Time evolving fluid from Vaidya spacetime

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Abstract

A time evolving fluid system is constructed on a timelike boundary hypersurface at finite cutoff in Vaidya spacetime. The approach used to construct the fluid equations is a direct extension of the ordinary Gravity/Fluid correspondence under the constrained fluctuation obeying Petrov type I conditions. The explicit relationships between the time dependent fluctuation modes and the fluid quantities such as density, velocity field and kinematic viscosity parameters are established, and the resulting fluid system is governed by a system of a sourced continuity equation and a compressible Navier-Stokes equation with non-trivial time evolution.

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1 Introduction

The AdS/CFT correspondence [1–3] has proven to be a powerful tool for the study of strongly coupled quantum field theories, especially for understanding their transport properties [4]. Hydrodynamics is the low-energy effective theory for slowly varying fluctuations around thermal equilibrium of any quantum field theory, it has been successfully applied to describe the quark-gluon plasma created in heavy-ion collisions [5]. Holography provides a simple prescription in which one can extract the properties from the classical perturbation equations of black holes [6, 7]. It has been shown that holographic plasmas near equilibrium are close to perfect fluids [8].

One of the important results from holographic hydrodynamics says that, for standard Einstein gravity with isotropic black hole horizons, the ratio of shear viscosity to entropy density \( \eta/s \) of the holographic quark-gluon plasmas tends to take a universal value \( \frac{1}{4\pi} \). However this value receives corrections due to higher curvature terms [11–13] or in the presence of translation symmetry breaking [14–16]. The value \( \frac{1}{4\pi} \) is known as the Kovtun-Son-Starinets (KSS) bound [9, 10]. One utilizes the violation of KSS bound to study the relationship between shear viscosity of boundary theories and thermodynamical phase structure of the bulk black holes [17, 18], and this promotes our understanding to the behaviors of bulk gravity solutions.

With the success of the holographic techniques, the hydrodynamic limits of boundary quantum field theories have been widely investigated, and were extend in [19, 20] for studying non-linear fluid dynamics and the higher order transport coefficients. Universality of hydrodynamics also appear in the second-order transport coefficients whose linear combination was proven to vanish both in conformal theory duality [21, 22] and non-conformal cases [23]. Again this analysis is corrected by higher curvature terms [24].

Actually, one can realize a horizon fluid which is governed by Damour-Navier-Stokes equation, and the seminal work could be dated back to the 1970’s [25]. In spite of the differences in approaches, the shear viscosity to entropy density ratio \( \eta/s \) can be evaluated in both cases [26, 27]. Efforts have been made to clarify that the Gravity/Fluid correspondence can be expressed in membrane paradigm language by considering a fictitious membrane at finite cutoff [28]. In analogy to the AdS/CFT duality, the radial position \( r_c \) of the holographic screen is related to the energy scale of the boundary quantum field theory, so the dependence of quantities on \( r_c \) is viewed as the renormalization group flow in the resulting fluid [29–31].

Instead of requiring an asymptotically AdS region, the author of [32] introduced an arbitrary finite cutoff \( r_c \) outside the horizon and construct the non-relativistic dual fluid on the hypersurface. With the Dirichlet boundary condition and the requirement of regularity at the future horizon, it was shown that any solution of the incompressible Navier-Stokes equation could be mapped to a unique solution of the vacuum Einstein equation [33]. More importantly the derivative expansions and the regularity conditions are shown to be mathematically equivalent to the near horizon expansions.
and Petrov type boundary condition respectively in Gravity/Fluid correspondence. In this construction, it is unnecessary to solve perturbed Einstein equation, making it mathematically much simpler and elegant [34]. Numerous generalizations and discussions about Gravity/Fluid correspondence have been carried out along this line of researches [35–46].

