Research Article

Magnetogravitodynamic Stability of Three Dimensional Streaming Velocities of Fluid Cylinder under the Effect of Capillary Force

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Abstract: The magnetohydrodynamic stability in a uniform cylinder of an incompressible inviscid fluid under the effect of self-gravitating, magnetic field and capillary forces is studied. The obtained results were studied theoretically and numerically. The dispersion relationship was obtained and the effect of the different parameters had been discussed. The behavior of the system in terms of whether stable or unstable had been studied. The uniform streaming has a destabilizing influence. We observe that the system gives an unstable situation where the streaming fluid under the effect of the capillary force and in the absence of the magnetic field. Also, the system gives an unstable situation where the streaming fluid under the effect of the self-gravitating and magnetic forces. But the system be more stable and the stability zone increase if the streaming fluid under the effect of the self-gravitating, magnetic and capillary forces. The curves are drawn to illustrate the areas of stability and instability.

Keywords: Capillary force, magnetohydrodynamic, self-gravitating, stability, streaming

INTRODUCTION

The electrodynamic stability of a dielectric self-gravitating streaming fluid cylinder with a different dielectric self-gravitating streaming fluid has been studied by Hasan (2017a). The magnetohydrodynamic stability of a gravitational medium with streams of variable velocity distribution for a general wave propagation in the present of the rotation forces was presented by Hasan (2017b). The stability of a fluid cylinder under the influence of the capillary force was studied in many researches (Yuen, 1968; Nayfeh and Hassan, 1971; Rayleigh, 1892). The magnetohydrodynamic (MHD) stability of an oscillating fluid with longitudinal magnetic field has been studied by Barakat (2015). Barakat (2016) studied self-gravitating stability of a fluid cylinder embedded in a bounded liquid, pervaded by magnetic field, for all symmetric and axisymmetric perturbation modes. The electrogravitational instability of an oscillating streaming fluid cylinder under the action of the self-gravitating, capillary and electrodynamic forces was presented by Hasan (2011), he used the Mathieu second order integro-differential equation in this model. The self-gravitating instability of a fluid cylinder pervaded by magnetic field and endowed with surface tension was investigated by Radwan and Hasan (2009). The instability of a self-gravitating fluid cylinder surrounded by a self-gravitating tenuous medium pervaded by transverse varying electric field is discussed under the combined effect of the capillary, self-gravitating and electric forces for all axisymmetric and nonaxisymmetric modes of perturbation was presented by Hasan (2012). Hasan and Abdelkhalek (2013) studied the Magneto-hydrodynamic stability criterion of self-gravitating streaming fluid cylinder under the combined effect of self-gravitating, magnetic and capillary forces.

DEFINITION OF THE PROBLEM

Research results were concluded in 2017. The study was conducted at Al-Azhar University.

Basic equations: We consider a fluid cylinder of radius $R$, with negligible motion with streaming velocity:

$$u_s = (V, W, U).$$

The internal and external magnetic fields:
where, $H_0$ represents the intensity of the magnetic field, $\alpha$ is an arbitrary parameter. We shall use the cylindrical polar coordinates $(r, \phi, z)$ system with the axis of the cylinder coinciding with the $z$-axis (Fig. 1). The fluid of the cylinder under the effects of the self-gravitating, magnetic forces and capillary forces. The surrounding tenuous medium of the fluid cylinder under the effects of the magnetic forces and self-gravitating only.

The basic equations of this model are given as follows:

$$
\begin{align*}
\rho \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} &= -\nabla P + \rho \nabla \mathbf{\phi} + \frac{\mu}{4\pi} (\nabla \times \mathbf{H}) \times \mathbf{H},
\n\nabla \cdot \mathbf{H} &= 0,
\n\frac{\partial \mathbf{H}}{\partial t} &= \nabla \times (\mathbf{u} \times \mathbf{H}),
\n\nabla \cdot \mathbf{\phi} &= -4\pi \rho G,
\end{align*}
$$

where, $\mathbf{u}$ is the velocity vector, $P$ is the kinetic pressure, $\mu$ is the magnetic field permeability coefficient and $G$ is the gravitational constant.

