Optical microfiber knot resonator (MKR) and its slow-light performance

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1. Introduction

There has been increasing interest in the development of optical microring resonators owing to their simple structure, compact size, and extensive applications in optical signal processing, communication, and active devices, such as optical filters, wavelength multiplexers, and lasers [1-3]. Planar waveguide microring resonators fabricated lithographically have been well developed, but suffer from larger internal and connection losses, higher cost and more complicated fabrication methods. Recently, research on low-loss micro- and nanofibers has opened up new opportunities for developing microphotonic devices such as resonators [4], couplers [5], and sensors [6]. As one of the basic functional elements, various structures of microfiber resonators, including loop, knot and coil, have been investigated [4, 7, 8] and extensively applied to optical filters [9], optical sensors [10], microfiber lasers [11], nonlinear resonators [12] and slow or fast light systems [13, 14]. Among these microfiber resonators, the microfiber knot resonator (MKR) has been regarded as one of the most attractive resonators [15-17], due to its many advantages, including easy fabrication, high stability, good compatibility with the available communication system, compactness, low loss, high Q value and high finesse. Since, Jiang et al. [4] firstly proposed the MKR, the resonator has been extensively applied to add-drop filters [9], miniature lasers [18], fast-light system [14], and so on. Recently, Xiao et al. proposed a new method to directly fabricate a MKR from a double-ended tapered fiber [15], which benefits the high finesse. We also proposed another approach to fabricate MKR with different structures [19], which might prompt the resonator to have a more extensive application.

In this paper, we investigate the MKRs with different structures and the slow-light performance of them from 6 sections. In Section 1, we make a simple introduction about microfiber resonators and their applications, and the structure of the whole paper. In Section 2, we give the mathematical expressions of the output light field with respect to the input one in the MKR, in the microfiber multi-knot resonator with a parallel structure, and in the microfiber multi-knot resonator with a serial structure. The slow-light performances of them are also investigated. It is found that a large slow-light time delay with a narrow bandwidth can be obtained in the microfiber multi-knot resonator with a parallel structure, while a slow-light time delay with a wide bandwidth can be obtained in the microfiber multi-knot resonator with a serial structure. In Section 3, we theoretically analyze and experimentally investigate how to design and fabricate a tapered microfiber with good optical and mechanical performance. In Section 4, we introduce a simple, polymer-microfiber-assisted approach to fabricating the silica MKRs with different structures. Comparing with other fabrication methods, this technique is quite simple and is easy to fabricate much more complicated multi-ring MKRs. In Section 5, we demonstrate a wide-bandwidth and zero-dispersion slow light in the microfiber double-knot resonator with a parallel structure based on an analogue of EIT through changing the correlated parameters, such as the coupling coefficients, and the diameters of the two knot rings. In Section 6, we make a conclusion about the whole paper.

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2. Theoretical study on slow light in different structures of optical microfiber knot resonators (MKRs)

2.1. A single-ring MKR

Figure 1 shows the schematic of a single-ring MKR. The theory of the MKR can be developed by combining the theory of ring resonators [20-23] with that of directional couplers [24-26]. The dependence of the output light fields, $E_3$ and $E_4$, on the input one $E_1$ (the detailed deduced process is given in Ref. [27].) can be expressed respectively as

$$E_3 = \frac{\sqrt{(1-r)(1-k)}}{1-j\sqrt{k(1-r)} \exp(j\beta L)} E_1,$$

$$E_4 = \left(\frac{j\sqrt{k(1-r)} + (1-r)\exp(j\beta L)}{1-j\sqrt{k(1-r)} \exp(j\beta L)}\right) E_1 = T E_1,$$

where $T$ is the amplitude transmission coefficient of the single-ring MKR, $k$ is the coupling coefficient, $r$ is the coupling loss coefficient, $L$ is the circumference of the ring resonator, and $\beta$ is the propagation constant given in Ref. [28]. The phase of the transmission amplitude can be expressed by

$$\phi_T = \text{Im} \left( \ln (T) \right).$$

And the group time delay is obtained by [29]

$$\tau_d = \frac{d\phi_T}{d\omega} = \frac{n_{\text{eff}}}{c} \left( \frac{d\phi_T}{d\beta} \right),$$

where $c$ is the speed of light in vacuum and $n_{\text{eff}}$ is the effective refractive index of the microfiber. And the group delay dispersion (GDD) is obtained by differentiating Eq. (4) with respect to the angular frequency $\omega$ [30] and can be expressed by

$$GDD = \frac{d^2\phi_T}{d\omega^2} = \frac{d\tau_d}{d\omega} = -\frac{\lambda^2}{2\pi c} \frac{d^2\tau_d}{d\lambda^2}.$$

