Hierarchy grey relational analysis using DEA and AHP
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Abstract
Purpose – This paper aims to apply an integrated data envelopment analysis (DEA) and analytic hierarchy process (AHP) approach to a multi-hierarchy grey relational analysis (GRA) model. Consistent with the most real-life applications, the authors focus on a two-level hierarchy in which the attributes of similar characteristics can be grouped into categories. Nevertheless, the proposed approach can be easily extended to a three-level hierarchy in which attributes might also belong to different sub-categories and further be linked to categories.

Design/methodology/approach – The procedure of incorporating the DEA and AHP methods in a two-level GRA may be broken down into a series of steps. The first three steps are under the heading of attributes and the latter three steps are under the heading of categories as follows: computing the grey relational coefficients of attributes for each alternative using the basic GRA model which further provides the required (output) data for an additive DEA model; computing the priority weights of attributes and categories using the AHP method which provides a priori information on the adjustments of attributes and categories in additive DEA models; computing the grey relational grades of attributes in each category for alternatives using an additive DEA model; converting the grey relational grades of attributes to the grey relational coefficients of categories; computing the grey relational grades of categories for alternatives using an additive DEA model; computing the dissimilarity grades of categories for the tied alternatives using an additive DEA exclusion model.

Findings – The proposed approach provides a more reasonable and encompassing measure of performance in a hierarchy GRA, based on which the overall ranking position of alternatives is obtained. A case study of a wastewater treatment technology selection verifies the effectiveness of this approach.

Originality/value – This research is a step forward to overcome the current shortcomings in a hierarchy GRA by extracting the benefits from both the objective and subjective weighting methods.

Keywords Data envelopment analysis, Analytic hierarchy process, Grey relational analysis, Hierarchical structures, Weighing

Paper type Research paper

1. Introduction
Grey relational analysis (GRA) is a multi-attribute decision-making (MADM) tool that provides a single measure of performance for each alternative with respect to a set of incommensurate attributes. Nevertheless, the traditional GRA is only limited to the situations with a single level of attributes, which might not entirely satisfy the need for increasingly complex MADM problems. In real-world applications, there are a great number of MADM activities which not only need to be represented by a set of attributes, but these
attributes might also belong to different categories constituting a hierarchical structure. Figure 1 illustrates a complex MADM problem into a system of hierarchies in which a set of alternatives lies at the lowest level, and attributes, categories and the overall objective of the decision are on the higher levels of this hierarchy, respectively. For example, the problems of selecting wastewater treatment plants (Zeng et al., 2007), renewable energy generation technologies (Sarucan et al., 2011), natural gas pipeline operation schemes (Jia et al., 2011), coal-fired power plants (Xu et al., 2011), biomass briquette fuel system schemes (Wang et al., 2015), weapon equipment systems (Guoqing and Lin, 2015), call center sites (Birgun and Gungor, 2014), firms demanding commercial credits (Ertug and Girginer, 2015), advertising spokesmen (Hsu and Su, 2008) and stock investments (Li et al., 2010).

These studies use the analytic hierarchy process (AHP) in a multi-level GRA, known as hierarchy GRA. AHP is a subjective data-oriented procedure that determines the relative priorities of attributes based on the formal expressions of decision makers’ preferences (Saaty, 1987). The application of AHP not only overcomes the drawback of assigning uniform weights to each attribute by GRA, but also incorporates the effects of attribute (sub) categories in the performance of alternatives. However, since the introduction of AHP in 1980, it has been a target of criticism due to its subjective nature of producing weights (Swim, 2001; Dyer, 1990). Therefore, hierarchy GRA may not result in the best ranking position for each alternative in comparison to all the other alternatives. This flaw can be corrected by integrating data envelopment analysis (DEA) in hierarchy GRA. DEA is an objective data-oriented approach that allows each alternative (known as a decision-making unit in the DEA terminology) to choose its own favorable system of weights to optimize its relative performance (Cooper et al., 2011). This flexibility in selecting the weights, on the other hand, may be undesirable for some decision makers because it may place an alternative in the best ranking position for unlikely weight combinations. By noting the problematic contradiction between objective weights in DEA and subjective weights in AHP, this research is intended to develop an integrated DEA and AHP approach in a multi-level GRA framework. Therefore, it can provide more reasonable and encompassing results for ranking alternatives in GRA. The integration of both the DEA and AHP methods in a single-level GRA can be found in Pakkar (2016a, 2016b). Pakkar (2016a) explores the tradeoff relationship between the objective weights obtained by DEA and the

