The curvaton field and the intermediate inflationary universe model

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Abstract

The curvaton in an intermediate inflationary universe model is studied. This study has allowed us to find some interesting constraints on different parameters that appear in the model.

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I. INTRODUCTION

An intermediate inflation model was introduced as an exact solution for a particular scalar field potential of the type $V(\phi) \propto \phi^{-\beta}$ [1], where $\beta$ is a free parameter. With this sort of potential, and with $\beta > 0$, it is possible in the slow-roll approximation to have a spectrum of density perturbations which presents a scale-invariant spectral index $n_s = 1$, i.e. the so-called Harrison-Zel’dovich spectrum of density perturbations, provided $\beta$ takes the value two [2]. Even though this kind of spectrum is disfavored by the current Wilkinson Microwave Anisotropy Probe (WMAP) data [3], the inclusion of tensor perturbations, which could be present at some point by inflation and parametrized by the tensor-to-scalar ratio $r$, the conclusion that $n_s \geq 1$ is allowed provided that he value of $r$ is significantly nonzero [4]. In fact, in ref. [5] was shown that the combination $n_s = 1$ and $r > 0$ is given by a version of the intermediate inflation model in which the scale factor varies as $a(t) \propto e^{(t/t_o)^{2/3}}$ and the slow-roll approximation was used.

The main motivation to study this sort of model becomes from string/M-theory. This theory suggests that in order to have a ghost-free action high order curvature invariant corrections to the Einstein-Hilbert action must be proportional to the Gauss-Bonnet (GB) term $\alpha$. GB terms arise naturally as the leading order of the $\alpha$ expansion to the low-energy string effective action, where $\alpha$ is the inverse string tension [7]. This kind of theory has been applied to possible resolution of the initial singularity problem [8], to the study of Black-Hole solutions [9], accelerated cosmological solutions [10]. In particular, very recently, it has been found [11] that for a dark energy model the GB interaction in four dimensions with a dynamical dilatonic scalar field coupling leads to a solution of the form $a(t) = a_0 \exp(A t f)$, where the universe starts evolving with a decelerated exponential expansion. Here, the constant $A$ becomes given by $A = \frac{2}{\kappa n}$ and $f = \frac{1}{2}$, with $\kappa^2 = 8\pi G$ and $n$ is a constant. In this way, the idea that inflation, or specifically, intermediate inflation, comes from an effective theory at low dimension of a more fundamental string theory is in itself very appealing.

The characteristic of the scalar potential $V(\phi)$ in this kind of model it does not present a minimum, so that the usual mechanism introduced to bring inflation to an end becomes useless. In fact, the standard mechanism is described by the stage of oscillations of the scalar field which is a essential part of the so called reheating mechanism, where most of the matter and radiation of the universe was created, via the decay of the inflaton field, while
the temperature grows in many orders of magnitude. It is at this point where the Big-Bang universe is recovered. Here, the reheating temperature, the temperature associated to the temperature of the universe when the Big-Bang model begins, is of particular interest. In this epoch the radiation domination begins, in which there exist a number of particles of different kinds.

The stage of oscillation of the scalar field is a essential part for the standard mechanism of reheating. Therefore a minimum in the inflaton potential is something crucial for the reheating mechanism. However, there are models where such a minimum does not exist, and thus the standard mechanism of reheating does not work\cite{12}. These models are known in the literature like non-oscillating models, or simple NO models\cite{13}. One of the mechanism of reheating in this kind of models is the introduction of the curvaton field\cite{14, 15}. Here, the decay of the curvaton field into conventional matter offers an efficient mechanism of reheating, and its field has the property whose energy density is not diluted during inflation so that the curvaton may be responsible for some or all the matter content of the universe at present. On the other hand, this field may also be the responsible for explaining the observed large scale structure of the universe.

In the context of intermediate inflation we would like to introduce the curvaton field as a mechanism to bring intermediate inflation to an end. Therefore, the main goal of the present paper is to implement the curvaton field into the intermediate inflationary scenario and see what consequences we may extract.

