Diffraction and correlations at the LHC: definitions and observables

V.A. Khoze, F. Krauss, A.D. Martin, M.G. Ryskin, and K.C. Zapp

1 Institute for Particle Physics Phenomenology, University of Durham, Durham, DH1 3LE
2 Petersburg Nuclear Physics Institute, Gatchina, St. Petersburg, 188300, Russia

Received: date / Revised version: date

Abstract. We note that the definition of diffractive events is a matter of convention. We discuss two possible “definitions”: one based on unitarity and the other on Large Rapidity Gaps (LRG) or Pomeron exchange. LRG can also arise from fluctuations and we quantify this effect and some of the related uncertainties. We find care must be taken in extracting the Pomeron contribution from LRG events. We show that long-range correlations in multiplicities can arise from the same multi-Pomeron diagrams that are responsible for LRG events, and explain how early LHC data can illuminate our understanding of ‘soft’ interactions.

PACS. PACS-key describing text of that key – PACS-key describing text of that key

1 Introduction

For many studies at hadron colliders it is important to unambiguously define what component of the inelastic cross section is selected. Traditionally observables such as the single-particle inclusive cross section and multiplicity distributions are given for non-diffractive events. These already serve as important input to understanding high-energy strong interactions and the tuning of Monte Carlos. On the other hand, it is not so clear to what extent non-diffractive processes can be disentangled. Recall, first, that inelastic diffraction is responsible for a sizeable part (say, about $0.2 - 0.3$) of the total $pp$ cross section; second, that the present LHC detectors do not have $4\pi$ geometry and do not cover the whole available rapidity interval. So the minimum-bias events account only for a part of the total inelastic cross section [1]. The extrapolation necessary to obtain the value of the total cross section is model dependent, and the uncertainties associated with this extrapolation will limit the accuracy of the total inelastic cross section measurements at the LHC.

Moreover, even when reconstructing the total cross section using the dedicated Totem detector [2] there is still an uncertainty due to incomplete knowledge of the low-mass single diffraction (SD) contribution. This, in turn, imposes restrictions on the accuracy of luminosity determination using the optical theorem, see for instance [3].

Note also that knowledge of the total inelastic cross section is important for the evaluation of quantities such as the number of interactions per bunch crossing, when a high luminosity of the LHC becomes available.

On the theory side, the situation is not so clear. At the moment the theoretical predictions for the total $pp$ cross section, $\sigma_{\text{tot}}$, at the LHC energy of 14 TeV differ by a factor of 2.5 in the range between 90 and 230 GeV; recent reviews and references can be found in [4,5]. Even models that are ideologically close [6–9], which incorporate (both eikonal and enhanced) absorptive corrections, differ in their predictions for $\sigma_{\text{tot}}$ at the LHC by about $30\%$ (covering the range of 90-130 mb), while the expectations for $\sigma_{\text{SD}}$ in these models differ by a factor of 1.7 (in the range 11-19 mb).

2 Definition of diffraction

At the outset, we have to say there is no unique definition of diffraction. It is a matter of convention only. Usually when talking about diffraction we mean events arising from Pomeron exchange, which experimentally we would like to associate with Large Rapidity Gaps (LRG). Unfortunately the situation is more complicated. LRG can also arise from the secondary Reggeon exchange or from simple fluctuations in the distribution of secondaries produced in the event. We amplify the problem by discussing two “definitions” of diffraction in detail. The second definition will be based on the association of Pomeron exchange with LRG (see Section 2.2), but, first we consider the unitarity-based definition which, at first sight, looks as if it may be unique.

2.1 First “definition” of diffraction

In analogy with optics, we could say that diffraction is “elastic” scattering caused, via unitarity, by the absorption of components of the wave functions of the incoming
protons. If we just consider elastic unitarity, then it gives an elastic amplitude of the form [10]

\[ A(b) = i(1 - \exp(-\Omega(b)/2)) \]  

in impact parameter, \(b\), space, where \(\Omega\) is the opacity or eikonal. The situation is sketched symbolically in Fig. 1(a).

Note, however, that as a rule the wave functions of incoming hadrons differ from the eigenstates corresponding to high energy scattering amplitude. Different components of the initial hadron wave function have different absorptive cross sections. As a consequence, the outgoing superposition of states will be different from the incident particle, so we will have inelastic, as well as elastic, diffraction.

To discuss inelastic diffraction, it is convenient to follow Good and Walker [11], and to introduce states \(\phi_k\) which diagonalize the \(A\) matrix. These, so-called diffraactive, eigenstates only undergo ‘elastic’ scattering. To account for the internal structure of the proton we, therefore, have to enlarge the set of intermediate states, from just the single elastic channel, and to introduce a multi-channel eikonal, see Fig. 1(b).

This definition of diffraction is good for elastic and quasi-elastic processes, where the size of the LRG is close to the whole available rapidity interval. But what about proton dissociation into high-mass systems? At first sight, it appears that we may account for it by simply enlarging the number of diffraactive eigenstates \(\phi_k\). Unfortunately, this does not work. To see this, we decompose the wave functions of the incoming protons in terms of parton distributions. At very small \(x\), where we sample the ‘sea’ parton distributions, we face a difficulty. The problem is double counting, which occurs when the ‘sea’ partons originating from the dissociation of the ‘beam’ and the ‘target’ protons overlap in rapidities, as shown in Fig. 2(a); see [12].

Thus, to avoid double counting, a reasonable convention for assigning the parton contributions would be to build up the Good-Walker diffractive eigenstates just from the valence distributions, and to attribute the sea distributions to Pomeron exchange. The multi-Pomeron exchange diagrams (such as those of Fig. 1(c)) describe, among other things, large rapidity gap (LRG) events. The last diagrams (such as those of Fig. 1(c)) describe, among other things, large rapidity gap (LRG) events. The last
diagrams (such as those of Fig. 1(c)) describe, among other things, large rapidity gap (LRG) events.