![Figure 1. The hierarchical structure of a complex MADM problem](image-url)
subjective weights obtained by AHP in a GRA methodology. This may result in various ranking positions for each alternative in comparison to the other alternatives. Pakkar (2016b) applies a pair of additive DEA models in a fuzzy multi-attribute GRA methodology to assess the overall performance of alternatives from both the optimistic and pessimistic perspectives. In this approach, the attribute weights obtained by additive DEA models are bounded by AHP. Nonetheless, as mentioned earlier, none of the proposed models consider the hierarchical structures of attributes. Simply treating all the attributes to be at the same level obviously ignores the hierarchical information and further leads up to invalid and unstable measures of performance assessment for alternatives. Therefore, the approach proposed in this research is a step forward to overcome the current shortcomings in a hierarchy GRA by extracting the benefits from both the objective and subjective weighting methods.

2. The proposed approach
As mentioned earlier, we focus on those MADM problems in which the attributes of similar characteristics can be grouped into different categories to construct a two-level hierarchy. The procedure of incorporating DEA and AHP in a two-level GRA may be broken down into the following steps (Figure 2).

2.1 Step 1: Computations at the level of attributes
- Computing the grey relational coefficients of attributes for each alternative using the basic GRA model which further provides the required (output) data for an additive DEA model;
- Computing the priority weights of attributes and categories using the AHP method which provides a priori information on the adjustments of attributes and categories in additive DEA models; and
- Computing the grey relational grades of attributes in each category for alternatives using an additive DEA model.

2.2 Step 2: Computations at the category level
- Converting the grey relational grades of attributes to the grey relational coefficients of categories;
- Computing the grey relational grades of categories for alternatives using an additive DEA model; and
- Computing the dissimilarity grades of categories for the tied alternatives using an additive DEA exclusion model.

Note that the idea of the two-level hierarchy is consistent with the most real-world applications. Nevertheless, the proposed approach can be easily extended to a three-level hierarchy in which attributes might also belong to different sub-categories and further be linked to categories (Appendix 1).

2.3 Basic grey relational analysis
Let \( y_{ik} \) be the value of attribute \( C_k \) \((k = 1, 2, \ldots, n)\) for alternative \( A_i \) \((i = 1, 2, \ldots, m)\) in an MADM problem. The term \( y_{ik} \) can be translated into the comparability value \( r_{ik} \) by using the following equations:
Figure 2. The flowchart of a two-level hierarchy GRA using DEA and AHP.
\[ r_{ik} = \frac{y_{ik}}{y_{k(\text{max})}} \forall i, k \text{ for desirable attributes} \quad (1) \]

\[ r_{ik} = \frac{y_{k(\text{min})}}{y_{ik}} \forall i, k \text{ for undesirable attributes} \quad (2) \]

where \( y_{k(\text{max})} = \max \{y_{1k}, y_{2k}, \ldots, y_{mk}\} \) and \( y_{k(\text{min})} = \min \{y_{1k}, y_{2k}, \ldots, y_{mk}\} \). Note that desirable attributes satisfy the property of “the larger the better” and undesirable attributes satisfy the property of “the smaller the better”. To eliminate the scale differences between all attributes, and moreover, ensure that all of them are in the same direction of change, equations (1) and (2) are used.

Now, let \( u_{0k} \) be the reference value for an ideal alternative, \( A_0 \), as follows:

\[ u_{0k} = \max \{r_{1k}, r_{2k}, \ldots, r_{mk}\} \quad (3) \]

Then the ideal alternative, \( A_0 \), can be defined as a virtual alternative which is characterized by a reference sequence of the maximum values of all attributes. To measure the degree of similarity of alternative \( A_i \) to the ideal alternative \( A_0 \), with respect to each attribute, the grey relational coefficient, \( \xi_{ik} \) (a distance function), can be calculated as follows:

\[ \xi_{ik} = \frac{\min \min_k |u_{0k} - r_{ik}| + \rho \max \max_k |u_{0k} - r_{ik}|}{|u_{0k} - r_{ik}| + \rho \max \max_k |u_{0k} - r_{ik}|} \quad (4) \]