Note that GDD is the counterpart of the group velocity dispersion (GVD), which also determines the spreading degree of the optical pulse.

2.2. Microfiber multi-knot resonator with a parallel structure

Let us first analyze the case of microfiber double-knot resonator with a parallel structure. As shown in Fig. 2, it consists of two rings labeled 1 and 2 that are realized by a microfiber. The smaller ring, MKR 2, is embedded into the larger one, MKR 1. The amplitude transmission coefficients of ring 2 and ring 1 can be obtained by
2. Microfiber double-knot resonator with a parallel structure

As shown in Fig. 2, it is comprised of two rings labeled 1 and 2. The overall amplitude transmission coefficient:

\[ T_2 = \frac{E_8}{E_5} = \frac{j\sqrt{k_2(1-r_2) + (1-r_2) \exp(j\beta L_2)}}{1 - j\sqrt{k_2(1-r_2) \exp(j\beta L_2)}}. \]  
\[ (6) \]

\[ T_1 = \frac{E_4}{E_1} = \frac{j\sqrt{k_1(1-r_1) + (1-r_1) T_2 \exp(j\beta L_1)}}{1 - j\sqrt{k_1(1-r_1) T_2 \exp(j\beta L_1)}}. \]  
\[ (7) \]

Where \( k_1, k_2 \) are the coupling coefficients, \( r_1, r_2 \) are the coupling loss coefficients and \( L_1, L_2 \) are the circumferences of ring 1 and ring 2, respectively. Note that \( T_1 \), in fact, is the overall amplitude transmission coefficient \( T \), i.e., \( T = T_1 \).

Similarly, the amplitude transmission coefficients of microfiber multi-knot resonator with a parallel structure can be calculated as

\[ T_n = \frac{j\sqrt{k_n(1-r_n) + (1-r_n) \exp(j\beta L_n)}}{1 - j\sqrt{k_n(1-r_n) \exp(j\beta L_n)}}; \]
\[ T_{n-1} = \frac{j\sqrt{k_{n-1}(1-r_{n-1}) + (1-r_{n-1}) T_n \exp(j\beta L_{n-1})}}{1 - j\sqrt{k_{n-1}(1-r_{n-1}) T_n \exp(j\beta L_{n-1})}}; \]
\[ \vdots \]
\[ T_2 = \frac{j\sqrt{k_2(1-r_2) + (1-r_2) T_3 \exp(j\beta L_2)}}{1 - j\sqrt{k_2(1-r_2) T_3 \exp(j\beta L_2)}}; \]
\[ T_1 = \frac{j\sqrt{k_1(1-r_1) + (1-r_1) T_2 \exp(j\beta L_1)}}{1 - j\sqrt{k_1(1-r_1) T_2 \exp(j\beta L_1)}}; \]
\[ T = T_1. \]  
\[ (8) \]

In Eq. (8), \( T_n, T_{n-1}, \ldots, T_2 \) and \( T_1 \) are the amplitude transmission coefficients of the \( n \)th, the \((n-1)\)th, \( \ldots, \) the second, and the first rings, respectively. The overall amplitude transmission coefficient \( T \) is equal to \( T_1 \).

2.3. Microfiber multi-knot resonator with a serial structure

Analogously, we firstly study the case of microfiber double-knot resonator with a serial structure. As shown in Fig. 3, it is comprised of two rings labeled 1 and 2. The overall amplitude transmission coefficient:

\[ T = T_1 T_2 \exp(j\beta L_{1,2}) \],  
\[ (9) \]

where

\[ T_1 = \frac{E_4}{E_1} = \frac{j\sqrt{k_1(1-r_1) + (1-r_1) \exp(j\beta L_1)}}{1 - j\sqrt{k_1(1-r_1) \exp(j\beta L_1)}}. \]  
\[ (10) \]
Note that $T_1$ and $T_2$ are the amplitude transmission coefficients of the first MKR and the second one, respectively, $L_{1,2}$ is the distance between them.

