A NEW TYPE OF MASSIVE SPIN-ONE BOSON:
AND ITS RELATION WITH MAXWELL EQUATIONS

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1. Introduction

The textbook understanding of quantum field theory states that a fermion and its associated antifermion have opposite relative intrinsic parity; and that a boson and its associated antiboson carry same relative intrinsic parity. No particles that do not fall within this understanding, coupled with the fact that no quantum field theory was known to exist that contradicted this canonical wisdom, has led to almost complete neglect of the fact that long ago Wigner [1] argued that space-time symmetries, and that these manifest in quantum constructs as projective representations, do allow for theories where a fermion and its associated antifermion have same relative intrinsic parity; and that a boson and its associated antiboson carry opposite relative intrinsic parity. The only scholarly footnote to the canonical wisdom in the widely read literature, see for example Refs. [7], that refers to Wigner’s observations is Appendix C of Weinberg’s latest monograph on the quantum theory of fields [8].

Recently [2], purely by accident (see Acknowledgements), in an attempt to understand an old work of Weinberg [9] and to investigate the possible

1Part of the work in Ref. [1] was done in collaboration with V. Bargmann and A. S. Wightman, as Wigner [1] points out and we have noted in Ref. [2]. Also similar work was done previously and independently, as we learned later while writing [3], by L. L. Foldy and B. P. Nigam [4, 5]. So, whenever we refer to Wigner-type bosons or theory, the reader may wish to keep this historical note in mind. However, it must be noted that the work of Ref. [1] is the most comprehensive and is within a much deeper and fundamental framework. I thank Zurab K. Silagadze for kindly bringing to my attention the work of Foldy and Nigam on the subject [6]. Also see Acknowledgements.
kinematical origin for the violation of P, CP, and other discrete symmetries [3], a Wigner-type quantum field theory was constructed for a spin-one boson. This new theory made it clear that *boson-antiboson relative intrinsic parity is not (necessarily and) uniquely determined by the representation space in which the theory is constructed.* For example, the (1/2, 1/2) representation space describes spin-one particles in which a particle and its antiparticle carry *same* relative intrinsic parity. On the other hand [2], the (1, 0) ⊕ (0, 1) representation space not only allows for the construction of a theory in which boson-antiboson relative intrinsic parity is same (“Weinberg’s theory” [9]) but also supports a construct in which boson-antiboson relative intrinsic parity is opposite (“our Wigner-type theory” [2]). This is a surprising result, particularly in view of the canonical wisdom that states that there is no physical distinction between the two representation spaces, i.e., (1/2, 1/2) and (1, 0) ⊕ (0, 1), for the description of spin-one particles. For an eloquent and forceful argument putting forward the canonical wisdom, one may refer to Weinberg’s paper [10] presented at the fiftieth anniversary of the famous 1939 Wigner’s paper [11] on the unitary representations of the inhomogeneous Lorentz group. In fact Weinberg [12] himself was surprised at the new theory and wrote “When I saw your paper I did not believe the result. ... I suspect that this is why you are getting such a surprising result. In all my work, I assume that states of given mass and spin are non-degenerate.”

The Wigner-type quantum theory of fields is a very natural consequence of space-time symmetries and the fundamental structure of quantum mechanics. The underlying physical principles for the standard quantum theory of fields and Wigner-type theory are the same — standard quantum theory is one of the classes of the Wigner’s general classification [1]. Our recent construction of a Wigner-type theory was purely accidental, as already noted, and it was indeed a very generous and knowledgeable referee from *Phys. Rev. Lett.* who brought Wigner’s 1962 work to my attention (see acknowledgements). *Now* that such a theory (i.e., the theory for a Wigner-type boson) exists and no bosons exist (as far as the nature has revealed its secrets to us so far) that are of Wigner type leads us to the conclusion that *either, in some future experiment (or as a theoretical inevitability in a much broader theoretical framework), we shall discover the*

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2At this time (September 1995) it is not known if (1/2, 1/2) representation space too can support a theory of the Wigner type.

3The canonical wisdom translated to the spin-one case under consideration can now be stated as: There is no physical distinction between the usual (See footnote[2]) (1/2, 1/2) representation space description and the Weinberg’s construct [9] in the (1, 0) ⊕ (0, 1) representation space.

4Even though in this presentation we confine our attention to spin one, the results that we report are valid for bosons of spin one and greater. See Ref. [2] for details.
spin-one Wigner boson that the new theory describes; or there is some deep underlying reason, yet to be discovered, that prohibits the existence of the spin-one Wigner boson as a physical reality.

Assuming that no subtle error of any significance exists in our or Weinberg's work presented in Refs. [2, 8, 9], the above conclusion is inescapable. Given this situation, we take the liberty of reviewing at this conference on “The Present Status of Quantum Theory of Light: A Symposium to Honour Jean-Pierre Vigier” the theory of the new type of boson and show that in the massless limit the theory suggests some very definite and fundamental modifications to Maxwell equations.

