Spin-Hall effect in two-dimensional mesoscopic hole systems

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The spin Hall effect in two dimensional hole systems is studied by using the four-terminal Landauer-Büttiker formula with the help of Green functions. We show that the heavy (light) hole spin Hall effect exists even when there are no correlations between the spin-up and -down heavy (light) holes and when the Γ-point degeneracy of the heavy hole and light hole bands is lifted due to the confinement or recovered by the strain. When only a heavy hole charge current without any spin polarization is injected from one lead, under right choice of lead voltages, one can get a pure heavy-(light-) hole spin current, combined with a possible impure light (heavy) hole spin current from two transverse leads. The spin Hall coefficients of both heavy and light holes depend on the Fermi energy, device size and the disorder strength. It is also shown that the spin Hall effect of two dimensional hole systems is much more robust than that of electron systems with the Rashba spin-orbit coupling and the spin Hall coefficients do not decrease with the system size but tend to some nonzero values when the disorder strength is smaller than some critical value.

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Recently spintronics has become one of the major focuses of the condensed matter physics.1 Generating and measuring spin current is still a primary task of this field. The spin Hall effect (SHE) is considered to be a hopeful way to produce the spin current. This effect, originally proposed as the extrinsic SHE as it requires spin-dependent impurity scattering, was first studied by Dyakonov and Perel’2 in the early seventies and lately by Hirsch3 and Zhang.4 It further evolves into the intrinsic SHE very recently as it appears in clean systems and bulk hole systems8 without any impurity scattering.

In 2DES, the intrinsic SO coupling comes from the Rashba9 or Dresselhaus10 terms which correlate the two spin states of electrons and lead to nonzero spin correlations ⟨σ+σ−⟩. Calculation based on Kubo formula found that the spin Hall coefficient (SHC) is a universal value e/8π in macroscopic 2DES with the Rashba SO coupling.5 The fate of the intrinsic SHE in the presence of disorder is an important problem and has raised a lot of controversies. On one hand, Burkov et al.11 and Schliemann and Loss12 pointed out that the SHE only survives at weak disorder scattering in macroscopic 2DES. On the other hand, more analytical and numerical calculations13,14,15,16 showed that the SHE vanishes even for any weak disorder.

Situations are quite different in the mesoscopic systems: using the four-terminal Landauer-Büttiker formula, Sheng et al.17 and Nikolić et al.18 showed that the SHC does not take a universal value in a finite mesoscopic 2DES, but depends on the magnitude of the SO coupling, the electron Fermi energy, and the disorder strength. Moreover, Sheng et al. showed that when the disorder is smaller than some critical values, the SHC does not decrease with system size but instead goes to some nonzero values. This robustness to the disorder in mesoscopic systems is suggested from the effect of the boundary effects (leads).15 Therefore a pure spin current (no charge current associated with the spin current) can be induced by unpolarized charge current due to SHE in mesoscopic 2DES,17,18,19

The SHE in macroscopic hole systems has also been studied. Murakami et al.8 predicted that the pure spin current can be obtained in p-type bulk semiconductors. They further suggested that this SHE comes from the Dirac magnetic mono-pole in momentum space because of the fourfold degeneracy at the Γ-point of the valence band. Then Murakami20 pointed out that the vortex correction that was reported to kill the SHE in 2DES14 is identically zero in the Luttinger model so that the SHE can survive the impurity scattering. This is verified by numerical simulation in bulk lattice Luttinger model very recently.21 On the other hand, Schliemann and Loss22 and Bernevig and Zhang23 studied the SHE in 2D hole gases in which the HH and the LH bands are no longer degenerate at the Γ-point (no mono-pole in this case). However, they considered the case where there are still correlations between spin-up and -down HH’s by introducing additionally the Rashba SO coupling in the hole systems. Therefore, whether the magnetic monopole is crucial for the SHE in hole systems is still unknown.

We notice that the study of the SHE in mesoscopic 2D hole systems is still absent. There are a lot of differences between 2DES and 2D hole systems: In 2DES, spin-up and -down electrons are always correlated via the Rashba or the Dresselhaus SO couplings. Nevertheless, in 2D hole systems the situation becomes much more complicated: For (001) quantum wells (QW’s) with small well
width where only the lowest subband is important, unless one adds an additional Rashba spin-orbit coupling, there are no direct or indirect spin correlations between spin-up and -down HH’s (LH’s), i.e., $\langle a_k^\sigma a_{k+}^{\bar{\sigma}} \rangle \equiv 0$ with $\sigma = \pm \frac{1}{2}$ for HH’s and $\pm \frac{1}{2}$ for LH’s. The spin-up HH’s (LH’s) are only coupled with the spin-down LH’s (HH’s). Adding strain can change the relative positions of the HH and LH bands and the $\Gamma$-point degeneracy can be recovered. It is interesting to see whether there is still SHE in the absence of any correlations between spin-up and -down HH’s (LH’s) and it is also interesting to see the role of magnetic mono-pole to the SHE in the hole systems. In this paper, we study these problems by using the four-terminal Landauer-Büttiker formula with the help of Green functions.

