Quantum teleportation of a spin-mapped Majorana zero mode qubit

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Quantum error correction is widely considered to be an essential ingredient for overcoming decoherence and achieving large-scale quantum computation [1–8]. Topological quantum computation based on anyons is a promising approach to achieve fault-tolerant quantum computing [9–15]. The Majorana zero modes in the Kitaev chain are an example of non-Abelian anyons where braiding operations can be used to perform quantum gates [14, 16–20]. Here we demonstrate in a superconducting quantum processor that the spin-mapped version of the Majorana zero modes can be used to perform quantum teleportation. The teleportation transfers the quantum state encoded on two-qubit Majorana zero mode states between two Kitaev chains, using only braiding operations. The Majorana encoding is a quantum-error-detecting code for phase flip errors, which is used to improve the average fidelity of the teleportation for six distinct states from 70.76 ± 0.35% to 84.60 ± 0.11%, well beyond the classical bound in either case.

Due to the presence of the inevitable interaction with the environment, one of the necessary ingredients of a large-scale quantum computer is a fault-tolerant way of storing and manipulating quantum information [1–8]. In fault-tolerant quantum computing, quantum error correction is employed in such a way such that when scaled up, it is possible to suppress logical errors to an arbitrarily small amount. One of the most attractive ways of performing fault-tolerant quantum computing is topological quantum computing [9–15]. In topological quantum computing, the quantum information is stored in the states of anyons, which have a non-trivial effect on the total state when they are interchanged. For non-Abelian anyons, their braiding can be used to construct elementary quantum gates that can be used for quantum computing. One of the attractive aspects of quantum computing based on anyons is that the resulting quantum gate is only dependent upon the topology of the braiding path. Thus small imperfections in the braiding can be tolerated as long as the operation is topologically equivalent. As with any quantum error correcting code, the logical states of the anyons form a subspace distinguishing the error-free space to those with errors. By energetically separating the states with errors, errors are suppressed via the topological gap.

One example of a non-Abelian anyon is the Majorana zero mode (MZM) [14, 16–20]. MZMs are zero energy excitations that occur typically in low-dimensional topological superconductors. Two physical systems where MZMs have been intensely investigated are fractional quantum Hall systems [21–24] and semiconductor nanowires [25–27]. A complementary approach to realizing MZMs in physical systems is to implement lattice models where they exist by construction. An elementary model that possesses MZMs is the Kitaev chain consisting of $N$ fermions with Hamiltonian $[16]$

$$\begin{align*}
H &= t \sum_{n=1}^{N-1} \left( -c_{n+1}^{\dagger} c_n - c_n^{\dagger} c_{n+1} + c_n c_{n+1}^{\dagger} + c_{n+1} c_n^{\dagger} \right),
\end{align*}$$

where $c_n$ is a fermionic annihilation operator on site $n$, and $t$ sets the scale of the gap energy. This model has a degenerate ground state, corresponding to the presence or absence of a pair of MZMs, and can be used to encode the state of a qubit. By braiding one of the MZMs with...
another, quantum gates on the encoded quantum information may be performed, thereby forming the basis for topological quantum computing.

When mapped to a spin model, the Hamiltonian \( \hat{H} \) takes the form of a one-dimensional Ising model, which has made it attractive to numerous proposals for simulating its equivalent dynamics. The spin-mapped model does not have the same topological properties as the original fermion Hamiltonian, but it does nevertheless have exactly the same energy structure. This means that it retains the same gap protection between the logical states and the error states, and an ability to detect particular types of errors. Xu, Pachos, and Guo implemented the spin version of the MZMs states in a Kitaev chain, and braiding of anyons was demonstrated to realize one qubit gates with imaginary time evolution [28, 29]. Several works have also demonstrated the path-independent nature of braiding anyons in the toric code [30, 31], another model possessing anyons. The protection of quantum information based on various quantum error correcting codes have been demonstrated in many past works [32–34]. In particular, Nigg, Blatt, and co-workers encoded one qubit using a two dimensional topological color code with trapped ions and performed logical gates and error syndrome detection [32]. Recently, Andersen, Wallraff and co-workers implemented a 7-qubit minimal instance of the surface code to perform an arbitrary single qubit error detection on one logical qubit [35]. However, to date we are not aware of any demonstration of a quantum algorithm involving more than one qubit encoded using any type of topological code.

In this paper, we investigate the feasibility of quantum computing with MZMs by performing a quantum teleportation [46] of a qubit encoded in the MZM states of the Kitaev chain on superconducting qubits. Figure 1 shows our experimental configuration and the relationship between the spin, fermion, and Majorana encodings. We realize four spin-mapped Kitaev chains using eight qubits of a superconducting qubit quantum processor. Each chain, consisting of two physical qubits, encodes a single logical qubit, corresponding to the MZM states of the Kitaev chain. In the teleportation, Alice is in possession of two of the Kitaev chains, and Bob holds the two other chains. The teleportation then transfers a single logical qubit, encoded as the MZM states, from Alice to Bob. One of the well-known issues of quantum computing based on MZMs of the Kitaev chain is that the braiding operations only allow for a discrete number of Clifford gates, which is insufficient for universal quantum computation [11, 12]. However, in the teleportation protocol, only Clifford gates are required, such that it can be completed entirely with braiding operations. In addition to demonstrating the feasibility of quantum computing with MZMs, we also show the error detecting capability of the MZMs. The redundant encoding of the qubits as MZMs allows us to detect when decoherence has removed the states from the logical MZM subspace. By using this as an error syndrome and postselecting on the error-free results, we are able to improve the fidelity of the teleportation significantly.

We first give a brief review of anyonic quantum computing with MZMs in the context of our experiment (see Supplementary and Refs. [11–13, 15, 18] for further details). Each fermion is written in terms of two Majoranas according to the definition

\[
\gamma_n,\ell = c_n + \frac{1}{2}\ell
\]

\[
\gamma_n,r = -ic_n + \frac{1}{2}r
\]

where \( n \) is an integer labeling the fermions, and the \( \ell, r \) label the two types of Majoranas, which correspond to

![Figure 1: Experimental configuration and encoding of quantum states in a Kitaev chain. (a) The superconducting quantum processor used in this study. There are 12 qubits in total in our superconducting quantum processor, from which we choose eight adjacent qubits labelled with \( Q_1 \) to \( Q_8 \) to perform the experiment. Qubits \( Q_1 \) to \( Q_4 \) are held by Alice. Qubits \( Q_5 \) to \( Q_8 \) are held by Bob. Pairs of qubits form a Kitaev chain (\( KC \)), each of which encode a single logical qubit. Each qubit couples to a resonator for state readout, marked by \( R_1 \) to \( R_8 \). After decoding, the resonators marked by “syn” are syndrome measurements to detect phase flip errors in the qubits. For each qubit, individual capacitively-coupled microwave control lines (XY) and inductively-coupled bias lines (Z) enable full control of qubit operations. An encoded qubit is teleported from \( KC_1 \) to \( KC_3 \). (b) Mapping between spin, fermions, and Majorana modes. The pairing of Majorana modes in the topologically non-trivial regime are indicated by the dotted ovals. In the topologically non-trivial phase, the Majorana Zero Modes (MZMs) are present at the ends of the chain. Logical qubit states \( |0\rangle_L, |1\rangle_L \) are formed by occupation or vacancy of the MZMs.](image)
FIG. 2: Majorana modes and their braiding operations. The six possible braiding operations for two Kitaev chains (KC), and the effect in terms of the logical states. The left- and right-most Majorana Zero Mode (MZM) on chains 1 and 2 are labeled by $\gamma^{(1,2)}_{l,r}$ respectively. We denote the Pauli operators for the underlying physical qubits by $\sigma^x, \sigma^y, \sigma^z$ and the higher level logical operators by $X, Y, Z$.

the real and imaginary part of the fermion operator, denoted by the left and right boxes in Fig. 1(b) respectively. Let us denote $|0_L\rangle$ a ground state of the Hamiltonian (1), taken as the state with no Majorana modes throughout the chain. The nature of the Kitaev Hamiltonian is such that applying the fermion creation operator

$$f^\dagger = \frac{1}{2}(\gamma_1, -i\gamma_N,)$$

consisting of the two Majorana edge modes at the ends of the lattice, produces another orthogonal degenerate state. These two states $|0_L\rangle$ and $|1_L\rangle \equiv f^\dagger |0\rangle$ are the MZM states and are used as the logical states for quantum computing. For example, a minimal implementation of the Kitaev chain Hamiltonian (1) consists of two fermions $N = 2$. Under a Jordan-Wigner mapping of the operators, the Hamiltonian takes a form $H = -t\sigma_1^x\sigma_2^z$ and the two MZM states are

$$|0_L\rangle = \frac{1}{\sqrt{2}}(|++\rangle + |--\rangle)$$

$$|1_L\rangle = \frac{1}{\sqrt{2}}(|++\rangle - |--\rangle).$$

To encode $M$ logical qubits, one then prepares $M$ Kitaev chains, each with the Hamiltonian (1). Let us label the MZMs from the $m$th chain as

$$\gamma^{(m)}_l \equiv \gamma_1^{(m)}$$

$$\gamma^{(m)}_r \equiv \gamma_N^{(m)},$$

such that we only label the left-most and right-most Majorana mode in the chain, which are the MZMs. An MZM, on the $m$th chain that is in the left- or right-most position $\sigma \in \{l, r\}$, can be braided with another labeled by $(m', \sigma')$ (Fig. 2). The effect of this is to apply the unitary braid operator $[49, 50]$, defined as

$$B_{(m,\sigma)(m',\sigma')} = e^{i\pi\gamma^{(m)}_\sigma\gamma^{(m')}_{\sigma'}}/4 = \frac{1}{\sqrt{2}}(1 + \gamma^{(m)}_\sigma\gamma^{(m')}_{\sigma'}).$$

For two logical qubits, there are four MZMs, and therefore there are $C_4^2 = 6$ possible braiding operations, including braids on the same qubit. Due to the non-Abelian nature of the MZMs, these produce gate operations on the MZM states. The possible gate operations on the MZM states by braiding are summarized in Fig. 2 (see Supplementary for details).

