Cabibbo-suppressed decays of the $\Omega_c^0$ — feedback to the $\Xi_c^+$ lifetime

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Abstract

We investigate a possible background of the type $\Omega_c^0 \to \Xi_c^+ \pi^-$ to the CLEO $\Xi_c^+$ lifetime measurement. This decay mode may lead to an overestimate of the $\Xi_c^+$ decay length and, therefore, increase the measured $\Xi_c^+$ lifetime. The branching ratio $\Gamma(\Omega_c^0 \to \Xi_c^+ \pi^-)/\Gamma(\Omega_c^0 \to \Omega^- \pi^+)$ is analyzed in the framework of the pole model and the modified current algebra. We find that the $\Omega_c^0 \to \Xi_c^+ \pi^-$ decay mode could not generate a substantial systematic error in the $\Xi_c^+$ lifetime measurement. Also, it cannot significantly reduce the disagreement between theoretical and experimental values of the $\Xi_c^+$ lifetime.

The lifetime measurements of charmed baryons are well known [1] to be very important in estimating and disentangling the different preasymptotic effects in the decays of charmed hadrons. They provide the most direct way to determine the weak mixing angles and to test the unitarity of the CKM matrix. The CP violation outside the kaon system can be studied, and one can also test our present knowledge of the QCD confinement inside hadrons.

The preasymptotic effects [2], like the inclusion of soft degrees of freedom (light quarks, gluons) generate nonperturbative power corrections, e.g. the destructive and/or constructive Pauli interference, and the W-exchange contribution, producing the diversity of lifetimes of charmed mesons [3–5] and baryons [6,7], which would, otherwise, be all equal in the asymptotic limit of infinitely heavy quark mass\(^1\).

\(^1\) It appears astonishig that decay rates of weak and radiative decays are described in terms of few basic quantities, e.g. quark masses, and hadronic expectation values of several leading local operators.
Inclusive hadronic decay rates and lifetimes were calculated a long time ago [3–7] by summing over all possible channels and integrating over some range of energies. A ‘practical’ version of the OPE is used in calculations, i.e. it is assumed that the coefficient functions can be found perturbatively and all nonperturbative effects reside in matrix elements. In real world, however, there are nonperturbative effects even at short distances, and the matrix elements are subject to perturbative corrections too.

Surprisingly enough, the theory works rather well, even in the charmed hadron sector, although the expansion parameter \( \sqrt{\mu^2 G(D)/m_c^2} \approx 0.5 \) is not really small (the corresponding parameter in beauty decays is \( \sqrt{\mu^2 G(B)/m_b^2} \approx 0.13 \)).

A systematic study of charmed baryon decays was performed a few years ago [8], with good agreement between theory and experiment. The theoretical predictions were rather stable to the uncertainties in the wave functions of heavy baryons and/or to the choice of the renormalization/factorization scale, except in the case of the \( \Xi_c^+ \) charmed baryon. It was not clear if the peculiar behavior of the \( \Xi_c^+ \) was a pure coincidence due to the wild cancellation of different preasymptotic effects, or some deeper understanding was missing. The theoretical result, \( \tau(\Xi_c^+)_{\text{th}} = 0.27 \) ps, for \( m_c = 1.35 \) GeV, \( \Lambda_{\text{QCD}} = 300 \) MeV, had to be compared with the experimental value, \( \tau(\Xi_c^+)_{\text{exp}} = (0.35 \pm 0.07) \) ps. The difference, at that time, was not so significant that one would have had to worry. However, it was clear that future more precise measurements could disturb an idyllic concordance between theory and experiment.

Fig. 1 shows the results of \( \Xi_c^+ \) lifetime experiments performed up to now. One can see that two new measurements with significantly improved accuracy, FOCUS [9] and CLEO [10], are above the previous world average 1\( \sigma \) margin, in the case of CLEO, even above the 2\( \sigma \) margin. By including these two new measurements the average has changed from 0.33 ps to 0.442 ps. In particular, FOCUS precisely measured the charmed-strange baryon \( \Xi_c^+ \) lifetime as

\[
\tau(\Xi_c^+) = 0.439 \pm 0.022 \pm 0.009 \text{ ps.} \tag{1}
\]

In the FOCUS spectrometer, which is well suited to reconstruct short-lived charmed decays, the charmed particles are produced as the product of the interaction between high energy-photons with \( \langle E \rangle \simeq 180 \) GeV in a segmented BeO target and an excellent vertex separation between the production and decay vertices is provided by two silicon vertex detectors.

