On the description of the turbulent flame acceleration with Kolmogorov law

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Abstract. A series of experiments on the flame propagation in a hydrogen–air mixtures in a cylindrical envelope of 4.5 m³ volume were carried out. Flame front propagation was recorded using ionization probes and video in the visible and infrared ranges. The flame propagation data interpretation using the Kolmogorov law has been applied. For the first time variation of turbulent energy dissipation rate per weight with combustion propagation was used. This approach allows the experimental data for mixtures with different compositions in non-spherical volumes to be described.

1. Introduction
The velocity of flame propagation is important factor for risk assessment. The character of the flame propagation is influenced by many factors: the way of combustion initiation, the concentration of the fuel mixture, the shape of the volume of the fuel mixture, the presence of obstacles in this volume, the features of these barriers, position of the initiation point in the volume of flammable gas. For adequate prediction of the nature of combustion in such conditions rather simple theoretical and empirical models are required to evaluate the burning characteristics in large, complex volumes. Models for quantitative assessment of the effects of gas explosions are different in a level of complexity [1].

Many studies have been performed on development of the combustion of hydrogen-air mixtures in large volumes [2–10]. For example, [7,9–11] show dependences of the position of the combustion front on time for various hydrogen-air mixtures in envelopes of volume from 5.27 to 300 m³. In [8] a preliminary analysis of the dependence of the position of the combustion front from time to time during the combustion of hydrogen–air mixtures with a hydrogen content of 22 to 54% (here and after percentage by volume) in the cubic volume of 1 m³ shell was conducted. Dependencies were approximated by power functions. The exponent is constantly changing from 1.04 and 1.16 at the initial stage of combustion propagation to 1.39 and 1.4, respectively, in the final phase for the selected mixtures. The small size of the experimental stand is not allowed the authors to obtain accelerating self-similar combustion mode and the non-self-similar acceleration of the combustion due to the development of turbulence combustion front and change of fractal dimension. In [6], the results of a large number of experiments with mixtures of a hydrogen content of from 28 to 44% in volume of 4 to 2094 m³ are presented. At some distance from the point of initiation of all the trajectories of the combustion front with good accuracy approaching a power function with an exponent of 1.5.
In [3] similar dependences were observed in methane–air and propane–air quiescent mixtures in volume of 550 m³. After a short section at a constant speed of propagation of combustion acceleration observed by a power law with an exponent of 1.5.

However, the values of exponent are different in other reports [12–19], and the exponent of 1.5 is applicable under strict conditions. In [8,12,15–19] acceleration of combustion is explained by the fractal structure of a flame. Fractal dimension equal to 7/3 (fractal excess—1/3) [19] is suitable for self-similar cases with exponent of 1.5. Other results show fractal excess values ranged from 0 to 0.48 and varying with combustion propagation.

In [2], based on a large number of experimental data on stoichiometric combustion of hydrogen–air mixtures in a spherical volume of 2 to 2000 m³ a model of flame acceleration based on the Kolmogorov law [20] with a constant turbulent energy dissipation rate per weight unit is suggested. The initial stage of combustion is not considered, and under steady self-similar turbulent spherical combustion theory and experiment show the propagation of the combustion front as a power law with an exponent of 1.5. The problems of the combustion of non-spherical volume, in a mixture with a composition different from stoichiometric remain open. Also, the statement of constant turbulent energy dissipation rate per weight unit is not directly following from the theory of Kolmogorov that be tested experimentally.

In contrast to the above investigations, which deal with the flame propagation in spherical and hemispherical volumes this article describes an experimental study of the propagation of lean and stoichiometric hydrogen–air mixtures combustion in a cylindrical volume of 4.5 m³ with the analysis of the results based on the Kolmogorov law. Improvement of the approach used in [2] with variation of turbulent energy dissipation rate per weight with combustion propagation is suggested.

