. In sub-block averaging (YL17b), the blocks are created by evenly dividing the maps into localized regions and identifying the peaks of the Gaussian fits in these blocks as the most probable values within the block. The theory of MHD turbulence provides ways to improve the procedures for magnetic field tracing. Below we discuss two ways of improving the alignment, and we view this as just the initial steps to an MHD turbulence theory-based automated procedure of velocity gradient tracing that employs machine learning (see Le Cun 1990).

The Moving Window (MW) approach is an attempt to employ sub-block averaging in a continuous rather than a discrete manner. From Figure 26, we see the alignment measure distribution of VChGs in a super-Alfvénic simulation Ms0.2Ma2.0 before block average. It is evident that most of the pixels are $AM = 0.6$ or above; however, one can observe a fluctuation of $\sim 0.3$ of AMs around the high AM points. As magnetic field provides a continuous representation, it is advantageous to move the block and provide calculations as the block moves along the magnetic field lines that it traces. The left panel of Figure 27 shows pictorially how we improve the alignment: when there is an abnormal gradient vector compared to the neighboring vectors, we rotate the abnormal vector so that a smooth field line is formed. Mathematically, the rotation can be handled by performing smoothing on both the cosines and sines of the raw gradient angle, which is a convolution of an averaging kernel with the raw cosine and sine data. We therefore pick a Gaussian kernel with a different width to test how good the moving window is in our synthetic map. After the smoothing, we apply sub-block averaging to the processed gradient angle. Figure 28 shows the strength of the moving window with respect to the smoothing strength. One can see that the alignment quickly rises until a window size of 0.75 pixels and becomes saturated after that. Similar to the sub-block averaging, one can estimate errors of the fitting in order to find the optimal moving window size.

Another possible improvement is to use the expectations of the MHD theory in terms of applying the Angle Constraint (AC). Observations necessarily use the global system of reference related to the mean magnetic field. The probability distribution of the magnetic field in terms of wave vectors parallel, $k_p$, and perpendicular, $k_\perp$, to the mean magnetic field is provided in Lazarian & Pogosyan (2012). This distribution predicts that the mean angle variations are the same at all scales. This variation can be determined with higher accuracy with the larger blocks and then can be used as a constraint when the direction of the gradients is established within noisy small sub-blocks. Our application of the procedure of AC provided a moderate improvement of the AM of 0.1 depending on the system’s $MA$. However, this procedure has minimal effect when $MA$ is larger than unity. We believe that AC can be a useful part of future algorithms of VChG calculation.

10.6. Dependence on the Alfvén Mach Numbers

For observational tracing of the magnetic field, it is important to know what to expect in terms of AM dependence on the Alfvén Mach number $MA$. Figure 29 provides these dependences for our simulations. We see that the alignment decreases as $MA$ approaches unity. We attribute this to the large angle variations of the magnetic field along the line of sight. As $MA$ increases, especially when the turbulence gets super-Alfvénic, it is important to remove the low spatial frequencies. The corresponding procedure was shown to work in LYL, but here we present the direct application of the VChGs to the data without any spatial filtering. The procedures to improve the AM for the VChGs are elaborated elsewhere.

Figure 26. Distribution of alignment measure between raw channel gradients and projected magnetic field in our super-Alfvénic example Ms0.2Ma2.0.

Figure 27. Illustration of how the Moving Window (left) and Angle Constraining (right) should be pictorially in our gradient technique.
11. Discussion

11.1. Comparison with Earlier Studies

The current study continues our series of research papers that are motivated by the modern understanding of the theory of MHD turbulence (see Brandenburg & Lazarian 2013 for a review), in particular, the notion of perpendicular cascade that is at the heart of this theory. Due to this cascade, which can be visualized as a hierarchy of turbulent eddies mixing magnetized plasmas perpendicular to the local direction of magnetic field (see Section 2), the gradients of velocity are expected to be maximal perpendicular to the local magnetic field, thus revealing its direction.