We consider a 2D hole system which is consisted of a square conductor of width $L$ attached with four ideal leads of width $L/2$ without any spin-orbit coupling (as illustrated in Fig. 1) in a (001) QW of width $a$. Due to the confinement of the QW, the momentum states along the growth direction ($z$-axis) are quantized. We take only the lowest subband for small well width. Then the Luttinger Hamiltonian in the momentum space reads with the matrix elements arranged in the order of $J_z = \frac{1}{2}$, $\frac{3}{2}$, $-\frac{1}{2}$ and $-\frac{3}{2}$:

$$H = \frac{1}{2m_0} \begin{pmatrix} P + Q & 0 & R & 0 \\ 0 & P - Q & 0 & R \\ R^\dagger & 0 & P - Q & 0 \\ 0 & R^\dagger & 0 & P + Q \end{pmatrix}$$

(1)

where $P \pm Q = (\gamma_1 \pm \gamma_2)(P_x^2 + P_y^2) + E_0^\pm$, and $R = -\sqrt{3}[\gamma_2(P_x^2 - P_y^2) - 2i\gamma_3 P_x P_y]$. It is noted that off-diagonal terms $R$ and $R^\dagger$ mix the HH with the LH. In real space the Luttinger Hamiltonian can be written in the tight-binding version as:

$$H = \sum_{i,j,\sigma = \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{1}{2}} [E_0^\pm - (\gamma_1 \pm \gamma_2)] t + \epsilon_{i,j}] a_{i,j,\sigma}^\dagger a_{i,j,\sigma}$$

$$+ \sum_{i,j,\sigma = \pm \frac{1}{2}} [\gamma_1 \pm \gamma_2] t [a_{i,j,\sigma}^\dagger a_{i,j,\sigma} + a_{i,j,\sigma}^\dagger a_{i,j,\sigma}]$$

$$+ \{ \sum_{i,j,\sigma = \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{1}{2}} [(-3\gamma_3) t (a_{i,j,\sigma}^\dagger a_{i,j,\sigma} + a_{i,j,\sigma}^\dagger a_{i,j,\sigma})]$$

$$- \frac{\sqrt{3}}{2} \gamma_3 t (a_{i,j,\sigma}^\dagger a_{i,j,\sigma} + a_{i,j,\sigma}^\dagger a_{i,j,\sigma})] + H.C.$$

(2)

where $i$ and $j$ denote the coordinates along the $x$- and $y$-axis; $\gamma_1$, $\gamma_2$ and $\gamma_3$ are the Luttinger coefficients, and $m_0/(\gamma_1 \pm \gamma_2)$ are the effective masses of the HH and the LH in the $x$-$y$ plane with $m_0$ representing the free electron mass; $t = -\hbar^2/2m_0 \alpha_0^2$ is the unit of energy with $\alpha_0$ standing for the “lattice” constant, and $(\gamma_1 \pm \gamma_2) t$ stands for the hopping energy; $E_0^\pm = (\gamma_1 \pm \gamma_2) \frac{\alpha^2}{\alpha_0^2} t$ is the first subband energy in the $z$ direction; $\epsilon_{i,j}$ accounts for the spin-independent disorder, which is a random value in the range $[-W/2, W/2]$.

Additionally

$$H_{\text{strain}} = \sum_{i,j,\sigma = \pm \frac{1}{2}, \pm \frac{3}{2}} \epsilon_{i,j}^\sigma a_{i,j,\sigma}^\dagger a_{i,j,\sigma}$$

(3)

is the strain Hamiltonian where $\epsilon_{i,j}^\sigma$ is the strain-induced energy with $\epsilon_{i,j}^\sigma \neq \epsilon_{i,j}^\bar{\sigma}$. Therefore, by adding strain, one may either further increase the separation between the HH and the LH or reduce it to recover the $\Gamma$-point degeneracy. Moreover, it is seen from the Hamiltonian that there is not any direct or indirect spin flip between the spin-up and -down HH’s or between the spin-up and -down LH’s in the Hamiltonian.