2.2 Second “definition” of diffraction

Another possibility is to call diffraction any process caused by Pomeron exchange, or to be more precise - by the exchange corresponding to the ‘rightmost vacuum singularity’ in the complex angular momentum plane. It was the old convention that any event with a LRG of the size \(\delta \eta > 3\) may be called “diffraction”, since here Pomeron exchange gives the major contribution. Unfortunately, again, the situation is more complicated. LRG, in the distribution of secondaries, may occur due to the Reggeon exchange, when the colour flow across the gap is absent and the parton wave function saves its initial coherence within the interval occupied by the Reggeon (that is across the LRG), see Section 3. In addition, a gap may also occur just due to fluctuations of secondaries generated during the hadronization process. Indeed, we show in Section 4 that events with LRG of size \(\delta \eta > 3\) do not unambiguously select diffractive events at LHC energies.

3 LRG from Reggeon and Pomeron exchange

First, we discuss the gaps caused by Reggeon exchange. In terms of Regge Field Theory (RFT) [13] this contribution is described by the Reggeon loop, see Fig. 2(c). If we neglect the multi-Reggeon diagrams, then the corresponding probability to get LRGs takes the form

\[ P_R(\delta \eta) = c_R \exp(\delta \eta (2\alpha_R - \alpha_P - 1)), \]

where \(\alpha_R\) is the trajectory of the secondary Reggeon, while \(\alpha_P\) is the intercept of the vacuum singularity (Pomeron).

On the other hand for the Pomeron loop, Fig. 2(b), the probability is

\[ P_P(\delta \eta) = c_P \exp(\delta \eta (\alpha_P - 1)). \]

Thus, secondary Reggeon exchange\(^3\) produces gaps with a correlation length \(l_R = -1/(2\alpha_R - \alpha_P - 1) \sim 1\), while the Pomeron loop provides a long range correlation with length \(l_P \sim O(1/(\alpha_P - 1))\).

In the framework of perturbative QCD, Pomeron exchange is given by the BFKL amplitude [14]. After accounting for the resummed Next-to-Leading Log (NLL) corrections [15–17] the expected BFKL Pomeron intercept is \(\alpha_P = 1 + \Delta_P \sim 1.3\). On the other hand, the present data on high-energy total cross sections are well described by a ‘soft’ Donnachie-Landshoff parametrization [18], corresponding to single Pomeron exchange (without the multi-Pomeron contributions) with an ‘effective’ \(\alpha_{DL}(0) - 1 = \Delta_{DL} = 0.08\).

There are two interpretations for the reduction in the rise of the cross section, \(\sigma_{tot} \propto s^\Delta\), from \(\Delta = 0.3\) to \(\Delta = 0.08\):

\(^1\) We emphasize that each multi-Pomeron exchange diagram describes several different processes, according to whether or not the individual Pomeron exchanges are ‘cut’. The LRG component corresponds to the case, where in some rapidity interval, the cuts go between the Pomerons, so that there are no particles produced in this rapidity interval.

\(^2\) We are not discussing here rapidity gaps caused by electroweak exchanges.

\(^3\) Here we put \(\alpha_R = 1/2\).
(a) Elastic amplitude

\[ \text{Im} A_{el} = \overbrace{\text{I}}_{k} = 1 - e^{-\Omega/2} = \sum_{n=1}^{\infty} \overbrace{\text{I}}_{\Omega/2} \]

(b) Inclusion of low-mass dissociation

\[ \text{Im} A_{ik} = \overbrace{\text{I}}_{k}^{i} = 1 - e^{-\Omega_{ik}/2} = \sum \overbrace{\text{I}}_{\Omega_{ik}/2} \]

(c) Inclusion of high-mass dissociation

\[ \Omega_{ik} = \overbrace{\text{I}}_{k}^{i} + \overbrace{\text{I}}_{k}^{i} + M + \overbrace{\text{I}}_{...} \]

Fig. 1. (a) The single-channel eikonal description of elastic scattering; (b) the multichannel eikonal formula which allows for low-mass proton dissociations in terms of diffractive eigenstates \(|\phi_i\rangle, |\phi_k\rangle\); and (c) the inclusion of the multi-Pomeron-Pomeron diagrams which allow for high-mass dissociation. In all these diagrams the exchanged lines represent Pomeron exchange.

Fig. 2. (a) A 'soft' high energy interaction in which the sea partons which originate from the dissociation of the colliding protons overlap in rapidity. The overlap illustrates the impossibility of achieving a unique definition of diffraction in terms of Good-Walker diffractive eigenstates; (b) is a corresponding Pomeron loop diagram. Plot (c) is a Reggeon loop diagram. There are four different cuts of the Pomerons in the loop of diagram (b): we may cut either the left or the right or neither or both of the Pomerons corresponding to processes of diagram (a) where the coherence of the partons in the central ('overlapping') rapidity region is destroyed or saved in the right or left parton shower. A cut of a Pomeron means that the coherence of the corresponding parton shower is destroyed. A rapidity gap occurs when neither Pomeron in the loop is cut.

- If it is caused by eikonal-like screening corrections, (1), that is by the rescatterings of the initial fast hadrons, while the intercept corresponding to an individual parton shower (the Pomeron) remains large \((\Delta \sim 0.3)\), then the major part of the LRG events are those events with the largest available rapidity gaps (the probability \(P_R(\delta \eta)\) grows with \(\delta \eta\)). Therefore, in this framework, we actually come back to low-mass, quasi-elastic, diffractive dissociation. Conversely, high-mass dissociation will give a small contribution only.

- Alternatively, if the effective Pomeron intercept becomes smaller due to rescattering (screening) between the internal partons inside an individual parton cascade (the BFKL ladder) (thought to be due to the so-called enhanced diagrams \([19]\)), then the probability of high-mass diffractive dissociation will be large. In addition, then the corresponding correlation length is expected to be of the order of the ‘renormalized’ \(1/\Delta \sim 10\).