The standard quantum teleportation circuit usually consists of a sequence of Hadamard and CNOT gates [51], which are not directly available by braiding operations. To match the gates that are available with braiding of MZMs as closely as possible, we use the modified teleportation scheme as shown in Fig. 3 (a) (see Methods and Supplementary for details).
FIG. 3: Quantum circuits for teleporting a Majorana Zero Mode (MZM) encoded qubit. (a) The logical quantum circuit which performs a modified quantum teleportation. (b) The braiding sequence for the MZMs that performs the quantum circuit in (a). (c) The corresponding qubit circuit of the MZM braiding sequence shown in (b). All measurements are performed in the $|0\rangle, |1\rangle$, which the exception of the measurement on qubit 6, where tomography (“tomo”) is performed. The measurements marked with “syn” are syndrome measurements, where single qubit phase errors are detected for a measurement outcome of $|1\rangle$. (d) The gate decompositions for the braiding, encoding, and decoding gates in (c). The sequence of qubit operations performed experimentally in this work corresponds to the circuit (c) with the decompositions (d). In all figures, $|\psi\rangle$ is the state to be teleported. Black lines connecting the quantum gates denote qubits, dark blue lines denote MZMs, and orange lines denote classical information transfer.

in a similar way to the standard teleportation circuit, except that the classically transmitted quantum correction (“classical correction” for short) is done according to the modified rules also shown in the classical circuit of Fig. 3(a). Using this modified teleportation circuit, the equivalent version with MZMs can be constructed entirely using the available braiding gates in Fig. 2. The one gate that is present in the circuit of Fig. 3(a) that is not present in Fig. 2 is the $X$-gate for the classical correction. No combination of the six braiding operations in Fig. 2 can produce a single qubit $X$-gate. However, by adding an extra ancilla MZM qubit ($m = 4$) prepared in the eigenstate with $X_4 = +1$, and applying the braiding operation for the logical $\sqrt{X_3}X_4$ twice, we can perform an $X_3$ gate. In this way all gates appearing in the teleportation circuit can be performed natively using only braiding operations (Fig. 3(b)).

Using a minimal implementation of the Kitaev chain with $N = 2$ fermions, and performing a Jordan-Wigner transformation, we convert the MZM teleportation circuit (Fig. 3(b)) into the equivalent 8 qubit version as shown in Fig. 3(c). In addition to the braiding operations that are required for the teleportation circuit, we require encoding and decoding operations to prepare the logical MZM qubit states of (4). The encoder takes an arbitrary qubit state and an auxiliary qubit in the state $|0\rangle$ and produces its associated logical MZM qubit state

$$U_{\text{enc}}|0\rangle(\alpha|0\rangle + \beta|1\rangle) = \alpha|0_L\rangle + \beta|1_L\rangle,$$

which can be performed using elementary gates and the definitions (4). Here $\alpha, \beta$ are arbitrary complex coefficients such that $|\alpha|^2 + |\beta|^2 = 1$. The gate decompositions for the braiding gates, encoder, and decoder are shown in Fig. 3(d).

With the auxiliary qubit in the state $|1\rangle$, the encoder produces the state

$$U_{\text{enc}}|1\rangle(\alpha|0\rangle + \beta|1\rangle) = \alpha|0_L\rangle + \beta|1_L\rangle,$$
where
\[
\begin{align*}
\hat{0}_L &= \sigma_1^L \hat{0}_L = \sigma_2^L \hat{0}_L = \frac{1}{\sqrt{2}} (|++\rangle + |--\rangle) \\
\hat{1}_L &= \sigma_1^L \hat{1}_L = -\sigma_2^L \hat{1}_L = \frac{1}{\sqrt{2}} (|--\rangle - |+-\rangle).
\end{align*}
\]

The states \(\hat{0}_L\), \(\hat{1}_L\) span an orthogonal subspace to that spanned by the logical MZM states, and are produced when any single qubit phase error \(\sigma_1\), \(\sigma_2\) occurs. Thus using the decoding circuit \(U_{dec} = U^{dec}_{iso}\) and examining the auxiliary qubit, one can detect whether a phase flip error has occurred on any of the qubits. This constitutes an error detecting code \([5, 52, 54]\), which can be used to passively improve the fidelity of the circuit by discarding any results where errors have occurred. Variations of such error detecting codes have been used to demonstrate protection of quantum information \([38, 39, 41–44]\).

A superconducting quantum processor \([55]\) is used to implement the quantum circuit of Fig. 3(c). The processor has 12 transmon qubits \([50]\) of the Xmon variety \([57]\), and 8 qubits among them are chosen in our experiment (see Methods for details). The average gate fidelities of single-qubit gates and the controlled-Z (CZ) gate are about 0.9994 and 0.985, respectively. The six input states of \(|0\rangle, |1\rangle, |+\rangle, |-\rangle, |+ i\rangle, |− i\rangle\), corresponding to pairs of eigenstates of the Pauli \(\sigma^x, \sigma^y, \sigma^y\) operators are prepared on qubit 2 as the input state for the teleportation. To perform the classical correction steps, we run four versions of the circuit with and without each of the X and Z classical correction gates. Then given a particular measurement outcome on qubits 2 and 4, the correct circuit for that outcome is selected. To perform the tomography measurement of the teleported state on qubit 6, we repeat the circuit by making measurements in the X, Y, Z basis such that the state can be tomographically reconstructed. Each of the circuit variants were run a total of 40000 times for statistics.

Figure 3 shows the fidelities of the teleportation for each of the six input states (blue bars). First we average over all measurement outcomes on qubits 1, 3, 5, 7, 8, which corresponds to ignoring all error syndrome measurements and any changes in the ancilla MZM qubit. We find the average fidelity of the six states is 70.76 ± 0.35%, which is above the 2/3 classical optimal state estimation bound for a qubit \([58]\) by 11 standard deviations. We have performed an explicit simulation of the circuit shown in Fig. 3(c) including dephasing and gate errors, and obtain good agreement between the experimentally obtained errors (see Supplementary for details). We note that the experiment further suffers from measurement readout errors, which are expected to further degrade the theoretical fidelities. From the operations on qubit 7 and 8 it is apparent that the final state should be in the state \(|00\rangle\), which is consistent with the fact that the role of these qubits are only to be in the \(X = 1\) eigenstate. We experimentally obtain the probability of getting the \(|00\rangle\) state is 97.98%, consistent with this expectation.

![Fidelities with and without error detection](image)

**FIG. 4: Fidelities of the teleportation with and without error syndrome detection.** The fidelity is calculated according to \(F = \langle \psi | \rho | \psi \rangle\), where \(|\psi\rangle = \{|0\rangle, |1\rangle, |+\rangle, |-\rangle, |+ i\rangle, |− i\rangle\\rangle\) are the ideal states to be teleported. The \(F = 2/3\) classical bound is shown as the dashed line. The error bars denote one standard deviation, deduced from propagated Poissonian counting statistics of the raw detection events.

In summary, we have performed a teleportation of a qubit encoded as the MZM states of the Kitaev chain. The teleportation circuit is performed entirely using braiding operations of the MZMs, including the quantum gates for classical correction. In our teleportation circuit we were careful to be faithful to the braiding process of the MZMs in the sense that no gate simplifications were performed in the circuit Fig. 3(c). This constitutes a demonstration of a non-trivial quantum circuit involving more than one topologically encoded qubit, in its spin-mapped counterpart. Numerous demonstrations of teleportation have been performed to date in qubit \([59, 60]\) and higher dimensional systems \([67, 71]\). In our experiment we encode the state of one qubit using two qubits (1 & 2) and teleport the encoded state to another pair of...
FIG. 5: Tomography of the final teleported state after using the error syndrome measurements. The initial state prepared on qubit 2 is (a) $|0\rangle$, (b) $|1\rangle$, (c) $|+\rangle$, (d) $|−\rangle$, (e) $|+i\rangle$, (f) $|−i\rangle$. Frames show ideal teleportation states, colored bars shows the experimentally determined state.

qubits (5 & 6). Teleportation of a topologically encoded qubit has not been performed before, to the best of our knowledge. Fidelities exceeding the classical bound were obtained, demonstrating that a non-trivial quantum information transfer was being achieved.

The MZM encoding allows for an error detection capability of phase flip errors, such that states marked with an error can be discarded. This naturally results in larger error bars due to less statistics from postselection, however it makes a significant difference in terms of the performance, violating the classical bound by a larger number of standard deviations. We note that bit flip errors cannot be guarded against using this type of encoding, since a single application of a $\sigma^z$ causes a logical error as evident from (4). Nevertheless, our experiment is a proof-of-principle demonstration that a non-trivial quantum computation with anyons should be feasible, with protection against single qubit phase errors. Phase error is one of the dominant sources of error in superconducting quantum circuits, which is the origin of the large improvement in fidelity with error detection. In addition to the passive error detection performed here, with the addition of a topological gap to energetically separate the logical space from the error space, errors could be actively suppressed, further improving the error protection.

**Methods**

**Modified teleportation circuit.** Our version of the teleportation circuit uses slightly different gates to the standard version of teleportation, such as that given in Ref. [51], where Hadamard and CNOT gates are used. Instead of these gates, we based our teleportation on the $\sqrt{X_1X_2}$ gate, which is implemented by braiding the right-
most MZM from the first chain with the left-most MZM of the second chain. This is most convenient type of gate because for the spin-mapped representation, this does not involve high order spin operations to be performed. For example, Eq. (S82) in the Supplementary is a second order operation, while Eq. (S60) in the Supplementary involves a product of many spin operators.

As with the standard teleportation circuit, there are primarily three steps: (i) preparation of an entangled qubit between Alice and Bob; (ii) measurement of Alice’s qubits in the Bell basis; (iii) classical correction at Bob, conditioned on Alice’s measurement outcome. In our circuit, the entanglement in (i) is prepared using a logical $\sqrt{X_1X_2}$ gate. The Bell measurement (ii) is performed by combining an entangling operation $\sqrt{X_2X_3}$ with a measurement in the local basis. Finally, as explained in the main text, the classical correction is performed by applying two $\sqrt{Z_3}$ gates to perform a $Z_3$ correction, two $\sqrt{X_3X_4}$ gates with an ancilla qubit set to $X_4 = 1$ to perform the $X_4$ correction.

