Efficient generation of distant atom entanglement

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We show how the entanglement of two atoms, trapped in distant separate cavities, can be generated with arbitrarily high probability of success. The scheme proposed employs sudden excitation of the atoms proving that the weakly driven condition is not necessary to obtain the success rate close to unity. The modified scheme works properly even if each cavity contains many atoms interacting with the cavity modes. We also show that our method is robust against the spontaneous atomic decay.

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Many quantum information tasks require an entanglement, especially an entanglement shared by distant atoms can play a very important role in quantum information processing. This is due to the fact that atomic states are ideal for quantum information storage. Therefore, a variety of schemes for entanglement of distant atoms have been proposed recently [1, 2, 3, 4, 5, 6, 7, 8, 9]. The schemes employ also photonic states providing fast quantum information transfer over long distances. Most of the schemes describe two cavities, each containing one trapped atom. The photons leaking out from the cavities are mixed at a beam splitter and detected [2, 3, 4, 5, 7, 8]. In those schemes, however, only two of the four Bell states can be used, and therefore, the success rate is less than 50% [10, 11]. Moreover, the success rate is lowered by the spontaneous atomic emissions. In most of the schemes the population of the excited state is considerable [3, 4, 5, 6] during the entangling operations and can therefore drastically lower the success rate as it has been proved in [12]. Only the proposal of Browne et al. [2] avoids all of the above problems. The whole operation is performed there in such a way that the population of the excited state is negligible thanks to the use of large detunings. Furthermore, the scheme uses the requirement of weak driving since a sudden excitation of the atoms limits the entanglement efficiency to 50% as is suggested in [3]. However, the condition makes it impossible to perform many operation requiring strong driving, and therefore, it can be difficult to use the entangled atoms in quantum computations. It is possible to change this condition by controlling laser intensity but then the entanglement operation time will be long.

In this paper a scheme is presented that employs sudden excitation to entangle two atoms with high success probability. The main idea of the scheme is to use a protocol which first prepares each cavity in one photon state, next creates maximally entangled state of both cavity fields detecting one photon decay from the cavities and finally maps the entangled state onto two distant atoms. The strong driving condition makes it possible to use the entangled distant atoms in various quantum information tasks. The setup consists of two cavities, a 50-50 beam splitter, two lasers ($L_A$ and $L_B$) and two detectors ($D_+$ and $D_-$) as depicted in figure 1. Quantum computations require also many qubits [13, 14] thus each cavity can contain up to $N$ atoms. Each atom is modeled by a three-level A system with one excited state $|2\rangle$ and two ground states $|0\rangle$ and $|1\rangle$. The energy level structure of the atom is shown in figure 2. The spontaneous decay rate of the excited state is denoted by $\gamma$. There are two transitions in the A-type atom. First of them, the $|1\rangle \leftrightarrow |2\rangle$ transition, is driven by classical laser field with the coupling strength $\Omega$. The frequency of the laser field is $\omega_L$. The second, the $|0\rangle \leftrightarrow |2\rangle$ transition, is coupled to the cavity mode with a frequency $\omega_{cav}$ and coupling strength $g$. Both the classical laser field and the quantized cavity mode are detuned from the corresponding transitions. The two detunings are given by $\Delta = (E_2 - E_1)/\hbar - \omega_L$ and $\Delta' = (E_2 - E_0)/\hbar - \omega_{cav}$. The evolution of the quantum system is conditional. During the time intervals when no photon decay is detected, the evolution is governed by the effective non-Hermitian Hamiltonian ($\hbar = 1$ here)}
FIG. 2: Level scheme of one of the Λ atoms interacting with the classical laser field and the quantized cavity mode.

