Supercavitation: Theory, experiment and scale effects

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Abstract. The article presents a number of dependences for reliable calculation of parameters required when performing supercavitation experiments. They include the equations and dependences for calculation of the shape and basic dimensions of cavities formed behind axisymmetric cavitators, influence of the free surface, cavity buoyancy, and cavitator lift, as well as compensation for certain scale effects.

1. Introduction

The intrinsic characteristics of supercavitation vehicles are very high flow velocities and large dimensions of the bodies. When performing experiments in cavitation tunnels or ducts, the test conditions are necessarily limited to smaller motion velocities and dimensions of the test bodies. The aim of this work is to present dependences for the experimental conditions that can be used effectively to calculate test conditions for experiments. To derive these dependences, asymptotic solutions are applied jointly with the nonlinear numerical calculation results based on the ideal incompressible liquid model, including calculation of flow around disks and cones within the framework of the symmetric Ryabushinsky scheme [1], as well as flows past thin cones with cavity closure on a disk.

2. Asymptotic solutions and practical dependencies

Small cavitation numbers $\sigma$ imply highly elongated cavities and comparatively small cavitators, as well as a weak dependence of the cavitator shape in the case of equal drag on the most part of the cavity form. In this case, the asymptotic second-order dependence of the cavity’s maximum radius $R_k$, half-length $L_k$, and aspect ratio $\lambda = L_k / R_k$ have the known following form but with values of $k$ and $\mu$ which are defined by dependencess on $\lambda$ [2]:

$$\lambda^2 = \frac{2\mu}{\sigma}, \quad R_k^2 = R_n^2 \frac{c_d}{k\sigma}, \quad L_k = \frac{R_n}{\sigma} \left( \frac{2c_d\mu}{k} \right)^{\frac{1}{2}}, \quad \mu = \ln \frac{\lambda}{\sqrt{e}}, \quad k = \left( 1 + \frac{2\ln 2/\sqrt{e}}{\ln \lambda^2} \right)^{-1}. \quad (1)$$

In equation (1), $\sigma = 2\Delta P / \rho U_\infty^2$ is the cavitation number, where $\Delta P$ is difference between ambient pressure $P_\infty$ and pressure inside the cavity $P_c$, $U_\infty$ is the incident flow velocity, $\rho$ is the mass density of the liquid, and $c_d$ is the cavitator drag coefficient within the ideal incompressible liquid model framework. The parameter $\mu$ characterizes the inertial properties of cavity sections while $k$ accounts for the small
transfer of energy between cavity sections. The approximations for these parameters, which preserve the first term of the asymptotic row and extend the applicable range of these formulas to \( \sigma < 0.2 \), are:

\[
\gamma > 1^\circ + 2^\circ, \sigma < 0.2 : \mu = \ln\left(\frac{\lambda + 0.9}{\sqrt{\epsilon}}\right) \text{ or } \mu = \ln\left(\frac{0.9\lambda + 1.3}{\sqrt{\epsilon}}\right) ; \quad \mu = \ln\left(\frac{1.3}{\sigma}\right) \text{ or } \mu = \ln\left(\frac{(1.8)}{\sigma}\right)^{1/2} - 0.65
\]

(2)

\[
\gamma > 10^\circ, \sigma < 0.2 : \quad k = \left[1 + \frac{2\ln 2 / \sqrt{\epsilon}}{\ln(0.6\lambda + 3)^2}\right]^{-1} ; \quad k = 1 - \frac{2\ln 2 / \sqrt{\epsilon}}{\ln(4/\sigma + 18)}
\]

(3)

These formulas are applicable to conical cavitators over a wide range of it’s semi-angles \( \gamma \) included disc. For slender cavitators \( \lambda \) corresponds to the cavity rear part after midsection. Accuracy of single-parameter approximations is \( \sim 3 + 5\% \), with two- parameters ones being \( \sim 2 + 3\% \). In view of the known principle of cavity section expansion independence [3], these dependencies can be used for approximate prediction of the steady and unsteady cavities and cavitators of different shapes.

