Quantitative study of a freely cooling granular medium

Pierre Deltour and Jean-Louis Barrat
Département de Physique des Matériaux (UMR CNRS 5586)
Université Claude Bernard - Lyon I, 69622 Villeurbanne Cedex

March 23, 2022

Pacs numbers: 47.50.d, 05.20.Dd, 47.55.Kf
Electronic mail: barrat@dpm.univ-lyon1.fr, pdeltour@dpm.univ-lyon1.fr
to be published in Journal de Physique I : Statistical Physics

Abstract

We present a numerical study of a two dimensional granular medium consisting of hard inelastic disks. The evolution of the medium throughout a cooling process is monitored. Two different types of instabilities (shearing and clustering instability) are found to develop in the system. The development of these instabilities is shown to be in qualitative and quantitative agreement with the predictions of linearized hydrodynamic theory.

1 Introduction

Granular fluids are dense media composed of elementary elements of macroscopic size, undergoing collisions in which their macroscopic energy is not conserved. In the last decade, the flow of this granular fluids has received a great attention from the physics community, both because these media offer a "simple" example of dissipative systems and because of their numerous industrial applications. Two factors make the behaviour of granular fluids very different from that of molecular fluids. Firstly, the macroscopic size of the particles implies that external fields (boundaries or gravity) have a much stronger effect on granular fluids. Secondly, the energy of the granular fluid
is not a conserved variable, since the heat dissipated in collisions can be considered as lost as far as the flow is concerned. These two effects are often difficult to disentangle in experiments, and also in the numerical simulations that aim at a realistic modelling of these experiments [1]. A different type of numerical simulations was initiated by several groups [2, 3, 4, 5, 6]. In this approach external fields are ignored, and only the dissipation is taken into account. This dissipation is moreover modelled in a very simplistic way, by describing the particles as monodisperse rigid hard spheres undergoing inelastic collisions. The dissipation is entirely specified by the restitution coefficient \( r \), where \( 1 - r \) is the fraction of the kinetic energy lost in a collision (in the center of mass frame).

The aim of these simulations is not to model actual experiments involving granular fluids. Instead, the models are used to assess the difference in behaviour induced by the dissipation between a granular fluid and its ”atomic” counterpart \((r = 1, \text{the hard sphere fluid.})\) In particular, the simulations can be used to investigate the validity of the hydrodynamic equations frequently used to describe granular flow in more realistic situations [7, 8]. As for atomic fluids, they also provide a direct way of measuring the equation of state and transport coefficients that enter these equations. These transport coefficients can then be compared to those obtained using ”granular kinetic theory” [9].

A particularly simple and instructive situation that can easily be studied in numerical simulations is the 2-dimensional ”cooling problem” first studied in [2, 5]. In their simulations, these authors start from an equilibrium configuration of an elastic \((r = 1)\) hard disc fluid. The behaviour of this fluid after introducing a nonzero restitution coefficient is followed using the standard molecular dynamics method for hard bodies [10], with a collision rule that takes the dissipation into account. The kinetic temperature (average kinetic energy per particle) decreases due to the inelasticity of the collisions, so that the fluid cools down as time increases. It was shown in [2, 5] that, depending on the system size, on the restitution coefficient and density, this cooling follows different routes. In the simplest case, (small systems or small dissipation) the fluid remains homogeneous at all temperatures. In larger systems or for larger dissipations, either the velocity field or the density field in the fluid develop instabilities and become inhomogeneous. It was also found by the same authors that the occurrence of such instabilities is in qualitative agreement with the predictions of a linear stability analysis of the hydrodynamic equations for granular fluids [7]. Finally, it was discovered in [3, 5] that in some cases, the cooling ends at a finite time due to a singularity in the system dynamics, which was shown to correspond to an
infinite number of collisions within a finite time. This singularity, observed both in 1 and 2 dimensions, was described as an "inelastic collapse" of the system.

