We consider the possibility that a neutron may disappear inside the nucleus, which will demonstrate the existence of baryon violating $\Delta B = 1$ interactions. It has recently been proposed that such a process may have an effect on the free neutron decay lifetime. We evaluate the widths for $n \to \chi$ and $n \to \chi \gamma$, with $\chi$ being a light dark matter particle emitted by a loosely bound neutron in various light nuclei. We find that, assuming a mass $m_\chi$ close to 938 MeV, the obtained width for $n \to \chi$ in $^{11}\text{Be}$ is much larger than the corresponding beta decay width. This suggests a severe limit on the possible decay channel of $n \to \chi \gamma$ for free neutron.

PACS numbers: 95.35.+d, 12.60.Jv 11.30Pb 21.60-n 21.60 Cs 21.60 Ev

I. INTRODUCTION

The neutron is one of the building blocks of matter. Along with the proton and electron it makes up most of the visible universe. Without it, complex atomic nuclei simply would not have formed. Although the neutron was discovered over eighty years ago and has been studied intensively thereafter, its precise lifetime is still an open question [1, 2]. The standard neutron decay mode is $\beta$ decay $n \to p + e^- + \bar{\nu}_e$ described by the matrix element

$$\mathcal{M} = gV_{ud}^{CKM}(1 - \lambda(1 - \gamma_5)n)\bar{\nu}_e(1 - \gamma_5)\nu, \quad \lambda = \frac{gA}{gV}$$

The theoretical estimate for the neutron lifetime thus obtained [3, 4] is given by

$$\tau_n = \frac{4908.7(1.9)}{|V_{ud}^{CKM}|^2(1 + 3\lambda^2)} s$$

The Particle Data Group (PDG) world average [5] of the parameter $\lambda$ is $-1.2723 \pm 0.002$. Adopting the PDG average $V_{ud}^{CKM} = 0.97417$ one gets

$\tau_n$ between 875.3 s and 891.2 s within 3$\sigma$.

There are two qualitatively different types of direct neutron lifetime measurements: bottle and beam experiments. In the first method, bottle experiments, ultracold neutrons are stored in a container for a time comparable to the neutron lifetime. The remaining neutrons that did not decay are counted and fit to an exponential decay, $e^{-t/\tau_n}$, leading to [5]:

$$t_n^{(bottle)} = (879.6 \pm 0.6) s$$

Recent measurements using trapping techniques [6, 7] yield a neutron lifetime within 2.0$\sigma$ of this average.

In the second, beam method, both the number of neutrons $N$ in a beam and the protons resulting from $\beta$ decays are counted, and the lifetime is obtained from the decay rate,

$$\frac{dN}{dt} = -\frac{N}{\tau_n}$$

This yields a considerably longer neutron lifetime; the average from the two beam experiments included in the PDG average [8, 9] is $\tau_n^{(beam)} = (888 \pm 2.0) s$. The discrepancy between the two results is 4.0$\sigma$. 

* e-mail address: vegados@uoi.gr
This suggests that either one of the measurement methods suffers from an uncontrolled systematic error, or there is a theoretical reason of why the two methods give different results, involving very interesting physics.

It is interesting to note that in the beam experiment the life time is longer.

In a recent paper\cite{10} the problem with the experimental measurement of neutron decay lifetime has addressed. They noted that, since in the beam experiment the decay is observed by detecting decay protons the lifetime they measure is related to the actual neutron lifetime by

$$\tau_{n}(\text{beam}) = \frac{\tau_{n}}{\text{Br}(n \rightarrow p + \text{anything})} \quad (5)$$

These authors suggest that the discrepancy can be explained by considering an extra channel in the beam experiment, which involves the emission of a dark fermion particle $\chi$, which goes undetected. Then they suggest a model which can give a branching ratio of 1\% to this new channel, while the standard channel covers only 99\%, thus settling the issue. It is, however, in their treatment necessary for this new particle to be a neutral Dirac like a particle with a mass slightly lighter the neutron. If it were a Majorana particle this mechanism could led in neutron-antineutron oscillations and could be in conflict with the non observation of such oscillations.

For a free neutron this cannot occur without the emission of another particle, e.g. a photon to conserve energy momentum, which at the same time provides an interesting experimental signature for the suggested mechanism.

Since the emitted particle is assumed not to carry any baryon number, this scenario is very interesting, since if true, it will demonstrate the existence of baryon number violating $\Delta \beta = 1$ interactions. This very interesting scenario seems, however, to be excluded from astrophysical data involving neutron stars \cite{11}, \cite{12}, \cite{13}. Such arguments, however, do not apply if there is a Coulomb repulsion between the dark fermions mediated by a dark photon \cite{14}. In any case the neutron star arguments apply only to neutrons bound by gravity and not by strong interactions.

