Automated Task Updates of Temporal Logic Specifications for Heterogeneous Robots

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Abstract—Given a heterogeneous group of robots executing a complex task represented in Linear Temporal Logic, and a new set of tasks for the group, we define the task update problem and propose a framework for automatically updating individual robot tasks given their respective existing tasks and capabilities. Our heuristic, token-based, conflict resolution task allocation algorithm generates a near-optimal assignment for the new task. We demonstrate the scalability of our approach through simulations of multi-robot tasks.

I. INTRODUCTION

Heterogeneous multi-robot systems consist of robots with different capabilities and are often created with a specific task in mind. However, if the task is changed during execution, especially if more requirements are added to the previous ones, there is a need for automated techniques that would allocate the task to the robots while maintaining the previous task and minimizing cost. For example, in humanitarian aid or disaster response situations, as new emergencies arise and timing is critical, automating the process for robots to interleave new tasks into their existing tasks without human input would 1) increase efficiency in the assignment process and 2) ensure that the teams are responding quickly.

In this paper, we address the problem of automatically updating robot behaviors given tasks encoded in Linear Temporal Logic (LTL) over an abstraction of the robot motion and capabilities. We assume each sub-task can be accomplished by a single robot, and that all tasks are defined over controllable atomic propositions. We also assume there exist continuous controllers that can implement the abstract behaviors in a way that ensures collision avoidance between the robots and guarantees that the continuous behaviors implement the abstract ones [1].

There exists a rich literature in addressing automated multi-robot task allocation and coalition formation [2]–[4]. Researchers have developed architectures to model robot capabilities and interactions, such as social networks [5]. Game-theoretic models represent systems as stochastic games by using Markov Decision Processes (MDPs) or partially observable MDPs [6]. While these methods can be shown to converge to a locally optimal policy, they do not scale for large numbers of agents, since the state space grows exponentially as more agents are introduced. The work in [7] addresses scalability for the multi-agent MDP problem, but does so by assuming special dependence structures among robots.

Dynamic coalition formation has been of particular interest, where autonomous robots cooperate to perform emerging time-varying tasks. Current methods include greedy approximate algorithms [8], particle swarm optimization [9], and evolutionary algorithms [10]. Another method is to use market-based algorithms for robots to form teams based on the bids they make for the specified task [11], [12]. This requires a leader that acts as a mediator for the group. To maintain a flat hierarchy while still allocating tasks in a near-optimal manner, [13] proposes a token-based framework, where tasks and resources are abstracted as tokens and passed locally among agents. Each agent decides whether to keep the token or to pass it to other agents. This decision requires information about how the token has been passed around to other agents. In this paper, we develop a token-based scheme for task allocation that is able to maintain near-optimal results without any token history information.

To increase the complexity of possible tasks, in recent years there has been a growing interest in multi-robot planning for task specifications written in temporal logics, which enables users to specify temporally extended tasks. In [14], the authors use Time Window Temporal Logic to address multi-robot planning with synchronization requirements. The authors of [15] encode Signal Temporal Logic specifications as mixed integer linear constraints to generate a plan for a heterogeneous team. The work in [16] presents an algorithm that allocates tasks to robots while simultaneously planning their actions. To avoid state-space explosion in their centralized planning, the authors sequentially link each robot model using switch transitions. In [17], the authors use reinforcement learning to synthesize plans over Markov decision processes under Linear Temporal Logic, and an auction-based algorithm assigns tasks to robots. All of these approaches assume that new tasks can only be directed towards unassigned robots.

There also exists literature that addresses the problem of rescheduling in response to unexpected disturbances. The authors of [18] propose a bargaining game approach to generate a real-time scheduling scheme for an Internet of Things-enabled job shop. The work in [19] presents an online hybrid contract-net negotiation protocol in response to unexpected disturbances in a shop. Agents place bids of their earliest finishing time, and a coordinator creates a new schedule accordingly. While these papers address task reallocation due to unexpected environment changes, to our knowledge, no work has been done to tackle the problem of
multirobot task distribution within the context of temporal logics, where new tasks are introduced to robots that are performing existing tasks.

