Self-Interference Suppression Based on Sampled-Data $\mathcal{H}^\infty$ Control for Baseband Signal Subspaces

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Abstract: In this paper, we propose a design method of self-interference cancelers for in-band full-duplex wireless relaying taking account of baseband signal subspaces. We model the relaying system with self-interference as a sampled-data feedback control system. Then we formulate the design problem as a sampled-data $\mathcal{H}^\infty$ control problem with a generalized sampler and a generalized hold. The problem can be reduced to a discrete-time $\ell^2$-induced norm optimization problem by explicitly considering the subspace spanned by baseband signals. Moreover, for implementation, we also adopt ideal uniform samplers and zero-order holds with digital filters and up/down samplers. Under these implementation constraints, we reformulate the problem as a standard discrete-time $\mathcal{H}^\infty$ control problem by using the discrete-time lifting technique. Simulation results are shown to illustrate the effectiveness of the proposed method.

Key Words: $\mathcal{H}^\infty$ optimization, in-band full-duplex relay, sampled-data control, self-interference suppression, wireless communication.

1. Introduction

A major obstacle in wireless communications is channel fading due to signal attenuation and multipath propagation of transmitted radio signals. One possible approach to cope with fading is to utilize cooperative diversity [1]. Cooperative diversity is achieved by placing a relay station that forwards transmitted radio signals from a source supporting communication between the source and destinations [2]. Traditionally, to avoid interference between the original transmitted signals and the forwarded signals, different wireless resources, such as time, frequency, and code, are used for the signals. However, because wireless resources are scarce, it is desirable to use the same resources by removing the interference with signal processing techniques. An in-band full-duplex relay station [3], which employs a common carrier frequency and performs simultaneous transmission and reception, has attracted attention as highly efficient wireless communication technology that offers dramatic performance gains [4]. Since the quality of the communication with in-band full-duplex relay stations totally depends on the performance of suppression of a particular type of interference, called self-interference [5], it is required to develop an effective self-interference cancellation technique for realizing such full-duplex relaying systems.

A wireless communication system with a relay station, which suffers from self-interference, is shown in Fig. 1. In the figure, radio waves are transmitted from the source, and one destination (Destination 1) directly receives the signal from the source. Another destination (Destination 2) cannot directly communicate with the source directly since it is far from the source. To relay radio waves, a relay station is attached between them and radio waves from the source are relayed from the station to the destination. At the same time, the radio waves from the relay station are fed back to the receiving antenna of the relay station directly or through reflection objects. Consequently, self-interference is caused in the relay station and may deteriorate the communication performance, and even worse, destabilize the relay station.

To overcome the problem, self-interference cancellation methods have been established. Typical cancelers are constructed by combining three cancellation stages: One is antenna cancellation [5] where the received power of the interference is suppressed by arranging antenna placement. The second cancellation is analog cancellation [6] where the interference is canceled out in the analog domain with an analog circuit. The last cancellation is digital cancellation [7] where the effect of self-interference is reduced with digital signal processing.
We focus on digital cancellation in this study. Typical digital cancellation schemes first compute the transfer function of the self-interference channel, use the knowledge of the intended transmit signal to estimate the effect of self-interference, and simply subtract the estimated self-interference signal from the received signal [8]. The existing methods operate well when the gain of the relay station is relatively small or self-interference is sufficiently suppressed by antenna and analog cancellation, otherwise the signal diverges because the stability is not theoretically guaranteed.

The authors have proposed a method of designing self-interference cancelers that theoretically guarantee the stability of the system and also suppress the continuous-time effects of self-interference, based on standard sampled-data $\mathcal{H}^\infty$ control [9]. The proposed method takes transmitted signals as filtered signals and minimizes the $\mathcal{H}^\infty$ norm (or the $L^2$-induced norm) of the corresponding error system between received and transmitted signals. In many cases, however, baseband signals used in communication belong to a subspace of $L^2$ and the subspace is intrinsically smaller than the entire space $L^2$. From this fact, the designed cancelers become conservative.