2. Experimental details
Experimental stand is a metal frame that confines the cylindrical envelope with diameter of 1.5 m and 2.4 m in height of high-density polyethylene (HDPE) 100 µm thick (figure 1). Top of cylinder was closed with thin rubber shell. Overpressure in the cylindrical envelope did not exceed 2 Pa. The envelope was filled with a hydrogen–air mixture of a predetermined composition at normal atmospheric pressure and temperature produced T ≈ 293 K. The greatest error of the mixture composition occurred for mixtures with hydrogen content of 12% and amounted to 3% of the hydrogen content, so the error of the mixture composition was 0.36%, so mixture of 12% hydrogen means 12±0.36% hydrogen content in mixture. Ignition was performed with explosion of thin nichrome wire 0.1 mm in diameter 10 mm in length, placed between two steel electrodes with a diameter of 4 mm at the bottom of the cylinder. The energy released on the wire was 5 J. The bottom of the structure is a solid aluminum sheet on a frame of steel strip. To measure the burning propagation speed the line of the ionization sensor (3) was used. For a video recording of the combustion propagation used camcorder VIDEOSPRINT/G-2 and Optronis CR600x2. For a video recording of the combustion propagation in IR specter used camcorder FLIR x6530sc. In all experiments, the distance from ignition point to the surface remained unchanged and amounted to 100 mm. Combustion front position was measured with accuracy of 6 mm (diameter of ionization probe electrode) which is 1.5% of minimal measured distance. Time was measured by oscilloscope with the accuracy of 100 µs (time of signal rise) which is 0.3% of time measured.

3. Experimental results
Experiments were carried out in hydrogen-air mixtures with a hydrogen content of from 12 to 30%. HDPE envelope torn after the flames propagation throughout the entire volume (figure 2) at hydrogen concentrations 20% and higher.
Figure 1. Experimental setup: 1—HDPE envelope; 2—ignition wire; 3—ionization gauges.

The sequence of frames shown in figure 2 shows that at the propagation of the burning in mixtures with hydrogen content of 20% and less the envelope does not rupture during the combustion propagation and, after a hemispherical stage, the propagation can be regarded as quasi-parallel. Quasi-parallel propagation of combustion front does not mean a planar flame front. This means that the front propagation is parallel to cylindrical envelope axis. In this case, the front can be markedly curved. Figure 3 is a graph showing the dependence of the burning front position on time for the various mixtures. In experiments with a hydrogen content of 12, 20 and 30% of the burning front position ionization sensors recorded, the data presented in figure 3 are an average of 2–3 or more experiments. For a mixture with hydrogen content of 15% position of the burning front was recorded by the video camera. Figure 3 also shows the original results and results of [6, 9, 11] for the stoichiometric hydrogen-air mixture that are in good correspondence.

Experimental points shown in figure 3 may be described by power-law dependences of the form $X = At^n$. As seen from the equation, the slope of the straight line corresponds to the degree of the argument, and the intersection point of the axis $t = 1$ corresponds to the logarithm of the factor $A$. All dependences presented in figure 3 show the change of inclination at the distance $X$ from 0.6 (12% of hydrogen) to 1.1 m (30% of hydrogen). Conditionally divide the acceleration of
Figure 2. Successive frames of combustion propagation and envelope shape during combustion of hydrogen-air mixture with a hydrogen content of 20% at time points 7, 27, 68 and 72 ms. Visible specter camera (top) and IR camera (bottom).

Figure 3. The burning front position as a function of time for different mixtures in comparison with other results: 1—[6] (three different experiments); 2—[9]; 3—[11].

...the burning front into two stages: a hemispherical burning front (3D) and quasi-parallel burning front (1D). These two stages are separated by a transitional stage.
Figure 4. Dependences of the flame front position on time at the hemispherical (a) and quasi-parallel stage (b).

3.1. Hemispherical flame front
Figure 4(a) shows the dependences of the burning front position on time for various mixtures at the initial stage of propagation. It is evident that throughout the range of the studied mixtures burning propagates with a small acceleration, and in all cases the exponent is close to 1. The factor increases monotonically with increasing hydrogen concentration in the investigated range of mixtures.

3.2. Quasi-parallel flame front
Figure 4(b) shows the dependences of the burning front position on time for the various mixtures at the quasi-parallel stage of propagation. It is seen that throughout the range of mixtures studied burning sharply accelerated, and in all cases the exponent is close to 3. Here, as in the case of hemispherical combustion propagation, the factor increases monotonically with increasing hydrogen concentration in the range of mixtures studied.

4. Discussion
In all the experiments, flame acceleration is observed. The flame is accelerated in two steps connected by transition segment. Flame propagation in all investigated mixtures at an initial stage obeys a power law with an exponent 1.1. At the final stage flame propagation obeys a power law with an exponent 3. In the transition segment the dependence varies smoothly, as can be viewed from the experimental points.

When globally spherical or hemispherical expanding flame propagates the combustion products force the gas flow directed radially from the ignition point. When reaching a certain size, initially smooth spherical flame front is curved and forms a cellular structure (figure 5(a)) [21, 22].

