Estimation of steady-state basic parameters of stars

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Abstract

From a minimum of total energy of celestial bodies, their basic parameters are obtained. The steady-state values of mass, radius, and temperature of stars and white dwarfs, as well as masses of pulsars are calculated. The luminosity and giromagnetic ratio of celestial bodies are estimated. All the obtained values are in a satisfactory agreement with observation data.

PACS: 64.30.+i; 95.30.-k; 97.10.-q

1 Introduction

It is agreed-upon to consider that pressure and density of a substance inside stars grow more or less monotonously approaching their centers. It is supposed even that an ultrahigh-density substance can exist near the center of some stars. Thus, it is considered that the basic parameters of stars, such as mass, radius and temperature, are more or less arbitrary.

However, density, mass, and temperature of a star can be determined coming from the requirement of a minimum of energy if a star exists in an equilibrium state.

The effect of gravity-induced electric polarization of plasma inside a star gives a basis for such a calculation.

Under the influence of gravity, the nuclei of plasma ”hang” on their electronic clouds. Therefore, they are always located right below these clouds. As a result, electrons form a stratum at the external surface of a plasma body, and inside a star each cell of plasma obtains a positive charge $\delta q$. This charge is very small and equal to $\frac{e}{R_{st}} \approx 10^{-19} e$ by the order of magnitude ($a_0$
is Bohr radius, $R_{st}$ is the radius of a star, and $e$ is the charge of an electron). However, it is sufficient, that at each point inside a star a gravitational force is cancelled by an electric force

$$m_i \vec{g} + \delta q \vec{E} = 0,$$

where $m_i$ is the mass of an ion, $\vec{g}$ is gravity acceleration, $\vec{E}$ is an electric field intensity created by charges $\delta q$

$$\text{div} \vec{E} = 4\pi \delta q n,$$

where $n$ is the electron density.

From here it follows that the gradient of pressure inside a star is equal to zero, and the substance has a steady value of density. For a star consisting of degenerate relativistic plasma, the temperature effects can be neglected. For a star consisting of hot nonrelativistic plasma, the absence of the gradient of pressure and density of a substance requires an assumption that temperature inside a star is constant. It concerns, certainly, only the basic core of stars consisting of electrically polarized plasma. There is a thin stratum at the surface of a star, where a substance exists in an usual atomic state. In this stratum, the gradient of pressure, the gradient of density, and the gradient of temperature are present. Because of a small thickness of this stratum (about $10^{-2} R_{st}$), its role in the energy balance of the whole star can be neglected.

Since the gravitational force is cancelled by the electric force in cores of stars, the gravitational energy is compensated by the electric energy inside a core as well \([\text{1}]\) (the gravitational energy is negative). There is an uncompensated part of energy of a gravitational field. It is the energy of gravitational field outside a star

$$E_G = -\frac{GM^2}{2R},$$

where $G$ is the gravitational constant, $M$ and $R$ are mass and radius of a star. Because plasma is electroneutral as a whole, one proton should be related to electron of the Fermi gas of plasma. The existence of one neutron per proton is characteristic of a substance consisting of light nuclei. The quantity of neutrons increases approximately to 1.8 per proton for the heavy-nuclei substance, i.e. to one electron in plasma a mass equal to $m_p \left(\frac{A}{Z}\right)$ should be related. Thus
\[ M = N m_p \left( \frac{A}{Z} \right). \] (4)

Here \( N \) is a full number of electrons in a star, \( m_p \) is the proton mass, \( A \) and \( Z \) are the mass number and charge of a nucleus, respectively. For hydrogen \( \left( \frac{A}{Z} \right) = 1 \). Therefore,

\[ 1 \leq \left( \frac{A}{Z} \right) < 2.8. \] (5)

The field Eq.(3) tends to compress a star as a whole.

The jump in electric polarization at the surface of a star core produces additional pressure inside a star \([1]\), since the jump in polarization is accompanied by the pressure jump. As this energy is considerably smaller than the gravitational one, it can be neglected for simplicity.

At high temperature the energy of the black radiation can be important in a general balance of the star energy

\[ E_r = \frac{4\sigma}{c} T^4 V \] (6)

where \( \sigma = 5.7 \cdot 10^{-5} \frac{\text{erg}}{\text{cm}^3 \text{K}^4} \) is the Stefan-Boltzmann constant, and \( V \) is the volume of a star.

