General Cubic Interacting Vertex for Massless Integer Higher Spin Fields

I.L. Buchbinder\(^{(a,b)}\), A.A. Reshetnyak\(^{(a,b)}\)

\(^{(a)}\)Center of Theoretical Physics, Tomsk State Pedagogical University, Kievskaya St. 60, 634061 Tomsk, Russia
\(^{(b)}\)National Research Tomsk State University, Lenin Av. 36, 634050 Tomsk, Russia

Abstract

We consider a massless higher spin field theory within the BRST approach and construct a general off-shell cubic vertex corresponding to irreducible higher spin fields of helicities \(s_1, s_2, s_3\). Unlike the previous works on cubic vertices, which do not take into account of the trace constraints, we use the complete BRST operator, including the trace constraints that describe an irreducible representation with definite integer helicity. As a result, we generalize the cubic vertex found in [14] and calculate the new contributions to the vertex, which contain additional terms with a smaller number space-time derivatives of the fields as well as the terms without derivatives.

1 Introduction

Study of the various aspects of the higher spin fields theory is one of the topical trends in modern high-energy theoretical and mathematical physics (for a review, see, e.g. [1], [2], [3], [4], [5], [6], [7] and the references therein). The current progress in higher spin field theory is closely related to a description of interactions between these fields. It is expected that taking account of interacting higher spin fields will open up the new perspectives of going beyond the Standard Model and contribute to the formation of new approaches to unifying the all fundamental interactions, including quantum gravity.

In this letter, we focus on some new aspects involving a cubic vertex for massless integer higher spin fields. The structure of cubic vertices has been investigated using different approaches in numerous works (see, e.g., the recent papers [9], [10], [11], [12], [13], [14], [15], [16] and the references therein); however, in these papers the vertex was constructed using fields subject the algebraic constraints which are not derived from the action principle. In our opinion, almost all of the results known to date on the structure of cubic vertices are

\(^{1}\)E-mail: joseph@tspu.edu.ru
\(^{2}\)E-mail: reshet@tspu.edu.ru
contained in a concise form in the work [14], where the cubic vertex was deduced using the BRST approach, and the results are consistent with the light-cone consideration [8], [4].

In general, a central object of BRST approach, the BRST operator is constructed using operator constraints which form a first-class gauge algebra. In the case under consideration, the corresponding constraints should include a dynamical on-shell condition $l_0$ and constraints $l_1$, $l_{11}$, responsible for divergences and traces, respectively. The explicit forms of $l_0$, $l_1$, $l_{11}$ are given by (11). However, in many cases (see, e.g., [14], [17] and references therein) the BRST charge is constructed, for the simplicity of calculations, without any trace constraints, with these constraints being imposed afterwards by hands. This consideration is certainly correct; however, the actual Lagrangian description of irreducible fields is achieved only after imposing the subsidiary conditions which are not derived from the Lagrangian. As a result, we face the problem of constructing a cubic vertex on the base of BRST operator including all the constraints. It is precisely this problem that is solved in this letter. One can expect the final cubic vertex to contain, as distinct from [14], some new terms containing the trace constraints.

The BRST approach to a Lagrangian description of various free higher spin field models in Minkowski and AdS spaces has been developed in numerous works (e.g., see [18], [19], [20] and the review [4]). The development of the BRST approach for calculating the cubic vertex was initiated by paper [21]. Now, we intend to present a complete solution of this problem and describe a general structure of the vertex, while taking into account all the constraints required to formulate an irreducible representation of the Poincaré group within the BRST approach.

The paper is organized as follows. Section 2 presents the basics of a BRST Lagrangian construction for free massless higher spin field, with all the constraints $l_0$, $l_1$, $l_{11}$ taken into account. In Section 3 we deduce a system of equations for a cubic (linear) deformation in fields of the free action (free gauge transformations). A solution for the deformed cubic vertices and gauge transformation is given in Section 4. The main result of the work is that the cubic vertex and deformed gauge transformations include both the constraint $l_1$ and the constraint $l_{11}$. Conclusion gives a final summary and comments.

We use the standard definition $\eta_{\mu\nu} = \text{diag}(+,-,...,-)$ for a metric tensor with Lorentz indices $\mu, \nu = 0, 1, ..., d-1$ and the respective notation $\epsilon(F)$, $gh(F)$, $[F, G]$, $[x]$ for the values of Grassmann parity and ghost number of a homogeneous quantity $F$, as well as the supercommutator and the integer part of a real-valued $x$.

2 BRST Lagrangian formulation for free higher spin fields

In this section, we briefly present the basic results of the BRST approach to free massless higher integer spin field theory. All these results will be used in the next section to describe a general structure of cubic interacting vertex.

As known, the unitary massless irreducible representations of Poincaré $ISO(1, d-1)$ group with integer helicities $s$ can be realized using the real-valued totally symmetric tensor

---

3It is worth pointing out the interacting higher spin field models (see [17] and the references therein) containing the cubic vertices, which possess the remarkable properties in quantum domain.
fields $\phi_{\mu_1...\mu_s}(x) \equiv \phi_{\mu(s)}$ under the following conditions
\[
\begin{align*}
(\partial^\nu \partial_\nu, \partial^{\mu_1}, \eta^\mu_{\mu_1\mu_2})& \phi_{\mu(s)} = (0, 0, 0) \\
(l_0, l_1, l_{11}, g_0 - d/2)|\phi\rangle &= (0, 0, 0, s)|\phi\rangle.
\end{align*}
\] (1)

Here the basic vectors $|\phi\rangle$ and the operators $l_0, l_1, l_{11}, g_0 - d/2$ are defined in the Fock space $\mathcal{H}$ with the bosonic oscillators $a_\mu, a_\nu^+, \{[a_\mu, a_\nu^+] = -\eta_{\mu\nu}\}$ in the form
\[
|\phi\rangle = \sum_{s \geq 0} \frac{1}{s!} \phi_{\mu(s)} \prod_{i=1}^{s} a_\mu^+ |0\rangle,
\] (2)

\[
(l_0, l_1, l_{11}, g_0) = (\partial^\nu \partial_\nu, -i a^\nu \partial_\nu, \frac{1}{2} a^\mu a_\mu, -\frac{1}{2} \{a_\mu^+, a^\mu\}).
\]