The neutron in the nucleus seems to behave differently, due to the nuclear binding. In certain cases it decays like in the $\beta$ decay, but the produced proton cannot escape due to nuclear binding, while a daughter nucleus appears with its charge increased by one unit is formed instead. Decays of well-bound nucleons (neutrons) into invisible particles have been searched by measuring $\gamma$ rays long time ago \cite{15,16}. In the model considered above the produced dark matter particle $\chi$, interacting very weakly, can escape. In this case energy-momentum can be conserved without the emission of additional particles, like the photon, and the decay width is expected to be much larger.

In this work we will consider the possibility that such a process may occur inside the nucleus:

$$A(N, Z) \rightarrow A(N-1, Z)^* + \chi \quad (6)$$

where $\chi$ is the dark matter fermion. Since $\chi$ is also supposed to be produced in the decay of the free neutron, it must be lighter than the neutron. In fact to explain the neutron life discrepancy it was necessary to assume \cite{10} its mass must be less than but very close to that of the neutron $939.565:\ $937.900 \text{ MeV} < m_\chi < 938.783 \text{ MeV}. \quad (7)$

where the lower and upper bounds come from the stability of $^9\text{Be}$ and the fact that the decay $\chi \rightarrow p + e^- + \bar{\nu}_e$ is forbidden, respectively.

So one has to search for nuclear systems that the neutron is loosely bound so that process\cite{6} can take place. If $m_\chi$ takes the lowest value an extra energy of 1.665 MeV is available compared to neutron emission.

Such nuclei actually exist. They are known as halo nuclei. They have a very peculiar structure. One or two neutrons are orbiting around the rest of the nucleons along large, by nuclear standards, orbits typically occupying loosely bound orbits in the next harmonic oscillator shell. These are odd mass nuclei, which can decay by neutron emission to typical light mass even "p-shell" nuclei. So, in principle, process\cite{6} can occur, if, as in the case considered above, the free neutron can decay to the channel $\chi$. There may be a number of targets with the right binding energy so that even excited states of the final nucleus can be reached. The simultaneous emission of photons is not necessary, but it may occur if $m_\chi < m_n$, but it is expected to be suppressed. Good candidates are the halo nuclei $^{11}\text{Be}$, $^{15}\text{C}$ and $^{19}\text{C}$. $^{11}\text{Li}$ may be considered but it has the disadvantage of beta delayed decay to $^9\text{Li}+d$, suggesting that it is a two neutron halo nucleus.

Before proceeding to the study of the baryon number violating process we will summarize sum facts about the nuclei, which can serve as targets.
One good candidate studied experimentally, see, e.g., ref. [17] and references there in, is \(^{11}\)Be. Its ground state is \(^{1/2}+\) and the first excited state is \(^{1/2}−\), which an unusual ordering from the point of view of the simple shell model. It seems that the \(^{1/2}+\) is composed of a major component of the form \(^{10}\)Be \(g.s \otimes 2s_{1/2}(n)\) and, possibly, of another one of the type \(^{10}\)Be\(^{2+} \otimes 1d_{5/2}(n)\). The first can decay into the ground state (\(g.s\)) of \(^{10}\)Be and the second to the first excited \(^{2+}\) state at excitation energy of 0.32 MeV.

On the theoretical side the situation is not clear. Variational Model approach [18] and models which vary the single particle energies via vibrational and rotational core couplings reproduce this level inversion in a systematic manner. Common to the success of these models is the inclusion of core excitation. Ab initio No-Core Shell Model calculations [19] have been unable to reproduce this level inversion, though a significant drop in the energy of the \(^{10}\)Be halo states show a considerable overlap with a valence neutron coupled to an excited \(^{10}\)Be\(^{(2+)}\) core, in addition to the \(^{10}\)Be\(^{(0+)}\)gs component. Despite decades of study, the extent of this mixing is not well understood, with both structure calculations and the interpretation of experimental results ranging from a few percent to over 50 percent core-excited component.

Experimentally, it is possible to gain quantitative insight into the mixed configuration of a state by studying reactions which provide observables that are sensitive to different components of the nuclear wave function [17]. By comparing the measured differential cross sections to those calculated theoretically, a spectroscopic factor \(S\) can be extracted, which reflects in an admittedly model-dependent way the extent to which the studied nuclear state resembles that used in the calculation. Spectroscopic factors reported from numerous direct-reaction studies, including one-neutron transfer [20–23], two-neutron transfer [24], neutron knockout [25], Coulomb breakup [26] , and re-analysis of the neutron-transfer data [27, 28], are summarized in Figure 1 of ref. [17], along with those from theoretical calculations. The average value for both the ground state and the excited \(^{2+}\) state is around 50%.

The next in importance is \(^{15}\)C. This has a \((1/2)^+\)gs, presumably of the form \(^{14}\)C \(g.s \otimes 2s_{1/2}(n)\) similar to the case of \(^{11}\)Be, discussed above.