Thus, the total energy of a star is a sum of the total energy of plasma and the energy of black radiation

\[ E_{\text{total}} = E_{\text{pl}} + E_r. \] (7)

### 2 A star consisting of hot nonrelativistic electron - nuclear plasma

The energy of plasma in a star is a sum of its potential energy \( U \) and kinetic energy \( E_{\text{kinetic}} \). According to the virial theorem the potential energy of a system of interacting charged particles and their kinetic energy are connected \([2, 3]\)

\[ U = -2E_{\text{kinetic}}. \] (8)
Because the ion mass is large, the chemical potential of ions is small and their kinetic energy can be neglected. Thus, electrons will make a main contribution into the kinetic energy of plasma.

We name plasma a hot one, if the temperature inside a star is more than the degeneration temperature of the electron gas and the electric interaction between particles of plasma can be neglected

$$\left(\frac{kT}{e^2}\right)^3 \gg n.$$  \hspace{1cm} (9)

Under this condition it is possible to consider the electron gas of plasma an ideal one, and write its equation of state as the ideal gas law

$$PV = NkT.$$ \hspace{1cm} (10)

Thus, the kinetic energy of plasma in a star

$$E_{\text{kinetic}} = \frac{3}{2}NkT,$$ \hspace{1cm} (11)

where $k$ is the Boltzmann constant.

Because gravity does not influence photons and acts on heavy ions only, the total energy of hot plasma is

$$E_{pl} = -\frac{GM^2}{2R} + \frac{3}{2}NkT,$$ \hspace{1cm} (12)

or, according to the virial theorem (Eq.(8)), the energy of plasma in a star

$$E_{pl} = -\frac{GM^2}{4R} = -\frac{3}{2}NkT$$ \hspace{1cm} (13)

and the total energy of a hot star is

$$E_{\text{total}} = -\frac{GM^2}{2R} + \frac{3}{2}NkT + \frac{4\sigma}{c}T^4V = -\frac{GM^2}{4R} + \frac{4\sigma}{c}T^4V$$ \hspace{1cm} (14)
3 Calculation of the corrections to the electron energy

This equation is valid for an electron gas at very high temperature, when the identity of particles is of no significance. At finite temperature the identity of particles leads to an extra stiffness of the electron gas than it follows from Eq. (10). On the other hand, electron gas in plasma will exhibit more softness under the influence of a Coulomb field of nuclei. We shall carry out the account of these corrections according to the methods described by Landau and Lifshits [2].

3.1 The correction on electron identity

The total energy of the electron gas is known [2]

\[ E_e = \int_0^\infty \varepsilon dN_e = \frac{2^{1/2} V m^{3/2}}{\pi^2 \hbar^3} \int_0^\infty \frac{\varepsilon^{3/2} d\varepsilon}{e^{(\varepsilon-\mu)/kT} + 1}, \]  

(15)

where \( \varepsilon = P^2/2m \), \( P \) and \( m \) are momentum and mass of an electron in the nonrelativistic gas.

According to the definition of a chemical potential of electrons [2],

\[ \mu = kT \ln N \left( \frac{2\pi \hbar^2}{mkT} \right)^{3/2}, \]  

(16)

and at high temperature

\[ e^{\mu/kT} << 1. \]  

(17)

Therefore, in this case the integral in Eq. (15) can be expanded into the power series \( e^{\mu/kT-\varepsilon/kT} \).

Keeping the first terms of series, an approximated expression of the free energy of the hot electron gas with the account of the correction on the identity of electrons is obtained

\[ F = F_{\text{ideal}} + N \frac{\pi^{3/2} e^{3/2} a_0^{3/2}}{4(kT)^{1/2}} n. \]  

(18)
3.2 The estimation of the influence of nuclei on the hot electron gas

The nuclei reduce pressure of electron gas apart from them, therefore, this correction to pressure must be negative. The estimation of this correlative correction can be carried out by the method offered by Debye and Hückel [2].

The energy of an charged particle inside plasma is equal to $e\varphi_e$, where $e$ is a charge of a particle, and $\varphi_e$ is the electric potential created by other particles on this particle. This potential in plasma is determined by the Debye law

$$\varphi = \frac{e}{r} e^{-\frac{r}{r_D}}.$$  \hspace{1cm} (19)

As in dense plasma nuclei form an ordered lattice [2], electrons play a role in screening charges only.