In this paper, on the premise of the perfect knowledge of the signal subspace spanned by baseband signals, we utilize the characteristic of the subspace for self-interference canceler design. The problem of the canceler design is formulated as a sampled-data $\mathcal{H}^\infty$ control problem with generalized samplers and holds. The difficulty here is that standard sampled-data $\mathcal{H}^\infty$ controller synthesis cannot be directly applied to the problem, because the input signals are restricted to the baseband subspace. The key observation to surmount the difficulty is that the baseband signals spanning the subspace are orthonormal in most digital communication systems. Utilizing the property, we show that the problem is reducible to a standard discrete-time $\mathcal{H}^\infty$ control problem. We also propose an implementation-aware design of the sampled-data $\mathcal{H}^\infty$ canceler for practical systems. Finally, we show simulation results to illustrate the effectiveness of the proposed method.

This paper builds on the preliminary version [10], where we have introduced the fundamental idea to design self-interference cancelers based on sampled-data $\mathcal{H}^\infty$ control for baseband signal subspaces. This paper provides more detailed description of in-band full-duplex wireless relaying systems compared with the paper [10], and furthermore, we derive the explicit solution of the optimal control problem in this paper.

The remainder of this paper is organized as follows. In Section 2.1, we give mathematical models of transmitters, receivers, and channels. In Section 2.2, we model the in-band full-duplex wireless relaying system considered in this paper. In Section 3.1, we formulate the self-interference suppression problem as a sampled-data $\mathcal{H}^\infty$ optimal control problem by explicitly considering the subspace spanned by baseband signals. In Section 3.2, we show that the formulated problem can be reduced to a discrete-time $\ell^2$-induced norm optimization problem. In Section 3.3, we consider implementation-aware design of the proposed canceler for implementable samplers and holds. In Section 4, we illustrate the effectiveness of the proposed method through simulation results. In Section 5, we offer concluding remarks.

### Notation

Throughout this paper, we use the following notation. The symbol $t$ denotes the argument of continuous-time, $n$ denotes the argument of discrete-time, $s$ denotes the argument of Laplace transform, and $z$ denotes the argument of $Z$ transform. These symbols are used to indicate whether a signal or a system is of continuous-time or discrete-time. We denote the identity matrix by $I$. For a vector $x$ and a matrix $A$, their transposes are denoted by $x^T$ and $A^T$. For a vector $x$, the Euclidean norm of $x$ is denoted by $\|x\|$. We denote by $L^2$ the Lebesgue space consisting of all square integrable functions on $[0, \infty)$ endowed with the inner product

$$\langle x, y \rangle_{L^2} \equiv \int_0^\infty y(t)^* x(t) dt,$$

and the norm

$$\|x\|_{L^2} \equiv \sqrt{\langle x, x \rangle_{L^2}}.$$ 

We also denote by $\ell^2$ the set of all square summable sequences on $\{0, 1, 2, \ldots\}$ endowed with the inner product

$$\langle x, y \rangle_{\ell^2} \equiv \sum_{n=0}^\infty y[n]^* x[n],$$

and the norm

$$\|x\|_{\ell^2} \equiv \sqrt{\langle x, x \rangle_{\ell^2}}.$$ 

The operator $e^{-Lt}$ with nonnegative real number $L$ denotes a continuous-time delay operator and the operator $z^{-l}$ with nonnegative integer $l$ denotes a discrete-time shift operator.

### 2. Full-Duplex Wireless Communication System

We focus on the communication system from a source (transmitter) to a destination (receiver) where a relay station is placed between them. We describe a mathematical model of the wireless relay communication system by using a standard and widely accepted models [11].

