A COMPLEXITY-BRIGHTNESS CORRELATION IN GAMMA-RAY BURSTS
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ABSTRACT

We observe strong correlations between the temporal properties of gamma-ray bursts (GRBs) and their apparent peak brightness. The strongest effect (with a significance level of ~10^-5) is the difference between the brightness distributions of simple bursts (dominated by a single smooth pulse) and complex bursts (consisting of overlapping pulses). The latter has a peak flux of ~1.5 photons cm^-2 s^-1, while the distribution of simple bursts is smooth down to the BATSE threshold. We also observe brightness-dependent variations in the shape of the average peak-aligned time profile (ATP) of GRBs. The decaying slope of the ATP shows time dilation when bright and dim bursts are compared, while the rising slope hardly changes. Both slopes of the ATP are deformed for weak bursts as compared to strong bursts. The interpretation of these effects is simple: a complex burst in which a number of independent pulses overlap in time appears to be intrinsically stronger than a simple burst. The BATSE sample of complex bursts then covers larger redshifts, where some cosmological factor causes the break in the peak brightness distribution. This break could correspond to the peak in the star formation rate that was recently shown to occur at a redshift of z ~ 1.5.

Subject headings: gamma rays: bursts — gamma rays: observations — methods: statistical

1. INTRODUCTION

Do the temporal properties of gamma-ray bursts (GRBs) have systematic trends that are dependent on brightness? The most intensively discussed and studied trend is the time dilation of weak bursts relative to strong bursts. This attention to time dilation effects is attributable to its possible cosmological interpretation (Paczynski 1992). The results of time dilation measurements differ for different groups: Norris et al. (1994), Fenimore & Bloom (1995), and Stern (1996) found time dilation, while Mitrofanov et al. (1996) and Lee & Petrosian (1997) did not find any significant effect.

Positive detections of time dilation are consistent with the simplest assumption that weak events are just redshifted (and therefore stretched) analogs of strong events. However, in the work of Stern, Poutanen, & Svensson (1997, hereafter SPS97), it was demonstrated that the situation is more complicated. Weak events are on average not only longer, but also more asymmetric. Furthermore, their average peak-aligned time profile (ATP) has a different shape from that of strong events. This difference can be quantified in terms of different stretched exponential (SE) indices.

Such brightness-dependent correlations are in contradiction to a straightforward interpretation of the temporal stretching as a product of cosmological time dilation. On the other hand, these correlations can easily be interpreted in terms of correlations between temporal properties and intrinsic peak luminosity. Indeed, GRBs are composed of individual asymmetric pulses with fast rise and slower exponential decay (FREDs). The ATP of simple GRBs consisting of one pulse (or a few pulses) is on average more asymmetric than the ATP for complex GRBs, in which a chaotic bunch of overlapping pulses can produce an arbitrary time profile and the asymmetry related to the individual pulses is washed out. If we assume that different elementary pulses originate in different regions associated with local events in the course of a global event, then the amplitudes of elementary pulses with overlapping arrival times sum up. Therefore, complex bursts consisting of many overlapping pulses are intrinsically brighter than simple events. A direct morphological classification performed by SPS97 confirmed this interpretation; complex bursts show systematically larger peak fluxes than simple GRBs.

Although such correlations are natural in intrinsic peak luminosity, we observe them in a narrow range (within one decade) of apparent peak brightness (e.g., peak count rate). This means that the distribution of GRBs over luminosity distance differs significantly from a power law. (In the case of a power law, intrinsically strong and intrinsically weak GRBs would be blended in the same proportion at any apparent brightness.) These correlations can put additional constraints on the spatial distribution of the GRB sources.

This work follows SPS97 in its main objectives. A larger statistics of GRBs is used, and, most importantly, the procedure for discriminating between simple and complex bursts is formalized and made more efficient. As a result of the latter improvement, the direct test for a complexity–peak brightness correlation gives a very significant and meaningful result, which is described in § 3. In § 4, we describe the results of our studies of the ATP shape as a function of brightness. These studies show further effects that are also associated with the complexity-brightness correlation. In § 5, we present a model demonstrating how the above correlations could occur using simple assumptions, and we discuss the issue of the GRB intrinsic luminosity function. In the Appendices, we present a detailed description of the data analysis for the ATP study, as well as a number of additional tests for possible systematic errors.

Some of the results presented here were also presented in a preliminary form in Stern, Svensson, & Poutanen (1997).

When using the terms “brightness,” “bright,” “dim,” “strong,” or “weak,” we always refer to the apparent peak brightness (peak flux or peak count rate), except in those cases when we clearly write intrinsic.
2. DATA ANALYSIS

This work is based on the data in the publicly available Compton Gamma Ray Observatory (CGRO) data archive at the Goddard Space Flight Center. Our sample includes bursts up to trigger number 6230, and contains 1395 events selected as useful for the complexity-brightness analysis and 1310 events useful for the ATP study. We use 0.064 s and 1.024 s time-resolution data from the Large Area Detectors (LAD). All the time profiles are constructed in 64 ms resolution, with a 1024 ms resolution extension if necessary. All background fits are made using the 1024 ms data, since they cover a wider time range, including the pretrigger history. We use the count rate summed over the four LAD energy channels covering the 25–1000 keV energy range when studying the behavior of the ATP, as well as the count rate summed over channels 2–3 (50–300 keV) in our complexity-brightness analysis (see SPS97). Backgrounds were subtracted using linear fits. A visual scan of all trigger events selected as useful for the complexity-brightness analysis and use their number to characterize complexity (Lestrade et al. 1996): where \( p \) is the criterion, the significance level is \( \alpha \).

3.1. The \( \chi^2 \) Separation between Simple and Complex Bursts

Unlike the visual test of SPS97, the \( \chi^2 \) test is aimed at extracting events dominated by a single FRED pulse rather than those consisting of a single FRED pulse. That is, the main peak was checked for how well it is fitted by the FRED pulse shape, while other, smaller peaks not affecting the fit were allowed for an event to be qualified as simple. Pulses were fitted with the parameterization from Norris et al. (1996): 

\[
C_{i,j}(t) = C_0 \exp \left[ -\left(1 - \frac{t - t_{\text{max}}}{\tau_v}\right)^q \right] ,
\]

where \( q \) was allowed to vary between 0.9 and 2.2, and \( t_{\text{max}} \) was allowed to vary within \( \pm 3 \) s around a direct estimate of the peak position. The three parameters, the peak count rate, \( C_0 \), the rise time, \( \tau_r \), and the decay time, \( \tau_d \), were free. The fitting time interval was \( t_{\text{max}} = 61/\tau_r, t_{\text{max}} + 61/\tau_d \). The fit for each event was performed for 64, 128, 256, and 512 ms bins. The maximum value of \( \chi^2 \) per degree of freedom (\( \chi^2_m \)) for these four variants of binning was used for the classification. All events with a peak count rate \( > 55 \) counts/64 ms were tested (dimmer events were discarded), and all of them were rescaled to a peak count rate (in energy channels 2 + 3) of 55 counts/64 ms. All events with \( \tau_r + \tau_d < 2 \) s were discarded, since the time resolution for short peaks at this small brightness is insufficient.