Most of the existing literature concentrates on the cases where the bulk gravity is consisted of a stationary spacetime, and consequently the resulting holographic fluid possesses a stationary, non-evolving density distribution. It is certainly of great interests to consider fluids with non-trivial time evolution in the framework of Gravity/Fluid correspondence. This will deepen our understanding on the non-equilibrium processes in the boundary field theory which, presumably may describe a system of holographic plasma. In [47], it is shown that non-equilibrium plasma behaves approximately like a perfect fluid and its stress tensor is determined by a single time-dependent function. The success of AdS/CFT on near-equilibrium cases motivates researchers to study thermalization of strongly coupled plasma. Ref. [48] investigated the dynamical time-dependent process in the boost invariant approximation in the context of AdS/CFT in hydrodynamics limit. It is known that the thermalization process in the dual gauge theory is related to the process of black hole formation in the bulk [49,50]. The holographic entanglement entropy which was initially proposed in static background is also generalized to study their evolution during thermalize process [51,52]. In the present work, we will adopt the Petrov type approach to construct a time-evolving fluid on a finite cutoff surface in a dynamical spacetime. The simplest dynamical spacetime is Vaidya spacetime, which describes the formation of a Schwarzschild-AdS like black hole out of a collapsing shell of null dust. According to [49], Vaidya spacetime can be used as a model dual to the equilibration process of the boundary theory. We find that the time-evolving fluid living on a finite cutoff surface in Vaidya spacetime obeys a sourced continuity equation and a forced Navier-Stokes equation, whose energy density and viscosity are both time-dependent.

2 General setup

Consider a \((d+2)\)-dimensional Einstein gravity with a negative cosmology constant and a null dust as matter source. The field equation reads

\[
R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu}, \quad \Lambda = - \frac{d(d+1)}{2\ell^2},
\]

where \(\ell\) represents the AdS radius and we will set \(\ell\) as well as the Einstein constant \(\kappa\) to unity in this paper. The stress-energy tensor of null dust source reads

\[
T_{\mu\nu} = \mu l_\mu l_\nu, \quad l^\mu l_\mu = 0.
\]

We focus ourselves on the planar Vaidya solution with the metric

\[
ds^2 = -f(r,u)du^2 + 2dudr + r^2 \sum_{i=1}^{d} dx_i^2, \quad f(r,u) = r^2 - \frac{m(u)}{r^{d-1}},
\]
where

\[ T_{uu} = \frac{d\partial_u m(u)}{2r^d} \]  

(3)

is the only non-vanishing component of the stress-energy tensor.

In the study of gravitational collapse, in order to characterize the formation of a black hole at the level of local time evolution, there is a generalized notion called apparent horizon which is distinct from the event horizon. The apparent horizon is a closed spacelike hypersurface of codimension two and is the boundary of the trapped surfaces for which the expansions along the two future directed normal null directions are negative. For Vaidya spacetime, the apparent horizon is located at

\[ r_h = \frac{m}{1/(d-1)}. \]

With the null-energy condition which implies \( \dot{m}(u) > 0 \), the radial position \( r_h \) of the apparent horizon always grows. In contrast, the event horizon is given by

\[ \lim_{u \to \infty} r_e = \frac{m}{1/(d-1)}. \]

We will assume that the mass function \( m(u) \) tends to a finite value \( M \) as \( u \to \infty \), so that the final state of the spacetime (2) corresponds to the geometry of Schwarzschild black hole with a planar topology. In the timelike asymptotic region, it is known that the apparent horizon and event horizon coincide in static spherical spacetime.

The physically meaningful profile for the mass function \( m(u) \) should be increasing from 0 to a finite value \( M \), which may be chosen to be

\[ m(u) = \frac{M}{2}(\tanh(u/v_0) + 1). \]  

(4)

This profile describes a geometry transition from a pure planar AdS spacetime to a Schwarzschild-AdS black brane. The parameter \( v_0 \) characterizes the thickness of the null dust shell which falls along \( u = 0 \). When \( v_0 \) tend to zero, the mass function becomes to a step function \( M \theta(u) \).

To construct holographic fluid living on the codimension-1 timelike hypersurface, we will always keep the intrinsic metric of the hypersurface fixed since we are mainly interested in the evolution of dual fluid caused by the process of the formation of bulk black hole. As a consequence, the near horizon limit which is usually taken in Petrov type I approach of Gravity/Fluid correspondence is not valid any more, because of the time dependences of both the event and apparent horizons. For this reason we take the holographic screen \( \Sigma_c \) at some finite cutoff \( r = r_c \) outside the apparent horizon. In view of the renormalization group interpretation, the radial direction corresponds to energy scale of the boundary theory, so the finite cutoff implies a finite energy scale, which is physically more realistic and may be reached by experiment.
3 Constraints on the hypersurface

