How spinodal decomposition influences observables at FAIR energies

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Abstract. The FAIR facility will make the region of high net-baryon densities experimentally accessible, where a first-order phase transition is conjectured. We investigate the dynamics of chiral symmetry breaking and the onset of confinement during a heavy-ion collision at large baryochemical potentials within a nonequilibrium chiral fluid dynamics model including effects of dissipation and noise. The order parameters are explicitly propagated and coupled to a fluid dynamically expanding medium of quarks. We demonstrate that the coupled system is strongly influenced by the nonequilibrium dynamics, leading only to a weak growth of the correlation length near the critical point. At the first-order phase transition, spinodal instabilities create domains in the order parameters and large spatial fluctuations in the baryon density within single events. As a consequence we find a clear enhancement of higher flow harmonics in coordinate space at the first-order phase transition in comparison with transitions through the crossover or critical point.

1. Introduction
The phase diagram of quantum chromodynamics (QCD) is investigated experimentally in heavy-ion collisions. The nature of the QCD phase transition has up to now only been revealed in the region of small baryochemical potential, where the transition between a hadron gas and a conjectured quark-gluon-plasma is not a phase transition but rather an analytic crossover \cite{1,2}. The existence of a critical point (CP) which may be inferred from mean-field effective models \cite{3} or functional approaches \cite{4,5} could not yet be confirmed in experiment. This will be especially interesting for the upcoming FAIR facility, where the region of large net-baryon densities will be accessible. If there is a CP and the system is near equilibrium, then one should find a clear enhancement in event-by-event fluctuations of conserved quantities like particle multiplicities or transverse momentum \cite{6}. Nevertheless, we know that an ideal thermalized system is impossible to create during the rapid dynamics of a particle collision.

Fluctuation signals are also important at a first-order phase transition adjacent to the CP. Here it has been demonstrated that due to spinodal decomposition spatial inhomogeneities can...
be amplified [7, 8], leading to nonstatistical fluctuations within single events. So the problems we aim to study within our dynamical model are twofold: first, whether signals or a CP can develop during the nonequilibrium dynamics of a heavy-ion collision and second, whether nonequilibrium effects are strong enough to enable the dynamical phase fragmentation at the first-order phase transition, creating some relevant signal.

2. Coupling of a quark fluid to chiral fields and Polyakov loop

Our model consists of two sectors: First, we have an ideal fluid of quarks and antiquarks which provides a locally equilibrated medium, as presumably created after the collision of two heavy nuclei. Second, we explicitly propagate fluctuations through the nonequilibrium equations of motion for the relevant order parameters, the sigma field and the Polyakov loop.

We start from the Polyakov-Quark-Meson model which exhibits the desired phase structure with a CP and a first-order phase transition on the mean-field level [9] and beyond [5]. The Lagrangian density reads

$$ L = \bar{q} \left[ i \left( \gamma^\mu \partial_\mu - i g_\sigma \gamma^0 A_0 \right) - g \sigma \right] q + \frac{1}{2} \left( \partial_\mu \sigma \right)^2 - U(\sigma) - U(\ell) - \Omega_{q\bar{q}}. $$

The physical process leading to the temperature-dependent damping coefficient $\eta_\sigma(T)$ is the decay of one sigma into two quarks. Therefore, around the CP where the mass of sigma vanishes, also $\eta_\sigma(T)$ is equal to zero, see Fig. 1. The stochastic noise term is assumed to be Gaussian and white, it is connected to the damping coefficient via a dissipation-fluctuation relation. In Eq. 2, $V_{\text{eff}}$ denotes the effective thermodynamic potential

$$ V_{\text{eff}} = U(\sigma) + U(\ell) + \Omega_{q\bar{q}}, $$

Figure 1. Damping coefficient for the sigma field as a function of temperature for a first-order and a CP scenario. We see that only around the CP, the damping vanishes. Here, the strength of the transition is tuned via the quark-meson coupling constant $g$. Figure from [11].
where the contribution $\Omega_{q\bar{q}}$ stems from integrating out the quarks degrees of freedom in the path integral formulation of the grand canonical partition function.

For the Polyakov loop, we use a phenomenological equation of motion, because the real-time dynamics is of $\ell$ is not yet understood, this quantity is defined only in Euclidean space-time. We interpret $\ell$ as an effective field and propose its nonequilibrium equation of motion as [11]

$$
\eta_\ell \partial_t \ell + \frac{\partial V_{\text{eff}}}{\partial \ell} = \xi_\ell .
$$

Although a rigorous argument for the fixing of the damping coefficient $\eta_\ell$ cannot be given, we tested different values and found that results are in general not sensitive to this. We finally set $\eta_\ell = 5/\text{fm}$. The noise term $\xi_\ell$ is also Gaussian and white, with a dissipation-fluctuation relation connecting it to $\eta_\ell$ [11].