II. THE FORMALISM

A. Neutron bound wave functions

We will consider the neutron as a bound state of three quarks in a color singlet s-state. The orbital part of the form:

\[
\Psi(R, \xi, \eta) = \Phi(R) \psi_0(\xi) \psi_0(\eta). \tag{8}
\]

The \(\psi_0(\xi)\) and \(\psi_0(\eta)\) are bound wave functions dependent on the relative internal variables, which, for simplicity, we will assume to be of the 0\(s\) harmonic oscillator type so that one can easily separate out the internal coordinates. Thus

\[
\psi_0(\xi) = \sqrt{\frac{1}{\pi \sqrt{\pi} (b_N)_{3/2}}} e^{-\frac{\xi^2}{2 b_N^2}}, \quad \xi = \frac{1}{\sqrt{2}}(x_1 - x_2). \tag{9}
\]

Similarly

\[
\psi_0(\eta) = \sqrt{\frac{1}{\pi \sqrt{\pi} (b_N)_{3/2}}} e^{-\frac{\eta^2}{2 b_N^2}}, \quad \eta = \frac{1}{\sqrt{6}}(x_1 + x_2 - 2x_3). \tag{10}
\]

The center of mass coordinate is taken to be:

\[
R = \frac{1}{\sqrt{3}}(x_1 + x_2 + x_3) \tag{11}
\]

\(b_N\) is the nucleon size parameter related to the nucleon radius \(R_N\) via the relation \(R_N^2 = (3/2)b_N^2\) with \(x_i, i = 1, 2, 3\) the quark coordinates. The functions \(\phi\(\xi\)\) and \(\phi\(\eta\)\) are normalized in the usual way.
The amplitude for neutron decay to a dark matter fermion

The process derived from the model of [10] is exhibited in Fig. 1. The amplitude associated with this process takes the form:

\[ \mathcal{M} = \frac{\lambda_4 \lambda_7}{m_\Phi^2} \psi_{0_3}(\xi) \psi_{0_3}(\eta) (2\pi)^3 \delta(p_1 + p_2 + p_3 - q), \]  

(12)

where \( p_i, i = 1, 2, 3 \) are the quark momenta and \( q \) the momentum of the outgoing dark matter particle. The Fourier transform of the amplitude in coordinate space becomes:

\[ \mathcal{M} = \frac{1}{(2\pi)^6} \frac{\lambda_4 \lambda_7}{m_\Phi^2} \psi_{0_3}(\xi) \psi_{0_3}(\eta) \int d^3 p_1 \int d^3 p_2 \int d^3 p_3 e^{i\mathbf{p}_1 \cdot \mathbf{x}_1} e^{i\mathbf{p}_2 \cdot \mathbf{x}_2} e^{i\mathbf{p}_3 \cdot \mathbf{x}_3} \delta(p_1 + p_2 + p_3 - q) \]

or

\[ \mathcal{M} = \frac{\lambda_4 \lambda_7}{m_\Phi^2} \psi_{0_3}(\xi) \psi_{0_3}(\eta) \delta(x_1 - x_3) \delta(x_2 - x_3) e^{i\mathbf{q} \cdot \mathbf{x}_3} \]

or

\[ \mathcal{M} = \frac{\lambda_4 \lambda_7}{m_\Phi^2} \psi_{0_3}(\xi) \psi_{0_3}(\eta) \delta(\sqrt{2} \xi) \delta(\sqrt{2} \eta) e^{i\mathbf{q} \cdot (-2\eta + R)}. \]

Thus we get:

\[ \mathcal{M} = \sqrt{3} \frac{1}{2\sqrt{2}} \frac{3\sqrt{3} \lambda_4 \lambda_7}{m_\Phi^2} |\psi(0)|^2 e^{i\frac{\mathbf{q} \cdot \mathbf{R}}{\sqrt{3}}}, \]

where the first factor is a color factor. Finally

\[ \mathcal{M} = \kappa_{\text{scale}} e^{i\mathbf{q} \cdot \mathbf{R}}, \quad \kappa_{\text{scale}} = \frac{9}{8}, \quad \mathbf{e} = |\psi(0)|^2 \frac{\lambda_4 \lambda_7}{m_\Phi^2}, \]

where

\[ |\psi(0)|^2 = \frac{1}{\pi \sqrt{\pi b_N^3}}, \quad b_N = \frac{\sqrt{2}}{3} R_N \]
is the baryon density at the origin. For a typical value of $R_N=0.8$ fm we obtain a value of $|\psi(0)|^2=0.005$ GeV$^5$. This is a bit smaller than the value of $\beta$ employed in the free neutron decay, i.e. $\beta=0.014$ GeV$^3$ obtained recently from lattice computation of proton decay matrix elements [29]. We will adopt the value of $b_N=0.5$ fm to be consistent with the larger value of $\beta=0.014$ GeV$^3$. Thus

$$\kappa_{\text{scale}} = \frac{9}{8} \frac{1}{\pi^\frac{1}{2} b_N (b_N m_\Phi)^2 \lambda_\psi \lambda_\chi}.$$  

