Shortcuts to adiabaticity in a time-dependent box

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A method is proposed to drive an ultrafast non-adiabatic dynamics of an ultracold gas trapped in a time-dependent box potential. The resulting state is free from spurious excitations associated with the breakdown of adiabaticity, and preserves the quantum correlations of the initial state up to a scaling factor. The process relies on the existence of an adiabatic invariant and the inversion of the dynamical self-similar scaling law dictated by it. Its physical implementation generally requires the use of an auxiliary expulsive potential. The method is extended to a broad family of interacting many-body systems. As illustrative examples we consider the ultrafast expansion of a Tonks-Girardeau gas and of Bose-Einstein condensates in different dimensions, where the method exhibits an excellent robustness against different regimes of interactions and the features of an experimentally realizable box potential.

Results

Dynamical invariants and self-similar dynamics. For a time-dependent box of width $\xi(t)$, the scaling laws governing the dynamics of the expanding eigenstates reported to date are associated with trajectories of the form...
Figure 1 | Time-modulation of the potential trap along a shortcut to adiabaticity. (A) Evolution of the width $\zeta(t)$ of a time-dependent box during a shortcut to different adiabatic expansions and a compression in a finite time $\tau$. (B) Frequency of the auxiliary external harmonic potential required to assist the self-similar dynamics, changing character from repulsive to attractive along the expansion process. The opposite sequence is required for a compression. The transient repulsive potential is responsible for the speed-up of the process.

$\zeta(t) = [at^2 + bt + c]^2$ (with $a, b, c$ real constants)22–24, which turn out to be unsuited for engineering a STA (see below). Nonetheless, given a time-dependent Hamiltonian $\hat{H}(t)$, it is possible to build a dynamical invariant $\hat{I}(t)$ such that

$$\frac{d\hat{I}(t)}{dt} = \frac{\partial\hat{I}(t)}{\partial t} + \frac{i}{\hbar} \{\hat{I}(t), \hat{H}(t)\} = 0,$$  

(1)

with spectral decomposition $\hat{I}(t) = \sum_n \lambda_n \langle \phi_n(t) | \phi_n(t) \rangle$ in terms of the set of eigenmodes $|\phi_n(t)\rangle$ with eigenvalues $\lambda_n$. This is a particularly useful basis to describe the time evolution of an initial state $\Psi$, by the superposition $\Psi(t) = \sum_n \exp(i\lambda_n t)|\phi_n(t)\rangle$, where the Lewis-Riesenfeld phase is given by $\lambda_n = \int_0^t \langle \phi_n(t') | i\hat{p}\hat{c}_x - \hat{H}(t') \rangle |\phi_n(t')\rangle dt'/\hbar$ and can be understood as the sum of the dynamical phase and the Aharonov-Anandan phase24. For a time-dependent box of width $\zeta(t)$ and initial width $\zeta(0) = \zeta_0$, a dynamical invariant exists24,

$$\hat{I} = \frac{1}{2m} \frac{\zeta^2(t)}{\zeta_0^2} \left( p - m \frac{\zeta(t)}{\zeta_0} \right)^2$$  

(2)

with eigenvectors $\langle x | \phi_n(t) \rangle = [2/\zeta(t)]^{1/2} \exp \left[ i \frac{m \zeta(t)}{2\hbar \zeta(t)} x^2 \right] \sin[n \pi x / \zeta(t)]$ and eigenvalues $\lambda_n = \frac{k_n^2}{2m}$, with $k_n = n \pi / \zeta_0$. The Lewis-Riesenfeld phase can be computed to be $\lambda_n(t) = -\frac{\hbar n^2 \pi^2}{2m \zeta_0^2} \eta(t)$ with

$$\eta(t) = \int_0^t \frac{d\eta'_n}{\zeta_0^2 / \zeta^2(t')}.$$  

The condition in Eq. (1) for $\hat{I}$ to be an invariant requires the box potential to be supplemented with an auxiliary harmonic term

$$U_{aux}(x,t) = -\frac{1}{2} m \frac{\zeta^2(t)}{\zeta_0^2} x^2,$$  

(3)

