Massive zero-metal stars: Energy production and mixing

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Abstract. Time-dependent nuclear network calculations at constant temperature show that for zero-metal stars \( \gtrsim 20 M_\odot \), i.e., massive Population III stars (Pop-III for short), has been studied since the pioneering work of Ezer et al. (1971) who followed the evolution of stars in the mass range 5–100 \( M_\odot \) from the pre-main sequence contraction phase until the exhaustion of hydrogen on the main sequence. This opened the field for many studies that were mainly concerned with later evolutionary stages (Cary, 1974; Castellani, 1983; El Eid et al., 1983; Ober et al., 1983; Klapp, 1983, 1984). Motivated by the still ongoing debate about the initial mass function of Pop-III stars, Marigo et al. (2001) present the most comprehensive study of zero-metal evolutionary models (0.7–100 \( M_\odot \)) starting from the ZAMS until the AGB in the case of low- and intermediate-mass stars, or to the onset of carbon burning in massive stars.

In earlier studies the authors were forced to make numerous assumptions about equilibria between the chemical species involved. The most recent studies by Marigo et al. (2001), however, incorporate nuclear networks without making any assumptions about equilibria. Nevertheless, even these recent state-of-the-art studies rely on some simplifications that are certainly fulfilled during the evolution of normal stars but are questionable in the case of massive zero-metal stars.

It is the aim of this letter to show that the following two assumptions widely used, namely

1. \( \beta \)-decay negligible against proton-capture on
2. instantaneous mixing across convective regions

are not met during the evolution of massive Pop-III stars and that proper modeling may alter the evolution of these objects.

2. Nuclear Network

It was first noted by Ezer (1961) that Pop-III stars more massive than about 20 \( M_\odot \) are supplied by a different mode of energy generation. Due to the lack of the elements C, N and O the CNO-cycle cannot generate the luminosity needed to halt contraction. This raises the temperature to the point where the 3\( \alpha \)-reaction produces enough carbon to initiate the CNO-cycle. As a result, the CNO-cycle operates at a much higher temperature, typically at \( 10^8 \) K. Since the energy generation in the CNO-cycle exhibits a strong temperature dependence the central temperatures of the more massive models up to 100 \( M_\odot \) do not exceed \( T_\odot = 1.3 \times 10^8 \) K (see e.g., El Eid et al., 1983; Marigo et al., 2001). At these temperatures, the CNO-cycle turn-over rate is governed by the beta-decay half-lives (Ezer et al., 1971).

This finding has one important consequence for modeling stellar evolution: one has to include the elements \( ^{13}\text{N} \) and \( ^{15}\text{O} \) (mono-cycle), \( ^{17}\text{F} \) (bi-cycle) and \( ^{18}\text{F} \) (tri-cycle) explicitly in time-dependent network calculations. In addition, for temperatures exceeding \( 10^8 \) K the proton capture \( ^{13}\text{N}(p,\gamma)^{14}\text{O} \) (followed by \( ^{14}\text{O}(\beta^+,\nu)^{14}\text{N} \)) supersedes the \( ^{13}\text{N}(\beta^+,\nu)^{13}\text{C} \) rate. Contrary to the findings of Klapp (1983) our calculations show a non-negligible effect on the energy generation for \( T > 10^8 \) K.

In order to estimate the described effects, we have performed time-dependent network calculations at constant temperature. Our reference calculation includes 15 chemical species: \(^1\text{H}, \; ^4\text{He}, \; ^{12}\text{C}, \; ^{13}\text{C}, \; ^{15}\text{N}, \; ^{14}\text{N}, \; ^{15}\text{N}, \; ^{14}\text{O}, \; ^{15}\text{O}, \; ^{16}\text{O}, \; ^{17}\text{O}, \; ^{18}\text{O}, \; ^{20}\text{Ne}, \; ^{22}\text{Ne}, \; ^{24}\text{Mg}, \; ^{26}\text{Mg}, \; ^{27}\text{Al}, \; ^{28}\text{Si}, \; ^{32}\text{S}, \; ^{34}\text{S}, \; ^{36}\text{Ar}, \; ^{38}\text{Ar}, \; ^{40}\text{Ca}, \; ^{43}\text{K}, \; ^{48}\text{Ca}, \; ^{56}\text{Fe} \).
$^{16}\text{O}$, $^{17}\text{O}$, $^{18}\text{O}$, $^{17}\text{F}$, $^{18}\text{F}$ and $^{19}\text{F}$ with initial abundances of $X_{\text{H}} = 0.77$, $X_{\text{He}} = 0.23$ and $X_{\text{other}} = 10^{-15}$, and the following reaction chains:

- 3α-process
- CNO tri-cycle
- $^{13}\text{N}(p,\gamma)^{14}\text{O}$ and $^{14}\text{O}(\beta^+,\nu)^{15}\text{N}$.