In this paper, a detailed and quantitative analysis of the instability of homogeneous cooling of granular fluids is attempted. Our aim is to compare quantitatively the predictions of granular kinetic theory and granular hydrodynamics to the results of molecular dynamics simulations of the cooling of an inelastic hard disk fluid. The paper is organized as follows. The main predictions of granular hydrodynamics concerning the cooling problem are briefly recalled. Computational details concerning the simulation are given in section 3. The different regimes occurring during the cooling are analyzed in section 4, and compared with the theoretical predictions. Our main focus will be on the growth rate of the density instability, that can be computed by monitoring the structure factor of the system as a function of time. Finally, the problem of the "inelastic collapse" is addressed in section 5, where a possible method for avoiding this singularity in the system dynamics is proposed.

2 Hydrodynamic analysis of the cooling problem

The hydrodynamic equations that have been proposed to describe granular flow [7] are based on mass and momentum conservation, and are very similar to the usual Navier-Stokes equations. The only modification is the appearance of a new term in the energy (or temperature) equation, accounting for the loss of energy in the collisions. These equations can be compactly written in the form

\[
\begin{align*}
\frac{D\rho}{Dt} &= -\rho \nabla \cdot \mathbf{v} \\
\frac{D\mathbf{v}}{Dt} &= -\nabla \cdot \bar{\mathbf{P}} \\
\frac{D\rho}{DT} &= -\nabla \cdot \mathbf{Q} - tr(\bar{\mathbf{P}} \mathbf{D}) - \gamma T^{3/2}
\end{align*}
\]

where \(D/Dt\) is the hydrodynamic derivative, \(\mathbf{D}\) the symmetrized velocity gradient tensor, \(\bar{\mathbf{P}}\) the stress tensor, \(\mathbf{Q}\) the heat flux and \(\gamma\) represented the rate of energy lost due to inelastic collisions. For a hard disk fluid, the equation of state and the expression of the various transport coefficients can be obtained from Jenkins and Richman kinetic theory [8]. These expressions
are recalled in Appendix A. The energy sink term, $\gamma T^{3/2}$, has also been written in the form appropriate for hard disks. $\gamma$ in that case is a function of the density and the restitution coefficient, which at least in the low density limit must be proportional to $\rho$ and $(1 - r)$. This can be understood from the following reasoning: the kinetic energy loss per particle per unit time is proportional to the collision frequency (i.e. to $\rho T^{1/2}$) and to the energy loss per collision $(1 - r)T$.

A trivial solution of the cooling problem formulated above corresponds to an homogeneously cooling fluid, with a uniform density, a vanishing velocity field, and a uniform temperature with an algebraic time decay

$$T(t) = T_0 \left(1 + \frac{t}{t_0}\right)^{-2}$$

Here $t_0 = 2\rho_0/(\gamma_0 T_0^{1/2})$ sets the time scale for temperature decay in the fluid. The linear stability of this homogeneous solution has been investigated in references [4, 11]. For completeness, the main steps of this analysis will be repeated here. The linearized equations that describe the evolution of a sinusoidal perturbation

$$\delta \rho = \delta \rho_k \exp(ik \cdot r)$$
$$\delta \mathbf{v} = \delta \mathbf{v}_k \exp(ik \cdot r)$$
$$\delta T = \delta T_k \exp(ik \cdot r)$$

around the homogeneous solution are

$$\frac{\partial \delta \rho_k}{\partial t} = -i \rho_0 (k \cdot \delta \mathbf{v}_k)$$
$$\frac{\rho_0 k \cdot \delta \mathbf{v}_k}{\partial t} = -ik^2 \left[\rho_0 p'(v_0) \delta T_k + T_0 \left(p'(v_0) + v_0 \left(\frac{\partial p'}{\partial v}\right)_0\right) \delta \rho_k - \mu_0 \left[k^2 (k \cdot \delta \mathbf{v}_k)\right]\right]$$
$$\frac{\rho_0 k_{\perp} \cdot \delta \mathbf{v}_k}{\partial t} = -\mu_0 \left[k^2 (k_{\perp} \cdot \delta \mathbf{v}_k)\right]$$

$$\left[\rho_0 \frac{\partial T_0}{\partial t} + \rho_0 \frac{\partial \delta T_k}{\partial t}\right] = -\kappa_0 k^2 \delta T_k - \rho_0 \delta \nabla \cdot (i\mathbf{k} \cdot \delta \mathbf{v}_k)$$
$$-\frac{3}{2} \gamma_0 T_0 \delta T_k - T_0 \frac{\partial}{\partial \nu} \left(\frac{\partial \gamma}{\partial \nu}\right)_0 \frac{\delta \rho_k}{\rho_0}$$

As for usual fluids, the transverse part of the velocity field completely decouples from the longitudinal part, and decays with time as $(1 + t/t_0)^{-k^2 T_0^{1/2}t_0/\rho_0}$. 

