Laser photons acquire circular polarization by interacting with a Dirac or Majorana neutrino beam

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A B S T R A C T
It is shown that for the reason of neutrinos being left-handed and their gauge-couplings being parity-violated, linearly polarized photons acquire their circular polarization by interacting with neutrinos. Calculating the ratio of linear and circular polarizations of laser photons interacting with either Dirac or Majorana neutrino beam, we obtain this ratio for the Dirac neutrino case, which is about twice less than the ratio for the Majorana neutrino case. Based on this ratio, we discuss the possibility of using advanced laser facilities and the T2K neutrino experiment to measure the circular polarization of laser beams interacting with neutrino beams in ground laboratories. This could be an additional and useful way to gain some insight into the physics of neutrinos, for instance their Dirac or Majorana nature.

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1. Introduction
Since their appearance, neutrinos have always been extremely peculiar. Their charge neutrality, near masslessness, flavor mixing, oscillation, type of Dirac or Majorana, and in particular parity-violating gauge-coupling have been at the center of a conceptual elaboration and an intensive experimental analysis that have played a major role in donating to mankind the beauty of the standard model for particle physics. Several experiments studying solar, atmospheric and reactor neutrinos in past several years provides strong evidences supporting the existence of neutrino oscillations [1]. This implies that neutrinos are not exactly massless although they chirally couple to gauge bosons. Therefore they cannot be exactly two-component Weyl fermions. Instead, it is tempting to think of the nature of neutrinos, they might be fermions of Dirac type or Majorana type [2]. Dirac and Majorana neutrinos have different electromagnetic properties [3], which can be used to determine which type of neutrinos exists. The well-known example is the neutrino-less double beta decay [4], still to be experimentally verified.

Some attention has been recently driven to study the neutrino interactions with laser beams; here we mention a few examples. The emission of $\nu\bar{\nu}$ pairs off electrons in a polarized ultra-intense electromagnetic (e.g., laser) wave field is analyzed in Ref. [5]. The electron–positron production rate has been calculated [6] using neutrinos in an intense laser field. In the frame of the standard model, by studying the elastic scattering of a muon neutrino on an electron in the presence of a linearly polarized laser field, multi-photon processes have been shown [7].

In this Letter, we quantitatively show and discuss linearly polarized photons acquire circular polarization by interacting with neutrinos, for the reason that neutrinos are left-handed and possess peculiar couplings to gauge bosons in a parity-violating manner. In particular we quantitatively calculate the circular polarization that a linearly polarized laser beam develops by interacting with a neutrino beam in ground laboratories. Moreover we show that using advanced laser facility one can possibly measure the circular polarization of laser photons interacting with a Dirac or Majorana neutrino beam produced by neutrino experiments, e.g., the Tokai-to-Kamioka (T2K) [8]. This could possibly provide an additional way to study some physics of neutrinos, for example, the nature of neutrinos, Dirac or Majorana type.

We recall that the similar photon–neutrino process was considered for the generation of circular polarizations of Cosmic Microwave Background (CMB) photons [9] interacting with cosmic background neutrinos (CNB), instead of photons [10] and electrons in the presence of magnetic field [11]. There the main calculation was done to obtain the power spectrum $C_{TT}^{\nu}$ of the circular polarization of CMB photons by the forward scattering between CMB photons and CNB neutrinos.

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2. Stokes parameters

The polarization of laser beam is characterized by means of the Stokes parameters: the total intensity \( I \), intensities of linear polarizations \( Q \) and \( U \), as well as the intensity of circular polarizations \( V \) indicating the difference between left- and right-circular polarizations intensities. The linear polarization can be represented by \( P \equiv \sqrt{Q^2 + U^2} \). An arbitrary polarized state of a photon \( |k| \) propagating in the \( z \)-direction, is given by

\[
|e\rangle = a_1 \exp(i\theta_1)|\epsilon_1\rangle + a_2 \exp(i\theta_2)|\epsilon_2\rangle,
\]
where linear bases \( |\epsilon_1\rangle \) and \( |\epsilon_2\rangle \) indicate the polarization states in the \( x \) - and \( y \) -directions, and \( \theta_{1,2} \) are initial phases. Quantum-mechanical operators in this linear bases, corresponding to Stokes parameters, are given by

\[
\hat{I} = |\epsilon_1\rangle\langle\epsilon_1| + |\epsilon_2\rangle\langle\epsilon_2|, \quad \hat{Q} = |\epsilon_1\rangle\langle\epsilon_1| - |\epsilon_2\rangle\langle\epsilon_2|, \quad \hat{U} = |\epsilon_1\rangle\langle\epsilon_2| + |\epsilon_2\rangle\langle\epsilon_1|.
\]

An ensemble of photons in a general mixed state is described by a normalized density matrix \( \rho_{ij} \equiv \langle \epsilon_i | \epsilon_j \rangle / \text{tr} \rho \), and the dimensionless expectation values for Stokes parameters are given by

\[
I = \langle \hat{I} \rangle = \text{tr} \hat{I}, \quad Q = \langle \hat{Q} \rangle = \text{tr} \hat{Q} = \rho_{11} - \rho_{22}, \quad U = \langle \hat{U} \rangle = \text{tr} \hat{U} = \rho_{12} + \rho_{21}, \quad V = \langle \hat{V} \rangle = \text{tr} \hat{V} = i \rho_{21} - i \rho_{12}.
\]