It is easy to extend to the case of microfiber multi-knot resonator with a serial structure, and in this case the amplitude transmission coefficient can be expressed by

$$T = \prod_{i=1}^{n} \left\{ \frac{j\sqrt{k_i(1-r_i)} + (1-r_i)\exp(j\beta L_i)}{1 - j\sqrt{k_i(1-r_i)}\exp(j\beta L_i)} \right\} \exp(j\beta L_{(i-1),i}) .$$

In Eq. (12), $L_{(i-1),i}$ is the distance between the $(i-1)$th and the $i$th rings and $L_{0,1}=0$.

**Figure 3.** Microfiber double-knot resonator with a serial structure.

2.4. Slow-light time-delay characteristics of microfiber multi-knot resonator with different structures

Assuming the input light field $E_1=1$, inserting Eq. (8) and Eq. (12) into Eq. (3), respectively, and then using Eq. (4), we can obtain the group time delay features of the microfiber multi-knot resonator with a parallel structure and the microfiber multi-knot resonator with a serial structure.

Figure 4(a) compares the group time delays in the single-ring MKR, the microfiber double-knot resonator with a parallel structure, and the microfiber three-knot resonator with a parallel structure. To see clearly, Fig. 4(a) only indicates the result around the wavelength of 1550 nm. The inset of Fig. 4(a) is the zoom-in spectrum of the group time delay of the single-ring MKR. It can be seen from Fig. 4(a) that the group time delay in the microfiber three-knot resonator with a parallel structure increases drastically, but at the same time the 3-dB bandwidth of it becomes narrower than that of the microfiber double-knot resonator with a parallel structure.

Figure 4(b) compares zoom-in spectrum of the group time delay in the single-ring MKR, the microfiber double-knot resonator with a serial structure, and the microfiber three-knot resonator with a serial structure at the wavelength around 1550 nm. The group time delay in the microfiber three-knot resonator with a serial structure does not change a lot, but the 3-dB bandwidth strongly increases.

To summarize, by designing the microfiber multi-knot resonator with a parallel structure, one can obtain large slow-light time delay with a narrow bandwidth; while one can obtain the slow-light time delay with a wide bandwidth by designing the microfiber multi-knot resonator with a serial structure.
3. Design and fabrication of tapered microfiber waveguide with good optical and mechanical performance

3.1. Theoretical analysis and simulation

In this section, we present the design and fabrication of tapered microfiber waveguide with good optical and mechanical performance. The tapered microfiber waveguide is fabricated by stretching a heated conventional SMF to form a structure with two transition regions and a uniform waist, as shown in Fig. 5. The diameter of the transition region decreases from the size of SMF down to the waist diameter. When the light propagates through the transition region, the light field distribution varies as the core and cladding diameters change along the fiber. Depending on the change rate of diameter, the energy transfers from the fundamental mode to a closest few higher order modes. Moreover, the number of the high order modes determines the loss. For the transition region with a slowly varying diameter, there always companies a less number of high order modes and thus a low loss of light. Therefore, one can minimize the microfiber loss by optimizing its profile parameters.

Based on the adiabaticity criteria [31], we use the Finite-difference Beam Propagation Method (FD-BPM) to simulate the light propagating in the tapered microfiber with different profile parameters. Firstly, the relationship between the light transmission of the tapered microfiber and the transition region length is simulated and shown in Fig. 6, one can see that when the uniform waist diameter is greater than 0.6 \( \mu m \) and the transition region length is longer than 5 mm, the loss becomes very small and can be neglected. Secondly, in order to analyze the dependence of the tapered microfiber optical loss on the waist diameter, in Fig. 7(a) we simulate the normalized transmission power of the tapered microfiber as a function of the uniform waist diameter. Finally, in the practical applications of the evanescent-field-based optical sensing, it is important to know the evanescent field...
distribution around the tapered microfiber. We therefore investigate the variation of the fractional energy of the evanescent field with the waist diameter of the tapered microfiber. The simulation result is plotted in Fig. 7(b), where the light wavelength is selected at 1550 nm as an example.

