Upper Bounds on Asymmetric Dark Matter Self Annihilation Cross Sections

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Abstract

Most models for asymmetric dark matter allow for dark matter self annihilation processes, which can wash out the asymmetry at temperatures near and below the dark matter mass. We study the coupled set of Boltzmann equations for the symmetric and antisymmetric dark matter number densities, and derive conditions applicable to a large class of models for the absence of a significant wash-out of an asymmetry. These constraints are applied to various existing scenarios. In the case of left- or right-handed sneutrinos, very large electroweak gaugino masses, or very small mixing angles are required.
1 Introduction

In the cosmological Standard Model, the present day baryon density $\Omega_B$ and the dark matter (DM) relic density $\Omega_{DM}$ are of similar order of magnitude: $\Omega_{DM} \approx 4.7 \times \Omega_B$. Typically one considers DM in the form of WIMPs (Weakly Interacting Massive Particles), which are in thermal equilibrium in the very early universe. Their stability can be ensured by a conserved quantum number of a discrete $\mathbb{Z}_N$ symmetry, such as $R$-parity in supersymmetric theories. Once the temperature falls below the WIMP mass, WIMP self annihilation processes and the expansion of the universe reduce the WIMP density until the self annihilation rate falls below the expansion rate of the universe, from where on the comoving WIMP density remains constant. Assuming that the present day WIMP relic density coincides with $\Omega_{DM}$ and that its mass is in the typical range of the order $10^{-100}$ GeV for cold DM and for new particles in models beyond the Standard Model (BSM), leads to a necessary WIMP self annihilation cross section of the order of weak interaction cross sections.

However, this origin of $\Omega_{DM}$ would be completely disconnected from $\Omega_B$, which is conventionally attributed to a baryon asymmetry $\eta_B$ originating from CP violating process in the early universe. Concrete calculations of $\Omega_{DM}$ show that it is quite sensitive to the WIMP self annihilation cross section and the WIMP mass, and varies over many orders of magnitude as function of the unknown parameters in BSM models attempting to explain the present DM relic density. Then the similar sizes of $\Omega_{DM}$ and $\Omega_B$ result from a numerical coincidence.

Asymmetric DM (ADM) (see [1–5] for some early discussions, and [6] for a review) is an attempt to explain the proximity of $\Omega_{DM}$ and $\Omega_B$. The particles $X$ forming the DM are assumed to be distinct from their antiparticles $\bar{X}$, and to carry a certain quantum number. The corresponding charge density of the universe is assumed to be related to baryon number through equilibrium processes in the early hot universe, such that the asymmetries $\eta_B$ and $\eta_X$ become related. If the $X - \bar{X}$ annihilation rate $\sigma_{XX}$ is sufficiently large, the resulting $X$ relic density will be determined exclusively by the asymmetry $\eta_X$. Then one obtains $\Omega_{DM} \approx \frac{M_X \eta_X}{M_p \eta_B} \Omega_B$ (where $M_p$ is the proton mass, and $M_X$ the mass of the $X$ particles), which gives the correct order if $\eta_X \approx \eta_B$.

In supersymmetric scenarios, the discrete $\mathbb{Z}_N$ symmetry responsible for the stability of the particles $X$ is equal to or related to $R$-parity. For instance, sneutrinos (left-handed or right-handed, mixtures thereof or mixtures with singlets) have been proposed as ADM [7–12]. Singlet extensions of the MSSM have been considered in [13,14]. Recently, higgsinos have been suggested in [15], since they possess a conserved quantum number before the electroweak phase transition. In all these cases, the dark matter asymmetry can be related to the baryon and/or lepton asymmetry through sphaleron, gauge and Yukawa induced process (see [16] and references therein). As long as the baryon/lepton asymmetry is generated at temperatures above the freeze out of the processes which transfer it to the ADM, this mechanism is independent from the precise CP and baryon/lepton number violating origin of the baryon asymmetry.

Of course, in the case of sizeable ADM couplings to Higgs or $Z$ bosons, the direct detection rate must be studied and must not exceed present bounds [17]. (See [18] for a discussion of the potential conflict between a sufficiently large $X - \bar{X}$ annihilation rate and a too large direct detection rate.) A too large direct detection rate can be avoided through
mass splittings leading to inelastic scattering \[12,15\]. Alternatively, one may consider that the \( X \) particles – after their density has frozen out to a value related to the baryon density – decay later into other essentially inert particles with very small couplings, which have not been in thermal equilibrium. Subsequently we will leave aside the problem of direct detection rates.