The time-evolution of the number operator \( D^{0}_{ij}(k) \) is governed by the Heisenberg equation

\[
\frac{d}{dt}D^{0}_{ij}(k) = i[H_I, D^{0}_{ij}(k)],
\]

where \( H_I \) is the interacting Hamiltonian of photons with other particles in the standard model. Calculating the expectation value of both sides of Eq. (11), one arrives at the following Quantum Boltzmann Equation (QBE) for the number operator of photons [13],

\[
(2\pi)^3 \delta^3(0)(2k^0) \frac{d}{dt} \rho_{ij}(x,k) = \frac{1}{2} \int dt \{ [H^0_I(t), D^0_{ij}(k)] - 1 \}
\]

3. Quantum Boltzmann equation for Stokes parameters

We express the laser field strength \( F_{\mu \nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \), and free gauge field \( A_\mu \) in terms of plane wave solutions in the Coulomb gauge [12],

\[
A_\mu(x) = \int \frac{d^3k}{(2\pi)^3} 2k^0 \delta_{ij} \langle \epsilon_i | e^{ik\cdot x} + \epsilon_j^\dagger(k) a_{ij}(k) e^{-ik\cdot x} \rangle,
\]

where \( \epsilon_{ij}(k) \) are the polarization four-vectors and the index \( i = 1, 2 \) representing two transverse polarizations of a free photon with four-momentum \( k \) and \( k^0 = |k| \), \( \epsilon_i \cdot \epsilon_j = 0 \) and \( \epsilon_i \cdot \epsilon_j = -\delta_{ij} \).

The creation operators \( a_i^\dagger(k) \) and annihilation operators \( a_i(k) \) satisfy the canonical commutation relation

\[
[a_i(k), a_j^\dagger(k')] = (2\pi)^3 2k^0 \delta_{ij} \delta^{(3)}(k-k'),
\]

The density operator describing an ensemble of free photons in the space of energy–momentum and polarization state is given by

\[
\hat{\rho}(x) = \frac{1}{\text{tr} \rho} \int \frac{d^3k}{(2\pi)^3} \rho_{ij}(x, k) a_i^\dagger(k) a_j(k),
\]

where \( \rho_{ij}(x,k) \) is the general density-matrix, analogous to Eqs. (3)–(6), in the space of polarization states with the fixed energy–momentum \( k \) and space–time “x”. The number operator of photons \( D^0_{ij}(k) \equiv a_i^\dagger(k) a_j(k) \) and its expectation value with respect to the density-matrix (9) is defined by

\[
\langle D^0_{ij}(k) \rangle = \text{tr} \hat{\rho} D^0_{ij}(k) = (2\pi)^3 \delta^3(0)(2k^0) \rho_{ij}(x,k).
\]
In the context of standard model $SU(2) \times U(1)$ for the electro-weak interactions, two Feynman diagrams representing the leading-order contribution to the interaction between photons and neutrinos are shown in Fig. 1. The leading-order interacting Hamiltonian is given by

$$H_0^f = \int d\mathbf{q} d\mathbf{q'} d\mathbf{p} d\mathbf{p'} (2\pi)^3 \delta^3 (\mathbf{q} + \mathbf{p'} - \mathbf{p} - \mathbf{q}) \times \exp[i(t(q'^0 + p'^0 - q^0 - p^0)] [b_0^\dagger a_s^\dagger, (M_1 + M_2) a_s b_l],$$

where

$$M_1 + M_2 = \frac{1}{8} \epsilon^2 g_w^2 \int \frac{d^4 l}{(2\pi)^4} D_{a\beta}(q - l) \tilde{\cal U}_l(q') \times \gamma^\alpha (1 - \gamma_5) S_F(l + p - p') \times [\bar{\psi}_s S_F(l + p) \psi_s + \bar{\psi}_s S_F(l - p) \psi_s] \times S_F(l) \gamma^\beta (1 - \gamma_5) \bar{\psi}_l(q).$$

$$D_{a\beta}$$ and $$S_F$$ are respectively $$W^\pm$$ gauge-boson and charged-lepton propagators. In Eq. (17), our notation $$d\mathbf{q} = d^4 q/[2(2\pi)^3]$$ for $$d\mathbf{p}$$ and $$d\mathbf{p'}$$.

5. Laser-photon circular polarization

From Eq. (17) for the photon–neutrino interacting Hamiltonian, we obtain the first forward scattering term of Eq. (12)

$$[H_0^f, D_{ij}^0(k)] = \int d\mathbf{q} d\mathbf{q'} d\mathbf{p} d\mathbf{p'} (2\pi)^3 \delta^3 (\mathbf{q} + \mathbf{p'} - \mathbf{p} - \mathbf{q}) \times (M_1 + M_2) \times (2\pi)^3 [b_0^\dagger b_l^\dagger a_s^\dagger a_l 2p^0 \delta_{ij} \delta^3 (\mathbf{k} - \mathbf{p}) - b_0^\dagger b_l^\dagger a_s^\dagger a_l 2p^0 \delta_{ij} \delta^3 (\mathbf{k} - \mathbf{p})].$$

