Investigation of the Hydraulic Characteristics of Capillary Elements of the Injector Head of Jet Engines under Conditions of Isothermal Flow of A Liquid

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Abstract. The article presents the results of an experimental study of the hydraulic characteristics of capillary elements of the injector head of jet engines in isothermal fluid flow and the proposed method of their calculation. The main geometric dimensions of the capillaries in the experiment were changed in the following range: Inner diameter from 0.16 to 0.36 mm, length from 4.3 to 158 mm and relative length from 25 to 614 and the inlet edge of the capillaries: sharp or smooth the leading edge. As the working fluid during the tests were distilled water, acetone and ethyl alcohol. Based on the results of a study of the dependences for calculation of ultimate losses in laminar and turbulent flow regimes in capillary tubes with smooth and sharp edges input. The influence of surface tension forces on loss of input on a sharp cutting edge. Experimentally confirmed the possibility of calculating the linear coefficient of hydraulic resistance of capillary tubes with a diameter of 0.16-0.36 mm in isothermal stable during the known dependencies that are valid for hydrodynamically smooth round tube.

1. Introduction
Jet thrusters as fluidic devices for supplying fuel into the combustion chamber using a round capillary tube of small diameter large relative lengths: inner diameter (0, 15-0, 35) mm and the relative length of more than 20. Round capillary tube compared to conventional round tubes have a number of features as in an isothermal fluid flow and flow with heat supply [1]. Therefore to calculate the hydraulic characteristics and the choice of the geometric dimensions of the capillary nozzle elements of the thrusters was an experimental study of the hydrodynamics of flow in capillary tubes and the method of calculation of their hydraulic characteristics in an isothermal liquid flow. The proposed calculation method of the capillary nozzle elements based on the known materials of calculation of hydraulic characteristics with the features identified by the authors in the experimental study of capillary tubes.

2. Facilities and experimental conditions
A hydrodynamic study was performed in capillaries made of stainless steel. A pilot study was conducted on the installation from cold flow. Install the provided pressure flow of the working fluid in the capillary with the aid of compressed air, whose pressure was controlled by pressure sensor, the flow rate of the working fluid was measured either by weighing or volumetric method using a level gauge, the filling time is fixed by an electric stopwatch. As working fluids in prolivka distilled water, acetone and ethyl alcohol. The main geometric dimensions of the capillaries during the experiment were changed in the following range: inner diameter from 0.16 to 0.36 mm, length from 4.3 to 158
mm and the relative length from 25 to 614. The capillaries were carried out with sharp or smooth edge of the input.

3. Hydraulic characteristic of the capillary
As the main hydraulic characteristic of the capillary was used, the flow characteristic in common to the hydrodynamics of the fluid flow [2]:

$$\Delta p = \left( \frac{\lambda}{d} + K \right) \frac{\rho u_0^2}{2},$$

(1)

where $\Delta p$ - the pressure drop in the capillary; $\rho$ - the density of the liquid; $u_0$ - the fluid velocity at the entrance to the capillary; $\lambda$ - linear coefficient of hydraulic resistance; $l$ and $d$ – the length and inner diameter of the capillary; $K$ - end loss.

The expression (1) can be converted with correlation:

$$\xi = \frac{\lambda}{d} + K,$$

(2)

$$\xi = \frac{1}{\mu^2},$$

(3)

$$\frac{\rho u_0^2}{2} = \frac{8\dot{m}^2}{\pi^2 \rho d^4},$$

(4)

to the next view:

$$\Delta p = 8 \frac{\xi \dot{m}^2}{\pi^2 \rho d^4} = 8 \frac{\dot{m}^2}{\pi^2 \mu^2 \rho d^4},$$

(5)

where $\dot{m}$ - mass flow rate of the liquid; $\xi$ – hydraulic resistance coefficient capillary; $\mu$ - flow coefficient capillary.

Linear coefficient of hydraulic resistance $\lambda$, generally, depends on the regisma flow - Reynolds number $Re$, roughness, wall temperature, etc. In the research process, as shown hereinafter, for the description of the hydrodynamics of fluid flow in the capillaries can be used known formulas to calculate the coefficient of hydraulic resistance in laminar flow - Poiseuille's formula [2], in turbulent flow regime (up to $Re = 10^4$) for hydrodynamically smooth pipes (the roughness is not more than ~ 1 µm)- the formula of Blasius [2].

4. Input loss
End for the pressure loss $K$ at the initial part of the capillary which the rebuilding of the velocity profile, and the acute front edge and the turbulent mode, in addition, the compression and expansion of the flow, and depend on the flow regime and type of inlet edge.

In laminar flow, due to the influence of the initial hydrodynamic phase, it is convenient to consider using functions $K_\xi$ from dimensionless capillary length $\tilde{z} = l/(d Re)$. In this regard, conducted the experimental determination $K_\xi$ of the capillary with a diameter of 0.16 mm with a sharp entrance edge and a smooth entrance. The results of the experiment are presented in figure 1a.