Besides this, there may be an interference contribution arising from the secondary Reggeon across the gap in the amplitude \(A\) and the Pomeron exchange in the amplitude
A* (and vice versa). Then the corresponding correlation length is \( l_{\text{int}} = 1/(1 - \alpha_R) \sim 2 \). Obviously, this additional effect will blur the two alternative pictures outlined above.

### 4 LRG from fluctuations

In addition, there is yet another effect, which will further obfuscate the picture: Obviously, a rapidity gap may also occur just due to fluctuations of secondaries generated during the hadronization process in an otherwise ‘perfectly inelastic’ event. We will first crudely estimate the probability of LRGs arising from fluctuations at the LHC, before we perform a more detailed Monte Carlo study of this possibility.

#### 4.1 Analytic estimate

If we assume an independent creation of secondaries in each rapidity interval, then the probability to get a gap during hadronization can be written as

\[
P_{\text{fluc}}(\delta \eta) = \frac{1}{l_f} \exp(-\delta \eta/l_f). \tag{4}
\]

The distribution over the gap size measured at the Tevatron by CDF [20] indicates that the correlation length \( l_f \sim 0.7 - 0.75 \).

To obtain the corresponding cross section, the probability (4) should be multiplied by the expected inelastic cross section \( \sigma_{\text{inel}} \sim 50 \text{ mb} \) and integrated over the chosen interval in \( \eta \) with a weight equal to the particle density \( dN/d\eta \). Thus, the cross section of the events with a gap \( \delta \eta \) larger than \( |\delta \eta|_{\text{min}} \equiv \Delta \eta \) is

\[
\sigma_{\text{fluc}}(\delta \eta > \Delta \eta) = \sigma_{\text{inel}} \int P_{\text{fluc}}(\delta \eta) \, d\delta \eta \, d\eta \, dN/d\eta. \tag{5}
\]

For an estimate at the LHC, we take \( l_f = 0.7 \) and \( dN/d\eta \sim 3 \). We use the ATLAS cuts [21] of \( p_\perp > 0.5 \text{ GeV} \) and \( |\eta| < 5 \). We find the probability

\[
P_{\text{fluc}}(\delta \eta > 3) = \exp(-3/0.7) \cdot (dN/d\eta) \cdot \Delta \eta \sim 0.25, \tag{6}
\]

which with \( \sigma_{\text{inel}} \sim 50 \text{ mb} \) gives \( \sigma_{\text{fluc}}(\delta \eta > 3) \sim 10 \text{ mb} \). This value should be compared with the expected cross section of diffractive double dissociation \( \sigma_{\text{DD}} \sim 10 \text{ mb} \), which according to the data from lower energy colliders, has a very flat energy dependence [20].

#### 4.2 Monte Carlo study of LRG from fluctuations

In order to obtain a somewhat more realistic estimate of fluctuation contributions to different event classes, inclusive QCD events were generated using the SHERPA 1.2.1 event generator [22]. This sample contains no diffractive events. On the matrix element level, partonic \( 2 \to 2 \) scattering processes are generated, which are supplemented by a Catani-Seymour dipole shower [23]. Multiple interactions are simulated using a model based on [24,25], which bases on independent (semi-)hard partonic scattering processes with decreasing \( p_\perp \). On the perturbative level, the only correlation between individual parton scatterings emerges through the rescaling of the parton distribution functions after each scatter and its parton shower has taken place. The parameters of the multiple interaction model where tuned to describe Tevatron underlying event data. SHERPA by default uses a cluster hadronisation [26], but, it can also can also hand over events to the Lund string fragmentation in PYTHIA [27]. Both models were tuned to reproduce LEP data\(^5\). The events were analysed using RIVET [28]. We use both models to get an estimate of the uncertainty. Our studies are performed at the hadron level without accounting for detector effects.

The probability for finding a gap larger than some \( \Delta \eta \) in an event was extracted as a function of the gap size for different gap definitions, hadronisation models, threshold \( p_\perp \), trigger conditions and beam energies. The default set up is as follows:

- the hadronic c.m.-energy is given by \( \sqrt{s_{pp}} = 7 \text{ TeV} \).
- all particles (charged and neutral) in \( |\eta| < 5 \) with a \( p_\perp \)-threshold of \( p_\perp, \text{cut} = 0.5 \text{ GeV} \) are considered;
- the gap is allowed to be anywhere in the ‘calorimeter acceptance’ \( \eta \in [-5, 5] \)
- no further trigger condition is required
- the simulation bases on inelastic \( 2 \to 2 \) scatters in QCD, with \( p_\perp > 2.8 \text{ GeV} \), supplemented with SHERPA’s default parton shower and cluster hadronisation.

Fig. 3 shows the gap probability for three different gap definitions, which are inspired by the ATLAS acceptance: The first (‘all’) is the default definition (all particles in \( |\eta| < 5 \), the gap can be anywhere), the second (‘central’) also takes all particles in \( |\eta| < 5 \) but requires the gap to be central (i.e. including \( \eta = 0 \)). In the third option (‘charged’) only charged particles in the tracking region \( |\eta| < 5 \) are considered and the gap, again, can be anywhere. The observed gap rates are generally sizable – in case of the ‘all’ definition as shown in Fig. 3 the probability for observing a rapidity gap larger than 3 units is around 25\% and the probability for a central gap with \( \Delta \eta > 3 \) is still sizable, around 10-15\%. Given the large inelastic QCD cross section these are worryingly large numbers.