4
The longitudinal part of the velocity field, the temperature and the density are coupled, and give rise to three modes that have an algebraic time dependence

\[ \delta \rho_k = \delta \tilde{\rho}_k [1 + t/t_0]^{\xi} \]  
\[ \delta v_k = \delta \tilde{v}_k [1 + t/t_0]^{\xi - 1} \]  
\[ \delta T_k = \delta \tilde{T}_k [1 + t/t_0]^{\xi - 2} \]

The exponents \( \xi(k) \) for the three modes are the three roots of the determinant of the following set of equations

\[ \delta \tilde{\rho}_k \left( \frac{\xi(\xi - 1)}{t_0^2} + k^2 T_0 \left[ p'(\nu_0) + \nu_0 \left( \frac{\partial p'}{\partial \nu} \right)_0 + \frac{\mu_0 k^2}{\rho_0 t_0} \xi \right] + \delta \tilde{T}_k \left[ k^2 \rho_0 p'(\nu_0) \right] \right) = 0 \]
\[ \delta \tilde{\rho}_k \left[ \frac{T_0}{t_0} \left( -2 + T_0^2 \nu \left( \frac{\partial^2}{\partial \nu^2} \right)_0 - \frac{\partial}{\partial \nu} \right) \frac{t_0}{\rho_0} - p'(\nu_0) \xi \right] + \delta \tilde{T}_k \left[ (\xi + 1) \frac{\rho_0}{t_0} + \kappa_0 k^2 \right] = 0 \]

A typical plot of the wavevector dependence of these three roots, together with the growth rate of the velocity perturbations, is shown in figure 1. It must be emphasized that the stability of velocity disturbances is determined by the comparison between the growth exponent of the disturbance with the value \(-1\) that characterizes the decay of the thermal velocity. Hence a growth exponent larger than \(-1\) for the transverse or longitudinal velocity fields is indicative of an instability of the macroscopic velocity. If the growth exponent of the longitudinal velocity field is larger than \(-1\), a corresponding instability in the density field will follow from equation 8.

This analysis yields to the prediction of three different possible behaviours of the system, depending on the value of the parameters and on the system size, that introduces a lower wavevector cutoff. If this lower cutoff corresponds to the line \(C\) of figure 1, the homogeneous solution will be linearly stable. This regime will be described as the homogeneous kinetic regime. If the lower cutoff moves to the abscissa indicated by line \(B\) in figure 1, the transverse velocity field will become unstable while the system remains homogeneous. In this "shearing" regime, first observed in reference 2, a shearing flow will develop in the system. Finally, for a lower cutoff corresponding to abscissa \(A\), an instability of the longitudinal velocity field and the corresponding instability in the density field will take place together with the shearing instability. In this "clustering" regime, the growth of density disturbances will yield to the formation of dense clusters of particles, as first observed in reference 12.

All three situations have already been observed in numerical simulations of the cooling in two dimensional granular fluids. The aim of the next sec-
tions will be to attempt a quantitative analysis of the behaviour of a cooling granular fluid, and to compare the results to the predictions summarized above.

3 Computational details

The model simulated in this work is in all respects similar to that studied in [2, 5]. The system is made up of $N$ hard inelastic disks of diameter $\sigma$, in a square cell of size $L$ with periodic boundary conditions. The cell size $L$ sets the lower cutoff in wavevector space, $k_{\text{min}} = 2\pi/L$. A standard cell-linked Molecular Dynamics algorithm for hard bodies [10] is used. In a first step, the system is equilibrated with a coefficient $r$ equal to unity. At time $t = 0$, inelasticity is switched on and cooling starts, with an initial temperature $T_0$. The restitution coefficient enters through a simple modification of the standard collision rule between hard disks, the velocities of the two disks after a collision being given by