The first paper that employed this theoretical argument to trace magnetic fields using velocity centroids was GL17. This study attempted a quantitative approach, i.e., it introduced the alignment measure AM that is used in subsequent publications, including this one. However, the actual start of the quantitative tracing of the magnetic fields with gradients relies on the block-averaging procedure introduced in YL17b. This procedure allowed both the pointwise direction of magnetic field to be determined and the uncertainty of this direction to be estimated. As a result, this procedure was used in all papers that followed. In this paper, the block averaging approach is used to identify the regions where the direction of gradients changes $90^\circ$ as a result of the effect of self-gravity. We noticed that the blocks over which this transition happens exhibit a much worse single direction fitting, which corresponds to a higher level of fitting errors. We showed that by measuring the level of the errors, it is possible to identify the regions of gravitational infall.

A sister technique tracing magnetic fields using SIGs was introduced in LYLC, where it was shown that it is possible to establish the optimal size of the block for averaging, providing the maximum resolution of the reliable magnetic field tracing. We employed this approach in the present paper.

The correlation between density gradients and magnetic fields was empirically revealed by Soler et al. (2013). The technique did not attempt to trace the magnetic field but was intended to establish the statistical correlation between the observed structures and the magnetic field direction. In terms of observational studies, the technique in Soler et al. (2013) relied on the comparison of the observed IGs and polarization, and the study answered the question whether the preferred orientation of density structures at a given column density is parallel or perpendicular to the magnetic field. Soler et al. (2013) introduced Histograms of Relative Orientation (HRO), which provided empirical insight into the probability of the IGs being aligned with the magnetic field depending on the density of the media. Note that if plotted in the pictorial plane, the orientation of the magnetic field and IG presents a rather chaotic pattern, which prevents one from tracing the magnetic field with the gradients. It is in the statistical sense that HRO establishes the change in the alignment with the ambient density. A detailed comparison of HRO (see also Soler et al. 2013, 2017; Soler & Hennebelle 2017) with our approach is provided YL17b.

The effect of the partial alignment of density gradients with the magnetic field in a diffuse medium is easy to understand from the point of view of the gradient theory in this paper. The velocity mixing motions corresponding to the MHD cascade in diffuse media project their properties on the density structures. Due to low sonic Mach numbers, such structures are preferentially elongated along the magnetic field, and velocity and density gradients behave similarly. However, the difference between density and velocity structures becomes significant as the Mach number increases (see Kowal et al. 2007). Thus, density gradients are inferior in tracing magnetic fields compared to velocity gradients.

Note that the procedures for calculating gradients in HRO do not include block averaging, and therefore the maps of density gradients obtained in that study exhibit chaotic variations of the magnetic fields, which make it impossible to trace magnetic fields even in low Mach number turbulence. By applying our block averaging procedure to densities, we can introduce the IG technique, which is different from HRO. Within this technique, one can determine both the direction of the density gradients and the uncertainty over the image plane. This information is complementary to that from HRO. The sensitivity of density gradients to shocks within the IG technique can be used to identify shocks in the interstellar medium, as discussed in YL17b.
The preferential alignment of filaments within H I channel maps with the direction of the magnetic field was established observationally in Clark et al. (2015). This work significantly resonated as it suggested the possibility of using this alignment to improve the separation of the CMB polarization from the galactic microwave foreground. In the work by Clark et al. (2015), the thickness of channels was empirically chosen to be 3 km s$^{-1}$, and the authors identified the structures with actual density filaments that exist in H I. Indeed, we were told (S. E. Clark 2017, private communication) of the correspondence of the H I filaments in channel maps with the filaments observed in dust, which suggests that the filaments are actual real physical objects rather than caustics. In this situation, it is necessary to study what is happening with the filaments as the channel map thickness increases. The theory in Lazarian & Pogosyan (2000) suggests that Clark et al. (2015) studied “thin channel maps,” where most of the structures should be due to velocity caustics. Indeed, assuming that the typical turbulent velocity dispersion is $\sim 10$ km s$^{-1}$ and that the turbulence is Kolmogorov type, i.e., $v_t \sim \ell_{\text{turb}}^{1/3}$, one can estimate that the turbulent motions at scales larger than $L/(10/3)^2 \approx 0.03L$, where $L$ is the injection scale, are in the “thin-slice regime” (Lazarian & Pogosyan 2000). Using the injection scale of $\sim 100$ pc obtained in Chepurnov et al. (2010), one concludes that on scales larger than 3 pc, the intensity of fluctuations in the channel maps is produced to a significant degree by velocity caustics rather than real physical entities, i.e., filaments. The calculations in Clark et al. (2015) are produced on the maps smoothed up to FWHM = 30$^\prime$. Based on the galactic rotation curve, the structures from the channel maps with velocity ranges $-56$ km s$^{-1}$ to $-\sim 10$ km s$^{-1}$ or 10 km s$^{-1}$ to $\sim 56$ km s$^{-1}$ mostly comprise intensity structures. However, the “velocity crowding effect” is expected to be more severe in channels maps at $-10$ km s$^{-1}$ to $\sim 10$ km s$^{-1}$, in which the structures in the channel maps are coming from contributions from clouds in different physical locations along the line of sight.