The geometry of the hypersurface $\Sigma_c$ is best characterized by its first and second fundamental forms. The first fundamental form is the projection tensor

$$h_{\mu\nu} = g_{\mu\nu} - n_\mu n_\nu,$$

where

$$n_\mu = \left(0, \frac{1}{\sqrt{f}}, 0, 0\right), \quad n^\mu = \left(\frac{1}{\sqrt{f}}, \sqrt{f}, 0, 0\right),$$

is a unit spacelike normal vector with $n_\mu n^\mu = 1$. Evidently one can identify $h_{\mu\nu}$ with the induced metric on $\Sigma_c$. More precisely, the induced metric $\gamma_{ab}$ on $\Sigma_c$ can be written as

$$\gamma_{ab} = h_{\mu\nu} e^\mu_a e^\nu_b,$$

where $e^\mu_a = \frac{\partial x^\mu}{\partial y^a}$ with $x^\mu = (u, r, x^I)$ and $y^a = (u, x^I)$. The second fundamental form is the extrinsic curvature

$$K_{\mu\nu} = \frac{1}{2} L_n h_{\mu\nu},$$

whose perturbations will be considered as the fundamental variables in the dual theory.

Projecting the field equations in the transverse and longitudinal directions, we get so-called a Hamiltonian and momentum constraints

$$\hat{R} + K^{ab} K_{ab} - K^2 = 2\Lambda - 2T_{\mu\nu} n^\mu n^\nu,$$

$$D_a (K^a_b - \gamma^a_b K) = T_{\mu\nu} n^\mu h^\nu_b,$$

where $\hat{R}$ is the Ricci scalar on the hypersurface $\Sigma_c$, $D_a$ is the covariant derivative compatible with induced metric. These constraints play essential roles in the construction of the fluid equations.

The definition of Brown-York stress tensor is

$$t_{ab} = h_{ab} K - K_{ab} - C h_{ab},$$

where the last term is a counter term to remove the divergence if the cutoff is taken at infinity. Here we take $C = 0$ since we consider only a finite cutoff. The Brown-York stress tensor is regarded as the stress-energy tensor of the dual fluid and mapped into polynomials in the extrinsic curvature through (8) on the boundary hypersurface. Therefore, we can take $t_{ab}$, instead of $K_{ab}$, as the fundamental dynamical quantity in the boundary theory. Then constraint equations are reformulated as

$$\mathcal{H} = \hat{R} + t^a_b t^b_a - \frac{t^2}{2} = 2\Lambda - 2T_{\mu\nu} n^\mu n^\nu,$$

$$\mathcal{P}_b = Da t^a_b = T_{\mu\nu} n^\mu h^\nu_b,$$

where $t^a_b = h^a_c e^b_c$.
where \( t = \gamma^{ab} t_{ab} \) is the trace of the Brown-York tensor.

Petrov type is a seminal proposal for classifying spacetime with certain symmetries. The Vaidya spacetime \((2)\) we are working in is of Petrov type D, which also belongs to Petrov type I. Taking the Vaidya spacetime \((2)\) as the initial data and demanding that the spacetime evolves no singularity after perturbation, the geometry of the perturbed spacetime in the vicinity of the hypersurface \( \Sigma_c \) should at least be of Petrov type I \([34]\). Thus we get the following the additional constraints

\[
P_{IJ} = l^\mu (m_I)^\nu l^\sigma (m_J)^\rho C_{\mu\nu\rho\sigma}|_{\Sigma_c} = 0, \tag{11}
\]

where \( C_{\mu\nu\rho\sigma} \) is the Weyl tensor of the bulk spacetime and the Newman-Penrose-like basis vector fields \( l^\mu, k^\mu, (m_I)^\mu \) are introduced, which obey

\[
l^2 = k^2 = 0, \quad (k, l) = 1, \quad (l, m_i) = (k, m_I) = 0, \quad (m_I, m_J) = \delta_I^J. \tag{12}
\]

In coordinates system \( x^\mu = (u, r, x^I) \), the basis vector fields restricted on the hypersurface \( \Sigma_c \) read

\[
m_I = \frac{1}{r_c} \partial_I, \quad l = \frac{1}{\sqrt{2}} \left( \frac{1}{\sqrt{f_c}} \partial_u - n \right), \quad k = -\frac{1}{\sqrt{2}} \left( \frac{1}{\sqrt{f_c}} \partial_u + n \right),
\]

where we have used the notation \( f_c = f_c(u) = f(r_c, u) \) for short. Inserting N-P basises into Petrov type condition \((11)\), we get

\[
P_{IJ} = \frac{1}{f_c} C_{uIuJ} + \frac{1}{\sqrt{f_c}} C_{uI(uJ(n))} + \frac{1}{\sqrt{f_c}} C_{uJI(n)} + C_{I(n)J(n)} = 0.
\]

The symmetric Petrov type I condition provides \( \frac{d(d+1)}{2} - 1 \) constraints over Brown-York stress tensor, where the extra \(-1\) is provided by the traceless condition for the Weyl tensor. Therefore the number of independent components for the Brown-York stress tensor reduces from \( \frac{(d+1)(d+2)}{2} \) to \( d+2 \), which is exactly the number of variables required to describe the energy density, pressure and velocity fields of a holographic fluid system. In the presence of matter source in the bulk, there are some extra degrees of freedom for the matter field, which will play the role of fluid source or external forces \([53, 54]\).