$$u_1' = u_1 - \frac{1}{2}(1 + r)[\hat{n} \cdot (u_1 - u_2)]\hat{n} \quad (15)$$

$$u_2' = u_2 + \frac{1}{2}(1 + r)[\hat{n} \cdot (u_1 - u_2)]\hat{n} \quad (16)$$

where the primes denote the quantities after collision and $\hat{n}$ is a unit vector along the centers line from particle 1 towards particle 2. The natural units in this problem are the particle mass $m$ and diameter, and the thermal energy at $t = 0$, i.e. $T_0$. The corresponding time unit is $\tau = (m/T)^{1/2}\sigma$. The state of the system is defined by three dimensionless numbers, which are the reduced size $L/\sigma$ (or equivalently the reduced cutoff $k_{\text{min}}^* = k_{\text{min}}\sigma$), the reduced density $\rho^* = \sigma^2 N/L^2$, and the restitution coefficient $r$.

The state of the fluid during the cooling was monitored by a systematic computation of coarse-grained (hydrodynamic) density and velocity fields. The coarse graining is obtaining here from a division of the system into 100 square subcells. Besides, statistical quantities characterizing the state of the system have also been systematically computed. These quantities are the momenta of the velocity distribution of individual particles, the pair correlation function $g(r)$ for interparticle distance, and the structure factor

$$S(k) = \frac{1}{N}\rho_k\rho_{-k} \quad (17)$$

This structure factor can be computed for all wavevectors compatible with the periodic boundary condition, of the form $(n_x, n_y)k_{\text{min}}$. As the system
is not in a stationary state, these quantities are time dependant. A large enough system is thus necessary to obtain reasonable statistics without time averaging. The values of $N$ investigated in this work vary from $N = 1600$ to $N = 10000$.

4 results

4.1 Kinetic regime

According to the analysis of section 1, the kinetic regime corresponding to a stable homogeneous cooling will be observed (at a given density and restitution coefficient) for small enough systems. Such a situation allows a clear testing of some of the hypothesis of the kinetic theory description of the granular fluid. In particular, the pair correlation function and velocity distribution can be compared to that of an elastic hard disk fluid throughout the cooling process. The temperature decay can be monitored and compared to the theoretical prediction (equation (4)), and the decay time $t_0$ (or equivalently the coefficient $\gamma(\rho)$) compared to the prediction of kinetic theory.

The pair correlation of an homogeneously cooling granular fluid after the temperature has dropped by a factor of 10 is shown in figure 2. This comparison shows that the local structure of the cooling granular medium (which determines its equation of state) remains essentially identical to that of an equilibrium fluid. The study of the velocity distribution function shows that this distribution remains maxwellian throughout the cooling.

This similarity between the structure and velocity distribution of the granular fluid and the usual hard disk fluid suggests that the kinetic theory of Jenkins [9] is applicable. This expectation is borne out by the study of the time dependance of the fluid temperature. As shown in figure 3, the temperature decay is perfectly described by equation (4). The density dependance of the decay time $t_0$ is compared in figure 4 to the prediction of kinetic theory (see appendix B). The agreement is extremely good, and suggests that all the transport coefficients appearing in the hydrodynamic equations can be estimated using this kinetic theory.

4.2 Shearing regime

If the restitution coefficient $r$ decreases or if the size of the system increases, the hydrodynamic theory predicts a regime in which transverse fluctuations of the velocity field are unstable. This regime is indeed observed in the sim-
ulations, as shown in figure 5. A shear flow that corresponds to the smallest wavevector compatible with the periodic boundary conditions develops in the system. In this regime, the total kinetic energy of the system (which in that case is not the temperature, since the system has developed an ordered flow pattern) appreciably deviates from equation (4) as shown in figure 6.

4.3 Clustered regime

For even larger systems or smaller restitution coefficients, the cooling granular fluid becomes inhomogeneous, as shown in figure 7. This spontaneous formation of density inhomogeneities (or clusters) was first observed in the simulations of the cooling problem by Goldhirsch and Zanetti and Young and McNamara [2, 3]. Two different explanations have been put forward to explain this cluster formation. The first one, found in [2], is to consider this cluster formation as a secondary instability of the shearing regime, due to the development of temperature and pressure gradients in the shearing regime. The second possible explanation is that cluster formation is directly related to the linear instability of the density modes predicted by hydrodynamic theory.