Processing of the obtained experimental results allowed us to obtain the following approximating dependence [3]:

$$K_\xi = 1 + 1.2[l - 0.61 \exp(-94.8 \tilde{z})]$$

(6)
which can also be represented as a function of dimensionless flow \( \bar{m} = m/\eta \) (where \( \eta \) - is the dynamic viscosity coefficient of fluid):

\[
K_z = 2,2 - 0,726 \cdot \exp(-74,5/\bar{m}).
\]  

(7)

According to (6), (7) just as in the case of a smooth inlet edge and for the case with a sharp entrance edge (figure 1 a) \( \bar{z} > 0.003 \) and \( \bar{m} < 260 \).

**Figure 1.** The results of determination of the limit losses in the capillaries in the laminar (a) and turbulent (b) flow regimes: a) 1, 2 - sharp entrance edge, 3, 4 - input smooth edge, 1, 3 - water, 2, 4 - alcohol; 5 - area of known results [3]; 6 - approximating the dependence (6); b) 1 - \( \text{We}=\infty \), 2 - \( \text{We}=100 \), 3 - \( \text{We}=20 \), 4 - \( Lp=1 \cdot 10^4 \); 5 - \( Lp=5 \cdot 10^3 \); 6 - \( Lp=20 \cdot 10^4 \); 7 - experimental data if \( \text{We}=\infty \) [4].

In turbulent flow regime limit the loss for a smooth entrance is small, due to only rebuild the velocity profile and can be taken into account using the proposed in [5] dependencies:

\[
K = 1 + 2,65\lambda
\]  

(8)

At the sharp front edge into the capillary as the jet is compressed during expiration the diaphragm and then expands to form an isolated annular cavity, in which the pressure is considered equal to the pressure in the compressed section of the jet and lower than the pressure in the place of adhesion of liquid to the wall. Using the Bernoulli equation and the Board for the mechanism of flow was obtained the formula for calculating entrance losses:

\[
K = 1 + \xi_0 = \frac{1}{\mu^2} - 2 \left( \frac{1}{\varepsilon} - 1 \right)
\]  

(9)

where \( \xi_0 \) - input loss; \( \varepsilon \) - the compression ratio of the jet efflux through the aperture.

As an isolated annular cavity has a phase interface, then the value of the input loss at small sizes of the capillaries should have an impact force of surface tension. Analysis of the results of experimental study of the influence of Weber numbers \( \text{We} \) for the coefficients of the diaphragm [2] shows that the relative change of the flow coefficient can be represented as:
where $\mu$ - the expense ratio in the absence of the influence of surface tension forces; $\mu^*$ - the ratio consumption in the presence of the influence of surface tension forces; $\bar{\mu}^* \Delta p$ - coefficient taking into account the change of flow rate of a diaphragm for reducing the effective differential pressure on it and determine by calculation; $\bar{\varepsilon}^* \sigma$ - coefficient taking into account the increase in the area of the compressed section of the jet efflux through the aperture and determined experimentally.

Taking into account the experimental values $\mu(Re)$, $\varepsilon(Re)$ and $\bar{\varepsilon}^*(We)$ [2, 4], where the Weber number $We=(u_0^2d\rho)/\sigma$, $\sigma$ - surface tension, calculated according to (9) and (10) change $K(Re,We)$. The results (figure 1 b) show good coincidence with experiment at $We \to \infty$ and significant influence of surface tension on entry-ney losses $We < 103$. The dotted lines show the change of the limit for loss of the capillary when you change the flow rate (Reynolds number $Re$ and the Laplace number $Lp=(\rho\sigma d)/\eta^2$).

5. Linear coefficient of hydraulic resistance

Due to the complexity of measuring the pressure drop on the section of stable flow, a linear coefficient of hydraulic resistance was determined by pressure drop over the capillary with the given limit losses according to the formulas (6)...(10). Thus the calculated diameter of the capillary is chosen so that $\lambda$ in laminar flow regime was consistent with the dependence of Poiseuille [2]:

$$\lambda = \frac{64}{Re}$$

The same method was used in [6], its validity is confirmed by good coincidence of experimental and calculated values of the coefficient of hydraulic resistance:

$$\xi = 16\pi \frac{\eta l}{m} + 2.2 - 0.726 \left( -74.5 \frac{\eta l}{m} \right)$$

for laminar regime. The deviation of experimental points from the calculated curve is observed in case of turbulence, which allows to determine the transition flow regime and to find critical Reynolds number:

$$Re_{cr} = \frac{4l}{\pi d m_{cr}}$$

The obtained experimental results confirmed (figure 2) applicability (1)...(5) for descriptions of the hydrodynamic characteristics of the capillaries, not only in laminar but also in turbulent (to $Re = 104$) flow regimes capillaries are hydrodynamically smooth pipes (the roughness less than ~ 1 µm) and amount is determined by the dependence of the Blasius [2]:

