Halftone Wave Front Control: Numerical Simulation and Laboratory Demonstration

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Abstract

High-contrast instruments are required for direct imaging of faint exoplanets around bright host stars. In high-contrast instruments, a wave front control system is needed to generate a dark hole by suppressing residual stellar speckles. However, the achievable contrast is limited by the phase quantization error (i.e., finite phase resolution) of wave front control devices, such as deformable mirrors or spatial light modulators. In this paper, we propose a halftone method for wave front control to improve the contrast using a wave front control device with quantized phase modulation. In a numerical simulation, the contrast was improved from 1.4 × 10−9 to 3.8 × 10−10 by halftone wave front control. In addition, we performed a laboratory demonstration in which a spatial light modulator was used for wave front control, and the contrast was improved from 2.2 × 10−7 to 6.0 × 10−8 for a phase resolution of 2π/256.

Unified Astronomy Thesaurus concepts: Coronagraphic imaging (313); Direct imaging (387); Exoplanets (498); Exoplanet detection methods (489); Astronomical techniques (1684)

1. Introduction

Characterization of exoplanets is one of the most important themes in astronomy and astrophysics. Thousands of exoplanets have been discovered by several observation methods, such as radial velocities, transits, and microlensing. Direct imaging of exoplanets is challenging because bright stellar light impedes the detection of the faint planetary light. For example, the brightness ratio between an Earth-like planet and a Sun-like star will reach 10−10. Therefore, a dedicated high-contrast instrument is required to suppress the stellar light strongly.

High-contrast instruments consist of a coronagraph and a wave front control (WFC) system. The coronagraph suppresses the stellar light diffracted by a telescope pupil. The WFC system generates a dark hole by suppressing the residual stellar speckles caused by amplitude and phase aberrations in the optical system. Several WFC algorithms have been proposed, from classical speckle nulling (Malbet et al. 1995; Trauger et al. 2004), to advanced methods, such as energy minimization and electric field conjugation (EFC) (Bordé & Traub 2006; Give’on et al. 2007). Extremely high contrasts of 3 × 10−10 over a 2% bandwidth, 6 × 10−10 over a 10% bandwidth, and 2 × 10−9 over a 20% bandwidth have been realized in the laboratory (Trauger et al. 2012).

The coronagraphic instrument for the Nancy Grace Roman Space Telescope, on which the advanced coronagraphs and the WFC system are mounted, will be an important pathfinder in the detection of Earth-like exoplanets (Kasdin et al. 2020). After the Roman Space Telescope, HabEx (Gaudi et al. 2020) and LUVOIR (The LUVOIR Team 2019) are planned for detecting biosignatures in the atmospheres of Earth-like exoplanets in habitable zones around Sun-like stars.

Deformable mirrors (DMs) are widely used for the development of WFC systems in high-contrast instruments. In addition, spatial light modulators (SLMs) have also been considered as WFC devices (Liu et al. 2015; Dou & Ren 2016). One advantage of SLMs is the large format of control pixels, which enables them to generate a huge dark hole on the order of several hundred λ/D on one side of the central star, where λ is the wavelength and D is the diameter of the telescope aperture, by correcting the aberrations with a high spatial frequency (Murakami et al. 2020). The achievable contrast of the dark hole will ultimately be limited by the quantization error of the phase modulation due to the finite phase resolution of the WFC devices (DMs or SLMs). The effect of the quantization error on the achievable contrast has been examined in several studies (Brown et al. 2003; Trauger et al. 2003; Ruane et al. 2020). It is expected that the large format will help to improve the achievable contrast of WFC devices with a low phase resolution. Dou & Ren (2016) proposed using the simulated annealing algorithm for improving the achievable contrast with a WFC device with a large format simulating an SLM. In addition, Pourcelot et al. (2021) studied the effect of the large format on the performance of an extreme adaptive optics system in a laboratory setting.

In this paper, we propose using the halftone method for WFC as an alternative approach to improve the achievable contrast, which is limited by the quantization error of the WFC device. Several halftone algorithms have been proposed for image processing to create pseudo-continuous color images by using dots of a finite number of colors. Floyd–Steinberg dithering is a halftone process based on the error diffusion method (Floyd & Steinberg 1976). Here, we propose using the halftone method to generate a pseudo-continuous phase map using a WFC device with a finite phase resolution. The halftone method has also been used in high-contrast techniques in different ways. For example, pseudo-continuous amplitude masks, which are based on the microdots technique, have been...
fabricated for the apodized pupil coronagraph and hybrid Lyot coronagraph (Martinez et al. 2009a, 2009b).

In Section 2, we describe the halftone method procedure used for the WFC system. We performed a numerical simulation and a laboratory demonstration of the halftone WFC. The results of the numerical simulation and the laboratory demonstration are reported in Sections 3 and 4. Several issues with the numerical simulation and the laboratory demonstration are discussed in Section 5. Finally, Section 6 summarizes our conclusion and future perspectives.