(13)

We find it convenient to indicate the size of the baryon number violating neutron conversion to a dark matter fermion by the dimensionless quantity $s = \left( \frac{\lambda_\psi \lambda_\chi}{(b_N m_\Phi)^2} \right)^2$ instead of the dimensionful parameter $\varepsilon^2$ used in [10]. Using the value of $(\lambda_\psi \lambda_\chi)/m_\Phi^2 = 6.7 \times 10^{-6}$ TeV$^{-2}$, i.e. the one employed in the free nucleon exotic decay to dark matter [10], we find that both are indeed very small:

$$s \approx 1.5 \times 10^{-24} \text{ or } \varepsilon^2 = 8.8 \times 10^{-27} \text{ GeV}^2$$

Either one can be used in bound as well as in free nucleon decay.

III. EXPRESSION FOR THE DECAY WIDTH

The decay width is given by the expression:

$$d\Gamma = \frac{1}{(2\pi)^3} d^3 q d^3 p_A \delta(q+p_A) \delta(\Delta - E_x + m_n - m_\chi - T) |\langle \text{ME} \rangle|^2$$  

(14)

where $p_A$ and $q$ are the momenta of the final nucleus and the outgoing dark matter particle $\chi$ respectively, with the latter’s mass being $m_\chi$ and its kinetic energy $T$. $\Delta$ is the difference of the ground state energies of the nuclei involved with the neutron mass separated out and $E_x$ the excitation energy of the populated final nuclear state. Finally ME (matrix element) is the invariant amplitude which will be given in the appendix. Thus

$$\Gamma = \frac{1}{\pi} \sqrt{2m_\chi(\Delta - E_x + m_n - m_\chi) m_\chi |\langle \text{ME} \rangle|^2} \approx \frac{1}{\pi} \sqrt{2m_\chi(\Delta - E_x + m_n - m_\chi) m_n |\langle \text{ME} \rangle|^2}$$  

(15)

or

$$\Gamma = \frac{81 a_N^3}{\pi^3 b_N^2} \left( \frac{\lambda_\psi \lambda_\chi}{(b_N m_\Phi)^2} \right)^2 \sqrt{2m_\chi(\Delta - E_x + m_n - m_\chi) m_n (A(N-1);A(N,Z))^2} \frac{f_{j,\ell}^2}{2j+1} (F_{n,\ell}(\alpha))^2,$$  

(16)

with $\alpha = \sqrt{2/3} \sqrt{2m_\chi(\Delta + m_n - m_\chi - E_x) a_N}$

The function $f_{j,\ell}^2$ is trivial, i.e. $f_{j,\ell}^2/(2j+1) \approx 1/2$. The form factors have been obtained analytically in the appendix but we illustrate their behavior in Fig. 2. We note that in the interesting case for $2s$ and $1d$ orbitals, the form factors are suppressed compared to unity at low values of $\alpha$. The needed nuclear CFP’s can be calculated in a shell model treatment or perhaps they can be extracted from other experiments as mentioned earlier.

IV. SOME RESULTS

Let us now assume a nuclear size parameter $a_N=1.5$ fm and a nucleon size parameter of $b_N=0.5$ fm. Let us also take the value of $(\lambda_\psi \lambda_\chi)/m_\Phi^2 = 6.7 \times 10^{-6}$ TeV$^{-2}$, i.e. the one employed in the sister free nucleon exotic decay to dark matter [10]. In the special case of halo nuclei considered here $\Delta = -B$ with $B > 0$ the binding energy of the decaying neutron, i.e. the process can take place so long as $B \leq m_n - m_\chi - E_x$, where $E_x$ is the excitation energy of the A(N-1,Z) system. Then Eq. (16) becomes:

$$\Gamma = 4.6 \times 10^{-15} \text{eV} \sqrt{\frac{m_n - B - m_\chi - E_x}{1 \text{ MeV}}} g, \ g = (A(N-1);A(N,Z))^2 f_{j,\ell}^2 (F_{n,\ell}(\alpha))^2$$  

(17)
FIG. 2: The form factors \((F_{n\ell}(\alpha))^2\) for 0s, 0p, 0d and 1s orbitals are plotted, indicated by thick solid, thin solid, short dashed and long dashed lines respectively. The vertical line corresponds to value of \(\alpha\) associated with available energy of 1 MeV

with

\[
\alpha = \sqrt{\frac{2}{3}} \sqrt{\frac{2m_n (m_n - B - m_\chi - Ex) a_N}{\hbar c}}
\]

(18)

Let us consider the invisible decay of a loosely-bound neutron in a nucleus. There are several nuclei to be considered as given in Table I. The deuteron and \(^9\)Be are stable isotopes with the half-life much longer than that of the universe. Then their small limits on the widths excludes the n decays to \(\chi\) with \(m_\chi \leq 937.3\) and \(m_\chi \leq 937.9\) MeV, respectively [10]. Then the \(\chi\) mass region is limited in a narrow region of \(937.9\) MeV \(\leq m_\chi \leq 939.56\) MeV.

