Deep inelastic $e - \tau$ and $\mu - \tau$ conversion in the NA64 experiment at the CERN SPS

Sergei Gninenko,1 Sergey Kovalenko,2 Serguei Kuleshov,2 Valery E. Lyubovitskij,2,3,4,5 and Alexey S. Zhevlakov4,6

1Institute for Nuclear Research, 117312 Moscow, Russia
2Departamento de Física y Centro Científico Tecnológico de Valparaíso-CCTVal, Universidad Técnica Federico Santa María, Casilla 110-V, Valparaíso, Chile
3Institut für Theoretische Physik, Universität Tübingen, Kepler Center for Astro and Particle Physics, Auf der Morgenstelle 14, D-72076, Tübingen, Germany
4Department of Physics, Tomsk State University, 634050 Tomsk, Russia
5Tomsk State Pedagogical University, 634061 Tomsk, Russia
6Matrosov Institute for System Dynamics and Control Theory SB RAS, Lermontov str. 134, 664033, Irkutsk, Russia

We study the Lepton Flavor Violating (LFV) $e(\mu) - \tau$ conversion in Deep Inelastic Scattering (DIS) of electron (muon) on fixed-target nuclei. Our model-independent analysis is based on the set of the low-energy effective four-fermion LFV operators composed of leptons and quarks with the corresponding mass scales $\Lambda_k$ for each operator. Using the estimated sensitivity of the search for this LFV process in events with large missing energy in the NA64 experiment at the CERN SPS, we derive lower limits for $\Lambda_k$ and compared them with the corresponding limits existing in the literature. We show that the DIS $e(\mu) - \tau$ conversion is able to provide a plenty of new limits as yet non-existing in the literature. We also analyzed the energy spectrum of the final-state $\tau$ and discussed viability of the observation of this process in the NA64 experiment and ones akin to it. The case of polarized beams and targets is also discussed.

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I. INTRODUCTION

The lepton flavor violation (LFV) is absent in the Standard Model (SM), if neutrinos are massless. Nowadays, nonzero neutrino masses and flavor mixing is a matter of experimental fact. LFV can be transmitted from the neutrino sector to the charged lepton one via the charged-current neutrino loop which, however is heavily suppressed by the small neutrino mass square-differences. On the other hand the possible high-scale physics beyond the SM (BSM) may contribute to the LFV in the charged lepton sector directly without the mediation of the neutrino loop. The high-scale BSM leaks into the low-energy theory via several universal effective non-renormalizable LFV operators parameterizing in a generic way all the possible UV realization of BSM. In what follows we specify these operators and estimate their possible contribution to the deep-inelastic $e - \tau$ and $\mu - \tau$ conversion

\[ e(\mu) + (A, Z) \rightarrow \tau + X \]  \hspace{1cm} (1)

of the initial electrons (muons) in the fixed-target with the atomic and mass numbers $Z$ and $A$, respectively.

The $e + p \rightarrow \tau + X$ process was searched for by the ZEUS Collaboration at HERA (DESY) in $e^+p$ collisions at a center-of-mass energy $\sqrt{s} \approx 300$ GeV. Theoretical study of the $e - \tau$, $\mu - e$, and $\mu - \tau$ conversion has been done before in Refs. [2]-[24].

II. THEORETICAL SETUP

We start with the low-energy effective Lagrangian relevant for two subprocesses of $e - \tau$ (ETC) and $\mu - \tau$ (MTC) conversion:

(1) on quarks

\[ e^- + q_i \rightarrow \tau^- + q_f, \quad \mu^- + q_i \rightarrow \tau^- + q_f \]  \hspace{1cm} (2)
(2) and on antiquarks

\[ e^- + \bar{q}_f \to \tau^- + \bar{q}_i, \quad \mu^- + \bar{q}_f \to \tau^- + \bar{q}_i. \]  

Its most general form up to the dominant dim=6 operators is

\[ \mathcal{L}_{\ell\tau} = \sum_{I,if,XY} \left( \Lambda_{\ell\tau}^{I_{if,XY}} \right)^{-2} \mathcal{O}_{\ell\tau}^{I_{if,XY}} + \text{H.c.}, \quad \ell = e, \mu, \]

where

- **S - type:** \[ \mathcal{O}_{S_{if,XY}}^{\ell\tau} = (\bar{\tau} P_X l)(\bar{q}_f P_Y q_i), \]
- **V - type:** \[ \mathcal{O}_{V_{if,XY}}^{\ell\tau} = (\bar{\tau} \gamma^\mu P_X l)(\bar{q}_f \gamma_\mu P_Y q_i), \]
- **T - type:** \[ \mathcal{O}_{T_{if,XX}}^{\ell\tau} = (\bar{\tau} \sigma^{\mu\nu} P_X l)(\bar{q}_f \sigma_{\mu\nu} P_X q_i). \]

are dim=6 operators with \( \ell = e^-, \mu^- \). In Eq. (4) the summation over \( I = S, V, T \), the quark flavors \( i, f = u, d, s, c, b, t \), and chiralities \( X, Y = L, R \) is implied. As usual \( P_L, R = (1 \mp \gamma_5)/2 \) are the chirality projection operators. The mass scales \( \Lambda_{\ell\tau}^{I_{if,XY}} \) set the strength of the low-energy effect of the corresponding operators. In total there are \( 360 = 6 \times 6 \times 10 \) operators for the 6 quark flavors for each quark field and 10 possible chirality combinations. These operators are subject to various already existing experimental constraints. Using these constraints we estimate the physics reach of the NA64 experiment \[25\] in the sense of the prospects of the observation of ETC and MTC \[1\] or improving the existing limits on the scales \( \Lambda_{\ell\tau}^{I_{if,XY}} \). Prospects for the experimental searching for \( \mu^- \tau \) conversion in different context have been previously discussed in Refs. \[2, 14, 15\] for the scalar operators.

### III. OBSERVABLES

First, we specify the kinematics. Let \( P, p, p', k, \) and \( k' \) be the momenta of initial nucleon, initial quark/final antiquark, final quark/initial antiquark, initial lepton, and final lepton, respectively. The set of invariant Mandelstam variables defining the kinematics of the quark/antiquark lepton scattering is given by

\[ \hat{s} = (k + p)^2 = (k + xP)^2, \]
\[ \hat{t} = (k - k')^2, \]
\[ \hat{u} = (k - p')^2, \]

obeying the condition \( \hat{s} + \hat{t} + \hat{u} = 0 \) for zero masses of quarks and nucleon in comparison with large value of initial lepton energy. Here \( x \) is the Bjorken variable (the fraction of the nucleon momentum carried by \( q_i \) or \( \bar{q}_i \)): \( x = Q^2/(q \cdot P) \). The inelasticity is \( y = (q \cdot P)/(k \cdot P) \). The set \( (\hat{s}, \hat{t}, \hat{u}) \) is related to the total energy \( s = (k + P)^2 \approx 2m_N E_\ell \), where \( m_N \) is the nucleon mass and \( E_\ell \) is the lepton beam energy

\[ \hat{s} = sx, \]
\[ \hat{t} = q^2 = -Q^2 = -sxy, \]
\[ \hat{u} = -sx(1 - y). \]