In Fig. 4 the dependence on the \( p_\perp \) cut and the hadronisation model is investigated. As expected, the gap probability depends strongly on the \( p_\perp \) cut. Since most particles are rather soft a moderate \( p_\perp \) cut removes a considerable fraction of the particles from the event and thus drastically increases the chances of observing a large rapidity gap. At high \( p_\perp \) the cluster and string fragmentation yield identical results (upper line in Fig. 4), but at lower \( p_\perp \) the difference between the two models becomes sizable. Recall, however, that the string model was only failed.

\(^5\) Note, though, that the tune of the Lund fragmentation performed was quite crude.
all particles in $|\eta| < 5$, all gaps
all particles in $|\eta| < 5$, central gaps
charged part. in $|\eta| < 2.5$, all gaps

Fig. 3. Probability for finding a rapidity gap larger than $\Delta \eta$ in an inclusive QCD event (cluster hadronisation) for different gap definitions. 'All gaps' refers to a scenario where the gap can be anywhere in the acceptance, while in 'central gaps' the gap is required to be central (i.e. $\eta = 0$ has to lie in the gap). The $p_\perp$ threshold is 500 MeV and no trigger condition was required, $\sqrt{s} = 7$ TeV.

cluster hadr.
Lund string frag.

Fig. 4. Probability for finding a rapidity gap (definition 'all') larger than $\Delta \eta$ in an inclusive QCD event for different threshold $p_\perp$. From top to bottom the thresholds are $p_\perp, \text{cut} = 1.0$, 0.5, 0.1 GeV. Note that the lines for cluster and string hadronisation lie on top of each other for $p_\perp, \text{cut} = 1.0$ GeV. No trigger condition was required, $\sqrt{s} = 7$ TeV.

crude tuned together with the SHERPA parton shower. The discrepancies seen in Fig. 4 should thus be regarded as an upper bound for the model uncertainty due to hadronisation effects. As an additional test, the cluster hadronisation model in SHERPA was employed with two different algorithms of assigning colour in the final state, to see if non-pertubative colour reconnection effects have a sizable impact. In these two algorithms, minimising the 'length' of the total colour flow in momentum space and random assignment, no significant change of the size of the difference between the cluster and the string fragmentation has been found. It is also noteworthy that the string hadronisation produces for low $p_\perp$ thresholds a distribution that is not exponential but has two components that resemble a Pomeron contribution at large gap sizes.

The gap probability decreases moderately with increasing beam energy (Fig. 5), since the multiplicity and mean
\( \sqrt{s} = 0.9 \text{ TeV} \)
\( \sqrt{s} = 7 \text{ TeV} \)
\( \sqrt{s} = 10 \text{ TeV} \)

Fig. 5. Beam energy dependence of the probability for finding a rapidity gap (definition 'all') larger than \( \Delta \eta \) in an inclusive QCD event (\( p_{\perp, \text{cut}} = 0.5 \text{ GeV} \), no trigger condition, cluster hadronisation).

Fig. 6. Probability for finding a rapidity gap (definition 'all') larger than \( \Delta \eta \) in an inclusive QCD event with different trigger conditions. Trigger 1 (2) requires at least one charged particle with \( p_{\perp} > 0.5 \) (1.0) GeV in the central region \( |\eta| < 2.5 \). The value of \( P(\Delta \eta) \) at \( \Delta \eta = 0 \) is the probability for an event to fulfill the trigger condition. Events were generated with cluster hadronisation and \( p_{\perp, \text{cut}} = 0.5 \text{ GeV} \) for \( \sqrt{s} = 7 \text{ TeV} \).

\( p_{\perp} \) increase with \( \sqrt{s} \). This translates directly into a lower probability for large gaps caused by fluctuations.

The additional trigger conditions investigated were to demand at least one charged particle with \( p_{\perp} > 0.5 \) (1.0) GeV in the central region \( |\eta| < 2.5 \) (‘trigger 1 (2)’). The dependence of the gap rate on these trigger conditions is shown in Fig. 6. The probability is defined such that the probability at \( \Delta \eta = 0 \) equals the probability for fulfilling the trigger condition. This leads to a downwards shift of the entire distribution. Otherwise, the trigger only affects the probabilities for very large gaps (\( \Delta \eta > 5 \)).

Given the large inclusive probabilities for rapidity gaps, fluctuations can also fake a signature with two large gaps which usually is attributed to double pomeron exchange (DPE) processes. The probabilities for events with two gaps above 2 or 3 units are given in Table 1. Due to the requirement of two large gaps these probabilities depend very strongly on the \( p_{\perp} \) cut. However, even relatively
small rates can be dangerous since the DPE cross section is much smaller than the inclusive QCD cross section. Furthermore, the invariant mass of the system between the gaps has contributions out to large masses (Fig. 7). The number of particles in the central system also has a wide distribution that does not fall off very quickly.

4.3 Implication of fluctuations

So, to study pure Pomeron exchange we either have to consider much larger gaps, where we are sure that the Pomeron dominates, or, in order to extract the pure diffractive Pomeron-loop contribution, to study the \( \delta \eta \) dependence of the cross section (of events with LRG) and to fit the data so as to subtract the part caused by the secondary Reggeons and/or by the fluctuations in the process of hadronization.

In principle this latter option sounds quite natural. However, first note, the probability to observe LRG due to fluctuations in hadronization is rather large; much larger than that expected from a simple Poissonian distribution. Moreover, in some models the \( \delta \eta \) dependence deviates from the simple exponential form predicted by Poissonian behaviour, so it is prohibitively hard to guarantee that Pomeron exchange is observed, rather than the details of the event. Indeed, given the crude analytical estimate and its order-of-magnitude confirmation by the MC study, it appears entirely possible that LRG of size \( \delta \eta \simeq 3 \) predominantly are caused by fluctuations.

Therefore, first, we have to select larger gaps with \( \delta \eta \sim 5 \) or more. Moreover, even fitting the gap size distribution by two (or more) exponents in some model for hadronization, we may obtain an exponent with a large correlation length see the lowest curve Fig. 4. If the diffractive contribution is about 5-10 mb this is not a problem.

