Analytical and Numerical Approach on Design of Cageless Open Differential Unit

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Abstract. A differential is a gear train that transmits an engine’s torque to the wheels. During a turn, the outer and inner wheels of the vehicle are forced to travel along paths of different radii. A differential allows the outer driving wheel to rotate at a faster speed when compared to the inner driving wheel during a turn. It is designed such that, increase in speed of one wheel is balanced by a decrease in speed of the other. This ensures that the vehicle can negotiate a turning without slipping. The gears in the differential are supported by a cage which results in its bulky appearance and heavy form. This causes a number of disadvantages such as difference in length of the driveshafts and offset center of mass of the system due to asymmetrical design. Such a cage also adds additional weight to the vehicle and increases fuel consumption as a consequence. Its elimination results in a more compact and lightweight configuration. This paper focuses on the complete methodology of designing and analyzing a cageless differential. Various materials were considered for the assembly and one with adequate properties of strength, wear resistance and other core parameters was selected. Gear ratios were obtained through theoretical calculation and the values were used as input for designing using Solidworks software. Finite element analysis of the gear train was carried out using ANSYS Workbench to test the structural integrity and durability of the system through various types of analysis such as linear static, fatigue and explicit dynamic analysis. The results proved that the design meets the desired functionality and is an improvement to the conventional type of differential gearbox.

1. Introduction
A differential is basically a set of gears and shafts which forms a gear train unit. The unit has one input and two outputs. The main function of a differential is to vary the power between the two output shafts depending on the traction available in the wheels [11]. Differentials are more common in everyday commute cars and in off-road vehicles, where as in some drag and race cars the differential unit is replaced by a spool gear unit for achieving over-steer characteristics [13]. Some types of differentials can be manipulated or controlled to act partially as spool gear unit using mechanical or electrical means. There are various types of differentials, and each serve a specific purpose. Open differential, limited slip differential, locking differential and torsen differential are few examples. This paper specifically focuses on open differential and methodology of devising a cageless open differential.

2. Differential
2.1. Open differential.

It consists of a pinion gear, which drives a ring gear. The ring gear is attached to a cage like structure, which in turn supports the spider gears with the help of a pin. These spider gears in turn drive the output side gears, ‘figure 1’.

![Components of a differential unit](image1)

**Figure 1.** Components of a differential unit

During motion, if the coefficient of friction between surface and tire is not uniform and similar between the left and right wheels there will be a change in traction. The tire with less traction begins to slip, ‘figure 2’. The differential varies the power to the left and right wheels and helps in overcoming the slip. A main disadvantage of open differential is that, it gives more power to the wheel with less traction, thus the wheel that slips initially rotates at a higher rpm and continues to slip rather than overcoming the situation.

During a turn, the outside wheel should trace a larger radius than the inside wheel, hence there is a need for outer wheel [11][13] to run at a higher rpm than the inside wheel for smooth handling, ‘figure 3’. This is accomplished by the differential. If a differential is not used, the tires will undergo slip, which in turn generates a lateral force on the wheel assembly components and may lead to failure.

![Tyre slip due to loss of traction](image2)

**Figure 2.** Tyre slip due to loss of traction

![Difference in velocity during a turn](image3)

**Figure 3.** Difference in velocity during a turn

2.2. Cageless Differential.

In a differential the cage acts as a support for the spider gears and ring gears, and acts like a coupler between them. But the cage is heavy and bulky, in turn increasing the weight of the vehicle. In a four-wheel drive car or an all-wheel drive car, there are 3 differentials, one in the front, one in the rear and another in the centre. Thus, a noticeable amount of weight in the drive train of a car is taken up by differentials[19].

The cageless differential reduces a considerable amount of weight from the unit with no compensation in performance and strength. In a cageless differential, the axis of spider gear and pin setup is overlapped with the plane formed by pitch circle of ring gear. The pin is supported in the ring gear with the help of two restriction brackets that prevents the pin slipping along the lateral direction. The output bevels act as support and holds the spider gears in position. These spider gears rest on the pin, thus supports it in position. The pin in turn supports the ring gear, ‘figure 4’.