We are now in position to estimate the widths in the case of the nuclei of interest. The relevant nuclear parameters are contained in the quantity \(g\) and are presented in Table I. Regarding the nuclear input in the case of \(^{11}\)Be we used an average spectroscopic factor \(\langle A(N-1,J_f;j(n);A(N,Z))^2 = 0.5\) both for transitions to the ground state as well the 0.38 MeV excited state [17]. In other words we assume that the \(^{11}\)Be state is composed of two equal parts of the form \(0^+(N-1,Z) \otimes 2s_1/2(n)\) and \(2^+(N-1,Z) \otimes 1d_5/2(n)\). The first will lead to the ground state, while the second to the excited \(2^+\) state. For the other nuclei a value of 0.88 was employed. The obtained results for a dark mater particle mass of 937.9 MeV are shown in Table I. The suppression of the transition to the \(2^+\) excited state of \(^{10}\)Be is due to the suppression of the d-wave form factor.

The decay half-life of \(^{11}\)Be is measured to be \(13.81 \pm 0.08\) s by counting the decay particle as a function of the time [30]. The width is \(3.3 \times 10^{-17}\) eV, which is much shorter than the evaluated width given by [17], unless the DM mass is very close to \(m_n - B_n\), as shown in Fig. 3.

Thus the decay to the DM with \(m_\chi \leq 939.06\) MeV is excluded. Note that this limit is very insensitive to the nuclear structure coefficient \(g\) given in Eq. (17). The lighter \(\chi\) with \(m_\chi \leq 937.9\) MeV is excluded by the long-lived \(^9\)Be [10]. On the other hand the heavier DM with \(m_\chi \geq 938.78\) MeV is excluded since the decay of \(\chi \to p + e^- + \bar{\nu}_e\) is forbidden [10].

Thus the decay of the bound neutron is excluded, and thus the suggested decay of the free neutron to \(\chi + \gamma\) is not likely. Note that \(\chi\) is assumed not to decay to \(p + e^- + \bar{\nu}_e\) [10] and thus, if we allow unstable \(\chi\), the mass region of 939.565-939.06 is not excluded.

The measured width of \(1.9 \times 10^{-16}\) eV for \(^{15}\)C also excludes the DM with \(m_\chi \leq 938.3\) MeV. The evaluated width is about 0.1 of the measured width of \(1.0 \times 10^{-14}\) eV for \(^{19}\)C with \(B_n=0.16\) MeV. Then one may study the possible DM with \(m_\chi \approx 939\) MeV. Recently n decays in nuclei are discussed [31].

As we have mentioned the radiation of a photon by the bound decaying neutron is not needed. We have,
FIG. 3: The experimental decay width $Γ_β$ for $^{11}$Be (thick line), the evaluated via Eq. (17) width $Γ_χ$ (squares), and the excluded mass-regions with A: by the $^{11}$Be decay width, B: by the decay to proton, and C: by the $^9$Be, respectively.

TABLE I: The expected widths for neutron decay into a dark matter particle $χ$ inside a nucleus for $m_χ = 937.9$ MeV. S stands for stable isotope

| nucleus | $B_{n}[\text{MeV}]$ | $J^π$ | $j_n$ | $T_{1/2}[s]$ | $g$ | $Γ[10^{-15}\text{eV}]$ |
|---------|----------------------|-------|-------|--------------|-----|------------------|
| $^2$H   | 2.225                | $1^+$ | $1s_{1/2}$ | $S$ | $-$ | $0$              |
| $^9$Be  | 1.665                | $(3/2)^-$ | $1p_{3/2}$ | $S$ | $-$ | $0$              |
| $^{11}$Be | 0.504              | $(1/2)^+$ | $2s_{1/2}$ | 13.8 | 0.150 | 0.75          |
| $^{11}$Be | 0.504              | $(1/2)^+$ | $1d_{5/2}$ | 13.8 | $6.05 \times 10^{-4}$ | $2.5 \times 10^{-3}$ |
| $^{13}$C | 1.218                | $(1/2)^+$ | $2s_{1/2}$ | 2.45 | 0.282 | 0.86          |
| $^{19}$C | 0.16                 | $(1/2)^+$ | $2s_{1/2}$ | 0.046 | 0.257 | 1.45          |

however, estimated in the appendix the branching ratio for such a process. It can be cast in the simple form:

$$\frac{Γ(J_i → J_f χ γ)}{Γ(J_i → J_f χ)} \approx 5.0 \times 10^{-7} ||σ||^2 \frac{(m_n - B - m_χ - E_x)^2}{1\text{MeV}^2}. \tag{19}$$

