Effective interactions of a light gravitino

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Abstract: We review some recent results on the effective interactions of a light gravitino with ordinary particles. In particular, we discuss on a simple example a novel aspect of the low-energy theorems for broken supersymmetry: in the effective lagrangian describing the goldstino couplings to matter, there are terms bilinear in the goldstino that, already at the lowest non-trivial order, are not entirely controlled by the supersymmetry-breaking scale, and introduce additional free parameters. We conclude by mentioning some phenomenological implications, including a lower bound on the gravitino mass from collider data.

The interest in the low-energy effective interactions of the gravitino (goldstino) in the spontaneously broken phase dates back to the early days of supersymmetry [1, 2, 3]. Since then, significant theoretical progress was made [4]. However, some aspects of the subject have been clarified only recently [5, 6], following a renewed interest (see, e.g., [7] and references therein) in the physics at energies much larger than the gravitino mass, but smaller than the masses of the other supersymmetric particles. This talk begins with some introductory remarks on spontaneously broken $\mathbb{N}=1$ supersymmetry and its low-energy limit, continues with the illustration of some recent theoretical developments [5, 6], and ends by mentioning some of their phenomenological implications, whose exploration has just started [8, 9].

1 The general framework

Most of the participants in this Workshop share the view that the fundamental theory lying beyond the Standard Model should exhibit a spontaneously broken $\mathbb{N}=1$ supersymmetry. Many dynamical questions on spontaneous supersymmetry breaking are still unanswered, but the ‘kinematical’ aspects of the problem are well understood [10, 11]. We begin our discussion by recalling the supergravity potential:

$$V = ||F||^2 + ||D||^2 - ||H||^2,$$

where $||F||^2$, $||D||^2$ and $||H||^2$ are positive-definite quantities, controlled by the auxiliary fields of the chiral, vector and gravitational supermultiplets, respectively. The microscopic scale of supersymmetry breaking and the gravitino mass are given by $\Lambda_S \equiv \langle ||F||^2 + ||D||^2 \rangle^{1/4}$ and $m_{3/2} \equiv \langle ||H|| \rangle/\sqrt{3}M_P$, respectively, where $M_P \equiv (8\pi G_N)^{-1/2} \simeq 2.4 \times 10^{18}$ GeV is the Planck mass. From the facts that we live in an approximately flat space-time and that no direct signal of supersymmetry has been detected so far,

$$\Lambda_S^2 \simeq \sqrt{3} m_{3/2} M_P,$$

with an extremely good accuracy. The VEVs of the auxiliary fields of the chiral and vector multiplets identify the goldstino, which provides the $\pm 1/2$ helicity components of the massive gravitino:

$$\tilde{G} = \langle F_i \rangle \psi^i + \langle D_a \rangle \lambda^a.$$

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The generic form of the supersymmetry-breaking mass splittings is

\[(\Delta m^2)_I \sim g_I \cdot \Lambda_S^2 \simeq g_I \cdot \left(\sqrt{3} m_{3/2} M_P\right),\]  

where \(g_I\) is the effective coupling of the goldstino multiplet to the sector ‘I’ of the spectrum. Even if we stick to the prejudice that these splittings are of the order of the electroweak scale, for supersymmetry to play a rôle in the solution of the hierarchy problem, this does not fix \(\Lambda_S\) (or, equivalently, \(m_{3/2}\)), since the couplings \(g_I\) are model-dependent. The dynamical origin of the scales \(\Lambda_S\) and \(m_{3/2}\) is still obscure, and different possibilities can be legitimately considered. In this talk we concentrate on the case where the interactions of the goldstino multiplet are much stronger than the gravitational ones, so that the gravitino is much lighter than all the other supersymmetric particles. Our goal is to discuss the low-energy limit, with the typical energies of the processes under consideration much larger than the gravitino mass, but smaller than the masses of all the other particles not belonging to the Standard Model.