All previous experiments, including that performed by FOCUS, are fixed-target experiments. CLEO performed the only colliding beam experiment. Therefore, it has different systematics and different backgrounds. In spite of the fact that the charmed baryon lifetimes are not measured as precisely as those of charmed mesons, CLEO and SELEX [13] recently measured \( \tau(\Lambda_c^+) \) to
Fig. 1. $\Xi_c^+$ lifetime experiments. The left (right) band is the $1\sigma$ PDG 2000 [11] (2002 [12]) world average. E687.93 is excluded from the PDG 2000 value and Accmor from the PDG 2002 value.

a precision of 5%. Other charmed baryons ($\Xi_c^+, \Xi_c^0, \Omega_c$) are measured with up to an uncertainty of 30%. CLEO’s measurement gives

$$\tau(\Xi_c^+) = 0.503 \pm 0.047 \text{ (stat.)} \pm 0.018 \text{ (syst.) ps.}$$

(2)

This result is obtained using an integrated luminosity of 9.0 fb$^{-1}$ of $e^+e^-$ annihilation data taken with the CLEO IV.V detector at the CESR. The data were taken at energies at and below the $\Upsilon(4S)$ resonance and include $\sim 11 \cdot 10^6 e^+e^- \rightarrow c\bar{c}$ events. The $\Xi_c^+$ is reconstructed from the $\Xi^-\pi^+\pi^-$ decay mode. Each $\Xi^-$ is reconstructed from $p\pi^-$. The assumption is that the $\Xi_c^+$ is produced at the primary event vertex and is not a decay product of another weakly decaying particle, e. g. $\Omega_c^0 \rightarrow \Xi_c^+\pi^-$. The measured $\Xi_c^+$ lifetime will be shifted towards a higher value.

The purpose of this letter is to examine the relevance of the $\Omega_c^0 \rightarrow \Xi_c^+\pi^-$ decay mode as a possible source of a systematic error for the $\Xi_c^+$ lifetime measurement. To this end, we study the ratio of two exclusive decay modes,

$$\eta = \frac{\Gamma(\Omega_c^0 \rightarrow \Xi_c^+\pi^-)}{\Gamma(\Omega_c^0 \rightarrow \Omega^-\pi^+)}$$

(4)

for the following reasons: the $\Omega_c^0 \rightarrow \Omega^-\pi^+$ process is expected to be one of the first and best measured $\Omega_c^0$ exclusive decays in the near future; therefore it is quite convenient to have the contribution of $\Omega_c^0 \rightarrow \Xi_c^+\pi^-$ normalized to the rate of $\Omega_c^0 \rightarrow \Omega^-\pi^+$ [14]; the $\Omega_c^0 \rightarrow \Omega^-\pi^+$ process has a factorizable
contribution only, which reduces theoretical uncertainties; uncertainties are further suppressed by considering ratios of exclusive decay widths.

**Ω⁰_c → Ξ⁺_cπ⁻ decay mode** In the Ω⁰_c → Ξ⁺_cπ⁻ decay, the decay happens in the light-quark sector and the pion emerges with a momentum of O(200 MeV) that can be considered 'reasonably' soft. Therefore, there is a similarity between this decay and the hyperon (ΔS = 1) decays for which the soft-pion limit technique with pole corrections was successfully applied [15] with the predictions for the branching fractions within 20% from experimental values. We believe that the soft-pion limit is equally applicable to the Ω⁰_c → Ξ⁺_cπ⁻ decay.

