A Flexible Approach for Checking Timed Automata on Continuous Time Semantics

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Abstract. Timed Automata (TA) are used to represent systems when the interest is the analysis of their behaviour as time progresses. Even if efficient model-checkers for Timed Automata exist, they have several limitations: 1. they are not designed to easily allow adding new Timed Automata constructs, such as new synchronization mechanisms or communication procedures; 2. they rely on a precise semantics for the logic in which the property of interest is expressed which cannot be easily modified and customized; 3. they do not easily allow using different solvers that may speed up verification in different contexts. This paper presents a novel technique to perform model checking of full Metric Interval Temporal Logic (MITL) properties on TA. It relies on the translation of both the TA and the MITL formula into an intermediate Constraint LTL over clocks (CLTLoc) formula which is verified through an available decision procedure. The technique is flexible since the intermediate logic allows the encoding of new TA constructs, as well as new semantics for the logic in which the property of interest is expressed, by just adding new CLTLoc formulae. Furthermore, our technique is not bound to a specific solver as the intermediate CLTLoc formula can be verified using different procedures.

1 Introduction

Timed Automata [2] (TA) are one of the most popular formalism used to specify how a system behaves over the time. There are various available tools supporting verification of TA which are based on different solving techniques, such as Difference Bounded Matrices [17], BDD and BDD-like structures [14,25] or SMT-solvers [5]. The majority of the tools provide some baseline functionalities that allow designers to analyze the system with respect to reachability problems (safety assessment) or to perform model-checking with temporal logics, mainly LTL, CTL and in very few cases fragments of Timed CTL [26]. The most famous ones are Kronos [26], the de facto standard tool Uppaal [21], RED [25] and MCMT [15]. The paradigm of time that is overwhelmingly adopted in practice is based on timed words [2]—i.e., sequences of states with an associated real-valued timestamp—and all the previous tools are founded on such semantics. A so-called signal-based semantics, where each instant of a dense temporal domain (typically \(\mathbb{R}_{\geq 0}\)) is associated with a state, can also be defined. Despite theoretical results demonstrate that signal-based formalisms have greater expressiveness than the ones based on time words, signal-based semantics has been so far confined mainly to theoretical investigations [3,12,24,18] and only seldom used in practice. Implemented
decision procedures that allow for the verification of TA with Metric Interval Temporal Logic (MITL) [3] specification over continuous signals are actually very recent [20]. The development of new decision procedures (such as those based on SMT-solvers) and of faster solvers has positively contributed to the development of feasible algorithms for solving verification problems of signal-based formalisms. In fact, [20,7] developed a decision procedure to solve the satisfiability of MITL formulae, a problem which is also very recently tackled by [13].

Even if a variety of tools supporting the analysis of TA and network of TA is available, they usually limit designers in different ways. 1. The existing model checkers provide designers with a fixed set of modeling constructs that are not easily modifiable and customizable. They usually involve (finite or infinite domain) discrete variables and different communication and synchronization features. For example, Uppaal provides designers with binary and broadcast synchronization primitives whereas RED offers sending/receiving communication features via finite FIFO channels. However, it is likely that new modeling requirements prompt the designers to formulate specific communicating features, type of synchronization or even data-structure such as queues, stacks, priority mechanisms and so on. Common model checkers are not designed to easily embed changes in their semantics. The addition of new semantics—e.g., ways in which the TA synchronize—or new constructs can possibly entail a significant variation of the underlying formalism adopted by the tools; and, on the other side, it can necessitate a deep knowledge of the internals of the tools. In many cases, the complexity of the software implementation, that is determined by many factors such as the architecture, the programming language used to implement the tool and the availability of documentation, hampers the development of new features. 2. All the tools that we have previously mentioned only deal with TA and do not allow designers to employ in the design other (timed) formalisms such as, for instance, Time Petri Nets [23,19]. This limits the capability of the tools that can only carry out a single-formalism verification. Multi-formalisms analysis [6], that can be used to design modular systems with heterogeneous component modeled each by means of the most suitable formalism, is therefore not possible. The lack of a way to plug-in a new definition for the semantics of the logic in which the property of interest is specified even worsens such limitation. 3. The existing model checkers are usually not solver-independent. They rely on a strong relation between the problem domain and the solution domain—i.e., the models to be verified and the input language that is used for model checking.

**Contribution.** This paper describes a novel technique to perform model checking of MITL properties on networks of TA which relies on a purely logic-based approach. The technique is exemplified in the diagram of Fig. 1. It uses an intermediate logical language in which both the MITL formula and the TA model are translated. Instead of a direct encoding of the model-checking problem into the language of the underlying solver, the procedure exploits an intermediate logical language similarly to the Java Byte code for Java program execution. The intermediate encoding is then processed by an appropriate solver. On the one hand, new TA constructs as well as other formalisms and semantics for the logic in which the property of interest is specified can be considered by proposing new encodings in the intermediate language. On the other hand, more efficient solvers can be developed for the intermediate language.
Specifically, the TA and the MITL formula to be checked are translated into a formula of CLTLoc metric temporal logic [10]. CLTLoc is a decidable extension of Linear Temporal Logic (LTL) which includes real-valued variables that behave like TA clocks. This intermediate language easily allows for considering different semantics of TA such as, for instance, the synchronization primitives that TA can use in a network. Moreover, different features of the TA modeling language can be introduced by simply adding or changing formulae in the CLTLoc encoding. The satisfiability of the intermediate CLTLoc formulae can be checked using different procedures: the one considered in this work concern a bounded approach based on SMT-solvers is available as part of the Z3 formal verification tool [10]. The proposed framework is implemented in a Java tool, called TACK (https://github.com/claudiomenghi/TACK) which takes as input a (networks of) TA described through a syntax that is compatible with the one of Uppaal and a MITL formula to be verified.

To evaluate the benefits of the framework we considered different synchronization primitives and liveness conditions on the TA executions and show that these constructs could be effectively encoded in TACK. To show the flexibility that is yielded by the decoupling between the model-checking problem and the resolution technique, different solvers are employed for verifying the intermediate CLTLoc encoding. Finally, the efficiency of the proposed technique is evaluated by comparing TACK with the tool presented in [20], considering the Fischer mutual exclusion protocol benchmark [1].

Related works. The Bounded model checking (BMC) problem for TA with MITL formulae has been considered and successfully solved for the first time in [20] and implemented in the Mtil0∞BMC tool [20]. Mtil0∞BMC tool is the first publicly available model-checker for TA that can handle MITL specifications over signals. The encoding presented in [20] is a translation of the BMC problem for TA into an SMT formula that belongs the quantifier-free first order real difference logic, a decidable fragment of the first order logic whose decision procedure is available in many SMT-solvers such as Z3 and Yices [11]. The underlying time semantics in [20] is based on the so called “super-dense” time that is also adopted in Uppaal to model execution traces of the TA over timed words. Super-dense time is a modeling abstraction that is useful to represent "fast" systems, namely those systems that react to events triggered by the environment, where they operate, with a negligible delay. With super-dense time, a TA is allowed to
fire more than one transition in the same (absolute) time instant, i.e., two transitions can be fired one after the other as time does not progress.

A proof of the relationship between T A and CL TL oc has been given in [9]. Despite the presented translation expresses the executions of a TA by means of CL TL oc formulae, the encoding was conceived to prove the language equivalence of TA and CL TL oc over timed words. The resulting CL TL oc formula is not intended to be implemented in a tool: it makes use of many additional clocks, besides those used by the TA, that would hinder the performances of any decision procedure elaborating the formula. This limitation fosters the definition of a new translation, especially when practical concerns are considered. The differences with the encoding described in [9] are radical: the encoding presented in this work extends to network of TA, whose traces has to be interpreted for the evaluation of MITL formulae, and it is not intended to prove language-equivalence.

Organization. Section 2 presents background notation. Section 3 introduces the continuous time semantics of TA. Section 4 describes how to convert a TA in CL TL oc and to check MITL formulae on TA. Section 5 concludes.

2 Background

We present the relevant definitions of TA (with integer-valued variables and synchronization), MITL, and CL TL oc. Additional information can be found in Appendix A1.

Timed automata. Let X be a finite set of clocks with values in \( \mathbb{R} \). \( \Gamma(X) \) is the set of clock constraints over \( X \) defined by the syntax \( \gamma := x \sim c | \lnot \gamma | \gamma \land \gamma \), where \( \sim \in \{<,=,\} \), \( x \in X \) and \( c \in \mathbb{N} \). Given a set of actions Act we define \( Act_\tau := Act \cup \{\tau\} \), where \( \tau \) is used to indicate a null action.

Definition 1. Given a set of atomic propositions AP, a set of clock X and a set of actions Act, a Timed Automaton is a tuple \( A = \langle AP, X, Act_\tau, Q, q_0, Inv, L, T \rangle \), where:

- \( Q \) is a finite set of states; \( q_0 \in Q \) is the initial state; \( Inv : Q \to \Gamma(X) \) is an invariant assignment function; \( L : Q \to \wp(AP) \) is a function labeling the states in \( Q \) with elements of \( AP \); \( T \subseteq Q \times Q \times \Gamma(X) \times Act_\tau \times \wp(X) \) is a finite set of transitions.

Function \( Inv \) associates each state \( q \) with an invariant in \( \Gamma(x) \). A transition \( t \in T \) is written as \( q \xrightarrow{\gamma_c,\alpha,S} q' \), where \((q, q', \gamma_c, \alpha, S)\) is an element of \( T \). Specifically, \( \gamma_c \) is the clock constraint the clock values must satisfy for the transition to be fired; \( \alpha \in Act \) is an action that labels the transition; the set \( S \in \wp(X) \) contains the clocks to be reset. Given a transition \( t \in T \) we use the notation \( t', t, t_y, t_\alpha, t_S \) to indicate the source \( q \), the destination \( q' \), the clock constraint \( \gamma_c \), the label \( \alpha \) and the set of clocks \( S \) to be reset of \( t \). Fig. 2(a) shows a simple example of TA. The constraints on the clocks represented within each state represent are the clock constraint associated by the invariant assignment function to that state. States are also labeled with the atomic propositions assigned by the function labeling. Consider a transition \( q \xrightarrow{\gamma_c,\alpha,S} q' \). The clock constraint \( \gamma_c \) is indicated in Fig. 2(a) as the “guard” of the transition. The action \( \alpha \) is indicated using the keyword “sync” since actions will be used to synchronize different TA. The clocks in \( S \) are indicated using the keyword “assign” meaning that they are
the broadcast synchronization receivers. We use the notation \(a\), \(a\!, a\#\) and \(a\@\) to denote broadcast synchronization sender, and \(a\@\) denotes broadcast synchronization receivers. We use the notation \(a\!, a\#\) and \(a\@\) to denote broadcast synchronization sender, and \(a\@\) denotes broadcast synchronization receivers.

Fig. 2. The TA in (a) has three states, \(l_0, l_1, l_2\), and one clock \(x\). The transition from \(l_2\) to \(l_0\) is labeled with guard \(x = 10\). When the transition is taken, clock \(x\) is reset—i.e., it is set to 0. State \(l_1\) is associated with invariant \(x \leq 5\). States \(l_0\) and \(l_2\) are labeled with atomic propositions \(a\) and \(c\), respectively. The TA in (b) is the same as the one of (a) except for the presence of discrete variable \(d\), which is set to 0, 1 or 2 depending on the transition taken.

assigned by the transition to the value 0. In Fig. 2(a) transitions are also labeled with identifiers \(\alpha, \beta, \lambda\) that will be used for explanation purposes.