Then, using the commutators of Eqs. (8) and (15), as well as the following expectation values of operators [13],

$$\langle a_{12} \ldots b_{12} \ldots \rangle = \langle a_{12} \ldots \rangle \langle b_{12} \ldots \rangle,$$

$$\langle a^\dagger_s (p') a_s (p) \rangle = 2 p^0 (2\pi)^3 \delta^3 (\mathbf{p} - \mathbf{p'}) \rho_{\nu}(\mathbf{x}, \mathbf{q}),$$

$$\langle b^\dagger_l (q') b_l (q) \rangle = (2\pi)^3 \delta^3 (\mathbf{q} - \mathbf{q'}) \delta_{ij} \frac{1}{2} n_\nu(\mathbf{x}, \mathbf{q}),$$

where $$\rho_{\nu}(\mathbf{x}, \mathbf{q})$$ is the local matrix density and $$n_\nu(\mathbf{x}, \mathbf{q})$$ is the local spatial density of neutrinos in the momentum state $$\mathbf{q}$$. Thus we define the neutrino distribution function $$n_\nu(\mathbf{x}, \mathbf{q})$$ and the average momentum $$\mathbf{q}$$ of neutrinos as

$$n_\nu(\mathbf{x}) = \int \frac{d^3 q}{(2\pi)^3} n_\nu(\mathbf{x}, \mathbf{q}),$$

$$\mathbf{q} = \frac{1}{n_\nu(\mathbf{x})} \int \frac{d^3 q}{(2\pi)^3} \mathbf{q} n_\nu(\mathbf{x}, \mathbf{q}).$$

For a neutrino beam we assume $$n_\nu(\mathbf{x}, \mathbf{q}) \sim \exp[-|\mathbf{q} - \mathbf{q}|/|\mathbf{q}|]$$, representing the most of neutrinos carry the momentum $$\mathbf{q} \approx \mathbf{q}$$. Then we arrive at

$$i\langle [H_0^f, D_{ij}^0(k)] \rangle \approx \frac{i}{16} \epsilon^2 g_w^2 \int d\mathbf{q} \rho_{\nu}(\mathbf{x}, \mathbf{q}) \delta_{ij}$$

$$\times \int \int dy \int dz \frac{1}{y - z} \tilde{\cal U}_l(q)(1 + \gamma_5)$$

$$\times [2 q \epsilon_{s'} \epsilon_s + 2 z (\epsilon_{s'} - \epsilon_s)] (1 + \gamma_5) \tilde{\cal U}_l(q).$$

Moreover, using dimensional regularization and Feynman parameterizations for four-momentum integration over $$l$$, we approximately obtain

$$i\langle [H_0^f, D_{ij}^0(k)] \rangle \approx \frac{1}{16} \frac{\epsilon^2 g_w^2}{4 \pi^2} \int d\mathbf{q} \rho_{\nu}(\mathbf{x}, \mathbf{q}) \delta_{ij}$$

$$\times \int \int dy \int dz \frac{1}{y - z} \tilde{\cal U}_l(q)(1 + \gamma_5)$$

$$\times [2 q \epsilon_{s'} \epsilon_s + 2 z (\epsilon_{s'} - \epsilon_s)] (1 + \gamma_5) \tilde{\cal U}_l(q).$$

where neutrino and photon energies are much smaller than $$W^\pm$$-boson mass $$M_W$$, i.e., $$E_\nu, E_\gamma \ll M_W$$.

The time evolution of Stokes parameter $$V$$ of linearly polarized laser-photons is determined by Eqs. (12) and (25),

$$\frac{dV(x, k)}{dt} \bigg|_D \approx \frac{1}{6} \frac{\epsilon^2 g_w^2}{(4 \pi)^2 M_W^2 k^0} \int d\mathbf{q} n_\nu(x, \mathbf{q}) \tilde{\cal U}_l(q)(1 + \gamma_5)$$

$$\times [2 q \epsilon_{s'} \epsilon_s - 2 z (\epsilon_{s'} - \epsilon_s)] \tilde{\cal U}_l(q).$$

where high-order terms suppressed at least by $$1/M_W^2$$ have been neglected. Using Eq. (16), we obtain

$$\frac{dV(x, k)}{dt} \bigg|_D \approx \frac{\sqrt{2}}{3 \pi k^0} G_F \int d\mathbf{q} n_\nu(x, \mathbf{q})$$

$$\times [2 q \epsilon_{s'} \epsilon_s - 2 z (\epsilon_{s'} - \epsilon_s)] \tilde{\cal U}_l(q).$$

where the Fermi and fine-structure constants are

$$G_F = \sqrt{2} \frac{g_w^2}{8 M_W^2} \approx 1.16 \times 10^{-5} (\text{GeV})^{-2},$$

$$\alpha = \frac{e^2}{4 \pi} = \frac{1}{137}.$$
implies that near to the source of high-energy muons (\(\mu^\pm\)) decay, the beam angle divergence \(\theta_{\text{div}} \sim m_{\mu^\pm}/E_{\mu^\pm} \ll 1\), where \(E_{\mu^\pm}(m_{\mu^\pm})\) is the muon energy (mass). This implies that near to the source of high-energy muons (\(\mu^\pm\)) decay, the momenta of neutrinos are around the averaged one \(\bar{q}\), given by Eq. (23). In this circumstance, we approximate Eq. (27) as

