CP from strings: ideas and problems

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CP has a natural embedding in superstring models as a gauge symmetry involving inversion of the compactified space. Hence the source of CP violation could be geometrical. Such models face the problem of how to suppress contributions to fermion electric dipole moments from softly-broken supersymmetry, as well as other problems of string phenomenology. Stringy symmetries are useful in evaluating models, and rule out some scenarios.

In this talk I will describe some aspects of CP violation within string models and low-energy supersymmetry (SUSY): the two are connected since it is extremely difficult to construct consistent string models without some supersymmetry. First I describe some established results which show that CP is an exact symmetry of the underlying theory, and has an elegant interpretation in geometrical terms. This motivates models of spontaneous CP violation through geometric moduli, which appear in the low-energy effective field theory as scalar singlets with nonrenormalisable couplings to Standard Model (SM) fields. I describe the “SUSY CP problem”, the fact that superpartner loops are expected to generate fermion electric dipole moments (EDM’s) in excess of experimental bounds, and discuss some possible solutions. I also remind the reader of the problems encountered by any attempt to describe details of low-energy physics within string theory. Finally I report a recent result which allows string model results to be interpreted without detailed calculation, and in many cases ruled out as sources of CP violation.

1 CP as gauge symmetry and as geometry

CP symmetry in four dimensions reverses the chirality and U(1) charges of fermions and the orientation of spacetime: surprisingly, if we consider the ten-dimensional spacetime and enlarged gauge group of heterotic or Type I string theory, CP can be embedded as a discrete subgroup of the gauge symmetry of the full theory. Fermion representations in the 10d Lorentz group must
be Majorana, since the fermion must live in the same irrep as its complex conjugate: in fact, \( \mathcal{N} = 1 \) string has Majorana-Weyl fermions. Gauge group representations must be such that each irrep containing a field with \( U(1) \) charge(s) \( q_a \) also contains a field with charge \(-q_a\): the irreps of \( E_8 \) and \( SO(32) \), among others, satisfy this condition automatically. Finally, the 4d parity transformation must correspond to a proper Lorentz transformation in 10d, which is achieved by taking CP to act by inversion on the 6-dimensional compact space, by a 6-dimensional Lorentz transformation with negative determinant \( \Gamma \). As a discrete gauge symmetry, CP is conserved even nonperturbatively in string interactions, hence CP-violating couplings including \( \theta_{QCD} \) are in principle calculable given a particular mechanism for symmetry-breaking at low energies.

An intriguing possibility is for CP to be broken by extra-dimensional geometry: if there is no inversion under which the 6-dimensional space, possibly including vector or tensor field backgrounds along the compact directions, is invariant, CP is violated spontaneously. One would expect low-energy couplings which depend on this geometry, for example quark Yukawas or soft SUSY-breaking terms, to be complex. In general, not enough is known about the 6d space for the low-energy parameters to be found explicitly (even the metric of Calabi-Yau spaces is in general unknown), but in simple, somewhat unrealistic examples this can be done, with interesting results which may have implications for models closer to the real world.

Orbifolds are the most intensively-studied examples of string compactification with \( \mathcal{N} = 1 \) supersymmetry: they are simply 2n-dimensional flat space with identifications under the action of a discrete group. For abelian orbifolds this group consists of six translation generators, which convert \( \mathbb{R}^6 \simeq \mathbb{C}^3 \) into the torus \( T^6 \), and a (product of) discrete rotations \( Z_N(\times Z_M) \) which acts as \( z^\alpha \mapsto e^{2\pi i k^\alpha / N(M)} z^\alpha \) on the three complex coordinates \( z^\alpha, \alpha = 1, 2, 3 \). Then CP acts as \( z^\alpha \mapsto z^\alpha \ast \) (which may be in general be combined with a 6-dimensional rotation). In any orbifold the antisymmetric tensor field \( B^{MN} \) in the compact directions is a free parameter which is easily seen to be CP-odd. For complex planes where the orbifold rotation is \( Z_2 \), the relative orientation of the two basis vectors of the torus (defined as 1 and \( \tau^\alpha \) when normalised to the overall radius \( R \)) can be CP-odd if \( \text{Re} \tau^\alpha \neq 0 \). By convention, these quantities are combined with other geometric parameters into the Kähler moduli \( T^\alpha = 2(R^{\alpha 2} + iB^\alpha) \) where \( B^\alpha = B^{MN} \), \( M = N - 1 = 2\alpha - 1 \), and the complex structure moduli \( U^\alpha \sim \tau^\alpha / i \), which transform into their complex conjugates under CP. Then in simple orbifold models, Yukawas and soft terms are relatively easy to compute as functions of \( T^\alpha \) and \( U^\alpha \). Unfortunately, none of these models gives a good account of the quark sector without further elaboration: either there are more than three generations, or the Yukawa couplings allowed by selection rules cannot give rise to a CKM phase (see e.g.\( ^a \)). Then either the extra generations must be decoupled in some way, or higher-order nonrenormalizable couplings involving charged scalar v.e.v.’s must play a role in generating the Yukawa textures (e.g.\( ^b \)): either alternative is likely to make the models more difficult, if not impossible, to compute reliably.

