Supplementary information to the manuscript:
Reconfigurable Josephson phase shifter

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Supplementary Figure 1. Sketch of the geometry of the phase shifter device from Fig. 2 with coordinates of the traps \( z_{Tj} \) and junctions \( z_{Ji} \) counted from \( J_2 \).

Samples
Studied devices contain planar JJ’s. They are made from bi-layer films with a 70 nm top Nb layer and thin non-superconducting metallic bottom layer. Devices in Figs. 1 (d), 2 and 3 are made using a paramagnetic CuNi alloy, the SQUID from Fig. 4 (a) - using pure Cu underlayer. We also tested other metals and just a single layer Nb film. All of them work in a similar manner and results do not depend on a specific material. Variable-thickness-bridge type JJ’s are made by cutting a narrow (~ 20 nm) groove in the top Nb layer by focused ion beam (FIB) etching, as sketched in Fig. 2 (b). Details of junction fabrication can be found elsewhere [18,23,27]. Planar junction properties were described in Ref. [52]. SQUID device, Fig. 4, was made by FIB milling of a rectangular loop in a similar manner and results do not depend on a specific material. Variable-thickness-bridge type JJ’s are made by cutting a narrow (~ 20 nm) groove in the top Nb layer by focused ion beam (FIB) etching, as sketched in Fig. 2 (b). Details of junction fabrication can be found elsewhere [18,23,27]. Planar junction properties were described in Ref. [52].

Experimental
Measurements are performed in a cryogen-free cryostat using a four-probe configuration. Magnetic field is applied perpendicular to the films. Vortex states are prepared in the following way. We start from the Meissner state by zero-field cooling of a device without bias current. Vortices are introduced either by applying current pulses, magnetic fields, or both, as described in Ref. [18]. Depending on the amplitude and the sign of current pulses, we can introduce either vortices, or antivortices as shown in Fig. 2 (c).

MFM imaging
Low-temperature MFM imaging is carried out on AttoCube scanning probe system (AttoDry 1000/SU) with a standard Co/Cr-coated cantilever (MESP, Bruker, 2.8 N/m spring constant). MFM images, shown in Figs. 1 (e) and (f) are made in a tapping mode at a fixed resonance frequency, \( \approx 87 \) kHz. The color scale represents the phase of tip oscillations: the black color corresponds to zero phase, brighter areas to a positive phase with the brightest level \( \approx +10^6 \). The positive phase shift indicates a repulsive force on the tip, which is caused by Meissner screening of the tip field by the superconductor. To trap a vortex, the tip was approached close enough to the hole so that inhomogeneous magnetic field of the tip locally introduced a vortex. The sign of the vortex depends on the direction of tip magnetization. Because the vortex is introduced by the tip field, the tip-vortex interaction is attractive, resulting in the dark contrast of the trapped vortex in a subsequent MFM phase map, shown in Fig. 1 (f). \( L_c(H) \) measurements, presented in Figs. 1 (h) and (i) are performed in the same MFM system with a retracted tip, in order not to induce extra distortion from the tip itself [29].

Numerical simulations
We use numerical fitting for extraction of JPS. Simulations presented by red lines in Figs. 1 (g), 2 (d-g) and 3 are done taking \( \varphi_v(x) \) from Eq. (1) with actual trap geometries \( (x_{0i}/L_x, z_{0i}/L_z, \Theta_{0i}) \) and using \( V_i \) as a fitting parameter. The critical current is calculated by maximization of integrated Josephson current, \( I = \left( I_0/L \right) \int_0^{L_z} \sin \left[ \left( \varphi(H) + \varphi_v \right) \right] dx \), where \( \varphi(H) \) represents the linear field-dependent phase gradient in the absence of vortices. Details of the formalism can be found in Ref. [29]. In all demonstrated cases such fitting allows unambiguous estimation of JPS, \( \Delta \varphi_v = -\sum_i V_i \Theta_{0i} \).

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