Within the BRST approach, the free dynamics of the field with definite helicity $s$ is described by the gauge invariant action depending on basic field $\phi_{\mu(s)}$ and twelve auxiliary fields $\phi_{1\mu(s-1)}$, ..., of lesser than $s$ ranks. All these fields are incorporated into the vector $|\chi\rangle_s$ and described by the action
\[
S_{0|s}[\phi, \phi_1, ...] = S_{0|s}[|\chi\rangle_s] = \int d\eta_0 \langle \chi | KQ |\chi\rangle_s,
\] (3)

where $\eta_0$ be a ghost field and $K$ be an operator defining the inner product. The action \[(3)\]
is invariant under the gauge transformations
\[
\delta|\chi\rangle_s = Q|\Lambda\rangle_s,
\] (4)

where the gauge parameter vector $|\Lambda\rangle_s$ is defined up to the gauge transformations
\[
\delta|\Lambda\rangle_s = Q|\Lambda^1\rangle_s, \quad \delta|\Lambda^1\rangle_s = 0.
\] (5)

Here the vectors $|\Lambda\rangle_s, |\Lambda^1\rangle_s$ are the vectors of zero-level and first-level gauge parameters of the abelian gauge transformations \[(5)\], which reflect the fact that the theory is the first-stage reducible gauge theory. The quantity $Q$ in \[(3)\] is the BRST operator, constructed on the base of the constraints $l_0, l_1, l_1^+, l_{11}, l_{11}^+ = \frac{1}{2} a^\mu a_\mu^+$ and contains the anticommuting ghost operators $\eta_0, \eta_1^+, \eta_1, \eta_{11}^+, \eta_{11}, \mathcal{P}_0, \mathcal{P}_1, \mathcal{P}_1^+, \mathcal{P}_{11}, \mathcal{P}_{11}^+$,

\[
Q = \eta_0 l_0 + \eta_1^+ l_1 + l_1^+ \eta_1 + \eta_{11}^+ \tilde{L}_{11} + \tilde{L}_{11}^+ \eta_{11} + \eta^+ \eta_1 \mathcal{P}_0
\] (6)

\[
= \eta_0 l_0 + \Delta Q + \eta_1^+ \eta_1 \mathcal{P}_0,
\] (7)

where
\[
(\tilde{L}_{11}, \tilde{L}_{11}^+) = (L_{11} + \eta_1 \mathcal{P}_1, L_{11}^+ + \mathcal{P}_{11}^+ \eta_1^+).
\] (8)

Here
\[
L_{11} = l_{11} + (b^+ b + h)b, \quad L_{11}^+ = l_{11}^+ + b^+
\] (9)

and $(\epsilon, gh)Q = (1, 1)$. The algebra of the operators $l_0, l_1, l_1^+, L_{11}, L_{11}^+, G_0$ looks like
\[
[l_0, l_1^{(\pm)}] = 0, \quad [l_1, l_1^+] = l_0 \quad \text{and} \quad [L_{11}, L_{11}^+] = G_0, \quad [G_0, L_{11}^+] = 2L_{11}^+
\] (10)
and their independent non-vanishing cross-commutators are \([l_1, L^+_{11}] = -l^+_1, [l_1, G_0] = l_1\).

The parameter \(h = h(s) = -s - \frac{d-4}{2}\). The ghost operators satisfy the following non-vanishing anticommutation relations

\[
\{\eta_0, \mathcal{P}_0\} = i, \quad \{\eta_i, \mathcal{P}^+_i\} = \{\eta^+_i, \mathcal{P}_i\} = \{\eta_{11}, \mathcal{P}^+_{11}\} = \{\eta^+_{11}, \mathcal{P}_{11}\} = 1. \tag{11}
\]

The theory under consideration is characterized by the spin operator \(\sigma\), which is defined as follows

\[
\sigma = G_0 + \eta^+_i \mathcal{P}_i - \eta_i \mathcal{P}^+_i + 2(\eta^+_i \mathcal{P}_{11} - \eta_i \mathcal{P}^+_{11}). \tag{12}
\]

Here \(G_0 = g_0 + 2b^+b + h\) with \(b, b^+ ([b, b^+] = 1)\) be auxiliary bosonic oscillators. The operator \(\sigma\) selects the vectors with definite spin value \(s\)

\[
\sigma(\mid \chi\rangle_s, \mid \Lambda\rangle_s) = (0, 0, 0), \tag{13}
\]

where the Grassmann parities and the ghost numbers of the above vectors are \((0, 0), (1, -1), (0, -2)\) respectively.

All the operators above act in a total Hilbert space with the inner product of the vectors depending on all oscillators and ghosts

\[
\langle \chi | \psi \rangle = \int d^d x \langle 0 | \chi^*(a, b; \eta_i, \mathcal{P}_i, \eta_{11}, \mathcal{P}_{11}) \psi(a^+, b^+; \eta^+_i, \mathcal{P}^+_i, \eta^+_{11}, \mathcal{P}^+_{11}) | 0 \rangle. \tag{14}
\]

The operators \(Q, \sigma\) are supercommuting and Hermitian with respect to the scalar product \(\langle 14 \rangle\) including the operator \(K\) (see e.g., \([19], [20]\))

\[
Q^2 = \eta^+_i \eta_{11} \sigma, \quad [Q, \sigma] = 0; \tag{15}
\]

\[
Q^+ K = K Q, \quad \sigma^+ K = K \sigma, \tag{16}
\]

\[
K = \sum_{n=0}^{\infty} \frac{1}{n!} (b^+)^n \langle 0 | b^n \prod_{i=0}^{n-1} (i + h(s)) \rangle \tag{17}
\]

The BRST operator \(Q\) is nilpotent on the subspace with zero eigenvectors for the spin operator \(\sigma\) \([13]\).