Working in the logical space, the entanglement preparation step produces the state

$$|E\rangle_{23} = \sqrt{X_2X_3}|00_L\rangle_{23} = \frac{1}{\sqrt{2}}(|00_L\rangle_{23} + i|11_L\rangle_{23}).$$

Logical qubit 1 is meanwhile prepared in the state

$$|\psi\rangle_1 = \alpha|0_L\rangle_1 + \beta|1_L\rangle_1.$$ (11)

Now applying the $\sqrt{X_1X_2}$ gate we have

$$\sqrt{X_1X_2}|\psi\rangle_1|E\rangle_{23}$$

$$= \frac{1}{2} (|00_L\rangle (\alpha|01_L\rangle - \beta|11_L\rangle) + i|01_L\rangle (\beta|0L\rangle + \alpha|1L\rangle)$$

$$-|00_L\rangle (\beta|0L\rangle - \alpha|1L\rangle) + i|01_L\rangle (\alpha|0L\rangle + \beta|1L\rangle)).$$ (13)

A measurement in the logical basis on the first two qubits collapses the state to four outcomes, and leaves logical qubit 3 in one of four possible states, as can be seen from (13). These can be corrected to the original state by applying the operations as summarized in Table M1.

**Experimental set-up.** In all experiments, the eight qubits (Fig. 1(a)) are chosen from a 12-qubit superconducting quantum processor. The processor has qubits lying on a 1D chain, and the qubits are capacitively coupled to their nearest neighbors. Each qubit has a microwave drive line (XY), a fast flux-bias line (Z) and a readout resonator. The single-qubit rotation gates are implemented by driving the XY control lines, and the CZ gate is implemented by driving the Z line using the “fast adiabatic” method (see the Supplementary Material for further experimental details).

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**Competing Interests**

The authors declare no competing interests.

**Author Contributions**

X. Z., T.B. and J.-W. P. conceived the research. H.-L. H., Y. Z., M.G., and X. Z. designed and performed the experiment. M.N., J.P.D. and T.B. conceived the theoretical scheme. H.-L. H. and Y. Z. analyzed the results. M.N. performed numerical simulations of the circuit. J.P.D. had great thoughts. H. D. and H. R. prepared the sample. Y. W. developed the programming platform for measurements. F.L., J.L., Y.X., and C.-Z.P. developed room-temperature electronics equipment. All authors contributed to discussions of the results and the development of the manuscript. X. Z. and J.-W. P. supervised the whole project.
TABLE M1: Classical correction required for the teleportation protocol.
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Supplementary Information for “Quantum teleportation of a spin-mapped Majorana zero mode qubit”

I. MAJORANA MODES IN THE KITAEV CHAIN

In this section we provide a brief review of Majorana modes in the Kitaev chain. We refer the reader to several excellent reviews for further details [5] [12] [13] [15] [18].

A. Definition of Majorana fermions

Consider a set of $N$ fermions, which can be described by standard fermionic anticommutation relations

\begin{equation}
\{c_{n}, c_{n'}^{\dagger}\} = 0
\end{equation}

\begin{equation}
\{c_{n}, c_{n'}^{\dagger}\} = \delta_{nn'},
\end{equation}

where $\delta_{nn'}$ is the Kronecker delta. We may rewrite the operators for creating and annihilating a fermion on site $n$ in terms of two Majorana fermions in following way

\begin{equation}
c_{n} = \frac{1}{2}(\gamma_{n,\ell} + i\gamma_{n,r})
\end{equation}

\begin{equation}
c_{n}^{\dagger} = \frac{1}{2}(\gamma_{n,\ell} - i\gamma_{n,r}).
\end{equation}

These equations can be solved for $\gamma_{\ell}$ and $\gamma_{r}$ resulting with definitions of Majorana fermions in terms of a single fermion

\begin{equation}
\gamma_{n,\ell} = c_{n} + c_{n}^{\dagger}
\end{equation}

\begin{equation}
\gamma_{n,r} = -ic_{n} + ic_{n}^{\dagger},
\end{equation}

According to the definition, Majorana fermions are purely real

\begin{equation}
\gamma_{n,\sigma} = \gamma_{n,\sigma}^{\dagger},
\end{equation}

where $\sigma \in \{\ell, r\}$. They share similarities with standard fermions with regard to their anti-commutation property

\begin{equation}
\{\gamma_{n,\sigma}, \gamma_{n',\sigma'}\} = 2\delta_{nn'}\delta_{\sigma\sigma'}.
\end{equation}

However, unlike standard fermions which obey the Pauli exclusion principle $c_{n}^{2} = (c_{n}^{\dagger})^{2} = 0$, Majorana fermions are their own anti-particle and we have

\begin{equation}
\gamma_{n,\sigma}^{2} = 1.
\end{equation}

B. Delocalized fermions

Under this formalism, it appears the concept of Majorana fermions is just an algebraic manipulation. The interesting aspect of utilizing the Majorana operators arises when we construct other types of fermions that are not necessarily the physical fermions $c_{n}$. Following the form of the fermion operators shown in (S3), new delocalized fermions can be defined using any pair of Majorana modes

\begin{equation}
f_{p} = \frac{1}{2}(\gamma_{n,\sigma} + i\gamma_{m,\nu})
\end{equation}

\begin{equation}
f_{p}^{\dagger} = \frac{1}{2}(\gamma_{n,\sigma} - i\gamma_{m,\nu}),
\end{equation}

where $\sigma, \nu \in \{\ell, r\}$. Here

\begin{equation}
p \rightarrow (n, \sigma, m, \nu)
\end{equation}

is a pairing label between two Majorana modes labeled by $(n, \sigma)$ and $(m, \nu)$. The fermion operator $f_{p}$ is constructed from two Majorana modes, which are potentially at different physical sites $n \neq m$, hence we call this a delocalized fermion. A particular pair $p$ always involves two different Majorana modes, such that $(n, \sigma) \neq (m, \nu)$, meaning that a pair with both $n = m$ and $\sigma = \nu$ is not allowed. It is possible however to have a pairing such that $n = m$ but $\sigma \neq \nu$, or $n \neq m$ but $\sigma = \nu$. The former is exactly the case of physical fermions as shown in (S3).

Given a set of $N$ fermions, and hence $2N$ Majorana modes, let us fix a particular pairing configuration labeled by (S11). Various examples of Majorana pairings are shown in Fig. [S1]. When establishing a pairing configuration, Majorana modes are never used twice, such that for different pairs $p \neq p'$, the underlying Majoranas are all different. Under these conditions, the anticommutation relations of the delocalized fermions (S10) can be evaluated as

\begin{equation}
\{f_{p}, f_{p'}^{\dagger}\} = \frac{1}{2}(\delta_{nn'}\delta_{\sigma\sigma'} + i\delta_{nm'}\delta_{\sigma\nu'} + i\delta_{n'm}\delta_{\sigma'\nu} - \delta_{nm}\delta_{\nu\nu'})
\end{equation}

\begin{equation}
= 0
\end{equation}

where the pairing label $p' \rightarrow (n', \sigma', m', \nu')$. The Kronecker delta functions simplify in (S12) because if $p = p'$ it implies that

\begin{equation}
(n, \sigma) = (n', \sigma') \neq (m, \nu) = (m', \nu'),
\end{equation}

but if $p \neq p'$ then it implies that

\begin{equation}
(n, \sigma) \neq (n', \sigma') \neq (m, \nu) \neq (m', \nu').
\end{equation}

Similarly we can evaluate

\begin{equation}
\{f_{p}, f_{p'}^{\dagger}\} = \frac{1}{2}(\delta_{nn'}\delta_{\sigma\sigma'} - i\delta_{nm'}\delta_{\sigma\nu'} + i\delta_{n'm}\delta_{\sigma'\nu} + \delta_{nm}\delta_{\nu\nu'})
\end{equation}

\begin{equation}
= \delta_{pp'}
\end{equation}

which shows that the delocalized fermions are fermion operators as claimed.

C. Majorana pairing Hamiltonian

To enforce a particular pairing configuration of Majorana modes, we must energetically stabilize the fermions
FIG. S1: Various Majorana mode pairing configurations. (a) Random, (b) topologically trivial, and (c) Kitaev chain pairings are shown.

FIG. S2: Energy spectrum of Majorana pairing Hamiltonian for \( N = 2 \) fermions. (a) Topologically trivial Hamiltonian (S17) as shown in Fig. S1(b). (b) Kitaev Hamiltonian (S24) as shown in Fig. S1(c). The Majorana mode labels are suppressed, and the occupancy of the Majorana modes are denoted by the red bars.

that are defined by (S10). For example, the Hamiltonian to enforce the regular fermion pairing (Fig. S1(b)) is given by

\[
H = t \sum_{n=1}^{N} c_n^\dagger c_n \tag{S16}
\]

where \( t \) is some energy constant. The eigenstates of this Hamiltonian are given by

\[
|j_1, \ldots, j_N\rangle = \prod_{n=1}^{N} (c_n^\dagger)^{j_n} |0\rangle \tag{S18}
\]

where \( j_n \in \{0,1\} \) labels the occupancy of the nth fermion. The energies of these states are

\[
E = t \sum_{n=1}^{N} j_n. \tag{S19}
\]

This can be rewritten in the Majorana language, where a fermion occupancy of the nth site means that the underlying Majorana modes are both occupied. Fig. S2(a) shows the spectrum for the example of \( N = 2 \).

