Calculation of the maximum endurance of a small unmanned aerial vehicle in a hover

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Abstract. Optimizing the flight performance, chiefly maximum endurance, of small unmanned aerial vehicles (SUAVs) relying on propellers powered by an electric motor/s as a form of propulsion is key challenge in SUAV development. Choosing the correct combination of propellers, electric motors and batteries is necessary and knowledge of characteristics of all three factors is of a great help. The paper presents a simple blade element model (BEM), which can be used to calculate the propeller performance characteristics. This model is further validated by measurements in the wind tunnel. The paper also presents measurements of electric motor and battery discharge characteristics. Finally, the paper presents a method of calculation of maximum endurance in hover.

1. Introduction
During the past decades, the amount of both military and civilian applications of small unmanned aerial vehicles (SUAVs) increased, and the focus of the development of SUAVs has shifted from development of the drivetrain to increasing its efficiency. Due to the limited amount of energy on board, often the main parameter to tune is the maximum endurance of the SUAV \cite{1}. This paper presents a method of calculating the maximum endurance in hover of an SUAV of a multirotor type, equipped with a low-Reynolds fixed-pitch propeller, brushless DC electric motor (BLDC motor) with a regulator and a battery. A modified blade-element model with Joukowsky theorem is used to calculate the propeller performance characteristics (PPCH) and the paper shortly discusses necessary corrections for this type of calculations. The performance characteristics of the BLDC motor with a regulator and discharge characteristics of the battery were measured prior to the calculation. The calculated results were then compared with measurement of the maximum endurance of a Yuneec Typhoon H hexacopter drone.

2. Blade element model (BEM)
There are several different models, which can be used to calculate the PPCH, with each having its pros and cons, as described in \cite{2}. BEM allows for rapid determination of PPCH, which allows e.g. optimization of the propeller geometry based on the flight configuration of the SUAV. Unfortunately, BEM models all assume quasi-static inflow conditions and cannot compensate for the dynamic behaviour of the propeller mounted on the SUAV \cite{3, 4}. This problem can be overcome by an implementation of the vortex models described in \cite{5} or \cite{6}. However, for purposes of this paper, in a hover under semi-ideal ambient conditions, it can be assumed that the inflow velocity is equal to zero (with no disturbances). Thus, only relatively small deviations of revolutions of each propeller can be expected, and the dynamics of the rotor do not significantly affect the results. Under these assumptions, the PPCH calculated by BEM can be used in the model.
2.1. Blade element model with Joukowsky modification

Principle of all BEM models is similar. The rotor is represented as a finite sum of elements with constant geometry, as shown in the figure 1. The element on the radius $r$ with the length of $dr$ has a constant chord length $c$, thickness $t$ and aerofoil. In a general form, if the propeller is subjected to a non-axial inflow, the forces acting on the blade element depend further on azimuthal position of the blade $\psi$ and thus discretisation along the azimuthal angle is needed as well – it is assumed that the forces acting on the blade element do not change during the increment of the azimuthal angle $d\psi$. Furthermore, the airflow over each element is considered two-dimensional with non-interference between neighbouring elements or different blades of the propeller is assumed. The validity of this assumption was proven e.g. in [7]. Each blade element is, as per Joukowsky theorem, represented as a system of vortices. The fundamental parameter for the calculation is circulation. The speed of the flow, induced by the propeller, can be calculated from the value of the circulation, i.e. circulation defines the speed of the flow. The relation of the local inflow velocity $W_1$ is shown in the figure 2. The inflow velocity $V$ is acting on all blade sections with the same magnitude. However, the circumferential component $U = r\Omega$ changes with the position of the blade element along the blade span and the velocity triangle alters as well.

![Figure 1. Division of a rotor into finite number of blade elements [8].](image1)

![Figure 2. Velocity triangle of Joukowsky theorem acting on a blade element [9].](image2)

The local inflow angle $\beta$ is defined as a subtraction of the pitch angle $\phi$ and the local angle of attack (AoA) $\alpha$. It can be shown, that the local inflow angle can be calculated as:

$$\beta = \tan^{-1}(V^*/U^*), \quad (1)$$

where $V^*$ is a sum of inflow velocity $V$ and axial component of the induced velocity $v_{ia}$ and $U^*$ is a sum of circumferential velocity and a tangential component of the induced velocity $v_{it}$. Even though the inflow velocity in hover is equal to zero, the axial component of the induced velocity can be non-zero, resulting in a non-zero local inflow angle $\beta$. Furthermore, it can be shown that both axial and rotational component of the local inflow velocity $W_1$ are a function of local circulation $\Gamma$. The value of local circulation can be derived as:

$$\Gamma = \frac{1}{2} c_L c \frac{U^*}{\cos(\beta)}. \quad (2)$$

The local circulation is directly proportional to the local chord length $c$ and local value of lift coefficient $c_L$. The local value of the lift coefficient is then depended on the local value of AoA and, therefore, an iterative solution to the equations is required. Also, the lift curve for each aerofoil is required as well.

