Tangential Momentum Accommodation Coefficient measurements for various materials and gas species

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Abstract. The tangential momentum accommodation coefficient was measured from the gaseous flow through a single microtube. The mass flow rate was measured by the constant volume technique, where the rate is related to time rate of pressure variation in a fixed volume tank. The measured mass flow rate was fitted by the theoretical mass flow rate expressed by the slip velocity at the surface to deduce the slip coefficient, which can be related to the tangential accommodation coefficient. The mean Knudsen number, which is determined as the mean pressure of the inlet and the outlet, was set to be below 0.32, where the second-order slip boundary condition was suggested to be valid. The measurement system was designed to allow using a microtube with large diameter of several hundred micrometers. Since low pressure environment was essential for large Knudsen number condition for the flow through such large microtube, a low leakage measurement system realized by applying the UHV technology is needed. Applicability of sub millimeters size microtubes allowed us to measure the tangential momentum accommodation coefficients on various materials. In this study, we measured the tangential momentum accommodation coefficients on an engineering metal surface for various gas species.

1. Introduction
In micro gaseous flows, Knudsen number \( \tilde{K}n \), which is defined as a ratio of the molecular mean free path \( \lambda \) to the characteristic length of the system \( L \), becomes large. In such so-called “high Knudsen number flows”, the collision number of gas molecules to surfaces can not be neglected compared to that to other gas molecules. The ratio of a surface area to a volume is also large. Those characteristics lead to the importance of gas-surface interaction in the micro thermal and flow fields. For precise numerical simulations, accurate gas-surface interaction model for a boundary condition is essential.

To illustrate overall gas-surface interaction without any detailed processes, the accommodation coefficient \( \alpha \) is the most widely used empirical parameter, which is also often used as a parameter for a gas-surface interaction models for numerical simulations. Related to flow properties, like a pressure loss of a fluid system, the tangential momentum accommodation coefficient \( \alpha_t \) is important, which is defined as,

\[
\alpha_t = \frac{\tau_i - \tau_r}{\tau_i}
\]

where \( \tau \) is tangential momentum of gas molecules and indices \( i \) and \( r \) represent incident and reflected molecules, respectively. The tangential momentum accommodation coefficient represents the
statistical characteristics of the mean exchange rate, efficiency, or probability of the tangential momentum. The energy (=thermal) accommodation coefficient, which describes the energy exchange by the similar definition, is also often used.

Especially for the machined and non-treated “as is” metal surfaces, often called engineering surfaces, the complete accommodation has been assumed unquestionably. However, the recent precise measurements on the energy accommodation coefficient suggested an incomplete thermal accommodation on metal surfaces. This result indicates the incomplete accommodation of gas molecules on the engineering surfaces, which leads to doubt the approximation of the tangential momentum accommodation coefficient as unity.

However, it is still not easy to measure the tangential momentum accommodation coefficients on an engineering metal surface of “bulk” materials, and few data were reported [1]. The gas flow through the stainless steel tube was studied by several researchers: Reference [2] measured by a single cylindrical tube with several inert gases, but the tangential momentum accommodation coefficient was not obtained. Reference [3] evaluated the conductance with a bundle of tubes. References [4] and [5] were studied under rarefied gas flow conditions, which were different in the ratio of the boundary layer thickness to the diameter of the tube compared to other researches.

In this paper, the mass flow rate through a microtube under the slip flow regime was measured by the constant volume method, by which the tangential momentum accommodation coefficient was calculated. The second-order effect of the velocity slip condition was considered in the analysis. A microtube with an engineering metal surface, which was manufactured from a bulk material, was employed by using the low pressure environment.