**Contributions:** In this paper, we propose a method for heterogeneous robots to respond to a new task given their capabilities and respective ongoing tasks while providing guarantees on task feasibility. The contributions are as follows: 1) a mathematical formulation of the new task distribution problem, 2) a framework for robots to automatically update their behavior based on both the new task that is introduced and the progress within their current one, allowing them to perform both tasks, and 3) a heuristic, token-based task allocation algorithm to determine the final task allocation assignment for the new task while minimizing overall cost.

II. PRELIMINARIES

A. Linear Temporal Logic

Let AP be a set of atomic propositions such that \( \pi \in AP \) is a Boolean variable. We use these propositions to capture robot capabilities. For example, \( \text{pick}_\text{up} \) can correspond to the robot performing a pick up action.

**Syntax:** An LTL formula [20] is defined recursively from atomic propositions \( \pi \in AP \) using the following grammar:

\[
\varphi ::= \pi \mid \neg \varphi \mid \varphi \lor \varphi \mid \varphi \land \varphi \mid \varphi U \varphi
\]

where \( \neg \) ("not") and \( \lor \) ("or") are Boolean operators. \( \land \) and \( U \) are the temporal operators "next" and "until", respectively. Using these basic operators, we can construct the additional logical operators conjunction \( \varphi \land \varphi \), implication \( \varphi \Rightarrow \varphi \), and equivalence \( \varphi \Leftrightarrow \varphi \), as well as the temporal operators eventually \( \Diamond \varphi \) and always \( \Box \varphi \).

**Semantics:** The semantics of an LTL formula \( \varphi \) are defined over a trace \( \sigma \), where \( \sigma = \sigma_0\sigma_1\sigma_2\ldots \) is an infinite sequence, and \( \sigma_p \) represents the set of \( AP \) that are True at position \( p \). We denote that \( \sigma \) satisfies LTL formula \( \varphi \) as \( \sigma \models \varphi \).

Intuitively, \( \sigma \models \Diamond \varphi \) if \( \varphi \) is True at the next state in the trace. To satisfy \( \varphi_1 U \varphi_2 \), \( \varphi_1 \) must stay True until \( \varphi_2 \) becomes True. The formula \( \Box \varphi \) is satisfied if \( \varphi \) is True at every position in \( \sigma \), and \( \sigma \models \Box \varphi \) if there exists a step in \( \sigma \) where \( \varphi \) is True. For a complete definition of the semantics of LTL, see [20].

B. Büchi Automata

A nondeterministic Büchi automaton can be constructed from an LTL formula such that an infinite trace is accepted by the Büchi automaton if and only if it satisfies the LTL formula [21]. A Büchi automaton is defined as a tuple \( B = (\Sigma, Z, z_0, \delta, F) \), where \( \Sigma \) is the alphabet of \( B \), \( Z \) is a finite set of states, \( z_0 \in Z \) is the initial state, \( \delta : Z \times \Sigma \rightarrow 2^Z \) is a transition function, and \( F \subseteq Z \) is a set of accepting states. A run of a Büchi automaton on an infinite word \( w = w_1w_2w_3\ldots \) is an infinite sequence of states \( z = z_0z_1z_2\ldots \) such that \( \forall i, w_i \in \Sigma \) and \( (z_{i-1}, w_i, z_i) \in \delta \). A run is accepting if and only if \( \text{inf}(z) \cap F \neq \emptyset \), where \( \text{inf}(z) \) is defined as the set of states that are visited infinitely often in \( z \) [20].

**Büchi intersection:** Given two LTL formulas, \( \varphi^1 \) and \( \varphi^2 \) over \( AP \), the intersection of their respective Büchi automata, \( B^1 \) and \( B^2 \), represents traces that satisfy both \( \varphi^1 \) and \( \varphi^2 \).