Let \(Int\) be a finite set of integer variables with values in \(\mathbb{Z}\) and \(\sim \in \{<, =\}\). \(Assign(Int)\) is the set of assignments of the form \(n := \text{exp}\), where \(n \in Int\) and \(\text{exp}\) is an arithmetic expression over the integer variables and elements of \(\mathbb{Z}\). \(\Gamma(Int)\) is the set of variable constraints \(\delta\) over \(Int\) defined as \(\delta := n \sim c\mid n \sim n'\mid \sim \delta\mid \delta \land \delta\), where \(n, n'\) are integer variables and \(c \in \mathbb{Z}\).

**Definition 2.** Given a set of atomic propositions \(AP\), a set of clocks \(X\), a set of actions \(\text{Act}\) and a set of integer variables \(\text{Int}\), a TA with Variables is a tuple \(\mathcal{A} = \langle AP, X, \text{Act}, \Int, Q, q_0, v_{var}, \text{Inv}, L, \tau\rangle\), where: \(Q\) is a finite set of states; \(q_0 \in Q\) is the initial state; \(v_{var}: \text{Int} \rightarrow \mathbb{Z}\) assigns each variable with a value in \(\mathbb{Z}\); \(\text{Inv}: Q \rightarrow \Gamma(X)\) is an invariant assignment function; \(L: Q \rightarrow \varphi(AP)\) is the labeling function; \(\tau \subseteq Q \times Q \times \Gamma(X)\times \Gamma(\text{Int}) \times \text{Act} \times \varphi(X)\times \varphi(\text{Assign}(\text{Int}))\) is a finite set of transitions.

A transition is written as \(q \xrightarrow{\gamma_{var}, \alpha, \text{Assign}(\text{Int})} q'\) where \(\gamma_{var}\) is a constraint of \(\Gamma(\text{Int})\) and \(A\) is a set of assignments from \(\text{Assign}(\text{Int})\). We use the notation \(t_{var}\) and \(t_{\text{Assign}}\) to indicate the variable constraint \(\gamma_{var}\) and the assignments \(A\) associated with a transition \(t\). An example of TA with Variables is presented in Fig. 2(b). The variable constraint \(\gamma_{var}\) is indicated in Fig. 2(a) in the “guard” of the transition. The assignments in \(A\) are indicated in the assignement of the transition and associate a variable with the assigned value.

When networks of TA are considered, the actions symbols that label the transitions are used to synchronize automata. The set of actions \(\text{Act}_t\) is then obtained as \(\text{Act}_t = \{\tau\} \cup \{\text{Act} \times \text{Sync}\}\) where \(\text{Sync}\) is a set of synchronization primitives and \(\tau\) indicates that no synchronization primitive is associated with the transition. In this work we consider \(\text{Sync} = \{!, \?, \#, @\}\) where the symbols ! and ? indicate that the TA emits and receives a message, respectively, # denotes a broadcast synchronization sender, and @ denotes the broadcast synchronization receivers. We use the notation \(a\!, a\#\) and \(a\@\) to...
indicating the element \((a, ?), (a, !), (a, \#)\) and \((a, \oplus)\) such that \((a, ?), (a, !), (a, \#)\), \((a, \oplus)\) is contained in the set \([Act \times Sync]\).

**Definition 3.** A network \(N\) of TA is a set \(N = \{A_1, A_2, \ldots A_k\}\) of TA defined over the same set of atomic propositions \(AP\), actions \(Act_r\), variables \(Int\) and clocks \(X\).

We will also use \(N = A_0 || A_1 || \ldots A_k\) to indicate a network \(N\) of TA.

**Metric Interval Temporal Logic** [3]. An interval \(I\) is a convex subset of \(R^2\) of the form \((a, b)\) or \((a, \infty)\), where \(a \leq b\) are non-negative integers; symbol \(\langle\) is either \((\) or \([\); symbol \(\rangle\) is either \(\rangle)\) or \(\rangle\).

The syntax of MITL formulae is defined by the grammar \(\phi ::= \alpha | \phi ∧ \phi | \neg \phi | \phi U \phi\), where \(\alpha\) are atomic formulae. Since we assume that MITL is used to specify properties of TA enriched with variables, atomic formulae \(\alpha\) are either propositions of \(AP\) or formulae of the form \(n - d\), where \(n \in \text{Int}, d \in \mathbb{Z}\) and \(\neg \in \{\, <, =\}\). In the following we use \(AP_r\) to indicate the universe of the atomic formulae of the form \(n - d\).

The semantics of MITL is defined w.r.t. signals. Denote \(Z^\text{Int}\) the set of total functions from \(\text{Int}\) to \(\mathbb{Z}\). A signal \(\tau\) is a total function \(M : R^2_0 \rightarrow \varphi(AP) \times Z^\text{Int}\). Given a signal \(M\), the semantics of an MITL formula is defined as follows.

\[
\begin{align*}
M, t \models p & \iff p \in P \text{ and } (P, v_{\text{var}}) = M(t) \\
M, t \models n - d & \iff v_{\text{var}}(n) - d \text{ and } (P, v_{\text{var}}) = M(t) \\
M, t \models \neg p & \iff M, t \models \neg \phi \\
M, t \models \phi \land \psi & \iff M, t \models \phi \text{ and } M, t \models \psi \\
M, t \models \phi U I \psi & \iff \exists t' > t, t' - t \in I, M, t' \models \psi \text{ and } \forall t'' \in (t, t') M, t'' \models \phi
\end{align*}
\]

An MITL formula \(\phi\) is satisfiable if there exists a signal \(M\), such that \(M, 0 \models \phi\). In this case, \(M\) is a model of \(\phi\).

**CLTLoc.** Constraint LTL over clocks (CLTLoc) [10] is a temporal logic where formulae are defined over a finite set of atomic propositions and a set of dense variables over \(R^2_0\) representing clocks. CLTLoc with counters [22] (in the following indicated as CLTLoc) extends CLTLoc by supporting expressions over arithmetical variables.

CLTLoc allows for two kinds of atomic formulae: over clock and arithmetical variables. Examples of atomic formula over clock and arithmetical variables are \(x < 4\), where \(x\) is a clock and \(n + m < 4\), where \(n\) and \(m\) are in \(\mathbb{Z}\), respectively. CLTLoc also exploits the \(X\) modality applied to integer variables, introduced in [16]: if \(n\) is an integer variable, the term \(X(n)\) represents the value of \(n\) in the next position in the execution.

Given a finite set of clocks \(X\) and a finite set of integer variables \(Int\), a CLTLoc formula is defined by the grammar: \(\phi ::= p | x \sim c | exp_1 \sim exp_2 | X(n) \sim exp | \phi \land \phi | \neg \phi | X \phi | \phi U \phi\), where \(p \in Ap, c \in \mathbb{N}, x \in X, exp, exp_1\) and \(exp_2\) are arithmetic expressions over the set \(Int\) and elements of \(\mathbb{Z}\), \(n \in \text{Int}\) and \(\sim\) is a relation in \(\{<, =\}\). \(X, U\) are the usual “next” and “until” operators of LTL. When applied to a variable, operator \(X\) represents the next value of the variable.

The strict linear order \((\mathbb{N}, <)\) is the standard representation of positions in time. The interpretation of clocks is defined through a clock valuation \(\sigma : \mathbb{N} \times X \rightarrow R^2_0\) assigning, for every position \(i \in \mathbb{N}\), a real value \(\sigma(i, x)\) to each clock \(x \in X\). A clock \(x\) measures
the time elapsed since the last time when \( x = 0 \), i.e., the last “reset” of \( x \). The semantics of the evolution of time adopted for CLTLoc, is strict, namely the value of a clock must strictly increase in two adjacent time positions, unless it is reset (i.e., for all \( i \in \mathbb{N}, x \in X \), it holds that \( \sigma(i + 1, x) > \sigma(i, x) \), unless \( \sigma(i + 1, x) = 0 \)). In this case, \( \sigma \) is called clock assignment. The initial value \( \sigma(0, x) \) may be any non-negative value. We also assume that a clock assignment is such that \( \sum_{i \in \mathbb{N}} \delta_i = \infty \), i.e., time is always progressing.

The interpretation of variables is defined by a mapping \( \iota : \mathbb{N} \times Int \rightarrow \mathbb{Z} \) assigning, for every position \( i \in \mathbb{N} \), a value in \( \mathbb{Z} \) to each variable of set \( \text{Int} \). Given a valuation \( \iota \) and a position \( i \), we indicate by \( \text{exp}(i, \iota) \) the evaluation of \( \text{exp} \) obtained by replacing each arithmetical variable \( n \in \text{Int} \) that occurs in \( \text{exp} \) with value \( \iota(i, n) \). An interpretation of CLTLoc, is a triple \((\pi, \sigma, \iota)\), where \( \sigma \) is a clock assignment, \( \iota \) is a valuation of variables and \( \pi : \mathbb{N} \rightarrow \text{gt}(\text{AP}) \) is a mapping associating a set of propositions with each position \( i \in \mathbb{N} \). The formal semantics for CLTLoc, is omitted, details are in Appendix A1.

A CLTLoc, formula \( \phi \) is satisfiable if there exist an interpretation \((\pi, \sigma, \iota)\) such that \((\pi, \sigma, \iota), 0 \models \phi \). In this case, \((\pi, \sigma, \iota)\) is a model of \( \phi \). It is easy to see that CLTLoc, is undecidable, as it can encode a 2-counter machine; however, in this work we only use a decidable subset of CLTLoc, where the domain of arithmetical variables is finite.

### 3 Continuous time semantics for Timed Automata

The behavior of TA over time is described by means of execution traces that define the evolution of the APs, variables and clocks of the automata changing their values because discrete transitions are taken or because time elapses. A formal definition of the semantics of (network of) TA has to consider the following aspects: a) how an automaton progresses over the time by performing discrete transitions (liveness conditions) and b) how the automata synchronize when transitions labeled with \(!, ?, \#\) and \( @ \) are fired.

We only discuss the semantics of a network of TA with variables. The semantics of a single TA can be obtained by considering a network with a single TA. The semantics of a network of TA without variables can be obtained by assuming that each TA in the network does not contain any variable. Without loss of generality, the semantics is defined assuming that state invariants are convex, as non-convex ones can be reduced to the convex case. In this paper we consider integer variables with finite domains.

Given a set of clocks \( X \), a clock valuation is a function \( v : X \rightarrow \mathbb{R}_{\geq 0} \). Given a clock constraint \( \gamma_c \in \Gamma(X) \), write \( v \vDash \gamma_c \) to indicate that the clock valuation satisfies \( \gamma_c \). Given \( t \in \mathbb{R} \), \( v + t \) denotes the clock valuation mapping clock \( x \) to value \( v(x) + t \), i.e., \( (v + t)(x) = v(x) + t \) for all \( x \in X \). Similarly, given a set of arithmetical variables \( \text{Int} \), a variable valuation is a function \( v_{\text{var}} : \text{Int} \rightarrow \mathbb{Z} \). Given a variable constraint \( \gamma_{v\text{ar}} \in \Gamma(\text{Int}) \), we write \( v_{\text{var}} \vDash \gamma_{v\text{ar}} \) when the variable valuation \( v_{\text{var}} \) satisfies \( \gamma_{v\text{ar}} \). Given a transition \( t = q \xrightarrow{\gamma_c,\gamma_{v\text{ar}},\alpha,S,A} q' \), we say that \( t \) is enabled when the clock valuation satisfies \( \gamma_c \) and the variable valuation satisfies \( \gamma_{v\text{ar}} \).

