Low rank approximation of tensors

S. Friedland\textsuperscript{1}, and V. Tammali\textsuperscript{2}

\textsuperscript{1}University of Illinois at Chicago, friedlan@uic.edu
\textsuperscript{2}University of Illinois at Chicago, vtamma2@uic.edu

In many applications such as data compression, imaging or genomic data analysis, it is important to approximate a given tensor by a tensor that is sparsely representable. For matrices, i.e. 2-tensors, such a representation can be obtained via the singular value decomposition, which allows to compute best rank $k$-approximations. For very big matrices a low rank approximation using SVD is not computationally feasible. In this case different approximations are available. It seems that variants of the CUR-decomposition are most suitable.

For $d$-mode tensors $T \in \otimes_{i=1}^{d} \mathbb{R}^{n_i}$, with $d > 2$, many generalizations of the singular value decomposition have been proposed to obtain low tensor rank decompositions. The most appropriate approximation seems to be best $(r_1, \ldots, r_d)$-approximation, which maximizes the $\ell_2$ norm of the projection of $T$ on $\otimes_{i=1}^{d} U_i$, where $U_i$ is an $r_i$-dimensional subspace $\mathbb{R}^{n_i}$. One of the most common methods is the alternating maximization method (AMM). It is obtained by maximizing on one subspace $U_i$, while keeping all other fixed, and alternating the procedure repeatedly for $i = 1, \ldots, d$. Usually, AMM will converge to a local best approximation. This approximation is a fixed point of a corresponding map on Grassmannians. We suggest a Newton method for finding the corresponding fixed point. We also discuss variants of CUR-approximation method for tensors. We compare numerically different approximation methods.

References

[1] S. Friedland and V. Tammali, Low rank approximation of tensors, arXiv:1410.6089.