whose frequency $\Omega(t) = \sqrt{\frac{\zeta^2(t)}{\zeta_0^2}}$ is dictated by the ratio of the acceleration of the trajectory $\zeta(t)$, and the trajectory itself. Note that $\zeta(t) > 0$, so that $\frac{\zeta^2(t)}{\zeta_0^2} > 0$, $\Omega(t)$ is purely imaginary, and the auxiliary term $U_{aux}$ is a repulsive harmonic potential. If $\zeta(0) = 0$, $\hat{H}(0), \hat{I}(0) = 0$ so $\hat{H}(0)$ and $\hat{I}(0)$ have common eigenstates. Further, if $\zeta(t) = 0$ holds as well, then $U_{aux}(x,0) = 0$ and an eigenstate $\Psi_n(x,0)$ of the box at $t = 0$ evolves into $\Psi_n(x,t) = \exp(i\lambda_n t)|\phi_n\rangle$, a key observation to engineer a STA as we shall see. We note that the experimental implementation of $U_{aux}(x,t)$ can be assisted by the same techniques used to create the box potential: the use of a blue-detuned laser25 or direct painting of the required trap26–28.

**Shortcuts to adiabaticity: inverting the dynamical scaling law.** We next discuss how to implement a non-adiabatic expansion of the box by a factor $\gamma(t) = \zeta(t)/\zeta_0$ in a given finite-time $\tau$ suppressing excitations in the final state. We shall impose the condition $U_{aux}(x,0) = U_{aux}(x,\tau) = 0$. As in the adiabatic case, in a STA the time evolution of an eigenmode of the initial box should reproduce an eigenmode of the final trap. As at $t = 0$, this can be enforced by imposing the condition $\zeta_0 = \zeta(t) = 0$. The set of boundary conditions at $t = 0, \tau$ excludes the possibility of a linear ramp, as well as the family of trajectories, $\zeta(t) = [at^2 + bt + c]^2$, considered so far in the literature22–24. However, it suffices to determine a polynomial ansatz for the trajectory $\zeta(t) = \sum_n \zeta_n t^n$, i.e. a scaling factor of the form $\gamma(t) = \frac{\zeta(t)}{\zeta_0} = 1 + |\gamma(t)|^{-1} s^2 [10 + 3s(2s - 5)]$, with $s = \tau/t$. This further determines the required time-dependent frequency of the auxiliary harmonic potential $U_{aux}(x,t)$ according to Eq. (3),

$$\Omega^2(t) = -\frac{\zeta(t)}{\zeta_0} = -\frac{\gamma(t) - 1}{\tau^2 / \gamma(t)}.$$  

(4)

The trajectory, displayed in Fig. 1 shows that during an expansion $U_{aux}(x,t)$ becomes an expulsive potential in an early stage ($t < \tau/2$), providing the speed-up required to achieve the STA in an arbitrary finite time $\tau$ (Bounds in the presence of perturbations will be discussed below). In a subsequent stage, $t > \tau/2$, $\Omega(t)$ changes sign and $U_{aux}(x,t)$ becomes a trapping potential, slowing down the expanding mode and reducing it to an eigenstate of the final Hamiltonian at $t = \tau$. Precisely the opposite behavior is exhibited during a fast nonadiabatic compression. Provided that an arbitrary $\Omega(t)$ dependence can be implemented, a STA has no lower bound for $\tau$ (notice however that $\Omega(t) \approx \tau^{-1}$). By contrast, the adiabatic condition

$$\max_{n,k} \left| \frac{\hbar \langle \phi_n(0) \vert \hat{c}_x \vert \phi_k(0) \rangle}{\langle E_n(t) - E_k(t) \rangle} \right| \ll 1,$$  

(5)

leads to the requirement $m n k \zeta(t) \zeta(t)/\hbar \ll 1$.