2. Halftone WFC Procedure

The achievable contrast of a high-contrast instrument equipped with a WFC system is estimated by

$$C = \pi \left( \frac{8h_{\text{rms}}}{N\lambda} \right)^2 = \frac{16\pi}{3} \left( \frac{h_{\text{res}}}{N\lambda} \right)^2, \quad (1)$$

where $h_{\text{rms}}$ is the rms error of a phase correction map to be applied to a WFC device, $h_{\text{res}}$ is the control resolution of the WFC device, and $N$ is the number of actuators or pixels of the WFC device across the beam diameter in the entrance pupil (Traub & Oppenheimer 2010; Ruane et al. 2020). It is expected that $h_{\text{rms}}$ and $h_{\text{res}}$, expressed in units of length, are roughly related by $h_{\text{rms}} = h_{\text{res}}/2\sqrt{3}$ (Ruane et al. 2020). Using the phase resolution in radians, $\sigma_{\text{res}} = 2\pi h_{\text{res}}/\lambda$, the achievable contrast can be written as

$$C = \frac{4}{3\pi} \left( \frac{\sigma_{\text{res}}}{N} \right)^2. \quad (2)$$

Equation (2) suggests that the achievable contrast depends on $\sigma_{\text{res}}$ and $N$. Thus, a WFC device with a higher phase resolution, $\sigma_{\text{res}}$, would lead to higher achievable contrast, $C$. In other words, a WFC device with a low phase resolution will significantly limit the achievable contrast because of the large residual of the WFC. Conversely, the achievable contrast would be improved by $N^{-2}$ by a larger number of actuators or pixels, $N$, in the WFC device. Here, we propose using the halftone method in the WFC algorithm, such as speckle nulling or EFC, to compensate for the degradation of the achievable contrast due to the low phase resolution, $\sigma_{\text{res}}$, with the large number of pixels, $N$, of the WFC device.

Figure 1 shows examples of phase correction maps that are applied to the WFC device. Bottom panels are close-up images of the central regions indicated by the red dashed squares in the top panels. The influence function of the WFC device is assumed to be a top-hat function, namely, simulating an ideal SLM. Figure 1(a) is an ideal phase correction map, which is not quantized by the phase resolution of the WFC device. The WFC device samples a telescope beam with $N$ pixels in one dimension, corresponding to the Nyquist region in a focal plane that can be up to $\pm N\lambda/2D$ from a central star. For the low phase resolution, that is, $2\pi/500$ in Figure 1(b), the phase map looks different from the ideal phase map in Figure 1(a). This is due to the quantization error of the WFC device, resulting in the degradation of the achievable contrast according to Equation (2).

The halftone WFC procedure is as follows. First, the ideal phase correction map to be applied to the WFC device is estimated by a wave front sensing method, as in the usual WFC algorithm. Next, the phase correction map is divided into subregions with a size of $N_{\text{sub}} \times N_{\text{sub}}$, and the mean value of the phase level over each subregion is calculated. In Figure 1(c), the size of the subregions is set to $4 \times 4$, as shown by the red dashed-line grid in the bottom-right panel. Finally, the halftone process is applied to each subregion by setting the phase value at each pixel to an appropriate quantized level to minimize the quantization error over the subregion. We
use the Floyd–Steinberg dithering algorithm (Floyd & Steinberg 1976) as the halftone process. In Figure 1(b) and (c), the phase resolution is set to $2\pi/500$.

To apply the proposed method, the WFC device needs to have a large format because the phase correction map is divided into subregions. The Nyquist region of the halftone WFC becomes $(N/N_{\text{sub}})\lambda/D \times (N/N_{\text{sub}})\lambda/D$, which is $N_{\text{sub}}$ times smaller than that without the halftone method. We define the Nyquist region as $N_{\text{req}}\lambda/D \times N_{\text{req}}\lambda/D$, namely, $N_{\text{req}} = N/N_{\text{sub}}$ in the example above. We used an SLM with $1440 \times 1050$ pixels for the laboratory demonstration. If the size of the subregions is set to $N_{\text{sub}} = 4$, the largest possible Nyquist region would be about $360 \lambda/D \times 260 \lambda/D$.

### 3. Numerical Simulation

We performed a numerical simulation of the halftone WFC. Figure 2 shows the optical setup. The optical parameters are summarized in Table 1. The WFC device is placed upstream of the coronagraph, which defines the entrance pupil plane. In the numerical simulation, we assume an unobscured circular telescope pupil of diameter $D$. We choose an eight-octant phase mask (8OPM; Murakami et al. 2008) as a focal plane coronagraphic mask. A Lyot stop of diameter $D_s \approx 0.83 D$ is placed at a reimaged pupil plane (Lyot plane). The residual speckle image of the coronagraph due to amplitude and phase aberrations is acquired by the detector in the final focal plane. The detector sampling in the focal plane is set to 4 pixels per $\lambda/D$. The amplitude and phase aberrations at the entrance pupil plane are set to 2.0% rms and $2\pi/100$ rad rms, respectively, both of which follow a $\rho^{-2.5}$ power law ($\rho$ is a spatial frequency).

