COSMIC STRINGS IN DILATON GRAVITY

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ABSTRACT

We examine the metric of an isolated self-gravitating abelian-Higgs vortex in dilatonic gravity for arbitrary coupling of the vortex fields to the dilaton. We look for solutions in both massless and massive dilaton gravity. We compare our results to existing metrics for strings in Einstein and Jordan-Brans-Dicke theory. We explore the generalization of Bogomolnyi arguments for our vortices and comment on the effects on test particles.

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1. Introduction.

Topological defects and other soliton structures have a wide application to many areas of physics. Cosmologists are interested in defects as possible sources for the density perturbations which seeded galaxy formation. String theorists are interested in defects not only as solutions of the low energy effective action, but as true solitons in the full non-perturbative theory which are required for consistency and inter-relation of the full spectrum of string theories.

A topological defect is a discontinuity in the vacuum, and in conventional field theory can be classified according to the topology of the vacuum manifold of the particular field theory being used to model the physical set up: disconnected vacuum manifolds give domain walls, non-simply connected manifolds, strings, and manifolds with non-trivial 2- and 3-spheres give monopoles and textures respectively. In this paper, we are concerned with defects associated with non-simply connected vacuum manifolds: cosmic strings[1]. The gravity of cosmic strings within the context of Einstein theory has been well explored, both in the case of ‘model’ strings, where the core of the string is modelled by a simplified energy-momentum tensor[2], and in the case of the fully coupled Einstein-Abelian-Higgs system[3,4]; with the result that the spacetime of a self-gravitating local string is found to be generically conical, with the angular deficit given by $8\pi G \mu$, $\mu$ being the energy per unit length of the vortex.

It seems likely however, that gravity is not given by the Einstein action, at least at sufficiently high energy scales, and the most promising alternative seems to be that offered by string theory, where the gravity becomes scalar-tensor in nature[5]. Scalar-tensor gravity is not new, it was pioneered by Jordan, Brans and Dicke[6], who sought to incorporate Mach’s principle into gravity. The implications of such actions on general Friedmann–Robertson–Walker cosmological models have been well explored[7,8], however, the implications for theories of structure formation have not been so well studied. Broadly speaking, there are two views on explaining structure formation – inflation or defects, the latter consisting of two subsets: cosmic string or texture[9] induced perturbations. While there is little to choose between these from the particle physics or large scale structure point of view, the implications of each of these theories for the perturbations of the microwave
background are distinct. However, calculations on the microwave background multipole moments do assume Einstein gravity[10], therefore it is interesting to question whether these conclusions are still valid in the context of scalar-tensor gravity. Even if the dilaton acquires a mass at a fairly high energy scale (with respect to the recombination temperature of the universe that is), at the core of a defect symmetry is restored and the physics is determined by the GUT scale, at which the dilaton might have rather different properties, impacting back on the cosmic microwave background.

Calculations involving radiation from a cosmic string network generally make use of a “worldsheet-approximation” in which the string is treated as an infinitesimally thin source which moves according to, and has an energy momentum tensor appropriate for a two-dimensional worldsheet governed by the Nambu action. That this action is appropriate for the local string has been convincingly argued in the absence of gravity[11,12], but as yet no proof exists in the presence of gravity. This is generally believed to be related to the problems of using distributional sources of codimension greater than one in general relativity[13]. Nonetheless, the fact that the self-gravitating infinite local vortex has a relatively small effect on spacetime lends credence to the worldsheet approximation for the string.

In the presence of a dilaton, the worldsheet approximation may no longer be appropriate. If the dilaton is massless, there is no reason to expect that the string will not have a long range effect on the dilaton, and even if the dilaton is massive, it introduces an additional length scale which may still have significant impact.

In this paper, we take a modest step towards resolving this issue by examining the gravi-dilaton field of a self-gravitating cosmic string in dilaton gravity. We consider a reasonably general form for the interaction with the dilaton, assuming that the abelian-Higgs lagrangian couples to the dilaton via an arbitrary coupling, $e^{2a\phi}L$, in the string frame. We consider both massive and massless dilatons. Our results for the massless dilaton are very similar to those of Gundlach, Ortiz and others [14], who considered cosmic strings in JBD theory. For the massive dilaton we find that, apart from an intermediate annular region, the long-range structure of the string is as for Einstein gravity, as might be expected. The main exception to this qualitative and expected picture is that for a
special value \((a = -1)\) of the coupling of the dilaton to the fields which constitute the vortex the dilaton effectively decouples from the string, showing little or no reaction to its presence. This occurs independent of whether the dilaton is massive, and independent of the specifics of the U(1) model, i.e. whether it is type I, II, or supersymmetric.

The layout of the paper is as follows: In the next section we review the Nielsen-Olesen vortex in the abelian-Higgs model. In section three we derive the main results of this paper, namely the gravitational and dilaton fields for the self-gravitating vortex in both massless and massive dilatonic gravity. In section four we consider Bogomolnyi bounds for the string, and show that these can only be saturated in the special case \(a = -1\). In this case, the dilaton effectively decouples from the string. In section five we consider the motion of test particles in the background of the string, and in section six we conclude.

2. The Abelian Higgs Vortex.

We start by briefly reviewing the U(1) vortex in order to establish notation and conventions. We take the abelian Higgs lagrangian

\[
\mathcal{L}[\Phi, A_a] = D_a \Phi^\dagger D^a \Phi - \frac{1}{4} \tilde{F}_{ab} \tilde{F}^{ab} - \frac{\lambda}{4} (\Phi^\dagger \Phi - \eta^2)^2
\]

where \(\Phi\) is a complex scalar field, \(D_a = \nabla_a + ieA_a\) is the usual gauge covariant derivative, and \(\tilde{F}_{ab}\) the field strength associated with \(A_a\). We use units in which \(\hbar = c = 1\) and a mostly minus signature. For cosmic strings associated with galaxy formation \(\eta \sim 10^{15}\text{GeV}\).