The outline of the paper goes as follow: in section II we give a brief description of the intermediate inflationary scenario. In section III the curvaton field is described in the kinetic epoch. Section IV describes the curvaton decay after its domination. Section V describe the decay of the curvaton field before it dominates. Section VI studies the consequences of the gravitational waves. At the end, section VII exhibits our conclusions.

II. INTERMEDIATE INFLATION MODEL

In order to describe intermediate inflationary universe models we start with the following field equations in a flat Friedmann-Robertson-Walker background

\[ 3 H^2 = \frac{\dot{\phi}^2}{2} + V(\phi), \]  

(1)
and
\[ \ddot{\phi} + 3H \dot{\phi} = -\frac{\partial V(\phi)}{\partial \phi}, \]  
(2)

where \( a \) is a scale factor, \( H \equiv \dot{a}/a \) is the Hubble factor, \( \phi \) is the standard inflaton field and \( V(\phi) \) is the effective scalar potential. Dots here mean derivatives with respect to the cosmological time, \( t \), and we use units in which \( 8\pi G = 8\pi/m_p^2 = c = \hbar = 1 \) (\( m_p \) is the Planck mass).

Exact solutions can be found for intermediate inflationary universe model where the scale factor \( a(t) \) expands as
\[ a(t) = \exp(At^f). \]  
(3)

Here \( f \) is a constant parameter with range \( 0 < f < 1 \) and \( A \) is a positive constant.

From Eqs.\((1)\), \((2)\) and \((3)\) the expressions for the scalar potential, \( V(\phi) \) and the scalar field, \( \phi(t) \), become
\[ V(\phi) = \frac{8A^2}{(\beta + 4)^2} \left[ \frac{\phi}{(2A\beta)^{1/2}} \right]^{-\beta} \left[ 6 - \frac{\beta^2}{\phi^2} \right], \]  
(4)

and
\[ \phi(t) = (2A\beta t^f)^{1/2}, \]  
(5)

respectively. Here, the parameter \( \beta \) is defined by \( \beta \equiv 4(f^{-1} - 1) \).

The Hubble parameter as a function of the inflaton field, \( \phi \), becomes
\[ H(\phi) = A f (2A\beta)^{\beta/4} \phi^{-\beta/2}. \]  
(6)

The form for the scale factor \( a \) expressed by Eq.\((3)\) also arises when we solve the field equations in the slow roll approximation, where a simple power law scalar potential is considered
\[ V(\phi) = \frac{48 A^2}{(\beta + 4)^2} (2A\beta)^{\beta/2} \phi^{-\beta}. \]  
(7)

Note that this kind of potential does not present a minimum. Also, the solutions for \( \phi(t) \) and \( H(\phi) \) obtained with this potential in the slow roll approximation are identical to those obtained in the exact solution, expressed by Eqs.\((5)\) and \((6)\).

The slow roll parameters \( \varepsilon \) and \( \eta \) are defined by \( \varepsilon = \frac{\dot{V}}{V^2} \) and \( \eta = V''/V \), respectively, where the prime denotes derivative with respect to the inflaton field \( \phi \). In our case they reduced to \( \varepsilon = \frac{\beta^2}{4\phi^2} \) and \( \eta = \frac{\beta(\beta+1)}{\phi^2} \), and its ratio, \( \varepsilon/\eta \) becomes \( \varepsilon/\eta = \frac{1}{2} \left( \frac{\beta}{\beta+1} \right) \). Note that \( \eta \) is always larger than \( \varepsilon \). Since \( \beta \) is positive, \( \eta \) reaches unity before \( \varepsilon \) does. In this way, we
may establish that the end of inflation is governed by the condition $\eta = 1$ more then $\varepsilon = 1$, from which we get, at the end of inflation $\phi^2_e = \beta (\beta + 1)$, for the inflaton field. From here on, the subscript $e$ is used to denote the end of the inflationary period.