A rather explicit method that can be used for this study is based on two simple logical steps. First, we start from a generic supergravity lagrangian, assuming that supersymmetry is spontaneously broken with a light gravitino, and we take the appropriate low-energy limit: \(M_P \to \infty\) with \(\Lambda_S\) fixed. In accordance with the supersymmetric equivalence theorem [3, 12], gravitational interactions are consistently neglected in this limit, and we end up with an effective (non-renormalizable) theory with linearly realized, although spontaneously broken, global supersymmetry, whose building blocks are the chiral and vector supermultiplets containing the light degrees of freedom, including the goldstino. Since no supersymmetric partners of ordinary particles have been observed yet, a situation of present interest is the one where the available energy is smaller than the supersymmetry-breaking mass splittings. In such a context, we can perform the second step and move to a ‘more effective’ theory, by explicitly integrating out the heavy superpartners in the low-energy limit. The only degrees of freedom left are then the goldstino and the Standard Model particles, and supersymmetry is non-linearly realized.

We now discuss what can be learnt by comparing the results obtained via the previous method with the conventional wisdom [1, 4] on non-linear realizations: on-shell scattering amplitudes with two goldstinos and additional matter or gauge particles are fully controlled, at the leading non-trivial order in the low-energy expansion, by the scale \(\Lambda_S\) of supersymmetry breaking (with no further reference to the supersymmetry-breaking mass splittings), and by the canonical energy-momentum tensor \(T_{\mu\nu}\) of the matter and gauge fields. We list here a number of interesting cases where this comparison can be done.

- The amplitudes involving two goldstinos and two photons: \(\tilde{G}\tilde{G}\gamma\gamma\). It was believed for some time that the processes described by these amplitudes could be relevant for stellar cooling, and may allow to put an interesting lower bound on \(m_{3/2}\). After some controversial results, however, it was found [5] that the amplitudes obtained via the explicit construction outlined above are equivalent to those obtained from the conventional effective lagrangian of the non-linear realization. The energy suppression is then so strong that only very weak bounds on \(m_{3/2}\) can be derived.

- The amplitudes involving two goldstinos and two matter fermions: \(\tilde{G}\tilde{G}ff\). These are the ones that have recently revealed [6] via some puzzling results, some previously unnoticed features of the low-energy theorems for supersymmetry: they will be discussed in detail, on a simple example, in the rest of this talk. Also the processes described by these amplitudes were believed to play a rôle in stellar cooling but, as we shall see, no interesting bound on \(m_{3/2}\) can be obtained either [6].

- The amplitudes involving two goldstinos, two matter fermions and a vector boson, for example the photon or the gluon: \(\tilde{G}\tilde{G}f\gamma\), \(\tilde{G}\tilde{G}fg\). They are the most important ones for phenomenology. In the case of heavy superpartners, they can lead to an absolute lower bound on \(m_{3/2}\) or, optimistically, to the best signals for the discovery of supersymmetry. At the end of this talk, we shall mention some recent results on the phenomenology of light gravitinos at \(e^+e^-\) and hadron colliders.
2 A puzzling result

Consider, for simplicity, a globally supersymmetric theory with only two chiral superfields, one describing the goldstino \( \tilde{G} \) and its complex spin-0 partner \( z \equiv (S + iP)/\sqrt{2} \), the other one describing a massless left-handed matter fermion \( f \) and its complex spin-0 partner \( \tilde{f} \). According to the standard formalism [3], and neglecting for the moment higher-derivative terms, the lagrangian is completely specified in terms of a superpotential \( w \) and a Kähler potential \( K \). Assume that, at the minimum of the potential, \( \langle z \rangle = 0 \), \( \langle \tilde{f} \rangle = \langle F^1 \rangle \neq 0 \) (consistently with matter conservation), and \( \langle F^0 \rangle \neq 0 \), where \( F^0 \) and \( F^1 \) denote the auxiliary fields associated with \( G \) and with \( f \), respectively.