Thus, the Debye radius is

$$r_D = \sqrt{\frac{kT}{4\pi e^2 n}}.$$  \hspace{1cm} (20)

At the small value of ratio $\frac{r}{r_D}$, the potential can be expanded into the series

$$\varphi = \frac{e}{r} - \frac{e}{r_D} + ...$$  \hspace{1cm} (21)

The following terms are converted into zero at $r = 0$. The first term of this series is the potential of the electron. The second term is the potential created by other particles on electron. Therefore, the energy of the electron gas in plasma created by the action of screened fields of other particle:

$$E_{corr} = -\frac{e^2}{r_D} N = 2e^3 \sqrt{\frac{4\pi}{kT}} N^{3/2}. $$  \hspace{1cm} (22)

By using of the known Gibbs-Helmholtz equation [2]

$$\frac{E}{T^2} = -\frac{\partial F}{\partial T T},$$  \hspace{1cm} (23)

we obtain the correction to the free energy
\[ F = F_{\text{ideal}} - N \frac{2e^3}{3} \sqrt{\frac{\pi n}{kT}}. \] (24)

### 3.3 The steady-state value of a star density

Accounting for these corrections on electron identity and influence of ions, the free energy of the hot electron gas acquires the form

\[ F = F_{\text{ideal}} + N \frac{\pi^{3/2} e^3 a_0^{3/2}}{4(kT)^{1/2}} n - N \frac{2e^3}{3} \frac{\pi^{1/2}}{(kT)^{1/2}} n^{1/2} \] (25)

From the equilibrium condition

\[ \left( \frac{\partial F}{\partial n} \right)_{N,T} = 0, \] (26)

we obtain a steady-state value of electron density in plasma of a star

\[ n_{st} = \frac{16}{9 \pi^2 a_0^3} \approx 2 \cdot 10^{23} \text{cm}^{-3}. \] (27)

For a star with a sufficiently high temperature, according to Eq.(8), it is energetically favourable to have this density.

### 3.4 The steady-state values of the mass, the radius and temperature of a star

According to Eq.(14), the total energy of a star

\[ E_{\text{total}} \approx -\frac{3}{2} kT n_{st} V + \frac{4 \sigma}{c} T^4 V. \] (28)

This function has a minimum at the temperature

\[ T_{st} = \left( \frac{3ckn_{st}}{32\sigma} \right)^{1/3} = \left( \frac{ck}{6\pi^2 \sigma a_0^3} \right)^{1/3} \approx 2.5 \cdot 10^7 \text{K}. \] (29)

According to the virial theorem Eq.(8), at an equilibrium state into the electron gas subsystem

\[ \frac{GM^2}{2R} = 3NkT. \] (30)
With the account of Eq.(27) and Eq.(29), we can calculate a steady-state value of radius of hot stars

\[ R_{st} = \left[ \left( \frac{3}{2} \right)^2 \left( \frac{\pi c k^4}{48\sigma G^3 m_p^6} \right)^{1/6} \right] \frac{a_0}{\left( \frac{4}{2} \right)} \simeq \frac{1.6}{\left( \frac{4}{2} \right)} R_\odot \]  \hspace{1cm} (31)

where \( R_\odot \) is the radius of the Sun.

The steady-state value of mass of hot stars

\[ M_{st} \simeq \left[ \frac{27}{4} \left( \frac{c}{3\pi \sigma G^3} \right)^{1/2} \left( \frac{k}{m_p \sqrt{2}} \right)^2 \right] m_p \left( \frac{A}{Z} \right)^2 \simeq \frac{1.1}{\left( \frac{4}{2} \right)^2} M_\odot , \]  \hspace{1cm} (32)

where \( M_\odot \) is the mass of the Sun.

4 The equilibrium condition of a white dwarf

When density of a substance is sufficiently high and temperature is insignificant, the electrons form a degenerate and relativistic Fermi gas. Thus, a star obtains the other equilibrium state.

It is known [2] that the electron gas is a relativistic one, when its Fermi momentum

\[ p_F = (3\pi^2)^{1/3} n^{1/3} \hbar > mc , \]  \hspace{1cm} (33)

i.e., at density of particles \( n > 10^{31} cm^{-3} \), which is characteristic for white dwarfs.