The long range-part coming from fluctuations is of the order of hundreds of \( \mu b \) (more than ten times less), but if the diffractive cross sections are smaller than 1 mb then they will be extremely difficult to isolate. In particular, the expected diffractive DPE cross sections are about 1 - 10 \( \mu b \), while from fluctuations (assuming, for example, the ATLAS rapidity cuts \([21]\)) we may obtain up to 0.5 mb for the production of a central system of mass squared \( M^2 \sim 10 \text{ GeV}^2 \), with gaps \( \delta \eta > 2 \) either side, see Fig. 7. This immediately implies that there is practically no chance to unambiguously select soft diffractive DPE events based on LRG triggered in the calorimeter interval \( |\eta| < 5 \). For this we would therefore need a larger \( \eta \) coverage, which could be achieved, for instance, by adding Forward Shower Counters \([29]\).

Bearing in mind all theses problems it seems natural to prefer the first definition of diffraction, based on elastic scattering of the diffractive eigenstates. Unfortunately there is no way to directly measure low-mass diffractive dissociation in the first years of LHC running. Note that for low-mass dissociation at high energies we deal with a very large rapidity gap \( \delta \eta \gg 5 \) and only the long range correlation, caused by the Pomeron exchange, survives. In this limiting case the two definitions of diffraction become equivalent to each other\(^6\).

\(^{6}\) Here, let us recall, just for completeness, the expression for single-proton dissociation, \( p \rightarrow X \) corresponding to the triple-Reggeon diagrams, that is the PPP (and PPR) diagram shown as the second term on the right hand side of Fig. 1(c). When the mass \( M_X \) of the system \( X \) is small in comparison with the initial energy \( \sqrt{s} \), the dominant contribution comes from the Pomeron exchange, and the \( M_X \) behaviour of the cross section takes the form

\[
\frac{d\sigma^{SD}}{dM_X^2} \propto (M_X^2)^{\alpha_k - 2\alpha_P}.
\] (7)
5 LRG in the early LHC runs

Of course, the correlation length $l_f$ of fluctuations at the LHC energies may be smaller than that measured at the Tevatron. Then the value of $\sigma_{\text{Pom}}(\delta \eta > \Delta \eta)$ would not be so large. This question should be studied experimentally in the first data runs of the LHC. Recall that the early LHC runs, which have relatively low luminosity, are well suited for diffractive processes where the expected cross sections are rather large, and where pile-up effects do not reduce the significance. Thus, we may select LRG events simply by using the ‘veto’ trigger, that is, by selecting events where in some interval $\delta \eta > \Delta \eta$ no particles are observed with $p_L > p_L \text{cut}$ in either the calorimeter or tracker.\(^7\)

The triple-Pomeron contribution (PPP term) has $\alpha_L = \alpha_P$, which, for $\alpha_P = 1$, leads to $d\sigma^{SD}/dM_X^2 \sim 1/M_X^2$, whereas for the PPR term, which may be important at smaller mass, yields $d\sigma^{SD}/dM_X^2 \sim 1/M_X^3$. Before the advent of the LHC, triple-Reggeon events were selected mainly by detecting forward protons with a large initial momentum fraction $x_L$ close to 1. From a theoretical viewpoint this is the same as the selection of LRG; the size of the gap $\delta \eta \approx \ln(1/(1 - x_L))$, and the conventional choice $x_L > 0.95$, are equivalent to $\delta \eta > 3$. At relatively low collider energies, from knowledge of the value of $x_L$, it was even possible to determine the mass of the diffractively produced system, $M_X = (1 - x_L)s$, and hence to distinguish the contributions of the different $N^*$ resonances. At LHC energies, we have no hope of reaching such a good accuracy in the measurement of $x_L$, and so we cannot study low-mass dissociation in this way. The only chance is to complement the LHC detectors by Forward Shower Counters (FSC) in order to veto the production of extra secondaries in the region close to the fragmentation of the incoming proton, and thus to suppress higher-mass dissociation.

\(^7\) To suppress fluctuations due to hadronization in non-diffractive events it would be better to have a very low $p_L \text{cut}$, however then the fluctuations in the calorimeter become important. In addition, it is not clear, a priori, how hadronization effects impact on diffractive events.

\(^8\) Note, however, that even in high luminosity runs, we may study diffractive processes by selecting LRG events using the ‘veto’ trigger. Of course, the efficiency of the ‘veto’ trigger becomes low at high luminosity, since quite often the gap will be filled by the secondaries produced in the ‘pile-up’ events. If we denote the mean number of inelastic pp interactions per bunch crossing by $n$, then we have the Poisson probability $P_n(0) = e^{-n}$ to have no additional ‘pile-up’ secondaries. Therefore the ‘veto’ trigger actually acts at an effective luminosity, $L_{\text{eff}} = L_0 e^{-n}$, which is much smaller than the true LHC luminosity $L_0$. On the other hand, the expected diffractive cross sections are rather large. For example, about $5 - 10$ mb for single diffractive dissociation and a few $\mu$b for Double-Pomeron-Exchange (DPE) events with two LRG. Thus, even in the case of a pile-up of $n \sim 10$ the cross sections are sufficiently large for the reduced luminosity, $L_{\text{eff}}$, to reveal diffractive processes.

5.1 Rapidity correlations

Let us work in terms of the second definition, where the word ‘diffraction’ means Pomeron exchange. We will show how the multi-Pomeron effect can be studied at the LHC, not only by selecting events with LRG, but also by measuring long-range rapidity correlations.

5.1.1 Rapidity correlations in multiplicity, $R_2(N)$

First note, from the AGK cutting rules [30], that multi-Pomeron diagrams describe not only the processes with LRG, but simultaneously also events with a larger density of secondaries (when a few Pomerons are cut). The simplest example is the four different cuts of the Pomeron loop diagram of Fig. 2(b); the processes described by this diagram are shown in Fig. 8. Therefore, in two-particle inclusive cross sections, we should observe the same long-range rapidity correlations, that is the same correlation length $l_P$, as in the LRG events.