However, typically the discrete \( \mathbb{Z}_N \) symmetry responsible for the stability of the particles \( X \) does not forbid \( X - X \) self annihilation processes, through the same couplings which transfer the baryon or lepton asymmetry to the \( X \) particles. Once \( X - X \) self annihilation processes are allowed, these processes can wash out the asymmetry \( \eta_X \).

This conclusion is too naive: A rough condition for the absence of a wash-out is to require that, at temperatures of the order of the DM mass, the rate of these processes is below the Hubble expansion rate. Subsequently we study this phenomenon quantitatively in the form of the coupled set of Boltzmann equations for the symmetric and antisymmetric dark matter number densities. We find that the upper bounds on \( \sigma_{XX} \) are extremely strong if one wishes to obtain a final \( X \) number density which is dominated by its asymmetry \( \eta_X \) such that \( \Omega_{DM} \) is related to \( \Omega_B \) as described above.

Interestingly it turns out that, under typical assumptions as a large \( X - \bar{X} \) annihilation rate, a fairly model independent upper bound on \( \sigma_{XX} \) (depending in a simple way on \( M_X \) and the \( s \)-wave or \( p \)-wave nature of the \( X - X \) annihilation process) can be derived from the only condition that the \( X \) asymmetry is not reduced by a large amount. The determination of this upper bound is the main result of this paper. Subsequently we apply it to sneutrino and higgsino ADM scenarios, and to the singlet extension of the MSSM proposed in \[14\] (in an approach similar to, but slightly different from \[19\]).

Our approach is based on the Boltzmann equations for the \( X \) and \( \bar{X} \) number densities in the presence of an \( X - \bar{X} \) asymmetry. Boltzmann equations in the presence of asymmetries have been considered previously e.g. in \[19\]-\[22\] where, however, the \( X - X \) annihilation rate \( \sigma_{XX} \) was assumed to vanish. We find that even a small \( X - X \) annihilation rate \( \sigma_{XX} \) can have a strong (negative) impact on the resulting \( X - \bar{X} \) asymmetry.

In the next Section we establish the Boltzmann equation for the \( X - \bar{X} \) asymmetry for non-vanishing \( \sigma_{XX} \), and clarify the assumptions allowing for its model-independent integration. Our main results are upper bounds on \( \sigma_{XX} \) depending on the tolerated dilution of the initial \( X - \bar{X} \) asymmetry. In Section 3 we study the consequences of this result for sneutrino and higgsino ADM scenarios. In Section 4 we consider the model with a term \( X^2 \Lambda H/\Lambda \) in the superpotential introduced in \[14\] where the expression for \( \sigma_{XX} \) is different from the previous scenarios. A summary and conclusions are given in Section 5.

## 2 Boltzmann equations for asymmetric dark matter

The common mass of the DM particles \( X \) and its antiparticle \( \bar{X} \) is denoted by \( m \). Their equilibrium number densities are assumed to differ by a chemical potential \( \mu \). Assuming Maxwell-Boltzmann statistics (justified for temperatures \( T < \sim m/3 \)), the equilibrium number densities are given by

\[
\begin{align*}
n_{X}^{eq} &= \frac{T}{2\pi^2} gm^2 K_2(m/T)e^{\mu/T}, \\
n_{\bar{X}}^{eq} &= \frac{T}{2\pi^2} gm^2 K_2(m/T)e^{-\mu/T}.
\end{align*}
\]
$g$ denotes the number of internal degrees of freedom of $X$, and $K_2$ the modified Bessel function of the second kind. We assume that the $X - \bar{X}$ asymmetry has been generated during periods before the one considered here, through processes (such as sphaleron processes) which have frozen out. Since we assume that the $X - \bar{X}$ asymmetry is related to the baryon asymmetry, $\mu/T$ is very small: $\mu/T \lesssim 10^{-9}$.

