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Precession and glitches in the framework of three-component model of neutron star

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Abstract. We consider the pulsar rotation assuming that the neutron star consists of crust component (which rotation is observed) and two core components. One of the core components contains pinned superfluid which can, for some reasons, suddenly inject small fraction of stored angular momentum in it. In the framework of this simple model the star can demonstrate glitch-like events together with long period precession (with period $1 - 10^4$ years).

1. Introduction

Some radio pulsars demonstrate periodic variations in pulse timing and beam shape. The most favorable explanation to these phenomena is the precession of the neutron star with periods $T_p \sim 10^2$ days \cite{1, 2}. There are pulsar characteristics and behavior features which can be interpreted as a manifestation of precession with much larger periods $T_p \sim 10^2 - 10^4$ years \cite{3, 4}. Another type of timing irregularities is pulsar glitches which are the sudden increase of pulsar angular velocity $\Omega$ with subsequent smooth recovery. Standard model relates these events with avalanche-like unpinning of superfluid neutron vortices pinned to the nuclei in the inner crust or proton flux tubes in the neutron star core. However, as it was first pointed out by Shaham \cite{5} and later confirmed by more detailed researches \cite{6, 7}, the pinning of neutron superfluid should dramatically decrease the period of precession making it very far from observed values. We propose a simple phenomenological model which allows the same pulsar to precess with a long period and demonstrate glitch-like timing features.

2. Three-components model

Let us assume that a neutron star consists of three dynamically distinguished components. For the sake of simplicity all components are supposed to rotate as rigid bodies.

The c-component is the outer component. It rotates with angular velocity $\vec{\Omega}_c = \vec{\Omega}$:

$$d_t M_c = \vec{K}_{\text{ext}} + \vec{N}_{rc} + \vec{N}_{gc},$$

where $\dot{M}_c = I_c \dot{\Omega} + I_c \epsilon \vec{\Omega}$ is the c-component angular momentum, $I_c$ is one of its principal moments of inertia. Tensor $\epsilon$ is the effective oblateness tensor. It describes the total departure of c-component mass distribution and electromagnetic field energy distribution from spherical symmetry. It is assumed that $\epsilon_{\alpha\beta} \ll 1$. Vector $\vec{K}_{\text{ext}}$ is the external electromagnetic torque.
Note that so-called anomalous torque is taken into account by tensor \( \dot{\epsilon} \) [8]. Therefore, torque \( \vec{K}_{ext} \) contains only the terms \( \propto \Omega^3 \).

The g-component is an inner component. It consists of normal matter characterized by moment of inertia \( I_g \) and rotating with angular velocity \( \vec{\Omega}_g \). It also contains superfluid matter with angular momentum \( \vec{L}_g \) pinned to the normal matter:

\[
d t \vec{M}_g = \vec{N}_{cg} + \vec{N}_{rg}, \quad d t \vec{L}_g = \vec{\Omega}_g \times \vec{L}_g,
\]

where \( \vec{M}_g = I_g \vec{\Omega}_g + \vec{L}_g \) is the total angular momentum of the g-component.

The r-component is the other inner component. It rotates with angular velocity \( \vec{\Omega}_r \):

\[
d t \vec{M}_r = \vec{N}_{cr} + \vec{N}_{gr}, \tag{3}
\]

where \( \vec{M}_r = I_r \vec{\Omega}_r \) is the angular momentum of the r-component, \( I_r \) is its moment of inertia. The components interact with each other by torques \( \vec{N}_{ij} \), where \( i, j = c, g, r \) and, obviously, \( \vec{N}_{ij} = -\vec{N}_{ji} \).

If \( \vec{R}_{ext} = 0 \) and \( \dot{\epsilon} = 0 \), system of equations (1)-(3) has a simple equilibrium solution: \( \vec{\Omega}_g = \vec{\Omega}_r = \vec{\Omega} = \) const, \( \vec{L}_g = L_g \vec{\epsilon}_\Omega \), where \( \vec{\epsilon}_\Omega = \vec{\Omega}/\Omega \). Further we will consider only small corrections to this equilibrium state. Introducing notations \( \vec{V}^\parallel = (\vec{\epsilon}_\Omega \cdot \vec{V}) \) and \( \vec{V}^\perp = \vec{V} - \vec{\epsilon}_\Omega (\vec{\epsilon}_\Omega \cdot \vec{V}) \) applied to arbitrary vector \( \vec{V} \) and holding only the linear in \( \vec{\mu}_{ij} \) terms one can rewrite system of equations (1)-(3) in the following form:

\[
\begin{align*}
\dot{\vec{\Omega}} &= \vec{R}_{cg}^\parallel + \vec{R}_{rc}^\parallel + S^\parallel, \tag{4} \\
\dot{\vec{\mu}}_{cg}^\parallel &= \left( \vec{\omega}_{cg}^\parallel - \vec{\Omega} \right) \times \vec{\mu}_{cg}^\parallel - \left[ \vec{\Omega} \times \vec{\mu}_{cg}^\parallel \right] + \vec{R}_{cg}^\parallel + \vec{R}_{rc}^\parallel - \vec{R}_{ge}^\parallel - \vec{R}_{gc}^\parallel - \vec{S}_{cg}^\parallel, \tag{5} \\
\dot{\vec{\mu}}_{cr}^\parallel &= \vec{R}_{cr}^\parallel + \vec{R}_{gr}^\parallel - \vec{R}_{rc}^\parallel - \vec{S}_{cr}^\parallel, \tag{6} \\
\vec{\omega}_{r}^\parallel &= 0, \tag{7} \\
\vec{\Omega} \vec{\epsilon}_\Omega &= \vec{R}_{gc}^\parallel + \vec{R}_{rc}^\parallel + \vec{S}_{cg}^\parallel, \tag{8} \\
\end{align*}
\]

where \( \vec{R}_{ij} = \vec{N}_{ij}/I_j, \vec{S} = \vec{R}_{ext}/I_c - \vec{\Omega} \times \vec{\epsilon}_\Omega, \vec{\omega}_r = \vec{L}_g/I_g \), and the point denotes the time derivative in the c-component frame of reference. We assume that vectors \( \vec{R}_{ij} \) linearized in \( \vec{\mu}_{ij} \) can be represented as

\[
\vec{R}_{ij} = -\left( \alpha_{ij} \vec{\mu}_{ij}^\parallel \vec{\epsilon}_\Omega + \beta_{ij} \vec{\mu}_{ij}^\perp + \gamma_{ij} [\vec{\epsilon}_\Omega \times \vec{\mu}_{ij}^\perp] \right), \tag{12}
\]

where \( \alpha_{ij}, \beta_{ij} \) and \( \gamma_{ij} \) are the phenomenologically introduced interaction constants.

3. Quasi-stationary rotation

Let us first assume that glitches do not occur. In this case, if the largest internal relaxation time-scale \( \tau_{rel} = \max (1/\alpha_{ij}, 1/\beta_{ij}, 1/\gamma_{ij}) \) is much smaller than the period of precession \( T_p \), and assumption (12) is valid, the neutron star will rotate in quasi-stationary regime. It means that, the departures of internal components rotation from c-component rotation are determined by instant values of \( \vec{\Omega} \) and \( \vec{\dot{\Omega}} \) and do not depend on prehistory (see detail in [9]). In other words, one can neglect time derivative terms in equations (5), (6), (9)-(11) because of their quadratic smallness. Solving equations (5), (6), (10) and (11) for \( \vec{\mu}_{cr}^\parallel, \vec{\mu}_{cg}^\perp \vec{\mu}_{cr}^\perp \) and \( \vec{\mu}_{cg}^\perp \), and substituting the obtained expressions into equations (4) and (8), one can obtain

\[
\vec{L}_{tot} \vec{\dot{\Omega}} = \vec{R}_{ext}^\parallel, \tag{13}
\]
I_\theta \dot{\vec{\Omega}} = \frac{(1 + \Gamma)(\vec{K}_{\text{ext}} - I_\theta \vec{\Omega} \times \dot{\vec{\Omega}}) + B \vec{e}_\theta \times (\vec{K}_{\text{ext}} - I_\theta \vec{\Omega} \times \dot{\vec{\Omega}})}{(1 + \Gamma)^2 + B^2}, \quad (14)

where \( \dot{I}_{\text{tot}} = I_c + I_g + I_r \) and in the weak interaction limit \( (\alpha_{ij}, \beta_{ij}, \gamma_{ij} \ll \Omega) \) equals to

\[
B \approx \frac{\beta_{gc} + \beta_{rc}}{\Omega} \ll 1, \quad \Gamma \approx \frac{\gamma_{gc} + \gamma_{rc}}{\Omega} \ll 1. \quad (15)
\]

One can note that according to equation (13) the neutron star is braked as if it is a solid body with moment of inertia \( \dot{I}_{\text{tot}} \) (which does not include the moment of inertia of pinned superfluid). It is a general feature of quasi-stationary approximation [10, section 2.3].

Equation (14) describes the change in the orientation of angular velocity vector \( \vec{\Omega} \) relative to the c-component. The first and the third terms on the right-hand side arise due to action of external torque \( \vec{K}_{\text{ext}} \), the second term makes vector \( \vec{\Omega} \) precess about a star principal axis and the fourth term describes the damping of the precession due to internal energy dissipation. Using equation (14), one can estimate precession period and precession damping time-scale as

\[
T_p \sim \frac{2\pi}{\epsilon \Omega}, \quad \tau_d \sim \frac{2\pi}{\epsilon \Omega B} \gg T_p, \quad (16)
\]

