Implications of WMAP Observations On the Population III Star Formation Processes

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ABSTRACT

In an earlier paper (Cen 2003) we pointed out the strong likelihood for the universal reionization to occur twice, giving rise to a much larger Thomson optical depth due to the intergalactic medium than that in the case of a single rapid reionization at $z \sim 6$. The latest Wilkinson Microwave Anisotropy Probe (WMAP) observations (Kogut et al. 2003) indicate that the universe indeed appears to have entered a significantly ionized state at a very high redshift. In light of this new development, we perform a more focused analysis of the Thomson optical depth in the context of the spatially flat, cosmological constant-dominated cold dark matter model constrained by WMAP observations.

While the current uncertainties on the observed Thomson optical depth are still relatively large, with $\tau_e = 0.17 \pm 0.04$ (68%) (Kogut et al. 2003), important implications on Pop III star formation processes at high redshift can already be inferred. We are able to draw four conclusions: (1) in the absence of a top-heavy initial stellar mass function (IMF) for Pop III metal-free stars and without a dramatic upturn in the star formation efficiency and ionizing photon escape fraction at high redshift ($z > 6$), we find $\tau_e \leq 0.09$; (2) with a top-heavy IMF for the Pop III metal-free stars and plausible star formation efficiency and ionizing photon escape fraction, it is expected that $\tau_e \leq 0.12$; (3) it is possible to reach $\tau_e = 0.15$, if the metal enrichment efficiency of the intergalactic medium by Pop III stars is very low thus Pop III era is prolonged; (4) to reach $\tau_e \geq 0.17$ requires either of the following two conditions: the cosmological model power index $n$ is positively tilted to $n \geq 1.03$, Pop III star formation in minihalos with molecular hydrogen cooling has an efficiency $c_s(H_2, III) > 0.01$ (with ionizing photon escape fraction greater than 30%). Thus, if the current observed value of Thomson optical depth withstands future data, we will have strong observational

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evidence that Pop III stars are massive and their formation efficiency may be much higher than current theoretical works suggest. Alternatively, there may be unknown, non-stellar ionizing sources at very high redshift.

Subject headings: cosmology: theory — intergalactic medium — reionization

1. Introduction

The cosmological reionization process is likely to be quite complex (e.g., Barkana & Loeb 2001; Madau 2002; Wyithe & Loeb 2003a; Mackey, Bromm, & Hernquist 2002; Cen 2003; Venkatesan, Tumlinson, & Shull 2003). It seems rather remarkable that we can now start observationally probing this process. A major milestone was laid down by the recent observations of high redshift quasars from the Sloan Digital Sky Survey (SDSS), indicating that the final reionization episode came to completion at \( z \sim 6 \) (e.g., Fan et al. 2001; Becker et al. 2001; Barkana 2002; Cen & McDonald 2002; Litz et al. 2002). Given the final reionization epoch and the fact that the density fluctuations at that epoch is sufficiently well constrained in the context of the standard cold dark matter model (Bahcall et al. 1999), it becomes possible to show (Cen 2003), under reasonable assumptions, that the universe has been reionized twice, first reionized at \( z \geq 15 \) by metal-free, massive Population III (Pop III) stars and second reionized at \( z = 6 \), provided that Pop III IMF is top-heavy. In this picture a larger Thomson optical depth, \( \tau_e \geq 0.07 \), was expected.

Another major milestone was set very recently by the WMAP polarization observations, which probes the ionization state of the gas in the universe at high redshift during the cosmological reionization period. The WMAP observations measured \( \tau_e = 0.17 \pm 0.04 \) (68%) (Kogut et al. 2003). One can immediately draw the conclusion that the universe did not just experience an impulsive reionization event at \( z \sim 6 \), which would have yielded \( \tau_e \sim 0.03 \), in the standard cosmological constant dominated cold dark matter model. Evidently, the universe became (and possibly had to stay) significantly ionized from a much higher redshift, probably from \( z \geq 15 \). In this Letter we compute the detailed reionization process and compare to WMAP observations, investigating what can be learned about Pop III star formation processes at high redshift.

