Effect of Viscoelastic Layer on Contact Fatigue Damage Accumulation in Rolling Friction

A R Meshcheryakova\textsuperscript{1,2}

\textsuperscript{1} Ishlinsky Institute for Problems in Mechanics RAS, Prospekt Vernadskogo, 101-1, Moscow, 119526, Russia
\textsuperscript{2} Moscow Institute of Physics and Technology (National Research University), 9 Institutskiy per., Dolgoprudny, Moscow Region, 141701, Russia
mif-almira@yandex.ru

Abstract. The cyclic rolling of an elastic sphere along a viscoelastic layer bonded to an elastic half-space is considered. For known distributions of normal and shear stresses, the internal stress state of the elastic half-space is calculated. The damage accumulation within the framework of the linear damage summation model was considered. The amplitude values of principal shear stress were calculated for various values of relative slippage and friction coefficient.

1. Introduction
Under cyclic loads, an inhomogeneous cyclic field of internal stresses arises in elastic material. For example, in railway transport the contact fatigue damage of rails is one of the most common defects of rails. The study of mechanism of the defect appearance in wheel-rail system and modeling of this process were conducted in [1], [2]. As a result, damage of the material accumulates in the subsurface layer [1]. The results of an analysis of the damage accumulation rate at wheel-rail rolling contact were presented in [2]. The calculation of the damage accumulation for elastomers was carried out in [3-5]. The effect of the thickness of viscoelastic layer, relative slippage and friction coefficient on the normal and shear stresses distribution under rolling conditions were calculated in [6]. For analysis of damage accumulation in wheel-rail contact model a macroscopic approach was used in [1, 2]. The friction modifiers and lubrication can be used in order to decrease the damage of rails. To study the influence of friction modifier on wheel-rail contact interaction the models with viscoelastic coated bodies are considered.

For an elastic sphere rolling over a viscoelastic layer bonded to an elastic half-space the normal and shear stress distribution were calculated for various surface layer properties and relative slippage in [6]. In addition to this problem the distribution of internal stresses in the half-space and the damage accumulation were calculated in this study.

2. Problem formulation
Let us consider the rolling of a sphere with a constant velocity $V$ over an elastic half-space bonded to the viscoelastic layer. A constant normal force $P$ is applied to the sphere. The contact scheme is shown in figure 1.
The problem is considered in a moving system of coordinates \((O, x, y, z)\), which is associated with a fixed system of coordinates \((O', x', y', z')\) as follows:

\[
x' = x + Vt, \quad y' = y, \quad z' = z
\]

(1)

The compliance of the viscoelastic layer in the normal direction is described by the modified Winkler model. In the tangential direction the mechanical properties of the layer are modeled by the Kelvin body. In the moving system of coordinates, the relationship between displacements and stresses of the viscoelastic layer is presented as follows:

\[
v(x, y) = h \left[ \frac{p(x, y)}{E_3^p} \right]^k, \tag{2}
\]

\[
u(x, y) - T_\varepsilon \frac{\partial u(x, y)}{\partial x} = \frac{h}{E_3^p} \left[ \tau(x, y) - T_\sigma V \frac{\partial \tau(x, y)}{\partial x} \right], \tag{3}
\]

where \(v(x, y), u(x, y)\) – normal and shear displacements of the viscoelastic layer, \(p(x, y), \tau(x, y)\) – normal and shear stresses, \(h\) – the layer thickness, \(E_3^p, E_3^t\) – the longitudinal modulus of elasticity of the layer in the normal and tangential directions, \(m\) – the modified Winkler model parameter \((0.1 \leq m \leq 1)\), \(T_\varepsilon\) and \(T_\sigma\) are the retardation and relaxation times, respectively.

In rolling, the contact region of the sphere and the base consists of stick and slip subregions, which boundaries are unknown in advance. In the slip subregion the relation between the normal and shear stresses is described by the Coulomb law:

\[
\left| \tau(x, y) \right| = \mu p(x, y), \quad (x, y) \in \Omega_s. \tag{4}
\]

In the stick subregion \(\Omega_s\), the tangential velocities of the sphere and the viscoelastic layer contacting points are equal [1]:

\[
\frac{\partial u(x, y)}{\partial x} = \Delta = \frac{\omega R - V}{V}, \quad (x, y) \in \Omega_s, \tag{5}
\]

where \(\Delta\) is the relative slippage.