What is peculiar about H I is still a subject for further investigation. Apparently, we do not observe any aligned features similar to filaments in our CO studies. At the same time, velocity gradients successfully trace magnetic fields both in H I and denser regions like CO. We defer the issue of the nature of the filaments in Clark et al. (2015) to future publications.

We feel that the advantage of the velocity gradient technique is that it is rooted in both the MHD turbulence theory (see Brandenburg & Lazarian 2013 for a review) and the theory of PPV statistics (see Lazarian et al. 2002; KLP16). It is important that the velocity gradients have a straightforward relation to the compressible MHD turbulence theory. For instance, using the VChGs, we explore the effect of the three fundamental modes of MHD turbulence, i.e., Alfvén, slow, and fast, on the analysis. These modes affect the properties of the VChGs differently, and our present paper opens a new avenue for the development of gradient analysis. In particular, we feel that similar to the techniques of separating Alfvén, slow, and fast modes using correlation anisotropy as in the synchrotron study in Lazarian & Pogosyan (2012) and subsequent studies of velocity correlations in Kandel et al. (2016, 2017b), the velocity gradients can be used to determine the regions with a prevalence of compressible or incompressible turbulence as well as determining the relative contribution of different modes. The corresponding procedures will be discussed elsewhere.

11.2. Toward a Unified Gradient Technique

The properties of the magnetic field in magnetized turbulence are well-established by a series of groundbreaking papers, starting with GS95. In fact, the application of turbulence theory to observations has been explored in a number of studies. In particular, using the analytical treatment of the PPV developed in LP00, Lazarian & Pogosyan (2004, 2006), and Kandel et al. (2017b), we provided the treatment of the anisotropies of the correlation functions of the channel map intensities and of the velocity centroids in Kandel et al. (2017b). These studies shed light on the properties of velocity gradients that we study in the present paper.

Our series of papers starting with GL17 and YL17a provide a way of tracking the magnetic field direction in realistically turbulent interstellar medium (see also LYLC, YL17b). We illustrated that the gradient technique is superior to the anisotropy tracing technique described in Section 6 in terms of tracing magnetic field directions. Nevertheless, these methods are complementary. Their synergy could provide both the detailed structure of the magnetic field through use of the velocity gradients as we discuss in the present paper and the information about the turbulence compressibility as it can be tested using the anisotropy of the correlation functions of the channel map intensities and velocity centroids (see Kandel et al. 2016, 2017a).