2. Thomson Optical Depth \( \tau_e \)

A spatially flat cold dark matter cosmological model with \( \Omega_M = 0.27, \Omega_b = 0.047, \Lambda = 0.73, H_0 = 72\text{km/s/Mpc}, n_s = 0.99 \) and \( \sigma_8 = 0.90 \), symbolized as LCDMAP model,
is used. This model is the best fit model constrained by WMAP observations with a fixed power-law index (Spergel et al. 2003). To set a plausible upper bound we also compute a model with $n_s = 1.03$ and $\sigma_8 = 1.0$, denoted as LCDMAP+ model, which is consistent with current WMAP data (Spergel et al. 2002) and maximizes the small-scale structures responsible for the reionization process. We employ an efficient method to compute the coupled evolution of gas and star formation in the high redshift universe, described in §4 of Cen (2003), which simultaneously computes the dynamic evolution of all gaseous phases including HII regions, HI regions and partially ionized regions. Such a careful treatment is necessary given the various short time scales involved at high redshift for important processes, including recombination and cooling. Throughout, Pop III stars are defined to be metal-free stars with a top-heavy initial mass function (IMF) and Pop III galaxies are defined to be galaxies where Pop III stars form. We define “minihalos” as those whose virial temperature is less than $\lesssim 1 \times 10^4$K where only $H_2$ cooling is possible in the absence of metals, and large halos as those with virial temperature above $\sim 1 \times 10^4$K capable of cooling via atomic lines.

The most important factors determining the ionization process are $\Psi \equiv c_s \times f_{\text{esc}}$ and $C$, where $c_s$ is the star formation efficiency, defined as the fraction of gas in halos formed into stars; $f_{\text{esc}}$ is the fraction of ionizing photons that escapes from galaxy halos and gets into the IGM; $C$ is the clumping factor of the intergalactic medium (IGM). Basically, $\Psi$ is the source of ionizing photons from stars in galaxies and $C$ determines the sink of ionizing photons due to the IGM. The competition between the two primarily dictates the ionization state of the IGM. The initial phase of ionization front propagating through neutral medium does not play a significant role near the end of the reionization process, and cosmological effects are unimportant throughout the reionization period. But all these processes are included in the calculation. Theoretically, $C$ is reasonably well determined, if we adopt the standard cold dark matter cosmological model. In addition, $C$ may be constrained directly by observations of quasar absorption spectra at redshifts close to $z = 6$ (e.g., Cen & McDonald 2002; Djorgovski et al. 2001; Cen & Haiman 2000). We set the IGM clumping factor at $z = 6$ to be 50.0, according to Gnedin & Ostriker (1997), which requires $C_{\text{halo}} = 702$ with the adopted cosmological model (see Cen 2003 for discussion). Results do not sensitively depend on $C$ at $z = 6$; for example, a change in $C$ at $z = 6$ by 25% only causes $\sim 5\%$ change in $\tau_e$. Both theoretically and observationally, our direct knowledge of $\Psi$ for high redshift galaxies is very little. However, as pointed out in Cen (2003), the fact that the universal reionization ends just at $z \sim 6$, as observations seem to indicate (e.g., Fan et al. 2001; Becker et al. 2001; Barkana 2002; Cen & McDonald 2002; Litz et al. 2002), tightly constrains $\Psi$, knowing $C$. Such a normalization point is proven to be very powerful and unambiguously determines $\Psi$ at $z \sim 6$ (see Figure 7 of Cen 2003) for Pop II stars, which are likely responsible for completing the long reionization process at $z \sim 6$. For all the models computed below,
the normalization at $z = 6$ requires $\Psi(II) \equiv c_s(II) \times f_{esc}(II) = 0.015 - 0.017$ for Pop II galaxies at $z \sim 6$. For models computed we assume $\Psi(II)$ to be constant with redshift, except indicated otherwise. On the other hand, $c_s(III)$ and $f_{esc}(III)$ are less constrained.