It is convenient to introduce number densities per comoving volume, $Y = n/s$, with $s$ the entropy density:

$$s = h_{\text{eff}}(T) \frac{2\pi^2}{45} T^3,$$

(2)

where

$$h_{\text{eff}}(T) = \sum_{\text{bosons}} g_i \left( \frac{T_i}{T} \right)^3 + \frac{7}{8} \sum_{\text{fermions}} g_i \left( \frac{T_i}{T} \right)^3$$

(3)

is a sum over the effective massless degrees of freedom. Furthermore one translates the time dependence of the number densities into a temperature dependence, using

$$H = \left( \frac{8\pi}{3} G \rho \right)^{1/2}$$

(4)

for the Hubble parameter in the radiation dominated epoch and

$$\rho = g_{\text{eff}}(T) \frac{\pi^2}{30} T^4$$

(5)

for the (relativistic) matter density. $g_{\text{eff}}$ is defined as in (3) with cubic powers replaced by quartic powers. Assuming a two-body final state and using the principle of detailed balance [23], the Boltzmann equations for $Y_X$ and $Y_{\bar{X}}$ as functions of $x \equiv m/T$ become

$$\frac{dY_X}{dx} = -\sqrt{\frac{\pi}{45G}} \frac{g_{s/2}^{1/2} m}{x^2} \left[ \langle \sigma_X v \rangle \left( Y_X^2 - Y_{eq}^2 \right) + \langle \sigma_{\bar{X}X} v \rangle \left( Y_{\bar{X}} Y_X - Y_{eq}^2 \bar{X} \right) \right],$$

(6)

$$\frac{dY_{\bar{X}}}{dx} = -\sqrt{\frac{\pi}{45G}} \frac{g_{s/2}^{1/2} m}{x^2} \left[ \langle \sigma_{X\bar{X}} v \rangle \left( Y_{\bar{X}}^2 - Y_{eq}^2 \right) + \langle \sigma_{X\bar{X}} v \rangle \left( Y_X Y_{\bar{X}} - Y_{eq}^2 \bar{X} \right) \right]$$

(7)

where the effective number of degrees of freedom $g_s$ is given by

$$g_{s/2} \equiv \frac{h_{eff}}{g_{eff}} \left( 1 + \frac{1}{3} \frac{T}{h_{eff}} \frac{dh_{eff}}{dT} \right).$$

(8)

Here we have assumed that the self annihilation cross sections satisfy $\sigma_{\bar{X}X} = \sigma_{XX}$, and $\langle \sigma v \rangle$ is the thermal average of the cross section times velocity given by

$$\langle \sigma v \rangle = \frac{1}{8m^4T^2K_2(m/T)} \int_{4m^2}^{\infty} \sigma(s) \left( s - 4m^2 \right) \sqrt{s} K_1(\sqrt{s}/T) \, ds.$$  

(9)

In most cases $\langle \sigma v \rangle$ can be expanded in powers of the relative velocity of the incoming particles. Then, the thermal average is approximated by an expansion in powers of $1/x$:

$$\langle \sigma v \rangle \simeq a + bx^{-1} + \mathcal{O} \left( x^{-2} \right).$$

(10)
It is useful to consider the difference and the sum of $Y_X$ and $Y_\bar{X}$ defined by $A$ and $Z$, respectively:

$$A = Y_X - Y_\bar{X}, \quad Z = Y_X + Y_\bar{X}. \quad (11)$$

Then the Boltzmann equation for $A$ becomes

$$\frac{dA}{dx} = -\sqrt{\frac{\pi}{45G}} \frac{g_*^{1/2} m}{x^2} \langle \sigma_{XX} v \rangle (ZA - Z^{eq} A^{eq}) \quad (12)$$

where, neglecting terms of $\mathcal{O} (\mu/T)^2$,

$$Z^{eq} = \frac{45g_* x^2 K_2(x)}{2\pi^4 h_{eff}(m/x)} ; \quad A^{eq} = \frac{\mu}{T} Z^{eq}. \quad (13)$$

For $\sigma_{XX} \ll \sigma_{X\bar{X}}$ we can neglect $\sigma_{XX}$ in the Boltzmann equation for $Z$, which becomes

$$\frac{dZ}{dx} = -\sqrt{\frac{\pi}{45G}} \frac{g_*^{1/2} m}{x^2} \langle \sigma_{X\bar{X}} v \rangle \frac{1}{2} (Z^2 - A^2 - Z^{eq} 2 + A^{eq} 2) . \quad (14)$$

Previously these Boltzmann equations were investigated in [19–22] under the assumption that $A$ remains constant, i.e. that the right hand side of (12) vanishes. If $\sigma_{XX}$ is large enough, the freeze-out temperature is low, and $Z \sim Z^{eq}$ to a very good approximation over a long period, and finally $Z_{t\to\infty} \sim A$ up to corrections studied in [19][22]. This is the desired result leading to a DM relic density determined by $A$ which, in turn, is supposed to be related to the baryon asymmetry.