We rewrite the fields in a way which makes manifest the physical degrees of freedom of the model:

\[
\Phi(x^\alpha) = \eta X(x^\alpha) e^{i\chi(x^\alpha)}
\]

\[
A_a(x^\alpha) = \frac{1}{e} \left[ P_a(x^\alpha) - \nabla_a \chi(x^\alpha) \right].
\]

where \(X, \chi\) and \(P_a\) are now real. In terms of these new variables, the lagrangian and equations of motion become

\[
\mathcal{L} = \eta^2 \nabla_a X \nabla^a X + \eta^2 X^2 P_a P^a - \frac{1}{4e^2} F_{ab} F^{ab} - \frac{\lambda \eta^4}{4} (X^2 - 1)^2
\]
\[ \Box X - P_a P^a X + \frac{\lambda \eta^2}{2} X(X^2 - 1) = 0 \quad (2.4a) \]
\[ \nabla_a F^{ab} + 2e^2 \eta^2 X^2 P^b = 0 \quad (2.4b) \]

Thus \( P_b \) is the massive vector field in the broken symmetry phase, \( F_{ab} = \nabla_a P_b - \nabla_b P_a \) its field strength, and \( X \) the residual real scalar field with which it interacts. \( \chi \) is not in itself a physical quantity, however, it can contain physical information if it is non-single valued, in other words, if \( \oint \nabla_a \chi dx^a = 2\pi n \) for some \( n \in \mathbb{Z} \). Continuity then demands (in the absence of non-trivial spatial topology) that \( X = 0 \) at some point on any surface spanning the loop - this is the locus of the vortex. Thus the true physical content of this model is contained in the fields \( P_a \) and \( X \) plus boundary conditions on \( P_a \) and \( X \) representing vortices.

The simplest vortex solution is the Nielsen-Olesen (NO) vortex[11], an infinite, straight static \( n = 1 \) solution with cylindrical symmetry. In this case, we can choose a gauge in which

\[ \Phi = \eta X_0(R) e^{i\phi} \quad ; \quad A_a = \frac{1}{e} [P_0(R) - 1] \nabla_a \phi \quad (2.5) \]

where \( R = \sqrt{\eta r} \), in cylindrical polar coordinates. The equations for \( X_0 \) and \( P_0 \) from (2.4) are

\[ -X_0'' - \frac{X_0'}{R} + \frac{P_0^2 X_0}{R^2} + \frac{1}{2} X_0 (X_0^2 - 1) = 0 \quad (2.6a) \]
\[ -P_0'' + \frac{P_0'}{R} + \beta^{-1} X_0^2 P_0 = 0 \quad (2.6b) \]

where a prime denotes \( \frac{d}{dR} \), and \( \beta = \lambda/2e^2 = m_X^2/m_P^2 \) is the Bogomolnyi parameter[15] (\( \beta = 1 \) corresponds to the vortex being supersymmetrizable). Note that in these rescaled coordinates, the string has width of order unity. This string has winding number one; for winding number \( N \), we replace \( \chi \) by \( N \chi \), and hence \( P \) by \( NP \). Figure 1 shows the Nielsen-Olesen solutions for \( X \) and \( P \) for a \( \beta = 1 \) winding number one string.

It is also useful to briefly review the self-gravitating NO vortex in Einstein gravity, as much of the formalism can be used directly in the next section. To include the self-gravity of the string, we require a metric which exhibits the symmetries of the source,
FIGURE (1): $X$ and $P$ for a $\beta = N = 1$ vortex.

namely, translational invariance along its length and rotational invariance around the core, i.e. cylindrical symmetry. The general cylindrically symmetric metric was given by Thorne [16]

$$ds^2 = e^{2(\gamma - \psi)}(dt^2 - dr^2) - e^{2\psi}dz^2 - \tilde{\alpha}^2 e^{-2\psi}d\phi^2$$

(2.7)

(where $\gamma$, $\psi$, $\tilde{\alpha}$ are independent of $z, \phi$). The string couples to this metric via its energy-momentum tensor

$$G_{ab} = 8\pi GT_{ab} = 8\pi G \left[ 2\eta^2 \nabla_a X \nabla_b X + 2\eta^2 X^2 P_a P_b - \frac{2\beta}{\lambda} F_{ac} F_b^c - \mathcal{L}_{g_{ab}} \right]$$

(2.8)

which can be seen to be boost invariant ($T^0_0 = T^z_z$). This in turn implies that for static metrics, $\gamma = 2\psi$, which we will assume from now on. Note that the expressions (2.7) and (2.8) appear in unrescaled coordinates, it proves to be convenient to rescale the coordinates so that the string width is of order unity, as in (2.6). To do this we set $R = \sqrt{\lambda} \eta r$ as before, $\alpha = \sqrt{\lambda} \eta \tilde{\alpha}$, and write the rescaled version of the energy and stresses $\hat{T}_{ab} = T_{ab}/(\lambda \eta^4)$

$$\hat{T}^0_0 = \mathcal{E} = e^{-\gamma} X^2 r^2 + \frac{e^{-\gamma} X^2 P^2}{\alpha^2} + \frac{\beta P^2}{\alpha^2} + (X^2 - 1)^2/4$$

(2.9a)

$$\hat{T}^R_R = -P_R = -e^{-\gamma} X^2 r^2 + \frac{e^{-\gamma} X^2 P^2}{\alpha^2} - \frac{\beta P^2}{\alpha^2} + (X^2 - 1)^2/4$$

(2.9b)
\[ \hat{T}_\theta^\theta = -\mathcal{P}_\theta = e^{-\gamma}X^2 - \frac{e^\gamma X^2 P^2}{\alpha^2} - \frac{\beta P^2}{\alpha^2} + (X^2 - 1)^2/4 \] (2.9c)

\[ \hat{T}_z^z = -\mathcal{P}_z = \hat{T}_0^0. \] (2.9d)

The Einstein equations can then be read off as [16]

\[ \alpha'' = -\epsilon \alpha e^\gamma (\mathcal{E} - \mathcal{P}_R) \] (2.10a)

\[ (\alpha \gamma')' = \epsilon \alpha e^\gamma (\mathcal{P}_R + \mathcal{P}_\theta) \] (2.10b)

\[ \alpha' \gamma' = \frac{\alpha \gamma'^2}{4} + \epsilon \alpha e^\gamma \mathcal{P}_R \] (2.10c)

where \( \epsilon = 8\pi G\eta^2 \) is the gravitational strength of the string. Also for future reference, the Bianchi identity gives

\[ \mathcal{P}_R' + (\mathcal{P}_R - \mathcal{P}_\theta)(\frac{\alpha'}{\alpha} - \frac{\gamma'}{2}) + \alpha' \mathcal{P}_R + \gamma' \mathcal{E} = 0. \] (2.11)

To zeroth order (flat space)

\[ \alpha = R , \quad \psi = \gamma = 0 , \quad X = X_0 , \quad P = P_0, \] (2.12)

and (2.11) gives

\[ (R\mathcal{P}_o R)' = \mathcal{P}_o \theta \] (2.13)

To first order in \( \epsilon = 8\pi G\eta^2 \) the string metric is given by[3,4]

\[ \alpha = \left[ 1 - \epsilon \int_0^R R(\mathcal{E}_0 - \mathcal{P}_o R)dR \right] R + \epsilon \int_0^R R^2(\mathcal{E}_0 - \mathcal{P}_o R)dR, \] (2.14a)

\[ \gamma = \epsilon \int_0^R R\mathcal{P}_o R dR. \] (2.14b)

where the subscript zero indicates evaluation in the flat space limit. Note that when the radial stresses do not vanish, there is a scaling between the time, \( z \) and radial coordinates for an observer at infinity and those for an observer sitting at the core of the string[4]. The
only case in which these stresses do vanish is when $\beta = 1$. In this case the field equations reduce to
\[
\begin{align*}
X' &= XP/\alpha \\
P' &= \frac{1}{2}\alpha(X^2 - 1) \\
\alpha' &= 1 - \epsilon[(X^2 - 1)P + 1] \\
\gamma &= 0
\end{align*}
\tag{2.15}
\]
a first order set of coupled differential equations as one might expect from the fact that the solution is supersymmetrizable.