Note that the energy of the expanding mode

$$\langle \hat{H}(t) \rangle_n = E_n(t) + \frac{m \zeta^2(t)}{12} \left( 2 - \frac{3}{n^2 \pi^2} \right),$$  

(6)

has two contributions, the first one being the adiabatic energy $E_n(t) = E_n(0) \zeta(t)^2/\zeta_0^2$, and the second one depending explicitly on $\zeta(t)$, so that $\langle \hat{H}(t) \rangle_n = E_n(t)$ given that a STA demands $\zeta(t) = 0$. This relation illustrates the fact that a STA is associated with a non-adiabatic evolution, which reproduces the adiabatic result at the end of the process. Moreover, STA work as well for excited states: the time evolution of the $n$-th eigenstate of the initial trap leads to the $n$-th eigenstate of the final trap at $t = \tau$. As a result, STA in boxes pave the way for fast population-preserving cooling in the following sense. Given a system described by the canonical ensemble with a density matrix $e^{-\beta \hat{H}} / \text{Tr} \left[ e^{-\beta \hat{H}} \right]$, where $\beta = 1/k_B T$, $k_B$ is the Boltzmann constant and $T$ is the temperature, the final temperature reads

$$T(t) = T(0) \frac{\gamma(t)}{\gamma(0)}.$$  

(7)

These results strictly hold for a box with infinite walls at $x \in [0, \zeta(t)]$. However, in the all-optical trap reported in29, the end-cap lasers
are ubiquitous in this type of scenario. The technique can be directly producing quantum transients associated with diffraction in time, which STA for expansions and compressions in a finite time, without insight of assisting the dynamics with an auxiliary potential to design a expansion (contraction) of the box. The numerical solution of potential in a length scale providing the box walls have a Gaussian profile which smooths the barriers, where the target states deviate from those of an idealized box.

We have further explored numerically the dynamics under “concatenated STA”, in which the overall expansion is splitted into a sequence of k STA with either constant expansion factor γ or constant box size increment between consecutive steps. The efficiency of the process exhibits a non-monotonic improvement with increasing k, suggesting a natural scenario where STA techniques could be combined with optimal quantum control.

Beyond adiabatic invariants: Shortcuts to adiabaticity in interacting many-body systems. Knowledge of the adiabatic invariants for a single particle in a time-dependent box has provided us with the insight of assisting the dynamics with an auxiliary potential to design STA for expansions and compressions in a finite time, without inducing quantum transients associated with diffraction in time, which are ubiquitous in this type of scenario. The technique can be directly applied to non-interacting gases and other many-body quantum fluids which can be mapped to non-interacting systems. It further suppresses the Talbot dynamics associated with quantum carpets woven by the density profile typically observed in boxes, and the question naturally arises as to its applicability to interacting systems. The presence of interactions, e.g. a two-body potential, hinders the exploitation of the superposition principle in terms of the eigenmodes of the Lewis-Riesenfeld invariant. However, we note that to design a STA it suffices to enforce a self-similar dynamics and ultimately no knowledge of adiabatic invariants is required. As a result, we next consider a broader family of many-body systems, confined in a box, defined by the Hamiltonian providing the box walls have a Gaussian profile which smooths the potential in a length scale σ, i.e. a box trap of the form

\[ U^{\text{box}}(x) = V_0 \left[ \exp \left( -\frac{x^2}{2\sigma^2} \right) + \exp \left( -\frac{(x-\xi(t))^2}{2\sigma^2} \right) \right]. \] (8)

A similar smoothing occurs in other physical realisations. Since this smoothing is expected to be the most significant deviation of laboratory potentials from the ideal infinite box, it is important to consider its effect on the self-similar dynamics required for a STA. This can be quantified by the overlap between the states resulting from the expansion in an idealized and realistic box trap, \( |\Psi_n(t)\rangle \) and \( |\Psi_{\text{aux}}(t)\rangle \) respectively. Clearly the role of σ decreases (increases) during a expansion (contraction) of the box. The numerical solution of the time-dependent Schrödinger equation for this box potential is shown in Fig. 2, where the STA is compared with both the polynomial and linear expansion of the box in the absence of \( U^{\text{aux}}(x,t) \). It is seen that the STA is the only successful strategy and that the process is robust even for a substantial smoothing of the potential barriers, where the target states deviate from those of an idealized box.