Let \( q \xrightarrow{\gamma_c,\gamma_{v\text{ar}},\alpha,S,A} q' \) be a transition \( t \) of a TA \( \mathcal{A} \) and \( v_{\text{var}}, v'_{\text{var}} \) be two assignments of variables in \( \text{Int} \). Moreover, let \( \text{exp}(v_{\text{var}}) \) be the value of \( \text{exp} \) obtained by replacing the occurrences of the variables in \( \text{exp} \) with the value defined by \( v_{\text{var}} \). We write \( v'_{\text{var}} \vDash u \)
Definition 4. Let \( N = \mathcal{A}_0 \parallel \mathcal{A}_1 \parallel \ldots \parallel \mathcal{A}_K \) be a network \( N \) of \( K \) TA. A configuration of \( N \) is a tuple \((1, v_{\text{var}}, v_c)\) where 1 is a vector \([q_1, \ldots, q_K]\) where \( q_1, \ldots, q_K \) are states of \( \mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_K \), \( v_{\text{var}} \) (resp., \( v_c \)) is a variable (resp., clock) evaluation for the set comprising all integer variables \( \mathbb{Int} \) (resp., clocks \( X \)) appearing in the TA of the network.

In the rest of the paper when network of TA are considered an automaton \( \mathcal{A}_k \) within the network will be indicated as \( \mathcal{A}_k = (AP, X, Act_k, Int, Q_k, q_{0_k}, v_{\text{var} k}, Inv_k, I_k, T_k) \).

We first introduce the notion of transition between configurations which describes how the configuration change by the firing of one or more discrete transitions of the TA in the network. It is allowed that some automata in the network take a transition while the remaining others do not fire a transition and keep their state unchanged. Firing a transition labeled with the null event \( \tau \) (i.e., a transition that does not synchronize) is however different from not taking a transition at all. To define the change between two configurations, we indicate that an automaton \( k \) does not perform any transition in \( T_k \) with the special symbol \( \not\rightarrow \).

We use the notation \( 1[k] \) to indicate the state of automaton \( \mathcal{A}_k \)—i.e., if \( 1[k] = q \), then automaton \( \mathcal{A}_k \) is in state \( q \), where we assume the states of each automaton to be numbered, with 0 indicating the initial state.

Definition 5. Let \( N = \mathcal{A}_0 \parallel \mathcal{A}_1 \parallel \ldots \parallel \mathcal{A}_K \) be a network \( N \) of \( K \) TA and \((1, v_{\text{var}}, v_c)\). \((1', v'_{\text{var}}, v')\) be two configurations, and let \( e \) be either a delay \( \delta \in \mathbb{R}_{>0} \) or a tuple \( \lambda = (\lambda_1, \lambda_2, \ldots, \lambda_K) \) such that for every \( 1 \leq k \leq K \) it holds that \( \lambda_k \in Act_k \cup \{\bot\} \), and there is \( 1 \leq i \leq K \) such that \( \lambda_i \neq \bot \). It holds that \((1, v_{\text{var}}, v) \xrightarrow{e} (1', v'_{\text{var}}, v')\) if the following conditions hold:

1. \( e = \lambda \) and
   (a) for each \( \lambda_k \neq \bot \) in \( \lambda \) there is a transition \( 1[k] \xrightarrow{\gamma_{\text{var}}, \lambda_k \cdot S, A} 1'[k] \) in \( \mathcal{A}_k \) whose guards hold (i.e., \( \gamma_{\text{var}} \models \gamma_{\text{var}} \)), the clocks in \( X \) are reset—i.e., \( v'(x) = 0 \) holds for all \( x \in S \)—and the assignments in \( A \) are performed—i.e., \( v'_{\text{var}} \models A(\gamma_{\text{var}}) \) holds;
   (b) for each \( \lambda_k = \bot \) in \( \lambda \), \( 1'[k] = 1[k] \);
   (c) for each clock \( x \in X \) (resp., integer variable \( n \in \mathbb{Int} \)), if \( x \) (resp., \( n \)) does not appear in any \( S \) (resp., it is not assigned by any \( A \)) of one of the transitions taken by \( \mathcal{A}_1, \ldots, \mathcal{A}_K \), then \( v'(x) = v(x) \) (resp., \( v'_{\text{var}}(n) = v_{\text{var}}(n) \));
   (d) the invariants of the states in the target configuration are satisfied—i.e., \( v' \models Inv(1'[k]) \) for all \( 1 \leq k \leq K \).
2. \( e = \delta, v' = v + \delta, v' \models Inv(1[k]) \) for all \( 1 \leq k \leq K \), \( 1'[k] = 1[k] \), and \( v'_{\text{var}} = v_{\text{var}} \).

In the rest of the paper, we indicate with \( \Lambda[k] \) the value \( \lambda_k \) of \( \Lambda = (\lambda_1, \lambda_2, \ldots, \lambda_K) \). We now provide a general notion of trace. We will later discuss restrictions on traces, according to various liveness and synchronization properties among automata.

Definition 6. Let \( N = \mathcal{A}_0 \parallel \mathcal{A}_1 \parallel \ldots \parallel \mathcal{A}_K \) be a network \( N \) of \( K \) TA, a trace is an infinite sequence \((1_0, v_{\text{var}0}, v_0), (1_1, v_{\text{var}1}, v_1), (1_2, v_{\text{var}2}, v_2), \ldots\) such that:
Formulation of the Semantics

**Strong transition liveness**
For every \( h \geq 0 \) and \( 0 < k \leq K \), there exists \( j > h \) such that \((1, j, \var v_{\text{var}, j}, v_j) \xrightarrow{A_j} (1_{j+1}, \var v_{\text{var}, j+1}, v_{j+1})\) belongs to the trace and \( A_j[k] \neq \_\).

**Weak transition liveness**
For every \( h \geq 0 \) there exist \( 0 < k \leq K \) and \( j > h \) such that \((1, j, \var v_{\text{var}, j}, v_j) \xrightarrow{A_j} (1_{j+1}, \var v_{\text{var}, j+1}, v_{j+1})\) belongs to the trace and \( A_j[k] \neq \_\).

**Strong guard liveness**
For every \( h \geq 0 \) and \( 0 < k \leq K \), there exist \( j > h \) and a configuration \((1, j, \var v_{\text{var}, j}, v_j)\) in the trace such that \(1[k]\) has an outgoing transition \(1[k] \xrightarrow{y, \var v_{\text{var} \in S.A}} 1'\) for which \( v_{\text{var}, j} \models \gamma v_{\text{var}}\) and \( v_j \models \gamma e\) hold.

**Weak guard liveness**
For every \( h \geq 0 \) there exist \( 0 < k \leq K \), \( j > h \), and a configuration \((1, j, \var v_{\text{var}, j}, v_j)\) in the trace such that \(1[k]\) has an outgoing transition \(1[k] \xrightarrow{y, \var v_{\text{var} \in S.A}} 1'\) for which \( v_{\text{var}, j} \models \gamma v_{\text{var}}\) and \( v_j \models \gamma e\) hold.

Table 1. Formal definition of different liveness properties for traces.

1. for all \( h \in \mathbb{N} \) it holds that \((1_h, \var v_{\text{var}, h}, v_h) \xrightarrow{e_0} (1_{h+1}, \var v_{\text{var}, h+1}, v_{h+1})\);
2. \( e_0 = \delta_0 \) for some \( \delta_0 \in \mathbb{R}_{\geq 0} \);
3. if \((1_h, \var v_{\text{var}, h}, v_h) \xrightarrow{e_0} (1_{h+1}, \var v_{\text{var}, h+1}, v_{h+1})\) \( \xrightarrow{e_{h+1}} (1_{h+2}, \var v_{\text{var}, h+2}, v_{h+2})\) holds, then \( e_h = \delta_h\), or \( e_{h+1} = \delta_{h+1} \) for some \( \delta_h, \delta_{h+1} \in \mathbb{R}_{\geq 0} \);
4. for all \( 1 \leq k \leq K \), it holds that \(1_0[k] = 0, v_0 \models \text{Inv}(1_0[k])\), for each \( x \in X \) it holds that \( v_0(x) = 0\), and for each \( n \in \text{Int} \), \( v_{\text{var}}(n) = v_{\text{var}, k}(n)\) also holds.

Condition 3 of Definition 8 states that there cannot be two consecutive configuration changes due to transitions taken.

We indicate a trace \((1_0, \var v_{\text{var}, 0}, v_0), e_0, (1_1, \var v_{\text{var}, 1}, v_1), e_1, (1_2, \var v_{\text{var}, 2}, v_2), e_2, \ldots\) as \((1_0, \var v_{\text{var}, 0}, v_0) \xrightarrow{e_0} (1_1, \var v_{\text{var}, 1}, v_1) \xrightarrow{e_1} (1_2, \var v_{\text{var}, 2}, v_2) \xrightarrow{e_2} \ldots\).

Definition 7. Given a trace \( \eta \) of the form \((1_0, \var v_{\text{var}, 0}, v_0) \xrightarrow{e_0} (1_1, \var v_{\text{var}, 1}, v_1) \xrightarrow{e_1} (1_2, \var v_{\text{var}, 2}, v_2) \xrightarrow{e_2} \ldots\) in which the sequence of delays is \( \delta_0, \delta_1, \ldots \in \mathbb{R}_{\geq 0}\), the signal \( M_\eta \) associated with trace \( \eta \) is the function \( M_\eta : \mathbb{R}_{\geq 0} \rightarrow \phi(\text{AP}) \times \mathbb{Z}^{\text{Int}}\) such that for all \( r \in \mathbb{R}_{\geq 0}\):

\[- \text{ if } r \leq \delta_0 \text{ then } M_\eta(r) = (\cup_{0 \leq k < h} L(1_0[k]), v_{\text{var}, 0}) \text{ holds};\]
\[- \text{ if } \sum_{i=0}^{h-1} \delta_i < r \leq \sum_{i=0}^{h} \delta_i + \delta_h \text{ for some } h \in \mathbb{N}_{\geq 0} \text{ then } M_\eta(r) = (\cup_{0 \leq k < h} L(1_0[k]), v_{\text{var}, h}).\]

**Liveness.** The proposed notion of trace allows for the possibility that, from a certain point on, no automaton takes a transition—i.e., only configuration changes of the form \((1_h, \var v_{\text{var}, h}, v_h) \xrightarrow{e_0} (1_{h+1}, \var v_{\text{var}, h+1}, v_{h+1})\) occur. Nevertheless, one is typically interested in “live” traces, in which some transition is eventually taken. Let us consider a trace \((1_0, \var v_{\text{var}, 0}, v_0) \xrightarrow{e_0} (1_1, \var v_{\text{var}, 1}, v_1) \xrightarrow{e_1} (1_2, \var v_{\text{var}, 2}, v_2) \xrightarrow{e_2} \ldots\), we have the following notions of liveness, whose formal definitions are given in Table 1.

- **Strong transition liveness:** at any time instant, it is true that eventually each automaton of the network performs a transition.
Formulation of the Semantics

Every configuration change \((I_k, v_{var,k}, v_k) \rightarrow (I_{k+1}, v_{var,k+1}, v_{k+1})\) in the trace is such that for every \(0 < k \leq K\) such that \(a! = \lambda_0[k]\), there exists exactly one \(0 < k' \leq K\) such that \(a? = \lambda_0[k']\), and vice-versa.

| Type              | Formulation of the Semantics                                                                 |
|-------------------|---------------------------------------------------------------------------------------------|
| Channels          | Every configuration change \((I_k, v_{var,k}, v_k) \rightarrow (I_{k+1}, v_{var,k+1}, v_{k+1})\) in the trace is such that for every \(0 < k \leq K\) such that \(a! = \lambda_0[k]\), there exists exactly one \(0 < k' \leq K\) such that \(a? = \lambda_0[k']\), and vice-versa. |
| Broadcast         | Every configuration change \((I_k, v_{var,k}, v_k) \rightarrow (I_{k+1}, v_{var,k+1}, v_{k+1})\) in the trace is such that for every \(0 < k \leq K\) such that \(a\# = \lambda_0[k]\), for every \(0 < k' \leq K\), with \(k' \neq k\), either \(a@ = \lambda_0[k']\), or it does not exist any transition \(1_a[k'] \rightarrow 1'\) in \(A_k\) such that \(v_{var,k} \models \gamma_{var}\) and \(v_k \models \gamma_c\). |

Table 2. Definition of different constraints on traces depending on synchronization primitives.

