Study of stability of mean-motion resonances in multiexoplanetary systems

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Abstract. Many exoplanetary systems have been found to harbour more than one planet. Some of them have commensurability in orbital periods of the planets (resonant-planet pair). The aim of this work is to analyse the stability of resonant-planet pair configuration in two multiexoplanetary systems which have two planets in near mean-motion resonances, i.e. Kepler-9 and HD 10180 systems. This work considers numerical and comparative empiric-analytical studies. Numerical studies are performed using the integrator package SWIFT with an integration time of 10 Myr. Results from numerical integrations indicate that all orbital solution sets of the systems are stable. Further numerical explorations also demonstrate that the systems are stable for small perturbations in the orbital elements and mass variations. Analyses of stability based on comparative empiric-analytical are done by applying a known stability criterion to all systems. We find that all systems tend to be stable.

1. Introduction
Exoplanets are known common throughout the universe. Some of them belong to multiplanetary systems. Study about their dynamical behaviours becomes one of the topic of interests in celestial mechanics, which are associated with orbit and stability of the systems. Orbital studies of planetary systems show commensurabilities among their orbits, known as resonances, which are also common in the Solar System. Near-resonances phenomena are also found in multiexoplanetary systems [1]. One of these resonances is two-planet mean-motion resonance (resonant-planet pair). Analyses of resonant-planet pair provide an idea about prediction of planets’ orbital configuration with respect to its stability.

There are two approaches to study the stability of resonant-planet pair configuration. First is numerical study using N-body integration. Numerical study calculates orbital evolution of a system during a given integration time or a time-scale, so that its stability can be decided. The stability of resonant-planet pair is indicated by evolution of orbital elements together with libration/circulation of the resonant arguments [2]. Second, the stability conditions of resonant-planet pair can be determined by applying a known empiric-analytical solution of the stability of three-body problem [3].

There are many known exoplanetary systems in near-resonances. This study focuses on Kepler-9 and HD 10180 systems. Kepler-9 is a single-star (1.07 Solar mass), known to have three planets in the system. Two outer planets (b and c) are claimed to be in 2:1 near-resonance [1]. Larger system, like HD 10180 system (having six confirmed planets) has two planets (d and e) which are in 3:1 near-resonance...
[1]. The HD 10180 star is also known a single-star (1.06 Solar mass). With current technology and methods, undiscovered planets in Kepler-9 and HD 10180 systems are believed to have smaller masses. Therefore, their perturbations to orbits of the known planets should be small.

2. Numerical studies
The SWIFT package [4] is commonly used in celestial mechanics studies. The package is designed to integrate a set of mutually gravitationally interacting bodies together with a group of test particles which feel the gravitational influence of the massive bodies but do not affect each other or the massive bodies. We used exoplanetary data consisting of the usual a set of six Keplerian orbital elements (semi-major axis $a$, eccentricity $e$, inclination $i$, longitude of ascending node $\Omega$, argument of pericenter $\omega$, mean anomaly $M$) that was then transformed into a set of Cartesian orbital elements to suit the SWIFT codes. We set values of Keplerian orbital elements when they were not available.

2.1. Data
Data for orbital elements and physical parameters of Kepler-9 and HD 20180 systems were taken from some references [5, 6]. Using TRADES software [5] two sets of orbital solutions for Kepler-9b and c are provided. The initial integration conditions for both of them were different, leading to the differences of their derived planet’s mass. As for set solution 1, they used TRADES with initial parameters and Transit Times from [7], while set solution 2 using TRADES+PSO+LM [5] with Transit Times from [8]. The given solutions include planetary mass $m$ in Jupiter mass ($M_{\text{Jup}}$), orbital period $T$, $e$, $\Omega$, $\omega$ and $M$. We eventually derived value of $a$ using the Kepler Third’s Law.

A set of orbital solutions for six confirmed planets in HD 10180 system are given [6]. Because of using radial velocity method, the planetary masses are considered as minimum ($m \sin i$). True masses are typically higher by about 15% due to the geometric effects of inclination. We derived value of $M$ using the given epoch and time of pericenter passage.