A visual examination of the fitting procedure showed that it is not perfect, but this is natural. Sometimes a bright GRB that looks like an apparently complex dense bunch of pulses gave a good \( \chi^2 \) when rescaled to low brightness, and sometimes a GRB that looked like a FRED had a bad \( \chi^2 \), not satisfying the parameterization (in such cases no human intervention occurred, of course). Nevertheless, in 90% of the events, the result of the \( \chi^2 \) test coincided with the visual impression, so we conclude that the above parameterization and the whole procedure work satisfactorily. Typically, the largest \( \chi^2_m \) was obtained for the widest binning, 512 ms.

The number of resolved events for which the classification into simple and complex was possible is 852 (of 1395 useful events). The distribution of \( \chi^2_m \) for these events is presented in Figure 1a.

3. DIFFERENT BRIGHTNESS DISTRIBUTIONS FOR SIMPLE AND COMPLEX BURSTS

It is comparatively easy to distinguish between complex and simple events for the bright GRBs. One can then introduce a numerical measure of complexity, e.g., the total length of up and down variations, normalized to the peak count rate. Alternatively, one can count runs up and down and use their number to characterize complexity (Lestrade et al. 1996). Where \( p \) is the criterion, the significance level is \( \alpha \).

The only possibility for extracting a “simple” subpopulation from the weak burst population is to use the canonical shape of a single pulse, which is more or less identifiable by the human eye or by a \( \chi^2 \) fit. The former approach was used by SPS97 in the form of a visual blind test. All events were rescaled to the same (low) brightness, adding proper Poisson noise. Then each rescaled event was classified by three test persons as “simple” (a single FRED pulse), “complex” (not a FRED pulse), or “unresolved” (usually too short to be confidently identified). Then the peak count rate distributions for simple and complex GRBs were compared. Two of three test persons showed that complex GRBs are systematically brighter at a significance level exceeding 0.01; the third test person showed the same effect at a significance level of 0.1. We now present results using the \( \chi^2 \) tests on a larger statistics of GRBs.

\[ \text{http://www.batse.msfc.nasa.gov/data/grb/catalog} \]
The classification of a burst as simple is \( \tau_d > \tau_r, \chi_m^2 < 1.45 \) (all other events satisfying \( \tau_d + \tau_r > 2 \) s are classified as complex).

### 3.2. Peak Count Rate Distributions for Simple and Complex GRBs

We construct differential peak count rate distributions (log \( N \)–log \( P \)) for simple and complex events separately. These distributions, corrected for the trigger efficiency (each class has its own efficiency), are presented in Figure 2. We plot the number of bursts as a function of the peak count rate, although the values for the peak flux from the current BATSE catalog corrected for the spacecraft orientation, number of triggered detectors, and reflection from the atmosphere could be a better measure of brightness. The effect of simple and complex GRBs in the BATSE catalog peak flux distributions is similar: the KS test gives a consistency level of \( 10^{-5} \) when using 1024 ms peak fluxes and \( 10^{-7} \) for 64 ms peak fluxes. Unfortunately, it is much more difficult to estimate the trigger efficiency as a function of the peak flux. The trigger efficiency is a much sharper and better defined function of the peak count rate than of the peak flux. This simplifies the problem of correcting for the trigger efficiency when using peak count rate units.

In addition to the trigger efficiency, there exists a bias for the brightness-dependent dead time for burst readouts. (The trigger during readout time is revised so that a new event brighter than the triggered event on a 64 ms timescale overwrites the first trigger, while a weaker event will be lost.) The effect can be accounted for by discarding from the sample all events that overwrite a previous trigger. We found that the effect is small enough (only 5% of our events are overwrites). Moreover, the effect is almost the same for simple and complex events (within the statistical accuracy) and does not affect the difference in their distributions. In order to save the statistics, we work with the full sample of useful bursts.

The difference in the behavior of the log \( N \)–log \( P \) distributions for the two classes of GRBs is striking. While the curve for simple events is consistent with a power law down to the trigger threshold, the complex class demonstrates an apparent break at a count rate exceeding the threshold by a factor of 3–5. If one shifts the “complex” distribution by a factor of \( 1/k < 1 \), discarding bursts moving below the threshold (i.e., \( P/k < 55 \) counts/64 ms), then the KS consistency level peaks at \( k = 2.1 \), which can be interpreted as meaning that complex events are \( \sim 2 \) times brighter on average. This factor can be even larger, depending on the behavior of the “simple” distribution below threshold.

The main difference in the count-rate distributions is concentrated in the low count rate range and can be described as a relative deficit of complex bursts, by a factor of \( \sim 2 \) near threshold. The low count rate range may contain systematic biases (threshold effects), and we must estimate them before drawing any conclusions. We have no reason to suspect that the rescaling procedure is a source of bias, since this procedure is trivial, and we can exactly account for the variation of noise when we rescale bursts to the same brightness. Another possible bias is associated with the errors of the background fits, but this bias has a sign opposite to the effect; weak bursts should give a higher \( \chi^2 \), because of nonzero residuals outside of the peak. The visual classification of SPS97, which is less sensitive to rescaling and insensitive to the background subtraction, gave a similar result.

We also cannot suspect systematic errors in the estimates of the peak count rates. The reasonable linearity of these estimates is demonstrated in Appendix A (see Fig. 8). A correlation between complexity and the peak count rate estimates could exist, but this is a 10% effect, while a factor...
of 2 bias is required to account for the effect. A more serious source of systematic error could be a different trigger efficiency for simple and complex events. Indeed, slow risers, which have lower trigger efficiency, mostly belong to the complex sample. In order to verify this, we calculated the trigger efficiency separately for simple and complex events as described in Appendix A (the distributions in Fig. 2 as well as the significance levels given above are already corrected for different trigger efficiencies).

The efficiency for complex events is actually lower, e.g., at a peak count rate of 55 counts/64 ms it is 0.69 for simple events and 0.56 for complex events. Nevertheless, the difference is negligible compared to that required to explain the effect; a correction by at least a factor of 2 applied only to complex weak bursts is required to make the distributions consistent. We can hardly admit as an explanation that this is the result of a huge unknown selective bias that affects only high-$\chi^2$ bursts but does not affect low-$\chi^2$ events. We suggest that the upper curve in Figure 2, representing presumably intrinsically strong events, extends to larger cosmological distances, where some effect associated with high redshifts becomes important. Then it is natural to suggest that the lower curve will show a similar behavior below the threshold.

Recently, Pendleton et al. (1997) found that a similar effect: separating GRBs into subclasses affects their log $N$–log $P$ distribution. The presence of a hard tail in the spectrum (significant emission above 300 keV) was used as a criterion for separation. It is possible that the “no high energy” and “high energy” subclasses of Pendleton et al. (1997) correlate with our simple and complex groups, respectively.