Similarly, for the delocalized fermions we can define a Majorana pairing Hamiltonian according to

\[
H = t \sum_{p=1}^{N} f_p^\dagger f_p, \tag{S20}
\]

where \( p \) runs over all \( N \) Majorana pairs, for example that defined in Fig. S1(a). The eigenstates are again defined by the occupancy of the new fermions

\[
|j_1, \ldots, j_N\rangle = \prod_{p=1}^{N} (f_p^\dagger)^{j_p} |0\rangle, \tag{S21}
\]

where \( |0\rangle \) is the ground state of (S20), and \( j_p \in \{0,1\} \) labels the occupancy of the \( p \)th pair. The energy spectrum is again given by

\[
E = t \sum_{p=1}^{N} j_p. \tag{S22}
\]

The Kitaev chain \[9\] is a particular example for the pairing configuration given in Fig. S1(c). In this case, the fermions are defined as

\[
f_n = \frac{1}{2} (\gamma_{n,r} + i\gamma_{n+1,l}), \tag{S23}
\]

where \( n \in [1, N-1] \) for this operator. Here a Majorana mode in the right box on site \( n \) is paired with another in the left box of site \( n + 1 \). The Kitaev chain Hamiltonian is then

\[
H = t \sum_{n=1}^{N} f_n^\dagger f_n
\]

\[
= t \frac{1}{2} \sum_{n=1}^{N-1} (1 + i\gamma_{n,r}\gamma_{n+1,l}). \tag{S24}
\]

Importantly, this Hamiltonian does not involve the Majorana pairing of the delocalized fermion corresponding to

\[
f_N = \frac{1}{2} (\gamma_{1,l} + i\gamma_{N,r}). \tag{S25}
\]

This means that this fermion costs zero energy to excite, and makes every state in the spectrum of (S24) doubly
For simplicity we denote in the main text the Majorana Hamiltonian. The Majorana modes $\gamma_n$ is the annihilation operator for the edge states of the Kitaev chain as $b_n^\dagger$. If we label the Majorana mode labelled by $(n, \sigma)$ on the $n$th site of the $m$th Kitaev chain as $c_n^{(m)}$, the Hamiltonian for the multiple chain is

$$H = \frac{t}{2} \sum_{n=1}^{N-1} (1 + i\gamma_n^{(m)} \gamma_{n+1}^{(m)})$$

$$= \frac{t}{2} \sum_{n=1}^{N-1} (1 - c_n^{(m)} c_n^{(m)\dagger}) - c_n^{(m)} c_{n+1}^{(m)\dagger} + c_n^{(m)\dagger} c_{n+1}^{(m)}$$

which up to a constant energy offset is the Hamiltonian from the main text. Here we denote the fermion annihilation operator on the $n$th site of the $m$th Kitaev chain as $c_n^{(m)}$. The MZMs on the $m$th Kitaev chain then occur on the first and last Majorana sites and we define the operator

$$f^{(m)} = \frac{1}{2} (\gamma_1^{(m)} + i\gamma_{N,r}^{(m)})$$

which destroys a MZM on the $m$th chain. Here henceforth use the notation

$$\gamma_\ell^{(m)} \equiv \gamma_{1,\ell}^{(m)}$$

$$\gamma_r^{(m)} \equiv \gamma_{N,r}^{(m)}.$$ 

The full set of $2^M$ logical states are built up by applying the creation operator $f^{(m)\dagger}$ on the ground state $|0\rangle$ contain zero Majorana modes.

D. Majorana zero modes

The doubly degenerate ground states of the Kitaev Hamiltonian $H_{\text{Kitaev}}$ have zero energy and form a pair of orthogonal states. Let us as usual take the ground state with the absence of any fermions $|0\rangle$ by $|0\rangle$. Then fermion operator $b_n^\dagger$ transforms this ground state into its degenerate pair, where the Majoranas on the end of the chain are occupied $f_n^\dagger |0\rangle$. These two states are used as the logical qubit states of the quantum computation, where

$$|0_L\rangle \equiv |0\rangle$$

$$|1_L\rangle \equiv f_n^\dagger |0\rangle.$$ 

For simplicity we denote in the main text $f = f_N$, which is the annihilation operator for the edge states of the Kitaev Hamiltonian. The Majorana modes $\gamma_1, \ell$ and $\gamma_{N,r}$ have zero energy and hence are called Majorana zero modes (MZMs). One Kitaev chain therefore encodes one logical qubit’s worth of information. In order to have multiple logical qubits, then multiple Kitaev chains are required. Labelling the Majorana mode labelled by $(n, \sigma)$ on the $n$th chain as $\gamma_n^{(m)}$, the Hamiltonian for the multiple chain case then reads

$$H = \frac{t}{2} \sum_{n=1}^{N-1} (1 + i\gamma_n^{(m)} \gamma_{n+1}^{(m)})$$

$$= \frac{t}{2} \sum_{n=1}^{N-1} (1 - c_n^{(m)} c_n^{(m)\dagger}) - c_n^{(m)} c_{n+1}^{(m)\dagger} + c_n^{(m)\dagger} c_{n+1}^{(m)},$$

which up to a constant energy offset is the Hamiltonian from the main text. Here we denote the fermion annihilation operator on the $n$th site of the $m$th Kitaev chain as $c_n^{(m)}$. The MZMs on the $m$th Kitaev chain then occur on the first and last Majorana sites and we define the operator

$$f^{(m)} = \frac{1}{2} (\gamma_1^{(m)} + i\gamma_{N,r}^{(m)})$$

which destroys a MZM on the $m$th chain. Here henceforth use the notation

$$\gamma_\ell^{(m)} \equiv \gamma_{1,\ell}^{(m)}$$

$$\gamma_r^{(m)} \equiv \gamma_{N,r}^{(m)}.$$ 

The full set of $2^M$ logical states are built up by applying the creation operator $f^{(m)\dagger}$ on the ground state $|0\rangle$ contain zero Majorana modes.

II. BRAIDING MAJORANA ZERO MODES

The MZMs are an example of non-Abelian anyons because their interchange causes a non-trivial effect on the ground state manifold. In this section we derive the effect of braiding of the Majorana zero modes on two Kitaev chains. It is sufficient to consider two Kitaev chains because we will consider the braiding of two MZMs to be the elementary process. The two MZMs can originate from the same Kitaev chain, or one MZM each from two Kitaev chains. This gives a total of 6 possible braidings of two MZMs, since each chain has two MZMs.

A. Braiding operator

Braiding two zero modes $\gamma_{n,\sigma}$ and $\gamma_{m,\nu}$ in a clockwise direction can be achieved by applying the operator

$$B_{(n,\sigma)(m,\nu)} = e^{\pi i \gamma_n^{(m)} \gamma_{m,\nu}} = \frac{1}{\sqrt{2}} (1 + \gamma_{n,\sigma} \gamma_{m,\nu}).$$ 

It is apparent that this performs a braiding operation via the transformation

$$B_{(n,\sigma)(m,\nu)} \gamma_{n,\sigma} B_{(n,\sigma)(m,\nu)}^\dagger = -\gamma_{m,\nu}$$

$$B_{(n,\sigma)(m,\nu)} \gamma_{m,\nu} B_{(n,\sigma)(m,\nu)}^\dagger = \gamma_{n,\sigma}.$$ 

When applied on the ground state manifold of the Kitaev chains, the braiding operators realize unitary operations on the MZM states. Consider for the purposes of this section that there are $M = 2$ Kitaev chains, such that the logical states are

$$|00_L\rangle \equiv |0\rangle$$

$$|10_L\rangle \equiv f^{(1)\dagger} |0\rangle$$

$$|01_L\rangle \equiv f^{(2)\dagger} |0\rangle$$

$$|11_L\rangle \equiv f^{(1)\dagger} f^{(2)\dagger} |0\rangle.$$ 

for simplicity. 


where \(|0\rangle\) is again the state with zero Majorana modes everywhere. The purpose of the following section will be to derive the effect of various braiding operators on the logical space of states \[S34\].

### B. Spin representation

For each braiding operator acting on logical space, an equivalent spin operator can be derived acting on corresponding physical space. This is done using Jordan-Wigner transformation to transform the Majorana variables to spin variables. We consider a layout of spins as shown in Fig. \[S3\]. The \(M\) Kitaev chains each with \(N\) fermions are arranged in a larger chains, in ascending order. We label the spin operators from 1 to \(NM\), the total number of fermions and spins in the mapping. In this case, the MZM can be transformed to spin variables according to

\[
\gamma^{(p)}_\ell = \left( \prod_{i=1}^{pN-N} \sigma^z_i \right) \sigma^x_{pN-N+1} \\
\gamma^{(p)}_r = \left( \prod_{i=1}^{pN-1} \sigma^z_i \right) \sigma^y_{pN},
\]

where \(p\) is the chain index. In the calculations below, we only consider two chains, and hence it is convenient to explicitly write the spin mapped MZM operators

\[
\gamma^{(1)}_\ell = \sigma^x_1 \\
\gamma^{(1)}_r = \left( \prod_{k=1}^{N-1} \sigma^y_k \right) \sigma^y_N = \sigma^z_1 \ldots \sigma^z_{N-1} \sigma^y_N \\
\gamma^{(2)}_\ell = \left( \prod_{k=1}^{N} \sigma^z_k \right) \sigma^x_{N+1} = \sigma^z_1 \ldots \sigma^z_N \sigma^x_{N+1} \\
\gamma^{(2)}_r = \left( \prod_{k=1}^{2N-1} \sigma^x_k \right) \sigma^y_{2N} = \sigma^z_1 \ldots \sigma^z_{2N-1} \sigma^y_{2N}
\]

(S35)

The logical states on which Jordan-Wigner transformed operators act are defined explicitly in a following way

\[
|0_L\rangle = \frac{1}{\sqrt{2}} \left(|+\ldots+\rangle + |\ldots-\rangle\right) \\
|1_L\rangle = \frac{1}{\sqrt{2}} \left(|+\ldots+\rangle - |\ldots-\rangle\right)
\]

(S37)

(S38)

C. Derivation of the six braiding gates in Fig. 2c

1. \(\gamma^{(1)}_\ell \rightleftharpoons \gamma^{(1)}_r\) braid: \(\sqrt{Z_1}\) gate

We can express this braiding operation in terms of spin operators by applying Jordan-Wigner transforma-

\[
B_{(1,\ell),(1,r)} = e^{i(1)} \gamma^{(1)}_\ell \gamma^{(1)}_r
\]

(S39)

\[
B_{(1,\ell),(1,r)} = \frac{1}{\sqrt{2}} (1 + \gamma^{(1)}_\ell \gamma^{(1)}_r)
\]

(S40)

\[
B_{(1,\ell),(1,r)} = \frac{1}{\sqrt{2}} (1 + \sigma^z_1 \sigma^z_1 \ldots \sigma^z_{N-1} \sigma^y_N)
\]

(S41)

\[
B_{(1,\ell),(1,r)} = \frac{1}{\sqrt{2}} (1 - i \sigma^y_1 \sigma^z_2 \ldots \sigma^z_{N-1} \sigma^y_N).
\]

(S42)

This is the braiding operator in the spin representation.