We compute seven models with varying parameters for Pop III star formation processes and cosmological parameters, listed in Table 1, to show the possible range for the Thomson optical depth due to the IGM at reionization. In Table 1, $c_s(H_2, III)$ indicates the star formation efficiency in minihalos with virial temperature less than 8,000K, where $H_2$ cooling dominates for metal-free gas; $c_s(HI, III)$ indicates the star formation efficiency in large halos with more efficient atomic cooling; $f_{crit}$ indicates the fractional amount of gas formed into Pop III stars at the transitional epoch from Pop III to Pop II (which is assumed to determine the metallicity of the IGM; see Cen 2003); $f_{esc}(III)$ is the mean fraction of ionizing photons produced by galaxies that escapes from the halos of Pop III galaxies and into the IGM; the last column $\tau_e$ is the integrated (from $z = 0$ to $z = 1000$) Thomson optical depth due to the IGM, where multiple entries correspond to respective multiple entries for $f_{esc}(III)$.

Models #1,2 represents the case where the IMF at high redshift is assumed to remain as the Salpeter (1955) function with the same low-mass cut-off $(0.1 \, M_\odot)$, as at $z = 6$, which is the likely minimum model in terms of $\tau_e$, for LCDMAP and LCDMAP+ model, respectively. Models #3,4 have a Pop III star formation efficiency in minihalos $c_s(H_2, III) = 0.002$, suggested by numerical simulations of Abel, Bryan & Norman (2002) with a reasonable star formation efficiency for Pop III galaxies with large halos of $c_s(HI, III) = 0.10$, for LCDMAP and LCDMAP+ model, respectively. Models #5,6 assume a higher Pop III star formation efficiency in minihalos $c_s(H_2, III) = 0.01$, for LCDMAP and LCDMAP+ model, respectively; in this case, ionizing photons from Pop III minihalos dominate over those from Pop III large halos for the first reionization process, regardless of the value of $c_s(HI, III)$. Model #7 is similar to Models #1, except that in the former we arbitrarily set the transition epoch from Pop III to Pop II at the time when a fraction $f_{crit} = 5 \times 10^{-4}$ of total gas has formed Pop III stars instead of $f_{crit} = 1 \times 10^{-4}$ in the latter set. We note that $f_{crit} = 1 \times 10^{-4}$ is derived based on the assumption that most of the Pop III metal-free massive stars experience supernovae and most of the ejected metals get out of galaxies to enrich the IGM (Oh et al. 2001; Cen 2003). If, for some reason, the fractional amount of metals enriching the IGM is significantly smaller than the amount of gas formed into massive stars, the efficiency of metal enrichment by Pop III stars would be lower; Model #7 serves to illustrate this possibility. Possible physical processes that may reduce the metal enrichment efficiency include a large fraction of Pop III stars being very massive stars ($M \geq 300 \, M_\odot$) which may collapse to black holes instead of exploding as supernovae (Rakavy, Shaviv, & Zinamon 1967; Bond, Arnett, & Carr 1984; Glatzel, Fricke, & El Eid 1985; Woosley 1986) or not all ejected metals are able to be transported to IGM. We note that Model #1 is close to the model used in Cen...
(2003), having perhaps more conventional (thus thought to be more reasonable) values for the relevant parameters. We also test a variant of Model #1 by using \( \Psi(II) \propto (1 + z)^{1/2} \) instead of being constant with redshift and find \( \tau_e = 0.08 \) for that case versus 0.07 for a constant \( \Psi(II) \) (Model #1).

From Table 1 we can draw several conclusions. First, without Pop III stars (Models #1,2), it is likely that \( \tau_e < 0.09 \) for both LCDMAP and LCDMAP+ models, inconsistent with WMAP results at \( \geq 2\sigma \) level (Kogut et al. 2003). The conclusions reached here with regard to the requirement of a top-heavy IMF for Pop III stars in order to explain the observed high \( \tau_e \) value are consistent with those by Wyithe & Loeb (2003b), Haiman & Holder (2003) and Sokasian et al. (2003). Second, for reasonable ranges for the star formation efficiency \( [c_s(HI, III) \leq 0.1, c_s(H_2, III) \leq 0.002] \) and ionizing photon escape fraction \( (f_{esc}(III) \leq 0.20) \) from Pop III galaxies, we expect that \( \tau_e \leq 0.12 \) for LCDMAP model (Model #3) and \( \tau_e \leq 0.15 \) for LCDMAP+ model (Model #4). Third, prolonging the Pop III era by making the metal enrichment of the IGM from Pop III galaxies less efficient increases the Thomson optical depth incrementally. Finally, while \( \tau_e = 0.17 \) is the mean value determined by WMAP observations, it seems hard to reach in the LCDMAP model. There are two possible ways to achieve a Thomson optical depth as high as \( \tau_e \geq 0.17 \): either the ionizing photon escape fraction is large \( (f_{esc}(III) \geq 0.3) \) and the underlying cosmological model has a blue power spectrum with a positive tilt to \( n \geq 1.03 \) (Model #5) or Pop III star formation efficiency in minihalos is substantially larger than current simulations seem to indicate, requiring \( c_s(H_2, III) > 0.01. \)

