Traceability of gravity acceleration measurement in calibration laboratories

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Abstract. Primary measurements of force, torque and pressure are directly influenced by local gravity acceleration, g. At present, g measurement in calibration laboratories is evaluated through absolute or relative measurements or through theoretical/empirical relations. Except for some absolute measurements, other methods are not traceable. As consequence, measurements of these quantities could be highly impacted by this lack in terms of accuracy and uncertainty. The Consultative Committee for Mass and related quantities (CCM) of CIPM and International Association of Geodesy (IAG) are currently working on a resolution strategy to ensure traceability to the SI for gravity measurements through the establishment of a global absolute gravity reference system. This network could become a useful tool to transfer the g measurement to calibration laboratories at international level. However, as well any other measurements, this possible solution should be established under the Quality Management system of accredited laboratories.

1. Introduction

Absolute gravity measurements play an important role in metrology since primary measurements of force, torque and pressure directly depend on gravity acceleration g. For example, forces generated by the dead-weight force calibration machine can be directly traceable from transfer standards calibrated in national force standard machines (path A) or through independent traceability path to the base SI units of mass, length, and time (path B). In the second case, the traceability is derived from measurements of mass and gravity acceleration, and the uncertainty associated with the generated force is calculated from the uncertainties associated with these measurements [1]. Another example can be found in the field of pressure measurements. In fact the pressure standards, usually above few kilopascal, are represented by the so-called “pressure balances”, which consist of a vertical piston freely rotating within a cylinder. The pressure to be measured is applied by a fluid to the base of the piston, generating an upward vertical force. This force is equilibrated by the gravitational downward force due to masses submitted to the local gravity and placed on the top of the piston [2]. Hence, the gravity acceleration directly affects the realization of pressure standards.

2. Traceability of g

Currently, at international level, traceability of gravity acceleration measurements is guaranteed only by five NMIs with declared CMCs: INRIM (Italy), METAS (Switzerland), FGI (Finland), NSC (Ukraine) and BEV (Austria), with absolute expanded uncertainties between 8.0×10⁻⁸ m/s² and 2.1×10⁻⁷ m/s². The Consultative Committee for Mass and related quantities (CCM) of CIPM and...
International Association of Geodesy (IAG) is working to ensure scientific excellence and measurement of the gravity acceleration traceable to the SI at the level of uncertainty of few microgals \(10^{-8} \text{ms}^{-2}\) according to the principles of the CIPM MRA for metrology (in particular for the realization of the new definition of the kilogram) and geodetic science (in particular for time variable gravity and gravity networks) [3]. Two distinct traceability paths for the measurements performed by absolute gravimeters are proposed: an independent traceability to the SI units of time and frequency and a calibration by comparison against a reference, as shown in figure 1. In the first case, the absolute ballistic gravimeter has independent traceability to the SI unit of time (frequency) through the calibration of the frequencies of the laser and reference clock and the uncertainties are calculated combining the contributions of uncertainty associated with these references, together with all other contributions of uncertainty. In the second path, the absolute gravimeter derives its traceability directly from a comparison with the gravimeter of a NMI or a DI having declared CMCs in the CIPM MRA or using a gravity value of a reference station (characterized and monitored by appropriate methods). The determination of uncertainty of the calibrated absolute gravimeter includes, in this case, the corresponding contributions of uncertainty and the bias obtained in the comparison.

![Figure 1. The traceability chain in gravimetry: independent traceability to the SI units of time and frequency (left) and calibration by comparison against a reference (right)](image)

Currently, NMIs with declared CMCs are represented in figure 1 by AG2, which refers to the absolute gravimeters within the CIPM-MRA framework with CMCs for measurements, not for calibration. AG1 represents the absolute gravimeters with CMCs for measurement and calibration that, in the future, could calibrate other gravimeters by comparison or through a reference station. At present, no institute has this kind of CMC. AG3 represents the gravimeters outside the CIPM-MRA framework that cannot guarantee the traceability according to ILAC P10 document [4] and ISO 17025:2017 [5].

3. Current situation in calibration laboratories
Measurements of gravity acceleration can be performed with absolute or relative gravimeters. Absolute gravimeters are typically ballistic gravimeters which measure the acceleration of small masses free falling in a vacuum. The mass includes a retroreflector and terminates one arm of a Michelson interferometer. By counting and timing the interference fringes, the acceleration of the mass can be measured. Absolute gravimeters are used for the calibration of relative gravimeters and for the monitoring of gravity anomalies [6]. Relative gravimeters are, instead, composed of a weight on a spring, and by measuring the amount by which the weight stretches the spring, local gravity can be measured. However, the strength of the spring must be calibrated by placing the instrument in different locations with a known value of gravity acceleration. Gravity acceleration measurements with such methods are usually performed in force and pressure calibration laboratories. Another approach is to use the typical theoretical/practical equations based on measurement database which entail accuracy and precision issues. Among the others, we can list:
1. the classical equation: \[ g = G \left( \frac{M}{r^2} \right) - \omega^2 r \cos^2 \Phi \] (1)
   where \( G = 6.673 \cdot 10^{-11} \text{Nm}^2/\text{kg}^2 \) is the Gravity constant, \( M = 5.97 \cdot 10^{24} \text{kg} \) is the Earth mass, \( r = 6.37 \cdot 10^6 \text{m} \) is the Earth radius, \( \omega = 7.29 \cdot 10^{-5} \text{rad/s} \) is the Earth rotation angular speed and \( \Phi \) is the latitude.