### 4 Non-relativistic hydrodynamic expansion

In order to construct the explicit fluid equations on the finite cutoff surface \( \Sigma_c \), there remains one more step which is the hydrodynamic expansion of the perturbed constraint equations. In static background spacetimes, this last step is often performed in the
near horizon limit. In view of the RG group flow approach, near horizon limit implies only the low energy modes of the perturbations contributes to the hydrodynamics description of the boundary system. However, in our case, we cannot do this because of the evolution of the horizons. Instead, what we need is to take the long wavelength limit on the finite cutoff surface, which involves contributions from degrees of freedom at relatively bigger energy scale whilst still maintaining the validity of the hydrodynamic explanation.

The line element corresponding to the induced metric on $\Sigma_c$ reads

$$ds^2_{d+1} = -f_c(u)du^2 + r^2_c \sum_{i=1}^{d} dx_i^2,$$

$$= -\frac{1}{\lambda^2} f_c(u(\tau))d\tau^2 + \frac{1}{\lambda} \sum_{i=1}^{d} dx_i^2,$$

where in the second line we have introduced a non-relativistic rescaling $\tau = \lambda u$, $x_i = \sqrt{\lambda} r_c x_I$, where $\lambda$ is taken to be a small dimensionless parameter. The effect of the rescaling $\tau = \lambda u$ sharpens the mass function (4), or effectively speeds up the formation of the black hole, but this will not affect the asymptotic behavior of the function $f(r, u)$. The different scalings in $u$ and $x_I$ allows us to interpret the limit $\lambda \rightarrow 0$ as both the non-relativistic and the long wavelength limit.

Now in coordinates $(\tau, x_i)$, we get the non-relativistic hydrodynamic expansion of the Hamiltonian constraint

$$\mathcal{H} = (t^\tau)^2 - 2t^i t^j h_{ij} + t^i t^j - \frac{t^2}{d} - 2\Lambda - 2T_{\mu\nu}n^\mu n^\nu,$$

as well as Petrov type I condition in terms of the Brown-York stress tensor,

$$P_{ij} = \frac{2 f_c}{\lambda^2} t^\tau t^j + \frac{t^2}{d^2} h_{ij} - \frac{t}{d^2} h_{ij} + t^\tau h_{ij} + \frac{2\lambda}{\sqrt{f_c}} \partial_\tau \left( \frac{t}{d} h_{ij} - t_{ij} \right)$$

$$- \frac{2\sqrt{f_c}}{\lambda} \partial_\mu (t^\mu h_{ij}) - t_{ik} t_{kj} + A_{ij} = 0,$$

where $A_{ij}$ represents the following contribution from the bulk matter and the cosmological constant,

$$A_{ij} = -\frac{1}{d} (T_{\nu\rho} n^\nu n^\rho - 2\Lambda + T + \frac{1}{f} T_{uu} - \frac{2}{\sqrt{f}} T_{uu} n^\rho) h_{ij} + T_{ij}.$$

On the background level, the Brown-York tensor on the finite cutoff surface can be
evaluated explicitly, yielding
\[
t_i = \frac{d\sqrt{f}}{r} t_i^\tau, \\
t_j = \left(\frac{1}{2\sqrt{f}} \partial_{\tau} f + \frac{(d-1)\sqrt{f}}{r} \right) h_{ij}^i, \\
t = d\left(\frac{1}{2\sqrt{f}} \partial_{\tau} f + \frac{d\sqrt{f}}{r} - \frac{\lambda \partial_{\tau} f}{2f^{3/2}}\right).
\]
All the constraint equations are automatically satisfied on the background level, since these equations are actually the field equations based on which we deduce the background solutions. As mentioned earlier, what matters in the hydrodynamic description is the long wavelength perturbation modes of the Brown-York tensor. So it necessary to consider expansions around the background values. The most general fluctuations can be very complicated, for simplicity, we restrict ourselves to the following polynomial expansion,
\[
t_{ab} = \sum_{n=1}^{\infty} \lambda^n t_{ab}^{(n)}, \tag{17}
\]
where we have intentionally taken the expansion parameter to be identical to the scaling parameter \(\lambda\), so that the perturbative limit \(\lambda \to 0\) is simultaneously the non-relativistic and long wavelength limit.