![FIG. 1: Schematic view of a four-terminal junction of width $L$ attached with four semi-infinite leads without any spin orbit coupling of width $L/2$.](image)

The spin dependent transmission coefficient from $\mu$ terminal with spin $\sigma$ to $\nu$ terminal with spin $\sigma'$ is calculated using the Green function method. $T_{\mu \nu} = \text{Tr}[\Gamma_{\mu}^R G_{\mu \nu}^R \Gamma_{\nu}^A G_{\nu \mu}^A]$, in which $\Gamma_{\mu}^R = i[\Sigma_{\mu}^R - \Sigma_{\mu}^A]$ represents the self-energy function for the isolated ideal leads. We choose the perfect ideal Ohmic contact between the leads and the semiconductor. $G_{\mu \nu}^R$ and $G_{\mu \nu}^A$, which can be obtained from $G^{(A)} = (E - H_C - \Sigma_{\mu}^{(A)})^{-1}$, are the retarded and advanced Green functions for the conductor, but with the effect from the leads included. Here $\Sigma_{\mu}^{(A)}$ represents the sum of the retarded (advanced) self-energies of the four leads.

We perform a numerical calculation when the leads are connected to isolated reservoirs at chemical potentials $E + V_{\mu}$ ($\mu = L, U, R$ and $D$), with the differences between each other caused by the applied gate voltages. The particle current going through the lead $\mu$ with spin $\sigma$ can be obtained by the Landauer-Büttiker formula $I_{\sigma \mu}^P = \frac{e}{h} \sum_{\nu \neq \mu, \sigma} [T_{\nu \mu}^{\sigma \sigma} V_{\nu} - T_{\nu \mu}^{\sigma \bar{\sigma}} V_{\nu}]$. The spin current is defined as $I_{s,\mu}^{H} = \frac{e}{h} \sum_{\nu \neq \mu} [T_{\nu \mu}^{\sigma \sigma} V_{\nu} - T_{\nu \mu}^{\bar{\sigma} \bar{\sigma}} V_{\nu}]$ for LH’s and

$$I_{s,\mu}^{L} = \frac{e}{h} (I_{s,\mu}^{H} - I_{s,\mu}^{L})$$

for LH’s in the lead $\mu$ and the SHC for the pure spin current is defined as

$$\sigma_{SH,\mu}^{H(L)} = \frac{I_{s,\mu}^{H(L)}}{(V_L - V_R)}$$

(4)

When the charge current of the hole $I_{h,\mu}^{H(L)} = e[I_{s,\mu}^{H(L)} + I_{s,\mu}^{L}] = 0$ and the spin current $I_{s,\mu}^{H(L)} \neq 0$ for the lead $\mu$, then $I_{s,\mu}^{H(L)}$ is a pure spin current, otherwise an impure one.
We drive a unit HH charge current without any spin polarization into the left lead \(L\) \((I_L^H = I_0 \text{ and } I_L^L = 0)\). In order to get pure spin currents of HH in the upper and down leads \((U \text{ and } D)\), one needs to find a set of suitable combinations of \(V_{\mu}\), which lead to \(I_{h,U}^H = I_{h,d}^L = 0\). This can be obtained by choosing \(V_R = 0\) for convenience, \(V_U = V_D\) due to the symmetry and \(V_U/V_L = (T_{UL}^{\pm \pm} + T_{UL}^{-\pm \pm})/(T_1 + T_2)\) with \(T_1 = T_{UL}^{\pm \pm} + T_{RU}^{\pm \pm} + T_{RU}^{-\pm \pm}\) and \(T_2 = T_{LU}^{\pm \pm} + T_{RU}^{\pm \pm} + T_{RU}^{-\pm \pm}\). And one obtains

\[
I_{U,L}^+ (I_{U,L}^-) \text{ when } U \text{ (} L\text{)} \text{ is the upper (} L\text{)} \text{ lead.}
\]

A pure HH spin current when \(I_{s,U}^H = \frac{2g_e}{\pi} 2V_L (T_1 T_{UL}^{-\pm \pm} + T_2 T_{UL}^{\pm \pm})/(T_1 + T_2) \neq 0\). This relation can be satisfied thanks to the phase shift provided by the last term of Hamiltonian \((1)\) when holes hop from site \((i,j)\) to site \((i+1, j \pm 1)\). Similarly one can obtain the pure LH spin current. The main results of our calculation are summarized in Figs. 2-4.