In a cosmological scenario, it is natural to suggest that the intrinsically strong subpopulation of BATSE GRBs extends to $1 < z < 3$, where the universe evolves strongly. The evolution at this epoch is clearly visible in the QSO redshift distribution (Hartwick & Schade 1990) and in the star formation rate curve (Madau et al. 1996; Abrahm 1997). In the neutron star merging scenario (Blinnikov et al. 1997), the GRB rate should be associated with the star formation rate (Lipunov et al. 1995; Prokhorov, Lipunov, & Postnov 1997). In this case, the break in the log $N$–log $P$ curve for complex GRBs should correspond to the peak in the star formation rate at $z \sim 1.5$. In the galactic halo model, a cutoff in the neutron star radial distribution could also account for the observed break.

Another interpretation is that the different log $N$–log $P$ distributions of simple and complex bursts could be associated with the possible existence of two different classes of events among the simple GRBs. One class could have a noncosmological origin and therefore a Euclidean log $N$–log $P$ distribution, which when added to the cosmological component could explain the effect. Extending the log $N$–log $P$ distribution using nontriggered bursts could provide the answer.

4. BRIGHTNESS-DEPENDENT CORRELATIONS IN THE AVERAGE TIME PROFILE

After confidently observing a complexity-brightness correlation, the ATP-brightness correlation can be considered to be a direct consequence. Nevertheless, the latter still provides an independent confirmation of the correlation tendency, completes the picture, and is interesting in itself. For this reason, we present the latest results of our ATP studies, together with a more detailed description than was given in SPS97.

4.1. On the Stretched Exponential Shape of the ATP

A stretched exponential (SE) shape of the ATP, $f(t) = \beta \exp (-|t/t_0|^\nu)$, was claimed by Stern (1996) for the decaying slope and was confirmed with successively higher statistics by Stern & Svensson (1996) and SPS97 (in the latter work, an SE shape of the rising slope of the ATP was also demonstrated). Both the rising and the decaying slopes of the ATP for the overall useful BATSE statistics (1310 events) are presented in Figure 3a. The high statistics demonstrates that the picture is more complicated than the idealized assumptions presented in SPS97. The rising and decaying sections have not only different time constants but also different shapes (i.e., different SE indices, $\nu$). The decaying ATP is perfectly described by an SE with $\nu = 0.37$, while the rising ATP is described by an SE with $\nu = 0.30$. Note that the whole “disorder” comes from the weakest bursts. If we remove them, both the rising and decaying ATPs are much closer to the canonical shape, with $\nu = \frac{1}{2}$ (Fig. 3a). The different ATP shape of the weakest bursts was noted by SPS97. The difference is significant, and cannot be a result of the trigger-selection effect. The interpretation of this is discussed below. For now we just note that:

1. An SE shape with $\nu = \frac{1}{2}$ for both slopes is still a good working hypothesis for stronger bursts.
2. For the full sample, the SE indices of the rising and decaying ATPs differ. The difference is real and significant ($\sim 4\sigma$ effect). However, both ATPs are still described by

![Figure 3](image-url)
SE’s. We demonstrate below that the difference in ATP shapes is a natural consequence of the asymmetric shape of elementary pulses.

Let us now consider the issue of whether the SE shape of the ATP is simply one successful fitting hypothesis among other comparable possibilities, or whether it is a natural, intrinsic feature of the ATP.

The log-log t plot of the ATP does not resemble a power law in any time interval. Nevertheless, Mitrofanov, Litvak, & Ushakov (1997) used a fitting expression with a power-law asymptotic, \( f(t) = \frac{t_0}{t + t_0} + t \), to fit the ATP. Introducing a third parameter, the general multiplier \( \beta \) (i.e., \( f(t) = \beta \left[ \frac{t_0}{t + t_0} + t \right] \)), the best fit for the decaying part of the ATP in the 0.125–216 s range gives \( t_0 = 3.62 \) s and \( \alpha = 1.33 \), with \( \chi^2 = 128 \) for 19 degrees of freedom (Fig. 3b), and with an unreasonably small value at \( t = 0 \): \( f(0) = \beta = 0.62 \). Since Mitrofanov et al. (1997) constructed the ATP in 1024 ms resolution (i.e., the \( t < 1 \) s region was lost) and only within the \( t < 20 \) s range, it is not surprising that they obtained a good fit in this narrow interval. However, for a wider time interval this parameterization is unacceptable.

Let us take as another example the log-normal distribution that is quite common in nature. It has the wrong asymptotic at \( t = 0 \), but we correct this by setting \( f(t) = \beta \exp \left[ -\log^2 \left( t/t_0 \right) / 2 \sigma^2 \right] \). As \( t > t_0 \) and \( f(t) = \beta \) at \( t < t_0 \), this semiartificial expression fits the ATP in the same range as above, with \( \chi^2 = 89 \) for 19 degrees of freedom (best-fit parameters \( t_0 = 0.20 \) s and \( \sigma = 2.10 \)), again with an unsatisfactory value at \( t = 0 \): \( f(0) = \beta = 0.6 \) (see Fig. 3b). For comparison, an SE fit of the same ATP gives \( \chi^2 = 4.1 \) at \( v = 0.37 \), \( t_0 = 1.12 \) s, and \( \beta = 1.04 \). For the reasonableness of the \( \chi^2 \) values, see Appendix B.

Most likely, there is no three-parameter expression that would fit the ATP on such a broad time interval except for the SE. Note that our SE fit is in fact a 2.5 parameter fit, i.e., the third parameter, the multiplier \( \beta \), only accounts for the uncertainty associated with the first 64 ms bin of the ATP, assumed to be close to 1. The best-fit value of \( \beta \) is actually always close to 1, and therefore we do not give the \( \beta \) values.

Summarizing the issue, we state that the stretched exponentiality of the ATP is probably not exact. However, with the existing statistics we do not see any statistically significant deviations. The SE is a natural distribution shape in very different classes of physical phenomena when a wide range of timescales are involved (Ching 1991; Jensen, Paladin, & Vulpiani 1992). Therefore, we believe that our choice of SE as the fitting expression is not only efficient but also meaningful.

The procedures for measuring the SE time constants of the ATP and estimating statistical errors follow SPS97. These are described in more detail in Appendix B.

4.2. Time Constants of the ATP as a Function of Peak Flux

We use the following parameterization of the ATP:

\[
I(t) = \beta \exp \left( -\frac{t}{t_r} \right)^{\nu} \]

where \( t_r \) and \( \nu \) are the time constants and the SE indices of the rising and decaying ATP, respectively. In our studies of the ATP behavior, we use two kinds of SE fits. The first is the simultaneous fit of both ATP slopes with SE’s of the same index, \( \nu = \frac{1}{2} \), and a common normalization factor, \( \beta \). This three-parameter fit \((t_r, t_d, \text{and } \beta)\) has the best statistical accuracy. The results of these SE fits are summarized in Table 1 and Figure 4.