2. A New Type of Spin-One Boson

Consideration of the space-time symmetries \((1, 0)\) and \((0, 1)\) matter fields Lorentz transform (“boost”) in the following fashion:

\[
(1, 0) : \quad \phi_R(p) = \exp \left( + \vec{J} \cdot \vec{\varphi} \right) \phi_R(\vec{0}) ,
\]

\[
(0, 1) : \quad \phi_L(p) = \exp \left( - \vec{J} \cdot \vec{\varphi} \right) \phi_L(\vec{0}) .
\]

\(\vec{J} = 3 \times 3\) angular momentum matrices with \(J_z\) diagonal.

\(\vec{\varphi} = \text{the boost parameter defined as}\)

\[
\cosh(\varphi) = \frac{E}{m}, \quad \sinh(\varphi) = \frac{|p|}{m}, \quad \vec{\varphi} = \frac{\vec{p}}{|p|}.
\]

\(\vec{p} = \) the three-momentum of the particle (of mass \(m\)).

Note: No “\(i\)” in the argument of exponentials that appear in the above equations.

Reason: \(K\), the generator of the boost, = \(\pm i\vec{J}\). The plus sign for the \((0, 1)\)-, and minus sign for \((1, 0)\)-, matter fields.

Under parity: \((1, 0) \Leftrightarrow (0, 1)\). Parity covariance \(\Rightarrow\) we introduce the \((1, 0) \oplus (0, 1)\) representation space spinor

\[
\psi(p) = \left( \begin{array}{c} \phi_R(p) \\ \phi_L(p) \end{array} \right).
\]

As the reader will soon discover, and as I learned from the generalization [13] of the work of Ryder [7], the wave equation satisfied by this \((1, 0) \oplus (0, 1)\) spinor is determined by the boost properties of the \(\phi_R(p)\) and \(\phi_L(p)\), given

\(^5\)Our notations and conventions are closest to those found in Ryder’s book [7] on quantum field theory. The reader not familiar with representations of the Lorentz group will find a very readable discussion in Chapter 2 of Ryder’s book.
in Eqs. (1) and (2), and the algebraic relation between these fields at zero momentum. In the past it had been argued (see p. 44 of Ryder’s book on quantum field theory [7]), and it does not matter which specific spin in the \((j,0) \oplus (0,j)\) representation space is under consideration, that “when a particle is at rest, one cannot define its spin as either left- or right-handed, so” for zero momentum “\(\phi_R(\vec{0}) = \phi_L(\vec{0})\).” That this is not true for the \((1/2,0) \oplus (0,1/2)\) Dirac field is manifest when we look at the explicit form of zero-momentum spinors. In canonical representation the argument is made in Ref. [2] for the \((j,0) \oplus (0,j)\) representation space; and in Weyl (also called chiral) representation [the representation in which \(\psi(\vec{p})\) is written in Eq. (4)] the reader should carefully follow Weinberg’s arguments on pp. 220-224 of his text [8] to obtain spin-1/2 zero-momentum spinors (given in Eqs. 5.5.35 and 5.5.36 of [8]) to arrive at the same conclusion as us for \(j = 1/2\).

Given Eqs. (1) and (2), the spinors at momentum \(\vec{p}\), \(\psi(\vec{p})\), are known if we specify the zero-momentum spinors \(\psi(\vec{0})\). From the work of Ref. [2] and the detailed study on the closely related subject contained in [8] it is clear that it is the choice of the zero-momentum spinors, i.e., specification of \(\psi(\vec{0})\), that must determine the parity, and other related structure, of the theory. In Ref. [2] we postulated that in the canonical representation the six-dimensional \((1,0) \oplus (0,1)\) representation space for a particle at rest can be spanned by the six zero-momentum spinors:

\[
\begin{align*}
\psi_{\text{canonical}}(\vec{p}) &= \frac{1}{\sqrt{2}} \begin{pmatrix} I & I \\ I & -I \end{pmatrix} \psi(\vec{p}) = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_R(\vec{0}) + \phi_L(\vec{0}) \\ \phi_R(\vec{0}) - \phi_L(\vec{0}) \end{pmatrix}.
\end{align*}
\]

The \(\psi(\vec{p})\) in the middle of the above equation refers to the Weyl representation spinor of Eq. (4) and \(I = 3 \times 3\) identity matrix.

\[
\begin{align*}
\psi(\vec{p}) &= \begin{pmatrix} m \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad u_0(\vec{0}) = \begin{pmatrix} 0 \\ m \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad u_{-1}(\vec{0}) = \begin{pmatrix} 0 \\ 0 \\ m \\ 0 \\ 0 \\ 0 \end{pmatrix}.
\end{align*}
\]

This observation was first made in the Summer of 1991 by C. Burgard [14] while trying to understand Ryder’s \textit{ab initio} derivation [7] of the Dirac equation.

