Problems with False Vacua in Supersymmetric Theories

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It has been suggested recently that in a consistent theory any Minkowski vacuum must be exactly
stable. As a result, a large class of theories that in ordinary treatment would appear sufficiently
long-lived, in reality make no sense. In particular, this applies to supersymmetric models in which
global supersymmetry is broken in a false vacuum. We show that in any such theory the dynamics
of supersymmetry breaking cannot be decoupled from the Planck scale physics. This finding poses
an obvious challenge for the idea of low-scale metastable (for example gauge) mediation.

Introduction. The stability of our vacuum is a question
of life and death, which must be faced if there exists a
lower energy state. After all, a metastable vacuum would
do, if sufficiently long-lived to comply with the physical
observations. The question appears an academic one, for
even if we live in a pre-apocalyptic age, we rest assured the
doomsday will not happen tomorrow. The real issue is whether or not a physical theory can guarantee us suf-
cient stability that we observe; if it does not, we must
discard it. In the standard view put forward in the classic
papers [1, 2], zero energy vacua can easily be metastable with a sufficiently long life-time. However, it was argued recently [3] that any Minkowski vacuum, which in the
standard treatment is regarded as metastable, in reality exhibits instability at an infinite rate, and becomes un-
physical. The only consistent Minkowski vacua are thus exactly stable ones, true even in the presence of lower
energy AdS vacua.

This puts severe constraints on any theory with false vacua. The typical situation where one naturally encounters such local vacua takes place in supersymmetric the-
ories. This Letter is devoted to this important question and we show that a large class of seemingly consistent (according to the standard treatment) metastable theories with spontaneously broken supersymmetry, in fact make no sense.

The vacuum that we live in can be taken to be
Minkowskian with an extraordinary accuracy, for even if non-vanishing, the cosmological constant is effectively zero at particle physics scales. Therefore, in what follows we shall work in the approximation of exact asymptotic Poincare-invariance.

Although the study of vacuum decay via tunneling has earlier history [4], a systematic study of this issue was
given in the classical work by Coleman [1]. He focused on a study of an O(4)- symmetric bounce that describes
materialization of a spherically-symmetric bubble of the true vacuum. A bubble describes a portion of the true
vacuum embedded in the false one, with the scalar field interpolating between the true and the false vacua across
a layer, which is called the bubble wall. In the so-called thin-wall approximation the bubble can be characterized
by a positive wall tension (energy per unit surface) T and the negative volume energy of the interior Vtrue. The
energy of a static bubble then can be approximated as

\[ M = 4\pi R^2 T - \frac{4}{3} \pi R^3 |V_{true}|. \]

(1)

Note that by energy conservation a critical bubble that
can be materialized in Minkowski space must have zero total energy, which fixes the size of the critical bubble to be \( R_c = \frac{3T}{|V_{true}|} \).

However, as shown by Coleman and De Luccia [2], in certain cases, when gravity effects are taken into ac-
tount, the materialization of the bubble never happens. This happens when the critical bounce has an infinite
action, or equivalently, the critical bubble an infinite radius. This effect is referred to as the Coleman-De Luccia (CDL) suppression and in the thin-wall approximation it leads to the bound

\[ 6\pi G_N T^2 \geq |V_{true}|. \]

(2)

where \( G_N \equiv 1/8\pi M_P^2 \) is the Newton’s constant and \( M_P \) is the Planck mass.

This bound can be directly read off from the expression of the bubble energy for finite \( M_P \) [5], which modifies the flat space expression (1) as

\[ M = 4\pi R^2 T \sqrt{1 + \frac{8\pi}{3} G_N |V_{true}| R^2} - \frac{4}{3} \pi R^3 |V_{true}| - 8\pi^2 G_N T^2 R^3. \]

(3)

The CDL bound is then recovered by demanding that the energy of the bubble be semi-positive definite, or equivalently, \( R_c \) infinite.
Since then, the standard view on the tunneling is that violation of the bound is not a problem, since although the vacuum becomes metastable, it can easily be sufficiently long lived. This view was rejected recently in [3], where it was pointed out that any theory that violates the CDL bound, and thus allows for an instability of Minkowski vacuum, is inconsistent. In other words, the CDL bound is not a stability condition, but rather a consistency requirement. In what follows, we shall refer to it as the "Minkowski-safety" condition.

