Research on analysis method of dynamic friction torque of high speed precision miniature bearings

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Abstract. High-speed precision miniature bearings are used in the gyro spindle of precision inertial navigators in the aerospace industry. The dynamic friction torque of the bearings is difficult to be accurately analyzed and calculated using existing methods because it is small and has many influencing factors. The special testing machine for dynamic friction torque testing of high-speed miniature bearings is used to test the dynamic friction torque of specific high-speed precision miniature bearings under various operating conditions. It is related to the speed, axial preload, and radial external load. The test data of bearing dynamic friction torque was obtained. On this basis, the least squares method of regression analysis theory is used to fit polynomial fitting of the dynamic friction torque of the tested bearing and the corresponding working condition parameters, and the regression equation for analyzing the dynamic friction torque of the tested bearing is obtained. The test results show that under different test conditions, the measured values of the bearing's dynamic friction torque are in good agreement with the calculated values using the regression equation. The obtained regression equation can be used to analyze the dynamic friction torque characteristics of the same bearing under other operating conditions. A reliable and effective method was proposed for quickly and easily obtaining accurate high-speed precision miniature bearing dynamic friction torque.

1. Introduction

High-speed precision miniature bearings are widely used in the support of gyro spindles in aerospace inertial navigation and other precision instruments [1]. The dynamic performance such as friction and wear, heat generation, temperature rise, and thermal deformation in the bearing depends on the dynamic friction torque and the changing state of the bearing during its operation. More importantly, the attitude of the gyro rotor in the inertial navigation system and the navigation accuracy of the navigation system will also be affected. More importantly, the attitude of the gyro rotor in the inertial navigator and the navigation accuracy of the navigator will be reduced, and the attitude and flight reliability of the aerospace vehicle in space will decrease [2]. Therefore, the bearing frictional torque is closely watched by scholars and engineers in related fields. Relevant test stands have been built and tested by scholars, and many theoretical models have been established and improved on bearing friction torque [3-5]. In the latest research, based on the calculation model of bearing friction torque, the research on bearing friction torque has been transformed into the application of relevant numerical analysis theory to predict the value of friction torque [6-9]. However, the miniature bearings and the high-speed operation of the bearings have not been concerned by relevant research institutes.
In summary, the existing theoretical analysis methods and calculation models of bearing friction torque cannot be directly applied to the analysis of high-speed precision miniature bearing friction torque. A special test system is still the most effective method to obtain the bearing friction torque. However, since the bearing test cost is too high in actual engineering, it is difficult to meet the needs of actual engineering. We have conducted a series of studies on the dynamic friction torque characteristics of high-speed miniature bearings[10-12]. However, it is difficult to accurately simulate the actual working conditions inside the high-speed precision miniature bearings in the theoretical analysis based on the calculation model in the early stage, and the friction torque that matches the actual project cannot be directly obtained. In this paper, we use a high-speed miniature bearing friction torque special test machine developed based on the principle of torque balance and balance weighing method to test the dynamic friction torque of the bearing. Based on the regression analysis theory, a method of curve fitting the obtained test data and obtaining the regression equation is proposed, which can more quickly and accurately obtain the dynamic friction torque during the high-speed precision miniature bearing operation.

2. High-speed miniature bearing dynamic friction torque testing machine

In this paper, a special test machine for dynamic friction torque of high-speed miniature bearings independently developed is used to measure the friction torque of high-speed precision miniature bearings under certain operating conditions. Bearing axial preload and radial external load loading, speed adjustment, and real-time monitoring and recording of friction torque and other functions can be realized by the test machine, the composition of the test machine is shown in Figure 1.

![Figure 1. Composition of miniature bearing friction torque testing machine.](image)

In Figure 1, the support base and the base constitute the support part of the testing machine, and respectively support the electric spindle and the electronic balance. The high-speed electric spindle technology is applied to the test machine, which simplifies the transmission system from the drive to the test bearing and ensures the accuracy of the test machine. The bearing is cantilevered by the extended end of the drive motor spindle. In Figure 2, the weight is connected by a nylon rope on the screw and placed horizontally on the tray of the electronic balance.

As shown in Figure 2, the bearings are mounted face-to-face on the cantilever end of the drive spindle. The speed of the test can be changed by adjusting the speed of the electric spindle. The axial fixation of the bearing is achieved by the left shoulder and the right end nut. The sleeve is tightly connected to the bearing outer ring, and no relative movement occurs. Test bearing operating conditions are inner ring rotation and outer ring fixed. The inner spacer, the outer spacer and the wave elastic ring are installed in the middle of the two bearings. The difference between the width of the inner and outer spacers is changed to change the compression amount of the wave elastic ring, so different axial preloads are applied to the tested bearing. The screws are installed under the sleeve, and
the radial external load of the tested bearing can be changed by connecting different weights with nylon ropes.