In order to characterize quantitatively this clustering regime, the structure factor $S(k, t)$ of the system has been computed as a function of time and wavevector. The corresponding data is shown in figure 8. The growth of the density inhomogeneities results in the appearance of a low wavevector peak in the structure factor, that rapidly increases with time. According to hydrodynamics, the time dependence of $S(k, t)$ should be algebraic, i.e.

$$S(k, t) = S(k, 0) \left(1 + \frac{t}{t_0}\right)^{2\xi(k)}$$ (18)

so that the ratio

$$\frac{\ln(S(k, t)) - \ln(S(k, 0))}{\ln \left(1 + \frac{t}{t_0}\right)} = 2\xi(k)$$ (19)

should be independent of time. This ratio is plotted in figure 9 as a function of wavevector for different times. $2\xi(k)$ seems to be reasonably independent of time, and its low wavevector value appears to be consistent with the prediction of linearized hydrodynamics. Hence the density instability can be interpreted as resulting from a linear instability of the homogeneous solution of the hydrodynamic equations. Note that it was recently observed
by McNamara and Young that the "clustering" fluid eventually develops for long times into an ordered flow pattern of the "shearing" type. This is also consistent with hydrodynamics, since the growth rate of the transverse velocity modes is positive. The description of the formation of this shearing flow in an inhomogeneous system, however, is beyond the possibilities of linearized hydrodynamics.

5 Inelastic collapse and how to avoid it

The inelastic collapse singularity was first observed by [6,3] in simulations of unidimensional inelastic system. This collapse can be described as the appearance of an infinite number of correlated collisions between a few particles, taking place in a finite time. The same phenomenon was observed in two dimensions by [5]. It was shown that in that case the correlated collisions take place between a small number of essentially aligned particles, so that the unidimensional situation is practically reproduced.

In order to avoid this inelastic collapse, a slightly modified collision rule between the particles can be introduced. At each collision, the relative velocity of the two particles is first computed according to the usual rule (equations 15 and 16), then rotated by a small (less than 5 degrees) random angle. This can be justified by invoking the unavoidable roughness of actual solid particles, conservation of angular momentum being (virtually) ensured by a transfer to the internal degrees of freedom of the particles. As to inelastic collapse, the aim of this modified collision rule is to hinder the formation of correlated particle lines that cause this singularity. Indeed, inelastic collapse was not observed in the simulations where this "random" collision rule was used, while under the same conditions a system following the "deterministic" collision rule always underwent inelastic collapse (figure 10). Hence inelastic collapse appears to be a pathology related to the use of purely specular collision rule between particles, rather than a characteristic of inelastic fluids.

6 Conclusion and perspectives

The main objective of this work was to assess the validity of the hydrodynamic description of granular fluids originally proposed by [4], and of the kinetic theory calculation of the associated transport coefficients. The study of the particularly simple "cooling fluid" case and of the associated instabilities provides an ideal benchmark for this description. The comparison
between numerical simulations and theoretical predictions in this simple case shows that the theory is quantitatively accurate. A similar conclusion was also reached in a recent study by McNamara and Young [3], who showed that the transitions between the different cooling regimes were correctly predicted by the theory.

The description of the inelastic collapse phenomenon observed by McNamara and Young is obviously beyond the possibilities of kinetic theory or hydrodynamics. It was shown that this phenomenon can easily be avoided by introducing a small amount of randomness in the collisions between particles, similar to what would be caused by the natural roughness of granular particles.

Obviously, a correct description of granular fluid cannot be achieved without a knowledge of the boundary conditions that must be used for the hydrodynamic equations. These conditions, and in particular those that correspond to the very important case of vibrating solid walls, are not known. Their determination, through the quantitative comparison of numerical simulation and theory, will be the subject of future work.