We conclude this section by demonstrating the asymptotically conical nature of the corrected metric. Note that since the string functions $X$ and $P$ rapidly fall off to their vacuum values outside the core, the integrals in (2.14) rapidly converge to their asymptotic, constant, values. Let
\[
\epsilon \int_0^R (E_0 - P_0 R) dR = A, \quad \epsilon \int_0^R R^2 (E_0 - P_0 R) dR = B \quad \text{and} \quad \epsilon \int_0^R R P_0 R = C \tag{2.16}
\]
then the asymptotic form of the metric is
\[
ds^2 = e^C [dt^2 - dr^2 - dz^2] - r^2 (1 - A + B/\sqrt{\lambda} r)^2 e^{-C} d\theta^2 \\
= d\hat{t}^2 - d\hat{r}^2 - d\hat{z}^2 - \hat{r}^2 (1 - A)^2 e^{-2C} d\theta^2 \tag{2.17}
\]
where
\[
\hat{t} = e^{C/2} t, \quad \hat{z} = e^{C/2} z, \quad \hat{r} = e^{C/2} (r + B/(1 - A)) \tag{2.18}
\]
This is seen to be conical with a deficit angle
\[
\Delta = 2\pi(A + C) = 2\pi \epsilon \int R E_0 dR = 16 \pi^2 G \int r T_0^0 d\tau = 8\pi G \mu \tag{2.19}
\]
where $\mu$ is the energy per unit length of the string. Notice that the deficit angle is independent of the radial stresses, but that there is a red/blue-shift of time between infinity and the core of the string if they do not vanish. Now let us examine the behaviour of the string with a dilaton present.
3. Cosmic strings in dilaton gravity.

We are interested in the behaviour of the isolated string metric (2.7) when the gravitational interactions take a form typical of low energy string theory [5]. In its most minimal form, string gravity replaces the gravitational constant, $G$, by a scalar field, the dilaton in a rather analogous fashion to that of Jordan, Brans and Dicke who were motivated by Mach’s principle. We take an empirical approach to cosmic strings in this background theory, not concerning ourselves with the origin of the fields that form the vortex, but inputting ‘by hand’ the abelian-Higgs lagrangian (2.1). To take account of the (unknown) coupling of the cosmic string to the dilaton, we choose

$$\hat{S} = \int d^4x \sqrt{-\hat{g}} \left[ e^{-2\phi} \left( -\hat{R} - 4(\hat{\nabla}\phi)^2 - \hat{\nabla}(\phi) \right) + e^{2a\phi}L \right]$$

(3.1)

where $\mathcal{L}$ is as in (2.3). This action is written in terms of the string metric, i.e. the metric which appears in the string sigma model. It proves useful to instead write the action in terms of the “Einstein” metric, which is defined via

$$g_{ab} = e^{-2\phi} \hat{g}_{ab}$$

(3.2)

in which the gravitational part of the action appears in the more familiar Einstein form:

$$S = \int d^4x \sqrt{-g} \left[ -R + 2(\nabla\phi)^2 - V(\phi) + e^{2(a+2)\phi} \mathcal{L}\{X, P, e^{2\phi}g\} \right]$$

(3.3)

where $V(\phi) = e^{2\phi}\hat{V}(\phi)$. Note however that this complicates the matter part of the lagrangian – a factor of $e^{-2\phi}$ being picked up each time $\hat{g}^{ab}$ is used:

$$T_{ab} = \frac{\delta \mathcal{L}[X, P, e^{2\phi}g]}{\delta g^{ab}} = 2\eta^2 e^{-2\phi} [\nabla_aX \nabla_bX + X^2 P_a P_b] - \frac{2\beta}{\lambda} e^{-4\phi} F_{ae} F_{b}{}^e - \mathcal{L}_{gab}$$

(3.4)

The “Einstein” equations are now

$$G_{ab} = \frac{1}{2} e^{2(a+2)\phi} T_{ab} + S_{ab}$$

(3.5)

where

$$S_{ab} = 2\nabla_a\phi \nabla_b\phi + \frac{1}{2} V(\phi) g_{ab} - (\nabla\phi)^2 g_{ab}$$

(3.6)
represents the energy-momentum of the dilaton, which has as its equation of motion

$$\Box \phi = \frac{1}{4} \frac{\partial V}{\partial \phi} + \frac{(a + 1)}{2} e^{2(a+2)\phi} \mathcal{L}[X, P, e^{2\phi}g] + e^{2(a+2)\phi} \left[ \frac{\beta}{2\lambda} F^2 e^{-4\phi} - \frac{\lambda\eta^2}{8} (X^2 - 1)^2 \right]$$

(3.7)

As before, we choose the Thorne metric (2.7), with \( \gamma = 2\psi \):

$$ds^2 = e^\gamma (dt^2 - dz^2 - dr^2) - \alpha^2 e^{-\gamma} d\theta^2$$

(3.8)

We will also rescale the coordinates again so that the string width is of order unity, \( R = \sqrt{\lambda \eta r} \), \( \alpha = \sqrt{\lambda \eta a} \), and the rescaled modified energy-momentum we redefine as:

$$\mathcal{E} = e^{2(a+2)\phi} \left[ e^{-2\phi} \left( e^{-\gamma} X'^2 + \frac{e^{\gamma} X^2 P^2}{\alpha^2} \right) + e^{-4\phi} \frac{\beta P^2}{\alpha^2} + (X^2 - 1)^2 / 4 \right]$$

(3.9a)

$$\mathcal{P}_R = e^{2(a+2)\phi} \left[ e^{-2\phi} \left( e^{-\gamma} X'^2 - \frac{e^{\gamma} X^2 P^2}{\alpha^2} \right) + e^{-4\phi} \frac{\beta P^2}{\alpha^2} - (X^2 - 1)^2 / 4 \right]$$

(3.9b)

$$\mathcal{P}_\theta = e^{2(a+2)\phi} \left[ e^{-2\phi} \left( -e^{-\gamma} X'^2 + \frac{e^{\gamma} X^2 P^2}{\alpha^2} \right) + e^{-4\phi} \frac{\beta P^2}{\alpha^2} - (X^2 - 1)^2 / 4 \right]$$