Let \( B^1 = (\Sigma_1, Z_1, z_0^1, \delta_1, F_1) \) and \( B^2 = (\Sigma_2, Z_2, z_0^2, \delta_2, F_2) \). Their intersection is defined as \( B^1 \cap B^2 = (\Sigma_1 \times \Sigma_2, Z_1 \times Z_2 \times \{0,1,2\}, \{(z_0^1, z_0^2, 0), \delta' \}, Z_1 \times Z_2 \times \{2\}) \). There is a transition on \( a \in \Sigma_1 \) if \( (r, q, a, (r', q', x')) \in \delta' \) and only if \( (r, a, r') \in \delta_1 \) and \( (q, a, q') \in \delta_2 \). The components \( x, x' \in \{0,1,2\} \) are determined by \( F_1 \) and \( F_2 \), the accepting conditions of the Büchi automata. It is insufficient to simply define the accepting states of \( B^1 \cap B^2 \) as \( F_1 \times F_2 \) even if the accepting states from both automata appear individually infinitely often, they may appear together only finitely many times. Thus, \( x, x' \in \{0,1,2\} \) ensures that accepting states from both \( B^1 \) and \( B^2 \) appear infinitely often together. For a detailed explanation on how to construct the intersection of two Büchi automata, see [22].

III. PROBLEM SETUP

A. Task Specification

A task is a set of LTL formulas \( \Phi = \{\varphi^1, \varphi^2, \ldots, \varphi^m\} \) for which the following properties hold:

- **Non-conflicting:** There exists a \( \sigma \) such that \( \forall j, k \in \sigma \models \varphi^j \land \varphi^k \). Intuitively, this means that the satisfaction of one sub-task must not violate any other sub-task.

- **Non-collaborative:** Every \( \varphi^i \) can be satisfied by a single robot.

**Example.** The task "pick up a box from room 2 and drop it off in room 3, pull the lever in room 3, and repeatedly scan and take a picture in room 1" can be encoded in LTL and decomposed into three sub-tasks:

\[
\varphi^1 = (\neg \text{drop off} \land \text{room 2} \land \text{pick up}) \quad (1)
\]

\[
\varphi^2 = \Diamond (\text{room 3} \land \text{pull lever}) \quad (2)
\]

\[
\varphi^3 = \Box \Diamond (\text{room 1} \land \text{scan} \land \text{use camera}) \quad (3)
\]

B. Robot Model

Each robot in the group has a set of capabilities related to the required tasks. We define a capability as a weighted transition system defined as a tuple \( \lambda = (AP, S, s_0, R, L, W) \), where

- \( AP \) is a set of atomic propositions
- \( S \) is a finite set of states
- \( s_0 \in S \) is the initial state
- \( R \subseteq S \times S \) is a transition relation where for all \( s \in S \) there exists \( s' \in S \) such that \((s, s') \in R \)
- \( L : S \rightarrow 2^{AP} \) is the labeling function such that \( L(s) \subseteq AP \) is the set of \( AP \) that are true in state \( s \)
- \( W : R \rightarrow \mathbb{R}_{\geq 0} \) is the cost function

Let there be a set of \( v \) capabilities, \( \Lambda = \{\lambda_1, \lambda_2, \ldots, \lambda_v\} \) where \( \lambda_k = (AP_k, S_k, s_{0,k}, R_k, L_k, W_k) \). We consider robots that are heterogeneous, where each robot has its own capability set \( \Lambda_i \subseteq \Lambda \).
Let $W$ be two states in $A$ that is partitioned into a set of regions. We describe the robot model (c) Fig. 1: Example of a motion model (a), a capability (b), and the robot model (c) such that

$$W \cdot L \cdot A \cdot R \cdot S \cdot \Sigma \cdot L$$

We assume all robots are moving in a shared workspace $(\Sigma, Z, z_0, \delta, F)$ such that $\Sigma = 2^{AP^r}$ is created from $AP^r$, the propositions in $\varphi$. Note that it is not necessary for $AP^i$ to be equivalent to $AP^r$. $AP^i \not\subseteq AP^r$ when a robot has additional capabilities that are not required for task $\varphi$. Similarly, $AP^r \not\subseteq AP^i$ when a robot does not have all the necessary capabilities required for task $\varphi$.