\[
\frac{dV(x, \mathbf{k})}{dt} \bigg|_D \approx \frac{\sqrt{2}}{6\pi k^0} G_F |q| n_\nu(x, \mathbf{q})
\]

\[
\times \left[ (\mathbf{q} \cdot \mathbf{e}_1 \mathbf{q} \cdot \mathbf{e}_1 - \hat{q} \cdot \mathbf{e}_2 \hat{q} \cdot \mathbf{e}_2) Q(x, \mathbf{k})
\right.
\]

\[
- \left( \hat{q} \cdot \mathbf{e}_1 \hat{q} \cdot \mathbf{e}_2 + \mathbf{q} \cdot \mathbf{e}_2 \mathbf{q} \cdot \mathbf{e}_1 \right) U(x, \mathbf{k}),
\]

(29)

where \(\hat{q} = \mathbf{q}/|\mathbf{q}|\) indicates the direction of neutrino beam, and \(n_\nu(x) \approx \text{const.}\) along the beam direction. On the analogy of averaged momentum \(\bar{q}\) of neutrino beam, \(\mathbf{k}\) should be understood as an averaged momentum of photons in a laser beam in the direction \(\hat{k} = \mathbf{k}/|\mathbf{k}|\), the Stokes parameters \(Q(x, \mathbf{k}), U(x, \mathbf{k})\) and \(V(x, \mathbf{k})\) are the functions averaged over the momentum distribution of photons in a laser pulse. As shown in Fig. 2, we select the \(\mathbf{k}\) along the \(\hat{z}\)-direction, and \(\vartheta\) and \(\phi\) spherical angles of \(\mathbf{q}\) with respect to the \(\mathbf{k}\) direction. Then Eq. (29) becomes

\[
\frac{dV(x, \mathbf{k})}{dt} \bigg|_D \approx \frac{\sqrt{2}}{6\pi k^0} G_F |q| n_\nu(x, \mathbf{q})
\]

\[
\times \left[ (\sin^2 \theta \cos 2\varphi) Q(x, \mathbf{k})
\right.
\]

\[
- \left( \sin^2 \theta \sin 2\varphi \right) U(x, \mathbf{k})
\]

\[
= \frac{\sqrt{2}}{6\pi k^0} G_F |q| \sin^2 \theta n_\nu(x, \mathbf{q}) Q(x, \mathbf{k}),
\]

(30)

where in the third line we set \(\phi = 0\) in the plane of \(\mathbf{q}\) and \(\mathbf{k}\). To discuss possible experimental relevance, we rewrite Eq. (30) as

\[
\Delta V \left( \frac{x, \mathbf{k}}{Q} \right) \left. \right|_D \approx \alpha G_F \left( \frac{\sqrt{2}|q|}{6\pi k^0} \right) \sin^2 \theta n_\nu(x, \mathbf{q}) \Delta t
\]

\[
= 2.37 \times 10^{-36} \left( \text{cm}^2 \right) \left( \frac{\bar{F}_e(x, \mathbf{q})}{k^0} \right) \sin^2 \theta \Delta t,
\]

(31)

where the averaged energy-flux of neutrino beam \(\bar{F}_e(x, \mathbf{q}) = c/|\mathbf{q}|n_\nu(x, \mathbf{q})\). Eq. (31) represents the ratio of circular and linear polarizations of laser beam interacting with the neutrino beam for a time interval \(\Delta t\).

To end this section, we would like to point out that the purely left-handed interaction (13) is the crucial reason why linearly polarized photons acquire circular polarizations by interacting with left-handed neutrinos, in contrast they do not acquire circular polarizations by interacting with electrons in the forward scattering terms of Eq. (12) [13]. If right-handed neutrinos were involved in the weak interaction, replacing \((1 - \gamma_5)/2\) by \((1 + \gamma_5)/2\) in Eq. (13), we would have obtained a contribution from right-handed neutrinos that completely cancels the circular polarization (30). This can be understood by the angular-momentum conservation in the photon–neutrino interaction (13), where neutrinos are in left-handed state (+), as a result, photons acquire the component of circular polarizations. This point will be further illustrated by the Compton scattering of photons and polarized electrons in the next section.

### 6. Polarized Compton scattering

It is shown [13] that the photon circular-polarization is not generated by the forward Compton scattering of linearly polarized photons on unpolarized electrons. The reason is that the contributions from left- and right-handed polarized electrons to photon circular-polarization exactly cancel each other. In the following, we show the circular polarization generated by a polarized Compton scattering: a photon beam scattering on a polarized electron beam, whose the number-density of incident left-handed electrons is not equal to the right-handed one. For the sake of simplicity, we describe the polarized electron beam by the net number-density of left-handed electrons \(\delta n_{\text{le}}\), and consider relativistic electrons \(E_e \gg m_e, E_e \gg E_Y\) and \(\mathbf{q} \gg \mathbf{k} \sim \mathbf{p}\), where \(\mathbf{q}\), \(\mathbf{p}\) and \(\mathbf{k}\) are the momenta of incident electrons, incident and scattered photons respectively (see Fig. 3). Following the calculations of Ref. [13], we obtain the time evaluation of the Stokes parameter \(V\) for the circular polarization of scattered photons,