![Diagram of miniature bearing loading assembly](image)

**Figure 2.** Schematic diagram of a miniature bearing loading assembly.

The principle of balance torque and balance weighing method are applied to the testing machine. The actual friction torque of the tested bearing is obtained by measuring the balance torque equal to the friction torque of the bearing and in the opposite direction. During the test, the inner ring of the tested bearing was driven to rotate at high speed by the electric spindle. The friction torque generated by the bearing makes the bearing outer ring and sleeve rotate in the same direction as the inner ring. The weight connected to the sleeve by screws and nylon rope will tend to be lifted. Because the weight of the measuring weight is large enough, the bearing outer ring and the sleeve will not actually rotate, and the weight will not be lifted completely. However, the displayed value on the precision electronic balance will be reduced. The reduction value is multiplied by the distance from the nylon rope to the center of the bearing, and the friction torque values inside the two sets of bearings tested are obtained. The data acquisition system records the reduction value of the electronic balance in real time, the computer of the testing machine performs numerical processing, and the dynamic friction torque during the running of the tested bearing is obtained. The high-speed miniature bearing dynamic friction torque test machine is shown in Figure 3.

![Testing machine setup](image)

**Figure 3.** High speed miniature bearing dynamic friction torque testing machine.
3. Analysis of dynamic friction torque test results

A miniature bearing produced by a bearing company was selected as the test object. During the test, the influence of operating parameters such as operating speed, axial preload and radial external load on the dynamic friction torque of the bearing was investigated. The tested bearings are angular contact ball bearings. The specific parameters are shown in Table 1.

Table 1. The parameters of the tested bearing.

| parameter                              | Value |
|----------------------------------------|-------|
| Outer diameter D/mm                    | 11    |
| Inner diameter d/mm                    | 4     |
| Ball diameter D_b/mm                   | 2     |
| Number of balls                        | 7     |
| Contact angle α/(°)                    | 24    |
| Inner groove curvature radius factor   | 0.53  |
| Outer groove curvature radius factor   | 0.57  |

According to the use conditions of the tested bearings, special instrument oil is used for bearing lubrication. Before testing the friction torque, the bearing was rotated for 15 hours to make the bearing enter normal working condition. During the test, the ambient temperature is room temperature, and the test conditions of the tested bearing are as follows[13-17]:

7 N is applied to the axial preload (P_a) and 0.16 N is applied to the radial outer load (P_r). At a speed of 5000-30000 r/min, the friction torque is tested after the speed is stable, as shown in Figure 4.

7 N is applied to the axial preload, 1.56 N, 0.75 N, and 0.16 N are applied to the radial external load. At a speed of 5000-30000 r/min, the friction torque is tested after the speed is stable, as shown in Figure 5.

1.56 N is applied to the radial external load, 7 N, 15 N, and 25 N are applied to the axial preload. At a speed of 5000-30000 r/min, the friction torque is tested after the speed is stable, as shown in Figure 6.

From the above test result chart, it can be analyzed that the dynamic friction torque of high-speed precision miniature bearings is affected by its speed, axial preload and radial external load. The value of the friction torque is positively related to each influencing factor, which is in line with the friction torque of angular contact ball bearings[18]. The influence of each working condition parameter on the dynamic friction torque is different. It is obvious that the dynamic friction torque of the bearing is greatly affected by the speed and the axial preload.
4. Numerical regression analysis of friction torque

According to the regression analysis theory, regression analysis is performed on the test results of the dynamic friction torque of the bearing, and a regression equation suitable for the calculation of the friction torque of the tested bearing is fitted. Bearing friction torque is most affected by speed and axial preload, so speed and axial preload are used as the main parameters of the regression equation. Exponential functions, logarithmic functions, and power functions are complicated to calculate. They are used to build multiple regression equations that make calculations too much. To facilitate engineering applications, polynomials are used to establish regression equations for experimental results[19].

4.1. Establishing the fitting formula

First a working condition was chosen to fit the formula. 7 N is used as the axial preload and 1.56 N is used as the radial external load. The speed \( n \) is used as the independent variable, and the friction torque \( M_D \) is used as the dependent variable. The fitted formula results are shown in Table 2 and Figure 7.

| No. | Function | Fitting formula | Correlation coefficient |
|-----|----------|-----------------|-------------------------|
| 1   | First-order equation | \( M_D = 7.861 \times 10^{-5} n - 0.0575 \) | 0.971 |
| 2   | Quadratic equation | \( M_D = 3.266 \times 10^{-3} n + 1.313 \times 10^{-5} n^2 + 0.2488 \) | 0.986 |
| 3   | Cubic equation | \( M_D = -4.941 \times 10^{-5} n + 6.750 \times 10^{-5} n^2 - 1.036 \times 10^{-13} n^3 + 0.575 \) | 0.989 |
| 4   | Quartic equation | \( M_D = -1.641 \times 10^{-4} n + 1.909 \times 10^{-4} n^2 - 6.216 \times 10^{-13} n^3 + 7.4 \times 10^{-18} n^4 + 0.908 \) | 0.982 |

Table 2 shows the expression of the fitted function and the correlation coefficient between the fitted data and the experimental data. In order to reduce the amount of calculation, according to the theory of numerical analysis, the appropriate low-order polynomial is selected by F-test. First, the F-test is used to analyze whether the error of the first-order equation and the quartic equation is allowed. If the error is allowed, the first-order polynomial is applied to the fitting formula; if not, the F-test is used to compare the quadratic equation and quartic equation until a suitable equation of the fourth order or less is selected.