The invariant amplitude for the decay of the initial baryon B_i(1/2⁺) to the final baryon B_f(1/2⁺) and a pion π^a, a = 1, 2, 3, is given by

\[ \langle B_f \pi^a | \mathcal{H}_W(0) | B_i \rangle = i \, \bar{u}_f (A - B \gamma_5) u_i, \tag{5} \]

with A and B to be determined using the standard nonleptonic weak hamiltonian

\[ \mathcal{H}_W = \sqrt{2} G_F V_{q_3 q_4} V_{q_1 q_2}^* (c_- O_- + c_+ O_+), \tag{6} \]

where \( O_\pm \) are local 4-quark operators

\[ O_\pm = (\bar{q}_{1L} \gamma_\mu q_{2L})(\bar{q}_{3L} \gamma^\mu q_{4L}) \pm (\bar{q}_{3L} \gamma_\mu q_{2L})(\bar{q}_{1L} \gamma^\mu q_{4L}), \tag{7} \]

with \((\bar{q}_{iL} \gamma_\mu q_{jL}) = \frac{1}{2} \bar{q}_i \gamma_\mu (1 - \gamma_5) q_j\), and \( V \)'s are the elements of the CKM matrix. The Wilson coefficients in the leading logarithmic approximation are given by

\[ c_\pm (\mu^2) \cong \left( \frac{\alpha_s(\mu^2)}{\alpha_s(M_W^2)} \right)^{d_\pm/2b}, \tag{8} \]

where \( b = \frac{1}{3} (11N_c - 2n_f) \), \( N_c \) and \( n_f \) being the number of colors and flavors, respectively. The quantities \( d_- = -2d_+ = 8 \) are proportional to the anomalous dimensions of the operators \( O_- \) and \( O_+ \).

![Diagram](image)

Fig. 2. Ω⁰_c is produced at the primary event (PE) vertex and decays into Ξ⁺_c.
In the approach of references [15,16] modified current algebra techniques were applied, i.e. the soft-pion amplitude (commutator term) was corrected for the soft-pion limit. The contribution coming from baryon poles is given as

\[ A^{CA} = A^{soft} + A^{corr} \]

\[ = \sqrt{2} f_\pi \left[ B_f \left[ Q^a, C^{PV}_W \right] B_i \right] \]

\[ - \sqrt{2} \sum_{B_n(\frac{1}{2}^-)} \left( m_{B_f} - m_{B_i} \right) \left( g_{A_{B_f}B_nB_i} B_n^* B_{B_i} + b_{B_fB_i} g_{A_{B_f}B_n} B_n^* B_{B_i} \right) \], \quad \text{(9)}

\[ B^{pole} = \sum_{B_n(\frac{1}{2}^+)} \frac{\sqrt{2}}{f_\pi} \left( m_{B_f} + m_{B_n} \right) g_{A_{B_f}B_nB_i} a_{B_nB_i} + \frac{m_{B_f} + m_{B_n}}{m_{B_f} - m_{B_n}} a_{B_fB_n} g_{A_{B_f}B_n} B_n^* B_{B_i} \right), \quad \text{(10)}

In the above equations the S- and P-wave amplitudes are calculated in the framework of the pole model. Using the Lehman-Symanzik-Zimmermann reduction formalism, the pion momentum is taken off shell. The pion field is related to the axial vector current via PCAC, and a complete set of states is inserted.

In (10), the baryon-baryon weak matrix elements \( b_{B_fB_i}^* \) and \( a_{B_fB_i} \) are defined as

\[ \langle B_f^{(1/2^-)} | C^{PV}_W | B_i \rangle = i b_{B_fB_i}^* u_j \Sigma_i, \quad \text{(11)} \]

\[ \langle B_f^{(1/2^+)} | C^{PV}_W | B_i \rangle = a_{B_fB_i} u_j \gamma_5 \Sigma_i, \quad \text{(12)} \]

and \( g_{A_{B_f}B_j}^{B_i} \) is the axial form-factor, related to the strong \( g_{B_fB_j}^{B_i} \) baryon-baryon-meson coupling through the generalized Goldberger-Treiman relation. The pion decay constant \( f_\pi \) is taken as 0.132 GeV. The weak matrix elements (11) and (12) and the axial form-factors are calculated inside the MIT bag model.