\[
\frac{dV(k)}{dt} \approx \frac{\delta n_{\text{le}} 3\pi \sigma_T}{4} \frac{m_e}{|\mathbf{q}|} \left( \frac{m_e}{|\mathbf{q}|} \right)^2 \int \frac{d\Omega_{\hat{p}}}{4\pi} \left[ \mathbf{q} \cdot \mathbf{e}_1(\hat{p}) \right] - \left[ \mathbf{q} \cdot \mathbf{e}_2(\hat{p}) \right] 
\]

\[
\left. \mathbf{q} \cdot \mathbf{e}_1(\hat{p}) \right|^2 - \left[ \mathbf{q} \cdot \mathbf{e}_2(\hat{p}) \right]^2 - 1 \left( \mathbf{e}_1 \rightarrow \mathbf{e}_2 \right) U(\hat{p}),
\]

(32)

which is proportional to \(\delta n_{\text{le}}\) the averaged number-density of left-handed polarized electrons, here \(\mathbf{q}\) is the averaged momenta of incident polarized electrons and \(\hat{p} = \mathbf{p}/|\mathbf{p}|\). This shows that linearly polarized photons scattering on polarized electrons acquire circular polarization, analogously with photon circular-polarization generated by linearly polarized photons scattering on left-handed neutrinos.

### 7. Majorana neutrinos

We turn to calculate the circular polarization of laser-photon beam due to its scattering with Majorana neutrinos [15],

\[
\psi_e^M(x) = \int \frac{d^4 q}{(2\pi)^3 \sqrt{2q^0}} \times \sum_{r=\pm} \left[ b_r(q) \mathcal{U}_r(q)e^{-i\mathbf{q} \cdot \mathbf{x}} + \lambda b_r^\dagger(q) \mathcal{V}_r(q)e^{i\mathbf{q} \cdot \mathbf{x}} \right],
\]

(33)
which are self-conjugated Dirac neutrinos (particle and anti-particle are identical), i.e., \( \psi^c_L(x) = \bar{\psi}^c_R(x) \) up to a phase \( \lambda \) \((|\lambda|^2 = 1)\), where the conjugated field \( \psi^c_M(x)^c \) is \( \gamma^0 \bar{\psi}^c_R(x)^c \). C = \( i \gamma^0 \gamma^2 \), and

\[
y^0 C \mathcal{V}^c_L(p) = \mathcal{U} \mathcal{E}^{-1}(p), \quad y^0 C \mathcal{V}^c_R(p) = \mathcal{V} \mathcal{E} \mathcal{K}^{-1}(p). \tag{34}\]

The interaction of Majorana neutrinos with charged particles via the \( W^\pm \) in the Standard Model SU(2) \( \times \) U(1) can be written as [15]

\[
\mathcal{E}^{\text{int}} = \frac{g_w}{2\sqrt{2}} \sum_{i=e,\mu,\tau} \bar{\mathcal{V}}^M \gamma^\mu (1 - y^5) \mathcal{V}_i W^+_{\mu}.
\]

\[
-\lambda y^\mu (1 + y^5) \mathcal{V}_i W^-_{\mu}. \tag{35}\]

where \( \psi^c \) is the conjugated left field. The first term is due to the left-handed Majorana neutrino, analogously to Eq. (13) for the left-handed Dirac neutrino, yielding Feynman diagrams in Fig. 1. The second term comes from the conjugated left-handed Majorana neutrino \( \psi^c_M(x)^c \) yielding "conjugated" Feynman diagrams, which are all the internal lines in Fig. 1 are replaced by their conjugated lines. The contribution from "conjugated" Feynman diagrams can be obtained by substitutions [15]:

\[
W^+ \rightarrow W^-, \quad \psi_e \rightarrow \psi^c_e, \quad y^5 \rightarrow -y^5, \quad g_w \rightarrow -\lambda g_w. \tag{36}\]

As a result, in Majorana neutrino case, the interacting Hamiltonian is

\[
H^I_1 = \int dq dq' dp dp' \{2 \pi^3 \delta^3 (q' + p' - p - q) \times \exp[ip \cdot q - p \cdot q'] \times [b^I a^I_1 (M_1 + M_2) a_{b_1} + a^I_1 b_I (M'_1 + M'_2) b_{b_1}] \}
\]

\[
\times \mathcal{V}_l (q') \gamma^\mu (1 + y^5) S_F (p' - p - l) \times [\gamma_5 S_F (-l - p) \gamma_5] \}
\times S_F (-l) \gamma^\mu (1 + y^5) V_l (q'). \tag{37}\]

Analogously to Eqs. (19)–(25), we obtain the contribution of "conjugated" part (38) to the forward scattering term \((|H^I_1, D^I_0(k)|_M)\).

\[
-\frac{1}{16} \frac{g^2}{4 \pi^2} \int dq dq' dp \{ \int dy \int dz \{ (1 - y^2 - z^2) V_l (q) (1 - y^5) \}
\times [2 y \bar{\psi} \gamma \gamma_5 \psi_R \psi_R \gamma_5 \] \times (3 y - 1) \} \rfloor. \tag{38}\]