Figures 1,2 show the reionization history and cumulative Thomson optical depth, respectively, for four representative models listed in Table 1. We see that without Pop III stars (Model #1) reionization event only occurs once at \( z \sim 6 \) and the reionization history is monotonic. With Pop III stars the first reionization occurs at \( z = 15 - 20 \) (Models #3,4). In the last case (Model #5) where Pop III star formation efficiency in minihalos is substantially larger than suggested by current simulations of Abel, Bryan & Norman (2002), 0.01 versus \( \sim 0.002 \), the first reionization occurs at \( z \sim 20 - 25 \). While both Model #4 and Model #5 have comparable total Thomson optical depth, \( \tau_e = 0.15 - 0.16 \), their respective reionization histories at early times \( (z \geq 15) \) are somewhat different. We find that for Models #4, minihalos and large halos contribute to 62% and 38% of all ionizing photons to reionize the universe at the first time, by which 9 ionizing photons per baryon have been produced; the corresponding numbers for Model #5 are 61%, 39% and 11 photons per baryon. In contrast, for Model #3, minihalos and large halos contribute to 20% and 80% of all ionizing photons to the first reionization and 6 ionizing photons per baryon have been produced by then. We see, as expected, that an earlier first reionization requires a larger number of ionizing photons per baryons and a larger contribution from minihalos appears likely if the first reionization
occurs at $z \geq 20$ in the context of the currently favored $\Lambda$ cosmology. It is possible that future CMB experiments such as Planck survey might be able to distinguish between these two scenarios (e.g., Holder et al. 2003). We will defer a more detailed analysis of this aspect to a separate paper.

3. Discussion

Since it appears that a significant fraction ($f_{esc}(III) \geq 0.1$) of ionizing photons produced by Pop III galaxies is required to escape from galaxies in order to sufficiently ionize the universe to be consistent with WMAP observations, it is worthwhile to check what kind of escape fraction is possible. Here we give an order of magnitude assessment of the situation. If a fraction, $c_s(III)$, of gas in a high redshift Pop III galaxy formed into very massive stars emitting at $1.6 \times 10^{48}$ hydrogen ionizing photons per solar mass of stars per second (e.g., Bromm, Kudritzki, & Loeb 2001), then we can obtain the ratio of the total number of hydrogen ionizing photons emitted during the $3 \times 10^6$ yrs of lifetime of the Pop III galaxy (assumed to be equal to the lifetime of the Pop III massive stars) to the number of baryons in the galaxy,

$$\frac{N_{ph}}{N_H} = 1.3 \times 10^5 c_s(III).$$

(1)

It can also be shown that the ratio of the number of recombinations of hydrogen atoms (assuming a temperature of $1.5 \times 10^4$ K at $z = 20$) to the number of baryons in the galaxy is

$$\frac{N_{rec}}{N_H} = 0.061 C_g,$$

(2)

where $C_g$ is the effective clumping factor of the gas in the galaxy, taking into account all possible effects including shielding and clumpiness. Thus, the ratio of the total number of ionizing photons to the total number of recombinations of hydrogen atoms during the lifetime of the galaxy at redshift $z = 20$ is

$$\frac{N_{ph}}{N_{rec}} = 2.1 \times 10^6 C_g^{-1} c_s(III).$$

(3)

The time it takes for the ionization front from very massive Pop III stars to break out is roughly

$$t_{IF} = \frac{4\pi r^3 n}{L_{ph}} = 6.8(\frac{M_s}{100 M_\odot})^{-1}(\frac{r}{1pc})^3(\frac{n}{10^4 cm^{-3}}) \text{ yrs},$$