In our earlier papers, we focused on velocity centroids as the measures of velocity that are available from observations; the present paper introduces new measures, namely, gradients within channel maps or VChGs and reduced velocity centroids gradients (RVCGs). These measures enhance the abilities of the gradient technique. In particular, both measures can analyze part of a spectral line, which has significant advantages, e.g., in the case of using the galactic rotation curve to get the magnetic field direction variation along the line of sight. Other ways of constructing measures of turbulent velocity from observed spectral lines are also possible. In YL17b, we showed advantages of using higher moments of velocity centroids when the observational data has low noise. Similarly, higher moments of reduced centroids were suggested for the RVCGs within this paper. The study of these different measures is another important avenue for gradient research. These measures are affected to different degrees by the fluctuations of density, and this opens a way to better disentangle the density and velocity effects in the gradient studies. As we discussed in detail in YL17b, the two fields have different properties, and the difference is important for, e.g., identifying the regions of collapse induced by self-gravity.

The application of the technique to different spectral lines allows one to study magnetic fields separately in different regions along the line of sight. This, in particular, is valuable for molecular clouds where by choosing the corresponding transitions, one can study a cloud’s magnetic fields in depth layer by layer.

These and other ways of gaining new information from gradients will be demonstrated elsewhere. In addition, velocity and IGS can be used together with the SIGs, and this was demonstrated in LYLC.
The procedures that we are developing for the VChGs are also applicable to the IGs, the VCGs, and the SIGs. In fact, block averaging makes our IG technique very different from that in Soler et al. (2013). Compared to the VCGs and the IGs, the VChGs include the advantages of both diagnostic tools, while being influenced by the interfering effects. In fact, as the thickness of the velocity channels increases, the VChGs gradually transfer to the IGs. However, compared the IGs, the VChGs contain much more information as the transfer from thin to thick gradients is taking place. At the same time, it is not right to think that VChGs completely overshadow the IGs. For instance, the IGs can be applied to dust emission, where there are no lines to study using the VChGs.

The VChGs and the RVCGs provide a useful tool for studying self-absorbing media (see GL17 for a discussion of the VCGs in self-absorbing media) and lines with complex structure, e.g., velocity lines have multiple peaks. Both the VChGs and the RVCGs allow using part of a spectral line. For strongly self-absorbing media, the useful parts could be the wings. At the same time, for lines with multiple peaks, the complexity of the line may present the complexity of the spatial structure of the emitting region. In the latter case, studies of the separate peaks can provide information on the magnetic field within a complex region. Detailed studies of the utility of the VChGs and the RVCGs as well as their complementary nature will be provided elsewhere.

11.3. Prospects for the Technique

The galactic rotation opens new prospects for studying the detailed structure of the magnetic field in the Milky Way. The position of the solar system within the galactic disk makes it impossible for us to use far-infrared polarimetry to study the magnetic fields of most molecular clouds. Indeed, for most of these studies, the line of sight inevitably crosses more than one cloud. This confusion, combined with the failure of grain alignment at large optical depths (see Lazarian 2007), makes a polarimetric study of most star formation hotbeds impossible. In comparison, the approach based on using gradients can employ the galactic rotation curve to separate the contribution from different clouds. Moreover, the velocity gradients can probe the magnetic connection of the diffuse gas and molecular clouds. Such studies are impossible with far-infrared polarimetry due to the signal from the diffuse media being too weak.

Velocity gradients add up differently compared to the Stokes parameters that are used in polarization. Therefore, to explore the magnetic field in the galactic plane where we can expect a significant direction variation along the line of sight, the VChGs and the RVCGs are advantageous compared to the VCGs. Using the former tools, one can subdivide the line into velocity segments of the order of the turbulent velocity dispersion, \( \sim 7 \text{ km s}^{-1} \), and calculate the magnetic field direction for these segments. Within these segments, the thin channel maps are used for calculating the VChGs and RVCGs. To compare with the polarization, we then can use the mock stock parameters similarly to how it was done in Clark et al. (2015) for the filaments.