As a result, we approximately obtain the time-evolution of the circular-polarization that laser-photons acquire after their interaction with a Majorana neutrino beam,

\[
\frac{dV(x,k)}{dt} \bigg|_M \approx \frac{dV(x,k)}{dt} \bigg|_D + \int dq \gamma^\mu (q) \mathcal{V}_l (q) (1 - y^5) \]

\[
\times \int \left[ (\gamma_5 q \cdot \psi_R - \gamma_5 q' \cdot \gamma_5 \psi_R) \right] \mathcal{E}_l (q), \tag{40}\]

The first term is the same as Eq. (27) with the replacement \( n_i (x, q) \rightarrow n^I_0 (x, q) \), and the rest comes from the so-called "conjugated" part, which is the contribution from conjugated left-handed Majorana neutrinos interacting with \( W^-_\mu \) in the second term of Eq. (35). From the identity

\[
\mathcal{V}_l (q') \gamma^\mu (1 \pm y^5) \mathcal{V}_l (q') = \mathcal{V}_l (q') \gamma^\mu (1 \mp y^5) \mathcal{V}_l (q). \tag{41}\]

Eq. (40) becomes

\[
\frac{dV(x,k)}{dt} \bigg|_M \approx \frac{dV(x,k)}{dt} \bigg|_D - \frac{1}{16} \frac{g^2}{4 \pi^2} \int dq \gamma^\mu (q) \mathcal{E}_l (q) (1 + 2 y^5) \]

\[
\times \left[ (\gamma_5 q \cdot \psi_R - \gamma_5 q' \cdot \gamma_5 \psi_R) \right] \mathcal{E}_l (q), \tag{42}\]

By using identities

\[
\mathcal{U}_l (q) \gamma^\mu \mathcal{U}_l (q) = 2 q^\mu \delta^3, \quad \frac{1}{2} \sum_r \bar{\mathcal{U}}_l (q) \gamma^\mu (1 \pm y^5) \mathcal{U}_l (q) = 2 q^\mu, \tag{43}\]

we approximately obtain

\[
\frac{dV(x,k)}{dt} \bigg|_M \approx 2 \frac{dV(x,k)}{dt} \bigg|_D, \tag{44}\]

indicating that the photon circular-polarization generated by the photon–neutrino (Majorana) scattering is about two times larger than that generated by the photon–neutrino (Dirac) scattering. This difference is due to the fact that in the standard model, see Eq. (13), there is only one species of the left-handed Dirac neutrino field, whereas the left-handed Majorana neutrino field has two species for it being a self-conjugated field, see Eq. (35). As a result, in the Majorana neutrino case, the interacting Hamiltonian \( H^I_1 \) of Eq. (37) has an additional "conjugated" part, compared with the interacting Hamiltonian \( H^I_2 \) of Eq. (17) in the Dirac neutrino case. These two parts in Eq. (37) have the same contribution to the V-parameter, which is equal to the contribution in the Dirac neutrino case. Thus, the V-contribution (44) from the photon and Majorana neutrino interaction is twice larger than the V-contribution (31) from photon and Dirac neutrino interaction.

8. Neutrino and laser-photon beams

Our result (31) shows that in order to produce a large intensity of photon circular-polarization for possible measurements, the intensities of neutrino and laser-photon beams should be large enough. In addition the interacting time \( \Delta t \) of two beams, i.e., the spatial dimension \( \Delta d \) of interacting spot of two beams (\( \Delta d \approx c \Delta t \))
should not be too small. This depends on the sizes of two beams at interacting point, assuming \(\sin^2 \theta \sim 1\). Based on Figs. 1 or 25 in Ref. [8] for the Tokai-Kamioka (T2K) neutrino experiment (ND280), the maximal neutrino flux \(\sim 10^{13} \text{cm}^{-2} \text{year}^{-1}\) locating at energies \(E_\nu \approx |q| \sim 0.5 \text{GeV}\), we estimate the mean energy-flux of muon neutrino beam,

\[
\bar{F}_\nu(x, q) \approx (|q|/m_e) c \sim 10^{-3} - 10^{-4} \text{GeV}/(\text{cm}^2 \cdot s).
\]  

(45)

The beam angle divergence \(\theta_{b\nu} \sim m_\mu/E_\nu \approx 10^{-2}\), where the muon energy \(E_\nu \approx E_\mu\). In addition, in the neutrino experiment (ND280), we estimate the spatial and temporal intervals \(\Delta d\) and \(\Delta t \approx \Delta t\), of interacting spot as \(\Delta d \sim 2R_0 + d \cdot \theta \sim 100 \text{cm}\) and \(\Delta t \approx \Delta d/c \sim 10^{-8} \text{s}\), where \(R_0\) is the beam radius near to the source of muon neutrinos and \(d = 2.8 \times 10^4 \text{cm}\) is the distance between the muon neutrino source and interacting spot with laser photons. Using optic laser beams, laser photon energy \(k^0 \sim \epsilon_\nu\) ev, we approximately obtain the ratio of Eq. (31)

\[
\frac{\Delta V}{Q} = \frac{\Delta V(x, k)}{Q(x, k)} \left|_D \right. \sim 10^{-23} \sin^2 \theta \Delta t / s,
\]  