### 1.1 CP as scalar v.e.v.’s

Supersymmetric string vacua have vanishing cosmological constant to all orders in perturbation theory, therefore the effective potential on the space of continuous parameters, including the moduli, is flat. Thus one can consider \( T^\alpha \) and \( U^\alpha \) as scalar fields massless before SUSY-breaking, which appear in the low-energy effective theory. After supersymmetry-breaking in a hidden sector by some nonperturbative mechanism such as gaugino condensation, the moduli (and in general the dilaton \( S \), which gives the gauge kinetic function at tree level) become stabilized in a nontrivial potential and receive \( F \)-terms, resulting in soft breaking in the observable

\(^a\)It was already noted that this inversion was a natural realization of CP.

\(^b\)The possibility also exists that CP is broken by discrete parameters of the compactification, which would not correspond to scalar degrees of freedom.
The moduli couple in general in a flavour-dependent way to MSSM matter, thus the expectation is for non-universality and CP violation in the soft terms (even if soft scalar masses are degenerate), which would likely lead to new physics signals beyond the “minimal flavour violation” benchmark. The apparent lack of such signals in CP and flavour-changing observables thus represents a challenge for such models.

2 Problems of CP violation in SUSY

This challenge is most marked for fermion EDM’s, where extremely sensitive experimental limits exist\(\textsuperscript{8}\): the SM contribution is negligible, but superpartner loops induce EDM’s two or more orders of magnitude above the limits, for soft terms of a few hundred GeV in magnitude with complex phases of order unity\(\textsuperscript{9}\). The soft terms involved are the gaugino masses \(M_i \lambda_i \lambda_i\), \(i = 1, 2, 3\), the scalar bilinear coupling \(B \mu H_U H_D\) (where \(\mu H_U H_D\) is the effective superpotential coupling generating Higgsino masses) and the scalar trilinear \(A\)-terms \(A^u_{ij} q_i \bar{c}_j H_U + (u \rightarrow d) + (q \rightarrow l)\) where the down quark and charged lepton terms have a similar form. The combinations \(BM^\ast_i \mu_i / |B|\) and \(A^\text{SCKM}_{11} M^i_1\) are invariant under phase redefinitions and appear directly in the expressions for loop-induced EDM’s. Here \(A^\text{SCKM}_{11}\) is the (11) element, for the up, down and lepton sectors, in the “super-CKM” basis where fermion masses and fermion-sfermion-gaugino couplings are diagonal. Then the phase of \(BM^\ast_i\) should be less than \(10^{-2}\), and the imaginary parts of \(A^\text{SCKM}_{11} M^i_1\) somewhat less than \(10^{-6}\), to satisfy experimental bounds, in the absence of cancellations which appear increasingly unlikely given improved bounds on the mercury EDM (see\(\textsuperscript{8}\) third reference). In the limit of universal soft terms, the problem remains but is less severe for the \(A\)-terms: the (11) element is now proportional to the small quark or lepton masses \(m_u,d,e\), automatically inducing a suppression of order \(10^{-5}\). Still, in almost all cases the soft phases have to be small, in contrast to the large Yukawa phases which are usually taken to generate CP violation in the quark sector. Radiative effects calculated by RG running can have an important effect on these bounds, since the \(A\)-terms obtain a large contribution proportional to gaugino masses, which “dilutes” the imaginary parts of \(A M^i_1\), on running from the GUT to the electroweak scale; the large “top” and “bottom” entries \(A^u_{33}\) also give contributions to the imaginary parts of \(A_{11}\) and \(B\mu\), which may be significant.