The curvature pressure due to the existence of the capillary force is:

$$
P_c = T (\nabla \cdot \mathbf{N}_s) \quad \text{where} \quad \mathbf{N}_s = \frac{\nabla F}{|\nabla F|}.
$$

Such that:

$$
F(r, \phi, z) = 0.
$$

whereas $T$ is the surface tension, $\mathbf{N}_s$ is the unit normal vector. Basic equations of the surrounding tenuous medium:

$$
\begin{align*}
\nabla \cdot \mathbf{H}^e &= 0 \quad (10)
\n\nabla \times \mathbf{H}^e &= 0 \quad (11)
\n\nabla \times \mathbf{\phi}^e &= 0 \quad (12)
\end{align*}
$$

Here $H_0, H_0^e$ are the intensity of the magnetic field and $\mathbf{\phi}, \mathbf{\phi}^e$ are the self-gravitating potentials, inside and outside the fluid cylinder.

### UNPERTURBED STATE

In the initial state, we get:

$$
P_0 = \frac{T}{R_0}
$$

is the surface pressure due to the capillary force (Chandrasekhar, 1981). In the initial state, the self-gravitating potentials $\mathbf{\phi}_0, \mathbf{\phi}^e_0$ satisfy:

$$
\begin{align*}
\nabla \times \mathbf{\phi}_0 &= -4\pi \rho G \quad (14)
\n\nabla \times \mathbf{\phi}^e_0 &= 0 \quad (15)
\end{align*}
$$

Where $
abla^2 = \left(\frac{\partial^2}{\partial r^2}, \frac{1}{r} \frac{\partial}{\partial r}, \frac{\partial^2}{\partial \phi^2}, \frac{\partial^2}{\partial z^2} \right)$.

If we put $\frac{\partial}{\partial \phi} = \frac{\partial}{\partial z} = 0$ cylindrical symmetries, we get solutions for the Eq. (14), (15) in the form:

$$
\begin{align*}
\mathbf{\phi}_0 &= -\pi G \rho r^2 + C_1 \quad (16)
\n\mathbf{\phi}^e_0 &= C_2 \ln r + C_3 \quad (17)
\end{align*}
$$

Where $C_1, C_2$ and $C_3$ are constants of integration.

In order to determine the values of these constants $C_1, C_2$ and $C_3$, we will use the conditions of the self-gravitational potential and its derivative, i.e., $\mathbf{\phi}_0 = \mathbf{\phi}^e_0$ and $\frac{\partial \phi}{\partial r} = \frac{\partial \phi}{\partial r}$ at $r = R_0$. Let $C_1 = 0$, since the potential inside the cylinder is zero, therefore:

$$
\begin{align*}
C_2 &= -2\pi G \rho R_0^2 \quad (18)
\nC_3 &= -\pi G \rho R_0^2 (1 + 2 \ln(\frac{1}{R_0})) \quad (19)
\end{align*}
$$

Then, we get the final solution for the Eq. (14), (15) in the form:
Perturbation theory leads to an expression for the desired solution in terms of a formal power series in some "small" parameter known as a perturbation series that quantifies the deviation from the exactly solvable problem, considers the effect of small disturbances for a small departure from the unperturbed state, then that the perturbed interface is described by equation:

\[ r = R_0 + \varepsilon(t)R_1, \]  

(23)

where,

\[ R_1 = e^{\sigma t + i(kz + m\phi)} \]  

(24)

\[ \varepsilon(t) = \varepsilon_0 e^{\sigma t} \]  

(25)

Here \( R_1, k, m \) represents the elevation of the surface wave, the wave number, the transverse wave number, respectively. \( \varepsilon(t) \) is the amplitude of the perturbation where \( \varepsilon_0 \) is the initial amplitude and \( \sigma \) is the temporal amplification.