In terms of resolving the spatial structure of the magnetic field using the galactic rotation curve, for the atomic hydrogen studies, the spatial resolution depends on the direction of study and the velocity range as well as on the turbulent velocity dispersion. The latter provides the grid size, \( \delta V \), that should be multiplied with the visual shear along the line of sight \((dV_{\text{gal}}/dz)^{-1}\) in the direction of the observations.

If the direction of the magnetic field is changing along the line of sight, the VChGs and the RVCGs can provide a coarse-grade picture of the changes in the magnetic field direction perpendicular to the line of sight. In this way, we can distinguish the variations of the regular magnetic field over the cells that are larger than \( V_t (dV_{\text{gal}}/dz)^{-1} \).

There is a particular domain where velocity gradients present the only way to trace magnetic fields. This is the case of high-velocity clouds. They are tenuous compared to the ISM along the line of sight, and therefore, any polarization associated with them is not possible to detect. However, these clouds are vivid in velocity space, which gives a way for either the VChGs or the RVCGs to map their magnetic fields.

Another domain where dust polarimetry fails to trace the magnetic field is related to circumstellar accretion disks. The dust there is aligned by radiative torques, which, according to the alignment theory in Lazarian & Hoang (2007), align grains in the vicinity of stars in respect to the radiation direction rather than the magnetic field direction (see also Tazaki et al. 2017). Therefore, in spite of the ability of ALMA to resolve such disks, it cannot really study their magnetic field structure. We expect that the velocity gradient will be able to trace the magnetic fields in circumstellar accretion disks.

The change in the channel thickness changes the relative contributions of the velocity and density fluctuations to the channel map. Taking the thickness of the channel map of the order of the velocity injection, one produces intensity-dominated channel maps, and therefore, the study of such maps using gradients is equivalent to the study using IGs.

It is also important that the VChGs can be applied to interferometric data in the situation when no single-dish observations are available. This extends the application of the technique to distant and extragalactic objects in an important way.

We stated above that the observational information on turbulence is available only in the system of the mean field. There are exceptions to this, however. For instance, if only a thin slice of turbulence is seen due to dust absorption effects (see Kandel et al. 2017a), the effects of the local system of reference may become important. Similarly, if the object under study and the turbulence scale are comparable, then the dispersion of the magnetic field is going to decrease with the scale.

The synergetic use of the polarimetry and the velocity gradients can be very beneficial. For instance, velocity gradients can be employed to study the magnetic fields in the disk of the galaxy, where traditional far-infrared polarimetry suffers from the effects of confusion as many clouds can be along the same line of sight. The galactic rotation curve can help isolate different clouds in the velocity space and allow their magnetic fields to be studied separately.

The available large ground-based telescopes and interferometers can provide much better resolution than the far-infrared telescopes on the balloons. The comparison of the polarimetry and velocity gradients can provide information on the regions of gravitational collapse as the velocity gradients will change their directions in such regions. As this low-resolution identification of gravity-dominated regimes is done, velocity gradients may be used to study the details of the magnetic field.
structure that are not available with the existing far-infrared polarimetry.

As different molecules are produced at different depths inside clouds, the velocity gradients can be used to study the 3D magnetic field structure of the clouds. This provides a new dimension to magnetic field studies.

It is important to note that far-infrared polarimetry and the velocity gradients do not provide identical information. First of all, the magnetic fields that are traced by the polarimetry are weighted by the dust density, the latter being proportional to the gas density. Within the VChG approach, the contribution of the density fluctuations is reduced, and the directions of magnetic field measured by the VChGs can potentially be closer to the actual projected magnetic field. Moreover, the addition of gradients along the line of sight is performed differently from the addition of the Stokes parameters in the case of polarimetry. This opens a way of probing the 3D magnetic structure by combining polarimetry and velocity gradients. We discuss this possibility in K. H. Yuen et al. (2018, in preparation).