There are many ways to get around this problem in SUSY, but none of them is strictly motivated within string theory. Perhaps the most obvious is to arrange for the scalar superpartners to be heavy\(\textsuperscript{14}\) (a few TeV is sufficient) suppressing the loop diagrams; this can be done without unacceptable fine-tuning of the electroweak sector if the third generation superpartners remain light\(\textsuperscript{14}\).

If we allow an extended gauge group, one simple solution is to embed MSSM into a parity-symmetric (left-right) SUSY model at the seesaw scale\(\textsuperscript{12}\). This model leads to hermitian Yukawas and \(A\)-terms, real gluino masses, \(\mu\) and \(B\mu\) terms so that it can solve both the SUSY CP and strong CP problems; many authors have pointed out that left-right symmetric models can emerge from strings, although it is not clear that left-right symmetry can be broken in the required way in these constructions.

“Approximate CP” is the proposal, motivated by spontaneous breaking of exact CP symmetry, that all complex phases are small: suppressed EDM’s are thus natural in the t’Hooft sense, but the KM phase is also taken\(\textsuperscript{14}\) to be small, thus one requires additional sources of CP violation in flavour-changing interactions\(\textsuperscript{15}\). This is possible, if somewhat complicated, to arrange in the \(K^0\) system, consistent with small phases, but the prediction for the \(B_d\) decay asymmetry \(a_{K^0} f_{K^\pm} f_{K^\mp}\) is small (< 0.1), thus approximate CP as usually formulated is ruled out by recent results\(\textsuperscript{16}\). However, it is worth noting that the effect of small Yukawa phases is strongly dependent on the

\(\textsuperscript{8}\)However, new contributions to EDM’s have recently been found\(\textsuperscript{13}\) that do not decouple in the heavy superpartner limit and may be dangerous for order (1) phases.
model of flavour (i.e. on the basis of quark states in which the phases are introduced). It is even possible to formulate a large KM phase as a small perturbation away from a CP-invariant theory, since it can result from Yukawa phases of order $10^{-3}$ in a democratic theory of flavour (where all Yukawa couplings are approximately equal): the Jarlskog parameter $J \simeq 2 \times 10^{-5}$ is then a more appropriate measure of CP violation in the SM. One can also reformulate approximate CP by imposing small imaginary parts, which would generically result in small soft phases and a large KM phase.

3 Problems of strings

We already noted that it is difficult to generate correct Yukawa textures reliably in three-generation heterotic models. “Pseudo-anomalous” U(1) groups, where the anomaly is cancelled by a Green-Schwarz mechanism, are common (see e.g.); in order to cancel the resulting F-I D-term, some charged scalars get v.e.v.’s, thus in the stabilized vacuum, nonrenormalizable superpotential terms may contribute significantly to the effective Yukawa couplings. The form of such terms is prohibitively hard to calculate, so one can do little more than estimate their magnitude: note that charged scalar v.e.v.’s may also be a source of CP violation. In the presence of many potential sources, a reasonable strategy is to evaluate each one in turn and only consider more complicated cases when the simple ones are ruled out. It is in this spirit that recent investigations of moduli as the source of CP violation have been carried out.

More serious and wide-reaching is the problem of vacuum selection, which encompasses both the choice between discrete (topologically different) compactifications and the stabilization of the many continuous flat directions. Without a model in which the SM or MSSM is obtained “cleanly” (i.e. without extra or exotic matter), one is usually reduced to calculating Yukawas and soft terms with a somewhat arbitrary assignment of the (MS)SM fields, thus apart from very general properties it is unclear how much credence should be given to the results.

Supersymmetry-breaking is another, related area of uncertainty: since it is necessarily nonperturbative in nature, it cannot be treated directly in string theory, thus one must use low-energy approximations which may hide some important features. The constraints that must be satisfied by any mechanism are to stabilize the moduli, dilaton and other scalars, with sufficiently large masses, and give soft masses to the superpartners consistent with bounds from electroweak symmetry-breaking, FCNC, etc.; in addition, one requires to the vacuum energy to vanish to good accuracy for the model to be self-consistent. To do all this turns out to be extremely difficult, such that one needs to introduce additional parameters, whose meaning in terms of string theory is unclear, and which must be set by hand to get reasonable results.