Flame propagates along the surface normal, the speed imparted to the gas during combustion, is also directed along the normal to the flame surface (figure 5(b)).

Thus, the spherical cellular flame generates shear flow heterogeneity, characterized by certain values of speed and spatial scale. The scale of the velocity fluctuations can be determined from the following equation given in [23]:

\[
    u_v = (2\sigma u_n u')^{1/2} \{1 - \sigma u_n / u' [1 - \exp(-u'/\sigma u_n)] \}^{1/2},
\]  

(1)
Figure 5. Schlieren picture of cellular flame front [22] (a) and scheme of the flame velocities [23] (b).

where \( u_v \) to be visible speed of flame, \( \sigma \) to be thermal expansion ratio, \( u_n \) to be normal burning velocity, \( u' \) to be velocity turbulent pulsations.

The spatial scale of the flame heterogeneities is determined by the solution of a spherical flame front stability problem with respect to the upper spherical harmonics. Spherical harmonics are special functions defined on the surface of a sphere. They are often employed in solving partial differential equations that commonly occur in science. The spherical harmonics are a complete set of orthogonal functions on the sphere, and thus may be used to represent functions defined on the surface of a sphere, just as circular functions (sines and cosines) are used to represent functions on a circle via Fourier series. Like the sines and cosines in Fourier series, the spherical harmonics may be organized by (spatial) angular frequency. The problem is solved in [24, 25]. The solution will not be rewritten, bring only the formulas used in the calculation.

The disturbance of the flame surface is defined by a function:

\[
\zeta = \frac{E}{u_n \sigma t} = f_4(t) \sin m \varphi P_n^m \cos \theta, \tag{2}
\]

where \( E \) to be disturbance amplitude, the dependence of the disturbance amplitude on the time is determined by function:

\[
f_4(t) = C_1 t^{\omega_1} + C_2 t^{\omega_2}, \tag{3}
\]

where \( \omega_1 \) and \( \omega_2 \) to be the roots of the characteristic equation:

\[
\omega^2 + a \omega + b = 0, \tag{4}
\]

where \( a = \frac{2n^2 + 4 \sigma n + 3n + 3}{\sigma n + n + 1}; \quad b = \frac{\left(1 - \sigma\right)n^3 + 2 \sigma n^2 + 3 \sigma^2 n + 3 \sigma n - n + 2 \sigma}{\sigma^2 n + \sigma n + \sigma}; \quad n \) to be harmonic number; \( \omega \) to be exponent of the harmonic disturbance amplitude growth. Disturbance increases if the equation has at least one positive root.

Limit of stability may be found with solution of the equation:

\[
b = \frac{(1 - \sigma)n^3 + 2 \sigma n^2 + 3 \sigma^2 n + 3 \sigma n - n + 2 \sigma}{\sigma^2 n + \sigma n + \sigma} = 0, \tag{5}
\]
Table 1. Values of dimensionless time \( q^* \) and exponent \( \omega \).

| Harmonic number | 12 | 13 | 14 | 15 | 16 |
|-----------------|----|----|----|----|----|
| \% H\(_2\)      | 12 | 1.607 | 64.8 | 1.840 | 65.3 | 2.074 | 66.1 | 2.308 | 67.3 |
| \% H\(_2\)      | 15 | 1.563 | 61.2 | 1.794 | 61.5 | 2.027 | 62.2 | 2.259 | 63.2 |
| \% H\(_2\)      | 20 | 1.456 | 59.0 | 1.681 | 58.9 | 1.907 | 59.3 | 2.134 | 60.0 |
| \% H\(_2\)      | 30 | 1.243 | 59.2 | 1.453 | 58.3 | 1.665 | 58.8 | 1.877 | 58.4 |

Table 2. Conditions of flame instability onset.

| \% H\(_2\) | n | \( \omega \) | \( q^* \) | t, s | \( R \), m | \( L_d \), m | \( L_M \), m | Re |
|------------|---|------------|---------|------|-------|-------|-------|----|
| 12         | 13 | 1.607 | 64.8 | 3.64 \times 10^{-2} | 2.35 \times 10^{-3} | 5.67 \times 10^{-3} | -2.89 \times 10^{-4} | 992 |
| 15         | 13 | 1.563 | 61.2 | 7.59 \times 10^{-3} | 1.67 \times 10^{-3} | 4.04 \times 10^{-3} | -2.06 \times 10^{-4} | 2330 |
| 20         | 14 | 1.681 | 59.8 | 1.08 \times 10^{-3} | 1.53 \times 10^{-3} | 1.53 \times 10^{-3} | -9.99 \times 10^{-4} | 2620 |
| 30         | 15 | 1.665 | 58.1 | 5.70 \times 10^{-5} | 1.53 \times 10^{-4} | 1.78 \times 10^{-5} | 508 |