#### A. Integral cross section of the \( \ell - \tau \) conversion

Now using effective four-fermion operators we calculate the integral cross sections for the ETC and MTC. The total cross section of the \( \ell - \tau \) conversion on a nucleus \[1\] can be approximated by the sum over the corresponding cross section on its constituent nucleons

\[ \sigma(\ell + (A, Z) \to \tau + X) = Z \sigma(\ell + p \to \tau + X) + (A - Z) \sigma(\ell + n \to \tau + X). \]
Here nucleon \( N = p, n \) cross section is

\[
\sigma(\ell + N \to \tau + X) = \sum_{i,f} \int_0^1 dx \int_0^1 dy \left[ \frac{d^2 \hat{s}}{dx dy} (\ell + q_i \to \tau + q_f) q_i^N(x, Q^2) + \frac{d^2 \hat{s}}{dx dy} (\ell + \bar{q}_f \to \tau + \bar{q}_i) \bar{q}_f^N(x, Q^2) \right],
\]

(11)

where \( q_i^N(x, Q^2) \) and \( \bar{q}_i^N(x, Q^2) \) are quark and antiquark PDFs, respectively. We will consider two nuclear targets: Fe with \( A = 56 \) and \( Z = 26 \) and Pb with \( A = 207 \) and \( Z = 82 \). Quark/antiquark PDFs depend on the resolution scale set by the square momentum transferred to the nucleon

\[
q^2 = -Q^2 = -(s - m_N^2 - m_i^2) xy \simeq -s xy,
\]

(12)

where \( m_N \) is the nucleon mass, \( x = Q^2/(q \cdot P) \) is Bjorken variable, \( y = (q \cdot P)/(k \cdot P) \) is inelasticity. Therefore, we should substitute \( Q^2 \) by \( s xy \) in Eq. (11). In the present paper we use quark PDFs from the CT10 next-to-next-to-leading order global analysis of QCD [26]. In fact, PDF fits using the standard CTEQ PDF evolution [27] but using the HOPPET \( \alpha_s \) running solution.

The elementary differential cross sections corresponding to the contact 4-fermion interactions in Eq. (4) are given by

\[
\frac{d^2 \hat{s}}{dx dy} (\ell + q_i \to \tau + q_f) = \sum_{I,XY} \frac{1}{x} \frac{\hat{s}_{I,XY}(y)}{s_{I,XY}} (\Lambda_{I,XY}^2),
\]

(13)

\[
\frac{d^2 \hat{s}}{dx dy} (\ell + \bar{q}_f \to \tau + \bar{q}_i) = \sum_{I,XY} \frac{1}{x} \frac{\hat{s}_{I,XY}(y)}{s_{I,XY}} (\Lambda_{I,XY}^2).
\]

(14)

Here \( f_{I,XY}(y) \) and \( g_{I,XY}(y) \) are functions related to the matrix elements of the effective operators (5)–(7). They are given in Appendix A.

Substituting (13), (14) into (11) and (10) we find

\[
\sigma(\ell + (A, Z) \to \tau + X) = \sum_{I,i,f,XY} Q_{I,i,XY}^A \frac{\rho_{I,i,XY}}{A_{I,i,XY}},
\]

(15)

with

\[
Q_{I,i,XY}^A = \frac{s}{64\pi} \int_0^1 dx \int_0^1 dy \left[ x f_{I,XY}(y) q_i^A(x, s xy) + x g_{I,XY}(y) \bar{q}_f^A(x, s xy) \right],
\]

(16)

where

\[
\begin{align*}
 u^A(x, Q^2) &= Z u^p(x, Q^2) + (A - Z) d^p(x, Q^2), \\
 d^A(x, Q^2) &= Z d^p(x, Q^2) + (A - Z) u^p(x, Q^2), \\
 u^A(x, Q^2) + d^A(x, Q^2) &= A \left( u^p(x, Q^2) + d^p(x, Q^2) \right), \\
 \bar{u}^A(x, Q^2) &= A \bar{u}^p(x, Q^2), \\
 \bar{d}^A(x, Q^2) &= A \bar{d}^p(x, Q^2), \\
 s^A(x, Q^2) &= s^A(x, Q^2) = A s^p(x, Q^2), \\
 c^A(x, Q^2) &= c^A(x, Q^2) = A c^p(x, Q^2), \\
 b^A(x, Q^2) &= b^A(x, Q^2) = A b^p(x, Q^2)
\end{align*}
\]

(17)

are the quark and antiquark PDFs in a nucleus \( A \). Numerical results for the double moments \( Q_{I,i,XY}^A \) are shown in Tables [II]–[IV] for Fe and Pb nuclear targets and for the electron and muon beams.
The dominant contribution to the inclusive $\ell + A$ cross section is due to the bremsstrahlung of leptons on nuclei, given by the formula \cite{28, 29}

$$
\sigma_{BS}(\ell + (A, Z) \rightarrow \ell + X) = 4 \alpha r_{\ell}^{2} Z^{2} \left[ \frac{7}{9} \log \left( \frac{183}{Z^{1/3} m_{\ell}} \right) \right]
$$

(18)

where $r_{\ell} = e^{2}/(4\pi \epsilon_{0} m_{\ell} e^{2})$ is the classical lepton radius: 2.818 fm (for $e$) and 0.0136 fm (for $\mu$). For specific beam and target we have numerically

$$
\sigma_{BS}(e + Fe \rightarrow e + X) = 0.129 \times 10^{5} \text{ GeV}^{-2},
$$

$$
\sigma_{BS}(e + Pb \rightarrow e + X) = 1.165 \times 10^{5} \text{ GeV}^{-2},
$$

$$
\sigma_{BS}(\mu + Fe \rightarrow \mu + X) = 0.692 \text{ GeV}^{-2},
$$

$$
\sigma_{BS}(\mu + Pb \rightarrow \mu + X) = 6.607 \text{ GeV}^{-2},
$$

(19)

which will be used in the following section for the extraction of the limits on $\mu(e) - \tau$ LFV form the expected sensitivity of the NA64 experiment.

### B. Energy Spectrum

An important characteristic helping to plan the $\ell - \tau$ conversion experiments is the energy spectrum of the final $\tau$-lepton defined as

$$
\mathcal{F}_{I}(E_{\ell}, E_{\tau}) = \frac{1}{\sigma_{I}(l + (A, Z) \rightarrow \tau + X)} \frac{d\sigma_{I}(l + (A, Z) \rightarrow \tau + X)}{dE_{\tau}},
$$

(20)

where $\sigma_{I}$ is the total cross section assuming the single operator $I = S, V, T$ dominance and the differential cross section is given by

$$
\frac{d\sigma_{I}}{dE_{\tau}} = \sum_{i,XY} \frac{M_{N}}{32\pi A_{i,XY}} \int_{0}^{1} dx \left[ x q_{i}^{A}(x, \mu^{2}) f_{I,XY}(1 - z) + x q_{g}^{A}(x, \mu^{2}) g_{I,XY}(1 - z) \right]
$$

(21)

with $\mu^{2} = sx(1 - z)$ and $z = E_{\tau}/E_{\ell}$ running from 0 to 1, which corresponds to $0 \leq E_{\tau} \leq E_{\ell} = s/(2M_{N})$. Here we use $d\sigma/dE_{\tau} = -(2M_{N}/s) (d\sigma/dz)$. Note, the quantity $\mathcal{F}_{I}(E_{\ell}, E_{\tau})$ is independent of the LFV scales $\Lambda_{I}$. It is also independent of the target nucleus, since we sum over all the initial quark flavors $i$. In Figs. 1-3 we plot the energy spectra $\mathcal{F}_{I}(E_{\ell}, E_{\tau})$ for $I = S, V, T$ disregarding the quark contributions subdominant in comparison with $u + d$ quark and antiquark contribution. For simplicity, for each type of the LFV operator with specific spin structure $I = S, V, T$ we suppose the same value of the coupling $\Lambda_{I}$ independent on the quark flavor. We use the following notations: $e_{F}$ and $\mu_{F}$ are the full contributions (including all species of quark and antiquarks) in case of the $E_{e} = 100$ GeV electron and $E_{\mu} = 150$ GeV muon beam, respectively; $e_{ud}$ and $\mu_{ud}$ are the respective $u + d$ contributions for the same values of energies of $e$ and $\mu$ beam.