III. THE CURVATON FIELD DURING THE KINETIC EPOCH

Neglecting the term $\frac{\partial V(\phi)}{\partial \phi}$ when compared with the friction term $3H\dot{\phi}$ in the field Eq. (2), the model enters to a new period which is called the ‘kinetic epoch’ or ‘kination’. In the following we will use the subscript (or superscript)’k’ to label different quantities at the beginning of this epoch. Note that during the kination epoch we have that $\dot{\phi}^2/2 > V(\phi)$ which could be seen as a stiff fluid since the relation between the pressure $P_\phi$ and the energy density $\rho_{\phi}$, corresponds to $P_\phi = \rho_{\phi}$.

In the kinetic epoch the field equations (1) and (2) becomes $3H^2 = \frac{\dot{\phi}^2}{2}$ and $\ddot{\phi} + 3H \dot{\phi} = 0$ where the latter equation could be solved and gives $\dot{\phi} = \dot{\phi}_k \left(\frac{a_k}{a}\right)^3$. This expression yields to

$$\rho_{\phi}(a) = \rho_{\phi}^k \left(\frac{a_k}{a}\right)^6,$$  \hspace{1cm} (8)

and the Hubble parameter becomes

$$H(a) = H = H_k \left(\frac{a_k}{a}\right)^3,$$  \hspace{1cm} (9)

where $H_k^2 = \frac{\rho_{\phi}^k}{3} \simeq \frac{\dot{\phi}_k^2}{6}$ is the value of the Hubble parameter at the beginning of the kinetic epoch.

The curvaton field obeys the Klein-Gordon equation and we will assume that the scalar potential associated to this field is given by $U(\sigma) = \frac{m^2 \sigma^2}{2}$, where $m$ is the curvaton mass.

Firstly, we assume that the energy density associated to the inflaton field, $\rho_{\phi}$, is the dominant component when compared with the curvaton energy density, $\rho_{\sigma}$. Secondly, the curvaton field oscillates around the minimum of its effective potential $U(\sigma)$. During the kinetic epoch the universe remains inflaton-dominated where the curvaton density evolves as a non-relativistic matter, i.e. $\rho_{\sigma} \propto a^{-3}$. The final stage corresponds to the decay of the curvaton field into radiation and thus the standard Big-Bang cosmology is recovered.

During the inflationary regime it is assumed that the curvaton field is effectively massless \[16, 17, 18, 19\]. In the same period the curvaton rolls down its potential until its kinetic energy is depleted by the exponential expansion. Its kinetic energy has almost vanished,
and it becomes frozen. The curvaton field assumes roughly a constant value, \( \sigma_\ast \approx \sigma_e \). Here, the subscript "\( \ast \)" refers to the epoch when the cosmological scale exit the horizon.

The hypothesis assumed here is that during the kinetic epoch the Hubble parameter decreases so that its value is comparable with the curvaton mass, i.e. \( m \simeq H \) (at this stage, the curvaton field becomes effectively massive). From Eq. (9), we obtain

\[
\frac{m}{H_k} = \left( \frac{a_k}{a_m} \right)^3,
\]

where the subscript 'm' stands for quantities at the time when the curvaton mass, \( m \), is of the order of \( H \) during the kinetic epoch.

In order to prevent a period of curvaton-driven inflation the universe must still be dominated by the inflaton field, i.e. \( \rho_\phi|_{a_m} = \rho_\phi^m \gg \rho_\sigma (\sim U(\sigma_e) \simeq U(\sigma_\ast)) \). This inequality allows us to find a constraint on the values of the curvaton field \( \sigma_\ast \). At the moment when \( H \simeq m \), we get that

\[
\frac{m^2 \sigma_\ast^2}{2\rho_\phi^m} \ll 1,
\]

which implies that the curvaton field \( \sigma_\ast \) satisfies the constraint \( \sigma_\ast^2 \ll 6 \), where we have used \( \rho_\phi^m = \rho_\phi^k \left( \frac{a_k}{a_m} \right)^6 = \rho_\phi^k \left( \frac{m}{H_k} \right)^2 \).