Acknowledgments

This work was supported by the Pole Scientifique de Modélisation Numérique at ENS-Lyon.
A Expressions for the transport coefficients and the equation of state

The Navier-Stokes like equations describing a granular fluid are:

\[
\begin{align*}
\frac{D\rho}{Dt} &= -\rho \nabla \cdot \mathbf{v} \\
\rho \frac{D\mathbf{v}}{Dt} &= -\nabla \cdot \overline{\mathbf{P}} \\
\rho \frac{DT}{Dt} &= -\nabla \cdot \mathbf{Q} - \text{tr}(\overline{\mathbf{P}} \overline{\mathbf{D}}) - \gamma T^{\frac{3}{2}}
\end{align*}
\]

\(D/Dt\) is the hydrodynamic derivative, \(\overline{D}\) the symmetrized velocity gradient tensor, \(\overline{P}\) the stress tensor, \(Q\) the heat flux and \(\gamma\) represents the rate of energy lost due to inelastic collisions. The definition for these quantities is:

\[
\begin{align*}
\frac{D}{Dt} &= \frac{\partial}{\partial t} + (\mathbf{v} \cdot \nabla) \\
D_{ij} &= \frac{1}{2} \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) \\
\overline{P} &= p_h \overline{T} - 2\mu \left( \overline{D} - \frac{1}{2} (\nabla \cdot \mathbf{v}) \overline{I} \right) \\
\mathbf{Q} &= -\kappa \nabla T
\end{align*}
\]

The various transport coefficients and equation of state are

\[
\begin{align*}
p_h &= p'(\nu)\rho T \\
\mu &= \mu'(\nu)T^{\frac{3}{2}}\rho \sigma \\
\kappa &= \kappa'(\nu)T^{\frac{3}{2}}\rho \sigma \\
\gamma &= \gamma'(\nu)\frac{2}{\sigma}
\end{align*}
\]

where \(p', \mu', \kappa', \gamma'\) are functions only of the solid fraction \(\nu = \rho/\rho_s\).

\[
p'(\nu) = \frac{2\nu + s_s}{s_s}
\]
\[ \mu'(\nu) = \left( \frac{\nu^2}{\sqrt{\pi}} + \frac{\sqrt{\pi}}{4} (\nu + s_*)^2 \right) \frac{1}{\nu s_*} \]

\[ \kappa'(\nu) = \left( \frac{1}{\sqrt{\pi}} + \frac{\sqrt{\pi}}{2} \left( \frac{3}{2} \nu + s_* \right)^2 \right) \frac{1}{\nu s_*} \]

\[ \gamma'(\nu) = \frac{8}{\sqrt{\pi}} (1 - r) \frac{\nu}{s_*} \]

where \( s_* \) is defined as

\[ s_*(\nu) = \frac{(1 - \nu)^2}{(1 - 7\nu/16)} \]

### B Enskog expansion

In the case of a hard core fluid, a semi-empirical modification of Boltzmann equation introduced by Enskog, widens the range of applicability of the kinetic approach to higher densities \([14]\). The Enskog approximation accounts for the finite size of the disks in the collision term of the Boltzmann equation. When two particles collide, their centers are separated by the diameter of the disks \( \sigma \). The collision term of Boltzmann equation should thus be multiplied by the probability of finding two particles separated by \( \sigma \) which is proportional to the pair correlation function evaluated at \( \sigma \). This correction will have an influence on all the transport coefficients. The validity of this approach was checked for the cooling rate \( \gamma \) in the kinetic regime.

Figure 2 shows successive snapshots of the pair correlation function in the kinetic regime. This function is essentially the same as in a hard core fluid at thermodynamic equilibrium even though the temperature has dropped by a factor 10 between the first and the last snapshot. Hence \( g(\sigma) \) is assumed to be given by the usual virial expression

\[ \frac{p_h}{\rho T} = 1 + 2\nu g(\sigma) \]

Introducing the equation of state of an 2d hard disks fluid \([E]\), the Enskog corrected cooling rate \( t_0 \) becomes:

\[ t_0 = t_{0\text{Boltzmann}} \frac{1}{g(\sigma)} \]

\[ = \left( \frac{1}{\sqrt{\pi}(1 - r)\nu T_0^{1/2}} \right) \frac{(1 - \nu)^2}{(1 - 7\nu/16)} \]