A useful “integral” quantity is the mean energy $\langle E_{\tau} \rangle_{I}$ of the final $\tau$ lepton defined as

$$
\langle E_{\tau} \rangle_{I} = \int_{0}^{E_{\ell}} dE_{\tau} E_{\tau} \mathcal{F}_{I}(E_{\ell}, E_{\tau}) \equiv E_{\ell} \sum_{i,XY} \frac{\tilde{Q}_{i,XY}^{A}}{\sum_{i,XY} Q_{i,XY}^{A}},
$$

(22)

where

$$
\tilde{Q}_{i,XY}^{A} = \frac{s}{64\pi} \int_{0}^{1} dx \int_{0}^{1} dzz \left[ q_{i}^{A}(x, sx(1 - z)) f_{I,XY}(1 - z) + q_{g}^{A}(x, sx(1 - z)) g_{I,XY}(1 - z) \right]
$$

(23)
separately for each of the operators $I = S, V, T$ in Eqs. (5)-(7). Here, as in case of Figs. 1-8 for each type of the LFV operator with specific spin structure $I = S, V, T$ in Eqs. (5)-(7) we use the same value of the coupling $\Lambda_I$ independent on the quark flavor. Note, that $E_\tau$ is independent on type of nucleus target because double moments of quark/antiquark $\tilde{Q}^{A}_{I_{ij},XY}$ and $Q^{A}_{I_{ij},XY}$ are both proportional to $A$.

For the definition of $Q^A$ see Eq. (10). Our predictions for $\tilde{Q}^{A}_{I_{ij},XY}$ and $\langle E_\tau \rangle_I$ are displayed in Tables XI-XII and in Tables XIII and XIV, respectively, for $\ell = e, \mu$ beams and Fe, Pb targets.

For the definition of $Q^{A}_I$ see Eq. (16). Our predictions for $\tilde{Q}^{A}_{I_{ij},XY}$ and $\langle E_\tau \rangle_I$ are displayed in Tables IX-XII and in Tables XIII and XIV, respectively, for $\ell = e, \mu$ beams and Fe, Pb targets.

For experiments searching for the LFV process (1) it is crucial that the missing energy in the decay of the final $\tau$-lepton be above some value. This is needed for the suppression of the typical backgrounds. For the NA64 experiment, this cutoff is preliminarily estimated to be in the range $10^{-30}$ GeV (the detailed simulation results will be reported elsewhere). Thus, the $E_\tau$ should be larger than the value. For example, for the cutoff of $10$ GeV, the mean energy $\langle E_\tau \rangle_I$ of the final $\tau$-lepton is significantly larger than this value for the contribution of all the operators (5)-(7) as one can seen from Tables XIII and XIV.

IV. LIMITS ON THE LFV SCALES

Here we derive limits on the mass scales $\Lambda$ of the LFV operators in Eqs. (5)-(7) and compare them with the corresponding limits existing in the literature.

A. Expected limits from NA64 experiment

The quantity of the interest in the planning measurements of the electron (muon)-tau lepton conversion is the ratio:

$$R_{\ell\tau} = \frac{\sigma(\ell + A \to \tau + X)}{\sigma(\ell + A \to \ell + X)}$$

(24)

where $\sigma(\ell + A \to \ell + X) \approx \sigma_{BS}(\ell + A \to \ell + X)$.

The physics reach of the NA64 experiment in this quantity is expected to be at the level of

$$R_{\ell\tau} \sim 10^{-13} - 10^{-12}.$$  

(25)

Assuming single operator dominance, the most optimistic value of (25) would result in the constraints on the LFV scales $\Lambda_{I_{ij},XY}$ shown in Tables VI-VIII. As seen from these tables the limits are in the ranges

$$S - \text{operators : } \Lambda^{\ell\tau}_S \geq 0.04 - 0.24 \text{ TeV} \,, \quad \Lambda^{\mu\tau}_S \geq 0.56 - 3.05 \text{ TeV} \,,$$

$$V - \text{operators : } \Lambda^{\ell\tau}_V \geq 0.05 - 0.44 \text{ TeV} \,, \quad \Lambda^{\mu\tau}_V \geq 0.78 - 5.60 \text{ TeV} \,,$$

$$T - \text{operators : } \Lambda^{\ell\tau}_T \geq 0.09 - 0.78 \text{ TeV} \,, \quad \Lambda^{\mu\tau}_T \geq 1.45 - 10.06 \text{ TeV} \,.$$  

(26)

The following comment is in order. Our limits shown in Tables VI-VIII have been derived from the best expected sensitivity (25) of the NA64 experiment and may look too optimistic. However, we note that the limits on $\Lambda$’s scale as $(R_{\ell\tau})^{1/4}$ and, therefore, for the less optimistic case of $R_{\ell\tau} \sim 10^{-12}$, or even worse, they will be comparable with the limits in our Tables and still valuable as they are obtained for the first time.

B. Limits from other experiments

In the literature there exist limits on many of the operators (5)-(7). As a reference point from the accelerator experiments let us mention the constraint from the ZEUS experiment [1] at HERA (DESY), which is

$$\Lambda^{\ell\tau}_{\text{ZEUS}} \geq 0.41 - 1.86 \text{ TeV}.$$  

(27)

These limits apply to the scales for all the operators in Eq. (5)-(7), but only for the first generation quarks $u, d$. Note that Ref. [1] used another basis of the four-fermion operators motivated by the leptoquark exchange, which we adjusted to our in Eqs. (5)-(7) and thus obtained (27) from the limits on the leptoquark mass in Ref. [1].
Limits on the operators (5)-(7) can be extracted from the decays:

\[
\begin{align*}
\tau &\to \ell + M^0, \\
B &\to \ell + \tau, \\
B &\to \ell^\pm + \tau^\mp + M,
\end{align*}
\]  

(28)

where \( M \) is a generic meson allowed by energy-momentum conservation. Using the existing experimental bounds in Ref. [15], the authors extracted limits from some of these processes on the scales of the scalar, pseudoscalar, vector and axial-vector \( \mu - \tau \) LFV effective operators, which differ from our basis (5)-(7). Thus, their limits translate to limits on linear combinations of our operators. Assuming no significant cancellations in these combinations we have the results of Ref. [15] translated to the scales of our operators:

- **S** - operators: \( A_{S_{dd,XY}}, A_{S_{ss,XY}} \geq 5.8 \text{ TeV}, A_{S_{us,XY}} \geq 5 \text{ TeV}, A_{S_{ud,XY}} \geq 1.9 \text{ TeV}, A_{S_{sd,XY}} \geq 4.7 \text{ TeV} \), (29)

- **V** - operators: \( \Lambda_{V_{dd,XY}}, \Lambda_{S_{ss,XY}} \geq 5.7 \text{ TeV}, \Lambda_{V_{us,XY}} \geq 4.8 \text{ TeV}, \Lambda_{V_{ud,XY}} \geq 1.8 \text{ TeV} \), (30)

As seen from Tables [VII] and [VIII] the DIS \( \mu - \tau \) conversion on nuclei [11] covers much wider set of the quark bilinears, providing limits on \( \Lambda_{I,f,XY} \), where \( i, f = u, d, s, c, b \) and \( I = S, V, T \). Our limits shown in Tables [VII-VIII] have been derived from the expected sensitivity [29] of the NA64 experiment. We conclude that the limits from the decays [28]-[29] on the set of \( \Lambda \) in [29]-[30] are typically about factor stronger than ours, but there are missing limits on the operators with many other combinations of quark flavors, neither there are limits on the \( T \)-operators, provided by the DIS \( \mu - \tau \) conversion on nuclei [11].