and in the following

$$H = \sum_k (\Delta - i\gamma)\sigma_{22}^{(k)} - \sum_k \Delta_r \sigma_{00}^{(k)} - i\kappa a^\dagger a$$

$$+ \sum_k (\Omega\sigma_{21}^{(k)} + ga\sigma_{20}^{(k)} + \text{H.c.}),$$

where $a$ denotes the annihilation operator for the cavity mode, $\kappa$ is the cavity decay rate, $\Delta_r = \Delta' - \Delta$ and $k$ indicates the atom. In expression (1) the flip operators $\sigma_{ij}^{(k)} \equiv |i\rangle\langle j|_k$, where $i, j = 0, 1, 2$, are also used. The evolution generated by (1) is interrupted by collapses corresponding to the action of the operator

$$C = \sqrt{\kappa}(a_A + a_B),$$

where $a_A$ and $a_B$ denote the annihilation operators for $A$ and $B$ cavity modes, respectively, and $\epsilon$ is equal to unity for the photon detection in $D_+$ and minus unity for the photon detection in $D_-$. The Hamiltonian takes a simpler form in the large detunings limit ($\Delta \gg \Omega$ and $\Delta' \gg g$), when the excited state can be adiabatically eliminated [14]. In order to avoid the lowering of the success probability by spontaneous atomic decay, it is necessary to assume that $\gamma \ll \Delta, \Delta'$ and $\gamma g^2/\Delta^2, \gamma \Omega^2/\Delta^2 \ll \kappa$. Then the Hamiltonian (1) passes into

$$H = -\sum_k (\Delta_r \sigma_{00}^{(k)} + z_1 \sigma_{11}^{(k)} + z_2 a^\dagger a \sigma_{00}^{(k)})$$

$$- \sum_k (z_3 a^\dagger a \sigma_{10}^{(k)} + \text{H.c.}) - i\kappa a^\dagger a,$$

where $z_1 = \Omega^2/\Delta$, $z_2 = g^2/\Delta'$ and $z_3 = \frac{1}{2} \Omega g (\Delta'^{-1} + \Delta^{-1})$. As mentioned above, it would be very useful to entangle two distant atoms within the strong driving limit. Therefore, we assume $z_3 \gg \kappa$. In order to further simplify the problem we now assume that there is only one atom inside each cavity. This allows us to assume that $\Omega = g$ and $\Delta_r = 0$. We will later extend the model over the case of many atoms. Under the conditions, the Hamiltonian (3) takes the form

$$H = -z a^\dagger a a^\dagger a \sigma_{00} - (za a a^\dagger a^\dagger a + \text{H.c.}) - i\kappa a^\dagger a.$$

The protocol needs two local operations. The first of them is to map the atomic state onto the cavity mode and the second of them is to map the photonic state in the atom. The local operations necessary to achieve the entanglement can be obtained via $e^{-iHt}$. Let us denote by $|jn\rangle \equiv |j\rangle \otimes |n\rangle$ a state of the system consisting of one atom in the state $|j\rangle$ trapped inside a cavity with $n$ photons. One can perform the two local operations by illuminating the atom for times $t_1$ and $t_2$

$$|10\rangle \rightarrow ie^{izt_1}e^{-\kappa t_1} |01\rangle,$$

$$|01\rangle \rightarrow ie^{izt_2}e^{-\kappa t_2} |10\rangle,$$

where $t_1 = \frac{\pi}{4} \left[\pi - \arctan\left(\frac{\Omega}{\kappa}\right)\right]$, $t_2 = \frac{\pi}{4} \arctan\left(\frac{\Omega}{\kappa}\right)$ and $\Omega_1 = \sqrt{4z^2 - \kappa^2}$. When the laser is turned off, the system’s evolution is given by

$$e^{-iHt} |10\rangle = |10\rangle,$$

$$e^{-iHt} |01\rangle = e^{izt}e^{-\gamma t} |01\rangle. \quad (4)$$

The state $|00\rangle$ always remains unchanged even if the laser is turned on.

At the beginning of the protocol, both cavity fields are empty and both atoms are prepared in the ground state $|1\rangle$. Thus, the initial state is given by $|10\rangle_A \otimes |10\rangle_B$. The protocol consists of four stages.

(i) In the first stage of the protocol the two atoms are illuminated by the lasers for the time $t_1$. On condition that no photon detection occurs during the time, the state becomes $|01\rangle_A \otimes |01\rangle_B$. Then, one should begin the detection stage which is the second stage of the protocol. However, if one collapse has been detected during the illuminating time $t_1$, the jump operator $C$ acts on the state. In this case, we obtain the entangled state of both cavity fields $(|00\rangle_A |01\rangle_B + \epsilon |01\rangle_A |00\rangle_B)$ and the detection stage is superfluous. It means that the third stage of the protocol should be started. The detection of two photons at the stage means that the entanglement procedure has failed.