Now, we will put a mathematical expression \( Y \) in the initial state for each of the following variables, \( P, \tilde{V}, \tilde{V}^{ex}, H, H^{ex} \) and \( N_4 \) in the form:

\[ Y(r, \phi, z) = Y_0(r) + Y_1(r, \phi, z) \]  

(26)

where \( Y_0 \) represent unperturbed quantity and \( Y_1 \) is a small increment of \( Y \) due to disturbances.

From Eq. (26), we can put the basic equations of motion (3)-(12) in the initial state as following:

\[ \rho \frac{\partial u}{\partial t} + (u_0 \cdot \nabla)u_1 = - \nabla P_1 + \rho \nabla \tilde{V}_1 + \frac{\mu}{4\pi} (H_0 \cdot \nabla)H_0 - \frac{\mu}{4\pi} \nabla (H_0 \cdot H_1) \]  

(27)

\[ \nabla \cdot u_1 = 0 \]  

(28)

\[ \nabla \cdot H_1 = 0 \]  

(29)

\[ \frac{\partial H_1}{\partial t} = (H_1 \cdot \nabla)u_1 - (u_0 \cdot \nabla)H_1 \]  

(30)

\[ \nabla \cdot \tilde{V}_1 = 0 \]  

(31)

\[ P_1 = \left( \frac{r}{R_0} \right) \left[ R_1 + \left( \frac{\partial^2 R_1}{\partial \phi^2} \right) + R_0 \left( \frac{\partial^2 R_1}{\partial z^2} \right) \right] \]  

(32)

\[ \nabla \cdot \tilde{H}_1^{ex} = 0 \]  

(33)

\[ \nabla \times \tilde{H}_1^{ex} = 0 \]  

(34)

\[ \nabla^2 \tilde{V}_1^{ex} = 0 \]  

(35)

According to the theory of wave function solutions, we can put \( Y_1(r, \phi, z, t) \) in the following form:

\[ Y_1(r, \phi, z, t) = y_1(r) e^{i(\sigma t + i(kz + m\phi))}. \]  

(36)

Substituting from Eq. (36) into Eq. (31) and Eq. (35), we obtain the second-order ordinary differential equations:

\[ \frac{d^2}{dr^2} V_1(r) + \frac{1}{r} \frac{d}{dr} V_1(r) - \left( k^2 + \frac{m^2}{r^2} \right) V_1(r) = 0. \]  

(37)

\[ \frac{d^2}{dr^2} V_1^{ex} (r) + \frac{1}{r} \frac{d}{dr} V_1^{ex} (r) - \left( k^2 + \frac{m^2}{r^2} \right) V_1^{ex} (r) = 0. \]  

(38)

Then, the solution of Eq. (31) and Eq. (35) can be written in the forms:

\[ \tilde{V}_1 = C_4 l_m (kr) e^{i(\sigma t + i(kz + m\phi))}. \]  

(39)

\[ \tilde{V}_1^{ex} = C_5 K_m (kr) e^{i(\sigma t + i(kz + m\phi))}. \]  

(40)

where, \( C_4 \) and \( C_5 \) are constants of integration \( l_m (kr) \) and \( K_m (kr) \) are the modified Bessel functions of the first and second kind of order \( m \). By using Eq. (36) in Eq. (27), we get:

\[ (\sigma + V \frac{\partial}{\partial r} + i mW \frac{1}{r} + ikU) u_1 - i \mu k H_0 \frac{1}{4\pi \rho} H_1 = - \nabla A \]  

(41)

where,

\[ A = \frac{p_4}{\rho} - \frac{1}{\varepsilon_0} \tilde{V}_1 + \frac{\mu}{4\pi \rho} (H_0 \cdot H_1) \]  

(42)