where \( |u_{0k} - r_{ik}| \) represents the absolute deviation of each alternative from the ideal alternative with respect to a particular attribute. Obviously, \( \xi_{ik} \) decreases when \( |u_{0k} - r_{ik}| \) increases and \( \xi_{ik} \) increases when \( |u_{0k} - r_{ik}| \) decreases. \( \min \min_k |u_{0k} - r_{ik}| \) and \( \max \max_k |u_{0k} - r_{ik}| \) are the minimum and maximum absolute deviations among all alternatives with respect to all attributes. \( \rho \in [0, 1] \) is the distinguishing coefficient, which adjusts the range of the grey relational coefficient. The smaller the \( \rho \) is, the greater is its distinguishing power. Generally it is taken as 0.5.

To find an aggregated measure of similarity of alternative \( A_i \) to the ideal alternative \( A_0 \), over all the attributes, the grey relational grade, \( \Gamma_i \), can be computed as follows:

\[ \Gamma_i = \sum_{k=1}^{n} w_k \xi_{ik} \quad (5) \]

Where \( w_k \) is the weight of attribute \( C_k \) and \( \sum_{k=1}^{n} w_k = 1 \). In practice, expert judgments using AHP are often used to obtain the weights of attributes. When such information is unavailable, equal weights seem to be a norm. In the next section, we show how the hierarchical structures of attributes can be incorporated in a traditional GRA method to constitute a two-level hierarchy in GRA.

### 2.4 Two-level grey relational analysis

The computational structure of a two-level GRA is illustrated in Figure 3. Suppose \( y_{ijk} \) is the value of attribute \( C_{jk} \) \((k = p, p + 1, \ldots, q)\) in category \( C_j (j = 1, 2, \ldots, n')\) for alternative \( A_i (i = 1, 2, \ldots, m) \) while \( 1 \leq p \leq q \leq n \). Using equations (1)-(5), the grey relational grade of attributes in category \( C_j \) for alternative \( A_i \), denoted as \( \Gamma_{ij} \), can be computed as follows:
\[
\Gamma_{ij} = \sum_{k=p}^{q} w_{jk} \xi_{ijk}
\]

where \(\xi_{ijk}\) and \(w_{jk}\) are the grey relational coefficient and the weight of attribute \(C_{jk}\) in category \(C_i^j\) for alternative \(A_i\).

Again, using equations (3)-(5) on \(\Gamma_{ij}\), the grey relational grade of categories for alternative \(A_i\), denoted as \(\Gamma_i^\prime\), is obtained as follows:

\[
\Gamma_i^\prime = \sum_{j=1}^{n} w_j \xi_{ij}
\]

**2.5 The analytic hierarchy process**

The AHP procedure for computing the priority weights of attributes and their categories may be broken down into the following steps:

**Step 1**: A decision maker makes a pairwise comparison matrix of different attributes of each category, denoted by \(B\) with the entries of \(b_{jk} (k = k' = p, p + 1, \ldots, q)\) while \(1 \leq p \leq q \leq n\). The comparative importance of attributes is provided by the decision maker using a rating scale. Saaty (1987) recommends using a 1-9 scale. In a similar way, a pairwise comparison matrix can be made to compare the importance of each category. This matrix is denoted by \(D\) with the entries of \(d_{jj} (j = j' = 1, 2, \ldots, n)\).

**Step 2**: The AHP method obtains the priority weights of attributes of each category by computing the eigenvector of matrix \(B\) (equation (8)), \(W_j = (w_{jp}, w_{jp+1}, \ldots, w_{jq})^T\), which is related to the largest eigenvalue, \(\gamma_{\text{max}}\):

\[
BW_j = \gamma_{\text{max}} W_j
\]

In a similar way, the priority weights of each category are obtained by computing the eigenvector of matrix \(D\) (equation (9)), \(W = (w_1, w_2, \ldots, w_n)^T\), which is related to the largest eigenvalue, \(\gamma_{\text{max}}\):

\[
DW = \gamma_{\text{max}} W
\]

To determine whether the inconsistency in a comparison matrix is reasonable, the random consistency ratio, \(C.R\), can be computed by the following equation:

\[
C.R = \frac{\gamma_{\text{max}} - N}{(N - 1)R.I}
\]

where \(R.I\) is the average random consistency index and \(N\) is the size of a comparison matrix.
2.6 Additive DEA models