(ii) In the second stage, one waits until the click is recorded in either of the detectors. During the detection time the lasers are turned off and the evolution is given by (4). Detection of one photon corresponds to the action of the jump operator $C$ and thus the state becomes $(|00\rangle_A |01\rangle_B + \epsilon |01\rangle_A |00\rangle_B)$. After the detection event, the third stage of the protocol should be started immediately.

(iii) The third stage is responsible for mapping and storage of the entangled state of both cavity fields in the state of the two atoms. This can be realized by turning the lasers on for the period of time $t_2$. After this operation the state is given by $(|00\rangle_A |10\rangle_B + \epsilon |10\rangle_A |00\rangle_B)$. If any collapse is detected during this stage the entangling process is unsuccessful.

(iv) If $D_+$ clicks, the whole protocol is over because $\epsilon = 1$. However, if $D_-$ clicks one has to remove the
phase shift factor using the Zeeman evolution. This is the objective of the fourth stage.

Finally, the entangled state of the two distant atoms is obtained. The probability that the protocol will be successful, under the strong driving condition, can be well approximated by

\[ P_{\text{suc}} = e^{-\alpha \pi} (2 - e^{-\alpha \pi}) , \]  

where \( \alpha = \kappa / z \). Figure 3 shows that the probability of success tends to unity with decreasing \( \alpha \). For reference, we also calculate numerically the probability of success using the quantum trajectory theory \([13,19]\). We use the non-Hermitian Hamiltonian \( H \) in all numerical computations. The parameters \((\Delta; \Delta'; \Omega; g; \gamma) / 2\pi = (300; 300; 25; 25; 0) \) MHz are chosen in such a way that all the aforementioned assumptions \((\Delta_r = 0; \Delta \gg \gamma; \Omega = g; \Delta \gg \Omega; z \gg \kappa \gg \gamma \Omega^2 / \Delta^2) \) are satisfied. We find that the average fidelity of the entanglement is about 0.99 and the average probability of success is about 0.94. The averages are taken over twenty thousand trajectories. Moreover, we set \( \gamma \) to zero to verify the analytical expression describing the success rate given by (5). In figure 3, the points show the average probability of success over twenty thousand trajectories calculated for different values of \( \alpha \). As one can see, the analytical results are in a remarkable agreement with the numerical solution. We have also investigated the influence of the spontaneous decay rate of the excited state on the entanglement. We plot in figure 4 the average probability of success as a function of the spontaneous decay rate. As evident from the figure, the probability of success decreases with increasing \( \gamma \). Figure 5 shows the influence of the decay rate on the entanglement fidelity. Surprisingly, an increase in the spontaneous decay rate leads to an improvement in the fidelity. This is due to the fact that the fidelity depends on the population of the excited state. If saturation parameters \((g^2 / \Delta^2, \Omega^2 / \Delta^2) \) are not small enough, the population can be lowered by a sufficiently high spontaneous decay rate.

Let us now consider the case of many atoms present in each cavity. All atoms, except the two distant ones that are to be entangled, can be in arbitrary states. Unfortunately, all atoms that are in state \(|0\rangle \) interact with the cavity mode. This makes it impossible to perform any operation on a separate atom without changing the other atoms' states. One can avoid the problem using the \( z_3 \gg z_2 \) condition, which is equivalent to the assumption \( \Omega \gg g \). It is also necessary to assume that \( \Delta_r = \Delta_1 \). In the above limits, the evolution generated by \( H \) can be used to obtain the necessary local operations. Let us denote the state of all atoms that are not intended to be entangled with a distant atom by \(|\Psi\rangle\). We assume that a number of the atoms in state \(|0\rangle \) which do not participate in the entanglement process is \( N_0 \). We can approximately perform a quantum state mapping by illuminating the atom for the time \( t_3 = \pi / (2z_3) \):

\[ |\Phi\rangle|10\rangle \rightarrow i e^{i \Delta_r (N_0+1)t_3} e^{-i \Delta_1 (N_0+1)z_3 t_3} e^{i N_0 z_3 t_3} |\Phi\rangle|01\rangle, \]

\[ |\Phi\rangle|01\rangle \rightarrow i e^{i \Delta_r (N_0+1)t_3} e^{-i \Delta_1 (N_0+1)z_3 t_3} |\Phi\rangle|10\rangle. \]