$$\lambda = \frac{0.3164}{(Re)^{1/4}}$$

In figure 2 shows the following notation of experimental points: 3 – capillary $d=0.26$ mm, $l/d=613$; 4-6 – capillary $d=0.35$ mm, $l/d=238$; 7, 8 – capillary $d=0.33$ mm, $l/d=438$; 9 – capillary $d=0.17$ mm, $l/d=232$; 10-12 – capillary $d=0.36$ mm, $l/d=211$; 13-15 – capillary $d=0.33$ mm, $l/d=158$; 16, 17 – capillary $d=0.35$ mm, $l/d=150$; 18-22 – capillary $d=0.16$ mm, $l/d=60$; 25, 26 – capillary $d=16$ mm, $l/d=26$; 3, 4, 7-11, 13, 16, 19, 25, 26 – sharp input edge; 5, 6, 12, 14, 15, 20-24 - smooth the input edge; 3-5, 7, 9, 10, 13, 14, 16, 20, 23, 25 - water; 6, 8, 11, 12, 15, 17, 18, 21 - acetone; 19, 22, 24, 26 - alcohol.
At $\text{Re} > 10^4$ (figure 2) the value of the linear coefficient of hydraulic resistance $\lambda$ is practically unchanged and amounts to (0.031...0.029). In the case of a sharp inlet edge, the loss of stability of laminar flow, determined by the change $\lambda$, occurs at $\text{Re} > 1.5 \cdot 10^3$; in the case of smooth input – $\text{Re} > (2.5...5) \cdot 10^3$, the transition to a developed turbulent flow – in $\text{Re}=3 \cdot 10^3$ and $\text{Re} = (4...6) \cdot 10^3$, respectively, even in the case of a smooth entry at $\text{Re} > 7 \cdot 10^3$ no plot with a laminar boundary layer. Similar to the transition region is obtained for changing the appearance of the flowing stream.

![Figure 2](image_url)

**Figure 2.** The dependence of the linear coefficient of hydraulic resistance of the capillaries under conditions of isothermal flow: 1 – the dependence of Poiseuille; 2 – the dependence of Blasius; 3...26 - experimental data obtained in prolivka capillaries

### 6. Flow of fluid from the capillary

The expiration of a short capillary with a sharp entrance edge has a number of features, due to the formation of an end of an isolated cavity. In the experiment determined the minimum relative length of capillary $l/d$ in which it ceases to affect the impact of the end of an isolated cavity on the discharge and the dependences for estimating the boundaries of the transition to stall at $\text{Re} < 10^4$ and cavitation $\text{Re} > 10^4$ the conditions of the flow through the cavitation number:

$$N_m = \xi \left( \frac{p_0 - p_1}{p_0 - p_2} \right) = \frac{1}{\mu^2}, \quad (15)$$

as well as the boundaries of the transition to shear mode expiry $\text{Re} > 10^4$:

$$N_s = \xi \left( \frac{p_0 - p_1}{p_0 - p_2} \right) = \left[ \frac{\mu^2 (2 + \xi \bar{\xi})}{\xi} \right]^{-1}. \quad (16)$$
In expressions (15), (16) $P_0$ and $P_2$ - the pressure at the inlet and at the outlet of the capillary, respectively; $p_{1\min}$ - the pressure in the narrow section of the capillary in the presence of an end of an isolated cavity. Comparison of calculated with experimental data [2, 4, 7] numbers of cavitation theoretically defined according to the relations (15) and (16), shows their satisfactory agreement.

Important when using jet schemes of mixing is to estimate the lower limits of applicability of these schemes - estimation of the minimum cost at which the jet is realized the regime of flow of the capillaries. The analysis shows that a jet is formed if the velocity head greater than the pressure created on the interface surface tension:

$$0.5 \rho u_0^2 > \frac{4 \sigma}{d} \quad \text{or} \quad We = \frac{\rho u_0^2 d}{\sigma} > 8 .$$

(17)

Experimental test performed when the capillary was poured with water, confirmed the validity of (17), however, showed that the degeneration of the jet occurs at smaller numbers $We$, and education with large numbers $We$.

7. Conclusion

Thus, a pilot study was proposed the method of calculation of hydraulic characteristics of capillary elements of mixing heads jet thrusters in an isothermal liquid flow, and also investigated the boundary cavitation and jetting regime of flow of the capillaries. Based on the results of a study of the dependences to calculate the end losses in laminar and turbulent flow regimes in capillary tubes with smooth and sharp edges input. The influence of surface tension forces on the input losses on the sharp front edge. Experimentally confirmed the possibility of calculating the linear coefficient of hydraulic resistance of capillary tubes with a diameter of 0.16...0.36 mm at isothermal stable during the known dependence of Poiseuille and Blasius, fair round for hydrodynamically smooth pipes (the roughness less than ~ 1 µm).

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