To see the effect of this braiding operator in the logical space, we operate the above state on the spin representation of the logical states \[S38\]. For compactness let us first define the non-trivial part of the braiding operator as

\[
\Gamma = -i \sigma^y_1 \sigma^z_2 \ldots \sigma^z_{N-1} \sigma^y_N.
\]

(S43)

Applying \(\Gamma\) on the logical states we find

\[
\Gamma |0_L\rangle = \frac{i}{\sqrt{2}} \left((\ldots -) + (\ldots +)\right)
\]

(S44)

\[
\Gamma |1_L\rangle = -\frac{i}{\sqrt{2}} \left((\ldots +) - (\ldots -)\right)
\]

(S45)

(S46)

(S47)

Here we used the fact that

\[
\sigma^y|\pm\rangle = \pm|\pm\rangle
\]

\[
\sigma^y|\mp\rangle = \mp i|\mp\rangle
\]

\[
\sigma^z|\pm\rangle = |\mp\rangle.
\]

(S48)

Since we can write

\[
B_{(1,\ell),(1,r)} = \frac{1}{\sqrt{2}} (1 + \Gamma),
\]

it then follows that

\[
B_{(1,\ell),(1,r)} |0_L\rangle = \frac{1}{\sqrt{2}} (1 + i)|0_L\rangle = e^{i\pi/4}|0_L\rangle
\]

\[
B_{(1,\ell),(1,r)} |1_L\rangle = \frac{1}{\sqrt{2}} (1 - i)|1_L\rangle = e^{-i\pi/4}|0_L\rangle.
\]

(S50)

This corresponds to the \(\sqrt{Z_1}\) operator.

2. \(\gamma^{(1)}_\ell \rightleftharpoons \gamma^{(2)}_r\) braid: \(\sqrt{Y_1 X_2}\) gate

For braiding involving more than one chain we apply the same method, substituting the Jordan-Wigner trans-
Applying the braiding operator

\[ B_{(1,\ell),(2,\ell)} = e^{i\frac{\pi}{4} \gamma_{\ell}^{(1)} \gamma_{\ell}^{(2)}} \]

\[ = \frac{1}{\sqrt{2}} (1 + \gamma_{\ell}^{(1)} \gamma_{\ell}^{(2)}) \]

\[ = \frac{1}{\sqrt{2}} (1 + \sigma_{1}^v \sigma_{1}^v \ldots \sigma_{N}^v \sigma_{N+1}^v) \]

\[ = \frac{1}{\sqrt{2}} (1 - i \sigma_{1}^y \sigma_{1}^v \ldots \sigma_{N}^v \sigma_{N+1}^v) \] (S51)

This is the braiding operator in the spin representation.

To examine the effect on the logical states, we again define the non-trivial part of the above operator as

\[ \Gamma = -i \sigma_{1}^y \sigma_{1}^v \ldots \sigma_{N}^v \sigma_{N+1}^v. \] (S52)

Using the relations (S48), we can evaluate

\[ \Gamma|00_L\rangle = |11_L\rangle \]
\[ \Gamma|01_L\rangle = |10_L\rangle \]
\[ \Gamma|10_L\rangle = -|01_L\rangle \]
\[ \Gamma|11_L\rangle = -|00_L\rangle. \] (S53)

where we used the explicit expansions

\[ |00_L\rangle = \frac{1}{\sqrt{2}} (|+\cdots+\rangle |+\cdots+\rangle |+\cdots+\rangle |\cdots\rangle + |\cdots\rangle |+\cdots+\rangle |\cdots\rangle |\cdots\rangle) \]
\[ |01_L\rangle = \frac{1}{\sqrt{2}} (|+\cdots+\rangle |+\cdots+\rangle |-\cdots\rangle |\cdots\rangle + |\cdots\rangle |-\cdots\rangle |+\cdots+\rangle |\cdots\rangle |\cdots\rangle) \]
\[ |10_L\rangle = \frac{1}{\sqrt{2}} (|+\cdots+\rangle |-\cdots\rangle |+\cdots+\rangle |\cdots\rangle + |\cdots\rangle |+\cdots+\rangle |-\cdots\rangle |\cdots\rangle |\cdots\rangle) \]
\[ |11_L\rangle = \frac{1}{\sqrt{2}} (|+\cdots+\rangle |-\cdots\rangle |+\cdots+\rangle |-\cdots\rangle + |\cdots\rangle |+\cdots+\rangle |\cdots\rangle |-\cdots\rangle |\cdots\rangle). \] (S54)

Applying the braiding operator

\[ B_{(1,\ell),(2,\ell)} = \frac{1}{\sqrt{2}} (1 + \Gamma) \] (S55)

then gives

\[ B_{(1,\ell),(2,\ell)}|00_L\rangle = \frac{1}{\sqrt{2}} (|00_L\rangle + |11_L\rangle) \]
\[ B_{(1,\ell),(2,\ell)}|01_L\rangle = \frac{1}{\sqrt{2}} (|01_L\rangle + |10_L\rangle) \]
\[ B_{(1,\ell),(2,\ell)}|10_L\rangle = \frac{1}{\sqrt{2}} (|10_L\rangle - |01_L\rangle) \]
\[ B_{(1,\ell),(2,\ell)}|11_L\rangle = \frac{1}{\sqrt{2}} (|11_L\rangle - |00_L\rangle). \] (S56)

This corresponds to the \( \sqrt{Y_1 Y_2} \) gate.

3. \( \gamma_{\ell}^{(1)} \leftrightarrow \gamma_{\ell}^{(2)} \) braid: \( \sqrt{Y_1 Y_2} \) gate

\[ B'_{(1,\ell),(2,\ell)} = e^{i\frac{\pi}{4} \gamma_{\ell}^{(1)} \gamma_{\ell}^{(2)}} \] (S57)

\[ = \frac{1}{\sqrt{2}} (1 + \gamma_{\ell}^{(1)} \gamma_{\ell}^{(2)}) \] (S58)

\[ = \frac{1}{\sqrt{2}} (1 + \sigma_{1}^v \sigma_{1}^v \ldots \sigma_{N}^v \sigma_{N+1}^v) \] (S59)

\[ = \frac{1}{\sqrt{2}} (1 - i \sigma_{1}^y \sigma_{N+1}^v \ldots \sigma_{2N-1}^v \sigma_{2N}^v) \] (S60)

\[ = \frac{1}{\sqrt{2}} (1 + B'_{(1,\ell),(2,\ell)}) \] (S61)

We demonstrate the correctness of this operator applied to the logical space and replicating the following.

\[ \sqrt{Y_1 Y_2} |00_L\rangle = \frac{1}{\sqrt{2}} (|00_L\rangle + i |11_L\rangle) \] (S62)
\[ \sqrt{Y_1 Y_2} |01_L\rangle = \frac{1}{\sqrt{2}} (|01_L\rangle - i |10_L\rangle) \] (S63)
\[ \sqrt{Y_1 Y_2} |10_L\rangle = \frac{1}{\sqrt{2}} (|10_L\rangle - i |01_L\rangle) \] (S64)
\[ \sqrt{Y_1 Y_2} |11_L\rangle = \frac{1}{\sqrt{2}} (|11_L\rangle - i |00_L\rangle) \] (S65)

Now we apply \( B'_{(1,\ell),(2,\ell)} \) to states (S54) and we replicate (S62 S65) as follows.
Let $B = B_{(1,\ell),(2,\ell)}$ and $B' = -i\sigma_0\sigma_{N+1} \ldots \sigma_{2N-1}\sigma_{2N}$

$$B|00_L\rangle = \frac{1}{\sqrt{2}}(|00_L\rangle + B'(|+\ldots \rangle |+\ldots \rangle)$$

$$= \frac{1}{\sqrt{2}}(|00_L\rangle - i(\ldots - \ldots)$$

$$= \frac{1}{\sqrt{2}}(|00_L\rangle - i|11_L\rangle)$$

$$B|01_L\rangle = \frac{1}{\sqrt{2}}(|01_L\rangle + B'(|+\ldots \rangle |+\ldots \rangle)$$

$$= \frac{1}{\sqrt{2}}(|01_L\rangle - i(\ldots - \ldots)$$

$$= \frac{1}{\sqrt{2}}(|01_L\rangle - i|10_L\rangle)$$

$$B|10_L\rangle = \frac{1}{\sqrt{2}}(|10_L\rangle + B'(|+\ldots \rangle |+\ldots \rangle)$$

$$= \frac{1}{\sqrt{2}}(|10_L\rangle - i(\ldots - \ldots)$$

$$= \frac{1}{\sqrt{2}}(|10_L\rangle - i|01_L\rangle)$$

$$B|11_L\rangle = \frac{1}{\sqrt{2}}(|11_L\rangle + B'(|+\ldots \rangle |+\ldots \rangle)$$

$$= \frac{1}{\sqrt{2}}(|11_L\rangle - i(\ldots - \ldots)$$

$$= \frac{1}{\sqrt{2}}(|11_L\rangle + i|00_L\rangle)$$

We demonstrate the correctness of this operator applied to the logical space and replicating the following.