Reaction rates are taken from NACRE (see Angulo et al., 1999). The calculations are performed utilizing the DAE solver LIMEX that is maintained by Ehrig et al. (1998).

Our results can be viewed in Fig. 1 where we have plotted the energy generation per $\rho X_{\text{H}}^2$ against the time in years for the two cases $T = 1.0 \times 10^8$ K (see Fig. 1a) and $T = 1.3 \times 10^8$ K (see Fig. 1b). Neglecting the beta-decay half-lives (dashed curves) has an impact on the energy generation of $\sim 10\%$ for $1.0 \times 10^8$ K and $\sim 65\%$ for $1.3 \times 10^8$ K. Whereas the proton capture on $^{13}\text{N}$ alters the energy generation at $1.0 \times 10^8$ K only slightly ($\sim 1\%$), it has a $\sim 30\%$ effect at $1.3 \times 10^8$ K (dotted-dashed curve). It is worth noting that the energy generation is only marginally ($< 1\%$) affected by both the second (named $^{17}$ON) and the third cycle (named $^{19}$FO). These cycles need only be included if one is interested in the relative abundances of the elements O and F. At constant temperature, the energy production never attains a constant value due to the ongoing feeding of freshly produced carbon via the 3α-process.

3. Mixing

Massive stars contain large convective cores in which mixing of chemical species occurs due to the turbulent convective motion. In normal stars, this process is very rapid compared to the slow changes of the chemical composition produced by nuclear reactions. Under these circumstances one can safely assume that the composition in a convective region always remains homogeneous, i.e., elements are instantaneously mixed over the convective region. The following arguments are put forward to show that the turnover time of the CNO-cycle at the high temperatures of massive Pop-III stars can be comparable to the timescale of turbulent mixing – or even shorter.

3.1. Mixing timescale

We show that the mixing timescale, $\tau_{\text{conv}}$, is of the order of 10 days in the case of both normal and zero-metal stars. An estimate of this timescale is easily derived from the standard mixing length theory (MLT, Böhm-Vitense, 1958) and the formulas we present resemble those given in Kippenhahn (1990, chap. 7). Let us assume a standard composition of $X_{\text{H}} = 0.77$ and $X_{\text{He}} = 0.23$, i.e., $\mu = 0.58$, and opacity due to electron scattering $\kappa = 0.2(1 + X_{\text{H}})$, i.e., $\kappa = 0.35 \text{cm}^2\text{g}^{-1}$. Even in the case of hot Pop-III stars, neglecting radiation pressure does not change this order of magnitude estimate and it suffices to assume a monoatomic ideal gas: $\delta = 1$, $c_p = 59/2\mu$. The mixing timescale is approximately given by the pressure scale height $H_p$ divided by the velocity of convective motion $v_{\text{conv}}$. With this:

$$\tau_{\text{conv}} \sim 10^8 \left( \frac{r}{R_{\odot}} \right)^2 \left( \frac{M_{\odot}}{m} \right) \sqrt{\frac{T}{10^8 \text{K}}} \left( \frac{10^{-4}}{\sqrt{\nabla - \nabla_e}} \right) \text{ s} \quad (1)$$

where $r$, $T$ and $m$ are the radius, temperature and mass, respectively, at one particular locus in a star. The quantity $\nabla - \nabla_e$ is the difference between the actual temperature gradient, $\nabla$, and $\nabla_e$ which describes the variation of $T$ in a mass element during its motion. Introducing two dimensionless variables:

$$U \sim 210^{-11} \left( \frac{m}{M_{\odot}} \right) \left( \frac{T}{10^8 \text{K}} \right)^{3/2} \times$$

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1. http://pntpm.ulb.ac.be/Nacre/
is primarily given by the slowest reaction of the cycle which stars is the turn-over time of the CNO-cycle. This time is a characteristic timescale for nuclear reactions in massive compact also the density goes down by a factor of ten.