Concerning the pole resonances, the only flavor structure that can be formed in an intermediate state of the \( \Omega^0 \rightarrow \Xi^*_c \pi^- \) decay is \( (dsc) \), (Fig.3). The main contribution to the S-wave amplitude comes from the commutator term in (14), providing a simple means of summing contributions from all intermediate states in the soft-pion limit. The correction to this term is dominated by \( (1/2^-) \) resonances, the lowest one being for our decay \( \Xi^0_c(2790) \) (denoted by \( \Xi^0_c \)). P-wave amplitudes are dominated by the lowest lying \( (1/2^+) \) baryon intermediate states. Since the charmed antitriplet - antitriplet axial form factors vanish, \( g_{A_{B_f}B_j}^{B_i(3)} = 0 \), the lowest lying \( \Xi^0_c \) resonance belonging to the charmed baryon antitriplet does not contribute. The main contribution to the P-wave amplitude comes from the \( \Xi^0_c \) baryon \( (1/2^+) \) state, belonging to the
Fig. 3. Pole diagrams for the $\Omega_c^0 \rightarrow \Xi_c^+ \pi^-$ decay mode: at the quark level and in terms of effective couplings.

charmed baryon sextet. Therefore we have

$$A^{CA} = \frac{1}{f_\pi} (\Xi_c^0 | \mathcal{H}_W^{PC} | \Omega_c^0) + \frac{1}{f_\pi} m_{\Omega_c^0} - m_{\Xi_c^0} \Xi_c^0 | \Xi_c^0 \rangle b_{\Xi_c^0 \Omega_c^0},$$

(13)

$$B^{\text{pole}} = \frac{1}{f_\pi} m_{\Xi_c^0} + m_{\Xi_c^+} \Xi_c^0 | \Xi_c^0 \rangle a_{\Xi_c^0 \Omega_c^0}.$$  

(14)

There is also a factorizable P-wave part of the $\Omega_c^0 \rightarrow \Xi_c^+ \pi^-$ amplitude, which can be expressed as

$$B^{\text{fact}} = \frac{G_F}{\sqrt{2}} V_{us} V_{ud} a_1 f_\pi (m_{\Omega_c^0} + m_{\Xi_c^0}) \Xi_c^0 | \Omega_c^0 \rangle,$$

(15)

where $a_1 = \frac{1}{3}(2c_+ + c_-)$. The decay rate for $\Omega_c^0 \rightarrow \Xi_c^+ \pi^-$ is then given by

$$\Gamma(\Omega_c^0 \rightarrow \Xi_c^+ \pi^-) = \frac{1}{4\pi m_{\Omega_c^0}} |A|^2 (E_{\Xi_c^+} + m_{\Xi_c^+}) + |B|^2 (E_{\Xi_c^+} - m_{\Xi_c^+}).$$

(16)

$\Omega_c^0 \rightarrow \Omega^- \pi^+$ decay mode. This mode is of the type $B_f(1/2^+) \rightarrow B_i(3/2^+) + \pi$ and its invariant amplitude is

$$\mathcal{M} = i q_\mu \bar{u}_f (B' - C \gamma_5) u_i.$$ 

(17)

The expression for the decay rate is

$$\Gamma(\Omega_c^0 \rightarrow \Omega^- + \pi^+) = \frac{|P_{\Xi_c^+}|^2 m_{\Omega_c^0}}{6\pi m_{\Xi_c^+}^2} |B|^2 (E_{\Xi_c^+} + m_{\Xi_c^+}) + |C|^2 (E_{\Xi_c^+} - m_{\Xi_c^+}) \rangle.$$ 

(18)

The $\Omega_c^0 \rightarrow \Omega^- \pi^+$ decay does not receive any pole contributions. There is only a factorizable P-wave amplitude contributing. This decay mode has already been calculated in the literature [17–19] by applying different quark models. We have recalculated it in the MIT bag model in order to have a consistent calculation of the ratio $\eta$ (4).
Table 1
Amplitudes ($\times 10^7$) and width (s$^{-1}$) for the $\Omega^0_c \to \Xi^+_c \pi^-$ decay mode. The invariant amplitude for the decay mode with the spin-1/2 particle in the final state is $i \bar{u}_f (A - B \gamma_5) u_i$, with dimensionless S- and P-wave amplitudes.