Our goal is to identify the effective four-fermion interaction involving two matter fermions and two goldstinos. Expanding around the vacuum, we can write:

\[
w = \hat{w}(z) + \ldots, \quad K = \hat{K}(z, \bar{z}) + \hat{K}(z, \bar{z}) |\tilde{f}|^2 + \ldots,
\]

where, here and in the following, the dots denote terms that are not relevant for our considerations. The spectrum can be easily derived from standard formulae [3]. The goldstino and the matter fermion remain massless, whilst all the spin-0 particles acquire in general non-vanishing masses. Moreover, the expansion of the lagrangian in (canonically normalized) component fields can be rearranged in such a way that all the interaction terms relevant for our calculation are expressed in terms of the mass parameters \( (m_S^2, m_P^2, m_f^2) \), associated with the spin-0 partners of the goldstino and of the matter fermion, and the scale \( \Lambda_S \) of supersymmetry breaking, without explicit reference to \( w \) and \( K \):

\[
\mathcal{L} = -\frac{1}{2\sqrt{2}F}[m_S^2 S + im_P^2 P]\hat{G}\hat{G} + \text{h.c.}] - \frac{\tilde{m}_f^2}{F} (\hat{f}^* \hat{G}\hat{f} + \hat{\tilde{f}} \bar{G}\bar{f}) - \frac{\tilde{m}_f^2}{F^2} \hat{G}\bar{f}\tilde{f} + \ldots.
\]

In (6), we have used two-component spinors. For simplicity, \( F \equiv \langle \sqrt{\frac{1}{2}}(K_{\bar{z}z})^{-1/2} \rangle \) (lower indices denote derivatives), which defines here the supersymmetry-breaking scale \( \Lambda_S = \sqrt{F} \), was assumed to be real.

Starting from (6), we take the limit of a heavy spin-0 spectrum, with \( (m_S, m_P, \tilde{m}_f) \) much larger than the typical energy of the scattering processes we would like to study. In this case, we can build an effective lagrangian for the light fields by integrating out the heavy states. As discussed in detail in [3], the crucial property of such an effective lagrangian is its dependence on the supersymmetry-breaking scale \( \Lambda_S \), without any further reference to the supersymmetry-breaking masses \( (m_S, m_P, \tilde{m}_f) \). This property is the result of subtle cancellations among different diagrams, corresponding to the contact interaction associated with the last term in eq. (6) and to \( \tilde{f} \) exchange, and agrees with previous results [1, 3, 4] on low-energy goldstino interactions. Focussing on the terms relevant for our calculation, we obtain

\[
\mathcal{L}_{\text{eff}}' = \frac{1}{F^2} \partial_\mu (f \hat{G}) \left[ \partial^\mu (\overline{f \hat{G}}) \right] + \ldots.
\]

Could we have derived (7) from the known results on non-linear realizations? To address this question, we recall that the standard literature on the subject [1, 3] prescribes an effective interaction of the form

\[
\mathcal{L}'_{\text{eff}} = \frac{i}{2F^2} \left[ \bar{G} \sigma^\mu \partial^\nu \hat{G} - (\partial^\mu \hat{G}) \sigma^\nu \bar{G} \right] T_{\nu \mu} + \ldots,
\]

where \( T_{\nu \mu} \) is the canonical energy-momentum tensor of the matter fermions,

\[
T_{\nu \mu} = \overline{f} \gamma_\nu \partial_\mu f + \ldots.
\]

Combining (6) with (8), we obtain a result that looks very different from (7):

\[
\mathcal{L}_{\text{eff}}'' = \frac{1}{F^2} \left( \overline{G} \sigma^\mu \partial^\nu \hat{G} \right) \left( \overline{\gamma_\nu \partial_\mu f} \right) + \ldots.
\]

To check that (7) and (10) are really inequivalent, we can concentrate on the process \( f \overline{\tilde{f}} \rightarrow \hat{G}\hat{G} \). The only non-vanishing amplitudes are those in which the two goldstinos have opposite chiralities. From
eq. (7), and in obvious notation, we obtain:

\[ a(L, R, L, R) = -\frac{(1 + \cos \theta)^2 s^2}{4F^2}, \quad a(L, R, R, L) = \frac{(1 - \cos \theta)^2 s^2}{4F^2}. \]

From eq. (10) we obtain instead:

\[ a'(L, R, L, R) = \frac{\sin^2 \theta s^2}{4F^2}, \quad a'(L, R, R, L) = -\frac{\sin^2 \theta s^2}{4F^2}. \]

We conclude that the effective interactions (7) and (10) give the same energy dependence, but different angular dependences (and total cross-sections). Surprisingly, the two approaches lead to different results.