For a degeneration, electron gas temperature must be

\[ T \ll \frac{mc^2}{k} \approx 10^{10} K. \]  \hspace{1cm} (34)

The energy of the relativistic electron gas is known [4]

\[ E = \frac{3}{4} \left( 3\pi^2 \right)^{1/3} h c N \left( \frac{N}{V} \right)^{1/3} . \]  \hspace{1cm} (35)

Therefore, the total energy of a white dwarf

\[ E_{total} = -\frac{GM^2}{2R} + \frac{3}{4} \left( 3\pi^2 \right)^{1/3} h c N \left( \frac{N}{V} \right)^{1/3} . \]  \hspace{1cm} (36)
As temperature is defined by the inequality Eq.(34), the temperature depending terms can be neglected. It is possible under the condition
\[ \frac{4\sigma}{c}T^4V << \frac{3}{4}(3\pi^2)^{1/3}\hbar cN\left(\frac{N}{V}\right)^{1/3}. \] (37)

As according to the virial theorem (Eq.(8))
\[ \frac{GM^2}{2R} = \frac{3}{2}\left(\frac{3\pi^2}{3}\right)^{1/3}\hbar cN\left(\frac{N}{V}\right)^{1/3}, \] (38)
the full number of electrons inside any white dwarf is fixed
\[ N_{dw} = \frac{M}{(\frac{4}{3})m_p} = \frac{3^{5/2}\pi^{1/2}}{2}\left(\frac{\hbar c}{G(\frac{4}{3})^2m_p^2}\right)^{3/2} \approx 3.2 \cdot 10^{58}. \] (39)
Thus, in the equilibrium state any white dwarf should have the steady-state value of mass, depending on world constants only and the ratio \((\frac{4}{3})^2\):
\[ M_{dw} = \left[\frac{3^{5/2}\pi^{1/2}}{2}\left(\frac{\hbar c}{Gm_p^2}\right)^{3/2}\right]m_p(\frac{4}{3})^2 \approx 12.5M_\odot. \] (40)

According to the inequality Eq.(33), the steady-state value of radius of a white dwarf
\[ R_{dw} < \left(\frac{9\pi}{4}\frac{N_{dw}}{m_p}\right)^{1/3}\frac{\hbar}{mc} \simeq 3 \cdot 10^{-2}R_\odot \] (41)
and the density of electrons
\[ n_{dw} = \frac{N_{dw}}{(\frac{4}{3})R_{dw}^3} >> 10^{30} \text{cm}^{-3}. \] (42)
This correlates well with Eq.(33).

According to Eq.(37)
\[ T_{dw} << \frac{mc^{3/2}}{(4\pi)^{1/2}\sigma^{1/4}\hbar^{3/4}} \simeq 3 \cdot 10^9 K. \] (43)

Which is consistent with Eq.(34).

The steady-state value of mass of a star (11), consisting from the relativistic electron gas, does not depend on mass of an electron. Due to this
fact, this equation is valid for stars, consisting of a degenerate gas of other relativistic Fermi particles, in particular, neutrons. Therefore, if we can consider a pulsar as a neutron star containing a small number of protons and electrons (with concentration more than $10^{-19}$), the equation Eq.(40) should determine the steady-state value of mass of a pulsar.

5 The mass-luminosity relationship

It seems possible to assume that temperature inside a star determines temperature at its surface. If the relation between a surface temperature $T$ and the steady-state values of inner temperature $T_{st}$ is constant for all stars

$$T = const \cdot T_{st},$$

(44)

it is possible to deduce a relationship between a visible luminosity of a star and its other parameters, in particular, mass.

As the luminosity of a star

$$L = 4\pi R^2 \frac{\alpha}{n} T^4,$$

(45)

using the obtained above expressions and after simple transformations, we obtain

$$\frac{L}{L_\odot} = \left( \frac{M}{M_\odot} \right)^{10/3},$$

(46)

where $L_\odot$ is the luminosity of the Sun.

6 The comparison of the calculated values with the observation data

The masses of stars can be measured with a considerable accuracy, if these stars compose a binary system. There are almost 300 double stars which masses are known with the required accuracy [3]. Among these stars there are stars of the main sequence, dwarfs, and pulsars. Approximately one half of them are visual binaries. Their masses are measured with the a high precision. Other half consists of spectroscopic binaries and eclipsing binaries.
For these stars the accuracy of mass measurement is a slightly worse. Nevertheless, we shall carry out a comparison of the calculated parameters of stars on the basis of all these measurements.