The argument of [3] is based on the fact that violation of the bound implies the existence of negative mass states in the Minkowski vacuum and renders the theory defined on such a vacuum senseless. This can be seen already from the expression of bubble energy above. Indeed, consider a theory that violates the CDL bound. This means that the theory allows a critical bubble of finite size $R_c$. Then, by continuity such a theory will allow bubbles of a bigger size that will have a negative mass. In order to illustrate this point, consider a bubble with $R \gg R_c$ that has zero velocity at some given moment of time (but experiences a non-zero acceleration that forces it to expand). Such a bubble has a negative mass, as it is obvious from the expressions (1) and (3). Any theory permitting such a bubble is a disaster. A consistent Poincare invariant theory cannot allow states of negative mass. The existence of such states makes the vacuum unstable with an infinite rate, because they can be nucleated (for example, pair-produced) in combination with positive mass states with an unbounded probability. The infinity appears due to the divergence in the phase space integration since a negative mass bubble can be pair produced in combination with a positive energy one with an arbitrary relative momentum.

The Euclidean bounces that describe materialization of such bubble pairs in general do not minimize the action, but this suppression is unimportant due to divergence of the phase space integral.

This argument tells us that a consistent theory in a gravity-decoupling limit cannot have a zero energy vaccum coexisting with a negative energy one. Consider, for example, a theory in which the negative energy vacuum survives in $M_P \to \infty$ limit, so that the effect of gravity on the bubble dynamics can be ignored. Then, according to (1), any bubble larger than the critical $R > R_c = 3T/|V_{\text{true}}|$ has a negative mass. Such a bubble is not a static solution of the equations of motion, but bubbles that materialize as a result of quantum tunneling need not be static. For example, the critical spherical bubble that carries zero energy is not static either.

In short, any theory that admits a Minkowski vacuum that violates the CDL bound in inconsistent. The purpose of this Letter is to apply this criterion to a class of supersymmetric theories in which supersymmetry breaking takes place in a false vacuum.

**Metastable supersymmetry breaking.** It is well known that in global supersymmetric theories the energy of the vacuum is semi-positive definite, being strictly positive for spontaneously broken supersymmetry and zero for unbroken one. On the other hand, in supergravity even the vacuum with spontaneously broken supersymmetry can have zero energy.

This fact enables one to consider an option of spontaneous supersymmetry breaking, in which the dynamics of supersymmetry breaking and its mediation is decoupled from gravity and survives in the limit $M_P \to \infty$. In such a case, it is usually assumed that the entire dynamics of supersymmetry breaking can be addressed in globally supersymmetric limit, with the role of supergravity corrections being reduced to a technical tool for fine tuning the vacuum energy to zero.

The possibility of tuning the vacuum energy to zero allows to consider scenarios in which spontaneous dynamical supersymmetry breaking in the global limit takes place in a false vacuum [6]. This scenario goes under the name of metastable supersymmetry breaking and has certain simplifications from the point of view of model building (e.g., avoiding constraints from the Witten index [7]). Naively, it seems that, since the vacuum energy has to be anyway tuned by supergravity, in the global limit the non-supersymmetric vacuum could be just a local minimum. We shall show that this intuition is false, and that any dynamical supersymmetry breaking vacuum that survives in the gravity-decoupling limit must be the true one.

In what follows, we start first with a general argument and then illustrate it on a concrete example.

**The general argument.** Let $X$ be a chiral superfield that spontaneously breaks supersymmetry through an expectation value of its $F$-term, $\bar{F}_X$. We shall assume that in global-supersymmetry limit supersymmetry is broken in a classically-stable false vacuum, where $\bar{F}_X = \Lambda^2$, and $X = X_0$, and that there also exists a true supersymmetry-preserving vacuum in which $\bar{F}_X = 0$. Since for our considerations what matters is the difference of scales, without loss of generality we can assume that in the true vacuum $X = 0$.