From Eq. (30), we get:

\[ (\sigma + V \frac{\partial}{\partial r} + i mW \frac{1}{r} + ikU) H_1 = i k H_0 u_1 \]  

(43)

By using Eq. (43) in Eq. (41), we obtain:
The scalar magnetic field intensity can be written as follows:

\( \Omega_A^2 = \frac{\mu k^2 \rho_0^2}{4 \pi \rho} \) \tag{45}

\( \Omega_A \) represent the Alfvén wave frequency. Take the divergence of the two sides of the Eq. (44) and by using \( \nabla \cdot \mathbf{u}_s = 0 \), we obtain:

\[ \nabla^2 A = 0 \] \tag{46}

Also, we can obtain the solution of Eq. (46) as follows:

\[ A = C_6 I_m(kr)e^{\left(\alpha r + i(kz + m\phi)\right)} \] \tag{47}

From Eq. (32) by using Eq. (24), we get the pressure surface \( P_{1s} \) in the initial state due to the capillary in the form:

\[ P_{1s} = \frac{\tau}{R_0}(1 - m^2 - (kR_0)^2)e^{\alpha r + i(kz + m\phi)} \] \tag{48}

The scalar magnetic field intensity \( H_1^{ex} \) are defined as:

\[ H_1^{ex} = \nabla \theta_1^{ex} \] \tag{49}

It follows that the scalar magnetic field intensity \( H_1^{ex} \) satisfy the Laplace’s equations:

\[ \nabla^2 \theta_1^{ex} = 0 \] \tag{50}

where \( \theta_1^{ex} \) is a scalar function, the solution of this Eq. (50) can be written as follows:

\[ \theta_1^{ex} = C_7 K_m(kr)e^{\left(\alpha r + i(kz + m\phi)\right)} \] \tag{51}

where \( C_7 \) is a constant of integration.

**BOUNDARY CONDITIONS**

- The gravitational potential and its derivative must be continuous across the fluid interface:

\[ \tilde{V}_1 + R_1 \frac{\partial V_0}{\partial r} = \tilde{V}_1^{ex} + R_1 \frac{\partial V_0^{ex}}{\partial r} \quad r = R_0 \] \tag{52}

\[ \frac{\partial V_1}{\partial r} + R_1 \frac{\partial^2 V_0}{\partial r^2} \bigg|_0 = \frac{\partial V_0^{ex}}{\partial r} \] \tag{53}

Substituting (20), (21), (24), (39) and (40) into (52), (53), then we will get the following two equations:

\[ C_4 I_m(x) = C_5 K_m(x) + 4\pi \rho G \] \tag{55}

We will solve the Eq. (54), (55) together to obtain the values of the constants \( C_4 \) and \( C_5 \), then:

\[ C_4 = 4\pi \rho G R_0 K_m(x) \] \tag{56}

\[ C_5 = 4\pi \rho G R_0 I_m(x) \] \tag{57}

where \( x \)is the dimensionless longitudinal wave number \( = kR_0 \). Also the dimensionless longitudinal wave number gives as the following equation:

\[ x = \frac{1}{R_0^2 m(x) K_m(x) - K^{'2} m(x) I_m(x)} \] \tag{58}

- At the unperturbed surface \( r = R_0 \), the normal component of the velocity vector \( u \) must be suitable with the velocity of the particles:

\[ u_{1r} = \frac{\partial R_1}{\partial r} + \left( u_0 \cdot \nabla \right) R_1. \] \tag{59}

From Eq. (24), (44) and (47) into (59), then we get:

\[ C_6 = \frac{-R_0\left[\left(\sigma + V \frac{\partial}{\partial r} + \frac{imW}{r} + ikU\right)^2 + \Omega_A^2\right]}{x \frac{d}{dx} m(x)} \] \tag{60}