Note that the \( T \)-operators operators for some quark flavor combinations can be constrained by the decays [29]. We have not found such limits in the literature and leave the study of this possibility for a future publication. It is also worth noting that the LFV decay \( \tau \to e(\mu) + \gamma \) does not constrains the operators (5)-(7) since the quark loop coupled to photon vanishes, if it is on-shell.

**V. POLARIZED LEPTON BEAM AND NUCLEAR TARGET**

Following Refs. [30, 31] we consider the lepton conversion for specific choice of spin configurations of incoming beam and nuclear target. For the studied energies of incoming leptons we may safely neglect quark and lepton masses. As shown in Refs. [30, 31] for \( m_N \ll E_\ell \) the orientation of the nucleon spin \( S \) is irrelevant, while there is a dependence on the helicity of the initial lepton \( \lambda \).

The differential cross section of the \( \ell - \tau \) conversion taking into account of the initial lepton helicity \( \lambda \) can be written as

\[
\sigma_{\text{pol}}(\ell + (A, Z) \to \tau + X) = \sum_{I,if,XY} \frac{Q^A_{I,if,XY}(\lambda)}{\Lambda^I_{if,XY}}
\]  

(31)

with

\[
Q^A_{I,if,XY}(\lambda) = \frac{s}{64\pi} \int_0^1 dx \int_0^1 dy \left[ x f_{I,XY}(y, \lambda) q_i^A(x, sxy) + x g_{I,XY}(y, \lambda) \bar{q}_j^A(x, sxy) \right],
\]  

(32)

where the functions \( f_{I,XY}(y, \lambda) \) and \( g_{I,XY}(y, \lambda) \) are given in Appendix [A].

As seen from Appendix, the choice of a particular helicity \( \lambda \) of the initial lepton allows one to eliminate the contributions of certain operators to the \( \ell - \tau \) flavor non-diagonal observables. In particular, the contribution of the operators \( O^T_{\bar{S}_{if,LR}}, O^T_{\bar{S}_{if,RR}}, O^T_{\bar{V}_{if,LR}}, O^T_{\bar{V}_{if,RR}}, O^T_{\bar{T}_{if,LR}} \) is absent in quark-lepton subprocesses at \( \lambda = +1 \) and antiquark-lepton subprocesses at \( \lambda = -1 \). Correspondingly, the the contribution of the operators \( O^T_{\bar{S}_{if,LL}}, O^T_{\bar{S}_{if,RL}}, O^T_{\bar{V}_{if,LL}}, O^T_{\bar{V}_{if,RL}}, O^T_{\bar{T}_{if,LL}} \) is absent in quark-lepton subprocesses at \( \lambda = -1 \) and antiquark-lepton subprocesses at \( \lambda = +1 \). Therefore, the experiments with polarized lepton beams could be useful for separating the contributions of different LFV operators and in this way to help us identify the underlying LFV theory.
Note that the averaging of the polarized cross section in Eq. (31) over $\lambda$ and $S$ reproduces our result for the unpolarized cross section [15]

$$\sigma(\ell + (A, Z) \to \tau + X) = \frac{1}{4} \sum_{\lambda, S} \sigma_{\text{pol}}(\ell + (A, Z) \to \tau + X)$$

and in agreement with Ref. [31].

VI. SUMMARY

We presented phenomenological analysis of the $e - \tau$ and $\mu - \tau$ conversion in electron (muon) scattering on fixed-target nuclei (Pb and Fe). Our analysis is based on the effective four-fermion lepton-quark LFV operators linked to individual mass scales characterizing underlying renormalizable high-scale LFV physics. In the present paper we have not considered its particular realization.

Using the expected sensitivity [25] of the NA64 experiment we derived lower limits for the LFV mass scales of the effective operators and compared them with the corresponding limits existing in the literature, derived from the LFV decays. We have shown that the process of $e(\mu) + (A, Z) \to \tau + X$ is going to provide a plenty of new limits as yet non-existing in the literature.

We predicted the spectrum of the final-state $\tau$-leptons helping one to assess the possibilities of discrimination of the signal from the typical backgrounds. We found that the mean $\tau$-lepton energy is significantly large than the required cutoff in the missing energy for all the effective operators [5]-[7]. In our opinion this result supports the viability of searching for the process [1] with the NA64 experiment, which will able to come up with new limits presented in Tables VI-VIII many of which do not yet exist in the literature. Currently, the NA64 searching for invisible decay of dark photon does not see background events with a large missing energy at the level $\sim 10^{-11}$ per 100 GeV electron on Pb target [32]. This result can be used to extract limits on $\Lambda$’s for the reaction of $e - \tau$ conversion which will be reported elsewhere.

We also examined the possible advantages of polarized beams and targets and showed that they can help distinguish the contributions of different LFV operators. We hope our results will be useful for planning experiments searching for Lepton Flavor Violation in accelerator experiments.

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Appendix A: Polarized and unpolarized matrix elements.

Here we give the definitions and the explicit forms of the functions $f_I$ and $g_I$ in Eqs. [13], [14] and [32] involved in the calculation of the square matrix elements of the operators in Eqs. [5]-[7].

For the case of unpolarized $\ell - \tau$ conversion we have

$$f_{S,LL}(y) = \text{Tr}[P_L \not{k} P_R \not{k}'] \text{Tr}[P_L \not{p} P_R \not{p}'] = y^2,$$

$$f_{S,RR}(y) = \text{Tr}[P_R \not{k} P_L \not{k}'] \text{Tr}[P_R \not{p} P_L \not{p}'] = y^2,$$

$$f_{S,LR}(y) = \text{Tr}[P_L \not{k} P_R \not{k}'] \text{Tr}[P_R \not{p} P_L \not{p}'] = y^2,$$

$$f_{S,RL}(y) = \text{Tr}[P_R \not{k} P_L \not{k}'] \text{Tr}[P_L \not{p} P_R \not{p}'] = y^2,$$