This we assert, not as a theorem, but as an essentially unavoidable conclusion made in an attempt to reconcile the apparently contradicting conclusions of Refs. [2] and [8, 9]. In principle, of course, there remains the possibility that something subtle is being missed by the authors of Ref. [2] or/and the author of [8].
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\[ v_{+1}(\vec{0}) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ m \end{pmatrix}, \quad v_0(\vec{0}) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ m \end{pmatrix}, \quad v_{-1}(\vec{0}) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ m \end{pmatrix}. \] (7)

The \( \sigma = 0, \pm 1 \) on \( u_\sigma(\vec{0}) \) and \( v_\sigma(\vec{0}) \) carry the meaning of the projection of spin on the z-axis, and the possibility of relative phases between various spinors (not important for the present considerations) are left implicit. On studying the C, P, and T properties of the associated wave equation, it turns out that the \( u- \) and \( v- \) spinors are related by the operation of Charge conjugation and carry opposite relative intrinsic parities [2].

Setting \( \vec{p} = \vec{0} \) in Eq. (5), of footnote [8], and comparing the resulting equation with Eqs. (6) and (7) we find that for the \( u- \) spinors \( \phi_R(\vec{0}) = +\phi_L(\vec{0}) \) and for the \( v- \) spinors \( \phi_R(\vec{0}) = -\phi_L(\vec{0}) \).

To sum up, therefore, we have the needed algebraic relation between \( \phi_R(\vec{p}) \) and \( \phi_L(\vec{p}) \) at zero momentum:

\[ \phi_R(\vec{0}) = \wp_{u,v} \phi_L(\vec{0}), \] (8)

with \( \wp_{u,v} = +1 \) for the \( u- \) spinors and \( \wp_{u,v} = -1 \) for the \( v- \) spinors.

The three \( u_\sigma(\vec{p}) \) and the three \( v_\sigma(\vec{p}) \) spinors, with \( \sigma = 0, \pm 1 \) representing the three spinorial degrees of freedom, are obtained by applying the \( (1,0) \oplus (0,1) \) boost implicit in definition (4) and the \( (1,0) \) and \( (0,1) \) boosts given in Eqs. (1) and (2). That is,

\[ u_\sigma(\vec{p}) = \begin{pmatrix} \exp(+\vec{J} \cdot \vec{\varphi}) & 0 \\ 0 & \exp(-\vec{J} \cdot \vec{\varphi}) \end{pmatrix} u_\sigma(\vec{0}), \] (9)

\[ v_\sigma(\vec{p}) = \begin{pmatrix} \exp(+\vec{J} \cdot \vec{\varphi}) & 0 \\ 0 & \exp(-\vec{J} \cdot \vec{\varphi}) \end{pmatrix} v_\sigma(\vec{0}); \] (10)

with \( \sigma = 0, \pm 1 \).

The reader will note that the six \( (1,0) \oplus (0,1) \) spinors thus obtained follow purely from the projective representations of the Lorentz group and the associated boosts, and the \( (1,0) \oplus (0,1) \) fields operator for the quantum description of these particles follows from the canonical arguments of translational invariance etc. [for details refer to any recent book on quantum

\[ ^9 \text{Note: we stay in the Weyl representation unless specifically indicated otherwise. For example, Eqs. (6) and (7) are written in canonical representation and this is explicitly noted right above these expressions.} \]
theory of fields, such as \[8\]}

\[\Psi(x) = \sum_{\sigma=0,\pm1} \int \frac{d^3p}{(2\pi)^3} \frac{1}{2\omega_p} \left[ u_{\sigma}(p^0) a_{\sigma}(p^\mu) e^{-ip\cdot x} + v_{\sigma}(p^0) b^\dagger_{\sigma}(p^\mu) e^{ip\cdot x} \right], \]

(11)

where \(\omega_p = \sqrt{m^2 + p^2}\); and \([a_{\sigma}(p^\mu), a^\dagger_{\sigma'}(p^\mu')] = \delta_{\sigma\sigma'}\delta(p - p')\); etc. and we leave implicit the possibility that the \(b^\dagger\) may carry a hidden numerical factor.

It is now a straightforward mathematical exercise to couple the right- and left-handed fields to obtain the free-field wave equation for the \((1,0) \oplus (0,1)\) spinors. Using Eq. (8) on the right-hand side of Eq. (1) to re-express \(\phi_R(\vec{0})\) in terms of \(\phi_L(\vec{0})\) and then using Eq. (2) to replace \(\phi_L(\vec{0})\) by \(\exp(J\cdot\bar{\varphi})\phi_L(p)\), and executing a similar exercise beginning with the right-hand side of Eq. (2), we obtain two coupled equations for \(\phi_R(p)\) and \(\phi_L(p)\). These two equations are then transformed into a single wave equation for the \((1,0) \oplus (0,1)\) spinor (4). This wave equation for the \((1,0) \oplus (0,1)\) spinor reads:

\[\left(\gamma_{\mu\nu} p^\mu p^\nu - \varphi_{\mu,\nu} m^2 I \right) \psi(p) = 0,\]

(12)

with

\[\gamma_{\mu\nu} p^\mu p^\nu = \begin{pmatrix} 0 & B + 2(J\cdot\bar{p})p^0 \\ B - 2(J\cdot\bar{p})p^0 & 0 \end{pmatrix},\]

(13)

where \(B = \eta_{\mu\nu} p^\mu p^\nu + 2(J\cdot\bar{p})(J\cdot\bar{p})\).