During the period where $Z \sim Z^{eq}$, (12) simplifies to

$$\frac{dA}{dx} = -\sqrt{\frac{\pi}{45G}} \frac{g_*^{1/2} m}{x^2} \langle \sigma_{X\bar{X}} v \rangle Z^{eq} (A - A^{eq}) , \quad (15)$$

which can be integrated with the usual initial condition $A_{in} \sim A^{eq}$ for $T \sim m$ or $x \sim 1$, and a given expression for $m\langle \sigma_{XX} v \rangle$. Note that it is $A_{in}$ which is assumed to be related to the baryon asymmetry. As in the usual case one finds that $A$ freezes out at a freeze-out temperature $T_f = m/x_f^A$, and $A_{\infty} \equiv A_{t\to\infty} \sim A^{eq}(x_f^A)$. However, since eq. (15) is linear in $A$, the ratio $R \equiv A_{\infty}/A_{in}$ is independent from $A_{in}$ and hence independent from $\mu/T$. The ADM paradigm requires that $R$ is not too small; otherwise $A_{\infty}$ is sensitive to $\langle \sigma_{XX} v \rangle$ as in usual DM scenarios, and $\Omega_{DM} \approx \Omega_B$ remains a numerical coincidence.

The dominant dependence on the parameters of the model originates from the combination $m\langle \sigma_{XX} v \rangle$ in (15), an additional weak dependence on $m$ arises from the effective number of degrees of freedom in $h_{eff}(m/x)$ in (13) and in $g_*$. For these we use the parametrization given in [24]. Now, $R$ can be computed for any given expression for $m\langle \sigma_{XX} v \rangle$. (We solved the coupled set (6,7) of Boltzmann equations, but for $\sigma_{XX} \lesssim 10^{-5}\sigma_{X\bar{X}}$ we have $Z \simeq Z^{eq}$ for the relevant range of $x$, and the result for $R$ is independent from $\sigma_{XX}$ as it is obvious from (15).

Assuming eq. (11), we will consider the two cases (a) $\langle \sigma_{XX} v \rangle \simeq a$ (s-wave annihilation) and (b) $\langle \sigma_{XX} v \rangle \simeq b/x$ (p-wave annihilation; in practice one may find a combination of both). The results for $R$ as function of $\log (ma)$ and $\log (mb)$ are shown in Fig. 1.
We see that within the cases (a) or (b) the dependence on \( m \) beyond the one in \( m \langle \sigma_{XX} v \rangle \) is negligibly small, and we can deduce upper bounds on \( m \langle \sigma_{XX} v \rangle \) as function of the tolerated reduction \( R \) of the asymmetry:

\[
R > 0.5: \quad ma \lesssim 5 \times 10^{-17} \text{ GeV}^{-1} \quad (\text{case (a)}), \quad mb \lesssim 1 \times 10^{-16} \text{ GeV}^{-1} \quad (\text{case (b)}),
\]

\[
R > 0.1: \quad ma \lesssim 1 \times 10^{-15} \text{ GeV}^{-1} \quad (\text{case (a)}), \quad mb \lesssim 5 \times 10^{-15} \text{ GeV}^{-1} \quad (\text{case (b)}). \quad (16)
\]

Clearly, if we require only a moderate reduction of the asymmetry \( A \), the freeze-out temperature \( T_A = m/x_A^f \) must not be far below \( m \), or \( x_A^f \) must not be too large. (Here we define \( x_A^f \) as the temperature where the expansion rate of the universe becomes larger than the rate of self-annihilation, i.e. \( H(x_A^f) = (n_{eq}^X - n_{eq}^{\bar{X}})\langle \sigma_{XX} v \rangle(x_A^f) \) which is solved numerically.) In Fig. 2 we show \( x_A^f \) as function of \( \log(ma) \) and \( \log(mb) \) and, indeed, \( x_A^f \) is well below 5 if the first of the conditions \( (16) \) is satisfied.

The requirement of a not too large value of \( x_A^f \) and the fact that \( \langle \sigma_{XX} v \rangle(x_A^f) \sim e^{x_A^f} \) explains why we obtain \( \langle \sigma_{XX} v \rangle \ll \langle \sigma_{XX} v \rangle \); the desired value of \( x_f \) in case of \( X - \bar{X} \) annihilation is rather of \( \mathcal{O}(20) \).

