On Secure Computation Over the Binary Modulo-2 Adder Multiple-Access Wiretap Channel

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Abstract

In this paper, the problem of securely computing a function over the binary modulo-2 adder multiple-access wiretap channel is considered. The problem involves a legitimate receiver that wishes to reliably and efficiently compute a function of distributed binary sources while an eavesdropper has to be kept ignorant of them. In order to characterize the corresponding fundamental limit, the notion of secrecy computation capacity is introduced. Although determining the secrecy computation capacity is challenging for arbitrary functions, it surprisingly turns out that if the function perfectly matches the algebraic structure of the channel, the secrecy computation capacity equals the computation capacity, which is the supremum of all achievable computation rates without secrecy constraints. Unlike the case of securely transmitting messages, no additional randomness is needed at the encoders nor needs the legitimate receiver any advantage over the eavesdropper. The results therefore show that the problem of securely computing a function over a multiple-access wiretap channel may significantly differ from the one of securely communicating messages.

Index Terms

Secure distributed computation, computation coding, multiple-access wiretap channel, physical layer security

I. INTRODUCTION

In their seminal work [1], Nazer and Gastpar lay the information-theoretic foundation of distributed computation over unreliable channels. The big difference between this approach and the standard theory dealing with reliable message transfer is that, in [1], the intended receiver decodes function values immediately from the channel output. In other words, the receiver does not care about individual messages and penalizes itself only when the function is incorrectly decoded.

In this regard, Nazer and Gastpar show that in many cases, the performance gain over separation-based computation strategies is proportional to the number of source terminals. In a separation-based strategy, the receiver first reliably decodes all individual messages and subsequently computes the sought function value. It is remarkable that the gains over separation-based strategies stem from a match between the desired function and the algebraic structure of the channel. Since the publication of [1], the results and ideas have been extended in many different ways [2]–[6].

Due to the trend towards large-scale decentralized networks consisting of many mutually distrusting terminals, security and integrity of computation results are of high priority in order to guarantee trustworthy operation. In this work, we therefore make a first attempt to extend the concept of computation coding [1] by taking information theoretic security aspects into account. In particular, we consider the problem of computing a function over the binary modulo-2 adder multiple-access wiretap channel (MAWC). The problem involves a legitimate receiver that wishes to reliably compute a function of distributed binary sources in the presence of an eavesdropper. To characterize the corresponding fundamental limit, we introduce the notion of secrecy computation capacity. Although determining the secrecy computation...
capacity for arbitrary functions is challenging, it turns out that if the function matches the algebraic structure of the modulo-2 adder MAWC, the secrecy computation capacity equals the computation capacity. Thus, the algebraic structure of the channel not only helps to efficiently compute the desired function but also to protect the transmitted source sequences against eavesdropping. It is noteworthy that, to achieve this, the source terminals do not need any additional source of randomness nor needs the legitimate receiver any advantage over the eavesdropper. This is in stark contrast to standard physical layer security results.

A. Related Work

Considering secure distributed computation, also known as secure multi-party computation, from an information theoretic (i.e., Shannon) perspective is still in its infancy. To the best of the authors’ knowledge there exist only some very recent results. For instance, Tyagi et al. introduce a new Shannon theoretic multiuser source model in [7] and [8], in order to characterize when a function is securely computable. In this context, they provide necessary and sufficient conditions for the existence of protocols that achieve this.

Within the standard secure multi-party computation model of [9], Lee and Abbe determine in [10] the least amount of randomness needed for securely computing a given function. This provides a novel notion of the complexity of a function for its secure computation. In the second part of that paper, the considerations are extended to a probabilistic source model for which the decoding error probability is required to vanish asymptotically in the block length.

In [11], Data et al. take a distributed source coding approach to the problem of securely computing the modulo-2 sum of two distributed binary sources. Similarly to [10], they assume the data to be drawn from some joint memoryless source and derive bounds on the amount of randomness and communication needed to asymptotically achieve secrecy. In [12], the results are extended to arbitrary functions.