3 Solution of the puzzle

To understand the origin of the discrepancy, we apply to the present case the superfield construction of the non-linear realization of \([1, 4]\). We define the superfield

\[ \Lambda_{\alpha}(x, \theta, \theta) \equiv \exp(\theta Q + \theta Q) \tilde{G}_{\alpha}(x) = \tilde{G}_{\alpha} + \sqrt{2} F \theta_{\alpha} + \frac{i}{\sqrt{2F}} (\tilde{G} \sigma^\mu \theta - \theta \sigma^\mu \tilde{G}) \partial_\mu \tilde{G}_{\alpha} + \ldots, \]

whose lowest component is the goldstino \( \tilde{G} \), and the superfield

\[ E_{\alpha}(x, \theta, \theta) \equiv \exp(\theta Q + \theta Q) f_{\alpha}(x) = f_{\alpha} + \frac{i}{\sqrt{2F}} (\tilde{G} \sigma^\mu \theta - \theta \sigma^\mu \tilde{G}) \partial_\mu f_{\alpha} + \ldots, \]

whose lowest component is the matter fermion \( f \). In the simple case under consideration, the goldstino couplings to matter in the non-linear realizations of \([1, 4]\) are described by the supersymmetric lagrangian

\[ \frac{1}{4F^2} \int d^4 \theta \, \Lambda^2 \tilde{G} \sigma^\mu \partial_\nu f, \]

which leads precisely to the result of eq. (10), as can be explicitly checked.

The crucial question is now the following: are there other independent invariants, besides (10), that can contribute to our effective interaction? The answer is positive \([6]\), since we can also write:

\[ \alpha \frac{1}{4F^2} \int d^4 \theta \, \Lambda E \tilde{G} f, \]

where \( \alpha \) is an arbitrary dimensionless coefficient. The new invariant (16) gives, among other things,

\[ \delta \mathcal{L}_{eff} = \alpha \frac{1}{4F^2} (\tilde{G} \sigma^\mu \partial_\nu f)(\tilde{G} \sigma^\mu \partial_\nu f) + \ldots. \]

From the contact interaction of eq. (17), we obtain the following amplitudes:

\[ \delta a'(L, R, L, R) = \alpha \frac{(1 + \cos \theta)s^2}{8F^2}, \quad \delta a'(L, R, R, L) = -\alpha \frac{(1 - \cos \theta)s^2}{8F^2}. \]

We may now wonder whether an appropriate linear combination of the two invariants can reproduce the result of eq. (11). Indeed, it is immediate to check that, with the special choice \( \alpha = -4 \), the combination \( \mathcal{L}'_{eff} + \delta \mathcal{L}'_{eff} \) reproduces the scattering amplitudes obtained from \( \mathcal{L}_{eff} \): the puzzle is solved!

As a first comment, we stress that there is no reason to believe that the result of eq. (11) is more fundamental than the standard result of eq. (10). Since two independent invariants can be constructed, there is just an ambiguity in the effective theory description, parametrized by the coefficient \( \alpha \) in eq. (17), which can be fixed only by the underlying fundamental theory. At the level of the linear realization, this
ambiguity is contained in the coefficients of higher-derivative operators, which are not included in the standard Kähler formulation of eq. (1).

In summary, the low-energy theorems of supersymmetry, as expressed in the standard literature on non-linear realizations \([1, 4]\), must be modified: in the low-energy effective lagrangian describing the goldstino couplings to matter there are terms that, contrary to previous expectations, are not entirely controlled by the supersymmetry breaking scale. At a second thought, this result is not as surprising as it may seem: there is a suggestive analogy with the textbook case of pion-nucleon scattering \([14]\), where the effective lagrangian consists of two independent terms, one fully controlled by the broken \(SU(2) \times SU(2)\) symmetry, the other one containing the axial coupling \(g_A\) as a free parameter.