Let us explain this in a bit more detail: If we were to cut $n$ Pomerons in a multi-Pomeron diagram, then we would get an event with multiplicity $n$ times larger than that generated by cutting just one Pomeron. The observation of a particle at rapidity $y_a$, say, has the effect of enlarging the weight of the contribution of diagrams with many Pomerons cut. For this reason the probability to observe another particle at quite a different rapidity $y_b$ becomes larger as well. This two-particle correlation can be observed experimentally via the ratio of inclusive cross sections

$$R_2 = \frac{\sigma_{\text{inel}} d^2\sigma/dy_a dy_b}{(d\sigma/dy_a)(d\sigma/dy_b)} - 1 = \frac{d^2N/dy_a dy_b}{(dN/dy_a)(dN/dy_b)} - 1,$$

where $dN/dy = (1/\sigma_{\text{inel}}) d\sigma/dy$ is the particle density. Without multi-Pomeron effects the value of $R_2$ exceeds zero only when the two particles are close to each other, that is when the separation $|y_a - y_b| \sim 1$ is not large. Such short-range correlations arise from resonance or jet production.

Table 1. Probabilities for having two large gaps for the two different hadronisation models at $\sqrt{s} = 7$ TeV. All particles (charged and neutral) in $|\eta| < 5$ were considered. No trigger condition was required, but there is no strong dependence on the trigger condition, since with two large gaps the central system is likely to be in the tracking region $|\eta| < 2.5$. 

| $p_L \text{cut}$ | $\delta \eta > 2$ cluster | $\delta \eta > 2$ string | $\delta \eta > 3$ cluster | $\delta \eta > 3$ string |
|----------------|----------------|----------------|----------------|----------------|
| 0.1 GeV       | 0.20 %         | 0.03 %         | 0.002 %        | 0.0001 %       |
| 0.2 GeV       | 1.1 %          | 0.12 %         | 0.04 %         | 0.007 %        |
| 0.5 GeV       | 17 %           | 5.7 %          | 1.5 %          | 0.25 %         |
| 1.0 GeV       | 55 %           | 56 %           | 19 %           | 19 %           |
However, a positive value of $R_2$ at large $|y_a - y_b|$ will indicate the presence of a long-range correlation arising from Pomeron loops. To see this, suppose that in a rapidity interval, which includes $y_a$ and $y_b$, there are $n$ cut Pomerons. That is, $n$ independent Multiple Interactions (MI) take place simultaneously in this interval. The total inelastic cross section

$$\sigma_{\text{inel}} = \sum_n \sigma_n,$$

(9)

the one-particle inclusive cross section

$$\frac{d\sigma}{dy} = \sum_n n \cdot \sigma_n \cdot a,$$

(10)

and the two-particle inclusive cross section

$$\frac{d^2\sigma}{dy_a dy_b} = \sum_n n^2 \cdot \sigma_n \cdot a^2,$$

(11)

where $a = dN^{(1)}/dy$ is the particle density in an individual MI or cut Pomeron. Then the two-particle correlation, $R_2$ of (8), takes the form

$$R_2 = \frac{\langle n^2 \rangle}{\langle n \rangle^2} - 1.$$

(12)

Since $\langle n^2 \rangle > \langle n \rangle^2$ for the exchange of a few Pomerons (i.e. a few MI), we expect $R_2 > 0$. Moreover, the rapidity interval occupied by these Pomerons controls the correlation length measured via $R_2$. To be explicit, the $R_2$ correlations measure the contributions due to those Pomeron loops which embrace both $y_a$ and $y_b$.

5.1.2 Rapidity correlations in $E_\perp$, $R_2(E_\perp)$

Besides the correlation $R_2(N)$ between the particle densities (that is, the multiplicities) in different rapidity bins, we may measure the correlations between the transverse momenta, $p_\perp$, of the secondaries (or correlations $R_2(E_\perp)$, of the transverse energy flow, $dE_\perp/d\eta$).

If a non-zero correlation $R_2$ arises from Multiple Interactions and if different MI do not depend on each other, then each Pomeron/MI (each parton shower) should have the same $p_\perp$ distribution. Hence the correlations in $E_\perp$ will be identical to the multiplicity correlations, since in this case

$$\frac{dE_\perp}{d\eta} = \sum_i \sqrt{m_i^2 + p_{i\perp}^2} \frac{dN_i}{d\eta},$$

(13)

and the $p_\perp$ distribution does not depend on the number of simultaneous Multiple Interactions.

On the other hand, a correlation can arise from resonance decay or from jet production (and fragmentation), in an individual MI. These correlations are of short-range. They will result in a relatively narrow peak in the $|y_a - y_b| \sim O(1)$ distribution. However, two high-$E_\perp$ jets, which balance each other in $p_\perp$, may be separated by a relatively large rapidity interval. The correlation due to the production of such a high-$E_\perp$ dijet system is revealed better in the $E_\perp$ flow. Moreover, a larger multiplicity at $\eta = y_a = y_{\text{jet1}}$ should lead to a larger transverse energy flow, and a larger mean $\langle p_\perp \rangle$, in the region occupied by the other jet, that is in the ‘away’ azimuthal region (with respect to the first jet) with $\eta = y_b = y_{\text{jet2}}$. We can always suppress these correlations, which originate from high-$E_\perp$ dijets, by studying correlations in the ‘transverse’ region in the azimuthal plane. The notation ‘away’ and ‘transverse’ is that of Field [31].