At the end of inflation, the ratio between the potential energies becomes

\[
\frac{U_e}{V_e} = \frac{m^2 \sigma_\ast^2}{6H_e^2} < 1,
\]

and, in this way, the curvaton energy becomes subdominant at the end of inflation. The curvaton mass should obey the constraint

\[
m^2 < H_e^2 = \frac{16 A^2}{(\beta + 4)^2} \left[ \frac{2A}{\beta + 1} \right]^{\beta/2},
\]

imposed by the fact that the curvaton field must be effectively massless during the inflationary era, and thus, \( m < H_e \). In the latter expression we have used the relation \( V_e = \frac{48A^2}{(\beta + 4)^2} (2A)^{\beta/2} \phi_e^{-\beta} \).

At the time when the mass of the curvaton field becomes important, i.e. when \( m \simeq H \), its energy density decays like a non-relativistic matter in the form \( \rho_\sigma = \frac{m^2 \sigma_\ast^2 a^3}{2a^3} \), since their potential and kinetic energy densities are comparable due to the curvaton is undergoing quasi-harmonic oscillations.
IV. CURVATON DECAY AFTER DOMINATION

The decay of the curvaton field may occur in two possible different scenarios. Firstly, if the curvaton field comes to dominate the cosmic expansion (i.e. \( \rho_\sigma > \rho_\phi \)), then, there must be a moment in which the inflaton and curvaton energy densities become equals. Let us assume that this happen when \( a = a_{eq} \), then, from Eqs.(8), (10) and using that \( \rho_\sigma \propto a^{-3} \) we get

\[
\frac{\rho_\sigma}{\rho_\phi} \bigg|_{a=a_{eq}} = \frac{m^2 \sigma^2 a^3}{2a^6 \rho^k_\phi} = \frac{m^2 \sigma^2 a^3}{6 H_k^2 a^6} = 1, \tag{14}
\]

which yields to \( H_k \left( \frac{a_k}{a_{eq}} \right)^3 = \frac{m^2}{6} \), where we have used the relation \( 3 H_k^2 = \rho^k_\phi \) together with Eq.(10).

From Eqs.(9), (10) and (14), we write a relation for the Hubble parameter, \( H(a_{eq}) = H_{eq} \), in terms of the curvaton parameters

\[
H_{eq} = H_k \left( \frac{a_k}{a_{eq}} \right)^3 = \frac{m \sigma^2}{6}. \tag{15}
\]

Since the decay parameter \( \Gamma_\sigma \) is constrained by nucleosynthesis, it is required that the curvaton field decays before nucleosynthesis, which means \( H_{nucl} \sim 10^{-40} < \Gamma_\sigma \) (in units of Planck mass \( m_p \)). We also require that the curvaton decay occurs after domination, i.e. \( \rho_\sigma > \rho_\phi \), and also for \( \Gamma_\sigma < H_{eq} \). Thus, we get a constraint on the decay parameter \( \Gamma_\sigma \), which is given by

\[
10^{-40} < \Gamma_\sigma < \frac{m \sigma^2}{6}. \tag{16}
\]

It is interesting to find constraints on the parameters appearing in our model by studying the scalar perturbations related to the curvaton field \( \sigma \). In general, we may say that the curvaton field creates the curvature perturbations in two separate stages. In the first stage, the quantum fluctuations during inflation are converted to classical perturbations characterized by a flat spectrum at horizon exit. Then, in the second period (at the time after inflation), the perturbations are converted into curvature perturbations. Differently, to the usual mechanism, the generation of curvature perturbations by the curvaton field require no assumptions about the nature of inflation, except by the requirement that the Hubble parameter remains practically constant.

During the time in which the fluctuations are inside the horizon, they obey the same differential equation of the inflaton fluctuations. We may conclude that they acquire the
amplitude $\delta \sigma \simeq H_*/2\pi$. On the other hand, outside of the horizon, the fluctuations obey the same differential equation as the unperturbed curvaton field and then, we expect that they remain constant during inflation. The Bardeen parameter, $P_\zeta$, whose observed value is about $2 \times 10^{-9}$, allows us to determine the value of the curvaton field, $\sigma_*$, in terms of the parameters $A$ and $\beta$. At the time when the decay of the curvaton field occurs the Bardeen parameter becomes

$$P_\zeta \simeq \frac{1}{9\pi^2} \frac{H_*^2}{\sigma_*^2}.$$  

The spectrum of fluctuations is automatically gaussian for $\sigma_*^2 \gg H_*^2/4\pi^2$, and is independent of $\Gamma_\sigma$. This feature will simplify the analysis in the space parameter of our model.