After some tedious calculations we find that the perturbed Hamiltonian constraint yields, at the first nontrivial order \(O(\lambda)\), the following equation,
\[
t_{\tau}^{(1)} = \frac{2r_c f^{2/3}}{-r_c \partial_{\tau} f + 2f_c} \delta^{ij} t_{ij}^{(1)} t_{\tau}^{(1)} + \frac{2f_c}{-r_c \partial_{\tau} f + 2f_c} t_{\tau}^{(1)} + \frac{d(r \partial_{\tau} f + 2(d-2)f_c) \partial_{\tau} f_c}{2(-r \partial_{\tau} f + 2f_c) f_c^{3/2}} + \frac{2 T_{\mu\nu} n^\mu n^\nu r \sqrt{f}}{-r \partial_{\tau} f + 2f_c}. \tag{18}
\]
Meanwhile, the perturbed Petrov type I condition at order \(O(\lambda)\) yields
\[
t_{j}^{(1)} = \frac{2r_c f^{3/2}}{r_c \partial_{\tau} f + (d-2)f_c} t_{k}^{(1)} t_{\tau}^{(1)} \delta_{ik} - \frac{2r_c f}{r_c \partial_{\tau} f + (d-2)f_c} \partial_{(k} t_{j)}^{(1)} \delta_{ik} + \frac{f_c}{r_c \partial_{\tau} f + (d-2)f_c} \delta_{j}^{(1)} + \frac{r_c \partial_{\tau} f + df_c}{r_c \partial_{\tau} f + (d-2)f_c} t_{\tau}^{(1)} \delta_{j}^{(1)}. \tag{19}
\]
Let us remind that apart from the contributions from the matter source and the background level metric functions, the last two equations are consisted purely of the first order perturbations of the Brown-York tensor. We will see in the next section that, by introducing an appropriate holographic dictionary, the momentum constraint imposed on \(t_{a}^{(1)}\) can be interpreted as the sourced continuity equation and the Navier-Stokes equation for a compressible fluid living on the finite cutoff surface \(\Sigma_c\), which subjects to a deviatoric stress and an external force term.
5 Holographic fluid in Vaidya spacetime

Now let us consider the momentum constraint imposed on the cutoff hypersurface $\Sigma_c$. Since the hypersurface is flat, the covariant divergence appearing in (10) can actually be rewritten as ordinary divergence, thus we have

$$\partial_a t^a_b = -T_{\mu b} n^\mu.$$  \hfill (20)

Inserting the relations (18) and (19) from the Hamiltonian constraint and Petrov type I condition into (20) and introducing the following holographic dictionary,

$$t^{(1)}_i = \frac{r_c \partial_{r_c} f + (d - 2)f_c}{2r_c f_c^{2/3}} v_i,$$

$$P = -\frac{f_c}{-r_c \partial_{r_c} f + 2f_c} v_k v_l \delta^{kl} + \frac{-2r_c^2 f_c^{3/2} \partial_{r_c} f}{d(r_c \partial_{r_c} f + (d - 2)f_c)(-r_c \partial_{r_c} f + 2f_c)} t^{(1)},$$

where $v_i$ and $P$ are to be understood as the velocity field and the pressure of the dual fluid respectively, the temporal component of momentum constraints at order $O(\lambda^0)$ becomes a sourced continuity equation

$$\partial_\tau \rho + \rho \delta^{ij} \partial_i v_j = Q,$$

while the spatial components turn out to be the Navier-Stokes equation at order $O(\lambda^1)$:

$$\partial_\tau v_i + v^j \partial_j v_i = -\frac{1}{\rho} \partial_i P + \nu \partial^j d_{ij} + f_i,$$

where the symmetric traceless tensor

$$d_{ij} = \partial_j v_i + \partial_i v_j - \frac{2}{d} \delta_{ij} \partial^k v_k,$$ \hfill (21)

represents the deviatoric stress, which depends only on the first derivatives of velocity field, and

$$\rho = \frac{r_c \partial_{r_c} f + (d - 2)f_c}{2r_c f_c^{3/2}},$$

$$Q = \frac{1}{f_c^{3/2}} (r_c \partial_{r_c} f - \frac{3 \partial_{r_c} f}{2f_c} + \frac{d + 2}{4r_c}) \partial_r f_c,$$

$$f_i = -\frac{Q}{\rho} v_i,$$

represent energy density, source in continuity equation and a body force acting on the fluid system. It is clear that the body force could be recognized as a linear resistance
force since it linearly proportional to the velocity field $v_i$. The kinematic viscosity $\nu$ of the fluid is determined solely by the background level metric function

$$\nu = \frac{r_c f_c}{r_c \partial_r f + (d-2)f_c}.$$  \hspace{3cm} (22)