![Cageless Differential setup](image4)
This possesses an increased concern on the strength of the gears and pin, as they take up the entire load. Hence extra care has to be taken in selection of materials for gears. The dimensions of gears are selected based on the strength calculations and are validated using finite element analysis.

![Figure 4. Cageless differential unit](image)

3. Material Selection

3.1. Factors of Material Selection.[2]

The materials used for gears may be broadly classified into metallic and non-metallic groups. Considering the availability of the material, method of manufacturing and the area of application, metallic materials appeared to be a more viable choice. The key purpose of the material selection process is to find a material with the right combination of physical properties that meet the desired functionality of the system. In order to identify a material to satisfy the requirements of the design, certain factors are prioritized over others to help determine the most suitable one for that particular application. In this case, the primary factors are selected by considering the parameters required to calculate gear strength and surface durability.

3.2. Characteristics.[1]

The material to be selected must be capable of withstanding the various failure modes that might occur in the system such as:

3.2.1. Bending Fatigue. This failure is the result of cyclic bending stresses that are concentrated at the root of the gear tooth, ‘figure 5’. A crack initiates at the surface of the root fillet due to fatigue and propagates into the gear tooth along a direction normal to the root fillet surface. The failure process follows three stages namely crack nucleation, crack propagation and ultimately leads to fracture.

![Figure 5. Crack propagation at tooth root](image)

3.2.2. Pitting. Metal to metal contact or defects generated due to low lubricant film thickness result in the formation of surface cracks which develop into pits. Inclusions in the gear materials which act as stress concentrators generate internal cracks in the material. Pits are formed when these cracks break through the surface and cause the material to separate [6], ‘figure 6’.
3.2.3. Wear. It involves the wearing off of material from the tooth of the mating gears due to a continuous abrasive action. Wear may occur due to a number of reasons, namely insufficient lubricating film thickness, removal of the hardened layer from surface hardened gears or inclusions in the oil. Continual wear of tooth root weakens the gear and leads to fracture, ‘figure 7’.

3.3. Materials considered. [1]

3.3.1. Cast Steel. This steel is known for its impact resistance features and is what makes it more desirable than cast iron. The combination of strength and ductility proves advantageous in mechanical and structural applications.

3.3.2. Carbon Steel. Carbon steels are preferred for their superior mechanical properties and corrosion resistance when compared to cast steels. Mostly, they are heat treated to obtain the best of their qualities and are quite economical. As the carbon content increases, the steel becomes harder and stronger, but at the cost of decrease in ductility.

3.3.3. Alloy Steel. Certain alloying combinations help in developing specific properties like fracture toughness of the material. The impact strength is also considerably improved by the alloy content. Similar to plain carbon steels, alloy steels are also preferred to undergo a suitable heat treatment process to enhance its properties.

3.4. Heat Treatment Processes. [3][5]
Materials can be heat treated to improve their properties to provide better performance in their intended environment. The process must be carefully selected based on the chemical properties of the material and its area of application. Heat treatment can be carried out before or after the material is to be machined. Values specified in table 1 are those of materials subjected to a particular heat treatment process found most suitable to that material.

3.4.1. Annealing. It is considered as a pre-hardening process generally carried out to soften the material to improve its machinability. It also alters its microstructure and refines the grain size. The process relieves stress in the material and improves its ductility.
3.4.2. Hardening. This process hardens all the sections of the gear. However, uniformity of hardness should not be assumed as the outer surface often cools at a faster rate than the inner. The final hardness achieved depends on the carbon percentage of the material. This process can be carried out either before or after the gear teeth are cut.

3.4.3. Carburizing. It is a heat treatment process performed to harden only the surface of the material. The inner material structure, with a carbon percentage less than 0.15 is also hardened to some extent. Gears hardened through this method are best suited for heavy-duty service like transmission gears and are more resistant to wear, pitting and fatigue failures.

3.4.4. Nitriding. Nitriding of steels can be used for gears which require a hard, wear resistant case with good fatigue strength and where some degree of corrosion resistance is preferred. Nitriding of gears is not recommended for applications where overload is likely to occur as they become badly crushed and pitted.