From expression (17), we may write

$$A = \left[ \frac{27\pi^2}{48} \sigma_*^2 (\beta + 4)^2 P_\zeta \left( \frac{\beta + 1}{2} - N_* \right)^{\beta/2} \right]^{2/3},$$  

where

$$N_* = \int_{t_*}^{t_e} H(t') \, dt' = \frac{1}{2\beta} (\phi_e^2 - \phi_*^2),$$

defines the number of the e-folds corresponding to the cosmological scales, i.e. the number of remaining inflationary e-folds at the time when the cosmological scale exits the horizon. Note that the parameter $\beta$ satisfies $\beta > 2 N_* - 1$ or equivalently $4 (2N_* + 3)^{-1} > f$.

Now, the constraint given by Eq. (13) becomes

$$m^2 < 9 \pi^2 \sigma_*^2 P_\zeta \left( 1 - \frac{2 N_*}{\beta + 1} \right)^{\beta/2},$$

and with the help of Eqs.(16) and (20) we may write

$$\Gamma_\sigma < \frac{\pi}{2} \sigma_*^3 P_\zeta^{1/2} \left( 1 - \frac{2 N_*}{\beta + 1} \right)^{\beta/4},$$

which gives an upper limits on $\Gamma_\sigma$ when the curvaton field decays after domination.

On the other hand, we give the constraints on the parameters $A$ and $\beta$ by using the Big Bang Nucleosynthesis (BBN) temperature $T_{BBN}$. We know that reheating occurs before the BBN, where the temperature is of the order of $T_{BBN} \sim 10^{-22}$ (in unit of $m_p$), and thus the reheating temperature has to satisfies the inequality $T_{reh} > T_{BBN}$. By using that $T_{reh} \sim \Gamma_\sigma^{1/2} > T_{BBN}$ we obtain the constraint

$$H_*^2 = 16 \left( \frac{A}{4 + \beta} \right)^2 \left[ \frac{2 A}{\beta + 1 - 2 N_*} \right]^{\beta/2} > (540 \pi^2)^{2/3} P_\zeta^{2/3} T_{BBN}^{4/3} \sim 10^{-33},$$
where we have taken the scalar spectral index $n_s$ close to one, and therefore $m \leq 0.1H^*$. Note that Eq.(22) is similar to that described in ref.[20]. Note also this constrain gives a lower limit for the parameters $A$ and $\beta$. On the other hand, following the same ref.[20], we could write an upper limit for the Hubble parameter $H^*$, which satisfies the inequality $H^* \leq 10^{-5}$.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig1}
\caption{Contour plot for the number $N_*$ of e-folds as a function of the parameters $\beta$ and $A$, fitted from the lower limit of the BBN temperature (see Eq.(22)). Lower values: the $N_*$ parameters correspond to darker regions and the contour levels are separated by the quantity $\Delta N_* = 40$.}
\end{figure}

In Fig.1 we plot contours curves corresponding to the same number of e-folds, $N_*$, and different combinations of the $\beta$ and $A$ parameters by fitting Eq.(22) in its lower limit. Here, we have taken $T_{BBN} \sim 10^{-22}$ (in units of $m_p$). From this plot, given a value of $N_*$, one can therefore constrain the values of $\beta$ and $A$ parameters. For instance, over the line $N_* = 60$ we could extract the values $\beta = 196$ and $A = 18$ and so for others values of these parameters. A similar graph is obtained when the upper limit, $H_* \leq 10^{-5}$, is used, except that the contour lines get bigger values for the $N_*$ parameter (see Fig.2).