The field \(\mid \chi\rangle_s\), the zero \(\mid \Lambda\rangle_s\) and the first \(\mid \Lambda^1\rangle_s\) level gauge parameters labeled by the symbol ”\(s\)” as eigenvectors of the spin condition in \([13]\) can be written in the form

\[
\mid \chi\rangle_s = \mid \phi\rangle_s + \eta^+_i \left( \mathcal{P}^+_1 | \phi_2\rangle_{s-2} + \mathcal{P}^+_{11} | \phi_{21}\rangle_{s-3} + \eta^+_i \mathcal{P}^+_i \mathcal{P}^+_{11} | \phi_{22}\rangle_{s-6} \right) \tag{18}
\]

\[
+ \eta^+_i \left( \mathcal{P}^+_1 | \phi_{31}\rangle_{s-3} + \mathcal{P}^+_{11} | \phi_{32}\rangle_{s-4} \right) + \eta_0 \left( \mathcal{P}^+_1 | \phi_4\rangle_{s-1} + \mathcal{P}^+_{11} | \phi_{11}\rangle_{s-2} \right. \\
+ \mathcal{P}^+_1 \mathcal{P}^+_{11} \left[ \eta^+_i | \phi_{12}\rangle_{s-4} + \eta^+_i | \phi_{13}\rangle_{s-5} \right],
\]

\[
\mid \Lambda\rangle_s = \mathcal{P}^+_1 \mid \xi\rangle_{s-1} + \mathcal{P}^+_{11} \mid \xi_{1}\rangle_{s-2} + \mathcal{P}^+_1 \mathcal{P}^+_{11} \left( \eta^+_i \mid \xi_{11}\rangle_{s-4} \right. \\
+ \eta^+_i \mid \xi_{12}\rangle_{s-5} \left. + \eta_0 \mathcal{P}^+_1 \mathcal{P}^+_{11} \mid \xi_{01}\rangle_{s-3} \right), \tag{19}
\]

\[
\mid \Lambda^1\rangle_s = \mathcal{P}^+_1 \mathcal{P}^+_{11} \mid \xi^1\rangle_{s-3}. \tag{20}
\]

One can prove that after imposing the appropriate gauge conditions and eliminating the auxiliary fields from equations of motion, the theory under consideration is reduced to Fronsdal form \([22]\) in terms of totally symmetric double traceless tensor field \(\phi_{\mu(s)}\) and traceless gauge parameter \(\xi_{\mu(s-1)}\).

Now we pass to the construction of the interacting theory.
3 Including the interaction: system of equations for cubic vertex

In this section we describe the general scheme of finding the cubic interaction vertices for the theory under consideration and derive the equations for these vertices.

Including the cubic interaction means the corresponding deformation of the free theory. For this aim we introduce three vectors \( \chi^{(i)}|s_i\), gauge parameters \( \Lambda^{(i)}|s_i\), \( \Lambda^{(i)(1)}|s_i\) with corresponding vacuum vectors \( |0\rangle^i\) and oscillators, where \( i = 1, 2, 3 \). Then one defines the deformed action and the deformed gauge transformations in the form

\[
S_{[1]|(s)_3}[\chi^{(1)}, \chi^{(2)}, \chi^{(3)}] = \sum_{i=1}^{3} S_{0|s_i} + g \int \prod_{e=1}^{3} \, d\eta^{(e)}_0 \left( \eta^{(e)} \langle \chi^{(e)} | K^{(e)} | V^{(3)} \rangle_{(s),3} + h.c. \right),
\]

(21)

\[
\delta_{[1]} \chi^{(i)}|s_i = Q_i^{(i)}|\Lambda^{(i)}|s_i - g \int \prod_{e=1}^{2} \, d\eta^{(i+e)}_0 \left( \eta^{(i+1)} \langle \Lambda^{(i+1)} | K^{(i+1)} | s_{i+1} \chi^{(i+2)} | K^{(i+1)} | s_{i+2} \rangle \right) + (i + 1 \leftrightarrow i + 2) |\tilde{V}^{(3)}\rangle_{(s),3},
\]

(22)

\[
\delta_{[1]} \Lambda^{(i)}|s_i = Q_i^{(i)}|\Lambda^{(i)(1)}|s_i - g \int \prod_{e=1}^{2} \, d\eta^{(i+e)}_0 \left( \eta^{(i+1)} \langle \Lambda^{(i+1)} | K^{(i+1)} | s_{i+1} \chi^{(i+2)} | K^{(i+1)} | s_{i+2} \rangle \right) + (i + 1 \leftrightarrow i + 2) |\tilde{V}^{(3)}\rangle_{(s),3},
\]

(23)

with some yet unknown vectors \( |V^{(3)}\rangle_{(s),3}, |\tilde{V}^{(3)}\rangle_{(s),3}, |\tilde{V}^{(3)}\rangle_{(s),3}\). Here \( S_{0|s_i} \) is the free action for the field \( \chi^{(i)}|s_i\), \( Q_i^{(i)}\) is the BRST charge corresponding to spin \( s_i\), \( i = 1, 2, 3 \), \( K^{(i)}\) is the operator \( K^{(i)}\) corresponding to spin \( s_i\), \( i = 1, 2, 3 \) and \( g \) is a coupling constant. Also we use the notation \( \langle s \rangle_3 \equiv (s_1, s_2, s_3) \) and convention \( \lfloor i + 3 \equiv i \rfloor \).

A concrete construction of the cubic interaction means finding the concrete vectors \( |V^{(3)}\rangle_{(s),3}, |\tilde{V}^{(3)}\rangle_{(s),3}, |\tilde{V}^{(3)}\rangle_{(s),3}\). For this aim we can use the set of fields, the constraints, ghost operators related with spins \( s_1, s_2, s_3 \) and the condition of gauge invariance of the deformed action under the deformed gauge transformations.

The integration over space-time coordinates is inherited from the inner product definition. A local dependence on space-time coordinates in the vertex \( |V^{(3)}\rangle \), \( |\tilde{V}^{(3)}\rangle \) and \( |\tilde{V}^{(3)}\rangle \) implies

\[
|V^{(3)}\rangle_{(s),3} = \prod_{j=1}^{3} \delta^{(d)}(x_1 - x_i) \langle V^{(3)} \rangle_{(s),3} \prod_{j=1}^{3} \eta^{(j)}_0 |0\rangle, \quad |0\rangle \equiv \otimes_{e=1}^{3} |0\rangle^e.
\]

(24)

The conservation law for the momenta associated with vertices \( |V^{(3)}\rangle, |\tilde{V}^{(3)}\rangle \) and \( |\tilde{V}^{(3)}\rangle \), holds true

\[
p_{\mu}^{(1)} + p_{\mu}^{(2)} + p_{\mu}^{(3)} = 0.
\]

(25)

This relation will be used in the next section for explicit finding the vertices.