We use speckle area nulling (SAN; Oya et al. 2015) as the WFC algorithm. We expect it is relatively easy to generate a large dark hole or a dark hole at a large distance with SAN compared with that of matrix-based algorithms, such as EFC, because SAN does not need a large amount of computer memory to implement WFC with a large format.

The results of the numerical simulations for five cases are summarized in Table 2. The number of pixels of the WFC device across the pupil diameter is set to $N = 512$, which is a realistic value for commonly available SLMs. The leftmost panel in Figure 3 is the final focal plane image before WFC. The target dark hole (solid red line) is a D-shaped region with an angular separation from the center to the inner edge of $1.5 \lambda/D$ and an outer radius of $60 \lambda/D$. The phase transitions of the 8OPM are excluded from the target dark-hole region, because algorithms based on speckle nulling cannot control speckles on these transitions, and planetary light is suppressed by the 8OPM on these transitions. We exclude a region within $1 \lambda/D$ from the phase transition lines from the target dark-hole region. The red dashed line in the leftmost panel in Figure 3 shows a region where the mean contrast is evaluated, which is reduced by $1 \lambda/D$ from the target dark-hole region. The 100 brightest speckles in the target region are selected for suppression at each iteration of WFC using the SAN algorithm. The magenta dashed square shows the Nyquist region of DMs with $64 \times 64$ actuators.

In case (a), we assume that the WFC device provides ideal continuous phase control, namely, the phase correction map is not quantized by the phase resolution. The mean contrast as a function of the iteration is shown by the red line in Figure 4 (left). The final contrast reaches $2.8 \times 10^{-10}$. In case (b), the phase resolution of the WFC device is assumed to be $2\pi/1024$, and the final contrast is degraded to $1.4 \times 10^{-10}$. In cases (c)–(e), we use the halftone method, in which the sizes of the subregions for the halftone WFC are set to (c) $2 \times 2$, (d) $3 \times 3$, and (e) $4 \times 4$. Consequently, the contrasts are improved by the halftone method and reach (c) $4.4 \times 10^{-10}$, (d) $3.8 \times 10^{-10}$, and (e) $4.0 \times 10^{-10}$.

The radial average contrasts are shown in Figure 4 (right). The black line shows the contrast before WFC, and the red, blue, green, magenta, and yellow lines show the contrasts for cases (a)–(e), respectively. Figures 3 and 4 and Table 2 show that the final contrasts for cases (c)–(e), with the halftone method, are better than that for case (b) (without the halftone method), and are comparable to that for the ideal case (a) with continuous phase control.

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**Table 1**

| Parameters            | Values or Descriptions |
|-----------------------|------------------------|
| Entrance pupil        | Unobscured circular pupil (diameter $D$) |
| Lyot stop             | $\approx 0.83 D$       |
| Sampling at the focal plane | 4 pixels per $\lambda/D$ |
| Coronagraph           | 8OPM                   |
| Amplitude aberration  | 2.0% rms               |
| Phase aberration      | $2\pi/100$ rad rms     |

*Note.* The wave front aberrations (both amplitude and phase) follow a $\rho^{-2.5}$ power law.
4. Laboratory Demonstration

4.1. Optical Setup

We performed a laboratory demonstration of the halftone WFC at the Facility for Coronagraphic Elemental Technologies (FACET) testbed (Murakami et al. 2020). Figure 5 shows the optical setup. An HeNe laser ($\lambda = 633$ nm), connected to a single-mode optical fiber (SMF) by an objective lens (OL), is used to simulate stellar light. The light emitted from the SMF is collimated by lens L1 (focal length 300 mm) in front of a high-contrast imaging simulator. As the WFC device, the SLM (SLM-100, Santec Corp.) is placed in the collimated beam and defines the entrance pupil plane. Polarizer P1 is placed in front of the SLM, and the transmission axis of P1 is set along the liquid crystal slow axis so that the SLM can be operated as a phase-only wave front modulator. The SLM is controlled by a driver connected to a computer. The gray level of the SLM is 10 bit ($2^{10}$) and the maximum phase stroke of the SLM is set to $2\pi$ at $\lambda = 633$ nm. Therefore, the phase resolution of WFC by the SLM is $2\pi/1024$, if the phase is linearly modulated as a function of the applied voltage. We assume that the influence function of the SLM is the ideal top-hat function. The entrance pupil plane is defined by the SLM and the entrance pupil is collimated by lens L1. The light transmitted through the SLM is collimated by lens L2 and is imaged onto a detector. The detector is a high-contrast imaging simulator.

Figure 3. Final focal plane images of the numerical simulation using the SAN algorithm. The leftmost panel is the coronagraphic image before WFC. The red cross indicates the center of the images. The target dark hole is a D-shaped region shown by the solid red line in the leftmost panel. Mean contrasts are evaluated over a D-shaped region, which is reduced by $1/\lambda/D$ from the target region, shown by the red dashed line in the leftmost panel. The magenta dashed square in the leftmost panel shows the Nyquist region of a $64 \times 64$ DM. (a)–(e) Images after WFC for cases (a)–(e) summarized in Table 2.