3.4.5. Quenching. The material is heated up to a particular temperature and then quenched in water or oil to achieve its full hardness. The quenched material must undergo additional heat treatment processes such as be ageing, tempering or stress relieving to achieve the proper toughness and final hardness.

3.4.6. Tempering. It is mostly preferred as a post-hardening process. Once hardened, the toughness of the material is decreased. Tempering increases the toughness and ductility of the material at the cost of its hardness.

3.5. Comparison.

SC46 (AISI 1020) is a grade of cast steel with high ductility, good machinability and weldability. However, its low carbon content makes it difficult to increase its hardness by heat treatment. This material is more suitable for light duty gears. S35C (AISI 1035) and S48C (AISI 1045) are carbon steels with the prime alloying element being carbon. S35C is more resistant to corrosion owing to the small percentage of chromium included in it. S48C having higher carbon content can be heat treated to achieve higher levels of hardness easily. It is preferred over S35C for the manufacture of bolts, gears, shafts and other machine parts [4].

Both alloy steels SCM440 (AISI 4140) and SNCM439 (AISI 4340), commonly available as EN19 and EN24 respectively, show superior properties when compared to carbon steels on account of the presence of alloying elements silicon and molybdenum. EN19 and EN24 are low alloy steels having similar carbon content and can be heat treated to increase hardness levels. The presence of chromium as well as nickel in EN24 makes it more corrosion resistant as well as gives it the capacity to withstand higher temperatures [5].

| S.no. | Material (JIS standard) | Hardness (BHN) | Allowable Hertz stress (kgf/mm²) | Allowable tooth bending stress (kgf/mm²) | Cost (Rs./kg) | Heat treatment |
|-------|-------------------------|----------------|----------------------------------|----------------------------------------|--------------|----------------|
| 1.    | SC46                    | 90             | 37                               | 14.2                                   | 60           | -              |
| 2.    | S35C                    | 150            | 45                               | 16.5                                   | 65           | Quenched tempered |
| 3.    | S48C                    | 180            | 49                               | 19                                     | 65           | Quenched tempered |
| 4.    | SCM440                  | 280            | 79                               | 32                                     | 70           | Carburized, quenched tempered and tempered |
EN24 in the heat-treated condition provides a yield strength of approximately 800MPa and an ultimate tensile strength of around 1000MPa which makes it more suitable for this application. When compared with other grades, it provides better properties at a similar density of 7.85 g/cm$^3$. This material is also readily available in the “T” condition which implies that it has been heat treated. Depending upon the hardness of the material, the necessity of pre-hardening processes like annealing and post-hardening process such as tempering must be considered.[9]

4. Calculation

4.1. Required gear ratio.[11][12]
The cageless differential is designed for an AWD all-terrain concept vehicle having a spool gear box and an open differential at rear and front axles respectively. The rear and front gear ratios are finalized based on the weight distribution and torque required for climbing slope angle of 30° and towing a 2-tonne tractor.

| Vehicle Specification | |
|------------------------|-----------------------------|
| Total Mass             | 230kg                       |
| Weight distribution    |                             |
|                       | Front: 45%                  |
|                       | Rear: 55%                   |
| Wheelbase              | 51 inches                   |
| Engine Specification   |                             |
|                       | 10 hp @ 3800 rpm            |
|                       | 19.1 Nm @ 2800 rpm          |
| Transmission Type      | CVT                         |
|                       | Lower ratio: 3.9:1          |
|                       | Higher ratio: 0.5:1         |
| Wheel diameter         | 22 inches                   |
| Centre of Gravity      | 20 inches                   |

Based on the above considerations the required front and rear gear ratios are determined as 4:1 and 4:1. Based on the size constraints the differential is designed for 2.7778:1 reduction ratio and 1.44 reduction is achieved before the propeller shaft.

4.2. Strength calculation.[7][10][15]
After reduction in CVT and rear transmission box the torque transmitted by the propeller shaft is 107.26 Nm.