(4)

which is clearly much shorter than the lifetime of the galaxy. In Equation (4) $M_s$ is the amount of Pop III stars formed, $r$ the size of the galaxy disk and $n$ the interstellar medium
Fig. 1.— shows the global mean of the hydrogen neutral fraction as a function of redshift for four models (Model #1: solid, blue; Model #3 with $f_{\text{esc}}(\text{III}) = 0.1$: dotted, green; Model #4 with $f_{\text{esc}}(\text{III}) = 0.3$: short dashed, red; Model #5 with $f_{\text{esc}}(\text{III}) = 0.2$: long dashed, black) in Table 1.
Fig. 2.— shows the cumulative Thomson scattering optical depth as a function of redshift for four models (Model #1: solid, blue; Model #3 with $f_{\text{esc}}(\text{III}) = 0.1$: dotted, green; Model #4 with $f_{\text{esc}}(\text{III}) = 0.3$: short dashed, red; Model #5 with $f_{\text{esc}}(\text{III}) = 0.2$: long dashed, black) in Table 1.
density. Thus, ionization of the gas inside the galaxy may be treated as instantaneous and the ratio in Equation (3) is an instructive gauge.

If \( c_\ast (HI, III) \geq 0.1 \), as appears to be required (as discussed in §2) in order to produce a large enough \( \tau_e (\geq 0.1) \) to be consistent with WMAP observations, then Equation (3) implies that \( C_g \) has to be less than \( 2.1 \times 10^5 \) in order for a significant fraction of ionizing photons to escape from galaxy halos. On one hand, if the clumping factor is dominated by gas in the halo with overdensity of order 100, \( C_g \) could be less than \( 2.1 \times 10^5 \). On the other hand, the gas on the disk or at radii much smaller than the virial radius would be much denser and their contribution to the clumping factor would critically depend on the density, geometry and self-shielding effects. A reliable estimate of the escape fraction for ionizing photons is unattainable without detailed modeling. Nevertheless, it appears that it is not entirely impossible to have a significant fraction of ionizing photons escape under special conditions. Possible favorable scenarios include a large fraction of star formation activities occur at a substantial height above the galactic disk and/or off-center within star formation sites embedded in very clumpy interstellar medium.

It is clearly seen from the above analysis that the smaller the value of \( c_\ast (III) \), the less likely that a significant fraction of ionizing photon can escape. Consequently, a high Pop III star formation efficiency may be the key to a high overall ionizing photon production rate in Pop III galaxies with respect to the receiving IGM, with the latter depending on the former in a probably nonlinear way. The question is then: how high can \( c_\ast (III) \) be?

The simulation by Abel, Bryan & Norman (2002; see also Bromm, Coppi, & Larson 2002) for a halo of mass \( 10^6 M_\odot \) with \( 6 \times 10^4 M_\odot \) baryons forms a massive star of mass \( 100 M_\odot \) at the center. This would give a star formation efficiency of 0.0017, taken at face value, which is much smaller than required value of order 0.01. For a discussion why Pop III star formation in minihalos may continue unimpeded throughout the Pop III era, see Cen (2003). The value of 0.1 required for Pop III stars in large halos to ionize the universe at \( z = 15 - 20 \) is still higher by a factor of ten. It should be pointed out that that the central protostar in the Abel et al. (2002) simulation appears to be still accreting matter rapidly at the end of their simulation, so it is possible that the stellar mass may grow substantially. We also note, however, that their simulated galaxy has a virial temperature below \( 1 \times 10^4 \) K hence does not possess the efficient atomic cooling. It is possible that, for galaxies with efficient atomic cooling, star formation efficiency may be much higher than what Abel et al. (2002) simulation indicates. Detailed simulations will be invaluable.

Finally, we note that, if star formation in cooling shells produced by exploding massive Pop III stars, the “Pop II.5” stars proposed by Mackey et al. (2003), is efficient, they may make a non-negligible contribution to the pool of ionizing photons. These stars might form at
off-center locations and thus might possess a relatively larger ionizing photon escape fraction. The counteracting factor is that these stars might be substantially less massive than the first generation metal-free stars, since the metallicity of the gas in the cooling shells may be quite high, hence are less efficient ionizing photon emitters.