![Figure 3](image.png)

Figure 4. Left: mean contrasts of the numerical simulation as a function of the iteration. Red, blue, green, magenta, and yellow lines are the results for cases (a)–(e), respectively, summarized in Table 2. Right: radial average contrast of each case. The black line shows the radial average contrast before WFC, and the other five lines are those for cases (a)–(e).

![Figure 4](image.png)

Table 2

Summary of the Numerical Simulation using the SAN Algorithm

| Case | Pixel Format $N \times N$ (pixels) | Subregion Size $N_{\text{sub}} \times N_{\text{sub}}$ (pixels) | Nyquist Region $N_{\text{nyq}} \times N_{\text{nyq}}$ ($\lambda/D$) | Phase Resolution $\sigma_{\text{res}}$ (rad) | Halftone | Mean Contrast |
|------|-----------------------------------|-----------------------------|-------------------------------------------------|------------------|---------|-------------|
| (a)  | 512 $\times$ 512                   | 1 $\times$ 1               | 512 $\times$ 512                                 | Continuous       | No      | $2.8 \times 10^{-10}$ |
| (b)  | 512 $\times$ 512                   | 1 $\times$ 1               | 512 $\times$ 512                                 | $2\pi/1024$      | No      | $1.4 \times 10^{-9}$ |
| (c)  | 512 $\times$ 512                   | 2 $\times$ 2               | 256 $\times$ 256                                | $2\pi/1024$      | Yes     | $4.4 \times 10^{-10}$ |
| (d)  | 512 $\times$ 512                   | 3 $\times$ 3               | 170 $\times$ 170                                | $2\pi/1024$      | Yes     | $3.8 \times 10^{-10}$ |
| (e)  | 512 $\times$ 512                   | 4 $\times$ 4               | 128 $\times$ 128                                | $2\pi/1024$      | Yes     | $4.0 \times 10^{-10}$ |

Note. The initial contrast before WFC is $9.3 \times 10^{-8}$. 

4. Laboratory Demonstration

4.1. Optical Setup

We performed a laboratory demonstration of the halftone WFC at the Facility for Coronagraphic Elemental Technologies (FACET) testbed (Murakami et al. 2020). Figure 5 shows the optical setup. An HeNe laser ($\lambda = 633$ nm), connected to a single-mode optical fiber (SMF) by an objective lens (OL), is used to simulate stellar light. The light emitted from the SMF is collimated by lens L1 (focal length 300 mm) in front of a high-contrast imaging simulator. As the WFC device, the SLM (SLM-100, Santec Corp.) is placed in the collimated beam and defines the entrance pupil plane. Polarizer P1 is placed in front of the SLM, and the transmission axis of P1 is set along the liquid crystal slow axis so that the SLM can be operated as a phase-only wave front modulator. The SLM is controlled by a driver connected to a computer. The gray level of the SLM is 10 bit ($2^{10}$) and the maximum phase stroke of the SLM is set to $2\pi$ at $\lambda = 633$ nm. Therefore, the phase resolution of WFC by the SLM is $2\pi/1024$, if the phase is linearly modulated as a function of the applied voltage. We assume that the influence function of the SLM is the ideal top-hat function. The entrance pupil plane is defined by the SLM and the entrance pupil is collimated by lens L1. The light transmitted through the SLM is collimated by lens L2 and is imaged onto a detector. The detector is a high-contrast imaging simulator.
Figure 5. Optical setup for the laboratory demonstration. The HeNe laser is used to simulate stellar light. The SLM defines the entrance pupil plane. The SLM is controlled by the driver connected to the PC. Polarizer P1 is used so that the SLM can be operated in phase-only modulation mode. Polarizer P2 is used to achromatize the pupil plane is reimaged by lenses L2 and L3 (focal length 200 mm). A circular pupil is placed in the reimaged entrance pupil plane, which is followed by the coronagraphic system shown in Figure 2. The diameter of the circular pupil ($D = 3.0$ mm) corresponds to 288 pixels of the SLM. Polarizer P2 is placed behind the 8OPM to filter polarization leakage for achromatizing the coronagraphic performance (Murakami et al. 2008, 2010) because the designed wavelength of the 8OPM deviates slightly from that used as the model star. A Lyot stop is placed in the reimaged pupil plane (the Lyot plane) formed by the next pair of lenses, L4 and L5 (focal length 200 mm). The diameter of the Lyot stop is set to 83% of the entrance pupil (i.e., $D_L = 2.5$ mm), which is comparable to that of the numerical simulation. Images in the final focal plane, which are formed by lens L6 (focal length 300 mm), are acquired by the CCD camera.

4.2. Experimental Results

We performed two experiments: dark-hole generations at a large distance over a small region and over a large region, both of which exceed the Nyquist region of a $64 \times 64$ DM.