When the laser is turned off, the evolution of the states
is given by

\[
e^{-i\Delta t}\Phi|10\rangle = e^{i\Delta rN_0t}\Phi|10\rangle,
\]

\[
e^{-i\Delta t}\Phi|01\rangle = e^{i(\Delta + z_2)(N_0 + 1)}e^{-\kappa t}\Phi|01\rangle.
\]

If the atom is in the ground state \(|0\rangle\) and the cavity field is empty, the population of the state always remains unchanged (even if laser is turned on)

\[
e^{-i\Delta t}\Phi|00\rangle = e^{i\Delta r(N_0 + 1)}t\Phi|00\rangle.
\]

Now let us present a modified scheme. The initial state is given by \(|\Phi\rangle_A|10\rangle_A \otimes |\Phi\rangle_B|10\rangle_B\). The new protocol consists of three steps.

(i) Illumination of chosen distant atoms (one from cavity \(A\) and one from cavity \(B\)) for the time \(t\). On condition that no photon decay has been recorded, the state becomes \(|\Phi\rangle_A|01\rangle_A \otimes |\Phi\rangle_B|10\rangle_B\). If one photon has been detected in time \(t\), the state is \(|\Phi\rangle_A|\Phi\rangle_B(001)A|01\rangle_B + e\theta(t_3 - t)\rangle A|00\rangle_B\) where \(\theta(t) = \exp[i\frac{\Delta t}{2}(N_0 - N_0)]\). In this case the third stage should be started.

(ii) Waiting for one photon decay. After detection the state becomes \(|\Phi\rangle_A|\Phi\rangle_B|00\rangle_A|01\rangle_B + e|01\rangle_A|00\rangle_B\).

(iii) Mapping and storage of the entangled state of both cavity fields in the two distant atoms by illuminating them for the time \(t_3\). The illuminating operations do not start simultaneously. The laser \(L_B\) is turned on after a delay \(t_\phi\). The state is then given by \(|\Phi\rangle_A|\Phi\rangle_B|(001)A|00\rangle_B\), where \(\phi = e\theta(2t_3 - t)\exp[-i\Delta r t_\phi]\) if photon decay has registered in the first stage and \(\phi = e\theta(t_3)\exp[-i\Delta r t_\phi]\) for photon detection in the second stage. We choose such a time \(t_3\) that \(\phi = 1\).

The probability of success of the modified protocol is given by expression [12] with \(\alpha = \kappa/z_3\). Finally, we have performed the numerical calculations for the parameters \((\Delta; \Delta'; \Omega; g; \kappa; \gamma)/2\pi = (1000; 1000.9; 30; 0.7; 0.001; 0.1)\ MHz which satisfy all the above assumptions \((\Delta_1 = \Delta_3; \Delta \gg \Omega \gg g; \Delta \gg \gamma; z_3 > \kappa > \gamma\Omega^2/\Delta^2)\). We assume that there are three atoms inside each cavity as in figure [14]. We have generated one thousand trajectories to compute the average of the probability of success and the average of the fidelity. For each trajectory we have generated random numbers \(N_{0A}, N_{0B} \in \{0, 1, 2\}\) and corresponding to them random initial states \(|\Phi\rangle_A, |\Phi\rangle_B \in \{|1\rangle|1\rangle|2\rangle, c_1|1\rangle|0\rangle|2\rangle + c_2|0\rangle|1\rangle|2\rangle, |0\rangle|1\rangle|0\rangle\rangle\}, where \(c_1\) and \(c_2\) are arbitrary complex amplitudes. We have obtained the fidelity of 0.99 and the success rate of 0.90.

In conclusion, we have presented a scheme to create an entangled state of two distant atoms. We have shown that the probability of success can be made arbitrarily close to unity without the weakly driven condition. The scheme works properly even if there are many atoms inside each cavity. We have also investigated the influence of the spontaneous decay rate of the excited state on the entanglement and we have found that with the increasing rate the probability of success is slightly lower and the entanglement fidelity is better.

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