#### 2.1 Transmitter, Receiver, and Channel Models

We first describe the input-output relationship of a transmitter. The encoder receives bit stream from the message source, creates a discrete-time two-dimensional signal $w[n] = [w_1[n] \ w_2[n]]^T$, which is called a symbol, and outputs a continuous-time two-dimensional signal

$$w(t) = \sum_{n=0}^\infty [w_1[n] \ w_2[n]]^T \phi(t - nT) \tag{1}$$

![Fig. 2 Transmitter](image-url)
with a pulse-shaping function $\phi \in L^2$, which is also called base-band signal, and a symbol period $T > 0$. Sinusoidal waves with a carrier frequency $f_c$ are then multiplied to $w(t)$ for the purpose of up-converting the signal, and finally a band-pass signal

$$w_0(t) \triangleq \sum_{n=0}^{\infty} (w_1[n] \cos(2\pi f_c t) + w_2[n] \sin(2\pi f_c t))\phi(t - nT)$$

is generated. Typically, the carrier frequency $f_c$ for mobile communications reaches several hundred megahertz (MHz). The frequency bandwidth of $\phi$ is in most cases assumed to be much smaller than $f_c$, and we follow this standard assumption in this paper.

The channel model is simply described by a linear combination of time-delay systems with attenuation [11], that is, the received signal is written as

$$z_0(t) = \sum_{l=1}^{L} a_l w_0(t - \tau_l)$$

with $a_l > 0$ and $\tau_l > 0$ for $l = 1, \ldots, L$. The parameters, namely, the number of multipass channels $L$, the attenuation factors $a_l$, and the delays $\tau_l$, totally depend on the environment around the communication system.

The structure of the receiver is shown in Fig. 3. The received band-pass signal $z_0(t)$ passes through a band-pass filter whose passband contains the interval $[f_c - f_\delta, f_c + f_\delta]$ where $f_\delta$ denotes the one-sided bandwidth of $\phi$. Note that effects of noise are not explicitly discussed in this paper, and hence $z_0(t)$ is not changed by the band-pass filter at all. The received signal $z_0(t)$ is again multiplied to the sinusoidal waves, and the output is described by

$$z(t) = \sum_{l=1}^{L} a_l w_0(t - \tau_l)$$

where $z(t)$ is called the equivalent low-pass representation [11] of the system from the transmitter to the receiver including channels. Because the high-frequency sinusoidal wave $\cos(2\pi f_c t)$ vanishes in (6), we do not need to explicitly consider the up-converting and down-converting operators for the analysis of the relationship between $w(t)$ and $z(t)$ employing the representation. We hereinafter use the equivalent low-pass representation in the following discussion.

The relationship (6) between $w(t)$ and $z(t)$ is described by

$$z(t) = \sum_{l=1}^{L} R_l w(t - \tau_l),$$

where $z(t)$ is the output of the low-pass filters whose passband contains the interval $[-f_\delta, f_\delta]$, LPF(·) is the operator of the low pass filters, and

$$R_l \triangleq \frac{a_l}{2} \begin{bmatrix} \cos(2\pi f_c \tau_l) & \sin(2\pi f_c \tau_l) \\ \sin(2\pi f_c \tau_l) & \cos(2\pi f_c \tau_l) \end{bmatrix}.$$
and the digital signal processor (DSP) is modeled by a linear
as well as self-interference suppression for the design of
larger than 1, and hence we should take account of the

formulation. As mentioned in the previous section, the original
system is shown in Fig. 6.