Figure 6 shows the experimental results of the dark hole generated at a large distance. The leftmost panel in Figure 6 is the coronagraphic image before WFC. The target dark-hole region is indicated by the solid red square in the leftmost panel and ranges from 5 to 15 $\lambda/D$ and from 35 to 45 $\lambda/D$ in the horizontal and vertical directions, respectively. Mean contrasts are evaluated over the red dashed square, which is reduced by 1 square shows the Nyquist region of the $64 \times 64$ DM. (a)-(d) Images after WFC (30 iterations) for cases (a)-(d) summarized in Table 3.

This target region is fully outside the Nyquist region of the $64 \times 64$ DM shown by the magenta dashed square. Mean contrasts are evaluated over the red dashed square, which is reduced by 1 $\lambda/D$ from the target region. In each iteration, the 10 brightest speckles are selected for suppression in the target region. The results of the laboratory demonstration for cases (a)-(d) are summarized in Table 3.

Figure 6(a) shows the residual speckle image after WFC without the halftone method. The contrast was $4.8 \times 10^{-8}$ after 30 iterations. The mean contrast as a function of the iteration is shown by the solid red line in Figure 7 (left). In Figure 6(b), we applied the halftone method, in which the size of the subregions was set to $N_{sub} = 3$ and the phase resolution of the SLM was $2\pi/1024$ per one voltage step. At the 30th iteration, the contrast was $4.6 \times 10^{-8}$, exhibiting no major improvement compared with case (a).

Next, we intentionally degraded the phase resolution to $2\pi/256$ and conducted similar experiments. The resultant image without the halftone method is shown in Figure 6(c). The final contrast at the 30th iteration was $2.2 \times 10^{-7}$. After 30 iterations, we used the halftone method and continued WFC for 30 more iterations. The final contrast was improved to $6.0 \times 10^{-8}$ (solid green line in Figure 7 (left)). In Figure 6(d), we used the halftone method from the first step of the original speckle image, and the final contrast reached $6.0 \times 10^{-8}$ at the 30th iteration.
The radial average contrasts of the experimental results are shown in Figure 7 (right). The black line is the contrast profile before WFC. The red, blue, green, and magenta lines are the contrasts for cases (a)–(d), respectively, after 30 iterations. We can see that the contrast was improved by the halftone method when the phase resolution was set to $2\pi/256$.

Figure 8 shows the results of the dark hole generated over a large region. In the leftmost panel, the target dark-hole region is shown by the red solid line whose inner and outer radii are $7\lambda/D$ and $45\lambda/D$, respectively. Mean contrasts are evaluated over the region indicated by the red dashed line, which is reduced by $1\lambda/D$ from the target region. The magenta dashed square shows the Nyquist region of the $64 \times 64$ DM. (a)–(d) Images after WFC for cases (a)–(d) summarized in Table 4.

The radial average contrasts for cases (a)–(d) are shown in Figure 7 (right). The black line is the contrast profile before WFC. The red, blue, green, and magenta lines are the contrasts for cases (a)–(d), respectively, after 30 iterations. We can see that the contrast was improved by the halftone method when the phase resolution was set to $2\pi/256$.

Figure 8 shows the results of the dark hole generated over a large region. In the leftmost panel, the target dark-hole region is shown by the red solid line whose inner and outer radii are $7\lambda/D$ and $45\lambda/D$, respectively. We exclude a region within $2\lambda/D$ from the phase transition lines of the 8OPM from the target region. The mean contrast is evaluated over the region indicated by the red dashed line, which is reduced by $1\lambda/D$ from the target region. In this demonstration, we select the 100 brightest speckles for suppression in the target region. The results of this demonstration are summarized in Table 4.

The mean contrasts after WFC were (a) $9.3 \times 10^{-8}$ with the phase resolution of $2\pi/1024$ without the halftone method, and (b) $4.8 \times 10^{-8}$ with the halftone method.
Figure 9. Left: mean contrasts of the laboratory demonstration of the dark hole generated over a large region as a function of the iteration. Red, blue, green, and magenta solid lines are results for cases (a)–(d). Right: radial average contrasts for cases (a)–(d). The black line is the contrast before WFC. In both graphs, dotted lines show the corresponding numerical simulation results.

Table 4
Summary of the Laboratory Demonstration of the Dark Hole Generated over a Large Region

| Case | Pixel Format $N \times N$ (pixels) | Subregion Size $N_{\text{sub}} \times N_{\text{sub}}$ (pixels) | Nyquist Region $N_{\text{nyq}} \times N_{\text{nyq}}$ ($\lambda/D$) | Phase Resolution $\delta_{\text{res}}$ (rad) | Halftone | Mean Contrast |
|------|--------------------------------|---------------------------------|---------------------------------|---------------------------------|--------|---------------|
| (a)  | 288 $\times$ 288               | 1 $\times$ 1                    | 288 $\times$ 288                | 2$\pi$/1024                      | No     | 9.3 $\times$ 10$^{-8}$ |
| (b)  | 288 $\times$ 288               | 3 $\times$ 3                    | 96 $\times$ 96                  | 2$\pi$/1024                      | Yes    | 1.3 $\times$ 10$^{-7}$ |
| (c)  | 288 $\times$ 288               | 1 $\times$ 1                    | 288 $\times$ 288                | 2$\pi$/256                       | No     | 1.3 $\times$ 10$^{-7}$ |
| (d)  | 288 $\times$ 288               | 3 $\times$ 3                    | 96 $\times$ 96                  | 2$\pi$/256                       | Yes    | 1.2 $\times$ 10$^{-7}$ |

Note. The initial contrast before WFC is $1.4 \times 10^{-6}$.