The product $G = (\Sigma, AP^i, Q, q_0, L^G, F^G)$, where

- $Q = S \times Z$ is a finite set of states
- $q_0 = (s_0, z_0)$ is the initial state
- $L^G$ is the labeling function such that $L^G((s, z)) = L(s) \cap AP^r$
- $\Delta$ is the transition function, where a transition exists from $(s, z)$ to $(s', z')$ if and only if $(s, s') \in R$, and $\exists \sigma \in \Sigma$ such that $\sigma = L^G(s')$ and $z' \in \delta(z, \sigma)$
- $W^G: R \rightarrow \mathbb{R}_{\geq 0}$ is the cost function such that for $q = (s, z)$ and $q' = (s', z')$, $W^G((q, q')) = W((s, s'))$
- $F^G = S \times F$ is a set of accepting states

Let path $q = q_0 \rightarrow q_1 \rightarrow q_2 \rightarrow ... \rightarrow q_\ell \rightarrow q_{\ell+1}...$ be an infinite sequence in $G$ that visits the states in $F^G$ infinitely often. The path is composed of a prefix – a finite trace – and a suffix – a cycle that repeats. A behavior $b_i$ of a robot is defined as the labels produced by $q$: $b_i = L^G(q_0) L^G(q_1) L^G(q_2) ... L^G(q_\ell) L^G(q_{\ell+1})...$. Given a prefix of length $\ell$, we define the cost of $b_i$ as:

$$c_i(b_i) = \sum_{r=0}^{\ell-1} W^G((q_r, q_{r+1})) \quad (4)$$

We allow the cost for each sub-task to be non-additive. That is, the sum of the costs of the behaviors for satisfying two individual sub-tasks may be more than the cost of the behavior for satisfying both. For a simple illustration, see Fig. 2.

**(C. Robot Behavior)**

Given a robot model $A_i$ and a desired behavior $\varphi$ captured using $B^\varphi$, we can synthesize a behavior for the robot such that it satisfies $\varphi$ by choosing an accepting trace in $G = A_i \times B^\varphi$ [20]. Let $A_i = (AP^i, S, s_0, R, L, W)$ and $B^\varphi = (\Sigma, Z, z_0, \delta, F)$ such that $\Sigma = 2^{AP^r}$ is created from $AP^r$, the propositions in $\varphi$. Note that it is not necessary for $AP^i$ to be equivalent to $AP^r$. $AP^i \not\subseteq AP^r$ when a robot has additional capabilities that are not required for task $\varphi$. Similarly, $AP^r \not\subseteq AP^i$ when a robot does not have all the necessary capabilities required for task $\varphi$.

The product $G = (\Sigma, AP^i, Q, q_0, L^G, F^G)$, where

- $Q = S \times Z$ is a finite set of states
- $q_0 = (s_0, z_0)$ is the initial state
- $L^G$ is the labeling function such that $L^G((s, z)) = L(s) \cap AP^r$
- $\Delta$ is the transition function, where a transition exists from $(s, z)$ to $(s', z')$ if and only if $(s, s') \in R$, and $\exists \sigma \in \Sigma$ such that $\sigma = L^G(s')$ and $z' \in \delta(z, \sigma)$
- $W^G: R \rightarrow \mathbb{R}_{\geq 0}$ is the cost function such that for $q = (s, z)$ and $q' = (s', z')$, $W^G((q, q')) = W((s, s'))$
- $F^G = S \times F$ is a set of accepting states