- **Weak transition liveness**: at any time instant, it is true that eventually at least one of the automata of the network performs a transition.

- **Strong guard liveness**: at any time instant, for each automaton the values of clocks and variables will eventually enable one of its transitions. Intuitively, this definition specifies that for each automaton one of its transitions is eventually enabled, but it does not force the transition to be taken.

- **Weak guard liveness**: at any time instant, there exists an automaton such that eventually the values of its clocks and variables will enable one of its transitions.

**Synchronization.** Section 2 introduced several qualifiers—!, ?, #, and @—for actions, with the goal of capturing different ways in which the automata of a network can synchronize. Qualifiers ! and ? are used to describe a so-called channel-based synchronization, whereas qualifiers # and @ describe a broadcast synchronization. Channel-based and broadcast synchronizations can be arbitrarily mixed in the same configuration change, but they must respect the following constraints, whose formalization is provided in Table 2.

- **Channel-based synchronization**: whenever the configuration changes due to \(A\) event, every “sending” (qualifier !) action is matched by exactly one corresponding “receiving” (qualifier ?) action on the same channel (e.g., \(a!\) and \(a?\)).

- **Broadcast synchronization**: if an automaton \(A_k\) executes a \(a\#\) transition, for every other automaton \(A_{k'}\) of the network, either it takes a transition labeled with \(a@\) or it does not exist any enabled transition for \(A_{k'}\) labeled with \(a@\) that is enabled.

Given a liveness notion, a synchronization paradigm and a network of TAs defined over the sets of events \(Act\), the set of clocks \(X\) and the set of integer variables \(Int\) where each \(A_k \in \mathcal{N}\) is defined as \(A_k = (AP, X, Act_k, Int_k, Q_k, q_{0,k}, v_{0,k}, Inv_k, L_k, T_k)\). We describe how to construct a CLTL formula \(\Phi_N = \phi_{clock} \land \phi_N \land \phi_{sync} \land \phi_{lv}\) whose models represent signals.

### 4 From timed automata to CLTL

This section describes how to convert a network of TAs into a CLTL formula. Let us consider a network \(\mathcal{N} = A_0 \parallel A_1 \parallel \ldots \parallel A_K\) of TA defined over the sets of events \(Act\), the set of clocks \(X\) and the set of integer variables \(Int\) where each \(A_k \in \mathcal{N}\) is defined as \(A_k = (AP, X, Act_k, Int_k, Q_k, q_{0,k}, v_{0,k}, Inv_k, L_k, T_k)\). We describe how to construct a CLTL formula \(\Phi_N = \phi_{clock} \land \phi_N \land \phi_{sync} \land \phi_{lv}\) whose models represent signals.
Using these atoms, a network of TA is encoded using the formulae of Figure 3.

| Formula | Meaning |
|---------|---------|
| \( \psi_1 := \bigwedge_{0 \leq k \leq \kappa} (1[k] = 0) \) | \( n \) is a variable of \( \kappa \) values |
| \( \psi_2 := \bigwedge_{n \in \text{Int}} \nu_{\text{var}}(n) \) | each variable \( \nu_{\text{var}} \) is assigned to its initial value |
| \( \psi_3 := \bigwedge_{0 \leq k \leq \kappa} (1[k] = q) \rightarrow \nu(1[k]) \) | each variable \( \nu \) is assigned to the initial value |
| \( \psi_4 := \bigwedge_{0 \leq k \leq \kappa} \varphi \in \varphi_{\text{var}} (q) \) | each variable \( \varphi \) is assigned to the initial value |
| \( \psi_5 := \bigwedge_{n \in \text{Int}} \psi \in \psi_{\text{var}}Q_0, 1[k] = q \) | each variable \( \psi \) is assigned to the initial value |
| \( \psi_6 := \bigwedge_{0 \leq k \leq \kappa} \nu(t[k] = t) \rightarrow (1[k] = t) \wedge \varphi_{\text{var}} \wedge \nu(1[k] = t) \wedge \varphi \wedge \varphi_{\text{syn}} \wedge \nu(1[t] = t) \) | each variable \( \psi \) is assigned to the initial value |

**Fig. 3. Encoding of the automaton.**

induced by the traces of \( \mathcal{N} \). Formulae \( \varphi_{\text{clock}}, \varphi_{\text{syn}}, \varphi_{\text{sync}} \) and \( \varphi_{\text{iv}} \) are used to encode a set of constraints on clocks used in \( \varphi_{\text{N}} \), the behavior of the network, a set of constraint on the firing of the transitions that depend on the synchronization mechanism, and the liveness constraints, respectively. Different semantics can be be considered by simply changing formulae \( \varphi_{\text{sync}} \) and \( \varphi_{\text{iv}} \).

Formula \( \varphi_{\text{clock}} \) is discussed in the Appendix since it only constraints clock assignments to ensures correctness of \( \varphi_{\text{N}} \).

**Encoding the network (\( \varphi_{\text{N}} \)).** An array \( k = [1_1, 1_2, \ldots, 1_k] \) of integer variables is used to encode the states of each automaton in the network. The value of each variable \( 1[k] \) is in the interval \([0, |Q_k| - 1]\), where \(|Q_k|\) is the cardinality of the set of the states of the TA \( \mathcal{A}_k \). We assume that each location is associated with a number in \([0, |Q_k| - 1]\) and that the initial state of each automaton is associated with the value 0. The variable \( 1_k \) is associated with the value \( q \in [0, |Q_k| - 1] \) if the automaton is in the location associated with the index \( q \) over the left-closed/right-open interval \([a_i, b_i] \), with \( I_i = (a_i, b_i) \). For each variable \( n \in \text{Int} \), we introduce a corresponding \( \text{CLTL}^c \) integer variable. An array \( t = [t_1, t_2, \ldots, t_k] \) of integer variables is used to encode the transitions of each TA in the network. The value of each variable \( t[k] \) is in the set \([0, |T_k| - 1] \cup \{\bot\}\), where \(|T_k|\) is the cardinality of the set of the transitions of the TA \( \mathcal{A}_k \) that are uniquely identified by an integer value and \( \gamma \) is a symbol representing the absence of firing of transitions in \( T_k \).

Using these atoms, a network of TA is encoded using the formulae of Figure 3:

- \( \psi_1 \) specifies that each automata is initially in its initial state.
- \( \psi_2 \) specifies that each variable \( n \) is assigned to its initial value \( \nu_{\text{var}}(n) \).
- \( \psi_3 \) specifies that the invariant of the initial state of each TA holds initially.
- \( \psi_4 \) specifies that if the TA \( \mathcal{A}_k \) is in its state \( q_k \), the invariant of \( q_k \) must hold in the next time instant. Indeed, either in the next time instant the automaton is in the state \( q_k \) or a transition is fired moving the automaton in \( q_k \). When the transition is fired, the invariant of the source state of the transition must not be violated. Additional details are in Appendix A2.
- \( \psi_5 \) specifies that an atomic proposition \( \psi \in \text{AP} \) holds if and only if at least one of the automaton is in a state in which that proposition holds.

Formula \( \varphi_6 \ldots \varphi_9 \) encode the transition relation of the automata in \( \mathcal{N} \). Specifically:
– Formula $\varphi_8$ encodes the effects of the execution of a transition of the automata. Let $t = q \xrightarrow{\gamma_i ; t \in I_i} q'$ be a transition of automaton $\mathcal{A}_i$ the followings are ensured:

1. If the formula holds at position $i$, in position $i + 1$ the automaton is in state $q'$, representing the fact that in the interval $I_i$ the state of the automaton is $q$.
2. $\phi_{\gamma_i}$ holding in $i + 1$ specifies that guard $\gamma_i$ holds.
3. $\phi_9$ specifies that each clock $x \in S$ is reset.
4. $\phi_{\text{var}}$ ensures that the condition on the integer variable is satisfied when a transition is taken.
5. $\phi_A$ encodes the effect of the assignments.
6. $\text{Inv}$ forces the invariant of the destination state of the transition to hold.

Formula $\varphi_9$ does not encode occurrences of symbols in $\text{Act}$ since they do not constrain configurations of the automata. They will be used to encode the synchronization mechanism.

– $\varphi_7$ specifies that if an automaton $\mathcal{A}_i$ changes its state a transition must be taken.

– $\varphi_6$ defines that a clock is reset only if a transition that resets the clock is performed, i.e., clocks can not be spontaneously reset.

– $\varphi_8$ specifies that, if the value of a variable changes, the automaton must have fired a transition that sets the value of that variable.

Formula $\varphi_N$ is defined as $\varphi_1 \land \varphi_2 \land G( \bigwedge_{3 \leq i < N} \varphi_i)$.

**Encoding the synchronization conditions ($\phi_{\text{sync}}$).** Table 3 presents the formulae to encode the channel based and broadcast synchronization.

– **Channel based synchronization.** It is encoded by the formula $\phi_{\text{sync}} := G(v_1 \land v_2)$. Formula $v_1$ specifies that any sending event $i.a!$ in an automaton $\mathcal{A}_i$ of the network must be matched by exactly one corresponding receiving event $j.a?$ in another au-
tomaton $\mathcal{A}_j$. Formula $\nu_2$ specifies that any receiving event $i.a?$ must be matched by exactly one corresponding sending event $j.a!$.

- **Broadcast synchronization.** It is encoded by the formula $\phi_{\text{sync}} := G(\nu_1 \land \nu_2)$. Formula $\nu_1$ specifies that if an event is sent, the other automata either they received it or they do not have any enabled transition that receives the event. Formula $\nu_2$ specifies that if an event is received someone has sent it.

**Encoding the liveness conditions ($\phi_{\text{liv}}$).** Table 4 presents different alternatives for formula $\phi_{\text{liv}}$ that encodes the liveness condition.

- **Strong transition liveness.** It states that globally finally at least one of the transitions of each automaton in $\mathcal{N}$ is fired.
- **Weak transition liveness.** It states that globally finally at least one of the transitions of one of the automata in $\mathcal{N}$ is fired.
- **Strong guard liveness.** It states always for each automaton at least one of the guards of the outgoing transitions of its current state must be enabled.
- **Weak guard liveness.** It states always there exist at least one of the automata that it is in one state that eventually satisfies the guards of one of its outgoing transitions.

**Checking the satisfaction of MITL formulae over TA.** The MITL formulae express properties on the value of the integer variables in $\text{Int}$ and the labels of locations in $\text{AP}$ over the time. The verification problem can then be formulated as follows.

**Definition 8.** Given a liveness notion, a synchronization paradigm, a network of TA $\mathcal{N} = \mathcal{A}_0 \parallel \mathcal{A}_1 \parallel \ldots \mathcal{A}_K$ and an MITL formula $\phi$, $\mathcal{N}$ satisfies $\phi$, i.e., $\mathcal{N} \models S \phi$, if, and only if, every trace $\eta$ of $\mathcal{N}$ that belongs to $S$ generates a signal $M_\eta$ such that $M_\eta \models \phi$.

Given a network of TA $\mathcal{N}$, a MITL formula $\psi$ and the CLTLloc translations $\Phi_\mathcal{N}$ and $\Phi_{\neg\psi}$ of $\mathcal{N}$ and $\neg\psi$, respectively, the model-checking problem $\mathcal{N} \models S \psi$ is reduced to the satisfiability of the $\Phi_\mathcal{N} \land \Phi_{\neg\psi}$ where $\Phi_{\neg\psi}$ is the encoding of the MITL formula $\neg\phi$ in CLTLloc. This encoding has been extended to deal with formulae of the form $n \sim d$. Details and remarks on the encoding and proof of correctness are in Appendixes A3-A4. They also include a discussion on how MITL are encoded in CLTLloc and how formulae of the form $n \sim d$ are managed.