\[ S_\delta : \begin{cases} y_1(t) \\ y_2(t) \end{cases} \mapsto \begin{cases} (y_1, \phi_0) \in \mathcal{L}^2 \\ (y_2, \phi_0) \in \mathcal{L}^2 \end{cases} \text{,} \quad (9) \]

where \( \phi_0(t) = \phi(t - nT) \), the digital-to-analog (D-A) converter
is modeled as the causal generalized hold

\[ H_\delta : \begin{cases} u_1[n] \\ u_2[n] \end{cases} \mapsto \begin{cases} \sum_{n=0}^{\infty} \phi_0(t) u_1[n] \\ \phi_0(t) u_2[n] \end{cases} \text{,} \quad (10) \]

and the digital signal processor (DSP) is modeled by a linear
time-invariant discrete-time controller \( K(z) \). By the block dia-
gram, we can see that the relay station with self-interference is a feedback
system. In practice, the gain of the power amplifier
is very high (e.g., \( a = 1000 \)) and the loop gain becomes much larger than 1, and hence we should take account of the stability
as well as self-interference suppression for the design of \( K(z) \).
To achieve these requirements, we formulate the design problem as a sampled-data \( \mathcal{H}_{\infty} \) control problem in the next section.

Remark: Two classes of relay station protocols can be found:
One is called AF, and the other is called decode-and-forward (DF) [12]. In the AF protocol, received signals are converted to discrete-time signals and then directly amplified. In the DF protocol, received signals are not only converted but also de-
coded into bit stream, and then the bit stream is encoded again and amplified. The most important difference between these protocols is to detect received signals or not. While in the DF protocol, the detection may reduce the risk of instability, the AF protocol requires much lower implementation complexity
than the DF protocol [12]. For the reason, we assume the relay
station operates with the AF protocol throughout this study.

3. Self-Interference Suppression via Sampled-Data \( \mathcal{H}_{\infty} \) Control

In this section, we formulate the design problem as a sampled-data \( \mathcal{H}_{\infty} \) control problem based on the mathemati-
cal model introduced in the previous section. We show that the problem can be reduced to a problem of discrete-time \( \ell^2 \)-induced norm optimization. Because it may be difficult to im-
plement the ideal generalized sampler and hold, we also propose implementation-aware design of the self-interference can-
celer.

3.1 Problem Formulation

The objective of self-interference suppression is to reduce the
error from the input \( w \) to the output \( u \) in Fig. 5. In communications, a certain amount of delay of the output \( u \) against the input
\( w \) is allowable, and accordingly we consider the delayed error

\[ e(t) = w(t - mT) - u(t) \text{,} \quad (11) \]

where \( m \) is a non-negative integer. The corresponding error
system is shown in Fig. 6.

We here prepare some mathematical notation for our problem
formulation. As mentioned in the previous section, the original
signal \( w \) is represented as a linear combination of \( \phi_0 \), assuming
that the effect of the channel from the source to the destination

is negligible. To characterize the original signal \( w \), we define
the signal subspace \( \mathcal{W} \subset \mathcal{L}^2 \) by

\[ \mathcal{W} \triangleq \text{span} \{ \phi_0, \phi_1, \ldots \} \text{,} \quad (12) \]

where \( \text{span} \{ \phi_0, \phi_1, \ldots \} \) is the space consisting of all finite linear combinations of \( \phi_0, \phi_1, \ldots \), and the overline denotes the closure
in \( \mathcal{L}^2 \).

Our design problem is then formulated as follows.

**Problem 1** Find a digital controller \( K(z) \) that stabilizes the
feedback system in Fig. 6 and achieves

\[ J < \gamma \text{,} \quad (13) \]

for a given \( \gamma > 0 \), where

\[ J \triangleq \sup_{w \in \mathcal{W}} \frac{\| u \|_{\mathcal{L}^2}}{\| w \|_{\mathcal{L}^2}} \text{.} \quad (14) \]

The design problem is a sampled-data \( \mathcal{H}_{\infty} \) suboptimal con-
trol problem, for which standard existing methods cannot be ap-
plied since the input signals are restricted to the subspace \( \mathcal{W} \).
It should be noted that these settings are quite different from
standard problem formulations in optimal sampled-data control
theory, and we cannot directly solve the problem by means of
existing approaches.