B. Paper Organization

This paper is organized as follows. Section II introduces the binary modulo-2 adder MAWC and provides the problem statement. In order to obtain some insights, in Section III we focus first on the noiseless case. The noisy case is then considered in Section IV which also contains a comparison with separation-based schemes. Section V concludes the paper.

C. Notational Remarks

Random variables are written as uppercase letters and their realizations as lowercase letters. A sequence \((X[1], \ldots, X[n])\) is written as \(X^n\) and it is considered as a column vector whenever multiplied by a matrix. For \(p \in [0, 1]\), \(H(p) = -p \log_2 p - (1 - p) \log_2 (1 - p)\) denotes the binary entropy function with the convention \(0 \log_2 0 = 0\). The Bernoulli distribution with parameter \(p \in [0, 1]\) is denoted as \(\text{Bern}(p)\), which means that any binary random variable \(X \sim \text{Bern}(p)\) takes on value 1 with probability \(p\). Addition modulo-2 is denoted as \(\oplus\) and \(\delta_{ij}\) denotes the Kronecker delta, which is 1 for \(i = j\) and 0 otherwise.

II. System Model and Problem Statement

Let \(S_1, \ldots, S_M\) be \(M\) binary memoryless sources drawn from a joint probability mass function \(P_{S_1 \ldots S_M}\). In the presence of an eavesdropper, the sources are communicated to a legitimate receiver over a noisy channel. Unlike the usual setup in which the legitimate receiver wishes to reliably reconstruct each individual source while keeping the eavesdropper ignorant of them [13]–[15], in this paper the legitimate receiver is interested in reliably and securely computing a Boolean function

\[
f : \{0, 1\}^M \to \{0, 1\}, \quad U = f(S_1, \ldots, S_M)
\]

of the sources, to which we refer as the desired function.
As illustrated in Fig. 1, we consider the particular toy scenario in which the channel between the sources and the destinations can be modeled as a memoryless binary modulo-2 adder multiple-access wiretap channel, which is characterized by the input-output relations

\begin{align}
Y &= X_1 \oplus \cdots \oplus X_M \oplus N_Y, \\
Z &= X_1 \oplus \cdots \oplus X_M \oplus N_Z.
\end{align}

Here and hereafter, $X_m \in \{0, 1\}$ is the channel input of source terminal $m$, $Y$ is the channel output seen by the legitimate receiver and $Z$ the output observed by the eavesdropper. The noise variables $N_Y \sim \text{Bern}(p)$ and $N_Z \sim \text{Bern}(q)$, for some $p, q \in [0, 1]$, are assumed to be independent of each other as well as independent of the channel inputs.

**Remark 1.** Note that each of the two multiple-access channels (MACs) in (1) is a modulo-2 adder followed by a binary symmetric channel (BSC).

For some $k \in \mathbb{N}$, $S_m^k$ denotes a length-$k$ sequence of independent and identically distributed samples of source $m$, $m = 1, \ldots, M$. In order to reliably compute at the legitimate receiver the sequence of corresponding function values, $U^k$, the source terminals employ a length-$n$ computation code defined as follows [1].

**Definition 1.** Given a fixed desired function $f$, a $(k, n)$ computation code for the binary modulo-2 adder MAWC consists of the following:

- Encoding functions
  \[
  \phi_m : \{0, 1\}^k \to \{0, 1\}^n, \quad m = 1, \ldots, M,
  \]
  each of which maps $k$ source symbols to a length-$n$ codeword (i.e., $\phi_m(s_m^k) = x_m^n$);

- A decoding function at the legitimate receiver
  \[
  \psi : \{0, 1\}^n \to \{0, 1\}^k,
  \]
  which maps each channel output sequence to a length-$k$ sequence of function values (i.e., $\psi(y^n) = \hat{u}^k$).