## 5 Discussion and conclusions

This paper presented a a flexible approach for checking TA with MITL considering a continuous time semantic. The technique relies on an intermediate artifact—i.e., a logic formula—in which both the model and the property are encoded. The intermediate artifact is then evaluated using suitable satisfiability checkers.
The approach successfully created a boundary between software engineers and formal method concerns. On the one hand, different semantics of TA can be developed by mapping the semantics into the intermediate language. On the other hand, the decision procedure for the intermediate language can independently be improved. Correctness is ensured as long as the intermediate language is not changed.

References

1. M. Abadi and L. Lamport. An old-fashioned recipe for real time. Transactions on Programming Languages and Systems, pages 1543–1571, 1994.
2. R. Alur and D. L. Dill. A theory of timed automata. Theoretical computer science, 126(2):183–235, 1994.
3. R. Alur, T. Feder, and T. A. Henzinger. The benefits of relaxing punctuality. Journal of the ACM (JACM), 43(1):116–146, 1996.
4. L. Baresi, M. M. Pourhashem Kallehbasti, and M. Rossi. How Bit-vector Logic Can Help Improve the Verification of LTL Specifications over Infinite Domains. In Symposium on Applied Computing, pages 1666–1673, New York, NY, USA, 2016. ACM.
5. C. Barrett, R. Sebastiani, S. A. Seshia, and C. Tinelli. Satisfiability Modulo Theories, volume 185 of Frontiers in Artificial Intelligence and Applications, chapter 26, pages 825 – 885.
6. M. M. Bersani, C. A. Furia, M. Pradella, and M. Rossi. Integrated modeling and verification of real-time systems through multiple paradigms. In Software Engineering and Formal Methods, SEFM, pages 13–22. IEEE, 2009.
7. M. M. Bersani, M. Rossi, and P. San Pietro. Deciding the satisfiability of MITL specifications. In International Symposium on Games, Automata, Logics and Formal Verification (GandALF), pages 64–78, 2013.
8. M. M. Bersani, M. Rossi, and P. San Pietro. An SMT-based approach to satisfiability checking of MITL. Information and Computation, 245:72–97, 2015.
9. M. M. Bersani, M. Rossi, and P. San Pietro. A logical characterization of timed regular languages. Theoretical Computer Science, 2016.
10. M. M. Bersani, M. Rossi, and P. San Pietro. A tool for deciding the satisfiability of continuous-time metric temporal logic. Acta Informatica, 53(2):171–206, 2016.
11. A. Biere and R. Bloem, editors. Yices 2.2. Springer International Publishing, 2014.
12. P. Bouyer, F. Chevalier, and N. Markey. On the expressiveness of TPTL and MTL. Information and Computation, 208(2):97–116, 2010.
13. T. Brihaye, G. Geeraerts, H.-M. Ho, and B. Monmege. Timed-automata-based verification of MITL over signals. In International Symposium on Temporal Representation and Reasoning (TIME 2017), page to appear,, May 2017.
14. R. E. Bryant. Graph-based algorithms for boolean function manipulation. IEEE Transactions on Computers, 35(8):677–691, Aug. 1986.
15. A. Carioni, S. Ghilardi, and S. Ranise. MCMT in the land of parametrized timed automata. In International Verification Workshop, VERIFY, pages 47–64, 2010.
16. S. Demri and D. D’Souza. An automata-theoretic approach to constraint LTL. Information and Computation, 205(3):380–415, 2007.
17. D. L. Dill. Timing assumptions and verification of finite-state concurrent systems. In International Workshop on Automatic Verification Methods for Finite State Systems, pages 197–212, New York, NY, USA, 1990. Springer-Verlag New York, Inc.
18. D. D’Souza and P. Prabhakar. On the expressiveness of mtl in the pointwise and continuous semantics. International Journal on Software Tools for Technology Transfer (STTT), 9(1):1–4, 2007.
19. C. A. Furia, D. Mandrioli, A. Morzenti, and M. Rossi. Modeling Time in Computing. EATCS Mon. in Theoretical Computer Science. Springer, 2012.
20. R. Kindermann, T. Junttila, and I. Niemelä. Bounded model checking of an mtiil fragment for timed automata. In Application of Concurrency to System Design (ACSD), pages 216–225. IEEE, 2013.
21. K. G. Larsen, P. Pettersson, and W. Yi. Uppaal in a nutshell. International Journal on Software Tools for Technology Transfer, 1(1):134–152, 1997.
22. F. Marconi, M. M. Bersani, M. Erascu, and M. Rossi. Towards the formal verification of data-intensive applications through metric temporal logic. In International Conference on Formal Engineering Methods, pages 193–209. Springer, 2016.
23. P. M. Merlin. A study of the recoverability of computing systems. PhD thesis.
24. J. Ouaknine and J. Worrell. Some recent results in metric temporal logic. In FORMATS, volume 5215 of LNCS, pages 1–13. Springer, 2008.
25. F. Wang. Symbolic verification of complex real-time systems with clock-restriction diagram. In Proceedings of the IFIP TC6/WG6.1 - 21st International Conference on Formal Techniques for Networked and Distributed Systems, FORTE '01, pages 235–250, Deventer, The Netherlands, The Netherlands, 2001. Kluwer, B.V.
26. S. Yovine. Kronos: A verification tool for real-time systems. (kronos user’s manual release 2.2). International Journal on Software Tools for Technology Transfer, 1:123–133, 1997.

Appendix

A1: Background

This Appendix contains additional remarks and details related with Section 2.

Timed automata. The following remark discusses inconsistent assignments in which a transition of a TA assigns a variable to multiple values.

Remark 1. An assignment $A \in \varphi(\text{Assign}(\text{Int}))$ might be inconsistent, i.e., a variable can be assigned to multiple values. For example, the assignment $A = \{x = 2, x = 3\}$ is inconsistent since two values are assigned to variable $x$. In this case, any transition defined with $A$ can not be fired.

The following remark discusses how local clocks and variables can be encoded in a TA.

Remark 2. Given a set of clock $X$ of $N$, a clock $x \in X$ is a local clock of an automaton $\mathcal{A}_i \in N$ if and only if $x$ is used in the invariants, guards or resets of $\mathcal{A}_i$ and it does not exist another automaton $\mathcal{A}_j \in N$, such that $\mathcal{A}_i \neq \mathcal{A}_j$, that uses the clock $x$ in its invariants, guards or resets. Given a set of variables $\text{Int}$ of $N$, a variable $v \in \text{Int}$ is a local variable of an automaton $\mathcal{A}_i \in N$ if and only if $v$ is used in the guards or resets of $\mathcal{A}_i$ and it does not exist another automaton $\mathcal{A}_j \in N$, such that $\mathcal{A}_i \neq \mathcal{A}_j$, that uses the variable $v$ in its guards or reset.

CLTLoc. In the following we presents a condition for CLTLoc, that ensures that time strictly progresses at the same rate for every clock. Specifically, to ensure that time strictly progresses at the same rate for every clock, $\sigma$ must satisfy the following
condition: for every position \( i \in \mathbb{N} \), there exists a “time delay” \( \delta_i > 0 \) such that for every clock \( x \in X \):

\[
\sigma(i + 1, x) = \begin{cases} 
\sigma(i, x) + \delta_i & \text{progress} \\
0 & \text{reset } x
\end{cases}
\]

We then present the semantics of CLTLoc\(_n\). Let \( x \) be a clock, \( n \) be a variable and \( c \) a constant in \( \mathbb{N} \), the semantic of CLTLoc\(_n\) at a position \( i \in \mathbb{N} \) over an interpretation \((\pi, \sigma, i)\) is defined as follows (standard LTL modalities are omitted):

\[
\begin{align*}
(\pi, \sigma, i), i \models p & \iff p \in \pi(i) \text{ for } p \in AP \\
(\pi, \sigma, i), i \models x \sim c & \iff \sigma(i, \alpha_1) \sim c \\
(\pi, \sigma, i), i \models \exp_1 \sim \exp_2 & \iff \exp_1(i, i) \sim \exp_2(i, i) \\
(\pi, \sigma, i), i \models X(n) \sim \exp & \iff (i + 1, n) \sim \exp(i, i) \\
(\pi, \sigma, i), i \models \sim \phi & \iff (\pi, \sigma, i), i \not\models \phi \\
(\pi, \sigma, i), i \models \phi \land \psi & \iff (\pi, \sigma, i), i \models \phi \text{ and } (\pi, \sigma, i), i \models \psi \\
(\pi, \sigma, i), i \models X(\phi) & \iff (\pi, \sigma, i), i + 1 \models \phi \\
(\pi, \sigma, i), i \models \mathcal{P}(\phi) & \iff (\pi, \sigma, i), i - 1 \models \phi \land i > 0 \\
(\pi, \sigma, i), i \models \phi \mathcal{U} \psi & \iff \exists j \geq i : (\pi, \sigma, i), j \models \psi \land \forall i \leq j, (\pi, \sigma, i), i \models \phi \\
(\pi, \sigma, i), i \models \phi \mathcal{S} \psi & \iff \exists 0 \leq j < i : (\pi, \sigma, i), j \models \psi \land \forall j < n \leq i (\pi, \sigma, i), n \models \phi
\end{align*}
\]

Modalities such as “eventually” (\( \mathcal{F} \)), “globally” (\( \mathcal{G} \)), and “release” (\( \mathcal{R} \)) are defined as usual.

### A2: Continuous time semantics for Timed Automata

This Appendix contains additional remarks and details related with Section 3.

We provide an additional remark on how \( \models_u \) deals with transitions in which a variable is assigned to multiple distinct values.

**Remark 3.** Relation \( \models_u \) does not hold for inconsistent transitions, i.e., when a variable is assigned to multiple distinct values. For example, if \( A = \{ x = 2, x = 3 \} \), it does not exist any assignment to the variable \( x \) such that \( x = 2 \) and \( x = 3 \).

We discuss how the introduced semantics allows multiple consecutive configuration changes due to delays.

Even under the liveness conditions defined above, the introduced semantics for networks of timed automata allows multiple consecutive configuration changes due to delays (i.e., sequences of the form \((I_h, v_{\text{var}, h}, v_h) \xrightarrow{\delta_h} (I_{h+1}, v_{\text{var}, h+1}, v_{h+1}) \xrightarrow{\delta_{h+1}} (I_{h+2}, v_{\text{var}, h+2}, v_{h+2}))

**Lemma 1.** Let \( N = \mathcal{A}_0 \parallel \mathcal{A}_1 \parallel \ldots \parallel \mathcal{A}_K \) be a network \( N \) of \( K \) TA and one of its trace \((I_0, v_{\text{var}, 0}, v_0), e_0, (I_1, v_{\text{var}, 1}, v_1), e_1, (I_2, v_{\text{var}, 2}, v_2), e_2, \ldots \) there exists a, equivalent trace \((I_0', v_{\text{var}, 0}', v_0'), e_0', (I_1', v_{\text{var}, 1}', v_1'), e_1', (I_2', v_{\text{var}, 2}', v_2'), e_2', \ldots \) of \( N \) such that for every \( h > 0 \) s.t. \( h \% 2 = 0 \) \( e_h = \delta_h \) and \( e_{h+1} = \Lambda_{h+1} \).
same variable. For example, if transitions synchronously if they are inconsistent, i.e., they assign multiple distinct values to the same variable.