4. Conclusions

Adopting the best fit standard cold dark matter model with a fixed power-law index by WMAP observations with $\Omega_M = 0.27$, $\Omega_b = 0.047$, $\Lambda = 0.73$, $H_0 = 72$ km/s/Mpc, $n_s = 0.99$ and $\sigma_8 = 0.90$ (Spergel et al. 2003) and based on the observed Thomson optical depth due to intergalactic medium by WMAP (Kogut et al. 2003), we are able to draw several relatively secure conclusions with regard to Pop III star formation processes at very high redshift.

(1) The combination of the normal Salpeter IMF for Pop III metal-free stars and the absence of a dramatic upturn in the star formation efficiency and/or ionizing photon escape fraction at high redshift ($z > 6$) would produce a Thomson optical depth due to IGM at reionization of $\tau_e \leq 0.09$, inconsistent with the observed $\tau_e = 0.17 \pm 0.04$ (68\%) (Kogut et al. 2003) at $\geq 2\sigma$ level.

(2) A top-heavy IMF for the Pop III metal-free stars and plausible star formation efficiency and ionizing photon escape, as gauged by the corresponding values for Pop II galaxies required in order to achieve the second reionization finale at $z \sim 6$ yield $\tau_e \leq 0.12$;

(3) In the event that the metal enrichment efficiency of the intergalactic medium by Pop III stars is very low thus Pop III era is prolonged, one may be able to obtain $\tau_e = 0.15$.

(4) It seems quite improbable to reach $\tau_e \geq 0.17$ even with very massive Pop III metal-free stars, unless (i) the cosmological model power index $n$ is positively tilted to $n \geq 1.03$ and/or (ii) Pop III star formation in minihalos with molecular hydrogen cooling has an efficiency $c_\ast (H_2, \text{III}) > 0.01$ (still requiring ionizing photon escape fraction greater than 0.3) or (iii) alternatively, there may be unknown, non-stellar ionizing sources at very high redshift in the context of the $\Lambda$CDM cosmology.

It is expected that the errorbars on the detected Thomson optical depth should shrink significantly with time using more data. Thus, if the current observed value of Thomson optical depth ($\tau_e \geq 0.09$ - currently $2\sigma$ lower bound) withstands future data, we will have strong observational evidence for the existence of massive metal-free Pop III stars. One possible outcome will be that $0.09 \leq \tau_e \leq 0.12$ ($1.25 - 2.0\sigma$ away from the current observed central value), in which case no drastic increase in the star formation efficiency in Pop III
galaxies would be needed. On the other hand, if $\tau_e \geq 0.17$ is firmed up in the future, there will be a degeneracy between possible competing scenarios. Future CMB experiments such as the Planck surveyor may be able to distinguish between them. On the theoretical side, more accurate calculations of Pop III metal-free star formation in both minihalos and large halos will be extremely useful to shed light on these vital parameters.

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Table 1. Summary of Star Formation Models

| #  | Model  | \(c_*(\text{H}_2, \text{III})\) | \(c_*(\text{HI, III})\) | \(f_{\text{crit}}\) | \(f_{\text{esc}}(\text{III})\) | \(\tau_e\) |
|----|--------|-------------------------------|-----------------|-----------------|-----------------|--------|
| 1  | LCDMAP | 0                             | 0               | 0               | -               | 0.07   |
| 2  | LCDMAP+ | 0                             | 0               | 0               | -               | 0.08   |
| 3  | LCDMAP | 0.002                         | 0.1             | \(1 \times 10^{-4}\) | (0.05,0.1,0.2,0.3) | (0.09,0.10,0.12,0.13) |
| 4  | LCDMAP+ | 0.002                         | 0.1             | \(1 \times 10^{-4}\) | (0.05,0.1,0.2,0.3) | (0.10,0.12,0.14,0.16) |
| 5  | LCDMAP | 0.01                          | 0.1             | \(1 \times 10^{-4}\) | (0.05,0.1,0.2,0.3) | (0.09,0.11,0.14,0.15) |
| 6  | LCDMAP+ | 0.01                          | 0.1             | \(1 \times 10^{-4}\) | (0.05,0.1,0.2,0.3) | (0.10,0.13,0.16,0.19) |
| 7  | LCDMAP | 0.002                         | 0.1             | \(5 \times 10^{-4}\) | (0.05,0.1,0.2,0.3) | (0.11,0.12,0.14,0.15) |