(46)

where the factor \(\sin^2 \theta\) is the order of unit. Eq. (46) represents the result for laser and neutrino beams (pulses) interacting once only. Suppose that by using mirrors or some laser facilities the laser beam can bent \(N\)-times in its traveling path, and all laser pulses are identical without interference, so that each laser pulse can interact with the neutrino beam \(N\)-times in its path, the ratio of Eq. (46) is approximately enhanced by a factor of \(N\),

\[
\frac{\Delta V}{Q} = \frac{\Delta V(x, k)}{Q(x, k)} \left|_D \right. \sim 10^{-23} \sin^2 \theta N \Delta t / s,
\]  

(47)

since \(\sin^2 \theta = \sin^2 (\pi - \theta)\) for back and forth laser beams in opposite directions. Suppose that mirrors that trap laser beams can reflect 99.999% of laser light for a narrow range of wavelengths and angles. The absorption coefficient of the mirror is then \(C_\alpha \sim 10^{-5}\), and the intensity \(Q\) of linearly polarized laser beam is reduced to \(Q \rightarrow Q(1 - C_\alpha)\) for each reflection, we approximately have \(N \sim \sum_{n=0}^{\infty} (1 - C_\alpha)^n \approx 1/C_\alpha\). In general, we express our result as

\[
\frac{\Delta V}{Q} = \frac{\Delta V(x, k)}{Q(x, k)} \left|_D \right. \sim 10^{-26} \left( \frac{4 \pi^2 \Delta t}{C_\alpha} \right) \left( \frac{\epsilon_\nu}{k_0} \right)^2
\]  

(48)

Based on the T2K neutrino experiment (ND280) and discussions presented in this Section, to have a significant probability of observing a circularly polarized photon (\(\sim \text{eV}\)), the intensity of incident linearly polarized photons of optic laser should at least be: \(Q_{\min} \sim 10^{26} \text{ev cm}^{-2} \text{s}^{-1} \sim 10 \text{MW cm}^{-2}\), which is not difficult to be achieved in present laser technologies. The larger intensity \(Q\) of incident laser beam in linear polarization is, the larger intensity of photons in circular polarizations should be observed. In addition, a laser or neutrino beam is made by sequent bunches (pulses) of photons or neutrinos, it should be important to synchronize two beams in such a way that the interacting probability of photons and neutrinos bunches is maximized.

For a comparison with Eq. (48), using Eq. (32) we estimate the quantity \(\Delta V/Q\) for a linearly polarized laser beam interacting with a polarized electron beam too. Considering the left-handed polarized electron beam with averaged density \(\delta \bar{n}_{L, e}\) and averaged momentum \(|q|\), we obtain

\[
\Delta V/Q \sim 10^{-10} \left( \frac{\bar{F}_e}{4 \pi^2 \text{GeV cm}^{-2} \text{s}^{-1}} \right) \left( \frac{\Delta t}{10^{-8} \text{s}} \right) \left( \frac{10^{-5}}{C_\alpha} \right) \left( \frac{k_0}{\epsilon_\nu} \right)^2,
\]  

(49)

where \(k_0\) is the averaged energy of laser photons, \(\Delta t\) indicates time interaction, the averaged velocity and energy flux of the left-handed polarized electron beam \(\bar{v}_e = |q|/m_e < 1\) and \(\bar{F}_e \equiv \langle |q|/m_e \rangle\).

In addition, Ref. [14] discussed the possibility of using intense muon neutrino beams, such as those available at proposed muon colliders, interacting with high powered lasers to probe the neutrino mass. The rate of photon–neutrino scattering (\(R_{\nu\gamma} \sim 1/\text{year}\) or \(3 \times 10^{-8} \text{s}\)) was estimated by considering dramatically short pulsed lasers with energies of up to \(1.6 \times 10^7 \text{ J}\) per pulse and very short pulse durations (\(\sim \text{s}\)), which is near to the critical intensity \(\sim 10^{28-29} \text{ W/cm}^2\) for the production of electron–positron pairs. Even in this extremal powered lasers, the total probability for photon–neutrino scattering is too small to be observed.

The main reason is that the interacting rate \(R_{\nu\gamma}\) is proportional to the photon–neutrino scattering cross-section \(\sigma_{\nu\gamma} \propto \alpha^2 (G_F m_e)^2\), which is very small.

To compare with the photon–neutrino scattering rate \(R_{\nu\gamma}\) obtained in Ref. [14], we estimate the rate of generating circular polarization of laser photons presented in this Letter. The quantity \(\Delta V/k\) of Eq. (31) represents the number of circularly polarized photons of energy \(|k| = k_0 \sim \epsilon_\nu\) ev per unit area (\(\text{cm}^{-2}\)), unit time (\(\text{s}^{-1}\)) in a laser pulse. Therefore the rate of generating circular polarization of laser photons can be estimated by

\[
R_{\nu\gamma} \approx \frac{\Delta V}{k_0} \sigma_{\text{laser}} f_{\text{pulse}} \tau_{\text{pulse}}.
\]  