Our assumptions will be that the mass scales $\Lambda$ and $X_0$ that determine particle masses and VEVs are small compared to the Planck mass $M_P$, and that masses of scalars in these two minima are much larger than the gravity-mediated soft mass,

$$m_X^2 \gg \frac{\Lambda^4}{M_P^2}. \quad (4)$$

This is a necessary condition for sub-dominance of gravity mediation. Usually, this follows from the condition $X_0 \ll M_P$, but we shall spell it out separately for clarity. This condition implies that the back-reaction through the effects of gravity on the expectation values in the vacua of interest is small.
Let us now switch on supergravity. The scalar potential becomes (we ignore possible $D$-terms)
\[
V(X) = e^{\frac{3X}{4}} \left( (K_X^\lambda)^{-1} F_X F_X^* - 3|W|^2/M_P^2 \right),
\]
where $K$ is the Kähler function, and upper (lower) index $X$ stands for derivative with respect to $X$ ($X^*$) respectively. In terms of superpotential $W$ and the Kähler metric the $F$-terms are given as
\[
F_X = (F_X^*)^* = D_X W \equiv W_X + K_X W/M_P^2.
\]
In order to have a supersymmetry-breaking Minkowski vacuum, the negative term in (5) must cancel the positive one. But since all VEVs are sub-Planckian, we have $K_X/M_P \sim X_0/M_P \ll 1$. So the tuning of the vacuum energy implies (up to corrections of order $X_0/M_P$) $W_X = \Lambda^2 = \sqrt{3}W/M_P$, which is accomplished by introducing the following constant in the superpotential,
\[
W_0 = \Lambda^2 M_P/\sqrt{3}.
\]
But now, the supersymmetry-preserving minimum appears to have a negative energy equal to
\[
V_{\text{true}} = -\Lambda^4.
\]
The two minima are separated by a barrier that is set by a scale $\Lambda^4$, and the length of the barrier is at most $X_0$. This situation badly violates the Coleman-De Luccia bound, since the tension of the domain wall is at most $T \sim \Lambda^4/m_X$. This is because the energy density in the wall is $\Lambda^4$ and the thickness is $m_X^{-1}$. So we have
\[
T^2/M_P^2 \ll |V_{\text{true}}|,
\]
and the Minkowski safety constraint cannot be satisfied.

**An explicit example.** Consider a simple case that reproduces the essential features of the model of [8] of metastable supersymmetry-breaking. Take two chiral superfields $X$ and $\Phi$, with the superpotential
\[
W = \frac{1}{2} X \Phi^2 - X \Lambda^2.
\]
Let us analyse the global supersymmetry case first. For a minimal Kähler $K = |X|^2 + |\Phi|^2$, the theory has a global supersymmetry-respecting vacuum with $X = 0$ and $\Phi^2 = 2\Lambda^2$, and a plateau for $\Phi = 0$ and $|X| > \Lambda$, along which $F_X = \Lambda^2$. In the supersymmetry-preserving vacuum the masses of the particles are $m_X^2 = 2\Lambda^2$ and $m_\Phi^2 = 2\Lambda^2$. Whereas along the supersymmetry-violating plateau, the masses of the real and imaginary components of the complex $\Phi$-scalar are split $m_{\Phi\pm}^2 = |X|^2 \pm \Lambda^2$, while the mass of the $X$-modulus is zero. Following [8], we shall now create a classically-stable local minimum on the plateau at some $X = X_0$. For this we have to modify the Kähler metric, in such a way, that the function $(K_X^\lambda)^{-1}$ has a minimum at some $X = X_0$. In [8] this was achieved by the perturbative renormalization of the Kähler (i.e. wave-function renormalization) due to the loops of chiral and gauge multiplets. The effect of this renormalization can be summed up in the following form:
\[
(K_X^\lambda)^{-1} = 1 - \epsilon_1 \log \left( \frac{|X|}{\Lambda} \right) + \epsilon_2 \log^2 \left( \frac{|X|}{\Lambda} \right),
\]
where, $\epsilon_1$ and $\epsilon_2$ are the one- and the two-loop factors respectively. So approximately $\epsilon_2 \sim \epsilon_1^2$. In order to have a metastable minimum, the parameters must be chosen in such a way that both coefficients are positive. In [8] this was achieved by the interplay of the gauge and Yukawa couplings, in the spirit of Witten’s inverted hierarchy idea [9]. In the global supersymmetric limit, the stable minimum then develops at:
\[
|X| = X_0 = \Lambda \exp \left( \frac{\epsilon_1}{2\epsilon_2} \right).
\]
The mass of the $X$-modulus in this minimum is, $m_X^2 = \epsilon_2\Lambda^4/X_0^2$, whereas the phase of $X$ is a Goldstone boson of a spontaneously broken $R$-symmetry. The masses of the real and imaginary components of the complex $\Phi$-scalar are split, $m_{\Phi\pm}^2 = X_0^2 \pm \Lambda^2$.