(14)

Here, \(\eta_{\mu\nu}\) is the flat space-time metric with the diagonal \((1,-1,-1,-1)\). The “0” on the diagonal represents a \(3\times3\) block of zeros. The off-diagonal terms are the \(3\times3\) block matrices.

From Eq. (13) we read off the following “gamma matrices” (not to be confused with the Dirac “gamma matrices”):

\[\gamma_{00} = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \gamma_{0i} = \gamma_{i0} = \begin{pmatrix} 0 & J_i \\ -J_i & 0 \end{pmatrix},\]

(15)

\[\gamma_{ij} = \gamma_{ji} = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} \eta_{ij} + \begin{pmatrix} 0 \\ \{J_i, J_j\} \end{pmatrix},\]

(16)

\(i\) and \(j\) run over a spatial index 1, 2, 3.

For the \((1,0) \oplus (0,1)\) representation space \(\psi(x) \equiv \psi(p) \exp(-i\varphi_{\mu,\nu} p\cdot x)\), and Eq. (12) requires:

\[\left(\gamma_{\mu\nu} \partial^\mu \partial^\nu + \varphi_{\mu,\nu} m^2 I \right) \psi(x) = 0.\]

(17)

10The procedure is in fact valid for any spin in the \((j,0) \oplus (0,j)\) representation space. In particular, when applied for \(j = 1/2\) one obtains the well-known Dirac equation.
This wave equation is identical to Steven Weinberg’s equation for the 
(1, 0) ⊕ (0, 1) representation space in all aspects except an important factor 
of \( \varphi_{u,v} \) attached to the mass term. It is this factor that leads to a 
fundamentally different, i.e., Wigner type, CPT structure in our theory. The 
CPT analysis of the theory is presented in Ref. [2] and is found to be very 
intricately related to the \( \varphi_{u,v} \) factor of our theory. The most important 
result that emerges from this analysis, and we simply quote it here, is that the operations of Charge conjugation and Parity anticommute in a quantum 
field theory built upon wave equation (17) and field operator (11) and thus 
results in the underlying spin-one boson and antiboson carrying opposite 
relative intrinsic parities. To the best of our knowledge the mathematical 
construction of Ref. [2], and further discussed here in its physical content, is 
the first explicit and non-trivial example of a quantum theory of the Wigner 
type. We look forward to further work on physical and mathematical impli-
cations of such a construct for future fundamental works in quantum field 
theory and possible experimental discovery of the Wigner-type bosons.

So far we have essentially emphasized the differences of our construct 
and that of Weinberg. We now discuss the aspects that are same (or similar) 
in both theories. The dispersion relations associated with both theories are 
determined by setting the determinant of 
\( (\gamma_{\mu\nu} p^\mu p^\nu - \varphi_{u,v} m^2 I) \) (for our 
theory), and the determinant of 
\( (\gamma_{\mu\nu} p^\mu p^\nu - m^2 I) \) (for Weinberg’s theory), equal to zero. The resulting equation is a 12th-order polynomial in 
\( (E, |p|, m) \) and results in the dispersion relations summarized in Table I.

| Dispersion Relation | Multiplicity | Interpretation   |
|---------------------|--------------|-----------------|
| \( E = + \sqrt{p^2 + m^2} \) | 3            | Causal, “particle”  
|                     |              | \( u_{\pm 1}(\vec{p}) \), \( u_0(\vec{p}) \) |
| \( E = - \sqrt{p^2 + m^2} \) | 3            | Causal, “antiparticle”  
|                     |              | \( v_{\pm 1}(\vec{p}) \), \( v_0(\vec{p}) \) |
| \( E = + \sqrt{p^2 - m^2} \) | 3            | Acausal, Tachyonic |
| \( E = - \sqrt{p^2 - m^2} \) | 3            | Acausal, Tachyonic |

There are two observations that we wish to make in regard to the tachy-
onic solutions:
1. In the \( m \to 0 \) limit all dispersion relations are non-tachyonic.
2. For \( m \neq 0 \), the the tachyonic solutions may be reinterpreted, on in-
   troducing a quartic self-coupling. The “negative mass squared” term
can then be interpreted as in the simplest versions of field theories with broken symmetry. Such an analysis [15] shows that the resulting theory describes four particles: two charged particles of mass \(m\) (of Wigner type in our theory and usual type in Weinberg’s theory), a (Goldstone-like) spin-one massless (Majorana-like) particle, and a massive (Majorana-like) spin-one particle of mass \(\sqrt{2|m|}\).