Let path $q = q_0 \rightarrow q_1 \rightarrow q_2 \rightarrow ... \rightarrow q_\ell \rightarrow q_{\ell+1}...$ be an infinite sequence in $G$ that visits the states in $F^G$ infinitely often. The path is composed of a prefix – a finite trace – and a suffix – a cycle that repeats. A behavior $b_i$ of a robot is defined as the labels produced by $q$: $b_i = L^G(q_0) L^G(q_1) L^G(q_2) ... L^G(q_\ell) L^G(q_{\ell+1})...$. Given a prefix of length $\ell$, we define the cost of $b_i$ as:

$$c_i(b_i) = \sum_{r=0}^{\ell-1} W^G((q_r, q_{r+1})) \quad (4)$$

We allow the cost for each sub-task to be non-additive. That is, the sum of the costs of the behaviors for satisfying two individual sub-tasks may be more than the cost of the behavior for satisfying both. For a simple illustration, see Fig. 2.
We define $P$ to be a partition of $\Phi^{new}$ such that the following properties hold for $p_x \in P$:

$$p_x \cap p_y = \emptyset, \quad \forall x \neq y$$  \hspace{1cm} (5)

$$\bigcup_x p_x = P$$

Given a set of new sub-tasks $\Phi^{new} = \{\varphi^1, \varphi^2, ..., \varphi^m\}$, we define for each robot $A_i$ the corresponding cost and satisfiability structures:

Cost $\Gamma_i$ is a function that maps $\varphi^{curr}_i \land \bigwedge_{j \in p_i} \Phi^{new}[j]$ to its corresponding cost, $c_i(b^{new})$, where $p_i \in P$ is the set of sub-tasks assigned to $A_i$.

Satisfiability $\zeta_i$ denotes which sub-tasks $A_i$ is able to perform:

$$\zeta_i[j] = \begin{cases} 1 & \exists b_i \text{ such that } b_i \models \varphi^j \\ 0 & \text{otherwise} \end{cases}$$

IV. Problem Statement

Let there be $n$ heterogeneous robots $A = (A_1, A_2, ..., A_n)$. Each robot $A_i$ has behavior $b^{curr}_i$ that satisfies its existing task specification $\varphi^{curr}_i$.

Given a new task $\Phi^{new} = \{\varphi^1, \varphi^2, ..., \varphi^m\}$ that is introduced during the robots’ execution of their current tasks, find a partition $P$, as defined in Eq. 5, to assign robots to new sub-tasks so that $b^{new}_i \models \varphi^{curr}_i \land \bigwedge_{j \in p_i} \Phi^{new}[j]$, subject to the following optimization criteria:

$$\arg\min_{p_i \in P} \sum_{i=1}^n c_i(b^{new}_i),$$  \hspace{1cm} (6)

We make the following assumptions about the system: collision avoidance is taken care of by low-level controllers; the sub-tasks are nonreactive, meaning that the robot behavior does not depend on external events; each sub-task can be satisfied by a single robot and does not require robot collaboration, as outlined in III-A.

A. Example

Consider a 2D environment with five rooms containing three robots. The set of all capabilities is $\Lambda = \{\lambda_{Mot}, \lambda_{arm}, \lambda_{scan}, \lambda_{camera}\}$. $AP_{arm}$ is an abstraction of a physical robot manipulator that is capable of grasping objects, such as boxes and levers. $AP_{scan}$ represents a robot’s ability to scan barcodes. Similarly, we abstract a robot’s camera as $AP_{camera}$, which denotes whether or not the robot is taking a picture.

Each robot has the following capabilities and tasks:

- Robot 1: “Scan in room 4, then scan in room 1”
  $$\varphi^{curr}_1 = \neg (room_1 \land scan) \lor (room_4 \land scan) \land \Diamond (room_1 \land scan)$$  \hspace{1cm} (7)
  $$\Lambda_1 = \{\lambda_{Mot}, \lambda_{arm}, \lambda_{scan}\}, s_0 = (room_2, 0, 0)$$

- Robot 2: “Repeatedly travel between rooms 2 and 5 and scan those rooms”
  $$\varphi^{curr}_2 = \Box \Diamond (room_2 \land scan) \land \Box \Diamond (room_5 \land scan)$$  \hspace{1cm} (8)
  $$\Lambda_2 = \{\lambda_{Mot}, \lambda_{arm}, \lambda_{camera}\}, s_0 = (room_1, 0, 0)$$

- Robot 3: “Take a picture in room 1 and pick up a box in room 4, in any order”
  $$\varphi^{curr}_3 = \Diamond (room_1 \land use\_camera) \land \Diamond (room_4 \land \text{pickup})$$  \hspace{1cm} (9)
  $$\Lambda_3 = \{\lambda_{Mot}, \lambda_{arm}, \lambda_{camera}\}, s_0 = (room_2, 0, 0)$$

The new task $\Phi^{new}$, provided in Eq. 1 - 3, is introduced while the robots are executing these tasks.

