The Application of Fuzzy Mathematics in the Diagnosis of Oil System Malfunction of Jet Engine
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Abstract. In this thesis we draw fuzzy mathematic method into the diagnosis of oil system malfunction of jet engine, bring forward setting up the fuzzy membership grade method which is based on the history data and expert preferential ordinal number, found the fuzzy diagnostic model, resolve to one or several malfunction, how nicety fast find cause malfunction really reason, and proffer scientific and feasible new method to the airplane malfunciton diagnosis service.

Introduction
Aircraft failure is a common problem for crew members, it is also hard to track in maintenance work. In actual use, accidents caused by airplane faults occur frequently because of design, manufacture, use, maintenance, etc. In the face of numerous facts and serious consequences caused by airplane faults, people have to focus on reducing and preventing failures and ensuring the reliability of equipment, systems and airplane. Once an airplane breaks down, or if there are signs of failure, it is timely to find out the reason of the failure and take corresponding maintenance measures, which is also called the diagnosis of airplane failure.

In the past, the diagnosis of airplane faults was always handled by physical detection and qualitative analysis, which contains valuable expert experience. However, there are many vague concepts and methods in airplane flight status, condition monitoring and technical diagnosis. Their fuzziness is embodied in the following three aspects:

1. The dynamic signals (vibration, noise, temperature, pressure, flow, etc.) of an airplane in flight carry abundant information about the state of the airplane. The classical diagnostic method is to obtain the characteristic parameters reflecting different physical meanings such as mean, variance, auto-correlation function, amplitude, phase, divergence and so on from the time and frequency domains through information processing. However, there are uncertainties in identifying and evaluating the status or fault of an airplane with these parameters, which are mainly manifested in two aspects: randomness and fuzziness. Random is caused by the uncertainty of causality. It can be analyzed by probability statistics and stochastic process. Fuzziness is due to the fact that things do not have exact meanings in nature and definite boundaries in quantity, which result in the appearance of the same or the other nature of things. The category of these attributes is not clear, and it is an uncertainty in the division of things. The only way to solve this kind of problem is through fuzzy mathematics.

2. During the whole process of airplane equipment from operation to damage, there is no clear boundary between states or faults. It is a gradual process from fault-free to fault-free. Besides sudden accidents, the "fault" and "non-fault" of airplane belong to two fuzzy sets. They are neither completely "intact" nor "fault", but in the intermediate state of "intact" and "fault". From the two extremes of "intact" and "fault", all intermediary states present the same or the other, and their boundaries are blurred. Therefore, by defining a clear boundary, or determining a fault threshold to judge whether an airplane is faulty, it will inevitably bring human subjective errors (including judgment errors) into the fault diagnosis results, and result in false reporting and missed detection of fault information.

3. People's observation about the symptoms of malfunction is not clear, and there are many sensory components. It is well known that a skilled technician can judge whether an airplane has a
fault or where it happened based on his experience, and this "experience" is a vague concept. For example, by looking, hearing, touching and instrument measurements, the concepts of "big noise", "strong vibration" and "high temperature" are formed in their minds to determine the airplane failure, the failure phenomena (or symptoms) are used to find out the cause of the failure and infer the location of the failure. So in the empirical method of fault diagnosis, its essence is also full of fuzziness.

In summary, it is impossible to solve the problem accurately and quickly only by qualitative decision-making, while the classical mathematical method seems powerless here. Therefore, it is necessary for us to use the theory and method of fuzzy mathematics to provide a scientific and feasible new method for airplane fault diagnosis and maintenance.

The Fuzzy Diagnostic Model about Plane Faults

**The Diagnosis Symptoms Vector and Reason Vector**

**Definition 1.** Suppose there are m symptoms belong to one fault, \( x_i \) (i = 1,2,...,m) denotes the ith estate variable, it takes 0 or 1, 1 means symptom occurred, 0 means symptom was not occurred, then call \( X = (x_1,x_2,...,x_m) \) diagnosis symptom fuzzy vector.

**Definition 2.** There are n reasons \( Y_j \) (j =1,2,...,n) occurred at the same time or independent which caused one fault occur, then call \( Y = (y_1,y_2,...,y_n) \) diagnosis reason vector.