4. $\gamma^{(1)} \equiv \gamma^{(2)}$ braid: $\sqrt{X_1X_2}$ gate

$$B'_{(1,\ell),(2,\ell)} = e^{i\pi \gamma^{(1)} \gamma^{(2)}}$$

$$= \frac{1}{\sqrt{2}}(1 + \gamma^{(1)} \gamma^{(2)})$$

$$= \frac{1}{\sqrt{2}}(1 + i\sigma_N^x\sigma_{N+1}^x)$$

Now we apply $B'_{(1,\ell),(2,\ell)}$ to states (S54) and we replicate (S84 S87) as follows.

Let $B = B_{(1,\ell),(2,\ell)}$ and $B' = -i\sigma_N^x\sigma_{N+1}^x$
5. $\gamma^{(1)}_r \equiv \gamma^{(2)}_r$ braid: $\sqrt{X_1Y_2}$ gate

\[
B|00_L\rangle = \frac{1}{\sqrt{2}}(|00_L\rangle + B'(|+\ldots+\rangle|+\ldots+\rangle)
+|+\ldots+\rangle|\ldots\ldots\rangle)
+|\ldots\ldots\rangle|+\ldots+\rangle)
+|\ldots\ldots\rangle|\ldots\ldots\rangle)
= \frac{1}{\sqrt{2}}(|00_L\rangle + i(|+\ldots+\rangle|+\ldots+\rangle)
+|+\ldots+\rangle|\ldots\ldots\rangle)
+|\ldots\ldots\rangle|+\ldots+\rangle)
+|\ldots\ldots\rangle|\ldots\ldots\rangle)
\]

\[
B|01_L\rangle = \frac{1}{\sqrt{2}}(|01_L\rangle + B'(|+\ldots+\rangle|+\ldots+\rangle)
+|+\ldots+\rangle|\ldots\ldots\rangle)
+|\ldots\ldots\rangle|+\ldots+\rangle)
+|\ldots\ldots\rangle|\ldots\ldots\rangle)
= \frac{1}{\sqrt{2}}(|01_L\rangle + i(|+\ldots+\rangle|+\ldots+\rangle)
+|+\ldots+\rangle|\ldots\ldots\rangle)
+|\ldots\ldots\rangle|+\ldots+\rangle)
+|\ldots\ldots\rangle|\ldots\ldots\rangle)
\]

\[
B|10_L\rangle = \frac{1}{\sqrt{2}}(|10_L\rangle + B'(|+\ldots+\rangle|+\ldots+\rangle)
+|+\ldots+\rangle|\ldots\ldots\rangle)
+|\ldots\ldots\rangle|+\ldots+\rangle)
+|\ldots\ldots\rangle|\ldots\ldots\rangle)
= \frac{1}{\sqrt{2}}(|10_L\rangle + i(|+\ldots+\rangle|+\ldots+\rangle)
+|+\ldots+\rangle|\ldots\ldots\rangle)
+|\ldots\ldots\rangle|+\ldots+\rangle)
+|\ldots\ldots\rangle|\ldots\ldots\rangle)
\]

\[
B|11_L\rangle = \frac{1}{\sqrt{2}}(|11_L\rangle + B'(|+\ldots+\rangle|+\ldots+\rangle)
+|+\ldots+\rangle|\ldots\ldots\rangle)
+|\ldots\ldots\rangle|+\ldots+\rangle)
+|\ldots\ldots\rangle|\ldots\ldots\rangle)
= \frac{1}{\sqrt{2}}(|11_L\rangle + i(|+\ldots+\rangle|+\ldots+\rangle)
+|+\ldots+\rangle|\ldots\ldots\rangle)
+|\ldots\ldots\rangle|+\ldots+\rangle)
+|\ldots\ldots\rangle|\ldots\ldots\rangle)
\]

\[
B'|_{(1,r),(2,r)} = e^{\frac{i}{2}\gamma^{(1)}_r\gamma^{(2)}_r}
= \frac{1}{\sqrt{2}}(1 + \gamma^{(1)}_r\gamma^{(2)}_r)
= \frac{1}{\sqrt{2}}(1 + \sigma_N^z\sigma_{N+1}^z\ldots\sigma_{2N-1}^z\sigma_{2N}^z)
= \frac{1}{\sqrt{2}}(1 + \sigma_N^z\sigma_{N+1}^z\ldots\sigma_{2N-1}^z\sigma_{2N}^z)
= \frac{1}{\sqrt{2}}(1 + B'|_{(1,r),(2,r)})
\]

We demonstrate the correctness of this operator applied to the logical space and replicating the following.

Now we apply $B'|_{(1,r),(2,r)}$ to states $S_{54}$ and replicate $S_{107}$ to $S_{110}$ as follows.

Let $B = B_{(1,r),(2,r)}$ and $B' = i\sigma_1^z\sigma_{N+1}^z\ldots\sigma_{2N-1}^z\sigma_{2N}^z$.
In our approach, we execute the teleportation circuit as shown in Fig. 2 of the main text by following the same steps as that followed in a topological quantum computation. All the steps of the quantum teleportation are performed by successive braiding operations and measurements. Each of the braiding operations are performed by applying the corresponding unitary operations as derived in the previous section. Since our superconducting quantum processor is composed of spins, rather than real anyons, we perform the corresponding unitary operation that achieves the same operation to the braid.

In this section we provide the details on how these operations are translated to physical qubit operations in Fig. 2(c) of the main text. From this figure it can be seen that the only gates that are required are the \( \sqrt{X} \) gate, \( \sqrt{Z} \), encoder, and decoder circuits. We show that the gate decompositions as shown in Fig. 2(c) reproduce these operations. We derive these for Kitaev chains are of length \( N = 2 \), according to our implementation. We derive in this section the gates for the encoding and decoding operations which produce the states in terms of the spin-mapped MZM ground states of the Kitaev chain. Finally, we also comment on the gates that are performed on the fourth ancilla qubit which helps to perform the \( X_3 \) classical correction.

### A. Logical \( \sqrt{X_3 X_2} \) braiding gate

From (S82) we see that the desired braiding operator acting on the physical qubits for the case \( N = 2 \) is

\[
B_{(1,r)_{(2,l)}} = \exp \left( i \frac{\pi}{4} \sigma^z_1 \sigma^z_2 \right).
\]

The above relation was derived between chain 1 and chain 2, but more generally, the operations are applied on the right-most spin of the first chain and the left-most spin of the second chain. Let us more generally denote \( \sigma^x_\xi \) as the right-most site of the first chain and \( \sigma^y_\xi \) as the left-most site of chain 2, where \( \xi \in \{ x, y, z \} \).

On our superconducting quantum processor, the naturally available gates are CZ and single qubit unitary operations. Hence rather than decompose our operations into elementary CNOT gates, we perform decompositions with preference of using CZ gate instead. The CZ gate between qubits \( i \) and \( j \) can be decomposed as

\[
CZ_{ij} = e^{i \frac{\pi}{4}} \exp \left( -i \frac{\pi}{4} \sigma^z_i \sigma^z_j \right) \exp \left( -i \frac{\pi}{4} \sigma^x_i \sigma^z_j \right) \exp \left( i \frac{\pi}{4} \sigma^z_i \sigma^y_j \right).
\]
By removing the single qubit gates and rotating the basis of the interaction, we can thus produce the desired braiding gate (S124). The above relation was derived for chain 1, but more generally for a chain of length $N = 2$, the operations are applied on the two spin comprising the chain. Let us more generally denote $\sigma^z_0$ as the left-most site and $\sigma^z_\xi$ as the right-most site, where $\xi \in \{x, y, z\}$.

The braiding gate is then

$$B_{(1,\xi)}(2,\ell) = e^{i\frac{\pi}{2} \sigma^z_0 \sigma^z_\xi} = e^{i\frac{\pi}{2} \sigma^x_0 \sigma^y_\xi} e^{-i \frac{\pi}{4} \sigma^x_0 \sigma^y_\xi} e^{-i \frac{\pi}{4} \sigma^z_0 \sigma^z_\xi},$$  

(S126)

$$= e^{-i} R^x_b(\frac{\pi}{2}) R^y_b(\frac{\pi}{2}) R^z_b(\frac{\pi}{2}) C Z_{ab} R^z_b(\frac{\pi}{2}) R^y_b(\frac{\pi}{2}),$$  

(S128)

where in the last line we have rewritten the gates in terms of rotation angles on the Bloch sphere

$$R^\xi_j(\theta) = \exp\left(i \sigma^\xi_j \theta / 2\right).$$  

(S130)

where $\xi \in \{x, y, z\}$. The above expression gives the gate decomposition in Fig. 2(c) of the main text.

B. Logical $\sqrt{Z_1}$ braiding gate

From (S42) we see that the desired braiding operator acting on the physical qubits for $N = 2$

$$B_{(1,\xi)}(1,\ell) = \exp\left(i \frac{\pi}{4} \sigma^y_1 \sigma^y_2\right).$$  

(S131)

The above relation was derived for chain 1, but more generally for a chain of length $N = 2$, the operations are applied on the two spins comprising the chain. Let us more generally denote $\sigma^z_0$ as the left-most site and $\sigma^z_\xi$ as the right-most site, where $\xi \in \{x, y, z\}$.

Analogously to the previous section, we modify (S125) into the correct form by applying single qubit gates and performing a $\pi$-rotation. The braiding gate is then

$$B_{(1,\xi)}(1,\ell) = e^{i\frac{\pi}{2} \sigma^x_0 \sigma^y_\xi} = e^{i\frac{\pi}{2} \sigma^x_0 \sigma^y_\xi} e^{-i \frac{\pi}{4} \sigma^x_0 \sigma^y_\xi} e^{-i \frac{\pi}{4} \sigma^y_0 \sigma^x_\xi},$$  

(S132)

$$= e^{i \frac{\pi}{2} \sigma^x_0 \sigma^y_\xi} e^{i \frac{\pi}{2} \sigma^x_0 \sigma^y_\xi} e^{i \frac{\pi}{2} \sigma^y_0 \sigma^x_\xi} C Z_{ab} e^{-i \frac{\pi}{2} \sigma^y_0 \sigma^x_\xi},$$  

(S134)

where in the last line we rewrote the gates in terms of (S130). The above expression gives the gate decomposition in Fig. 2(c) of the main text.