(1 - \sigma)n^3 + 2\sigma n^2 + 3\sigma^2 n + 3\sigma n - n + 2\sigma = 0. \quad (6)

To account the effect of chemical reactions kinetics and transport processes for higher spherical harmonics disturbances which wavelength is comparable with the width of the flame front, the techniques proposed by Markstein [26] is used. The total effect of the transport processes and reaction kinetics can be described by a constant linking change in the normal flame velocity with the curvature of its front:

\[ u = u_n(1 + \frac{L_M}{\Lambda}), \quad (7) \]

where \( L_M \) to be Markstein length, \( \Lambda \) to be the radius of curvature of the flame front.

First, the disturbance of all wavelengths are attenuated—the relative magnitude of the perturbation decreases. Attenuation of various harmonics occurs with different rates. After some time, the amplitude of a harmonic passes through a minimum and starts to increase. The dimensionless time of the harmonic disturbance growth beginning is determined by the equation:

\[ q^* = \frac{u_n \sigma t}{L_M} = \frac{n(n+1)(2n+1)}{\sigma n + n + 1} \frac{\omega + n + 1}{\omega(2\omega + a - 1)}. \quad (8) \]

Then begin to grow adjacent harmonics—flame instability develops. Within the range of mixtures with hydrogen content of 12 to 30% the values of exponent \( \omega \) and dimensionless time \( q^* \) are shown in table 1. The minimum dimensionless time values are highlighted with bold.

The harmonic number, the value of the growth exponent, dimensionless time and corresponding radius of the flame, the scale of disturbances and Markstein length for mixtures we are interested are presented in table 2.

Figure 6(a) shows the dependences of the ratio of instability time scale to the Kolmogorov time on the flame front radius. The only harmonic corresponding instability developed first of all was considered. With the increase of the flame radius the instabilities of neighboring harmonics
Figure 6. The ratios of temporal (a) and spatial (b) scales of instabilities to Kolmogorov scales as functions of the flame front radius.

arise that contribute to the flow pattern. However, to determine the radius of the flame at which the flow ahead the front becomes turbulent, it is enough to consider the disturbance (harmonic number), which first begin to grow. Transfer of turbulent velocity fluctuations described by Kolmogorov [20] in the form of

$$\langle [v(M') - \bar{v}(M)]^2 \rangle = c \varepsilon^{3/2} L^{3/2}, \quad (9)$$

where $\bar{v}(M')$ and $\bar{v}(M)$ to be the average values of the turbulent velocity fluctuations in the points $M'$ and $M$ correspondingly, $c$ to be constant close to 1, $\varepsilon$ to be turbulent energy dissipation specific rate, $L$ to be distance between points $M'$ and $M$.

The thickness of the turbulent pulsations transfer layer is determined by equation

$$L_{\text{tr}} = \frac{v'}{8 \varepsilon}. \quad (10)$$

Figure 6(b) shows the dependences of the ratio of turbulent pulsations transfer layer thickness to the Kolmogorov scale on the radius of the flame front.

Thus, the flame front with a radius of 16 mm and higher characterized disturbed cellular surface. This structure forces tangential components of the flame front and the flow velocity in the flame vicinity. Thus, the radial velocity of the flame front as a whole is small enough for a flame front passes the disturbance scale for a time exceeding Kolmogorov time scale in 15 times or more. This disturbance scale exceeds the scale of the Kolmogorov vortex in 40 or more times.

For all the mixtures thickness of turbulent fluctuations transfer layer exceeds the size of the Kolmogorov vortex 7 times or more. Thus, it can be argued that the velocity fluctuations generated by the cellular flame front form the structure of a homogeneous isotropic turbulence in the burned gas and layer of fresh gas ahead the flame front.