According to the F-test, the error of the quadratic equation is allowed at a significance level . Therefore, the fitting formula of the friction torque of the tested bearing can be expressed as

\[
M_D = An + Bn^2 + C
\]  

(1)

Where, \( n \) is the speed, \( B \) and \( C \) are fitting coefficients, \( M_D \) is the friction torque.

Similarly, the same method is used to determine the functional form for different axial preloads, which will not be repeated here. The function fitting results of the tested bearing under the radial external load of 1.56 N are shown in Table 3.

| No. | Function | Fitting formula | Correlation coefficient |
|-----|----------|-----------------|-------------------------|
| 1   | 56 2 43 672 10 1 105 10 1 7 10 \( \cdot P_a \) | \( A \) | 0.984 |
| 2   | 91 1 2 91.165 10 3.718 10 7.647 10 \( \cdot P_a \) | \( B \) | 0.989 |
| 3   | 20.07792 0.00359 0.6184 \( \cdot P_a \) | \( C \) | 0.971 |

Get the unified regression equation of the friction torque of the tested bearing:

\[
M_D = A_0 + B_0 n + C_0
\]  

(2)

Where, \( n \) is the speed, \( A_0, B_0 \), and \( C_0 \) are fitting coefficients.
\begin{equation}
M_D = (3.672 \times 10^3 P_a - 1.105 \times 10^4 P_a^2 - 1.7 \times 10^4)n
+ (-1.165 \times 10^5 P_a + 3.718 \times 10^{11} P_a^2
+ 7.647 \times 10^7) n^2 - 0.07792 P_a
+ 0.00359 P_a^2 + 0.6184
\end{equation}

Table 3. Calculation formula of friction torque fitted under different axial preloads.

| Axial preload(N) | Fitting formula |
|------------------|-----------------|
| 7                | $M_D = 3.266 \times 10^3 n + 1.313 \times 10^{-9} n^2 + 0.249$ |
| 15               | $M_D = 1.319 \times 10^4 n - 1.464 \times 10^{-9} n^2 + 0.258$ |
| 25               | $M_D = 5.701 \times 10^4 n + 1.757 \times 10^{-9} n^2 + 0.916$ |

\(^a \text{r } / \text{min as the unit of } n \text{ and g.cm as the unit of } M_D\)

4.2. Test verification

According to the theory of regression analysis, the consistency between the fitted values of the regression equation and the actual experimental data under different working conditions is verified by the size of the parameters such as sum squared residual and correlation coefficient. The sum square residual and correlation coefficient calculation formula is as follows[20]:

\begin{equation}
SSE = \sum_{i=1}^{n} (x_i - y_i)^2
\end{equation}

\begin{equation}
R = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n} (x_i - \bar{x})^2 \sum_{i=1}^{n} (y_i - \bar{y})^2}}
\end{equation}

Where, SSE is the sum of squared residuals, R is the correlation coefficient, $x_i$ is the test value, $\bar{x}$ is the average value of the test value, $y_i$ is the fitted value, $\bar{y}$ is the average value of the fitted value.

Figure 7 is a comparison of the data curves between the actual experimental measured values and the predicted values of the regression equation.

![Figure 7. Comparison of measured values with regression equation predictions](image_url)
It can be seen from Figure 7 that the fitted value of the friction torque regression equation in this paper is basically consistent with the actual experimental measurement value. When the axial preload is 10 N, the sum squared residual is 0.0786. When the axial preload is 20 N, the sum squared residual does not exceed 0.3699. Therefore, the regression equation obtained in this paper is suitable for the calculation of the dynamic friction torque of the high-speed precision miniature bearings tested, and can obtain more accurate dynamic friction torque values under different operating conditions. The method in this paper can also be used to calculate the friction torque of other high-speed precision miniature bearings under different working conditions.

5. Conclusion
(1) Based on the analysis of the dynamic friction torque measurement results of high-speed precision miniature bearings under certain operating conditions, and based on regression theory, a regression equation for calculating the dynamic friction torque of high-speed precision miniature bearings was fitted using polynomials, and experimental verification was performed. It is shown that this regression equation is suitable for the calculation of the friction torque under different operating conditions of the tested bearing.

(2) An empirical formula for calculating the dynamic friction torque of high-speed precision miniature bearings is established in this paper, which can quickly and accurately calculate the friction torque inside the bearings under different working conditions, greatly reducing the test workload. It provides a reliable and effective method for the analysis of bearing dynamic friction torque.

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