For the most interesting case of s-state neutron we obtain:

$$\frac{Γ(J_i → J_f χ γ)}{Γ(J_i → J_f χ)} \approx 3.0 \times 10^{-6} \frac{(m_n - B - m_χ - E_x)^2}{1\text{MeV}^2}. \tag{20}$$

The width for the radiative free neutron decay is given [10] by:

$$Γ(n → γχ) = \frac{g^2}{8\pi} 4\pi α \left(1 - \frac{m_χ^2}{m_n^2}\right)^3 \frac{e^2}{(m_n - m_χ)^2} m_n. \tag{21}$$
where $\varepsilon = 9.38 \times 10^{-14}$ GeV. Here $\alpha$ is the fine structure constant, not to be confused with that of Eq. (18). Then one obtains:

$$\Gamma(n \to \gamma\chi) \approx \frac{g^2}{8\pi} \frac{8\varepsilon^2}{m_n} (m_n - m_\chi) \approx 4 \times 10^{-27} (m_n - m_\chi),$$

(22)

which almost the same with the $7 \times 10^{-27}$ MeV estimated in ref. [10] to fit the free neutron life time. We thus see that the radiative width for a bound neutron is of the same order with that estimated for a free neutron.

V. DISCUSSION AND REMARKS

From our estimates in table I it appears that the expected widths for baryon number violating neutron decay to a dark matter particle inside the nucleus are much larger than that expected in the case of the free neutron [10], provided that its mass is around 938 MeV. Widths such as those produced here seem to be detectable in future nuclear physics experiments. The spontaneous production of a low energy photon, which is not required here, but could offer a good experimental signature. It is, however, expected to be of the same order as that estimated in the case of a free neutron and, thus, quite hard to detect.

It is finally remarked that the experimental width for $^{11}$Be is smaller by an order of magnitude than that evaluated for $n \to \chi$ with $m_\chi \leq 939$ MeV in $^{11}$Be, and thus the free neutron decay of $n \to \chi \gamma$ with $m_\chi = 937.90$ Mev - 938.78 of ref. [10] can hardly be the major channel to account for the neutron lifetime discrepancy between the ”’bottle’” and ”’beam’” experiments.

[1] F. E. Wietfeldt and G. L. Greene, Rev. Mod. Phys 83, 1173 (2011).
[2] G. L. Greene and P. Geltenbort, Scientific American 314, 36 (2016).
[3] W. J. Marciano and A. Sirlin, Phys. Rev. Lett. 96, 032002 (2006), arXiv:hep-ph/0510099 [hep-ph].
[4] J. C. Hardy and I. S. Towner, Phys. Rev. C 91, 025501 (2015), arXiv:1411.5987 [nucl-ex].
[5] C. Patrignani et al. (Particle Data Group), Chin. Phys. C 40, 1000001 (2016).
[6] A. P. Serebrov et al., neutron Lifetime Measurements with the Big Gravitational Trap for Ultracold Neutrons, arXiv:1712.05663 [nucl-ex].
[7] R. W. Pattie et al., measurement of the Neutron Lifetime Using an Asymmetric Magneto-Gravitational Trap and In Situ Detection, arXiv:1707.01817 [nucl-ex].
[8] J. Byrne and P. G. Dawber, Europhys. Lett 33, 187 (1996).
[9] A. T. Yue, M. S. Dewey, D. M. Gilliam, G. L. Greene, A. B. Laptev, J. S. Nico, W. M. Snow, and F. E. Wietfeldt, Phys. Rev. Lett 111, 222501 (2013), arXiv:1309.2623 [nucl-ex].
[10] B. Fornal and B. Grinstein, Dark Matter Interpretation of the Neutron Decay Anomaly, arXiv:1801.01124.
[11] T. F. Motta, P. A. M. Guichon, and A. W. Thomas, J. Phys. G 45, no. 5, 05LT01 (2018), arXiv:1802.08427 [nucl-th].
[12] S. R. D. Z. D. McKeen, Ann E. Nelson, Neutron stars exclude light dark baryons, arXiv:1802.08244 [hep-ph].
[13] G. Baym, D. H. Beckand, P. Geltenbort, and J. Shelton, Coupling neutrons to dark fermions to explain the neutron lifetime anomaly is incompatible with observed neutron stars, arXiv:1802.08282 [hep-ph].
[14] J. M. Cline and J. M. Cornell, Dark decay of the neutron, arXiv:1803.04961 [hep-ph].
[15] H. Ejiri, Phys. Rev. C 48, 48 (1993).
[16] R. Hazama, H. Ejiri, K. Fushimi, and H. Ohsumi, Phys. Rev. C 49, 2407 (1994).
[17] K. Schmitt et al., Phys. Rev. Lett 108, 192701 (2012), arXiv:1203.3081 [nucl-ex].
[18] T. Otsuka, N. Fukunishi, and H. Sagawa, Phys. Rev. Lett. 70, 1385 (1993).
[19] Forssen, P. Navratil, W. E. Ormand, and E. Caurier (Particle Data Group), Chin. Phys. C 71, 9412049 (2005), arXiv:nucl-th/041204.
[20] D. Auton, Nucl. Phys. A 157, 305 (1970).
[21] B. Zwieglinski et al., Nucl. Phys. A 315, 124 (1979).
[22] J. S. Winfield et al., Nucl. Phys. A 683, 48 (2001).
[23] S. Fortier et al., Phys. Lett. B 461, 22 (22).
[24] G.-B. Liu and H. T. Fortune, Phys. Rev. C 42, 167 (1990).
[25] T. Aumann et al., Phys. Rev. Lett 84, 35 (2000).
[26] R. Palit et al., Phys. Rev. C 68, 034318 (2003).
[27] N. K. Timofeyuk and R. C. Johnson, Phys. Rev. C 59, 1545 (1999).
particle is an spin state \( m_a \) where \( m_e \) will be defined below.