The average probability or error of a $(k, n)$ computation code is defined as

\[
P_e^{(n)} := \mathbb{P}[\hat{U}^k \neq U^k],
\]

whereas the information about the source sequences leaked to the eavesdropper is measured by

\[
I(S_m^k; Z^n), \quad m = 1, \ldots, M.
\]
which we combine to the single constraint
\[
L^{(n)} := I(S^k_1; Z^n) + \cdots + I(S^k_M; Z^n) .
\]

Definition 2. For some given desired function, a rate \( R := k/n \) is said to be an achievable secrecy computation rate if there exists a sequence of \( (nR, n) \) computation codes such that
\[
\lim_{n \to \infty} P_e^{(n)} = 0 \quad \text{and} \quad \lim_{n \to \infty} L^{(n)} = 0 .
\]

Definition 3. For some given desired function, the secrecy computation capacity is defined as
\[
C_{sc} := \sup \{ R : R \text{ is an achievable secrecy computation rate} \}.
\]

Since the problem is challenging for arbitrary \( f \), in this paper we focus on securely computing the modulo-2 sum of the source symbols:
\[
f(s_1, \ldots, s_M) = s_1 \oplus \cdots \oplus s_M.
\]

III. THE NOISELESS CASE

First, in order to fix ideas and obtain insight, in this section we consider the noiseless case (i.e., \( p = q = 0 \)), which results in the channel outputs
\[
Y = Z = X_1 \oplus X_2 \oplus \cdots \oplus X_M .
\]
For independent \( \text{Bern}(1/2) \) sources, we obtain the following surprising result.

Theorem 1. Let the sources be independently \( \text{Bern}(1/2) \) distributed and let the desired function be the modulo-2 sum. Then, the secrecy computation capacity is \( C_{sc} = 1 \) function values per channel use.

Proof: To prove the theorem, we will make use of the following special case of Forney’s Crypto Lemma.

Lemma 1 (Forney [10]). Let \( V \) and \( W \) be independent binary random variables with \( W \) distributed according to \( \text{Bern}(1/2) \). Then, \( U := V \oplus W \) is distributed according to \( \text{Bern}(1/2) \) and statistically independent of \( V \).

(Achievability). Transmitting the source samples uncoded results in the channel output sequences
\[
Z^k = Y^k = S^k_1 \oplus \cdots \oplus S^k_M = U^k
\]
and thus in \( P_e^{(n)} \equiv 0 \). In order to analyze the leakage, we show that each term of (2) equals zero. Consider
\[
I(S^k_m; Z^k) = kH(S_m) - kH(S_m|Z) = kH(S_m) - kH(S_m|S_m \oplus W) ,
\]
\( m = 1, \ldots, M \), with
\[
W := S_1 \oplus \cdots \oplus S_{m-1} \oplus S_{m+1} \oplus \cdots \oplus S_M = S_l \oplus W'
\]
for some \( l \neq m \). Notice that due to \( S_l \sim \text{Bern}(1/2) \), Lemma 1 implies that \( W \) is distributed according to \( \text{Bern}(1/2) \) as well. But if \( W \sim \text{Bern}(1/2) \), then Lemma 1 also guarantees that the independence of \( S_m \) and \( W \) implies the independence of \( S_m \) and \( Z = S_m \oplus W \), which allows us to conclude \( H(S_m|S_m \oplus W) = H(S_m) \). Inserting this into (3) results in \( I(S^k_m; Z^k) \equiv 0 \). As this applies to all \( m = 1, \ldots, M \), we finally have \( L^{(n)} \equiv 0 \).
(Converse). If we allow the encoders to fully cooperate, then the sum rate of the MAC in (1a) cannot exceed $\max_{X_1, \ldots, X_M} I(X_1, \ldots, X_M; Y)$, where $P_{X_1, \ldots, X_M}$ denotes the joint distribution of the channel inputs. With or without secrecy constraint, we have
\[
I(U; ˆU) \leq (a) \leq I(X_1, \ldots, X_M; Y) = H(Y) - H(Y|X_1, \ldots, X_M) = H(U) - H(U|S_1, \ldots, S_M) \leq (b) H(U) \leq 1,
\]
which is a tight upper bound in our case. Note that (a) follows from the data processing inequality and (b) from the fact that $U$ is a function of $S_1, \ldots, S_M$.