(3.9c)

In terms of these variables, the full equations of motion for the gravitating vortex in dilaton gravity are

$$\alpha'' = -\alpha e^\gamma \tilde{V}(\phi) - \epsilon \alpha e^\gamma (\mathcal{E} - \mathcal{P}_R)$$

(3.10a)

$$(\alpha \gamma)' = -\alpha e^\gamma \tilde{V}(\phi) + \epsilon \alpha e^\gamma (\mathcal{P}_R + \mathcal{P}_\theta)$$

(3.10b)

$$\alpha' \gamma = -\frac{1}{2} \alpha e^\gamma \tilde{V}(\phi) + \frac{\alpha \gamma'^2}{4} + \alpha \phi'^2 + \epsilon \alpha e^\gamma \mathcal{P}_R$$

(3.10c)

$$(\alpha \phi')' = \frac{\alpha e^\gamma}{4} \frac{\partial \tilde{V}}{\partial \phi} + \epsilon \frac{(a + 1)}{2} \alpha e^\gamma \mathcal{E} - \frac{1}{2} \epsilon \alpha e^\gamma (\mathcal{P}_R + \mathcal{P}_\theta)$$

(3.10d)

$$\frac{1}{\alpha} (\alpha X')' = -2(a + 1) X' \phi' + \frac{XP^2}{\alpha^2} e^{2\gamma} + \frac{1}{2} X (X^2 - 1) e^{\gamma + 2\phi}$$

(3.10e)

$$\alpha \left( \frac{P'}{\alpha} \right)' = -\gamma' P' - 2a \phi' P' + \beta^{-1} X^2 P e^{\gamma + 2\phi}$$

(3.10f)

where \( \epsilon = \eta^2 / 2 \) now defines the gravitational strength of the string, and \( \tilde{V} = V/\lambda \eta^2 \) represents the dilaton potential in units natural to the vortex. The Bianchi identity (2.11) becomes

$$\epsilon (\alpha e^\gamma \mathcal{P}_R)' = \epsilon \alpha' e^\gamma \mathcal{P}_\theta + \frac{1}{2} \epsilon \alpha \gamma' e^\gamma [\mathcal{P}_R - \mathcal{P}_\theta - 2\mathcal{E}] - \alpha' \phi'^2 - (\alpha \phi'^2)' + \frac{1}{2} \epsilon \alpha e^\gamma \phi' \frac{\partial V}{\partial \phi}$$

(3.11)
We start by examining the case \( V(\phi) \equiv 0 \), i.e. a massless dilaton, as this ought to be qualitatively the same as a cosmic string in Brans-Dicke gravity.

3.1 Massless dilatonic gravity.

In the case that the dilaton is massless the equations (3.10) are rather reminiscent of the pure Einstein gravity vortex (2.10), however, there is one crucial difference - the constraint equation (3.10c) now contains an \( \alpha \phi'^2 \) term, and unless \( a = -1 \), \( \alpha \phi' \) will definitely be nonzero. In order to explore this solution, let us first consider the “wire approximation”, namely

\[
\alpha e^\gamma \mathcal{E}(R) = \hat{\mu} \delta(R) ; \quad \mathcal{P}_R = \mathcal{P}_\theta = 0 \tag{3.12}
\]

where \( \hat{\mu} = \mu / 4\pi \epsilon \) represents the energy per unit length of the cosmic string in units natural to the vortex, and is of order unity. (Recall that \( \epsilon \) sets the gravitational strength of the string.) In this case, eqns.(3.10) are readily integrated to give

\[
\alpha(R) = (1 - \epsilon \hat{\mu}) R \tag{3.13a}
\]

\[
\gamma(R) = 0 \tag{3.13b}
\]

\[
\phi(R) = \frac{\epsilon \hat{\mu}(a + 1)}{2(1 - \epsilon \hat{\mu})} \ln R \tag{3.13c}
\]

but now we find a contradiction – the constraint (3.10c) is no longer satisfied unless \( a = -1 \). It is worth examining what has gone wrong here. The wire model is an approximate version of the stress-energy tensor which usually works well in Einstein gravity since the integral

\[
\int_0^\infty \alpha e^\gamma (\mathcal{P}_R + \mathcal{P}_\theta) = 0,
\]

which is no longer necessarily true in the presence of the dilaton. A Bogomolnyi solution in flat space or Einstein gravity has the property that \( \mathcal{P}_R = \mathcal{P}_\theta \equiv 0 \), therefore the fact that we cannot consistently use the wire approximation for these variables (unless \( a = -1 \)) is an indication that a Bogomolnyi argument cannot exist unless \( a = -1 \).

Instead, let us examine consistent vacuum solutions to (3.10) which should represent asymptotic spacetimes for the string. Setting \( \mathcal{E} = \mathcal{P}_R = \mathcal{P}_\theta = 0 \) in (3.10) gives as the general solution:

\[
\alpha = dR + b \tag{3.14a}
\]

\[
\gamma = \gamma_0 + \frac{c}{d} \ln(dR + b) \tag{3.14b}
\]

11
\[ \phi = \phi_0 + \frac{f}{2d} \ln(dR + b) \]  

(3.14c)

where \( f = \pm \sqrt{4dc - c^2} \) from (3.10c). This gives a Levi-Civita[17] solution for the metric. (Note that if \( \phi \) is constant, we have \( c = 0 \) or \( 4d \), corresponding to the vacuum general relativistic solutions.) The constants \( b, c, d, f \) are given by integrating (3.10) and to order \( \epsilon \) are

\[ d = 1 - A, \quad b = B, \quad c = 0, \quad f = \frac{1}{2}(a + 1)(A + C) = \frac{1}{2}(a + 1)\epsilon \hat{\mu} \]  

(3.15)

where \( A, B, C \) are given in (2.16). We can therefore see that \( c \) cannot remain zero, and to order \( \epsilon^2 \), \( c = \frac{1}{4}(a + 1)^2 \epsilon^2 \hat{\mu}^2 \). So, unlike the Einstein self-gravitating vortex, the dilaton vortex for \( a \neq -1 \) has a strong gravitational effect far from the core, albeit an \( O(\epsilon^2) \) one:

\[ ds^2 \approx \hat{r}^2[a^2\epsilon^2 \hat{\mu}^2/2 + (1 - \epsilon^2)\hat{r}^2/2d\theta^2] \]

(3.16)

where \( \hat{r} \) etc. were defined in (2.18) and we have set \( (\sqrt{\lambda\eta})(a + 1)^2 \epsilon^2 \hat{\mu}^2/4 \simeq e^{-\epsilon^2} \simeq 1 \). This metric agrees with Gundlach and Ortiz[14], who derived the metric for a Jordan-Brans-Dicke cosmic string. In the string frame,

\[ ds^2 = e^{2\phi} ds^2 = \hat{r}^2[a^2\epsilon^2 \hat{\mu}^2/2 + (1 - \epsilon^2)\hat{r}^2/2d\theta^2] \]

(3.17)

which is almost, but not quite, a conformally rescaled cone. Note [14] that the radius at which non-conical effects become important is when \( R \simeq \sqrt{\lambda\eta}e^{(a+1)^2\epsilon^2 \hat{\mu}^2} \) or \( r \simeq \sqrt{\lambda\eta}e^{(a+1)^2\epsilon^2 \hat{\mu}^2} \), therefore, for a typical GUT string, \( r = O(10^{100\text{billion}}) \) i.e. well beyond any reasonable cosmological scale.