12
The values of the cooling rate found in the simulation are compared in figure 4 with this prediction.

References

[1] H.J. Hermann, Physica A, **191**, 263, (1992)
[2] I. Goldhirsch and G. Zanetti, Phys. Rev. Letters, **70**, 1619, (1993)
[3] S. McManara et W.R. Young, Phys. Fluids A, **4**, (3), (1992)
[4] S. McManara et W.R. Young, Phys. Fluids A, **5**, (1), (1993)
[5] S. McManara et W.R. Young, Phys. Rev. E, **50**, R28, (1994)
[6] B. Bernu et R. Mazighi, J. Phys. A: Math. Gen., **23**, 5745, (1990)
[7] P. K. Haff, J. Fluid Mech., **134**, 401, (1983)
[8] C.S. Campbell, Annu. Rev. Fluid Mech., **22**, 57, (1990)
[9] J.T. Jenkins and M.W. Richman, Phys. Fluids, **28**, 3585, (1985)
[10] M. P. Allen et T. E. Tidesley, *Computer simulation of Liquids*, Oxford University Press, (1987)
[11] S. McManara, Phys. Fluids A, **5**, (12), (December 1993)
[12] M.A. Hopkins and M.Y. Louge, Phys.Fluids A, **3**, 47, (1990)
[13] S. McManara et W.R. Young, submitted to Phys. Rev. E, , , (1995)
[14] P. Résibois et M. De Leener, *Classical Kinetic Theory of Fluids*, John Wiley and Sons, (1977)
[15] D. Henderson, Mol.Phys., **30**, 971, (1975)
**figure 1:** The decay rate of the velocity field disturbances computed by solving equation (4) for \( \rho = 0.2, r = 0.9 \) is shown in this figure as a function of wavevector. The dashed line indicates the decay rate of the transverse modes. The black and grey lines correspond respectively to the real and imaginary parts of the decay rate of the three longitudinal modes.

**figure 2:** Pair correlation functions \( g(r) \) computed for the initial (in black shifted up by 0.5) and final (in grey) configurations in an homogeneously cooling system \((N = 1600, d = 0.8, r = 0.98)\). The temperature drops by a factor of ten without changes in the pair correlation function.

**figure 3:** Evolution of the square root of the inverse temperature versus \( t \) in an homogeneously cooling system \((r = 0.99, d = 0.1, N = 1600)\). The solid line corresponds to the hydrodynamic prediction in the kinetic regime.

**figure 4:** Enskog corrected value of the decay time calculated as a function of density for \( r = 0.98 \), compared to the values obtained in the simulations \((N = 1600)\).

**figure 5:** Velocity field in the shearing regime after the granular medium has spontaneously developed a flow pattern corresponding to the lowest wave vector compatible with the boundary conditions. \((N=1600, d=0.1, r=0.92)\)

**figure 6:** square root of the inverse of the temperature versus \( t \) in the shearing regime. The solid line extrapolates towards the first moments of the run. There is a substantial deviation from this kinetic regime fit. \((N = 1600, d = 0.1, r = 0.92)\)

**figure 7** Final configuration (141 collisions per particle) of a simulation in the cluster regime. \((N = 1600, d = 0.25, r = 0.6)\).

**figure 8:** Evolution of the structure factor during the clusters formation \((N = 1600, d = 0.5, r = 0.4)\). The curves from the bottom are separated by 10 collisions per particle. Note the large increase of the structure factor in the long wavelength limit \((k \to 0)\).

**figure 9:** Growth exponent of the density field disturbance, obtained from the simulation using equation (19) \((N=10000, d=0.5, r=0.9)\) The different symbols correspond to different times. The prediction of the hydrodynamic description of the instability is the solid line.

**figure 10** A system with \( N = 1600, d = 0.25, r = 0.25 \) obeying the specular collision rule collapses after 3.77 collisions per particle. The grey particles are those involved in the last two hundred collisions. The aligned particles represent more than 99 % of the grey particles. Under the same conditions, a system obeying the modified collision rule does not undergo collapse after 125 collisions per particle.
This figure "fig5b.gif" is available in "gif" format from:

http://arxiv.org/ps/cond-mat/9609184v1
2 * grow exponent of the density disturbance