2. Theory
The tangential momentum accommodation was measured from the mass flow rate through a single microtube [6].

The constant volume method [7-10], in which the mass flow rate was related to the pressure change of the outlet tank through the equation of state under an ideal gas approximation, was employed. The mass flow rate was obtained by

\[
Q_m = \frac{V \cdot \delta P}{RT \cdot \tau} \tag{2}
\]

where \(V, R, T, \delta P, \tau\) are volume, the specific gas constant, temperature, pressure change, measurement time, respectively. The mass flow rate through a microtube is theoretically obtained in frame of the Navier-Stokes equation with the second-order velocity slip boundary condition as [11, 12],

\[
Q_m = \frac{\pi D^4 \Delta P P_m}{128 \mu R T L} \left( 1 + 8 A_1 K n_m + 16 A_2 K n_m^2 \frac{P_m}{P} + 1 \ln P \right) \tag{3}
\]

where \(D, \Delta P, P_m, \mu, L, K n_m, P\) are the diameter of a microtube, the pressure difference from the inlet to the outlet, the mean pressure of the inlet and the outlet, the viscosity coefficient, the length of a microtube, the mean Knudsen number based on the mean pressure of the inlet and the outlet, and the pressure ratio of the inlet to the outlet, respectively. Experimentally measured mass flow rates are normalized to \(S\) as follows:

\[
S = \frac{Q_m}{\left( \frac{\pi D^4 \Delta P P_m}{128 \mu R T L} \right)} = 1 + 8 A_1 K n_m + 16 A_2 K n_m^2 \frac{P_m}{P} + 1 \ln P \tag{4}
\]
The normalized mass flow rates $S$ are plotted as a function of the mean Knudsen number $K_n$ and fitted with theoretical expression of equation (4) by the least-square method. The coefficient $A_1$ is function of the slip coefficient and also of the tangential momentum accommodation coefficient. Therefore, the tangential accommodation coefficient can be deduced from the procedure. A scattering model is used to relate the slip coefficient and the tangential momentum accommodation coefficient. Following the Maxwell scattering model, the tangential momentum accommodation coefficient $\alpha_t^M$ becomes as follows:

$$
\sigma_p = \sqrt{\frac{\pi}{2}} \frac{2 - \alpha_t^M}{\alpha_t^M} \tag{5}
$$

While, it was suggested [13, 14] that this model gave inconsistent slip velocity coefficient for the diffuse reflection with the analytical result. The model by Sharipov [15] based on the correct result for full accommodation by the variational approach to the BGK kinetic equation was proposed as follows:

$$
\sigma_p = \frac{2 - \alpha_t^S}{\alpha_t^S} \left( \sigma_p(\alpha_t^S = 1) - 0.1211(1 - \alpha_t^S) \right) \tag{6}
$$

where $\sigma_p(\alpha_t^S=1)=1.016$. In this study, the Maxwell diffuse-specular scattering model and the model by Sharipov were employed, which gave quantitatively different results but not qualitatively.

For the measurement accuracy of the obtained tangential momentum accommodation coefficient, the uncertainties in the mass flow measurement by the constant volume method caused from the non-isothermal effect, the outlet tank volume, temperature and the linear fitting process, those in the theoretical mass flow rate from the inner diameter the length of the microtube, temperature, and pressure, and that from the fitting procedure of the measured mass flow rate by that from the theory were considered [6]. The outlet tank volume was measured from the pressure change induced by changing the volume in a syringe connected to the tank [6] for the precise measurement. Since the error arose mainly from the inner diameter of the microtube, it was estimated from the mass flow rate in the hydrodynamic regime using the regression analysis. The total measurement uncertainty was evaluated at most 2.3%. The uncertainties were evaluated on each experiment, and were reflected in the results.

3. Experimental setup

The experimental setup was explained elsewhere [6]. The schematic image of the experimental setup is shown in figure 1. Two tanks connected with a single microtube were set to slightly different pressures, which caused a pressure-driven flow through the microtube.

The microtubes of SUS304 with the nominal inner diameter of $D=0.29$mm (Ohbakiko) were employed. The inner surface of the microtube was estimated as an engineering surface. Three different tubes were applied to eliminate individual differences; S1, S2, and S3. The microtubes were evacuated for a long time in advance of measurement without any special surface treatment; i.e. “as is” condition of an engineering surface. The length of the microtubes were $L=50.0$mm, which were much greater than the inner diameter allowing to neglect the inlet and the outlet effects.