This is reminiscent of metric of the global string[18], a system which has very strong asymptotic effects and was for some time thought to be singular [19]. The effect of the global string also becomes evident at very large radii \( (e^{1/\epsilon}) \), however, unlike the metric (3.16) the global string metric is actually non-static and has an event horizon at finite distance from the core [20].
Note that the back-reaction of the linearized solution \((3.16)\) on the vortex fields is to alter the long-range fall-off of the \(X\) and \(P\) fields:

\[
1 - X \simeq \exp\left\{ -R^{1+(a+1)\epsilon\hat{\mu}/4} \right\}
\]

\[
P \simeq \exp\left\{ -R^{1+(a+1)\epsilon\hat{\mu}/4}/\sqrt{\beta} \right\}
\]  

which could be interpreted as a thickening of the core by a factor \((1 + (a + 1)\epsilon\hat{\mu}/4)\).

Now let us consider the special case \(a = -1\). In this case we see that (setting \(\gamma = \phi = 0\) at the core) \(\gamma = -2\phi\), and \((3.10c)\) implies that \(\gamma \to 0\) rapidly outside the core. In this case we see that to leading order, \(\gamma\) takes its Einstein form, and the back-reaction of the dilaton on the vortex fields serves only to perturb slightly the solution for the Einstein self-gravitating vortex. Thus for \(a = -1\), the cosmic string is essentially the same as its Einstein gravity cousin. It therefore has the metric \((2.17)\) and \(e^{2\phi} = e^{-\gamma} \to e^{-C}\) which gives in the string frame

\[
\begin{align*}
\tilde{d}s^2 &= dt^2 - dr^2 - dz^2 - \tilde{\alpha}_E^2 e^{-2\gamma_E} d\theta^2 \\
&\sim_{r \to \infty} dt^2 - dr^2 - dz^2 - (1 - \epsilon\hat{\mu})^2 r^2 d\theta^2
\end{align*}
\]  

i.e. there is no red/blue-shift of time between the core and infinity in the string frame, no matter what the value of \(\beta\).

Finally, let us consider \(\beta = 1\). In this case, to linear order \(\mathcal{P}_R = \mathcal{P}_\theta = 0\), and \(\gamma = \phi = 0\), which we suspect to be the case to all orders, and indeed, the Bogomolnyi system \((2.15)\) can be shown to provide the solution to the fully self-gravitating string in this case.

Before moving on to the massive dilaton, it is worth emphasising that the \(a = -1\) massless dilatonic cosmic string has no long range effects (other than the deficit angle), and merely shifts the value of the dilaton between the core and infinity by a constant of order \(\epsilon\). For the special case \(\beta = 1\), there is no effect at all on the dilaton field.

### 3.2 Massive dilatonic gravity

In the absence of a preferred potential to take for the dilaton, we will use \(\tilde{V}(\phi) = 2M^2\phi^2\), where \(M = m/\sqrt{\lambda\eta}\) is the ratio of the dilaton mass to Higgs mass. Of course, we
do not expect that this will be the exact form of the dilaton potential, however, a quadratic approximation will be valid provided $\phi$ remains close to the minimum of the potential. For a GUT string we expect $10^{-11} \leq M \leq 1$, representing a range for the unknown dilaton mass of 1TeV - $10^{15}$GeV. The dilaton equation (3.10d), then becomes

$$(\alpha\phi')' = \alpha e^\gamma M^2 \phi + \epsilon \frac{(a+1)}{2} \alpha e^\gamma \mathcal{E} - \frac{1}{2} \epsilon \alpha e^\gamma (\mathcal{P}_R + \mathcal{P}_\theta)$$

(3.20)

Once again, we begin by considering the wire model for the string which again gives $\alpha(R)$ and $\gamma(R)$ as in (3.13a,b). However, the presence of the mass term in (3.20) now alters the form of the dilaton; integrating (3.20) for the wire model gives

$$\phi_w = -\frac{1}{2} (a+1) \epsilon \hat{\mu} K_0(MR)$$

(3.21)

where $K_0$ is the modified Bessel function. In this case, the constraint equation (3.10c) is satisfied for $R > M$, but for $R < M$ we once again require $O(\epsilon^2)$ corrections, this is not really surprising since this is within the Compton radius of the dilaton and we might expect a behaviour analogous to that of the massless dilaton. However, since these corrections are only significant for $R \simeq e^{1/\epsilon^2}$, we can in this case safely ignore them. At the string boundary, we have that $\phi \sim \frac{1}{2}(a+1) \epsilon \hat{\mu} \ln M = O(\epsilon)$, hence the quadratic approximation for the potential appears to be justified.

For an extended source, we may solve (3.20) implicitly using its Green’s function:

$$\phi = -\frac{1}{2} \epsilon K_0(MR) \int_0^R I_0(MR')R' \left[ (a+1) \mathcal{E}(R') - (\mathcal{P}_R(R') + \mathcal{P}_\theta(R')) \right] dR'$$

$$- \frac{1}{2} \epsilon I_0(MR) \int_R^\infty K_0(MR')R' \left[ (a+1) \mathcal{E}(R') - (\mathcal{P}_R(R') + \mathcal{P}_\theta(R')) \right] dR'$$

(3.22)

$$\simeq -\frac{1}{2} (a+1) \epsilon \hat{\mu} K_0(MR) \text{ for } R > 1, \ M \ll 1$$

which, unlike the massless dilaton case, is now in agreement with the wire model estimate. A plot of $\phi(R)$ is illustrated in figure 2 for the $\beta = 1$ vortex shown in figure 1 with various values of $M$.