\section{Curvaton Decay Before Domination}

For the second scenario, we assume that the decay of the curvaton field happens before this dominates the cosmological expansion. In this way, we need that the curvaton decays before its energy density becomes greater than the inflaton one. Additionally, the mass
FIG. 2: Contour plot for the number \( N_\ast \) of e-folds as a function of the parameters \( \beta \) and \( \lambda \), fitted from the upper limit, \( H_\ast \leq 10^{-5} \). Lower values the \( N_\ast \) parameters correspond to darker regions and the contour levels are separated by the quantity \( \Delta N_\ast = 20 \).

of the curvaton is non-negligible when compared with the Hubble expansion rate \( H \), so that we could use \( \rho_\sigma \propto a^{-3} \). We may say that the curvaton field decays at a time when \( \Gamma_\sigma = H(a_d) = H_d \) and then from Eq. (9) we get

\[
\Gamma_\sigma = H_d = H_k \left( \frac{a_k}{a_d} \right)^3,
\]

where ‘ \( d \)’ stands for quantities at the time when the curvaton decays.

If we allow the decaying of the curvaton field after its mass becomes important, (so that \( \Gamma_\sigma < m \)) and before that the curvaton field dominates the expansion of the universe (i.e., \( \Gamma_\sigma > H_{eq} \)), we may write a new constraint, given by

\[
\frac{\sigma^2}{6} < \frac{\Gamma_\sigma}{m} < 1,
\]

which results in being the same as that obtained in ref. [16].

In the second scenario, the curvaton decays at the time when \( \rho_\sigma < \rho_\phi \). If we define the \( r_d \) parameter as the ratio between the curvaton and the inflaton energy densities, evaluated at the time in which the curvaton decay occurs, i.e. at \( a = a_d \) and for \( r_d \ll 1 \) the Bardeen parameter results [15, 21]
\[ P_\zeta \simeq \frac{r_d^2 H^2}{16\pi^2 \sigma^2_*}. \]  

(25)

Defining \( r_d = \left. \frac{\rho_\sigma}{\rho_\phi} \right|_{a = a_d} \), from which we get that \( r_d = \frac{m^2 \sigma^2_2 a^3_m a^3}{6 H^4_k a^4_k} \), where we have used \( \rho_\sigma(a) = \frac{m^2 \sigma^2}{2} \left( \frac{a}{a_d} \right)^3 \) and \( \rho_\phi(a) = \rho_\phi \left( \frac{a}{a_d} \right)^6 \), and using expressions (10) and (23) we obtain

\[ r_d = \frac{m \sigma^2_2}{6 \Gamma_\sigma}. \]  

(26)

From expressions (25) and (26) we find that \( \sigma^2_2 \simeq 576 \pi^2 \frac{P_\zeta r^2}{m^2 H^2} \) and using that

\[ H^2_* = \frac{V_*}{3} = 16 \left( \frac{A}{4 + \beta} \right)^2 \left[ \frac{2 A}{\beta + 1 - 2 N_*} \right]^{\beta/2}, \]  

we get

\[ \sigma^2_* = 36 \pi^2 \frac{P_\zeta \Gamma_\sigma^2 (\beta + 4)^2}{m^2 A^2} \left[ \frac{\beta + 1 - 2 N_*}{2 A} \right]^{\beta/2}. \]  

(27)

Thus, expression (24) becomes

\[ 18 \pi^2 \frac{P_\zeta \Gamma_\sigma^2 (\beta + 4)^2}{m^2 A^2} \left[ \frac{\beta + 1 - 2 N_*}{2 A} \right]^{\beta/2} \cdot \frac{\Gamma_\sigma}{m} < 1, \]  

from which we could write the inequality

\[ \Gamma_\sigma < \frac{1}{18 \pi^2 P_\zeta (\beta + 4)^2} \left[ \frac{2 A}{\beta + 1 - 2 N_*} \right]^{\beta/2}. \]  

(28)

We see that this inequality for \( \Gamma_\sigma \) depends on the free parameters, \( A \) and \( \beta \), characteristic of the intermediate inflationary universe model.