Result of the BEM calculation is propeller performance characteristics – dependence of thrust coefficient $c_T$, power coefficient $c_N$ and efficiency $\eta$ on the advance ratio $\lambda$. Thrust and power coefficients can be defined as follows:

$$c_T = \frac{T}{\rho n^2 D^2}; \quad (3)$$
\[ T \] and \( N \) are thrust or power consumed by the propeller, while \( n \) is its revolution and \( D \) is the diameter of the propeller. Advance ratio is a ratio between tangential speed of the tip of the blade and the inflow velocity. It can be expressed as:

\[ \lambda = \frac{\nu}{nD} \]  

The efficiency is, in this case, a ratio between thrust and power coefficient, multiplied by the advance ratio:

\[ \eta = \frac{c_T}{c_N} \lambda. \]  

BEM with Joukowsky modification is further detailed in [10].

2.2. Impact of Reynolds number to input aerofoil data for BEM calculation

The non-constant distribution of both aerofoil thickness on a propeller with general geometry and local inflow velocity along the blade span, the value of local Reynolds number can change significantly. This phenomenon is demonstrated in figure 3. As shown in the figure 4, for large Reynolds numbers (500000+) this is not a problem. However, for Reynolds numbers below 50000 the drag of the aerofoil starts considerably increasing and the aerofoil polars can change significantly even with a relatively small change of the Reynolds number. At Reynolds number of 50000, a distortion can be observable. At some point, the drag of the aerofoil increases significantly, indicating stall. However, after further increase of the AoA, the flow over the aerofoil fully recovers and then the aerofoil polar behaves as expected. This highly atypical behaviour could be attributed to convergence problems. At these very low Reynolds numbers, similar behaviour was observed on e.g. Hofsaess Espada aerofoil [11]. Design of aerofoil, which can operate efficiently at these low Reynolds numbers is described in [12].

![Figure 3](image1.png)  
**Figure 3.** Calculated distribution of Reynolds numbers along the blade span for two different rotational speeds.

![Figure 4](image2.png)  
**Figure 4.** Aerofoil polars of Clark-Y as calculated by XFOIL.

2.3. Calculation of input aerofoil polars

Input Aerofoil data, namely aerofoil lift curve and polar are critical for the correctness of the BEM calculation. However, as demonstrated in previous chapter, obtaining them is perhaps the most difficult step of the calculation. While it is possible to measure each polar, it is impractical and expensive due to the sheer amount of polars required. As demonstrated in [9] or in [13], XFOIL or CFD can be used to approximate the aerofoil polars well. Comparison is shown in figure 5, where both XFOIL and CFD fail to correctly estimate the behaviour close to the stall conditions, otherwise the fit seems to be good enough.
Figure 5. Extended data of NACA 4412 aerofoil to ±180°, $AR = \infty$ and $Re < 10^6$.

Often the input polars need to be extended to include post-stall data. While it is possible to obtain the data numerically, typically the extrapolation is accurate enough. Different methods of extrapolation are discussed in [14]. All calculations presented in this paper included the Viterna-Corrigan extrapolation [15].

2.4. Corrections to the calculation

Different corrections to the BEM model are described in [8] or [16]. Typically, Mach number correction, stall delay corrections, rotational augmentation corrections and surface roughness corrections should be performed before running the BEM calculation. As a general rule of thumb, it is always advisable to measure the propeller performance at least in a static regime, because even with the corrections, the BEM typically overestimates the propeller performance in static regime (advance ratio $\lambda$ equal to 0) by about 10-15%. This overestimation can be improved by introducing a tip-loss factor to the calculation, as discussed in [2].

2.5. PPCH as an input for maximum endurance calculations

To decrease the calculation time of the maximum endurance, the PPCH can be inputted as a map of thrust and power coefficient for different revolutions to compensate for effects of different Reynolds numbers. The thrust and power coefficient for required rpm can be calculated as an interpolation between different maps. The example of such map is shown in the figure 6. The thrust coefficient is plotted against advance ratio $\lambda$ and incidence angle $\theta$. In hover, both the incidence angle and advance ratio are equal to zero. In regimes, where thrust coefficient is negative, the propeller no longer produces thrust, but is braking. This regime is not suitable for propeller operation.

Figure 6. Map of thrust coefficient.
3. Motor performance characteristics
For purposes of this paper, an SUAV powered with an BLDC electric motor is calculated. For SUAV to sustain hover under aforementioned assumptions, the propeller needs to maintain its revolutions. However, as the battery discharges, the voltage in the system drops and the controller needs to increase the throttle (or PWM) on all motors. Furthermore, the propeller is directly mounted to the electric motor and its revolution are equal with no losses in transmission. The efficiency of the electric motor $\eta_{mot}$ is defined as a ratio between output mechanical power $P_{OUT}$ and consumed electrical power $P_{IN}$. Measured motor efficiency for different input voltages is shown in figure 7. The initial drops in the efficiency are caused by the heating effects. The measurement started with cold motor and constant load. As the motor heated up, the internal resistance increased as well and at the same PWM, both the efficiency and speed dropped. Then, the load on the motor was decreased and the typical motor efficiency was measured.