where \( \epsilon \) is characteristic oblateness. The external torque \( \vec{K}_{\text{ext}} \) tends to minimize the energy loss caused by electromagnetic radiation. It leads to increase or decrease of precession amplitude depending on the orientation of star dipole moment relative to the principal axes [11]. One can see that expression for precession period \( T_p \) does not contain \( L_g \). This seems to contradict to Shaham who has obtained that \( T_p \) equals rather to \( L_g/\dot{I}_{\text{tot}} \Omega \) [5]. The reason for this discrepancy is that we allow the g-component (which containing the pinned superfluid) to move relative to the c-component which rotation is observed. It is required that \( \beta_{cg} \ll \Omega \) to ensure the validity of expressions (16). The Shaham’s result can be reproduced by passing to limit \( \beta_{cg} \to \infty \) in equations (9)-(11) so that \( \mu_{cg}^+ \to 0 \).

4. Glitch-like event
Since superfluid is pinned, the lag between rotations of superfluid and normal fractions of g-component increases during the retardation of star rotation. When critical lag is reached the superfluid unpins and then repins such that some small fraction of its angular momentum is transferred from superfluid into the normal fraction. In present work we do not specify the physical mechanism of glitch triggering. However, we assume that the glitch occurs and then relaxes at time-scales much less the the time-scales of quasi-stationary evolution. It allows us to neglect the effects of external torque and star oblateness considering these processes.

The solution of equations (4)-(6) for \( \Omega \) with the following initial conditions: \( \Omega = \Omega_0, \mu_{cr} = 0, \mu_{cg} = \Delta L_g/I_g \) and zeroth \( S^i \) has the form

\[
\Omega(t) = \Omega_0 + \Delta \Omega \left(1 - e^{-p_+ t} - Q(1 - e^{-p_- t})\right), \quad (17)
\]

where \( \Delta \Omega = \Delta \Omega_\infty/(1 - Q), \Delta \Omega_\infty = \Delta L_g/\dot{I}_{\text{tot}}, Q = (\dot{I}_{\text{tot}} \alpha_{cg} - I_c p_+)/(\dot{I}_{\text{tot}} \alpha_{cg} - I_c p_-) \) and the coefficients \( p_+ \) and \( p_- \) \((p_+ > p_-)\) are the roots of equation

\[
p^2 - (\alpha_{cg} + \alpha_{rg} + \alpha_{cr} + \alpha_{gr} + \alpha_{gc} + \alpha_{rc}) p + \n
\quad + (\alpha_{gc} + \alpha_{rg} + \alpha_{cg}) \cdot (\alpha_{cr} + \alpha_{gr} + \alpha_{rc}) + (\alpha_{rc} - \alpha_{rg}) \cdot (\alpha_{gr} - \alpha_{gc}) = 0. \quad (18)
\]
The meaning of parameters $\Delta \Omega$, $\Omega_\infty$ and $Q$ becomes clear if one looks at the sketch of the solution in figure 1. We suppose that the interaction between the $c$- and $g$-components is the strongest one ($\alpha_{cg} \gg \alpha_{rc}, \alpha_{rg}$). In this case,

$$p_+ \approx \left(1 + \frac{I_g}{I_c}\right) \alpha_{cg}, \quad p_- \approx \frac{I_{tot}}{I_c + I_g} (\alpha_{cr} + \alpha_{gr}) \quad \text{and} \quad Q \approx \frac{I_r}{I_{tot}}.$$ (19)

If one wants to relate solution (17) with observed pulsar glitches, then $1/p_+$ and $1/p_-$ should be interpreted as glitch growth ($<30 \text{ s}$ [12]) and relaxation ($1 - 10^2 \text{ days}$ [13]) times respectively. One can see that the glitch growth rate is proportional to $\alpha_{cg}$ while the subsequent relaxation is governed by the $r$-component. Thus, on the one hand, the interaction between the $c$- and $g$-components should be strong enough to ensure rapid spin-up. But simultaneously it should be not too much strong, namely $\beta_{cg} \ll \Omega$, to long period precession could exist. It is natural to assume that $\alpha_{cg} \sim \beta_{cg}$.

5. Discussion

Let us speculate a little about possible nature of formally introduced components. The $c$-component can be associated with neutron star crust and a part of core charged particles which is strongly coupled with the crust ($I_c \sim 10^{-2} - 10^{-1} I_{ns}$). The role of $g$-component can be performed by tangles of closed flux tubes which could be formed after protons became superconductive. The part of core neutron superfluid which is not pinned to the $g$-component and a part of normal matter coexisting with it and weakly coupled with $c$-component ($I_r \sim I_{ns}$). The interaction coefficients values which are needed to make the model able to explain the observed glitches time-scales can be estimated with expressions (19). The weakest side of the presented simple model is the value of recovery fraction $Q$. According to (19) it is of the order of unity. It is not so bad for young pulsars but mature ones demonstrate rather recovery fractions several orders less than unity [13]. However, we believe that it is a consequence of the oversimplifying of the model and further researches will allow us to avoid this discrepancy.

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