![Figure 6](image)

**Figure 6.** Normalized transmission power depends on the transition region length of tapered microfiber. The open circle line, the solid circle line, the open up-triangle line and the solid up-triangle line represent four tapered microfibers with uniform waist diameters of 0.4 µm, 0.5 µm, 0.6 µm and 0.8 µm, respectively.

![Figure 7](image)

**Figure 7.** (a) Normalized transmission power of the tapered microfiber as a function of the uniform waist diameter. The open circle line and the solid circle one correspond to the incident light wavelengths of 650 nm and 1550 nm, respectively. (b) Dependence of the fractional power of the evanescent field on the tapered microfiber waist diameter for the incident light wavelength of 1550 nm.

Based on the analysis and simulations mentioned above, it can be concluded from Figs. 6 and 7 that the tapered microfiber with low loss, strong evanescent field and relatively shorter transition region should have two longer than 5-mm-length transition regions and a uniform waist whose diameter is larger than 600 nm and less than 1 µm. Additionally, it should be noted that the taper shape of the transition region also affects the microfiber properties. In our simulation process, we assume that the taper shape of the transition region is a decaying-exponential shape, which has been demonstrated in some experiments [32, 33].

### 3.2. Fabrication and performance of the tapered microfiber

The schematic experimental setup for the tapered microfiber fabrication is shown in Fig. 8. It mainly comprises of the microfiber heater (MHI FIBHEAT200, US), two translation stages with high precision stepper motors (FL110TA600, China), two fiber holders on the translation stages and the computer control system.
Figure 8. Schematic experimental setup for tapered microfiber fabrication.

Figure 9. (a) Profile of the tapered microfiber we fabricated. Two inserted microscope images are a partial transition region and a uniform waist of the tapered microfiber, respectively. (b) The transmission spectra of the ASE light when it passes through the un-tapered SMF (the open circle line), the tapered microfiber in air (the solid circle line) and on the MgF$^2$ substrate (the cross line), respectively.

The profile of the tapered microfiber we fabricated is shown in Fig. 9(a). The total length of the tapered microfiber is 80 mm. The length of the transition region with an approximate decaying-exponential shape is about 30 mm and the length of the uniform waist with a diameter of 1 $\mu$m is about 20 mm. Two microscope images inserted in Fig. 9(a) show the partial transition region and the uniform waist, respectively. Note that the tapered microfiber has low surface roughness. In order to survey the optical performance of the tapered microfiber during the fabrication, ASE and OSA are connected with the two ends of the fiber.

The optical spectra are shown in Fig. 9(b). The open circle line, the solid circle line and the cross line represent the transmission spectra of the ASE light when it passes through the un-tapered SMF, the tapered microfiber in air and on the MgF$^2$ substrate, respectively. The loss at 1550 nm is about 0.05 dB in air. It is seen that the loss of the tapered microfiber is very low in air, while it increases up to 0.8 dB when the tapered microfiber is moved onto the MgF$^2$ substrate. The reason of the loss increasing is that a small quantity of evanescent filed energy is diverted to the MgF$^2$ surface and is lost when the microfiber is moved on the MgF$^2$ substrate.