To compute the grey relational grade of attributes in a particular category for each alternative, an additive DEA model can be developed in which all the grey relational coefficients, \( \xi_{ijk} \), are treated as outputs. This model is similar to the additive model in Cooper et al. (1999) without explicit inputs as follows:

\[
P_{oj} = \max \sum_{k=p}^{q} w_{jk} s_{jk} \\
\text{s.t.} \sum_{i=1}^{m} \lambda_{ij} \xi_{ijk} - s_{jk} = \xi_{oij} \quad \forall k, \\
\sum_{i=1}^{m} \lambda_{ij} = 1 \\
s_{jk}, \lambda_{ij} \geq 0,
\]

while \( 0 \leq P_{oj} \leq 1 \), and \( 1 - P_{oj} \) indicates the grey relational grade, \( \Gamma_{oj} (o = 1,2,\ldots, m, j = 1, 2, \ldots, n') \), of attributes in category \( C'_j \) for alternative under assessment \( A_o \) (known as a decision-making unit in the DEA terminology). \( s_{jk} \) is the slack variable of attribute \( C_{jk} (k = p, p + 1,\ldots, q) \) in category \( C_j \), expressing the difference between the performance of a composite alternative and the performance of the assessed alternative with respect to each attribute. In other words, \( s_{jk} \) identifies a shortfall in the attribute value of \( C_{jk} \) of category \( C'_j \) for alternative \( A_o \). Obviously, when \( P_{oj} = 0 \), alternative \( A_o \) is considered as the best alternative in comparison with all the other alternatives in category \( C'_j \). \( w_{jk} \) is the priority weight of attribute \( C_j \) of category \( C'_j \), which is defined out of the internal mechanism of DEA using AHP, and \( \lambda_{ij} \) is the weight of alternative \( A_i (i = 1, 2, \ldots, m) \) in category \( C'_j \). The convexity constraint in Model (11) meets the assumption of variable returns-to-scale frontier for an additive model. Similarly, we can develop a model to obtain the grey relational grade of categories for each alternative as follows:

\[
P_o = \max \sum_{j=1}^{n'} \sum_{i=1}^{m} w_{ij} s_{j} \\
\text{s.t.} \sum_{i=1}^{m} \lambda_{ij} \xi_{ij} - s_{j} = \xi_{oj} \quad \forall j, \\
\sum_{i=1}^{m} \lambda_{ij} = 1 \\
s_{j}, \lambda_{ij} \geq 0,
\]

while \( 0 \leq P_o \leq 1 \), and \( 1 - P_o \) indicates the grey relational grade, \( \Gamma'_o (o = 1,2,\ldots, m) \), of categories for alternative under assessment \( A_o \). \( s_{j} \) is the slack variable of category \( C'_j \), \( w_{ij} \) is the priority weight of category \( C'_j \), obtained by AHP, and \( \lambda_{ij} \) is the weight of alternative \( A_i (i = 1, 2, \ldots, m) \). One should notice that the additive DEA models bounded by AHP does not necessarily yield results that are different from those obtained from the original additive DEA models (Charnes et al., 1985). In particular, it does not increase the power of discrimination between the considerable number of alternatives which form the best practice-frontier. The alternatives on this frontier are usually ranked in the first place by obtaining the grey relational grades of 1. To eliminate the ties that occur for the best alternatives, we propose model (13) that is similar to the additive DEA exclusion (or super-efficiency) model in Du et al. (2010) without explicit inputs:
\[ \alpha_o = \min \sum_{j=1}^{n} w_j t_j \]

\[ s.t. \sum_{i=1,i \neq o}^{m} \lambda_i x_{ij} \geq x_{oj} - t_j \quad \forall j, \]

\[ \sum_{i=1,i \neq o}^{m} \lambda_i = 1 \]

\[ t_j, \lambda_i \geq 0, \]

(13)

After removing alternative \( A_o \) from the best practice frontier of model (12), we need to decrease the grey relational coefficients of categories for alternative \( A_o \) to reach the frontier constructed by the remaining alternatives. Note that the value of objective function, \( \alpha_o \), can be considered as a dissimilarity grade between alternative \( A_o \) and the remaining alternatives. \( t_j \) is a slack variable representing a decrease in the grey relational coefficient of category \( C_j \) for alternative \( A_o \) to reach the frontier.