The Reynolds number of the transition from hemispherical to quasi-parallel stage is from $4.8 \times 10^4$ to $1.2 \times 10^6$. According to [2], the free spherical or cylindrical laminar flames from a weak source of ignition in the absence of any significant external influences become unstable, turbulent and self-accelerating at Reynolds numbers $10^3 - 10^5$. The lowest Reynolds numbers observed during the flame front propagation a mixture containing 12% of hydrogen. The value of the Reynolds number equal to 2400 is achieved at a distance of 40 mm from the ignition point. Thus, we can assume that the flame is turbulent in the entire observed region. According to [12] Peclet
number \((Pe= vL/\chi,\) where \(L\) to be distance to ignition source, \(v\) to be combustion front velocity, \(\chi\) to be thermal diffusivity\) to be higher than critical one \(Pe_c \approx 300\) for acceleration of combustion front. In all our experiments Peclet number reaches the critical value at the distances less than 10 mm from the ignition source. Thus, the laminar burning is missed in the observed segment, and the acceleration of the flame can not be explained by the laminar-turbulent transition or other instabilities onset. More data about the distance and the speed at which a transition from hemispherical to quasi-parallel combustion is presented hereinafter. As in all the experiments, the observed burning speed does not exceed 20% of the speed of sound in the initial mixture, the combustion process can be considered deeply subsonic. Pressure and temperature is also kept constant. From this it follows that the drastic change in the mechanism of propagation of the combustion can not be expected. In this case, the deviation of the exponent of 6% in the case of the hemispherical and by 2% in the case of quasi-parallel burning propagation can be considered as caused by measurement error and total nonsphericity/non-planarity of flame front.

The approach of flame propagation data interpretation using the Kolmogorov law is described in detail in [2]. Initially, the mixture is quiescent, the only significant release of energy occurs when the chemical reaction of hydrogen oxidation by atmospheric oxygen. Consequently, the turbulent energy dissipation \(\varepsilon\) is proportional to the specific heat release rate during the oxidation of hydrogen with atmospheric oxygen. In [2] the value of \(\varepsilon\) is considered to be constant, which may be acceptable in the formulation of the experiments described in [6], when the combustion develops a spherical envelope with the ignition in the center. Constant \(\varepsilon\) leads to the acceleration of the flame according to a power law with an exponent of 1.5. In the general case, the value of \(\varepsilon\) is not necessarily constant and does depend on the characteristics of the combustion front and boundary conditions. The only significant condition—specific rate of heat release should change a little on the Kolmogorov vortex scale \(\lambda_0 = (\nu^3/\varepsilon)^{1/4}\), and for the lifetime of the Kolmogorov vortex \(\tau_0 = (\nu/\varepsilon)^{1/2}\), where \(\nu\) \((\text{m}^2/\text{s})\) to be kinematic viscosity of the initial mixture.

Consider a cylindrical shell filled with a combustible mixture of hydrogen and oxygen. The minimum Froude number:

\[
Fr = \frac{v^2}{gL},
\]

where \(v\) to be flame velocity, \(g\) to be acceleration of gravity, is observed for the mixture with a hydrogen content of 12% and amounts to 0.75, well above the critical value \(Fr^* = 0.11\). This means that the “ascent” of hot gas cloud is negligible compared with the propagation of the burning front. Assuming a fixed center of gravity \(M = 0\) at the initial stage of hemispherical burning propagation it is convenient to use spherical coordinates \(L \sim R\), then equation 9 can be rewritten as

\[
\dot{R} = (\varepsilon(R)/R)^{1/3}.
\]

From here one can get the dependence of the specific heat release rate on the distance from the source of initiation:

\[
\varepsilon(R) = \dot{R}^3/R.
\]

Since the experimental data on the hemispherical stage of flame propagation shows the general relationship between the position and a visible flame front velocity in the next form:

\[
R(t) = At^{1.1},
\]

\[
\dot{R}(t) = 1.1At^{0.1},
\]

where \(t\) \((\text{s})\) to be the time from initiation of combustion, \(A\) \((\text{m/s}^{1.1})\) to be factor depending on the hydrogen concentration in the mixture, then

\[
\varepsilon(R) = 1.331A^2t^{-0.8} = 1.331A^{30/11}R^{-8/11}.
\]
The physical sense of the obtained dependence means lower specific heat release rate with increasing distance from the source of initiation. The overall rate of energy release is growing, but the mass involved in the movement is growing faster. Therefore, the acceleration of the combustion front is lower than in the self-similar case at a constant specific energy release rate.