- At the perturbed interface \( r = R_0 \), the jump of the normal component of the magneticfield is zero:

\[ N_2 \cdot H - N_2 \cdot H^{ex} = 0. \] \tag{61}

Eq. (61) leads to,

\[ H_{3r} - H_{3r}^{ex} = iKR_1 H_0 (1 - \alpha). \] \tag{62}

Substituting from Eq. (43), (44), (47), (49) and (51) into (62), then we obtain:

\[ C_7 = \frac{i \omega H_0}{K_m} \] \tag{63}

**DISPERSION RELATION**

The dispersion relation can be written as follows:

\[ P_{1s} = P_1 + R_1 \frac{\partial P_0}{\partial r} + \frac{\mu}{4\pi} \left( H_0 \cdot \frac{d}{dr} H_B \right) - \frac{\mu}{4\pi} (H_0 \cdot H_B)^{ex}. \] \tag{64}
The dimensionless dispersion relation is:

\[
P_{15} + \frac{\mu}{4\pi \rho} (H_0 \cdot H_2)^x = \rho (A + \nabla_T) + R_1 \frac{\partial p}{\partial T} \tag{65}
\]

From Eq. (2), (22), (24), (39), (47), (48), (49), (50) (51), (60) and (61) into the Eq.(65), we get the dispersion relation in the form:

\[
\frac{V^2 x^2}{R_0^2} I''_m(x) + \frac{2Vx}{R_0} \left( \sigma + \frac{imW}{R_0^2} + iku \right) I'_m(x)
- \frac{imW}{R_0^2} I_m(x)
+ (\sigma + \frac{imW}{R_0^2} + iku)^2 I_m(x)
= \frac{T}{\rho R_0^2} \left( 1 - m^2 - x^2 \right) x I'_m(x) + 4\pi \rho x G l'_m(x) \left( l_m(x) K'_m(x) - \frac{1}{2} \right) + \frac{\mu \rho^2}{4\pi \rho^2 \rho_0^2} \left( -x^2 I_m(x) + \frac{a^2 x^2 I'_m(x) K'_m(x)}{K''_m(x)} \right) \tag{66}
\]

The dimensionless dispersion relation is:

\[
\sigma = \tilde{\sigma} + \tilde{\sigma} - \tilde{\varphi} x \frac{I'_m(x)}{l_m(x)} + \frac{1}{(l_m(x))^2} \left[ \frac{\varphi^2 x^2 (l'_m(x))^2}{l_m(x)} - I''_m(x) \right]
+ M \left( 1 - m^2 - x^2 \right) x I'_m(x) + x I'_m(x) \left( l_m(x) K'_m(x) - \frac{1}{2} \right)
- \tilde{\varphi} I_m(x) + \gamma \left( -x^2 I_m(x) + \frac{a^2 x^2 I'_m(x) K'_m(x)}{K''_m(x)} \right) \right]^{1/2}, \tag{67}
\]

where,

\[
\sigma = \frac{\sigma}{(4\pi \rho G)^{1/2}}, \quad \tilde{\sigma} = \frac{-iku}{(4\pi \rho G)^{1/2}}, \quad \tilde{\varphi} = \frac{-imW}{(4\pi \rho G)^{1/2}}.
\]

\[
\tilde{\varphi} = \frac{\nu}{(4\pi \rho G)^{1/2}}, \quad M = \frac{\tau}{4\pi \rho G R_0^2}, \quad \gamma = \frac{\mu \rho^2}{16\pi \rho^2 G R_0^2}.
\]

**RESULTS AND DISCUSSION**

**CAPILLARY INSTABILITY**

Suppose the magnetic field vanishes and the streaming fluid under the effect of the capillary force only. In this case, we can write the dispersion relationship as follows:

\[
\sigma = \tilde{\sigma} + \tilde{\sigma} - \tilde{\varphi} x \frac{I'_m(x)}{l_m(x)} + \frac{1}{(l_m(x))^2} \left[ \varphi^2 x^2 (l'_m(x))^2 - I''_m(x) \right]
+ M \left( 1 - m^2 - x^2 \right) x I'_m(x)
+ x I'_m(x) \left( l_m(x) K'_m(x) - \frac{1}{2} \right)
- \tilde{\varphi} I_m(x) \right]^{1/2}. \tag{68}
\]