Compared to the GRB statistics used by SPS97 (912 events), the temporal stretching of the decaying slope for weak events is now more pronounced and significant, \( t_{a,b}/t_d = 1.89 \pm 0.68 \) (90% confidence interval), comparing samples 9 and 7 (see Table 1). The corresponding rejection level for the null hypothesis (i.e., no temporal stretching) has now increased to 0.99995 (assuming a normal distribution for the deviations of the values). The temporal stretching of the rising slope also increased slightly, but this is still at a low significance level. The resulting variation of the asymmetry (the ratio \( t_d/t_r \)) decreased slightly. Comparing samples 1 and 9, we get \( (t_d/t_r)_{\text{dim/bright}} = 1.48 \pm 0.33 \) (90% confidence interval). The significance level for this correlation remains the same as for the SPS97 sample, \( \sim 0.02 \). The variation of \( t_r + t_d \) with peak flux has a statistical significance of \( 4 \times 10^{-4} \); the “dim/bright” ratio becomes \( (t_r + t_d)/t_d \) \( = 1.71 \pm 0.02 \) (sample 9 compared to sample 7).

The results presented so far were obtained with a fixed SE index, \( v = \frac{1}{2} \). We now present a second fitting variant. As mentioned above, the ATP for the whole sample indicates a larger \( v \) for the decaying slope and a lower \( v \) for the rising slope. Despite the fact that the difference results from the contribution of weak GRBs, it would be interesting to check the behavior of the time constants when \( v \) is fixed to different values for the two slopes. Unfortunately, we cannot then use a common \( \beta \) for both slopes, since this would lead to unacceptable values of \( \chi^2 \). Therefore, we performed inde-

TABLE 1

| No. | Peak Flux | \( N \) | \( t_r \) | \( t_d \) | \( t_r + t_d \) | \( t_d/t_r \) | \( \chi^2 \) |
|-----|-----------|--------|--------|--------|-------------|-------------|--------|
| 1   | 12–∞      | 84     | 0.29 ± 0.08 | 0.36 ± 0.07 | 0.65 ± 0.13 | 1.23 ± 0.17 | 7.6    |
| 2   | 3–12      | 282    | 0.35 ± 0.04 | 0.51 ± 0.06 | 0.87 ± 0.09 | 1.47 ± 0.11 | 6.9    |
| 3   | 1.75–3    | 239    | 0.37 ± 0.05 | 0.64 ± 0.08 | 1.01 ± 0.12 | 1.74 ± 0.21 | 4.8    |
| 4   | 1–1.75    | 358    | 0.45 ± 0.05 | 0.73 ± 0.07 | 1.17 ± 0.11 | 1.63 ± 0.11 | 21     |
| 5   | 0.7–1     | 196    | 0.43 ± 0.06 | 0.74 ± 0.10 | 1.17 ± 0.15 | 1.70 ± 0.15 | 22     |
| 6   | 0–0.7     | 151    | 0.38 ± 0.06 | 0.77 ± 0.11 | 1.15 ± 0.17 | 2.00 ± 0.20 | 29     |
| 7   | 7.5–∞     | 145    | 0.29 ± 0.05 | 0.38 ± 0.06 | 0.67 ± 0.10 | 1.33 ± 0.14 | 11     |
| 8   | 5–∞       | 209    | 0.29 ± 0.04 | 0.41 ± 0.05 | 0.71 ± 0.09 | 1.40 ± 0.12 | 9      |
| 9   | 0.8–2.5   | 659    | 0.43 ± 0.04 | 0.72 ± 0.06 | 1.15 ± 0.09 | 1.68 ± 0.09 | 25     |

Note.—Time constants \( t_r \) and \( t_d \) are given for the SE simultaneous fit with \( v = \frac{1}{2} \) to the prepeak and postpeak average time profiles, respectively. Peak fluxes in 64 ms resolution (photons cm\(^{-2}\) s\(^{-1}\)) are taken from the BATSE database and are measured in channels 2 and 3. \( N \) is the number of bursts in the given brightness interval. The fitting time interval is \( 0.5 < |t^{1/2}| < 5 \). The errors and the \( \chi^2 \) values (for 33 formal degrees of freedom) are estimated as described in Appendix B.
demonstrated that the behaviors of these time constants with peak flux are the same. The variation of the asymmetry, \( t_d/t_r \), is even slightly larger than for the first fitting variant.

To test for spectral redshift of weak bursts as the possible explanation of the asymmetry variation, we measured the asymmetry in separate LAD energy channels. If GRBs were more asymmetric at higher energies, then the spectral redshift of the softer, more symmetric component to energies below the observational band could cause an asymmetry.

| Channel | \( t_r \) | \( t_d \) | \( t_d/t_r \) |
|---------|---------|--------|------------|
| 1       | 0.50    | 0.70   | 1.40       |
| 2       | 0.42    | 0.56   | 1.33       |
| 3       | 0.35    | 0.42   | 1.20       |
| 4       | 0.20    | 0.27   | 1.35       |
| 1-4     | 0.39    | 0.53   | 1.35       |

Note.—Time constants \( t_r \) and \( t_d \) of the averaged time profiles are given for the 280 brightest events in separate LAD energy channels. The value of the index \( \nu \) was fixed at \( \frac{1}{2} \). Relative errors for \( t_r \) and \( t_d \) are 12%; for \( t_d/t_r \), they are 8%.

4.3. Variations of the Stretched Exponential Index with Peak Flux

Figure 5 presents the decaying and rising slopes for samples 1 + 2, 3 + 4, and 5 + 6. The SE indices for these samples are given in Table 3. Except for the apparent time dilation effect, the deformation of the weakest ATP is clearly visible. In part, this deformation results from brightness-dependent biases (short, weak bursts have smaller trigger efficiency, and the peak searching scheme is unable to find the sharp peaks of longer events; see also Fig.

| Number | Peak Flux | \( N \) | \( \nu_r \) | \( \nu_d \) |
|--------|-----------|--------|------------|------------|
| 1 + 2   | 3 - 200   | 366    | 0.338 ± 0.027 | 0.322 ± 0.026 |
| 3 + 4   | 1 - 3     | 597    | 0.316 ± 0.020 | 0.359 ± 0.023 |
| 5 + 6   | 0 - 1     | 347    | 0.252*        | 0.411 ± 0.034 |
| All     | 0 - 200   | 1310   | 0.300 ± 0.017 | 0.371 ± 0.021 |

Note.—\( \nu_r \) and \( \nu_d \) are best-fit SE indices for the three-parameter (free \( \beta \), \( \nu, \nu_r, \nu_d \)) fit in the 0.5 < \( t^{1/3} \) < 6 interval.

For sample 5 + 6, the fit was done in the 1 < \( t^{1/3} \) < 6 time interval because of ATP deformations that were too strong within 1 s. Errors were not estimated.
9 in Appendix A). These biases suppress the ATP within 1 s from the peak, while its effect beyond 1 s is not so strong.