In any case, at this stage negative mass squared may be considered to provide the sought-after physical origin of the “\(-m^2\)” that appears in the simplest versions of field theories with broken symmetry. Or, in absence of this re-interpretation one may argue that these solutions violate the original input on the mass parameter via equation (3), and hence may be considered physically inadmissible (without perhaps interactions that make it possible to re-interpret the “\(-m^2\)” as above!). While we do not suspect that usual interactions can induce transitions between the \(E = \pm \sqrt{p^2 + m^2}\) and \(E = \pm \sqrt{p^2 - m^2}\) sectors, we do not know of any proof on the subject.

3. The Massless Limit and Maxwell Equations

That the massless limit is well behaved for the representation space under consideration was shown in the sixties by Weinberg [9]. So to obtain the massless limit of the theory under consideration, one may wish to set \(m = 0\) in (12), or equivalently in (17), and argue that \(\eta_{\mu\nu}p^\mu p^\nu \phi_R(\vec{p}) = 0\) and \(\eta_{\mu\nu}p^\mu p^\nu \phi_L(\vec{p}) = 0\) for a massless particle, to obtain:

\[
2 \vec{J} \cdot \vec{p} \left( \vec{J} \cdot \vec{p} + p^0 I \right) \phi_R(\vec{p}) = 0, \tag{18}
\]

\[
2 \vec{J} \cdot \vec{p} \left( \vec{J} \cdot \vec{p} - p^0 I \right) \phi_L(\vec{p}) = 0; \tag{19}
\]

and since it is known (see, for example, [13]) that

\[
(\vec{J} \cdot \vec{p} + p^0 I) \phi_R(\vec{p}) = 0 \tag{20}
\]

\[
(\vec{J} \cdot \vec{p} - p^0 I) \phi_L(\vec{p}) = 0, \tag{21}
\]

are indeed free Maxwell equations one may claim that we have obtained Maxwell equations in the limit \(m = 0\) of our, or that of Weinberg’s, theory. **No**, this is not so. The reason, as we already noted a few years ago [16], is that Maxwell equations (20) and (21) do not follow from Equations (18) and (19) because the matrix \(2 \vec{J} \cdot \vec{p}\) is non-invertible. The Determinant \(2 \vec{J} \cdot \vec{p}\) identically vanishes.

The analysis of the massless limit is not yet complete, but at this stage one can already note that all solutions of (Maxwell) Eqs. (20) and (21) are solutions of Eqs. (18) and (19), but there may exist solutions that do
not satisfy Maxwell equations but still are solutions of Eqs. (18) and (19). Another point to note is that (Maxwell) Eqs. (20) and (21) are first order in space-time derivatives. Eqs. (18) and (19) are of second order in space-time derivatives. Hence at least additional boundary conditions are to be satisfied. Any departures, therefore, from Maxwell equations will only be expected, or are most likely, for phenomenon that involve strong fields and/or strongly varying fields.

To complete the story we note that in the beginning of this section we assumed \( \eta_{\mu\nu} p^\mu p^\nu \phi_R(\vec{p}) = 0 \) and \( \eta_{\mu\nu} p^\mu p^\nu \phi_L(\vec{p}) = 0 \) for a massless particle. There is no justification to invoke this assumption a priori. Therefore, the Maxwell equations (20) and (21) should, rigorously speaking, be replaced, instead of (18) and (19), by:

\[
\begin{align*}
\left( \eta_{\mu\nu} p^\mu p^\nu I + 2 \vec{J} \cdot \vec{p} \left( \vec{J} \cdot \vec{p} + p^0 I \right) \right) \phi_R(\vec{p}) &= 0, \\
\left( \eta_{\mu\nu} p^\mu p^\nu I + 2 \vec{J} \cdot \vec{p} \left( \vec{J} \cdot \vec{p} - p^0 I \right) \right) \phi_L(\vec{p}) &= 0.
\end{align*}
\]

In reference to the above equations and Eqs. (18), (19), (20), and (21), it seems important to observe that these equations have solutions only if the appropriate dispersion-relation determining determinant vanishes. These determinants are:

\[
\begin{align*}
\text{Determinant} \left( \vec{J} \cdot \vec{p} \pm p^0 I \right) &= \mp E \left( \vec{p}^2 - E^2 \right) \quad (24) \\
\text{Determinant} \left( 2 \vec{J} \cdot \vec{p} \left( \vec{J} \cdot \vec{p} \pm p^0 I \right) \right) &= \text{(identically) 0}, \quad (25) \\
\text{Determinant} \left( \eta_{\mu\nu} p^\mu p^\nu I + 2 \vec{J} \cdot \vec{p} \left( \vec{J} \cdot \vec{p} \pm p^0 I \right) \right) &= -\left( \vec{p}^2 - E^2 \right)^3. \quad (26)
\end{align*}
\]

A comparison of the dispersion relations implied by setting the above determinants to zero with the dispersion relations for \( m \neq 0 \) case, tabulated in Table I, again indicates Eqs. (22) and (23) as the sole candidate for the massless limit of our (or, Weinberg’s) theory.