(A1)
In case of polarized lepton beam and target the corresponding functions are

\[ f_{V,LL}(y) = \text{Tr}\left[ \gamma^\mu P_L \not{k} \gamma^\nu P_L \not{k}' \right] \text{Tr}\left[ \gamma_\mu P_L \not{p} \gamma_\nu P_L \not{p}' \right] = 4, \]
\[ f_{V,RR}(y) = \text{Tr}\left[ \gamma^\mu P_R \not{k} \gamma^\nu P_R \not{k}' \right] \text{Tr}\left[ \gamma_\mu P_R \not{p} \gamma_\nu P_R \not{p}' \right] = 4, \]
\[ f_{V,LR}(y) = \text{Tr}\left[ \gamma^\mu P_L \not{k} \gamma^\nu P_L \not{k}' \right] \text{Tr}\left[ \gamma_\mu P_R \not{p} \gamma_\nu P_R \not{p}' \right] = 4(1-y)^2, \]
\[ f_{V,RL}(y) = \text{Tr}\left[ \gamma^\mu P_R \not{k} \gamma^\nu P_R \not{k}' \right] \text{Tr}\left[ \gamma_\mu P_L \not{p} \gamma_\nu P_L \not{p}' \right] = 4(1-y)^2, \]  

\begin{equation}
(A2)
\end{equation}

\[ f_{T,LL}(y) = \text{Tr}\left[ \sigma^{\mu\nu} P_L \not{k} P_R \sigma^{\alpha\beta} \not{k}' \right] \text{Tr}\left[ \sigma_{\mu\nu} P_L \not{p} P_R \sigma_{\alpha\beta} \not{p}' \right] = 16(2-y)^2, \]
\[ f_{T,RR}(y) = \text{Tr}\left[ \sigma^{\mu\nu} P_R \not{k} P_L \sigma^{\alpha\beta} \not{k}' \right] \text{Tr}\left[ \sigma_{\mu\nu} P_R \not{p} P_L \sigma_{\alpha\beta} \not{p}' \right] = 16(2-y)^2 \]  

\begin{equation}
(A3)
\end{equation}

and

\[ g_{S,LL}(y) = \text{Tr}\left[ P_L \not{k} P_R \not{k}' \right] \text{Tr}\left[ P_L \not{p} P_R \not{p}' \right] = y^2, \]
\[ g_{S,RR}(y) = \text{Tr}\left[ P_R \not{k} P_L \not{k}' \right] \text{Tr}\left[ P_R \not{p} P_L \not{p}' \right] = y^2, \]
\[ g_{S,LR}(y) = \text{Tr}\left[ P_L \not{k} P_R \not{k}' \right] \text{Tr}\left[ P_R \not{p} P_L \not{p}' \right] = y^2, \]
\[ g_{S,RL}(y) = \text{Tr}\left[ P_R \not{k} P_L \not{k}' \right] \text{Tr}\left[ P_L \not{p} P_R \not{p}' \right] = y^2, \]  

\begin{equation}
(A4)
\end{equation}

\[ g_{V,LL}(y) = \text{Tr}\left[ \gamma^\mu P_L \not{k} \gamma^\nu P_L \not{k}' \right] \text{Tr}\left[ \gamma_\mu P_L \not{p} \gamma_\nu P_L \not{p}' \right] = 4(1-y)^2, \]
\[ g_{V,RR}(y) = \text{Tr}\left[ \gamma^\mu P_R \not{k} \gamma^\nu P_R \not{k}' \right] \text{Tr}\left[ \gamma_\mu P_R \not{p} \gamma_\nu P_R \not{p}' \right] = 4(1-y)^2, \]
\[ g_{V,LR}(y) = \text{Tr}\left[ \gamma^\mu P_L \not{k} \gamma^\nu P_L \not{k}' \right] \text{Tr}\left[ \gamma_\mu P_R \not{p} \gamma_\nu P_R \not{p}' \right] = 4, \]
\[ g_{V,RL}(y) = \text{Tr}\left[ \gamma^\mu P_R \not{k} \gamma^\nu P_R \not{k}' \right] \text{Tr}\left[ \gamma_\mu P_L \not{p} \gamma_\nu P_L \not{p}' \right] = 4, \]  

\begin{equation}
(A5)
\end{equation}

\[ g_{T,LL}(y) = \text{Tr}\left[ \sigma^{\mu\nu} P_L \not{k} \sigma^{\alpha\beta} P_R \not{k}' \right] \text{Tr}\left[ \sigma_{\mu\nu} P_L \not{p} \sigma_{\alpha\beta} P_R \not{p}' \right] = 16(2-y)^2, \]
\[ g_{T,RR}(y) = \text{Tr}\left[ \sigma^{\mu\nu} P_R \not{k} \sigma^{\alpha\beta} P_L \not{k}' \right] \text{Tr}\left[ \sigma_{\mu\nu} P_R \not{p} \sigma_{\alpha\beta} P_L \not{p}' \right] = 16(2-y)^2. \]  

\begin{equation}
(A6)
\end{equation}

In case of polarized lepton beam and target the corresponding functions are

\[ f_{S,LL}(y,\lambda) = \text{Tr}\left[ P_L \not{k} (1+\gamma_5 \lambda) P_R \not{k}' \right] \text{Tr}\left[ P_L \not{p} P_R \not{p}' \right] = y^2(1+\lambda), \]
\[ f_{S,RR}(y,\lambda) = \text{Tr}\left[ P_R \not{k} (1+\gamma_5 \lambda) P_L \not{k}' \right] \text{Tr}\left[ P_R \not{p} P_L \not{p}' \right] = y^2(1-\lambda), \]
\[ f_{S,LR}(y,\lambda) = \text{Tr}\left[ P_L \not{k} (1+\gamma_5 \lambda) P_R \not{k}' \right] \text{Tr}\left[ P_R \not{p} P_L \not{p}' \right] = y^2(1+\lambda), \]
\[ f_{S,RL}(y,\lambda) = \text{Tr}\left[ P_R \not{k} (1+\gamma_5 \lambda) P_L \not{k}' \right] \text{Tr}\left[ P_L \not{p} P_R \not{p}' \right] = y^2(1-\lambda), \]  

\begin{equation}
(A7)
\end{equation}

\[ f_{V,LL}(y,\lambda) = \text{Tr}\left[ \gamma^\mu P_L \not{k} (1+\gamma_5 \lambda) \gamma^\nu P_L \not{k}' \right] \text{Tr}\left[ \gamma_\mu P_L \not{p} \gamma_\nu P_L \not{p}' \right] = 4(1+\lambda), \]
\[ f_{V,RR}(y,\lambda) = \text{Tr}\left[ \gamma^\mu P_R \not{k} (1+\gamma_5 \lambda) \gamma^\nu P_R \not{k}' \right] \text{Tr}\left[ \gamma_\mu P_R \not{p} \gamma_\nu P_R \not{p}' \right] = 4(1-\lambda), \]
\[ f_{V,LR}(y,\lambda) = \text{Tr}\left[ \gamma^\mu P_L \not{k} (1+\gamma_5 \lambda) \gamma^\nu P_L \not{k}' \right] \text{Tr}\left[ \gamma_\mu P_R \not{p} \gamma_\nu P_R \not{p}' \right] = 4(1-y)^2(1+\lambda), \]
\[ f_{V,RL}(y,\lambda) = \text{Tr}\left[ \gamma^\mu P_R \not{k} (1+\gamma_5 \lambda) \gamma^\nu P_R \not{k}' \right] \text{Tr}\left[ \gamma_\mu P_L \not{p} \gamma_\nu P_L \not{p}' \right] = 4(1-y)^2(1-\lambda), \]  

\begin{equation}
(A8)
\end{equation}

\[ f_{T,LL}(y,\lambda) = \text{Tr}\left[ \sigma^{\mu\nu} P_L \not{k} (1+\gamma_5 \lambda) \sigma^{\alpha\beta} P_R \not{k}' \right] \text{Tr}\left[ \sigma_{\mu\nu} P_L \not{p} \sigma_{\alpha\beta} P_R \not{p}' \right] = 16(2-y)^2(1+\lambda), \]
\[ f_{T,RR}(y,\lambda) = \text{Tr}\left[ \sigma^{\mu\nu} P_R \not{k} (1+\gamma_5 \lambda) \sigma^{\alpha\beta} P_L \not{k}' \right] \text{Tr}\left[ \sigma_{\mu\nu} P_R \not{p} \sigma_{\alpha\beta} P_L \not{p}' \right] = 16(2-y)^2(1-\lambda). \]  