The estimate of the significance level for the shape variation is difficult, because trigger selection and Poisson biases give deformations of the same sign as (but still much smaller than) observed. SPS97 estimated the significance level of the deformation of the decaying slope for the weakest sample as 0.05, after correcting for the above biases and the possible error of the background subtraction. We do not make such an estimate here, because the deformations of the ATP must appear as a direct consequence of the much more significant complexity-brightness correlation described above.

5. ON THE ISSUE OF THE INTRINSIC LUMINOSITY FUNCTION

When separating GRBs into brightness groups using their peak fluxes, we assumed that the peak fluxes give us an approximate measure of the distance (a standard-candle approximation). The fact that we see correlations between the shape of the ATP and apparent peak brightness tells us that the peak luminosity has a rather broad distribution and correlates with the ATP. What could serve as a better standard candle?

Some researchers (e.g., Petrosian & Lee 1996) suggest that the total energy fluence is a better standard candle than the peak luminosity, arguing that this would be more physical. However, if we accept that something like the pulse avalanches of Stern & Svensson (1996) takes place, then a dispersion in the fluence of a few orders of magnitude seems natural. The GRB itself emits just a fluctuating fraction of the total available energy, and probably there are many events in which an observer sees no GRB. The pulse avalanche model describes the transmission of energy from a main reservoir as a highly unstable, near-critical process, where in the idealized case of exact criticality and infinite available energy the fluence distribution should become a power law.

A better candidate for the standard candle would be a single pulse, regardless of whether it alone constitutes a burst or appears as one of the pulses in a complex bursts. Again, the distribution of the pulse peak amplitudes (fluxes) within one GRB seems narrower than the distribution of the pulse fluences (for studies of the fluence distribution of pulses in GRBs, see Li & Fenimore 1996); otherwise, a negative correlation between the pulse duration and its amplitude within one GRB would be visible. Does any correlation exist within one event? The visual impression of simple and complex events in this example (see Fig. 6) appears only as a result of the correlation between temporal properties of GRBs and their intrinsic luminosity (if we discard as an explanation an evolutionary effect (the ratio of the median peak flux of the distributions is 3.7)).

6. SUMMARY AND CONCLUSIONS

The effects we confidently detect can be summarized as follows:

1. Complex bursts have systematically larger peak count rates than simple bursts. Their peak count rate distribution has an apparent break at a count rate of \( \sim 200 \) counts/64 ms, which exceeds the threshold by a factor of 4.
2. Weaker bursts have a stronger asymmetry between the rising and decaying slopes of the average time profile.
3. The weakest bursts have a different shape of the average time profile, which can be described as a larger SE index (at least for the decaying slope).
4. The general temporal stretching of the ATP for weak bursts is complicated by the deformations of the ATP resulting from the two previous effects.

Effects (2) and (3) appear as direct consequences of effect (1). Effect (4) can be a superposition of both cosmological time dilation (Paczynski 1992) and the intrinsic luminosity-duration correlation (Brainerd 1994). Effects (1)–(3) can appear only as a result of the correlation between temporal properties of GRBs and their intrinsic luminosity (if we discard as an explanation an evolution of GRB temporal properties with time that seems unnatural). The large value of these effects indicates that the intrinsic luminosity function is a wide one, comparable with the apparent brightness dispersion from the distance distribution. Therefore, the log \( N \)-log \( P \) curve can be a convolution of two functions of comparable dispersions.

![Diagram](image-url)
Both the most confident and the most informative effect is the first one. There is no obvious source for such a high near-threshold bias (selective for complex events) to account for this effect. There is, however, a natural phenomenon that could account for it. As mentioned above, this could be the evolution of the star production rate. We must accept this as a possible explanation if we accept the merging of neutron star binaries as the source of GRBs.

It seems that we succeeded in extracting an intrinsically strong subsample from the BATSE GRBs, covering distances to redshifts of $z \sim 1.5$, where evolutionary effects should be strong. A cosmological fit with an evolutionary factor for GRBs based on the measured star formation rate and with a model luminosity function similar to that presented in § 5 is a matter for a separate work. The crucial data needed to confirm this point of view could be provided by the search for untriggered bursts in the continuous BATSE data records. This search is ongoing (Kommers et al. 1997) and is worth intensifying.

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APPENDIX A

DATA ANALYSIS

A1. BACKGROUND FITTING

The background fitting was based on a visual scan of all BATSE bursts. This work cannot be done automatically because there exist many events with complex backgrounds contaminated by non-Poisson features that can mimic the contribution of a GRB. In fact, each individual burst requires a researcher’s decision on how to fit it or whether it should be discarded. To avoid subjective biases for weak events, we followed these rules:

1. All fits are linear (since the background often demonstrates a nonpolynomial behavior, and a higher order polynomial fit could be unstable). Linear fits were made over one or two fitting windows, which were set in quiet time intervals, having a good $\chi^2$. We also set the observational windows to avoid background features not associated with the burst. In the case of a smooth curved background with several burst episodes in the event, we set a few fitting windows including quiet time intervals between burst episodes, so the background was approximated by a broken line.

2. Each feature separated by a wide time interval from the main peak should be tested to determine whether it came from the same direction as the main peak. To compare directions, we made a linear fit to the count rate in each of eight LADs in the time interval covering the feature. We then maximized the total reduced $\chi^2$ in the time interval, varying the time resolution, and calculated the eight-component vector $\chi^2$. This vector should be approximately parallel to that of the main peak; otherwise, the feature should be avoided when setting the observational windows. The procedure turned out to be an efficient way to clean bursts from unrelated fluctuations.

3. We adopted a default set of windows: a fitting window ($-120 \pm 70$ s, where the boundaries are given relative to the trigger time), an observational window ($-70 \pm 200$ s), and a second fitting window ($+200 \pm 250$ s). If possible, we used the default set. If the background has a moderate curvature, it should not give a bias, since it has a random sign. This rule reduces a possible subjective bias. A narrower time interval between the fitting windows was allowed if the burst was apparently short or if the background was strongly curved.

4. We discarded all events for which we were unable to make a confident conclusion that we did not lose or contaminate the signal at 100 s after and 50 s before the highest peak. These contaminations could be due to wide data gaps, strong solar flares, rapid variations of the background, or chaotic variations with bad $\chi^2$ from the wrong direction.