2. the Geodetic Reference System Formula (IGF 1967)
   \[ g = 9.780327(1 + 0.0053024 \sin^2 \Phi - 0.0000058 \sin^2 2\Phi) \] (2)

3. and the practical equation, which keeps into account the altitude \( h \).
   \[ g = 9.780318(1 + 0.0053024 \sin^2 \Phi - 0.0000058 \sin^2 2\Phi) - 3.086 \cdot 10^{-6}h \] (3)

4. In the end, we can cite the web portal “Gravity Information system” of PTB (Physikalisch-Technische Bundesanstalt) that provides the local gravity information at specified geographic position and topographic height [7].

### 4. Risks
Gravity acceleration measurements with absolute gravimeters can be traceable only with those laboratories with declared CMCs, as previously stated. All other measurements performed with absolute or relative gravimeters, even if accurate and precise and validated through periodic international comparisons, and the calculation methods, do not guarantee a defined traceable paths following the requirements of ILAC P10 document [4] and ISO 17025:2017 [5].

#### 4.1 Experimental evaluation of calculation methods
Evaluations of \( g \) through theoretical/practical equations are based on simplified Earth models that do not take into account the real local mass distribution. However, calculation of \( g \) on database information implies an interpolation process that raises an uncertainty calculation issues. In order to test the accuracy of such approaches, a comparison between the \( g \) values obtained through the calculations of equations (1-4) with experimental values measured by an absolute traceable gravimeter in different Italian sites was performed. The sites are 43, spread across Italy as shown in figure 2, with an altitude ranging from the sea level to 2820 m (Pizzi Deneri Observatory).

![Figure 2. Italian sites (red dots) with experimental measurements of gravity acceleration (left) and relative differences between theoretical/practical equations and experimental \( g \) values (right) ](image)

The measurements of local gravity acceleration were performed by means of INRIM absolute gravimeter described in [6], during a period of about 40 years [8]. The comparison between experimental and calculated values highlighted that theoretical/practical equations are not able to
correctly take into account the variation of \( g \) with altitude. As a matter of fact, the relative differences between experimental and calculated values according to equations (2) and (3) above 1000 m are as high as \( 6 \cdot 10^{-4} \), whereas below 1000 m, the relative differences are comprised within \( \pm 150 \cdot 10^{-6} \). Moreover, the values obtained from equation (1) diverge quite a lot from the experimental values, so that below 500 m, the relative differences are systematically negative by a factor of about \( 4-5 \cdot 10^{-4} \). Finally, the values obtained from the PTB website show a better agreement with the experimental values: relative differences are comprised within \( \pm 60 \cdot 10^{-6} \) with a standard deviation of \( 18 \cdot 10^{-6} \), compatible with the declared uncertainty.

5. IAG Resolution
To ensure the traceability to \( g \), IAG is currently working for the establishment of a global absolute gravity reference system, based upon the international comparisons of absolute gravimeters, where gravity reference is available at any time, to be transferred to the calibration laboratories [9]. Another aim is to link the reference stations to the International Terrestrial Reference System by co-location with space-geodetic techniques and to initiate the replacement of the International Gravity Standardization Net 1971 (IGSN71) and the latest International Absolute Gravity Base Station Network 1986 (IAGBN) by the new Global Absolute Gravity Reference System, as shown in figure 3. In this way, the traceability to SI units could be defined and guaranteed at international level for calibration laboratories.

![Figure 3](image)

**Figure 3.** The IGSN 1971 (left) and the IAGBN 1986 (right)

6. References
[1] EURAMET cg-4 2011 Calibration Guide: Uncertainty of Force Measurements, 2nd edn
[2] EURAMET cg-3 2011 Calibration Guide: Calibration of Pressure Balances, 1st edn
[3] CCM 2015 CCM – IAG Strategy for Metrology in Absolute Gravimetry
[4] ILAC-P10:01/2013. ILAC Policy on the Traceability of Measurement Results
[5] ISO/IEC 17025:2017 General requirements for the competence of testing and calibration laboratories
[6] Agostino GD, Desogus S, Germak A, Origlia C, Quagliotti D, Berrino G, Corrado G, D’Errico V and Ricciardi G 2008 The new IMGC-02 transportable absolute gravimeter: measurement apparatus and applications in geophysics and volcanology *Annals of Geophysics* 51(1) pp 39-49
[7] PTB website [http://www.ptb.de/cartoweb3/SISproject.php](http://www.ptb.de/cartoweb3/SISproject.php)
[8] INRiM technical reports from 1977 to 2016
[9] IAG 2015 IAG Resolution No. 1 for the definition and realization of an International Height Reference System (IHRS) and No. 2 for the establishment of a global absolute gravity reference system