On the possibility to observe neutron dark decay in nuclei [31] M. Pftzner and K. Riisager,

[28] A. Deltuva, Phys. Rev. C 79, 054603 (2009).
[29] E. S. Y. Aoki, T. Izubuchi and A. Soni, Phys. Rev. D 96, 014506 (2017), arXiv:1705.01338 [hep-lat].
[30] D. Alburger and G. Engelfertink, Phys. Rev C 2, 1594 (1970).
[31] M. Pftzner and K. Riisager, On the possibility to observe neutron dark decay in nuclei, arXiv:1803.01334 [nucl-ex].

VI. APPENDIX EVALUATION OF THE NUCLEAR MATRIX ELEMENT

We need the structure of the initial \( A(N, Z) \) nucleus and the structure of the \( A(N - 1, Z)^+ \) final nucleus. The essential information is contained in the CFP (coefficient of fractional parentage)

\[
\langle A(N - 1,) J_f; j(n); A(N, Z) \rangle
\]

which separates out the interacting neutron indicated by the quantum numbers \( n, \ell, j \) and essentially gives the overlap involving the non interacting nucleons or spectroscopic factor. This can be obtained by a nuclear structure calculation or in some cases extracted from experiment in reaction involving a knock out neutron. Then the matrix element involved is:

\[
ME = \kappa_{\pi a} \langle A(N - 1,) J_f; j(n); A(N, Z) \rangle \langle J_f M_f - m j m_j | J_l M_l \rangle m_e \tag{23}
\]

where \( m_e \) will be defined below.

A. The elementary transition matrix element

Let us suppose that the decaying nucleon is in a shell model state \( n, \ell, j m(\mathbf{R}) \) state and the outgoing dark matter particle is an spin state \( m_e \). We must first evaluate the me

\[
\text{me} = \langle n, \ell, j m | e^{-i \mathbf{R}} | m_e \rangle = 4\pi \sum_{\ell', m'} (i)^\ell \langle n, \ell, j m | j_{\ell'} \left( q r / \sqrt{3} \right) Y_{m'}^\ell (\hat{r}) m_e \rangle \left( Y_{m'}^\ell (\hat{q}) \right)^* \]

where \( j_{\ell'}(z) \) is a spherical Bessel function and \( Y_{m'}^\ell (\hat{r}) \) the usual spherical harmonic. Using the standard angular momentum re-coupling we obtain a contribution only when \( \ell = \ell', m' = m_e - m \). i.e.:

\[
\text{me} = 4\pi f_{j,\ell} 2\sqrt{2} a_N \sqrt{a_N} F_{n,\ell} \left( \sqrt{2/3} q a_N \right) \langle j, m, \ell, m_e - m | 1/2 m_s \rangle (-1)^\ell \left( Y_{m_e - m}^\ell (\hat{q}) \right)^* ,
\]

\[
f_{j,\ell} = \begin{cases} \sqrt{\ell + 1}, & j = \ell + 1/2 \\ \sqrt{\ell}, & j = \ell - 1/2 \end{cases}
\]

\( a_N \) is the nuclear harmonic oscillator parameter and

\[
F_{n,\ell} \left( \sqrt{2/3} q a_N \right) = \int_0^\infty dx \psi_{n,\ell}(x) j_{\ell} \left( q \sqrt{2} a_N x / \sqrt{3} \right) , \quad x = \frac{R}{a_N \sqrt{2}}
\]

(dimensionless “form factor”).

\[
F_{00}(\alpha) = \frac{1}{2} \sqrt{\pi} e^{-\frac{\alpha^2}{4}}
\]

\[
F_{00}(\alpha) = -\frac{1}{2} \left( \alpha^2 - 2 \right) \left( 1 - \alpha F \left( \frac{\alpha}{2} \right) \right) - 2 \alpha F \left( \frac{\alpha}{2} \right) + 1 - \frac{2 \sqrt{\frac{\alpha}{4}}}{\sqrt{\pi}}
\]

\[
F_{0d} = -\frac{\sqrt{\pi} e^{-\frac{\alpha^2}{4}} \alpha^2 (\alpha^4 - 16\alpha^2 + 20)}{8 \sqrt{15}}
\]

\[
F_{1s} = -\frac{\sqrt{\pi} e^{-\frac{\alpha^2}{4}} (\alpha^2 - 3)}{2 \sqrt{6}}
\]
B. The invariant amplitude squared