This fitting procedure gives as small a bias as possible when summing up signals from different GRBs to get the average time profile, despite the fact that it is not accurate when dealing with individual events. To estimate the magnitude of possible errors introduced by the fitting procedure, we made the following test. For each event where it was possible ($\sim 75\%$ of all events), we selected testing windows between our fitting windows, avoiding regions with GRB signals, and measured the residuals of our fits in the testing windows. The distribution of residuals for 469 weak and medium events is shown in Figure 7. The distribution of residuals is reasonably symmetric, the average value for the residual is $+0.084$ counts/64 ms, and the 1 $\sigma$ variance is 1.43 counts/64 ms. This is an argument that we have no significant systematic error (that exceeds the statistical error). The statistical error for the average residual is 0.11 counts/64 ms. For comparison, the typical background in channels 2 and 3 is 300 counts/64 ms, and the peak count rate for the weakest events is $\sim 50$ counts/64 ms. The error introduced into the average time profile is inversely proportional to the peak count rate. For medium and bright events, we can neglect the fitting uncertainty; it is much less than $10^{-3}$. The exception is the weakest group, where the 1 $\sigma$ error in the relative averaged residual is $1.5 \times 10^{-3}$.

A2. SELECTION OF THE HIGHEST PEAK

Although direct peak selection using the highest 64 ms bin works well for bright events, it suffers from Poisson noise for weak events. This technique simply takes the highest Poisson fluctuation as the burst’s highest peak. Nevertheless, since we adopted the 64 ms resolution approach, we should not use a different time resolution for weak bursts. As a compromise, we...
developed a hybrid scheme that combines a search for the peak interval in a lower time resolution and a search for statistically significant deviations in this interval with a higher time resolution.

First we determine the shortest timescale, $\Delta t_j = 64 \text{ ms} \times 2^j$, $j = 0, \ldots, 4$, where a GRB has statistically significant variations between neighboring time bins (a 7 $\sigma$ threshold, which corresponds to a 5 $\sigma$ threshold for a deviation from the average). We then search for the highest peak centered at bin number $k$, using the time resolution $\Delta t_j$, and calculating the peak flux as $\max \left( \sum_i c(i) \exp \left( -\frac{(k - i)^2}{2\sigma^2} \right) \right)$, where $k$ and $i$ are indices of 64 ms bins, and $c(i)$ is the count rate in the $i$th bin.

Then if $j \geq 1$, we make a second iteration, searching for statistically significant excess over average within the brightest $\Delta t_j$ interval using a shorter timescales. The significance threshold, $h_l (l < j)$, is reduced at this step. The thresholds were optimized empirically when tuning the scheme by rescaling strong bursts to the weakest, adding corresponding Poisson noise, selecting

![Graph](image.png)

**Fig. 7.**—Residuals of our background fits in testing windows located in time intervals where the contribution of the GRB is invisible. The background fits were obtained using fitting windows (see text).

![Graph](image.png)

**Fig. 8.**—Test of the procedure for the peak count rate estimate. (a) Expected vs. measured count rate scattering plot obtained with bright events longer than 2 s, rescaled to lower brightness. Note that the peak count rate estimate is based on the 64 ms resolution. (b) Distributions of the measured peak count rates at fixed expected count rate $P$: $P = 55$ counts/64 ms; $P = 110$ counts/64 ms; $P = 200$ counts/64 ms. Bursts that did not pass the simulated trigger are not included. For description of rescaling and triggering procedures, see Appendix A.3.
the highest peak, and comparing the resulting peak amplitude and position with the true values. With thresholds of $h_0 = 4.0 \sigma$, $h_1 = 3.2 \sigma$, $h_2 = 2.0 \sigma$, and $h_3 = 1.2 \sigma$ above the average count rate in $\Delta t_j$, we obtain a reasonable linearity between the expected and the measured count rates and still preserve the 64 ms resolution.

A test of this procedure by rescaling strong events to a given value of the peak count rate is presented in Figure 8. The rescaling procedure is described in Appendix A3. Only those events that passed our simulated trigger after rescaling were included in the test distributions. One can see from Figure 8a that we have achieved a reasonable linearity between the measured and the expected peak count rates (compare with a similar plot in Fig. 2 of in ’t Zand & Fenimore 1994). The systematic bias in the peak count rate for the weakest events is within 3%, and the relative error is $\sim 35\%$ (FWHM) for $P = 55$ counts/64 ms and $\sim 22\%$ for $P = 110$ counts/64 ms. There are non-Gaussian tails toward higher values associated with Poisson fluctuations. However, they do not exceed a few percent of the peak integral. If one increases the thresholds, these tails will disappear, but a nonlinear bias of peaks toward lower values will appear. The thresholds have been set so as to have a reasonable average linearity of peak count rate estimates for weak events. (Note that direct selection of the highest bin in 64 ms resolution systematically overestimates the peak amplitude by a factor of 2 for the weakest events). The errors in the peak position can be characterized as follows: in 35% of the cases, the error does not exceed one 64 ms bin; with 47% probability the error is within 0.128 s, with 83% probability it is within 1 s, and in 6% of the cases the error exceeds 3 s.

A3. ESTIMATION OF THE TRIGGER EFFICIENCY

We estimated the trigger efficiency assuming that GRBs with near-threshold count rates do not differ in their temporal properties from stronger GRBs (which is not exactly true). The procedure consists of the following steps:

1. For a given peak count rate $P_0$, randomly sample one of the stronger bursts with a peak count rate of $P > P_0$ from the same class (i.e., simple or complex) for which we are going to estimate the trigger efficiency and subtract the background, thus extracting a pure signal.
2. Randomly sample another strong event of arbitrary class in order to use its linear background fit.
3. Rescale the signal by a factor of $P_0/P$, and distribute it between the two detectors with the largest projected areas for a randomly sampled direction of the burst. Distribute the signal proportionally to their projected areas. This step does not take into account reflection from atmosphere, which causes more uniform exposure of the detectors, thus enhancing the trigger efficiency. This procedure also ignores the dependence of the detector response matrix on the projection angle. This dependence also enhances the trigger efficiency. Thus, our procedure slightly underestimates the trigger efficiency. It also neglects the probability of triggering three detectors for weak bursts.
4. Add a new linear background from two arbitrary detectors of the event sampled at step 2 and a corresponding Poisson noise (taking into account that some noise is already present in the rescaled signal).
5. Try the trigger procedure to the rescaled burst as it is programmed in BATSE (with the 5.5 $\sigma$ threshold that was used most of the observational time; see online BATSE catalog).

The fraction of rescaled bursts triggered with this procedure is the trigger efficiency. Note that with such a procedure, the trigger efficiency is a function of the expected count rate.

A4. ROBUSTNESS OF THE ATP: TEST FOR BRIGHTNESS-DEPENDENT BIASES

Brightness-dependent errors in the ATP induced by Poisson noise are of three kinds:

1. Errors in the peak count rate estimate that is used as the normalization of a time profile.
2. Errors in the peak position.
3. Trigger selection effects that remove short or short-spike-dominated events from the sample.

We estimated the brightness-dependent deformations of the ATP by rescaling strong events to low peak count rates and applying simulated trigger selection to the rescaled sample. The parent sample included 353 GRBs with the highest peak count rates of all morphologies and durations. The results are presented in Table 4 and in Figure 9.