Remark 4. Two transitions \( t_1 \) and \( t_2 \) of two automata of the network cannot be fired synchronously if they are inconsistent, i.e., they assign multiple distinct values to the same variable. For example, if transitions \( t_1 \) and \( t_2 \) assign, respectively, values 2 and 3 to variable \( x \), they cannot be synchronously fired.

Figure 4 shows a trace of the automaton depicted in Fig. 2(b) that consists of various time transitions (represented for convenience with a small vertical bar over the arrows connecting the configurations) and three discrete transitions associated with events \( e_1 \), \( e_2 \) and \( e_3 \) fired in sequence one after the other. To facilitate readability, transitions are marked in the figure with \( t(e_1) \), \( t(e_2) \) and \( t(e_3) \). Each discrete and time transition corresponds to a unique position in the CLTLoc\(_c\) model. The first area below the trace shows, the discrete positions \( i \) of the CLTLoc\(_c\) model. In the second segment, for each position \( i \), the values of the variables representing location 1, variable \( d \) and the value of clock \( x \) are shown (a dot is a positive value progressing in a monotonic manner with respect to the previous one). At position 4, clock \( x \) is evaluated with the constraint \( x < 5 \) and reset at the same time, because of the occurrence of the discrete transition labeled with

\[
\begin{array}{cccccccccc}
1 & = & 1_0 & 1_0 & 1_2 & 1_1 & 1_1 & 1_2 & 1_2 & 1_0 \\
\delta = & 0 & 0 & 0 & 0 & 2 & 2 & 2 & 1 & 0 \\
x = & 0 & 0.7 & 3.2 & 4.5 & 10 \\
\end{array}
\]

Fig. 4. Interpretation of atom in \( \varphi_{N'} \).

Proof. The equivalence of a finite sequence of delays \( (1_{t_1}, v_{var,t_1}, v_{t_1}) \overset{\delta_t}{\rightarrow} (1_{t_2}, v_{var,t_2}, v_{t_2}) \) with a single delay \( (1_h, v_{var,t}, v_{t}) \) is obvious, and any sequence of delays can be replaced by a single delay generating a trace \( (1'_0, v'_{var,0}, v'_0), e'_1, (1'_1, v'_{var,1}, v'_1), e'_1, (1'_2, v'_{var,2}, v'_2), e'_2, \ldots \) of \( N \) such that for every \( h > 0 \) s.t. \( h \% 2 = 0 \), \( e'_h = \delta_h \) and \( e'_{h+1} = \Lambda_{h+1} \).
A3: From timed automata to CLTLoc

This Appendix contains additional remarks and details related with Section 4. Specifically, it focuses on how TA are encoded in CLTLoc.

First, we discuss formula $\phi_{\text{clock}}$ which ensure correctness w.r.t. CLTLoc,clocks in the rest of the formulae of the encoding.

Encoding constraints over clocks ($\phi_{\text{clock}}$). Differently from TA, clocks in CLTLoc, formulae cannot be tested and reset at the same time. For instance, while it is possible that a transition in a TA has guard $x > 5$ and resets clock $x$, in CLTLoc, simultaneous test and reset would yield a contradictory formula, as testing $x > 5$ and resetting $x$ in the same position equals to formula $x > 5 \land x = 0$. Therefore, for each clock $x \in X$, two clocks $x_0$ and $x_1$ are introduced in formula $\Phi_N$ to represent clock $x$ of the automaton. An additional boolean variable $x_i$ keeps track, in any discrete time position, of which clock $x_0$ or $x_1$ is the “active” CLTLoc,clock. Clocks $x_0$ and $x_1$ are never reset at the same time and their resets alternate. If $x_i = 0$ (resp., $x_i = 1$) at position $i$ of the model of $\Phi_N$ then $x_0$ (resp., $x_1$) is the active clock at $i$ and $\sigma(i, x_0)$ (resp., $\sigma(i, x_1)$) is the value used to evaluate the clock constraints at $i$. If the reset of $x$ has to be represented at $i$, clock $x_1$ (resp., $x_0$) is set to 0 and the value $x_i$ in position $i + 1$ is set to 1 (resp., 0)—i.e., the active clock is switched.

$$\phi_1 := \bigwedge_{x \in X} (x_0 = 0 \land x_1 > 0 \land x_i = 0)$$

$$\phi_2(j) := \bigwedge_{x \in X} (x_j = 0) \land \bigwedge_{x \in X} (x_1 = j \land (x_j > 0) \land x_i = 0) \land (x_i \equiv 0)$$

Fig. 5. Encoding of the clocks of the automaton.
A Flexible Approach for Checking Timed Automata on Continuous Time Semantics

\[
x := 0 \quad x > 5, x := 0 \quad x < 1 \quad x = 3, x := 0
\]

\[
x_0 = 0 \quad x_0 = 0 \quad x_0 = 1 \quad x_0 = 1
\]

\[
x_1 = 0 \quad x_1 < 1 \quad x_1 = 3
\]

\[
x_0 = 0 \quad x_0 > 5 \quad x_0 = 0
\]

\[
\sigma(3, x_1) = 0 \quad \sigma(5, x_1) < 1 \quad \sigma(5, x_1) = 3
\]

\[
\sigma(0, x_0) = 0 \quad \sigma(3, x_0) < 5 \quad \sigma(5, x_0) = 0
\]

Fig. 6. Representation of tests and resets of clock \(x\) by means of the two copies \(x_0\) and \(x_1\).

Figure 5 shows the formulae \(\phi_1\) and \(\phi_2\) that are used to define \(\phi_{\text{clock}}\). Formula \(\phi_1\) specifies that initially, the active clock is \(x_0\). In position 0 variable \(x_c\) is equal to 0 (indicating that \(x_0\) is the active clock), \(x_0\) is also equal to 0 and \(x_1\) has an arbitrary value greater than zero. Formula \(\phi_2\) specifies that if \(x_j\) is reset it cannot be reset again before 8 discrete positions. The first row shows the sequence of operations on \(x\) over 8 discrete positions. The first row shows the sequence of tests and resets on clock \(x\) over 8 discrete positions. The third row shows the constraints on \(\sigma\) that are enforced by the operations on \(x\).

We provide additional details on how the formulae in the network are generated.

**Encoding the network (\(\phi_S\)).** Since every clock \(x\) is represented in the encoding by two clocks \(x_1\) and \(x_2\) represented in the formulae of Table 3 as follows:

- every clock constraint of the form \(x \sim c\) contained in the invariant is translated by means of the following CLTLoc formula: \(\phi_{x \sim c} := ((x_0 \sim c) \land (x_c = 0)) \lor ((x_1 \sim c) \land (x_c = 1))\);
- every clock constraint of the form \(x \sim c\) contained in the guard is translated by means of the following CLTLoc formula: \(\phi_{x \sim c} := ((x_0 \sim c) \land (x_c = 0)) \lor ((x_1 \sim c) \land (x_c = 1))\);
- the assignments performed by the transition are applied to the non active clock. More precisely, if a clock \(x\) is reset the following formula \(\phi_{x \sim c} := ((x_1 \sim c) \land (x_c = 0)) \lor ((x_0 \sim c) \land (x_c = 1))\) is added to \(\phi_S\).
Formula $\varphi_5$ is the core of the encoding since it describes in a CLTLc formula the effects of firing transitions of a TA. We use the transition $\lambda$ from state $l_2$ to $l_0$ in Fig. 2(b) as example to illustrate the encoding. Consider the portion of the trace produced when transition $\lambda$ is fired and described in Fig. 7. From Lemma 1 it has the form...

This change in the configuration is encoded in $\varphi_5$ by the following formula:

$$t[0] = \varphi_5 \rightarrow (l_2 = l_2 \land \varphi_{\gamma var} \land X(l_0 = l_0 \land \varphi_\gamma \land \varphi_s \land \varphi_A \land Inv^*(t[0]))$$

where $\varphi_{\gamma var}$, $\varphi_s$, $\varphi_A$ and $Inv^*(t[0])$ are defined as follows:

- $\varphi_{\gamma var} : d = 1$
- $\varphi_\gamma : (x_1 = 0 \land x_v = 0) \lor (x_0 = 0 \land x_v = 1)$
- $\varphi_A : d = 0$
- $\varphi_S : (x_0 = 1 \land x_v = 0) \lor (x_1 = 1 \land x_v = 1)$
- $Inv^*(t[0]) : (x_1 \leq 8 \land x_v = 0) \lor (x_0 \leq 8 \land x_v = 1)$

Consider Fig. 7 and the position $i = j$. In position $j$, $t[0]$ is associated with the value $\lambda$ meaning that at the next time instant the transition $\lambda$ is going to be performed. Formula $\varphi_{\gamma var}$ specifies that variable $d$ is equal to the value $\varphi$, which is the constraint on the variables for the transition to be performed. Note that the constraint on the variables can be checked at time instant $j$ since the values of the variables do not change between positions $j$ and $j + 1$. Viceversa, the constraint on the clocks must be checked at time instant $j + 1$ since the values of the clocks change between positions $j$ and $j + 1$. Specifically, formula $\varphi_\gamma$ forces that the value of the active clock $x$ (the one specified by variable $x_v$) to satisfy the guard in position $j + 1$. Formula $\varphi_A$ forces the integer variable $d$ to be assigned at value 0 as specified in the assignment of the transition. Formula $\varphi_S$ forces the non active clock $x_1$ to be reset. Note that this will be the active clock at
time instant \( j + 2 \) as specified in formula \( \phi_{\text{clock}} \). Formula \( Inv(t') \) forces the non active clock \( x_1 \) to satisfy the invariant of the destination state in position \( j + 1 \). Indeed, the new assignment of clock \( x_1 \) (obtained by reset) should not cause a violation of the invariant.

Note that the fact that an invariant is not violated when a transition is performed is ensured by formula \( \varphi_4 \) of Fig. 5. This formula specifies that if an automaton is in a state \( q \), the invariant of that state must be specified in the next position. In the example of Fig 5 at position \( j \) the automaton is in state \( l_2 \), thus the invariant (which checks the active clocks) is ensured at position \( j + 1 \). Removing the next operator from the right side of the implication in formula \( \varphi_4 \) would lead to a contradiction. Specifically, in time instant \( j + 1 \), the invariant of state \( l_0 \) would be checked on the active clocks which is not correct since when the system enters \( l_0 \) the non active clocks must be checked as specified by \( Inv(t') \).

We now outline the proof of the correctness of the encoding.

**Proof of correctness.** To prove correctness we need to show that the traces of network \( N = A_0 \parallel A_1 \parallel \ldots \parallel A_k \) of automata correspond to the models of the CLTLoc, generated from the network, i.e., Given a network \( N = A_0 \parallel A_1 \parallel \ldots \parallel A_k \) of TA and the CLTLoc formula \( \Phi_N \) generated from the TA using the procedure described in Section 4, the traces of a network \( N = A_0 \parallel A_1 \parallel \ldots \parallel A_k \) of automata correspond to the models \( \Phi_N \). Formally,

**Proposition 1.** Given a \( N = A_0 \parallel A_1 \parallel \ldots \parallel A_k \) of TA and the CLTLoc, formula \( \Phi_N \) generated from the TA using the procedure described in Section 4, a trace \( \eta \) is a trace of \( Na \) if and only if there exists a model \((\pi, \sigma, i) \) of \( \Phi_N \) such that \((\pi, \sigma, i) \models \eta \).

Since for Lemma 1, given a network \( N \) of \( K \) TA for every trace \((1_0, v_{\text{var}, 0}, v_0), e_0, (1_1, v_{\text{var}, 1}, v_1), e_1, (1_2, v_{\text{var}, 2}, v_2), e_2, \ldots \) there exists a trace \((1_{0'}^{\prime}, v_{\text{var}, 0'}, v_0'), e_0', (1_{1'}^{\prime}, v_{\text{var}, 1'}, v_1'), e_1', (1_{2'}^{\prime}, v_{\text{var}, 2'}, v_2'), e_2', \ldots \) of \( N \) such that for every \( h > 0 \) such that \( h \% 2 = 0 \), \( e_h' = \delta_h \) and \( e_{h+1}' = \Lambda_h \) that generates the same signal, in the following we only consider traces that have this form.