\begin{equation}
(A9)
\end{equation}
and

\[ g_{S,LL}(y, \lambda) = \text{Tr}\left[ P_L \gamma_k (1 - \gamma_5 \lambda) P_R \gamma_{k'} \right] \text{Tr}\left[ P_L \gamma_{\mu} P_R \gamma_{\mu'} \right] = y^2 (1 - \lambda), \]
\[ g_{S,RR}(y, \lambda) = \text{Tr}\left[ P_R \gamma_k (1 - \gamma_5 \lambda) P_L \gamma_{k'} \right] \text{Tr}\left[ P_R \gamma_{\mu} P_L \gamma_{\mu'} \right] = y^2 (1 + \lambda), \]
\[ g_{S,LR}(y, \lambda) = \text{Tr}\left[ P_L \gamma_k (1 - \gamma_5 \lambda) P_R \gamma_{k'} \right] \text{Tr}\left[ P_R \gamma_{\mu} P_L \gamma_{\mu'} \right] = y^2 (1 - \lambda), \]
\[ g_{S,RL}(y, \lambda) = \text{Tr}\left[ P_R \gamma_k (1 - \gamma_5 \lambda) P_L \gamma_{k'} \right] \text{Tr}\left[ P_L \gamma_{\mu} P_R \gamma_{\mu'} \right] = y^2 (1 + \lambda), \quad (A10) \]

\[ g_{V,LL}(y, \lambda) = \text{Tr}\left[ \gamma^\mu P_L \gamma_{\mu} (1 - \gamma_5 \lambda) P_R \gamma_{\mu'} \right] \text{Tr}\left[ \gamma^\mu P_L \gamma_{\mu} P_R \gamma_{\mu'} \right] = 4 (1 - y)^2 (1 - \lambda), \]
\[ g_{V,RR}(y, \lambda) = \text{Tr}\left[ \gamma^\mu P_R \gamma_{\mu} (1 - \gamma_5 \lambda) P_L \gamma_{\mu'} \right] \text{Tr}\left[ \gamma^\mu P_R \gamma_{\mu} P_L \gamma_{\mu'} \right] = 4 (1 + y)^2 (1 + \lambda), \]
\[ g_{V,LR}(y, \lambda) = \text{Tr}\left[ \gamma^\mu P_L \gamma_{\mu} (1 - \gamma_5 \lambda) P_R \gamma_{\mu'} \right] \text{Tr}\left[ \gamma^\mu P_L \gamma_{\mu} P_R \gamma_{\mu'} \right] = 4 (1 - \lambda), \]
\[ g_{V,RL}(y, \lambda) = \text{Tr}\left[ \gamma^\mu P_R \gamma_{\mu} (1 - \gamma_5 \lambda) P_L \gamma_{\mu'} \right] \text{Tr}\left[ \gamma^\mu P_R \gamma_{\mu} P_L \gamma_{\mu'} \right] = 4 (1 + \lambda), \quad (A11) \]

\[ g_{T,LL}(y, \lambda) = \text{Tr}\left[ \sigma^{\mu\nu} P_L \gamma_{\mu} (1 - \gamma_5 \lambda) \sigma^{\alpha\beta} P_R \gamma_{\alpha} \gamma_{\beta} \gamma_{k'} \right] \text{Tr}\left[ \sigma^{\mu\nu} P_L \gamma_{\mu} \gamma_{\nu'} \sigma^{\alpha\beta} P_R \gamma_{\alpha} \gamma_{\beta} \gamma_{k'} \right] = 16 (2 - y)^2 (1 - \lambda), \]
\[ g_{T,RR}(y, \lambda) = \text{Tr}\left[ \sigma^{\mu\nu} P_R \gamma_{\mu} (1 - \gamma_5 \lambda) \sigma^{\alpha\beta} P_L \gamma_{\alpha} \gamma_{\beta} \gamma_{k'} \right] \text{Tr}\left[ \sigma^{\mu\nu} P_R \gamma_{\mu} \gamma_{\nu'} \sigma^{\alpha\beta} P_L \gamma_{\alpha} \gamma_{\beta} \gamma_{k'} \right] = 16 (2 - y)^2 (1 + \lambda), \quad (A12) \]

where \( \lambda \) is the helicity of the initial lepton.

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TABLE I: Double moments of quark PDF $Q_i^{A,XY}$ (in GeV$^2$) with $f = u, d, s, c, b$ and $i$ specified in the Table. The case of a Fe target and an electron beam with $E_e = 100$ GeV.

| $(I_iXY)$ | $Q_i^{A,XY}$ | $(I_iXY)$ | $Q_i^{A,XY}$ |
|-----------|--------------|-----------|--------------|
| $S$ operators |
| $(S_{uXY})$ | 3.82 | $(S_{dXY})$ | 4.07 |
| $(S_{sXY})$ | 0.74 | $(S_{cXY})$ | 0.21 |
| $(S_{bXY})$ | 0.006 |
| $V$ operators |
| $(V_{uLL/RR})$ | 43.83 | $(V_{uLR/RL})$ | 29.51 |
| $(V_{dLL/RR})$ | 46.23 | $(V_{dLR/RL})$ | 22.46 |
| $(V_{sLL/RR})$ | 5.85 | $(V_{sLR/RL})$ | 5.85 |
| $(V_{cLL/RR})$ | 1.41 | $(V_{cLR/RL})$ | 1.41 |
| $(V_{bLL/RR})$ | 0.02 | $(V_{bLR/RL})$ | 0.02 |
| $T$ operators |
| $(T_{uLL/RR})$ | 453.52 | $(T_{uLR/RR})$ | 484.37 |
| $(T_{sLL/RR})$ | 81.84 | $(T_{cLL/RR})$ | 19.21 |
| $(T_{bLL/RR})$ | 0.23 |

TABLE II: The same as in Table I but for a muon beam with $E_{\mu} = 150$ GeV.

| $(I_iXY)$ | $Q_i^{A,XY}$ | $(I_iXY)$ | $Q_i^{A,XY}$ |
|-----------|--------------|-----------|--------------|
| $S$ operators |
| $(S_{uXY})$ | 5.64 | $(S_{dXY})$ | 6.01 |
| $(S_{sXY})$ | 1.12 | $(S_{cXY})$ | 0.35 |
| $(S_{bXY})$ | 0.02 |
| $V$ operators |
| $(V_{uLL/RR})$ | 64.30 | $(V_{uLR/RL})$ | 30.21 |
| $(V_{dLL/RR})$ | 67.81 | $(V_{dLR/RL})$ | 33.07 |
| $(V_{sLL/RR})$ | 8.84 | $(V_{sLR/RL})$ | 8.84 |
| $(V_{cLL/RR})$ | 2.32 | $(V_{cLR/RL})$ | 2.32 |
| $(V_{bLL/RR})$ | 0.07 | $(V_{bLR/RL})$ | 0.07 |
| $T$ operators |
| $(T_{uLL/RR})$ | 665.88 | $(T_{uLR/RR})$ | 710.69 |
| $(T_{sLL/RR})$ | 123.48 | $(T_{cLL/RR})$ | 31.57 |
| $(T_{bLL/RR})$ | 0.79 |
TABLE III: The same as in Table III but for a Pb and an electron beam with $E_e = 100$ GeV.