Let us now take into the account the supergravity corrections. In order for these corrections to have a negligible effect on supersymmetry breaking (and in particular, for gauge-mediation to dominate) we have to demand that the condition (4) be satisfied. Our general argument at the end of the previous subsection then applies and the vacuum energy difference ends up violating the CDL condition. The theory is inconsistent, as argued in [3].

It is easy to convince ourselves that the inconsistency we are discovering is not an artifact of a particular form of the Kähler metric, but rather is fundamental for any form that creates a metastable vacuum in globally supersymmetric limit, which is not separated from the supersymmetric vacuum by a Planck distance. Replacing the Kähler in (11) by any smooth functional dependence that creates a minimum around $X_0$ gives the same result. This is obvious from the fact that an expansion around the point $X_0$ recovers all the features of the above example. We leave it up to the reader to try different forms.

The only way out would be to either violate the condition (4) or eliminate the globally-supersymmetric vacuum altogether.

**Summary and Outlook.** We reach a surprisingly strong conclusion: in false-vacuum supersymmetry breaking, the contribution of gravity cannot be ignored. This finding imposes a severe constraint on supersymmetry-breaking model building, and in particular creates an obvious obstacle for phenomenologically acceptable low scale gauge-mediated metastable supersymmetry breaking.
This is not necessarily a no-go theorem, and with some imagination one could attempt to build a working model of this sort. For example, in the above case one may give up on perturbative calculable corrections to Kähler and instead rely on Planck scale corrections for creating a stable minimum at $X_0 \sim M_P$. One may even arrange a low-scale gauge mediation by coupling $X$ to a messenger superfield $Q$, with a bare mass $M_Q$, 

$$ (X - M_Q)\bar{Q}Q, \tag{13} $$

and fine tuning the latter, in such a way that in a stable minimum $X_0 - M_Q \ll M_P$. But such a construction defeats the cause, since it gives up both the calculability in terms of low energy parameters as well as the naturalness for which low scale mediation is invoked to start with.

Putting aside such options, supersymmetry-breaking must either take place in a true vacuum (such as in the O’Raifeartaigh model) or be of high-scale. The latter possibility happens naturally in perturbative theories for which low scale mediation is invoked to start with.

Our findings are still in full agreement with the general proof that, in supergravity Minkowski vacua, the energy is semi-positive definite [11]. The starting condition of these theorems is an existence of a well-defined Hilbert space and of a conserved supercharge. What we are showing is that, although in broken supersymmetry naively one can have negative energy states, these not only destroy the Hilbert space, but the vacuum itself. In other words, Minkowski vacuum cannot allow the negative energy states irrespectively of supersymmetry.

In short, one can say that the observation of [3] makes the requirement of energy-positivity in any sensible Minkowski vacuum obvious, since it shows that any such vacuum admitting negative mass objects is not a metastable state, but is rather infinitely short lived, and thus has no meaningful description in terms of a Hilbert space. Such are the vacua which violate CDL bounds, and must be excluded.

One last comment, likely to be obvious to the reader. Strictly speaking, our vacuum is not exactly Minkowski, but this only matters in the case when the size of the critical bubble becomes comparable to the Hubble horizon. In the case under study, when one worries about ending up in the universe with a large cosmological constant (even on present particle physics scales), this clearly plays a negligible role and does not affect our results.

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