(b) $1.3 \times 10^{-7}$ with the phase resolution of $2\pi$/1024 with the halftone method, (c) $1.3 \times 10^{-7}$ with the phase resolution of $2\pi$/256 without the halftone method, (d) $1.2 \times 10^{-7}$ with the phase resolution of $2\pi$/256 with the halftone method. The mean contrasts as a function of the iteration are shown by the solid lines in Figure 9 (left).

Figure 9 (right) shows radial average contrasts for cases (a)–(d). The black solid line is the contrast profile before WFC. Within about 5 $\lambda/D$ from the center, the intensity acquired by the CCD camera was saturated. As can be seen in Figure 9 (right), we could not observe an improvement of the contrast by the halftone method for both of the phase resolutions $2\pi$/256 and $2\pi$/1024.

5. Discussion

In Sections 5.1 and 5.2, we discuss the results of the numerical simulation and laboratory demonstration. The numerical simulation and the laboratory demonstration used the SAN algorithm. In Section 5.3, we also present results of the numerical simulation of the halftone method using a matrix-based algorithm, that is, EFC.

5.1. Numerical Simulation

The numerical simulation results suggest that the halftone method can improve the contrast of the dark hole using the WFC device with a quantized phase resolution. The contrasts obtained by the halftone method were roughly comparable for cases (c)–(e), regardless of the size of the subregions. However, there were slight differences in the contrasts achieved for these cases (Figure 4). In future work, it will be interesting to consider optimizing the control parameters of the halftone process, such as the subregion size for a given target dark-hole region, amplitude and phase rms errors, the type of coronagraph, and pixel sampling over the telescope beam.

The wide-field version of the final focal plane images in Figures 3(a) and 3(d) are shown in Figure 10 (left). Images obtained using the halftone method contain diffraction patterns, which were probably caused by the halftone process. The positions of the diffraction patterns depended on the subregion size, $N_{\text{sub}}$. This diffraction pattern could spread the energy of the planetary light over a wide field in the focal plane, and could decrease the Strehl ratio of the planetary point-spread function (PSF). Figure 10 (right) shows the evaluated Strehl ratios as a function of the iteration for all five cases in Table 2. The Strehl ratios were high, within 0.9966–0.9972 for all of these cases. Thus, we concluded that the diffraction patterns did not affect the quality of the planetary PSF.

5.2. Laboratory Demonstration

In the laboratory demonstration, the contrast was not improved by the halftone method for the phase resolution of $2\pi$/1024 in the first experiment shown in Figure 7, and for both $2\pi$/256 and $2\pi$/1024 in the second experiment shown in Figure 9. To evaluate the potential achievable contrast of the halftone WFC, we performed additional numerical simulations, in which the initial contrasts before WFC were set to be comparable to those in the laboratory demonstration. The results of the numerical simulations are plotted as dotted lines in Figures 7 and 9, together with the experimental results (solid lines).
The mean contrasts corresponding to the first experiment at the 60th iteration were (a) $2.1 \times 10^{-8}$ with a phase resolution of $2\pi/1024$ without the halftone method, (b) $1.0 \times 10^{-9}$ with a phase resolution of $2\pi/1024$ with the halftone method, (c) $4.5 \times 10^{-7}$ with a phase resolution of $2\pi/256$ without the halftone method, and (d) $2.9 \times 10^{-5}$ with a phase resolution of $2\pi/256$ with the halftone method. In case (c), the experimental contrast ($2.2 \times 10^{-7}$) was comparable with the simulated one ($4.5 \times 10^{-7}$). However, the other experimental results were all worse than the corresponding simulated results.

By contrast, the mean contrasts of the numerical simulation corresponding to the second experiment at the 50th iteration were (a) $4.4 \times 10^{-9}$ with the phase resolution of $2\pi/1024$ without the halftone method, (b) $4.0 \times 10^{-9}$ with the phase resolution of $2\pi/1024$ with the halftone method, (c) $3.1 \times 10^{-8}$ with the phase resolution of $2\pi/256$ without the halftone method, and (d) $1.8 \times 10^{-8}$ with the phase resolution of $2\pi/256$ with the halftone method.

Comparing the experimental and simulated results suggested that the achievable contrasts in the laboratory are limited to roughly $5 \times 10^{-8}$ for the first experiment and roughly $1 \times 10^{-7}$ for the second experiment due to the environment of the laboratory testbed, such as atmospheric turbulence, mechanical vibrations, and incoherent light due to imperfect polarization control and imperfect optical components. Therefore, it is necessary to investigate why the experimental contrasts are limited to these values, which will be our priority for improving the achievable contrast in the laboratory.