Based on the assembly constrains the maximum allowable outside diameter of ring gear is limited 102mm. Hence the PCD of 100 is chosen with reduction ratio of 2.353:1 is designed with 2.5 module. But the 2.5 module did not satisfy the required bending strength. On further iteration with different module and ratio, the reduction ratio of 2.7778:1 with 4 module is chosen.

| Differential Specification | |
|-----------------------------|-----------------------------|
| Type                        | Open Differential           |
| Ratio                       | 2.778:1                     |
| Ring gear                   | 25 teeth; 4 module; 100mm PCD; 15mm face width |
| Pinion gear                 | 9 teeth; 4 module; 36mm PCD; 15mm face width |
| Spider and Output gears     | 20 teeth; 2 module; 40mm PCD |

4.2.1. Straight bevel Geometry.[15]
Cone Angle
\[ \delta = \tan^{-1}\left(\frac{z_1}{z_2}\right) = 19.8 \text{ deg} \]

Cone distance

\[ R = 0.5m_r\sqrt{z_1^2 + z_2^2} \quad [15] \]

\[ R = 0.5 \times 4\sqrt{9^2 + 25^2} = 53.1413 \text{mm} \]

4.2.2. Bevel gear strength.

Bevel Gear Forces

Tangential Force, \( F_t = \frac{2\pi F}{d_{av}} \) [10]

\[ d_{av} = d_p - b \sin \alpha \]

\[ d_{av} = 36 - 15 \sin 20 = 30.87 \text{mm} \]

\[ F_t = \frac{2 \times 107.26 \times 10^3}{30.87} = 6949.2 \text{ N} \]

Radial force, \( F_r = F_t \times \tan \alpha \times \cos \delta \) [10]

\[ = 6949.2 \times \tan 20 \times \cos 19.8 = 2379.79 \text{ N} \]

Axial force, \( F_a = F_t \times \tan \alpha \times \sin \delta \) [10]

\[ = 6949.2 \times \tan 20 \times \sin 19.8 = 856.730 \text{ N} \]

Design stress

Compressive stress, \([\sigma_c] = C_B H B k_{cl} \text{N/mm}^2 \) [15]

\[ [\sigma_c] = 25 \times 290 \times 1 = 7250 \text{ kgf/cm}^2 \]

\[ = 7.11 \times 10^2 \text{ N/mm}^2 \]

Bending stress, \([\sigma_b] = \frac{1.4 \rho B}{n_k \sigma} \sigma_{-1} \text{N/mm}^2 \)

\[ [\sigma_b] = \frac{1.4 \times 1}{1.2 \times 1.1} \times 480 = 510 \text{ N/mm}^2 \]
Induced stress Calculation

Compressive stress, \( \sigma_c = \frac{0.72}{R-0.5b} \left[ \sqrt{\left( \frac{1}{E}\right)^2 \frac{F[M_r]}{b \ell b}} \right]^{1/2} \) N/mm² [15]

\[ [M_r] = k_o k_k \cdot M_t \]
\[ = 1 \times 1.2 \times 107.26 \]
\[ = 128712 \text{ N mm} \]

\[ \sigma_c = \frac{0.72}{53.1413 - 0.5 \times 15} \left[ \sqrt{\left( \frac{2.7778^2 + 1}{2.7778 \times 15} \frac{450[128712]}{15 \times 4.338}} \right]^{1/2} \]

\[ = 94.35 \text{ N/mm}^2 < 711 \text{ N/mm}^2 \]

Bending stress, \( \sigma_b = \frac{8N}{(R-0.5b)^2b_m\times b_c} \) N/mm²

\[ = \frac{53.1413 \sqrt{2.7778^2 + 1}}{(53.1413 - 0.5 \times 15)^2 15 \times 4 \times 0.338} \]

\[ = 489.6 \text{ N/mm}^2 < 510 \text{ N/mm}^2 \]

Modified Lewis equation with AGMA standards [17]

Bending stress, \( \sigma_b = \frac{2 \pi K_d K_m K_h K_x}{d_p b m J K_o K_x} \) N/mm² [17]

\[ = \frac{2 \times 107.260 \times 2.7778 \times 1 \times 1.5 \times 1}{36 \times 4 \times 15 \times 0.22 \times 1} \]

\[ = 208.8 \text{ N/mm}^2 \]