The next step is to obtain \(|\text{ME}|^2\) average over the initial m-sub-states and sum over the final m-sub-states. The result is

\[
\langle|\text{ME}|^2\rangle = \left(\kappa_{\text{scale}} \langle A(N-1,J_f;j(n);A(N,Z)\rangle 4\pi f_{J_fJ_s}2\sqrt{2}a_N\sqrt{a_N}\frac{1}{J_s+j}\sum_{M_s,m_s}|J_fM_f\rangle - m jm|J_sM_s\rangle)^2 \langle j(m,\ell,m_s-m)1/2m_s\rangle^2 Y_{m_s-m}^\ell(\hat{q})(Y_{m_s}^\ell(\hat{q}))^* \right) ^2
\]

\[
= \left(\kappa_{\text{scale}} \langle A(N-1,J_f;j(n);A(N,Z)\rangle 4\pi f_{J_fJ_s}2\sqrt{2}a_N\sqrt{a_N}\frac{1}{J_s+j}\sum_{M_s,m_s}|J_fM_f\rangle - m jm|J_sM_s\rangle)^2 \langle j(m,\ell,m_s-m)1/2m_s\rangle^2 \langle \sqrt{2/3}q(n)\rangle \right) ^2 \frac{2}{2J+1} \frac{1}{4\pi} \tag{24}
\]

C. Neutron decay with photon emission

We will now consider that the neutron before its decay emits a photon via its magnetic moment in a two step process, i.e.

\[
J_i \lim_{n \to \gamma n} J_n \lim_{n \to \chi f}
\]

In this case we have:

\[
\text{ME}^2 = \text{ME}_{n \to \gamma n}^2 \frac{1}{\delta E_n} \text{ME}_{n \to \chi}^2
\]

One finds

\[
\text{ME}_{n \to \gamma n}^2 = \left(\frac{g_s}{2}\right)^2 4\pi \alpha \frac{a^2}{3}\langle|\sigma||\rangle^2
\]

where \(k\) is the photon momentum and \(|\langle|\sigma||\rangle|\) is the reduced ME of the spin normalized to \(\sqrt{6}\) for s-states. For the d-state we have \(d_{s/2} \to d_{s/2} = \sqrt{2}\) and \(d_{s/2} \to d_{3/2} = -\sqrt{2}\).

Here \(\text{ME}_{n \to \chi}^2\) is the matrix element calculated above without the photon emission. We thus find for the differential width for photon emission:

\[
d\Gamma = \text{ME}_{n \to \chi}^2 \frac{1}{4\pi^3} \left(\frac{g_s}{2}\right)^2 4\pi \alpha \langle|\sigma||\rangle^2 \frac{1}{2\kappa^2} \frac{1}{m_n} \frac{2}{m_{n-B-m_{\chi}-E_x-k}} \frac{1}{2k} \sqrt{2m_{\chi}(n_B+m_{\chi}-E_x-k)}
\]

The total photon width for the neutron decay for photon emission is given by:

\[
\Gamma(J_i \to J_f\gamma) = \text{ME}_{n \to \chi}^2 \frac{1}{4\pi^3} \left(\frac{g_s}{2}\right)^2 4\pi \alpha \langle|\sigma||\rangle^2 \frac{1}{2\kappa^2} \frac{1}{m_n^2} \frac{32}{3} \frac{315}{m_{n-B-m_{\chi}-E_x}^2}
\]

we thus obtain:

\[
\frac{\Gamma(J_i \to J_f\gamma)}{\Gamma(J_i \to J_f\chi)} = \frac{1}{4\pi^2} \left(\frac{g_s}{2}\right)^2 4\pi \alpha \langle|\sigma||\rangle^2 \frac{1}{2\kappa^2} \frac{32}{3} \frac{315}{m_{n-B-m_{\chi}-E_x}^2}
\]

Or

\[
\frac{\Gamma(J_i \to J_f\gamma)}{\Gamma(J_i \to J_f\chi)} \approx 5.0 \times 10^{-7} \langle|\sigma||\rangle^2 \frac{(m_{n-B-m_{\chi}-E_x})}{1\text{MeV}^2}
\]

For the most interesting case of s-state neutron we obtain:

\[
\frac{\Gamma(J_i \to J_f\gamma)}{\Gamma(J_i \to J_f\chi)} \approx 3.0 \times 10^{-6} \frac{(m_{n-B-m_{\chi}-E_x})}{1\text{MeV}^2}
\]