V. Approach

The approach is as follows: each robot determines how much of the current task it has already accomplished. There are two reasons for this: 1) so that the robot does not repeat completed portions of the task (thus reducing cost), and 2) so that if the new task conflicts only with completed portions of the current task, the robot does not deem the new task as impossible to achieve. The robot then synthesizes the corresponding behavior for the new sub-tasks based on its capabilities and the remaining current task. It calculates the cost of performing different feasible combinations of sub-tasks (Sec. V-A). To determine the assignment of tasks that minimizes the overall cost for the robots, we develop a token-based, conflict resolution task allocation algorithm. Robots pass around an assignment token and assign themselves to tasks based on the cost of the corresponding behavior (Sec. V-B).

A. Synthesis of Robot Behavior

Algorithm 1: Synthesize Behavior

```
Input: $A_i, z_i, B^{curr}_i, \varphi^k$
Output: $b^{new}_i, c_i(b^{new}_i)$
1 $B^k := \text{LTL2BUCHI}(\varphi^k)$
2 $B^{curr}_i := \text{FIND_REACHABLE_BUCHI}(z_i, B^{curr}_i)$
3 $B^k_i := \text{CREATE_BUCHI_INTERSECT}(B^k, B^{curr}_i)$
4 $G = A_i \times B^k_i$
   // Let $F^G := \text{the set of accepting states in } G$
   // Let $q_i := \text{initial state of } G$
5 $b^{new}_i, c_i(b^{new}_i) := \text{DIJKSTRA}(G, q_i, F^G)$
6 if $b^{new}_i = \emptyset$ then
7   $c_i(b^{new}_i) := \infty$
```

Given a new task, each robot runs Alg. 1 to automatically synthesize a new behavior. We transform the LTL formula $\varphi^k$ into Büchi automaton $B^k$ using Spot [21] (line 1).

To synthesize a behavior for a robot that would cause it to perform both its current task and $\varphi^k$, we first determine what the robot needs to do to complete its current task. To do so, we calculate the reachable portion of $B^{curr}_i$ from the state the robot is in when the new task is introduced, denoted as $z_i$. To generate the reachable portion of $B^{curr}_i$, the function $\text{FIND_REACHABLE_BUCHI}$ calculates the forward reachable set [23] and removes any non-reachable states.

The function $\text{CREATE_BUCHI_INTERSECT}$ finds the intersection of $B^{curr}_i$, the current task remaining, and $B^k$
representing the new task (line 3). The alphabets \( \Sigma_{\text{curr}} = 2^{A_{\text{curr}}} \), \( \Sigma_k = 2^{A_{pk}} \) of the respective B"uchi automata might not be equivalent, since the task specifications \( \varphi \) may require different capabilities and thus be defined over different \( APs \).

Borrowing from the definition in Sec. 2-B, the B"uchi intersection \( B_i \) has the alphabet \( \Sigma_i = 2^{A_{\text{curr}}(i)} \). Given \( \sigma \in \Sigma_i \), a transition \( (r, q, x), \sigma, (r', q', x') \) \( \in \delta_i \) if and only if \( (r, \sigma \cap A_{\text{curr}}(i), r') \in \delta_{\text{curr}} \) and \( (q, \sigma \cap A_{pk}, q') \in \delta_{pk} \). All other elements in the tuple \( B_i \) remain the same as defined in Sec. 2-B.

In line 5, the robot calculates the minimum cost behavior \( b_{i, new} \) by using Dijkstra’s shortest path algorithm to find the minimum cost path through \( A_i \times B_i \) to an accepting state and an accepting cycle. \( b_{i, new} = \emptyset \) when the robot is unable to perform \( \varphi_i \). This occurs either if the robot does not have the capabilities to satisfy the new task, or if the current remaining task and the new task conflict with each other.