We can state that down to sample 5 of Table 1, deformations of the ATP are negligible (the count rate 110 counts/64 ms is near the boundary between samples 5 and 6), and only for sample 6 (75 counts/64 ms) are they significant. Therefore, we can measure the variation of the ATP slopes without the rescaling procedure, which would have erased some information for the strong samples.

APPENDIX B

STRETCHED EXPONENTIAL FITS AND THEIR ERRORS

As noted in Stern (1996), the main problem when fitting ATPS is the correlation of deviations in different time bins (the ATP is the sum of more or less smooth but very different curves). This means that one cannot rely on deviations of individual profiles in a $\chi^2$ fit, especially when estimating the accuracy of this fit. In principle, a proper solution of the problem should exist, and it should require an overall correlation matrix implemented in the maximum-likelihood method, but we expect that such a solution is not an easy one. In Stern (1996), the errors were estimated using a number of smaller samples of bursts. The variance of the time constants derived from the smaller samples was then rescaled to larger samples as $1/\sqrt{N}$, where $N$ is the number of bursts in a sample. With a limited statistics, such a procedure can give only a very approximate estimate of the errors, and it is very unreliable if one uses it to estimate the statistical significance level of observed effects.
Deformations of the decaying (upper histograms) and rising (lower histograms) slopes of the ATP after rescaling of the parent sample of bright bursts. Thickest histograms: the ATP of the parent sample. Medium thick histograms: the ATP of the parent sample, rescaled to a peak count rate of 150 counts/64 ms. Thin histograms: the ATP of the parent sample, rescaled to a peak count rate of 75 counts/64 ms. The ATP of the parent sample rescaled to 250 counts/64 ms is almost indistinguishable from the parent sample.

Following SPS97, we estimate the statistical errors using the pulse avalanche model simulations. Here we present further details of that procedure. For a description of the pulse avalanche model, see Stern & Svensson (1996). The general timescale in the model is defined mainly by an upper cutoff for the pulse width distribution. Another parameter that was varied is the criticality index, $\mu$, that defines the Poisson average of baby pulses per parent pulse in the avalanche. At supercritical values of $\mu$, the process diverges. By varying this index, some finer features such as the stretched exponential index and the rise/decay asymmetry of the average time profile could be tuned. However, we varied $\mu$ mainly in order to test a possible model dependency of the statistical errors.

We found that the rise/decay asymmetry of the ATP can be described better when we introduced a "global envelope," that is, an external time dependence for the criticality index, $\mu$, such that the Poisson average of baby pulses per parent pulse in the avalanche. At supercritical values of $\mu$, the process diverges. By varying this index, some finer features such as the stretched exponential index and the rise/decay asymmetry of the average time profile could be tuned. However, we varied $\mu$ mainly in order to test a possible model dependency of the statistical errors.

To fit the ATP, one needs to split the ATP into a number of bins. This number should not be too large, or else neighboring bins will be too strongly correlated and the value of $\chi^2$ or other likelihood estimators will be completely meaningless. We choose equidistant binning in the $t^{1/3}$ scale, each bin being 0.25 s$^{1/3}$ wide. Correlations are still strong, but with a wider binning we could lose some information. Our value of $\chi^2$ is still not usable as a direct estimator of the quality of the fit, but

TABLE 4

| Peak Counts | $t_r$ | $t_d$ | Efficiency |
|-------------|------|------|------------|
| Parent sample | 0.32 | 0.43 | 1.00 |
| 250 .......... | 0.31 | 0.42 | 0.97 |
| 150 .......... | 0.31 | 0.42 | 0.83 |
| 110 .......... | 0.32 | 0.44 | 0.71 |
| 75 .......... | 0.33 | 0.47 | 0.57 |

**Note:** Test for robustness of ATP slopes against brightness-dependent effects. The parent sample consists of 353 GRBs, with peak count rate (in 64 ms resolution) in the interval 500–∞. It was then rescaled to the count rates displayed in the first column. Note that all time constants are correlated, since all have the same parent sample. Therefore, the errors of Table 1 are not applicable here, and the time constants for different peak count rates have small dispersion. When comparing with Table 1, one can use the approximate coefficient 0.0072 to translate peak count rates into peak fluxes with units photons s$^{-1}$ cm$^{-2}$. 

**FIG. 9.** Deformations of the decaying (upper histograms) and rising (lower histograms) slopes of the ATP after rescaling of the parent sample of bright bursts. Thickest histograms: the ATP of the parent sample. Medium thick histograms: the ATP of the parent sample, rescaled to a peak count rate of 150 counts/64 ms. Thin histograms: the ATP of the parent sample, rescaled to a peak count rate of 75 counts/64 ms. The ATP of the parent sample rescaled to 250 counts/64 ms is almost indistinguishable from the parent sample.
using the model we can calculate the distribution of $\chi^2$ for many intrinsically good (i.e., giving a good SE at high statistic) ATPs, and then define the actual effective number of degrees of freedom and the renormalization coefficient for $\chi^2$.

When fitting an ATP for a sample of $N$ bursts, we must know how the deviations in each time bin are distributed for many independent samples. We calculated such distributions using $N \times K$ simulation runs, which produce $K$ independent samples of $N$ events each (typically $K = 500$ and $N$ vary from 100 to 1000). We found that the deviations are excellently described by a continuous Poisson distribution, or, in other terms, by a gamma distribution, $\Phi(x, a) = a^x e^{-a} / \Gamma(x + 1)$, where $x$ is the distributed value and $a$ is a parameter equivalent to the Poisson average (traditionally, gamma distribution is used with $a$ as the distributed value and $x = a$ as a parameter).

We found that the Poisson average $a$ in bin $j$ for all studied cases can be parameterized as $a_j = p_j N \xi(p_j) \phi(p_j, N)$, where $p_j$ is the ATP value in the bin (averaged over $N \times K$ events), and $\xi$ and $\phi$ are slowly varying functions: $\xi$ varies between 3 and 5, $\phi(x) = 1$ at $x > 2$ and smoothly decreases at $x < 2$. Then the standard maximum likelihood procedure was applied with an estimator $\chi^2 = -2 \sum_j \ln [\Phi(p_j - h_j, a_j)/\Phi(a_j, a_j)]$, where $h_j$ is our hypothesis: $h_j = \beta \exp \left(-t/t_0\right)$. At large values of $a$, this expression coincides with the traditional $\chi^2$ estimator.