Since the metric function $f_c(\tau)$ is time-dependent (no dependence on boundary spatial coordinates), the energy density as well as the kinematic viscosity in this dual fluid system are both time dependent, and hence the continuity equation implies compressibility rather than incompressibility as one often finds in static background spacetimes.

In the late time limit $\tau \to \infty$, the mass function $m(u(\tau))$ becomes a constant $M$, the corresponding background spacetime becomes static, which is the final state of the collapsing matter. In this limit, the density of the holographic fluid becomes constant and the continuity equation becomes that of an incompressible unsourced fluid. Meanwhile, one can see that the external force term $f_i$ vanishes in this limit. However, comparing the kinematic viscosity (22) in the late time limit with the result obtained in [46] for the static case, which reads

$$\nu = \frac{r_c \sqrt{f_c}}{r_c \partial_r f + (d-2)f_c},$$ \hspace{3cm} (23)

one can find that there is a slight difference between the two results. The reason behind this difference lies in that the induced metric (13) we choose contains an extra time-scaling factor $f_c(\tau)$. In principle, this factor can be absorbed by a second rescaling of the timelike coordinate via $\sqrt{f_c(\tau)}d\tau = dt$. Performing this second rescaling we get

$$\partial_\tau \to \sqrt{f_c(\tau)} \partial_t, \quad v_i \to \sqrt{f_c}v_i.$$ 

Therefore, reformulating the resulting Navier-Stokes equation in terms of the new time coordinate $t$ we will get the new kinematic viscosity as given in (23). It should be emphasized that the second time rescaling can only be done in the temporal asymptotics $u \to \pm \infty$ without introducing extra complexities. In the process of the collapse, there is no obvious reason of doing the second time rescaling and so we should stick to (22) as the expression for kinematic viscosity.

The thermodynamics of dynamical spacetimes is still an open question, since there are some difficulties in defining thermodynamic quantities like temperature, entropy etc. which are based on equilibrium state of system. Therefore, kinematic viscosity to entropy density ratio in our case remains unclear. A possible way to solve this problem is to consider the proposal made in [55, 56], i.e. making use of the unified first law derived from the field equation which is also valid on apparent horizons.

6 Conclusion

Motivated by studies on non-equilibrium problems of dual field theory within the context of AdS/CFT, we constructed the Gravity/Fluid correspondence in time-dependent
dynamical bulk spacetime. The Vaidya geometry as the simplest lab has been adopted, which describes the process of formation of a Schwarzschild-AdS black hole in AdS spacetime.

On a fixed, timelike hypersurface at finite cutoff, the second fundamental form, i.e. extrinsic curvature is regarded as the fundamental quantity of the dual theory. An equivalent set of variables, i.e. the Brown-York stress-energy tensor, may be chosen to describe the dual fluid. Under the Petrov type I conditions, it is shown that the remaining unconstrained independent components of the Brown-York stress-energy tensor obey a system of equations which can be identified to the continuity equation and the compressible Navier-Stokes equation.

According to the AdS/CFT, the non-equilibrium phenomenon of quark-gluon plasma can be described by dual gravity geometry. In this paper, we also obtained the time-dependent energy density and kinematic viscosity of the non-relativistic dual fluid by using Petrov type approach in the context of Gravity/Fluid correspondence. Unlike usual works with static backgrounds, where the energy densities are either constants which naturally lead to incompressibility conditions or hypersurface spatial coordinate dependent due to anisotropy \([57,58]\), which correspond to compressible but stationary fluids, in the present case, the density of the fluid evolves with time and the continuity equation indicates that the fluid is both compressible and sourced. Meanwhile, the kinematic viscosity is also dependent on both time coordinate and the radial cutoff \(r_c\). The dependence on time is inherited from the dynamical time-dependent bulk space-time, and the \(r_c\)-dependence should be considered as an effective energy scale. The Navier-Stokes equation also receives a time-dependent external force density which can be regarded as linear resistance force.

Acknowledgment

This work is supported by the National Natural Science Foundation of China under Grant No. 11575088 and No. 11605137.

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