In Fig. 2 a pure HH spin current is generated in the \(U \text{ and } D\) leads of a \(12 \times 12\) square conductor made from the unstrained QW, where the \(\Gamma\)-point degeneracy of the HH and LH is lifted. In Fig. 2(a), all the particle currents in these two leads are plotted as functions of the Fermi energy. It is seen that \(I_{U}^H = I_{D}^H = -I_{U}^L = -I_{D}^L\), leading to \(I_{s,U}^H = -I_{s,D}^H\). Therefore pure HH spin currents with opposite spin polarizations are obtained in the absence of any correlations between the spin-up and -down HH’s and the \(\Gamma\)-point degeneracy. The SHC \(\sigma_{SH,U}^H\) of the upper lead is also plotted in the same panel. One finds that the SHC depends strongly on the Fermi energy. It is interesting to see that when the energy \(E\) is high enough, LH’s produce an impure spin current as shown in Fig. 2(b), where \(I_{D}^H = I_{D}^L = -I_{U}^L = -I_{D}^L\), and hence \(I_{s,U}^H = I_{s,D}^H \neq 0\). One can also obtain a pure LH spin current combined with an impure HH spin current in both \(U \text{ and } D\) leads when a unit HH charge current without any spin polarization injected into the left lead \(L\), by using the condition \(I_{h,U}^H = I_{h,D}^L = 0\) as shown in Fig. 3 for the same conductor. It is seen from Fig. 3(a) that \(I_{U}^H = I_{D}^H = -I_{U}^L = -I_{D}^L\), which result in the pure LH spin currents with opposite spin polarizations in the \(U \text{ and } D\) leads. The SHC of the LH again varies with the Fermi energy. Moreover, from Fig. 3(b) one has \(I_{U}^H = I_{D}^H \neq -I_{U}^L = -I_{D}^L\) which lead to an impure HH spin current with \(I_{s,U}^H = I_{s,D}^H \neq 0\).

Therefore, both HH and LH pure spin currents can be obtained through the suitable combination of the applied voltages. This provides us a unique way to manipulate the hole spin currents. Moreover, if one further allows only the HH charge current through the right lead \(R\), then pure spin currents of HH and LH can be obtained at the same time in the \(U \text{ and } D\) leads.

In order to check the robustness of the SHC of the hole system, we plot in Fig. 4(a) the HH SHC of the \(U\) lead of a \(12 \times 12\) conductor with and without strain versus the strength of disorder \(W\) over 5000 random disorder configurations. The strain is chosen to recover the \(\Gamma\)-point degeneracy of the HH and LH bands (and thus the magnetic mono-pole). For the strain free and strain applied cases, \(E = 16|t|\) and \(20|t|\) to ensure that the energy always sits around \(1/4\) of the HH bandwidth from the \(\Gamma\)-point. It is seen that for strain free case, the SHC is negative when \(W\) is small and then becomes positive. It reaches to the maximum near \(W = 30|t|\), and then decreases to zero when \(W\) is larger than \(100|t|\). When the \(\Gamma\)-point degeneracy is recovered by the strain, the SHC...
rescale), much smaller than the value of HH’s. Therefore the SHC of hole systems is much more robust than electron ones. This originates from the stronger intrinsic SO coupling of the hole systems. We further examine the size effect by calculating the SHC for the HH as a function of the system size \( L \) under different disorder strengths in the cases with and without strain. As seen in Figs. 4(b) and (c), similar to the case of electron systems with the Rashba SO coupling,\(^{17}\) the SHC does not decrease with the size but goes to some nonzero value when the disorder strength is smaller than some critical value. However, when the disorder is large enough, then the SHC goes to zero with the size. We have also calculated the SHC of LH’s and come to the same conclusion.

In summary, we have performed a mesoscopic investigation of the SHE for holes in four-terminal (001) QW’s with a small well width so that only the lowest subband is populated, the correlations between the spin-up and -down HH’s (or LH’s) are totally absent and the \( \Gamma \)-point degeneracy between the HH and LH bands are lifted unless a certain strain is applied. We find that the SHE still exists. Moreover a pure HH (LH) spin current can be generated combined with a possible impure LH (HH) spin current, when a HH charge current without any spin polarization is injected from one lead. The SHE’s for both HH and LH depend on the Fermi energy, device size and the disorder strength. We also find that the SHE of holes does not need the mono-pole from the \( \Gamma \)-point degeneracy. We show that the SHE of 2D hole systems are much more robust than that in 2DES. This is consistent with the latest study in bulk systems.\(^{21}\) The SHC does not decrease with the system size but tends to some nonzero value when the disorder strength is smaller than some critical value, similar to the electron case but where there are direct correlations between the spin-up and -down electrons.

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