AGMA Equation [8]

Bending stress, \( \sigma_b = \frac{1000W_{t} F_t G_{x} G_{y}}{b \times m Y_{f} \gamma_{f}} \) N/m² [8]

Allowable bending stress, \( \sigma_{FP} = \frac{61 \gamma_{t} Y_{f}}{S_f K_d G_{y}} \) [8]

Bending stress, \( \sigma_b = 16 \text{N/mm}^2 \)

Allowable bending stress, \( \sigma_{FP} = 26.36 \text{N/mm}^2 \)

4.2.3. Contact stress

Contact Stress Equation, \( \sigma_H = \frac{1000W_{t}}{b d Z_i} K_a K_v K_h \beta Z_x Z_{xc}^{1/2} \) N/m² [8]
Allowable contact stress, $\sigma_{hp} = \frac{\alpha Z_{vt} Z_w}{S_{H/Ko} L_z}$ [8]

Contact Stress, $\sigma_H = 2.12kN/m^2$

Allowable contact stress, $\sigma_{hp} = 3.0 kN/m^2$

5. Modelling
Using the gear parameter values obtained in strength calculations, the designing of gears and assembly is done in Solidworks, ‘figure 8’, ‘figure 9’.

![Image](https://example.com/figure8.png)

(a) (b)

**Figure 8.** Sketching and part modelling of gear

![Image](https://example.com/figure9.png)

**Figure 9.** Exploded view of assembly

6. Analysis
After designing the unit based on the gear parameters obtained in calculations, finite element analysis was done to check the integrity of the assembly. Being a setup in which each and every component mutually supports each other and contributes for the strength of rest of the parts, analyzing the unit part wise was not advisable [21]. Hence the whole unit was analyzed together with all the parts. Static structural, fatigue and explicit dynamic analysis were done on the unit using ANSYS workbench. Nonlinear material properties were considered for static structural and fatigue analysis, and explicit material properties was considered for explicit dynamic analysis [23]. Mesh size of 4mm, linear element order and explicit physics preference was used. Medium span angle, and slow transition was fixed.[14][16]

6.1. Static structural. [18][22]
The input pinion is given a moment of 126 Nm, ‘figure 10’. Frictionless support is given in areas where bearings are used, ‘figure 11’. The output shafts are fixed in order to depict a worst-case scenario, ‘figure 12’.

A factor of safety of 1.6 is achieved. Maximum stress is induced in the pin and ring gear interface which is as expected. But the stress values induced were within the safety limits.
6.2. Fatigue

This analysis is done to find whether the unit withstands repeated and reversible loads without undergoing failure [20]. Stress life analysis type with Soderberg mean stress theory is used because the material used is a ductile material and the deciding criteria for failure is yield strength. The loading is set to fully reversed depicting the reversal of loads in transmission system when a car is put in reverse gear.
6.3. Explicit dynamic analysis.
A cylindrical coordinate system with respect to input pinion is created. This is used when giving a input velocity to the pinion gear. The end time is set to 0.0002 seconds. The velocity of 44 rad/s is given to the input pinion, ‘figure 18’. The output shafts are fixed to depict the worst-case scenario.

![Figure 18. Input velocity](image1)

![Figure 19. Deformation](image2)

![Figure 20. Factor of safety](image3)

![Figure 21. Equivalent stress](image4)
7. Conclusion
A maximum stress of 404.479 MPa in the pinion and ring gear interface and a maximum stress of 226 MPa in the spider and output gear interface was induced respectively. The static structural analysis results indicated a stress of 291 MPa (figure 16) in the tooth root of pinion gear which corresponds to the value of 210 MPa obtained through analytical calculations. Thus, the concept of cageless differential was devised and designed based on the parameters obtained through analytical calculation and validated through numerical methods. The implementation of a cageless differential will significantly decrease the weight of the conventional drive train and thereby increase the overall performance of the vehicle owing to its more compact and lightweight structure.

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