\[ \begin{aligned} \hat{H} &= \sum_{i=1}^{N} \left[ -\frac{\hbar^2}{2m} \Delta \mathbf{q}_i + U^{\text{max}}(\mathbf{q}_i,t) \right] + \epsilon \sum_{i<j} \mathbf{V}(\mathbf{q}_i - \mathbf{q}_j) \end{aligned} \] (9)

where \( \mathbf{q}_i \in \mathbb{R}^D \), \( \Delta \mathbf{q}_i \) is the Laplace operator in dimension D, the auxiliary term is now given by

\[ U^{\text{aux}}(\mathbf{q},t) = \frac{1}{2} \frac{\zeta(t)}{\xi(t)} |\mathbf{q}|^2. \] (10)

and the two-body interaction potential obeys \( \mathbf{V}(\mathbf{q}) = \lambda \mathbf{V}(\mathbf{q}) \), e.g. for the Fermi-Huang pseudopotential describing s-wave scattering in ultracold gases, \( \lambda = D \). For a hard-wall box, \( r_c = |\mathbf{q}| \in [0,\xi(t)] \); we shall relax below this approximation and consider realistic potential boxes as those created in all-optical setups. The case \( D = 1 \) corresponds to a box with one-wall moving (the symmetric case in which both walls move in opposite directions can be obtained by a Duru transformation). For \( D = 2,3 \) cylindrical and spherical symmetry is assumed. Without loss of generality, we choose the dimensionless time-dependent coupling constant \( \epsilon = \epsilon(t) \) to satisfy \( \epsilon(0) = 1 \). A stationary state \( \Psi(t) = \Psi(\mathbf{q}_1, ..., \mathbf{q}_N) \) of N particles and chemical potential \( \mu \) follows for \( t > 0 \) the evolution

\[ \Psi(t) = \gamma^{-\frac{N}{2}} \exp \left[ -\frac{1}{\hbar} \sum_{j=1}^{N} \frac{m|\mathbf{q}_j|^2}{2} - \frac{\mu |\mathbf{q}_j|}{\hbar} \right] \Psi(\mathbf{q}_1, ..., \mathbf{q}_N; 0). \] (11)

with the boundary conditions \( \Psi(t) = 0 \) for \( |\mathbf{q}_i| = \xi(t) \) (i = 1,...,N, in addition to \( \Psi(t) = 0 \) for \( |\mathbf{q}_i| = 0 \) in \( D = 1 \)), as long as

\[ \epsilon(t) = \gamma(t)^{\frac{N}{2} - 2}, \] (12)

which can be implemented exploiting a Feshbach resonance or modulating the transverse confinement in anisotropic systems. The self-similar dynamics in a STA leads to a scaling of all local correlation functions. In particular the density of a given many-body state follows the law \( n(\mathbf{q},t) = \int d\mathbf{q}_1 ... d\mathbf{q}_N n(\mathbf{q}_1, ..., \mathbf{q}_N; t) / \gamma(t)^D \). By contrast, non-local correlation functions exhibit a non-trivial dynamics. The one-body reduced density matrix \( \rho_1(\mathbf{q},q; t) = N \int d\mathbf{q}_2 ... d\mathbf{q}_N \Psi(\mathbf{q}_1, ..., \mathbf{q}_N; t)^\dagger \Psi(\mathbf{q}_1, ..., \mathbf{q}_N; t) \) of a state obeying Eq. (11), follows the scaling law \( \rho_1(\mathbf{q},q; t) = \exp \left[ -\frac{m|q|^2 - (\mathbf{q})^2}{2\gamma(t)} \right] \rho_1(\gamma,\gamma; 0) \), analogous to that observed under harmonic confinement. The additional phase factor induces a major distortion of the momentum distribution, \( n(k,t) = \int d\mathbf{q} \rho_1(\mathbf{q},q; t) \exp \left[ -\frac{|k - \mathbf{q}(\mathbf{q},q; t)\|^2}{2} \right] \). However, a STA ensures that at \( t = \tau \) the Lewis-Riesenfeld phase factor vanishes, so that the final state exhibits the same correlations of the initial state scaled by a factor \( \gamma(t) \).
\begin{align}
\theta_t(q,q';\tau) &= \frac{1}{\gamma(t)} \theta_t\left( \frac{q}{\gamma(t)}, \frac{q'}{\gamma(t)}, 0 \right), \\
n(k,\tau) &= \gamma(t)^D n(\gamma(t)k,0). \tag{13}
\end{align}

**Examples.** In the following we shall illustrate different aspects of shortcuts to adiabaticity in some paradigmatic models.