Due to the modulo-2 additivity of the channel along with the fact that the desired function perfectly matches this algebraic structure, the source sequences behave like one-time pads protecting each other. Thus, the algebraic structure of the channel not only helps to efficiently compute the desired function at the legitimate receiver but also to protect the source sequences against eavesdropping. A remarkable fact is that the source terminals do not need any additional source of randomness nor needs the legitimate receiver any advantage over the eavesdropper. This is in stark contrast to standard physical layer security problems in which a legitimate receiver wishes to securely decode messages. For instance, when the objective is to securely communicate messages over a MAWC, without local randomness the achievable secrecy rate region would be an empty set [13]–[15].

Remark 2. Note that the coding strategy used in the proof of Theorem 1 achieves perfect secrecy. Furthermore, the converse part of the proof implies that for the considered scenario, the secrecy computation capacity equals the computation capacity $C_c$. The latter is defined as the supremum over all achievable computation rates (i.e., without secrecy constraints) $\overline{C}$.

IV. THE NOISY CASE

Now, in this section we extend our considerations to the noisy case in which parameters $p$ and $q$ can be chosen arbitrary (see (1)). The joint source distribution can be arbitrary as well.

A. Computation Capacity vs. Secrecy Computation Capacity

Before presenting the main result of this paper, we recap a result of [1], which provides the computation capacity of the binary Modulo-2 adder MAC (1a).

Theorem 2 (Nazer · Gastpar [1]). Let $f$ be the modulo-2 sum of the source symbols. Then, the computation capacity of the binary modulo-2 adder MAC (1a) is given by
\[
C_c = \frac{C}{H(U)} = \frac{1 - H(p)}{H(U)},
\]
where $C$ denotes the capacity of a BSC with crossover probability $p$.

For the achievability part of the proof, Nazer and Gastpar employ random linear code ensembles for source compression and channel coding. By following their approach, we are able to extend Theorem 1 to the following.

Theorem 3. Let $f$ be the modulo-2 sum and the joint source distribution be arbitrary. Then, the secrecy computation capacity of the binary modulo-2 adder MAWC is
\[
C_{sc} = C_c = \frac{1 - H(p)}{H(U)}.
\]
Proof: (Achievability). Let $C = 1 - H(p)$ denote the capacity of a BSC with crossover probability $p \in [0, 1]$.

- **Code construction:** Generate two matrices $A \in \{0, 1\}^n \times \ell$ and $B \in \{0, 1\}^{\ell \times k}$, each entry drawn uniformly at random, with

\[
kH(U) < \ell < nC.
\]

Reveal $A$ and $B$ to the source terminals, the legitimate receiver, and the eavesdropper.

- **Encoding:** Given $s_m^k$ at source terminal $m$, transmit

\[
X_m^n = \phi_m(s_m^k) = ABs_m^k,
\]

where all operations are carried out modulo-2.

With this encoding rule, the legitimate receiver observes the sequence of channel output symbols

\[
Y^n = X^n_1 \oplus \cdots \oplus X^n_M \oplus N^n_Y = ABs^n_1 \oplus \cdots \oplus ABs^n_M \oplus N^n_Y = AB(S^n_1 \oplus \cdots \oplus S^n_M) + N^n_Y.
\]

Effectively, (6) is a BSC with crossover probability $p$. The random linear code induced by generator matrix $A$ therefore has the objective to protect $BU^n_k$ against the channel noise $N^n_Y$, whereas the linear code induced by $B$ is used to compress $U^n_k$ to its entropy. As long as condition (4) is fulfilled, there exist decoding functions $\psi' : \{0, 1\}^n \to \{0, 1\}^\ell$ and $\psi'' : \{0, 1\}^\ell \to \{0, 1\}^k$ such that for arbitrary $\varepsilon > 0$ and $n$ large enough, the average probabilities of error (averaged over $A$ and $B$) fulfill $P(\psi'(Y^n) \neq BU^n_k) < \frac{\varepsilon}{2}$ and $P(\psi''(BU^n_k) \neq U^n_k) < \frac{\varepsilon}{2}$. This was shown in [1] based on results from [17] and [18]. Thus, defining the decoding function in Definition [1] as

\[
\psi(y^n) := (\psi'' \circ \psi')(y^n),
\]

by means of the union bound we have $P_e(n) < \varepsilon$ as long as $R = \frac{k}{n} < \frac{C}{H(U)}$ and $n$ sufficiently large.