So, to sum up our analysis of the massless limit of our (or that of Weinberg’s theory), we conclude that: Present theoretical arguments suggest that in strong fields, or high-frequency phenomenon, Maxwell equations may not be an adequate description of nature. Whether this is so can only be decided by experiment(s). Similar conclusions, in an apparently very different framework, have been independently arrived at by M. Evans [17] and communicated to the author.

\[11\]What sets the scale that determines “strong” in the above statement? This question requires a precise answer, and we wish to take up this subject in the future. But for the moment we shall assume that a definite scale can be defined, or at least that the question can be answered in a specific experimental set up.
4. Concluding Remarks

First, we showed that in the $(1,0) \oplus (0,1)$ representation space there exist not one but two theories for charged particles. In the Weinberg construct, the boson and its antiboson carry same relative intrinsic parity, whereas in our construct the relative intrinsic parities of the boson and its antiboson are opposite. These results originate from the commutativity of the operations of Charge conjugation and Parity in Weinberg’s theory, and from the anti-commutativity of the operations of Charge conjugation and Parity in our theory. We thus claim that we have constructed a first non-trivial quantum theory of fields for the Wigner-type particles. Second, the massless limit of both theories seems formally identical and suggests a fundamental modification of Maxwell equations. At its simplest level, the modification to Maxwell equations enters via additional boundary condition(s).

References

1. E. P. Wigner, in Group Theoretical Concepts and Methods in Elementary Particle Physics – Lectures of the Istanbul Summer School of Theoretical Physics, 1962, edited by F. Gürsey; (Gordon and Breach, 1964). Also see: Z. K. Silagadze, Yad. Fiz. 55, 707 (1992) [Sov. J. Nucl. Phys. 55, 392 (1992)].
2. D. V. Ahluwalia, M. B. Johnson, and T. Goldman, Phys. Lett. B 316, 102 (1993).
3. D. V. Ahluwalia, “Theory of Neutral Particles: McLennan-Case Construct for Neutrino, its Generalization, and a Fundamentally New Wave Equation,” Int. J. Mod. Phys. A (in press). LANL HEP archive: hep-th/9409134.
4. L. L. Foldy, Phys. Rev. 102, 568 (1956).
5. B. P. Nigam and L. L. Foldy, Phys. Rev. 102, 1410 (1956). Typographical error: In the right hand side of Eq. (37) of this reference $U_s(\theta)$ should be corrected to read $U_s(\theta = 0)$.
6. Z. K. Silagadze, private communication (August, 1994)
7. L. H. Ryder, Quantum Field Theory, G. Sterman, An Introduction to Quantum Field Theory (Cambridge University Press, Cambridge University Press, Cambridge, U.K., 1993); M. Kaku, Quantum Field Theory: A Modern Introduction (Oxford University Press, Oxford, U.K., 1993); C. Itzykson and J.-B. Zuber Quantum Field Theory (McGraw-Hill Inc., U.S.A, 1980); and B. Hatfield, Quantum Field Theory of Point Particles and Strings, (Addison-Wesley Publishing Company, California, U.S.A., 1992).
8. S. Weinberg, The Quantum Theory of Fields, Vol. I (Foundations) (Cambridge University Press, Cambridge, U.K., 1995).
9. S. Weinberg, Phys. Rev. 133 , B1318 (1964); 134, B882 (1964). Also see, H. Joos, Forts. Phys. 10, 65 (1962).

Considerations of particles that are self-charge conjugate leads to a yet another theory in the $(1,0) \oplus (0,1)$ representation space. This construction appears in Ref. [3].

Despite the fact that the $\nu_{\mu,\tau}$ factor appears only in the mass term one cannot claim that Weinberg’s theory and our theory have the same physical content in the massless limit. The differences may arise from how the zero momentum $(1,0) \oplus (0,1)$ spinors are chosen in the two theories. This certainly is true for the differences in the two theories for massive particles. This aspect requires further study. Rigorously speaking, the “rest spinors” and “zero-momentum spinors” must be distinguished while speaking of the massless limit. For massless particles there are no rest spinors.
Acknowledgements

This an unusually long acknowledgement for an anonymous referee who has inspired and taught me in many ways. So I begin with a few personal comments in the nature of an introduction. I am deeply aware of the unusual nature of this acknowledgement and the criticism that it may draw, but to keep its contents in my files will be unfair to my fellow students of physics for many reasons.