**The Fuzzy Diagnosis Matrix**

**Definition 3.** Denote fuzzy membership grade of the ith fault symptom \( x_i \) to the jth fault symptom \( y_j \) by \( \mu_{ij}(x_i)=r_j \) (i=1,2,...,m; j=1,2,...,n), then call \( R = (r_{ij})_{mn} \) fuzzy diagnosis matrix, that is

\[
R = \begin{pmatrix}
  r_{11} & r_{12} & \cdots & r_{1n} \\
  r_{21} & r_{22} & \cdots & r_{2n} \\
  \vdots & \vdots & \ddots & \vdots \\
  r_{m1} & r_{m2} & \cdots & r_{mn}
\end{pmatrix},
\]

where fuzzy membership grade is decided by the following methods:

1. Confirming the experience membership grade \( v_{ij} \) by experiment data:

\[
v_{ij} = \frac{\text{numbers of the ith symptom belongs to the jth reason}}{\text{the total numbers of the ith symptom appears}}
\]  \hspace{1cm} (1)

2. Initial membership grade value \( s_{ij} \) determined by method of expert preferential ordinal number: a certain fault symptom \( x_i \) is caused by n possible fault reasons \( y_1,y_2,...,y_n \). Aiming at \( x_i \), contrasting two different elements in n reasons, denoting the one which can easily arouse fault symptoms by a first priority, then an expert obtain \( n(n-1)/2 \) first priority comparisons. Supposed that M experts make comparisons following the above method, and get \( Mn(n-1)/2 \) first priority comparisons. In these \( Mn(n-1)/2 \) comparisons, priority number is \( t_j \), denoted \( t_k = \max\{t_j \mid j=1,2,...,n\} \), the initial membership grade of fault symptom \( x_i \) to reason \( y_j \) is
Suppose expert experiment weight is \( w_1 \), experiment data weight is \( w_2 \), where \( w_1, w_2 \geq 0 \), and \( w_1 + w_2 = 1 \). Then, the comprehensive fuzzy membership degree of experiment data and expert experiment is

\[
r_{ij} = w_1 s_{ij} + w_2 y_{ij} \quad (i=1,2,\ldots,m; j=1,2,\ldots,n)
\]

Therefore, we can construct fuzzy diagnosis matrix.

The Fuzzy Diagnosis Theory

Just as following process:

- Experience, Stat., experiment, datum
- Constructs subject degree \( \mu_{ij}(x_i) \)
- Sets up diagnosis matrix \( R \)
- Decide synthesizing algorithms \( \oplus \)
- Find fuzzy equation \( Y = X \oplus R \)
- Diagnosis result

Suppose vector is \( X = (x_1, x_2, \ldots, x_m) \) and fault vector is \( Y = (y_1, y_2, \ldots, y_n) \). The fuzzy compositional transformation is

\[
Y \Delta X \circ R = (x_1, x_2, \ldots, x_m) \circ \begin{pmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ r_{m1} & r_{m2} & \cdots & r_{mn} \end{pmatrix}
\]

It can be concluded that the diagnostic fault cause vector is

\[
Y = (y_1, y_2, \ldots, y_n)
\]

The Fuzzy Diagnosis Model

\[
M(\cdot, +), y_j = \sum_{i=1}^m x_i r_{ij}, j = 1, 2, \ldots, n.
\]

The Fuzzy Diagnosis Principle

With the most subjection degree principle to conclude, it is to say for the given diagnosis reason vector \( Y = (y_1, y_2, \ldots, y_n) \), then \( Y_i \) is the \( t \)th fault reason from \( Y_i = \max\{ y_j | j = 1,2,\ldots,n \} \).

The Fault Diagnosis Case

Fault diagnosis of large lubrication consumption of WP-Jet engine.

The Diagnosis Symptoms

\( x_1 \) (lubricating oil accumulated in the lower nozzle of the turbine), \( x_2 \) (lubricating oil at the inlet of the compressor and the lower skin of the front tank), \( x_3 \) (lubricating oil marks on the tail of the
airplane), \(x_4\) (white smoke from back load reducing chamber), \(x_5\) (exhaust hole of centrifugal ventilator discharges oil beads), \(x_6\) (purple smoke from ejector tube), \(x_7\) (great oil consumption in flight).