Now, we have to analyze the leakage. As in the proof of Theorem [1] we consider each term of $L(n)$ separately. Towards this end, first note that (5) defines a deterministic encoder, which therefore has the following two properties:

- Source sequences are uniquely determined by codewords:

\[
H(S_m^k | X_m^n) = 0, \quad m = 1, \ldots, M.
\]

- The encoding functions are injective:

\[
H(X_m^n | S_m^k) = 0, \quad m = 1, \ldots, M.
\]

Expanding the $m$-th term of $L(n)$ as

\[
I(s_m^k; Z^n) = I(s_m^k, X_m^n; Z^n) - I(X_m^n; Z^n | S_m^k) = I(X_m^n, Z^n) + I(s_m^k; Z^n | X_m^n) - I(X_m^n, Z^n | S_m^k)
\]

it can be easily seen that due to (7) and (8), we have

\[
I(s_m^k; Z^n) = I(X_m^n; Z^n).
\]

Thus, to show that $I(s_m^k; Z^n)$ vanishes with increasing $n$ is equivalent to showing that $I(X_m^n; Z^n)$ vanishes.

As the entries of matrix $A$ are independently generated according to the Bern(1/2) distribution, it can be easily proven that the entries of each codeword, $X_m^n$, are independently distributed according to Bern(1/2) as well and that all codewords are pairwise independent [19, Sec. 6.2]. Therefore, following along similar lines as in the proof of Theorem [1] it can be verified that $Z^n$ is statistically independent of
$X^n$. But if $Z^n$ is independent of $X^n_m$, then $I(X^n_m; Z^n) \equiv 0$. As $m$ was arbitrarily chosen, we thus have $L(n) \equiv 0$.

(Converse). For the average probability of error, $P_e^{(n)}$, to vanish with increasing block length, with or without a secrecy constraint every computation code has to fulfill

$$kH(U) \leq I(U^k; \hat{U}^k)$$

$^{(a)} \leq I(X^n_1, \ldots, X^n_M, Y^n)$$

$\leq \max_{P_{X^n_1\ldots X^n_M}} I(X^n_1, \ldots, X^n_M, Y^n)$$

$= n(1 - H(p))$,

where (a) is due to the data processing inequality. Combining the left hand side of (9) with (10) results in the upper bound $R = \frac{k}{n} \leq \frac{1 - H(p)}{H(U)}$, which is tight in our case.

Remark 3. It has to be emphasized that the secrecy computation capacity of Theorem 3 is independent of the MAC between the source terminals and the eavesdropper (i.e., independent of $q$). Note also that perfect secrecy is achieved.

Surprisingly, the sequence of linear random codes that achieves the computation capacity also achieves the secrecy computation capacity. No additional source of randomness is needed at the encoding or decoding side as the codewords itself act as one-time pads.

B. Comparison with Separation-Based Computation

Consider the case $M = 2$ and let $(S_1, S_2)$ be a doubly symmetric source with joint probability mass function

$$P_{S_1S_2}(s_1, s_2) = \frac{1}{2}(1 - \theta)\delta_{s_1s_2} + \frac{1}{2}\theta(1 - \delta_{s_1s_2}),$$

for some $\theta \in (0, 1)$. By means of this explicit example, in this subsection we compare Theorem 3 with the secrecy computation rate that is achievable with a separation-based coding scheme. A separation-based scheme first distributively compresses the source sequences into messages and then uses a capacity achieving MAC code in order to reliably reconstruct the messages at the legitimate receiver. Once $\hat{S}_1^k$ and $\hat{S}_2^k$ are known to the legitimate receiver it computes $\hat{U}^k = \hat{S}_1^k \oplus \hat{S}_2^k$, resulting in an estimate of the sequence of function values.