We may now write down the asymptotic solution for the cosmic string to order $\epsilon$ as:

$$ds^2 = e^{\gamma E} \left[ dt^2 - dr^2 - dz^2 \right] - \alpha^2 e^{-\gamma E} d\theta^2$$

$$e^{2\phi} = e^{-(a+1) \epsilon \hat{\mu} K_0(MR)}$$

(3.23)
The dilaton field produced by the vortex

Figure (2): A plot of the dilaton field generated by a $\beta = N = 1$ vortex for various values of the dilaton mass. The factor $(a + 1)\epsilon \hat{\mu}$ has been scaled out of the dilaton. Note the reciprocal dependence of the dilaton fall-off on the mass, compared to the logarithmic dependence of the amplitude.

Thus the spacetime is asymptotically conical in both string and Einstein frames.

Now consider $a = -1$. In this case the dilaton is very strongly damped to zero outside the core:

$$\phi \approx \frac{\epsilon M}{2} \left[ I_0(MR) \int_0^\infty K_0'(MR)R^2\mathcal{P}_R - K_0(MR) \int_0^R I_0'(MR)R^2\mathcal{P}_R \right]$$

(3.24)

to a good approximation $\phi = 0$ outside the core, irrespective of $M$, and therefore in both the Einstein and the string frames there is a red or blue shift between the core and infinity.

Finally, if $\beta = 1$, we once again have $\gamma = \phi = 0$, and (2.15) gives the first order equations of motion which this system satisfies.

4. Bogomolnyi bounds for dilatonic cosmic strings

The results of the previous section suggest that $a = -1$ is a rather special point. Usually, for $\beta = 1$, the Bogomolnyi limit, the equations of motion for the cosmic string
simplify – they become first order – and the vortex saturates an energy bound determined by the winding number of the vortex [21]. For the dilatonic vortex, this delicate balance appears to be destroyed, except in the special case $a = -1$. In this section we would like to formalise this by presenting an energetic argument that a topological bound can be saturated if and only if $\beta = -a = 1$.

Since the cosmic string is cylindrically symmetric, and we do not a priori wish to make any assumptions about the global behaviour of the spacetime, we use an energy tailored to the system at hand – the C-energy introduced by Thorne[16]:

$$E_c = 4\pi \left[ \gamma - \ln \frac{\partial \tilde{\alpha}}{\partial r} \right] = 4\pi [\gamma - \ln \alpha']$$  \hspace{1cm} (4.1)

modified slightly to allow for the absence of the Newton constant, $G$. This energy can in turn be represented as the integral of the zeroth component of a covariantly conserved C-momentum vector:

$$E_c = \int \tilde{\alpha} e^\gamma P^0 dr = \int \alpha e^\gamma \tilde{P}^0 dR$$  \hspace{1cm} (4.2)

where

$$\tilde{P}^0 = \frac{1}{2\pi \alpha e^\gamma} \frac{\partial E_c}{\partial R} = \frac{2}{\alpha e^\gamma} \left[ \gamma' - \frac{\alpha''}{\alpha'} \right]$$

$$= \frac{2}{\alpha'} \left[ e\mathcal{E} + \frac{1}{4} \gamma' e^{-\gamma} + e^{-\gamma} \phi'^2 + \frac{1}{2} \tilde{V}(\phi) \right]$$  \hspace{1cm} (4.3)

Clearly every term in $\tilde{P}^0$ is positive semi-definite, and all vanish only in flat space, the latter three vanishing if $\phi = \gamma = 0$. Now consider $\mathcal{E}$, we may rewrite this as

$$\mathcal{E} = e^{2(a+1)\phi} \left\{ e^{-\gamma} \left[ X' - e^\gamma \frac{XP}{\alpha} \right]^2 + \left[ \frac{P'}{\alpha} e^{-\phi} - \frac{1}{2} e^\phi (X^2 - 1) \right]^2 \right\}$$

$$+ (\beta - 1) \frac{P'^2}{\alpha^2} e^{-2\phi} + \frac{1}{\alpha} \left[ (X^2 - 1)P' \right]^2$$  \hspace{1cm} (4.4)

In order to get a ‘topological’ value for the C-energy, we need $(\gamma - \ln \alpha')$ to be expressed in terms of $X$ and $P$; alternatively, we require $\tilde{P}^0$ to be a total derivative. For $a = -1$, $\beta = 1$,
\( \phi = \gamma = 0 \) and (2.15) implies that all terms in \( \hat{P}^0 \) vanish except for the last expression in equation (4.4) for \( \mathcal{E} \). We thus obtain

\[
E_c = \int \frac{2\epsilon}{\alpha'} [(X^2 - 1)P]' = -2\int (\ln [1 - \epsilon[(X^2 - 1)P + 1]])' = -2 \ln(1 - \epsilon) = \epsilon + \ldots = \eta^2 + O(\eta^4)
\]

– the topological bound. For a string with winding number other than one, we replace \( P \) by \( NP \) and hence \( E_c \) becomes \(-2 \ln(1 - N\epsilon) = N\eta^2 + O(\eta^4)\).

For \( \beta \neq 1 \) it is immediately clear that this topological bound cannot be saturated due to the presence of the \((\beta - 1)P^2\) term in the integral. Similarly, if \( a \neq -1 \), the equation of motion for \( \phi \) shows that \( \phi' \) must be nonzero due to the presence of the \((a + 1)\mathcal{E}\) term on the right hand side of (3.10d), hence \( \hat{P}^0 \) is strictly greater than \( 2\epsilon\mathcal{E}/\alpha' \), and once again, the topological bound cannot be saturated.

Therefore, by considering a fully covariant relativistic definition of energy for cylindrically symmetric systems, we have shown that there exists a topological ‘bound’ for the energy of the vortices, in a rather analogous fashion to the topological quantity originally derived by Bogomolnyi[15] for flat space vortices, and this bound is saturated only for \( \beta = -a = 1 \).

5. Geodesics

In this section we discuss the motion of test particles following geodesics in the spacetimes presented in section three. According to experimental tests [22] any theory describing gravity has to verify the Weak Equivalence Principle (WEP). This principle states that any path through spacetime of a freely falling neutral test body is independent of its structure and composition. Therefore gravity has to couple in the same way to massive test particles and to photons. The obvious way is coupling directly to the metric in the string frame, which is what one usually does in scalar-tensor theories. Clearly, since the string and Einstein frames are related by a conformal transformation, null geodesics will be the same in either frame, but the geodesics of massive particles will be different.
We begin by commenting on the massive dilaton. Here the metric is given by (2.17) outside the Compton radius of the dilaton, and is therefore conical. Geodesics are therefore the same as for the Einstein cosmic string, and indeed, since the corrections within the Compton radius of the dilaton are extremely small \(O(\epsilon^2)\), the geodesics throughout the whole spacetime in the Einstein frame are essentially the same as for the Einstein self-gravitating string.