### 3. Case study

In this section, we present the application of the proposed approach to assess the performance of four wastewater treatment technology alternatives: anaerobic/anoxic/oxic (\( A_1 \)), triple oxidation ditch (\( A_2 \)), anaerobic single oxidation ditch (\( A_3 \)) and

| Goal | Categories | Attributes | 1 | 2 | 3 | 4 |
|------|------------|------------|---|---|---|---|
| C’1 economic category | \( C_{11} \) capital cost (\( \times 10^4 \) RMB) | 13,762 | 12,080 | 12,375 | 11,870 |
| | \( C_{12} \) O\&M cost (\( \times 10^4 \) RMB) | 7,612 | 8,747 | 8,126 | 8,233 |
| | \( C_{13} \) land area (\( \times 10^4 \) m\(^2\)) | 9.88 | 11.78 | 11.93 | 9 |
| | \( C_{24} \) removal efficiency of nitrous and phosphorous pollutants | G (0.7) | M (0.5) | E (0.9) | M (0.5) |
| Wastewater treatment technology selection | \( C_{25} \) sludge disposal effect | P (0.3) | G (0.7) | G (0.7) | P (0.3) |
| | \( C_{26} \) stability of plant operation | G (0.7) | E (0.9) | E (0.9) | G (0.7) |
| | \( C_{27} \) maturity of technology | E (0.9) | G (0.7) | G (0.7) | P (0.3) |
| | \( C_{38} \) professional skills required for operation and maintenance | M (0.5) | E (0.9) | G (0.7) | M (0.5) |

| Linguistic values | Quantity |
|-------------------|----------|
| Excellent (E) | 0.9 |
| Good (G) | 0.7 |
| Moderate (M) | 0.5 |
| Poor (P) | 0.3 |
| Very Poor (VP) | 0.1 |

Table I. Data for wastewater treatment technology selection

Table II. Linguistic values scale
sequencing batch reactor (A₄) with respect to eight attributes which are grouped into three attribute categories. Table I presents the required data as adopted from Zeng et al. (2007). Note that some attributes are provided by the numerical values and some are by the quantification of the linguistic values of experienced decision makers based on Table II (Zeng et al., 2007). Capital cost, operation and maintenance (O & M) cost and land area are undesirable attributes while the other attributes are desirable. These data are turned into the comparability sequence by using equations (2) and (3) as presented in Table III. Using Equation (4), all grey relational coefficients for attributes are

| Categories | Attributes | Alternative technologies |
|------------|------------|--------------------------|
|            |            | A₀ | A₁ | A₂ | A₃ | A₄ |
| C₁         | C₁₁        | 1  | 0.86 | 0.98 | 0.96 | 1  |
|            | C₁₂        | 1  | 1   | 0.87 | 0.94 | 0.93|
|            | C₁₃        | 1  | 0.91 | 0.76 | 0.75 | 1  |
|            | C₁₄        | 1  | 0.78 | 0.56 | 1   | 0.56|
|            | C₁₅        | 1  | 0.43 | 1   | 1   | 0.43|
|            | C₁₆        | 1  | 0.78 | 1   | 1   | 0.78|
|            | C₁₇        | 1  | 1   | 0.78 | 0.78 | 0.33| C₃     | C₁₈        | 1  | 0.56 | 1   | 0.78 | 0.56|

Table III.
Results of grey relational generation for wastewater treatment technology selection

| Categories | Attributes | Alternative technologies |
|------------|------------|--------------------------|
|            |            | A₁ | A₂ | A₃ | A₄ |
| C₁         | C₁₁        | 0.705 | 0.944 | 0.893 | 1 |
|            | C₁₂        | 1   | 0.72 | 0.848 | 0.827|
|            | C₁₃        | 0.788 | 0.583 | 0.573 | 1 |
|            | C₁₄        | 0.604 | 0.432 | 1   | 0.432|
|            | C₁₅        | 0.37 | 1   | 1   | 0.37|
|            | C₁₆        | 0.604 | 1   | 1   | 0.604|
|            | C₁₇        | 1   | 0.604 | 0.604 | 0.333|
| C₃         | C₁₈        | 0.432 | 1   | 0.604 | 0.432|