![Figure 7. Measured efficiency of an electric motor.](image)

4. Battery discharge characteristics
For purposes of this paper, a nickel-manganese-cobalt (NMC) Li-Ion battery was assumed. During the discharge process, the voltage on the battery drops. The rate at which it drops is shown in the figure 8 in battery discharge characteristics.

The maximum voltage on commercially available battery cells is 4.25 V. Also, if the voltage on cell drops below 2 V, the battery suffers a permanent damage. Therefore, a cut-off voltage is typically selected, at which the battery is indicated as empty. There are two factors contributing to the value of cut-off voltage: battery protection and minimal revolutions, at which the SUAV can sustain flight. Choosing a correct value of cut-off voltage can significantly affect the maximum endurance of an SUAV. E.g. if the SUAV was equipped with a battery presented in figure 8 and the cut-off voltage was set to 3.1 V because of e.g. motor limitations, at larger loads it would not be possible to drain all the available capacity of the battery. I.e. not only the battery has limited energy available, this amount can be proportionally decreased by incorrect value of cut-off voltage.

![Figure 8. Measured battery discharge characteristics.](image)
Other factors, which can affect the battery performance are ambient conditions (temperature), investigated e.g. in [17], or number of discharge cycles of each cell during their life cycle, as described in [18].

5. Maximum endurance in hover

The hover conditions can be solved by solving the equations of motion (SUVAT) simplified for a single axis problem (no disturbances and perfect symmetry assumptions). The scheme of the calculation for a given time step with a cut-off voltage of 3.2 V is shown in figure 9.

At first, the initial parameters need to be set (number of propellers, propeller performance input maps, weight of the SUAV, initial rpm, etc.). For given parameters, thrust and power coefficient is calculated. Then, the thrust of the propeller is adjusted for the obstacles in the downwash (mounting legs of the SUAV). After that, the consumed power by the propeller is corrected for motor efficiency at given voltage and PWM. If the motor cannot produce the required power, the thrust of the propeller is recalculated for the maximum available power. Then, the battery voltage drop is calculated. “Physics” represents the calculation of SUVAT. Furthermore, while it is possible to find the required revolutions of the propeller by directly solving this system, it is much easier to use controller with multiple PID controllers to stabilize the system. Because of the high instability at the beginning of the calculation, which depends on the “quality” of the initial guess of the revolutions, the maximum endurance is calculated only after the SUAV stabilization. The result of such calculation is shown in figure 10.

In the first phase, the stabilization can be seen. Then the current in the system proportionally increases, as the voltage decreases. The overall amount of capacity drained is non-linear and as the battery is drained more and more, the efficiency of the motor decreases, causing the increase rate of the consumed energy.
6. Comparison of measured and calculated results
The measurement of maximum endurance of Yuneec Typhoon H, presented in [19] and showed in figure 11, served as the source for the validation of the computational program. Yuneec Typhoon H is a hexarotor with fixed-pitch propellers. The measurement was performed at different altitudes with variable weight. The results are presented in the table 1.

![Figure 11. Yuneec Typhoon H [19].](image)

| Altitude, m.a.s.l. | Weight, g | Measured, mm:ss | Calculated, mm:ss |
|-------------------|-----------|-----------------|------------------|
| 120               | 1,950     | 19:30           | 19:33            |
| 183               | 1,950     | 19:00           | 19:20            |
| 1,000             | 1,950     | 18:00           | 18:47            |
| 370               | 2,165     | 18:10           | 17:33            |

In most cases, the calculation overpredicted the overall maximum endurance. However, with the increased maximum weight caused the calculation to significantly lower the maximum endurance, more than it did in real life. Furthermore, the drop of the maximum endurance was more significant with the increased altitude as the calculation suggested. The resulting maximum endurance values seems to be within margin of error considering the simplifications of the model.

7. Conclusion
The paper presents a simple method of combining measured or calculated characteristics of different components of an SUAV of a multirotor type to calculate its maximum endurance. The propeller performance characteristics calculation via Joukowsky BEM method was presented, along with references to necessary corrections. Also, the motor power curves for different voltages at different RPM were discussed. Finally, battery discharge characteristics were introduced as well, with highlight of the importance of cut-off voltage.

The model itself is simple, though despite all simplifications, it produces relatively good results. However, it is computationally expensive. Furthermore, any dynamical changes in the inflow velocity vector are outside of the scope of this model. Corrections for incident inflow to BEM calculation are highly advisable.

Comparison of measured and calculated results was promising. In most scenarios, the calculation slightly overestimates the propeller performance, but the results are within expected range.

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