4. A Simple, Polymer-Microfiber-Assisted Approach to Fabricating the Silica Microfiber Knot Resonator

4.1. Basic principle of fabrication

The schematically fabrication process is illustrated in Fig. 10, which can be divided into four steps. (1) Using the high-temperature tapered-drawing technique, one fabricates a silica microfiber with a diameter of micrometer scale from the standard single mode fiber (SMF). As a result, there is a tapered-drawing region which connects the silica microfiber to the SMF at each end of the silica microfiber. Cutting one end of the silica microfiber makes it become a freestanding end. As shown in Fig. 10(a), a polymer microfiber with a diameter of tens micrometers, drawn from solvent polymers [34], is tailored to a suitable length. Then let’s manually wind a knot ring with a diameter of several millimeters from the above polymer microfiber and adhere one end of the knot ring to the freestanding end of the silica microfiber. (2) The polymer microfiber knot ring is driven to the silica microfiber with the assistance of a tapered-drawing fiber probe until the polymer microfiber is completely drawn out of the knot ring. As shown in Fig. 10(b), in this case, the knot ring is only composed of the silica microfiber. (3) As shown in Fig. 10(c), continuously drawing the polymer microfiber thus finely tunes the diameter of the
knot ring under an optical microscope. (4) Departing the polymer microfiber from the freestanding end of the silica microfiber and adhering the other silica microfiber produced in the step (1) to it, as shown in Fig. 10(d), one finally fabricates a silica MKR. Note that the zoom-in image in Fig. 10(d) is the intertwined overlap at the contact area.

![Figure 10. (a)-(d) Schematic of the fabrication process.](image)

### 4.2. Experimental results and discussions

Experimentally, we fabricated a MKR with the above mentioned technique. Figure 11(a) shows an optical microscope image of the knot resonator, in which the polymer microfiber ring has being driven to the freestanding end of the silica microfiber. As one can see in Fig. 11(a), the bold line is the polymer microfiber with a diameter of about 12 µm, while the slim line is the silica microfiber with a diameter of about 1 µm. Figure 11(b) shows a zoom-in optical microscope image of the knot region in Fig. 11(a). It is easy to see that the polymer microfiber and the silica microfiber are adhered together firmly. This is attributed to the strong van der Waals and electrostatic forces between them. The final knot resonator consisted of the silica microfiber is shown in Fig. 11(c). The diameter of the fabricated MKR is about 409 µm. Figure 11(d) shows a zoom-in optical microscope image of the knot region in Fig. 11(c). To see the spectral property of this MKR, a broadband amplified spontaneous emission (ASE) light was launched into the MKR and the spectrum from the output port was collected. As an example, Fig. 11(e) shows the transmission spectrum in the wavelength range of 1548-1554 nm, where an extinction ratio of about 5 dB is obtained.

![Figure 11. Optical microscope images of (a) the MKR located in the adhered region between the polymer microfiber and the silica microfiber on an MgF2 substrate; (b) the zoom-in knot region in (a); (c) the final fabricated MKR; (d) the zoom-in knot region in (c); (e) Transmission spectrum of the fabricated MKR.](image)
Using this method, we can also fabricate microfiber multi-knot resonator with different structures readily. Figure 12(a) shows an optical microscope image of the microfiber double-knot resonator with a serial structure. The diameter of the silica microfiber is about 2 µm, and the diameters of the small ring and the big ring are about 680 µm and 723 µm, respectively. A corresponding transmission spectrum is given in Fig. 12(b). The whole transmission spectrum is the overlapped consequence of the two transmission spectra produced by the two knot rings. Figure 12(c) shows an optical microscope image of the microfiber double-knot resonator with a parallel structure. The diameter of the silica microfiber is about 2 µm and the diameters of the small ring and the big ring are about 788 µm and 1605 µm, respectively. A corresponding transmission spectrum is given in Fig. 12(d). Note that the whole transmission spectrum isn’t simply an overlapped consequence of the two transmission spectra produced by the two knot rings.

5. Wide-bandwidth and zero-dispersion slow light in MKRs with a two-ring parallel connection structure based on an analogue of electromagnetically induced transparency