On a quasi-parallel stage of burning propagation it is convenient to use cylindrical coordinates $L \sim h$, we obtain

$$\dot{h} = (\varepsilon(h)h)^{1/3}. \quad (17)$$

From here one can get the dependence of the specific heat release rate on the distance from the source of initiation:

$$\varepsilon(h) = \dot{h}^3/h. \quad (18)$$

As the experimental data on a quasi-parallel stage of burning propagation shows the general relationship between the position and a visible combustion front velocity as

$$h(t) = Bt^3, \quad (19)$$
$$\dot{h}(t) = 3Bt^2, \quad (20)$$

where $B \ (m/s^3)$ to be factor depending on the hydrogen concentration in the mixture,

$$\varepsilon(h) = 27B^2t^3 = 27Bh. \quad (21)$$

The physical sense of the obtained dependence is a linear increase of the specific heat release rate with increasing distance from the source of initiation. The total rate of energy release is growing faster than weight of involved substance. Therefore, the acceleration of the combustion front is higher than in the self-similar case at a constant specific energy release rate.

Figure 7 shows the comparison of flame propagation with constant (a) and variable (b) turbulent energy dissipation rate per weight. Such results demonstrate that acceleration of combustion in hydrogen-air mixtures may be described with the Kolmogorov law modified with variation of turbulent energy dissipation rate per weight with combustion propagation.
5. Conclusions

- Experimental data for propagation of combustion in a hydrogen–air mixture with hydrogen content of from 12 to 30% in cylindrical envelopes with volume of 4.5 m³ for initiating combustion energy of 5 J are obtained.
- For the first time variation of turbulent energy dissipation rate per weight with combustion propagation was used. This approach allows the experimental data for mixtures with different compositions in non-spherical volumes to be described using Kolmogorov theory.

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References

[1] Vinnem J E 2013 Principles, Modelling and Applications of QRA Studies (Offshore Risk Assessment vol 1) (New York: Springer Science & Business Media)
[2] Gostintsev Y A, Shatskikh Y V, Shulenin Y V and Fortov V E 2008 Russ. J. Phys. Chem. B 2 437–41
[3] Bradley D, Cresswell T M and Puttock J S 2001 Combust. Flame 124 551–9
[4] Breitung W et al 2005 Nucl. Eng. Des. 235 253–70
[5] Dorofeev S B, Sidorov V P, Dvoënishnikov A E and Breitung W 1996 Combust. Flame 104 95–110
[6] Gostintsev Y A, Istratov A G and Shulénin Y V 1988 Combust., Explos. Shock Waves 24 563–9
[7] Groethe M, Merilo E, Colton J, Chiba S, Sato Y and Iwabuchi H 2007 Int. J. Hydrogen Energy 32 2125–33
[8] Kim W K, Mogi T and Dobashi R 2013 J. Loss Prev. Process Ind. 26 1501–5
[9] Petukhov V A, Bublik N P, Gusev P A, Gutkin L D and Solntsev O I 2016 High Temp. 54 99–104
[10] Sato Y, Iwabuchi H, Groethe M, Merilo E and Chiba S 2006 J. Power Sources 159 144–8
[11] Molkov V V, Makarov D V and Schneider H 2007 Int. J. Hydrogen Energy 32 2198–205
[12] Kim W K, Mogi T, Kuwana K and Dobashi R 2015 Proc. Combust. Inst. 35 2051–8
[13] Akkerman V, Law C K and Bychkov V 2011 Phys. Rev. E 83 1–6
[14] Karlin V and Sivashinsky G 2007 Proc. Combust. Inst. 31 1 1023–30
[15] Liberman M A, Ivanov M F, Peil O E, Valiev D M and Eriksson L E 2004 Phys. Fluids 16 2476–82
[16] Pan K L and Fursenko R 2008 Phys. Fluids 20 094107
[17] Wada Y and Kuwana K 2013 J. Loss Prev. Process Ind. 26 392–5
[18] Wu F, Jomaas G and Law C K 2013 Proc. Combust. Inst. 34 937–45
[19] Gostintsev Y A, Fortov V E and Shatskikh Y V 2004 Dokl. Phys. Chem. 397 141–4
[20] Kolmogorov A N 1941 Dokl. Akad. Nauk SSSR 30 299–303
[21] Bradley D 1999 Philos. Trans. R. Soc., A 357 3567–81
[22] Yang S, Saha A, Wu F and Law C K 2016 Combust. Flame 171 112–8
[23] Karlovitz B, Dennistone Jr D W and Wells F E 1951 Chem. Phys. 19 541–7
[24] Istratov A G and Librovich V B 1966 J. Appl. Mech. Tech. Phys. 7 43–54
[25] Aslanov S K 1965 Combust., Explos. Shock Waves 1(3) 43–6
[26] Markstein G H 1951 J. Aeronaut. Sci. 18(3) 199–209