Turn now to invariance of the action \( S_{[1]} \) with respect to deformed transformations \( \delta_{[1]} \chi^{(i)}|s_i\), \( i = 1, 2, 3 \). It yields to the following equations in the zeroth and first orders in \( g \) (the second order is not required for finding the cubic vertex):

\[
g^0 : \quad Q_i^{(i)}Q_i^{(i)}|\Lambda^{(i)}|s_i = \eta^{(i)}_1 + \eta^{(i)}_1 \sigma^{(i)} |\Lambda^{(i)}|s_i \equiv 0, \quad i = 1, 2, 3,
\]

(26)

\[
g^1 : \quad \int \prod_{e=1}^{3} \, d\eta^{(e)}_0 \langle \Lambda^{(j)} | K^{(j)} | s_{j+1} \chi^{(j+1)} | K^{(j+1)} | s_{j+2} \rangle |Q(V^{3}, \tilde{V}^{3}) = 0,
\]

(27)
where

\[ Q(V^3, \bar{V}^3) = \sum_{k=1}^{3} Q^{(k)}|V^{(3)}\rangle_{(s)3} + Q^{(j)}\left(|V^{(3)}\rangle_{(s)3} - |\bar{V}^{(3)}\rangle_{(s)3}\right), \quad j = 1, 2, 3. \] (28)

The conservation of the form of the gauge transformations for the fields \( |\chi^{(i)}\rangle_{s_i} \), under the gauge transformations \( \delta|\chi^{(i)}\rangle_{s_i} \), with the parameters \( |\Lambda^{(i)}\rangle_{s_i} \), leads to the similar equations

\[
g^{(i)}: \quad Q^{(i)}Q^{(i)}|\Lambda^{(i)}\rangle_{s_i} = \eta^{(i)}_{11} + \eta^{(i)}_{11} \sigma^{(i)}|\Lambda^{(i)}\rangle_{s_i} \equiv 0, \quad i = 1, 2, 3, \quad (29)
\]

\[
g^{(1)}: \quad \int \prod_{e=1}^{2} d\eta_0^{(e)} s_{j+1} (\Lambda^{(j+1)}_1 \Lambda^{(j+1)}_1) |s_{j+2} \rangle \langle \chi^{(j+2)}_1 \Lambda^{(j+2)}_1 (|Q(\Lambda^{(j+2)}_1 \Lambda^{(j+2)}_1) - Q^{(j+2)}|\bar{V}^{(3)}\rangle) = 0 \quad (30)
\]

where

\[ Q(\bar{V}^3, \bar{V}^3) = \sum_{k=1}^{3} Q^{(k)}|\bar{V}^{(3)}\rangle_{(s)3} + Q^{(j)}\left(|\bar{V}^{(3)}\rangle_{(s)3} - |\bar{V}^{(3)}\rangle_{(s)3}\right), \quad j = 1, 2, 3. \] (31)

Let us pay attention that the last term in the relation (30) can be omitted since it proportional to zero order in \( g \) of right hand side of the equation of motion \( Q^{(j+2)}|\chi^{(j+2)}\rangle \) and hence can be eliminated by field redefinition in free action.

Taking into account the above deformed gauge transformations we, should make sure that these transformations still form the closed algebra, i.e. the following relation should be fulfilled

\[ [\delta^{\Lambda_1}_{[1]}, \delta^{\Lambda_2}_{[1]}]|\chi^{(i)}\rangle = -g \delta^{\Lambda_3}_{[1]}|\chi^{(i)}\rangle, \] (32)

where the Grassmann-odd gauge parameter \( \Lambda_3 \) should be a function of the parameters \( \Lambda_1, \Lambda_2, \Lambda_3 = \Lambda_3(\Lambda_1, \Lambda_2) \). The validity of the relation (32) is verified by explicit calculations

\[
[\delta^{\Lambda_1}_{[1]}, \delta^{\Lambda_2}_{[1]}]|\chi^{(i)}\rangle = -g \left\{ \left( \int \prod_{e=1}^{2} d\eta_0^{(i+e)} (\langle \Lambda^{(i+1)}_2 | K (|\Lambda^{(i+2)}_1 | K Q^{(i+2)}
\right.
\]

\[
+ (i + 1 \leftrightarrow i + 2) - (\Lambda_1 \leftrightarrow \Lambda_2) \bigg) \bigg| \bar{V}^{(3)} \bigg) \bigg) \right\} \bigg| \bar{V}^{(3)} \bigg) \bigg)
\]

\[
+ g^2 \left\{ \int \prod_{e=2}^{3} d\eta_0^{(i+e)} \left[ \int \prod_{f=1}^{2} d\eta_0^{(i+2+f)} (\langle \Lambda^{(i+1)}_2 | K (|\Lambda^{(i+1)}_1 | \chi^{(i)}_0 \rangle \langle \bar{V}^{(3)} | \Lambda^{(i)} \rangle K^3
\right.
\]

\[
+ (i \leftrightarrow i + 1) + (i + 1 \leftrightarrow i + 2) \bigg) - (\Lambda_1 \leftrightarrow \Lambda_2) \bigg) \bigg| \bar{V}^{(3)} \bigg) \bigg) \right\} \bigg| \bar{V}^{(3)} \bigg) \bigg)
\]

for \( K^3 \equiv \otimes_{i=1}^{3} K^{(i)} \). The relation (33) allows to find the parameter \( \Lambda_3 \) in cubic approximation in the form

\[ |\Lambda_3 \rangle \sim \int \prod_{e=1}^{2} d\eta_0^{(i+e)} (\langle \Lambda^{(i+1)}_2 | K (|\Lambda^{(i+2)}_1 | K (i + 1 \leftrightarrow i + 2) \bigg) - (\Lambda_1 \leftrightarrow \Lambda_2) \bigg) \bigg| \bar{V}^{(3)} \bigg) \bigg). \] (34)

The condition of closure of the algebra of deformed gauge transformations provides the fact that a set of the generators of deformed gauge transformations, encoded in (22) will remain
complete, without appearance of new generators. In addition, the commutator \((32)\) is the condition which restricts the form of the vertices \(\hat{V}^{(3)}\).