Argon, nitrogen and oxygen were employed as test gases. Pressures were measured by temperature controlled capacitance manometers (MKS Baratron 627B Series; accuracy ±0.15%). The leakage was less than 0.1% of the mass flow rate through a microtube. The pressure ratio was set from 3 to 5, and the mean Knudsen number $K_n$ was less than 0.32, where the velocity slip condition of the second-order was valid [16, 17]. The highest pressure condition in the inlet tank among all measurement was about 500Pa.
4. Results and Discussions
The inner diameters of the microtubes were estimated from the mass flow rate in the hydrodynamic regime. The obtained values are listed in Table 1. They slightly differ among gas species due to the difference in the adsorption coefficients. For evaluation of the mass flow rate, the estimated inner diameter for each microtube and each gas species was used. The difference among the gas species might be attributed to the adsorption on the inner surface; however, it is still under investigation.

Table 1. Estimated inner diameter of the microtubes in [μm].

| microtube | Ar       | N₂       | O₂       |
|-----------|----------|----------|----------|
| S1        | 298.2±0.2| 297.7±0.2| 297.6±0.2|
| S2        | 298.3±0.2| 298.1±0.2| 297.8±0.2|
| S3        | 296.6±0.2| 296.0±0.2| 296.1±0.2|

The obtained mass flow rates was normalized and plotted against the mean Knudsen number. The normalized data were well fitted by quadratic functions; equation (4). The typical example of the result for S1 microtube with argon is shown in Figure 2. The determination coefficients of the fitted curves were larger than 0.99 for all conditions. The second-order effect was confirmed to be small for all cases, which indicates the mean Knudsen number range was appropriate for the slip flow assumption.

The tangential momentum accommodation coefficients were calculated for two models, and plotted in Figure 3. The obtained values seem to show some tendency between the microtubes of S1, S2, and S3, which could be an individual difference of engineering surfaces. However, these discrepancies are small. Considering the measurement error, it is concluded that the difference is not significant and negligible. The obtained values of the tangential momentum accommodation coefficients were quite close to unity, but clearly less than unity. The accurate measurement allowed us to measure the small discrepancy of the tangential momentum accommodation coefficient from the unity. Thus, the accommodation even for tangential momentum is incomplete on the non-treated “as is” engineering surface of SUS304.
This result is consistent with the report on the thermal accommodation coefficient $\alpha_E$ [18]; $\alpha_E=0.95$ for argon and $\alpha_E=0.87$ for nitrogen. Thus, the common assumption of complete accommodation on engineering surfaces must be inappropriate. Comparing the accommodation coefficients for the tangential momentum with that for the energy, there are some discrepancies. Therefore, it is also inappropriate to employ the Maxwell diffuse-specular scattering model, where all the accommodation coefficients are assumed to be the same.

The effect of gas species on the tangential momentum accommodation coefficient was negligibly small on the engineering surface, which is different from the tendency for non-metal surfaces already shown in the literatures [13]. However, this might be because the difference in the molecular weight was too small among argon, nitrogen, and oxygen molecules. Further measurement is required to investigate on the effect.

![Figure 2](image)

**Figure 2.** Typical example of the obtained normalized mass flow rate against the mean Knudsen number (Argon, S1 microtube).

![Figure 3](image)

**Figure 3.** Obtained tangential momentum accommodation coefficients.
5. Conclusion
The tangential momentum accommodation coefficient was successfully obtained from the mass flow rate measurement through a microtube with an engineering metal surface of SUS304, which was manufactured from a bulk material, in the slip flow regime. The obtained tangential momentum accommodation coefficients for argon, nitrogen, and oxygen molecules were less than unity, which is inconsistent with the assumption of full accommodation on metal surfaces often used in the numerical simulations.

The measurement on other metal, polymer, glass and quartz surfaces with gases of much larger range in the molecular weight, like He and Xe, would be the future work.

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