We shall first consider the evolution of correlations in a one-dimensional cloud of ultracold bosons in the limit of hard-core contact interactions, this is, in the Tonks-Girardeau (TG) regime\textsuperscript{35}. This system, as well as its lattice-version, has become a favorite test-bed to study the breakdown of thermalization and adiabaticity\textsuperscript{36,37}. Its many-body ground state is given by the Bose-Fermi mapping\textsuperscript{36},

\[ \Psi_{TG}(q_1, \ldots, q_N) = \frac{1}{\sqrt{N!}} \prod_{1 \leq i < j \leq N} \epsilon(q_j - q_i) \det_{j=1}^{\infty} \left[ \Psi_j(q_k) \right], \]

where \( \epsilon(q) = 1 (\ell 1) \) if \( x > 0 (\ell < 0) \) and \( \epsilon(0) = 0 \). In a STA, the self-similar dynamics is inherited from the single-particle orbitals \( \Psi_j(q_k, t) \) whence it follows that no tuning of interactions is required. Its density exhibits a scaling law for all \( t \). The same holds true for the entanglement entropy with respect to a bipartition \([0, a] \). The self-similar dynamics breaks down for the momentum distribution, which can be computed efficiently\textsuperscript{38}, and we shall focus on its evolution along a STA. Different snapshots are depicted in Fig. 3A, and confirm that during an expansion the cloud is accelerated during the interval \([0, t/2]\) and slowed down during \([t/2, t]\). The reverse sequence, is observed in a fast frictionless compression. The axis are scaled up by the expansion factor \( \gamma(t) \) in such a way that for an adiabatic dynamics, curves at different times collapse into a single curve. Along a STA, the width and mean of the momentum distribution do not remain constant and change along the process.

A similar distortion of correlations, known as dynamical fermionization, occurs in the dynamics of a cloud suddenly released from an arbitrary trap\textsuperscript{40}. Under ballistic dynamics the asymptotic momentum distribution in a 1D expansion evolves to that of the dual system, a spin-polarized Fermi gas. In particular for a cloud released from a box the exact time evolution is not self-similar\textsuperscript{40} but dynamical fermionization is observed\textsuperscript{41,42}. However, under a self-similar scaling law, the asymptotic \( n(k) \) maps to the density profile of the initial state\textsuperscript{43} and no dynamical fermionization occurs. This is the case of relevance to STA, where the dynamical scaling law in Eq. (11) holds. (We note that the case of the initial harmonic confinement is singular in that the free expansion is self-similar and that the single-particle eigenstates can be written in terms of Hermite polynomials, which are eigenfunctions of the continuous Fourier transform. As a result the asymptotic momentum distribution can be related to both the initial density profile and the momentum distribution of non-interacting fermions. See [5] for a discussion of STA in harmonic traps.) Moreover, this distortion of correlations is not restricted to expansion processes. Along a STA, this is shown in Fig. 3 for both expansions (A) and compressions (B). This is a spurious effect for the purpose of STA, which is to reproduce the adiabatic result in a finite short time. Indeed, the distortion induced during the first half of the STA associated with the accelerated expansion or compression, is compensated in the second half of the dynamics, in such a way that the correlations of the initial state are reconstructed at \( t = \tau \) and scaled by a factor \( \gamma(t) \).