3. Hydrodynamic forces at the cavitator

For cones with semi-angles \( 0 < \gamma \leq 90^\circ \), \( \beta(\gamma) = \gamma^{\circ} \left(90^\circ\right)^{-1} \), the drag coefficient \( c_d = c_{do} + \kappa_\sigma \sigma \) is:

\[
c_{do} = \frac{(\pi \beta)^2}{2} \ln\left[\frac{1.74}{\pi \beta} \left(1 - 0.45 \sqrt{\beta + 2 \beta}\right)\right] - 0.87\beta^{1.7} , \quad \kappa_\sigma = \left(0.46 + 0.39 \beta + \frac{0.015}{\beta^{1.36} + 0.028}\right)
\]

(4)

4. Axisymmetric cavity shape and dimensions

The simplest boundary-value problem for extended axisymmetric cavities [2] has the following form:

\[
\mu_u \frac{d^2 R^2}{dx^2} + \sigma(x) = 0, \quad \frac{dR^2}{dx} \bigg|_{x=0} = R_n \left(\frac{2(c_d - k \sigma)}{k \mu_u}\right)^{1/2} , \quad R^2 \bigg|_{x=0} = R_n^2 , \quad \mu_u = \left(\frac{2\sqrt{c_{d*}}}{\sqrt{c_{d*}} - k \sigma + \sqrt{c_{d*}}}\right)^2
\]

(5)

Here \( \mu_u - \mu \) for \( c_{d*} \approx 0.82 \) is incorporated for a slight adjustment. Equation (5) is applicable for disk-type and slender cavitators. Assume a cavity \( R = R(x) \) with the maximum radius \( R_k \), midsection coordinate \( L_m \), half-length \( L_k \) (length of a cavity section after the midsection), and the cavity total length \( L_c \). Then at \( \sigma = \text{const} \) the solution for equation (6) is derived from the following equations:

\[
R^2 = R_n^2 + R_n \left[\frac{2(c_d - k \sigma)}{k \mu_u}\right]^{1/2} , \quad R_k = R_n \left(\frac{c_d}{k \sigma}\right)^{1/2} , \quad L_m = \frac{R_n}{\sigma} \left[\frac{2 \mu_u (c_d - k \sigma)}{k}\right]^{1/2}
\]

(6)

\[
L_k = \frac{R_n}{\mu_u} \left(\frac{2 \mu_u c_d}{k}\right)^{1/2} , \quad L_c = \frac{R_n}{\sigma} \left(\frac{2 \mu_u c_d}{k}\right)^{1/2} \left[\sqrt{c_d - k \sigma} + \sqrt{c_{d*}}\right]
\]

5. Free-surface effect

The free-surface influence on a cavity form is modeled by known model of the influence of mirror-image flow around a cavity, which is simulated by a source and a drain within the small cavitator model framework. Here new more simple approach of the model realization is proposed. The problem solution is reduced to solving the four relationships in equation (7) for the parameters with subscript \( h \), where parameters with subscript \( o \) correspond to a cavity without accounting for the free-surface effect. The free-surface effect is accounted for by deriving a corrected value of cavitation number \( \sigma_h \) for \( L_k \), the cavity half-length, \( R_k \), the maximum radius, \( \mu \), and \( H \), the depth. In view of small variations of values \( \mu_h \) and \( H_h \), even the first approximation for \( \sigma_h \) is quite accurate, starting from the first two equations (8) for \( \bar{L}_{kh} \) and \( \bar{R}_{kh} \) at values \( \mu_o \) and \( H_o \):
\[ \bar{T}_{kh} = 1 + \frac{1 - \bar{H}_h}{2 \left( \bar{H}_h (1 - 2 \ln 2 \bar{H}_h - 4 \mu_h) - 1 \right)}, \quad \bar{R}_{kh}^2 = \frac{8 \mu_h \bar{H}_h \bar{T}_{kh} (1 - \bar{T}_{kh})}{\bar{L}_{kh} - \bar{H}_h}, \]

\[ \mu_h = \ln \left( \frac{1}{\sqrt{e}} \left( 0.9 \frac{\mu_h \bar{T}_{kh}}{\bar{R}_{kh}} \lambda_0 + 1.3 \right) \right), \] (7)

\[ \bar{H}_h = \bar{R} \sqrt{\frac{\mu_h}{\bar{H}_h}} : \sigma_h = \frac{2 \mu_h}{\lambda_0^2}; \quad \bar{L}_{kh} = \sqrt{\frac{\mu_h \bar{L}_{kh}}{\mu_h \bar{L}_{ko}}}, \quad R_{kh} = \frac{R_k}{R_{ko}}, \quad \bar{H} = \frac{H}{L_{ko}} \]

6. Cavity axis deformations

Cavity axis distortions due to gravity \((h_g)\) and to cavitator lift force \((h_o)\) are described by the equations (8), where \(c_{dy}\) is cavitator lift force [3]:

\[ \frac{dh_h}{dx} = \frac{1}{R^2(x) U_{\infty}} \int_0^x R^2(x) dx, \quad \frac{dh_o}{dx} = \frac{c_{dy} R_n^2}{2 R^2(x)}, \quad \frac{U_{\infty}^2}{g R_n^2 c_d} > 11 \] (8)

Ranges of small deviations from the circular shape of cavity sections are estimated by inequalities (8), which are valid for a much wider range for disk-type and thin cavitators with some limitations for a disk with a nonzero angle of attack.