Table IV.
Results of grey relational coefficients for attributes

| Goal | Categories | Weights | Attributes | Weights |
|------|------------|---------|------------|---------|
| Prioritizing attributes and categories | C¹ economic category | 0.6371 | C₁₁ capital cost | 0.6371 |
| | | | C₁₂ O&M cost | 0.1052|
| | | | C₁₃ land area | 0.2581|
| C² technical category | 0.2581 | C₂₃ removal efficiency of nitrous and phosphorous pollutants | 0.2271|
| | | | C₂₅ sludge disposal effect | 0.1904|
| | | | C₂₆ stability of plant operation | 0.2483|
| | | | C₂₇ maturity of technology | 0.3345|
| | | | C₂₈ professional skills required for operation and maintenance | 1|
| C³ administrative category | 0.1052 | C₃₉ professional skills required for operation and maintenance | 1|

Table V.
The priority weights of attributes and categories obtained by AHP
computed to provide the required (output) data for the additive DEA model (11) as shown in Table IV.

Note that grey relational coefficients depend on the distinguishing coefficient $\rho$, which here is 0.50. Table V depicts the hierarchical structure of attributes for wastewater treatment technologies and the corresponding priority weights in the AHP model as constructed by Zeng et al. (2007).

For the attributes and categories shown in Table V, four comparison matrices need to be elicited from the decision maker–three for computing the weights of attributes with respect

### Table VI. Pairwise comparison matrix at category level

| Categories | $C_1$ | $C_2$ | $C_3$ | Weights |
|------------|-------|-------|-------|---------|
| $C_1$      | 1     | 3     | 5     | 0.6371  |
| $C_2$      | 1/3   | 1     | 3     | 0.2581  |
| $C_3$      | 1/5   | 1/3   | 1     | 0.1052  |

**Notes:** $\gamma_{\text{max}} = 3.0379; \ C.R = 0.0327$

### Table VII. Grey relational grades with respect to each category

| Categories | $A_1$ | $A_2$ | $A_3$ | $A_4$ |
|------------|-------|-------|-------|-------|
| $C_1$      | 1.000 | 0.648 | 0.656 | 1.000 |
| $C_2$      | 1.000 | 0.688 | 1.000 | 0.393 |
| $C_3$      | 0.333 | 1.000 | 0.418 | 0.333 |

### Table VIII. Grey relational coefficients with respect to each category

| Categories | $A_1$ | $A_2$ | $A_3$ | $A_4$ |
|------------|-------|-------|-------|-------|
| $C_1$      | $\Gamma_1$ = 1 | 1 | 0.845 | 0.851 | 1 |
| $C_2$      | 1 | 0.871 | 1 | 0.562 |
| $C_3$      | 0.432 | 1 | 0.604 | 0.432 |

### Table IX. Overall grey relational grades for alternatives

| Alternatives | $P_o$ | $\Gamma_o = 1 - P_o$ | Rank |
|--------------|-------|-----------------------|------|
| $A_1$        | 0     | 1                     | 1    |
| $A_2$        | 0     | 1                     | 1    |
| $A_3$        | 0     | 1                     | 1    |
| $A_4$        | 0.157 | 0.843                 | 4    |

### Table X. Dissimilarity grades for alternatives using additive DEA exclusion

| Alternatives | $\alpha$ values | Rank |
|--------------|-----------------|------|
| $A_1$        | 0.219           | 2    |
| $A_2$        | 0.656           | 3    |
| $A_3$        | 0.009           | 1    |
to each category and one for estimating the priority weights of categories with respect to the problem goal. Table VI shows the results of the pairwise comparison matrix at the category level with respect to the goal which further are used in the additive DEA model (12) and the additive DEA exclusion model (13).

Obtaining the results of grey relational coefficients and the priority weights of attributes, the additive DEA model (11) can be run. Table VII shows the results of running model (11) that computes the grey relational grade of attributes in each category for the alternative under assessment.

Again, using equation (4), the grey relational grades of each category are turned into the grey relational coefficients for that category as shown in Table VIII.

The overall grey relational grade for the alternative under assessment is obtained from the additive DEA model (12) as shown in Table IX. Since alternatives $A_1$, $A_2$ and $A_3$ are placed in the best ranking positions, the additive DEA exclusion model (13) is run to create a unique rank order among these alternatives. As indicated in Table X, the three alternatives $A_1$, $A_2$ and $A_3$ are ranked 2, 3 and 1, based on the minimum grade of dissimilarity, respectively. Therefore, the anaerobic single oxidation ditch ($A_3$) is selected as the optimal alternative for the studied municipal wastewater treatment technologies.