| $A_{\text{fact}}$ | $A_{\text{soft}}$ | $A_{\text{corr}}$ | $A_{\text{tot}}$ | $B'_{\text{fact}}$ | $B_{\text{pole}}$ | $B_{\text{tot}}$ | $\Gamma$ (s$^{-1}$) |
|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|
| 0                | 2.87             | 0.25             | 3.12             | 7.47             | -45.16           | -37.69           | 4.50$\cdot 10^9$ |

Table 2
Amplitudes ($\times 10^7$) and widths (s$^{-1}$) for the $\Omega^0_c \to \Omega^- \pi^+$ decay mode. The invariant amplitude for the decay mode with the spin-3/2 particle in the final state is $i q_\mu \bar{u}_f (B' - C \gamma_5) u_i$, with P- and D-wave amplitudes having units GeV$^{-1}$.

| $B'$ (GeV$^{-1}$) | $C$ (GeV$^{-1}$) | $\Gamma$ (s$^{-1}$) |
|------------------|------------------|------------------|
| 13.75            | 0                | 2.89$\cdot 10^{11}$ |

**Numerical results and discussions.** As we have already stated before, all our form factors, decay constants and matrix elements have been calculated in the MIT bag model. The calculations have been performed using the following parameter set: $\mu = 1$ GeV, $\Lambda_{\text{QCD}} = 200$ MeV, particle masses are taken to be PDG average values [12], and MIT bag model parameters have the same values as in [20].

For the $\Omega^0_c \to \Xi^+_c \pi^-$ decay mode, we have the S-wave amplitude which is given by the current algebra term and a pole correction of 10%. The P-wave amplitude has a factorizable contribution and a large pole contribution. Note from (16) that the P-wave amplitude is suppressed by a small kinematical factor, making contributions from S- and P-wave amplitudes of the same order of magnitude. The results of the calculation are summarized in Table 1. In the $\Omega^0_c \to \Omega^- \pi^+$ decay the only nonvanishing contribution is from the factorizable part of the P-wave ($B'$) amplitude, the D-wave ($C$) amplitude is zero (Table 2). Finally, the ratio of partial decay rates two exclusive decay modes of $\Omega^0_c$ considered in this letter is

$$\eta = \frac{\Gamma(\Omega^0_c \to \Xi^+_c \pi^-)}{\Gamma(\Omega^0_c \to \Omega^- \pi^+)} = \frac{2.96 \cdot 10^{-15}}{1.90 \cdot 10^{-13}} \text{ GeV} = 0.016.$$

The uncertainties, of order 10%, are connected with the scale $\mu$ at which the Wilson coefficients are evaluated, whereas the variation of $\Lambda_{\text{QCD}}$ from 200 MeV to 300 MeV leads to 15% larger value of $\eta$.

The ratio of partial decay rates (19) shows that the branching ratios for the Cabibbo-suppressed decays of $\Omega^0_c$ are at most at the level of a percent. The apparent dilatation of the $\Xi^+_c$ baryon path due to the described cascade of weak decays from the initially formed $\Omega^0_c$ baryon is quite small and certainly insufficient to explain the discrepancy of the recent $\Xi^+_c$ lifetime measurements [9,10] and theoretical calculations [8]. This result is altogether not so surprising, al-
though reassuring given that in exclusive decays there is always a possibility of a large pole contribution.

Finally, it is worth mentioning that the improved knowledge on Cabibbo-suppressed decays of singly charmed baryons may have other important implications on the understanding of the $\Xi_c^+$ lifetime. As shown in [21], it is possible to obtain a model-independent prediction of this lifetime once a reliable estimate of the decay rate of inclusive Cabibbo-suppressed decays of $\Lambda_c^+$ is available. Therefore, a more systematic and detailed approach to the Cabibbo-suppressed decays of singly charmed baryons is called for from both the experimental side, as a way of reducing systematic errors, and the theoretical side, as a way of obtaining model-independent predictions of the $\Xi_c^+$ lifetime.

With the calculated level of the contribution of Cabibbo-suppressed $\Omega_c^0$ decays, it is clear that this form of the systematic error in the determination of the $\Xi_c^+$ lifetime cannot provide an explanation of the present disaccord between theory and experiment. To achieve agreement, a new layer of theoretical analysis will have to be uncovered and new experimental data will have to be compiled.

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