3.2 Equivalent Transformation

The key observation to surmount the difficulty is that
\( \{ \phi_0, \phi_1, \ldots \} \) are orthonormal in most digital communication sys-
tems. For example, the square root raised cosine pulse, which
is widely employed in practical systems, satisfies the orthonor-
mality [13]. We will show that the design problem for the system
in Fig. 6 can be equivalently transformed into a discrete-
time \( \ell^2 \)-induced norm optimization problem for the system
in Fig. 7 provided that the property of orthonormality holds. We
hereafter assume that the system in Fig. 7 is stable.

First, we show the following lemma.

**Lemma 1** Assume the feedback system in Fig. 7 is internally
stable. Then we have \( e \in \mathcal{W} \) for any \( w \in \mathcal{W} \).

**Proof:** Since the delay \( mT \) is an integer multiple of \( T \), the
delayed input \( w(t - mT) \) is an element of \( \mathcal{W} \). Moreover, because
the system is internally stable and $y$ is the output of $\mathcal{H}_d$, $u$ is also an element of $\mathcal{W}$. Since $\mathcal{W}$ is a vector space we have $e(t) = w(t - mt) - u(t) \in \mathcal{W}$.

From Lemma 1, we can describe $e(t)$ by

$$e(t) = \sum_{n=-\infty}^{\infty} e_d[n] \phi_n(t).$$  \hspace{1cm} (15)

We have the following theorem.

**Theorem 1** Consider the sampled-data error system in Fig. 6 and the discrete-time error system in Fig. 7. If $\{\phi_0, \phi_1, \ldots\}$ is an orthonormal system in $\mathcal{W}$, then the two systems are equivalent in the sense that

$$J = J_d,$$

where

$$J_d = \sup_{w \in L^2} \frac{\|e\|_{L^2}}{\|w\|_{L^2}}.$$  \hspace{1cm} (17)

**Proof:** Since $\mathcal{W}$ is a closed subspace of $L^2$, $\mathcal{W}$ is a Hilbert space. Since $\text{span}\{\phi_0, \phi_1, \ldots\}$ is dense in $\mathcal{W}$, $\{\phi_0, \phi_1, \ldots\}$ is a complete orthonormal system in $\mathcal{W}$. Thus, for any $w \in \mathcal{W}$, Perseval’s theorem [14] gives

$$\|w\|_L^2 = \|w_d\|_L^2,$$

where $w_d[n] = \langle w, \phi_n \rangle_L$. Similarly, we have

$$\|e\|_L^2 = \|e_d\|_L^2.$$  \hspace{1cm} (18)

Then $w \in \mathcal{W}$ is a necessary and sufficient condition for $w_d \in L^2$ and hence we have $J = J_d$.

From the theorem, we see that it suffices to solve the following problem.

**Problem 2** Find a digital controller $K(z)$ that stabilizes the feedback system in Fig. 7 and achieves

$$J_d < \gamma.$$  \hspace{1cm} (20)

Problem 2 is actually a discrete-time $L^2$-induced norm optimization problem.

Finally, we give formulas for the $\mathcal{H}^\infty$ design of the controller. Define the transfer functions $G_{e_{d,w}}(z), G_{y_{d,w}}(z), G_{e_{d,u}}(z),$ and $G_{y_{d,u}}(z)$ by

\[
G_{e_{d,w}}(z) = S_d e^{-mt} \mathcal{H}_d, \\
G_{y_{d,w}}(z) = S_e \mathcal{H}_d, \\
G_{e_{d,u}}(z) = -S_d \mathcal{H}_d, \\
G_{y_{d,u}}(z) = S_e D(s) d \mathcal{H}_d.
\]

(21)

Let the channel transfer function $D(s)$ be

$$D(s) = \sum_{i=0}^{L} R_i e^{-\tau_i s}$$

\hspace{1cm} (22)

with matrices $R_i$ and scalars $\tau_i > 0$. We then have the following proposition.