4. Conclusions
In many MADM cases, it makes sense to group attributes hierarchically, while different weights may be assigned to different attributes and their own categories to reflect their relative priorities. The standard GRA model is not able to reflect such hierarchical structures, as they assume that all the attributes use the same weights. To cope with this problem, scholars have adopted the application of AHP in GRA, known as hierarchy GRA, where attributes are constructed hierarchically and different weights can be used at different levels. However, the subjective process of producing weights in AHP may not place each alternative in its best light in comparison with all the other alternatives. To overcome this issue, we integrate the two variants of DEA models in hierarchy GRA. Since we use both the DEA and AHP methods in a multi-level GRA framework, more reasonable and encompassing results can be provided for assessing the performance of alternatives. Finally, the usefulness of the proposed approach is demonstrated using a real case study of the hierarchy system of wastewater treatment technology selection.

References
Birgun, S. and Gungor, C. (2014), “A multi-criteria call center site selection by hierarchy grey relational analysis”, *Journal of Aeronautics and Space Technologies*, Vol. 7 No. 1, pp. 45-52.
Charnes, A., Cooper, W.W., Golany, B., Seiford, L. and Stutz, J. (1985), “Foundations of data envelopment analysis for pareto-koopmans efficient empirical production functions”, *Journal of Econometrics*, Vol. 30 No. 1, pp. 91-107.
Cooper, W.W., Park, K.S. and Pastor, J.T. (1999), “RAM: a range adjusted measure of inefficiency for use with additive models, and relations to other models and measures in DEA”, *Journal of Productivity Analysis*, Vol. 11 No. 1, pp. 5-42.
Cooper, W.W., Seiford, L.M. and Zhu, J. (Eds) (2011), *Handbook on Data Envelopment Analysis*, Vol. 164, Springer Science & Business Media, Berlin.
Du, J., Liang, L. and Zhu, J. (2010), “A slacks-based measure of super-efficiency in data envelopment analysis: a comment”, *European Journal of Operational Research*, Vol. 204 No. 3, pp. 694-697.
Dyer, J.S. (1990), “Remarks on the analytic hierarchy process”, *Management Science*, Vol. 36 No. 3, pp. 249-258.
Ertuğ, Z.K. and Girginer, N. (2015), “Evaluation of banks’ commercial credit applications using the analytic hierarchy process and grey relational analysis: a comparison between public and private banks”, South African Journal of Economic and Management Sciences, Vol. 18 No. 3, pp. 308-324.

Guoqing, H. and Lin, S. (2015), “Effectiveness evaluation analysis of weapon equipment system based on weighted gray relational degree”, Information Engineering, Vol. 4, pp. 33-37.

Hsu, P.F. and Su, Y.H. (2008), “Optimal selection of advertising spokesman using an analytic hierarchy process and grey relational analysis”, Journal of Information and Optimization Sciences, Vol. 29 No. 6, pp. 1067-1083.

Jia, W., Li, C. and Wu, X. (2011), “Application of multi-hierarchy grey relational analysis to evaluating natural gas pipeline operation schemes”, in Shen G. and Huang, X. (Eds), Advanced Research on Computer Science and Information Engineering, Springer, Berlin, Heidelberg, Vol. 152, pp. 245-251.

Li, H.Y., Zhang, C. and Zhao, D. (2010), “Stock investment value analysis model based on AHP and gray relational degree”, Management Science and Engineering, Vol. 4 No. 4, pp. 1-6.

Pakkar, M.S. (2016a), “Multiple attribute grey relational analysis using DEA and AHP”, Complex & Intelligent Systems, Vol. 2 No. 4, pp. 243-250.

Pakkar, M.S. (2016b), “An integrated approach to grey relational analysis, analytic hierarchy process and data envelopment analysis”, Journal of Centrum Cathedra, Vol. 9 No. 1, pp. 71-86.

Saaty, R.W. (1987), “The analytic hierarchy process-what it is and how it is used”, Mathematical Modelling, Vol. 9 No. 3, pp. 161-176.

Sarucan, A., Baysal, M.E., Kahraman, C. and Engin, O. (2011), “A hierarchy grey relational analysis for selecting the renewable electricity generation technologies”, Lecture Notes in Engineering and Computer Science, Vol. 2191 No. 1, pp. 1149-1154.