7. Modelling problems – scale effects

Both calculated and experimental results, including those depicted in Fig 1, imply that application of cavitators with a small angle of attack, in view of inequalities (8), allows one to provide a quite accurate modelling by the Froude number \(Fr = U_{\infty} / g R_n\), which produces a nearly axisymmetric cavity with the maximum radius and length corresponding to a weightless liquid.

Figure 1 shows a photo of an experimental cavity [4] behind a cone with a semi-angle of \(\gamma = 20^\circ\), which profile is made parallel to the free border by means of the cavitator angle of attack \(\alpha = 5^\circ\).

![Figure 1](image)

**Figure 1.**

a) Experimental cavity behind a cone \((\gamma = 20^\circ, d = 5cm, \alpha = 5^\circ)\) in the water duct at depth \(H = 10cm\) moving with velocity \(U_{\infty} = 9.75 m / s\) for \(\sigma = 0.039\) [4].

b) Calculated form of experimental cavity with normalization by \(R_n\):

- - - experimental cavity shape calculated via equations (5, 7, 8): \(\alpha = 5^\circ\)
- - - axisymmetric part of the cavity via equations (5, 7): \(Fr = \infty, \alpha = 0^\circ\)

The results of processing of this photo and calculations of the cavity shape and dimensions via equations (6) and then (7) for \(Fr = 0\) with account of the free surface influence only yield the
corrected cavitation number \( \sigma_h \approx 0.0486 \) (vs. experimental \( \text{exp:} \) 0.039), as well as cavity dimensions with account of the free surface effect: \( R_k = 0.057m \) (exp: 0.057), \( L_c = 0.90m \) (exp: 0.88).

The way of compensation of scale effects by the Froude number with help of lift on a slender cavitator seems to be the only option for conducting experiments in water ducts. Typical experimental curves are characterized by the minimal gas loss values at the intermediate stage between the gas loss processes via the chaotic flow in cavity wake and via vertical hollow tubes cords. Lift forces acting on a cavity, in particular, lift force at cavitator with a nonzero angle of attack, drastically affect the flow vortex system, which controls gas loss processes. The experimental studies confirm that, in case the lift force at cavitator exceeds the hydrostatic forces acting on a cavity, an additional pair of hollow vortex tubes appears in the cavity rear part. In this case, the gas loss vs. cavitator angle of attack curve exhibits two minimum and one maximum peaks [5]. If the above forces are equal, the transverse momentum of the cavity wake is zero, while the maximum gas loss, which is possible within the small cavitator model framework, is controlled by equation (9), \( \eta = 1 - \sqrt{1 - 2\sigma / c_d} : 

\[ c_{d_y}^* = 2 \left[ \frac{4}{3} \eta^2 \left( 1 - \frac{\eta}{3} \right) \right] \left( \frac{c_d}{k} \right)^{1.5} \frac{\sqrt{2\mu g R_k}}{\sigma^2 U_{\infty}^2} = 2 \left[ \frac{4}{3} \left( \frac{\beta}{2} \frac{c_d}{k} \right)^{1.5} \frac{\sqrt{2\mu g R_k}}{\sigma^2 U_{\infty}^2} \right] = 3 \left( \frac{2\mu}{3} \frac{g R_k}{c_d} \right)^{1.5} \frac{\sqrt{2\mu g R_k}}{\sigma^2 U_{\infty}^2} \] (9)

In the particular case of cavitator having a half-cavity shape \( \eta = 1, c_d = k \sigma \): \( \alpha^* = \frac{2}{3}\sqrt{2\mu g R_k} \); \( \Gamma = \frac{3}{4} \pi \alpha R_k U_{\infty} \); \( \Gamma_k = 2\alpha R_k U_{\infty} \) (10)

8. Conclusions
The presented dependences are recommended for reliable estimation of the basic dimensions and shapes of cavities for a prompt conduction of cavitation experiments.

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