(50)

where \(\tau_{\text{pulse}}\) is the time duration of a laser pulse, the effective area of photon–neutrino interaction is represented by the laser-beam size \(\sigma_{\text{laser}}\) being smaller than the neutrino-beam size \(\Delta d\), and the laser repetition rate \(f_{\text{pulse}}\) is the number of laser pulses per second. To have more efficiency, we assume that laser and neutrino beams are synchronized and the \(f_{\text{pulse}}\) is equal to the repetition rate of neutrino beam \(f_{\text{bunch}}\), which is the number of neutrino bunches per second. In contrast with the rate of photon–neutrino scattering \(R_{\nu\gamma} \sim \alpha^2 (G_F m_e)^2\), the rate \(R_{\nu\gamma}\) of Eq. (50) linearly depends on \(\alpha G_F\) via the \(\Delta V/k\) of Eq. (31). This implies that the rate \(R_{\nu\gamma}\) of Eq. (50) should be much larger than the photon–neutrino scattering rate \(R_{\nu\gamma}\) considered in Ref. [14].

Combining Eqs. (48) and (50), we obtain

\[
R_{\nu\gamma} \approx 10^{-26} \left( \frac{\bar{F}_e}{4 \pi^2 \text{GeV cm}^{-2} \text{s}^{-1}} \right) \left( \frac{\Delta t}{10^{-8} \text{s}} \right) \left( \frac{10^{-5}}{C_\alpha} \right) \left( \frac{\epsilon_\nu}{k_0} \right)^2 \left( \frac{N\gamma}{k_0} \right) \times \left( \frac{f_{\text{pulse}}}{k_0} \right) \times \left( \frac{f_{\text{pulse}}}{k_0} \right) \times \left( \frac{f_{\text{bunch}}}{k_0} \right) \times \left( \frac{f_{\text{bunch}}}{k_0} \right)
\]  

(51)

where \(N\gamma\) is the number of photons in a laser pulse

\[
N\gamma = \frac{Q(k)}{k_0} \sigma_{\text{laser}} \tau_{\text{pulse}} = \frac{e_{\text{pulse}}}{k_0},
\]  

(52)

and the total energy of a laser pulse \(e_{\text{pulse}} = Q(k)\sigma_{\text{laser}}\tau_{\text{pulse}}\). The averaged power of linearly polarized laser beam is approximately given by \(P_{\text{laser}} = f_{\text{pulse}}\epsilon_{\text{pulse}}\), then Eq. (51) becomes

\[
R_{\nu\gamma} \approx 10^{-26} \left( \frac{\bar{F}_e}{4 \pi^2 \text{GeV cm}^{-2} \text{s}^{-1}} \right) \left( \frac{\Delta t}{10^{-8} \text{s}} \right) \left( \frac{10^{-5}}{C_\alpha} \right) \left( \frac{f_{\text{pulse}}}{k_0} \right) \times \left( \frac{f_{\text{bunch}}}{k_0} \right) \times \left( \frac{f_{\text{bunch}}}{k_0} \right).
\]  

(53)
As a result, with a neutrino beam $\bar{F}_\nu \sim 10^4 \text{ GeV cm}^{-2} \text{s}^{-1}$ and a linearly polarized laser beam of energy $k_0 \sim \text{eV}$ and power $P_{\text{laser}} \simeq 10 \text{ MW}$, the rate of generating circularly polarized photons $R_V \sim 1/\text{s}$ ($\sim 3 \times 10^7/\text{year}$). This rate should be large enough for observations.

9. Conclusion and remark

We show and discuss the reason why a linearly polarized photon acquires its circular polarization by interacting with a neutrino is due to the fact that the neutrino is left-handed and possesses chiral gauge-coupling to gauge bosons. Calculating the ratio of linear and circular polarizations of photons interacting with either Dirac or Majorana neutrinos, we obtain that this ratio in the case of Dirac neutrinos is about twice less than the ratio in the case of Majorana neutrinos. Based on quantitative value of this ratio, we discuss the possibility of using advanced laser facilities and the T2K neutrino experiment to measure the circular polarization of laser beam interacting with neutrino beam in ground laboratories.

In Ref. [14], the rate of neutrino–photon scattering was calculated by the Feynman diagram of one fermion-loop with exchanging a $Z_\mu^0$-boson (neutral current channel). This rate is very small because the obtained cross-section of photon–neutrino is the order of $(\alpha G_F m_\nu)^2$, and it was concluded that the total probability for photon–neutrino scattering is too small to be observed (having an event rate below 1/year), even at PW laser powers. In our case, the time evaluation of the $V$-parameter (12) for the photon circular polarization can be simply written as

$$\frac{dV}{dt} = [H_0^V, V] + \int dt \left[H_0^V, \left[H_0^V, V\right]\right],$$

where the leading non-trivial contribution comes from the forward scattering amplitude $[H_0^V, V]$, which is the order of $(\alpha G_F Q)$, instead of $(\alpha G_F Q)^2$. Whereas the second term $\int dt \left[H_0^V, \left[H_0^V, V\right]\right]$ corresponds to the high-order contributions. This implies that our results (44) and (48) of generating circular polarization of laser beam by interacting with neutrino beam might be experimentally measurable. This could be an additional and useful way to gain some insight into the physics of neutrinos, for instance their Dirac or Majorana nature, moreover their mixing with the right-handed sterile neutrino [16].

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