We next turn our attention to the design of STA for a BEC in time-dependent box trap, where different strategies can be adopted depending on the dimensionality and the regime of interactions. The time-dependent Gross-Pitaevskii equation (TDGPE) governs the evolution of the normalized condensate wavefunction \( \Phi(q, t) \),

\[ i\hbar \partial_t \Phi(q, t) = \left[ -\frac{\hbar^2}{2m} \Delta_q + U_{\text{ext}}(q, t) + g_D \Phi(q, t)^2 \right] \Phi(q, t), \quad |q| \in [0, \xi(t)]. \tag{14} \]

for which adiabaticity conditions have been reported\textsuperscript{39}. The ansatz

\[ \Phi(t) = \gamma(t)^{-\frac{1}{2}} \exp \left[ \frac{m q^2}{2\hbar} \frac{d}{dt} - i \frac{\mu(t)}{\hbar} \right] \Phi[\xi(t)/\gamma(t); 0], \tag{15} \]

satisfies the TDGPE provided that

\[ \eta(t) = \int dt' \frac{d}{\gamma(t')} \Omega^2(t') = -\frac{d}{\gamma(t)} \frac{\gamma(t)}{\gamma(t) - 1} \frac{g_D(0)}{\gamma(t)}, \tag{16} \]

These relations constitute the box analogue of the well-known Castin-Dum-Kagan-Surkov-Shyapnikov relations in harmonic traps\textsuperscript{41,43}. The two-dimensional case is special since the scaling law holds when \( g_{2D}(t) \) is kept constant.
Figure 4 | Shortcut to an adiabatic expansion of a Bose-Einstein condensate.

The upper row shows the space-time evolution of a quasi-1D BEC (n(x,t)/\gamma(t)) while the lower row corresponds to a quasi-2D (n(x,t)/\gamma(t)) BEC cloud. The shortcut to adiabaticity (left) is compared with a polynomial (center) and a linear ramp (right) of the box width $\xi(t)$ in the absence of the auxiliary harmonic potential ($\sigma(\xi(0)=2/10, g_D=10^{-6}/m_\text{Bo}^{-2}$, and $\gamma=10$).

Figure 4 is a set of numerical solutions of the time-dependent Gross-Pitaevskii equation that illustrate the robustness of the STA for realistic BEC experiments. We consider a box trap with Gaussian barriers and in all numerical simulations interactions are kept constant, i.e. $g_D(t) = g_D(0)$, deviating from the ideal prescription in Eq. (16). The top row illustrates the dynamics for a quasi-1D BEC. The (one-body) fidelity between the resulting state $\Phi(t)$ and the ground state of the final box $\Phi_x$ is $F_{\gamma=10} = \left(\left|\langle\Phi(t)|\Phi_x\rangle\right|^2\right) = 0.911$. For smaller values of $\gamma$, the fidelity is even higher $F_{\gamma=5} = 0.999$ as expected, given that implementation of the exact STA requires a smaller tuning of $g_1$. The bottom row shows the dynamics of a quasi-2D cloud, which requires no interaction tuning in a STA, but is more sensitive to the smoothness of the box boundaries, $F_{\gamma=5} = 0.988$.

It is noteworthy that in the Thomas-Fermi regime, the kinetic term contribution can be neglected and it is possible to induce an exact self-similar dynamics (and a STA) exclusively with the help of an external field. Then, the scaling ansatz is a solution of the TDGPE provided that

$$\eta(t) = \int^t dt' \frac{d^2}{\gamma'(t')} g_D(t') = g_D(0).$$

This regime is particularly robust against the smooth boundaries of physically realizable box potentials. The simulations correspond to the most delicate regime with moderate mean-field interactions, both far from the non-interacting and Thomas-Fermi limits.

**Discussion**

In conclusion, we have presented a method to drive an ultrafast dynamics in a time-dependent box trap which reproduces the adiabatic result at the end of the evolution. The method is assisted by an auxiliary external harmonic potential which provides the speed-up and is applicable to a large family of both non-interacting and interacting many-body systems supporting dynamical scaling laws, where it not only leads to a robust expansion of the density but also preserves the non-local correlation functions of the initial state, up to an expansion factor. The proposal is applicable to realistic box potentials and can be implemented in the laboratory with well-established technology. Its applications range over all scenarios requiring a shortcut to adiabaticity, i.e., probing strongly correlated phases, preventing decoherence, the effect of perturbations and atomic losses. The method can be directly applied as well to ultrafast population-preserving cooling methods, quantum heat engines and refrigerators providing an alternative to the paradigmatic model of a quantum piston.
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Author contribution

A.d.C. initiated the project, developed the theoretical analysis, and prepared the manuscript. Both authors carried out the numerical simulations and interpretation of the numerical data.

Additional information

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