The parameters are presented in Table 1. Inclinations for both systems are unknown, so we set the values to zero. Unknown values of $\Omega$ are also set to zero.

| Planets     | $a$ (au) | $e$     | $\Omega$ (°) | $\omega$ (°) | $M$ (°) | $m$ ($M_{\text{Jup}}$) |
|-------------|----------|---------|---------------|--------------|--------|------------------------|
| Kepler-9b#  | 0.144    | 0.131 (0.058) | -            | 18.9 (356.1) | 333.8 (3.8) | 0.246 (0.137) |
| Kepler-9c#  | 0.230    | 0.119 (0.068) | 1.6 (~0)     | 102.9 (167.6) | 7.5 (307.4) | 0.169 (0.094) |
| HD 10180d*  | 0.129    | 0.131    | -             | 325.0        | 37.5    | 0.038**                |
| HD 10180e*  | 0.270    | 0.051    | -             | 147.0        | 168.8   | 0.081**                |

Values in parenthesis are for the set solution 2 [5]. # [5], * [6], ** $m \sin i$

2.2. Integration scheme
We did not include the rotational motions of the bodies and only took planets that have been confirmed in http://exoplanetarchive.ipac.caltech.edu/ on January 28, 2016. The inner planet’s mass in each systems was added to mass of the host star for the efficiency of computing times. We have done numerical integrations for both systems and set an integration time of 10 Myr. For each system one set of the planets’ orbital solutions was integrated. For the purpose of numerical explorations, we use the second solution [5] for Kepler-9 system, while for HD 10180 system we added 15% mass to the known planetary mass.

3. Empiric-analytical study
Among empiric-analytical studies for determining stability of multiplanetary system, including Hill’s stability, there is a new solution to clarify the stability of resonant-planet pair system, considering that the system follows three-body problem [3]. The criterion is given in (1)
\[ \Gamma = \frac{a_2(1-e_2)}{a_1(1+e_1)} - 2.4[\max(\mu_1, \mu_2)]^{1/3} \left( \frac{a_2}{a_1} \right)^{1/2} + 1.15 > 0, \]  

where \( \Gamma \) is a stability parameter, \( \mu \) is ratio of planetary to the star mass, and indices 1 and 2 refer to inner and outer planets. This criterion discriminates stable and unstable regions, although numerical studies [3] still found a “grey” region between them, meaning that it can be stable or unstable. The above criterion has been applied successfully [9] to other systems, i.e. one two-planet sytem and one multiplanetary system. We applied Petrovich criterion (1) to examine whether or not the resonant-planet pairs in Kepler-9 and HD 10180 systems are stable.

4. Results and discussion

4.1. Evolution of orbital elements and resonant arguments

We describe evolution of orbital elements and the resonant arguments (\( \theta \) and \( \Phi \)) of the resonant-planet pairs. Following [2] the resonant arguments for 2:1 resonance: \( \theta_{1,2} = 2\lambda_2 - \lambda_1 - \omega_{1,2}, \Phi = 4\lambda_2 - 2\lambda_1 - \Omega_1 - \Omega_2 \), and for 3:1 resonance: \( \theta_{1,2} = 3\lambda_2 - \lambda_1 - 2\omega_{1,2}, \Phi = 3\lambda_2 - \lambda_1 - \Omega_1 - \Omega_2 \), with \( \lambda = \omega + M \) and \( \omega = \omega + \Omega \). Subscripts 1 and 2 refer to inner and outer planets of the pair.

Figures 1 and 2 show orbital evolutions of the resonant-planet pairs of Kepler-9 set solutions 1 and 2, while figure. 3 is that for HD 10180 system. We see that the resonant-planet pairs remain “bound” to their host stars. The evolution of their orbital elements are limited in narrow range values, indicating
that the systems are indeed in orbital commensurability. Argument of pericenters circulate as the nodes of pericenter of the planet pairs evolve regularly. We found that their configurations remain unchange for 10 Myr. Set solution 2 of Kepler-9 is more stable than that for set solution 1. Additional 15% planetary masses to the HD10180 system do not much affect the system. Figure 4 depicts evolution of resonant arguments of HD 10180 system which shows clear episodic patterns. Less noticeable patterns are taken place for Kepler-9 system. This means that HD 10180 system is more dynamic.

4.2. Petrovich stability
We applied Petrovich criterion (1) to Kepler-9 system set solutions 1 and 2, also to HD 10180 system with minimum mass solution. Kepler-9 system set 2 and HD 10180 system have Γ-values greater than zero, but Kepler-9 system set 1 has value below zero (see figure 5). According to Petrovich’s work, the inner resonant planet of Kepler-9 system set 1 might be plunged into the host star as μ₁ > μ₂.

It is obvious in figure. 5 that HD 10180 system is located in stable region with 3:1 resonance line, while Kepler-9 system is located in “grey” region with 2:1 resonance line. However, Kepler-9 system set 2 tends to be stable (Γ > 0).

5. Conclusions
Our work finds that resonant-planet pairs in Kepler-9 set 2 and HD 10180 systems are dynamically stable for at least 10 Myr. Numerical simulations showed that Kepler-9 set 2 appears more “bound” than that for set 1. Verification using Petrovich criterion supports this numerical result. In addition, HD 10180 systems is more dynamic, considering the clearly seen of episodic patterns in the evolution of resonant arguments. Additional planetary masses do not much affect the dynamics of HD 10180 system.

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