Also, the vertices should satisfy the spin conditions as the consequence of the spin equation \((13)\) for each sample \(|\chi^{(i)}\rangle_{s_i}, |\Lambda^{(i)}\rangle_{s_i}, |\Lambda^{(i)1}\rangle_{s_i};\)

\[
\sigma^{(i)}\left(\hat{V}^{(3)}\rangle_{s_3}, \hat{V}^{(3)}\rangle_{s_3}, \hat{V}^{(3)}\rangle_{s_3}\right) = 0, \quad (35)
\]

providing the nilpotency of total BRST operator \(Q^{tot} \equiv \sum_i Q^{(i)}\) when evaluated on the vertices due to the equations \((26)\) and \(\{Q^{(i)}, Q^{(j)}\} = 0\) for \(i \neq j\). Indeed, from the definition of the vertices \(\hat{V}^{(3)}\rangle_{s_3}\) \((21)\) it follows from the completeness of the inner product that the spin numbers \(s_i\) for the oscillators from each vector \(s_i|\chi^{(i)}\rangle\) are equal to the spin numbers \(s_1, s_2, s_3\) of the oscillators in the each monomial from 3-vector \(\hat{V}^{(3)}\rangle_{s_3}\).

The equations \((35)\) and \((28)\), \((31)\):

\[
Q(V^3, \hat{V}^3) = 0, \quad Q(\hat{V}^3, \hat{V}^3) = 0, \quad (36)
\]

together with the form of the commutator of the gauge transformations \((33)\) determine the cubic interacting vertices for irreducible massless totally symmetric higher spin fields.

4 General solution for cubic vertices

In this section we will construct the general solution for the cubic vertex.

Further, for simplicity, we assume that \(\hat{V}^{(3)}\rangle_{s_3} = |V^{(3)}\rangle_{s_3} = \hat{V}^{(3)}\rangle_{s_3}. \) Then, the equations \((27)\), \((30)\) and the operators \((28)\), \((31)\) take the form

\[
\begin{align*}
\{ & s_2 \langle \Lambda^{(1)} K^{(1)} \rangle_{s_2} \langle \chi^{(2)} K^{(2)} \rangle_{s_3} \langle \chi^{(3)} K^{(3)} \rangle_{s_3} |Q(V^3, V^3) = 0, \langle V^3, V^3) = 0 \Rightarrow Q^{tot}|V^{(3)}\rangle_{s_3} = 0. \quad (37)
\end{align*}
\]

Remind that \(Q^{tot} \equiv \sum_i Q^{(i)}\).

We look for a general solution of the equation \((37)\) in the form of products of specific operators, homogenous in oscillators. First, they include the different \(Q^{tot}\) - BRST closed forms \(L^{(i)}_{k_i}, i = 1, 2, 3, k_i = 1, \ldots, s_i\) and \(Z\) constructed from ones \(L^{(i)}\), \(Z\) known from \([14]\), where \(L^{(i)}\) are the linear in oscillators and \(Z\) is cubic in oscillators,

\[
\begin{align*}
L^{(i)}_{k_i} &= (L^{(i)})^{k_i-2} \left( (L^{(i)})^2 - \frac{i_{k_i}^!}{2(k_i-2)!} \eta^{(i)+}_1 [2P_0^{(i+1)} + 2P_0^{(i+2)} - P_0^{(i)}] \right), \\
L^{(i)} &= (P^{(i)+}_1 - P^{(i)+}_2) a^{(i)+} - i (P^{(i)+}_0 - P^{(i)+}_2) \eta^{(i)+}_1, \\
Z &= L^{(12)}_{11} + L^{(3)} + L^{(23)}_{11} + L^{(1)} + L^{(13)} + L^{(2)}.
\end{align*}
\]

Here we have used the relations \((12)\) - \((14)\) below, \(p^{(i)}_{0} = -i \partial^{(i)} \) and

\[
L^{(i)+}_{11} = a^{(i)+} a^{(i)+} - \frac{1}{2} P^{(i)+}_1 + \eta^{(i)+}_1 - \frac{1}{2} P^{(i)+}_2 + \eta^{(i)+}_2.
\]

Second, these operators involve the new two-, four-, ..., \([s_i/2]\) forms in powers of oscillators, corresponding to the trace operators

\[
U^{(s_i)}_{j_i} (\eta^{(i)+}_1, P^{(i)+}_1) := (\hat{L}^{(i)+})^{(j_i-2)} \left( (\hat{L}^{(i)+})^2 - j_i (j_i - 1) \eta^{(i)+}_1 P^{(i)+}_1 \right), \quad i = 1, 2, 3. \quad (41)
\]

\(^4\)In principle, the equations \((27)\), \((30)\) can be solved without these simplifying assumption however, it complicates the consideration.
First of all we have to check that the operators \( (38) \) and \( (40) \) are closed relatively operator \( Q^{\text{tot}} \) which is extension of the BRST operator in works \([3] , [4]\) by the operators responsible for the traces. Indeed, the \( Q^{\text{tot}} \) BRST closeness for the operator \( L^{(i)} \) is reduced to the fulfillment of the equations at the terms linear in \( \eta_{11}^{(i)+} \)

\[
\tilde{L}^{(i)}_{11} L^{(i)} |0\rangle = \left( - \frac{1}{2} \frac{1}{2} \eta_{11}^{(i)+} + \frac{1}{2} \right) \langle \hat{L}^{(i)}_{11} |0\rangle = 0, \quad (42)
\]

\[
\tilde{L}^{(i)}_{11} (L^{(i)})^2 |0\rangle = \left( - \frac{1}{2} \frac{1}{2} \eta_{11}^{(i)+} + \frac{1}{2} \right) \langle \hat{L}^{(i)}_{11} |0\rangle = 0.
\]

The last relations and ones for \((L^{(i)})^k\) do not vanish under the sign of inner products and justify the introduction of the forms \( (38) \). By the same reason, the any power of the form \( Z \) is not BRST-closed as well.

Let us prove now BRST-closeness of the new forms \((11)\). To see that it is sufficient to present the vertex as \( |V^{(3)}\rangle_{(s)3} = U_{j_1}^{(s)} |X^{(3)}\rangle_{(s)3-2j_1} \) with some \( |X^{(3)}\rangle_{(s)3-2j_1} \) where \( \langle \varepsilon, gh \rangle |X^{(3)}\rangle = (1, 3) \) and check the closeness for \( i \neq k \) and for \( j = 1, 2 \)

\[
\eta^{(i)+}_{11} \hat{L}^{(i)}_{11} U_{j_k}^{(s)} |X^{(3)}\rangle_{(s)3-2j_1} = \left( \sum_{\nu=0}^{\nu} \left( \hat{L}^{(i)}_{11} \right)^{\nu} \right) \langle \sigma^{(i)} \eta^{(i)+}_{11} + 2 \eta^{(i)+}_{11} \nu^{(i)} \rangle |X^{(3)}\rangle_{(s)3-2j_1} = 0, \quad (46)
\]

\[
\eta^{(i)+}_{11} \hat{L}^{(i)}_{11} U_{j_k}^{(s)} |X^{(3)}\rangle_{(s)3-4i} = \left( \{ \sigma^{(i)} \eta^{(i)+}_{11}, \tilde{L}^{(i)}_{11} \} - 2 \eta^{(i)+}_{11} \tilde{L}^{(i)+}_{11} \right) \langle \sigma^{(i)} \eta^{(i)+}_{11} + 2 \eta^{(i)+}_{11} \nu^{(i)} \rangle |X^{(3)}\rangle_{(s)3-4i} = 0.
\]