Fig. 2: Stability diagram for a system having the particulars:

\[ \tilde{\varphi} = \tilde{\sigma} = \tilde{\sigma} = 0.01, m = 0, \gamma = 0 \]

Fig. 3: Stability diagram for a system having the particulars:

\[ \tilde{\varphi} = \tilde{\sigma} = \tilde{\sigma} = 0.02, m = 0, \gamma = 0 \]

Fig. 4: Stability diagram for a system having the particulars:

\[ \tilde{\varphi} = \tilde{\sigma} = \tilde{\sigma} = 0.06, m = 0, \gamma = 0 \]

Now, we begin drawing some neutral curves stability problem:

From Fig. 2, in this domain \[ 0.0101 \leq x \leq 1.0101 \], we get stability states. While otherwise domin, we get unstable state.
MAGNETOGRAVITODYNAMIC STABILITY

Suppose that the streaming fluid under the effect of the self-gravitating and magnetic forces. In this case, we can write the dimensionless dispersion relationship as follows:

\[ \sigma = \hat{W} + \hat{U} - \hat{V} x \frac{l'_m(x)}{l_m(x)} + \frac{1}{(l_m(x))^2} \left[ \hat{V}^2 x^2 \frac{(l'_m(x))^2}{l_m(x)} \right] - l''_m(x) \]

\[ + x l'_m(x) \left( l_m(x)K_m(x) - \frac{1}{2} \right) - \hat{W} \hat{V} l_m(x) \]

\[ + \gamma \left( -x^2 l''_m(x) + \frac{a^2 x^2 l'_m(x)K_m(x)}{K'_m(x)} \right)^{1/2} \]

From Fig. 7, in this domain \(0.0101 \leq x \leq 1.0501\), \(0.0101 \leq x \leq 1.0301\), \(0.0101 \leq x \leq 1.0401\), \(0.0101 \leq x \leq 1.0301\), \(0.0101 \leq x \leq 1.0601\), \(0.0101 \leq x \leq 1.0501\), \(0.0101 \leq x \leq 1.0401\), \(0.0101 \leq x \leq 1.0301\), we get unstable state. While otherwise domin, we get stability states.

From Fig. 8, in this domain \(0.0101 \leq x \leq 1.0501\), \(0.0101 \leq x \leq 1.0501\), \(0.0101 \leq x \leq 1.0501\), \(0.0101 \leq x \leq 1.0501\), \(0.0101 \leq x \leq 1.0501\), \(0.0101 \leq x \leq 1.0501\), \(0.0101 \leq x \leq 1.0501\), \(0.0101 \leq x \leq 1.0501\), we get unstable state. While otherwise domin, we get stability states.

From Fig. 9, in this domain \(0.0101 \leq x \leq 1.0501\), \(0.0101 \leq x \leq 1.0501\), \(0.0101 \leq x \leq 1.0501\), \(0.0101 \leq x \leq 1.0501\), \(0.0101 \leq x \leq 1.0501\), \(0.0101 \leq x \leq 1.0501\), \(0.0101 \leq x \leq 1.0501\), \(0.0101 \leq x \leq 1.0501\), we get unstable state. While otherwise domin, we get stability states.