In the next Sections we study the implications of the upper bounds on \( \sigma_{XX} \) for various models for supersymmetric ADM.
3 Sneutrino and higgsino ADM

Left-handed sneutrinos or mixtures of left- and right-handed sneutrinos (or singlets) have been proposed as ADM in [7–12]. Clearly, left-handed sneutrinos \( \tilde{\nu}_L \) can self-annihilate through processes of the form \( \tilde{\nu}_L + \tilde{\nu}_L \rightarrow \nu_L + \nu_L \) by the exchange of electroweak gauginos in the t-channel. Electroweak gauginos are the binos with mass \( M_1 \), and winos with mass \( M_2 \). The corresponding expression for \( \langle \sigma_{\tilde{\nu}_L\tilde{\nu}_L} v \rangle \) can be obtained from [25], and reads in the limit \( M_1, M_2 \gg m \)

\[
\langle \sigma_{\tilde{\nu}_L\tilde{\nu}_L} v \rangle \approx \frac{g_2^4}{16\pi} \left( 1 - \frac{3}{2x} \right) \left( \frac{\tan^2 \theta_w}{M_1} + \frac{1}{M_2} \right)^2
\]

where \( g_2 \) is the SU(2) gauge coupling and \( \theta_w \) the weak mixing angle. If one assumes universal gaugino masses at the GUT scale, \( M_1 \) and \( M_2 \) are related by \( M_1 \approx M_2/2 \) and, with \( \tan^2 \theta_w \approx 0.3 \), \( \text{(17)} \) simplifies to

\[
\langle \sigma_{\tilde{\nu}_L\tilde{\nu}_L} v \rangle \approx \frac{g_2^4}{8\pi} \left( 1 - \frac{3}{2x} \right) \frac{1}{M_2^2}.
\]

From Fig. 2, the first term leads to stronger constraints, and applying the conservative bound \( R > 0.1 \) (case (a)) from \( \text{(16)} \) leads to

\[
M_2 \gtrsim 3 \times 10^7 \text{ GeV} \times \left( \frac{m}{100 \text{ GeV}} \right)^{1/2}
\]

which excludes gaugino masses of the order of the electroweak scale. (Even for such large gaugino masses the sneutrino-antisneutrino annihilation rate would remain large due to processes with slepton exchange in the t-channel.)
If the ADM $X$ consists in a mixture of left-handed sneutrinos $\tilde{\nu}_L$ and right-handed sneutrinos or other electroweak singlets, the result depends on the $\tilde{\nu}_L$ component of $X$ or the mixing angle $\sin \delta$ where $X = \tilde{\nu}_L \sin \delta + \ldots$:

$$\langle \sigma_{XX} v \rangle \simeq \frac{g_4^4 \sin^4 \delta}{8\pi} \left( 1 - \frac{3}{2x} \right) \frac{1}{M_2^2}.$$  \hspace{1cm} (20)

Now the same argument leads to

$$\sin^2 \delta \lesssim 3.3 \times 10^{-6} \times \frac{M_2}{\sqrt{m \cdot 100 \text{ GeV}}}.$$  \hspace{1cm} (21)

Hence, for $M_2 \approx m \approx 100$ GeV, $\sin \delta$ has to be very small independently from constraints from direct DM detection experiments – or, for mixing angles $\sin \delta \approx 1$, one is lead back to (19).

Higgsinos $\tilde{h}_u$, $\tilde{h}_d$ with masses $m \sim 200 - 1000$ GeV as ADM were proposed recently in [15]. Before the electroweak phase transition (where the Higgs vevs develop), higgsinos can be considered as mass eigenstates. Through sphaleron, gauge and Yukawa interactions at high temperature, a higgsino asymmetry $A_{\tilde{h}_u} \sim A_{\tilde{h}_d}$ proportional to the baryon asymmetry is generated [15, 16]. If the higgsinos are the LSPs (lightest supersymmetric particles) and other sparticles are sufficiently heavy, this asymmetry could survive until today [15] provided the $\tilde{h}_u\tilde{h}_u$ (and $\tilde{h}_d\tilde{h}_d$) self annihilation rates are sufficiently small.

However, higgsinos have the same couplings to electroweak gauginos as sneutrinos, and can again self-annihilate into $H_u, H_d$ (to be considered as eigenstates before the electroweak phase transition) through $t$-channel exchange of bino's and wino's. This time the scattering process is a $p$-wave process (case (b) in [16]) and we obtain from (26) (again for $M_1$, $M_2 \gg m$)

$$\langle \sigma_{\tilde{h}_u\tilde{h}_u} v \rangle = \langle \sigma_{\tilde{h}_d\tilde{h}_d} v \rangle \simeq \frac{3g_4^4}{8\pi} \left( \frac{\tan^2 \theta_w}{M_1} + \frac{1}{M_2} \right)^2.$$  \hspace{1cm} (22)

Applying again the conservative bound $R > 0.1$ (case (b)) from (16) leads to the same constraint on $M_2$ as in (19), which is close to the estimated bound on gaugino masses given in [15] from the same argument.