For arbitrary \( j > 2 \) we have

\[
\eta^{(i)+}_{11} \hat{L}^{(i)}_{11} U_{j_k}^{(s)} |X^{(3)}\rangle_{(s)3-2j_1} = \left( \sum_{\nu=0}^{\nu} \left( \hat{L}^{(i)}_{11} \right)^{\nu} \right) \langle \sigma^{(i)} \eta^{(i)+}_{11} + 2 \eta^{(i)+}_{11} \nu^{(i)} \rangle |X^{(3)}\rangle_{(s)3-2j_1} = 0.
\]

In the relations \((45)-(48)\) we have used the supercommutators which linear in \( \hat{L}^{(i)+}_{11} \)

\[
\eta^{(i)+}_{11} \left[ \hat{L}^{(i)}_{11}, \tilde{L}^{(i)+}_{11} \right] = \eta^{(i)+}_{11} \left\{ (\sigma^{(i)} + 2) - 2 \nu^{(i)} \eta^{(i)} \right\} = \sigma^{(i)} \eta^{(i)+}_{11} - 2 \eta^{(i)+}_{11} \nu^{(i)} \eta^{(i)}, \quad (49)
\]

then, polynomial in \( \hat{L}^{(i)+}_{11} \)

\[
\eta^{(i)+}_{11} \left[ \hat{L}^{(i)}_{11}, (\hat{L}^{(i)+}_{11})^{j_i} \right] = \sum_{\nu=0}^{\nu} \left( \hat{L}^{(i)+}_{11} \right)^{\nu} \eta^{(i)+}_{11} \left[ \hat{L}^{(i)}_{11}, \hat{L}^{(i)+}_{11} \right] (\hat{L}^{(i)+}_{11})^{j_i-\nu-1},
\]

\[
\sum_{\nu=0}^{\nu} \left( \hat{L}^{(i)+}_{11} \right)^{\nu} \sigma^{(i)} \eta^{(i)+}_{11} (\hat{L}^{(i)+}_{11})^{j_i-\nu-1} - 2 j_i (\hat{L}^{(i)+}_{11})^{j_i-1} \eta^{(i)+}_{11} \nu^{(i)} \eta^{(i)}.
\]

\[
= (j_i - 1) \sigma^{(i)} \eta^{(i)+}_{11} (\hat{L}^{(i)+}_{11})^{j_i-1} - 2 j_i (\hat{L}^{(i)+}_{11})^{j_i-1} \eta^{(i)+}_{11} \nu^{(i)} \eta^{(i)}.
\]
Besides we took into account an independence of $|X^{(3)}\rangle_{(s)3-2j_i}$ on the momenta $P_{11}^{(i)+}$ and the fact that

$$\sigma^{(i)}(\eta_{11}^{(i)+}(\hat{L}_{11}^{(i)+})^{j_i-1}|X^{(3)}\rangle_{(s)3-2j_i} = 0.$$  \hspace{1cm} (52)

As the result, the general solution for the equation (37) for cubic vertex has the form

$$|V^{(3)}\rangle_{(s)3} = |V^{M(3)}\rangle_{(s)3} + \sum_{(j_1,j_2,j_3)>0} U_{j_1}^{(s_1)} U_{j_2}^{(s_2)} U_{j_3}^{(s_3)} |V^{M(3)}\rangle_{(s)3-2(j)3},$$

where the vertex $|V^{M(3)}\rangle_{(s)3-2(j)3}$ was defined in Metsaev’s paper [14] with account for (24) but with modified forms $L_{k_i}$, (38) and $Z_j$ instead of $Z^j$ (39)

$$V^{M(3)}_{(s)3-2(j)3} = \sum_k Z_{1/2\{s-2J-k\}}^3 \prod_{i=1}^k L_{s_i-2j_i-1/2(s-2J-k)},$$

$$(s, J) = (\sum_i s_i, \sum_i j_i),$$

and is numerated by the natural parameter $k$ subject to the equations

$$s - 2J - 2s_{\min} \leq k \leq s - 2J, \quad k = s - 2J - 2p, \quad p \in \mathbb{N}_0.$$  \hspace{1cm} (56)

Here the quantity $Z_j$ is determined, e.g. for $j = 1$ as follows

$$Z \prod_{j=1}^3 L_{k_j}^{(j)} = Z \prod_{j=1}^3 L_{k_j}^{(j)} - \sum_{i=1}^3 \frac{b^{(i)+}}{h^{(i)}} \left[[\hat{L}^{(i)}_{11}, Z], L^{(i)}\right] \prod_{j=1}^3 L_{k_j-\delta_{ij}},

+ \sum_{i\neq e} k_i k_e \frac{b^{(i)+} + b^{(e)+}}{h^{(i)} h^{(e)}} \left[[\hat{L}^{(i)}_{11}, Z], L^{(i)}\right] \prod_{j=1}^3 L_{k_j-\delta_{ij}}

- \sum_{i\neq e \neq o} k_i k_e k_o \frac{b^{(i)+} + b^{(e)+} + b^{(o)+}}{h^{(i)} h^{(e)} h^{(o)}} \left[[\hat{L}^{(i)}_{11}, \hat{L}^{(e)}_{11}, Z], L^{(i)}\right] \prod_{j=1}^3 L_{k_j-\delta_{ij}}.$$  \hspace{1cm} (54)

For $j > 1$ the expressions for $Z_j$ may be derived in the similar way.