6.1 Masses of celestial bodies

According to these data, the distribution of masses of hot stars is described by the equality

\[ < M > = (2.98 \pm 0.25)M_\odot. \]  \hspace{1cm} (47)

It is in satisfactory agreements with the calculated steady-state value of mass of a star Eq. (32) by the order of magnitude. Graphically this distribution is shown in Fig.1. According to the binary star tables [5], the distribution of white dwarfs masses is described by

\[ < M > = (0.96 \pm 0.05)M_\odot. \]  \hspace{1cm} (48)

This distribution is shown in Fig. 2. In this figure, the distribution of pulsar masses [6] is shown. It is described by

\[ < M > = (1.40 \pm 0.02)M_\odot. \]  \hspace{1cm} (49)

It is possible to conclude that with the account of the corrections on the factor \( \left( \frac{A}{Z} \right) \), the results of calculations of the steady-state values of masses are in agreement with measured data.

6.2 The mass-luminosity relationship

The luminosity of stars from binary systems, depending on their masses are shown in Fig. 3. The results of more precise measurements of visual binaries are marked by squares. The data for spectroscopic and eclipsing binaries are marked by triangles and points, respectively. The line corresponds to the calculated dependence Eq. (46). It can be seen from this figure, that in the logarithmic scale the calculated dependence is in satisfactory agreement with the observed data.
7 The gyromagnetic ratio of stars

The effect of gravity-induced electric polarization \([1]\) is characteristic for all celestial bodies, consisting of a substance in a plasma state. The value of this polarization does not depend on plasma properties: no matter if it is a relativistic one or not and if it has a degeneration or not. As \(\text{div}\vec{E} = 4\pi\rho\) and \(\text{div}\vec{g} = -4\pi G\gamma\) and according to Eq.(1), the gravity-induced density of the volume electrical charge \(\rho\) is related to the density of substance \(\gamma\) by the relation \([1]\):

\[
\rho = \sqrt{G\gamma}.
\] (50)

Therefore a rotation of a celestial body about its axis must induce a magnetic field. If one assumes that the electrically polarized core of a body occupies its entire volume and if one neglects the existence of a surface stratum, where the substance is in an atomic state, that the gyromagnetic ratio (the relation of a magnetic moment of a body to its angular momentum) will obtain a steady value \([1]\):

\[
\vartheta = \frac{\sqrt{G}}{3c}.
\] (51)

It is possible to consider the above assumption is quite acceptable for large celestial bodies, such as stars, and less acceptable for planets, where the mantle can be rather large. However, the detailed calculation for the Earth gives the result which is in agreement with the measured value with in the accuracy of the factor of two \([4]\). The values of giromagnetic ratio for all celestial bodies (for which they are known today) are shown in Fig.4.

The data for planets are taken from \([7]\), the data for stars are taken from \([8]\), and for pulsars - from \([9]\). Therefore, for all celestial bodies - for planets and their satellites, for Ap-stars and several pulsars - the calculated value Eq.(51) with a logarithmic precision quite satisfactorily agrees to measurements, when moments itself change within the limits of more than 20 orders.
Figure 1: The mass distribution of stars from the binary systems [5]. Data for visual, spectroscopic, and eclipsing binaries are shown separately. On the abscissa, the logarithm of the star mass over the Sun mass is shown.
Figure 2: The mass distribution of white dwarfs and pulsars from the binary systems [3, 4]. On the abscissa, the logarithm of the star mass over the Sun mass is shown.
Figure 3: The luminosity of stars from binary systems, depending on their masses are shown. The results of more precise measurements of visual binaries are marked by squares. The data for spectroscopic and eclipsing binaries are marked by triangles and points, respectively. The line corresponds to the calculated dependence Eq.(46). On the ordinate, the logarithm of the star luminosity over the Sun luminosity is shown. On the abscissa, the logarithm of the star mass over the Sun mass is shown.
Figure 4: The observed values of the magnetic moments of celestial bodies vs. their angular momenta. On the ordinate, the logarithm of the magnetic moment over $Gs \cdot cm^3$ is plotted; on the abscissa the logarithm of the angular momentum over $erg \cdot s$ is shown. The solid line illustrates Eq.(51). The dash-dotted line is the fitting of the observed values.
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