4 The $\Delta W \sim XXHL/\Lambda$ model

Left-handed sneutrinos and higgsinos as ADM tend to violate bounds on direct DM detection cross sections (unless mass splittings are introduced leading to inelastic scattering [12, 15]). Moreover, as we have seen in the previous section, constraints from sufficiently small self annihilation cross sections are very strong. These constraints would be alleviated, if the asymmetry is transferred to lighter essentially inert particles. A simple model of that kind has been proposed in [14], where a gauge singlet scalar superfield $X$ and a superpotential

$$\Delta W = \frac{1}{\Lambda} XXH_u L_i$$  \hspace{1cm} (23)

are introduced. (Here $H_u$ denotes a Higgs superfield, and $L_i$ any left-handed lepton superfield.) This nonrenormalizable interaction can originate from integrating out heavy
vector-like sterile neutrinos or electroweak doublets [14] with mass $\sim \Lambda$, typically $\gtrsim 1$ TeV. Another singlet chiral superfield $X$ should be introduced to allow for a supersymmetric Dirac mass term $M_X$ for the fermionic components $\psi_X\psi_X$, e.g. via a NMSSM-like singlet $S$ with $\langle S \rangle \neq 0$ and a coupling $\lambda SXX$ in the superpotential.

$\psi_X, \bar{\psi}_X$ would carry lepton number $\pm 1$, respectively. The superpotential (23) breaks the usual R parity, but preserves a $Z_4$ symmetry which allows for the decay of the usual LSP into $\psi_X\bar{\psi}_X$ [14]. At high temperature where the processes induced by (23) are in equilibrium together with sphaleron, gauge and Yukawa interactions, these imply an asymmetry $A_X \sim 35\%$ of the baryon asymmetry [14,19], the precise value depending on whether top quarks and squarks are still in equilibrium when the interactions from (23) decouple. Assuming a sufficiently rapid $\psi_X - \bar{\psi}_X$ annihilation rate and $M_X \sim 11 - 13$ GeV, the relic density is then automatically of the correct order.

After electroweak symmetry breaking, the superpotential (23) gives rise to an interaction of the form

$$\frac{\nu_u}{\Lambda} \psi_X\psi_X \bar{\nu}_i.$$  \hspace{1cm} (24)

(Subsequently we omit the neutrino/sneutrino index $i$.) The sneutrino $\tilde{\nu}$ does not have to be the LSP; the LSP can be the lightest neutralino $\tilde{\chi}$. Then an on-shell sneutrino $\tilde{\nu}$ would decay via the usual vertex $g\tilde{\nu}\tilde{\chi}\nu$ – where $g$ is of the order of electroweak gauge couplings, if $\tilde{\chi}$ is dominantly bino-like – into $\tilde{\chi}$ plus a neutrino $\nu$. At energies below the sneutrino mass $m_{\tilde{\nu}}$, integrating out the sneutrino leads to an effective four Fermi interaction

$$\frac{g\nu_u}{m_{\tilde{\nu}}^2 \Lambda} \psi_X\psi_X \tilde{\chi}\nu.$$  \hspace{1cm} (25)

At energies above $M_{\tilde{\chi}}$, (25) allows for the scattering process $\psi_X\psi_X \rightarrow \tilde{\chi}\nu$. However, assuming $M_{\tilde{\chi}} > 2M_X$, the lightest neutralino $\tilde{\chi}$ is not stable. Since $\tilde{\chi}$ is a Majorana Fermion, (25) leads to its decay into $\psi_X\psi_X \nu$ and $\bar{\psi}_X\bar{\psi}_X \bar{\nu}$ with corresponding branching ratios of 50%. The latter case leads to the scattering process

$$\psi_X\psi_X \rightarrow \bar{\psi}_X\bar{\psi}_X \bar{\nu}\bar{\nu}.$$  \hspace{1cm} (26)

As in the case of sneutrinos and higgsinos, this ADM self annihilation process can have potentially disastrous consequences for the remaining asymmetry. In [14], the rate for this process has been estimated by integrating out both the sneutrino $\tilde{\nu}$ and the lightest neutralino $\tilde{\chi}$ with the result that, for $m_{\tilde{\nu}} \sim M_{\tilde{\chi}} \sim 100$ GeV and $\Lambda \gtrsim 1$ TeV, it would go out of equilibrium (drop below the Hubble expansion rate) for decoupling temperatures $T_D$ somewhat above $M_X$, in which case the asymmetry would hardly be washed out.