Finally, we derive a constraint for the parameters \( A \) and \( \beta \) by using the BBN temperature \( T_{BBN} \). Since, the reheating temperature satisfies the bound \( T_{reh} > T_{BBN} \), and also \( \Gamma_\sigma > T_{BBN}^2 \), we get

\[ H^2_* = 16 \left( \frac{A}{4 + \beta} \right)^2 \left[ \frac{2 A}{\beta + 1 - 2 N_*} \right]^{\beta/2} > \left( 960 \pi^2 \right)^{2/3} P_\zeta^{2/3} T_{BBN}^{4/3} \sim 10^{-33}, \]  

where, as before, we have used the scalar spectral index \( n_s \) closed to one. Note that this constrain is similar to that obtained when the curvaton field decays before domination, as expressed by Eq. (22).

\[ VI. \ \text{CONSTRAINTS FROM GRAVITATIONAL WAVES} \]

In the same way that we have a constraint for \( \Gamma_\sigma \) parameter, we could restrict the value of the curvaton mass, but now using tensor perturbations. In this kind of model the corresponding gravitational wave amplitude can be written as

\[ h_{GW} \simeq C_1 H_*, \]  

(31)
where $C_1$ is an arbitrary constant.

Note that in this case we could take $H \ll 10^{-5}$, meaning that inflation may take place at an energy scale smaller than the grand unification. In this way, this is an advantage of the curvaton approach when compared with the single inflaton field scenario.

Now, from the approximated Friedmann Eqs. we have $H^2 = V_*/3$, and thus we may write for the gravitational wave amplitude

$$h_{GW}^2 \simeq 16 C_1^2 \left( \frac{A}{\beta + 4} \right)^2 \left( \frac{2A}{\beta + 1 - 2 N_*} \right)^{\beta/2}, \quad (32)$$

where we have used Eq.(27).

From Eqs.(13) and (32) we get the inequality

$$m^2 < \frac{h_{GW}^2}{C_1^2} \left( \frac{\beta + 1 - 2 N_*}{\beta + 1} \right)^{\beta/2}, \quad (33)$$

which gives an upper limit for the curvaton mass.

If we consider that $h_{GW}$ of the order of $10^{-5}$, and we take $C_1 \simeq 10^{-5}$ and $\beta = 250$, and if we take the number of e-fold to be $N_* = 60$, then we find that the above equation gives the following upper limit for the curvaton mass (in units of $m_p$)

$$m < 10^{-18}. \quad (34)$$

This value is closed to that considered in ref.[20].

Since after inflation the inflaton field follows an equation of state which is almost stiff the spectrum of relic gravitons presents a characteristic in which the slope grows with the frequency (spike) for models that re-enter the horizon during this epoch. This means that at high frequencies the spectrum forms a spike instead of being of flat as in the case of radiation dominated universe[24]. Therefore, high frequency gravitons re-entering the horizon during the kinetic epoch may disrupt BBN by increasing the Hubble parameter. This problem can be avoided if the following constraint is required on the density fraction of the gravitational wave[25]

$$I \equiv h^2 \int_{k_{BBN}}^{k_*} \Omega_{GW}(k) \, d \ln k \simeq 2 h^2 \epsilon \Omega_\gamma(k_0) h_{GW}^2 \left( \frac{H_*}{H} \right)^{2/3} \leq 2 \times 10^{-6}, \quad (35)$$

where $\Omega_{GW}(k)$ is the density fraction of the gravitational wave with physical momentum $k$, $k_{BBN}$ is the physical momentum corresponding to the horizon at BBN, $\Omega_\gamma(k_0) = 2.6 \times$
$10^{-5}h^{-2}$ is the density fraction of the radiation at present on horizon scales. Here, $\epsilon \sim 10^{-2}$ and $h = 0.73$ is the Hubble constant in which $H_0$ is in units of 100 km/sec/Mpc. The parameter $\tilde{H}$ represents either $\tilde{H} = H_{eq}$, when the curvaton decays after domination, or $\tilde{H} = H_d$, if the curvaton decays before domination.

For the first scenario, the decay of the curvaton field happens after that this field dominates the cosmological expansion. In this way, the constraint on the density fraction of the gravitational wave, expressed by Eq.(35), becomes

$$\frac{m}{\sigma_*^2} \gtrsim \left( \frac{P_\zeta}{4 \times 10^5} \right)^2 \sim 10^{-28},$$

where we have used expressions (15), (31) and $C_1 \sim 10^{-5}$.