At the boundaries of the stick and slip subregions the conditions of the continuity for shear stresses and displacements functions must be satisfied.

3. Method of solution

3.1. Normal and shear stresses

Assuming that the shear stresses do not influence the normal stress, their calculation process was divided in two parts. The method of evaluation of normal stress distribution is described in details in [7, 8]. The normal stress distribution in contact region is calculated using the method of successive approximations.
The shear stress distribution in a contact region was calculated by the strip method, in which the three-dimensional contact problem reduces to the set of two-dimensional contact problems with different boundary conditions [8], [10]. For each strip we obtain the shear stress distribution and the configuration of the stick and slip subregions. The detailed description of the solution method and the analysis of normal and shear stresses distribution for various layer viscosity and relative slippage are presented in [6].

### 3.2. Internal stresses

For internal stresses calculation the method based on boundary element method is used. To find the components of stress tensor at any point of half-space, summation is performed over all loaded surface elements. The following expression to calculate $\sigma_i$, at the point with coordinates $(x, y, z)$ is used:

$$\sigma_i(x, y, z) = \sum_{i=1}^{M} \left[ p(x_i, y_i) \left[ \sigma_{\xi}^p(\xi, \eta, x, y, z) d\xi d\eta - \tau(x_i, y_i) \left[ \sigma_{\xi}^r(\xi, \eta, x, y, z) d\xi d\eta \right] \right] \right]$$

where $p(x, y)$ and $\tau(x, y)$ are normal and shear stresses, $\sigma_{\xi}^p$, $\sigma_{\xi}^r$ are determined from Boussinesque solution for concentrated normal force and Cerruti solution for tangential force [11]. Other components of the stress tensor are calculated in the similar manner.

### 3.3. Damage accumulation model

To study the damage accumulation in the elastic half-space the linear summation model of damage is used [11]. In this model the previously accumulated damage doesn’t change the increment of damage function.

Damage at the point $(x, y, z)$ of elastic half-space is characterized by the function $Q(x, y, z, t)$, which depends on the amplitude values of internal stresses at this point. Material is damaged when the value of the function becomes equal to the threshold value. In the case of a normalized function, the threshold value is equal to 1.

We assume that the model of damage accumulation is associated with the principal shear stress and selected number of cycles. In accordance with the linear damage summation model the following equation is used to calculate the damage accumulation rate [8]:

$$q(x, y, z, t) = \frac{\partial Q(x, y, z, t)}{\partial t} = \frac{c}{E} \left[ \frac{\Delta \tau_{\max}(x, y, z, t)}{\Delta \tau_{\max}(x, y, z, t)} \right]^m$$

where $E$ is the elastic modulus of the half-space material, $\Delta \tau_{\max}(x, y, z)$ is the amplitude value of the principal shear stress at the point $(x, y, z)$, $c$ and $m$ are the constants, which are determined from the experimental data. Using eq. (7) damage at an arbitrary point of half-space that has accumulated over $N$ cycles is calculated as follows:

$$Q(x, y, z, N) = \int_0^N q(x, y, z, n) dn + Q_0(x, y, z)$$

where $N$ is the number of cycles, $q(x, y, z, n)$ is the damage accumulation rate and $Q_0(x, y, z)$ is the initial damage of material. The initial damage could be taken equal to zero at calculations.

The amplitude value of principal shear stresses is calculated as the difference between its maximum and minimum values at a fixed depth $z$:

$$\Delta \tau_{\max}(z_0) = \max_{x, y} \left[ \tau_{\max}(x, y, z_0) \right] - \min_{x, y} \left[ \tau_{\max}(x, y, z_0) \right]$$

The principal shear stresses are determined from the following equation [6]:

$$\tau_{\max}(x, y, z) = \frac{1}{2} \left( \sigma_1(x, y, z) - \sigma_3(x, y, z) \right)$$

where $\sigma_1$, $\sigma_2$, $\sigma_3$ are the maximum and minimum principal stresses at a point $(x, y, z)$. The principal stresses $\{\sigma_1, \sigma_2, \sigma_3\}$ are the solutions of the following equation:
\[
\begin{bmatrix}
\sigma_x - \sigma & \tau_{xy} & \tau_{xz} \\
\tau_{xy} & \sigma_y - \sigma & \tau_{yz} \\
\tau_{xz} & \tau_{yz} & \sigma_z - \sigma
\end{bmatrix} = 0, 
\]  \hspace{1cm} (11)

if at each point the condition \(\sigma_3 < \sigma_2 < \sigma_1\) is satisfied.