**Proposition 1** The matrices in (21) are given by

\[
G_{e_{d,w}}(z) = z^{-m} I, \\
G_{y_{d,w}}(z) = I, \\
G_{e_{d,u}}(z) = -I, \\
G_{y_{d,u}}(z) = a \sum_{i=0}^{L} R_i \sum_{n=0}^{\infty} \langle \phi(t - \tau'_i), \phi(t - nT) \rangle_L z^{-n+N-1},
\]

\hspace{1cm} (23)

where an integer $N_i$ and $\tau'_i > 0$ satisfy the relation $\tau_i = N_i T + \tau'_i$.

**Proof:** From the definition of the generalized sampler and hold, and the orthonormality of $\phi_m$, we have

$$G_{e_{d,w}}(z) = z^{-m} I, \quad G_{y_{d,w}}(z) = I, \quad G_{e_{d,u}}(z) = -I.$$  \hspace{1cm} (24)

From the assumption of causality of the generalized sampler and hold, letting the impulse response matrices of $S_d R_i e^{-\tau_i s} \mathcal{H}_d$ be $d_i[n]$, we have

$$d_i[n] = R_i \langle \phi(t - \tau_i), \phi(t - nT) \rangle_L,$$

\hspace{1cm} (25)

Hence

$$G_{y_{d,u}} = S_d D(s) d \mathcal{H}_d,$$

$$= a \sum_{i=1}^{L} R_i \sum_{n=0}^{\infty} \langle \phi(t - \tau'_i), \phi(t - nT) \rangle_L z^{-n+N-1} I.$$  \hspace{1cm} (26)

From Proposition 1, we have the solution to Problem 2.

**Solution:** Consider the discrete-time system and the controller

$$\begin{bmatrix} z_d[n] \\ y_d[n] \end{bmatrix} = \begin{bmatrix} G_{e_{d,w}}(z) & G_{y_{d,w}}(z) \\ G_{e_{d,u}}(z) & G_{y_{d,u}}(z) \end{bmatrix} \begin{bmatrix} w_d[n] \\ u_d[n] \end{bmatrix},$$

\hspace{1cm} (27)

in which the transfer matrices are given by (23). For the closed-loop system, the $\mathcal{H}^\infty$ suboptimal controller achieving (20) is the solution to Problem 2. The $\mathcal{H}^\infty$ suboptimal controller can be easily found by standard discrete-time $\mathcal{H}^\infty$ control as long as the problem is feasible. Note that $G_{y_{d,u}}$ in (23) is a finite-dimensional system because the pulse-shaping function $\phi$ has a finite support in practical systems.

### 3.3 Implementation-Aware Controller Design

In practice, the generalized sampler and hold considered above are hard to implement in real systems. Here we give a controller design method taking account of the implementation based on the previously stated framework.

The generalized hold $\mathcal{H}_d$ is approximated by

$$\mathcal{H}_d \approx \mathcal{H}_d(\mathcal{P}(z)) \uparrow N$$  \hspace{1cm} (28)

with the zero-order hold $\mathcal{H}_d$ with its sampling period $h$ defined by

$$\mathcal{H}_d : \begin{bmatrix} u_d[n] \\ u_{d+1}[n] \end{bmatrix} \mapsto \begin{bmatrix} \sum_{i=0}^{\infty} 1_{[0,N]}(t - nh) u_{d+1}[n] \\ \sum_{i=0}^{\infty} 1_{[0,N]}(t - nh) u_{d+1}[n] \end{bmatrix}_{t \in [0,\infty)},$$

\hspace{1cm} (29)

where $I_{[0,h]}(t)$ is the indicator function associated with the interval $[0,h]$, the upsampler $\uparrow N$ with its upsampling ratio $N = T/h$ defined by

$$\uparrow N : [u_d[n]]_{n=0}^{\infty} \mapsto \underbrace{u_d[0], 0, \ldots, 0, u_d[1], 0, \ldots, 0}_{N-1},$$