The number of pixels across the beam diameter was $N = 288$ in the laboratory demonstration. However, the SLM used in the experiment had $1440 \times 1050$ pixels, and many more pixels were available for WFC. For example, if the light beam at the entrance pupil becomes too large to sample with 1000 pixels across the beam diameter, and the size of the subregions for the halftone WFC is set to $N_{\text{sub}} = 3$, the system could generate dark holes larger than $100 \lambda/D$ on one side of the central star.

In the laboratory demonstration and the numerical simulation, the influence function of the SLM was assumed to be an ideal top-hat function without any crosstalk between adjacent pixels. However, it is important to consider the effect of the deviation of the influence function from the ideal top-hat function on the achievable contrast. In addition, it might be possible to use other WFC algorithms that take into consideration a practical influence function with substantial crosstalk between pixels. For DMs, by contrast, the influence function would be a single Gaussian or a sum of Gaussians instead of the top-hat function (Ruane et al. 2020). It would also be interesting to examine the halftone WFC using the DMs considering a Gaussian-like influence function.

### 5.3. Application of the Halftone Method Using the EFC

We also performed a numerical simulation of the halftone method using the EFC algorithm (Give’on et al. 2007). Here, we briefly describe the procedure of the halftone method used for the EFC algorithm. For simplicity, we assume that the electric field of residual speckles can be measured perfectly. First, we divide the pixel format of the WFC device into subregions for the halftone method. Next, the control matrix of the coronagraphic system is modeled for the EFC algorithm. The size of the control matrix depends on the number of subregions (i.e., $N_{\text{sub}} \times N_{\text{sub}}$). For constructing the control matrix of the EFC algorithm, we need to poke $N_{\text{sub}} \times N_{\text{sub}}$ subregions and to measure intensity changes on the detector.

Without the halftone method, we need to poke all $N \times N$ pixels ($N = 288$ in the case of the laboratory demonstration). However, a single poke will cause an imperceptible intensity change of roughly $1/80,000$, which cannot be resolved by the 16 bit CCD camera used in the demonstration. In the case of the halftone method, we need to poke only $N_{\text{sub}} \times N_{\text{sub}}$ subregions as mentioned above. For the control parameters shown in case (d) of Table 3, for example, $96 \times 96$ subregions must be poked, resulting in an intensity change of roughly $1/9000$. We expect that this level of the intensity change can be detected by the 16 bit CCD camera.

After constructing the control matrix, the phase correction map to be applied to the WFC device is derived by performing a pseudo inversion of the control matrix to the electric field vector of the speckle field. The influence function of the WFC device is now assumed to be the top-hat function, which is an approximate model of the SLM. Finally, the halftone method is applied to each subregion of the phase correction map derived by EFC, as in the procedure based on the SAN algorithm.

The optical parameters assumed in the numerical simulation are set to be identical to the ones in the previous simulation summarized in Table 1. The leftmost panel of Figure 11 is the
The final focal plane image before WFC. The solid red line shows the target dark-hole region whose inner and outer radii are $3 \lambda/D$ and $8 \lambda/D$, respectively. The results of the numerical simulation are summarized in Table 5. In cases (a)–(c), the number of pixels across the pupil diameter is set to $N = 64$. In case (a), we assume an ideal WFC device with continuous phase modulation that does not use the halftone method. Figure 11(a) shows the final focal plane image after WFC. The mean contrast is evaluated over the region indicated by the red dashed line in the leftmost panel of Figure 11, which is the region reduced by $1 \lambda/D$ from the target region. (a)–(d) Images after WFC (10 iterations) for cases (a)–(d) summarized in Table 5.

Figure 11. Final focal plane images acquired by the EFC algorithm. The leftmost panel shows the image before WFC. The target dark-hole region is indicated by a solid red line whose inner and outer radii are $3 \lambda/D$ and $8 \lambda/D$, respectively. Mean contrasts are evaluated over the region indicated by the red dashed line, which is reduced by $1 \lambda/D$ from the target region. (a)–(d) Images after WFC (10 iterations) for cases (a)–(d) summarized in Table 5.

Figure 12. Left: mean contrasts of the numerical simulation as a function of the iteration assuming the EFC. Red, blue, green, and magenta lines are results for cases (a)–(d) in Table 5. Right: radial average contrast of each case.

Table 5

| Case | Pixel Format | Subregion Size | Nyquist Region | Phase Resolution | Halftone | Mean Contrast |
|------|--------------|----------------|----------------|------------------|----------|--------------|
|      | $N \times N$ (pixels) | $N_{sub} \times N_{sub}$ (pixels) | $N_{nyq} \times N_{nyq}$ ($\lambda/D$) | $\sigma_{res}$ (rad) |          |              |
| (a)  | 64 $\times$ 64 | 1 $\times$ 1 | 64 $\times$ 64 | Continuous | No        | $5.3 \times 10^{-11}$ |
| (b)  | 64 $\times$ 64 | 1 $\times$ 1 | 64 $\times$ 64 | $2\pi/1024$ | No        | $2.9 \times 10^{-9}$ |
| (c)  | 64 $\times$ 64 | 4 $\times$ 4 | 16 $\times$ 16 | $2\pi/1024$ | Yes       | $3.5 \times 10^{-9}$ |
| (d)  | 256 $\times$ 256 | 4 $\times$ 4 | 64 $\times$ 64 | $2\pi/1024$ | Yes       | $6.1 \times 10^{-11}$ |

Note. The initial contrast before WFC is $5.8 \times 10^{-6}$.