A more detailed analysis of the ADM self annihilation processes has been performed in [19]. There it was pointed out that, for $m_{\tilde{\nu}} \sim M_{\tilde{\chi}} \sim 100$ GeV and $\Lambda \gtrsim 1$ TeV, the dominant ADM self annihilation process is real $\tilde{\chi}$ production through the interaction (25), and the corresponding cross section was given.

Subsequently the authors in [19] estimated $T_D$ by equating the ADM self annihilation rate with the Hubble expansion rate. They tolerated a considerable wash-out of the asymmetry and/or a Boltzmann suppression for $T_D < M_X$, and studied the necessary relations between the final values for $A_\infty$ and $Z_\infty$ defined in (11) (or $r_\infty = (Z_\infty - A_\infty)/(Z_\infty + A_\infty)$), the
DM mass $M_X$ and the lightest neutralino mass $M_\chi$ required for a desired DM relic density. Clearly, in most of this parameter space (after a considerable wash-out of the asymmetry and/or Boltzmann suppression), the desired DM relic density is no longer simply related to the baryon asymmetry in contrast to ADM paradigm.

Here we ask the question under which conditions this does not happen, i.e. under which conditions the initial asymmetry $A_X$ determines essentially the DM relic density. As before we assume that the $\psi_X - \bar{\psi}_X$ annihilation rate is sufficiently large, such that we can assume $Z \sim Z^{eq}$ in (12) leading to (13).

If the threshold for the process (26) would be $s > 4M_\chi^2$, we could apply our previous formulas. However, for the (dominant) annihilation process via real $\tilde{\chi}$ (+ neutrino) production, the threshold is $s > M_\chi^2 > 4M_X^2$. As a consequence the thermal average of the cross section times velocity in (15) depends in a more complicated way on $M_X$, $M_\chi$ and notably on the temperature or $x = M_X/T$ such that the expansion (10) is no longer applicable. First, we use the cross section $\sigma_{XX}(s)$ for the process $\psi_X \bar{\psi}_X \rightarrow \tilde{\chi} \bar{\tilde{\chi}}$ from [19]:

$$\sigma_{XX}(s) = \frac{\kappa^2 M_\chi^4}{256\pi} \frac{1}{s} \left( \frac{s}{M_\chi^2} - 1 \right)^2$$

(27)

where $\kappa = \frac{A_{\text{in}}}{\Lambda m_p}$. Then, using (9) with $M_\chi^2$ as lower threshold of the integral (without expanding in $M_X/M_\chi$ as in [12]), we obtain

$$\langle \sigma_{XX} v \rangle = \frac{\kappa^2}{16\pi} \left( \frac{M_\chi}{M_X} \right)^2 \left( \frac{M_\chi^2}{M_X} - 4M_X^2 \right) K_2 \left( \frac{M_\chi}{M_X} x \right) + \frac{6M_\chi M_X K_3 \left( \frac{M_\chi}{M_X} x \right)}{x^2 K_2^2 (x) \bar{\nu}} \right).$$

(28)

For $M_X \ll M_\chi$, $\langle \sigma_{XX} v \rangle$ is proportional to $M_\chi^{7/2} e^{-M_\chi/T}$ like the collision term evaluated in [19]. The Boltzmann suppression $\sim e^{-M_\chi/T}$ is an obvious consequence of the threshold $s > M_\chi^2$ required for real $\tilde{\chi}$ production. Subsequently we integrate the Boltzmann equation (15) numerically employing (28) for $\langle \sigma_{XX} v \rangle$, which allow us to study $R \equiv A_{\infty}/A_{\text{in}}$ as before. Now, however, $R$ depends in a more complicated way on the parameters $\kappa$, $M_\chi$ and $M_X$ of the model. On the other hand, the initial asymmetry is quite well known in this class of models, $A_{\text{in}} \sim 0.35 B$ (where $B$ is the baryon asymmetry), and finally we must obtain

$$\frac{\Omega_{DM}}{\Omega_B} = \frac{A_{\infty} M_X}{B m_p} \sim 4.7$$

(29)

which determines $M_X$ in terms of $A_{\infty}$ or $R$:

$$M_X \sim (12.5 \text{ GeV})/R.$$ 

(30)

Hence, once the correct ADM relic density is imposed, the free parameters of the model are $\kappa = \frac{A_{\text{in}}}{\Lambda m_p}$, $M_\chi$ and $M_X$ or $R$. In order to clarify the correlations between these parameters, we show $R$ in the range $R = 1 \ldots 0.1$ ($M_X = 12.5 \ldots 125$ GeV) as function of $M_\chi$ for various values of $\kappa \leq 10^{-5}$ GeV$^{-2}$ in Fig. 3 (Note that all curves continue horizontally along $R = 1$ beyond their upper end.)