**Proof.** \( (\Rightarrow) \) Given a network of automata \( N = A_0 \parallel A_1 \parallel \ldots \parallel A_k \) and a trace \( \eta \) of the form \((1_0, v_{\text{var}, 0}, v_0) \overset{\delta_0}{\rightarrow} (1_1, v_{\text{var}, 1}, v_1) \overset{\Lambda_1}{\rightarrow} (1_2, v_{\text{var}, 2}, v_2), \ldots \) of \( N \) where \( M_i \) indicates the corresponding signal, there is a model \((\pi, \sigma, i) \) of \( \Phi_N \) such that \((\pi, \sigma, i) \models \eta \). This means that there exist a function \( \rho : \mathbb{N} \rightarrow \mathbb{R}_{\geq 0} \) such that for every \( i \geq 0 \) and \( \rho(i) \leq j < \rho(i+1) \) the following conditions hold:

- for every \( n \in \text{Int} \), \( M_i(j) = (P, v_{\text{var}}) \) and \( \alpha(i, n) = v_{\text{var}}(n) \);
- for every \( ap \in AP \), \( M_i(j) = (P, v_{\text{var}}) \) and \( ap \in \pi(i) \Rightarrow ap \in P \).

We prove that a model \((\pi, \sigma, i) \) of \( \Phi_N \) that satisfies the conditions of function \( \rho \) can be effectively constructed. First, we show how the proposed model \((\pi, \sigma, i) \) satisfies the conditions of function \( \rho \) in \( i = 0 \). Then we show that for every configuration change \((1_i, v_{\text{var}, i}, v_i) \overset{\delta_i}{\rightarrow} (1_{i+1}, v_{\text{var}, i+1}, v_{i+1}) \) the conditions of function \( \rho \) are ensured.

1. Consider the initial values \((1_0, v_{\text{var}, 0}, v_0) \). The proposed encoding forces the TA to be in their initial states, variables assigned to their initial values and AP to be true or false depending on the states in which the automata are located. This is ensured by
formulae $\varphi_1$, $\varphi_2$ and $\varphi_3$. Furthermore, formulae $\varphi_2$ and $\varphi_3$ ensure that the invariant initially holds and it must also hold in position 1. This shows that the model $(\pi, \sigma, i)$ satisfies the conditions of function $\rho$ in $i = 0$.

2. for every change $(1_i, v_{var,i}, v_i) \xrightarrow{\delta_i} (1_{i+1}, v_{var,i+1}, v_{i+1})$ a model of $\Phi_N$ is obtained by considering the left hand side of the implications in formulae $\varphi_6$, $\varphi_7$, $\varphi_8$ and $\varphi_9$ as false. Note that, $\varphi_8$ and $\varphi_9$ forces the variables and AP to maintain their values (i.e., values of variables and APs do not spontaneously change) and time simply progresses. This shows that conditions of function $\rho$ holds for $i$ and $i + 1$ by the generated models when transitions of type are performed $(1_i, v_{var,i}, v_i) \xrightarrow{\delta_i} (1_{i+1}, v_{var,i+1}, v_{i+1})$.

3. for every change $(1_i, v_{var,i}, v_i) \xrightarrow{\delta_i} (1_{i+1}, v_{var,i+1}, v_{i+1})$ the left hand side of the implication of every formula $\varphi_6$ which is obtained by a transition of the TA that was fired to cause the configuration to change from $(1_i, v_{var,i}, v_i)$ to $(1_{i+1}, v_{var,i+1}, v_{i+1})$. This, forces the state of the automata to change, applies the changes in the variables, resets the clocks. Note that formula $\gamma_i$ and $\gamma_i \varphi$ are satisfied since guards are satisfied by the provided trace. Furthermore, formulae $\varphi_7$, $\varphi_8$ and $\varphi_9$ force state of the automata, variables and clock to not spontaneously change between the CLTLoc, positions $i$ and $i + 1$. This shows that conditions of function $\rho$ holds for $i$ and $i + 1$ by the generated models when transitions of type are performed $(1_i, v_{var,i}, v_i) \xrightarrow{\delta_i} (1_{i+1}, v_{var,i+1}, v_{i+1})$.

($\Leftarrow$) Given a network of automata $N = A_0 \parallel A_1 \parallel \ldots \parallel A_K$ and a model $(\pi, \sigma, i)$ of $\Phi_N$, there is a trace $\eta$ of the form $(1_0, v_{var,0}, v_0) \xrightarrow{\delta_0} (1_1, v_{var,1}, v_1) \xrightarrow{\delta_1} (1_2, v_{var,2}, v_2) \ldots$ of $N$ such that $(\pi, \sigma, i) \models \eta$. This means that there exist a function $\rho : \mathbb{N} \rightarrow \mathbb{R}_{\geq 0}$ such that the signal $M_\eta$ associated with $\eta$ ensures the following conditions for every $i \geq 0$ and $\rho(i) \leq \rho(i + 1)$:

- for every $n \in \text{Int}$, $M_\eta(j) = (P, v_{var})$ and $\iota(i, n) = v_{var}(n)$;
- for every $ap \in AP$, $M_\eta(j) = (P, v_{var})$ and $ap \in \pi(i) \Rightarrow ap \in P$.

The prove is performed by constructing a trace $(1_0, v_{var,0}, v_0) \xrightarrow{\delta_0} (1_1, v_{var,1}, v_1) \xrightarrow{\delta_1} (1_2, v_{var,2}, v_2) \ldots$ that satisfies the previous conditions.

1. $(1_0, v_{var,0}, v_0)$ is chosen such that $1_0$ for every $0 < k \leq K$, $1_0(k) = 0$, i.e., every automaton is in its initial state. Function $v_{var,0}$ is such that for every $n \in \text{Int}$, $v_{var,0}(n) = v_{var,0}(n)$, i.e., every variable is associated to its initial value. For all $x \in X$, $v_0(x) = 0$. This ensures that the signal $M_\eta$ satisfies the conditions for $i = 0$ and $j = 0$.

2. for every $i$ such that $\iota(i, n) = \iota(i + 1, n)$ and for all $ap \in AP, ap \in \pi(i) \Rightarrow ap \in \pi(i + 1)$, a change of the configuration of type $(1_i, v_{var,i}, v_i) \xrightarrow{\delta_i} (1_{i+1}, v_{var,i+1}, v_{i+1})$ is constructed as follows: $1_i = 1_{i+1}, v_{var,i} = v_{var,i+1}$ and for all $x \in X, v_{i+1}(x) = v_i(x) + c$ where $c = \sigma(i + 1) - \sigma(i)$. This model is a valid portion of trace of $N$ since the automata remain in their states and time progressing does not cause any of the invariants of the automata are violated due to the condition $\varphi_4$, encoding. The signal $M_\eta$ satisfies the conditions since between $i$ and $i + 1$ the values of the variables and of the atomic propositions do not change.
3. for every \( i \) such that \( t(i, n) \neq v_{var}(n) \) or it does not hold that for all \( ap \in AP, ap \in \pi(i) \Rightarrow ap \in \pi(i+1) \), the left hand side of one of the implications generated by formula \( \varphi_6 \) must be true. A change of the configuration \( (l_i, v_{var,i}, v_i) \xrightarrow{\varphi} (l_{i+1}, v_{var,i+1}, v_{i+1}) \) where \( \Lambda_i = (\Lambda_i^1, \Lambda_i^2, \ldots, \Lambda_i^k) \) can be obtained as follows:

- \( \Lambda_i \) is such that the values of integer variables before at position \( i \), specified by function \( v_{var,i} \) satisfies the guards of the performed transitions in \( \Lambda_i \). The existence of such transition is ensured by the fact that the left hand side of one of the implications generated by formula \( \varphi_6 \) must be true.

- \( \Lambda_i \) is such that every (and only) the integer variable \( n \) with \( t(i+1, n) \neq t(i, n) \) are assigned to value \( t(i+1, n) \) in the assignement \( \Lambda \) in one of the transitions in \( \Lambda_i \). The existence of this set of transitions is ensured by the CLTLoc\( \varphi \) formulae \( \varphi_8 \) that force integer variables to not spontaneously be changed.

- \( \Lambda_i \) is such that for every clock \( x \) the guards of the transitions in \( \Lambda_i \) does not violate the clock constraint where the value of the clock \( x \) is \( v_{j+1}(x) = v_i(x) + c \) where \( c = \sigma(i+1) - \sigma(i) \). The existence of such transition is ensured by the fact that the left hand side of one of the implications generated by formula \( \varphi_6 \) must be true.

- \( \Lambda_i \) is such that every (and only) the clocks \( c \) with \( \sigma(i+1, n) = 0 \) are in the set \( S \) of some of the transitions in \( \Lambda_i \). The existence of this set of transitions is ensured by the CLTLoc\( \varphi \) formulae \( \varphi_8 \) that force clocks to not spontaneously be reseted.

This shows that is possible to construct a trace \( \eta \) models \( (\pi, \sigma, \iota) \), i.e., the signal \( M_\eta \) models \( (\pi, \sigma, \iota) \).

**A4: Checking the satisfaction of MITL formulae over TA**

This Appendix contains additional remarks and details related with Section 4. Specifically, it discusses how the CLTLoc\( \varphi \) formulae are obtained from the MITL formula and how TAs are used by the model checking algorithm.

**Reducing model-checking to CLTLoc satisfiability** Given a network \( N \) and the corresponding set of traces \( S \), let \( \mu \) be the mapping between the traces \( \eta \) of \( N \) and the signals \( M_\eta \) and let \( M_{\eta, S} \) be the set \( \{\mu(\eta) | \eta \in T\} \) of all the signals \( M_\eta \) such that \( \eta \in S \). Also, let \( M_\varphi \) be the set \( \{M | M, 0 \models \psi\} \) of all the signals that are models of MITL formula \( \psi \). By standard language-theoretical arguments, \( N \models_S \psi \) reduces to verifying the inclusion \( M_{\eta, S} \subseteq M_\varphi \), or equivalently, \( M_{\eta, S} \cap M_{\neg \psi} = \emptyset \).

Let \( \Phi_N \) and \( \Phi_\varphi \) be two CLTLoc, formulae representing, respectively, signals in \( M_{\eta, S} \) and signals in \( M_\varphi \), for a generic MITL formula \( \psi \) defined by atomic propositions in \( AP \) and arithmetical constraints over \( Int \). Hence, checking \( M_{\eta, S} \cap M_{\neg \psi} = \emptyset \) reduces to the satisfiability of \( \Phi_N \land \Phi_{\neg \psi} \), that is:

\[
N \models_S \psi \text{ iff } \Phi_N \land \Phi_{\neg \psi}.
\]  

(1)

Formulae \( \Phi_N \) and \( \Phi_\varphi \) of Eq. (1), for any MITL formula \( \psi \), can be effectively computed.
CLTLoc encoding of MITL signals [8] shows how to build a CLTLoc formula \( \Phi_\eta \) from a MITL formula \( \psi \) such that set \( M_\psi \) is represented by the models of \( \Phi_\psi \) (hence, the satisfiability of \( \psi \) can be derived from the satisfiability of \( \Phi_\psi \)). Mapping a continuous-time signal \( M \) to a denumerable sequence of elements is done by partitioning \( M \) into infinitely many finite intervals, each one representing a portion of \( M \). Let \( I \) be an interval of the form \((a, b)\), with \( a < b \), and \( I_0, I_1, \ldots \) be a denumerable set of adjacent intervals (i.e., \( a_i + 1 = b_i \) for all \( i > 0 \)) covering \( \mathbb{R}_{\geq 0} \), i.e., such that \( \bigcup_{i \geq 0}(I_i \cup \{a_i\}) = \mathbb{R}_{\geq 0} \), with \( a_0 = 0 \). Every position \( i \) in a CLTLoc \( \eta \) model captures the “configuration of \( M \)” in the interval \( I_i \) and it is such that for all pairs of time instants \( i, i' \) in interval \( I_i \) it holds that \( M(i) = M(i') \).