Now consider the massless dilaton. In the string frame the metric is given by eq.(3.17) and the radial motion of a test particle in a plane transverse to the string, \(d\bar{z} = 0\), is given by:

\[
\dot{\hat{r}}^2 + \frac{h^2}{\hat{r}^2(1+\nu)} + \frac{k}{\hat{r}^\nu(1+\nu)} = \frac{E^2}{\hat{r}^{2\nu(\nu+1)}}
\] (5.1)

where \(\nu = (a + 1)\epsilon\hat{\mu}/2\), and the dot denotes a derivative with respect to the proper time along a timelike geodesic, or an affine parameter for photons. The parameter \(k\) is either one or zero, representing either a massive particle or photon respectively. \(E\) and \(h\) are constants of the motion representing energy and angular momentum respectively, and are given by:

\[
E = \hat{g}_{tt}\dot{t} = \hat{r}^\nu(\nu+1)\dot{\hat{r}}
\]

\[
h = (1 - \epsilon\hat{\mu})\hat{g}_{\theta\theta}\dot{\theta} = (1 - \epsilon\hat{\mu})^{3\hat{r}^{2-\nu^2}+\nu}\dot{\hat{\theta}}
\] (5.2)

For \(a = -1\), \(\nu = 0\), and irrespective of whether the dilaton is massive or massless, the geodesics are qualitatively the same as for the Einstein cosmic string. Indeed, from (5.1) one sees that the radial motion of a geodesic is the same as the classical trajectory of a unit mass particle of energy \(\frac{E^2}{2}\), with an effective potential given by:

\[
V_{\text{eff}} = \frac{h^2}{2\hat{r}^2} + \frac{k}{2}
\] (5.3)

which is an identical effective potential to that of a particle moving in flat space. (The presence of the \(\epsilon\hat{\mu}\) terms in the definition of \(h\) shows that the spacetime is not globally flat, but conical.) All non-static trajectories therefore escape to infinity, and satisfy

\[
\dot{\hat{r}} \geq \frac{h}{\sqrt{E^2 - k}}
\] (5.4)
In addition, there exist static trajectories for massive particles: \( \hat{r} = \hat{r}_0, E = 1 \).

Now consider \( a \neq -1 \). For comparison with the effective potential (5.3), it is useful to redefine the radial coordinate \( \hat{r} \) via

\[
\rho = \frac{\hat{r}^{\nu^2 + \nu + 1}}{\nu^2 + \nu + 1}
\]

which gives the \( \rho \)-radial motion as that of a unit mass particle of energy \( \frac{E^2}{2} \), with an effective potential given by:

\[
U_{\text{eff}} = \frac{h^2}{2[(\nu^2 + \nu + 1)\rho]^{2(1-\nu)}} + \frac{k}{2}[(\nu^2 + \nu + 1)\rho]^{\frac{\nu(\nu+1)}{\nu^2 + \nu + 1}}
\]  

(5.6)

Since \( \nu = O(\epsilon) \), to leading order this is

\[
U_{\text{eff}} \simeq \frac{h^2}{2(1+2\nu)\rho^{2(1-\nu)}} + \frac{k}{2} \rho^\nu.
\]  

(5.7)

First consider \( \nu > 0 \), i.e. \( a > -1 \). For massless particles, \( U_{\text{eff}} \simeq V_{\text{eff}} \), and thus photons escape to infinity, however, note that as \( \hat{r} \to \infty \), \( g_{tt} = \hat{r}^{\nu(\nu+1)} \to \infty \), hence these photons will be infinitely redshifted. (Note that this will happen in either frame, although the red-shifting in the Einstein frame occurs at a rate proportional to \( \epsilon^2 \) rather than \( \epsilon \).)

For massive particles, \( U_{\text{eff}} \) is now a potential well, (see figure 3) hence all trajectories of massive particles are bounded, however, for \( \rho \ll e^{1/\nu} \), \( U_{\text{eff}} \simeq V_{\text{eff}} \) hence trajectories approaching ‘close’ to the cosmic string (i.e. on all scales of cosmological interest) behave as if in a conical spacetime. Such orbits will be highly eccentric, and have an outer bound of \( \hat{r} = O(E^{1/\nu}) \). Note that there are no static geodesics in this case, all particles initially at rest will be attracted to the string by an acceleration of order \( \epsilon/r_0 \).

If \( \nu < 0 \), i.e. \( a < -1 \), then \( U_{\text{eff}} \) is once more a scattering potential and all particles escape to infinity. Since \( g_{tt} \to 0 \) in this case, photons will now be infinitely blue-shifted. Once again there are no static solutions to the massive particle geodesic equation, this time the particles are repelled from the string.

In the Einstein frame, the photon trajectories are identical to that of the string frame, but now all the massive particle trajectories are bound, as can be seen by removing all the terms involving \( \nu \) from (5.6).
6. Discussion

In this paper, we have derived the metric for \( U(1) \) local cosmic strings in dilaton gravity both with and without a potential for the dilaton. The (unknown) coupling of the abelian-Higgs model to the dilaton is accounted for by coupling the Lagrangian to the gravitational sector by an arbitrary \( e^{2a\phi} \) factor.

For a massless dilaton, the results are qualitatively the same as those of Gundlach and Ortiz [14], who considered cosmic strings in JBD theory. Essentially, the metric is the same as the usual cosmic string, i.e. conical, in the Einstein frame, and conformally conical in the string frame on scales of cosmological interest. However, on the very large scale, \( (r \sim \sqrt{\lambda e^{(a+1)\cdot r^*}}) \), there is additional curvature, and the spacetime is not asymptotically locally flat in either frame. The exception is the special case \( a = -1 \), in which the metric is conical in either frame, and the dilaton is shifted in the core relative to infinity, the direction of the shift depending on whether the cosmic string is type I or II, no alteration in the dilaton occurring for the boundary between types I and II: \( \beta = 1 \).

For a massive dilaton, as expected, the metric asymptotes a conical metric, in both
string and Einstein frames, however, the string does generate a dilaton ‘cloud’, approximately of width $m_H/m_\phi$, which is schematically depicted in figure 3, for $a \neq -1$. For $a = -1$ the dilaton is only perturbed away from its vacuum value in the core of the string, and for $\beta = 1$, it is not affected at all.

