Novel magnetoinductance effects in Josephson Junction Arrays:
A single-plaquette approximation

Sergei Sergeenko
Departamento de Física, CCEN, Universidade Federal da Paraíba,
Cidade Universitária, 58051-970 João Pessoa, PB, Brazil
(Dated: February 2, 2008)

Abstract

Using a single-plaquette approximation, novel magnetoinductance effects in Josephson junction arrays (JJAs) are predicted, including the appearance of steps in the temperature behavior of magnetic susceptibility. The number of steps (as well as their size) is controlled by the kinetic inductance of the plaquette whose field dependence is governed by the Abrikosov vortices penetrating superconducting regions of the array. The experimental conditions under which the predicted effects should manifest themselves in artificially prepared JJAs are discussed.

PACS: 74.25.Ha; 74.50.+r; 74.80.-g
INTRODUCTION

Many unusual magnetic properties of Josephson junctions (JJs) and their arrays (JJAs) continue to attract attention of both theoreticians and experimentalists (for recent reviews on the subject see, e.g. [1-5] and further references therein). In particular, among the numerous spectacular phenomena recently discussed and observed in JJAs we would like to mention the dynamic temperature reentrance of AC susceptibility [2] (closely related to paramagnetic Meissner effect [3,4]) and avalanche-like magnetic field behavior of magnetization [5,6] (well described by the theory of self-organized criticality [7]). It is also worth mentioning the recently observed geometric quantization [8] and flux induced oscillations of heat capacity [9] in artificially prepared JJAs as well as recently predicted flux driven temperature oscillations of thermal expansion coefficient [10] both in JJs and JJAs. At the same time, successful adaptation of the so-called two-coil mutual-inductance technique to impedance measurements in JJAs provided a high-precision tool for investigation of the numerous magnetoinductance (MI) related effects in Josephson networks [11-14]. To give just a few recent examples, suffice it to mention the MI measurements [12] on the JJAs of periodically repeated Sierpinski gaskets which have clearly demonstrated the appearance of fractal and Euclidean regimes for non-integer values of the frustration parameter, and theoretical predictions [13] regarding a field-dependent correction to the sheet inductance of the proximity JJA with frozen vortex diffusion. Besides, recently [14] AC magnetoimpedance measurements performed on proximity-effect coupled JJA on a dice lattice revealed unconventional behaviour resulting from the interplay between the frustration $f$ created by the applied magnetic field and the particular geometry of the system.

By analyzing the influence of MI on magnetization of SIS-type JJAs in the mixed Abrikosov state within a single-plaquette approximation, in the present paper we predict yet another interesting effect: the appearance of temperature steps in the behavior of susceptibility due to vortices induced magnetic field dependence of the kinetic inductance of the array. We also discuss the conditions under which the predicted effects can be observed experimentally in artificially prepared ordered arrays of unshunted junctions.
THE MODEL

Strictly speaking, to study the influence of MI effects on temperature behavior of magnetization and susceptibility in realistic arrays, one would need to analyze in detail the flux dynamics in these arrays. However, assuming a well-defined periodic structure of the array (which is actually the case in most experiments [2,6,8,12,15]), to achieve our goal it is sufficient to study just a single unit cell (plaquette) of the array. (It is worth noting that the single-plaquette approximation has proved successful in treating the temperature reentrance phenomena of AC susceptibility in ordered 2D-JJA [2,8,15].) Recall that the unit cell is a loop containing four identical Josephson junctions. If we apply an external magnetic field \( B \) normally to the plaquette, then the total magnetic flux \( \Phi(B) \) threading the four-junction superconducting loop is given by \( \Phi(B) = BS + L(T, B)I(B) \) where \( L(T, B) \) is the loop magnetoinductance (see below), \( S \) the projected area of the loop, and the circulating current in the loop reads \( I(B) = I_C(T)\sin(\phi(B)) \), where \( \phi(B) = \frac{\pi}{2}[n + \Phi(B)/\Phi_0] \) is the gauge-invariant superconducting phase difference across the \( i \)th junction (\( n \) is an integer and by symmetry we assume that \( \phi_1 = \phi_2 = \phi_3 = \phi_4 \equiv \phi \)), \( I_C(T) \) is the field-independent critical current of the junction, and \( \Phi_0 \) is the magnetic flux quantum.