Are (13) and (14) the only independent invariants that contribute to the effective four-fermion coupling under consideration, or are there others? After a straightforward but tedious classification, it was found in \([1]\) that, assuming matter conservation, the parametrization of the most general on-shell amplitudes involving two goldstinos \(G\) and two massless matter fermions \(f\) does not require any additional invariant.

It is of course interesting to generalize the above framework by including gauge interactions. At the level of local four-fermion operators, the previous result is not affected. In particular \([1]\), there are no \(d = 6\) local supersymmetric operators contributing to \(e^+e^- \rightarrow GG\) in the limit of vanishing electron mass and negligible selectron mixing. If present, these operators would have been characterized by a dimensionful coupling \(M^2/F^2\), where \(M\) is an independent mass scale, possibly arising from the underlying fundamental theory, and the process \(e^+e^- \rightarrow GG\) could have been used to extract interesting lower bounds on \(m_{3/2}\) from supernova cooling, but this is not the case.

### 4 Phenomenological implications

When extended to observable processes and realistic models, the previous results have other important phenomenological implications. As discussed in \([1]\), powerful processes to search for a light gravitino \(\tilde{G}\) (when the supersymmetric partners of the Standard Model particles and of the goldstino are above threshold) are \(e^+e^- \rightarrow GG\gamma\) and \(q\bar{q} \rightarrow GG\gamma\), which would give rise to a distinctive (photon + missing energy) signal. The first process can be studied at \(e^+e^-\) colliders such as LEP or the proposed NLC, the second one at hadron colliders such as the Tevatron or the LHC. At hadron colliders, we can also consider the partonic subprocesses \(q\bar{q} \rightarrow G\bar{G}g, qg \rightarrow g\bar{G}G, \bar{q}g \rightarrow \bar{G}G\bar{G}\) and \(gg \rightarrow g\bar{G}G\), all contributing to the (jet + missing energy) signal.

Consider, for definiteness, the reaction \(e^+e^- \rightarrow GG\gamma\). As before, we can start from a linear realization with pure \(F\)-breaking, neglecting higher-derivative terms and selectron mixing. Also in this case, explicit integration of the heavy superpartners gives results \([8]\) that differ from those obtained \([3]\) in the non-linear realization of \([1, 4]\). For a model-independent study, we would need the general form of the low-energy effective interactions, allowed by the non-linearly realized supersymmetry, that may contribute to the relevant amplitudes at leading order. This was not known at the time of the Workshop, but a definite theoretical prescription for such an investigation has become available in the meantime \([16]\) (the same paper has also confirmed the results of \([3]\), and found the explicit general form of the coupling of two on-shell goldstinos to a single photon). In the absence of a general phenomenological analysis, we can use the results of \([8]\) and derive from the present LEP data the tentative lower bound \(m_{3/2} > 10^{-5}\) eV. Because of the strong and universal power-law behaviour of the cross-section, always proportional to \(s^3/|F|^4\), and the absence of accidental zeroes, this bound is expected to be rather stable with respect to variations of the parameters characterizing the most general non-linear realization. However, should a signal show up at LEP or at future linear colliders, having the general expression of the cross-section would be very important, since a detailed analysis of the photon spectrum would offer the unique opportunity of distinguishing among possible fundamental theories.

At hadron colliders, the analysis is more complicated, since different signals can be considered, with several partonic subprocesses contributing, and the background is more severe. However, the prospects for the present Tevatron data are at least as good as for LEP, since the higher available energy is more than enough to compensate for the more difficult experimental environment \([1]\). The present Tevatron
bound is estimated to be $m_{3/2} > 2.7 \times 10^{-5}$ eV, and the LHC should be sensitive to values of $m_{3/2}$ up to $1.2 \times 10^{-3}$ eV.

In summary, high-energy colliders are by far the best environment to test the possible existence of a very light gravitino. In our opinion, it would be important to provide our experimental colleagues with a model-independent framework for such a search, and we hope to complete this project soon.

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