We need two kinds of fits, depending on the aim. If we are interested in the shape of the ATP slope, we use a three-parameter fit, $f(t) = \beta \exp \left(-|t/t_0|\right)$, where $\beta$, $\nu$, and $t_0$ are free parameters, with the additional requirement that $\beta$ is close to 1, which is fulfilled in all reasonable cases. With pulse avalanche simulation runs (500 $\times$ 300) and (500 $\times$ 1000), we found that the error in

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{scatter.png}
\caption{Scatter diagram for best-fit values of SE index, $\nu$, and time constant, $t_0$, for pulse avalanche simulations}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{chi2.png}
\caption{Examples of $\chi^2$ distributions for simulated samples of GRBs. \textit{Thick histogram}: Three-parameter fit of the decaying slope (free $\beta$, $\nu$, and $t_0$), 1000 events in the sample, 15 formal degrees of freedom. \textit{Thin histogram}: Three-parameter simultaneous fit of both slopes of the ATP (free $t_\nu$, $t_0$, and a common $\beta$; $\nu$ is fixed to $\frac{3}{2}$), 300 events in the sample, 33 formal degrees of freedom.}
\end{figure}
the best-fit estimate for $v$ is $\sigma_v/\nu = 0.049 \times (1000/N)^{1/2}$, where $N$ is the number of events in the sample. The cross-correlation of $t_0$ and $v$ for a pulse avalanche simulation run ($200 \times 750$) is shown in Figure 10. One can see that it is very strong and that the error for $t_0$ is large for a free $v$.

However, if we are interested in the behavior of time constants as a function of brightness, we should fix $v$. Indeed, we can compare time constants only with a hypothesis that the ATPs have intrinsically the same shape (i.e., the same SE index $v$). Our studies demonstrate that this hypothesis is not exactly true (see §4). Nevertheless, the measurement of $t_0$ with constant $v$ is still the best that can be done. The deformation of the slope, if moderate, gives a second-order error. Therefore, we measure $t_0$ with a two-parameter fit.

In principle, one can set $\beta = 1$ and make a one-parameter fit, but the first bin includes all possible biases from the finite resolution and the Poisson noise. We therefore excluded the first two bins ($t^{1/3} < 0.5$) from the fit, treating $\beta$ as a free parameter. The upper limit for the fitting range was set to $t^{1/3} = 5$ (i.e., $t = 125$ s).

If we had two slopes of the profile, prepeak and postpeak, we fitted them simultaneously with a different time constant, $t_r$ (prepeak, rising slope), $t_d$ (postpeak, decaying slope), and a common $\beta$. Performing a number of model runs with different parameters, we found the accuracy of the stretched exponential fits to be almost model-independent. The standard deviation for the time constant does not change by more than 5% for different parameters, and scales as $1/\sqrt{N}$, depending on the number of events in the sample. The accuracy slowly increases when the fitting time interval is extended (see Table 5). We chose the widest interval, which is the default for the results presented elsewhere in the paper.

For the sum and ratio of time constants for prepeak and postpeak slopes, we have

$$\sigma(t_r + t_d) = 0.196 \frac{100}{\sqrt{N}},$$

$$\sigma(t_d/t_r) = 0.135 \frac{100}{\sqrt{N}}.$$  

Note that the relative accuracy for the sum of the time constants is close to that for one constant, while the accuracy for their ratio is considerably better. This is the result of the strong correlation between the two slopes, a circumstance that favors the measurement of shape-brightness correlations and complicates the measurement of a time-dilation effect.

As mentioned above, the formal values of the $\chi^2$ have no direct interpretation; the Pearson criterion does not work in this case because of strong correlations along the ATP. This could be interpreted as the effective (unknown) number of degrees of freedom being smaller than the numbers of bins. In fact, the distribution of $\chi^2$ becomes wider for a larger number of events in the sample. In Figure 11 we present two distributions of $\chi^2$ for simulated samples that could be used for an approximate evaluation of the goodness of the $\chi^2$ obtained in the ATP fittings of real GRB samples.

### REFERENCES

Abraham, R. 1997, Nature, 387, 850
Blinnikov, S. I., Novikov, I. D., Perevodchikova, T. V., & Polnarev, A. G. 1984, Soviet Astron. Lett., 10, 177
Brainerd, J. J. 1994, ApJ, 428, L1
Ching, E. 1991, Phys. Rev. A, 44, 3622
Fenimore, E. E., & Bloom, J. S., 1995, ApJ, 453, 25
Hartwick, F. D. A., & Schade, D. 1990, ARA&A, 28, 437
in ’t Zand, J. J. M., & Fenimore, E. E. 1994, in Proc. 2d Huntsville Gamma-Ray Burst Workshop, ed. G. J. Fishman, J. J. Brainerd, & K. Hurley (New York: AIP), 692
Jensen, M. H., Paladini, G., & Vulpiani, A. 1992, Phys. Rev. A, 45, 7214
Kommers, J. M., Lewin, W. H. G., Kouveliotou, C., van Paradijs, J., Pendleton, G. N., Meegan, C. A., & Fishman, G. J. 1997, ApJ, 491, 704
Lee, T. T., & Petrosian, V. 1997, ApJ, 474, 37
Lestrade, J. P. 1994, ApJ, 429, L5
Li, H., & Fenimore, E. E. 1996, ApJ, 469, L115
Lipunov, V. M., Postnov, K. A., Prokhorov, M. E., Panchenko, I. E., & Jorgensen, H. 1995, ApJ, 454, 593
Madau, P., Ferguson, H. C., Dickinson, M. E., Giavalisco, M., Steidel, C. C., & Fruchter, A. 1996, MNRAS, 283, 1388
Mitrofanov, I. G., Chernenko, A. M., Pozanenko, A. S., Briggs, M. S., Paciesas, W. S., Fishman, G. J., Meegan, C. A., & Sagdeev, R. Z. 1996, ApJ, 459, L70
Norris, J. P., Nemiroff, R. J., Bonnell, J. T., Scargle, J. D., Kouveliotou, C., Paciesas, W. S., Meegan, C. A., & Fishman, G. J. 1996, ApJ, 459, 393
Norris, J. P., Nemiroff, R. J., Scargle, J. D., Kouveliotou, C., Fishman, G. J., Meegan, C. A., Paciesas, W. S., & Bonnell, J. T. 1994, ApJ, 424, 540
Pacyński, B. 1992, Nature, 355, 521
Pendleton, G. N., et al. 1997, ApJ, 489, 175
Petrosian, V., & Lee, T. T. 1996, ApJ, 467, L29
Prokhorov, M. E., Lipunov, V. M., & Postnov, K. A. 1997, in Proc. XXXII Rencontres de Moriond (Les Arcs, France), in press (preprint astro-ph/9704039)
Stern, B. E. 1996, ApJ, 464, 1111
Stern, B. E., Poutanen, J., & Svensson, R. 1997a, ApJ, 489, L41 (SPS97)
Stern, B. E., & Svensson, R. 1996, ApJ, 469, L109
Stern, B. E., Svensson, R., & Poutanen, J. 1997b, in 2d INTEGRAL Work-shop: The Transparent Universe (ESA SP-382) (Paris: ESA), 473

### TABLE 5

| Fitting Interval | $\sigma(t_{p+d})/(100N)^{1/2}$ |
|------------------|-------------------------------|
| $0.125 < |t| < 8$ | 0.252 |
| $0.125 < |t| < 27$ | 0.219 |
| $0.125 < |t| < 64$ | 0.205 |
| $0.125 < |t| < 125$ | 0.201 |