In turn, the loop magnetoinductance reads \( L(T, B) = L_g + L_k(T, B) \) where \( L_g = \mu_0a \) is the geometric contribution with \( a \) being the perimeter of the loop, and \( L_k(T, B) = \mu_0\lambda^2(T, B)/a \) is the so-called kinetic contribution with \( \lambda(T, B) \) being a properly defined London penetration depth. In what follows, we shall assume that the array is in the mixed Abrikosov state which means that the field dependence of \( L_k \) is due to vortices penetrating superconducting regions of the array. More specifically, for \( B_{c2} \gg B > B_{c1} \), the kinetic magnetoinductance follows the linear field dependence (dictated by the corresponding dependence of the penetration depth [16-18]) \( L_k(T, B) - L_k(T, 0) \propto B \).

As usual, the net magnetization and susceptibility of the plaquette are given by

\[
M(T, B) = -\frac{1}{V} \left( \frac{\partial \mathcal{H}}{\partial B} \right)
\]

and

\[
\chi(T, B) = \frac{\partial M}{\partial B}
\]

respectively, where

\[
\mathcal{H} = J(T)[1 - \cos(\phi(B))] + \frac{\Phi^2}{2L(T, B)}
\]
is the Hamiltonian of the system which describes the tunneling (first term) and magnetoinductive (second term) contributions to the total energy of the plaquette. Here, \( J(T) = \left( \Phi_0 / 2\pi \right) I_C(T) \) is the Josephson coupling energy, and \( V \) the sample’s volume.

**RESULTS AND DISCUSSION**

To properly address the influence of true MI effects on magnetic properties of the array, in what follows we assume that for all applied fields \( L(T, B) I(B) \gg BS \). In this approximation, the net susceptibility of the plaquette will depend on applied magnetic field only via the universal parameter \( \beta_L(T, B) = 2\pi I_C(T) L(T, B) / \Phi_0 \), namely

\[
\chi(T, B) \simeq -\chi_0(T) \left( \frac{\partial \beta_L}{\partial f} \right)^2 \left( \cos \beta_L - \frac{1}{2} \beta_L \right)
\]

(4)

where \( \chi_0(T) = I_C(T) S^2 / (\pi \Phi_0 V) \), and \( f = BS / \Phi_0 \) is the frustration parameter.

Let us analyze the obtained results. Figure 1 shows the behavior of the normalized susceptibility \( \chi_L(T, B) \equiv \chi(T, B) / [\chi_0(T)(\partial \beta_L / \partial f)^2] \) as a function of \( \beta_L(T, B) \) according to Eq. (4). Notice the appearance of characteristic minima (steps). They correspond to the number of flux quanta that can be screened by the critical currents in single plaquette.

To further discuss the predicted behavior of susceptibility, we need to specify the explicit temperature and field dependencies of \( \beta_L(T, B) \). As usual [8,10], for the explicit temperature dependence of the Josephson critical current

\[
I_C(T) = I_C(0) \left( \frac{\Delta(T)}{\Delta(0)} \right) \tanh \left( \frac{\Delta(T)}{2k_B T} \right)
\]

(5)

we will use the analytical approximation of the gap parameter (valid for all temperatures) [19], \( \Delta(T) = \Delta(0) \tanh \left( 2.2 \sqrt{\frac{L(T)}{T}} \right) \) with \( \Delta(0) = 1.76k_B T_C \).

At the same time, as was mentioned before, the temperature and field dependencies of the total inductance \( L(T, B) \) are governed by the vortices driven contribution to the kinetic inductance [16-18], that is \( L_k(T, B) = L_k(T, 0)[1 + B / B_{c1}(T)] \) with \( L_k(T, 0) = \mu_0 \lambda^2(T, 0)/a = L_k(0, 0)/(1 - T^2/T_C^2) \) and \( B_{c1}(T) = B_{c1}(0)(1 - T^2/T_C^2) \) assuming a two-fluid model expression for the temperature dependence of the penetration depth [10].

Figure 2 shows the temperature dependence of the normalized MI induced susceptibility \( \chi(T, B) / \chi_0(0) \) for different values of \( \beta_L(0, B) \). Similar to Figure 1, we see the appearance of
FIG. 1: The normalized magnetoinductance induced contribution to susceptibility of a single plaquette $\chi_L(T, B)$ as a function of the universal parameter $\beta_L(T, B)$, according to Eq.(4).

well-developed flux-induced temperature steps at some discrete values of the critical parameter $\beta_L(0, B)$. The analysis of Eqs. (4) and (5) near $T_C$ reveals that the number of steps $n(B)$, their length $\Delta T(B)$ and height $\Delta \chi(B)$ depend on the value of $\beta_L(0, B)$ as follows

$$n(B) = \frac{2}{\pi} \beta_L(0, B),$$

$$\Delta T(B) = \left[ \frac{\pi}{\beta_L(0, B)} \right] T_C,$$

and

$$\Delta \chi(B) = \pi \left[ \frac{\partial}{\partial f} \left( \frac{2\pi}{\sqrt{\beta_L(0, B)}} \right) \right]^2 \chi_0(0)$$

(8)
FIG. 2: Theoretically predicted dependence of the normalized susceptibility on reduced temperature for discrete values of the universal parameter $\beta_L(0, B)$ (from bottom to top): $\beta_L(0, B) = \pi, 2\pi, 4\pi,$ and $6\pi$.