Given \( b_{i, new} \), each robot calculates its satisfiability \( \zeta_i \) and cost \( \Gamma_i \), as outlined in Alg. 2. In lines 2-4, the robot determines if it can perform both its current task and the \( j \)th new sub-task. If it can, we set \( \zeta_i[j] = 1 \) and include the corresponding cost in \( \Gamma_i \).

After calculating \( \zeta_i \), the robot synthesizes the behavior for each combination of sub-tasks it can do (lines 7-11). It does this because the cost for each sub-task is non-additive (as explained in Sec. 3-C). The combinations of sub-tasks are determined based on \( \zeta_i \).

**Algorithm 2: Compute Satisfiability and Cost**

**Input**: \( A_i, z_i, B_{i, curr}, \Phi_{new} \)

**Output**: \( \zeta_i, \Gamma_i \)

1. \( \zeta_i := 0, \Gamma_i := \emptyset \)
   // for individual sub-tasks

2. for \( j \in \{\Phi_{new}\} \)
   3. \( b_{i, new}, c_i(b_{i, new}) := \text{SYNTHESIZE_BEHAVIOR}(A_i, z_i, B_{i, curr}, \Phi_{new[j]}) \)
   4. \( \Gamma_i[j] := c_i(b_{i, new}) \)
   5. if \( b_{i, new} \neq \emptyset \) then
      6. \( \zeta_i[j] := 1 \)

7. // for combinations of sub-tasks

8. for \( k \in 2^{Y_i} \)
   9. if \( |k| > 1 \) then
      10. \( \varphi^k := \bigwedge_{j \in k} \Phi_{new[j]} \)
      11. \( b_{i, new}, c_i(b_{i, new}) := \text{SYNTHESIZE_BEHAVIOR}(A_i, z_i, B_{i, curr}, \varphi^k) \)
      12. \( \Gamma_i[k] := c_i(b_{i, new}) \)

**B. Task Allocation**

We introduce a token-based heuristic algorithm to determine a near-optimal allocation for the new task, as shown in Alg. 3. The token is \( \alpha \), the global assignment vector of length \( m \) where \( \alpha_j \) corresponds to the robot that has been assigned to \( \Phi_{new[j]} \), \( \alpha \) is initialized to be a zero vector.

Each robot \( A_i \) assigns itself to the sub-tasks that it can perform and have not yet been assigned to any other robot (lines 3-5). The algorithm includes a conflict resolution scheme when the sub-tasks that \( A_i \) can perform have already been assigned (lines 6-16). In this case, for each robot \( A_k \) with conflicts, the algorithm looks at the overlap between the tasks already assigned to \( A_k \) and the tasks \( A_i \) can do, which is provided by its satisfiability vector \( \zeta_i \) (lines 10-15). The function \( \text{UPDATE_ASSIGNMENT} \) iterates through every combination of overlapping assignments for the \( A_i \) and \( A_k \), finds the one with the minimum cost, and updates \( \alpha \).

In this algorithm, at each iteration of the conflict resolution, a robot only compares possible conflicting assignments with one other robot. Although we cannot guarantee optimality of the final task allocation assignment, it significantly reduces the computation time. Our algorithm has complexity \( O(2^m n) \), compared to the optimal algorithm, which checks every \( \binom{n}{m} \) combinations and has complexity \( O(n^m) \). In addition, because the token is passed to every robot, we can guarantee that the algorithm will find an assignment for every sub-task if one exists.