It would be most interesting to observe how the different MI depend on each other. Indeed, we would not expect to have independent MI, each of which are producing strongly interacting secondaries in a limited volume of configuration space. In an event with many MI, where the density of secondaries is large, it is natural to expect a larger $p_\perp$ (due to the rescattering of the secondaries and a stronger absorption of low $p_\perp$ hadrons; in other words,
due to a larger saturation momentum\(^9\), \(Q_s\). This effect will result in a larger value of \(\langle p_{\perp} \rangle\), or of the \(E_\perp\) flow, measured at \(\eta = y_b\), in events with a larger particle density at \(\eta = y_a\). In terms of the \(E_\perp\) flow, a correlation larger than that given by \(R_2(N)\) would reflect the growth of \(Q_s\) or \(\langle p_{\perp} \rangle\) as the particle density, increases. In other words we expect that \(R_2(E_\perp) > R_2(N)\).

At the LHC such long-range correlations can be measured in the ATLAS or CMS calorimeters which cover quite a large rapidity interval, \(-5 < \eta < 5\). The observation of similar rapidity distributions in both LRG events and \(R_2\) correlations would be an argument in favour of the effects being due to Pomeron exchange (that is, diffraction).

### 5.2 Events with multiple rapidity gaps

At the LHC, it would be informative to study not only the events with one LRG, but also events with two (or a few) LRG\(^{32}\). However, this would require a significantly larger coverage in rapidity space. The gap survival probability \(S_{\text{el}}\) corresponding to eikonal rescatterings (i.e. the probability that the gaps will not be filled by secondaries produced in an additional inelastic interaction of the incoming fast protons) depends very weakly on the number of LRG in an event. On the other hand, the main absorptive effect may be due to enhanced rescattering involving intermediate partons. But, the number of different enhanced diagrams increases with the number of gaps, and so, in this case, we may expect a stronger suppression of multi-gap events. The simplest example is the Central Diffractive Production of some system \(X\) of mass \(M_X\) separated on both sides from the other secondaries by LRG, or even the ‘exclusive’ process \(pp \rightarrow p \oplus X \oplus p\) (where the \(\oplus\) signs indicate LRG). Such reactions are usually called Double-Pomeron-Exchange (DPE) processes. It was demonstrated in\(^{12}\) that the predictions for DPE cross sections depend strongly on the details of the model. Therefore the detailed study of multi-gap events with much larger gaps at the LHC would select the most realistic model of ‘soft’ physics, and, moreover, constrain the values of the parameters used in the model.

A more detailed discussion of soft physics at the LHC, and the qualitative features that are essential for a realistic model of ‘soft’ interactions, was presented in\(^{10}\).

### 6 Conclusions

If we use the convention that diffractive events are just the ‘elastic’ scattering of ‘Good-Walker’ eigenstates, then we consider only elastic scattering and low-mass diffractive dissociation of the proton, processes which are not immediately observable at the LHC.

From a wider viewpoint, any process due to Pomeron exchange may be called diffractive. In general, such processes lead to Large Rapidity Gaps (LRG) in the distribution of secondary hadrons. However, the probability to obtain a gap without Pomeron exchange is not negligible; the gap can simply arise from fluctuations in the hadronization process. The Monte Carlo studies presented in this paper show that, with the present rapidity acceptances and \(p_{\perp}\) cuts of the LHC detectors, up to \(\sim 0.5\) mb of the diffractive cross section can be mimicked by fluctuations\(^{10}\) which have nothing to do with Pomeron exchange. This is not a serious background if the cross section of the diffractive process that we are studying is \(\sim 10\) mb or larger, but it will pose a problem for studying the so-called double-Pomeron exchange (DPE) events, where the expected cross sections are \(\sim 10\) \(\mu\)b.

In Ref.\(^{33}\) CMS\(^{11}\) claim observation of inclusive diffraction at \(\sqrt{s} = 900\) and 2360 GeV. The events have low multiplicity in the detector and/or have relatively low light-cone momentum, \(E \pm p_{\perp} < 8\) GeV. In order to describe these events CMS use the single-diffractive dissociation option in the PYTHIA or Phojet Monte Carlos. The contribution from the non-diffractive component (due to fluctuations) is about 3 – 4 times lower than that observed in the lowest multiplicity or the lowest light-cone momentum bins. However, note that PYTHIA uses ‘string’ hadronization whereas for ‘cluster’ hadronization the probability of fluctuations with \(\Delta \eta \sim 3\) is more than 3 times larger both with \(p_{\perp,\text{cut}} = 0.1\) GeV and with \(p_{\perp,\text{cut}} = 0.5\) GeV, see Fig. 4. Therefore it is possible that the main contribution, to the ‘diffractive’ events observed by CMS, may be actually due to fluctuations. It would be interesting to study experimentally the dependence of the cross section on the size of the rapidity gap, \(\Delta \eta\), and on the value of \(p_{\perp,\text{cut}}\), in order to constrain the model used for hadronization, and hence to be able to select true diffractive events.

Recall, for processes driven by multi-Pomeron exchange, that the same diagram describes LRG events and events with an enlarged multiplicity of secondaries. Therefore, comparing the long-range correlations between multiplicities (and between transverse energy) with that observed in LRG events, it is possible to confirm that the effect is due to Pomeron-exchange (and not due to fluctuations).

### Acknowledgements

We thank Risto Orava and Andrew Pilkington for valuable discussions. MGR would like to thank the IPPP at the University of Durham for hospitality. VAK thanks the participants of the Diffraction at LHC meeting at\(^{10}\) As seen in fig. 4, with \(p_{\perp,\text{cut}} = 0.5\) GeV the probability to have a gap \(\delta \eta > 5\) is between 0.01 (string hadronisation) and 0.1 (cluster hadronisation). With the inelastic cross section \(\sigma_{\text{inel}} \sim 50\) mb this leads to \(0.5 - 5\) mb of LRG caused by fluctuations.

\(^{11}\) We thank the referee for pointing us to this note, which actually appeared after the submission of our manuscript.