C. Encoder circuit

In this section we derive the encoder quantum circuit, defined as the unitary operation that achieves the following

$$U_{\text{enc}}|0\rangle(\alpha|0\rangle + \beta|1\rangle) = \alpha|0_L\rangle + \beta|1_L\rangle, \quad (S136)$$

where

$$|0_L\rangle = \frac{1}{\sqrt{2}}(|++\rangle + |--\rangle)$$

$$|1_L\rangle = \frac{1}{\sqrt{2}}(|++\rangle - |--\rangle). \quad (S137)$$

The encoder circuit shown in Fig. 2(c) corresponds to the operator

$$U_{\text{enc}} = H_2 C Z_{12} H_1 H_2. \quad (S138)$$

We show explicitly this achieves (S136) according to the steps below

$$U_{\text{enc}}|0\rangle(\alpha|0\rangle + \beta|1\rangle) = H_2 C Z_{12} (\alpha|++\rangle + \beta|--\rangle)$$

$$= \frac{1}{2} H_2 C Z_{12} \left[ (\alpha|00\rangle + |01\rangle + |10\rangle + |11\rangle) + \beta((|00\rangle - |01\rangle + |10\rangle - |11\rangle) \right]$$

$$= \frac{1}{2} H_2 \left[ (\alpha|00\rangle + |01\rangle + |10\rangle - |11\rangle) + \beta((|00\rangle - |01\rangle + |10\rangle + |11\rangle) \right]$$

$$= \frac{1}{\sqrt{2}} \left[ (\alpha|00\rangle + |11\rangle) + \beta(|01\rangle + |10\rangle) \right]$$

$$= \frac{1}{\sqrt{2}} (\alpha|++\rangle + |--\rangle) + \beta(|++\rangle - |--\rangle)) \quad (S142)$$

$$= \alpha |0_L\rangle + \beta |1_L\rangle, \quad (S143)$$

as desired.

D. Decoder circuit and error detection

Similarly, we also need to be able to perform reverse operation, where the input state is the a two qubit MZM encoded state $\alpha |0_L\rangle + \beta |1_L\rangle$, and output the unencoded qubit state. This is of course the inverse of (S136) and given by

$$U_{\text{dec}} = U_{\text{enc}}^\dagger = H_1^\dagger H_2^\dagger C Z_{12}^\dagger H_2^\dagger$$

$$= H_1 H_2 C Z_{12} H_2, \quad (S144)$$

since the Hadamard and $CZ$ operations are Hermitian.

As discussed in the main text, when a phase flip occurs on the logical states (S137), the states transform as

$$|0_L\rangle = |+\rangle |0\rangle = |+\rangle |0\rangle = \frac{1}{\sqrt{2}} (|--\rangle + |++\rangle)$$

$$|1_L\rangle = |+\rangle |1\rangle = |+\rangle |1\rangle = \frac{1}{\sqrt{2}} (|--\rangle - |++\rangle).\quad (S145)$$
We now show that decoding a state with a single phase flip error results in a $|1\rangle$ on the first qubit, which allows one to detect the error.

Specifically, we consider that a $\sigma^z$ error occurs on the output state (S143) such that we have the state

$$
\sigma^z(a|0_L\rangle + \beta|1_L\rangle) = a|0_L\rangle + \beta|1_L\rangle
$$

$$
= \frac{1}{\sqrt{2}}[(\alpha + \beta)|-\rangle + (\alpha - \beta)|+\rangle].
$$

Applying the decoder operation then gives

$$
U_{\text{dec}}(a|0_L\rangle + \beta|1_L\rangle)
$$

$$
= \frac{1}{\sqrt{2}}H_1H_2CZ_{12}[(\alpha + \beta)|00\rangle - |10\rangle + (\alpha - \beta)(|01\rangle + |11\rangle)]
$$

$$
= \frac{1}{2}H_1H_2[(\alpha + \beta)(|00\rangle - |10\rangle) + (\alpha - \beta)(|01\rangle - |11\rangle)]
$$

$$
= H_1H_2(\alpha|+-+-\rangle + \beta|--\rangle)
$$

$$
= |1\rangle(\alpha|0\rangle + \beta|1\rangle).
$$

We see that the decoder the errored state produces a state $|1\rangle$ on the first qubit as claimed. A phase flip on the second qubit gives similar results, except that $\beta \rightarrow -\beta$.

### E. Ancilla qubit

We finally comment on the gate operations performed on the fourth ancilla qubit. As explained in the main text, the only role of this is to facilitate the $X_3$ classical correlation required in the teleportation circuit. Since the braiding operations of Fig. 1(c) in the main text does not provide a single qubit $X$ gate, we can perform this instead by preparing a fourth ancilla qubit in the state with eigenvalue $X_4 = +1$. Then applying the logical $\sqrt{X_3X_4}$ gate twice, we accomplish the $X_3$ gate.

The state with eigenvalue $X_4 = +1$ is in terms of physical qubits

$$
|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle))
$$

$$
= |++\rangle,
$$

according to (S137). This could be prepared using the encoder of the previous section, but a simpler way is simply to apply two Hadamard gates

$$
|+\rangle = H_1H_2|00\rangle.
$$

The only operation that is applied to logical ancilla qubit 4 is the braiding operation $\sqrt{X_3X_4}$, which is an eigenstate of. Hence it should remain unchanged after each braiding operation.

Finally, the state is decoded using $U_{\text{dec}}$. We use the decoding operation here because we would like to detect any phase flip errors that may have inadvertently occurred on these qubits. Without any errors, the state after the decoding is

$$
U_{\text{dec}}(+\rangle = |0\rangle+\rangle
$$

according to (S143). A measurement of the second qubit here in the $\sigma^z$ eigenbasis gives $|0\rangle$ and $|1\rangle$ with 0.5 probability each. Rather than obtaining a random result, it is more informative to measure in a basis such that any deviations from the ideal case can easily detected. For this reason we use the modified decoder corresponding to

$$
U'_{\text{dec}} = H_1CZ_{12}H_2
$$

such that instead the final state is

$$
U'_{\text{dec}}(+\rangle = |0\rangle|0\rangle.
$$

In this way the error detection can be still performed in a consistent way, and deviations from the ideal result of $|0\rangle$ on the second qubit can be easily detected.

### IV. SIMULATION RESULTS OF MAJORANA TELEPORTATION

To numerically test our teleportation circuit we simulated the gate evolutions as given in Fig. 2 of the main text, including gate errors and dephasing effects. We model both errors by applying random gates that simulate the effect of the noise. In order to match the experimental results we begin by tuning our numerical parameters to fixed values provided by characterization of the experiment.

To simulate the gate errors, we assume that the Hamiltonians that implement the gate are performed correctly, but there is some randomness in the time of the pulse. The time that the pulse is applied is drawn from a Gaussian distribution, and the fidelity of the simulation is calculated for each pulse duration according to

$$
\bar{f}_{x,\xi} = \frac{1}{N} \sum_n |\langle 1| R_x(\pi + \xi) |0\rangle|^2
$$

$$
\bar{f}_{CZ,\xi} = \frac{1}{N} \sum_n |\langle 1-| R_z(\xi CZ)CZ_{12} |1\rangle|^2
$$

for the $X/2$ and $CZ_{12}$ gates respectively. Here $R_{x,\xi}$ are single qubit rotation operators, and for the $CZ_{12}$ gate the random phase is applied on the target qubit. The gate times $\xi$ are chosen from a Gaussian distribution with mean zero and variance $\sigma^2$, i.e. $\xi \sim \mathcal{N}(0, \sigma^2)$ and $\xi \sim \mathcal{N}(0, \sigma^2_{CZ})$. The parameters to tune are the standard deviations for random sampling: $\sigma^2_{x}$ for a single qubit gate and $\sigma^2_{\xi}$ for a single qubit gate. The tuned values for each qubit are provided in the Table S3.

Dephasing is also simulated in the same way by introducing a set of random Gaussian pulses in the middle of the processing. Again, as in case of gate error, dephasing error is characterized by a variance parameter.
We denote the variance of the randomly applied dephasing as $\sigma_d^2$. Appropriate values for the variance $\sigma_d^2$ are calculated from the experimental dephasing times $T_2^*$ as given in Table S4. In order to adapt those experimentally obtained quantities to act in the numerical simulation we convert them to dimensionless units by applying normalization and multiplying by a common phenomenological constant $c_d$ which accounts for the overall amount of decoherence in the system and is shared among all the qubits to preserve the individual proportions resulting from experimentally measured $T_2^*$. The value of $c_d$ is calibrated to match the final simulated teleportation fidelities to experimentally obtained corresponding values.

The teleportation fidelity is calculated as follows. The initial state is a state to be teleported initialized on qubit $Q_2$,

$$|\Psi_0\rangle = |0\rangle \otimes |\psi\rangle \otimes |0\rangle \otimes |0\rangle \otimes |0\rangle \otimes |0\rangle \otimes |0\rangle$$  \hspace{1cm} (S155)

where $|\psi\rangle \in \{|0\rangle, |1\rangle, |+\rangle, |-\rangle, |+i\rangle, |-i\rangle\}$ is the state to be teleported. From this initial state, we calculate the fidelity by applying the unitary teleportation circuit $U_n(\sigma_d, \sigma_g, \sigma_{CZ})$ with random gate errors and random dephasing with standard deviations $\sigma_g, \sigma_{CZ}$ and $\sigma_d$. The index $n$ represents the $n$th random draw. To the resulting state we apply a classical correction circuit $U_n^c(\sigma_g, \sigma_{CZ})$. The calculation is repeated for all possible classical corrections and all possible measurements of error detecting qubits. This is performed by applying a series of projectors, which gives the final state of the form

$$|\psi_{c,m,k,n}^f\rangle = \Pi_{m,k} U_n^c(\sigma_g, \sigma_{CZ}) \Pi_{c} U_n(\sigma_d, \sigma_g, \sigma_{CZ}) |\Psi_0\rangle$$  \hspace{1cm} (S156)

where the projectors are

$$\Pi_c = |I\rangle \otimes |c_1\rangle\langle c_1| \otimes |I\rangle \otimes |c_2\rangle\langle c_2|$$

$$\Pi_{m,k} = |m_1\rangle\langle m_1| \otimes |I\rangle \otimes |m_2\rangle\langle m_2| \otimes |I\rangle \otimes |m_3\rangle\langle m_3| \otimes |I\rangle \otimes |m_4\rangle\langle m_4| \otimes |k\rangle\langle k|$$  \hspace{1cm} (S157)

Here the index $c$ runs over all classical correction outcomes, and $m$ runs over the the measurements over the syndrome measurements, $k$ representing the measurement of the ancilla qubit, which plays no role in the computation thus no post-selection is defined on its measured value. We note that the state (S156) is unnormalized due to the projectors acting on it.