From Fig. 10, in this domain \(0.0101 \leq x \leq 1.0501\), \(0.0101 \leq x \leq 1.0501\), \(0.0101 \leq x \leq 1.0501\), \(0.0101 \leq x \leq 1.0501\), \(0.0101 \leq x \leq 1.0501\), \(0.0101 \leq x \leq 1.0501\), \(0.0101 \leq x \leq 1.0501\), \(0.0101 \leq x \leq 1.0501\), we get unstable state. While otherwise domin, we get stability states.
MAGNETOGRAVITODYNAMIC CAPILLARY STABILITY

If the streaming fluid under the effect of the self-gravitating, magnetic forces and the capillary force. In this case, dispersion relationship identical with the general dispersion relation (67):

\[ \sigma = \sqrt{\frac{\gamma}{\rho_x} \frac{I_m'(x)}{I_m(x)} + \frac{1}{2} \left( \frac{\rho^2 x^2 (I_m'(x))^2}{I_m(x)} - I_m'(x) \right) + \frac{M (1 - m^2 - x^2) x I_m'(x)}{I_m(x) K_m(x)} + \frac{\gamma (-x^2 I_m(x))}{K_m(x)}}{\sqrt{[\gamma - x^2 I_m(x) \alpha^2 x^2 I_m'(x) K_m(x)]}}^{1/2} \]

From Fig. 10, in this domain \((0.0101 \leq x \leq 0.7301), (0.0101 \leq x \leq 0.7601), (0.0101 \leq x \leq 0.7901), (0.0101 \leq x \leq 0.8201)\), we get stability states. While otherwise domin, we get unstable state.

From Fig. 11, in this domain \((0.0901 \leq x \leq 0.8201), (0.0801 \leq x \leq 0.8501), (0.0801 \leq x \leq 0.8701), (0.0801 \leq x \leq 0.8901)\), we get stability states. While otherwise domin, we get unstable state.

From Fig. 12, in this domain \((0.0901 \leq x \leq 0.8801), (0.0801 \leq x \leq 0.9101), (0.0801 \leq x \leq 0.9301), (0.0701 \leq x \leq 0.9401)\), we get stability states. While otherwise domin, we get unstable state.

From what we had before, we observe that the streaming velocity has a destabilizing influence, which is consistent with all previous studies. We observed that with the increase of the M values, the system gives an unstable situation and with the continuous increase of the M values and \(\gamma = 0\), the system be more unstable and the stability zone decreases (Fig. 1 to 6). Also, we observe that under the effect of the self-gravitating and magnetic forces with the increase of the \(\gamma\) values, the system gives an unstable situation and with the continuous increase of the \(\gamma\) values and \(M = 0\), the system be more unstable and the stability zone decreases (Fig. 7 to 9). Finally, we conclude that in the increase of M values with the existence of the \(\gamma\) variable, the system be more stable and with the continuous increase of the M values with the existence of the \(\gamma\) variable, the system be more unstable and the stability zone increase (Fig. 10 to 12). In addition, we found that the best value for the streaming velocities is less than one, this leads the system to be more stable, this is compatible with the results of Hasan and Abdelkhalek (2013).

CONCLUSION

In this study, we have examined the influence of the existence of self-gravitating, magnetic field and
capillary forces in the stability magnetohydrodynamic in a uniform cylinder of an incompressible inviscid fluid. After we obtained the dispersion relation, we plotted \((x - \sigma)\) plane and studying the effect of different variables on the process stability.

The following is a general summary of the study in this paper:

- The streaming velocity has a destabilizing influence.
- In the absence of the magnetic field and the streaming fluid under the effect of the capillary force only, we observe that the increase of the \(M\) values, the system gives an unstable situation and with the continuous increase of the \(M\) values, the system be more unstable and the stability zone decreases.
- If the streaming fluid under the effect of the self-gravitating and magnetic forces, we observe that the increase of the \(\gamma\) values, the system gives an unstable situation and with the continuous increase of the \(\gamma\) values, the system be more unstable and the stability zone decreases.
- If the streaming fluid under the effect of the self-gravitating, magnetic and capillary forces, We conclude that in the increase of \(M\) values with the existence of the \(\gamma\) variable, the system be more stable and with the continuous increase of the \(M\) values with the existence of the \(\gamma\) variable, the system be more stable and the stability zone increase.

**CONFLICT OF INTEREST**

The authors declare that they have no competing interest.

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