As we may assume that the quarks equilibrate on a significantly smaller time scale than the order parameters, we describe their evolution via an ideal energy-momentum tensor $T_{\mu\nu}^q = (e + p)u^\mu u^\nu - pg_{\mu\nu}$. Pressure, energy density and quark density $n$ are obtained from the standard thermodynamic relations

$$
p(\sigma, \ell, T, \mu) = -\Omega_{q\bar{q}} ,
$$

$$
e(\sigma, \ell, T, \mu) = T \frac{\partial p}{\partial T} + \mu \frac{\partial p}{\partial \mu} - p ,
$$

$$
n(\sigma, \ell, T, \mu) = \frac{\partial p}{\partial \mu} .
$$

Energy and momentum of the fields are not conserved if the temperature changes like it does in an expanding system. However, we can account for conservation of energy and momentum in the coupled system by introducing source terms for $\sigma$ and $\ell$ respectively, which are equal to the divergence of $T_{\mu\nu}^q$,

$$
\partial_\mu T_{\mu\nu}^q = S_\nu^\sigma + S_\nu^\ell .
$$

We could demonstrate in [11, 12] that this ensures conservation of the total energy throughout the numerical simulations. For systems at finite chemical potential, we also have to ensure quark number conservation which is achieved by the continuity equation

$$
\partial_\mu n^\mu = 0 .
$$

### 3. The correlation length in an expanding medium

The correlation length $\xi$ is the key quantity for a CP as it is directly related to the strength of event-by-event fluctuations, the proposed observable in heavy-ion experiments. However, certain effects will clearly weaken the growth of $\xi$ in a nonequilibrium dynamical situation. The simplest aspect is the finite size of the small system which poses a natural limit to the correlation length. Furthermore, one has to consider critical slowing down. If a system out of equilibrium approaches a CP, relaxation times become infinitely large. This has been studied in [13] within a simple phenomenological model, where the growth of $\xi$ near the QCD CP has been estimated to a maximum of $1.5 - 2.5$ fm.

For our simulation we initialize an ellipsoidal droplet of quark matter at a high temperature and afterwards let the system expand according to the equations of motion for fields and fluid. We then extract the correlation length from an averaging procedure over the inverse mass of the system in the central region of the fireball. One might also think of other procedures how to calculate $\xi$ as has been discussed in [14]. The results are shown in Fig. 2, for both a CP...
scenario and an evolution through the first-order phase transition. We find a clear peak around the critical temperature, where the correlation length grows up to 1.5 fm. For the first-order phase transition, it remains small at around 0.2 − 0.4 fm with only two minor peaks when the system is near the transition temperature, the first time as a result of the cooling and the second time due to reheating. We see that although we are at the CP, the expected signals might not be as clearly visible as they would be in equilibrium. Furthermore, the significant enhancement is limited to a finite amount of time, so depending on where the system’s freeze out happens, one might not obtain a measurable effect.

4. Anisotropic flow at the first-order phase transition
We focus now on the behavior of the system which evolves through the first-order phase transition at large net-baryon densities. We initialize a droplet of quark matter with appropriate initial conditions and let it expand and cool through the phase boundary. In [15], we found that this leads to strong non-statistical multiplicity fluctuations which do not occur at the CP or crossover. Due to spinodal instabilities that occur during the evolution, the high temperature phase dynamically fragments into small droplets, creating fluctuations in the azimuthal baryon number distribution. In contrast to the event-by-event fluctuations at the CP, this is clearly a single event signal. For a set of many events, one may average the flow harmonics in coordinate space \( v_n \) and obtain a clear enhancement in the higher harmonics compared to the CP evolution, see Fig. 3.

Although we have shown that interesting dynamical effects can be expected at the first-order phase transition, we want to investigate in the future how this might produce signals in the detector. At the moment, our model clearly lacks a hadronic phase for the low temperature regime. This could be implemented via a hadronic afterburner or a simple hadronic freeze-out. Then it would be interesting to study flow in momentum space and see if some effect can also be found here. For signals of a CP, we then aim at performing an event-by-event analysis in order to provide some realistic estimate of the expected signal.
Figure 3. Average values of the harmonic coefficients in coordinate space from the azimuthal net-baryon number distribution. At the first-order phase transition we find a strongly anisotropic expansion, while small coefficients indicate a homogeneous spherical shape at the CP.

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