In part, the purpose of this long addenda in the form of an acknowledgement to the manuscript is to document and acknowledge the significant contributions of this anonymous referee from Phys. Rev. Lett. in the construction of the Wigner-type quantum theory of fields. That the construction of a Wigner-type quantum theory of fields was purely accidental will also become apparent in the process of reading the two reports (from the same referee) that follow. To fully appreciate the impact of the referee on our work the reader should read Ref. [2] concurrently. While the physics these reports contain is important, it is equally important how that physics weaves with history and personal affection. Having talked about phases, projective representations, and given proofs of some important theorems, and having asked many questions, the referee suddenly seems overtaken by his affection for the man from whom he must have learned much of all this and writes “I am not professor Wigner (he has been 90 this month; let him in peace).”

I treat these reports as little monographs and these little monographs have provided me much guidance in the construction of the theory that I reviewed here. These reports are exceptional in their clarity, unsurpassed in their generosity, and exhibit a deep affection that their author holds for physics and the giants in his field. I reproduce these reports here not only to document the contributions that this referee made to my work, but also in the hope that future generations will be inspired and guided by the content and style of these reports when they write their reviews for the manuscripts of their colleagues. I remain deeply thankful to so unusual a referee. The very existence of this referee gives me hope for the future
of physics in these difficult cultural times when so little of support exists for fundamental science and it is so difficult to find a true mentor in the classic sense of this word. For the roughly six-month period during which I worked on the revision of the manuscript the referee became my mentor, and perhaps my collaborators too gained from the knowledge and good advice of the referee.

The text of the two reports that appears here is the unedited exact version, and all spellings are left as in the original to preserve complete flavour of the original reports. For example at one place we have “Majorana articles” (instead of Majorana particles); at other particles appears as “particules.” Similarly all punctuation, spacing after commas, italicized letters and boldface letters are reproduced as they appear in the original.

First Report:

This paper cannot be published in any physical journal. Indeed it presents an interesting idea but this idea is not new: for instance the possibility to have boson anti-boson relative parity is presented (with an equivalent, but different representation of the Lorentz group for particle states) in the E.P. Wigner (the physicist who has introduced parity in quantum mechanics in 1928) contribution to the volume: Group Theoretical concepts and methods in Elementary particle Physics. F. Gursey editor, Gordon and Breach, New-York 1964.

In this paper Wigner shows that to have opposite relative parity for boson anti-boson one has to pay a price: the doubling of the number of states, i.e. a new type of degeneracy (referred to as “Wigner type” by the knowledgeable physicists). So the author should refer to this Wigner paper and since he does not claim a doubling of the usual number of states he has to explain where is the discrepancy with Wigner. If he succeeds that will be very interesting to publish it. Unhappily there is not enough information in the manuscript to know if the author may succeed. For instance the author should write explicitly the Charge Conjugation operator $U(C)$ in his field theory.

Here are comments and questions for helping the author:

1) In the traditional notation $(j,0) \oplus (0,j)$ for the finite dimensional reducible representation of the connected Lorentz group, the parity operation exchanges these inequivalent irreducible components. So to use equation (2) (taken from your thesis) you have to give more precisions on what you

\[ \text{No other referee report was received by the editors of Phys. Rev. Lett. and despite a strong recommendation from the referee to publish the paper the manuscript was rejected by the editors. The manuscript, with the sole addition of an acknowledgement to the anonymous referee, was then submitted to Phys. Lett. B where it was independently reviewed and accepted without any revisions.} \]
have done: the parity operator cannot be block diagonal in the direct sum; probably you have already taken a symmetric and antisymmetric combinations of these two subspaces which carry inequivalent representations of the connected Lorentz group? The reader needs to know in detail. Redactional details: your equation (3) contains no information as long as you do not give $t', x'$ as function of $t, x$! Why do you call spinors the vectors $(1, 0)$ and $(0, 1)$ which correspond to $\vec{E} \pm i \vec{B}$ in electromagnetism (they were introduced by Helmoltz).

2) You do quote Wigner paper of 1939 on the inhomogeneous Lorentz group, but it is irrelevant here (e.g. in this beautiful and very important mathematical paper, Wigner studied the unitary representations of the full group, including time reversal, although he did know that in physics time reversal has to be represented by an antiunitary operator: see his paper of 1932 in Göttingen Nachrichten). The important fact to remember from this 1939 paper (or from Wigner’s earlier book) is that in quantum physics, the relativity group is realized by a projective representation, i.e. a representation up to a factor. The phase of the unitary operator representing a discrete operation is therefore arbitrary; and you are completly right to emphasize after your equation (6) that there is a “global phase factor”: to neglect it, as you suggest, might be throwing the baby out with the water of the bath tub. The way out is simple: independently of the arbitrary phase of the parity operator, the relative phase $\epsilon$ between particle and anti particle states is not arbitrary and is naturally defined as:

$$U(P)U(C) = \epsilon U(C)U(P) \quad (1)$$

It is a simple exercise to prove $\epsilon^2 = 1$ by using associativity of the group law. Indeed multiplying (1) on the right you obtain $U(P)U(C)^2 = \epsilon U(C)U(P)U(C)$ and using (1) again $U(P)U(C)^2 = \epsilon^2 U(C)^2 U(P) = \epsilon^2 U(C)U(C)^2$.