The general vertex (52) contains besides the known modified term (51) the new terms. These new vertices have the following structure. First, they contain the linear terms in powers of trace operators $U_{j_i}^{(s_i)} = \hat{L}_{11}^{(i)+}$ at least for one higher spin field copy

$$\sum_{(j_1,j_2,j_3)>0} (\hat{L}_{11}^{(1)+})^{j_1} (\hat{L}_{11}^{(2)+})^{j_2} (\hat{L}_{11}^{(3)+})^{j_3} |V^{M(3)}\rangle_{(s)3-2(j)3},$$

Note, that in comparison with cubic vertex constructed in [14], the general cubic vertex contains the degree 2 homogeneous polynomials in the oscillators $\hat{L}_{11}^{(1)+}, \hat{L}_{11}^{(2)+}, \hat{L}_{11}^{(3)+}$, which depend in addition on $b^{(1)+}, b^{(2)+}, b^{(3)+},$ oscillators respectively. Second, the quantity $U_{j_i}^{(s_i)}$ (11) for $j_i \geq 2$ depends on the product $\eta_{11}^{(i)+} P_{11}^{(i)+}$ with spin value 4.

Therefore, for even $s_i, i = 1, 2, 3$ the vertex for $j_i = [s_i/2]$ will contain the $s_i/2$ traces for the initial fields $|\phi\rangle_{s_i}, \prod_i T_{r_{s_i/2}}|\phi\rangle^{(i)}$, without any derivatives

$$\nabla^{(3)}_{(s)3} = \prod_{i=1}^3 U_{[s_i/2]}^{(s_i)} = \prod_{i=1}^3 (\hat{L}_{11}^{(i)+})^{(j_i-2)} \{(\hat{L}_{11}^{(i)+})^2 - j_i(j_i - 1)\eta_{11}^{(i)+} P_{11}^{(i)+}\}.$$  \hspace{1cm} (58)
In case of one, two or all odd values of the helicities \( s_1, s_2, s_3 \), the vertices with minimal number of derivatives will contain respectively one, two or three derivatives:

\[
V_{1(s)_3}^{(3)} = U_{[s_1-1/2]}^{(s_1)} U_{[s_2/2]}^{(s_2)} U_{[s_3/2]}^{(s_3)} L_1^{(1)},
\]

\[
V_{2(s)_3}^{(3)} = U_{[s_1-1/2]}^{(s_1)} U_{[s_2-1/2]}^{(s_2)} U_{[s_3/2]}^{(s_3)} L_1^{(1)} L_2^{(2)},
\]

\[
V_{3(s)_3}^{(3)} = \prod_{i=1}^{3} U_{[s_i-1/2]}^{(s_i)} \left\{ \prod_{i=1}^{3} L^{(i)} + Z \right\}.
\]

For \( d > 4 \) the number of independent (parity invariant) vertices in each \( |V^{M(3)}_{(s)_3-2(j)_3}\) enumerated by \( k \) is equal to \( (s_{\text{min}} + 1) \), whereas for \( d = 4 \) it reduced to 2, i.e. \( k = s - 2J, s - 2J - 2s_{\text{min}} \) due to proportionality of the vertices for \( k: s - 2J - 2s_{\text{min}} < k < s - 2J \), to the terms with d’Alambert operator \( \nabla^2 \) which can be removed by the field redefinitions (see [8] for the explanation). For equal helicities, \( s_1 = s_2 = s_3 \) there is the cubic vertex \( (53) \) for self-interacting higher spin fields \( \phi_{\mu(s)} \).

Emphasize, that the including the constraints \( L_{11} \) responsible for the traces into BRST operator means that the standard condition of vanishing double traces of the fields is fulfilled only on-shell as the consequence of free equations of motion. Off-shell the (double) traces of the fields do not vanish. As the result, the trace constraints come into the cubic vertices. To pass from our completely irreducible formulation to the formulation, where the trace constraints are not included into BRST charge, we should use the free equations of motion.

5 Conclusion

To summarize, we have constructed generic cubic vertices for an irreducible gauge-invariant Lagrangian formulation of massless totally-symmetric higher spin fields with arbitrary helicities \( s_1, s_2, s_3 \) in \( d \)-dimensional Minkowski space-time. The construction is realized within the BRST approach to higher spin field theories, and, unlike all the previous works, every constraint that determines an irreducible massless higher spin representation has been taken into account on equal footing in a complete BRST operator.

To find cubic vertices being consistent with a deformed gauge invariance, we have followed an additive deformation of classical actions for three copies of the higher spin fields and the gauge transformations for the fields and gauge parameters, while demanding for the deformed action to be invariant in a linear approximation with respect to the coupling constant \( g \), and for the gauge algebra to be closed on a deformed mass shell up to the second order in \( g \). These requirements lead to a system of generating equations for the cubic vertices, containing the total BRST invariance operator condition \( Q(V^3, \bar{V}^3) = 0 \), \( Q(\tilde{V}^3, \tilde{\bar{V}}^3) = 0 \) \([28]\), \([31]\), the spin condition \( (33) \), and the condition \( (32) \) for the gauge algebra closure. The cubic vertex, in the particular case of coinciding vertices, \( V^3 = \tilde{V}^3 = \bar{V}^3 \), satisfies the equations \( (35) \), \( (37) \), and their solution is found using a set of BRST-closed forms, which include the modified forms \( (38) \), \( (39) \), constructed from ones in \([14]\), and the new forms \( (11) \) related to the trace operator constraints, having dependence on additional oscillators, \( b_{\mu}^{(i)+}, \eta_{11}^{(i)+}, \mathcal{P}_{11}^{(i)+} \). As a result, admissible cubic vertices for irreducible fields may involve terms with less space-time derivatives in comparison with \([14]\).

A general solution to the equation \( (37) \) for a cubic vertex is presented by \( (53) \) and contains, in addition to the familiar vertices (containing derivatives) given in \([14]\), some new
terms containing traces in the BRST-closed operators $U_{j_i}(s_i)$ of the rank $j_i = 1, 2, \ldots, [s_i/2]$ without derivatives. For even helicities $s_i$, $i = 1, 2, 3, \ldots$, the vertex for $j_i = [s_i/2]$ in (53) thereby contains $s_i/2$ traces for the initial fields $|\phi\rangle_{s_i}$ without any derivatives. In the case of one, two, or all odd values of helicities, $s_1, s_2, s_3, \ldots$, the vertices with a minimal number of derivatives will contain one (59), two (60) or three (61) derivatives, respectively. The results of [14] can be obtained from our results if one switches off all the terms in the cubic vertex (53) that contain the operators $\hat{L}_{i1}^{(i)+}$ and $\eta_{i1}^{(i)+} F_{(i)+}$, responsible for the trace constraints.