![FIGURE (4): A representation of the dilaton field surrounding the cosmic string for $a \neq -1$.](image)

Although it is beyond the scope of this paper to derive the effective action of the cosmic string, the results do support a Nambu approximation for the string, since they show that the metric is little affected on cosmological length scales, and remains approximately flat locally (unlike the global string [18]). Damour and Vilenkin [23] have recently explored the impact of a massive dilaton on string networks using a model for the interactions which modifies the Nambu approximation by making the mass per unit length interact with the (massive) dilaton. In other words, the worldsheets act as sources for the dilaton which has a mass $m_\phi$. They concluded that a TeV mass dilaton was incompatible with a GUT string network. Our results largely back up this calculation, but with one important caveat:
The model used by Damour and Vilenkin makes no reference to the details of the dilaton coupling to the particle physics model producing the strings, the abelian-Higgs lagrangian, \textit{i.e. their coupling is independent of our variable }$a$. Therefore, one should renormalize their calculations by factors of $(a + 1)$. This means that the conclusion that a TeV mass dilaton is incompatible with string theories of structure formation is only valid if $a$ is not close to $-1$. For $a = -1$, such as might be the case if the fields composing the string are derived from heterotic string theory or the NS-NS sector of type II string theory for example, there will be little dilatonic radiation from the cosmic string network, and hence a much weaker constraint.

To sum up: the gravitational field of a cosmic string in dilaton gravity is surprisingly close to that of an Einstein cosmic string on cosmological distance scales. However, it is the microwave background rather than cosmological observations, that provides the tightest constraint on the cosmic string theory of structure formation. If the strings couple to the dilaton directly ($a = -1$), then such constraints are identical to those derived in Einstein gravity. However, if the string couples with $a$ different from $-1$, then the constraints of Damour and Vilenkin [23] apply, and a ‘low’ (i.e. close to electroweak) mass for the dilaton rules out the cosmic string scenario of galaxy formation.

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References.

[1] R.H.Brandenberger, \textit{Modern Cosmology and Structure Formation} astro-ph/9411049.
M.B.Hindmarsh and T.W.B.Kibble, \textit{Rep. Prog. Phys.} \textbf{58} 477 (1995). hep-ph/9411342
A.Vilenkin and E.P.S.Shellard, \textit{Cosmic strings and other Topological Defects} (Cambridge Univ. Press, Cambridge, 1994).

[2] A.Vilenkin, \textit{Phys. Rev.} D\textbf{23} 852 (1981).
J.R.Gott III, \textit{Ap. J.} \textbf{288} 422 (1985).
W.Hiscock, *Phys. Rev.* D31 3288 (1985).
B.Linet, *Gen. Rel. Grav.* 17 1109 (1985).

[3] D.Garfinkle, *Phys. Rev.* D32 1323 (1985).

[4] R.Gregory, *Phys. Rev. Lett.* 59 740 (1987).

[5] E.Fradkin, *Phys. Lett.* 158B 316 (1985).
C.Callan, D.Friedan, E.Martinec and M.Perry, *Nucl. Phys.* B262 593 (1985).
C.Lovelace, *Nucl. Phys.* B273 413 (1985).

[6] P.Jordan, *Zeit. Phys.* 157 112 (1959).
C.Brans and R.H.Dicke, *Phys. Rev.* 124 925 (1961).

[7] G.Veneziano, *Phys. Lett.* 265B 287 (1991).
A.A.Tseytlin and C.Vafa, *Nucl. Phys.* B372 443 (1992). [hep-th/9109048]
A.Tseytlin, *Int. J. Mod. Phys.* D1 223 (1992). [hep-th/9203033]
D.Goldwirth and M.Perry, *Phys. Rev.* D49 5019 (1994). [hep-th/9308023]
E.Copeland, A.Lahiri and D.Wands, *Phys. Rev.* D50 4868 (1994). [hep-th/9406216]

[8] J.D.Barrow and K.Maeda, *Nucl. Phys.* B341 294 (1990).
A.Burd and A.Coley, *Phys. Lett.* 267B 330 (1991).
J.D.Barrow, *Phys. Rev.* D47 5329 (1993). *Phys. Rev.* D48 3592 (1993).
A.Serna and J.Alimi, *Phys. Rev.* D53 3074 (1996). [astro-ph/9510139]
S.Kolitch and D.Eardley, *Ann. Phys.* (N.Y.) 241 128 (1995).
C.Santos and R.Gregory, *Cosmology in Brans-Dicke theory with a scalar potential*, gr-qc/9611063.

[9] N.Turok, *Phys. Rev. Lett.* 63 2625 (1989).

[10] A.Albrecht, D.Coulson, P.Ferreira and J.Magueijo, *Phys. Rev. Lett.* 76 1413 (1996). [astro-ph/9505030]
N.Crittenden and N.Turok, *Phys. Rev. Lett.* 75 2642 (1995). [astro-ph/9505120]
R.Durrer, A.Gangui and M.Sakellariadou, *Phys. Rev. Lett.* 76 579 (1996). [astro-ph/9507033]

[11] H.B.Nielsen and P.Olesen, *Nucl. Phys.* B61 45 (1973).

[12] D.Forster, *Nucl. Phys.* B81 84 (1974).
K.I.Maeda and N.Turok, *Phys. Lett.* 202B 376 (1988).
R.Gregory, *Phys. Lett.* **206**B 199 (1988). *Phys. Rev.* **D43** 520 (1991).

[13] R.Geroch and J.Traschen, *Phys. Rev.* **D36** 1017 (1987).

[14] C.Gundlach and M.Ortiz, *Phys. Rev.* **D42** 2521 (1990).

L.O.Pimental and A.N.Morales, *Rev. Mex. Fis.* **36** S199 (1990).

M.E.X.Guimaraes, [gr-qc/9610007](http://arxiv.org/abs/gr-qc/9610007).

[15] E.B.Bogomolnyi, *Yad. Fiz.* **24** 861 (1976) [Sov. J. Nucl. Phys. **24** 449 (1976)]

[16] K.S.Thorne, *Phys. Rev.* **138** 251 (1965).

[17] T.Levi-Civita, *Atti Acc. Lincei. Rend.* **28** 101 (1919).

[18] A.G.Cohen and D.B.Kaplan, *Phys. Lett.* **215**B 67 (1988).

[19] R.Gregory, *Phys. Lett.* **215**B 663 (1988).

G.Gibbons, M.Ortiz and F.Ruiz, *Phys. Rev.* **D39** 1546 (1989).

[20] R.Gregory, *Phys. Rev.* **D54** 4955 (1996). [gr-qc/9606002](http://arxiv.org/abs/gr-qc/9606002)

[21] B.Linet, *Phys. Lett.* **124**A 240 (1987).

A.Comtet and G.Gibbons, *Nucl. Phys.* **B299** 719 (1988).

[22] R.Eötvös, V. Pekár, E. Fekete, *Ann. der Phys.* **68** 11 (1922).

R.H.Dicke, *Am. J. Phys.* **28** 344 (1960).

V.B.Braginsky, V.I. Panov, *Sov. Phys. JETP* **34** 463 (1972).

[23] T.Damour and A.Vilenkin, *Cosmic strings and the string dilaton*, [gr-qc/9609067](http://arxiv.org/abs/gr-qc/9609067).