These analytical expressions confirm the predicted correlation (seen in Fig.2) between increase of the number of steps (as well as decrease of their size) and magnetoinductance parameter $L(0, B) \propto \beta_L(0, B)$.

To test experimentally the predicted here effects, two-dimensional arrays of unshunted $Nb-AlO_x-Nb$ Josephson junctions with the following working parameters [8,15] can be used: lattice spacing $a = 46\mu m$ (loop area $S = a^2$), critical current $I_C(4.2K) \simeq 150\mu A$ for each junction as well as geometric inductance of the plaquette $L_g = \mu_0a \simeq 64pH$, producing $\beta_L(0, 0) \simeq 30$.

In summary, the influence of magnetoinductance effects on the temperature behavior of magnetization of the overdamped SIS-type JJAs in the Abrikosov mixed state was studied theoretically. Within a single-plaquette approximation, novel phenomenon was predicted which should manifest itself through appearance of steps in the temperature behavior of susceptibility. The number of steps is totally controlled by the vortices driven field-dependent kinetic inductance of the plaquette.

This work was supported by the Brazilian agency CAPES.
[1] S. Sergeenkov, In: A. Narlikar, Editor, *Studies of High Temperature Superconductors*, vol. 50, Nova Science, New York (2006), p. 229.

[2] F.M. Araujo-Moreira, P. Barbara, A.B. Cawthorne and C.J. Lobb. In: A. Narlikar, Editor, *Studies of High Temperature Superconductors*, vol. 43, Nova Science, New York (2002), p. 227.

[3] M.S. Li, *Phys. Rep.* 376 (2003), p.133.

[4] J.R. Kirtley, A.C. Mota, M. Sigrist and T.M. Rice, *J. Phys.: Condens. Matter* 10 (1998), p. L97.

[5] E. Altshuler and T.H. Johansen, *Rev. Mod. Phys.* 76 (2004), p.471.

[6] S.M. Ishikaev, E.V. Matizen, V.V. Ryazanov, V.A. Oboznov and A.V. Veretennikov, *JETP Lett.* 72 (2000), p.26.

[7] H.J. Jensen, *Self Organized Criticality: Emergent Complex Behavior in Physical and Biological Systems*, Cambridge University Press (1998).

[8] S. Sergeenkov and F.M. Araujo-Moreira, *JETP Lett.* 80 (2004), p.580.

[9] O. Bourgeois, S. E. Skipetrov, F. Ong and J. Chaussy, *Phys. Rev. Lett.* 94 (2005), p.057007.

[10] S. Sergeenkov, G. Rotoli, G. Filatrella and F.M. Araujo-Moreira, *Phys. Rev. B* 75 (2007), p.014506.

[11] P. Martinoli and C. Leeman, *J. Low Temp. Phys.* 118 (2000), p.699.

[12] R. Meyer, S.E. Korshunov, Ch. Leemann and P. Martinoli, *Phys. Rev. B* 66 (2002), p.104503.

[13] S.E. Korshunov, *Phys. Rev. B* 68 (2003), p.094512.

[14] M. Tesei, R. Theron and P. Martinoli, *Physica C* 437-438 (2006), p.328.

[15] F.M. Araujo-Moreira, P. Barbara, A.B. Cawthorne and C.J. Lobb, *Phys. Rev. Lett.* 78 (1997), p.4625.

[16] Dong-Ho Wu and S. Sridhar, *Phys. Rev. Lett.* 65 (1990), p.2074.

[17] Klaus Halterman, Oriol T. Valls and Igor Zutić, *Phys. Rev. B* 63 (2001), p.180405(R).

[18] R. Prozorov, D.D. Lawrie, I. Hetel, P. Fournier and R.W. Giannetta, *Phys. Rev. Lett.* 93 (2004), p.147001.

[19] S. Sergeenkov, *JETP Lett.* 76 (2002), p.170.