**The Fault Reasons**

\(y_1\) (unsealed unidirectional valve for oil intake and outlet of oil pump), \(y_2\) (one-way valve of radiator interface is not sealed), \(y_3\) (membrane rupture of centrifugal valve), \(y_4\) (high pressure in bearing casing chamber), \(y_5\) (low efficiency of three-stage oil return pump).

| Reason j | symptom i | \(y_1\) | \(y_2\) | \(y_3\) | \(y_4\) | \(y_5\) |
|----------|-----------|--------|--------|--------|--------|--------|
| \(x_1\)  |           | 3      | 2      | 0      | 0      | 0      |
| \(x_2\)  |           | 1      | 2      | 0      | 1      | 0      |
| \(x_3\)  |           | 0      | 1      | 3      | 0      | 0      |
| \(x_4\)  |           | 3      | 0      | 0      | 5      | 1      |
| \(x_5\)  |           | 2      | 0      | 3      | 5      | 0      |
| \(x_6\)  |           | 3      | 0      | 3      | 0      | 2      |
| \(x_7\)  |           | 0      | 2      | 0      | 0      | 3      |

**Constructing Diagnosis Matrix**

From experience membership grade \(v_i = \frac{\text{numbers of the } i\text{th symptom belongs to the } j\text{th reason}}{\text{the total numbers of the } i\text{th symptom appears}}\),

We get Empirical Diagnostic Matrix

\[
V = \begin{bmatrix}
0.60 & 0.40 & 0 & 0 & 0 \\
0.25 & 0.50 & 0 & 0.25 & 0 \\
0 & 0.25 & 0.75 & 0 & 0 \\
0.33 & 0 & 0 & 0.56 & 0.11 \\
0.20 & 0 & 0.30 & 0.50 & 0 \\
0.37 & 0 & 0.38 & 0 & 0.25 \\
0 & 0.40 & 0 & 0 & 0.60
\end{bmatrix}
\]

From expert selection, we obtain initial diagnostic matrix

\[
S = \begin{bmatrix}
0.75 & 0.25 & 0 & 0 & 0 \\
0.20 & 0.70 & 0 & 0.10 & 0 \\
0 & 0.20 & 0.80 & 0 & 0 \\
0.25 & 0 & 0 & 0.67 & 0.08 \\
0.12 & 0 & 0.20 & 0.68 & 0 \\
0.22 & 0 & 0.65 & 0 & 0.13 \\
0.22 & 0.10 & 0.10 & 0 & 0.58
\end{bmatrix}
\]

Setting \(W_2 = 0.4\), from \(r_{ij} = 0.6s_{ij} + 0.4v_{ij}\), the fuzzy diagnosis matrix can be obtained as following:
\[
R = \begin{bmatrix}
0.69 & 0.31 & 0 & 0 & 0 \\
0.22 & 0.62 & 0 & 0.16 & 0 \\
0 & 0.22 & 0.78 & 0 & 0 \\
0.28 & 0 & 0 & 0.62 & 0.10 \\
0.15 & 0 & 0.24 & 0.61 & 0 \\
0.28 & 0 & 0.54 & 0 & 0.18 \\
0.13 & 0.22 & 0.06 & 0 & 0.59
\end{bmatrix}
\]

The Fault Diagnosis Analyses

Inputs the symptom vector \( \tilde{X} \), we can get fault reason vector \( \tilde{Y} \) from the model \( \tilde{Y} = \tilde{X} \circ R \), then get the diagnosis results with most subjection degree principle. For example: input \( \tilde{X} = (0 \ 1 \ 1 \ 0 \ 0) \), it is to say \( x_3, x_6 \) occurred, then \( \tilde{Y} = (0.13 \ 0.44 \ 0.84 \ 0 \ 0.59) \) from the most subjection degree principle, we know \( y_j = \max\{y_j | j = 1, 2, ..., 5\} \), the diagnosis results is the third reason: “membrane rupture of centrifugal valve”.

Reference

[1] Song Zhao Gong. Aeroengine representative fault analysis. Beijing: aviation spaceflight university press, 1993.

[2] Wu Jinpei. Fuzzy diagnosis theory and useness, Beijing: Beijing technology press, 1995.

[3] Ding Xiao Li. The selection of highway tunnel pavement evaluation based on the fuzzy mathematics theory under. Highway Engineering, 2017, 42(5).

[4] Zang Chaoping. The faulty diagnosis methods about circumgyrate machine libration, China engineering machine, 1993.

[5] Meng Xiuyan Wang Zhiliang Deng Rongrong, An affective model based on fuzzy theory, Journal of communication and computer, 2008.

[6] Zhang Zhonghai Ou Changjin Ding Jianbo, Research on fuzzy diagnosis method for air brake of diesel locomotive, Mechanical and Electrical engineering magazine, 2011,28(3).