Section [2] presents an extended version of MITL where atomic formulae can be added specific constraints to the CL TL possible assignment to all the elements in the domain of a MITLformula, over the continuous-time. For every \( (i.e., a_n \in \{\leq, =\} \). According to [8], special CLTLoc atoms are introduced to represent the signal defined by the atomic propositions and arithmetical formulae, occurring in a MITL formula, over the continuous-time. For every \( p \in AP \), the value of \( p \) over open and non empty intervals of \( \mathbb{R}_{\geq 0} \), is represented by proposition \( \overrightarrow{p} \), called rest of \( p \); if \( \overrightarrow{p} \) holds at position \( i \) of the CLTLoc \( \eta \), then atom \( p \) holds in \( I_i \). The value of \( p \) in every point \( a_i \) is represented with a special proposition \( \overrightarrow{d}_p \), called first of \( p \). If \( \overrightarrow{d}_p \) holds in \( i \), then atom \( p \) holds in time instant \( a_i \). Extending [8] to atomic formulae of the form \( n \sim d \) is quite straightforward. For every \( n \sim d \in AP \), the value of \( n \sim d \) over open and non empty intervals \( I_i \) and in the singular points \( a_i \) is represented by proposition \( n \sim d \) and \( \overrightarrow{d}_n \), respectively. Formulae \( \overrightarrow{d}_n \) and \( n \sim d \) are predicate abstractions of the arithmetical constraint \( n \sim d \), occurring in \( \psi \).

CLTLoc encoding of network signals Formula \( \Phi_N \) represents the signals in \( M \) that derives from traces \( \eta \in S \). By Def. [7] every trace \( \eta \) can be associated with a signal \( M_\eta \) which is left-open/right-closed: \( M_\eta \) can be decomposed into its initial value \( M_\eta(0) \) and an infinite set of intervals \((0, a_1], (a_1, a_2], (a_2, a_3], \ldots \) such that for every \( t_1, t_2 \in (a_i, a_{i+1}] \) there exists \( a_0 \in (a_i, a_{i+1}) \) such that \( M_\eta(t_1) = M_\eta(t_2) \). Formula \( \Phi_N \) is built by adding specific constraints to the CLTLoc formula representing the traces \( \eta \) of network \( N \) to express restrictions on the propositions \( \overrightarrow{d}_p \) and \( \overrightarrow{d}_n \), for every \( \beta \) in \( AP \) or \( AP \). In particular, \( \Phi_N \) must enforce that the signals represented by means of \( \overrightarrow{d}_p \) and \( \overrightarrow{d}_n \) are left-open/right-closed, as it follows from Def. [7] and that the value of \( \overrightarrow{d}_n \) and \( n \sim d \), for all \( n \sim d \in AP \), is determined by the value of \( n \) in the CLTLoc model representing \( \eta \). Despite signal \( M_\eta \) specifies a valuation \( \nu_{\text{var}} \), defining the exact assignment for every variable \( n \in Int \), for all the time instants of \( \mathbb{R}_{\geq 0} \), formula \( \Phi_N \) does not represent every possible assignment to all the elements in the domain of \( n \). In \( \Phi_N \), only the formulae \( n \sim d \) that appear in the MITL formula \( \psi \) are considered because the value of \( \psi \) only depends to the value of its subformulae. The value \( \overrightarrow{d}_n \) and \( n \sim d \) is determined by means of the value of variable \( n \) in the CLTLoc model of \( \Phi_\eta \) that, at position \( i \), equals to \( \nu_{\text{var}}(i, n) \).

The additional formulae that contribute to the definition of \( \Phi_N \) are gathered into formula \( \Phi_{\text{sig}} \). Formula \( \Phi_{\text{sig}} \) is built with the formulae in Table [5] and it is the conjunction
respectively:

\[ \begin{align*}
\mu_1 &:= \bigwedge_{a \in AP} \nexists a \iff a \\
\mu_2 &:= \bigwedge_{(a,d) \in AF} (n \ni a \iff n \ni d) \\
\mu_3 &:= \bigwedge_{a \in AP} \nexists a \iff a \\
\mu_4 &:= \bigwedge_{(a,d) \in AF} (n \ni d \iff n \ni d) \\
\mu_5 &:= \bigwedge_{a \in AP} \nexists a \iff a \\
\mu_6 &:= \bigwedge_{(a,d) \in AF} (n \ni d \iff X^n a) \\
\end{align*} \]

Table 5. Formulae encoding the selected relation between the network of TA and the signal.

\[ \Phi_I := \Phi_{\text{lock}} \land \Phi_{\text{N}} \land \Phi_{\text{sync}} \land \Phi_{\text{lub}} \land \varphi_{\text{sig}}. \]

(2)

All the formulae \( \mu_1 - \mu_6 \) are here explained in detail by showing how a signal \( \mu \) can be obtained from a CLTLoc interpretation that is model of \( \varphi_{\text{sig}} \). Let \((\pi, \sigma, \iota)\) be a CLTLoc interpretation over \( [\nexists a \mid a \in AP] \) and \( [\nexists d \mid n \ni d \mid n \ni d \in AP,] \) such that \((\pi, \sigma, \iota) \models \varphi_{\text{sig}} \). Let \( I_0, I_1, \ldots \) be an infinite sequence of adjacent intervals of the form \((a_i, b_i),\) with \( a_i = \sum_{j=0}^{i-1} \delta_j \) and \( b_i = \sum_{j=0}^{i+1} \delta_j \) and \( \delta_j \) is the time delay between positions \( j \) and \( j + 1 \) determined by \((\pi, \sigma, \iota)\) (see Sect. 2). A signal \( \mu \) is such that for all \( i \in \mathbb{N} \) there exists a pair \((P_i, v_i)\), where \( P_i \) and \( v_i \) is a subset of \( AP \) and a mapping in \( \mathbb{Z}^{\mathbb{N}} \), respectively, it holds that \( M(t) = (P_i, v_i) \) for all \( t \in I_i \).

The initial position \( 0 \) of \( M \) is linked to the initial position of the CLTLoc interpretation \((\pi, \sigma, \iota)\) by means of formulae \( \mu_1 \) and \( \mu_2 \):

- \( \mu_1 \). Initially, label \( a \) holds only if, and only if, one of the TA of the network is in a state labeled with \( a \).
- \( \mu_2 \). Initially, formula \( \nexists n \ni a \) holds if, and only if, the initial variable assignment satisfies the formula \( n \ni d \).

The equivalences \(3\) and \(4\) define the relationship between the interval \( I_i \) of \( M \) and the \( i \)-th position of the CLTLoc model, for any \( i \geq 0 \).

\[ a \in P_i \iff (\pi, \sigma, \iota), i \models \nexists a \]  

\[ v_i(n) \ni d \iff (\pi, \sigma, \iota), i \models n \ni d \]  

for all \( a \in AP \) and \( n \ni d \in AP \). \(3\) and \(4\) are enforced through \( \mu_3 \) and \( \mu_4 \) of \( \varphi_{\text{sig}} \), respectively:

- \( \mu_3 \): label \( a \) holds over the interval \( I_i \), i.e., \( \nexists a \) holds at position \( i \), if and only if \( a \) holds at position \( i \).
- \( \mu_4 \): formula \( n \ni d \) holds over the interval \( I_i \), i.e., \( n \ni d \) holds at position \( i \), if and only if the formula \( n \ni d \) holds at position \( i \).

Definition\(1\) prescribes that the signal associated with a trace can be decomposed into left-open/right-closed intervals. Such signals adhere to the following property: for all \( i \in \mathbb{N} \) it holds that for all \( t \in I_i = (a_i, b_i) \) then \( M(t) = m \) if \( M(b_i) = m \), for some \( m \). This property is reflected on the CLTLoc interpretation \((\pi, \sigma, \iota)\) at position \( i \) through the following constraints:

\[ (\pi, \sigma, \iota), i \models \nexists a \iff (\pi, \sigma, \iota), i + 1 \models \nexists a \]  

(5)
Fig. 8. Relationship between a trace and the MITL signal derived by Formula $\varphi_{\text{sig}}$.

\[(\pi, \sigma, \iota), i \models n \sim d \iff (\pi, \sigma, \iota), i + 1 \models \uparrow_{n-d} \quad (6)\]

for all $a \in AP$ and $n \sim d \in AF$. (5) and (6) are enforced through $\mu_5$ and $\mu_6$ of $\varphi_{\text{sig}}$, respectively:

- $\mu_5$: if label $a$ holds in an interval $(a, b)$—i.e., $\uparrow_a$ is true—$a$ also holds in $b$—i.e., $\uparrow_b$ is true.
- $\mu_6$: if $n \sim d$ holds in an interval $(a, b)$—i.e., $n \sim d$ is true—$n \sim d$ also holds in $b$—i.e., $\uparrow_{n-d}$ is true.

Example 1. Figure 8 shows the relation between a portion of a trace of the automaton in Fig. 2(b) and the MITL signals referring to labels $a$ and $c$ and subformula $d = 2$, given by formula $\varphi_{\text{sig}}$. The signal of formula $d = 2$ (and not for $d$) is shown because it is assumed that $d = 2$ appears in the MITL formula $\varphi$ that is evaluated over the automaton traces—e.g., a formula such as $G(d = 2 \rightarrow a)$—therefore the atomic formulae $\uparrow_{d=2}$ and $\uparrow_{d=2}$ appear in CLTLoc, formula $\varphi_N$ translating $\varphi$. The trace is depicted at the top of the figure and shows the firing of the first two transitions labeled with events $e_1$ and $e_2$. The second and the third dashed segments from the top show the value of some of the relevant CLTLoc atoms satisfying formulae $\varphi_N$ and $\varphi_{\text{sig}}$. More precisely, the second and third segments enumerate the first nine discrete positions of the CLTLoc model.
In particular, the first row shows the value of variable $l$ defining the location of the automaton; the notation is simplified as notation $l_j$ is abbreviating $l[0]=l$ (the network consists of one automaton). The second row contains the value of variable $d$, that initially is 0 and then changes to 2 and to 1 because of the assignments performed by the transitions labeled with $e_1$ and $e_2$, respectively. From the third row to the sixth one all positions are labeled only with the CLTL$_{oc}$ formulae that hold therein (if a position is empty then none of the considered formulae holds there). The formulae, in the form $\rightarrow\leftarrow$ and $\rightarrow$, refer to labels $a$ and $c$ and to the MITL formula $d = 2$. For instance, at position 4, $\rightarrow a$ and $d = 2$ hold whereas $\rightarrow c$, $\rightarrow c$, $\rightarrow a$ and $\rightarrow d = 2$ are false. The last segment shows the signal related to labels $a$ and $c$ and to formula $d = 2$ that are built according to the value of CLTL$_{oc}$ atoms $\rightarrow$ and $\rightarrow$ specified in the third segment.

**Remark 5.** The trace of the automaton depicted in Fig. 8 consists of three delay and two discrete transitions, but it is represented with a CLTL$_{oc}$ model that consists of 8 different positions. According to [8], it is in fact possible to fragment a bounded interval of time into a finite number of smaller adjacent intervals, each one represented by means of a position in the CLTL$_{oc}$ model of the formula.