There are numerous directions for application and development of the suggested approach, such as finding cubic vertices for irreducible massless half-integer higher spin fields on flat backgrounds; for massive integer and half integer higher spin fields, for higher spin fields with mixed symmetry of indices, for higher spin supersymmetric fields, where the vertices should include any powers of traces. The construction under consideration can be generalized to find cubic vertices for irreducible higher spin fields on anti-de-Sitter backgrounds, having in mind the impossibility of making a flat limit for many of the cubic vertices in anti-de-Sitter spaces [23], [24]. One should also note the problems of constructing the fourth and higher vertices within the BRST approach. The interesting results in this direction were obtained in [24]. We plan to address all of the mentioned problems in our forthcoming works.

Acknowledgements  The authors are grateful to R.R. Metsaev and V.A. Krykhtin for useful comments. The work was partially supported by the Ministry of Education of Russian Federation, project No FEWF-2020-003.

References

[1] M.A. Vasiliev, Higher Spin Gauge Theories in Various Dimensions, Fortsch. Phys. 52 (2004) 702, [arXiv:hep-th/0401177].

[2] M.A. Vasiliev, Higher spin gauge theories in any dimension, Comptes Rendus Physique, 5 (2004) 1101, [arXiv:hep-th/0409260].

[3] X. Bekaert, S. Cnockaert, C. Iazeolla, M. A. Vasiliev, Nonlinear higher spin theories in various dimensions, in Higher spin gauge theories: Proceedings, 1st Solvay Workshop : Brussels, Belgium, 12-14 May, 2004, 132-197 [arXiv:hep-th/0503128].

[4] A. Fotopoulos, M. Tsulaia, Gauge Invariant Lagrangians for Free and Interacting Higher Spin Fields. A Review of the BRST formulation, Int. J. Mod. Phys. A 24 (2008) 1, [arXiv:0805.1346 [hep-th]].

[5] X. Bekaert, N. Boulanger, P. Sundell, How higher spin gravity surpasses the spin two barrier: no-go theorems versus yes-go examples, Rev. Mod. Phys. 84 (2012) 987, [arXiv:1007.0435 [hep-th]].

[6] V.E. Didenko, E.D. Skvortsov, Elements of Vasiliev theory, [arXiv:1401.2975 [hep-th]].

[7] M.A. Vasiliev, higher spin Theory and Space-Time Metamorphoses, Lect. Notes Phys. 892 (2015) 227, [arXiv:1404.1948 [hep-th]].
8] R.R. Metsaev, Cubic interaction vertices for massive and massless higher spin fields, Nucl. Phys. B 759 (2006) 147, [arXiv:hep-th/0512342].

9] R. Manvelyan, K. Mkrtchyan, W. Ruhl, General trilinear interaction for arbitrary even higher spin gauge fields, Nucl. Phys. B 836 (2010) 204, [arXiv:1003.2877 [hep-th]].

10] R. Manvelyan, K. Mkrtchyan, W. Ruhl, A generating function for the cubic interactions of higher spin fields, Phys. Lett. B 696 (2011) 410, [arXiv:1009.1054 [hep-th]].

11] E. Joung, M. Taronna, Cubic interactions of massless higher spins in (A)dS: metric-like approach, Nucl.Phys.B 861 (2012) 145, [arXiv:1110.5918[hep-th]].

12] M. Vasiliev, Cubic Vertices for Symmetric higher spin Gauge Fields in (A)dS_d, Nucl. Phys. b 862 (2012) 341, [arXiv:1108.5921[hep-th]].

13] R.R. Metsaev, Cubic interaction vertices for fermionic and bosonic arbitrary spin fields, Nucl. Phys. B 859 (2012) 13 [arXiv:0712.3526 [hep-th]].

14] R.R. Metsaev, BRST-BV approach to cubic interaction vertices for massive and massless higher spin fields, Phys. Lett. B 720 (2013) 237, [arXiv:1205.3131 [hep-th]].

15] M.V. Khabarov, Yu.M. Zinoviev. Cubic interaction vertices for massless higher spin supermultiplets in d=4, JHEP 02 (2021) 167, [arXiv:2012.00482[hep-th]].

16] I.L. Buchbinder, V.A. Krykhtin, M. Tsulaia, D. Weissman, Cubic Vertices for \(\mathcal{N} = 1\) Supersymmetric Massless Higher Spin Fields in Various Dimensions, Nucl.Phys. B 967 (2021) 115247, [arXiv:2103.08231 [hep-th]].

17] E.D. Skvortsov, T.Tran, M. Tsulaia, Quantum Chiral Higher Spin Gravity, Phys. Rev. Lett. 121 (2018) 031601, [arXiv:arXiv:1805.00048 [hep-th]].

18] A. Pashnev, M. Tsulaia, Description of the higher massless irreducible integer spins in the BRST approach, Mod. Phys. Lett. A 13 (1998) 1853, [arXiv:hep-th/9803207].

19] I.L. Buchbinder, A. Pashnev, M. Tsulaia, Lagrangian formulation of the massless higher integer spin fields in the AdS background, Phys. Lett. B 523 (2001) 338, [arXiv:hep-th/0109067].

20] I.L Buchbinder, A.A. Reshetnyak, General Lagrangian Formulation for Higher Spin Fields with Arbitrary Index Symmetry. I. Bosonic fields, Nucl. Phys. B 862 (2012) 270, [arXiv:1110.5044[hep-th]].

21] I.L Buchbinder, A. Fotopoulos, A.C. Petkou, M. Tsulaia, Constructing the cubic interaction vertex of higher spin gauge fields, Phys. Rev. D 74 (2006) 105018, [arXiv:hep-th/0609082].

22] C. Fronsdal, Massless Fields with Integer Spin, Phys. Rev. D 18 (1978) 3624.

23] E.S. Fradkin, M.A. Vasiliev, On the Gravitational Interaction of Massless Higher Spin Fields, Phys. Lett. B 189, (1987) 89.

24] E.S. Fradkin, M.A. Vasiliev, Cubic Interaction in Extended Theories of Massless Higher Spin Fields, Nucl. Phys. B 291 (1987) 141.
[25] P. Dempster, M. Tsulaia, On the Structure of Quartic Vertices for Massless Higher Spin Fields on Minkowski Background, Phys. Rev. D 86 (2012) 025007, [arXiv:1203.5597[hep-th]].