4. Results and discussion

For normal and shear stress distributions calculated with the method described in [6] numerical calculation of the internal stresses in the elastic half-space is performed. The distribution of the principal shear stress in the half-space is calculated taking into account the presence of viscoelastic layer. Figure 2 shows the dependence of the principal shear stress distribution from dimensionless coordinate \(x/a\), where \(a\) is the contact radius, for various relative slippage.

![Figure 2. The principal shear stresses for various relative slippage at \(y = 0\): \(\mu = 0.3, T_o/T_s = 40 \) and \(\Delta = 0.0025 \) (a), \(\Delta = 0.0045 \) (b), \(\Delta = 0.0065 \) (c).](image)

Figure 3 illustrates the distribution of the principal shear stress in half-space for various friction coefficients.

![Figure 3. The principal shear stresses for various friction coefficients at \(y = 0\) \(T_o/T_s = 40, \Delta = 0.0045\) and \(\mu = 0.3 \) (a), \(\mu = 0.5 \) (b), \(\mu = 0.7 \) (c).](image)

The results show that with an increase of friction coefficient and relative slippage, the subregion with the maximum values of principal shear stresses is located near the surface of the elastic half-space.

The damage distribution in the half-space is calculated using eq. (7) and (8). Figure 4 illustrates the effect of relative slippage on damage related to the number of cycles and material parameter \(c\) for various values of parameter \(m\). Damage accumulated at the interface increases with the growth of relative slippage. The influence of relative slippage on damage is greater in subsurface layer of elastic half-space, having thickness which is equal to \(a/2\). For large values of depth \(z\), the damage has the same value for various relative slippage.
5. Conclusion

The normal and shear stresses were calculated for a sphere rolling over a viscoelastic layer bonded to an elastic half-space. The stress state of the elastic half-space was calculated. The location of subregion with amplitude values of principal shear stress was determined. The damage accumulated in the subsurface layer of half-space was calculated under rolling friction conditions for the selected criterion of damage accumulation and given number of cycles. The effect of relative slippage and friction coefficient on the amplitude values of principal shear stress and damage value was analyzed.

Acknowledgments

This work was financially supported by RFBR according to the research project № 17-58-52030.

References

[1] Goryacheva I G, Soshenkov S N and Torskaya E V 2013 Modelling of wear and fatigue defect formation in wheel–rail contact Vehicle System Dynamics 51 6 pp. 767–783
[2] Goryacheva I G, Dobychin M N and Torskaya E V 2004 Simulation of the conditions of contact fatigue damages of the rolling surface Contact-Fatigue Damages of Wheels of the Freight Cars, (Zakharov S M Ed Moscow: Intekst) pp. 58–97
[3] Goryacheva I G and Torskaya E V 2010 Modeling of fatigue wear of a two-layered elastic half-space in contact with periodic system of indenters Wear 268 11-12 pp. 1417–1422
[4] Torskaya E V 2016 Modeling of fatigue damage of coated bodies under frictional loading Physical Mesomechanics 19 3 pp. 291–297
[5] Makushkin A P Stress–strain state of elastic layer in contact with spherical indentor 1990 Sov. J. Fric. Wear (Engl. Transl.) 11 (3) pp. 423–434
[6] Goryacheva I G and Miftakhova A R 2019 Modelling of the viscoelastic layer effect in rolling contact Wear 430 pp 256–262
[7] Goryacheva I G 1979 Plane and axisymmetric contact problems for rough elastic bodies J. Appl. Math. Mech. 43 pp. 99–105
[8] Goryacheva I G 1998 Contact Mechanics in Tribology (Dordrecht: Kluwer Academic Publishers) p. 344
[9] Haines D J and Ollerton E 1963 Contact stress distribution on elliptical contact surfaces to radial and tangential forces Pros. Inst. Mech. Engrs. 95.
[10] Kalker J J 1990 Three-Dimensional Elastic Bodies in Rolling Contact (Kluwer, Dordrecht)
[11] Johnson K L 1985 Contact Mechanics (Cambridge: Cambridge University Press)