In order to study the slow-light characteristics at the windows of the EIT-like in the microfiber double-knot resonator with a parallel structure. Firstly, we theoretically simulate the transparency window of the EIT-like [35] with the variation of the coefficients $k_2$ and $k_1$. Assuming the input light field $E_1$=1 and using Eq. (7) one can calculate the dependence of the transmission coefficient of the microfiber double-knot resonator with a parallel structure on wavelengths for varying coupling coefficient $k_2$. Numerical results are shown in Fig. 13(a), where, as an example, we set the parameters to $k_1$=0.8, $r_1$=$r_2$=0.1, $d$=2 µm, $D_1$=1600 µm, $D_2$=800 µm, $n_{eff}$=$n_1$=1.45 and $n_2$=1. It is easy to see from Fig. 13(a) that, when the coupling coefficient $k_2$=0 (meaning that there isn’t coupling action at the knot region of MKR 2, the light field $E_5$ converts to the light field $E_7$ completely), the whole microfiber double-knot resonator with a parallel structure is equivalent to a single-ring MKR with a circumference of $L_1+L_2$. According to the definition of the free spectral range (FSR) of the resonator $\Delta \lambda = \lambda^2 / (n_{eff} \cdot L)$, the longer the circumference of the resonator is, the narrower the FSR is. So when $k_2$=0, the FSR shown in the Fig. 13(a) is narrowest. However, when the coupling coefficient $k_2$ isn’t equal to zero, a narrow transmission peak, the so-called EIT-like window, is produced between two wide transmission peaks due to the overlap of transmission spectra. Moreover, as the coupling coefficient $k_2$ increases, the induced transparency window becomes narrower and narrower. When the coupling coefficient $k_2$=1.0, the induced transparency window vanishes. This is due to the fact that, in this case, the light field $E_5$ converts to $E_8$ completely, except that there is 10% light energy loss (because $r_2$=0.1), the microfiber double-knot resonator with a parallel structure is equivalent to a single-ring MKR. Based on this characteristic, one can create narrow line-width lasers, filters, sensors and so on, through modulating the coupling coefficient $k_2$ and the circumferences of the two rings.

When we vary the coupling coefficient $k_1$ and keep the other parameters $k_2$=0.8, $r_1$=$r_2$=0.1, $d$=2 µm, $D_1$=1600 µm, $D_2$=800 µm, $n_{eff}$=$n_1$=1.45 and $n_2$=1 unchanged, we calculate the dependence of the transmission
coefficient of the microfiber double-knot resonator with a parallel structure on wavelengths. The numerical results are shown in Fig. 13(b). One can see that when the coupling coefficient $k_2=0$, the FSR of the resonator is broadest. The reason is that, in this case, there isn’t coupling action at the coupling region of MKR 1, the input light field $E_1$ converts to $E_3$ completely, thus the whole microfiber double-knot resonator with a parallel structure is equivalent to a sing-ring MKR with a circumference of $L_2$. As the coupling coefficient $k_1$ increases, the extinction ratio of the output spectrum increases, but the FSR of the induced transparency window between two wide transmission peaks doesn’t change. When the coupling coefficient $k_1$ increases to 1, the input light field $E_1$ converts to $E_4$ completely, except that there is 10% light energy loss (because $r_1=0.1$). Therefore, the output spectrum is a horizontal line and the transmission coefficient $T$ is equal to 0.9.

Based on the analysis mentioned above, one can modulate the bandwidth and the extinction ratio of the induced transparency window by changing the coupling coefficients $k_1$ and $k_2$ simultaneously. On the other hand, as we know that group time delay is supposed to be observed at the resonance of the transmission spectrum of the resonator [13]. So, in order to study the characteristic of the group time delay, we simulate the transmission coefficient, the phase, the group time delay and the GDD with the variation of the coupling coefficient $k_2$ at the induced transparency window. The corresponding simulated results are shown in Fig. 14.

![Figure 13](image1.png)

**Figure 13.** Dependence of the transmission coefficient of MKRs with a two-ring parallel connection structure on wavelengths for varying coupling coefficients (a) $k_2$ ($k_1=0.8$) and (b) $k_1$ ($k_2=0.8$), the other parameters are chosen to $k_1=0.8$, $r_1=r_2=0.1$, $d=2 \, \mu m$, $D_1=1600 \, \mu m$, $D_2=800 \, \mu m$, $n_{et}=n_1=1.45$ and $n_2=1$.