In your theory, you should define explicitly the charge conjugation op erator $U(C)$. Does it commute or anticommute with $U(P)$?

3) In the frame of quantum field theory do not forget that operators which correspond to physical identity, as for instance those representing $C^2$, $P^2$, $(CP)^2$, $T^2$, $(CPT)^2$ etc... are not a multiple of the identity operator, but their value depends on the superselection sector on which they act. The values of the square of the antiunitary operators are well fixed on each superselection sector; (this is not the case of the unitary operators such as $P^2$ for the non vacuum sectors, but the relative value between different sectors might be well defined). The proof uses again the associativity of the group law. Let $V = UK$ be an antiunitary operator: in a basis it is the product of a unitary operator and of the complex conjugation $K$. We assume that the restriction to a superselection sector of the square of this antiunitary operator is a multiple of the identity: $V^2 = \omega I$; since
$V^2$ is unitary, $\omega$ is a phase: $\overline{\omega}\omega = 1$. Then $V^3 = \omega V = V\omega = \overline{\omega}V$; so $\omega = \overline{\omega} = \pm 1$. For instance, in usual quantum field theory, without the “Wigner type” degeneracy, $V(CPT)^2 = 1$ on the bosonic sector and $-1$ on the fermionic sector. This implies that $V(CPT)^2\psi = -\psi V(CPT)^2$ for any fermion field $\psi$.

4) There is arbitrariness in defining parity of a single field without interaction, but this arbitrariness can be completely reduced with the interactions. Indeed to measure the parity of a particle you must interact (by a parity conserving interaction!) with the particle, either to produce it or by the study of its spontaneous decay (the case interesting for you?). Do you wish that these particles have electromagnetic interactions: write their electric current if they have an electric charge. If not they can still interact by an induced magnetic moment and, for spin 1 particle, a quadrupole moment. Or do you expect these mesons with absolutely no electromagnetic interaction?

5) Finally a serious study will lead you to rise the fundamental question of $CTP$. What do you expect about the $CTP$ behaviour of your particles? That is the type of question you must be able to answer if you want to make an interesting contribution to present particle physics. Indeed if you want their interaction to violate $CTP$ invariance, you have to reconstruct all present day physics!

I am not professor Wigner (he has been 90 this month; let him in peace). But I wished to help you; that is why I wrote this long report asking you many questions.

P.S. While writing this report I have no access to a collection of Physical Reviews of the sixties. In one of them, if my memory is faithful, you will find a paper by T.D. Lee and G.C. Wick which deals with the discrete symmetries $C, P, T$ in quantum field theory and give the values of the square of their representing operators on different superselection rules sectors.

**Second Report**

The first named author has appreciated my exceptionally long report. He has read and well assimilated the literature I suggested. Congratulations!

This very new version of the manuscript has now three authors and carries a very well chosen title. Indeed Bargmann, Wightman and Wigner had studied, this subject forty years ago, in an unpublished book (several chapters were distributed as preprints). The authors explain well the scope of their paper. They have made a thorough construction of the field theory of a non usual Wigner type; that is completely new and all given references are relevant. *This paper should be published.*

However the authors have missed an important point: in quantum theories, symmetry groups are implemented through **projective** representations. As the authors rightly write (page 7 and footnote 2) one can ignore an
overall phase factor (except in Majorana theory), but a crucial argument of parity should not be obscured by conventions. In my report, I recalled the proof that for the operators $C, P, T$ and their products, there are signs of $\epsilon$ independent from conventions: on each superselection rule sector, the value $\epsilon 1$ of the square of the antiunitary operators: $T^2$, $(PT)^2$, $(CT)^2$, $(CPT)^2$ and the sign in $PC = \epsilon CP$ which indicates the commutation or the anticommutation of $P$ and $C$; of course, these last two possibilities correspond respectively to same or opposite parity for particles and antiparticles. To stress this important point, the authors should show the anticommutation of $P$ and $C$ in the theory they develop and compute the value of the squares of the four antiunitary operators on the bosonic and fermionic states. Those characterize the Wigner’s types.

In his letter presenting this new version, the first named author points out that for zero mass particles the situation is different (specialy for Majorana articles). He is completely right and I am looking forward for the announced paper. I want to bring to his attention that this was understood in Tiomno thesis (princeton, around 1947-48), unpublished I believe. Either Tiomno (in Brazil) or Wightman, in Princeton, could give him more details.