The final focal plane image before WFC. The solid red line shows the target dark-hole region whose inner and outer radii are $3 \lambda/D$ and $8 \lambda/D$, respectively.

The results of the numerical simulation are summarized in Table 5. In cases (a)–(c), the number of pixels across the pupil diameter is set to $N = 64$. In case (a), we assume an ideal WFC device with continuous phase modulation that does not use the halftone method. Figure 11(a) shows the final focal plane image after WFC. The mean contrast is evaluated over the region indicated by the red dashed line in the leftmost panel of Figure 11, which is the region reduced by $1 \lambda/D$ from the target dark hole. Figure 12 (left) shows mean contrasts for cases (a)–(d) as a function of the iteration. The radial average contrasts for cases (a)–(d) are shown in Figure 12 (right), together with that before WFC. For the simulation in case (a), the final contrast reached $5.3 \times 10^{-11}$ after 10 iterations.

When the WFC device has a quantized phase resolution of $2\pi/1024$, in case (b), the mean contrast was degraded to...
Next, in case (c), we applied the halftone method to the WFC device with a phase resolution of \(2\pi/1024\). The final contrast after 10 iterations was \(3.5 \times 10^{-9}\), which was slightly worse than, but comparable to, the final contrast in case (b). This was because the achievable contrasts in these cases were limited by the phase resolution of the WFC device. Although the halftone method did not improve the achievable contrast, applying the halftone method to the EFC algorithm may still be useful to reduce the size of the control matrix of the EFC procedure, and would save computer resources, such as memory.

In case (d), we conducted the numerical simulation assuming a larger number of pixels of \(N = 256\) across the beam diameter, and applying the halftone method with a subregion size of \(N_{\text{sub}} = 4\). In case (d), the size of the control matrix for the EFC procedure is comparable to that of case (a) and (b), that is, there are \(N = 64\) pixels across the beam diameter and the halftone method is not used. The contrast reached \(6.1 \times 10^{-11}\), which was comparable to that of case (a), in which an ideal WFC device with continuous phase modulation is assumed. Importantly, the contrast in case (d) became much better than that of case (b), in which the WFC device with the same quantized phase resolution was used without the halftone method.

### 6. Conclusion

We proposed a high-contrast imaging technique that uses a coronagraph and halftone WFC to improve the achievable contrast. The WFC system generates a dark hole by suppressing residual speckles from the coronagraph. The halftone method helps to improve the achievable contrast, which is otherwise limited by the phase resolution of the WFC device.

In the numerical simulation using the SAN algorithm, the contrast was limited to \(1.4 \times 10^{-9}\) by the assumed phase resolution of \(2\pi/1024\). By using the halftone method, we improved the contrast to \(3.8 \times 10^{-10}\), which was comparable to the achievable contrast with ideal continuous phase modulation.

We performed a laboratory demonstration by introducing the halftone WFC to the 8OPM coronagraph. As the WFC device, we used an SLM with a phase resolution of \(2\pi/1024\). The contrast was not improved despite the use of the halftone method. To investigate the effect of the halftone method, we intentionally degraded the phase resolution to \(2\pi/256\). As shown in Figure 7, the final contrast was improved to \(6.0 \times 10^{-8}\) compared with \(2.2 \times 10^{-7}\) without the halftone method in the case of the dark hole generated at a large distance over a small region. Even with the halftone method, the experimental results did not reach the contrast level demonstrated by the numerical simulations. We suppose that the achievable contrast in the laboratory may be limited due to the environment of the laboratory testbed, such as atmospheric turbulence, mechanical vibrations, and incoherent light due to imperfect polarization control and imperfect optical components.

One advantage of the halftone method is that the size of the control matrix for the EFC algorithm can be reduced. We also carried out a numerical simulation of the halftone method using EFC. We achieved a contrast of \(3.5 \times 10^{-9}\), which was comparable to that of \(2.9 \times 10^{-9}\) obtained without the halftone method. Both results were obtained assuming a WFC device with a pixel format of \(64 \times 64\) over a light beam at the entrance pupil. Importantly, the size of the control matrix could be reduced to that corresponding to the WFC device of \(16 \times 16\) pixels by introducing the halftone method.

The numerical simulation and the laboratory demonstration reported in this paper suggest that the halftone method can achieve an extremely high contrast on the order of \(10^{-10}\), which could be used for detecting an Earth-like exoplanet around a Sun-like star, even with a WFC device with a low phase resolution. The halftone method is also expected to save computer resources, such as memory, for matrix-based WFC algorithms, such as EFC.

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