| $(i_{XY})$ | $Q_{i_{LL},XY}^S$ | $(i_{XY})$ | $Q_{i_{LL},XY}^A$ |
|------------|-----------------|------------|-----------------|
| $S$ operators | | | |
| $(S_{u_{XY}})$ | 13.58 | $(S_{d_{XY}})$ | 15.61 |
| $(S_{s_{XY}})$ | 2.73 | $(S_{c_{XY}})$ | 0.77 |
| $(S_{b_{XY}})$ | 0.02 | | |
| $V$ operators | | | |
| $(V_{u_{LL}/RR})$ | 155.11 | $(V_{u_{LR}/RL})$ | 73.38 |
| $(V_{d_{LL}/RR})$ | 177.78 | $(V_{d_{LR}/RL})$ | 85.47 |
| $(V_{s_{LL}/RR})$ | 21.63 | $(V_{s_{LR}/RL})$ | 21.63 |
| $(V_{c_{LL}/RR})$ | 5.20 | $(V_{c_{LR}/RL})$ | 5.20 |
| $(V_{b_{LL}/RR})$ | 0.08 | $(V_{b_{LR}/RL})$ | 0.08 |
| $T$ operators | | | |
| $(T_{u_{LL}/RR})$ | 1610.52 | $(T_{d_{LL}/RR})$ | 1856.30 |
| $(T_{s_{LL}/RR})$ | 302.50 | $(T_{c_{LL}/RR})$ | 71.00 |
| $(T_{b_{LL}/RR})$ | 0.85 | | |

TABLE IV: The same as in Table III but for a muon beam with $E_\mu = 150$ GeV.

| $(i_{XY})$ | $Q_{i_{LL},XY}^S$ | $(i_{XY})$ | $Q_{i_{LL},XY}^A$ |
|------------|-----------------|------------|-----------------|
| $S$ operators | | | |
| $(S_{u_{XY}})$ | 20.03 | $(S_{d_{XY}})$ | 23.02 |
| $(S_{s_{XY}})$ | 4.14 | $(S_{c_{XY}})$ | 1.28 |
| $(S_{b_{XY}})$ | 0.07 | | |
| $V$ operators | | | |
| $(V_{u_{LL}/RR})$ | 227.61 | $(V_{u_{LR}/RL})$ | 108.12 |
| $(V_{d_{LL}/RR})$ | 266.74 | $(V_{d_{LR}/RL})$ | 125.80 |
| $(V_{s_{LL}/RR})$ | 32.67 | $(V_{s_{LR}/RL})$ | 32.67 |
| $(V_{c_{LL}/RR})$ | 8.57 | $(V_{c_{LR}/RL})$ | 8.57 |
| $(V_{b_{LL}/RR})$ | 0.25 | $(V_{b_{LR}/RL})$ | 0.25 |
| $T$ operators | | | |
| $(T_{u_{LL}/RR})$ | 2365.21 | $(T_{d_{LL}/RR})$ | 2723.18 |
| $(T_{s_{LL}/RR})$ | 456.45 | $(T_{c_{LL}/RR})$ | 116.71 |
| $(T_{b_{LL}/RR})$ | 2.92 | | |
TABLE V: Lower limits for LFV mass scales $\Lambda_{i,f,XY}$ (in TeV) of the operators in Eq. (4) with $f = u,d,s,c,b$ and $i$ specified in the Table. The case of a Fe target and an electron beam with $E_e = 100$ GeV.

| $(IiXY)$ | $\Lambda_{u,f,XY}$ | $(IiXY)$ | $\Lambda_{d,f,XY}$ |
|----------|-------------------|----------|-------------------|
| (SuXY)   | 0.23              | (SdXY)   | 0.24              |
| (SsXY)   | 0.15              | (ScXY)   | 0.11              |
| (SbXY)   | 0.05              |          |                   |

$S$ operators

| $(SuLL/RR)$ | 0.43 | $(SuLR/RL)$ | 0.35 |
| $(SdLL/RR)$ | 0.44 | $(SdLR/RL)$ | 0.36 |
| $(SsLL/RR)$ | 0.26 | $(SsLR/RL)$ | 0.26 |
| $(ScLL/RR)$ | 0.18 | $(ScLR/RL)$ | 0.18 |
| $(SbLL/RR)$ | 0.06 | $(SbLR/RL)$ | 0.06 |

$V$ operators

| $(TuLL/RR)$ | 0.77 | $(TdLL/RR)$ | 0.78 |
| $(TsLL/RR)$ | 0.50 | $(TkLL/RR)$ | 0.34 |
| $(TbLL/RR)$ | 0.12 |          |   |

$T$ operators

| $(TuLL/RR)$ | 9.90 | $(TdLL/RR)$ | 10.06 |
| $(TsLL/RR)$ | 6.50 | $(TkLL/RR)$ | 4.62 |
| $(TbLL/RR)$ | 1.84 |          |   |

TABLE VI: The same as in Table V but for a muon beam with $E_\mu = 150$ GeV.

| $(IiXY)$ | $\Lambda_{u,f,XY}$ | $(IiXY)$ | $\Lambda_{d,f,XY}$ |
|----------|-------------------|----------|-------------------|
| (SuXY)   | 3.00              | (SdXY)   | 3.05              |
| (SsXY)   | 2.01              | (ScXY)   | 1.50              |
| (SbXY)   | 0.72              |          |                   |

$S$ operators

| $(VuLL/RR)$ | 5.52 | $(VuLR/RL)$ | 4.57 |
| $(VdLL/RR)$ | 5.60 | $(VdLR/RL)$ | 4.67 |
| $(VsLL/RR)$ | 3.36 | $(VsLR/RL)$ | 3.36 |
| $(VcLL/RR)$ | 2.41 | $(VcLR/RL)$ | 2.41 |
| $(VbLL/RR)$ | 1.00 | $(VbLR/RL)$ | 1.00 |

$V$ operators

| $(TuLL/RR)$ | 9.90 | $(TdLL/RR)$ | 10.06 |
| $(TsLL/RR)$ | 6.50 | $(TkLL/RR)$ | 4.62 |
| $(TbLL/RR)$ | 1.84 |          |   |
TABLE VII: The same as in Table [V] but for a Pb target and an electron beam with $E_e = 100$ GeV.

| $(i_{iXY})$ | $\Lambda_{t,XY}$ | $(i_{iXY})$ | $\Lambda_{t,XY}$ |
|-------------|-----------------|-------------|-----------------|
| $S$ operators |
| $(SuXY)$ | 0.18 | $(SdXY)$ | 0.19 |
| $(SsXY)$ | 0.12 | $(ScXY)$ | 0.09 |
| $(ShXY)$ | 0.04 | |
| $V$ operators |
| $(VuLL/RR)$ | 0.34 | $(VuLR/RL)$ | 0.28 |
| $(VdLL/RR)$ | 0.35 | $(VdLR/RL)$ | 0.29 |
| $(VsLL/RR)$ | 0.21 | $(VsLR/RL)$ | 0.21 |
| $(VcLL/RR)$ | 0.15 | $(VcLR/RL)$ | 0.15 |
| $(VbLL/RR)$ | 0.05 | $(VbLR/RL)$ | 0.05 |
| $T$ operators |
| $(TuLL/RR)$ | 0.61 | $(TdLL/RR)$ | 0.63 |
| $(TsLL/RR)$ | 0.40 | $(TcLL/RR)$ | 0.28 |
| $(TbLL/RR)$ | 0.09 | |

TABLE VIII: The same as in Table [VII] but for a muon beam with $E_\mu = 150$ GeV.