**Algorithm 3: Task allocation for \( \Phi_{new} \)**

**Input**: \( \zeta_1, \zeta_2, \ldots, \zeta_n, \Gamma_1, \Gamma_2, \ldots, \Gamma_n, m := |\Phi_{new}| \)

**Output**: \( \alpha \)

1. \( \alpha := 0 \)

2. for \( i \in \{1, \ldots, n\} \) do
   3. for \( j \in \{1, \ldots, m\} \) do
      4. if \( \zeta_i[j] = 1 \) and \( \alpha[j] = 0 \) then
         5. \( \alpha[j] := i \)

6. // Conflict resolution

7. \( \text{satis}_i := \{p \mid \zeta_i[p] = 1\} \)

8. for \( j \in \{1, \ldots, m\} \) do
   9. if \( \zeta_i[j] = 1 \) and \( \alpha[j] \neq i \) then
      10. \( k := \alpha[j] \)
      11. if \( k \notin \text{compared} \) then
          12. \( \text{assigned}_i := \{p \mid \alpha[p] = i\} \)
          13. \( \text{assigned}_k := \{p \mid \alpha[p] = k\} \)
          14. \( \text{conflicts} := \text{satis}_i \cap \text{assigned}_k \)
          15. \( \alpha := \text{UPDATE_ASSIGNMENT}(\text{assign}_i, \text{assign}_k, \text{conflicts}, \Gamma_i, \Gamma_k) \)
      16. \( \text{compared} := \text{compared} \cup \{k\} \)

VI. RESULTS AND EVALUATION

We demonstrate the effectiveness of our synthesis framework by showing the changes to the robot behaviors for the example in Sec. IV-A. Furthermore, we compare the results of our token-based algorithm to the optimal assignment for different team and task sizes.

A. Simulation of Robot Behavior

For the example in IV-A, the final task allocation assignment is \( \alpha = [1, 1, 2] \), meaning that Robot 1 is tasked to complete the first two sub-tasks (Eq. 1, 2), and Robot 2 is tasked to complete the third sub-task (Eq. 3). Fig. 3 shows the updated behavior of Robot 1 after being assigned, mid-execution, the new sub-tasks. For reference, its current task is to scan first in room 4 and then scan in room 1 (Eq. 7).
Observe that the robot interleaves the current and new tasks rather than performing them sequentially - before completing its current task by scanning in room 1, it performs part of the new task by picking up a box in room 2.

Fig. 3: Updated behavior for Robot 1. The robot starts in room 2, and its original trajectory for the current task is drawn in orange. It receives the new task at the star and updates its behavior based on the sub-tasks it assigned itself. The new behavior is shown in blue. The colored circles indicate the action the robot takes.

B. Task Allocation Performance

We compare our token-based allocation scheme with the optimal algorithm, which produces the task assignment with the minimum cost by checking \( \binom{n}{m} \) different assignments. We show the cost of the behavior of the final assignment and the computation time of the two algorithms.

We varied the number of robots from 1 to 20 with 10 fixed new sub-tasks (Fig. 4). Similarly, we varied the number of new sub-tasks from 1 to 8 with 5 robots (Fig. 5). For each of these scenarios, we ran 30 simulations, randomizing the robots’ capabilities and current tasks each time. The simulations ran on a 2.5 GHz quad-core Intel Core i7 CPU.

In both cases, the optimal algorithm’s computation time grows exponentially. The computation time of our task allocation algorithm grows much slower while also maintaining little to no sub-optimality in the final allocation results.

VII. CONCLUSION

We present an approach for robots to automatically distribute a new task while still satisfying their current tasks. Each robot determines if it can satisfy both its current task and the new sub-tasks and resynthesizes its behavior accordingly. We provide a heuristic token-based task distribution algorithm to determine the final task assignment for the new task. The algorithm is scalable and provides a near-optimal assignment that minimizes overall cost.

In future work, we will consider new tasks that are reactive, which will require robots to be able to adapt their behavior at runtime, and a method to model the dynamic and possibly adversarial environment. We also plan to introduce new tasks that require collaboration between robots, which will add complexity to both the synthesis of new behaviors and the allocation of tasks.

Fig. 4: Comparison of overall cost (a) and computation time (b) between our algorithm and the optimal algorithm when varying the number of robots. The error bars represent the min/max values of the simulations.

Fig. 5: Comparison of computation time (a) and cost (b) between our algorithm and the optimal algorithm when varying the number of new tasks. The error bars represent the min/max values of the simulations.