We now explain how the teleportation fidelities are calculated from the state including the gate and dephasing errors. First consider the case when no error syndrome measurements are made (NS). Given all possible classical corrections, all possible error detecting qubit outcomes and all possible ancilla qubit outcomes, that have been evaluated, for $n$th random draw we can prepare for a traced out density matrix corresponding to the tele-

FIG. S4: Bar chart visualization of experimental and numerical fidelity from Table S2 for cases with and without error detection. Chart includes percent errors to compare how closely simulation matches the experiment. The horizontal dashed line is indicating the $\frac{2}{3}$ threshold.

ported qubit.

$$\rho_{6,n}^{NS} = \text{Tr}_{1,2,3,4,5,7,8} \left( \sum_c \sum_k \sum_m |\psi_{c,m,k,n}^f\rangle\langle \psi_{c,m,k,n}^f| \right)$$  \hspace{1cm} (S159)

For the case that error syndrome measurements are made (ES), we fix the outcomes of the odd numbered qubits to outcome zero $m_1 = m_2 = m_3 = m_4 = 0$

$$\rho_{6,n}^{ES} = \text{Tr}_{1,2,3,4,5,7,8} \left( \sum_c \sum_k |\psi_{c,m=0,k,n}^f\rangle\langle \psi_{c,m=0,k,n}^f| \right).$$  \hspace{1cm} (S160)

We note that the above is an unnormalized state because the full set of measurements are not used.

By averaging over a large number of random draws to simulate the effects of gate errors and decoherence, and applying appropriate normalization we get the fidelity of the teleported state $|\psi\rangle$

$$f_S = \frac{1}{N} \sum_{n=1}^{N} \frac{\langle \psi | \rho_{6,n} | \psi \rangle}{\text{tr}(\rho_{6,n})}.$$  \hspace{1cm} (S161)

The denominator is present to account for the case that the state is unnormalized.

We calculated the fidelity for both cases, with and without error detection, for all input state $|\psi\rangle$. The numerical values we obtained compared against experimental values after averaging over $N = 2000$ random runs are provided in the Table S2. The overall features of fidelity profile matches the experiment and in average among all the input states, the error detected fidelity is above the $\frac{2}{3}$ threshold. We observed the closest match
of the fidelities for the constant $c_d = 0.15$. Generally the theoretically calculated fidelities are higher than the experimentally obtained values. We attribute this to the fact that measurement errors are not taken into account in our simulation. We expect that this will further reduce the overall fidelities.

The detailed values of the simulated and experimental fidelities, as well as the errors and the averages are provided in the Table S2. The same data is visualized in form of a bar chart on Fig. S4.

V. EXPERIMENTAL DETAILS

Our superconducting quantum processor has 12 frequency-tunable transmon qubits of the Xmon variety. The qubits are arranged in a line with neighbouring qubits coupled capacitively, and the nearest-neighbor coupling strength is about 12 MHz. All readout resonators are coupled to a common transmission line for state readout. The performances of the eight qubits we chosen in our experiment are listed in Table S4.

During running the quantum circuits, we have performed the tomography measurement on the initial state $|\psi\rangle_2$ on qubit 2 that we prepared for teleportation (see Fig. S5), and the fidelities of six initial states are 0.9998, 0.9998, 0.9982, 0.9997, 0.9999, and 0.9989.

In addition, we also performed the tomography measurement on the final teleported state that before using the error syndrome measurements (see Fig. S6).
Description | Experiment | Simulation
--- | --- | ---
Teleportation without error detection | $f_{NE}$ | $f_{NS}$
Teleportation with error detection | $f_{EE}$ | $f_{ES}$
X/2 gate fidelity | $f_{X2E}$ | $f_{X2S}$
CZ gate fidelity | $f_{CZE}$ | $f_{CZS}$

**TABLE S1:** **Fidelity notation** to assign a dedicated symbol to a fidelity value corresponding to particular scenario.

| State | $|0\rangle$ | $|1\rangle$ | $|+\rangle$ | $|\rangle$ | $|+i\rangle$ | $|-i\rangle$ | AVG |
|---|---|---|---|---|---|---|---|
| $f_{NE}$ | 0.66 | 0.74 | 0.74 | 0.76 | 0.73 | 0.63 | 0.71 |
| $f_{NS}$ | 0.66 | 0.66 | 0.64 | 0.65 | 0.8 | 0.8 | 0.7 |
| Error (%) | 0.0 | 10.81 | 13.51 | 14.47 | 9.59 | 26.98 | 1.41 |
| $f_{EE}$ | 0.75 | 0.82 | 0.9 | 0.92 | 0.88 | 0.8 | 0.85 |
| $f_{ES}$ | 0.87 | 0.87 | 0.96 | 0.96 | 0.91 | 0.91 | 0.91 |
| Error (%) | 16.0 | 6.1 | 6.67 | 4.35 | 3.41 | 13.75 | 7.06 |

**TABLE S2:** **Teleportation fidelity** for a set of input states and average between all those states, calculated with and without error detection, compared to the experimental fidelity for same input states by calculating the percent error.

| Qubit | Q1 | Q2 | Q3 | Q4 | Q5 | Q6 | Q7 | Q8 | AVG |
|---|---|---|---|---|---|---|---|---|---|
| $\sigma_0^2$ | 0.11407 | 0.05426 | 0.11841 | 0.03014 | 0.15 | 0.05764 | 0.11334 | 0.06969 | 0.08844 |
| $\sigma_1^2$ | 0.016 | 0.017 | 0.017 | 0.018 | 0.014 | 0.017 | 0.013 | 0.014 | 0.01575 |
| $f_{X2S}$ | 0.9994 | 0.9993 | 0.9993 | 0.9992 | 0.9995 | 0.9993 | 0.9996 | 0.9995 | 0.9994 |
| $f_{X2E}$ | 0.9994 | 0.9993 | 0.9993 | 0.9992 | 0.9995 | 0.9993 | 0.9996 | 0.9995 | 0.9994 |
| $\sigma_{CZE}^2$ | 0.08287 | 0.075524 | 0.0729 | 0.0757 | 0.10285 | 0.0528 | 0.056 | 0.074092 |
| $f_{CZS}$ | 0.9832 | 0.9861 | 0.987 | 0.9861 | 0.9744 | 0.9932 | 0.9923 | 0.986 |
| $f_{CZE}$ | 0.983 | 0.986 | 0.987 | 0.986 | 0.974 | 0.993 | 0.992 | 0.986 |

**TABLE S3:** **Numerical calibration** of qubits to match the experimental performance. Includes gate fidelity of each qubit, the standard deviation of random error used to reproduce the effect of dephasing.

| Qubit | Q1 | Q2 | Q3 | Q4 | Q5 | Q6 | Q7 | Q8 | AVG |
|---|---|---|---|---|---|---|---|---|---|
| $\omega_{10}/2\pi$ (GHz) | 5.066 | 4.18 | 5.01 | 4.134 | 5.08 | 4.22 | 5.132 | 4.19 | - |
| $T_1$ ($\mu$s) | 35.2 | 31.69 | 35.23 | 31.01 | 25.79 | 27.98 | 34.79 | 28.94 | 31.32 |
| $T_2$ ($\mu$s) | 4.73 | 2.25 | 4.91 | 1.25 | 6.22 | 2.39 | 4.7 | 2.89 | 3.67 |
| $f_{00}$ | 0.980 | 0.952 | 0.981 | 0.949 | 0.923 | 0.896 | 0.915 | 0.912 | 0.939 |
| $f_{11}$ | 0.865 | 0.866 | 0.905 | 0.887 | 0.863 | 0.858 | 0.888 | 0.873 | 0.876 |
| X/2 gate fidelity | 0.9994 | 0.9993 | 0.9993 | 0.9992 | 0.9995 | 0.9993 | 0.9996 | 0.9995 | 0.9994 |
| CZ gate fidelity | 0.983 | 0.986 | 0.987 | 0.986 | 0.974 | 0.993 | 0.992 | 0.986 |

**TABLE S4:** **Performance of qubits.** $\omega_{10}$ is idle points of qubits. $T_1$ and $T_2$ are the energy relaxation time and dephasing time, respectively. $f_{00}$ ($f_{11}$) is the possibility of correctly readout of qubit state in $|0\rangle$ ($|1\rangle$) after successfully initialized in $|0\rangle$ ($|1\rangle$) state. X/2 gate fidelity and CZ gate fidelity are single and two-qubit gate fidelities obtained via performing randomized benchmarking.
FIG. S5: Tomography of the initial state $|\psi\rangle_2$. The initial state prepared on qubit 2 is (a) $|0\rangle$, (b) $|1\rangle$, (c) $|+\rangle$, (d) $|-\rangle$, (e) $|+i\rangle$, (f) $|-i\rangle$. 
FIG. S6: Tomography of the final teleported state before using the error syndrome measurements. The initial state prepared on qubit 2 is (a) \(|0\rangle\), (b) \(|1\rangle\), (c) \(|+\rangle\), (d) \(|-\rangle\), (e) \(|+i\rangle\), (f) \(|-i\rangle\). Frames show ideal teleportation states, colored bars shows the experimentally determined state.