| $(i_{iXY})$ | $\Lambda_{t,XY}$ | $(i_{iXY})$ | $\Lambda_{t,XY}$ |
|-------------|-----------------|-------------|-----------------|
| $S$ operators |
| $(SuXY)$ | 2.35 | $(SdXY)$ | 2.45 |
| $(SsXY)$ | 1.58 | $(ScXY)$ | 1.17 |
| $(ShXY)$ | 0.56 | |
| $V$ operators |
| $(VuLL/RR)$ | 4.31 | $(VuLR/RL)$ | 3.57 |
| $(VdLL/RR)$ | 4.46 | $(VdLR/RL)$ | 3.71 |
| $(VsLL/RR)$ | 2.65 | $(VsLR/RL)$ | 2.65 |
| $(VcLL/RR)$ | 1.90 | $(VcLR/RL)$ | 1.90 |
| $(VbLL/RR)$ | 0.78 | $(VbLR/RL)$ | 0.78 |
| $T$ operators |
| $(TuLL/RR)$ | 7.74 | $(TdLL/RR)$ | 8.01 |
| $(TsLL/RR)$ | 5.13 | $(TcLL/RR)$ | 3.65 |
| $(TbLL/RR)$ | 1.45 | |
TABLE IX: Double moments of quark PDF $\tilde{Q}_{i,f,XY}^A$ (in GeV$^2$) with $f = u, d, s, c, b$ and $i$ specified in the Table. The case of a Fe target and an electron beam with $E_e = 100$ GeV.

| $iXY$ | $\tilde{Q}_{i,f,XY}^A$ | $iXY$ | $\tilde{Q}_{i,f,XY}^A$ |
|-------|----------------|-------|----------------|
| $SuXY$ | 0.97 | $SdXY$ | 1.03 |
| $SsXY$ | 0.18 | $ScXY$ | 0.05 |
| $SbXY$ | 0.001 | |
| $VuLL/RR$ | 23.08 | $VuLR/RL$ | 14.12 |
| $VdLL/RR$ | 24.43 | $VdLR/RL$ | 15.31 |
| $VsLL/RR$ | 3.28 | $VsLR/RL$ | 3.28 |
| $VcLL/RR$ | 0.72 | $VcLR/RL$ | 0.72 |
| $VbLL/RR$ | 0.005 | $VbLR/RL$ | 0.005 |
| $TuLL/RR$ | 282.06 | $TdLL/RR$ | 301.39 |
| $TsLL/RR$ | 49.55 | $TcLL/RR$ | 10.79 |
| $TbLL/RR$ | 0.06 | |

TABLE X: The same as in Table IX but for a muon beam with $E_\mu = 150$ GeV.

| $iXY$ | $\tilde{Q}_{i,f,XY}^A$ | $iXY$ | $\tilde{Q}_{i,f,XY}^A$ |
|-------|----------------|-------|----------------|
| $SuXY$ | 1.42 | $SdXY$ | 1.52 |
| $SsXY$ | 0.28 | $ScXY$ | 0.08 |
| $SbXY$ | 0.003 | |
| $VuLL/RR$ | 33.82 | $VuLR/RL$ | 20.75 |
| $VdLL/RR$ | 35.79 | $VdLR/RL$ | 22.48 |
| $VsLL/RR$ | 4.94 | $VsLR/RL$ | 4.94 |
| $VcLL/RR$ | 1.18 | $VcLR/RL$ | 1.18 |
| $VbLL/RR$ | 0.02 | $VbLR/RL$ | 0.02 |
| $TuLL/RR$ | 413.67 | $TdLL/RR$ | 441.89 |
| $TsLL/RR$ | 74.58 | $TcLL/RR$ | 17.53 |
| $TbLL/RR$ | 0.24 | |
TABLE XI: The same as in Table [IX] but for a Pb target and an electron beam with $E_e = 100$ GeV.

| $(i_{iXY})$ | $Q_{i_{iXY}}^S$ | $(i_{iXY})$ | $Q_{i_{iXY}}^A$ |
|-------------|-----------------|-------------|----------------|
| $S$ operators |
| $(S_{uXY})$ | 3.44 | $(S_{dXY})$ | 3.96 |
| $(S_{sXY})$ | 0.68 | $(S_{cXY})$ | 0.18 |
| $(S_{bXY})$ | 0.004 | |
| $V$ operators |
| $(V_{uLL/RR})$ | 81.73 | $(V_{uLR/RL})$ | 50.33 |
| $(V_{dLL/RR})$ | 93.87 | $(V_{dLR/RL})$ | 58.45 |
| $(V_{sLL/RR})$ | 12.13 | $(V_{sLR/RL})$ | 12.13 |
| $(V_{cLL/RR})$ | 2.67 | $(V_{cLR/RL})$ | 2.67 |
| $(V_{bLL/RR})$ | 0.02 | $(V_{bLR/RL})$ | 0.02 |
| $T$ operators |
| $(T_{uLL/RR})$ | 1001.40 | $(T_{dLL/RR})$ | 1155.30 |
| $(T_{sLL/RR})$ | 183.18 | $(T_{cLL/RR})$ | 39.88 |
| $(T_{bLL/RR})$ | 0.22 | |

TABLE XII: The same as in Table [XI] but for a muon beam with $E_\mu = 150$ GeV.

| $(i_{iXY})$ | $Q_{i_{iXY}}^S$ | $(i_{iXY})$ | $Q_{i_{iXY}}^A$ |
|-------------|-----------------|-------------|----------------|
| $S$ operators |
| $(S_{uXY})$ | 5.06 | $(S_{dXY})$ | 5.82 |
| $(S_{sXY})$ | 1.03 | $(S_{cXY})$ | 0.29 |
| $(S_{bXY})$ | 0.02 | |
| $V$ operators |
| $(V_{uLL/RR})$ | 119.80 | $(V_{uLR/RL})$ | 74.00 |
| $(V_{dLL/RR})$ | 137.52 | $(V_{dLR/RL})$ | 85.79 |
| $(V_{sLL/RR})$ | 18.26 | $(V_{sLR/RL})$ | 18.26 |
| $(V_{cLL/RR})$ | 4.35 | $(V_{cLR/RL})$ | 4.35 |
| $(V_{bLL/RR})$ | 0.07 | $(V_{bLR/RL})$ | 0.07 |
| $T$ operators |
| $(T_{uLL/RR})$ | 1469.25 | $(T_{dLL/RR})$ | 1693.52 |
| $(T_{sLL/RR})$ | 275.69 | $(T_{cLL/RR})$ | 64.82 |
| $(T_{bLL/RR})$ | 0.89 | |

TABLE XIII: $\langle E_\tau \rangle_I$ (in GeV) for different LFV operators in the case of an electron beam with $E_e = 100$ GeV.

| Operator | $\langle E_\tau \rangle_I$ |
|----------|----------------|
| $S$ operators | 25 |
| $V$ operators | 57 |
| $T$ operators | 62 |

TABLE XIV: The same as in Table [XIII] but for a muon beam with $E_\mu = 150$ GeV.

| Operator | $\langle E_\tau \rangle_I$ |
|----------|----------------|
| $S$ operators | 38 |
| $V$ operators | 86 |
| $T$ operators | 93 |
FIG. 1: Spectrum of $\tau$ lepton for $S$ operators.

FIG. 2: Spectrum of $\tau$ lepton for $V$ operators.

FIG. 3: Spectrum of $\tau$ lepton for $T$ operators.