For an estimate of $\tau_{\text{conv}}$ at the center of a star the first two equations can be even further reduced, since $m = 4/3\pi \rho c r^3$:

$$\tau_{\text{conv}} \sim 10^6 \left( \frac{M_{\odot}}{m} \right)^{1/3} \left( \frac{100 \, \text{g cm}^{-3}}{\rho_c} \right)^{2/3} \left( \frac{T_c}{10^8 \, \text{K}} \right)^{1/2} \times \left( \frac{7 \times 10^{-4}}{\sqrt{\nabla - \nabla_e}} \right)^{8/3}$$

For a typical example, consider a $20M_{\odot}$ Pop-III star. There we have $T_c \sim 10^8 \, \text{K}$, $\rho_c \sim 100 \, \text{g cm}^{-3}$. Let us choose a mass coordinate: $m = 1M_{\odot}$, hence $U \sim 4 \times 10^{-10}$ from Eq. (6). For reasonable values of $W$, $W \sim 1 \ldots 100$ it follows that $(\nabla - \nabla_e)^{1/2} \sim 7 \times 10^{-4}$. Note that the assumption $U \ll (\nabla - \nabla_e)^{1/2} \ll W$ is fulfilled. With Eq. (6) we finally arrive at

$$\tau_{\text{conv}} \sim 10 \, \text{days}.$$  

For a star containing metals the central temperature is by a factor of three smaller but as normal stars are less compact also the density goes down by a factor of ten. Thus $\tau_{\text{conv}}$ changes by a factor of two or so. Similar results are obtained at other loci of the convective core and for stellar masses in the range 20–100 $M_{\odot}$.

3.2. Nuclear timescale

A characteristic timescale for nuclear reactions in massive stars is the turn-over time of the CNO-cycle. This time is primarily given by the slowest reaction of the cycle which is $^{14}\text{N}(p,\gamma)^{15}\text{O}$. Even in the high temperature regime ($T = 1.3 \times 10^9 \, \text{K}$) where proton-captures become comparable to $\beta$-decays this estimate holds within a factor of two. Hence,

$$\tau_{\text{nuc}} \sim \tau_{n14pg} = \frac{A_H}{X_H \rho \lambda_{14pg}(T)},$$

where $A_H$, $X_H$, and $\rho$ are the atomic mass of hydrogen, the mass abundance of hydrogen and the density, respectively (see [Clayton, 1983]). The reaction rate $\lambda_{14pg}(T)$ measured in $\text{cm}^3 \text{Mol}^{-1} \text{s}^{-1}$ is only a function of temperature. Taking this reaction rate again from NACRE (see [Angulo et al., 1999]):

$$\lambda_{14pg}(T) = 4.83 \times 10^7 \, T_9^{-2/3} \times \exp(3.1 \times 10^{-2} T_9 - 9.5 \times 10^{-2} T_9^{3/2}) \times \log(T_9^{3/2} \rho),$$

where $T_9 = T / 10^9 \, \text{K}$, $\rho$ is the density in $\text{g cm}^{-3}$.

Fig. 2. Lines of constant $\tau_{\text{nuc}}$ at $X_H = 0.77$. In the region above the dashed line the mixing time is slow against the nuclear timescale.

4. Conclusions

Performing time-dependent nuclear network calculations at constant temperature we show that in the case of massive Pop-III stars ($\gtrsim 20M_{\odot}$) neglecting the $\beta$-decay against proton-capture leads to a considerable error in the energy generation rate. In addition, the reaction $^{14}\text{N}(p,\gamma)^{15}\text{O}$ cannot be omitted for temperatures exceeding $10^8 \, \text{K}$.

Moreover, the nuclear timescale of massive Pop-III stars can be very short (order of hours) compared to the timescale of convective mixing which is of the order of 10 days. Therefore, instantaneous mixing which is well justified in normal stars may introduce large errors in evolutionary calculations of zero-metal stars.

Contrary to normal stars massive Pop-III stars have not forgotten their nuclear history since equilibrium
abundances between chemical species are never attained. Hence, evolutionary calculation must start on the pre-main sequence well before the onset of nuclear reactions.

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