When the decay of the curvaton field happens before that this field dominates, the constraint on the density fraction of the gravitational wave given by Eq.(35), becomes

$$\frac{m^2}{{\Gamma_\sigma}^{1/4}} \gtrsim 6 \times 10^{-5} P_\zeta \sim 10^{-13},$$

where we have used Eqs.(23) and (25).

Another set of bounds could be put forward by considering the decay rate of the curvaton field $\sigma$, which, in a very particular case could be considered to be $\Gamma_\sigma = g^2 m$ ref.[20], where $g$ is the coupling of the curvaton to its decay products. The allowed range for the coupling constant in this case becomes given by the expression

$$\max \left( \frac{T_{BBN}}{m^{1/2}}, m \right) \lesssim g \lesssim \min \left( 1, \frac{m \sigma_*^3}{T_{BBN}^2} \right),$$

where the inequality $m \lesssim g$ is due to gravitation decay. For the curvaton decays before domination and $T_{reh} > T_{BBN}$ this constrain gives an upper limit given by $g < m\sigma_*^3/T_{BBN}^2$, and when the curvaton decays after domination, a lower limit is obtain given by $T_{BBN} m^{-1/2} < g$.

VII. CONCLUSIONS

We have introduced the curvaton field in the intermediate inflationary universe model. We have describe the curvaton reheating in which we considered two cases. In the first case the curvaton dominates the universe before it decays. Here, we have arrived to Eq.(18), which represents an interesting constraint for the $A$ parameter that appears in the scale factor (see Eq.(33)). In the second case the curvaton decays before domination. Here, we have found a restriction for the $\Gamma_\sigma$ parameter, as shown by Eq.(29).
In the context of the curvaton scenario, reheating does occur at the time when the curvaton decays, but only in the period when the curvaton dominates. In contrast, if the curvaton decays before its density dominates the universe, reheating occurs when the radiation due to the curvaton decay manages to dominate the universe.

During the epoch in which the curvaton decays after its dominates \( \rho_\sigma > \rho_\phi \), the reheating temperature is the order of \( 3 \times 10^{-21} \) (in units of \( m_p \)), since the decay parameter \( \Gamma_\sigma \propto T_{rh}^2 \), where \( T_{rh} \) represents the reheating temperature. Here, we have used Eq. (21), with \( \sigma_* \sim 10^{-9} \), \( N_* = 60 \) and \( \beta = 250 \). We should note that this upper limit for \( T_{rh} \), could be modified. Now, by using Eqs. (34) and (38) we obtain that \( 10^{-13} \lesssim g \lesssim 10^{-1} \), and from Eq. (21) we get that \( H_d/\Gamma_\sigma \sim 10^{-18} g^{-2} \). If the decay of the curvaton field happens after domination, then it is found the range \( 10^{-13} \lesssim g \lesssim 10^{-9} \). In this way since \( T_{reh} \sim g m^{-1/2} \), the allowed range for the reheating temperature becomes \( 10^{-22} \lesssim T_{reh} \lesssim 10^{-18} \) (in units of \( m_p \)). Note that, the bounds given by Eqs. (36) and (37) may truncate further the range for the Hubble parameter expressed by \( 10^{-17} \leq H_* \leq 10^{-5} \) and Eq. (38).

Now, let us choose \( \sigma_* \sim 1 \) (following ref. [20]), \( N_* = 60 \) and \( \beta = 250 \). From expression (38) the range for \( g \) becomes \( 10^{-13} \lesssim g \lesssim 1 \), and since \( H_d/\Gamma_\sigma \sim \sigma_*^2 g^{-2} \gtrsim 1 \), the curvaton decay it produced at or after domination. Therefore, with \( T_{reh} \sim g m^{-1/2} \), the allowed range for the reheating temperature becomes given by \( 10^{-22} \lesssim T_{reh} \lesssim 10^{-9} \) (in units of \( m_p \)). The constraints on the density fraction of Gravitational Waves suggest \( g \sim 1 \) [20]. In this case, we obtain that the reheating temperature becomes of the order of \( T_{rh} \sim 10^{-9} \) (in units of \( m_p \)), which seriously challenges gravitino constraints [26].

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