\(^9\) In Reggeon Field Theory these effects are described by enhanced multi-Pomeron diagrams.
CERN for encouraging discussions. This work was supported by the Federal Programme of the Russian State RSGSS-3628.2008.2.

References

1. V. A. Khoze, A. D. Martin and M. G. Ryskin, Phys. Lett. B 679, 56 (2009) [arXiv:0906.4876 [hep-ph]].
2. TOTEM Collaboration: V. Berardi et al., CERN-LHCC-2004-002, Jan 2004; CERN-LHCC-2004-020, Jun 2004.
3. V. A. Khoze, A. D. Martin, R. Orava and M. G. Ryskin, Eur. Phys. J. C 19, 313 (2001) [arXiv:hep-ph/0010163].
4. M. M. Block, Phys. Rept. 436, 71 (2006) [arXiv:hep-ph/0606215].
5. R. Fiore, L. L. Jenkovszky, R. Orava, E. Predazzi, A. Prokudin and O. Selyugin, Int. J. Mod. Phys. A 24, 2551 (2009) [arXiv:0810.2902 [hep-ph]].
6. M. G. Ryskin, A. D. Martin and V. A. Khoze, Eur. Phys. J. C 54, 199 (2008) [arXiv:0710.2494 [hep-ph]].
7. A. Gotsman, E. Levin, U. Maor and J. Miller, Eur. Phys. J. C 57, 689 (2008).
8. A. B. Kaidalov and M. G. Poghosyan, arXiv:0909.5156 [hep-ph].
9. S. Ostapchenko, arXiv:1003.0196 [hep-ph].
10. for a recent review see M. G. Ryskin, A. D. Martin, V. A. Khoze and A. G. Shuvaev, J. Phys. G 36, 093001 (2009) [arXiv:0907.1374 [hep-ph]].
11. M.L. Good and W.D. Walker, Phys. Rev. 120, 1857 (1960).
12. M. G. Ryskin, A. D. Martin and V. A. Khoze, Eur. Phys. J. C 54, 199 (2008) [arXiv:0710.2494 [hep-ph]].
13. V.N. Gribov, Sov. Phys. JETP 26, 414 (1968).
14. V.S. Fadin, E.A. Kuraev, and L.N. Lipatov, Phys. Lett. B 60, 50 (1975);
E.A. Kuraev, L.N. Lipatov, and V.S. Fadin, Zh. Eksp. Teor. Fiz. 71, 840 (1976) [Sov. Phys. JETP 44, 443 (1976)]; ibid. 72, 377 (1977) [45, 199 (1977)];
I.I. Balitsky and L.N. Lipatov, Yad. Fiz. 28, 1597 (1978) [Sov. J. Nucl. Phys. 28, 822 (1978)].
15. V.S. Fadin and L.N. Lipatov, Phys. Lett. B 429, 127 (1998).
G. Camici and M. Ciafaloni, Phys. Lett. B 430, 349 (1998).
16. G.P. Salam, JHEP 9807, 019 (1998), Act. Phys. Pol. B 30, 3679 (1999);
M. Ciafaloni, D. Colferai and G.P. Salam, Phys. Lett. B 452, 372 (1999), Phys. Rev. D 60, 114036 (1999).
17. V. A. Khoze, A. D. Martin, M. G. Ryskin and W. J. Stirling, Phys. Rev. D 70, 074013 (2004) [arXiv:hep-ph/0406135].
18. A. Donnachie and P. V. Landshoff, Phys. Lett. B 296, 227 (1992) [arXiv:hep-ph/9209205].
19. J.L. Cardy, Nucl. Phys. B 75, 413 (1974);
A.B. Kaidalov, L.A. Ponomarev and K.A. Ter-Martirosyan, Sov. J. Nucl. Phys. 44 (1986) 468.
20. T. Affolder et al, PRL 87, 141802 (2001)
21. G. Aad et al. [ATLAS Collaboration], arXiv:0901.0512 [hep-ex];
G. Aad et al. [ATLAS Collaboration], Phys. Lett. B 688, 21 (2010) [arXiv:1003.3124].
22. T. Gleisberg, S. Hoche, F. Krauss, M. Schonherr, S. Schumann, F. Siegert and J. Winter, JHEP 0902 (2009) 007 [arXiv:0811.4622 [hep-ph]].
23. S. Schumann and F. Krauss, JHEP 0803 (2008) 038 [arXiv:0709.1027 [hep-ph]].
24. T. Sjostrand and M. van Zijl, Phys. Rev. D 36 (1987) 2019.
25. S. Alekhi et al., arXiv:hep-ph/0601012.
26. J. C. Winter, F. Krauss and G. Soff, Eur. Phys. J. C 36 (2004) 381 [arXiv:hep-ph/0311085].
27. T. Sjostrand, L. Lonnblad, S. Mrenna and P. Z. Skands, arXiv:hep-ph/0308153.
28. A. Buckley, arXiv:0809.4638 [hep-ph].
29. USCMS Collaboration: M. Albrow et al., arXiv:0811.0120 [hep-ex].
30. V.A. Abramovsky, V.N. Gribov and O.V. Kancheli, Sov. J. Nucl. Phys. 18, 308 (1973).
[hep-ex]
31. R. D. Field [CDF Collaboration], in Proc. of the APS/DPF/DPB Summer Study on the Future of Particle Physics (Snowmass 2001) ed. N. Graf, In the Proceedings of APS / DPF / DPB Summer Study on the Future of Particle Physics (Snowmass 2001), Snowmass, Colorado, 30 Jun - 21 July 2001, pp P501 [arXiv:hep-ph/0211192].
32. V. A. Khoze, A. D. Martin and M. G. Ryskin, Eur. Phys. J. C 23, 311 (2002) [arXiv:hep-ph/0111078].
33. CMS report no: PAS-FWD-10-001