The shape of these curves can be understood as follows: Near the upper end, $R \sim 1$, $M_X \sim 12.5$ GeV and practically no wash-out takes place by construction, since $\langle \sigma_{XX} v \rangle$ is
too small. For fixed $\kappa$, $\langle \sigma_{XX} v \rangle$ increases for decreasing $M_\tilde{\chi}$ due to $\langle \sigma_{XX} v \rangle \sim e^{-M_\tilde{\chi}/T}$, and below some critical value of $M_\tilde{\chi}$, $\langle \sigma_{XX} v \rangle$ is large enough such that $R$ starts to decrease.

If a sizeable wash-out takes place ($R \lesssim 0.5$), it stops when the annihilation rate falls below the Hubble expansion rate, i.e. for a given value of $\langle \sigma_{XX} v \rangle$. For fixed $\kappa$, this implies a certain fixed value for $e^{-M_\tilde{\chi}/T_D}$ or $M_\tilde{\chi}/T_D$. Using $x_D = M_X/T_D$ and (30) one easily derives $R \sim 1/(x_D M_\tilde{\chi})$, which explains the decrease of $R$ for increasing $M_\tilde{\chi}$ in this range.

Fig. 3 allows to identify the regions in parameter space in which $R$ is not too small ($R > \sim 0.5$), i.e. where $\Omega_{DM}$ follows naturally from $\Omega_B$ as in the ADM paradigm. If the coefficient $\kappa$ relatively is large, $\kappa \sim 10^{-5}$ GeV$^{-2}$ or $m_\tilde{\nu} \sim 100$ GeV and $\Lambda \sim 1$ TeV, one needs $M_\tilde{\chi} \gtrsim 300$ GeV. If $\tilde{\chi}$ is lighter, $M_\tilde{\chi} \sim 100$ GeV, one needs a very small value of $\kappa \lesssim 10^{-9}$ GeV$^{-2}$, hence correspondingly large values of $m_\tilde{\nu}$ and/or $\Lambda$.

### 5 Conclusions

If the dark matter asymmetry is related to the baryon (or lepton) asymmetry, some couplings must necessarily relate these sectors. The same couplings can lead to dark matter self annihilation processes, which can wash out the corresponding asymmetry $A$. A rough condition for the absence of a wash-out is to require that, at temperatures of the order of the DM mass, the rate of these processes is below the Hubble expansion rate. In Section 2 we have studied the corresponding set of Boltzmann equations quantitatively (assuming a two-body final state). Requiring a modest wash-out ($A_{\infty}/A_{\in} \gtrsim 0.1$), the upper bounds on $m_\langle \sigma_{XX} v \rangle$ are very strong, and can be deduced from Fig. 1 if $\langle \sigma_{XX} v \rangle \sim a$ or $\langle \sigma_{XX} v \rangle \sim b/x$.

If the ADM consists in sparticles with couplings to electroweak gauginos (left-handed...
sneutrinos or higgsinos), it follows that the electroweak gauginos must be extremely heavy such that supersymmetry does not solve the hierarchy problem. If the ADM consists in particles like right-handed sneutrinos which mix weakly with left-handed sneutrinos, the mixing angle must be very small, see eq. (21).

In different models for ADM, the dominant ADM self annihilation process may be kinematically possible only for $s$ above a threshold larger than $(2M_X)^2$, in which case it becomes Boltzmann suppressed and eq. (10) is no longer valid. An example is the popular $\Delta W \sim XXHL/\Lambda$ model of [14]. Also in this case, the numerical integration of the Boltzmann equation for the asymmetry allowed us to specify the range of parameters where the wash-out of the asymmetry remains modest.

In the past, the constraints following from the absence of a wash-out of the asymmetry have sometimes been neglected or underestimated (notably in the case of sneutrinos); we hope that the present work helps to clarify the relevance of ADM self annihilation processes and the resulting conditions on corresponding models.

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