For this scenario it is shown in [1] that the best possible computation rate (i.e., without secrecy constraint) achievable with separation is

$$R = \frac{1}{2} \left( \frac{1 - H(p)}{H(\theta)} \right).$$

The rate can be achieved with Körner-Marton compression for $U$ \cite{20} in combination with time-sharing. Compared with Theorem 2 this rate is only half the computation capacity. Because of time-sharing, however, when adding secrecy constraints the other source sequences do not act as one-time pads any longer so that local randomness has to be used at the encoders in order to confuse the eavesdropper.

Theorem 4. Let $M = 2$, the joint source distribution be as in (11), $f$ be the modulo-2 sum, $p, q \in [0, 1]$, and $\theta \in (0, 1)$. Then, for the binary modulo-2 adder MAWC, the best secrecy computation rate achievable with separation is

$$R = \frac{1}{2} \left( \frac{H(q) - H(p)}{H(\theta)} \right)^+, \quad (13)$$

where $(x)^+ := \max\{0, x\}$, $x \in \mathbb{R}$.

\footnote{Note that for the two MACs given in \cite{1}, time-sharing is optimal.}
Proof: As in the achievability part of the proof of Theorem 3, the source terminals use the same linear random code for compressing $U$ to its entropy $H(U) = H(\theta)$. In [20], Körner and Marton show that this is optimal for the joint source distribution given in (11). Now, using time-sharing, the legitimate receiver alternately observes the channel outputs
\begin{align*}
Y' &= X_1 \oplus N_Y \quad \text{and} \quad Y'' = X_2 \oplus N_Y \quad (14)
\end{align*}
while the eavesdropper sees
\begin{align*}
Z' &= X_1 \oplus N_Z \quad \text{and} \quad Z'' = X_2 \oplus N_Z \quad (15)
\end{align*}
Thus, for each channel use we effectively have a binary symmetric wiretap channel of secrecy capacity
\begin{align*}
(C(p) - C(q))^+ &= (1 - H(p) - (1 - H(q)))^+ \\
&= (H(q) - H(p))^+,
\end{align*}
where $C(q)$ denotes the capacity of the BSCs in (14) and $C(p)$ the capacity of the BSCs in (15), respectively. Thus, using standard wiretap coding allows $P_e^n$ to be driven to zero as long as
\begin{align*}
k2H(\theta) < n(H(q) - H(p))^+,
\end{align*}
which provides the rate in (13) and thus the theorem.

A closer look at (13) reveals the following.

Corollary. The secrecy computation rate achievable with separation is nonzero if and only if the MAC between the source terminals and the eavesdropper (i.e., (1b)) is noisier than the MAC between the source terminals and the legitimate receiver (i.e., (1a)). That is, if and only if $H(p) < H(q)$.

Remark 4. For $p, q \in [0, 1/2]$, $H(p) < H(q)$ if and only if $p < q$ and for $p, q \in (1/2, 1]$ if and only if $p > q$.

After comparing (13) with (12), we conclude that separation-based computation schemes generally suffer from imposing a secrecy constraint. In order to keep the source sequences secret from the eavesdropper, wiretap coding is needed and therefore local randomness at the encoding side. This generally further reduces the achievable computation rate.

V. CONCLUSION

We have considered the problem of securely computing a function of distributed sources over the modulo-2 adder MAWC. Instead of individual source samples, the legitimate receiver is interested in reliably decoding a function of the sources from the channel output. To characterize the corresponding limits, we have introduced the notion of secrecy computation capacity and determined it for a function that perfectly matches the structure of the channel. Unlike standard results in physical layer security, no additional randomness is needed in order to confuse the eavesdropper.

Future work includes extensions to more general functions and MAWCs. On the other hand, the leakage in (2) might be replaced by another secrecy criterion such as $L^n = I(U^n; Z^n)$. This criterion is less restrictive as it prohibits the eavesdropper only from knowing anything about the function that is computed at the legitimate receiver.
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