Swim, L.K. (2001), “Improving decision quality in the analytic hierarchy process implementation through knowledge management strategies”, Ph.D., The University of Oklahoma.

Wang, Z., Lei, T., Chang, X., Shi, X., Xiao, J., Li, Z., He, X., Zhu, J. and Yang, S. (2015), “Optimization of a biomass briquette fuel system based on grey relational analysis and analytic hierarchy process: a study using cornstalks in china”, Applied Energy, Vol. 157 No. 1, pp. 523-532.

Xu, G., Yang, Y.P., Lu, S.Y., Li, L. and Song, X. (2011), “Comprehensive evaluation of coal-fired power plants based on grey relational analysis and analytic hierarchy process”, Energy Policy, Vol. 39 No. 5, pp. 2343-2351.

Zeng, G., Jiang, R., Huang, G., Xu, M. and Li, J. (2007), “Optimization of wastewater treatment alternative selection by hierarchy grey relational analysis”, Journal of Environmental Management, Vol. 82 No. 2, pp. 250-259.
Figure A1.
The flowchart of a three-level hierarchy GRA using DEA and AHP.
Appendix 2. The glossary of modeling symbols

$y_{jk}$ is the value of attribute $C_k$ ($k = 1, 2, \ldots, n$) for alternative $A_i$ ($i = 1, 2, \ldots, m$).

$r_{ik}$ is the comparability value of attribute $C_k$ for alternative $A_i$.

$y_{(\text{max})}$ is the maximum value of attribute $C_k$.

$y_{(\text{min})}$ is the minimum value of attribute $C_k$ in a basic GRA model.

$u_{ok}$ is the reference value for a virtual ideal alternative, $A_o$.

$j_{ik}$ is the grey relational coefficient of attribute $C_k$ for alternative $A_i$.

$r$ is the distinguishing coefficient.

$U_i$ is the grey relational grade for alternative $A_i$ in a basic GRA model.

$w_{jk}$ is the weight of attribute $C_k$ in a basic GRA model.

$y_{ijk}$ is the value of attribute $C_{jk}$ ($k = p, p + 1, \ldots q$) in category $C_{0j}$ ($j = 1, 2, \ldots, n'$) for alternative $A_i$ while $1 \leq p \leq q \leq n$.

$\xi_{ijk}$ is the grey relational coefficient of attribute $C_{jk}$ in category $C_{0j}$ for alternative $A_i$.

$U_{ij}$ is the grey relational grade of attributes in category $C_{0j}$ for alternative $A_i$.

$j_{ij}$ is the grey relational coefficient of category $C_{0j}$ for alternative $A_i$.

$w_j$ is the weight of category $C_{0j}$ obtained by AHP.

$C_{0i}$ is the grey relational grade of categories for alternative $A_i$.

$b_{jk'}$ is the $k - k' (k = p, p + 1, \ldots q)$ element of the pairwise comparison matrix for attributes, denoted by $B$, with respect to category $C_{0j}$.

$d_{j'}$ is the $j - j' (j = j' = 1, 2, \ldots, n')$ element of the pairwise comparison matrix for categories, denoted by $D$, with respect to the problem goal.

$\gamma_{\text{max}}$ is the largest eigenvalue.

$R/I$ is the average random consistency index.

$N$ is the size of a comparison matrix.

$C.R$ is the random consistency ratio.

$1 - P_{oj}$ is the grey relational grade, $\Gamma_{oj}$ ($o = 1, 2, \ldots, m, j = 1, 2, \ldots, n'$), of attributes in category $C_{0j}$ for alternative under assessment $A_o$.

$S_{oj}$ is the slack variable of attribute $C_{jk} (k = p, p + 1, \ldots q)$ in category $C_{0j}$.

$\lambda_{oj}$ is the weight of alternative $A_i$ in category $C_{0j}$.

$1 - P_{o}$ is the grey relational grade, $\Gamma_{o}$ ($o = 1, 2, \ldots, m$), of categories for alternative under assessment $A_o$.

$S_j$ is the slack variable of category $C_{0j}$.

$\lambda_j$ is the weight of alternative $A_i$ ($i = 1, 2, \ldots, m$).

$\alpha_o$ is a dissimilarity grade between alternative $A_o$ and the remaining alternatives in the additive DEA exclusion (or super-efficiency) model.

$t_j$ is a slack variable of category $C_{0j}$ in the additive DEA exclusion model.

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