4 Results from stringy symmetry

Still, one can use the special properties of string models to diagnose whether some specific scenarios of CP violation are viable, without detailed calculation. Many simple heterotic compactifications have a duality group $\Gamma$ acting on the geometric moduli: this means that doing string perturbation theory on one background \{\(T, U\)\} gives the same result as on another background \{\(\gamma(T), \gamma(U)\)\}, where \(\gamma(T(U))\) is the action of a group element \(\gamma\) on \(T(U)\) (suppressing the $\alpha$ labels). If we restrict attention to group elements which do not mix complex planes, then we always have three separate $\text{SL}(2, \mathbb{Z})$ groups acting on the $T^\alpha$ as

\[
\gamma(a^\alpha, b^\alpha, c^\alpha, d^\alpha) : T^\alpha \mapsto \frac{a^\alpha T^\alpha - i b^\alpha}{i a^\alpha T^\alpha + d^\alpha}, \quad a^\alpha d^\alpha - b^\alpha c^\alpha = 1 \quad (1)
\]

for integer $a, b, c, d$, generated by $T \mapsto 1/T$ and $T \mapsto T + i$. Then, the low-energy physics at one v.e.v. $\langle T \rangle = T_0$ should be the same as at $\langle T \rangle = \gamma(T_0)$. Calculations of $T$-dependent gauge and
Yukawa couplings are consistent with this symmetry, called modular invariance. The low-energy supergravity effective is invariant under this replacement as expected, when matter fields are also given appropriate transformation properties. All possible values of $T$, and therefore all possible values of Yukawas and soft terms for a particular string model, can be reached by a modular transformation from precisely one point inside the fundamental domain $\mathcal{F}$, depicted in Fig. 1.

Then, given the exact CP symmetry of the underlying theory, it is relatively simple to show that moduli v.e.v.'s on the boundary of $\mathcal{F}$ cannot be a source of CP violation, since they are modular images of their own complex conjugates (under $T \mapsto 1/T$ or $T \mapsto T + i$), therefore describe backgrounds for which physics is unchanged by CP transformation. If SM matter fields are in twisted sectors, which are localised at fixed points of the $\mathbb{Z}_N \times \mathbb{Z}_M$ group, then they mix with each other under duality, ensuring that the total effective Lagrangian is invariant: this mixing, which is unavoidably nonabelian, makes it somewhat subtle to prove the result, particularly in the case of models with more than three generations where some twisted matter decouples.

If one assumes that the extra states become heavy in the original “twist” basis without mixing with the three light generations, then the CKM matrix might not in principle be a modular invariant smooth function of $T$, and CP-violating quantities might not vanish on $\mathcal{F}$, because the modular transformation would not act unitarily on the light generations. Then one would have to switch to a different basis of light states after modular transformation, for which the CKM matrix was a different function of $T$; but the value of the new function at $\langle T \rangle = \gamma(T_0)$ would be equal to the value of the old function for $\langle T \rangle = T_0$, confirming that physics is unchanged under duality. In fact, in any reasonable situation, mass matrices in the twist basis must have off-diagonal entries, and thus are diagonalised by modulus-dependent matrices. As a result, the mass eigenstates are formally invariant under duality: thus, the couplings of the Lagrangian written in the mass eigenstate basis are smooth $\text{SL}(2, \mathbb{Z})$ invariant functions of $T$, and the CP-odd part of any such function vanishes on the boundary of $\mathcal{F}$. The dilaton may be a source of CP violation consistent with modular symmetry, but it cannot play a significant role in the observed effects, since its couplings cannot induce CP violation in flavour-changing terms. The construction of modular invariant eigenstates may have interesting consequences for flavour in...
models with twisted matter.

This result is significant because most models of modulus stabilization consistent with modular invariance result either in real $\langle T \rangle$ or in exactly these boundary values, (with, however, some exceptions [16]). It has been suggested that complex values of $T$ on the unit circle could form part of a solution of the SUSY CP problem, since $F^T$ then vanishes in most cases, but then there must be additional sources of CP violation in the model, which may or may not give additional contributions to soft terms. If $\langle T \rangle$ is sufficiently close to the boundary, then CP violation in both Yukawas and soft terms is expected to be small, motivating approximate CP; however, as remarked above, this scenario is likely ruled out without some special type of flavour structure.

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