Although synergistic with polarimetry, velocity gradients present an independent way of studying magnetic fields that does not require polarimetric information. Naturally, testing the velocity gradients with as much polarimetric data is advantageous to gain more confidence in the new technique. Eventually, velocity gradients should be used on their own, however.

We have identified (see YL17b) the 90° change in the relative orientation of the velocity gradients and magnetic field directions in the case of self-gravitational collapse. Other situations, such as when regular flows dominate turbulence, can be present. Therefore, it is important to provide more numerical studies of velocity gradients in expanding H I regions, supernova explosions, etc., in order to see whether one should expect the change in the direction of velocity gradients in other astrophysical settings. If this happens, it is important to test our approach based on calculating the uncertainties of the fitting of velocity gradients as a way of identifying the change.

Studies of microwave foregrounds are extremely important in the attempts to detect and explore the polarization-induced enigmatic cosmological B-modes. Separating this polarization from the polarization arising from the galactic foreground requires a breakthrough in understanding galactic magnetic fields. Velocity gradients, e.g., the VChGs and RVCGs, as well as SIGs provide an independent way of mapping magnetic fields. The corresponding information can be used independently to predict the foreground polarization, or preferably, as a prior for polarization studies.

12. Summary

In this paper, we have shown that a new measure, i.e., gradients calculated within velocity channel maps (VChGs), can trace the magnetic field in both diffuse media and molecular clouds. The VChGs of the thin channel maps carry information about turbulent velocities, while the VChGs of the thick maps carry information about the turbulent densities. The essence of the emerging technique is to vary the channel thickness to get complementary information on both magnetic fields and shocks. As velocity gradients are more direct tracers of the magnetic field, most of the paper is devoted to the VChGs calculated for the thin velocity channels. We compared the abilities of the VChGs and another new measure, Reduced Velocity Centroid Gradients (RVCGs), which also traces magnetic fields with spectroscopic data. In particular:

1. We demonstrated the alignments of VChGs obtained with basic MHD modes, i.e.,
   (a) the VChGs from Alfvén modes are perpendicular to the magnetic field, and demonstrate the highest alignment;
   (b) the VChGs from slow modes are also perpendicular to the magnetic field but show a somewhat reduced alignment compared to that from Alfvén modes; their alignment drops faster as the channel width increases;
   (c) the VChGs from the fast modes are parallel to the magnetic field.
2. We showed that the VChGs are more powerful in tracing the magnetic field directions than the channel map correlation functions that we proposed earlier, namely, the VChGs provide more detailed information about the magnetic field and can trace the magnetic field for both supersonic and subsonic turbulence.
3. We applied the VChGs to the observational HI data and compared the tracing of the VChGs of the magnetic field with the Planck polarity data.
4. We found that the RVCGs are comparable in their performance to the VChGs, and both techniques can be used to trace the magnetic field in diffuse interstellar gas and its interface with molecular clouds, molecular clouds, high-velocity clouds, etc.
5. We believe that the VChGs are synergetic with other ways of studying the magnetic field, in particular, with far-infrared polarimetry. Nevertheless, it is an independent way of studying magnetic fields, which can be used to trace magnetic fields on its own.
6. We claim that VChGs and RVCGs can trace magnetic fields in situations when traditional far-infrared polarimetry fails, e.g., due to the failure of dust to be aligned or due to the confusion effect typical in the study of molecular clouds at low galactic latitudes.
7. We demonstrated the advantages of the synergistic use of the different types of gradients (e.g., synchrotron intensity, spectral line intensity), paving the way for a new gradient technique of studying the magnetic field ecosystem, shocks, and self-gravitational collapse. This technique can provide the magnetic field structure, which is valuable for disentangling the galactic foregrounds and CMB polarization.

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Note added in proof: Our recent research has shown that the ability of the of velocity gradients to trace the 3D distribution of the
perpendicular to the line of sight component of magnetic field is complementary to the ability of 3D studies of magnetic field using gradients of polarized synchrotron emission that we are currently working on.

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