Note that in Fig. 14 the parameters of the theoretical model are chose to $r_1=r_2=0.1$, $d=2 \, \mu m$, $D_1=1600 \, \mu m$, $D_2=800 \, \mu m$, $n_{et}=n_1=1.45$ and $n_2=1$ which are identical with the parameters given in Fig. 13. When the coupling coefficients $k_1=0.02$, and $k_2$ increases from 0.1 to 0.7, the induced transparency window firstly becomes lower gradually and then disappears, which can be easily seen in Fig. 14(a). Especially, when the coupling coefficient $k_2$ is equal to 0.5, the transmission coefficient $T$ reduces to 0.48 with a flat wavelength bandwidth of about 82.7 pm. Accordingly, as shown in Fig. 14(c), the group time delay at the corresponding induced transparency window increases gradually with the increasing of the coupling coefficient $k_2$ from 0.1 to 0.7. And it reaches to about 72.4 ps with a flat wavelength bandwidth of about 82.7 pm, the corresponding FWHM is about 228 pm, when the coupling coefficient $k_2$ is equal to 0.5. The reason for this phenomenon is that the phase of the transmission light field is modulated effectively at the resonance by resonator, which makes $d\phi/d\omega$ increase rapidly. It can be seen from Fig. 14(b) that, with the increasing of the coupling coefficient $k_2$, the absolute value of the slope rate is increased correspondingly at the resonant wavelength of around 1549.8 nm. Therefore, from Eq. (4), one can see that the group time delay increases greatly. When the coupling coefficient $k_2$ is equal to 0.5, the slope rate of the phase is almost a straight line at a small wavelength range of around 1549.8 nm. Thus, the group time delay is a constant at this wavelength range, which can be seen from Fig. 14(c). On the other hand, using Eq. (5), one can calculate the GDD with respect to the variation of wavelengths. It can be seen from Fig. 14(d) that the GDD at the same induced transparency window occurs the corresponding change with the increasing of the coupling coefficient $k_2$ from 0.1 to 0.7. And it becomes zero with a wavelength bandwidth of about 82.7 pm which locates at the same wavelength range of the flat group time delay, when the coupling coefficient $k_2$ is equal to 0.5. Thus, through modulating the suitable values of the coupling coefficients $k_2$ and $k_1$, one can obtain a wide-bandwidth and zero-dispersion group time delay at the induced transparency window in the microfiber double-knot resonator with a parallel structure. We believe that the slow light with a characteristic of the wide bandwidth and zero dispersion at the induced transparency window in the microfiber double-knot...
A resonator with a parallel structure will have significant potential applications in optical buffers, data delay lines and optical memories, etc.

![Figure 14](image)

**Figure 14.** Dependence of the transmission coefficient (a), the phase (b), the group time delay (c), the GDD (d) at the induced transparency window in the microfiber double-knot resonator with a parallel structure on wavelengths for various coupling coefficients \(k_2\), the other parameters are chosen to \(k_1=0.02\), \(r_1=r_2=0.1\), \(d=2\) \(\mu\)m, \(D_1=1600\) \(\mu\)m, \(D_2=800\) \(\mu\)m, \(n_{eff}=n_1=1.45\) and \(n_2=1\).

### 6. Conclusions

In conclusion, firstly, we investigate the theoretical models of the sing-ring MKR, the microfiber multi-knot resonator with a parallel structure and the microfiber multi-knot resonator with a serial structure and numerically simulate the group time delay spectra of these MKRs with different structures. The numerical result indicates that by designing the microfiber multi-knot resonator with a parallel structure, one can obtain large slow-light time delay with a narrow bandwidth. By designing the microfiber multi-knot resonator with a serial structure, one can obtain the slow-light time delay with a wide bandwidth. Secondly, we investigate the design and fabrication of tapered microfiber waveguide with good optical and mechanical performance theoretically and experimentally. The result demonstrates that the tapered microfiber with low loss, strong evanescent field and relatively shorter transition region should have two longer than 5-mm-length transition regions and a uniform waist whose diameter is larger than 600 nm and less than 1 \(\mu\)m, the taper profile of the transition region should be a decaying-exponential shape. Thirdly, we present a simple, polymer-microfiber-assisted approach to fabricating the silica microfiber knot resonator. Using this technique, we have successfully fabricated several kinds of MKRs with different structures, such as those with a double-knot serial structure and with a double-knot parallel structure. Comparing with other fabrication methods, this technique is quite simple and is easy to fabricate much more complicated multi-ring MKRs. Finally, a wide-bandwidth and zero-dispersion slow light in the microfiber double-knot resonator with a parallel structure based on an analogue of EIT is demonstrated through changing the correlated parameters, such as the coupling coefficients, and the diameters of the two knot rings.

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