\hspace{1cm} (30)

and a digital filter $\mathcal{P}(z)$. This structure is widely used for pulse-shaping [15], where $\mathcal{P}(z)$ is called a band-limiting filter. The filter $\mathcal{P}(z)$ for the hold is designed such that its impulse response is equal to the sampled values of the baseband pulse $\phi$. More precisely, $\mathcal{P}(z)$ is designed according to

$$\mathcal{P}(z) = \sum_{n=0}^{\infty} \langle \phi(nh) \rangle z^{-n} I.$$  \hspace{1cm} (31)
We assume that the support of $\phi(t)$ is finite and the support length is an integer multiple of $h$, that is, there exists a natural number $N_p$ such that the support length of $\phi(t)$ is $N_p h$. By this assumption, the filter $P(z)$ becomes a finite impulse response (FIR) filter.

The generalized sampler is also approximated by

$$\tilde{S}_h \triangleq (\downarrow N)P(z)S_h$$

(32)

with the ideal (2-dimensional) uniform sampler $S_h$ with its sampling period $h > 0$ defined by

$$S_h : \begin{Bmatrix} y_1(t) \\ y_2(t) \end{Bmatrix}_{t \in [0,\infty)} \mapsto \begin{Bmatrix} y_1(nh) \\ y_2(nh) \end{Bmatrix}_{n = 0,1,2,...}.$$  (33)

the downampler $\downarrow N$ with its upsampling ratio $N$ defined by

$$\downarrow N : \{y[n]\}_{n=0,1,2,...} \mapsto \{y[0], y[N], y[2N], \ldots\}.$$  (34)

When the generalized sampler and hold in Fig. 7 are replaced by $\tilde{S}_h$ and $\tilde{H}_G$, the closed loop system becomes a multi-rate system but one can obtain the optimal controller $K(z)$ via a standard discrete-time $\mathcal{H}_\infty$ optimal control method using the technique of discrete-time lifting [16].

4. Simulation

We here show simulation results of self-interference suppression to illustrate the effectiveness of the proposed method.

Throughout the simulation, the sampling period $h$ is normalized to 1, the normalized carrier frequency $f_c$ is set to be 10000, the symbol period $T$ is 5 (consequently the upsampling ratio $N$ is 5), the process delay $\eta$ is 5, the baseband modulation is taken as quadrature phase shift keying (QPSK), the pulse-shaping function $\phi$ is set to be a root-raised cosine pulse with its roll-off factor 0.2, and the self-interference channel $D(s) = 0.5e^{-Ts}I$.

Although we have ignored the noise in the analysis, we assume additive white Gaussian noise at the relay station in the simulation.

In Fig. 8, we show the absolute values of the error signal $|e(t)| \in \mathbb{R}^2$ with a log-scale vertical axis where signal-to-noise ratio (SNR) is 10 dB. The solid and dashed lines represent the error signals with the proposed implementation-aware approach and the standard sampled-data optimal control approach [9], respectively. Moreover, Fig. 9 shows the mean of $\|e(t)\|$ versus SNR for the proposed approach and the sampled-data optimization control approach. From the figures, we can see that the error is significantly reduced with the proposed method compared with the existing method. In Fig. 10, we show the bit error rate (BER) versus SNR at the relay output in order to evaluate the quality of the relayed signal, while the signal detection is not performed at the relay output in the actual system. The figure implies that the quality of the signal is improved by taking account of the baseband subspace.

5. Conclusion

In this paper, we have proposed a new method for self-interference suppression taking account of the baseband signal subspace. The problem is first formulated as a sampled-data $\mathcal{H}_\infty$ control problem with a generalized sampler and a generalized hold. The formulated problem can be equivalently reduced to a discrete-time $\ell^2$-induced norm optimization problem. Considering implementation constraints, we reformulate the problem as a standard discrete-time $\mathcal{H}_\infty$ control problem. Simulation results are shown to illustrate the effectiveness of the proposed method.

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